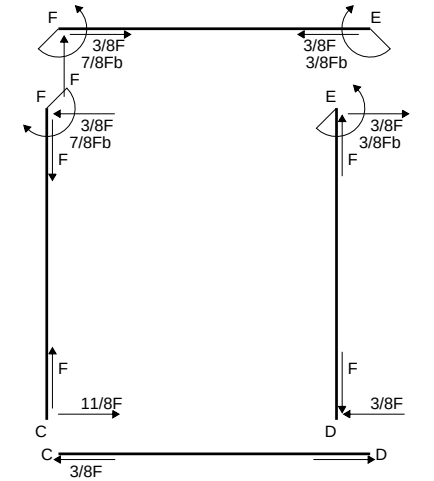
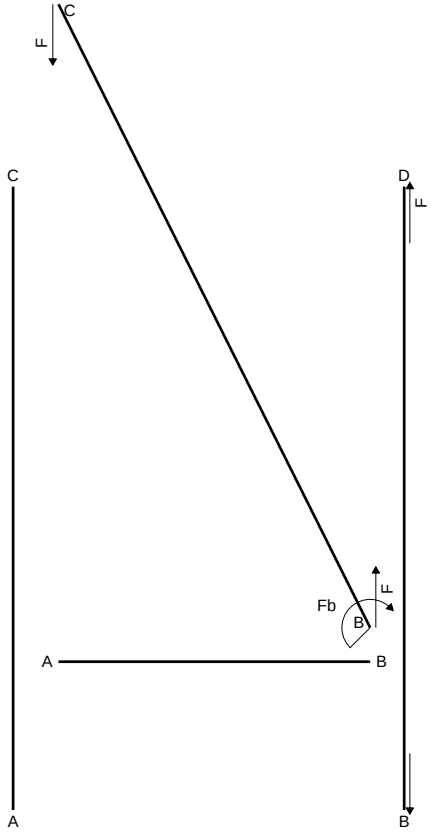
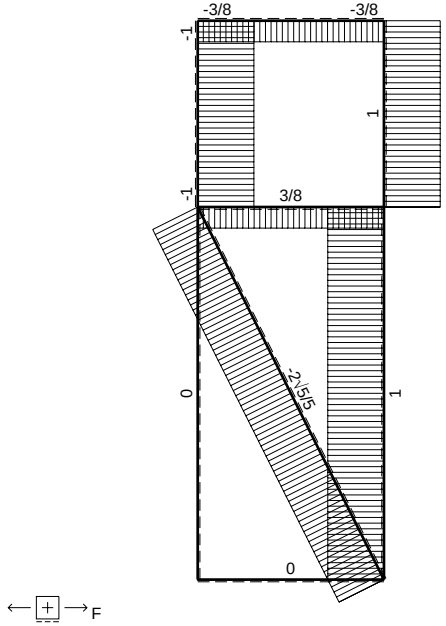
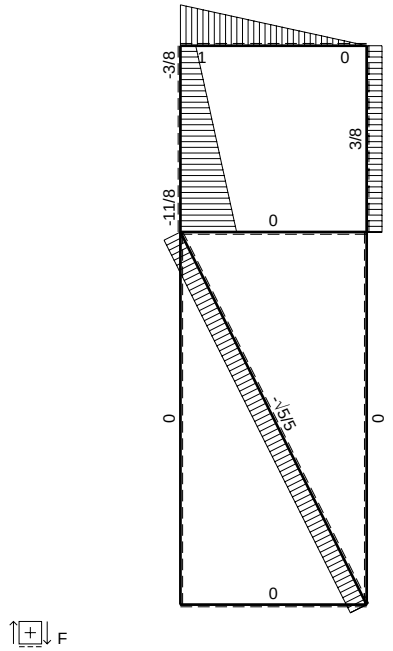
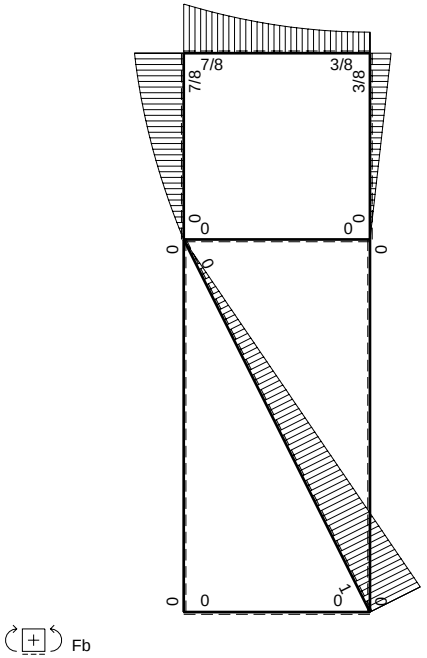

F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
1
0
-2√5/5
1
0
0
F


-3/8
1
0
3/8
-11/8
0
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

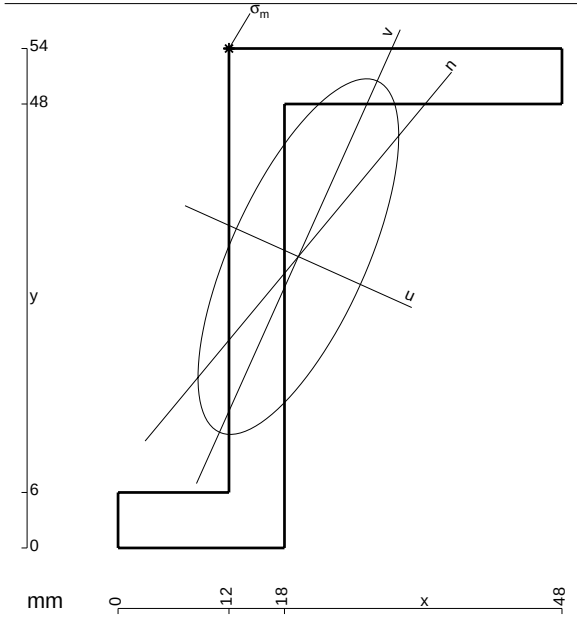
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

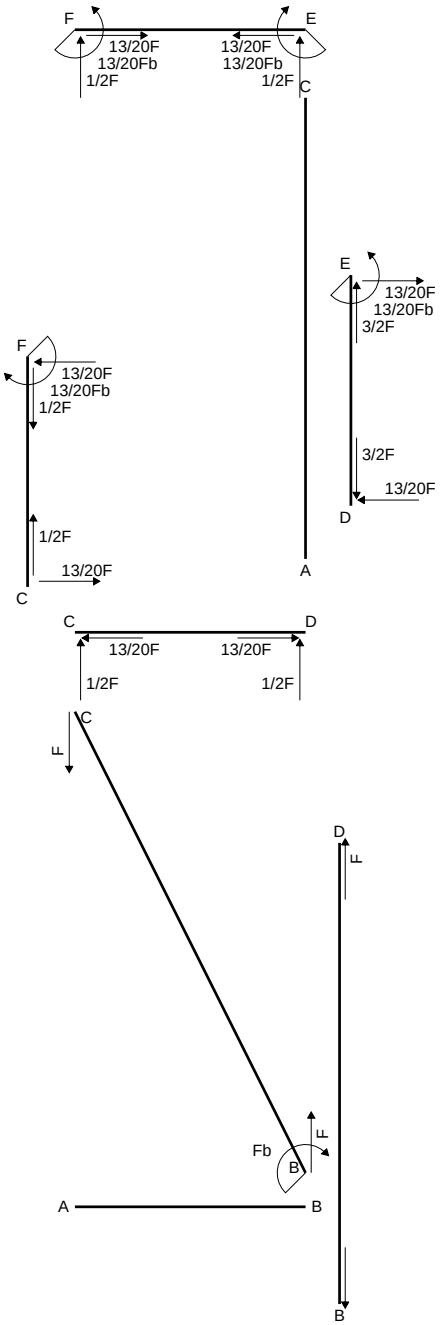
$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

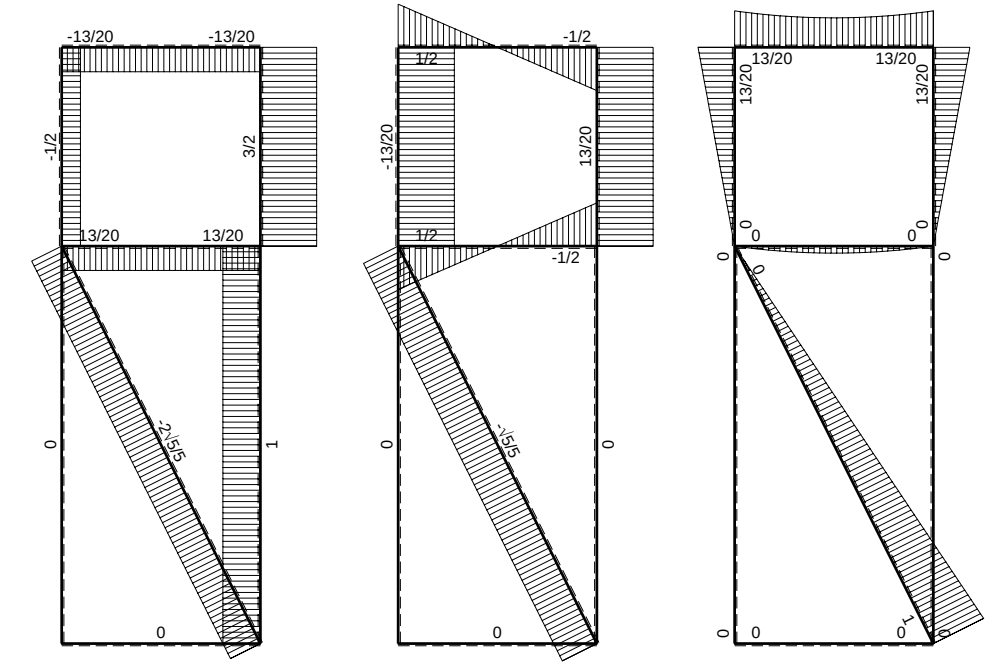


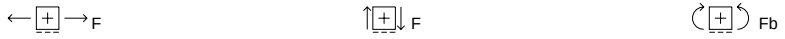
A = 576. mm^2
J_x = 212976. mm^4
J_y = 67824. mm^4
J_{xy} = 81648. mm^4
J_u = 249641. mm^4
J_v = 31159. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4221
c = cosα = .9122
s = sinα = -.4097
x_g = 19.5 mm
y_g = 31.5 mm
N = -1637. N
T_y = -818.4 N
M_x = 750300. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -16.06 mm
v_m = 17.45 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.1 N/mm^2

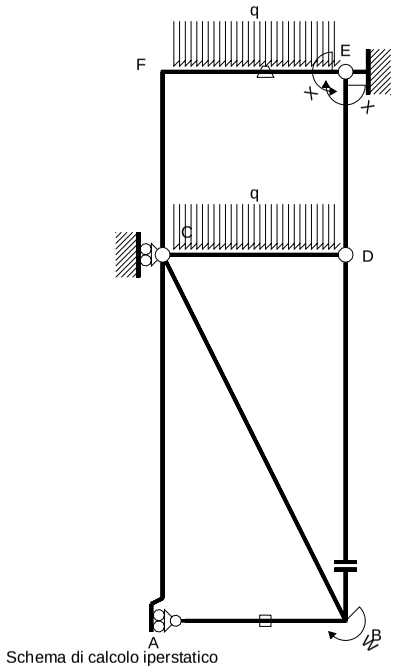
CI09.xxxx.001

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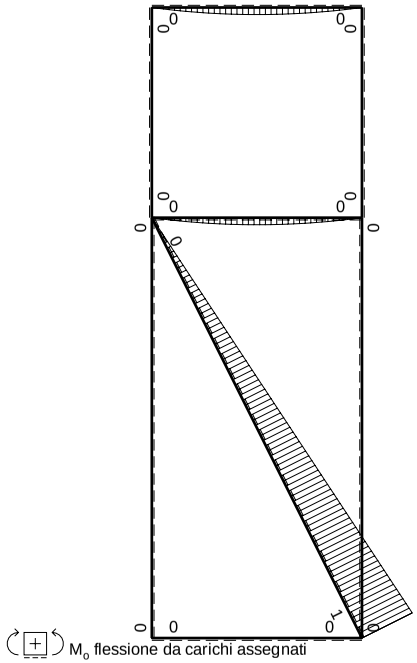

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

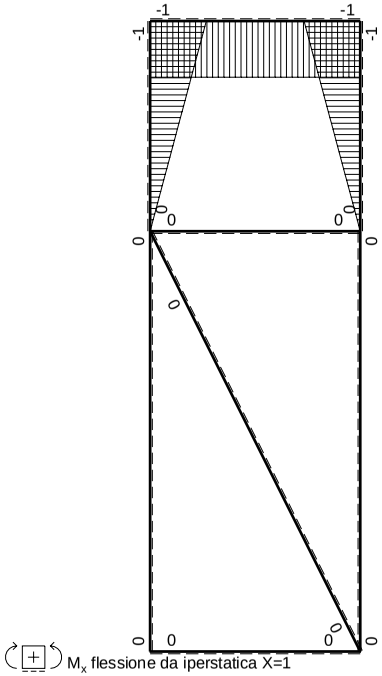

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 957766 CI09.xxxx.002

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

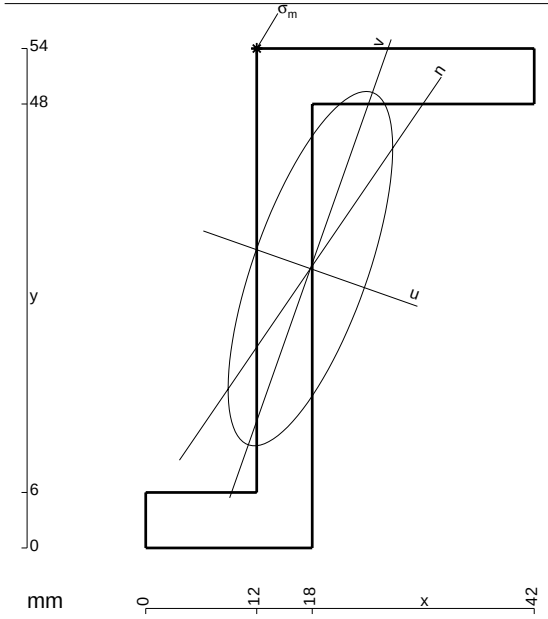
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

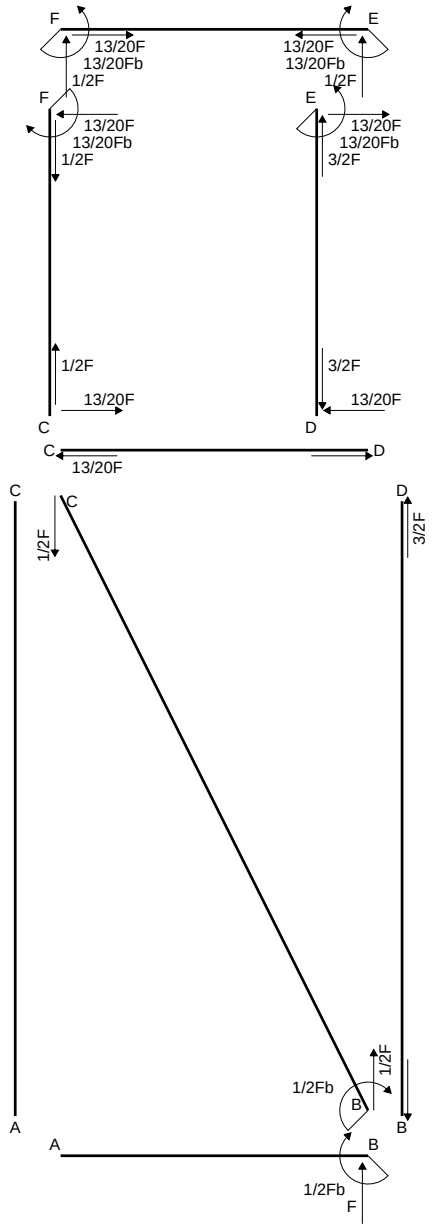
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

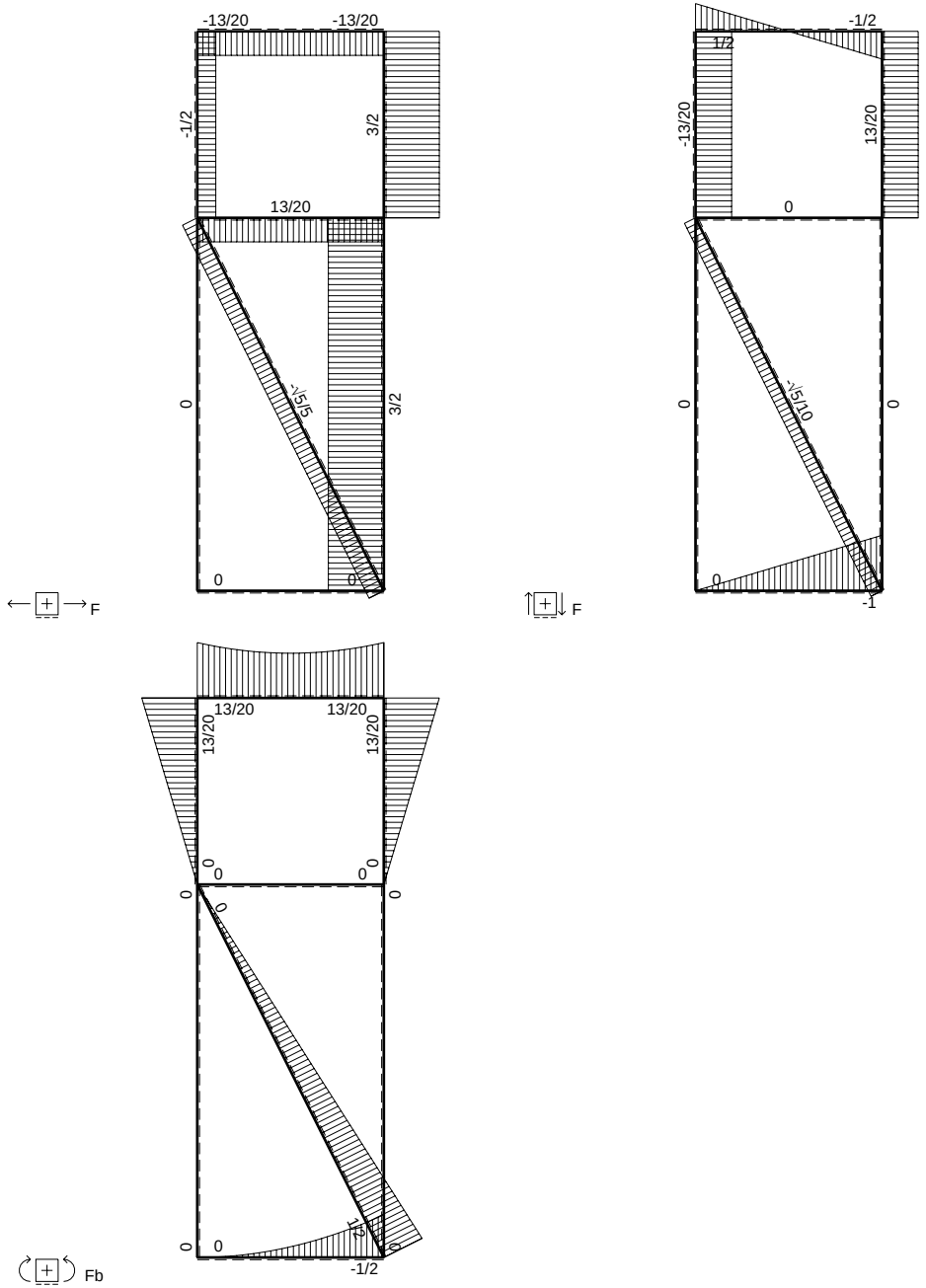


A = 540. mm^2
J_x = 198266. mm^4
J_y = 42746. mm^4
J_{xy} = 62554. mm^4
J_u = 220304. mm^4
J_v = 20709. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3387
$c = \cos\alpha$ = .9432
$s = \sin\alpha$ = -.3323
x_g = 17.8 mm
y_g = 30.2 mm
N = -1422. N
T_y = -711.1 N
M_x = 715500. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -13.38 mm
v_m = 20.52 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -219.1 N/mm^2

CI09.xxxx.002

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi \theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 218144 CI09.xxxx.003

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

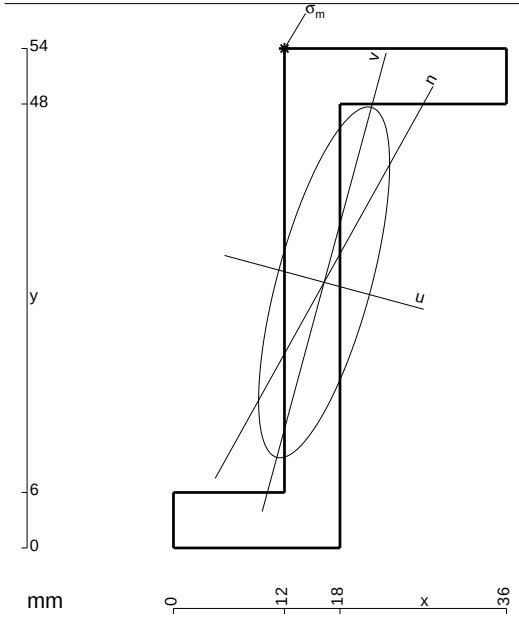
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

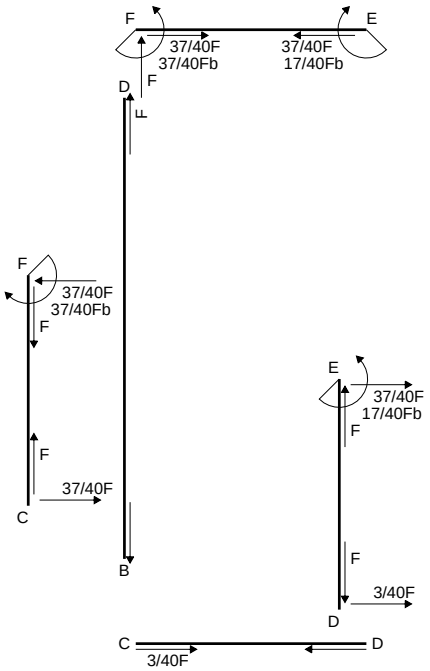
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

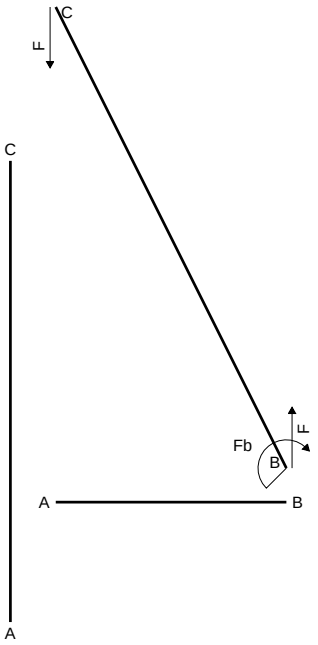


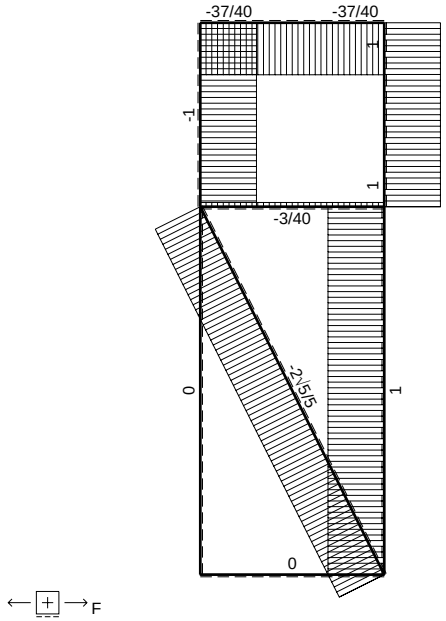
A = 504. mm^2
J_x = 181471. mm^4
J_y = 25303. mm^4
J_{xy} = 45545. mm^4
J_u = 193783. mm^4
J_v = 12991. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2640
c = cosα = .9653
s = sinα = -.2610
x_g = 16.29 mm
y_g = 28.71 mm
N = -1398. N
T_y = -1075. N
M_x = 684775. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -10.74 mm
v_m = 23.29 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.9 N/mm^2

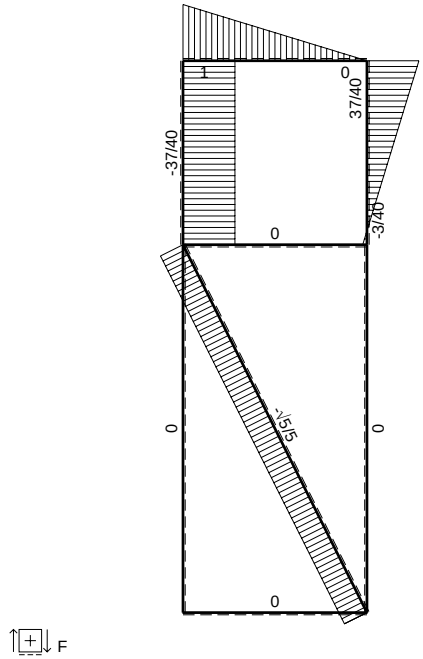
CI09.xxxx.003

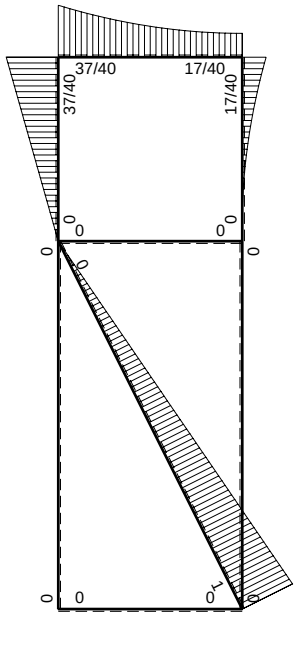
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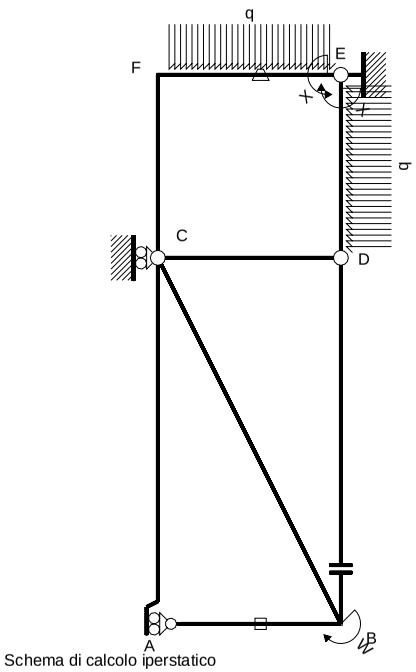

F
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37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
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17/40Fb
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F
37/40F
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A
B
A

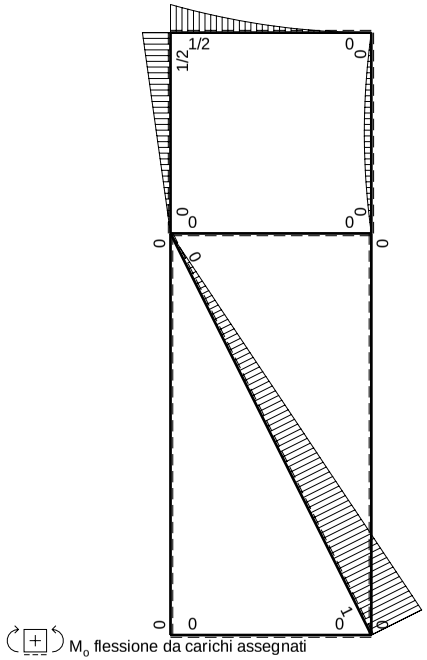

-37/40
-37/40
1
-1
1
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-2√5/5
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37/40
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-3/40
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-√5/5
0
0
F

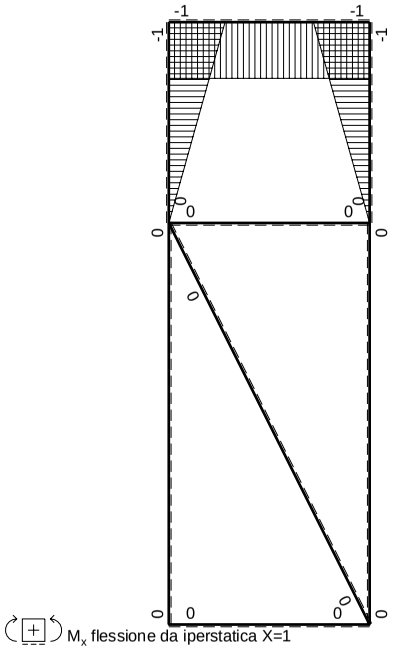

37/40
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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213350 CI09.xxxx.004

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

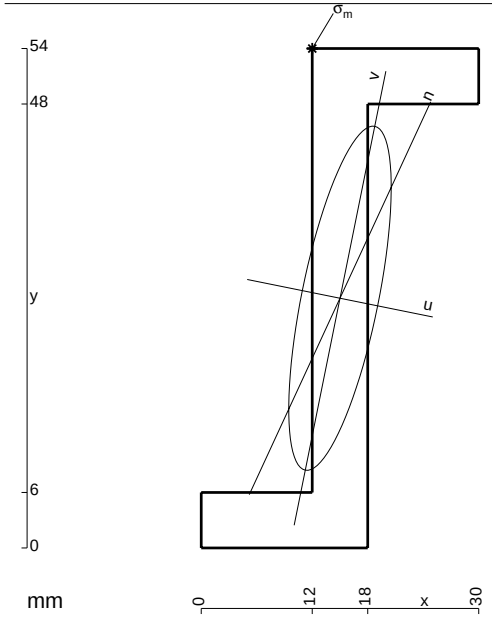
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 213350 CI09.xxxx.004

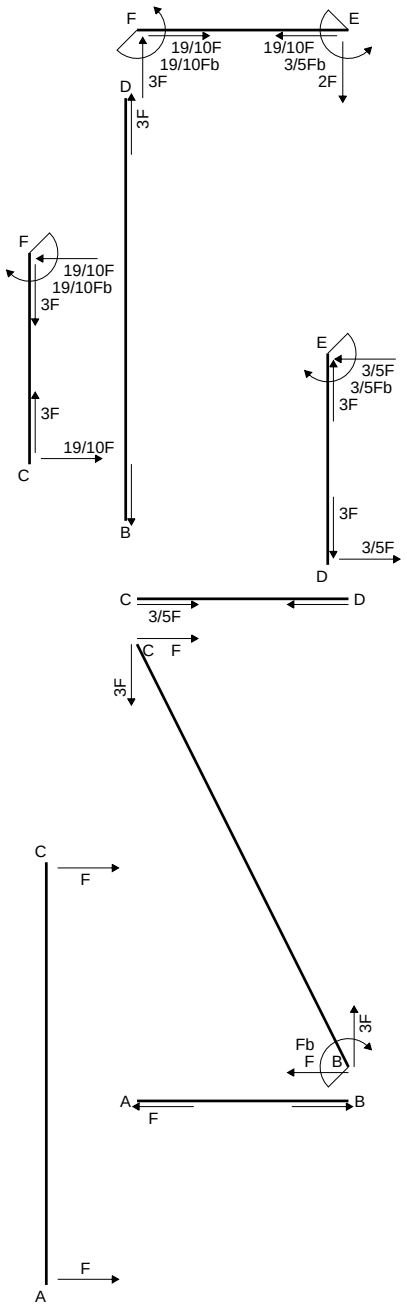


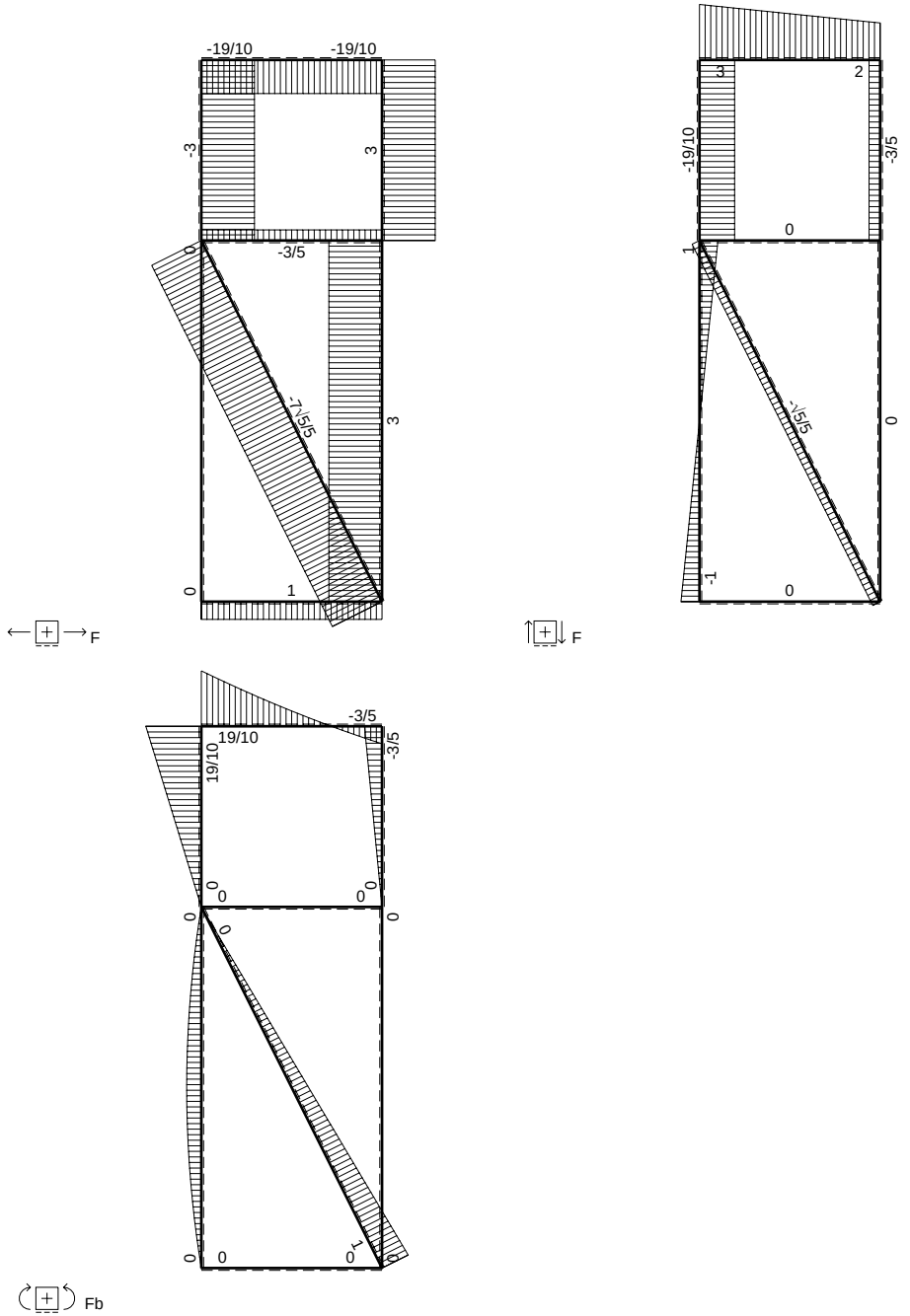
A = 468. mm^2
J_x = 162108. mm^4
J_y = 14364. mm^4
J_{xy} = 31104. mm^4
J_u = 168389. mm^4
J_v = 8083. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1993
$c = \cos\alpha$ = .9802
$s = \sin\alpha$ = -.1979
x_g = 15. mm
y_g = 27. mm
N = -1127. N
T_y = -563.5 N
M_x = 667800. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -8.285 mm
v_m = 25.87 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -238.5 N/mm^2

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CI09.xxxx.004

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F
E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
19/10Fb
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D
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3/5F
C
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3F
C
F
Fb
F
B
3F
A
B
F
F
A


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3
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F
3
2
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-3/5
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1
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19/10
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0
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0
0
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0
1
0
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							3/5Fb

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

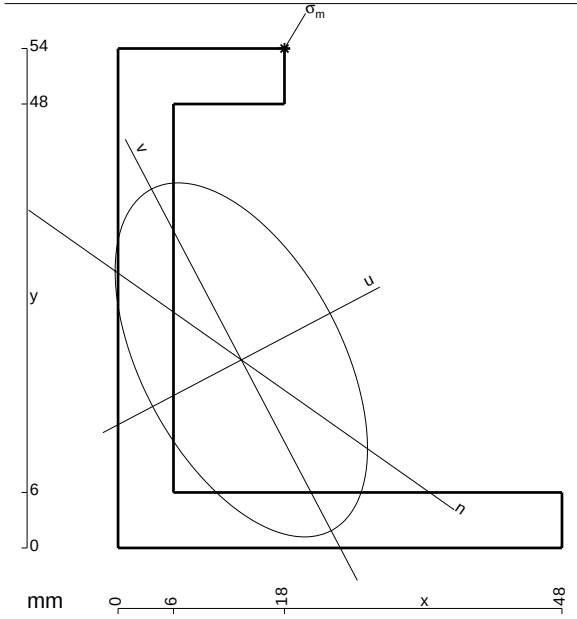
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

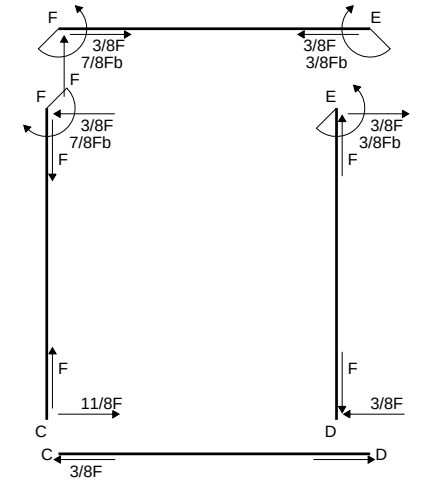
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

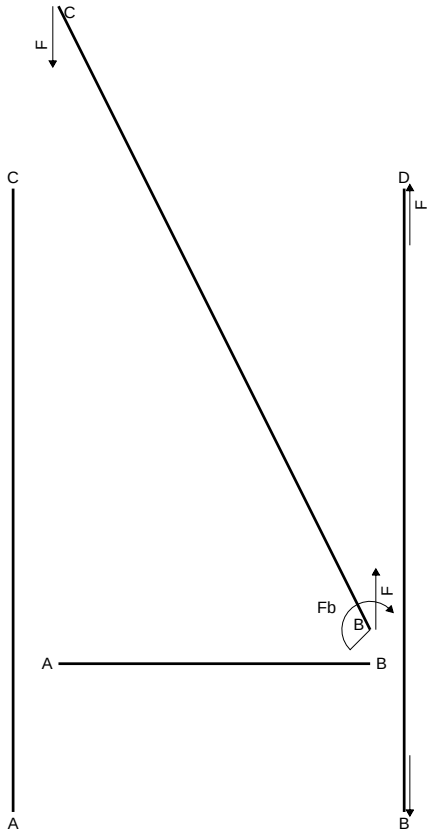


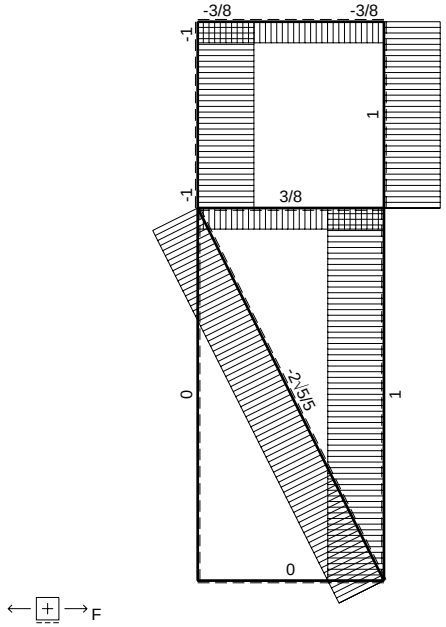
A = 648. mm^2
J_x = 237528. mm^4
J_y = 120672. mm^4
J_{xy} = -84960. mm^4
J_u = 282212. mm^4
J_v = 75988. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4842
$c = \cos\alpha$ = .8851
$s = \sin\alpha$ = .4655
x_g = 13.33 mm
y_g = 20.33 mm
N = -2610. N
T_y = -1653. N
M_x = 942210. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 19.8 mm
v_m = 27.62 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -199.9 N/mm^2

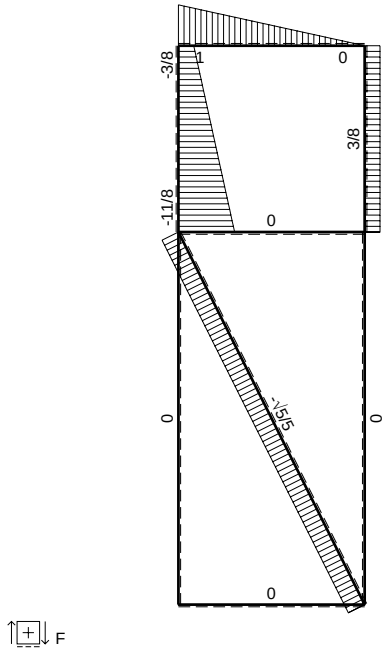
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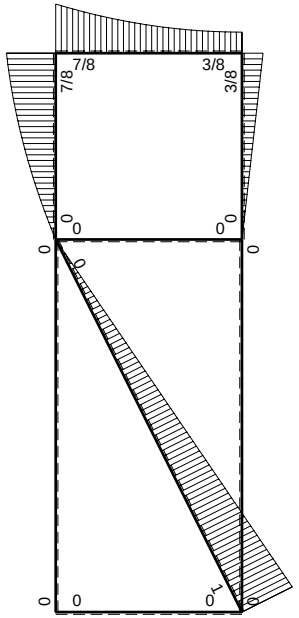
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F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
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B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
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3/8
-11/8
0
-√5/5
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F


7/8
7/8
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Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

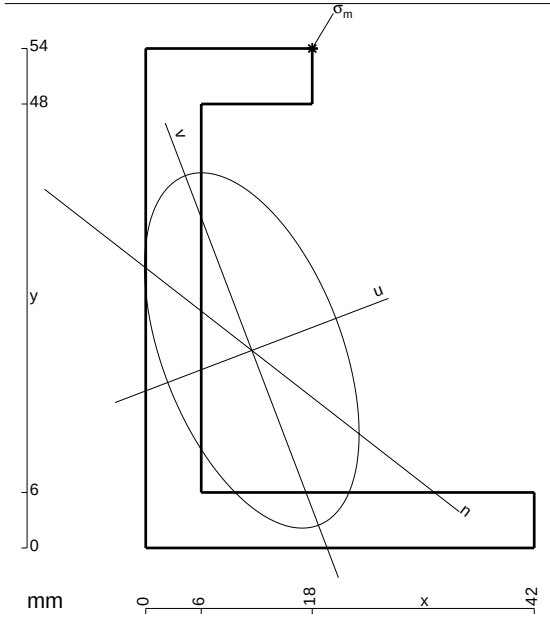
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

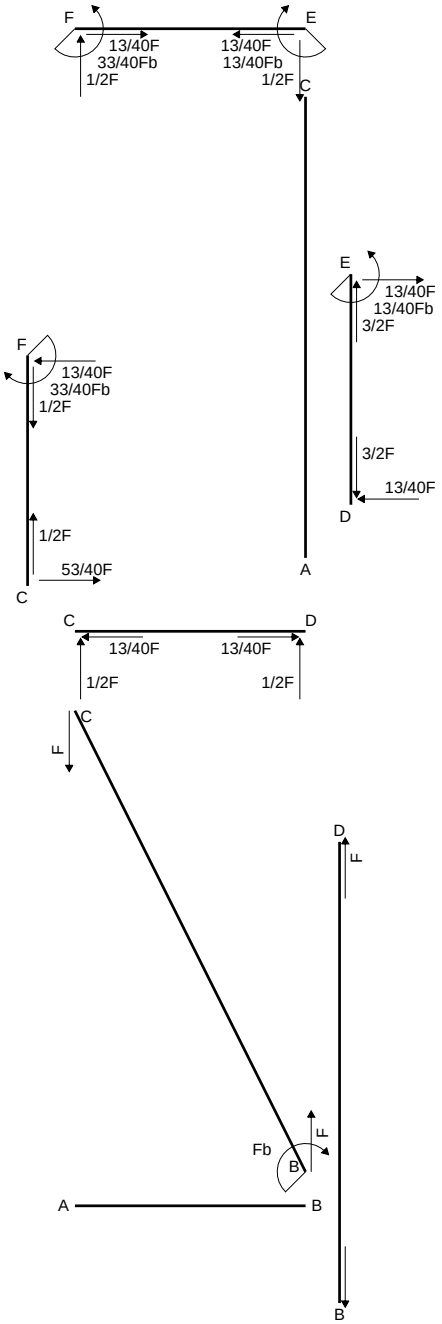
$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

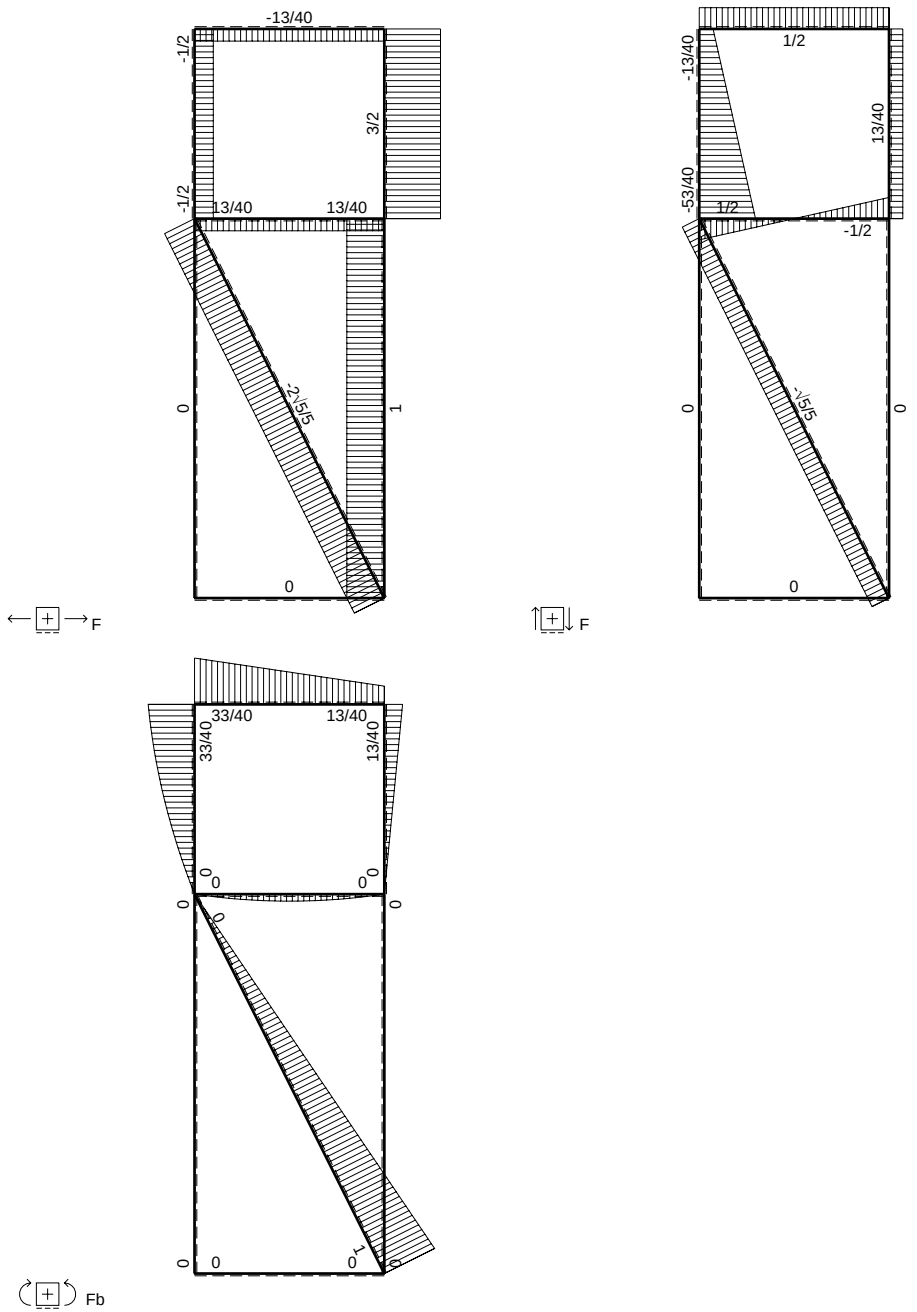


A = 612. mm^2
J_x = 225968. mm^4
J_y = 82341. mm^4
J_{xy} = -64038. mm^4
J_u = 250373. mm^4
J_v = 57936. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3641
c = cosα = .9344
s = sinα = .3561
x_g = 11.47 mm
y_g = 21.35 mm
N = -1440. N
T_y = -720. N
M_x = 966000. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 17.73 mm
v_m = 28.18 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.2 N/mm^2

CI09.xxxx.006

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
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1/2F
C
F
D
F
Fb
B
F
A
B
B


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F
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-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XbEJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	XbEJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3XbEJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XbEJ
	iperstatica $X=W_{EF}$						-13/40Fb	

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PROCEDIMENTO E RISULTATI 248501 CI09.xxxx.007

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

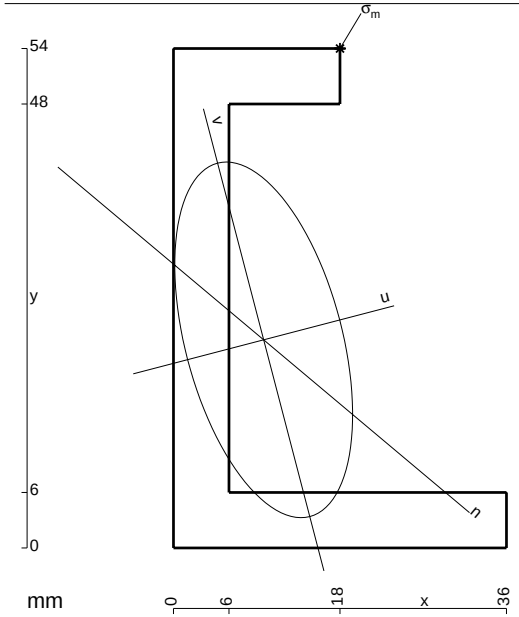
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 248501 CI09.xxxx.007

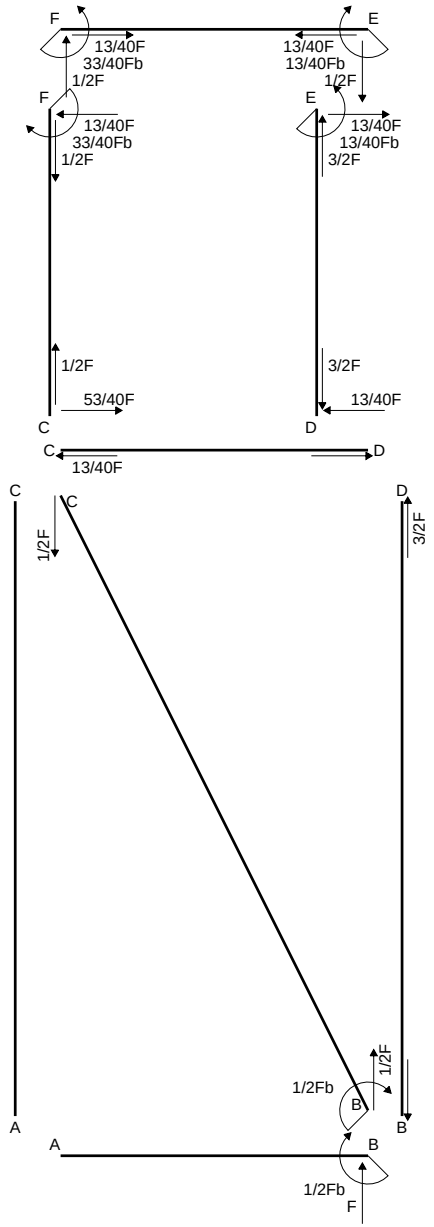


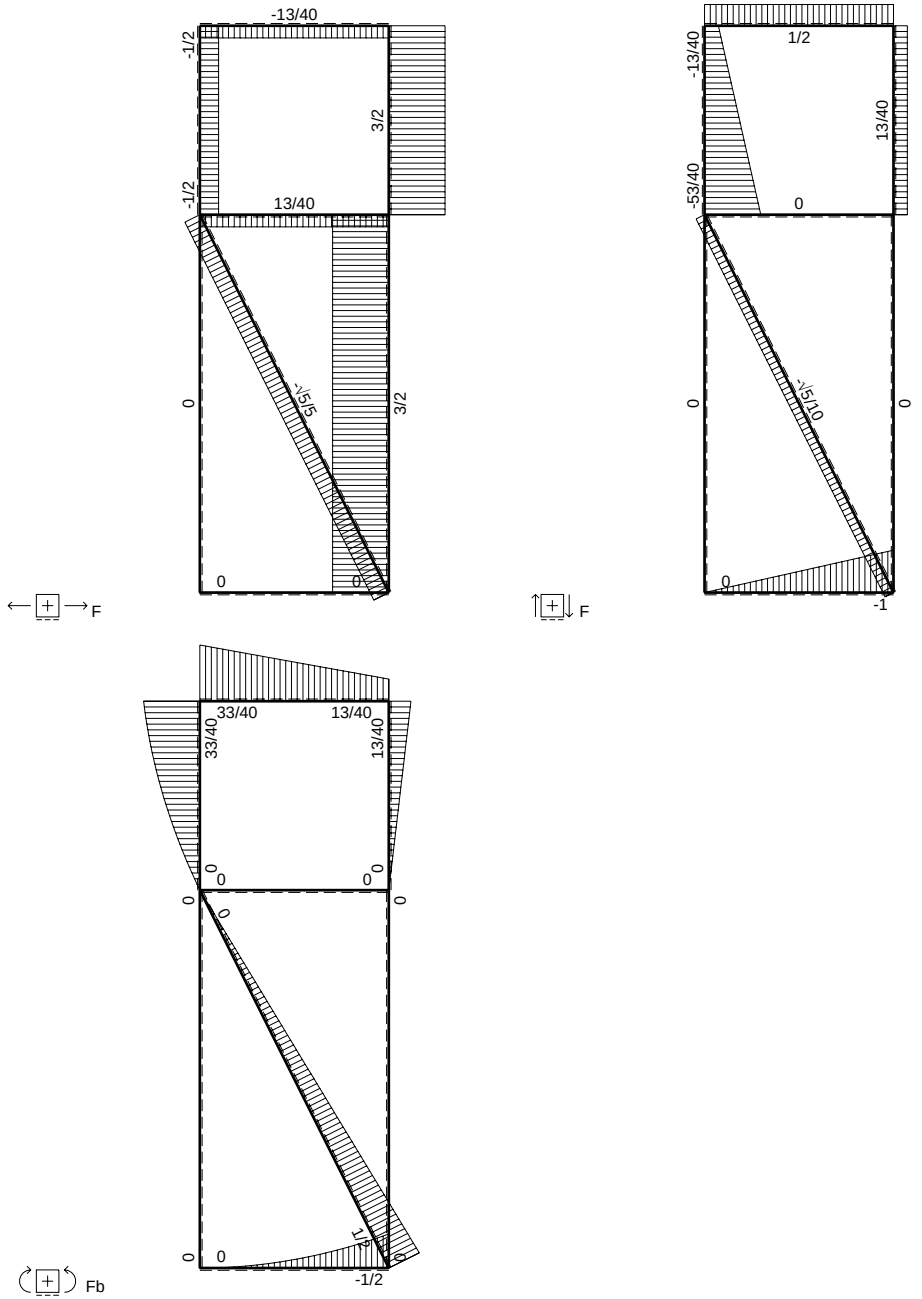
A = 576. mm^2
J_x = 212976. mm^4
J_y = 53244. mm^4
J_{xy} = -44712. mm^4
J_u = 224640. mm^4
J_v = 41580. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2552
$c = \cos\alpha$ = .9676
$s = \sin\alpha$ = .2524
x_g = 9.75 mm
y_g = 22.5 mm
N = -1386. N
T_y = -693.2 N
M_x = 992000. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 15.93 mm
v_m = 28.4 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.7 N/mm^2

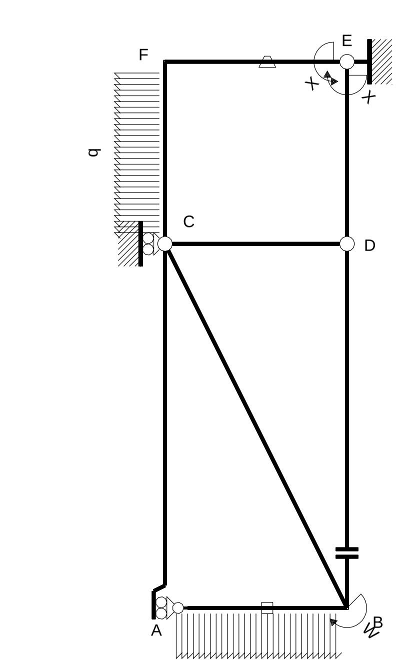
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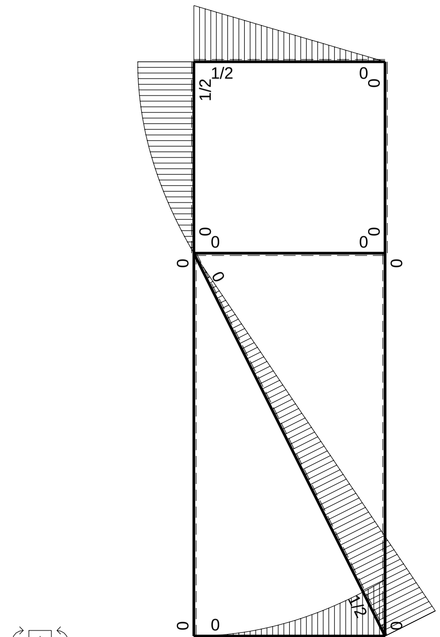
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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B
A
B
1/2Fb
F

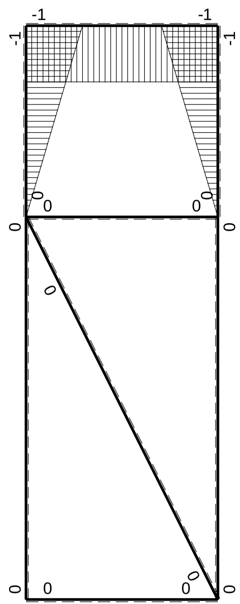

-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-√2/2
0
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	-Fb/EJ	$-1/2Fx$	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

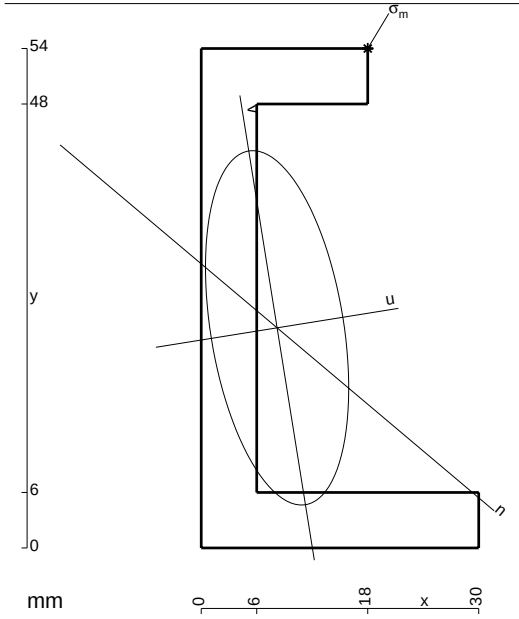
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

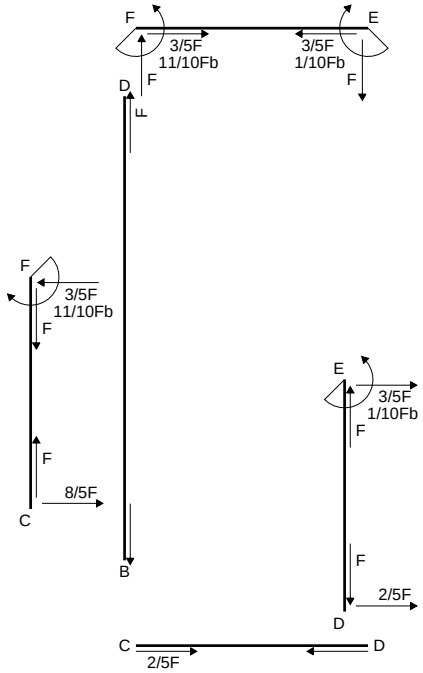
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

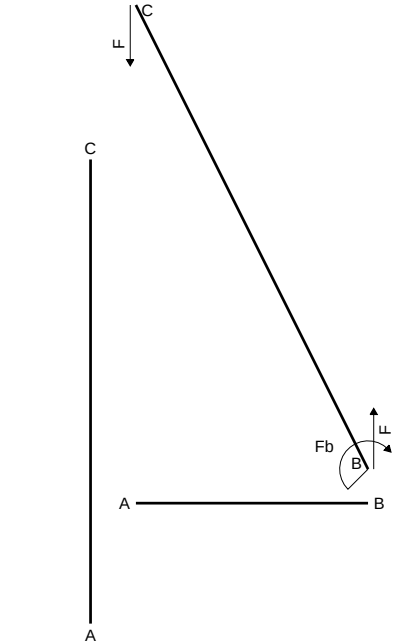


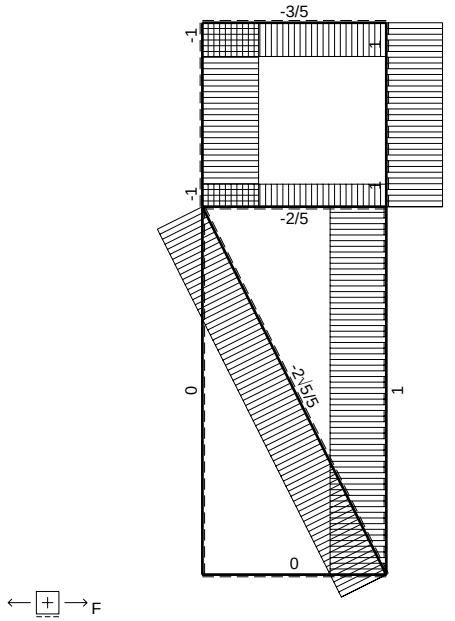
A = 540. mm^2
J_x = 198266. mm^4
J_y = 32378. mm^4
J_{xy} = -27302. mm^4
J_u = 202644. mm^4
J_v = 28000. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1590
$c = \cos\alpha$ = .9874
$s = \sin\alpha$ = .1583
x_g = 8.2 mm
y_g = 23.8 mm
N = -925. N
T_y = -601.3 N
M_x = 1037850. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 14.46 mm
v_m = 28.27 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.5 N/mm^2

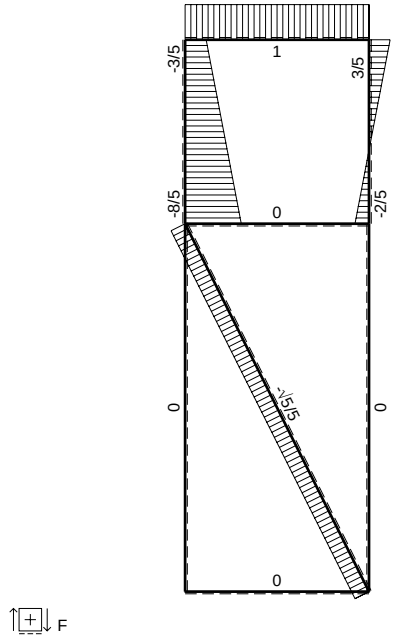
CI09.xxxx.008

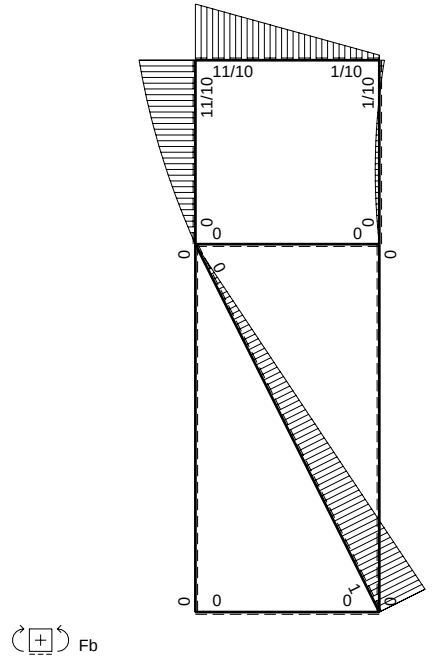
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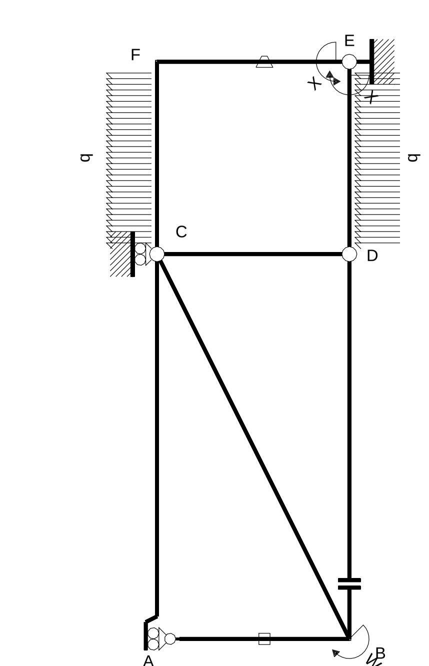

F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A

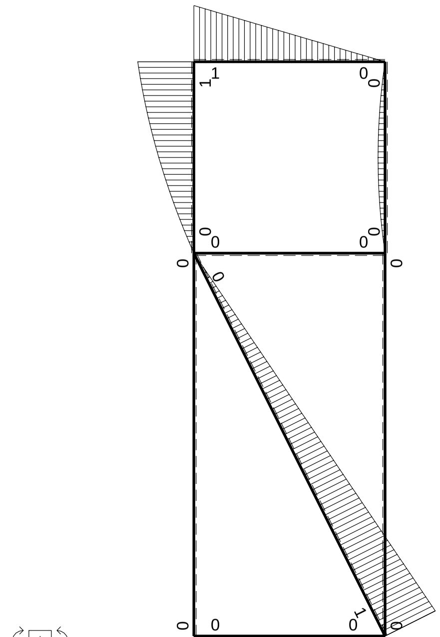

-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
0
F

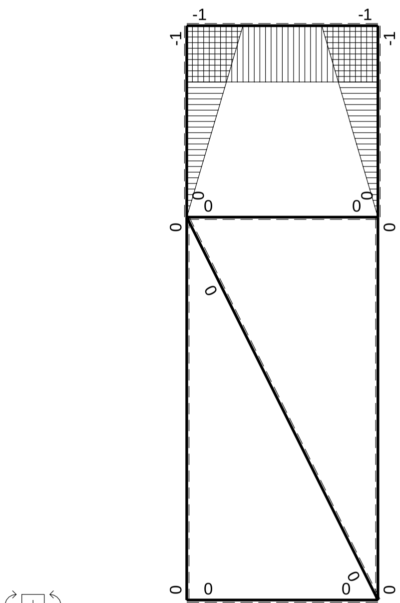

11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$1/6Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

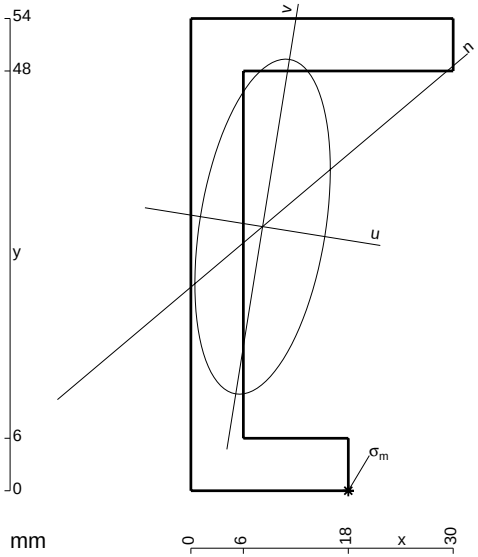
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

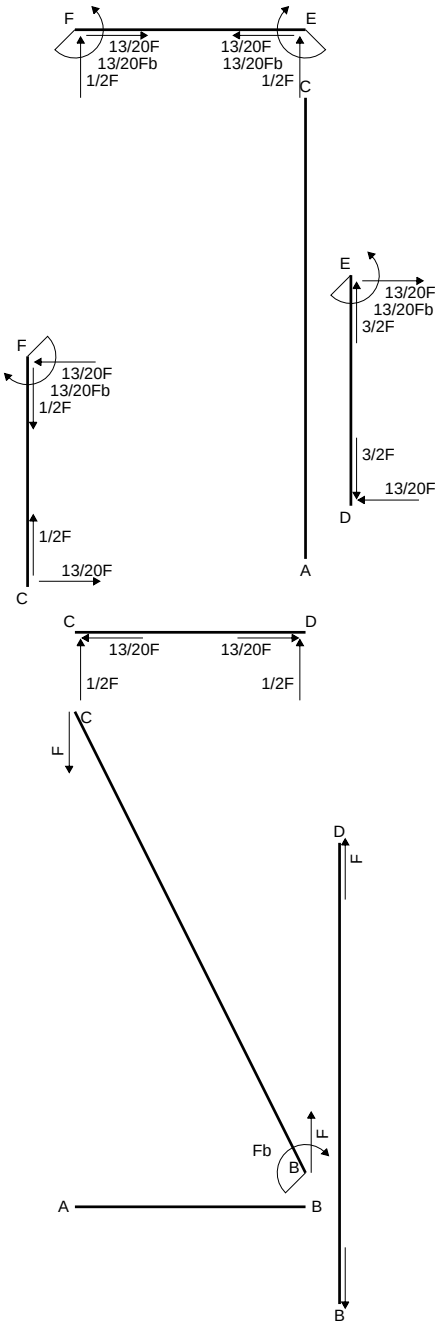
$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

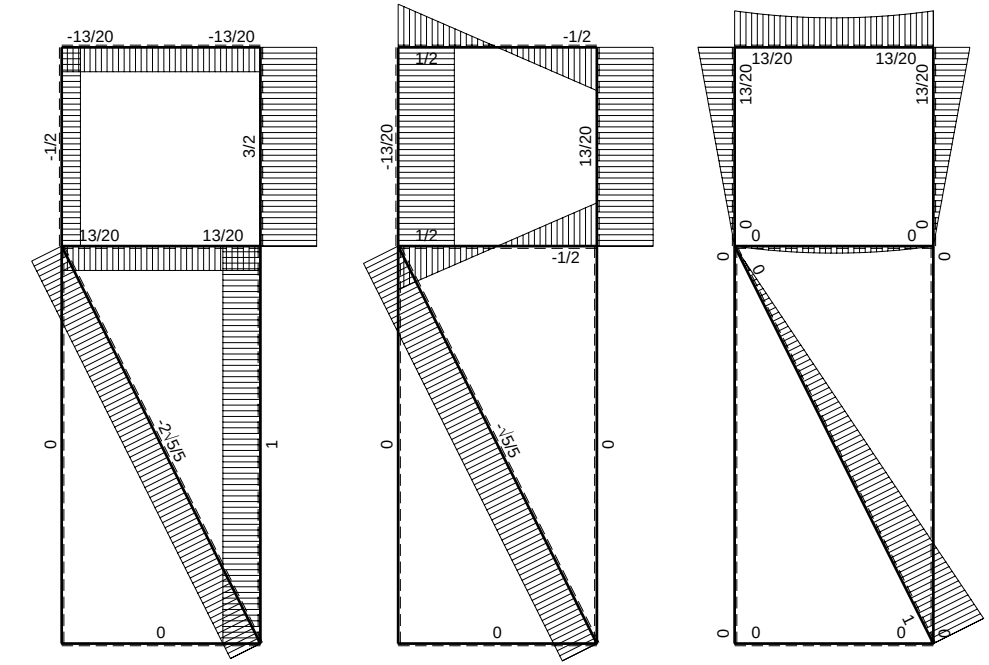


A = 540. mm^2
J_x = 198266. mm^4
J_y = 32378. mm^4
J_{xy} = 27302. mm^4
J_u = 202644. mm^4
J_v = 28000. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.1590$
$c = \cos\alpha = .9874$
$s = \sin\alpha = -.1583$
x_g = 8.2 mm
y_g = 30.2 mm
N = -828. N
T_y = 1380. N
M_x = 1092960. Nmm
x_m = 18. mm
u_m = 14.46 mm
v_m = -28.27 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 238.4 N/mm^2

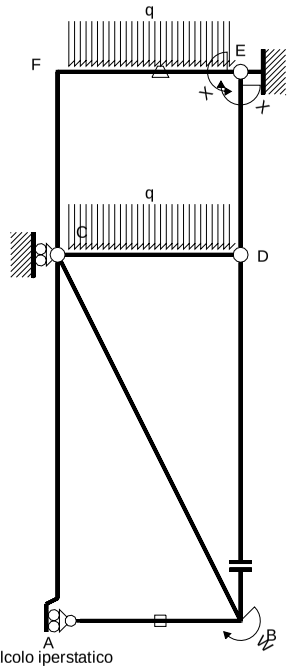
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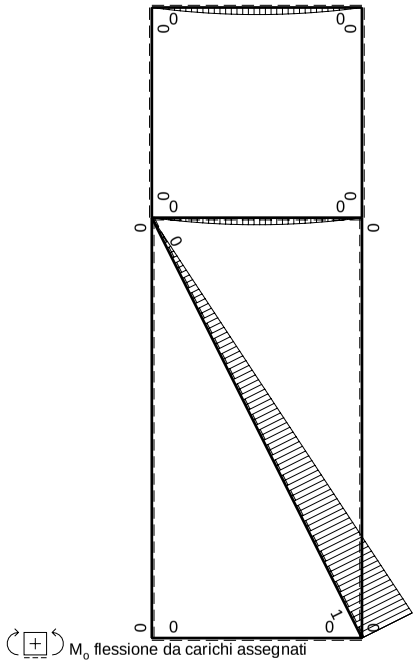

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
13/20
-1/2
3/2
1/2
0
1
-√5/5

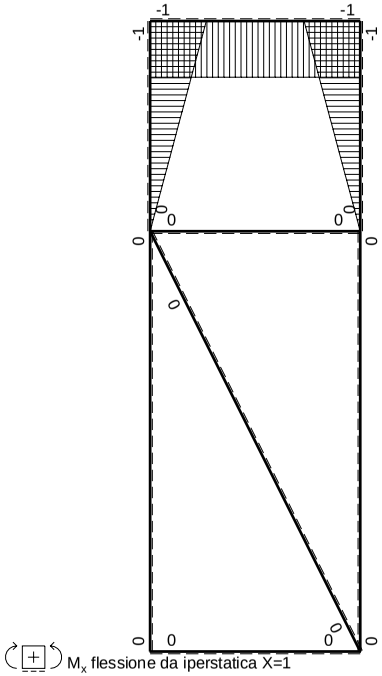

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 937106 CI09.xxxx.010

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

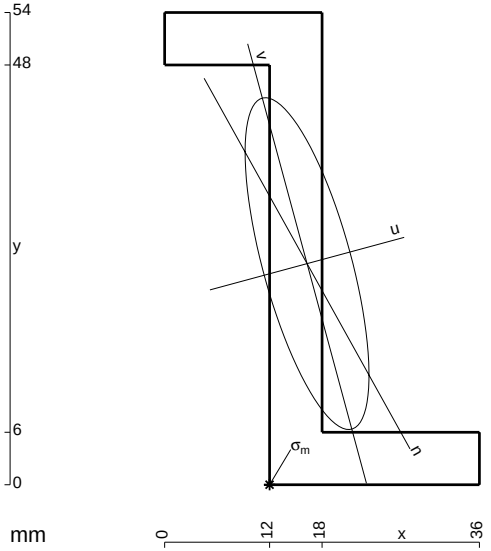
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

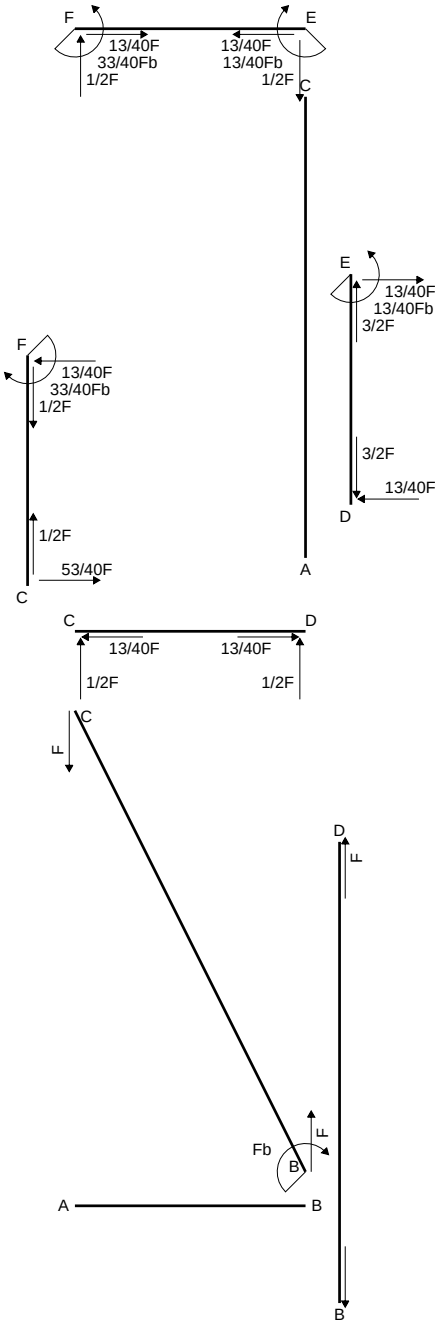
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

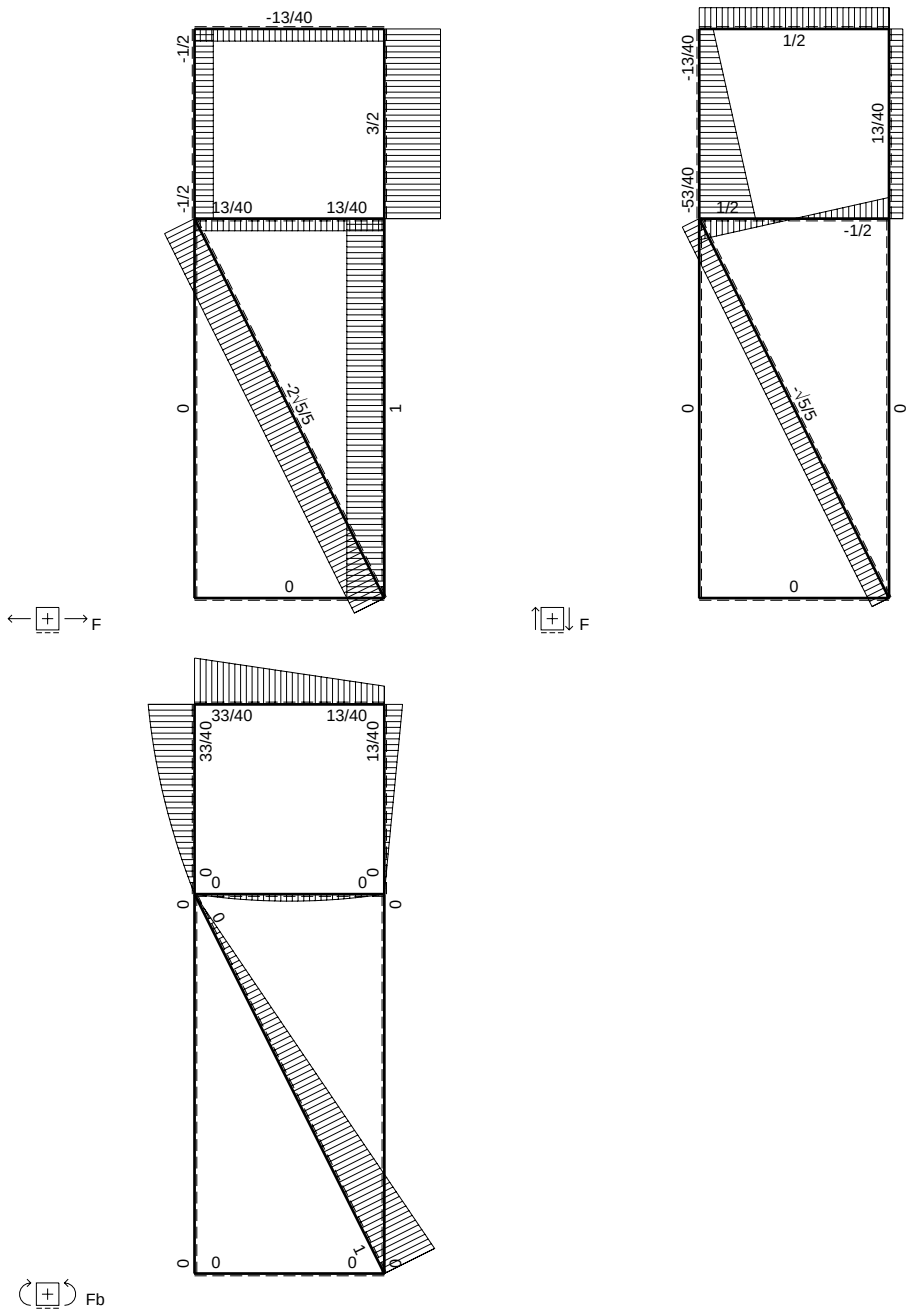


A = 504. mm^2
J_x = 181471. mm^4
J_y = 25303. mm^4
J_{xy} = -45545. mm^4
J_u = 193783. mm^4
J_v = 12991. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2640
$c = \cos\alpha$ = .9653
$s = \sin\alpha$ = .2610
x_g = 16.29 mm
y_g = 25.29 mm
N = -1431. N
T_y = -715.5 N
M_x = 608000. Nmm
x_m = 12. mm
u_m = -10.74 mm
v_m = -23.29 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 198.8 N/mm^2

CI09.xxxx.010

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
-2√5/5
0
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XkJ/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	XbJ/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3XkJ/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XkJ/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 265724 CI09.xxxx.011

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

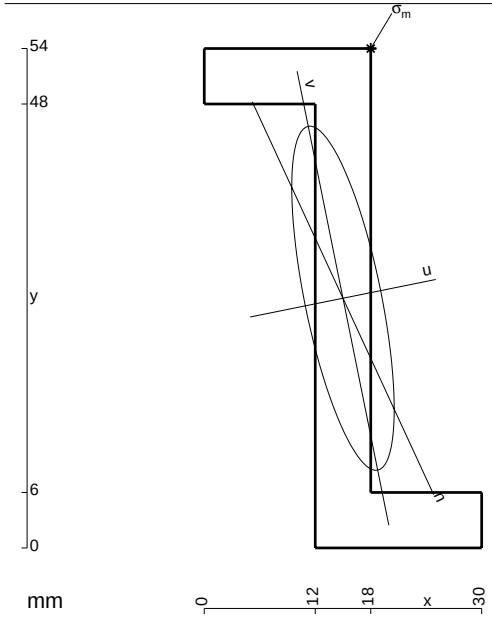
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

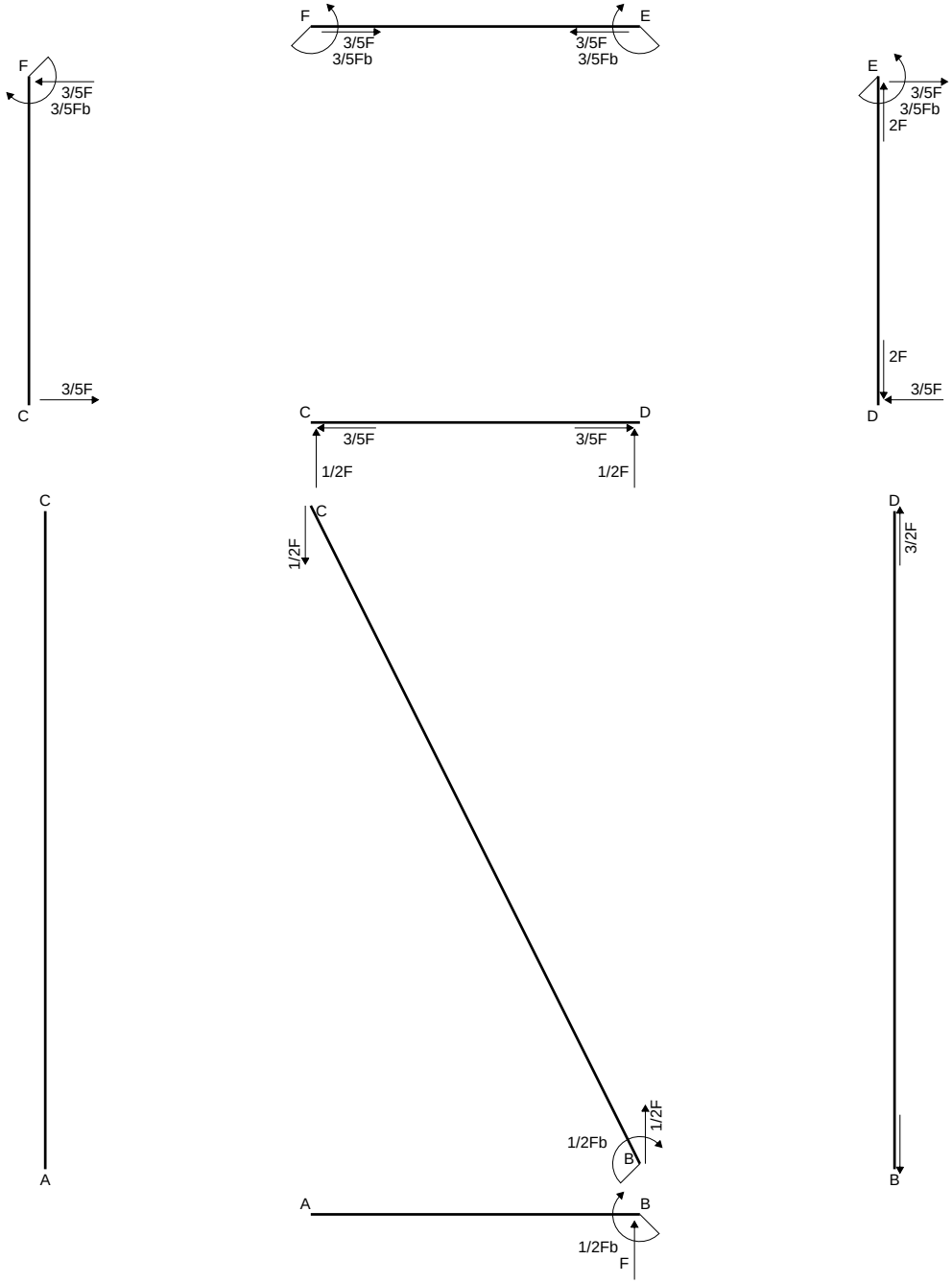
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

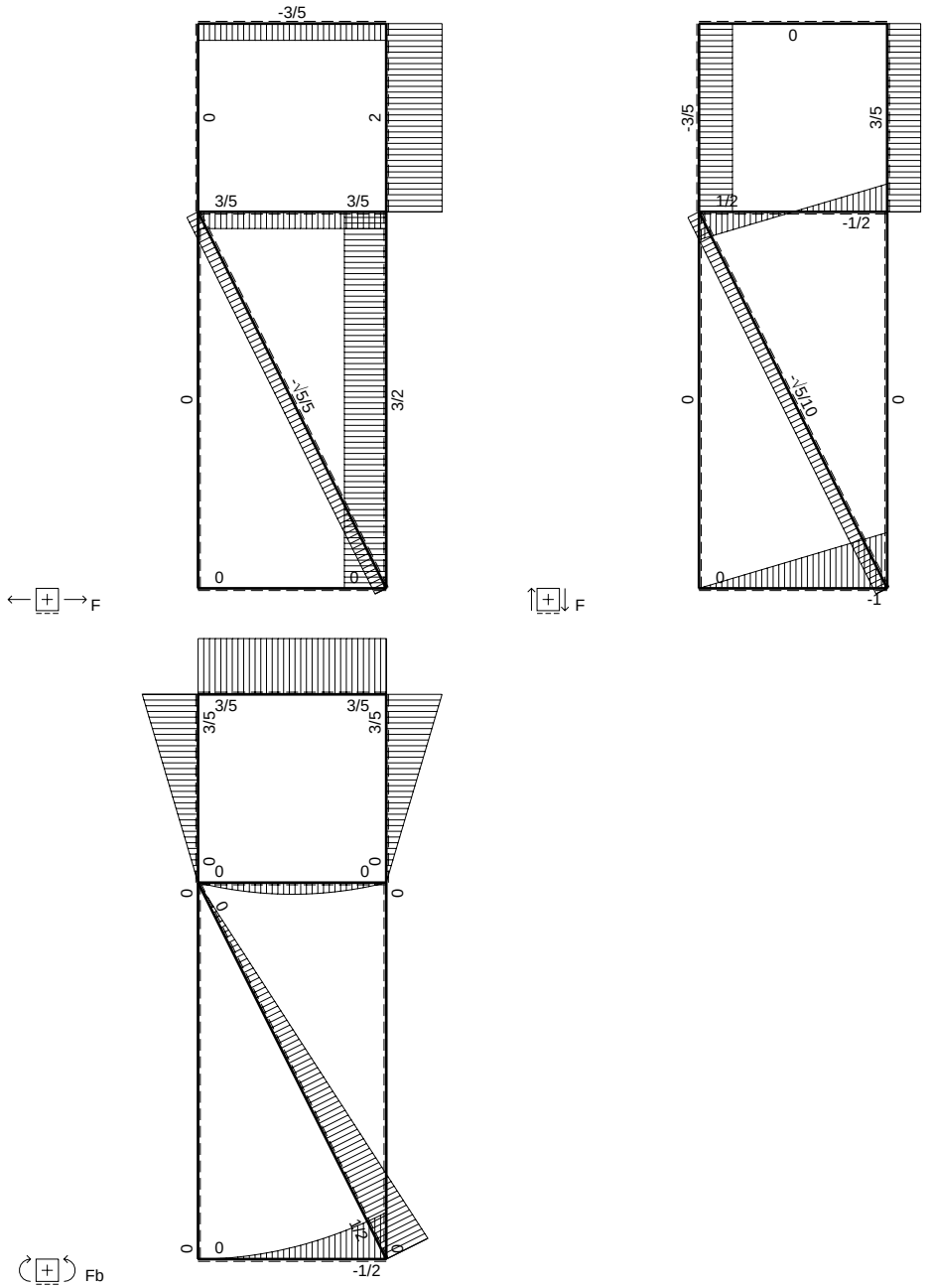


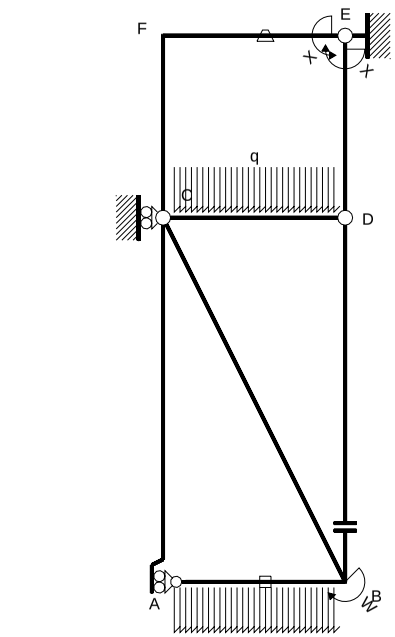
A = 468. mm^2
J_x = 162108. mm^4
J_y = 14364. mm^4
J_{xy} = -31104. mm^4
J_u = 168389. mm^4
J_v = 8083. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1993
$c = \cos\alpha$ = .9802
$s = \sin\alpha$ = .1979
x_g = 15. mm
y_g = 27. mm
N = -1279. N
T_y = -639.5 N
M_x = 586300. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 8.285 mm
v_m = 25.87 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -210. N/mm^2

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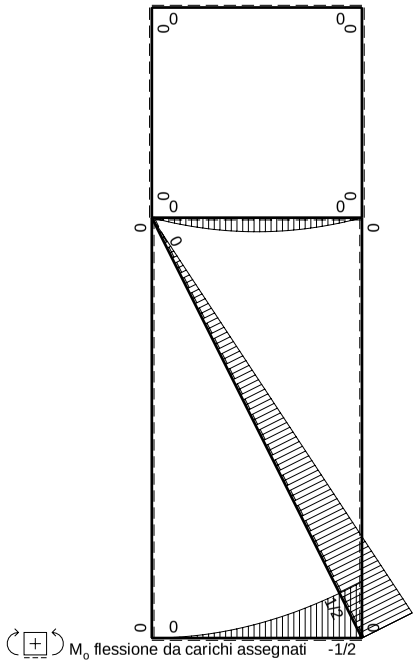
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
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1/2F
1/2Fb
B
A
B
1/2Fb
F

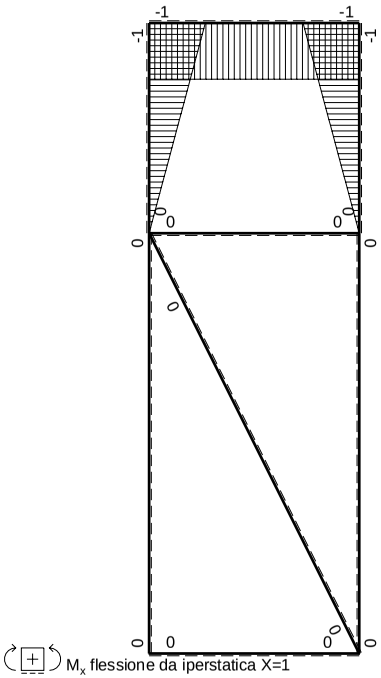

-3/5
0
2
3/5
3/5
0
3/2
-3/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
0
-3/10
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

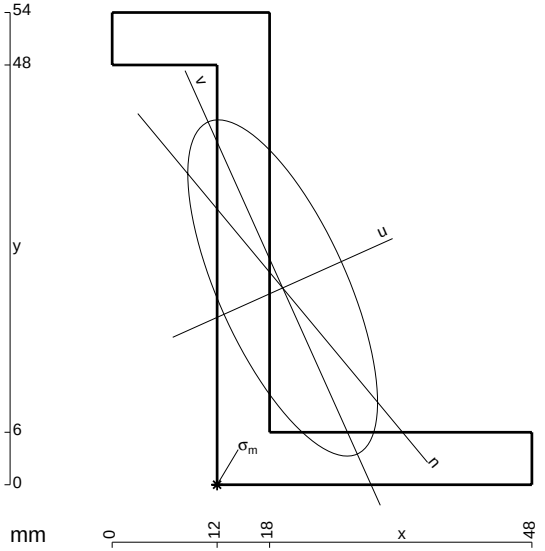
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

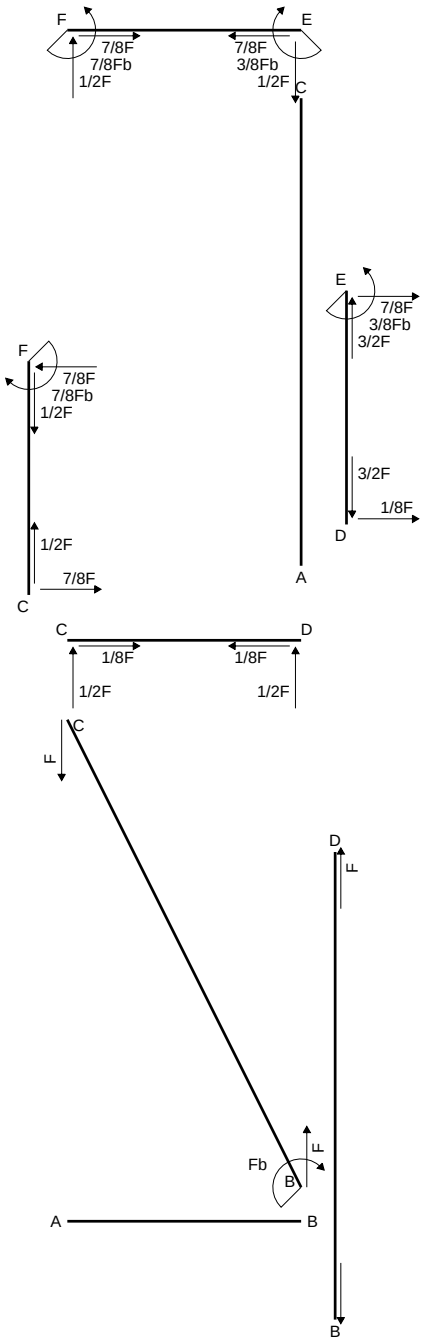
$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

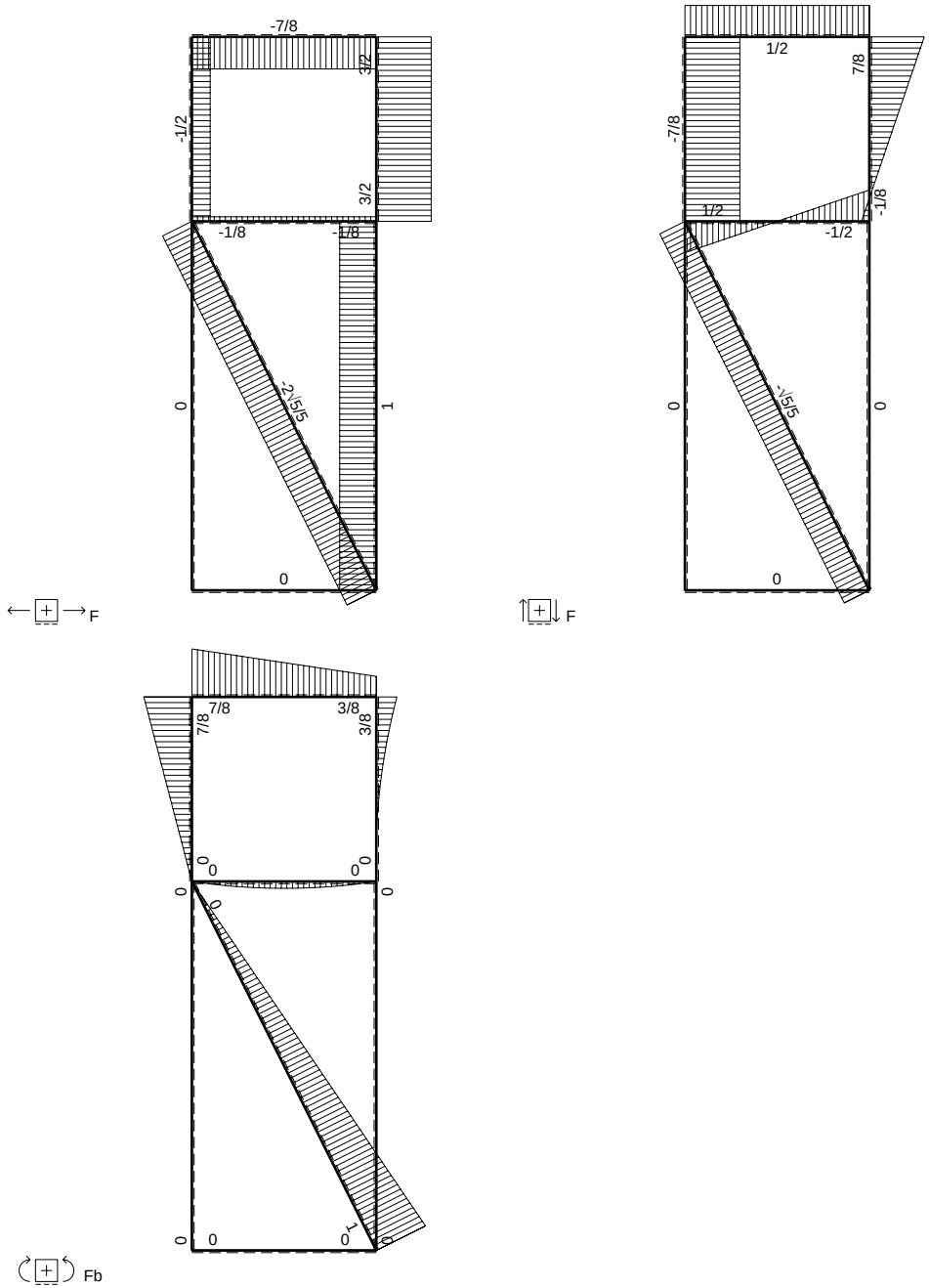


A = 576. mm^2
J_x = 212976. mm^4
J_y = 67824. mm^4
J_{xy} = -81648. mm^4
J_u = 249641. mm^4
J_v = 31159. mm^4
$\alpha = artg(2J_{xy}/(J_y - J_x))/2$ = .4221
$c = \cos\alpha$ = .9122
$s = \sin\alpha$ = .4097
x_g = 19.5 mm
y_g = 22.5 mm
N = 5660. N
T_y = 1698. N
M_x = 764100. Nmm
x_m = 12. mm
u_m = -16.06 mm
v_m = -17.45 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 219.9 N/mm^2

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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
-2√5/5
0
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
-√5/5
0
0
F
7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984264 CI09.xxxx.013

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

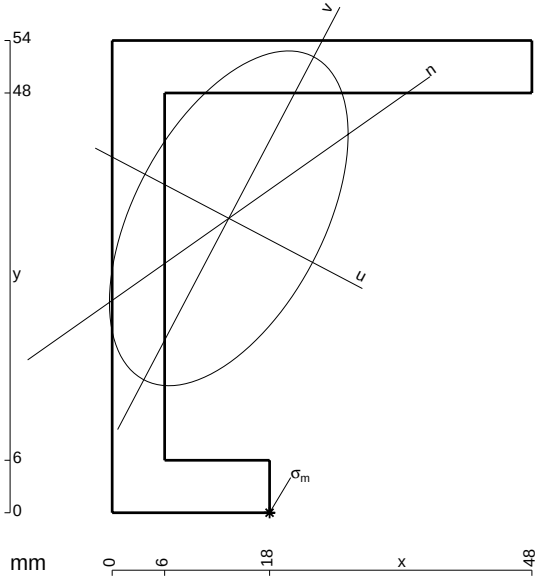
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

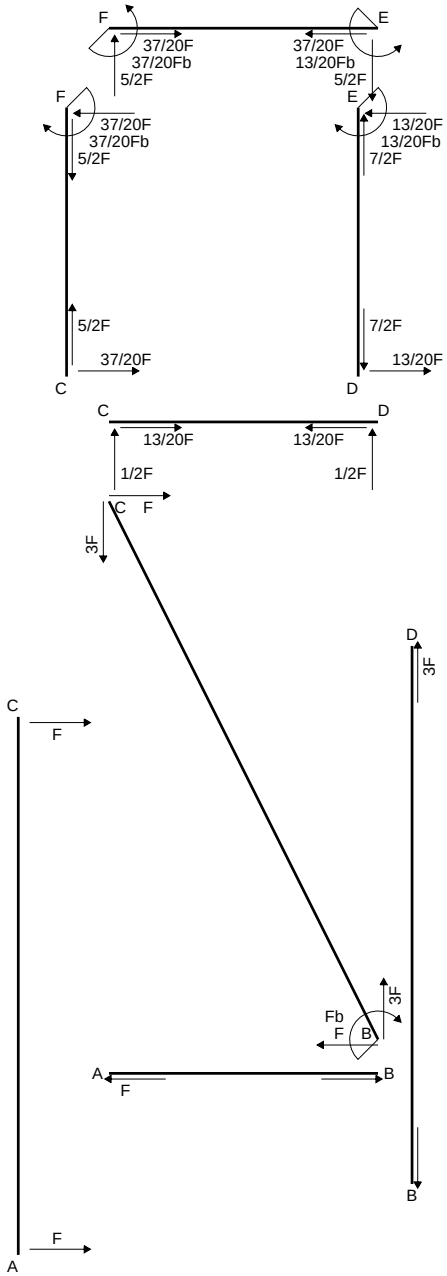
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

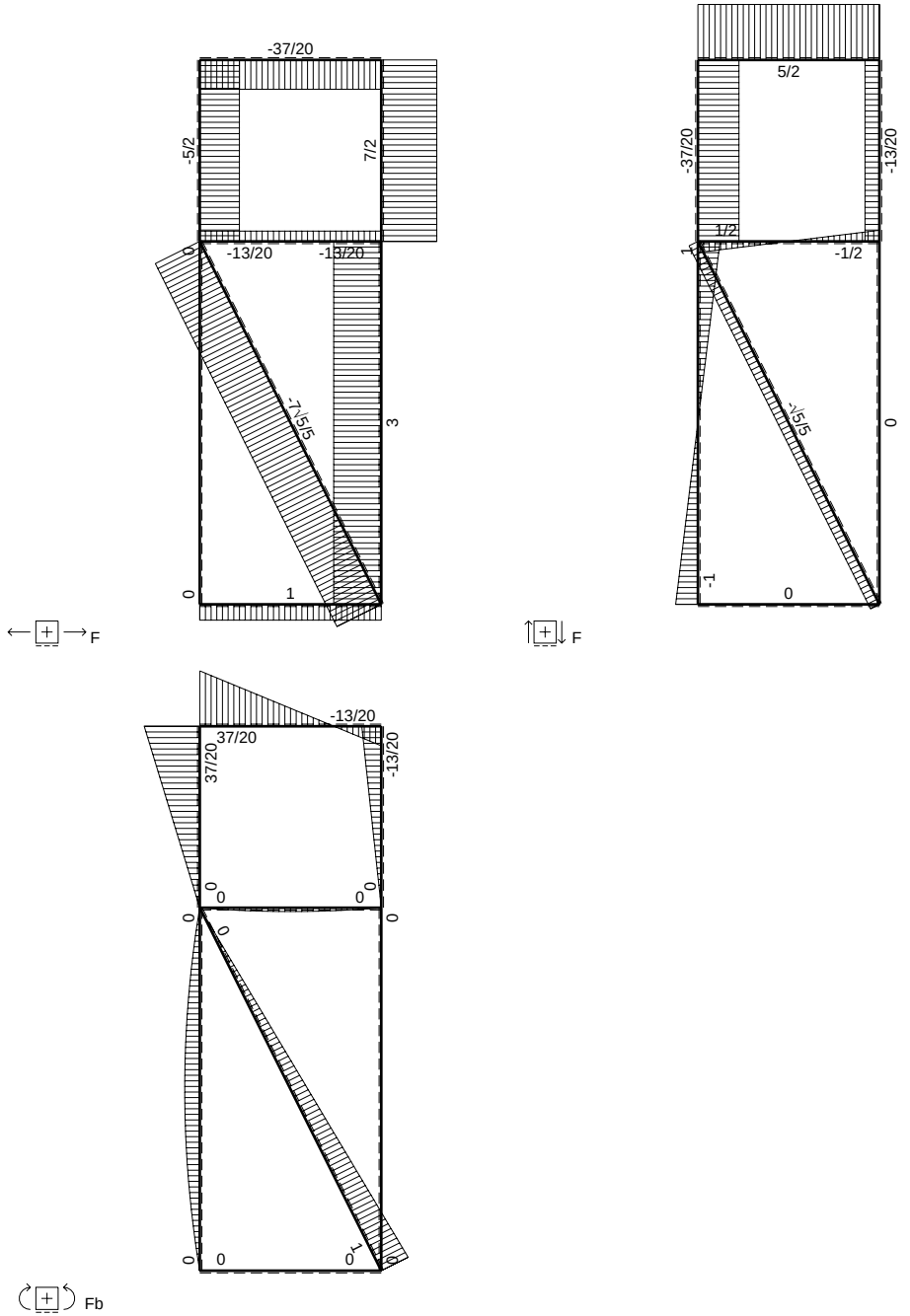


A = 648. mm^2
J_x = 237528. mm^4
J_y = 120672. mm^4
J_{xy} = 84960. mm^4
J_u = 282212. mm^4
J_v = 75988. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4842
$c = \cos\alpha$ = .8851
$s = \sin\alpha$ = -.4655
x_g = 13.33 mm
y_g = 33.67 mm
N = -2039. N
T_y = -1020. N
M_x = 1117200. Nmm
x_m = 18. mm
u_m = 19.8 mm
v_m = -27.62 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 229.2 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
7/2F
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C
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3F
D
3F
C
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3F
Fb
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F
F
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B


-37/20
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-37/20
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F
-13/20
37/20
37/20
-13/20
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0
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0
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0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x²/b²	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x²/b²		
CD b	0	1/2Fx-1/2qx²	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx²	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	1-2x/b+x²/b²	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x²/b²		
	totali						-13/12Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

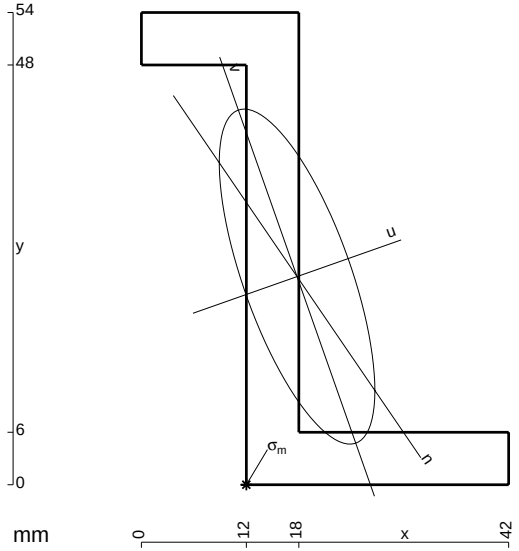
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

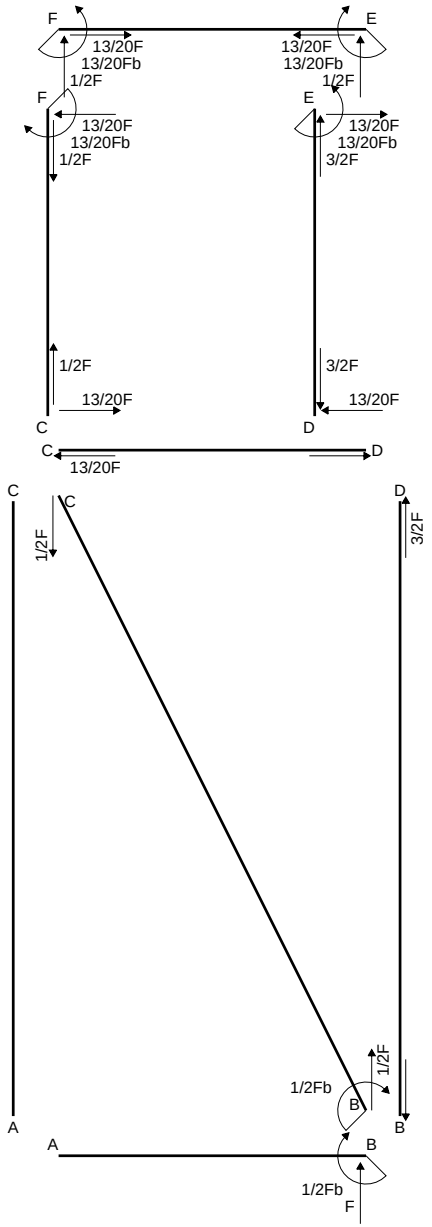
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

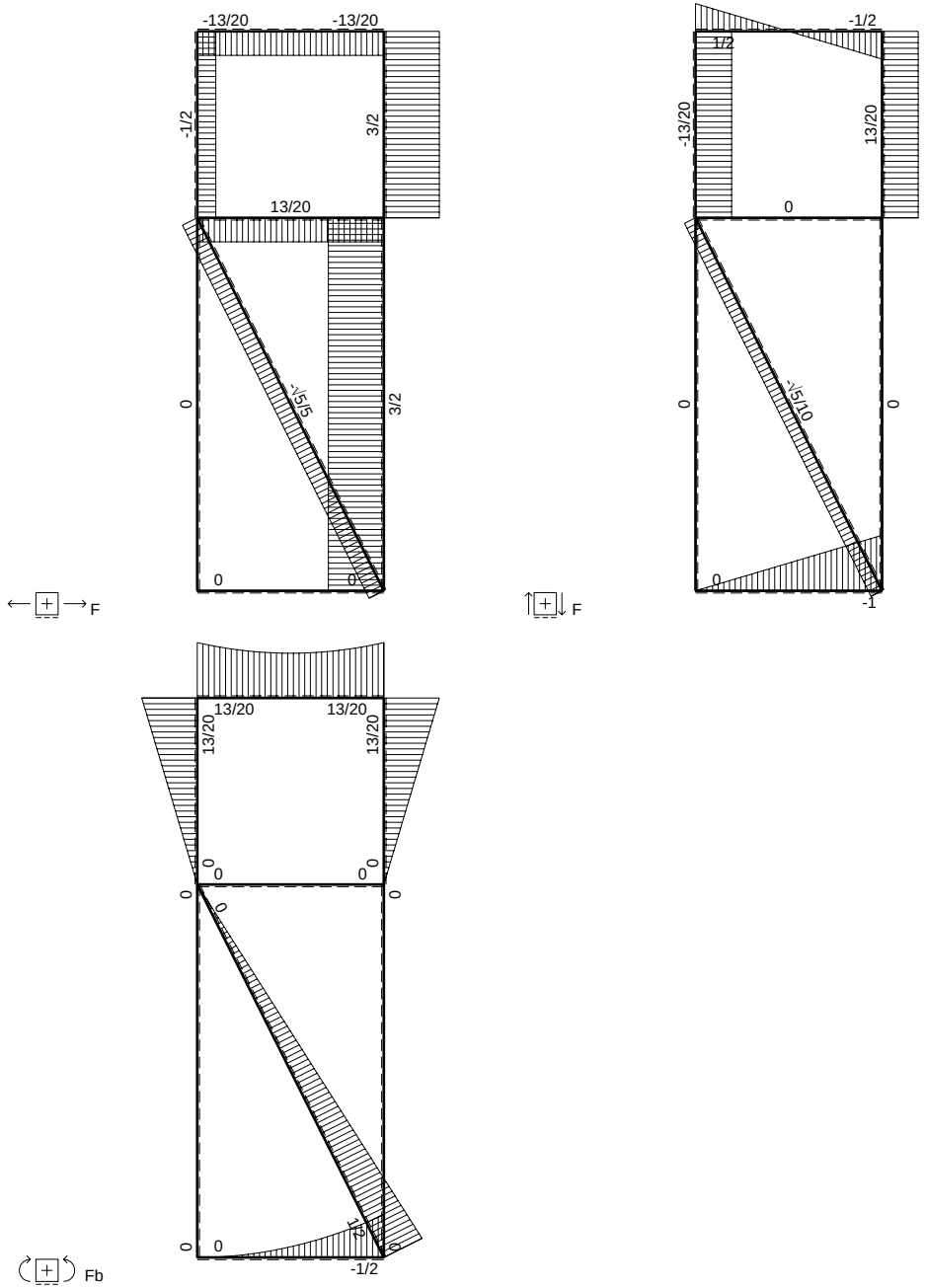


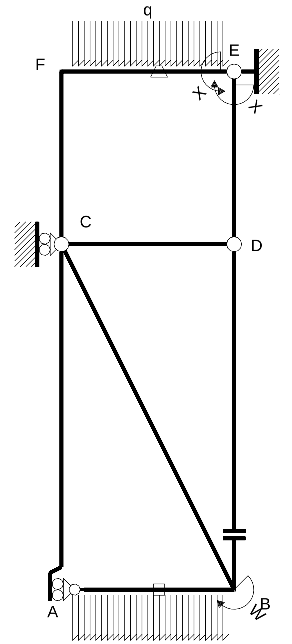
A = 540. mm^2
J_x = 198266. mm^4
J_y = 42746. mm^4
J_{xy} = -62554. mm^4
J_u = 220304. mm^4
J_v = 20709. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3387
$c = \cos\alpha$ = .9432
$s = \sin\alpha$ = .3323
x_g = 17.8 mm
y_g = 23.8 mm
N = -1499. N
T_y = 2025. N
M_x = 794205. Nmm
x_m = 12. mm
u_m = -13.38 mm
v_m = -20.52 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 237.5 N/mm^2

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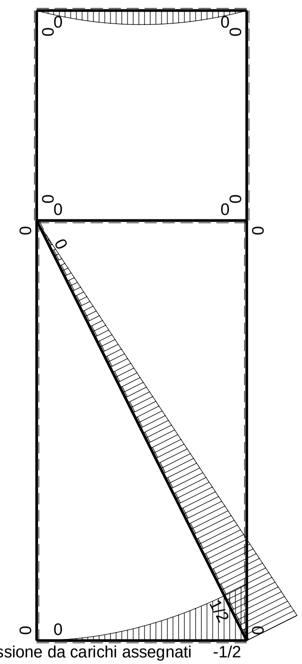
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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

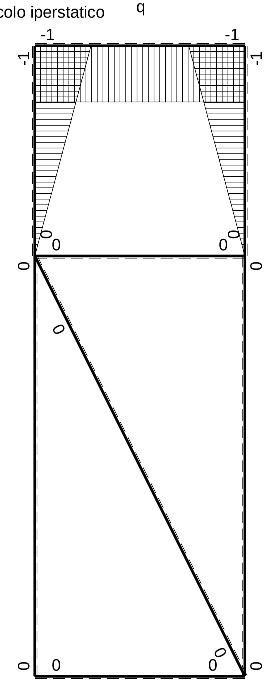

-13/20
13/20
-1/2
3/2
0
-√5/5
F
1/2
-√5/10
-1
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati -1/2



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

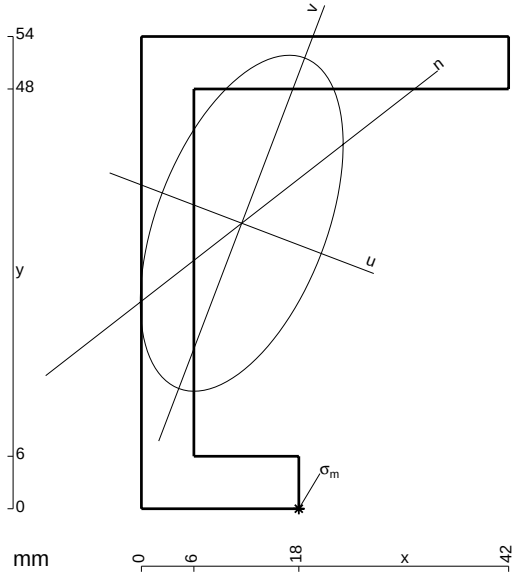
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 268862 CI09.xxxx.015

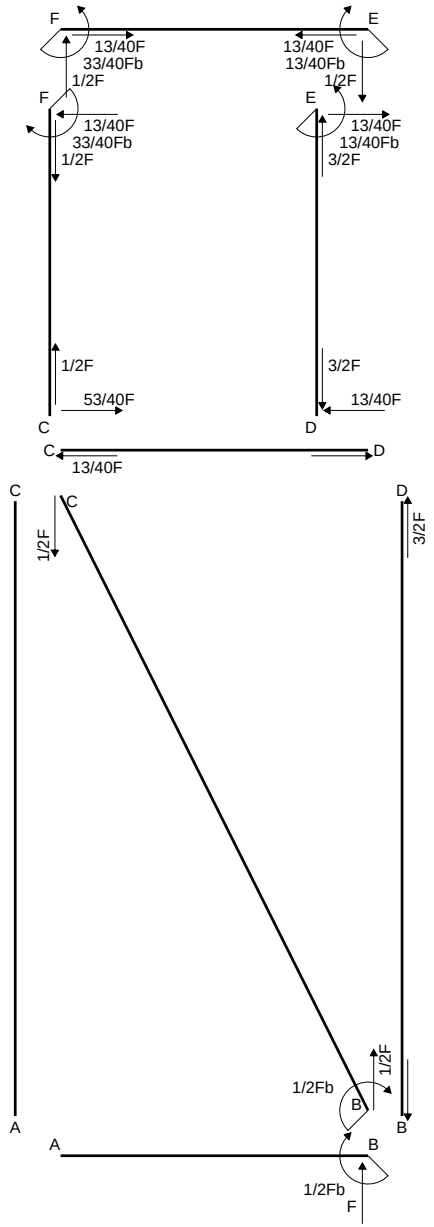


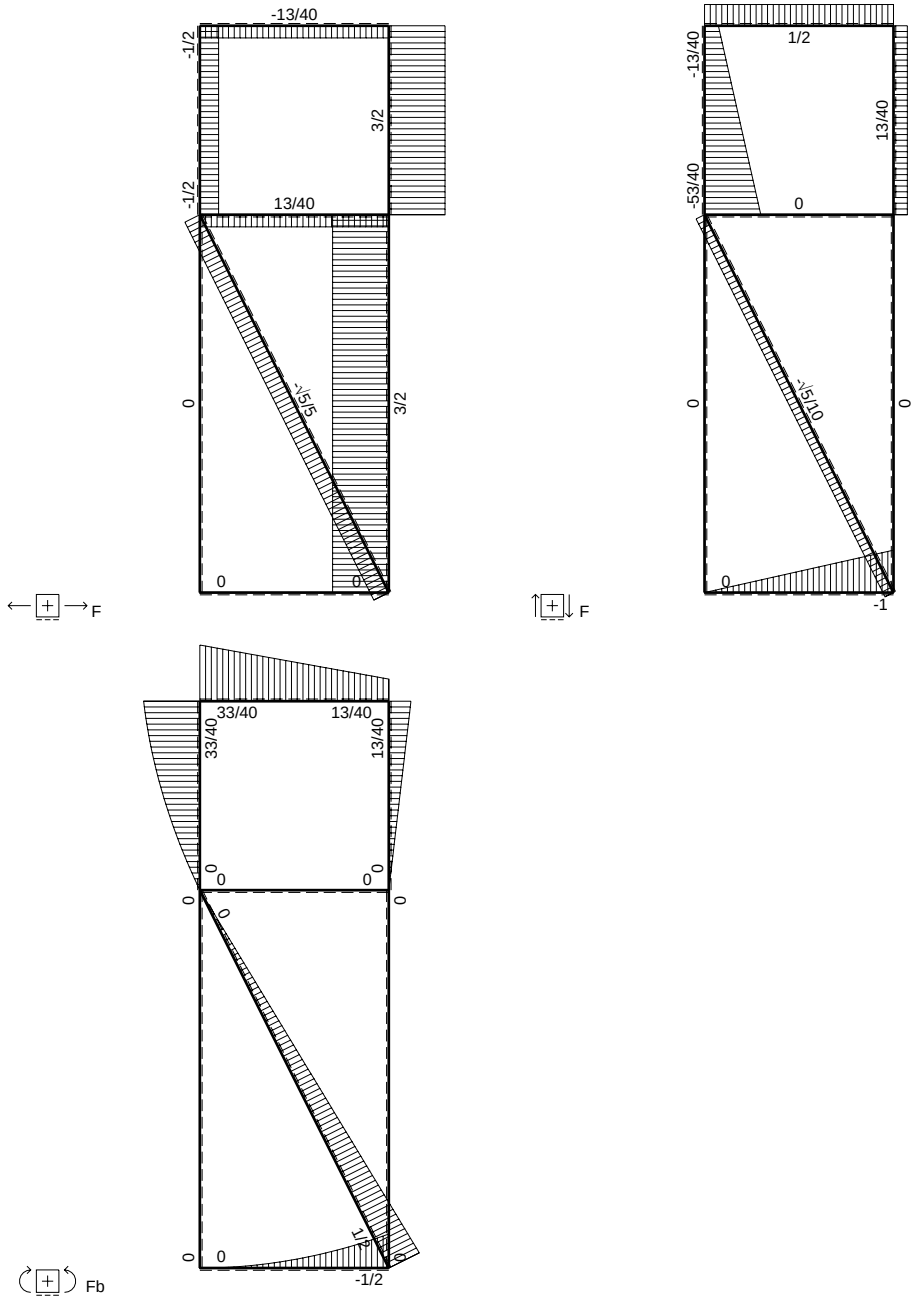
A = 612. mm^2
J_x = 225968. mm^4
J_y = 82341. mm^4
J_{xy} = 64038. mm^4
J_u = 250373. mm^4
J_v = 57936. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3641
$c = \cos\alpha$ = .9344
$s = \sin\alpha$ = -.3561
x_g = 11.47 mm
y_g = 32.65 mm
N = 3660. N
T_y = 1586. N
M_x = 904020. Nmm
x_m = 18. mm
u_m = 17.73 mm
v_m = -28.18 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 199.6 N/mm^2

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
1/2F
3/2F
1/2Fb
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-√2/2
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984261 CI09.xxxx.016

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

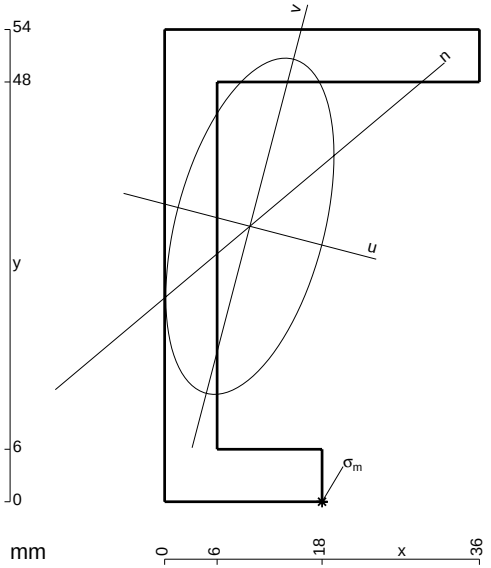
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

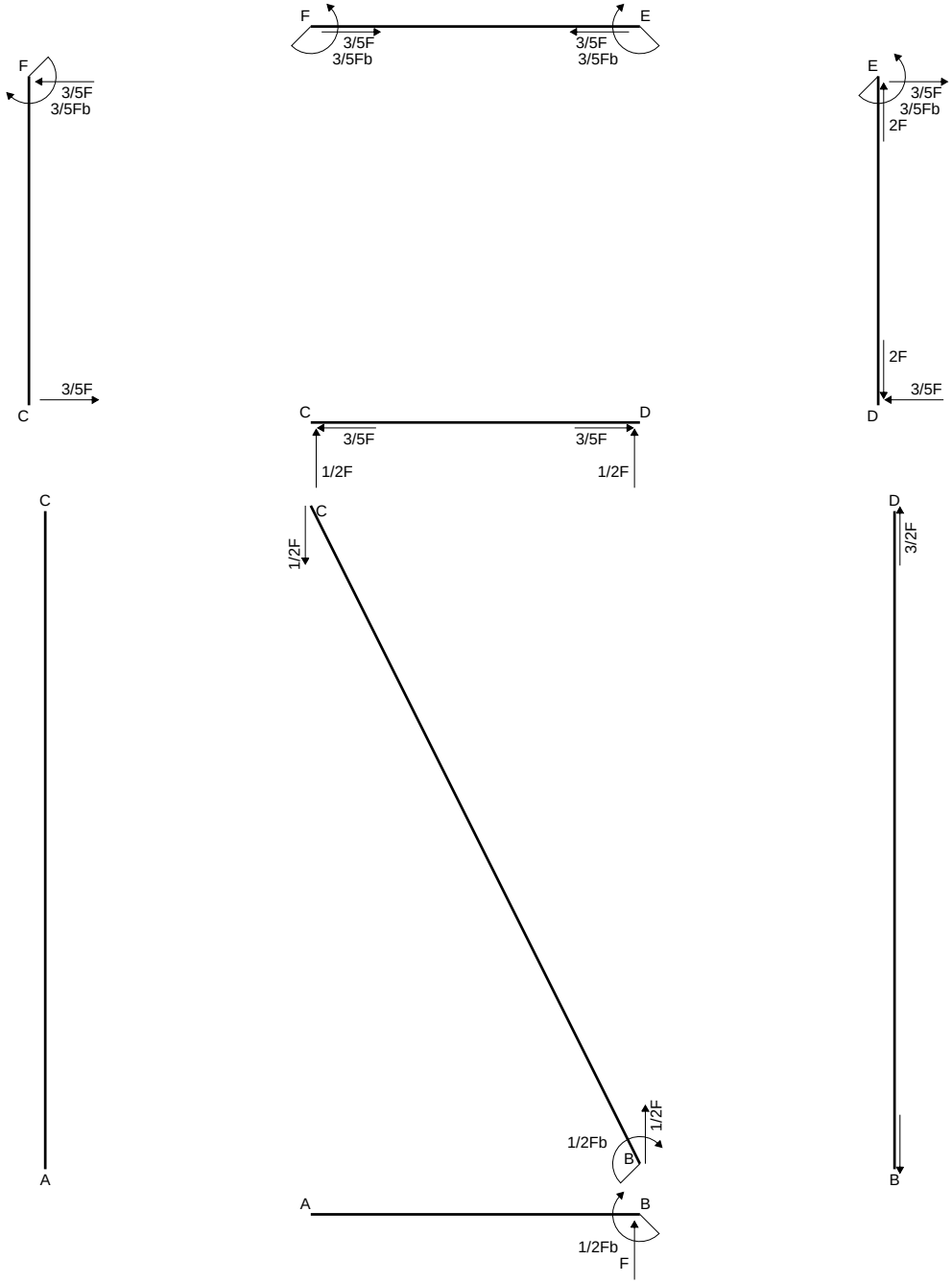
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

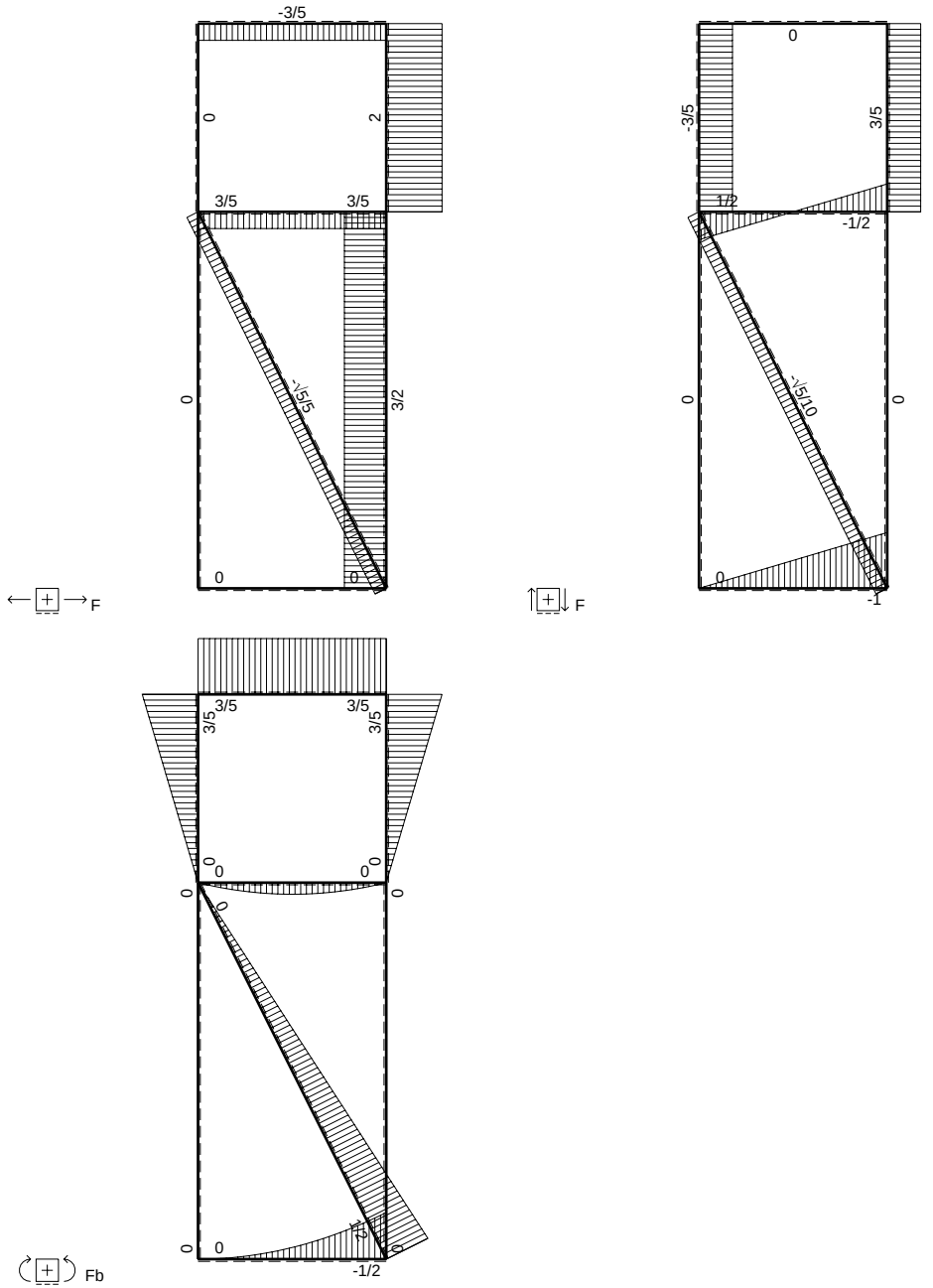


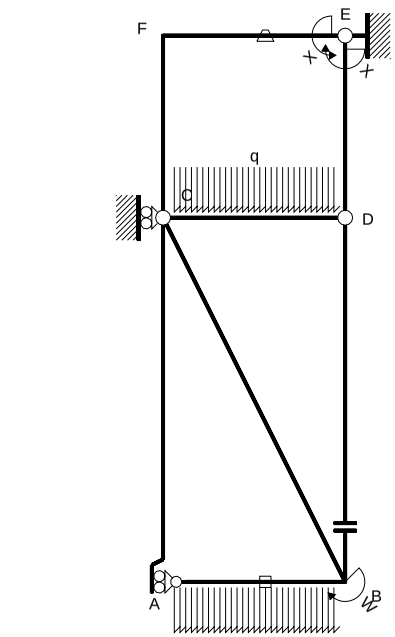
A = 576. mm^2
J_x = 212976. mm^4
J_y = 53244. mm^4
J_{xy} = 44712. mm^4
J_u = 224640. mm^4
J_v = 41580. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.2552$
$c = \cos\alpha = .9676$
$s = \sin\alpha = -.2524$
x_g = 9.75 mm
y_g = 31.5 mm
N = -630.5 N
T_y = 970. N
M_x = 960300. Nmm
x_m = 18. mm
u_m = 15.93 mm
v_m = -28.4 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 209.3 N/mm^2

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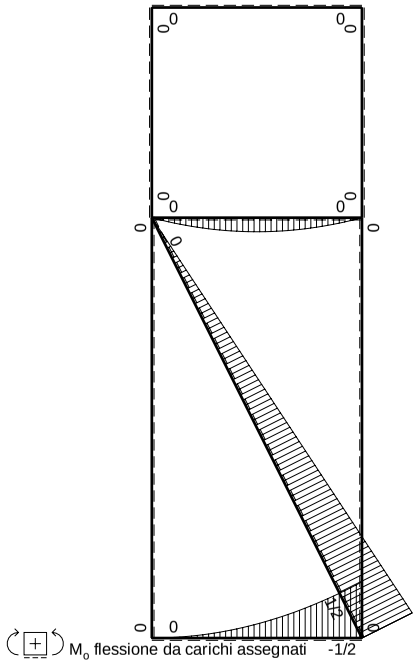
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
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B
A
B
1/2Fb
F

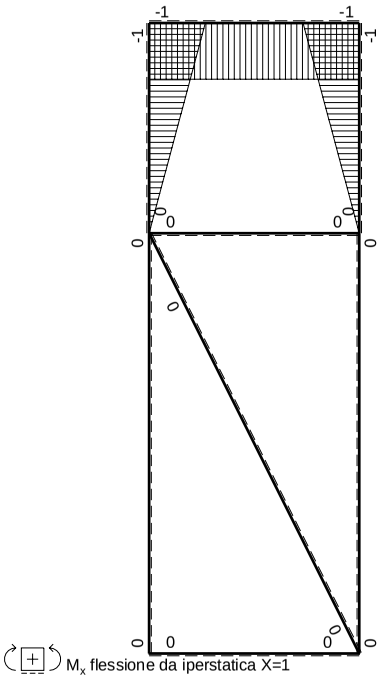

-3/5
0
2
3/5
3/5
0
3/2
-3/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-3/5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

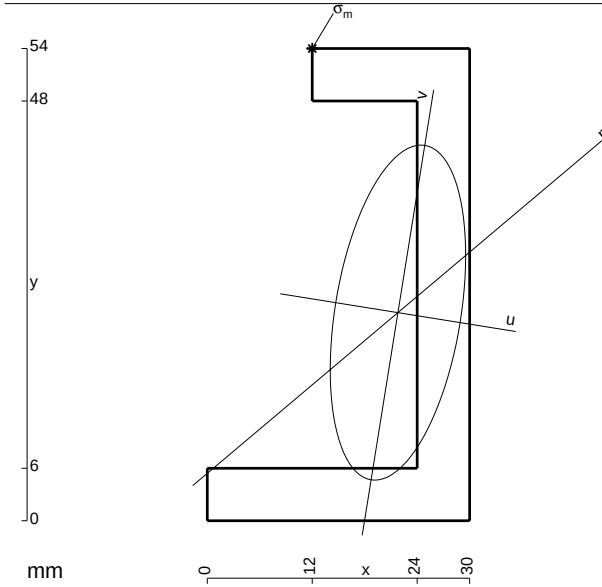
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

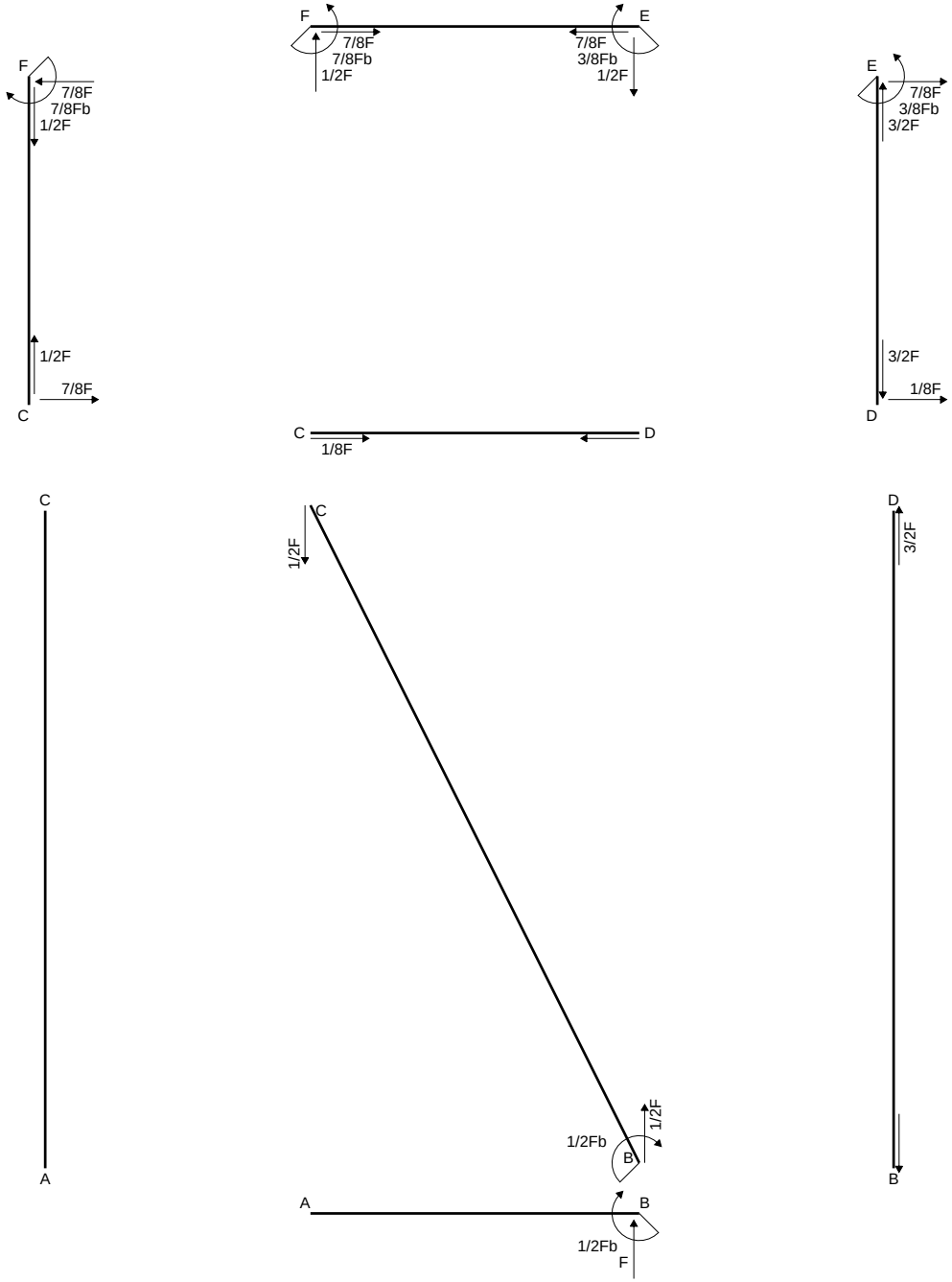
$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

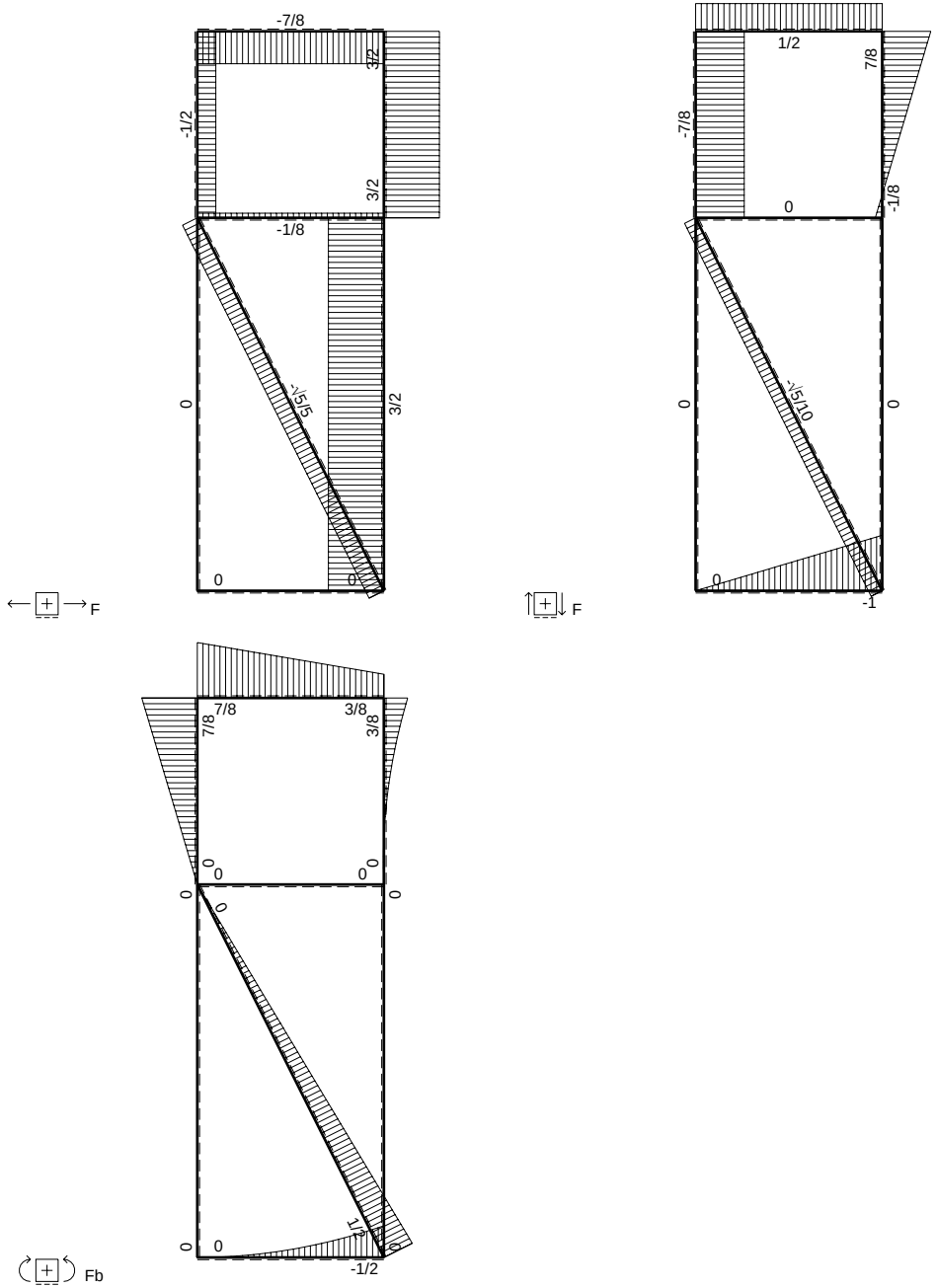


A = 540. mm^2
J_x = 198266. mm^4
J_y = 32378. mm^4
J_{xy} = 27302. mm^4
J_u = 202644. mm^4
J_v = 28000. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1590
c = cosα = .9874
s = sinα = -.1583
x_g = 21.8 mm
y_g = 23.8 mm
N = -1542. N
M_x = 986880. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -14.46 mm
v_m = 28.27 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.5 N/mm^2

CI09.xxxx.017

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F
7/8F
7/8Fb
1/2F
1/2F
7/8F
C
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
D
1/8F
E
7/8F
3/8Fb
3/2F
3/2F
1/8F
D
C
A
C
1/2F
1/2Fb
B
1/2F
A
B
1/2Fb
F
D
3/2F
B


-7/8
3/2
-1/2
3/2
-1/8
0
-√5/5
3/2
0
0
F
1/2
7/8
-7/8
0
-1/8
-√5/10
0
0
-1
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
-√2/2
0
-1/2
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [-\ x]_o^b\ \theta$

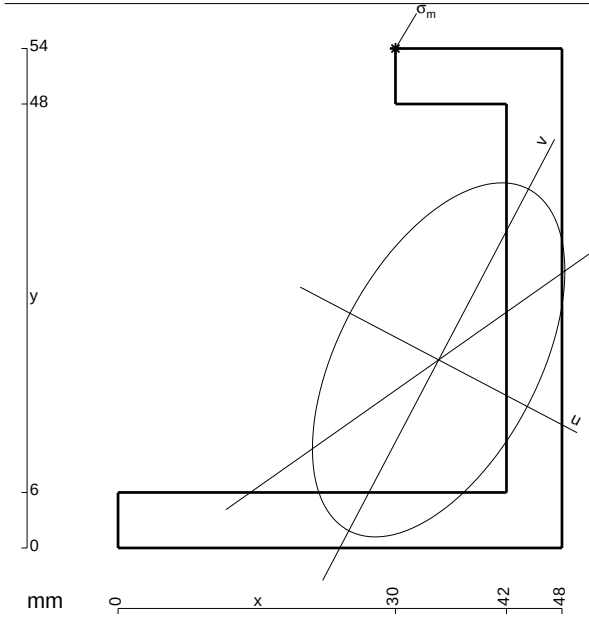
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

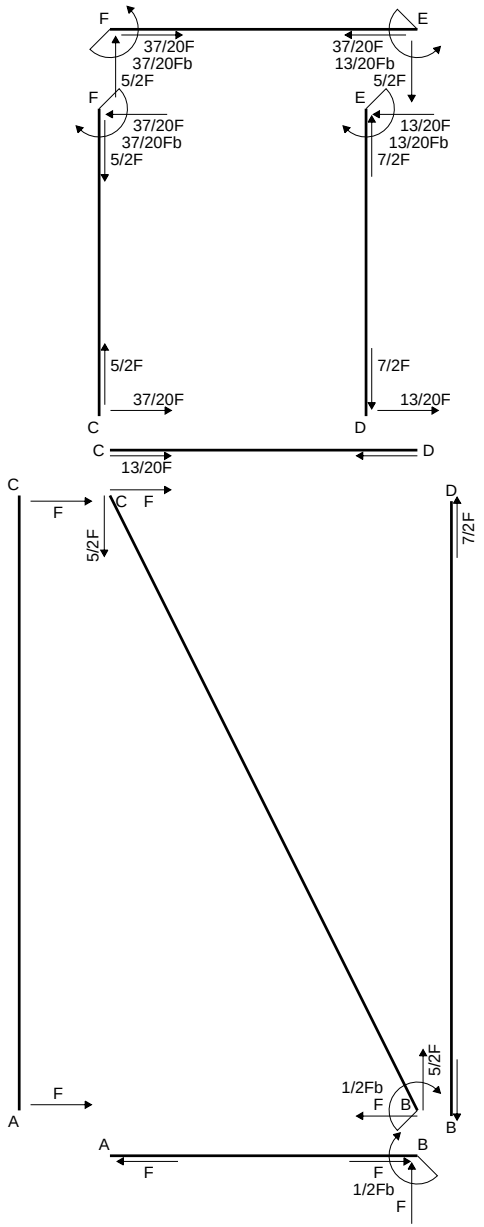
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

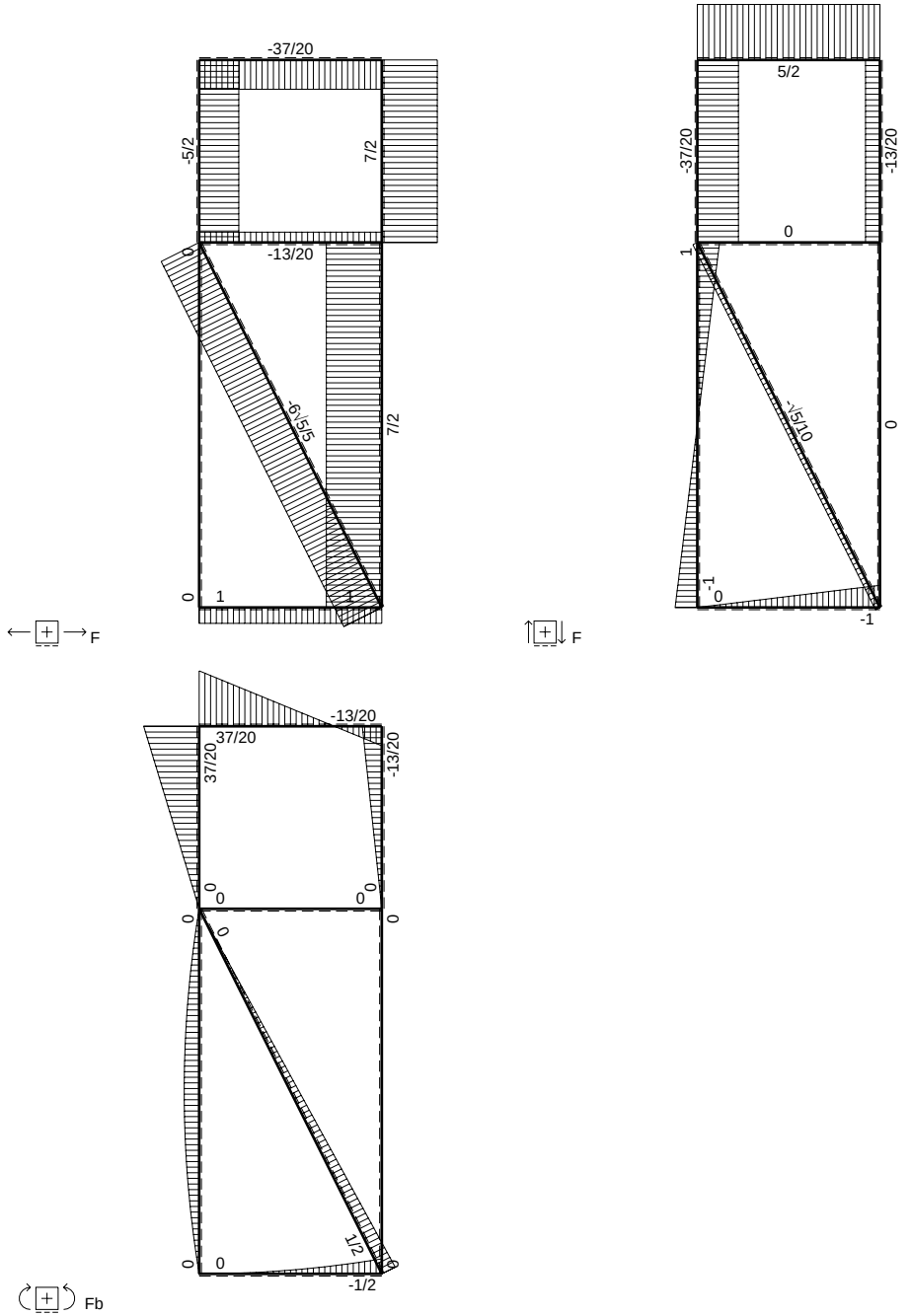


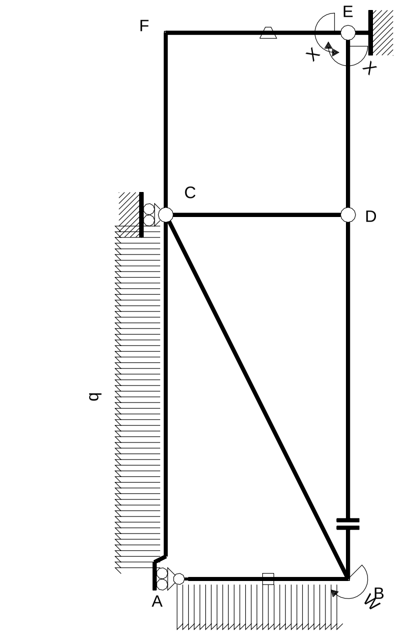
A = 648. mm^2
J_x = 237528. mm^4
J_y = 120672. mm^4
J_{xy} = 84960. mm^4
J_u = 282212. mm^4
J_v = 75988. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4842
$c = \cos\alpha$ = .8851
$s = \sin\alpha$ = -.4655
x_g = 34.67 mm
y_g = 20.33 mm
N = -1601. N
T_y = 915. N
M_x = 1088850. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = -19.8 mm
v_m = 27.62 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -228.9 N/mm^2

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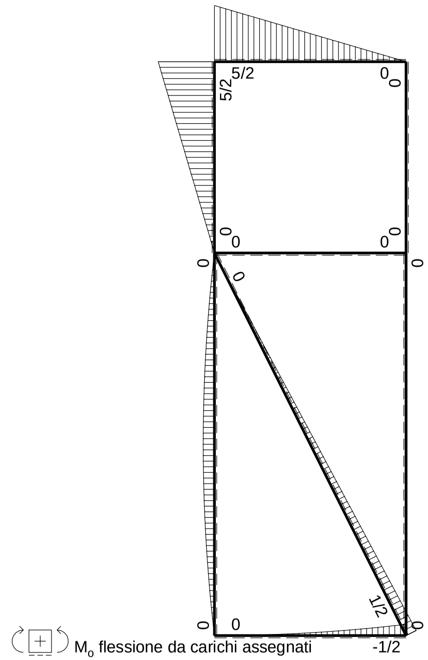
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F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb

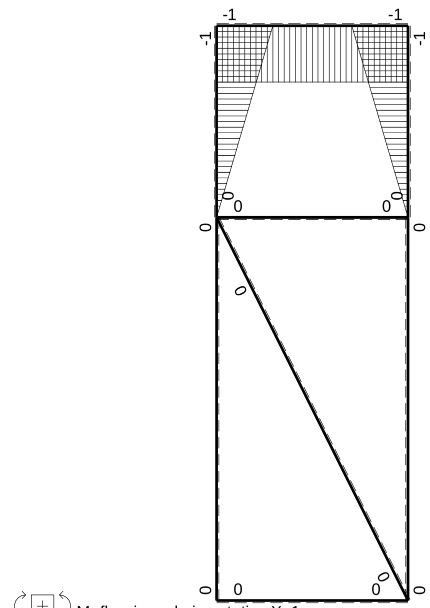

-37/20
-5/2
7/2
-13/20
-5\5/5
1
0
F
5/2
-37/20
-13/20
-5\5/10
-1
Fb
-1/2



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

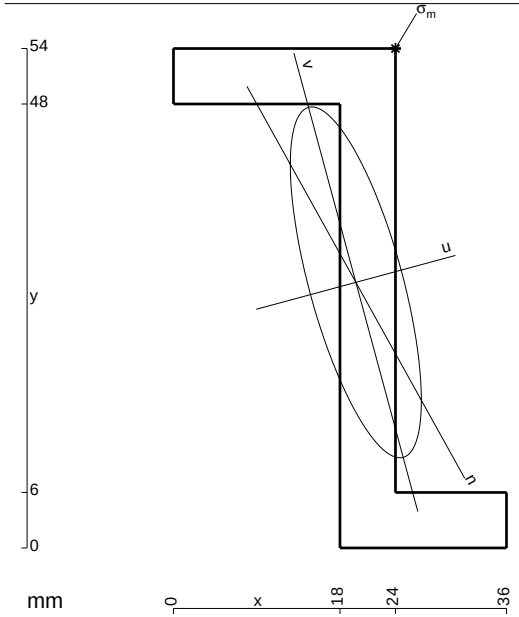
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

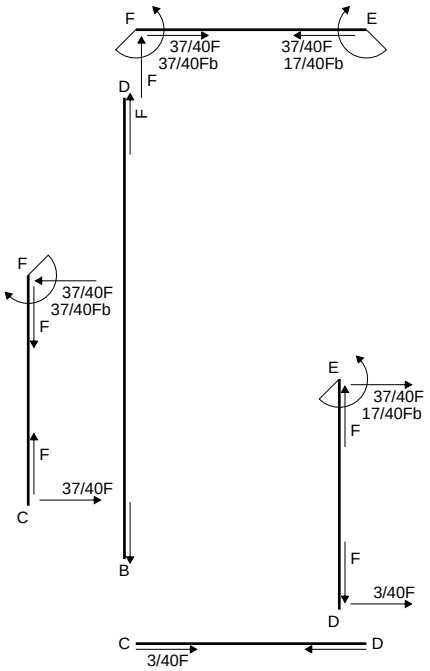
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

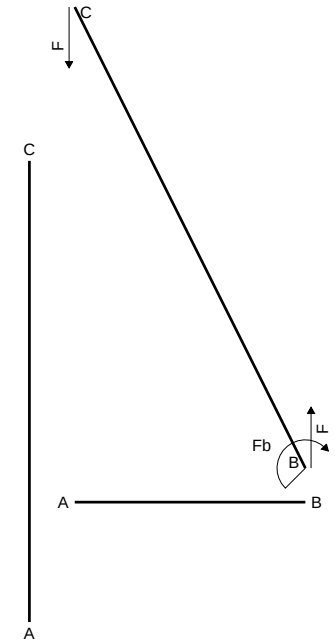


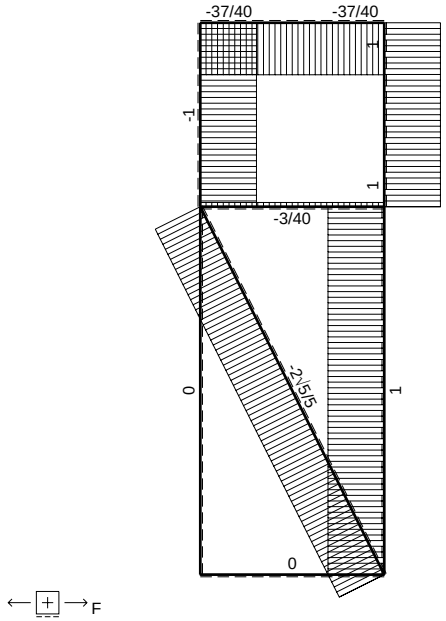
A = 504. mm^2
J_x = 181471. mm^4
J_y = 25303. mm^4
J_{xy} = -45545. mm^4
J_u = 193783. mm^4
J_v = 12991. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2640
$c = \cos\alpha$ = .9653
$s = \sin\alpha$ = .2610
x_g = 19.71 mm
y_g = 28.71 mm
N = -1325. N
T_y = -980.5 N
M_x = 705960. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 10.74 mm
v_m = 23.29 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -236.8 N/mm^2

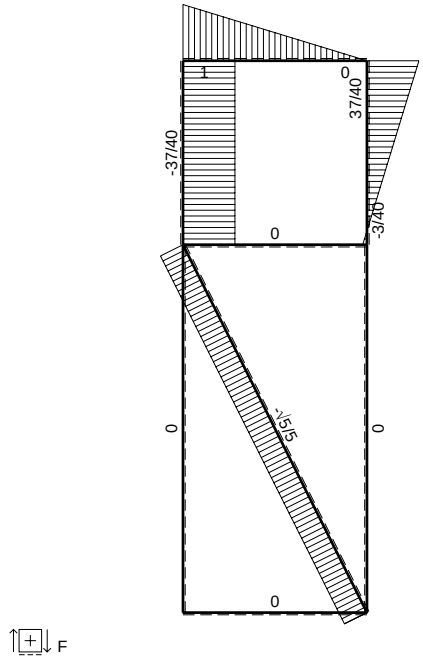
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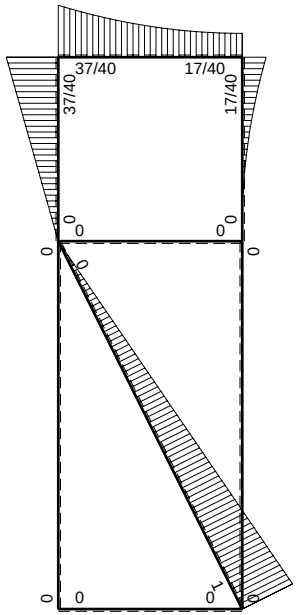
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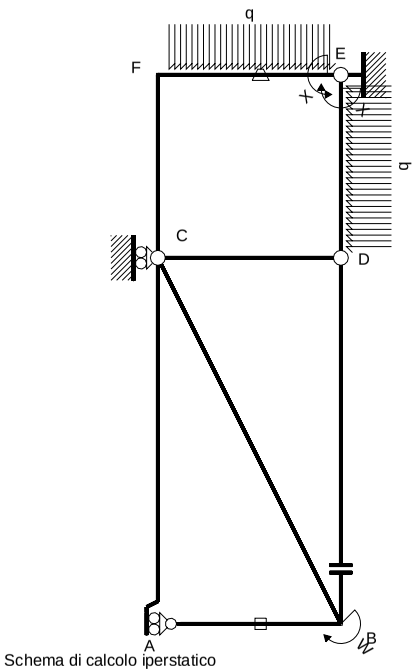

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
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37/40F
17/40Fb
F
F
37/40F
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Fb
F
B
A
B
A

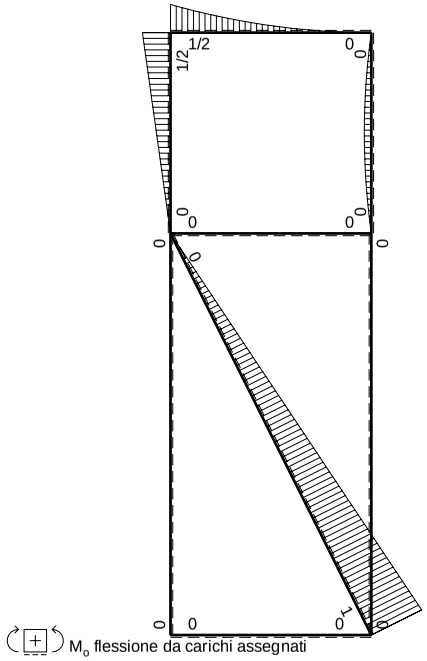

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-37/40
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-1
1
-3/40
-2√5/5
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1
0
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
F

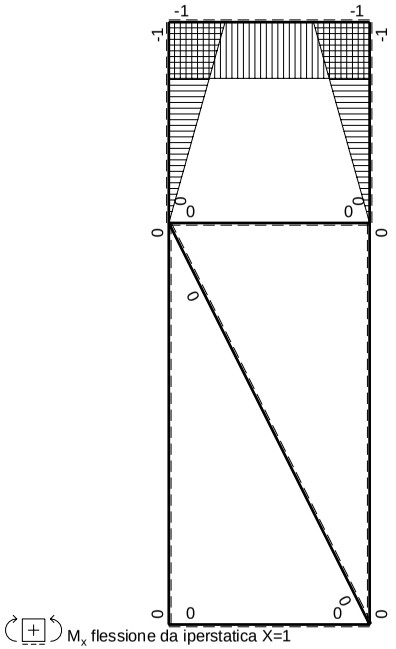

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 269138 CI09.xxxx.020

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

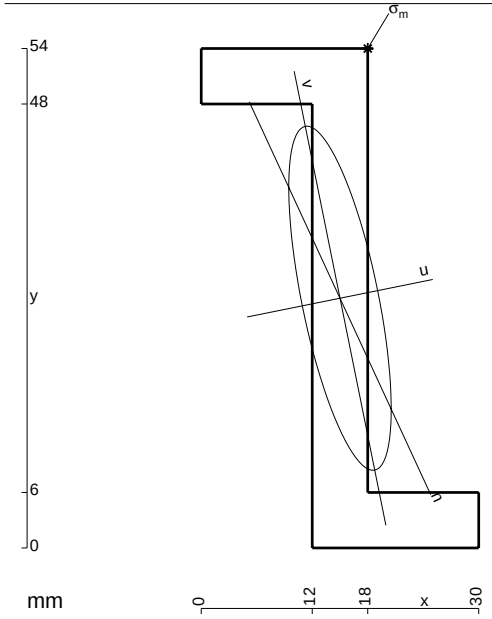
$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

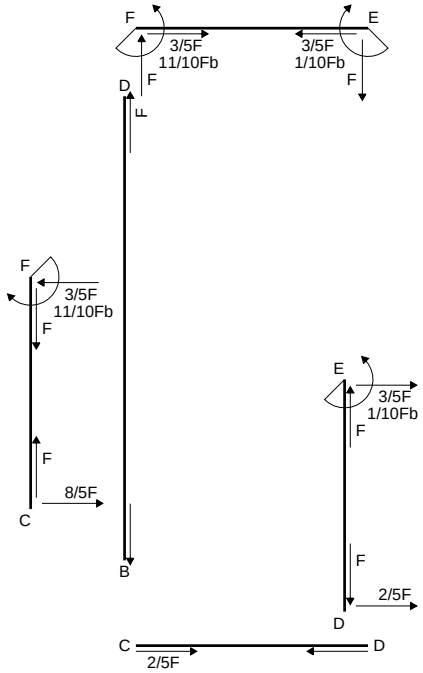
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

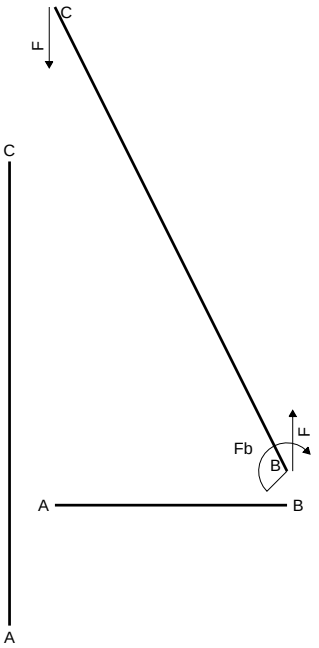


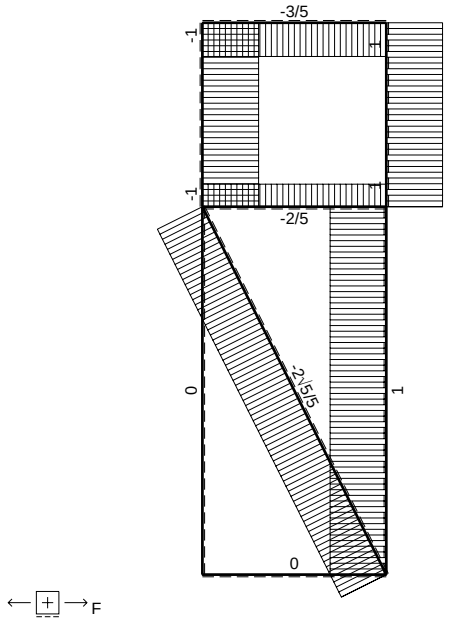
A = 468. mm^2
J_x = 162108. mm^4
J_y = 14364. mm^4
J_{xy} = -31104. mm^4
J_u = 168389. mm^4
J_v = 8083. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1993
c = cosα = .9802
s = sinα = .1979
x_g = 15. mm
y_g = 27. mm
N = -1306. N
T_y = -652.9 N
M_x = 554800. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 8.285 mm
v_m = 25.87 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -198.9 N/mm^2

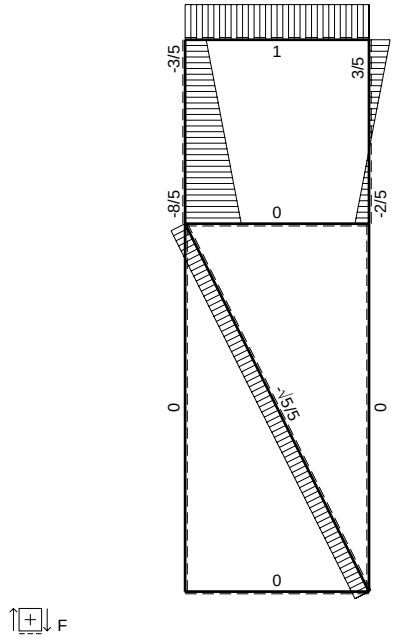
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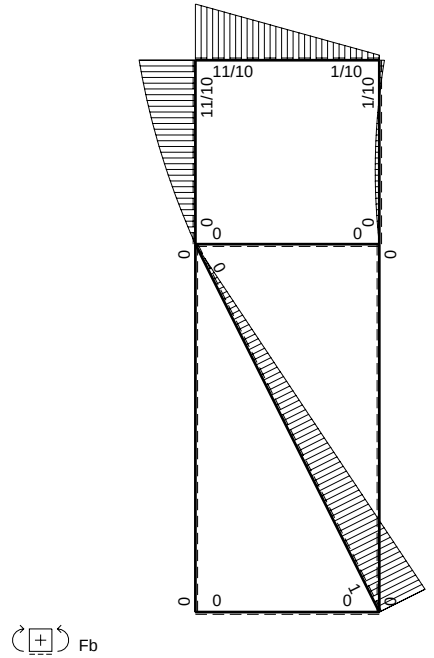
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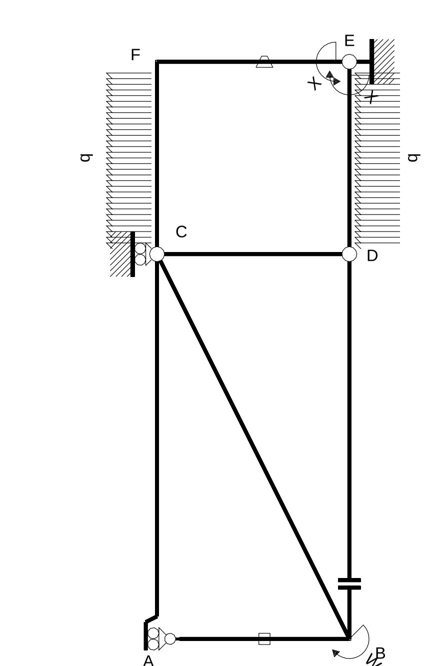

F
E
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11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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E
3/5F
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A

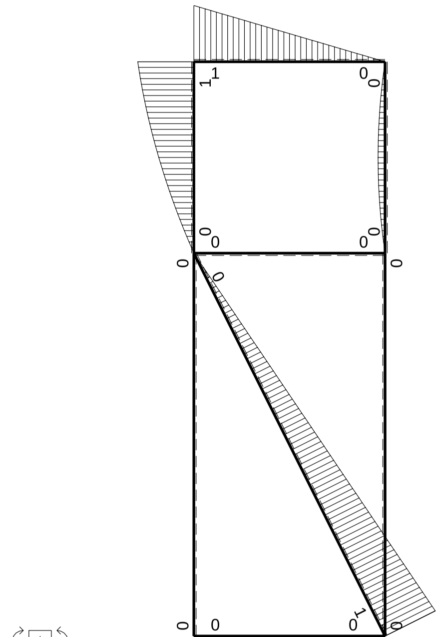

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-1
1
-2/5
0
-2/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5
0
0
F

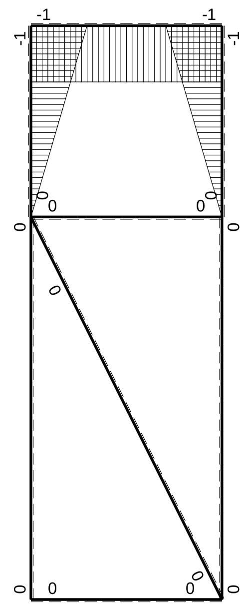

11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 248038 CI09.xxxx.021

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$1/6Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

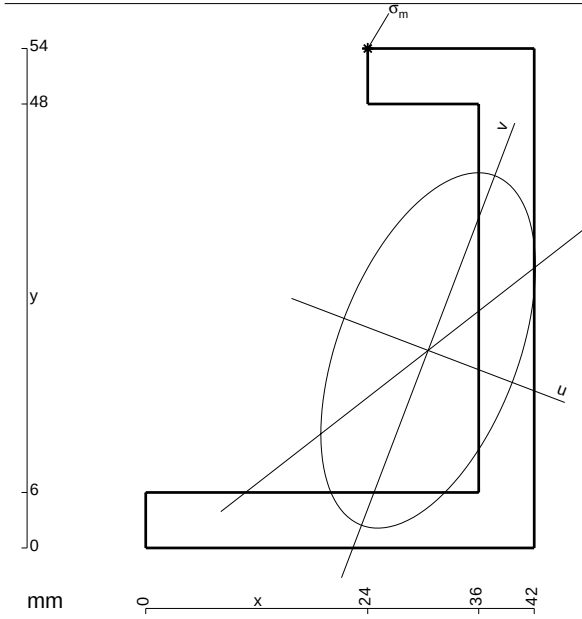
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

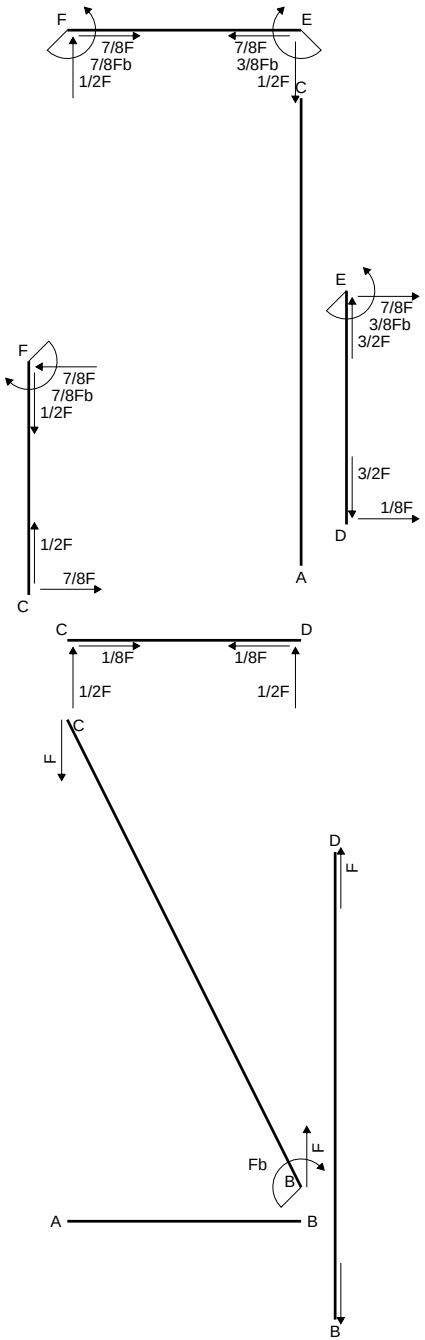
$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

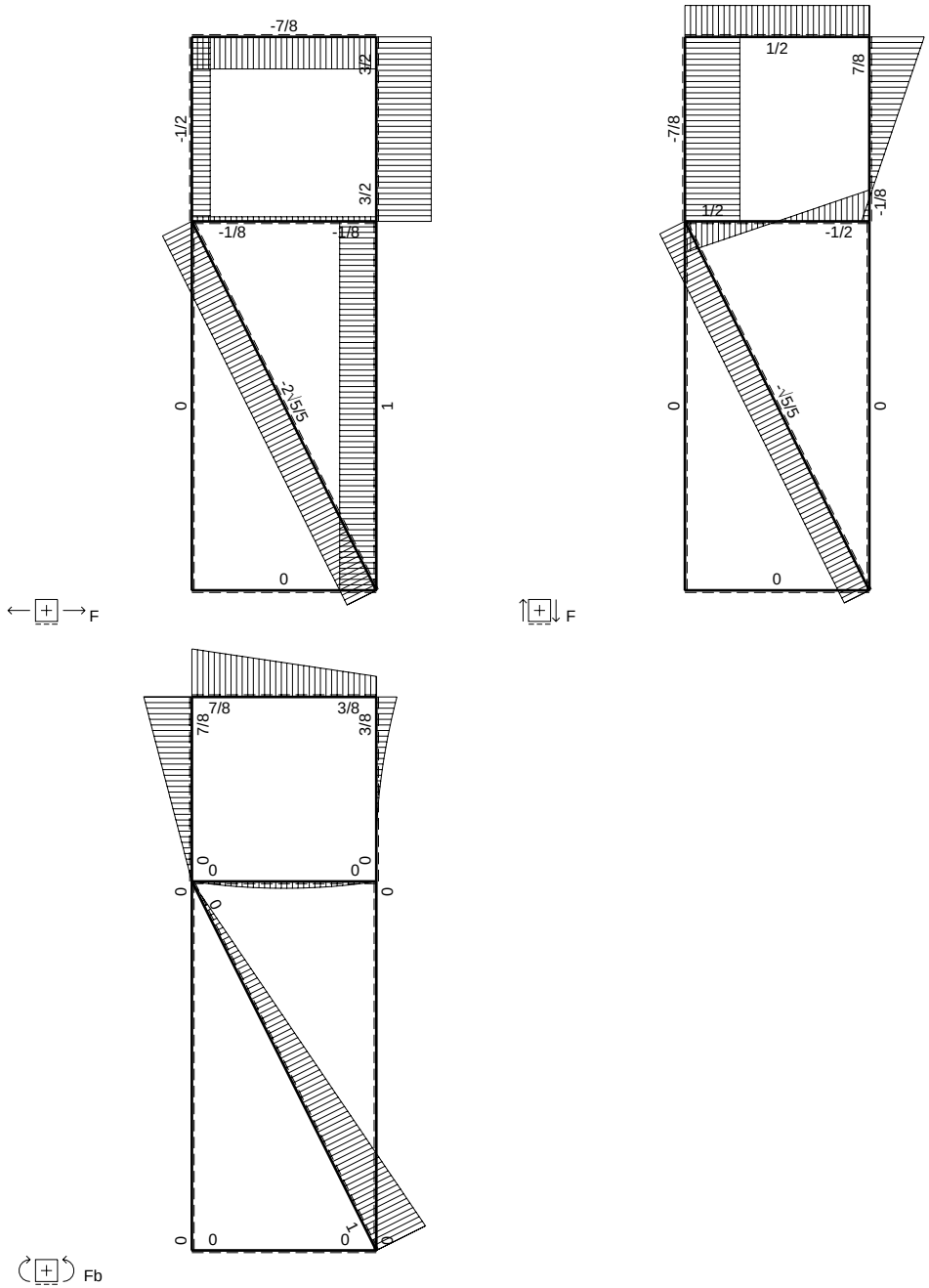


A = 612. mm^2
J_x = 225968. mm^4
J_y = 82341. mm^4
J_{xy} = 64038. mm^4
J_u = 250373. mm^4
J_v = 57936. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3641
$c = \cos\alpha$ = .9344
$s = \sin\alpha$ = -.3561
x_g = 30.53 mm
y_g = 21.35 mm
N = -2130. N
T_y = -1278. N
M_x = 960630. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = -17.73 mm
v_m = 28.18 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.2 N/mm^2

CI09.xxxx.021

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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
-2√5/5
0
1
0
F
1/2
7/8
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-1/8
1/2
-1/2
-√5/5
0
0
F
7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264292 CI09.xxxx.022

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [-\ x]_o^b\ \theta$

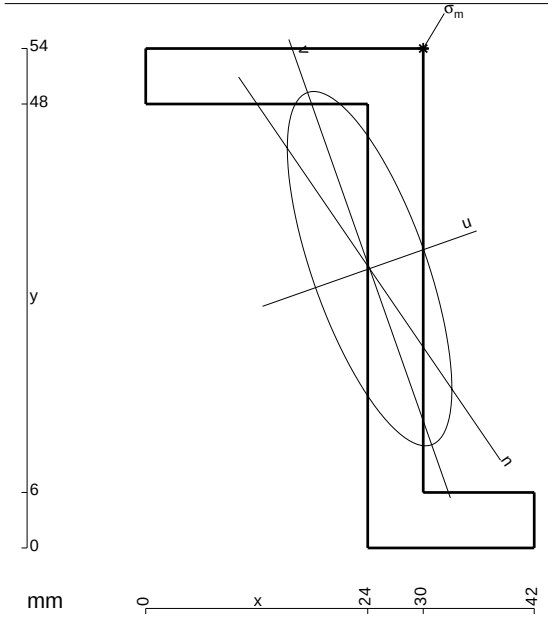
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

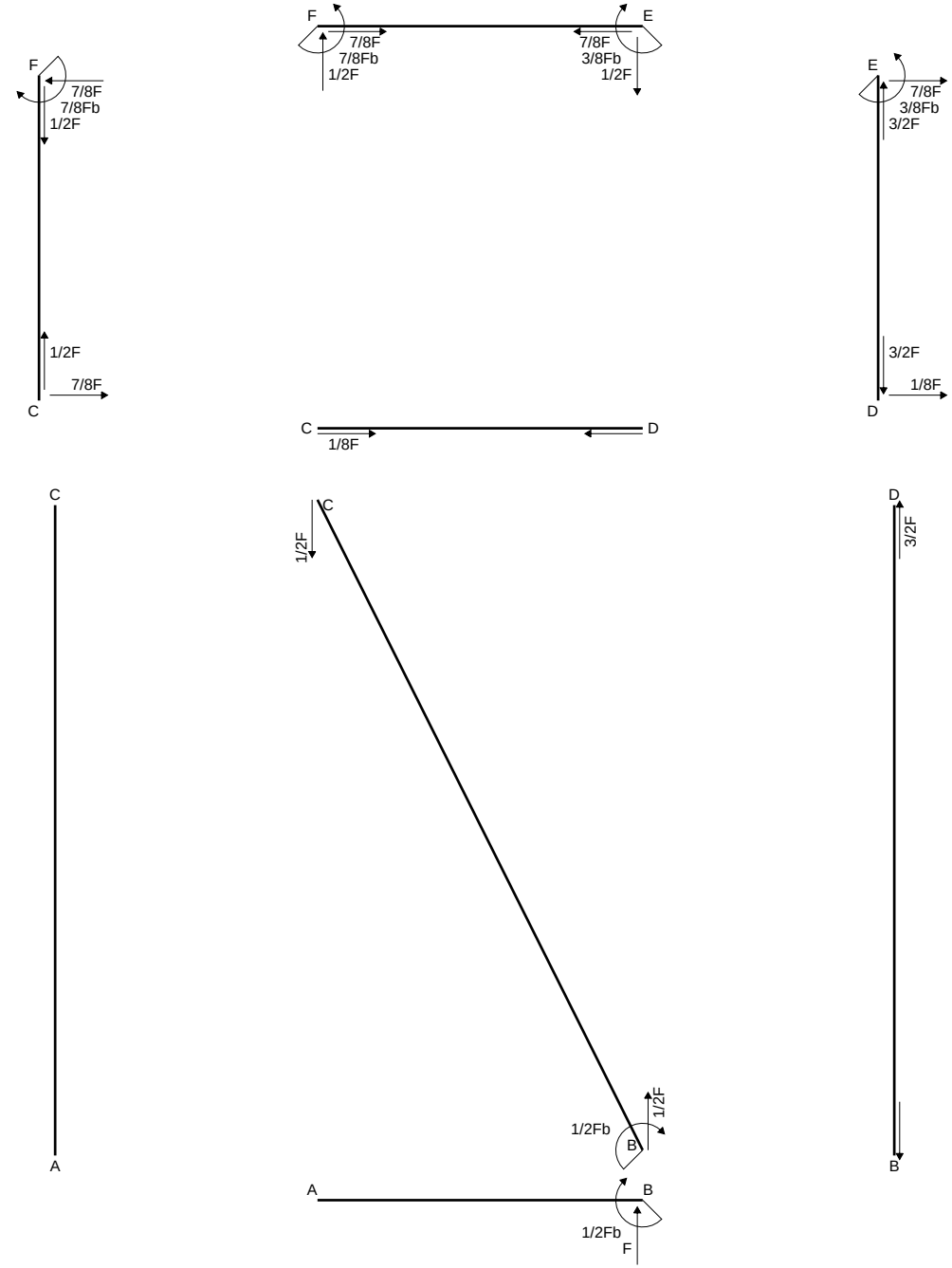
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

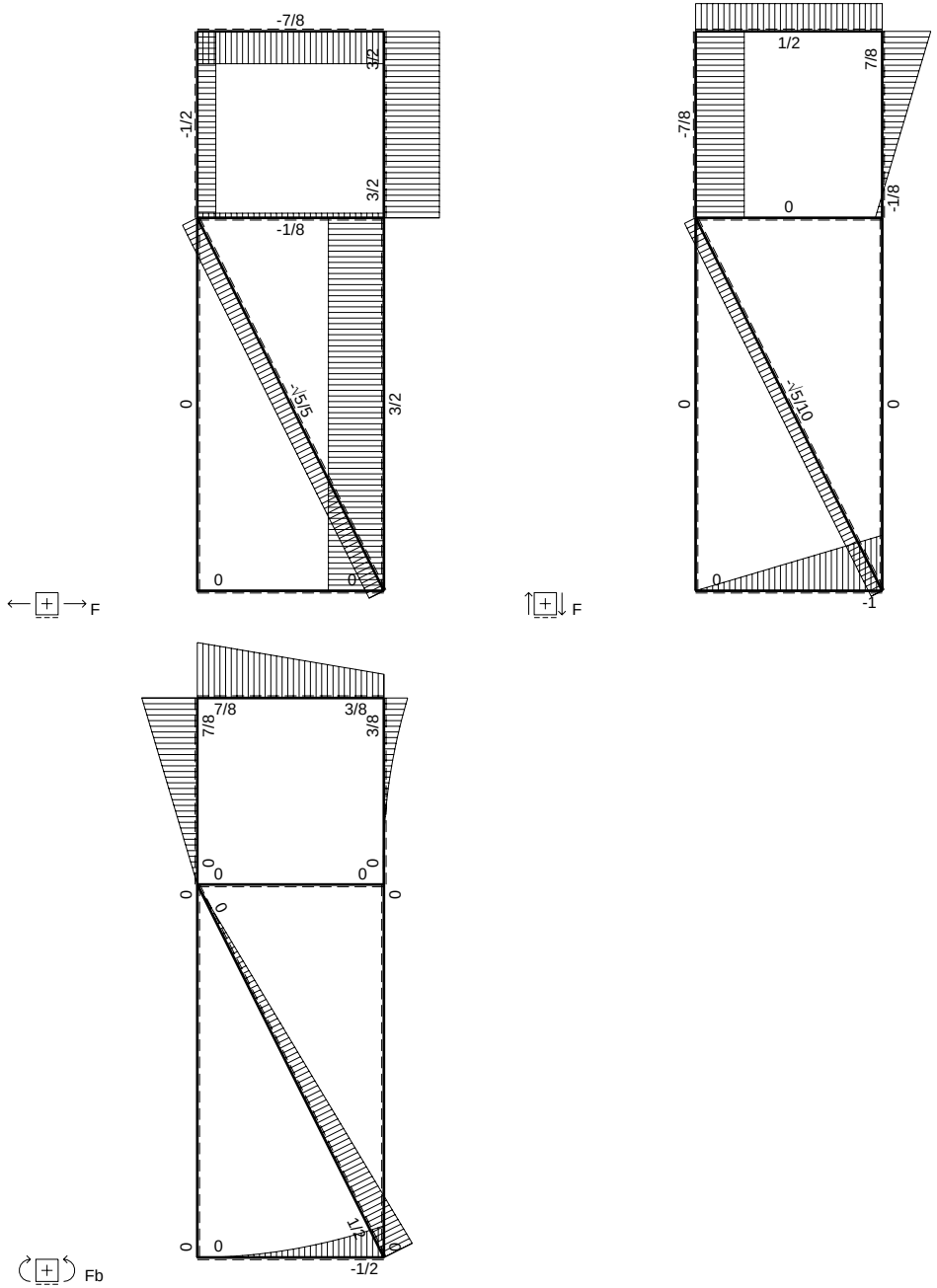


A = 540. mm^2
J_x = 198266. mm^4
J_y = 42746. mm^4
J_{xy} = -62554. mm^4
J_u = 220304. mm^4
J_v = 20709. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3387
$c = \cos\alpha$ = .9432
$s = \sin\alpha$ = .3323
x_g = 24.2 mm
y_g = 30.2 mm
N = -1422. N
T_y = -711.1 N
M_x = 715500. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 13.38 mm
v_m = 20.52 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.1 N/mm^2

CI09.xxxx.022

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F
7/8F
7/8Fb
1/2F
1/2F
7/8F
C
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
D
1/8F
E
7/8F
3/8Fb
3/2F
3/2F
1/8F
D
C
A
C
1/2F
1/2Fb
B
1/2F
A
B
1/2Fb
F
D
3/2F
B


-7/8
3/2
-1/2
3/2
-1/8
0
-√5/5
3/2
0
0
F
1/2
7/8
-7/8
0
-1/8
-√5/10
0
0
-1
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
-√2/2
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	$-Fb/EJ$	$-1/2Fx$	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L_{EF}^{Xo} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 3/4\ \ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [-\ x]_o^b\ \theta$

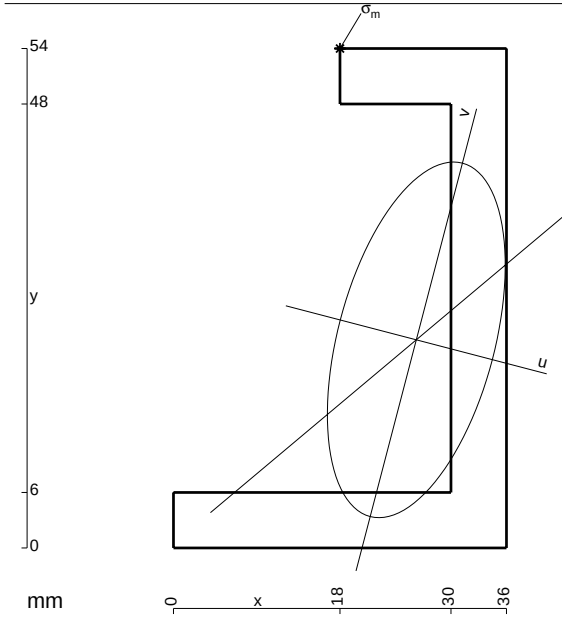
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ \ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

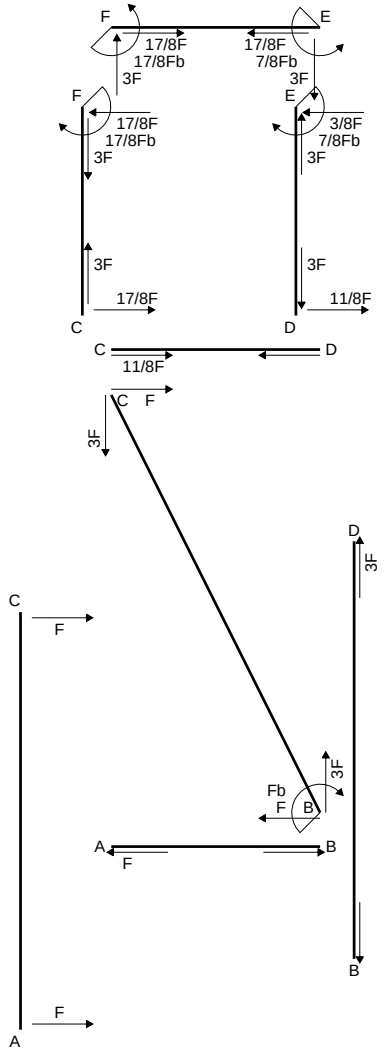
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

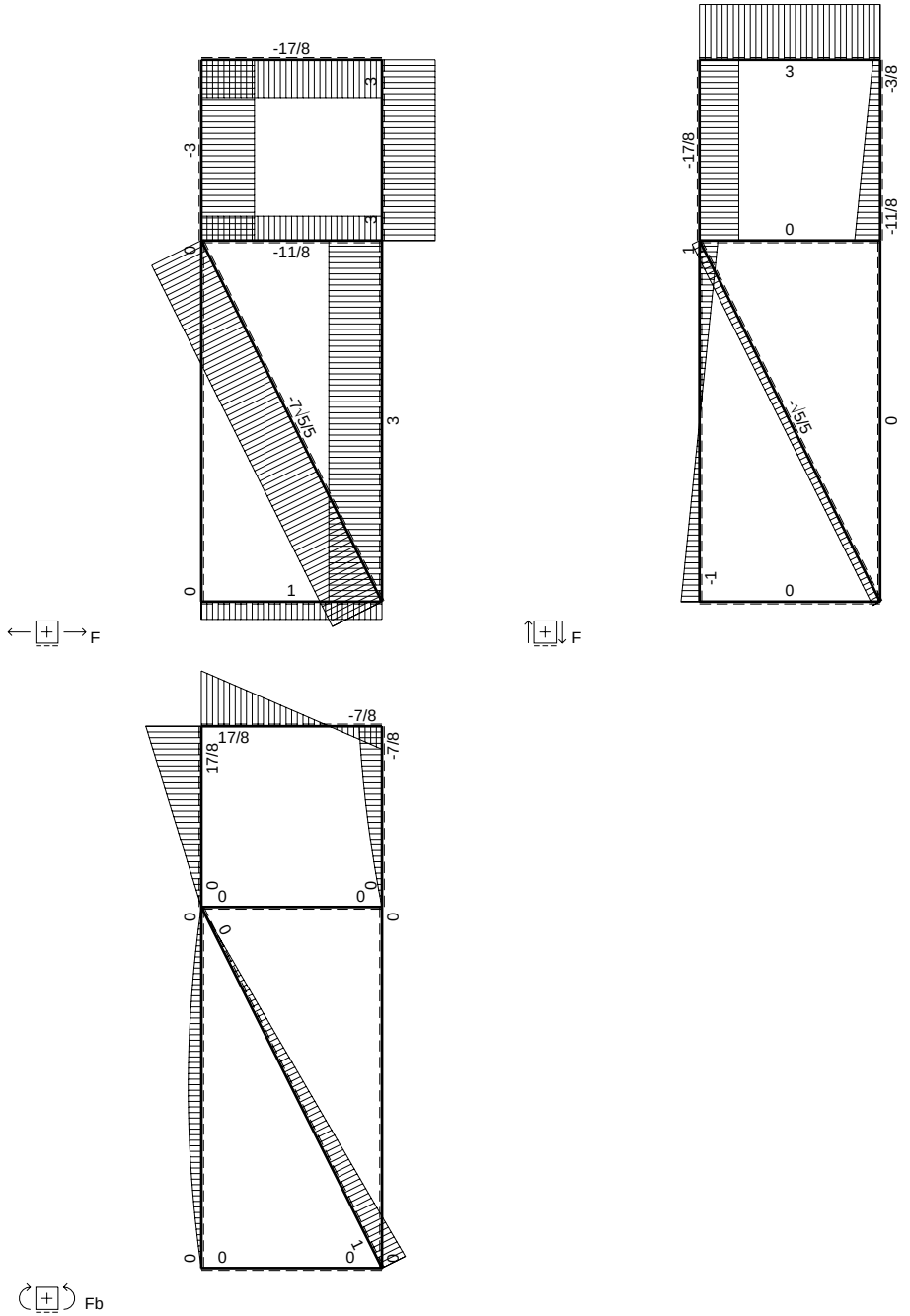


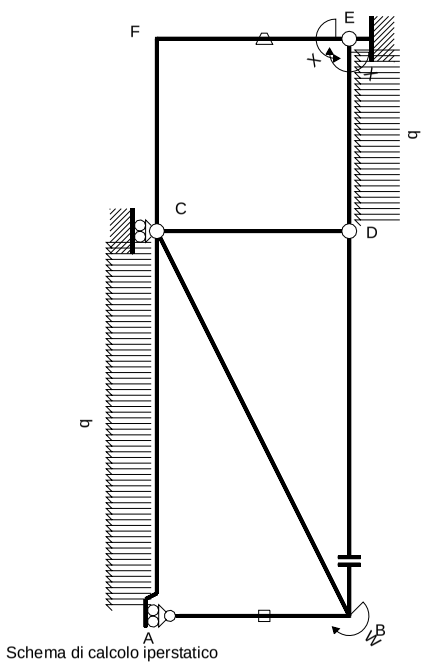
A = 576. mm^2
J_x = 212976. mm^4
J_y = 53244. mm^4
J_{xy} = 44712. mm^4
J_u = 224640. mm^4
J_v = 41580. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.2552$
$c = \cos\alpha = .9676$
$s = \sin\alpha = -.2524$
x_g = 26.25 mm
y_g = 22.5 mm
N = -2100. N
T_y = 1200. N
M_x = 1029000. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = -15.93 mm
v_m = 28.4 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v = -229.1\ N/mm^2$

CI09.xxxx.023

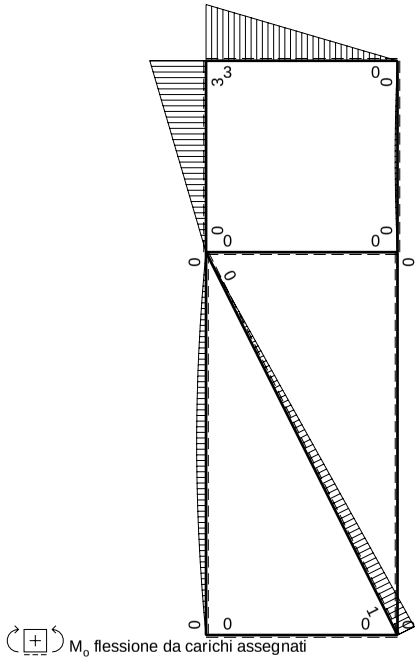
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
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D
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C
F
3F
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F
B
A
B
F
B
A
F

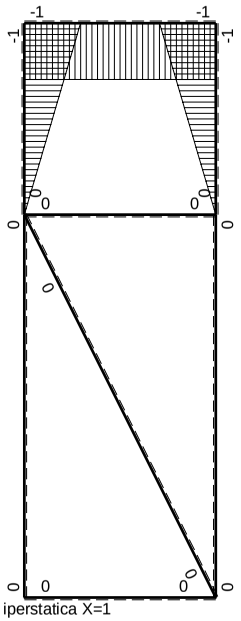

-17/8
-3
-11/8
-7√5/5
3
0
1
F
3
-3/8
-17/8
-11/8
0
-1
-7√5/5
F
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
				totali			-35/24Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 267066 CI09.xxxx.024

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

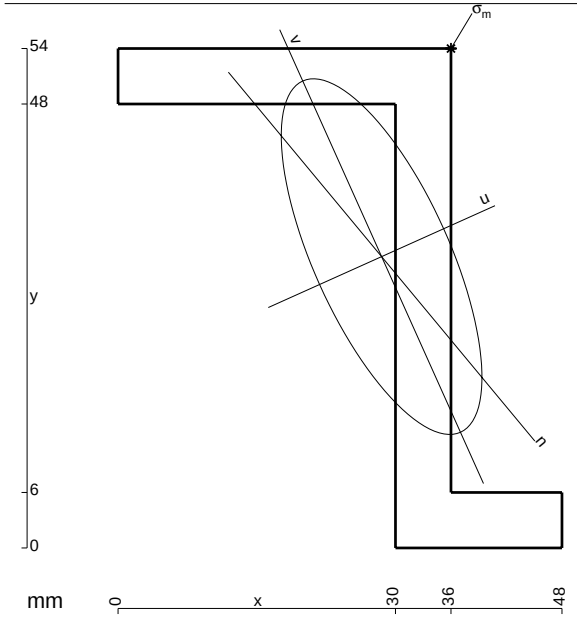
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

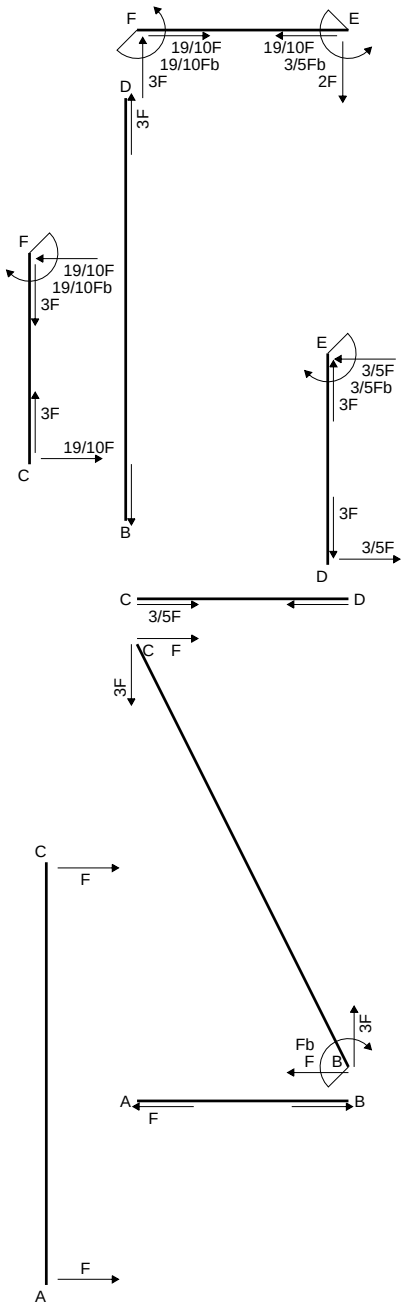
$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

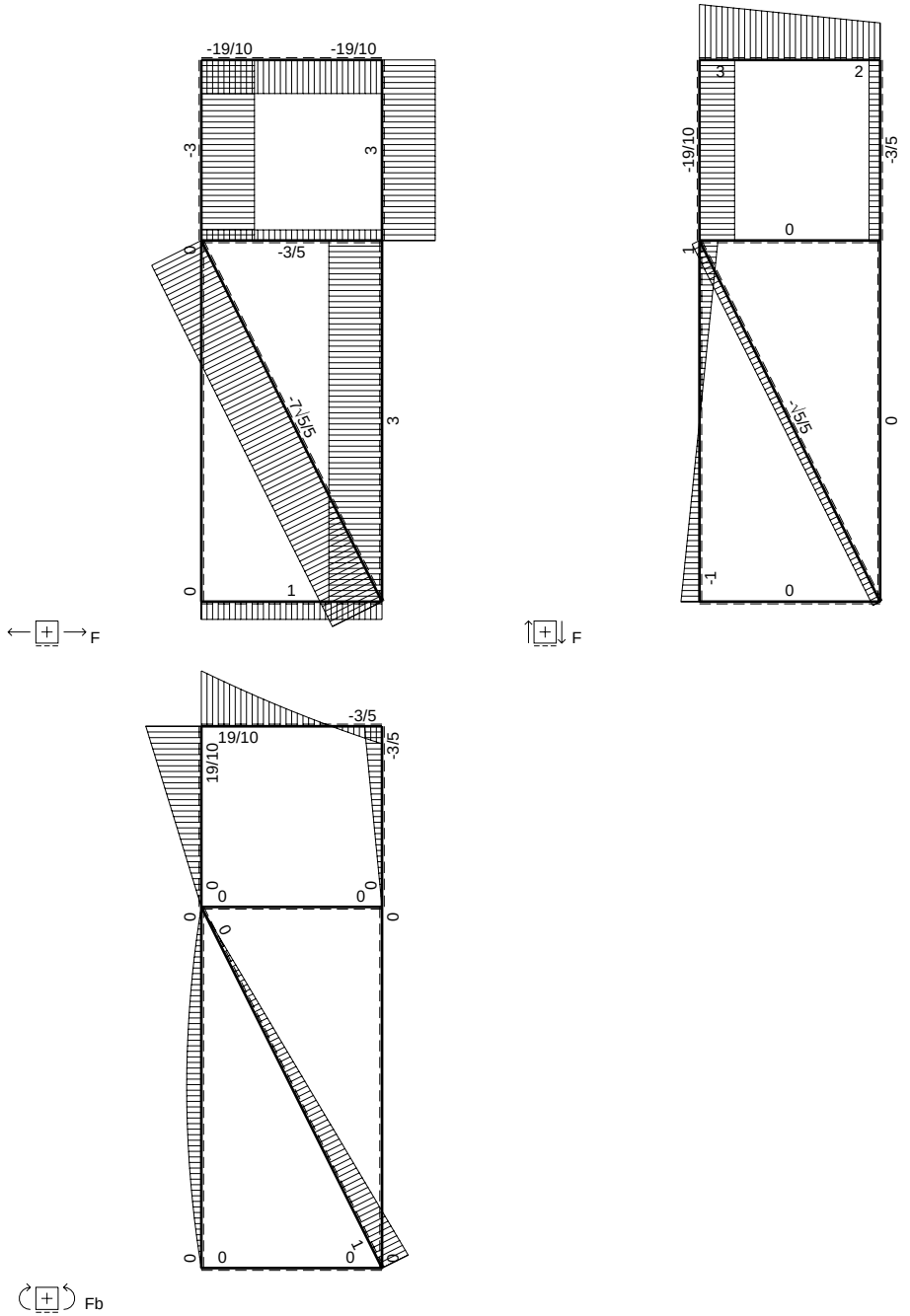


A = 576. mm^2
J_x = 212976. mm^4
J_y = 67824. mm^4
J_{xy} = -81648. mm^4
J_u = 249641. mm^4
J_v = 31159. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4221
c = cosα = .9122
s = sinα = .4097
x_g = 28.5 mm
y_g = 31.5 mm
N = -2280. N
T_y = -1615. N
M_x = 855950. Nmm
x_m = 36. mm
y_m = 54. mm
u_m = 16.06 mm
v_m = 17.45 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -239.3 N/mm^2

CI09.xxxx.024

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F
E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
B
3F
3/5F
D
C
D
3/5F
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A


-19/10
-19/10
-3
3
-3/5
0
3
-7\5/5
1
F
3
2
-19/10
-3/5
0
1
-1\5/5
-1
0
F
-3/5
19/10
19/10
-3/5
0
0
0
0
0
0
0
0
0
0
Fb

16.04.26

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

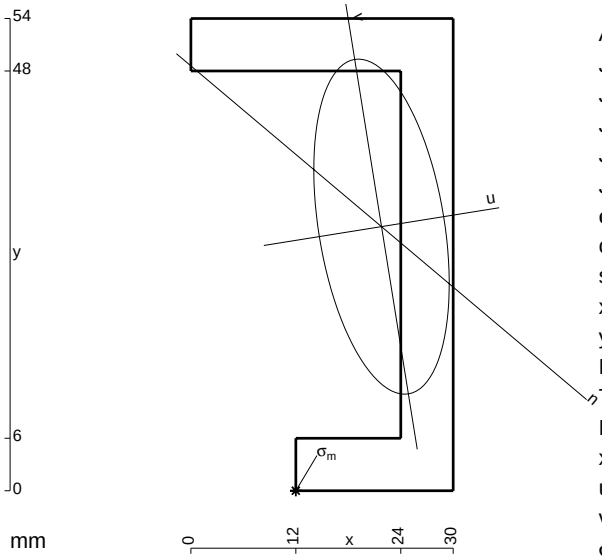
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

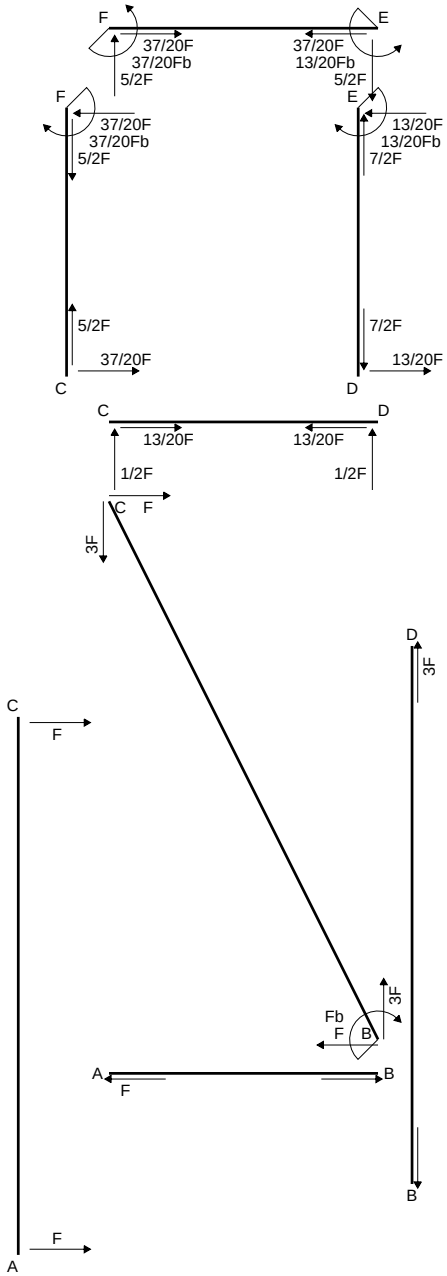
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

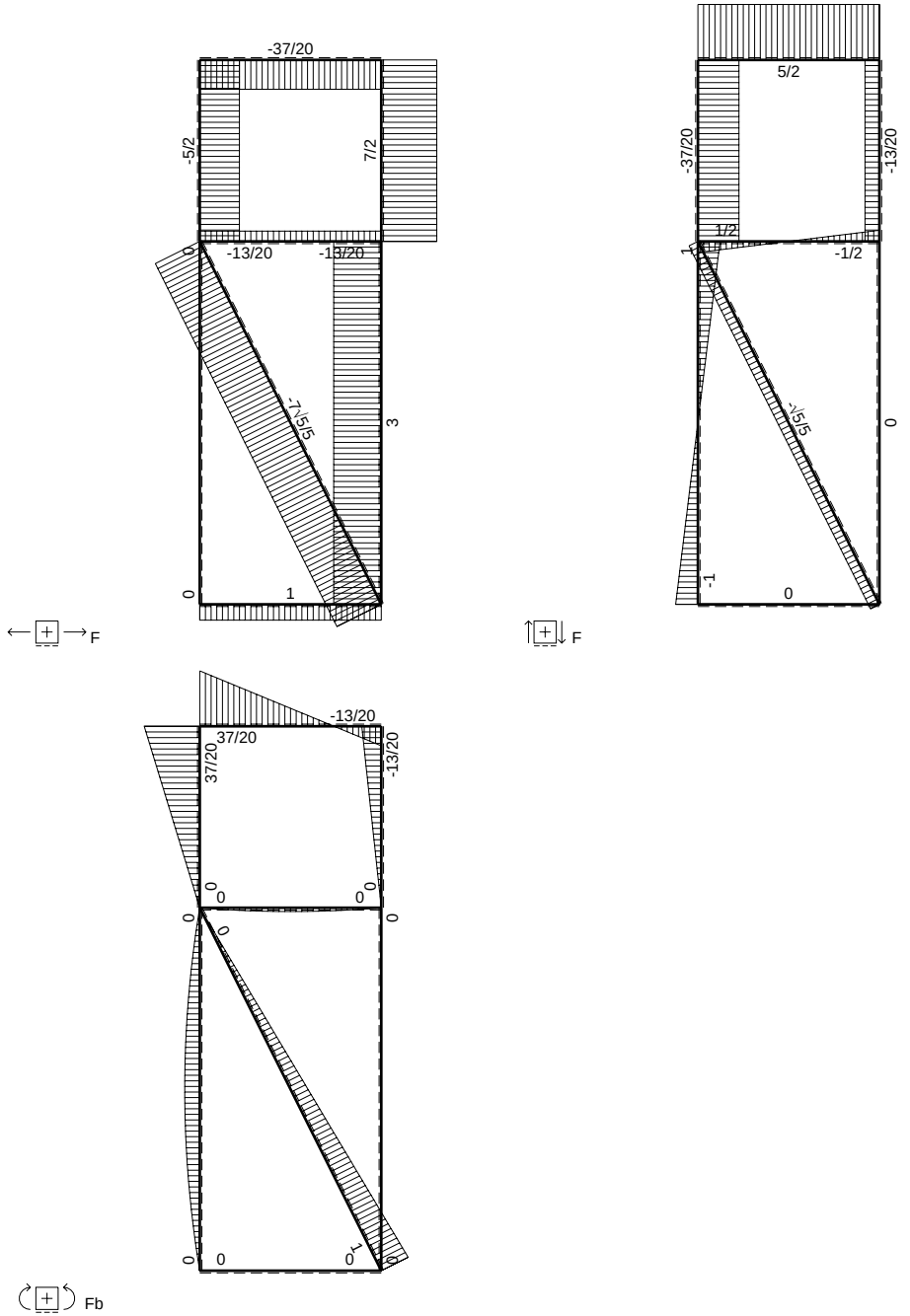


A = 540. mm^2
J_x = 198266. mm^4
J_y = 32378. mm^4
J_{xy} = -27302. mm^4
J_u = 202644. mm^4
J_v = 28000. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1590
c = cosα = .9874
s = sinα = .1583
x_g = 21.8 mm
y_g = 30.2 mm
N = -1615. N
T_y = 2550. N
M_x = 920550. Nmm
x_m = 12. mm
u_m = -14.46 mm
v_m = -28.27 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = 199.1 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
7/2F
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C
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C
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D
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C
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Fb
F
B
3F
A
B
F
F
A
B


-37/20
-5/2
7/2
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3
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F
5/2
-37/20
-13/20
1/2
-1/2
-7/5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x²/b²	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x²/b²		
CD b	0	1/2Fx-1/2qx²	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx²	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	1-2x/b+x²/b²	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x²/b²		
	totali						-13/12Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

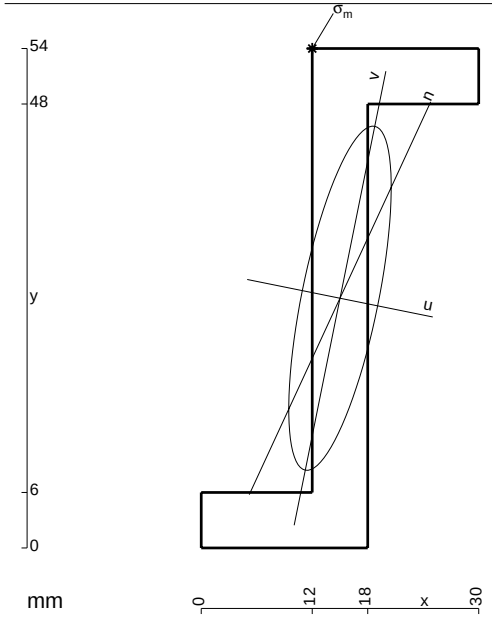
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

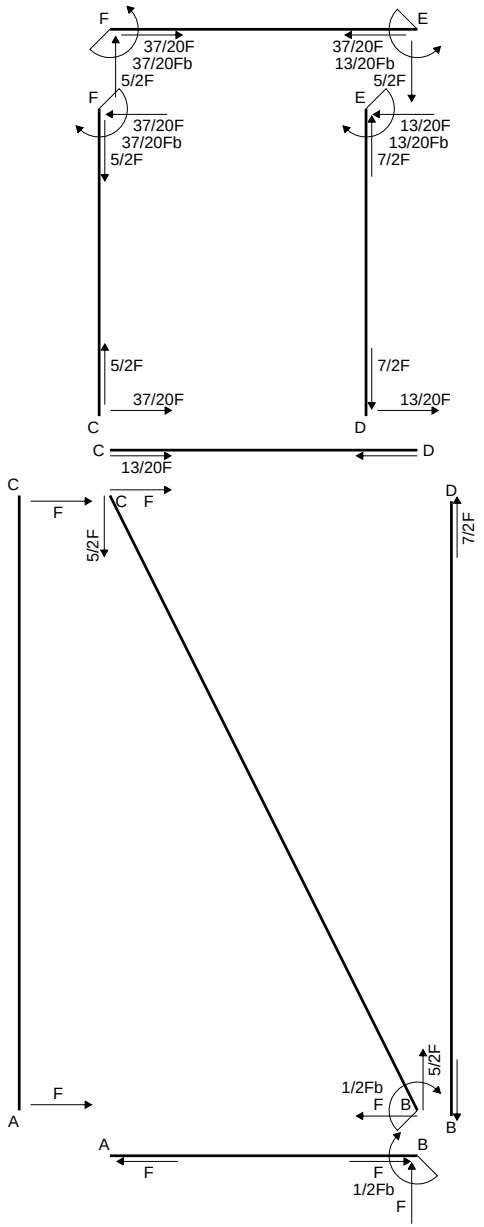
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

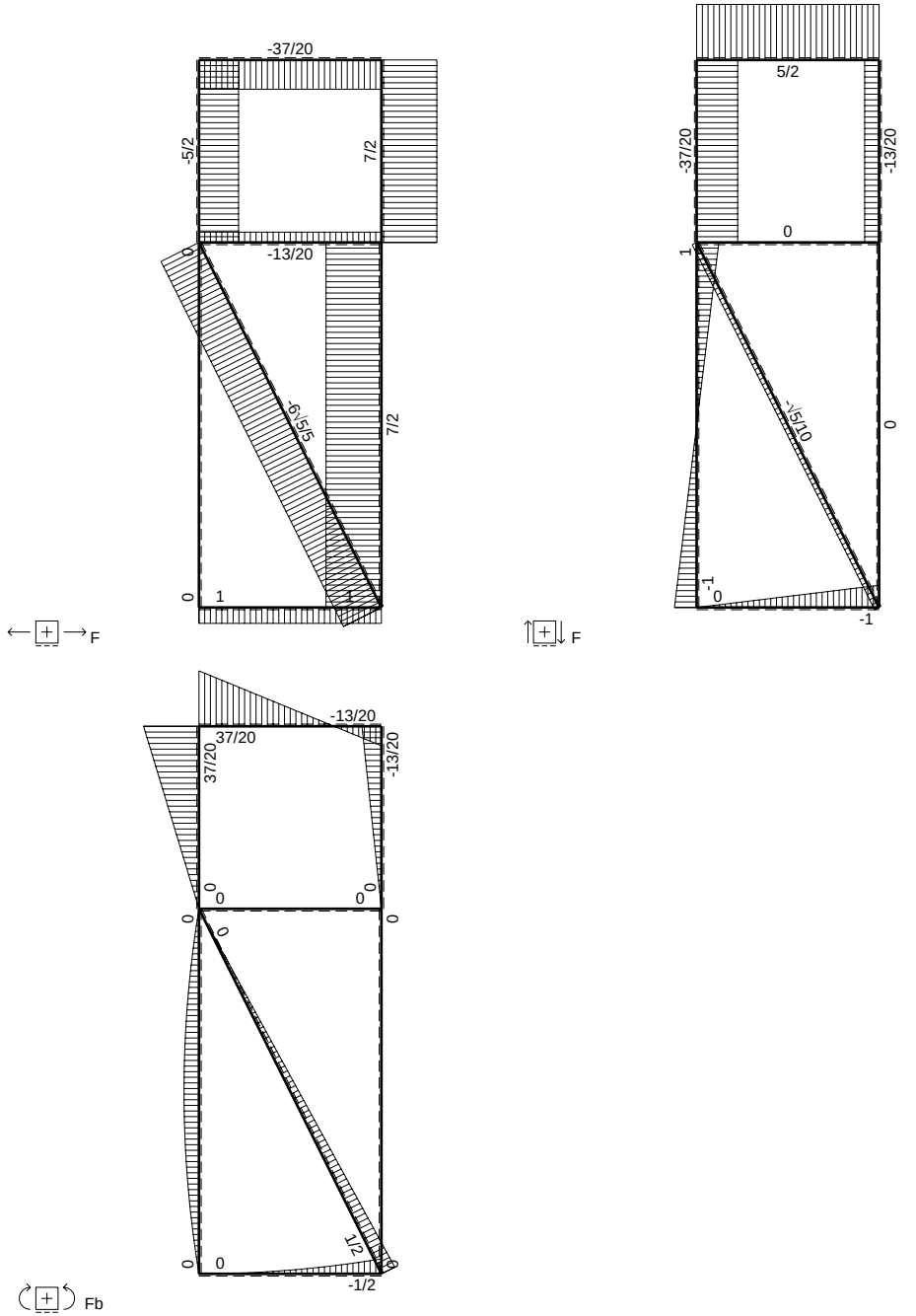


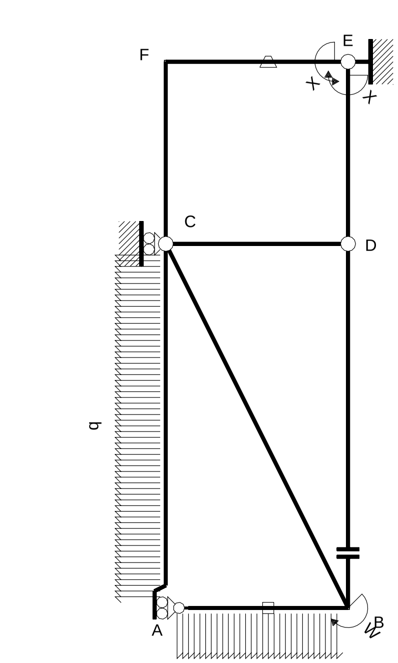
A = 468. mm^2
J_x = 162108. mm^4
J_y = 14364. mm^4
J_{xy} = 31104. mm^4
J_u = 168389. mm^4
J_v = 8083. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1993
$c = \cos\alpha$ = .9802
$s = \sin\alpha$ = -.1979
x_g = 15. mm
y_g = 27. mm
N = -1300. N
T_y = -962. N
M_x = 577200. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -8.285 mm
v_m = 25.87 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -206.8 N/mm^2

CI09.xxxx.026

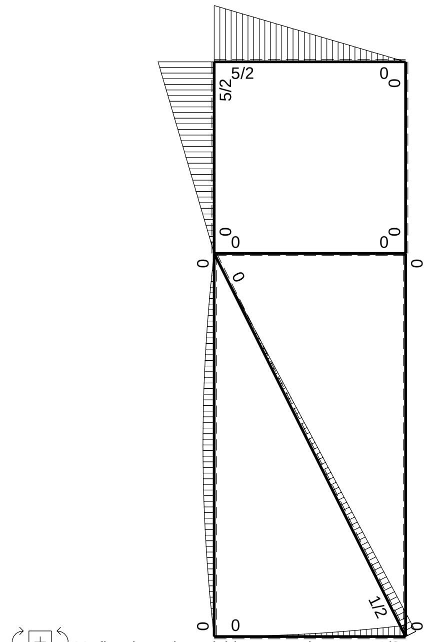
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F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb

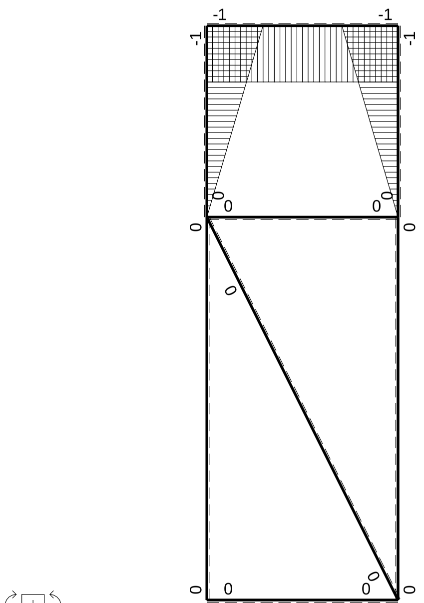

-37/20
-5/2
7/2
-13/20
-5\5/5
F
5/2
-37/20
-13/20
-5\5/10
-1
37/20
-13/20
1/2
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	$-1+x/b$	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

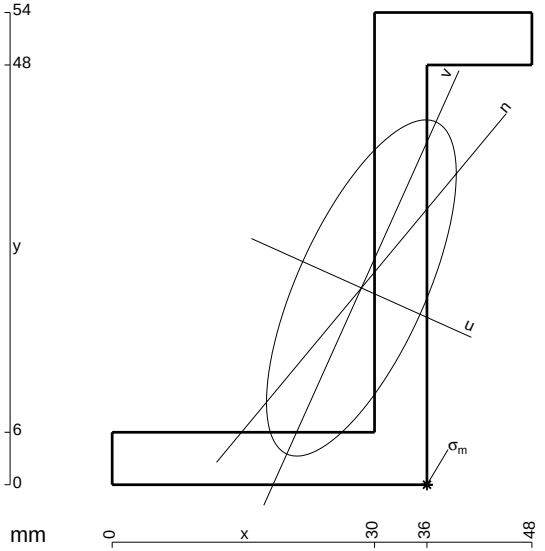
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

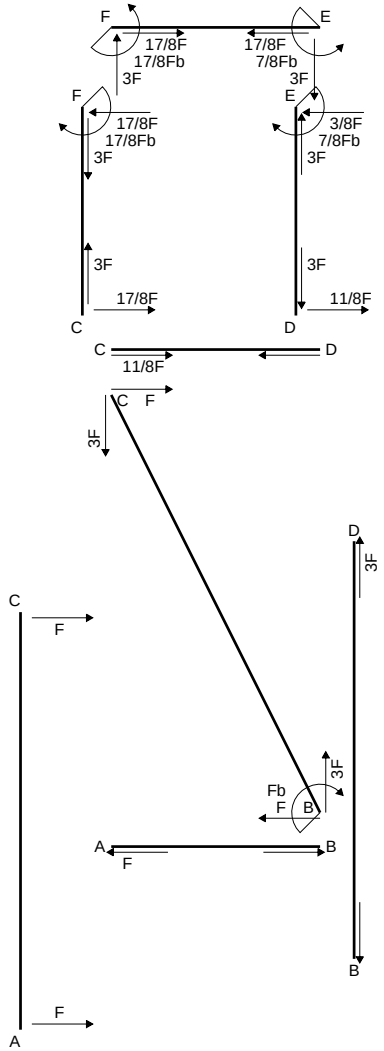
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

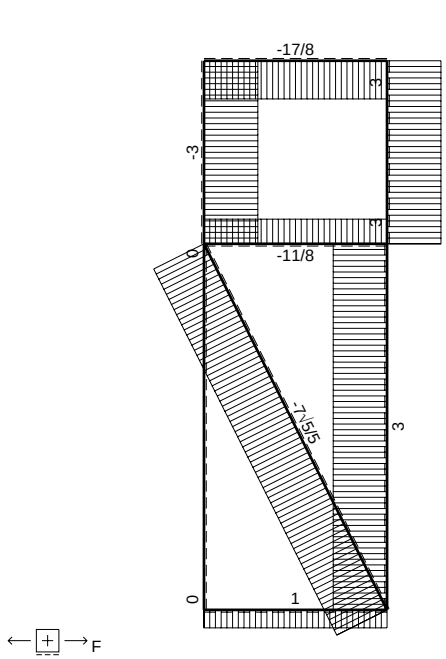


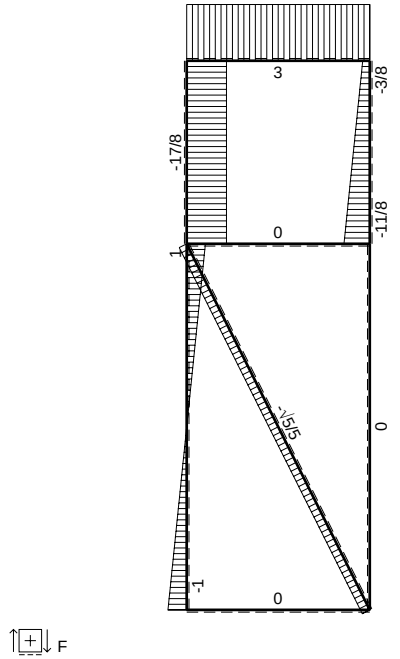
A = 576. mm^2
J_x = 212976. mm^4
J_y = 67824. mm^4
J_{xy} = 81648. mm^4
J_u = 249641. mm^4
J_v = 31159. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4221
$c = \cos\alpha$ = .9122
$s = \sin\alpha$ = -.4097
x_g = 28.5 mm
y_g = 22.5 mm
N = -1258. N
T_y = 1700. N
M_x = 805120. Nmm
x_m = 36. mm
u_m = 16.06 mm
v_m = -17.45 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 219.2 N/mm^2

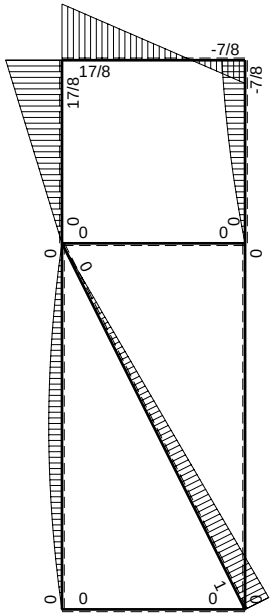
CI09.xxxx.027

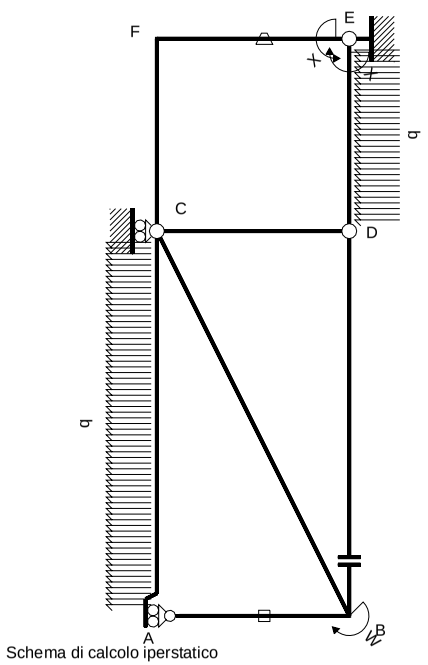
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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
B
F
A

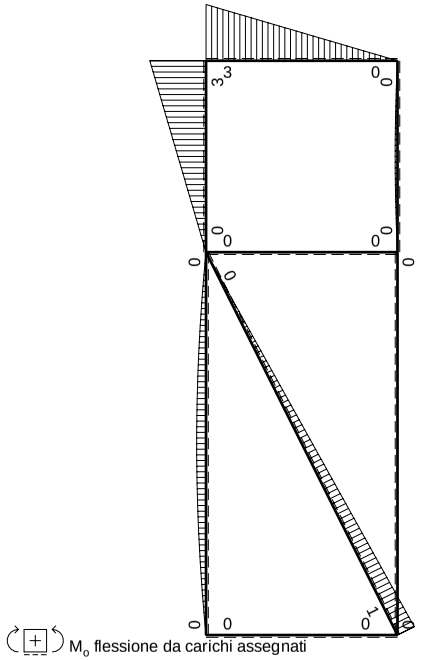

-17/8
-3
-11/8
-7\5/5
3
0
1
F


3
-3/8
-17/8
-11/8
0
1
-7\5/5
-1
F

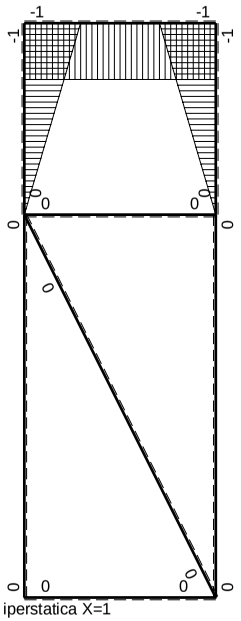

-7/8
17/8
-7/8
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
				totali			$-35/24Fb^2/EJ$	5/3Xb/EJ
				iperstatica $X=W_{EF}$			7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264395 CI09.xxxx.028

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

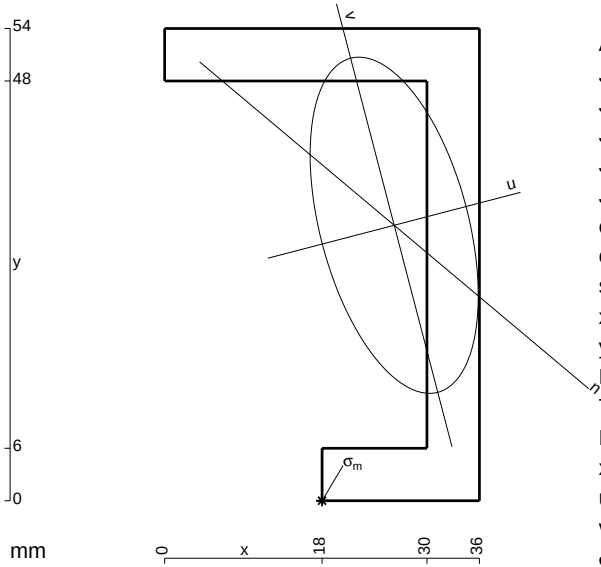
$= (-3\ b +3/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

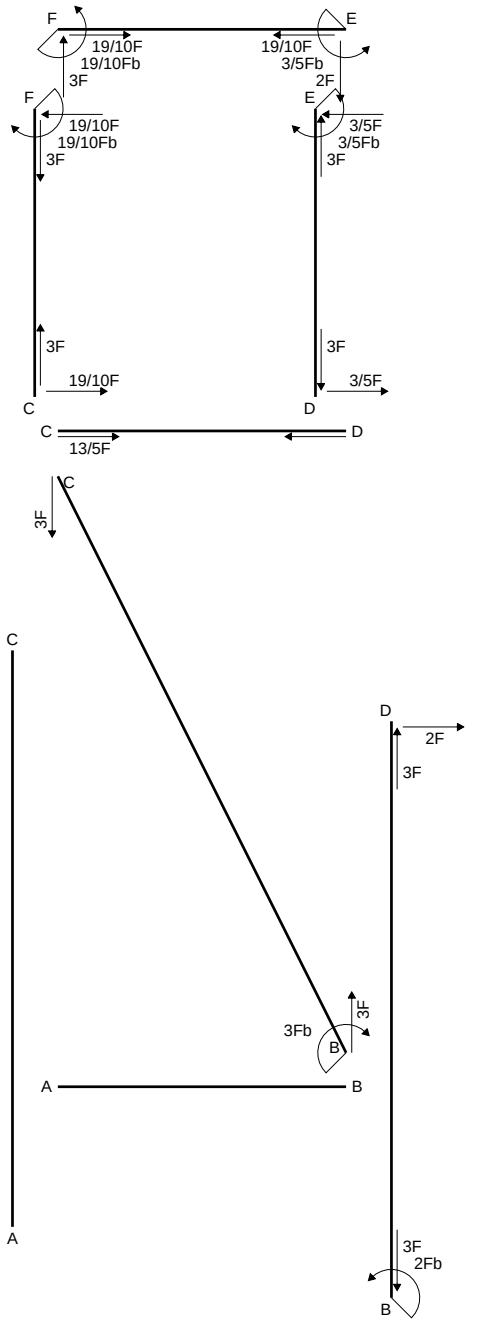
$= (-\ b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

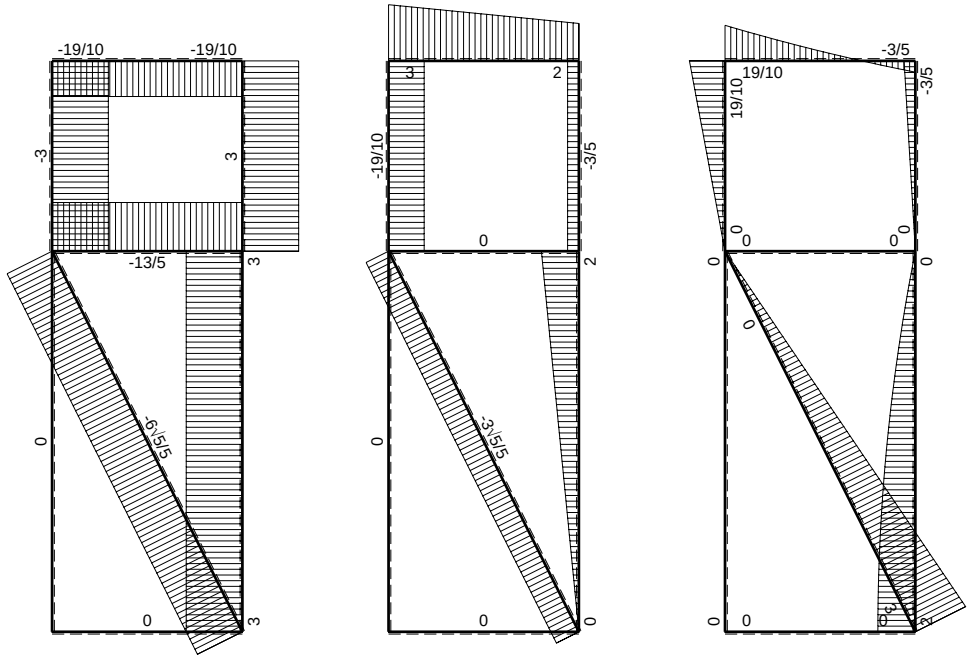


A = 576. mm^2
J_x = 212976. mm^4
J_y = 53244. mm^4
J_{xy} = -44712. mm^4
J_u = 224640. mm^4
J_v = 41580. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2552
$c = \cos\alpha$ = .9676
$s = \sin\alpha$ = .2524
x_g = 26.25 mm
y_g = 31.5 mm
N = -1551. N
T_y = 2190. N
M_x = 1054850. Nmm
x_m = 18. mm
u_m = -15.93 mm
v_m = -28.4 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 228.4 N/mm^2

CI09.xxxx.028

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F
E
19/10F
19/10Fb
3F
19/10F
3/5Fb
2F
F
E
19/10F
19/10Fb
3F
3/5F
3/5Fb
3F
3F
19/10F
C
D
3F
3/5F
C
D
13/5F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-19/10
-19/10
-3
3
3
-13/5
0
-6√5/5
0
3
3
-19/10
2
-3/5
0
2
-3√5/5
0
0
19/10
-3/5
0
0
0
0
0
0
0
0

F F Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							3/5Fb

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 268891 CI09.xxxx.029

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

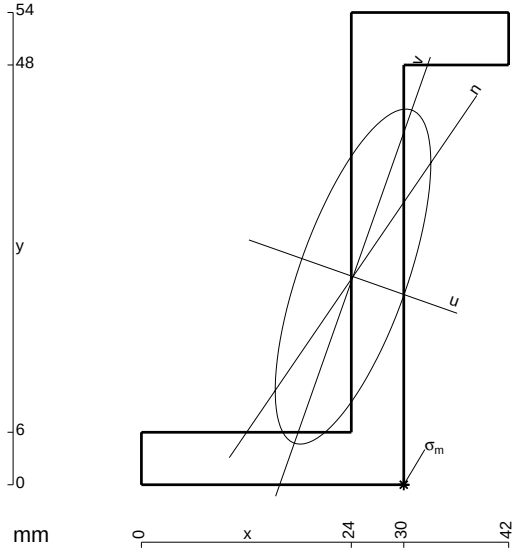
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

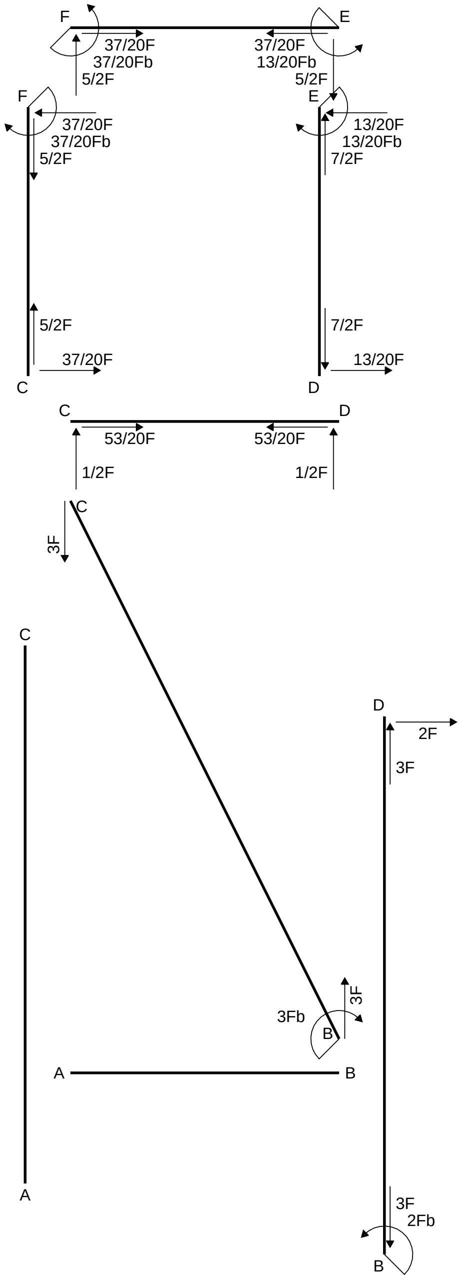
$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

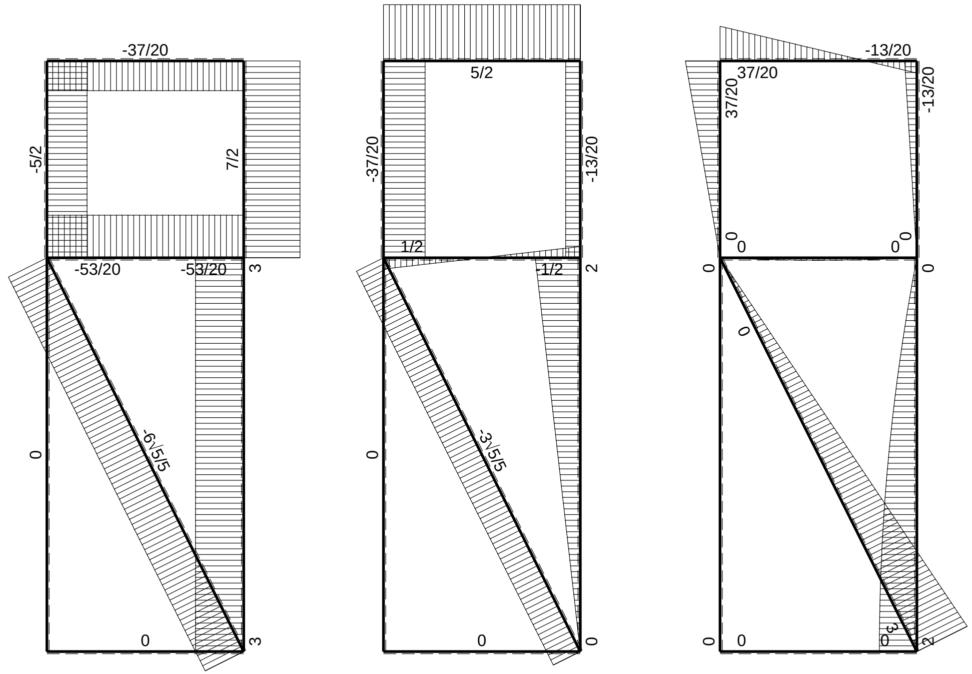


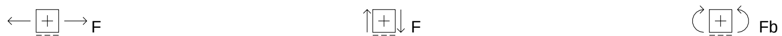
A = 540. mm^2
J_x = 198266. mm^4
J_y = 42746. mm^4
J_{xy} = 62554. mm^4
J_u = 220304. mm^4
J_v = 20709. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3387
c = cosα = .9432
s = sinα = -.3323
x_g = 24.2 mm
y_g = 23.8 mm
N = -992.8 N
T_y = -496.4 N
M_x = 799200. Nmm
x_m = 30. mm
u_m = 13.38 mm
v_m = -20.52 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.9 N/mm^2

CI09.xxxx.029

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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
C
D
53/20F
53/20F
1/2F
1/2F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-37/20
-5/2
7/2
-53/20
-53/20
3
0
-6√5/5
0
3
5/2
-37/20
-13/20
1/2
-1/2
2
0
-3√5/5
0
0
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
3
2


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi\theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 267675 CI09.xxxx.030

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

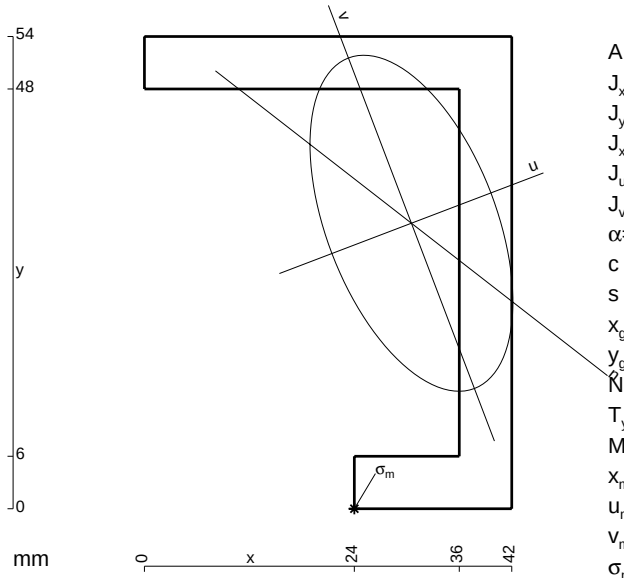
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

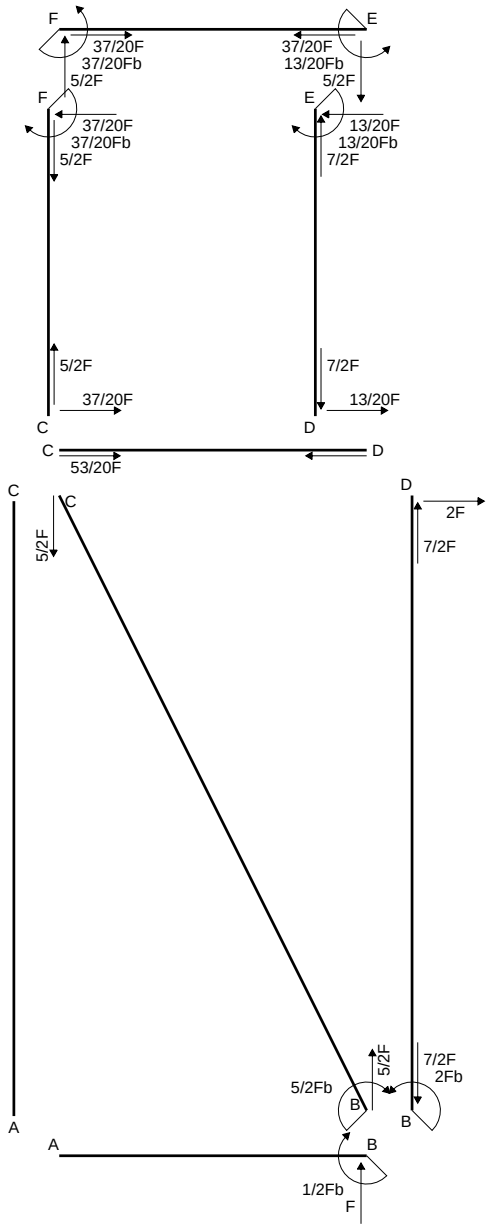
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

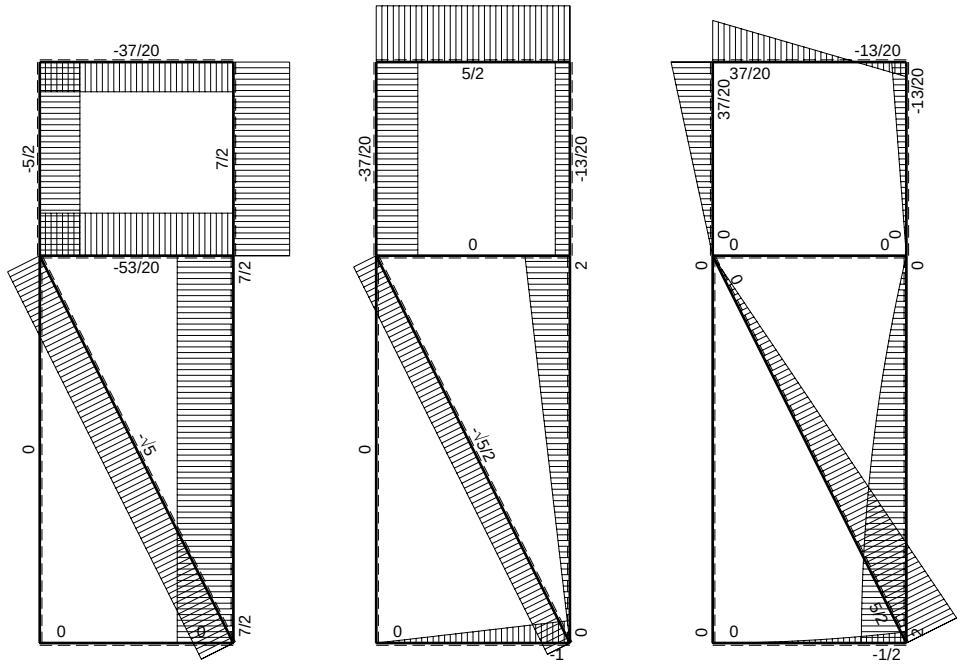


A = 612. mm^2
J_x = 225968. mm^4
J_y = 82341. mm^4
J_{xy} = -64038. mm^4
J_u = 250373. mm^4
J_v = 57936. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3641
c = cosα = .9344
s = sinα = .3561
x_g = 30.53 mm
y_g = 32.65 mm
N = -2227. N
T_y = -1114. N
M_x = 946200. Nmm
x_m = 24. mm
u_m = -17.73 mm
v_m = -28.18 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = 199. N/mm^2

CI09.xxxx.030

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F
E
37/20F
37/20Fb
5/2F
13/20Fb
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
53/20F
2F
7/2F
5/2F
5/2Fb
2Fb
A
B
1/2Fb
F


-37/20
-5/2
7/2
-53/20
0
-5/2
5/2
-13/20
2
-1
37/20
-1/2


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$5/2Fb-\sqrt{5}/2Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 248738 CI09.xxxx.031
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

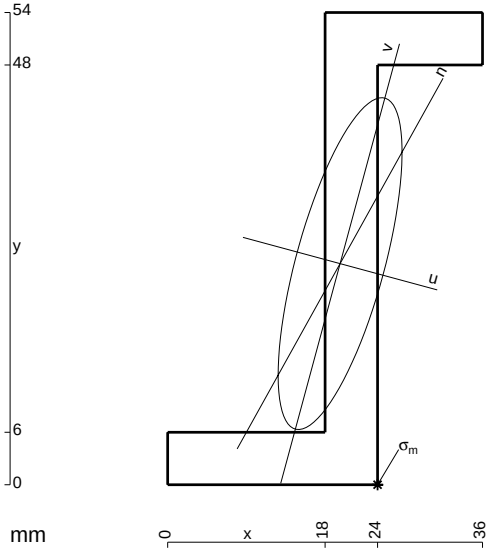
$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 248738 CI09.xxxx.031

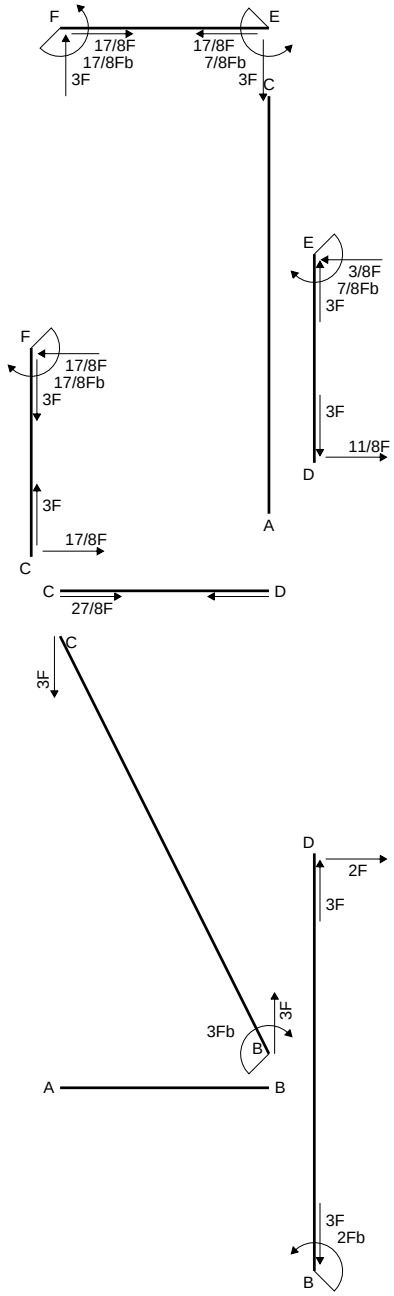


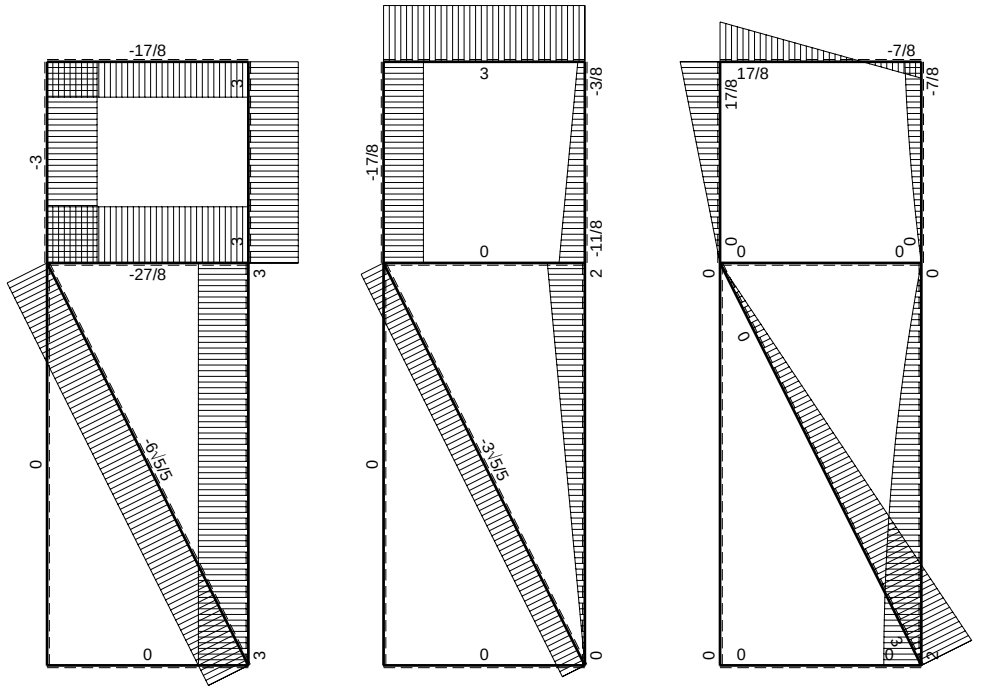
A = 504. mm^2
J_x = 181471. mm^4
J_y = 25303. mm^4
J_{xy} = 45545. mm^4
J_u = 193783. mm^4
J_v = 12991. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = -.2640
c = cosα = .9653
s = sinα = -.2610
x_g = 19.71 mm
y_g = 25.29 mm
N = -1386. N
T_y = -693.2 N
M_x = 635500. Nmm
x_m = 24. mm
u_m = 10.74 mm
v_m = -23.29 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 208. N/mm^2

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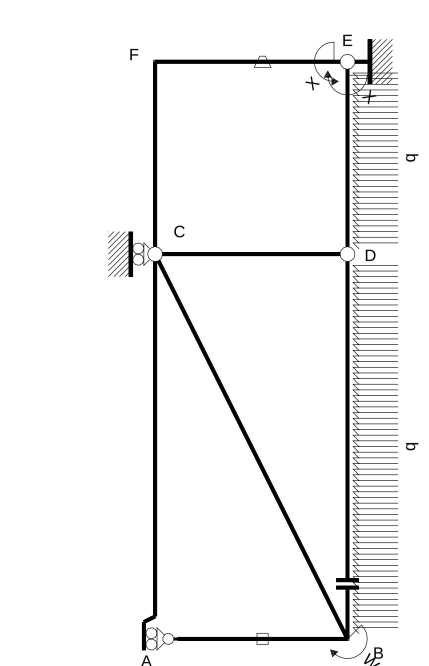
CI09.xxxx.031

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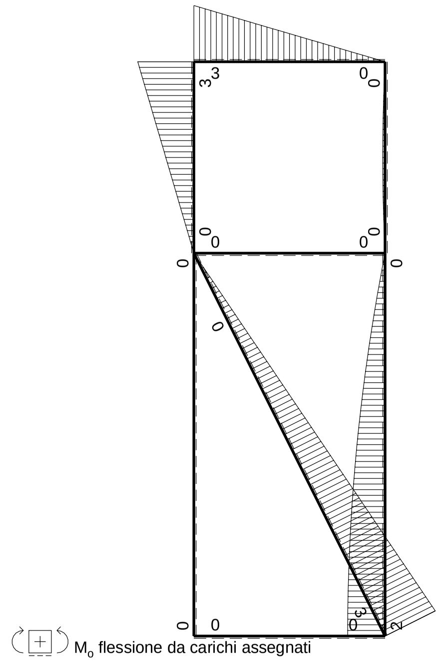

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
C
E
3/8F
7/8Fb
3F
F
17/8F
17/8Fb
3F
3F
11/8F
D
3F
A
17/8F
C
C
D
27/8F
C
3F
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
B


-17/8
3
-3
3
-27/8
3
0
-6.15/5
3
0
0
-17/8
3
-3/8
0
-11/8
2
-3.15/5
0
0
-7/8
17/8
17/8
-7/8
0
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0
0
0
3.15/5
0

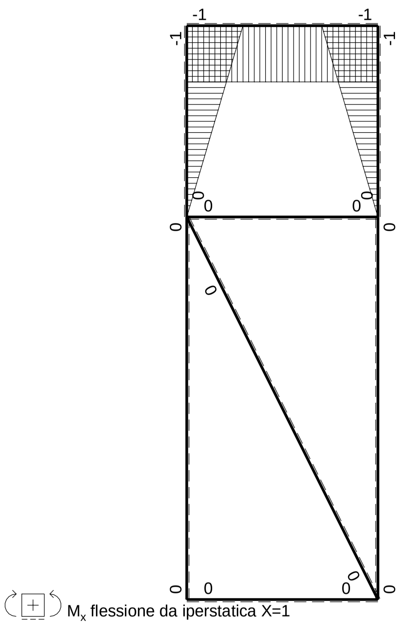

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 269783 CI09.xxxx.032

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
	totali						$-35/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

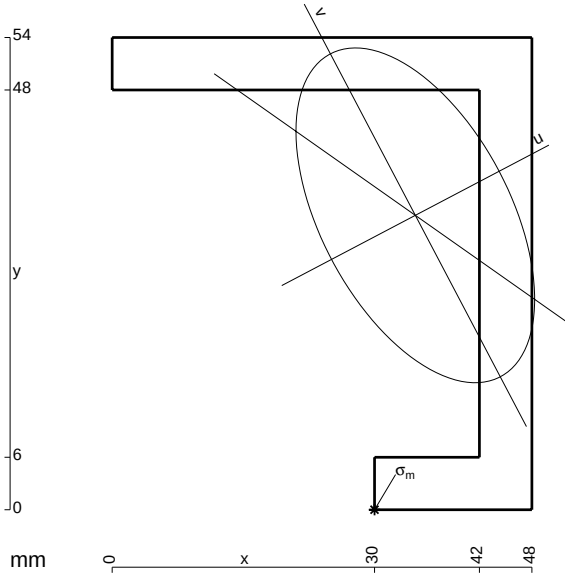
$= (-3\ b +3/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

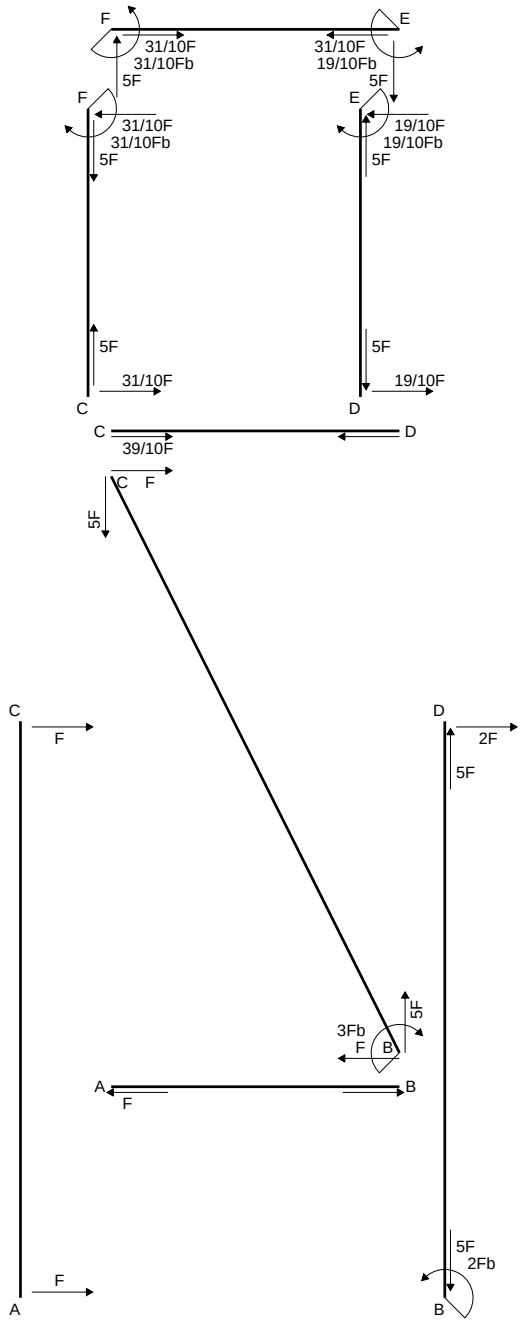
$= (-\ b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

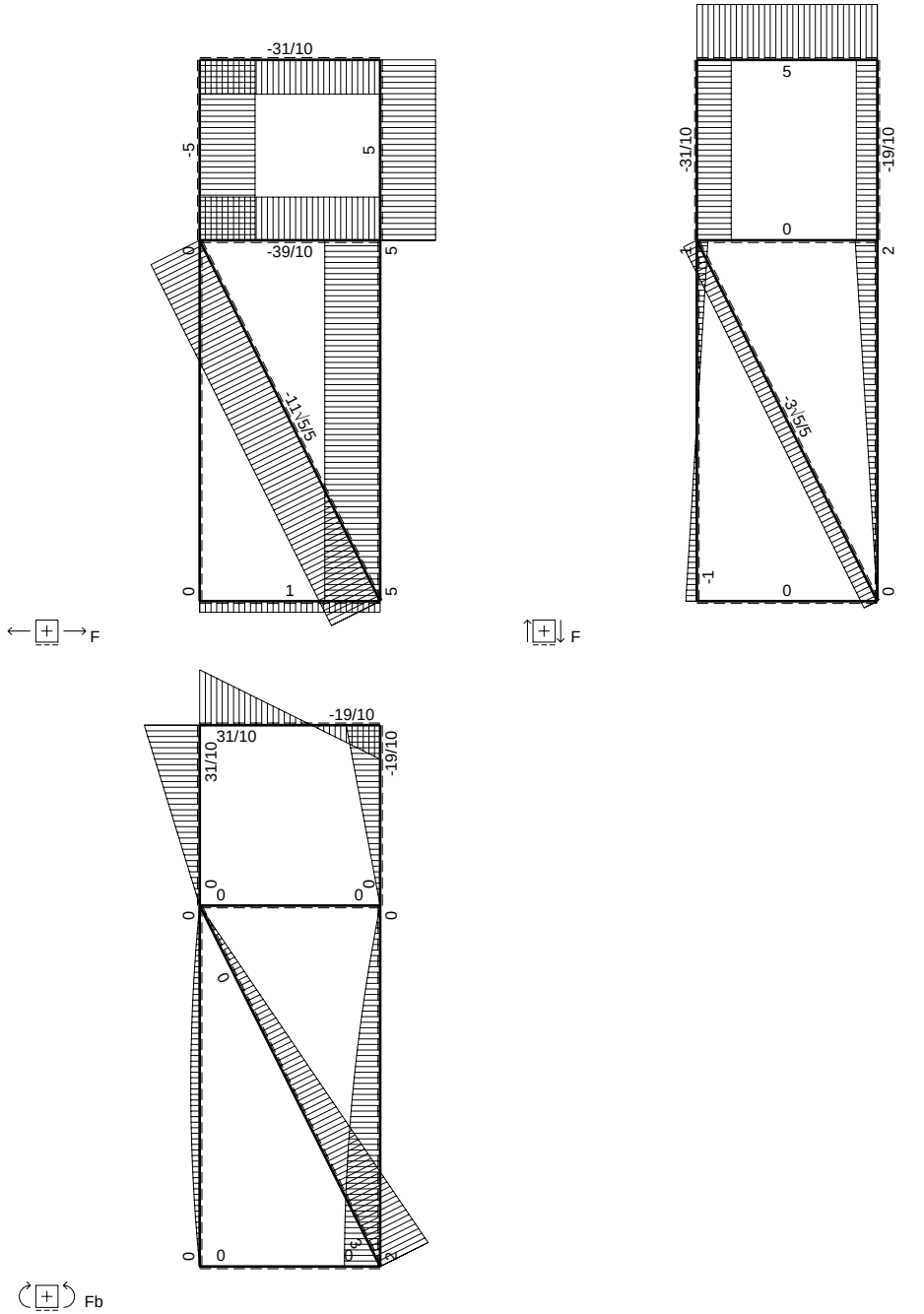


A = 648. mm^2
J_x = 237528. mm^4
J_y = 120672. mm^4
J_{xy} = -84960. mm^4
J_u = 282212. mm^4
J_v = 75988. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4842
c = cosα = .8851
s = sinα = .4655
x_g = 34.67 mm
y_g = 33.67 mm
N = -2120. N
T_y = -1060. N
M_x = 1066500. Nmm
x_m = 30. mm
u_m = -19.8 mm
v_m = -27.62 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 218.5 N/mm^2

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F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
C
D
39/10F
2F
3Fb
2Fb
A
B


-31/10
-39/10
-11√5/5
-3√5/5
-19/10
31/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
totali							-19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						19/10Fb	

Sviluppi di calcolo iperstatica

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16.04.26

PROCEDIMENTO E RISULTATI 248373 CI09.xxxx.033

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

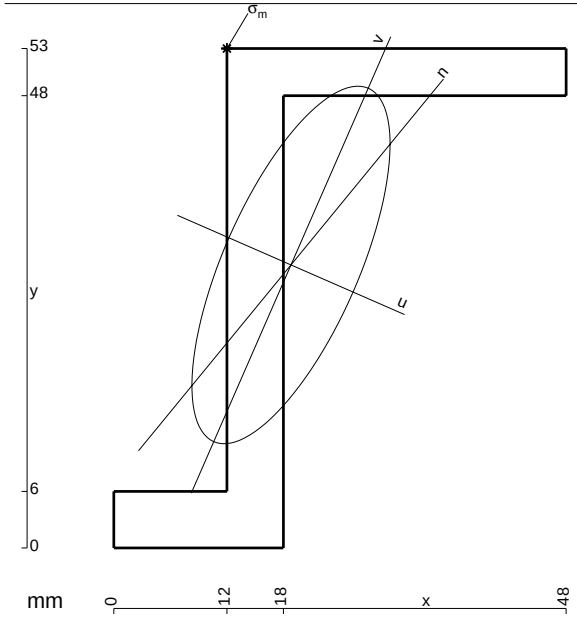
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

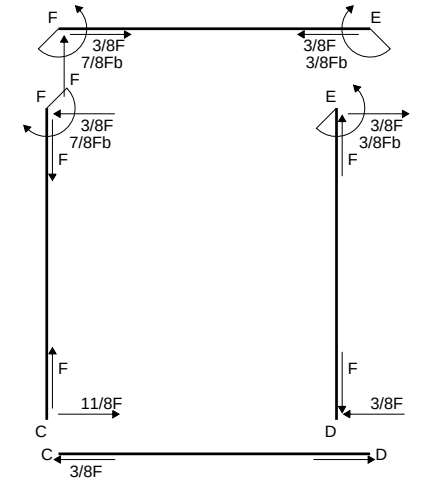
$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

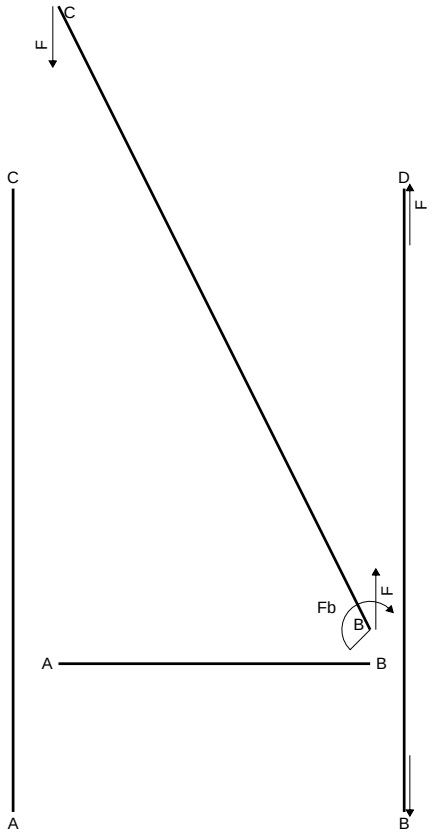


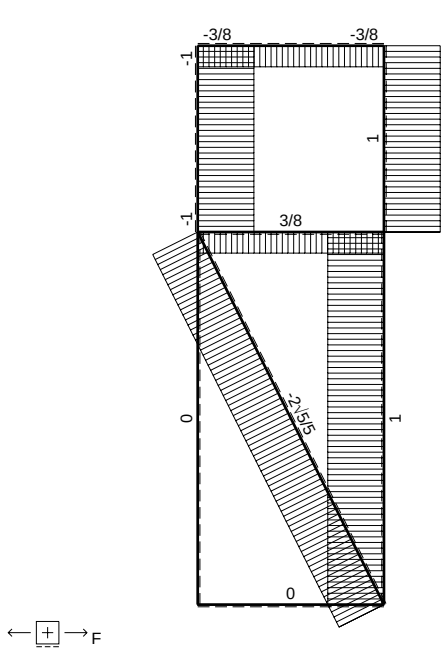
A = 540. mm^2
J_x = 194387. mm^4
J_y = 59702. mm^4
J_{xy} = 72778. mm^4
J_u = 226199. mm^4
J_v = 27891. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4121
$c = \cos\alpha$ = .9163
$s = \sin\alpha$ = -.4005
x_g = 18.8 mm
y_g = 30.03 mm
N = -2550. N
T_y = -1581. N
M_x = 758880. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -15.43 mm
v_m = 18.32 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.2 N/mm^2

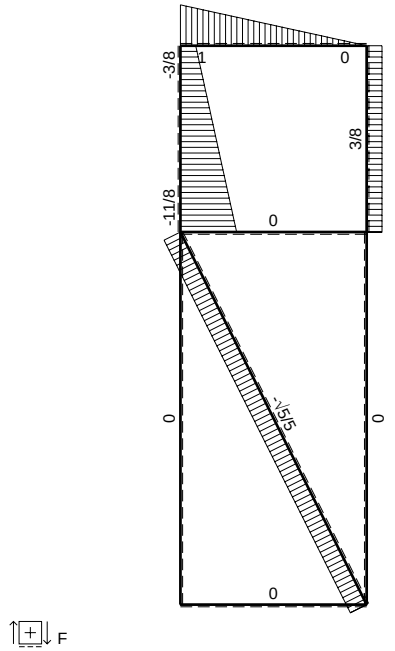
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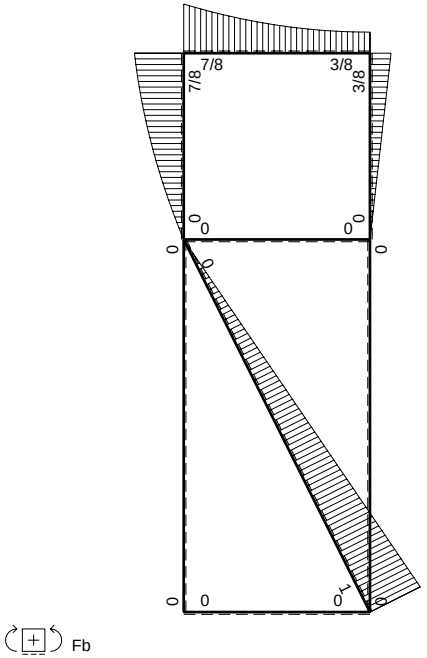
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F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

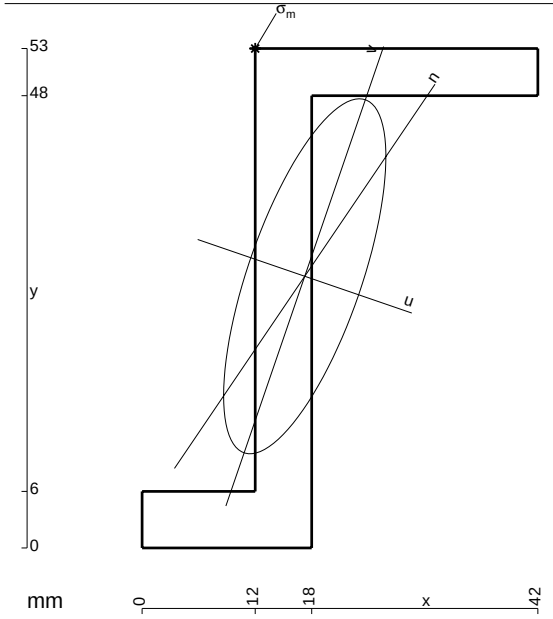
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

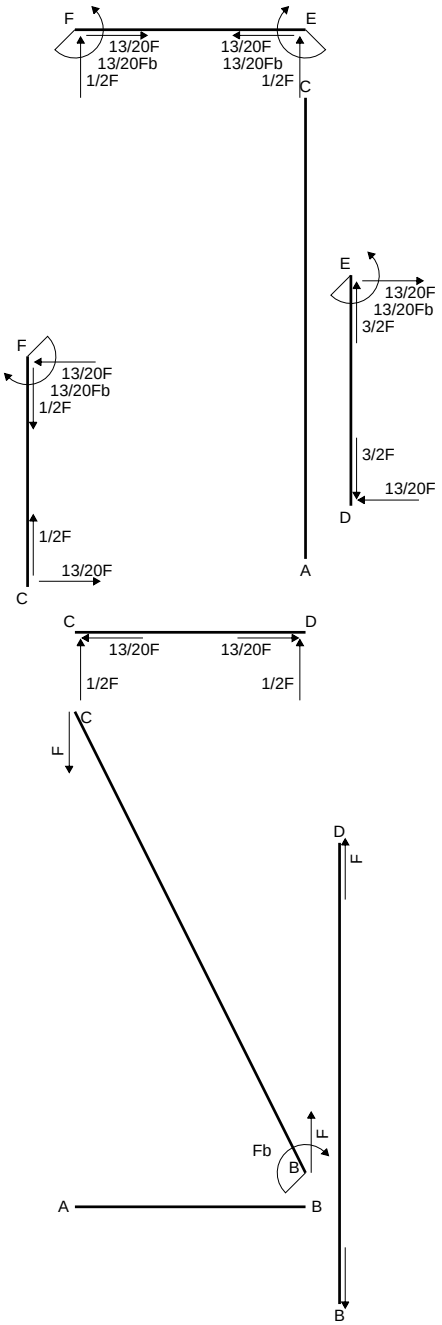
$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

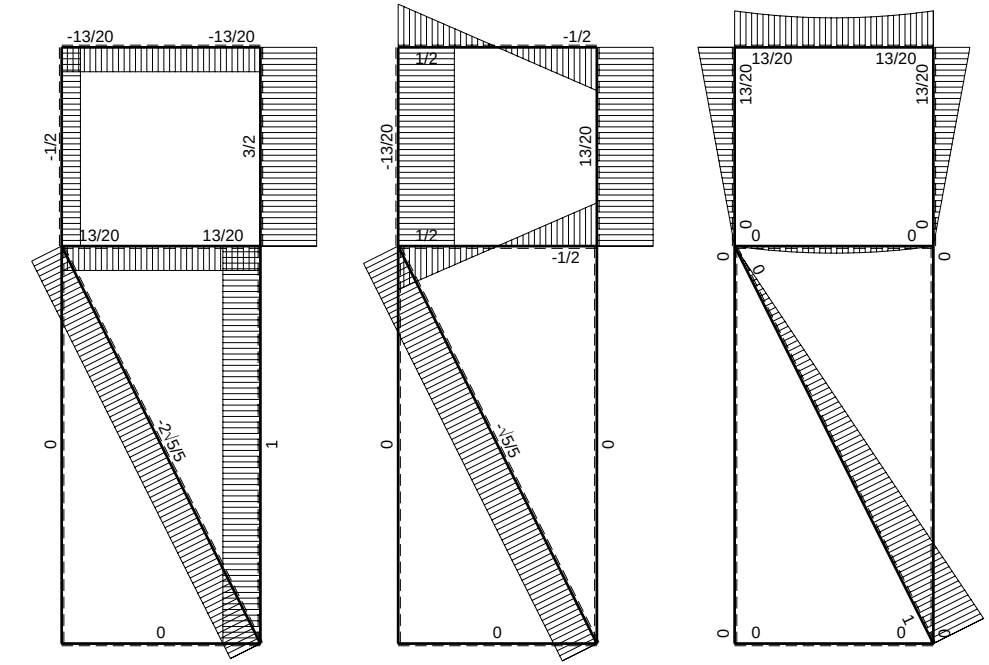


A = 510. mm^2
J_x = 181019. mm^4
J_y = 37808. mm^4
J_{xy} = 55745. mm^4
J_u = 200159. mm^4
J_v = 18668. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3307
$c = \cos\alpha$ = .9458
$s = \sin\alpha$ = -.3247
x_g = 17.26 mm
y_g = 28.83 mm
N = -1288. N
T_y = -644. N
M_x = 734400. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -12.82 mm
v_m = 21.15 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -239.8 N/mm^2

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F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-√5/5


F
F
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

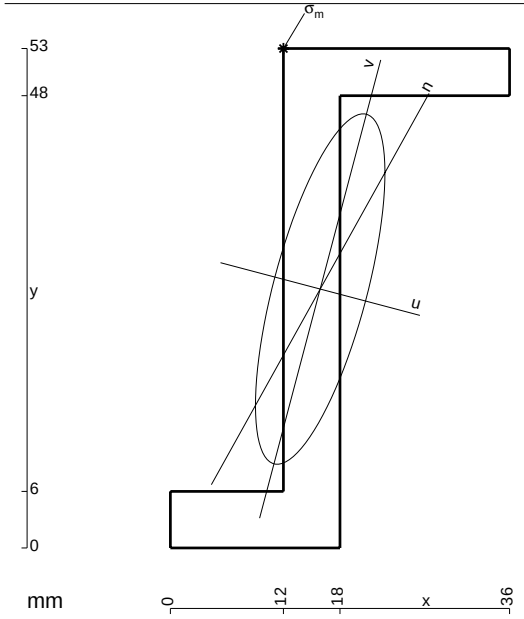
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 301451 CI09.xxxx.035

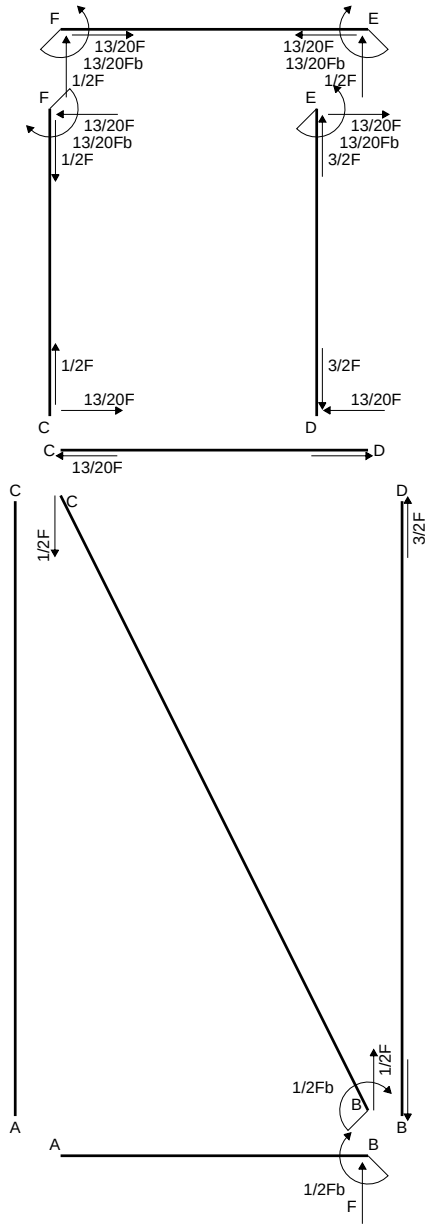


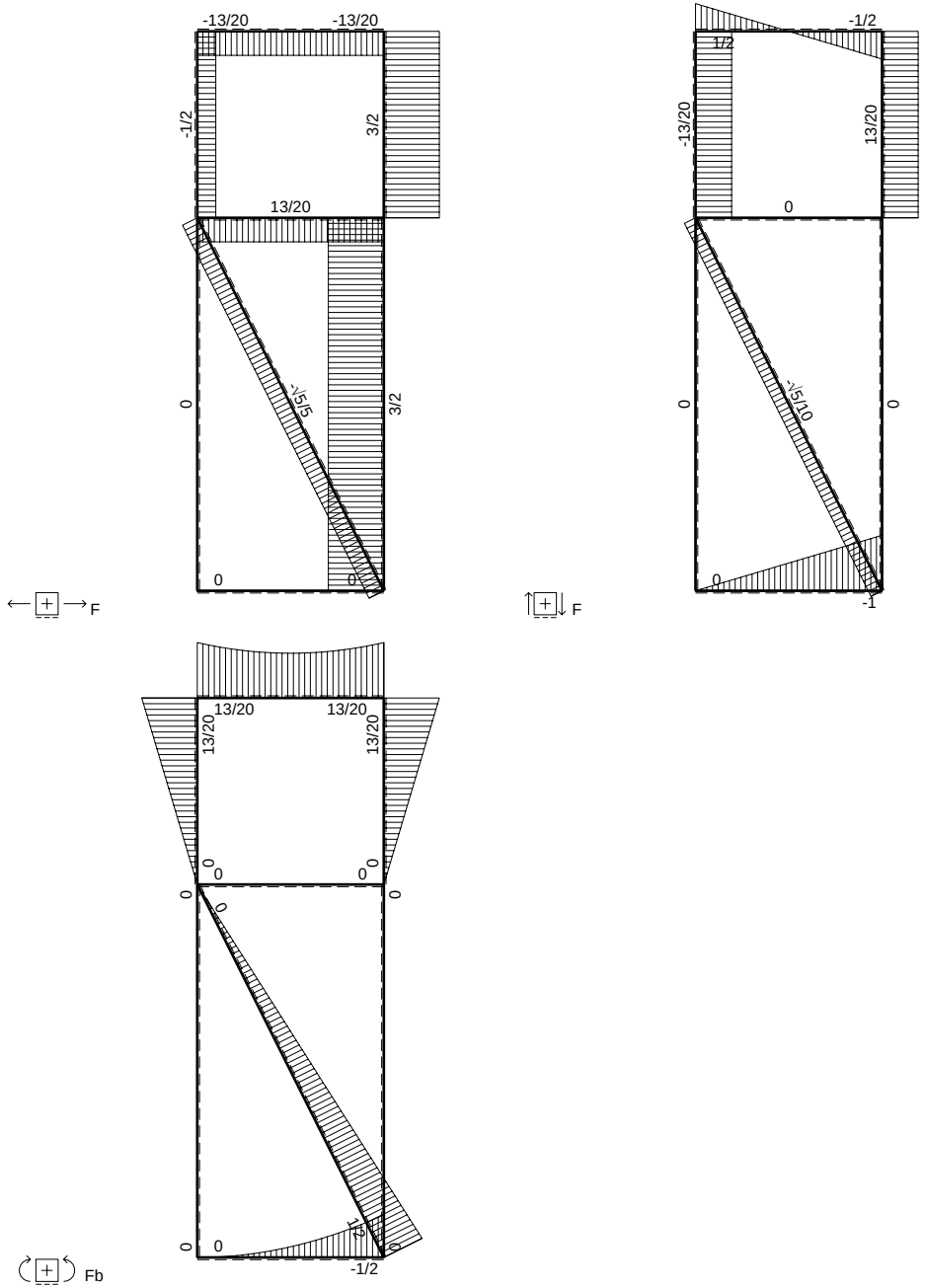
A = 480. mm^2
J_x = 165988. mm^4
J_y = 22651. mm^4
J_{xy} = 40727. mm^4
J_u = 176751. mm^4
J_v = 11888. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2584
$c = \cos\alpha$ = .9668
$s = \sin\alpha$ = -.2555
x_g = 15.9 mm
y_g = 27.48 mm
N = -912.3 N
T_y = -456.2 N
M_x = 561000. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -10.29 mm
v_m = 23.68 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -198.7 N/mm^2

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
F
1/2
-√5/10
-1
13/20
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
totali							$13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

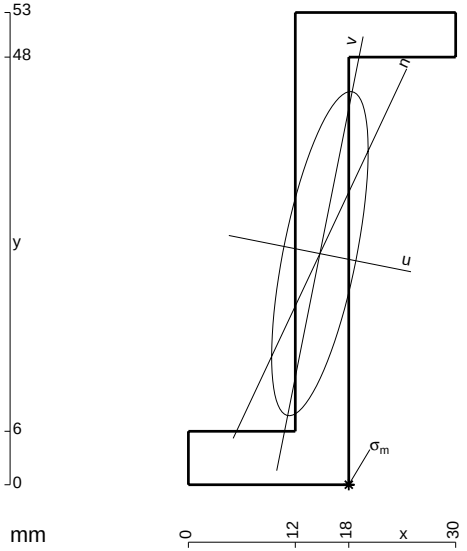
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

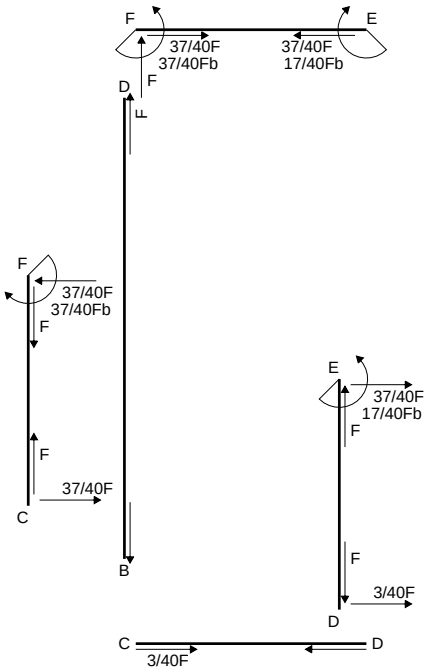
$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

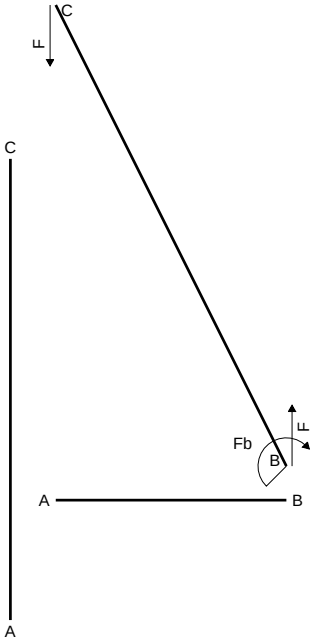


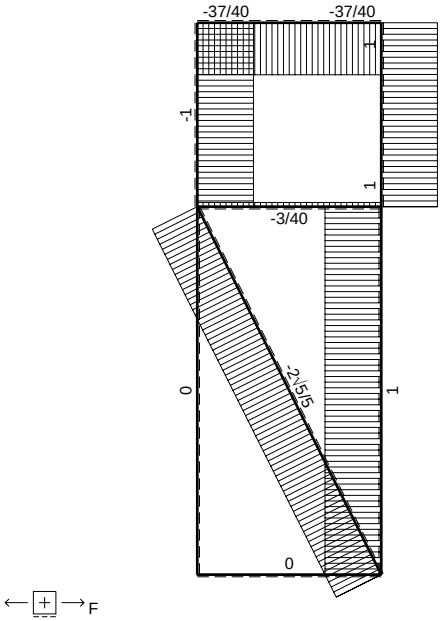
A = 450. mm^2
J_x = 148960. mm^4
J_y = 13204. mm^4
J_{xy} = 28128. mm^4
J_u = 154557. mm^4
J_v = 7607. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1964
c = cosα = .9808
s = sinα = -.1952
x_g = 14.76 mm
y_g = 25.94 mm
N = 2175. N
T_y = 942.5 N
M_x = 556075. Nmm
x_m = 18. mm
u_m = 8.24 mm
v_m = -24.81 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = 209.9 N/mm^2

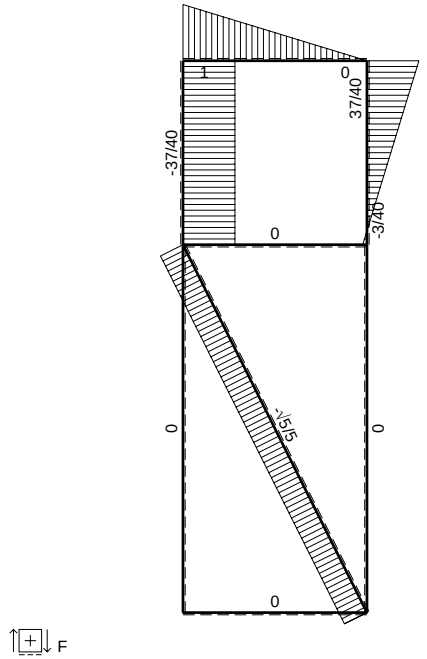
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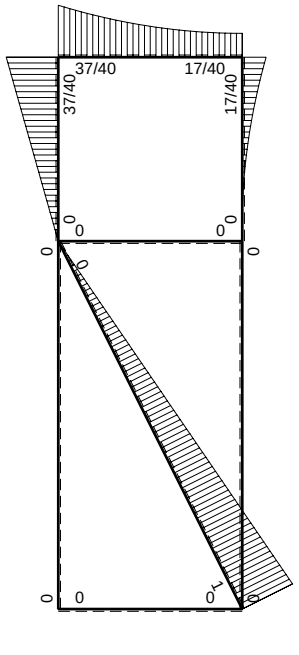
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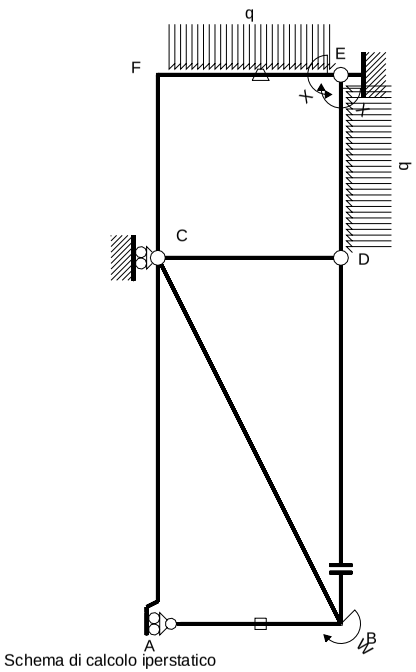

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
D
3/40F


C
F
C
Fb
F
B
A
B
A

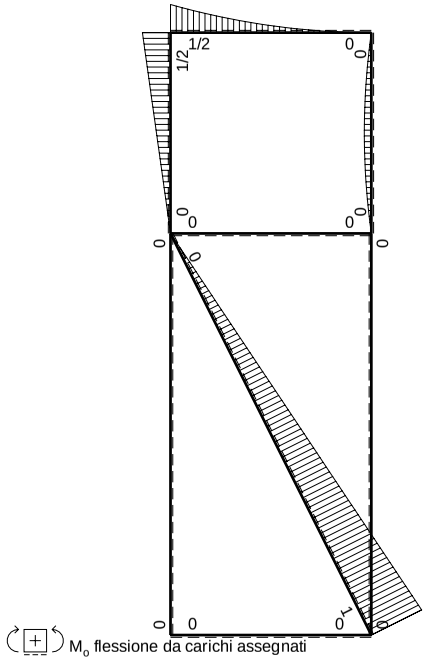

-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
1
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
0
F

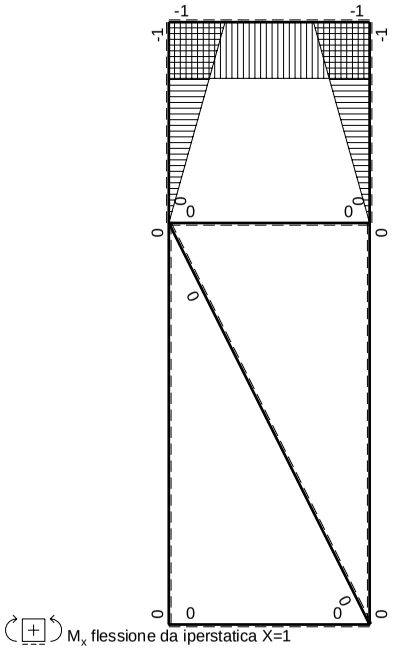

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 270236 CI09.xxxx.037

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

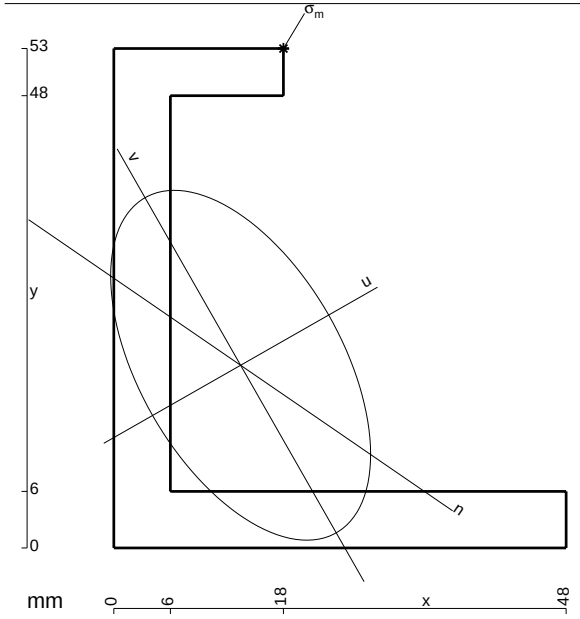
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

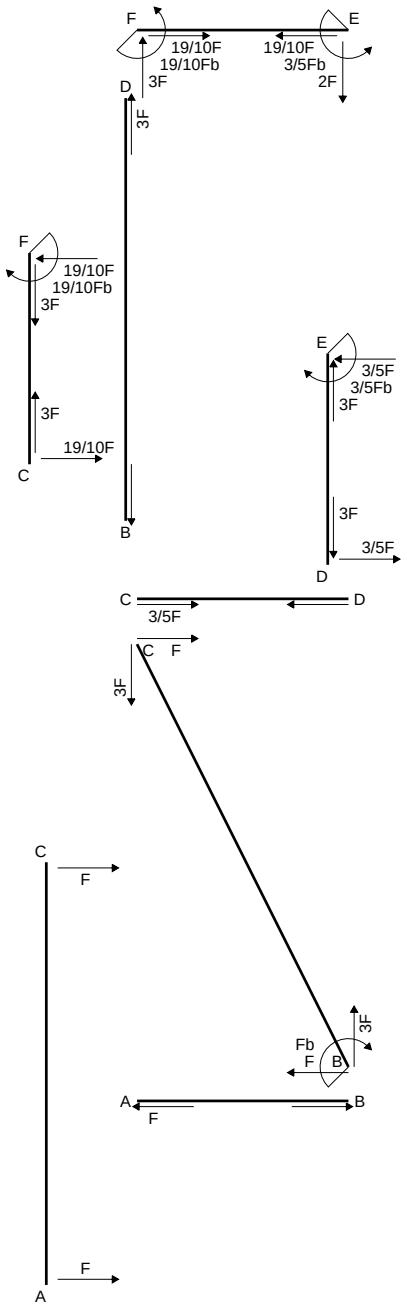
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

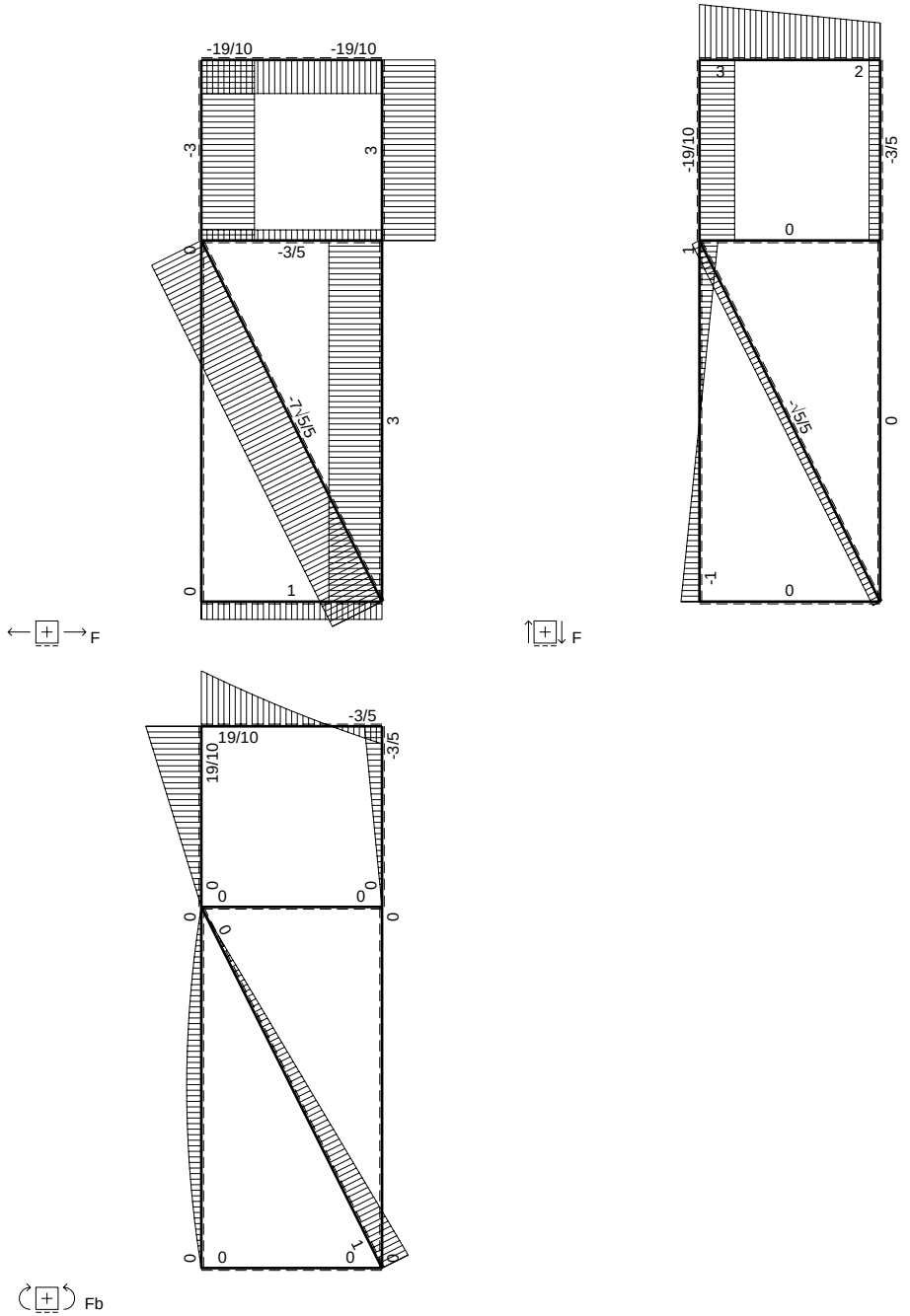


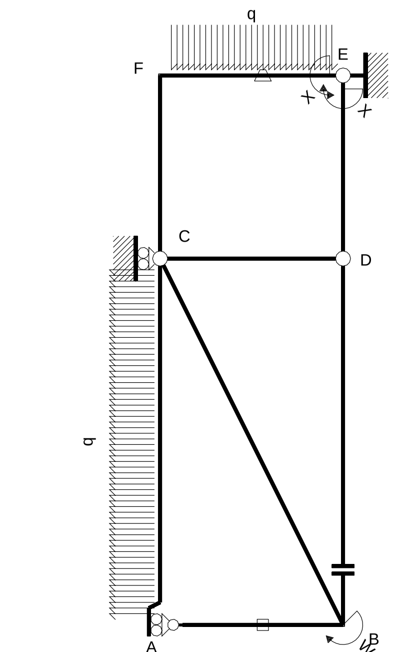
A = 630. mm^2
J_x = 217160. mm^4
J_y = 119838. mm^4
J_{xy} = -82299. mm^4
J_u = 264108. mm^4
J_v = 72891. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .5184
$c = \cos\alpha$ = .8686
$s = \sin\alpha$ = .4955
x_g = 13.46 mm
y_g = 19.39 mm
N = -1368. N
T_y = -684.2 N
M_x = 948600. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 20.6 mm
v_m = 26.95 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.1 N/mm^2

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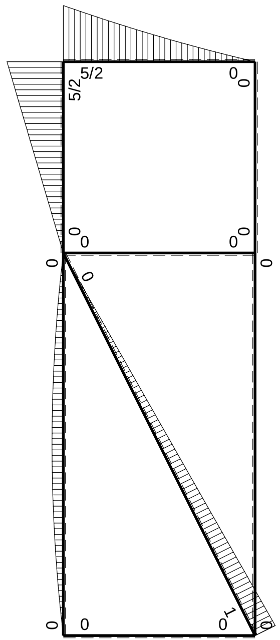
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F
E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
B
3F
3/5F
D
C
D
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Fb
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F
A

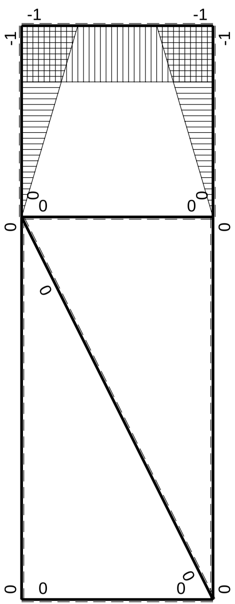

-19/10
-19/10
-3
3
-3/5
0
-7\5/5
3
0
1
F
-19/10
3
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-3/5
0
1
-1\5/5
0
-1
0
F
-3/5
19/10
19/10
-3/5
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

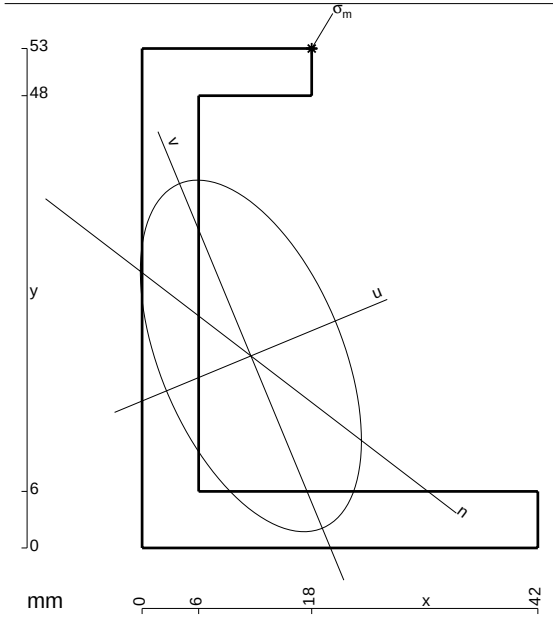
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

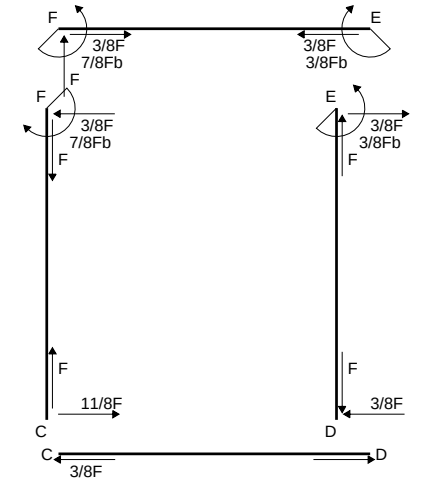
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

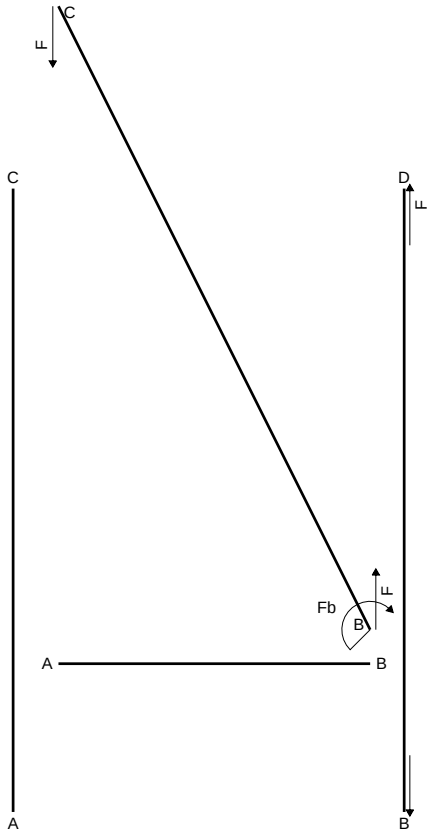


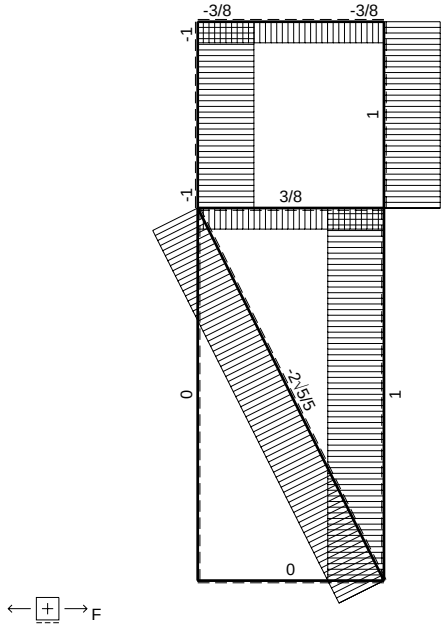
A = 594. mm^2
J_x = 206801. mm^4
J_y = 81741. mm^4
J_{xy} = -62565. mm^4
J_u = 232726. mm^4
J_v = 55816. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3928
$c = \cos\alpha$ = .9238
$s = \sin\alpha$ = .3828
x_g = 11.55 mm
y_g = 20.38 mm
N = -2280. N
T_y = -1444. N
M_x = 953040. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 18.45 mm
v_m = 27.67 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.1 N/mm^2

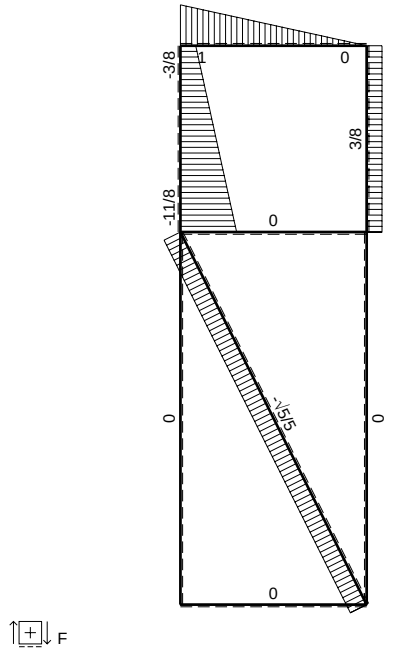
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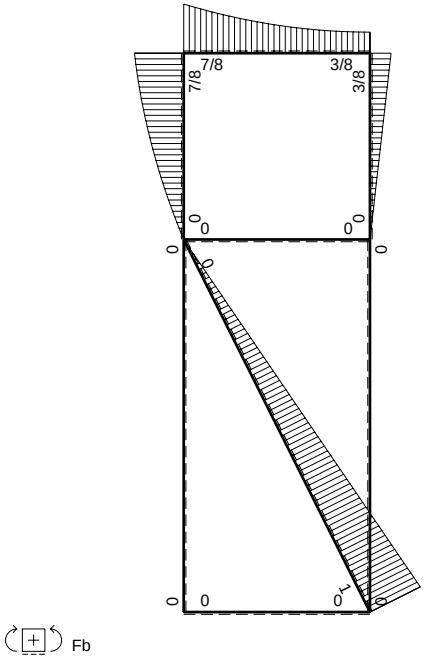
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F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
F
3/8F
C
D
C
D
3/8F


C
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F
Fb
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A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
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-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

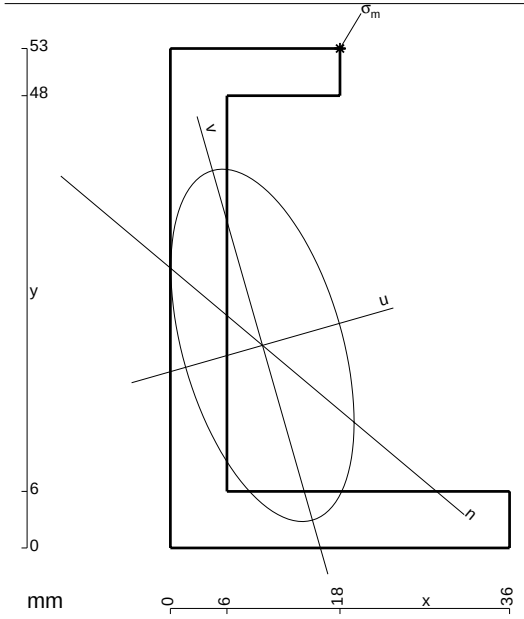
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

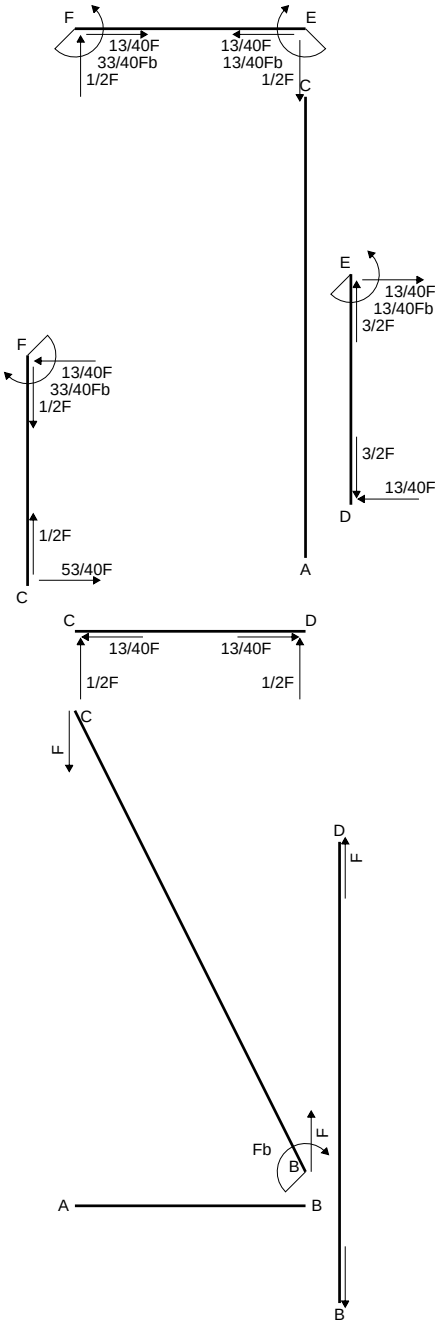
$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

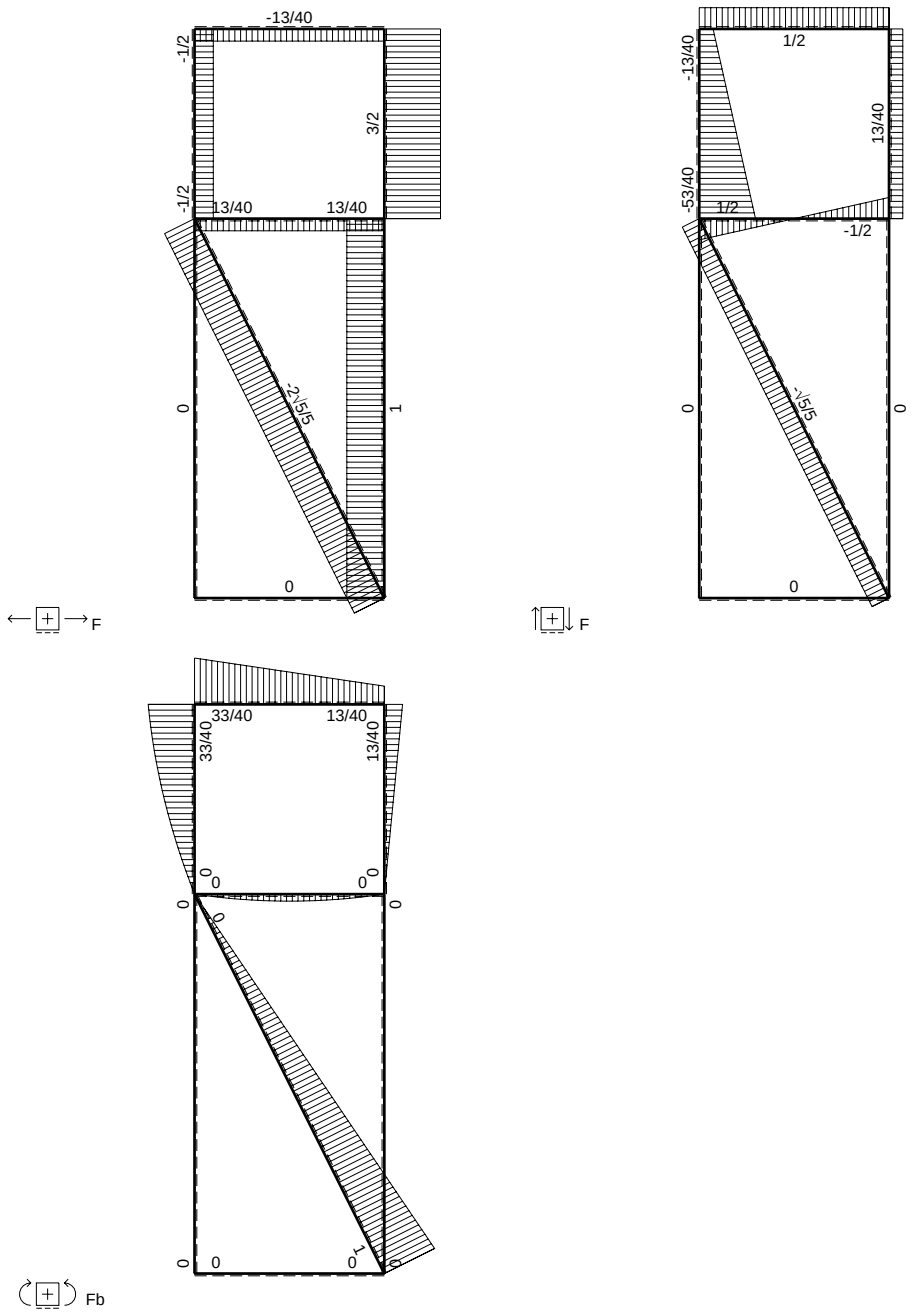


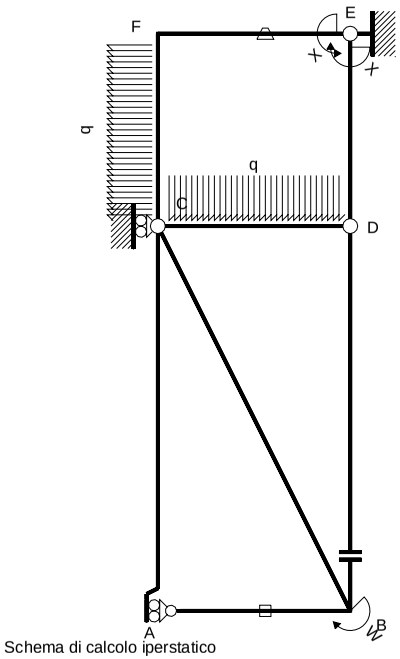
A = 558. mm^2
J_x = 195119. mm^4
J_y = 52748. mm^4
J_{xy} = -44280. mm^4
J_u = 207767. mm^4
J_v = 40099. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2782
$c = \cos\alpha$ = .9615
$s = \sin\alpha$ = .2747
x_g = 9.774 mm
y_g = 21.5 mm
N = -1243. N
T_y = -621.6 N
M_x = 973000. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 16.56 mm
v_m = 28.03 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -238.8 N/mm^2

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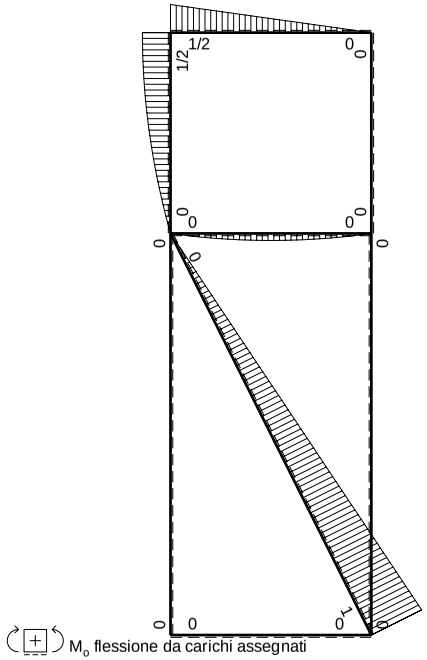
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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B

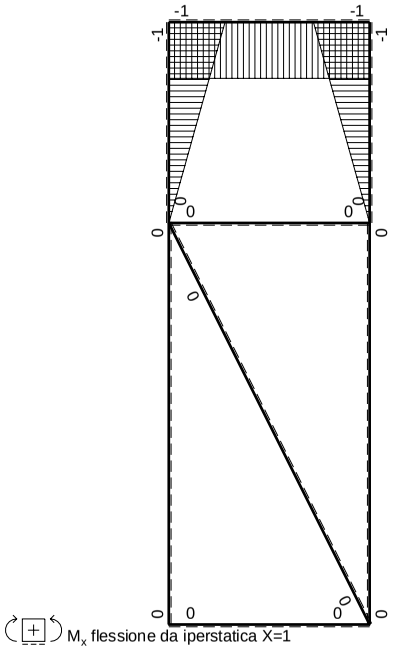

-13/40
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-1/2
3/2
13/40
13/40
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0
F
-13/40
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13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264363 CI09.xxxx.040

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

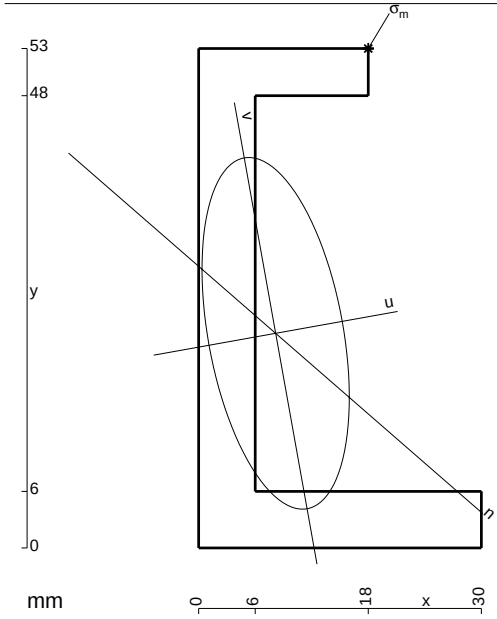
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

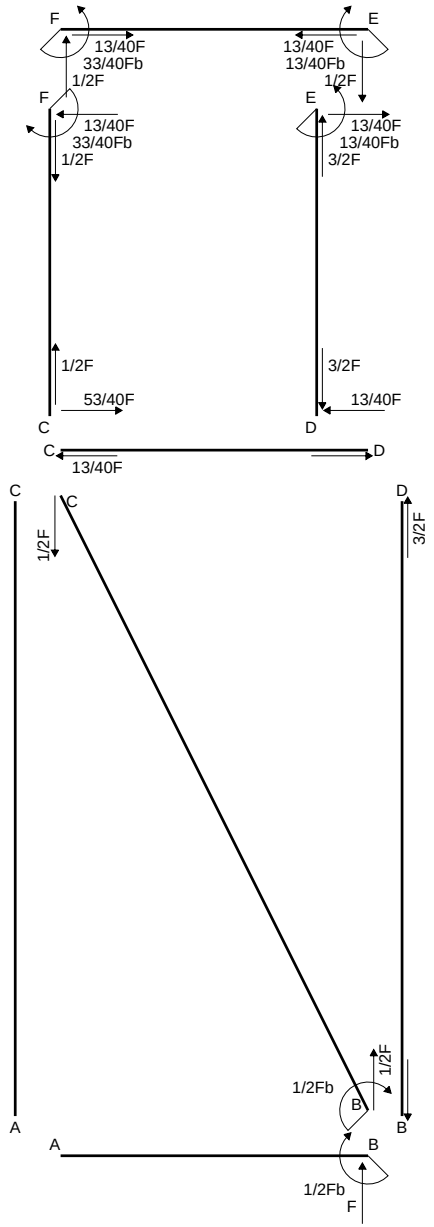
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

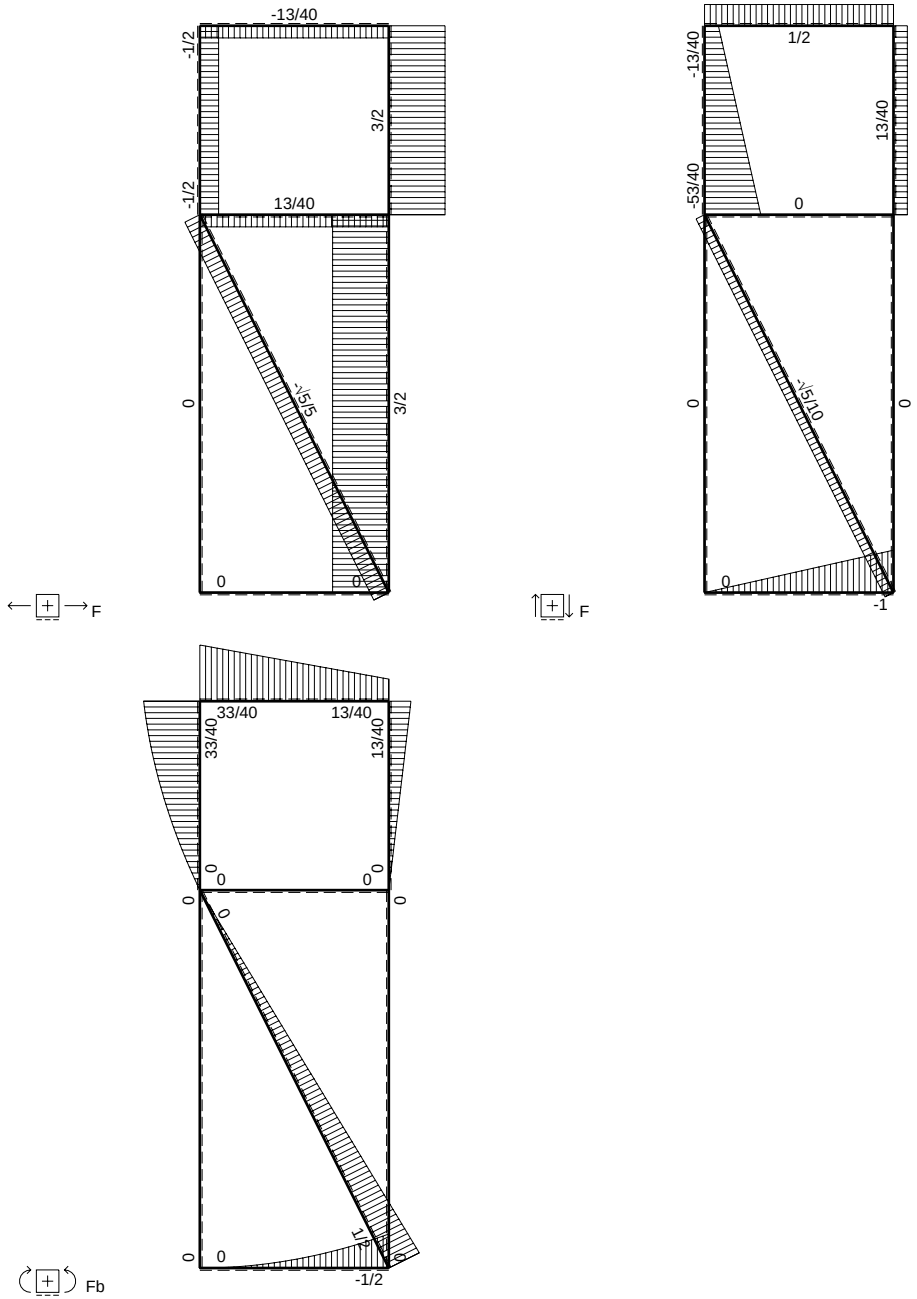


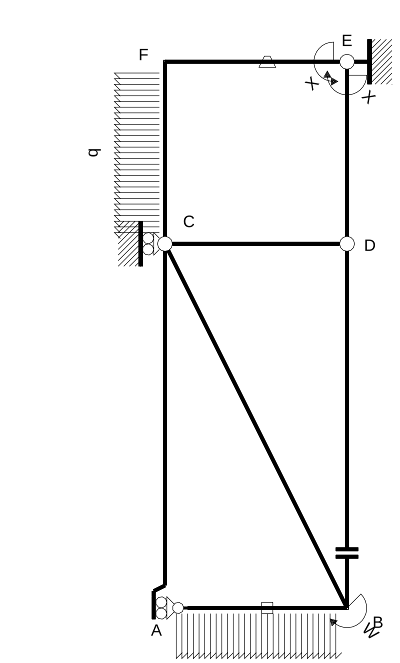
A = 522. mm^2
J_x = 181840. mm^4
J_y = 31881. mm^4
J_{xy} = -27745. mm^4
J_u = 186808. mm^4
J_v = 26912. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1772
$c = \cos\alpha$ = .9843
$s = \sin\alpha$ = .1763
x_g = 8.172 mm
y_g = 22.78 mm
N = -1923. N
T_y = -961.5 N
M_x = 795500. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 15. mm
v_m = 28.02 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -199.3 N/mm^2

CI09.xxxx.040

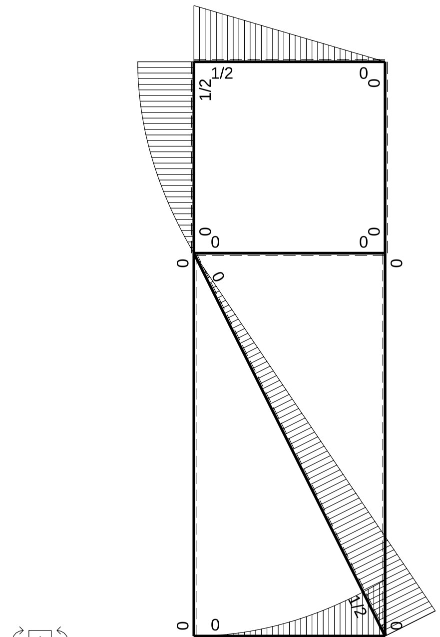
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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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B
A
B
1/2Fb
F

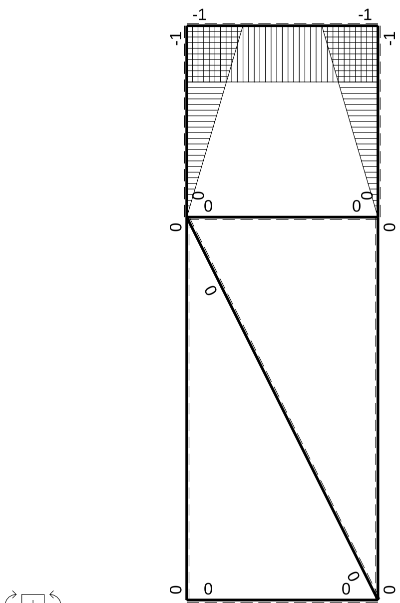

-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
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13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$13/24Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

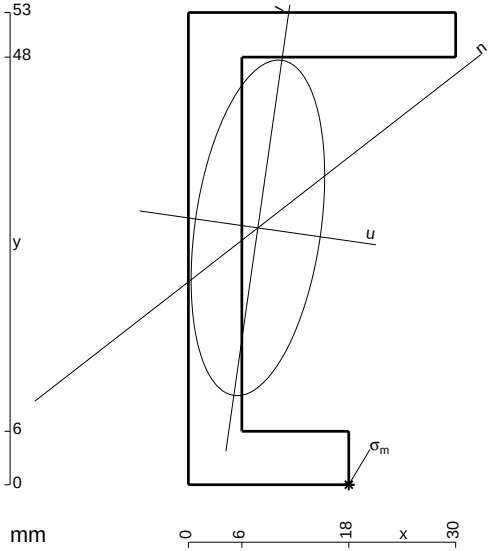
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

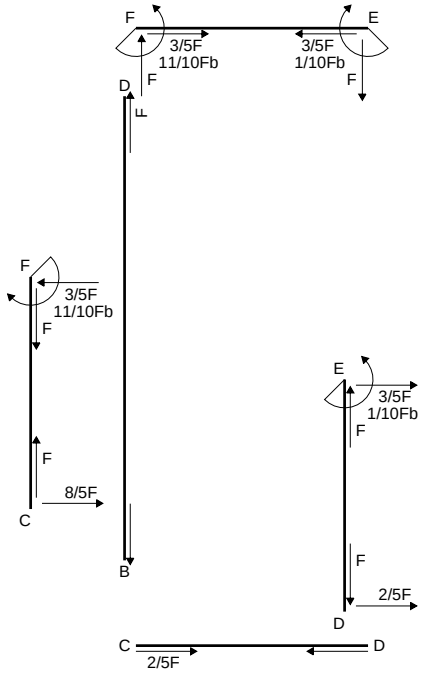
PROCEDIMENTO E RISULTATI 991322 CI09.xxxx.041

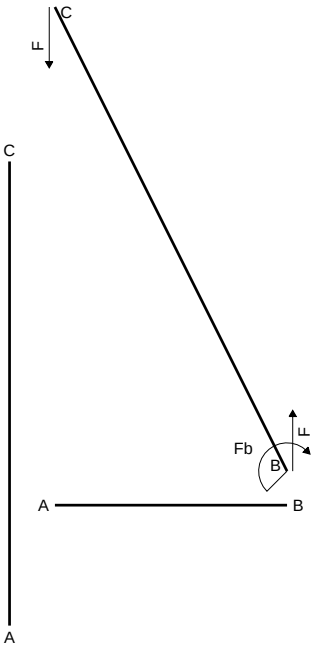


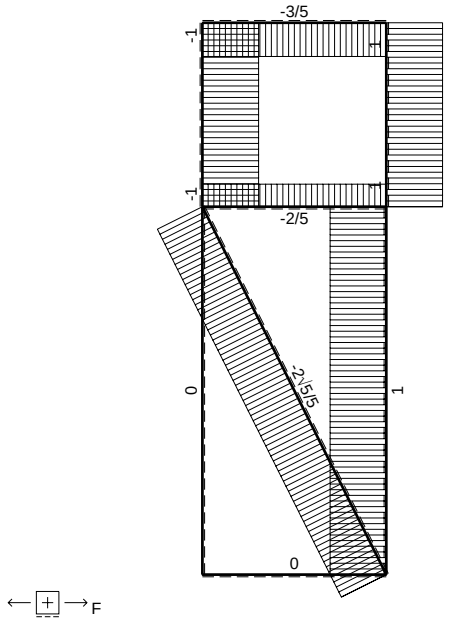
A = 510. mm^2
J_x = 181019. mm^4
J_y = 28660. mm^4
J_{xy} = 22270. mm^4
J_u = 184207. mm^4
J_v = 25471. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.1422$
$c = \cos\alpha = .9899$
$s = \sin\alpha = -.1417$
x_g = 7.8 mm
y_g = 28.83 mm
N = -926.3 N
T_y = 1425. N
M_x = 940500. Nmm
x_m = 18. mm
u_m = 14.18 mm
v_m = -27.09 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 209.3 N/mm^2

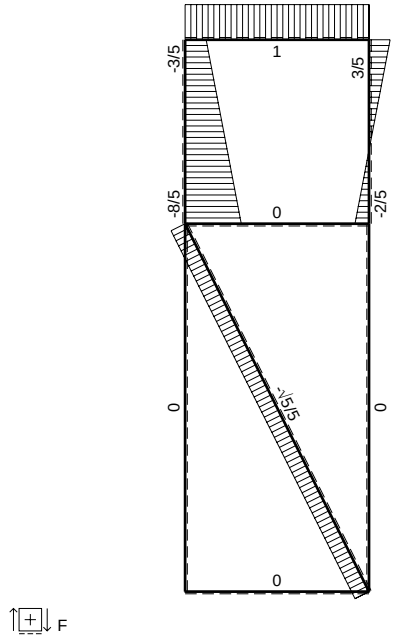
CI09.xxxx.041

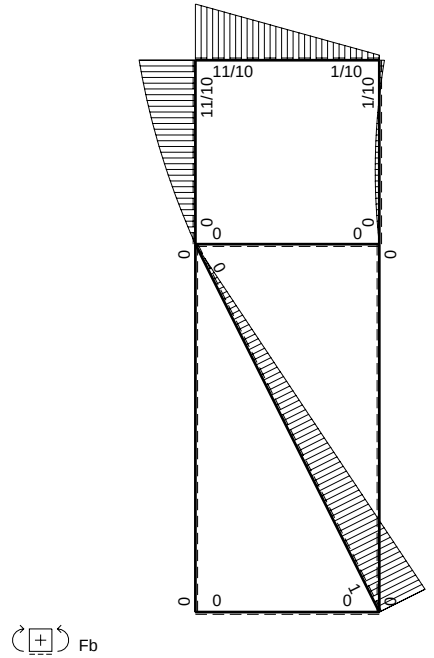
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F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
A
Fb
F
B
A
B


-3/5
-1
1
-1
1
-2/5
0
-2/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5
0
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3XLb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb^2/EJ	XLb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx^2	0	-Fb+3/2Fx-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-3/8+0)Fb^2/EJ	1/3XLb/EJ
CF b	x/b	-3/2Fx+1/2qx^2	0	-3/2Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						1/6Fb^2/EJ	5/3XLb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

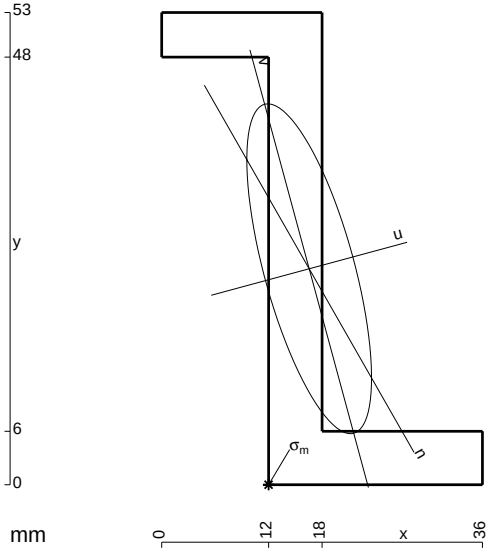
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

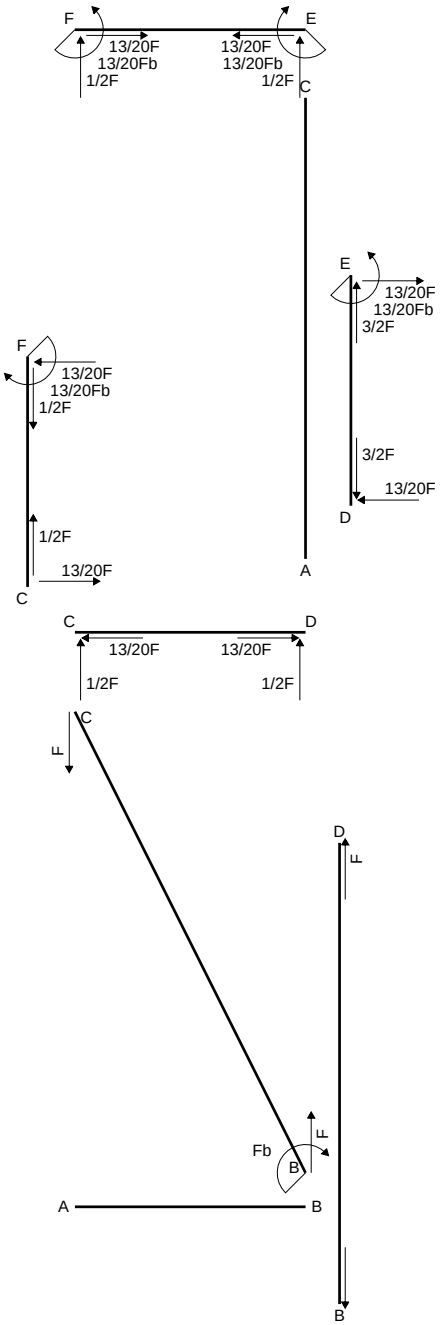
$= (-1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$

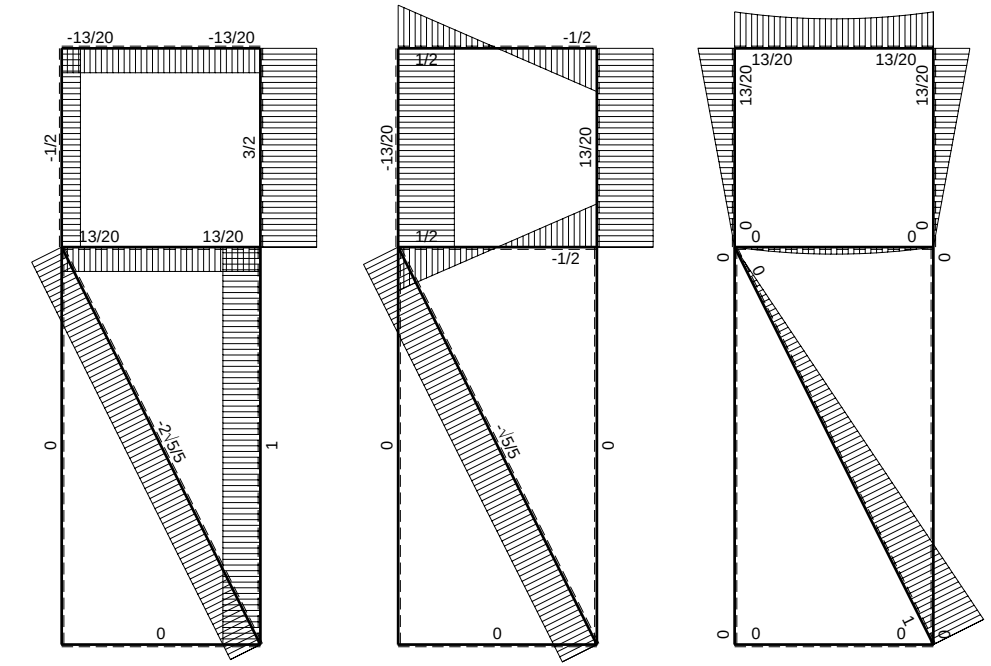


A = 486. mm^2
J_x = 166610. mm^4
J_y = 23826. mm^4
J_{xy} = -41708. mm^4
J_u = 177900. mm^4
J_v = 12536. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2644
$c = \cos\alpha$ = .9653
$s = \sin\alpha$ = .2613
x_g = 16.56 mm
y_g = 24.24 mm
N = -798. N
T_y = 1330. N
M_x = 643720. Nmm
x_m = 12. mm
u_m = -10.73 mm
v_m = -22.21 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 219.9 N/mm^2

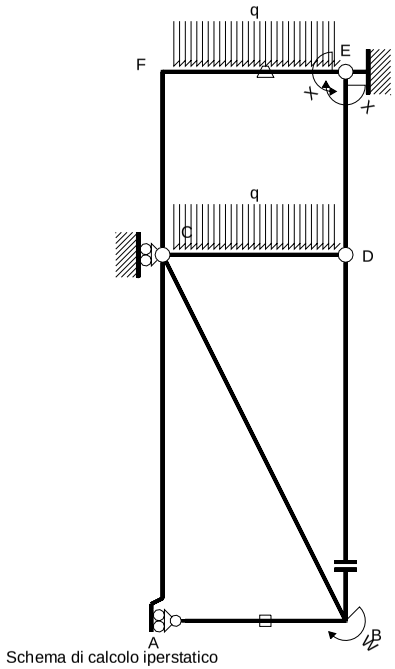
CI09.xxxx.042

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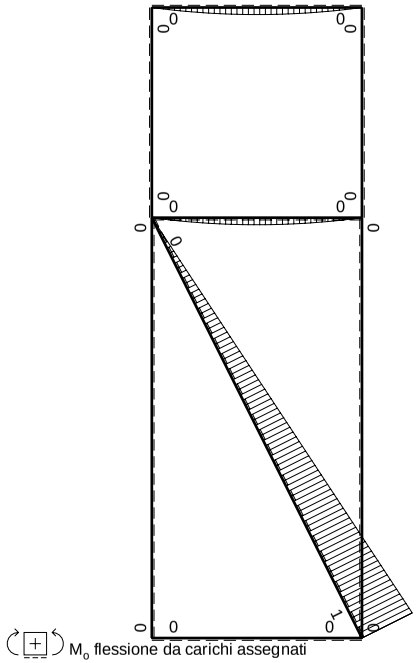

F
E
13/20F
13/20Fb
1/2F
13/20F
13/20Fb
1/2F
C
E
13/20F
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F
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13/20Fb
1/2F
3/2F
13/20F
D
1/2F
13/20F
C
A
C
D
13/20F
13/20F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-13/20
-13/20
-1/2
3/2
13/20
13/20
-2√5/5
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0
0
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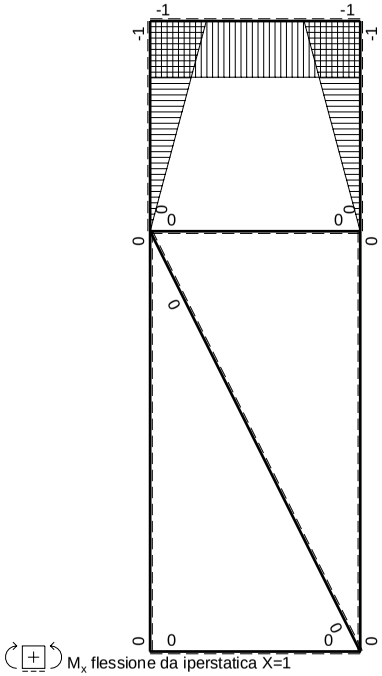

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 311832 CI09.xxxx.043

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

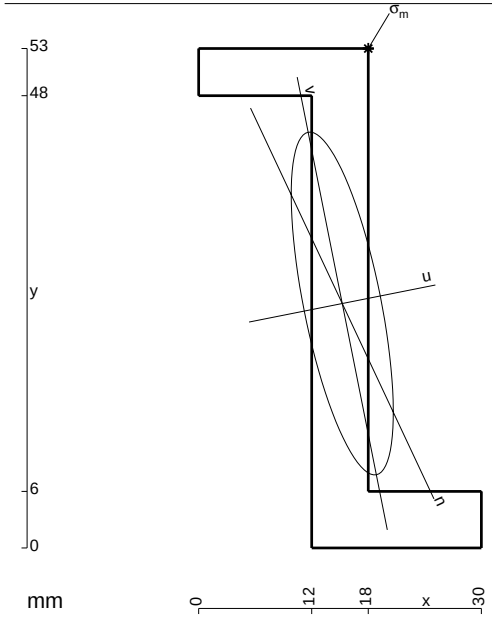
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 311832 CI09.xxxx.043

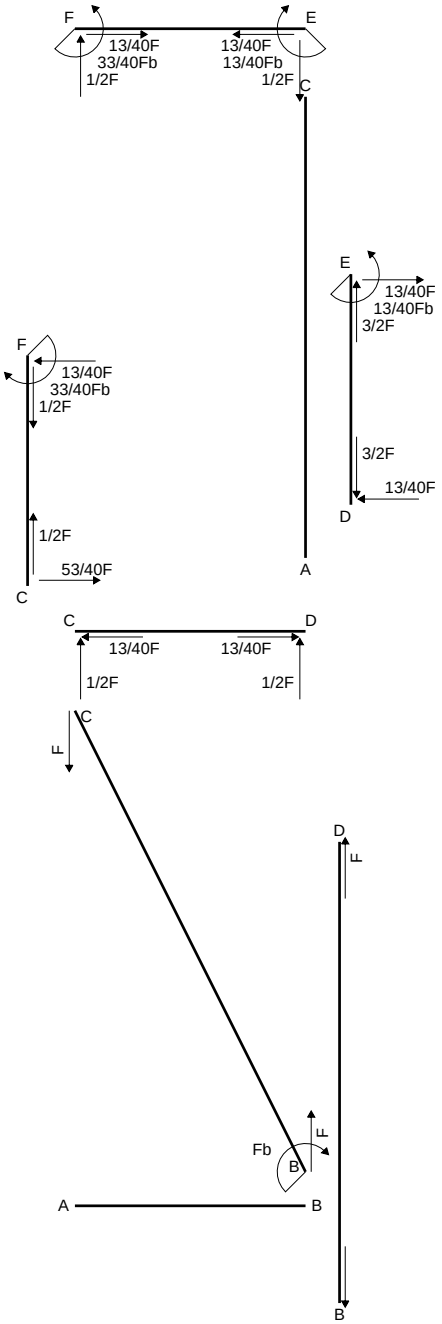


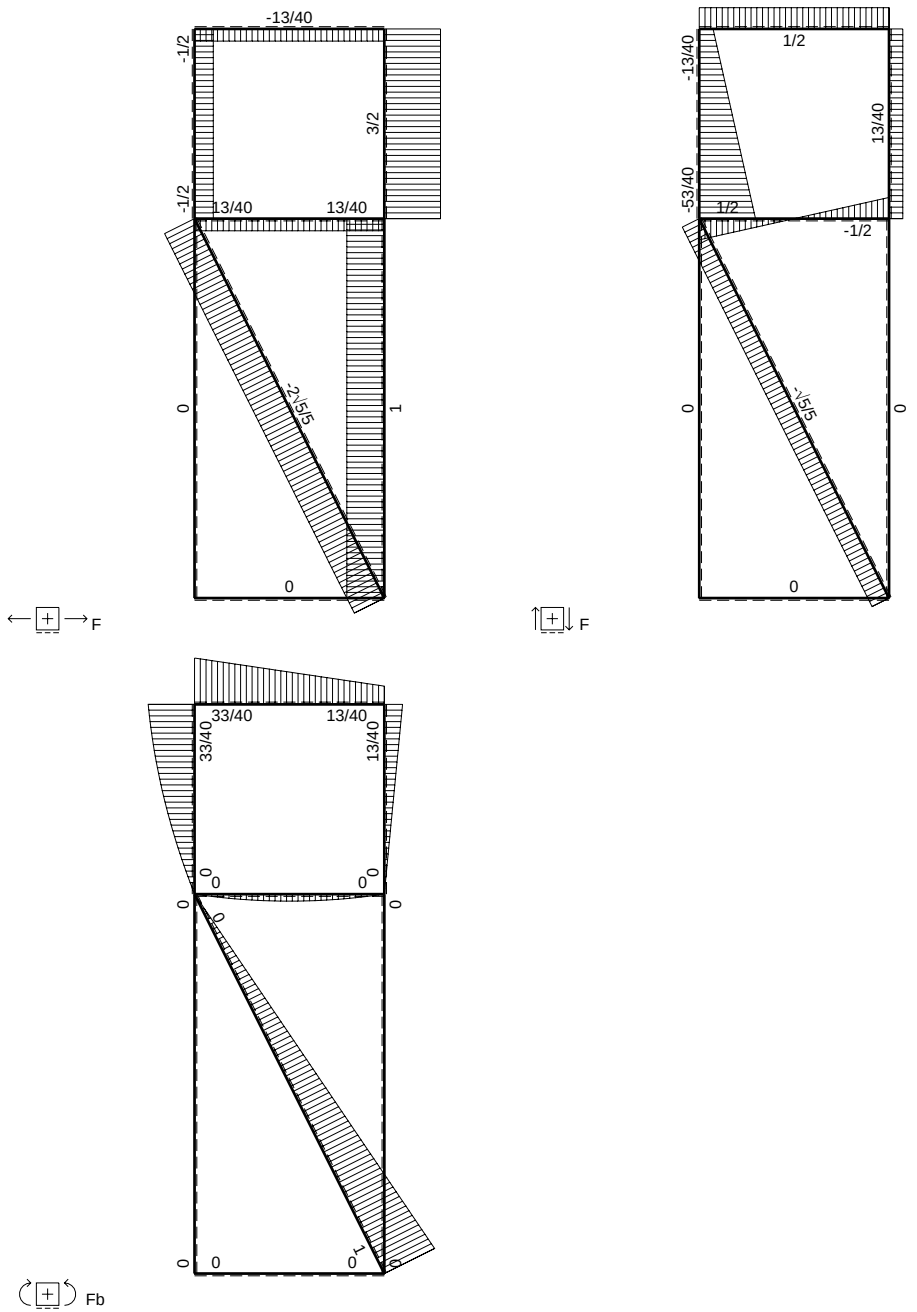
A = 450. mm^2
J_x = 148960. mm^4
J_y = 13204. mm^4
J_{xy} = -28128. mm^4
J_u = 154557. mm^4
J_v = 7607. mm^4
$\alpha = artg(2J_{xy}/(J_y - J_x))/2$ = .1964
$c = \cos\alpha$ = .9808
$s = \sin\alpha$ = .1952
x_g = 15.24 mm
y_g = 25.94 mm
N = -1145. N
T_y = -572.4 N
M_x = 614400. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 7.988 mm
v_m = 26. mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.8 N/mm^2

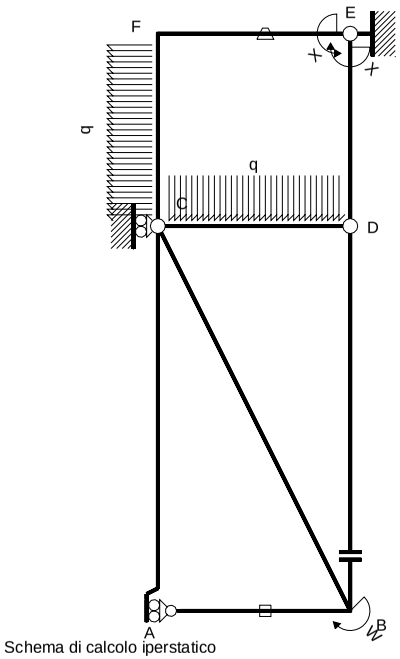
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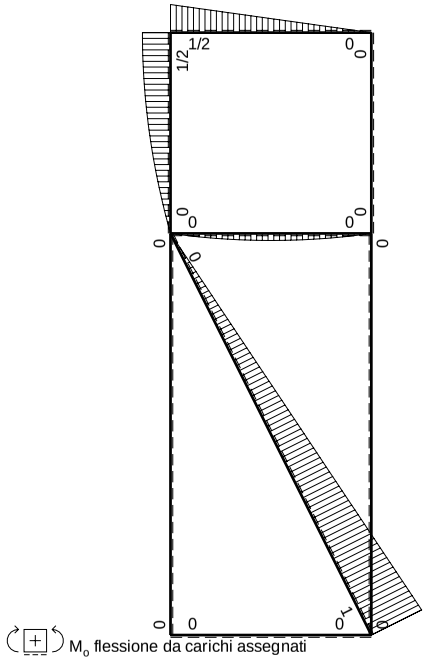
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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
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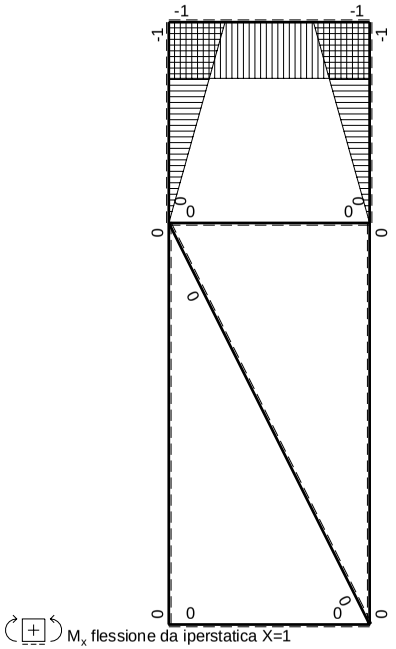

-13/40
-1/2
-1/2
3/2
13/40
13/40
-2√5/5
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F
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-53/40
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-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

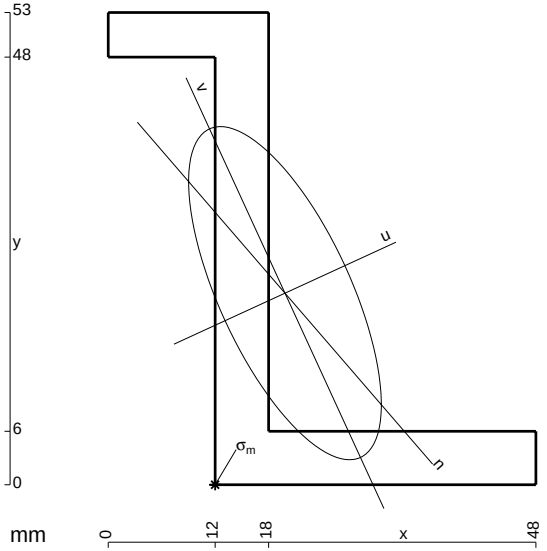
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

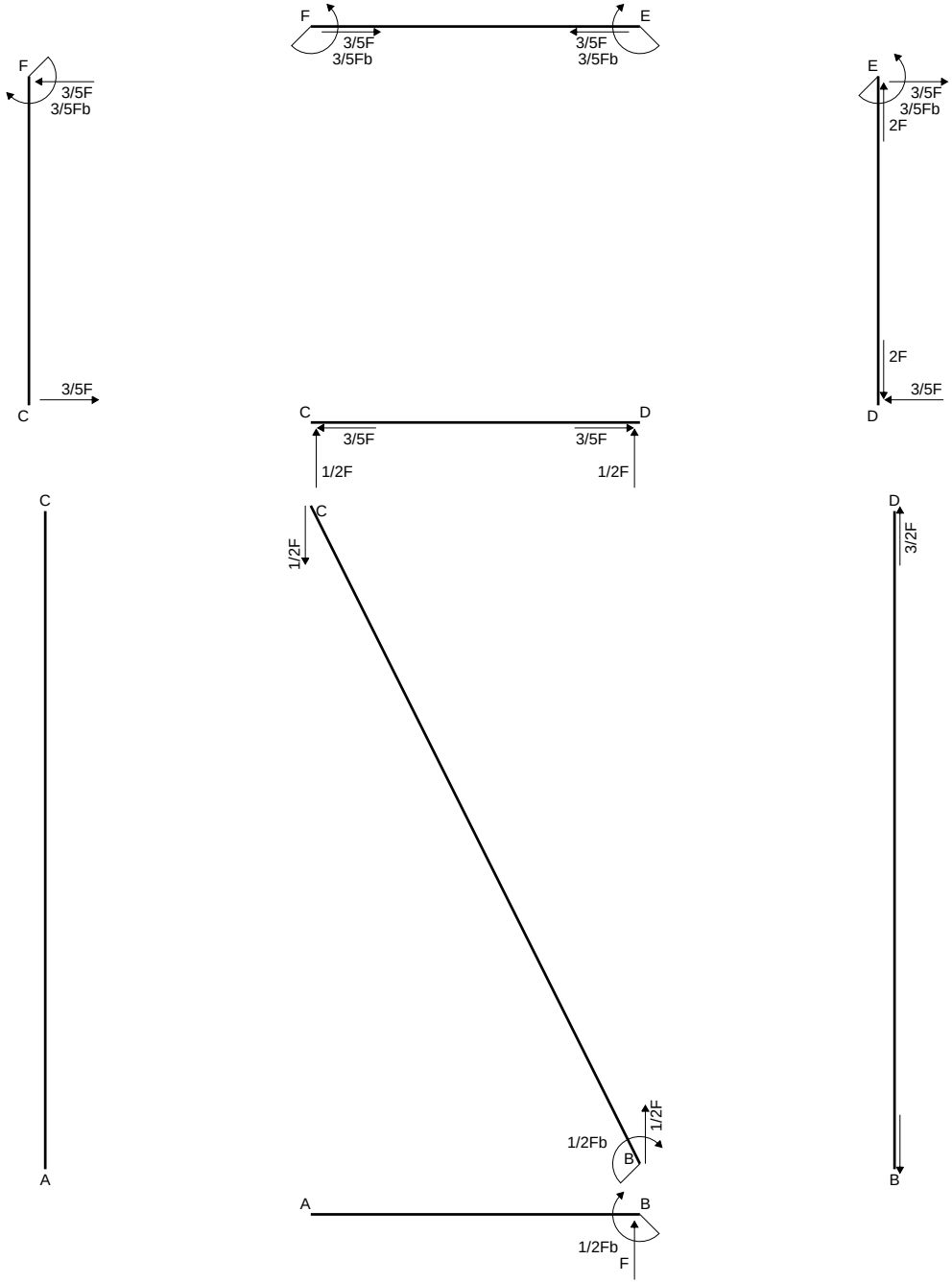
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

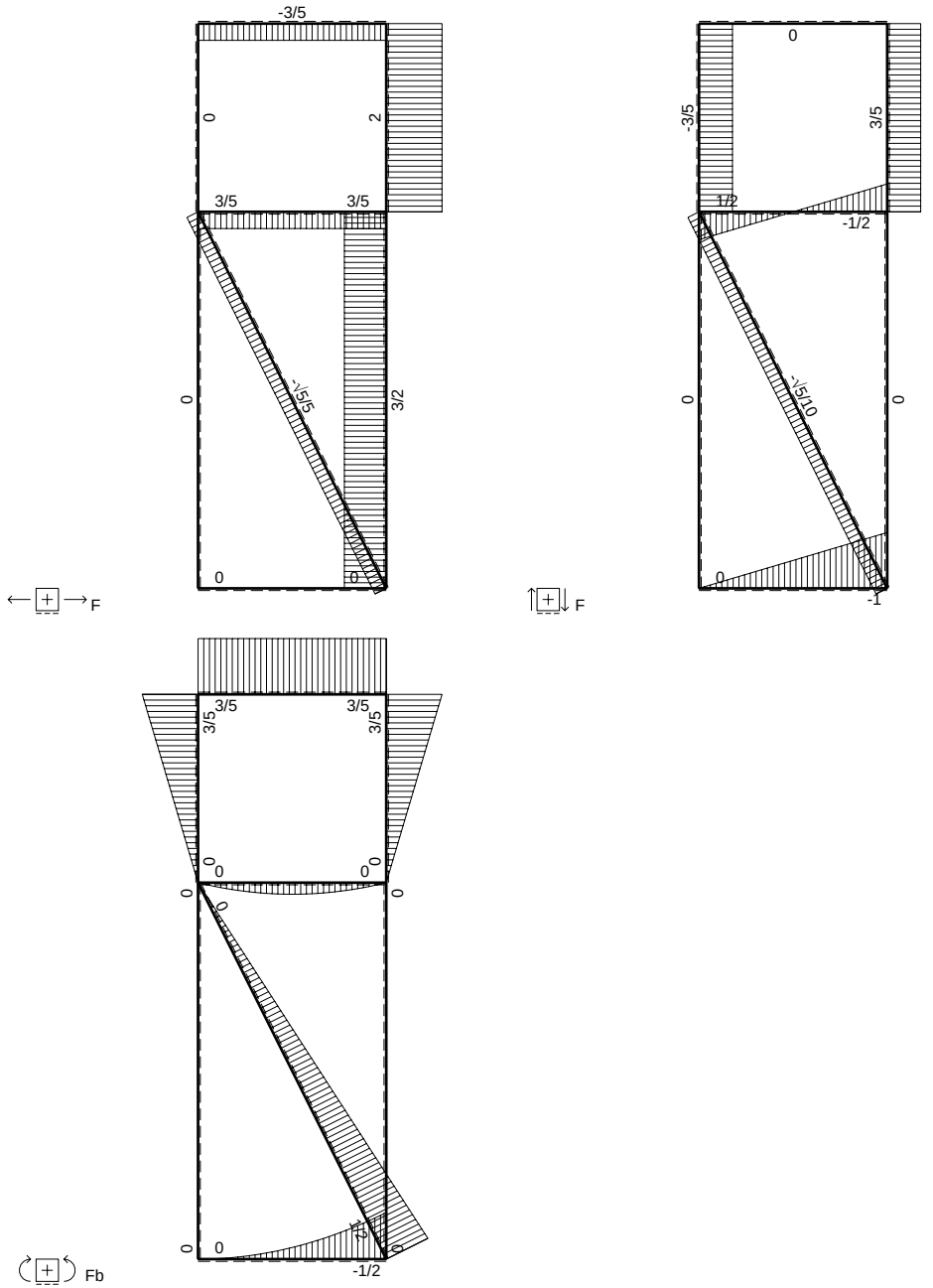


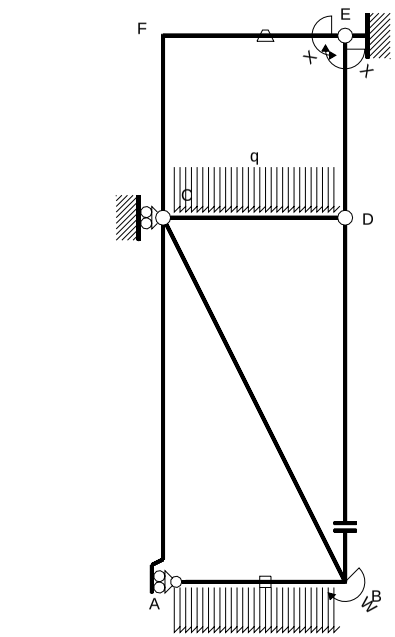
A = 558. mm^2
J_x = 195119. mm^4
J_y = 65290. mm^4
J_{xy} = -75600. mm^4
J_u = 229850. mm^4
J_v = 30558. mm^4
α= artg($2J_{xy}/(J_y-J_x)$)/2 = .4306
c = cosα = .9087
s = sinα = .4175
x_g = 19.84 mm
y_g = 21.5 mm
N = -1494. N
T_y = -746.8 N
M_x = 851700. Nmm
x_m = 12. mm
u_m = -16.1 mm
v_m = -16.26 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.4 N/mm^2

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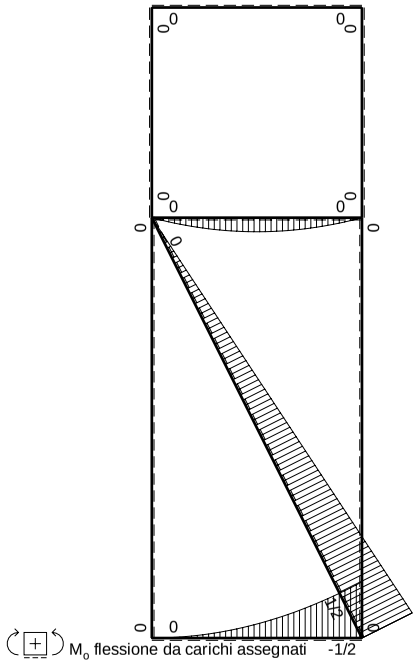
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
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B
A
B
1/2Fb
F

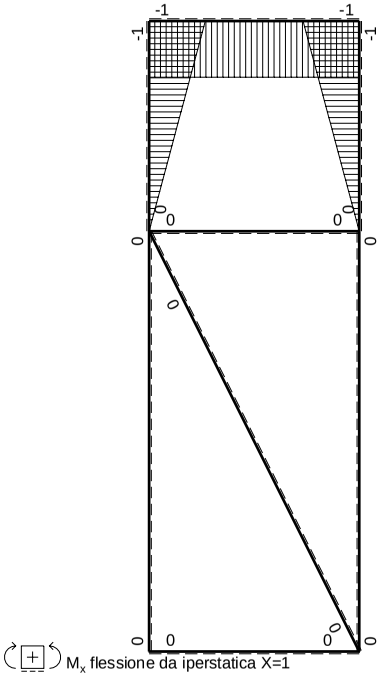

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3/5
3/5
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3/2
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0
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3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
0
0
-√2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

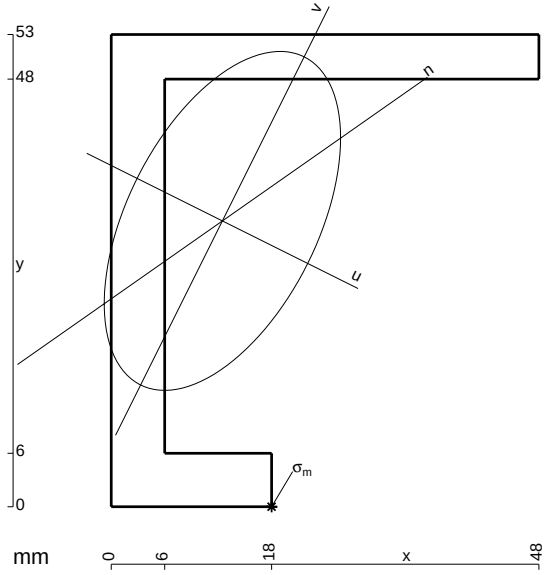
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

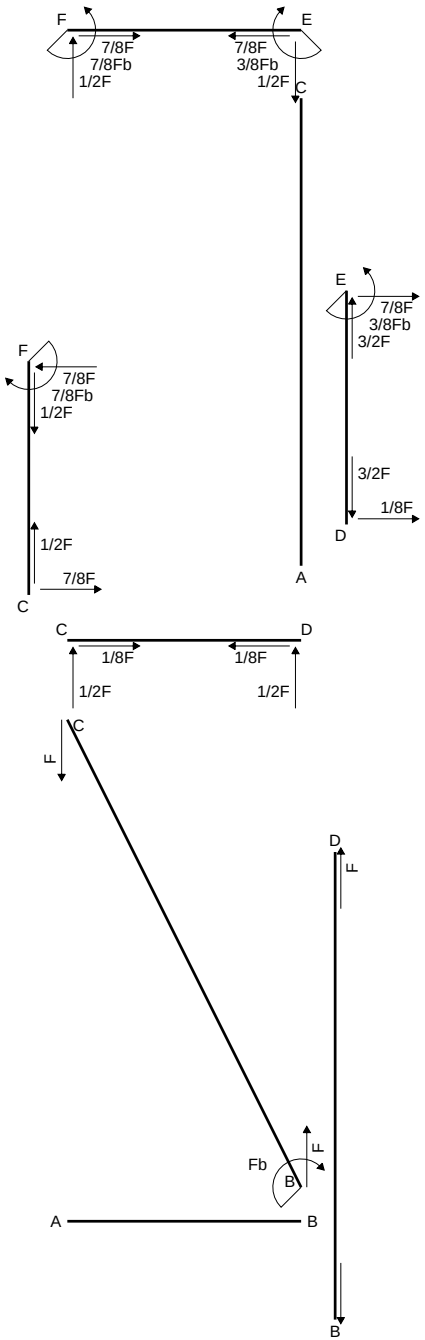
$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

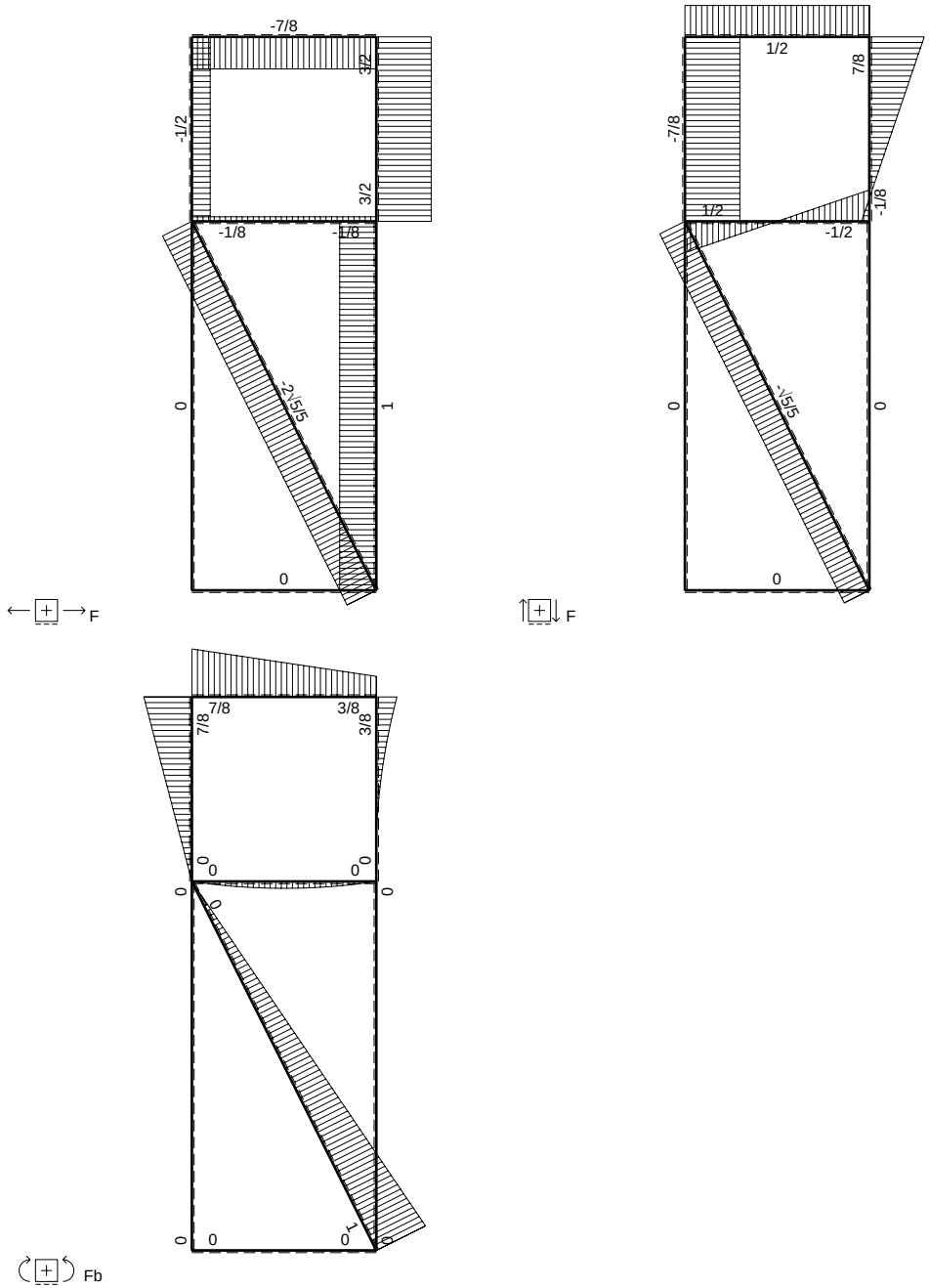


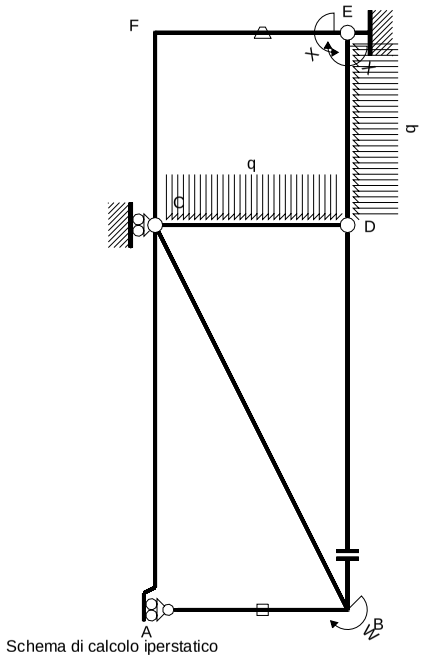
A = 600. mm^2
J_x = 217132. mm^4
J_y = 105558. mm^4
J_{xy} = 73993. mm^4
J_u = 254012. mm^4
J_v = 68678. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4624
$c = \cos\alpha$ = .8950
$s = \sin\alpha$ = -.4461
x_g = 12.48 mm
y_g = 32.08 mm
N = 5320. N
T_y = 1596. N
M_x = 877800. Nmm
x_m = 18. mm
u_m = 19.25 mm
v_m = -26.25 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 199.8 N/mm^2

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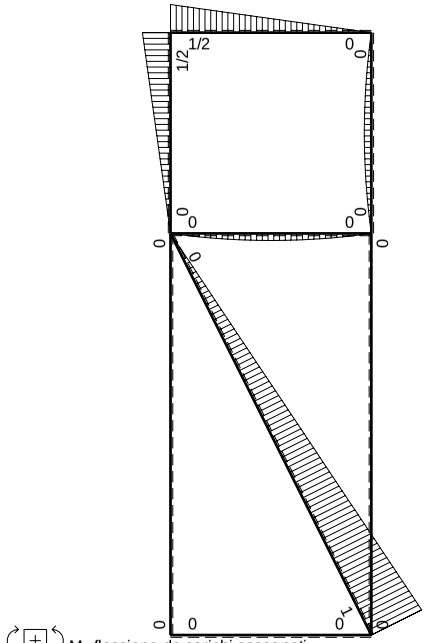
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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
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B

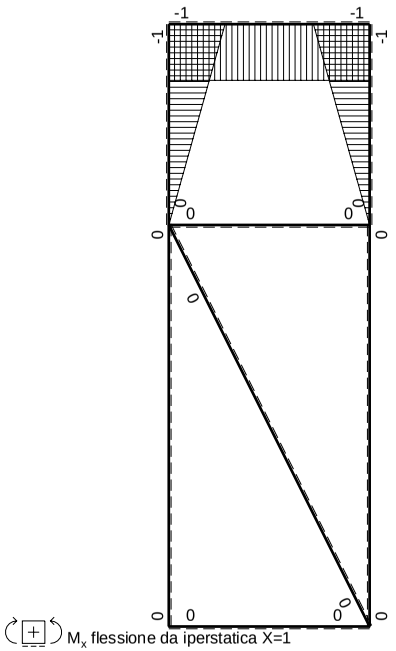

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-7/8
-1/8
1/2
-1/2
-√5/5
0
0
F
7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					5/8Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 265892 CI09.xxxx.046

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

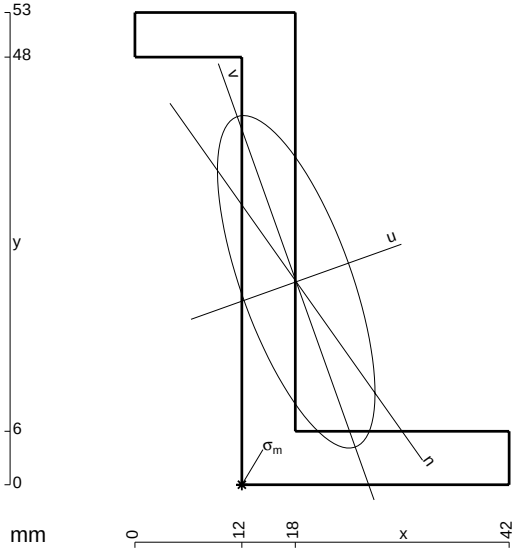
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

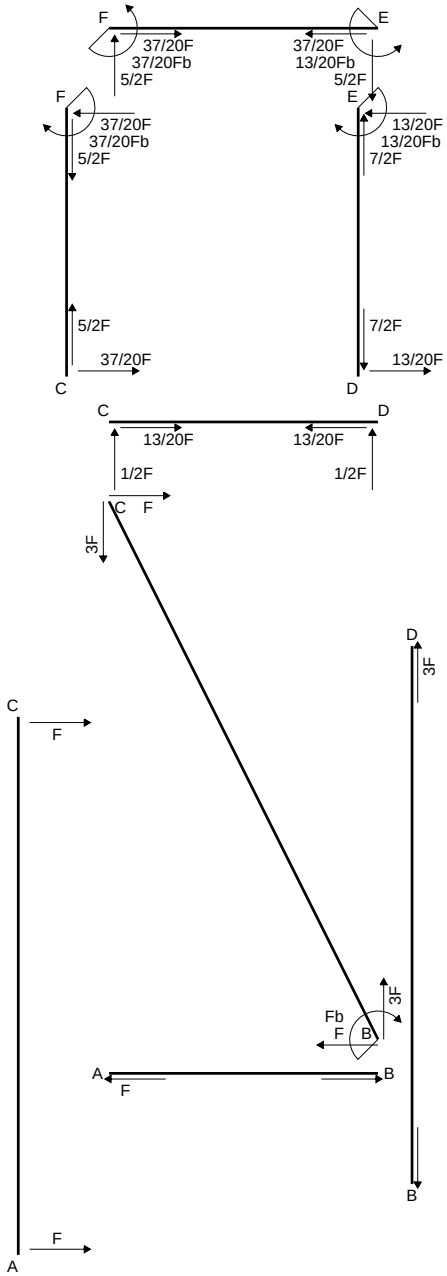
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

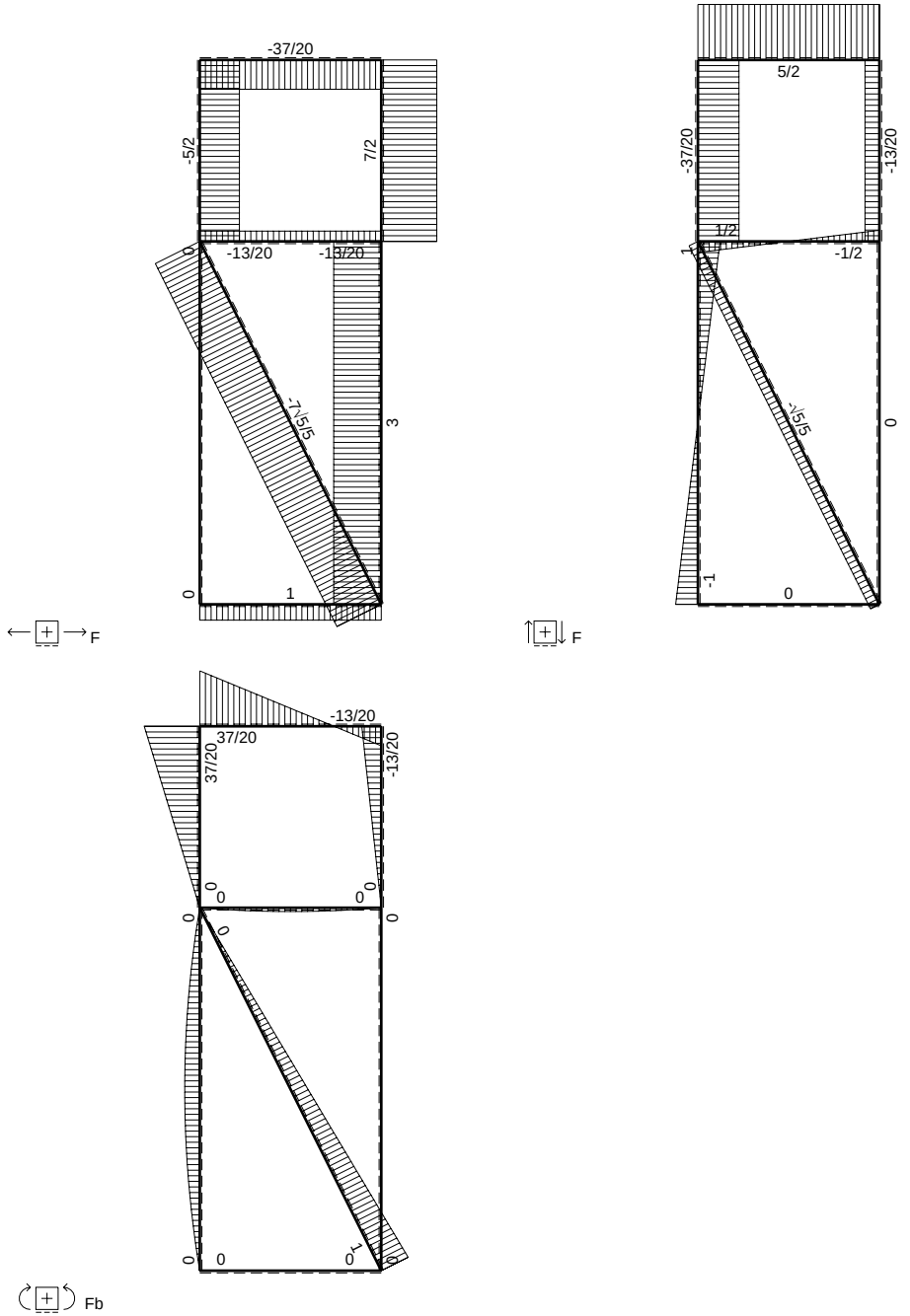


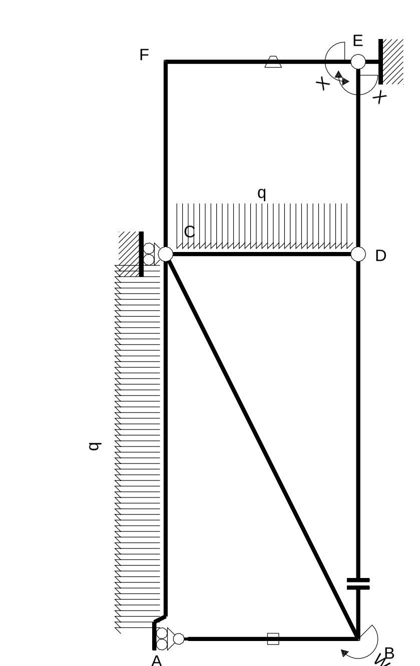
A = 522. mm^2
J_x = 181840. mm^4
J_y = 40818. mm^4
J_{xy} = -57687. mm^4
J_u = 202431. mm^4
J_v = 20227. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3428
c = cosα = .9418
s = sinα = .3362
x_g = 18.1 mm
y_g = 22.78 mm
N = -1020. N
T_y = -509.8 N
M_x = 672600. Nmm
x_m = 12. mm
u_m = -13.4 mm
v_m = -19.4 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 208.6 N/mm^2

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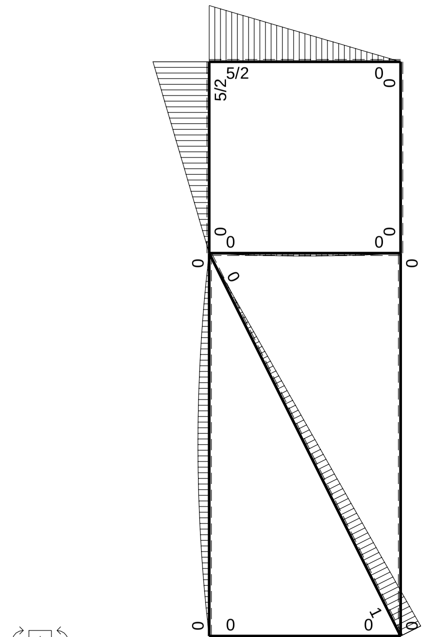
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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
7/2F
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13/20F
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B

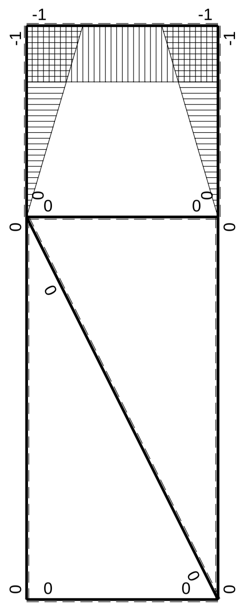

-37/20
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7/2
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0
-7/5/5
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F
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1/2
-1/2
-7/5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

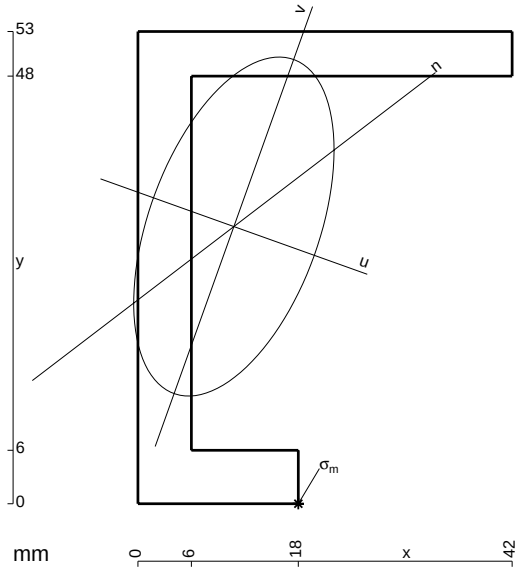
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

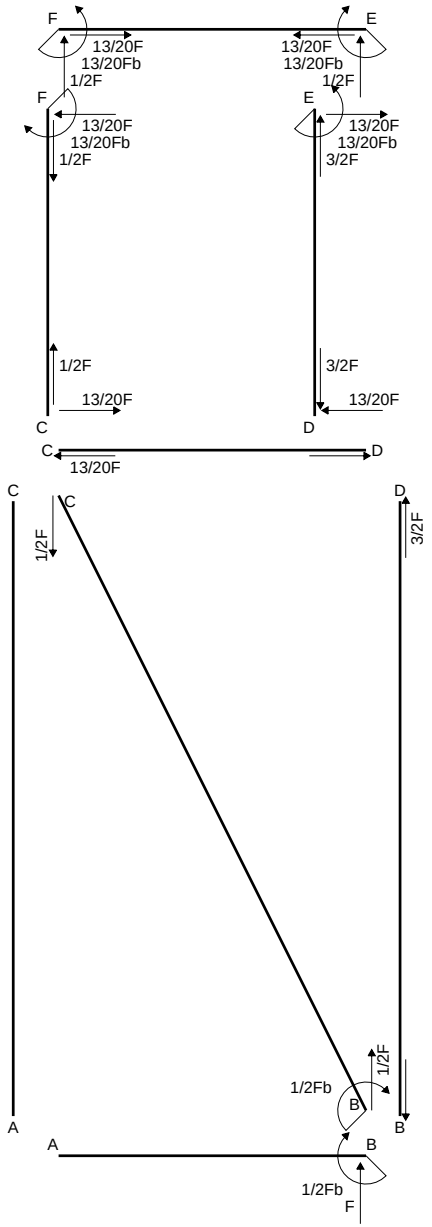
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

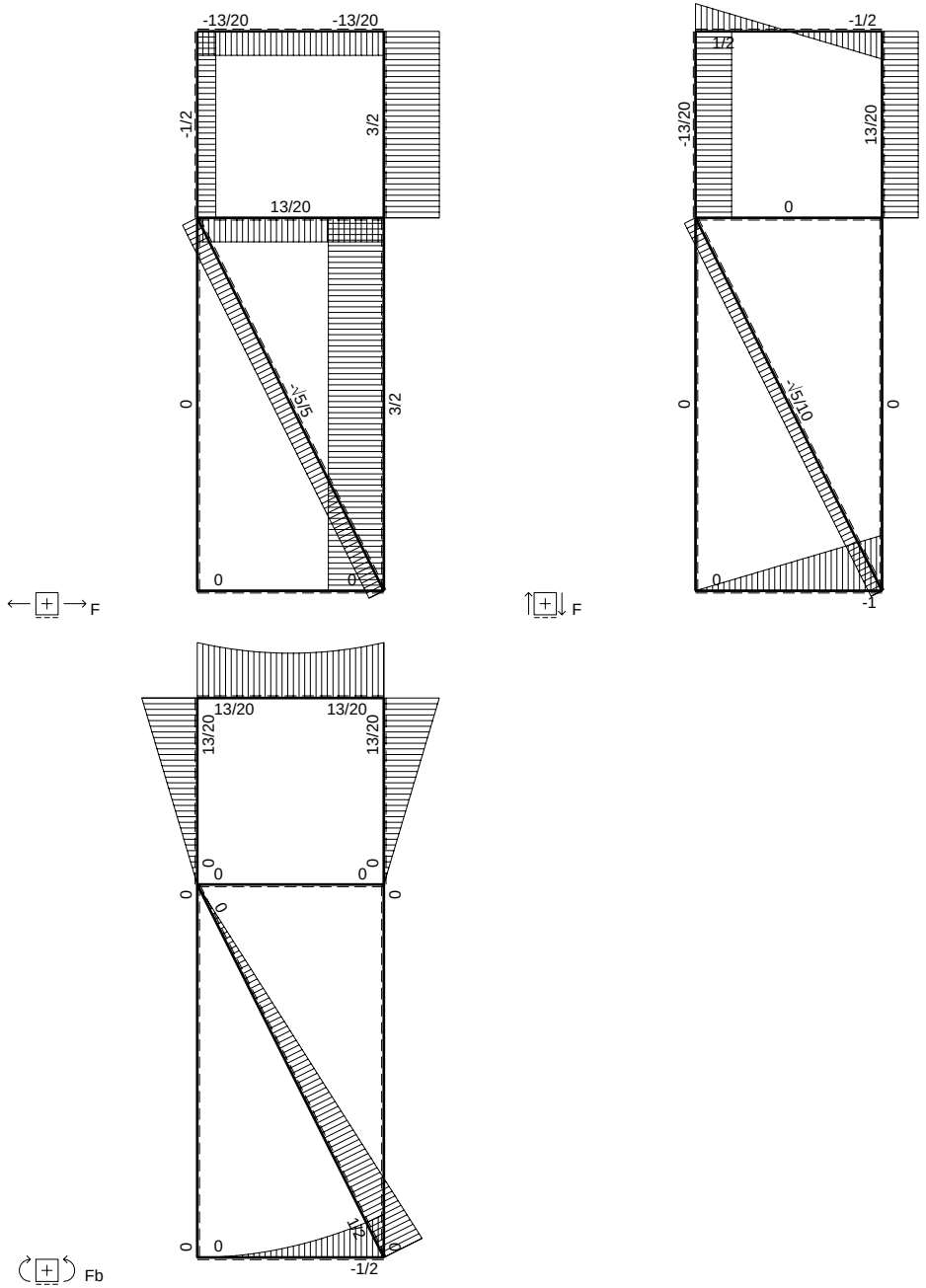


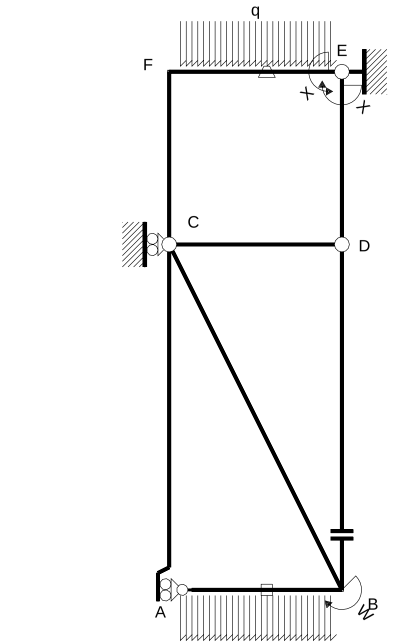
A = 570. mm^2
J_x = 206355. mm^4
J_y = 72071. mm^4
J_{xy} = 55077. mm^4
J_u = 226055. mm^4
J_v = 52372. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3435
c = cosα = .9416
s = sinα = -.3368
x_g = 10.77 mm
y_g = 31.11 mm
N = -1610. N
T_y = 2175. N
M_x = 997890. Nmm
x_m = 18. mm
u_m = 17.29 mm
v_m = -26.86 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 219.7 N/mm^2

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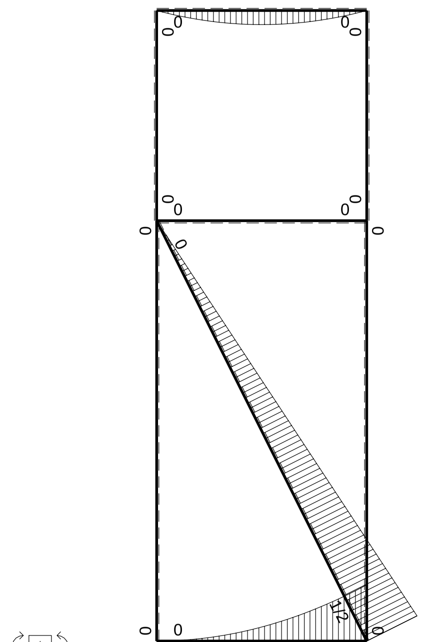
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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

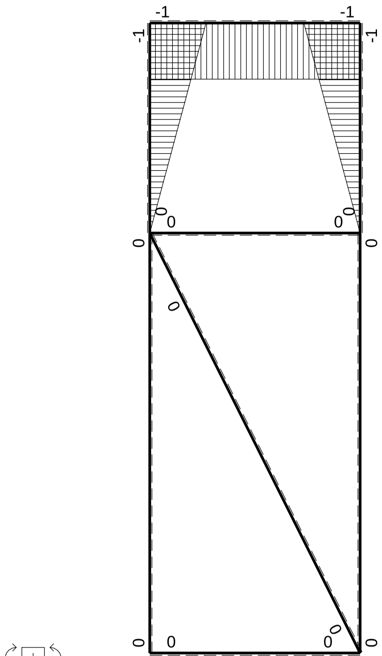

-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

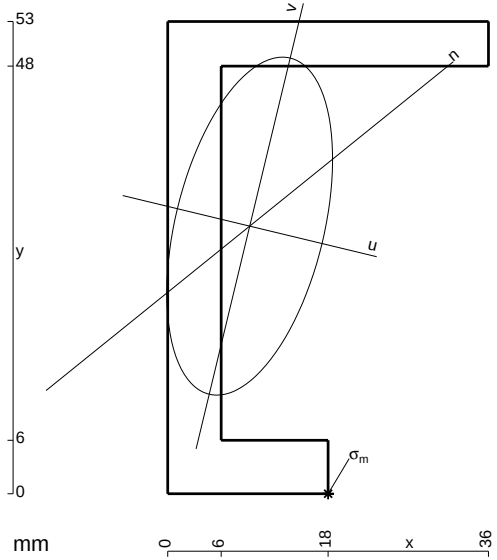
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

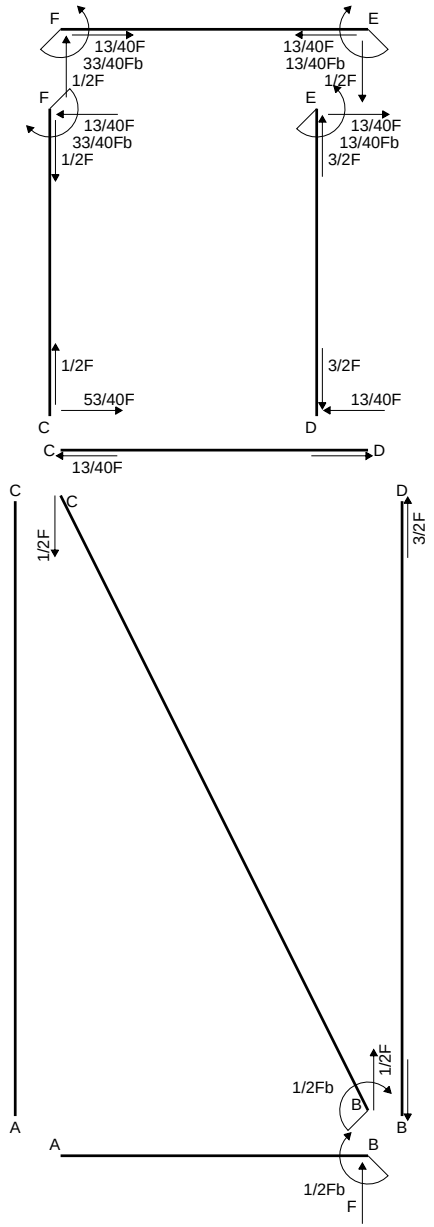
$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

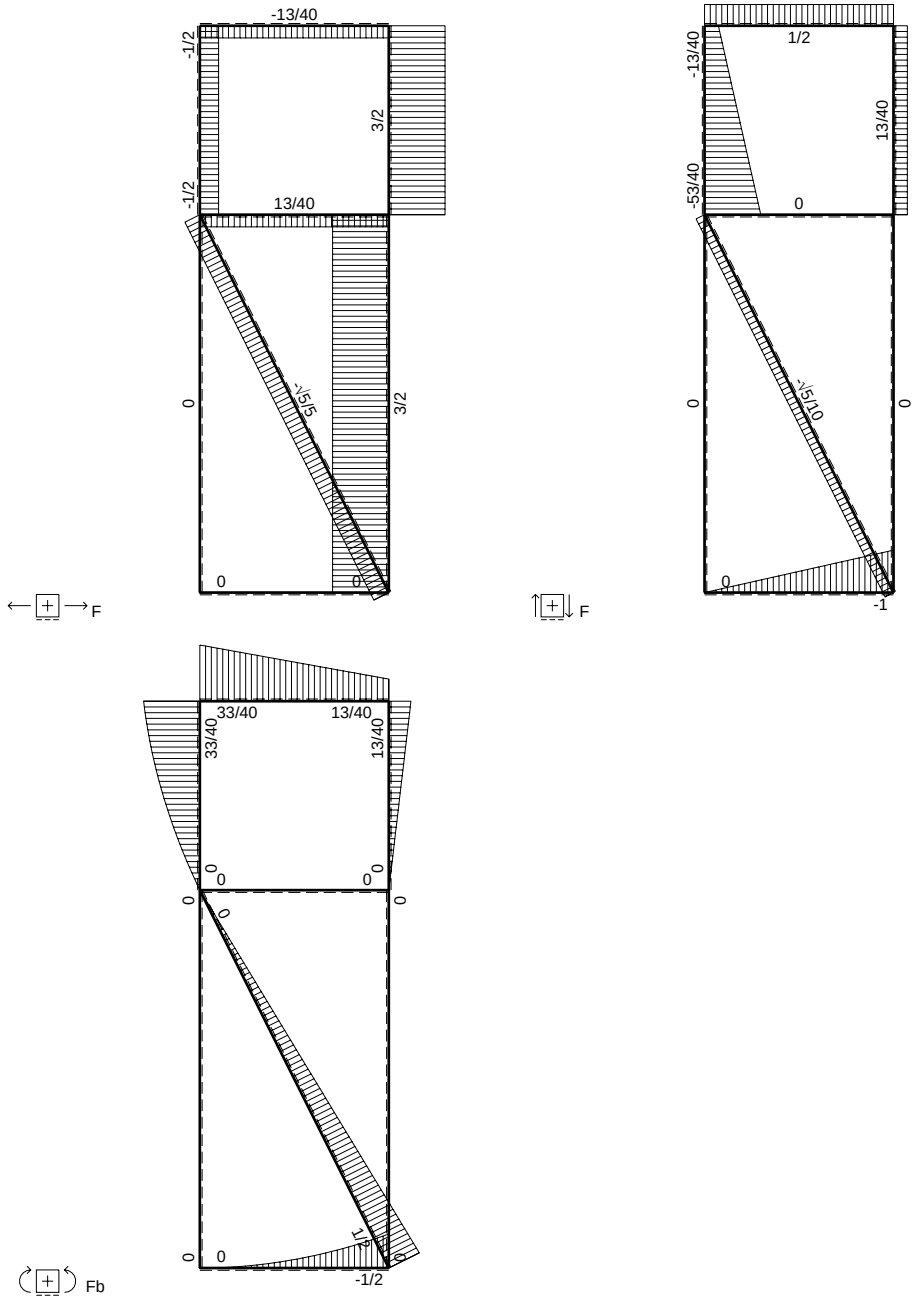


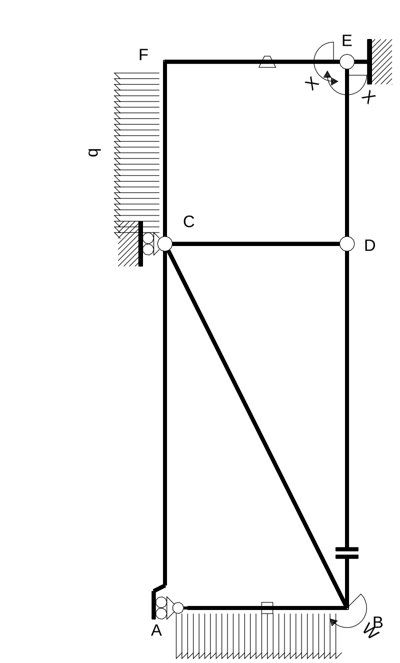
A = 540. mm^2
J_x = 194387. mm^4
J_y = 46742. mm^4
J_{xy} = 37742. mm^4
J_u = 203476. mm^4
J_v = 37654. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y - J_x))/2$ = -.2363
c = cosα = .9722
s = sinα = -.2341
x_g = 9.2 mm
y_g = 30.03 mm
N = 3450. N
T_y = 1495. N
M_x = 986700. Nmm
x_m = 18. mm
u_m = 15.59 mm
v_m = -27.14 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 230. N/mm^2

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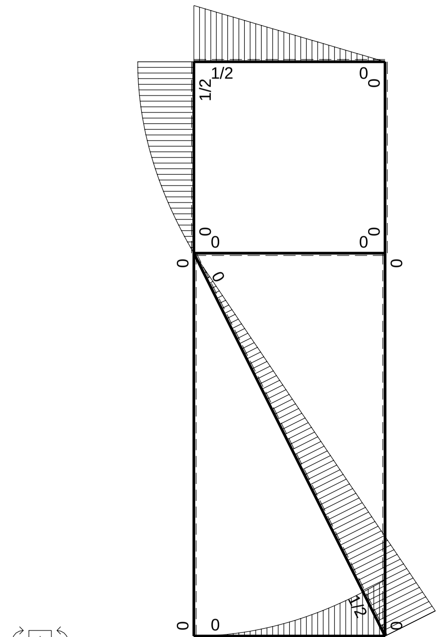
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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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B
A
B
1/2Fb
F

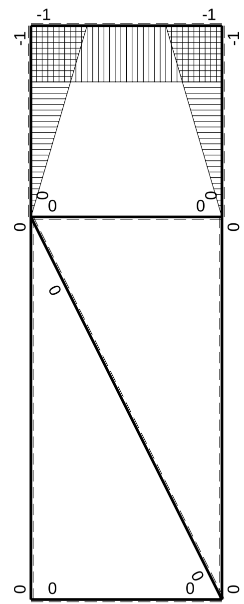

-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
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13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	-Fb/EJ	$-1/2Fx$	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

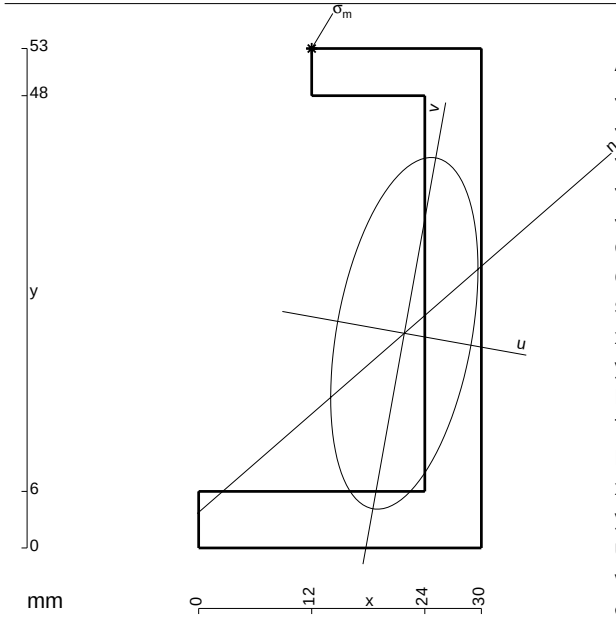
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 265373 CI09.xxxx.049

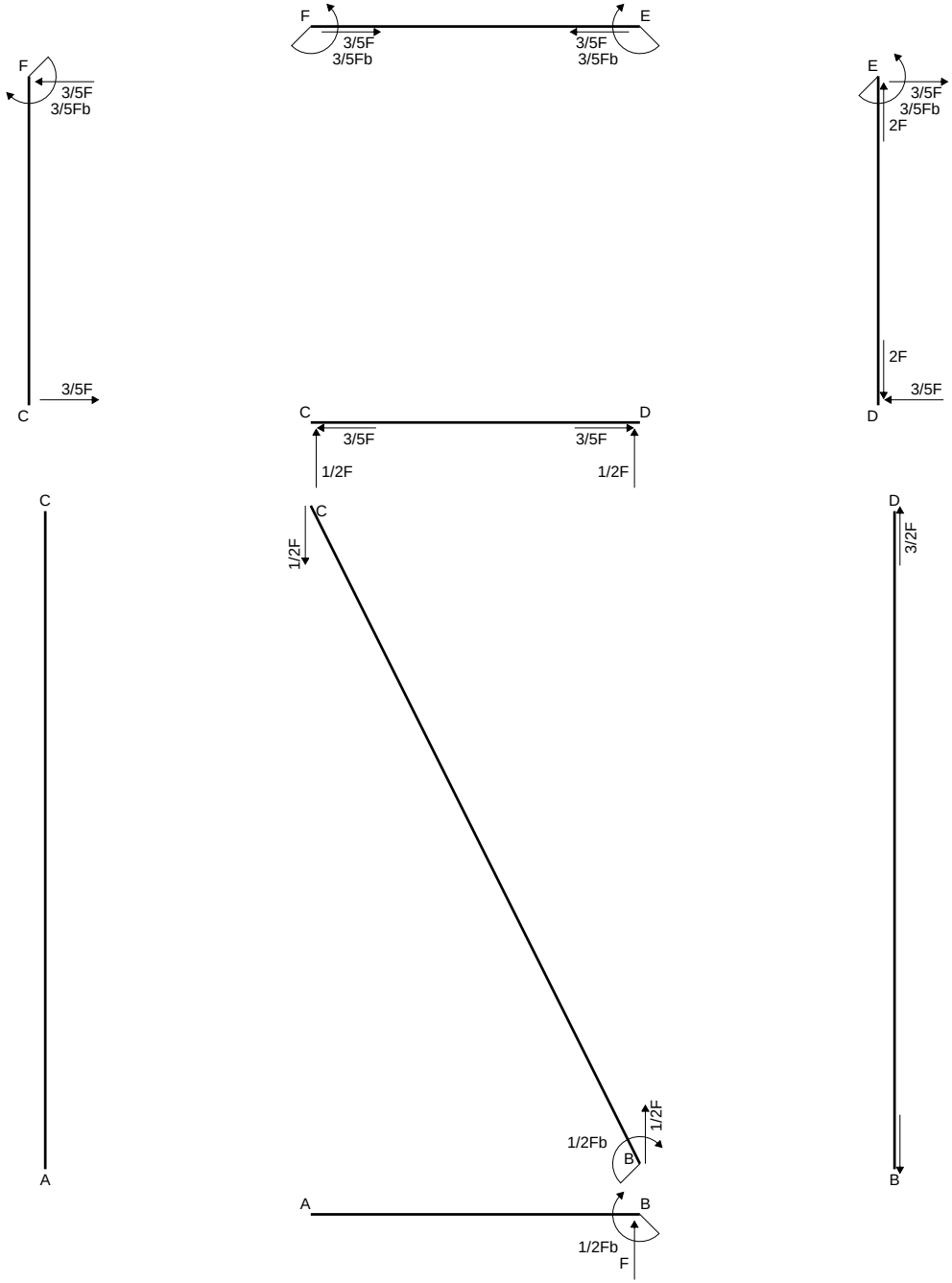


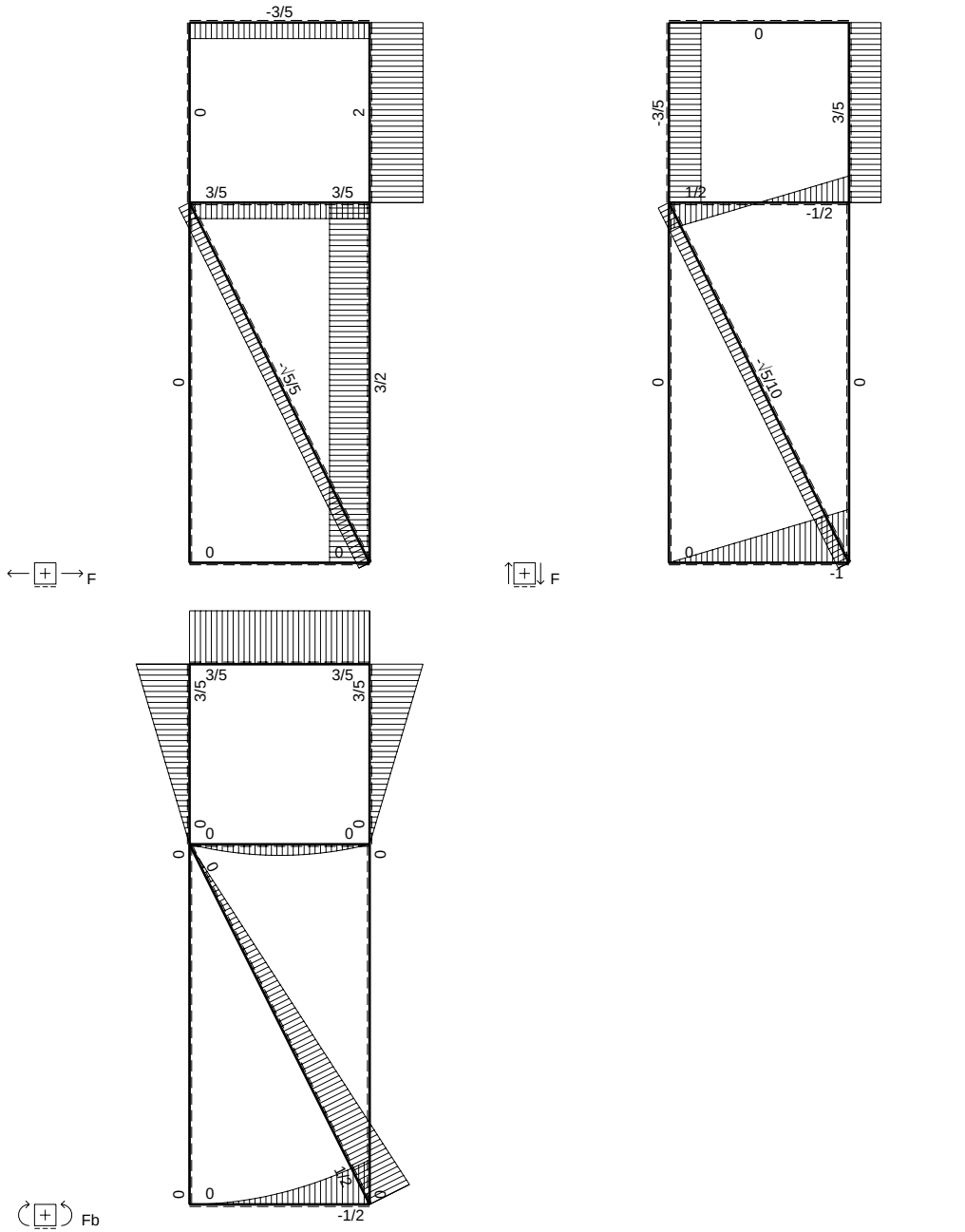
A = 522. mm^2
J_x = 181840. mm^4
J_y = 31881. mm^4
J_{xy} = 27745. mm^4
J_u = 186808. mm^4
J_v = 26912. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1772
c = cosα = .9843
s = sinα = -.1763
x_g = 21.83 mm
y_g = 22.78 mm
N = -835. N
T_y = -542.8 N
M_x = 964425. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -15. mm
v_m = 28.02 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -238.8 N/mm^2

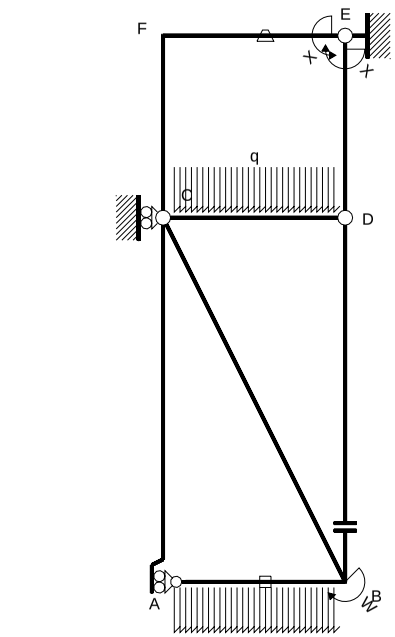
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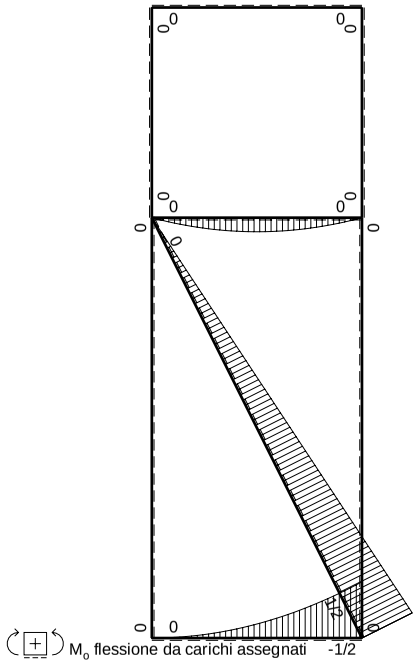
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
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B
A
B
1/2Fb
F

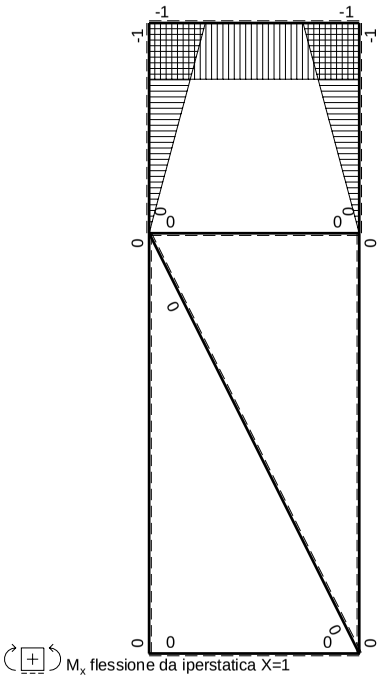

-3/5
0
2
3/5
3/5
0
3/2
-3/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
0
-3/10
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

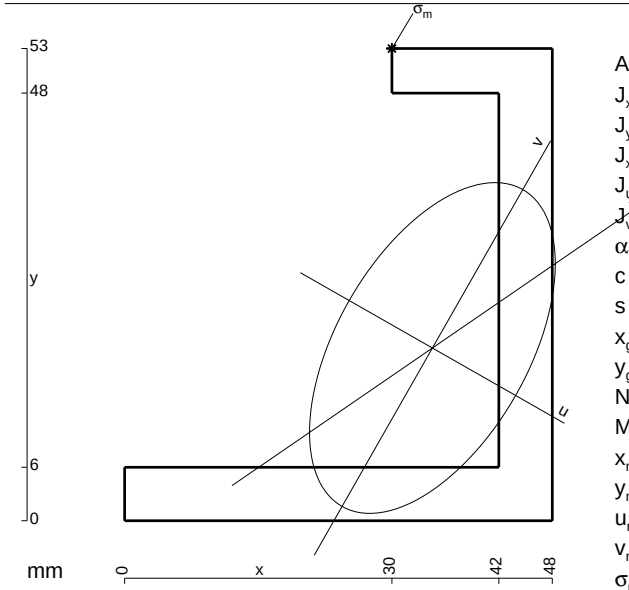
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

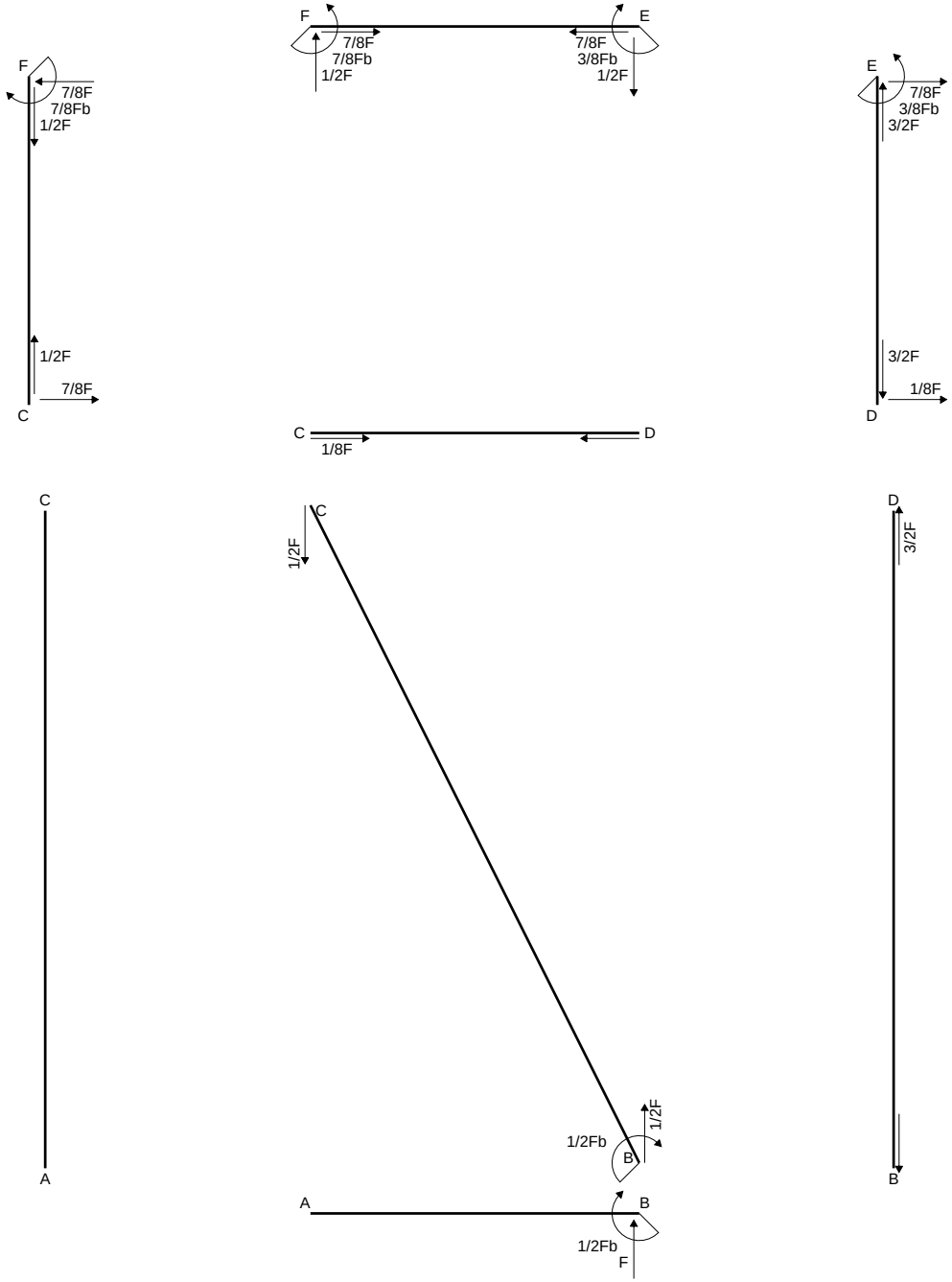
$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

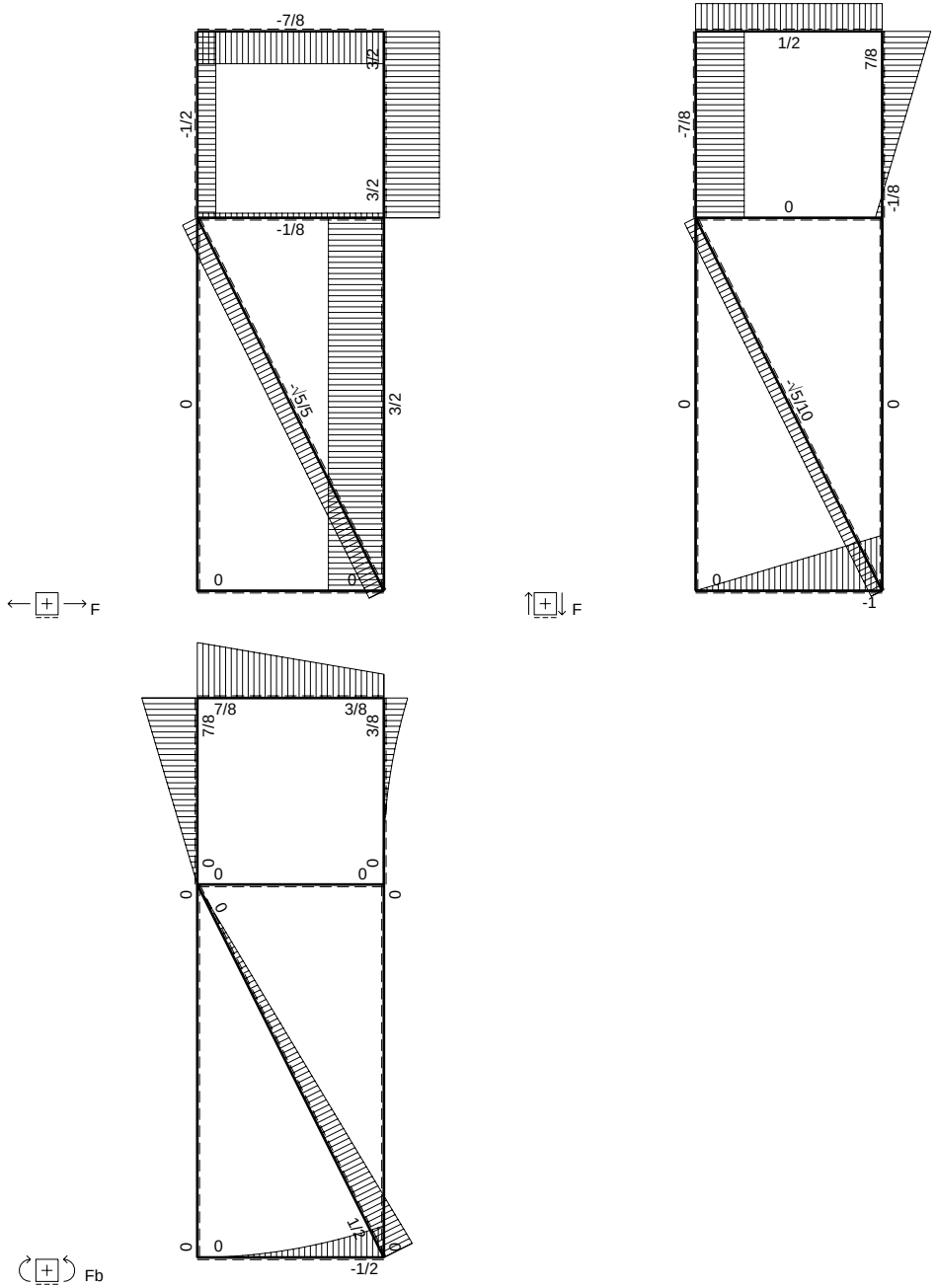


A = 630. mm^2
J_x = 217160. mm^4
J_y = 119838. mm^4
J_{xy} = 82299. mm^4
J_u = 264108. mm^4
J_v = 72891. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.5184
$c = \cos\alpha$ = .8686
$s = \sin\alpha$ = -.4955
x_g = 34.54 mm
y_g = 19.39 mm
N = -2316. N
M_x = 856920. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = -20.6 mm
v_m = 26.95 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -199.6 N/mm^2

CI09.xxxx.050

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F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
3/2F
1/8F
C
D
1/8F
A
B
1/2Fb


-7/8
3/2
-1/2
-1/8
0
-√5/5
F
1/2
7/8
-1/8
-√5/10
-1
7/8
3/8
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	$-Fb/EJ$	$-1/2Fx$	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

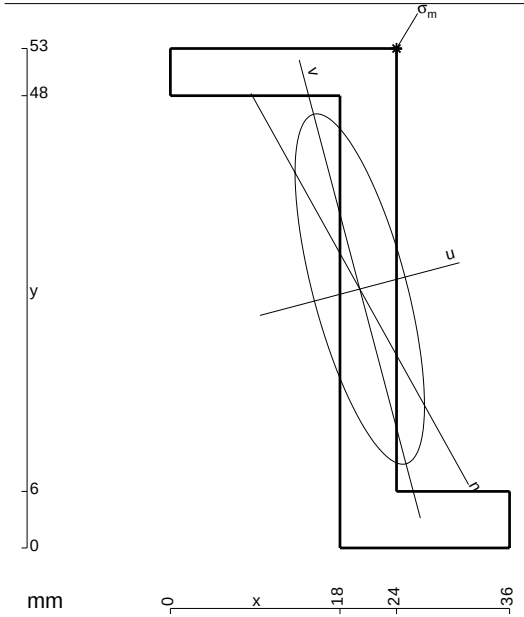
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

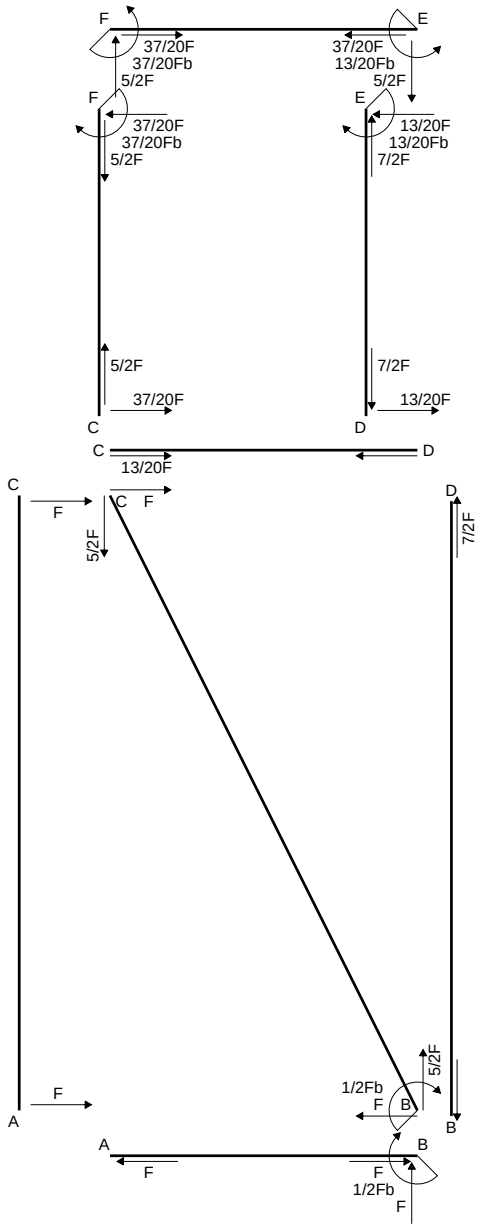
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

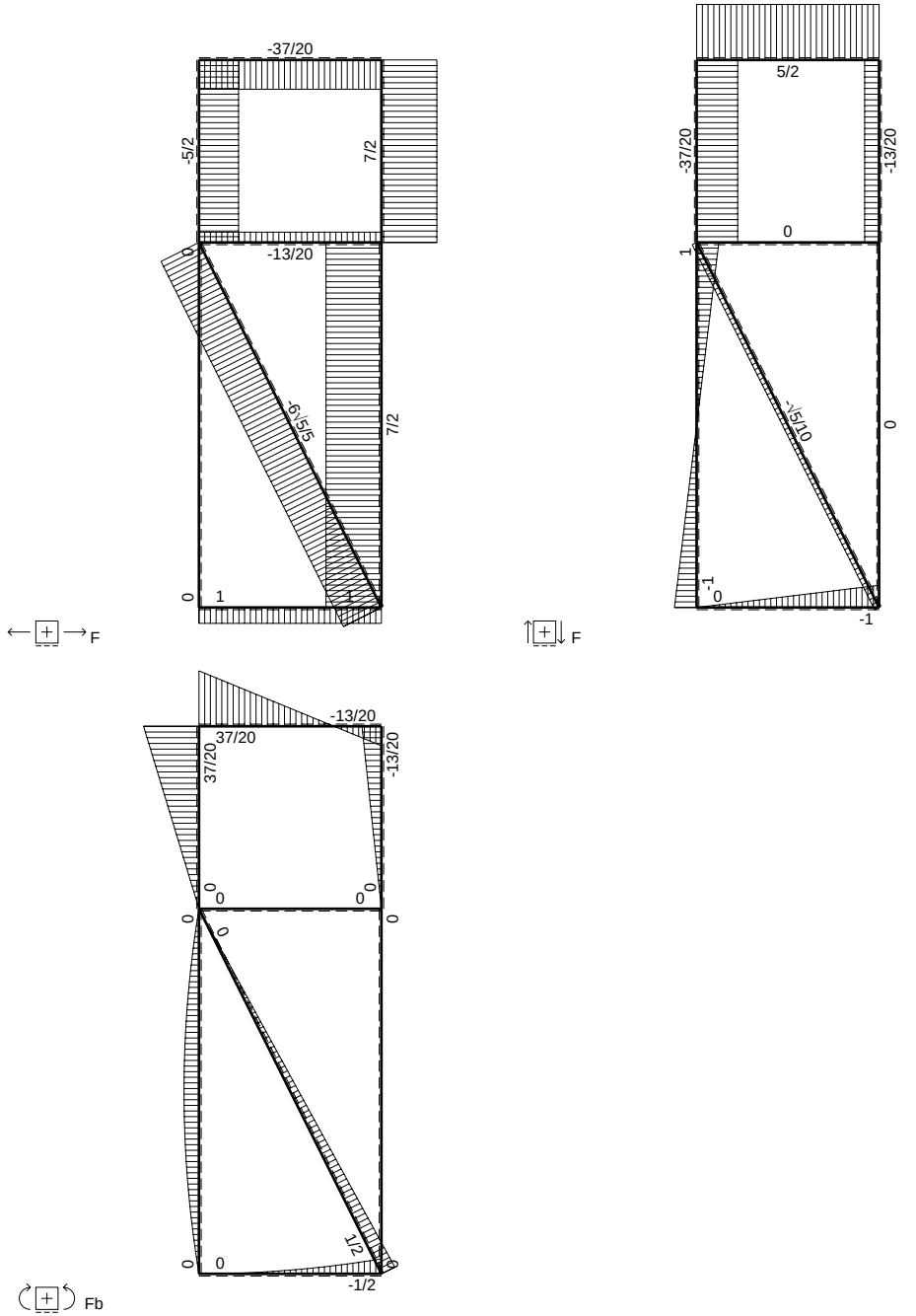


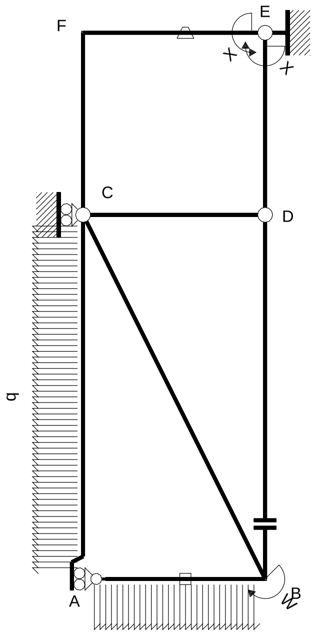
A = 480. mm^2
J_x = 165988. mm^4
J_y = 22651. mm^4
J_{xy} = -40727. mm^4
J_u = 176751. mm^4
J_v = 11888. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2584
c = cosα = .9668
s = sinα = .2555
x_g = 20.1 mm
y_g = 27.48 mm
N = -1470. N
T_y = 840. N
M_x = 588000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 10.29 mm
v_m = 23.68 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.3 N/mm^2

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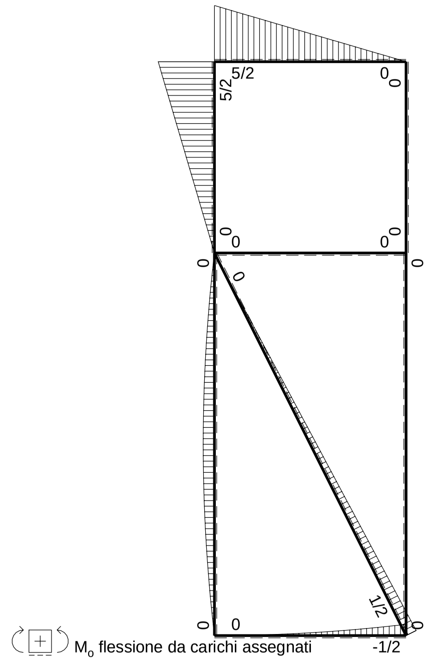
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F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb

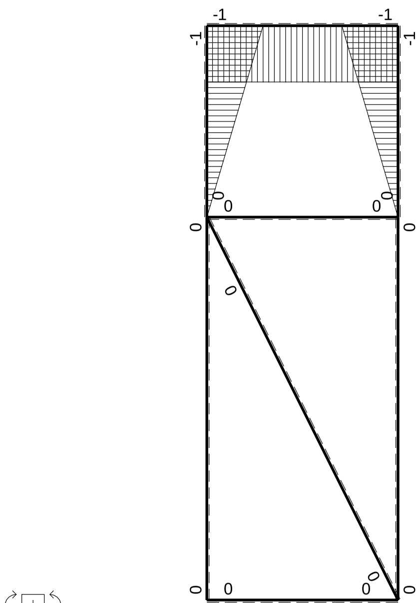

-37/20
-5/2
7/2
-13/20
-5\√5/5
F
5/2
-37/20
-13/20
-√5/10
37/20
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	$-1+x/b$	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

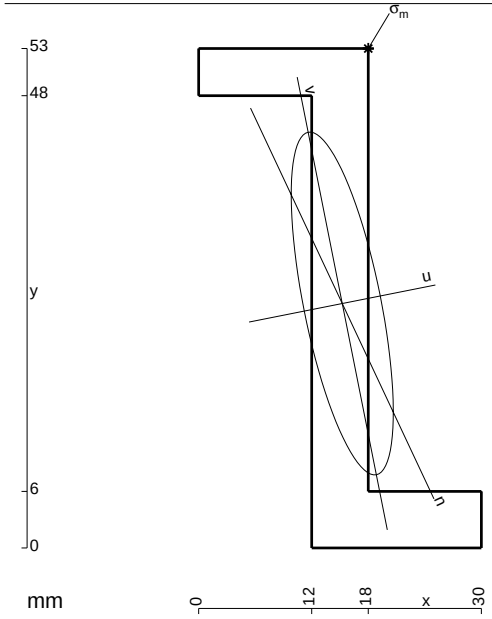
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

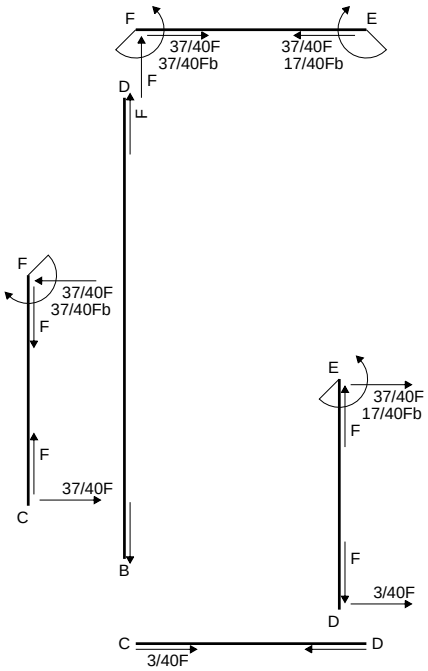
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

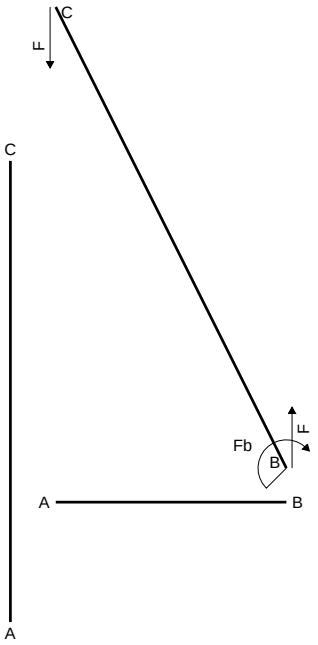


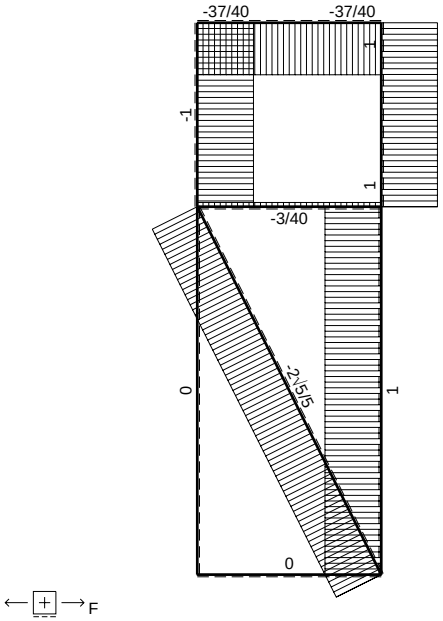
A = 450. mm^2
J_x = 148960. mm^4
J_y = 13204. mm^4
J_{xy} = -28128. mm^4
J_u = 154557. mm^4
J_v = 7607. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1964
$c = \cos\alpha$ = .9808
$s = \sin\alpha$ = .1952
x_g = 15.24 mm
y_g = 25.94 mm
N = -1775. N
T_y = -1314. N
M_x = 577940. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 7.988 mm
v_m = 26. mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -217.7 N/mm^2

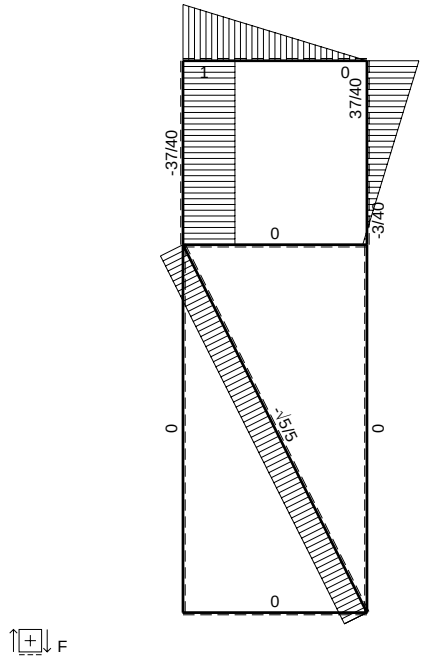
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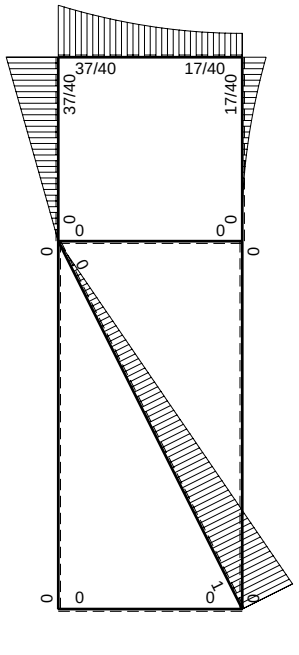
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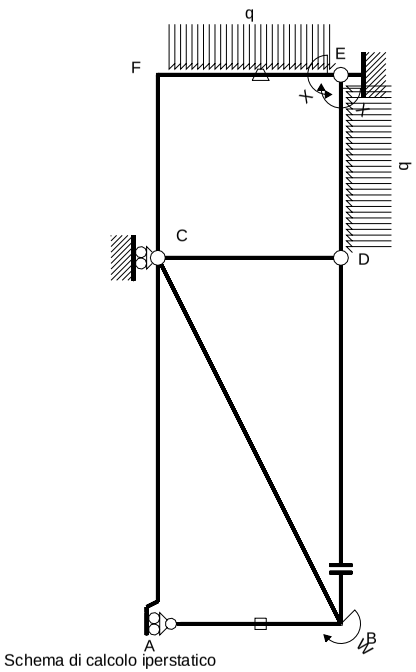

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
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D


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Fb
F
B
A
B
A

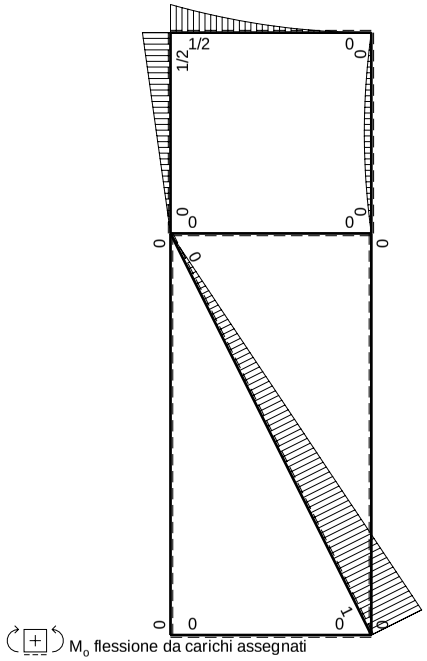

-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
1
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
0
F

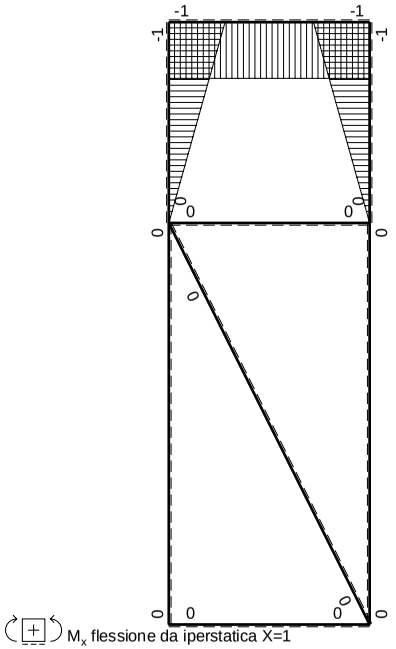

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
				totali			17/24Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 267850 CI09.xxxx.053

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

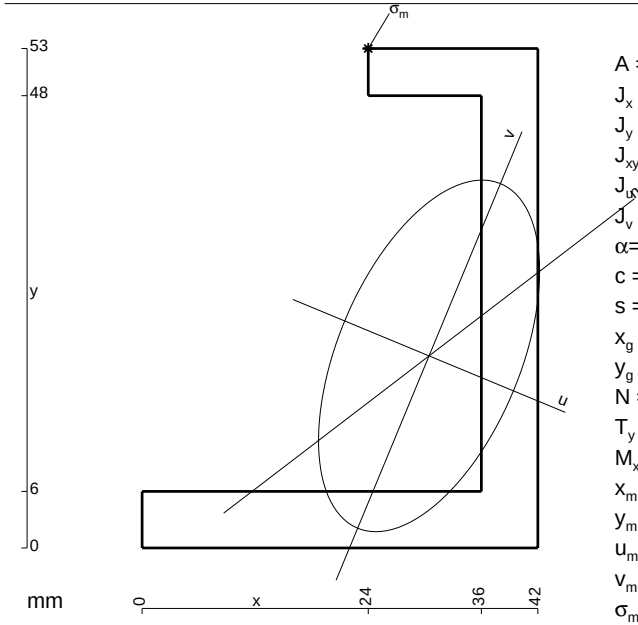
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

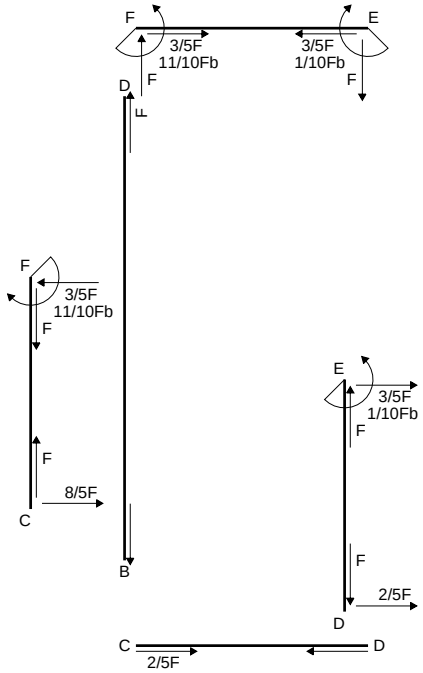
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

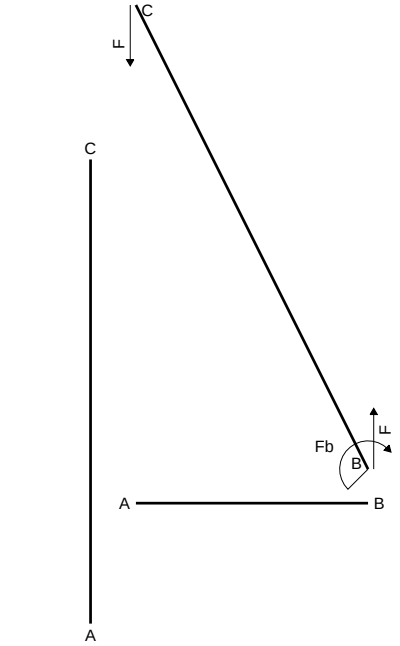


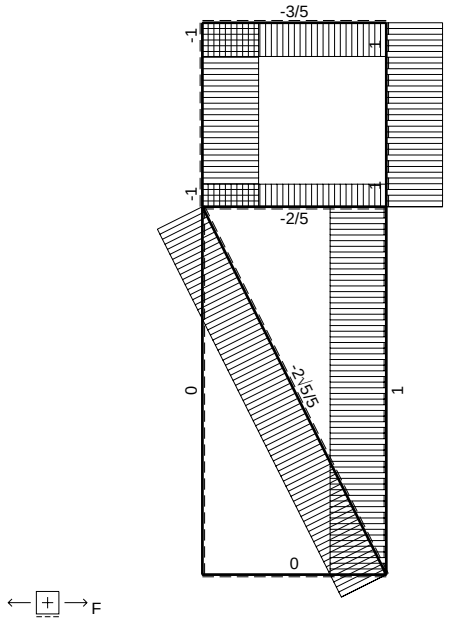
A = 594. mm^2
J_x = 206801. mm^4
J_y = 81741. mm^4
J_{xy} = 62565. mm^4
J_u = 232726. mm^4
J_v = 55816. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3928
c = cosα = .9238
s = sinα = -.3828
x_g = 30.45 mm
y_g = 20.38 mm
N = -1789. N
T_y = -894.4 N
M_x = 960000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = -18.45 mm
v_m = 27.67 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.9 N/mm^2

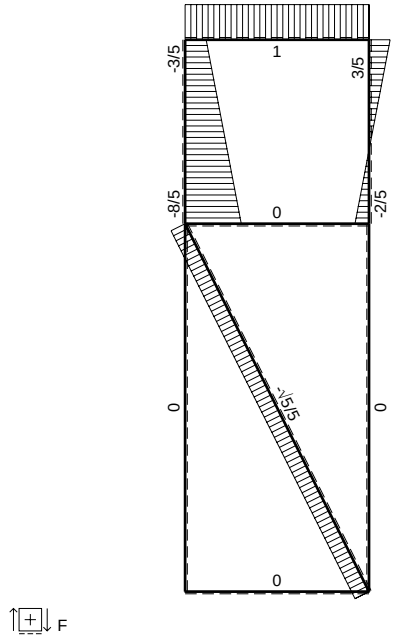
CI09.xxxx.053

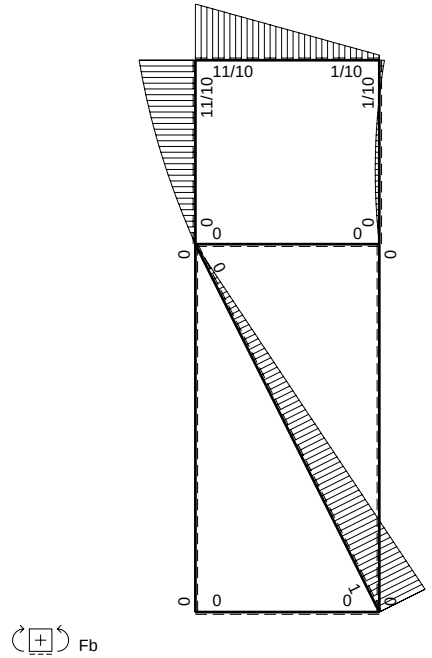
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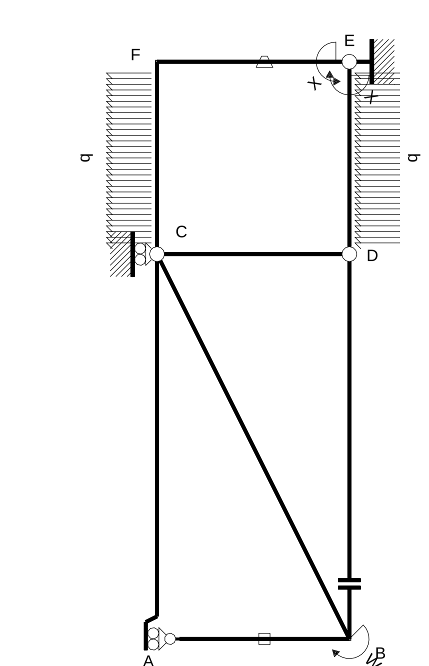

F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A

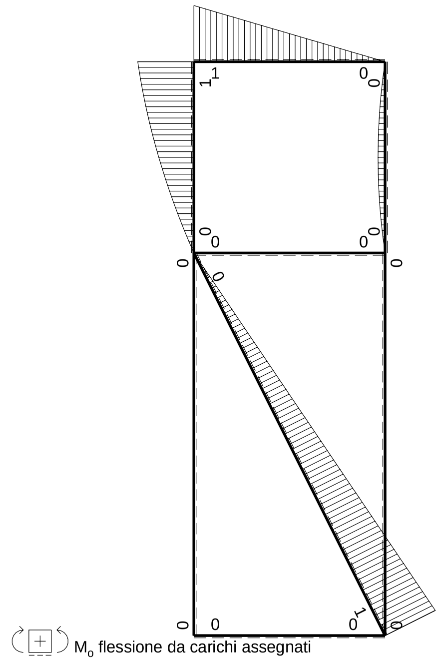

-3/5
-1
1
-1
1
-2/5
0
-2/5
1
0
F

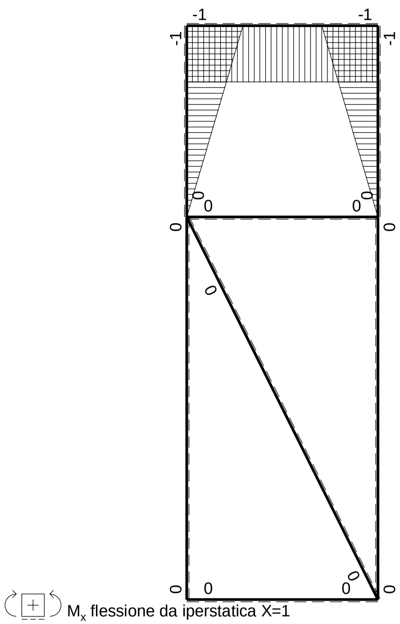

-3/5
1
3/5
-8/5
0
-2/5
0
-2/5
0
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico


M_o flessione da carichi assegnati


M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 267178 CI09.xxxx.054

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$1/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

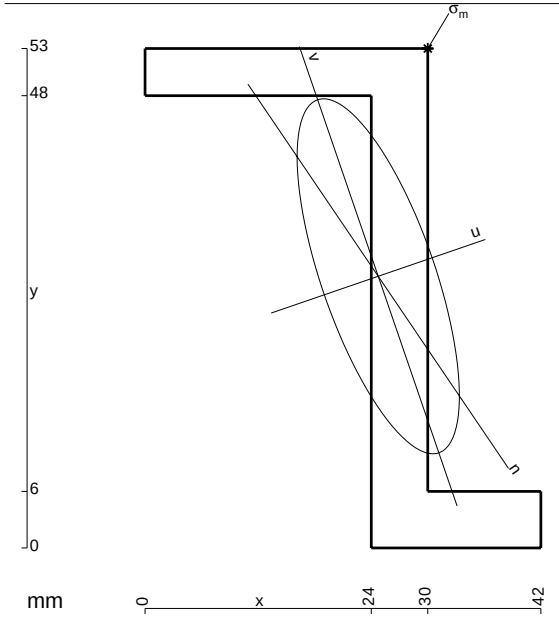
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

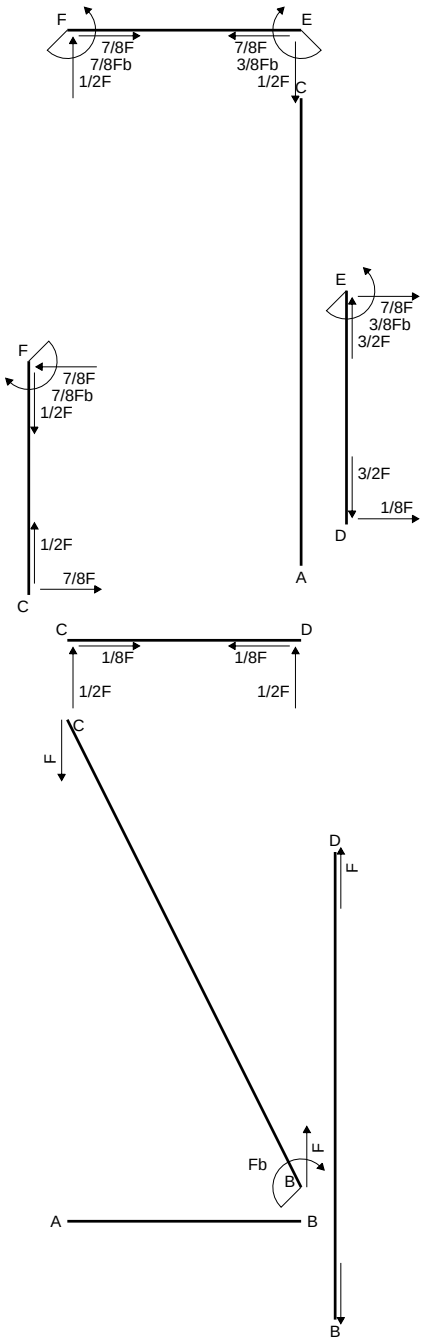
$= (-1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$

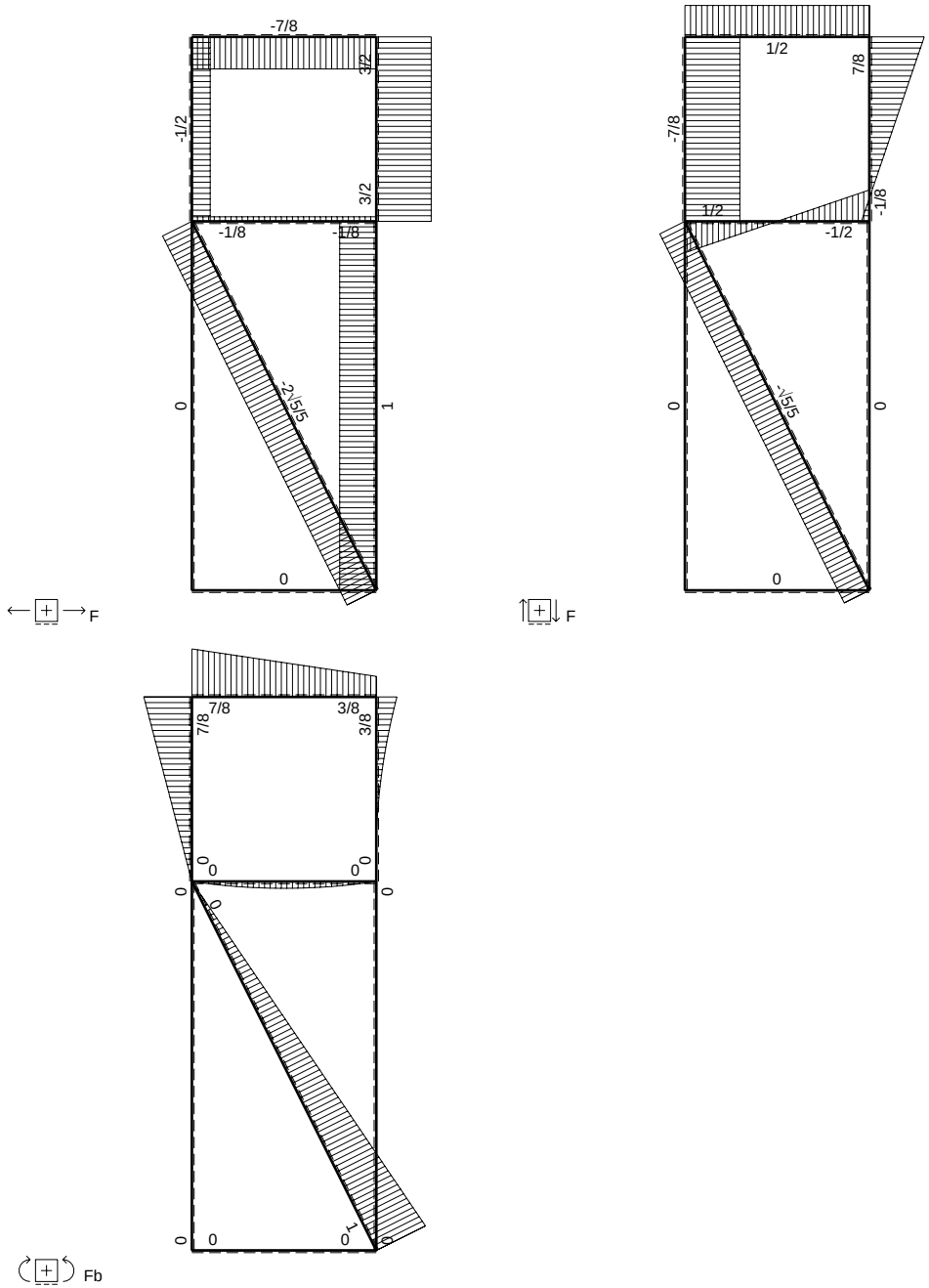


A = 510. mm^2
J_x = 181019. mm^4
J_y = 37808. mm^4
J_{xy} = -55745. mm^4
J_u = 200159. mm^4
J_v = 18668. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3307
$c = \cos\alpha$ = .9458
$s = \sin\alpha$ = .3247
x_g = 24.74 mm
y_g = 28.83 mm
N = -1310. N
T_y = -786. N
M_x = 734910. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 12.82 mm
v_m = 21.15 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -240. N/mm^2

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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
-√5/5
0
1
0
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
-√5/5
0
0
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 263281 CI09.xxxx.055

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$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L_{EF}^{Xo} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

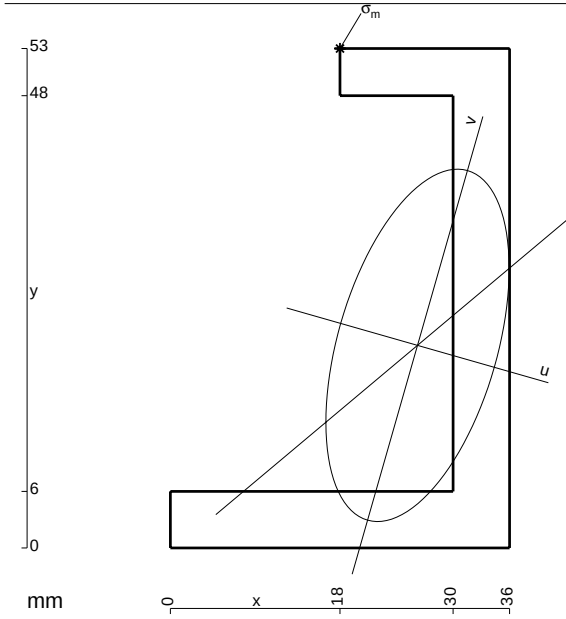
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ \ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

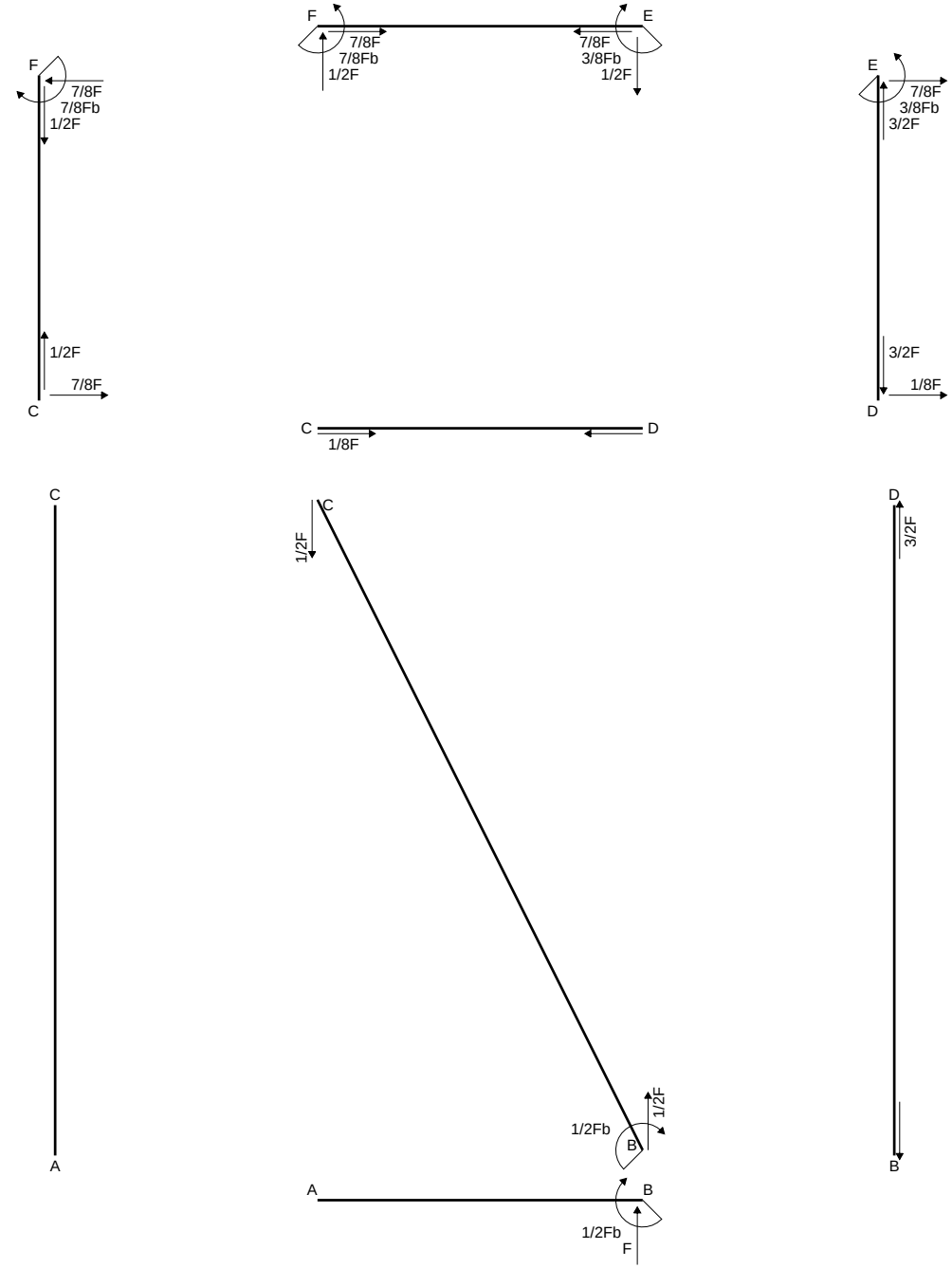
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

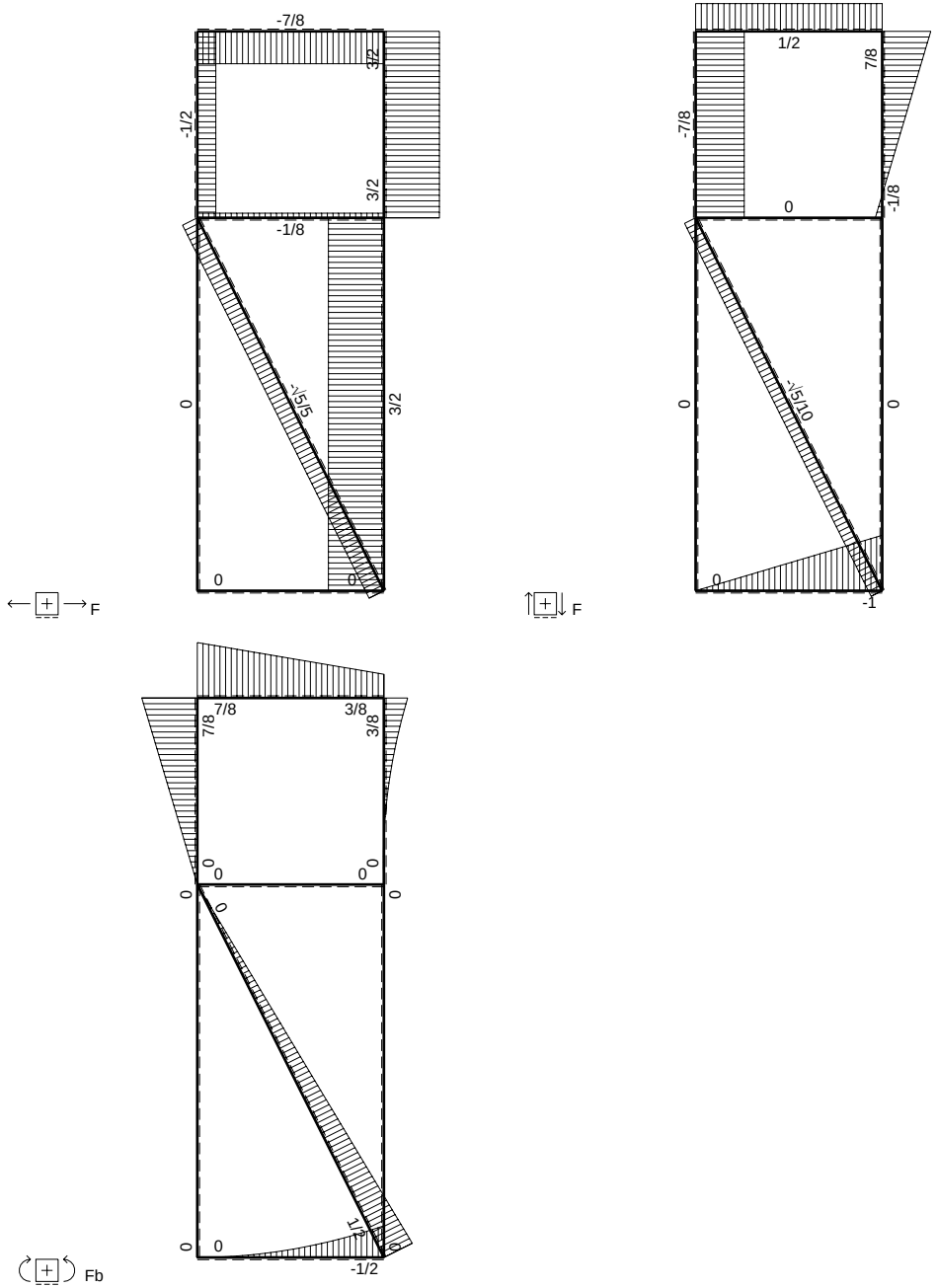


A = 558. mm^2
J_x = 195119. mm^4
J_y = 52748. mm^4
J_{xy} = 44280. mm^4
J_u = 207767. mm^4
J_v = 40099. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2782
c = cosα = .9615
s = sinα = -.2747
x_g = 26.23 mm
y_g = 21.5 mm
N = -1315. N
T_y = -657.4 N
M_x = 808500. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = -16.56 mm
v_m = 28.03 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -198.9 N/mm^2

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F
7/8F
7/8Fb
1/2F
1/2F
7/8F
C
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
D
1/8F
E
7/8F
3/8Fb
3/2F
3/2F
1/8F
D
C
A
C
1/2F
1/2Fb
B
1/2F
A
B
1/2Fb
F
D
3/2F
B


-7/8
3/2
-1/2
3/2
-1/8
0
-√5/5
3/2
0
0
F
1/2
7/8
-7/8
0
-1/8
0
-√5/10
0
0
-1
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
-1/2
Fb

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Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

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16.04.26

PROCEDIMENTO E RISULTATI 253104 CI09.xxxx.056

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

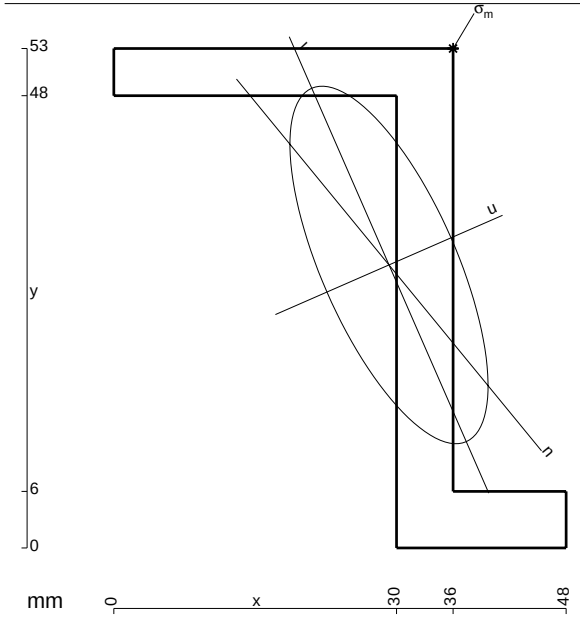
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

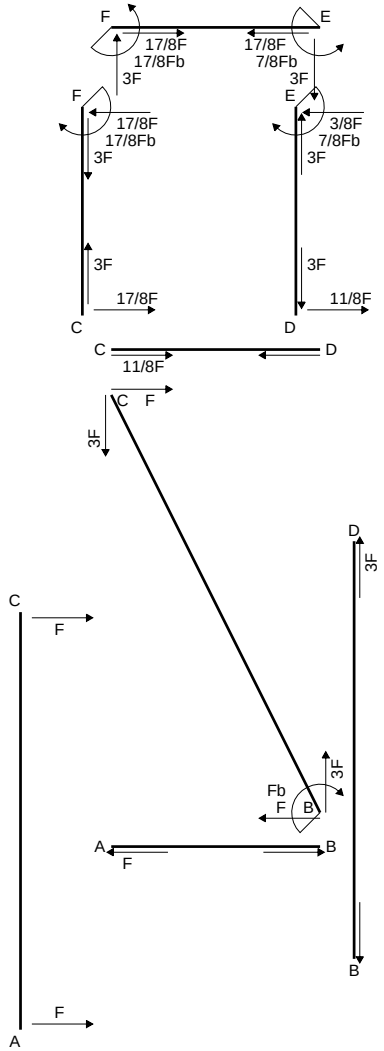
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

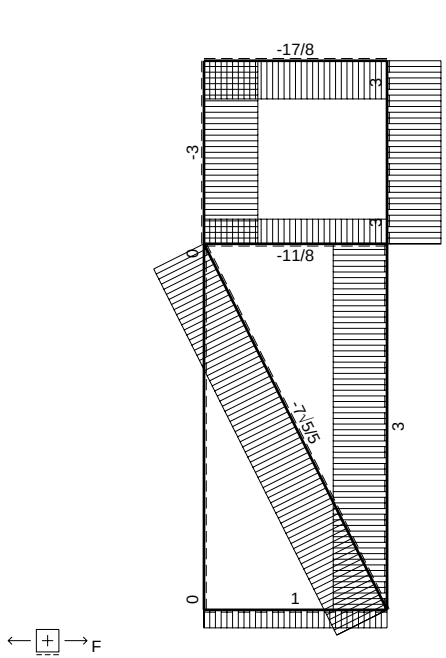


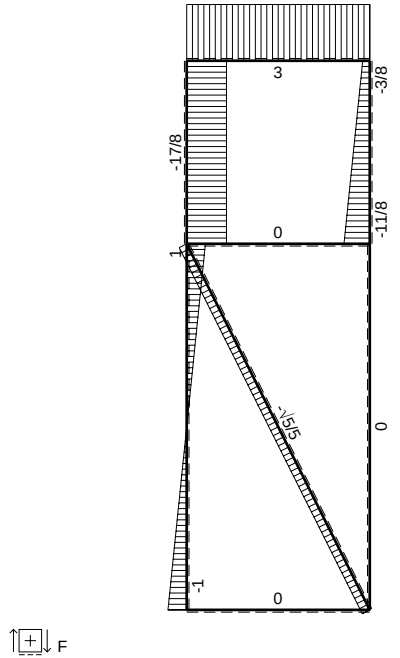
A = 540. mm^2
J_x = 194387. mm^4
J_y = 59702. mm^4
J_{xy} = -72778. mm^4
J_u = 226199. mm^4
J_v = 27891. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4121
c = cosα = .9163
s = sinα = .4005
x_g = 29.2 mm
y_g = 30.03 mm
N = -1190. N
T_y = 680. N
M_x = 702100. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 15.43 mm
v_m = 18.32 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.9 N/mm^2

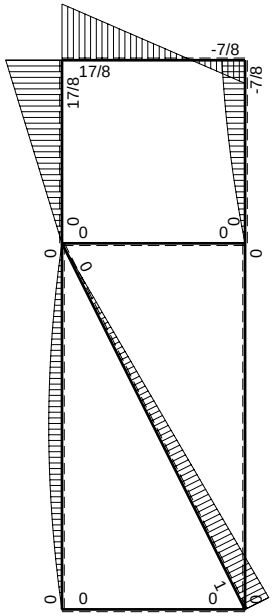
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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
B
F
A


-17/8
-3
-11/8
-7√5/5
3
0
1
F


3
-3/8
-17/8
-11/8
0
-7√5/5
-1
F


-7/8
17/8
-7/8
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^{2}b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
	totali						-35/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

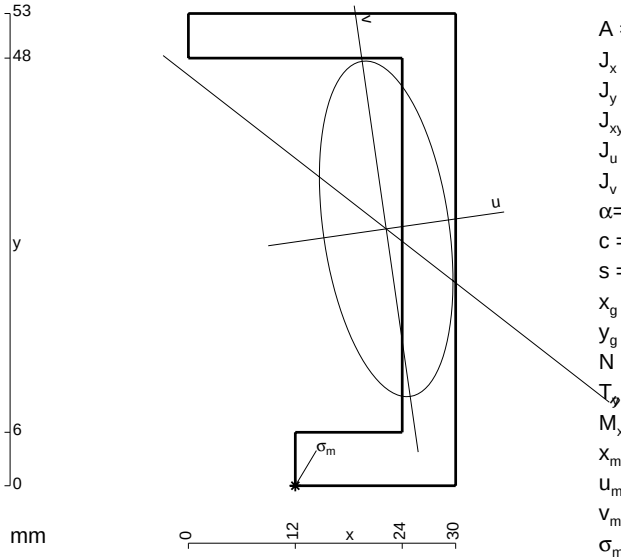
$= (-3\ b +3/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

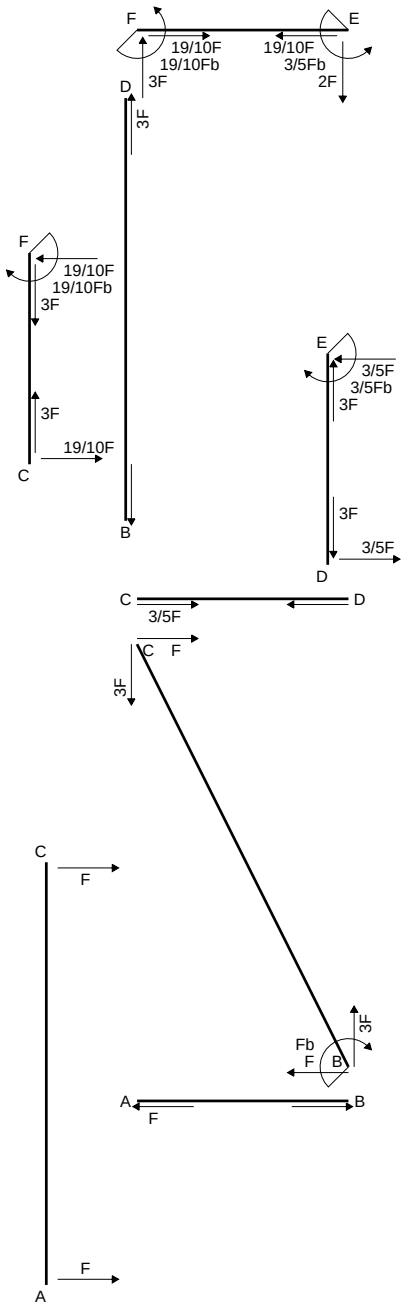
$= (-\ b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

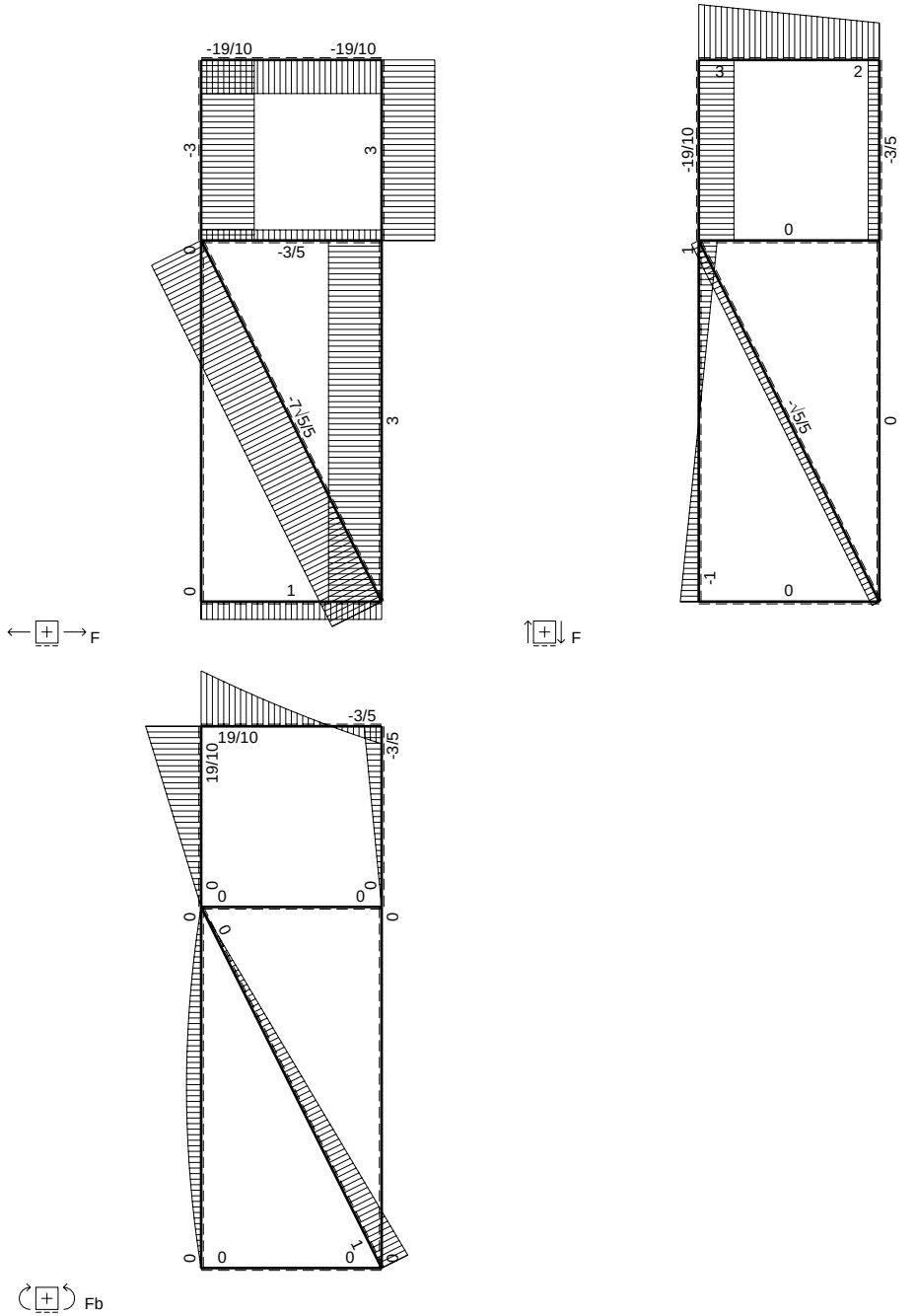


A = 510. mm^2
J_x = 181019. mm^4
J_y = 28660. mm^4
J_{xy} = -22270. mm^4
J_u = 184207. mm^4
J_v = 25471. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1422
$c = \cos\alpha$ = .9899
$s = \sin\alpha$ = .1417
x_g = 22.2 mm
y_g = 28.83 mm
N = -1594. N
T_y = 2250. N
M_x = 988125. Nmm
x_m = 12. mm
u_m = -14.18 mm
v_m = -27.09 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 218.7 N/mm^2

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F
E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
B
3F
3/5F
D
C
D
3/5F
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A


-19/10
-19/10
-3
3
-3/5
0
3
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F
-19/10
3
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-3/5
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-1/5/5
-1
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-3/5
19/10
19/10
-3/5
0
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0
0
0
0
0
0
0
0
0
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx²	-Fb/EJ	-2Fx-1/2Fx²/b	Fb/EJ	1	(-7/6+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx²	Fb/EJ	-5/2Fb+3Fx-1/2Fx²/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
totali							-Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							3/5Fb

PROCEDIMENTO E RISULTATI 215079 CI09.xxxx.058

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

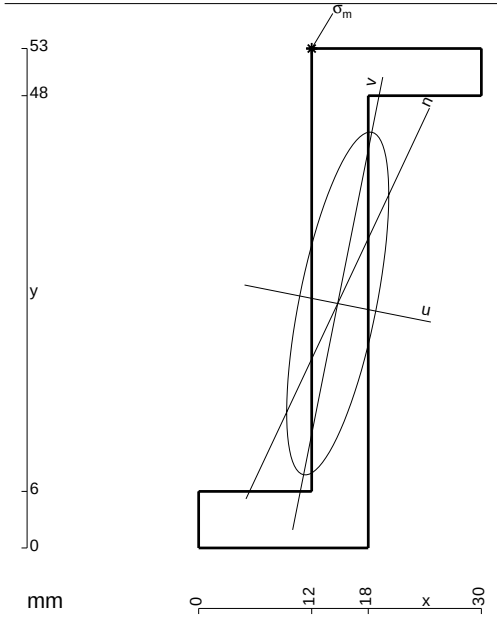
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

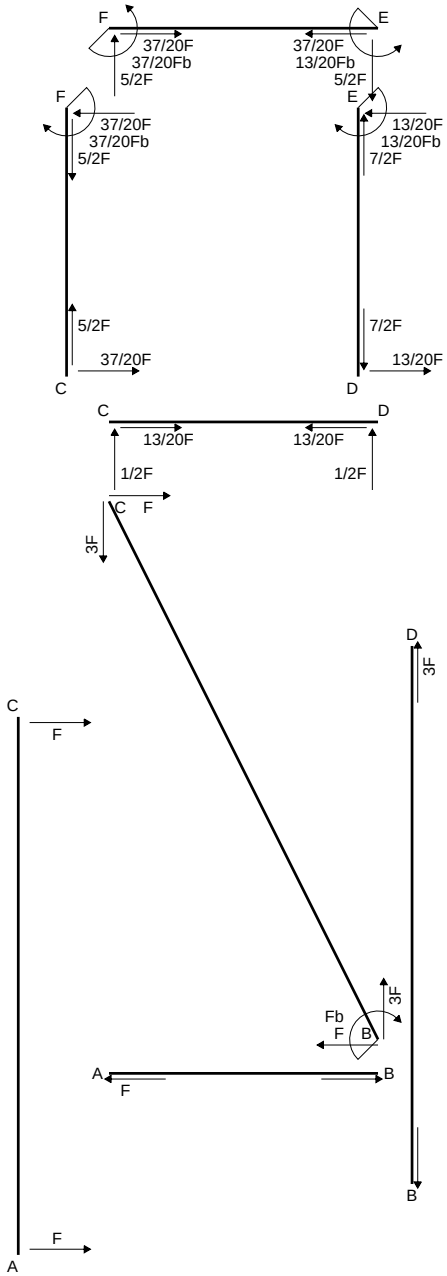
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

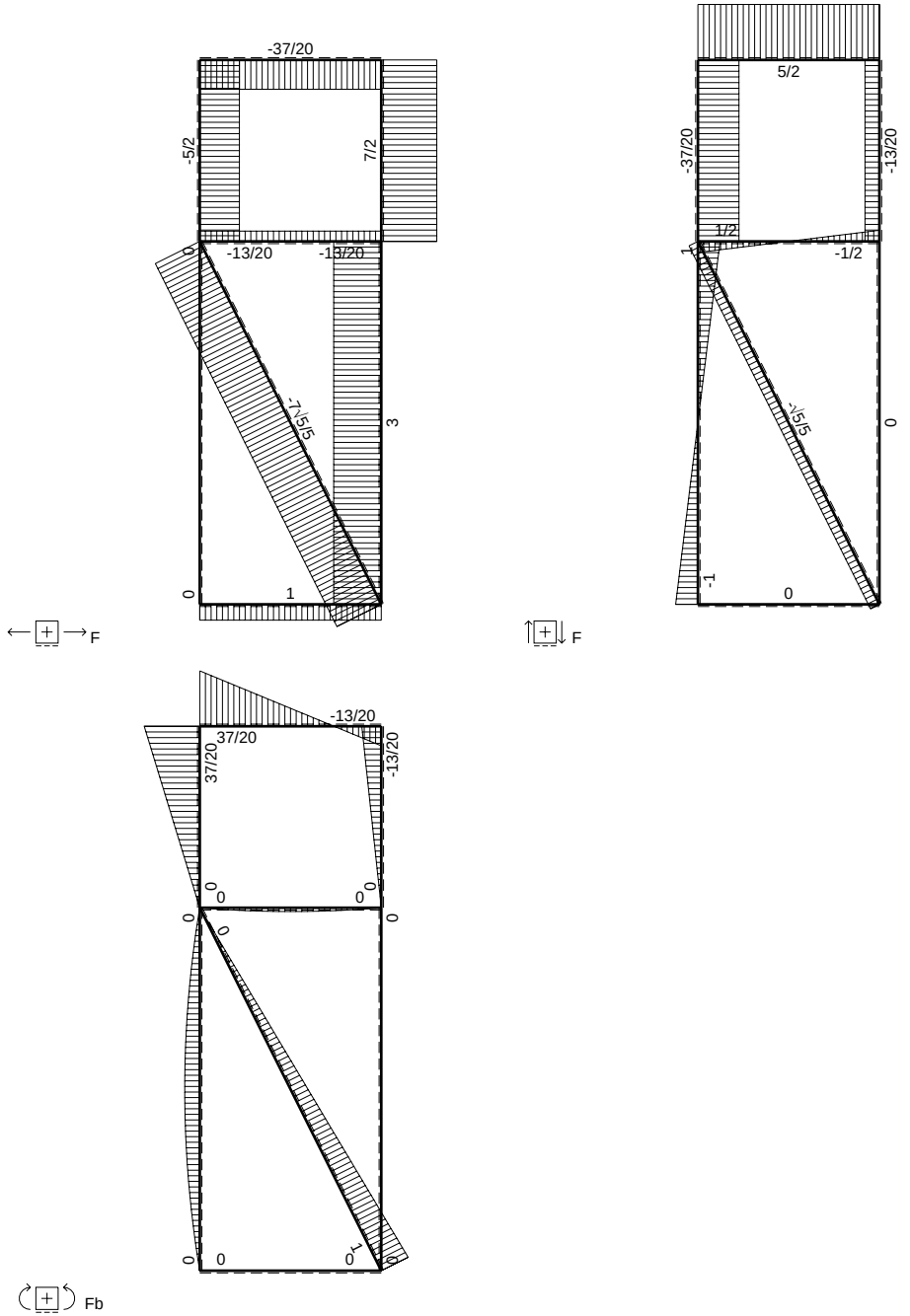


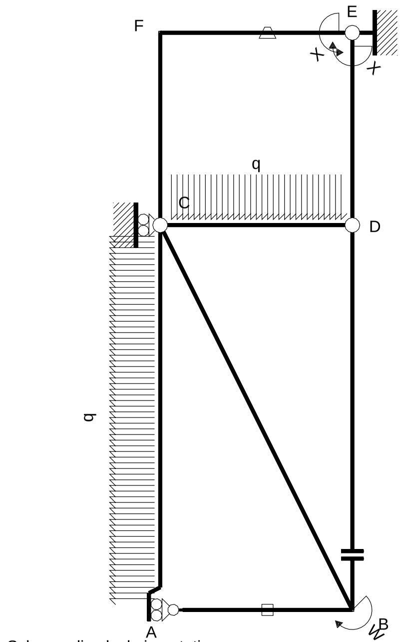
A = 450. mm^2
J_x = 148960. mm^4
J_y = 13204. mm^4
J_{xy} = 28128. mm^4
J_u = 154557. mm^4
J_v = 7607. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = -.1964
c = cosα = .9808
s = sinα = -.1952
x_g = 14.76 mm
y_g = 25.94 mm
N = -1440. N
T_y = -912. N
M_x = 601920. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -7.988 mm
v_m = 26. mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -225.9 N/mm^2

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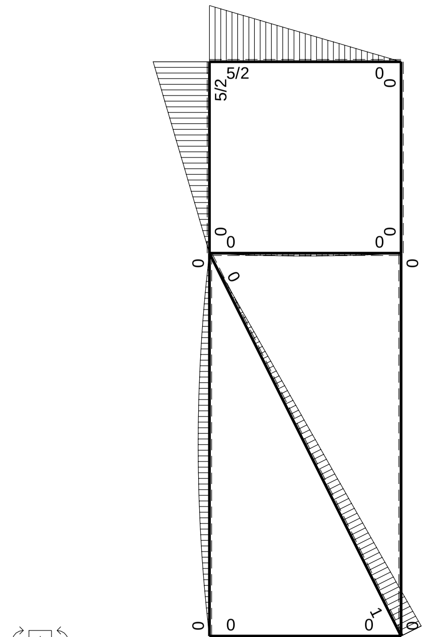
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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
7/2F
37/20F
13/20F
C
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
F
Fb
F
B
3F
A
B
F
F
A
B

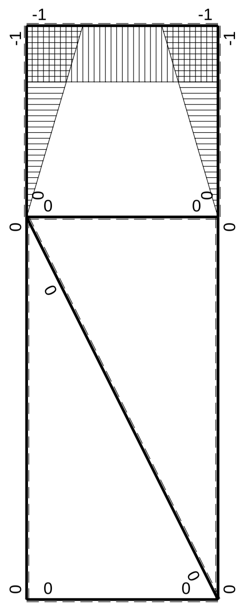

-37/20
-5/2
7/2
-13/20
-13/20
0
-7/5/5
3
0
1
F
5/2
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-13/20
1/2
-1/2
-7/5/5
0
-1
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F
-13/20
37/20
37/20
-13/20
0
0
0
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0
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0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

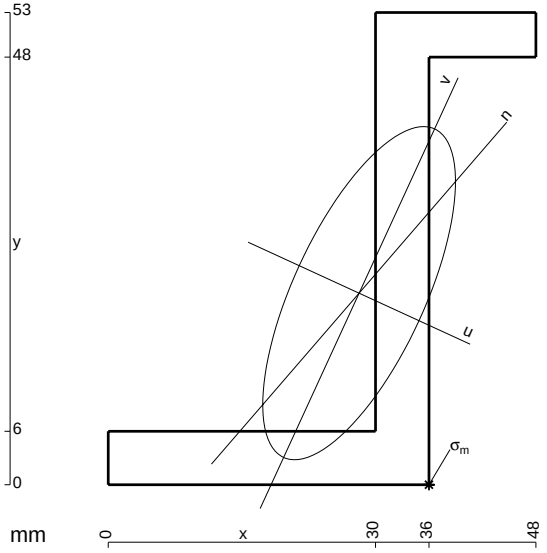
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

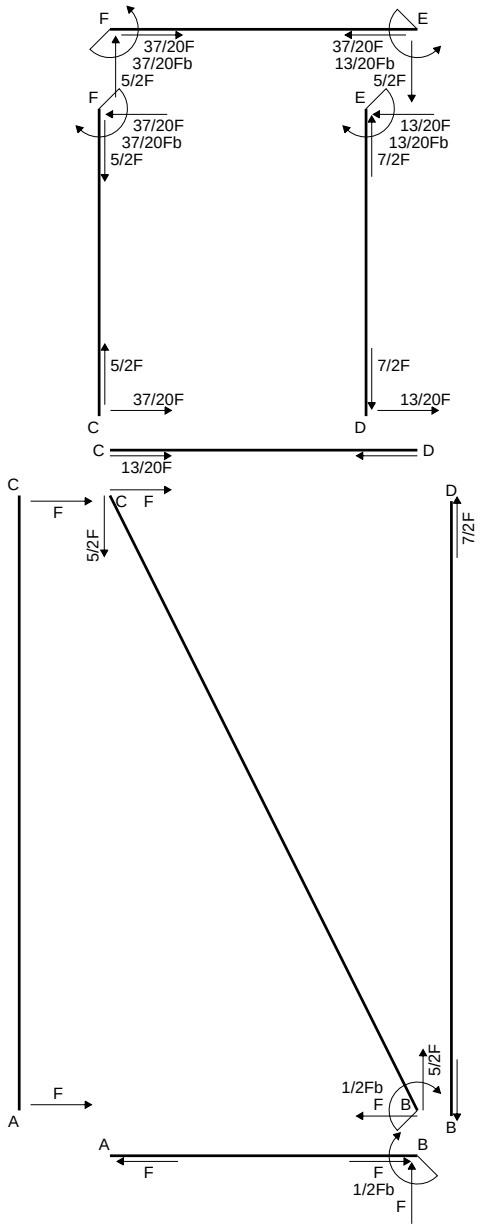
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

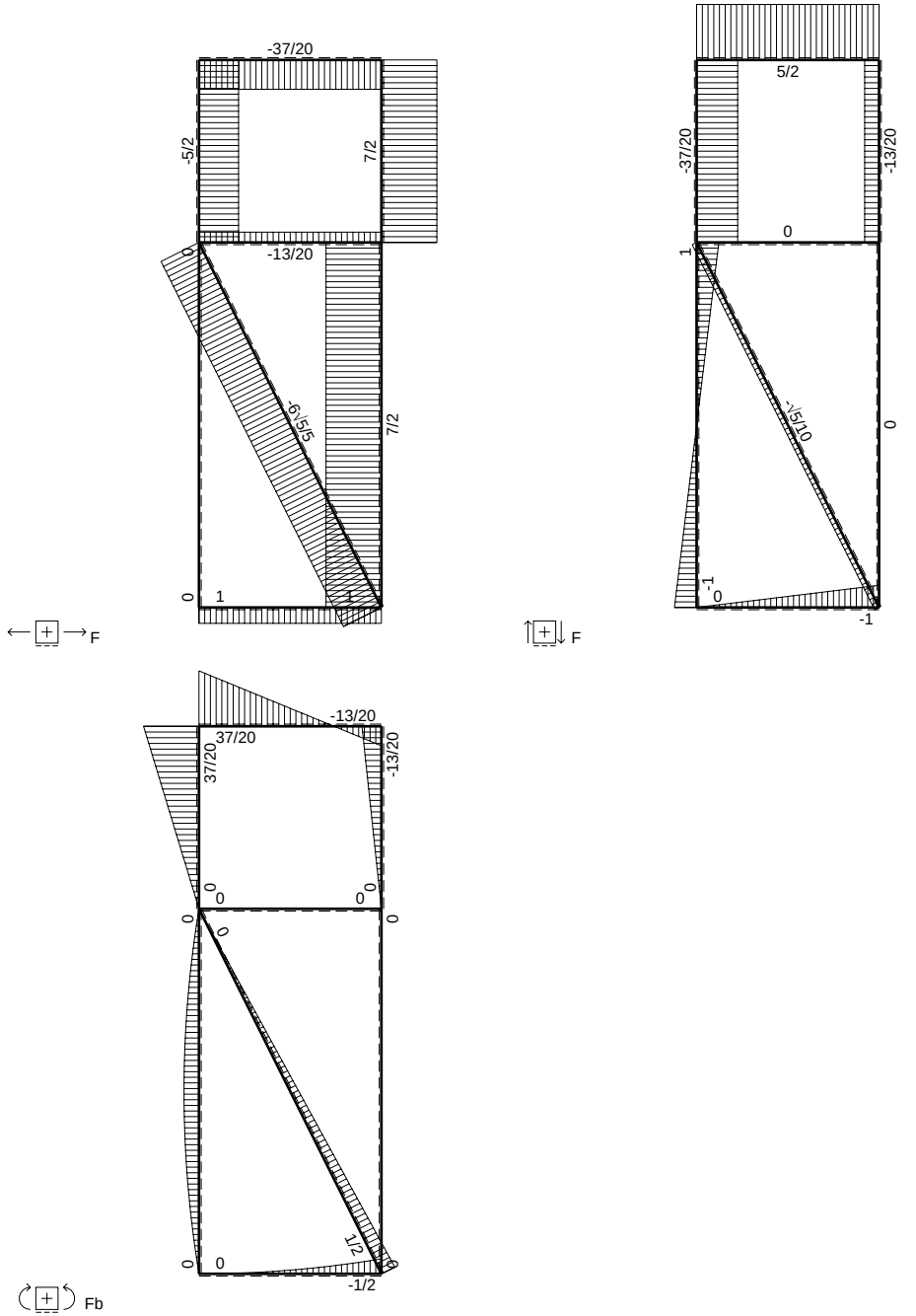


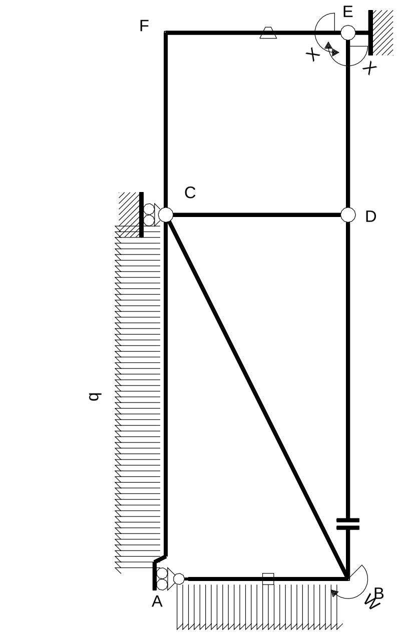
A = 558. mm^2
J_x = 195119. mm^4
J_y = 65290. mm^4
J_{xy} = 75600. mm^4
J_u = 229850. mm^4
J_v = 30558. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4306
c = cosα = .9087
s = sinα = -.4175
x_g = 28.16 mm
y_g = 21.5 mm
N = -1203. N
T_y = 1625. N
M_x = 841750. Nmm
x_m = 36. mm
u_m = 16.1 mm
v_m = -16.26 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 237.1 N/mm^2

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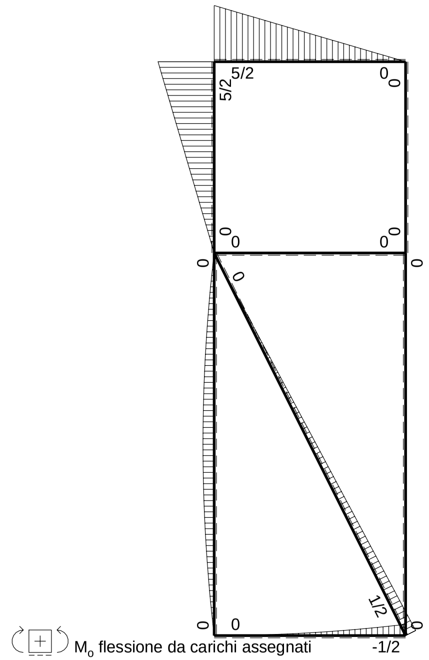
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F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
13/20F
C
D
A
B
1/2Fb

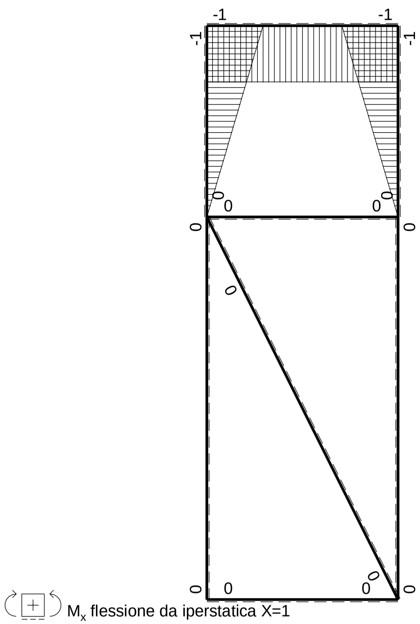

-37/20
-5/2
7/2
-13/20
-5\5/5
F
5/2
-37/20
-13/20
-5/10
-1
37/20
1/2
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

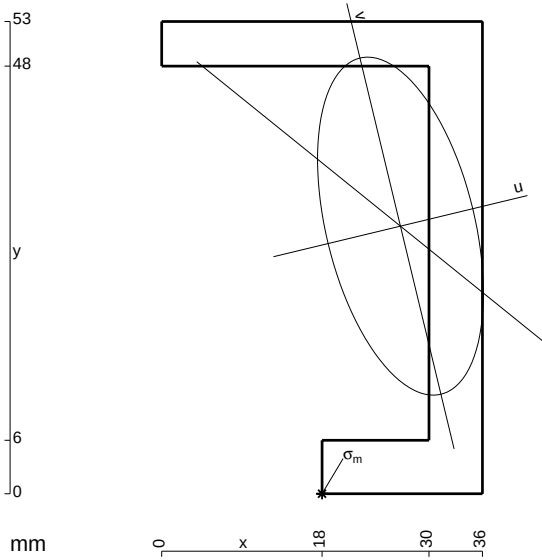
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

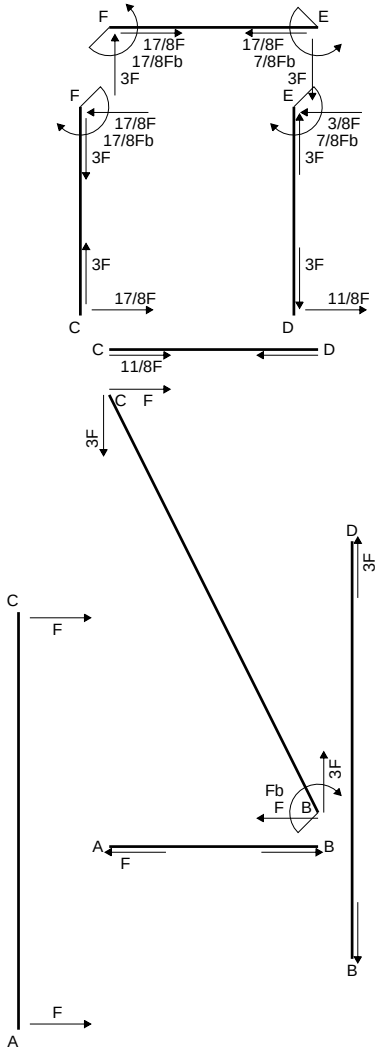
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

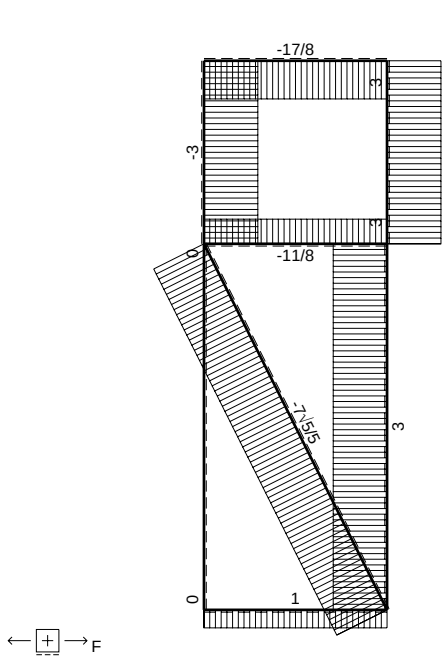


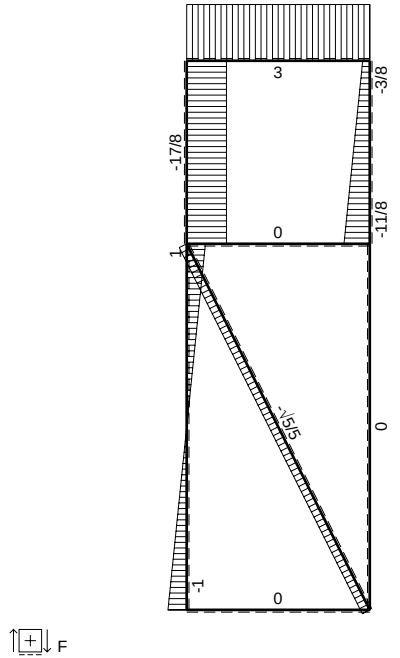
A = 540. mm^2
J_x = 194387. mm^4
J_y = 46742. mm^4
J_{xy} = -37742. mm^4
J_u = 203476. mm^4
J_v = 37654. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2363
$c = \cos\alpha$ = .9722
$s = \sin\alpha$ = .2341
x_g = 26.8 mm
y_g = 30.03 mm
N = -2424. N
T_y = 3275. N
M_x = 896695. Nmm
x_m = 18. mm
u_m = -15.59 mm
v_m = -27.14 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 198.7 N/mm^2

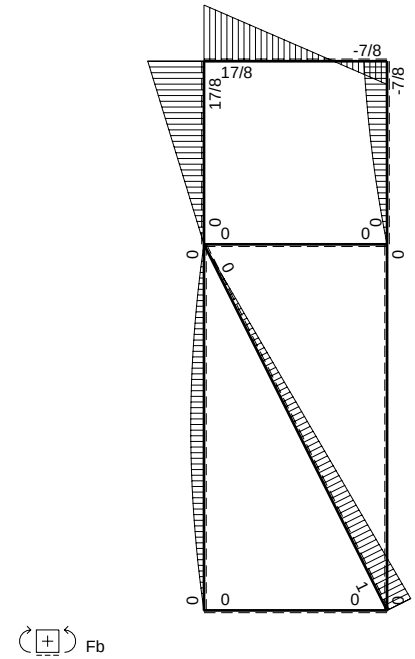
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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
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17/8Fb
3F
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7/8Fb
3F
3F
3F
17/8F
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16.04.26

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb²/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx²/b	0	1-2x/b+x²/b²	(-1+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx²/b	0	x²/b²		
	totali						-35/24Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

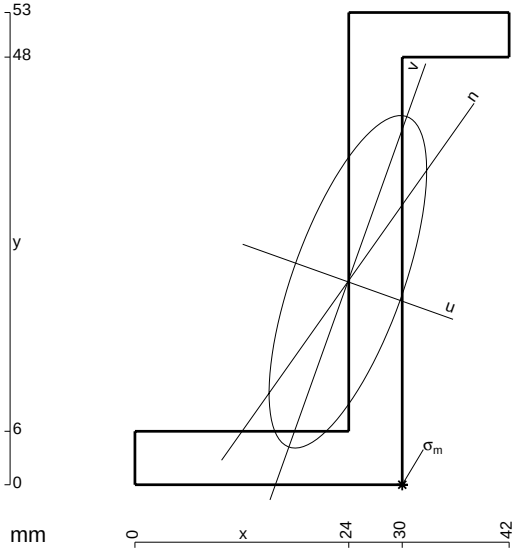
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

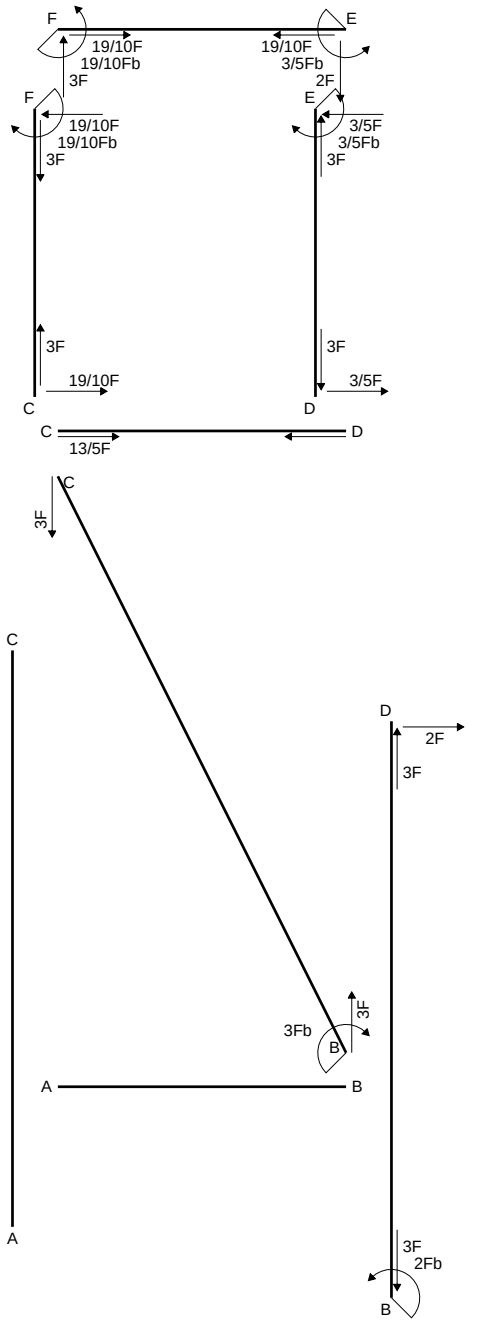
$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

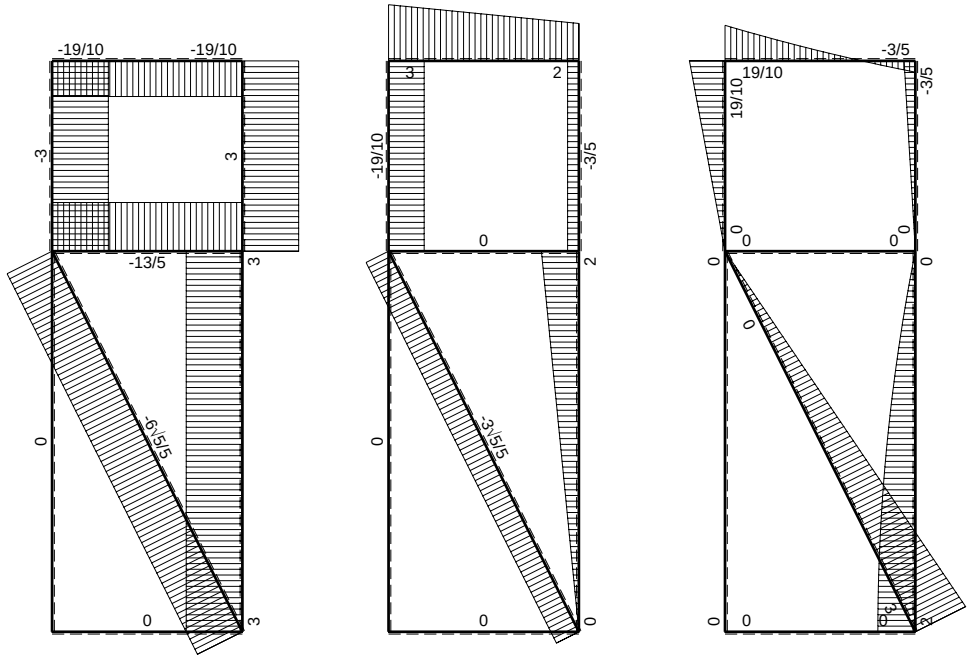


A = 522. mm^2
J_x = 181840. mm^4
J_y = 40818. mm^4
J_{xy} = 57687. mm^4
J_u = 202431. mm^4
J_v = 20227. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.3428$
$c = \cos\alpha = .9418$
$s = \sin\alpha = -.3362$
x_g = 23.9 mm
y_g = 22.78 mm
N = -1700. N
T_y = 2400. N
M_x = 680000. Nmm
x_m = 30. mm
u_m = 13.4 mm
v_m = -19.4 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 209.6 N/mm^2

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F
E
19/10F
19/10Fb
3F
19/10F
3/5Fb
2F
F
E
19/10F
19/10Fb
3F
3/5F
3/5Fb
3F
3F
19/10F
C
D
3F
3/5F
C
D
13/5F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-19/10
-19/10
-3
3
3
-13/5
0
-6√5/5
0
3
3
2
-19/10
-3/5
0
2
-3√5/5
0
0
19/10
-3/5
0
0
0
0
0
0
0

F
F
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
totali							-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							3/5Fb

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

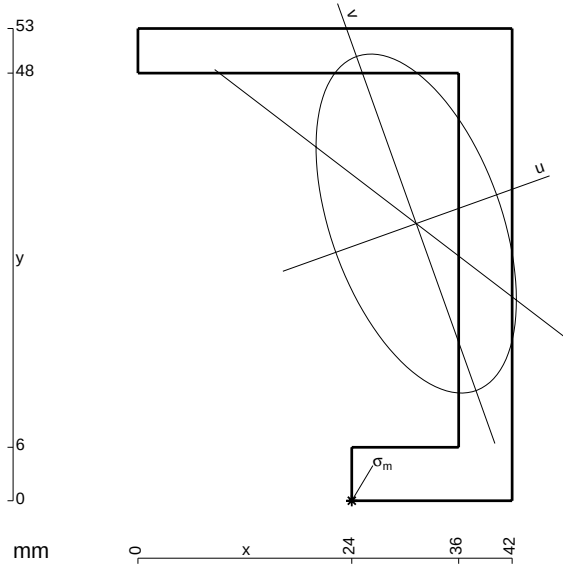
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

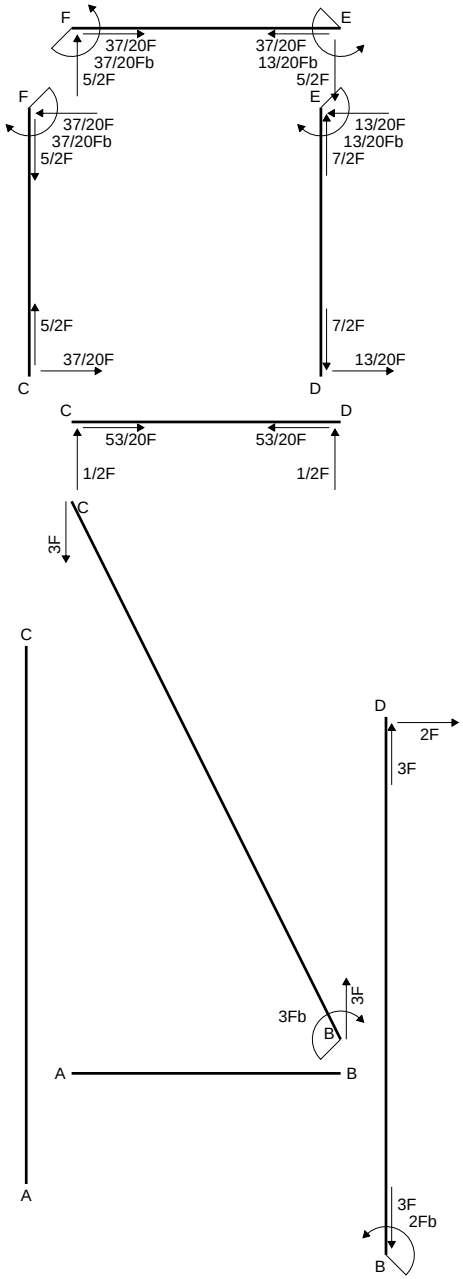
$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

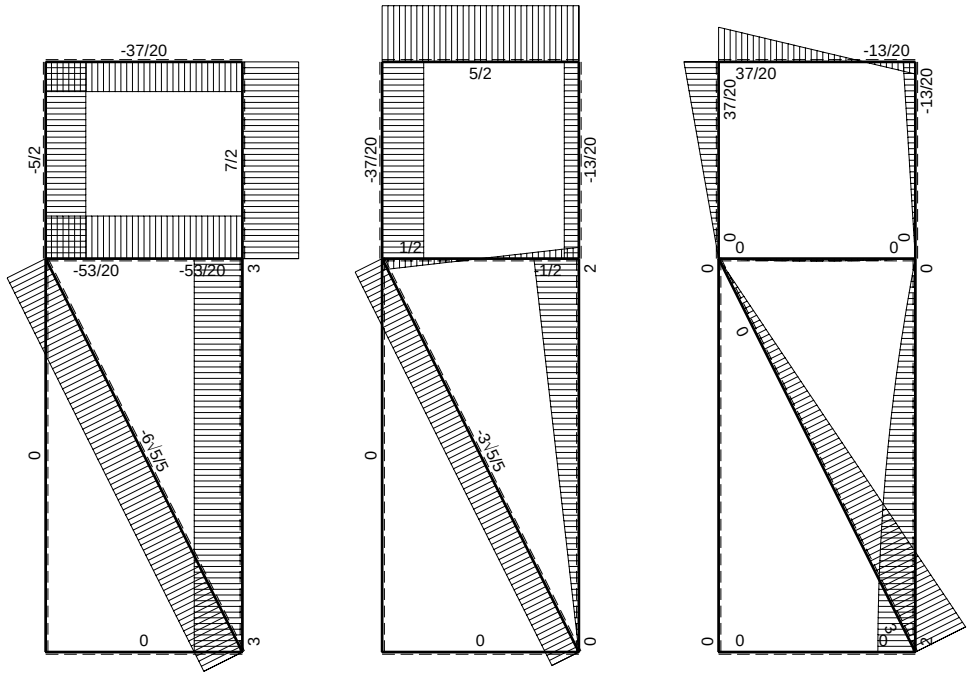


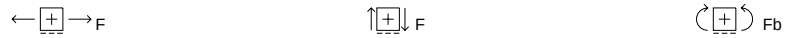
A = 570. mm^2
J_x = 206355. mm^4
J_y = 72071. mm^4
J_{xy} = -55077. mm^4
J_u = 226055. mm^4
J_v = 52372. mm^4
α= artg($2J_{xy}/(J_y-J_x)$)/2 = .3435
c = cosα = .9416
s = sinα = .3368
x_g = 31.23 mm
y_g = 31.11 mm
N = -2012. N
T_y = -1006. N
M_x = 990000. Nmm
x_m = 24. mm
u_m = -17.29 mm
v_m = -26.86 mm
σ_m = N/A-Mcv/J_u-Msu/J_v = 217.3 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
C
D
53/20F
53/20F
1/2F
1/2F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-37/20
-5/2
7/2
-53/20
-53/20
3
0
3
-6√5/5
0
5/2
-37/20
-13/20
1/2
1/2
2
-3√5/5
0
0
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
3√5
0


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx²	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

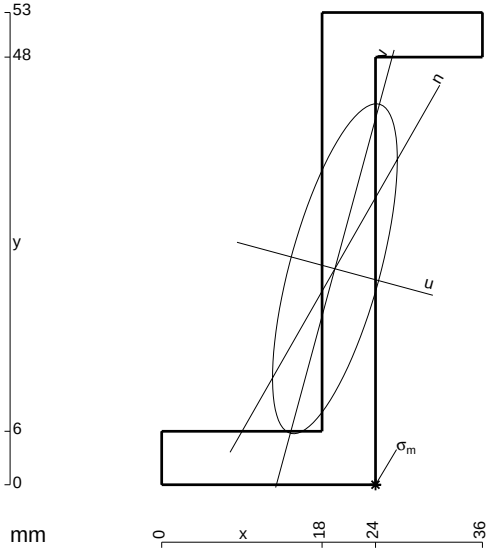
$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 308763 CI09.xxxx.063

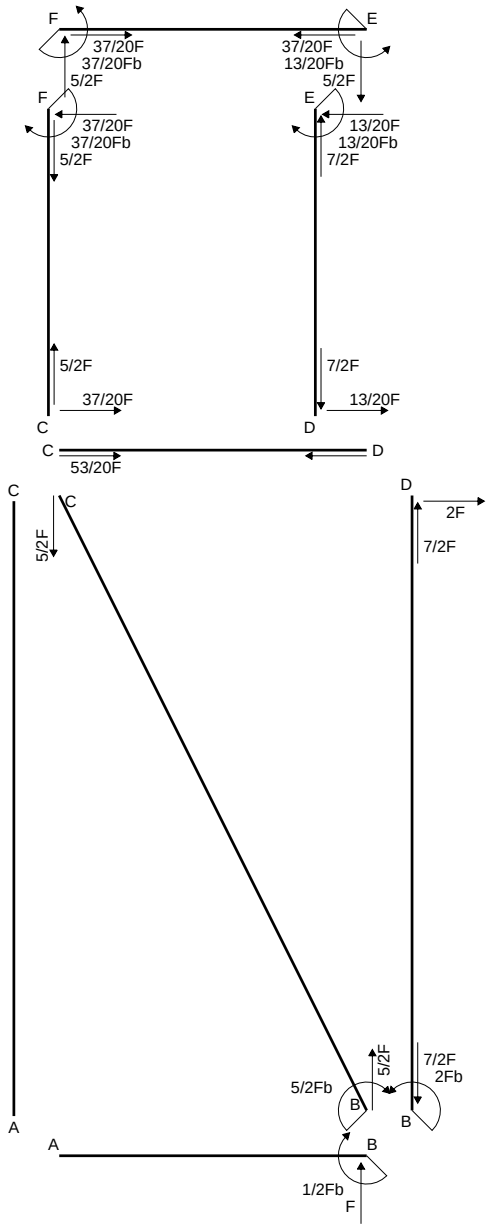


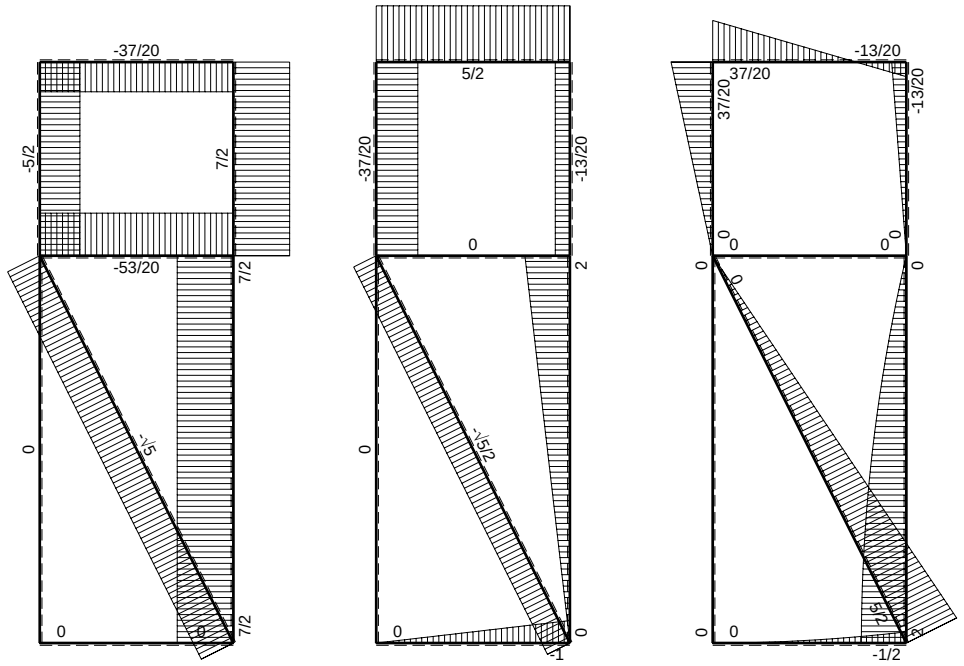
A = 486. mm^2
J_x = 166610. mm^4
J_y = 23826. mm^4
J_{xy} = 41708. mm^4
J_u = 177900. mm^4
J_v = 12536. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2644
c = cosα = .9653
s = sinα = -.2613
x_g = 19.44 mm
y_g = 24.24 mm
N = -1234. N
T_y = -617.2 N
M_x = 662400. Nmm
x_m = 24. mm
u_m = 10.73 mm
v_m = -22.21 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 225.4 N/mm^2

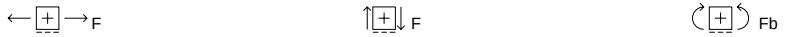
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F
E
37/20F
37/20Fb
5/2F
13/20Fb
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
53/20F
2F
7/2F
5/2F
5/2Fb
2Fb
A
B
1/2Fb
F


-37/20
-5/2
7/2
-53/20
0
-5/2
5/2
-13/20
2
-1
37/20
-1/2


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$5/2Fb-\sqrt{5}/2Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

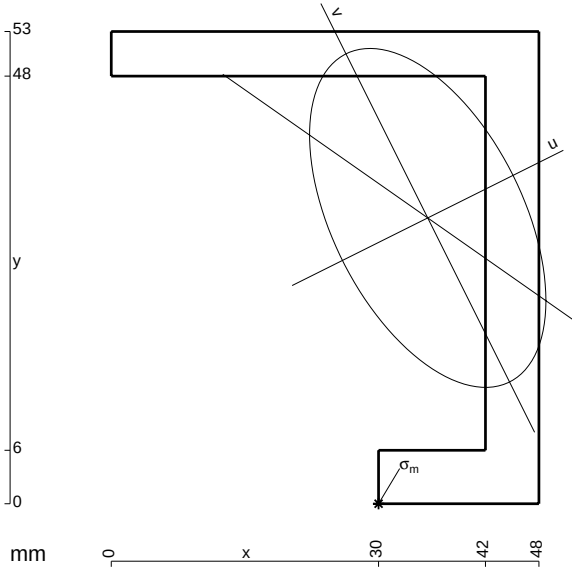
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

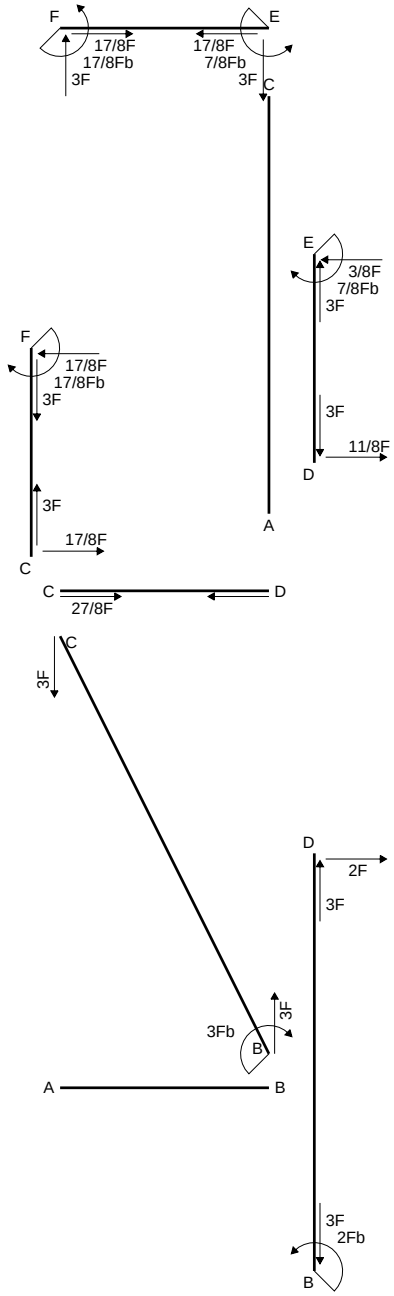
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

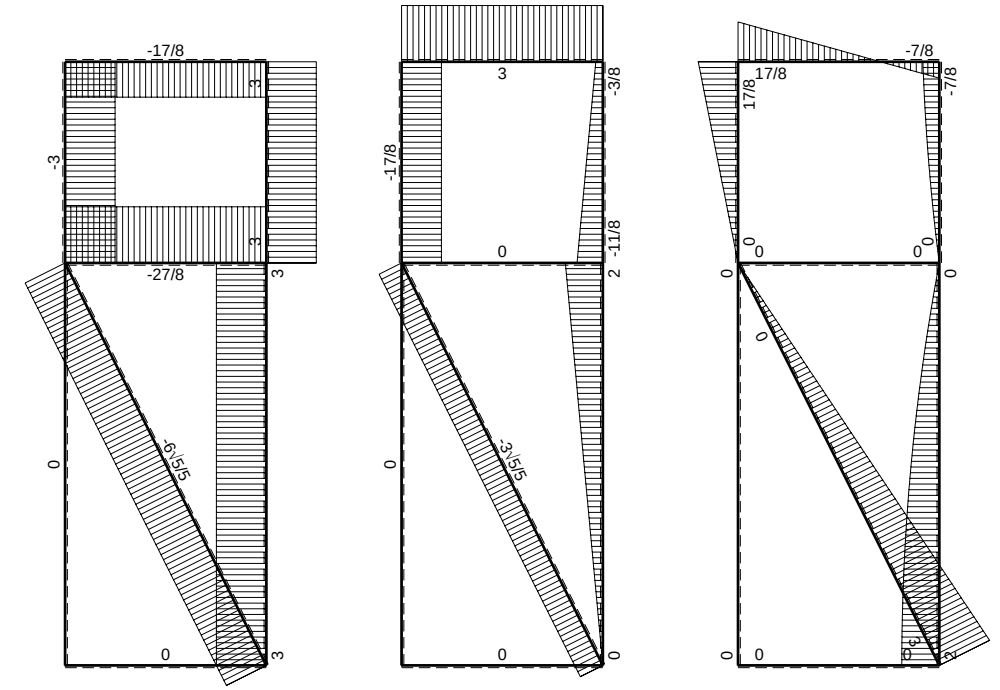


A = 600. mm^2
J_x = 217132. mm^4
J_y = 105558. mm^4
J_{xy} = -73993. mm^4
J_u = 254012. mm^4
J_v = 68678. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4624
$c = \cos\alpha$ = .8950
$s = \sin\alpha$ = .4461
x_g = 35.52 mm
y_g = 32.08 mm
N = -1945. N
T_y = -972.7 N
M_x = 1109250. Nmm
x_m = 30. mm
u_m = -19.25 mm
v_m = -26.25 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 238. N/mm^2

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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
C
E
3/8F
7/8Fb
3F
F
17/8F
17/8Fb
3F
3F
11/8F
D
3F
A
17/8F
C
C
D
27/8F
C
3F
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
B


-17/8
3
-3
3
-27/8
3
0
-6.5/5
3
0
0
-17/8
3
-3/8
0
-11/8
2
-3.5/5
0
0
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
3.5/5
0


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^{2}b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3XkJ/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	XkJ/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3XkJ/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
totali							-35/24Fb^2/EJ	5/3XkJ/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

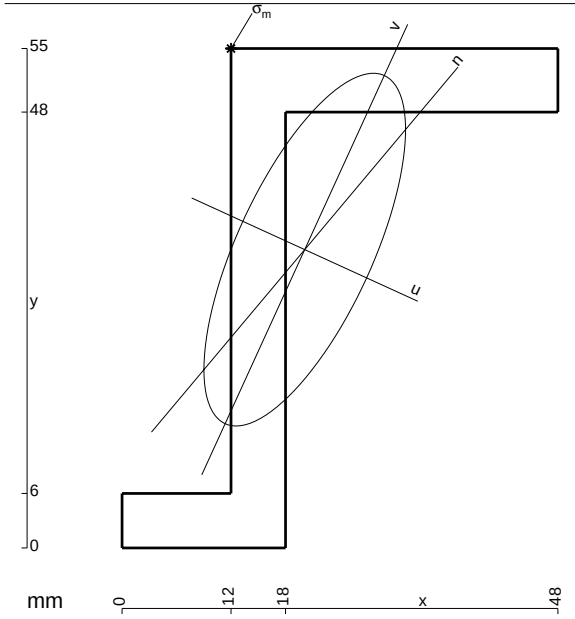
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

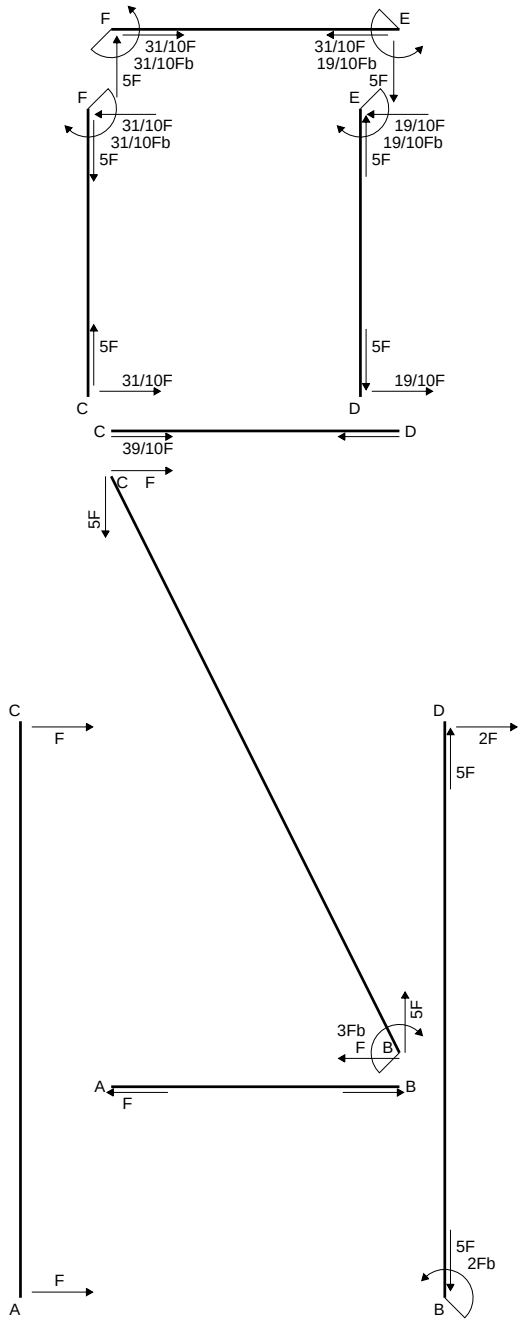
$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

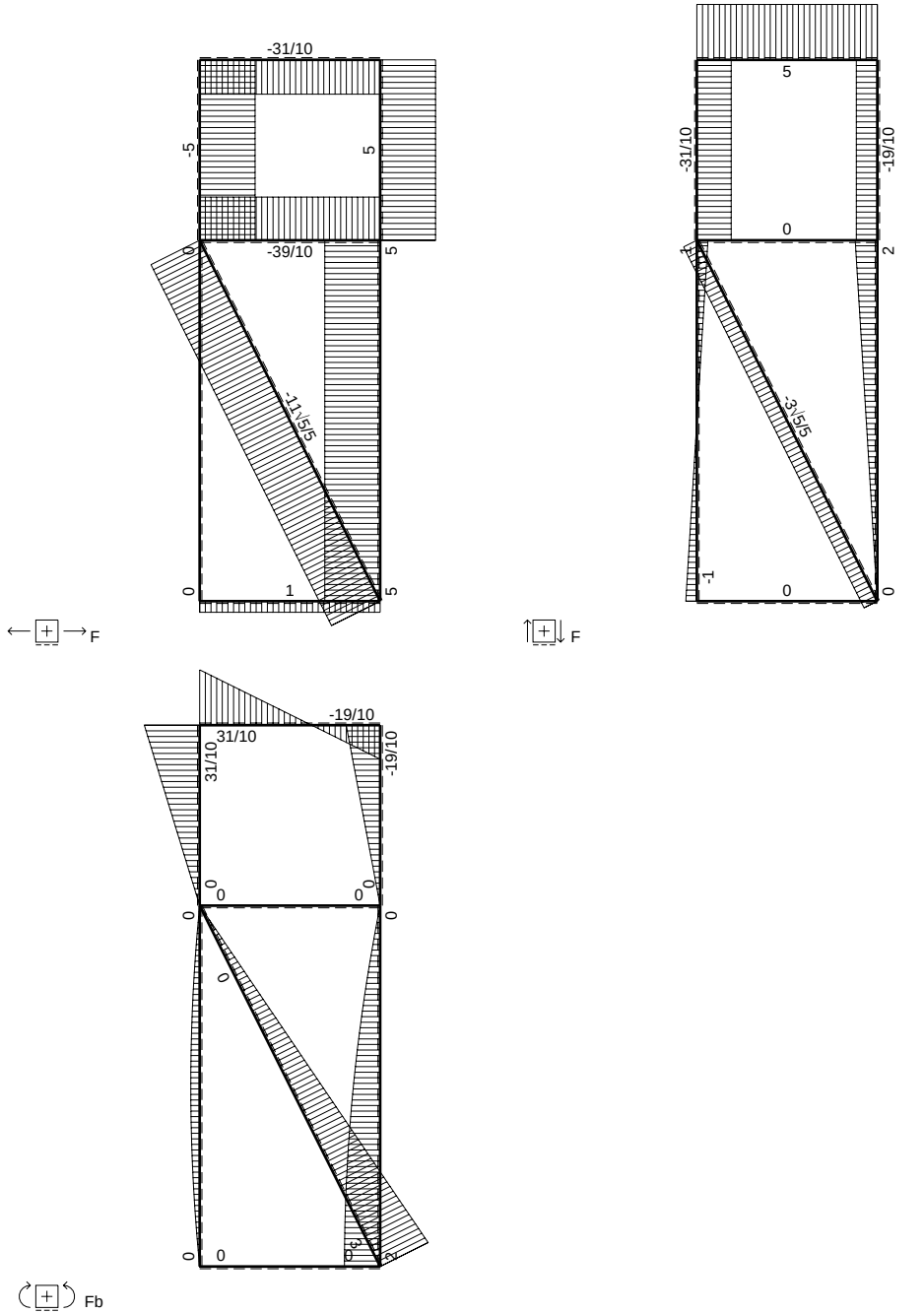


A = 612. mm^2
J_x = 230903. mm^4
J_y = 75448. mm^4
J_{xy} = 89831. mm^4
J_u = 271965. mm^4
J_v = 34385. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4288
c = cosα = .9095
s = sinα = -.4157
x_g = 20.12 mm
y_g = 32.85 mm
N = -1181. N
T_y = -590.3 N
M_x = 765600. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -16.59 mm
v_m = 16.77 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -198.4 N/mm^2

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F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
C
D
39/10F
2F
3Fb
2Fb
A
B


-31/10
-39/10
-11√5/5
-3√5/5
-19/10
31/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
totali							-19/6Fb^2/EJ	5/3Xb/EJ
iperstatica $X=W_{EF}$							19/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 981229 CI09.xxxx.066

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

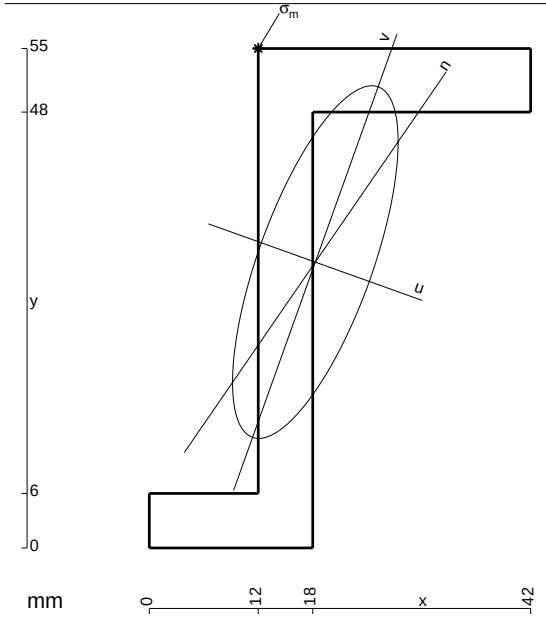
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

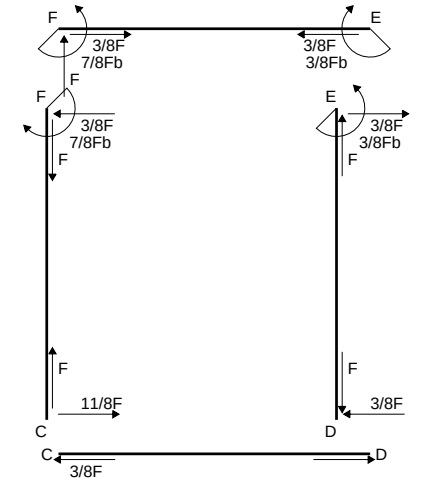
$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

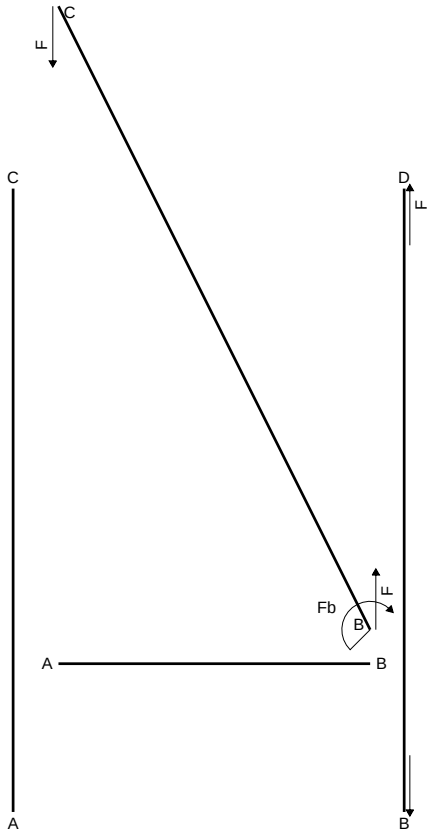


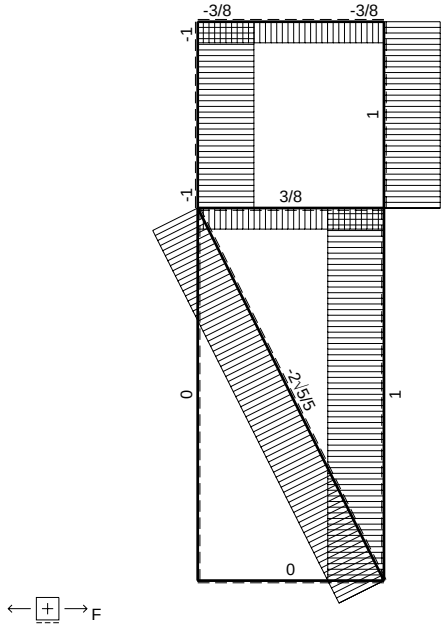
A = 570. mm^2
J_x = 215051. mm^4
J_y = 47402. mm^4
J_{xy} = 68907. mm^4
J_u = 239738. mm^4
J_v = 22715. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3440
c = cosα = .9414
s = sinα = -.3373
x_g = 18.28 mm
y_g = 31.48 mm
N = -1850. N
T_y = -1147. N
M_x = 711140. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -13.85 mm
v_m = 20.02 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -205.4 N/mm^2

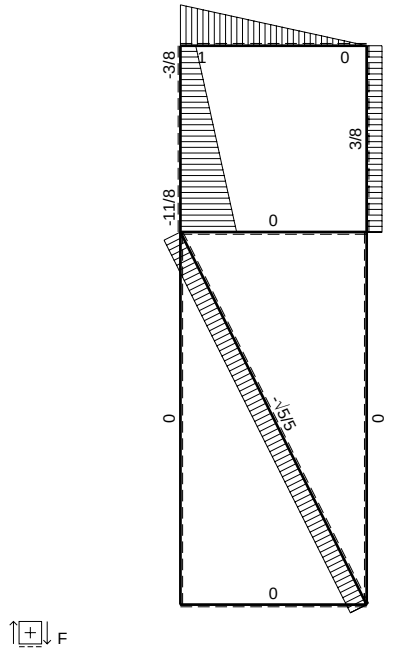
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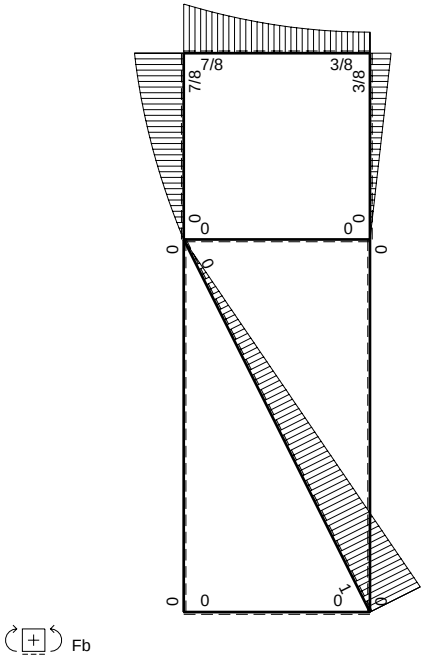
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F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

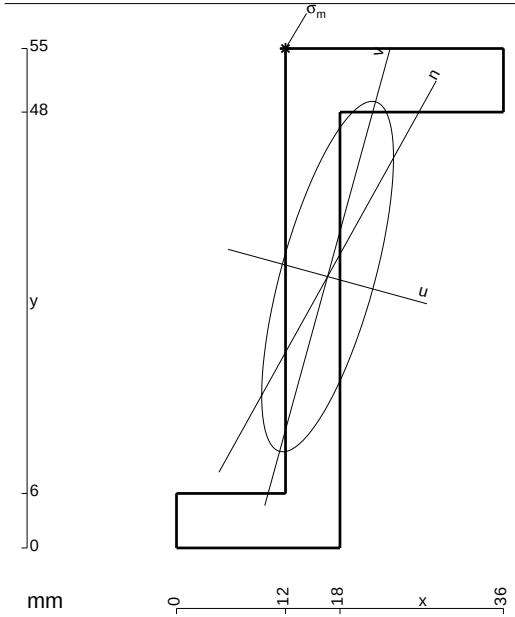
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

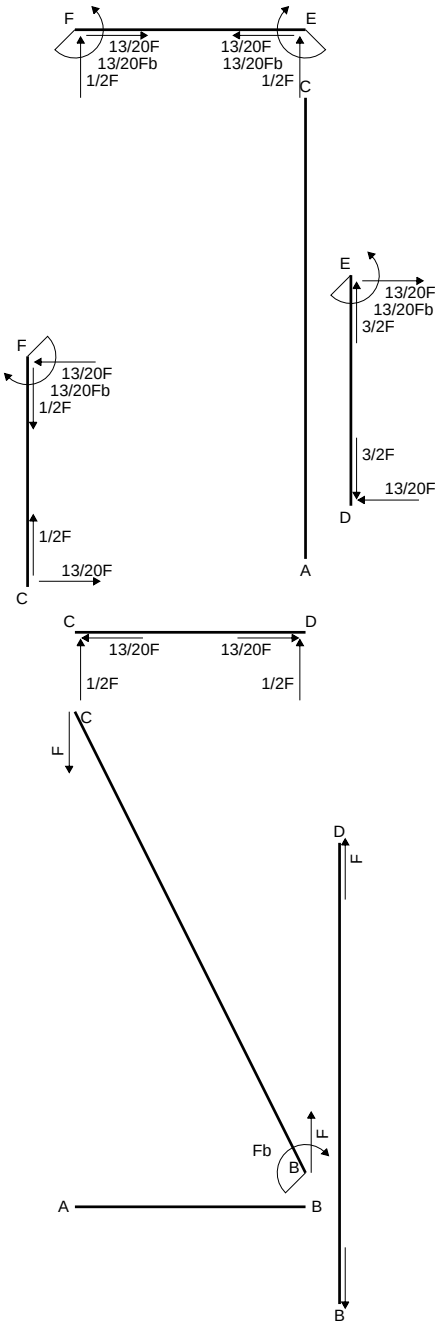
$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



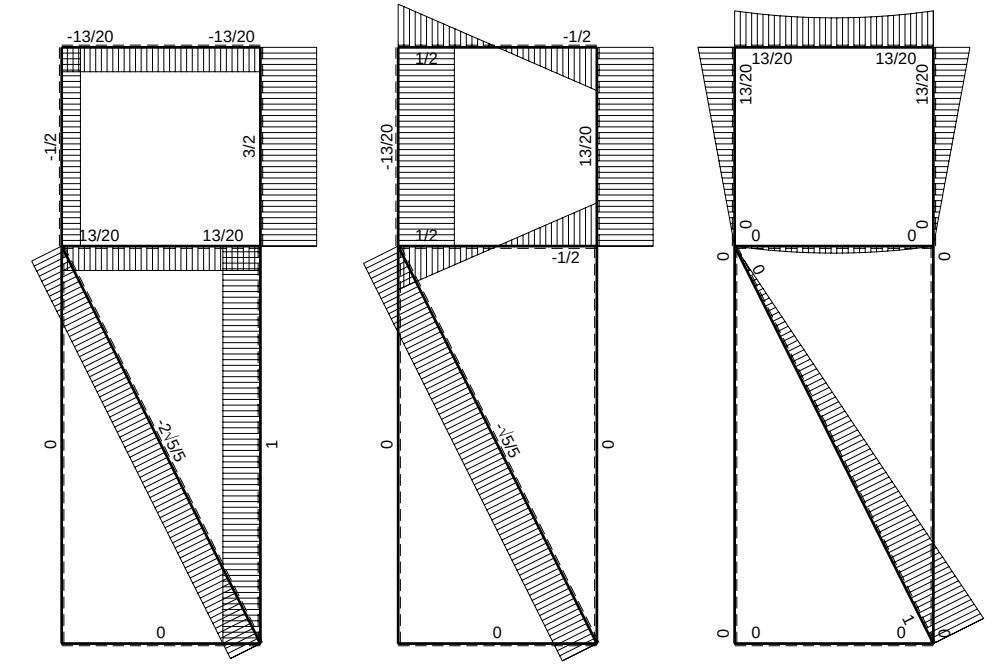
A = 528. mm^2
J_x = 196705. mm^4
J_y = 27818. mm^4
J_{xy} = 50102. mm^4
J_u = 210450. mm^4
J_v = 14073. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2677
$c = \cos\alpha$ = .9644
$s = \sin\alpha$ = -.2646
x_g = 16.64 mm
y_g = 29.89 mm
N = -939.1 N
T_y = -469.6 N
M_x = 693000. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -11.12 mm
v_m = 22.99 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.6 N/mm^2

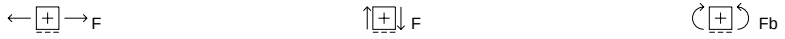
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Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	Xb/EJ	$(1/12+1)Fb^2/EJ$
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

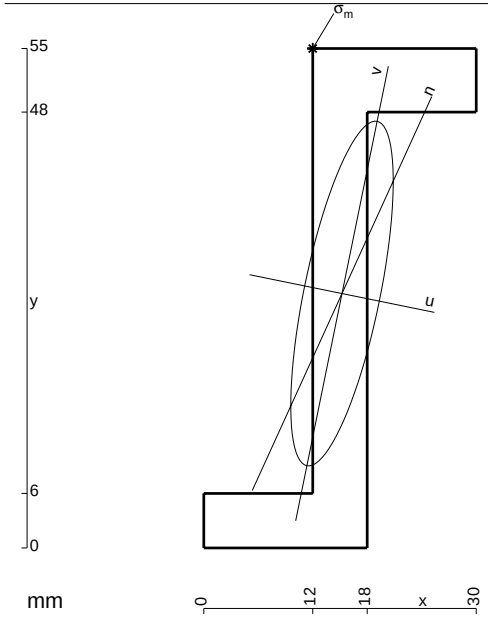
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

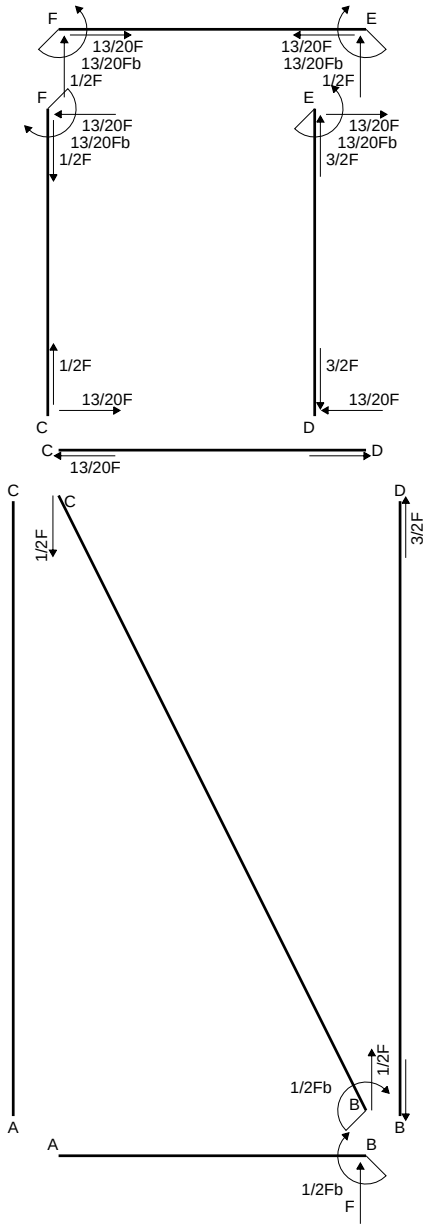
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

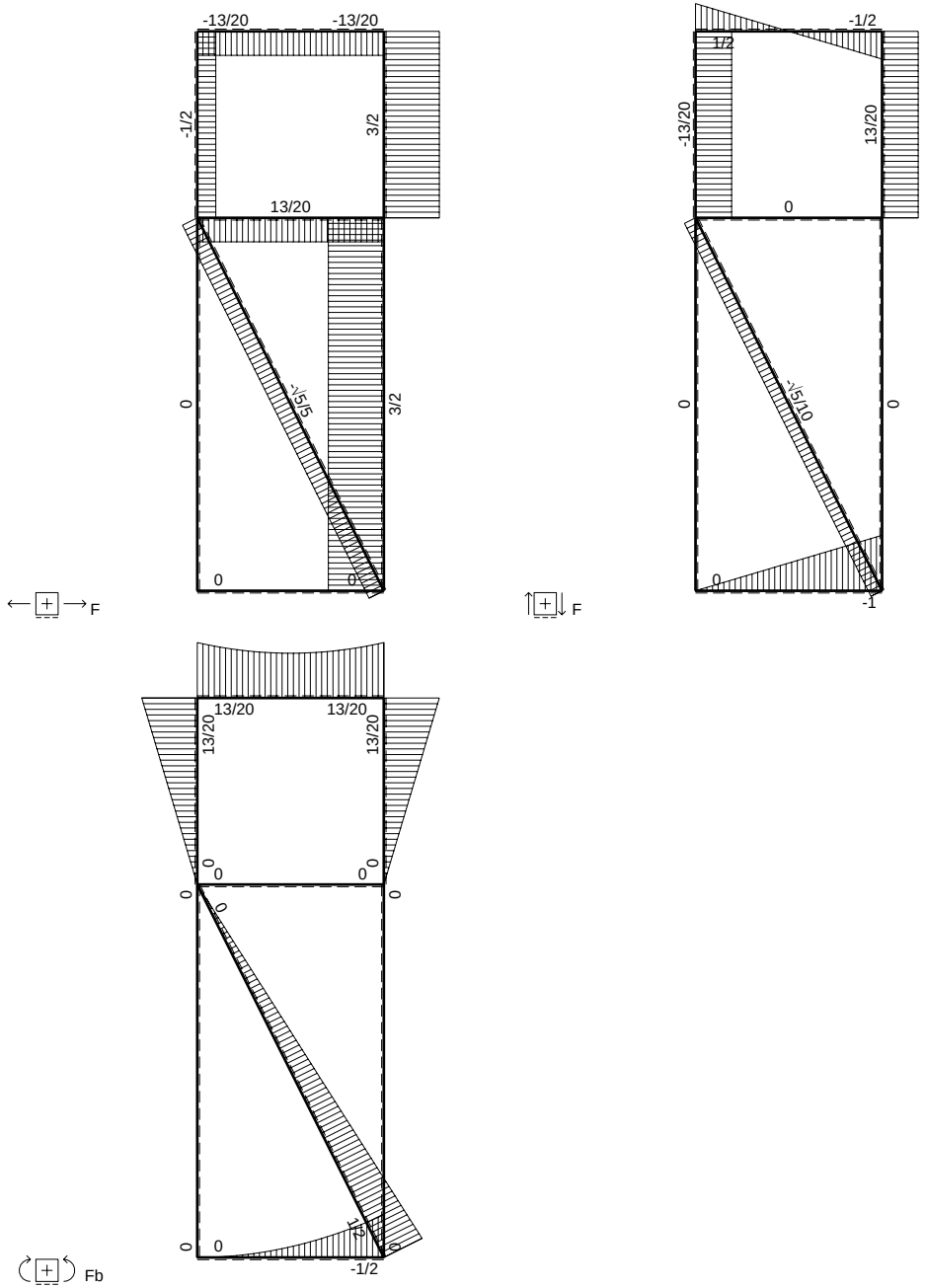


A = 486. mm^2
J_x = 175218. mm^4
J_y = 15474. mm^4
J_{xy} = 33964. mm^4
J_u = 182139. mm^4
J_v = 8553. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2010
c = cosα = .9799
s = sinα = -.1997
x_g = 15.22 mm
y_g = 28.02 mm
N = -858.7 N
T_y = -429.3 N
M_x = 672000. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -8.545 mm
v_m = 25.79 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.1 N/mm^2

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
F
1/2
-√5/10
-1
13/20
Fb
-1/2

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

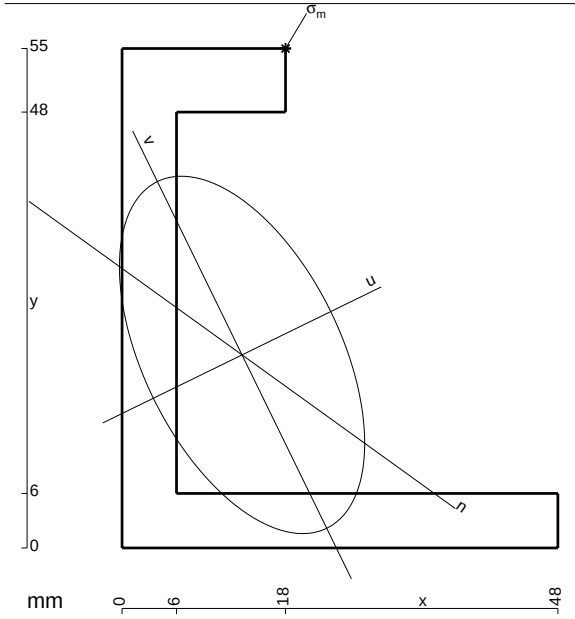
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 247885 CI09.xxxx.069

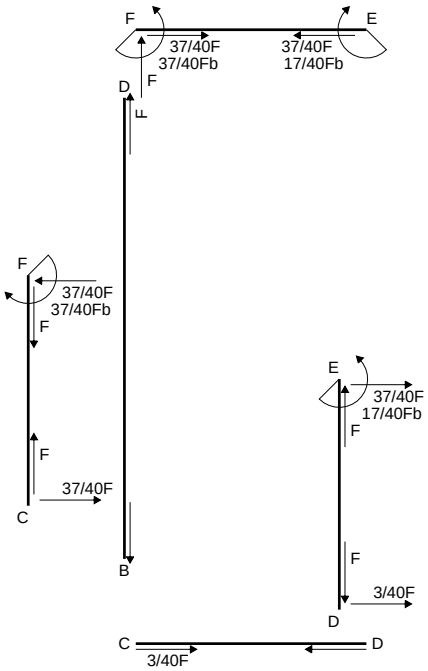


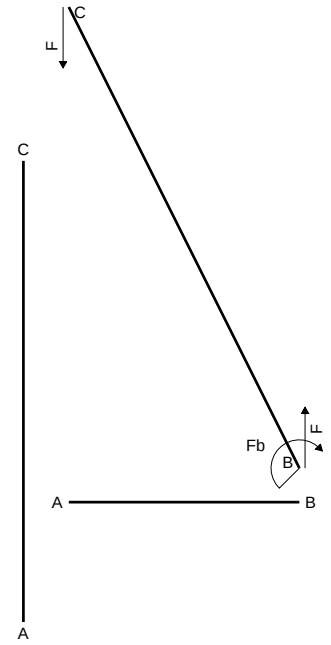
A = 666. mm^2
J_x = 257974. mm^4
J_y = 121487. mm^4
J_{xy} = -87553. mm^4
J_u = 300738. mm^4
J_v = 78723. mm^4
$\alpha = artg(2J_{xy}/(J_y - J_x))/2$ = .4544
$c = \cos\alpha$ = .8985
$s = \sin\alpha$ = .4389
x_g = 13.22 mm
y_g = 21.26 mm
N = -1677. N
T_y = -1290. N
M_x = 1240980. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 19.11 mm
v_m = 28.22 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -239.4 N/mm^2

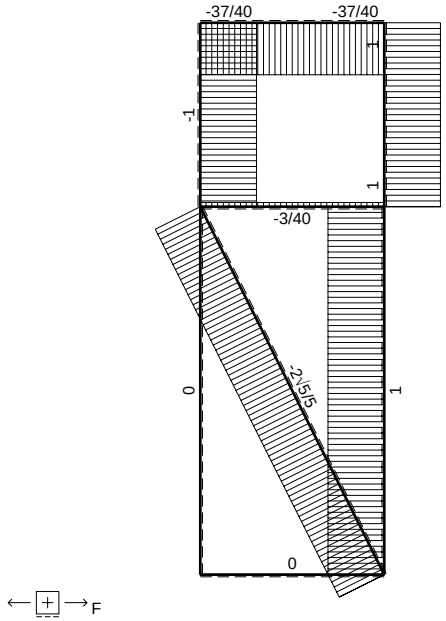
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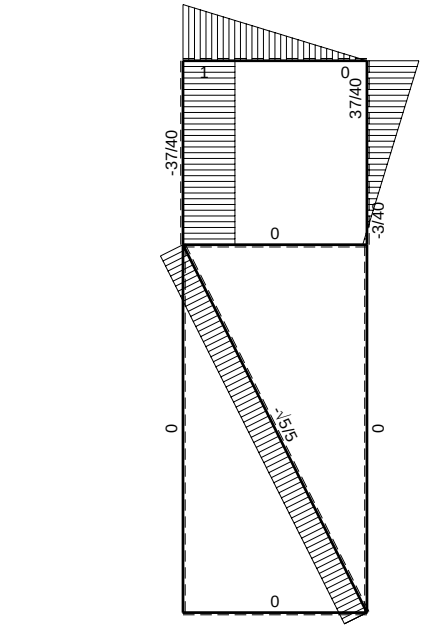
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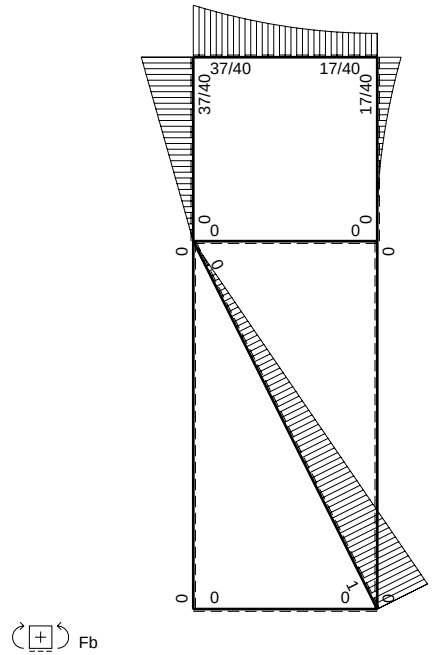
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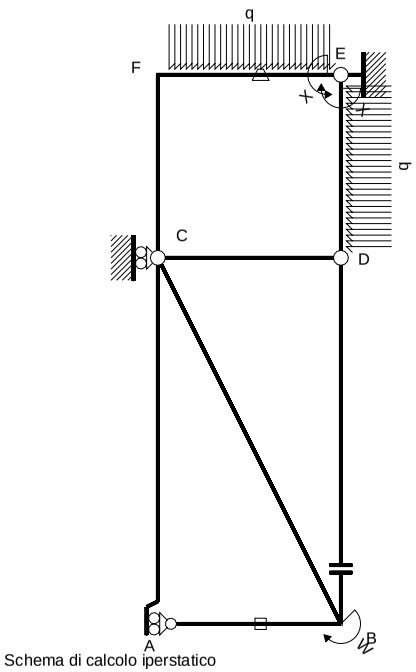

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
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C
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Fb
F
B
A
B
A

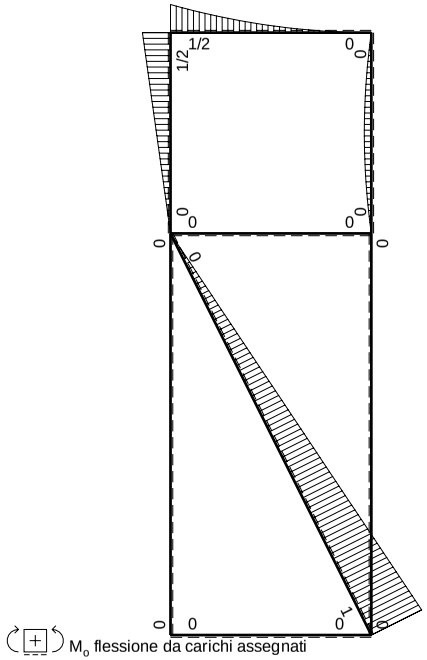

-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
1
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
0
F

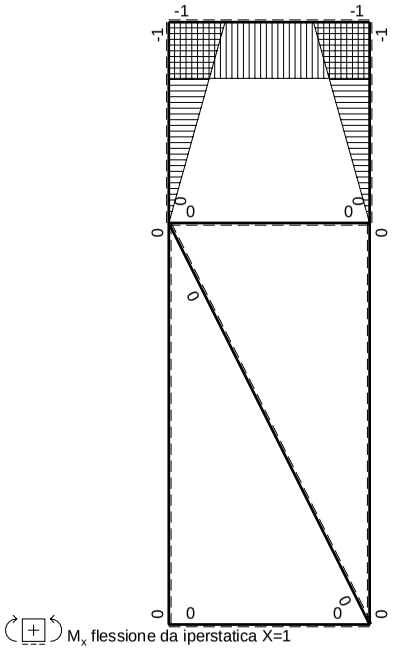

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 265084 CI09.xxxx.070

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

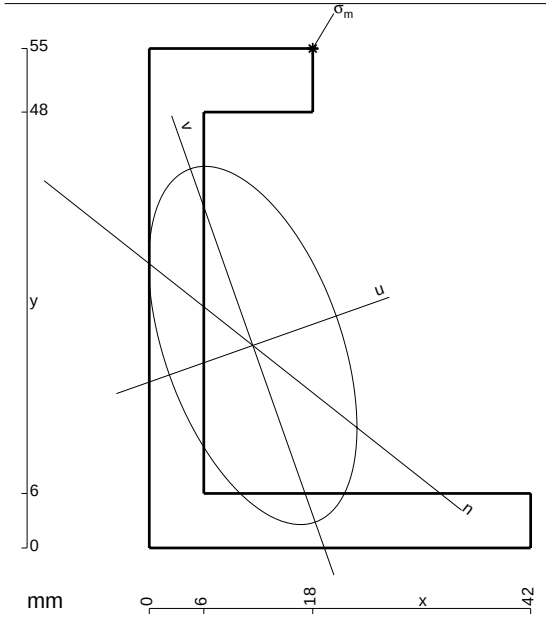
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

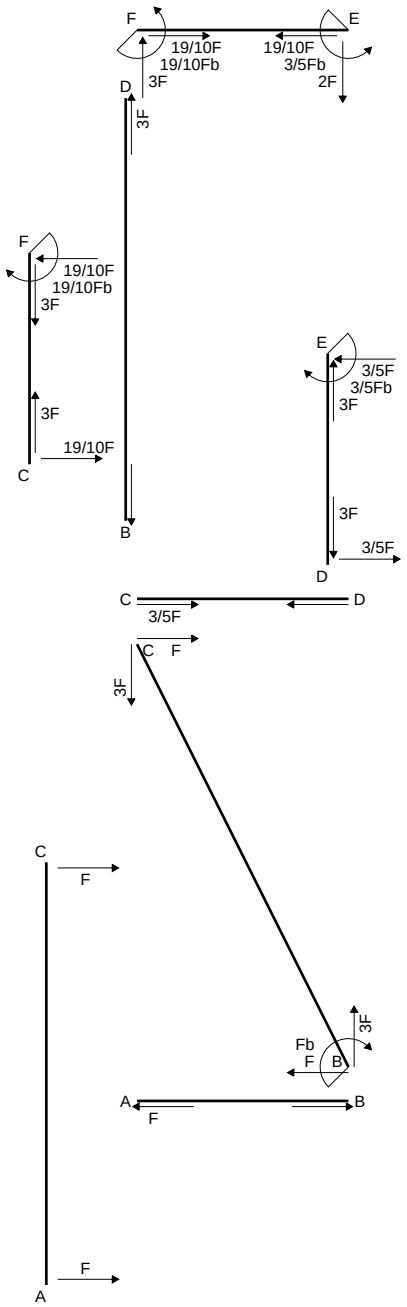
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

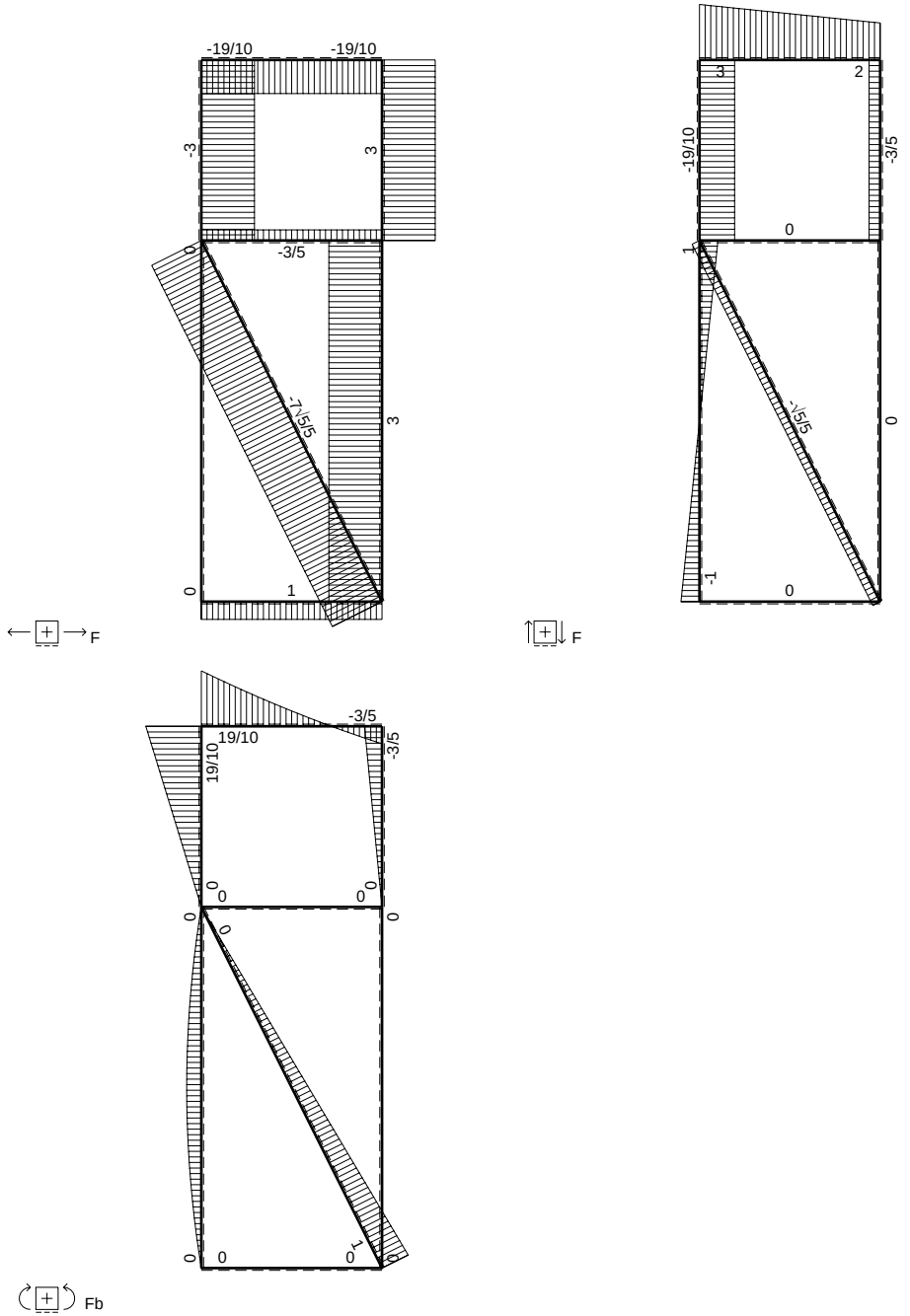


A = 630. mm^2
J_x = 245181. mm^4
J_y = 82933. mm^4
J_{xy} = -65470. mm^4
J_u = 268304. mm^4
J_v = 59811. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3395
c = cosα = .9429
s = sinα = .3330
x_g = 11.4 mm
y_g = 22.3 mm
N = -2290. N
T_y = -1145. N
M_x = 998400. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 17.11 mm
v_m = 28.64 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -199.2 N/mm^2

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F
E
19/10F
19/10Fb
19/10F
3/5Fb
D
3F
2F
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
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19/10F
C
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3/5F
C
D
3/5F
C
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3F
C
F
Fb
F
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3F
A
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F
F
A


-19/10
-19/10
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-3/5
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-19/10
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-1\5/5
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-1
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19/10
-3/5
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0
Fb

16.04.26

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx²	-Fb/EJ	-2Fx-1/2Fx²/b	Fb/EJ	1	(-7/6+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx²	Fb/EJ	-5/2Fb+3Fx-1/2Fx²/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
totali							-Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							3/5Fb

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

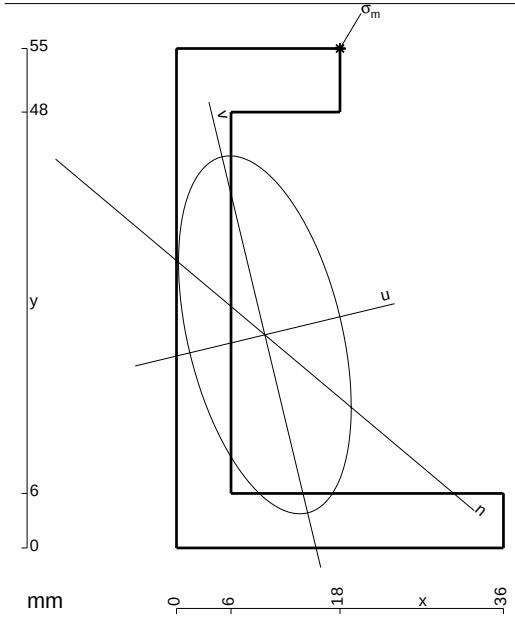
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

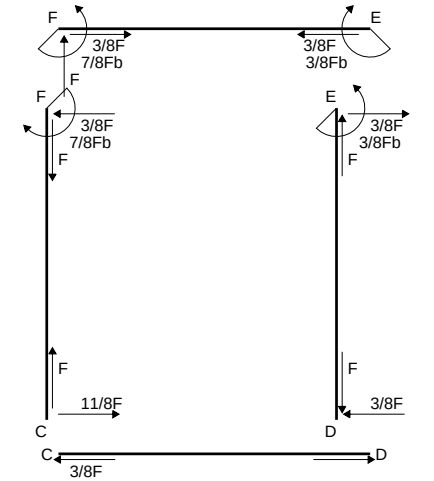
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

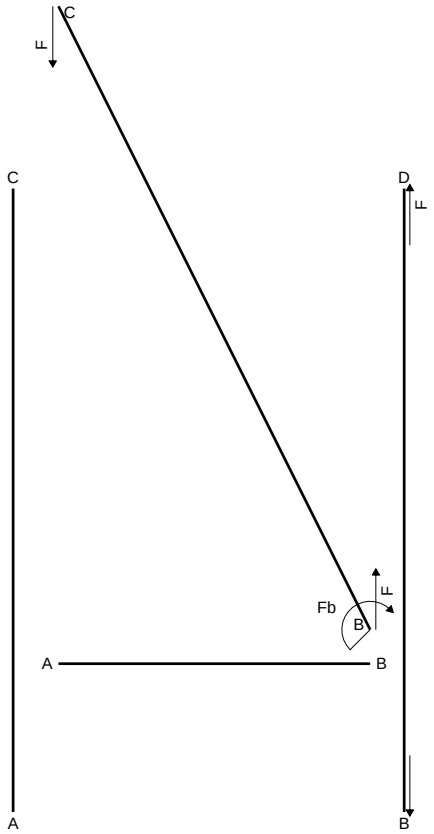


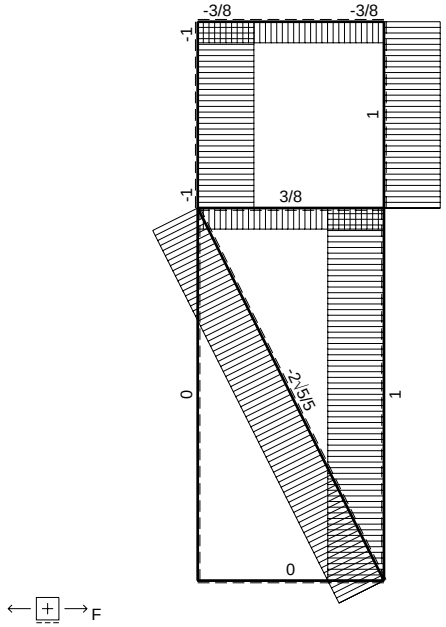
A = 594. mm^2
J_x = 230851. mm^4
J_y = 53740. mm^4
J_{xy} = -45131. mm^4
J_u = 241688. mm^4
J_v = 42903. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2357
c = cosα = .9724
s = sinα = .2335
x_g = 9.727 mm
y_g = 23.47 mm
N = -3810. N
T_y = -2413. N
M_x = 1013460. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 15.41 mm
v_m = 28.73 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -208.5 N/mm^2

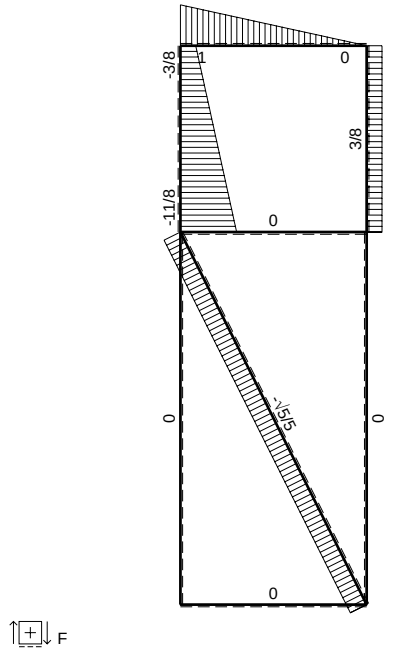
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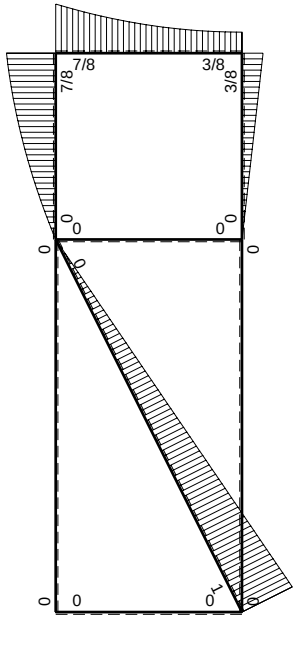
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F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
3/8F
C
D
C
D
3/8F


C
F
C
D
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Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
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0
F


7/8
7/8
3/8
3/8
0
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0
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0
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0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

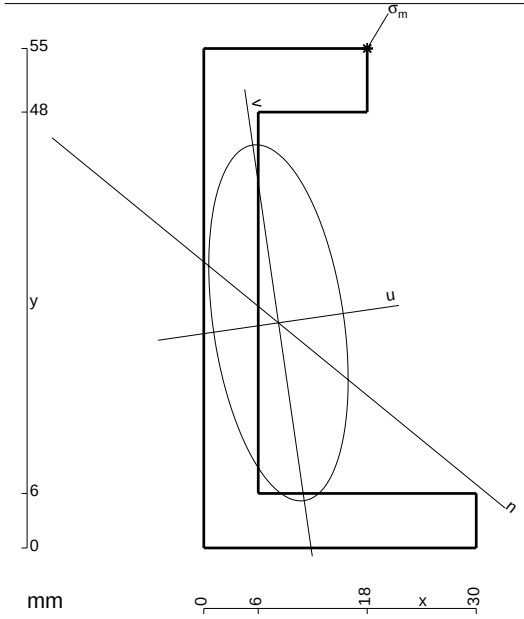
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

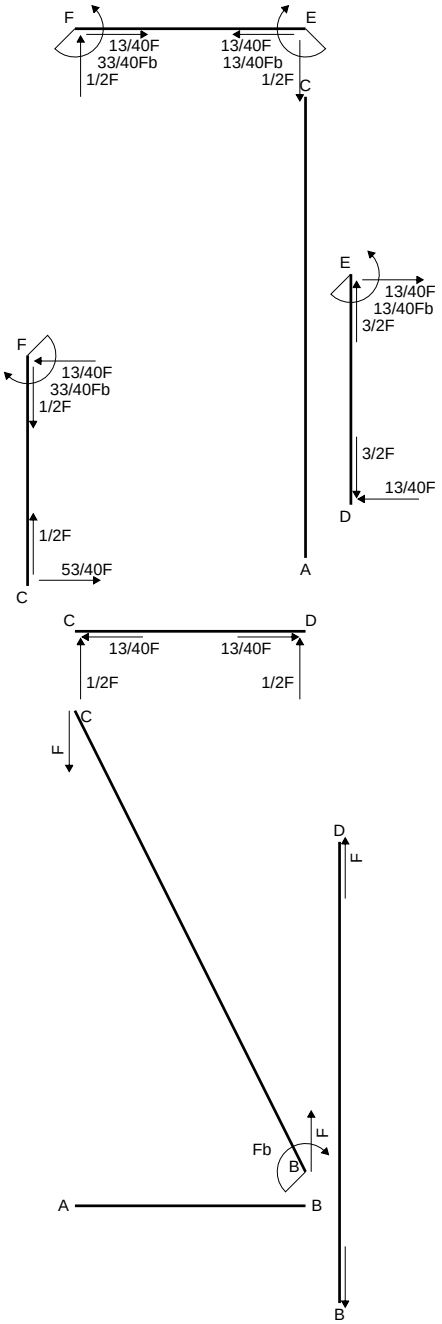
$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

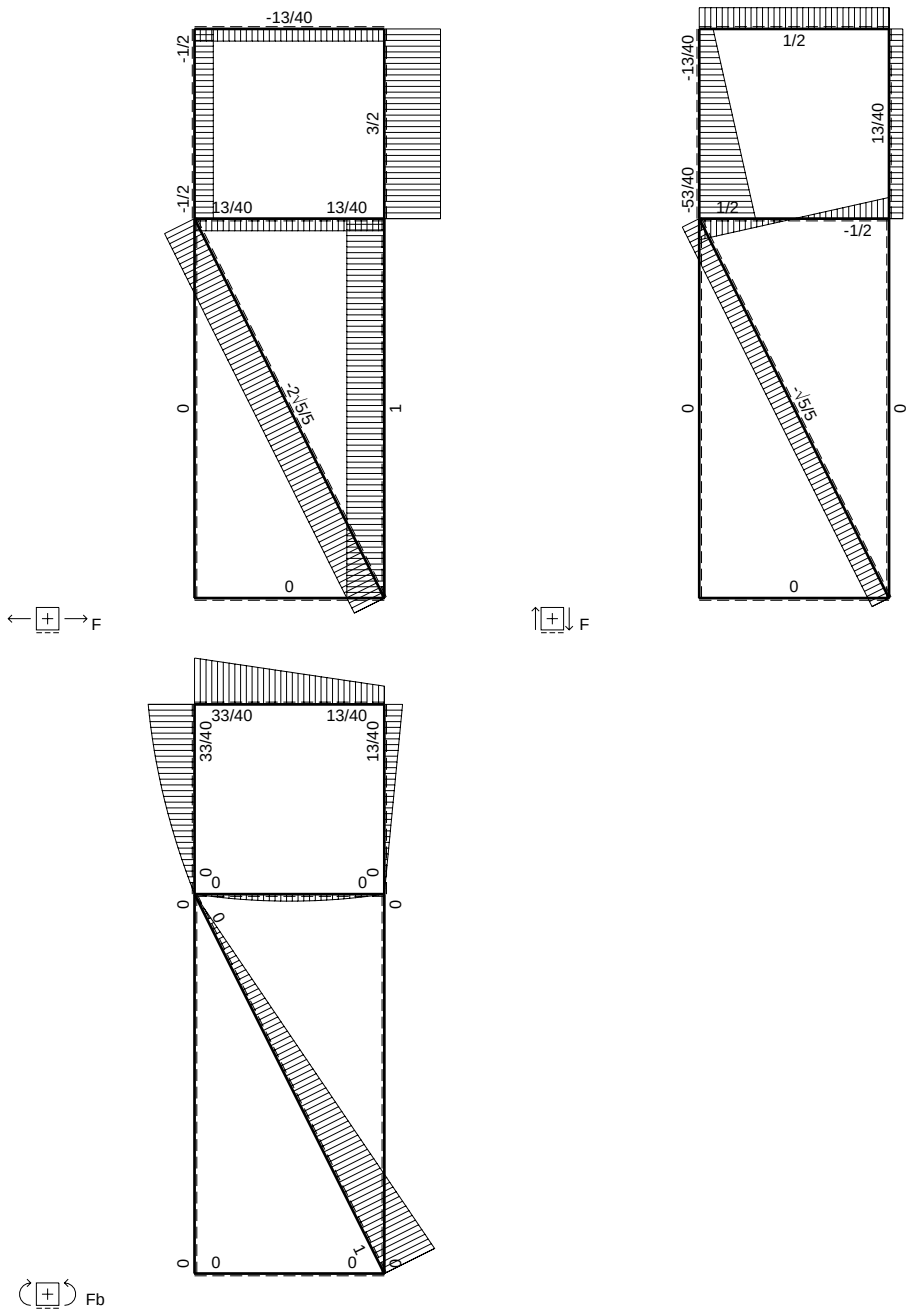


A = 558. mm^2
J_x = 214685. mm^4
J_y = 32876. mm^4
J_{xy} = -26875. mm^4
J_u = 218575. mm^4
J_v = 28986. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1437
$c = \cos\alpha$ = .9897
$s = \sin\alpha$ = .1432
x_g = 8.226 mm
y_g = 24.79 mm
N = -2120. N
T_y = -1060. N
M_x = 1090200. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 14. mm
v_m = 28.5 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.9 N/mm^2

CI09.xxxx.072

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
-2√5/5
0
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XbEJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-FbEJ	-1/2Fx	FbEJ	1	$(-1/4+1)Fb^2/EJ$	XbEJ
FE b	1	$-1/2Fb+1/2Fx$	FbEJ	$-1/2Fb+1/2Fx$	FbEJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3XbEJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XbEJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264989 CI09.xxxx.073

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

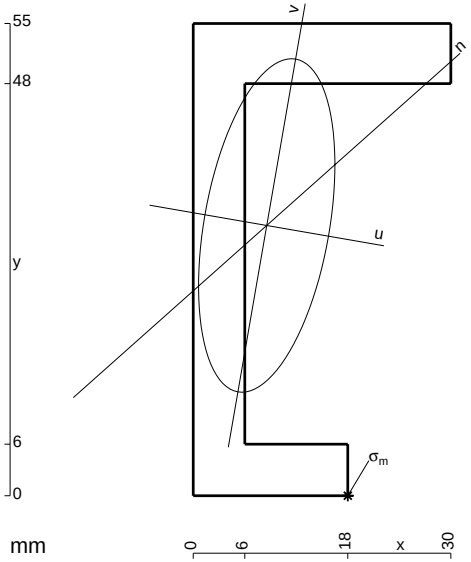
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

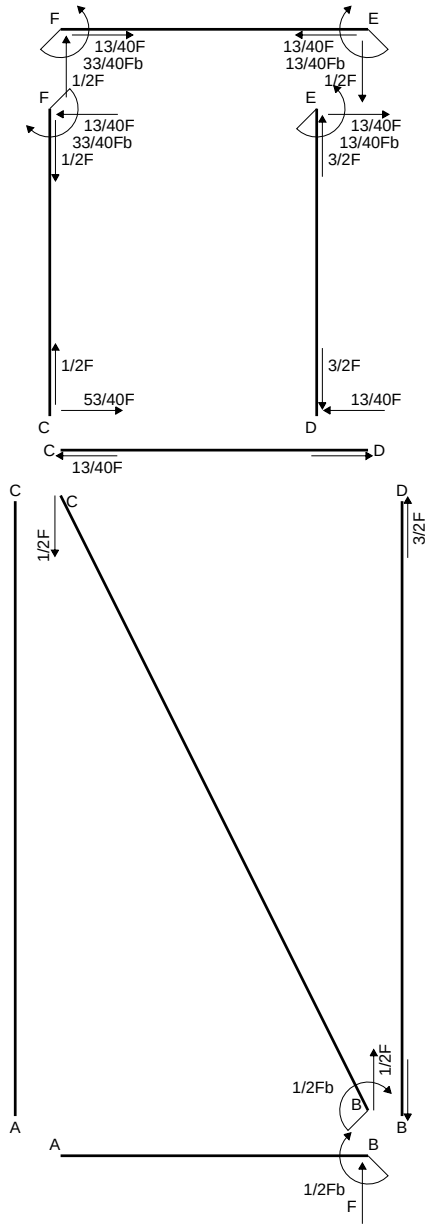
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

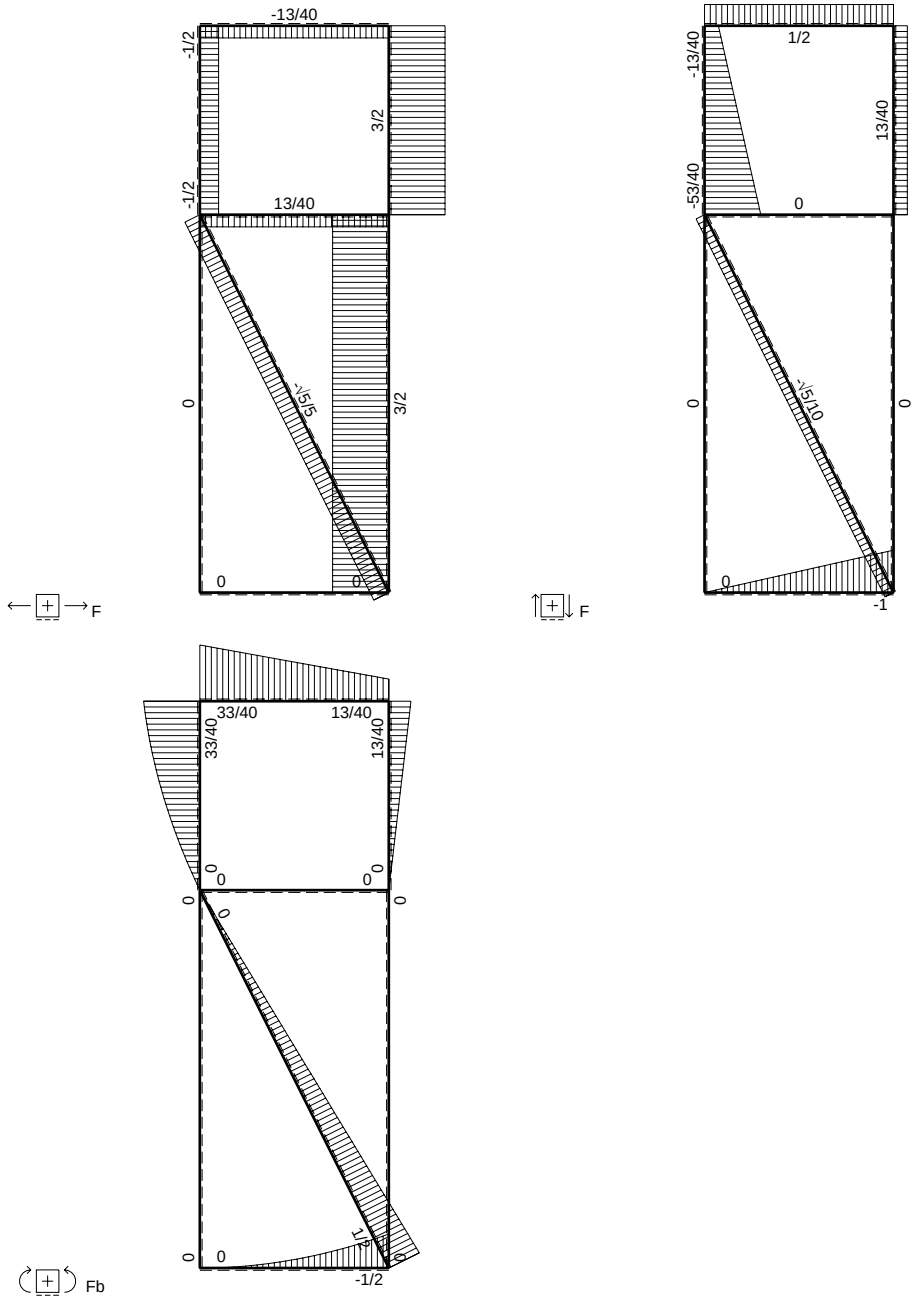


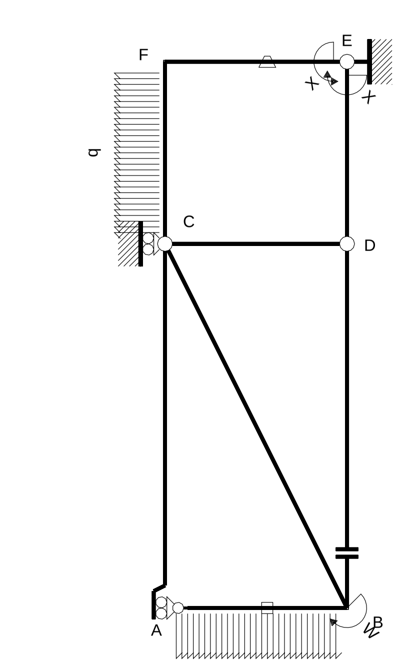
A = 570. mm^2
J_x = 215051. mm^4
J_y = 35943. mm^4
J_{xy} = 31999. mm^4
J_u = 220596. mm^4
J_v = 30398. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = -.1716
$c = \cos\alpha$ = .9853
$s = \sin\alpha$ = -.1707
x_g = 8.558 mm
y_g = 31.48 mm
N = -1950. N
T_y = -974.9 N
M_x = 1090000. Nmm
x_m = 18. mm
u_m = 14.68 mm
v_m = -29.4 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 229.6 N/mm^2

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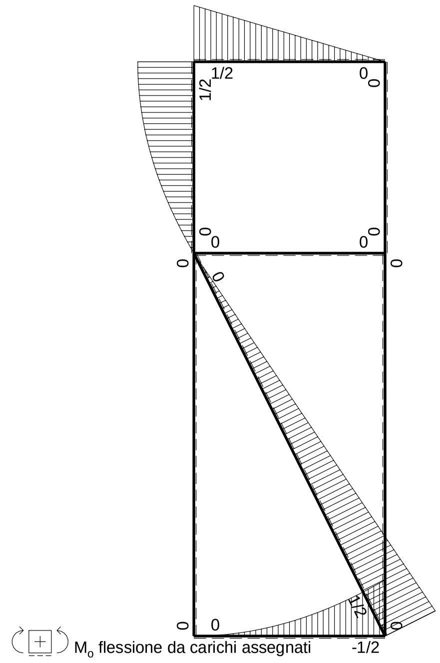
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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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B
A
B
1/2Fb
F

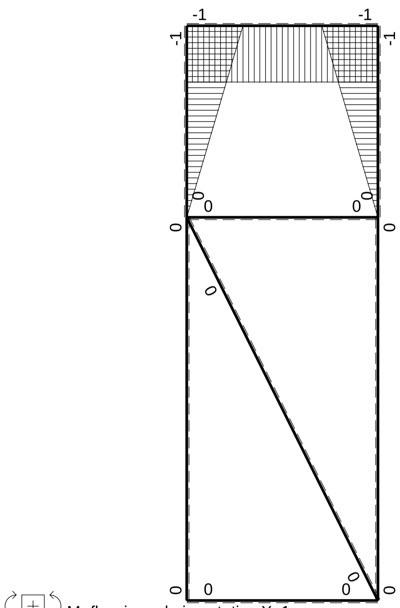

-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 274704 CI09.xxxx.074

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

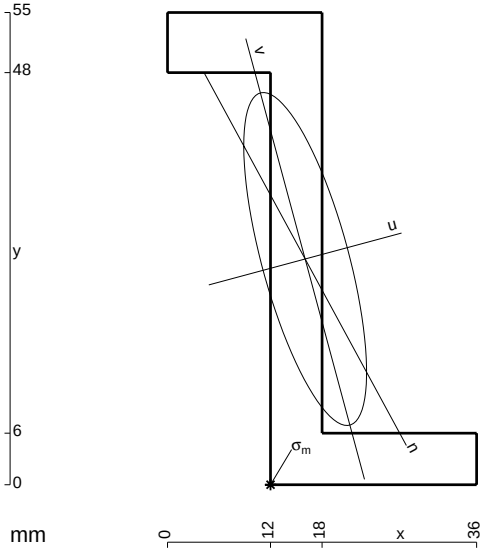
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

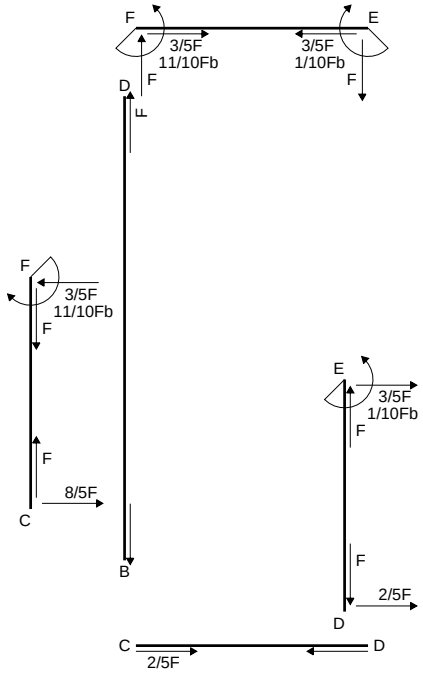
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

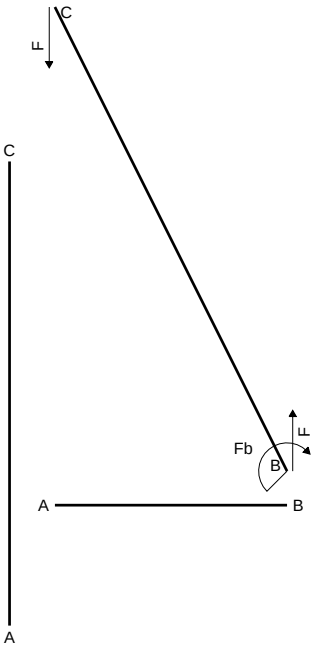


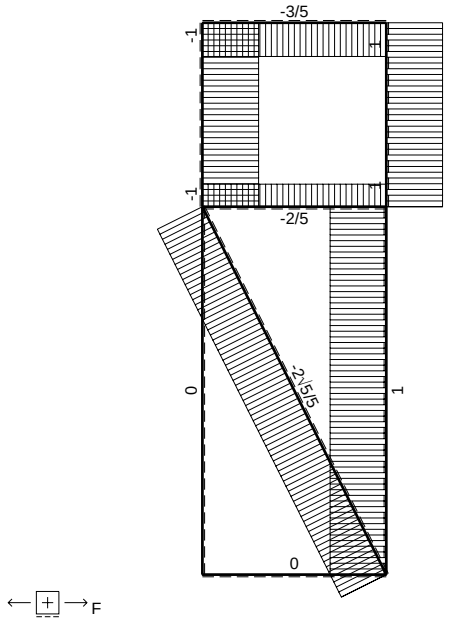
A = 522. mm^2
J_x = 196305. mm^4
J_y = 26711. mm^4
J_{xy} = -49244. mm^4
J_u = 209567. mm^4
J_v = 13450. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2631
$c = \cos\alpha$ = .9656
$s = \sin\alpha$ = .2600
x_g = 16.03 mm
y_g = 26.29 mm
N = -549.3 N
T_y = 845. N
M_x = 752895. Nmm
x_m = 12. mm
u_m = -10.73 mm
v_m = -24.34 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.6 N/mm^2

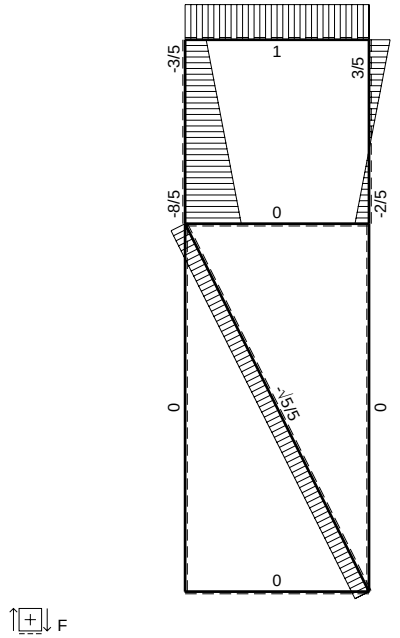
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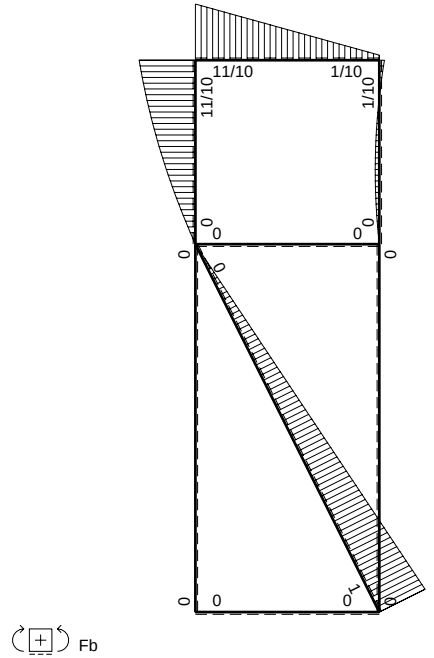
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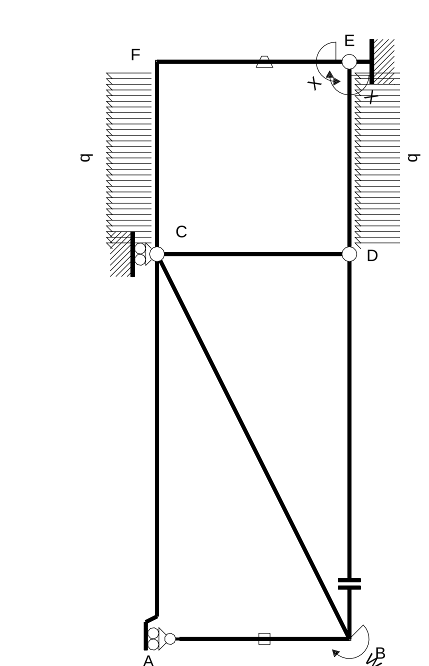

F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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E
3/5F
1/10Fb
F
F
8/5F
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Fb
F
B
A
B
A

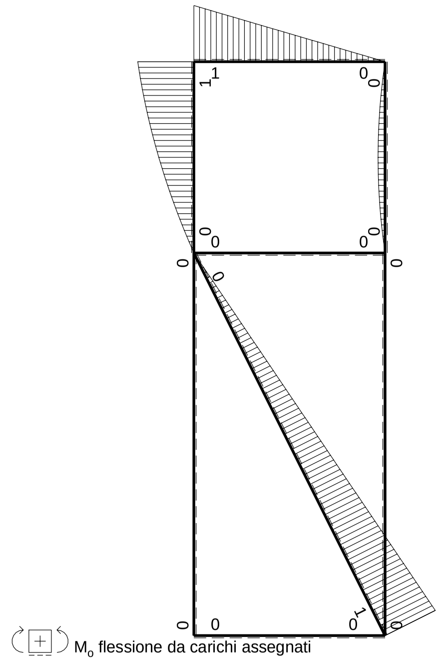

-3/5
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-1
1
-2/5
0
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1
0
F


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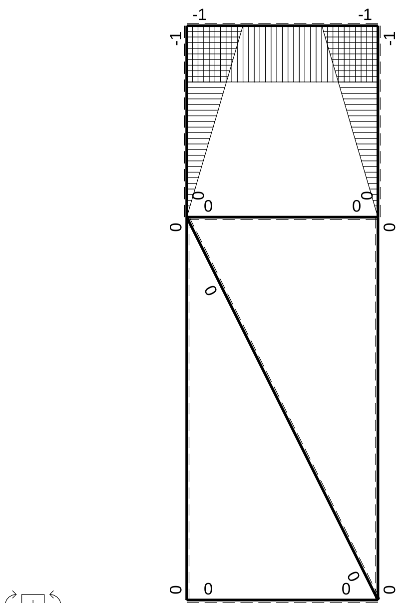

11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$1/6Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

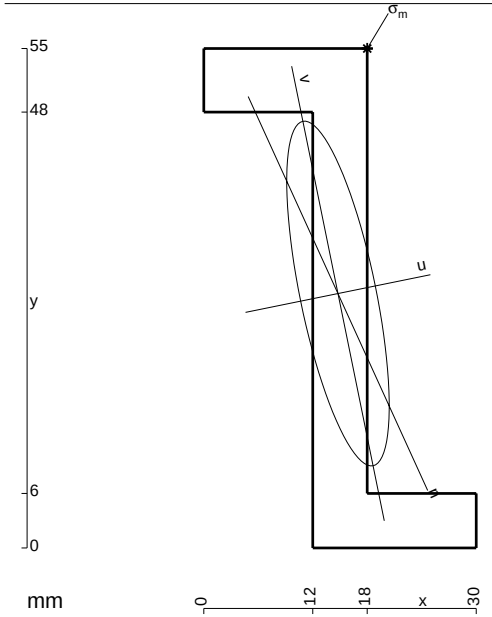
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

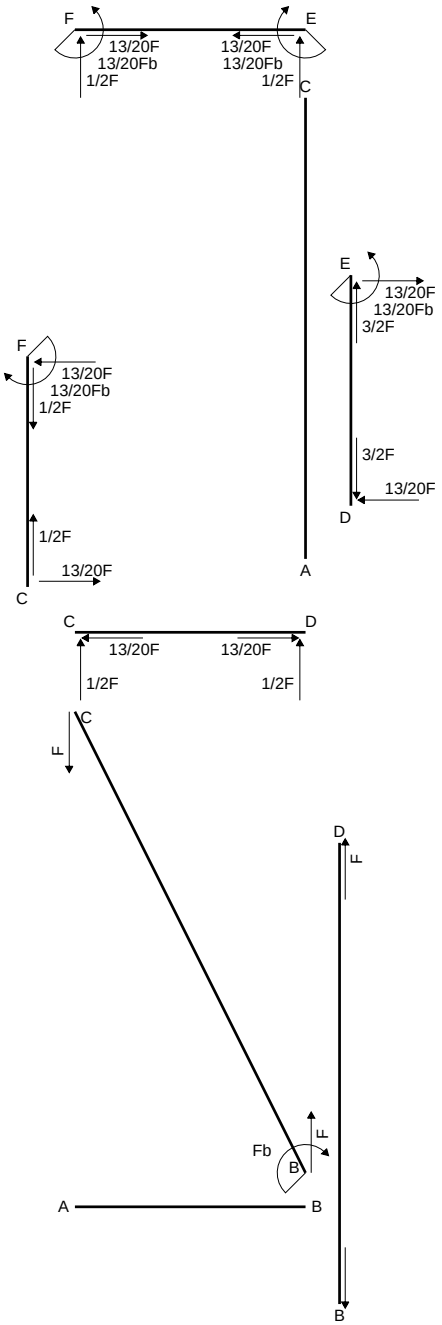
$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$

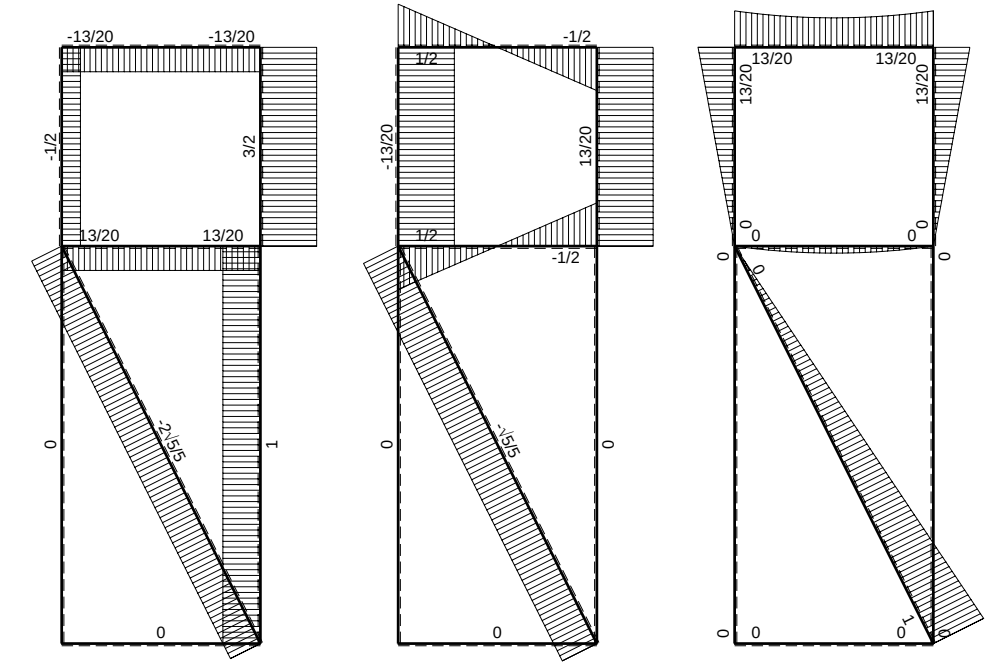


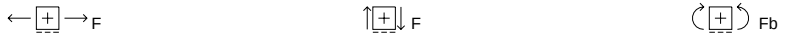
A = 486. mm^2
J_x = 175218. mm^4
J_y = 15474. mm^4
J_{xy} = -33964. mm^4
J_u = 182139. mm^4
J_v = 8553. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2010
$c = \cos\alpha$ = .9799
$s = \sin\alpha$ = .1997
x_g = 14.78 mm
y_g = 28.02 mm
N = -910. N
T_y = -546. N
M_x = 580580. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 8.545 mm
v_m = 25.79 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -198.3 N/mm^2

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F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5


F
F
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

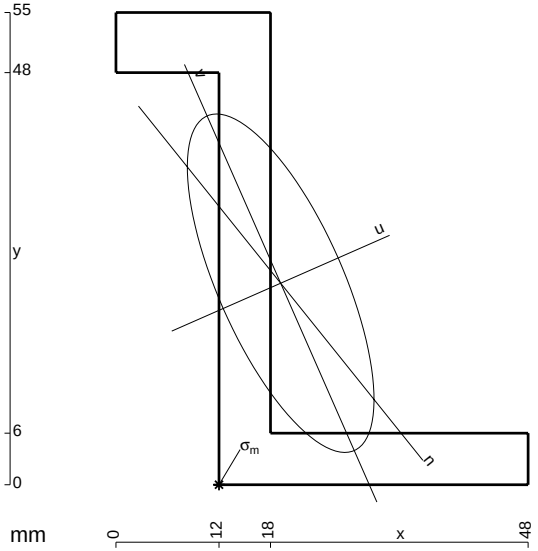
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

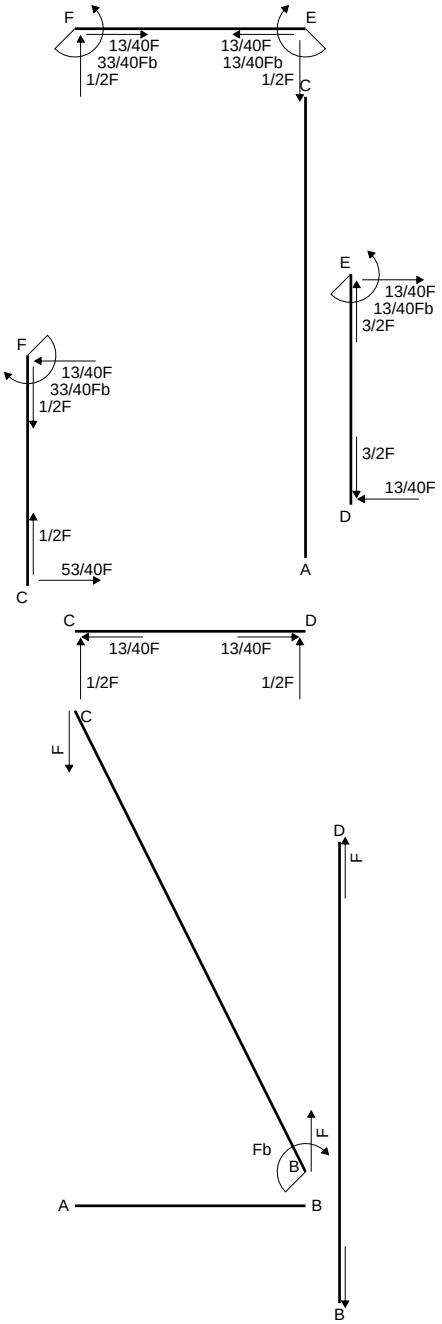
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

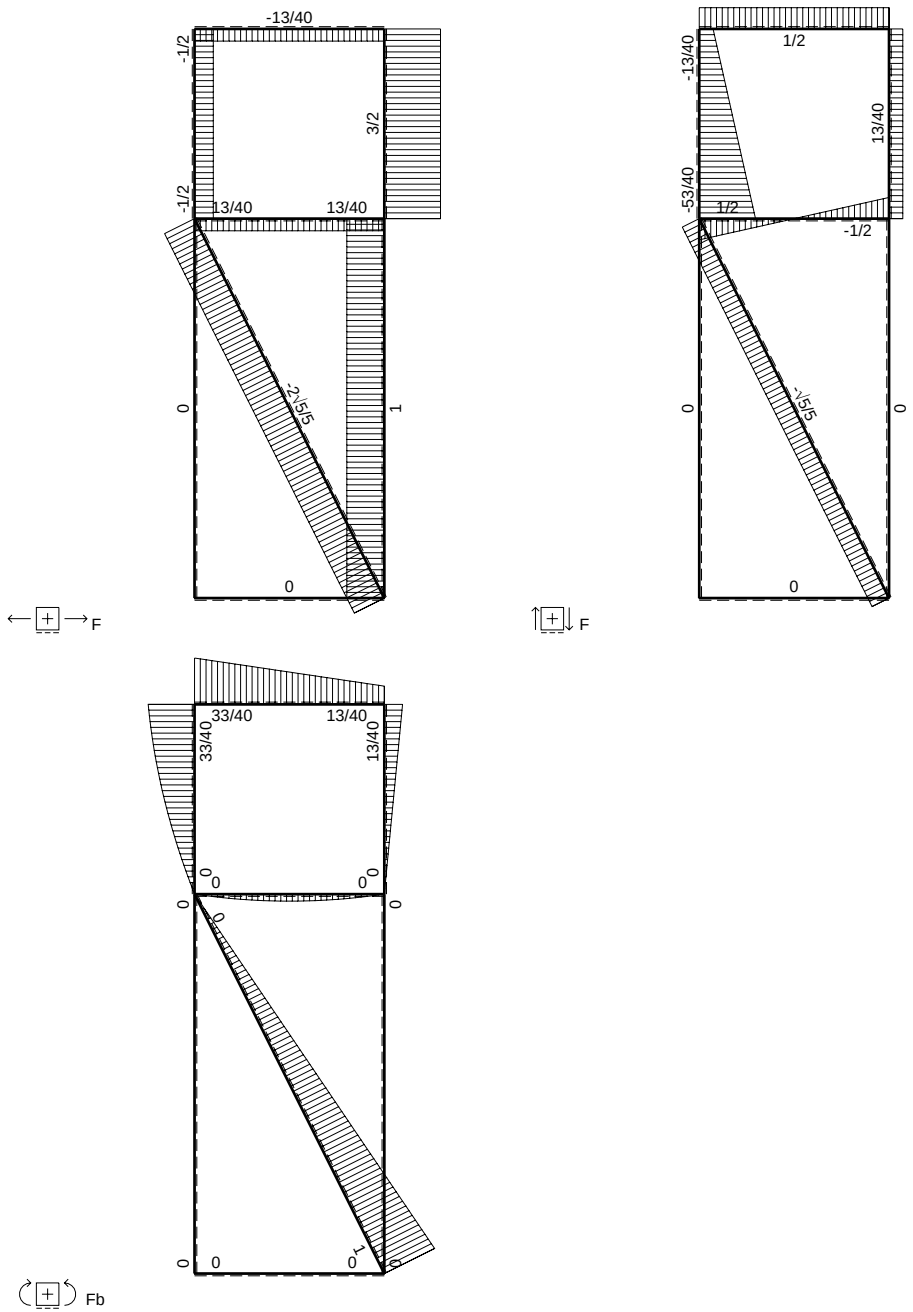


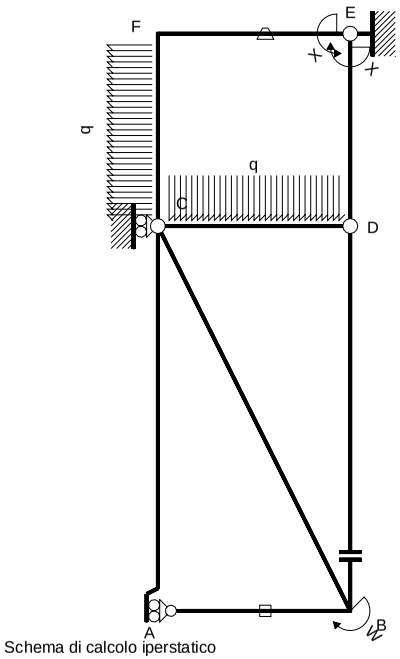
A = 594. mm^2
J_x = 230851. mm^4
J_y = 70234. mm^4
J_{xy} = -87513. mm^4
J_u = 269319. mm^4
J_v = 31766. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4141
$c = \cos\alpha$ = .9155
$s = \sin\alpha$ = .4024
x_g = 19.18 mm
y_g = 23.47 mm
N = -1145. N
T_y = -572.4 N
M_x = 793600. Nmm
x_m = 12. mm
u_m = -16.02 mm
v_m = -18.6 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 209.3 N/mm^2

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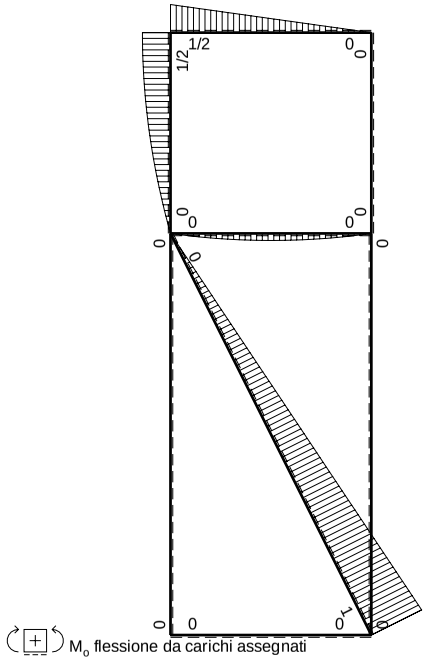
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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B

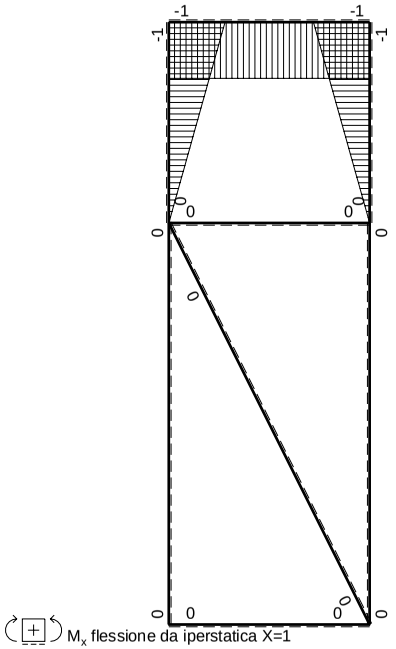

-13/40
-1/2
-1/2
3/2
13/40
13/40
-2√5/5
0
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 265715 CI09.xxxx.077

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

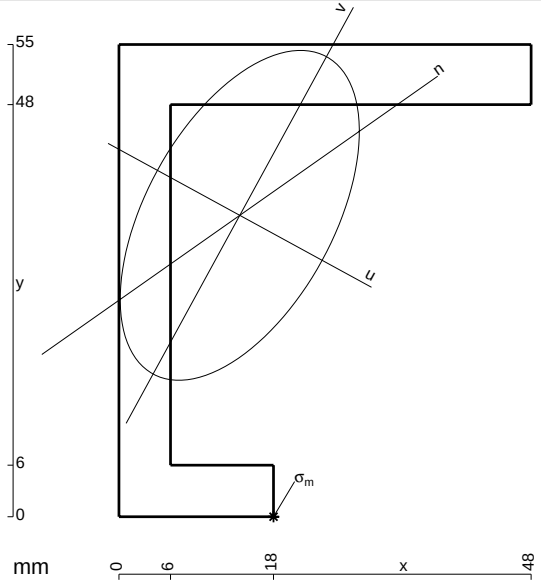
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

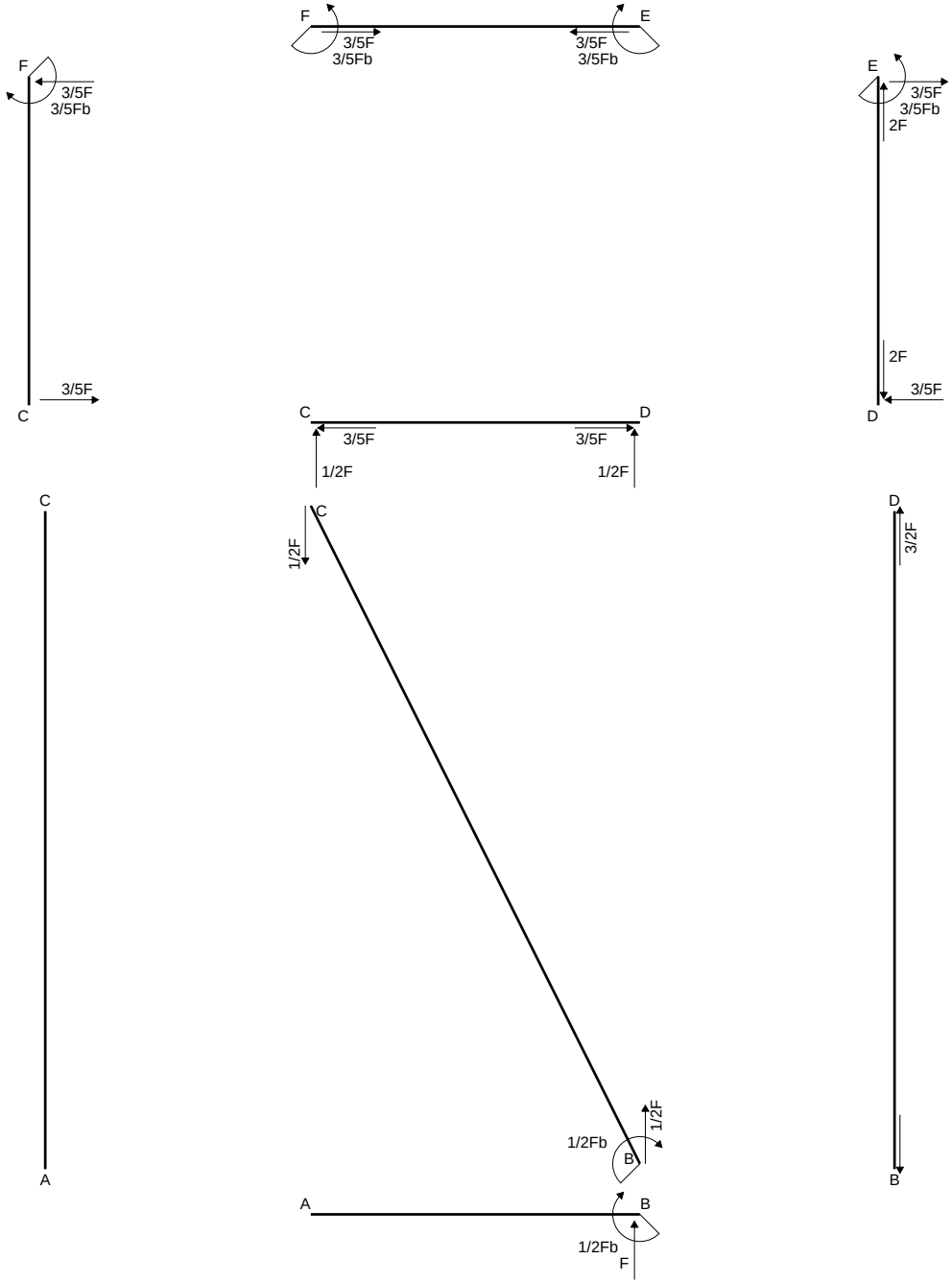
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

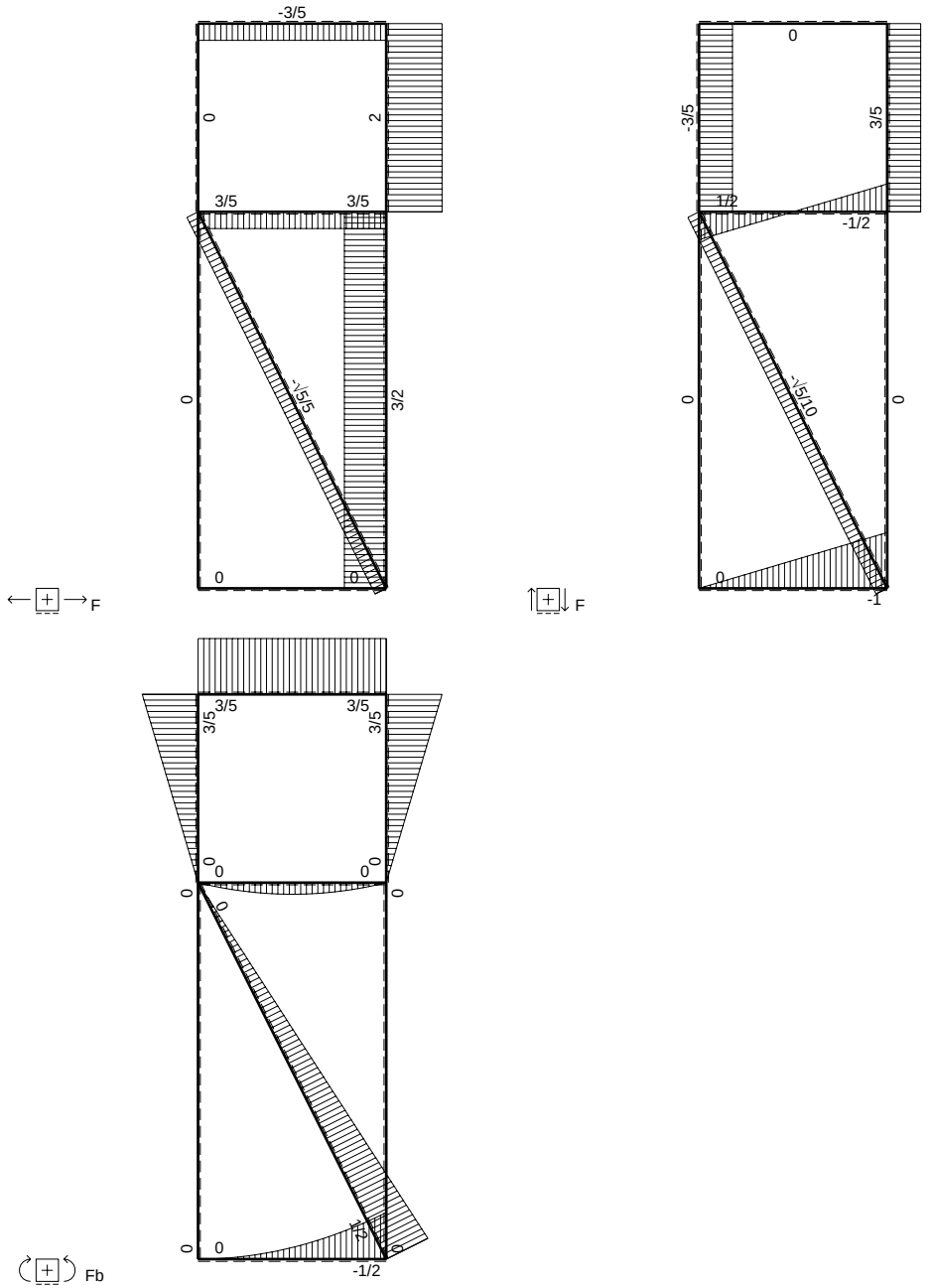


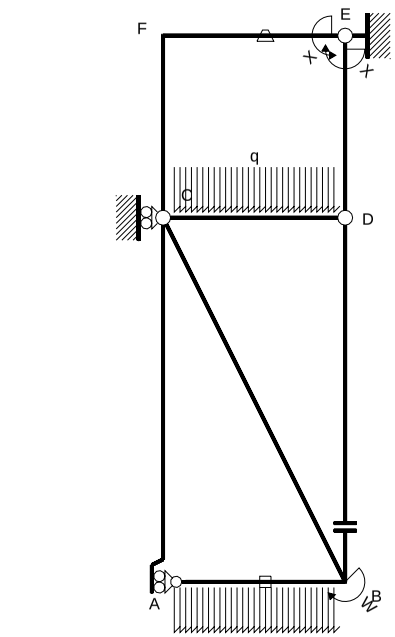
A = 696. mm^2
J_x = 256929. mm^4
J_y = 134973. mm^4
J_{xy} = 94891. mm^4
J_u = 308745. mm^4
J_v = 83156. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.4998$
$c = \cos\alpha = .8777$
$s = \sin\alpha = -.4793$
x_g = 14.07 mm
y_g = 35.1 mm
N = -1512. N
T_y = -755.8 N
M_x = 1115400. Nmm
x_m = 18. mm
u_m = 20.27 mm
v_m = -28.93 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 219.9 N/mm^2

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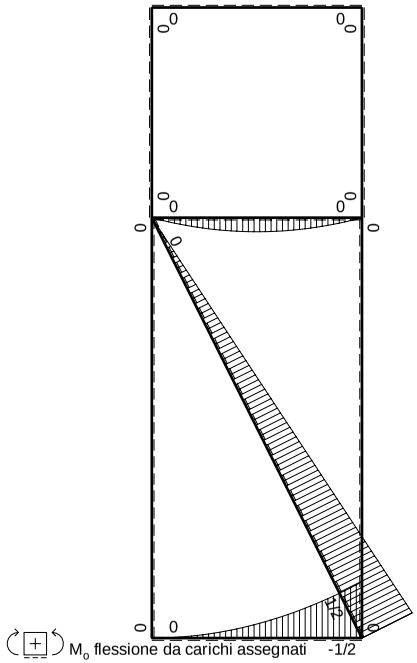
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
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A
A
B
1/2Fb
F

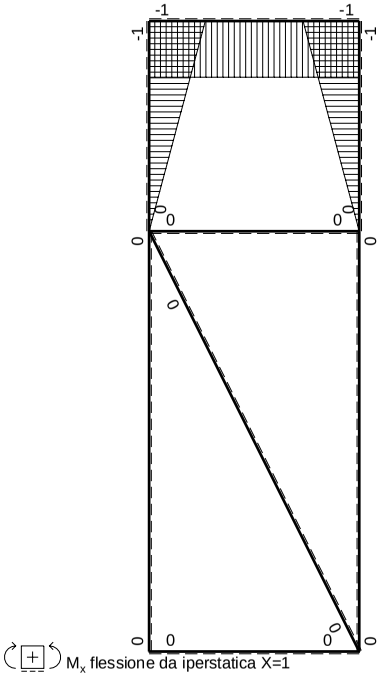

-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
0
0
-√2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

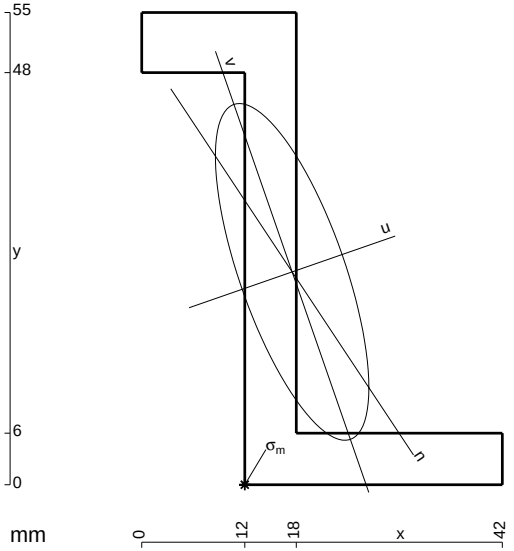
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

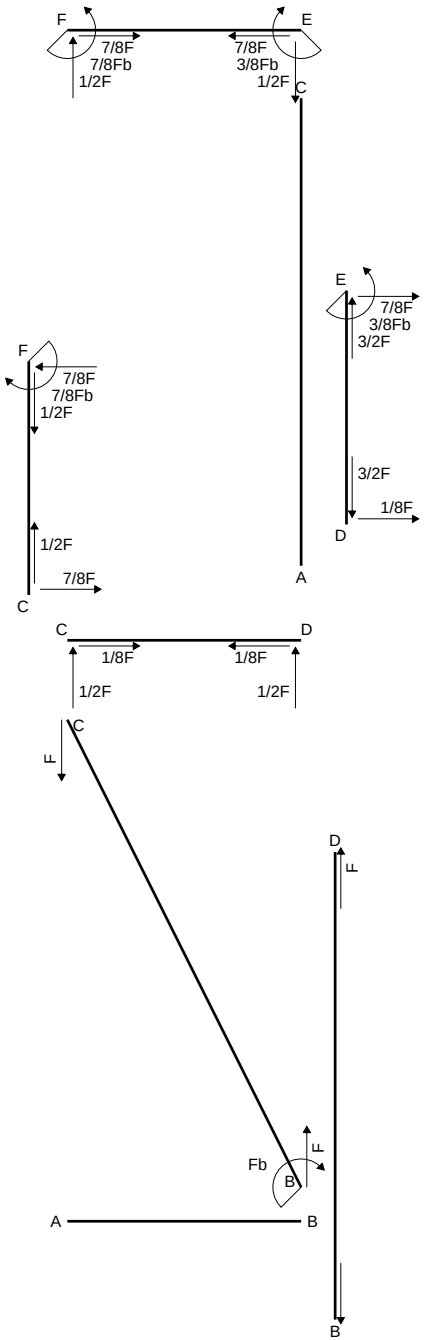
$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

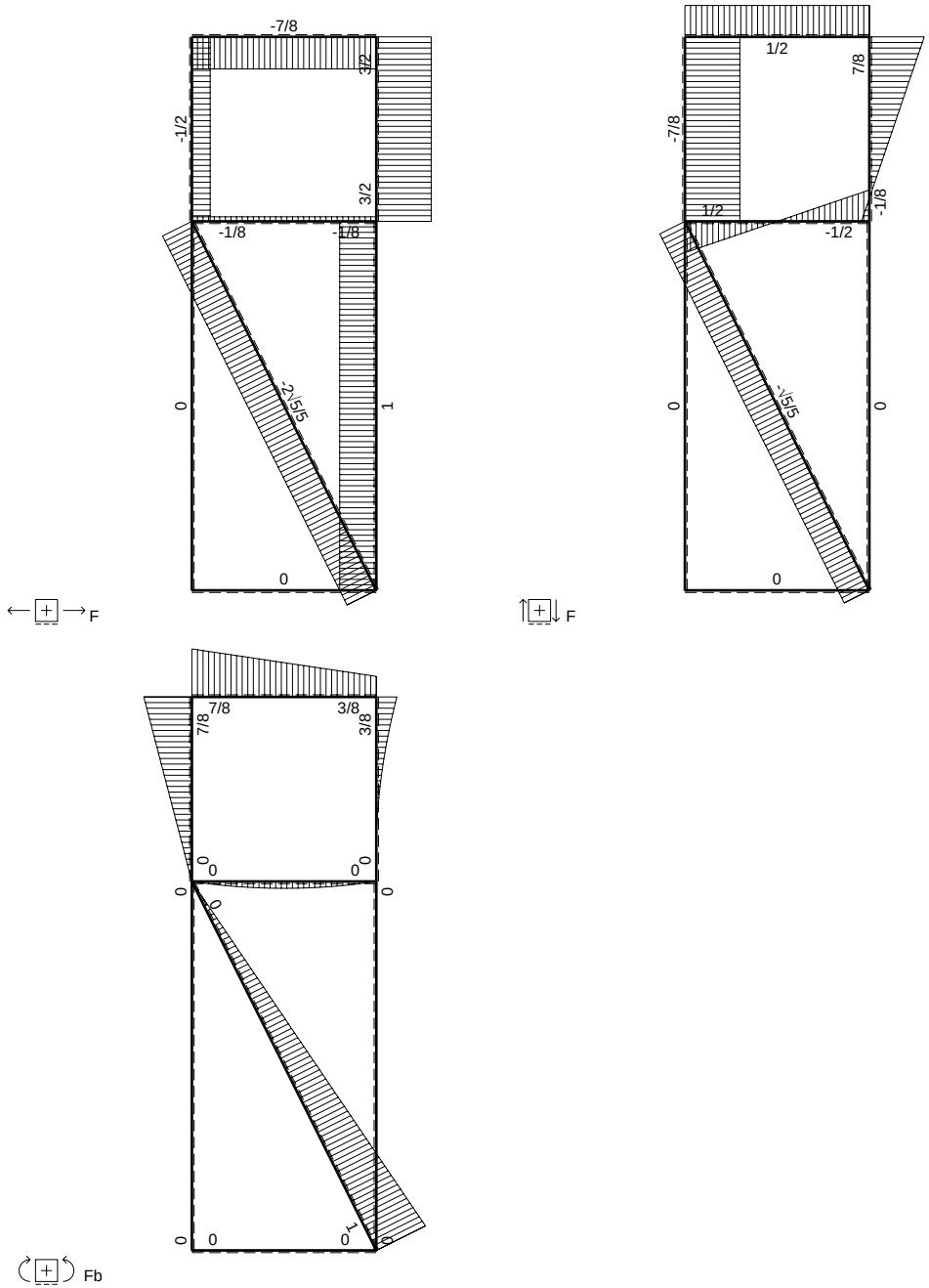


A = 558. mm^2
J_x = 214685. mm^4
J_y = 44581. mm^4
J_{xy} = -67260. mm^4
J_u = 238066. mm^4
J_v = 21201. mm^4
$\alpha = artg(2J_{xy}/(J_y - J_x))/2$ = .3346
$c = \cos\alpha$ = .9446
$s = \sin\alpha$ = .3283
x_g = 17.52 mm
y_g = 24.79 mm
N = 3620. N
T_y = 1086. N
M_x = 760200. Nmm
x_m = 12. mm
u_m = -13.35 mm
v_m = -21.6 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 228.8 N/mm^2

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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
-√5/5
0
1
0
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
-√5/5
0
0
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264526 CI09.xxxx.079

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

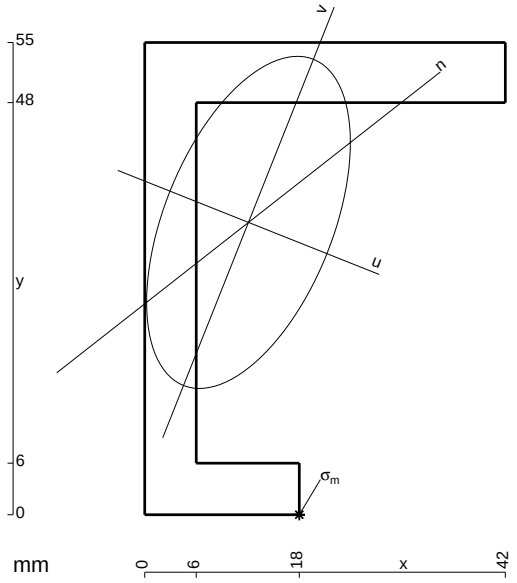
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

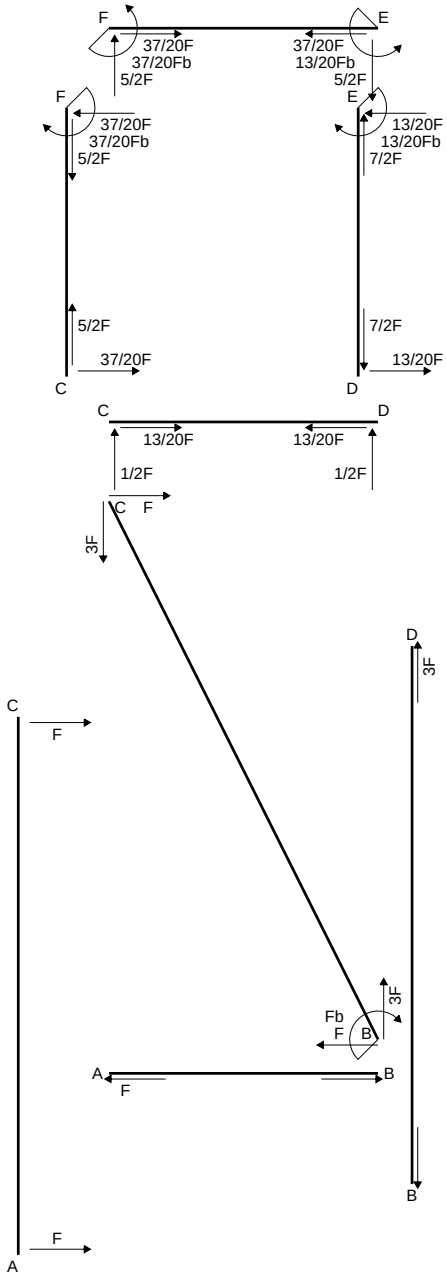
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

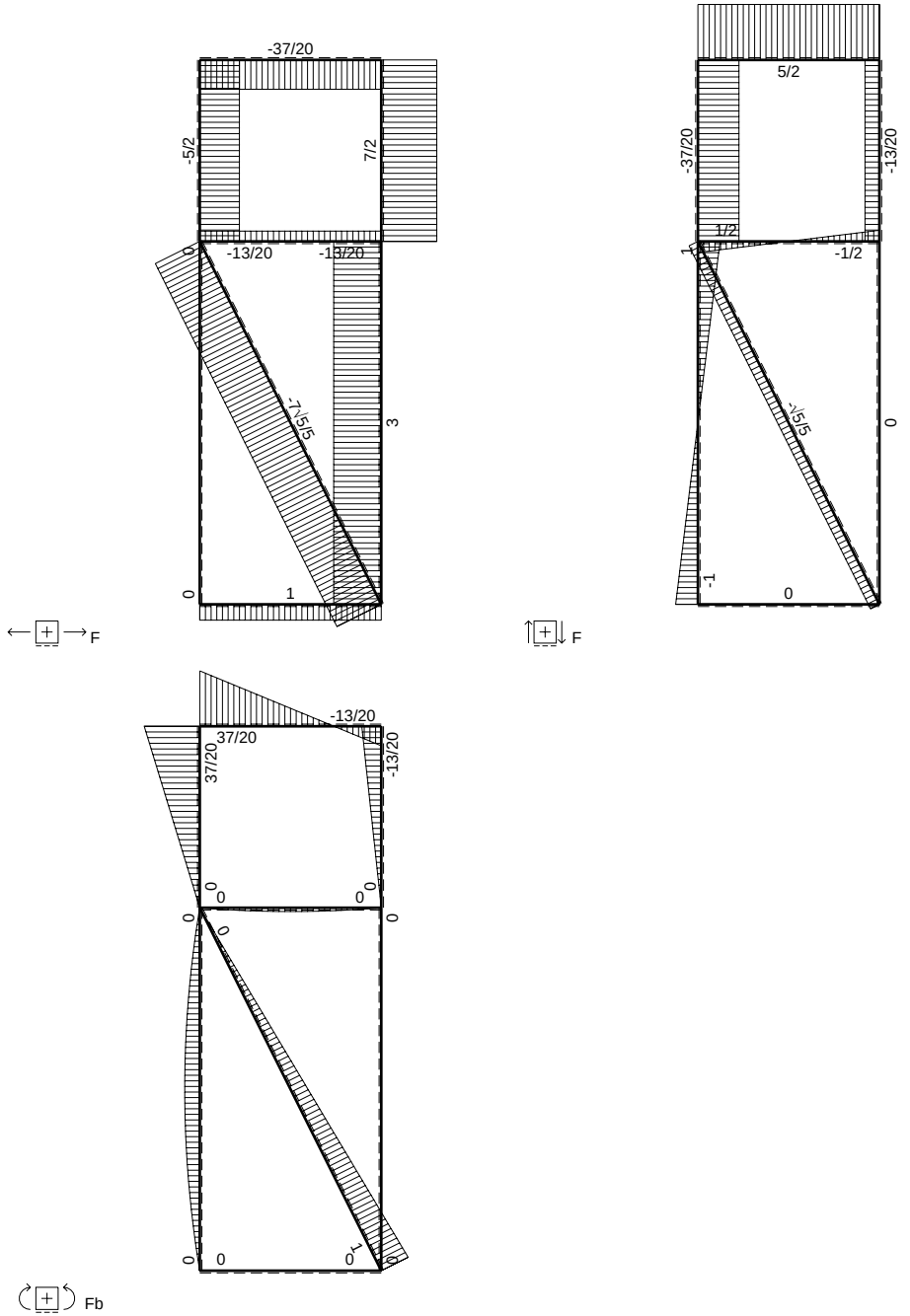


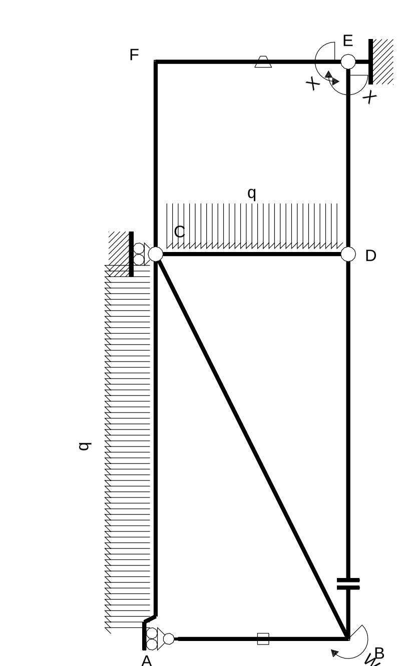
A = 654. mm^2
J_x = 244740. mm^4
J_y = 92084. mm^4
J_{xy} = 72222. mm^4
J_u = 273493. mm^4
J_v = 63331. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3789
c = cosα = .9291
s = sinα = -.3699
x_g = 12.08 mm
y_g = 34.05 mm
N = -1422. N
T_y = -711.1 N
M_x = 1176600. Nmm
x_m = 18. mm
u_m = 18.09 mm
v_m = -29.45 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.9 N/mm^2

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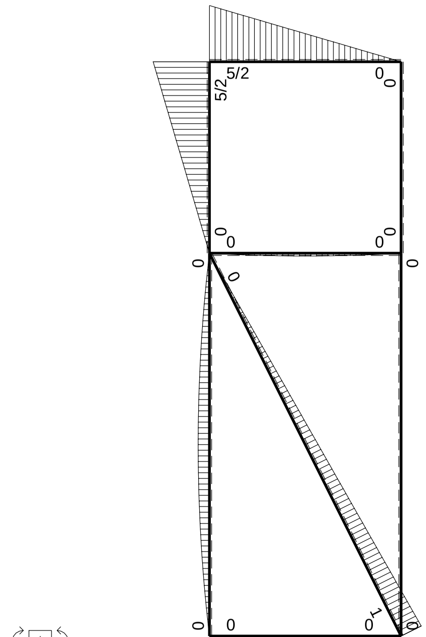
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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
7/2F
37/20F
13/20F
C
D
C
D
13/20F
13/20F
1/2F
1/2F
C
F
3F
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3F
C
F
Fb
F
B
3F
A
B
F
B
F
A

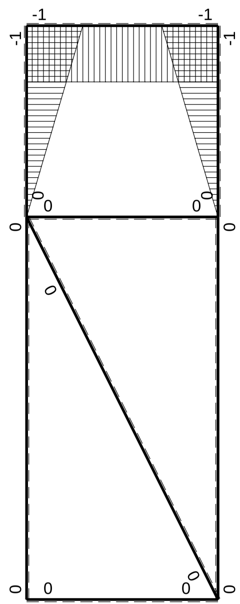

-37/20
-5/2
7/2
-13/20
-13/20
0
-7/5/5
3
0
1
F
5/2
-37/20
-13/20
1/2
-1/2
-7/5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

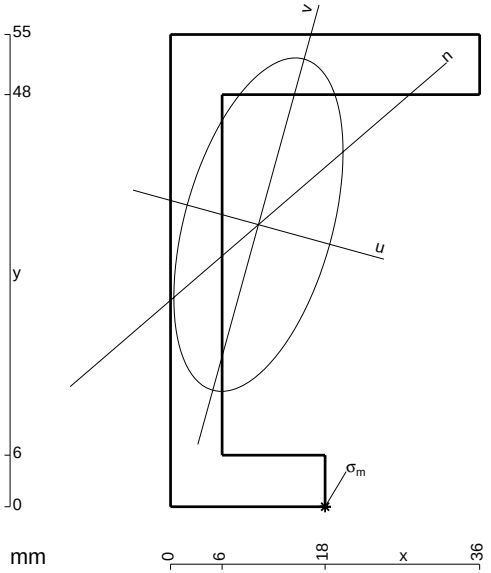
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

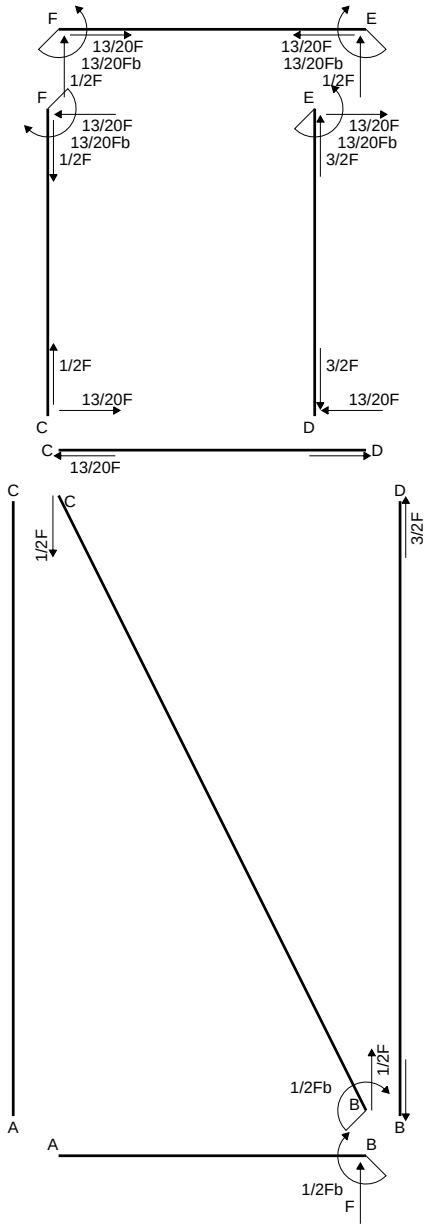
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

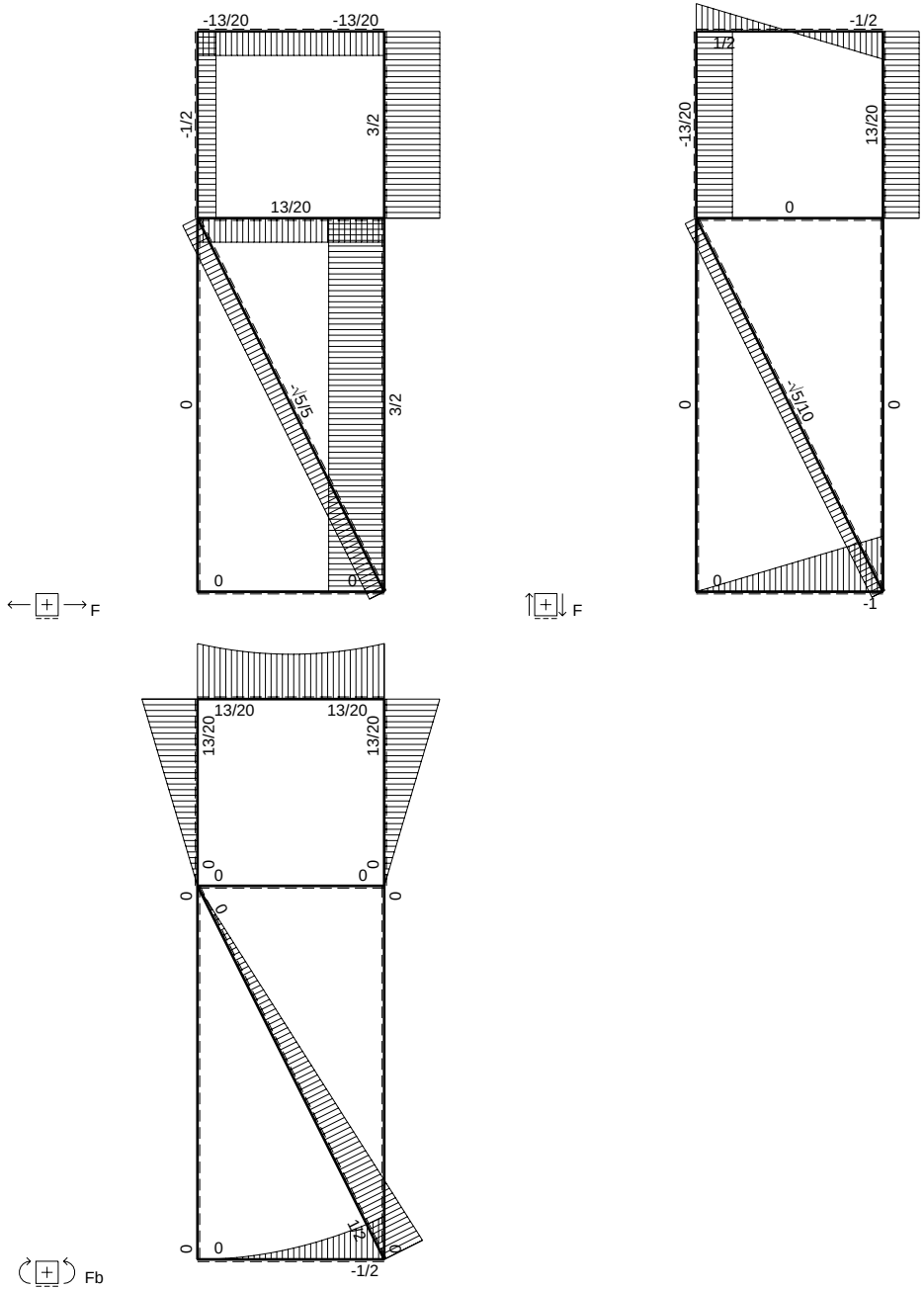


A = 612. mm^2
J_x = 230903. mm^4
J_y = 59438. mm^4
J_{xy} = 51141. mm^4
J_u = 244998. mm^4
J_v = 45343. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.2689$
$c = \cos\alpha$ = .9641
$s = \sin\alpha = -.2657$
x_g = 10.24 mm
y_g = 32.85 mm
N = -2461. N
T_y = 3325. N
M_x = 959595. Nmm
x_m = 18. mm
u_m = 16.21 mm
v_m = -29.61 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = 199. N/mm^2

CI09.xxxx.080

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
totali							$13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

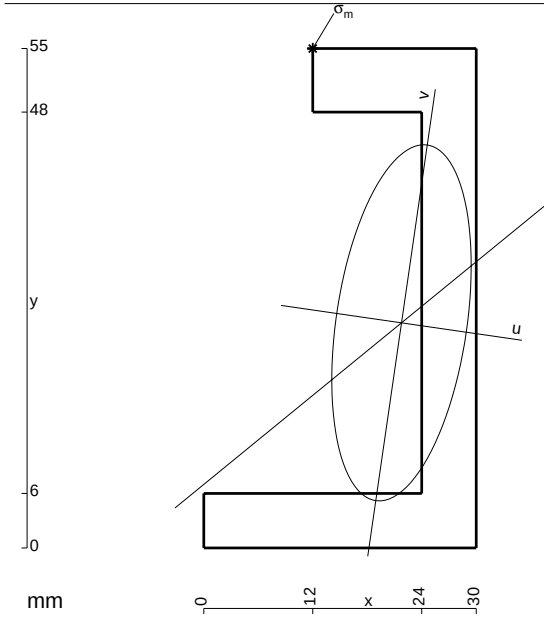
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 962891 CI09.xxxx.081

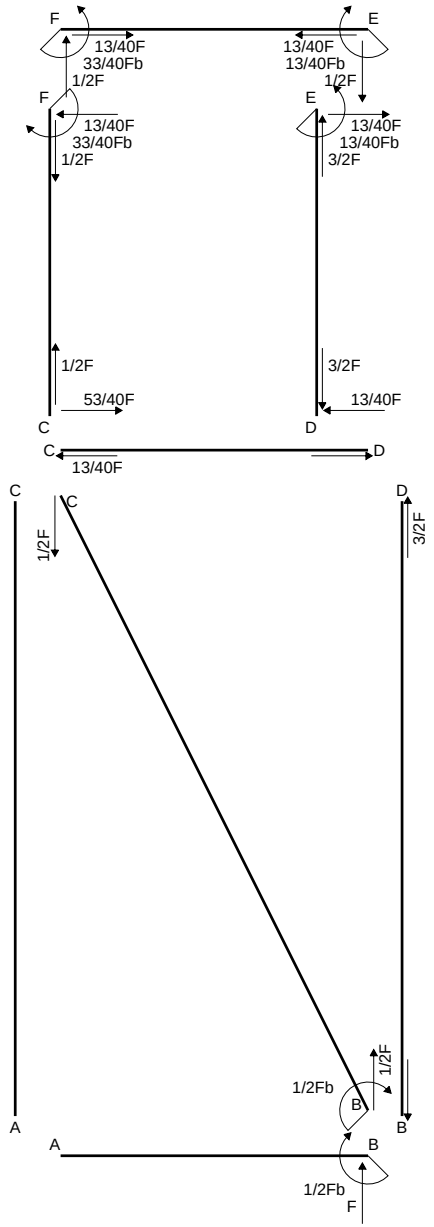


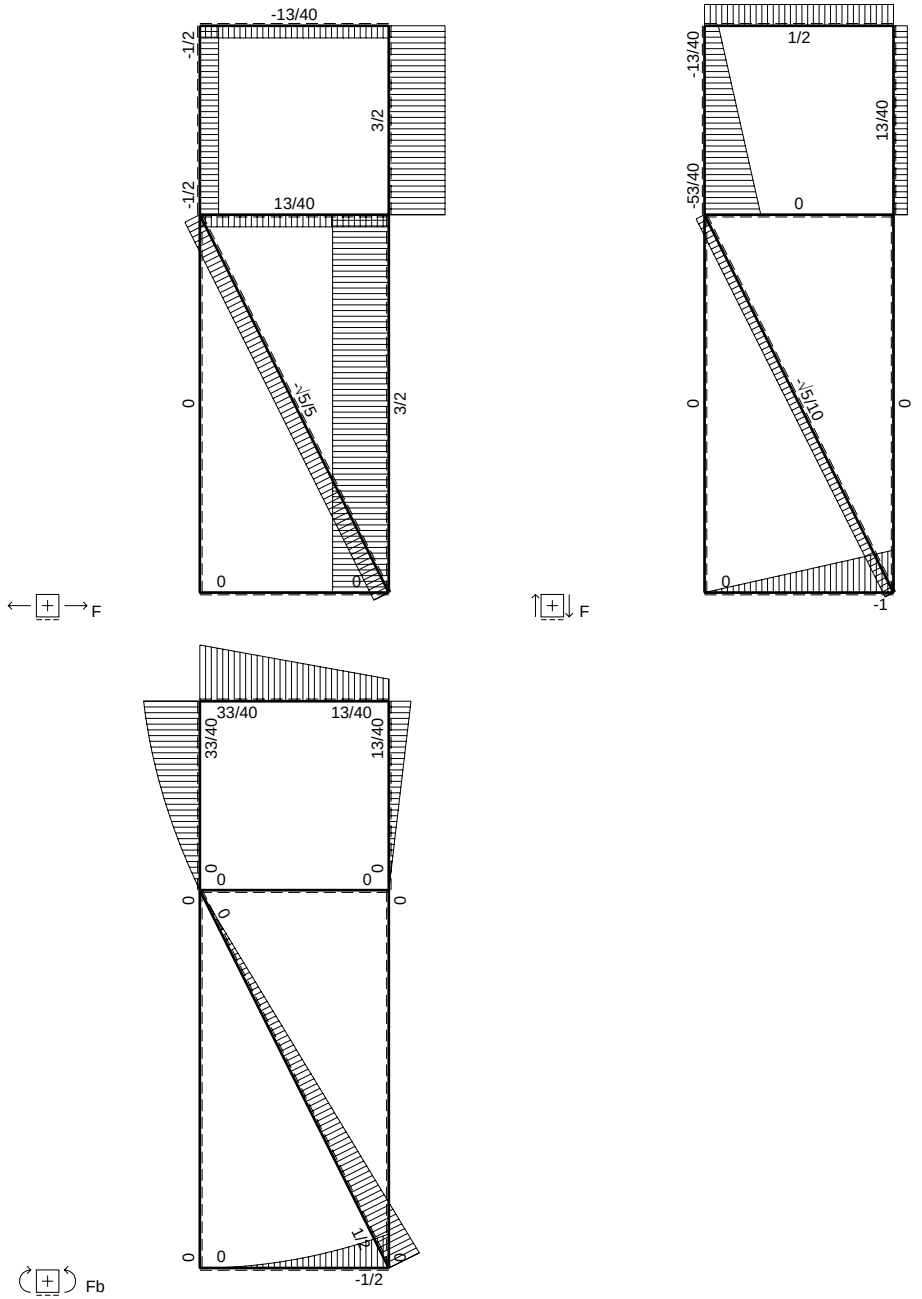
A = 558. mm^2
J_x = 214685. mm^4
J_y = 32876. mm^4
J_{xy} = 26875. mm^4
J_u = 218575. mm^4
J_v = 28986. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1437
$c = \cos\alpha$ = .9897
$s = \sin\alpha$ = -.1432
x_g = 21.77 mm
y_g = 24.79 mm
N = -2464. N
T_y = -1895. N
M_x = 1034670. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -14. mm
v_m = 28.5 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.5 N/mm^2

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
1/2F
3/2F
1/2Fb
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

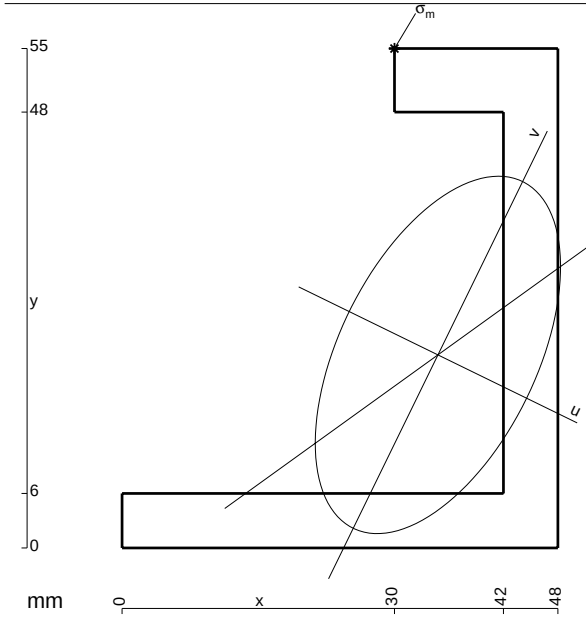
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

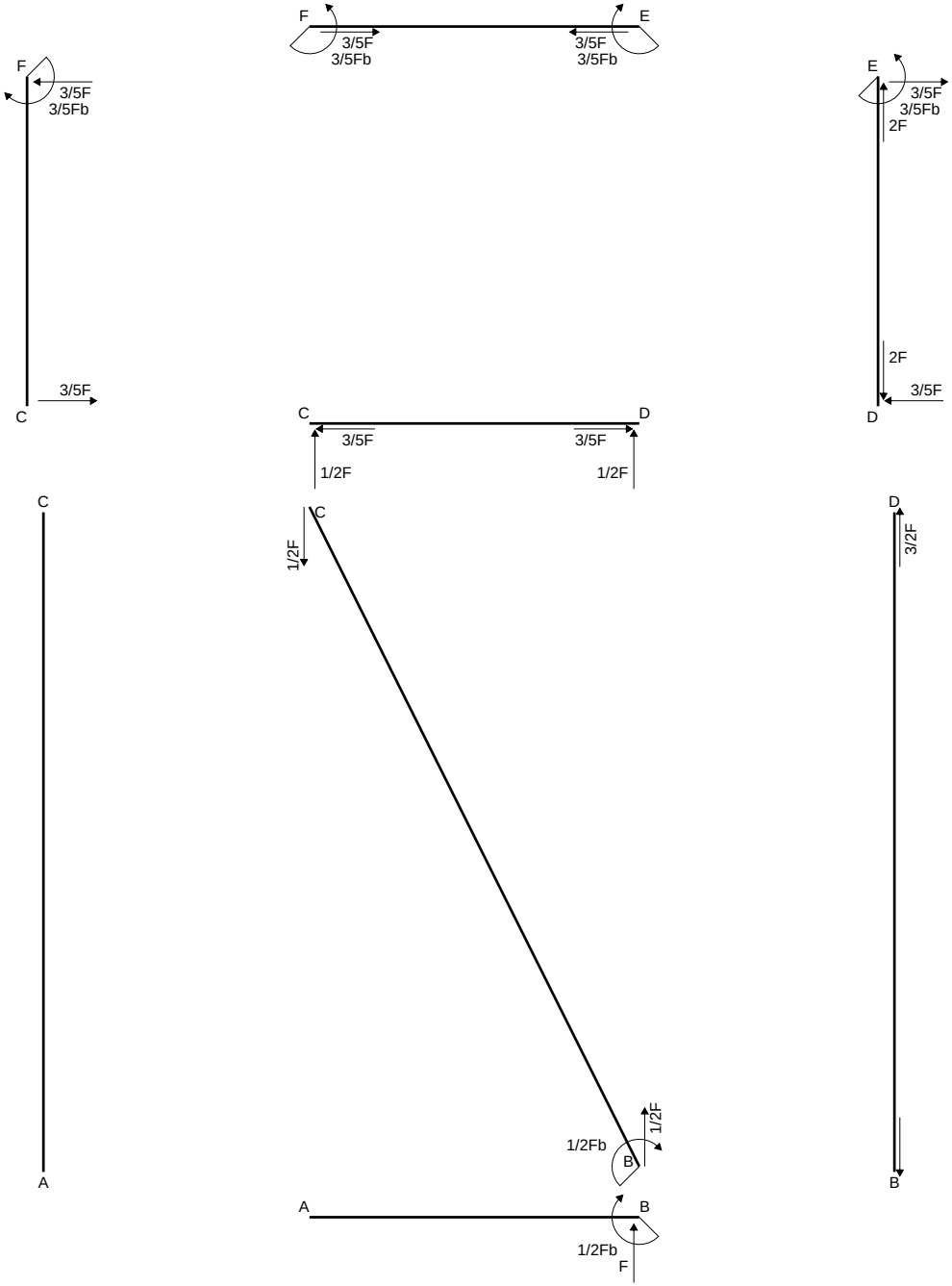
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

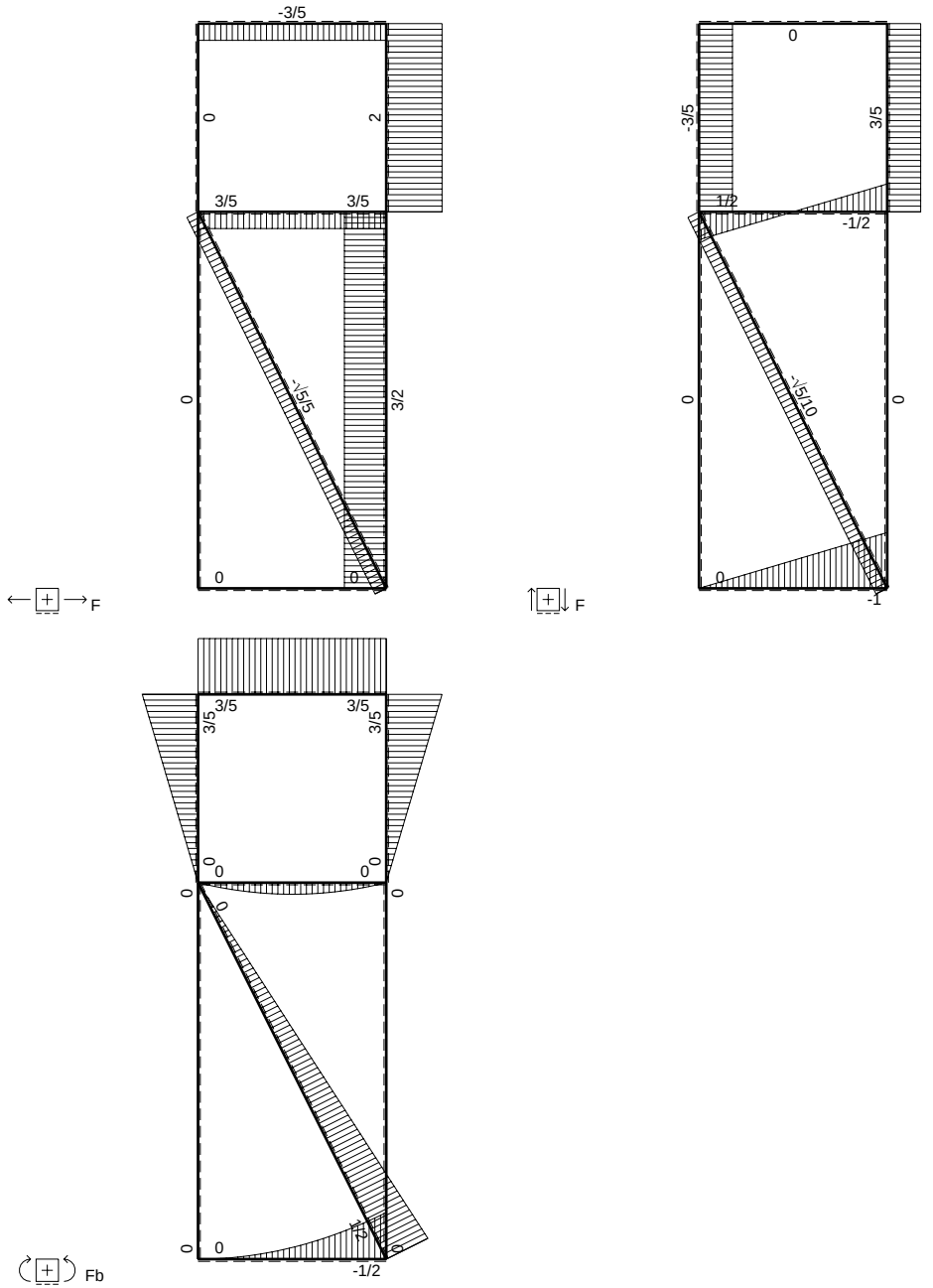


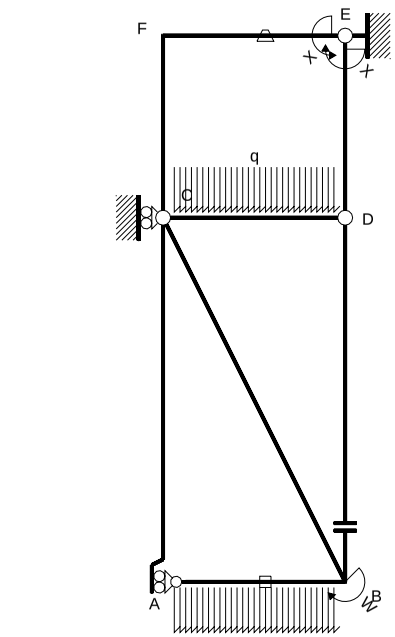
A = 666. mm^2
J_x = 257974. mm^4
J_y = 121487. mm^4
J_{xy} = 87553. mm^4
J_u = 300738. mm^4
J_v = 78723. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4544
c = cosα = .8985
s = sinα = -.4389
x_g = 34.78 mm
y_g = 21.26 mm
N = -1500. N
T_y = -975. N
M_x = 1138500. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = -19.11 mm
v_m = 28.22 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -219.5 N/mm^2

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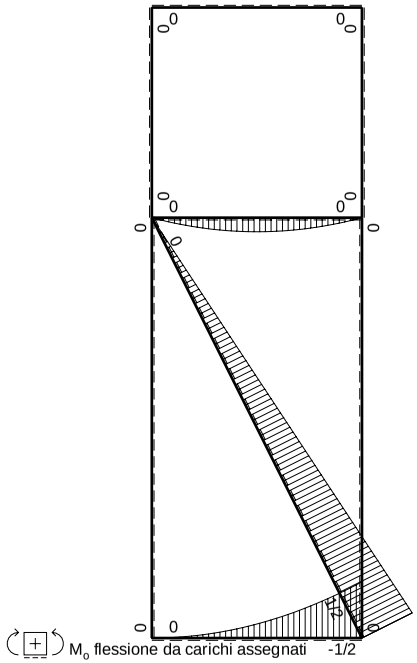
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
A
B
1/2Fb
F

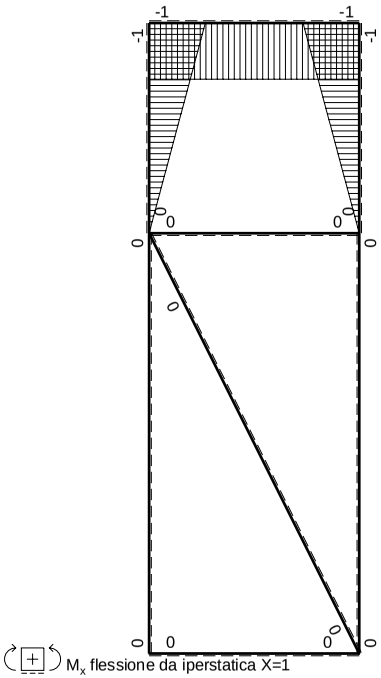

-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
0
-√2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

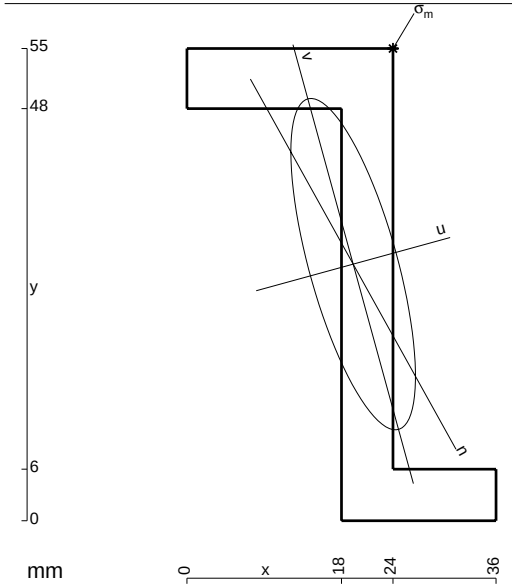
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1\ )\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1\ )\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 528. mm^2

J_x = 196705. mm^4

J_y = 27818. mm^4

J_{xy} = -50102. mm^4

J_u = 210450. mm^4

J_v = 14073. mm^4

$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2677

$c = \cos\alpha$ = .9644

$s = \sin\alpha$ = .2646

x_g = 19.36 mm

y_g = 29.89 mm

N = -1446. N

M_x = 723000. Nmm

x_m = 24. mm

y_m = 55. mm

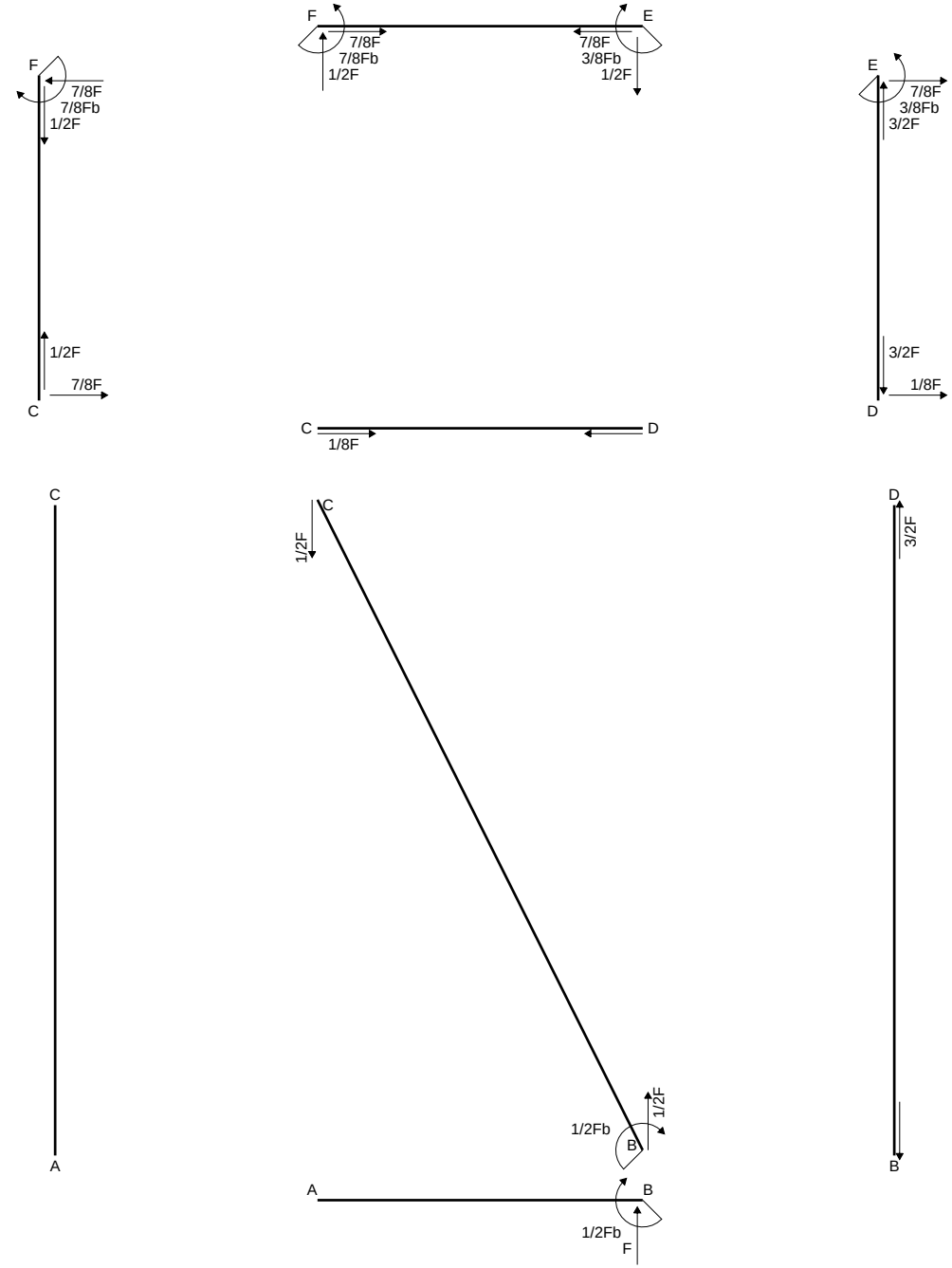
u_m = 11.12 mm

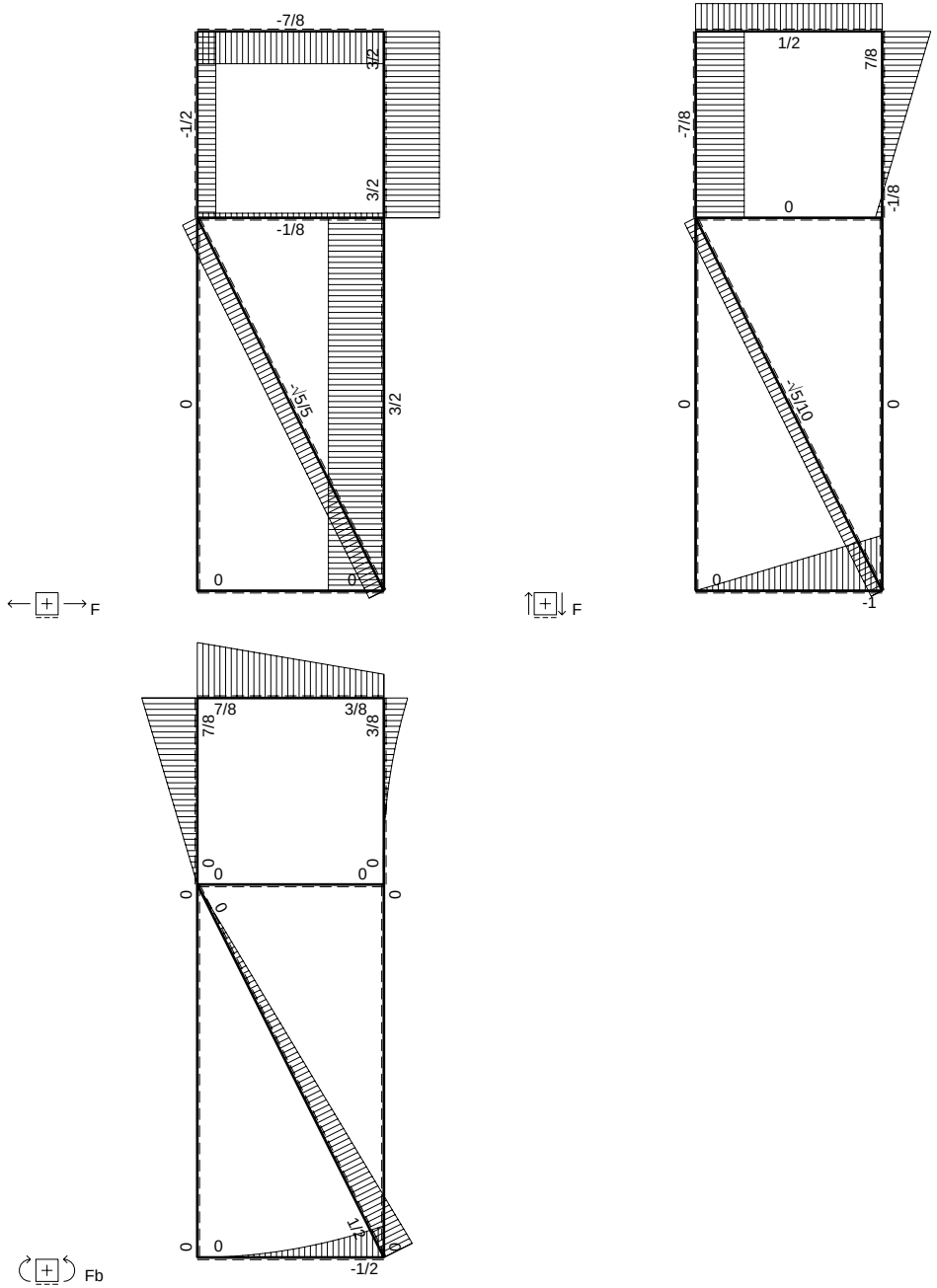
v_m = 22.99 mm

$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -230. N/mm^2

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F
7/8F
7/8Fb
1/2F
1/2F
7/8F
C
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
D
1/8F
E
7/8F
3/8Fb
3/2F
3/2F
1/8F
D
C
A
C
1/2F
1/2Fb
B
1/2F
A
B
1/2Fb
F
D
3/2F
B


-7/8
3/2
-1/2
3/2
-1/8
0
0
-√5/5
3/2
0
F
1/2
7/8
-7/8
0
-1/8
0
0
-√5/10
0
-1
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
-√5/2
0
-1/2
Fb

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Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	$-Fb/EJ$	$-1/2Fx$	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ \ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ \ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

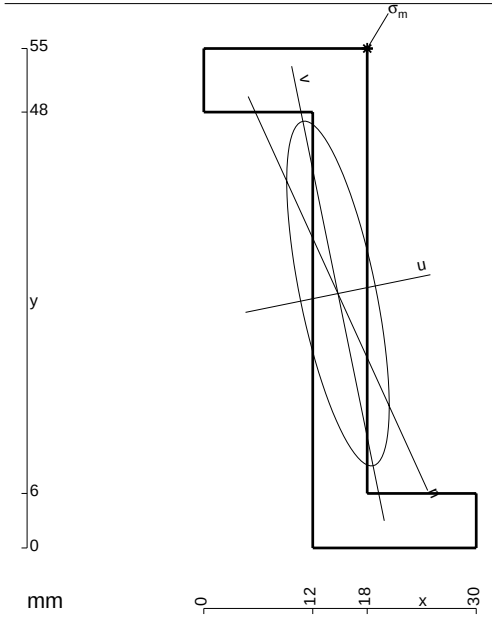
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ \ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

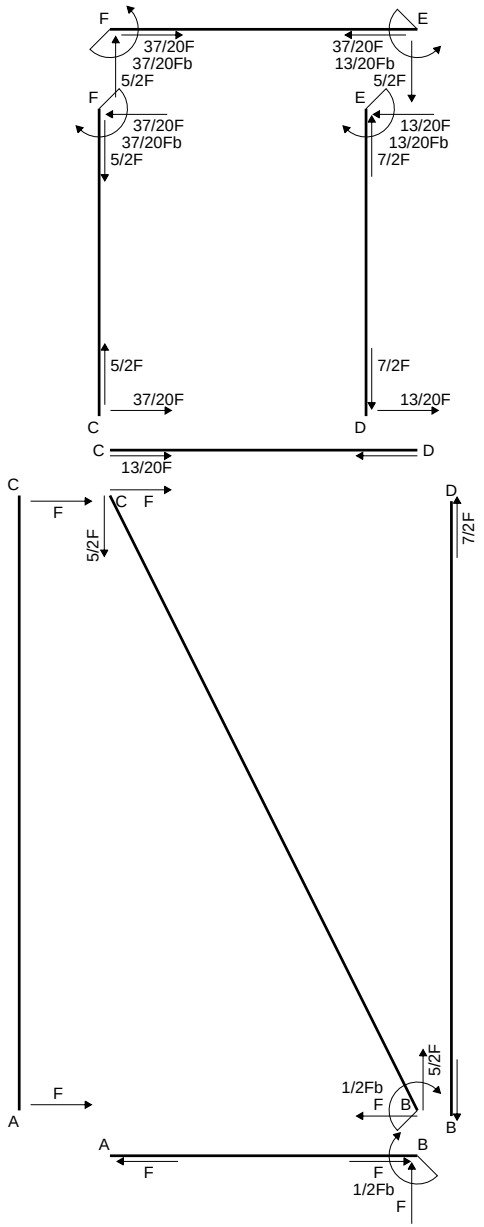
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

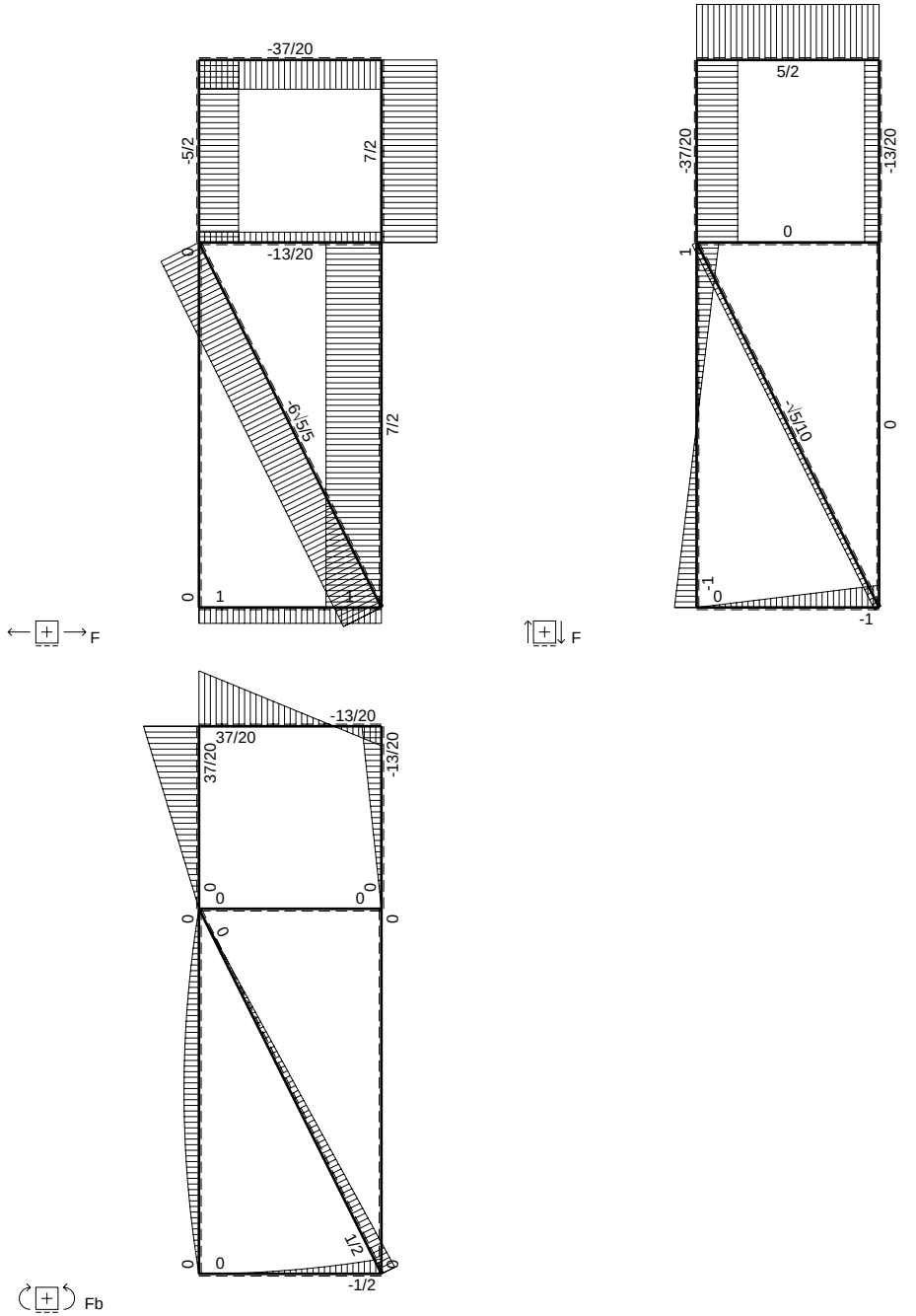


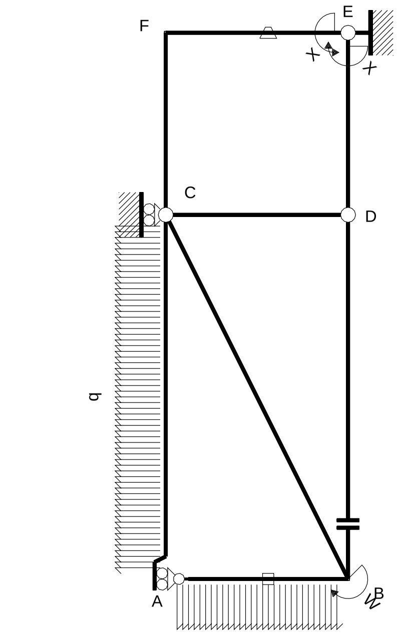
A = 486. mm^2
J_x = 175218. mm^4
J_y = 15474. mm^4
J_{xy} = -33964. mm^4
J_u = 182139. mm^4
J_v = 8553. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2010
c = cosα = .9799
s = sinα = .1997
x_g = 14.78 mm
y_g = 28.02 mm
N = -1295. N
T_y = 740. N
M_x = 699300. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 8.545 mm
v_m = 25.79 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -239.2 N/mm^2

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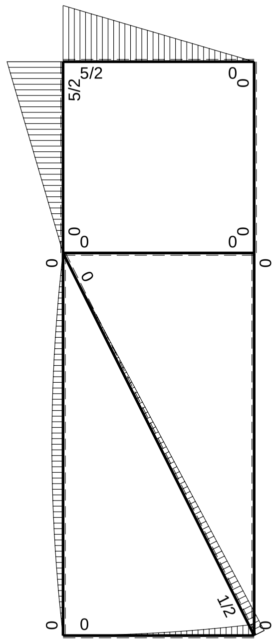
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F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb

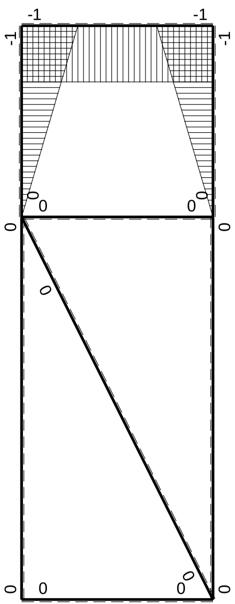

-37/20
-5/2
7/2
-13/20
-5\5/5
F
5/2
-37/20
-13/20
-5\5/10
-1
37/20
1/2
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

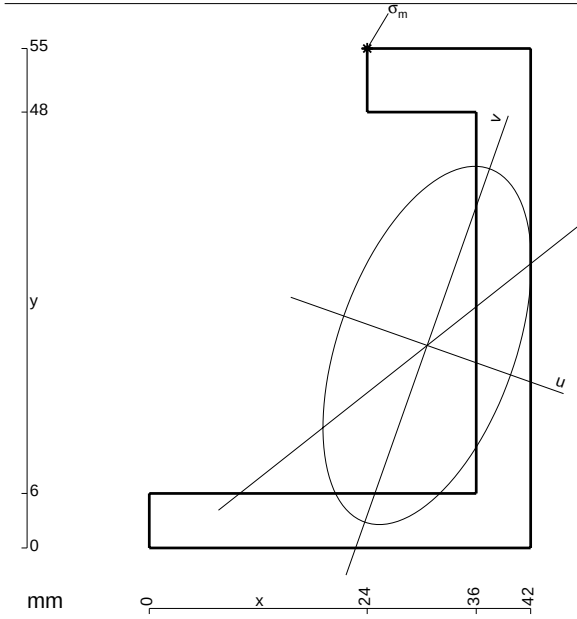
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

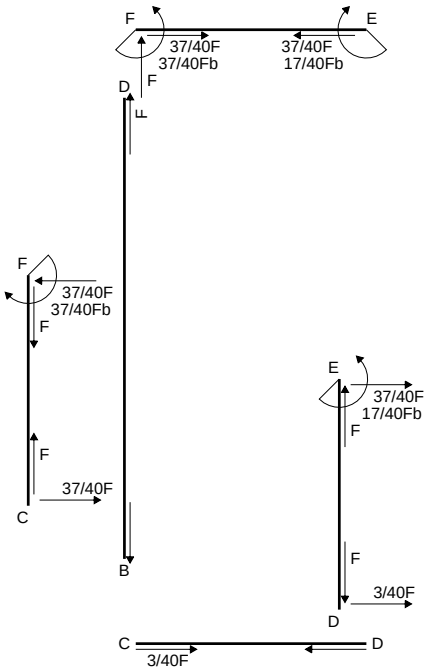
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

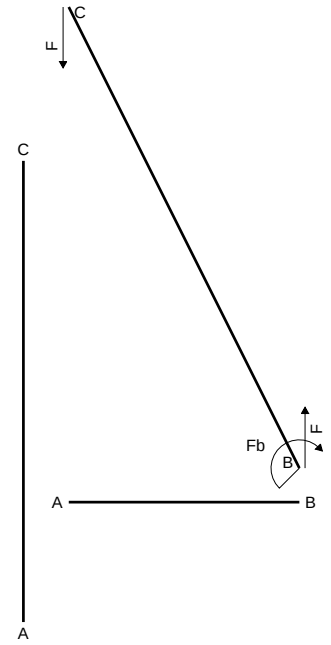


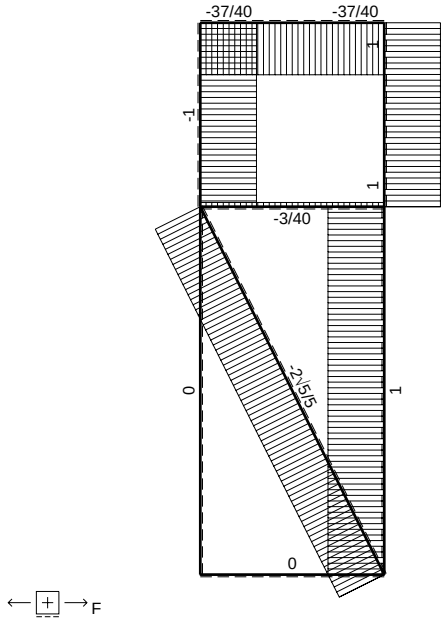
A = 630. mm^2
J_x = 245181. mm^4
J_y = 82933. mm^4
J_{xy} = 65470. mm^4
J_u = 268304. mm^4
J_v = 59811. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3395
c = cosα = .9429
s = sinα = -.3330
x_g = 30.6 mm
y_g = 22.3 mm
N = -2325. N
T_y = -1721. N
M_x = 997890. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = -17.11 mm
v_m = 28.64 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -199.2 N/mm^2

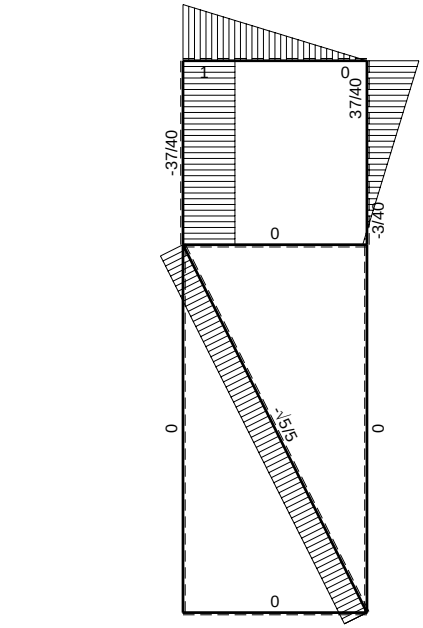
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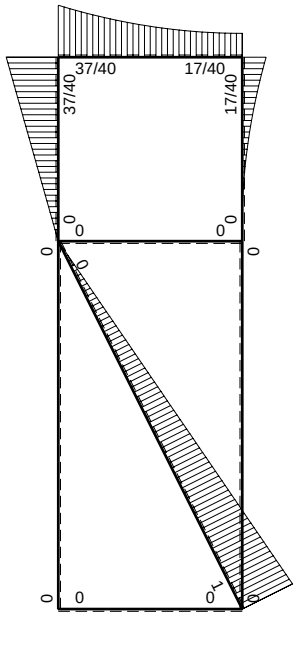
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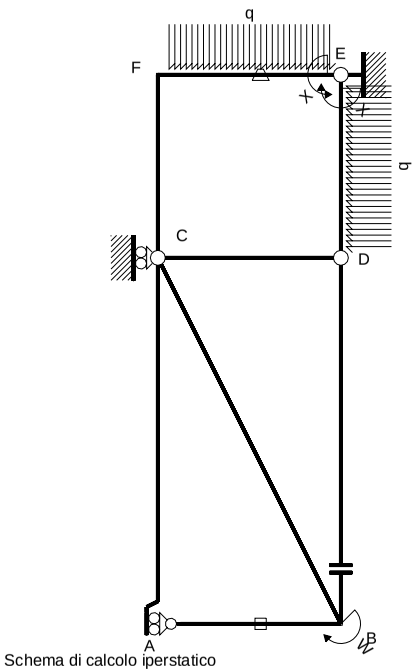

F
E
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37/40Fb
37/40F
17/40Fb
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37/40Fb
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17/40Fb
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A
B
A

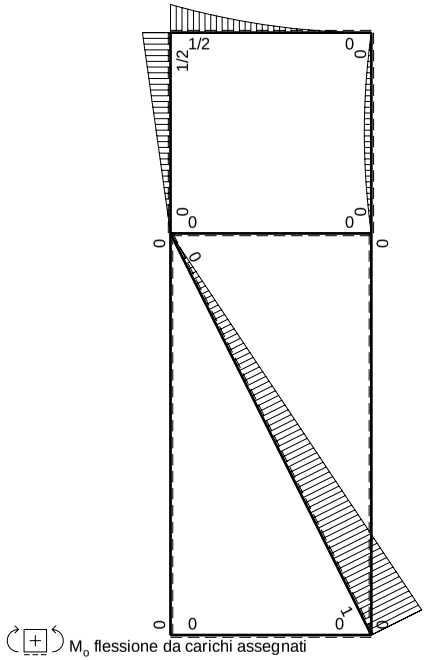

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-3/40
-2√5/5
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1
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1
F


1
0
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-37/40
-3/40
0
-√5/5
0
0
0
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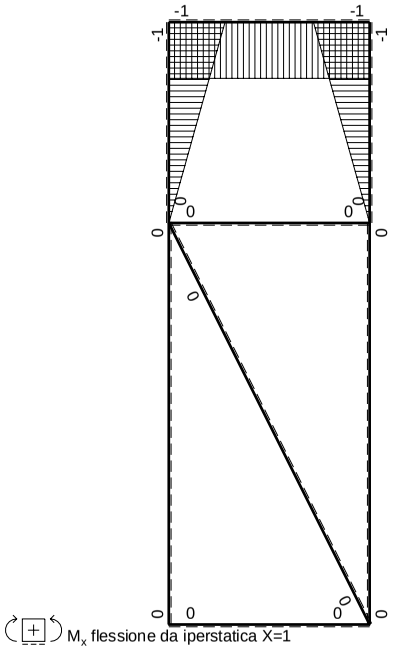

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 265477 CI09.xxxx.086

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

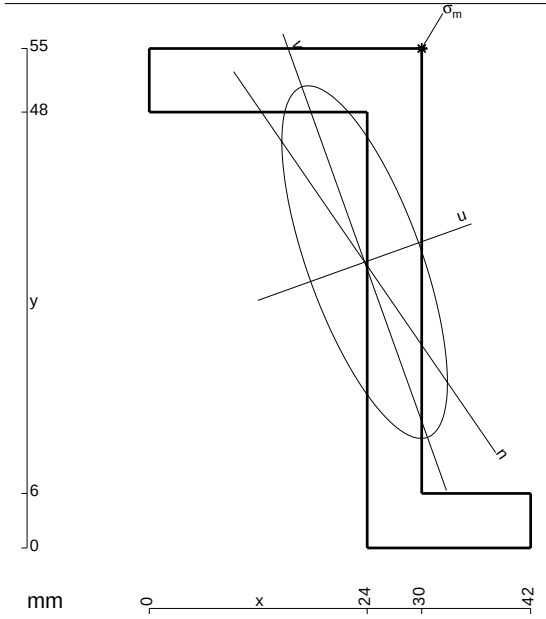
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

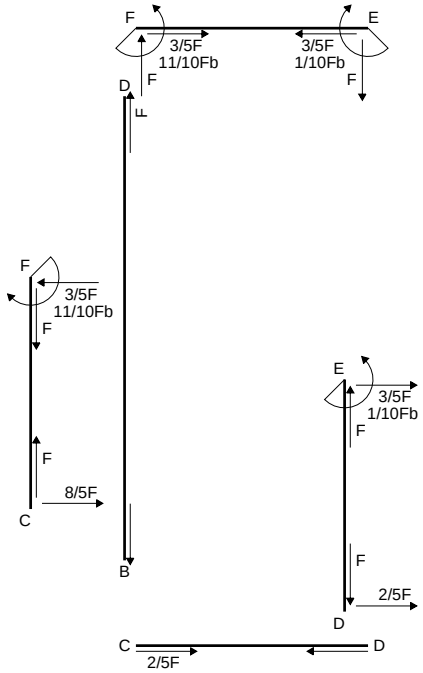
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

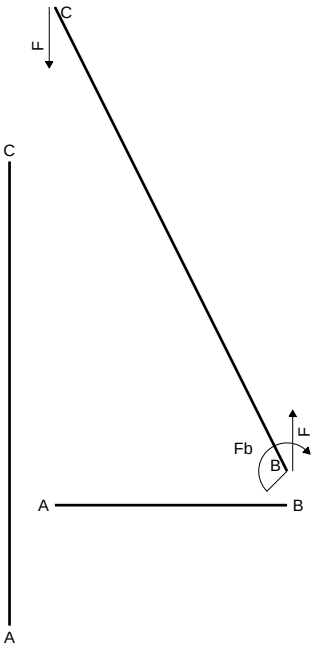


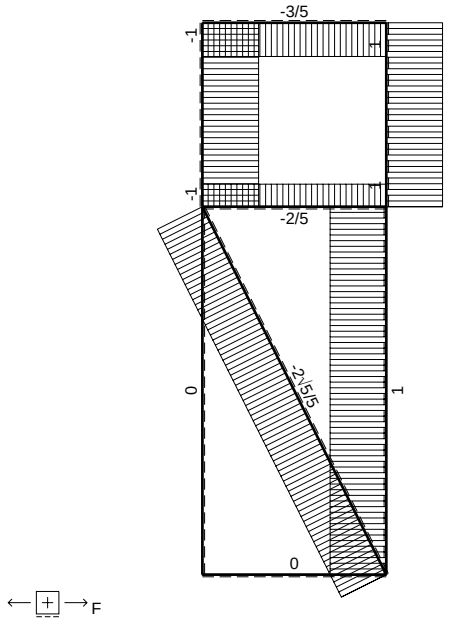
A = 570. mm^2
J_x = 215051. mm^4
J_y = 47402. mm^4
J_{xy} = -68907. mm^4
J_u = 239738. mm^4
J_v = 22715. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3440
c = cosα = .9414
s = sinα = .3373
x_g = 23.72 mm
y_g = 31.48 mm
N = -1055. N
T_y = -527.7 N
M_x = 731600. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 13.85 mm
v_m = 20.02 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.8 N/mm^2

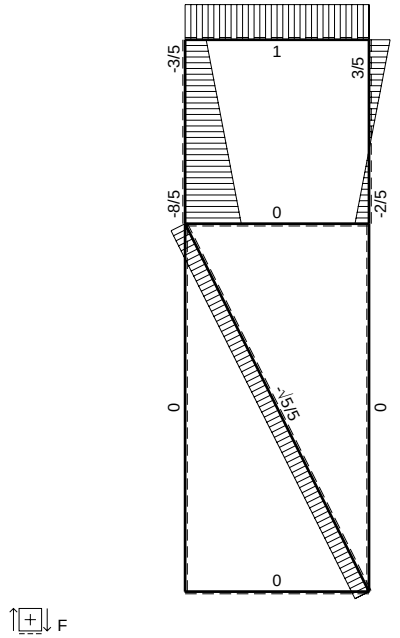
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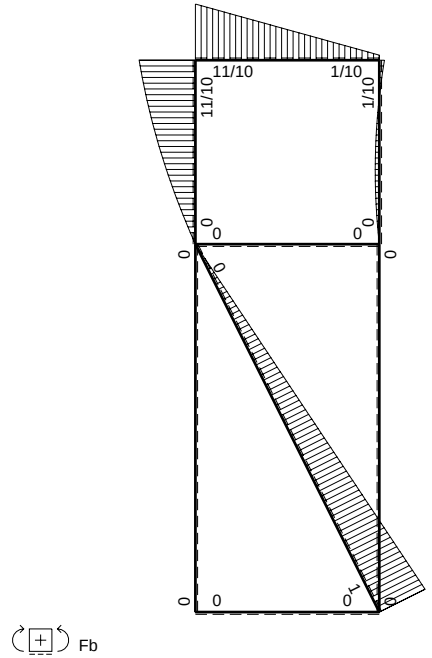
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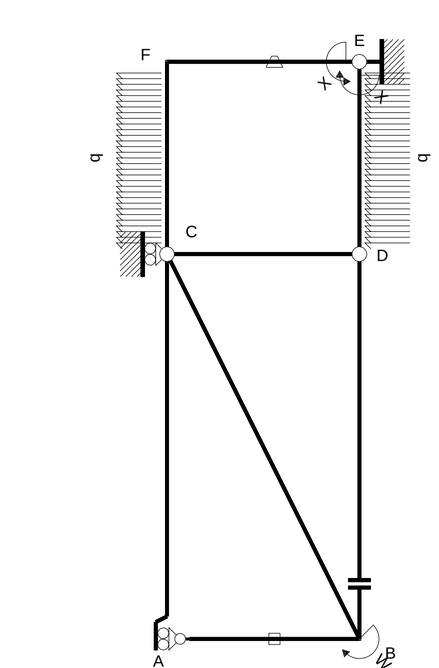

F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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3/5F
1/10Fb
F
F
8/5F
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Fb
F
B
A
B
A

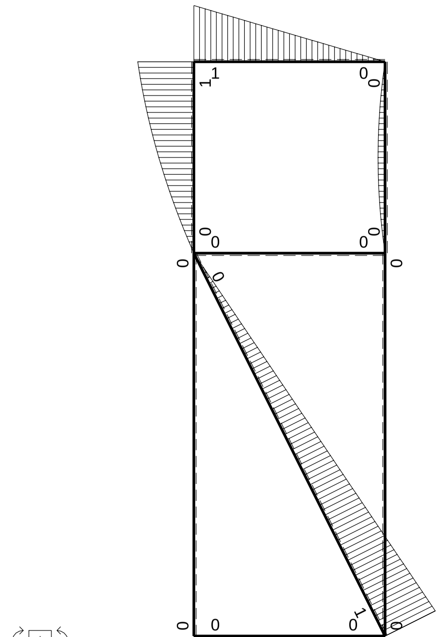

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-1
1
-2/5
0
-2/5
1
0
F


-3/5
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3/5
-8/5
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-2/5
0
-2/5
0
0
F

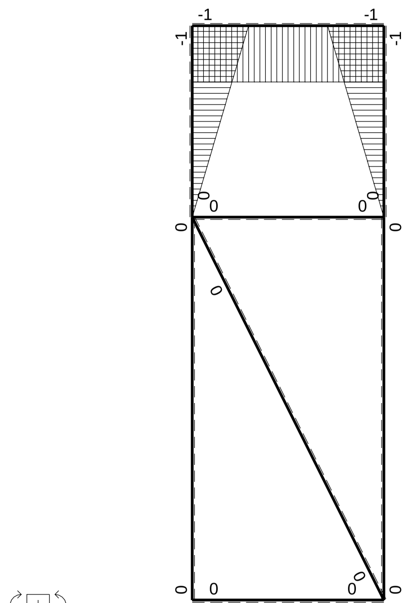

11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$1/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

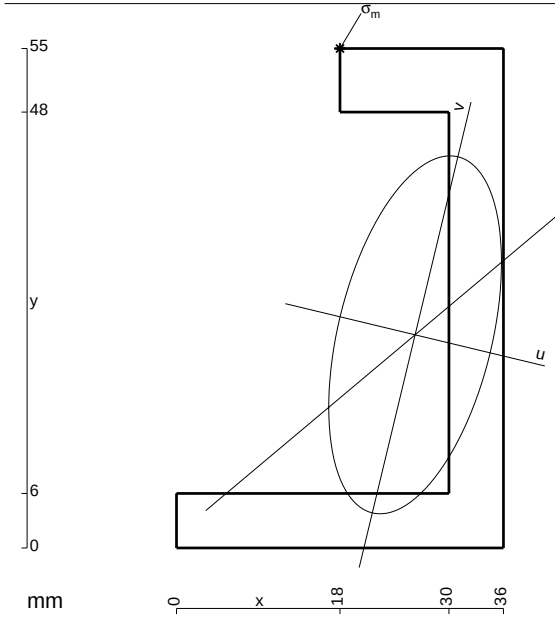
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

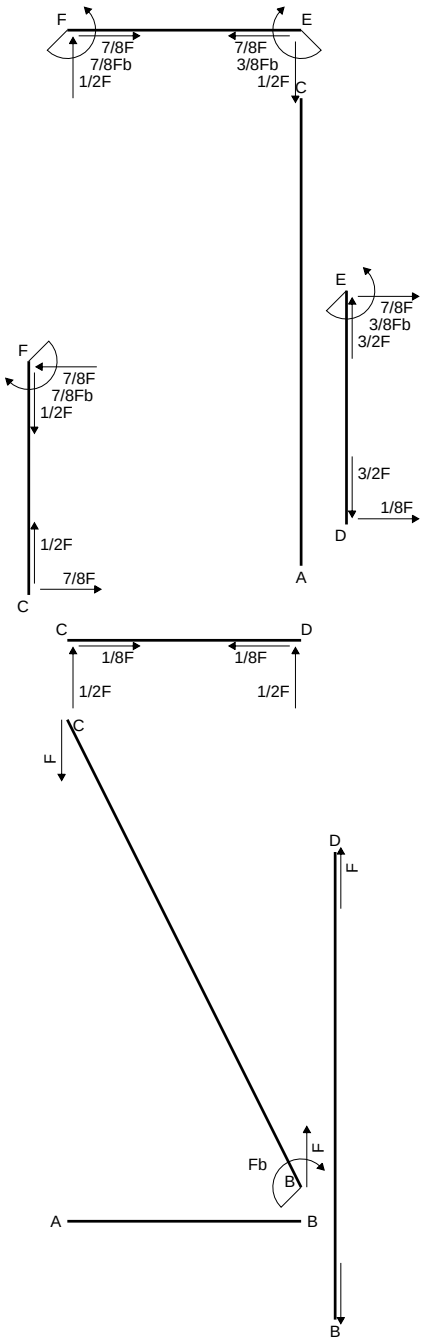
$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

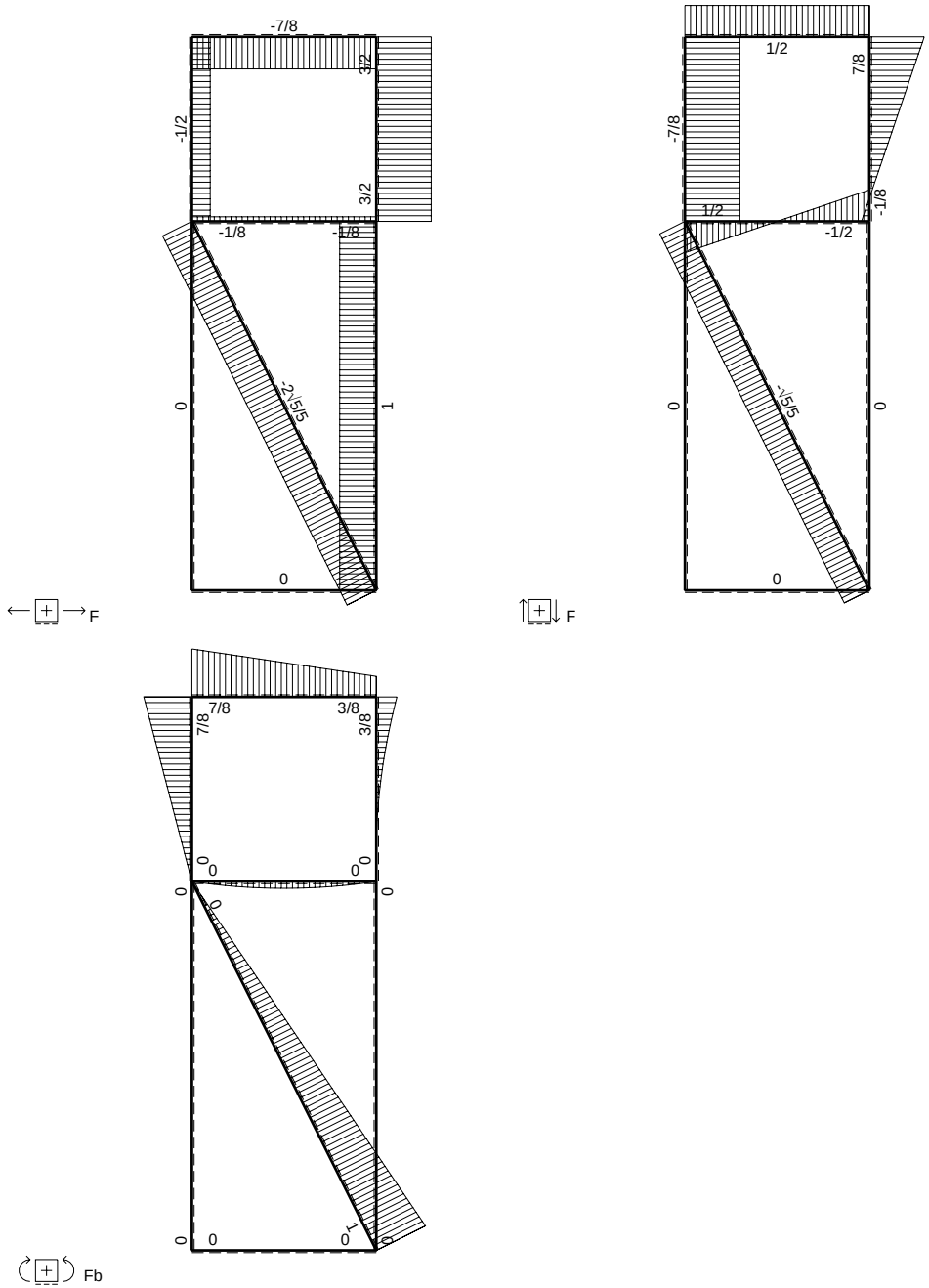


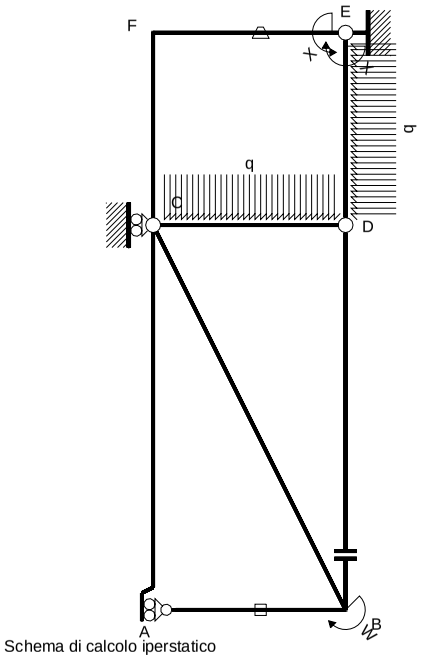
A = 594. mm^2
J_x = 230851. mm^4
J_y = 53740. mm^4
J_{xy} = 45131. mm^4
J_u = 241688. mm^4
J_v = 42903. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2357
c = cosα = .9724
s = sinα = -.2335
x_g = 26.27 mm
y_g = 23.47 mm
N = -1500. N
T_y = -900. N
M_x = 1089000. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = -15.41 mm
v_m = 28.73 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.7 N/mm^2

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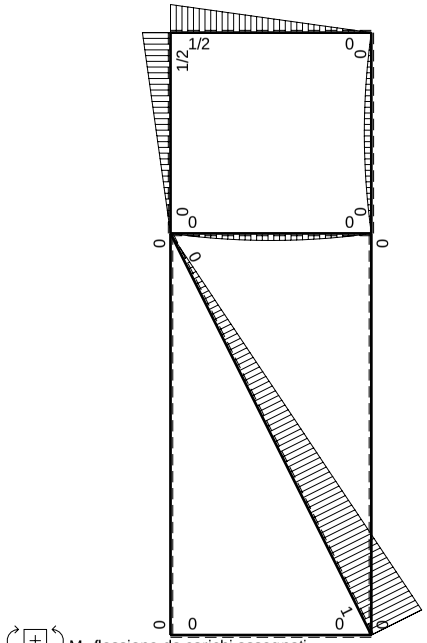
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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B

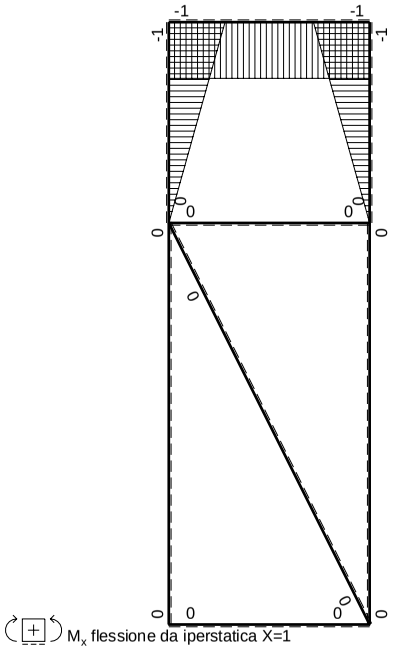

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3/2
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0
F
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-1/8
1/2
-1/2
0
-√5/5
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					5/8Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 263194 CI09.xxxx.088

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

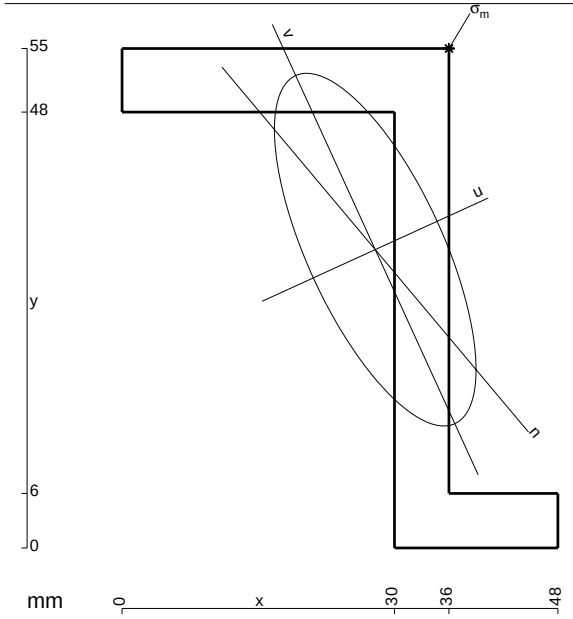
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

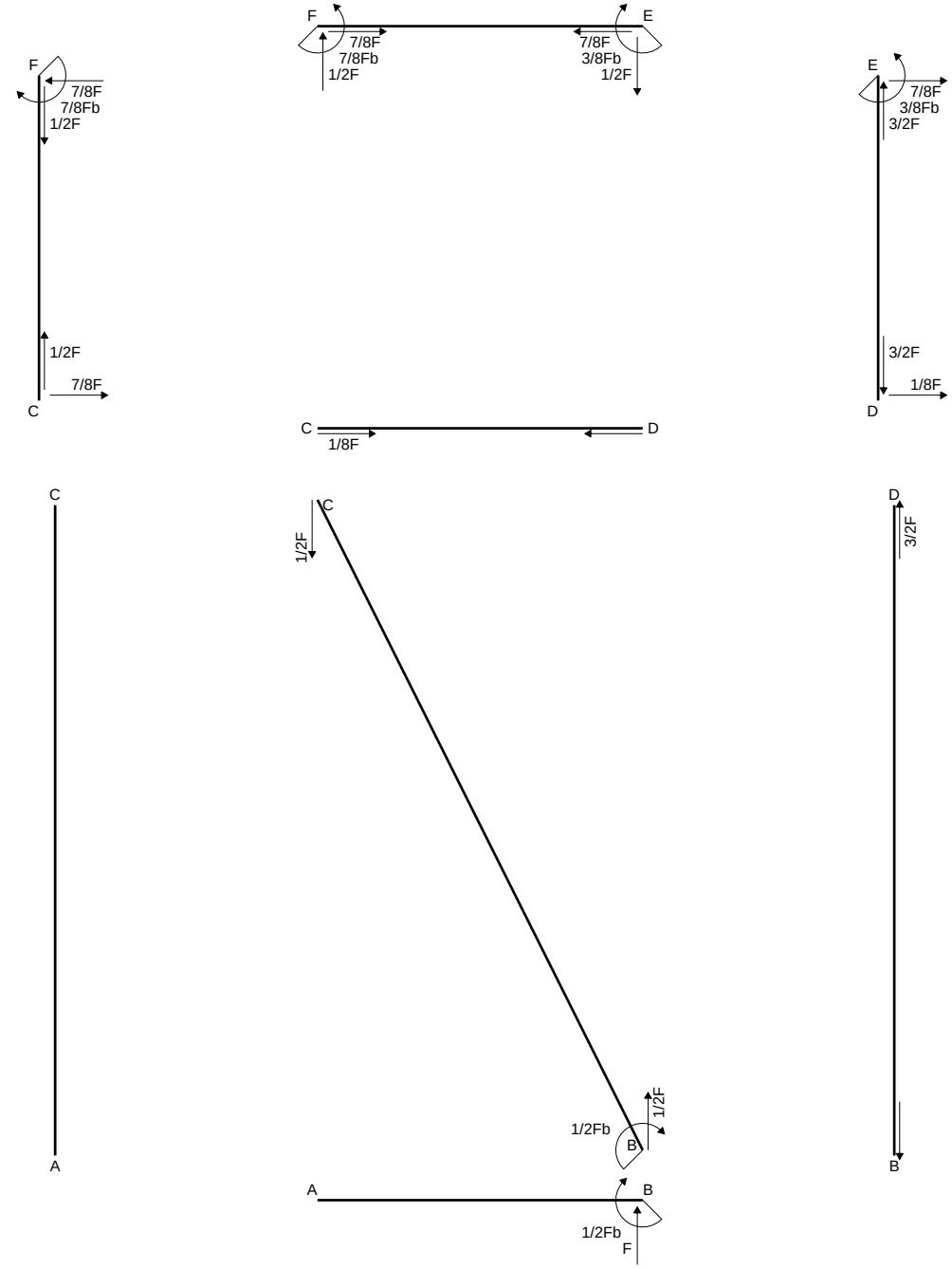
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

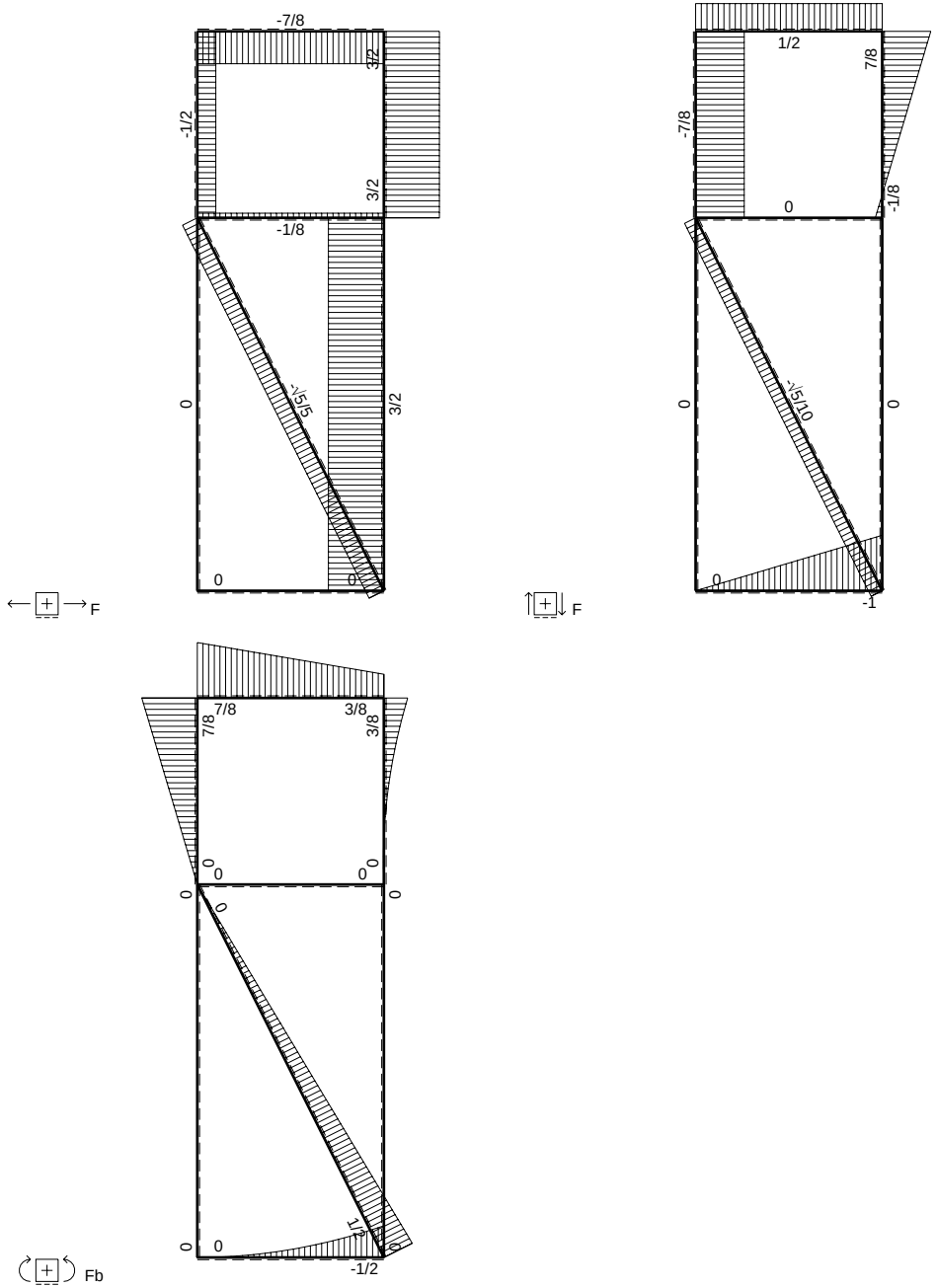


A = 612. mm^2
J_x = 230903. mm^4
J_y = 75448. mm^4
J_{xy} = -89831. mm^4
J_u = 271965. mm^4
J_v = 34385. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4288
c = cosα = .9095
s = sinα = .4157
x_g = 27.88 mm
y_g = 32.85 mm
N = -1127. N
T_y = -563.5 N
M_x = 882000. Nmm
x_m = 36. mm
y_m = 55. mm
u_m = 16.59 mm
v_m = 16.77 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -228.2 N/mm^2

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F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
3/2F
1/8F
C
D
1/8F
A
B
1/2Fb


-7/8
3/2
-1/2
-1/8
0
-√5/5
1/2
7/8
-1/8
-√5/10
-1
7/8
3/8
-1/2
F
Fb

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Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	$-Fb/EJ$	$-1/2Fx$	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
totali							$5/8Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$								-3/8Fb

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

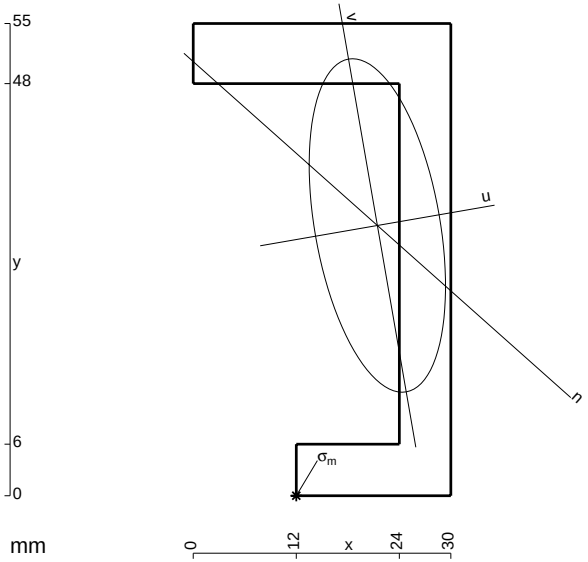
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

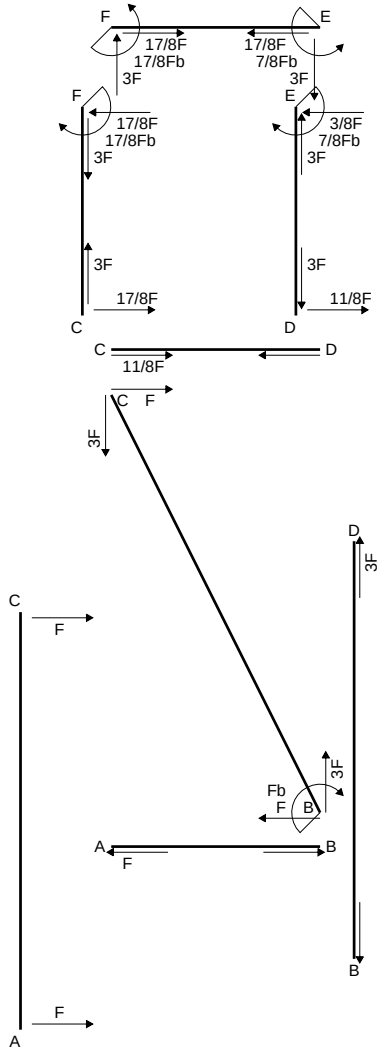
PROCEDIMENTO E RISULTATI 218770 CI09.xxxx.089

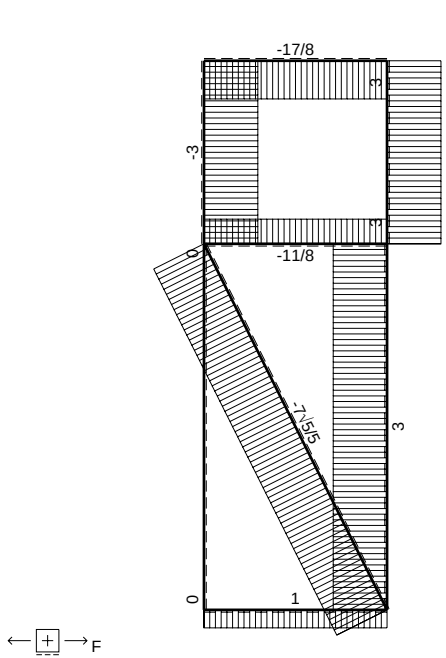


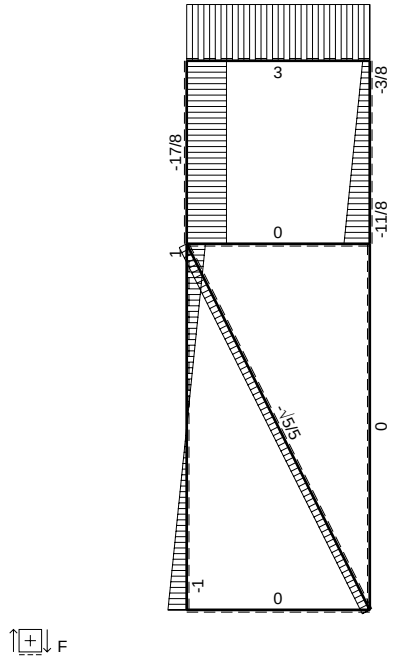
A = 570. mm^2
J_x = 215051. mm^4
J_y = 35943. mm^4
J_{xy} = -31999. mm^4
J_u = 220596. mm^4
J_v = 30398. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1716
$c = \cos\alpha$ = .9853
$s = \sin\alpha$ = .1707
x_g = 21.44 mm
y_g = 31.48 mm
N = -870. N
T_y = -1523. N
M_x = 1126650. Nmm
x_m = 12. mm
u_m = -14.68 mm
v_m = -29.4 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.3 N/mm^2

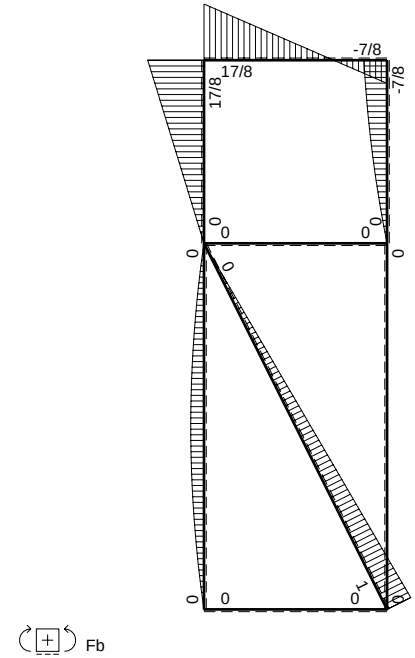
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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
B
F
A


-17/8
-3
-11/8
0
1
3
-7√5/5
F


3
-3/8
-17/8
-11/8
0
1
-1
-√5/5
F


-7/8
17/8
-7/8
0
0
0
0
0
0
0
0
1
Fb

16.04.26

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb²/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx²/b	0	1-2x/b+x²/b²	(-1+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx²/b	0	x²/b²		
	totali						-35/24Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

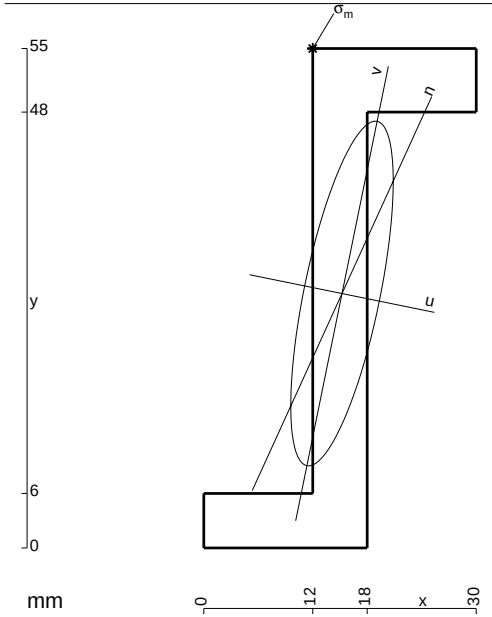
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2]_o^b\ Fb\ 1/EJ$

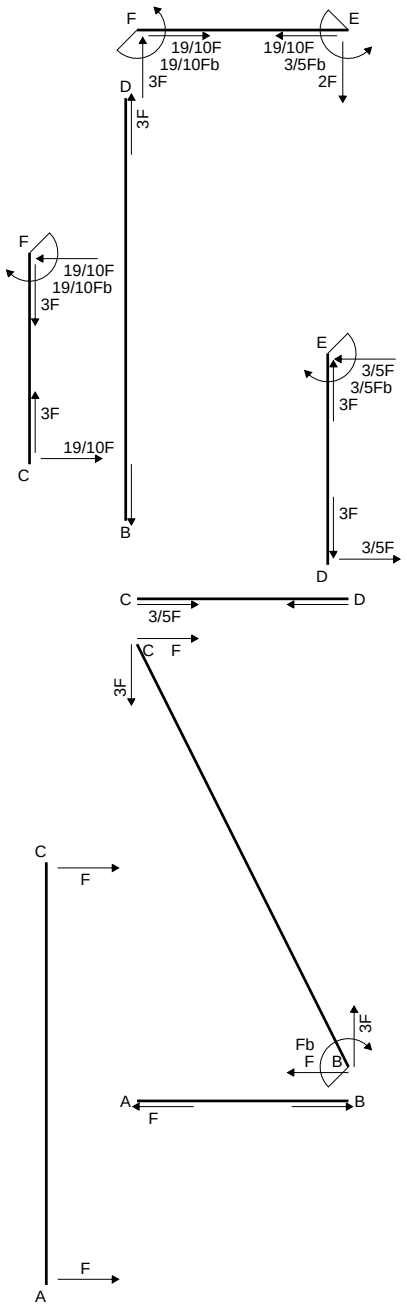
$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

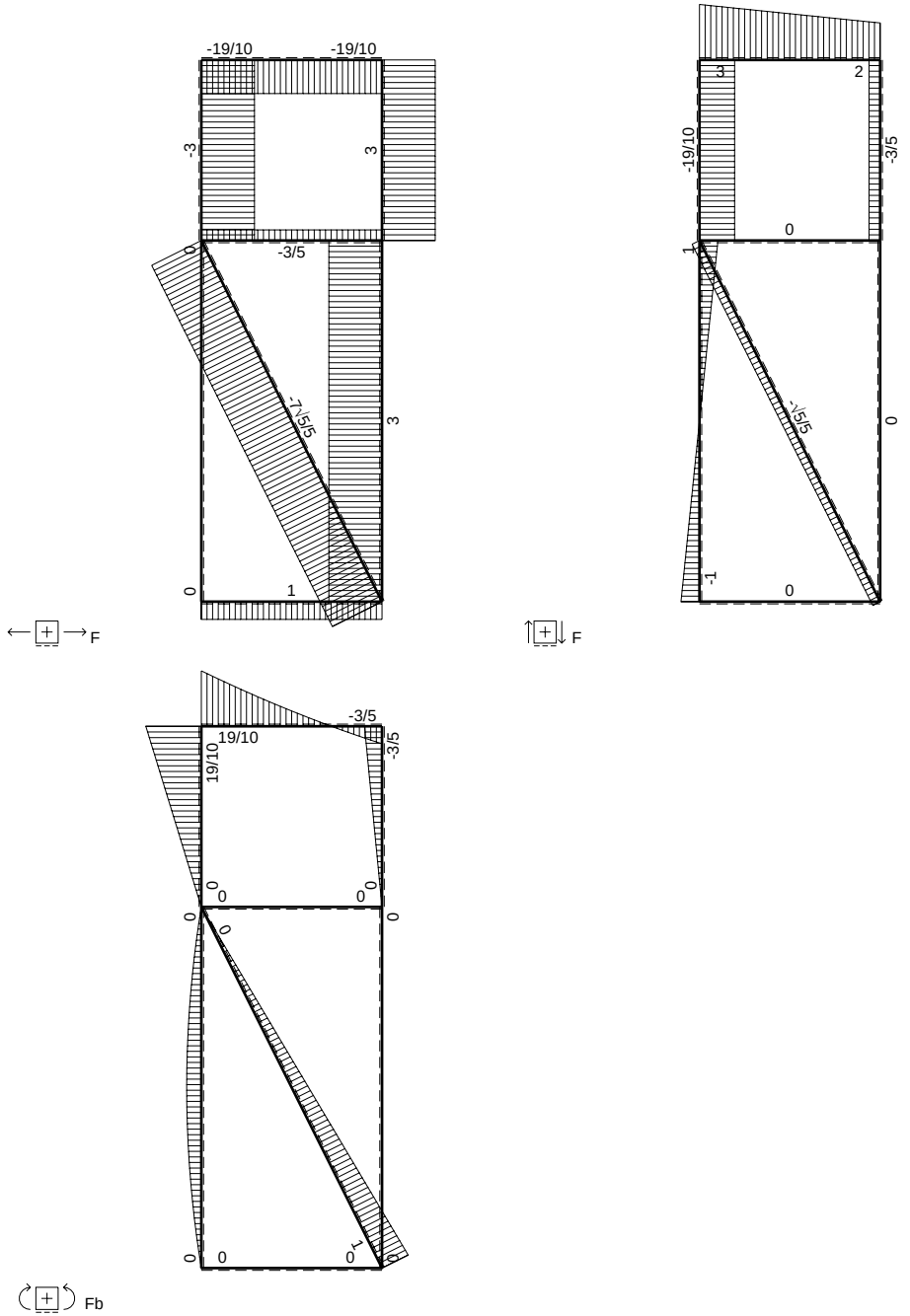


A = 486. mm^2
J_x = 175218. mm^4
J_y = 15474. mm^4
J_{xy} = 33964. mm^4
J_u = 182139. mm^4
J_v = 8553. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2010
c = cosα = .9799
s = sinα = -.1997
x_g = 15.22 mm
y_g = 28.02 mm
N = -2070. N
T_y = -1466. N
M_x = 571838. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -8.545 mm
v_m = 25.79 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -197.7 N/mm^2

CI09.xxxx.090

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F
E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
3F
3/5F
B
D
C
D
3/5F
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A


-19/10
-19/10
-3
3
-3/5
0
-7\5/5
3
0
1
F
-19/10
3
2
-3/5
0
1
-1\5/5
0
-1
0
F
-3/5
19/10
19/10
-3/5
0
0
0
0
0
0
0
0
0
0
Fb

16.04.26

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

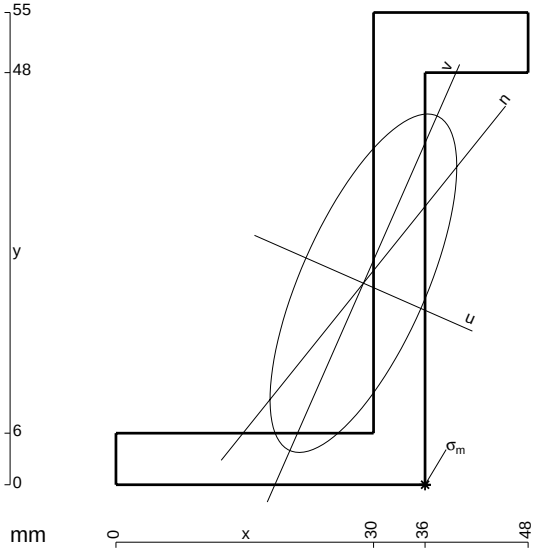
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

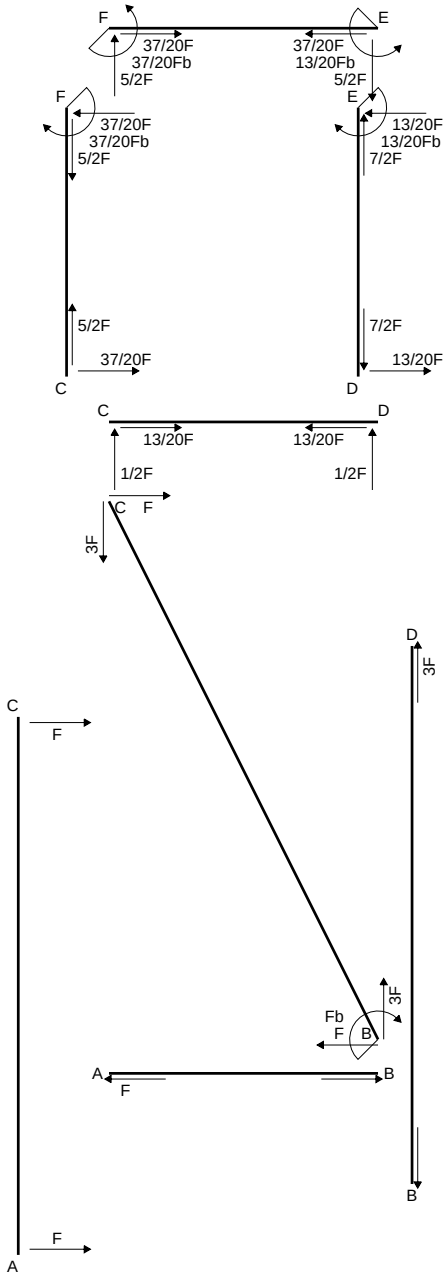
$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

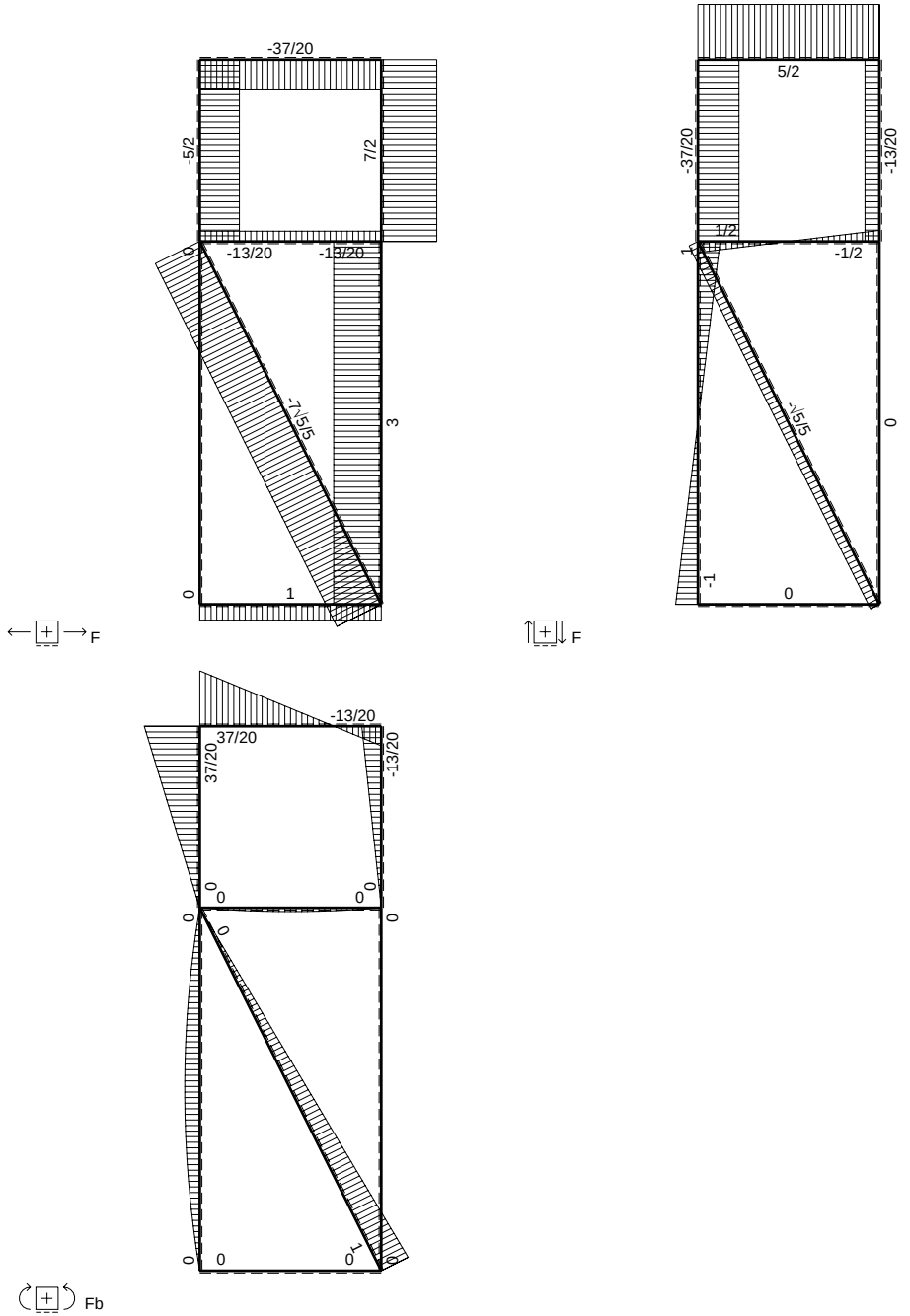


A = 594. mm^2
J_x = 230851. mm^4
J_y = 70234. mm^4
J_{xy} = 87513. mm^4
J_u = 269319. mm^4
J_v = 31766. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4142
c = cosα = .9155
s = sinα = -.4024
x_g = 28.82 mm
y_g = 23.47 mm
N = -1900. N
T_y = 3000. N
M_x = 798000. Nmm
x_m = 36. mm
u_m = 16.02 mm
v_m = -18.6 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 209.2 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
7/2F
37/20F
13/20F
C
D
C
D
13/20F
13/20F
1/2F
1/2F
C
F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
F
A
B


-37/20
-5/2
7/2
-13/20
-13/20
0
-7/5/5
3
0
1
F
5/2
-37/20
-13/20
1/2
-1/2
-7/5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	1/2Fx-1/2qx²	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx²	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

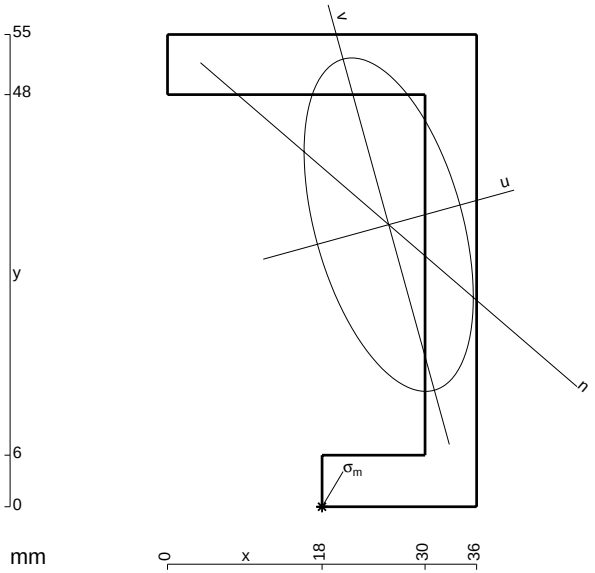
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

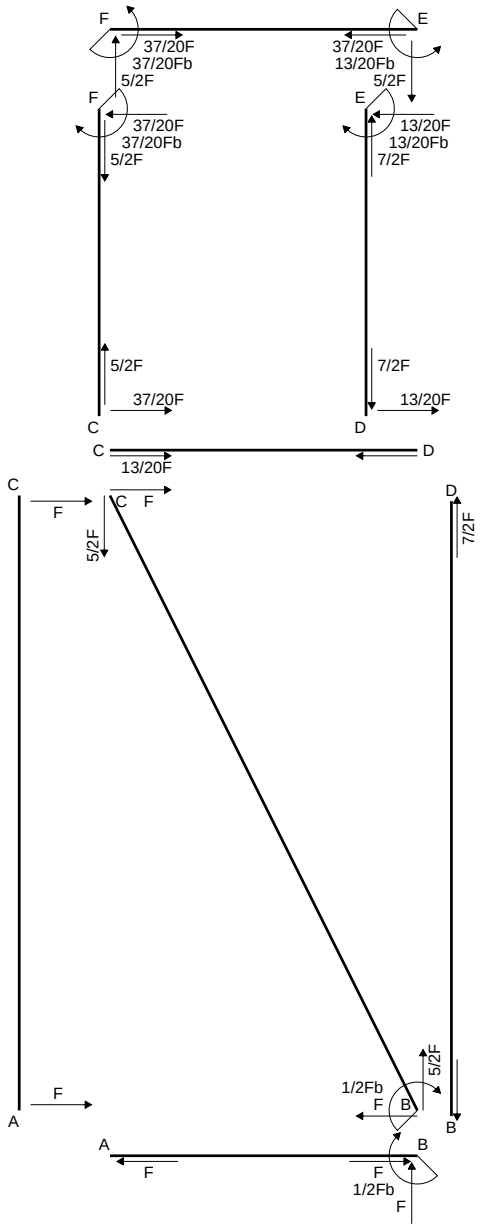
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

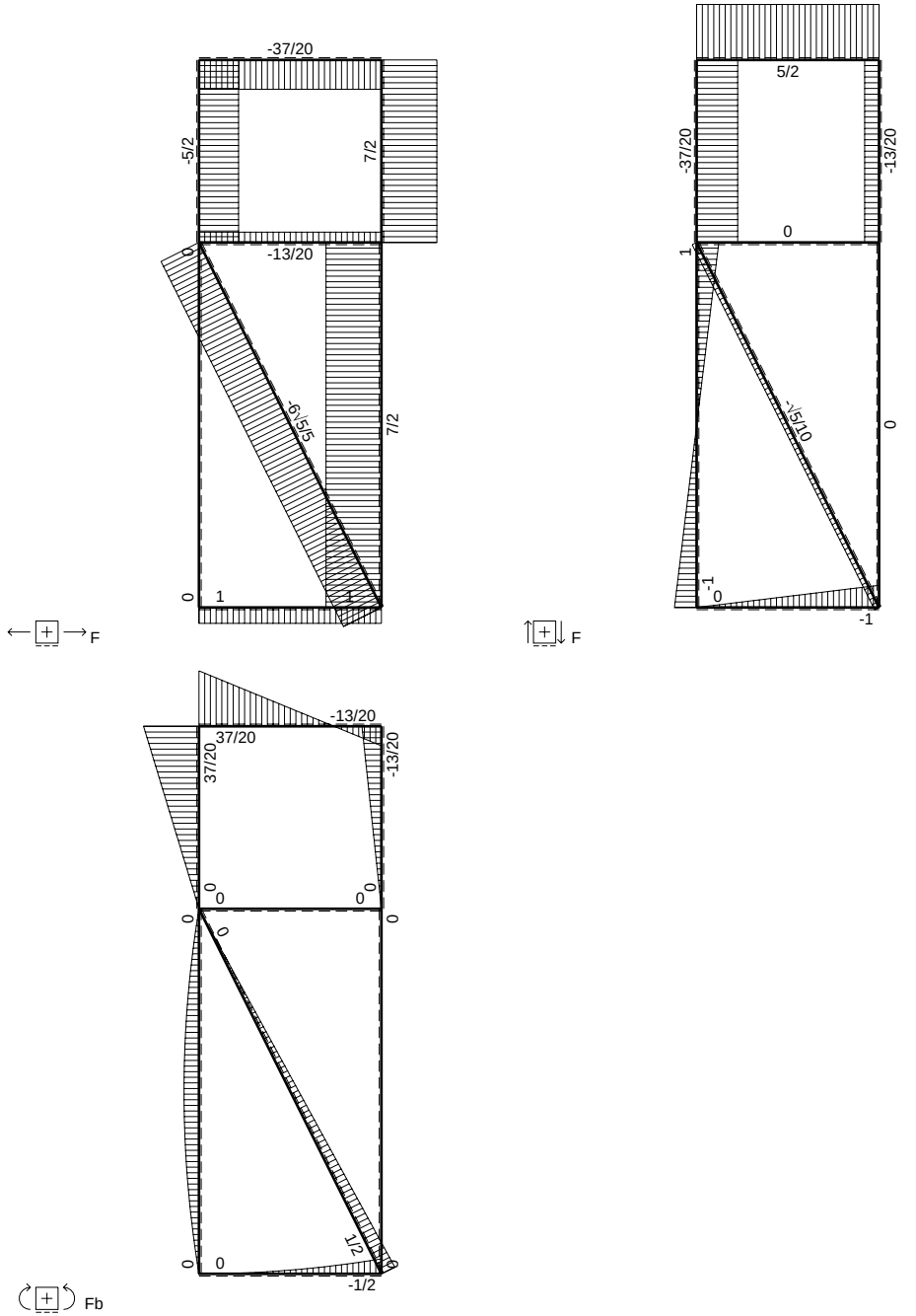


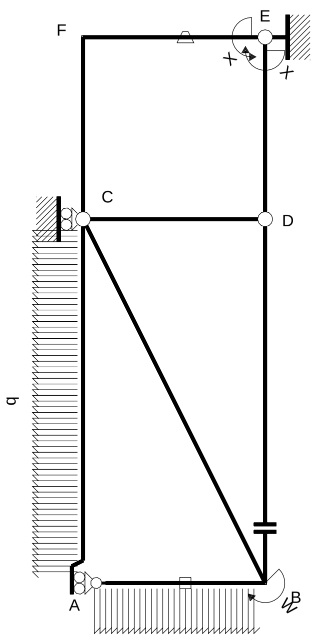
A = 612. mm^2
J_x = 230903. mm^4
J_y = 59438. mm^4
J_{xy} = -51141. mm^4
J_u = 244998. mm^4
J_v = 45343. mm^4
α= artg($2J_{xy}/(J_y-J_x)$)/2 = .2689
c = cosα = .9641
s = sinα = .2657
x_g = 25.76 mm
y_g = 32.85 mm
N = -2294. N
T_y = 3100. N
M_x = 1055240. Nmm
x_m = 18. mm
u_m = -16.21 mm
v_m = -29.61 mm
σ_m = N/A-Mcv/J_u-Msu/J_v = 219.5 N/mm^2

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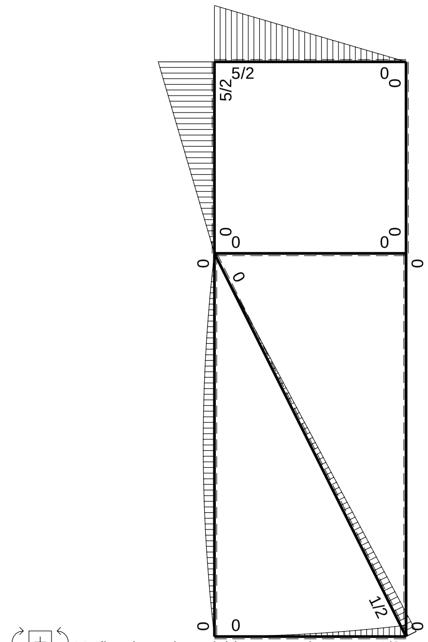
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F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb

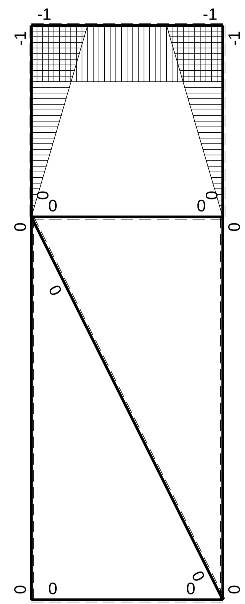

-37/20
-5/2
7/2
-13/20
-5\5/5
F
5/2
-37/20
-13/20
-5\5/10
37/20
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

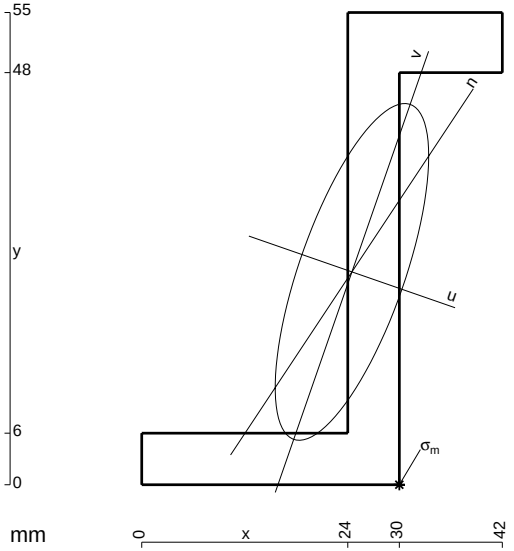
$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

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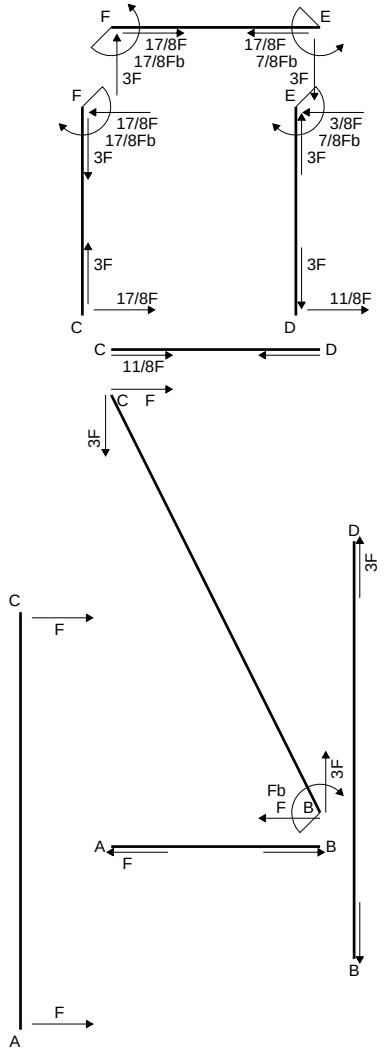


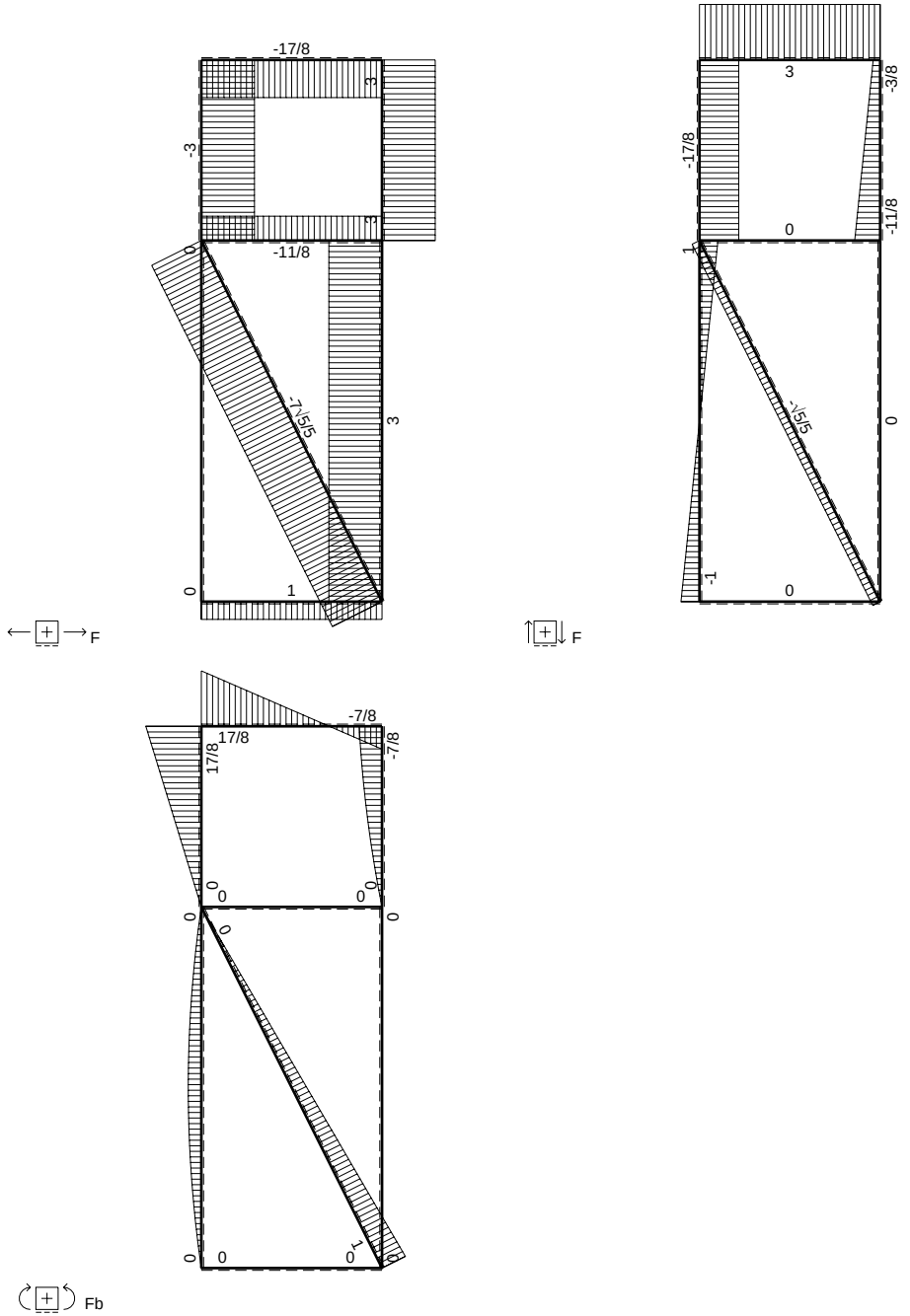
A = 558. mm^2
J_x = 214685. mm^4
J_y = 44581. mm^4
J_{xy} = 67260. mm^4
J_u = 238066. mm^4
J_v = 21200. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3346
c = cosα = .9446
s = sinα = -.3283
x_g = 24.48 mm
y_g = 24.79 mm
N = -1591. N
T_y = 2150. N
M_x = 795500. Nmm
x_m = 30. mm
u_m = 13.35 mm
v_m = -21.6 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 229.8 N/mm^2

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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
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11/8F
C F
3F
D
3F
C
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3F
Fb
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B
F
B
A
F


-17/8
-3
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-11/8
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-3/8
-17/8
-11/8
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1
-7\5/5
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-1
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F
-7/8
17/8
17/8
-7/8
0
0
0
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0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb²/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx²/b	0	1-2x/b+x²/b²	(-1+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx²/b	0	x²/b²		
	totali						-35/24Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

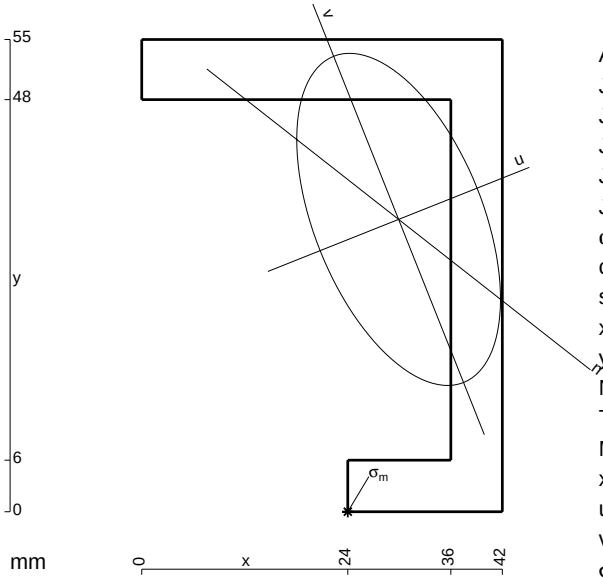
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

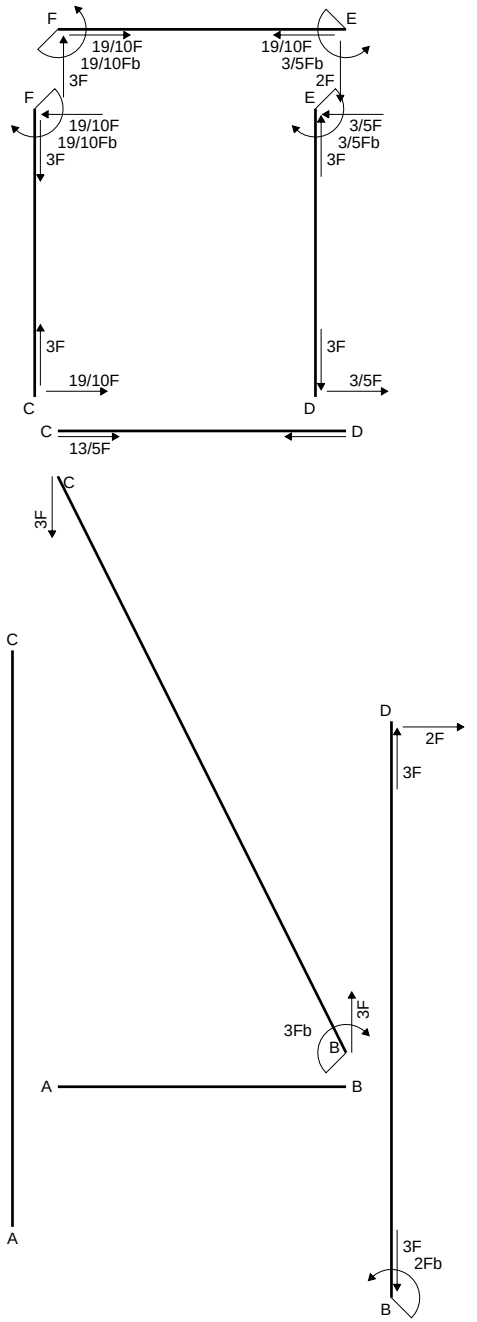
$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

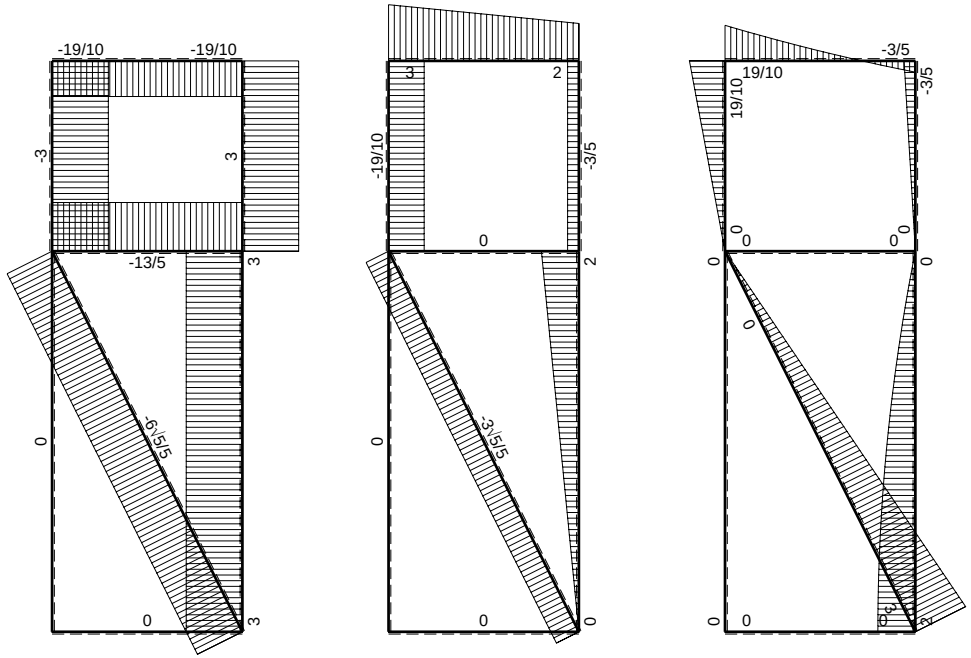


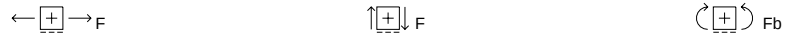
A = 654. mm^2
J_x = 244740. mm^4
J_y = 92084. mm^4
J_{xy} = -72222. mm^4
J_u = 273493. mm^4
J_v = 63331. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3789
$c = \cos\alpha$ = .9291
$s = \sin\alpha$ = .3699
x_g = 29.92 mm
y_g = 34.05 mm
N = -2189. N
T_y = 3090. N
M_x = 1181930. Nmm
x_m = 24. mm
u_m = -18.09 mm
v_m = -29.45 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.8 N/mm^2

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F
E
19/10F
19/10Fb
3F
19/10F
3/5Fb
2F
F
E
19/10F
19/10Fb
3F
3/5F
3/5Fb
3F
3F
19/10F
C
D
3F
3/5F
C
D
13/5F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-19/10
-19/10
-3
3
3
-13/5
0
-6√5/5
0
3
3
2
-19/10
-3/5
0
2
-3√5/5
0
0
19/10
-3/5
0
0
0
0
0
0
0
0


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							3/5Fb

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

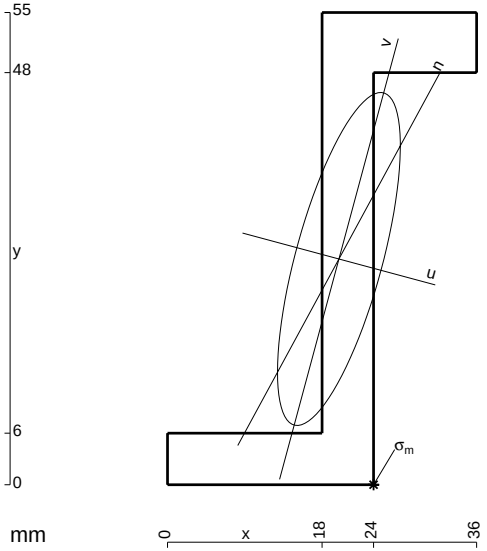
$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 264038 CI09.xxxx.095

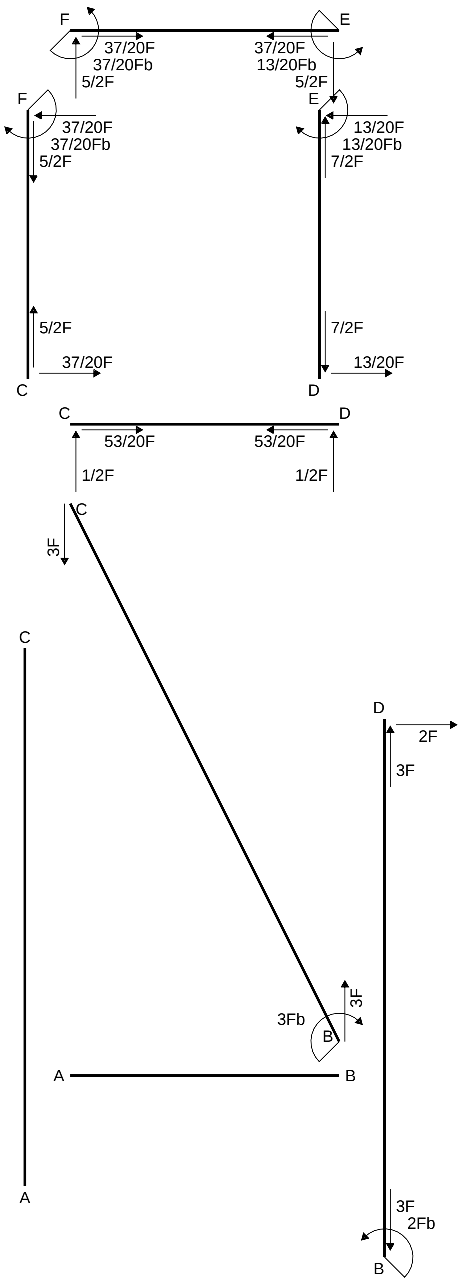


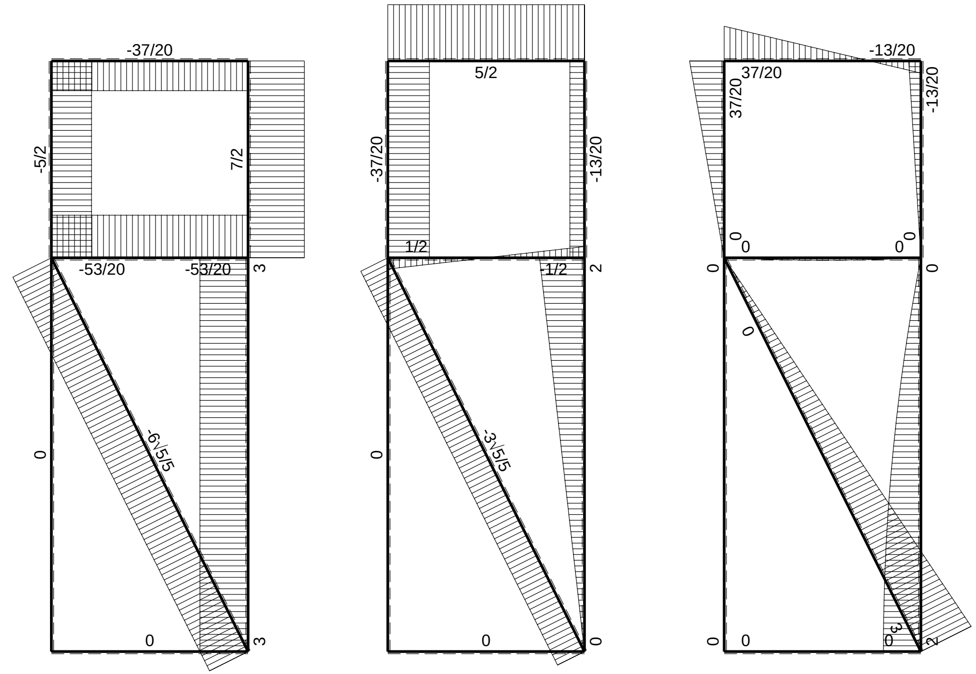
A = 522. mm^2
J_x = 196305. mm^4
J_y = 26711. mm^4
J_{xy} = 49244. mm^4
J_u = 209567. mm^4
J_v = 13450. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2631
$c = \cos\alpha$ = .9656
$s = \sin\alpha$ = -.2600
x_g = 19.97 mm
y_g = 26.29 mm
N = -966. N
T_y = -483. N
M_x = 626400. Nmm
x_m = 24. mm
u_m = 10.73 mm
v_m = -24.34 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 198.4 N/mm^2

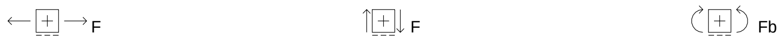
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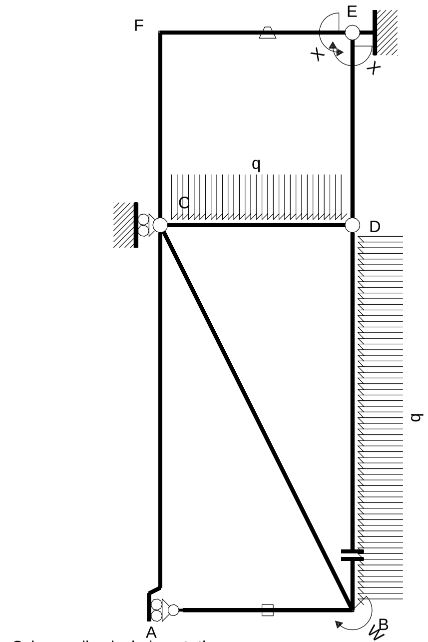
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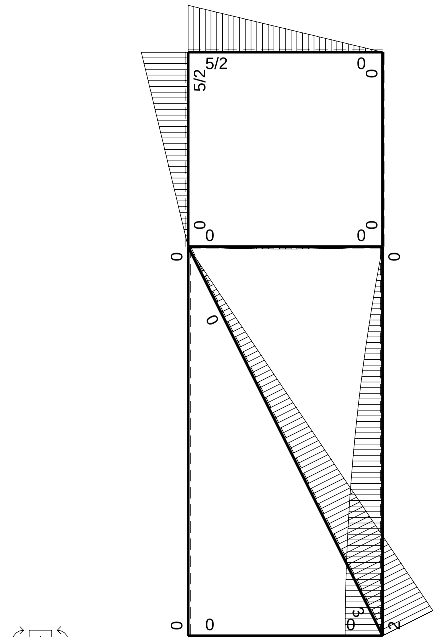

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
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5/2F
13/20F
13/20Fb
7/2F
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7/2F
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3F
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3F
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3F
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2Fb
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-37/20
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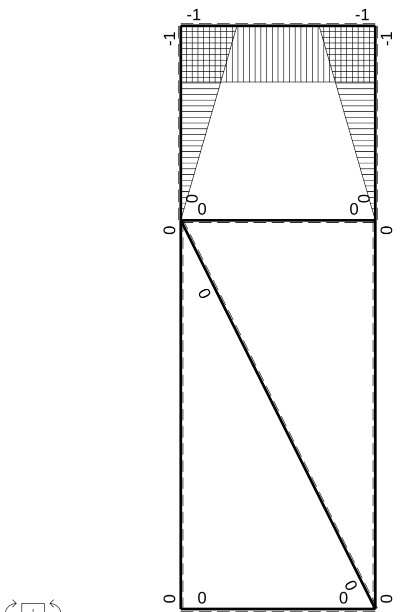

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

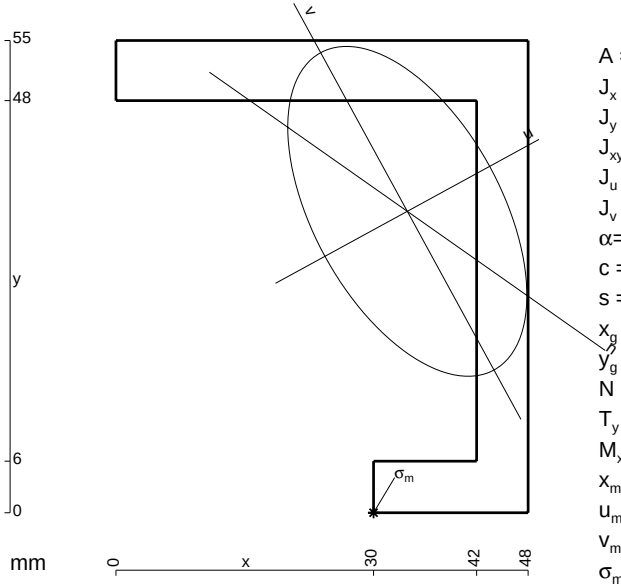
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

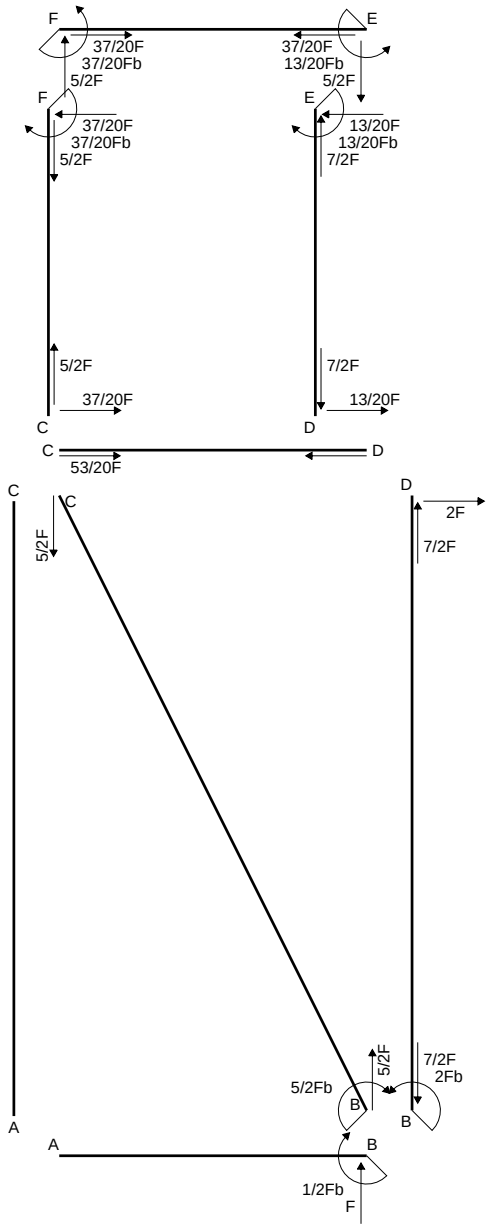
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

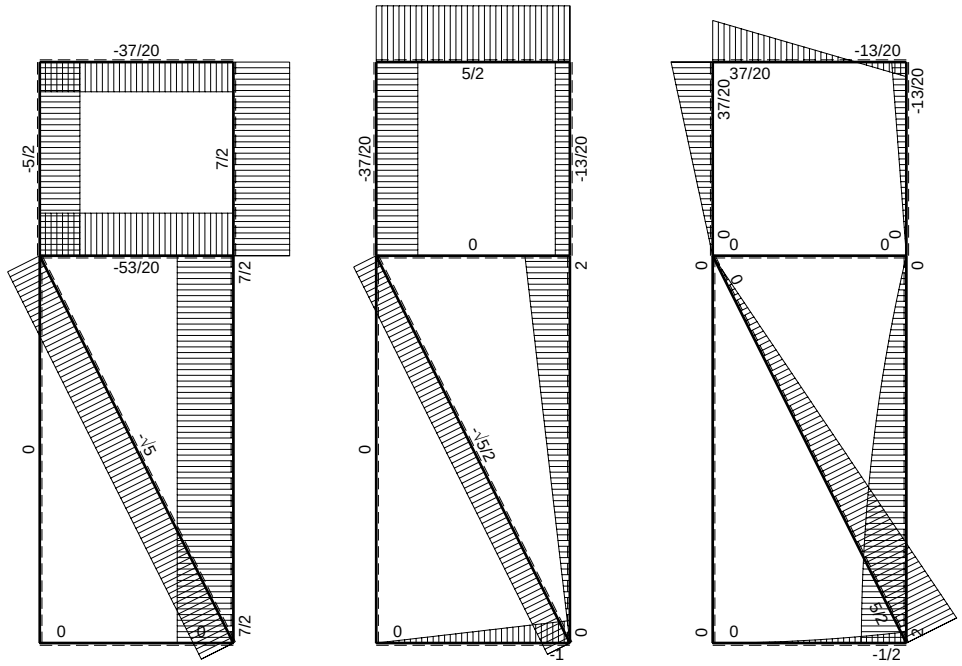


A = 696. mm^2
J_x = 256929. mm^4
J_y = 134973. mm^4
J_{xy} = -94891. mm^4
J_u = 308745. mm^4
J_v = 83156. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4998
c = cosα = .8777
s = sinα = .4793
x_g = 33.93 mm
y_g = 35.1 mm
N = -1529. N
T_y = -764.7 N
M_x = 1060200. Nmm
x_m = 30. mm
u_m = -20.27 mm
v_m = -28.93 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = 208.9 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
13/20Fb
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
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53/20F
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B
1/2Fb
F


-37/20
-5/2
7/2
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-5/2
5/2
-13/20
2
-1
37/20
-1/2


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$5/2Fb-\sqrt{5}/2Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

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PROCEDIMENTO E RISULTATI 267428 CI09.xxxx.097

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

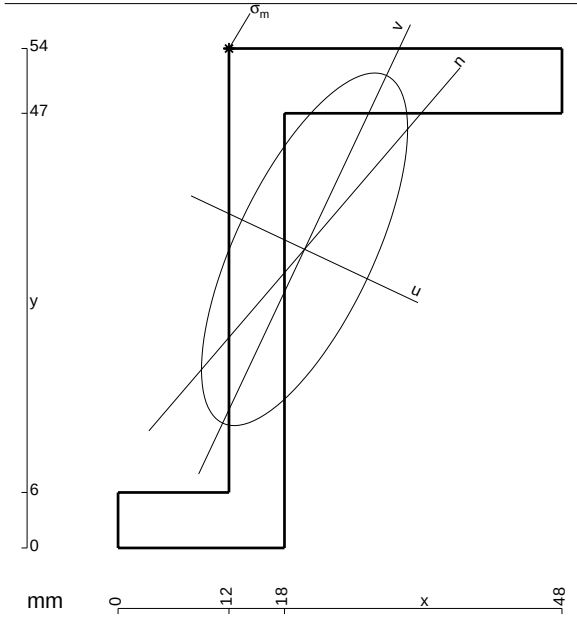
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

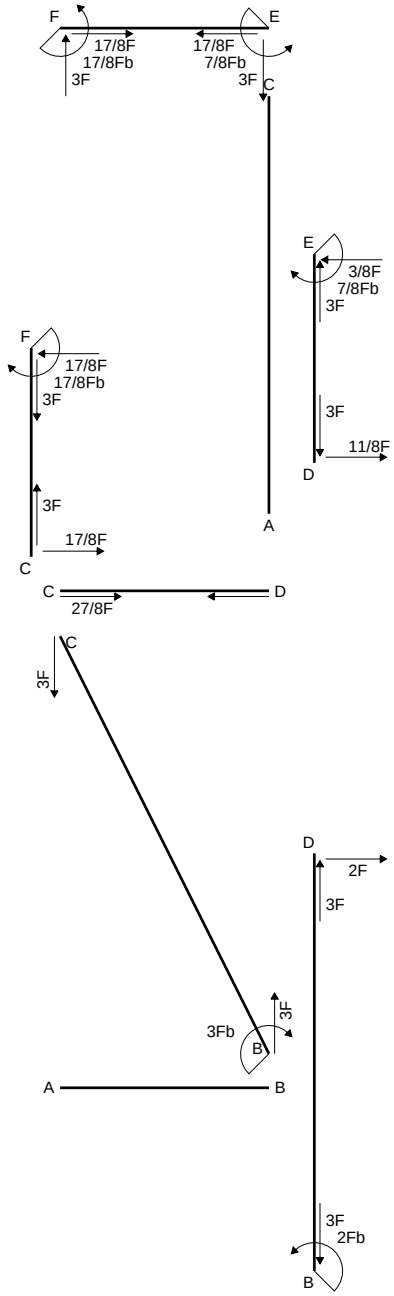
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

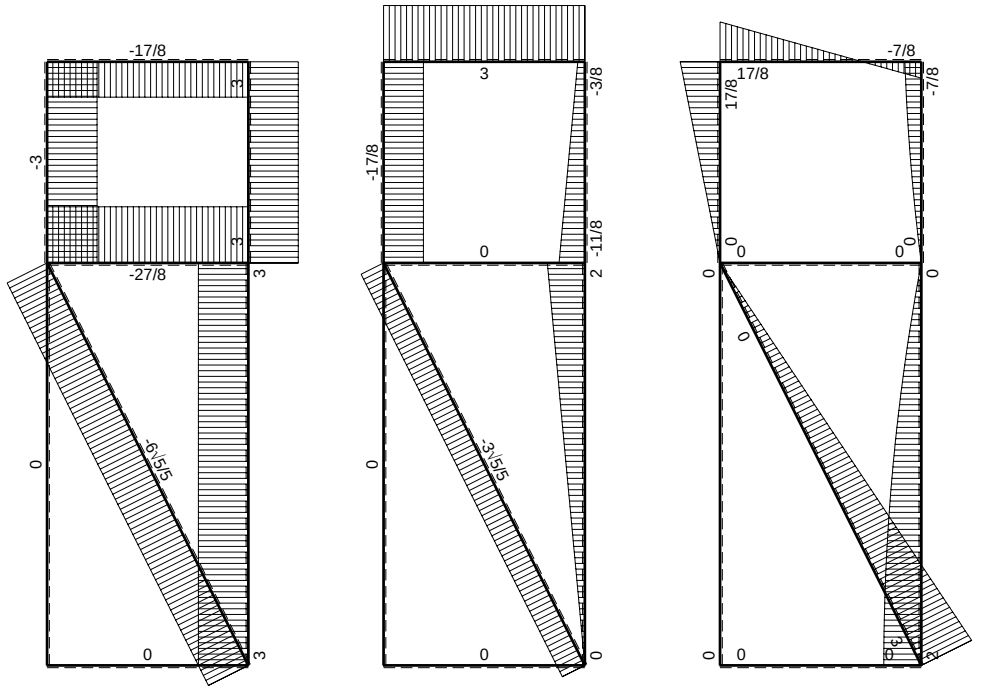


A = 606. mm^2
J_x = 220278. mm^4
J_y = 75271. mm^4
J_{xy} = 87807. mm^4
J_u = 261647. mm^4
J_v = 33902. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4403
c = cosα = .9046
s = sinα = -.4262
x_g = 20.17 mm
y_g = 32.29 mm
N = -1140. N
T_y = -570.2 N
M_x = 816000. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -16.64 mm
v_m = 16.16 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -218.2 N/mm^2

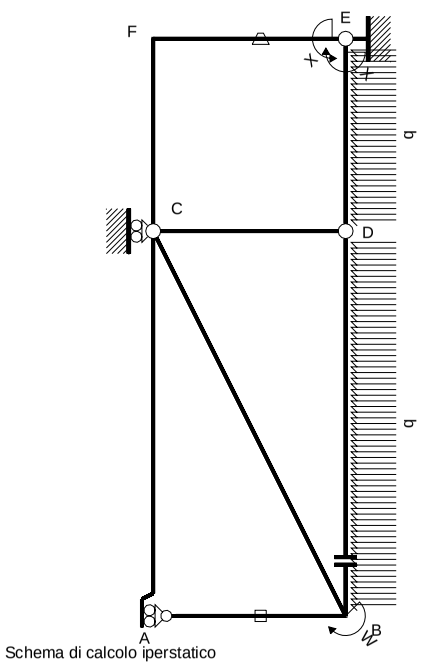
CI09.xxxx.097

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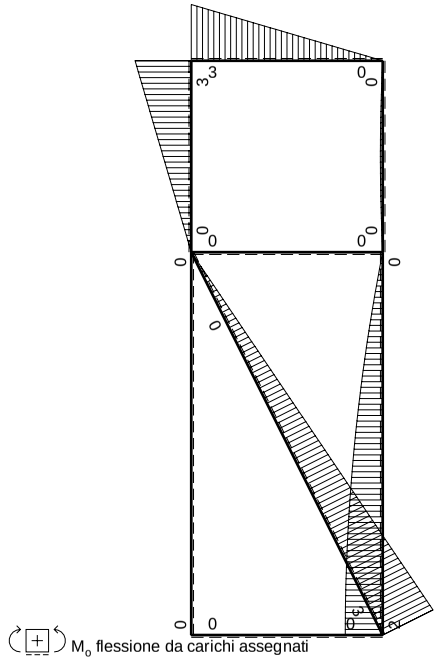

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
C
E
3/8F
7/8Fb
3F
F
17/8F
17/8Fb
3F
3F
11/8F
D
3F
A
17/8F
C
C
D
27/8F
C
3F
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
B


-17/8
3
-3
3
-27/8
3
0
-6√5/5
3
0
0
-17/8
3
-3/8
0
-11/8
2
-3√5/5
0
0
-7/8
17/8
17/8
-7/8
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0
0
0
0
0
0

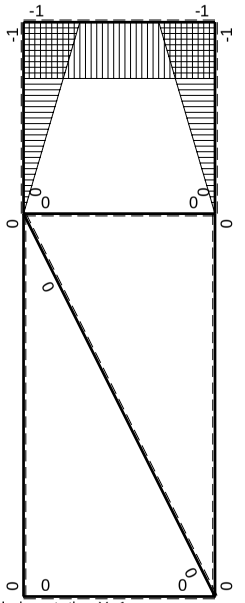

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
				totali			-35/24Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264872 CI09.xxxx.098

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

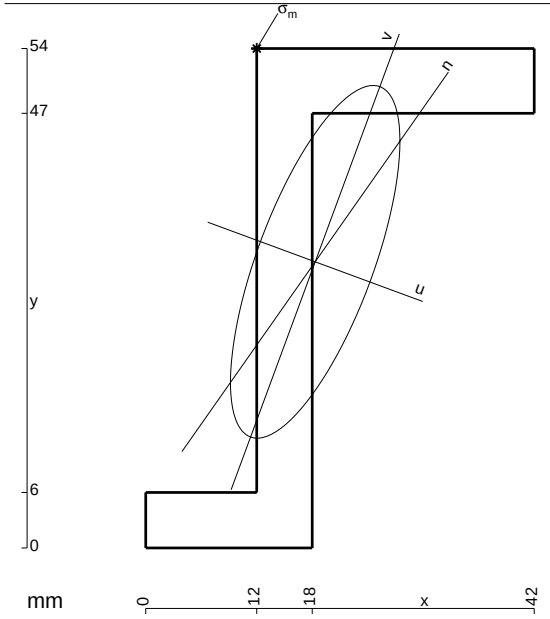
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2]_o^b\ Fb\ 1/EJ$

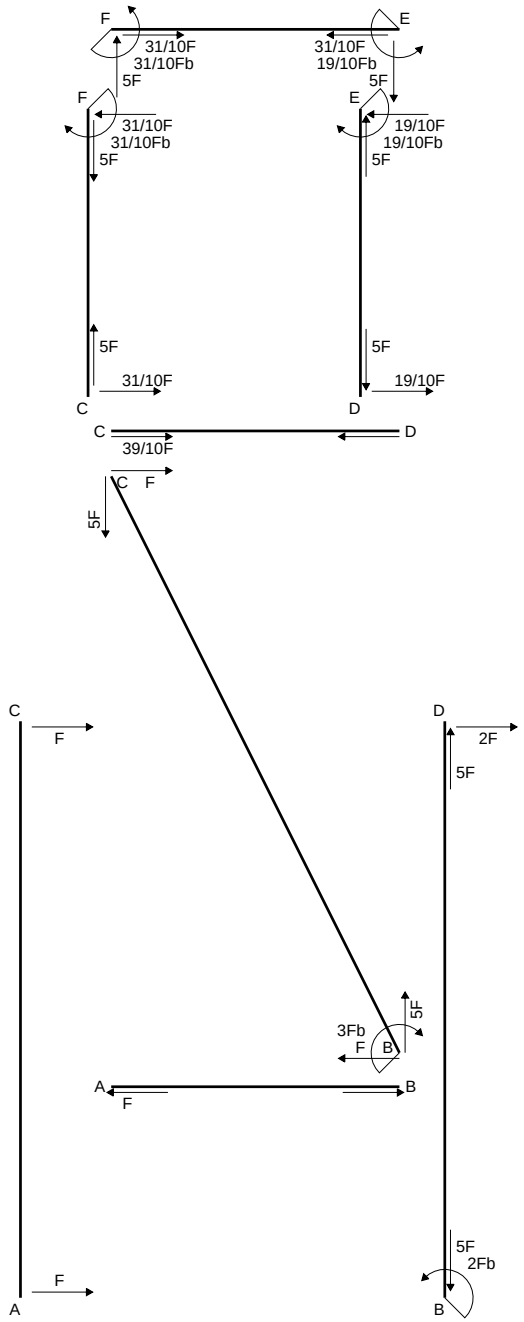
$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

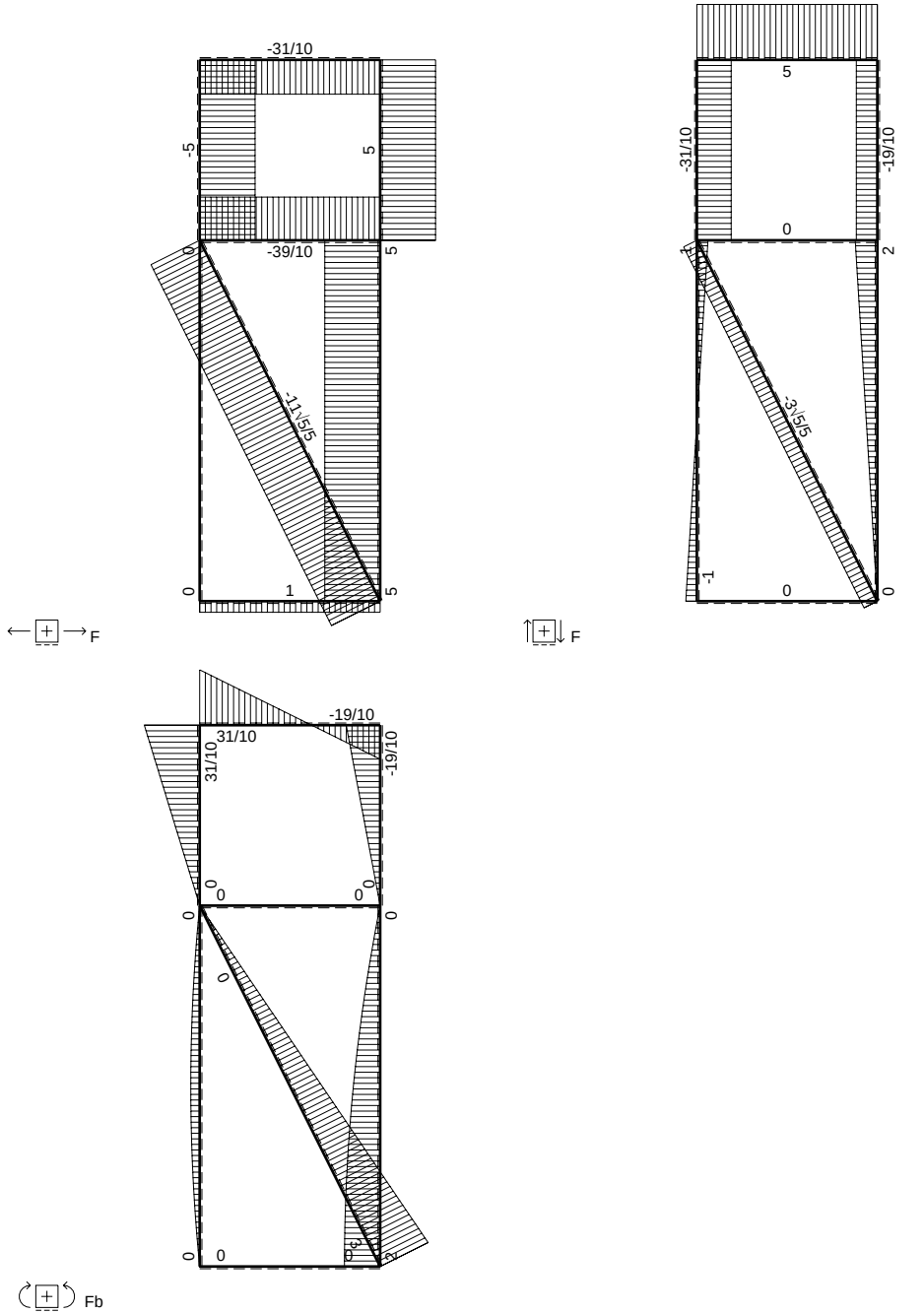


A = 564. mm^2
J_x = 205146. mm^4
J_y = 47319. mm^4
J_{xy} = 67404. mm^4
J_u = 230014. mm^4
J_v = 22451. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3534
$c = \cos\alpha$ = .9382
$s = \sin\alpha$ = -.3461
x_g = 18.32 mm
y_g = 30.94 mm
N = -1020. N
T_y = -509.8 N
M_x = 775200. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -13.91 mm
v_m = 19.45 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -229.6 N/mm^2

CI09.xxxx.098

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F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
19/10Fb
C
D
39/10F
2F
3Fb
2Fb
A
B


-31/10
-39/10
-11√5/5
-3√5/5
-19/10
31/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
totali							-19/6Fb^2/EJ	5/3Xb/EJ
iperstatica $X=W_{EF}$							19/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264047 CI09.xxxx.099

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

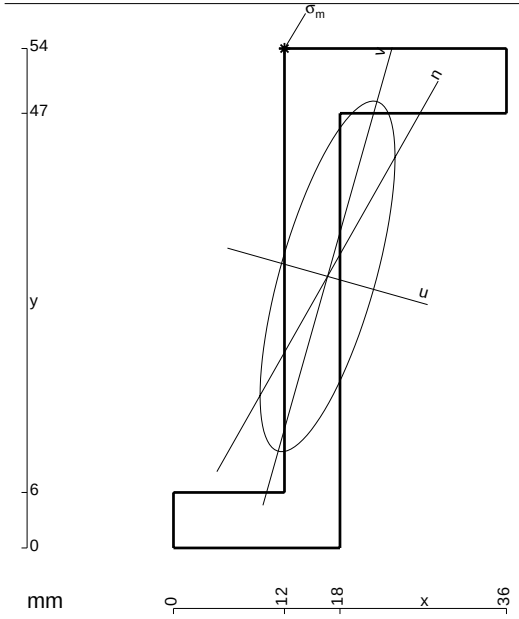
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

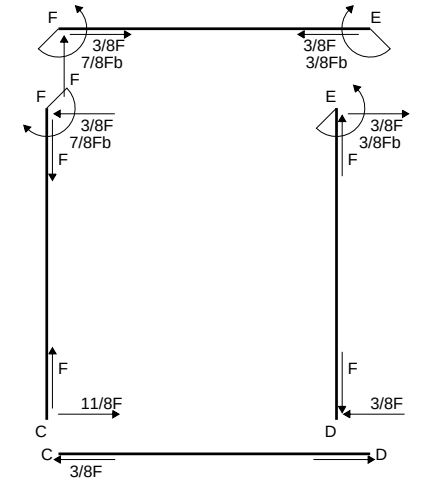
$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

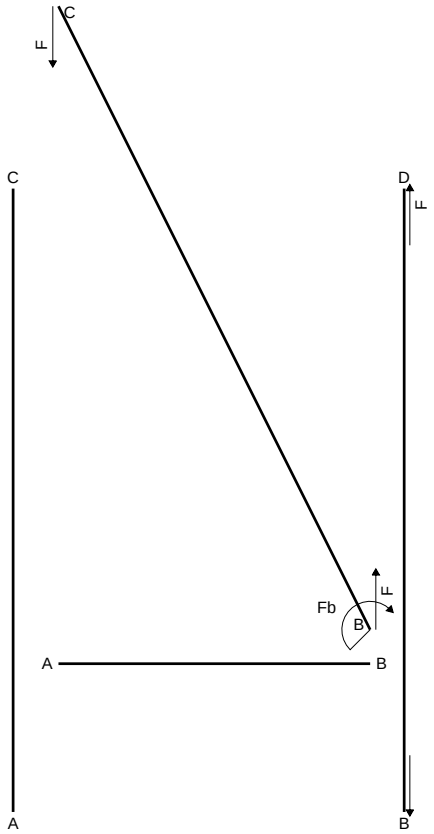


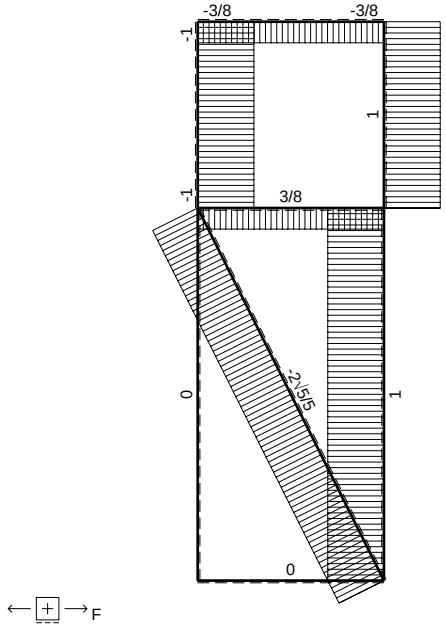
A = 522. mm^2
J_x = 187606. mm^4
J_y = 27784. mm^4
J_{xy} = 49043. mm^4
J_u = 201455. mm^4
J_v = 13935. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2752
c = cosα = .9624
s = sinα = -.2718
x_g = 16.66 mm
y_g = 29.36 mm
N = -1600. N
T_y = -992. N
M_x = 714240. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -11.18 mm
v_m = 22.45 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -235.3 N/mm^2

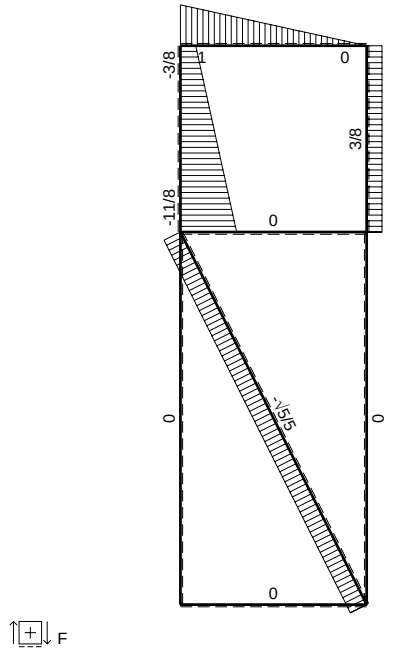
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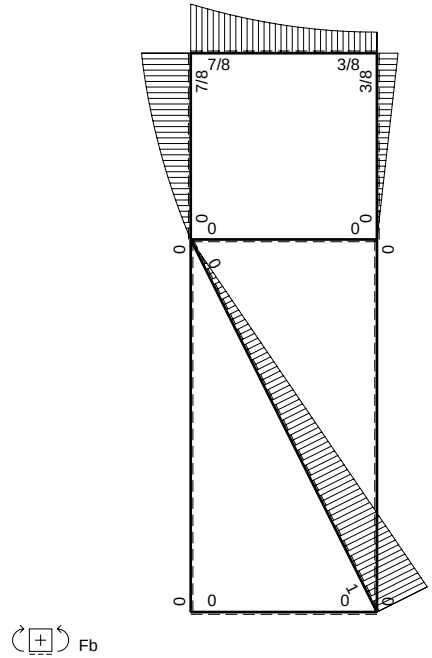
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F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
-2√5/5
1
1
0
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

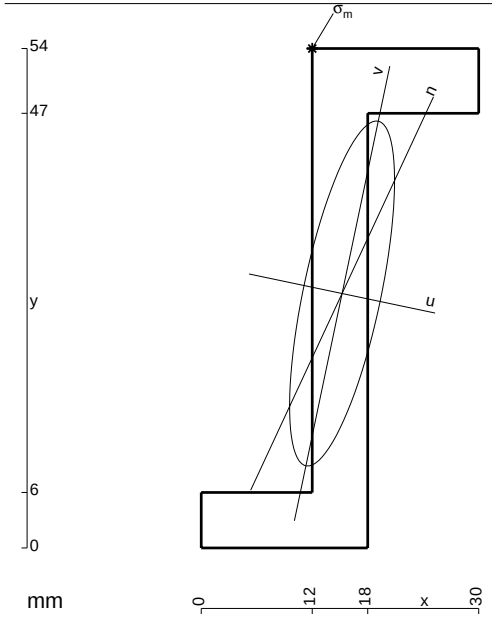
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

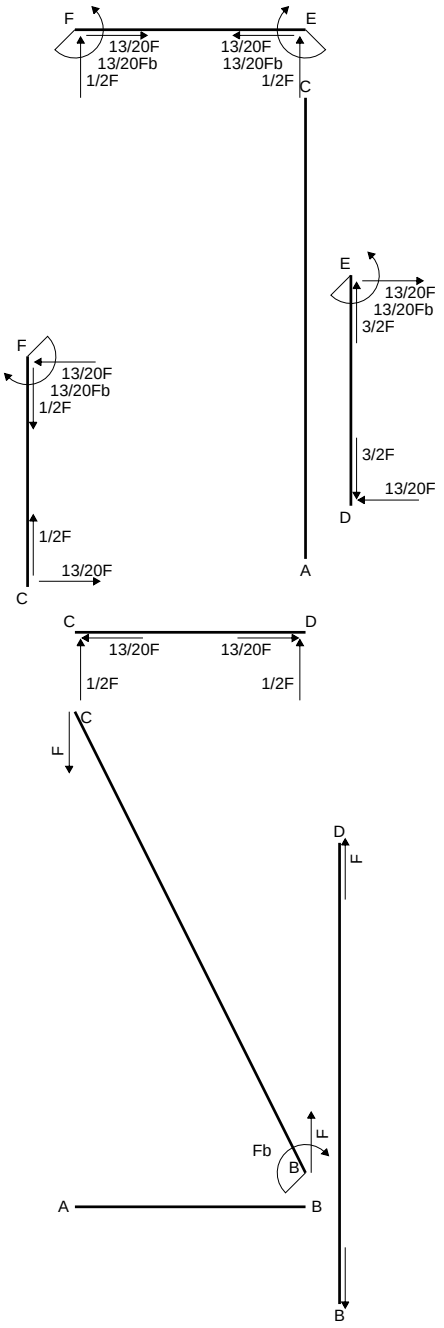
$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

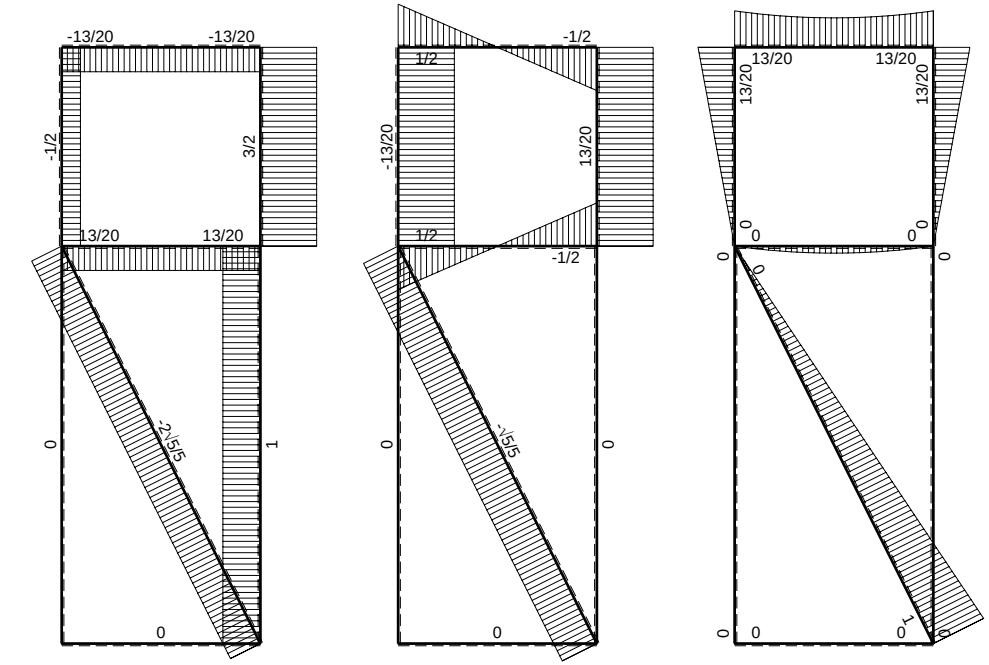


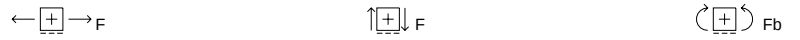
A = 480. mm^2
J_x = 167026. mm^4
J_y = 15456. mm^4
J_{xy} = 33263. mm^4
J_u = 174004. mm^4
J_v = 8477. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = -.2068
c = cosα = .9787
s = sinα = -.2053
x_g = 15.23 mm
y_g = 27.51 mm
N = -1324. N
T_y = -661.9 N
M_x = 562400. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -8.595 mm
v_m = 25.26 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -199.7 N/mm^2

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F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

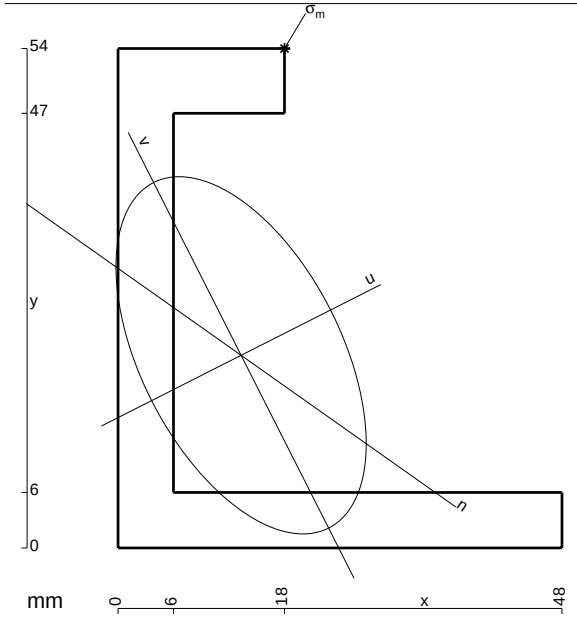
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 264522 CI09.xxxx.101

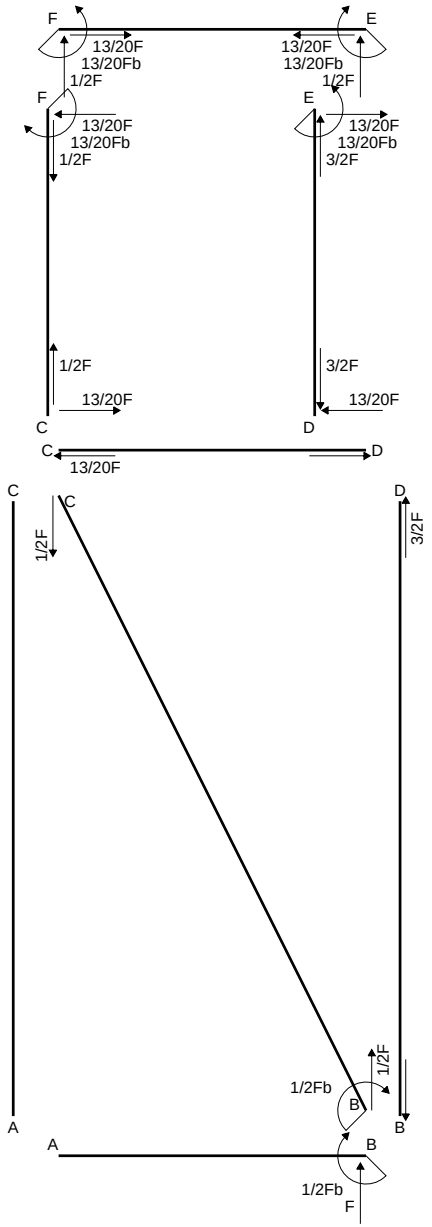


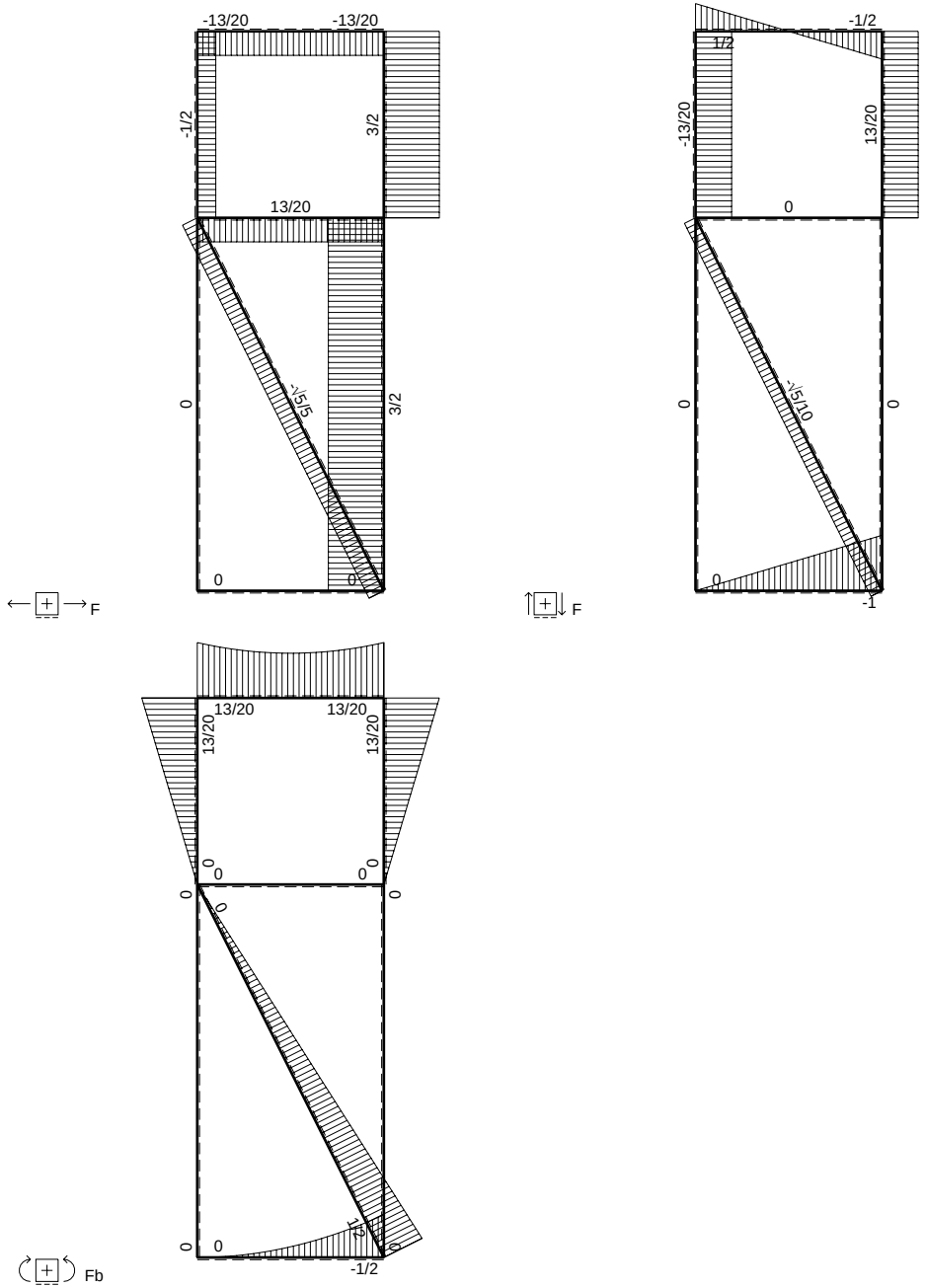
A = 660. mm^2
J_x = 246224. mm^4
J_y = 120837. mm^4
J_{xy} = -85387. mm^4
J_u = 289462. mm^4
J_v = 77600. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4687
$c = \cos\alpha$ = .8921
$s = \sin\alpha$ = .4518
x_g = 13.31 mm
y_g = 20.83 mm
N = -2290. N
T_y = -1145. N
M_x = 1049600. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 19.17 mm
v_m = 27.48 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.5 N/mm^2

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CI09.xxxx.101

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi \theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

PROCEDIMENTO E RISULTATI 268257 CI09.xxxx.102

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

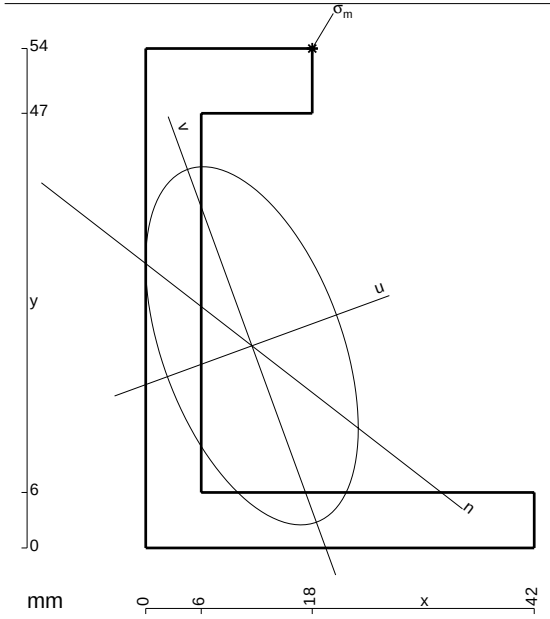
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

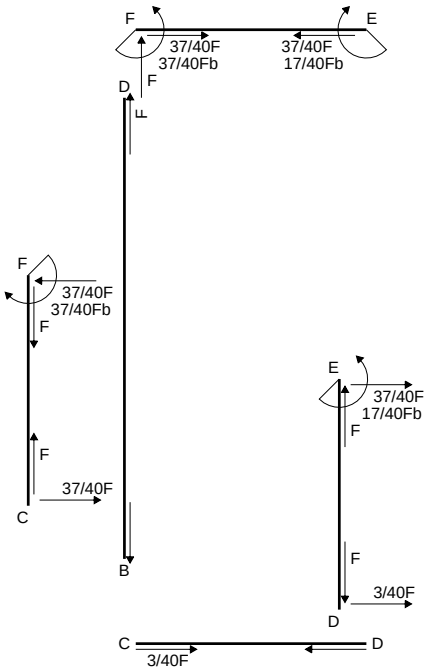
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = 13/12\ Fb^2/EJ$

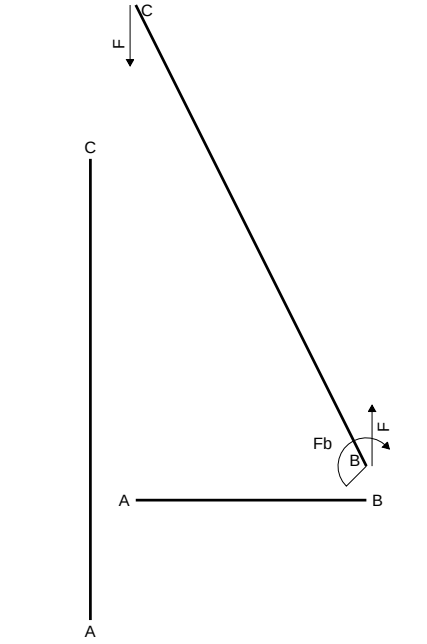


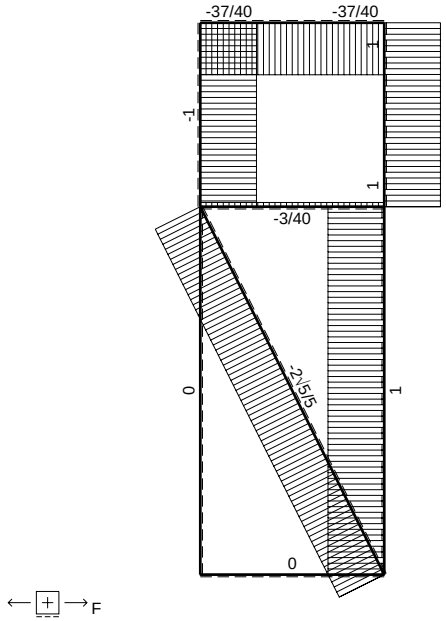
A = 624. mm^2
J_x = 234015. mm^4
J_y = 82488. mm^4
J_{xy} = -63875. mm^4
J_u = 257348. mm^4
J_v = 59155. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3502
$c = \cos\alpha$ = .9393
$s = \sin\alpha$ = .3431
x_g = 11.48 mm
y_g = 21.86 mm
N = -2379. N
T_y = -1830. N
M_x = 1070550. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 17.15 mm
v_m = 27.96 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.6 N/mm^2

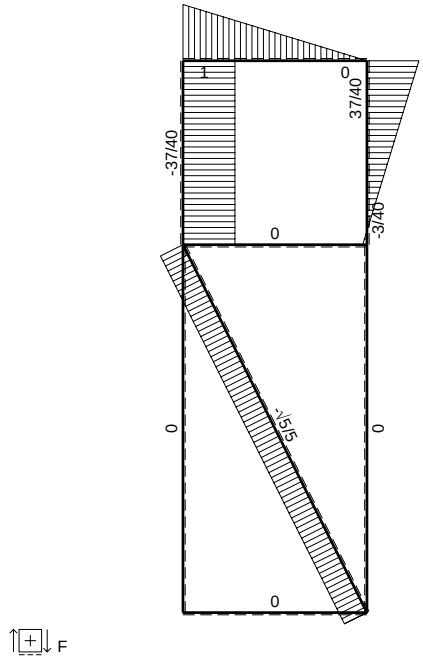
CI09.xxxx.102

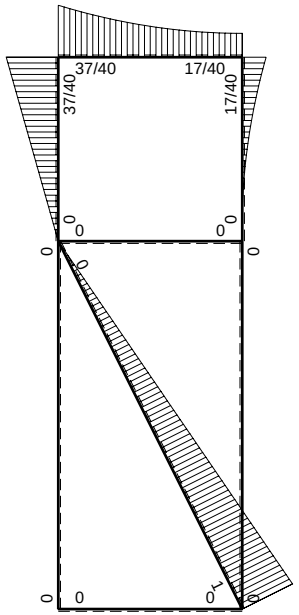
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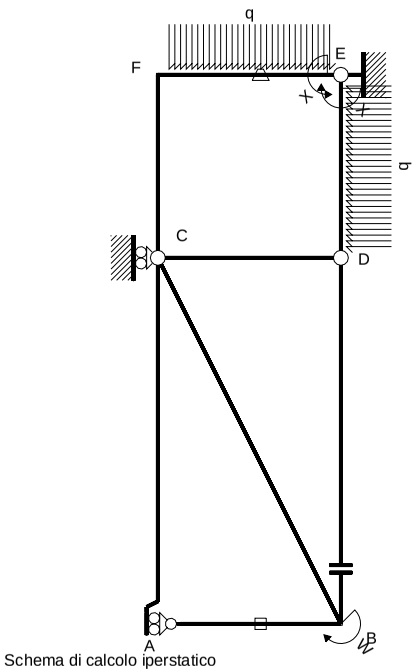

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A
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A

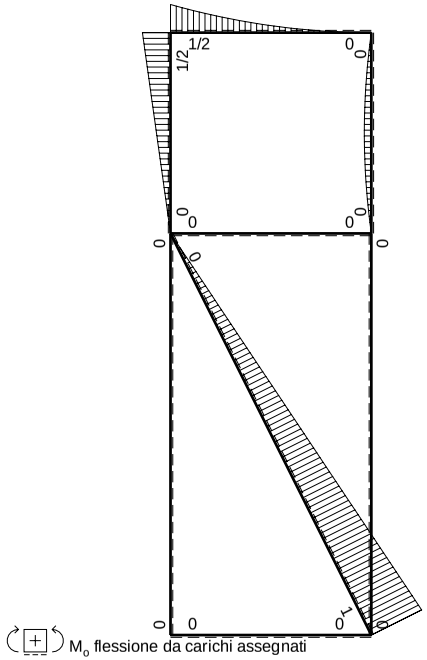

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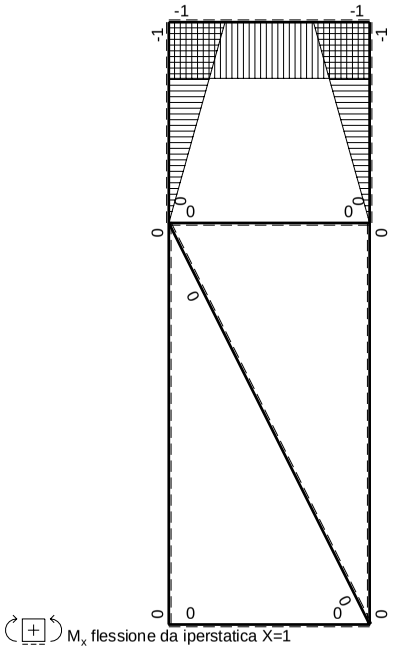

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Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
				totali			17/24Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 956343 CI09.xxxx.103

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ \ Fb^2/EJ$

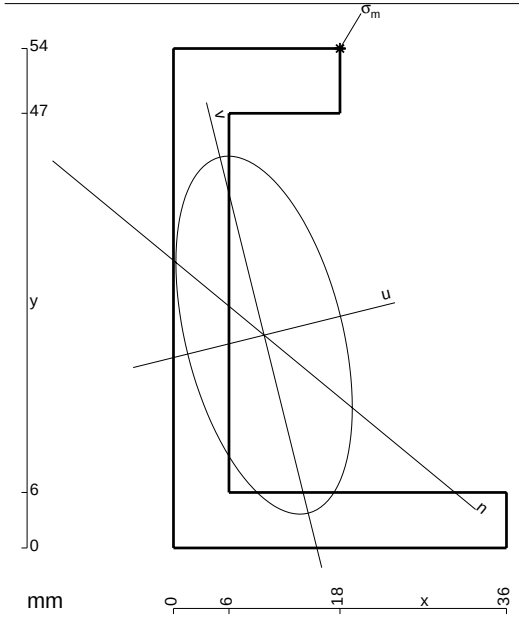
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 956343 CI09.xxxx.103

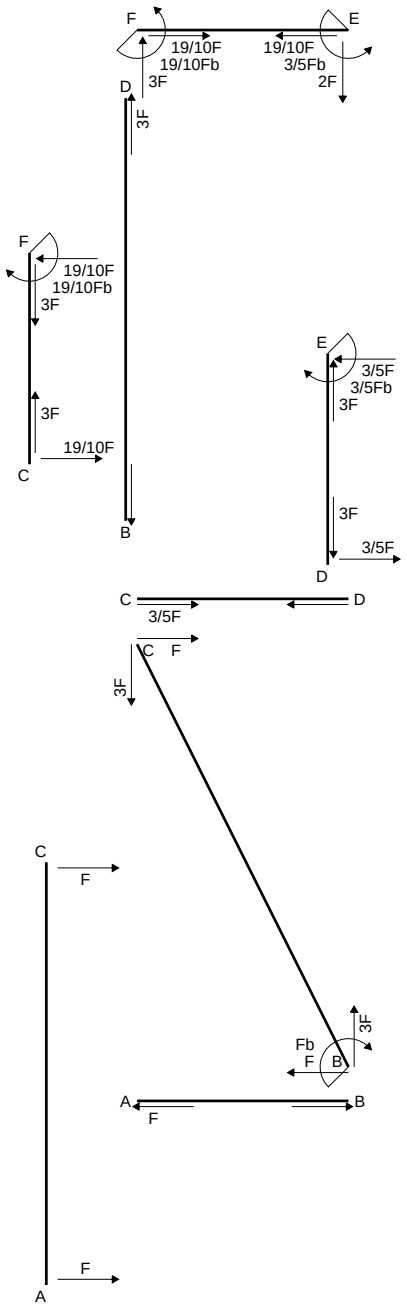


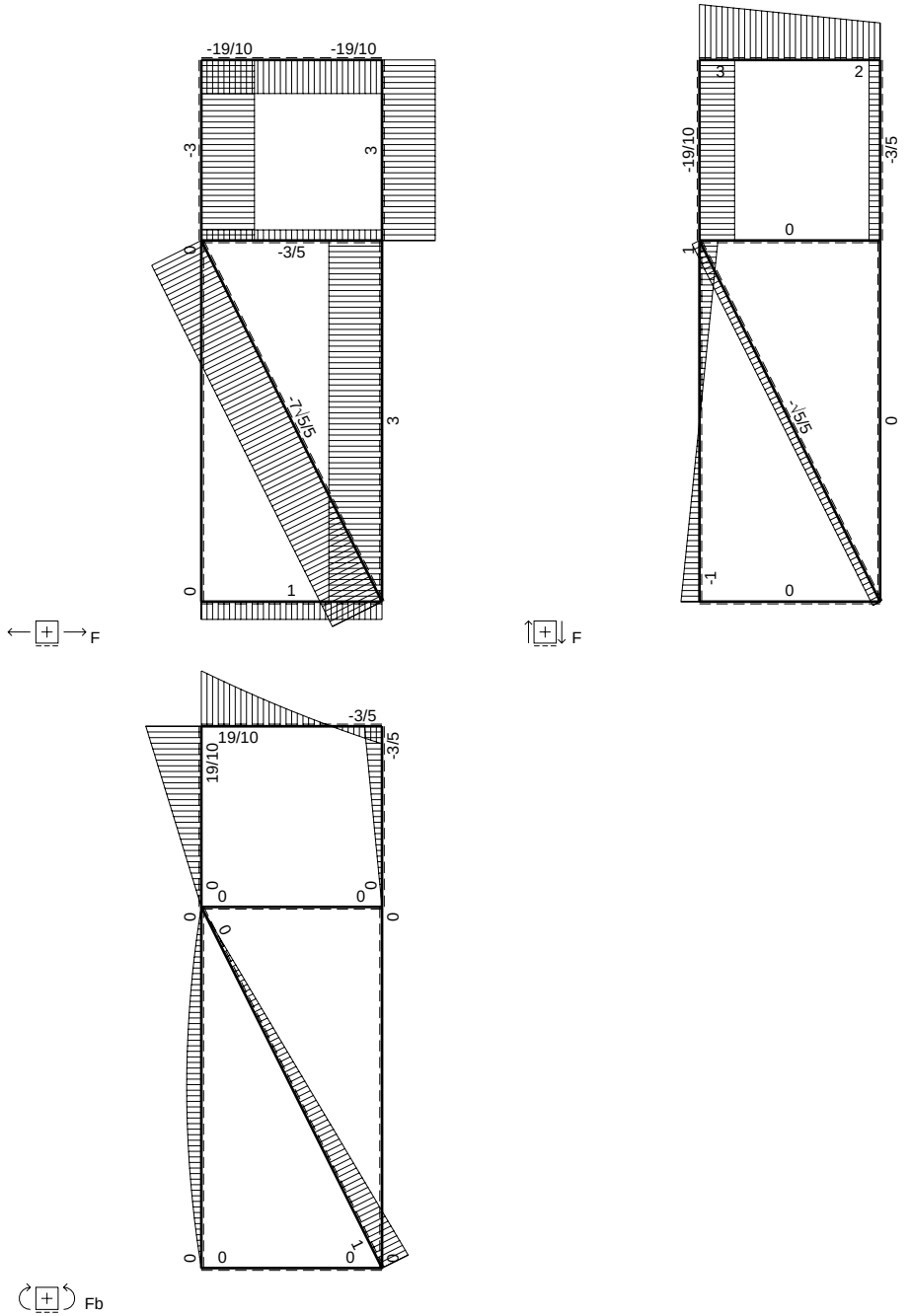
A = 588. mm^2
J_x = 220324. mm^4
J_y = 53448. mm^4
J_{xy} = -44051. mm^4
J_u = 231238. mm^4
J_v = 42533. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2429
$c = \cos\alpha$ = .9707
$s = \sin\alpha$ = .2405
x_g = 9.796 mm
y_g = 23.01 mm
N = -2012. N
T_y = -1006. N
M_x = 1102500. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 15.42 mm
v_m = 28.11 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.6 N/mm^2

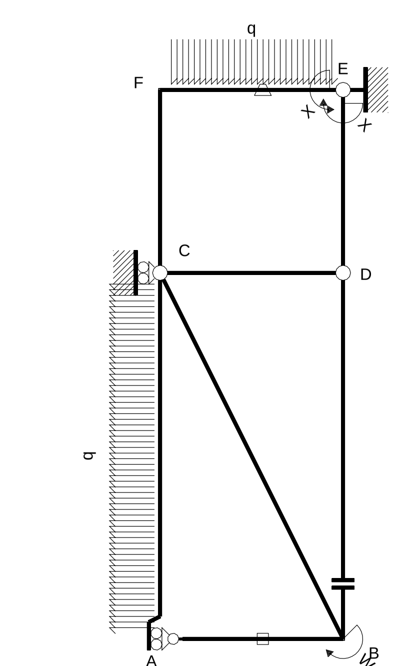
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CI09.xxxx.103

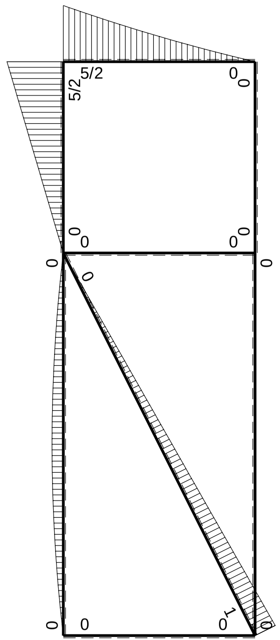
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


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3/5Fb
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2F
D
3F
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19/10F
19/10Fb
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E
3/5F
3/5Fb
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19/10F
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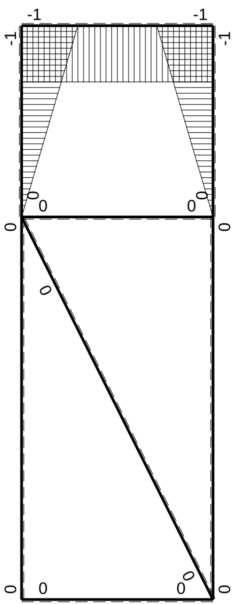

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19/10
19/10
-3/5
0
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0
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0
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0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
totali							-Fb^2/EJ	5/3Xb/EJ
iperstatica $X=W_{EF}$							3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

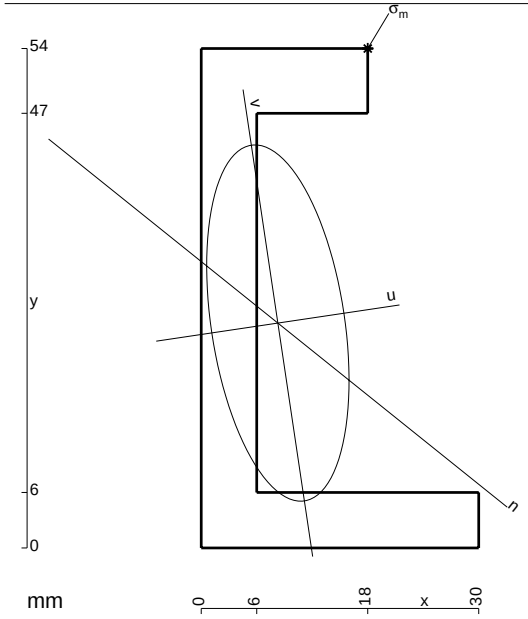
$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

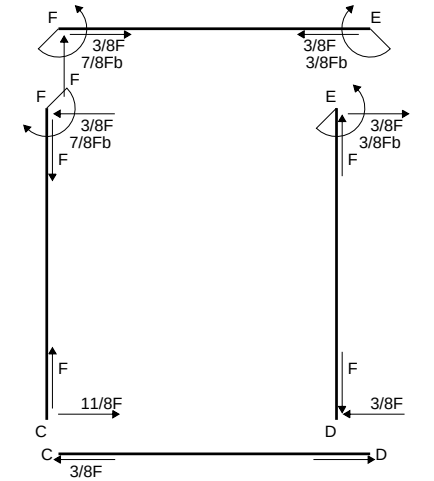
PROCEDIMENTO E RISULTATI 212572 CI09.xxxx.104

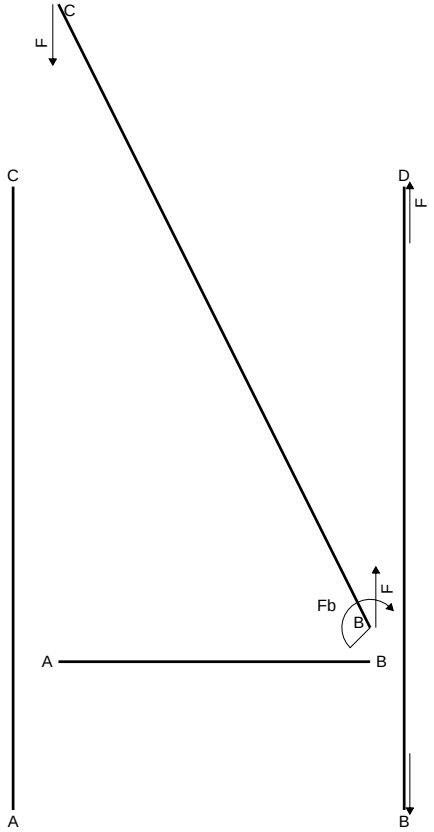


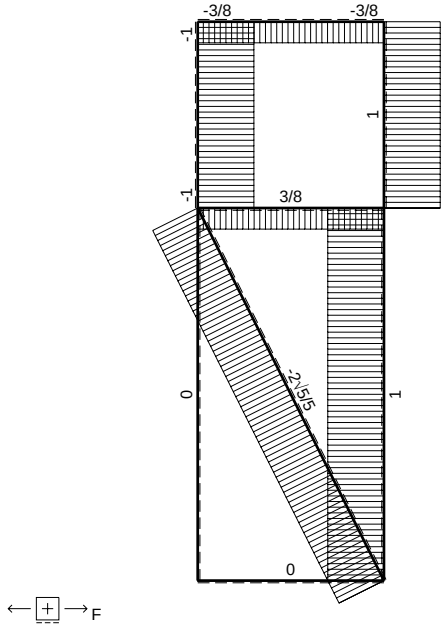
A = 552. mm^2
J_x = 204861. mm^4
J_y = 32692. mm^4
J_{xy} = -26245. mm^4
J_u = 208773. mm^4
J_v = 28780. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1480
c = cosα = .9891
s = sinα = .1474
x_g = 8.283 mm
y_g = 24.32 mm
N = -3390. N
T_y = -2147. N
M_x = 1137910. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 13.99 mm
v_m = 27.93 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -238.2 N/mm^2

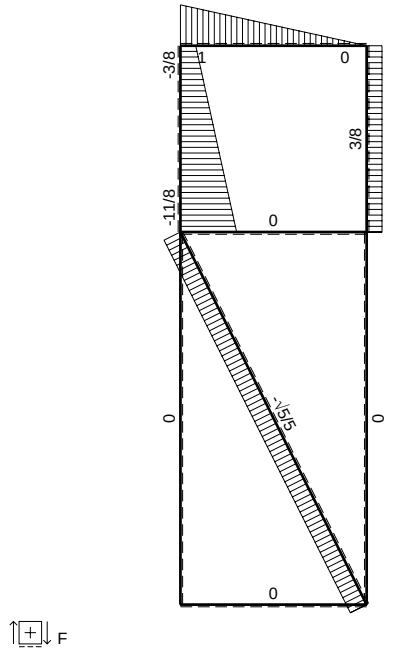
CI09.xxxx.104

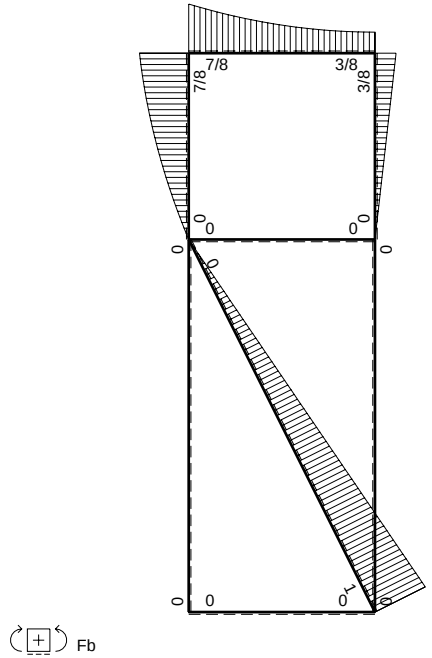
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3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

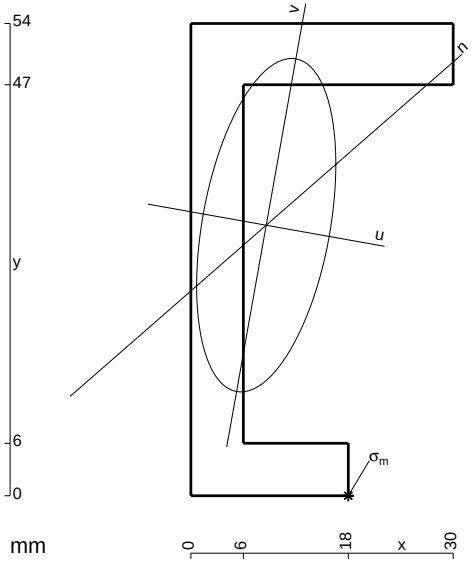
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

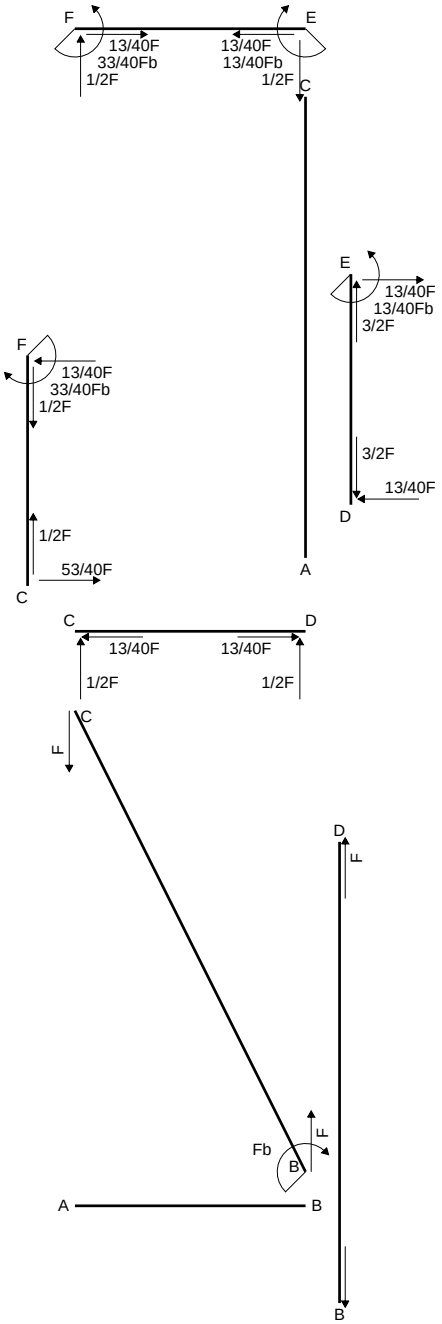
$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

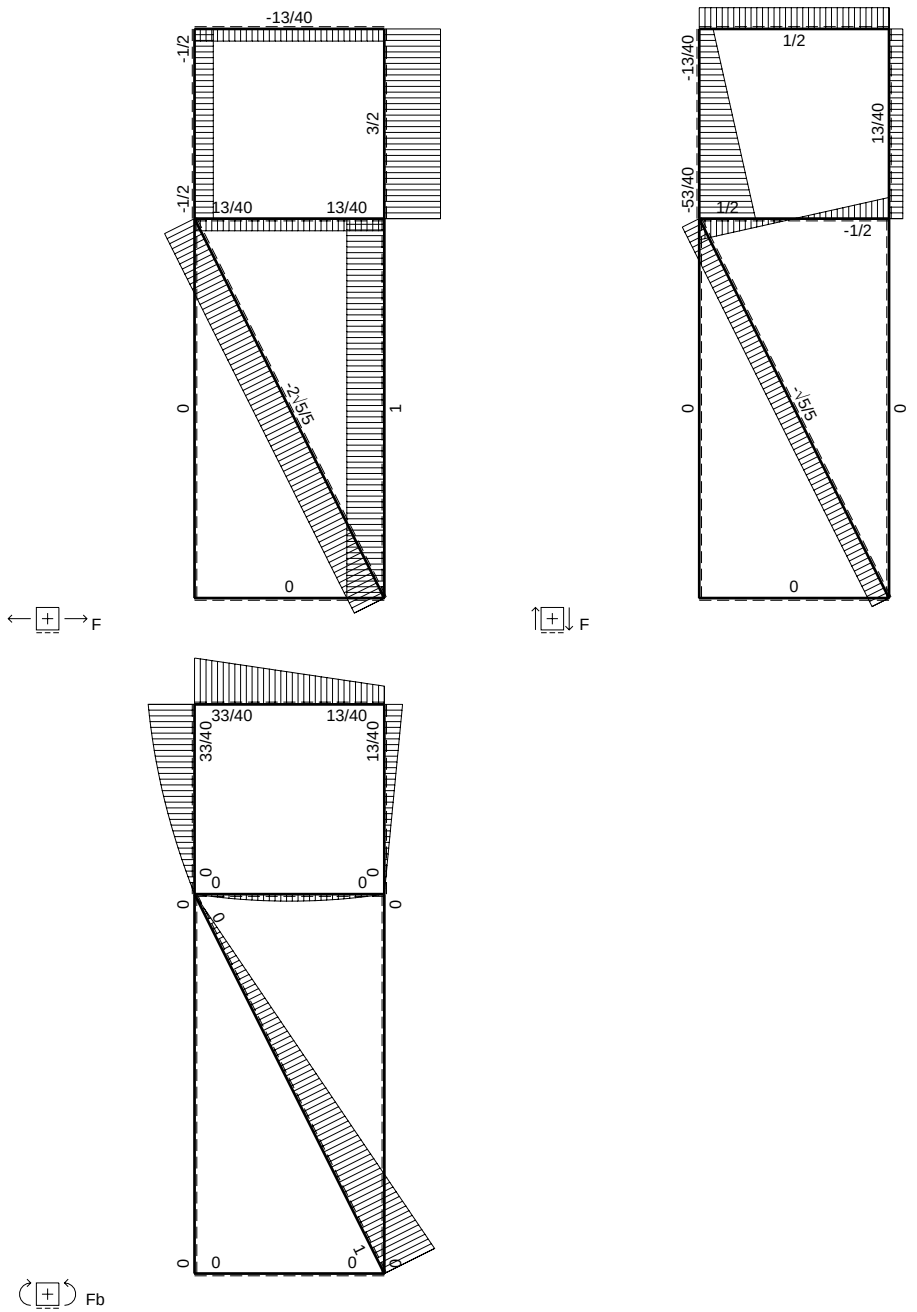


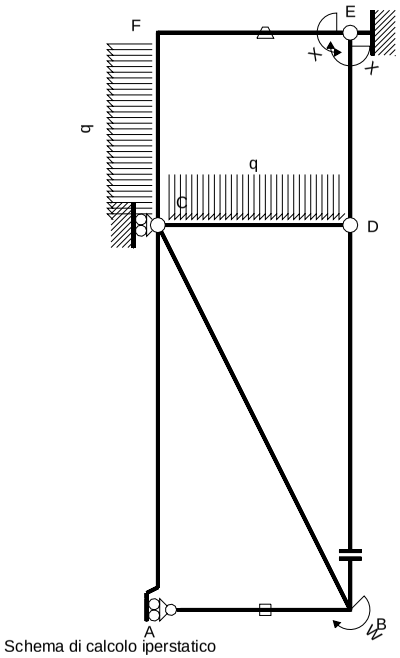
A = 564. mm^2
J_x = 205146. mm^4
J_y = 35737. mm^4
J_{xy} = 31198. mm^4
J_u = 210708. mm^4
J_v = 30175. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1764
$c = \cos\alpha$ = .9845
$s = \sin\alpha$ = -.1755
x_g = 8.617 mm
y_g = 30.94 mm
N = -1440. N
T_y = -720. N
M_x = 917700. Nmm
x_m = 18. mm
u_m = 14.67 mm
v_m = -28.81 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 199.3 N/mm^2

CI09.xxxx.105

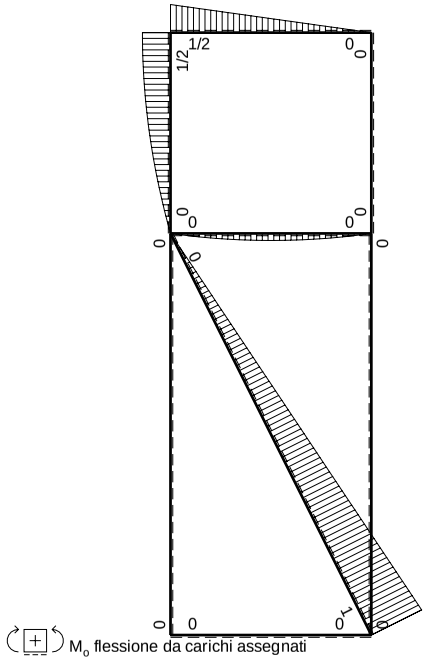
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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
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C
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Fb
F
B
A
B
B

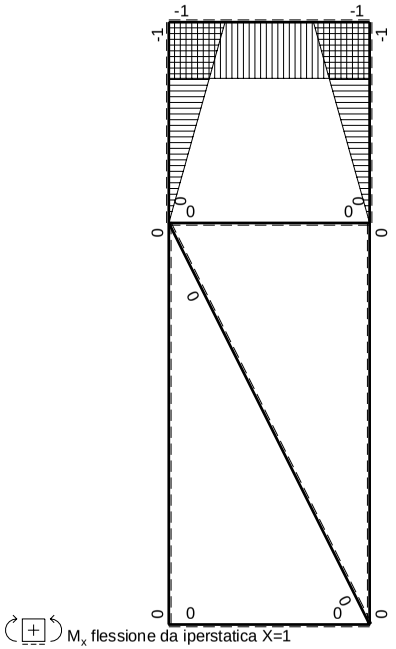

-13/40
-1/2
-1/2
3/2
13/40
13/40
-2√5/5
0
1
0
F
-13/40
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13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

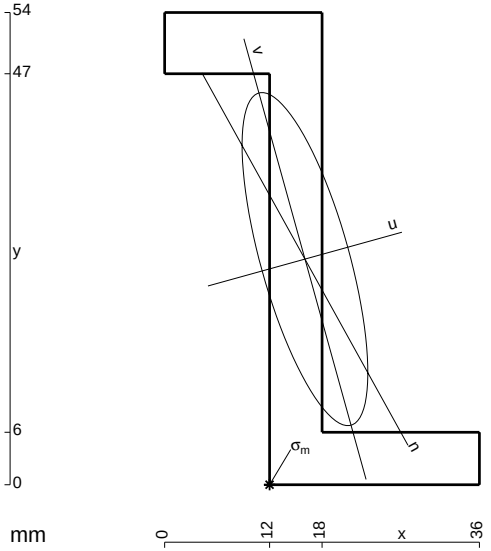
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 266252 CI09.xxxx.106

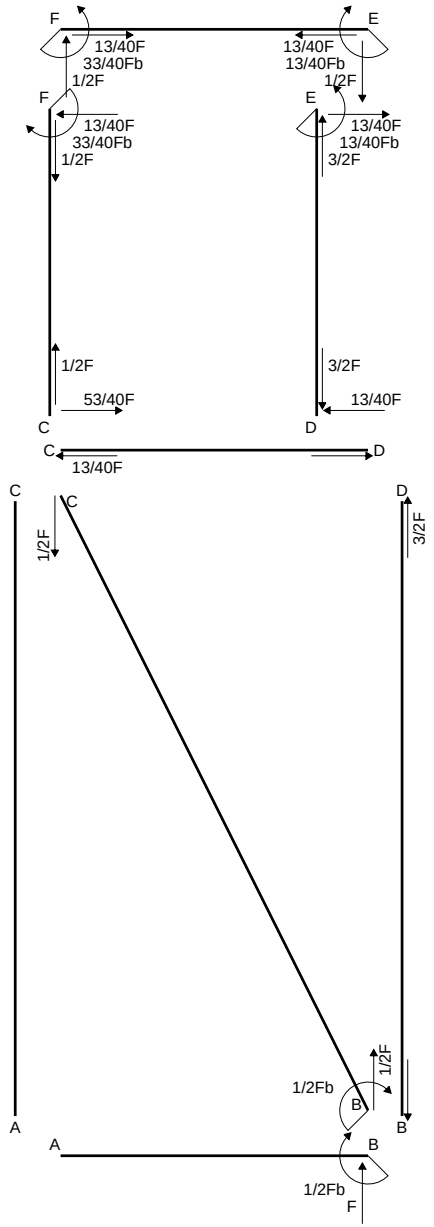


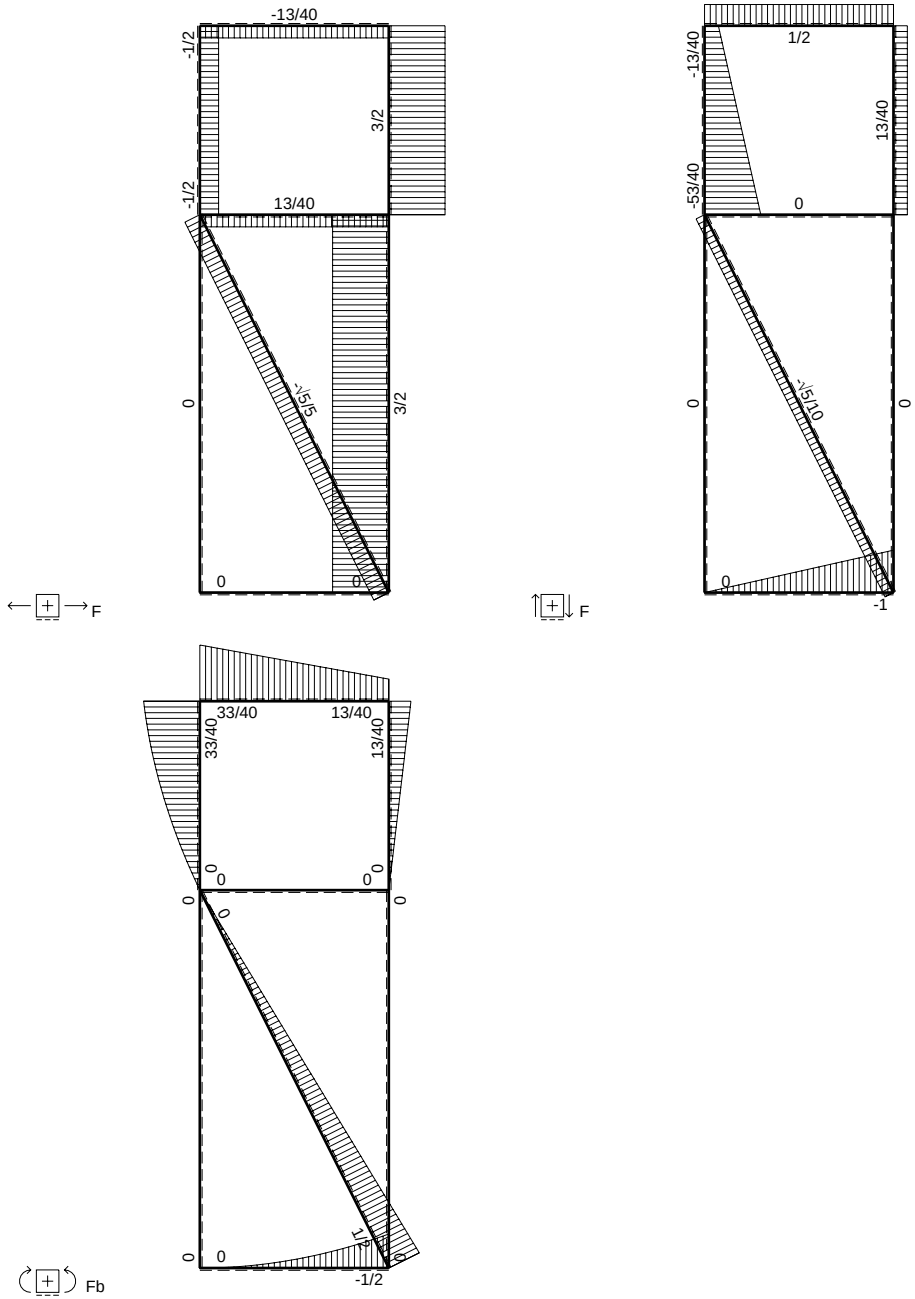
A = 516. mm^2
J_x = 187256. mm^4
J_y = 26687. mm^4
J_{xy} = -48223. mm^4
J_u = 200625. mm^4
J_v = 13317. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2704
c = cosα = .9637
s = sinα = .2672
x_g = 16.05 mm
y_g = 25.8 mm
N = -948.1 N
T_y = -474. N
M_x = 636000. Nmm
x_m = 12. mm
u_m = -10.79 mm
v_m = -23.78 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = 208.5 N/mm^2

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CI09.xxxx.106

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
1/2F
3/2F
1/2Fb
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
0
F
-13/40
1/2
13/40
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0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
0
0
0
√2/2
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 226461 CI09.xxxx.107

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

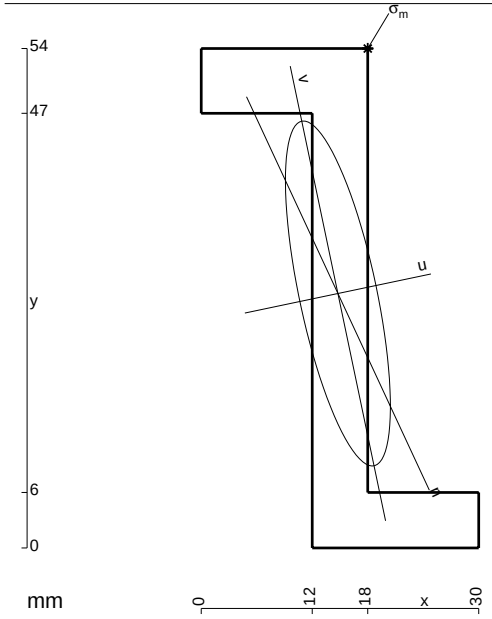
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

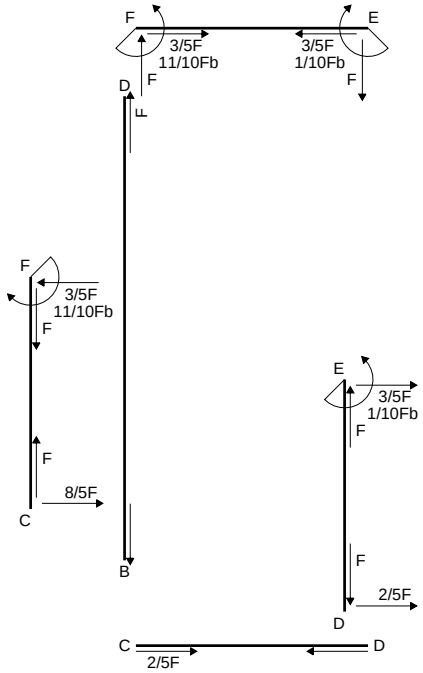
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

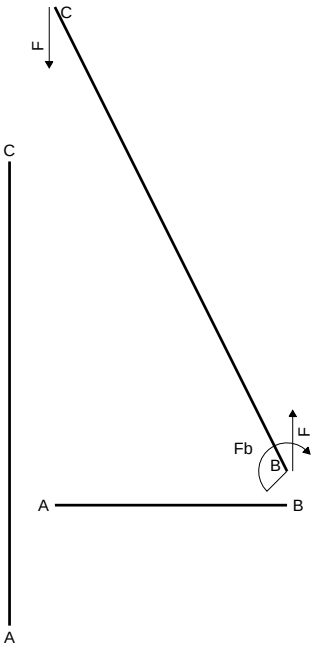


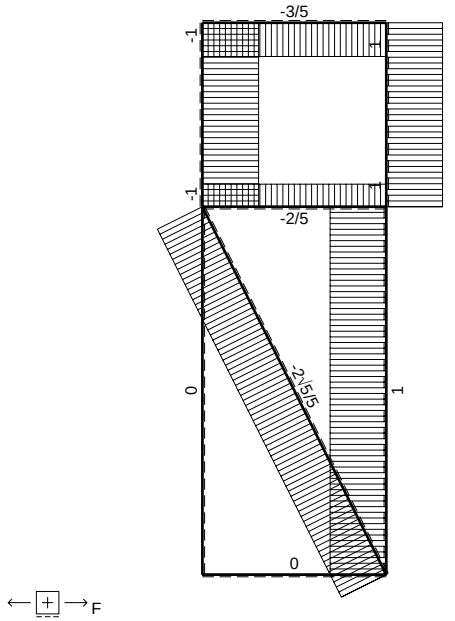
A = 480. mm^2
J_x = 167026. mm^4
J_y = 15456. mm^4
J_{xy} = -33263. mm^4
J_u = 174004. mm^4
J_v = 8477. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2068
$c = \cos\alpha$ = .9787
$s = \sin\alpha$ = .2053
x_g = 14.77 mm
y_g = 27.51 mm
N = -590. N
T_y = -383.5 N
M_x = 623040. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 8.595 mm
v_m = 25.26 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.4 N/mm^2

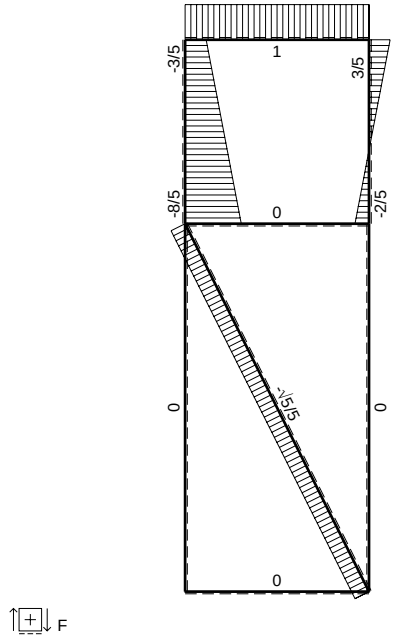
CI09.xxxx.107

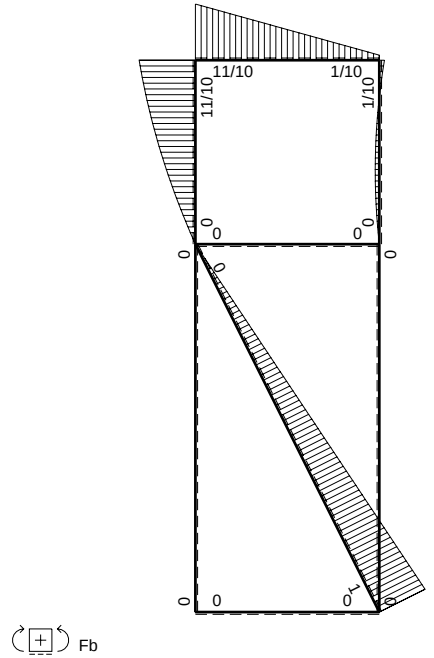
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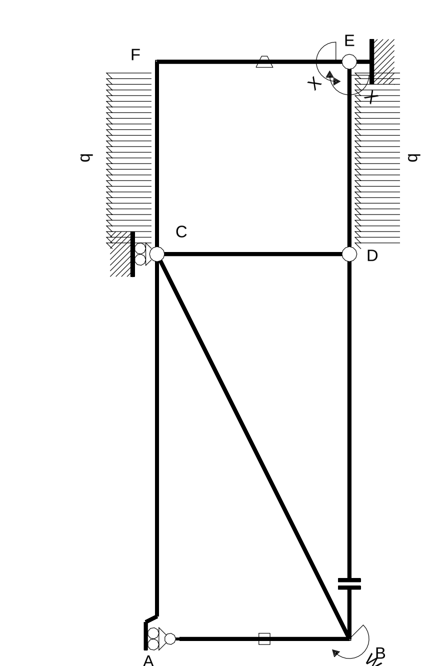

F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A

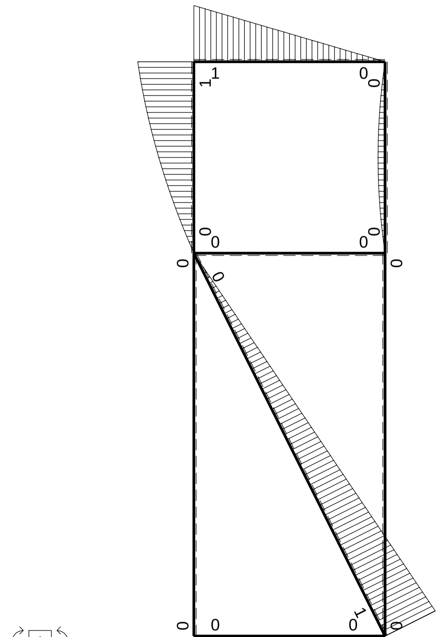

-3/5
-1
1
-1
1
-2/5
0
-2/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5
0
0
F

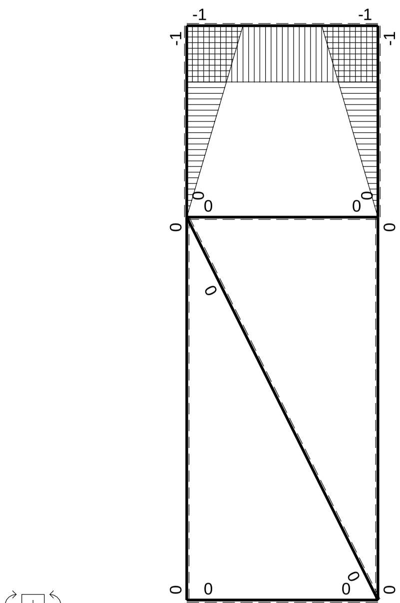

11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$1/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

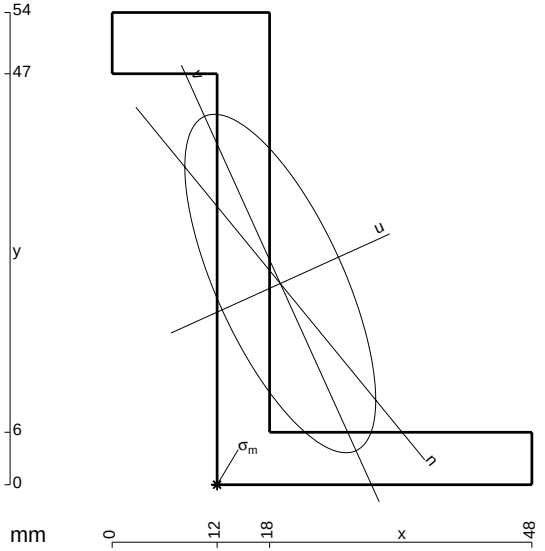
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

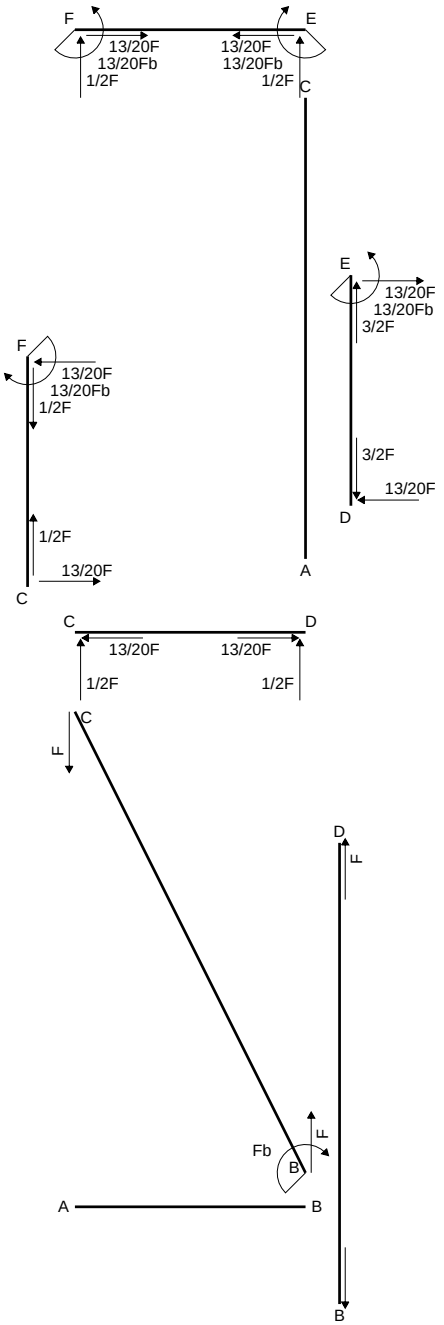
$= (-1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$

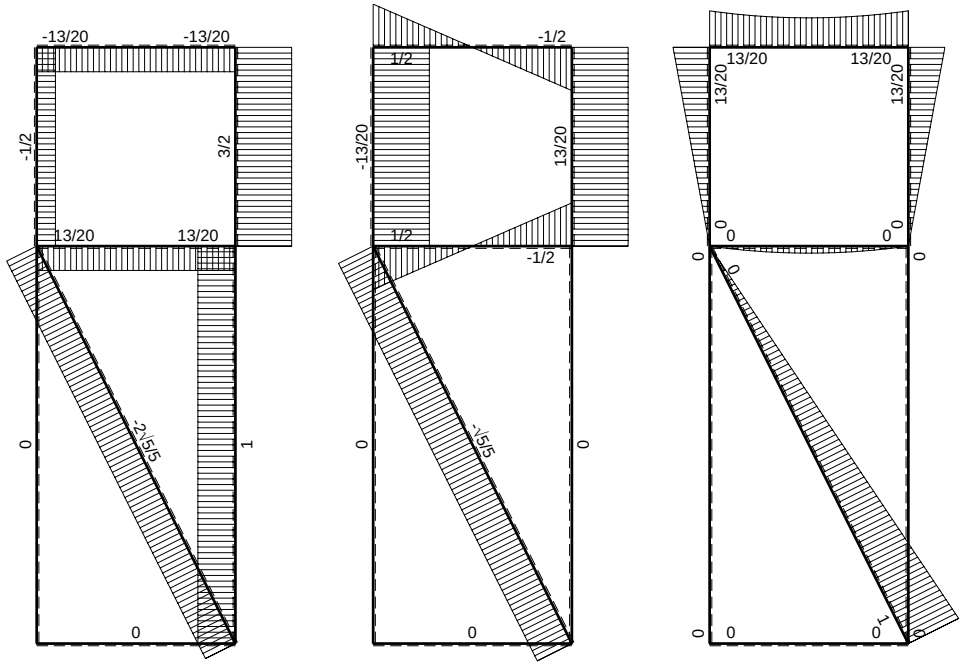


A = 588. mm^2
J_x = 220324. mm^4
J_y = 70110. mm^4
J_{xy} = -85615. mm^4
J_u = 259108. mm^4
J_v = 31327. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4253
$c = \cos\alpha$ = .9109
$s = \sin\alpha$ = .4126
x_g = 19.22 mm
y_g = 23.01 mm
N = -672. N
T_y = 1120. N
M_x = 837760. Nmm
x_m = 12. mm
u_m = -16.08 mm
v_m = -17.98 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 229.2 N/mm^2

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F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

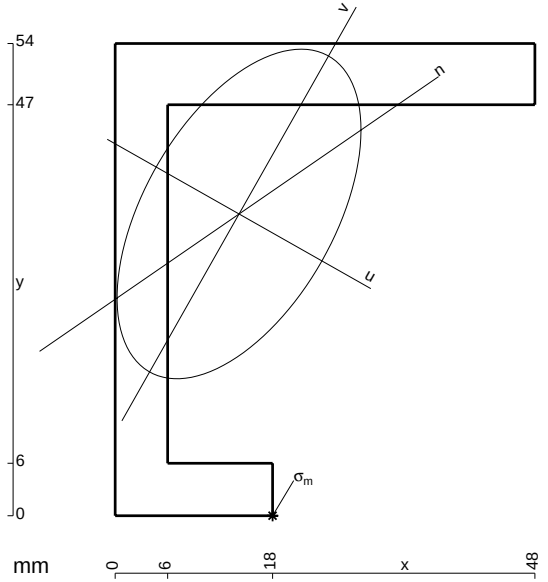
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

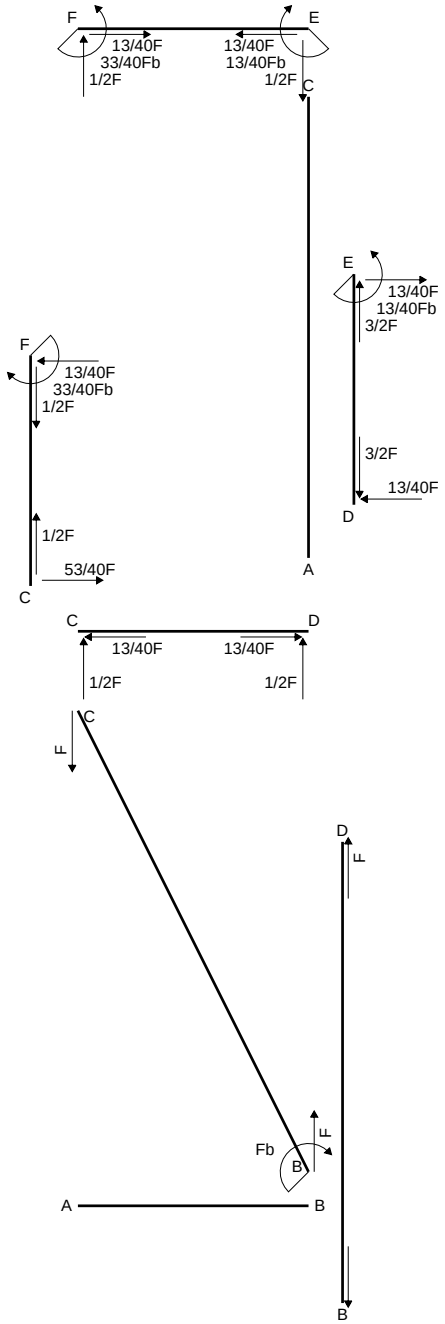
$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

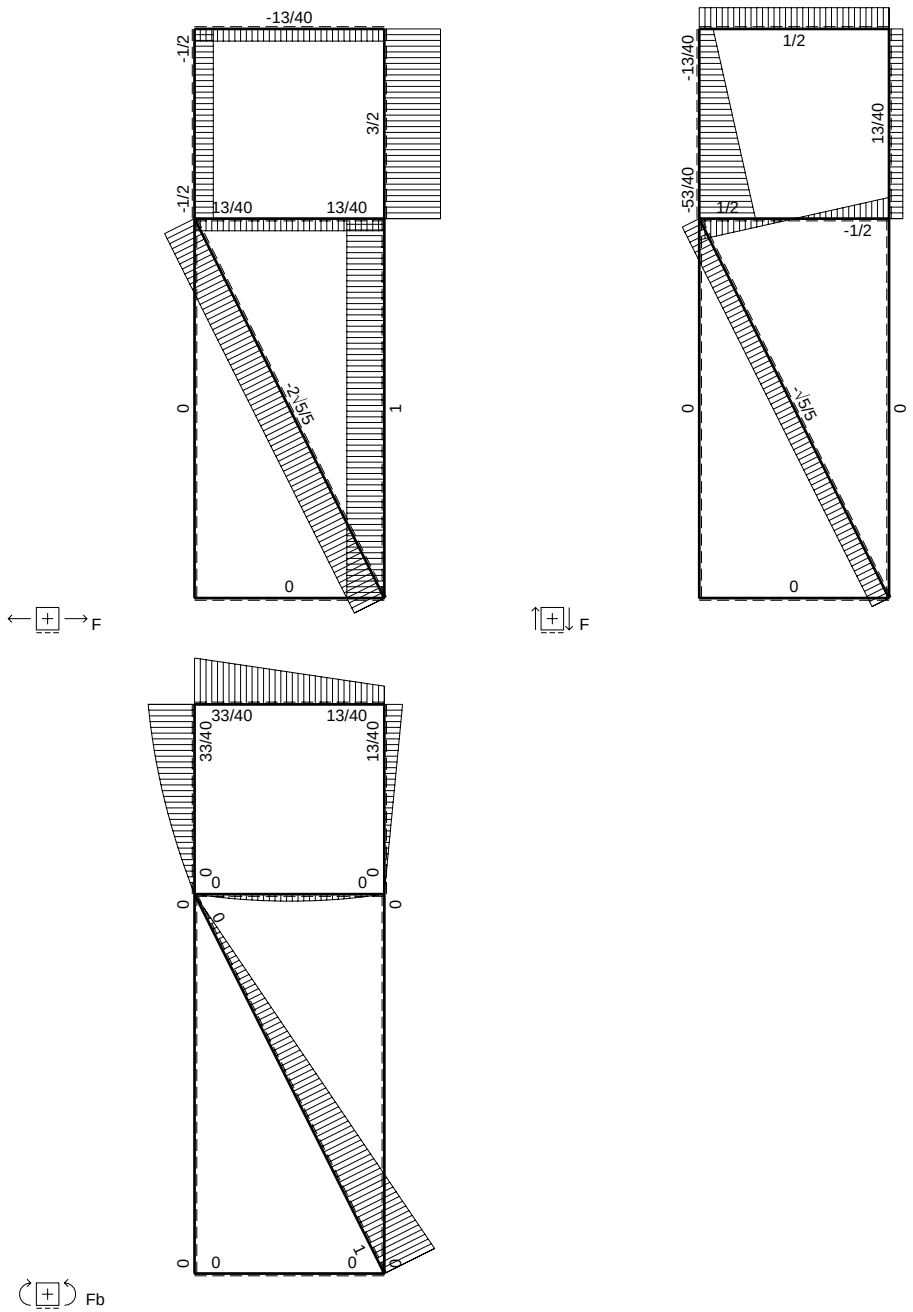


A = 690. mm^2
J_x = 245079. mm^4
J_y = 134213. mm^4
J_{xy} = 92417. mm^4
J_u = 297413. mm^4
J_v = 81879. mm^4
$\alpha = artg(2J_{xy}/(J_y - J_x))/2$ = -.5153
$c = \cos\alpha$ = .8702
$s = \sin\alpha$ = -.4928
x_g = 14.17 mm
y_g = 34.51 mm
N = -1467. N
T_y = -733.4 N
M_x = 1180800. Nmm
x_m = 18. mm
u_m = 20.34 mm
v_m = -28.14 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.6 N/mm^2

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
F
13/40F
33/40Fb
1/2F
3/2F
13/40F
D
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
3/2
-1/2
13/40
13/40
-2√5/5
0
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XbEJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-FbEJ	-1/2Fx	FbEJ	1	$(-1/4+1)Fb^2/EJ$	XbEJ
FE b	1	$-1/2Fb+1/2Fx$	FbEJ	$-1/2Fb+1/2Fx$	FbEJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3XbEJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XbEJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 215080 CI09.xxxx.110

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

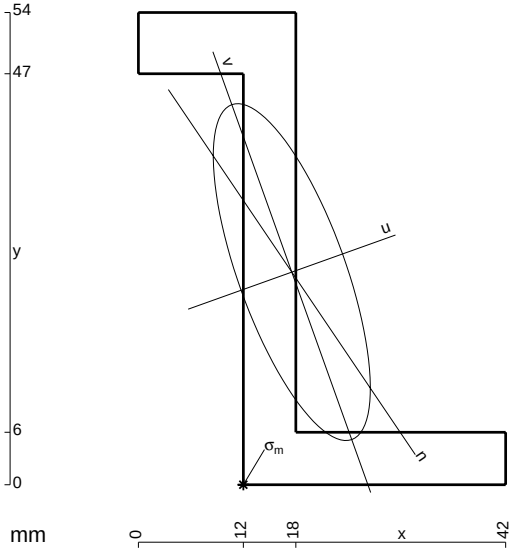
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 215080 CI09.xxxx.110

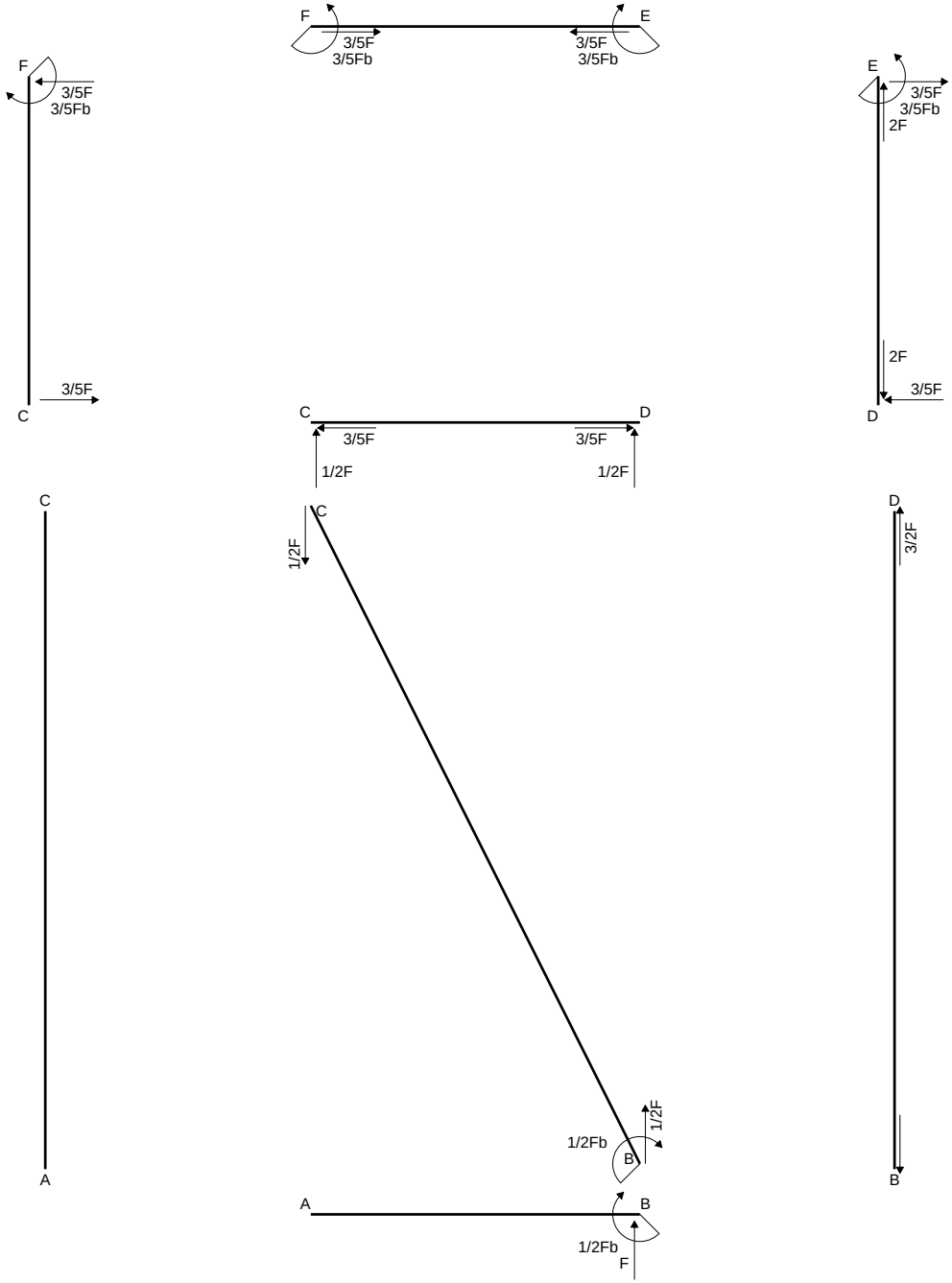


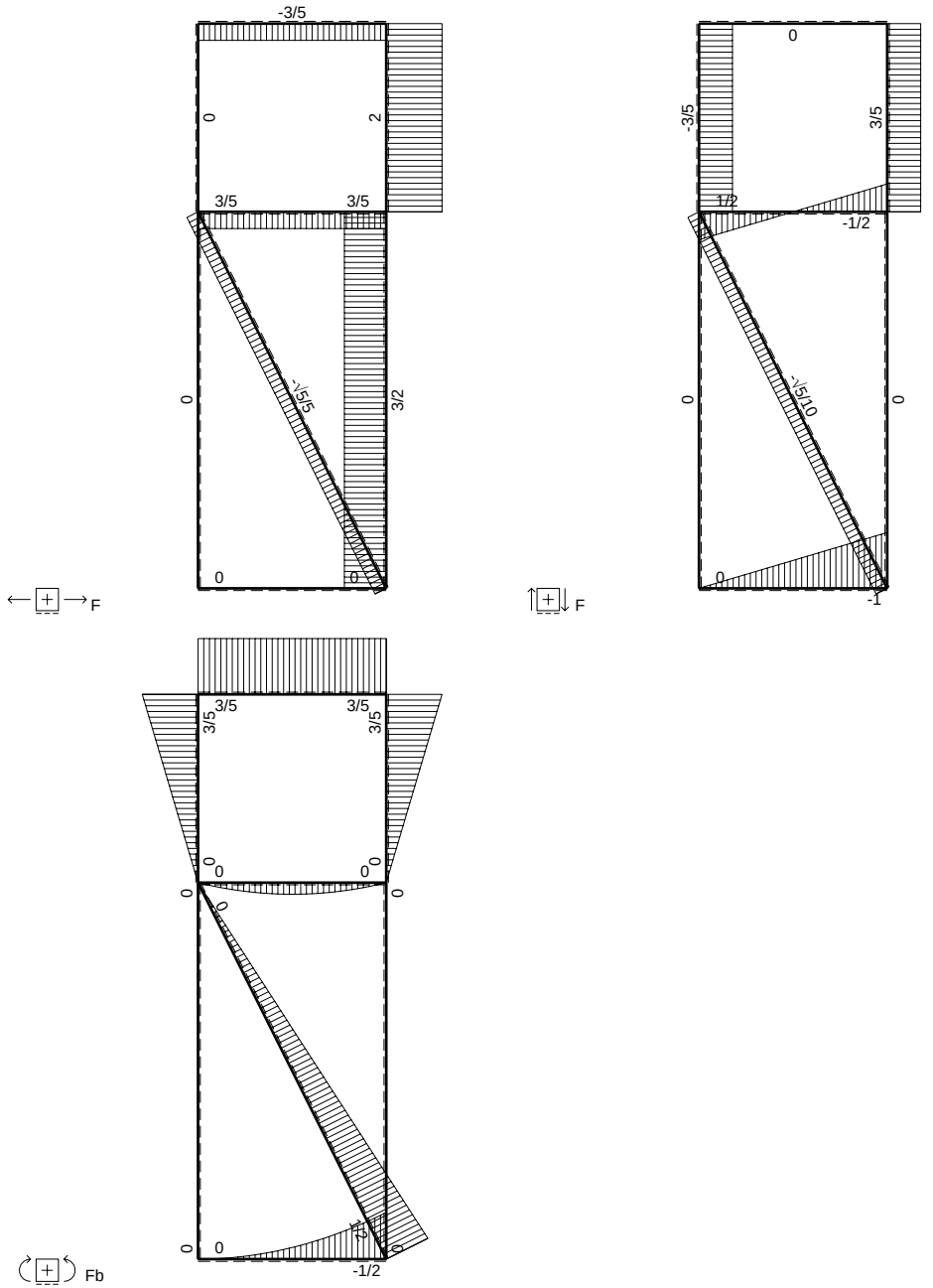
A = 552. mm^2
J_x = 204861. mm^4
J_y = 44525. mm^4
J_{xy} = -65837. mm^4
J_u = 228430. mm^4
J_v = 20956. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3438
$c = \cos\alpha$ = .9415
$s = \sin\alpha$ = .3370
x_g = 17.54 mm
y_g = 24.32 mm
N = -1574. N
T_y = -787.1 N
M_x = 668800. Nmm
x_m = 12. mm
u_m = -13.41 mm
v_m = -21.02 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 199.4 N/mm^2

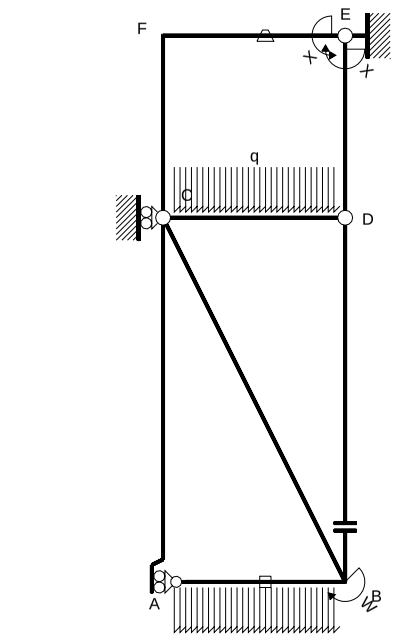
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CI09.xxxx.110

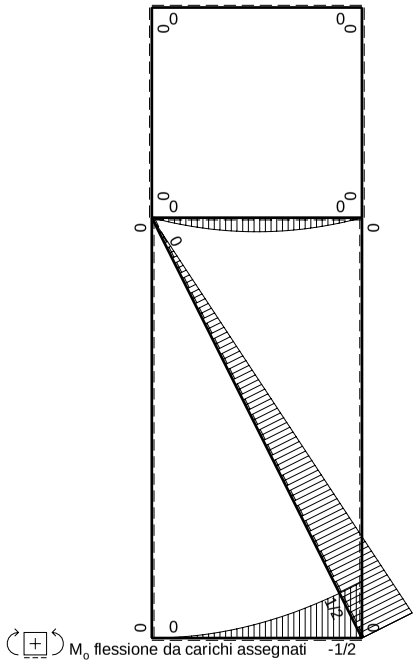
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
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B
A
B
1/2Fb
F

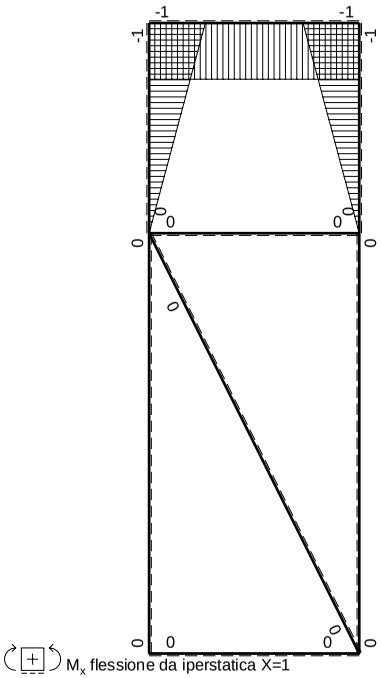

-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

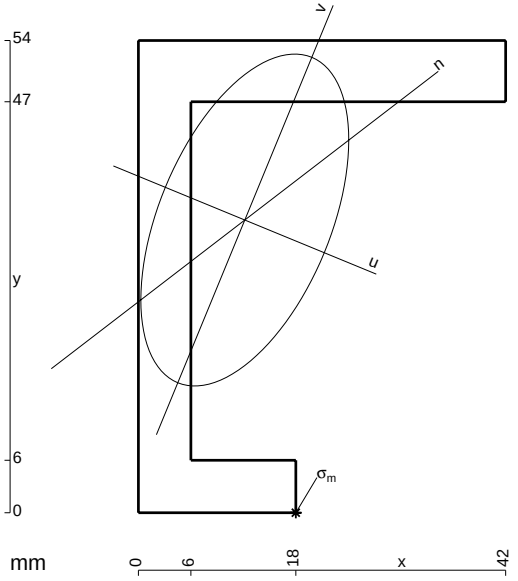
$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 266284 CI09.xxxx.111

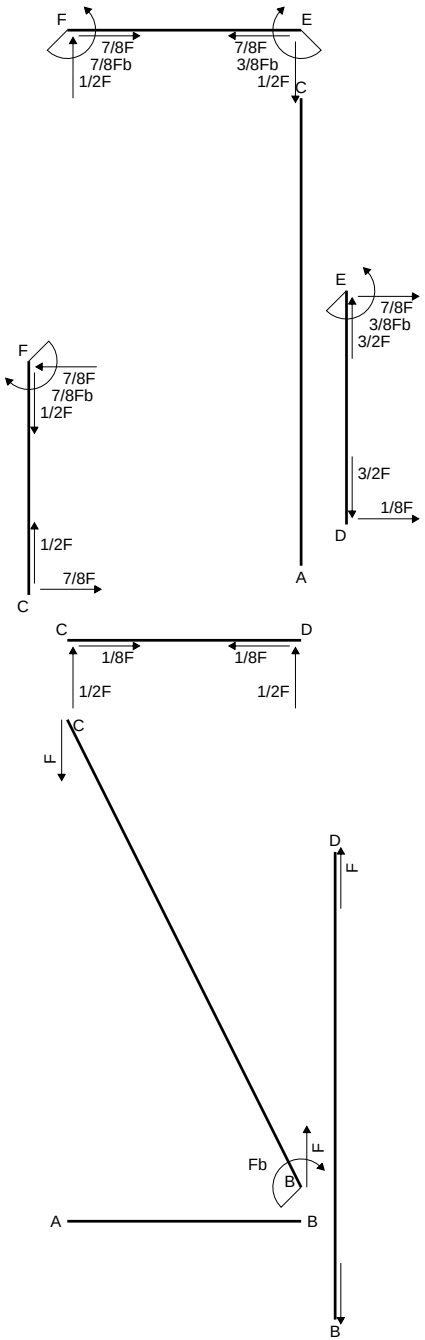


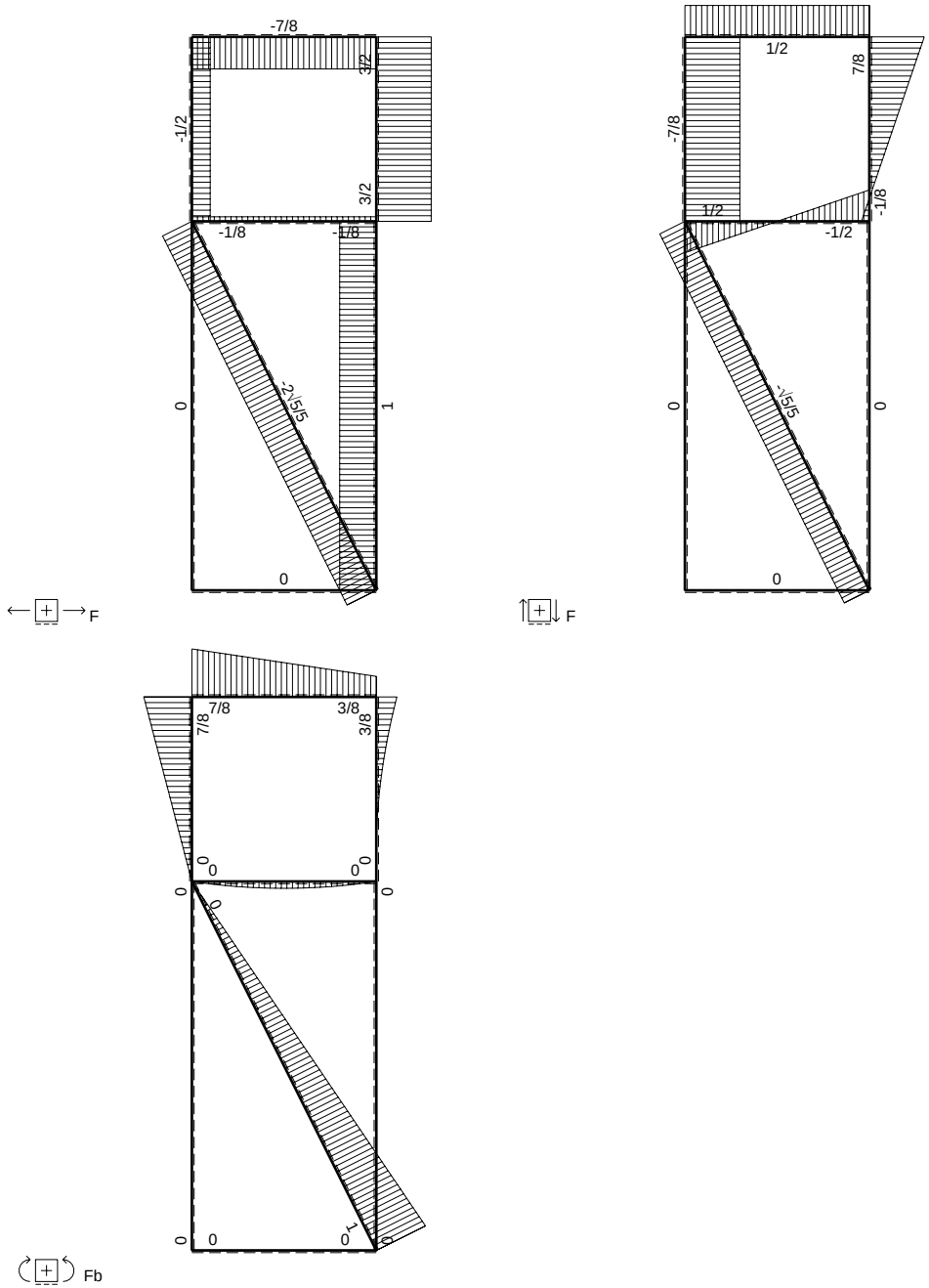
A = 648. mm^2
J_x = 233472. mm^4
J_y = 91566. mm^4
J_{xy} = 70365. mm^4
J_u = 262446. mm^4
J_v = 62591. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3906
$c = \cos\alpha$ = .9247
$s = \sin\alpha$ = -.3808
x_g = 12.17 mm
y_g = 33.47 mm
N = 7600. N
T_y = 2280. N
M_x = 934800. Nmm
x_m = 18. mm
u_m = 18.14 mm
v_m = -28.73 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 209.5 N/mm^2

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CI09.xxxx.111

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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 270433 CI09.xxxx.112

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

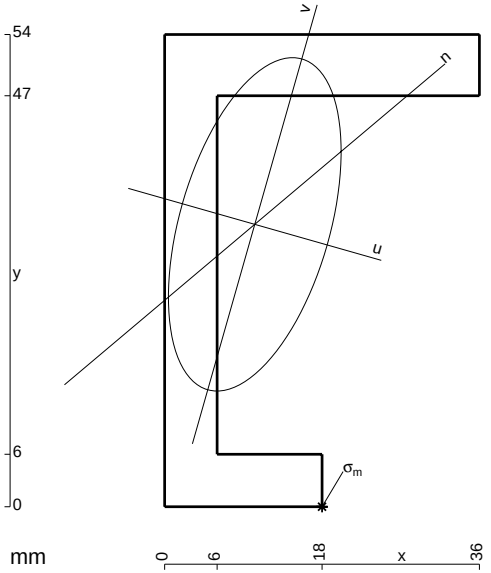
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

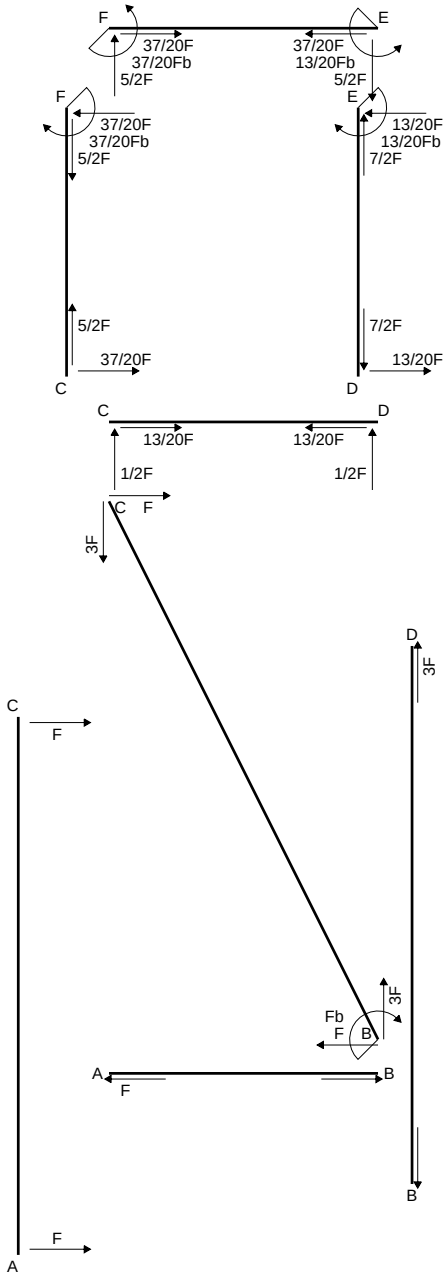
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

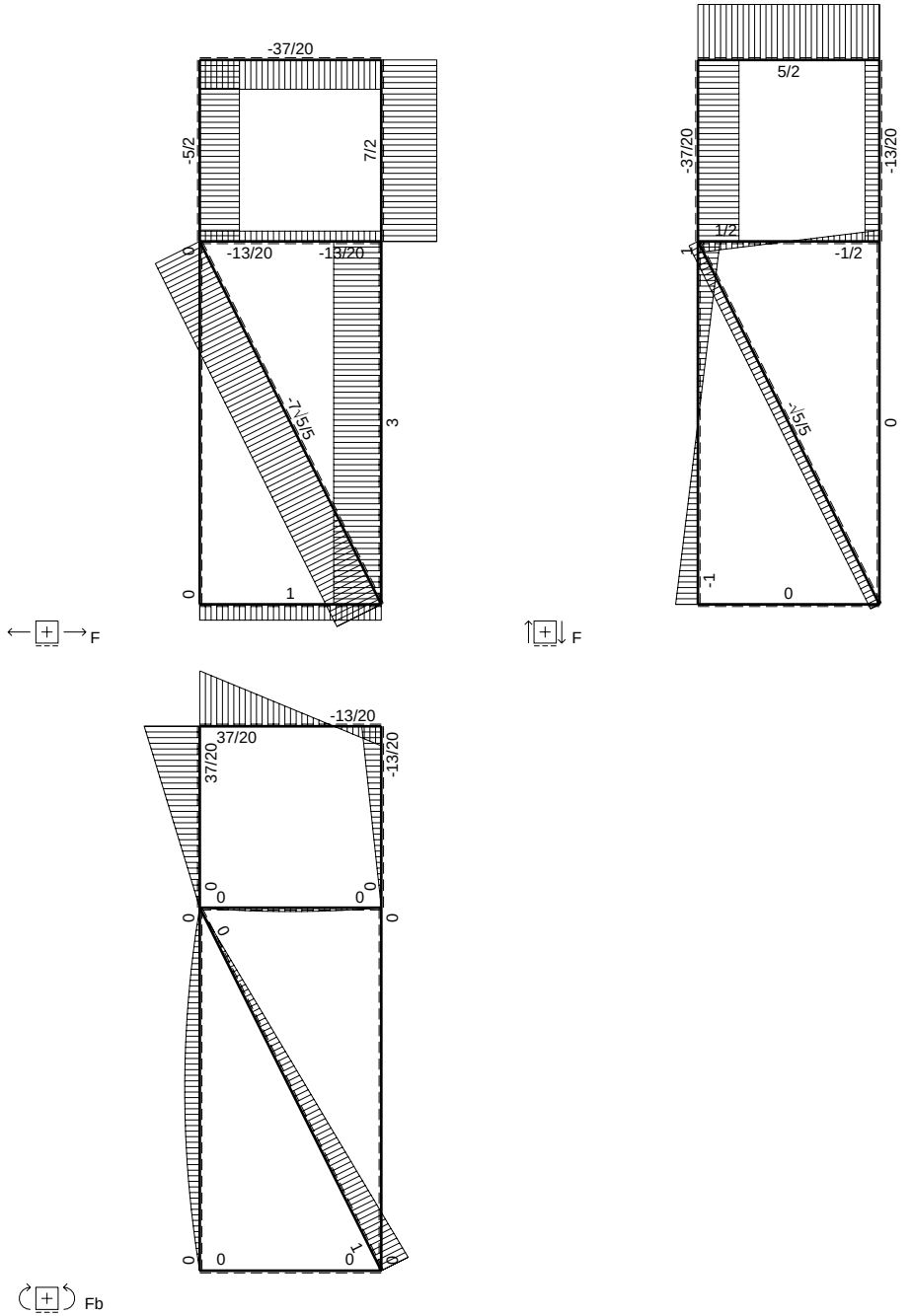


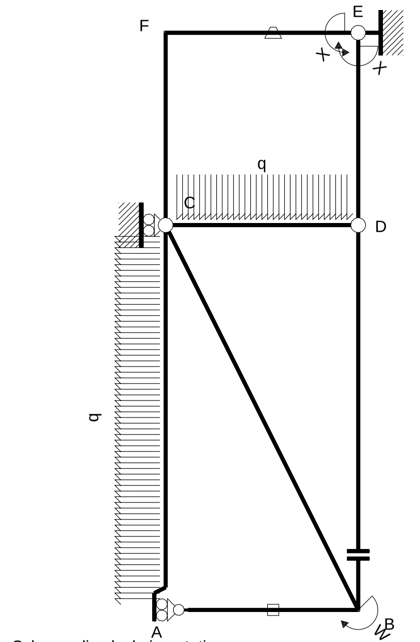
A = 606. mm^2
J_x = 220278. mm^4
J_y = 59103. mm^4
J_{xy} = 49845. mm^4
J_u = 234447. mm^4
J_v = 44934. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2770
$c = \cos\alpha$ = .9619
$s = \sin\alpha$ = -.2734
x_g = 10.31 mm
y_g = 32.29 mm
N = -2039. N
T_y = -1020. N
M_x = 1026000. Nmm
x_m = 18. mm
u_m = 16.23 mm
v_m = -28.96 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 219.9 N/mm^2

CI09.xxxx.112

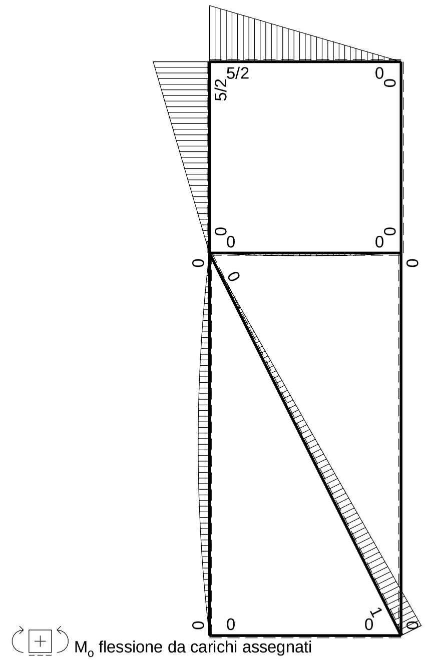
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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
7/2F
37/20F
13/20F
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1/2F
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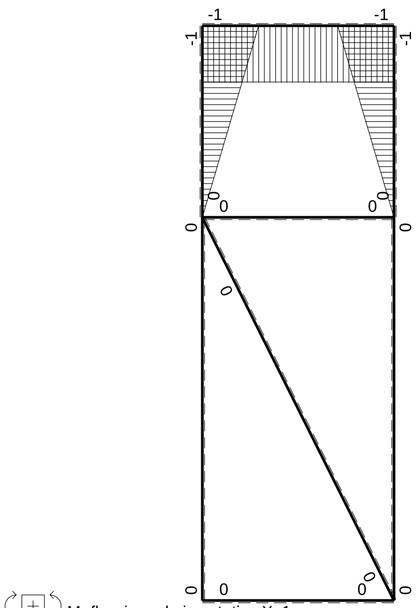

-37/20
-5/2
7/2
-13/20
-13/20
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3
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1
F
5/2
-37/20
-13/20
1/2
-1/2
-7.5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

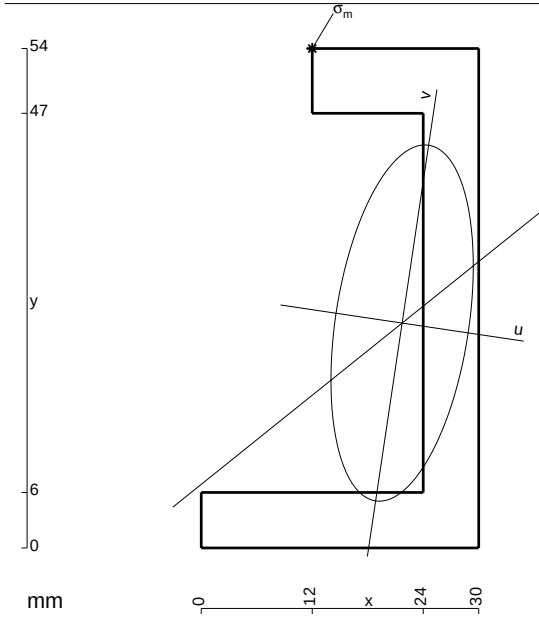
$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 265736 CI09.xxxx.113

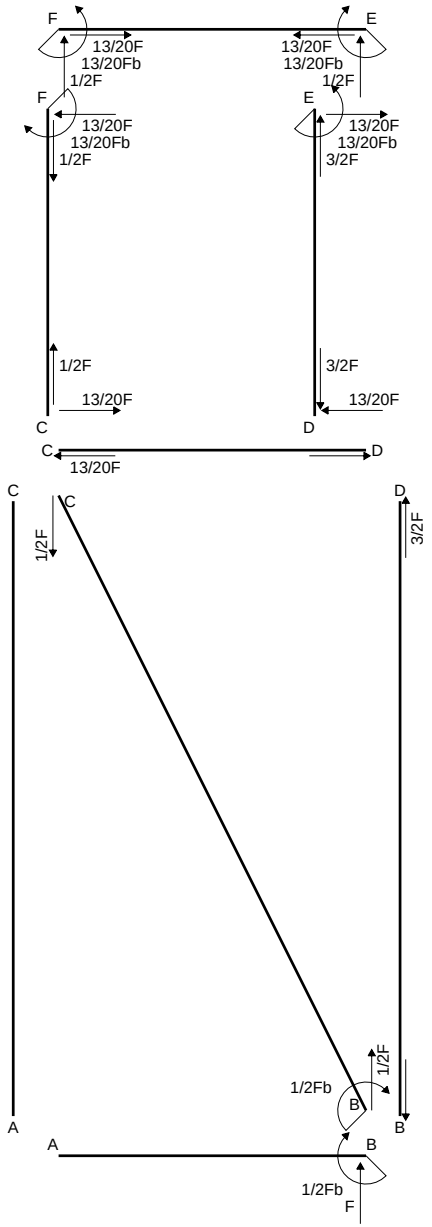


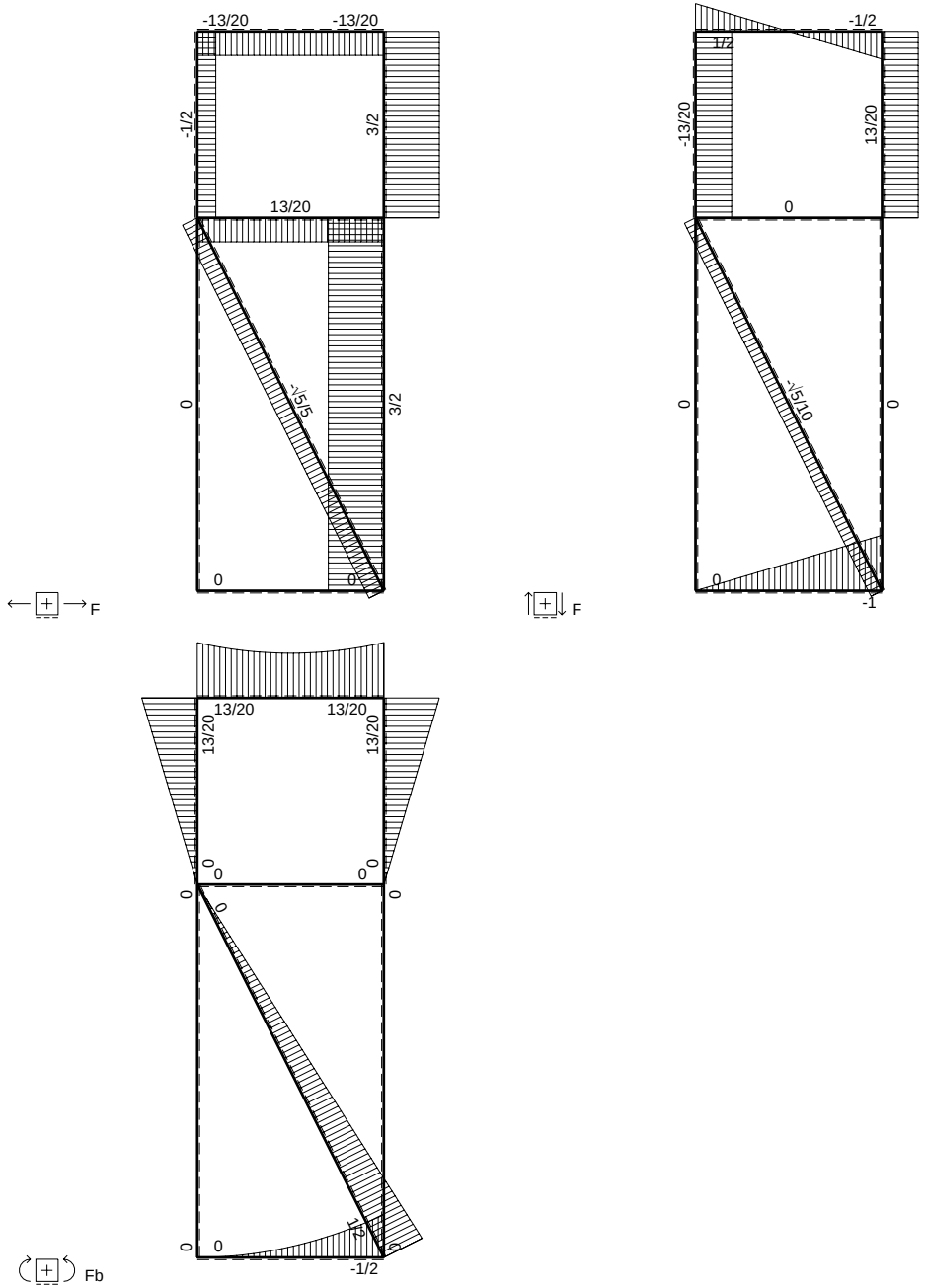
A = 552. mm^2
J_x = 204861. mm^4
J_y = 32692. mm^4
J_{xy} = 26245. mm^4
J_u = 208773. mm^4
J_v = 28780. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1480
c = cosα = .9891
s = sinα = -.1474
x_g = 21.72 mm
y_g = 24.32 mm
N = -3025. N
T_y = -2239. N
M_x = 1096870. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -13.99 mm
v_m = 27.93 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -229.2 N/mm^2

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
1/2
-1
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi θ$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+θ)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

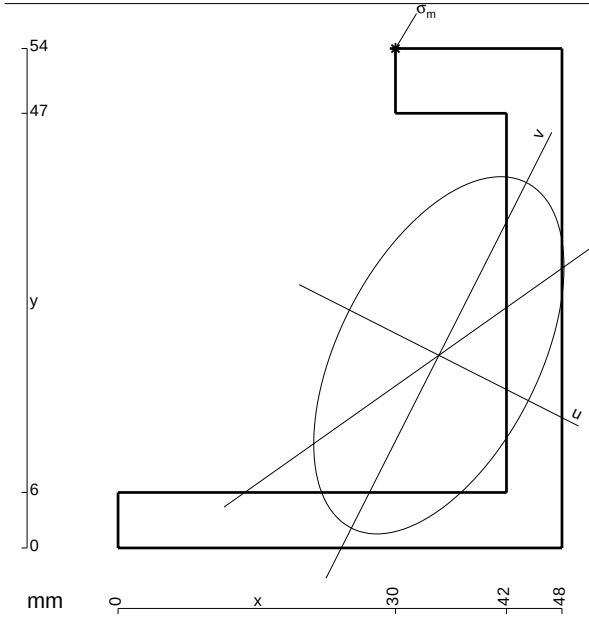
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 939153 CI09.xxxx.114

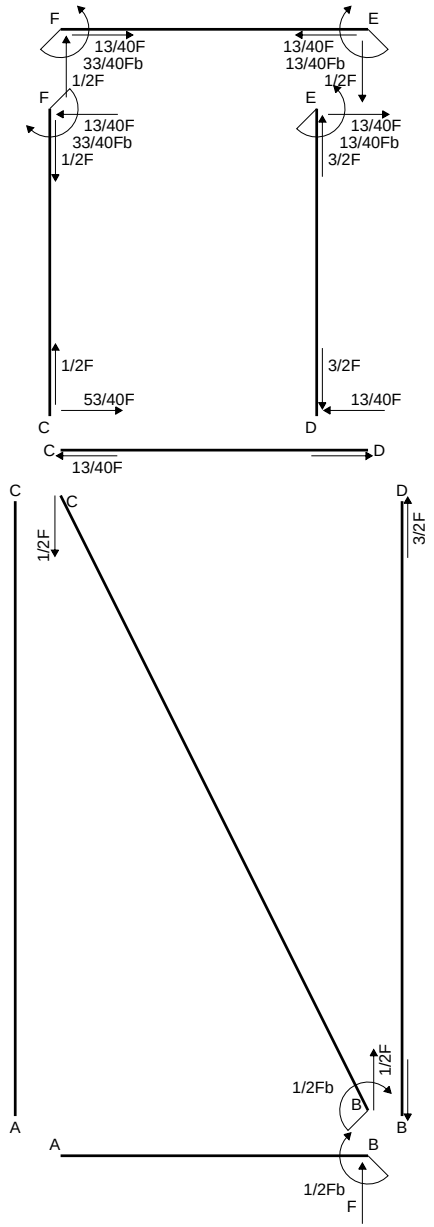


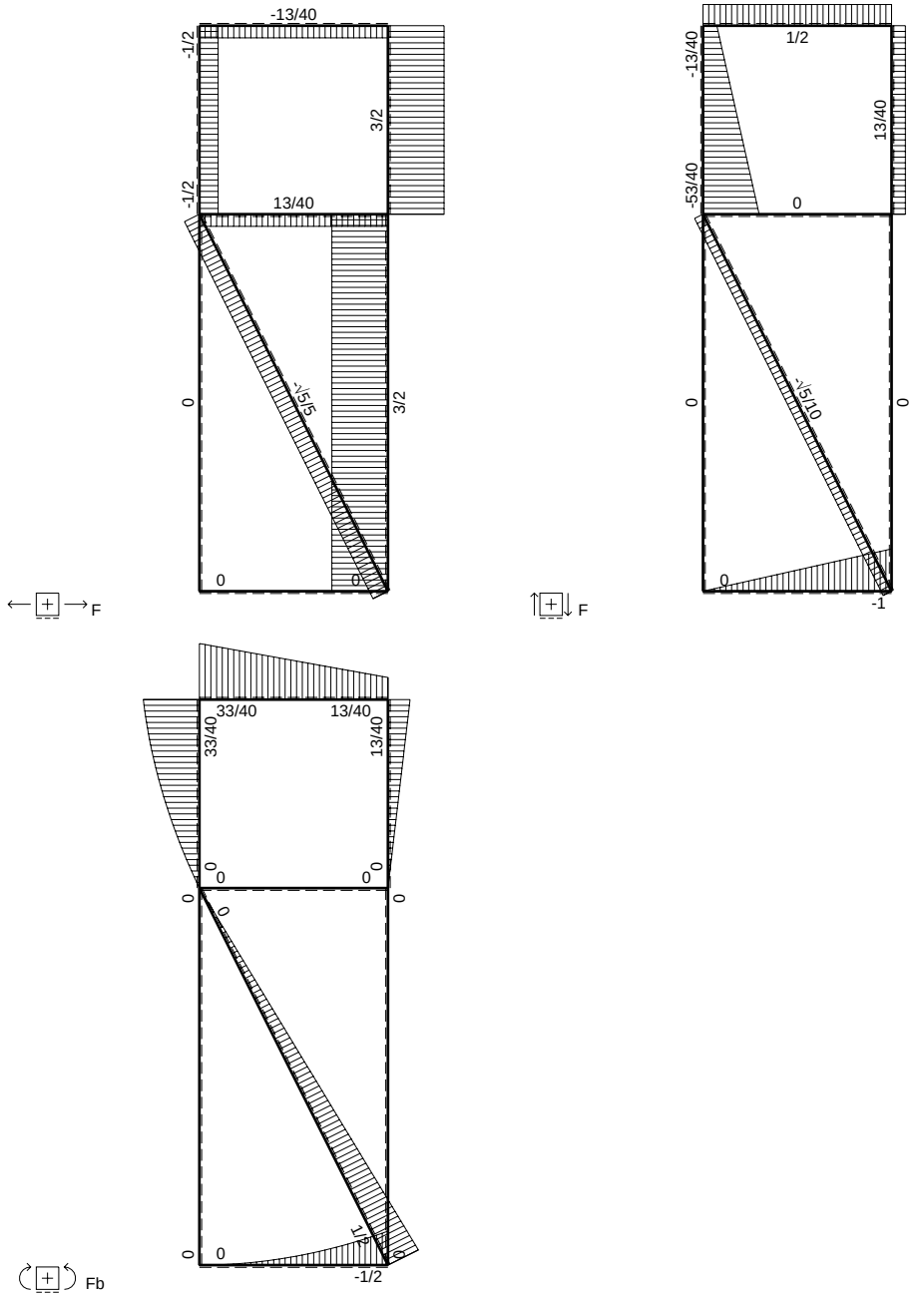
A = 660. mm^2
J_x = 246224. mm^4
J_y = 120837. mm^4
J_{xy} = 85387. mm^4
J_u = 289462. mm^4
J_v = 77600. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4687
c = cosα = .8921
s = sinα = -.4518
x_g = 34.69 mm
y_g = 20.83 mm
N = -2269. N
T_y = -1745. N
M_x = 1202310. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = -19.17 mm
v_m = 27.48 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -239.4 N/mm^2

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
1/2F
3/2F
1/2Fb
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-√2/2
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264484 CI09.xxxx.115

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

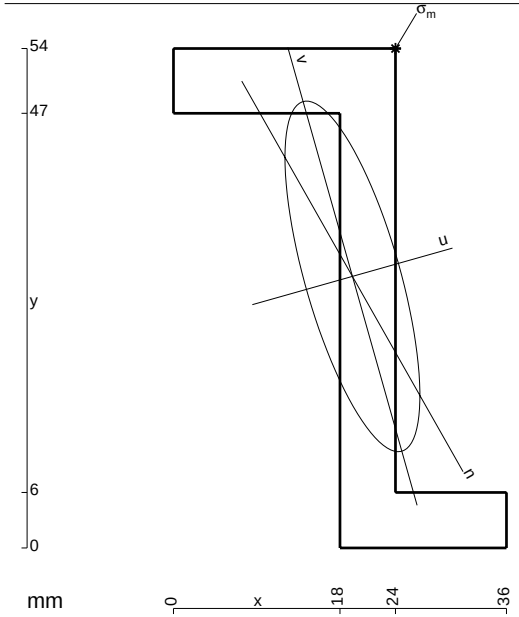
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

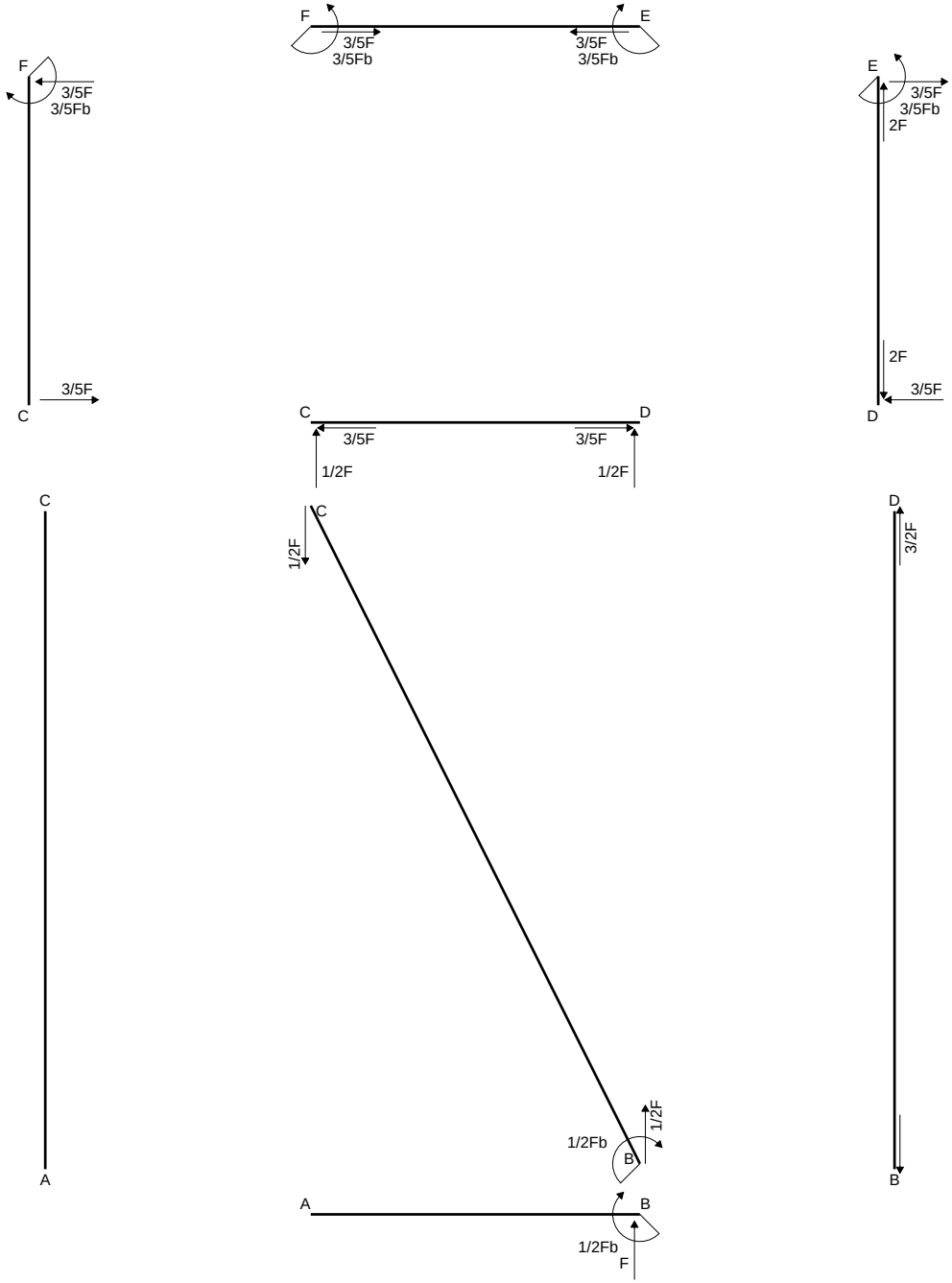
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

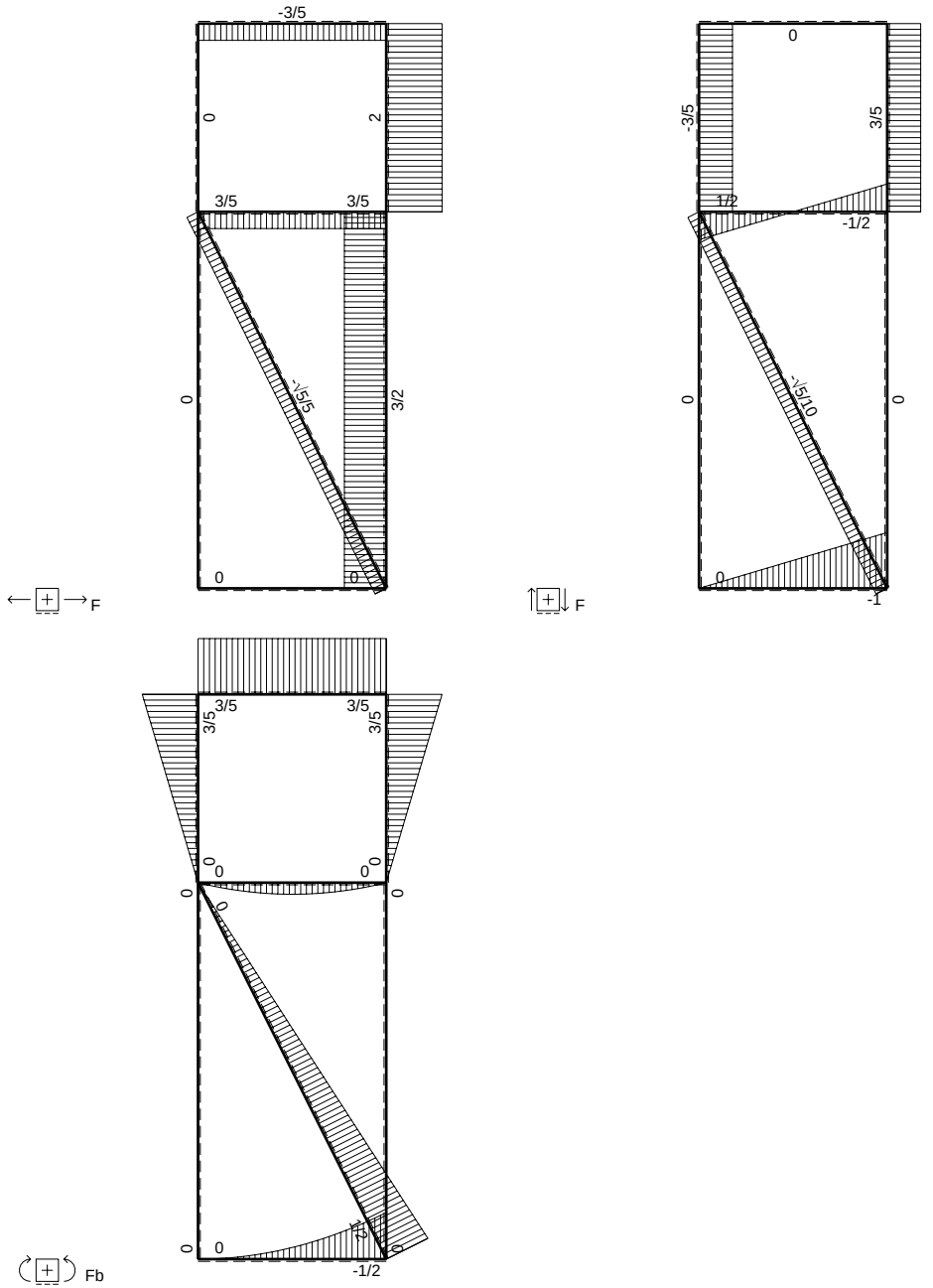


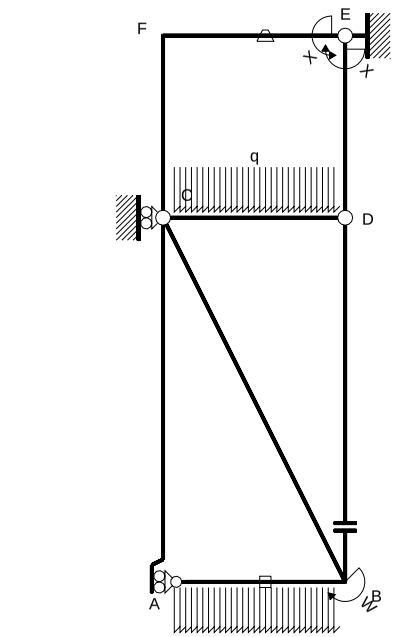
A = 522. mm^2
J_x = 187606. mm^4
J_y = 27784. mm^4
J_{xy} = -49043. mm^4
J_u = 201455. mm^4
J_v = 13935. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2752
$c = \cos\alpha$ = .9624
$s = \sin\alpha$ = .2718
x_g = 19.34 mm
y_g = 29.36 mm
N = -645. N
T_y = -419.3 N
M_x = 606623. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 11.18 mm
v_m = 22.45 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -198.5 N/mm^2

CI09.xxxx.115

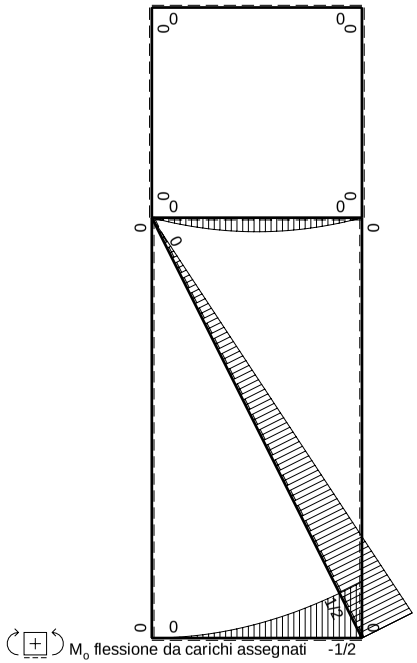
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
3/5F
C
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F

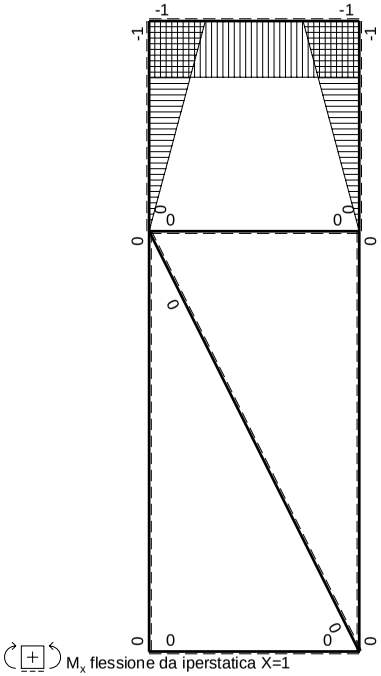

-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
0
-√2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

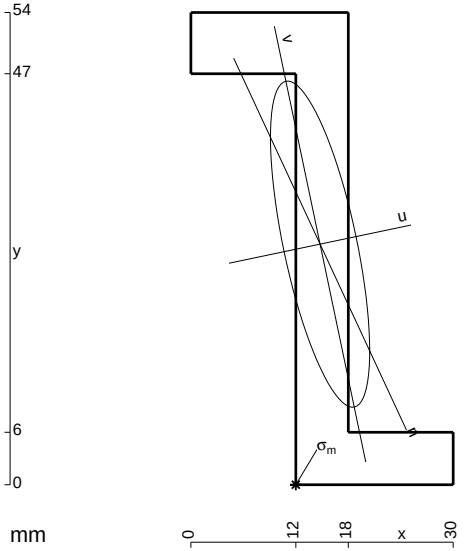
$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 990867 CI09.xxxx.116

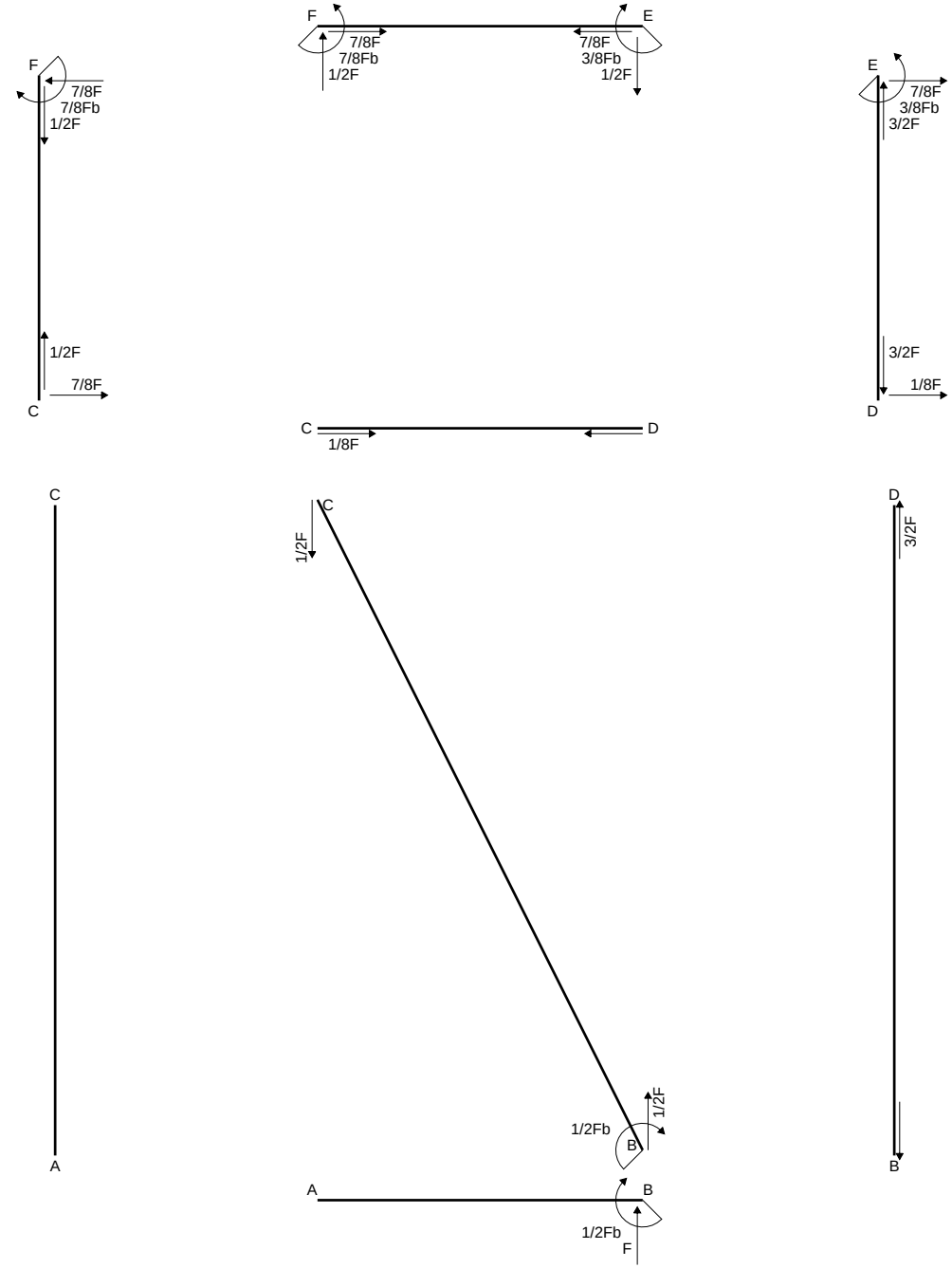


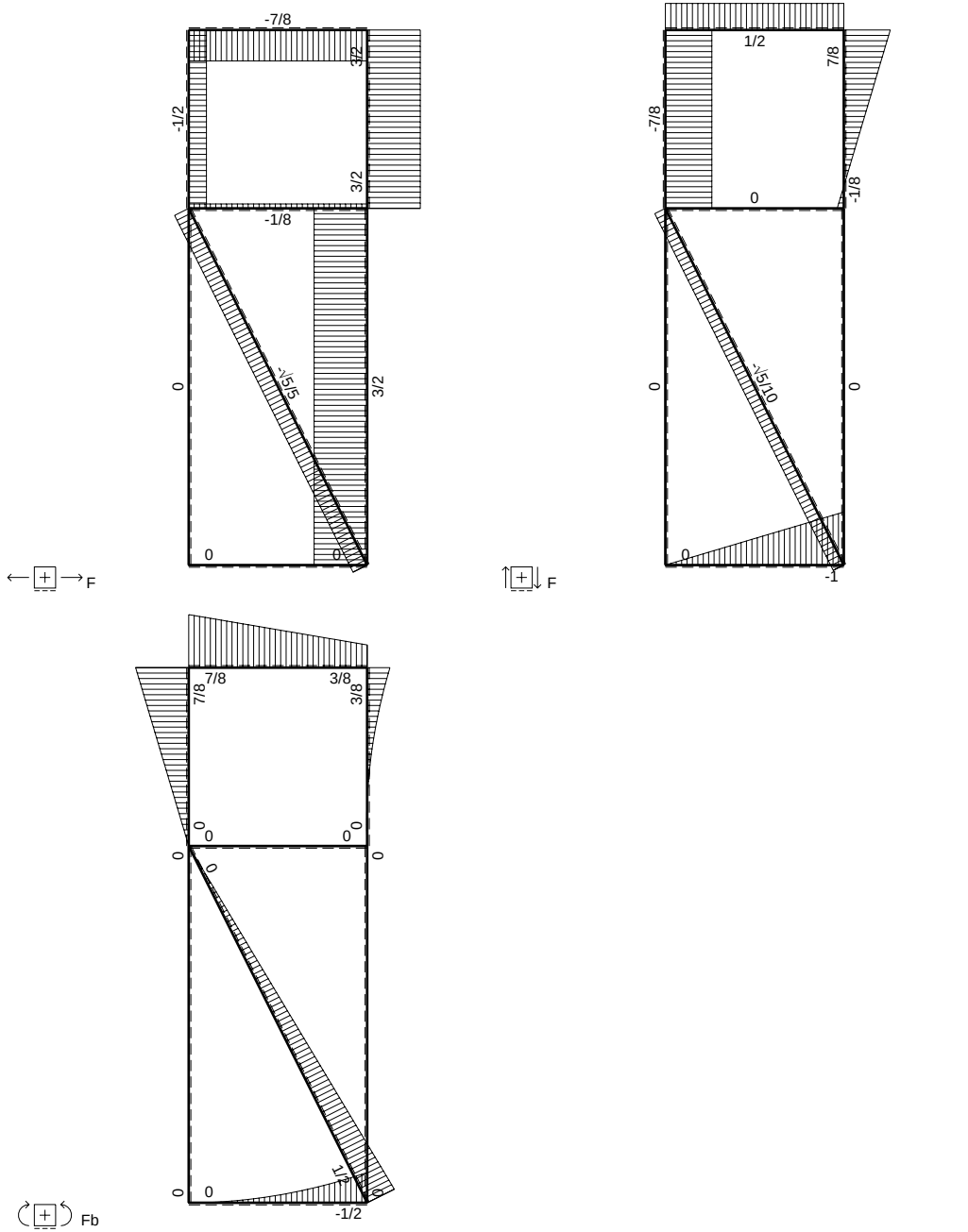
A = 480. mm^2
J_x = 167026. mm^4
J_y = 15456. mm^4
J_{xy} = -33263. mm^4
J_u = 174004. mm^4
J_v = 8477. mm^4
$\alpha = artg(2J_{xy}/(J_y - J_x))/2$ = .2068
$c = \cos\alpha$ = .9787
$s = \sin\alpha$ = .2053
x_g = 14.77 mm
y_g = 27.51 mm
N = 3200. N
T_y = 960. N
M_x = 576000. Nmm
x_m = 12. mm
u_m = -8.365 mm
v_m = -26.36 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 208.8 N/mm^2

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CI09.xxxx.116

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F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
3/2F
1/8F
C
D
A
B
1/2Fb


-7/8
3/2
-1/2
-1/8
0
-√5/5
1/2
7/8
-√5/10
F
Fb
7/8
3/8
-1/2

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 247897 CI09.xxxx.117

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [-\ x]_o^b\ \theta$

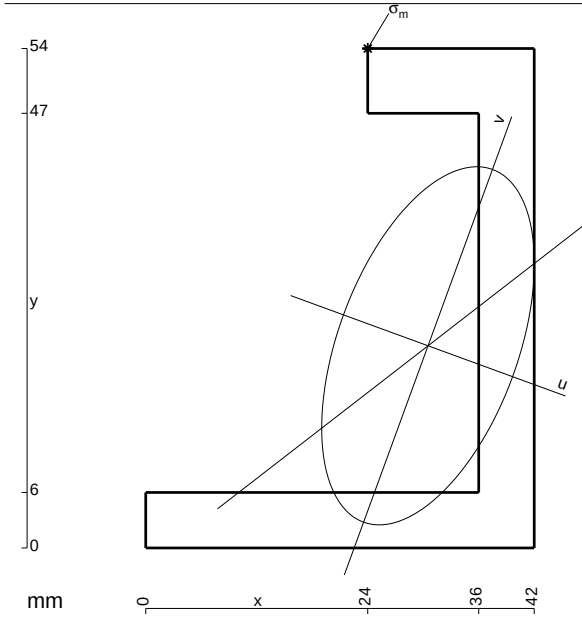
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

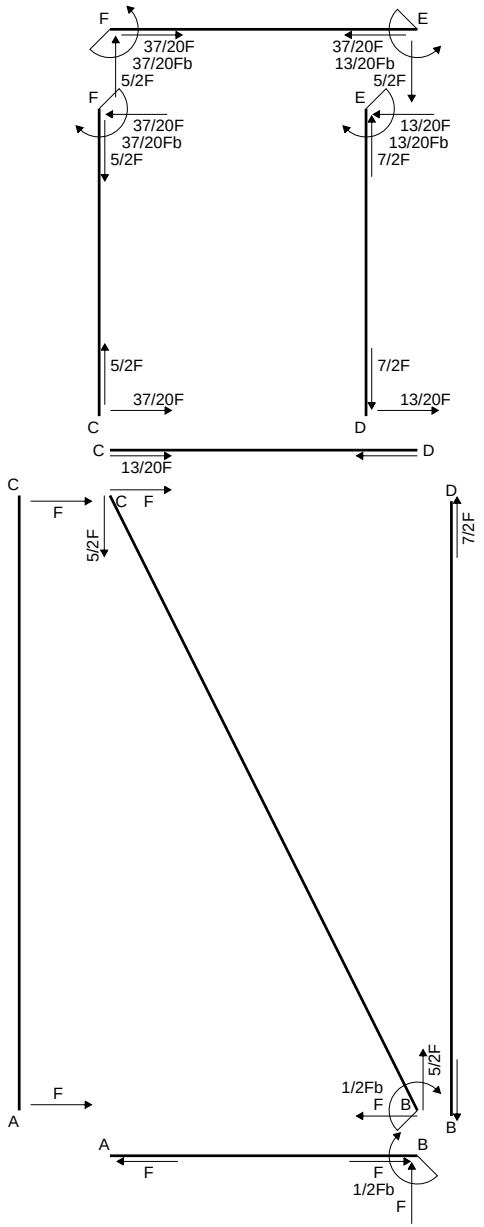
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

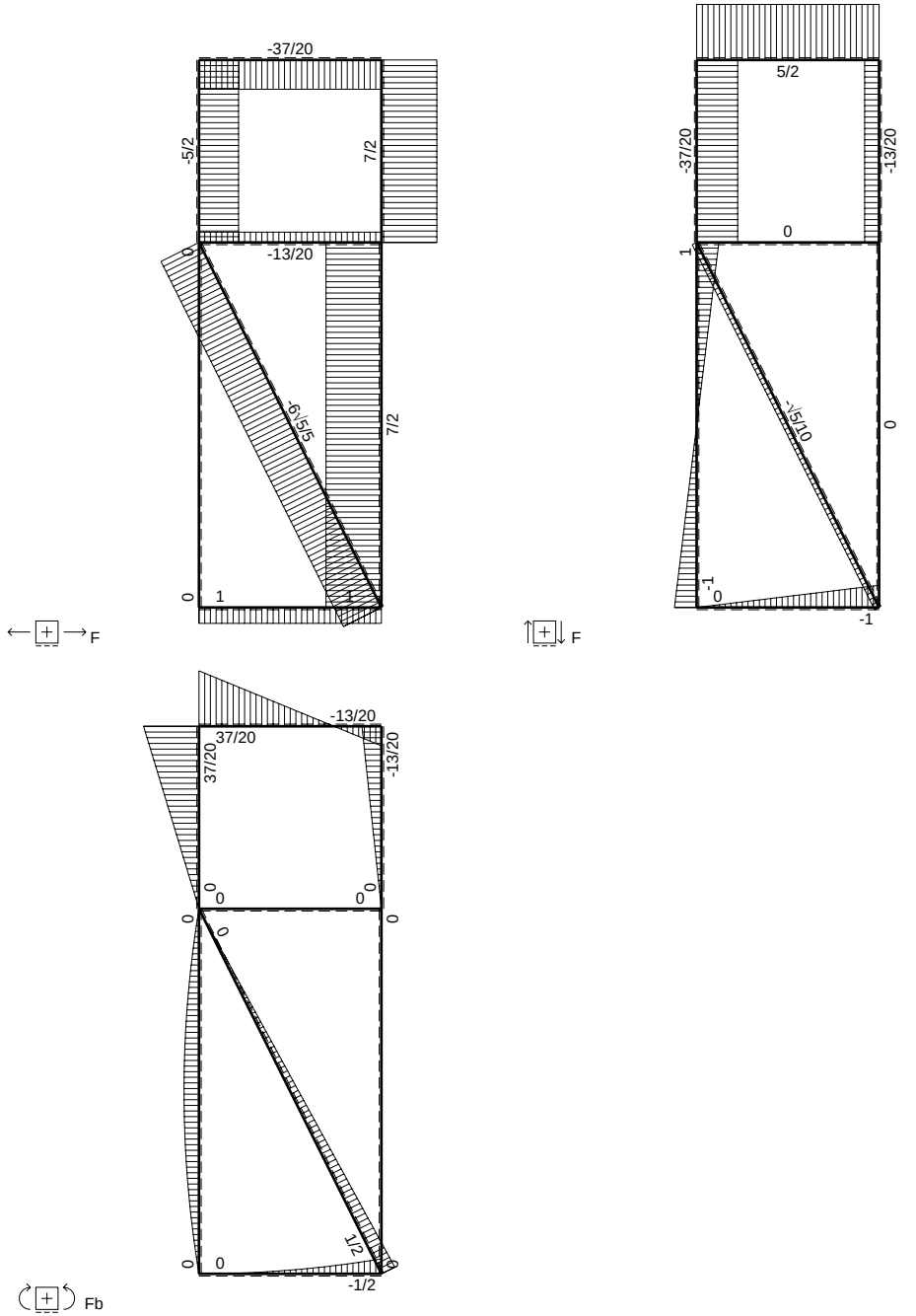


A = 624. mm^2
J_x = 234015. mm^4
J_y = 82488. mm^4
J_{xy} = 63875. mm^4
J_u = 257348. mm^4
J_v = 59155. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = -.3502
c = cosα = .9393
s = sinα = -.3431
x_g = 30.52 mm
y_g = 21.86 mm
N = -1680. N
T_y = 960. N
M_x = 1075200. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = -17.15 mm
v_m = 27.96 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.4 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb


-37/20
-5/2
7/2
-13/20
-5\5/5
F
5/2
-5\10
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 247861 CI09.xxxx.118

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$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{Xo} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

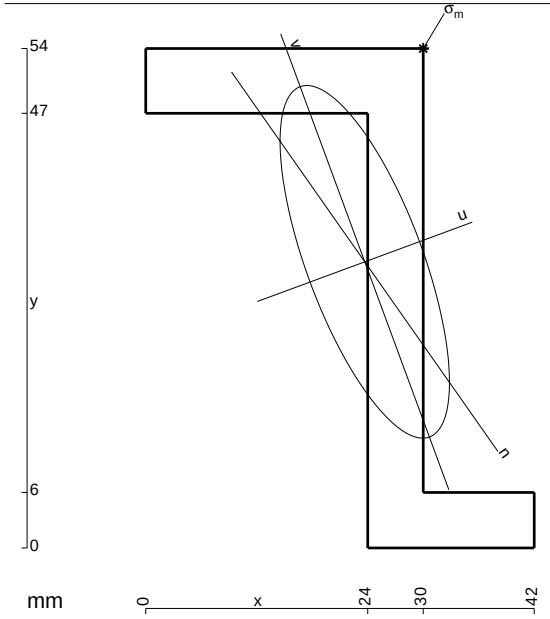
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

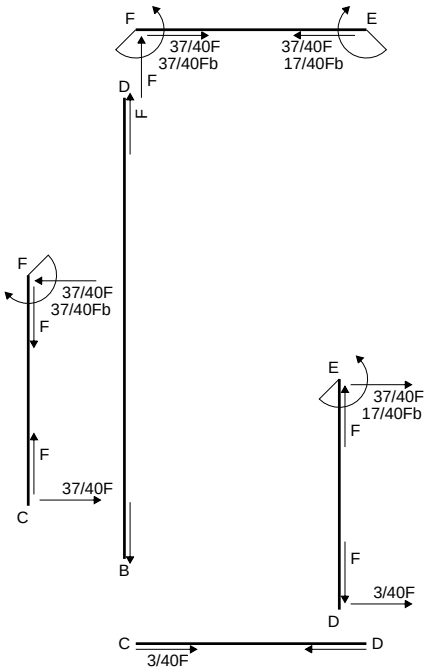
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

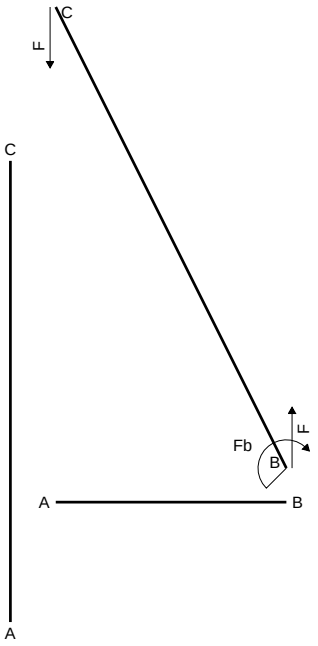


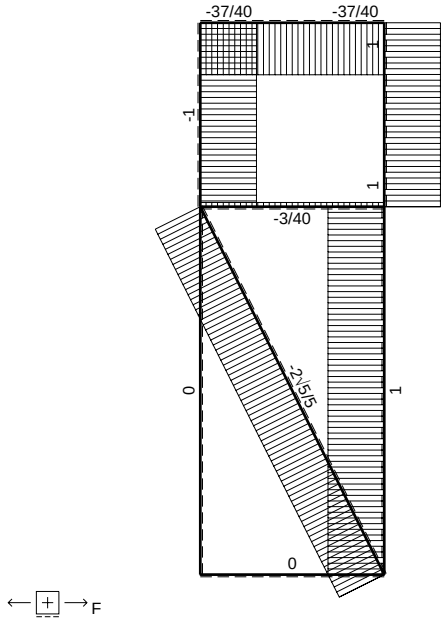
A = 564. mm^2
J_x = 205146. mm^4
J_y = 47319. mm^4
J_{xy} = -67404. mm^4
J_u = 230014. mm^4
J_v = 22451. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3534
c = cosα = .9382
s = sinα = .3461
x_g = 23.68 mm
y_g = 30.94 mm
N = -1525. N
T_y = -1129. N
M_x = 767380. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 13.91 mm
v_m = 19.45 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -228.2 N/mm^2

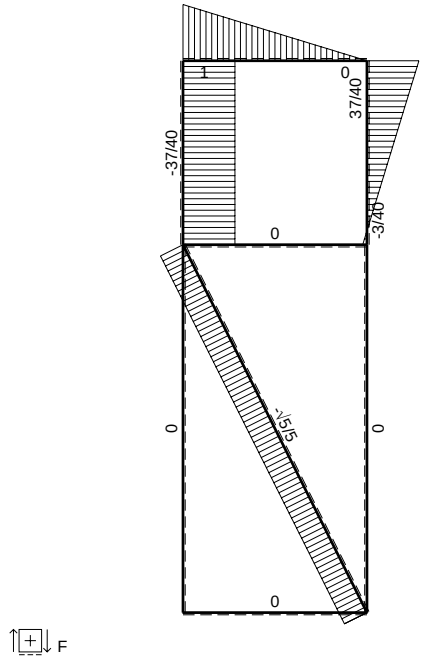
CI09.xxxx.118

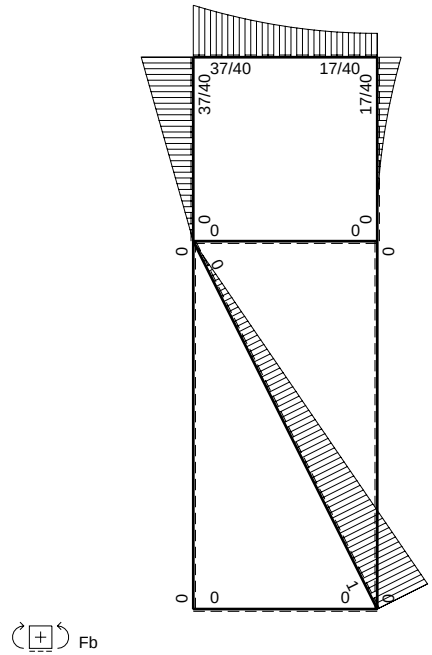
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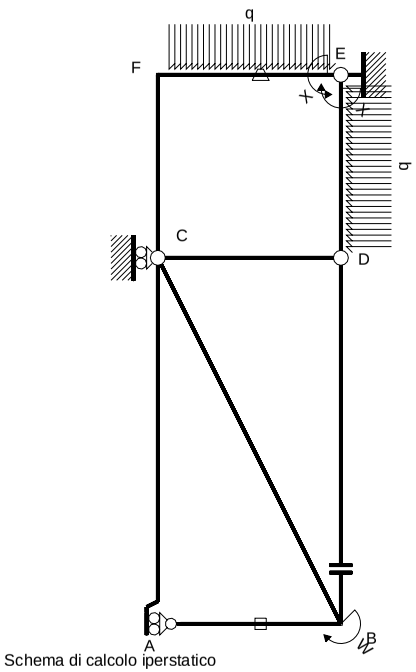

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
3/40F
D


C
F
C
Fb
F
B
A
B
A

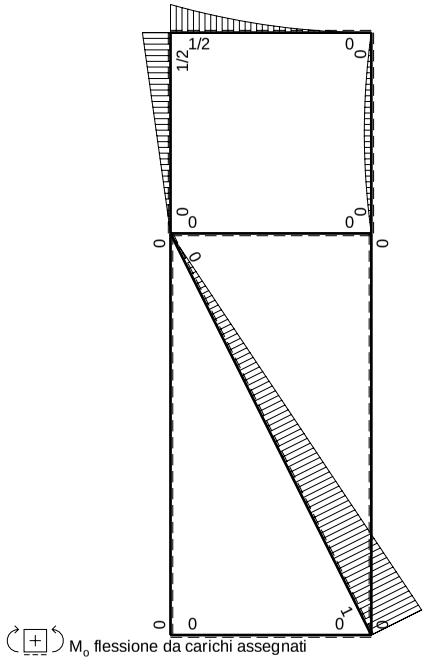

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-1
1
-3/40
-2√5/5
0
1
0
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
F

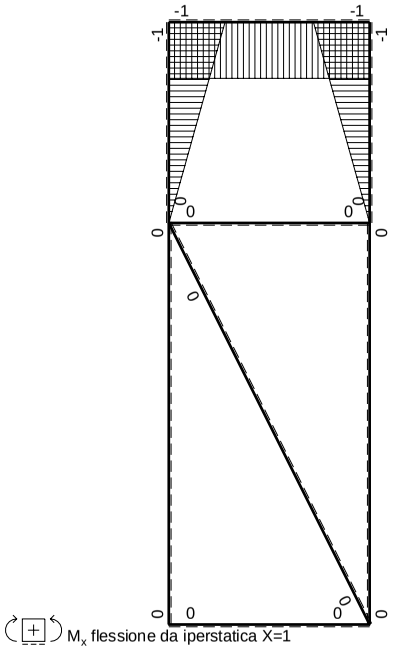

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 215531 CI09.xxxx.119

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ \ Fb^2/EJ$

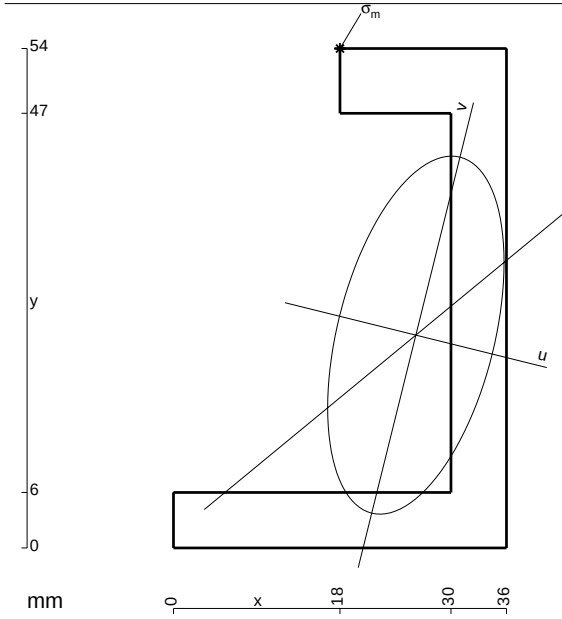
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 215531 CI09.xxxx.119

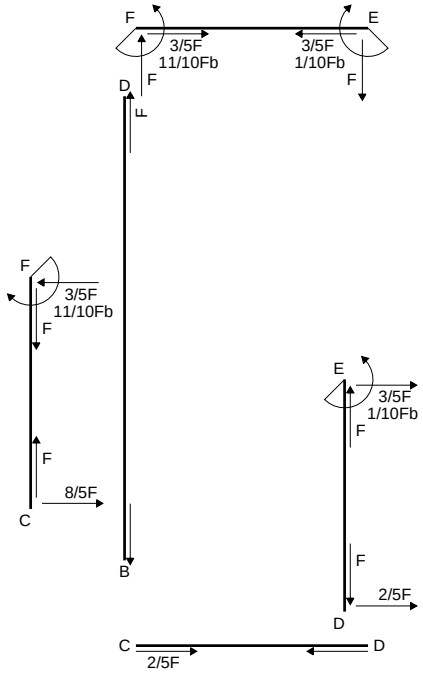


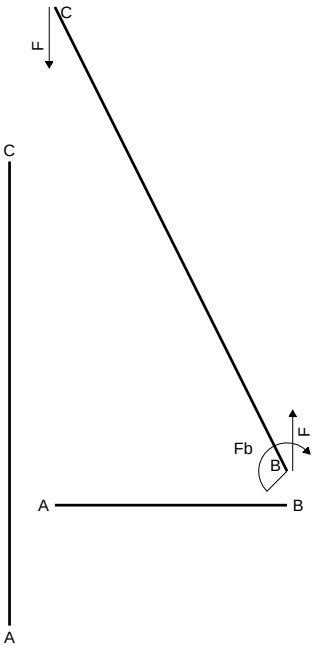
A = 588. mm^2
J_x = 220324. mm^4
J_y = 53448. mm^4
J_{xy} = 44051. mm^4
J_u = 231238. mm^4
J_v = 42533. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2429
$c = \cos\alpha$ = .9707
$s = \sin\alpha$ = -.2405
x_g = 26.2 mm
y_g = 23.01 mm
N = -1431. N
T_y = -715.5 N
M_x = 1152000. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = -15.42 mm
v_m = 28.11 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -238.8 N/mm^2

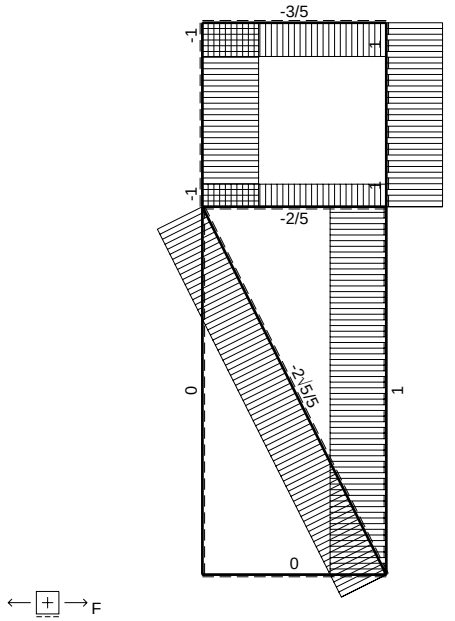
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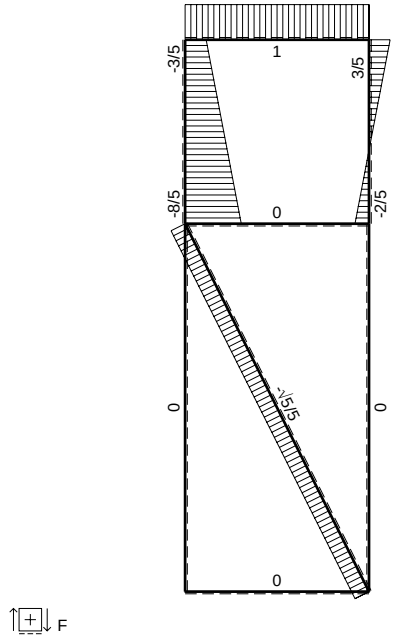
CI09.xxxx.119

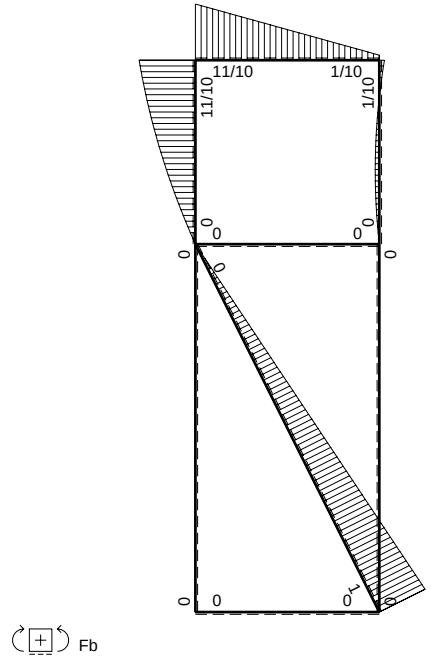
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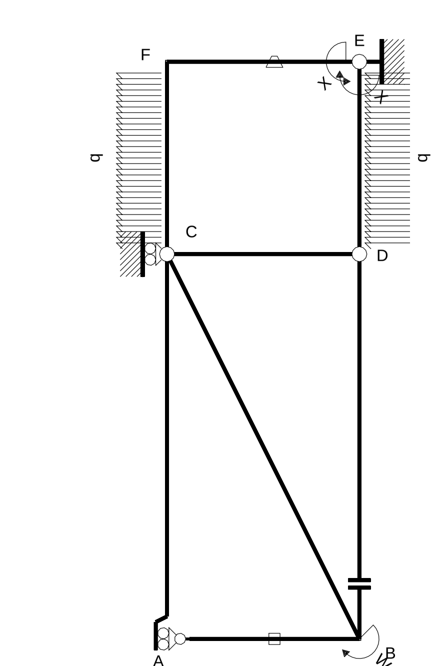

F
E
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11/10Fb
3/5F
1/10Fb
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F
F
F
3/5F
11/10Fb
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E
3/5F
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A

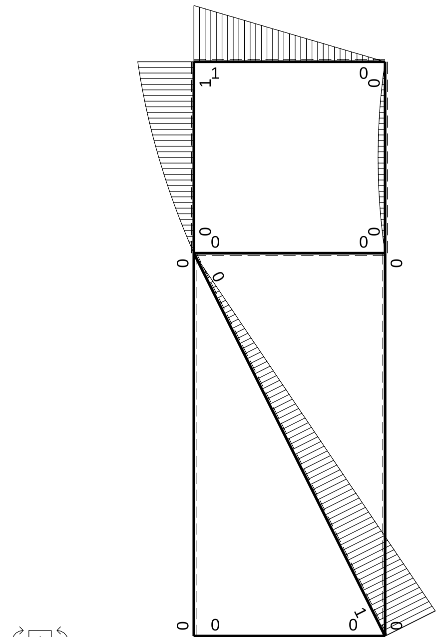

-3/5
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-2/5
0
-2/5
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0
F


-3/5
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3/5
-8/5
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-2/5
0
-2/5
0
0
F

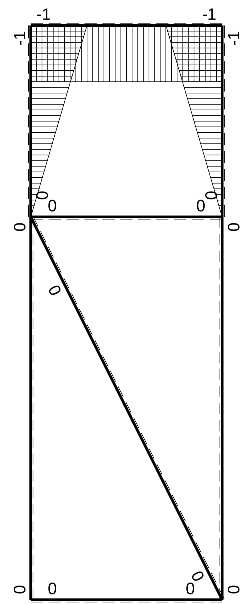

11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$1/6Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

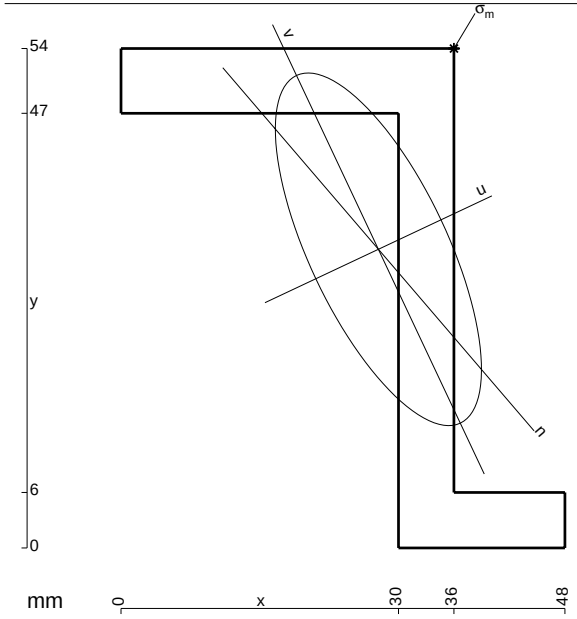
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

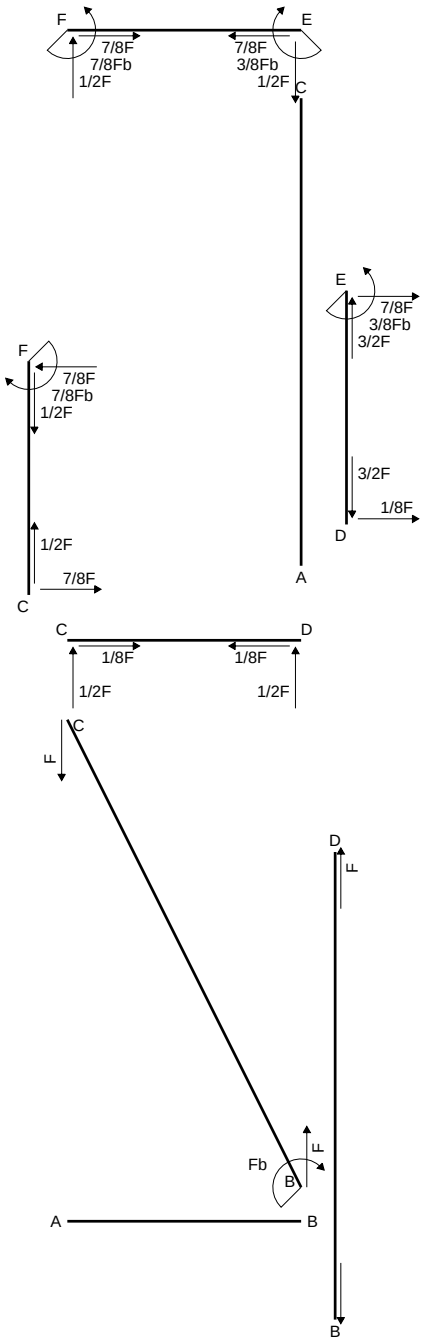
$= (-1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

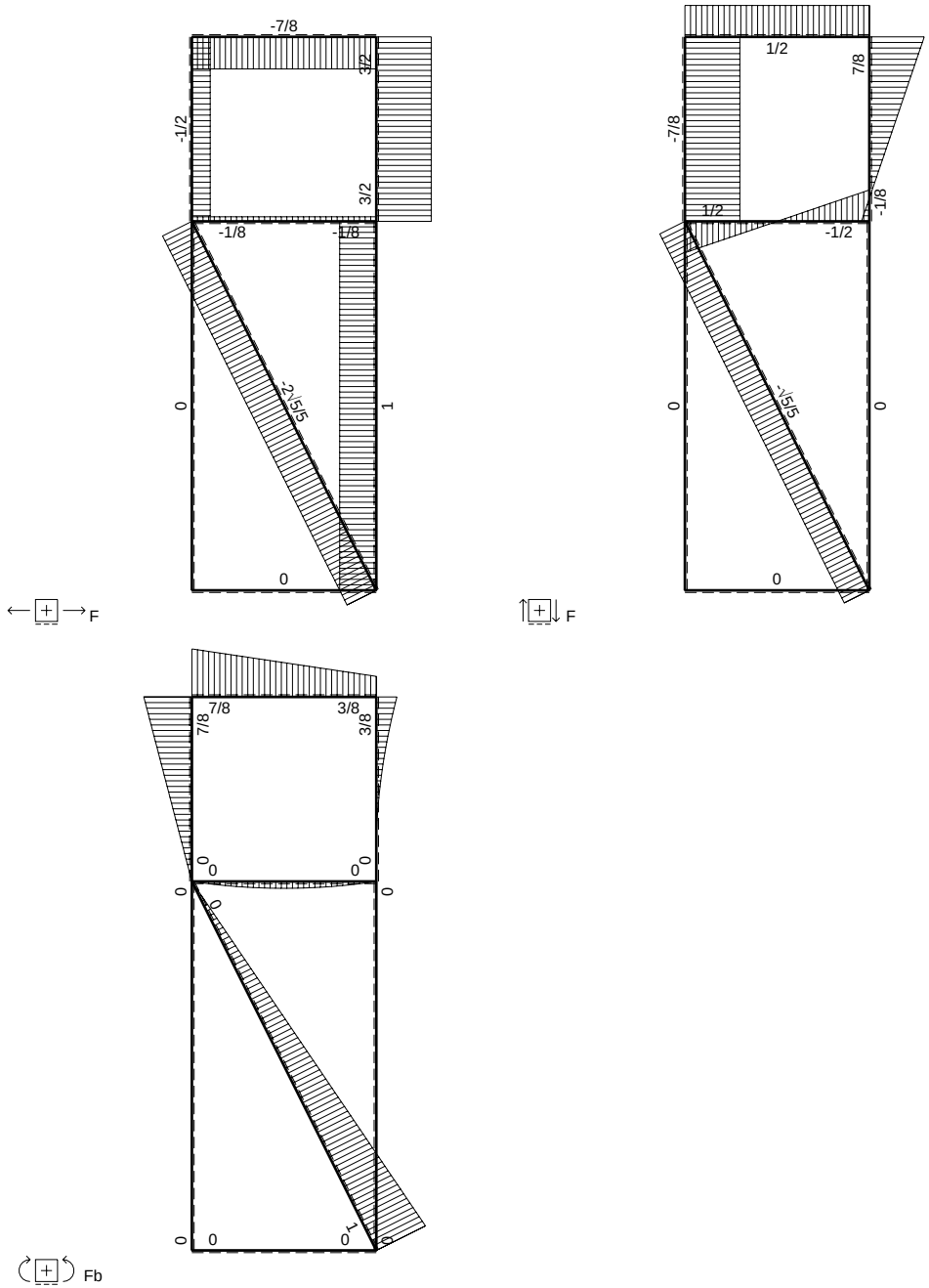


A = 606. mm^2
J_x = 220278. mm^4
J_y = 75271. mm^4
J_{xy} = -87807. mm^4
J_u = 261647. mm^4
J_v = 33902. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4403
$c = \cos\alpha$ = .9046
$s = \sin\alpha$ = .4262
x_g = 27.83 mm
y_g = 32.29 mm
N = -1770. N
T_y = -1062. N
M_x = 739860. Nmm
x_m = 36. mm
y_m = 54. mm
u_m = 16.64 mm
v_m = 16.16 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -199. N/mm^2

CI09.xxxx.120

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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
-2√5/5
0
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
-√5/5
0
0
F
7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2$/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2$/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 237158 CI09.xxxx.121

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

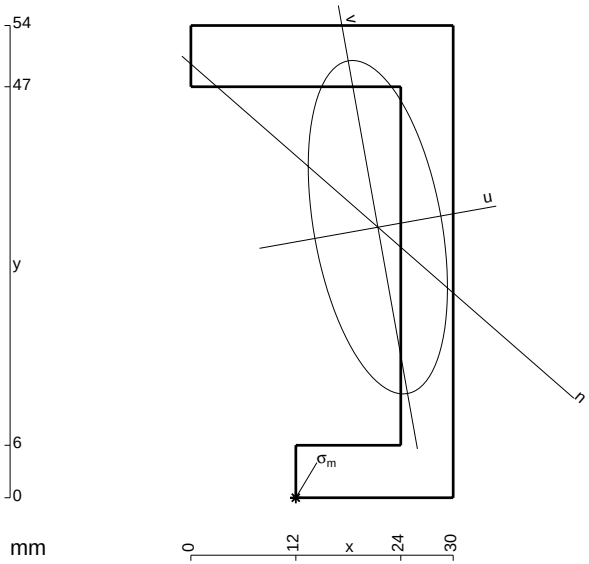
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 237158 CI09.xxxx.121

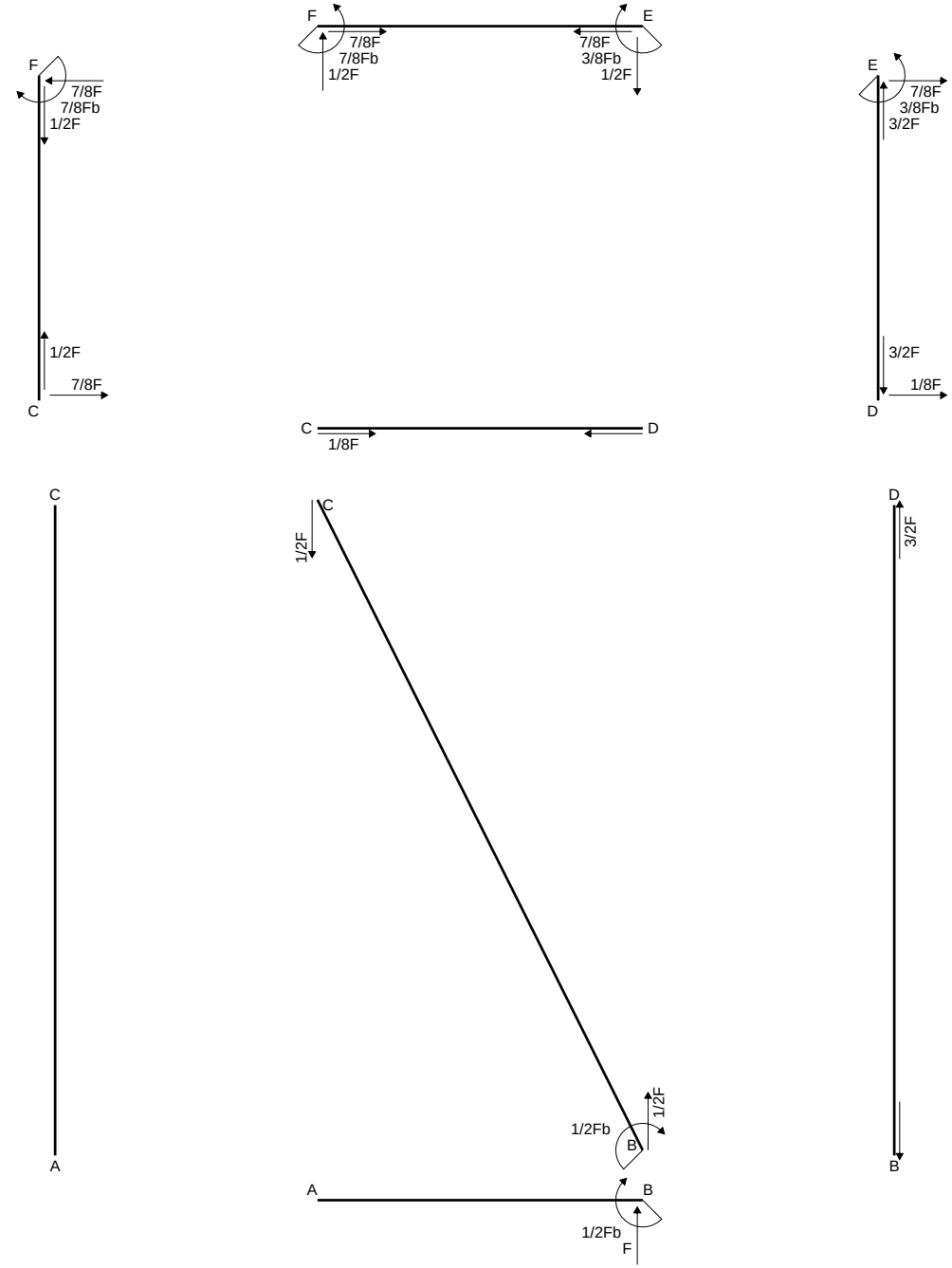


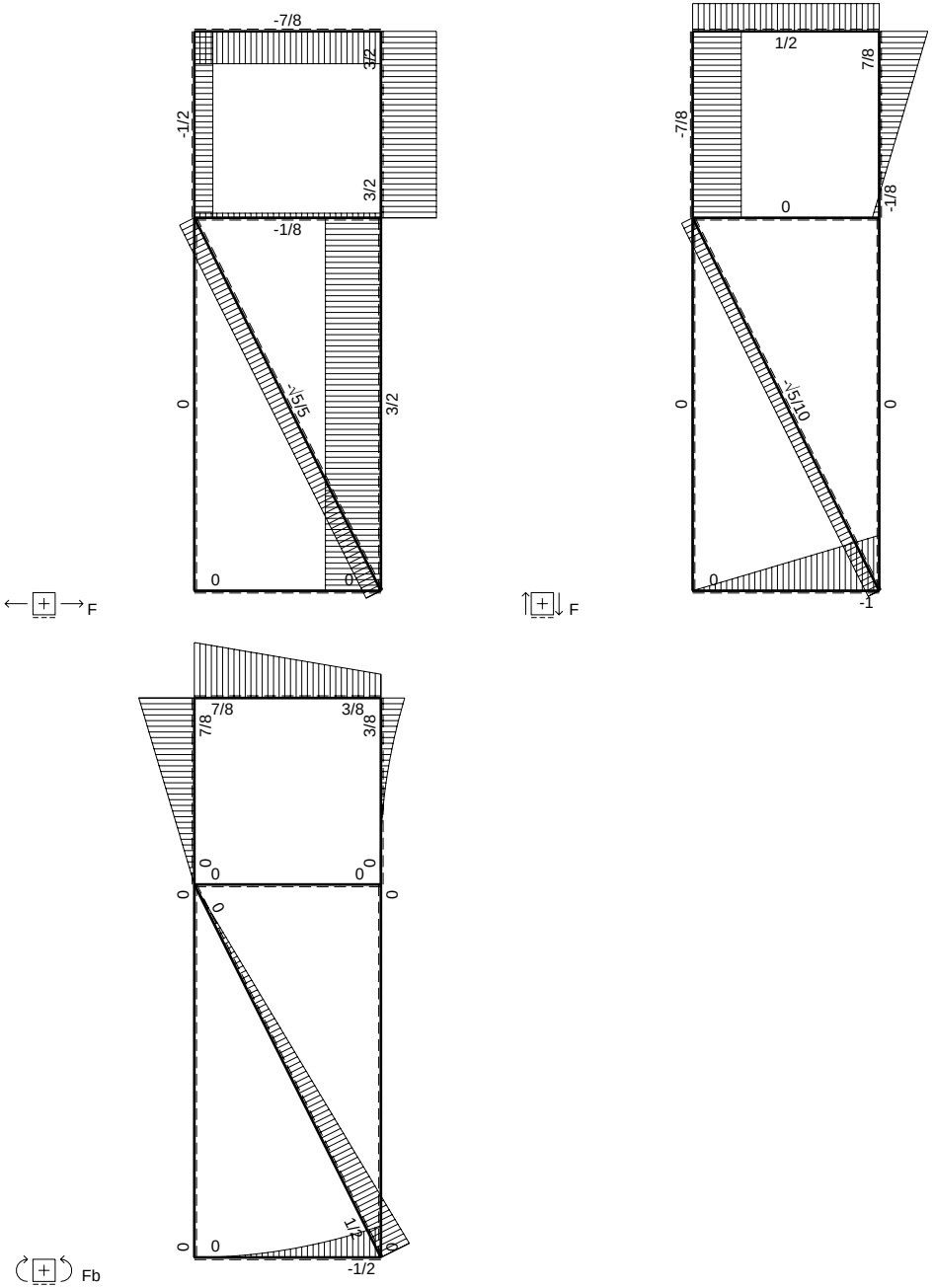
A = 564. mm^2
J_x = 205146. mm^4
J_y = 35737. mm^4
J_{xy} = -31198. mm^4
J_u = 210708. mm^4
J_v = 30175. mm^4
α= artg($2J_{xy}/(J_y-J_x)$)/2 = .1765
c = cosα = .9845
s = sinα = .1755
x_g = 21.38 mm
y_g = 30.94 mm
N = -2120. N
T_y = -1060. N
M_x = 971700. Nmm
x_m = 12. mm
u_m = -14.67 mm
v_m = -28.81 mm
σ_m = N/A-Mcv/J_u-Msu/J_v = 209.9 N/mm^2

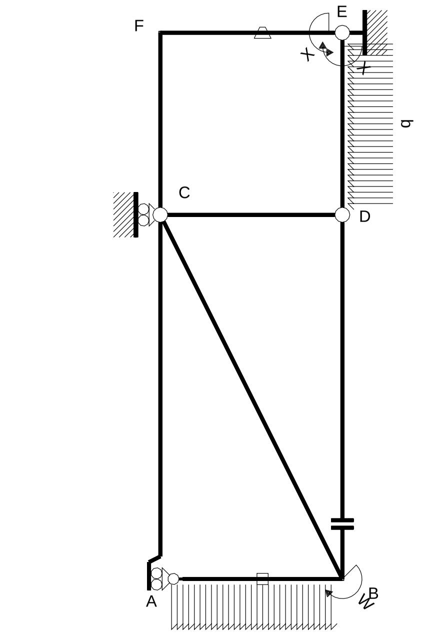
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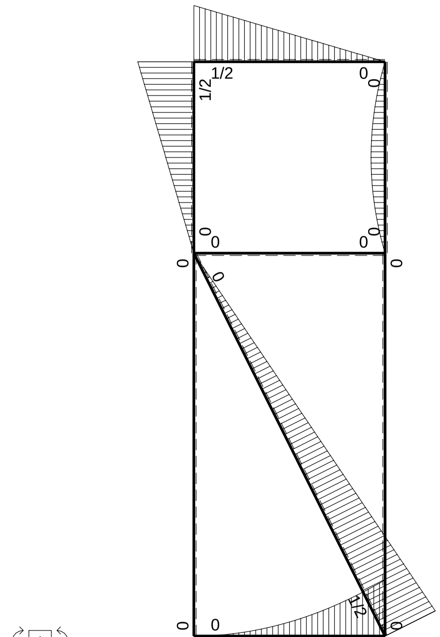
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F
7/8F
7/8Fb
1/2F
1/2F
7/8F
C
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
D
1/8F
E
7/8F
3/8Fb
3/2F
3/2F
1/8F
D
C
A
C
1/2F
1/2Fb
B
1/2F
A
B
1/2Fb
F
D
3/2F
B

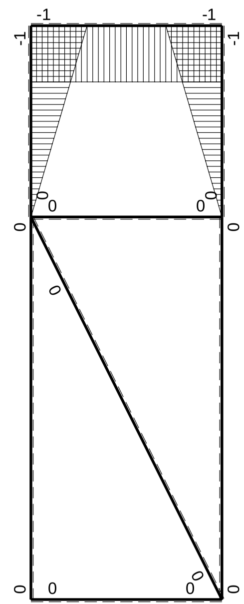

-7/8
3/2
-1/2
3/2
-1/8
0
-√5/5
3/2
0
0
F
1/2
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-7/8
0
-1/8
-√5/10
0
0
-1
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
-√5/2
0
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ \ = 3/4\ \ Fb^2/EJ$

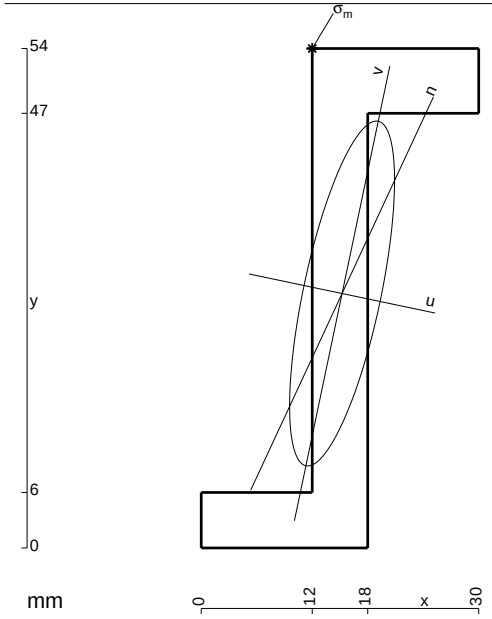
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 264611 CI09.xxxx.122

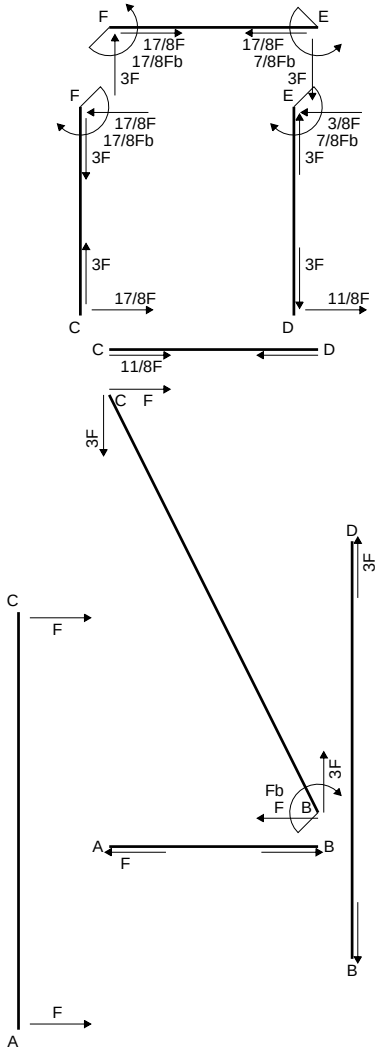


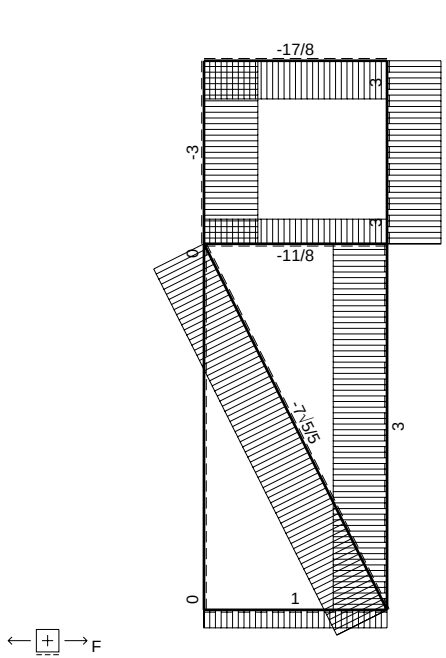
A = 480. mm^2
J_x = 167026. mm^4
J_y = 15456. mm^4
J_{xy} = 33263. mm^4
J_u = 174004. mm^4
J_v = 8477. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = -.2068
c = cosα = .9787
s = sinα = -.2053
x_g = 15.23 mm
y_g = 27.51 mm
N = -1374. N
T_y = 785. N
M_x = 618188. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -8.595 mm
v_m = 25.26 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.4 N/mm^2

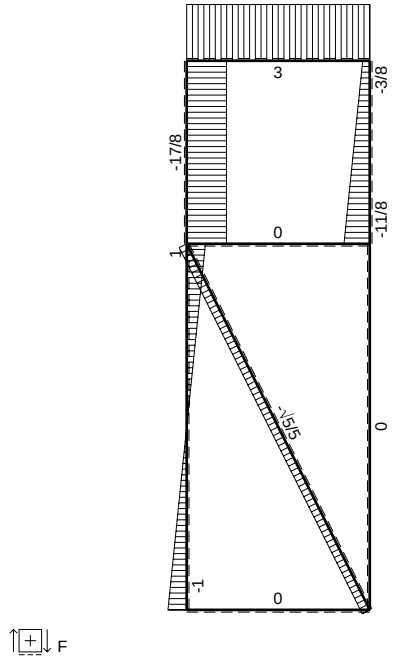
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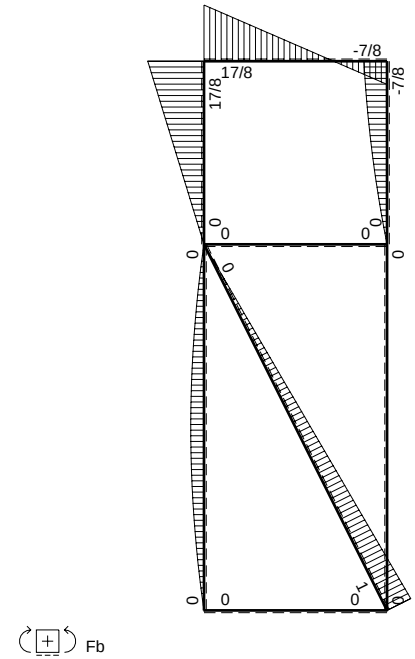
CI09.xxxx.122

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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
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7/8Fb
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3F
C
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3F
Fb
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B
A
B
F
A
F
B


-17/8
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3
0
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F


3
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0
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0
-1
0
F


-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb²/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx²/b	0	1-2x/b+x²/b²	(-1+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx²/b	0	x²/b²		
	totali						-35/24Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

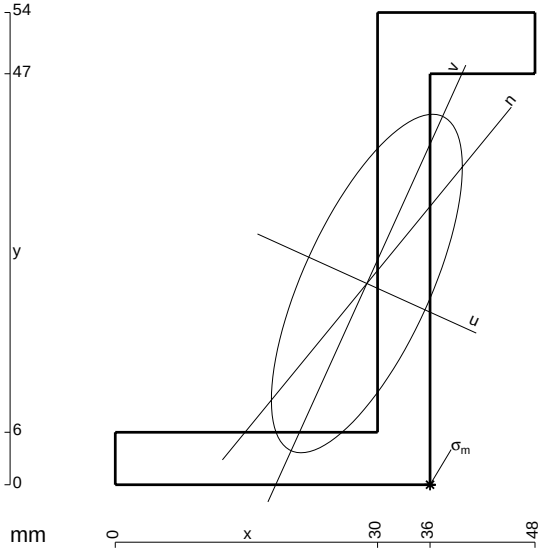
$= (-3\ b +3/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

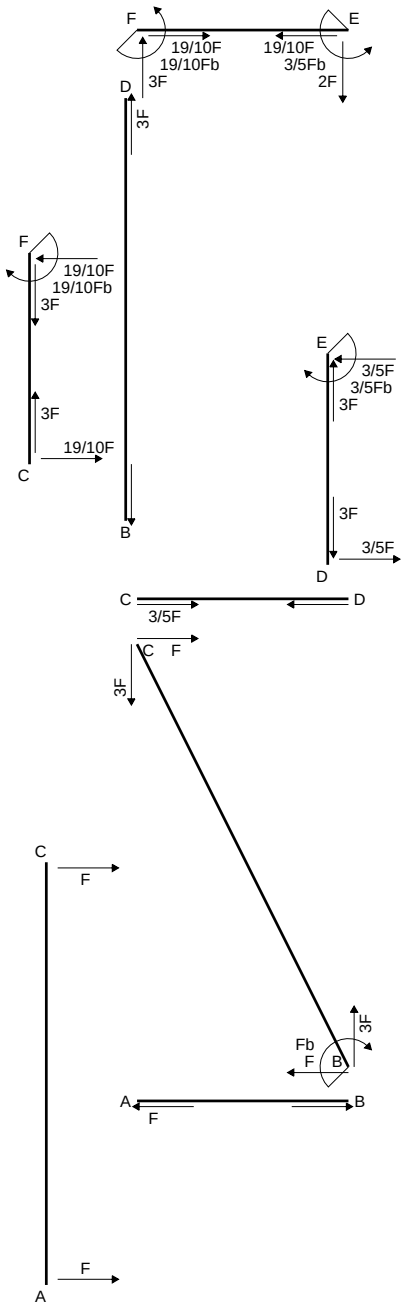
$= (-\ b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

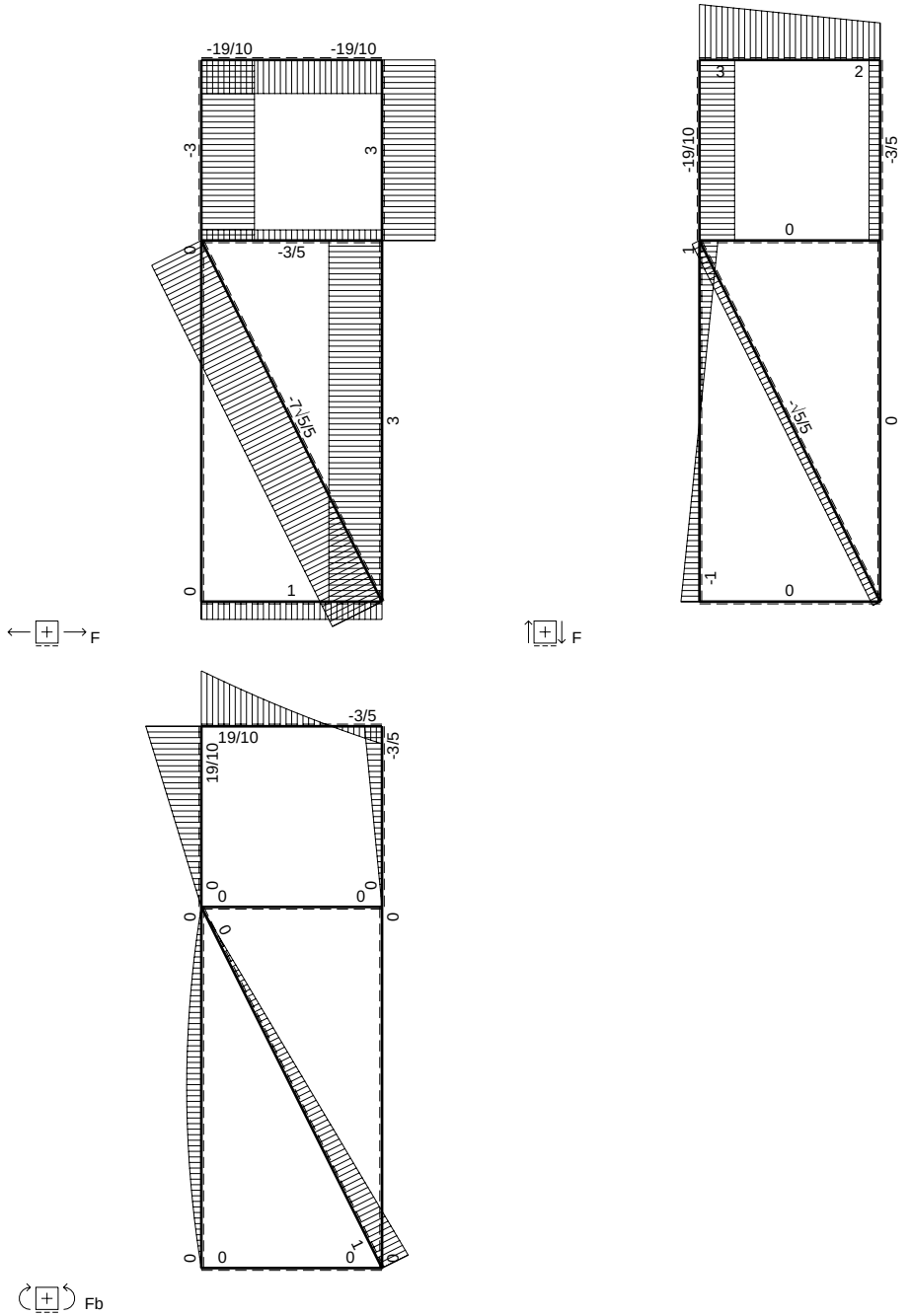


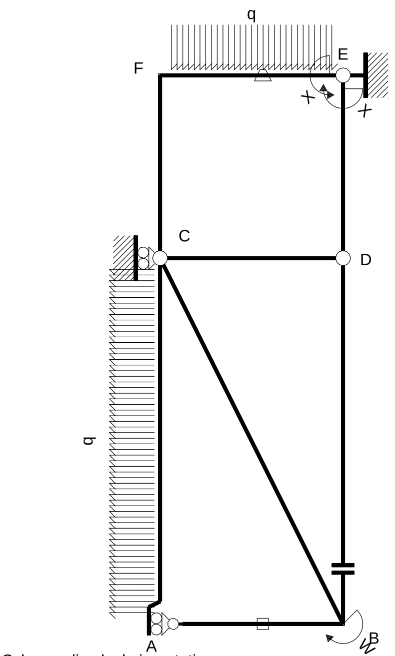
A = 588. mm^2
J_x = 220324. mm^4
J_y = 70110. mm^4
J_{xy} = 85615. mm^4
J_u = 259108. mm^4
J_v = 31327. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = -.4253
c = cosα = .9109
s = sinα = -.4126
x_g = 28.78 mm
y_g = 23.01 mm
N = -1721. N
T_y = 2430. N
M_x = 843413. Nmm
x_m = 36. mm
u_m = 16.08 mm
v_m = -17.98 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 229. N/mm^2

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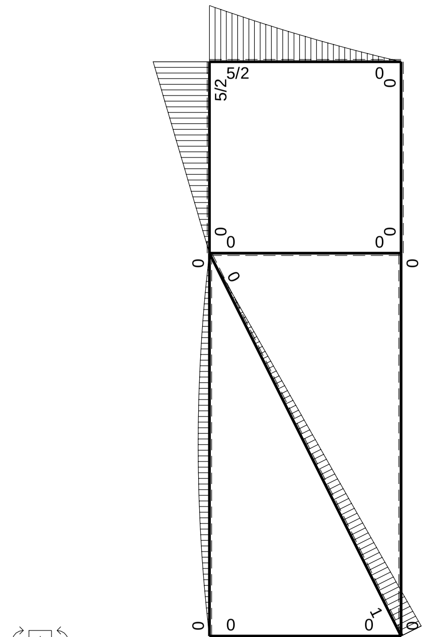
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F
E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
B
3F
3/5F
D
C
D
3/5F
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A

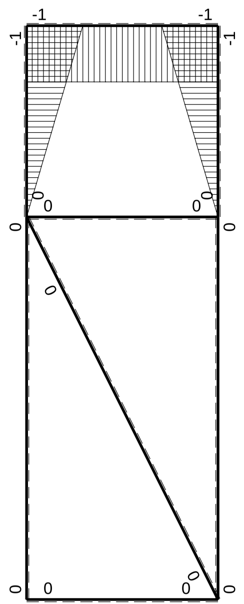

-19/10
-19/10
-3
3
-3/5
0
0
-7.5/5
3
1
F
3
2
-19/10
-3/5
0
1
-1
-7.5/5
0
-1
0
F
-3/5
19/10
19/10
-3/5
0
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

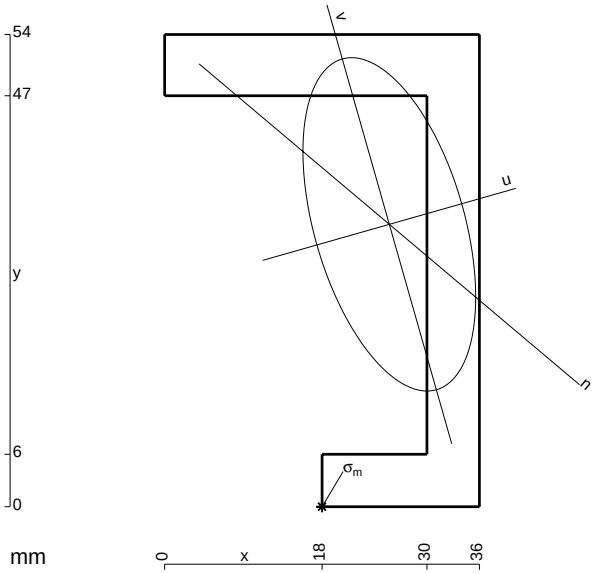
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

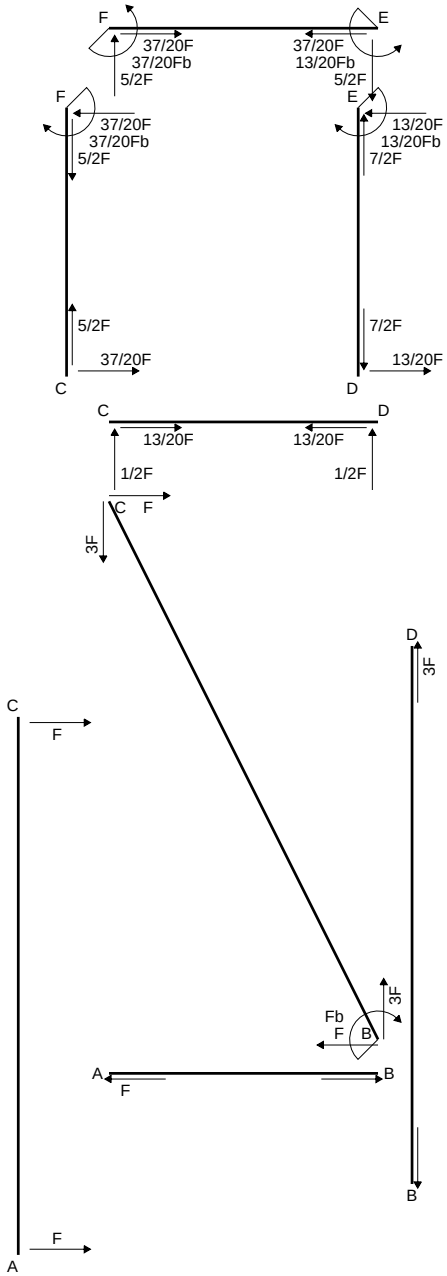
$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

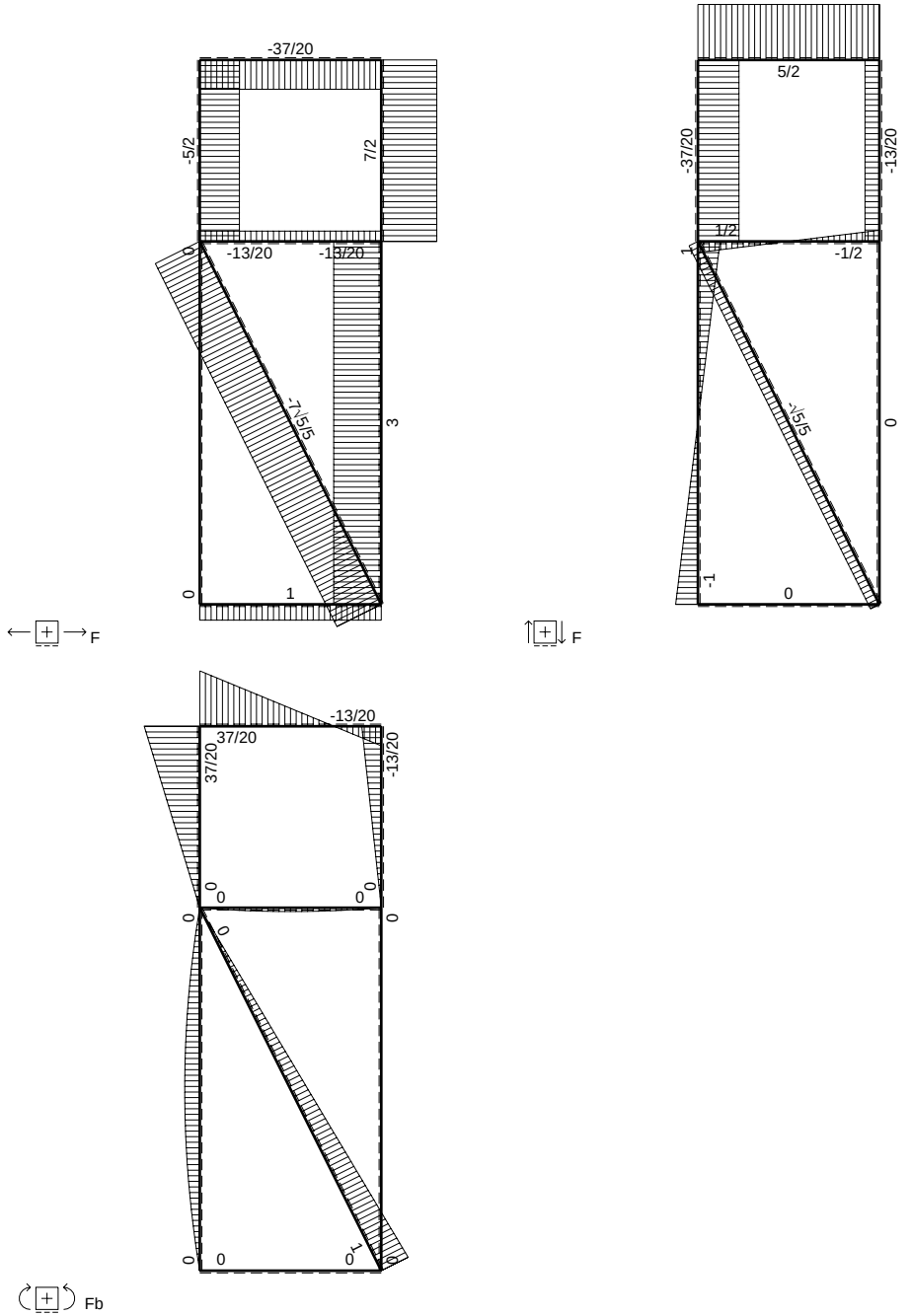


A = 606. mm^2
J_x = 220278. mm^4
J_y = 59103. mm^4
J_{xy} = -49845. mm^4
J_u = 234447. mm^4
J_v = 44934. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2770
c = cosα = .9619
s = sinα = .2734
x_g = 25.69 mm
y_g = 32.29 mm
N = -2109. N
T_y = 3330. N
M_x = 1117770. Nmm
x_m = 18. mm
u_m = -16.23 mm
v_m = -28.96 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.7 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
7/2F
37/20F
13/20F
C
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
F
Fb
F
B
3F
A
B
F
F
A
B


-37/20
-5/2
7/2
-13/20
-13/20
0
3
-7/5/5
0
1
F
5/2
-37/20
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1/2
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-1
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-7/5/5
F
-13/20
37/20
37/20
-13/20
0
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0
0
0
0
0
0
0
0
1
Fb

Schema di calcolo iperstatico

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	1/2Fx-1/2qx²	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx²	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

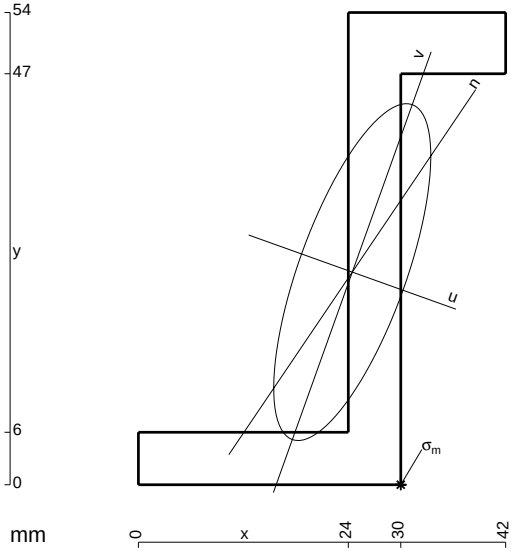
$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 267320 CI09.xxxx.125

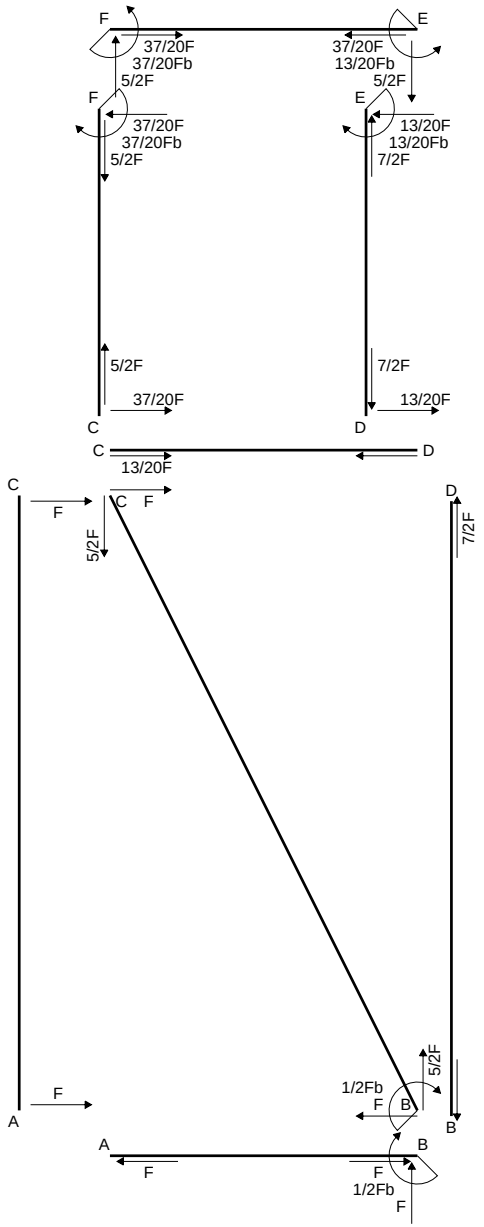


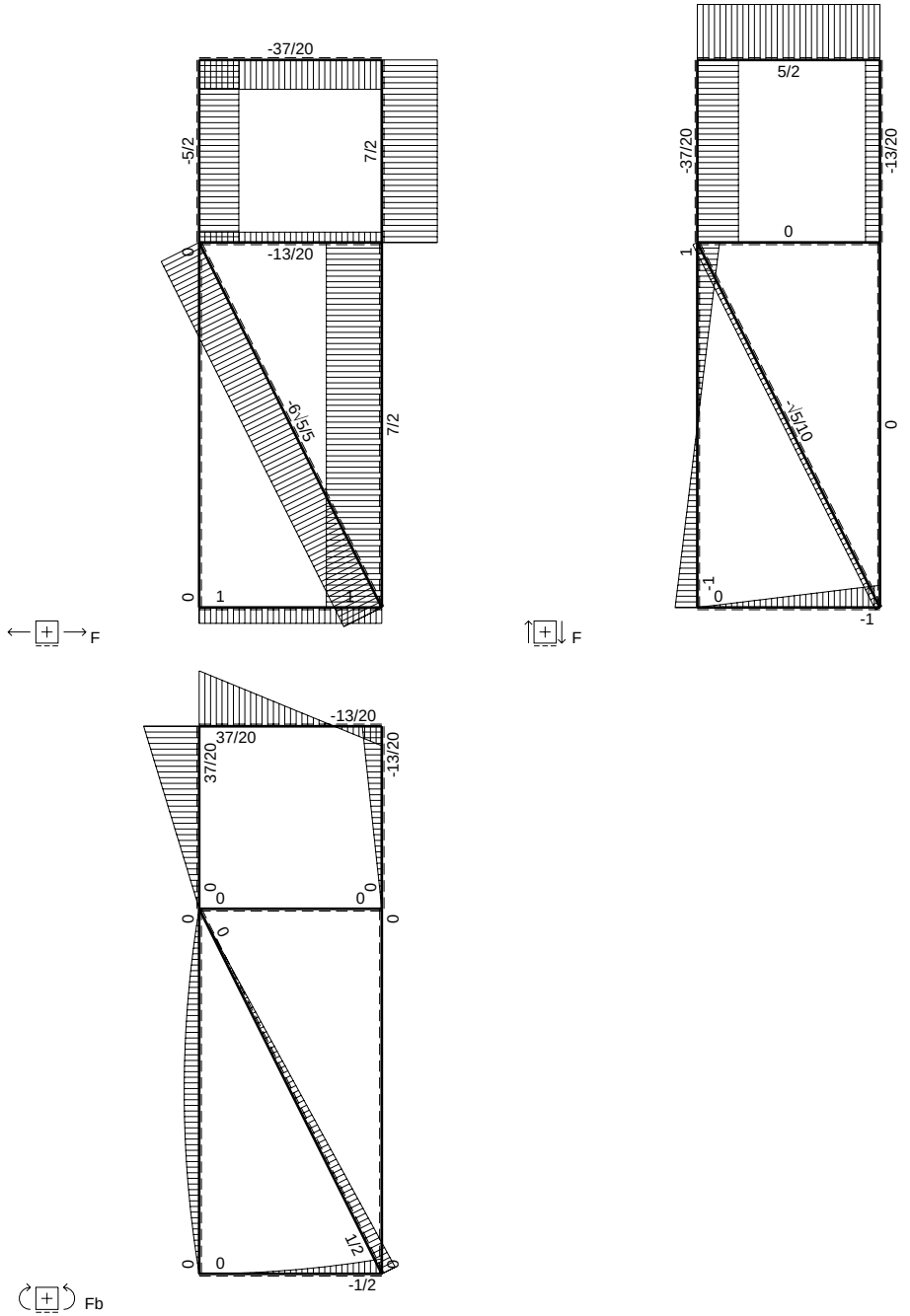
A = 552. mm^2
J_x = 204861. mm^4
J_y = 44525. mm^4
J_{xy} = 65837. mm^4
J_u = 228430. mm^4
J_v = 20956. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3438
c = cosα = .9415
s = sinα = -.3370
x_g = 24.46 mm
y_g = 24.32 mm
N = -1166. N
T_y = 1575. N
M_x = 664335. Nmm
x_m = 30. mm
u_m = 13.41 mm
v_m = -21.02 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 198.8 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb


-37/20
-5/2
7/2
-13/20
-5\5/5
F
5/2
-37/20
-13/20
-5/10
37/20
1/2
-1/2
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{Xo} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

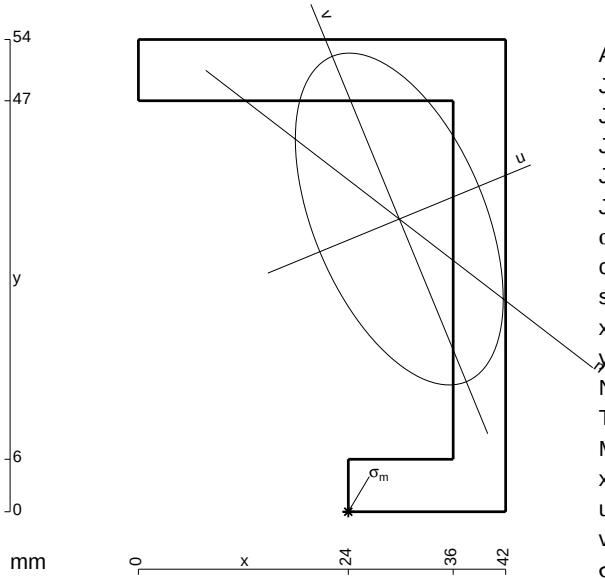
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

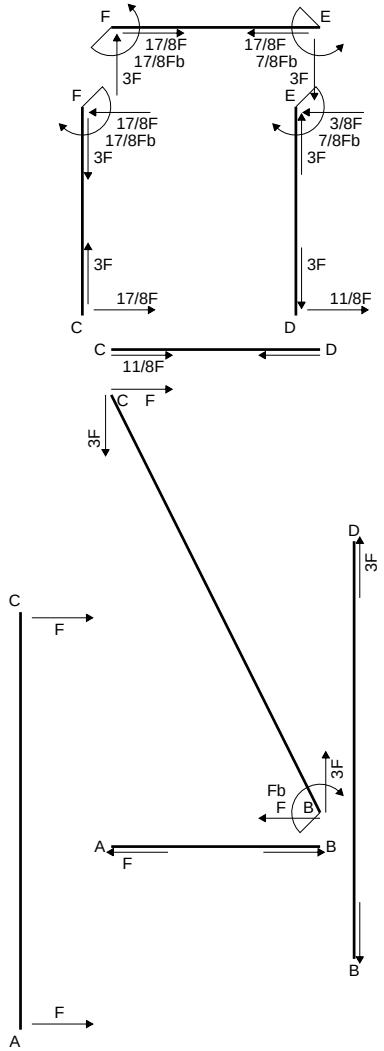
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

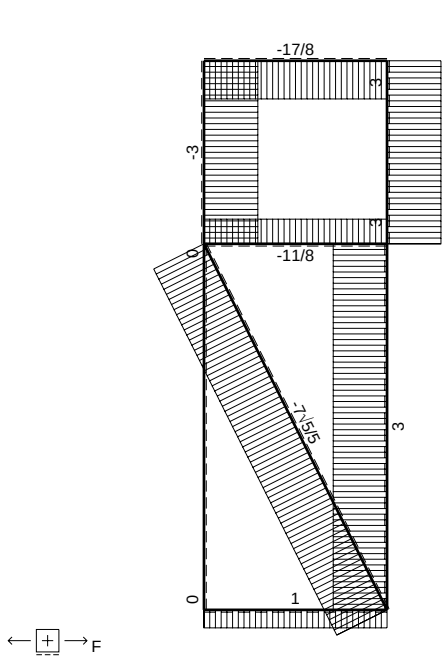


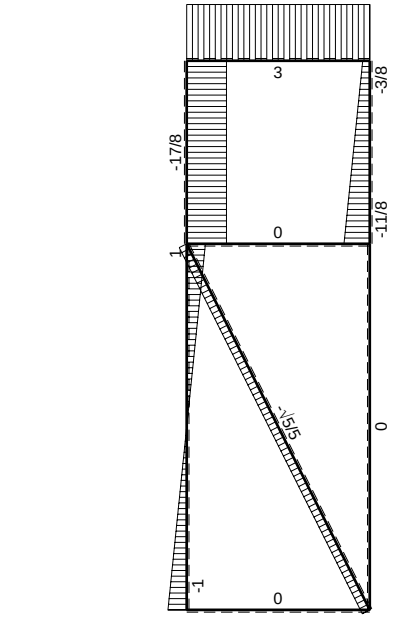
A = 648. mm^2
J_x = 233472. mm^4
J_y = 91566. mm^4
J_{xy} = -70365. mm^4
J_u = 262446. mm^4
J_v = 62591. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3906
$c = \cos\alpha$ = .9247
$s = \sin\alpha$ = .3808
x_g = 29.83 mm
y_g = 33.47 mm
N = -1665. N
T_y = 2250. N
M_x = 999000. Nmm
x_m = 24. mm
u_m = -18.14 mm
v_m = -28.73 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 208.8 N/mm^2

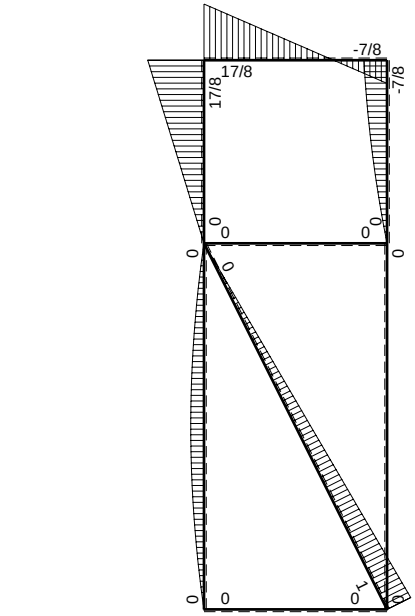
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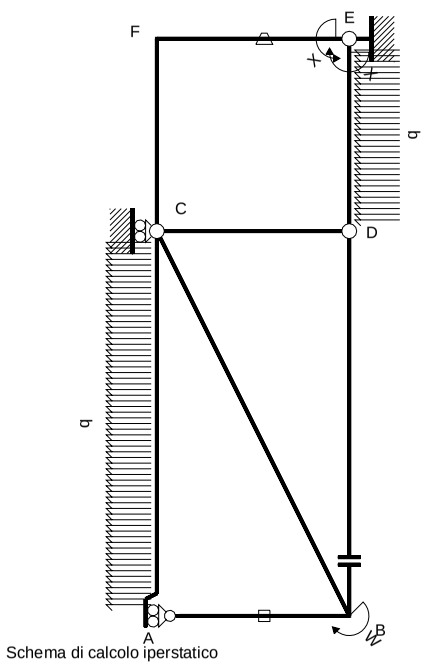
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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
A
F
B

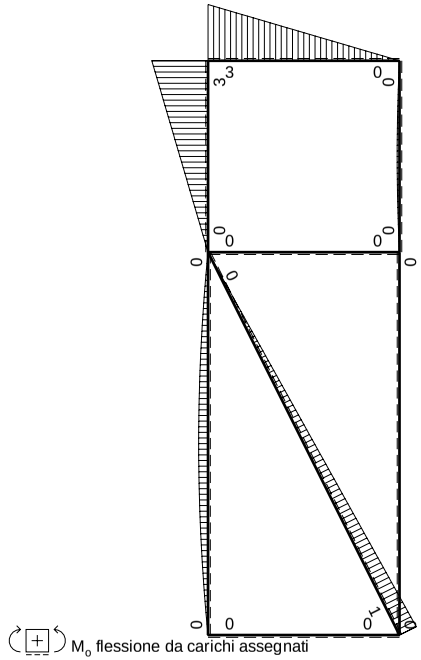

-17/8
-3
-11/8
0
1
3
-7√5/5
F


3
-3/8
-17/8
-11/8
0
1
-1
-√5/5
F

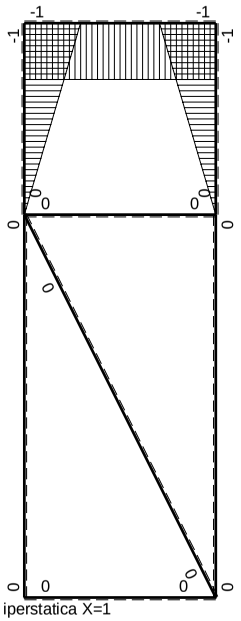

-7/8
17/8
-7/8
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
				totali			-35/24Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 240551 CI09.xxxx.127

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

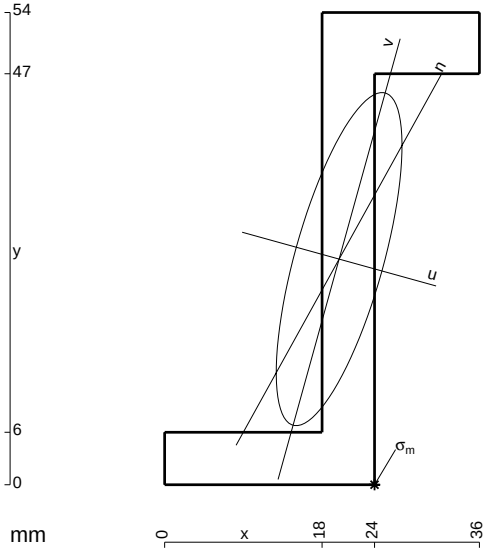
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

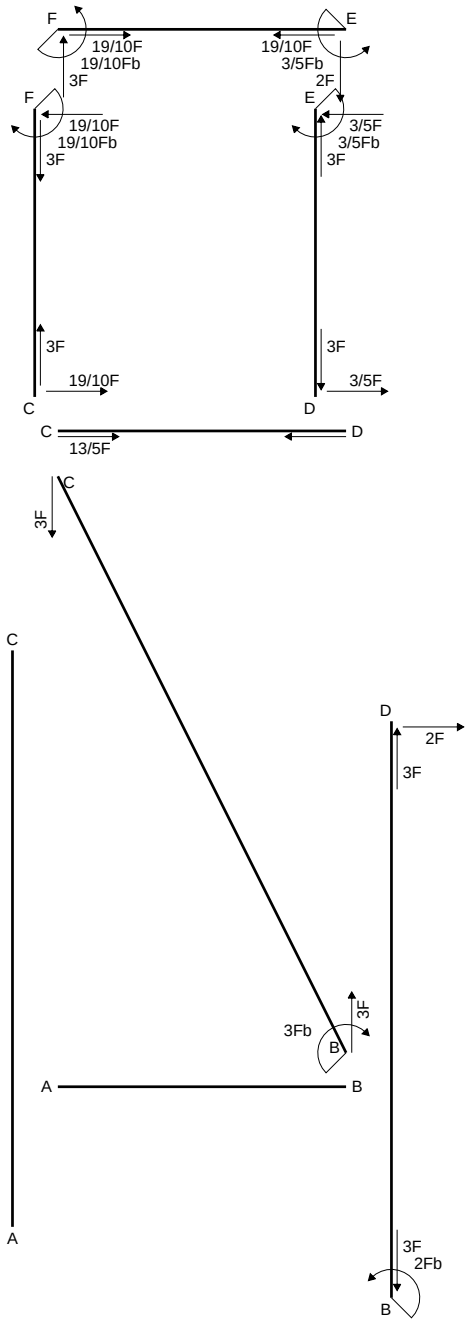
$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

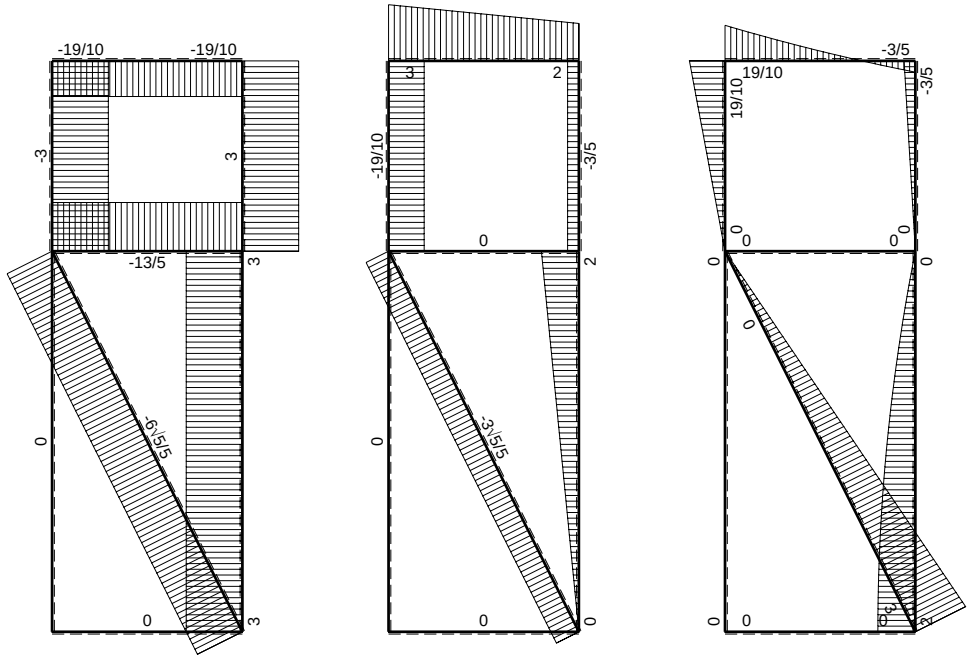


A = 516. mm^2
J_x = 187256. mm^4
J_y = 26687. mm^4
J_{xy} = 48223. mm^4
J_u = 200625. mm^4
J_v = 13317. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.2704$
$c = \cos\alpha$ = .9637
$s = \sin\alpha = -.2672$
x_g = 19.95 mm
y_g = 25.8 mm
N = -1041. N
T_y = 1470. N
M_x = 666400. Nmm
x_m = 24. mm
u_m = 10.79 mm
v_m = -23.78 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 218.4 N/mm^2

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F
E
19/10F
19/10Fb
3F
19/10F
3/5Fb
2F
F
E
19/10F
19/10Fb
3F
3/5F
3/5Fb
3F
3F
19/10F
C
D
3F
3/5F
C
D
13/5F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-19/10
-19/10
-3
3
3
-13/5
0
-6√5/5
0
3
3
2
-19/10
-3/5
0
2
-3√5/5
0
0
19/10
-3/5
0
0
0
0
0
0
0
0


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							3/5Fb

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

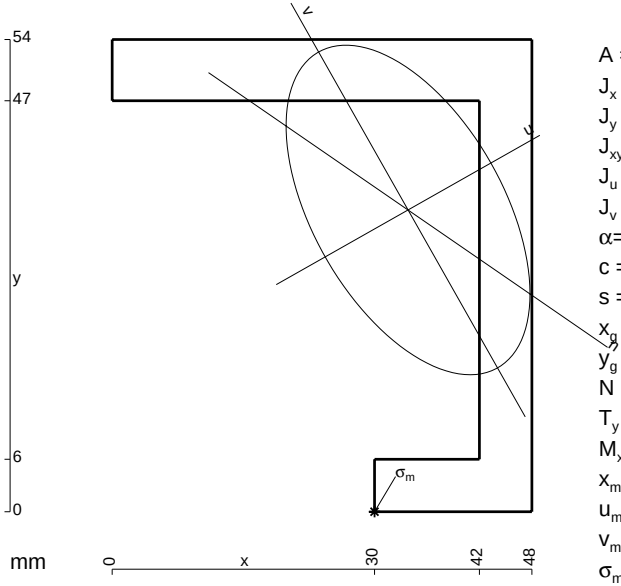
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

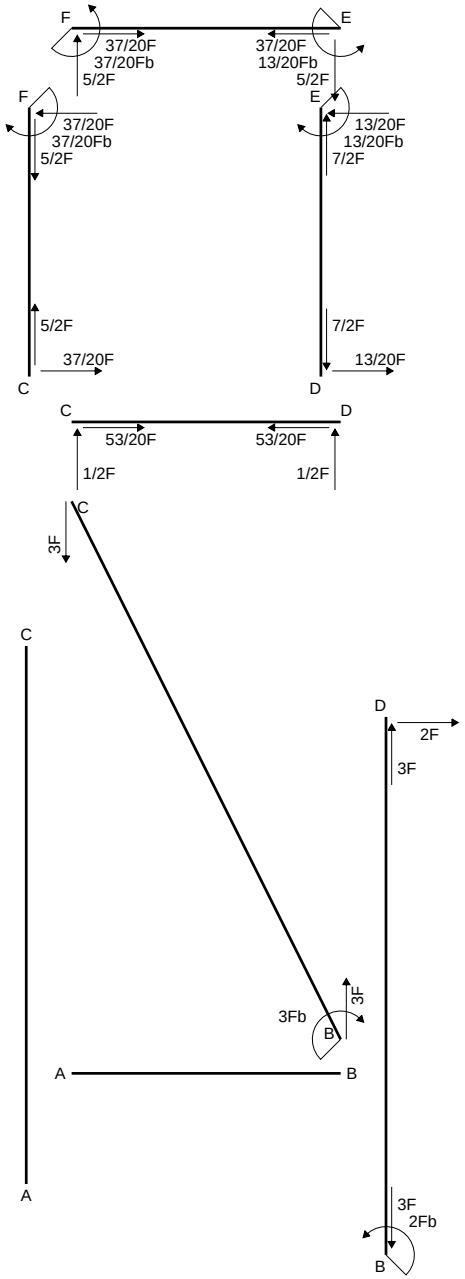
$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

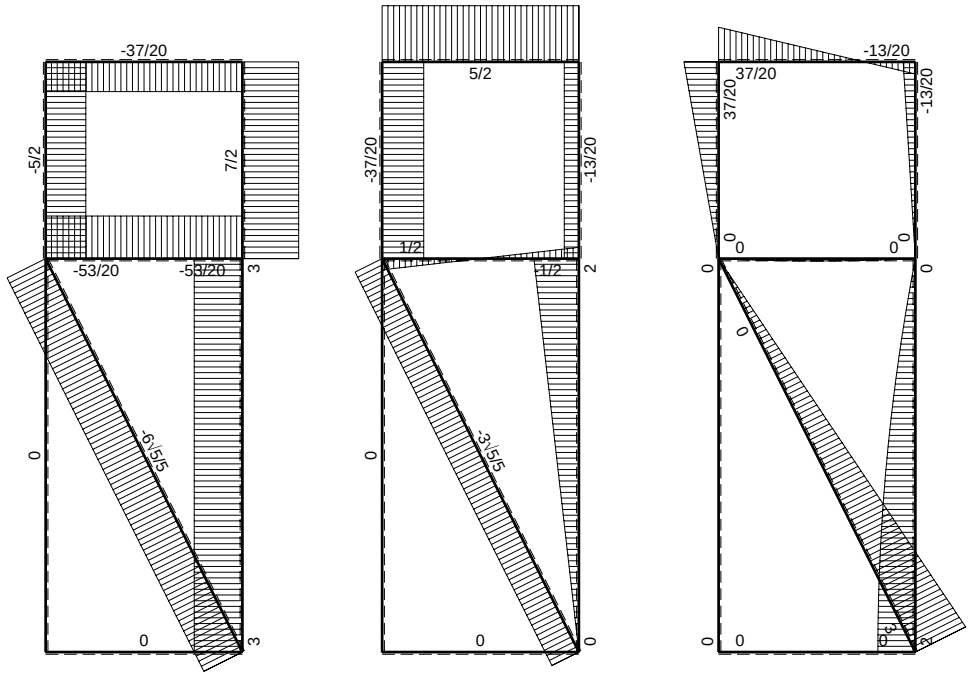


A = 690. mm^2
J_x = 245079. mm^4
J_y = 134213. mm^4
J_{xy} = -92417. mm^4
J_u = 297413. mm^4
J_v = 81879. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .5153
c = cosα = .8702
s = sinα = .4928
x_g = 33.83 mm
y_g = 34.51 mm
N = -1476. N
T_y = -737.9 N
M_x = 1122000. Nmm
x_m = 30. mm
u_m = -20.34 mm
v_m = -28.14 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 227.6 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
C
D
53/20F
53/20F
1/2F
1/2F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-37/20
-5/2
7/2
-53/20
-53/20
3
0
-6√5/5
0
3
5/2
-37/20
-13/20
1/2
1/2
2
-3√5/5
0
0
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
-3√5
0


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx²	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

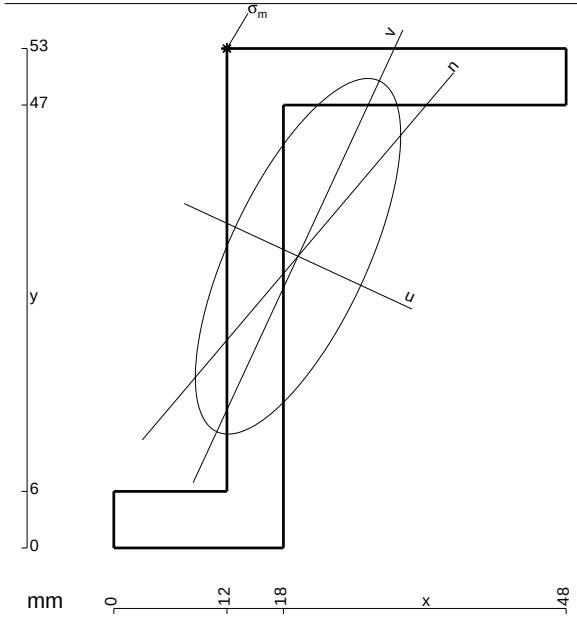
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

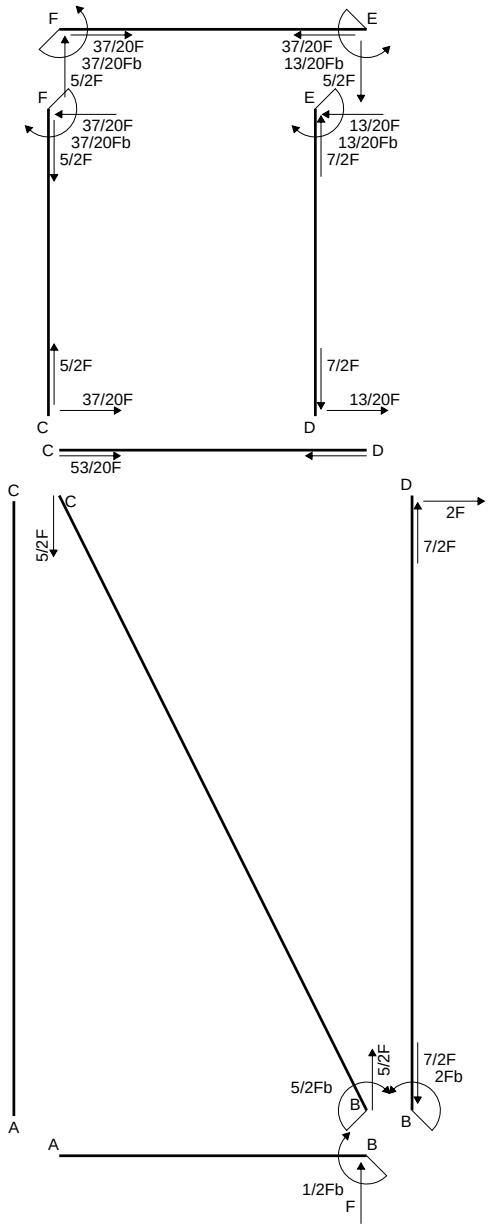
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

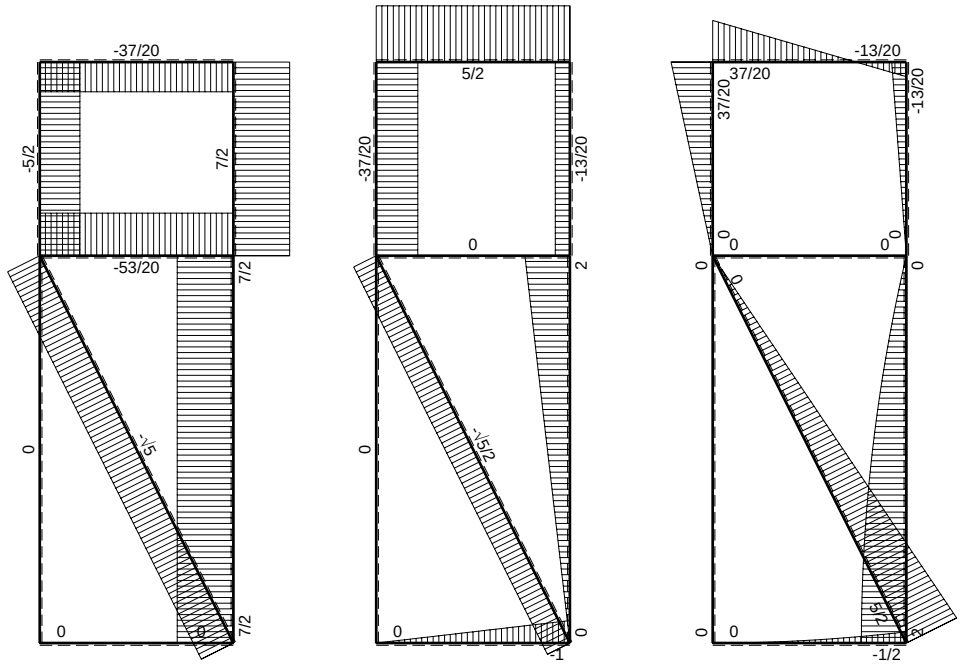


A = 570. mm^2
J_x = 203061. mm^4
J_y = 67683. mm^4
J_{xy} = 79827. mm^4
J_u = 240034. mm^4
J_v = 30710. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4337
c = cosα = .9074
s = sinα = -.4203
x_g = 19.55 mm
y_g = 30.95 mm
N = -1046. N
T_y = -523.2 N
M_x = 819000. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -16.11 mm
v_m = 16.83 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -234.6 N/mm^2

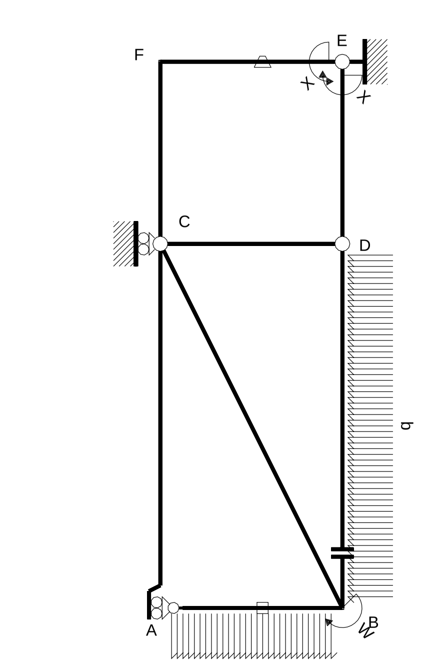
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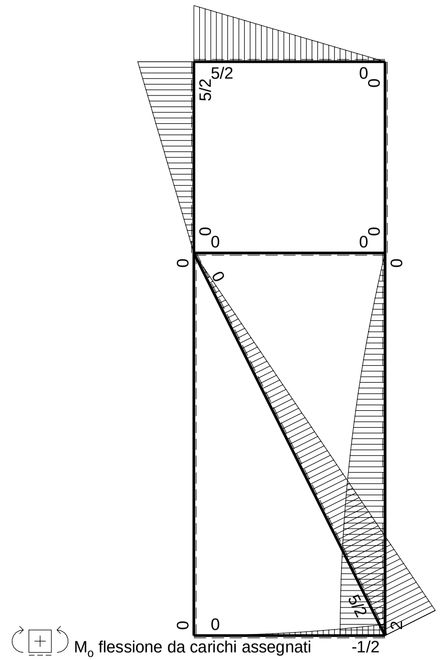

F
E
37/20F
37/20Fb
5/2F
13/20Fb
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
53/20F
2F
7/2F
5/2F
5/2Fb
2Fb
A
B
1/2Fb
F


-37/20
-5/2
7/2
-53/20
0
-5/2
5/2
-13/20
2
-1
37/20
-1/2

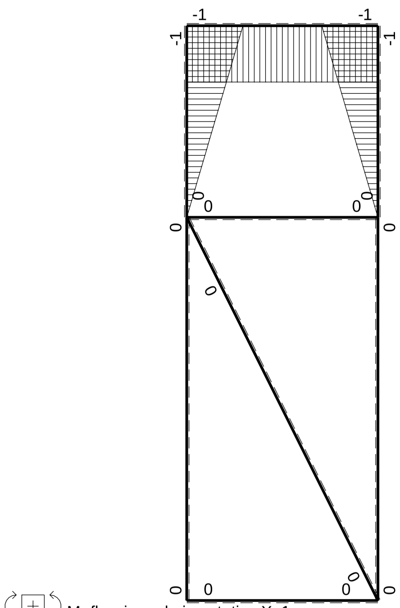

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	5/2Fb-√5/2Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

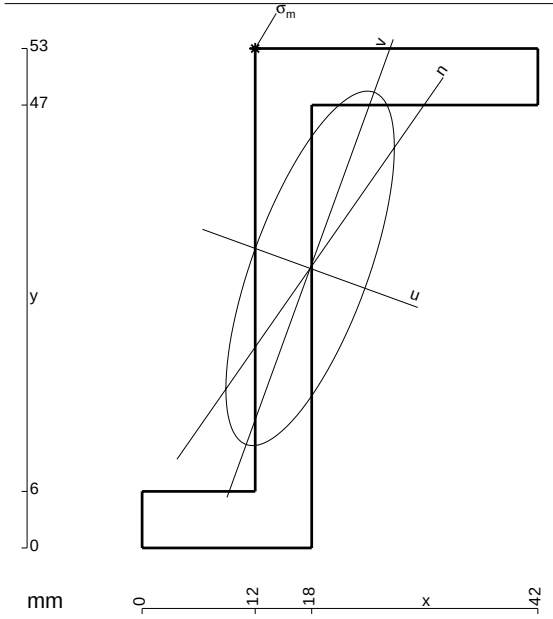
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

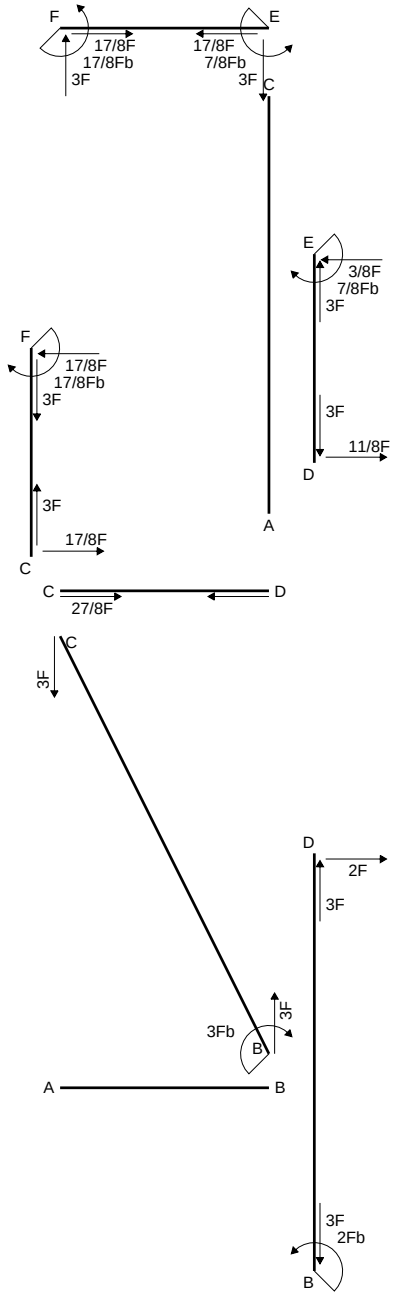
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

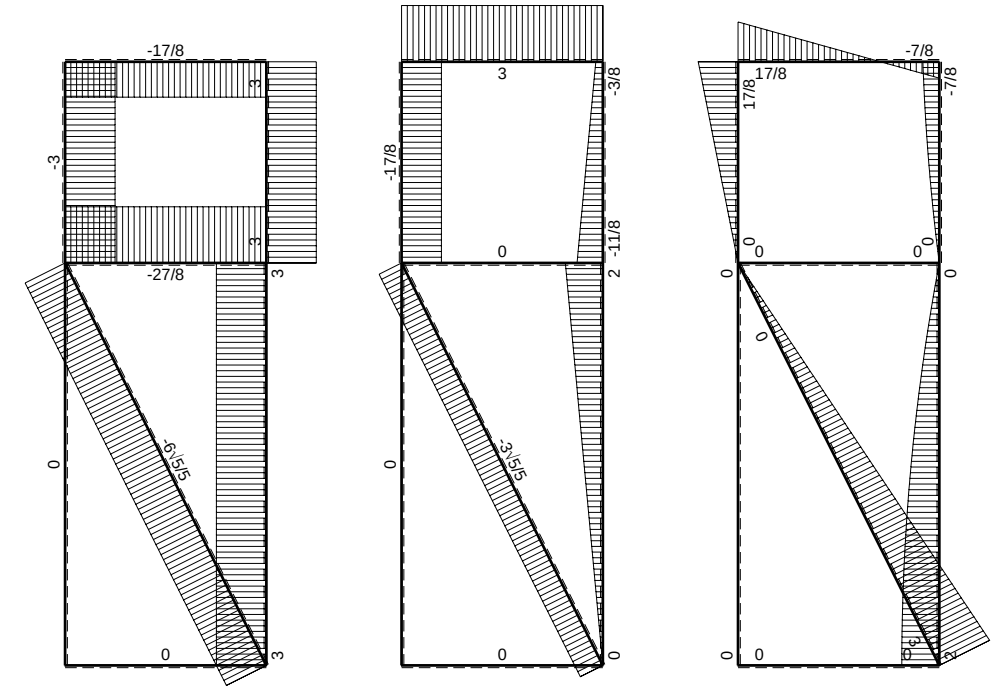


A = 534. mm^2
J_x = 189011. mm^4
J_y = 42681. mm^4
J_{xy} = 61197. mm^4
J_u = 211231. mm^4
J_v = 20461. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3483
$c = \cos\alpha$ = .9400
$s = \sin\alpha$ = -.3413
x_g = 17.83 mm
y_g = 29.67 mm
N = -1521. N
T_y = -760.3 N
M_x = 629000. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -13.44 mm
v_m = 19.94 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -199.7 N/mm^2

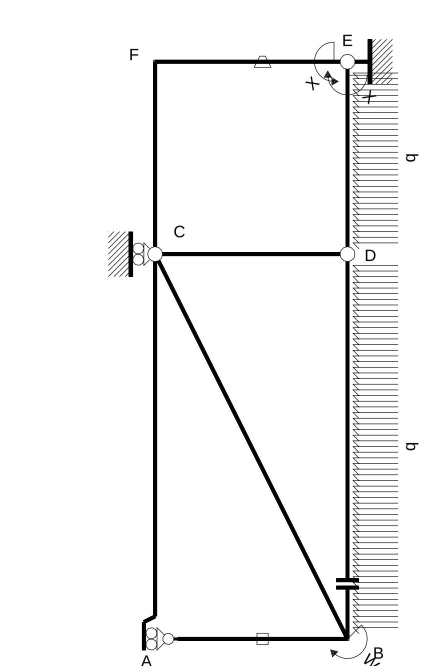
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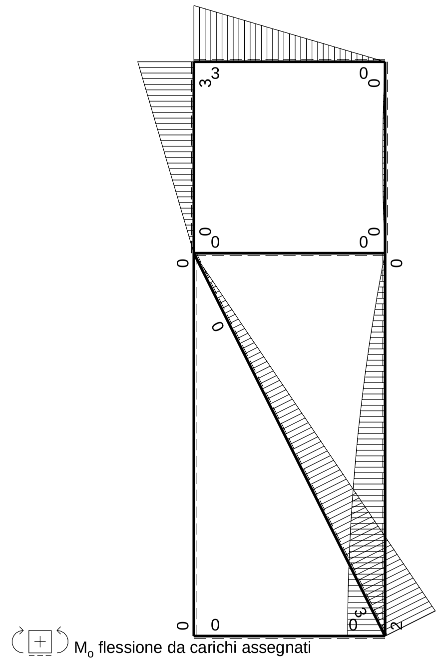

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
C
E
3/8F
7/8Fb
3F
F
17/8F
17/8Fb
3F
3F
11/8F
D
3F
A
17/8F
C
C
D
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D
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3F
3Fb
3F
B
A
B
3F
2Fb
B

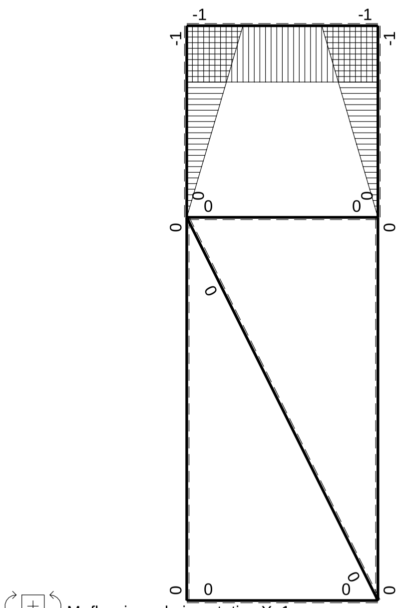

-17/8
3
-3
3
-27/8
3
0
-6√5/5
3
0
0
-17/8
3
-3/8
0
-11/8
2
-3√5/5
0
0
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
3√5/5


F
F
Fb



Schema di calcolo iperstatico


M_o flessione da carichi assegnati


M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 268787 CI09.xxxx.131

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
totali							$-35/24Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							7/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

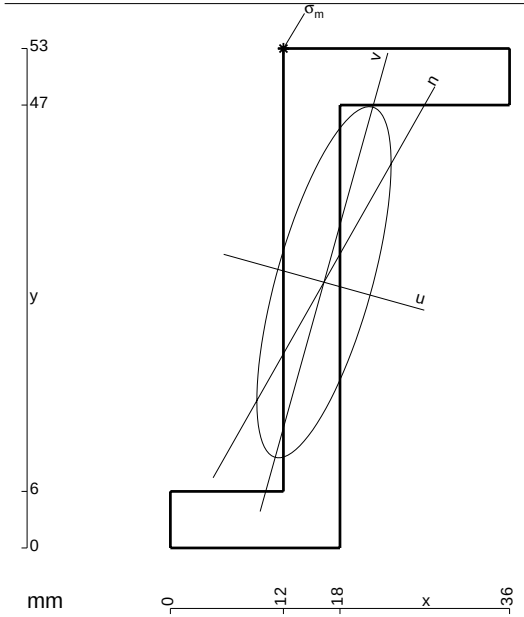
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

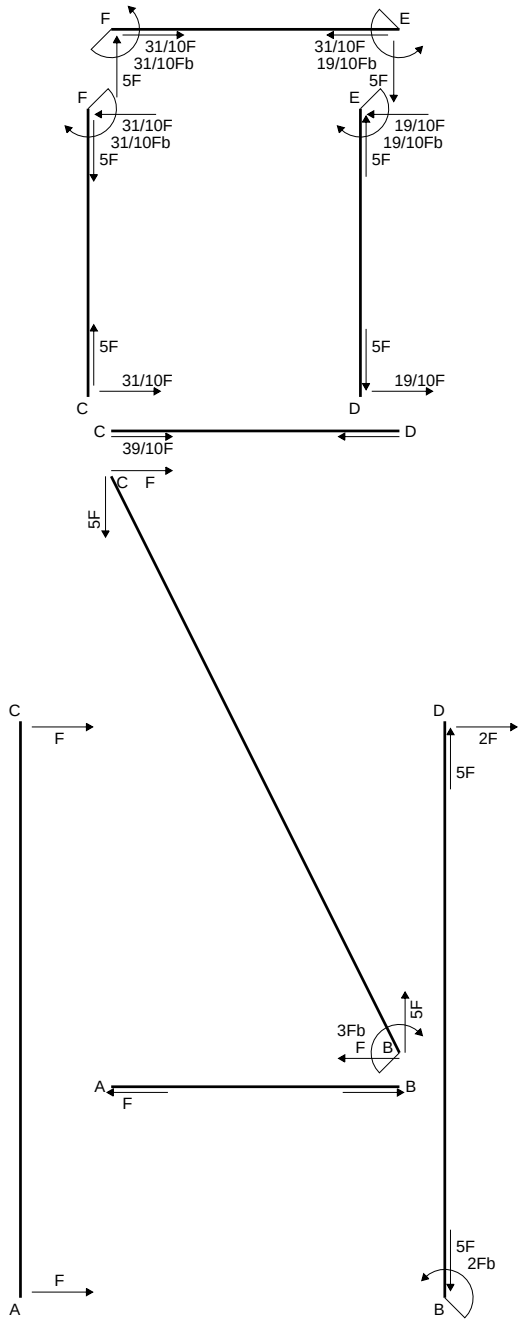
$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

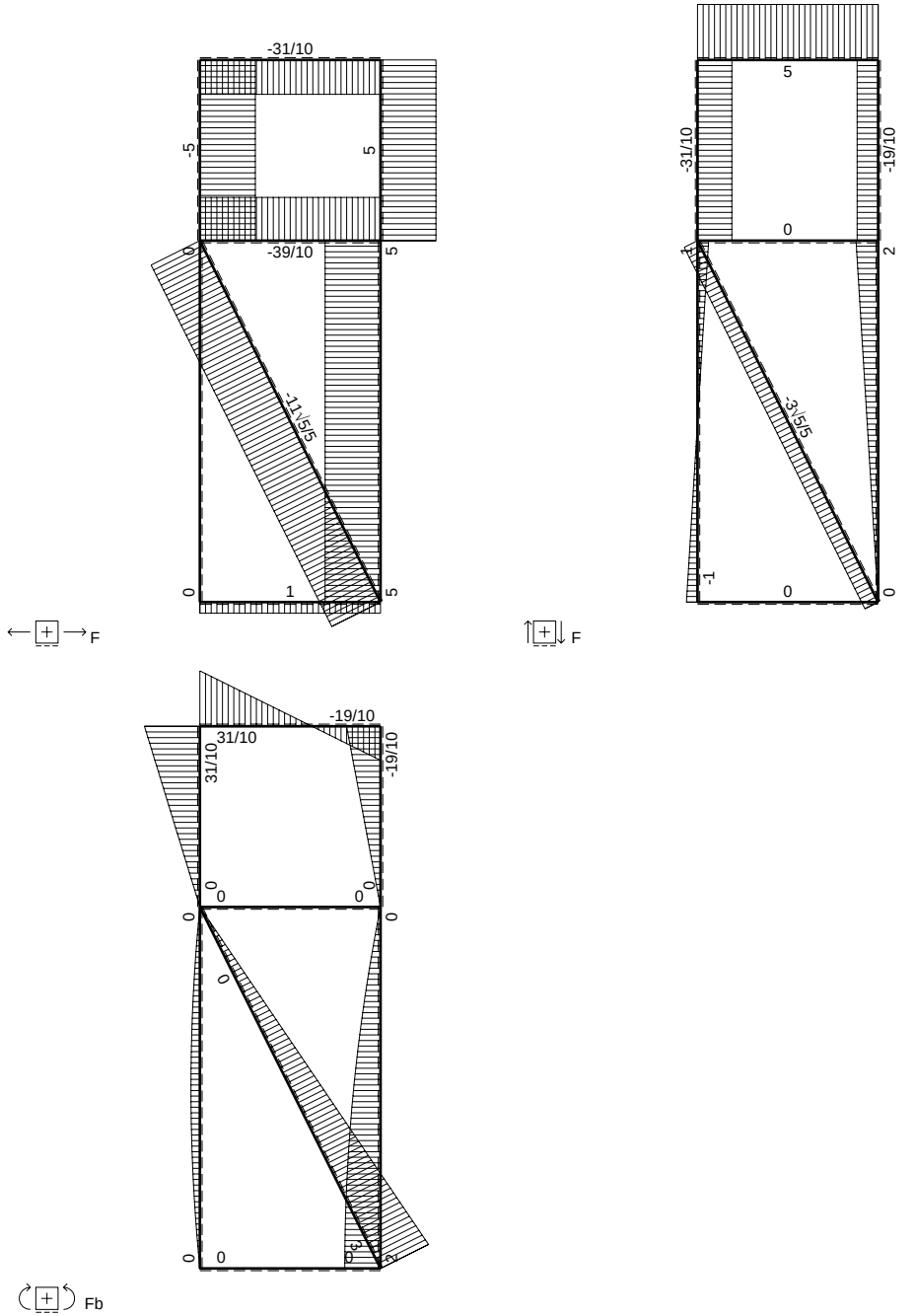


A = 498. mm^2
J_x = 172946. mm^4
J_y = 25275. mm^4
J_{xy} = 44583. mm^4
J_u = 185362. mm^4
J_v = 12859. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2716
c = cosα = .9633
s = sinα = -.2683
x_g = 16.3 mm
y_g = 28.2 mm
N = -1342. N
T_y = -670.8 N
M_x = 600000. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -10.8 mm
v_m = 22.74 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -208.8 N/mm^2

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F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
19/10Fb
C
D
39/10F
2F
3Fb
2Fb
A
B


-31/10
-39/10
-11√5/5
-3√5/5
-19/10
31/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
totali							-19/6Fb^2/EJ	5/3Xb/EJ
iperstatica $X=W_{EF}$							19/10Fb	

Sviluppi di calcolo iperstatica

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PROCEDIMENTO E RISULTATI 242068 CI09.xxxx.132

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

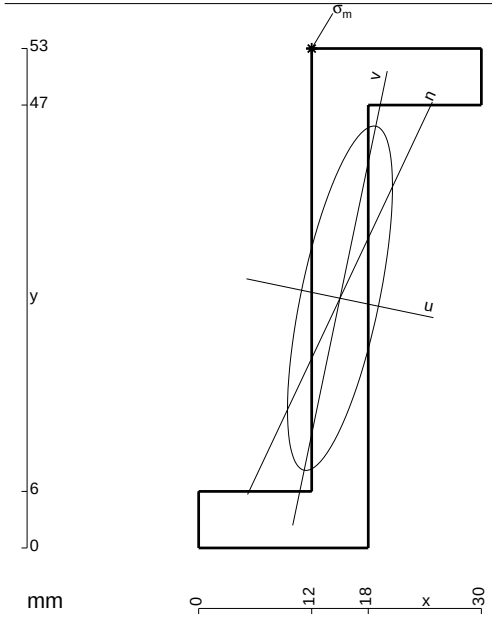
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

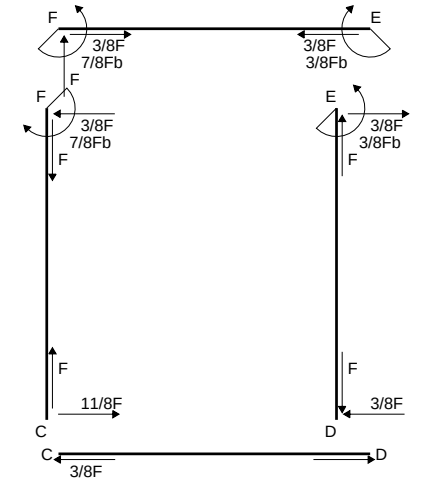
$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

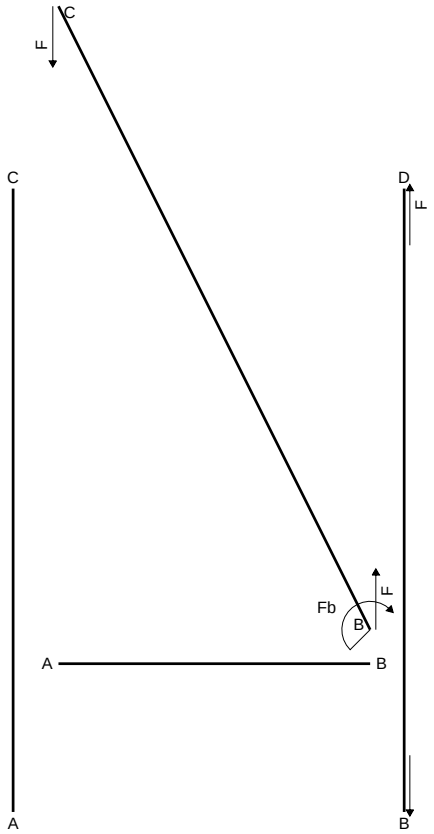


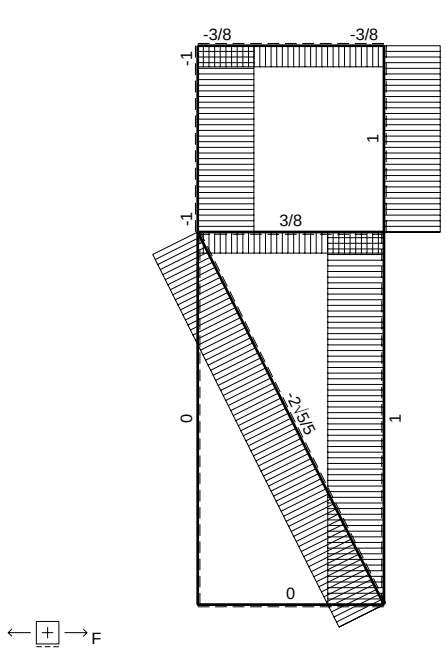
A = 462. mm^2
J_x = 154395. mm^4
J_y = 14346. mm^4
J_{xy} = 30456. mm^4
J_u = 160731. mm^4
J_v = 8010. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2051
c = cosα = .9790
s = sinα = -.2037
x_g = 15. mm
y_g = 26.5 mm
N = -2150. N
T_y = -1333. N
M_x = 586520. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -8.335 mm
v_m = 25.33 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.5 N/mm^2

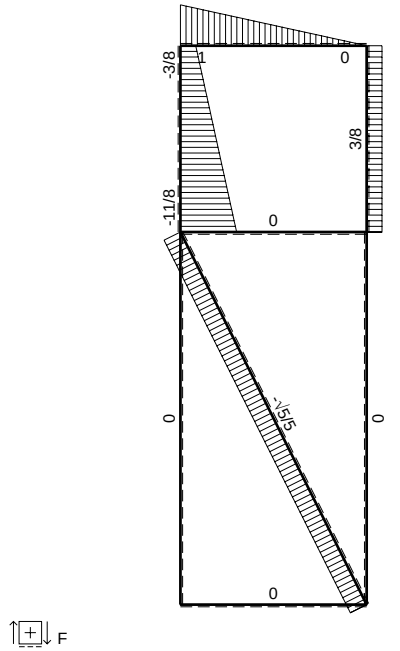
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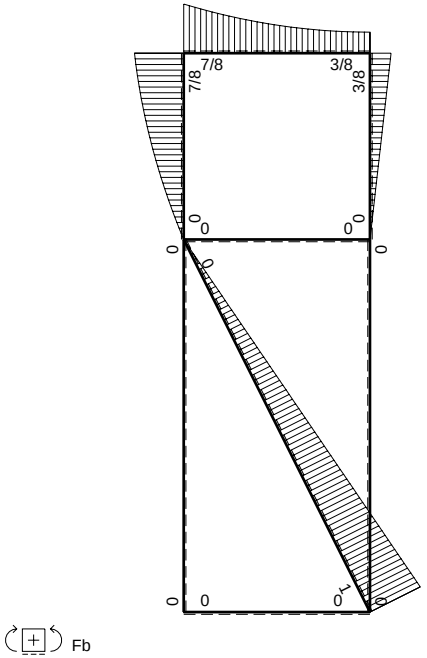
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F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

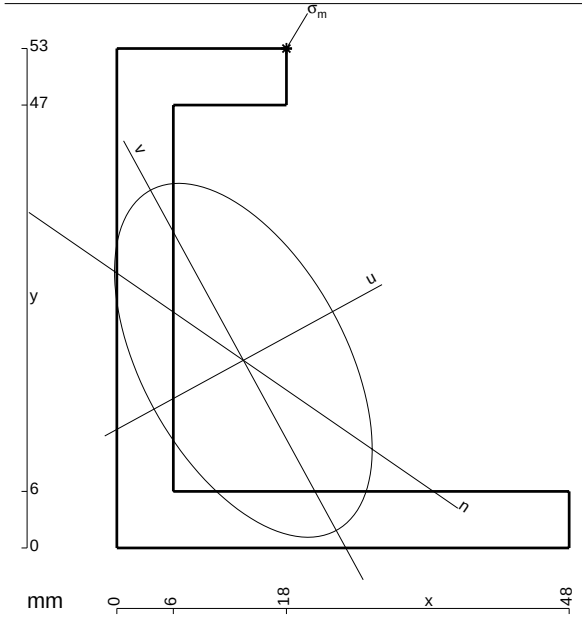
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 269676 CI09.xxxx.133

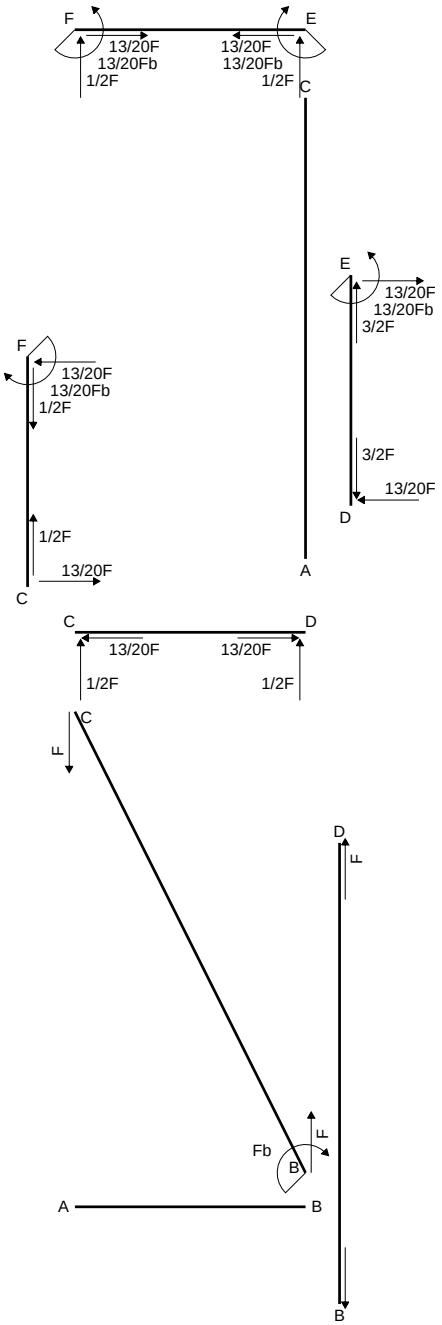


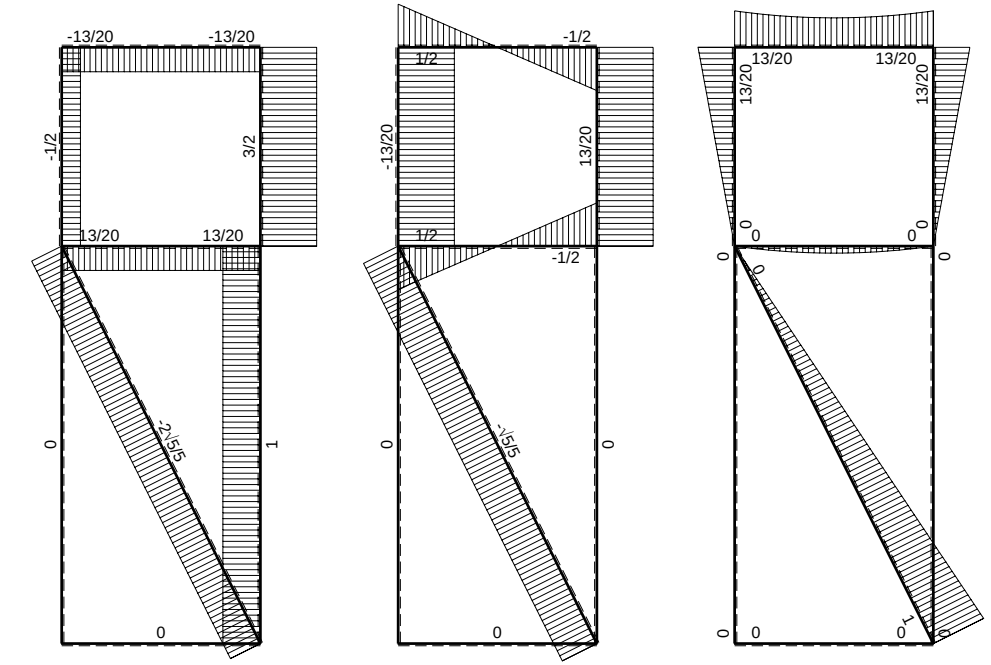
A = 642. mm^2
J_x = 226469. mm^4
J_y = 120007. mm^4
J_{xy} = -82782. mm^4
J_u = 271657. mm^4
J_v = 74819. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4997
c = cosα = .8777
s = sinα = .4791
x_g = 13.43 mm
y_g = 19.91 mm
N = -1968. N
T_y = -983.9 N
M_x = 1056000. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 19.87 mm
v_m = 26.85 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -229. N/mm^2

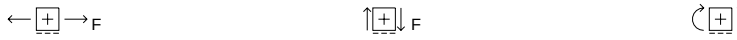
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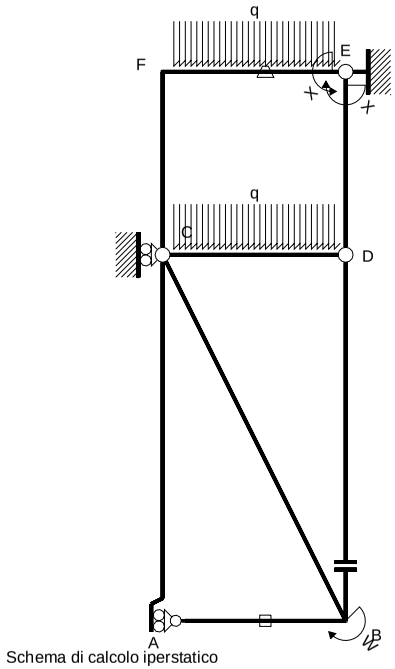
CI09.xxxx.133

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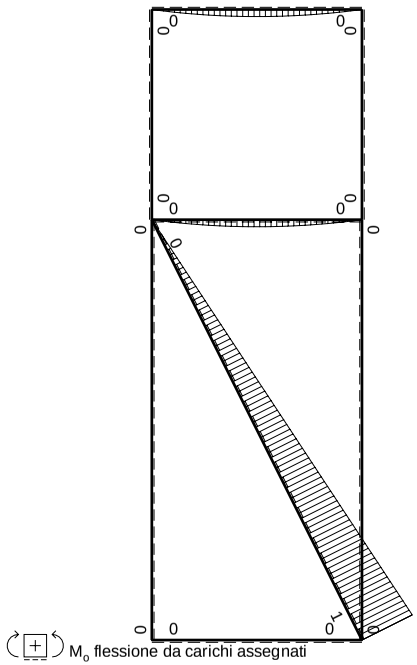

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-√5/5

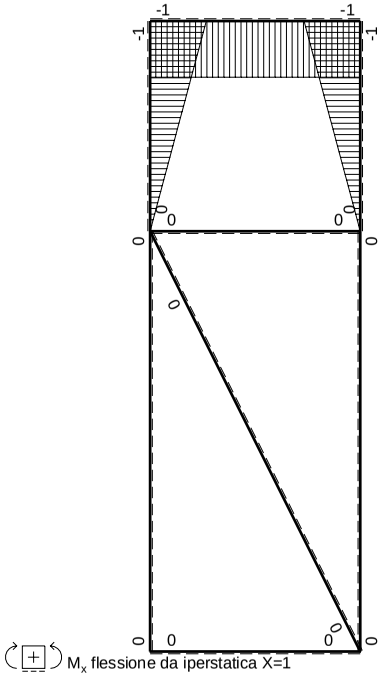

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264909 CI09.xxxx.134

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

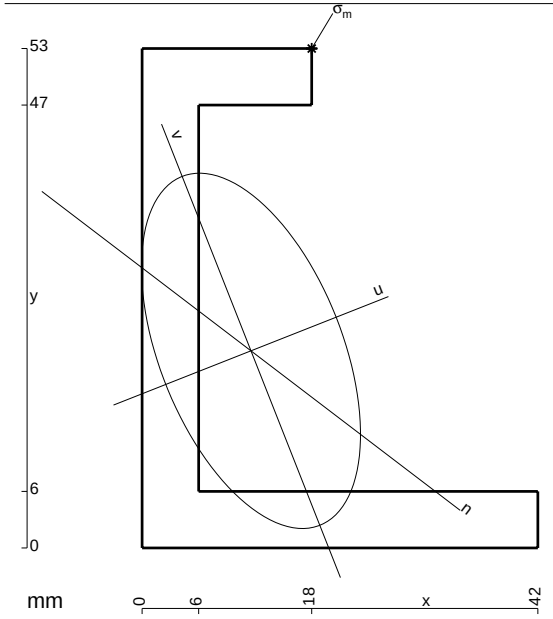
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

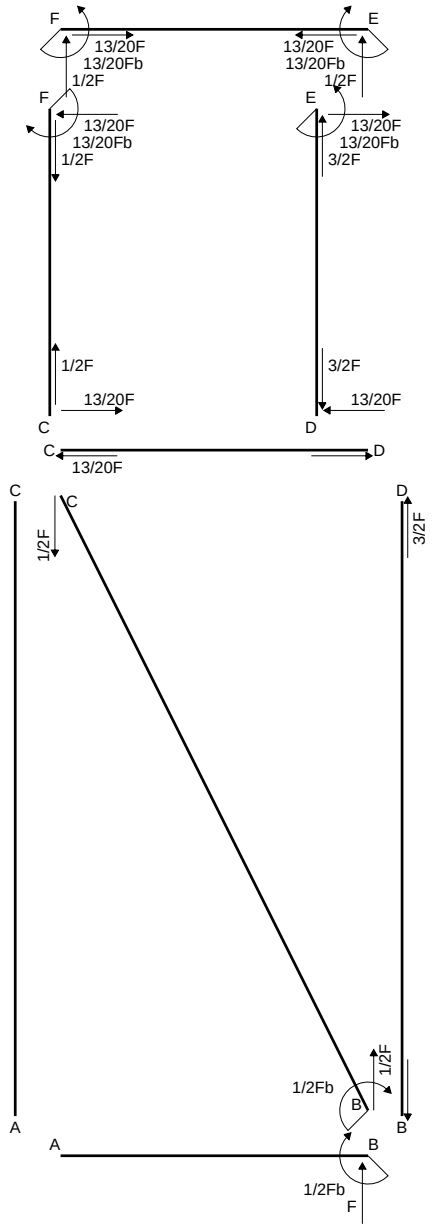
PROCEDIMENTO E RISULTATI 264909 CI09.xxxx.134

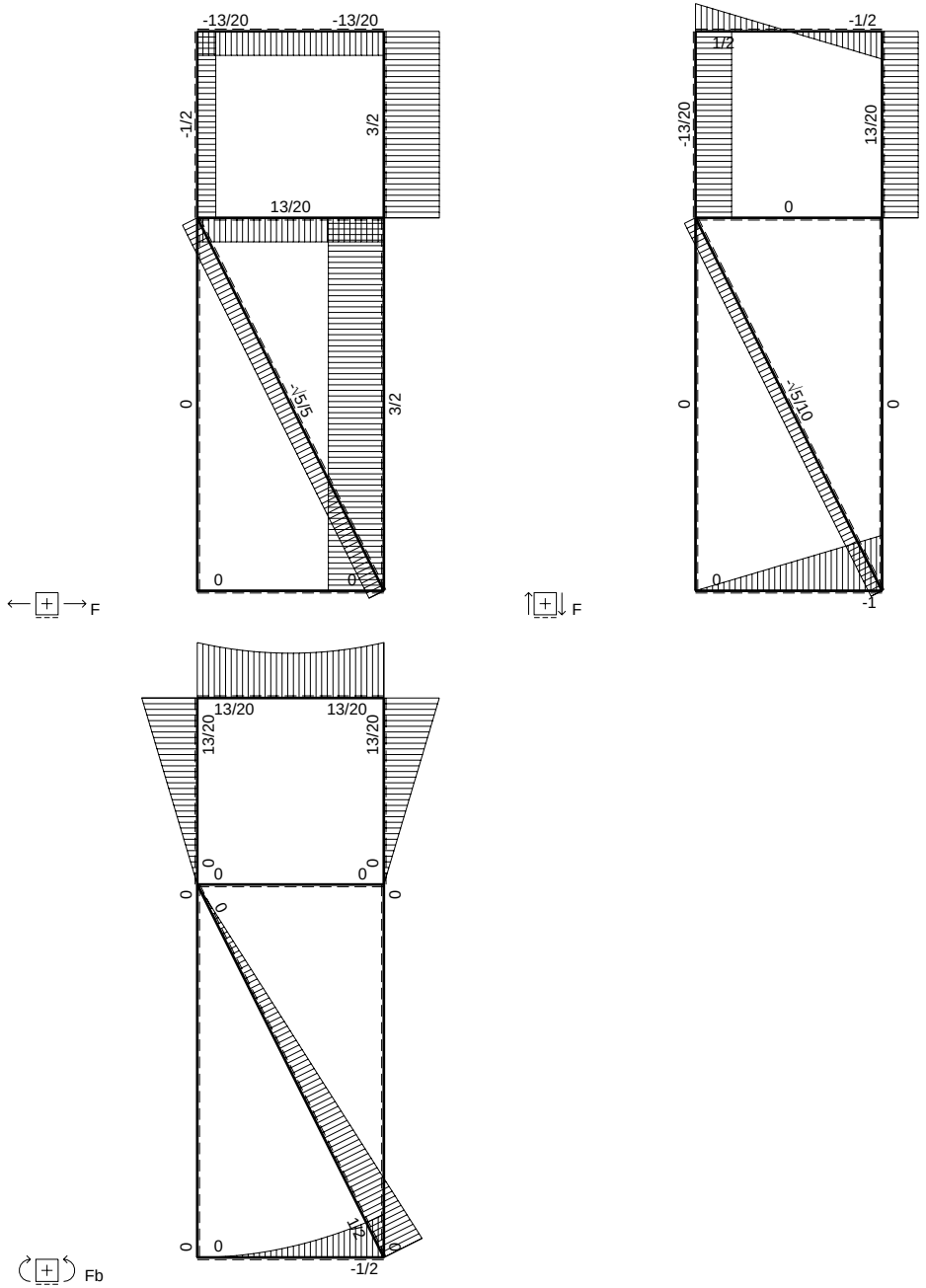


A = 606. mm^2
J_x = 215454. mm^4
J_y = 81888. mm^4
J_{xy} = -62420. mm^4
J_u = 240083. mm^4
J_v = 57259. mm^4
$\alpha = artg(2J_{xy}/(J_y - J_x))/2$ = .3758
$c = \cos\alpha$ = .9302
$s = \sin\alpha$ = .3670
x_g = 11.55 mm
y_g = 20.92 mm
N = -1878. N
T_y = -939.1 N
M_x = 1071000. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 17.77 mm
v_m = 27.48 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -239.1 N/mm^2

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi \theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

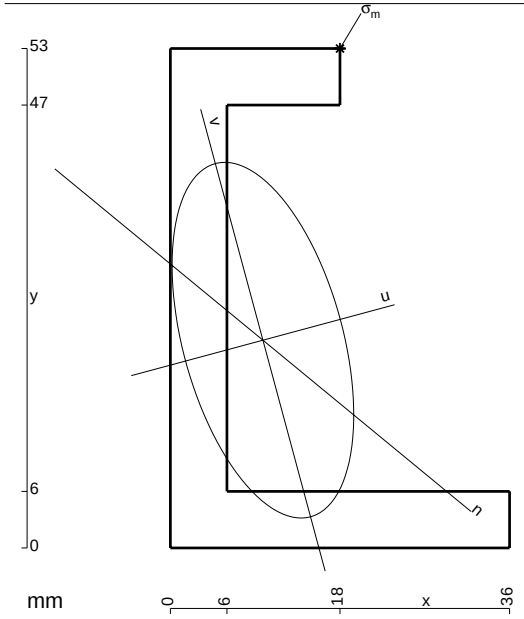
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

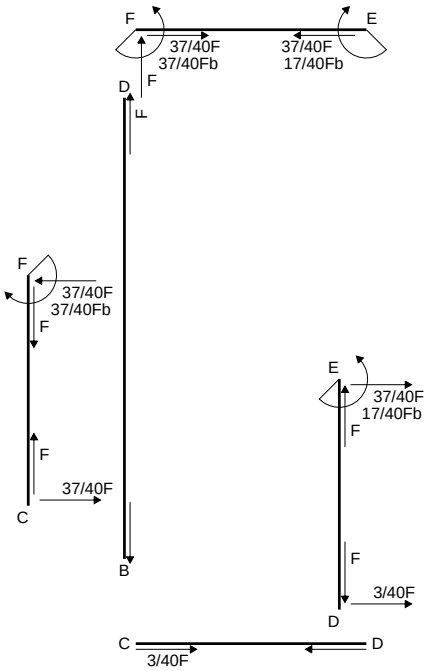
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = 13/12\ Fb^2/EJ$

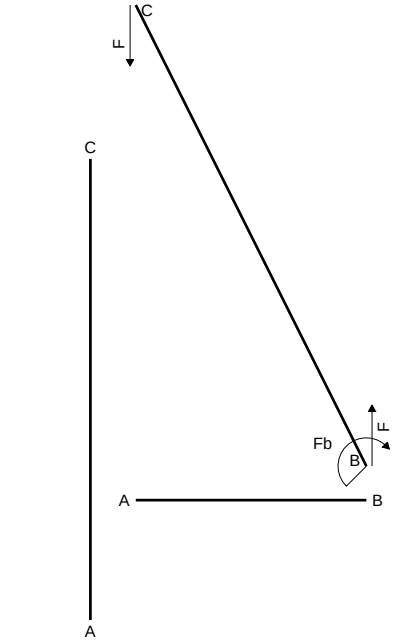


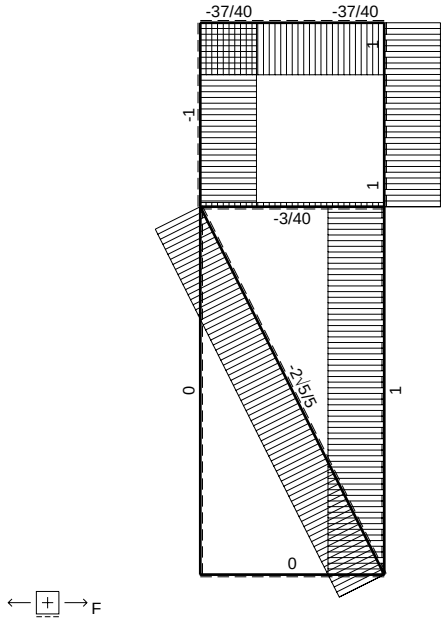
A = 570. mm^2
J_x = 203061. mm^4
J_y = 52950. mm^4
J_{xy} = -43600. mm^4
J_u = 214806. mm^4
J_v = 41205. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2631
c = cosα = .9656
s = sinα = .2601
x_g = 9.821 mm
y_g = 22.05 mm
N = -1586. N
T_y = -1220. N
M_x = 872300. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 15.95 mm
v_m = 27.76 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -199.4 N/mm^2

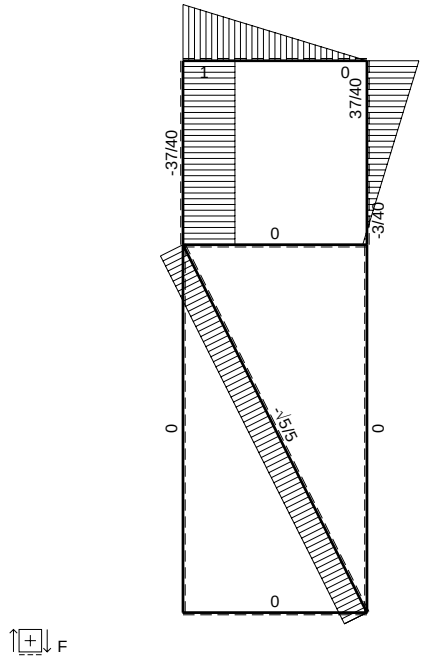
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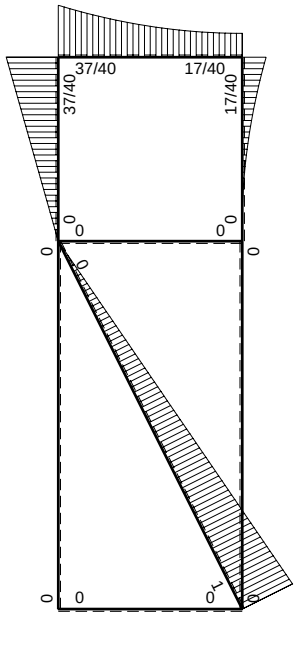
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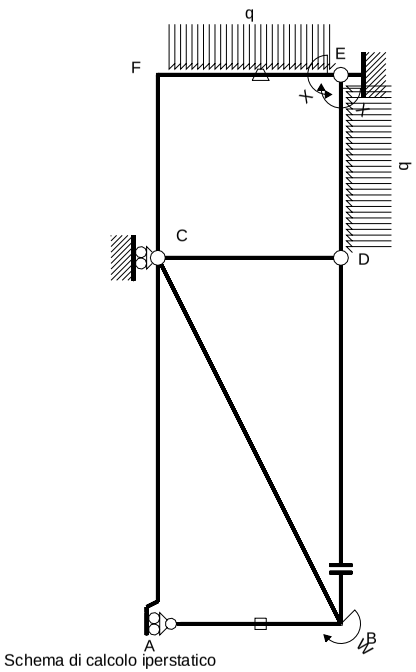

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
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37/40F
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
D
3/40F


C
F
C
Fb
F
B
A
B
A

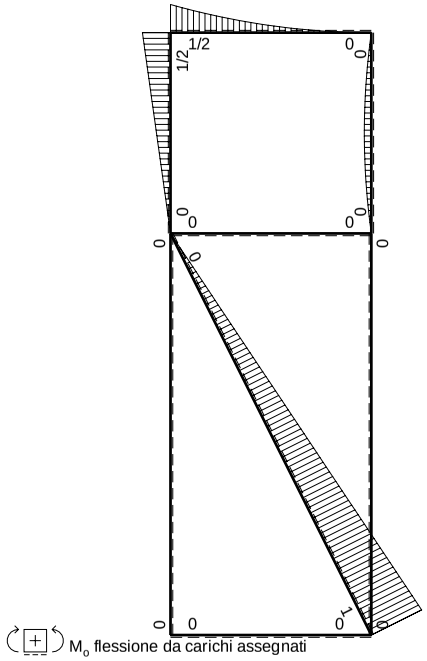

-37/40
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1
-1
1
-3/40
-2√5/5
0
1
0
1
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
0
F

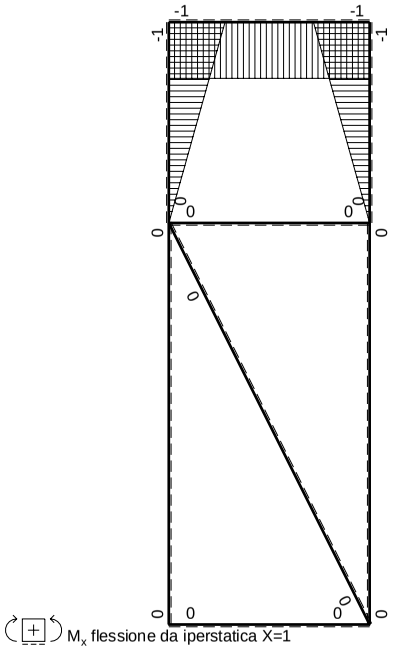

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ \ Fb^2/EJ$

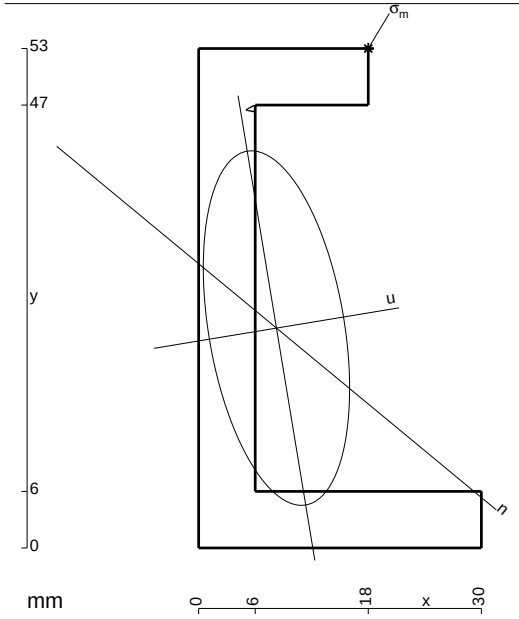
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 274782 CI09.xxxx.136

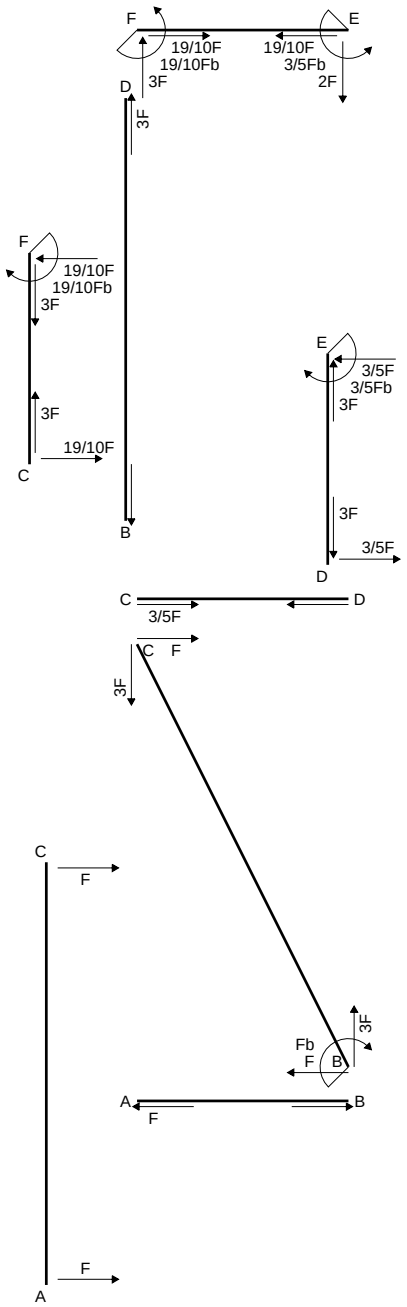


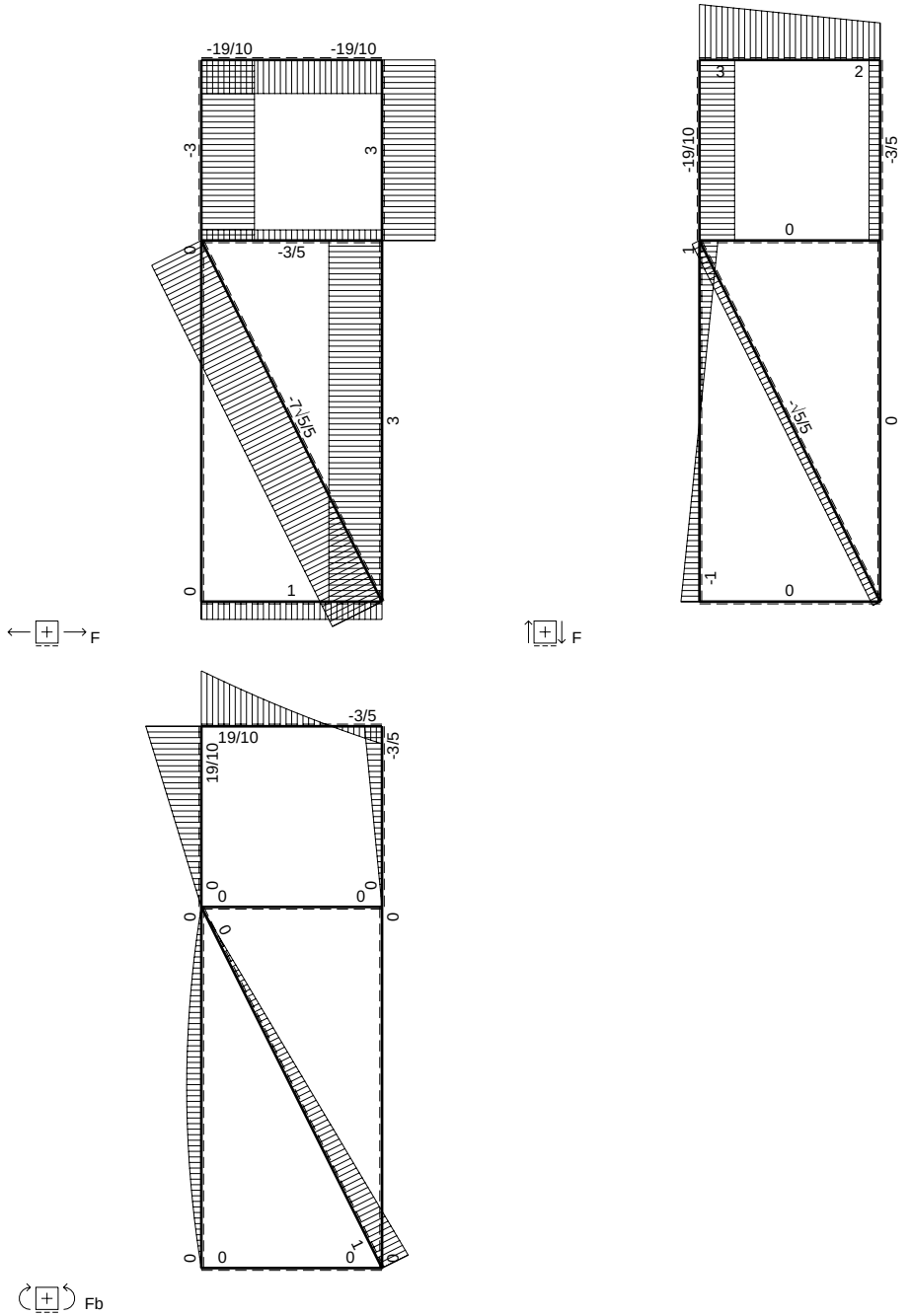
A = 534. mm^2
J_x = 189011. mm^4
J_y = 32196. mm^4
J_{xy} = -26635. mm^4
J_u = 193412. mm^4
J_v = 27796. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1637
$c = \cos\alpha$ = .9866
$s = \sin\alpha$ = .1630
x_g = 8.258 mm
y_g = 23.33 mm
N = -1386. N
T_y = -693.2 N
M_x = 914500. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 14.45 mm
v_m = 27.68 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.2 N/mm^2

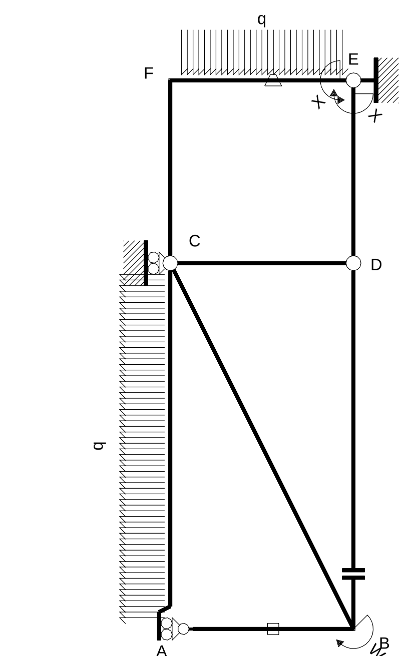
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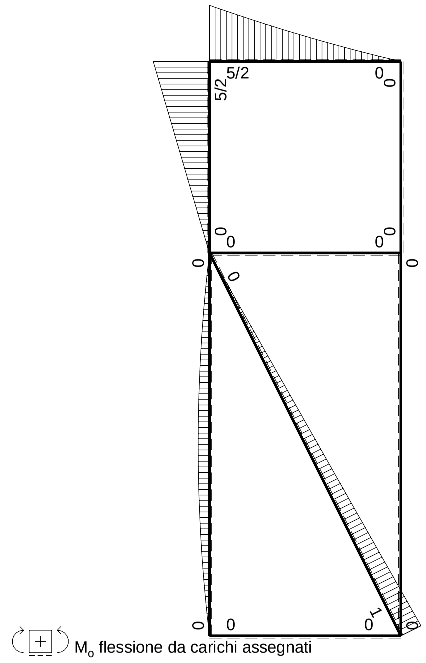
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F
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19/10Fb
19/10F
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3F
2F
D
3F
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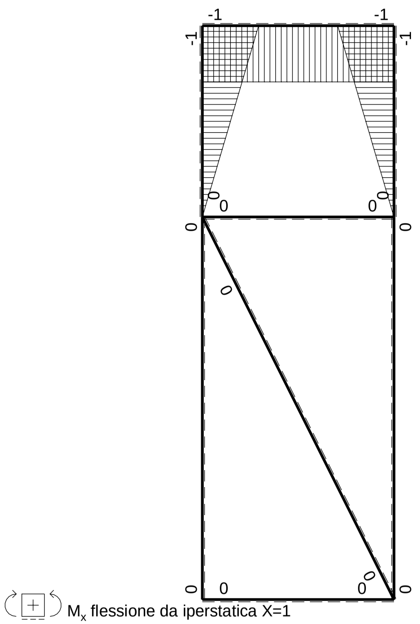

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0
1
-1\5/5
0
-1
0
F
-3/5
19/10
19/10
-3/5
0
0
0
0
0
0
0
0
0
1
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

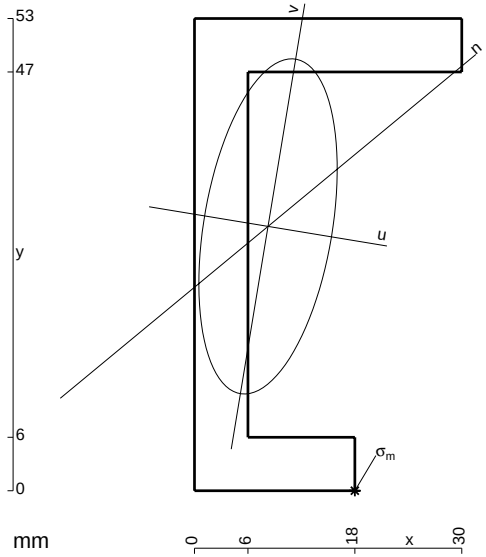
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

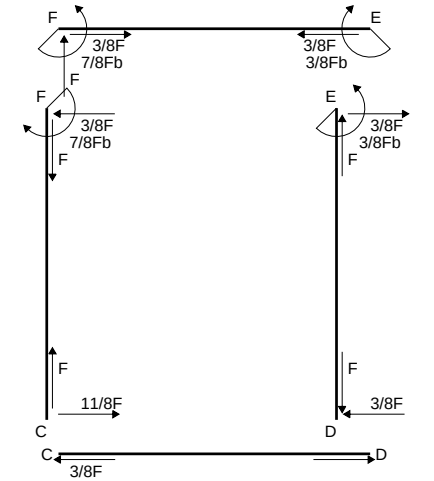
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

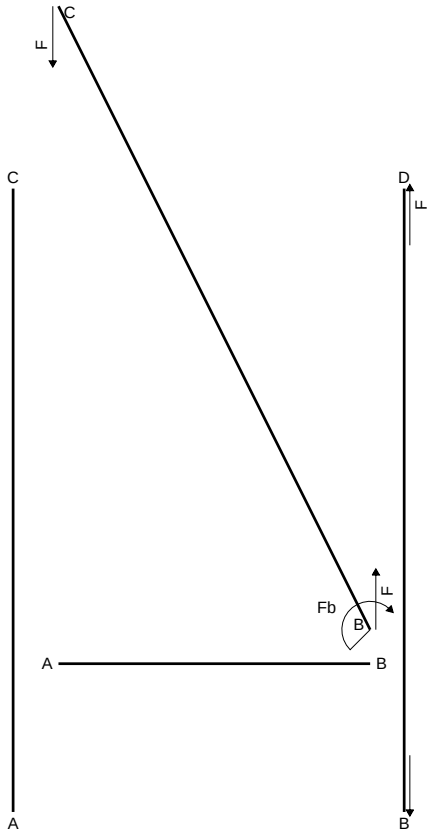


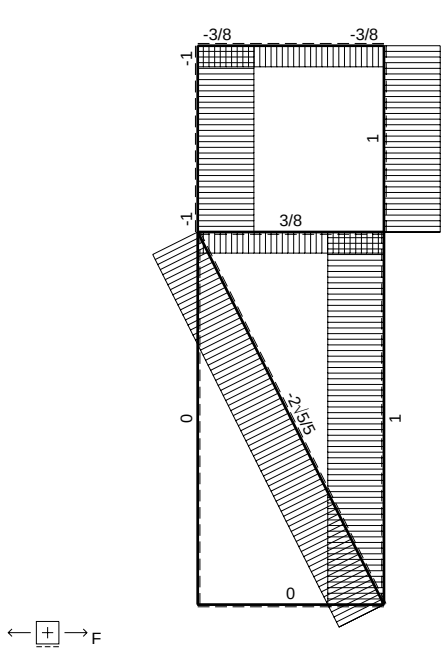
A = 534. mm^2
J_x = 189011. mm^4
J_y = 32196. mm^4
J_{xy} = 26635. mm^4
J_u = 193412. mm^4
J_v = 27796. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1637
$c = \cos\alpha$ = .9866
$s = \sin\alpha$ = -.1630
x_g = 8.258 mm
y_g = 29.67 mm
N = -1577. N
T_y = 2490. N
M_x = 977740. Nmm
x_m = 18. mm
u_m = 14.45 mm
v_m = -27.68 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 218. N/mm^2

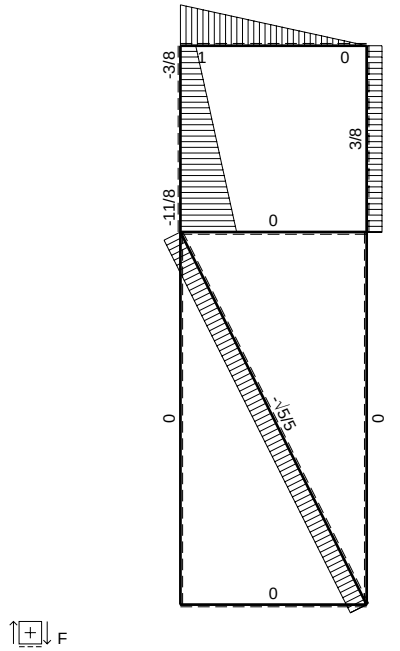
CI09.xxxx.137

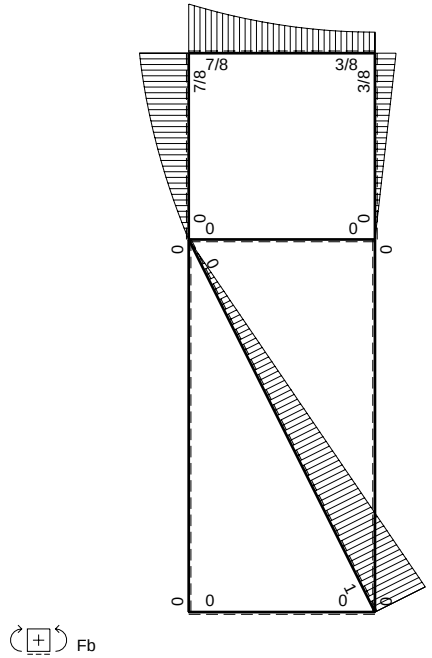
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

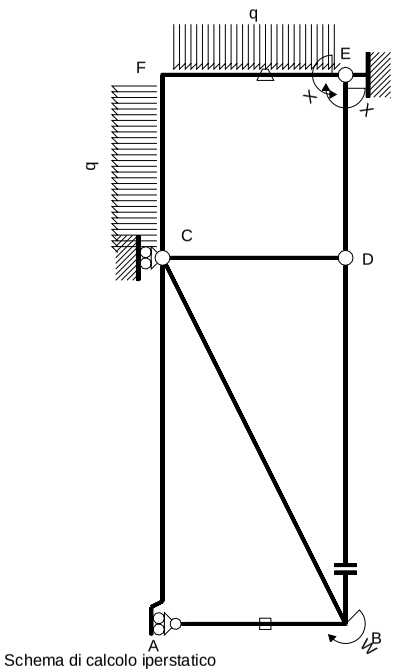

F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
F
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F
B
A
B
A
B

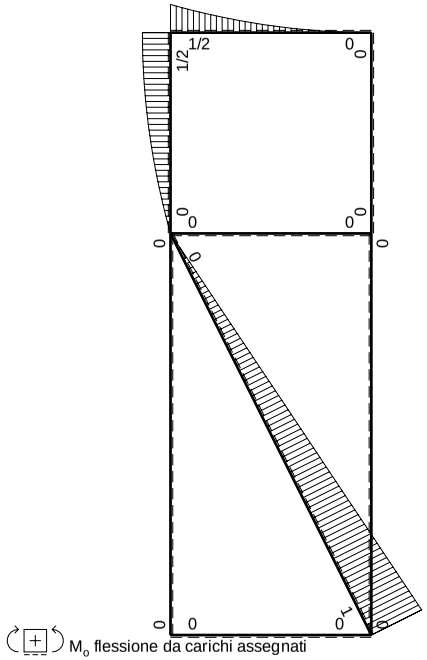

-3/8
-3/8
-1
-1
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3/8
0
-2√5/5
1
1
0
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
0
F

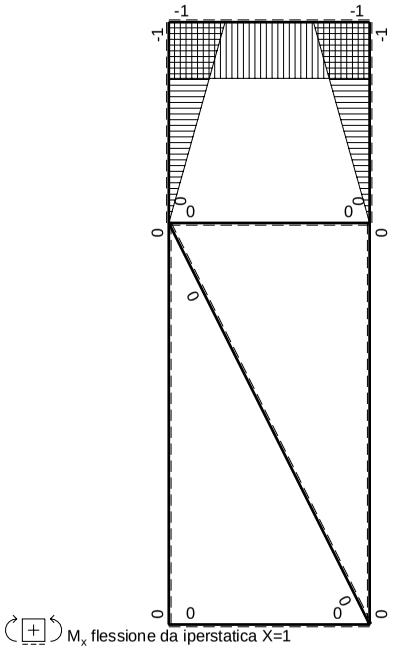

7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-$1/2qx^2$	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-$1/2qx^2$	0	-1/2Fb+1/2Fx+1/2Fx^2/b-$1/2qx^3$/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+$1/2qx^2$	0	-Fx^2/b+$1/2qx^3$/b	0	x^2/b^2		
		totali					5/8Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 310226 CI09.xxxx.138

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

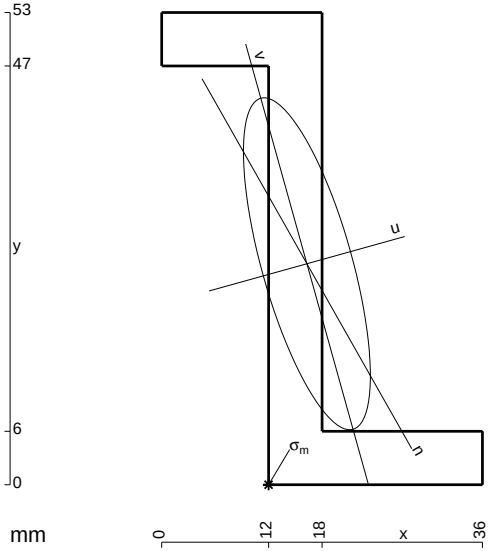
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 310226 CI09.xxxx.138

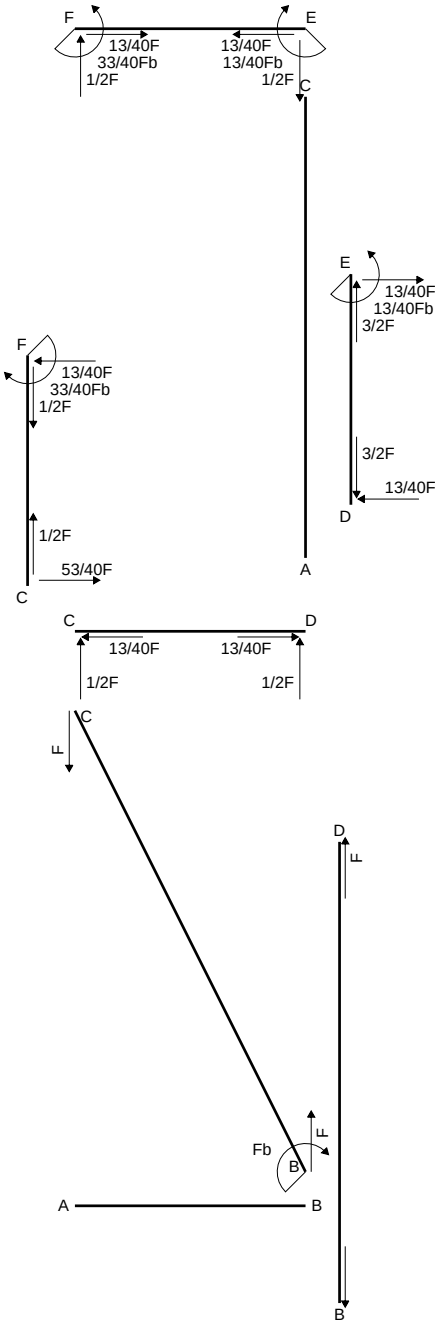


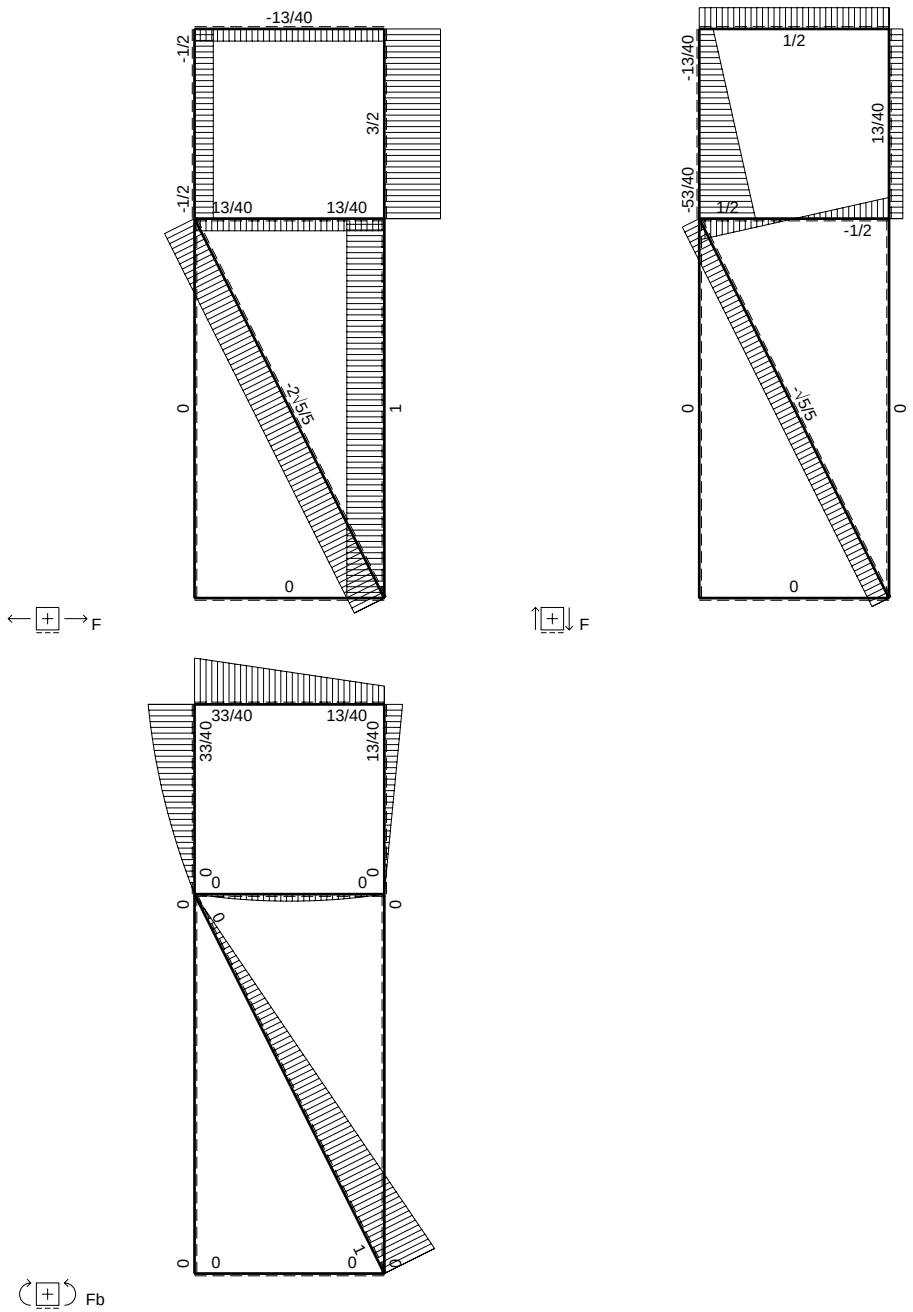
A = 498. mm^2
J_x = 172946. mm^4
J_y = 25275. mm^4
J_{xy} = -44583. mm^4
J_u = 185362. mm^4
J_v = 12859. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2716
c = cosα = .9633
s = sinα = .2683
x_g = 16.3 mm
y_g = 24.8 mm
N = -912.3 N
T_y = -456.2 N
M_x = 673200. Nmm
x_m = 12. mm
u_m = -10.8 mm
v_m = -22.74 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 229.4 N/mm^2

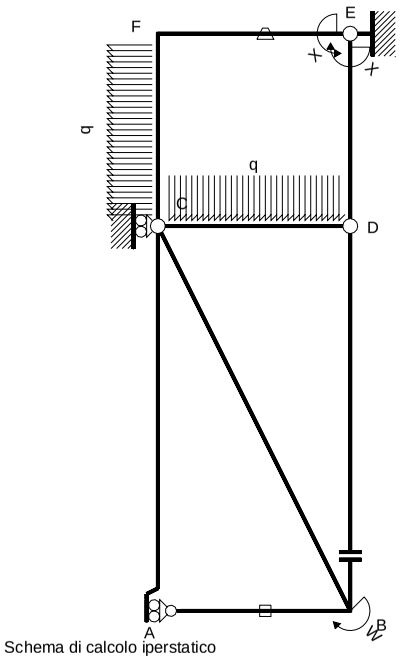
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CI09.xxxx.138

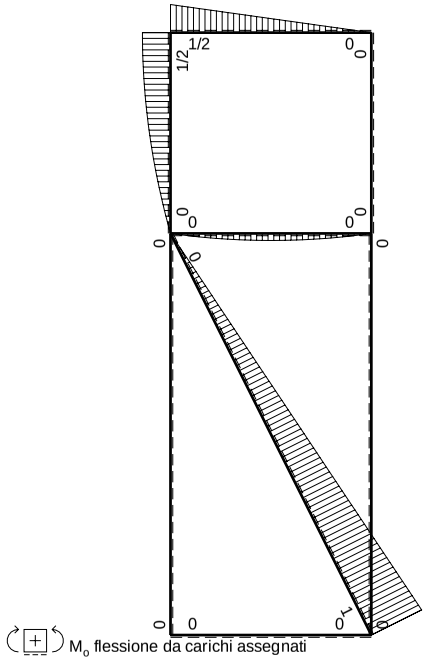
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
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Fb
B
F
A
B
B

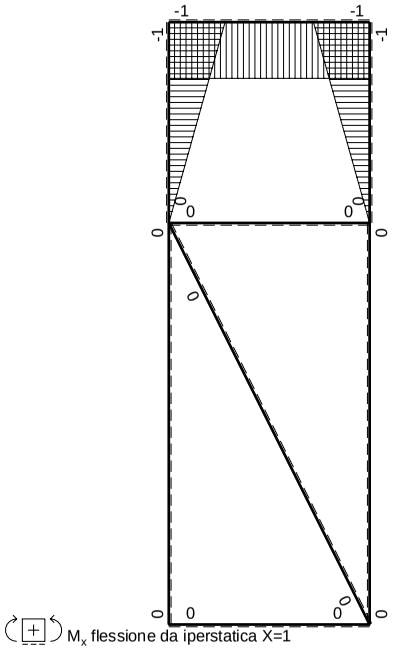

-13/40
-1/2
-1/2
3/2
13/40
13/40
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0
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 263941 CI09.xxxx.139

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

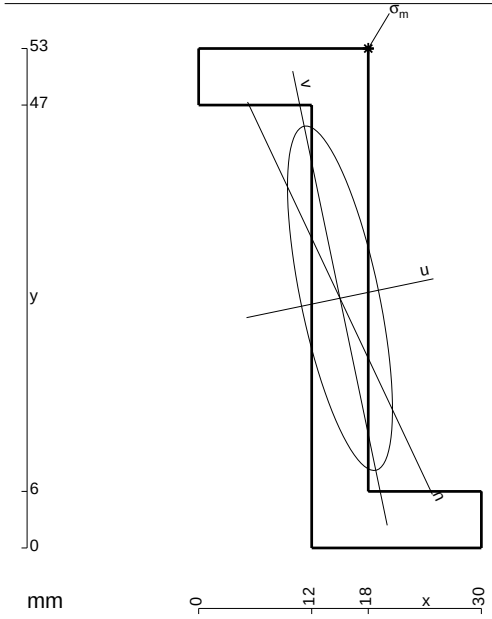
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 263941 CI09.xxxx.139

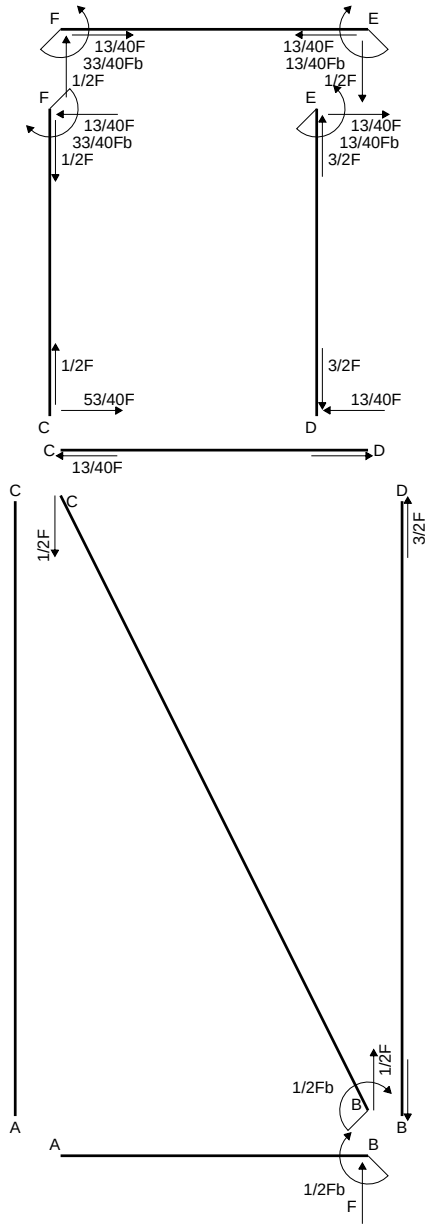


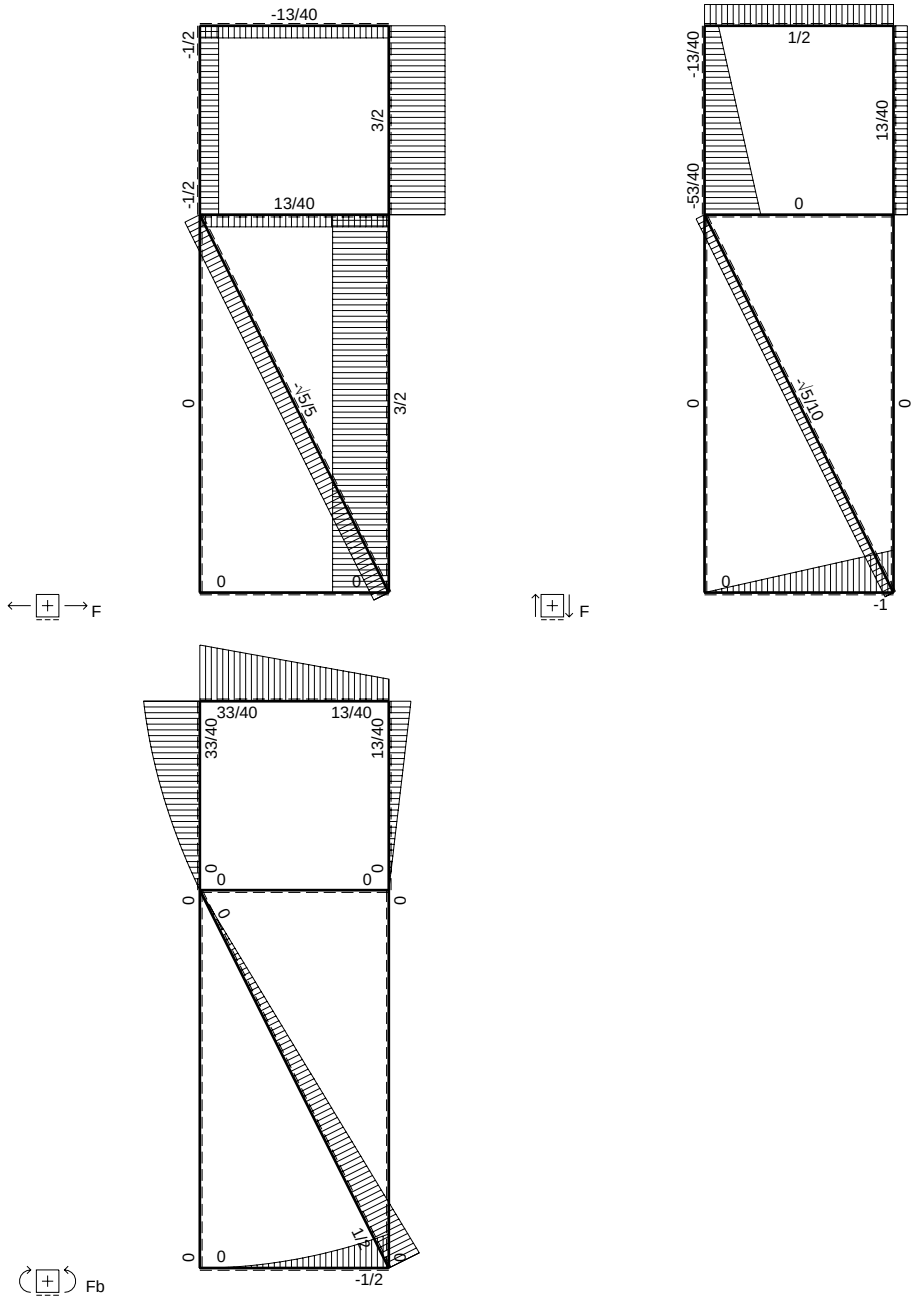
A = 462. mm^2
J_x = 154395. mm^4
J_y = 14346. mm^4
J_{xy} = -30456. mm^4
J_u = 160731. mm^4
J_v = 8010. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = .2051
$c = \cos\alpha$ = .9790
$s = \sin\alpha$ = .2037
x_g = 15. mm
y_g = 26.5 mm
N = -822.9 N
T_y = -411.4 N
M_x = 644000. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 8.335 mm
v_m = 25.33 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -237.7 N/mm^2

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
1/2F
3/2F
1/2Fb
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
0
0
-1/2
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

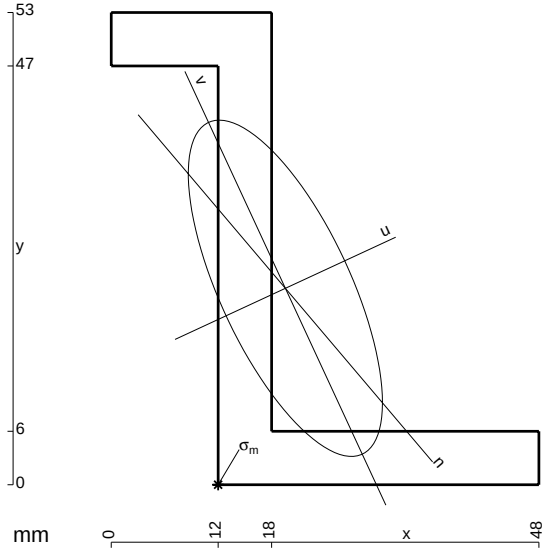
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

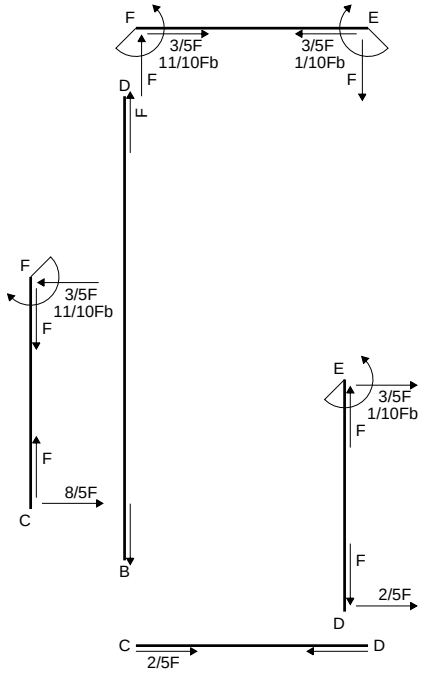
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

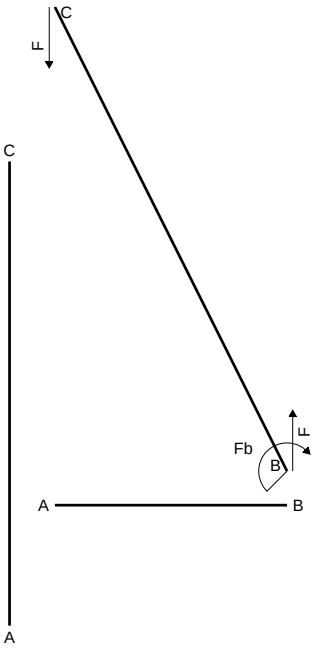


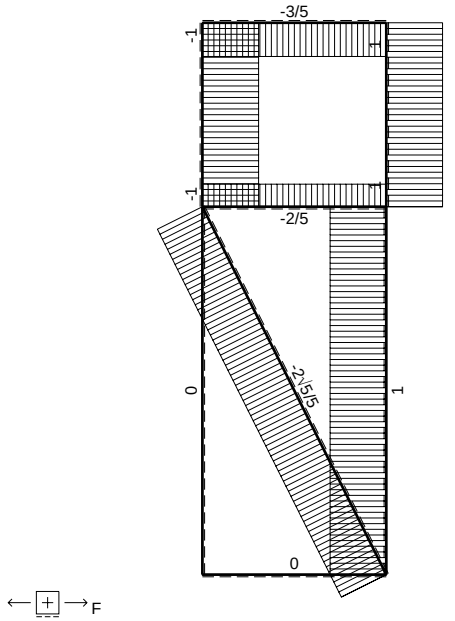
A = 570. mm^2
J_x = 203061. mm^4
J_y = 67683. mm^4
J_{xy} = -79827. mm^4
J_u = 240034. mm^4
J_v = 30710. mm^4
α= artg($2J_{xy}/(J_y-J_x)$)/2 = .4337
c = cosα = .9074
s = sinα = .4203
x_g = 19.55 mm
y_g = 22.05 mm
N = -754. N
T_y = 1160. N
M_x = 708180. Nmm
x_m = 12. mm
u_m = -16.11 mm
v_m = -16.83 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = 199.9 N/mm^2

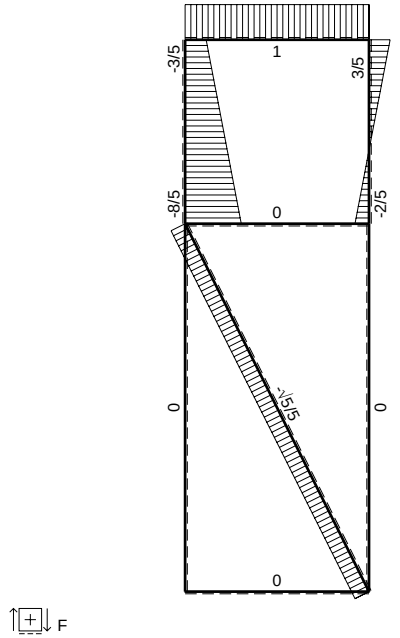
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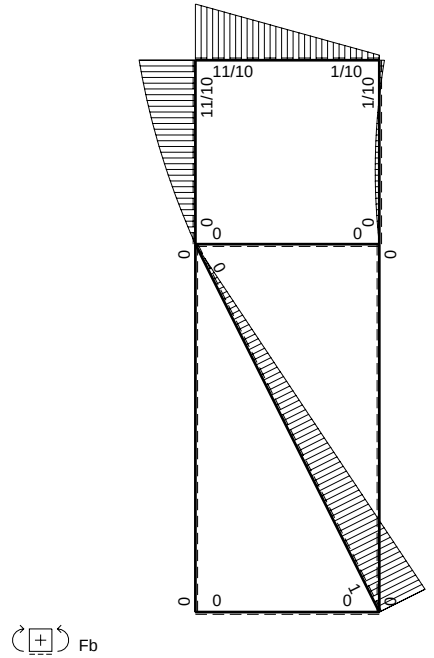
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F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5
0
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx^2	0	-Fb+3/2Fx-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-3/8+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3/2Fx+1/2qx^2	0	-3/2Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						1/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

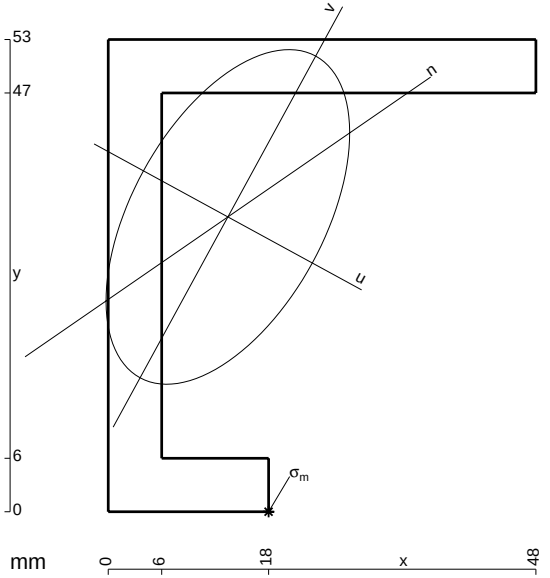
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

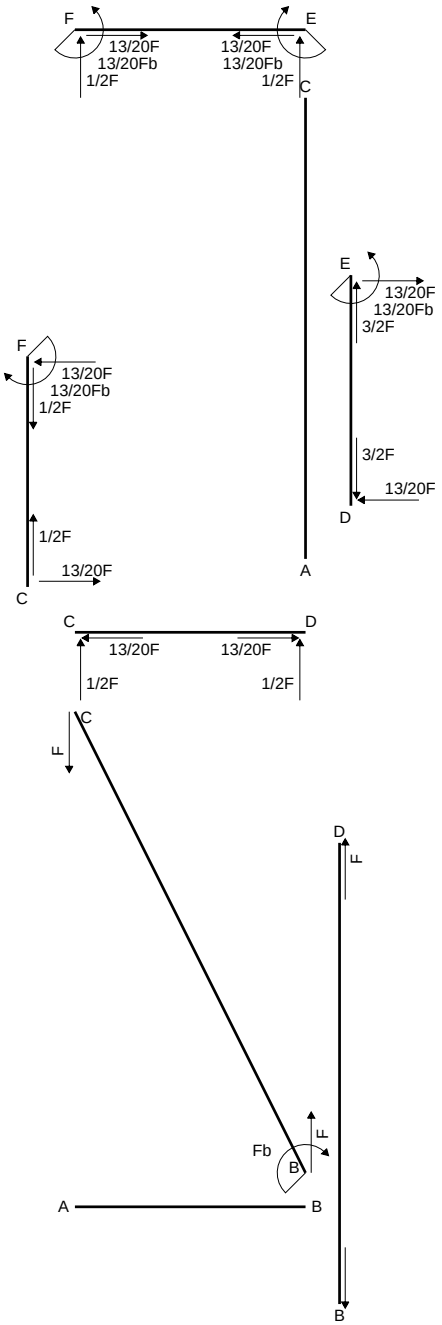
$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

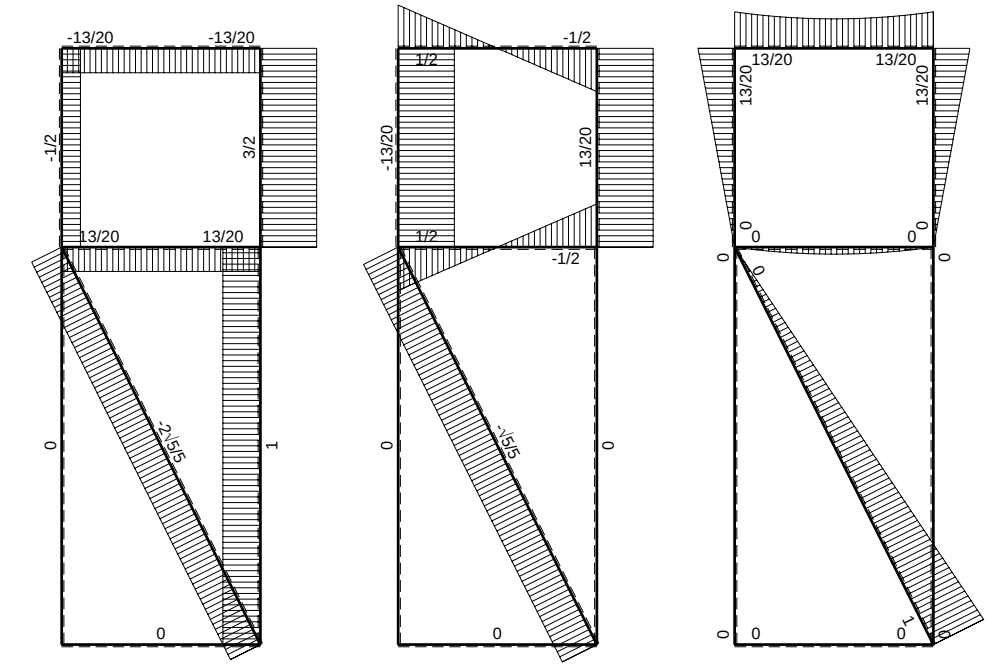


A = 642. mm^2
J_x = 226469. mm^4
J_y = 120007. mm^4
J_{xy} = 82782. mm^4
J_u = 271657. mm^4
J_v = 74819. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4997
$c = \cos\alpha$ = .8777
$s = \sin\alpha$ = -.4791
x_g = 13.43 mm
y_g = 33.09 mm
N = -1350. N
T_y = 2250. N
M_x = 990000. Nmm
x_m = 18. mm
u_m = 19.87 mm
v_m = -26.85 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 209.7 N/mm^2

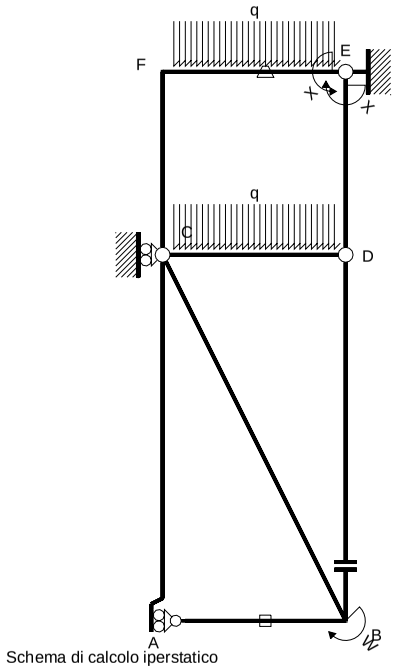
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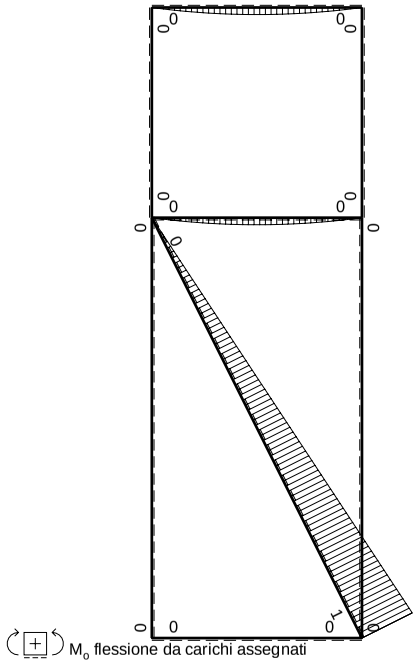

F
E
13/20F
13/20Fb
1/2F
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13/20Fb
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C
E
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0
0
13/20
13/20
0
0
0
0
0
0
1
0

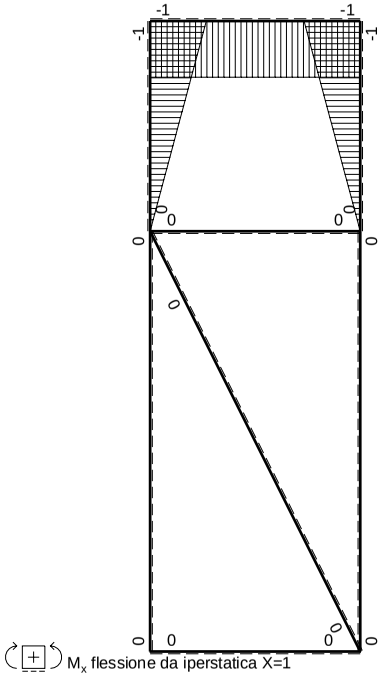

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 218087 CI09.xxxx.142

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

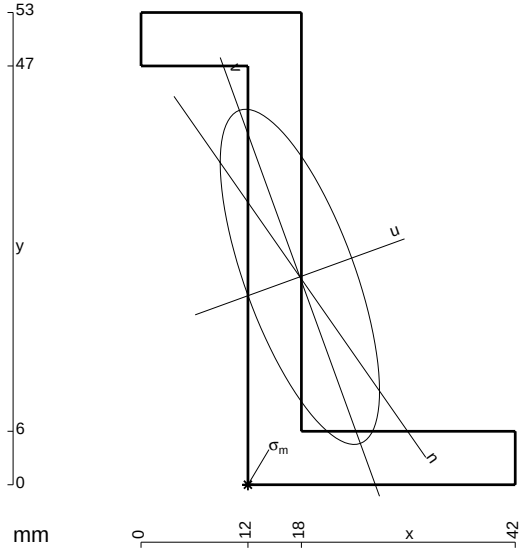
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

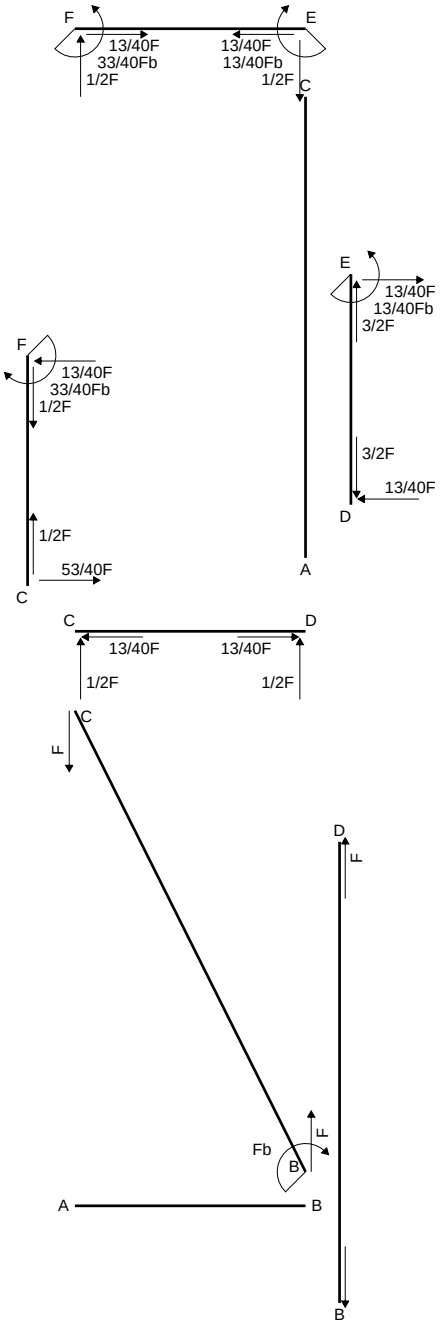
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

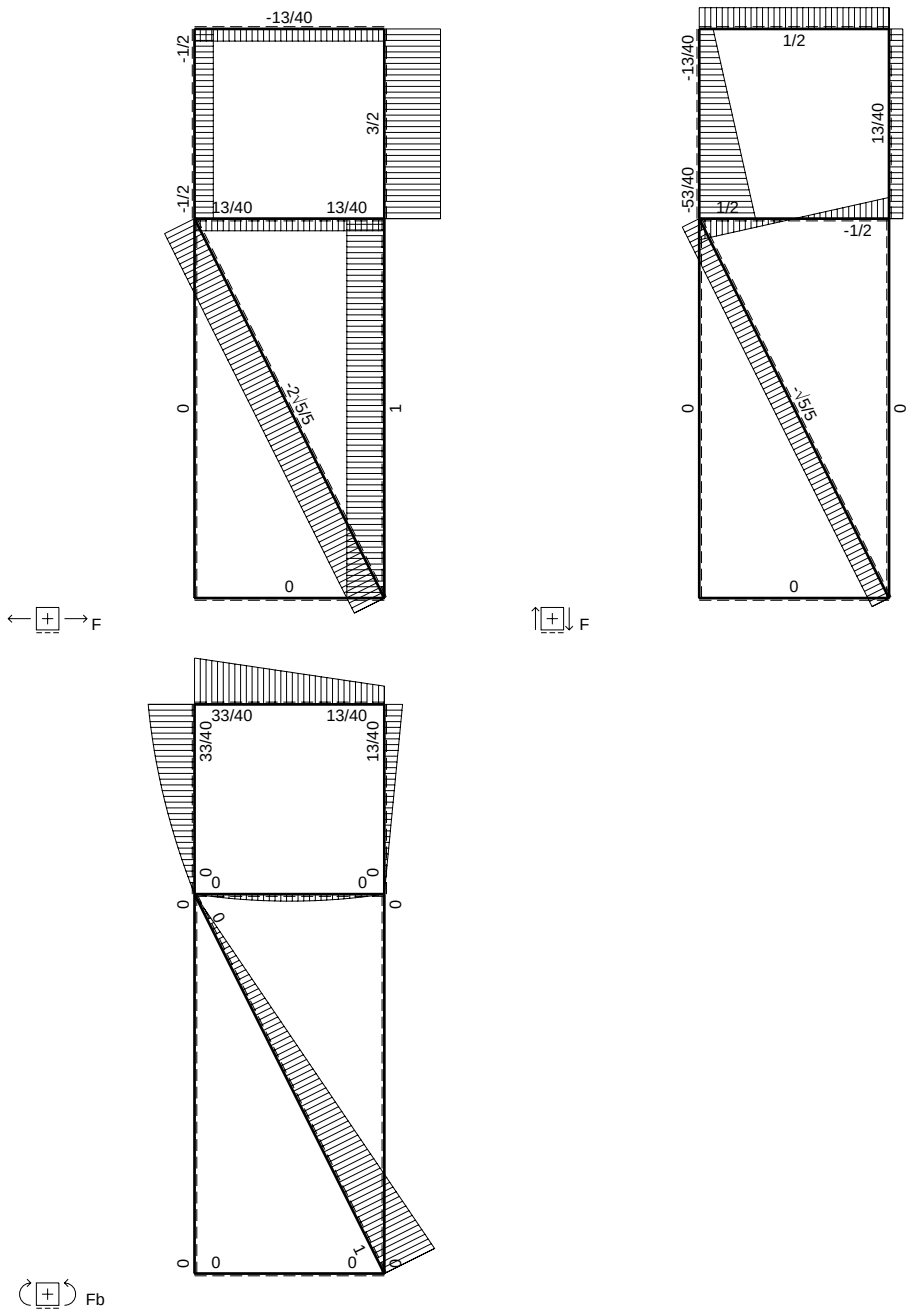


A = 534. mm^2
J_x = 189011. mm^4
J_y = 42681. mm^4
J_{xy} = -61197. mm^4
J_u = 211231. mm^4
J_v = 20461. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3483
c = cosα = .9400
s = sinα = .3413
x_g = 17.83 mm
y_g = 23.33 mm
N = -1440. N
T_y = -720. N
M_x = 708400. Nmm
x_m = 12. mm
u_m = -13.44 mm
v_m = -19.94 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 219. N/mm^2

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
-2√5/5
0
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

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Schema di calcolo iperstatico

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

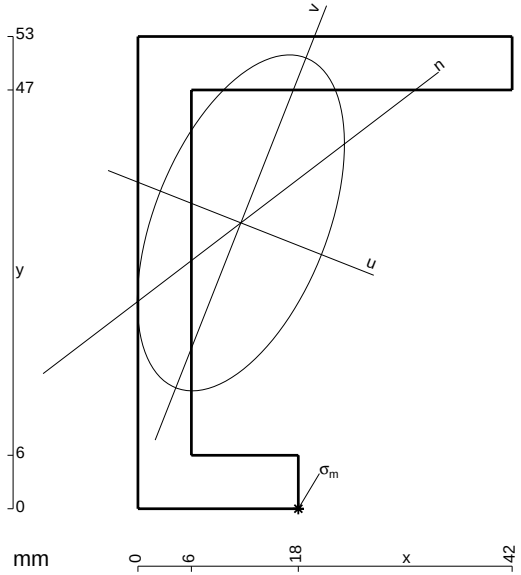
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

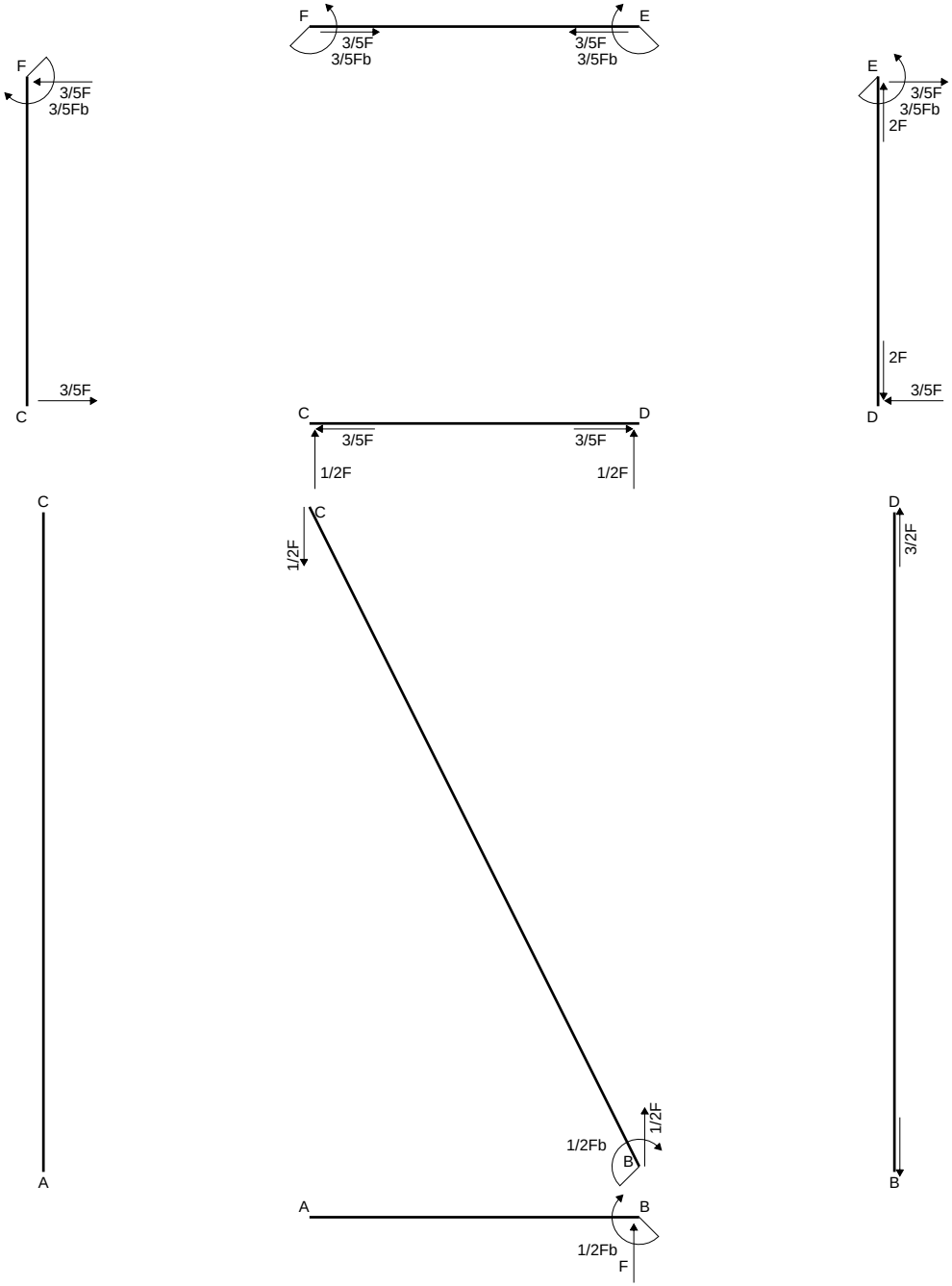
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

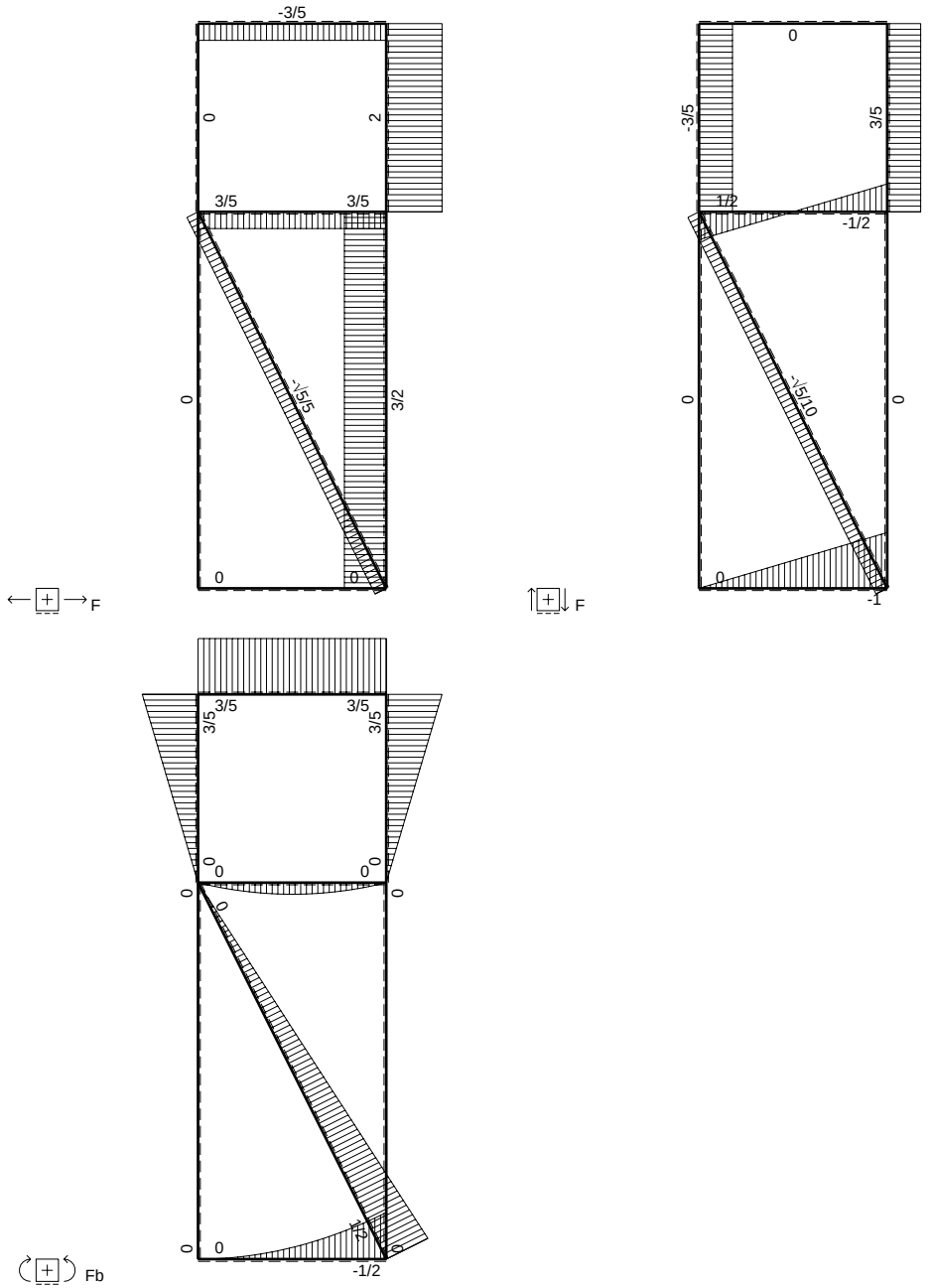


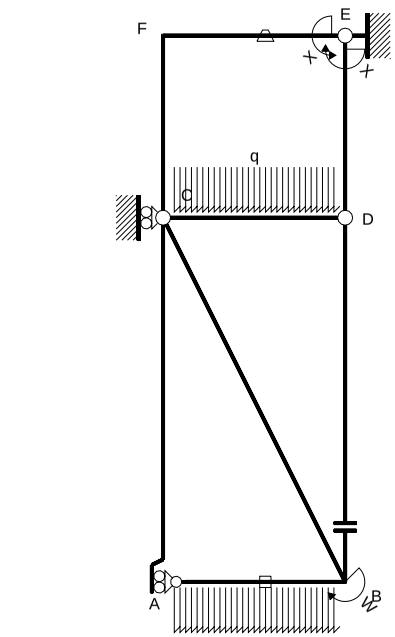
A = 606. mm^2
J_x = 215454. mm^4
J_y = 81888. mm^4
J_{xy} = 62420. mm^4
J_u = 240083. mm^4
J_v = 57259. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3758
c = cosα = .9302
s = sinα = -.3670
x_g = 11.55 mm
y_g = 32.08 mm
N = -1968. N
T_y = -983.9 N
M_x = 1056000. Nmm
x_m = 18. mm
u_m = 17.77 mm
v_m = -27.48 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 229.5 N/mm^2

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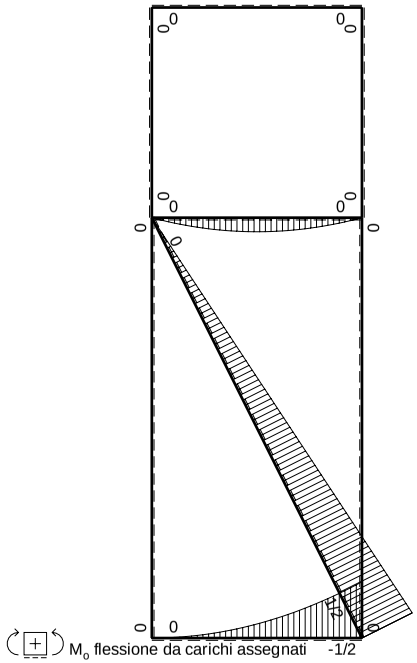
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
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C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F

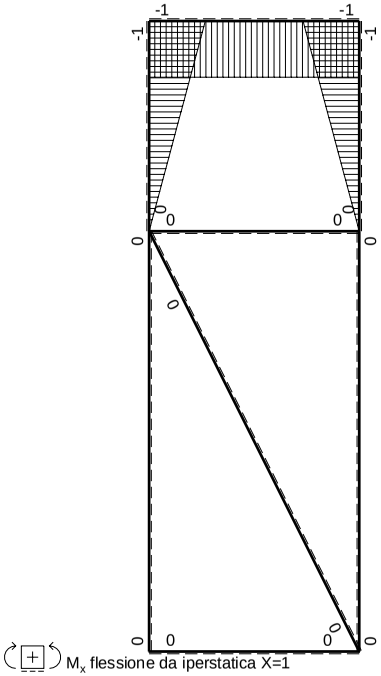

-3/5
0
2
3/5
3/5
0
3/2
-3/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-3/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
0
-1/2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

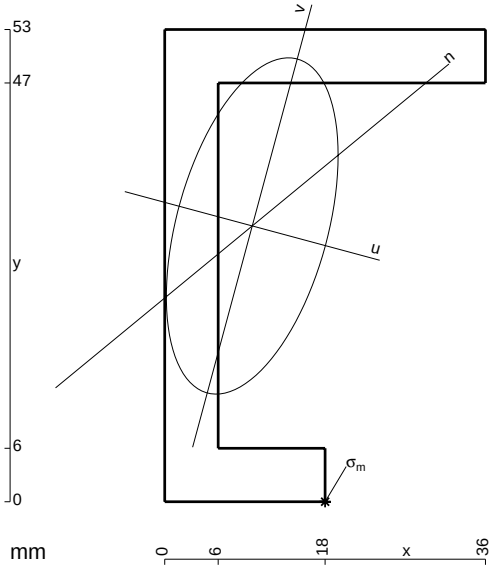
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

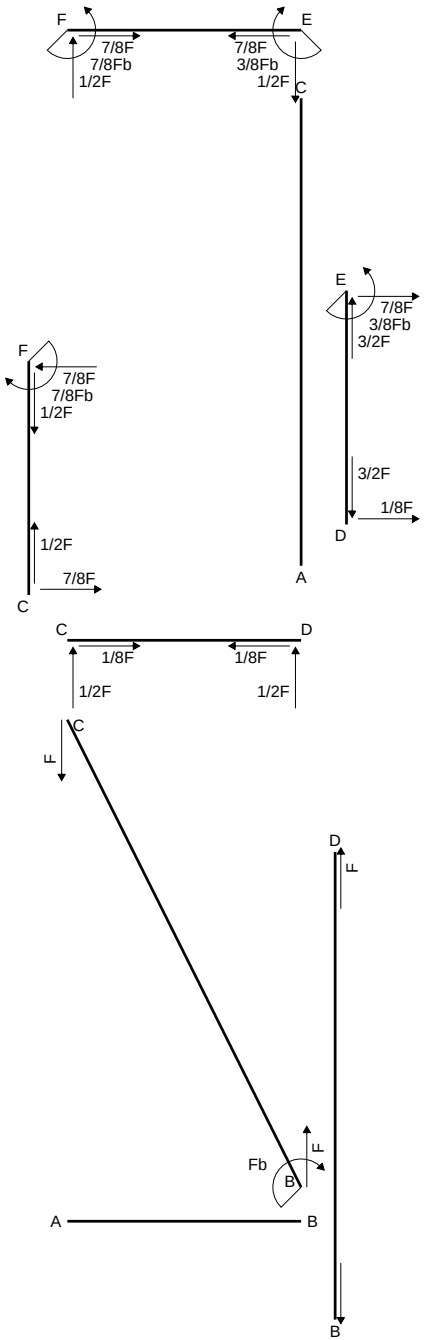
$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

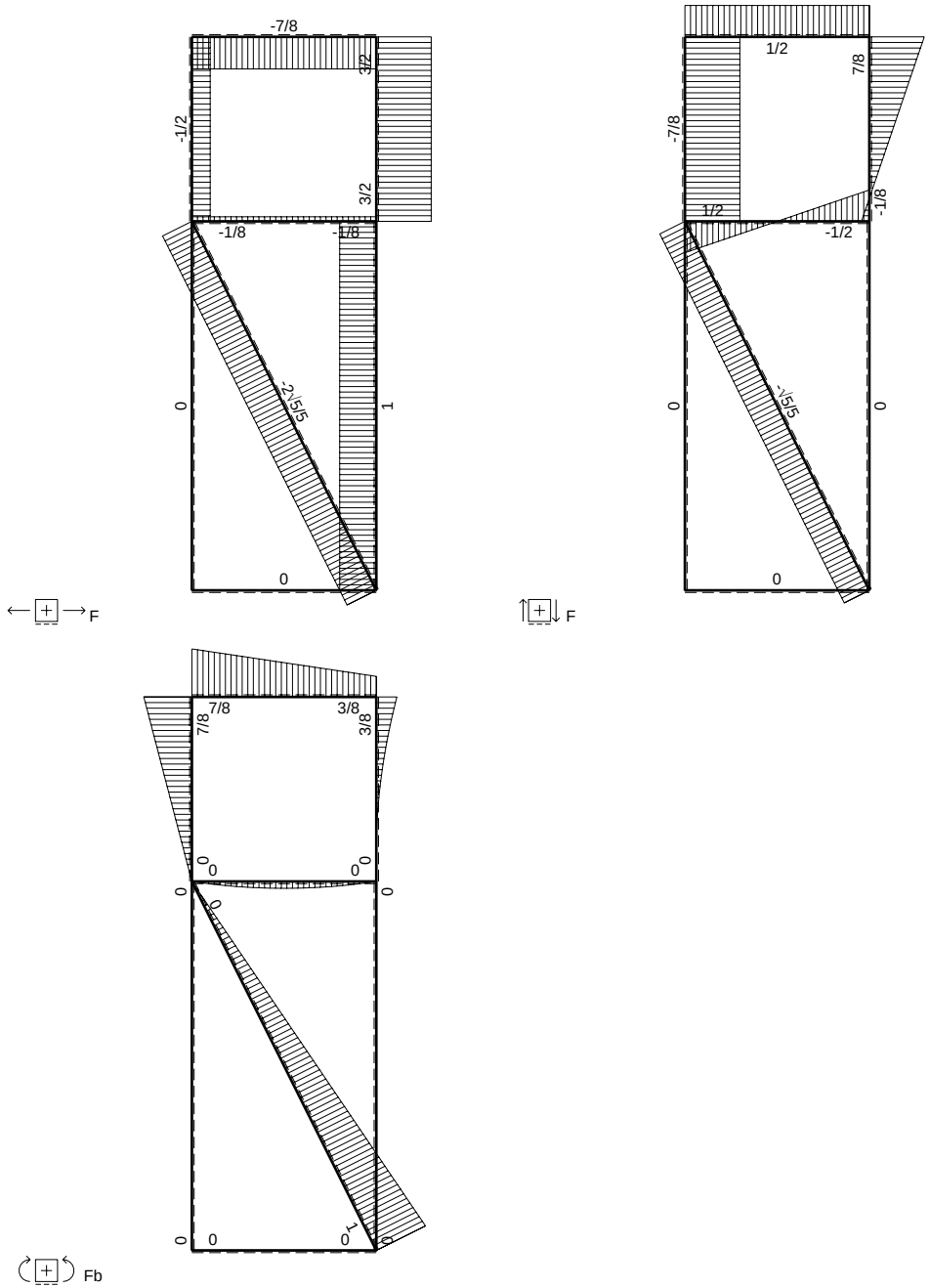


A = 570. mm^2
J_x = 203061. mm^4
J_y = 52950. mm^4
J_{xy} = 43600. mm^4
J_u = 214806. mm^4
J_v = 41205. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2631
c = cosα = .9656
s = sinα = -.2601
x_g = 9.821 mm
y_g = 30.95 mm
N = 6620. N
T_y = 1986. N
M_x = 1012860. Nmm
x_m = 18. mm
u_m = 15.95 mm
v_m = -27.76 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 240. N/mm^2

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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-2√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 274905 CI09.xxxx.145

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [-\ x]_o^b\ \theta$

$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ Fb^2/EJ$

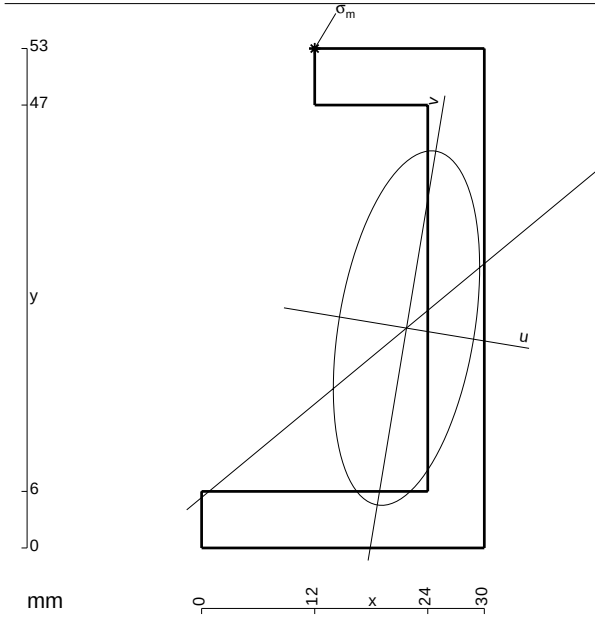
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 274905 CI09.xxxx.145

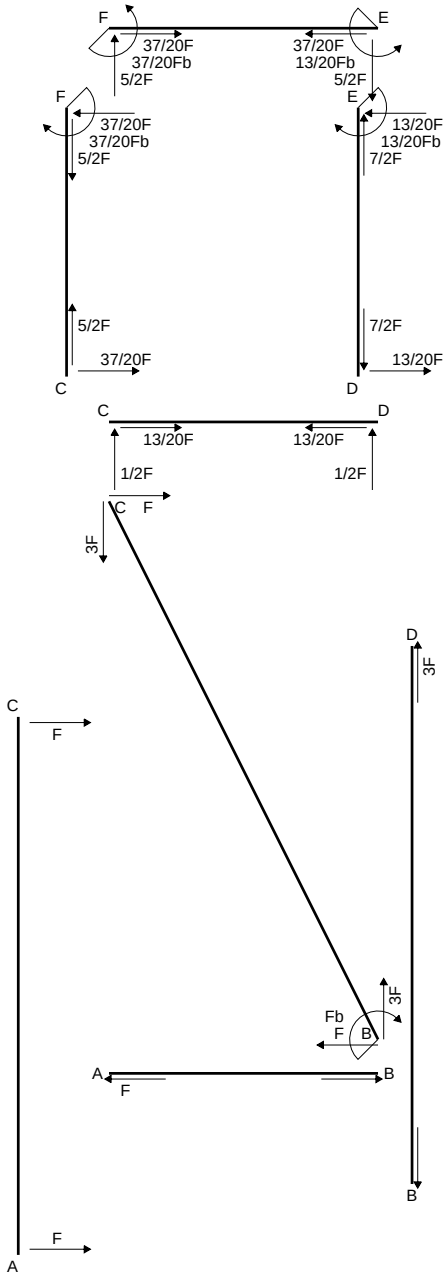


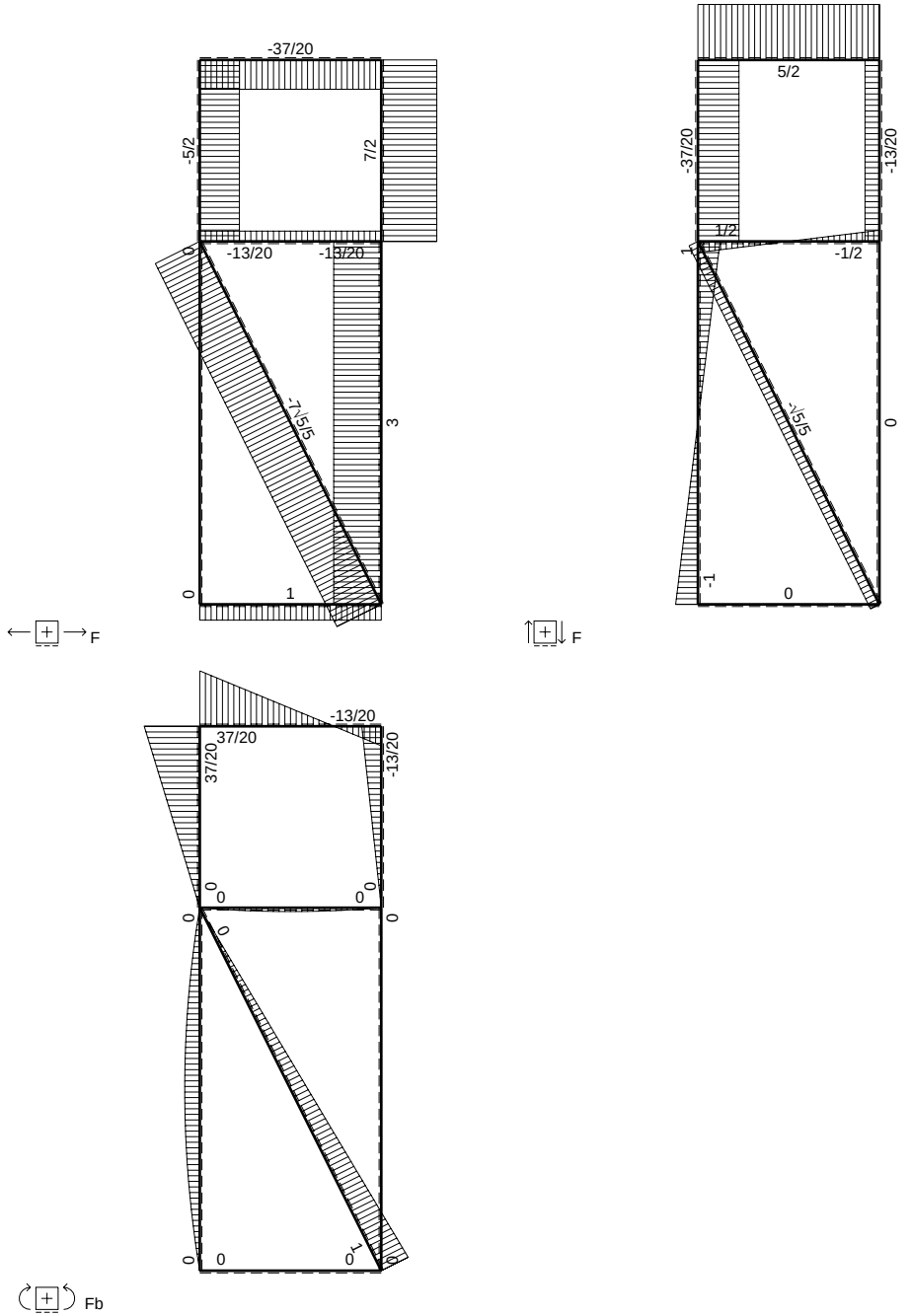
A = 534. mm^2
J_x = 189011. mm^4
J_y = 32196. mm^4
J_{xy} = 26635. mm^4
J_u = 193412. mm^4
J_v = 27796. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1637
$c = \cos\alpha$ = .9866
$s = \sin\alpha$ = -.1630
x_g = 21.74 mm
y_g = 23.33 mm
N = -1413. N
T_y = -706.6 N
M_x = 869000. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -14.45 mm
v_m = 27.68 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -199. N/mm^2

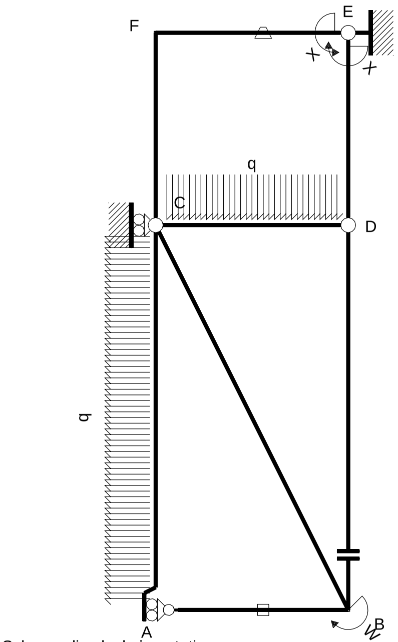
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CI09.xxxx.145

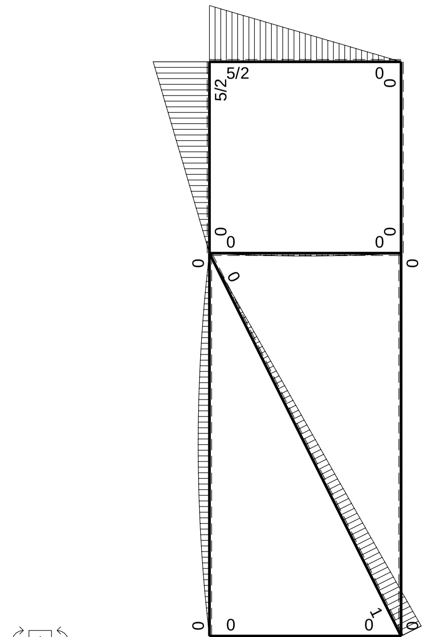
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
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A
B
F
F
A
B

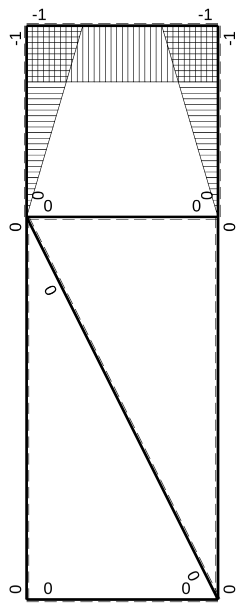

-37/20
-5/2
7/2
-13/20
-13/20
0
3
-7.5/5
0
1
F
5/2
-37/20
-13/20
1/2
-1/2
-1
0
-7.5/5
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 264372 CI09.xxxx.146

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

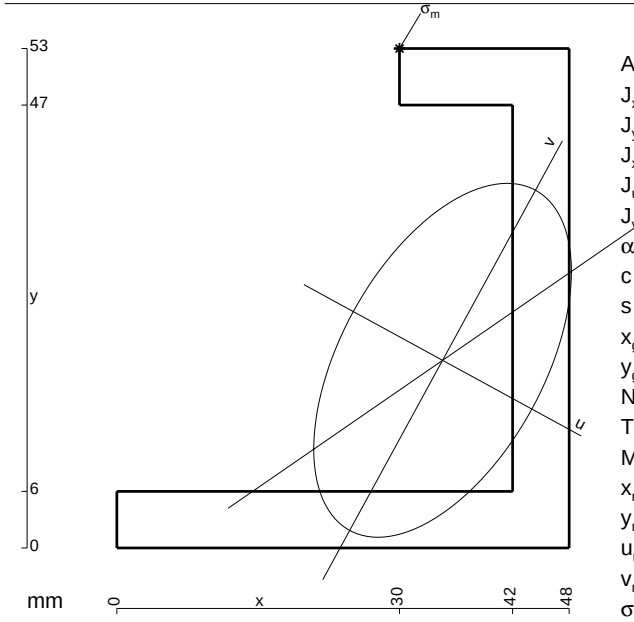
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

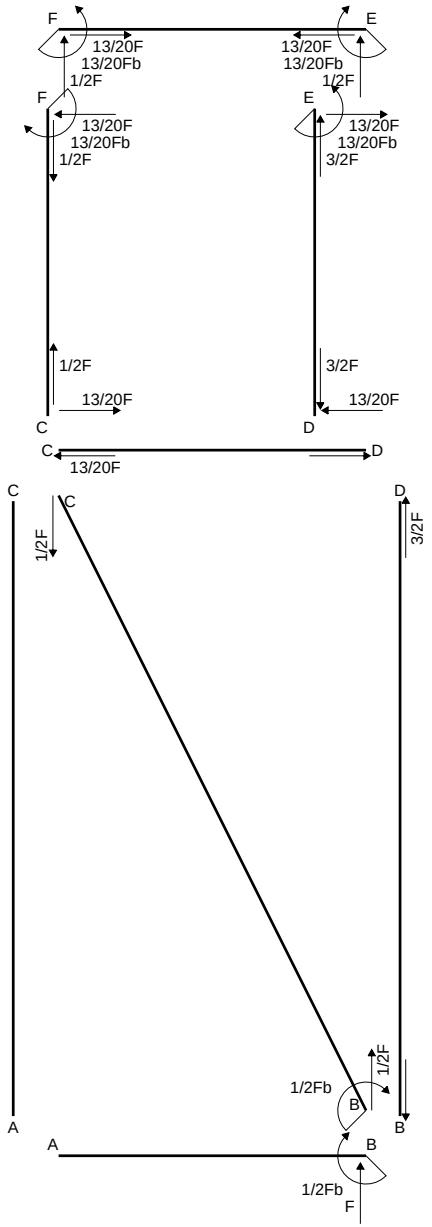
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

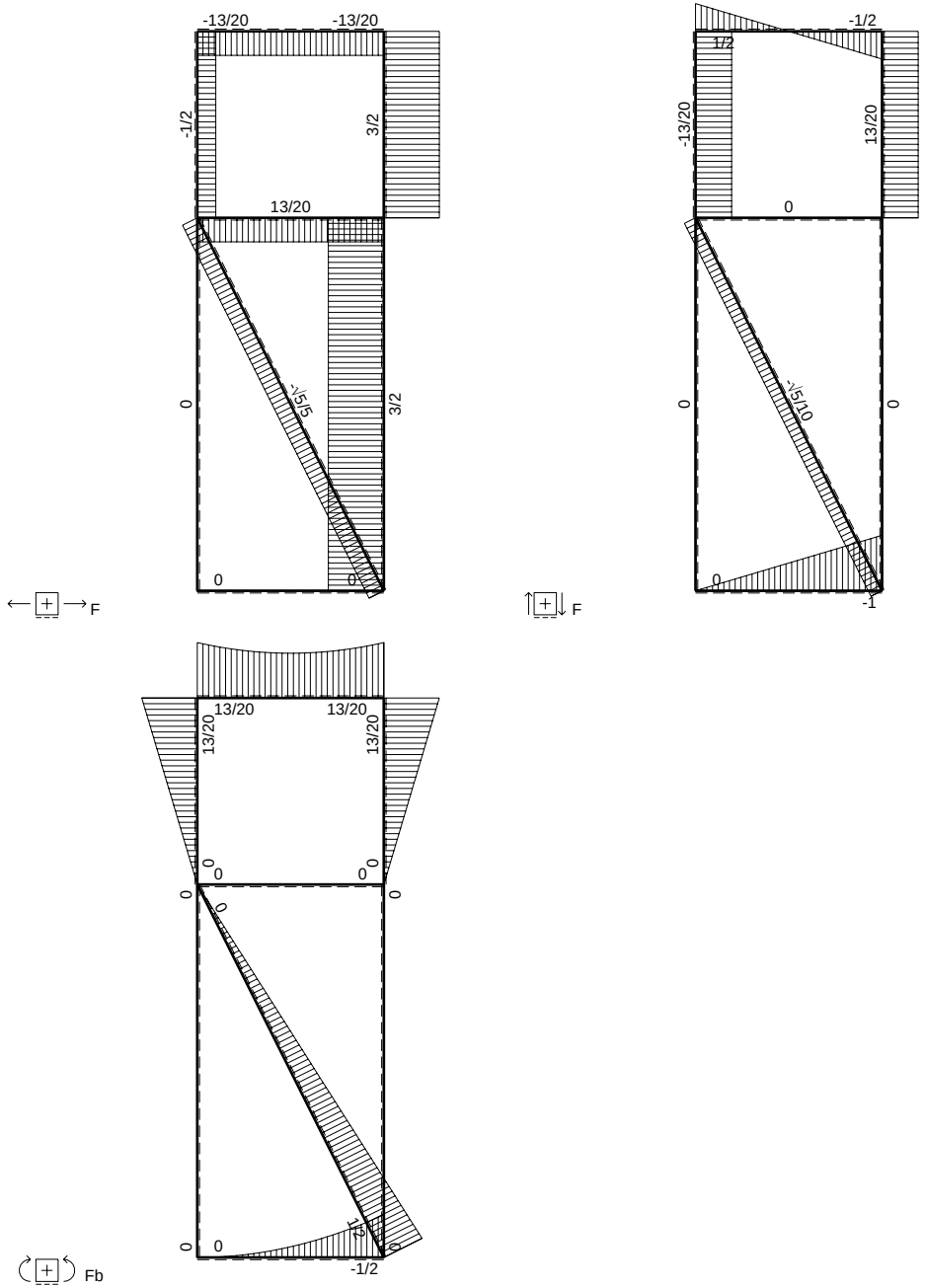


A = 642. mm^2
J_x = 226469. mm^4
J_y = 120007. mm^4
J_{xy} = 82782. mm^4
J_u = 271657. mm^4
J_v = 74819. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.4997$
$c = cos\alpha$ = .8777
$s = sin\alpha$ = -.4791
x_g = 34.57 mm
y_g = 19.91 mm
N = -2200. N
T_y = -1628. N
M_x = 960520. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = -19.87 mm
v_m = 26.85 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -209. N/mm^2

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi \theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
totali							$13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

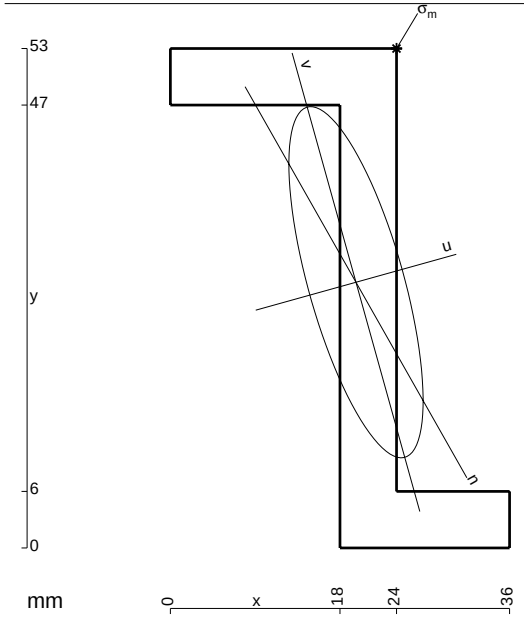
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

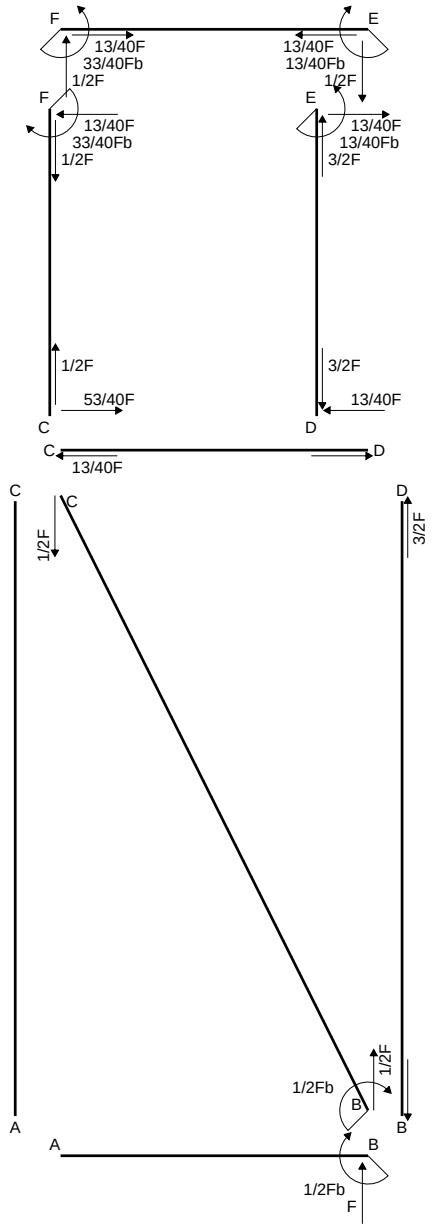
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

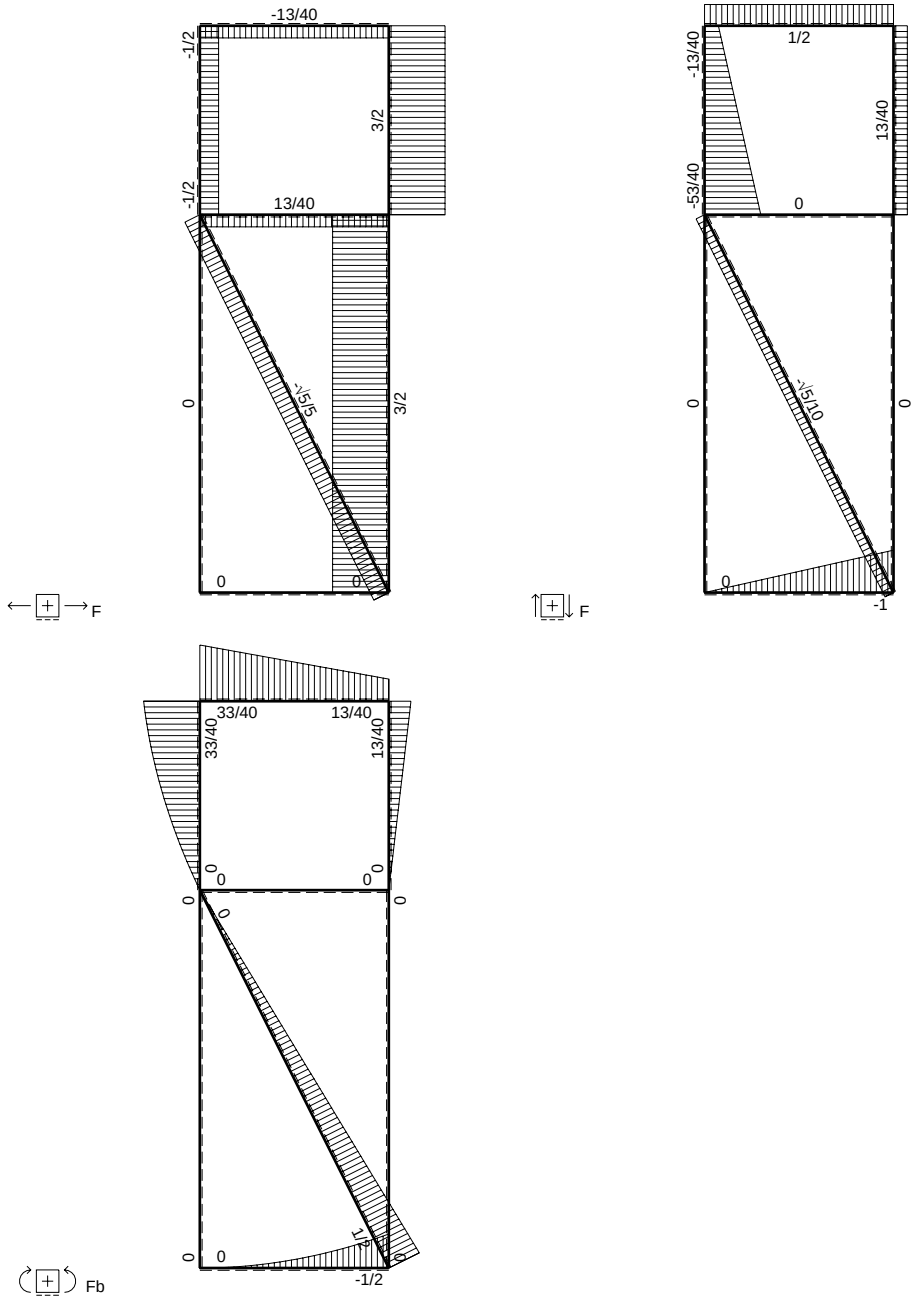


A = 498. mm^2
J_x = 172946. mm^4
J_y = 25275. mm^4
J_{xy} = -44583. mm^4
J_u = 185362. mm^4
J_v = 12859. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2716
$c = \cos\alpha$ = .9633
$s = \sin\alpha$ = .2683
x_g = 19.7 mm
y_g = 28.2 mm
N = -1021. N
T_y = -785. N
M_x = 632710. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 10.8 mm
v_m = 22.74 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.4 N/mm^2

CI09.xxxx.147

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
1/2F
3/2F
1/2Fb
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
0
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 267798 CI09.xxxx.148

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

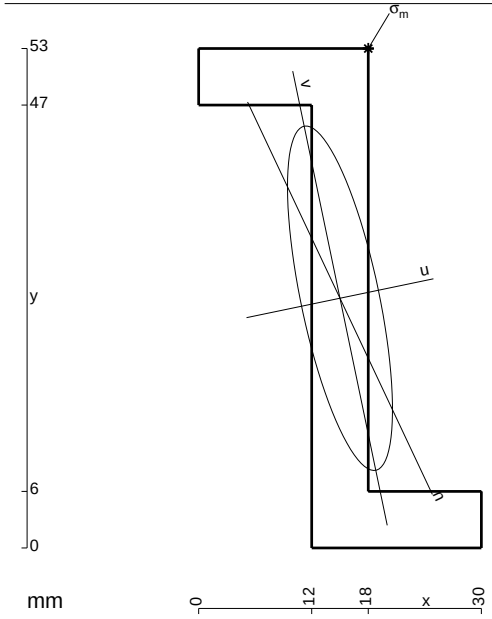
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

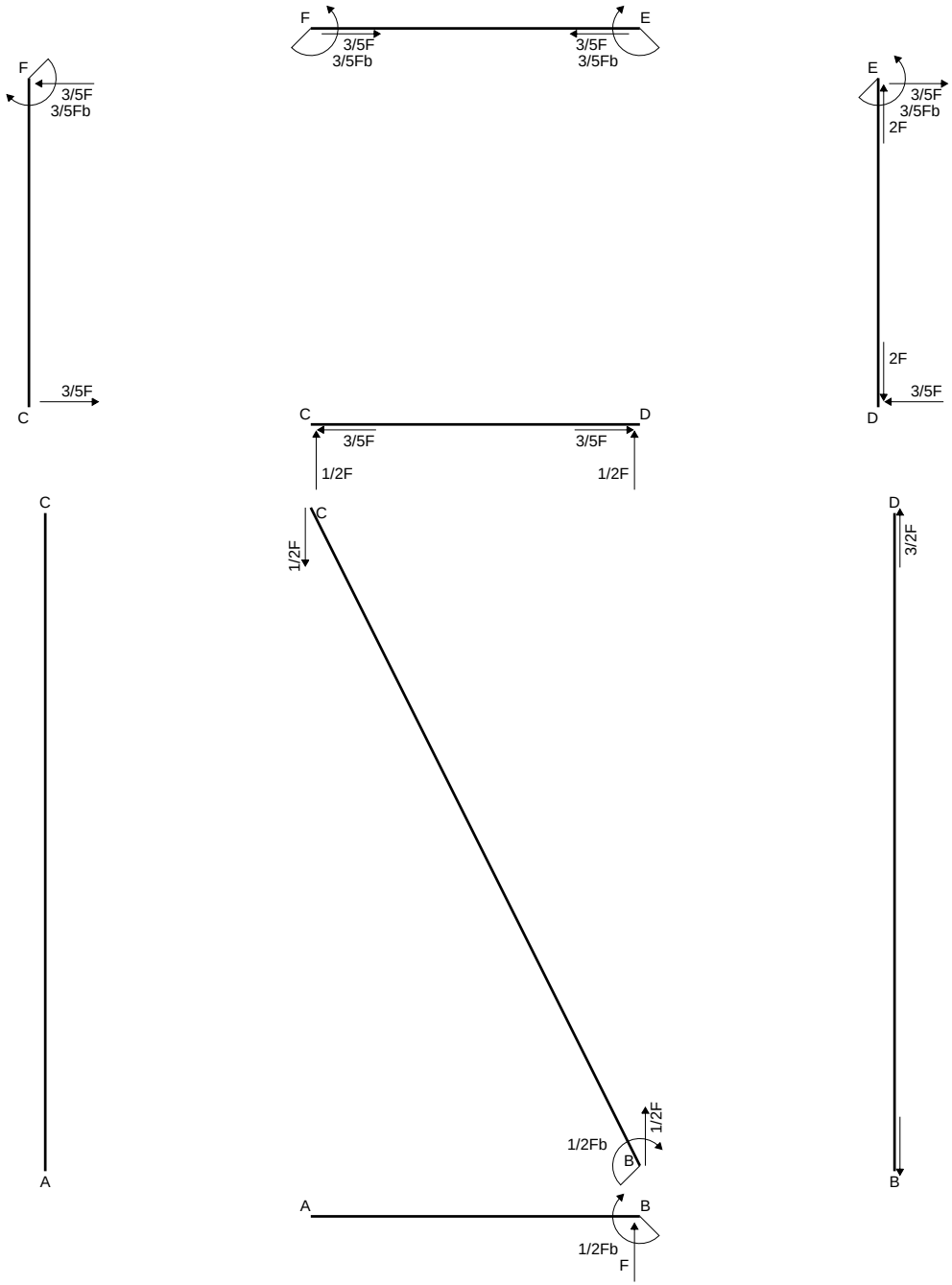
PROCEDIMENTO E RISULTATI 267798 CI09.xxxx.148

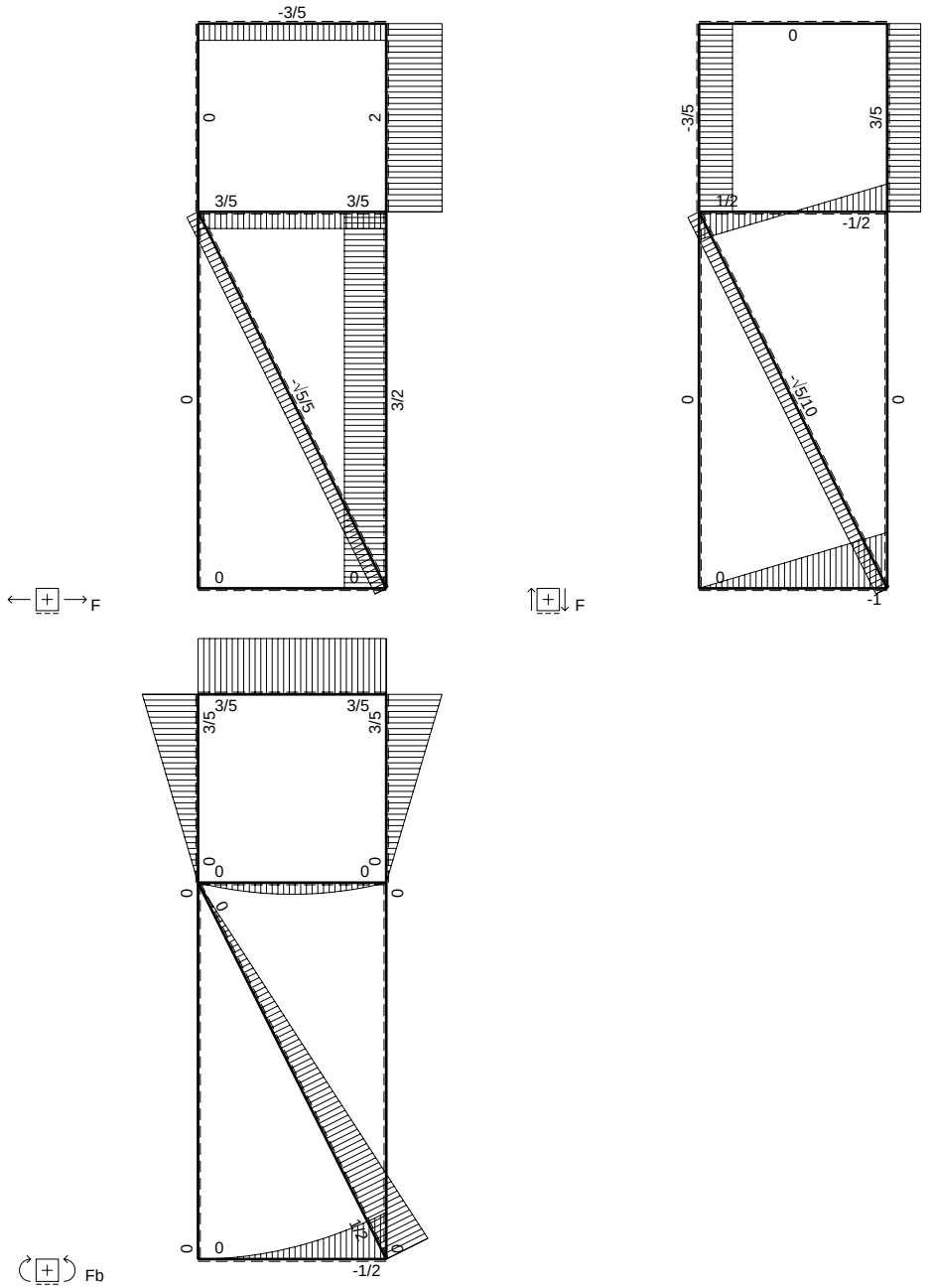


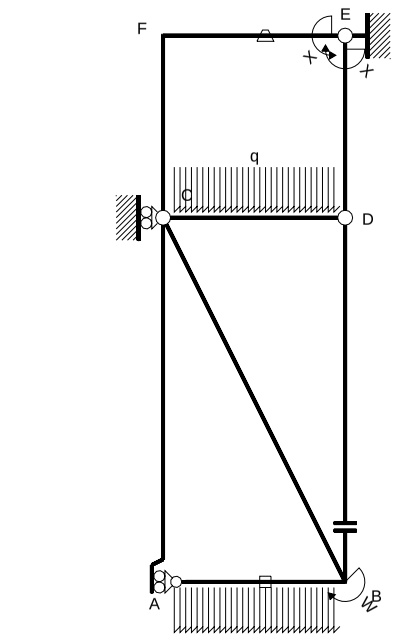
A = 462. mm^2
J_x = 154395. mm^4
J_y = 14346. mm^4
J_{xy} = -30456. mm^4
J_u = 160731. mm^4
J_v = 8010. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2051
c = cosα = .9790
s = sinα = .2037
x_g = 15. mm
y_g = 26.5 mm
N = -570. N
T_y = -370.5 N
M_x = 620730. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 8.335 mm
v_m = 25.33 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -228.6 N/mm^2

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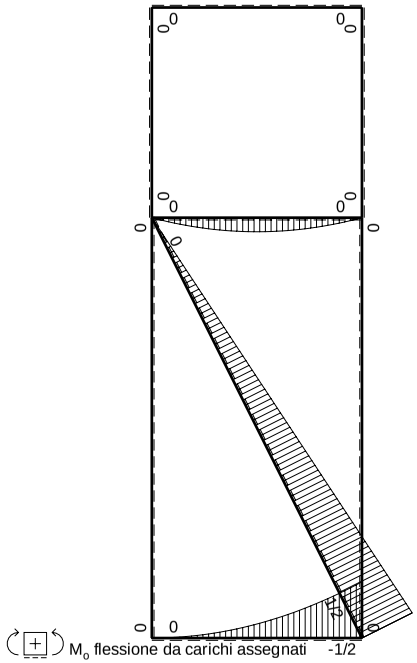
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
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C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F

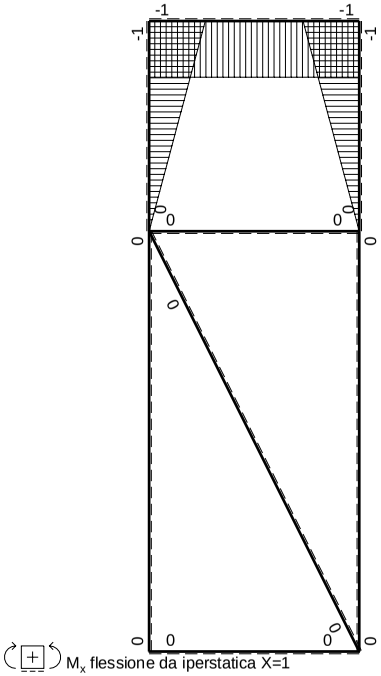

-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
0
0
-√2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

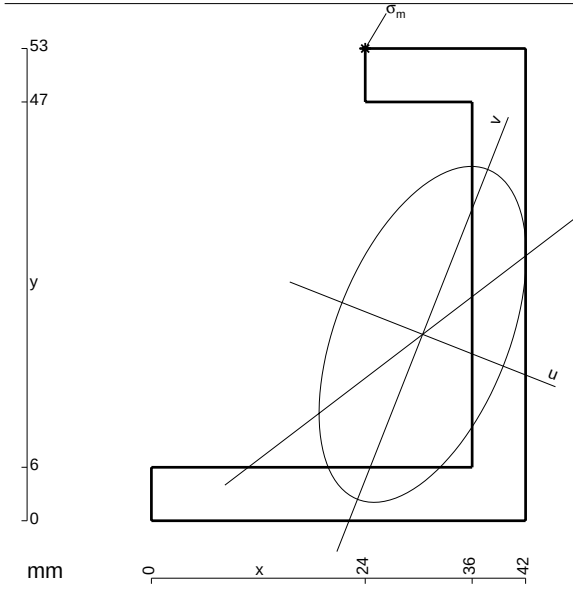
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

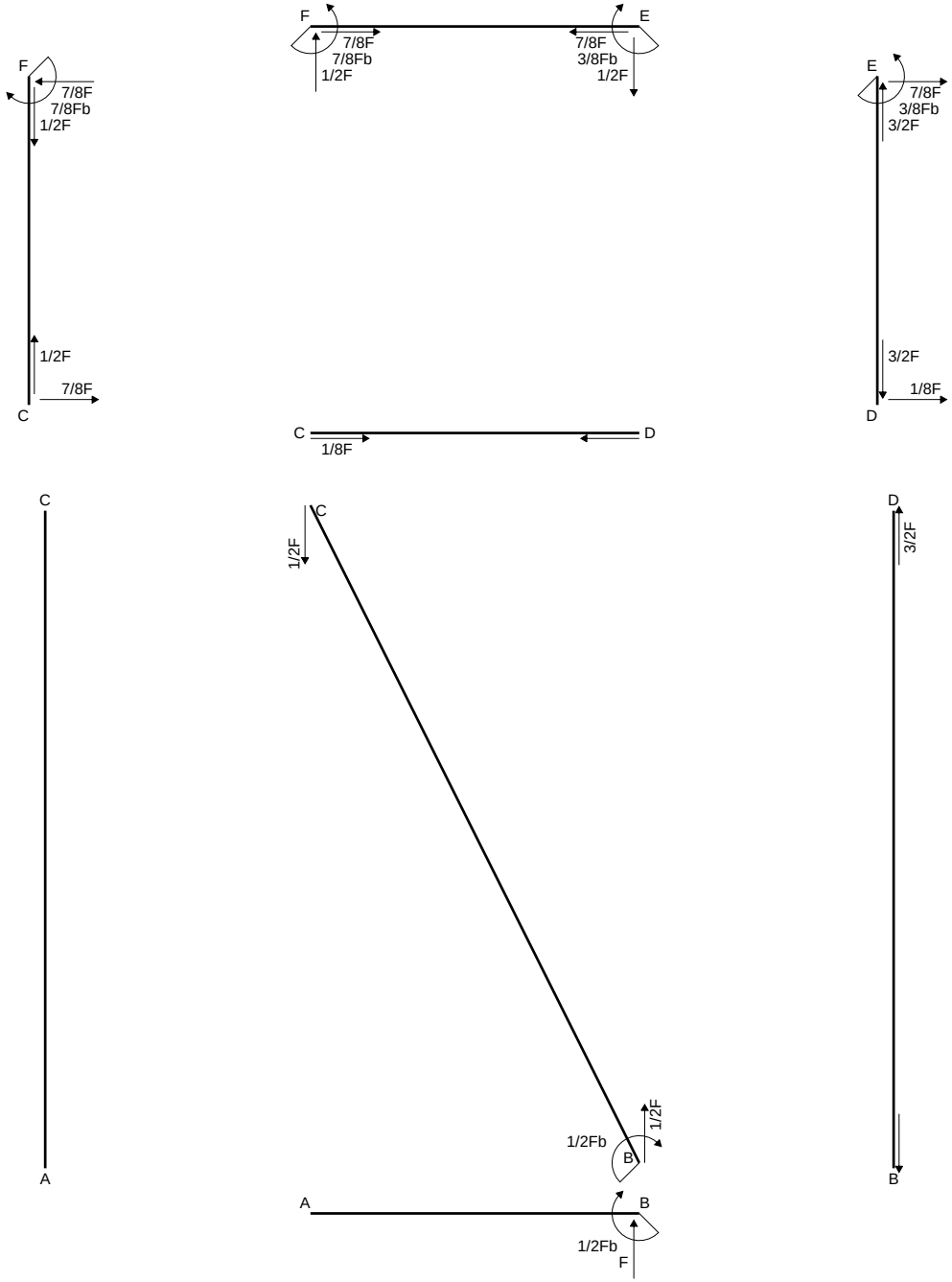
$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

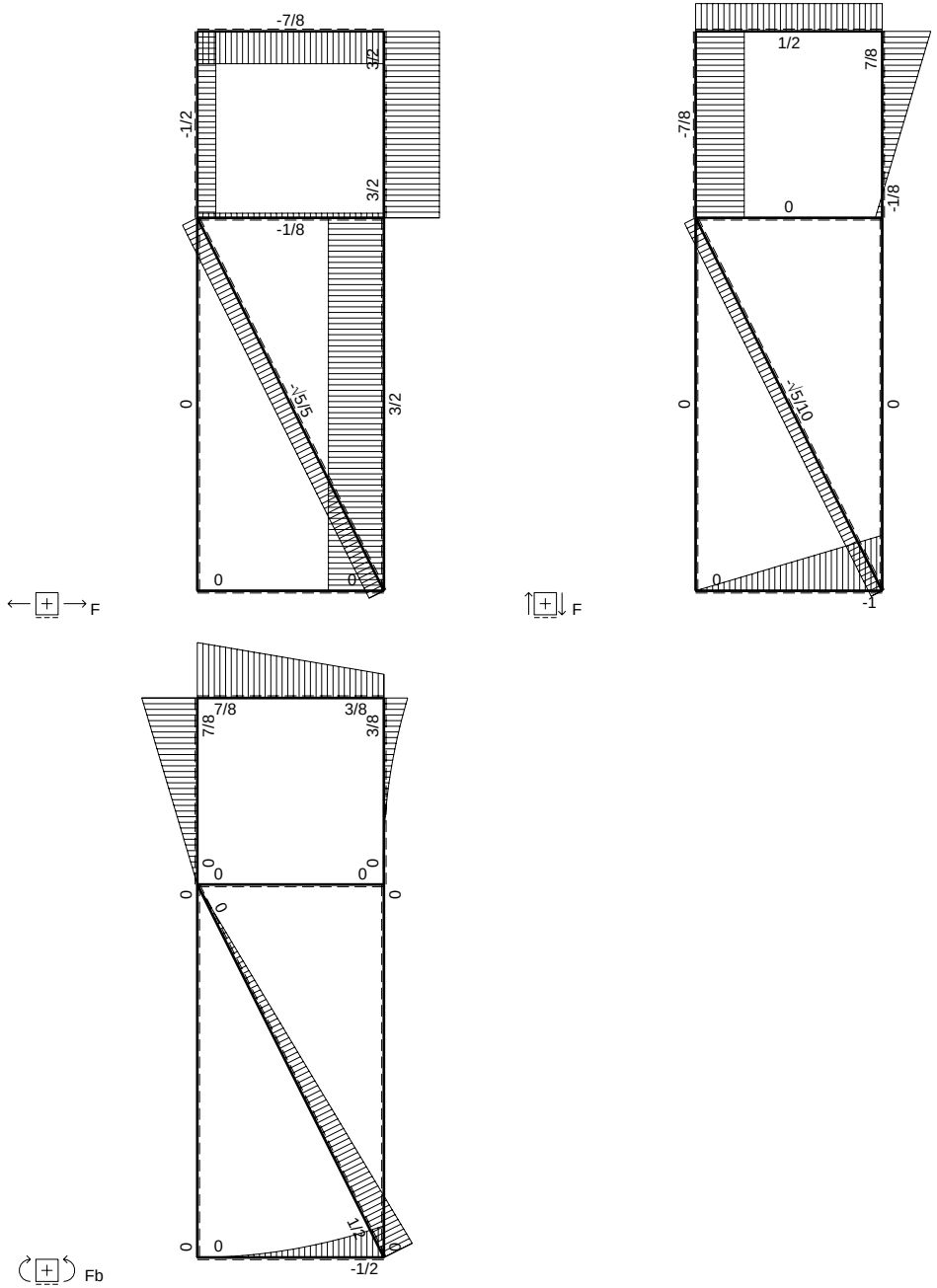


A = 606. mm^2
J_x = 215454. mm^4
J_y = 81888. mm^4
J_{xy} = 62420. mm^4
J_u = 240083. mm^4
J_v = 57259. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3758
$c = \cos\alpha$ = .9302
$s = \sin\alpha$ = -.3670
x_g = 30.45 mm
y_g = 20.92 mm
N = -1536. N
M_x = 1075200. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = -17.77 mm
v_m = 27.48 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -239.5 N/mm^2

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F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
3/2F
1/8F
C
D
1/8F
A
B
1/2Fb


-7/8
3/2
-1/2
-1/8
0
-√5/5
F
1/2
7/8
-1/8
-√5/10
7/8
3/8
Fb
-1/2

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ \ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ \ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

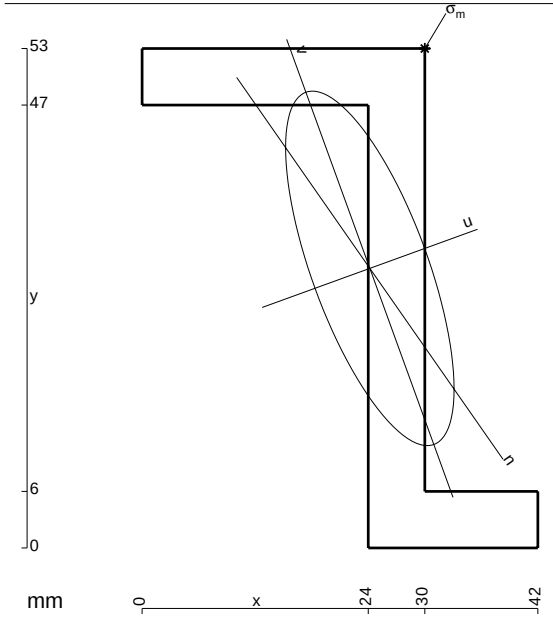
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ \ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

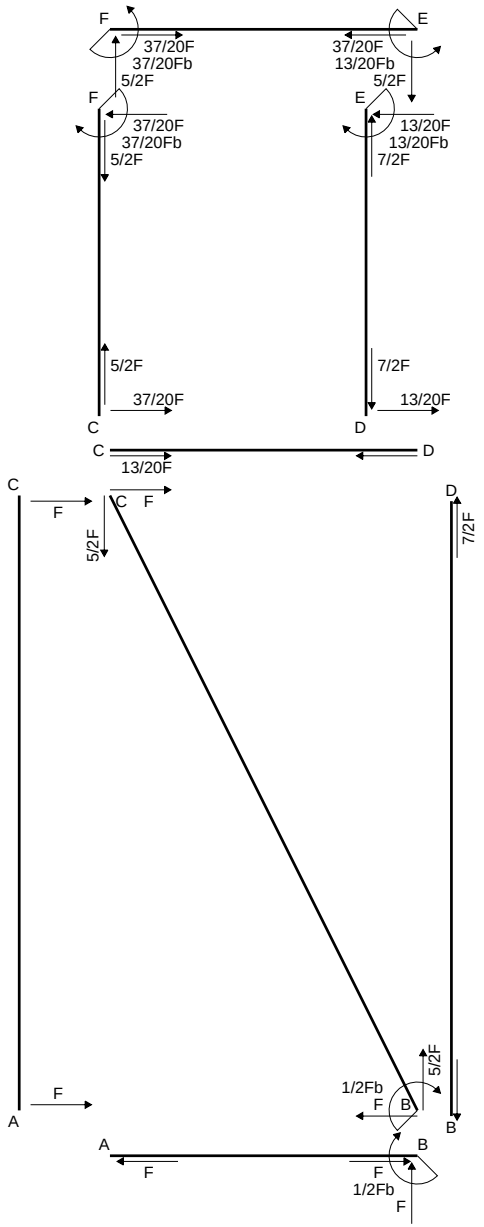
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

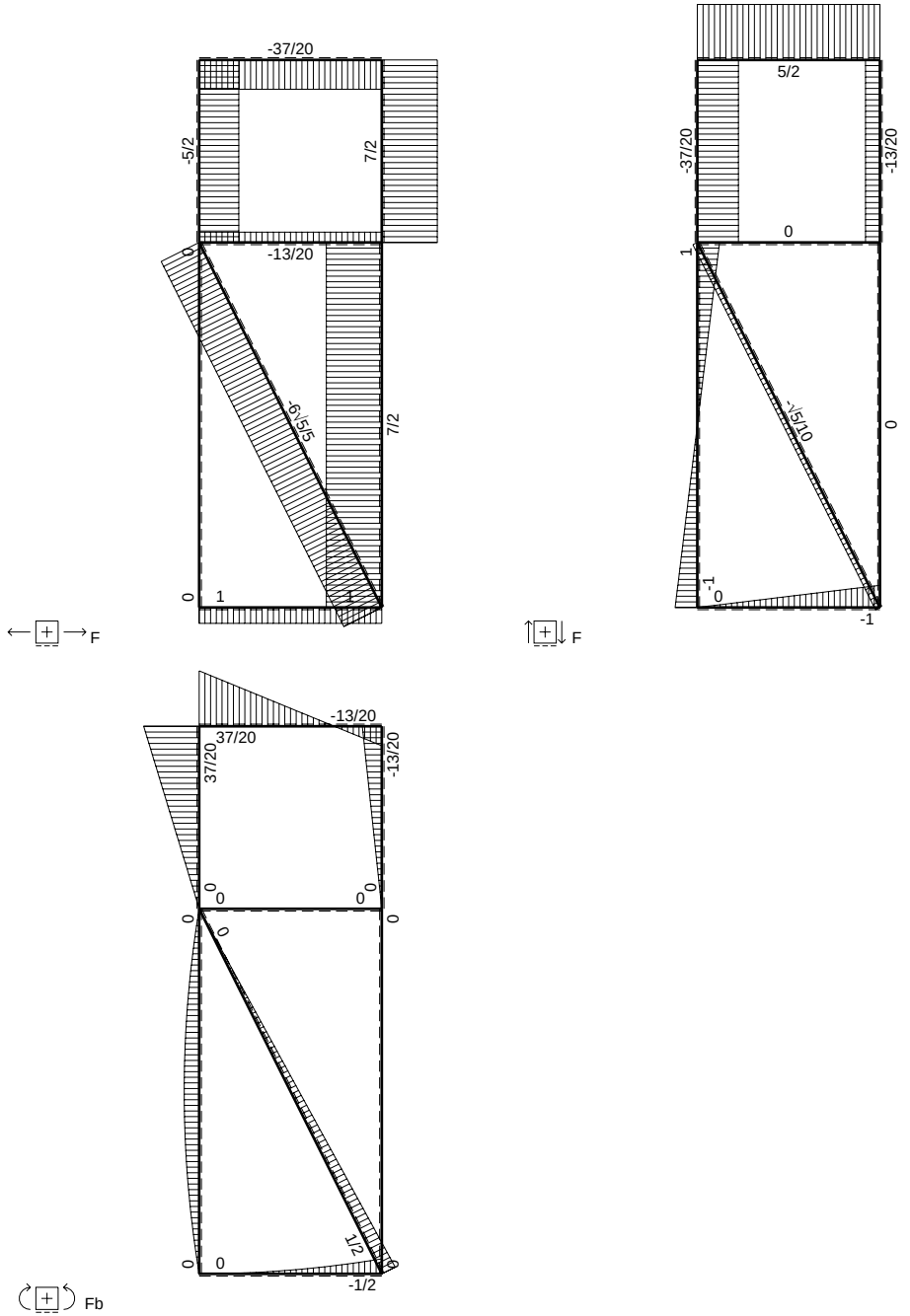


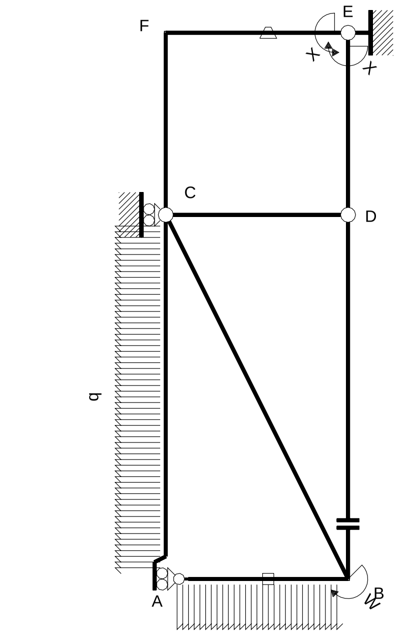
A = 534. mm^2
J_x = 189011. mm^4
J_y = 42681. mm^4
J_{xy} = -61197. mm^4
J_u = 211231. mm^4
J_v = 20461. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3483
$c = \cos\alpha$ = .9400
$s = \sin\alpha$ = .3413
x_g = 24.17 mm
y_g = 29.67 mm
N = -1698. N
T_y = 970. N
M_x = 628075. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 13.44 mm
v_m = 19.94 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -199.7 N/mm^2

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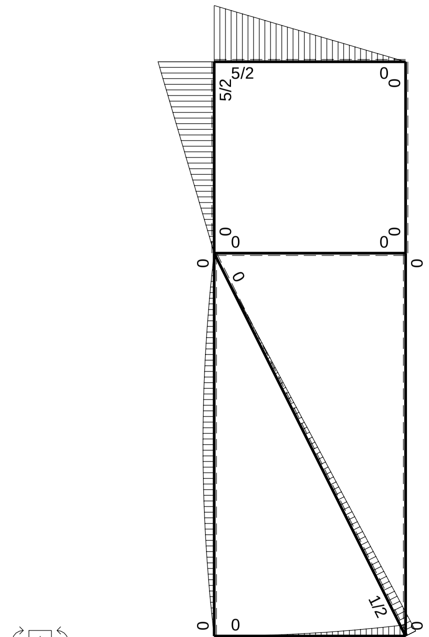
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F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb

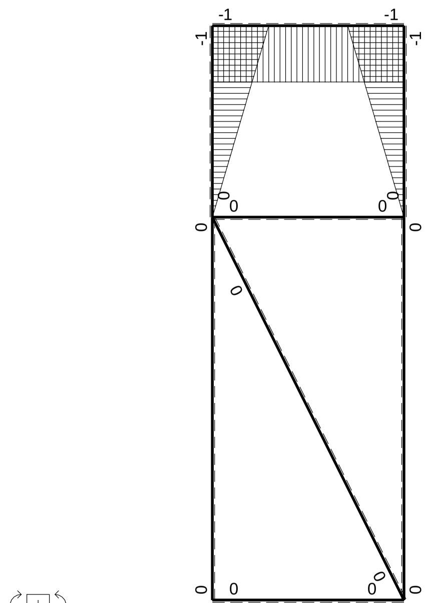

-37/20
-5/2
7/2
-13/20
-5\5/5
F
5/2
-37/20
-13/20
-5/10
-1
37/20
1/2
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

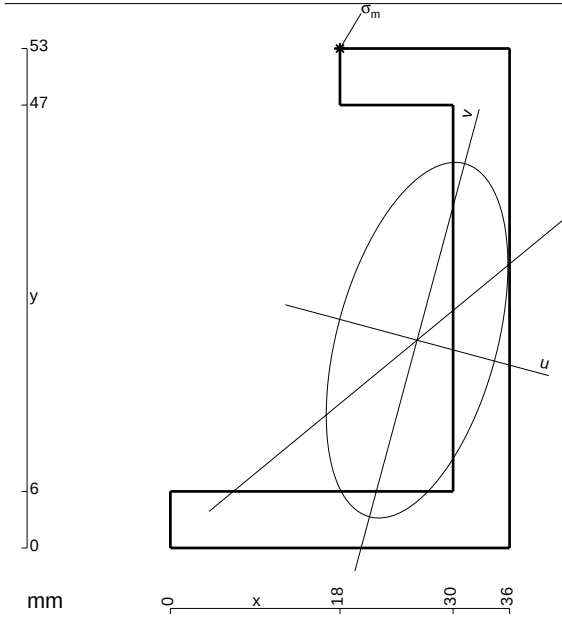
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

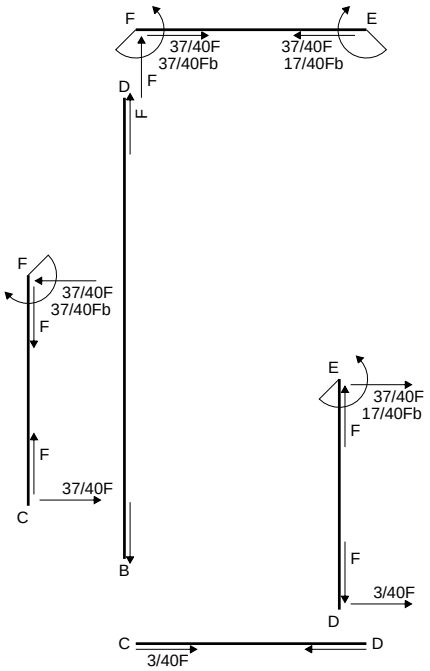
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

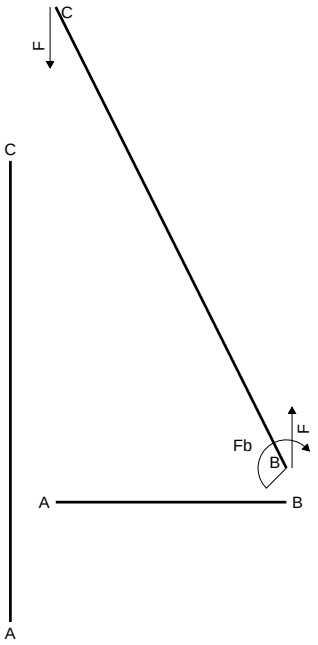


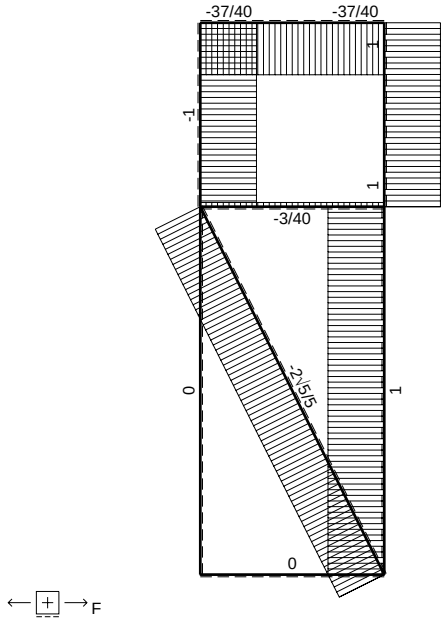
A = 570. mm^2
J_x = 203061. mm^4
J_y = 52950. mm^4
J_{xy} = 43600. mm^4
J_u = 214806. mm^4
J_v = 41205. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2631
c = cosα = .9656
s = sinα = -.2601
x_g = 26.18 mm
y_g = 22.05 mm
N = -3050. N
T_y = -2257. N
M_x = 902800. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = -15.95 mm
v_m = 27.76 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -208.9 N/mm^2

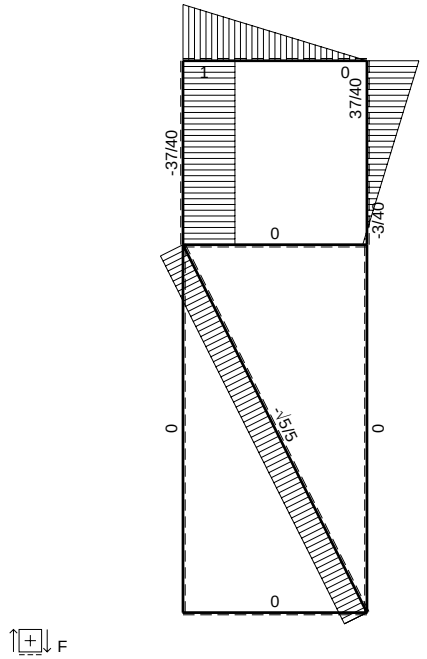
CI09.xxxx.151

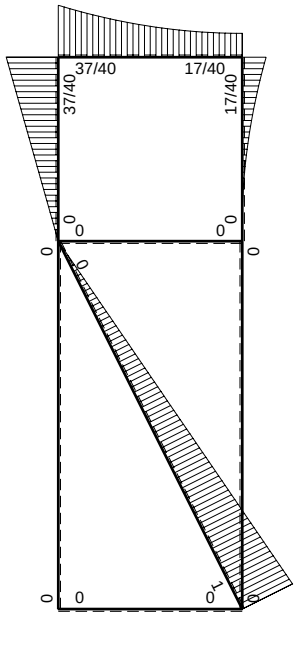
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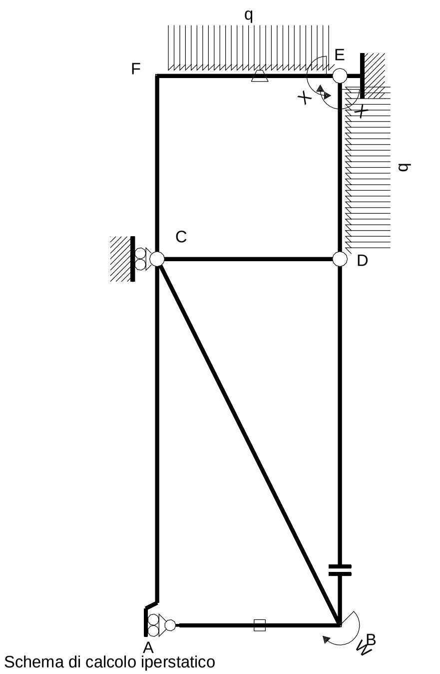

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
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37/40F
17/40Fb
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37/40F
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Fb
F
B
A
B
A

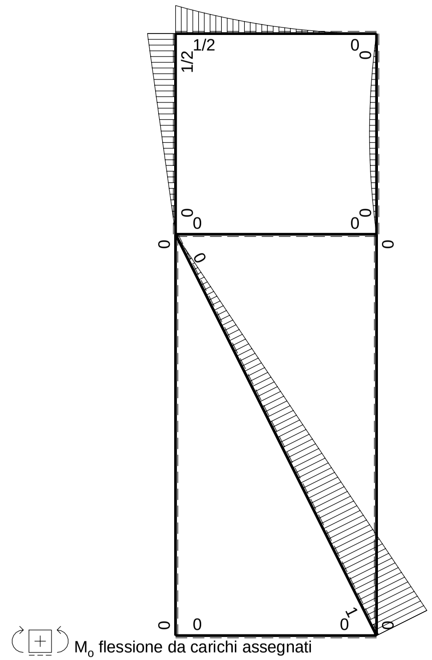

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-1
1
-3/40
0
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1
0
F


1
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37/40
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0
-√5/5
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0
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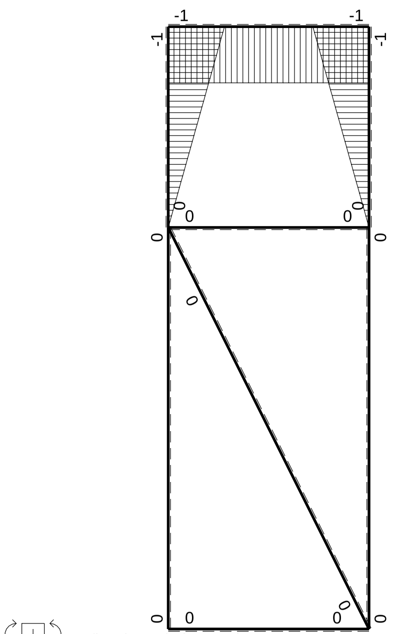

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
totali							$17/24Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-17/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

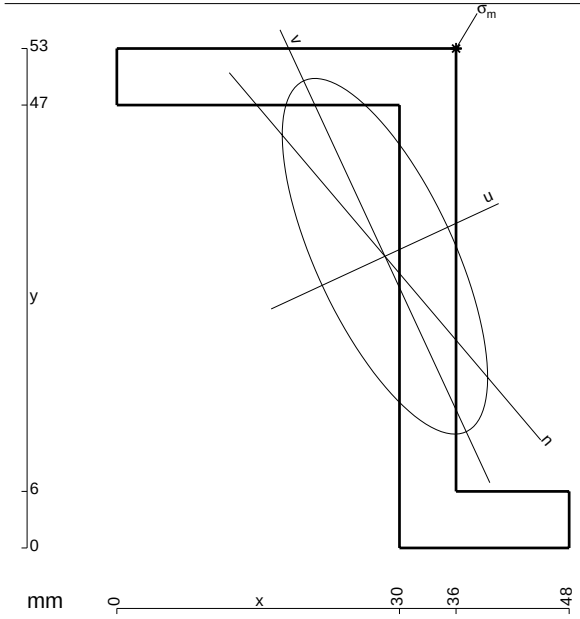
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

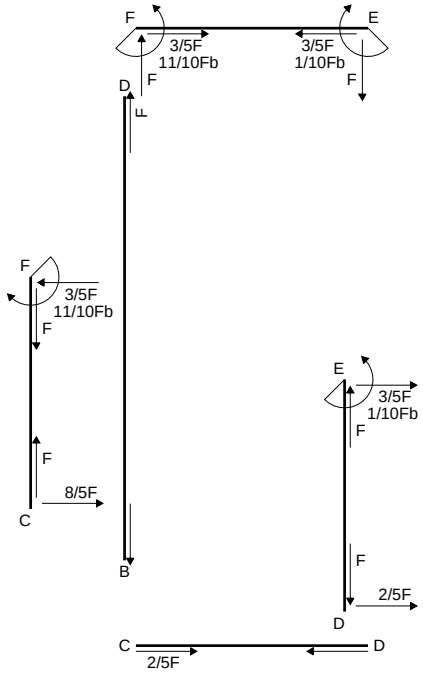
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

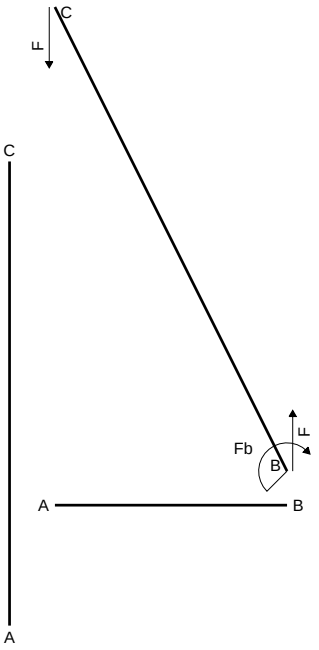


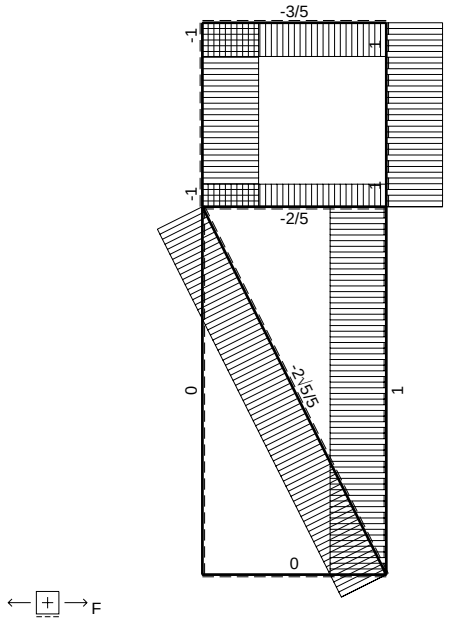
A = 570. mm^2
J_x = 203061. mm^4
J_y = 67683. mm^4
J_{xy} = -79827. mm^4
J_u = 240034. mm^4
J_v = 30710. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4337
$c = \cos\alpha$ = .9074
$s = \sin\alpha$ = .4203
x_g = 28.45 mm
y_g = 30.95 mm
N = -1547. N
T_y = -773.7 N
M_x = 761200. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 16.11 mm
v_m = 16.83 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -219. N/mm^2

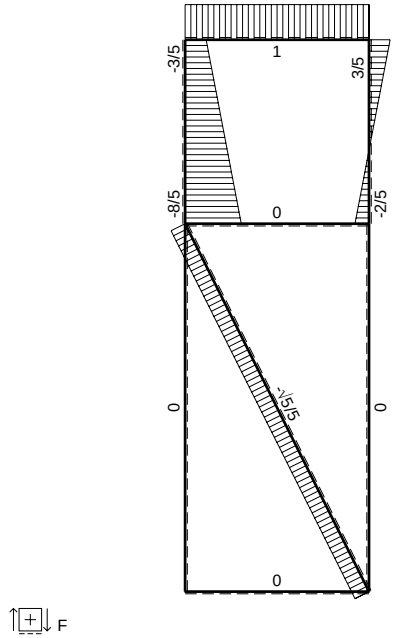
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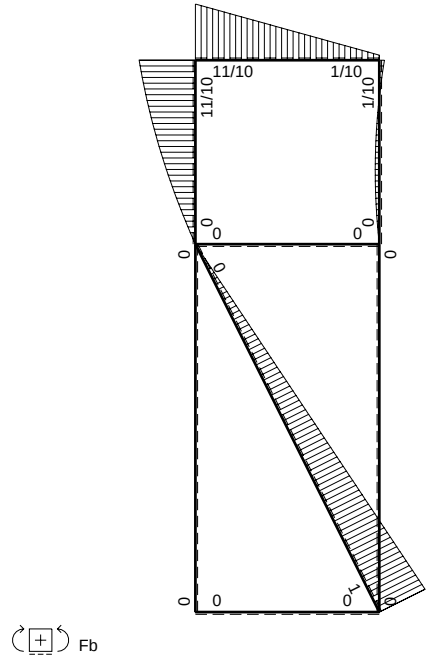
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F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
1/10Fb
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F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
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F


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F


11/10
1/10
11/10
1/10
0
0
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0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3XMb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb²/EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx²	0	-Fb+3/2Fx-1/2qx³/b	0	1-2x/b+x²/b²	(-3/8+0)Fb²/EJ	1/3XMb/EJ
CF b	x/b	-3/2Fx+1/2qx²	0	-3/2Fx²/b+1/2qx³/b	0	x²/b²		
	totali						1/6Fb²/EJ	5/3XMb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

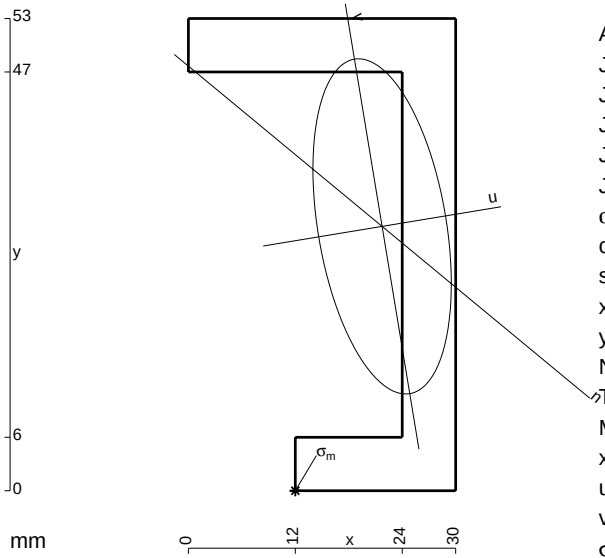
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

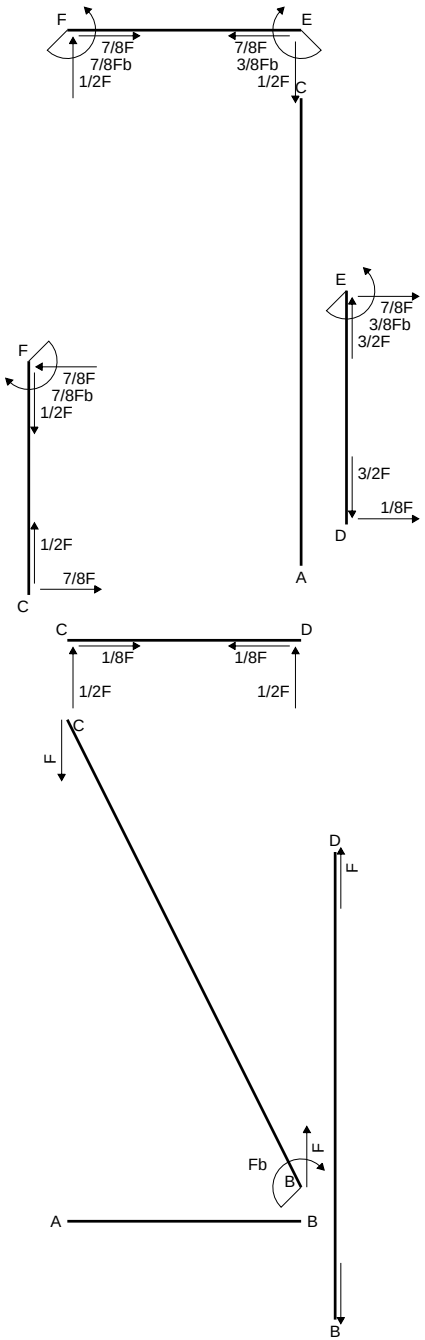
$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

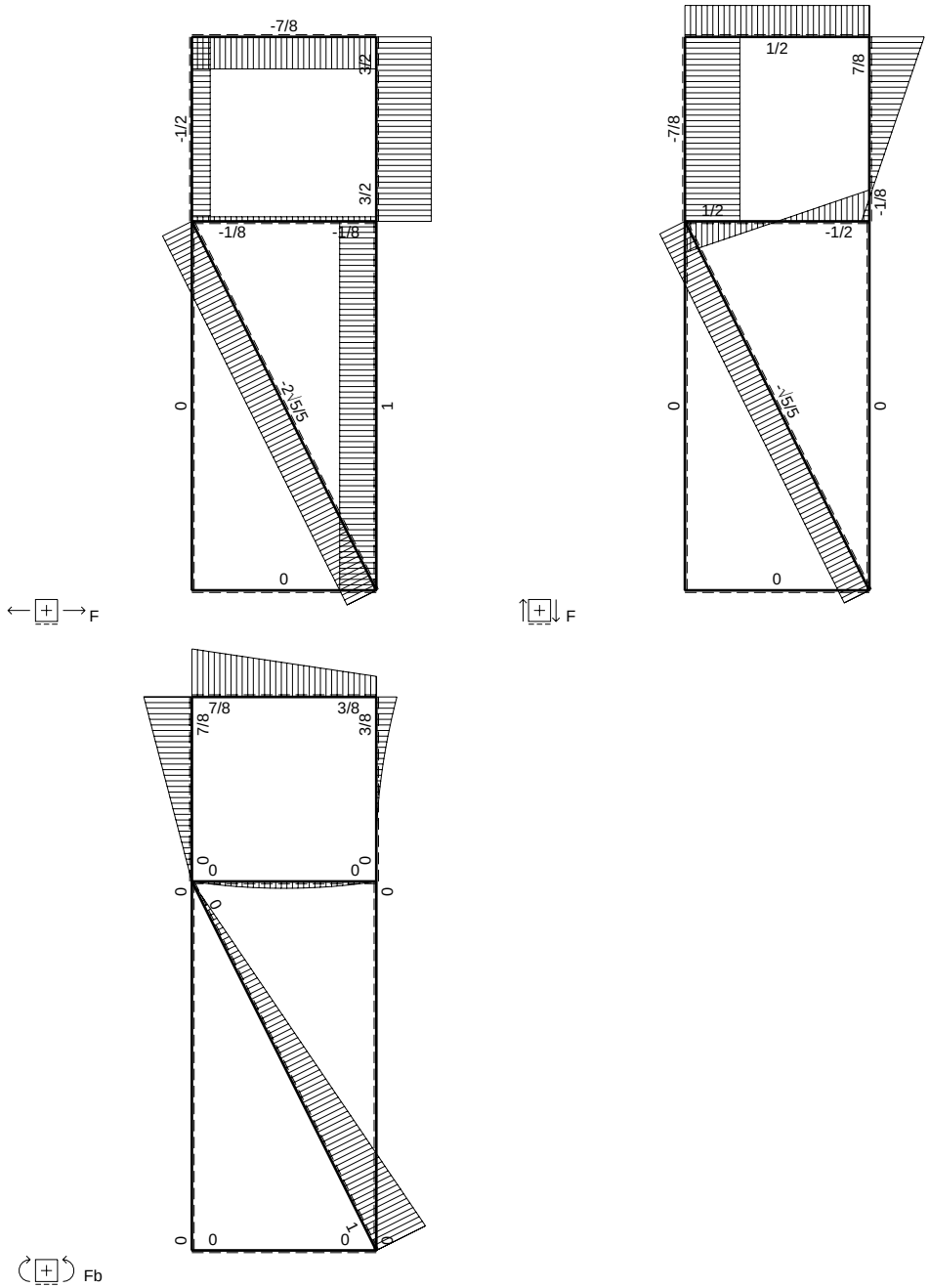


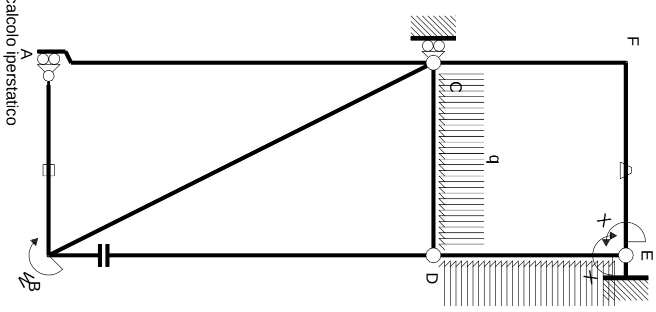
A = 534. mm^2
J_x = 189011. mm^4
J_y = 32196. mm^4
J_{xy} = -26635. mm^4
J_u = 193412. mm^4
J_v = 27796. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1637
$c = \cos\alpha$ = .9866
$s = \sin\alpha$ = .1630
x_g = 21.74 mm
y_g = 29.67 mm
N = -1164. N
T_y = 1940. N
M_x = 1024320. Nmm
x_m = 12. mm
u_m = -14.45 mm
v_m = -27.68 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 229.3 N/mm^2

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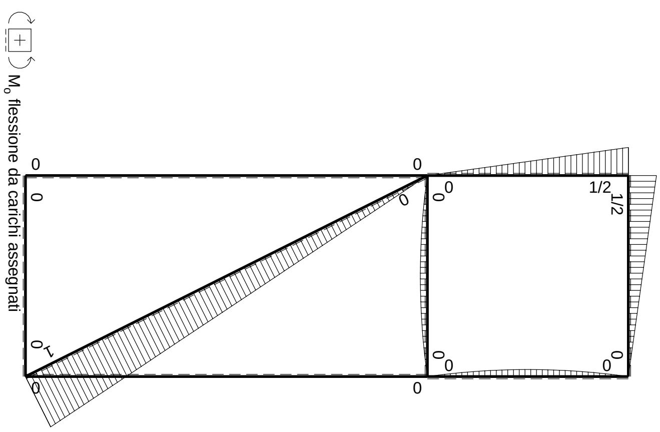
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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
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Fb
F
B
A
B
B

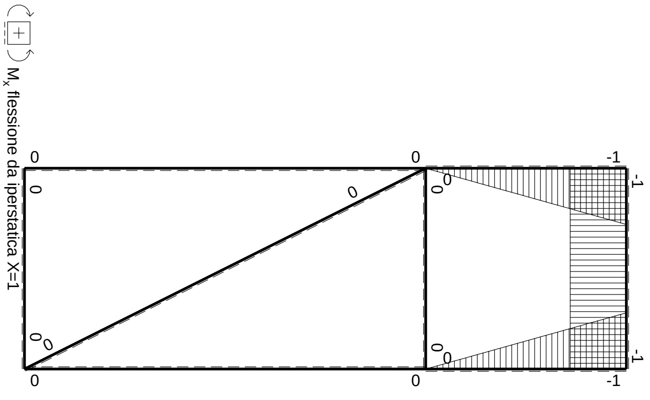

-7/8
3/2
-1/2
3/2
-1/8
-1/8
-2√5/5
0
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
-√5/5
0
0
F
7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

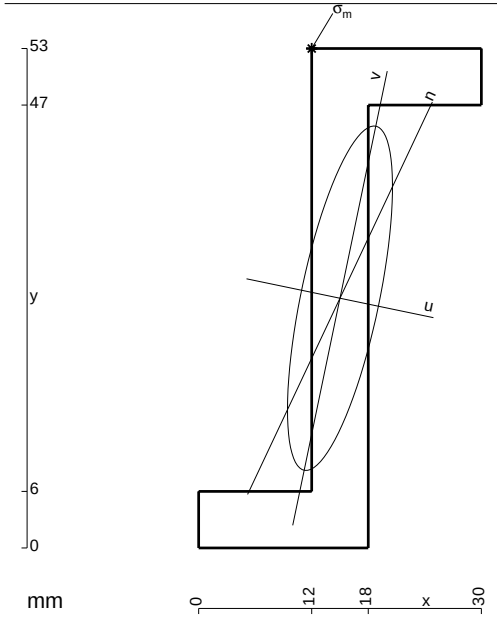
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

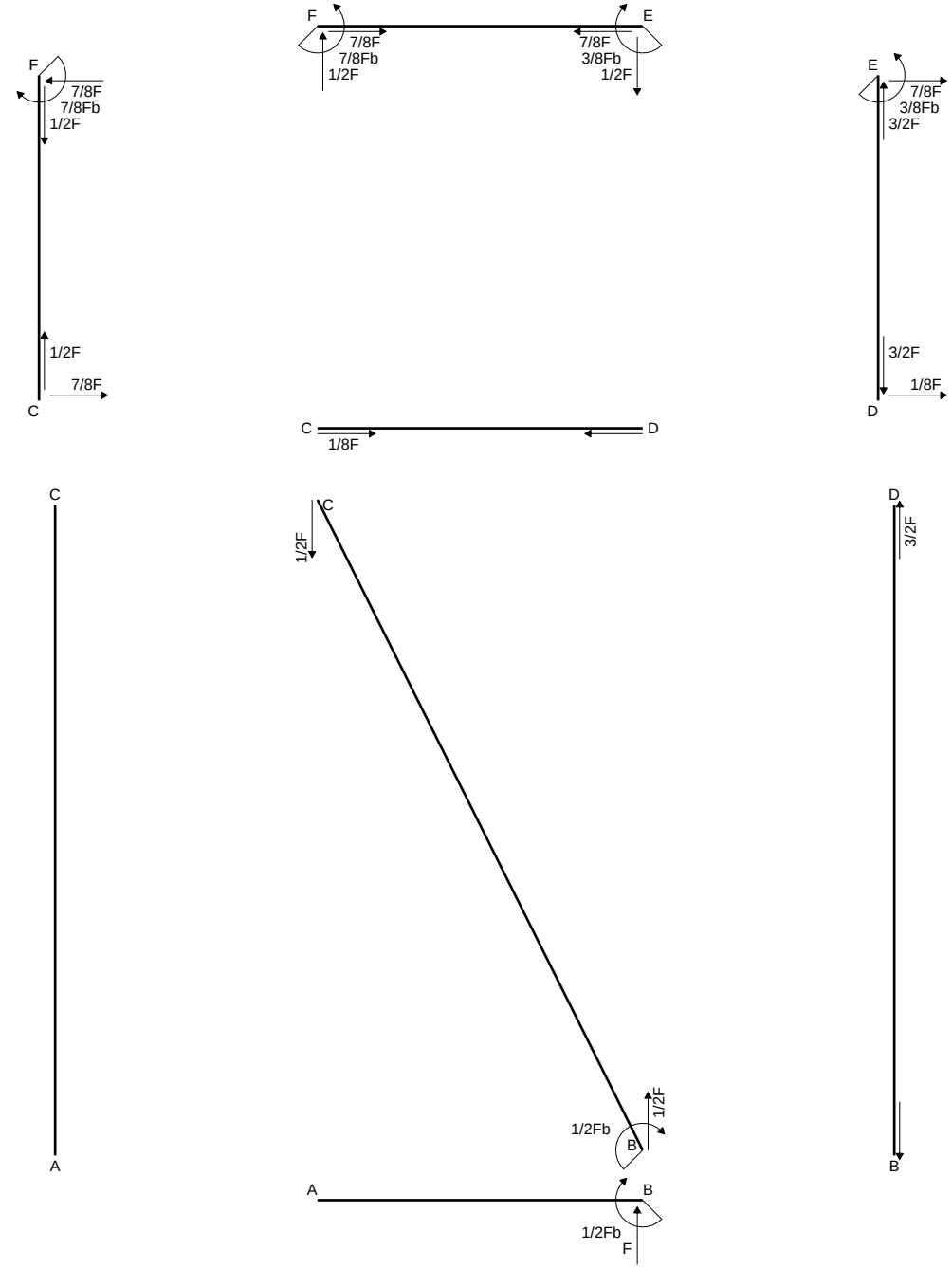
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

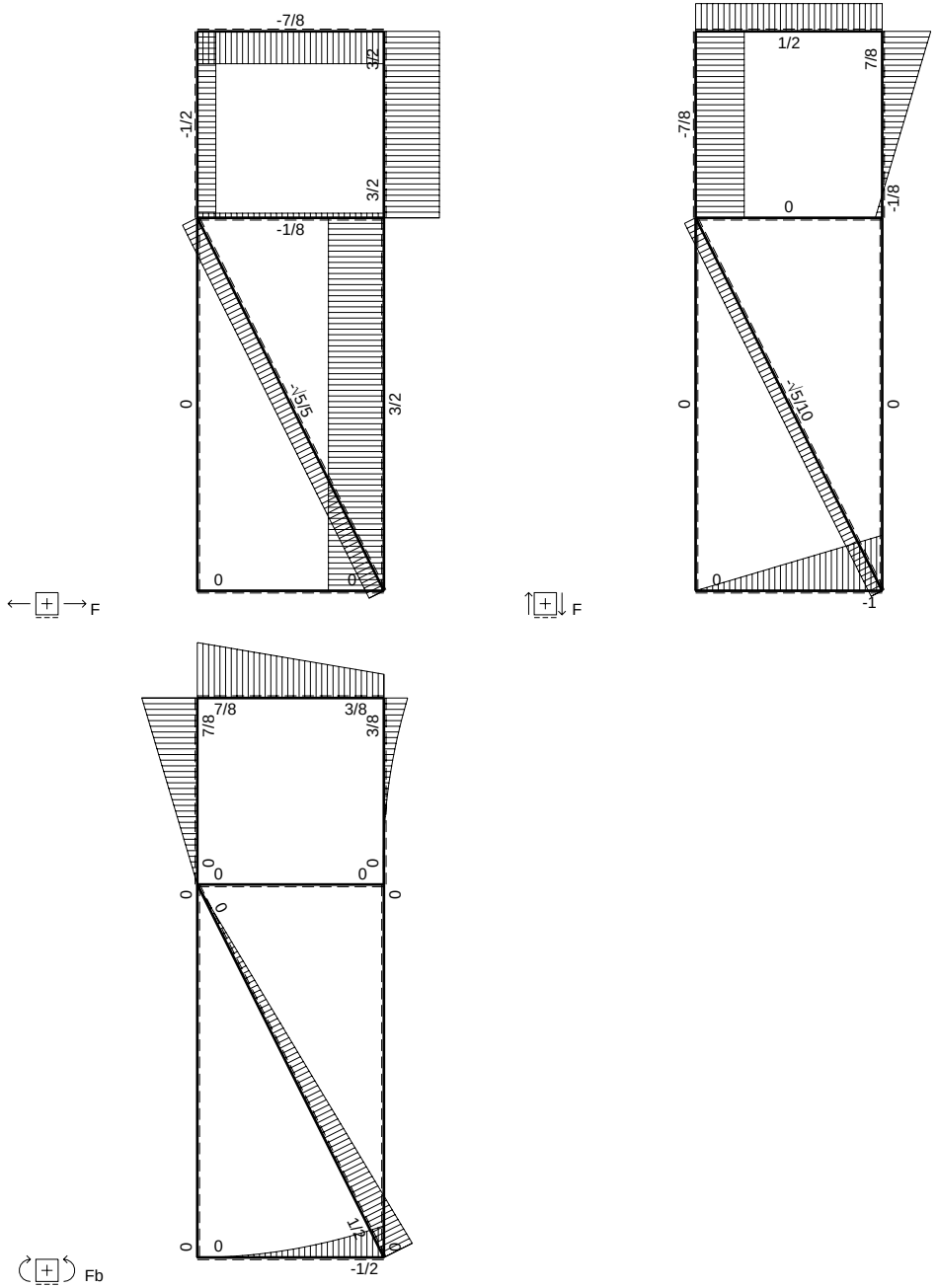


A = 462. mm^2
J_x = 154395. mm^4
J_y = 14346. mm^4
J_{xy} = 30456. mm^4
J_u = 160731. mm^4
J_v = 8010. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2051
c = cosα = .9790
s = sinα = -.2037
x_g = 15. mm
y_g = 26.5 mm
N = -1136. N
T_y = -568. N
M_x = 647700. Nmm
x_m = 12. mm
y_m = 53. mm
u_m = -8.335 mm
v_m = 25.33 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -239.7 N/mm^2

CI09.xxxx.154

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F
7/8F
7/8Fb
1/2F
1/2F
7/8F
C
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
D
1/8F
E
7/8F
3/8Fb
3/2F
3/2F
1/8F
D
C
A
C
1/2F
1/2Fb
B
1/2F
A
B
1/2Fb
F
D
3/2F
B


-7/8
3/2
-1/2
3/2
-1/8
0
-√5/5
3/2
0
0
F
1/2
7/8
-7/8
0
-1/8
-√5/10
0
0
-1
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
-√2/2
0
-1/2
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

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Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

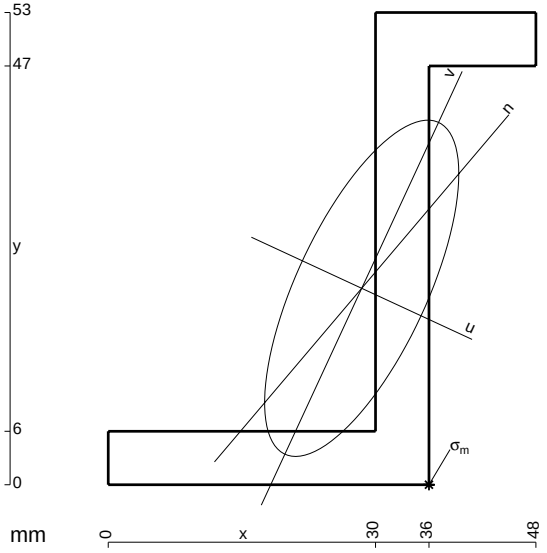
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

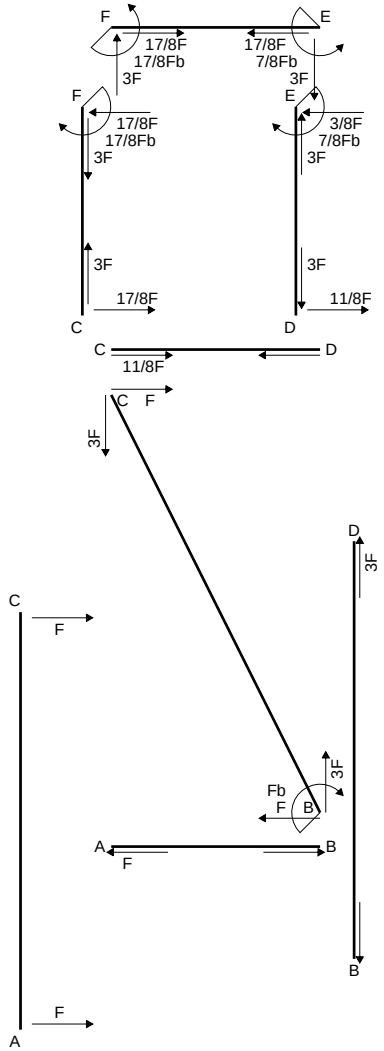
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

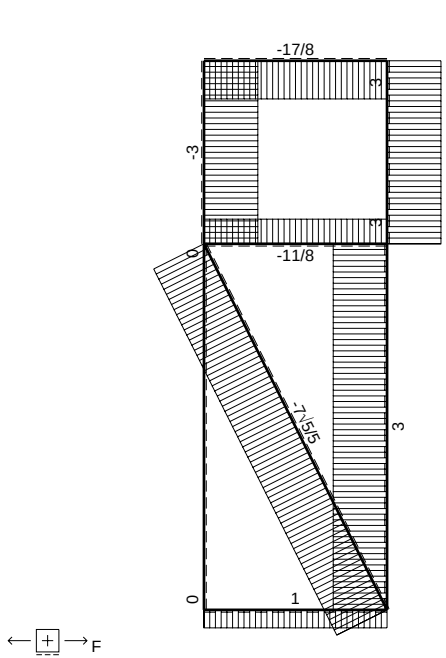


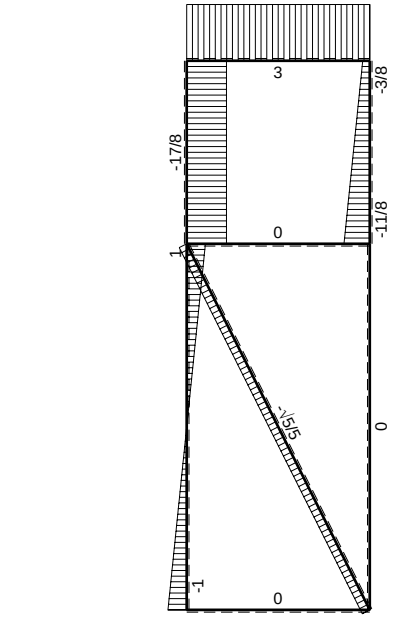
A = 570. mm^2
J_x = 203061. mm^4
J_y = 67683. mm^4
J_{xy} = 79827. mm^4
J_u = 240034. mm^4
J_v = 30710. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4337
$c = \cos\alpha$ = .9074
$s = \sin\alpha$ = -.4203
x_g = 28.45 mm
y_g = 22.05 mm
N = -735. N
T_y = -1286. N
M_x = 707438. Nmm
x_m = 36. mm
u_m = 16.11 mm
v_m = -16.83 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 199.7 N/mm^2

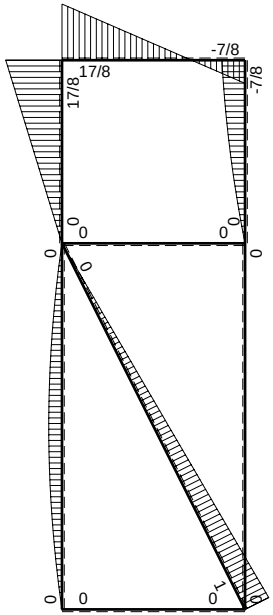
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Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
	totali						-35/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

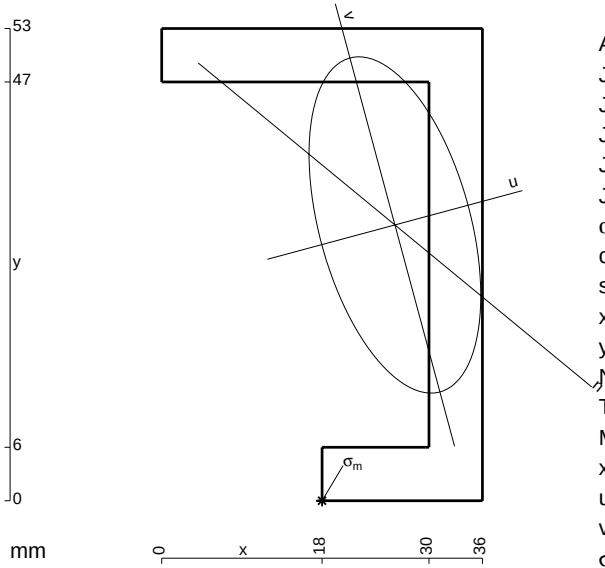
$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 986254 CI09.xxxx.156

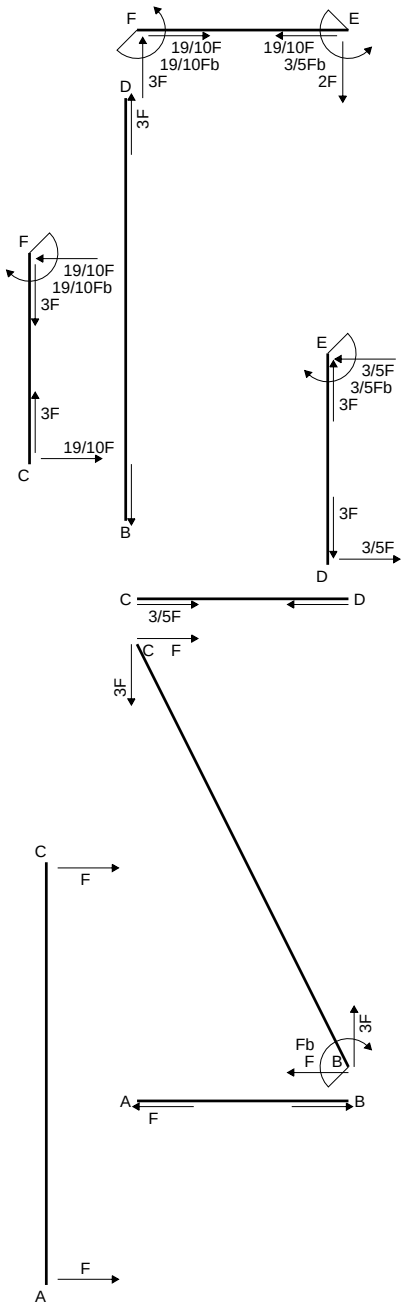


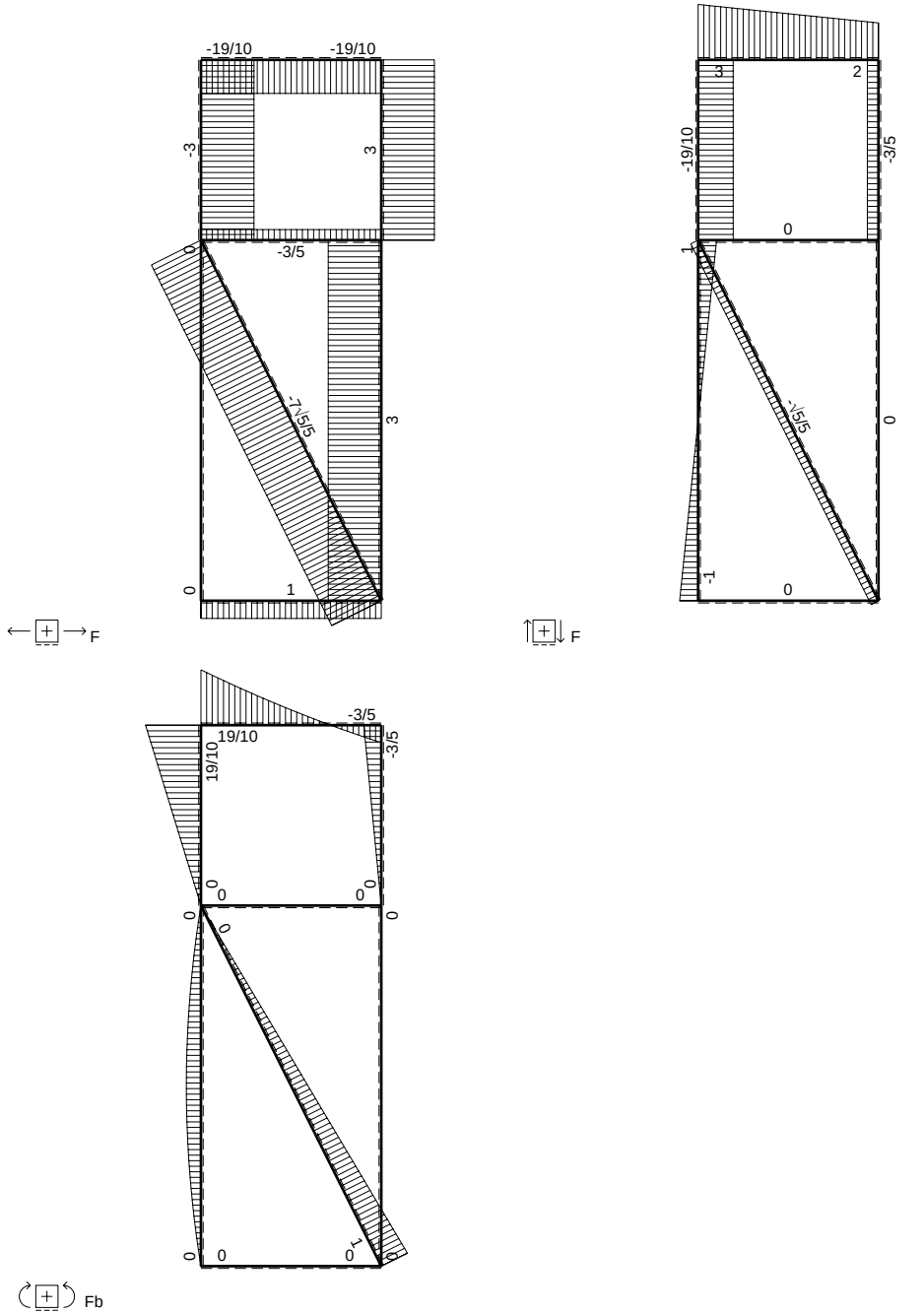
A = 570. mm^2
J_x = 203061. mm^4
J_y = 52950. mm^4
J_{xy} = -43600. mm^4
J_u = 214806. mm^4
J_v = 41205. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2631
c = cosα = .9656
s = sinα = .2601
x_g = 26.18 mm
y_g = 30.95 mm
N = -1594. N
T_y = 2250. N
M_x = 940313. Nmm
x_m = 18. mm
u_m = -15.95 mm
v_m = -27.76 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 209.2 N/mm^2

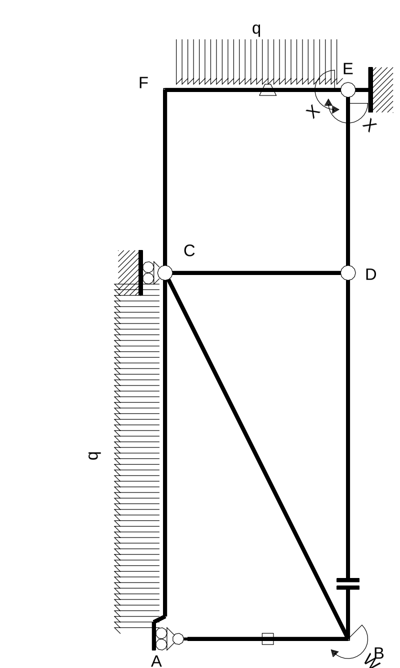
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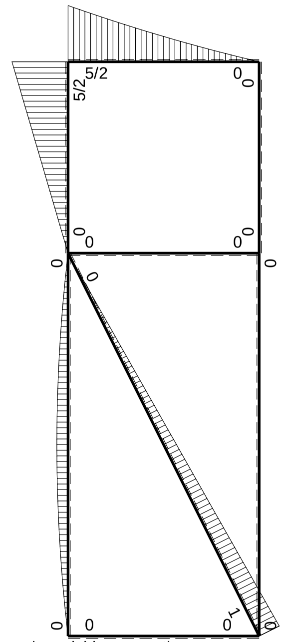
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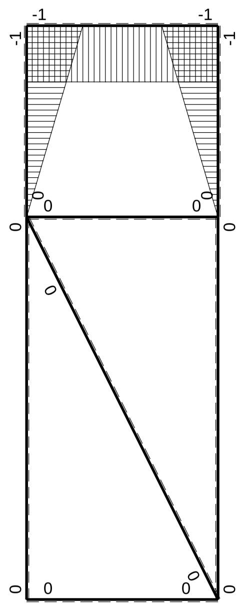

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19/10
-3/5
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1
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

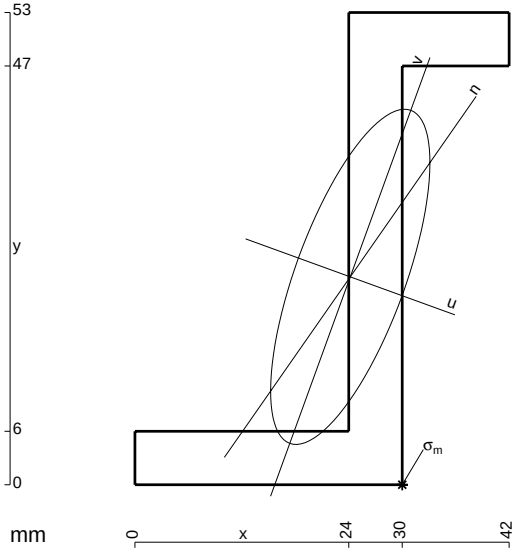
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

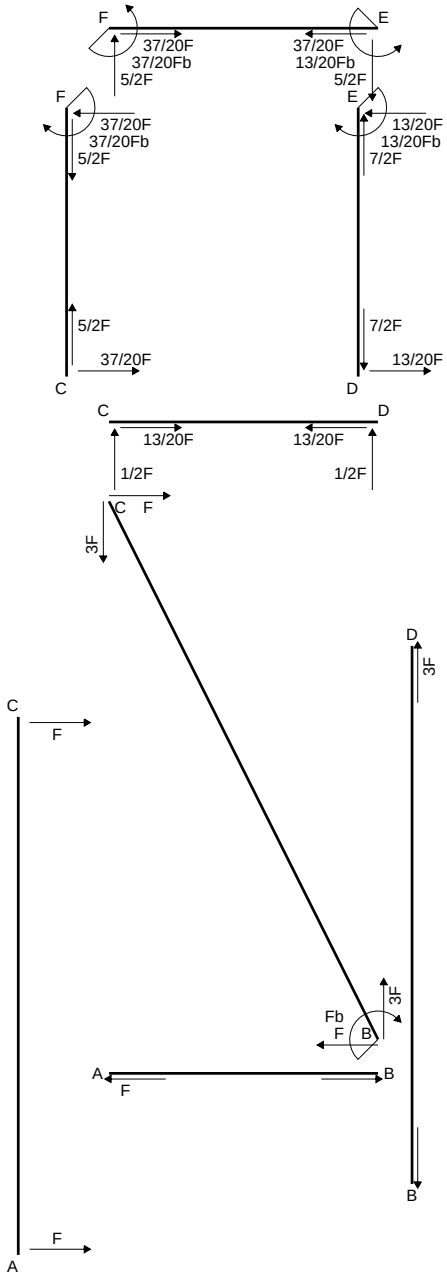
$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

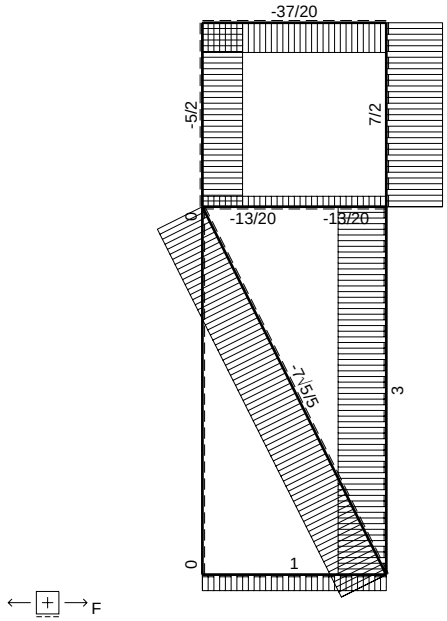


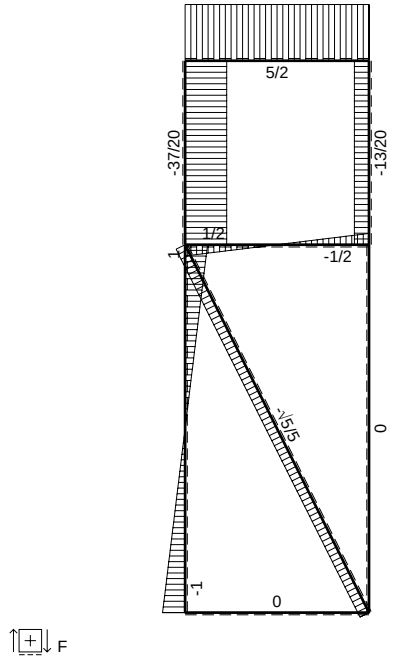
A = 534. mm^2
J_x = 189011. mm^4
J_y = 42681. mm^4
J_{xy} = 61197. mm^4
J_u = 211231. mm^4
J_v = 20461. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3483
c = cosα = .9400
s = sinα = -.3413
x_g = 24.17 mm
y_g = 23.33 mm
N = -1140. N
T_y = 1800. N
M_x = 706800. Nmm
x_m = 30. mm
u_m = 13.44 mm
v_m = -19.94 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 219.1 N/mm^2

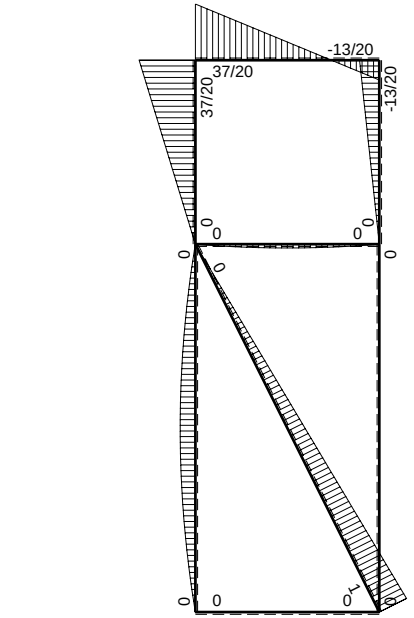
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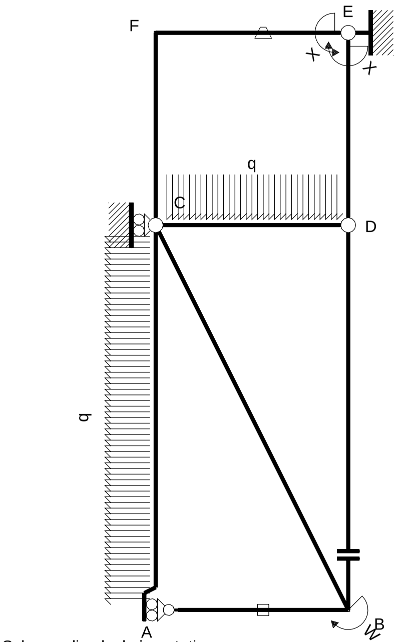
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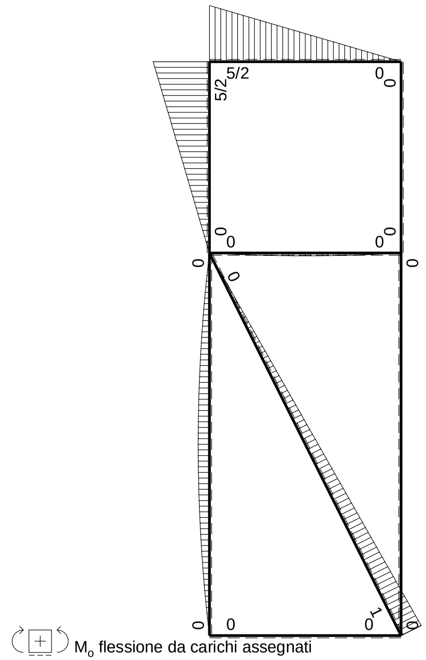

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-1
-5/5
0
-1
0
F

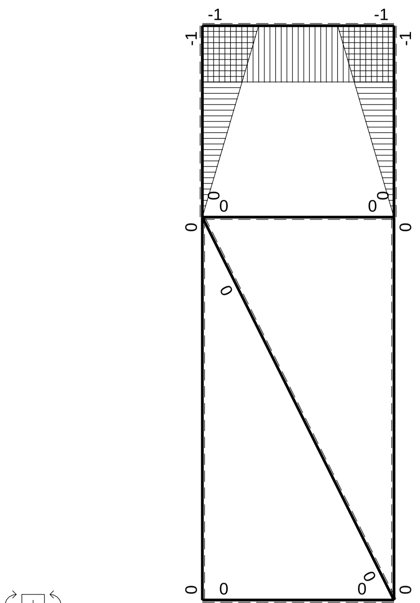

-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

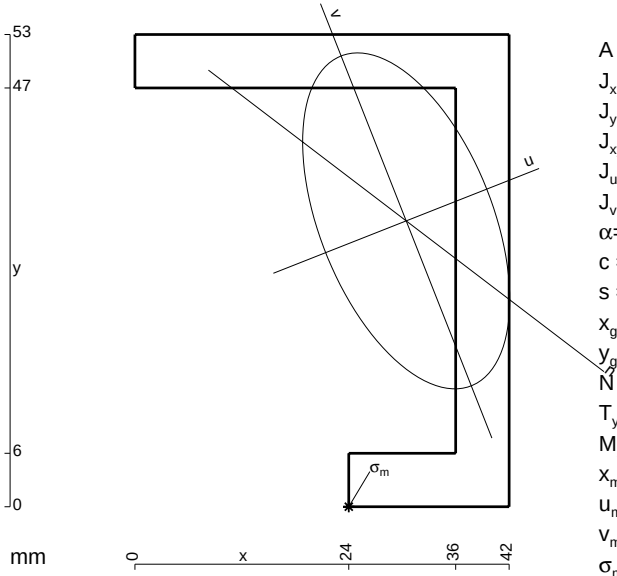
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

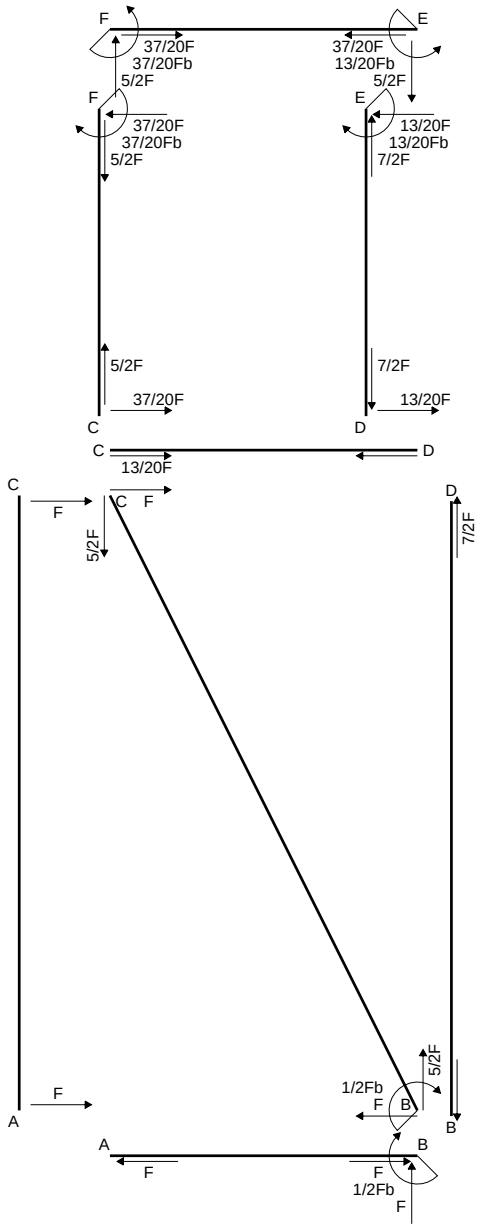
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

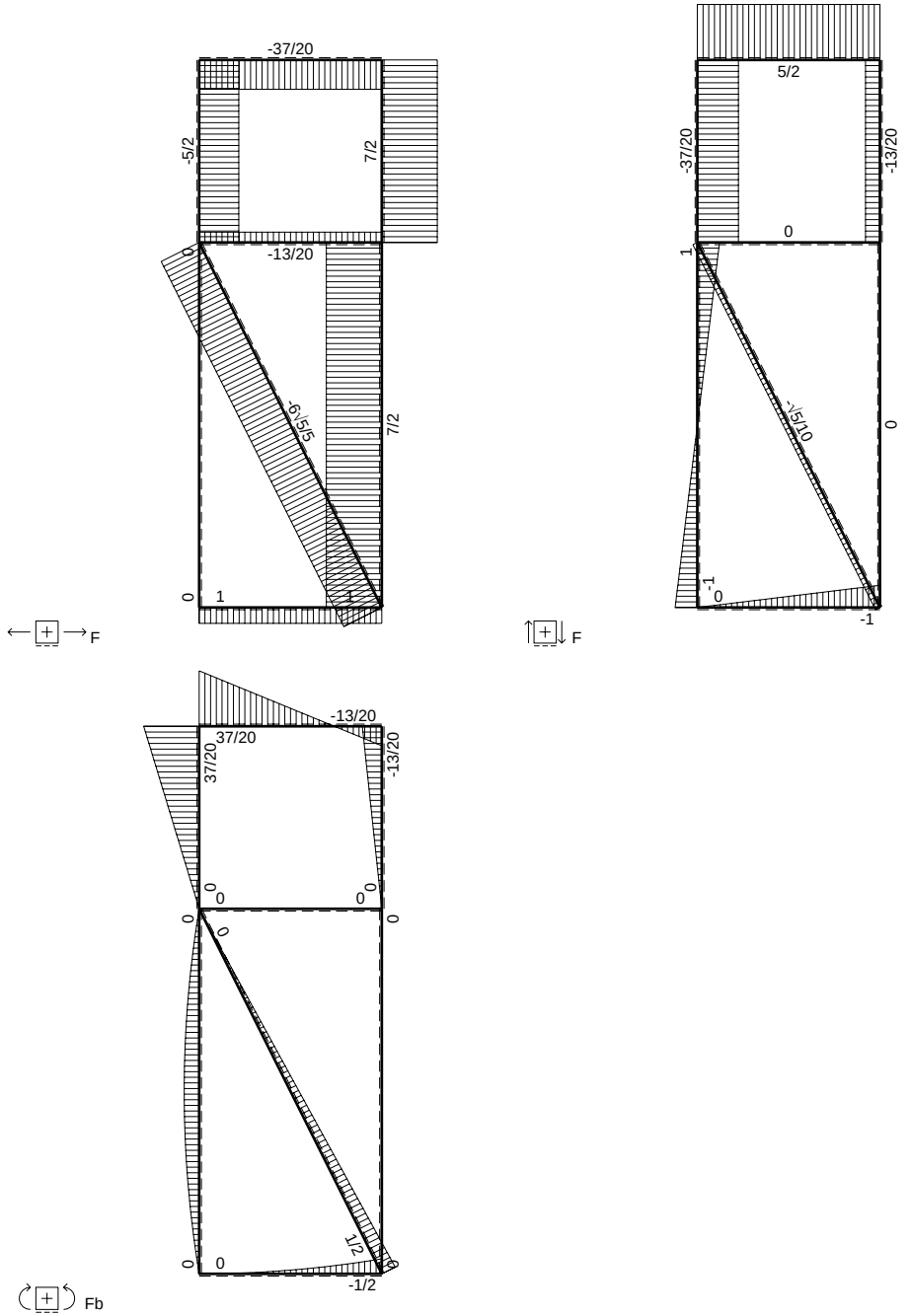


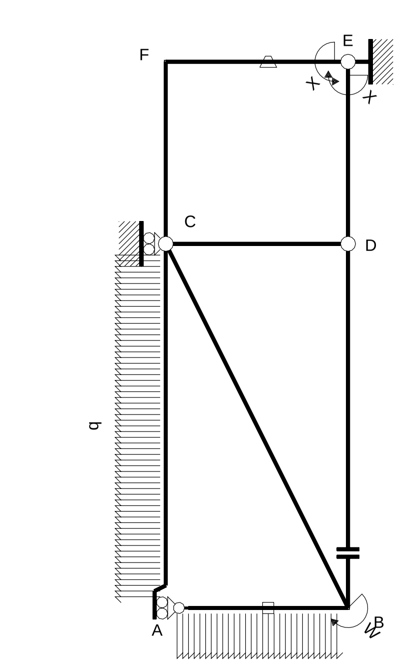
A = 606. mm^2
J_x = 215454. mm^4
J_y = 81888. mm^4
J_{xy} = -62420. mm^4
J_u = 240083. mm^4
J_v = 57258. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3758
c = cosα = .9302
s = sinα = .3670
x_g = 30.45 mm
y_g = 32.08 mm
N = -1591. N
T_y = 2150. N
M_x = 1050060. Nmm
x_m = 24. mm
u_m = -17.77 mm
v_m = -27.48 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 228.8 N/mm^2

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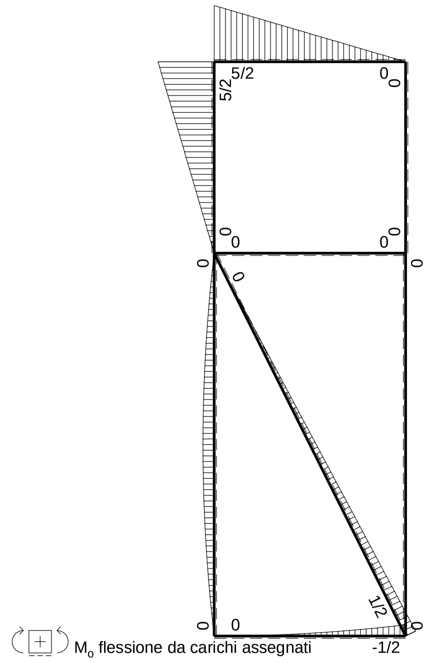
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F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb

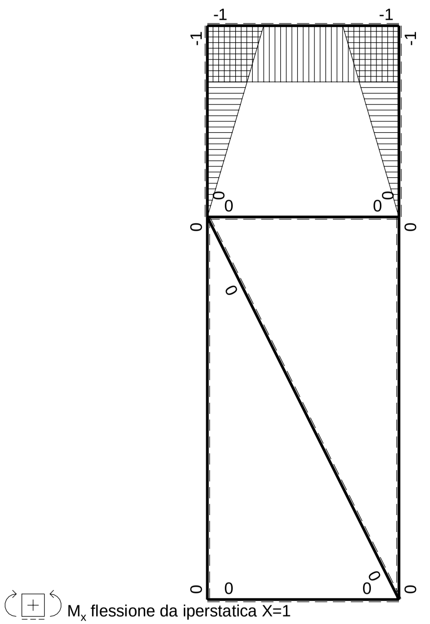

-37/20
-5/2
7/2
-13/20
-5\√5/5
1
F
5/2
-37/20
-13/20
-5\√5/10
-1
0
37/20
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

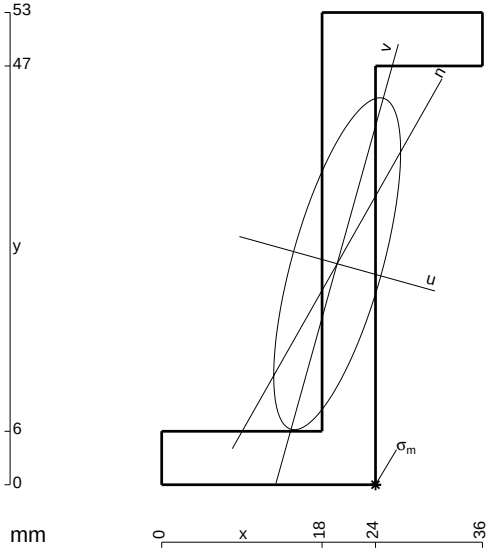
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

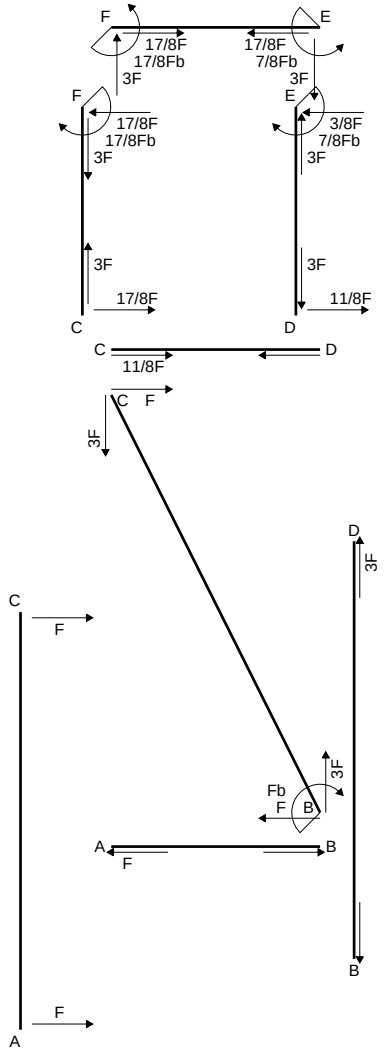
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

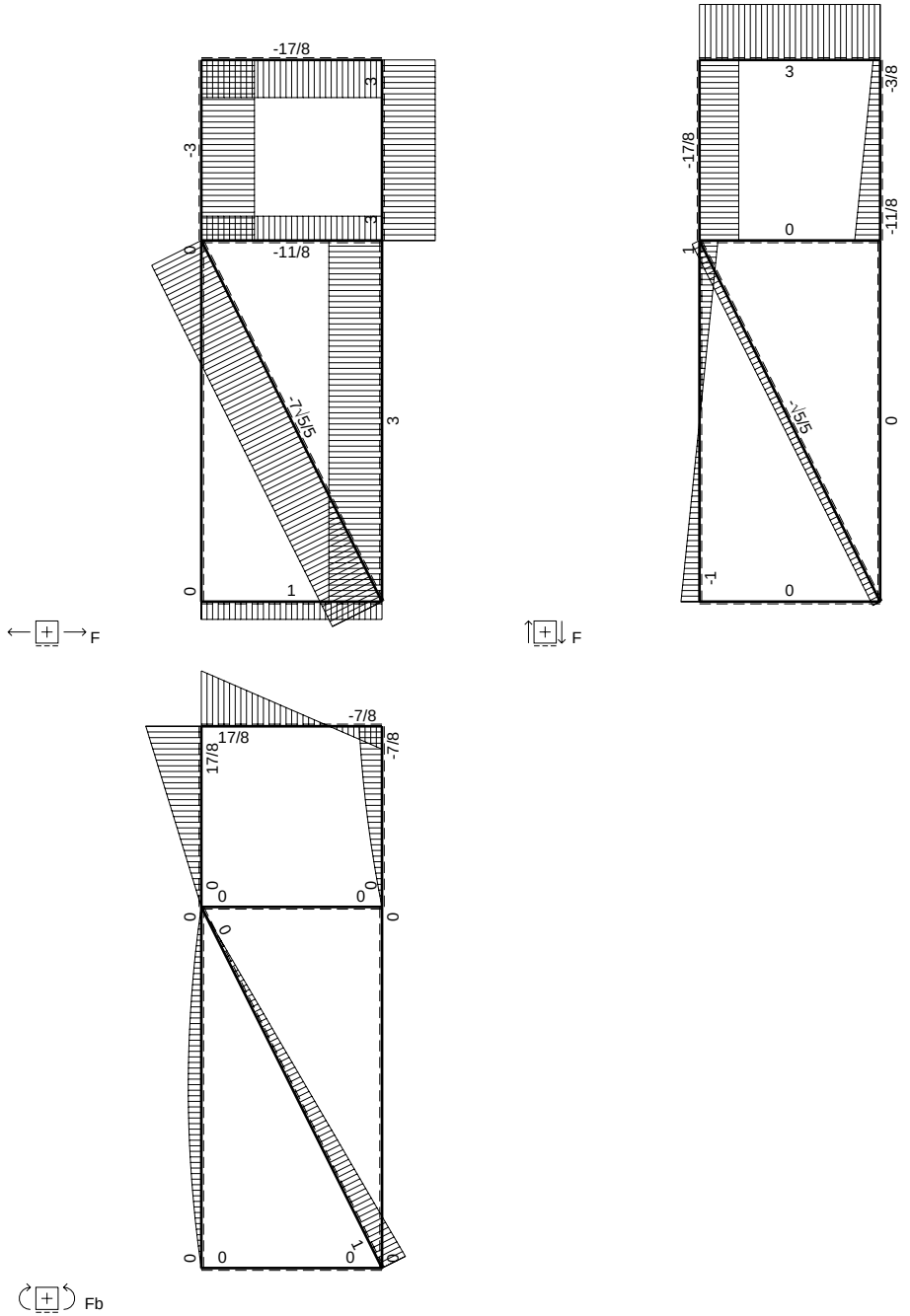


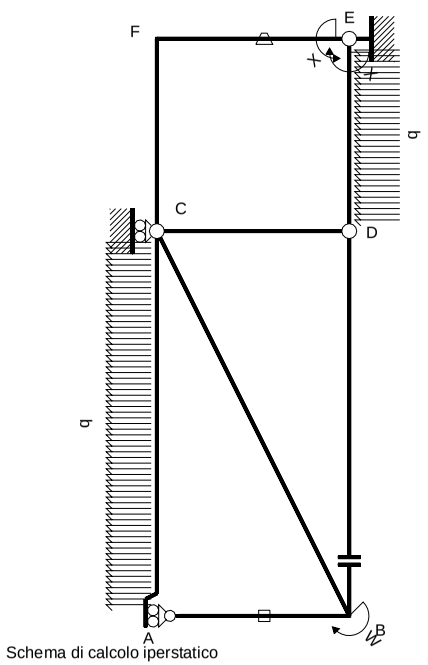
A = 498. mm^2
J_x = 172946. mm^4
J_y = 25275. mm^4
J_{xy} = 44583. mm^4
J_u = 185362. mm^4
J_v = 12859. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2716
c = cosα = .9633
s = sinα = -.2683
x_g = 19.7 mm
y_g = 24.8 mm
N = -999. N
T_y = 1350. N
M_x = 699300. Nmm
x_m = 24. mm
u_m = 10.8 mm
v_m = -22.74 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = 238.2 N/mm^2

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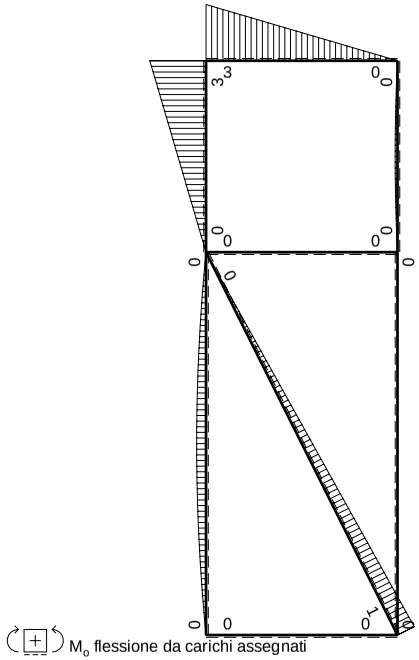
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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
A
F
B

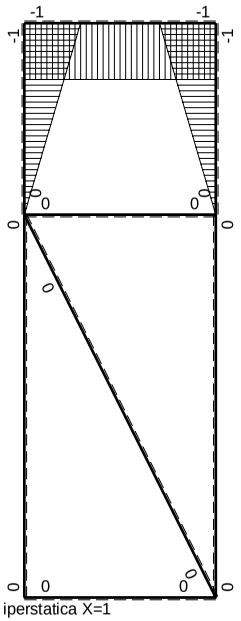

-17/8
-3
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-11/8
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-7√5/5
3
0
1
F
-17/8
3
-3/8
-11/8
0
-7√5/5
0
-1
0
F
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
				totali			-35/24Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 236686 CI09.xxxx.160

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

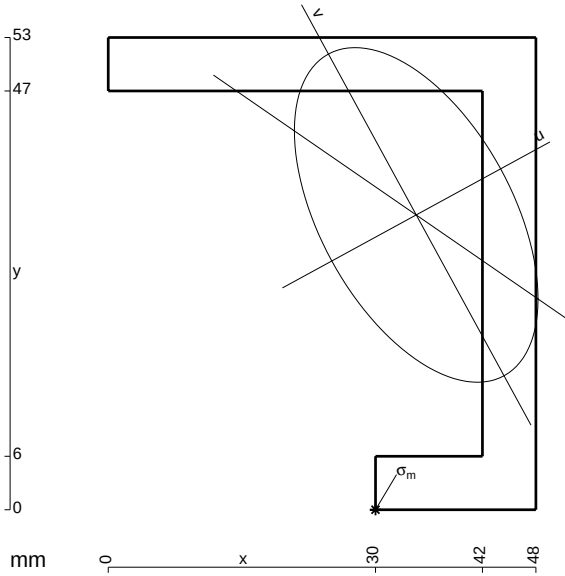
$= (-3\ b +3/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

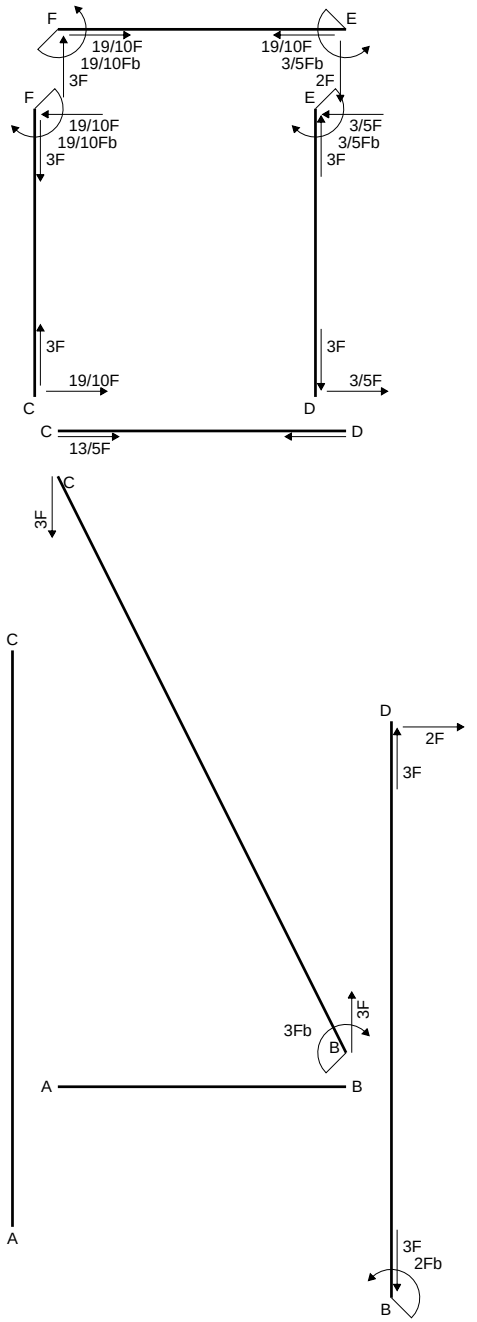
$= (-\ b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

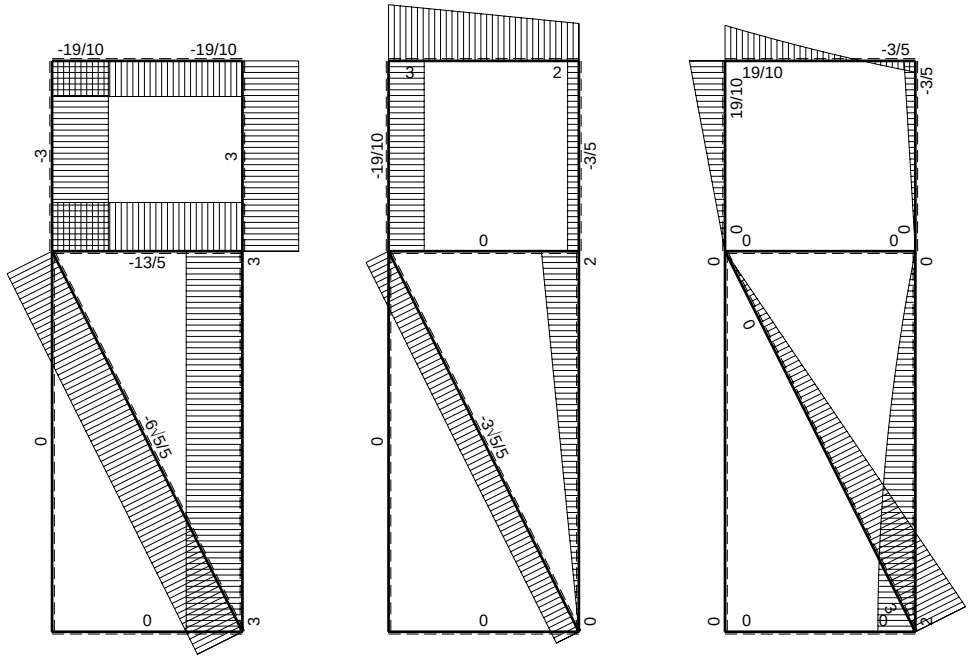


A = 642. mm^2
J_x = 226469. mm^4
J_y = 120007. mm^4
J_{xy} = -82782. mm^4
J_u = 271657. mm^4
J_v = 74819. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4997
c = cosα = .8777
s = sinα = .4791
x_g = 34.57 mm
y_g = 33.09 mm
N = -2571. N
T_y = 3630. N
M_x = 951363. Nmm
x_m = 30. mm
u_m = -19.87 mm
v_m = -26.85 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 199.6 N/mm^2

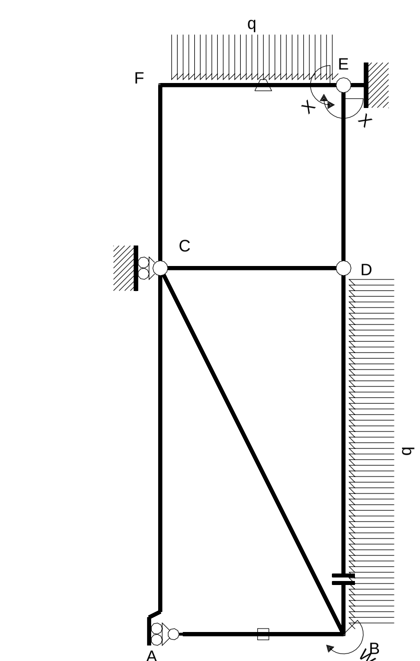
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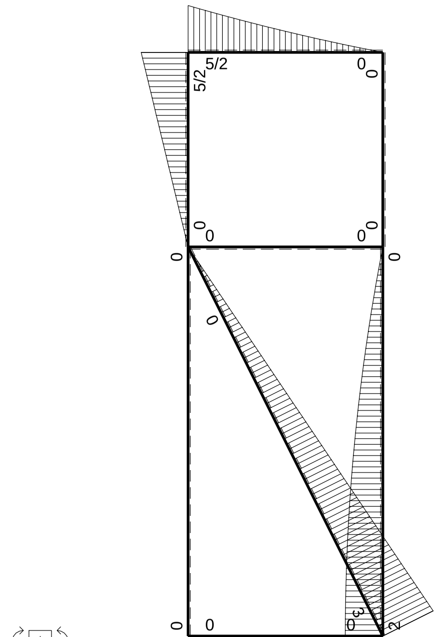

F
E
19/10F
19/10Fb
3F
19/10F
3/5Fb
2F
F
E
19/10F
19/10Fb
3F
3/5F
3/5Fb
3F
3F
19/10F
C
D
3F
3/5F
C
D
13/5F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-19/10
-19/10
-3
3
3
-13/5
0
-6√5/5
0
3
3
2
-19/10
-3/5
0
2
-3√5/5
0
0
19/10
-3/5
0
0
0
0
0
0
0
0

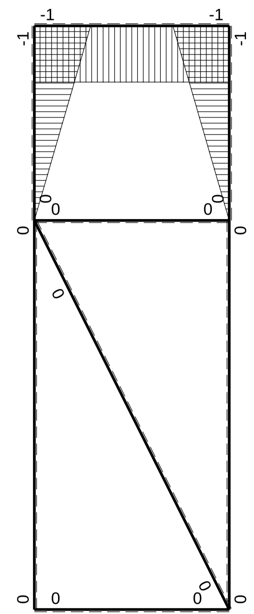
F F Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

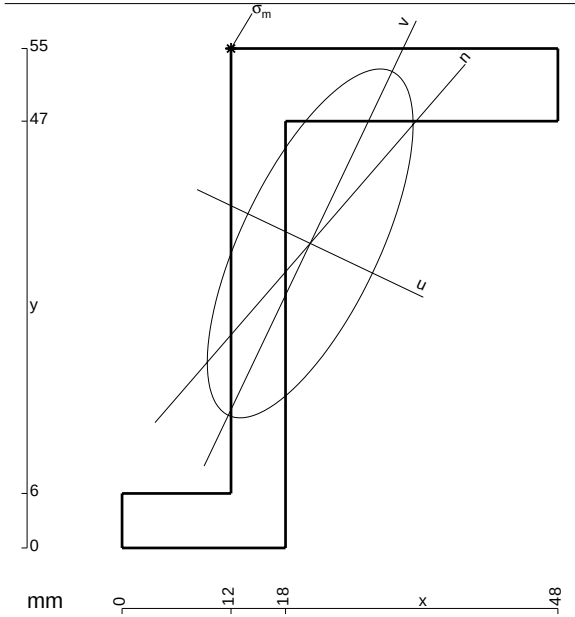
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

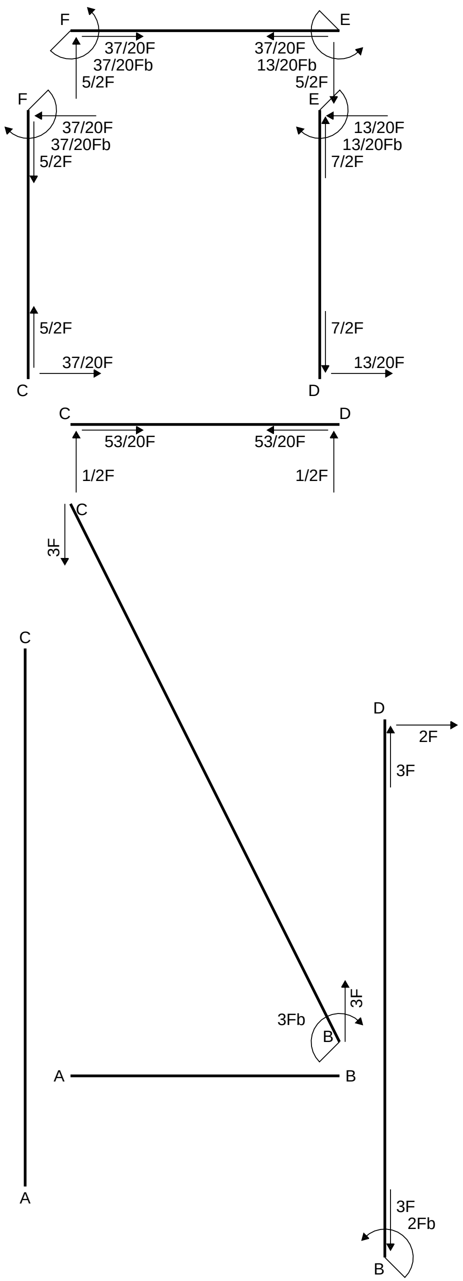
$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

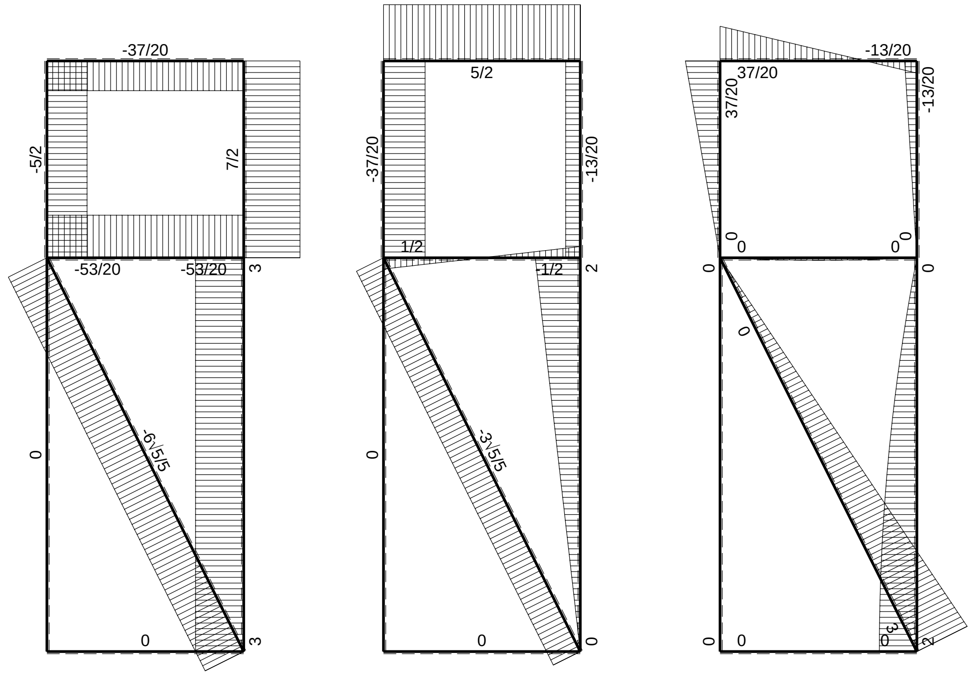


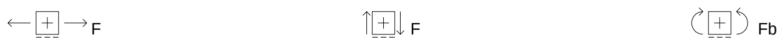
A = 642. mm^2
J_x = 237041. mm^4
J_y = 82444. mm^4
J_{xy} = 95227. mm^4
J_u = 282393. mm^4
J_v = 37091. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4445
$c = \cos\alpha$ = .9028
$s = \sin\alpha$ = -.4300
x_g = 20.72 mm
y_g = 33.54 mm
N = -1771. N
T_y = -885.5 N
M_x = 831600. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -17.1 mm
v_m = 15.63 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -209.2 N/mm^2

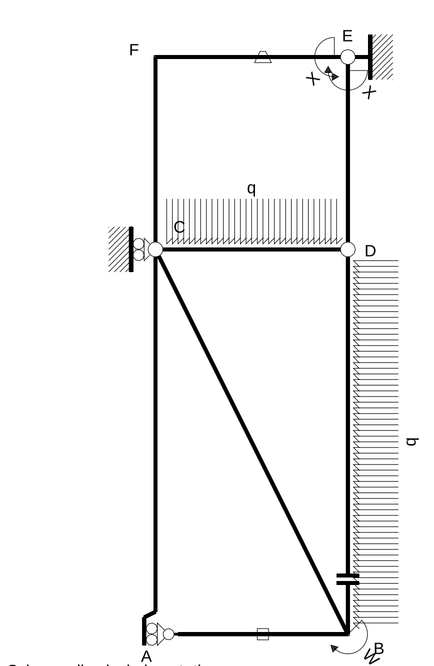
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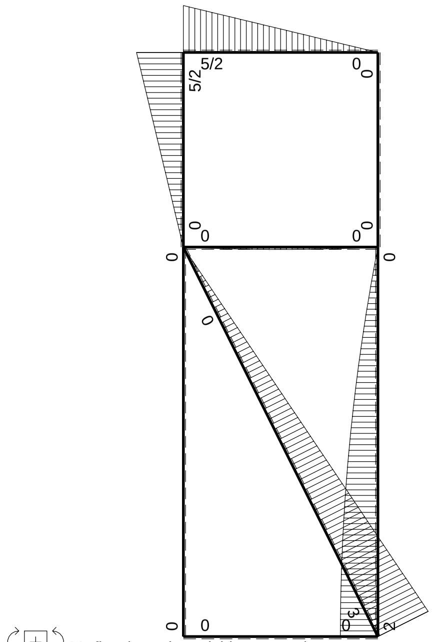

F
E
37/20F
37/20Fb
5/2F
13/20Fb
13/20F
7/2F
C
D
53/20F
1/2F
3F
2F
3Fb
A
B
2Fb


-37/20
-5/2
7/2
-53/20
3
0
-6√5/5
5/2
-37/20
-13/20
1/2
-1/2
2
-3√5/5
-13/20
37/20

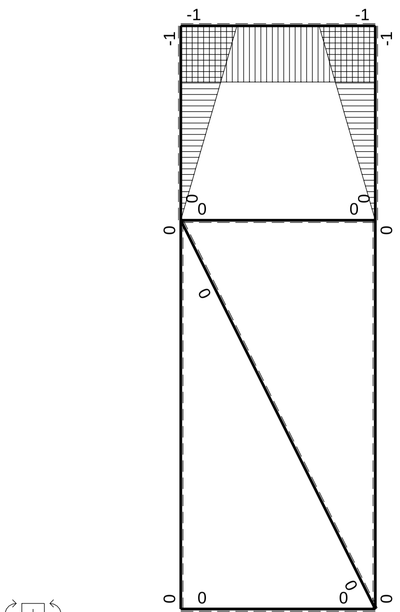

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

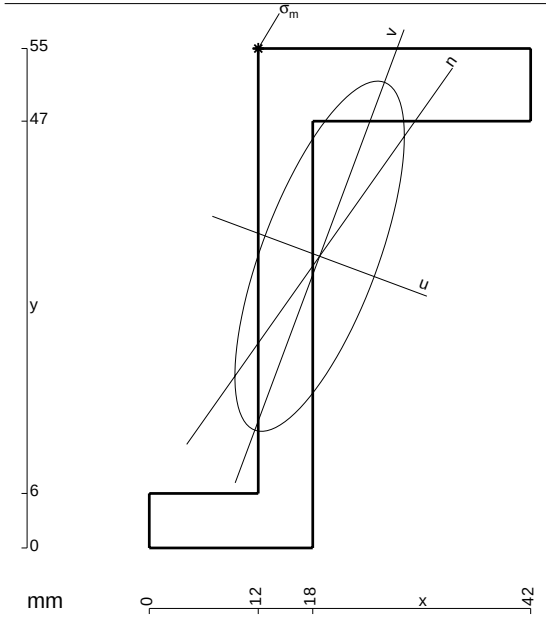
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

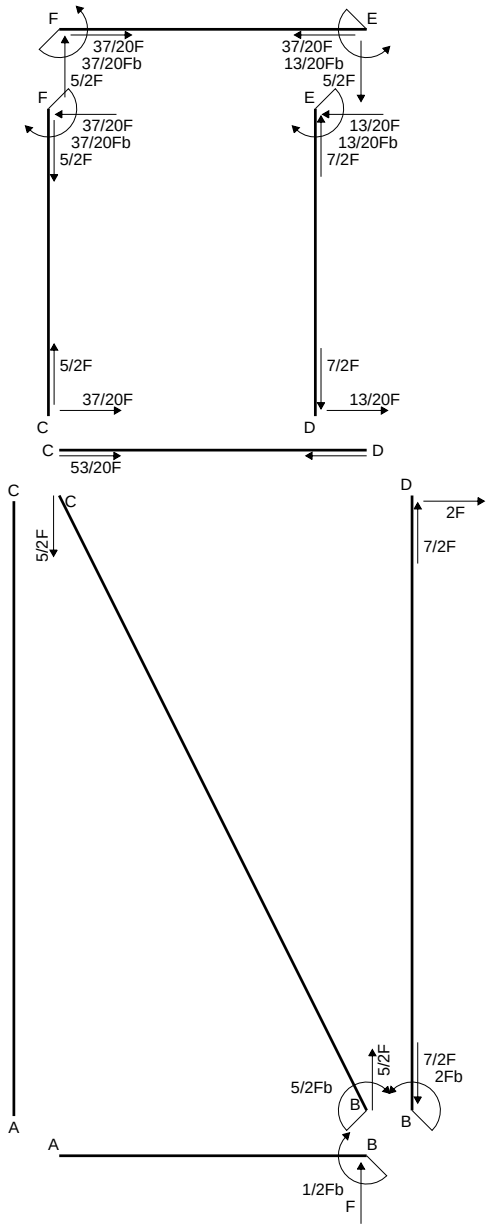
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

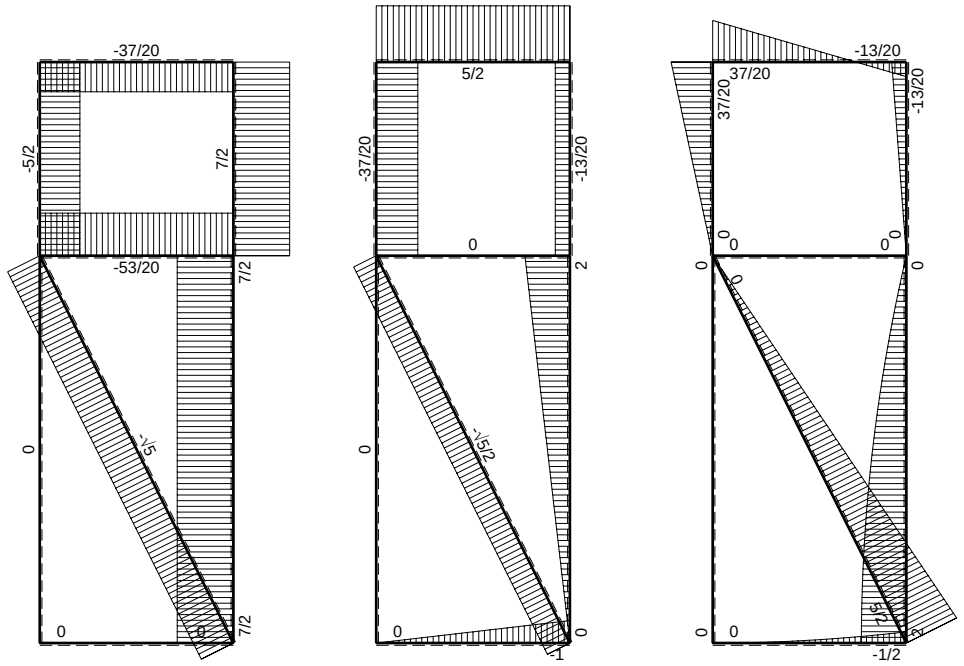


A = 594. mm^2
J_x = 220965. mm^4
J_y = 51715. mm^4
J_{xy} = 73230. mm^4
J_u = 248250. mm^4
J_v = 24429. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3567
c = cosα = .9371
s = sinα = -.3492
x_g = 18.76 mm
y_g = 32.13 mm
N = -1503. N
T_y = -751.3 N
M_x = 772800. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -14.32 mm
v_m = 19.07 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -216.3 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
13/20Fb
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
53/20F
2F
7/2F
5/2F
5/2Fb
2Fb
A
B
1/2Fb
F


-37/20
-5/2
7/2
-53/20
0
-5/2
5/2
-13/20
2
-1
37/20
-1/2


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$5/2Fb-\sqrt{5}/2Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 264603 CI09.xxxx.163

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

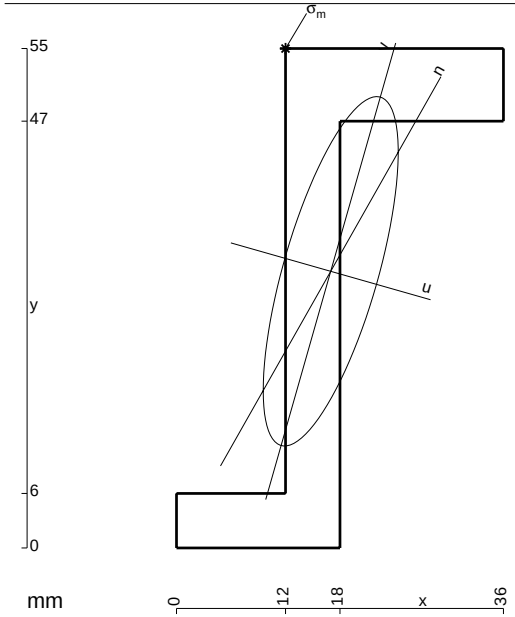
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

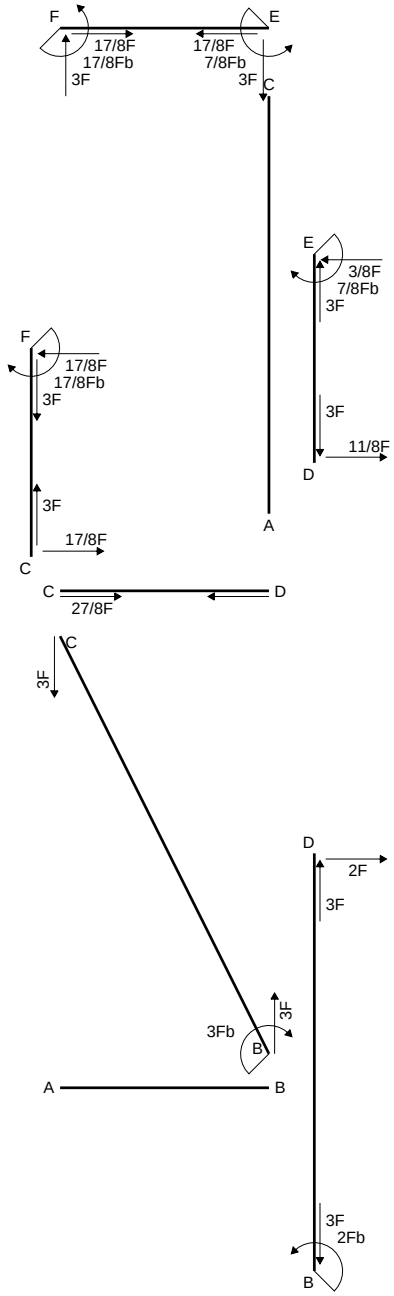
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

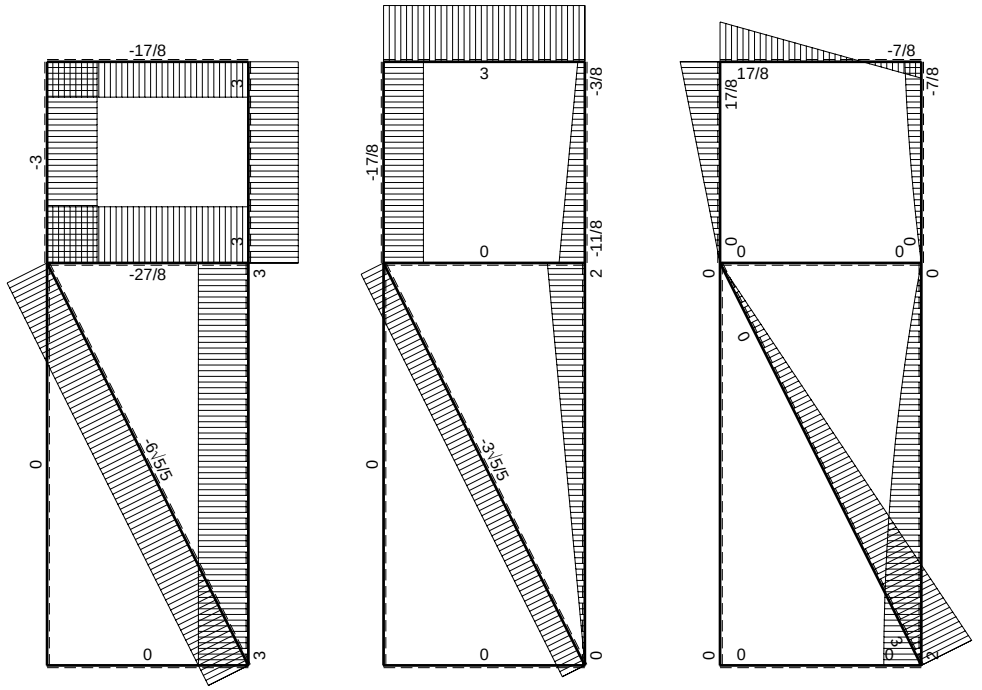


A = 546. mm^2
J_x = 202107. mm^4
J_y = 30174. mm^4
J_{xy} = 53280. mm^4
J_u = 217279. mm^4
J_v = 15002. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = -.2774
c = cosα = .9618
s = sinα = -.2739
x_g = 16.98 mm
y_g = 30.47 mm
N = -1319. N
T_y = -659.6 N
M_x = 737500. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -11.51 mm
v_m = 22.23 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.9 N/mm^2

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F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
C
E
3/8F
7/8Fb
3F
F
17/8F
17/8Fb
3F
3F
11/8F
D
A
3F
17/8F
C
C
D
27/8F
C
3F
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
B


-17/8
3
-3
3
-27/8
3
0
-6√5/5
0
3
-17/8
3
-3/8
0
-11/8
2
-3√5/5
0
0
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
0
0
3


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^{2}b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
totali							-35/24Fb^2/EJ	5/3Xb/EJ
iperstatica $X=W_{EF}$							7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

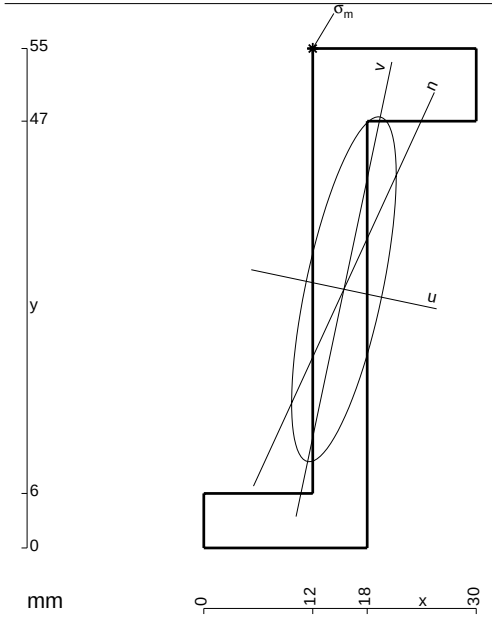
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

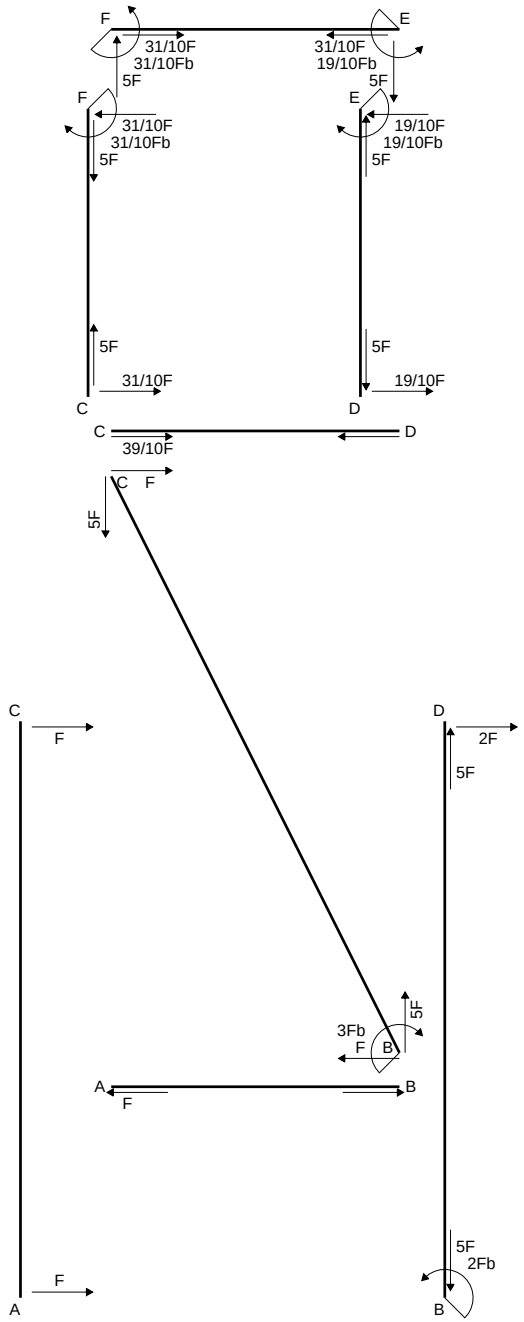
$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

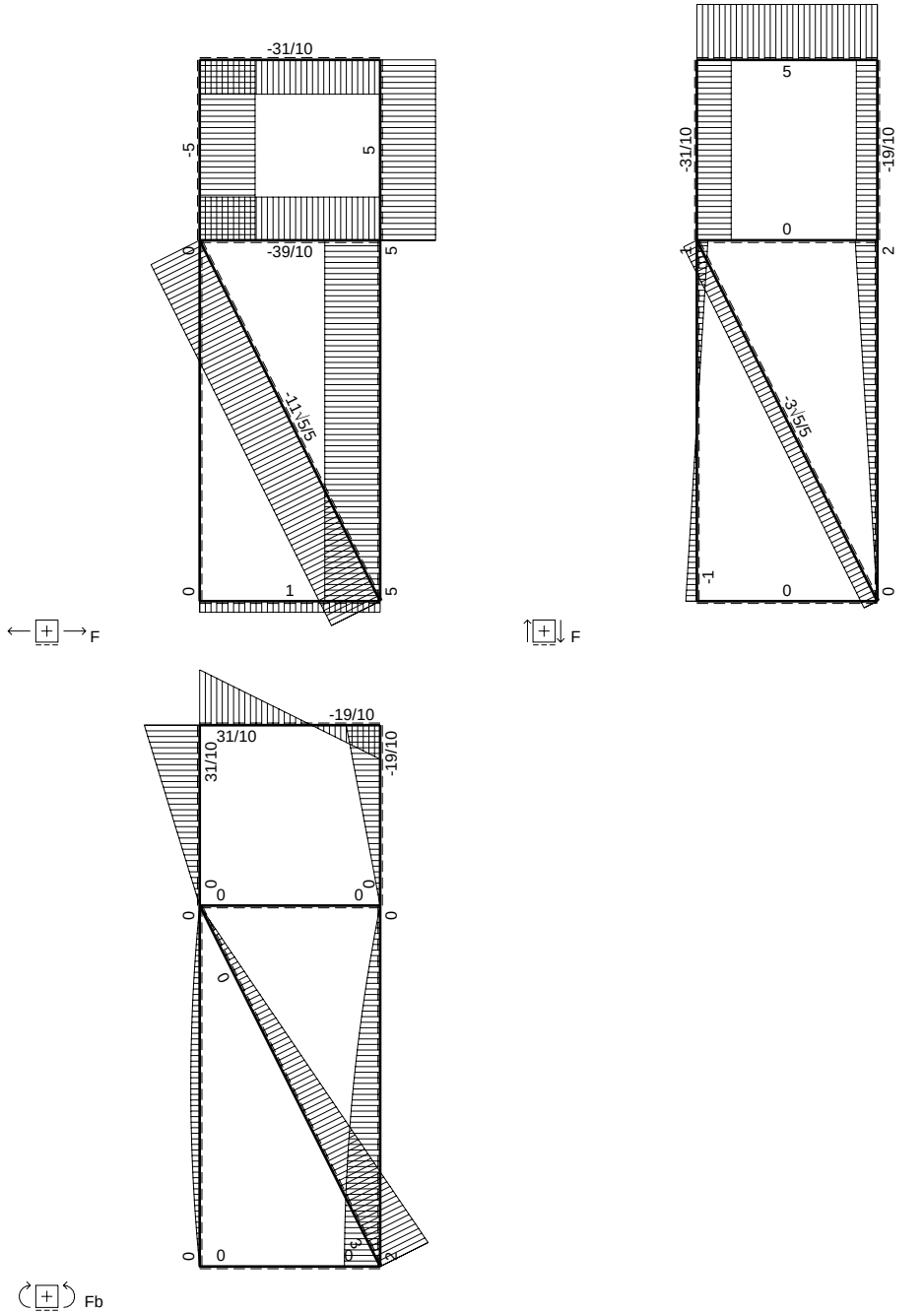


A = 498. mm^2
J_x = 179663. mm^4
J_y = 16520. mm^4
J_{xy} = 35967. mm^4
J_u = 187241. mm^4
J_v = 8943. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.2076$
$c = \cos\alpha = .9785$
$s = \sin\alpha = -.2061$
x_g = 15.43 mm
y_g = 28.49 mm
N = -1154. N
T_y = -576.9 N
M_x = 696600. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -8.825 mm
v_m = 25.23 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v = -235.9\ N/mm^2$

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F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
C
D
39/10F
2F
2Fb
3Fb
A
B


-31/10
-39/10
-11√5/5
-3√5/5
-19/10
31/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
totali							-19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						19/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 265269 CI09.xxxx.165

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

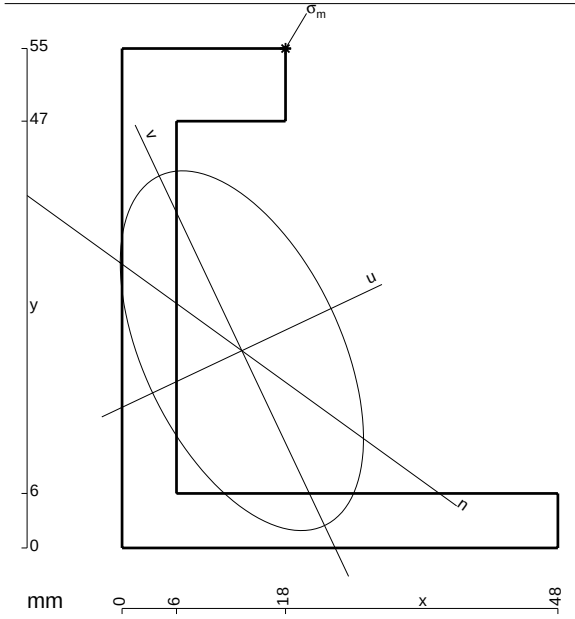
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

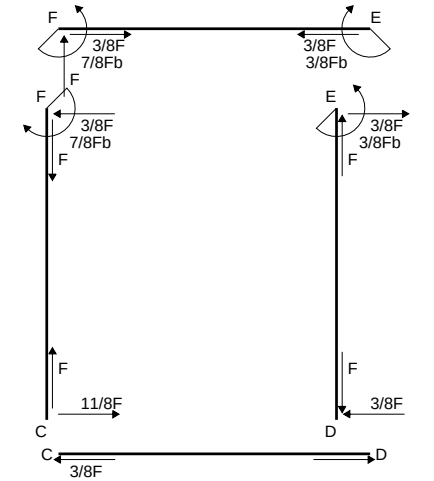
$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

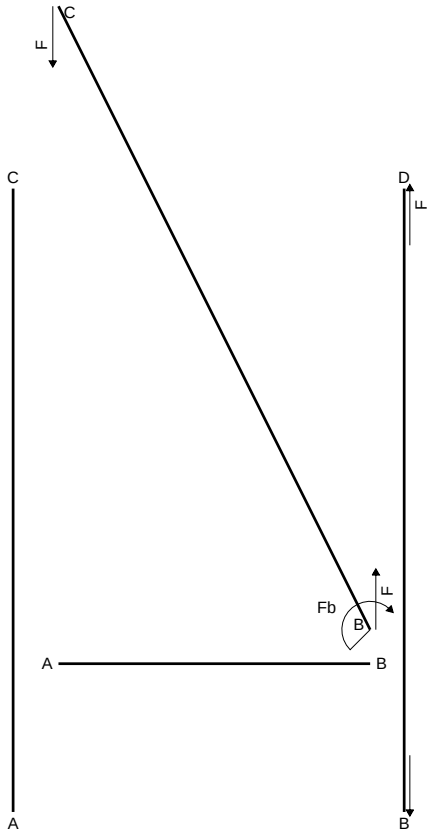


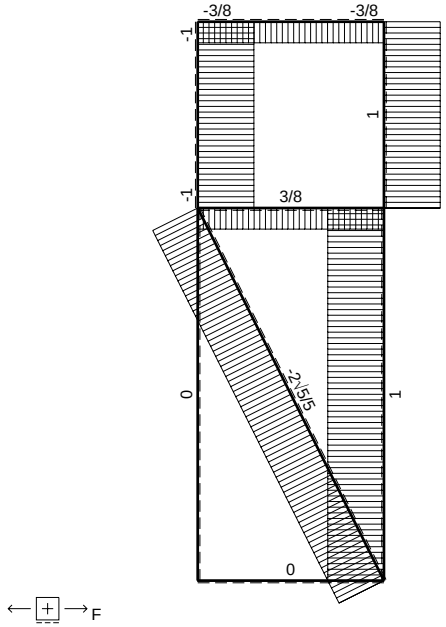
A = 678. mm^2
J_x = 266093. mm^4
J_y = 121648. mm^4
J_{xy} = -87929. mm^4
J_u = 307658. mm^4
J_v = 80083. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4416
c = cosα = .9041
s = sinα = .4274
x_g = 13.19 mm
y_g = 21.72 mm
N = -2950. N
T_y = -1829. N
M_x = 1060820. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 18.57 mm
v_m = 28.03 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -196.8 N/mm^2

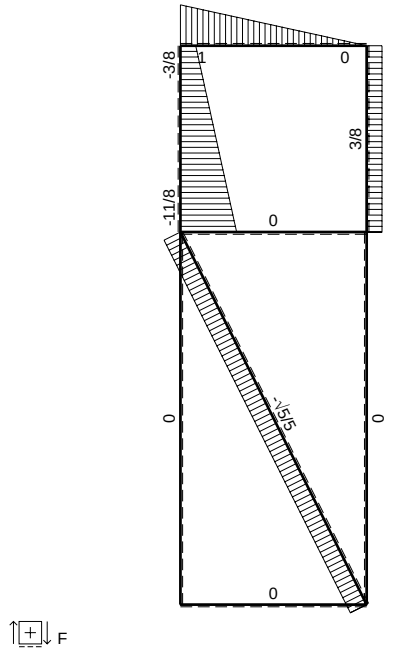
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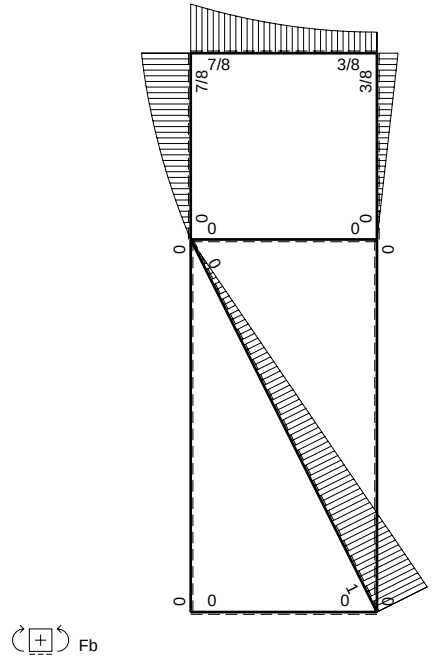
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F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
-2√5/5
1
1
0
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

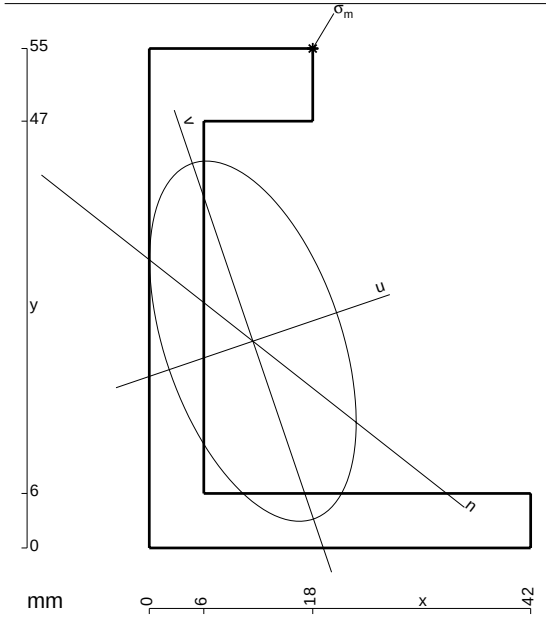
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

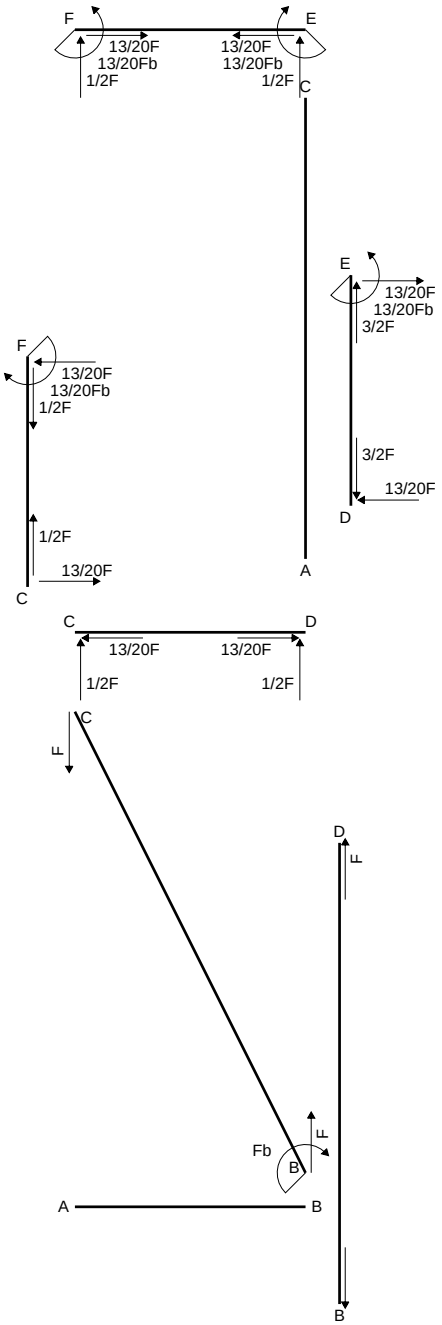
$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

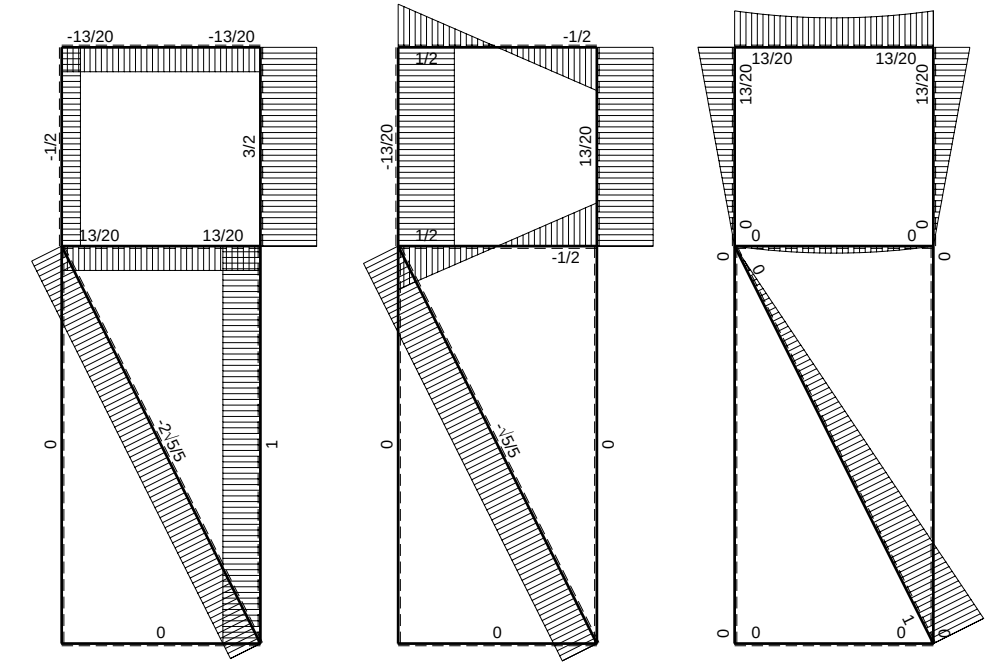


A = 642. mm^2
J_x = 252660. mm^4
J_y = 83081. mm^4
J_{xy} = -65292. mm^4
J_u = 274886. mm^4
J_v = 60856. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3281
c = cosα = .9467
s = sinα = .3222
x_g = 11.41 mm
y_g = 22.77 mm
N = -1610. N
T_y = -805. N
M_x = 1116000. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 16.62 mm
v_m = 28.39 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.8 N/mm^2

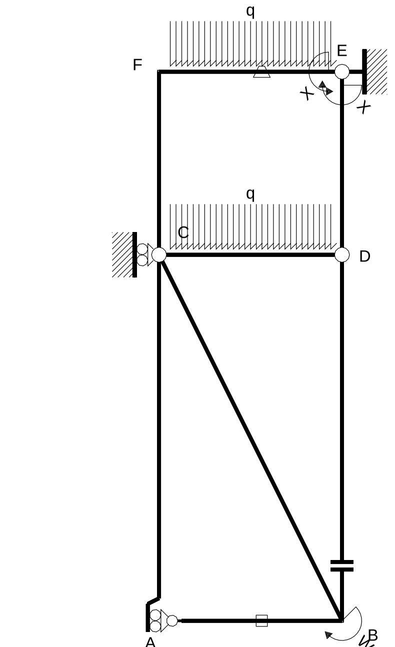
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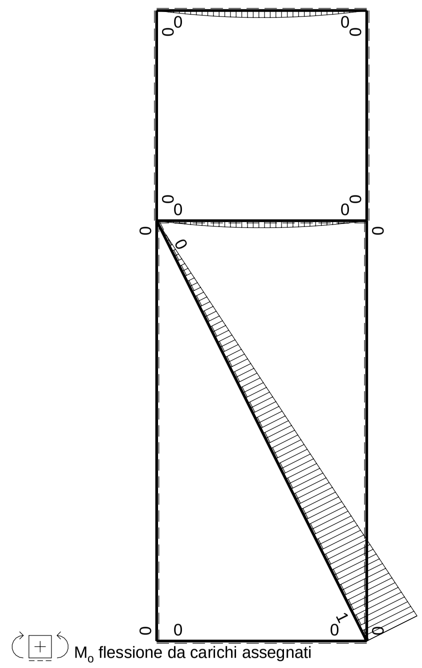

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

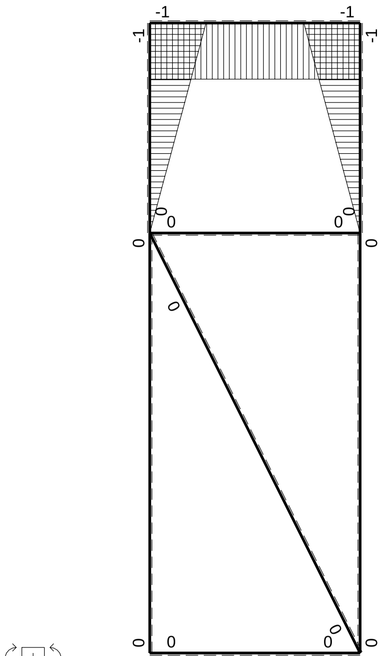

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

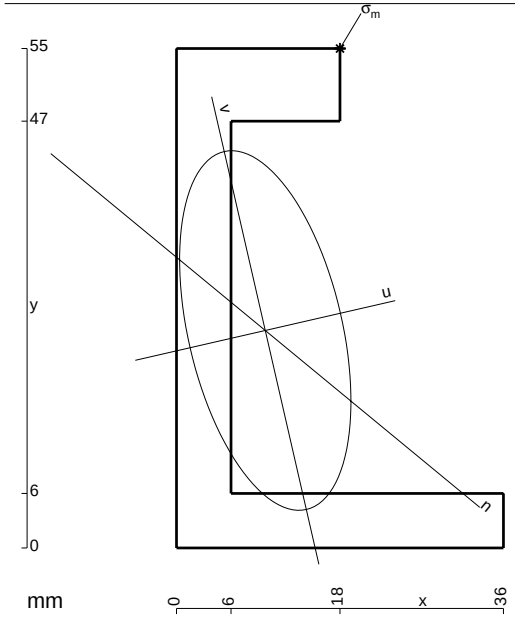
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

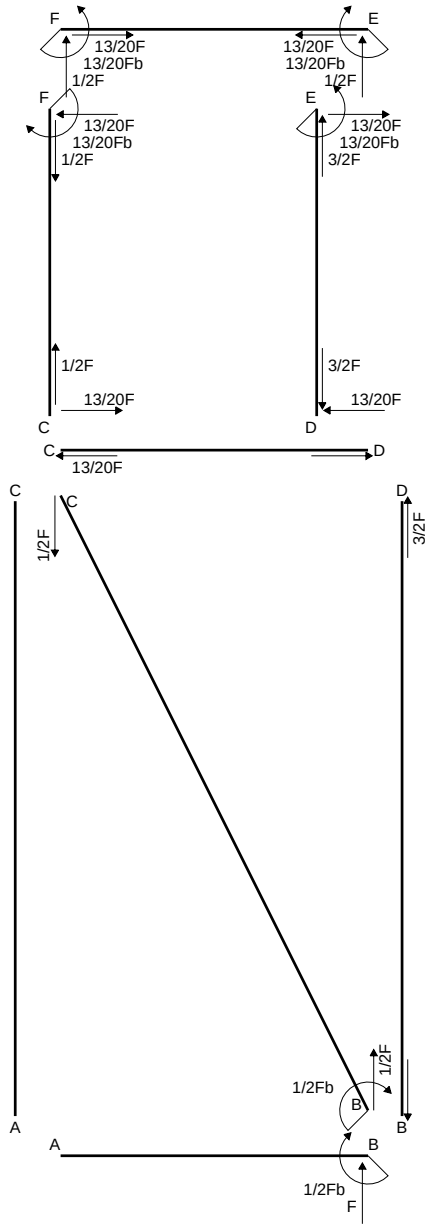
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

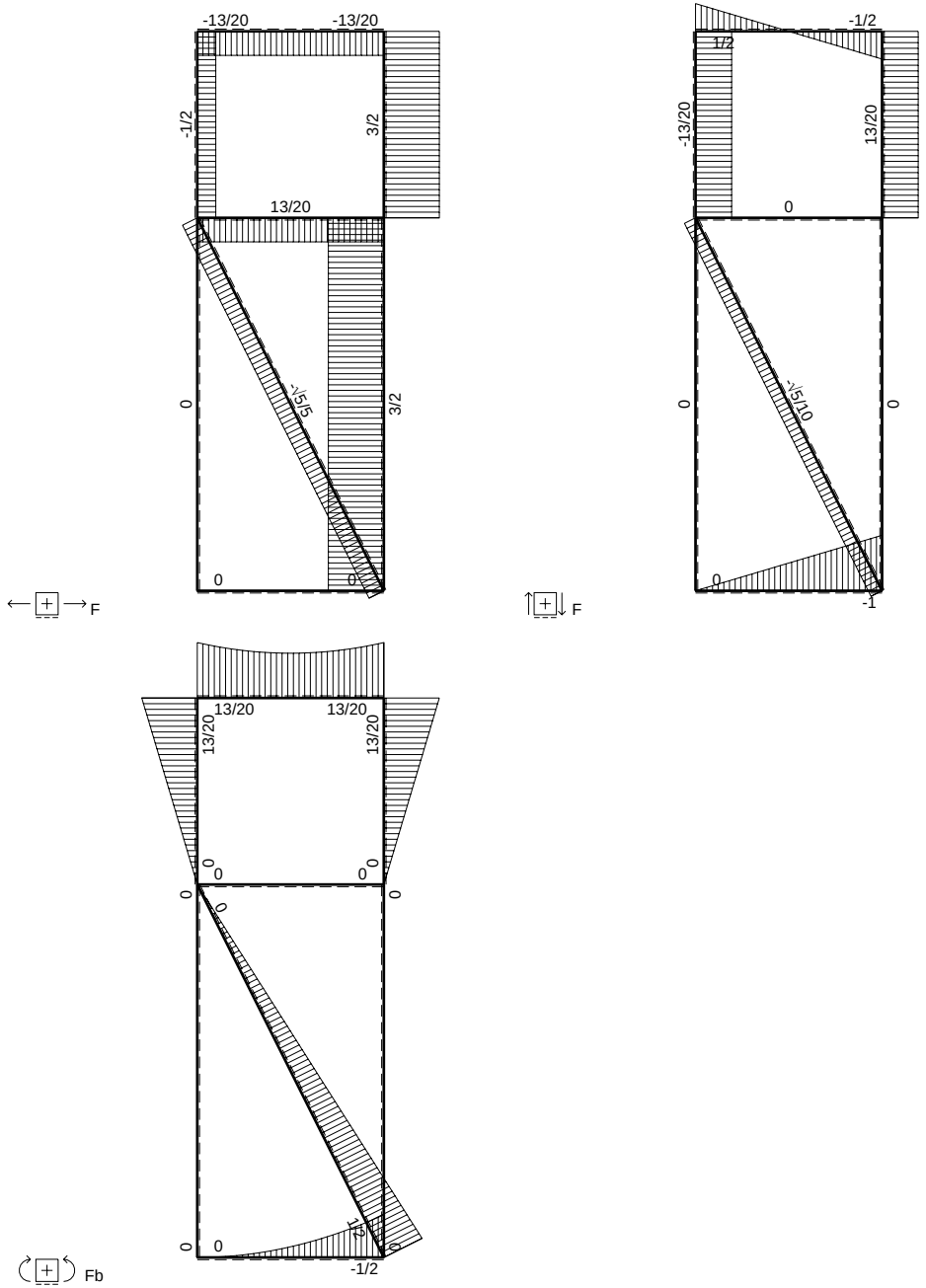


A = 606. mm^2
J_x = 237644. mm^4
J_y = 53945. mm^4
J_{xy} = -44489. mm^4
J_u = 247851. mm^4
J_v = 43738. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2255
$c = \cos\alpha$ = .9747
$s = \sin\alpha$ = .2236
x_g = 9.772 mm
y_g = 23.95 mm
N = -1556. N
T_y = -778.2 N
M_x = 1148400. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 14.96 mm
v_m = 28.43 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -218.8 N/mm^2

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
F
1/2
-√5/10
-1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
totali							$13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

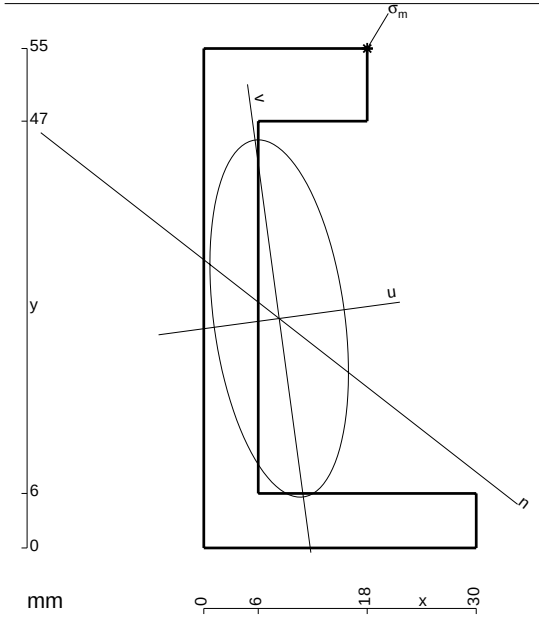
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

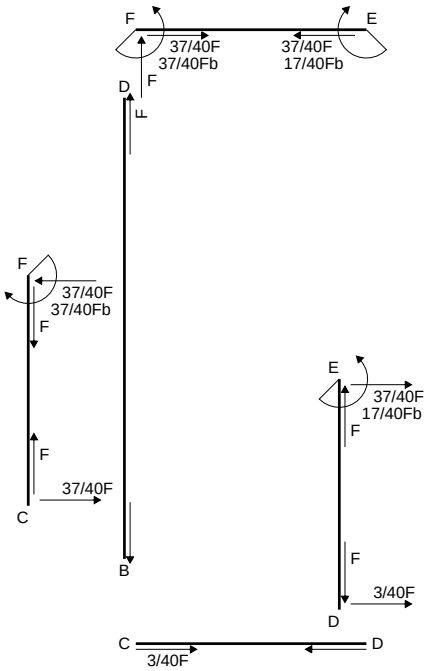
$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

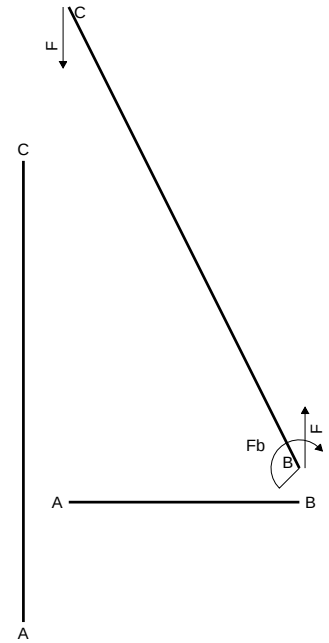


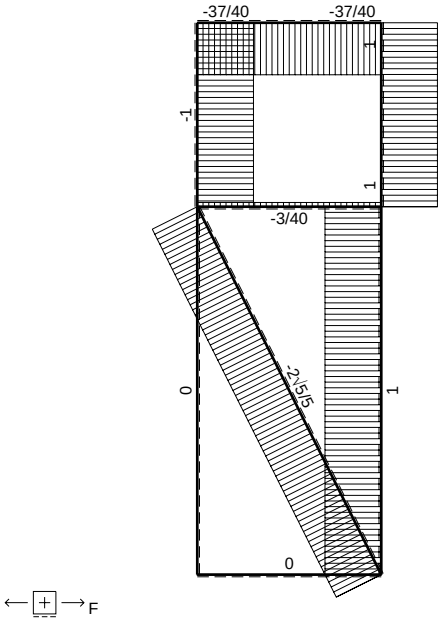
A = 570. mm^2
J_x = 220745. mm^4
J_y = 33187. mm^4
J_{xy} = -25868. mm^4
J_u = 224247. mm^4
J_v = 29685. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .1346
$c = \cos\alpha$ = .9910
$s = \sin\alpha$ = .1342
x_g = 8.305 mm
y_g = 25.27 mm
N = -1742. N
T_y = -1340. N
M_x = 1219400. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 13.6 mm
v_m = 28.16 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -229.7 N/mm^2

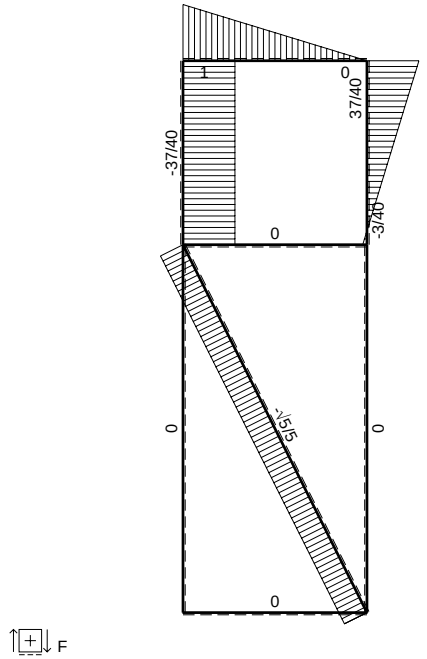
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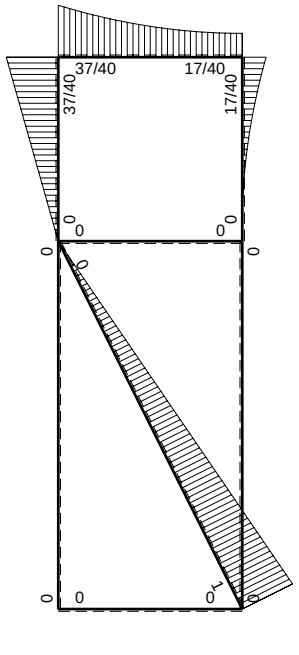
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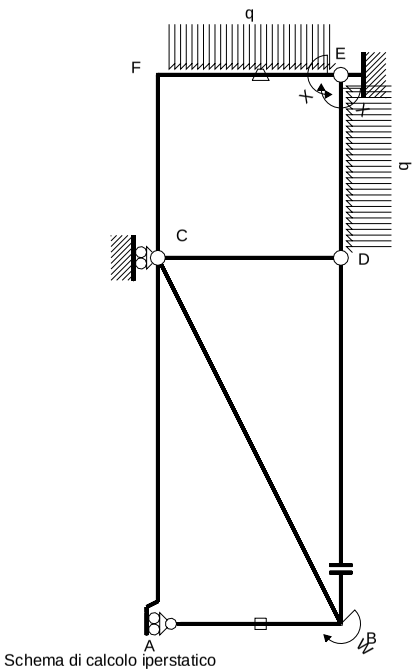

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
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Fb
F
B
A
B
A

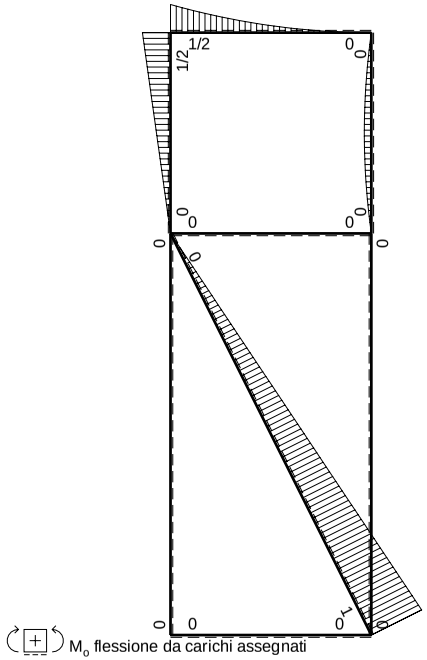

-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
1
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
F

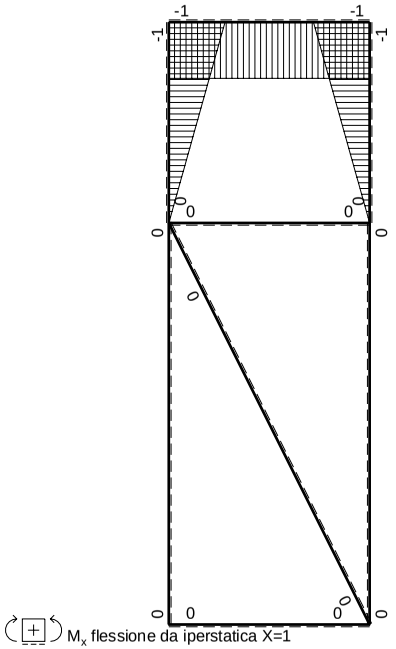

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 268284 CI09.xxxx.169

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

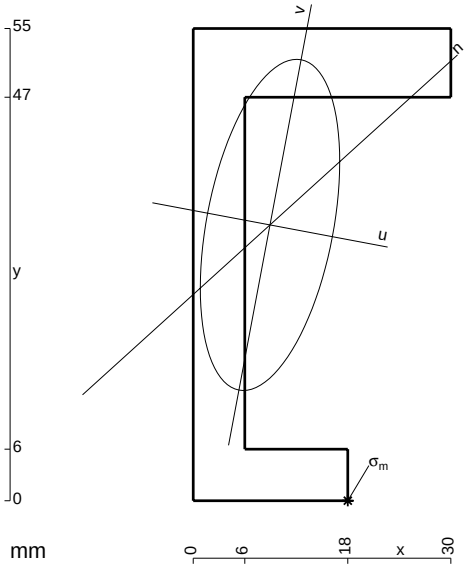
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

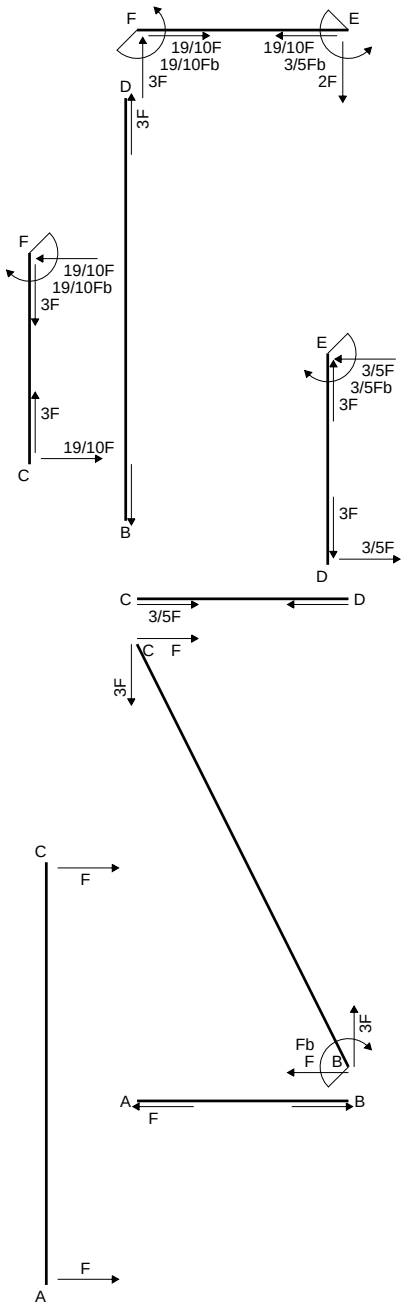
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

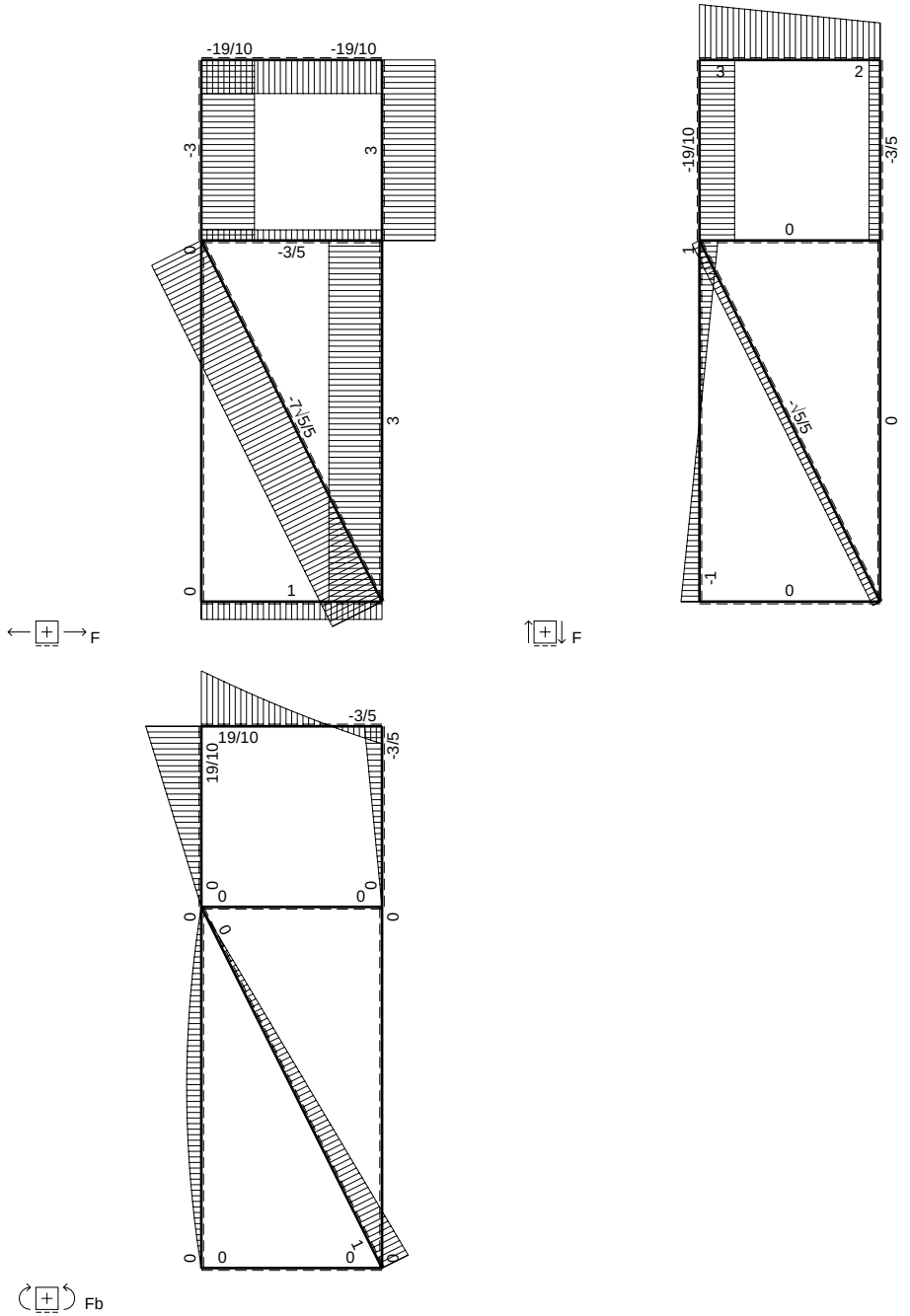


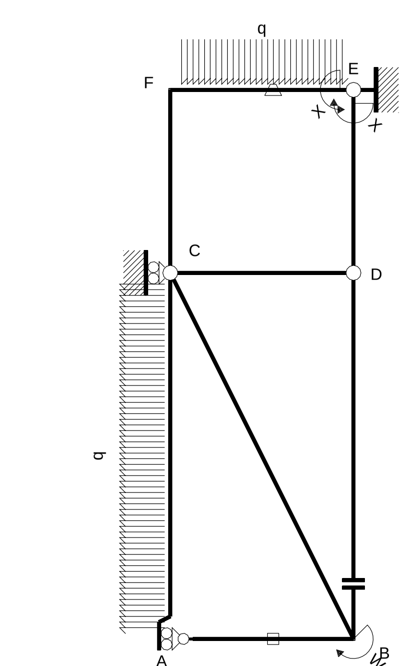
A = 594. mm^2
J_x = 220965. mm^4
J_y = 39148. mm^4
J_{xy} = 35483. mm^4
J_u = 227644. mm^4
J_v = 32469. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.1861$
$c = \cos\alpha = .9827$
$s = \sin\alpha = -.1850$
x_g = 8.939 mm
y_g = 32.13 mm
N = -1368. N
T_y = -684.2 N
M_x = 1132200. Nmm
x_m = 18. mm
u_m = 14.85 mm
v_m = -29.9 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.6 N/mm^2

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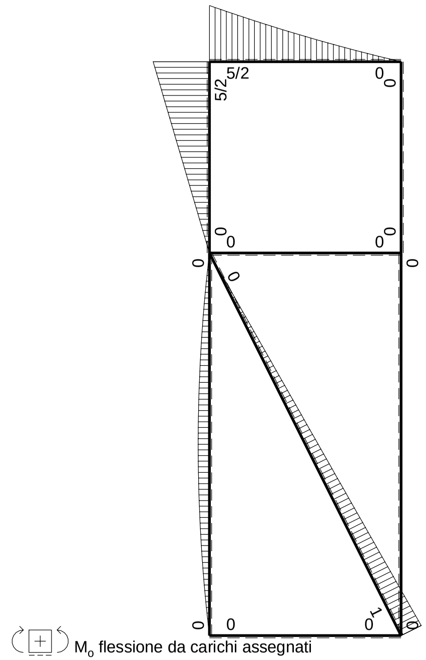
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E
19/10F
19/10Fb
19/10F
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3F
2F
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3F
F
19/10F
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E
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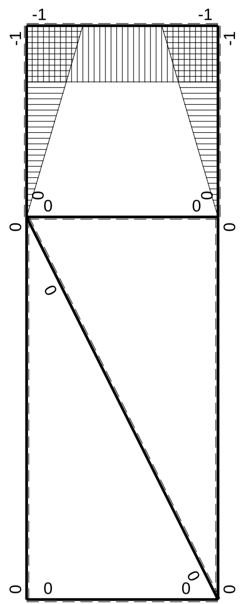

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19/10
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0
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0
0
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0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

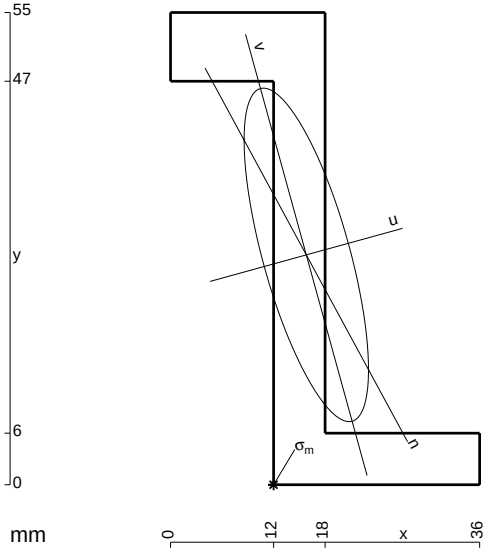
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

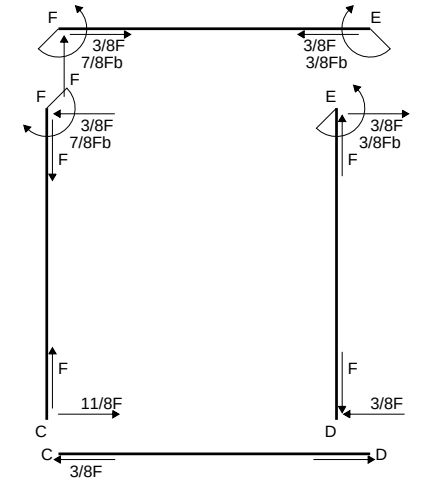
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

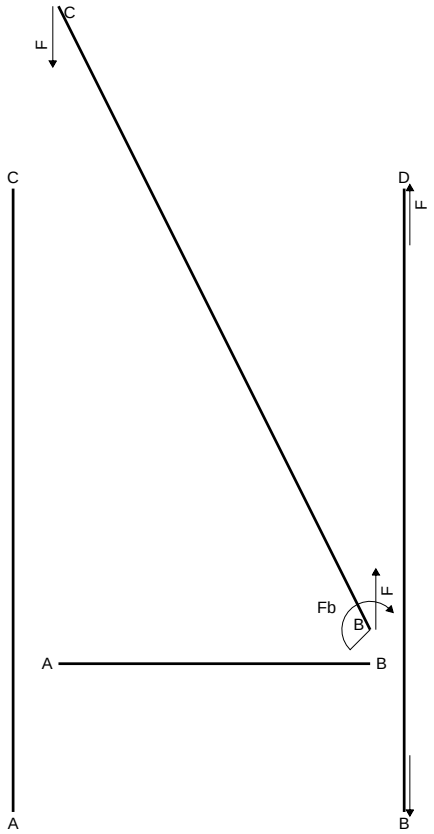


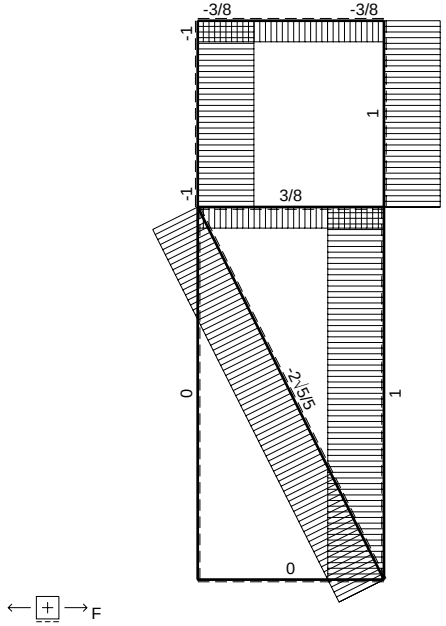
A = 534. mm^2
J_x = 201582. mm^4
J_y = 28037. mm^4
J_{xy} = -51741. mm^4
J_u = 215837. mm^4
J_v = 13782. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2688
$c = \cos\alpha$ = .9641
$s = \sin\alpha$ = .2656
x_g = 15.81 mm
y_g = 26.77 mm
N = -1634. N
T_y = 2580. N
M_x = 637260. Nmm
x_m = 12. mm
u_m = -10.78 mm
v_m = -24.8 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 200. N/mm^2

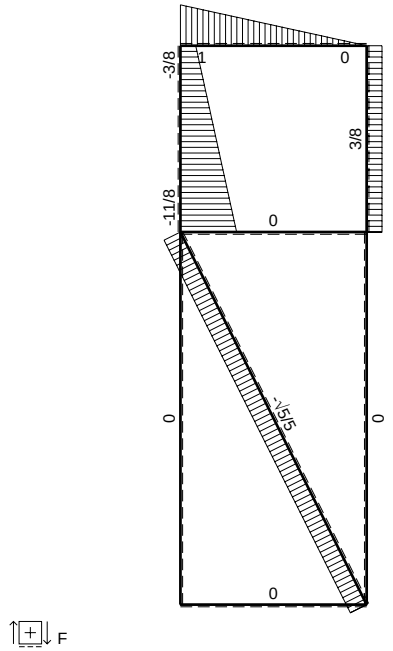
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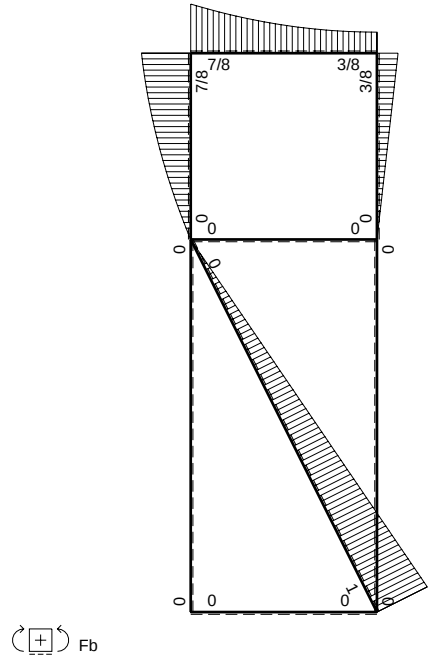
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Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 5/6\ Fb^2/EJ$

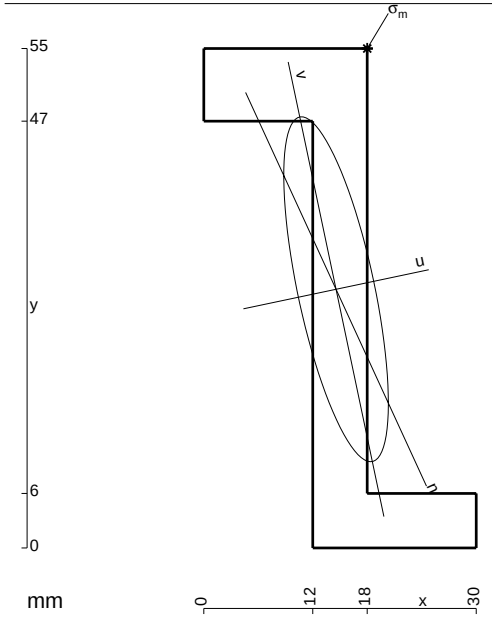
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

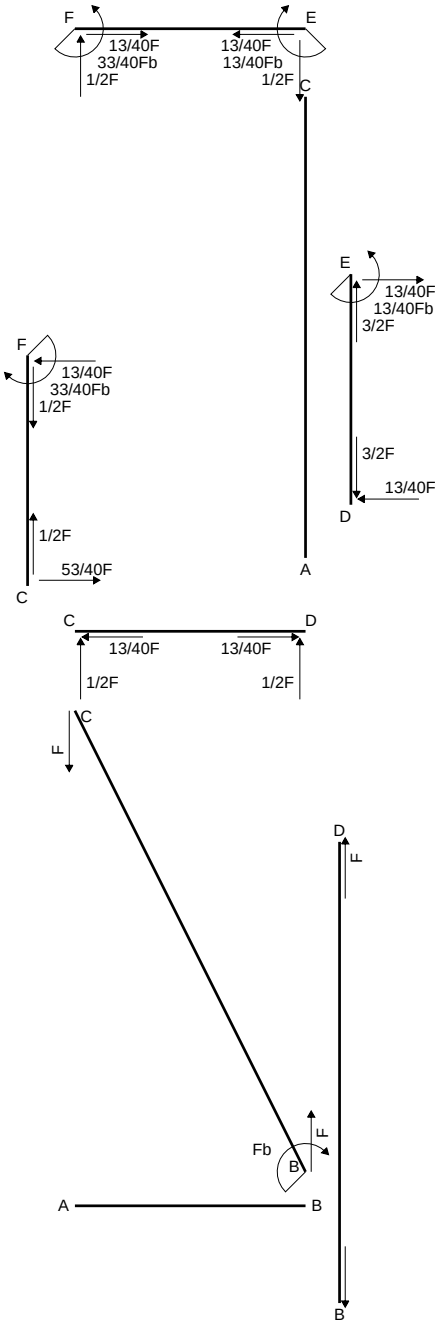
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

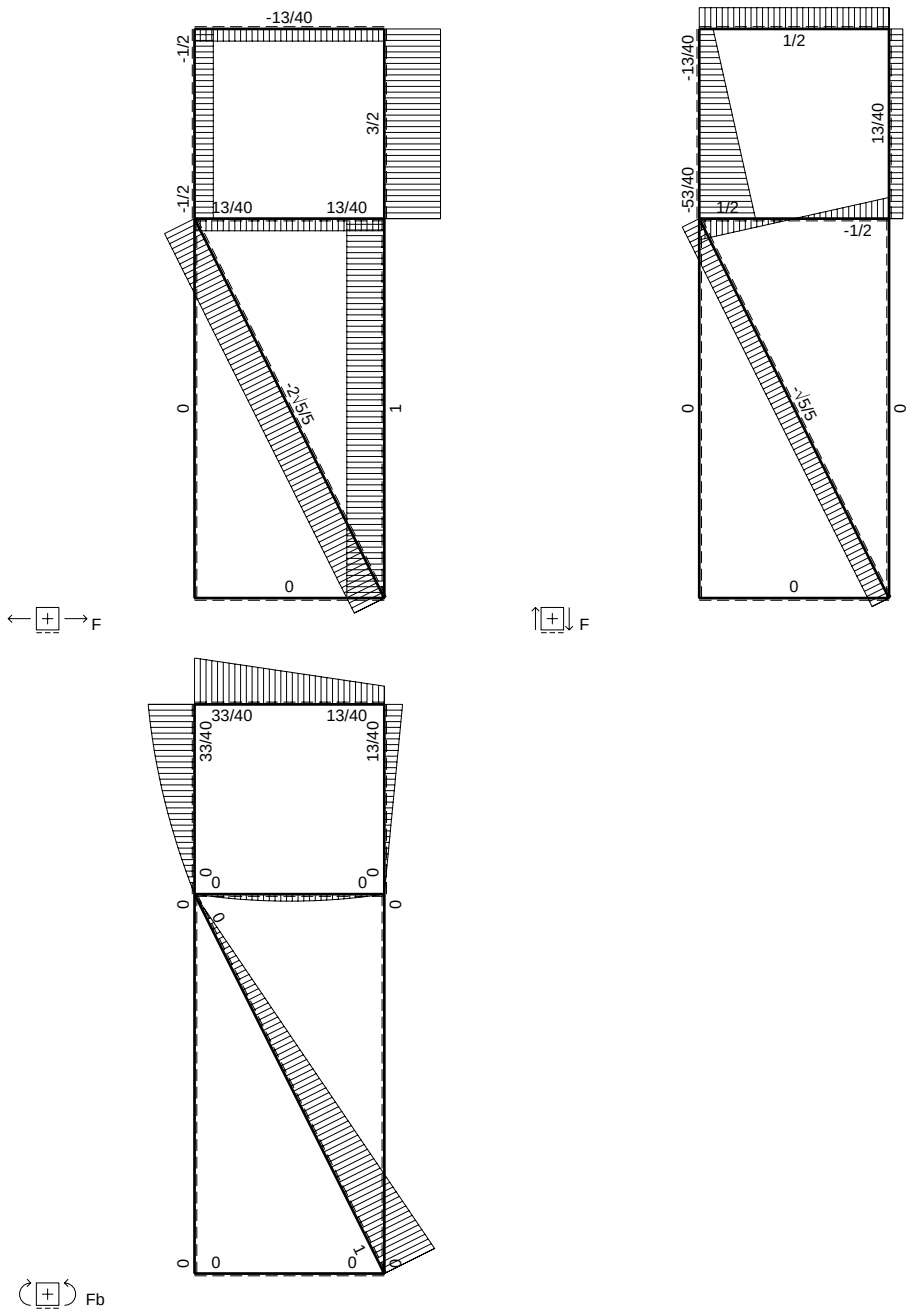


A = 498. mm^2
J_x = 179663. mm^4
J_y = 16520. mm^4
J_{xy} = -35967. mm^4
J_u = 187241. mm^4
J_v = 8943. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2076
$c = \cos\alpha$ = .9785
$s = \sin\alpha$ = .2061
x_g = 14.57 mm
y_g = 28.49 mm
N = -1315. N
T_y = -657.4 N
M_x = 617400. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 8.825 mm
v_m = 25.23 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -209.7 N/mm^2

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
-2√5/5
0
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XbEJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-FbEJ	-1/2Fx	FbEJ	1	$(-1/4+1)Fb^2/EJ$	XbEJ
FE b	1	$-1/2Fb+1/2Fx$	FbEJ	$-1/2Fb+1/2Fx$	FbEJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3XbEJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XbEJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 309547 CI09.xxxx.172

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

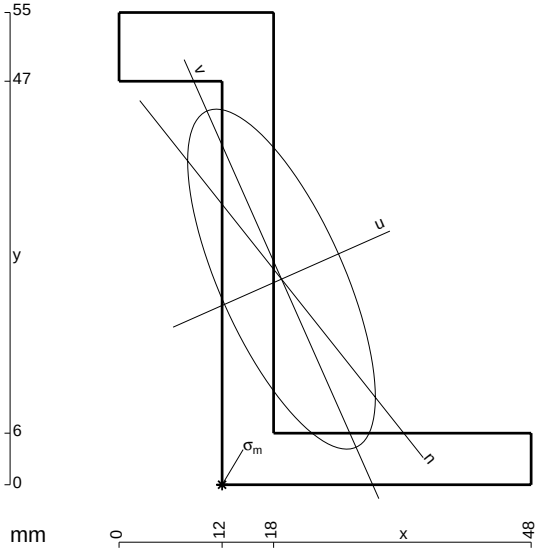
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

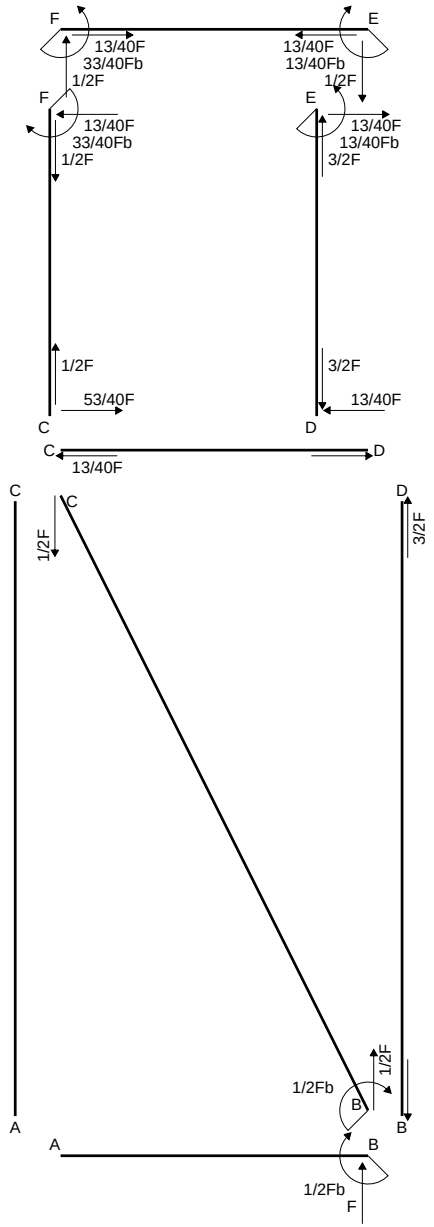
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

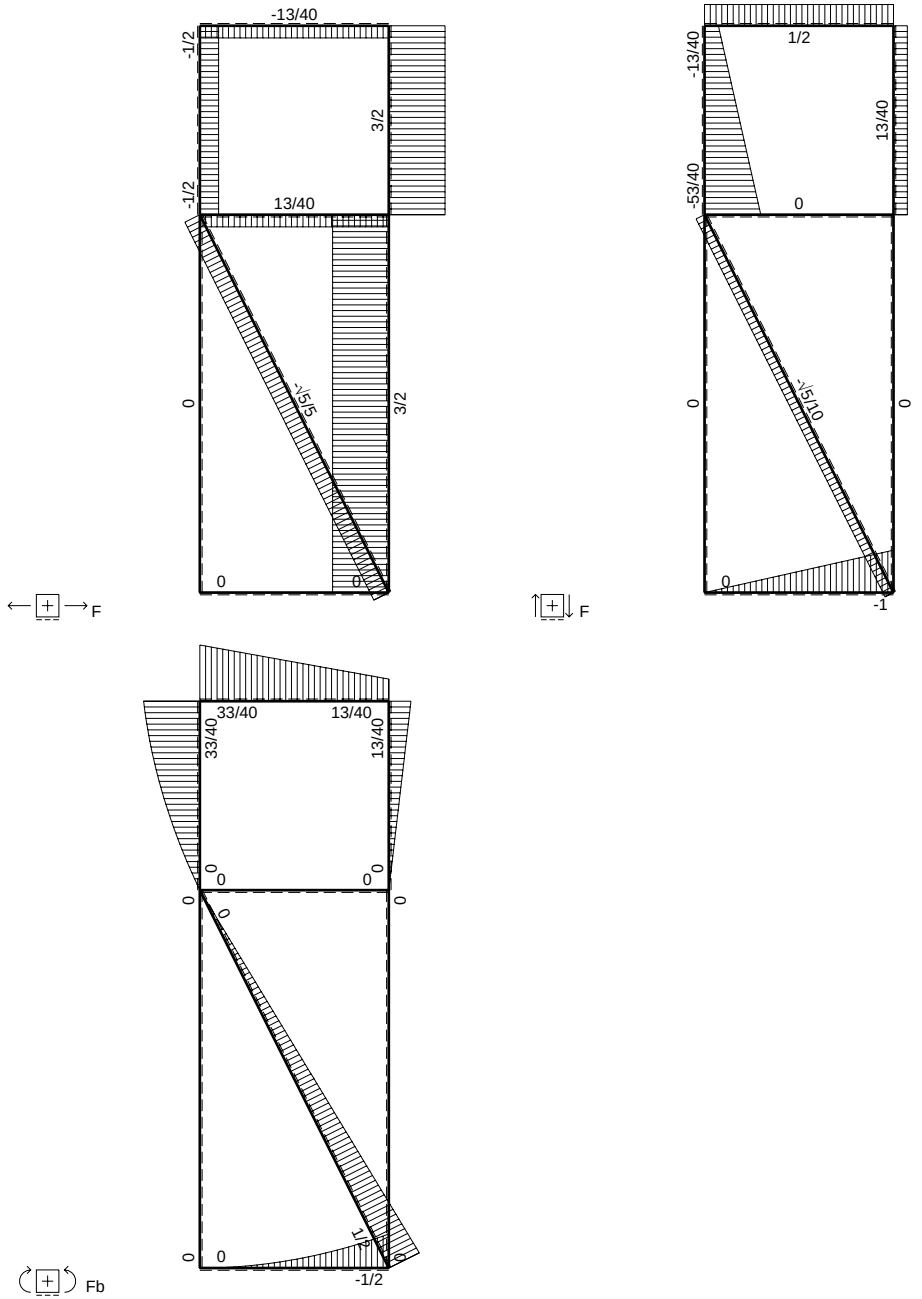


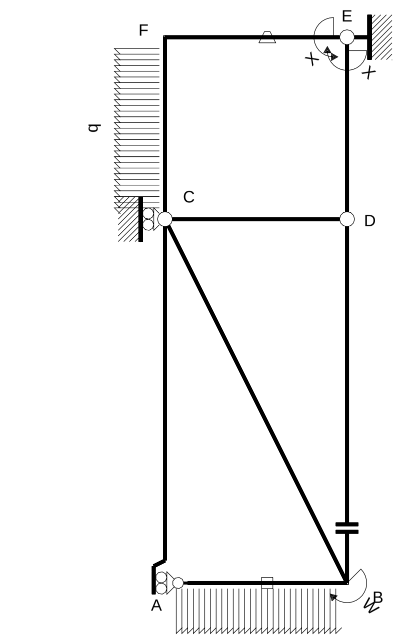
A = 606. mm^2
J_x = 237644. mm^4
J_y = 72422. mm^4
J_{xy} = -91239. mm^4
J_u = 278115. mm^4
J_v = 31952. mm^4
$\alpha = \text{artg}(2J_{xy}/(J_y-J_x))/2$ = .4175
c = cosα = .9141
s = sinα = .4055
x_g = 18.92 mm
y_g = 23.95 mm
N = -1619. N
T_y = -809.5 N
M_x = 832600. Nmm
x_m = 12. mm
u_m = -16.04 mm
v_m = -19.08 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 219. N/mm^2

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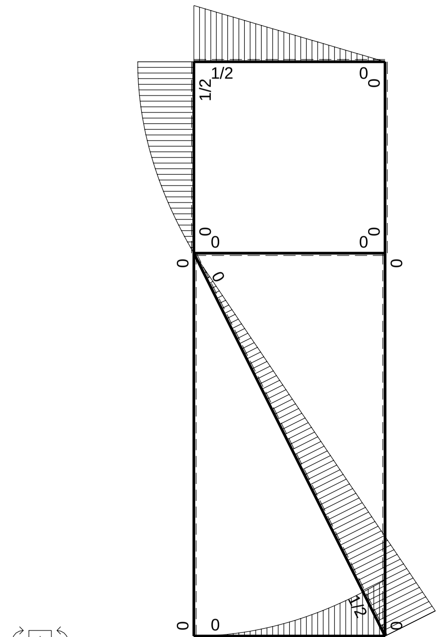
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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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B
A
B
1/2Fb
F

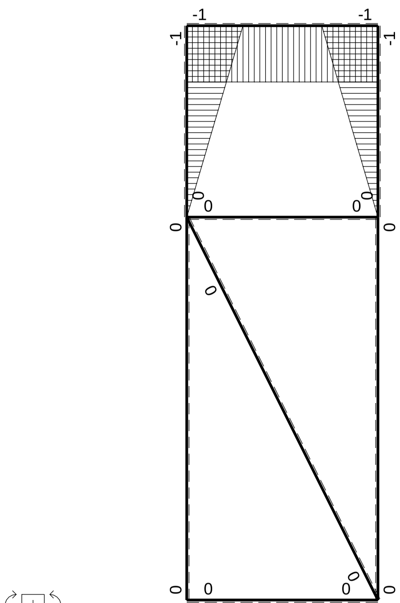

-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-√2/2
0
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 275579 CI09.xxxx.173

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{Xo} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

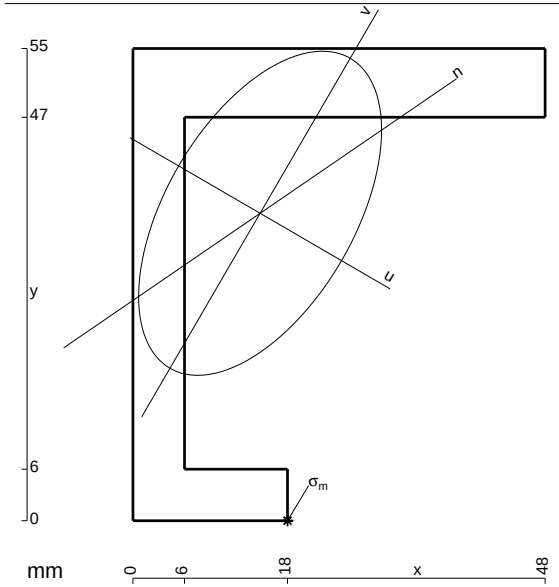
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 275579 CI09.xxxx.173

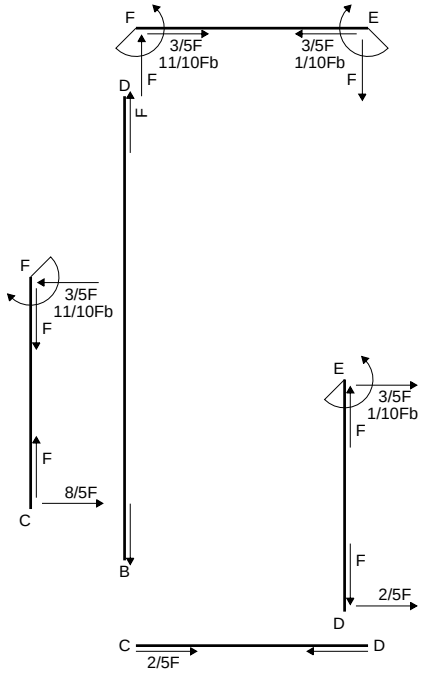


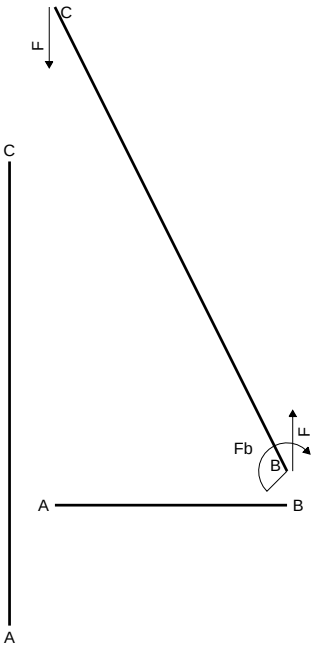
A = 738. mm^2
J_x = 263019. mm^4
J_y = 147770. mm^4
J_{xy} = 101240. mm^4
J_u = 321886. mm^4
J_v = 88903. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.5267
$c = \cos\alpha$ = .8645
$s = \sin\alpha$ = -.5027
x_g = 14.8 mm
y_g = 35.81 mm
N = -926.3 N
T_y = 1425. N
M_x = 1175630. Nmm
x_m = 18. mm
u_m = 20.76 mm
v_m = -29.35 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 229.4 N/mm^2

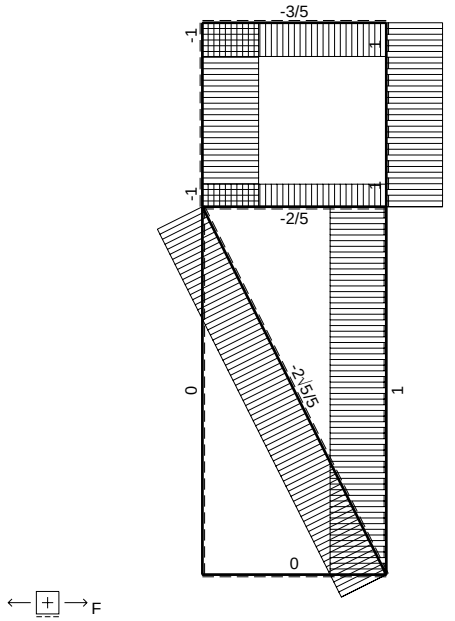
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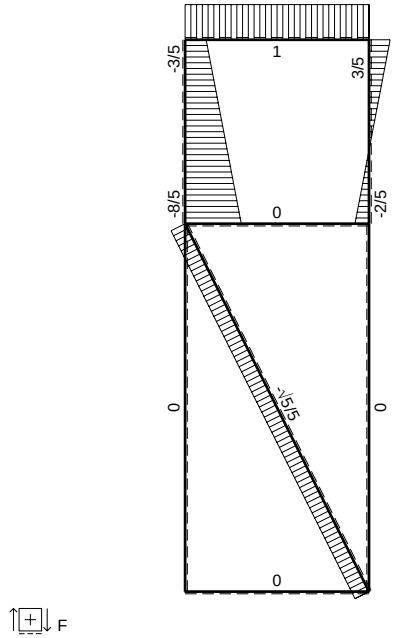
CI09.xxxx.173

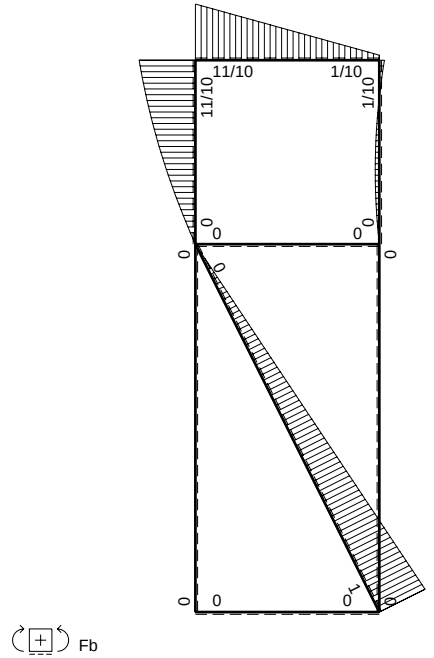
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F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5
0
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx^2	0	-Fb+3/2Fx-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-3/8+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3/2Fx+1/2qx^2	0	-3/2Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						1/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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PROCEDIMENTO E RISULTATI 215204 CI09.xxxx.174

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

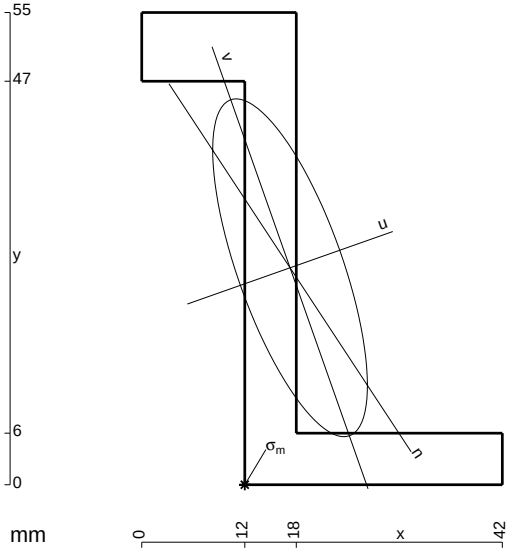
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

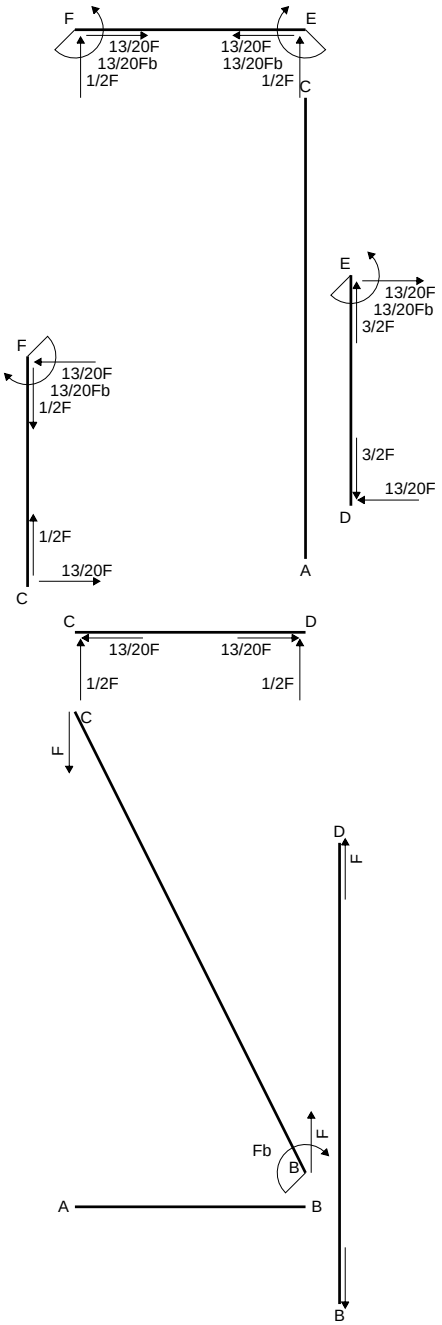
$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

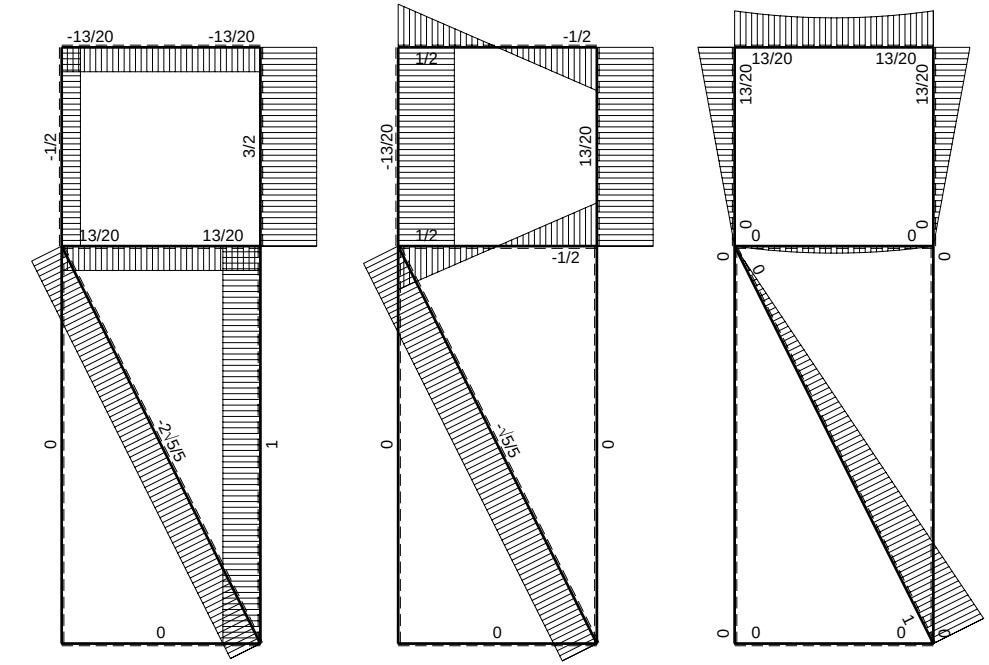


A = 570. mm^2
J_x = 220745. mm^4
J_y = 46283. mm^4
J_{xy} = -70332. mm^4
J_u = 245567. mm^4
J_v = 21462. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3393
$c = \cos\alpha$ = .9430
$s = \sin\alpha$ = .3328
x_g = 17.27 mm
y_g = 25.27 mm
N = -834. N
T_y = 1390. N
M_x = 825660. Nmm
x_m = 12. mm
u_m = -13.38 mm
v_m = -22.07 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 239.9 N/mm^2

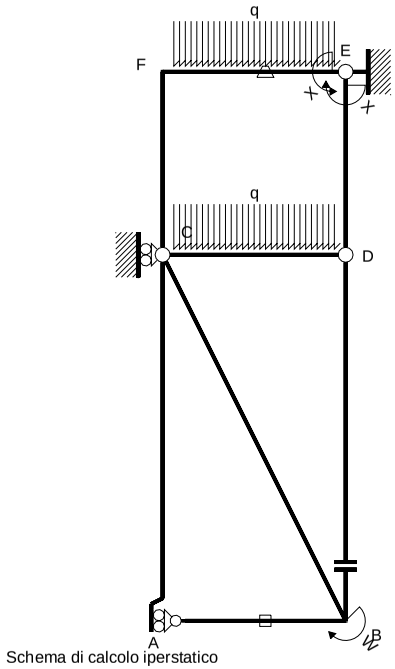
CI09.xxxx.174

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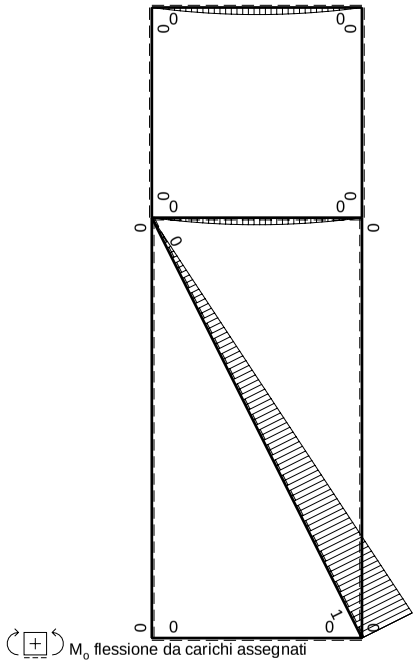

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

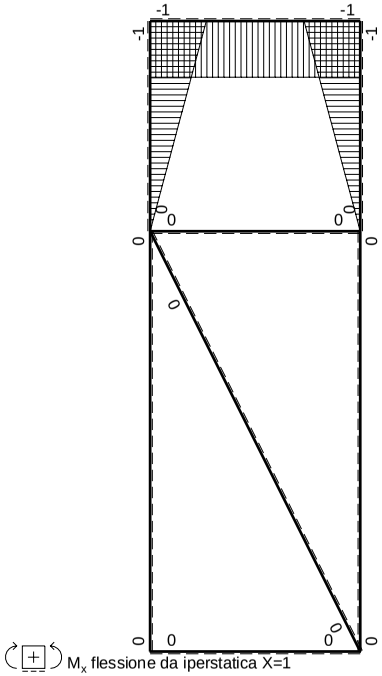

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 265726 CI09.xxxx.175

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

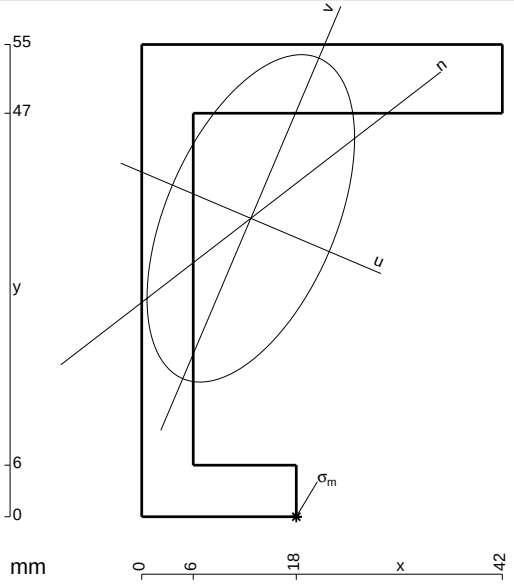
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

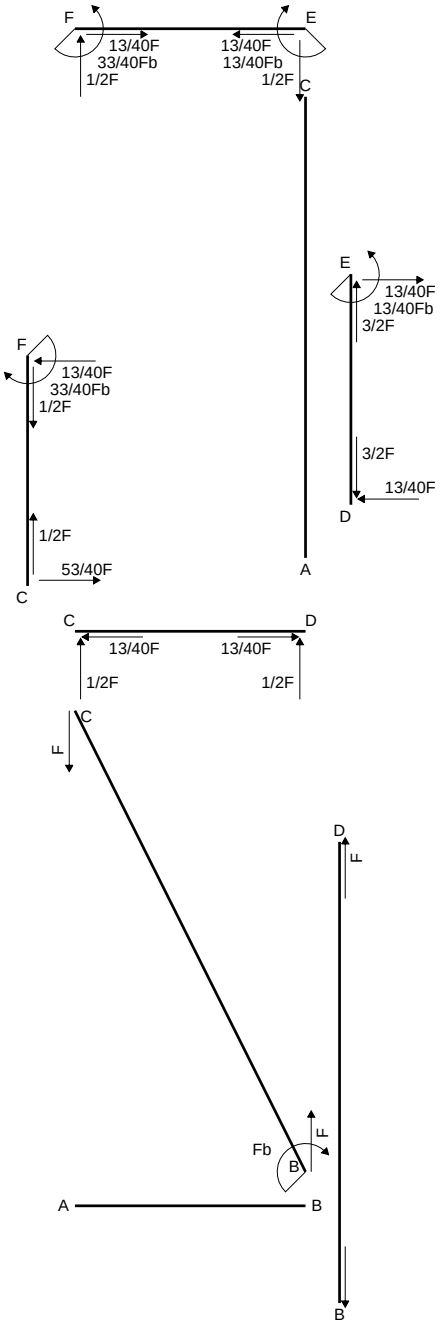
$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

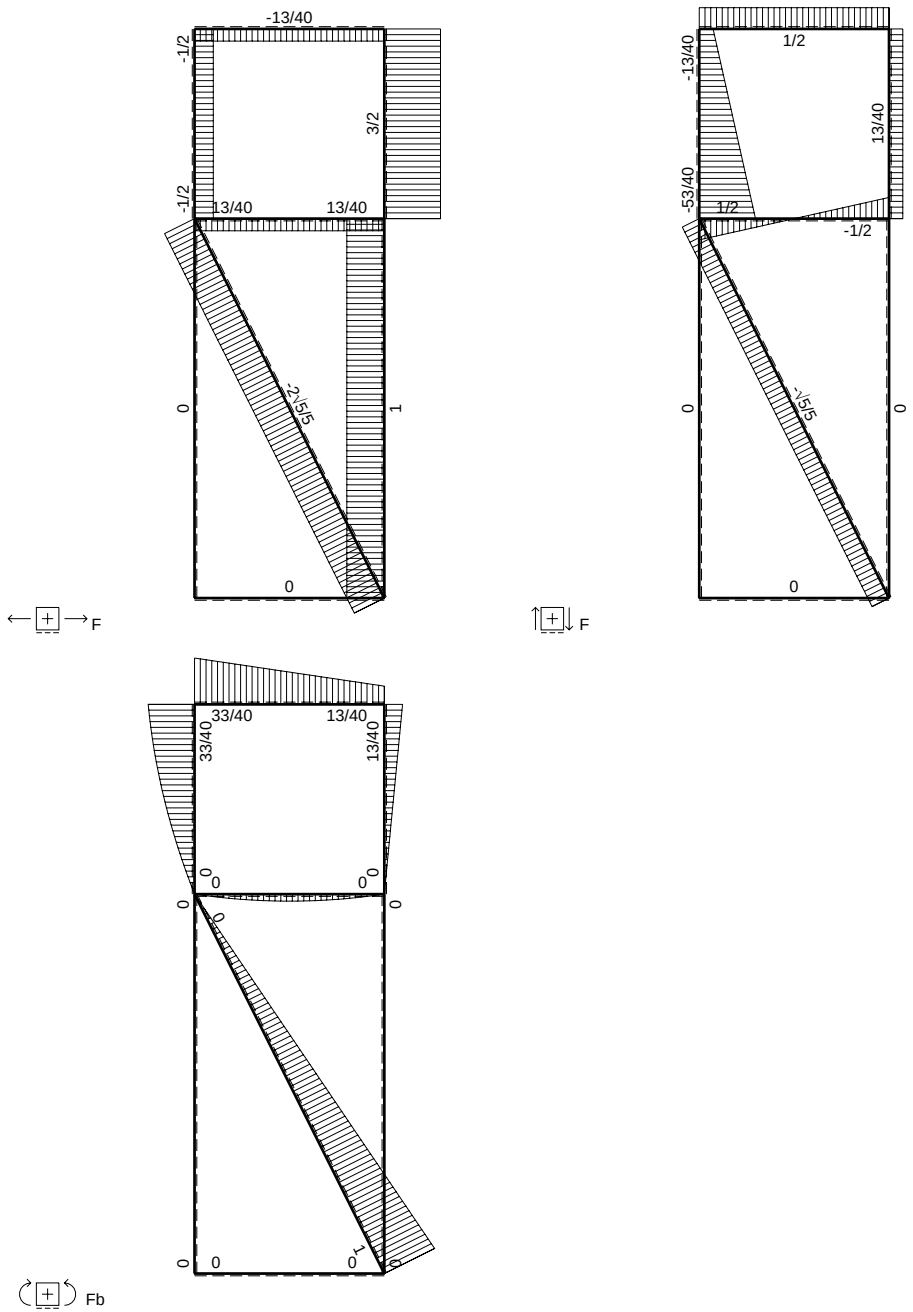


A = 690. mm^2
J_x = 250916. mm^4
J_y = 100818. mm^4
J_{xy} = 77691. mm^4
J_u = 283887. mm^4
J_v = 67847. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4013
c = cosα = .9205
s = sinα = -.3907
x_g = 12.7 mm
y_g = 34.75 mm
N = -1529. N
T_y = -764.7 N
M_x = 991800. Nmm
x_m = 18. mm
u_m = 18.45 mm
v_m = -29.92 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 199.4 N/mm^2

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
-2√5/5
0
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XbEJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-FbEJ	-1/2Fx	FbEJ	1	$(-1/4+1)Fb^2/EJ$	XbEJ
FE b	1	$-1/2Fb+1/2Fx$	FbEJ	$-1/2Fb+1/2Fx$	FbEJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3XbEJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XbEJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 270371 CI09.xxxx.176

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

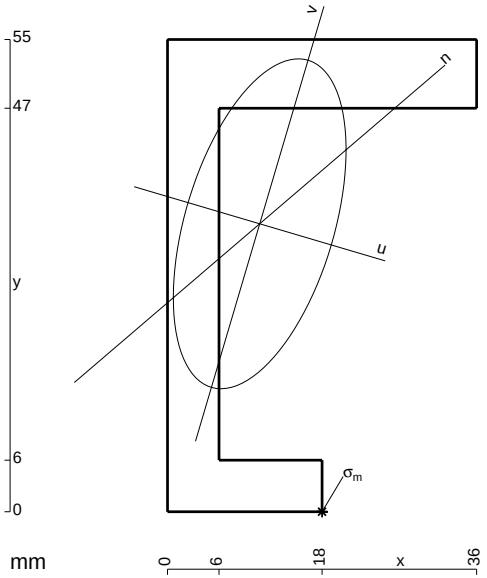
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

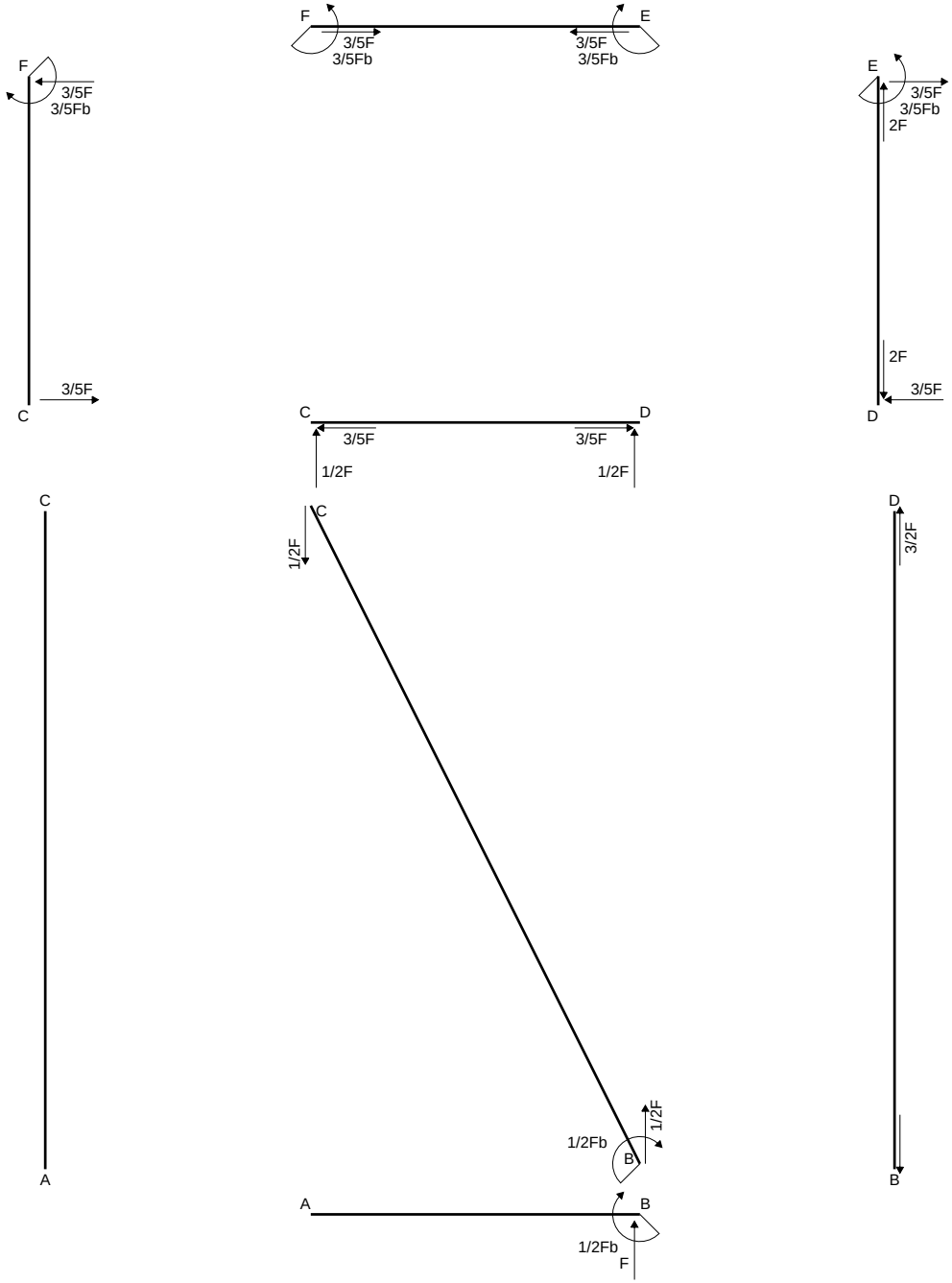
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

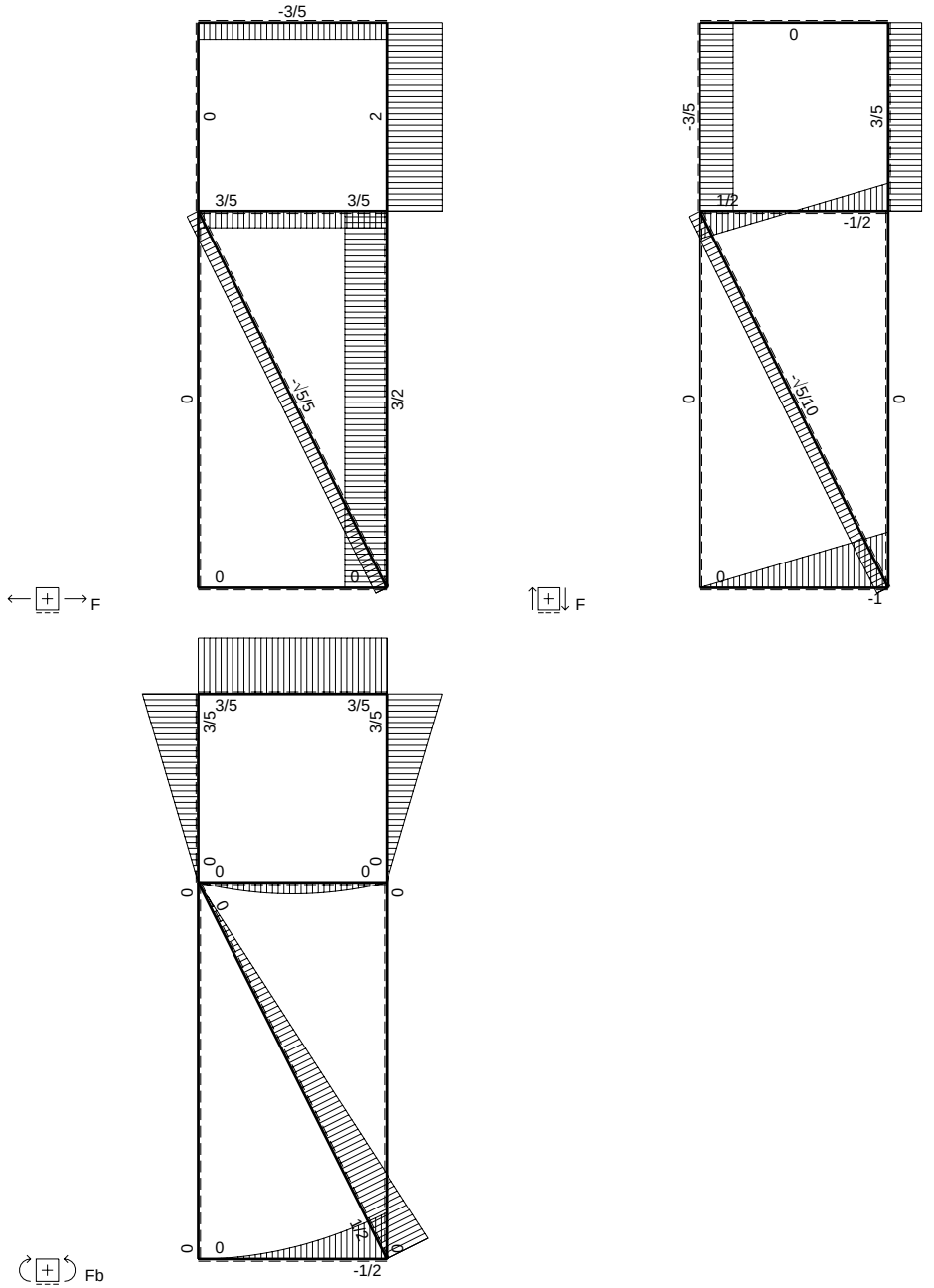


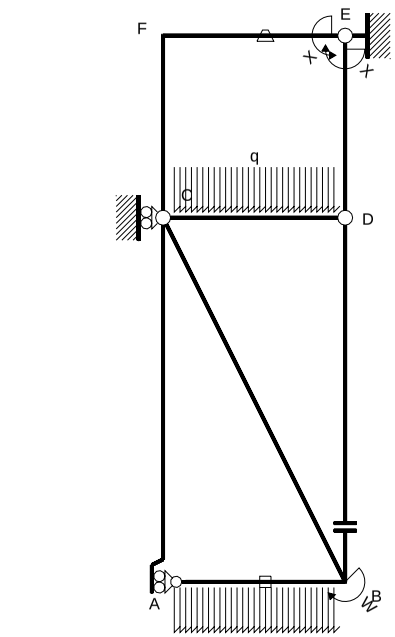
A = 642. mm^2
J_x = 237041. mm^4
J_y = 65002. mm^4
J_{xy} = 55650. mm^4
J_u = 253473. mm^4
J_v = 48570. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.2871
c = cosα = .9591
s = sinα = -.2832
x_g = 10.74 mm
y_g = 33.54 mm
N = -1458. N
T_y = -729. N
M_x = 1010600. Nmm
x_m = 18. mm
u_m = 16.46 mm
v_m = -30.11 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = 209.9 N/mm^2

CI09.xxxx.176

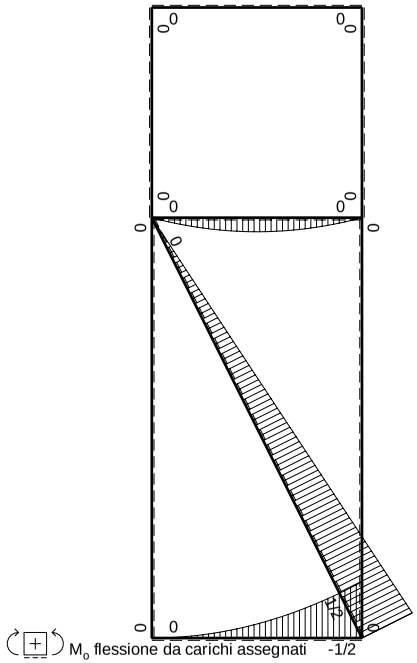
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
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1/2F
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1/2Fb
B
A
B
1/2Fb
F

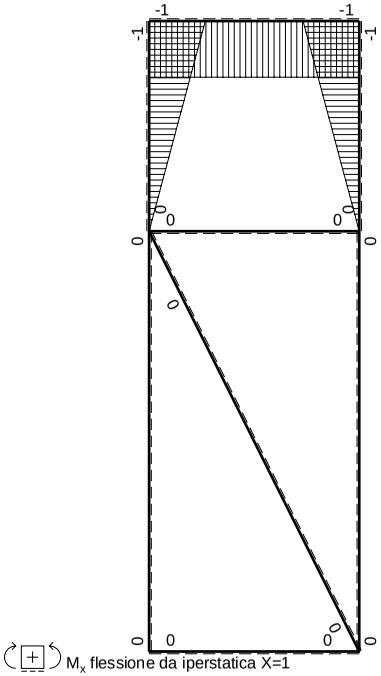

-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

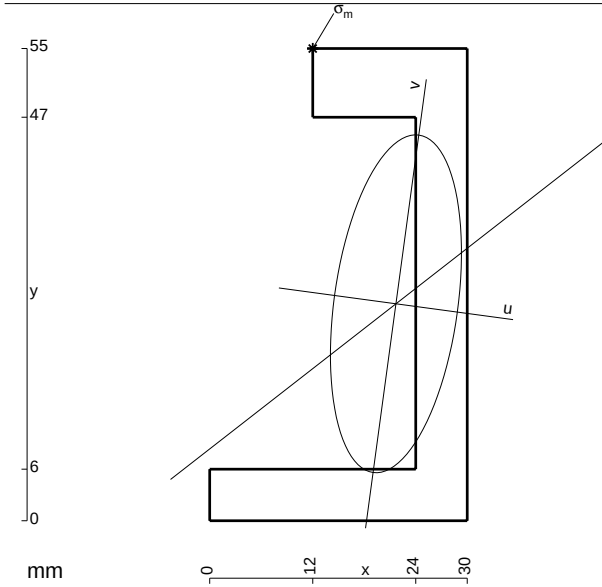
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

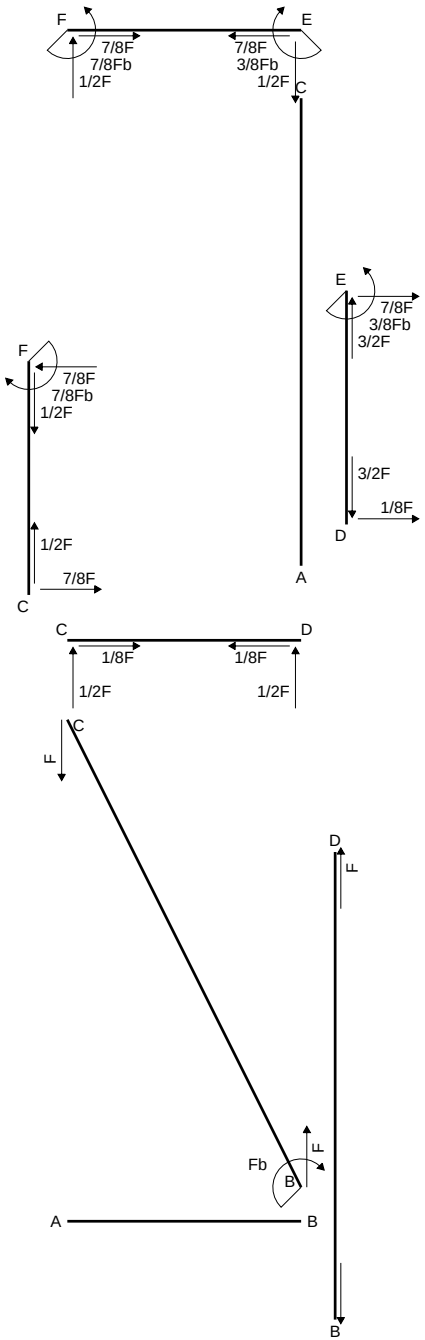
$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

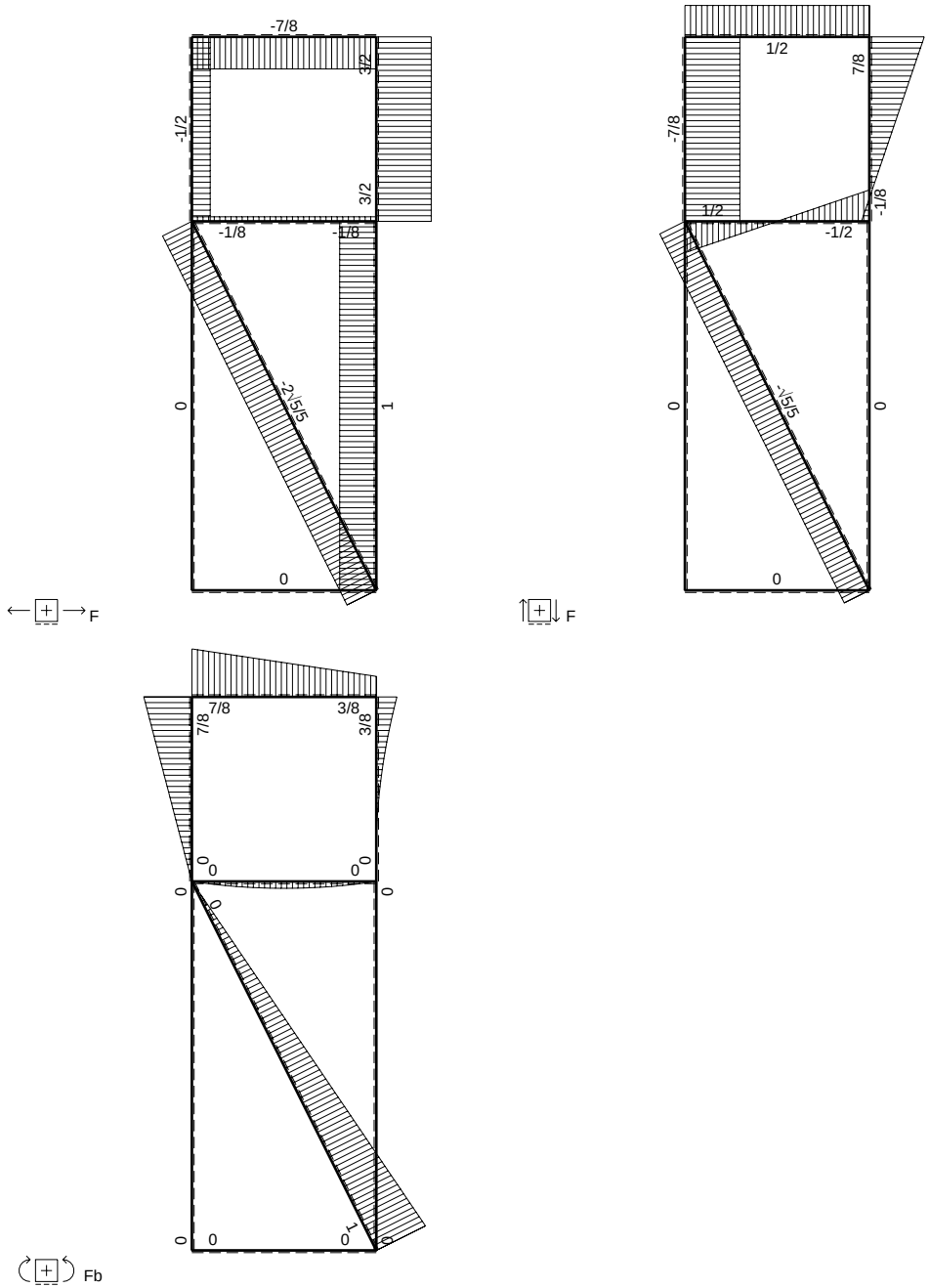


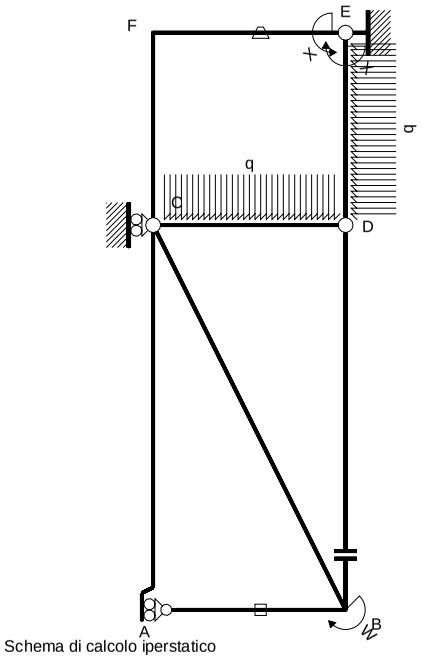
A = 570. mm^2
J_x = 220745. mm^4
J_y = 33187. mm^4
J_{xy} = 25868. mm^4
J_u = 224247. mm^4
J_v = 29685. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.1346
c = cosα = .9910
s = sinα = -.1342
x_g = 21.69 mm
y_g = 25.27 mm
N = -1764. N
M_x = 1164240. Nmm
x_m = 12. mm
y_m = 55. mm
u_m = -13.6 mm
v_m = 28.16 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -219.5 N/mm^2

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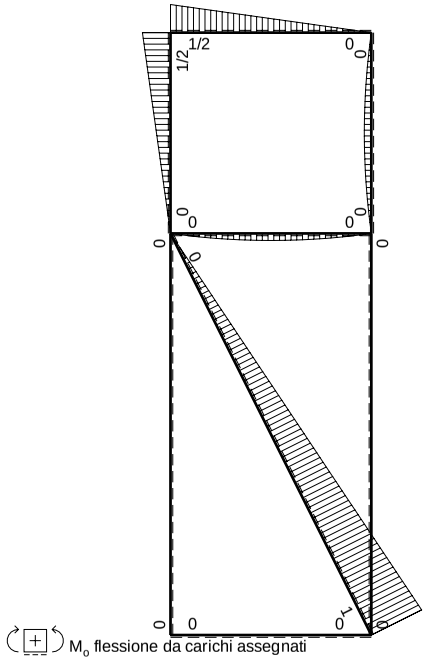
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F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B

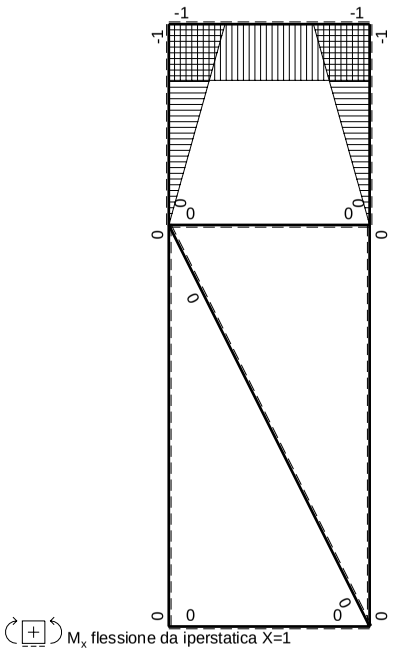

-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-2√5/5
1
0
0
F
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7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
0
0
F
7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
				totali			5/8Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

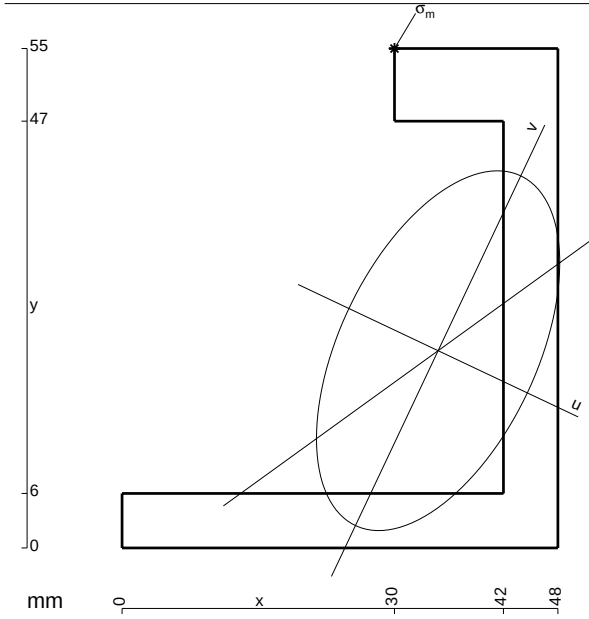
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

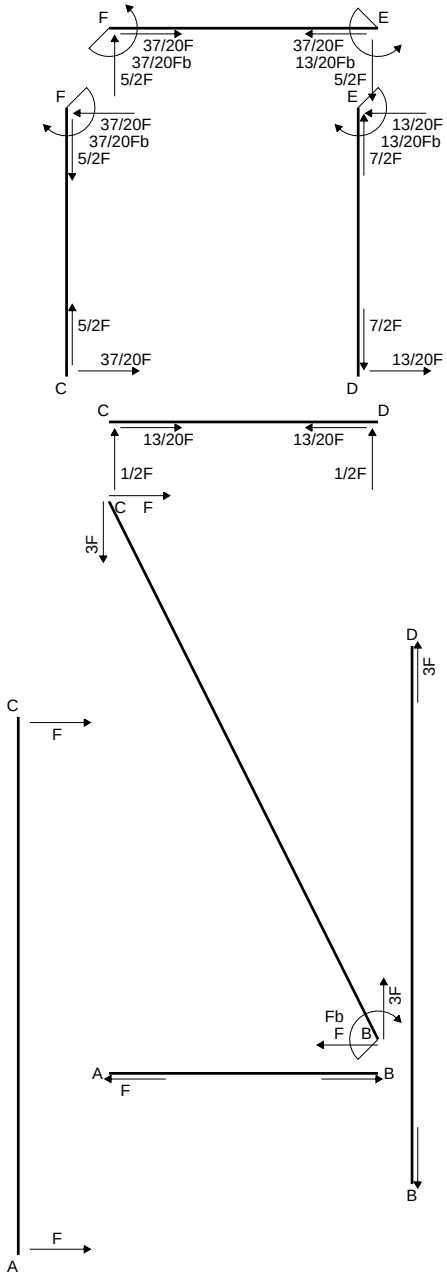
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

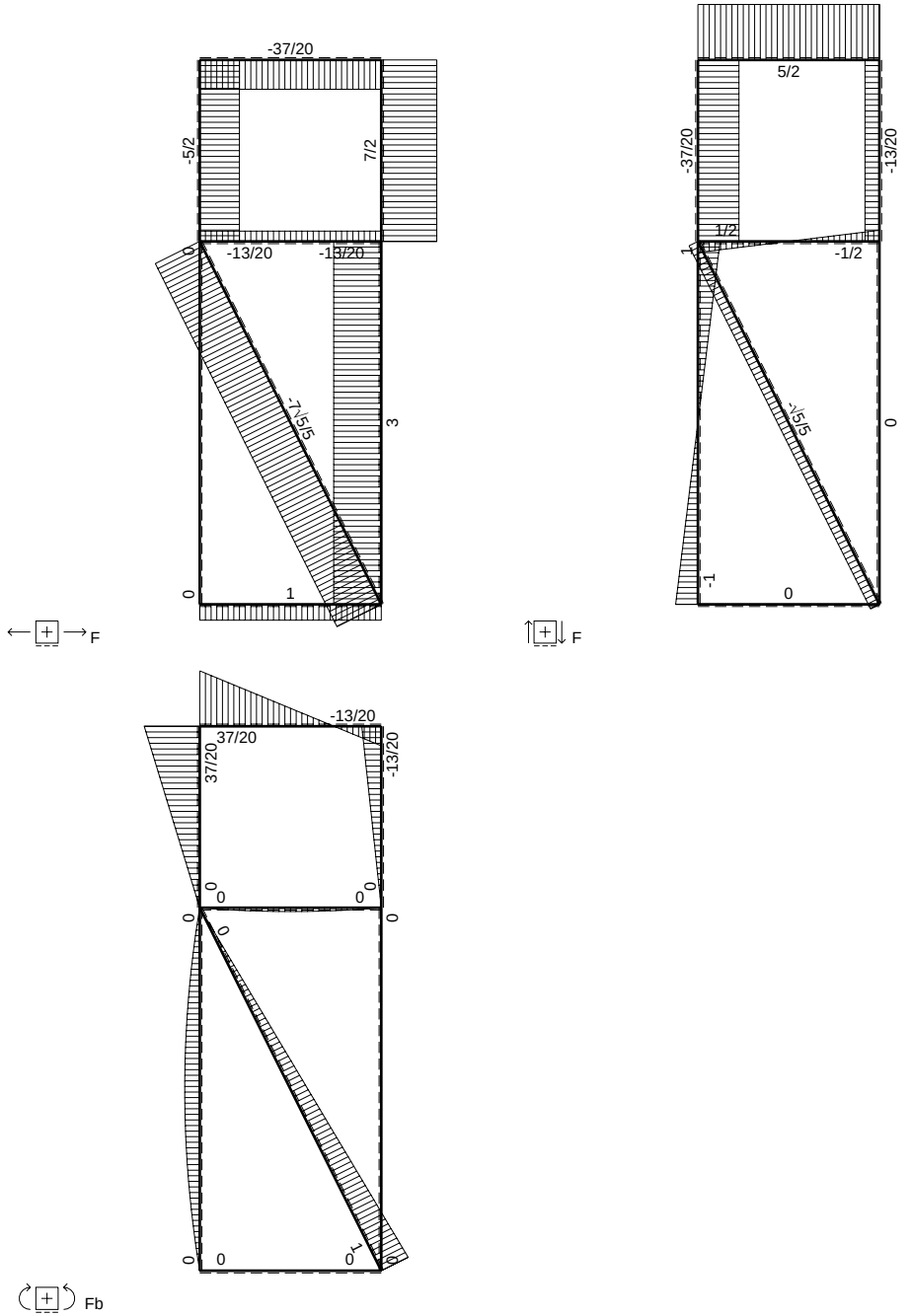


A = 678. mm^2
J_x = 266093. mm^4
J_y = 121648. mm^4
J_{xy} = 87929. mm^4
J_u = 307658. mm^4
J_v = 80083. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.4416
c = cosα = .9041
s = sinα = -.4274
x_g = 34.81 mm
y_g = 21.72 mm
N = -1601. N
T_y = -800.5 N
M_x = 1253000. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = -18.57 mm
v_m = 28.03 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v$ = -229.7 N/mm^2

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F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
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C
F
3F
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3F
C
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Fb
F
B
3F
A
B
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F
A
B


-37/20
-5/2
7/2
-13/20
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3
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1
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5/2
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-5/5/5
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	1/2Fx-1/2qx²	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx²	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{Xo} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

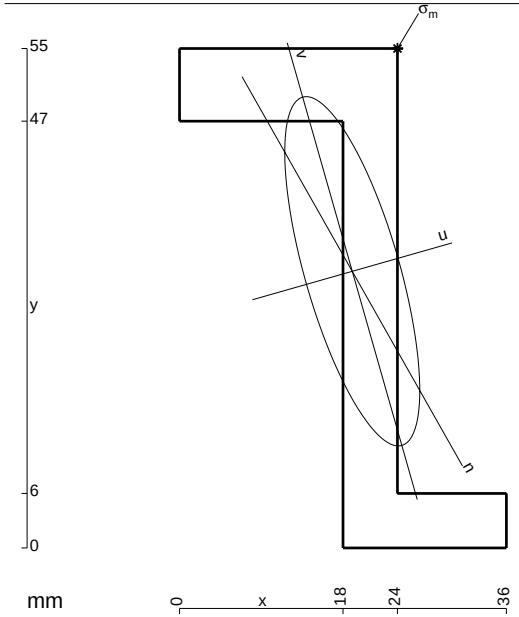
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

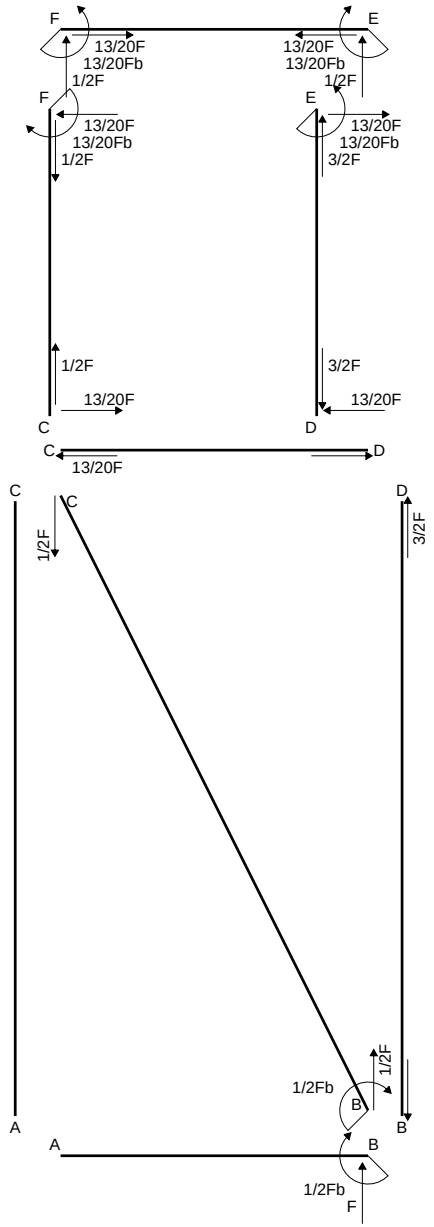
$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

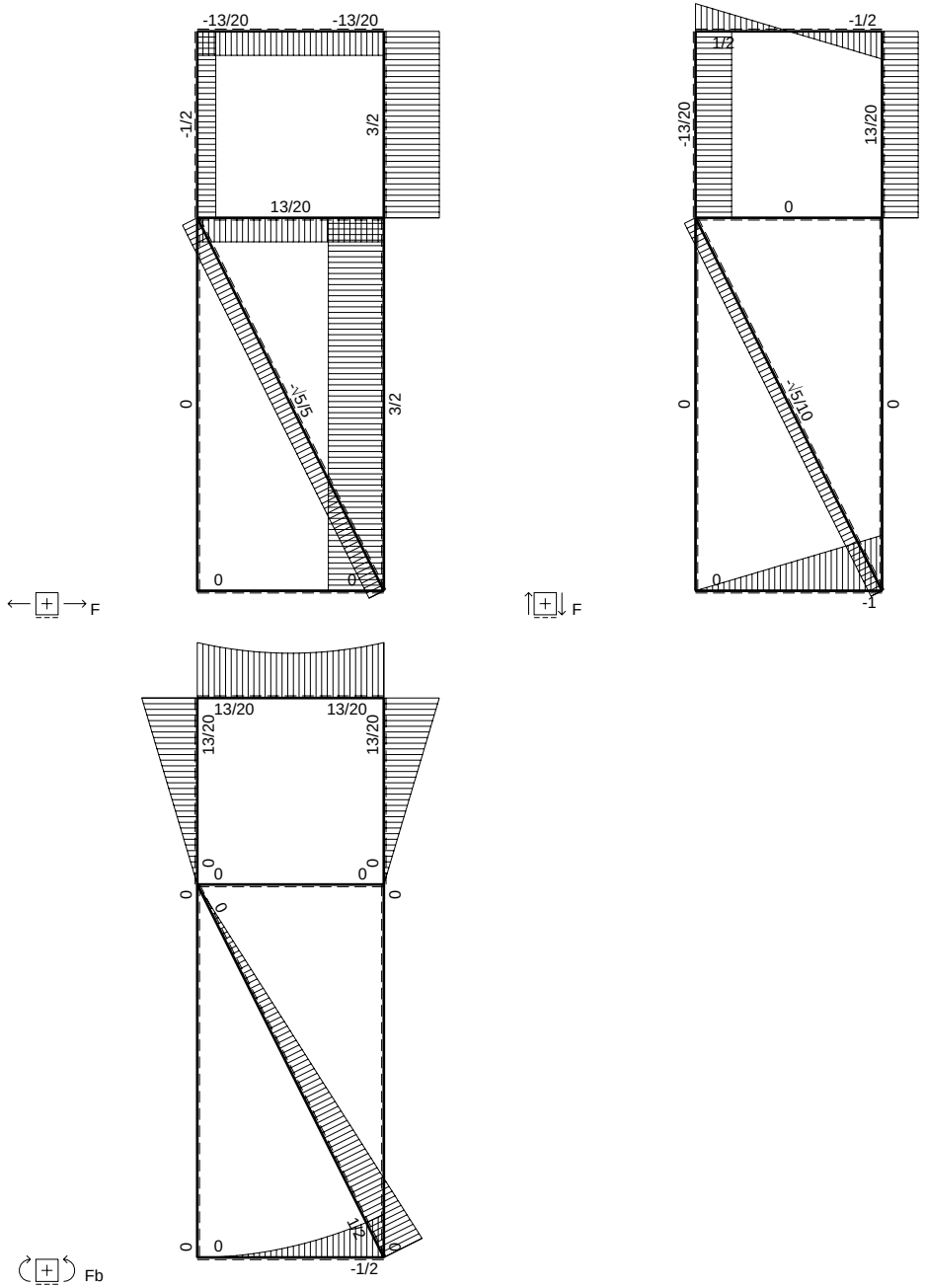


A = 546. mm^2
J_x = 202107. mm^4
J_y = 30174. mm^4
J_{xy} = -53280. mm^4
J_u = 217279. mm^4
J_v = 15002. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .2774
$c = \cos\alpha$ = .9618
$s = \sin\alpha$ = .2739
x_g = 19.02 mm
y_g = 30.47 mm
N = -1400. N
T_y = -1036. N
M_x = 766640. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 11.51 mm
v_m = 22.23 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -239. N/mm^2

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F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
0
-√5/5
F
1/2
-13/20
13/20
-√5/10
-1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi \theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
totali							$13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

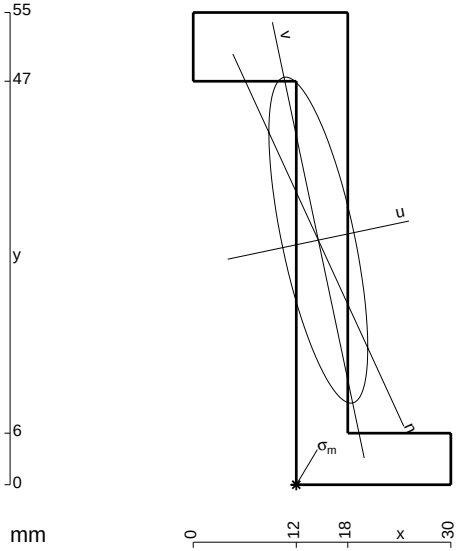
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

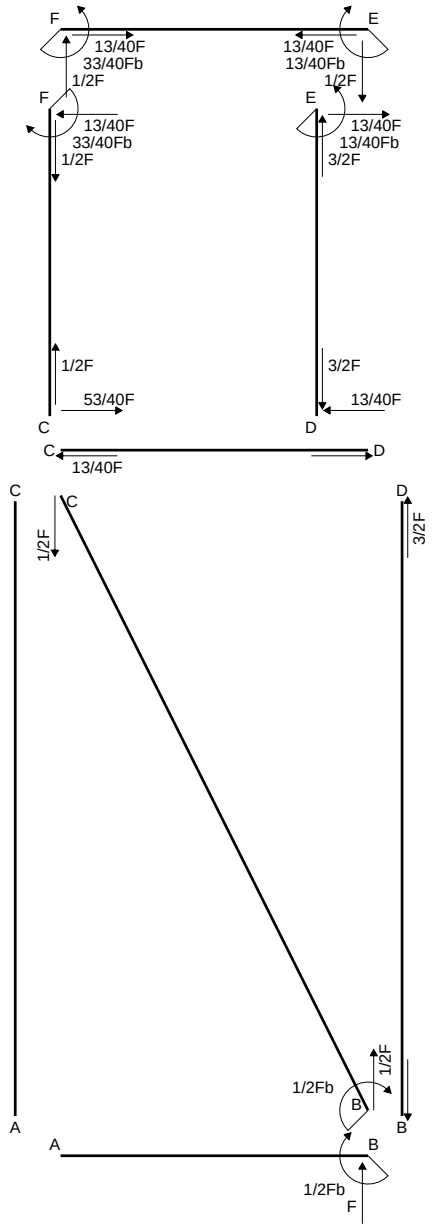
$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

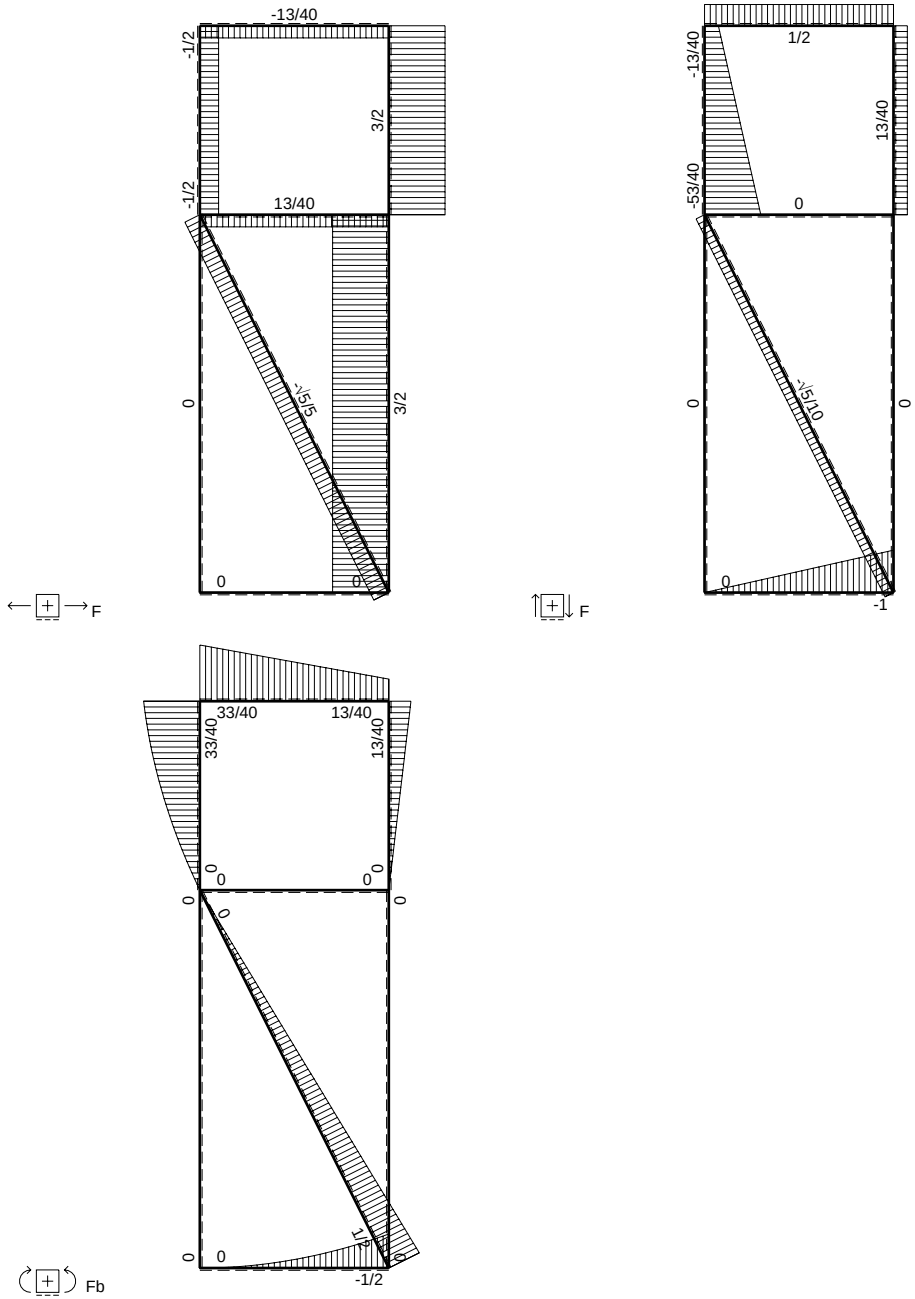


A = 498. mm^2
J_x = 179663. mm^4
J_y = 16520. mm^4
J_{xy} = -35967. mm^4
J_u = 187241. mm^4
J_v = 8943. mm^4
α= artg($2J_{xy}/(J_y-J_x)$)/2 = .2076
c = cosα = .9785
s = sinα = .2061
x_g = 14.57 mm
y_g = 28.49 mm
N = 3390. N
T_y = 1469. N
M_x = 572910. Nmm
x_m = 12. mm
u_m = -8.384 mm
v_m = -27.35 mm
σ_m = N/A-Mcv/J_u-Msu/J_v = 199.4 N/mm^2

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F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
1/2F
3/2F
1/2Fb
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

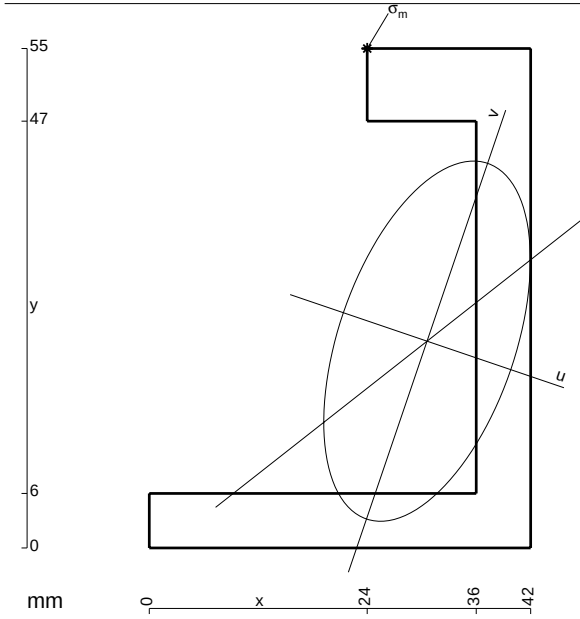
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

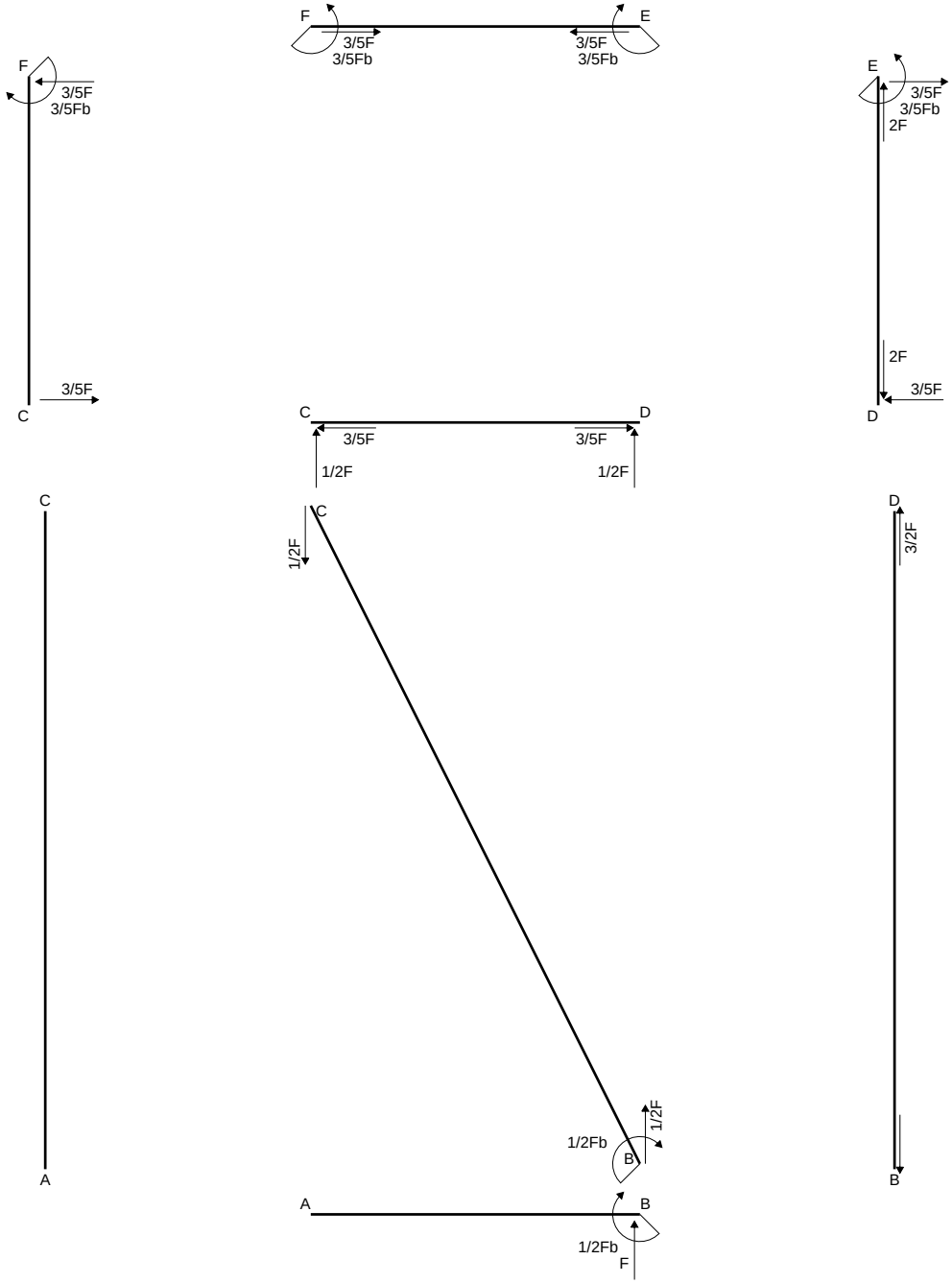
$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

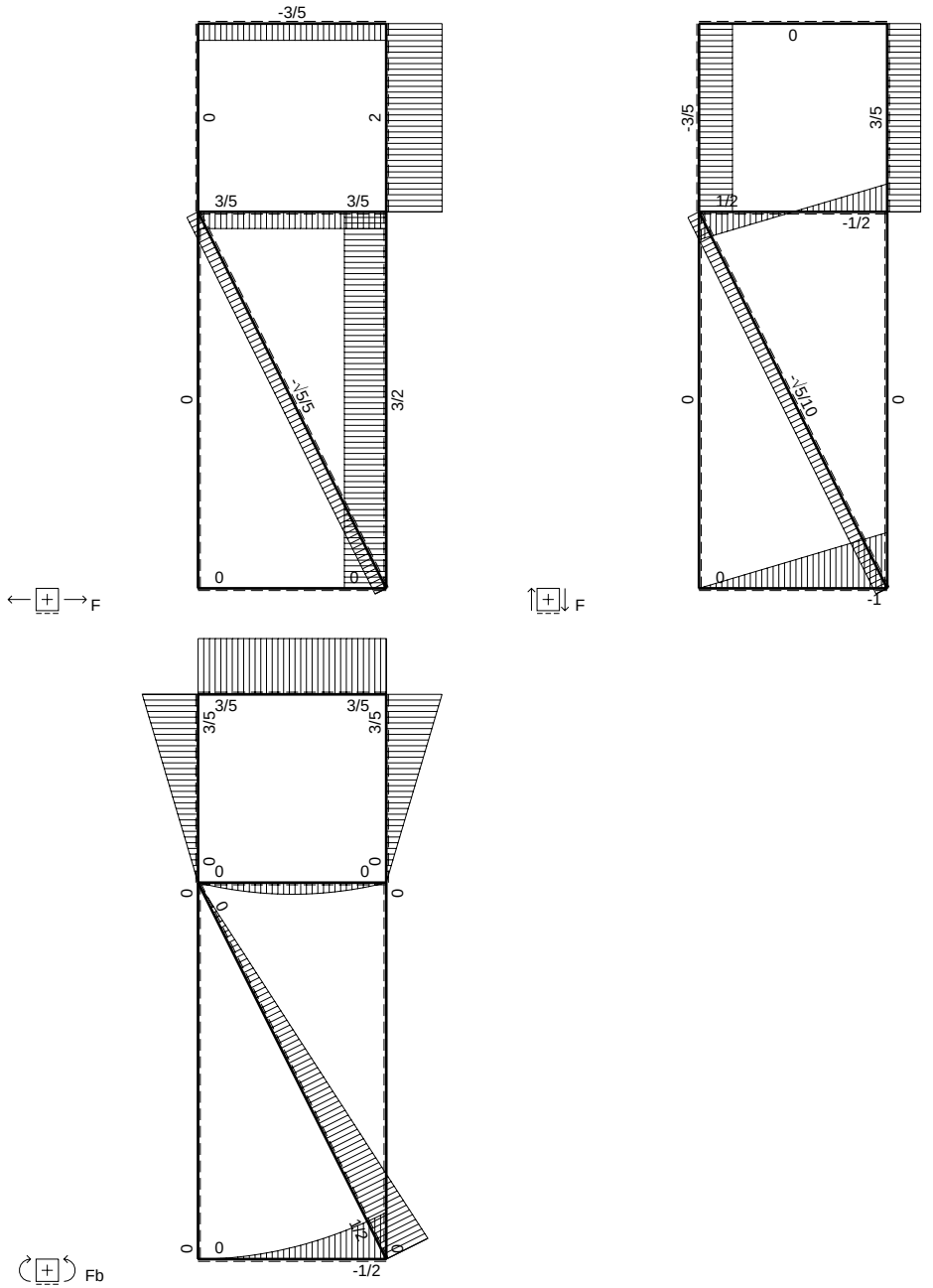


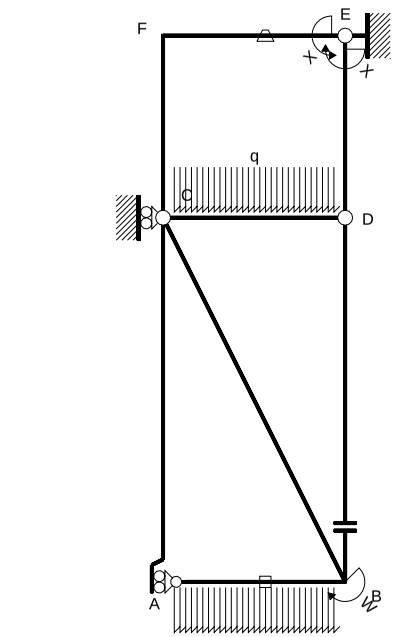
A = 642. mm^2
J_x = 252660. mm^4
J_y = 83081. mm^4
J_{xy} = 65292. mm^4
J_u = 274886. mm^4
J_v = 60856. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = -.3281
$c = \cos\alpha$ = .9467
$s = \sin\alpha$ = -.3222
x_g = 30.59 mm
y_g = 22.77 mm
N = -1610. N
T_y = -1047. N
M_x = 1115730. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = -16.62 mm
v_m = 28.39 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -209.8 N/mm^2

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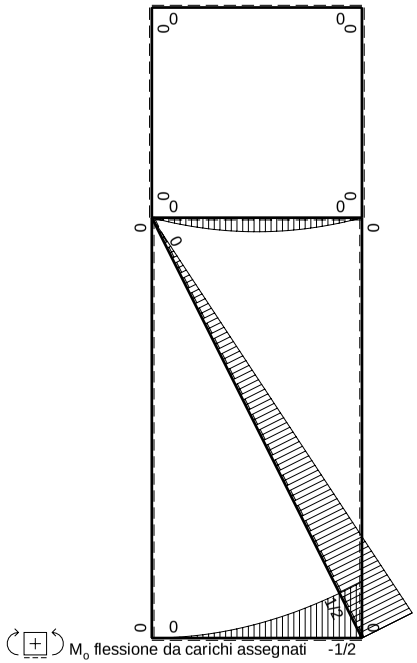
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F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
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3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F

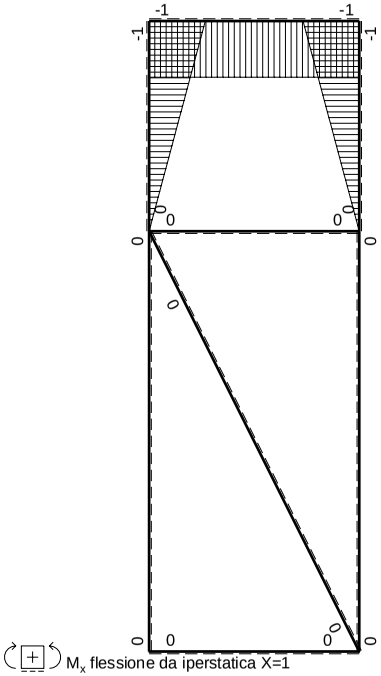

-3/5
0
2
3/5
3/5
0
3/2
-3/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-3/5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

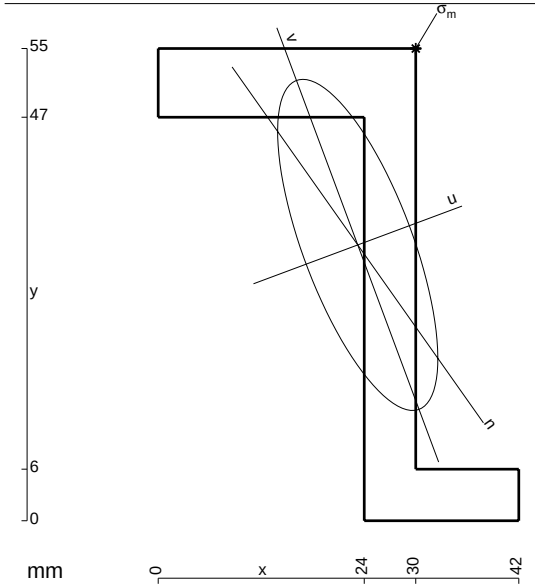
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1\ )\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1\ )\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

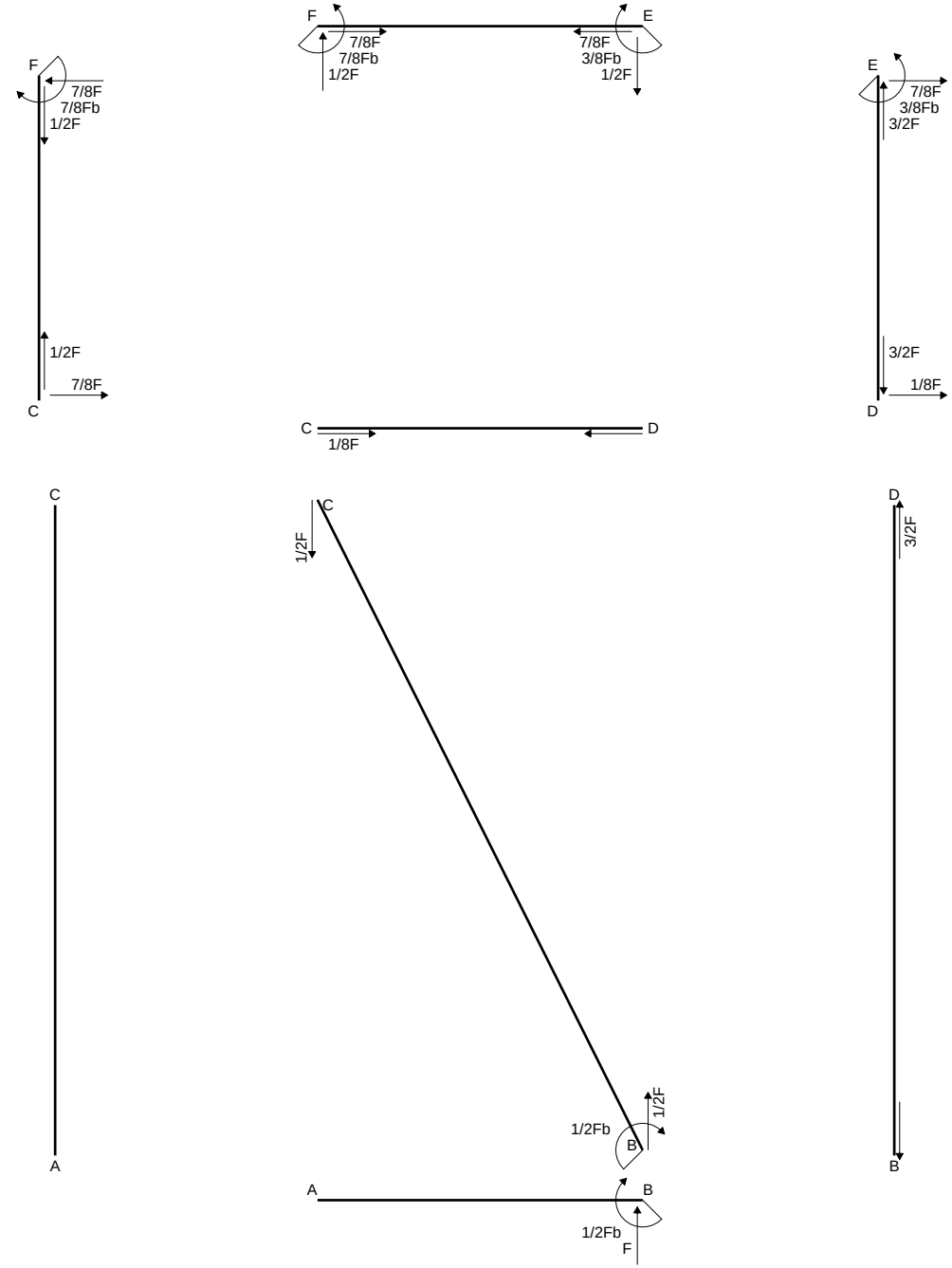
$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

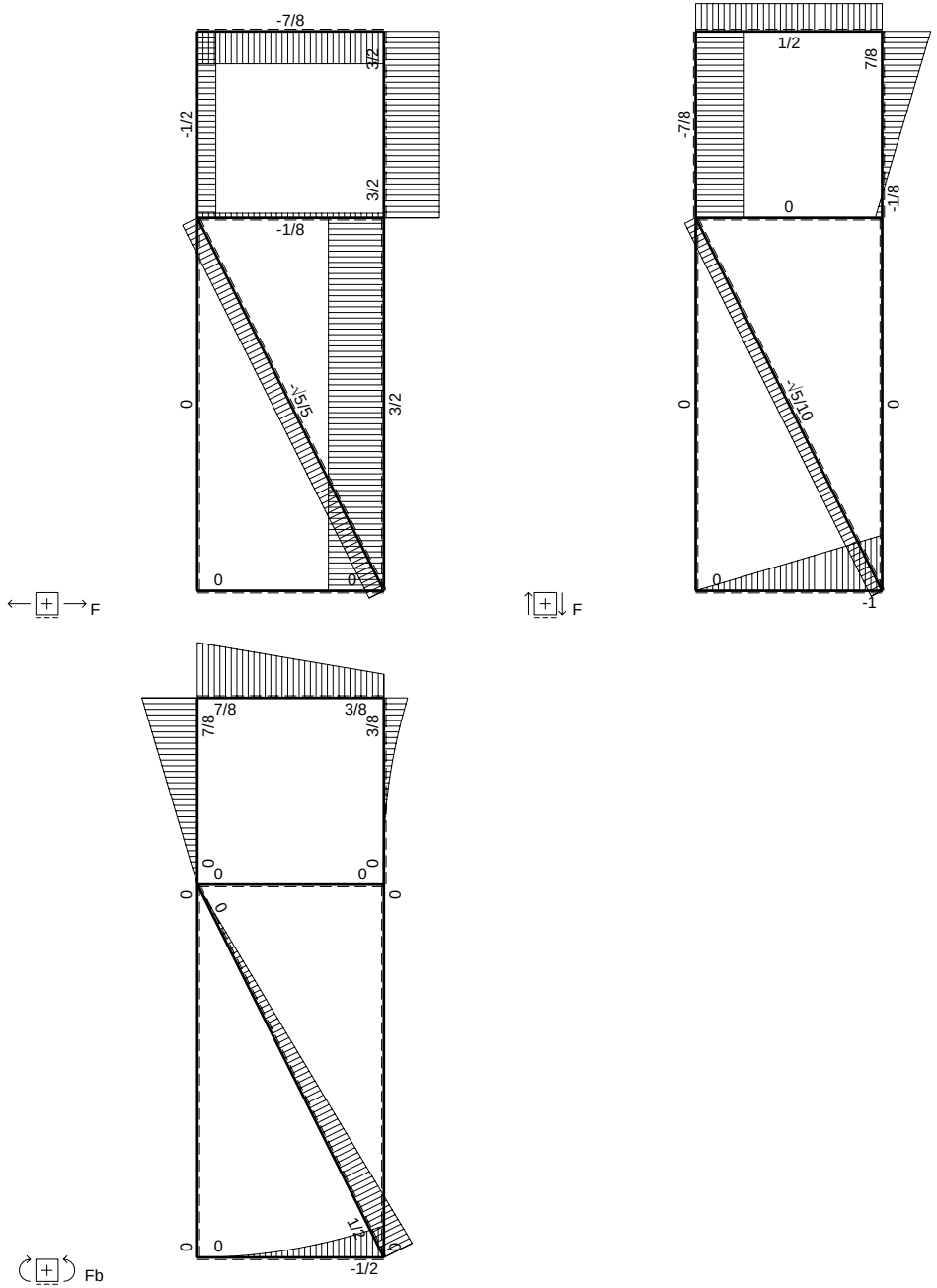


A = 594. mm^2
J_x = 220965. mm^4
J_y = 51715. mm^4
J_{xy} = -73230. mm^4
J_u = 248250. mm^4
J_v = 24429. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .3567
c = cosα = .9371
s = sinα = .3492
x_g = 23.24 mm
y_g = 32.13 mm
N = -1704. N
M_x = 783840. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 14.32 mm
v_m = 19.07 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -219.7 N/mm^2

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F
7/8F
7/8Fb
1/2F
1/2F
7/8F
C
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
D
1/8F
E
7/8F
3/8Fb
3/2F
3/2F
1/8F
D
C
A
C
1/2F
1/2Fb
B
1/2F
A
B
1/2Fb
F
D
3/2F
B


-7/8
3/2
-1/2
3/2
-1/8
0
-√5/5
3/2
0
0
F
1/2
7/8
-7/8
0
-1/8
0
-√5/10
0
0
-1
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	$-Fb/EJ$	$-1/2Fx$	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
totali							$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

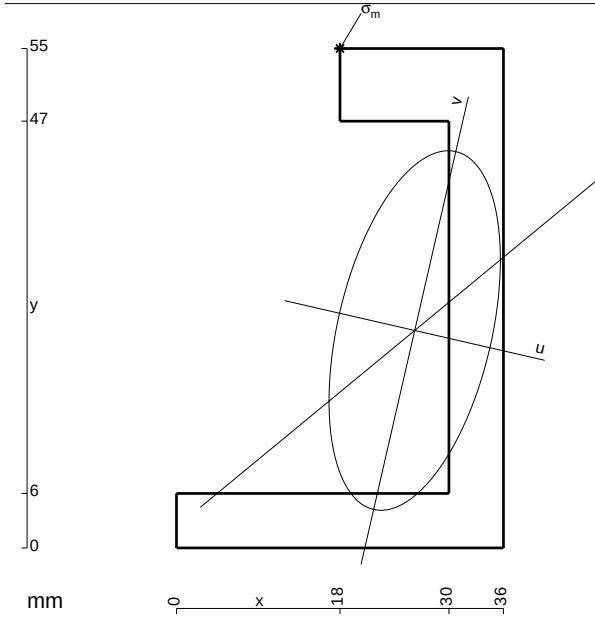
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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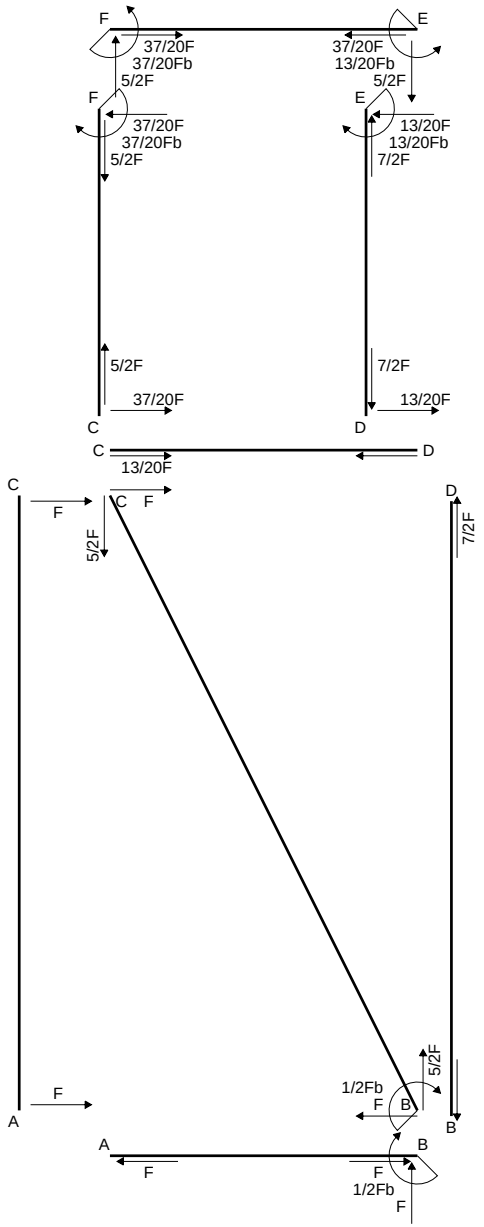


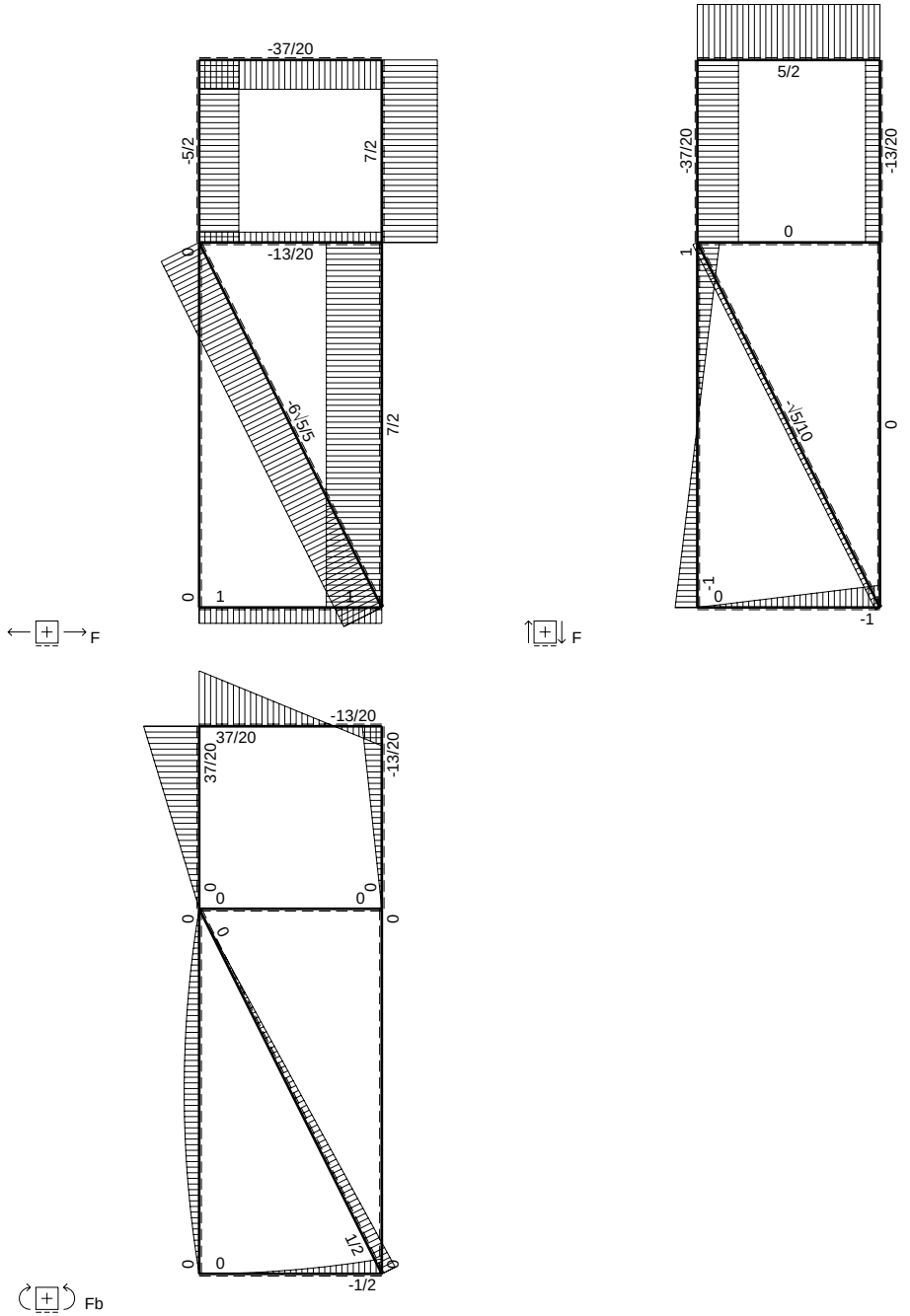
A = 606. mm^2
J_x = 237644. mm^4
J_y = 53945. mm^4
J_{xy} = 44489. mm^4
J_u = 247851. mm^4
J_v = 43738. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2 = -.2255$
$c = \cos\alpha = .9747$
$s = \sin\alpha = -.2236$
x_g = 26.23 mm
y_g = 23.95 mm
N = -2398. N
T_y = 1370. N
M_x = 1198750. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = -14.96 mm
v_m = 28.43 mm
$\sigma_m = N/A - Mcv/J_u - Msu/J_v = -229.7\ N/mm^2$

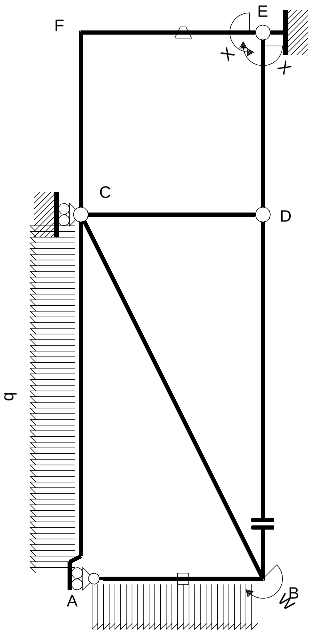
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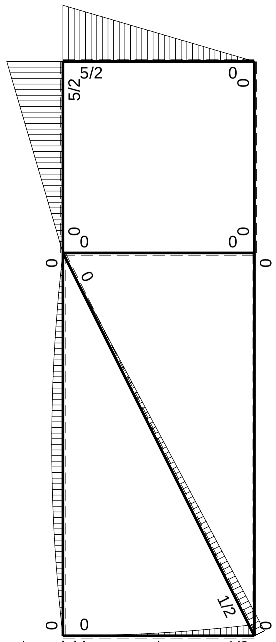
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F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb

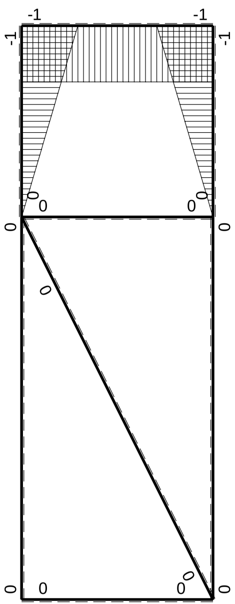

-37/20
-5/2
7/2
-13/20
-5\5/5
F
5/2
-37/20
-13/20
-5/10
37/20
1/2
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

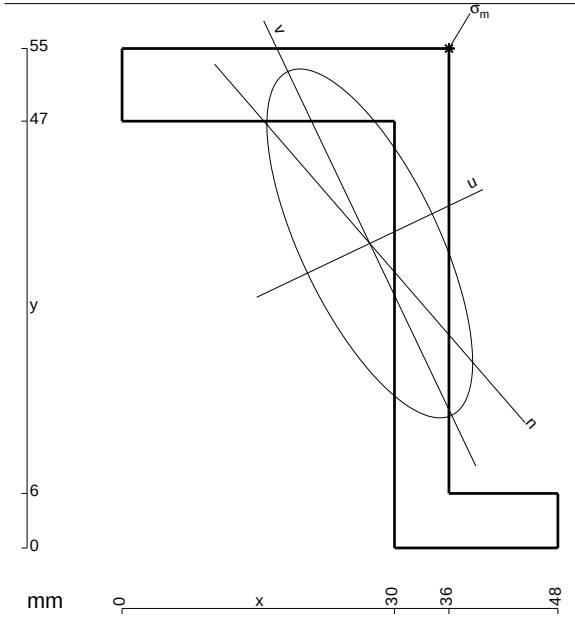
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 642. mm^2
J_x = 237041. mm^4
J_y = 82444. mm^4
J_{xy} = -95227. mm^4
J_u = 282393. mm^4
J_v = 37091. mm^4
$\alpha = artg(2J_{xy}/(J_y-J_x))/2$ = .4445
$c = \cos\alpha$ = .9028
$s = \sin\alpha$ = .4300
x_g = 27.28 mm
y_g = 33.54 mm
N = -2375. N
T_y = -1758. N
M_x = 949050. Nmm
x_m = 36. mm
y_m = 55. mm
u_m = 17.1 mm
v_m = 15.63 mm
$\sigma_m = N/A-Mcv/J_u-Msu/J_v$ = -239.3 N/mm^2

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