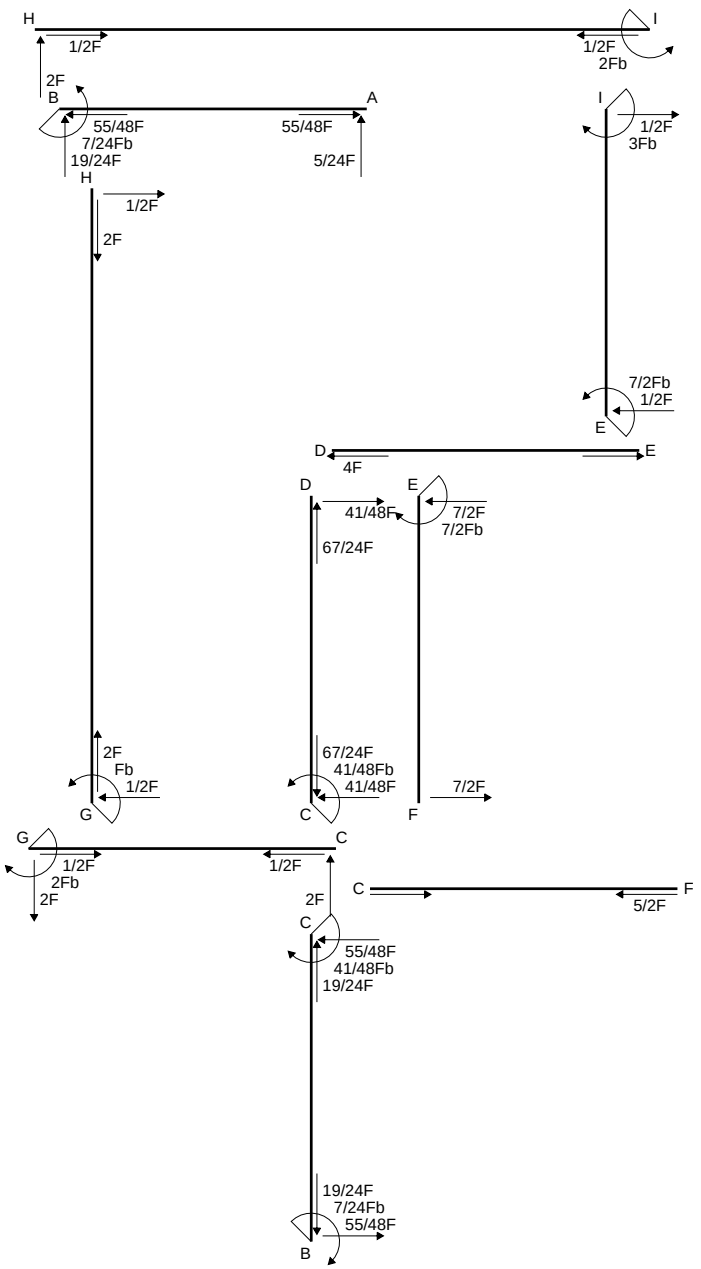
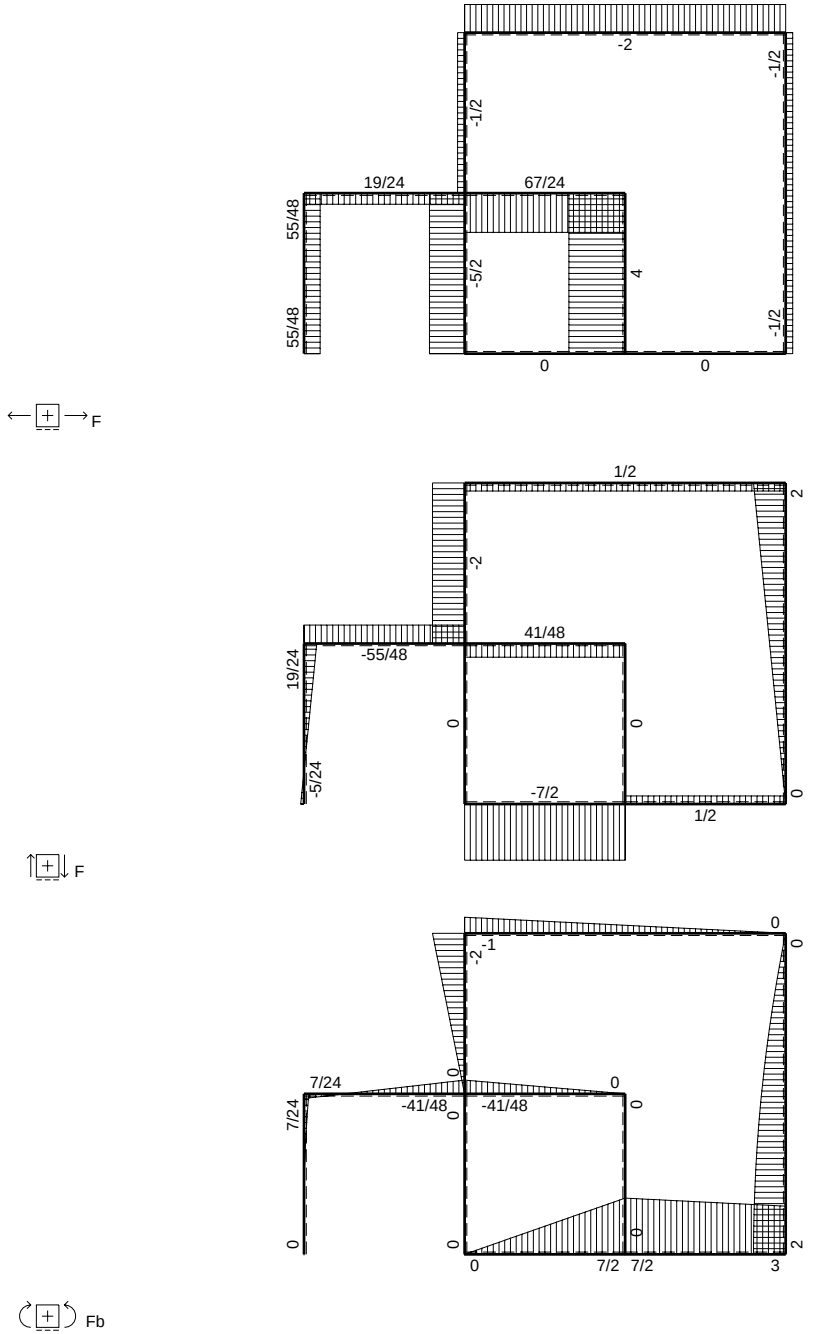
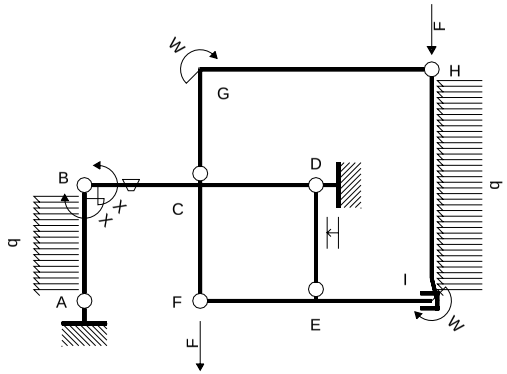




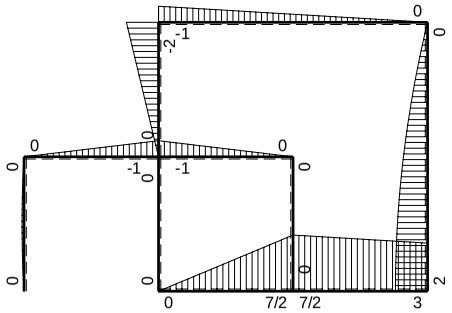





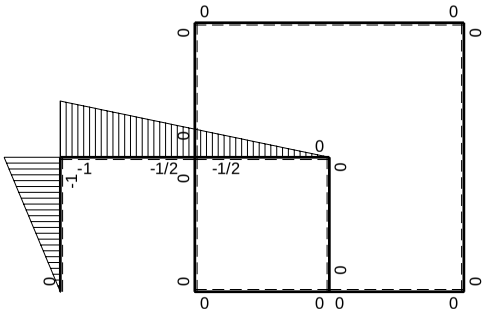

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Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 249087 CI01.xxxx.001

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $7/24Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

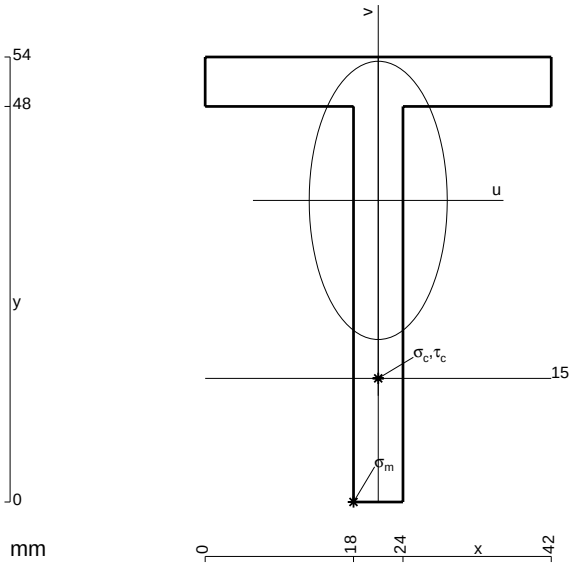
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

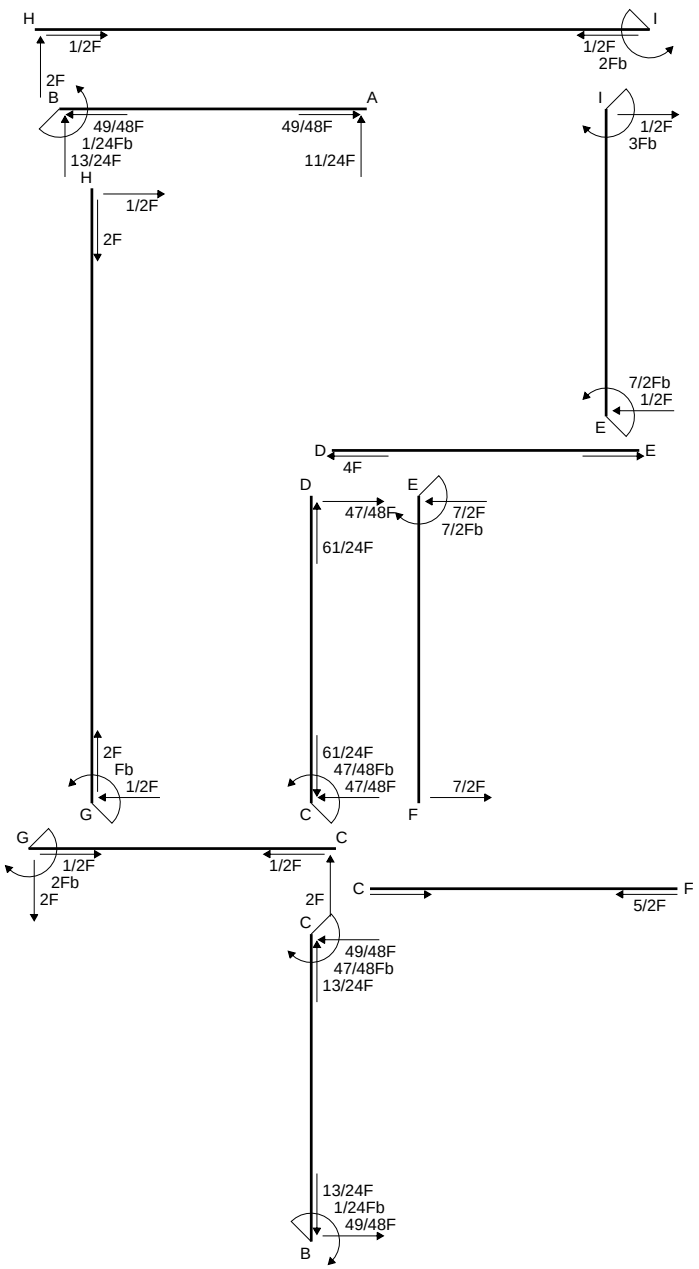
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





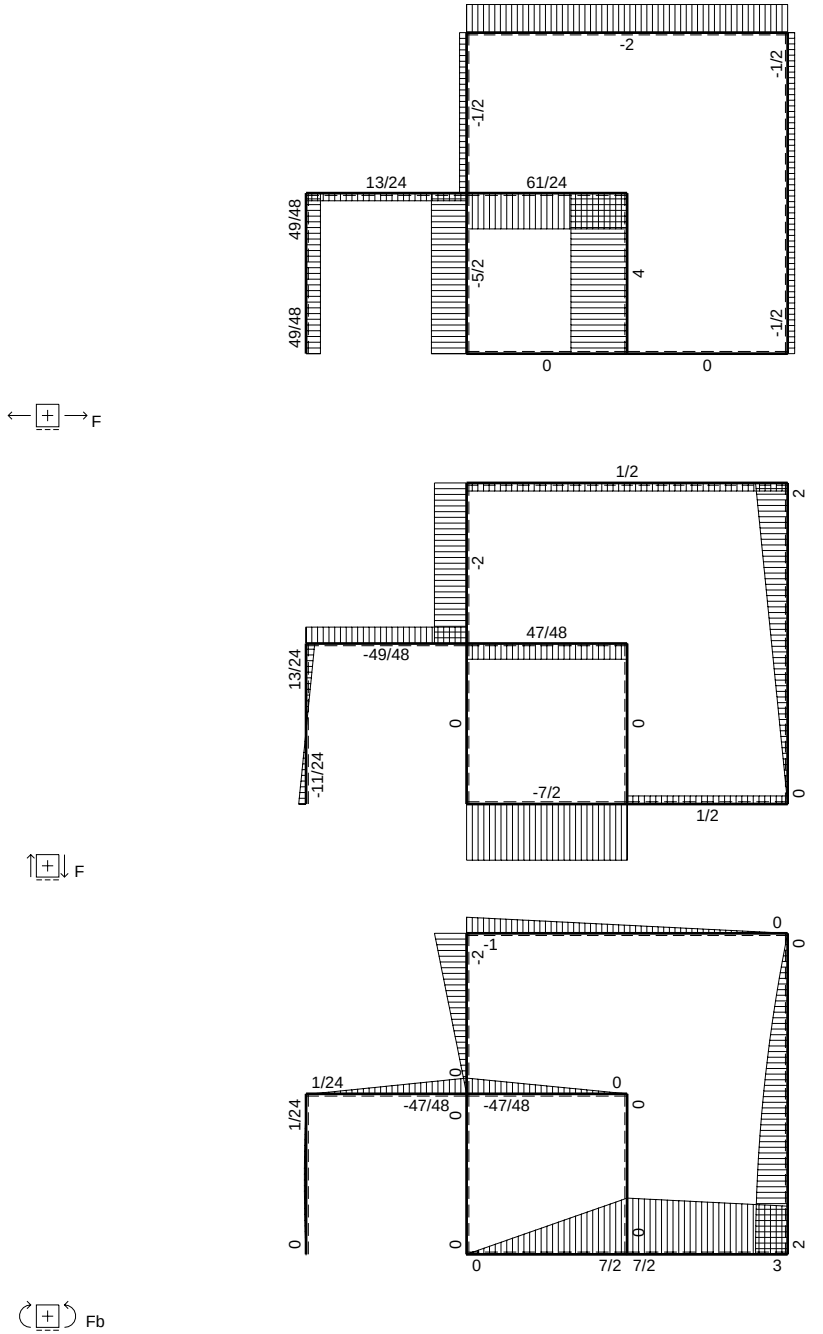


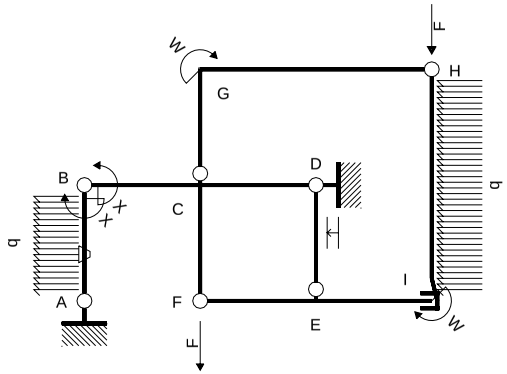

A = 540. $mm^2$
$J_u$ = 154030. $mm^4$
$J_v$ = 37908. $mm^4$
$y_g$ = 36.6 mm
$T_y$ = -1785. N
$M_x$ = 874650. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -36.6 mm
$\sigma_m = -Mv/J_u$ = 207.8 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 15. mm
$v_c$ = -21.6 mm
$\sigma_c = -Mv/J_u$ = 122.7 $N/mm^2$
$\tau_c$ = 5.058 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123. $N/mm^2$
$S^*$ = 2619. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
I
49/48F
1/24Fb
13/24F
49/48F
11/24F
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
D
E
47/48F
7/2F
7/2Fb
61/24F
2F
Fb
1/2F
G
61/24F
47/48Fb
47/48F
C
7/2F
F
G
C
1/2F
2Fb
2F
1/2F
C
F
5/2F
2F
C
49/48F
47/48Fb
13/24F
13/24F
1/24Fb
49/48F
B

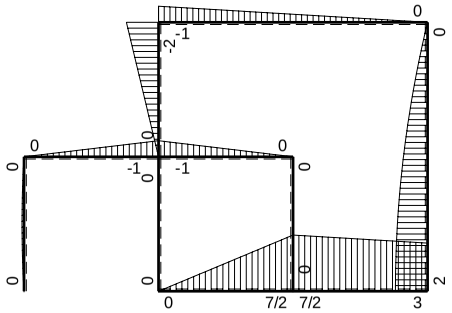




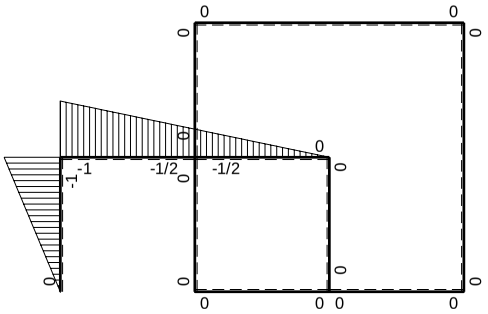

-2
-1/2
-1/2
13/24
61/24
49/48
-5/2
4
49/48
-1/2
0
0
F
1/2
2
-2
47/48
-49/48
13/24
0
0
-11/24
-7/2
0
1/2
F
0
0
-2
-1
0
1/24
0
1/24
-47/48
-47/48
0
0
0
0
0
2
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 957766 CI01.xxxx.002

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PROCEDIMENTO E RISULTATI 957766 CI01.xxxx.002

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | $-x/b$ | $-1/2Fx+1/2qx^2$ | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$ | $Fx/EJ$ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | $1-x/b$ | $1/2Fx-1/2qx^2$ | $Fb/EJ$ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-Fx$ | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | $Fb-Fx$ | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-Fb+Fx$ | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | $Fx$ | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-7/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/2Fx$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $2Fb-2Fx$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+1/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $1/2Fx$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-7/2Fb+1/2Fx$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/24Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | $-1/24Fb$ | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b ( x/b )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

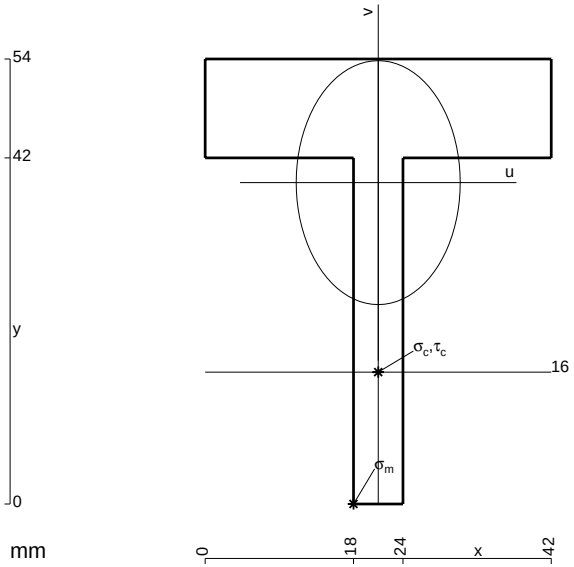
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

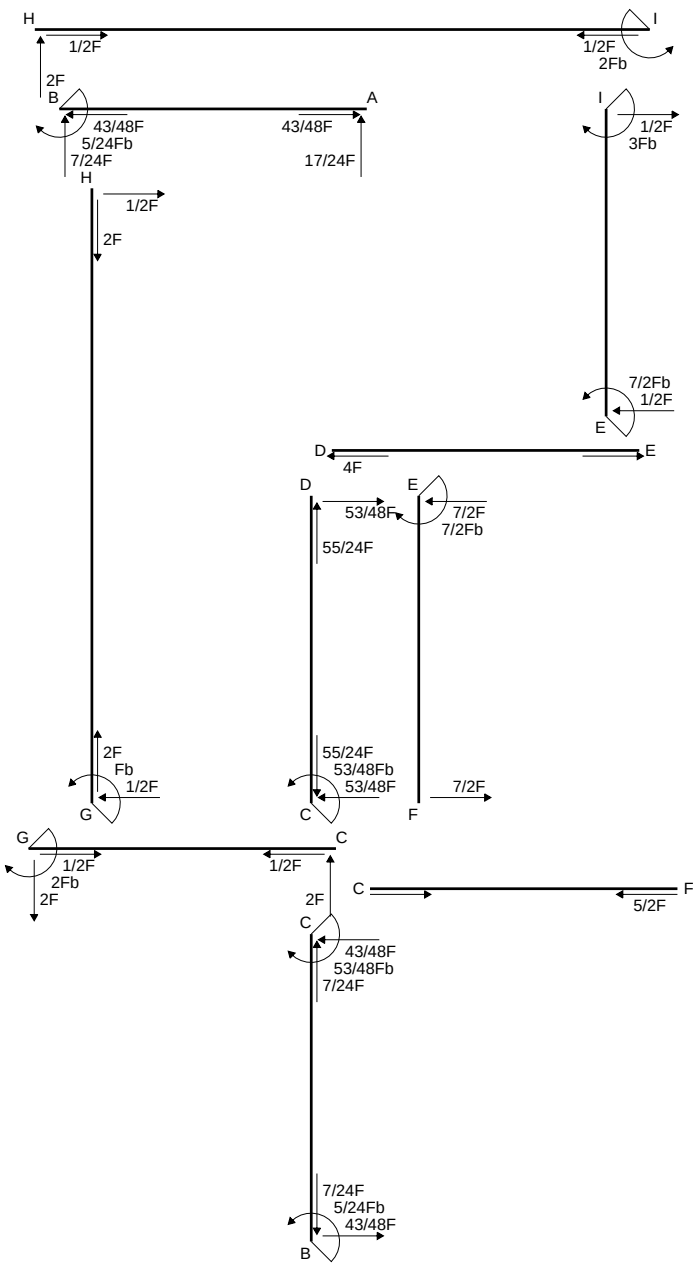
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$



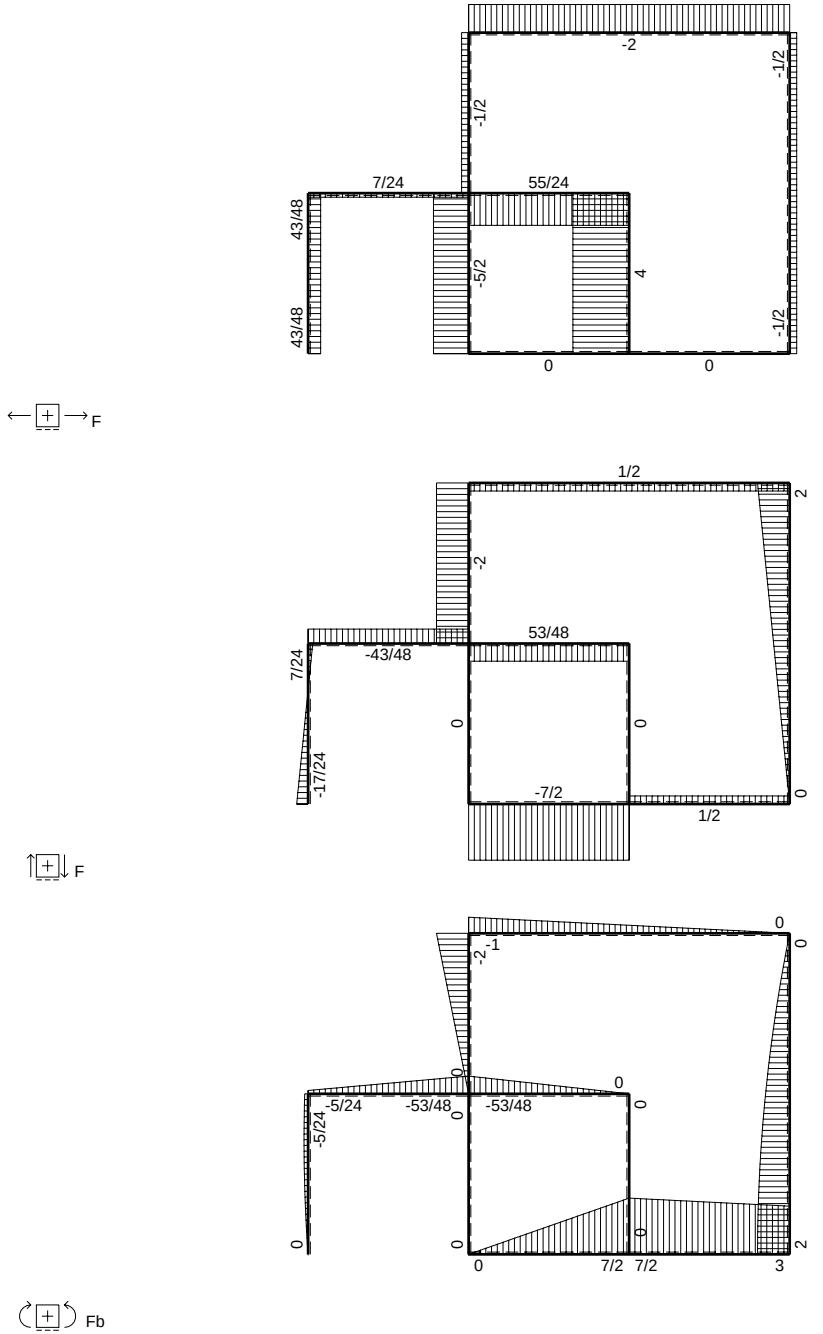


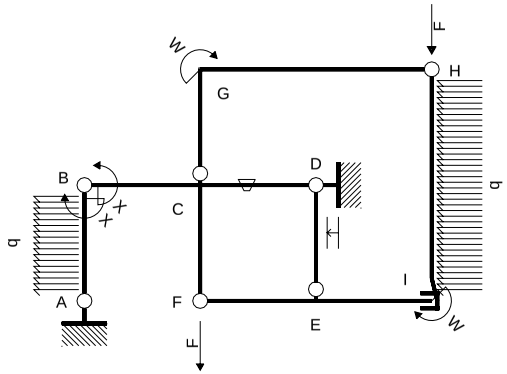



A = 756. $mm^2$
$J_u$ = 165564. $mm^4$
$J_v$ = 74844. $mm^4$
$y_g$ = 39. mm
$T_y$ = -1715. N
$M_x$ = 926100. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -39. mm
$\sigma_m = -Mv/J_u$ = 218.2 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -23. mm
$\sigma_c = -Mv/J_u$ = 128.7 $N/mm^2$
$\tau_c$ = 5.138 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129. $N/mm^2$
$S^*$ = 2976. $mm^3$

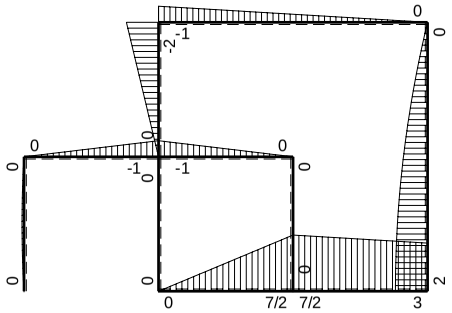

H
I
1/2F
1/2F
2Fb
2F
B
A
43/48F
43/48F
5/24Fb
7/24F
17/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
D
E
53/48F
7/2F
7/2Fb
55/24F
2F
Fb
1/2F
55/24F
53/48Fb
53/48F
7/2F
G
C
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
5/2F
C
43/48F
53/48Fb
7/24F
7/24F
5/24Fb
43/48F
B



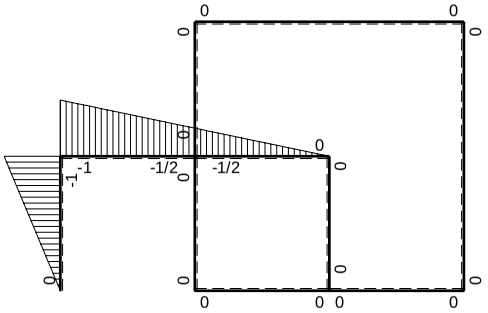

F
-2
-1/2
-1/2
7/24
55/24
43/48
-5/2
4
43/48
-1/2
0
0
F
1/2
2
-2
53/48
-43/48
7/24
0
0
-17/24
-7/2
0
1/2
Fb
0
0
-2
1
0
0
0
-5/24
-53/48
-53/48
-5/24
0
0
0
0
0
2
0
7/2
7/2
3




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 218144 CI01.xxxx.003

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PROCEDIMENTO E RISULTATI 218144 CI01.xxxx.003

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-5/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 5/24Fb | |

Sviluppi di calcolo iperstatica


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$L_{AB}^{XX} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BA}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{XX} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L_{AB}^{Xo} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

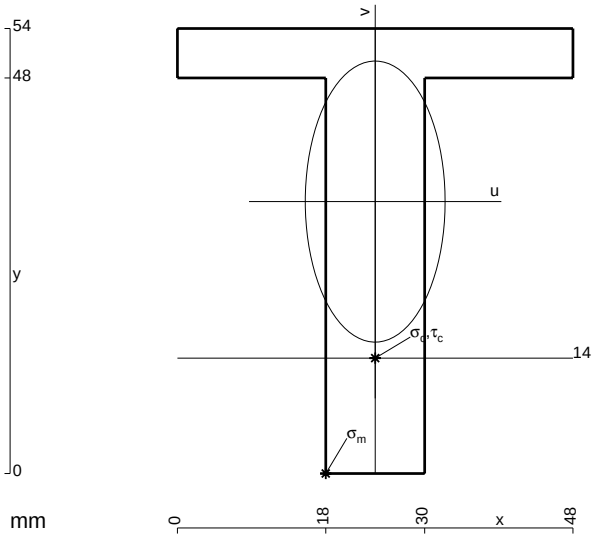
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

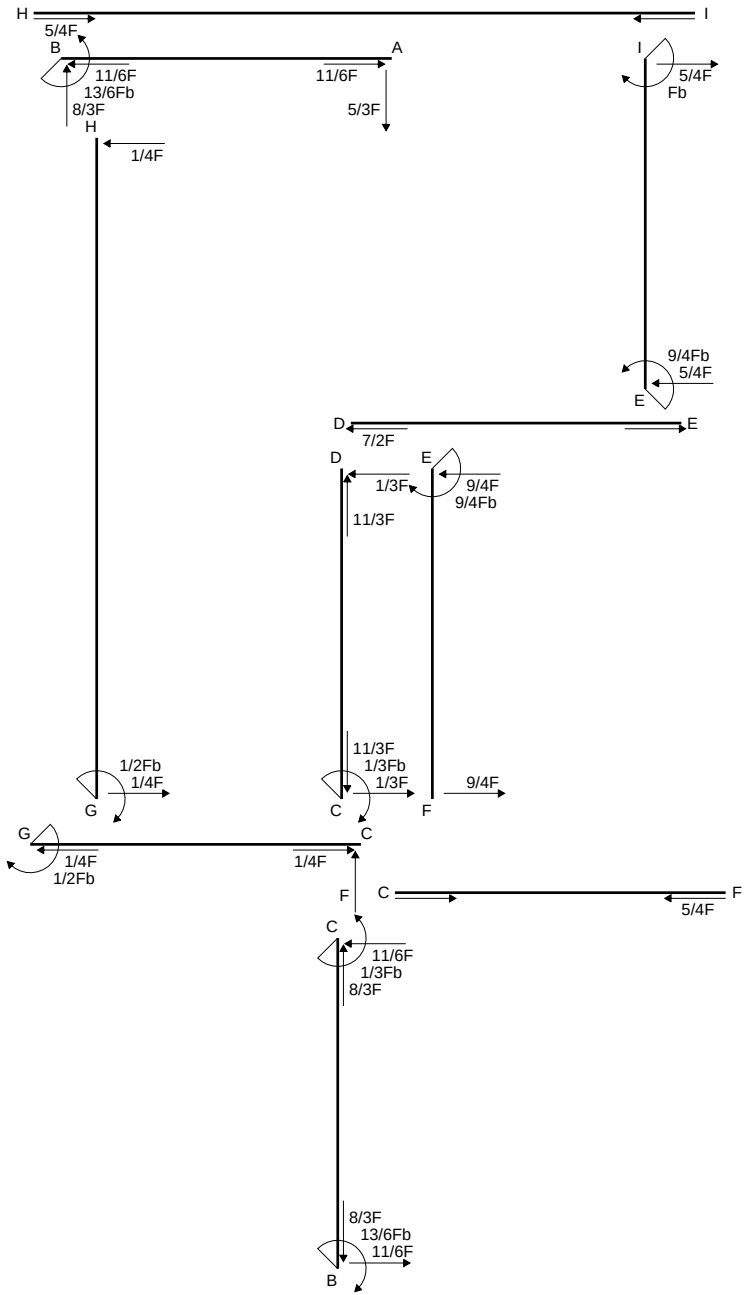


PROCEDIMENTO E RISULTATI 218144 CI01.xxxx.003



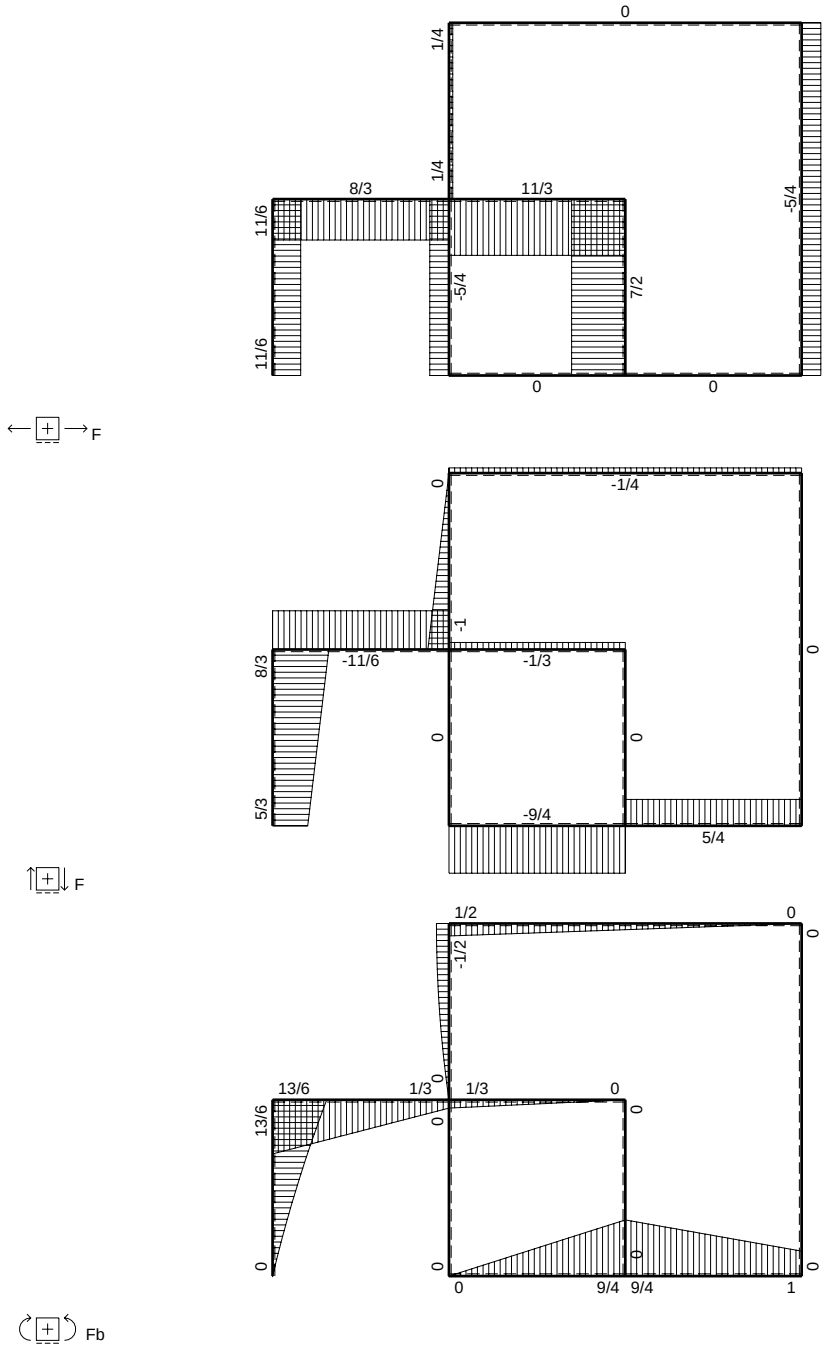

A = 864. $mm^2$
$J_u$ = 251424. $mm^4$
$J_v$ = 62208. $mm^4$
$y_g$ = 33. mm
$T_y$ = -3010. N
$M_x$ = 1745800. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -33. mm
$\sigma_m = -Mv/J_u$ = 229.1 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 14. mm
$v_c$ = -19. mm
$\sigma_c = -Mv/J_u$ = 131.9 $N/mm^2$
$\tau_c$ = 4.358 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.1 $N/mm^2$
$S^*$ = 4368. $mm^3$

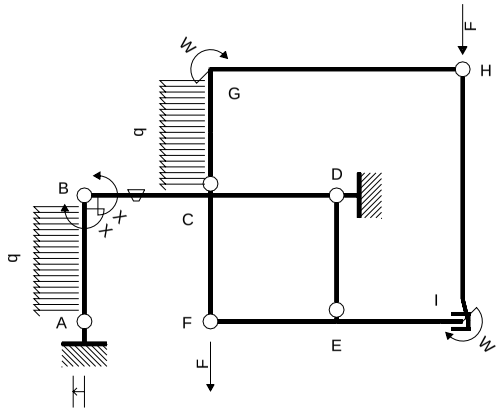
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
5/4F
B
A
I
11/6F
13/6Fb
8/3F
11/6F
5/3F
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
1/3F
9/4F
9/4Fb
11/3F
1/2Fb
1/4F
G
11/3F
1/3Fb
1/3F
9/4F
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
C
11/6F
1/3Fb
8/3F
8/3F
13/6Fb
11/6F
B

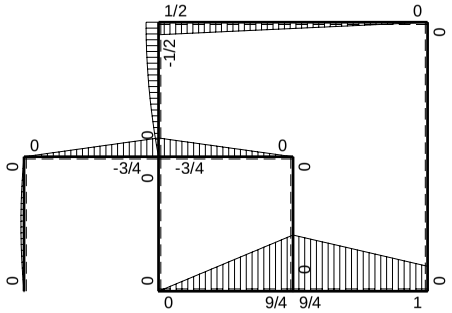




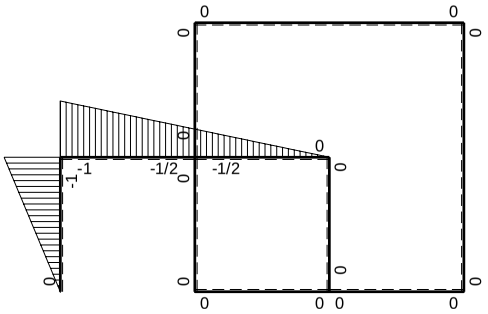

0
1/4
1/4
8/3
11/3
-5/4
11/6
-5/4
7/2
11/6
0
0
F
0
-1/4
-1
8/3
-11/6
-1/3
0
0
0
0
5/3
-9/4
5/4
F
1/2
0
0
-1/2
0
13/6
1/3
1/3
0
13/6
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 213350 CI01.xxxx.004

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PROCEDIMENTO E RISULTATI 213350 CI01.xxxx.004

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

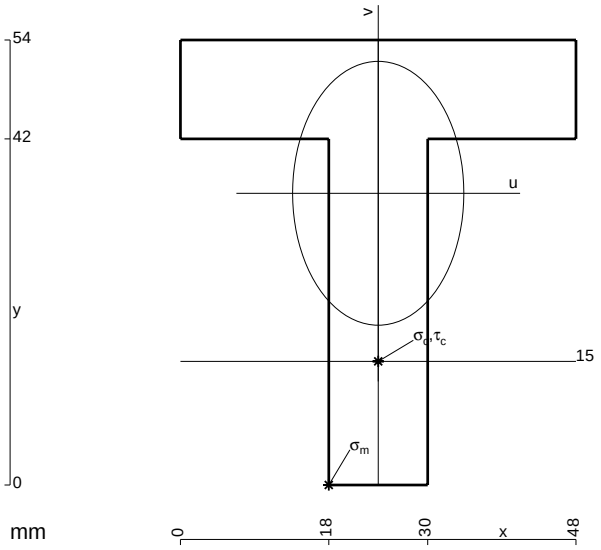
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

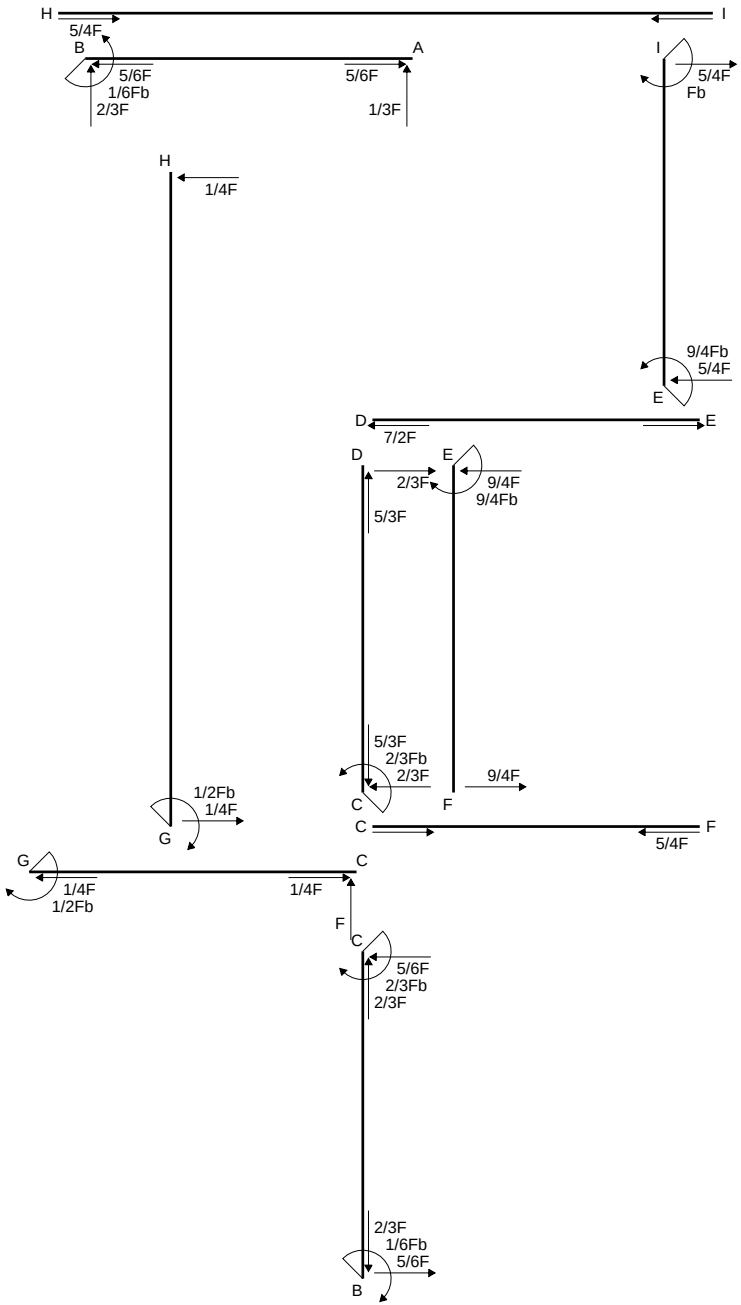
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





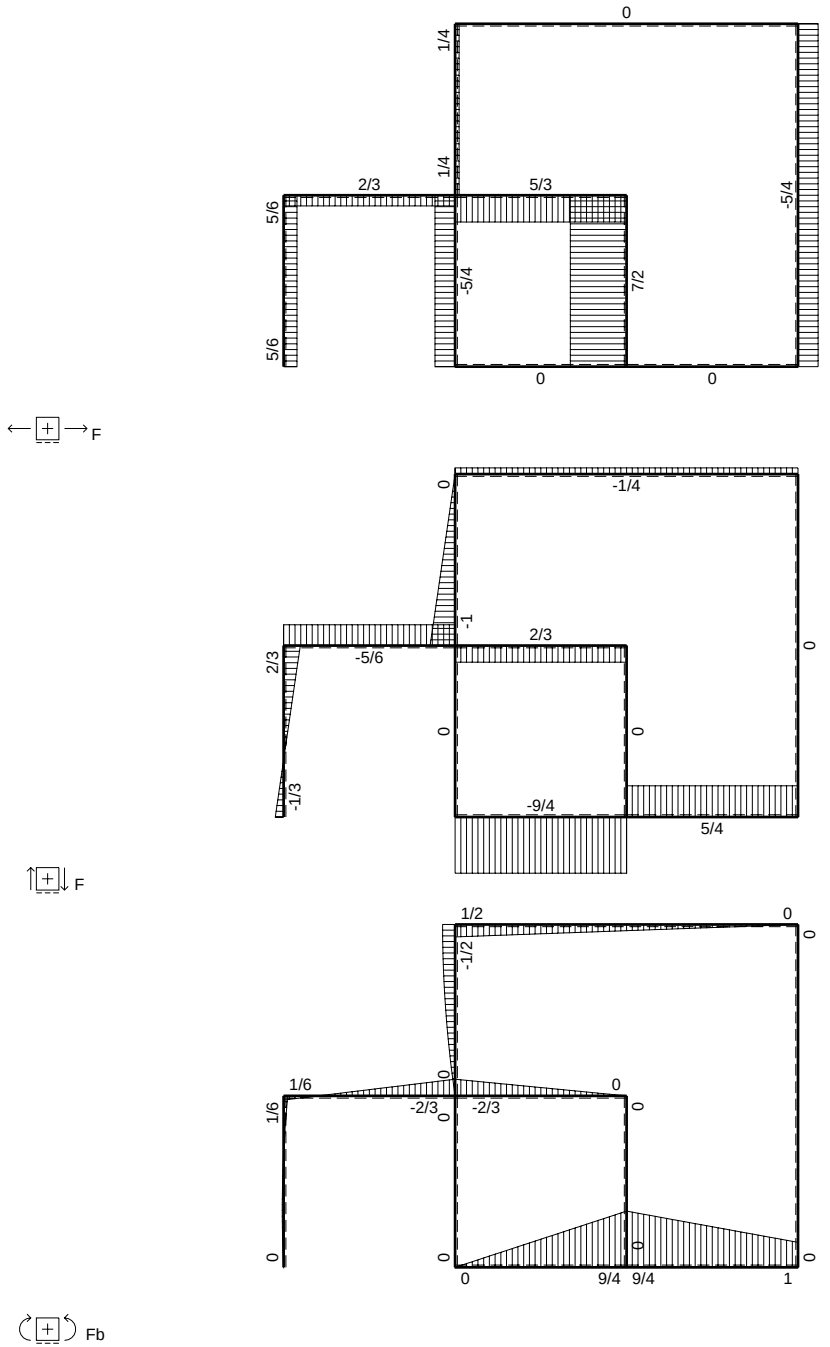


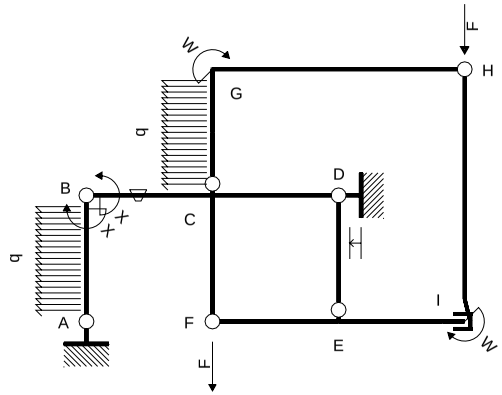

A = 1080. $mm^2$
$J_u$ = 276955. $mm^4$
$J_v$ = 116640. $mm^4$
$y_g$ = 35.4 mm
$T_y$ = -2970. N
$M_x$ = 1871100. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -35.4 mm
$\sigma_m = -Mv/J_u$ = 239.2 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -20.4 mm
$\sigma_c = -Mv/J_u$ = 137.8 $N/mm^2$
$\tau_c$ = 4.488 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138. $N/mm^2$
$S^*$ = 5022. $mm^3$


H
I
5/4F
B
A
5/6F
1/6Fb
2/3F
5/6F
1/3F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
2/3F
9/4F
9/4Fb
5/3F
5/3F
2/3Fb
2/3F
9/4F
C
F
1/2Fb
1/4F
G
C
F
5/4F
G
C
1/4F
1/4F
1/2Fb
F
C
5/6F
2/3Fb
2/3F
2/3F
1/6Fb
5/6F
B

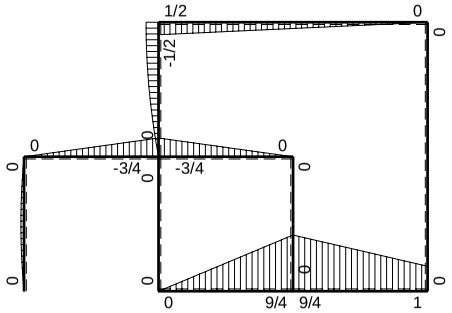




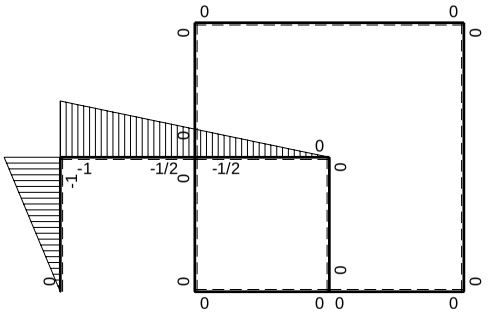

F
F
Fb
0
1/4
1/4
2/3
5/3
-5/4
5/6
-5/4
7/2
5/6
0
0
-1/4
0
-1
2/3
2/3
-5/6
0
0
0
-1/3
-9/4
5/4
1/2
0
0
-1/2
1/6
0
0
1/6
-2/3
-2/3
0
0
0
0
0
0
9/4
9/4
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265319 CI01.xxxx.005

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 265319 CI01.xxxx.005

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

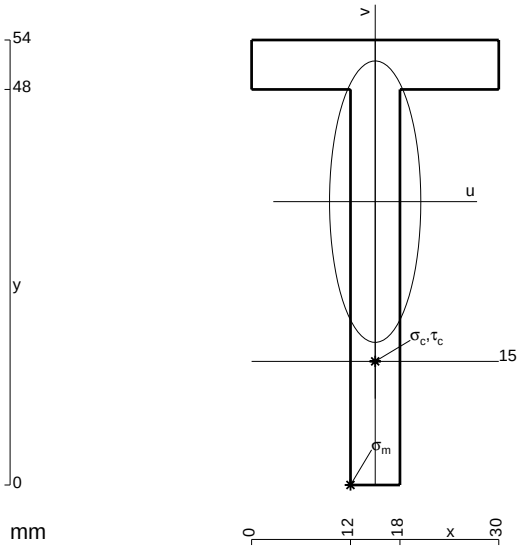
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

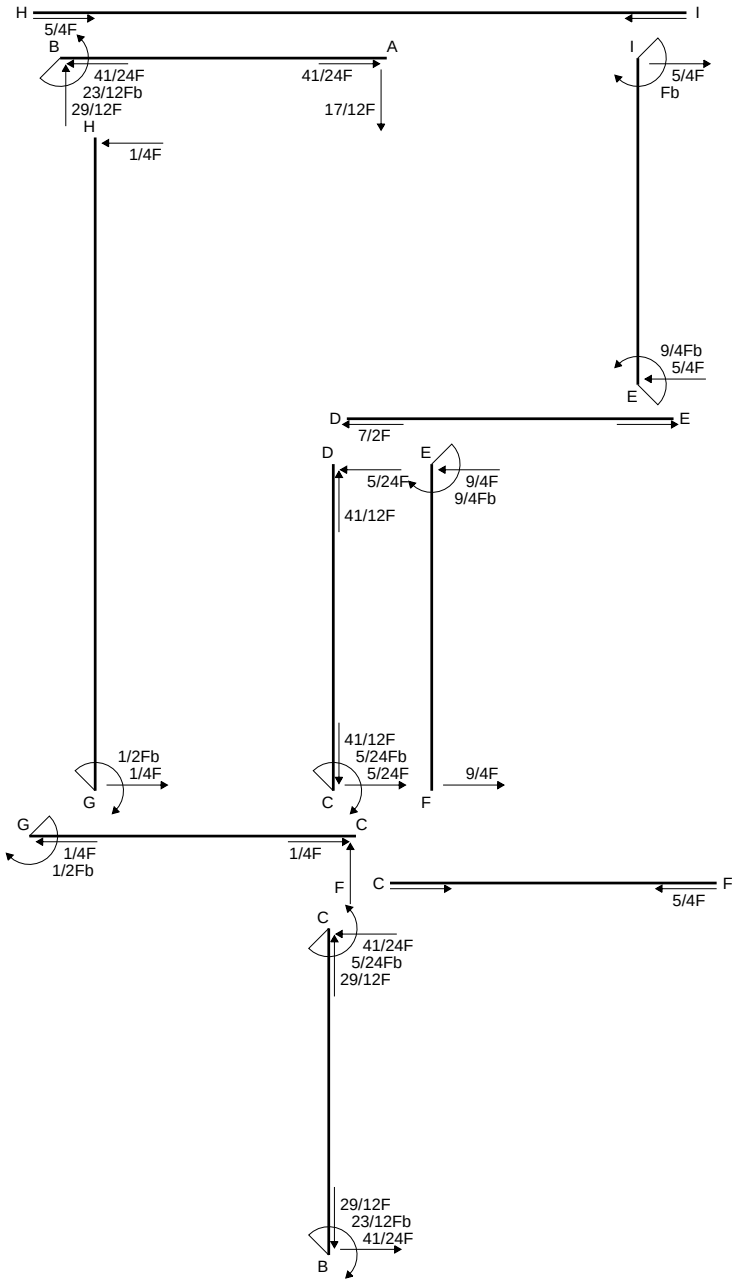
$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





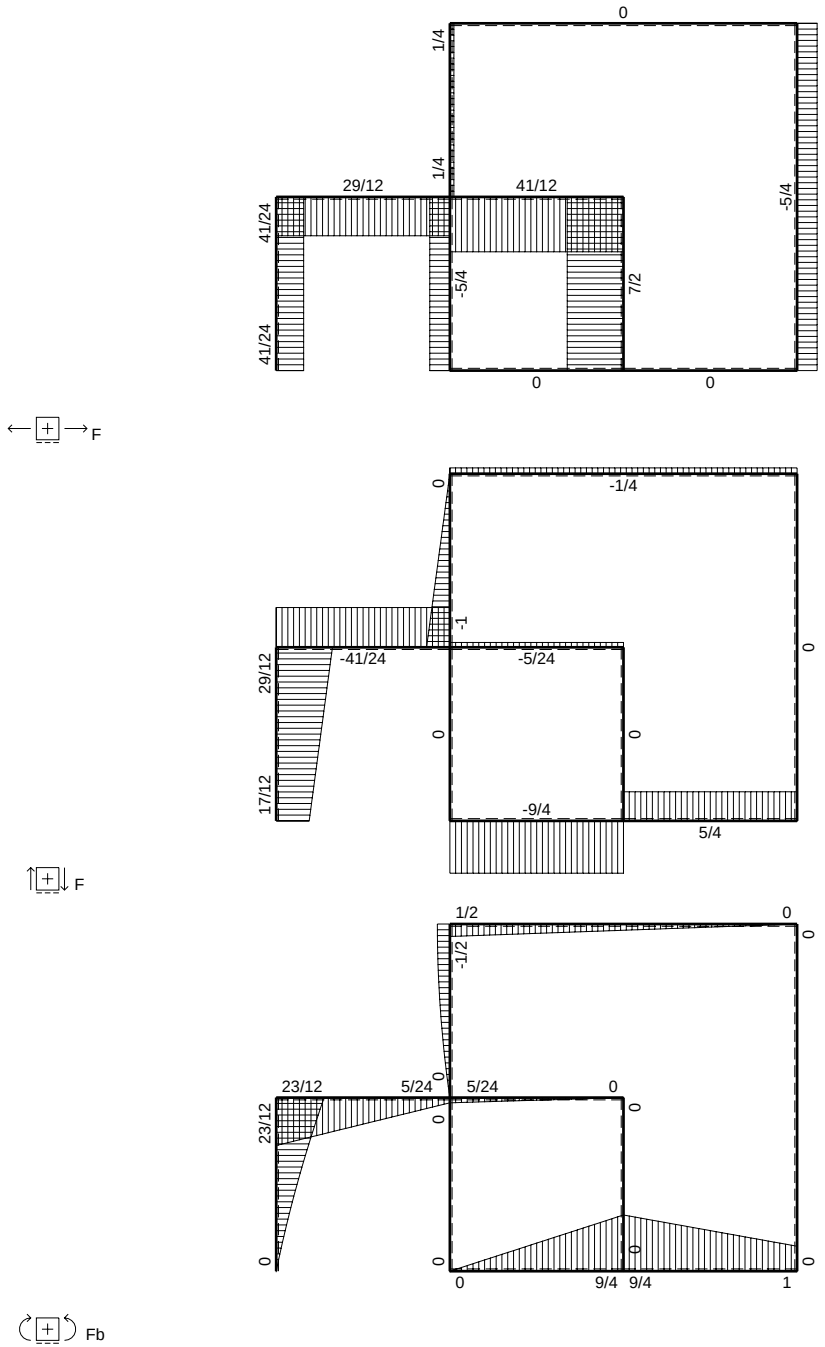


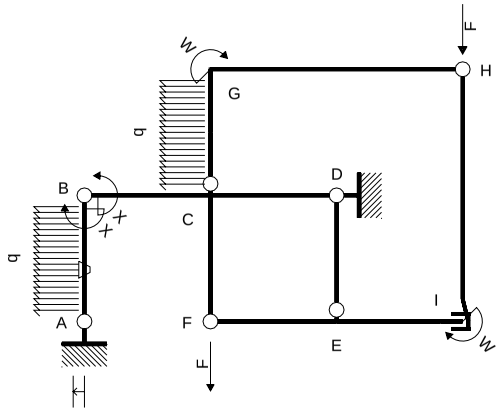

A = 468. $mm^2$
$J_u$ = 136587. $mm^4$
$J_v$ = 14364. $mm^4$
$y_g$ = 34.38 mm
$T_y$ = -1170. N
$M_x$ = 783900. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -34.38 mm
$\sigma_m = -Mv/J_u$ = 197.3 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 15. mm
$v_c$ = -19.38 mm
$\sigma_c = -Mv/J_u$ = 111.3 $N/mm^2$
$\tau_c$ = 3.454 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 111.4 $N/mm^2$
$S^*$ = 2420. $mm^3$


H
I
5/4F
B
A
41/24F
23/12Fb
29/12F
41/24F
17/12F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
5/24F
9/4F
9/4Fb
41/12F
1/2Fb
1/4F
41/12F
5/24Fb
5/24F
9/4F
G
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
C
41/24F
5/24Fb
29/12F
29/12F
23/12Fb
41/24F
B

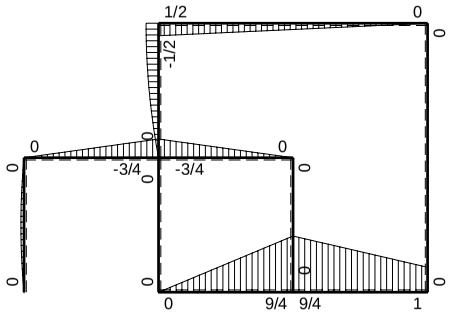




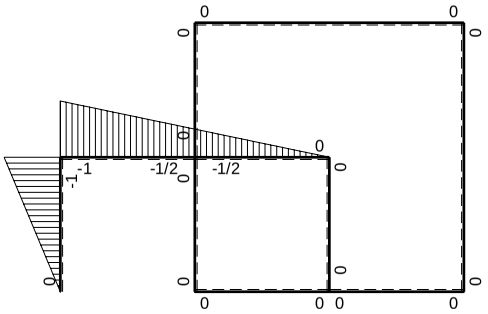

0
1/4
1/4
29/12
41/12
-5/4
41/24
41/24
-5/4
7/2
0
0
F
0
-1/4
-1
0
29/12
-41/24
-5/24
0
0
17/12
-9/4
5/4
F
1/2
0
0
-1/2
0
23/12
5/24
5/24
0
23/12
0
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 299327 CI01.xxxx.006

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 299327 CI01.xxxx.006

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2 ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/12\ b )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b ( x/b )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3 ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b ]_o^b\ \theta$

$= (1/6\ b -1/8\ b )\ Fb\ 1/EJ\ + (1/2\ b )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3 ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b )\ Fb\ 1/EJ\ + (- b +1/2\ b )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

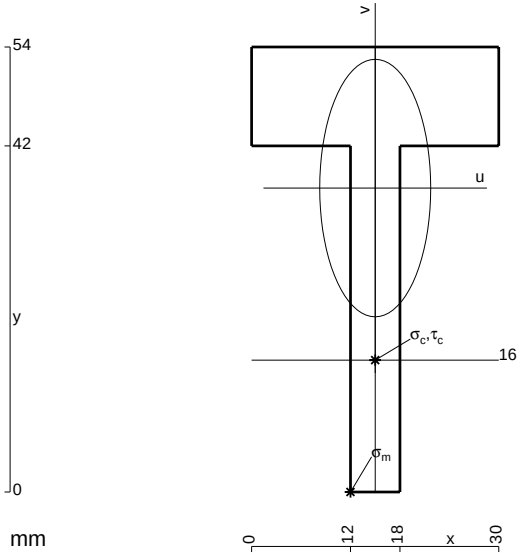
$= (3/8\ b -3/8\ b +1/8\ b )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

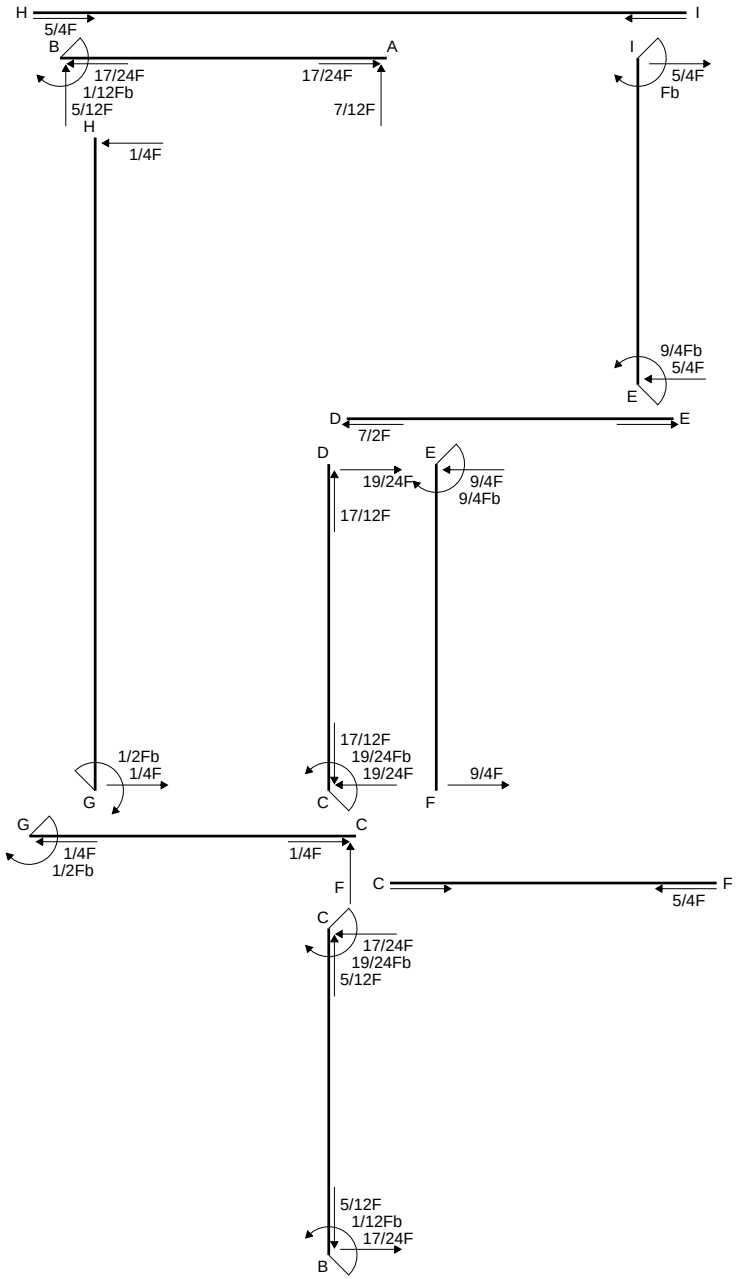
$= (1/8\ b )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$



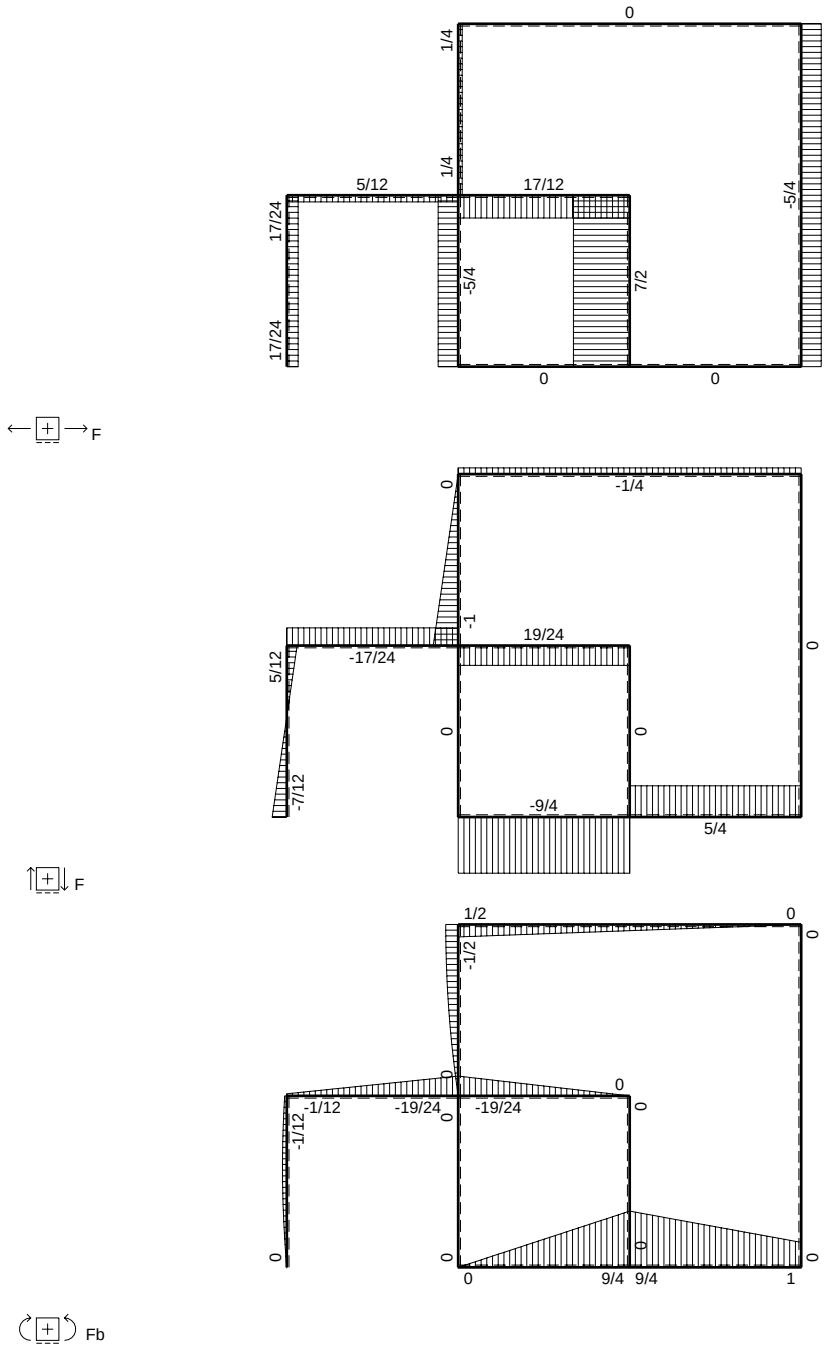





A = 612. $mm^2$
$J_u$ = 149427. $mm^4$
$J_v$ = 27756. $mm^4$
$y_g$ = 36.88 mm
$T_y$ = -1170. N
$M_x$ = 842400. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -36.88 mm
$\sigma_m = -Mv/J_u$ = 207.9 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 16. mm
$v_c$ = -20.88 mm
$\sigma_c = -Mv/J_u$ = 117.7 $N/mm^2$
$\tau_c$ = 3.618 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.9 $N/mm^2$
$S^*$ = 2773. $mm^3$

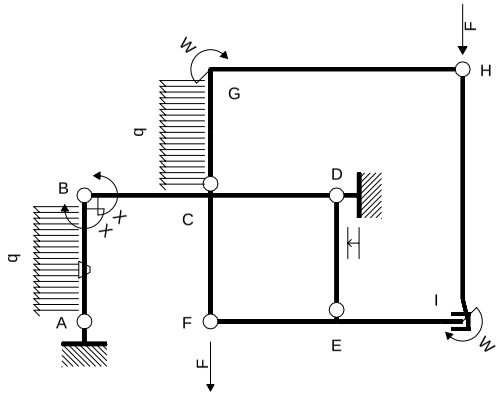




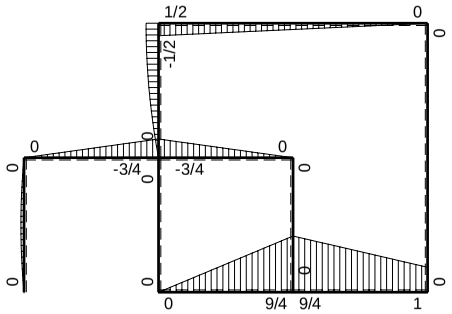





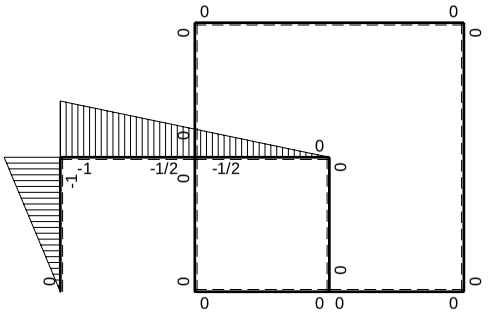

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Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248501 CI01.xxxx.007

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PROCEDIMENTO E RISULTATI 248501 CI01.xxxx.007

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

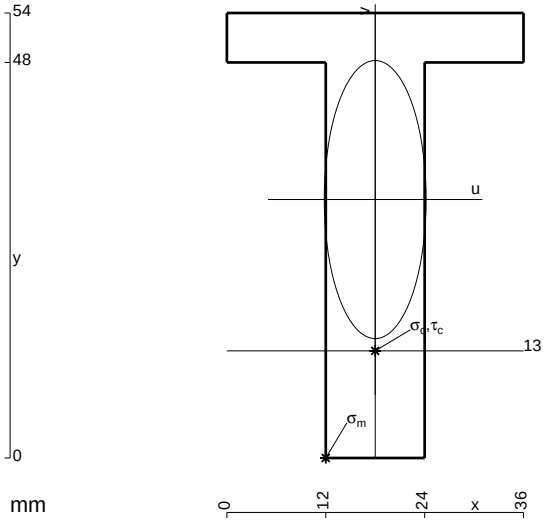
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

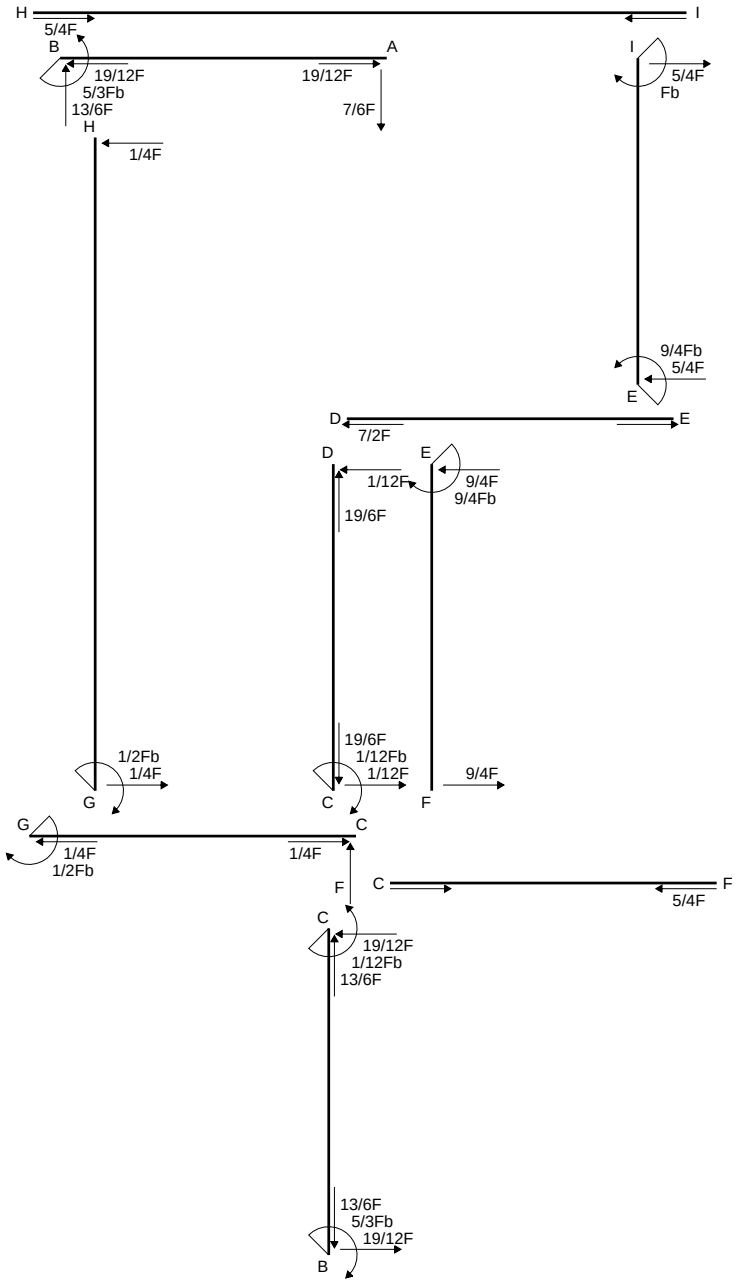
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





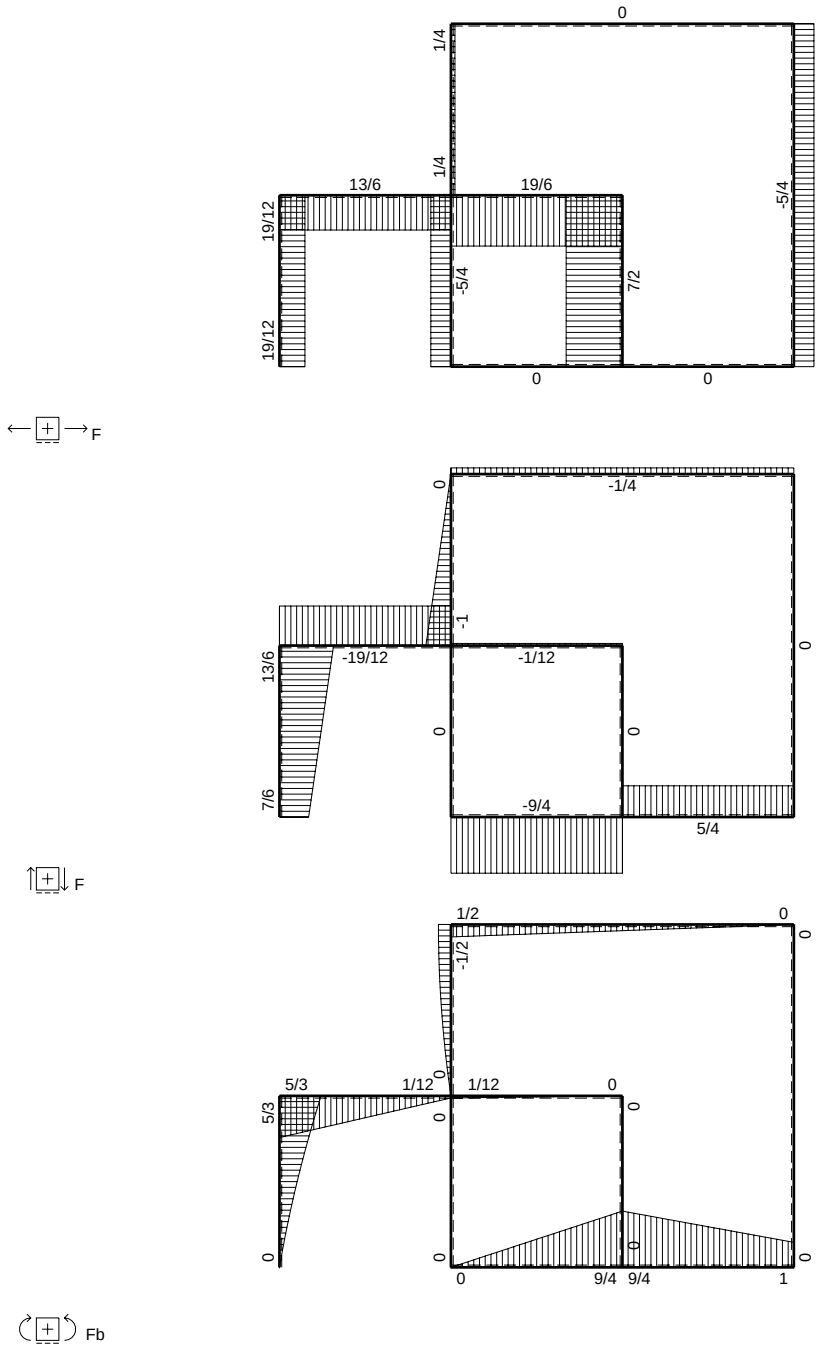


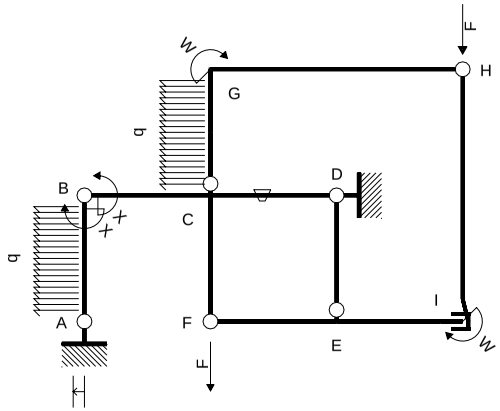

A = 792. $mm^2$
$J_u$ = 225759. $mm^4$
$J_v$ = 30240. $mm^4$
$y_g$ = 31.36 mm
$T_y$ = -2070. N
$M_x$ = 1573200. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -31.36 mm
$\sigma_m = -Mv/J_u$ = 218.6 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 13. mm
$v_c$ = -18.36 mm
$\sigma_c = -Mv/J_u$ = 128. $N/mm^2$
$\tau_c$ = 2.964 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.1 $N/mm^2$
$S^*$ = 3879. $mm^3$


H
I
5/4F
B
A
I
19/12F
19/12F
5/4F
5/3Fb
Fb
13/6F
7/6F
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
1/12F
9/4F
9/4Fb
19/6F
19/6F
1/2Fb
1/12Fb
1/4F
1/12F
9/4F
G
C
F
G
C
1/4F
1/4F
1/2Fb
F
C
F
5/4F
C
19/12F
1/12Fb
13/6F
13/6F
5/3Fb
19/12F
B

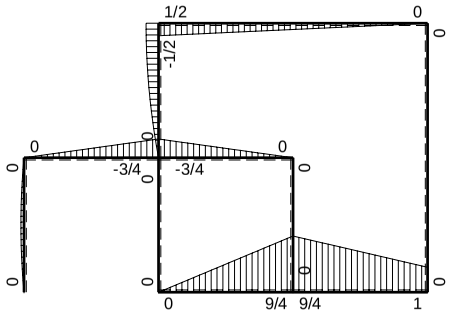




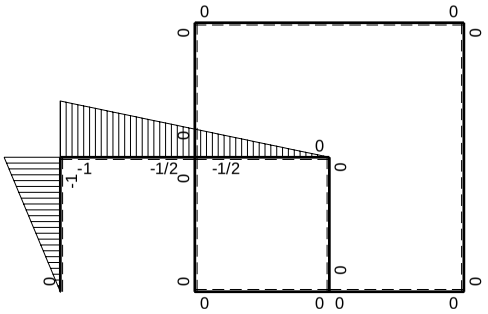

0
1/4
1/4
13/6
19/6
-5/4
19/12
-5/4
7/2
19/12
0
0
F
0
-1/4
-1
13/6
-19/12
-1/12
0
0
0
7/6
-9/4
5/4
F
1/2
0
0
-1/2
0
5/3
1/12
1/12
0
5/3
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267042 CI01.xxxx.008

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PROCEDIMENTO E RISULTATI 267042 CI01.xxxx.008

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-3/4Fx$ | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | $3/4Fb-3/4Fx$ | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | $3/4Fx$ | Fb/EJ | $3/8Fx^2/b$ | $1/2Fx/EJ$ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-9/4Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-9/4Fx$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $1/2Fb-1/4Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-1/4Fx$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-9/4Fb+5/4Fx$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $5/3Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

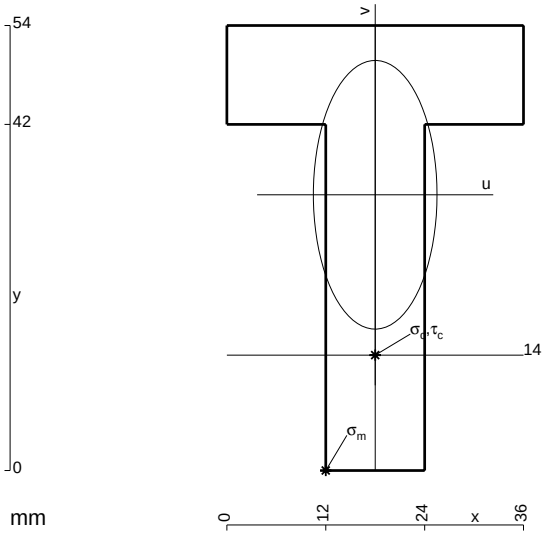
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

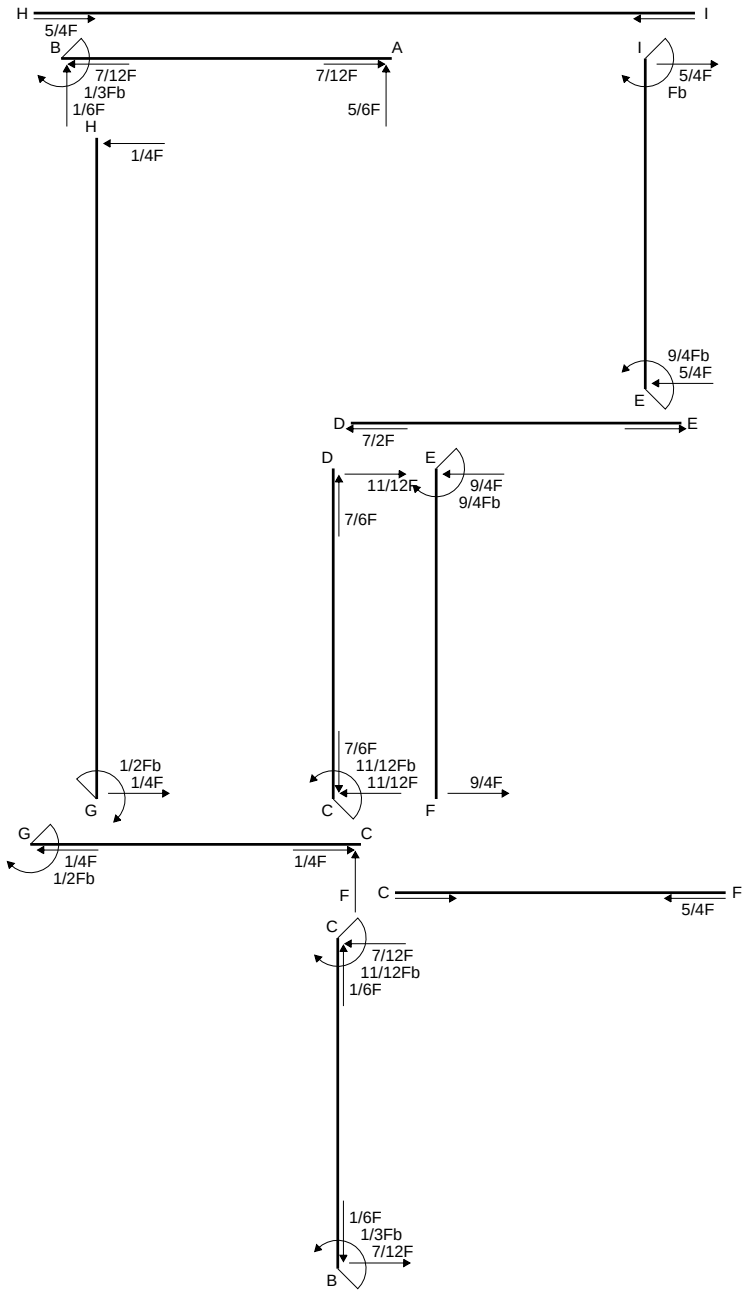


PROCEDIMENTO E RISULTATI 267042 CI01.xxxx.008



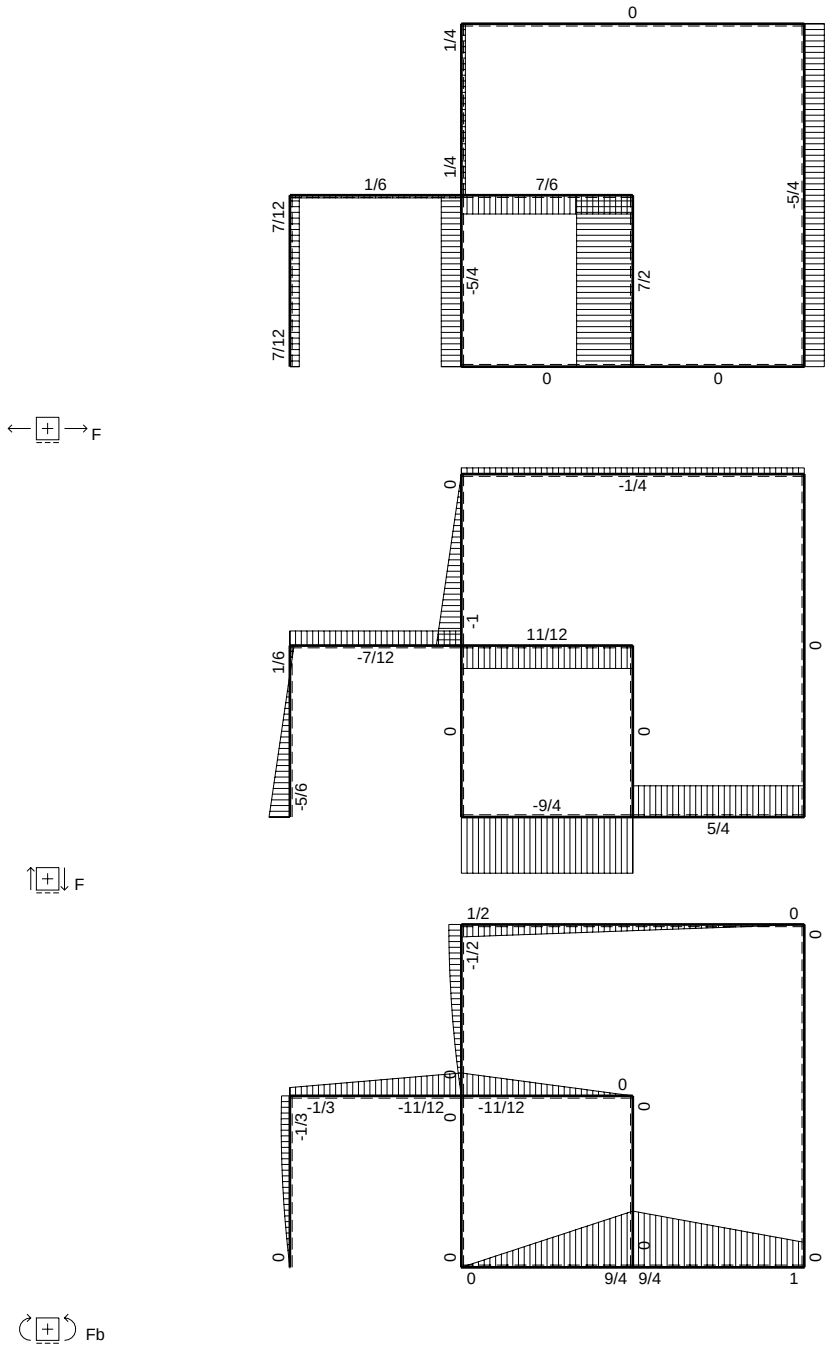

A = 936. $mm^2$
$J_u$ = 248849. $mm^4$
$J_v$ = 52704. $mm^4$
$y_g$ = 33.46 mm
$T_y$ = -2093. N
$M_x$ = 1694930. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -33.46 mm
$\sigma_m = -Mv/J_u$ = 227.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -19.46 mm
$\sigma_c = -Mv/J_u$ = 132.6 $N/mm^2$
$\tau_c$ = 3.115 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.7 $N/mm^2$
$S^*$ = 4446. $mm^3$

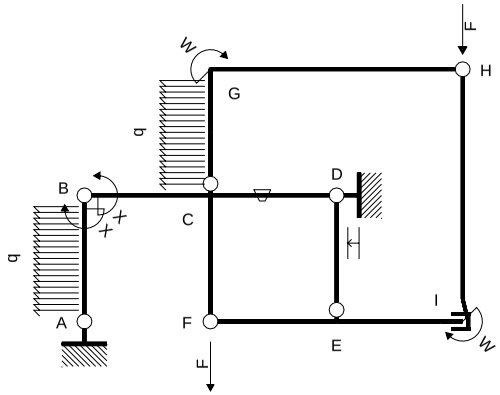
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H
I
5/4F
B
A
7/12F
1/3Fb
1/6F
7/12F
5/6F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
11/12F
9/4F
9/4Fb
7/6F
1/2Fb
1/4F
G
7/6F
11/12Fb
11/12F
C
F
9/4F
G
C
1/4F
1/4F
1/2Fb
F
C
F
5/4F
C
7/12F
11/12Fb
1/6F
1/6F
1/3Fb
7/12F
B

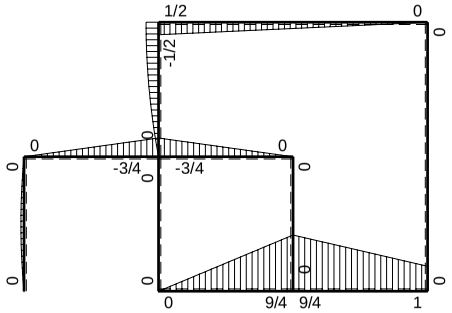




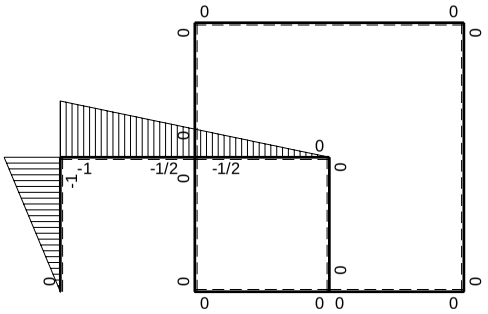

F
F
Fb
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1/4
1/4
1/6
7/6
-5/4
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5/4
1/2
0
0
-1/2
0
-1/3
-11/12
-11/12
-1/3
0
0
0
0
0
0
9/4
9/4
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 266299 CI01.xxxx.009

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PROCEDIMENTO E RISULTATI 266299 CI01.xxxx.009

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | $1/2Fx/EJ$ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/3Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | $1/3Fb$ | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

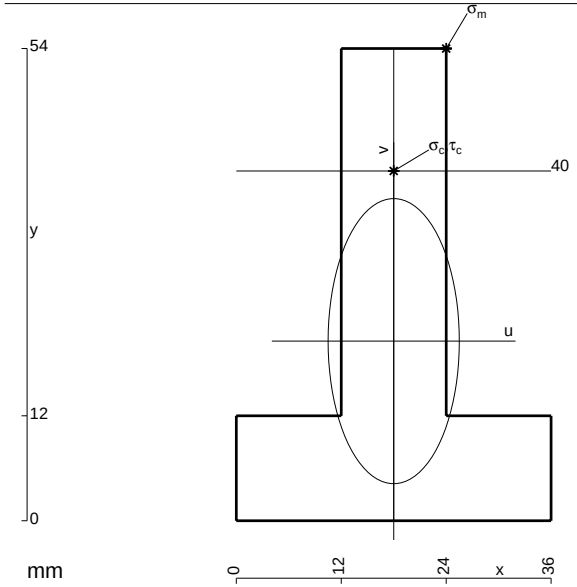
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

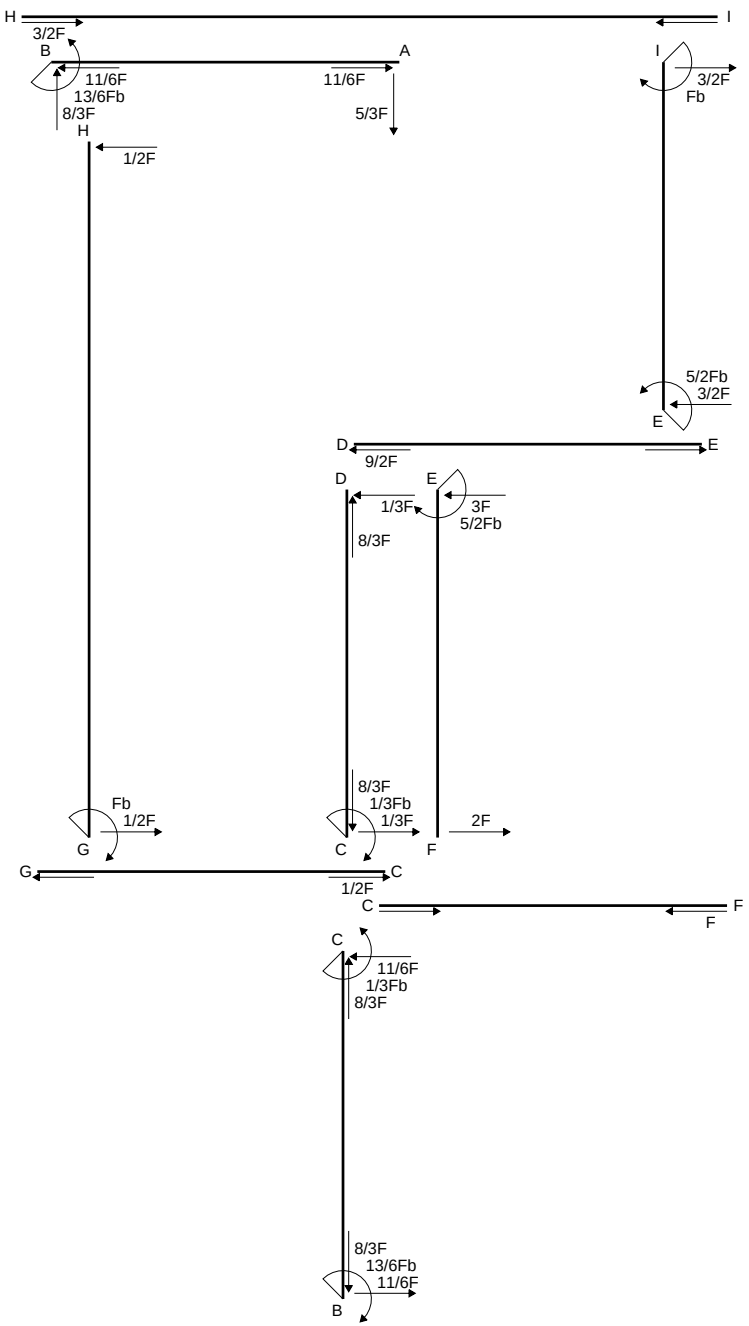


PROCEDIMENTO E RISULTATI 266299 CI01.xxxx.009



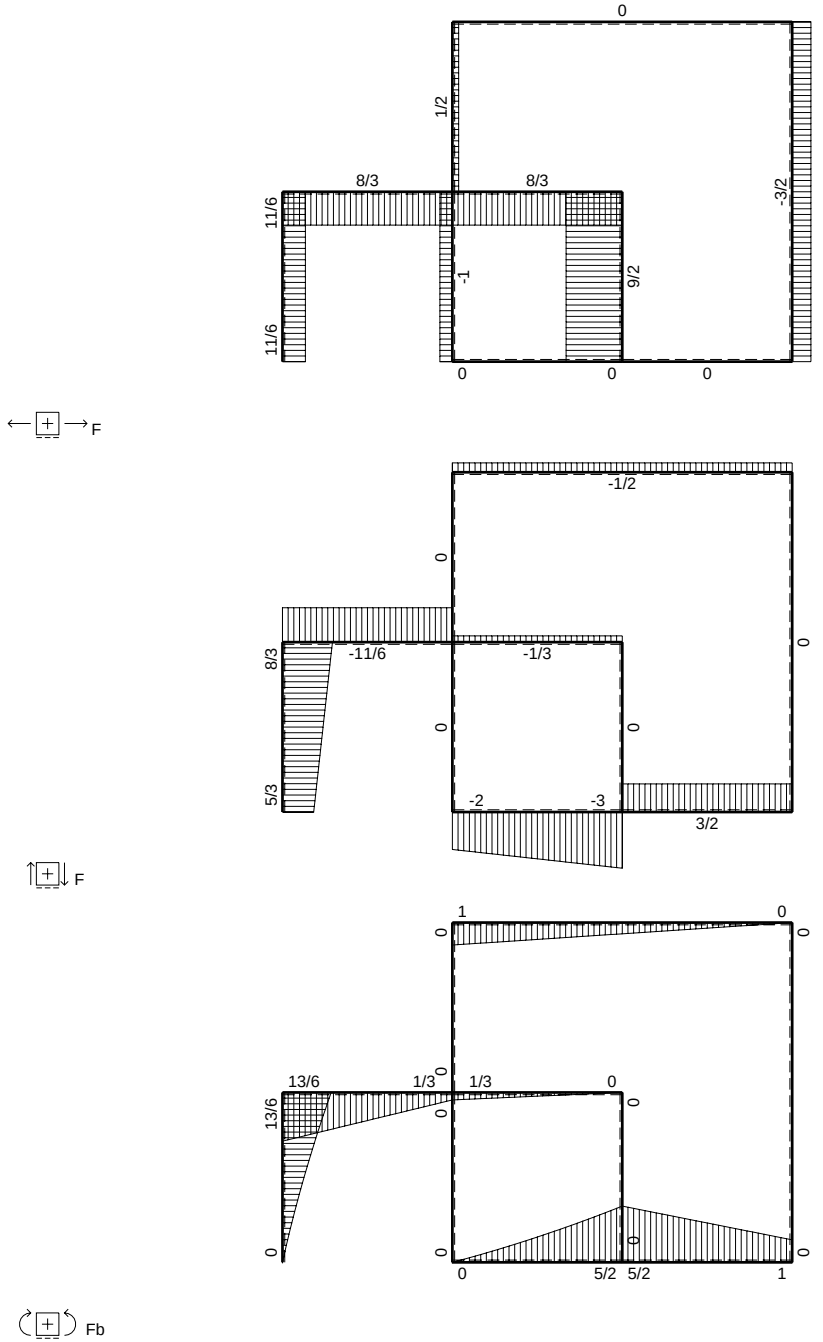

A = 936. $mm^2$
$J_u$ = 248849. $mm^4$
$J_v$ = 52704. $mm^4$
$y_g$ = 20.54 mm
$T_y$ = -2093. N
$M_x$ = 1778630. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 33.46 mm
$\sigma_m = -Mv/J_u$ = -239.2 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 40. mm
$v_c$ = 19.46 mm
$\sigma_c = -Mv/J_u$ = -139.1 $N/mm^2$
$\tau_c$ = 3.115 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.2 $N/mm^2$
$S^*$ = 4446. $mm^3$

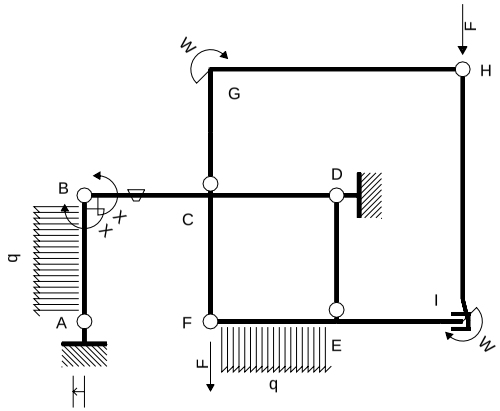
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H
I
3/2F
B
A
11/6F
13/6Fb
8/3F
11/6F
5/3F
I
3/2F
Fb
H
1/2F
5/2Fb
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D
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F
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11/6F
B

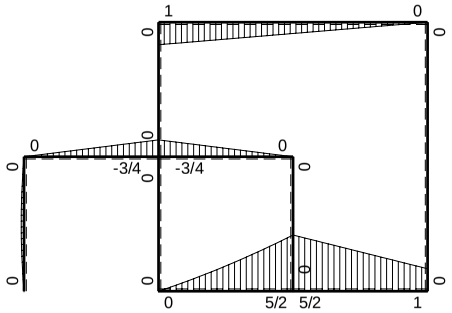




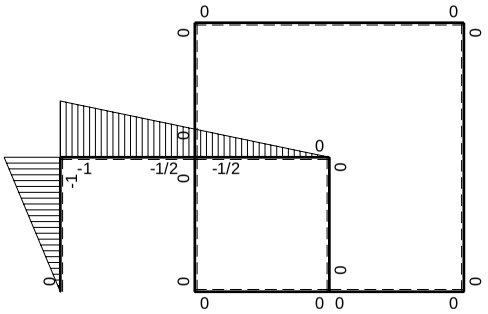

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-3
3/2
Fb
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0
0
13/6
1/3
1/3
0
13/6
0
0
0
0
0
0
5/2
5/2
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 937106 CI01.xxxx.010

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 937106 CI01.xxxx.010

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

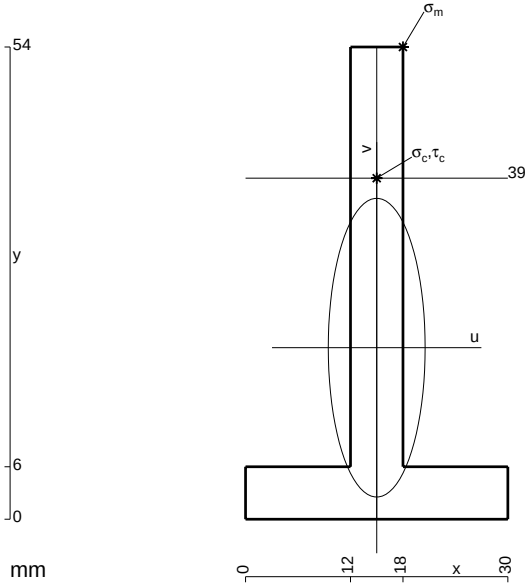
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

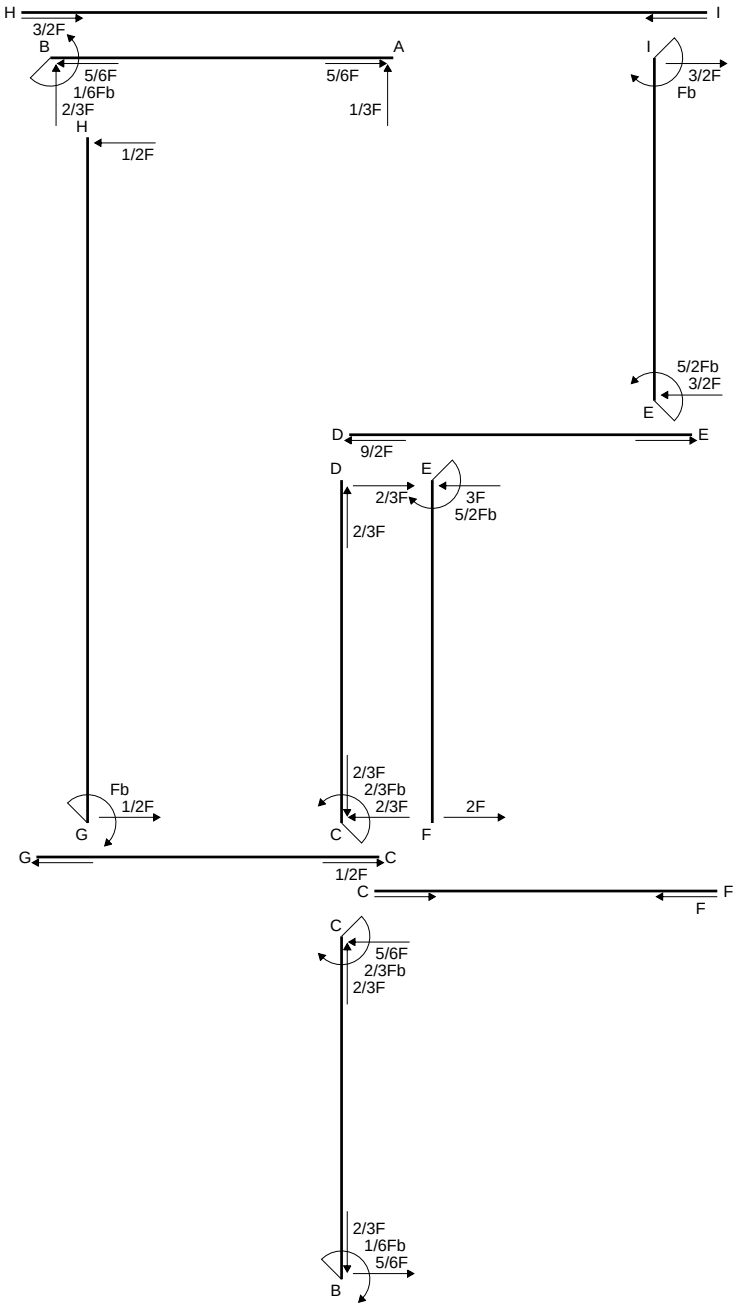
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





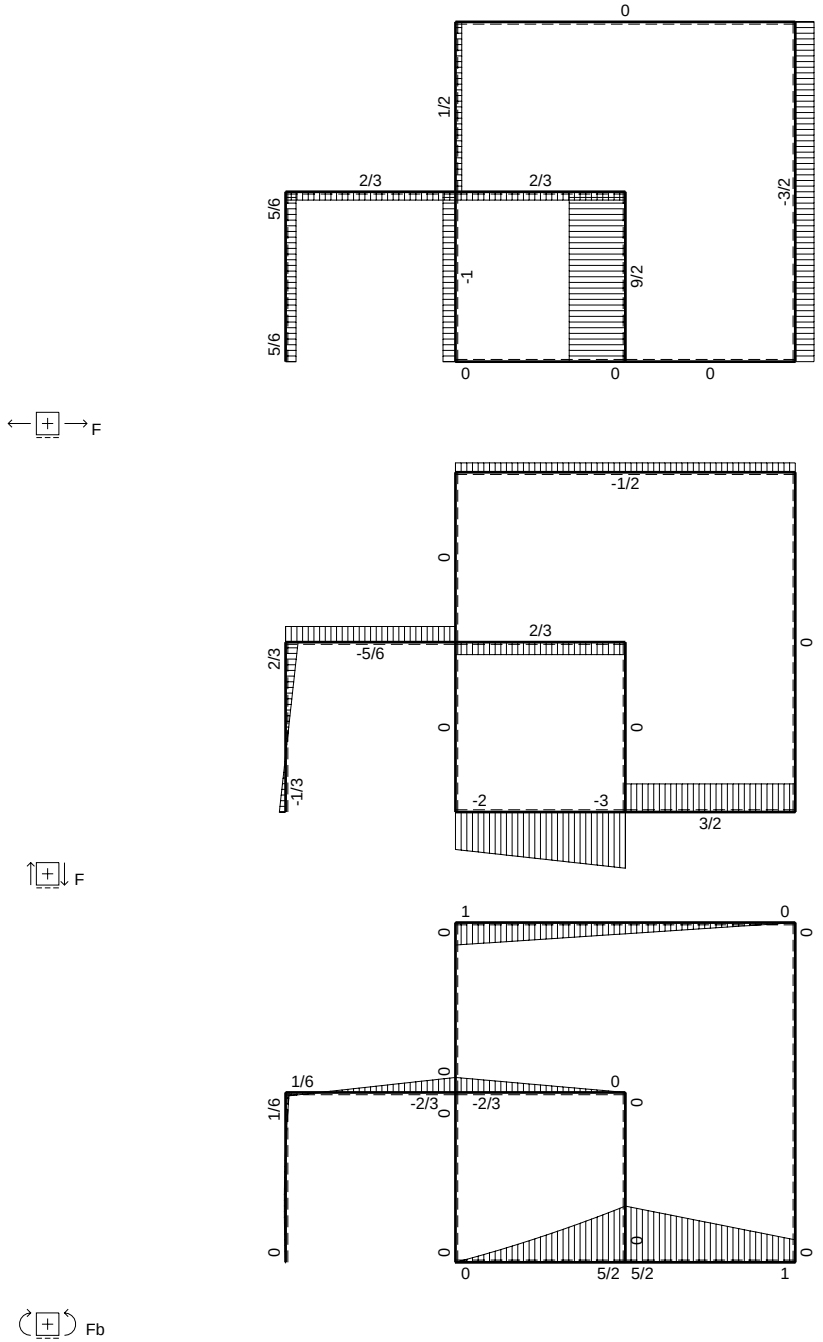


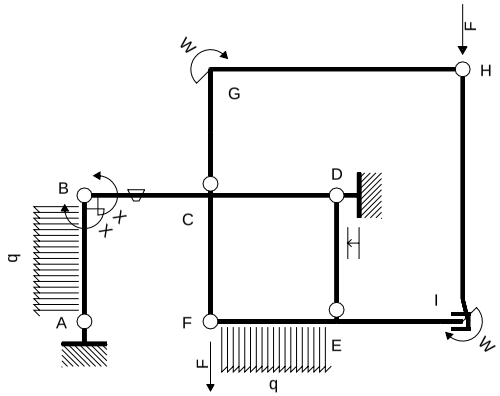

A = 468. $mm^2$
$J_u$ = 136587. $mm^4$
$J_v$ = 14364. $mm^4$
$y_g$ = 19.62 mm
$T_y$ = -2100. N
$M_x$ = 787500. Nmm
$x_m$ = 18. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 34.38 mm
$\sigma_m = -Mv/J_u$ = -198.2 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 39. mm
$v_c$ = 19.38 mm
$\sigma_c = -Mv/J_u$ = -111.8 $N/mm^2$
$\tau_c$ = 6.2 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 112.3 $N/mm^2$
$S^*$ = 2420. $mm^3$


H
I
3/2F
B
A
5/6F
1/6Fb
2/3F
5/6F
1/3F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
2/3F
3F
5/2Fb
2/3F
2/3F
2/3Fb
2/3F
Fb
1/2F
G
C
F
2F
G
C
1/2F
C
F
F
C
5/6F
2/3Fb
2/3F
2/3F
1/6Fb
5/6F
B

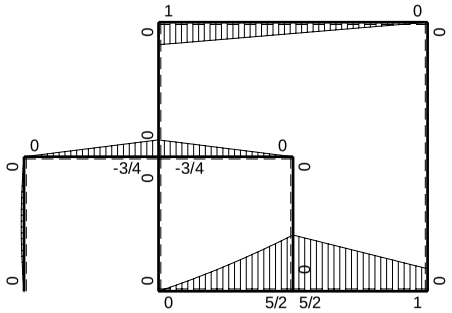




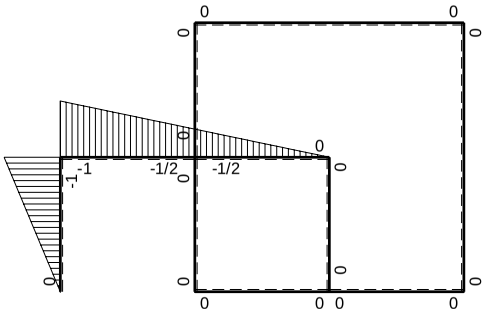

F
0
1/2
2/3
2/3
-3/2
5/6
-1
9/2
5/6
0
0
0
F
-1/2
0
2/3
-5/6
2/3
0
0
0
0
-1/3
-2
-3
3/2
F
1
0
0
0
1/6
0
-2/3
-2/3
1/6
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265724 CI01.xxxx.011

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 265724 CI01.xxxx.011

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

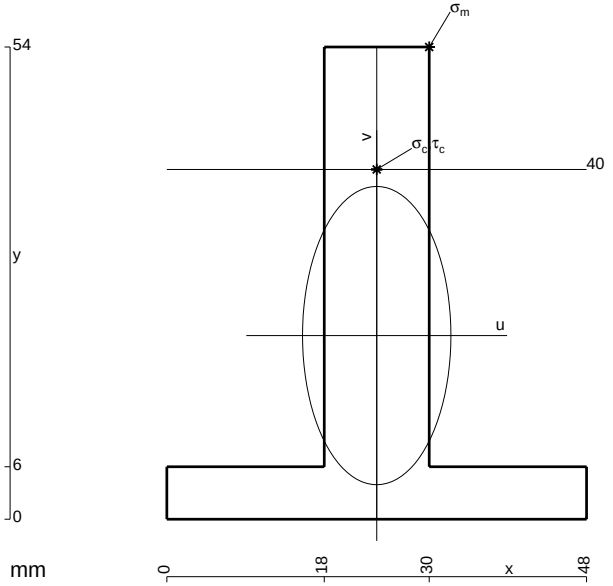
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

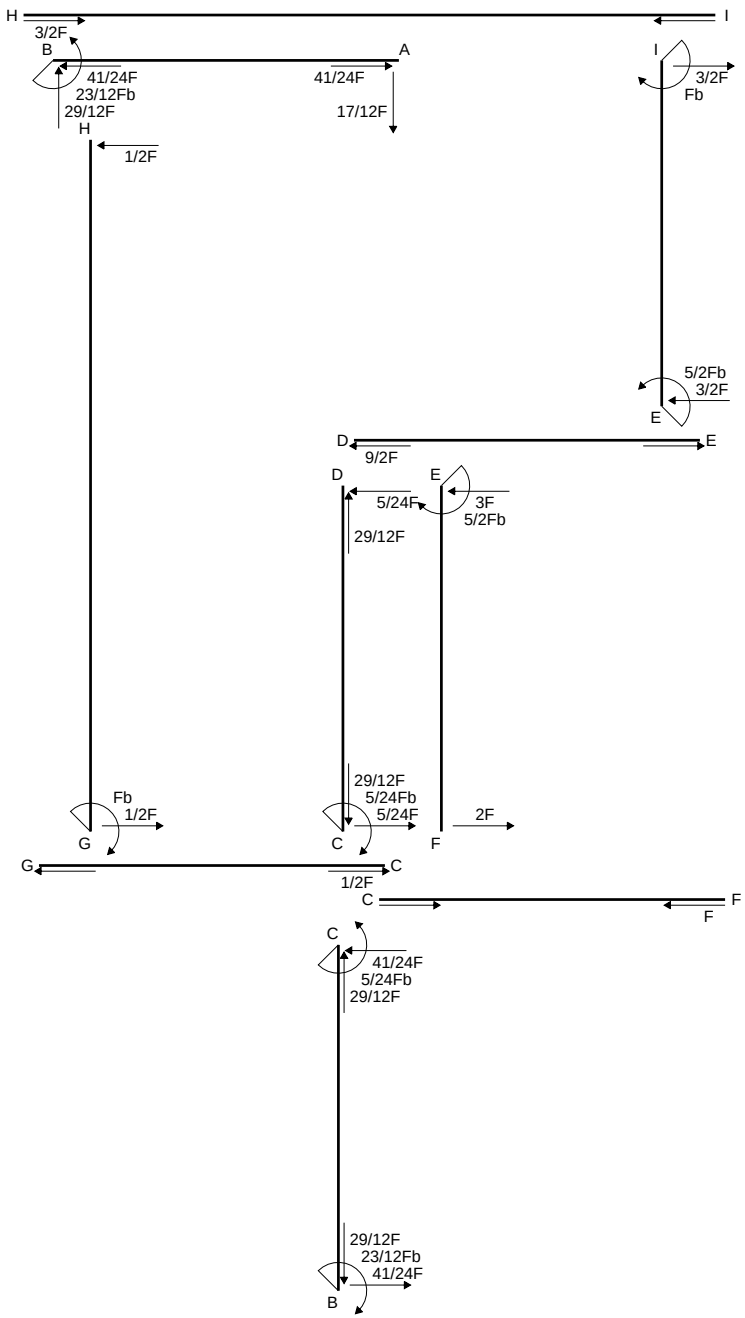
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





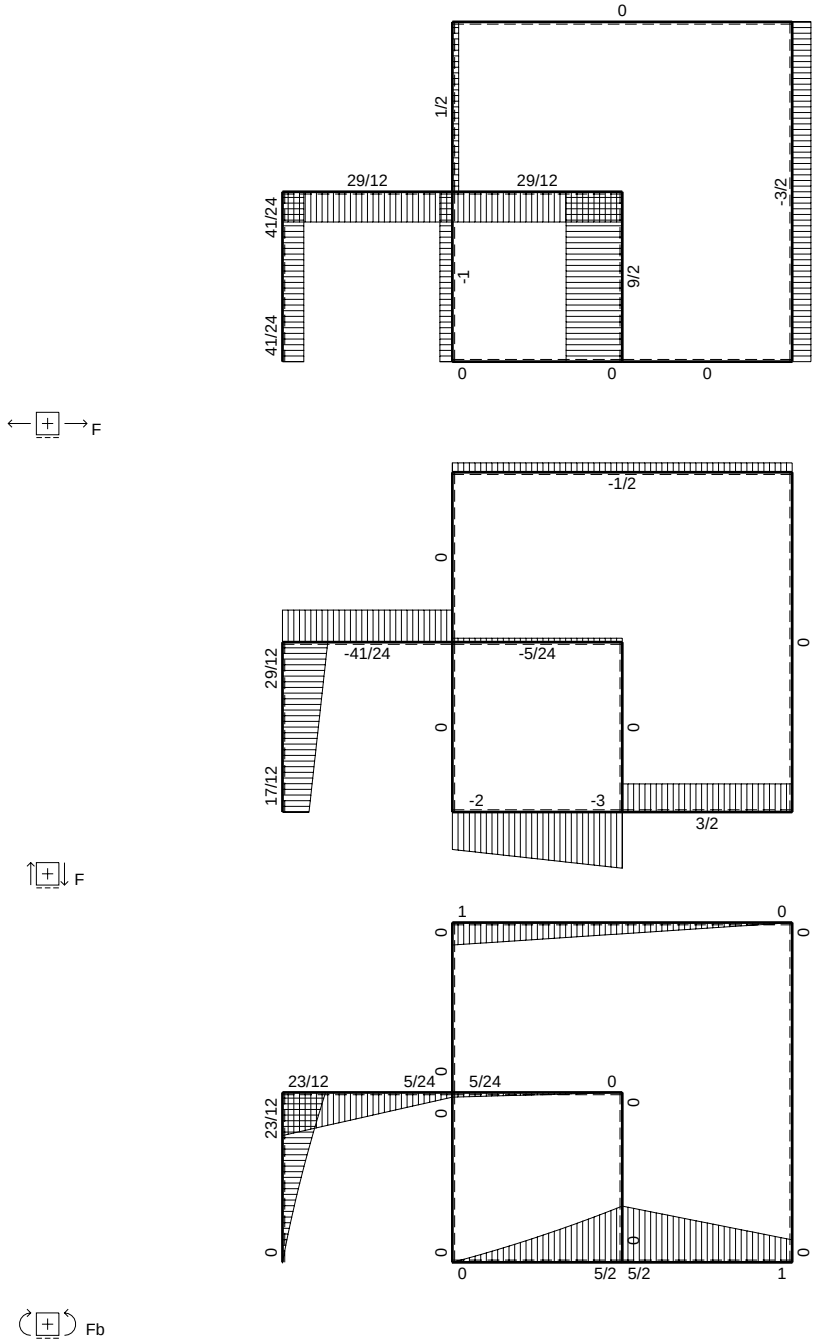


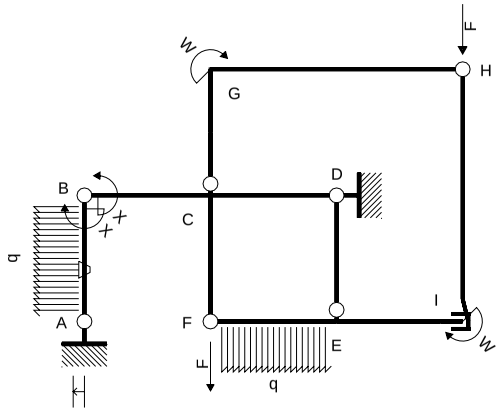

A = 864. $mm^2$
$J_u$ = 251424. $mm^4$
$J_v$ = 62208. $mm^4$
$y_g$ = 21. mm
$T_y$ = -3900. N
$M_x$ = 1592500. Nmm
$x_m$ = 30. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 33. mm
$\sigma_m = -Mv/J_u$ = -209. $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 40. mm
$v_c$ = 19. mm
$\sigma_c = -Mv/J_u$ = -120.3 $N/mm^2$
$\tau_c$ = 5.646 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.7 $N/mm^2$
$S^*$ = 4368. $mm^3$


H
I
3/2F
B
A
41/24F
23/12Fb
29/12F
41/24F
17/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
5/24F
3F
5/2Fb
29/12F
Fb
1/2F
29/12F
5/24Fb
5/24F
2F
G
C
F
G
C
1/2F
C
F
F
C
41/24F
5/24Fb
29/12F
29/12F
23/12Fb
41/24F
B

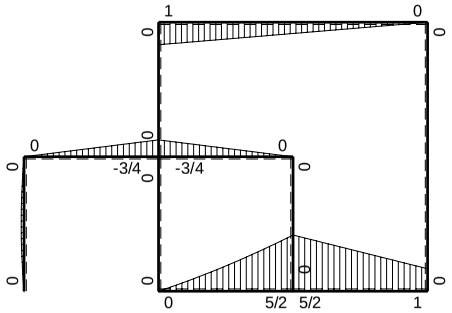




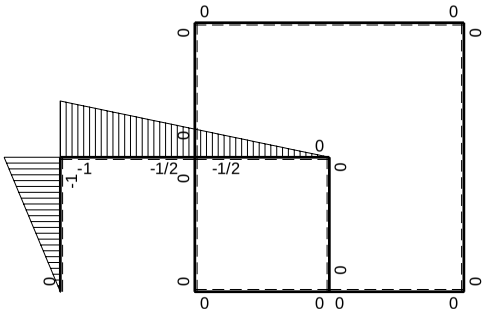

0
1/2
29/12
29/12
-3/2
41/24
-1
9/2
41/24
0
0
0
F
-1/2
0
29/12
-41/24
-5/24
0
0
0
17/12
-2
-3
3/2
F
1
0
0
0
0
23/12
5/24
5/24
0
0
23/12
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248385 CI01.xxxx.012

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 248385 CI01.xxxx.012

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

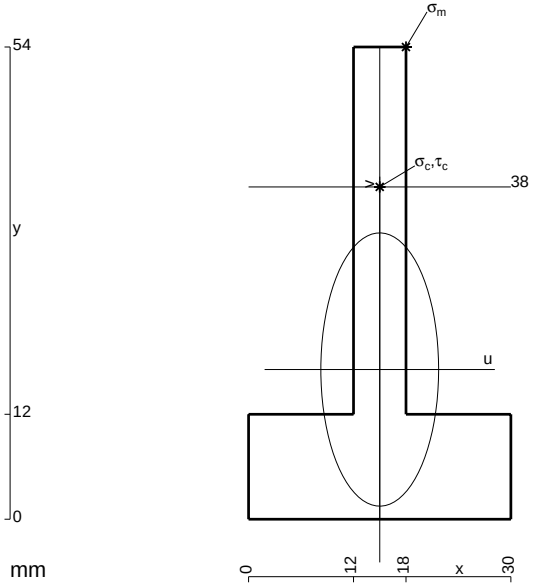
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

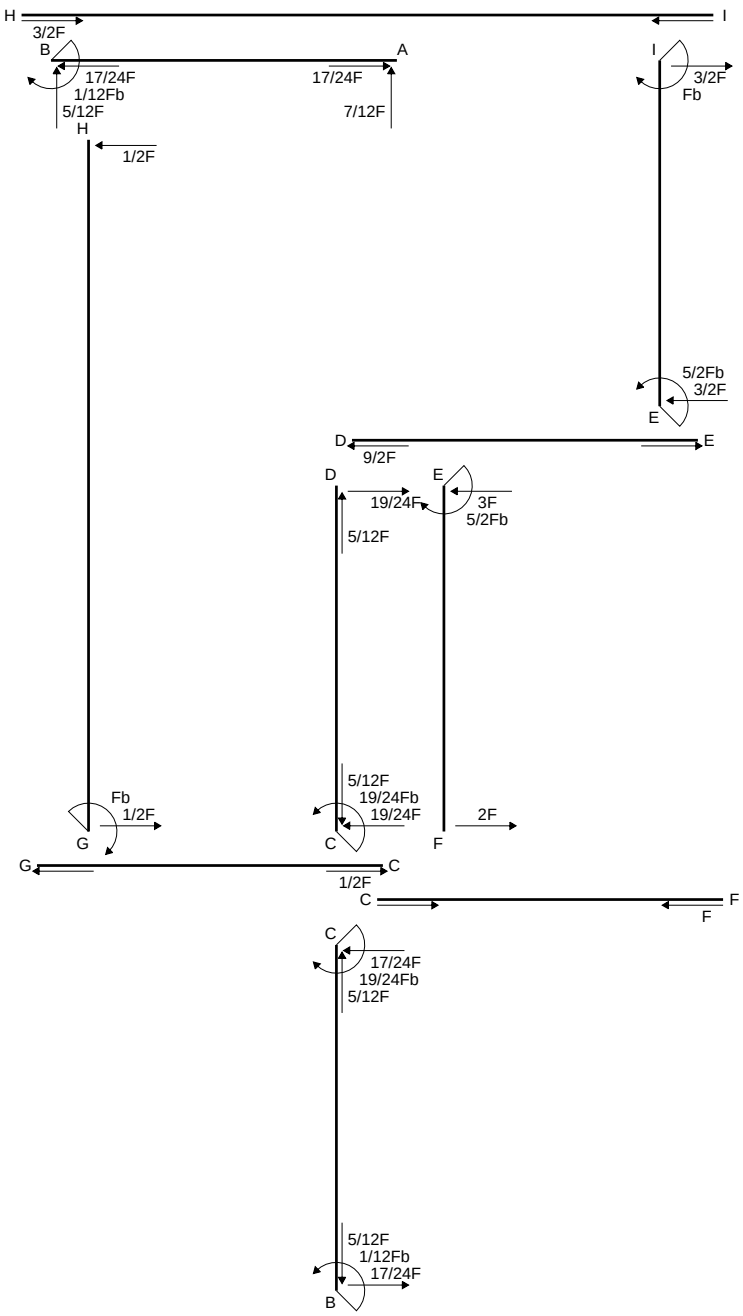


PROCEDIMENTO E RISULTATI 248385 CI01.xxxx.012



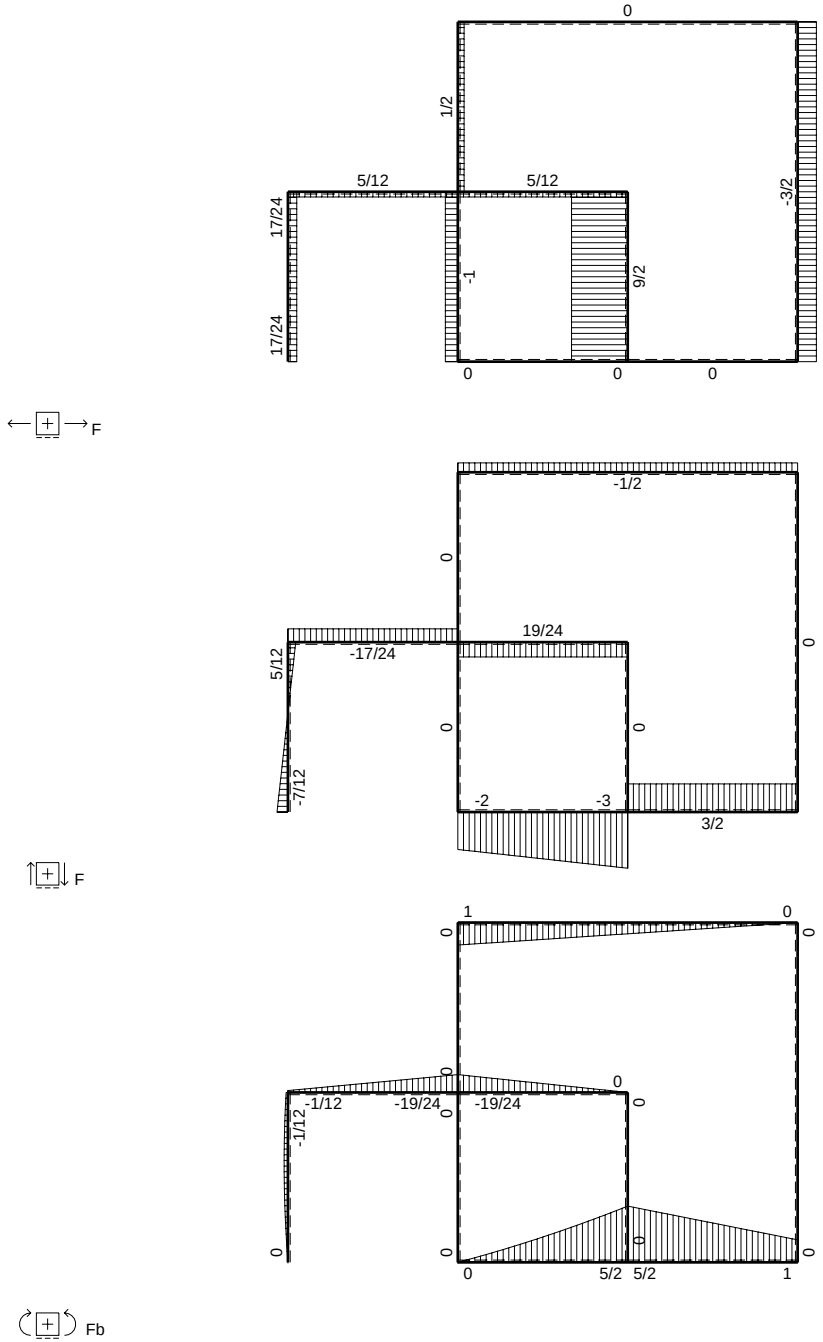

A = 612. $mm^2$
$J_u$ = 149428. $mm^4$
$J_v$ = 27756. $mm^4$
$y_g$ = 17.12 mm
$T_y$ = -1980. N
$M_x$ = 891000. Nmm
$x_m$ = 18. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 36.88 mm
$\sigma_m = -Mv/J_u$ = -219.9 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 38. mm
$v_c$ = 20.88 mm
$\sigma_c = -Mv/J_u$ = -124.5 $N/mm^2$
$\tau_c$ = 6.123 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 125. $N/mm^2$
$S^*$ = 2773. $mm^3$

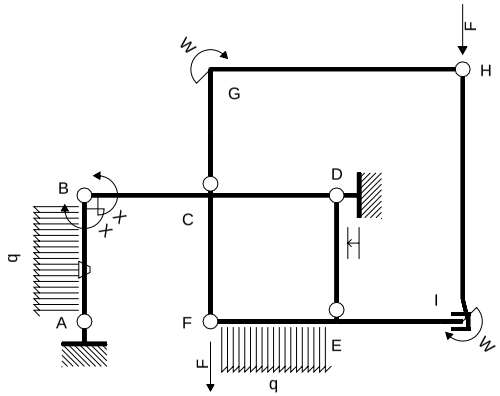
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H
I
3/2F
B
A
17/24F
1/12Fb
5/12F
17/24F
7/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
19/24F
3F
5/2Fb
5/12F
Fb
1/2F
5/12F
19/24Fb
19/24F
2F
G
C
F
G
C
1/2F
C
F
F
C
17/24F
19/24Fb
5/12F
5/12F
1/12Fb
17/24F
B

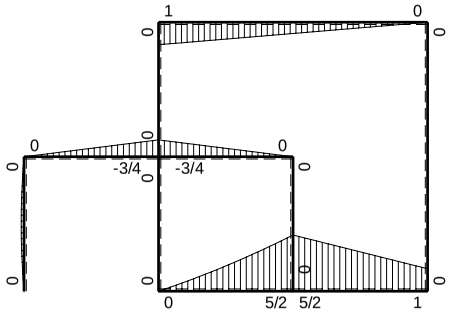




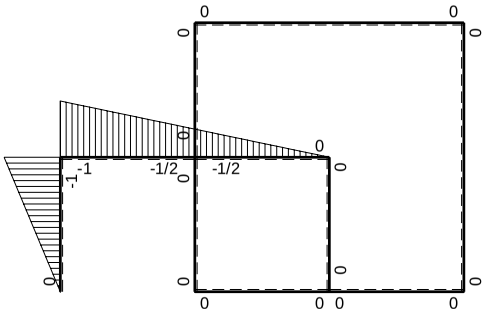

0
1/2
5/12
5/12
-3/2
17/24
-1
9/2
17/24
0
0
0
F
-1/2
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19/24
-17/24
5/12
0
0
0
-7/12
-2
-3
3/2
F
1
0
0
0
0
-1/12
-19/24
-19/24
-1/12
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

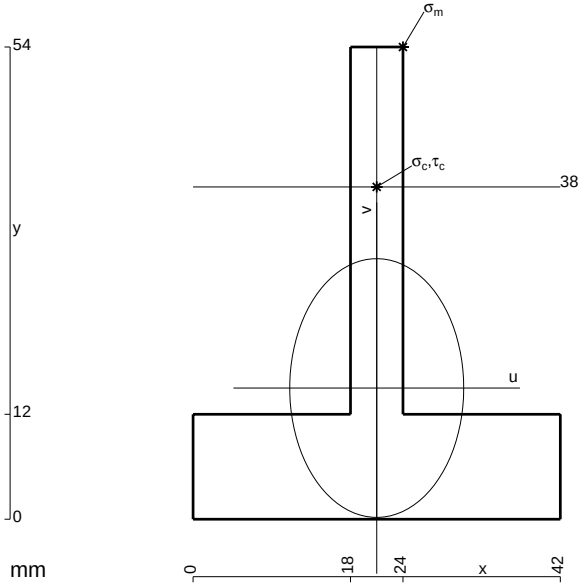
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

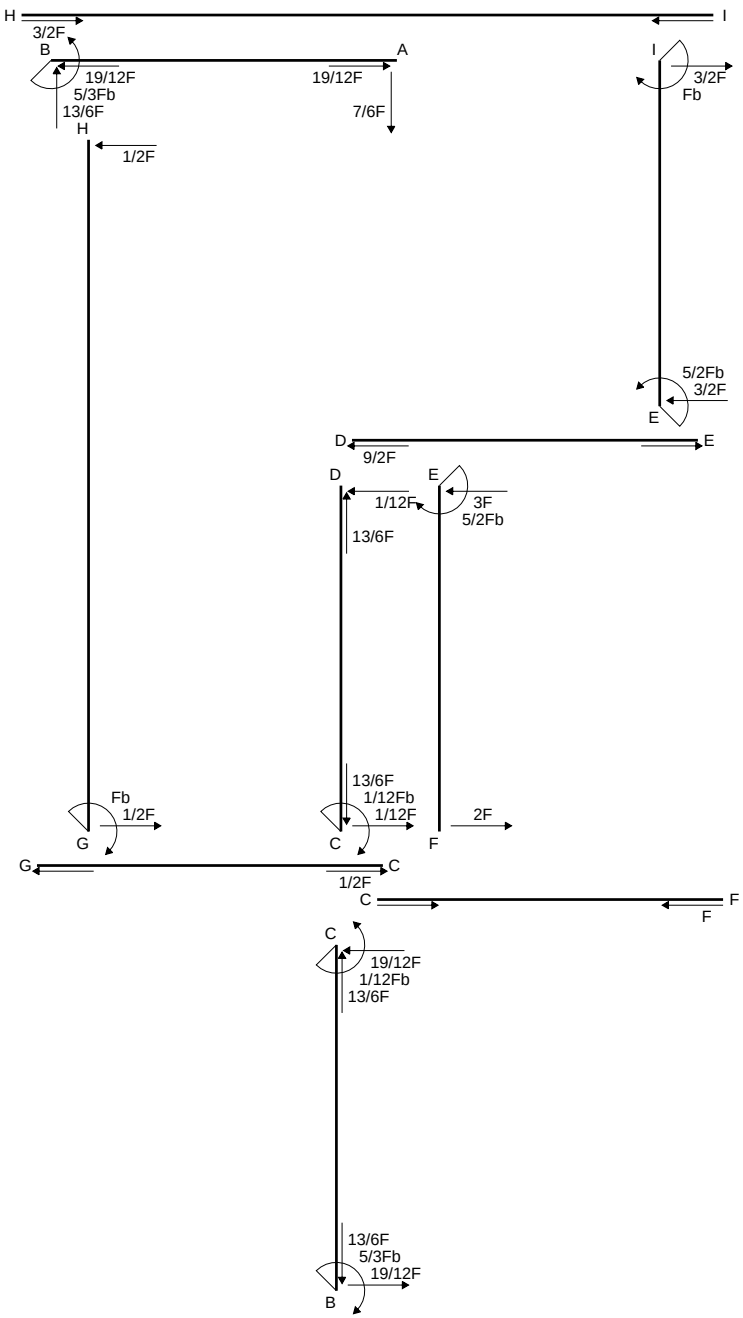


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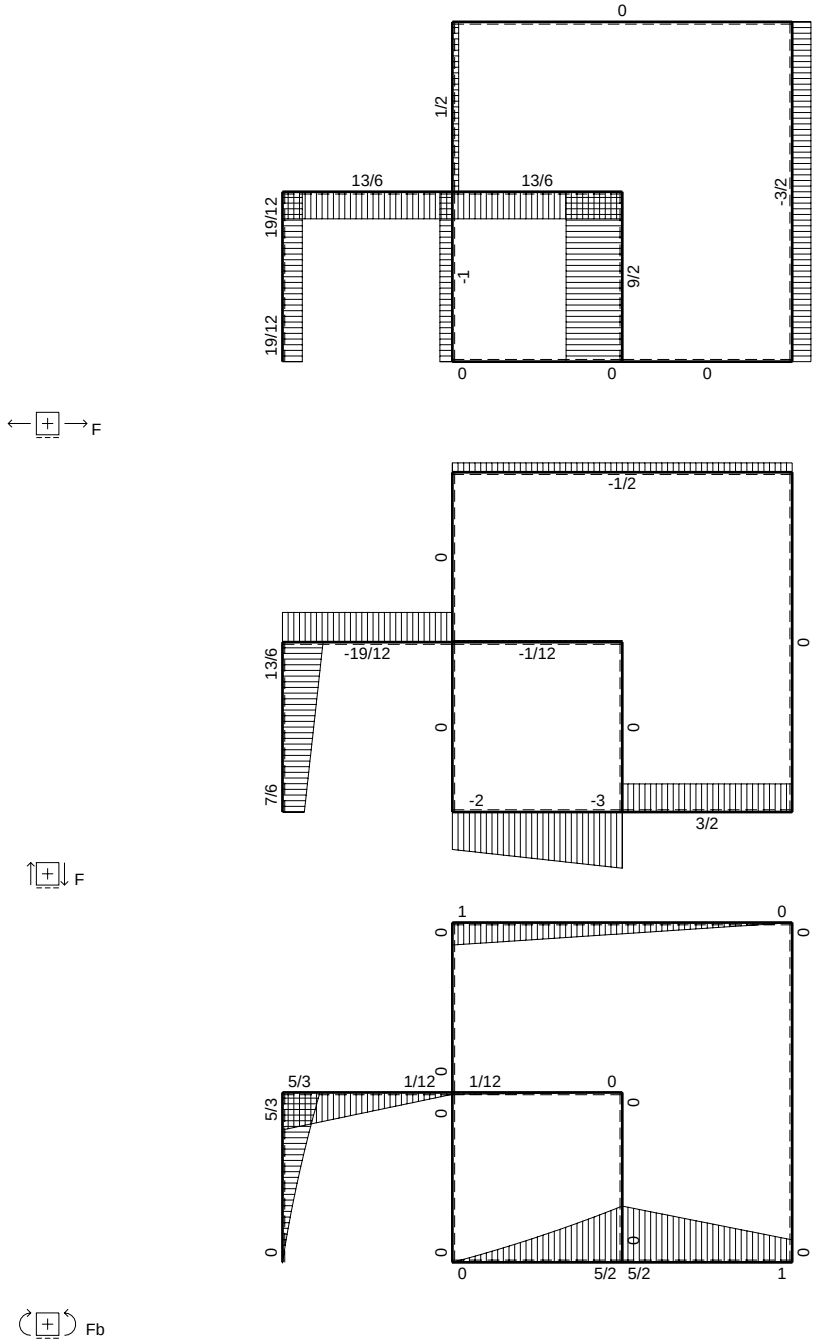

A = 756. $mm^2$
$J_u$ = 165564. $mm^4$
$J_v$ = 74844. $mm^4$
$y_g$ = 15. mm
$T_y$ = -2010. N
$M_x$ = 971500. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 39. mm
$\sigma_m = -Mv/J_u$ = -228.8 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 38. mm
$v_c$ = 23. mm
$\sigma_c = -Mv/J_u$ = -135. $N/mm^2$
$\tau_c$ = 6.022 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.4 $N/mm^2$
$S^*$ = 2976. $mm^3$

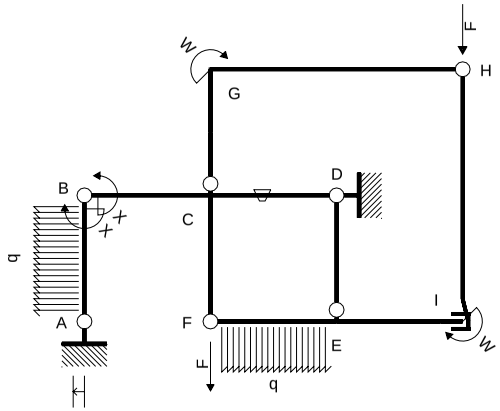
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H
I
3/2F
B
A
19/12F
5/3Fb
13/6F
19/12F
7/6F
H
1/2F
I
3/2F
Fb
5/2Fb
3/2F
E
D
E
9/2F
D
E
1/12F
3F
5/2Fb
13/6F
13/6F
1/12Fb
1/12F
2F
Fb
1/2F
G
C
F
G
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5/3Fb
19/12F
B

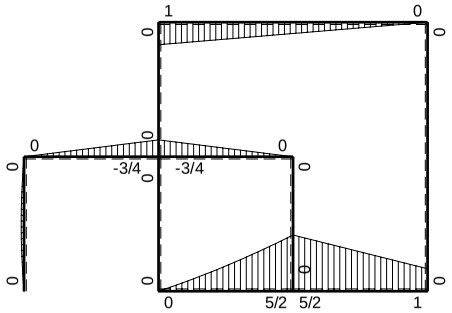




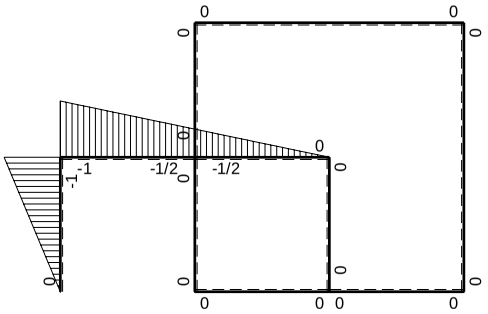

0
1/2
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13/6
-3/2
19/12
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19/12
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0
13/6
-19/12
-1/12
0
7/6
0
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-2
-3
3/2
F
1
0
0
0
5/3
1/12
1/12
0
5/3
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $5/3Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

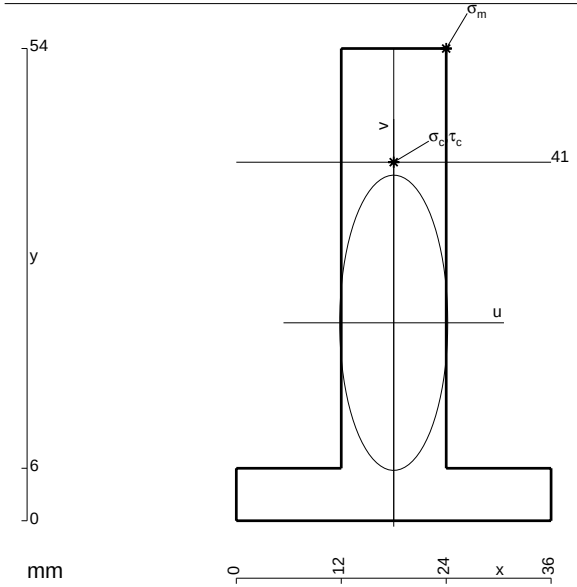
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

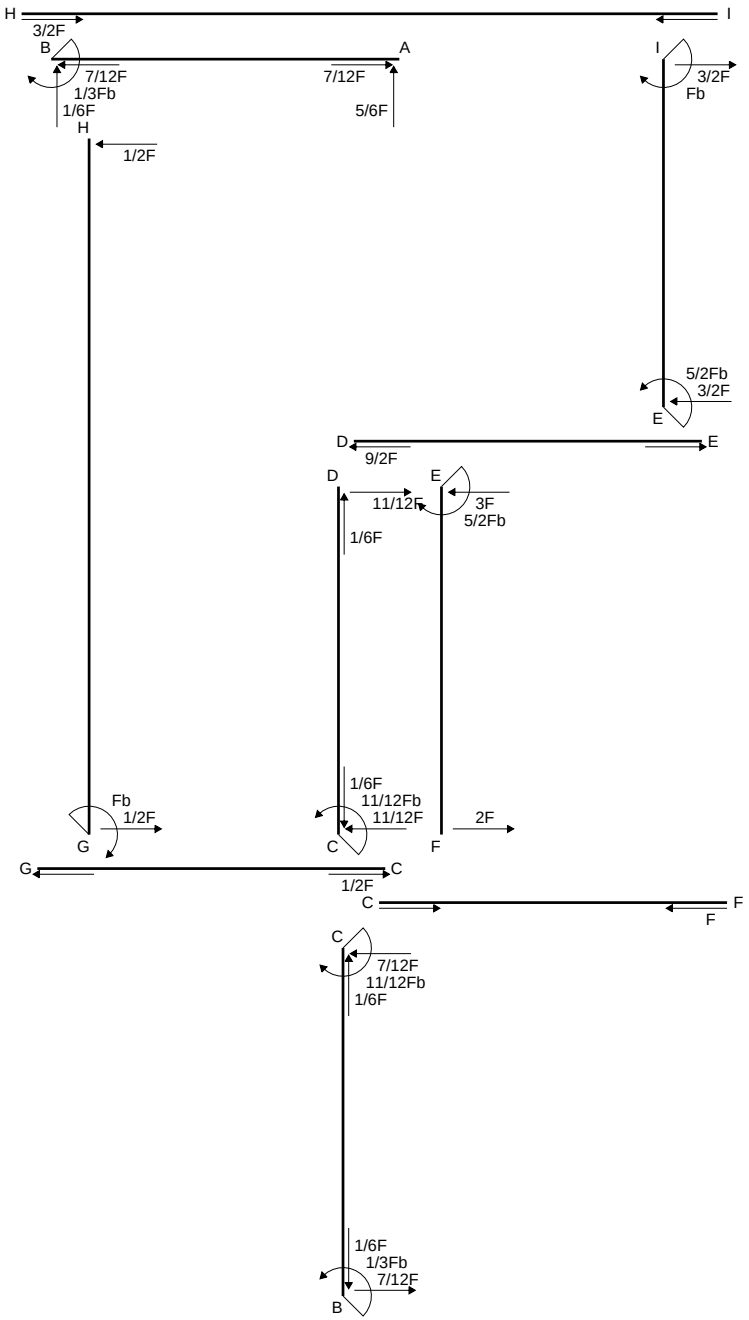


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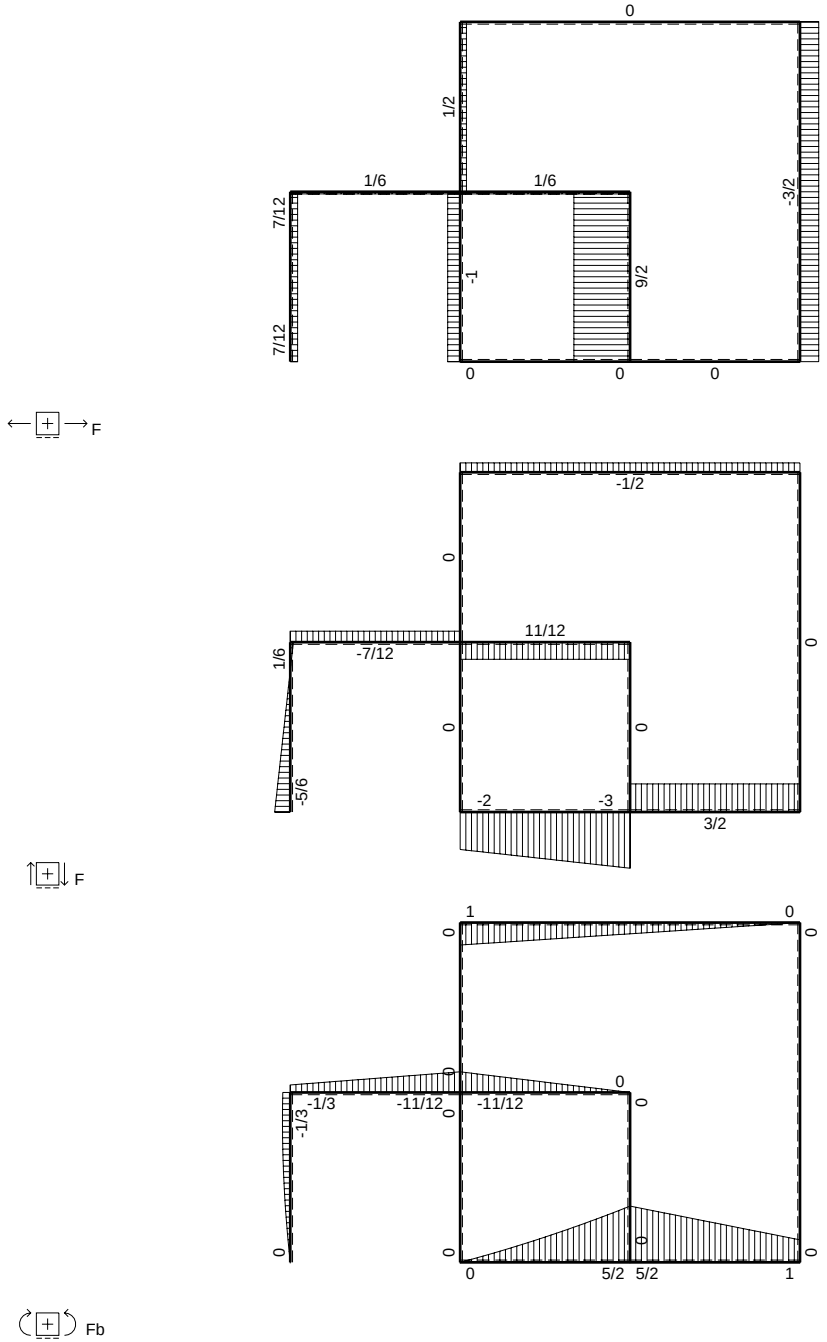

A = 792. $mm^2$
$J_u$ = 225759. $mm^4$
$J_v$ = 30240. $mm^4$
$y_g$ = 22.64 mm
$T_y$ = -3270. N
$M_x$ = 1716750. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 31.36 mm
$\sigma_m = -Mv/J_u$ = -238.5 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 41. mm
$v_c$ = 18.36 mm
$\sigma_c = -Mv/J_u$ = -139.6 $N/mm^2$
$\tau_c$ = 4.682 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.9 $N/mm^2$
$S^*$ = 3879. $mm^3$

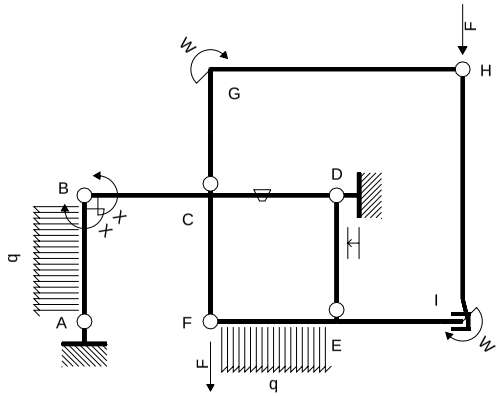
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H
I
3/2F
B
A
7/12F
7/12F
1/3Fb
1/6F
5/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
11/12F
3F
5/2Fb
1/6F
1/6F
11/12Fb
11/12F
Fb
1/2F
2F
G
C
F
G
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1/6F
1/3Fb
7/12F
B

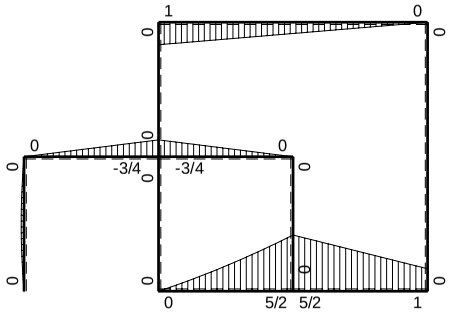




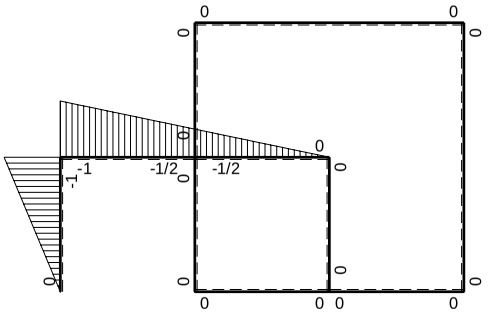

0
1/2
1/6
1/6
-3/2
7/12
-1
9/2
7/12
0
0
0
F
-1/2
0
11/12
1/6
-7/12
0
0
0
-5/6
-2
-3
3/2
F
1
0
0
0
0
-1/3
-1/3
-11/12
-11/12
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

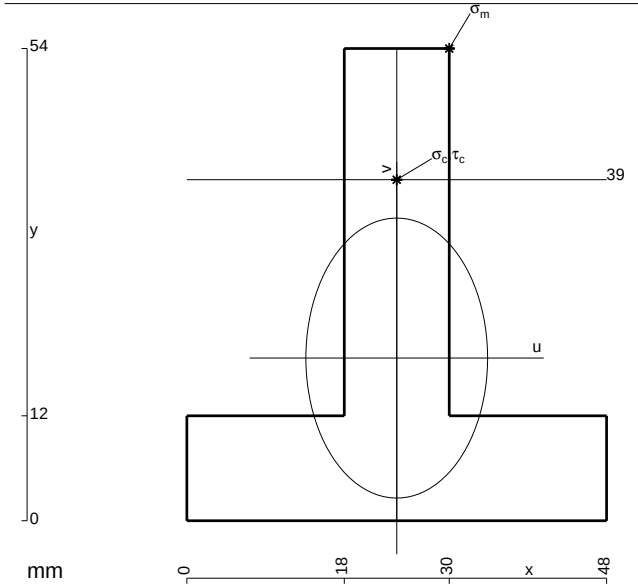
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

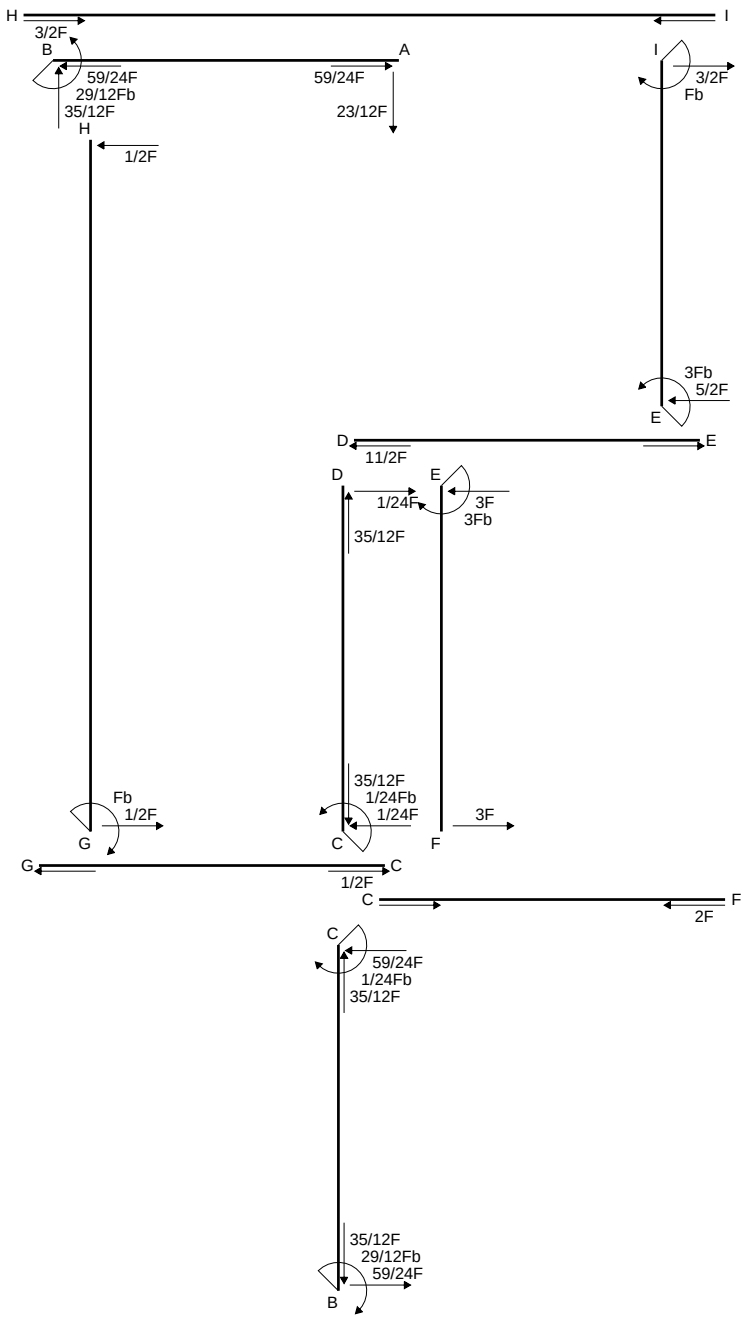


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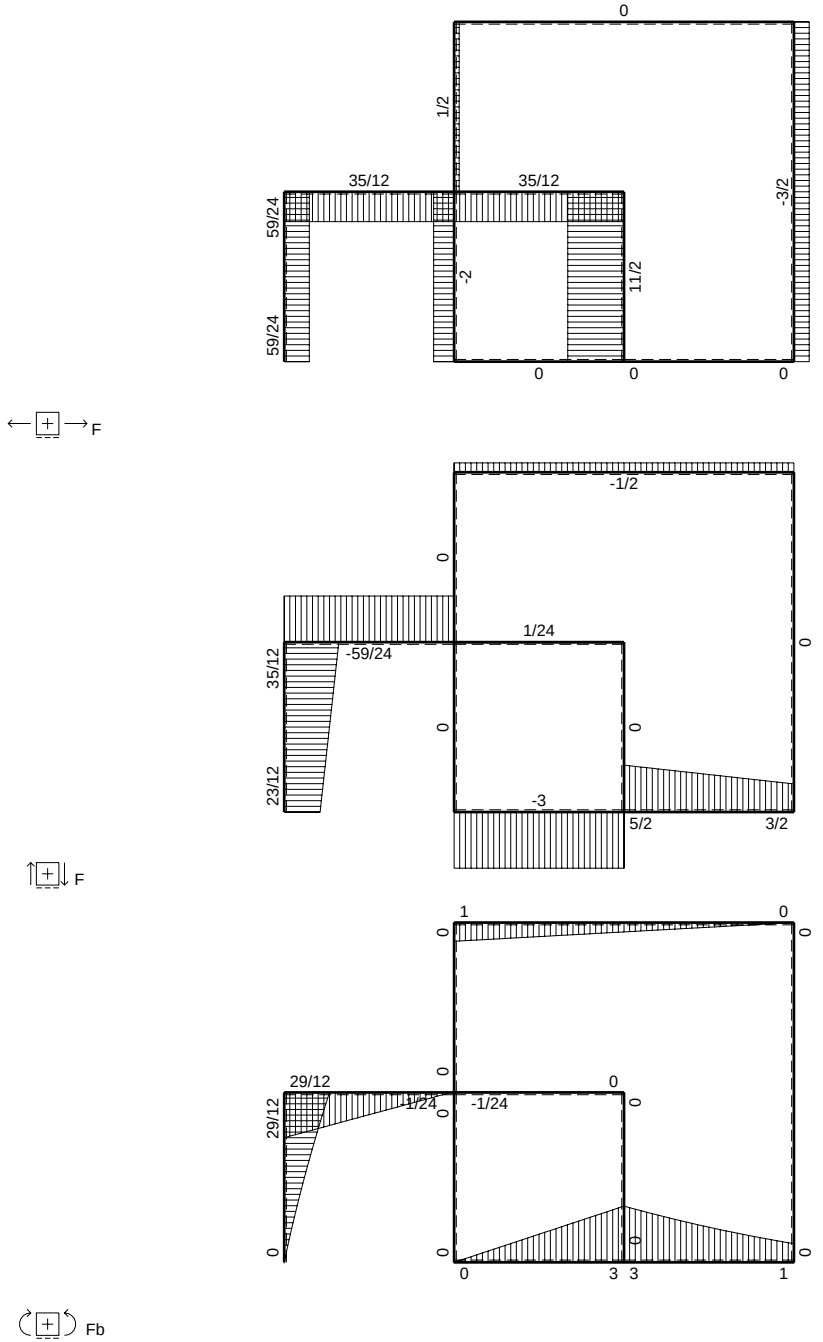

A = 1080. mm$^2$
$J_u$ = 276955. mm$^4$
$J_v$ = 116640. mm$^4$
$y_g$ = 18.6 mm
$T_y$ = -2790. N
$M_x$ = 1557750. Nmm
$x_m$ = 30. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 35.4 mm
$\sigma_m = -Mv/J_u$ = -199.1 N/mm$^2$
$x_c$ = 24. mm
$y_c$ = 39. mm
$v_c$ = 20.4 mm
$\sigma_c = -Mv/J_u$ = -114.7 N/mm$^2$
$\tau_c$ = 4.216 N/mm$^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115. N/mm$^2$
$S^*$ = 5022. mm$^3$

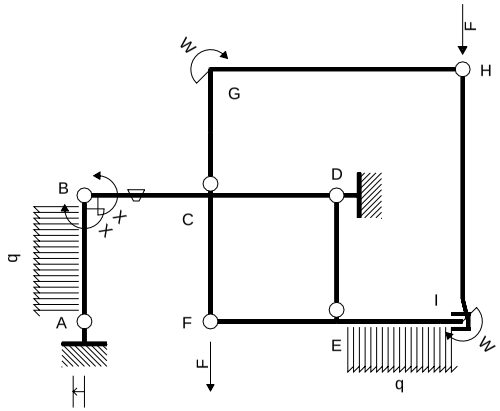
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H
I
3/2F
B
A
59/24F
29/12Fb
35/12F
59/24F
23/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
1/24F
3F
3Fb
35/12F
35/12F
1/24Fb
1/24F
3F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
2F
C
59/24F
1/24Fb
35/12F
35/12F
29/12Fb
59/24F
B

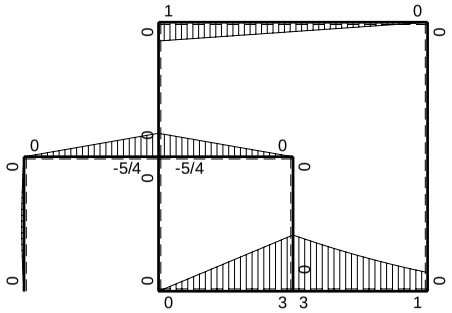




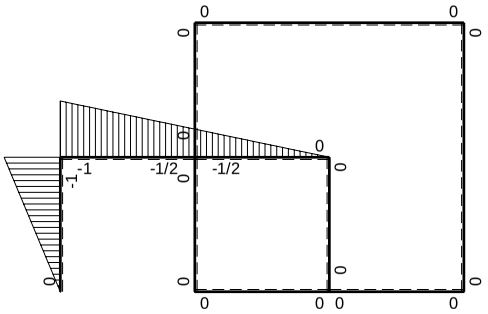

0
1/2
35/12
35/12
-3/2
59/24
-2
11/2
59/24
0
0
0
F
-1/2
0
1/24
35/12
-59/24
0
0
0
23/12
-3
5/2
3/2
F
1
0
0
0
0
29/12
0
29/12
1/24
-1/24
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 984261 CI01.xxxx.016

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 984261 CI01.xxxx.016

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $29/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -29/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

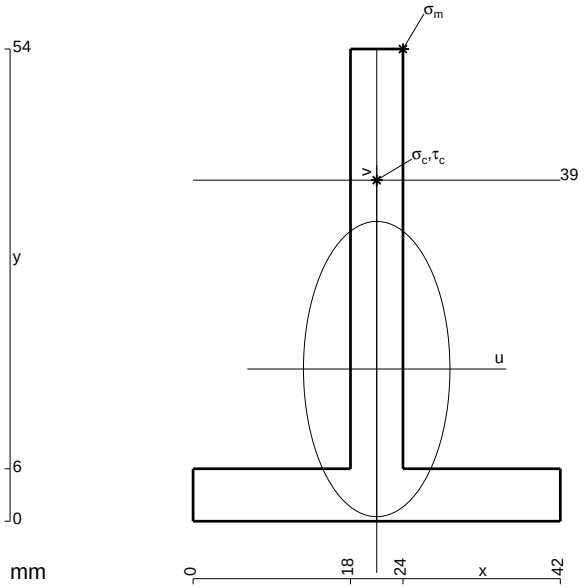
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

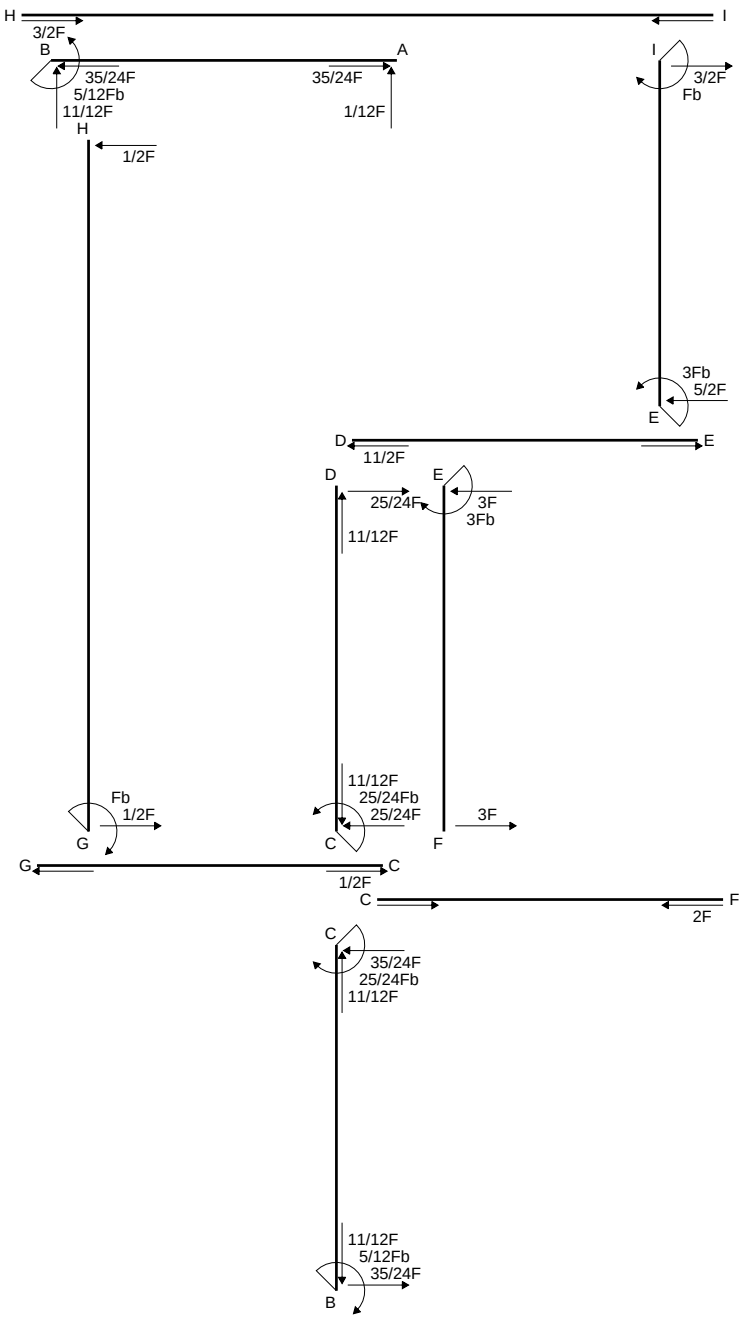


PROCEDIMENTO E RISULTATI 984261 CI01.xxxx.016



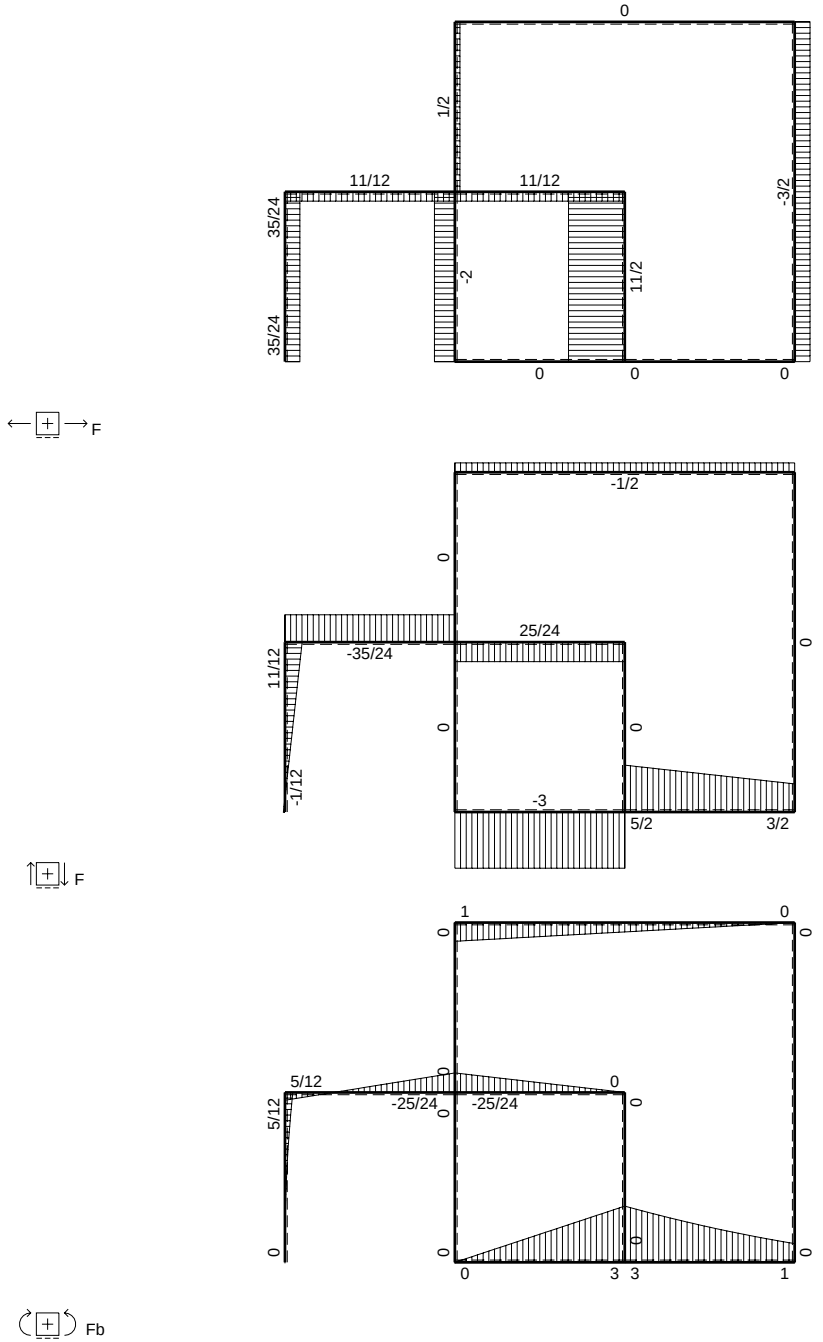

A = 540. $mm^2$
$J_u$ = 154030. $mm^4$
$J_v$ = 37908. $mm^4$
$y_g$ = 17.4 mm
$T_y$ = -1200. N
$M_x$ = 864000. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 36.6 mm
$\sigma_m = -Mv/J_u$ = -205.3 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 39. mm
$v_c$ = 21.6 mm
$\sigma_c = -Mv/J_u$ = -121.2 $N/mm^2$
$\tau_c$ = 3.401 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.3 $N/mm^2$
$S^*$ = 2619. $mm^3$

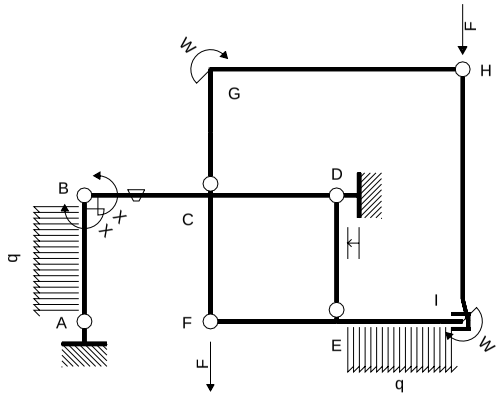
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
3/2F
B
A
35/24F
5/12Fb
11/12F
35/24F
1/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
25/24F
3F
3Fb
11/12F
Fb
1/2F
11/12F
25/24Fb
25/24F
3F
G
C
F
G
C
1/2F
C
F
2F
C
35/24F
25/24Fb
11/12F
11/12F
5/12Fb
35/24F
B

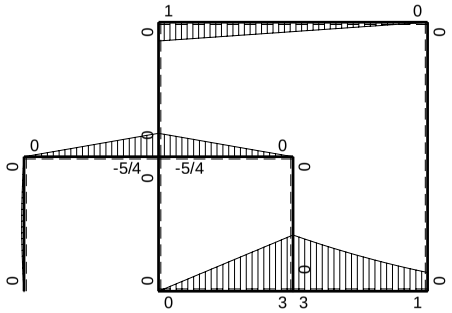




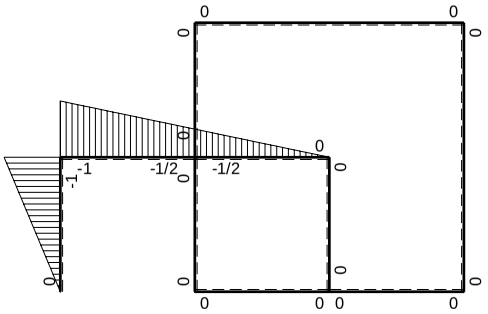

F
0
1/2
11/12
11/12
-3/2
35/24
-2
11/2
35/24
0
0
0
F
-1/2
0
25/24
-35/24
11/12
0
0
0
-1/12
-3
5/2
3/2
Fb
1
0
0
0
5/12
0
-25/24
-25/24
5/12
0
0
0
0
0
0
3
3
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 226747 CI01.xxxx.017

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $5/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/12Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 226747 CI01.xxxx.017

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

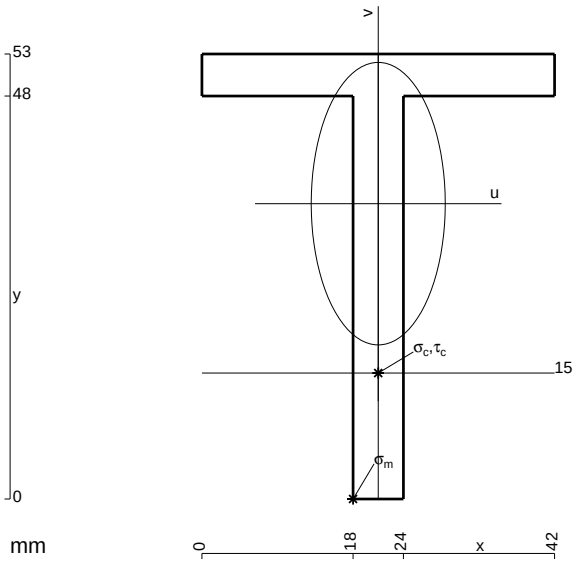
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

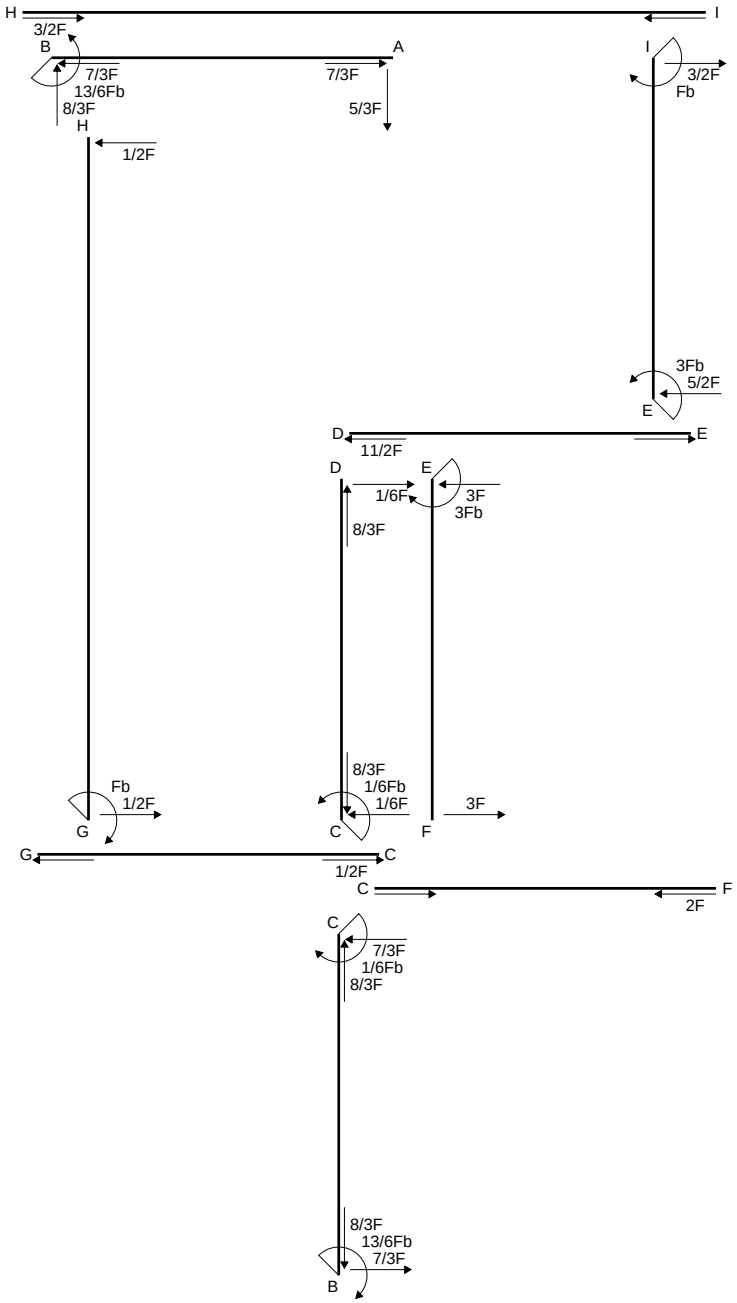
$= (5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





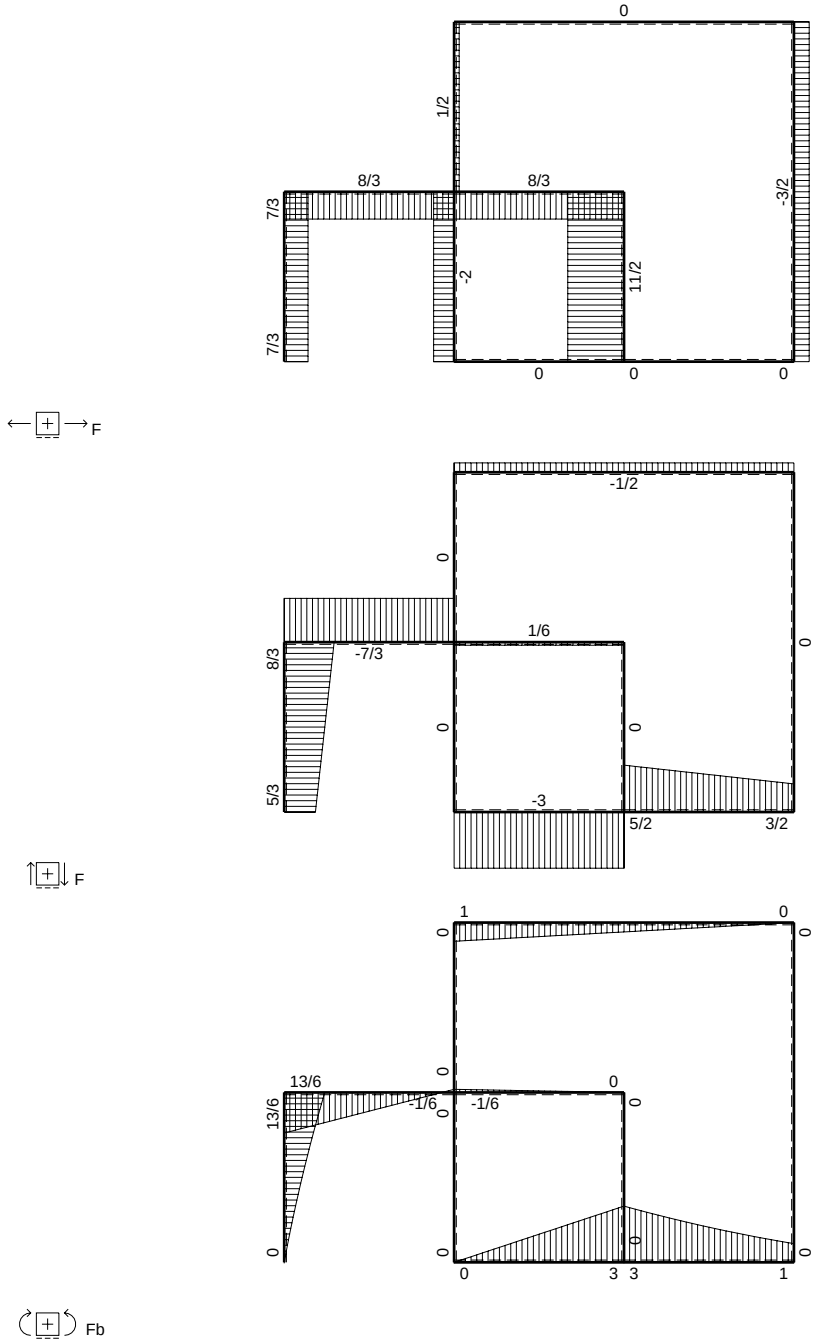


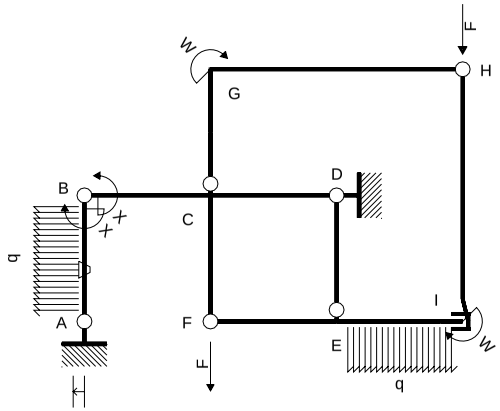

A = 498. $mm^2$
$J_u$ = 141019. $mm^4$
$J_v$ = 31734. $mm^4$
$y_g$ = 35.17 mm
$T_y$ = -1170. N
$M_x$ = 865800. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -35.17 mm
$\sigma_m = -Mv/J_u$ = 216. $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 15. mm
$v_c$ = -20.17 mm
$\sigma_c = -Mv/J_u$ = 123.9 $N/mm^2$
$\tau_c$ = 3.444 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124. $N/mm^2$
$S^*$ = 2491. $mm^3$


H
I
3/2F
B
A
7/3F
13/6Fb
8/3F
7/3F
5/3F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
1/6F
3F
3Fb
8/3F
Fb
1/2F
G
8/3F
1/6Fb
1/6F
C
F
3F
G
C
1/2F
C
F
2F
C
7/3F
1/6Fb
8/3F
8/3F
13/6Fb
7/3F
B

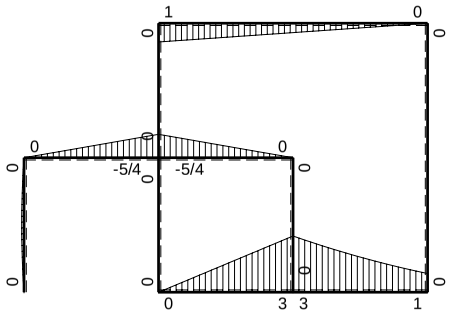




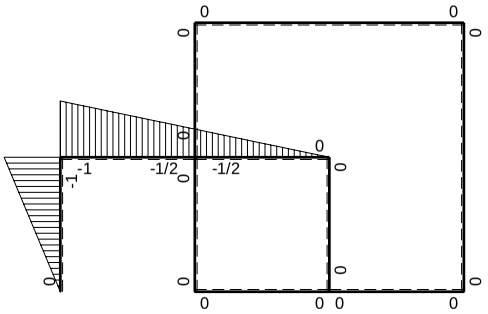

0
1/2
8/3
8/3
-3/2
7/3
-2
11/2
7/3
0
0
0
F
-1/2
0
1/6
8/3
-7/3
0
0
0
5/3
-3
5/2
3/2
F
1
0
0
0
0
13/6
0
13/6
-1/6
-1/6
0
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 986512 CI01.xxxx.018

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 986512 CI01.xxxx.018

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b ( x/b )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

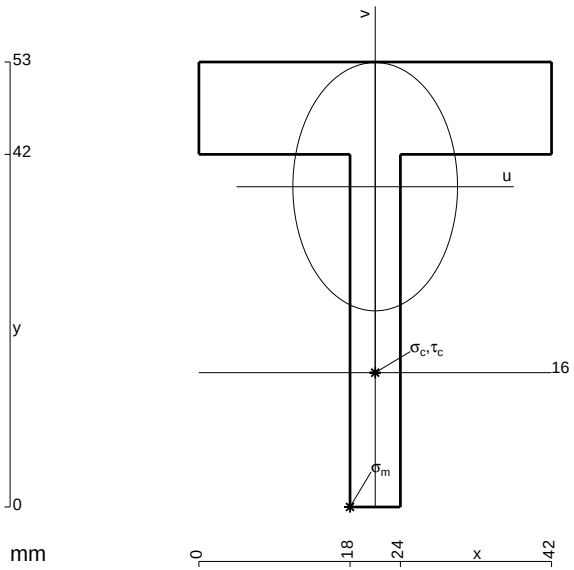
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

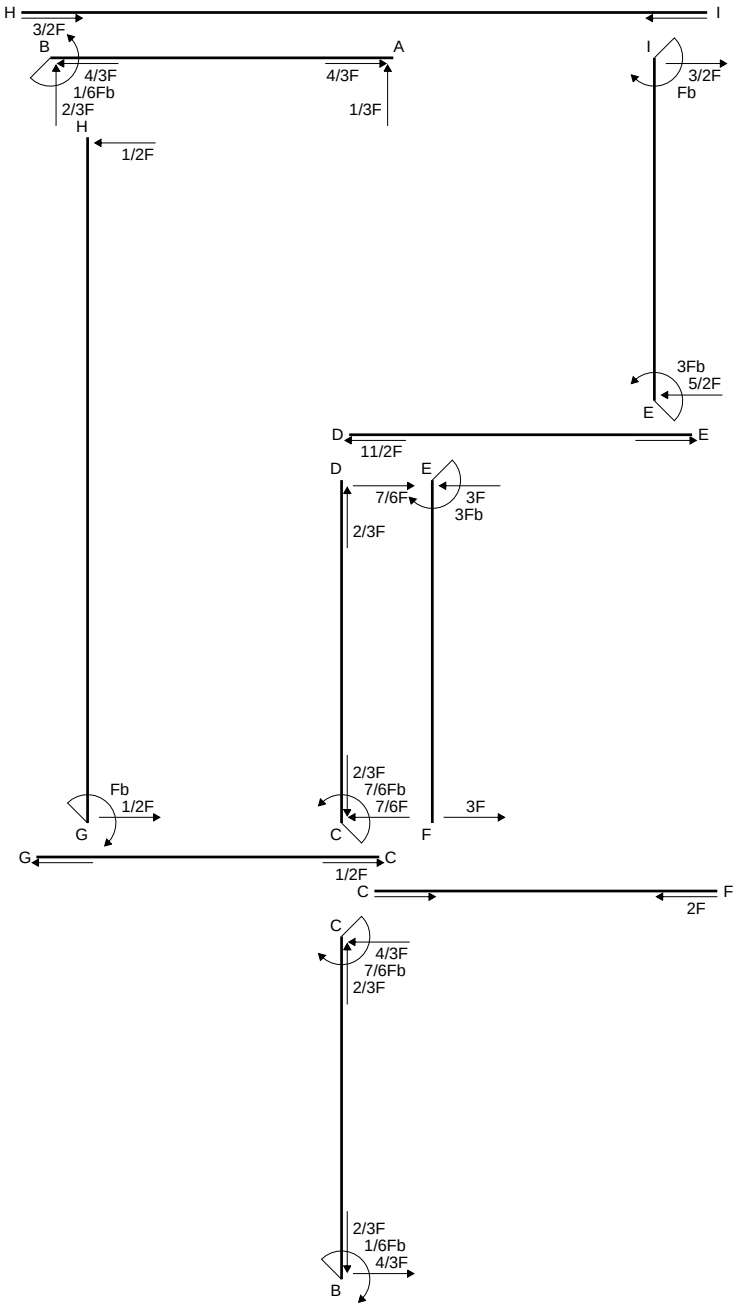
$= (5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





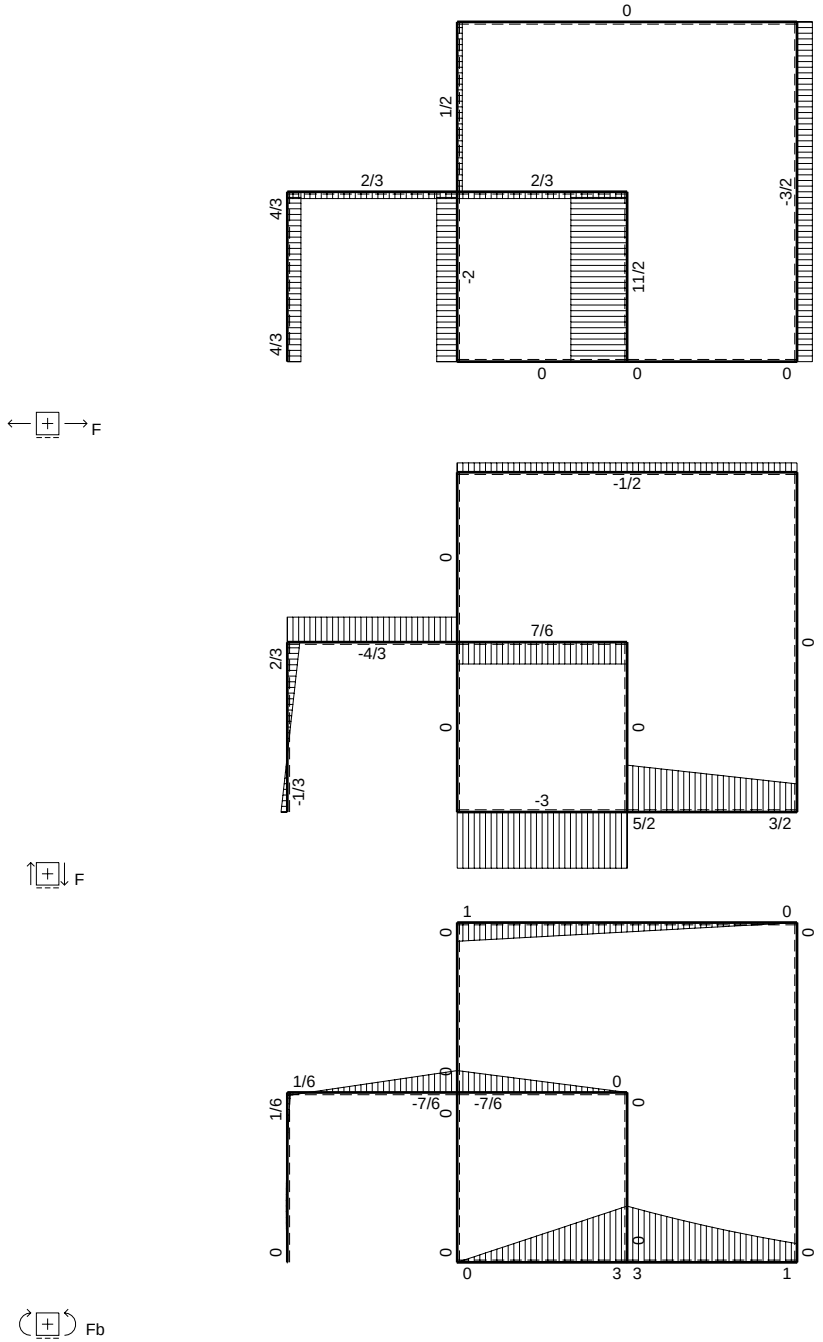


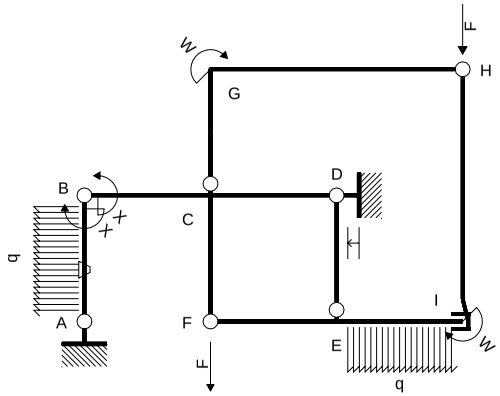

A = 714. $mm^2$
$J_u$ = 156210. $mm^4$
$J_v$ = 68670. $mm^4$
$y_g$ = 38.15 mm
$T_y$ = -1170. N
$M_x$ = 924300. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -38.15 mm
$\sigma_m = -Mv/J_u$ = 225.7 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -22.15 mm
$\sigma_c = -Mv/J_u$ = 131. $N/mm^2$
$\tau_c$ = 3.613 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.2 $N/mm^2$
$S^*$ = 2894. $mm^3$


H
I
3/2F
B
A
I
4/3F
4/3F
3/2F
1/6Fb
Fb
2/3F
1/3F
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
7/6F
3F
3Fb
2/3F
2/3F
Fb
7/6Fb
1/2F
7/6F
3F
G
C
F
G
C
1/2F
C
F
2F
C
4/3F
7/6Fb
2/3F
2/3F
1/6Fb
4/3F
B

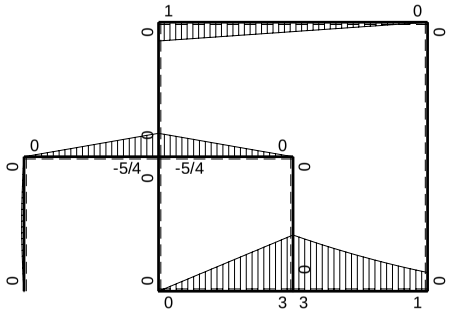




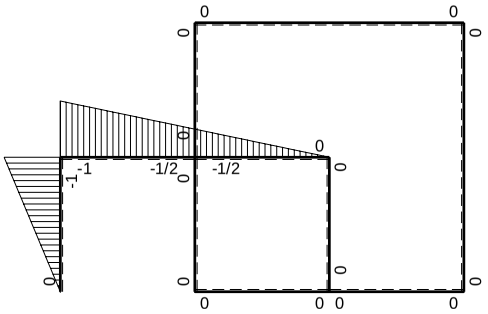

0
1/2
2/3
2/3
-3/2
4/3
-2
11/2
4/3
0
0
0
F
-1/2
0
7/6
2/3
-4/3
0
0
0
-1/3
-3
5/2
3/2
F
1
0
0
0
1/6
0
0
-7/6
-7/6
1/6
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265064 CI01.xxxx.019

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 265064 CI01.xxxx.019

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

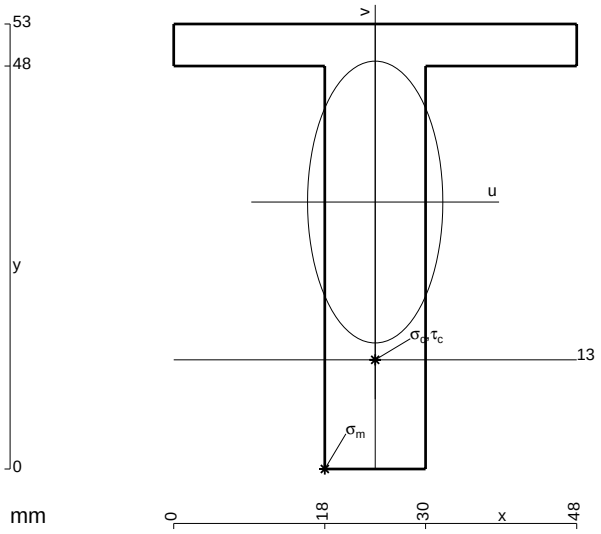
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

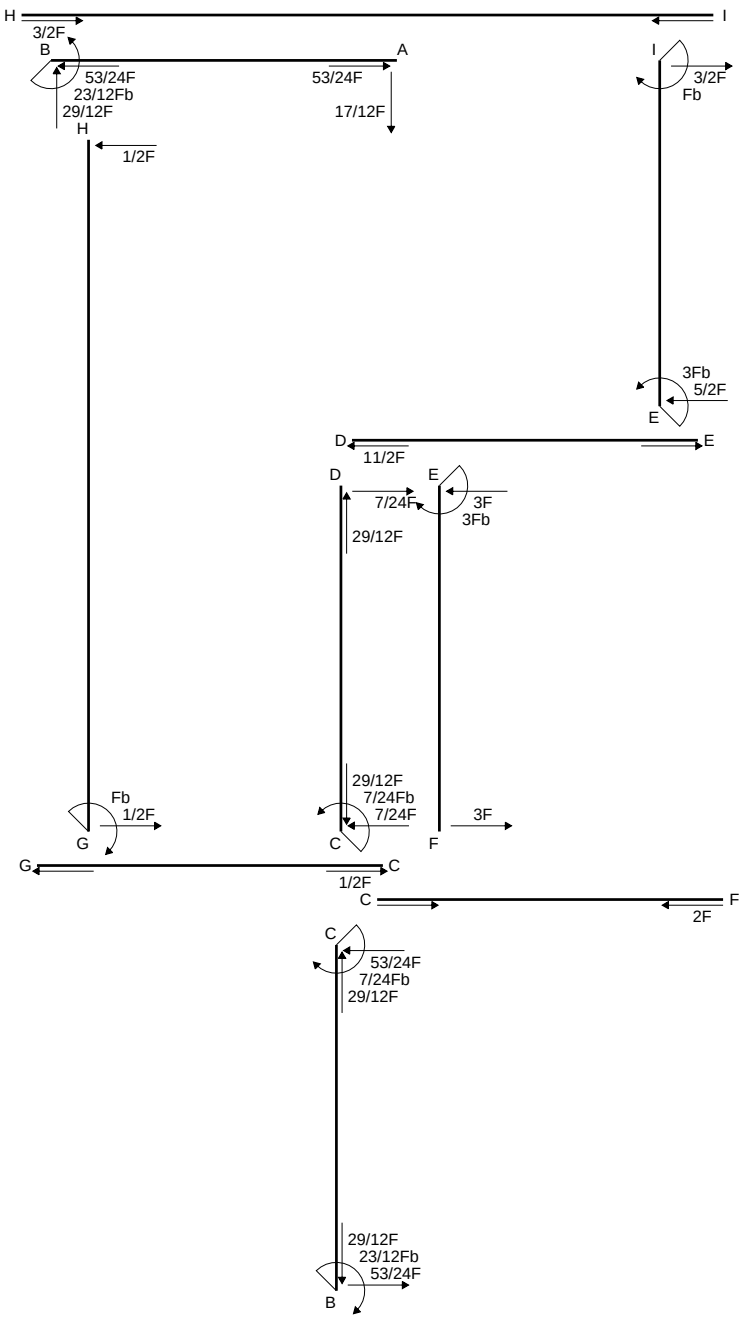
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





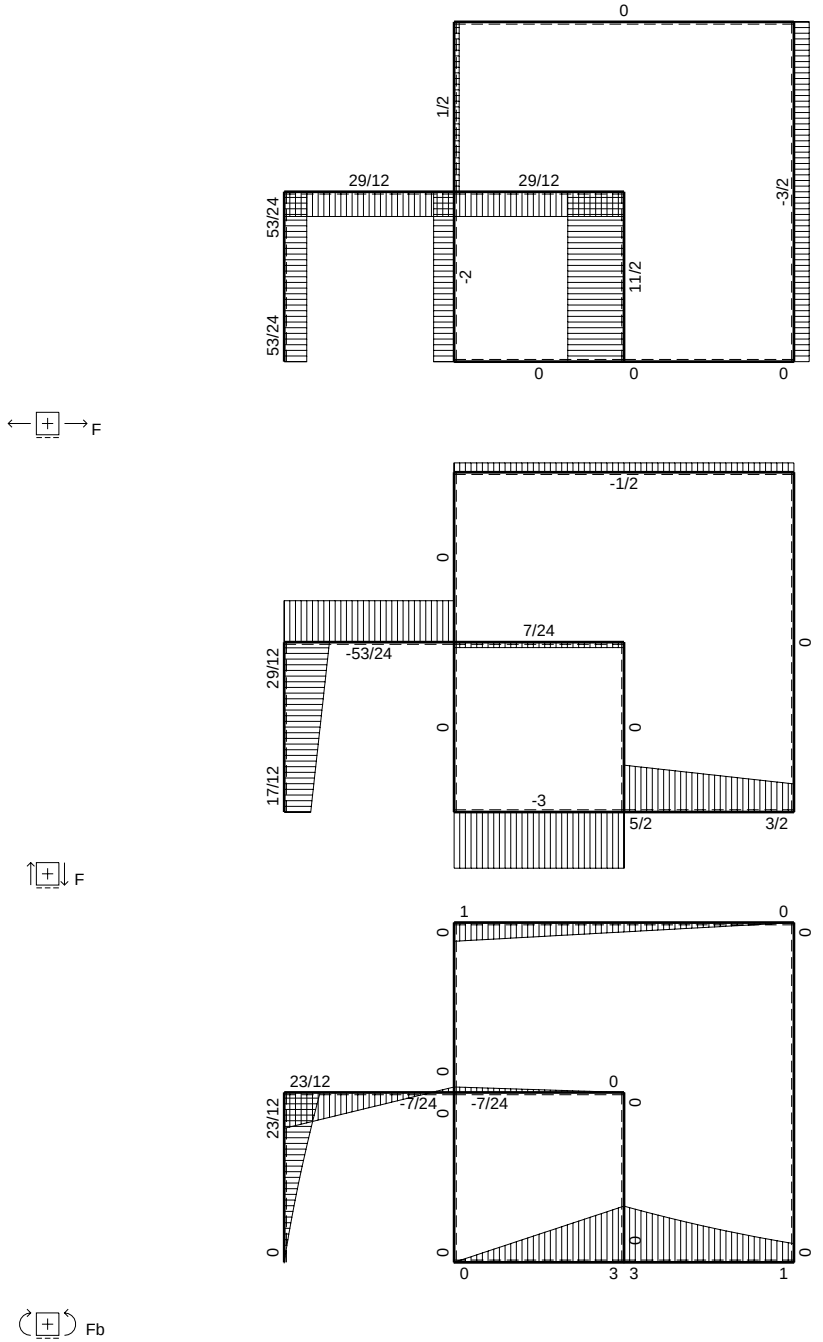


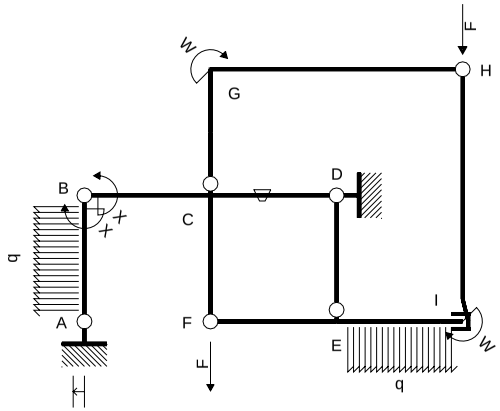

A = 816. $mm^2$
$J_u$ = 230061. $mm^4$
$J_v$ = 52992. $mm^4$
$y_g$ = 31.79 mm
$T_y$ = -2070. N
$M_x$ = 1718100. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -31.79 mm
$\sigma_m = -Mv/J_u$ = 237.4 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 13. mm
$v_c$ = -18.79 mm
$\sigma_c = -Mv/J_u$ = 140.4 $N/mm^2$
$\tau_c$ = 2.959 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.4 $N/mm^2$
$S^*$ = 3946. $mm^3$


H
I
3/2F
B
A
53/24F
23/12Fb
29/12F
53/24F
17/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
7/24F
3F
3Fb
29/12F
29/12F
7/24Fb
7/24F
Fb
1/2F
G
C
F
3F
G
C
1/2F
C
F
2F
C
53/24F
7/24Fb
29/12F
29/12F
23/12Fb
53/24F
B

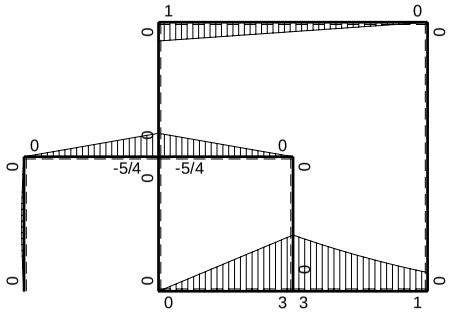




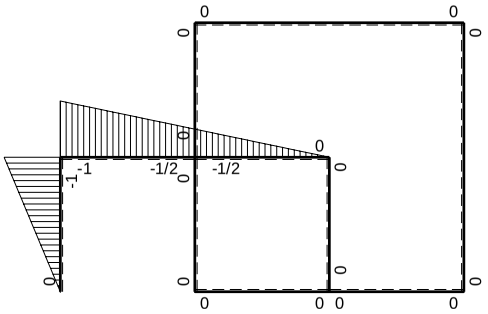

0
1/2
29/12
29/12
-3/2
53/24
-2
11/2
53/24
0
0
0
F
-1/2
0
7/24
29/12
-53/24
0
0
0
17/12
-3
5/2
3/2
F
1
0
0
0
0
23/12
0
23/12
-7/24
-7/24
0
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 269138 CI01.xxxx.020

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

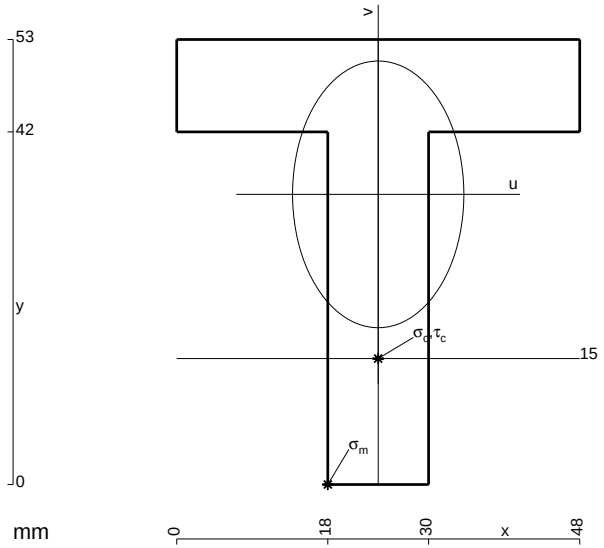
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/24\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 11/24\ Fb^2/EJ$

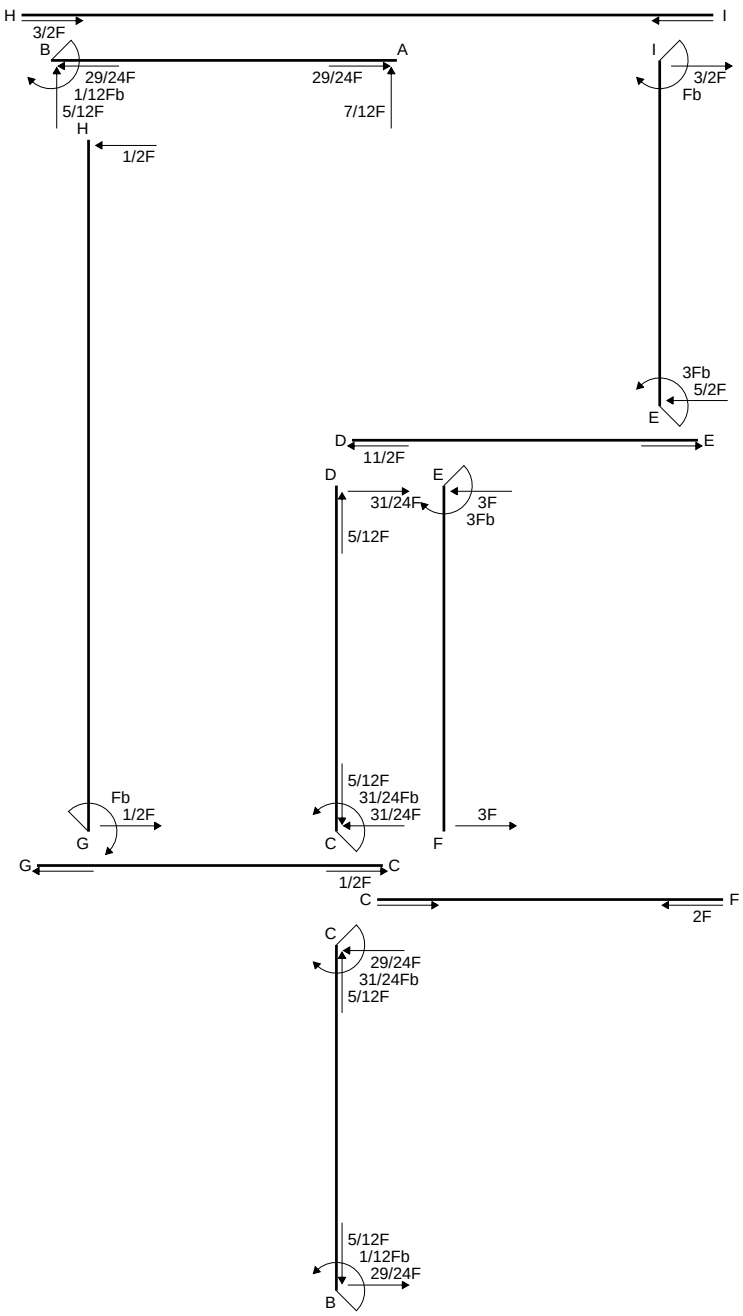


PROCEDIMENTO E RISULTATI 269138 CI01.xxxx.020



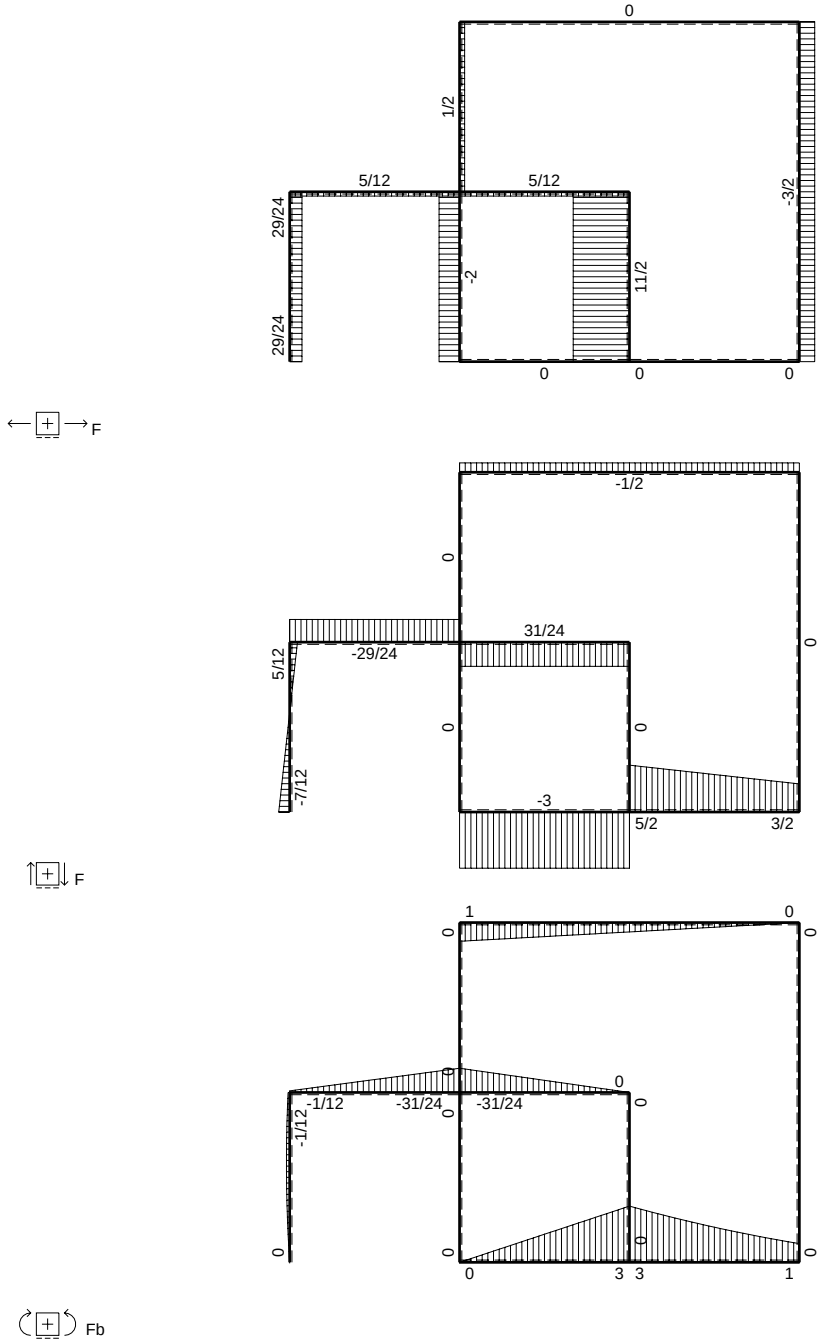

A = 1032. $mm^2$
$J_u$ = 260495. $mm^4$
$J_v$ = 107424. $mm^4$
$y_g$ = 34.56 mm
$T_y$ = -3420. N
$M_x$ = 1504800. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -34.56 mm
$\sigma_m = -Mv/J_u$ = 199.6 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -19.56 mm
$\sigma_c = -Mv/J_u$ = 113. $N/mm^2$
$\tau_c$ = 5.329 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 113.4 $N/mm^2$
$S^*$ = 4870. $mm^3$

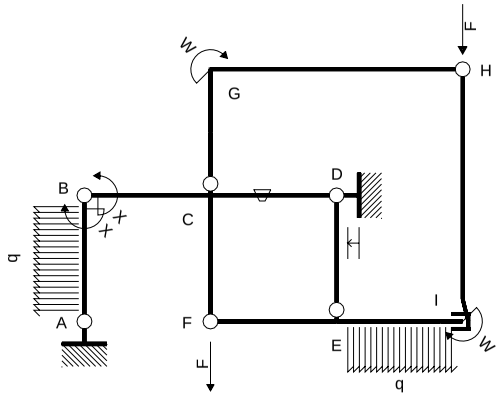
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
3/2F
B
A
29/24F
1/12Fb
5/12F
29/24F
7/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
31/24F
3F
3Fb
5/12F
5/12F
31/24Fb
31/24F
Fb
1/2F
G
C
F
3F
G
C
1/2F
C
F
2F
C
29/24F
31/24Fb
5/12F
5/12F
1/12Fb
29/24F
B

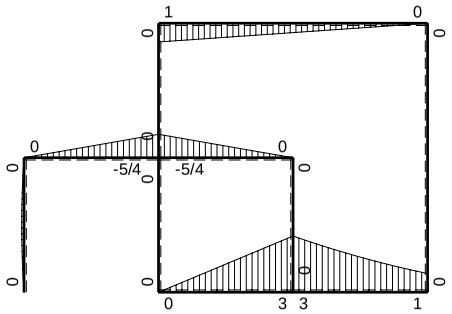




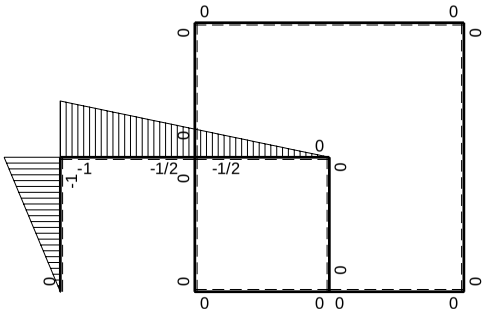

0
1/2
5/12
5/12
-3/2
29/24
-2
11/2
29/24
0
0
0
F
-1/2
0
31/24
5/12
-29/24
0
0
-7/12
-3
5/2
3/2
F
1
0
0
0
0
-1/12
-31/24
-31/24
-1/12
0
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248038 CI01.xxxx.021

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-5/4Fb+5/4Fx$ | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | $1/2Fx/EJ$ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 248038 CI01.xxxx.021

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

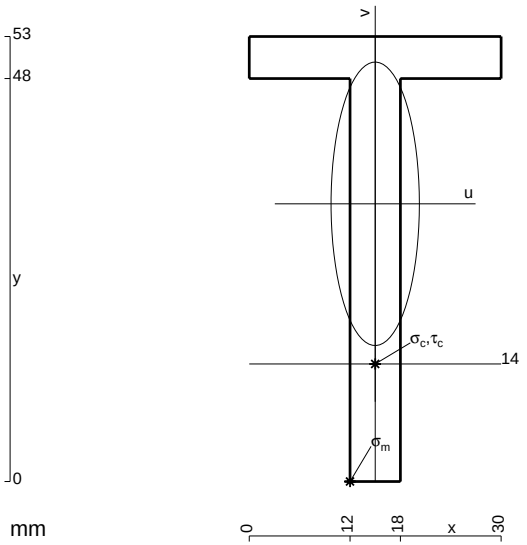
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

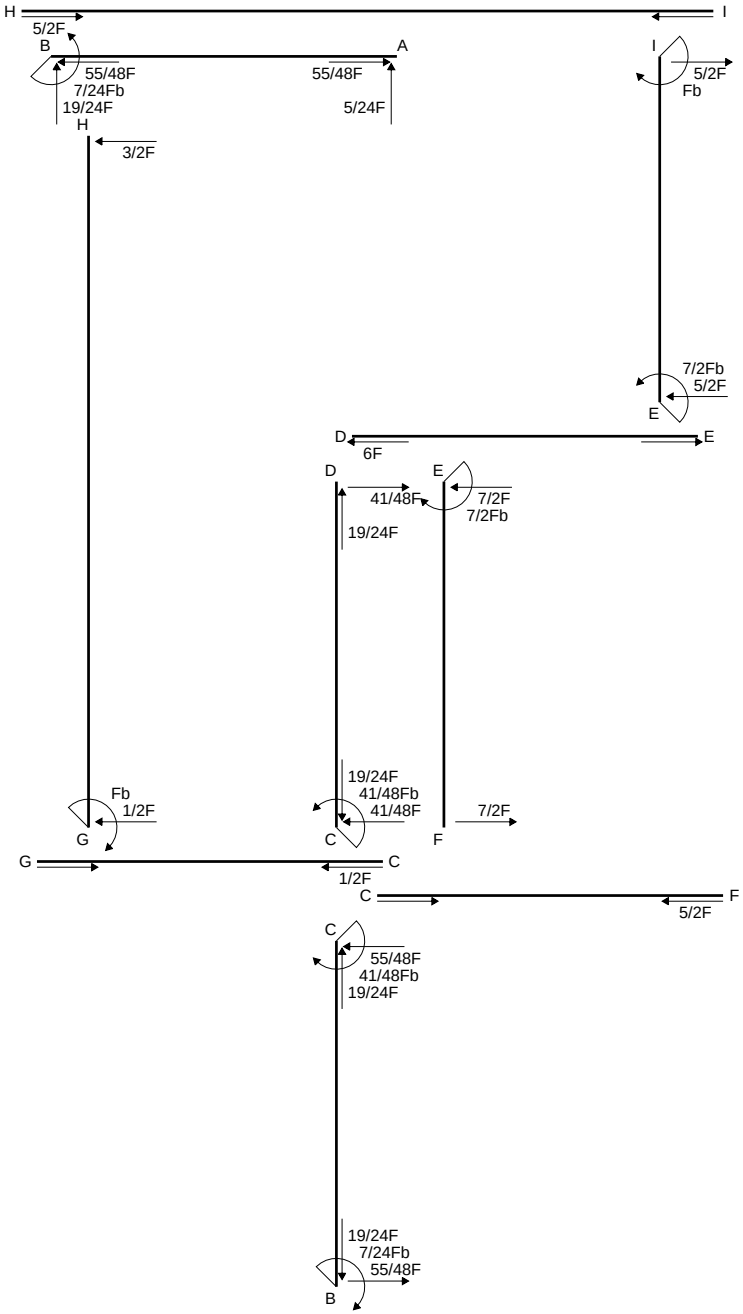
$= (5/24\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 248038 CI01.xxxx.021

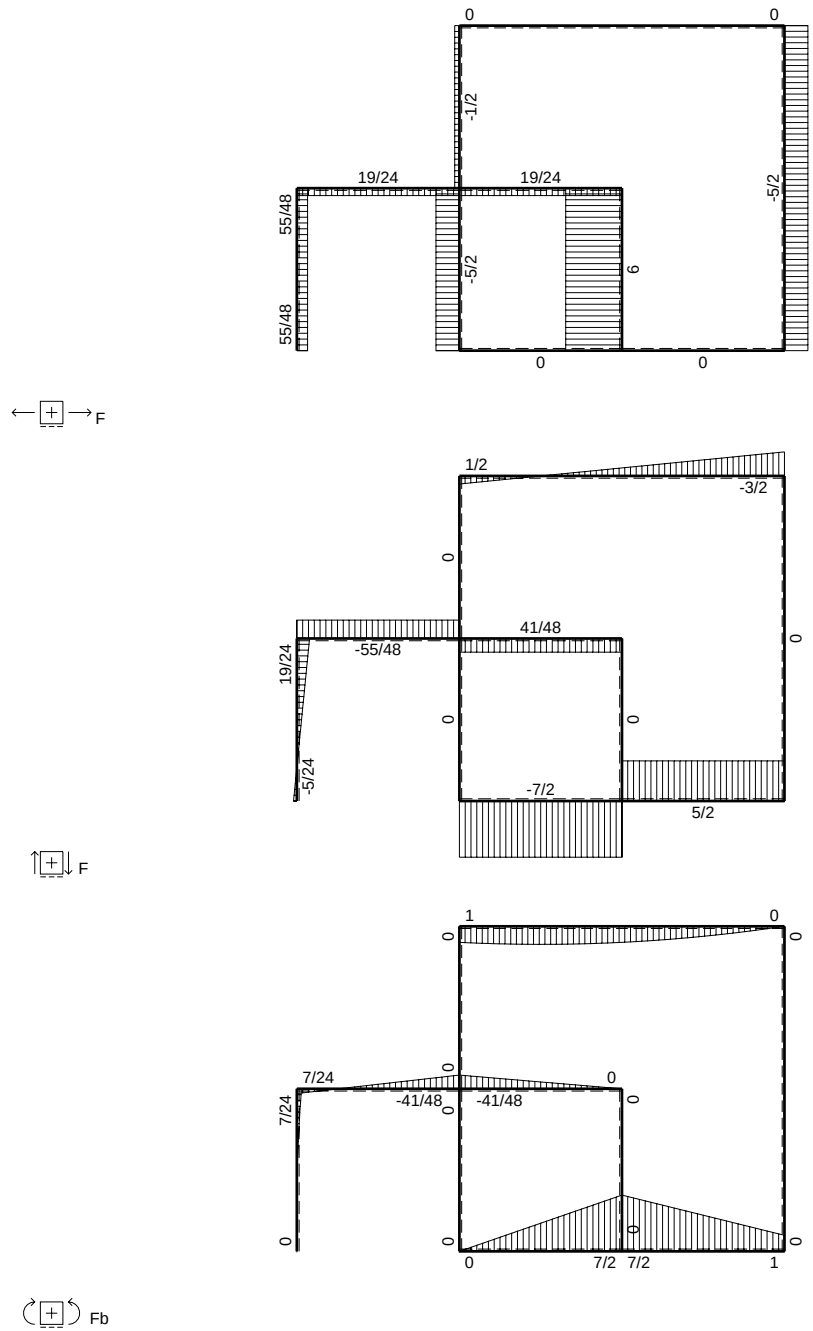


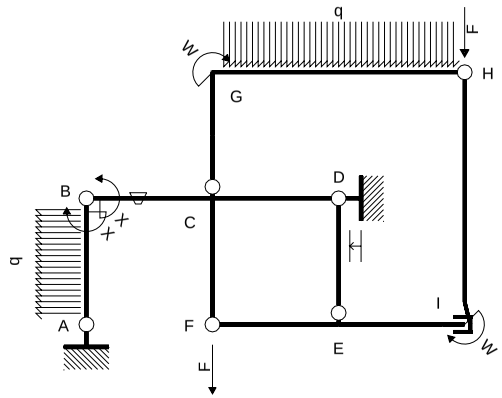

A = 438. $mm^2$
$J_u$ = 124871. $mm^4$
$J_v$ = 12114. $mm^4$
$y_g$ = 33.08 mm
$T_y$ = -1650. N
$M_x$ = 792000. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -33.08 mm
$\sigma_m = -Mv/J_u$ = 209.8 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 14. mm
$v_c$ = -19.08 mm
$\sigma_c = -Mv/J_u$ = 121. $N/mm^2$
$\tau_c$ = 4.824 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 121.3 $N/mm^2$
$S^*$ = 2190. $mm^3$


H
I
5/2F
B
A
55/48F
7/24Fb
19/24F
55/48F
5/24F
H
3/2F
I
5/2F
Fb
7/2Fb
5/2F
E
D
E
6F
D
E
41/48F
7/2F
7/2Fb
19/24F
19/24F
41/48Fb
41/48F
Fb
1/2F
7/2F
G
C
F
G
C
1/2F
C
F
5/2F
C
55/48F
41/48Fb
19/24F
19/24F
7/24Fb
55/48F
B

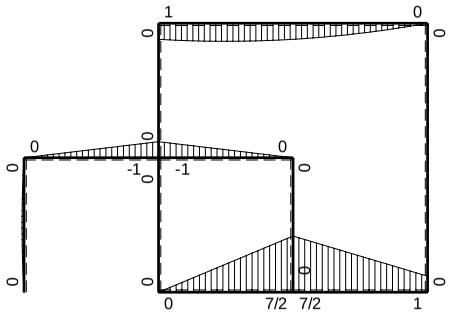




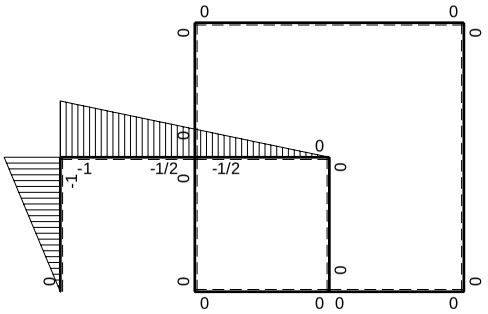

0
0
-1/2
19/24
19/24
-5/2
55/48
-5/2
6
55/48
0
0
F
1/2
-3/2
0
41/48
-55/48
19/24
0
0
0
-5/24
-7/2
5/2
F
1
0
0
0
7/24
0
0
-41/48
-41/48
7/24
0
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264292 CI01.xxxx.022

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 264292 CI01.xxxx.022

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $7/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

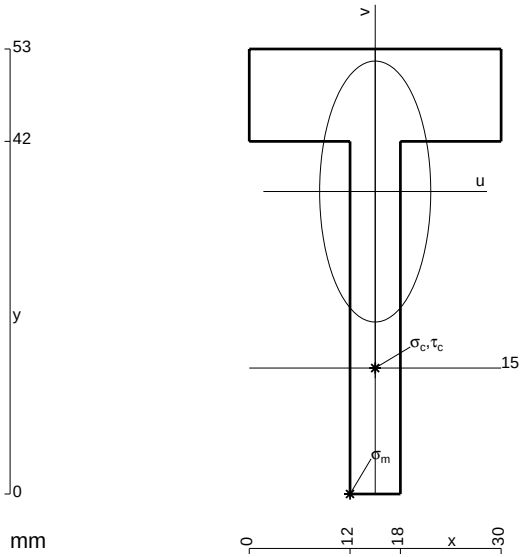
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

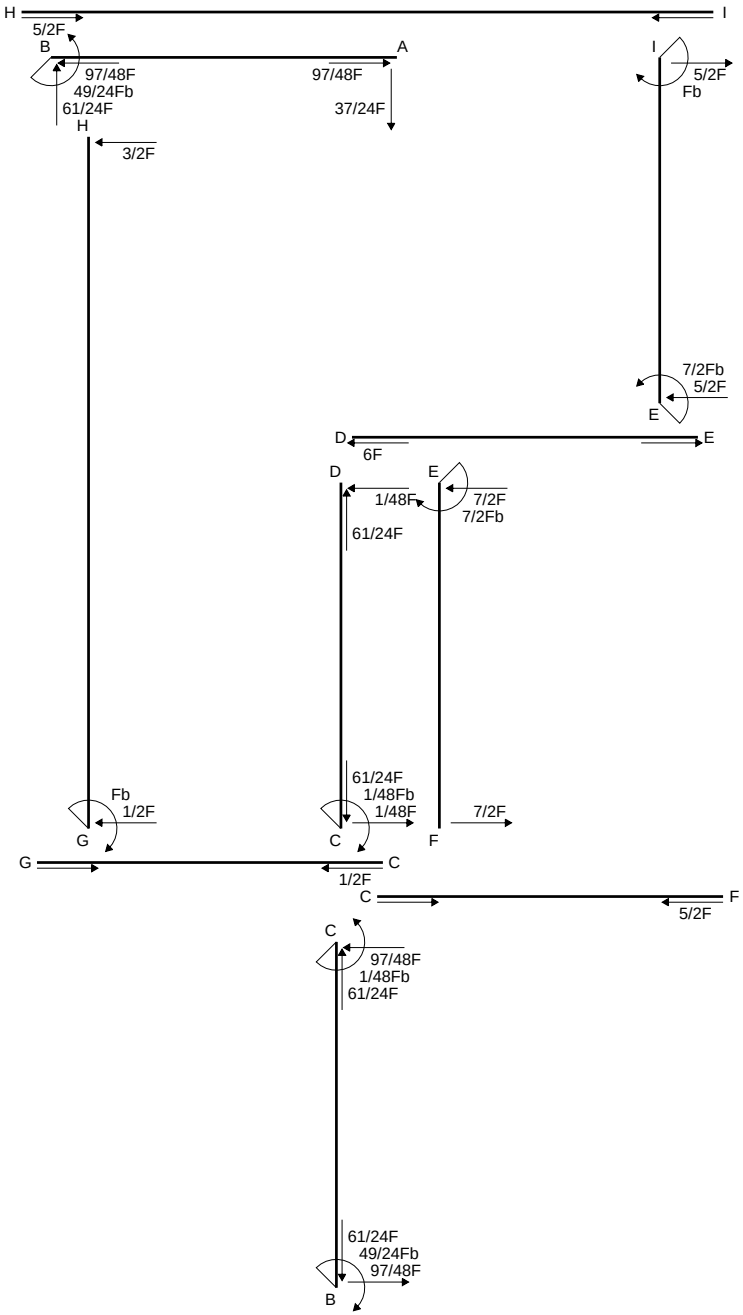
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 264292 CI01.xxxx.022

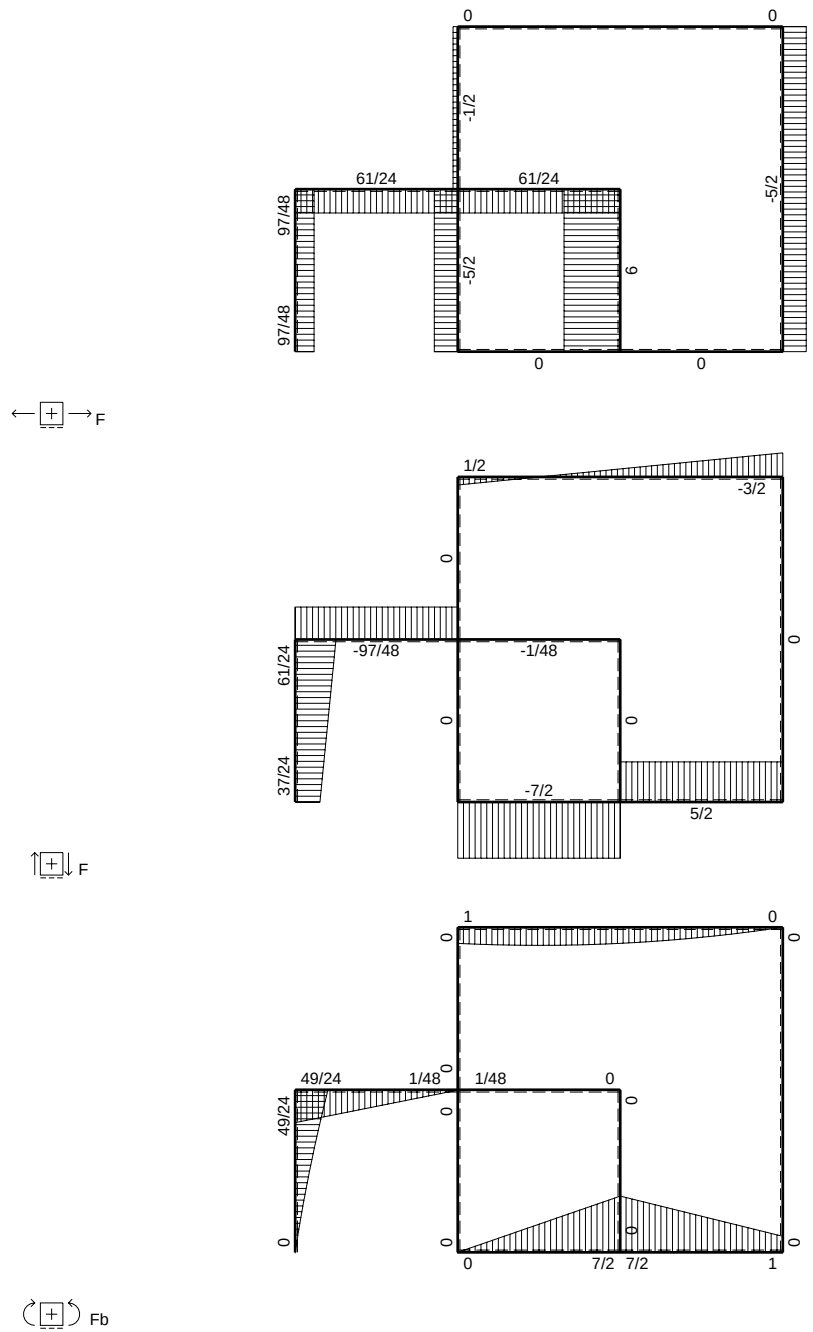


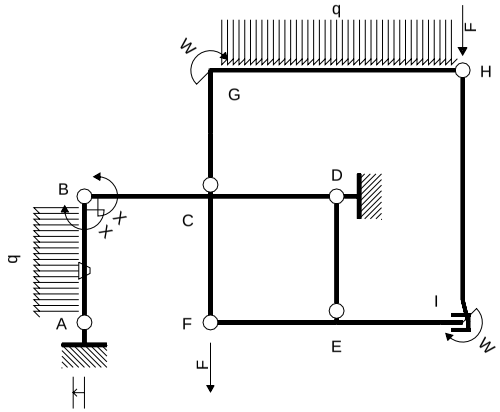

A = 582. $mm^2$
$J_u$ = 140714. $mm^4$
$J_v$ = 25506. $mm^4$
$y_g$ = 36.03 mm
$T_y$ = -1645. N
$M_x$ = 855400. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -36.03 mm
$\sigma_m = -Mv/J_u$ = 219. $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 15. mm
$v_c$ = -21.03 mm
$\sigma_c = -Mv/J_u$ = 127.8 $N/mm^2$
$\tau_c$ = 5.002 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.1 $N/mm^2$
$S^*$ = 2567. $mm^3$


H
I
5/2F
B
A
97/48F
49/24Fb
61/24F
97/48F
37/24F
I
5/2F
Fb
H
3/2F
7/2Fb
5/2F
E
D
E
6F
D
E
1/48F
7/2F
7/2Fb
61/24F
61/24F
1/48Fb
1/48F
7/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
5/2F
C
97/48F
1/48Fb
61/24F
61/24F
49/24Fb
97/48F
B

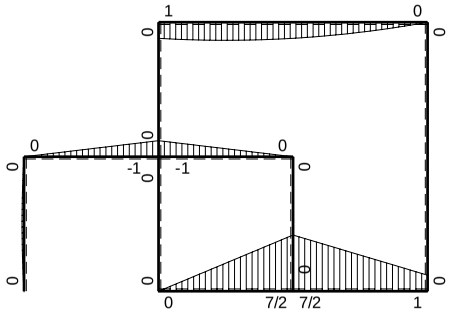




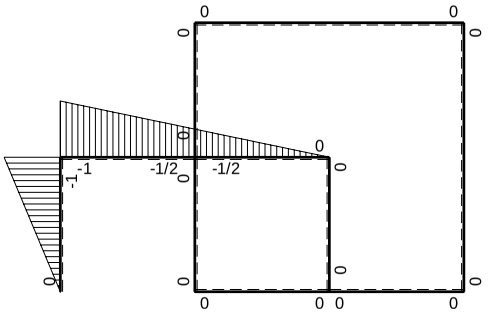

0
0
-1/2
61/24
61/24
-5/2
97/48
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6
97/48
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0
F
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0
0
61/24
-97/48
-1/48
0
37/24
-7/2
5/2
F
1
0
0
0
0
0
49/24
1/48
1/48
0
49/24
0
0
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268210 CI01.xxxx.023

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PROCEDIMENTO E RISULTATI 268210 CI01.xxxx.023

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $49/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -49/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b ( x/b )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

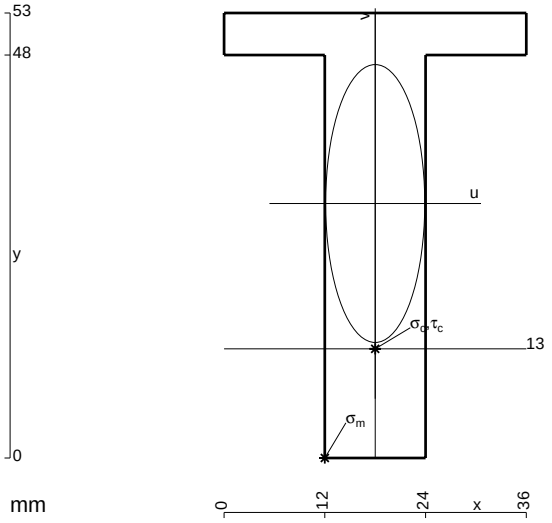
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

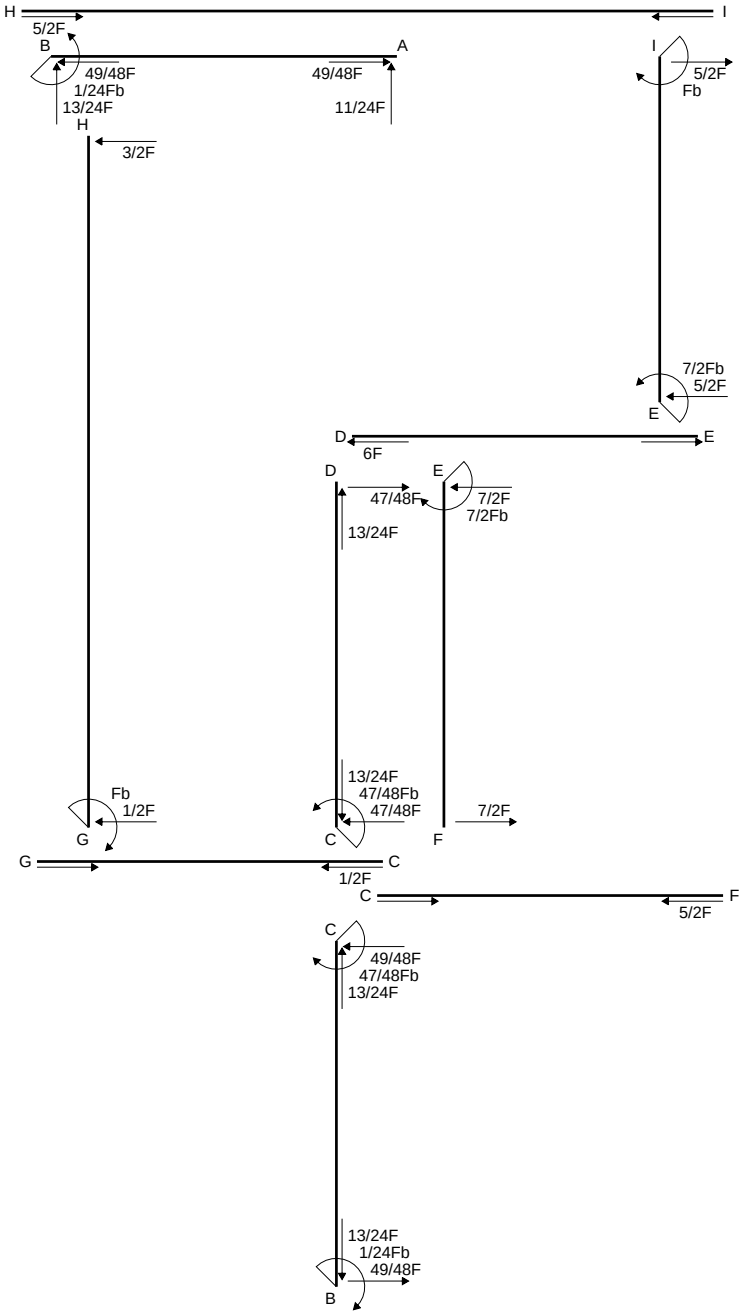
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





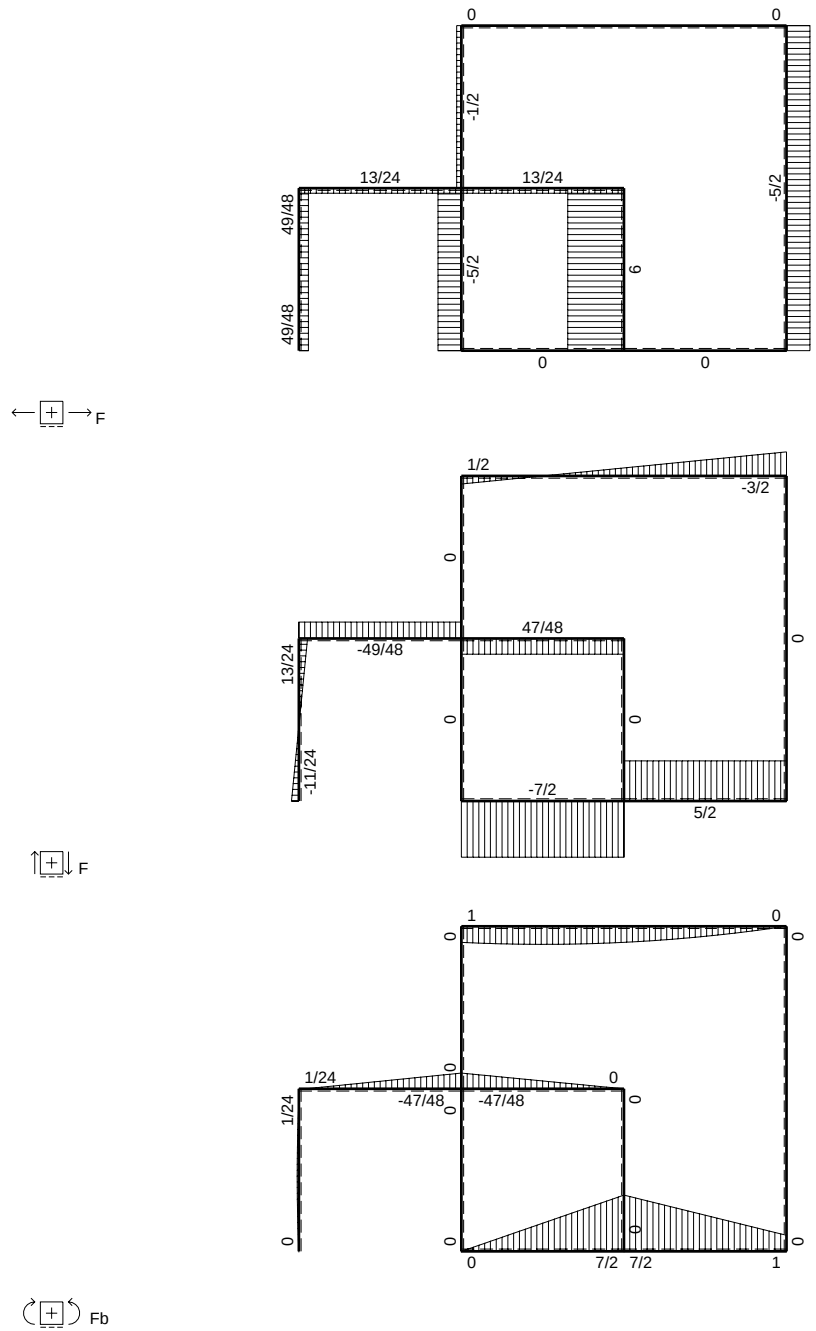


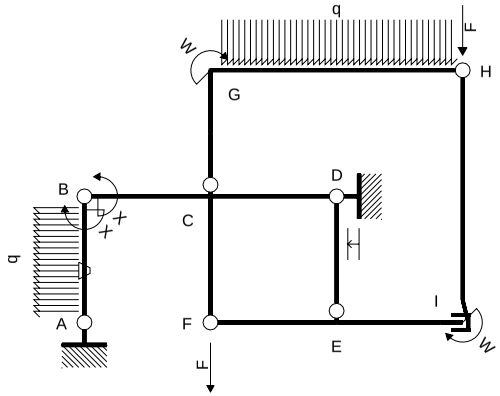

A = 756. $mm^2$
$J_u$ = 207276. $mm^4$
$J_v$ = 26352. $mm^4$
$y_g$ = 30.31 mm
$T_y$ = -2730. N
$M_x$ = 1556100. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -30.31 mm
$\sigma_m = -Mv/J_u$ = 227.5 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 13. mm
$v_c$ = -17.31 mm
$\sigma_c = -Mv/J_u$ = 129.9 $N/mm^2$
$\tau_c$ = 4.077 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.1 $N/mm^2$
$S^*$ = 3714. $mm^3$


H
I
5/2F
B
A
49/48F
1/24Fb
13/24F
49/48F
11/24F
I
5/2F
Fb
H
3/2F
7/2Fb
5/2F
E
D
E
6F
D
E
47/48F
7/2F
7/2Fb
13/24F
13/24F
47/48Fb
47/48F
Fb
1/2F
7/2F
G
C
F
G
C
1/2F
C
F
5/2F
C
49/48F
47/48Fb
13/24F
13/24F
1/24Fb
49/48F
B

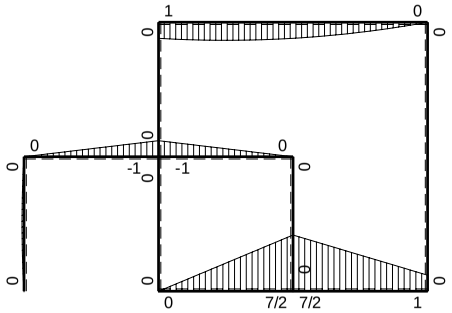




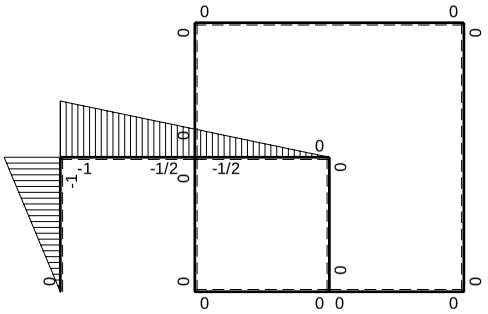

0
0
-1/2
13/24
13/24
-5/2
49/48
-5/2
6
49/48
0
0
F
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-3/2
0
47/48
-49/48
13/24
0
0
0
-11/24
-7/2
5/2
F
1
0
0
0
1/24
0
0
-47/48
-47/48
1/24
0
0
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267066 CI01.xxxx.024

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 267066 CI01.xxxx.024

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b ( x/b )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

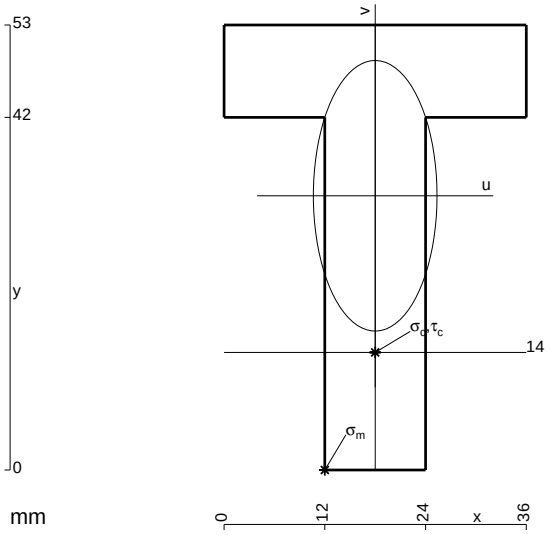
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

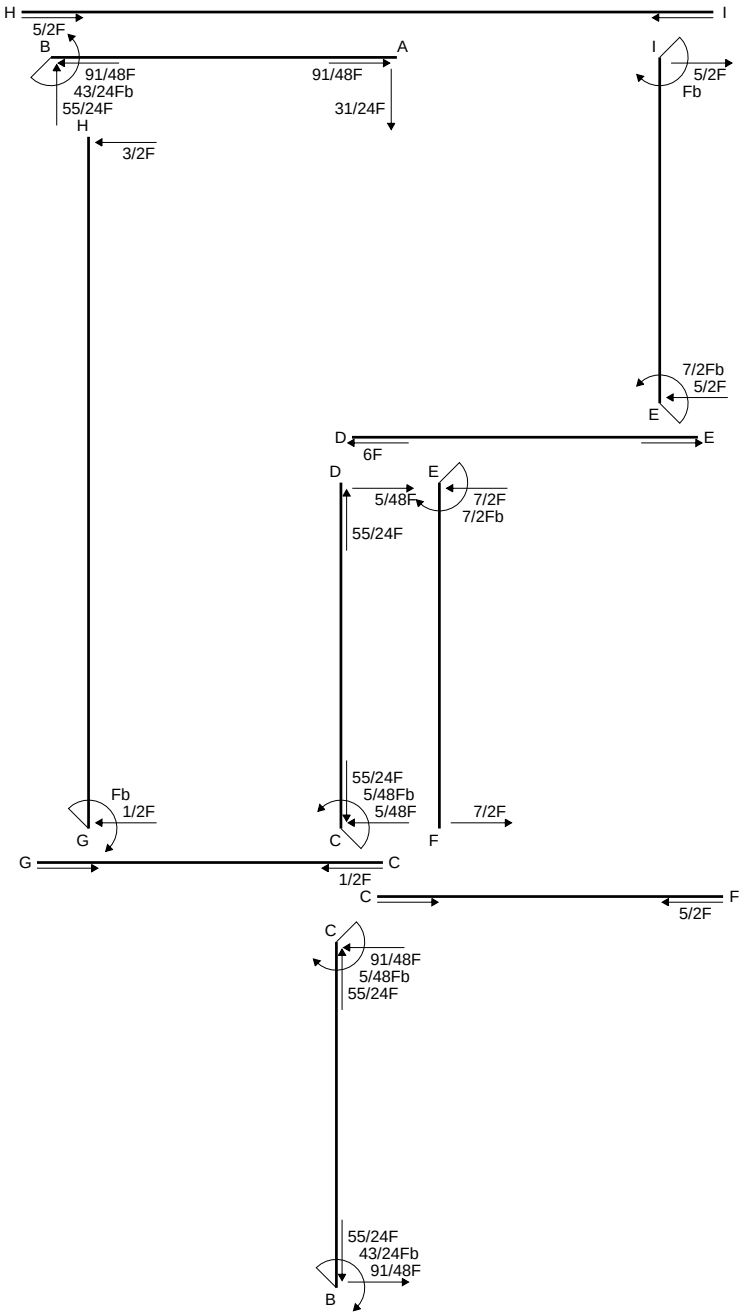
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





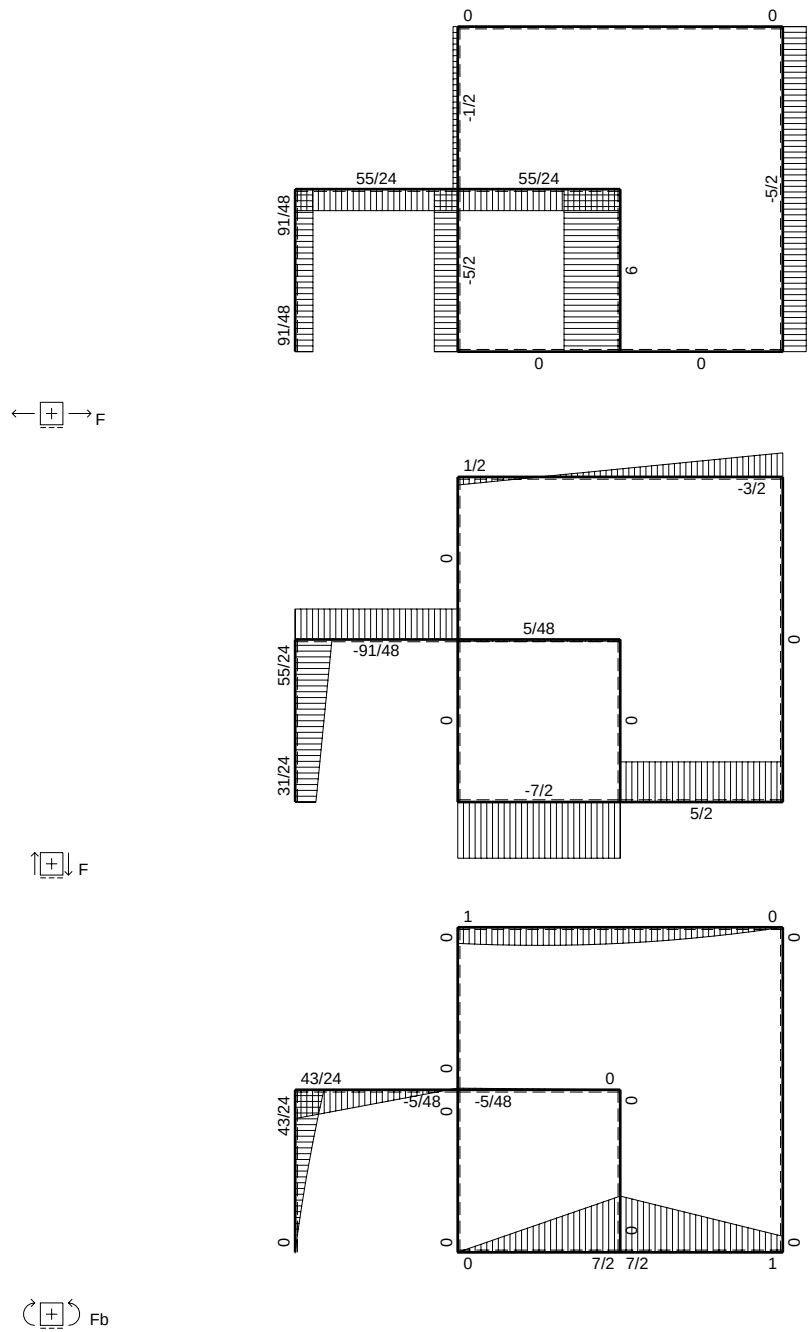


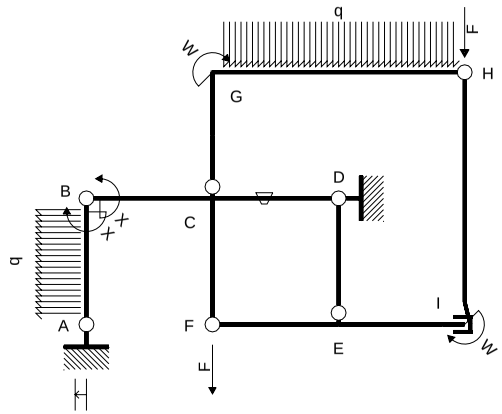

A = 900. $mm^2$
$J_u$ = 233812. $mm^4$
$J_v$ = 48816. $mm^4$
$y_g$ = 32.66 mm
$T_y$ = -2800. N
$M_x$ = 1708000. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -32.66 mm
$\sigma_m = -Mv/J_u$ = 238.6 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -18.66 mm
$\sigma_c = -Mv/J_u$ = 136.3 $N/mm^2$
$\tau_c$ = 4.302 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.5 $N/mm^2$
$S^*$ = 4311. $mm^3$


H
I
5/2F
B
A
91/48F
43/24Fb
55/24F
91/48F
31/24F
I
5/2F
Fb
H
3/2F
7/2Fb
5/2F
E
D
E
6F
D
E
5/48F
7/2F
7/2Fb
55/24F
55/24F
5/48Fb
5/48F
Fb
1/2F
7/2F
G
C
F
G
C
1/2F
C
F
5/2F
C
91/48F
5/48Fb
55/24F
55/24F
43/24Fb
91/48F
B

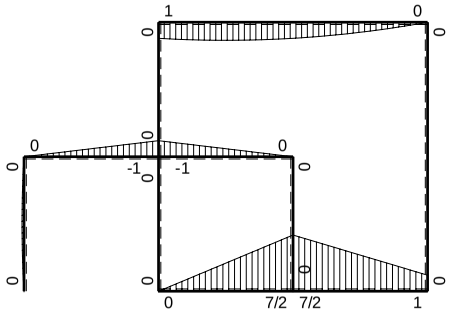




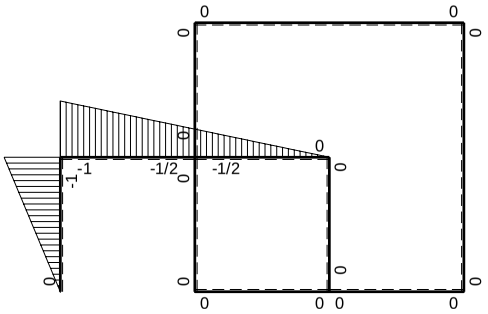

0
0
-1/2
55/24
55/24
-5/2
91/48
-5/2
6
91/48
0
0
F
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-3/2
0
5/48
-91/48
55/24
0
0
0
31/24
-7/2
5/2
F
1
0
0
0
0
43/24
0
43/24
-5/48
-5/48
0
0
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267821 CI01.xxxx.025

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 267821 CI01.xxxx.025

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | Fx | Fb/EJ | $1/2Fx^2/b$ | $1/2Fx/EJ$ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $43/24Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | -43/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

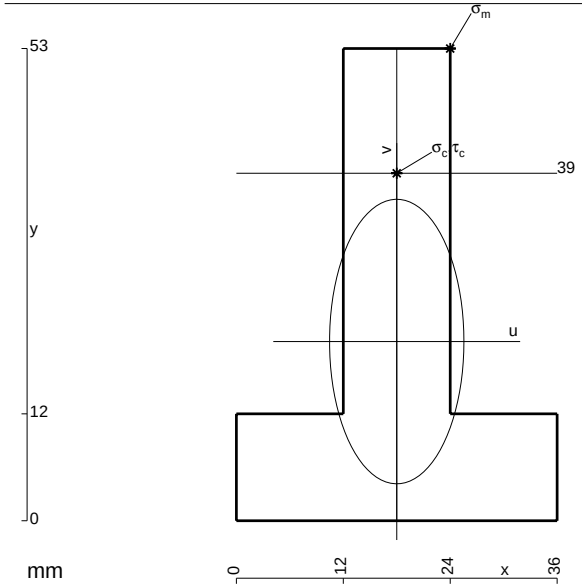
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

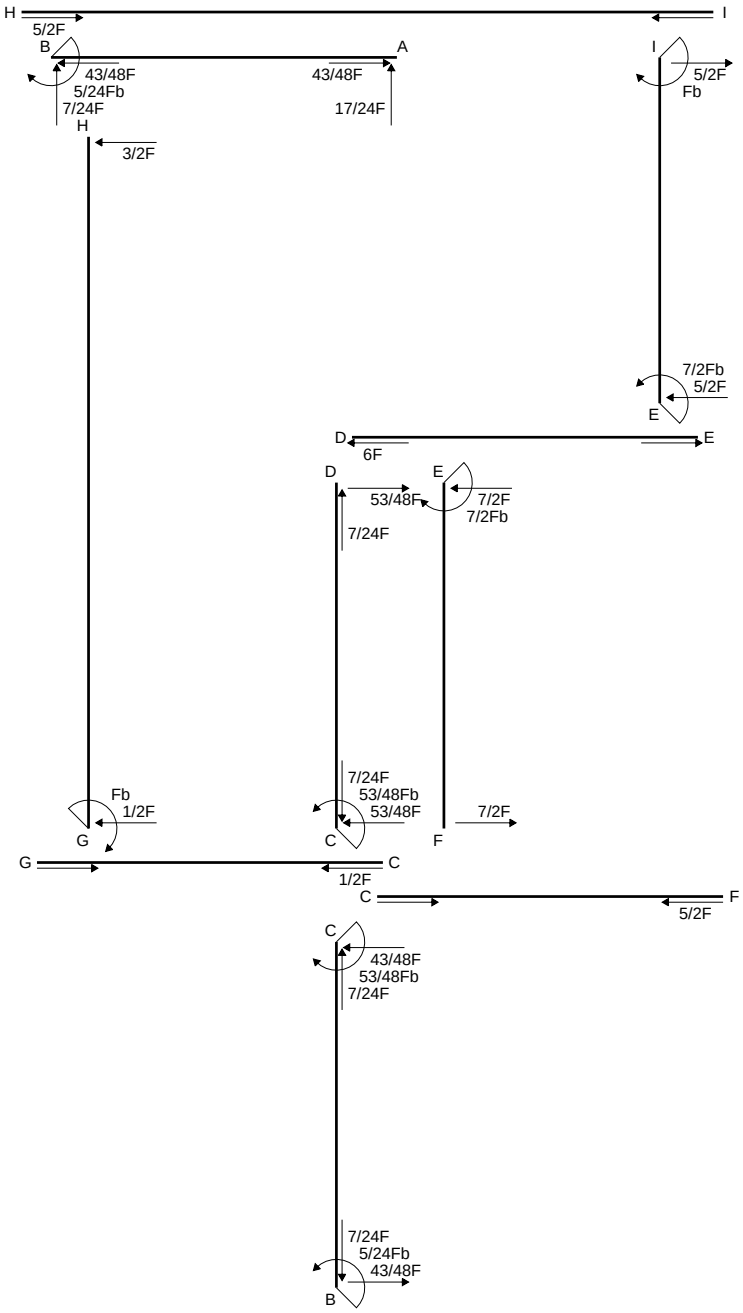


PROCEDIMENTO E RISULTATI 267821 CI01.xxxx.025



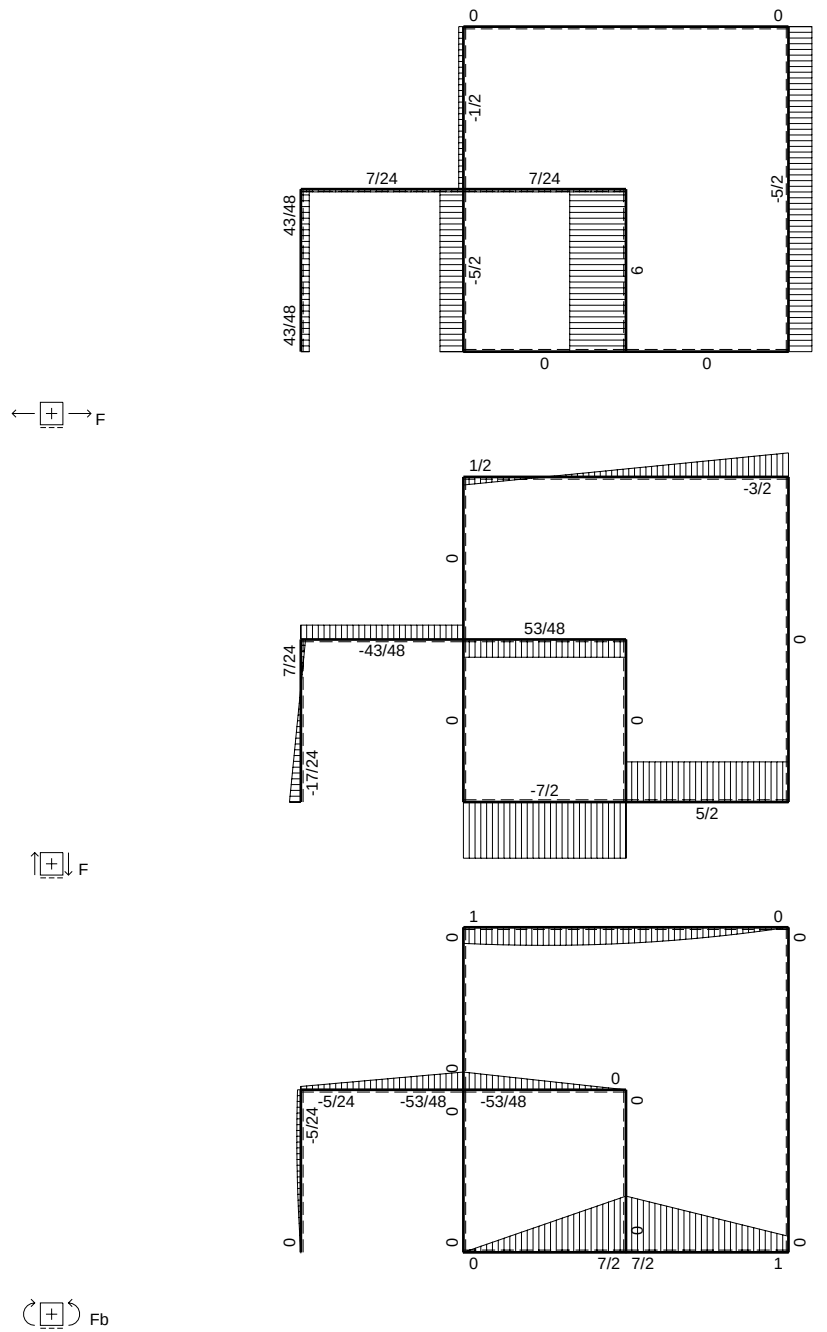

A = 924. $mm^2$
$J_u$ = 235641. $mm^4$
$J_v$ = 52560. $mm^4$
$y_g$ = 20.11 mm
$T_y$ = -2170. N
$M_x$ = 1432200. Nmm
$x_m$ = 24. mm
$y_m$ = 53. mm
$u_m$ = 6. mm
$v_m$ = 32.89 mm
$\sigma_m = -Mv/J_u$ = -199.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 39. mm
$v_c$ = 18.89 mm
$\sigma_c = -Mv/J_u$ = -114.8 $N/mm^2$
$\tau_c$ = 3.338 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115. $N/mm^2$
$S^*$ = 4349. $mm^3$

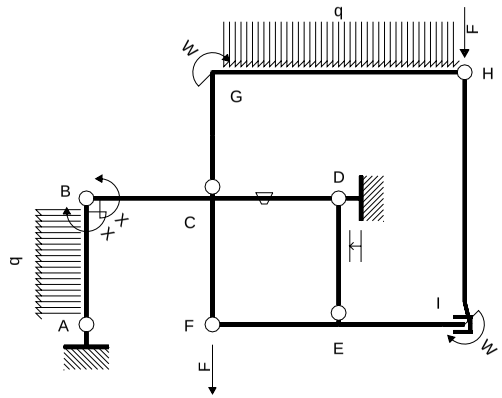
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
5/2F
B
A
43/48F
5/24Fb
7/24F
43/48F
17/24F
I
5/2F
Fb
H
3/2F
7/2Fb
5/2F
E
D
E
6F
D
E
53/48F
7/2F
7/2Fb
7/24F
7/24F
53/48Fb
53/48F
Fb
1/2F
7/2F
G
C
F
G
C
1/2F
C
F
5/2F
C
43/48F
53/48Fb
7/24F
7/24F
5/24Fb
43/48F
B

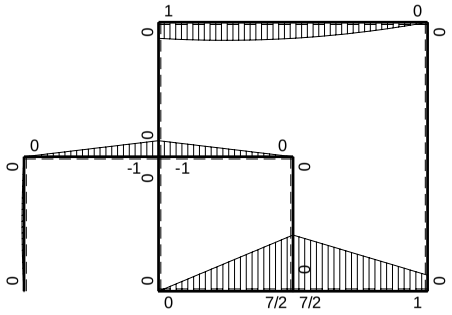




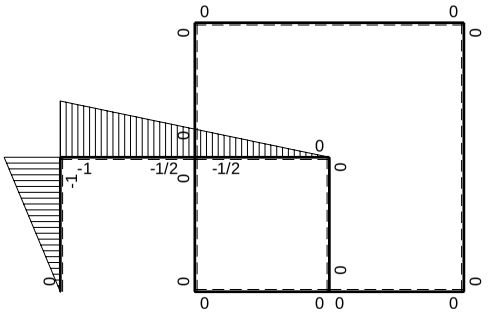

0
0
-1/2
7/24
7/24
-5/2
43/48
-5/2
6
43/48
0
0
F
1/2
-3/2
0
53/48
-43/48
7/24
0
0
0
-17/24
-7/2
5/2
F
1
0
0
0
0
-5/24
-53/48
-53/48
0
-5/24
0
0
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 218343 CI01.xxxx.026

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PROCEDIMENTO E RISULTATI 218343 CI01.xxxx.026

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | Fx | Fb/EJ | $1/2Fx^2/b$ | $1/2Fx/EJ$ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-5/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 5/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

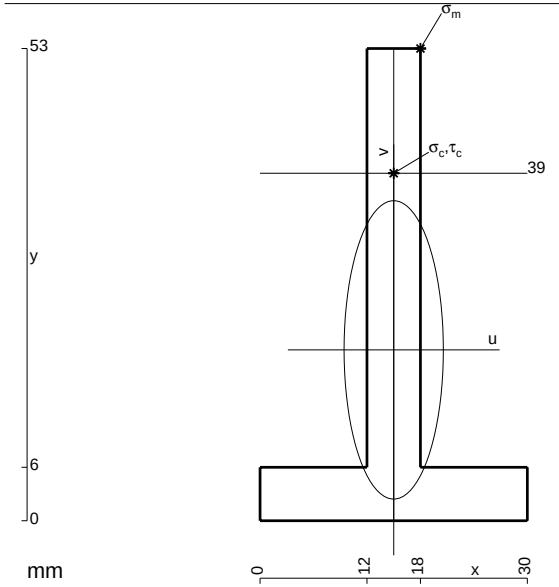
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

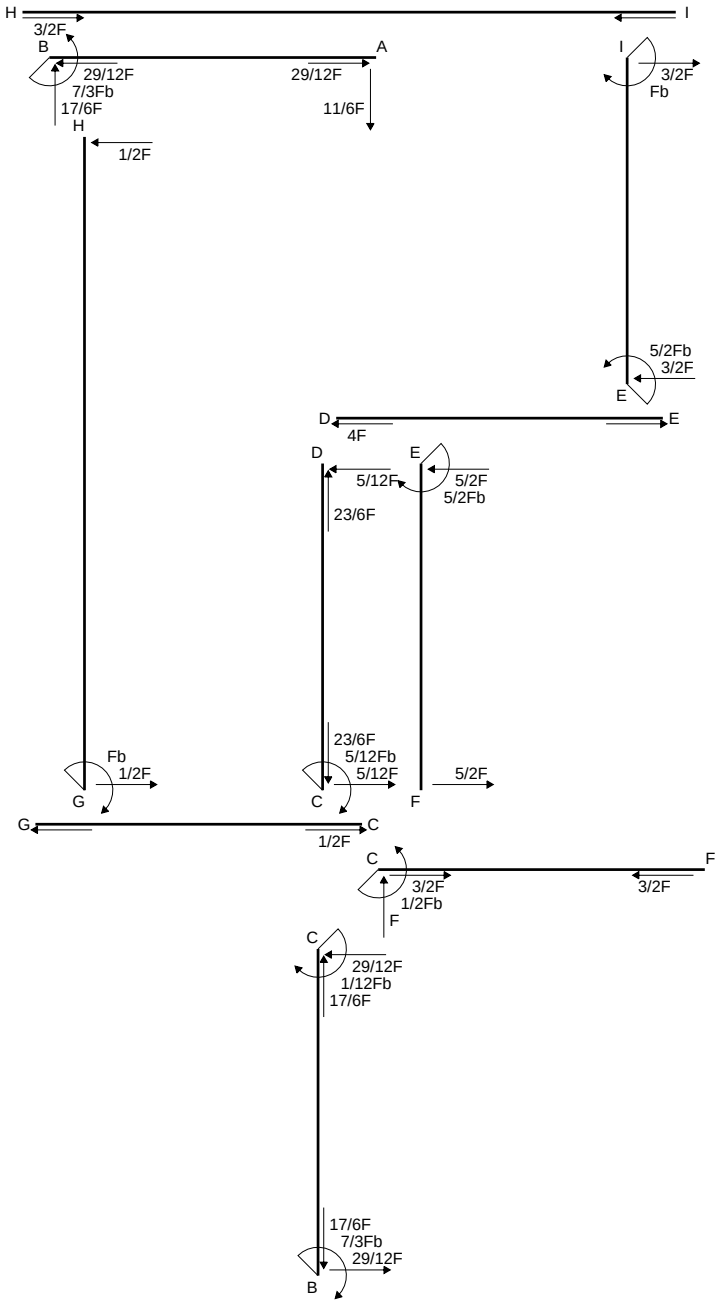
$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 218343 CI01.xxxx.026

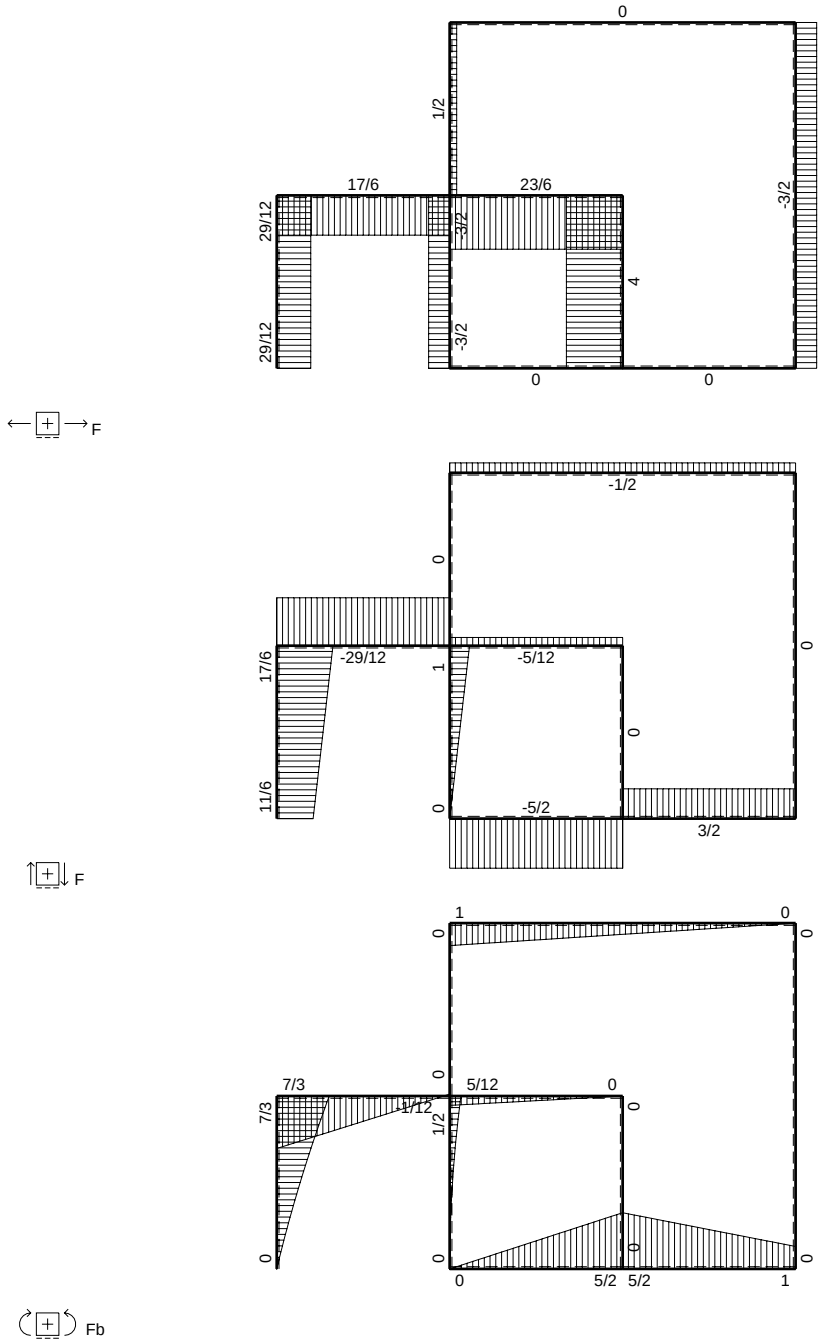


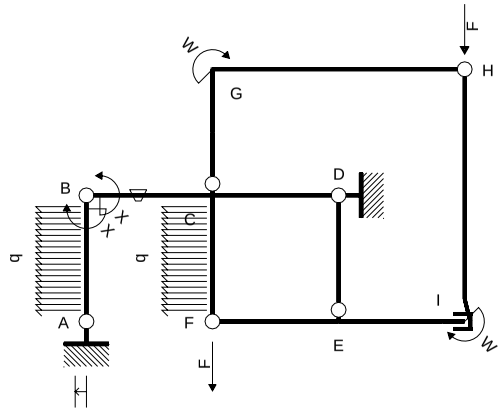

A = 462. $mm^2$
$J_u$ = 129608. $mm^4$
$J_v$ = 14346. $mm^4$
$y_g$ = 19.18 mm
$T_y$ = -1120. N
$M_x$ = 784000. Nmm
$x_m$ = 18. mm
$y_m$ = 53. mm
$u_m$ = 3. mm
$v_m$ = 33.82 mm
$\sigma_m = -Mv/J_u$ = -204.6 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 39. mm
$v_c$ = 19.82 mm
$\sigma_c = -Mv/J_u$ = -119.9 $N/mm^2$
$\tau_c$ = 3.245 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.1 $N/mm^2$
$S^*$ = 2253. $mm^3$


H
I
3/2F
B
A
29/12F
7/3Fb
17/6F
29/12F
11/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
D
E
5/12F
5/2F
5/2Fb
23/6F
23/6F
5/12Fb
5/12F
5/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
3/2F
3/2F
1/2Fb
F
C
29/12F
1/12Fb
17/6F
17/6F
7/3Fb
29/12F
B

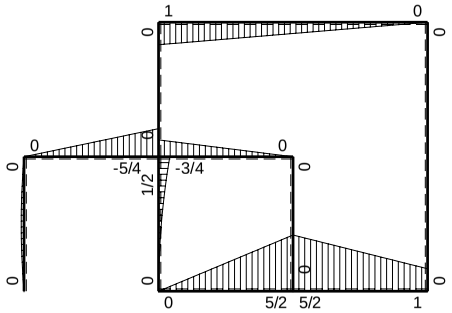




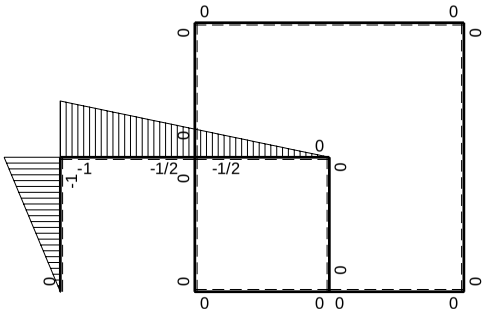

0
1/2
17/6
23/6
-3/2
29/12
-3/2
4
29/12
-3/2
0
0
F
-1/2
0
17/6
-29/12
1
-5/12
0
11/6
0
-5/2
3/2
F
1
0
0
0
0
7/3
5/12
0
7/3
1/12
1/2
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 271093 CI01.xxxx.027

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $7/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/3Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 271093 CI01.xxxx.027

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

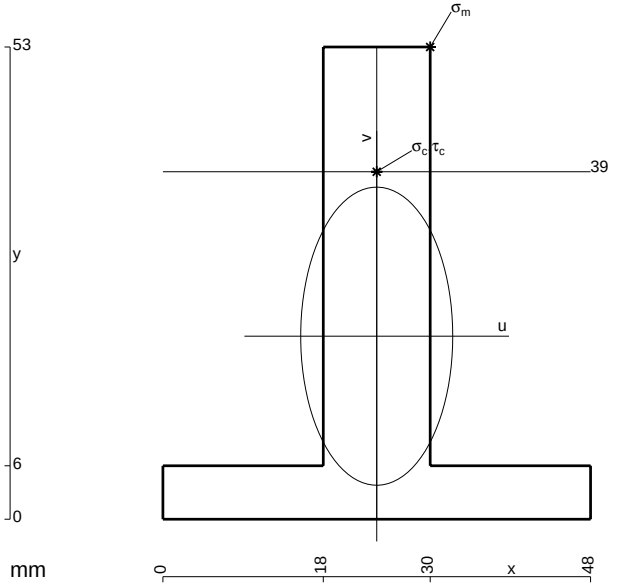
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

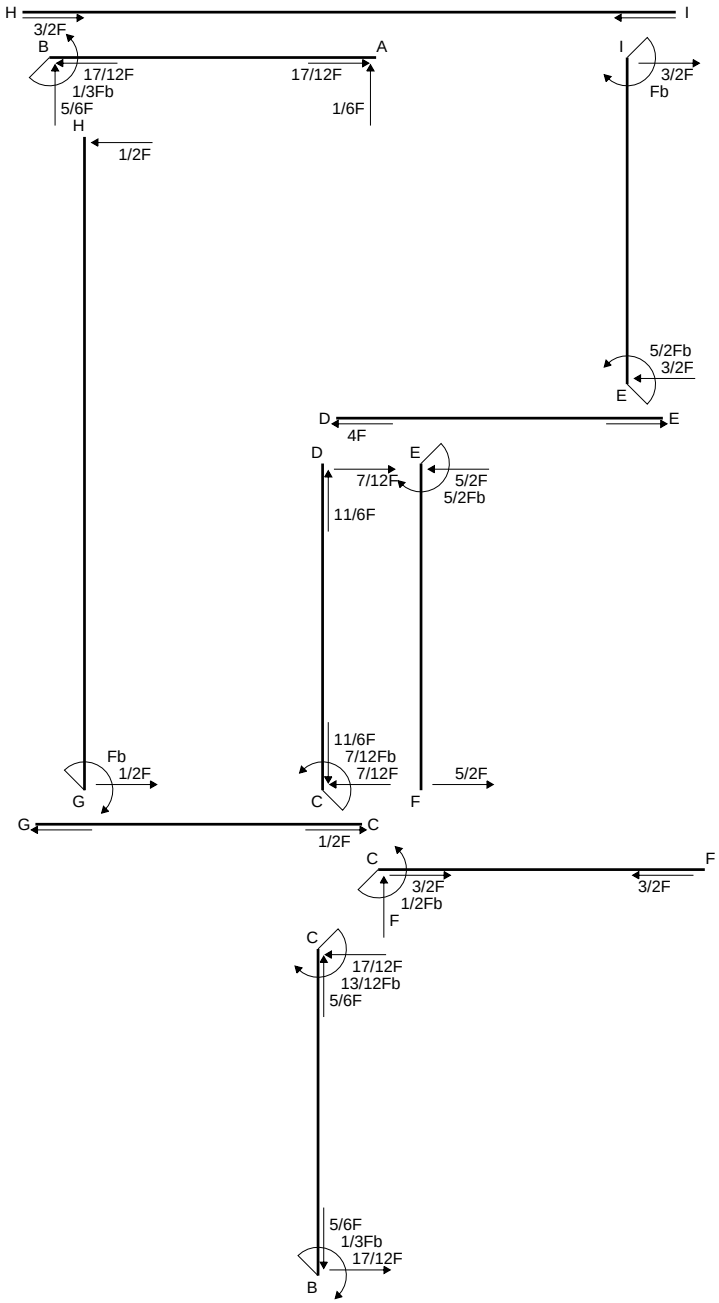


PROCEDIMENTO E RISULTATI 271093 CI01.xxxx.027



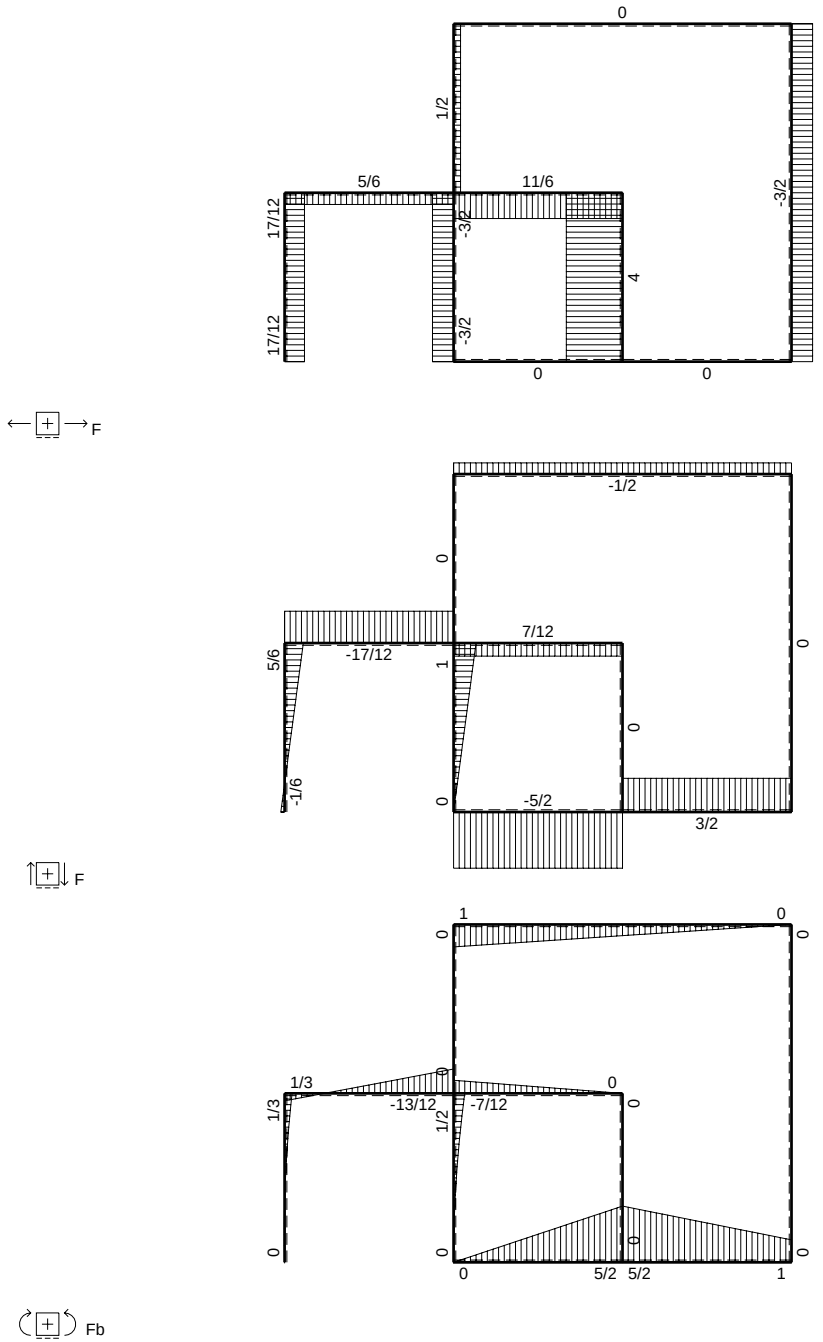

A = 852. $mm^2$
$J_u$ = 238569. $mm^4$
$J_v$ = 62064. $mm^4$
$y_g$ = 20.54 mm
$T_y$ = -2175. N
$M_x$ = 1609500. Nmm
$x_m$ = 30. mm
$y_m$ = 53. mm
$u_m$ = 6. mm
$v_m$ = 32.46 mm
$\sigma_m = -Mv/J_u$ = -219. $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 39. mm
$v_c$ = 18.46 mm
$\sigma_c = -Mv/J_u$ = -124.5 $N/mm^2$
$\tau_c$ = 3.249 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.7 $N/mm^2$
$S^*$ = 4277. $mm^3$

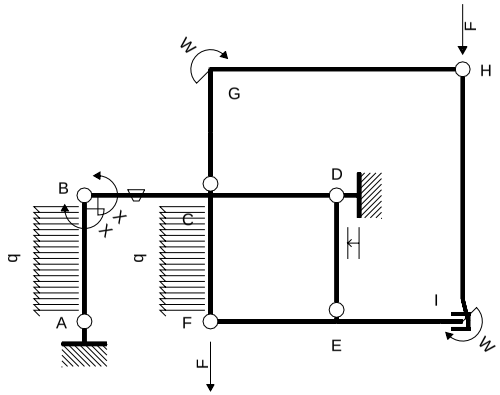
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H
I
3/2F
B
A
17/12F
1/3Fb
5/6F
17/12F
1/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
D
E
7/12F
5/2F
5/2Fb
11/6F
11/6F
7/12Fb
7/12F
Fb
1/2F
G
C
F
5/2F
G
C
1/2F
C
F
3/2F
3/2F
1/2Fb
F
C
17/12F
13/12Fb
5/6F
5/6F
1/3Fb
17/12F
B

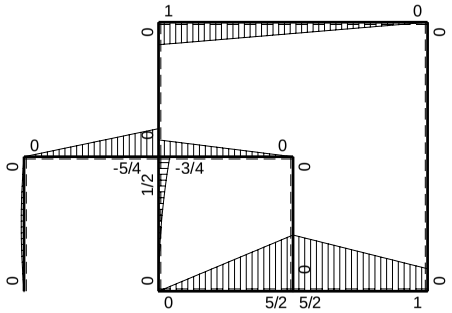




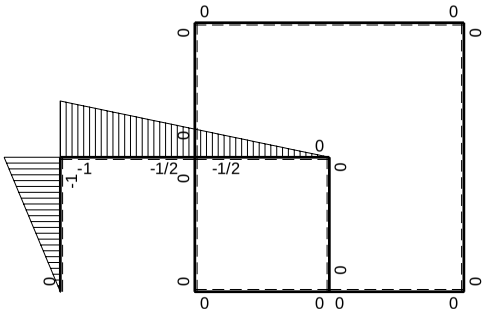

0
1/2
5/6
11/6
-3/2
17/12
-3/2
4
17/12
-3/2
0
0
F
-1/2
0
7/12
5/6
-17/12
1
0
0
-1/6
0
-5/2
3/2
F
1
0
0
0
1/3
0
0
-13/12
-7/12
1/3
1/2
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264395 CI01.xxxx.028

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

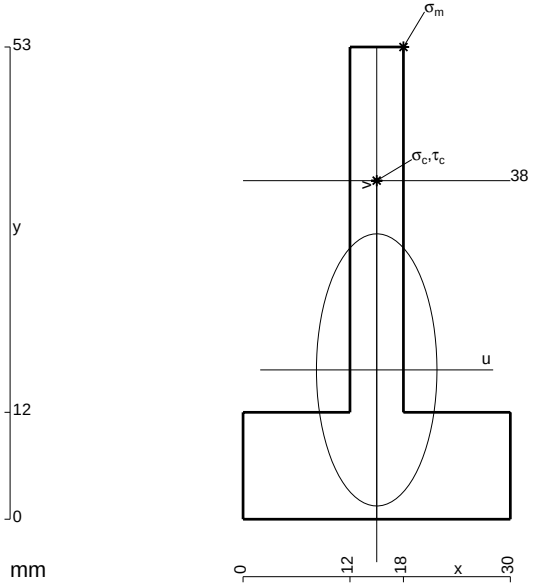
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

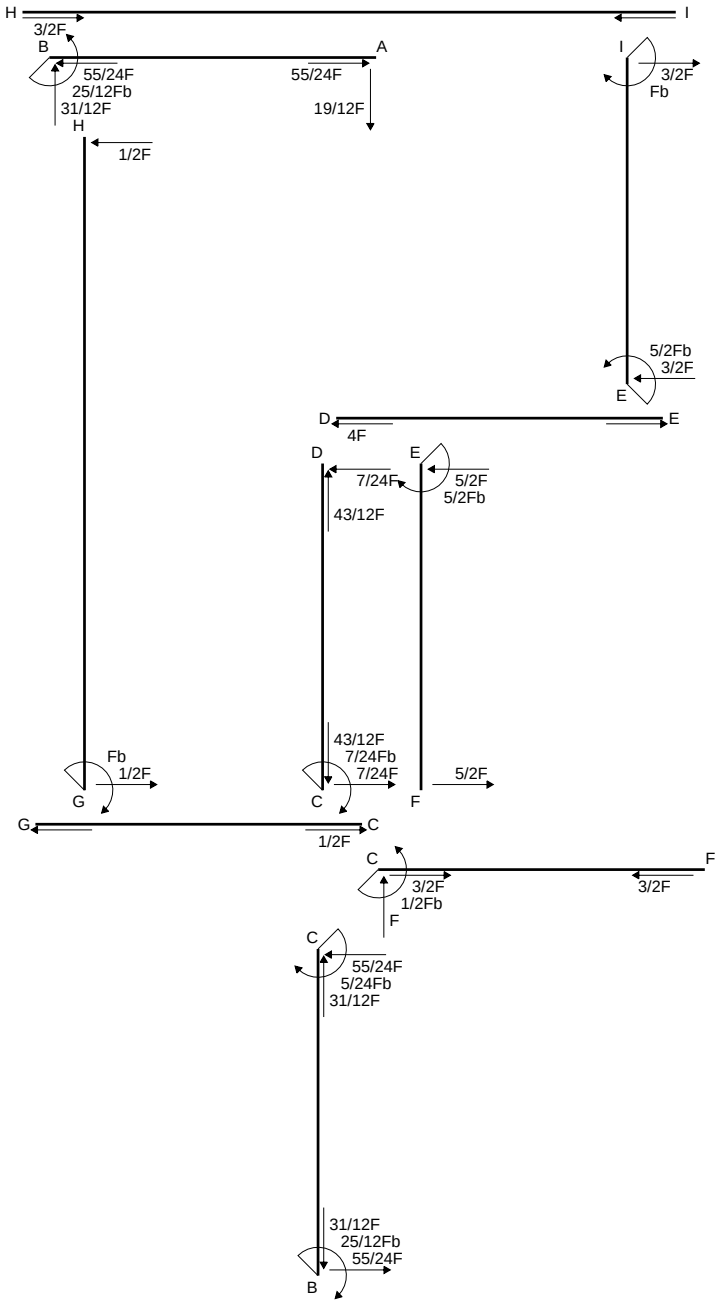
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 264395 CI01.xxxx.028

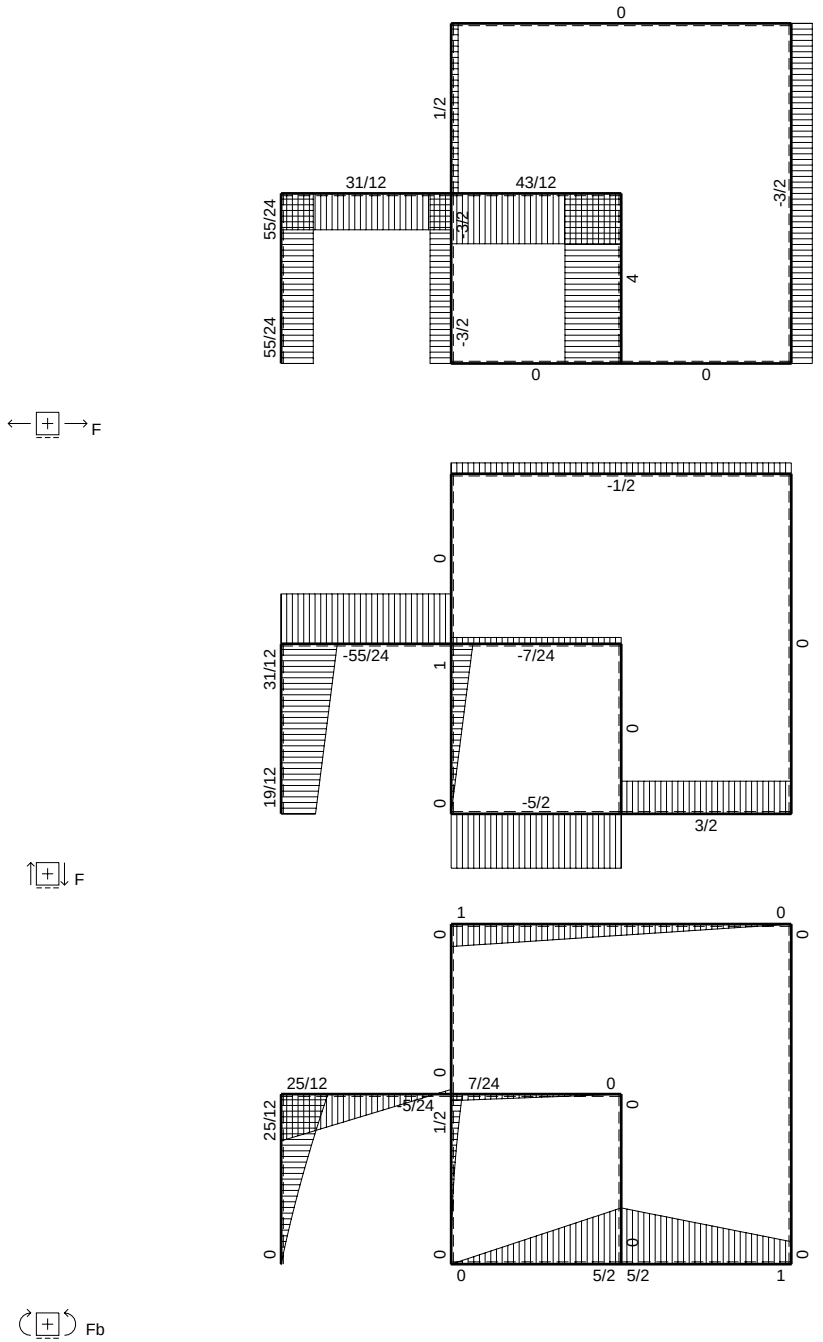


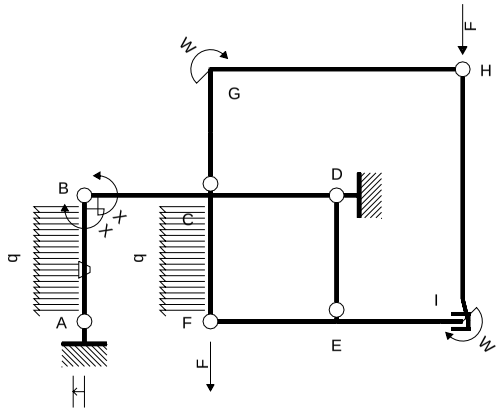

A = 606. $mm^2$
$J_u$ = 141406. $mm^4$
$J_v$ = 27738. $mm^4$
$y_g$ = 16.76 mm
$T_y$ = -1125. N
$M_x$ = 888750. Nmm
$x_m$ = 18. mm
$y_m$ = 53. mm
$u_m$ = 3. mm
$v_m$ = 36.24 mm
$\sigma_m = -Mv/J_u$ = -227.8 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 38. mm
$v_c$ = 21.24 mm
$\sigma_c = -Mv/J_u$ = -133.5 $N/mm^2$
$\tau_c$ = 3.43 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.6 $N/mm^2$
$S^*$ = 2587. $mm^3$


H
I
3/2F
B
A
55/24F
25/12Fb
31/12F
55/24F
19/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
D
E
7/24F
5/2F
5/2Fb
43/12F
43/12F
7/24Fb
7/24F
Fb
1/2F
G
C
F
5/2F
G
C
1/2F
C
F
3/2F
3/2F
1/2Fb
F
C
55/24F
5/24Fb
31/12F
31/12F
25/12Fb
55/24F
B

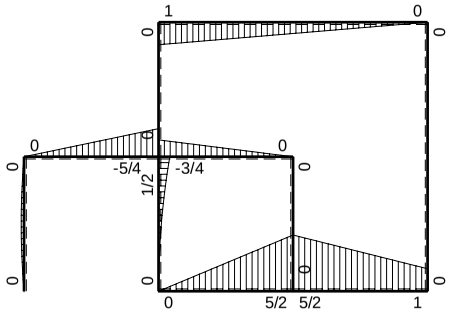




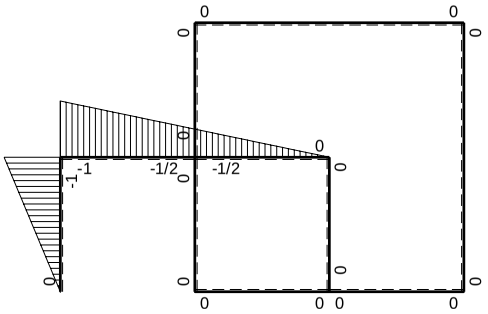

F
0
1/2
31/12
43/12
-3/2
55/24
-3/2
4
55/24
-3/2
0
0
F
-1/2
0
31/12
-55/24
1
-7/24
0
0
19/12
0
-5/2
3/2
F
1
0
0
0
0
25/12
7/24
0
25/12
-5/24
1/2
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268891 CI01.xxxx.029

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PROCEDIMENTO E RISULTATI 268891 CI01.xxxx.029

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $25/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -25/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

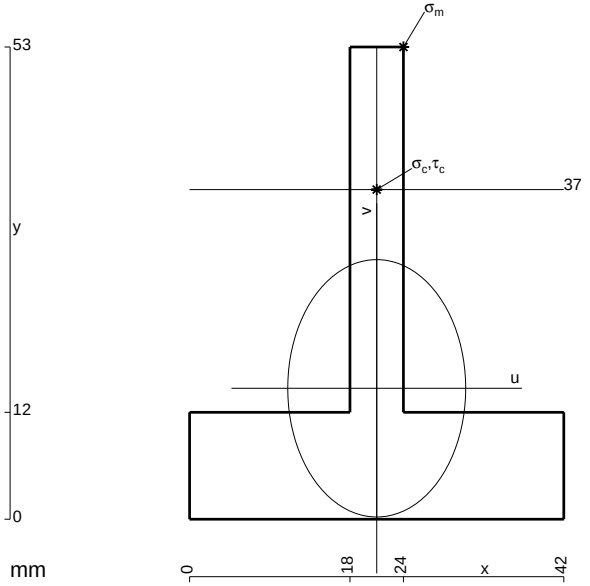
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

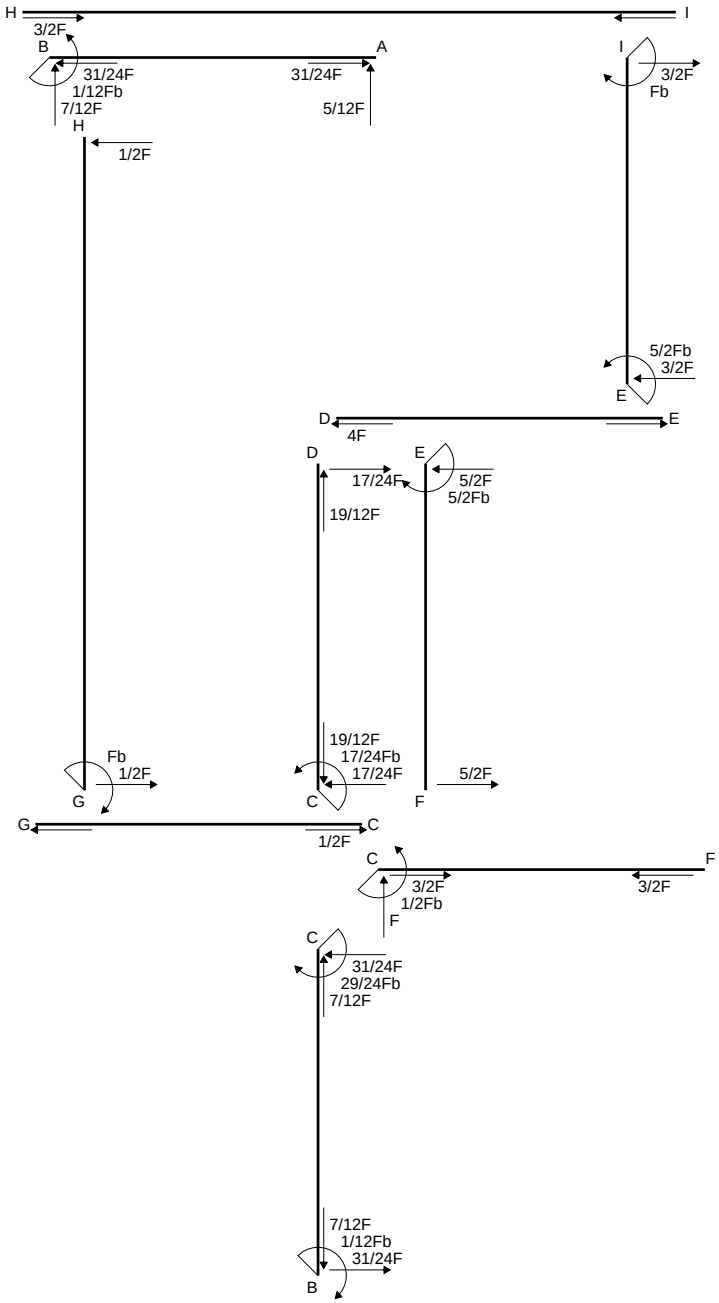
$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





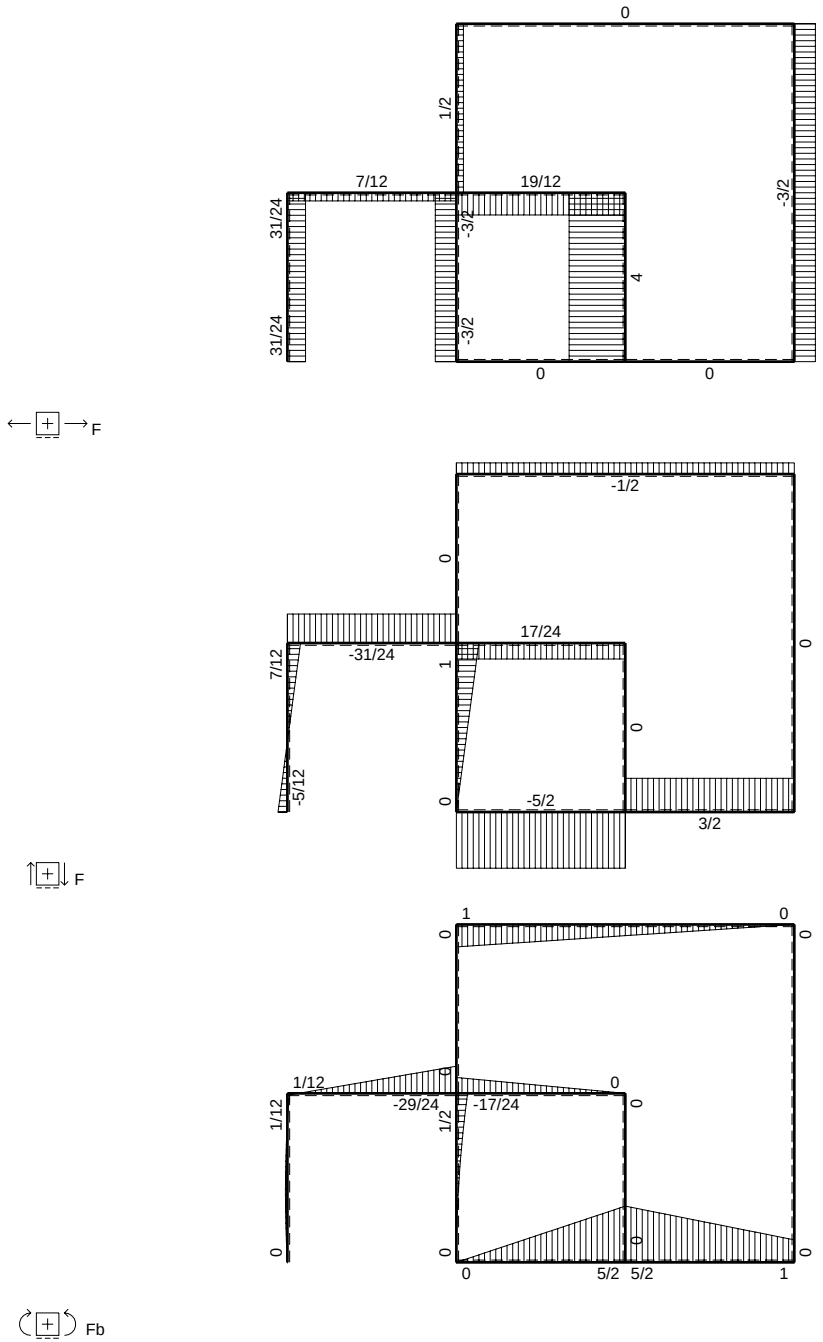


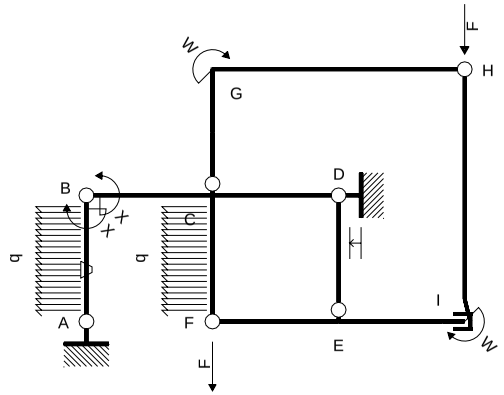

A = 750. $mm^2$
$J_u$ = 156599. $mm^4$
$J_v$ = 74826. $mm^4$
$y_g$ = 14.69 mm
$T_y$ = -1175. N
$M_x$ = 975250. Nmm
$x_m$ = 24. mm
$y_m$ = 53. mm
$u_m$ = 3. mm
$v_m$ = 38.31 mm
$\sigma_m = -Mv/J_u$ = -238.6 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 37. mm
$v_c$ = 22.31 mm
$\sigma_c = -Mv/J_u$ = -138.9 $N/mm^2$
$\tau_c$ = 3.639 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.1 $N/mm^2$
$S^*$ = 2910. $mm^3$


H
I
3/2F
B
A
31/24F
1/12Fb
7/12F
31/24F
5/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
D
E
17/24F
5/2F
5/2Fb
19/12F
19/12F
17/24Fb
17/24F
Fb
1/2F
5/2F
G
C
F
G
C
1/2F
C
F
3/2F
3/2F
1/2Fb
F
C
31/24F
29/24Fb
7/12F
7/12F
1/12Fb
31/24F
B

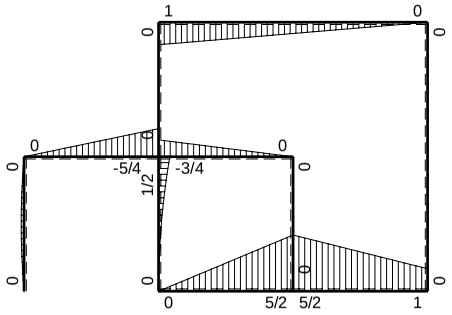




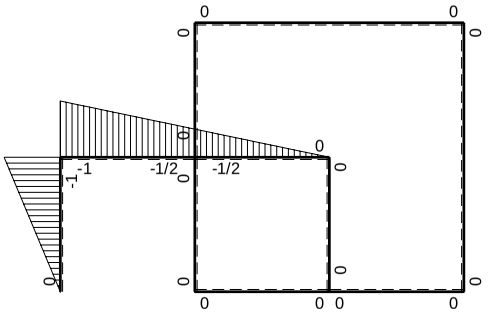

F
0
1/2
7/12
19/12
-3/2
31/24
-3/2
4
31/24
-3/2
0
0
F
-1/2
0
17/24
-31/24
7/12
1
0
0
-5/12
-5/2
3/2
F
1
0
0
0
1/12
0
1/12
-29/24
-17/24
1/2
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267675 CI01.xxxx.030

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b)\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

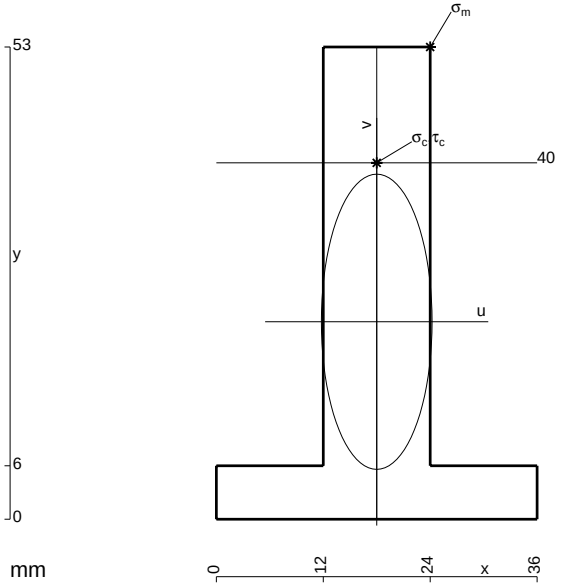
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

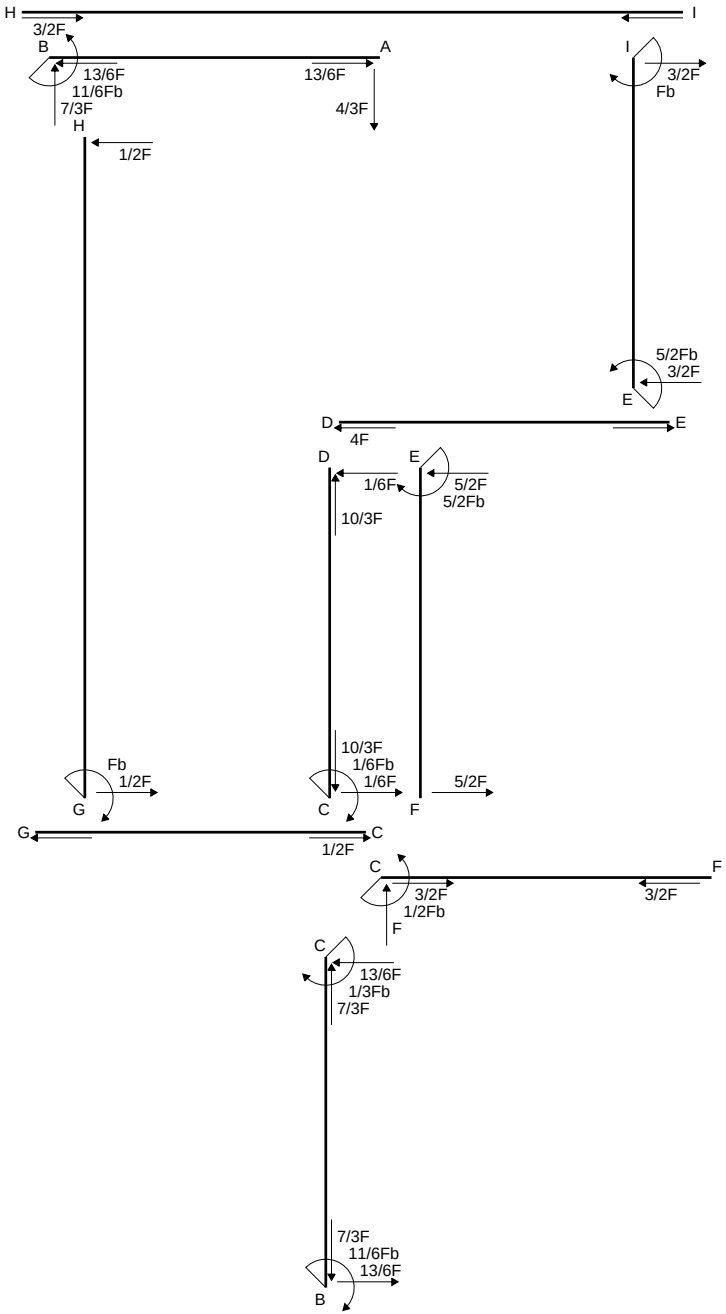
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





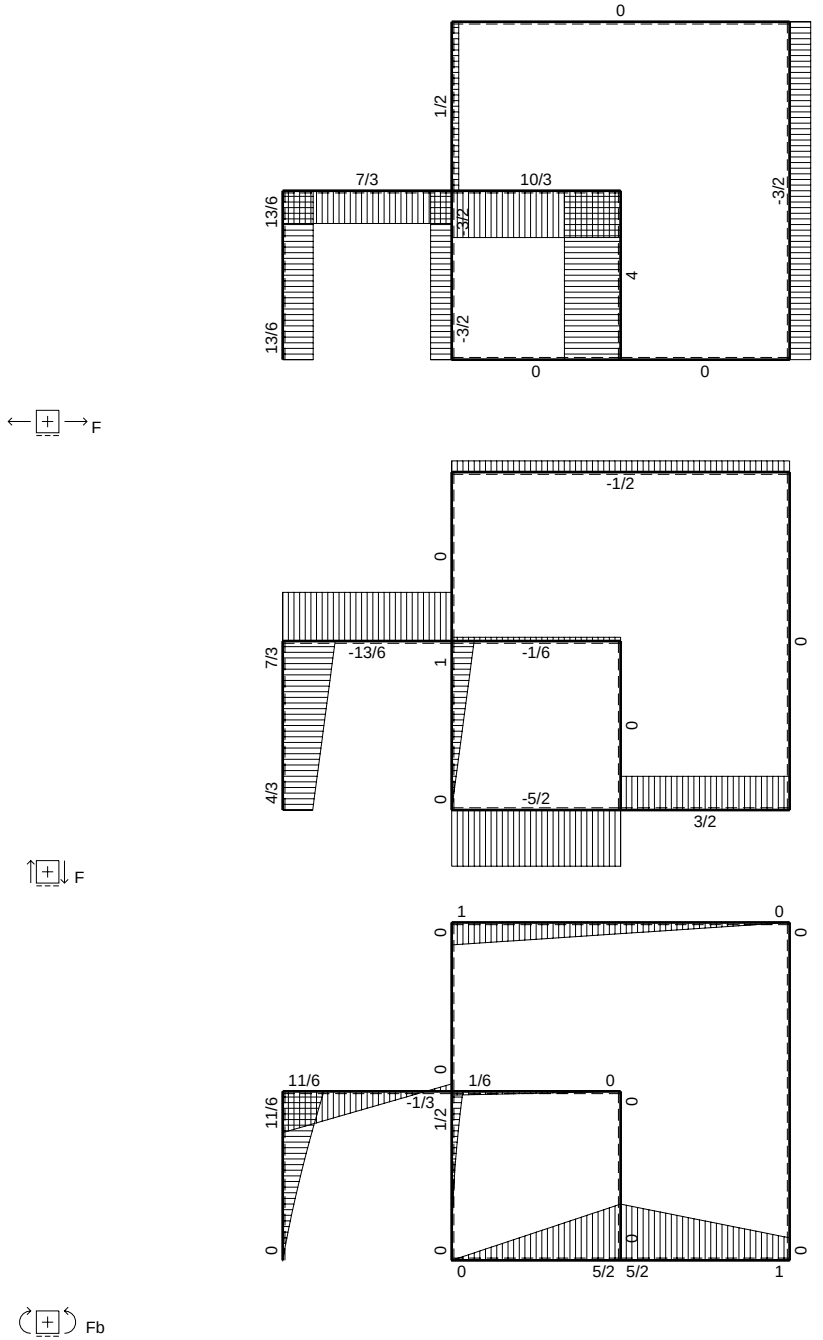


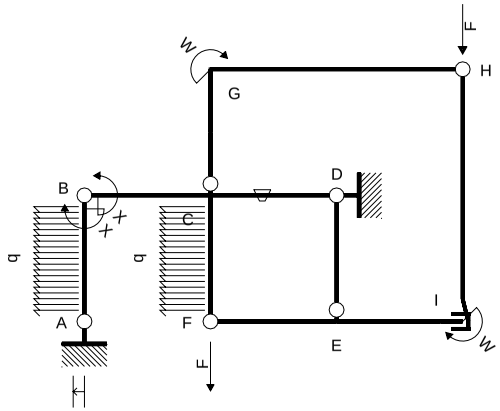

A = 780. $mm^2$
$J_u$ = 214152. $mm^4$
$J_v$ = 30096. $mm^4$
$y_g$ = 22.16 mm
$T_y$ = -3150. N
$M_x$ = 1386000. Nmm
$x_m$ = 24. mm
$y_m$ = 53. mm
$u_m$ = 6. mm
$v_m$ = 30.84 mm
$\sigma_m = -Mv/J_u$ = -199.6 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 40. mm
$v_c$ = 17.84 mm
$\sigma_c = -Mv/J_u$ = -115.5 $N/mm^2$
$\tau_c$ = 4.654 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.7 $N/mm^2$
$S^*$ = 3797. $mm^3$


H
I
3/2F
B
A
13/6F
11/6Fb
7/3F
13/6F
4/3F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
D
E
1/6F
5/2F
5/2Fb
10/3F
10/3F
1/6Fb
Fb
1/2F
1/6F
5/2F
G
C
F
G
C
1/2F
C
F
3/2F
3/2F
1/2Fb
F
C
13/6F
1/3Fb
7/3F
7/3F
11/6Fb
13/6F
B

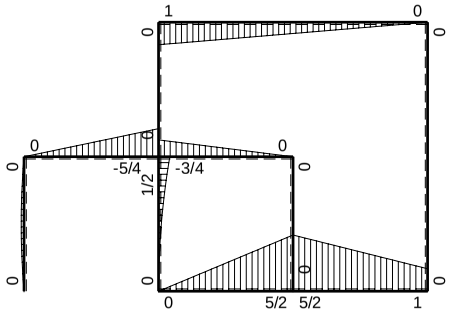




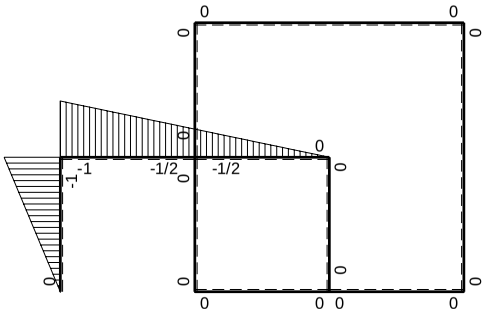

0
1/2
7/3
10/3
-3/2
13/6
-3/2
4
13/6
-3/2
0
0
0
F
-1/2
0
7/3
-13/6
1
-1/6
0
0
4/3
0
-5/2
3/2
F
1
0
0
0
0
11/6
1/6
0
11/6
-1/3
1/2
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248738 CI01.xxxx.031

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PROCEDIMENTO E RISULTATI 248738 CI01.xxxx.031

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $11/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -11/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

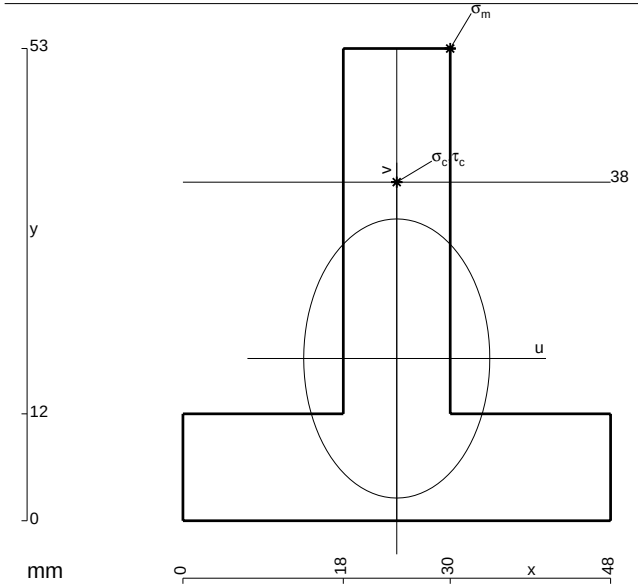
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

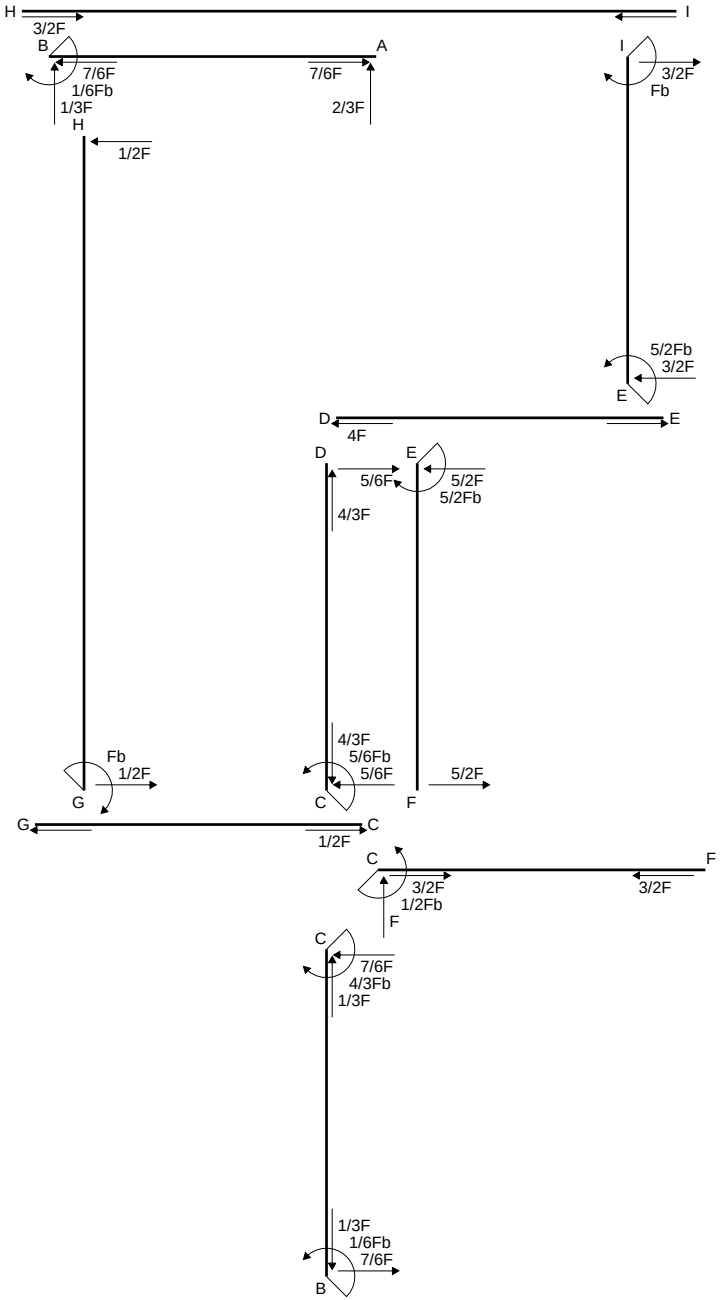


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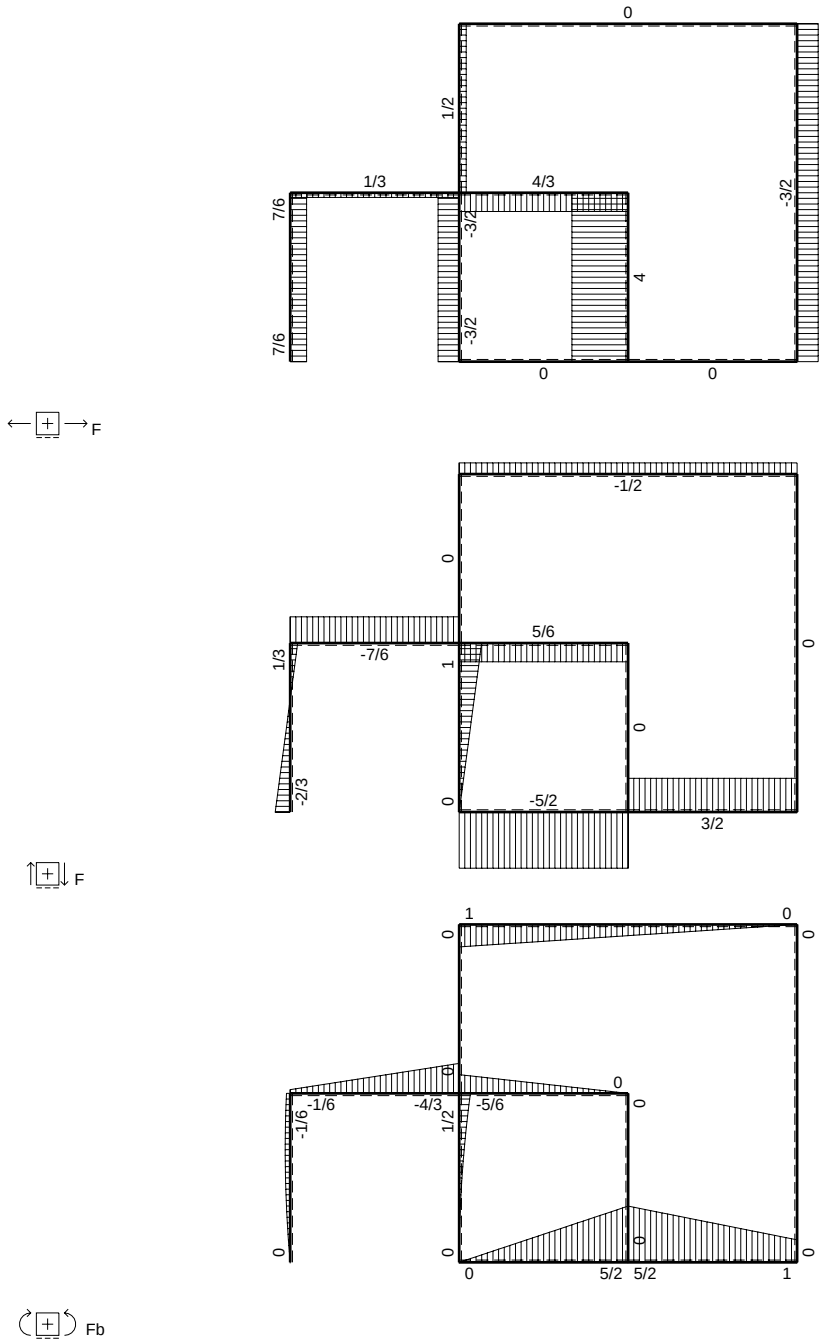

A = 1068. $mm^2$
$J_u$ = 262174. $mm^4$
$J_v$ = 116496. $mm^4$
$y_g$ = 18.21 mm
$T_y$ = -3275. N
$M_x$ = 1572000. Nmm
$x_m$ = 30. mm
$y_m$ = 53. mm
$u_m$ = 6. mm
$v_m$ = 34.79 mm
$\sigma_m = -Mv/J_u$ = -208.6 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 38. mm
$v_c$ = 19.79 mm
$\sigma_c = -Mv/J_u$ = -118.7 $N/mm^2$
$\tau_c$ = 5.114 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119. $N/mm^2$
$S^*$ = 4913. $mm^3$

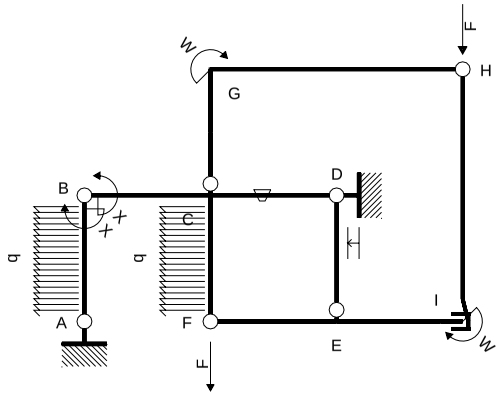
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H
I
3/2F
B
A
7/6F
1/6Fb
1/3F
7/6F
2/3F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
D
E
5/6F
5/2F
5/2Fb
4/3F
4/3F
5/6Fb
5/6F
5/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
3/2F
3/2F
1/2Fb
F
C
7/6F
4/3Fb
1/3F
1/3F
1/6Fb
7/6F
B

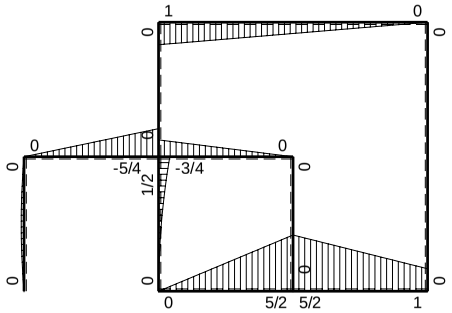




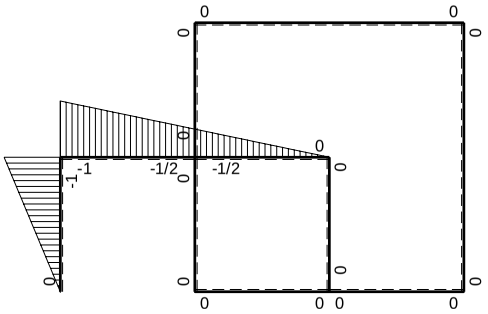

0
1/2
1/3
4/3
-3/2
7/6
-3/2
4
7/6
-3/2
0
0
0
F
-1/2
0
5/6
1/3
-7/6
1
0
0
-2/3
0
-5/2
3/2
F
1
0
0
0
0
0
-1/6
-4/3
-5/6
1/2
-1/6
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 269783 CI01.xxxx.032

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PROCEDIMENTO E RISULTATI 269783 CI01.xxxx.032

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

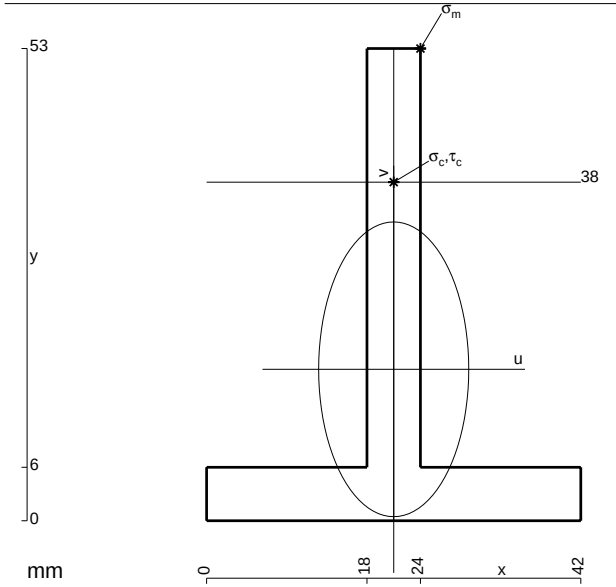
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

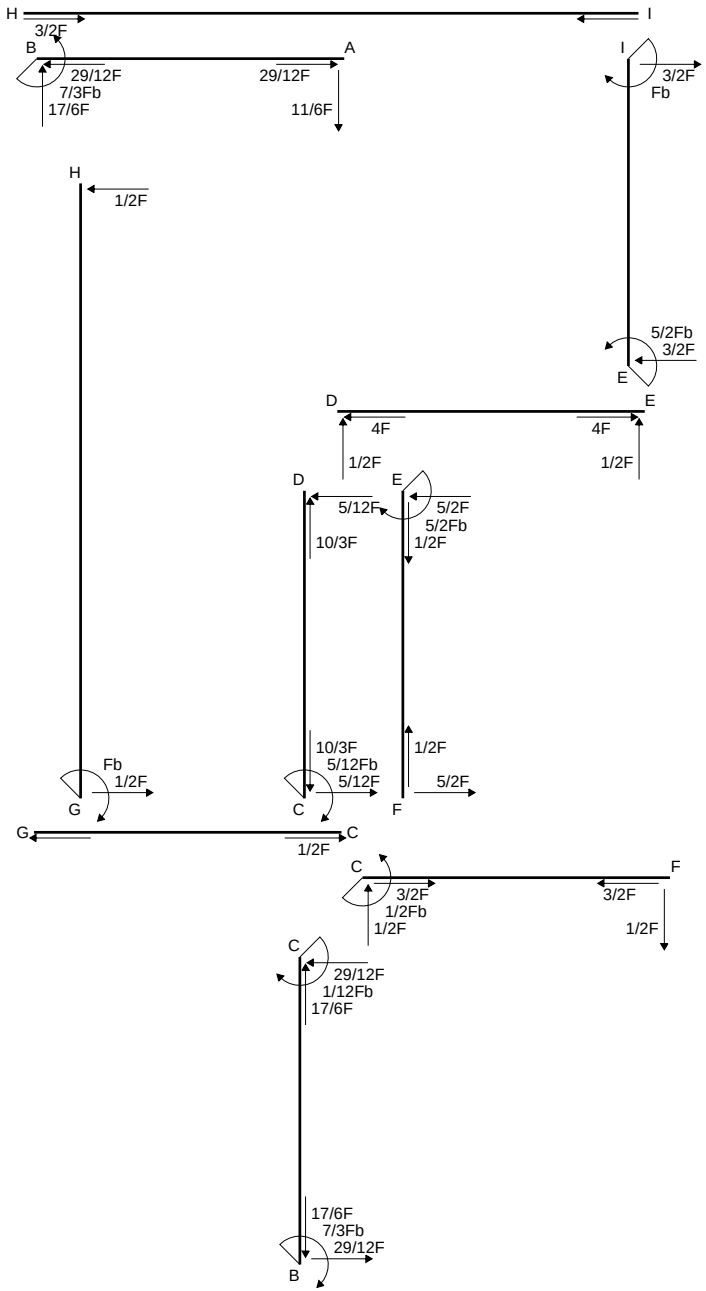


PROCEDIMENTO E RISULTATI 269783 CI01.xxxx.032



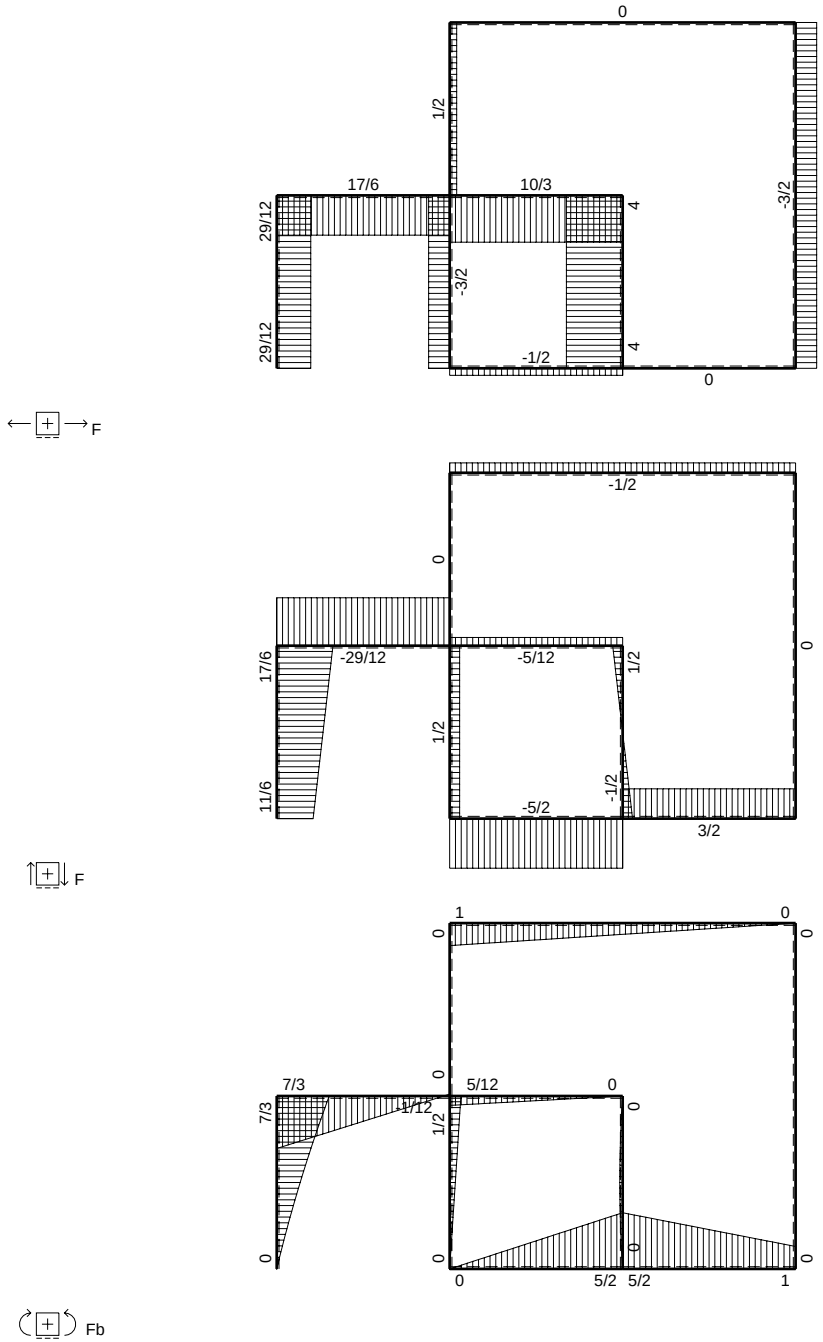

A = 534. $mm^2$
$J_u$ = 146122. $mm^4$
$J_v$ = 37890. $mm^4$
$y_g$ = 16.99 mm
$T_y$ = -1700. N
$M_x$ = 884000. Nmm
$x_m$ = 24. mm
$y_m$ = 53. mm
$u_m$ = 3. mm
$v_m$ = 36.01 mm
$\sigma_m = -Mv/J_u$ = -217.8 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 38. mm
$v_c$ = 21.01 mm
$\sigma_c = -Mv/J_u$ = -127.1 $N/mm^2$
$\tau_c$ = 4.975 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.4 $N/mm^2$
$S^*$ = 2566. $mm^3$

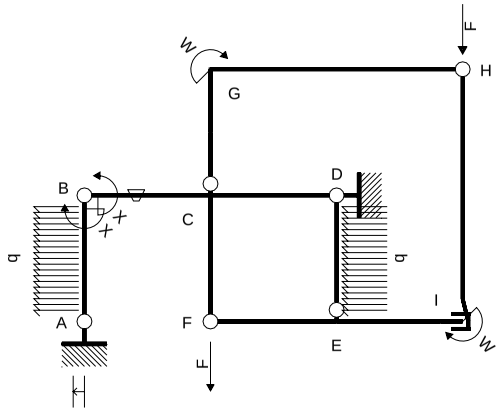
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
3/2F
I
B
A
29/12F
7/3Fb
17/6F
29/12F
11/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
4F
1/2F
1/2F
D
E
5/12F
5/2F
5/2Fb
10/3F
1/2F
10/3F
5/12Fb
5/12F
1/2F
5/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
3/2F
1/2Fb
1/2F
3/2F
1/2F
C
29/12F
1/12Fb
17/6F
17/6F
7/3Fb
29/12F
B

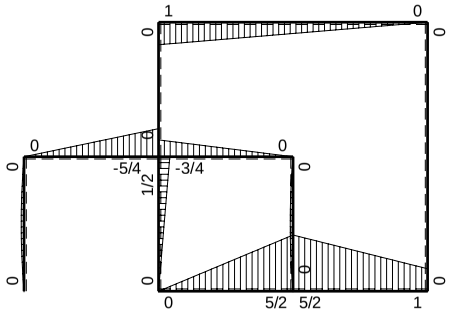




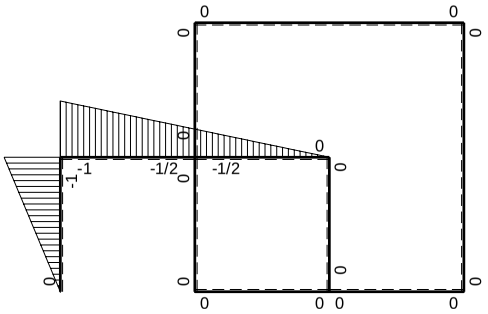

0
1/2
17/6
10/3
-3/2
29/12
4
-3/2
29/12
-1/2
4
0
F
-1/2
0
0
17/6
-29/12
-5/12
1/2
1/2
11/6
-1/2
-5/2
3/2
F
1
0
0
0
0
7/3
5/12
0
7/3
-1/12
1/2
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248373 CI01.xxxx.033

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $7/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

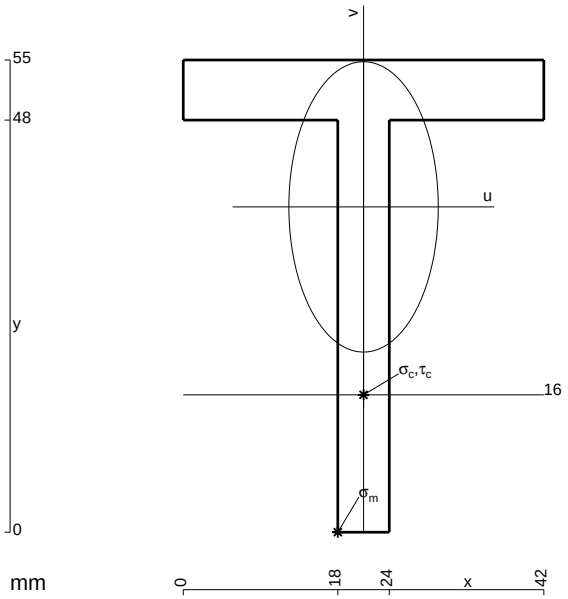
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

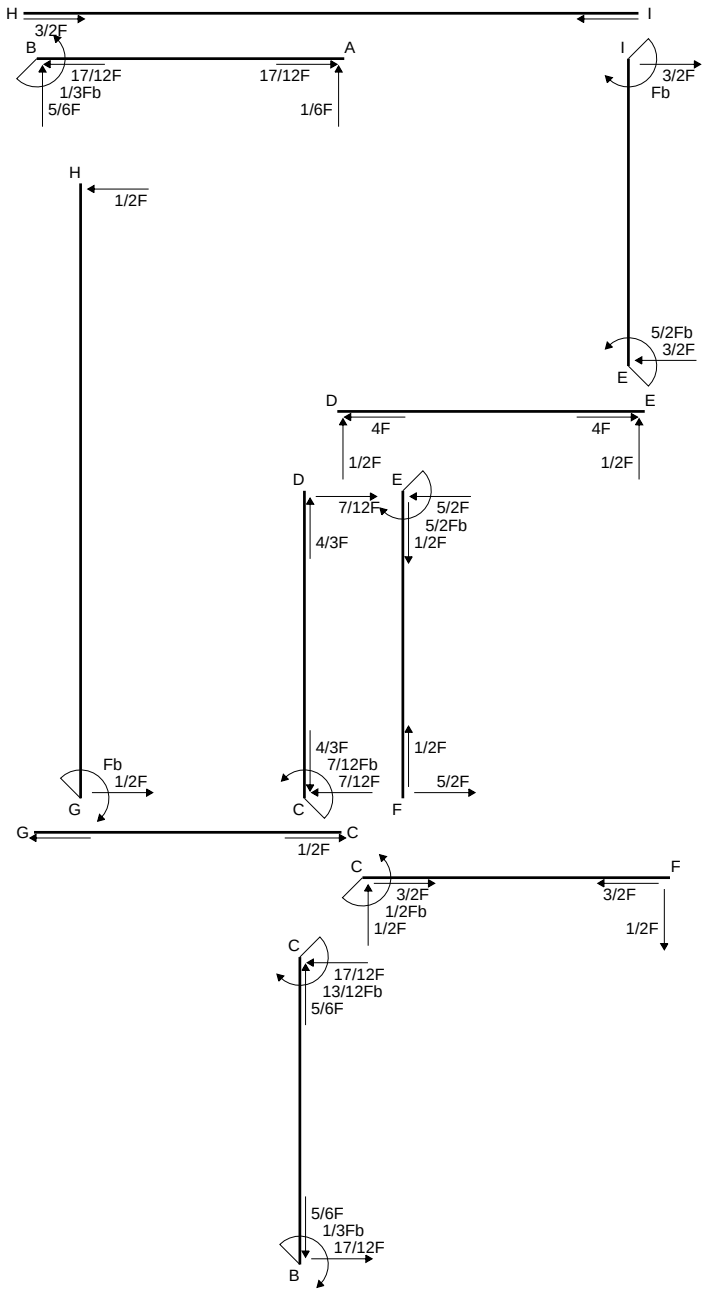
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





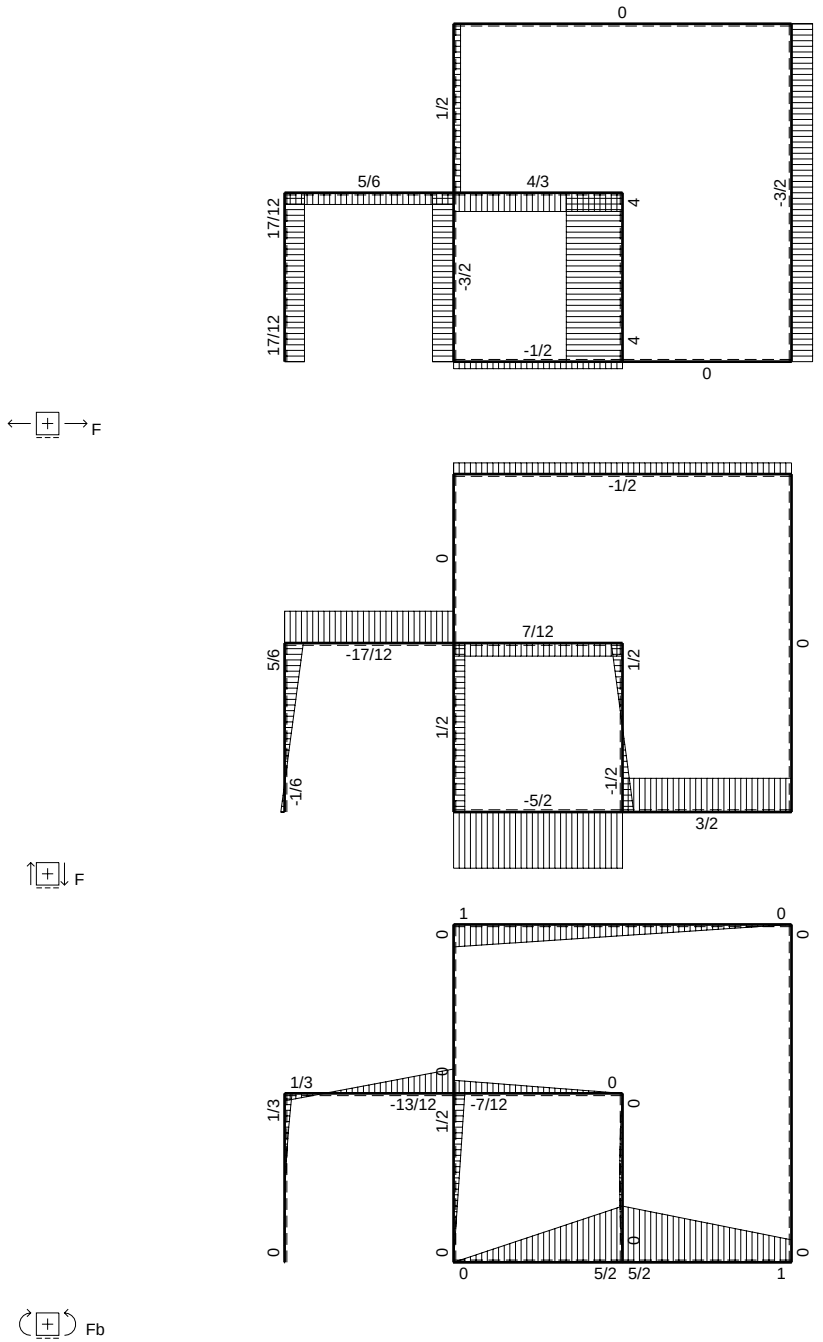


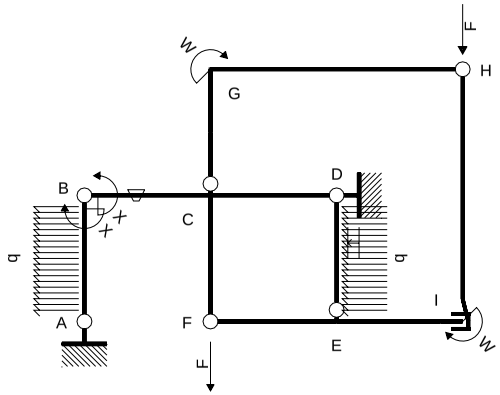

A = 582. $mm^2$
$J_u$ = 166519. $mm^4$
$J_v$ = 44082. $mm^4$
$y_g$ = 37.89 mm
N = -345. N
$T_y$ = -1725. N
$M_x$ = 1000500. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -37.89 mm
$\sigma_m = N/A - Mv/J_u$ = 227.1 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -21.89 mm
$\sigma_c = N/A - Mv/J_u$ = 130.9 $N/mm^2$
$\tau_c$ = 4.954 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 131.2 $N/mm^2$
$S^*$ = 2870. $mm^3$


H
I
3/2F
B
A
17/12F
1/3Fb
5/6F
17/12F
1/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
4F
1/2F
1/2F
D
E
7/12F
5/2F
5/2Fb
4/3F
1/2F
4/3F
7/12Fb
7/12F
1/2F
5/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
3/2F
1/2Fb
1/2F
3/2F
1/2F
C
17/12F
13/12Fb
5/6F
5/6F
1/3Fb
17/12F
B

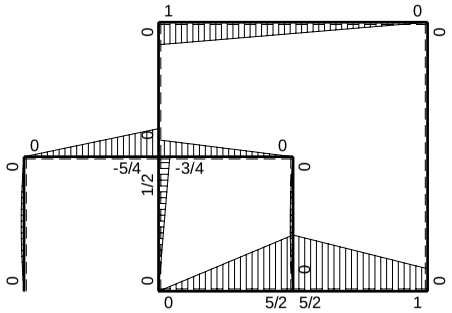




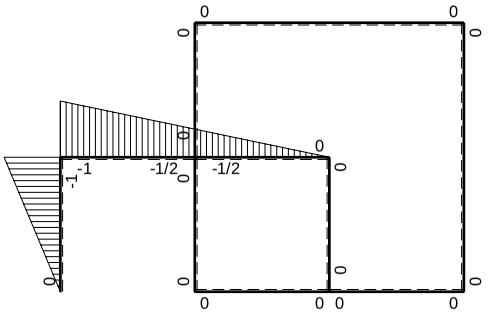

0
1/2
5/6
4/3
-3/2
17/12
4
-3/2
17/12
-1/2
4
0
F
-1/2
0
7/12
5/6
-17/12
1/2
0
1/2
-1/6
-1/2
-5/2
3/2
F
1
0
0
0
1/3
0
1/3
-13/12
-7/12
0
1/2
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264857 CI01.xxxx.034

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PROCEDIMENTO E RISULTATI 264857 CI01.xxxx.034

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

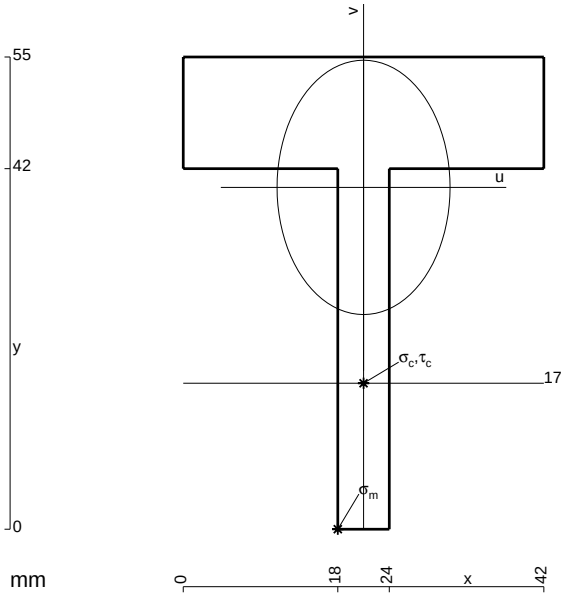
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

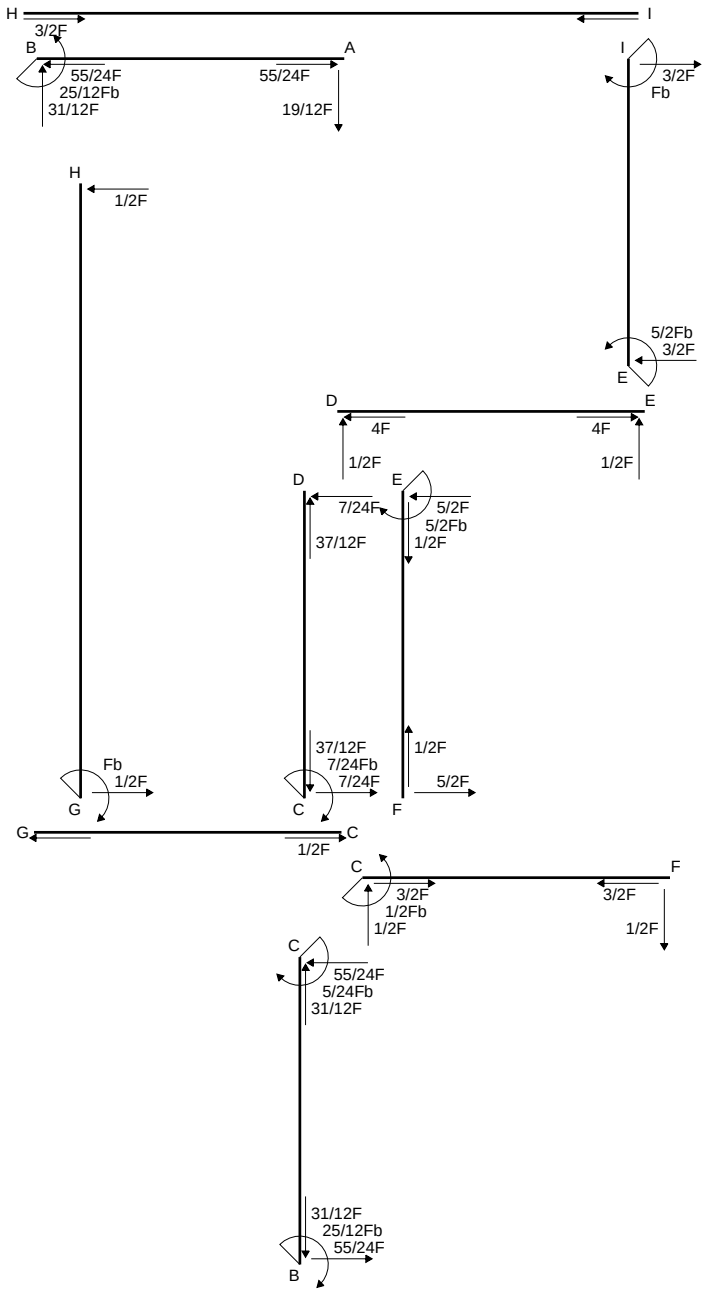


PROCEDIMENTO E RISULTATI 264857 CI01.xxxx.034



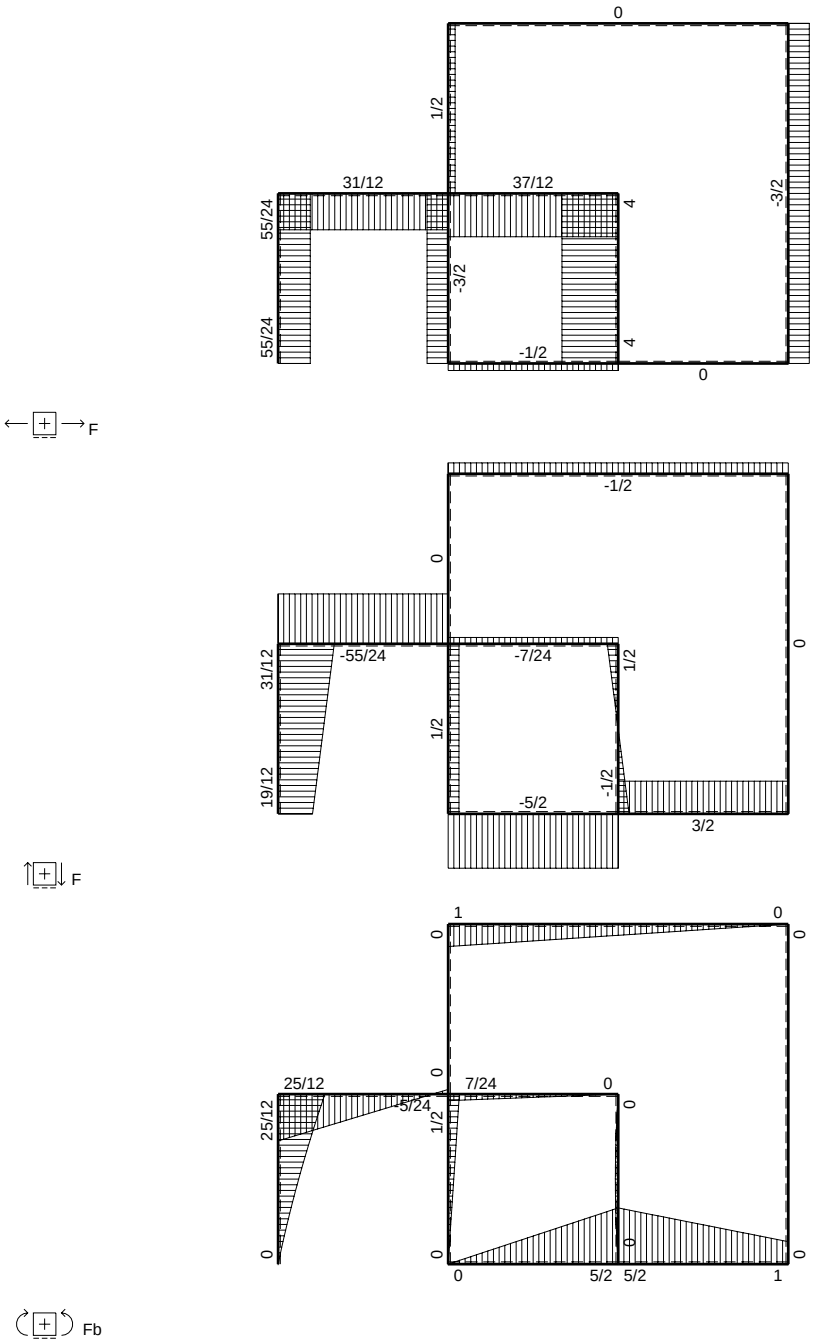

A = 798. $mm^2$
$J_u$ = 175127. $mm^4$
$J_v$ = 81018. $mm^4$
$y_g$ = 39.82 mm
N = -335. N
$T_y$ = -1675. N
$M_x$ = 1055250. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -39.82 mm
$\sigma_m = N/A - Mv/J_u$ = 239.5 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 17. mm
$v_c$ = -22.82 mm
$\sigma_c = N/A - Mv/J_u$ = 137.1 $N/mm^2$
$\tau_c$ = 5.092 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137.3 $N/mm^2$
$S^*$ = 3194. $mm^3$

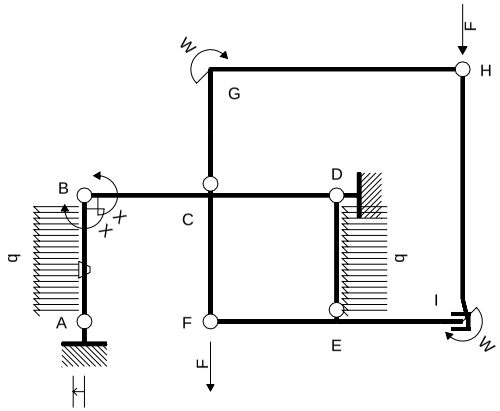
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
3/2F
I
B
A
55/24F
25/12Fb
31/12F
55/24F
19/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
4F
1/2F
1/2F
D
E
7/24F
5/2F
5/2Fb
37/12F
1/2F
37/12F
7/24Fb
7/24F
1/2F
5/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
3/2F
1/2Fb
1/2F
3/2F
1/2F
C
55/24F
5/24Fb
31/12F
31/12F
25/12Fb
55/24F
B

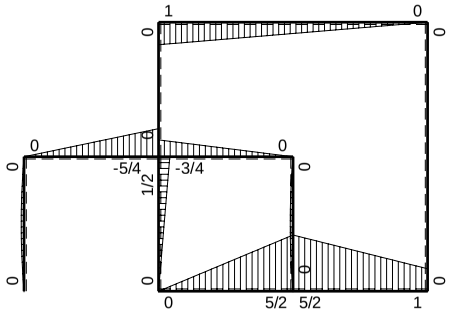




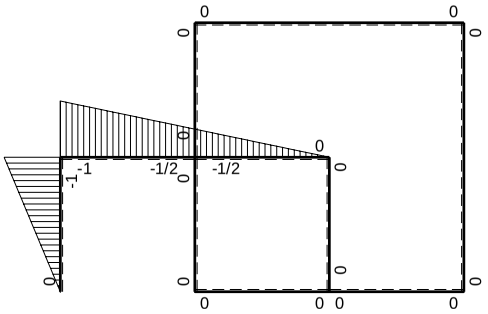

0
1/2
31/12
37/12
-3/2
55/24
4
-3/2
55/24
-1/2
4
0
F
-1/2
0
0
31/12
-55/24
-7/24
1/2
1/2
19/12
-1/2
-5/2
3/2
F
1
0
0
0
0
0
25/12
7/24
0
-5/24
25/12
1/2
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 301451 CI01.xxxx.035

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PROCEDIMENTO E RISULTATI 301451 CI01.xxxx.035

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $25/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -25/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

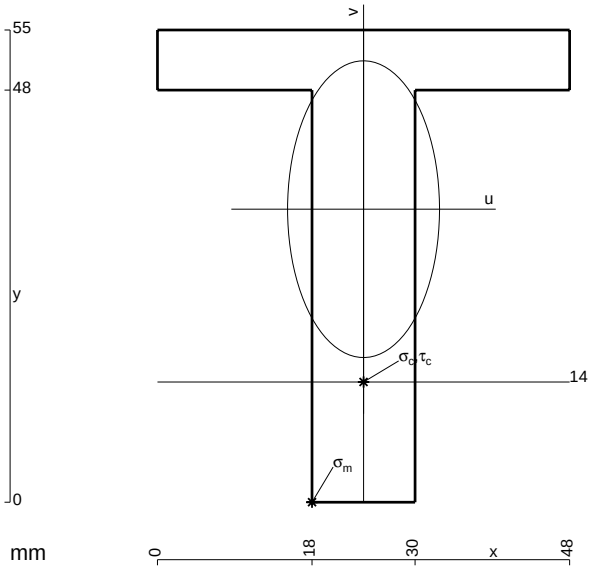
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

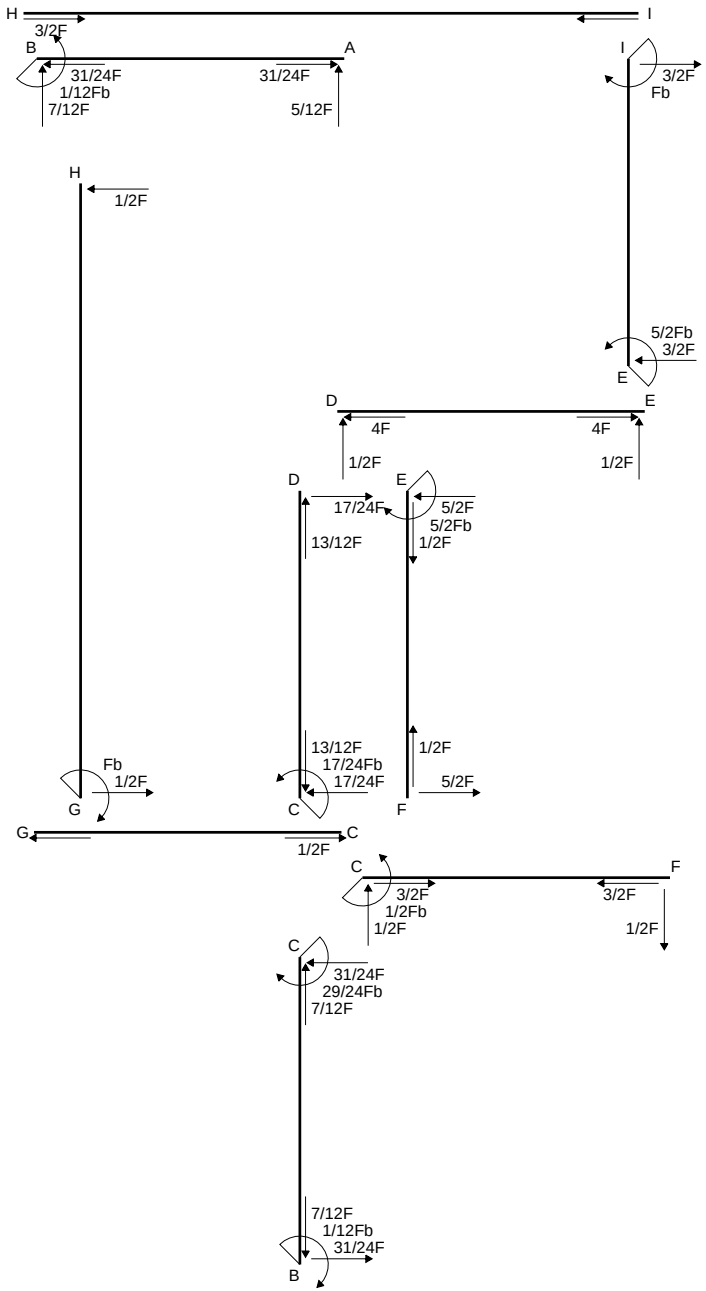
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





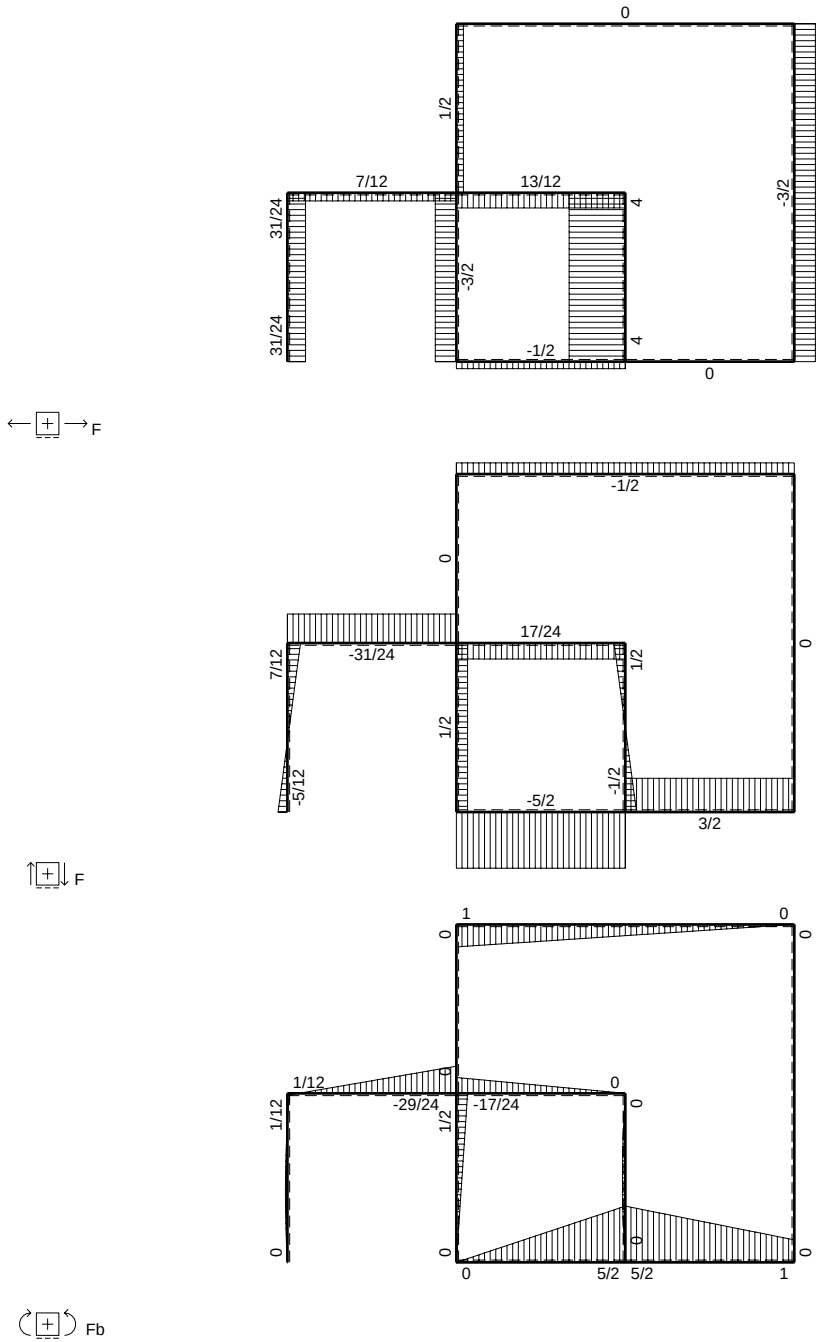


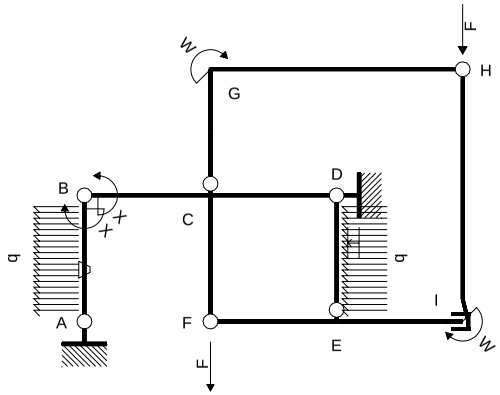

A = 912. $mm^2$
$J_u$ = 272448. $mm^4$
$J_v$ = 71424. $mm^4$
$y_g$ = 34.13 mm
N = -475. N
$T_y$ = -2375. N
$M_x$ = 1591250. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -34.13 mm
$\sigma_m = N/A - Mv/J_u$ = 198.8 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 14. mm
$v_c$ = -20.13 mm
$\sigma_c = N/A - Mv/J_u$ = 117.1 $N/mm^2$
$\tau_c$ = 3.311 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 117.2 $N/mm^2$
$S^*$ = 4558. $mm^3$


H
I
3/2F
B
A
31/24F
1/12Fb
7/12F
31/24F
5/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
4F
1/2F
1/2F
D
E
17/24F
5/2F
5/2Fb
13/12F
1/2F
13/12F
17/24Fb
17/24F
1/2F
5/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
3/2F
1/2Fb
1/2F
3/2F
1/2F
C
31/24F
29/24Fb
7/12F
7/12F
1/12Fb
31/24F
B

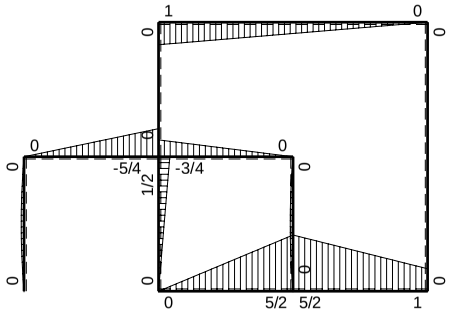




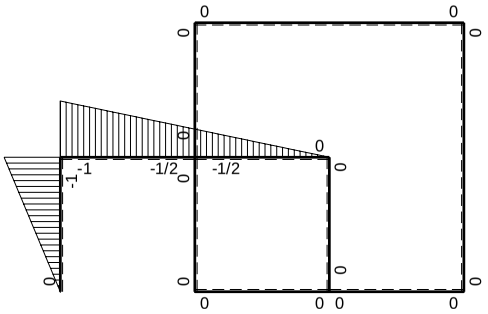

0
1/2
7/12
13/12
-3/2
31/24
4
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31/24
4
-1/2
0
F
-1/2
0
17/24
0
7/12
-31/24
1/2
1/2
-5/12
-1/2
-5/2
3/2
F
1
0
0
0
1/12
0
0
-29/24
-17/24
1/12
1/2
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265095 CI01.xxxx.036

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PROCEDIMENTO E RISULTATI 265095 CI01.xxxx.036

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

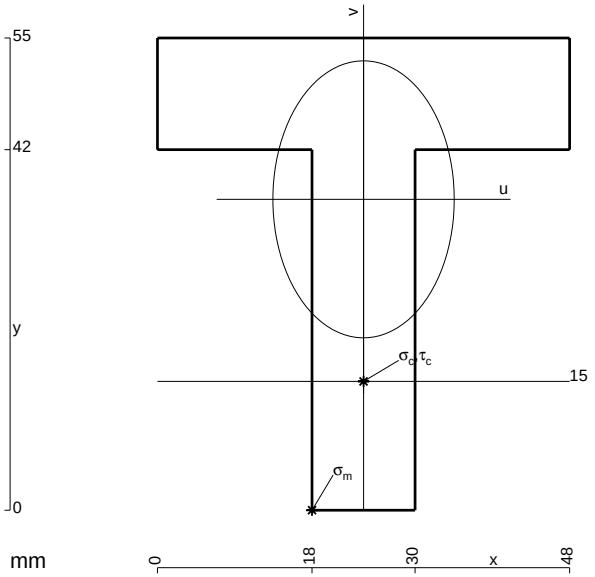
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

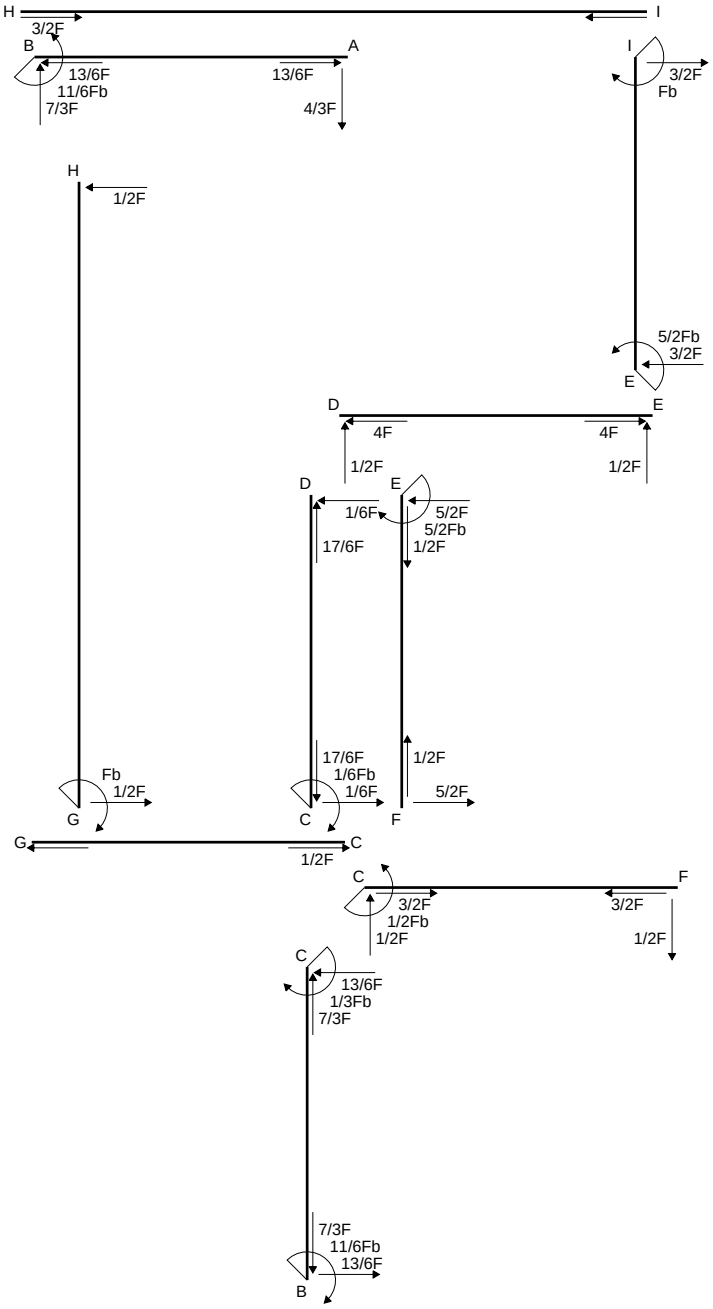


PROCEDIMENTO E RISULTATI 265095 CI01.xxxx.036



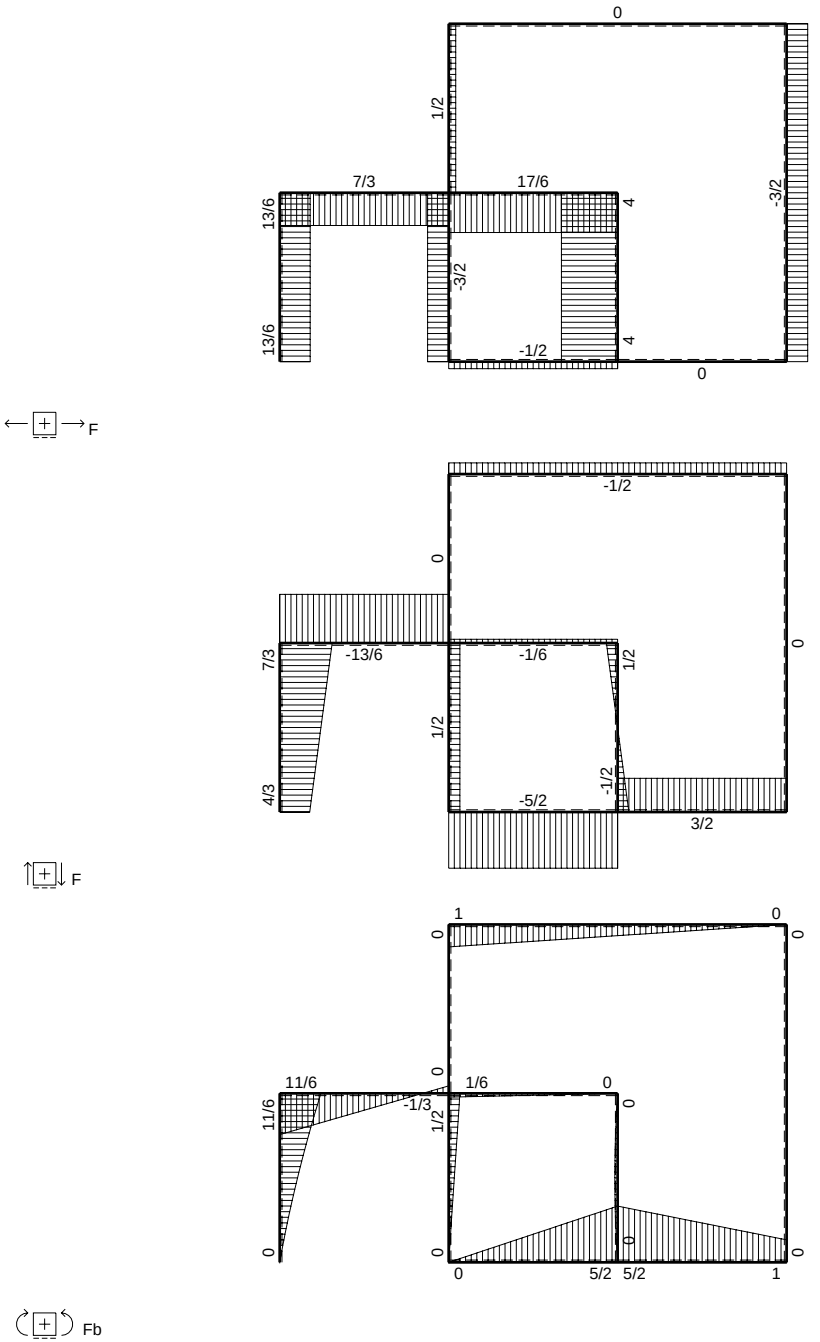

A = 1128. $mm^2$
$J_u$ = 293725. $mm^4$
$J_v$ = 125856. $mm^4$
$y_g$ = 36.21 mm
N = -470. N
$T_y$ = -2350. N
$M_x$ = 1692000. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -36.21 mm
$\sigma_m$ = N/A-Mv/$J_u$ = 208.2 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -21.21 mm
$\sigma_c$ = N/A-Mv/$J_u$ = 121.8 $N/mm^2$
$\tau_c$ = 3.446 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.9 $N/mm^2$
$S^*$ = 5168. $mm^3$

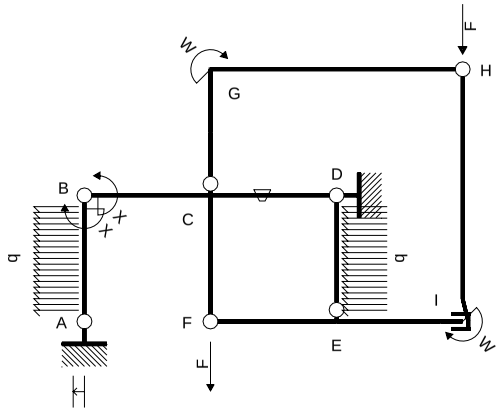
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H
I
3/2F
B
A
13/6F
11/6Fb
7/3F
13/6F
4/3F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
4F
1/2F
1/2F
D
E
1/6F
5/2F
5/2Fb
17/6F
1/2F
17/6F
1/6Fb
1/6F
1/2F
5/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
3/2F
1/2Fb
1/2F
3/2F
1/2F
C
13/6F
1/3Fb
7/3F
7/3F
11/6Fb
13/6F
B

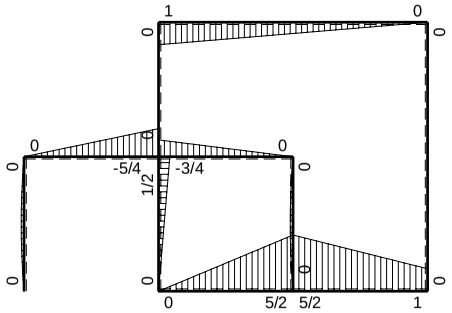




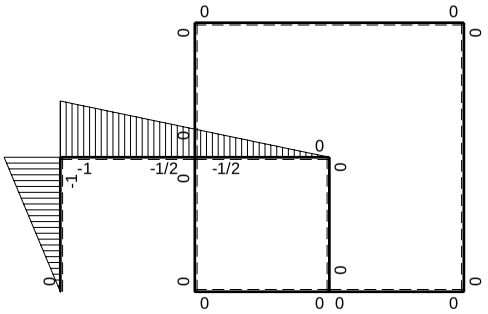

F
0
1/2
7/3
17/6
13/6
4
-3/2
-3/2
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4
-1/2
0
F
-1/2
0
7/3
-13/6
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1/2
0
1/2
4/3
-1/2
-5/2
3/2
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0
11/6
1/6
0
11/6
-1/3
1/2
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 270236 CI01.xxxx.037

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PROCEDIMENTO E RISULTATI 270236 CI01.xxxx.037

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | $3/4Fx$ | Fb/EJ | $3/8Fx^2/b$ | $1/2Fx/EJ$ | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-5/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+1/2Fx$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-5/2Fb+3/2Fx$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $11/6Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | -11/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

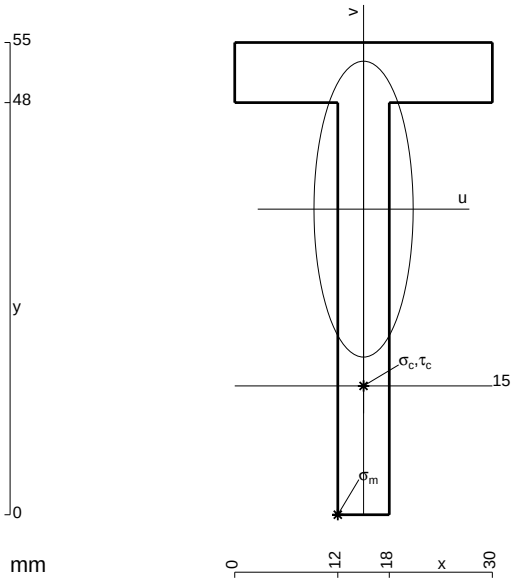
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$



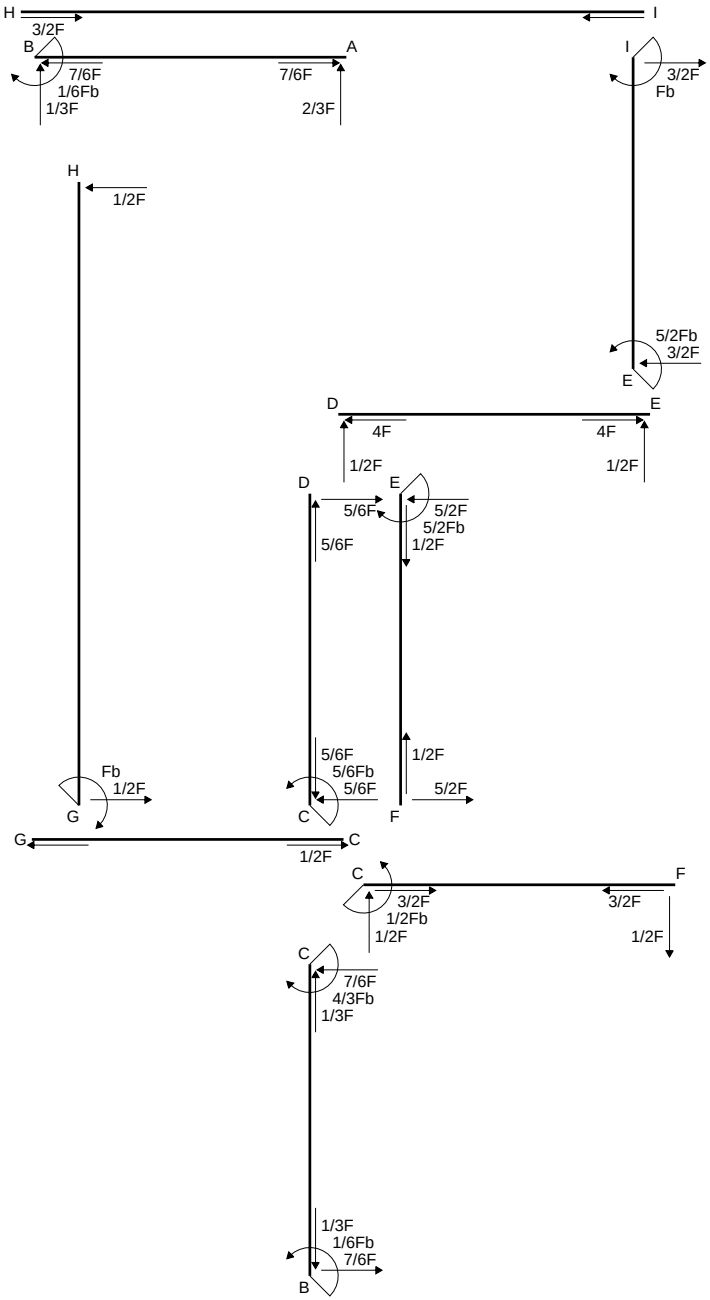
PROCEDIMENTO E RISULTATI 270236 CI01.xxxx.037




mm

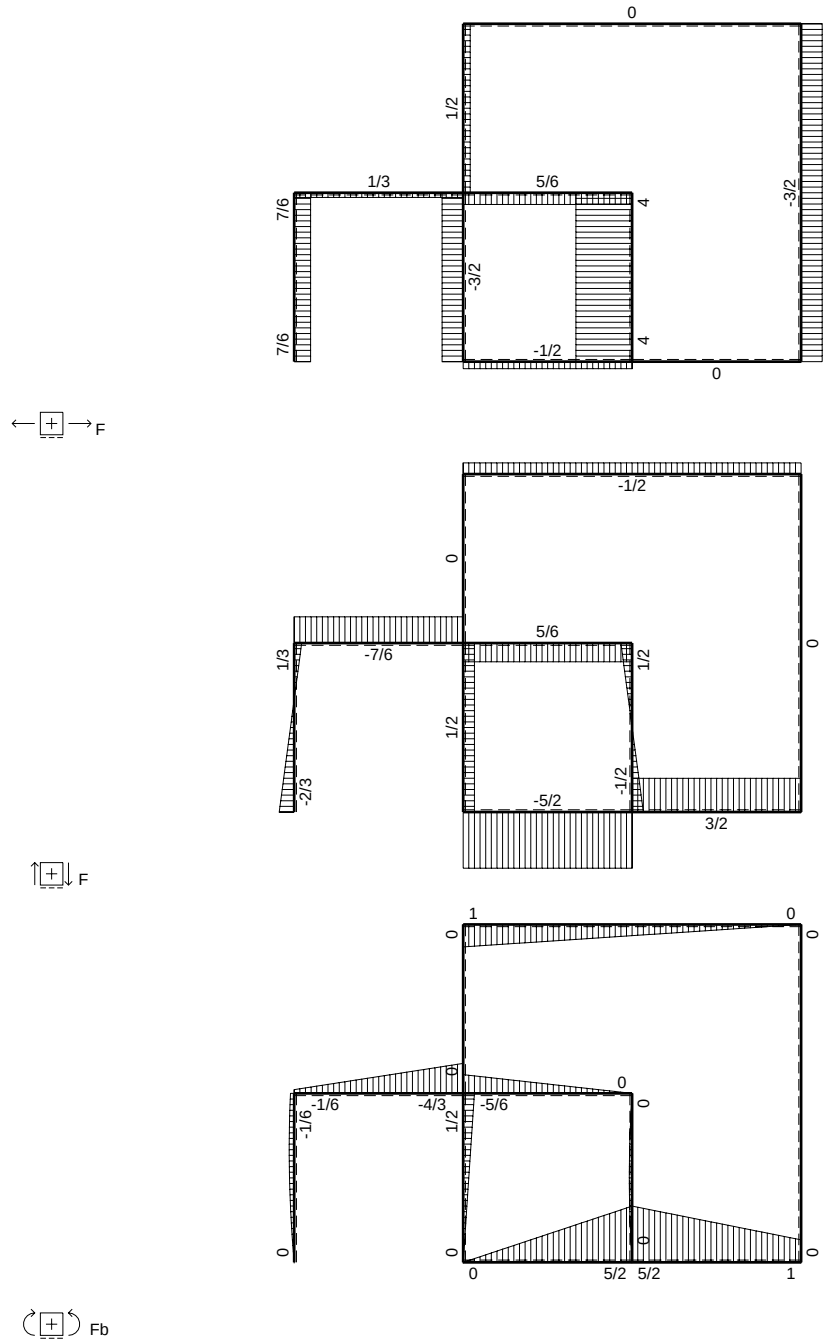
A = 498. $mm^2$
$J_u$ = 147997. $mm^4$
$J_v$ = 16614. $mm^4$
$y_g$ = 35.6 mm
N = -240. N
$T_y$ = -1200. N
$M_x$ = 912000. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -35.6 mm
$\sigma_m = N/A - Mv/J_u$ = 218.9 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 15. mm
$v_c$ = -20.6 mm
$\sigma_c = N/A - Mv/J_u$ = 126.4 $N/mm^2$
$\tau_c$ = 3.417 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.6 $N/mm^2$
$S^*$ = 2529. $mm^3$

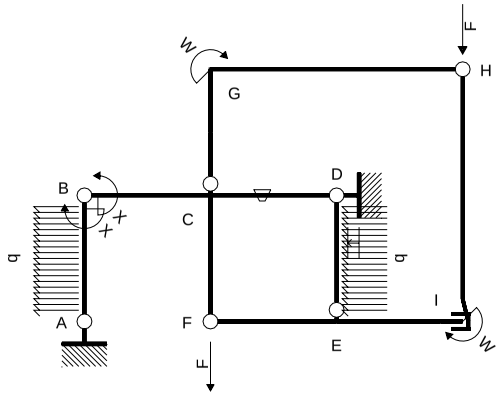
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H
I
3/2F
B
A
7/6F
1/6Fb
1/3F
7/6F
2/3F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
4F
4F
1/2F
1/2F
D
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1/2Fb
1/2F
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1/2F
C
7/6F
4/3Fb
1/3F
1/3F
1/6Fb
7/6F
B

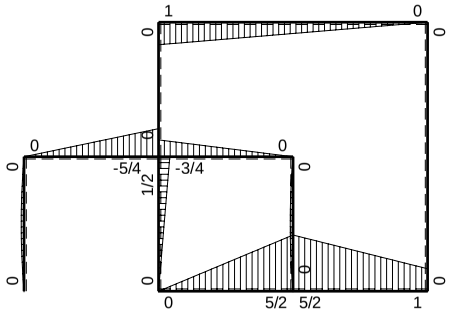




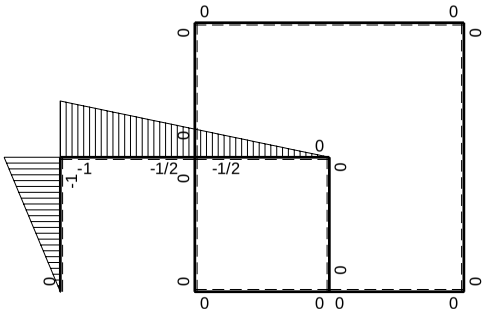

0
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5/6
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7/6
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7/6
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0
5/6
1/3
-7/6
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0
1/2
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-1/2
-5/2
3/2
F
1
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0
0
0
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-1/6
-4/3
-5/6
1/2
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267751 CI01.xxxx.038

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PROCEDIMENTO E RISULTATI 267751 CI01.xxxx.038

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -5/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

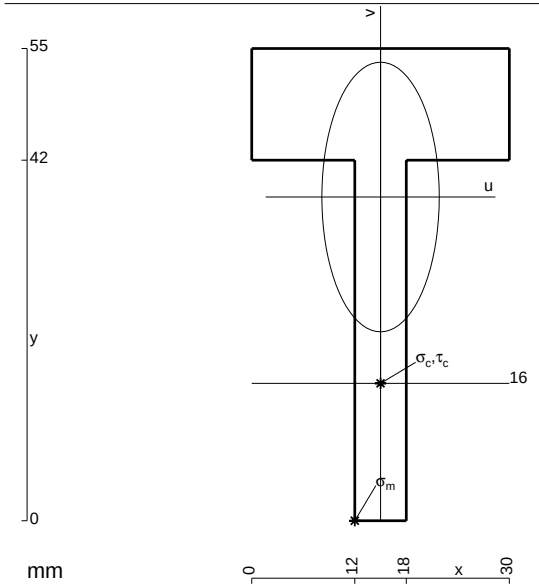
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

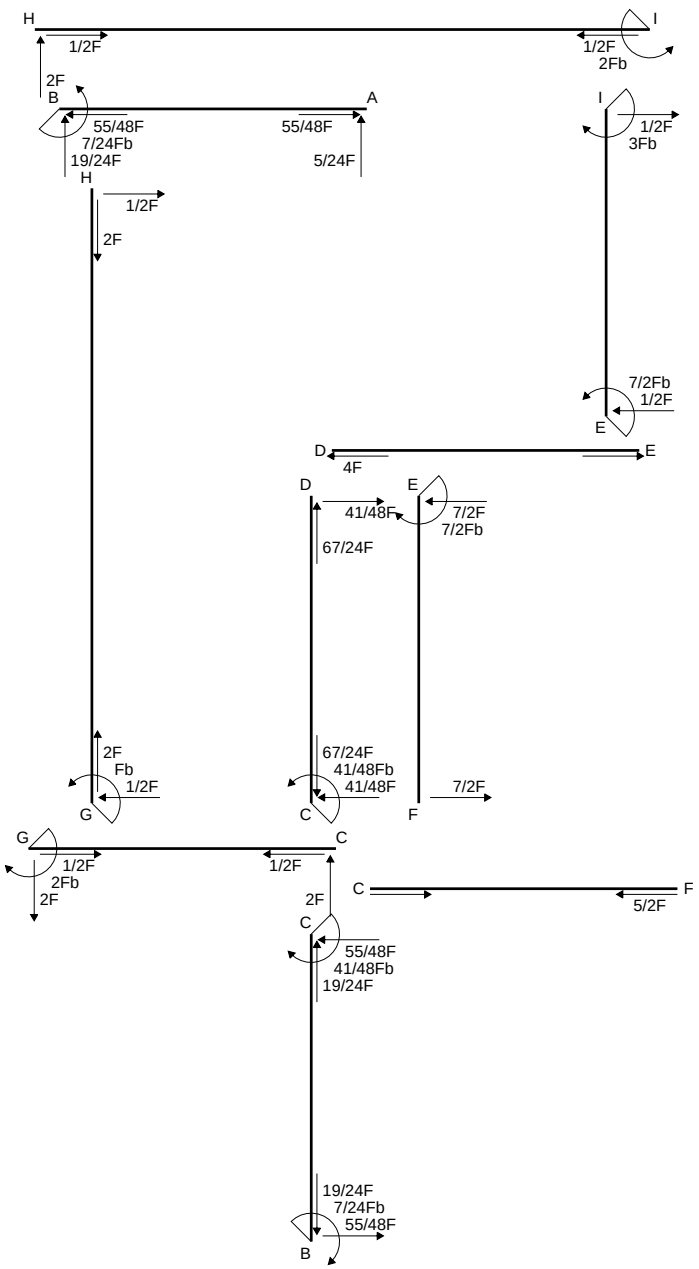


PROCEDIMENTO E RISULTATI 267751 CI01.xxxx.038

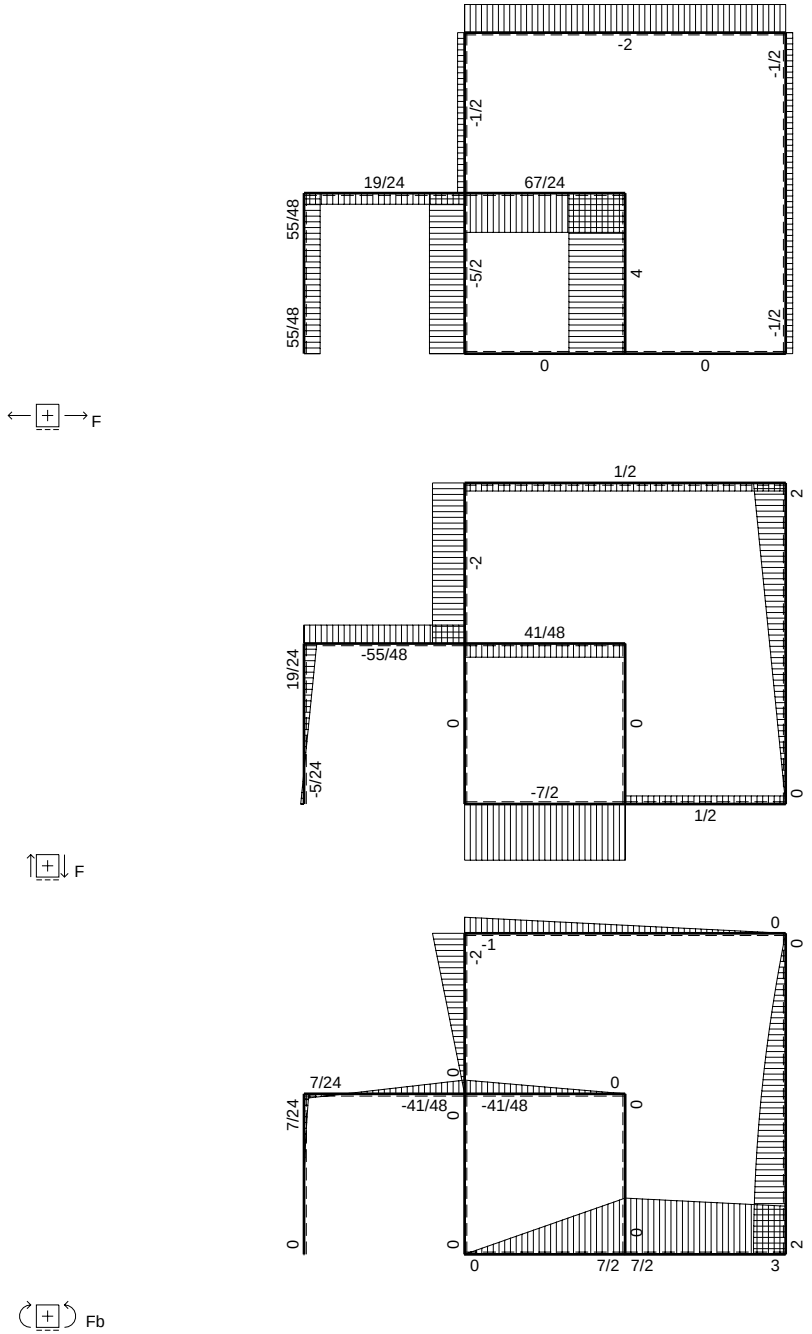



A = 642. $mm^2$
$J_u$ = 158306. $mm^4$
$J_v$ = 30006. $mm^4$
$y_g$ = 37.71 mm
N = -235. N
$T_y$ = -1175. N
$M_x$ = 951750. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -37.71 mm
$\sigma_m$ = N/A-Mv/$J_u$ = 226.3 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 16. mm
$v_c$ = -21.71 mm
$\sigma_c$ = N/A-Mv/$J_u$ = 130.1 $N/mm^2$
$\tau_c$ = 3.528 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.3 $N/mm^2$
$S^*$ = 2852. $mm^3$

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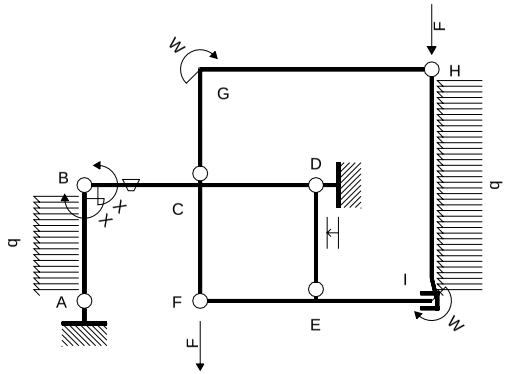




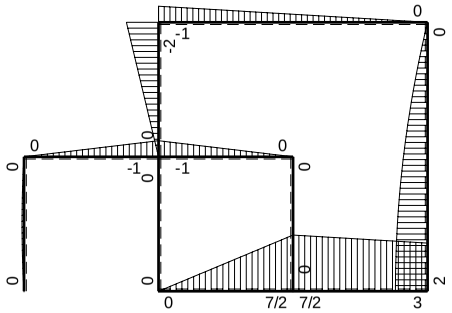





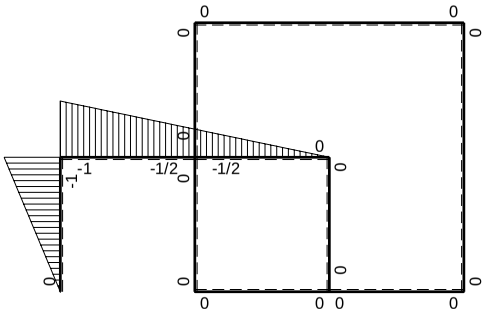

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Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 270578 CI01.xxxx.039

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PROCEDIMENTO E RISULTATI 270578 CI01.xxxx.039

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $7/24Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

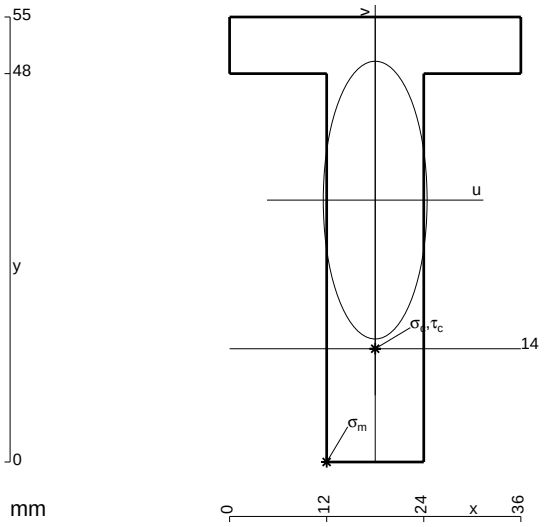
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

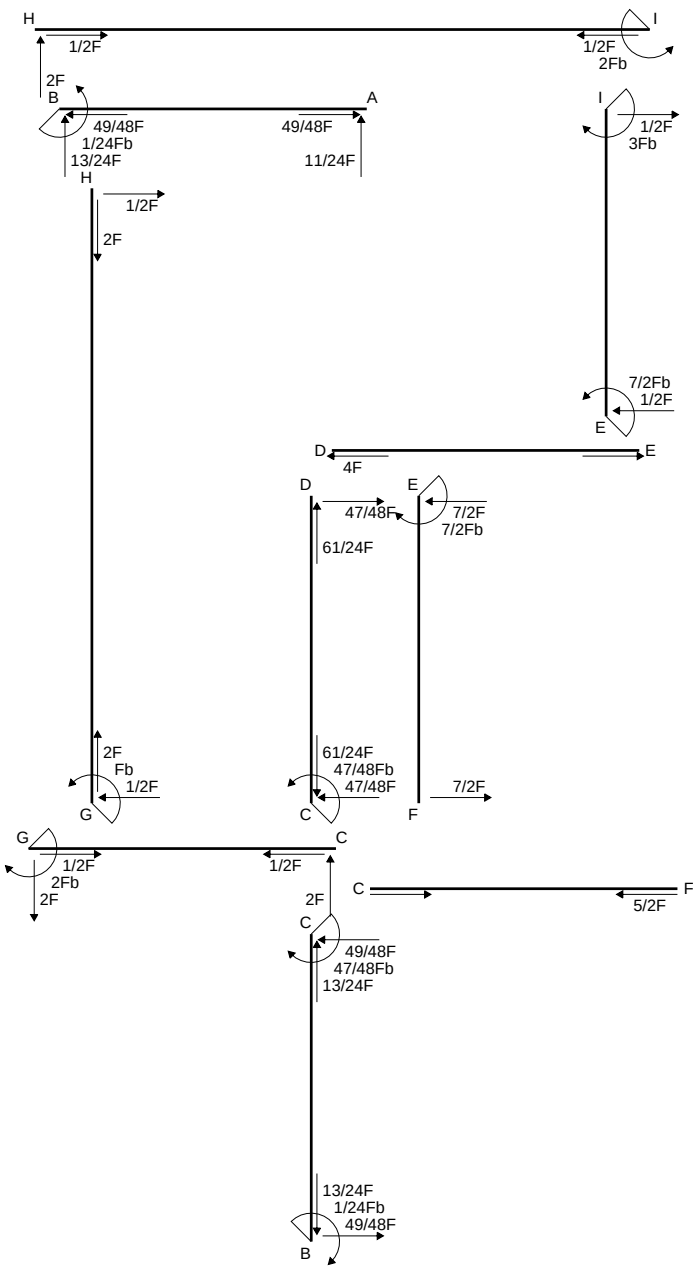


PROCEDIMENTO E RISULTATI 270578 CI01.xxxx.039

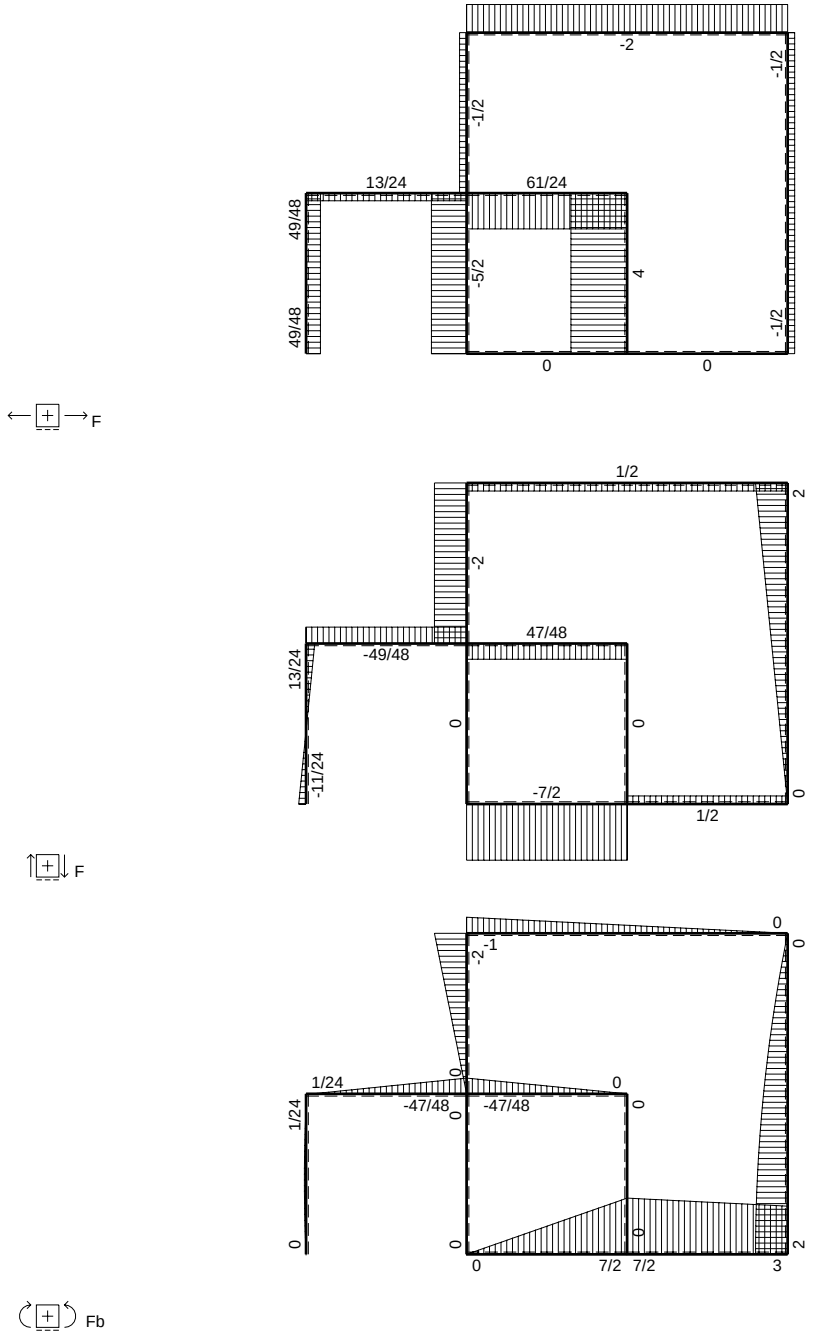



A = 828. $mm^2$
$J_u$ = 244195. $mm^4$
$J_v$ = 34128. $mm^4$
$y_g$ = 32.37 mm
$T_y$ = -2100. N
$M_x$ = 1785000. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -32.37 mm
$\sigma_m = -Mv/J_u$ = 236.6 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -18.37 mm
$\sigma_c = -Mv/J_u$ = 134.3 $N/mm^2$
$\tau_c$ = 3.054 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.4 $N/mm^2$
$S^*$ = 4262. $mm^3$

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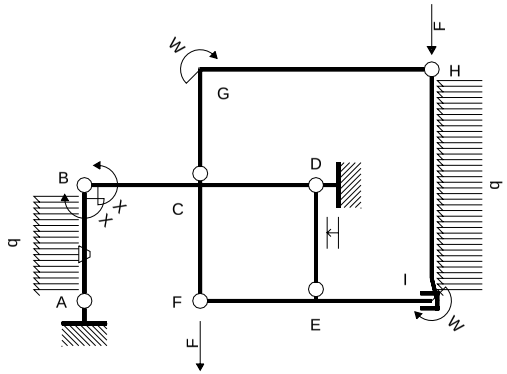




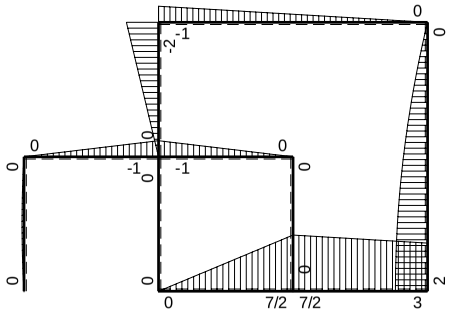





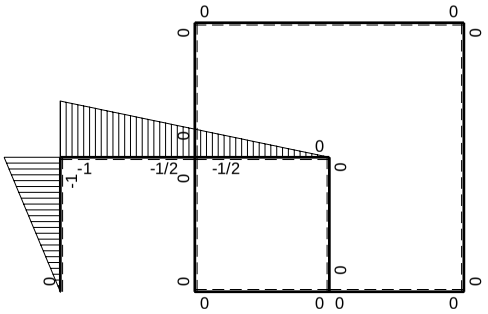

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Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264363 CI01.xxxx.040

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PROCEDIMENTO E RISULTATI 264363 CI01.xxxx.040

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b ( x/b )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

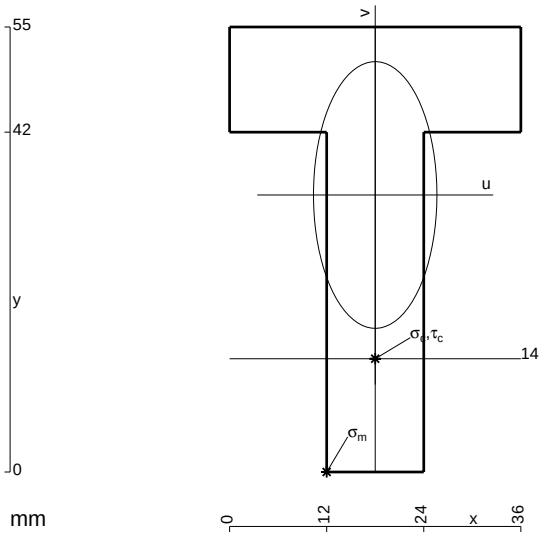
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

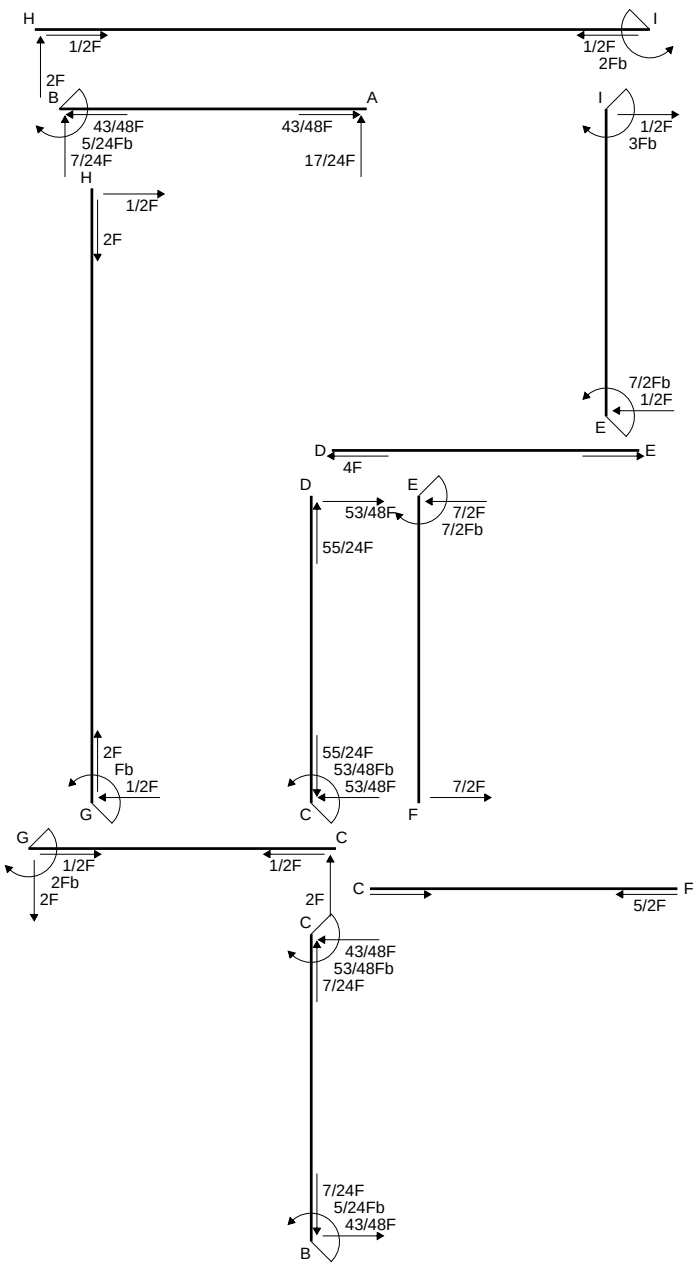
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





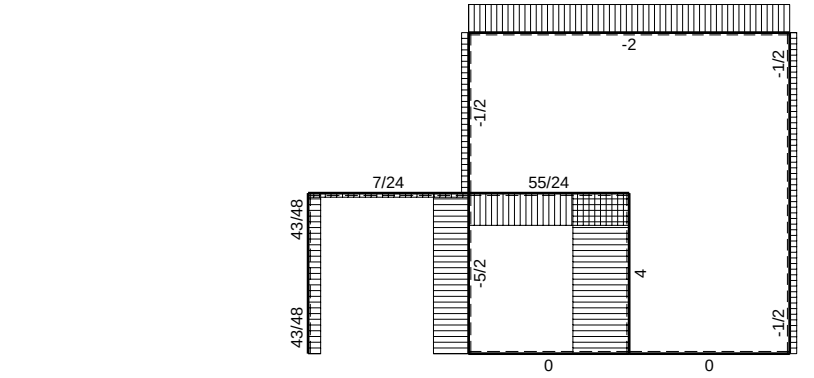



A = 972. $mm^2$
$J_u$ = 264196. $mm^4$
$J_v$ = 56592. $mm^4$
$y_g$ = 34.24 mm
$T_y$ = -3395. N
$M_x$ = 1527750. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -34.24 mm
$\sigma_m = -Mv/J_u$ = 198. $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -20.24 mm
$\sigma_c = -Mv/J_u$ = 117. $N/mm^2$
$\tau_c$ = 4.901 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.4 $N/mm^2$
$S^*$ = 4576. $mm^3$

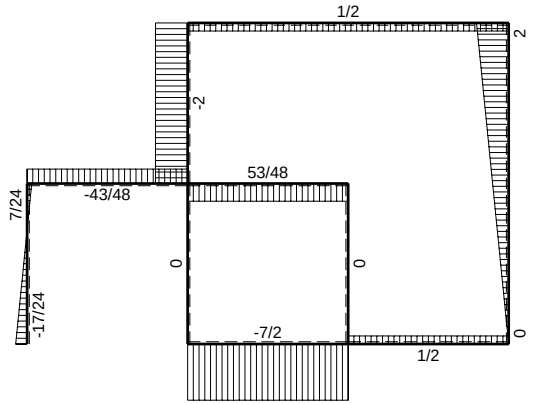

H
I
1/2F
1/2F
2Fb
2F
B
A
43/48F
43/48F
5/24Fb
7/24F
17/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
D
E
53/48F
7/2F
7/2Fb
55/24F
2F
Fb
1/2F
G
55/24F
53/48Fb
53/48F
C
7/2F
F
G
C
1/2F
2Fb
2F
1/2F
C
F
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2F
C
43/48F
53/48Fb
7/24F
7/24F
5/24Fb
43/48F
B



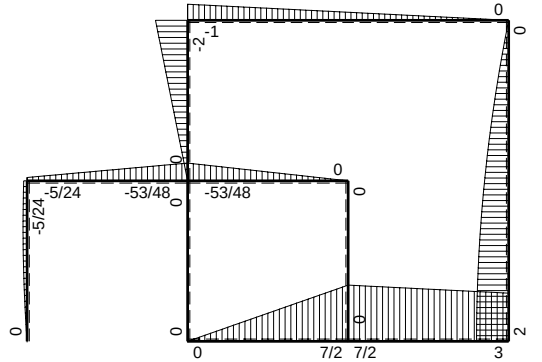



-2
-1/2
-1/2
7/24
55/24
43/48
-5/2
4
43/48
-1/2
0
0

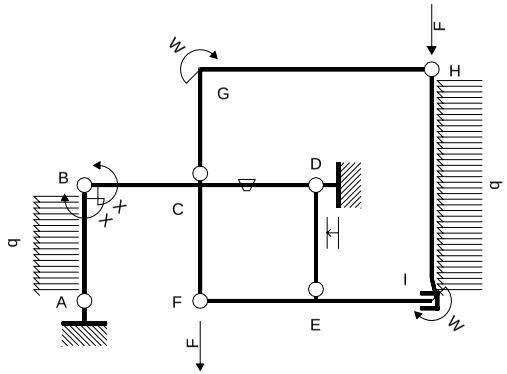
F


1/2
2
-2
53/48
-43/48
7/24
0
0
-17/24
-7/2
0
1/2

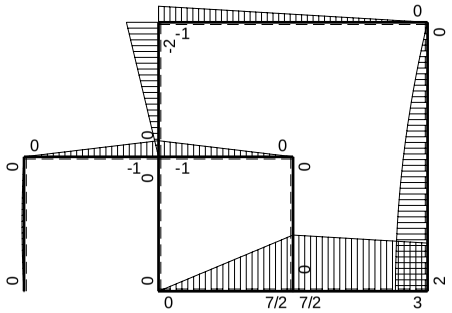
F


0
0
-2
1
0
0
0
-5/24
-53/48
-53/48
-5/24
0
0
0
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2
0
7/2
7/2
3

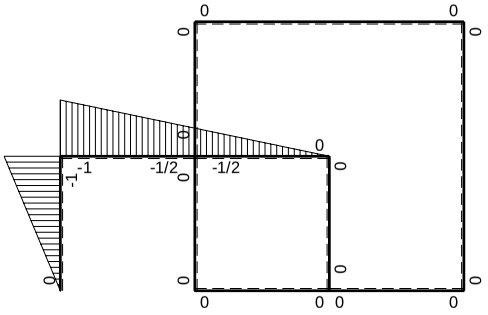
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 991322 CI01.xxxx.041

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | $-x/b$ | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | $1-x/b$ | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-Fx$ | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | $Fb-Fx$ | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-Fb+Fx$ | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | $Fx$ | $Fb/EJ$ | $1/2Fx^2/b$ | $1/2Fx/EJ$ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-7/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/2Fx$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $2Fb-2Fx$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+1/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $1/2Fx$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-7/2Fb+1/2Fx$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-5/24Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | $5/24Fb$ | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

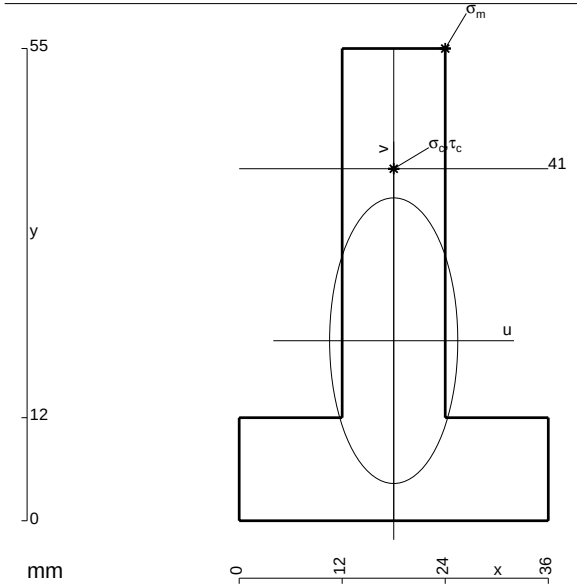
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

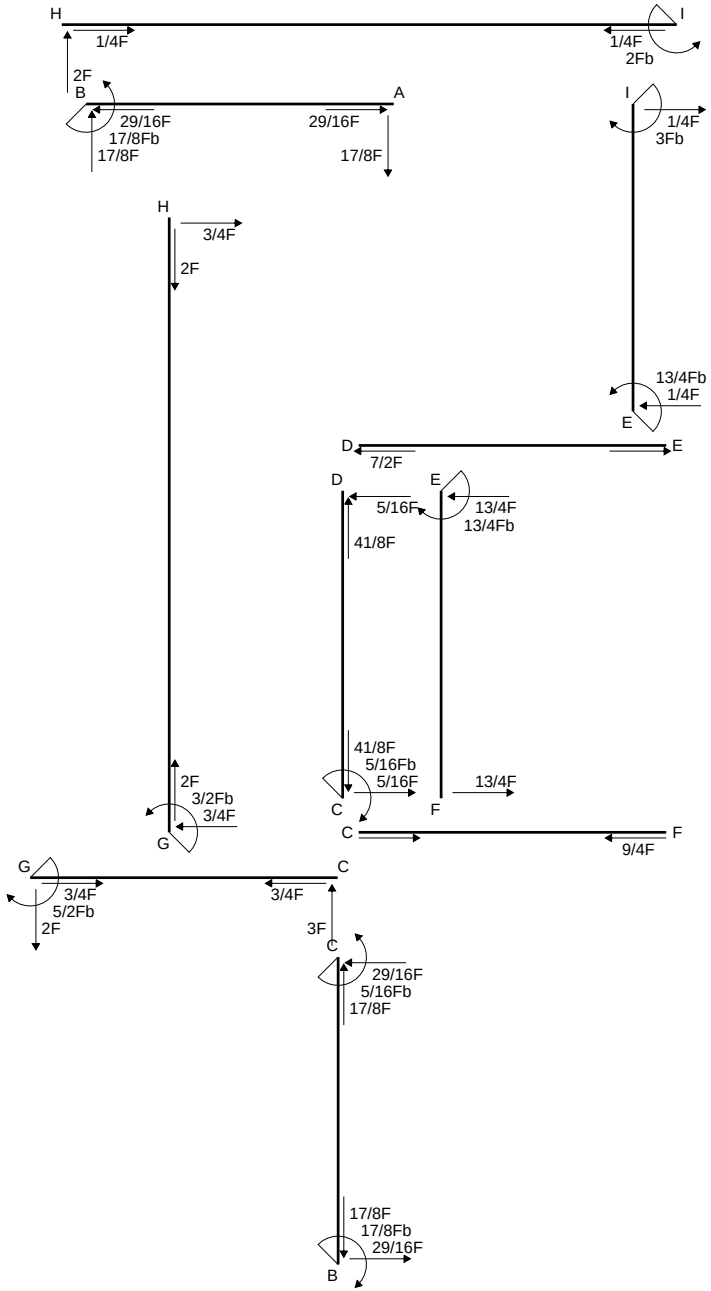


PROCEDIMENTO E RISULTATI 991322 CI01.xxxx.041



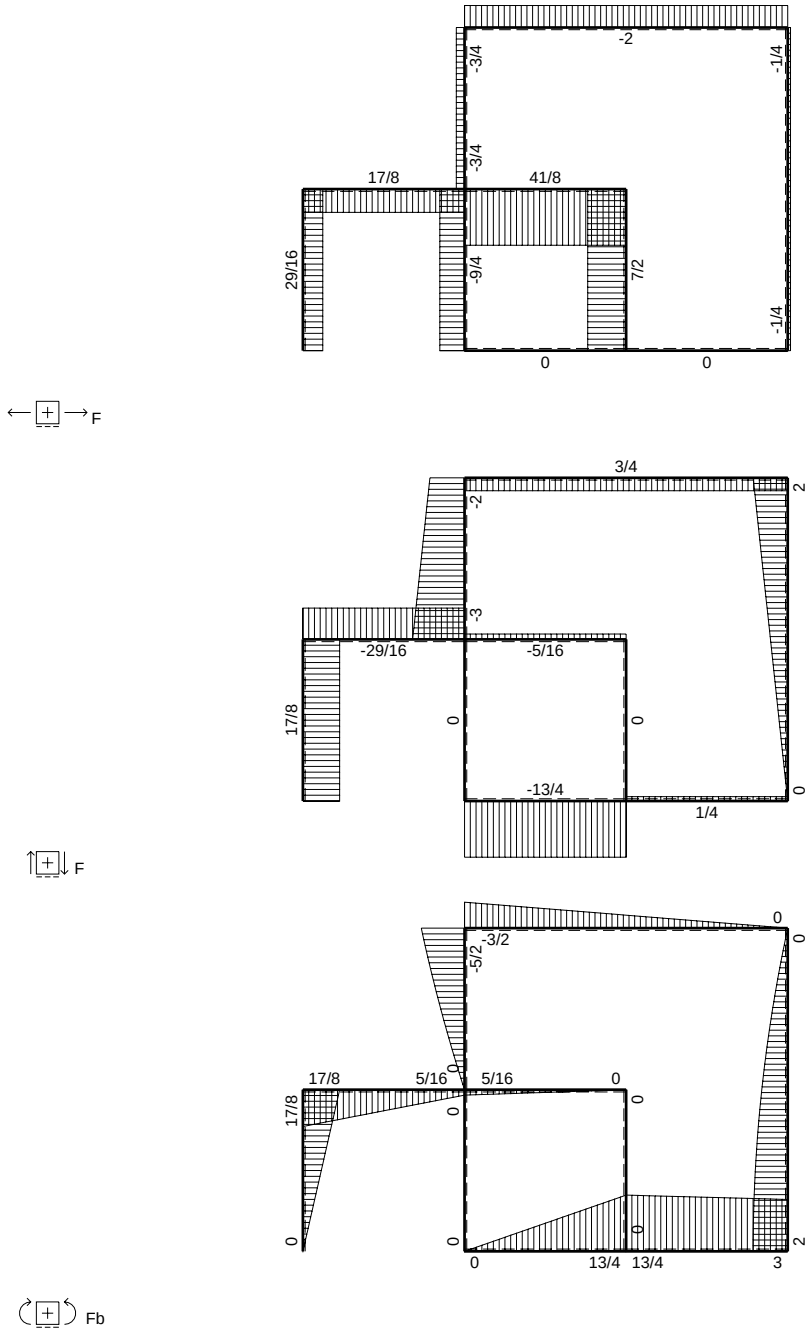

A = 948. $mm^2$
$J_u$ = 262515. $mm^4$
$J_v$ = 52848. $mm^4$
$y_g$ = 20.97 mm
$T_y$ = -3290. N
$M_x$ = 1612100. Nmm
$x_m$ = 24. mm
$y_m$ = 55. mm
$u_m$ = 6. mm
$v_m$ = 34.03 mm
$\sigma_m = -Mv/J_u$ = -209. $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 41. mm
$v_c$ = 20.03 mm
$\sigma_c = -Mv/J_u$ = -123. $N/mm^2$
$\tau_c$ = 4.743 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.3 $N/mm^2$
$S^*$ = 4541. $mm^3$

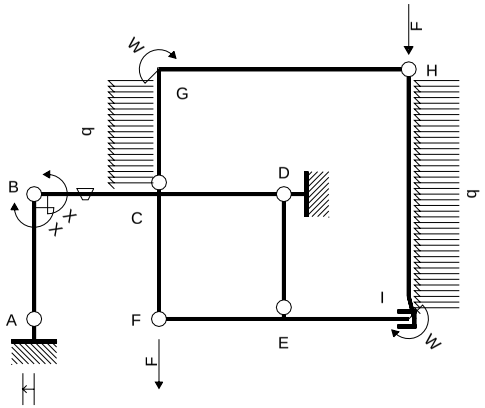
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H
I
1/4F
1/4F
2Fb
2F
B
A
29/16F
17/8Fb
17/8F
29/16F
17/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
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13/4F
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41/8F
41/8F
5/16Fb
5/16F
13/4F
C
F
C
F
9/4F
2F
3/2Fb
3/4F
G
G
C
3/4F
3/4F
5/2Fb
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3F
C
29/16F
5/16Fb
17/8F
17/8F
17/8Fb
29/16F
B

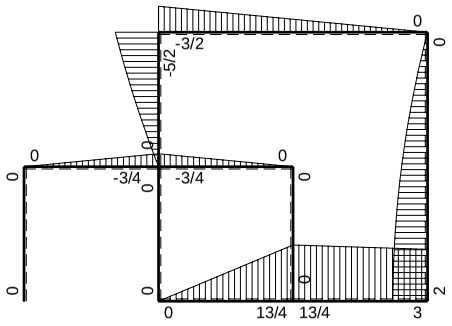




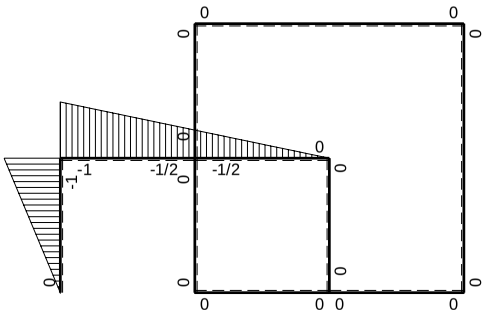

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3/4
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2
-3
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-5/16
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0
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0
1/4
F
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-3/2
0
-5/2
17/8
5/16
5/16
0
17/8
0
0
0
0
0
0
13/4
13/4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 263623 CI01.xxxx.042

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PROCEDIMENTO E RISULTATI 263623 CI01.xxxx.042

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $17/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -17/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

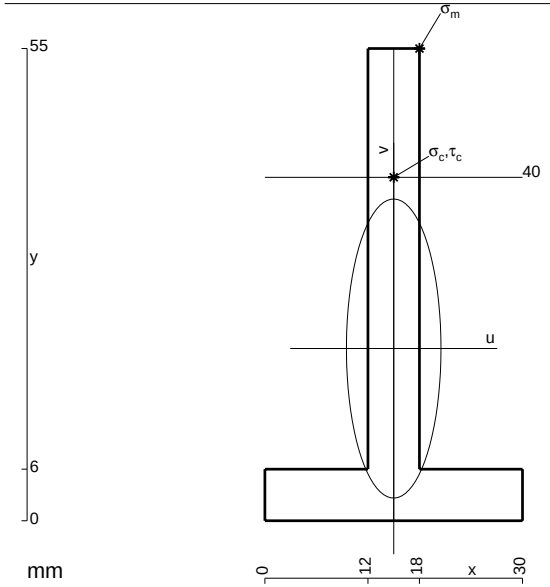
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

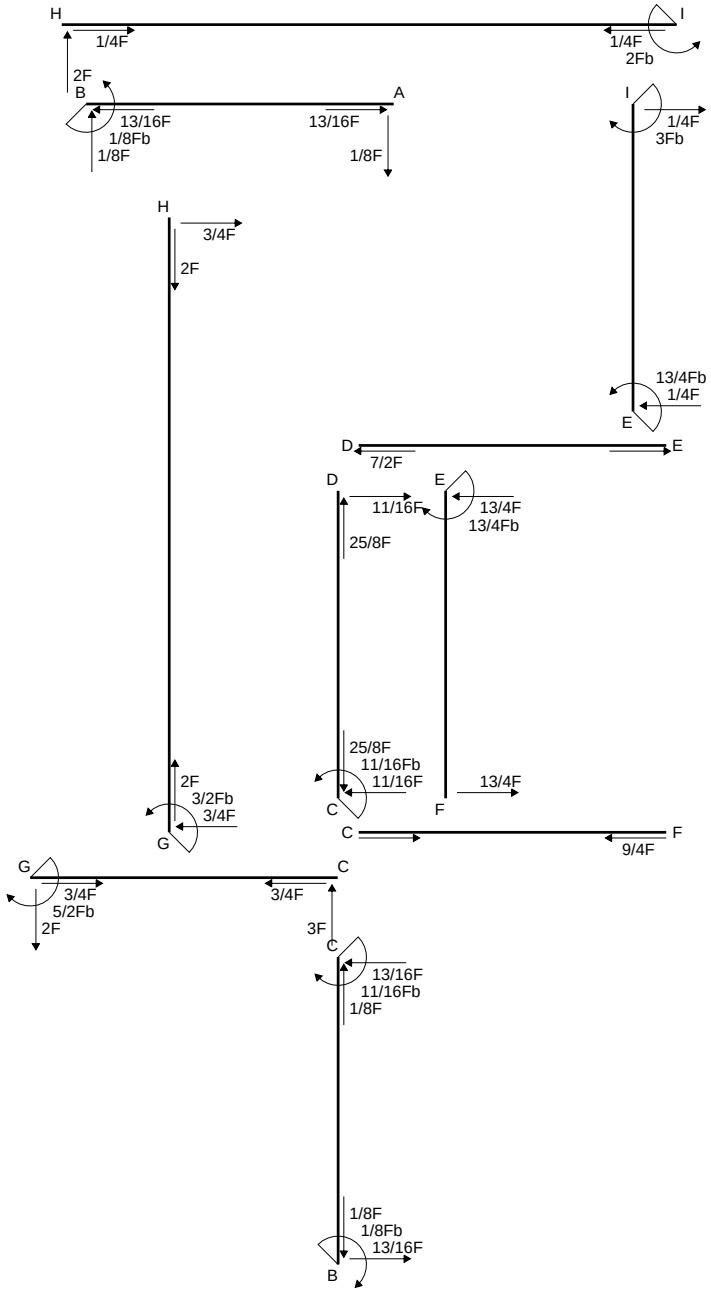


PROCEDIMENTO E RISULTATI 263623 CI01.xxxx.042



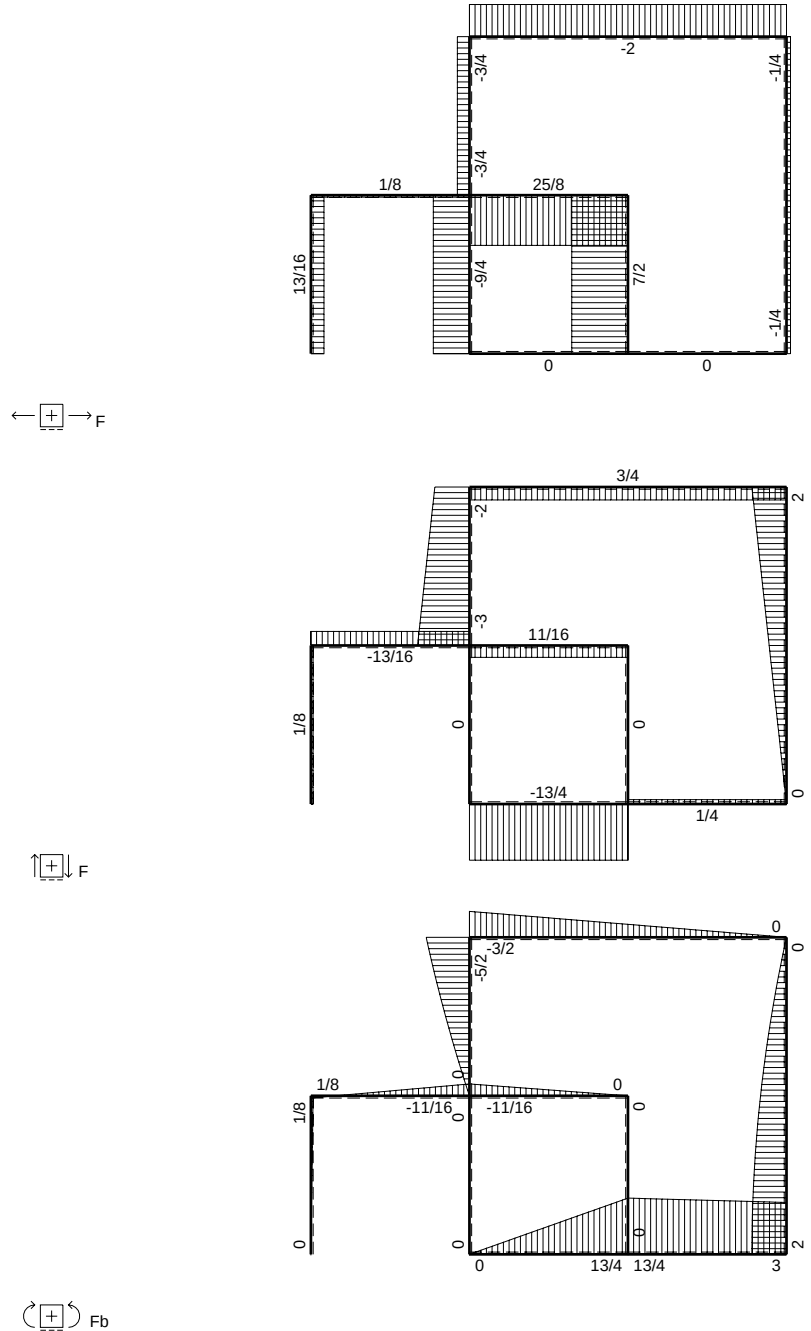

A = 474. $mm^2$
$J_u$ = 143796. $mm^4$
$J_v$ = 14382. $mm^4$
$y_g$ = 20.06 mm
$T_y$ = -1658. N
$M_x$ = 895050. Nmm
$x_m$ = 18. mm
$y_m$ = 55. mm
$u_m$ = 3. mm
$v_m$ = 34.94 mm
$\sigma_m = -Mv/J_u$ = -217.5 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 40. mm
$v_c$ = 19.94 mm
$\sigma_c = -Mv/J_u$ = -124.1 $N/mm^2$
$\tau_c$ = 4.745 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.4 $N/mm^2$
$S^*$ = 2470. $mm^3$

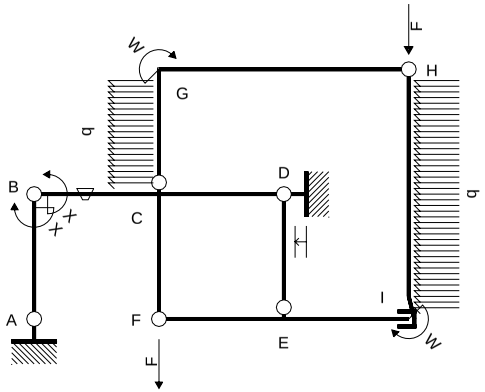
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
1/4F
1/4F
2Fb
2F
B
A
13/16F
1/8Fb
1/8F
13/16F
1/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
11/16F
13/4F
13/4Fb
25/8F
25/8F
11/16Fb
11/16F
13/4F
C
F
2F
3/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
5/2Fb
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3F
C
13/16F
11/16Fb
1/8F
1/8F
1/8Fb
13/16F
B

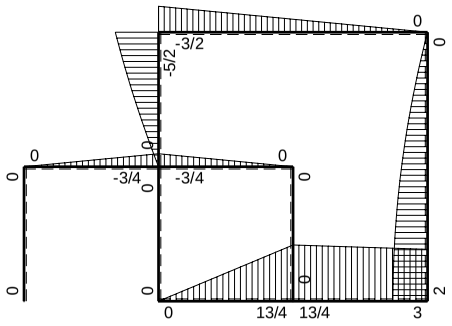




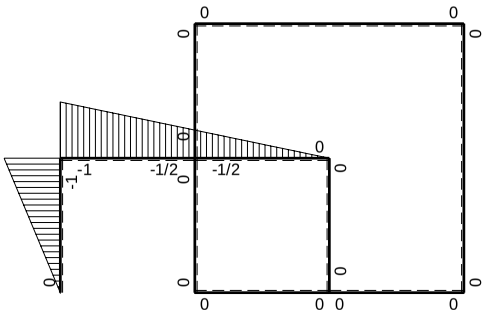

-2
-3/4
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-3/4
1/8
25/8
13/16
-9/4
7/2
-1/4
0
0
F
3/4
-2
2
-3
11/16
-13/16
1/8
0
0
-13/4
0
1/4
F
0
-3/2
0
-5/2
0
1/8
0
1/8
-11/16
-11/16
0
0
0
0
0
2
0
13/4
13/4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 311832 CI01.xxxx.043

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 311832 CI01.xxxx.043

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

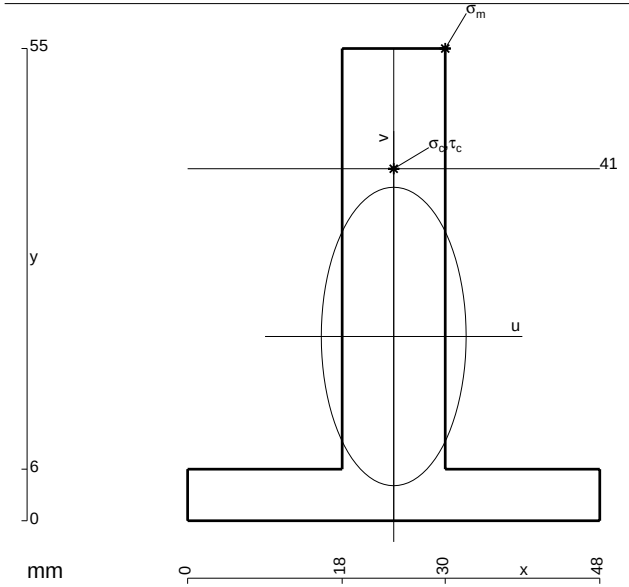
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

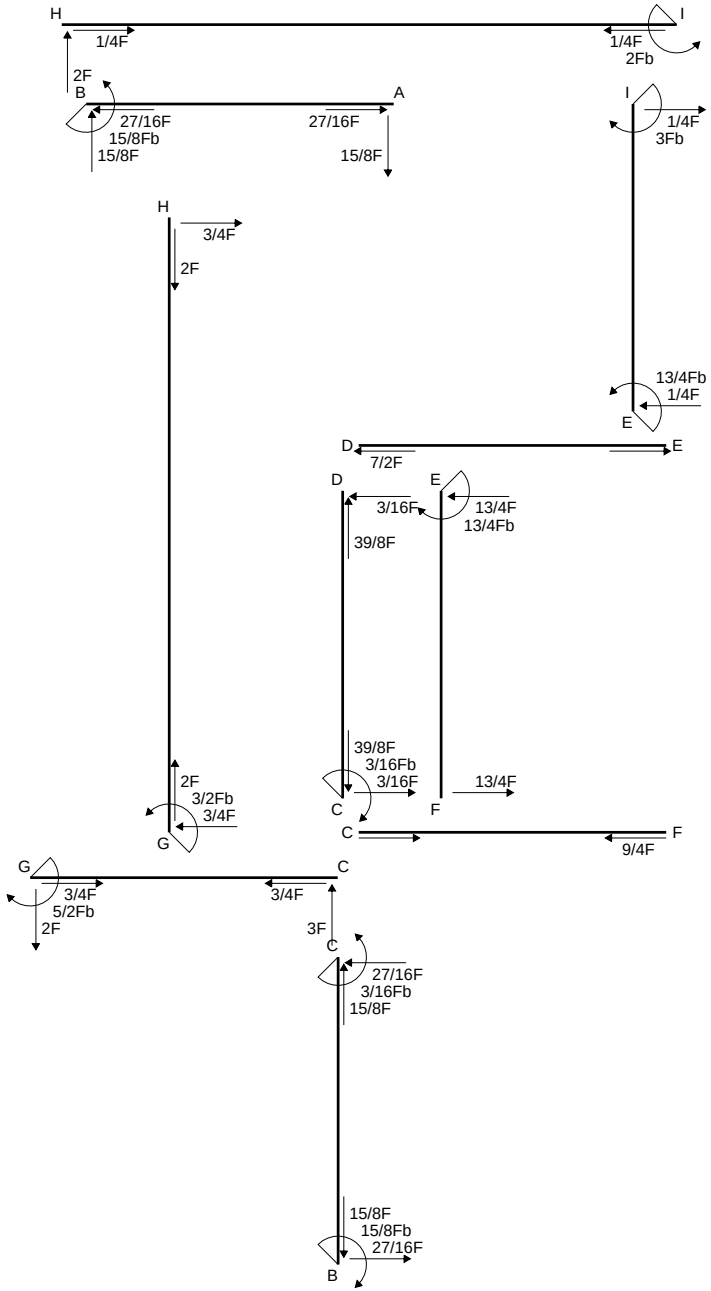


PROCEDIMENTO E RISULTATI 311832 CI01.xxxx.043



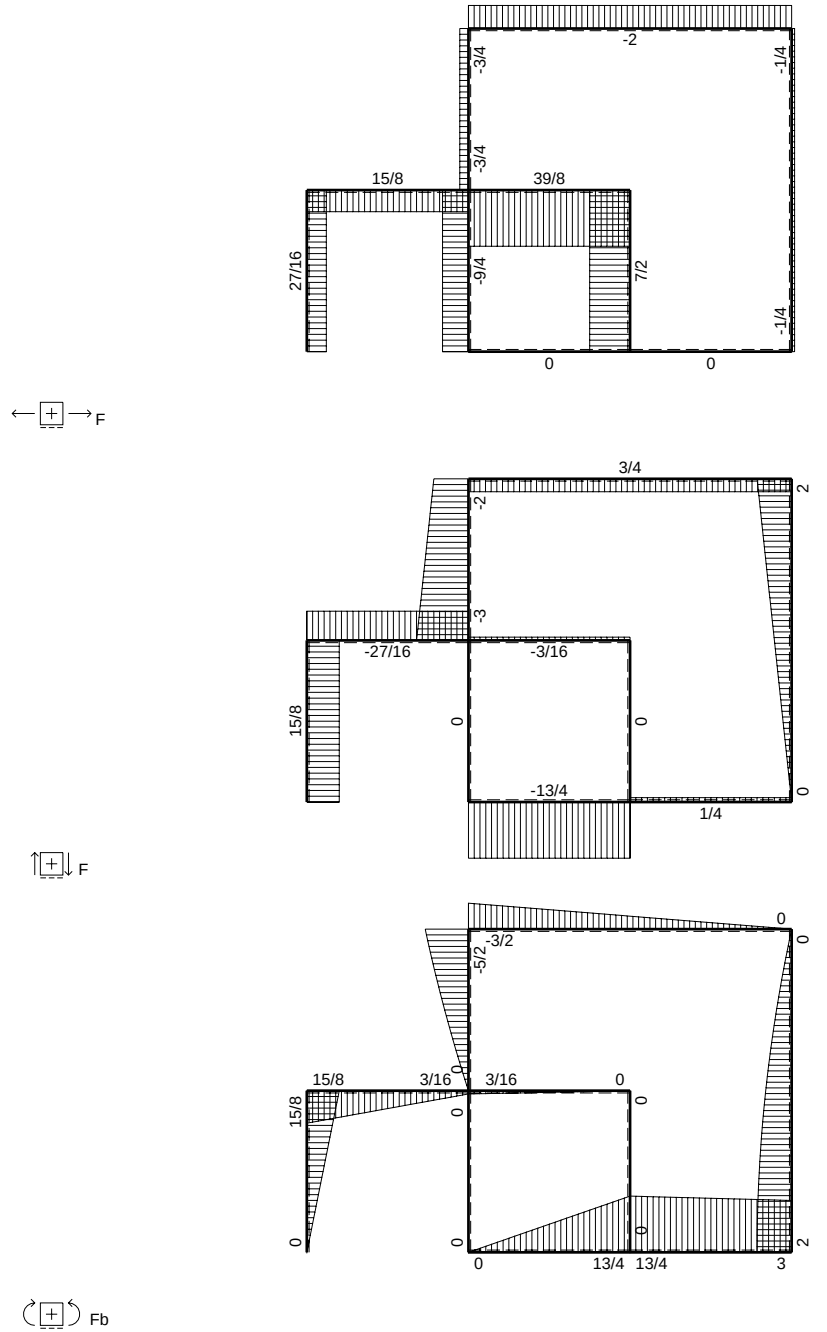

A = 876. $mm^2$
$J_u$ = 264708. $mm^4$
$J_v$ = 62352. $mm^4$
$y_g$ = 21.46 mm
$T_y$ = -3120. N
$M_x$ = 1809600. Nmm
$x_m$ = 30. mm
$y_m$ = 55. mm
$u_m$ = 6. mm
$v_m$ = 33.54 mm
$\sigma_m = -Mv/J_u$ = -229.3 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 41. mm
$v_c$ = 19.54 mm
$\sigma_c = -Mv/J_u$ = -133.6 $N/mm^2$
$\tau_c$ = 4.38 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.8 $N/mm^2$
$S^*$ = 4459. $mm^3$

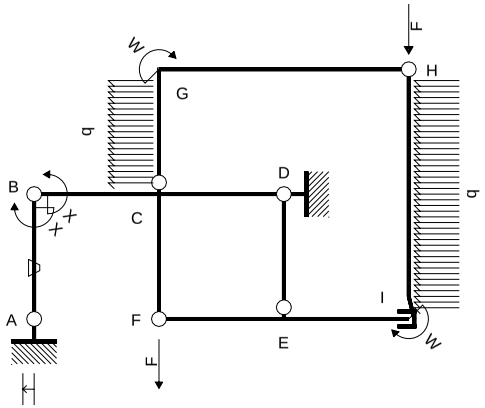
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H
I
1/4F
1/4F
2Fb
2F
B
A
27/16F
15/8Fb
15/8F
27/16F
15/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
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E
7/2F
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3/16F
13/4F
13/4Fb
39/8F
39/8F
3/16Fb
3/16F
13/4F
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C
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15/8Fb
27/16F
B

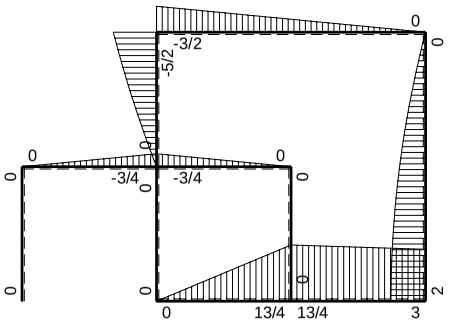




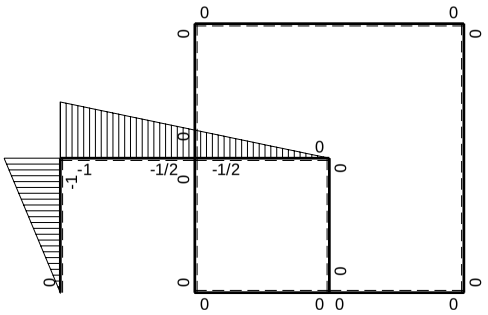

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-3/4
15/8
39/8
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F
3/4
-2
2
-3
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15/8
0
0
-13/4
0
1/4
F
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0
-5/2
15/8
3/16
3/16
0
15/8
0
0
0
0
0
0
13/4
13/4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 238953 CI01.xxxx.044

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 238953 CI01.xxxx.044

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $15/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -15/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

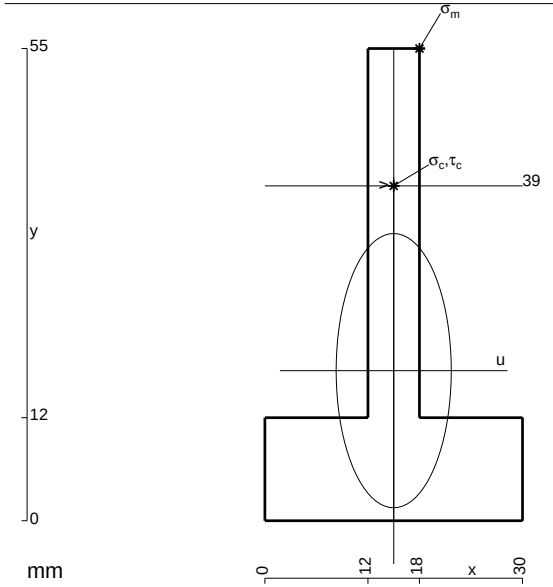
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

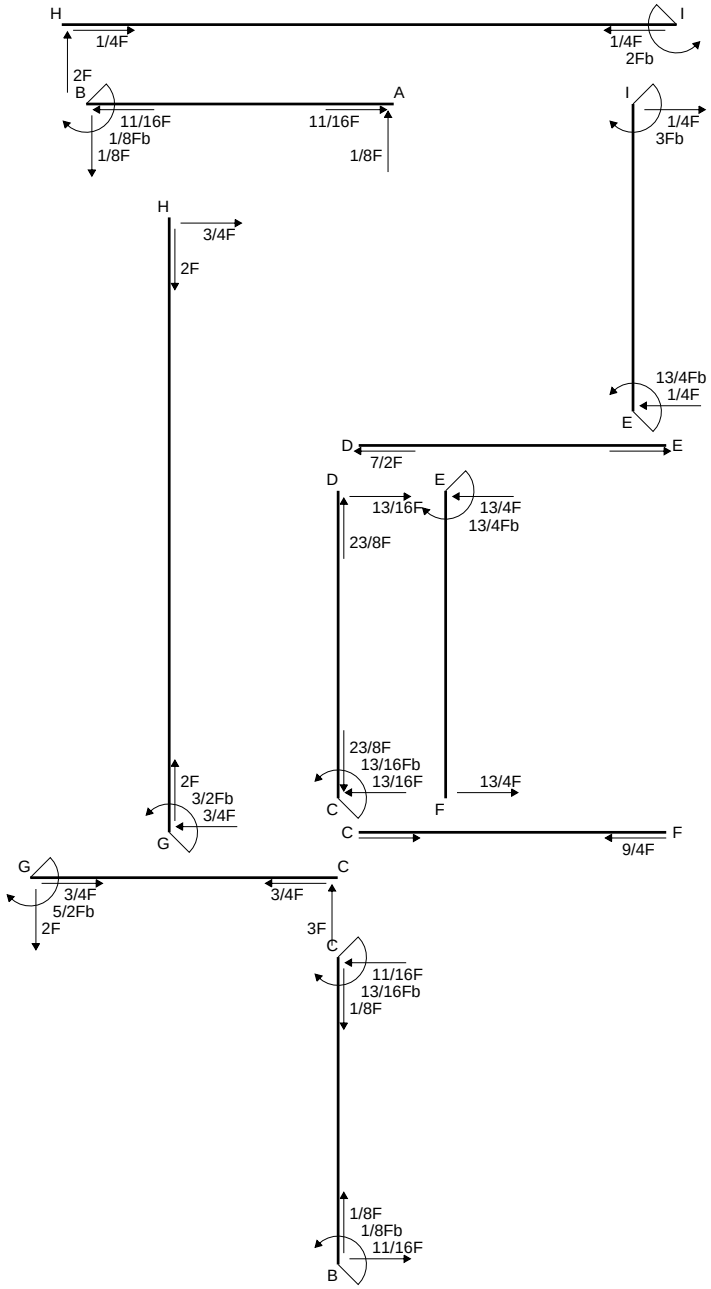


PROCEDIMENTO E RISULTATI 238953 CI01.xxxx.044



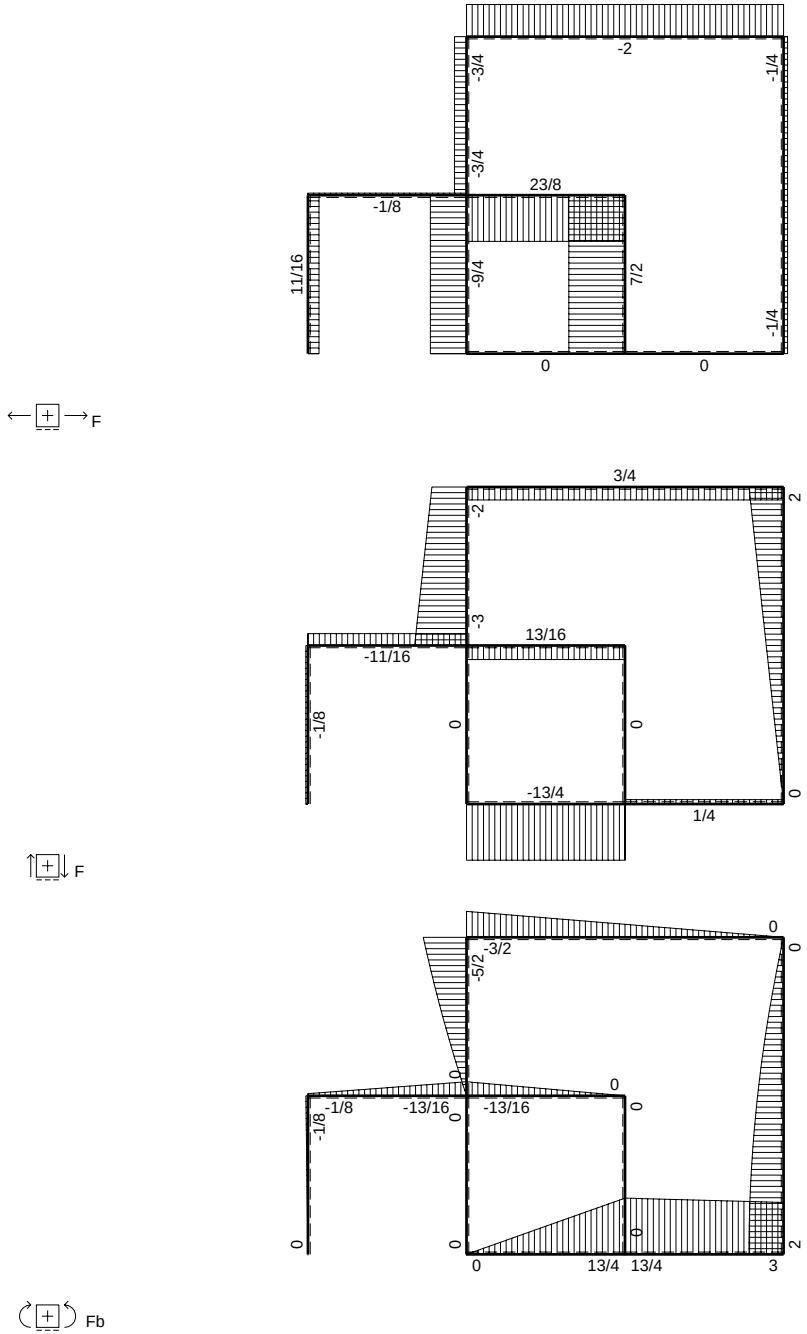

A = 618. $mm^2$
$J_u$ = 157731. $mm^4$
$J_v$ = 27774. $mm^4$
$y_g$ = 17.48 mm
$T_y$ = -1593. N
$M_x$ = 1003280. Nmm
$x_m$ = 18. mm
$y_m$ = 55. mm
$u_m$ = 3. mm
$v_m$ = 37.52 mm
$\sigma_m = -Mv/J_u$ = -238.6 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 39. mm
$v_c$ = 21.52 mm
$\sigma_c = -Mv/J_u$ = -136.9 $N/mm^2$
$\tau_c$ = 4.769 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137.1 $N/mm^2$
$S^*$ = 2834. $mm^3$

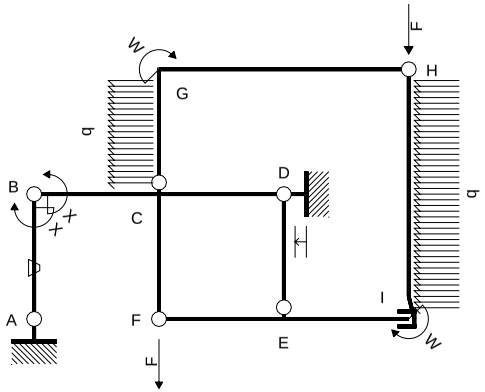
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
1/4F
1/4F
2Fb
2F
B
A
11/16F
1/8Fb
1/8F
11/16F
1/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
13/16F
13/4F
13/4Fb
23/8F
23/8F
13/16Fb
13/16F
13/4F
C
F
2F
3/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
5/2Fb
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3F
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11/16F
13/16Fb
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1/8F
1/8Fb
11/16F
B

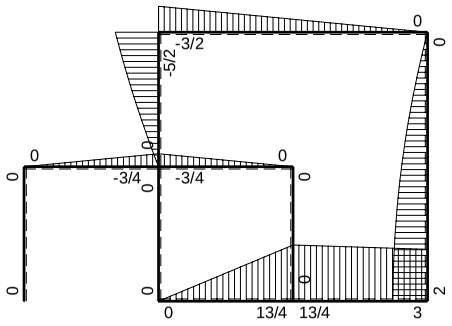




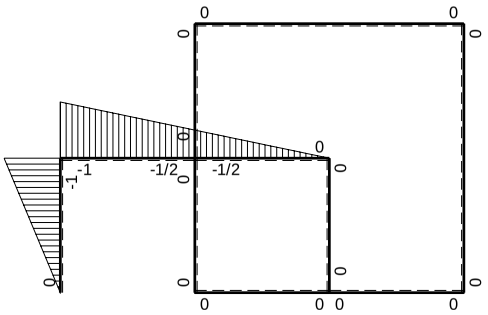

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-1/4
-3/4
23/8
-1/8
11/16
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7/2
-1/4
0
0
F
3/4
-2
2
-3
13/16
-11/16
-1/8
0
0
-13/4
0
1/4
F
0
-3/2
0
-5/2
0
0
-1/8
-13/16
-13/16
0
0
0
0
0
2
0
13/4
13/4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267087 CI01.xxxx.045

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PROCEDIMENTO E RISULTATI 267087 CI01.xxxx.045

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

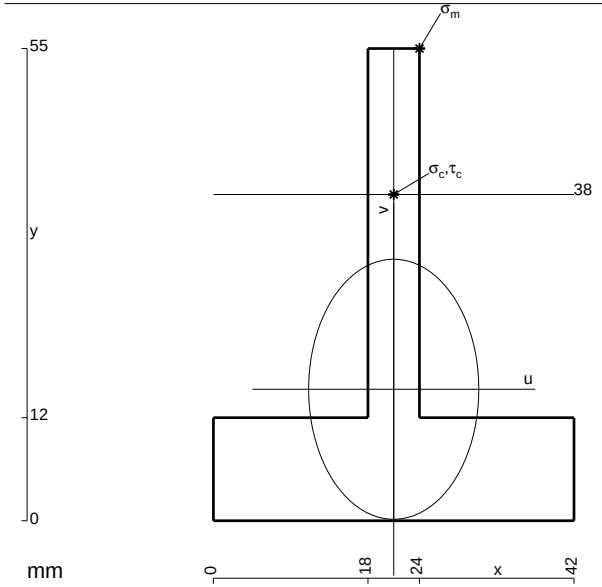
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

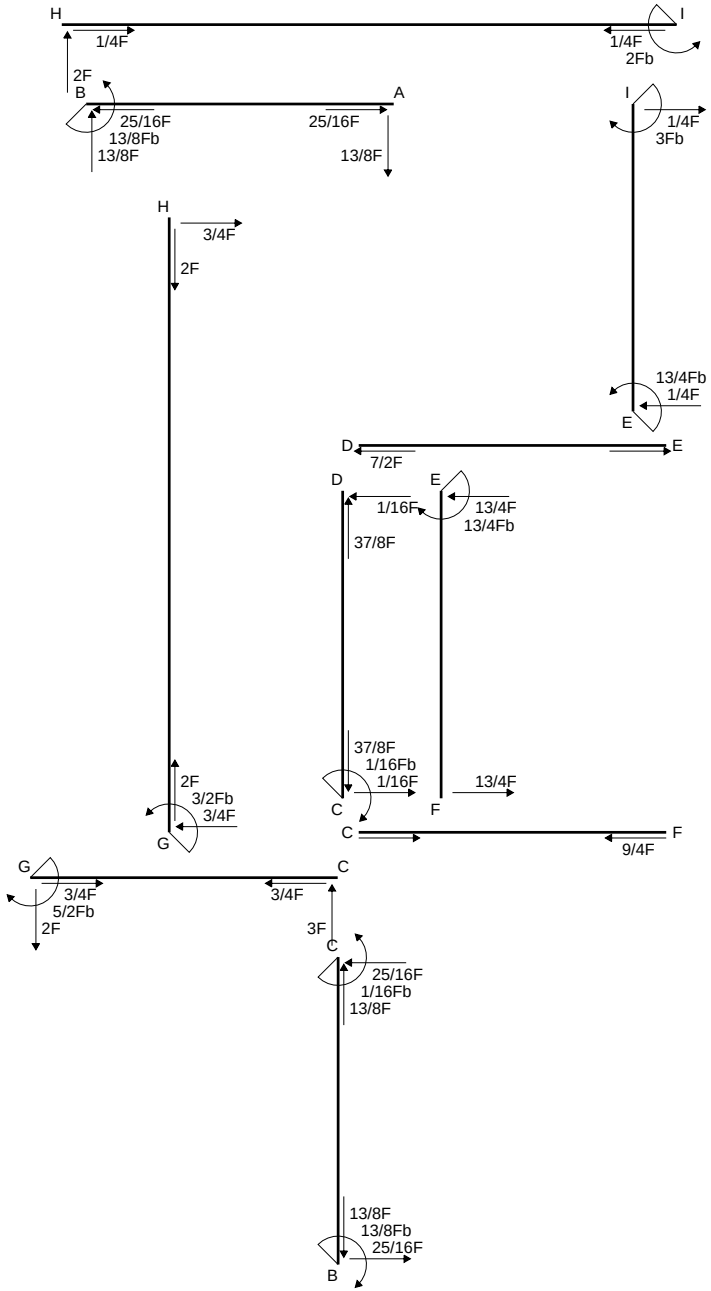
$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





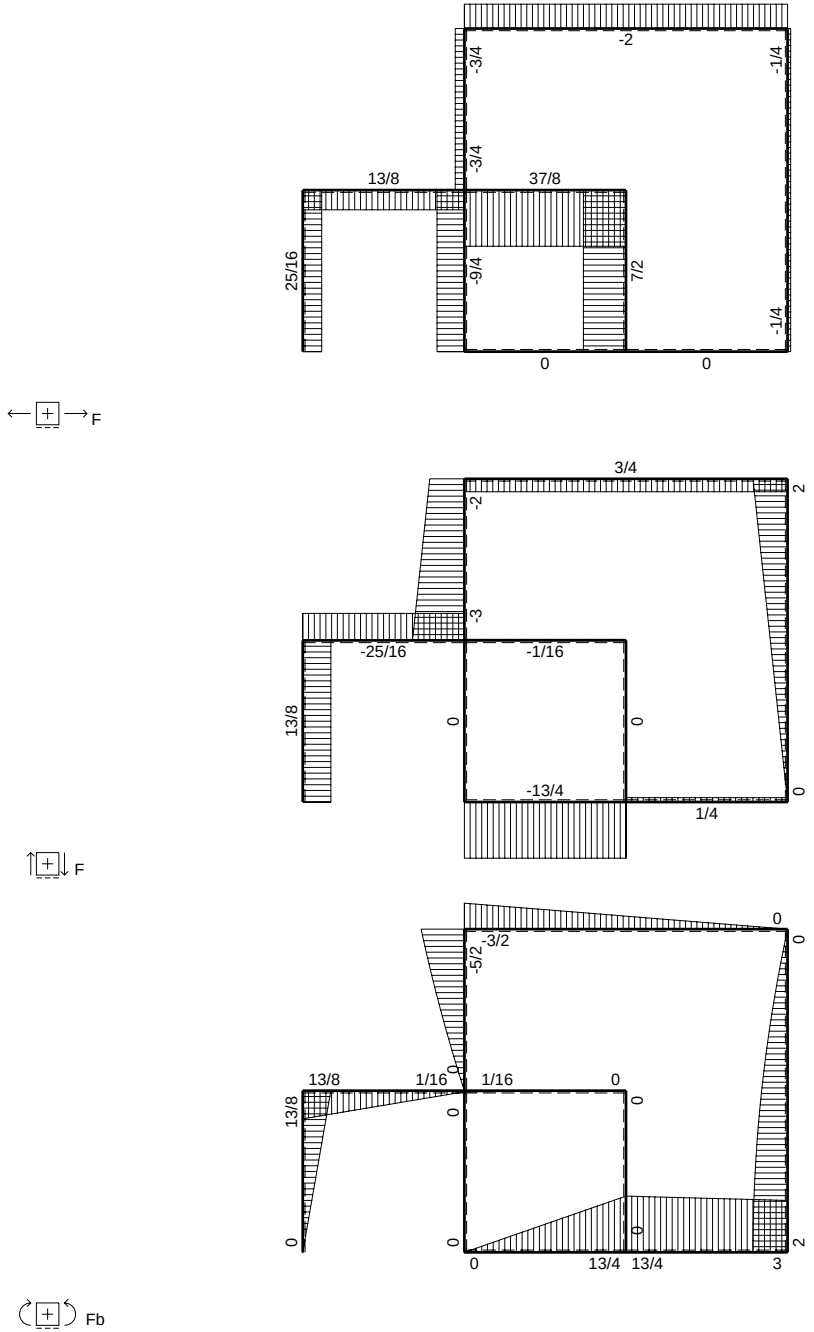


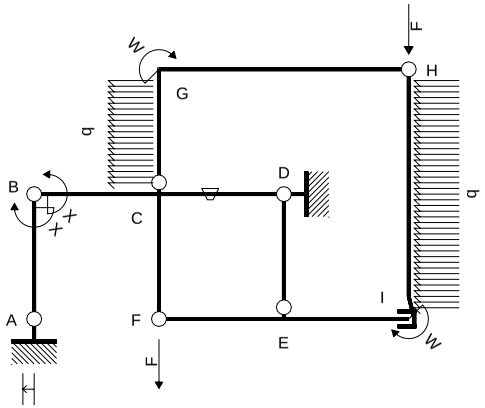

A = 762. $mm^2$
$J_u$ = 174852. $mm^4$
$J_v$ = 74862. $mm^4$
$y_g$ = 15.31 mm
$T_y$ = -1300. N
$M_x$ = 871000. Nmm
$x_m$ = 24. mm
$y_m$ = 55. mm
$u_m$ = 3. mm
$v_m$ = 39.69 mm
$\sigma_m = -Mv/J_u$ = -197.7 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 38. mm
$v_c$ = 22.69 mm
$\sigma_c = -Mv/J_u$ = -113. $N/mm^2$
$\tau_c$ = 3.942 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.2 $N/mm^2$
$S^*$ = 3181. $mm^3$


H
I
1/4F
1/4F
2Fb
2F
B
A
I
25/16F
13/8Fb
13/8F
25/16F
13/8F
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
1/16F
13/4F
13/4Fb
37/8F
37/8F
1/16Fb
1/16F
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2F
3/2Fb
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F
C
F
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C
3/4F
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13/8Fb
25/16F
B

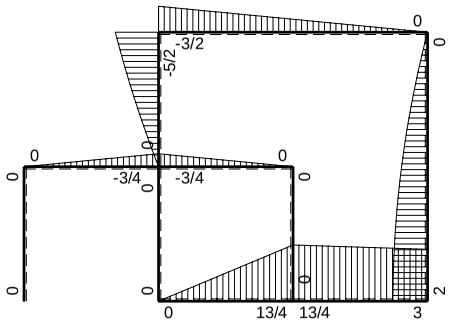




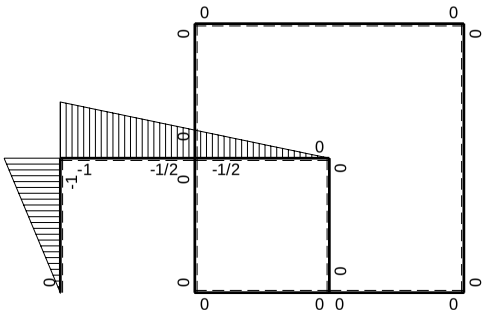

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F
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2
-3
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0
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13/8
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1/16
0
13/8
0
0
0
0
0
2
0
13/4
13/4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265892 CI01.xxxx.046

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PROCEDIMENTO E RISULTATI 265892 CI01.xxxx.046

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

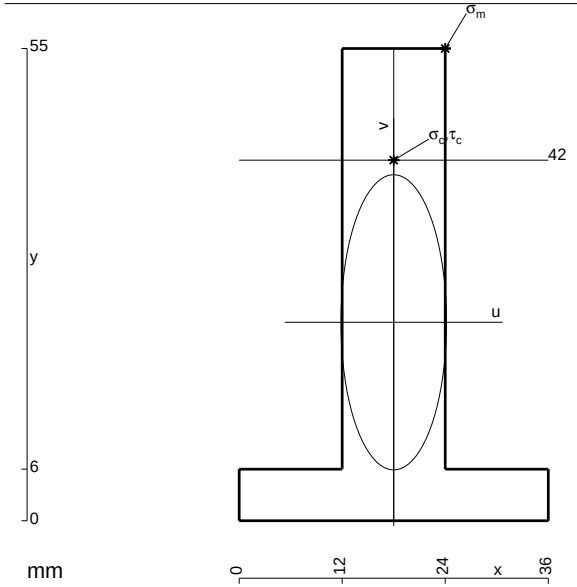
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

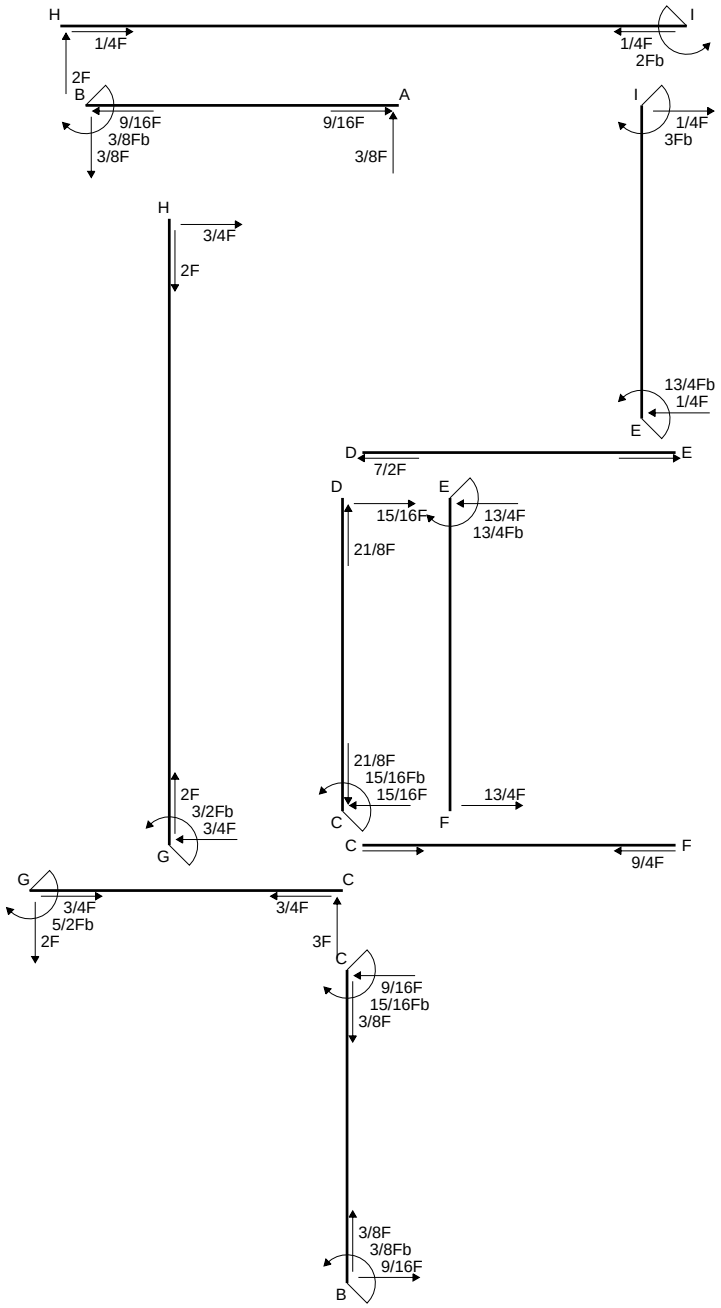


PROCEDIMENTO E RISULTATI 265892 CI01.xxxx.046



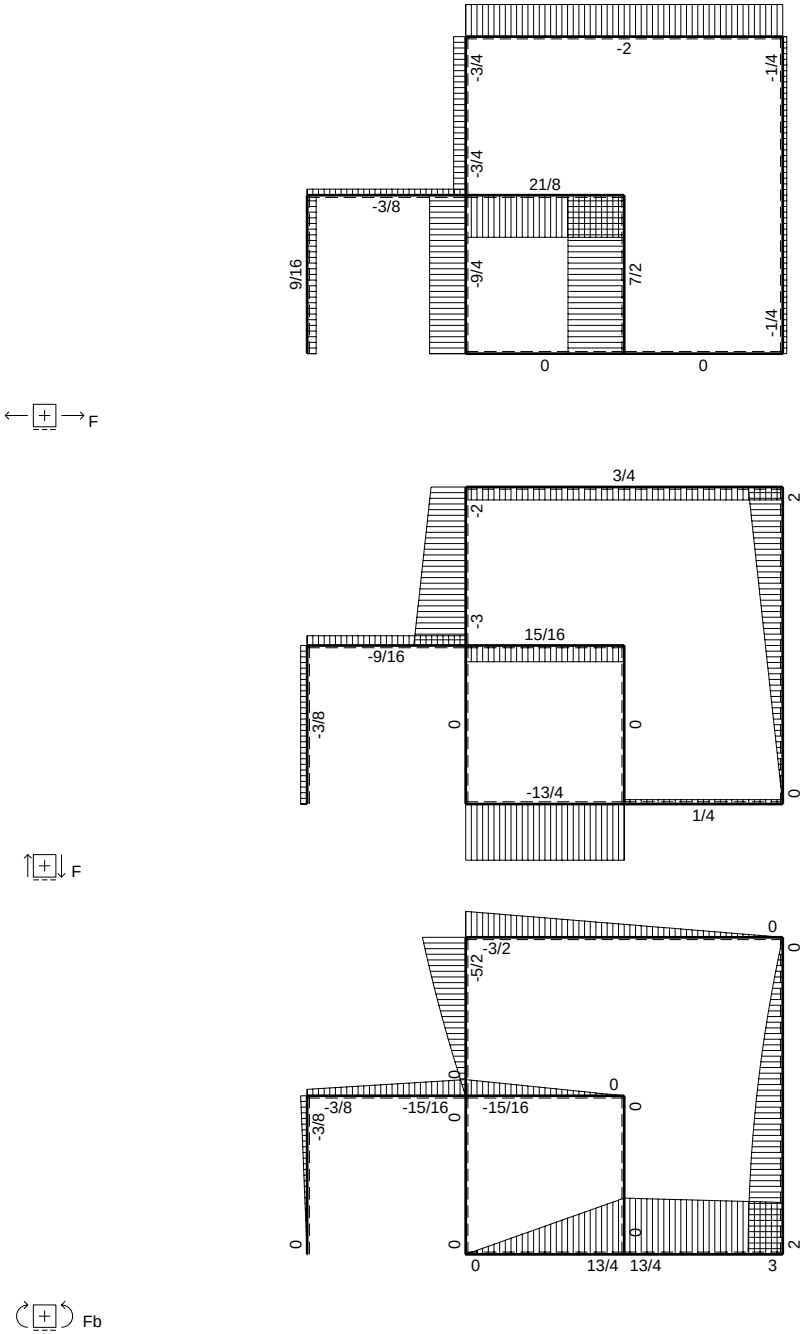

A = 804. $mm^2$
$J_u$ = 237762. $mm^4$
$J_v$ = 30384. $mm^4$
$y_g$ = 23.11 mm
$T_y$ = -2145. N
$M_x$ = 1544400. Nmm
$x_m$ = 24. mm
$y_m$ = 55. mm
$u_m$ = 6. mm
$v_m$ = 31.89 mm
$\sigma_m = -Mv/J_u$ = -207.1 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 42. mm
$v_c$ = 18.89 mm
$\sigma_c = -Mv/J_u$ = -122.7 $N/mm^2$
$\tau_c$ = 2.978 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.8 $N/mm^2$
$S^*$ = 3961. $mm^3$

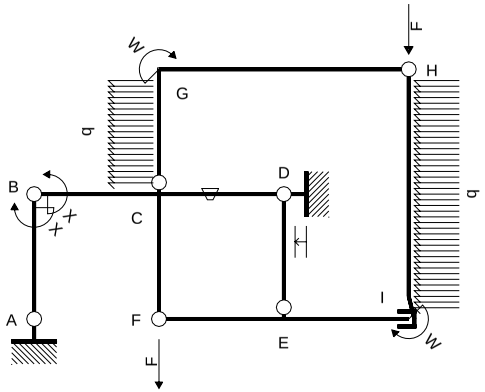
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H
1/4F
I
1/4F
2Fb
2F
B
A
9/16F
3/8Fb
3/8F
9/16F
3/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
15/16F
13/4F
13/4Fb
21/8F
21/8F
15/16Fb
15/16F
13/4F
C
F
2F
3/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
5/2Fb
2F
3F
C
9/16F
15/16Fb
3/8F
3/8F
3/8Fb
9/16F
B

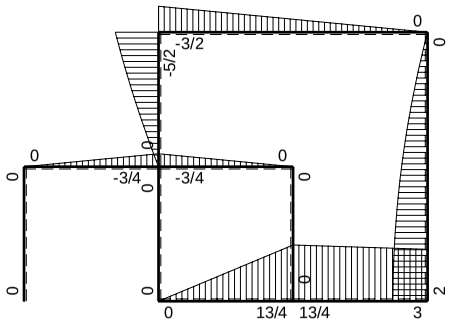




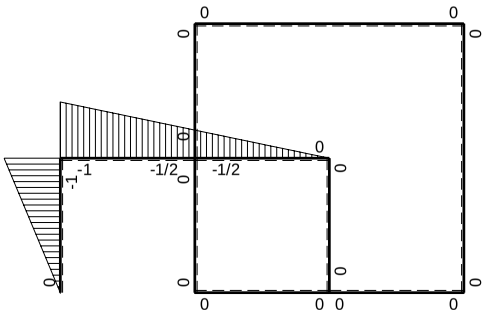

F
-2
-3/4
-1/4
-3/4
21/8
-3/8
9/16
-9/4
7/2
-1/4
0
0
F
3/4
2
-2
-3
15/16
-9/16
-3/8
0
0
-13/4
0
1/4
F
0
0
-3/2
-5/2
0
0
-3/8
-15/16
-15/16
0
0
-3/8
0
0
0
0
13/4
13/4
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264983 CI01.xxxx.047

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PROCEDIMENTO E RISULTATI 264983 CI01.xxxx.047

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-3/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 3/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

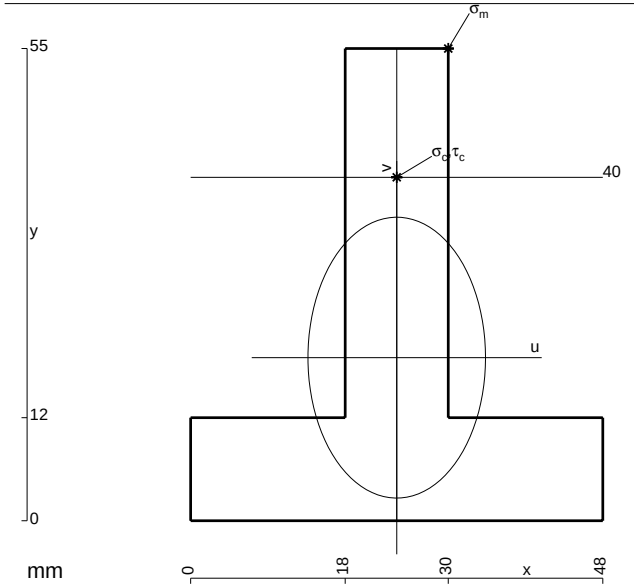
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$

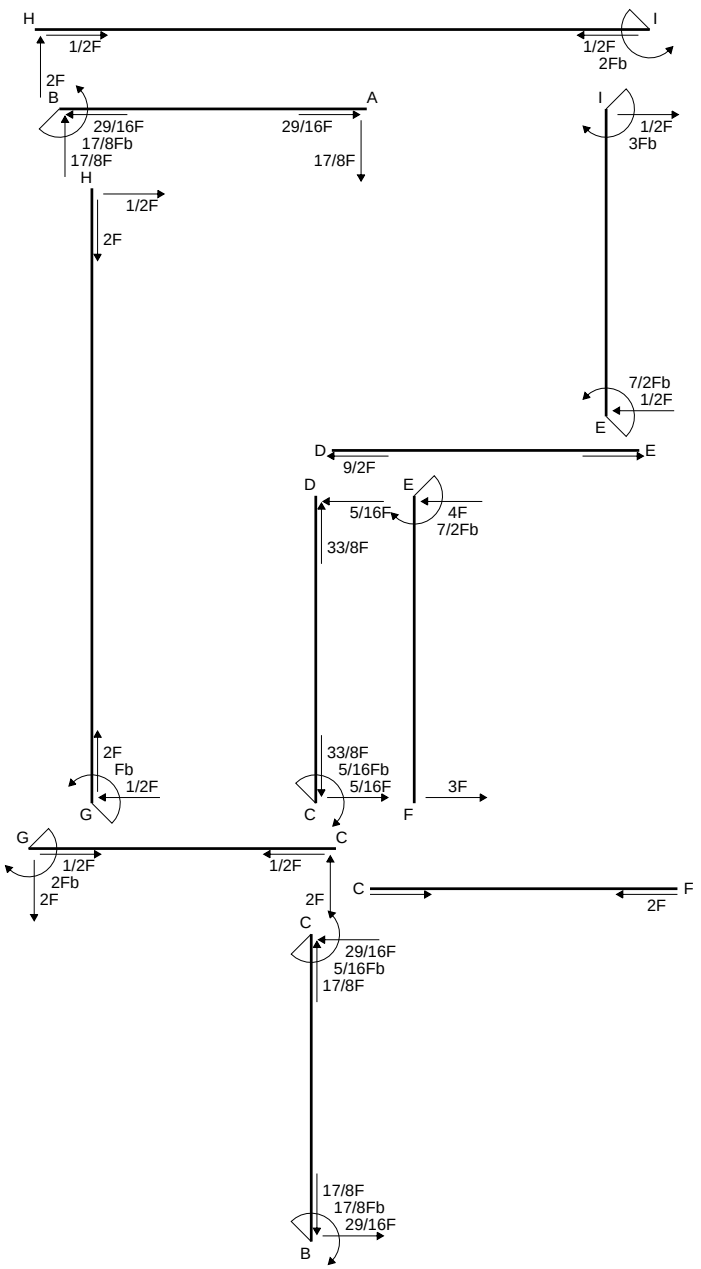


PROCEDIMENTO E RISULTATI 264983 CI01.xxxx.047



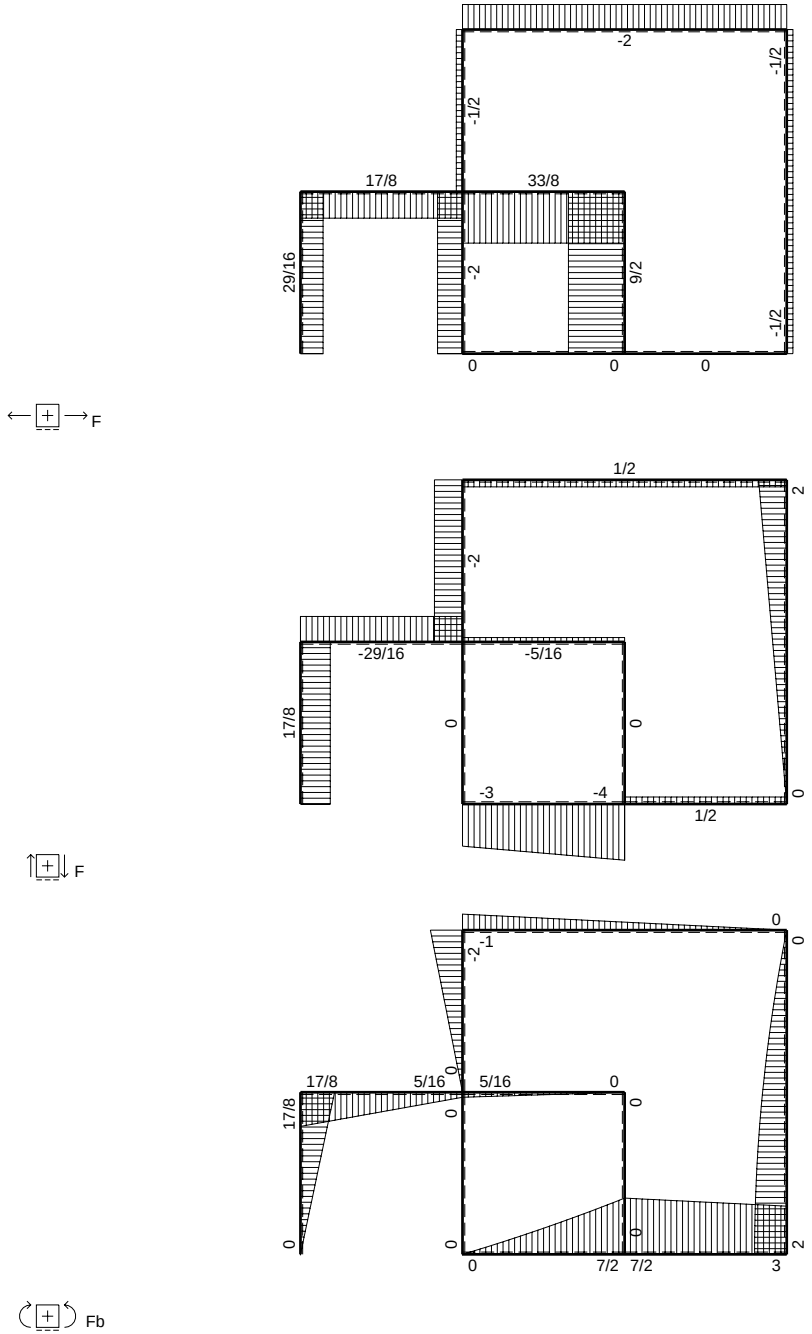

A = 1092. $mm^2$
$J_u$ = 292252. $mm^4$
$J_v$ = 116784. $mm^4$
$y_g$ = 18.99 mm
$T_y$ = -2340. N
$M_x$ = 1778400. Nmm
$x_m$ = 30. mm
$y_m$ = 55. mm
$u_m$ = 6. mm
$v_m$ = 36.01 mm
$\sigma_m = -Mv/J_u$ = -219.1 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 40. mm
$v_c$ = 21.01 mm
$\sigma_c = -Mv/J_u$ = -127.8 $N/mm^2$
$\tau_c$ = 3.424 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128. $N/mm^2$
$S^*$ = 5131. $mm^3$

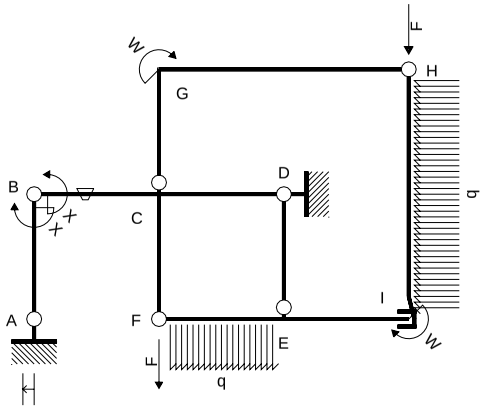
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H
I
1/2F
1/2F
2Fb
2F
B
A
29/16F
17/8Fb
17/8F
29/16F
17/8F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
5/16F
4F
7/2Fb
33/8F
2F
Fb
1/2F
G
33/8F
5/16Fb
5/16F
C
3F
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
2F
C
29/16F
5/16Fb
17/8F
17/8F
17/8Fb
29/16F
B

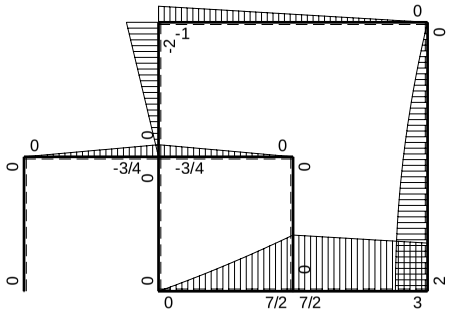




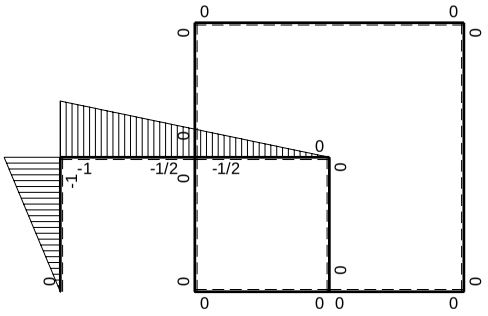

F
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17/8
33/8
29/16
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9/2
-1/2
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0
F
1/2
2
-2
-29/16
-5/16
17/8
0
0
0
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-4
1/2
0
F
0
0
-2
-1
0
17/8
5/16
5/16
0
17/8
0
0
0
0
0
2
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 269177 CI01.xxxx.048

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PROCEDIMENTO E RISULTATI 269177 CI01.xxxx.048

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $17/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -17/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

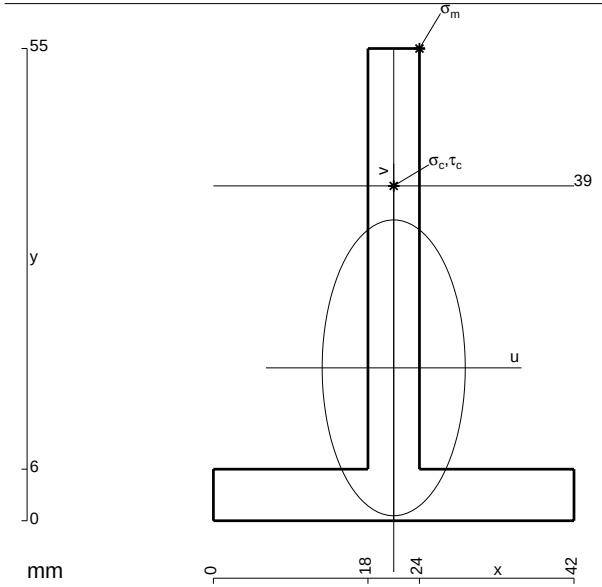
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

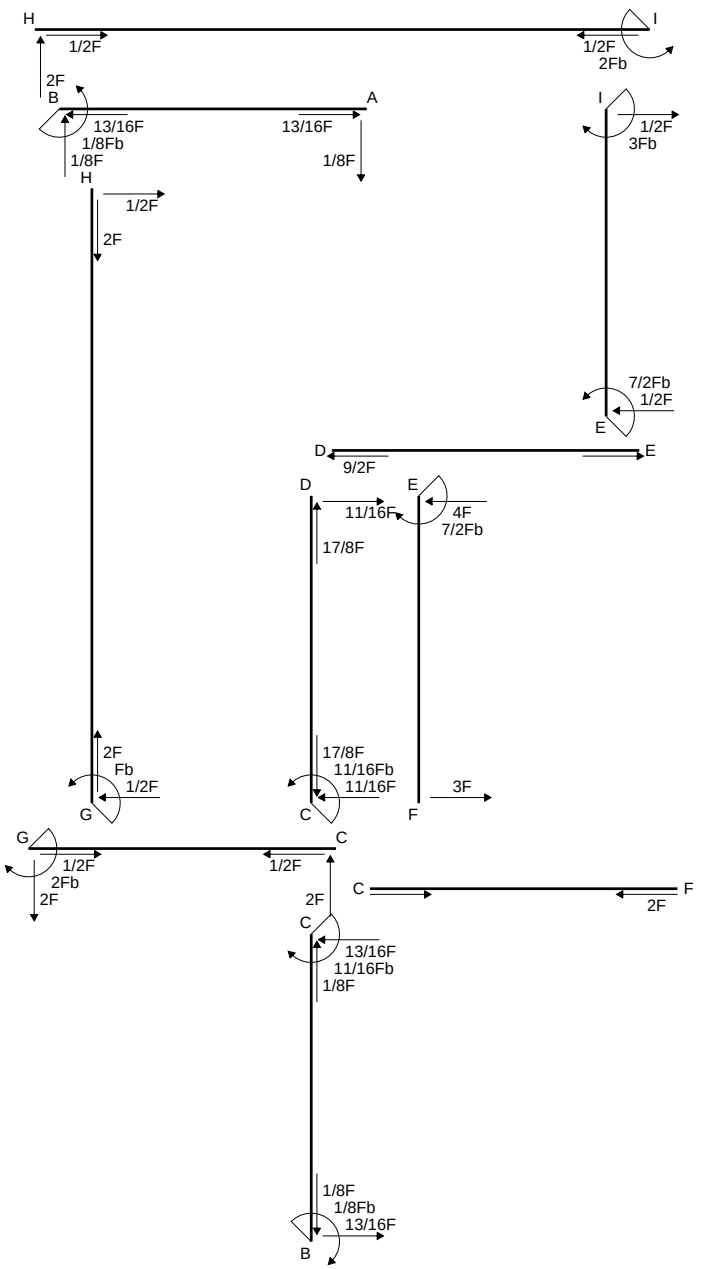
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





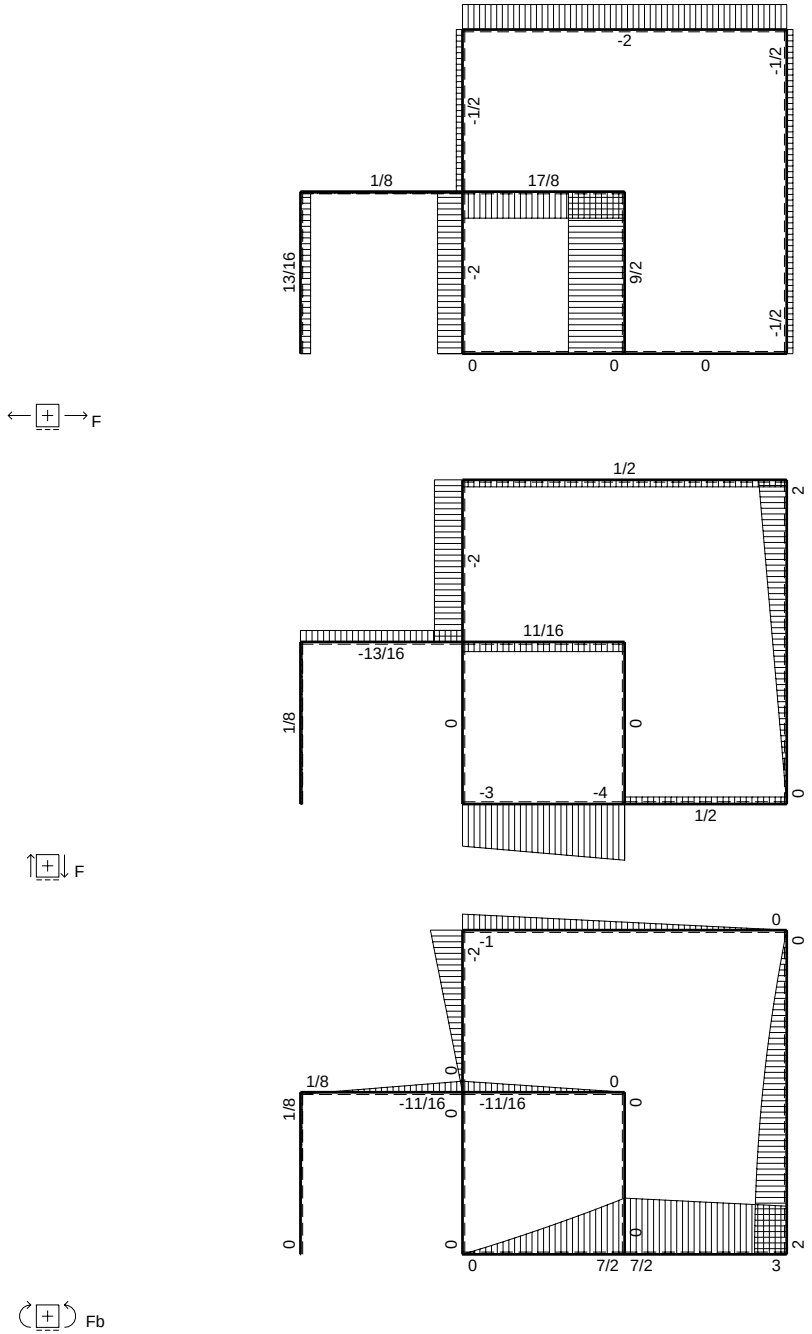


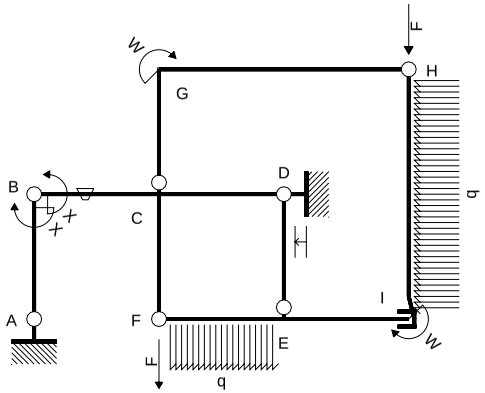

A = 546. $mm^2$
$J_u$ = 162198. $mm^4$
$J_v$ = 37926. $mm^4$
$y_g$ = 17.81 mm
$T_y$ = -1400. N
$M_x$ = 992250. Nmm
$x_m$ = 24. mm
$y_m$ = 55. mm
$u_m$ = 3. mm
$v_m$ = 37.19 mm
$\sigma_m = -Mv/J_u$ = -227.5 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 39. mm
$v_c$ = 21.19 mm
$\sigma_c = -Mv/J_u$ = -129.6 $N/mm^2$
$\tau_c$ = 4.032 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.8 $N/mm^2$
$S^*$ = 2802. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
I
13/16F
1/8Fb
1/8F
13/16F
1/8F
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
11/16F
4F
7/2Fb
17/8F
2F
Fb
1/2F
G
17/8F
11/16Fb
11/16F
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3F
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G
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1/2F
2Fb
2F
1/2F
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1/8F
1/8Fb
13/16F
B

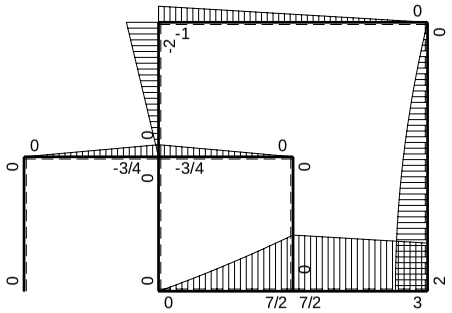




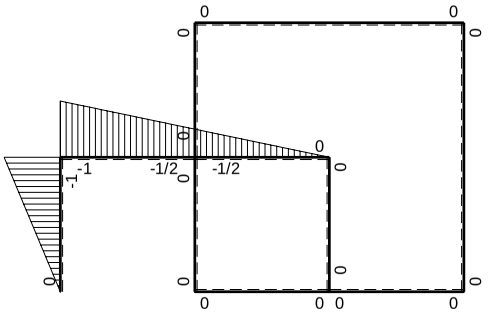

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-1/2
1/8
17/8
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9/2
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1/2
2
-2
11/16
-13/16
1/8
0
0
-3
-4
0
1/2
F
0
0
-2
-1
1/8
0
0
-11/16
-11/16
1/8
0
0
0
0
0
2
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265373 CI01.xxxx.049

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 265373 CI01.xxxx.049

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

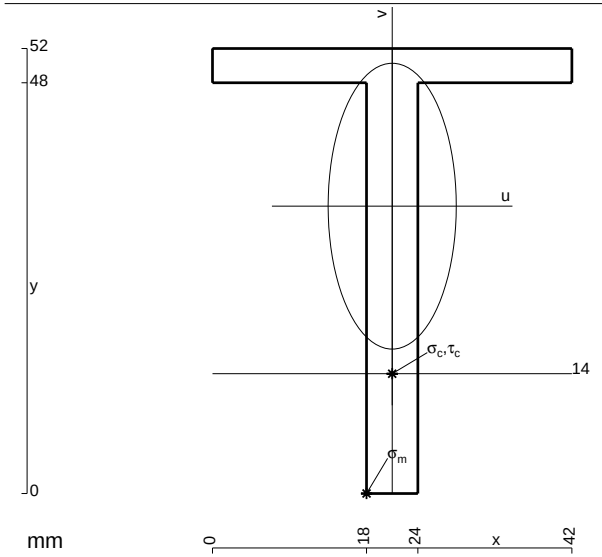
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

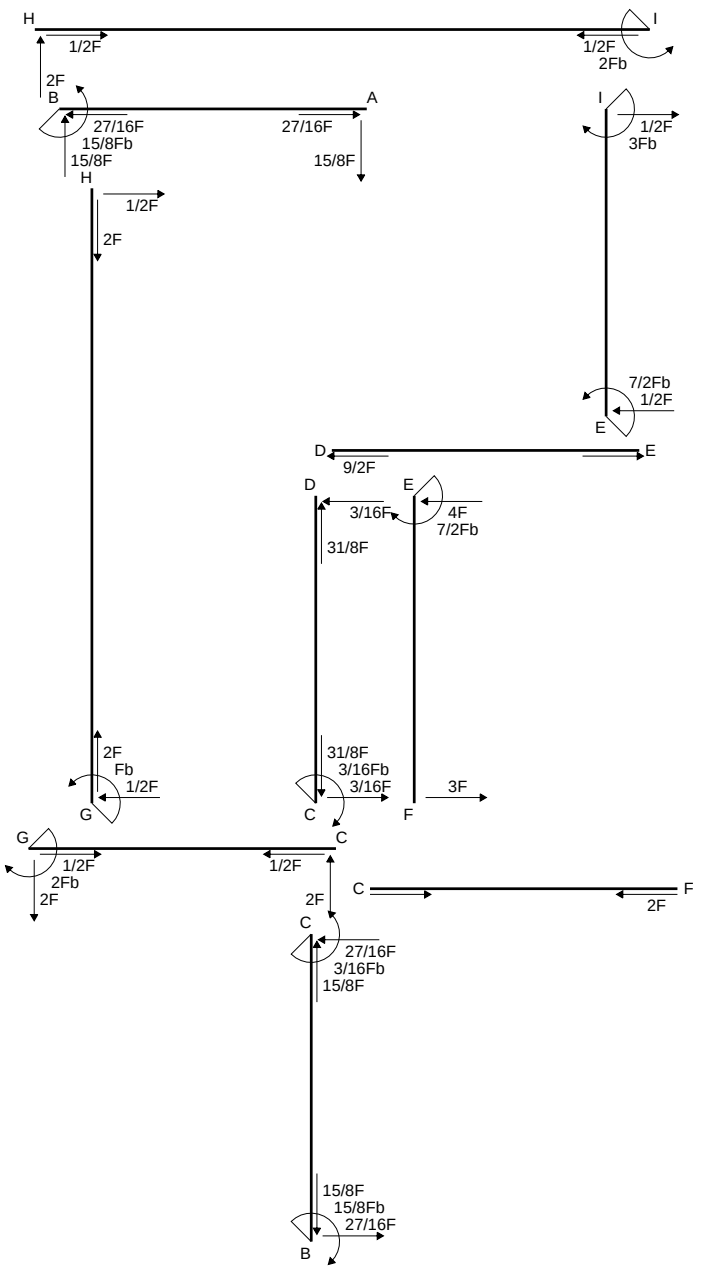


PROCEDIMENTO E RISULTATI 265373 CI01.xxxx.049



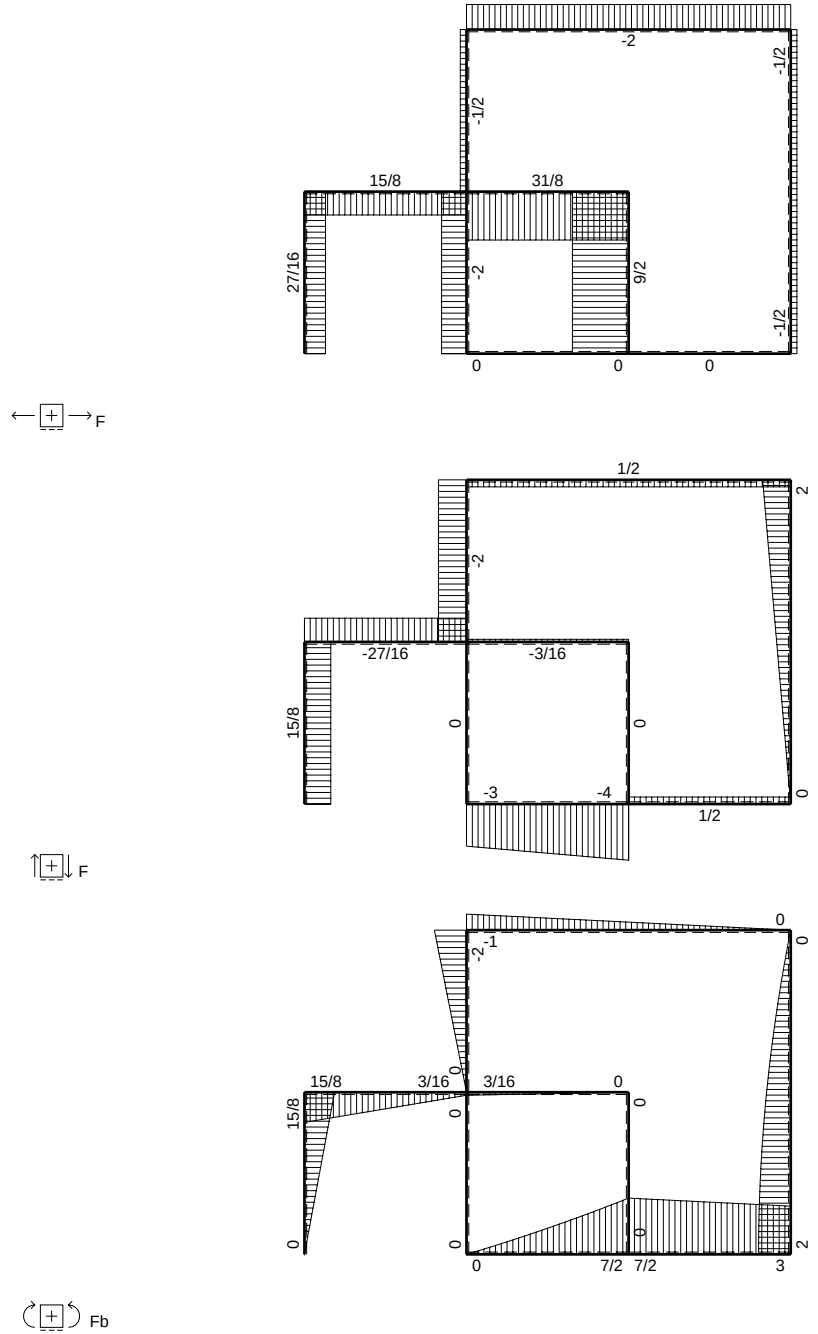

A = 456. $mm^2$
$J_u$ = 127247. $mm^4$
$J_v$ = 25560. $mm^4$
$y_g$ = 33.58 mm
$T_y$ = -1280. N
$M_x$ = 907200. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -33.58 mm
$\sigma_m = -Mv/J_u$ = 239.4 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 14. mm
$v_c$ = -19.58 mm
$\sigma_c = -Mv/J_u$ = 139.6 $N/mm^2$
$\tau_c$ = 3.743 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 139.7 $N/mm^2$
$S^*$ = 2233. $mm^3$

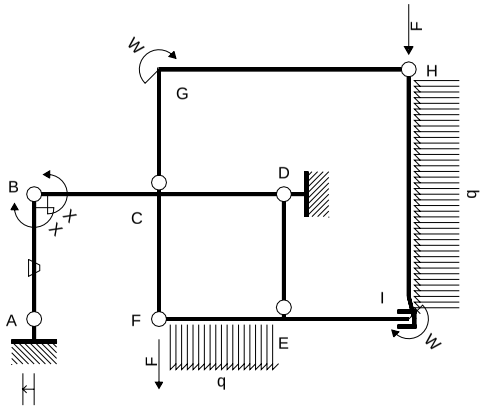
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H
I
1/2F
1/2F
2Fb
2F
B
A
I
27/16F
15/8Fb
15/8F
27/16F
15/8F
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
3/16F
4F
7/2Fb
31/8F
2F
Fb
1/2F
31/8F
3/16Fb
3/16F
3F
G
C
F
G
C
1/2F
2Fb
2F
1/2F
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F
2F
2F
C
27/16F
3/16Fb
15/8F
15/8F
15/8Fb
27/16F
B

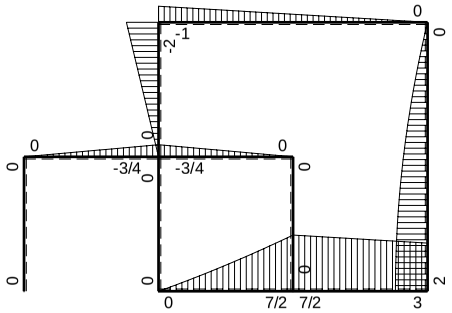




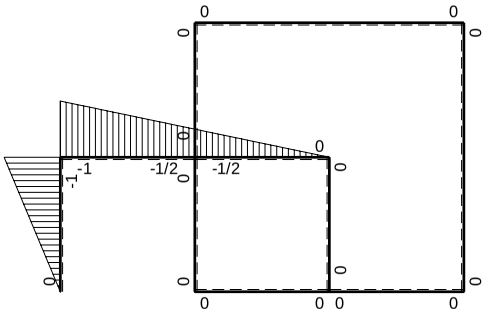

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-1/2
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15/8
31/8
27/16
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F
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-2
2
-27/16
-3/16
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0
-3
-4
0
1/2
F
0
0
-2
-1
15/8
3/16
3/16
0
15/8
0
0
0
0
0
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 266093 CI01.xxxx.050

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PROCEDIMENTO E RISULTATI 266093 CI01.xxxx.050

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $15/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -15/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

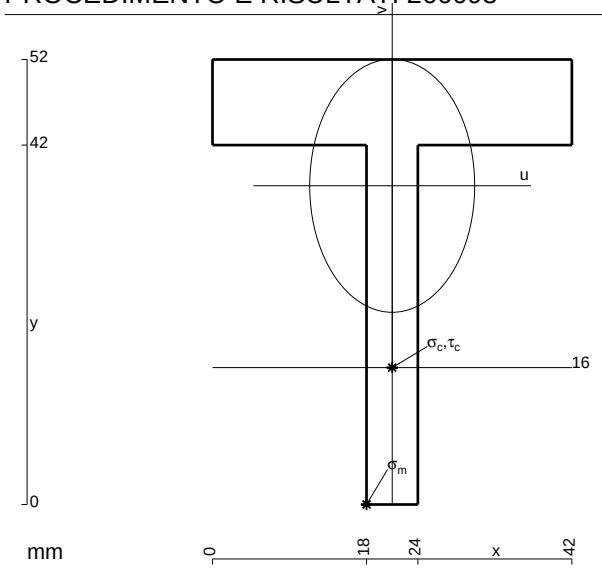
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

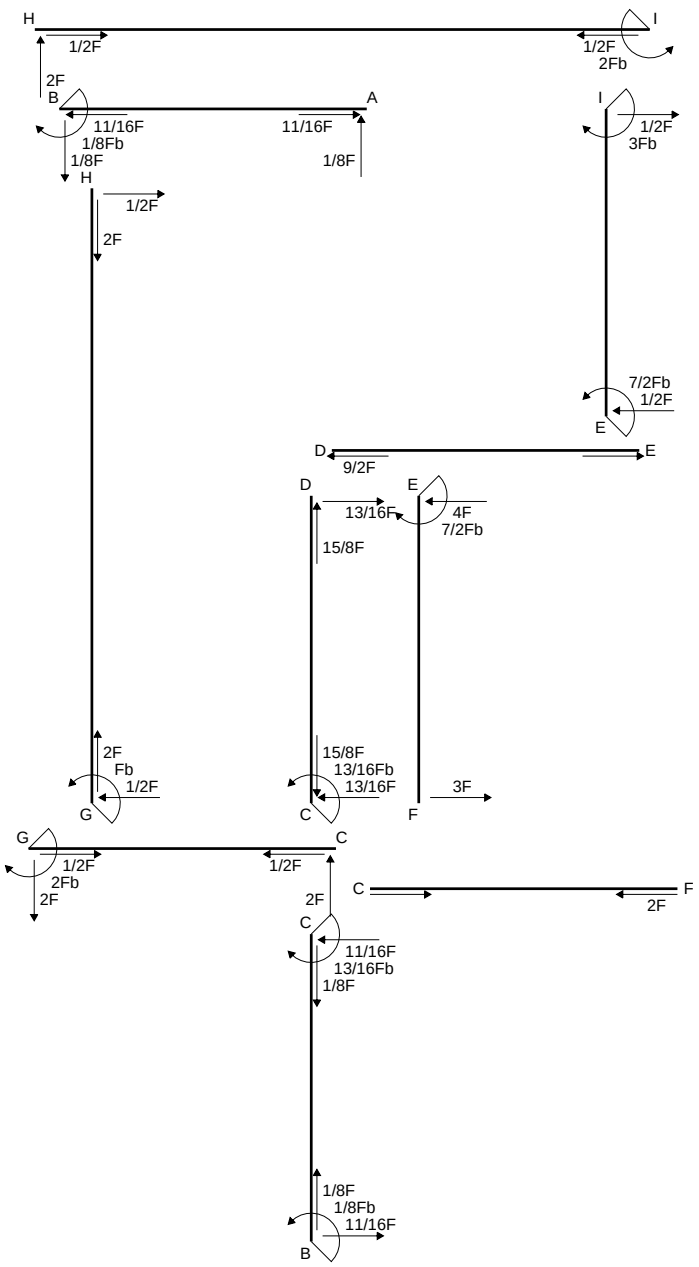
$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





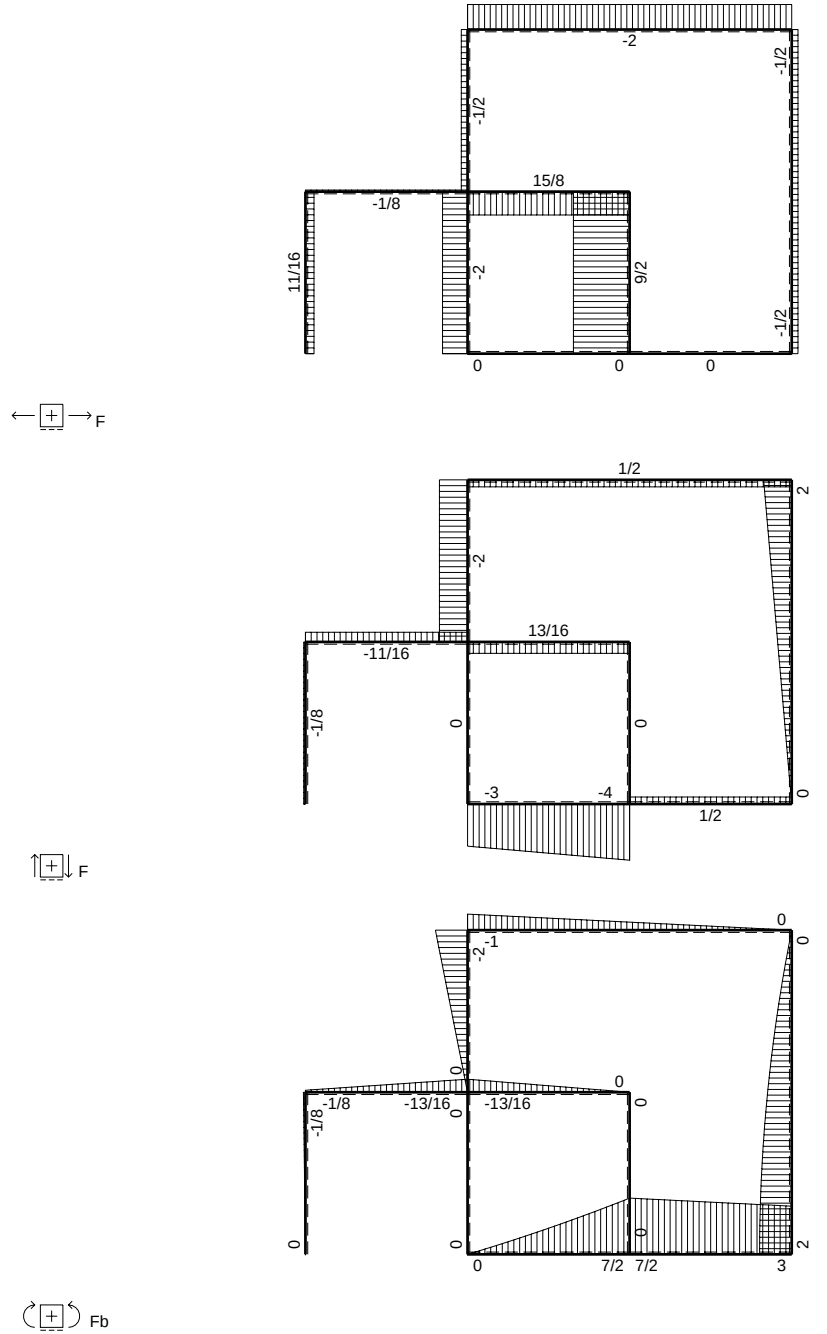


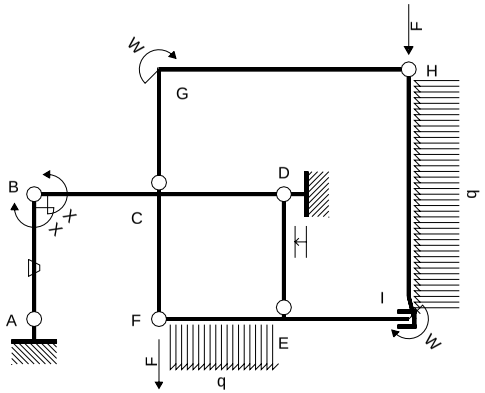

A = 672. $mm^2$
$J_u$ = 147014. $mm^4$
$J_v$ = 62496. $mm^4$
$y_g$ = 37.25 mm
$T_y$ = -2080. N
$M_x$ = 782600. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -37.25 mm
$\sigma_m = -Mv/J_u$ = 198.3 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -21.25 mm
$\sigma_c = -Mv/J_u$ = 113.1 $N/mm^2$
$\tau_c$ = 6.621 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 113.7 $N/mm^2$
$S^*$ = 2808. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
I
11/16F
11/16F
1/2F
1/8Fb
3Fb
1/8F
1/8F
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
13/16F
4F
7/2Fb
15/8F
2F
Fb
1/2F
15/8F
13/16Fb
13/16F
3F
G
C
F
G
C
1/2F
2Fb
1/2F
2F
C
F
2F
C
11/16F
13/16Fb
1/8F
1/8F
1/8Fb
11/16F
B

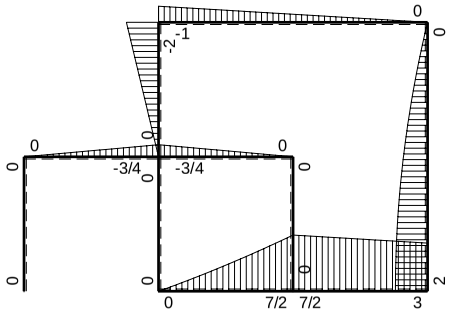




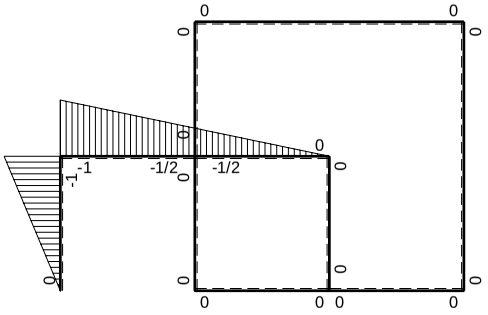

-2
-1/2
-1/2
15/8
-1/8
11/16
-2
9/2
-1/2
0
0
0
F
1/2
2
-2
13/16
-11/16
-1/8
0
0
-3
-4
0
1/2
F
0
0
-2
-1
0
0
-1/8
-13/16
-13/16
0
0
0
0
0
0
0
7/2
7/2
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 310123 CI01.xxxx.051

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PROCEDIMENTO E RISULTATI 310123 CI01.xxxx.051

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

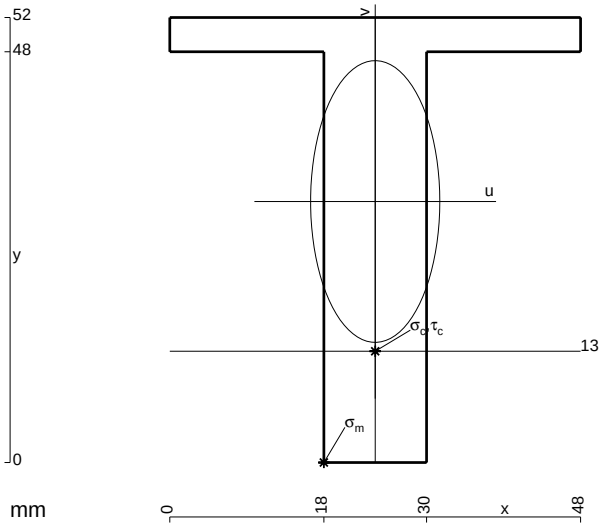
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$








$A = 768.\ mm^2$

$J_u = 208192.\ mm^4$

$J_v = 43776.\ mm^4$

$y_g = 30.5\ mm$

$T_y = -3480.\ N$

$M_x = 1431150.\ Nmm$

$x_m = 18.\ mm$

$u_m = -6.\ mm$

$v_m = -30.5\ mm$

$\sigma_m = -Mv/J_u = 209.7\ N/mm^2$

$x_c = 24.\ mm$

$y_c = 13.\ mm$

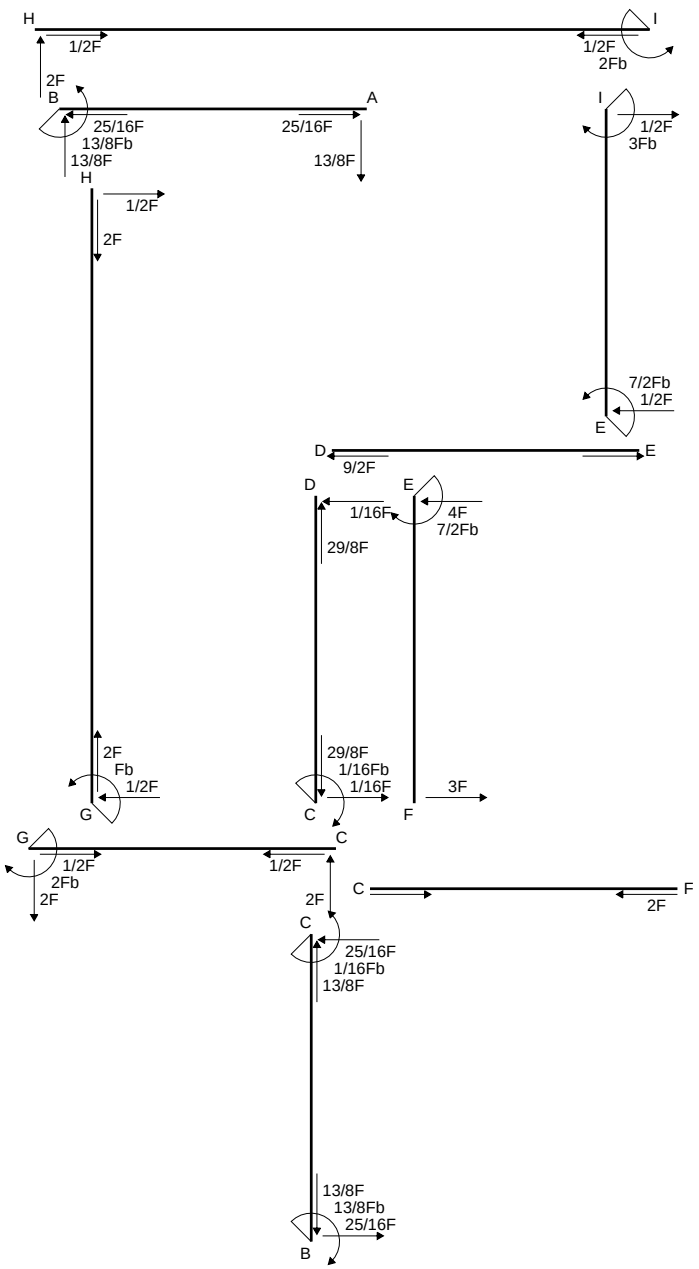
$v_c = -17.5\ mm$

$\sigma_c = -Mv/J_u = 120.3\ N/mm^2$

$\tau_c = 5.215\ N/mm^2$

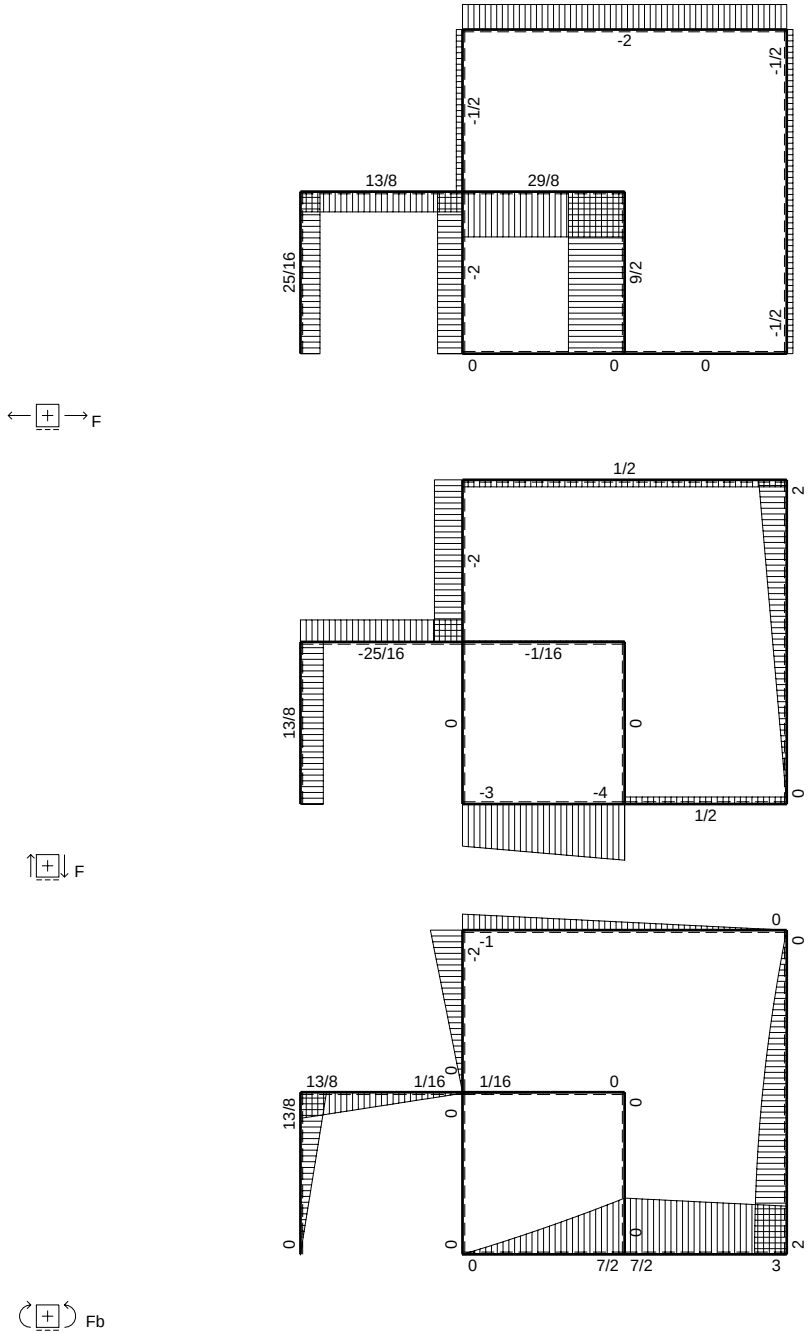
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 120.6\ N/mm^2$

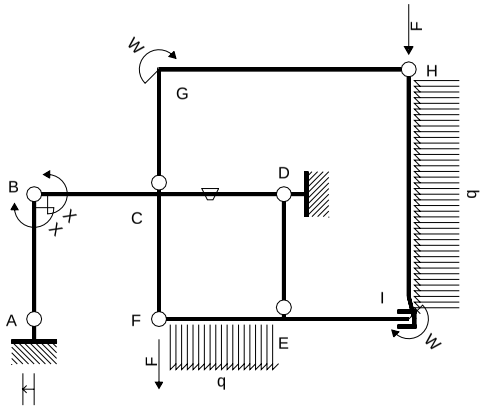
$S^* = 3744.\ mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
25/16F
13/8Fb
13/8F
25/16F
13/8F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
1/16F
4F
7/2Fb
29/8F
2F
Fb
1/2F
G
29/8F
1/16Fb
1/16F
C
3F
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
2F
C
25/16F
1/16Fb
13/8F
13/8F
13/8Fb
25/16F
B

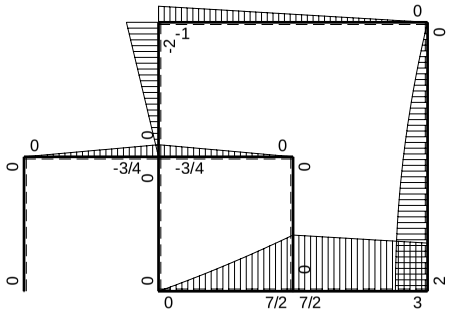




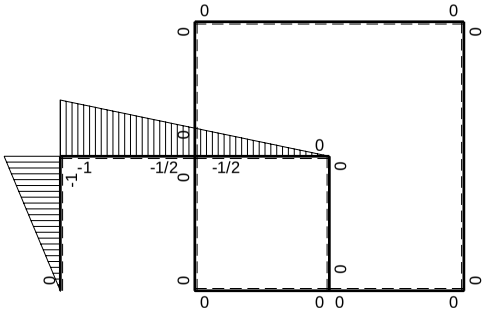

-2
-1/2
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13/8
29/8
25/16
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9/2
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F
1/2
2
-2
-25/16
-1/16
13/8
0
0
-3
-4
0
1/2
F
0
0
-2
-1
13/8
1/16
1/16
0
0
13/8
0
0
0
0
0
2
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264851 CI01.xxxx.052

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 264851 CI01.xxxx.052

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

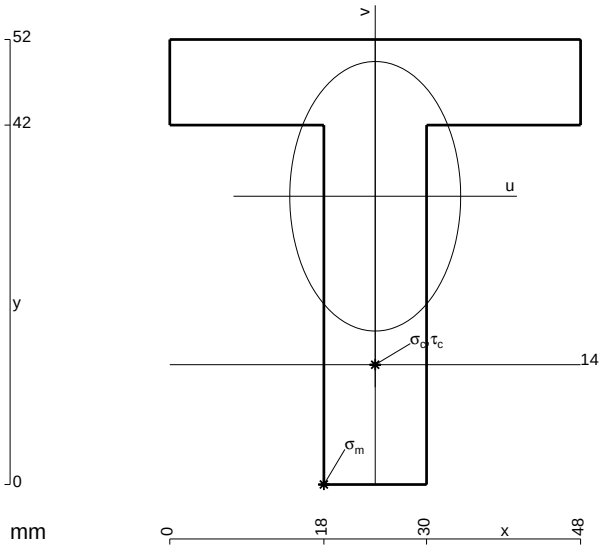
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

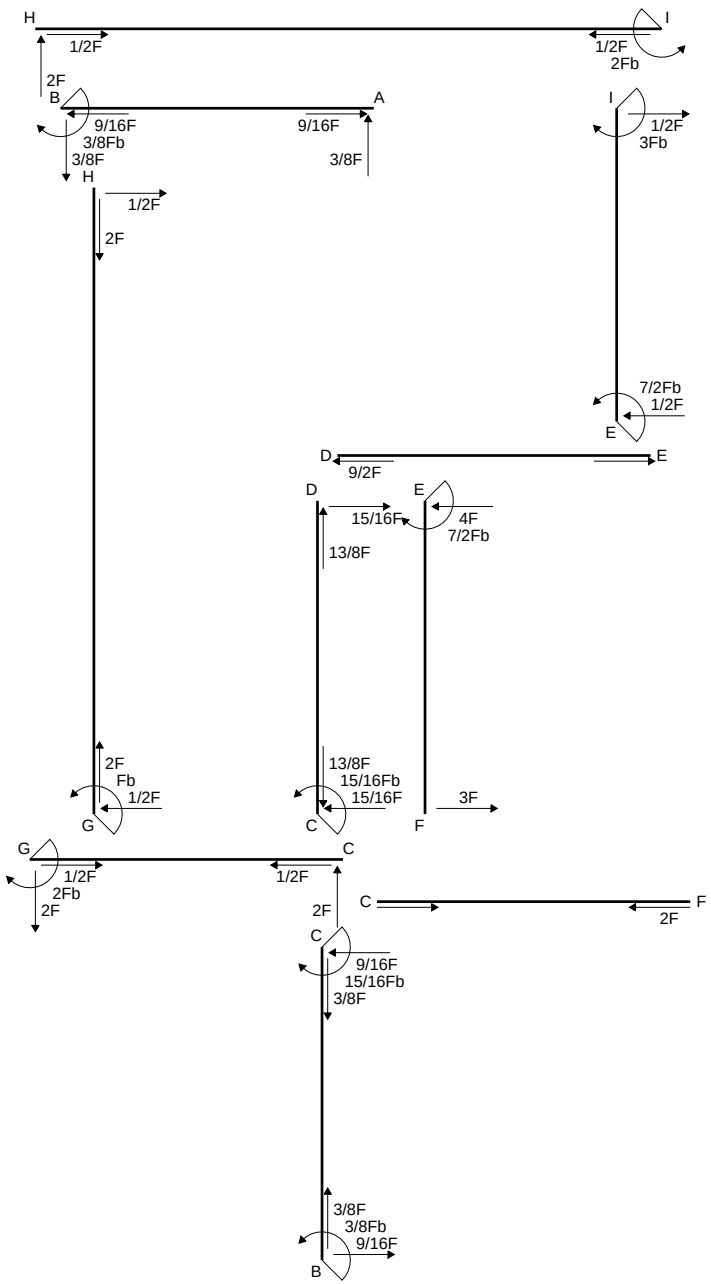


PROCEDIMENTO E RISULTATI 264851 CI01.xxxx.052



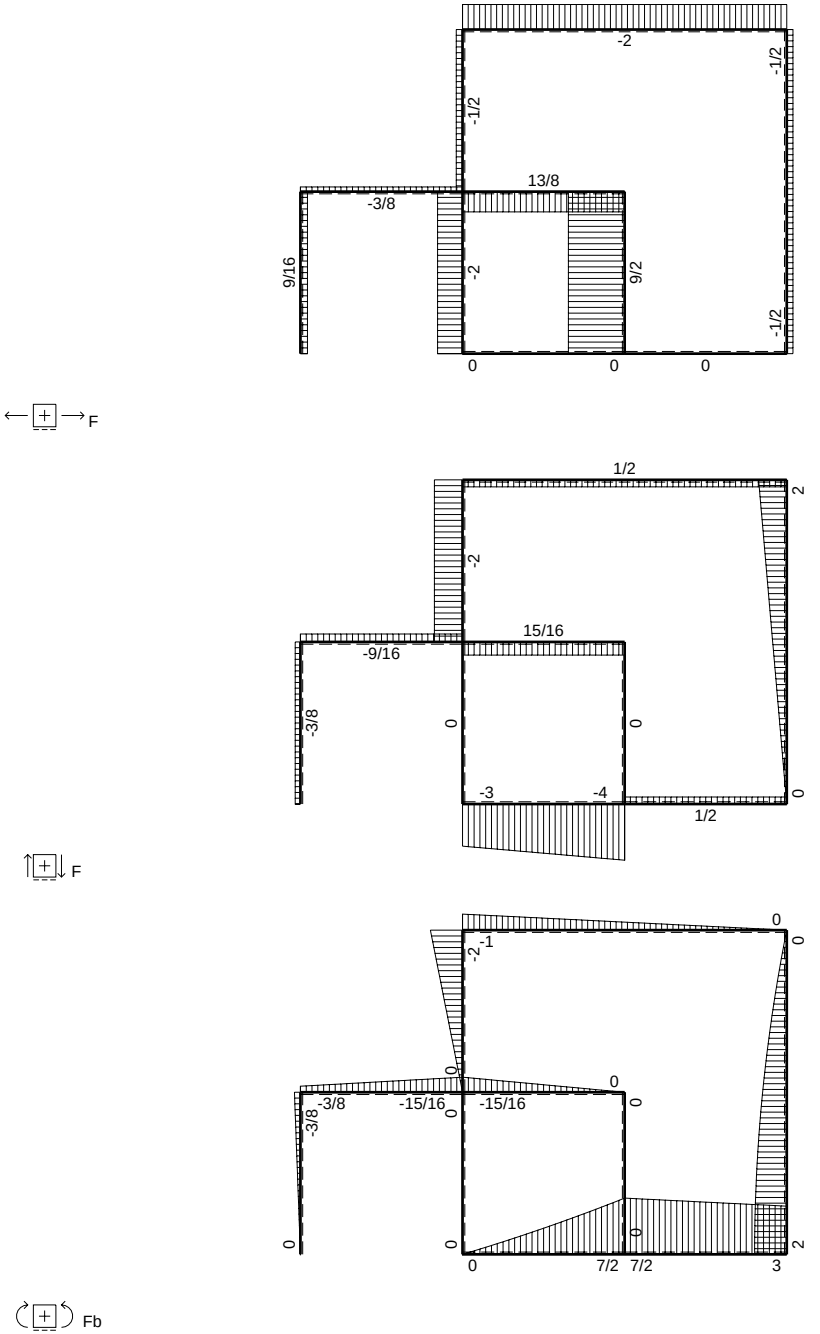

A = 984. $mm^2$
$J_u$ = 244285. $mm^4$
$J_v$ = 98208. $mm^4$
$y_g$ = 33.68 mm
$T_y$ = -3560. N
$M_x$ = 1588650. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -33.68 mm
$\sigma_m = -Mv/J_u$ = 219. $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 14. mm
$v_c$ = -19.68 mm
$\sigma_c = -Mv/J_u$ = 128. $N/mm^2$
$\tau_c$ = 5.444 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.4 $N/mm^2$
$S^*$ = 4483. $mm^3$

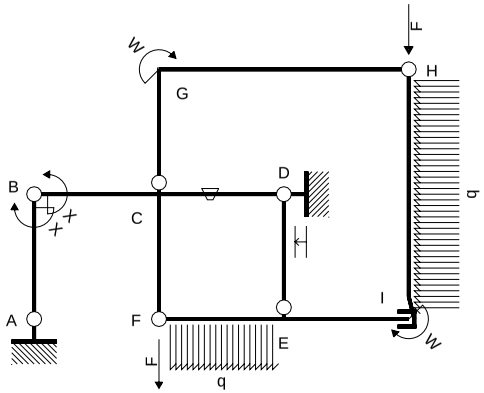
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
1/2F
1/2F
2Fb
2F
B
A
9/16F
3/8Fb
9/16F
3/8F
3/8F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
15/16F
4F
7/2Fb
13/8F
2F
Fb
1/2F
13/8F
15/16Fb
15/16F
3F
G
C
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
2F
C
9/16F
15/16Fb
3/8F
3/8F
3/8Fb
9/16F
B

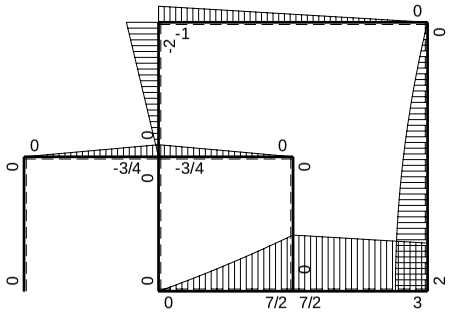




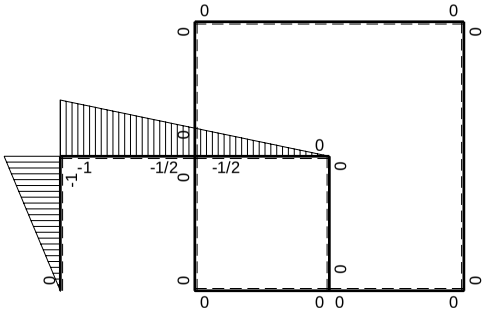

F
-2
-1/2
-1/2
13/8
-3/8
9/16
-2
9/2
-1/2
0
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2
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15/16
-9/16
-3/8
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0
0
-2
-1
0
0
0
-3/8
-3/8
-15/16
-15/16
0
0
0
0
0
0
7/2
7/2
3
2




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267850 CI01.xxxx.053

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-3/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 3/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 267850 CI01.xxxx.053

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

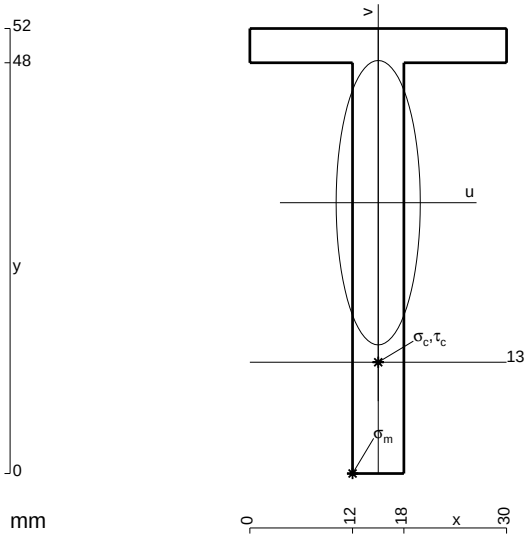
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

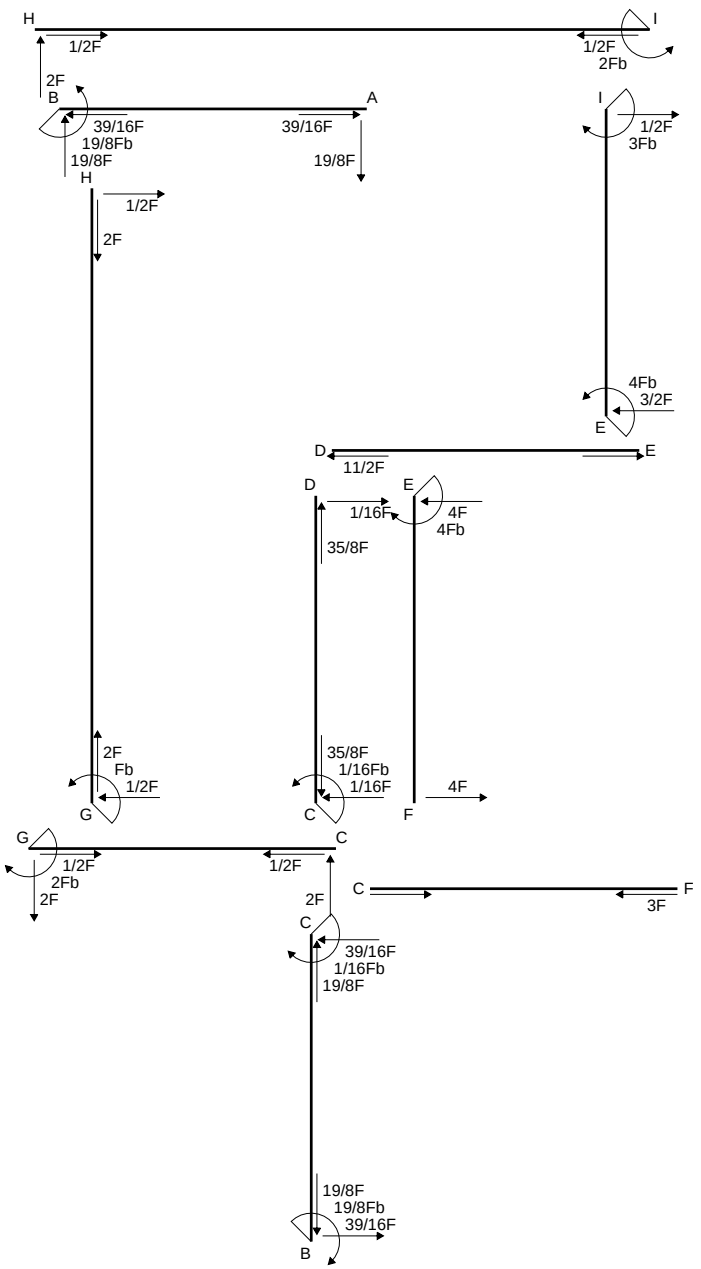


PROCEDIMENTO E RISULTATI 267850 CI01.xxxx.053



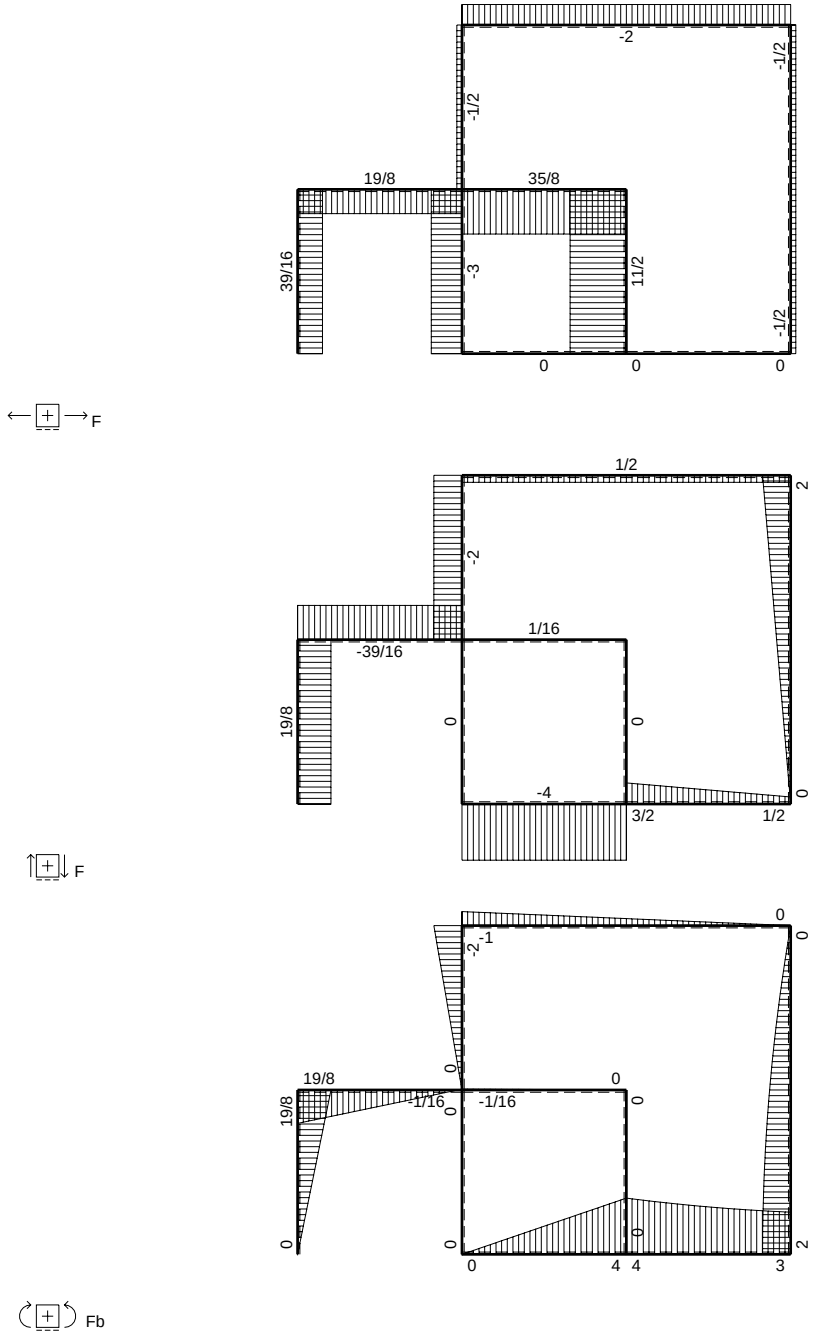

A = 408. $mm^2$
$J_u$ = 112717. $mm^4$
$J_v$ = 9864. $mm^4$
$y_g$ = 31.65 mm
$T_y$ = -1680. N
$M_x$ = 808500. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -31.65 mm
$\sigma_m = -Mv/J_u$ = 227. $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 13. mm
$v_c$ = -18.65 mm
$\sigma_c = -Mv/J_u$ = 133.8 $N/mm^2$
$\tau_c$ = 4.872 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134. $N/mm^2$
$S^*$ = 1961. $mm^3$

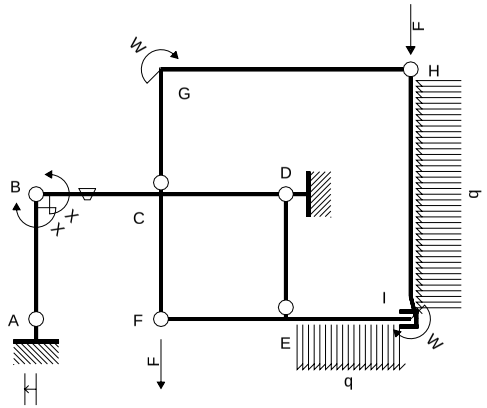
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H
I
1/2F
1/2F
2Fb
2F
B
A
I
39/16F
39/16F
1/2F
19/8Fb
3Fb
19/8F
19/8F
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
1/16F
4F
4Fb
35/8F
2F
Fb
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G
35/8F
1/16Fb
1/16F
C
4F
F
G
C
1/2F
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2Fb
2F
C
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C
39/16F
1/16Fb
19/8F
19/8F
19/8Fb
39/16F
B

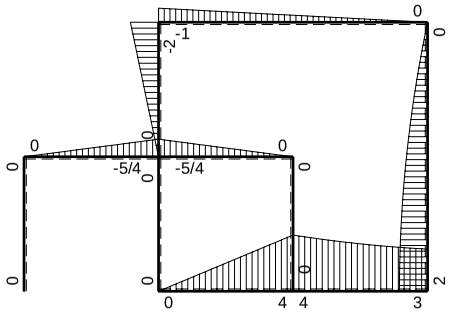




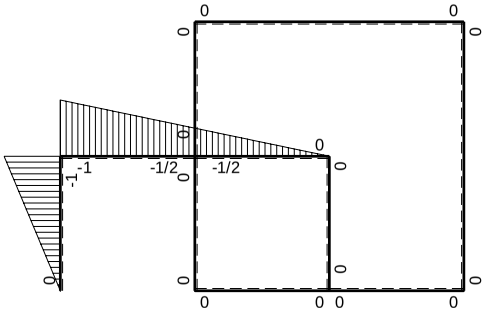

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35/8
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1/2
2
-2
1/16
-39/16
19/8
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0
0
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3/2
1/2
F
0
0
-2
-1
0
19/8
0
19/8
1/16
0
-1/16
0
0
0
0
0
0
4
4
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267178 CI01.xxxx.054

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PROCEDIMENTO E RISULTATI 267178 CI01.xxxx.054

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $19/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -19/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

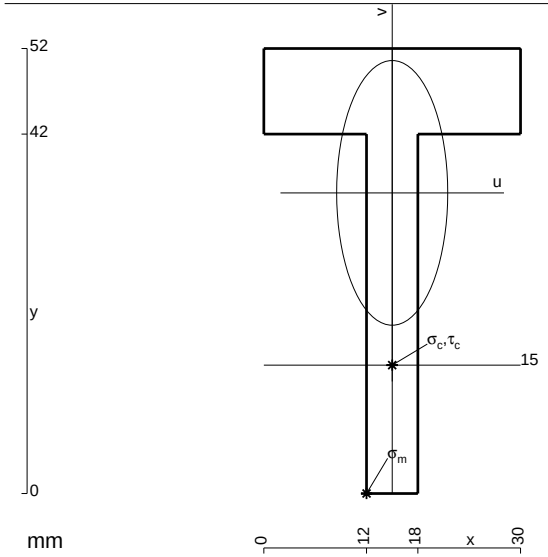
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

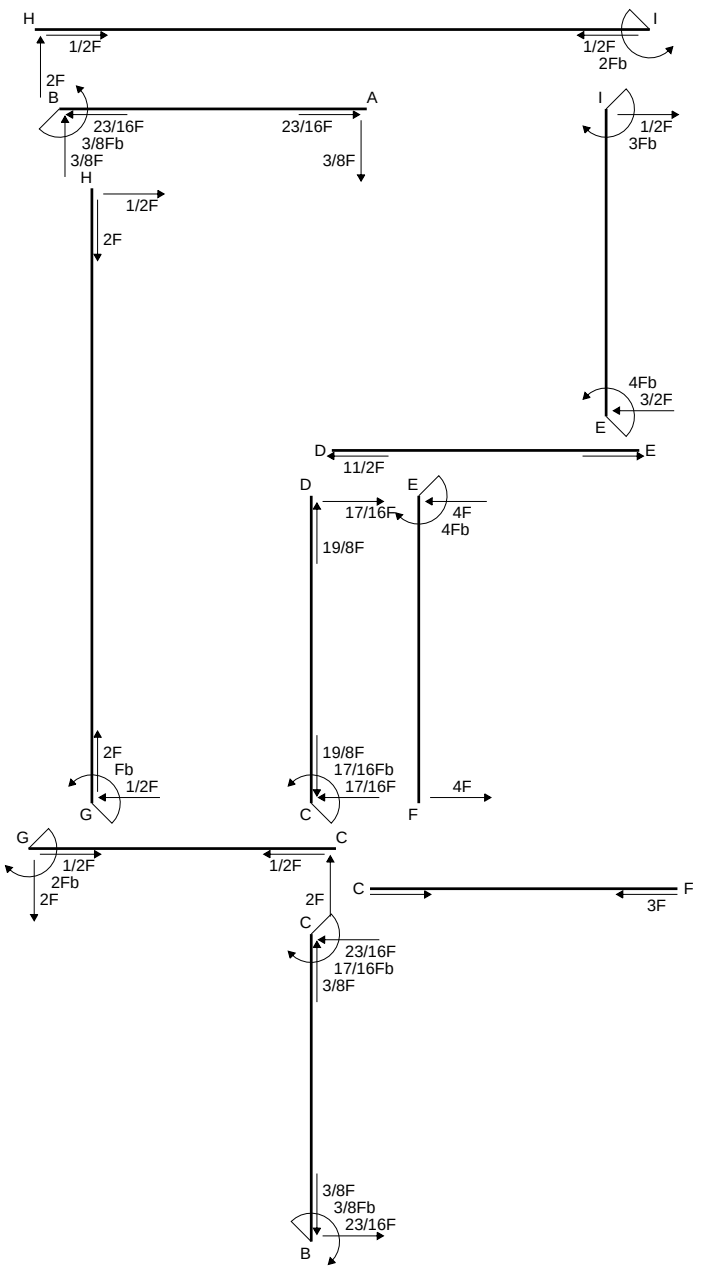
$= (5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$



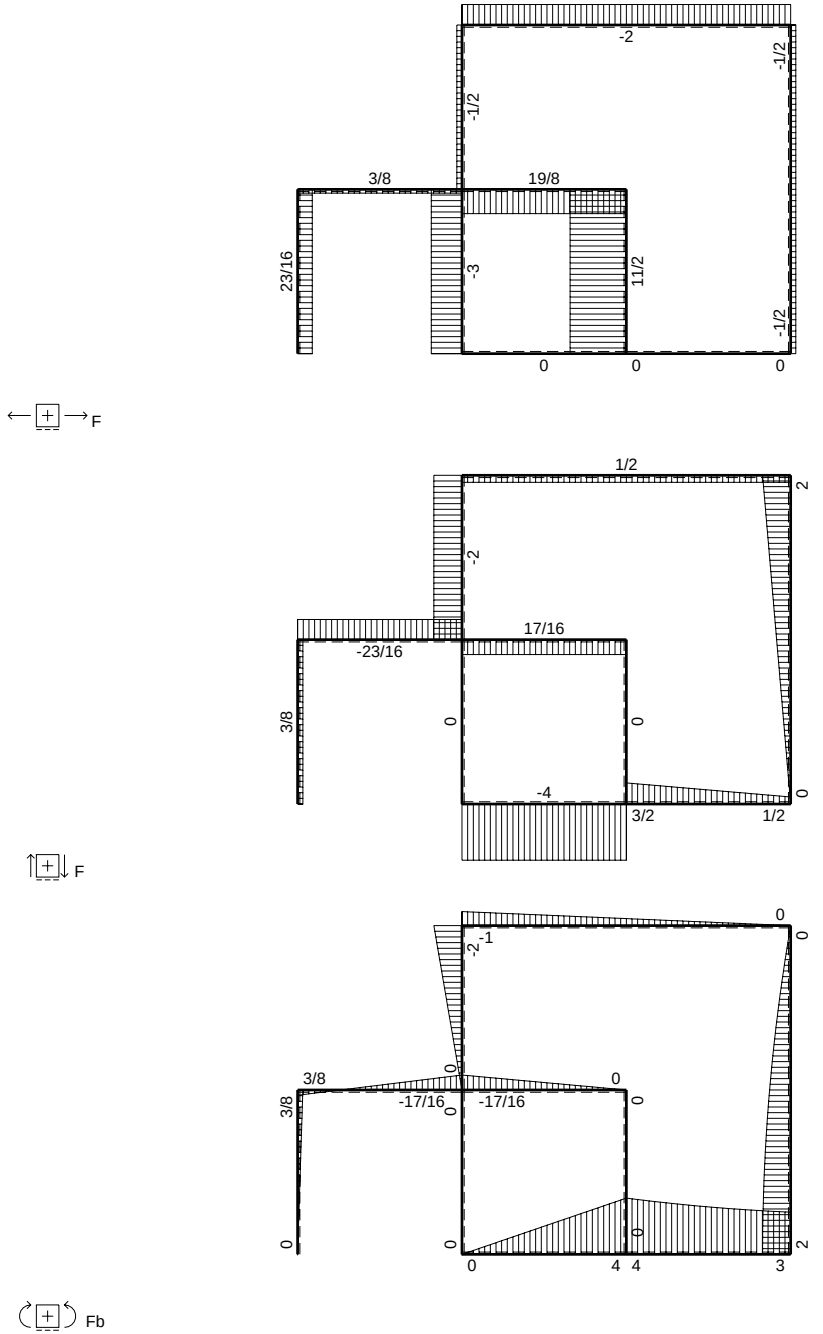





A = 552. $mm^2$
$J_u$ = 132127. $mm^4$
$J_v$ = 23256. $mm^4$
$y_g$ = 35.13 mm
$T_y$ = -1480. N
$M_x$ = 888000. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -35.13 mm
$\sigma_m = -Mv/J_u$ = 236.1 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 15. mm
$v_c$ = -20.13 mm
$\sigma_c = -Mv/J_u$ = 135.3 $N/mm^2$
$\tau_c$ = 4.642 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 135.5 $N/mm^2$
$S^*$ = 2487. $mm^3$

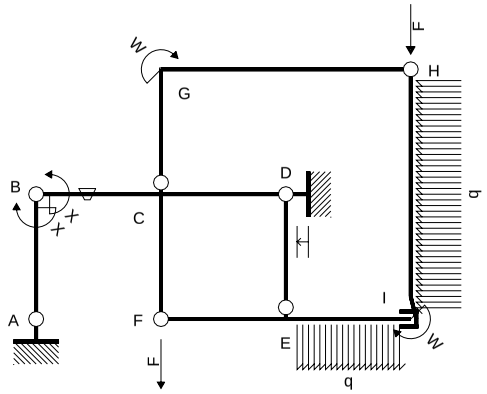




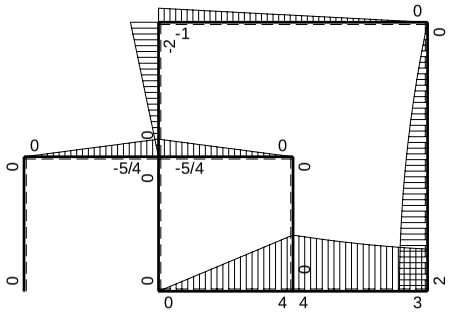





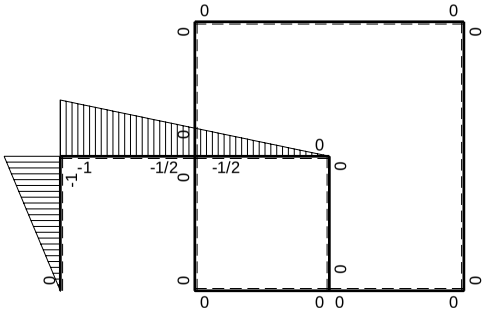

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Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 263281 CI01.xxxx.055

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $3/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -3/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

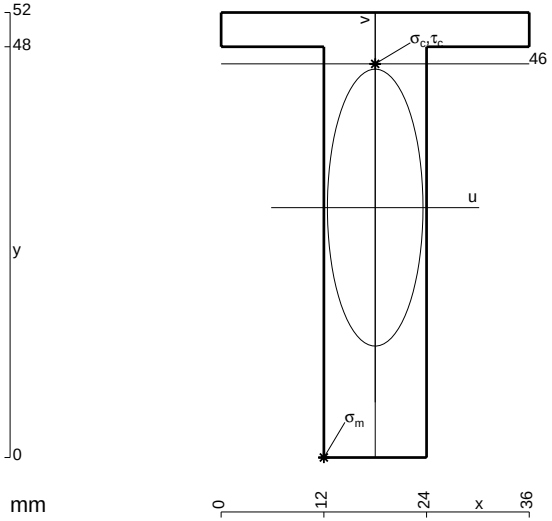
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

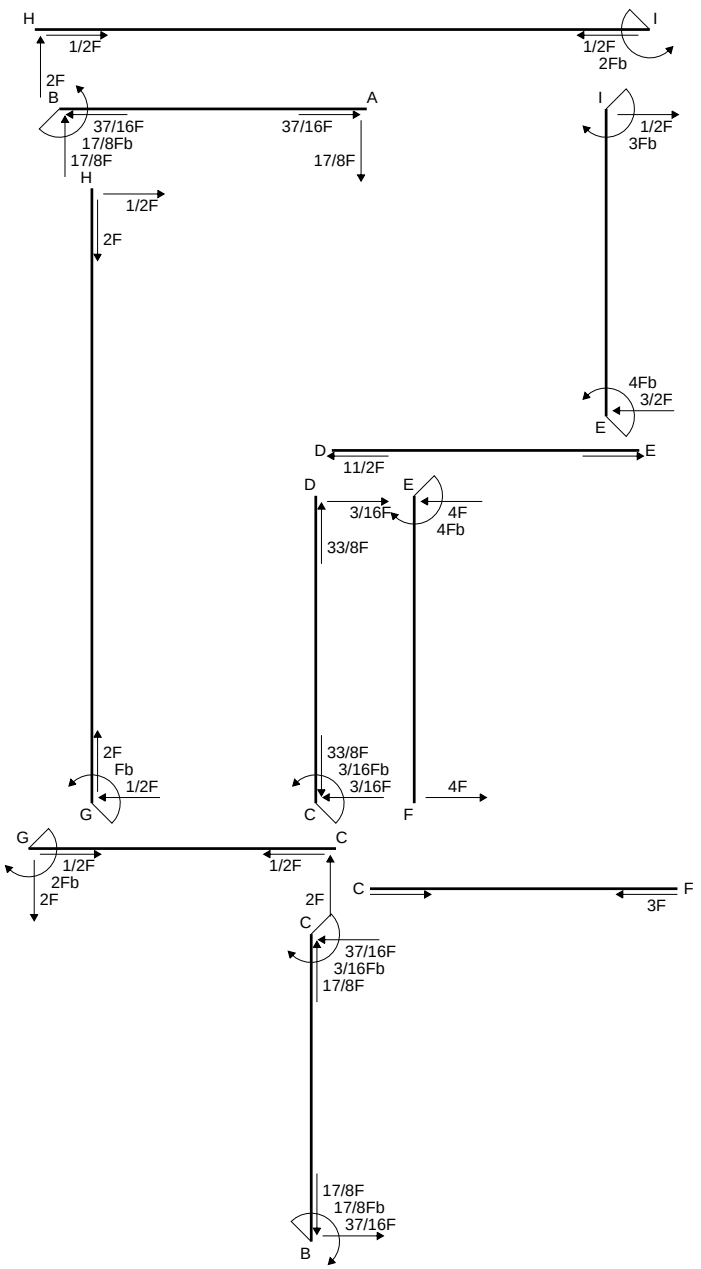
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





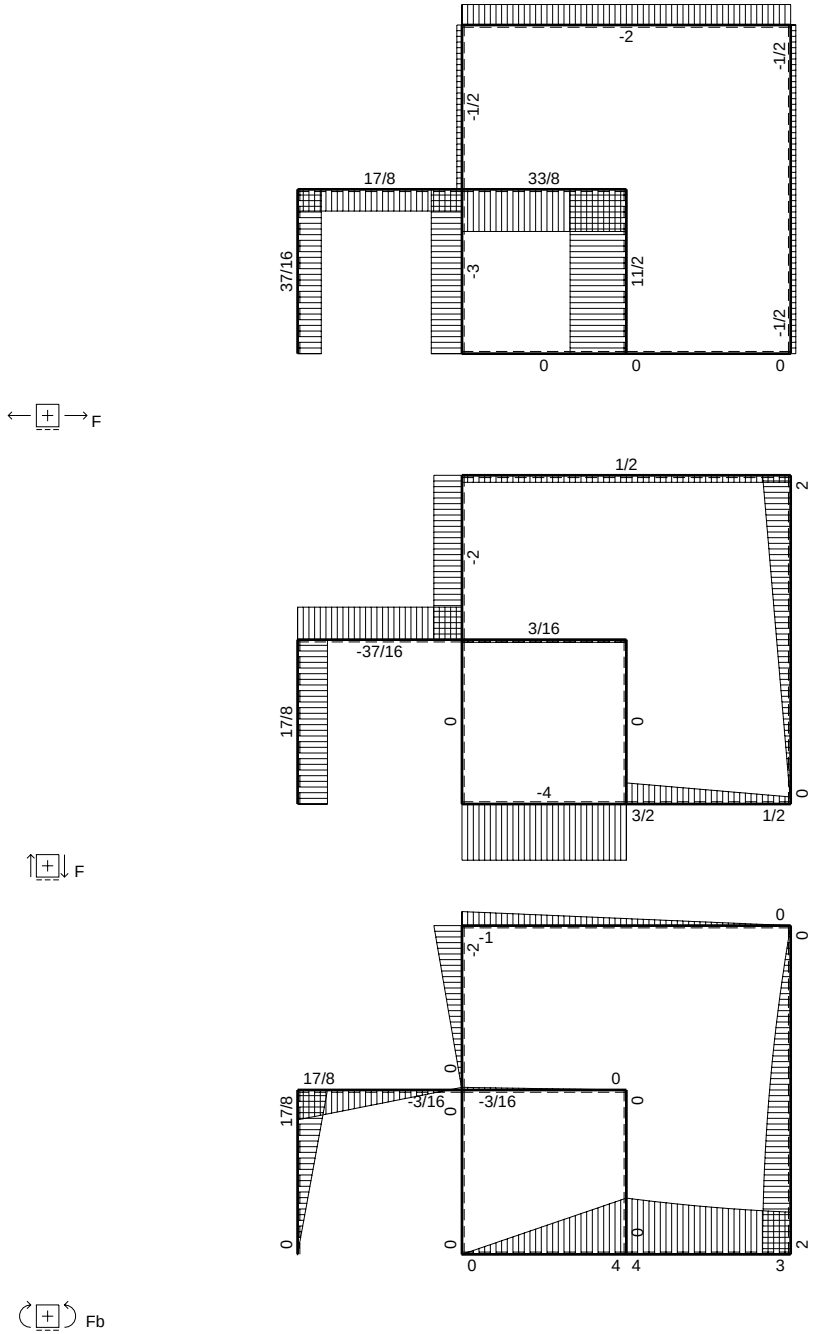


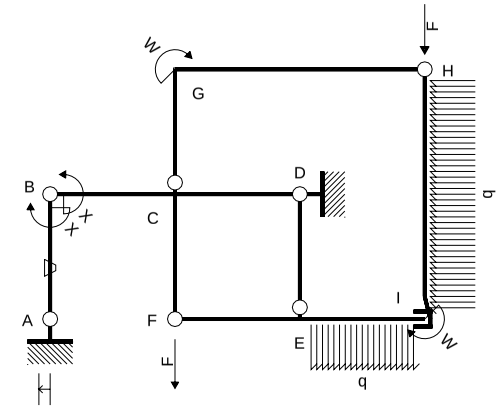

A = 720. $mm^2$
$J_u$ = 188659. $mm^4$
$J_v$ = 22464. $mm^4$
$y_g$ = 29.2 mm
$T_y$ = -2000. N
$M_x$ = 1280000. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -29.2 mm
$\sigma_m = -Mv/J_u$ = 198.1 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 46. mm
$v_c$ = 16.8 mm
$\sigma_c = -Mv/J_u$ = -114. $N/mm^2$
$\tau_c$ = 3.023 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 114.1 $N/mm^2$
$S^*$ = 3422. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
37/16F
17/8Fb
17/8F
37/16F
17/8F
I
1/2F
3Fb
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
3/16F
4F
4Fb
33/8F
2F
Fb
1/2F
G
33/8F
3/16Fb
3/16F
C
4F
F
G
C
1/2F
2Fb
2F
1/2F
C
F
3F
2F
C
37/16F
3/16Fb
17/8F
17/8F
17/8Fb
37/16F
B

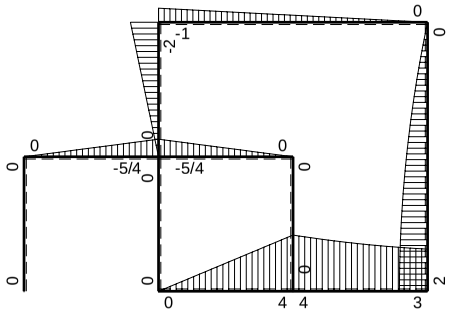




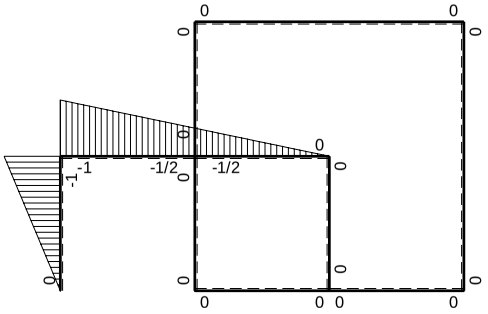

-2
-1/2
-1/2
17/8
33/8
37/16
-3
11/2
-1/2
0
0
0
F
1/2
2
-2
3/16
-37/16
17/8
0
0
0
-4
3/2
1/2
F
0
0
-2
-1
0
17/8
0
-3/16
-3/16
17/8
0
0
0
0
0
0
4
4
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 253104 CI01.xxxx.056

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-5/4Fx$ | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-5/4Fb+5/4Fx$ | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | $5/4Fx$ | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+1/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $17/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -17/8Fb | |

Sviluppi di calcolo iperstatica


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b ( x/b )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

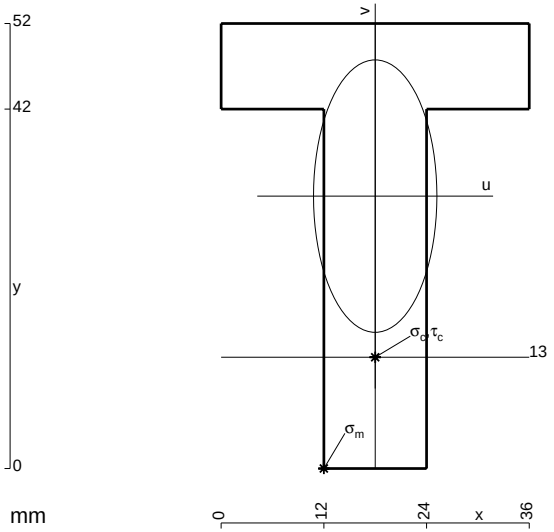
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

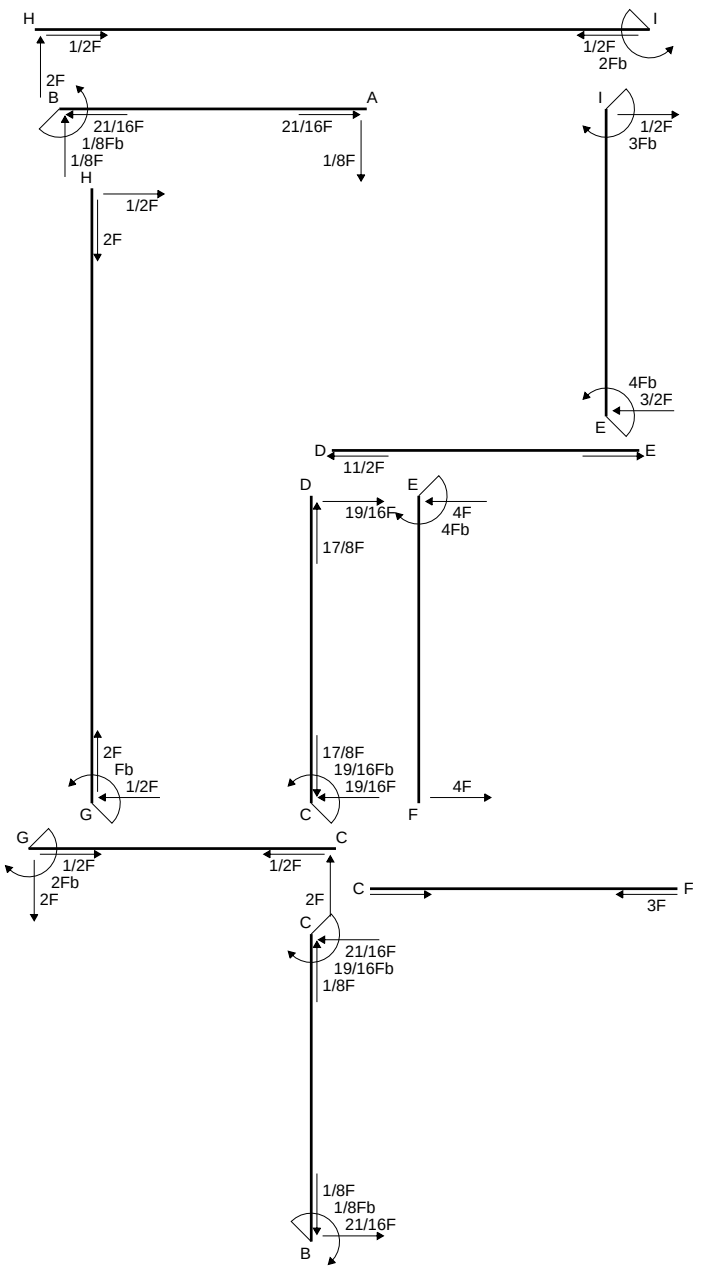
$= (5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





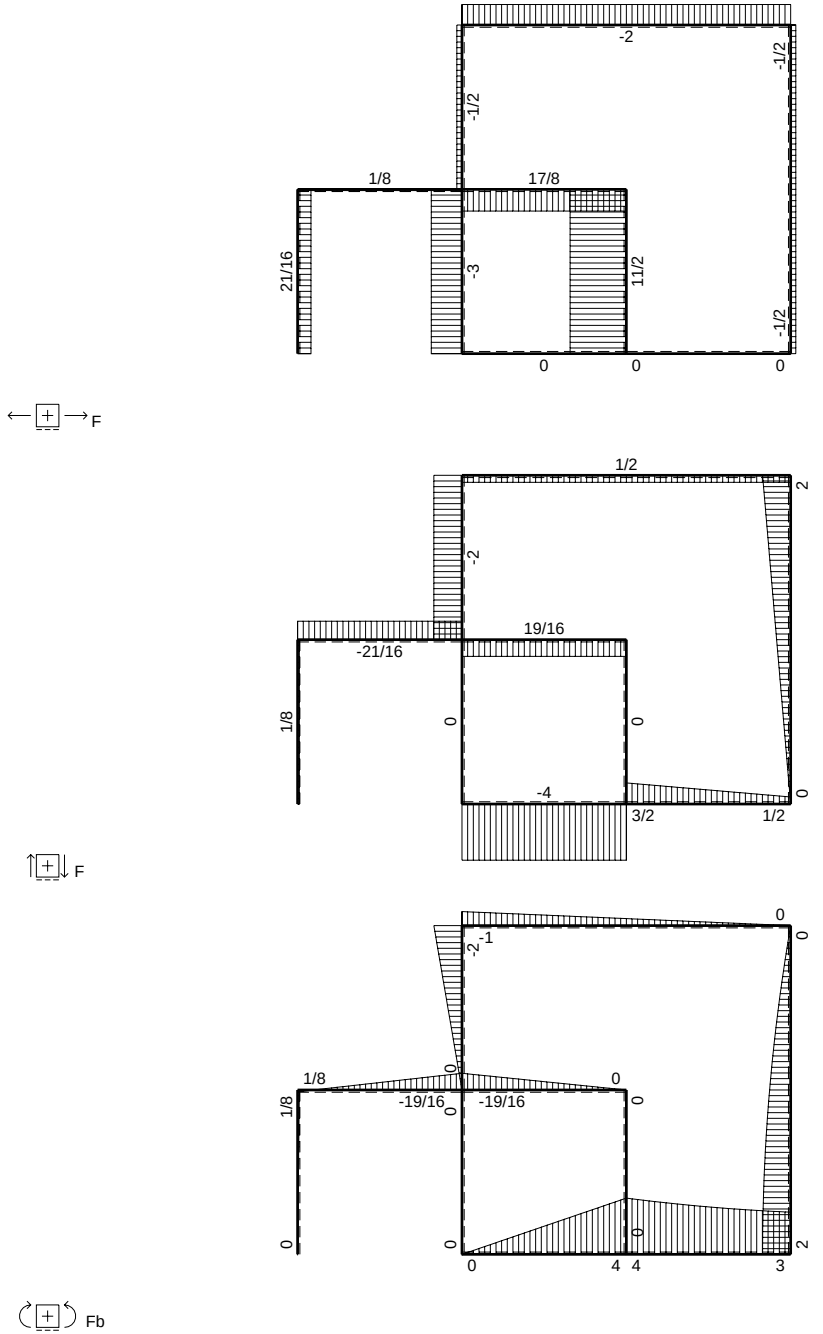


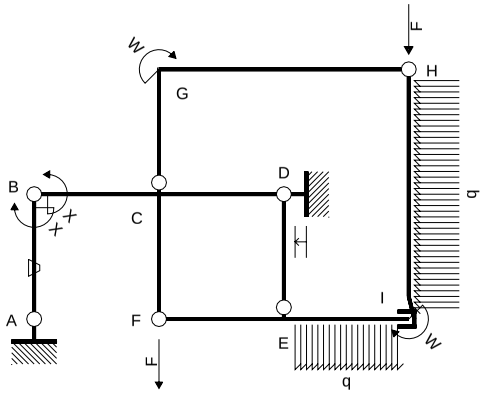

A = 864. $mm^2$
$J_u$ = 219048. $mm^4$
$J_v$ = 44928. $mm^4$
$y_g$ = 31.83 mm
$T_y$ = -2120. N
$M_x$ = 1441600. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -31.83 mm
$\sigma_m = -Mv/J_u$ = 209.5 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 13. mm
$v_c$ = -18.83 mm
$\sigma_c = -Mv/J_u$ = 123.9 $N/mm^2$
$\tau_c$ = 3.187 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 124.1 $N/mm^2$
$S^*$ = 3952. $mm^3$


H
1/2F
I
1/2F
2Fb
2F
B
A
21/16F
1/8Fb
1/8F
21/16F
1/8F
I
1/2F
3Fb
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
19/16F
4F
4Fb
17/8F
2F
Fb
1/2F
G
17/8F
19/16Fb
19/16F
C
4F
F
G
C
1/2F
2Fb
2F
1/2F
C
F
3F
2F
C
21/16F
19/16Fb
1/8F
1/8F
1/8Fb
21/16F
B

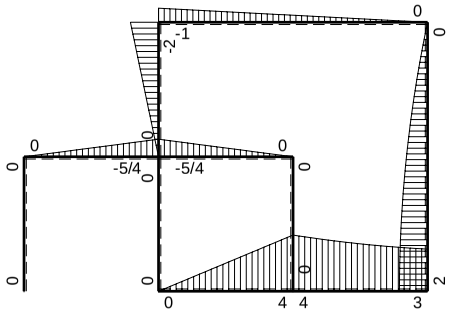




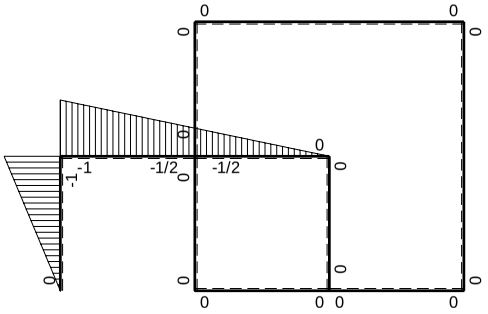

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1/8
17/8
21/16
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11/2
-1/2
0
0
0
F
1/2
2
-2
19/16
-21/16
1/8
0
0
-4
0
3/2
1/2
F
0
0
-2
-1
1/8
0
0
1/8
-19/16
-19/16
0
0
0
0
0
2
0
4
4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 992702 CI01.xxxx.057

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 992702 CI01.xxxx.057

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/8Fb | |

Sviluppi di calcolo iperstatica


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

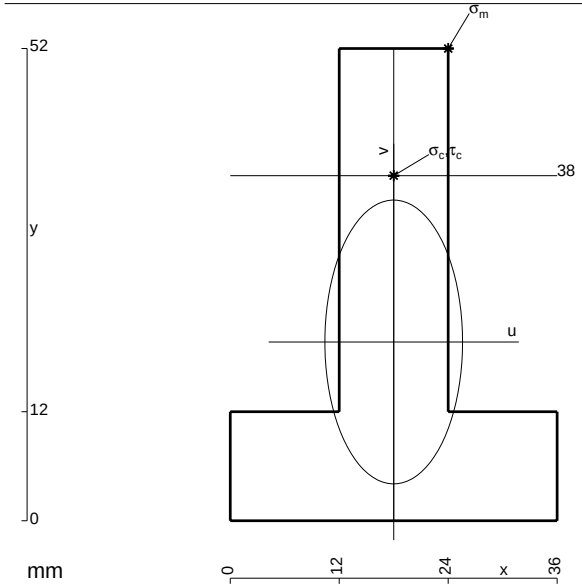
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

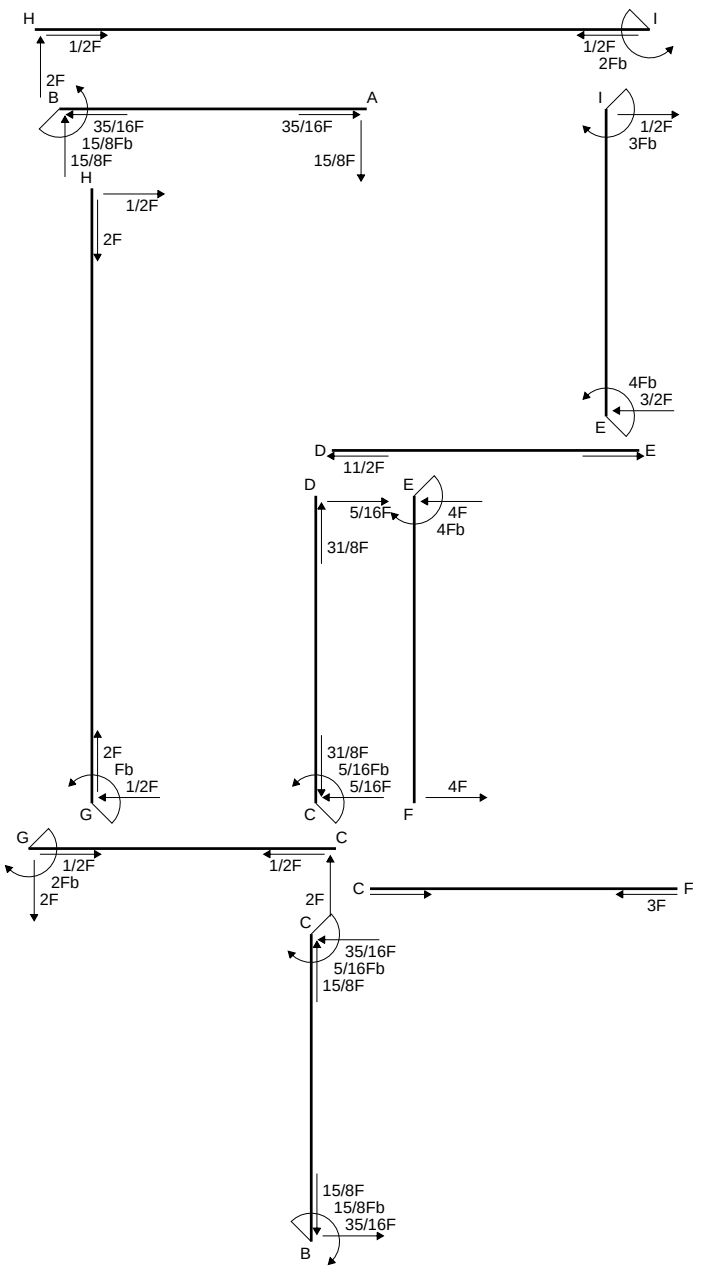
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





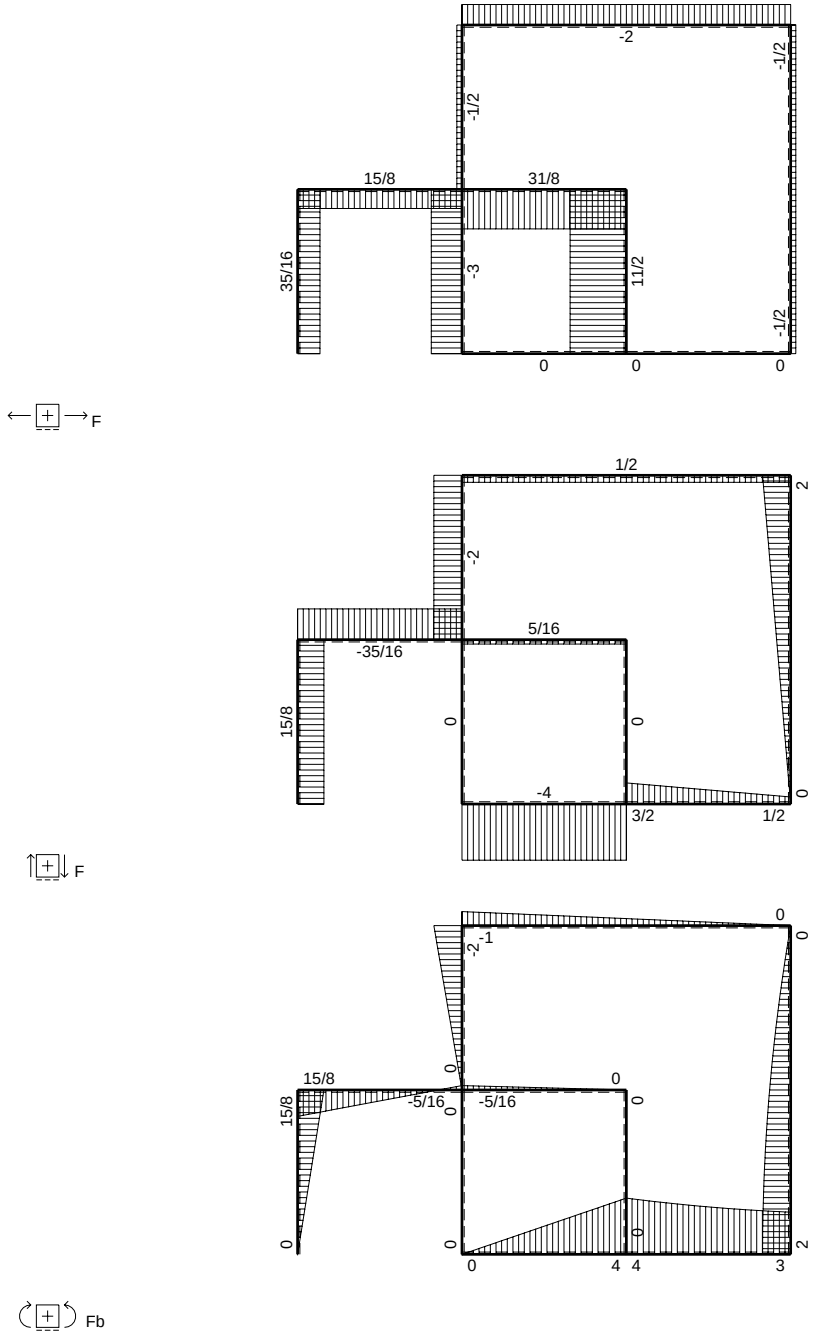


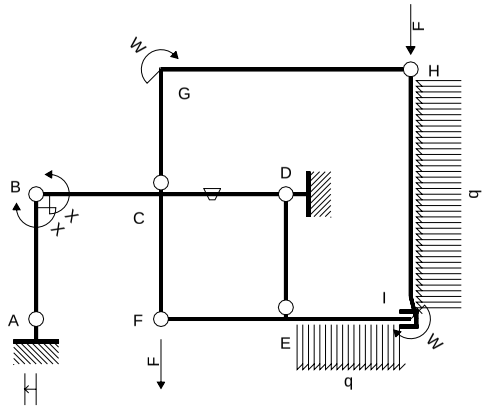

A = 912. $mm^2$
$J_u$ = 222885. $mm^4$
$J_v$ = 52416. $mm^4$
$y_g$ = 19.68 mm
$T_y$ = -2040. N
$M_x$ = 1489200. Nmm
$x_m$ = 24. mm
$y_m$ = 52. mm
$u_m$ = 6. mm
$v_m$ = 32.32 mm
$\sigma_m = -Mv/J_u$ = -215.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 38. mm
$v_c$ = 18.32 mm
$\sigma_c = -Mv/J_u$ = -122.4 $N/mm^2$
$\tau_c$ = 3.244 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.5 $N/mm^2$
$S^*$ = 4253. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
35/16F
15/8Fb
15/8F
35/16F
15/8F
I
1/2F
3Fb
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
5/16F
4F
4Fb
31/8F
2F
Fb
1/2F
G
31/8F
5/16Fb
5/16F
C
4F
F
G
C
1/2F
2Fb
2F
1/2F
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5/16Fb
15/8F
15/8F
15/8Fb
35/16F
B

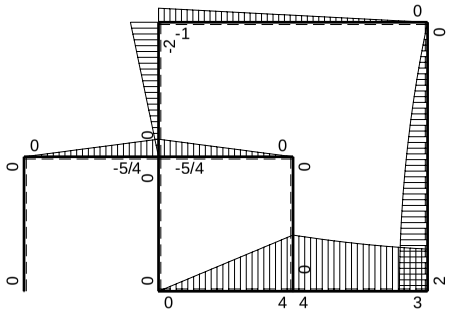




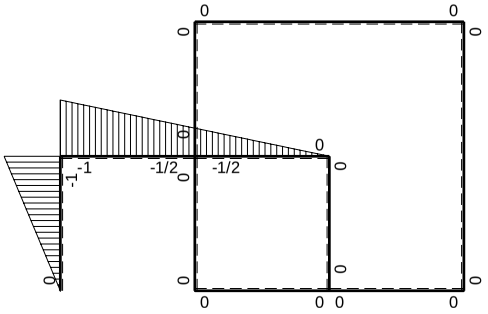

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15/8
31/8
35/16
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F
1/2
2
-2
5/16
-35/16
15/8
0
0
0
-4
3/2
1/2
Fb
0
0
-2
-1
0
15/8
0
0
-5/16
-5/16
15/8
0
0
0
0
0
2
0
4
4
3




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 215079 CI01.xxxx.058

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $15/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -15/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 215079 CI01.xxxx.058

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

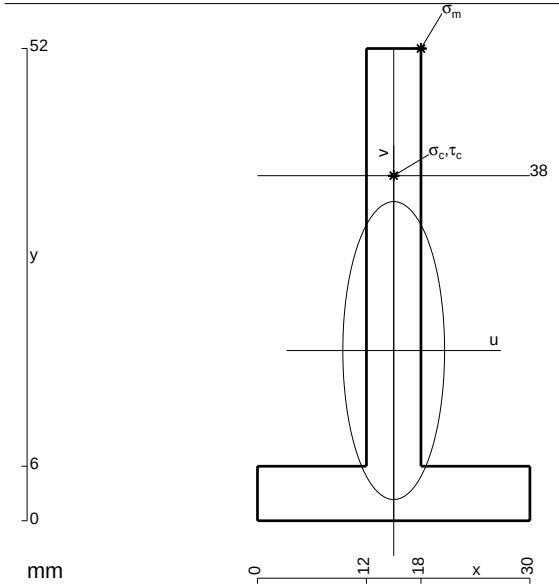
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

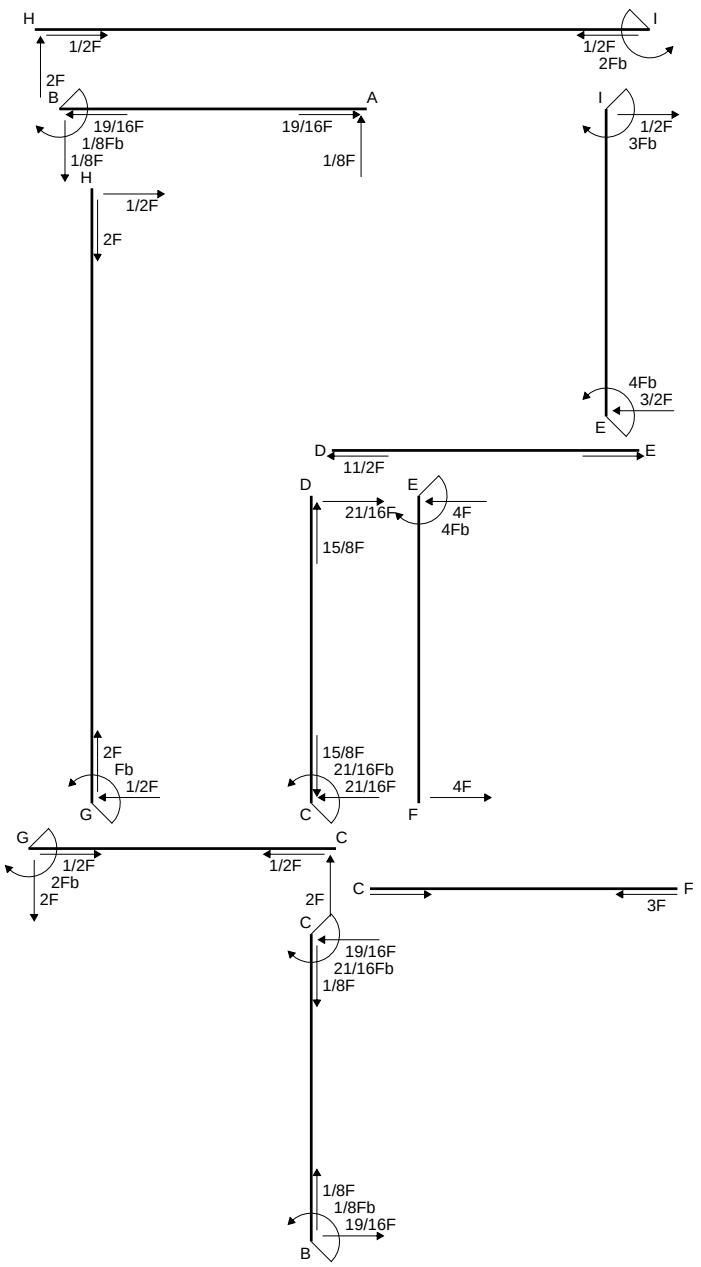
$= (5/24\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 11/24\ Fb^2/EJ$





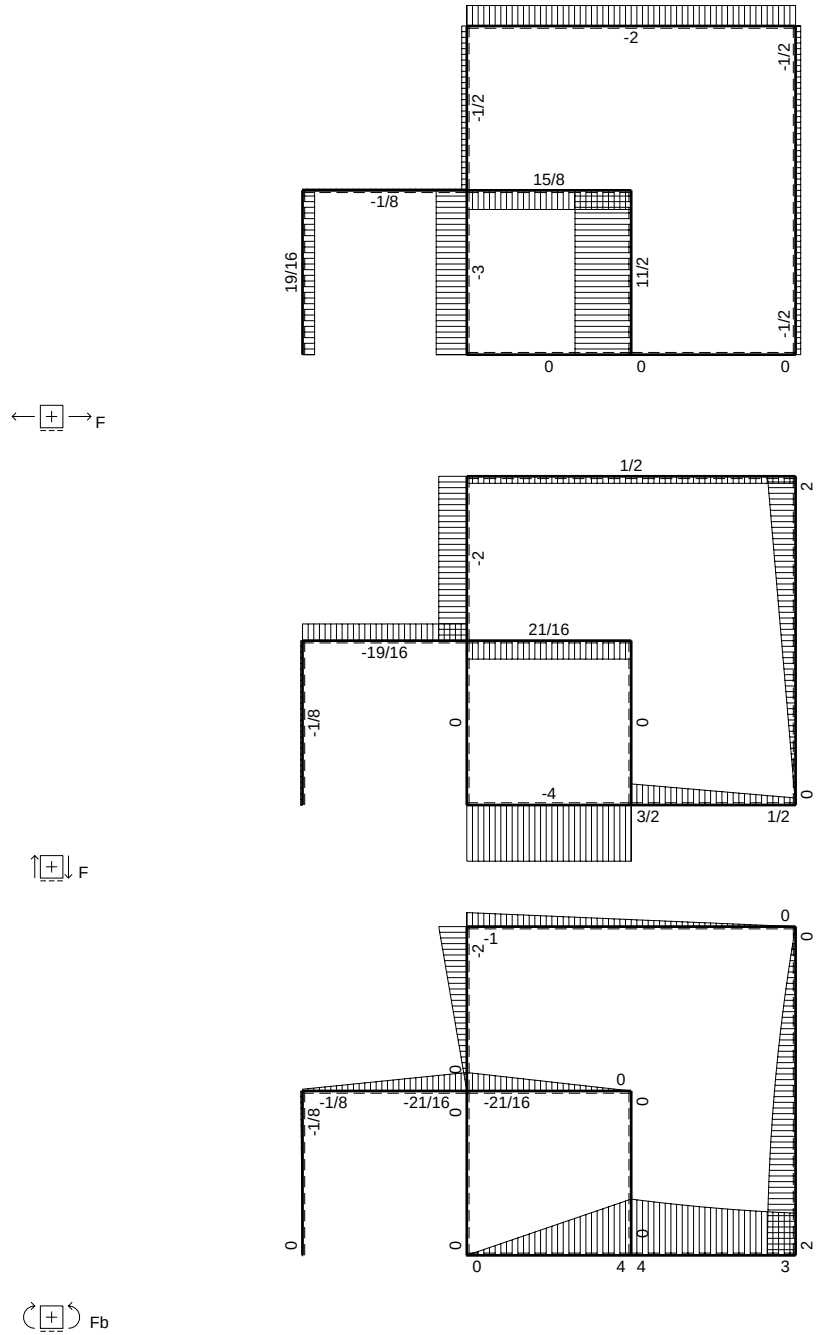


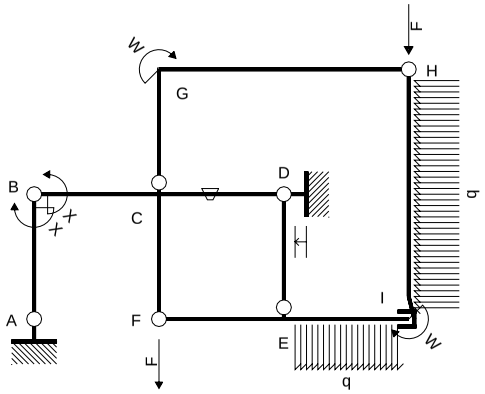

A = 456. $mm^2$
$J_u$ = 122856. $mm^4$
$J_v$ = 14328. $mm^4$
$y_g$ = 18.74 mm
$T_y$ = -1080. N
$M_x$ = 831600. Nmm
$x_m$ = 18. mm
$y_m$ = 52. mm
$u_m$ = 3. mm
$v_m$ = 33.26 mm
$\sigma_m = -Mv/J_u$ = -225.2 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 38. mm
$v_c$ = 19.26 mm
$\sigma_c = -Mv/J_u$ = -130.4 $N/mm^2$
$\tau_c$ = 3.232 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.5 $N/mm^2$
$S^*$ = 2206. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
I
19/16F
19/16F
1/2F
1/8Fb
3Fb
1/8F
1/8F
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
21/16F
4F
4Fb
15/8F
2F
Fb
1/2F
15/8F
21/16Fb
21/16F
4F
G
C
F
G
C
1/2F
1/2F
2Fb
2F
C
F
3F
2F
C
19/16F
21/16Fb
1/8F
1/8F
1/8Fb
19/16F
B

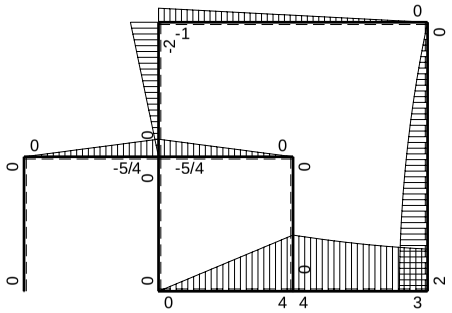




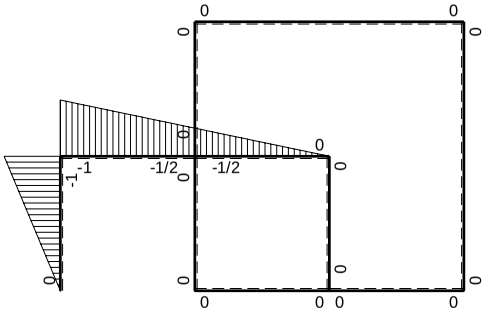

-2
-1/2
-1/2
15/8
-1/8
19/16
-3
11/2
-1/2
0
0
0
F
1/2
2
-2
21/16
-19/16
-1/8
0
0
0
-4
3/2
1/2
F
0
0
-2
-1
0
0
-1/8
-21/16
-21/16
0
0
0
0
0
0
0
4
4
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 266797 CI01.xxxx.059

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 266797 CI01.xxxx.059

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

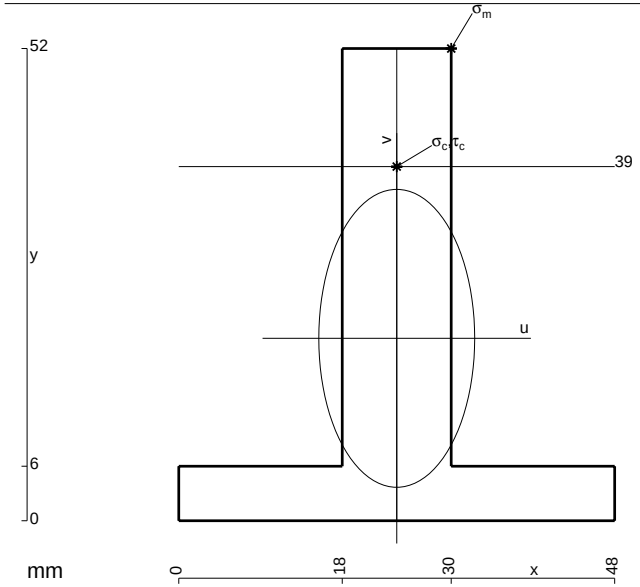
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

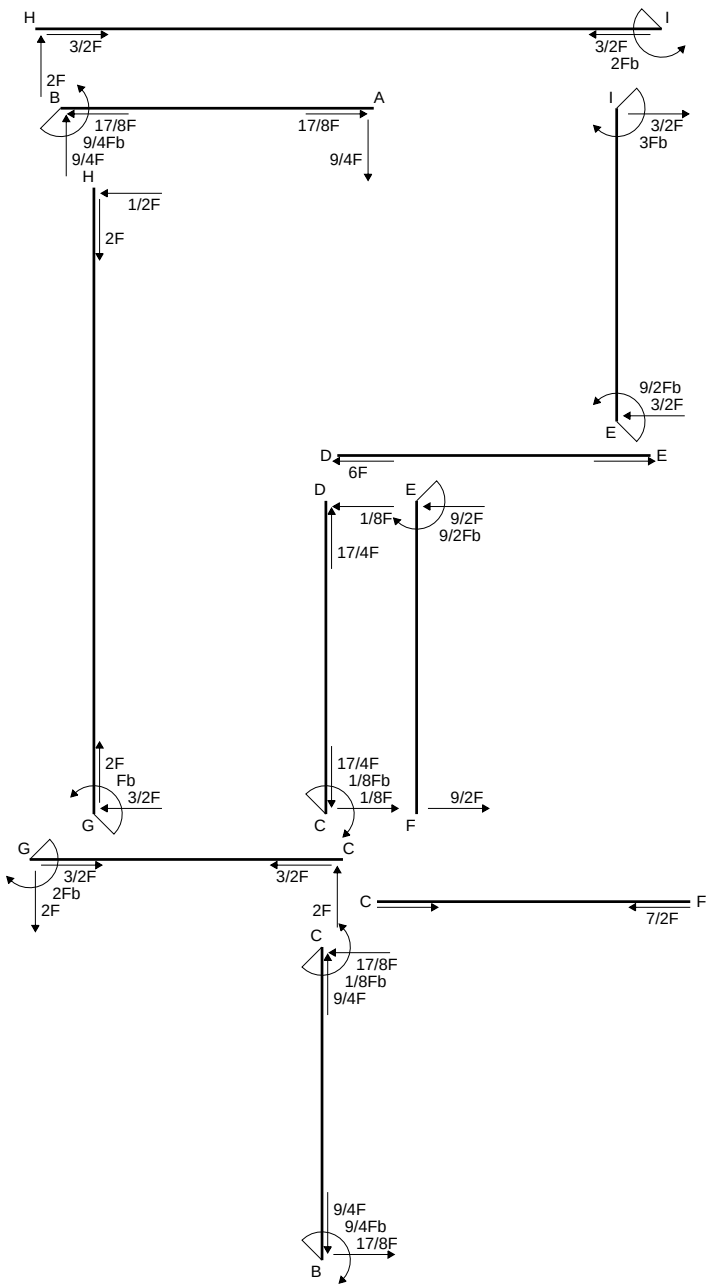
$= (5/24\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$



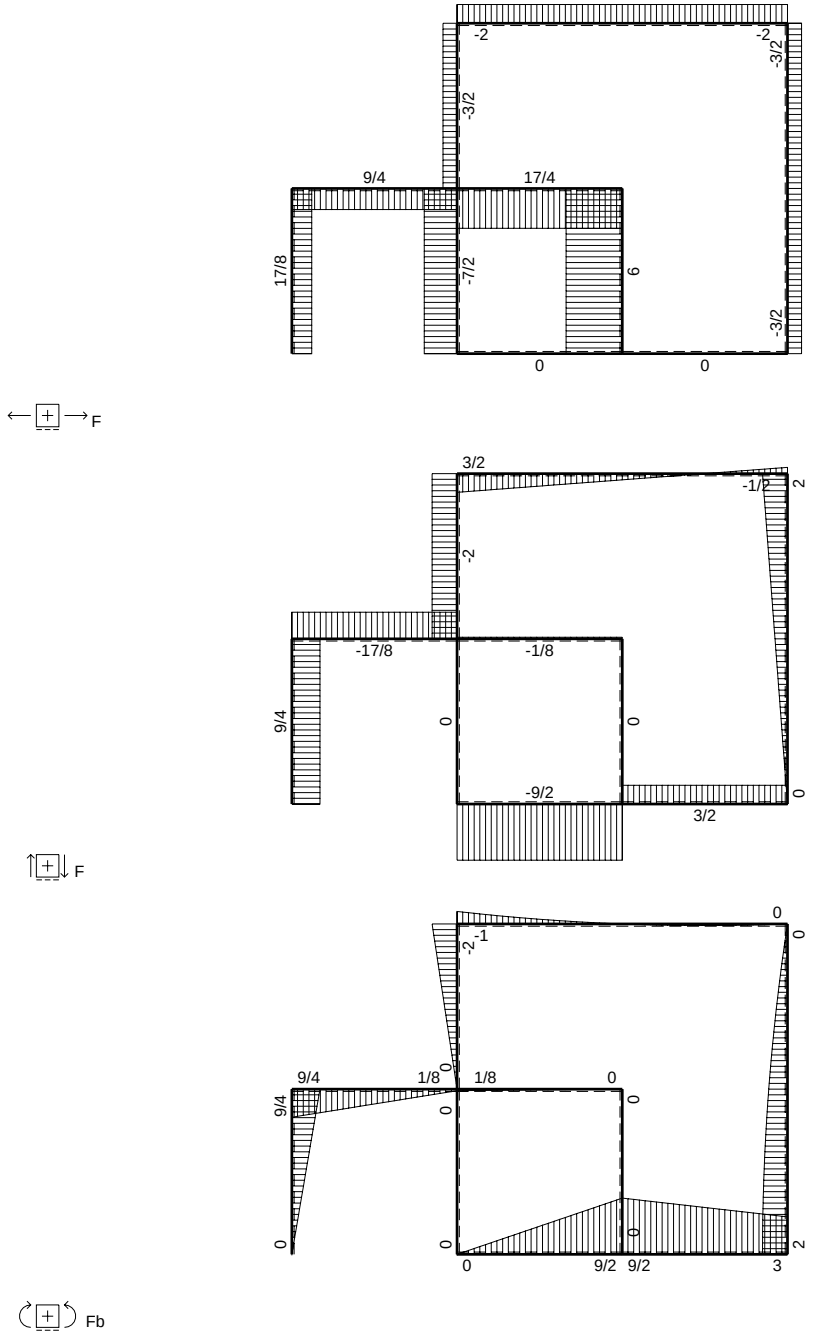





A = 840. $mm^2$
$J_u$ = 226138. $mm^4$
$J_v$ = 61920. $mm^4$
$y_g$ = 20.09 mm
$T_y$ = -2080. N
$M_x$ = 1684800. Nmm
$x_m$ = 30. mm
$y_m$ = 52. mm
$u_m$ = 6. mm
$v_m$ = 31.91 mm
$\sigma_m = -Mv/J_u$ = -237.8 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 39. mm
$v_c$ = 18.91 mm
$\sigma_c = -Mv/J_u$ = -140.9 $N/mm^2$
$\tau_c$ = 3.039 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 141. $N/mm^2$
$S^*$ = 3965. $mm^3$

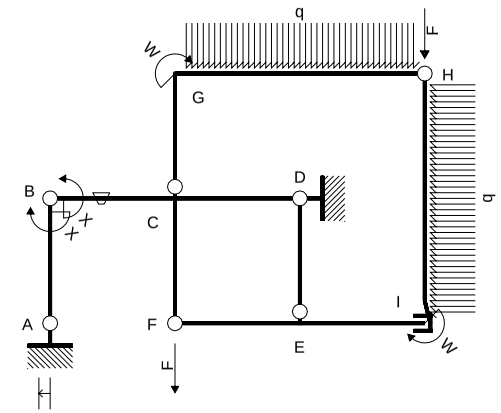




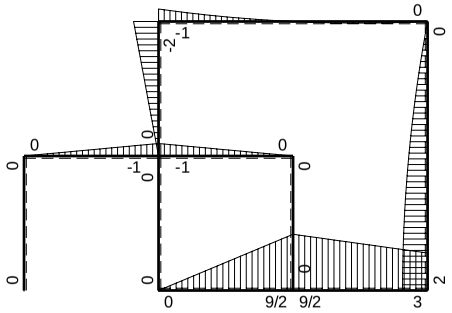





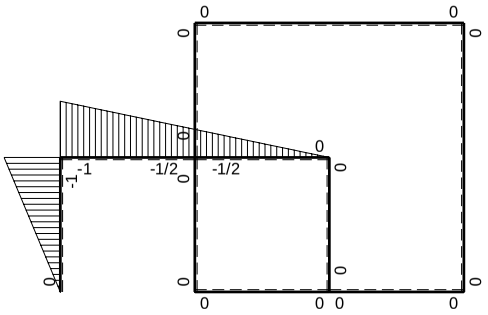

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268175 CI01.xxxx.060

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 268175 CI01.xxxx.060

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/2Fb-9/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $9/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -9/4Fb | |

Sviluppi di calcolo iperstatica


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

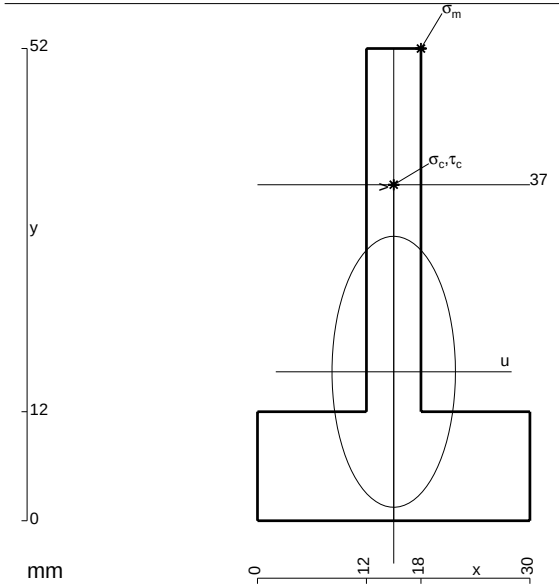
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

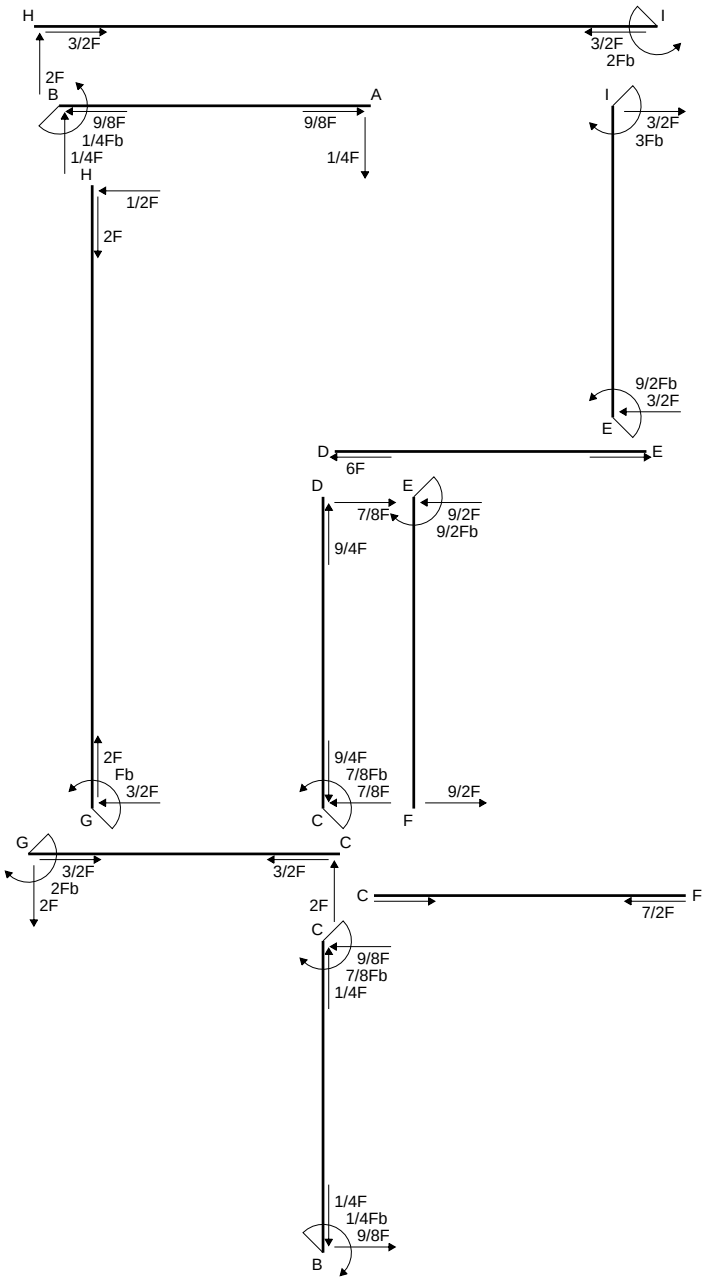
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$



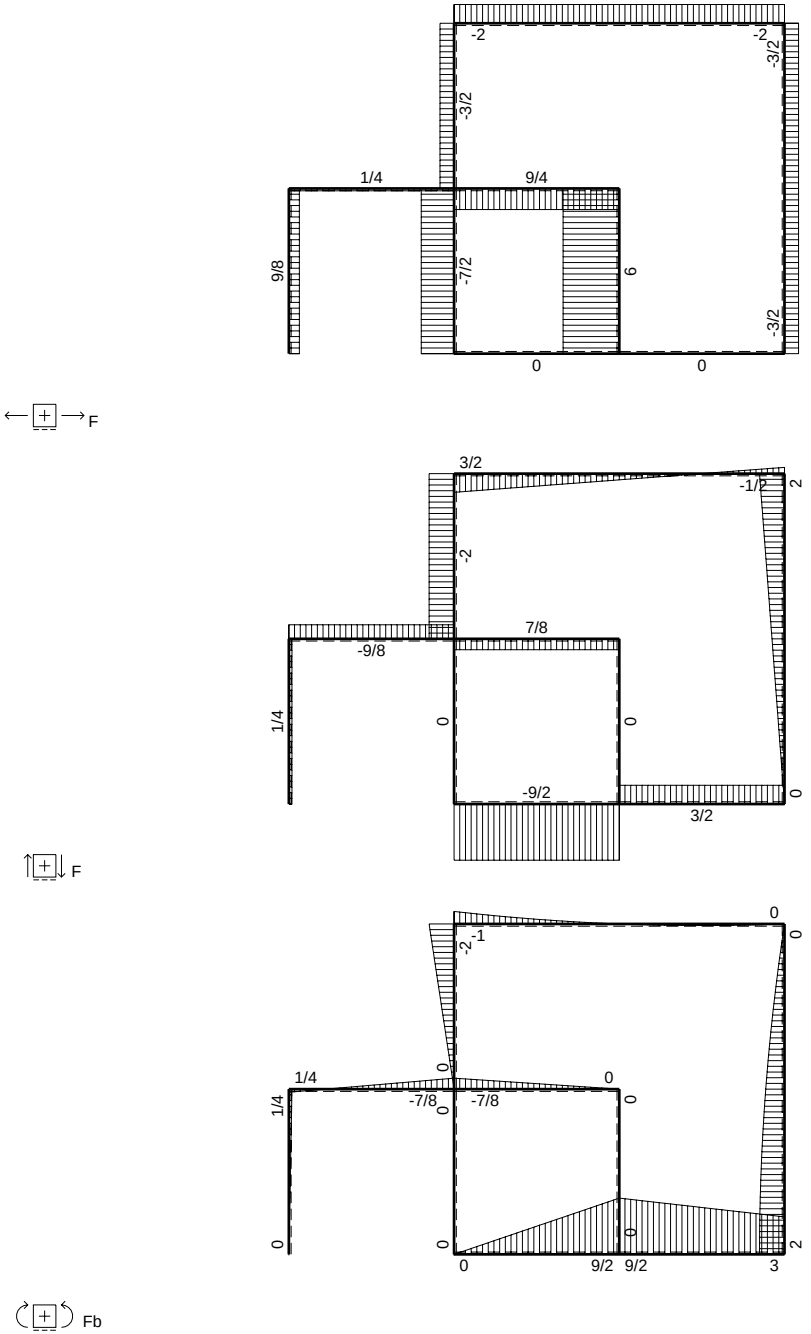





A = 600. $mm^2$
$J_u$ = 133664. $mm^4$
$J_v$ = 27720. $mm^4$
$y_g$ = 16.4 mm
$T_y$ = -1710. N
$M_x$ = 735300. Nmm
$x_m$ = 18. mm
$y_m$ = 52. mm
$u_m$ = 3. mm
$v_m$ = 35.6 mm
$\sigma_m = -Mv/J_u$ = -195.8 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 37. mm
$v_c$ = 20.6 mm
$\sigma_c = -Mv/J_u$ = -113.3 $N/mm^2$
$\tau_c$ = 5.392 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.7 $N/mm^2$
$S^*$ = 2529. $mm^3$

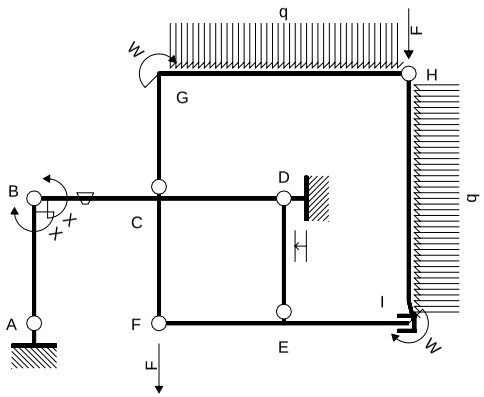




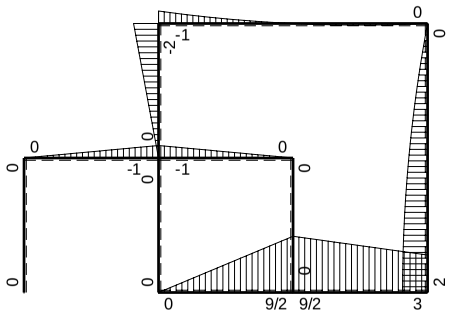





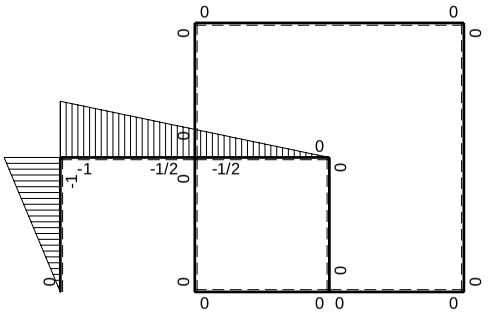

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Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264259 CI01.xxxx.061

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/2Fb-9/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

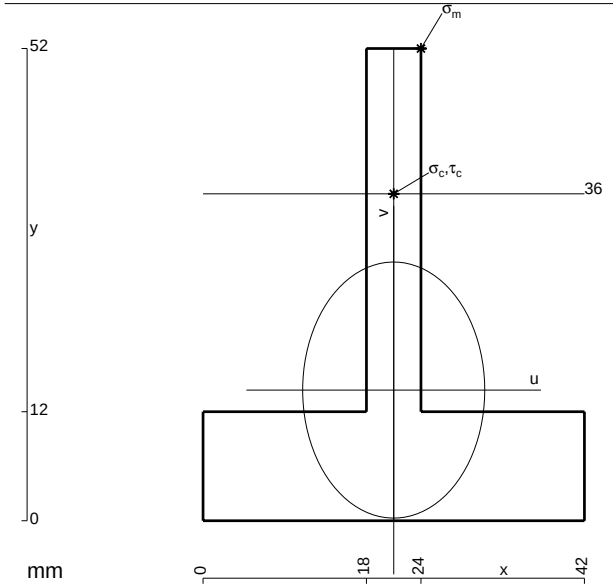
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

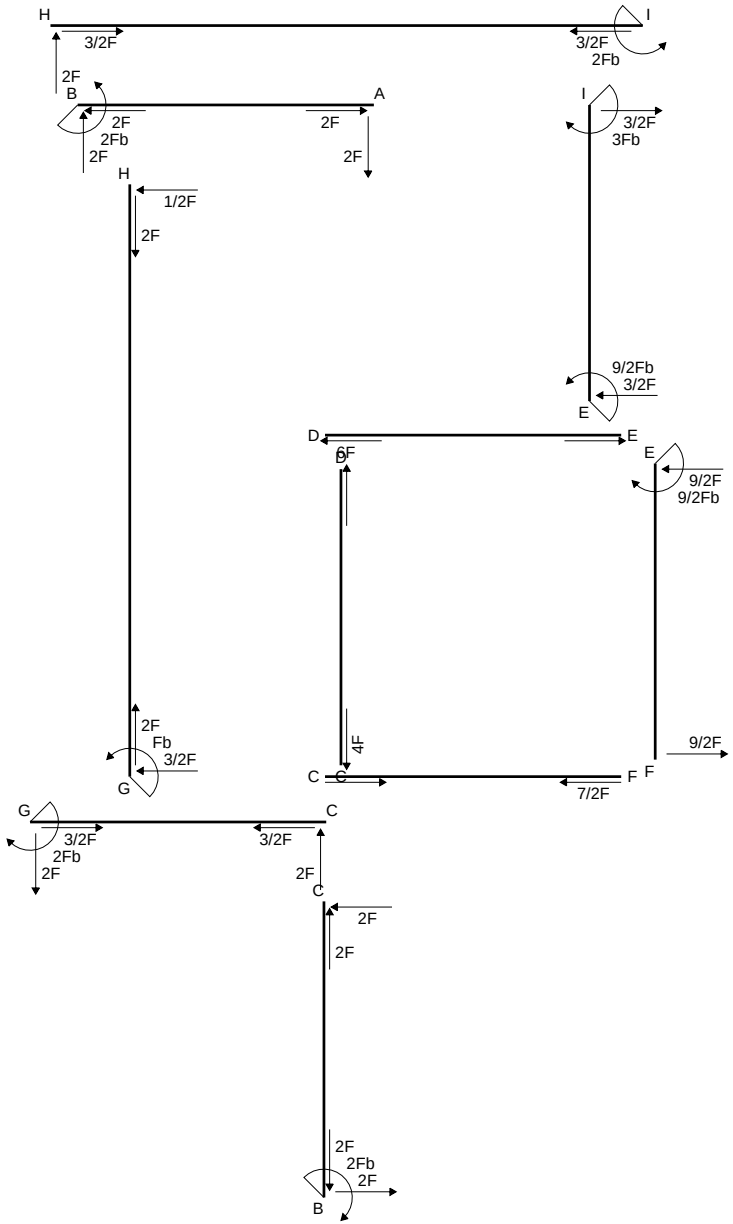


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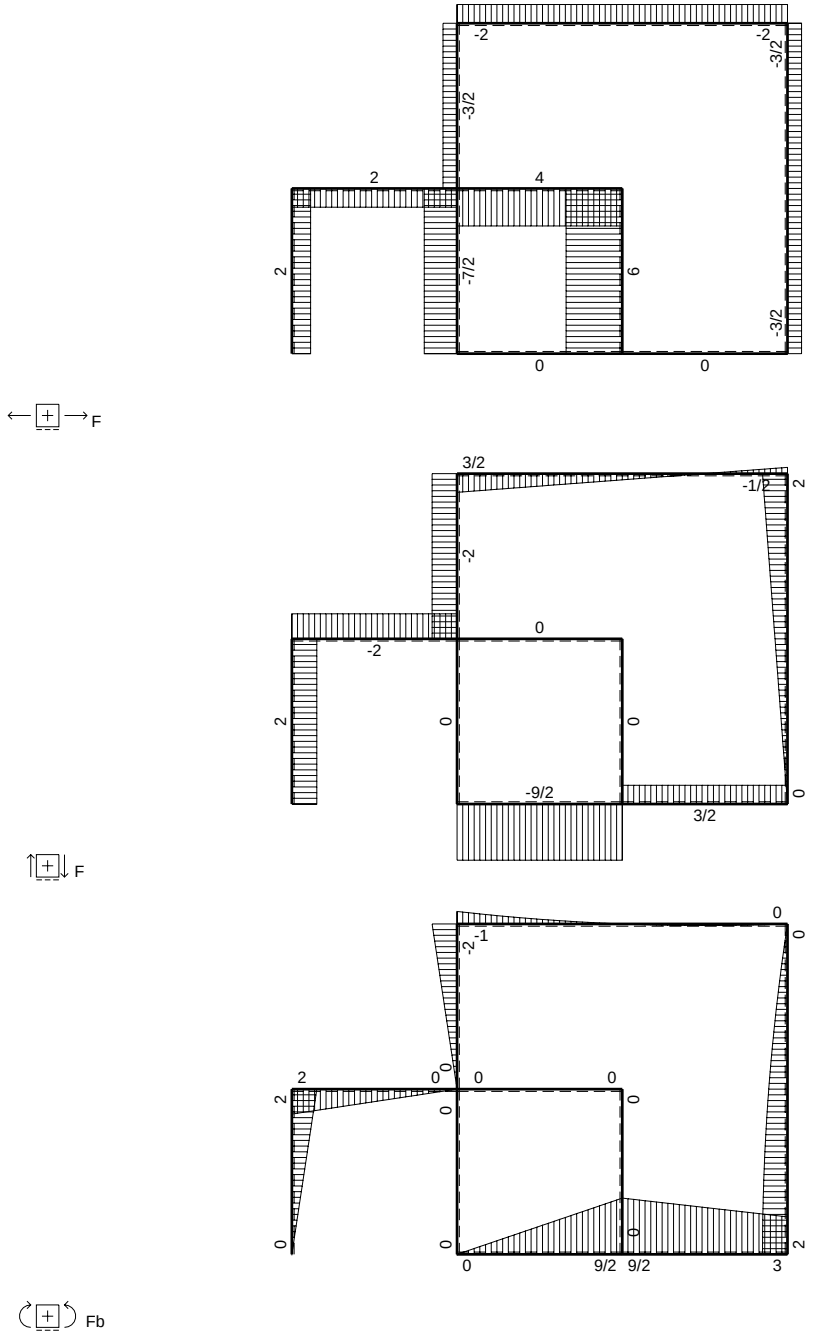

A = 744. $mm^2$
$J_u$ = 147953. $mm^4$
$J_v$ = 74808. $mm^4$
$y_g$ = 14.39 mm
$T_y$ = -1755. N
$M_x$ = 824850. Nmm
$x_m$ = 24. mm
$y_m$ = 52. mm
$u_m$ = 3. mm
$v_m$ = 37.61 mm
$\sigma_m = -Mv/J_u$ = -209.7 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 36. mm
$v_c$ = 21.61 mm
$\sigma_c = -Mv/J_u$ = -120.5 $N/mm^2$
$\tau_c$ = 5.62 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.9 $N/mm^2$
$S^*$ = 2843. $mm^3$

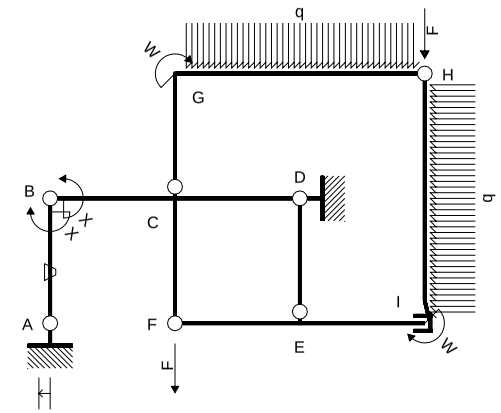
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H
3/2F
I
3/2F
2Fb
2F
B
A
2F
2Fb
2F
2F
I
3/2F
3Fb
H
1/2F
2F
9/2Fb
3/2F
E
D
E
6F
9/2F
9/2Fb
2F
Fb
3/2F
G
4F
9/2F
C
C
F
7/2F
G
C
3/2F
3/2F
2Fb
2F
2F
C
2F
2F
2F
2Fb
2F
B

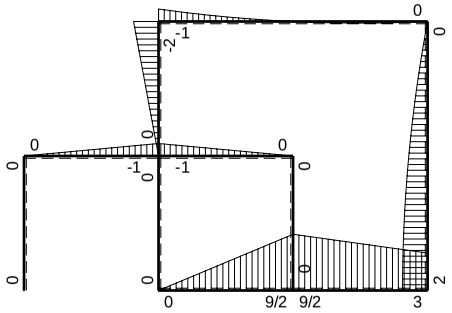




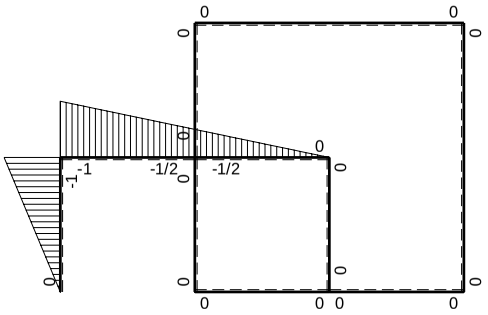

F
-2
-2
-3/2
-3/2
2
4
-7/2
2
6
-3/2
0
0
F
3/2
-1/2
2
-2
0
-2
2
0
0
0
-9/2
3/2
F
0
0
-2
-1
0
0
2
0
0
0
0
0
0
0
0
0
0
2
0
9/2
9/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 270256 CI01.xxxx.062

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PROCEDIMENTO E RISULTATI 270256 CI01.xxxx.062

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/2Fb-9/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

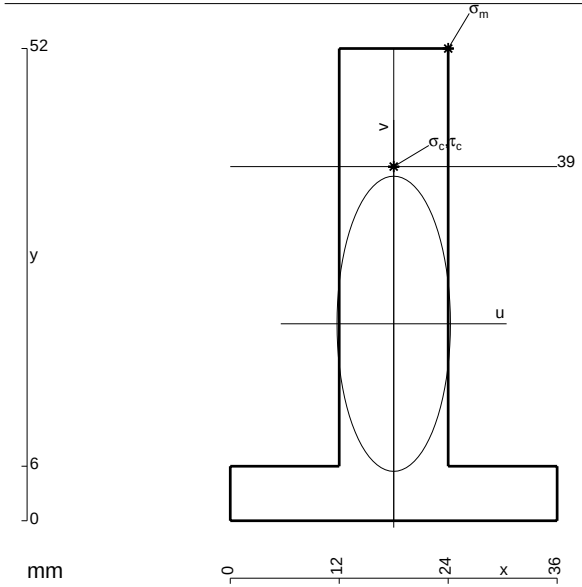
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

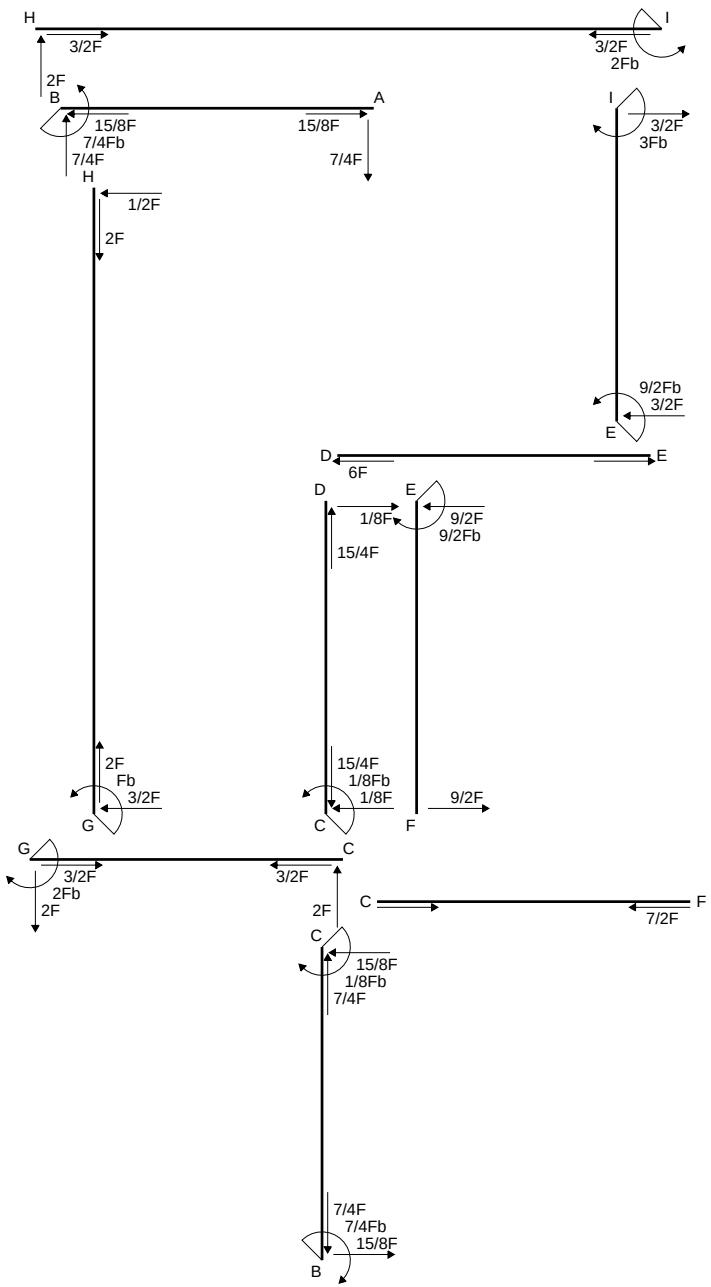


PROCEDIMENTO E RISULTATI 270256 CI01.xxxx.062



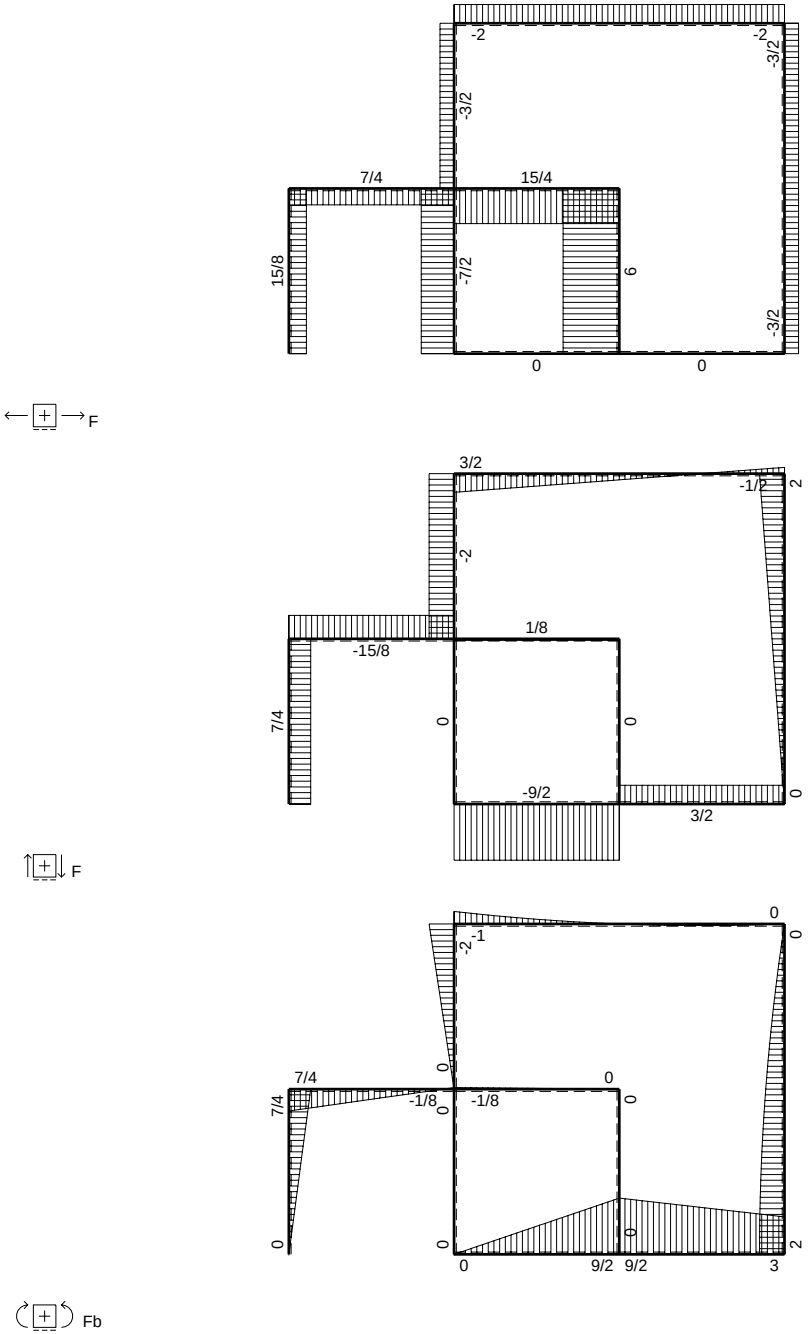

A = 768. $mm^2$
$J_u$ = 202933. $mm^4$
$J_v$ = 29952. $mm^4$
$y_g$ = 21.69 mm
$T_y$ = -2880. N
$M_x$ = 1468800. Nmm
$x_m$ = 24. mm
$y_m$ = 52. mm
$u_m$ = 6. mm
$v_m$ = 30.31 mm
$\sigma_m = -Mv/J_u$ = -219.4 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 39. mm
$v_c$ = 17.31 mm
$\sigma_c = -Mv/J_u$ = -125.3 $N/mm^2$
$\tau_c$ = 4.393 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.5 $N/mm^2$
$S^*$ = 3715. $mm^3$

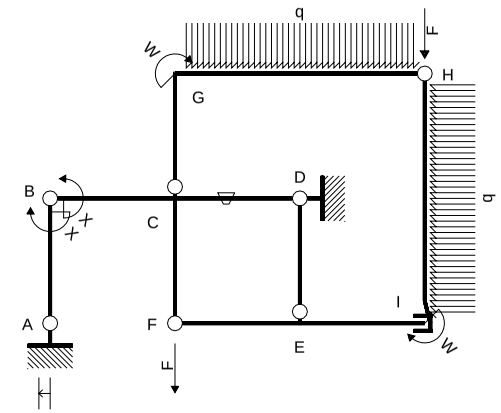
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H
I
3/2F
3/2F
2Fb
2F
B
A
15/8F
7/4Fb
7/4F
15/8F
7/4F
I
3/2F
3Fb
H
1/2F
2F
9/2Fb
3/2F
E
D
E
6F
D
E
1/8F
9/2F
9/2Fb
15/4F
2F
Fb
3/2F
G
15/4F
1/8Fb
1/8F
C
9/2F
F
G
C
3/2F
3/2F
2Fb
2F
2F
C
F
7/2F
C
15/8F
1/8Fb
7/4F
7/4F
7/4Fb
15/8F
B

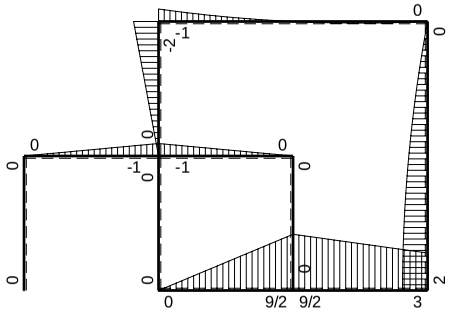




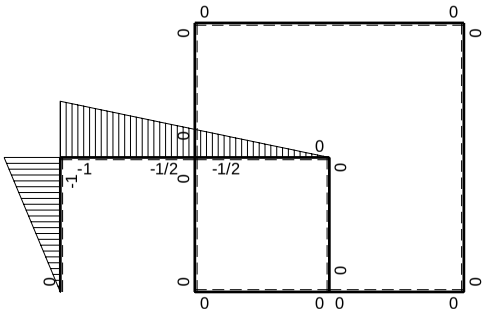

-2
-2
-3/2
-3/2
7/4
15/4
15/8
-7/2
6
-3/2
0
0
F
3/2
-1/2
2
-2
1/8
-15/8
7/4
0
0
-9/2
0
3/2
F
0
0
-2
-1
0
7/4
0
-1/8
-1/8
7/4
0
0
0
0
0
0
9/2
9/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 308763 CI01.xxxx.063

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PROCEDIMENTO E RISULTATI 308763 CI01.xxxx.063

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/2Fb-9/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $7/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

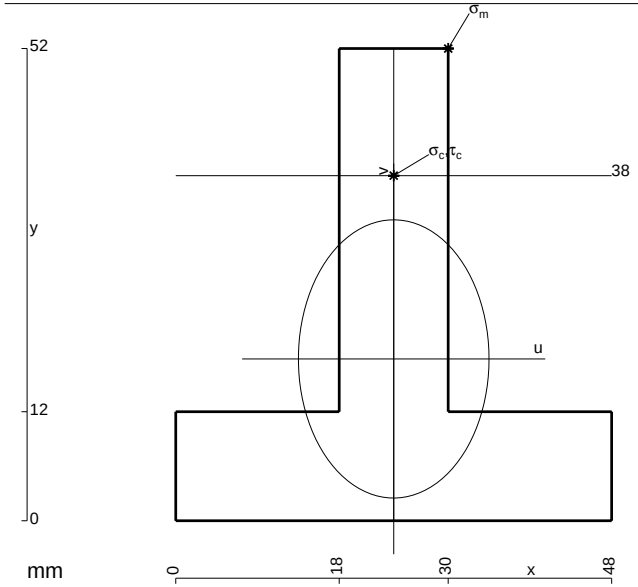
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

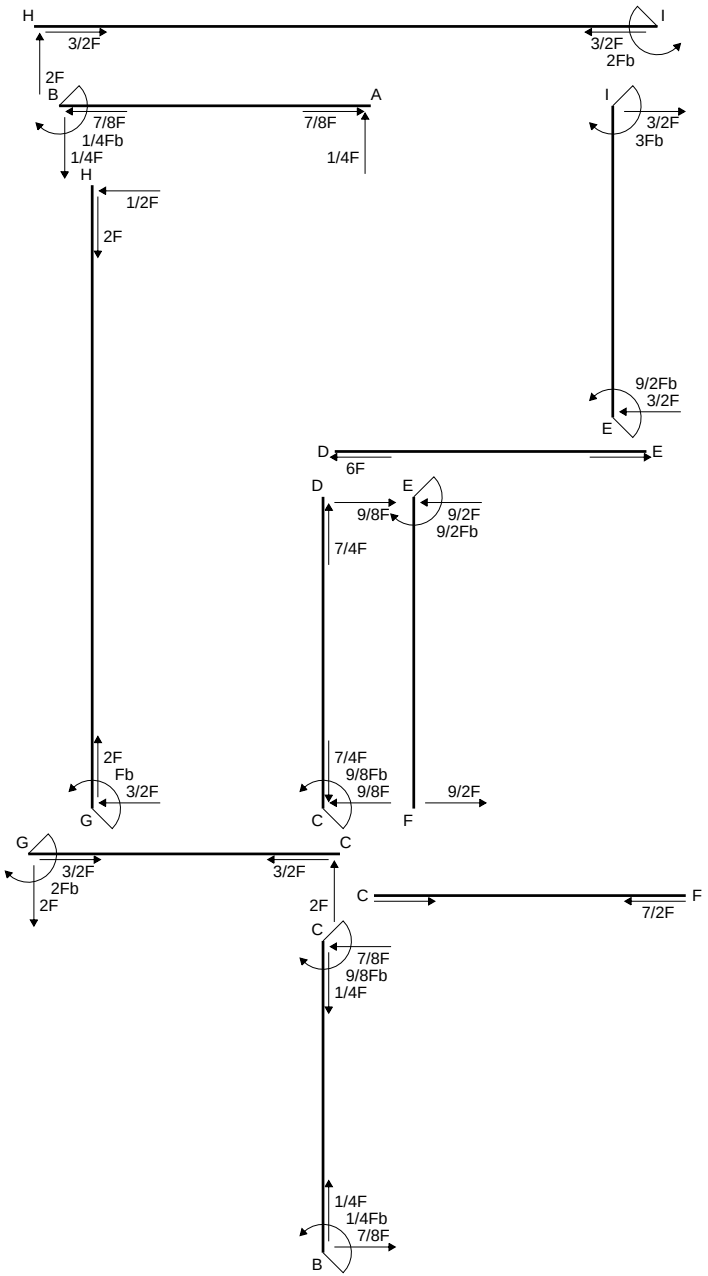
$= (1/6\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$



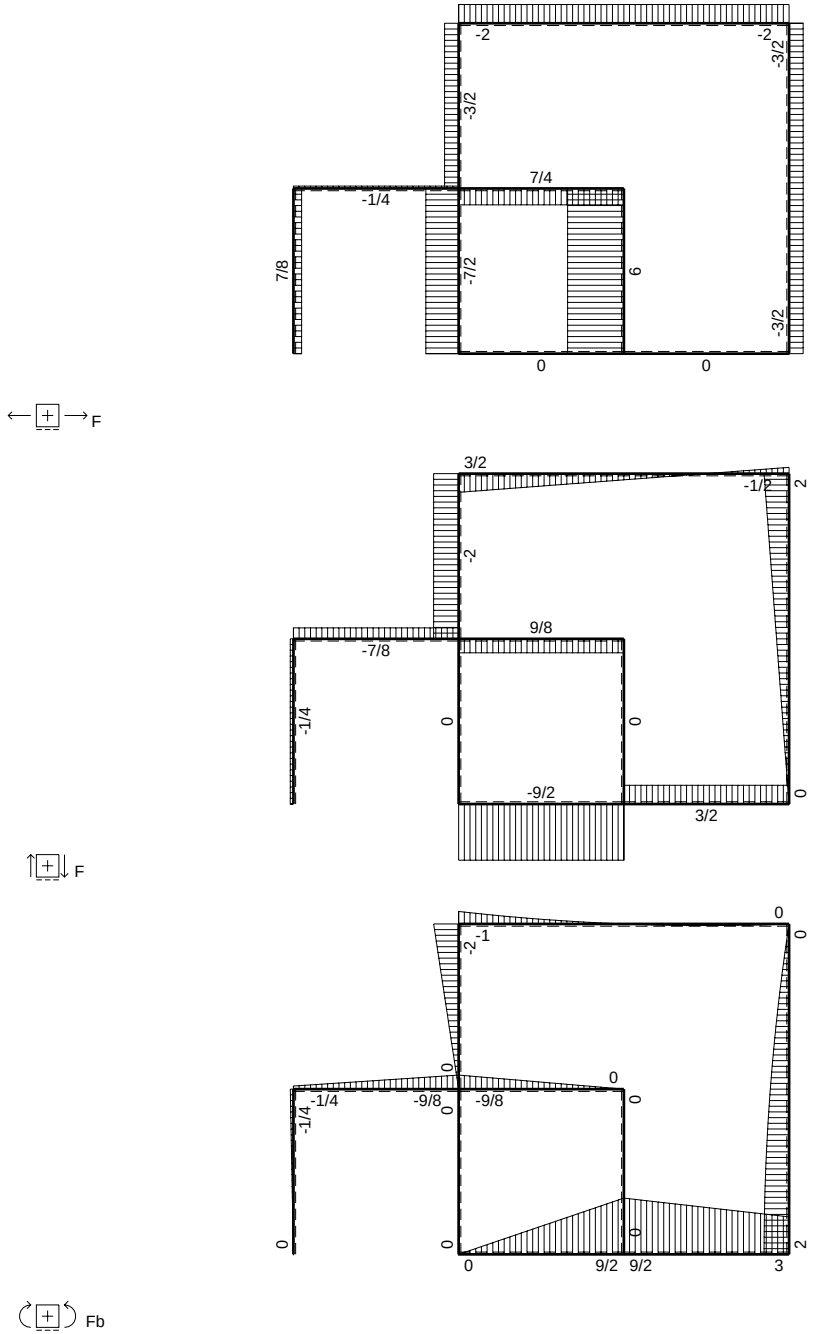





A = 1056. $mm^2$
$J_u$ = 247901. $mm^4$
$J_v$ = 116352. $mm^4$
$y_g$ = 17.82 mm
$T_y$ = -3015. N
$M_x$ = 1658250. Nmm
$x_m$ = 30. mm
$y_m$ = 52. mm
$u_m$ = 6. mm
$v_m$ = 34.18 mm
$\sigma_m = -Mv/J_u$ = -228.6 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 38. mm
$v_c$ = 20.18 mm
$\sigma_c = -Mv/J_u$ = -135. $N/mm^2$
$\tau_c$ = 4.628 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.2 $N/mm^2$
$S^*$ = 4567. $mm^3$

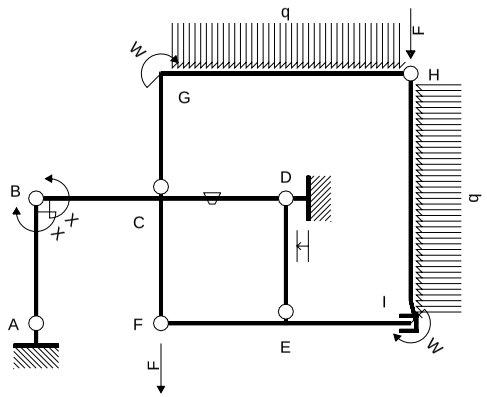




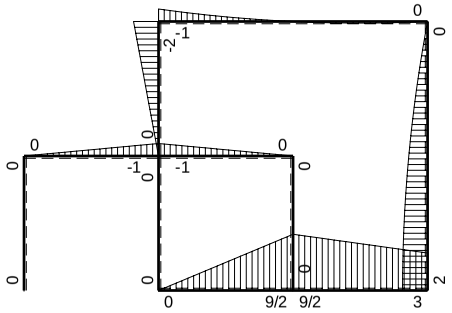





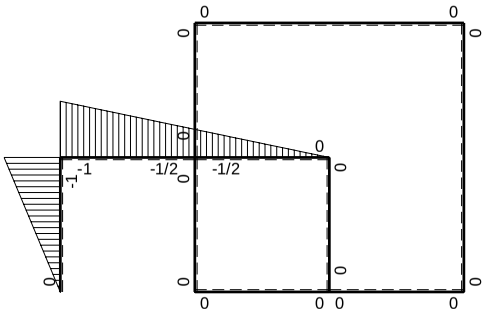

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Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 239027 CI01.xxxx.064

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PROCEDIMENTO E RISULTATI 239027 CI01.xxxx.064

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/2Fb-9/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

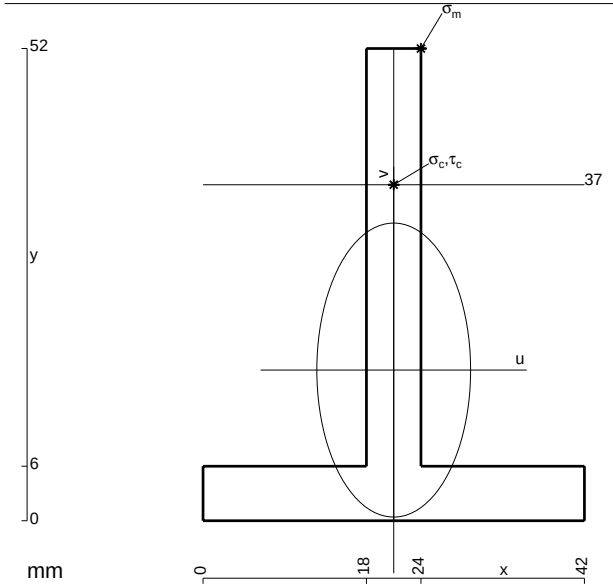
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$

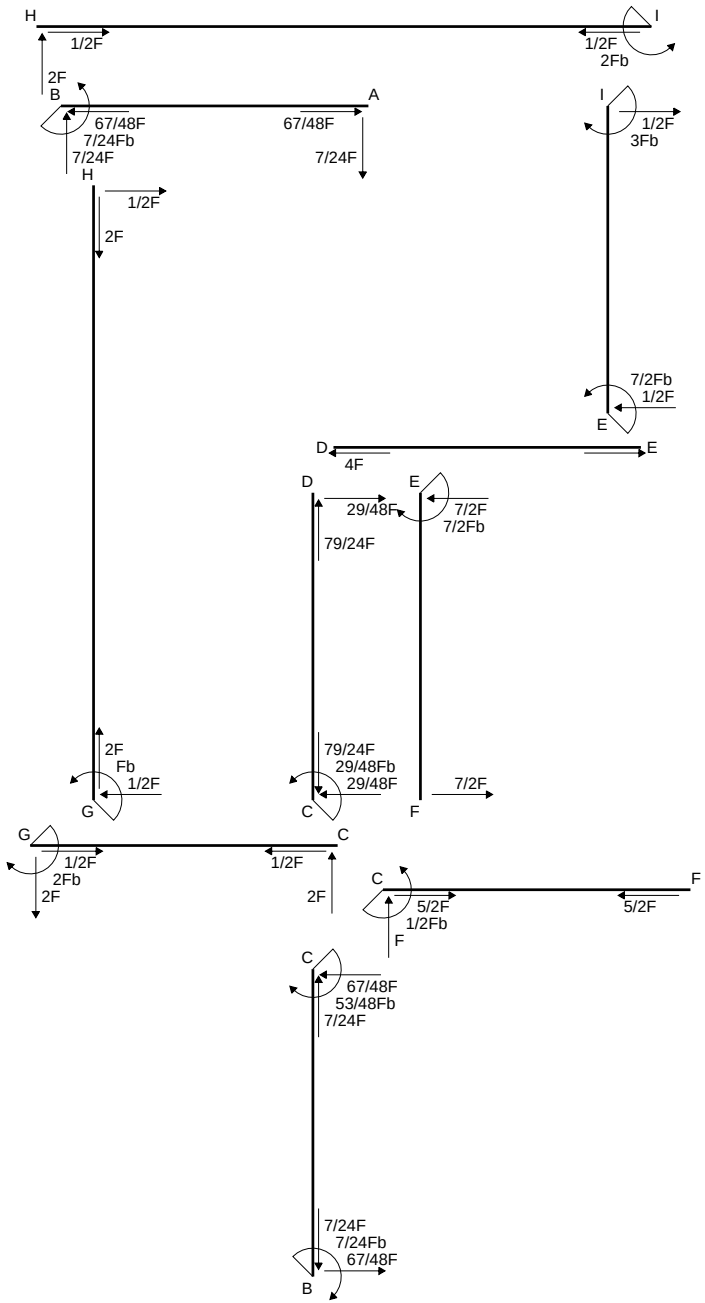


PROCEDIMENTO E RISULTATI 239027 CI01.xxxx.064



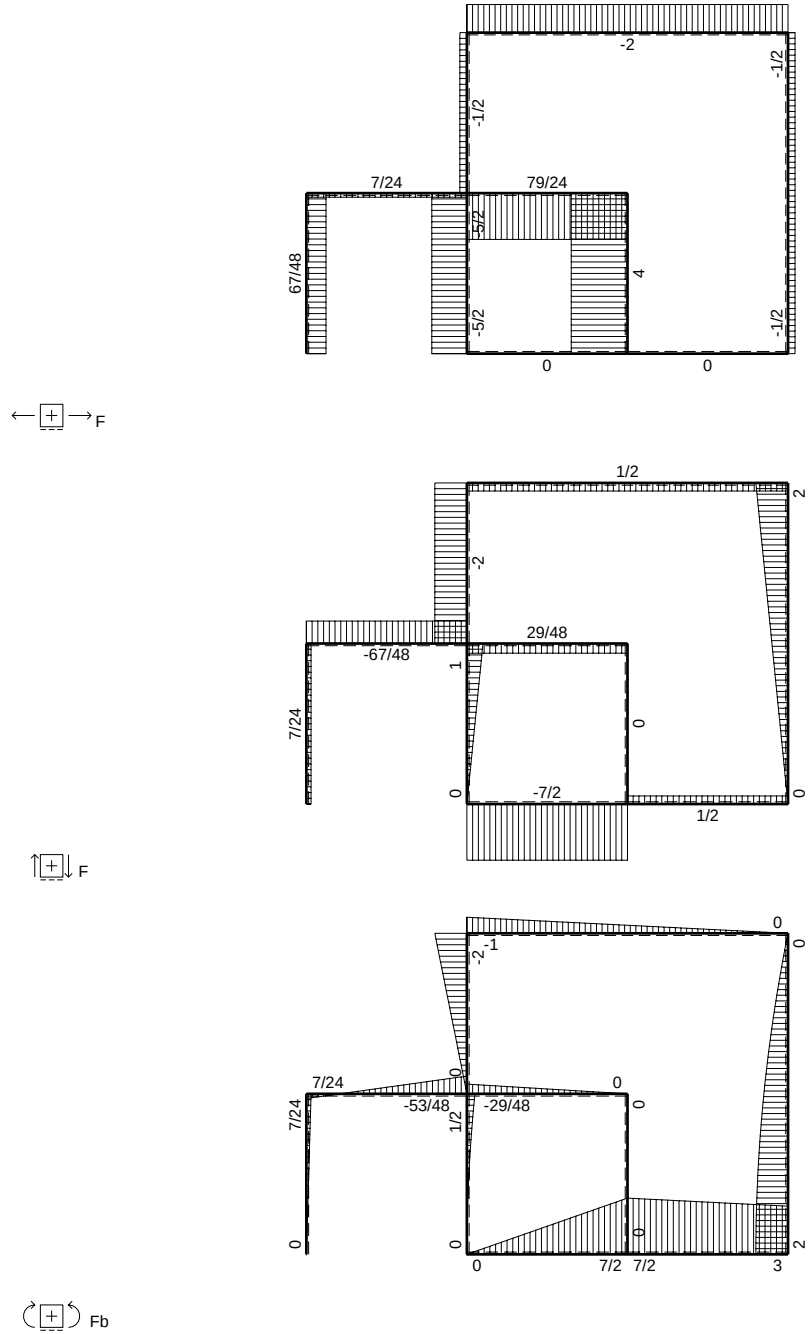

A = 528. $mm^2$
$J_u$ = 138472. $mm^4$
$J_v$ = 37872. $mm^4$
$y_g$ = 16.59 mm
$T_y$ = -1530. N
$M_x$ = 918000. Nmm
$x_m$ = 24. mm
$y_m$ = 52. mm
$u_m$ = 3. mm
$v_m$ = 35.41 mm
$\sigma_m = -Mv/J_u$ = -234.7 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 37. mm
$v_c$ = 20.41 mm
$\sigma_c = -Mv/J_u$ = -135.3 $N/mm^2$
$\tau_c$ = 4.626 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.5 $N/mm^2$
$S^*$ = 2512. $mm^3$

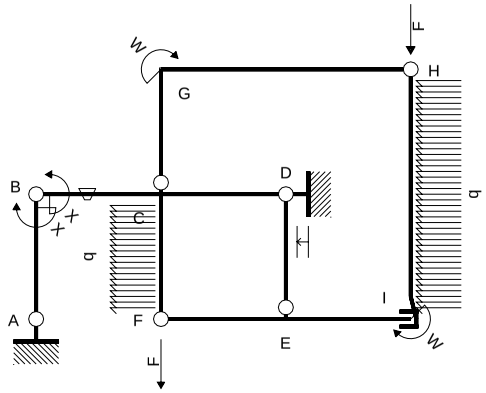
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
1/2F
1/2F
2Fb
2F
B
A
67/48F
67/48F
7/24Fb
7/24F
7/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
D
E
29/48F
7/2F
7/2Fb
79/24F
2F
Fb
1/2F
G
79/24F
29/48Fb
29/48F
C
7/2F
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
5/2F
5/2F
1/2Fb
F
C
67/48F
53/48Fb
7/24F
7/24F
7/24Fb
67/48F
B

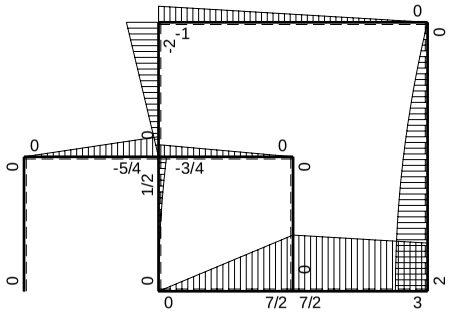




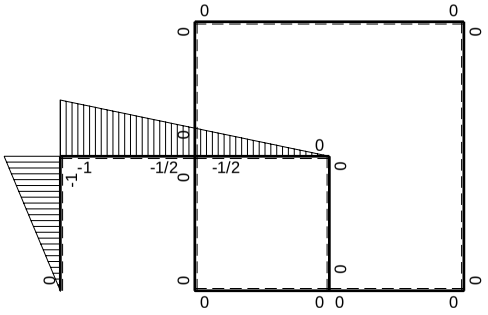

F
-2
-1/2
-1/2
7/24
79/24
5/2
67/48
4
-5/2
-1/2
0
0
F
1/2
2
-2
29/48
-67/48
1
7/24
0
0
-7/2
0
1/2
F
0
0
-2
-1
0
7/24
0
-53/48
-29/48
1/2
7/24
0
0
0
0
0
2
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 269546 CI01.xxxx.065

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PROCEDIMENTO E RISULTATI 269546 CI01.xxxx.065

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $7/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

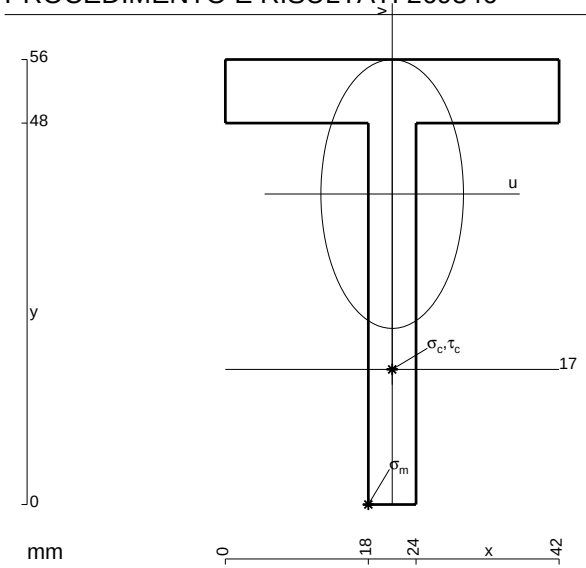
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

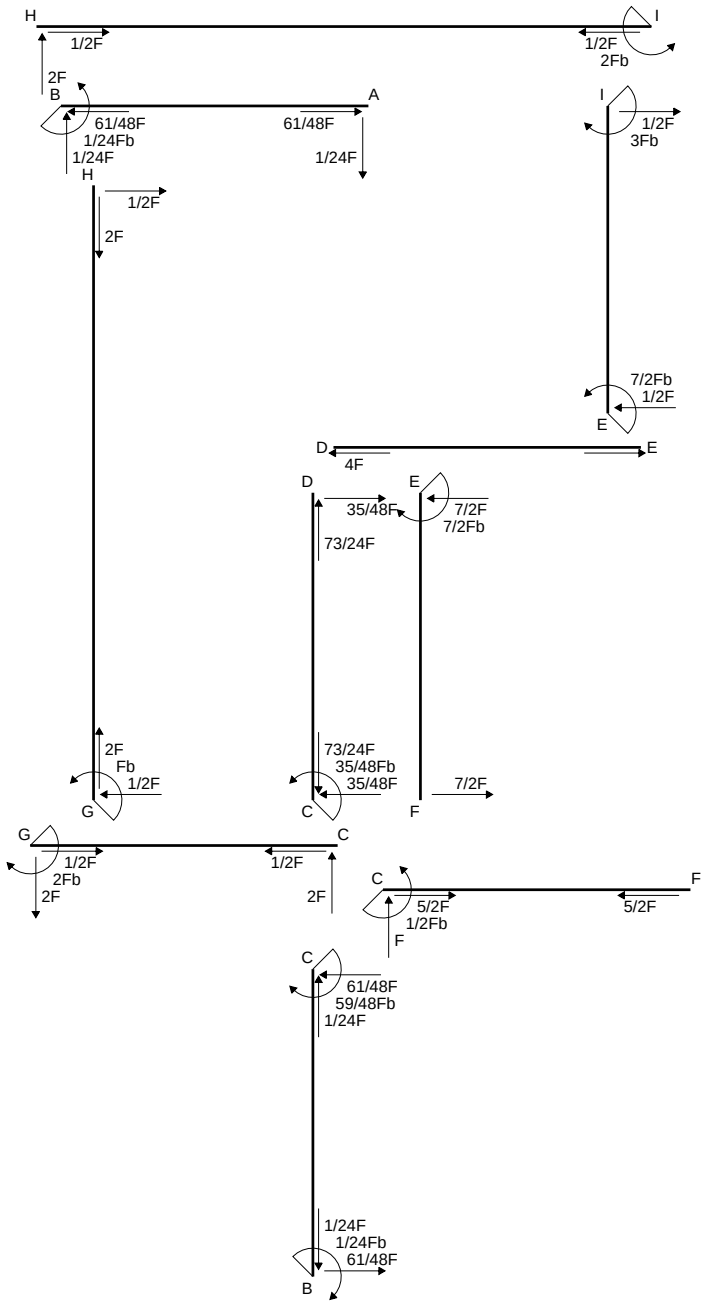
$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





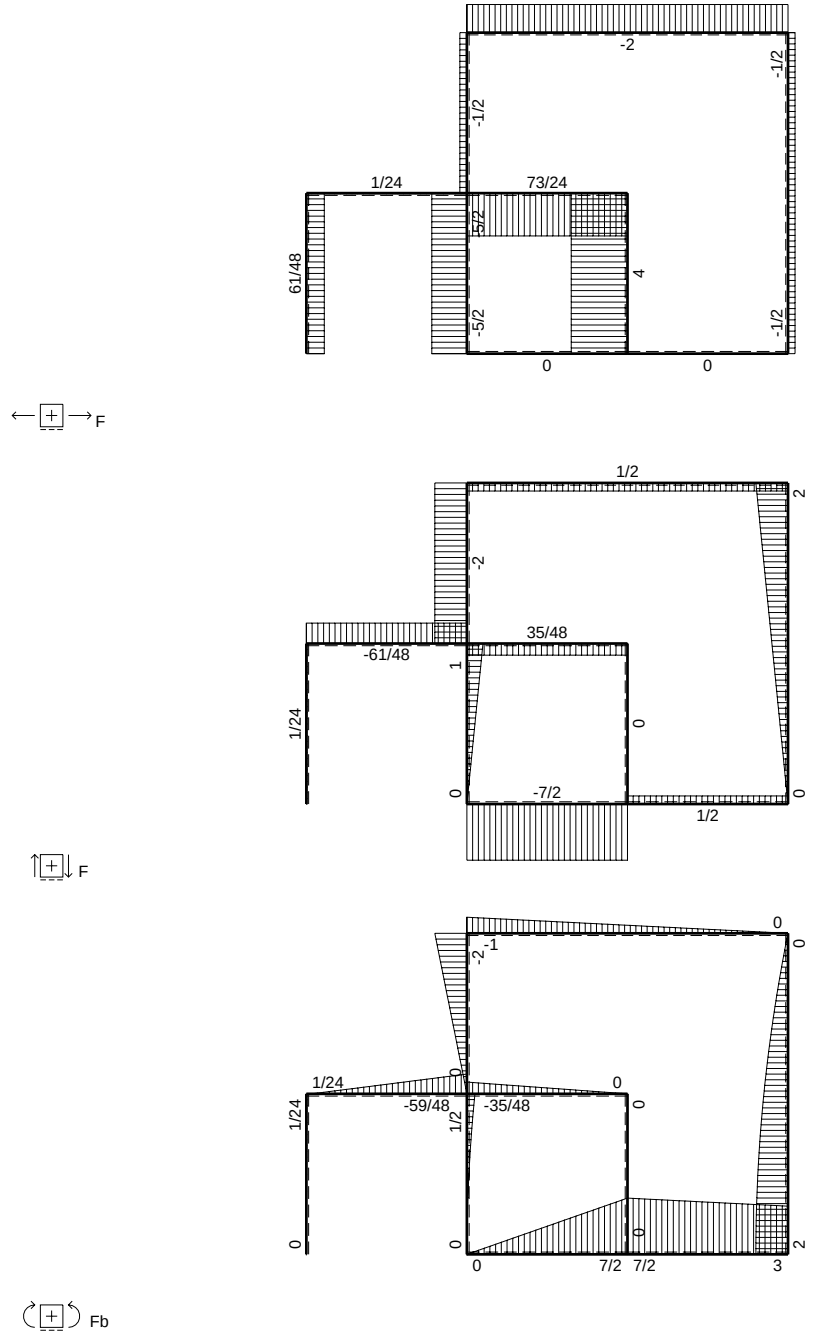


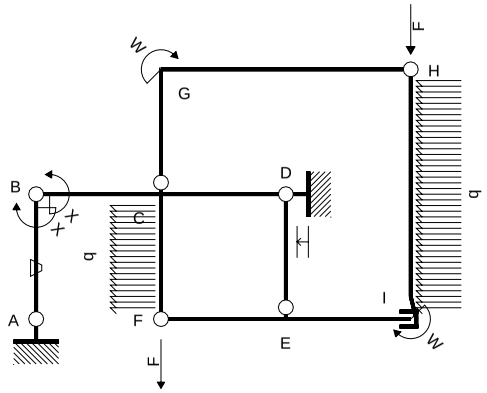

A = 624. $mm^2$
$J_u$ = 178668. $mm^4$
$J_v$ = 50256. $mm^4$
$y_g$ = 39.08 mm
$T_y$ = -1295. N
$M_x$ = 893550. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -39.08 mm
$\sigma_m = -Mv/J_u$ = 195.4 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 17. mm
$v_c$ = -22.08 mm
$\sigma_c = -Mv/J_u$ = 110.4 $N/mm^2$
$\tau_c$ = 3.768 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 110.6 $N/mm^2$
$S^*$ = 3119. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
61/48F
1/24Fb
1/24F
61/48F
1/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
D
E
35/48F
7/2F
7/2Fb
73/24F
2F
Fb
1/2F
G
73/24F
35/48Fb
35/48F
C
7/2F
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
5/2F
1/2Fb
5/2F
F
C
61/48F
59/48Fb
1/24F
1/24F
1/24Fb
61/48F
B

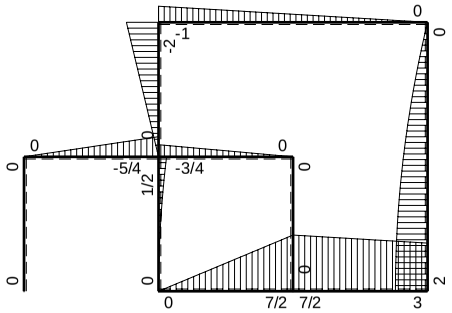




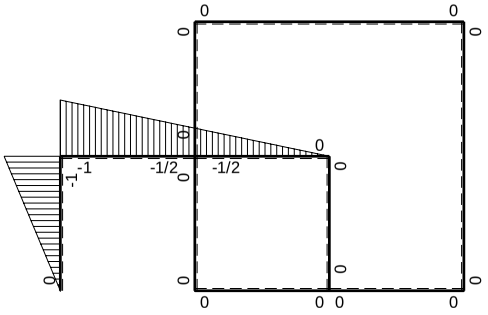

F
-2
-1/2
-1/2
1/24
73/24
5/2
61/48
4
-5/2
-1/2
0
0
F
1/2
2
-2
35/48
-61/48
1
1/24
0
0
-7/2
1/2
0
Fb
0
0
-2
-1
1/24
0
0
-59/48
-35/48
1/2
1/24
0
0
0
0
0
7/2
7/2
3
2




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 981229 CI01.xxxx.066

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PROCEDIMENTO E RISULTATI 981229 CI01.xxxx.066

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

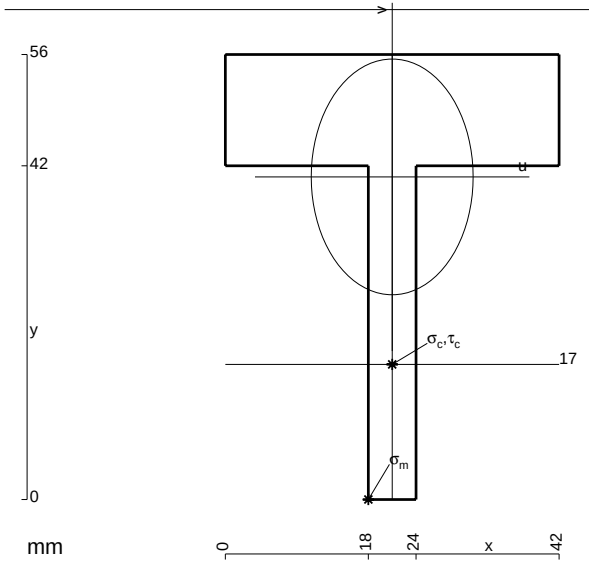
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

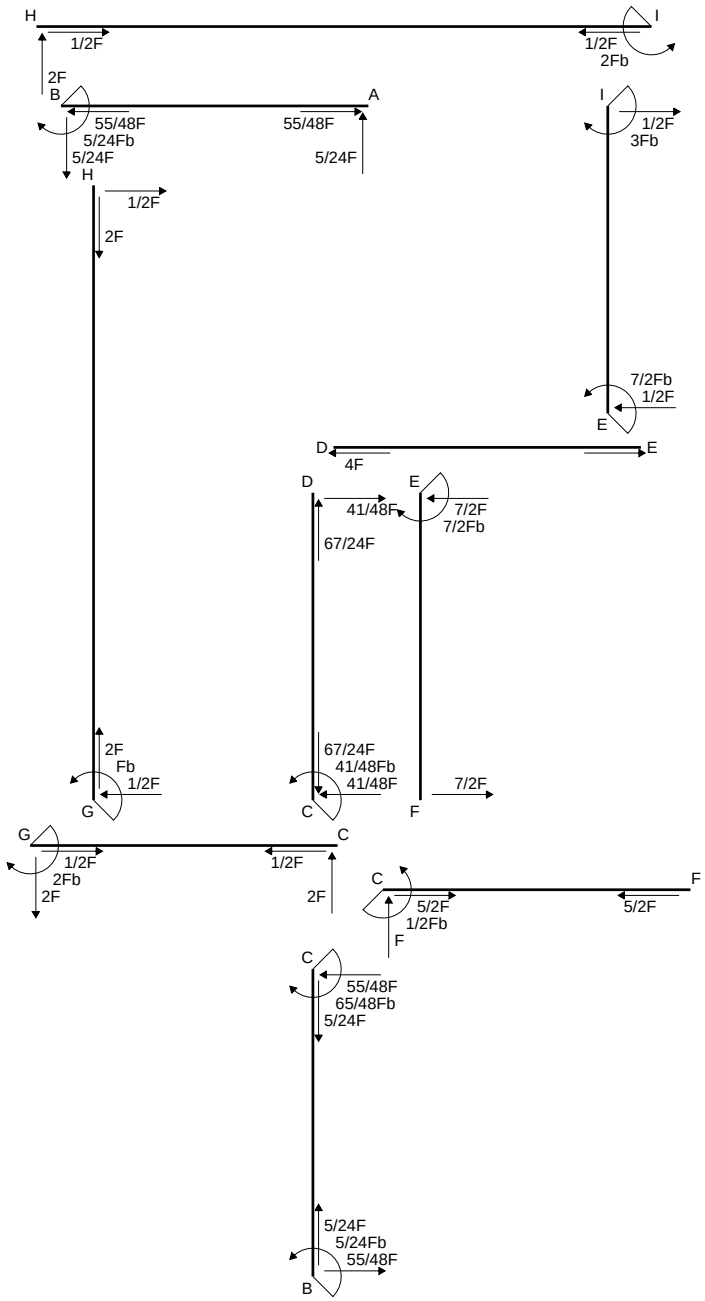
$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





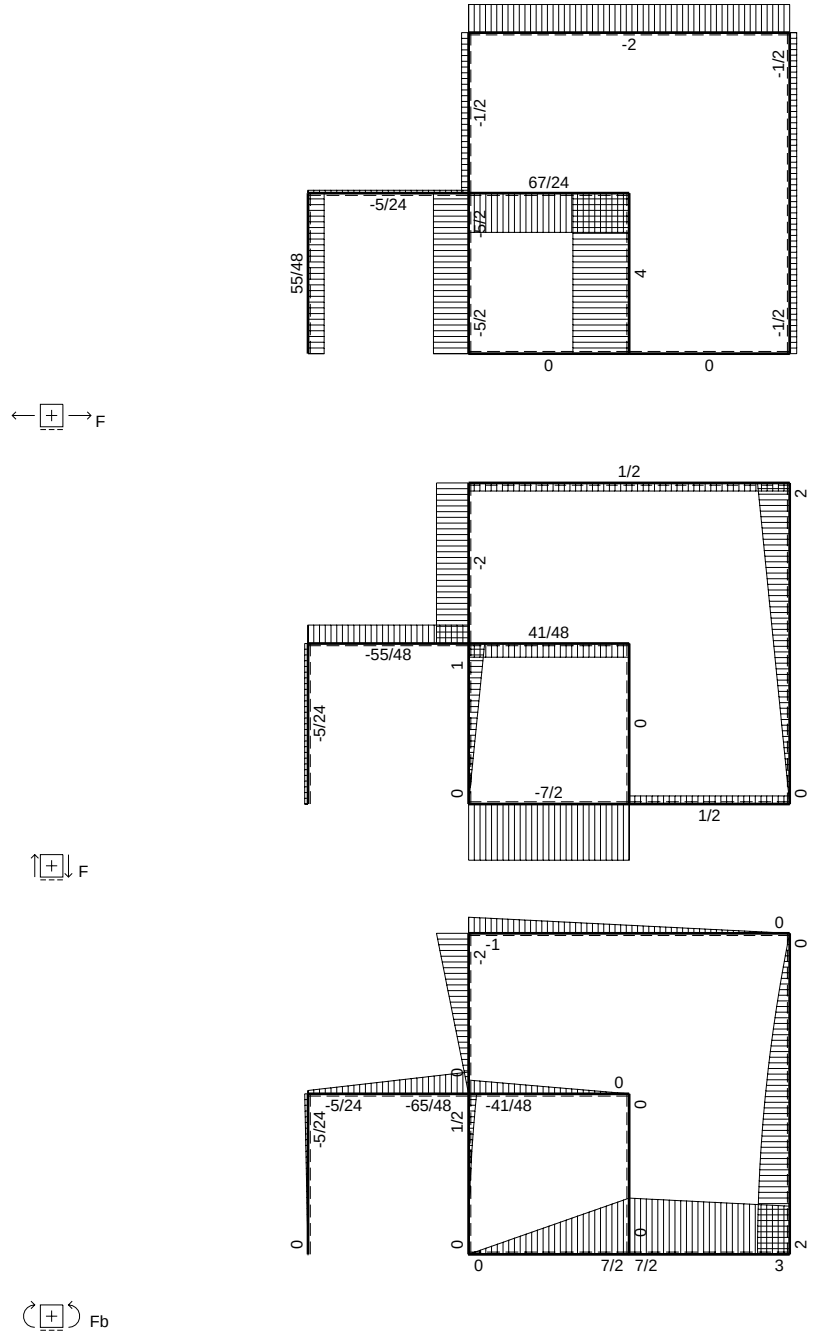


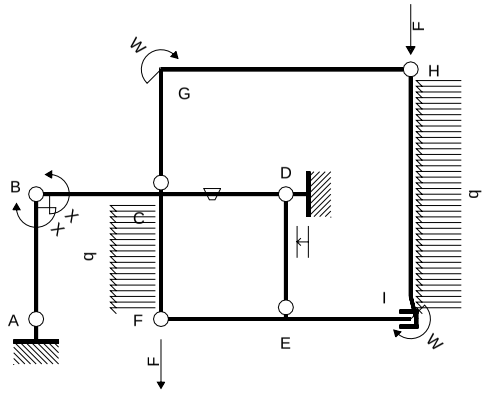

A = 840. $mm^2$
$J_u$ = 184946. $mm^4$
$J_v$ = 87192. $mm^4$
$y_g$ = 40.6 mm
$T_y$ = -1295. N
$M_x$ = 945350. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -40.6 mm
$\sigma_m = -Mv/J_u$ = 207.5 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 17. mm
$v_c$ = -23.6 mm
$\sigma_c = -Mv/J_u$ = 120.6 $N/mm^2$
$\tau_c$ = 3.821 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 120.8 $N/mm^2$
$S^*$ = 3274. $mm^3$


H
1/2F
I
1/2F
2Fb
2F
B
A
55/48F
5/24Fb
5/24F
55/48F
5/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
D
E
41/48F
7/2F
7/2Fb
67/24F
2F
Fb
1/2F
G
67/24F
41/48Fb
41/48F
C
7/2F
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
5/2F
1/2Fb
5/2F
F
C
55/48F
65/48Fb
5/24F
5/24F
5/24Fb
55/48F
B

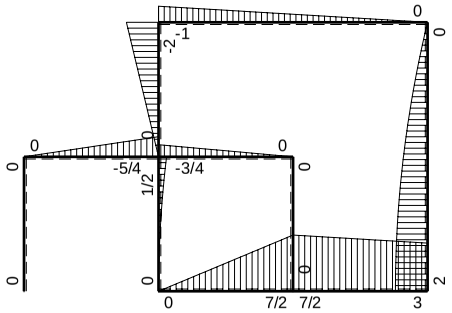




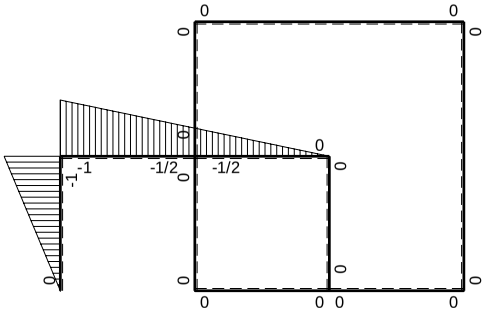

F
-2
-1/2
-1/2
67/24
-5/24
5/2
55/48
4
-5/2
-1/2
0
0
F
1/2
2
-2
41/48
-55/48
1
-5/24
0
0
-7/2
0
1/2
Fb
0
0
-2
-1
0
0
-5/24
-65/48
1/2
-41/48
-5/24
0
0
0
0
0
7/2
7/2
3
2




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268131 CI01.xxxx.067

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PROCEDIMENTO E RISULTATI 268131 CI01.xxxx.067

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-5/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 5/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

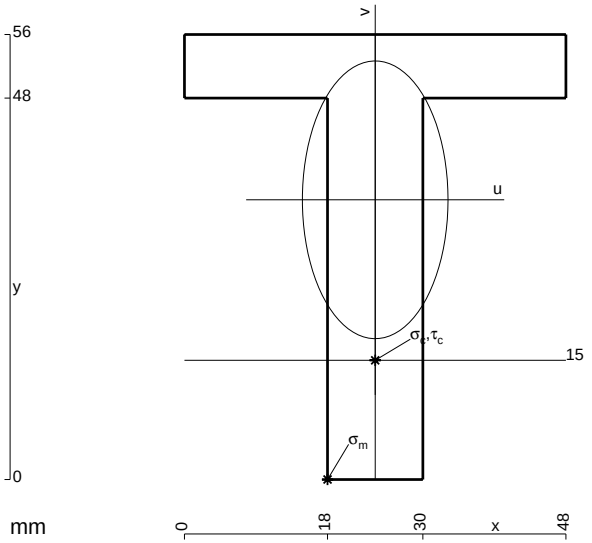
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

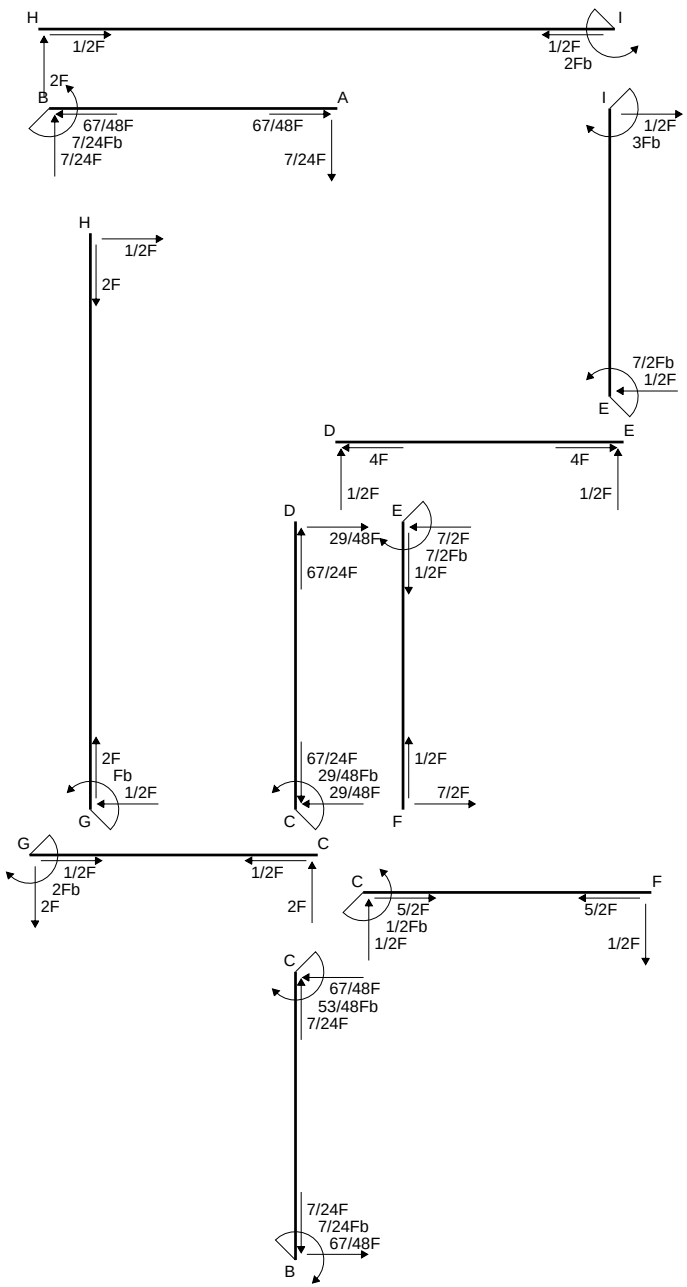
$= (1/8\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 268131 CI01.xxxx.067

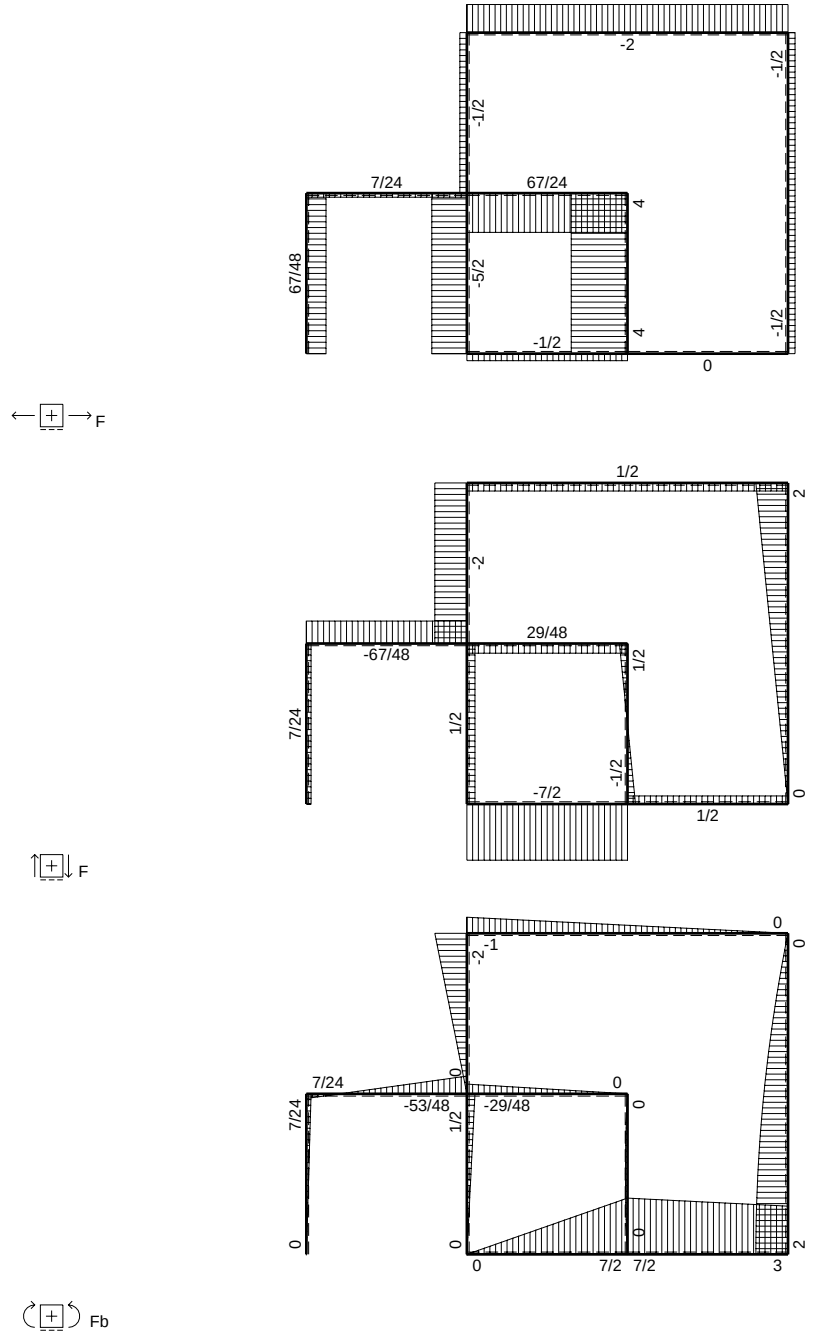


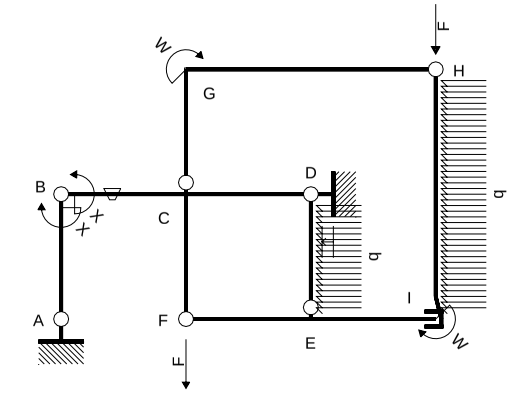

A = 960. $mm^2$
$J_u$ = 293274. $mm^4$
$J_v$ = 80640. $mm^4$
$y_g$ = 35.2 mm
$T_y$ = -2345. N
$M_x$ = 1829100. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -35.2 mm
$\sigma_m = -Mv/J_u$ = 219.5 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -20.2 mm
$\sigma_c = -Mv/J_u$ = 126. $N/mm^2$
$\tau_c$ = 3.322 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.1 $N/mm^2$
$S^*$ = 4986. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
67/48F
7/24Fb
7/24F
67/48F
7/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
4F
1/2F
1/2F
D
E
29/48F
7/2F
7/2Fb
1/2F
67/24F
2F
Fb
1/2F
G
67/24F
29/48Fb
29/48F
C
1/2F
7/2F
F
G
C
1/2F
1/2F
2Fb
2F
2F
C
F
5/2F
1/2Fb
1/2F
5/2F
1/2F
C
67/48F
53/48Fb
7/24F
7/24F
7/24Fb
67/48F
B

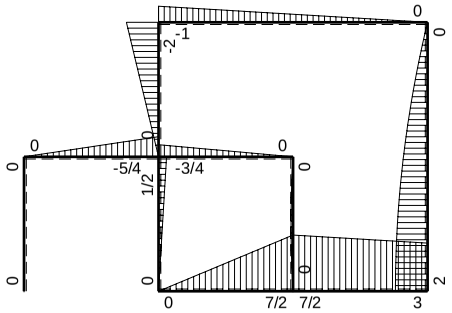




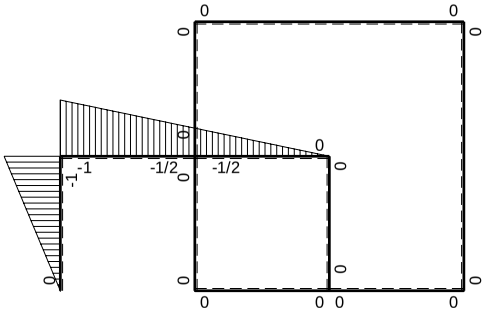

F
-2
-1/2
-1/2
7/24
67/24
4
67/48
-5/2
4
-1/2
-1/2
0
F
1/2
2
-2
29/48
-67/48
1/2
7/24
1/2
-1/2
-7/2
0
1/2
F
0
0
-2
-1
0
7/24
0
0
-53/48
-29/48
7/24
1/2
0
0
0
0
0
2
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 263971 CI01.xxxx.068

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PROCEDIMENTO E RISULTATI 263971 CI01.xxxx.068

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $7/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

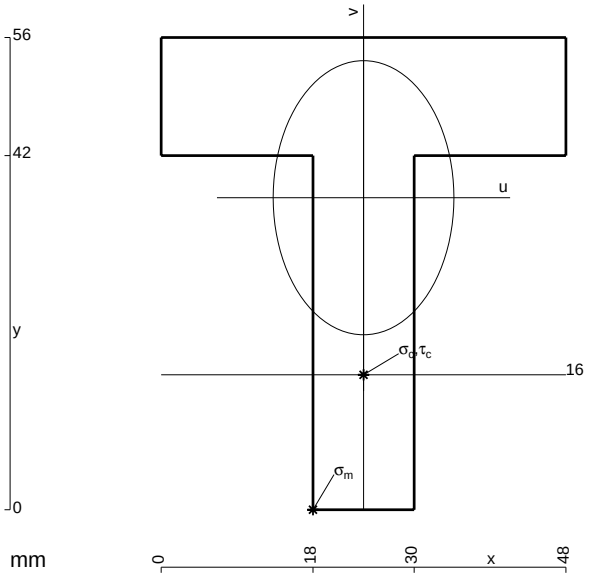
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

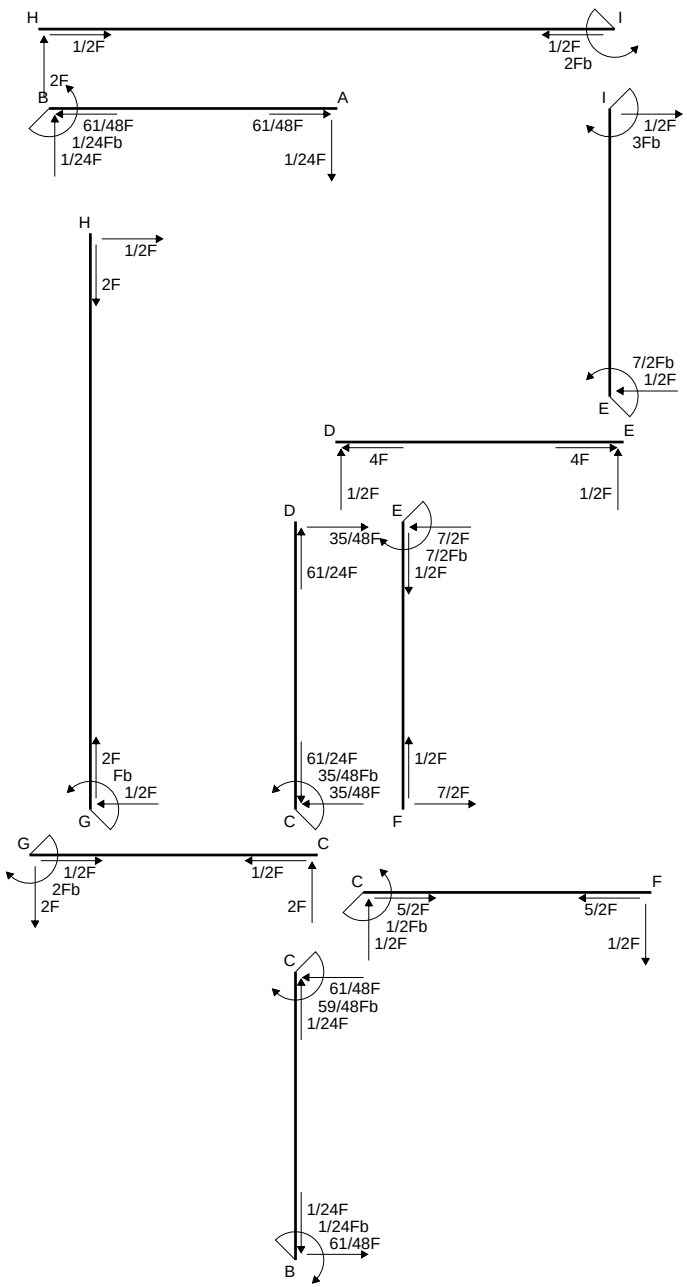
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





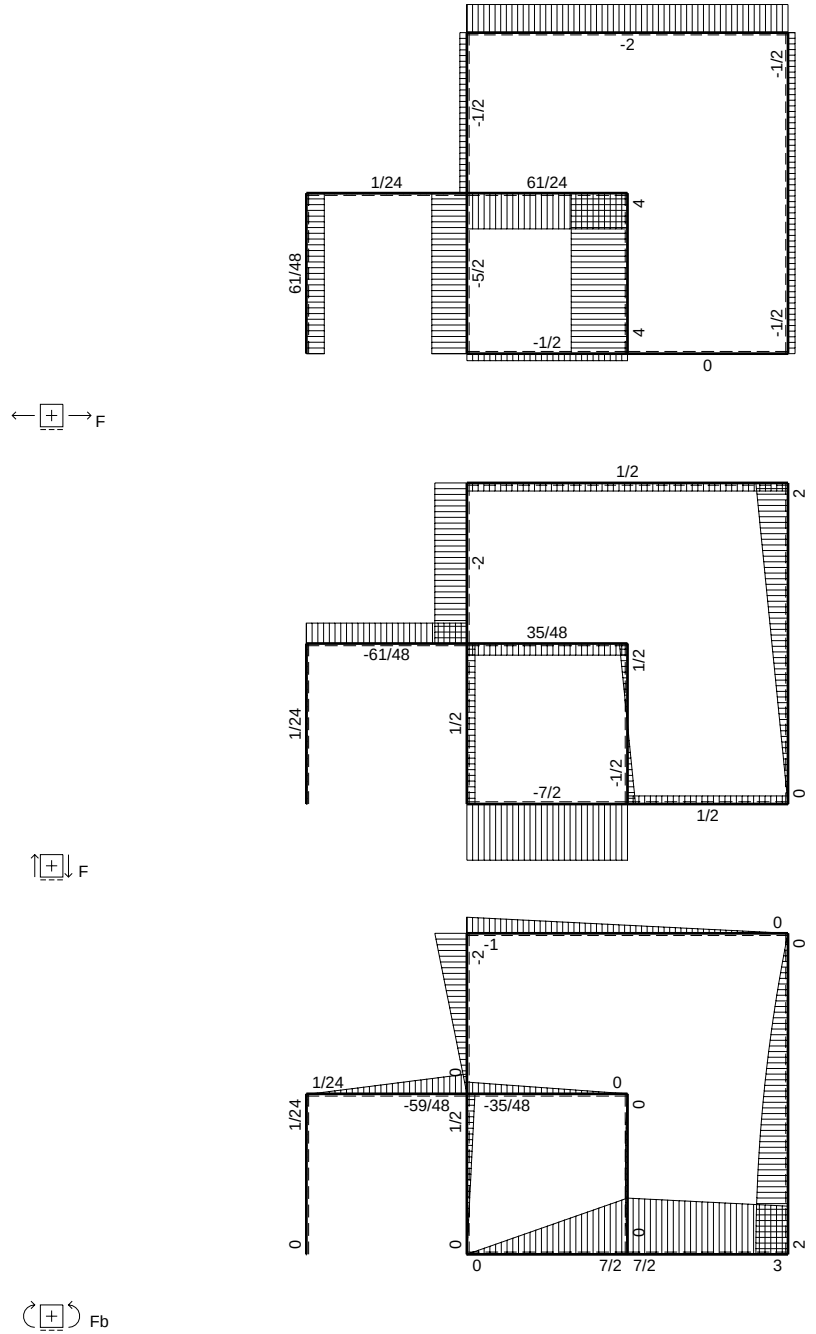


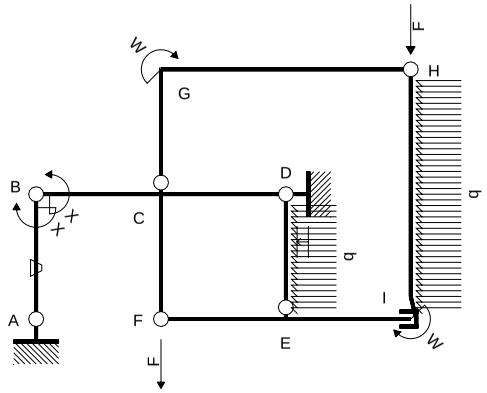

A = 1176. $mm^2$
$J_u$ = 310856. $mm^4$
$J_v$ = 135072. $mm^4$
$y_g$ = 37. mm
N = -335. N
$T_y$ = -2345. N
$M_x$ = 1922900. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -37. mm
$\sigma_m = N/A - Mv/J_u$ = 228.6 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 16. mm
$v_c$ = -21. mm
$\sigma_c = N/A - Mv/J_u$ = 129.6 $N/mm^2$
$\tau_c$ = 3.5 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.8 $N/mm^2$
$S^*$ = 5568. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
61/48F
1/24Fb
1/24F
61/48F
1/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
4F
1/2F
1/2F
D
E
35/48F
7/2F
7/2Fb
61/24F
1/2F
2F
Fb
1/2F
G
61/24F
35/48Fb
35/48F
C
1/2F
7/2F
F
G
C
1/2F
1/2F
2Fb
2F
2F
C
F
5/2F
1/2Fb
1/2F
5/2F
1/2F
C
61/48F
59/48Fb
1/24F
1/24F
1/24Fb
61/48F
B

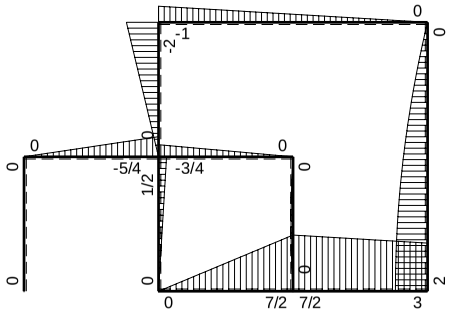




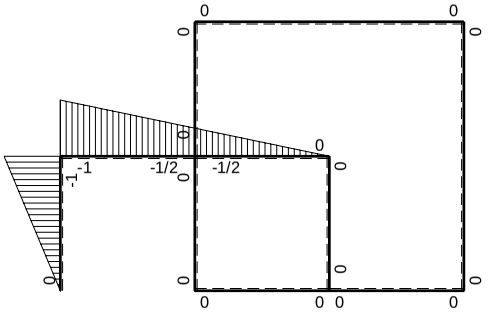

F
-2
-1/2
-1/2
1/24
61/24
4
61/48
-5/2
4
-1/2
-1/2
0
F
1/2
2
-2
35/48
-61/48
1/2
1/24
1/2
-1/2
-7/2
0
1/2
F
0
0
-2
-1
1/24
0
-59/48
-35/48
1/24
1/2
0
0
0
0
0
2
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 247885 CI01.xxxx.069

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PROCEDIMENTO E RISULTATI 247885 CI01.xxxx.069

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-5/4Fx$ | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | $3/4Fx$ | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

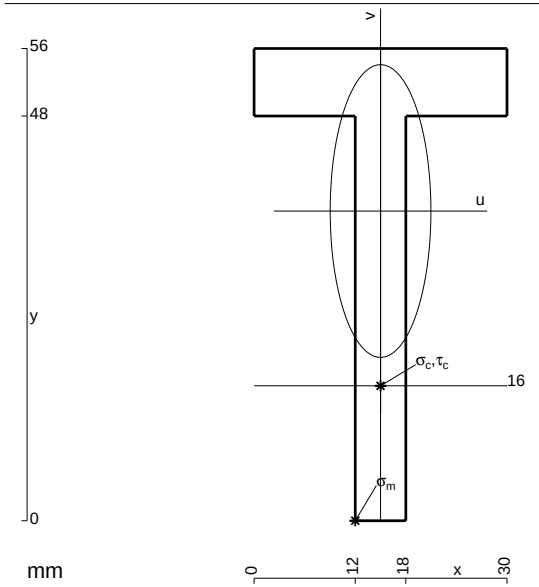
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

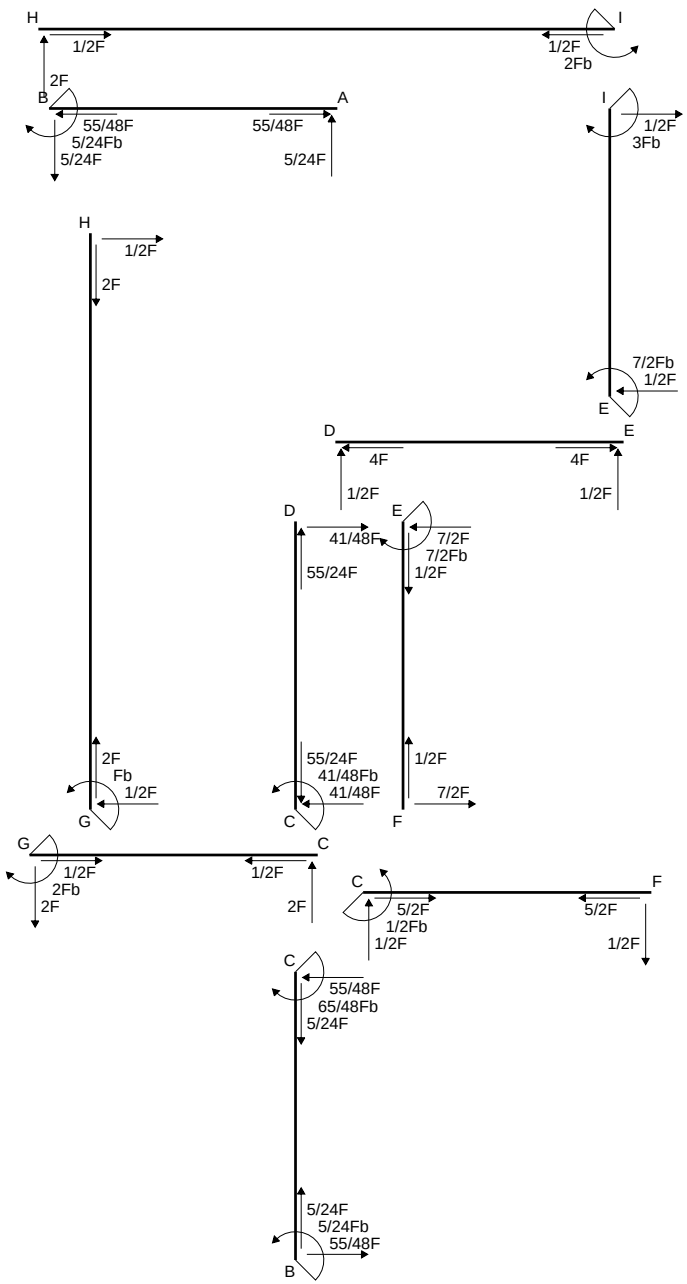


PROCEDIMENTO E RISULTATI 247885 CI01.xxxx.069



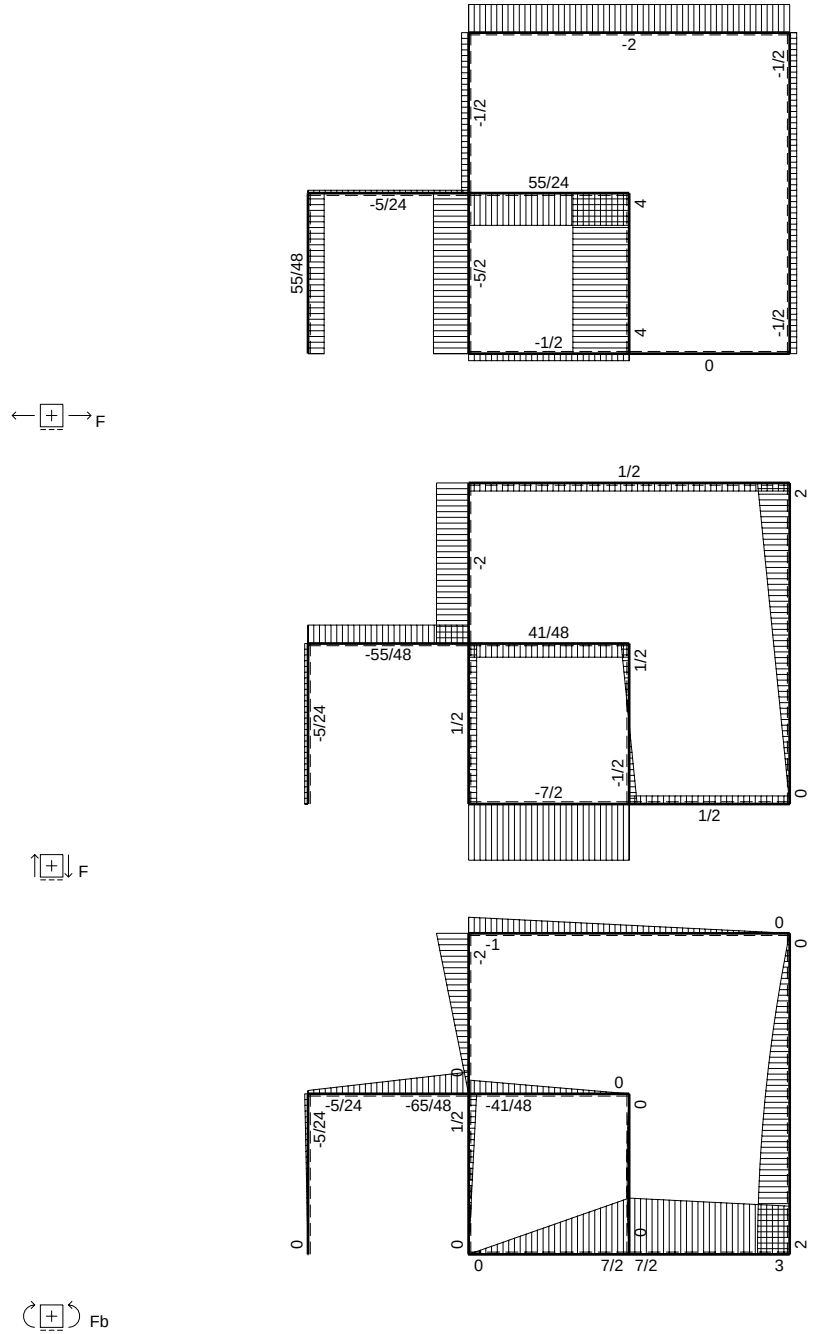

A = 528. $mm^2$
$J_u$ = 159209. $mm^4$
$J_v$ = 18864. $mm^4$
$y_g$ = 36.73 mm
N = -170. N
$T_y$ = -1190. N
$M_x$ = 1035300. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -36.73 mm
$\sigma_m = N/A - Mv/J_u$ = 238.5 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 16. mm
$v_c$ = -20.73 mm
$\sigma_c = N/A - Mv/J_u$ = 134.5 $N/mm^2$
$\tau_c$ = 3.436 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.6 $N/mm^2$
$S^*$ = 2758. $mm^3$

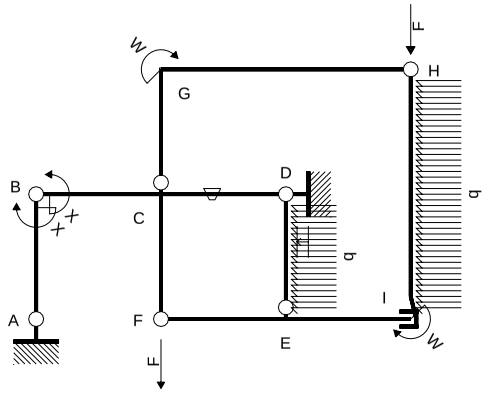
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H
1/2F
I
1/2F
2Fb
2F
B
A
55/48F
5/24Fb
5/24F
55/48F
5/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
4F
1/2F
1/2F
D
E
41/48F
7/2F
7/2Fb
1/2F
55/24F
2F
Fb
1/2F
G
55/24F
41/48Fb
41/48F
C
1/2F
7/2F
F
G
C
1/2F
1/2F
2Fb
2F
2F
C
F
5/2F
1/2Fb
1/2F
5/2F
1/2F
C
55/48F
65/48Fb
5/24F
5/24F
5/24Fb
55/48F
B

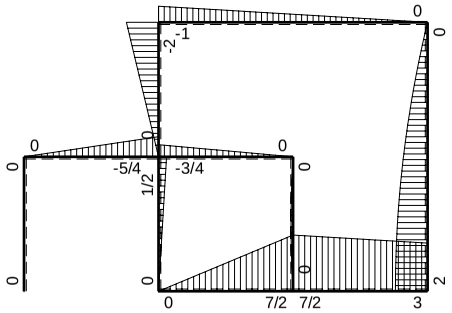




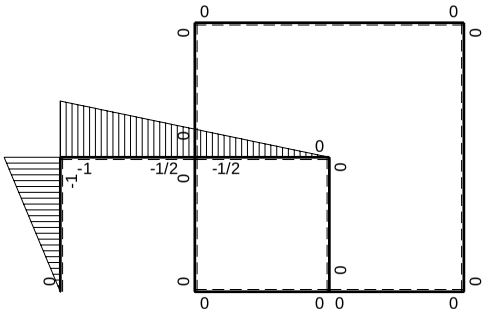

F
-2
-1/2
-1/2
55/24
-5/24
4
55/48
-5/2
4
-1/2
-1/2
0
F
1/2
2
-2
41/48
-55/48
1/2
-5/24
1/2
-1/2
-7/2
0
1/2
Fb
0
0
-1
-2
0
0
-5/24
-5/24
-65/48
1/2
-41/48
0
0
0
0
0
0
7/2
7/2
3
2




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265084 CI01.xxxx.070

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PROCEDIMENTO E RISULTATI 265084 CI01.xxxx.070

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-5/4Fx$ | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 7/2Fb-7/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -7/2Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-5/24Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 5/24Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

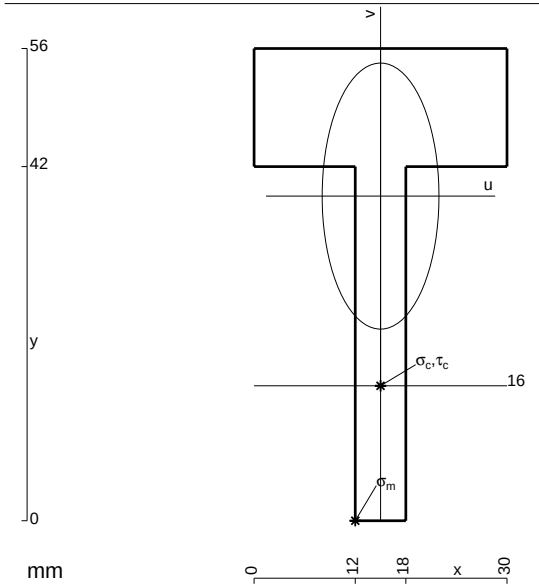
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

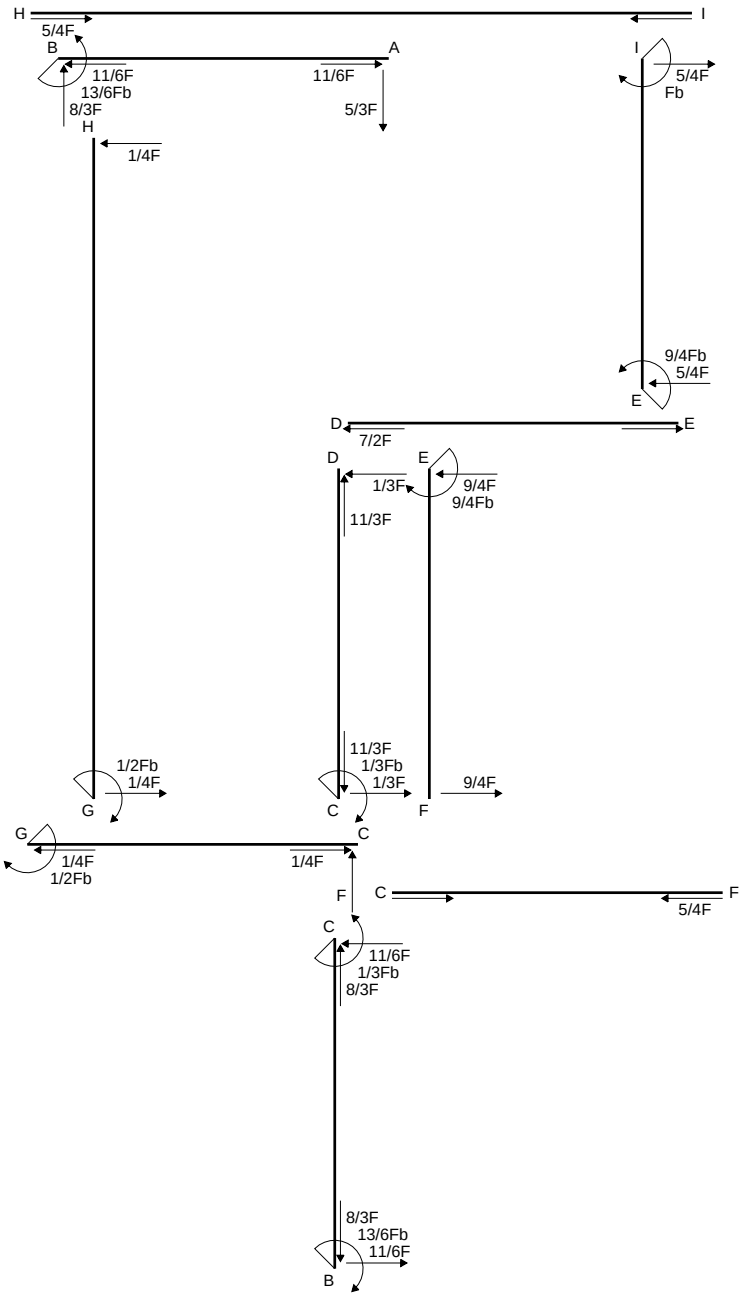


PROCEDIMENTO E RISULTATI 265084 CI01.xxxx.070



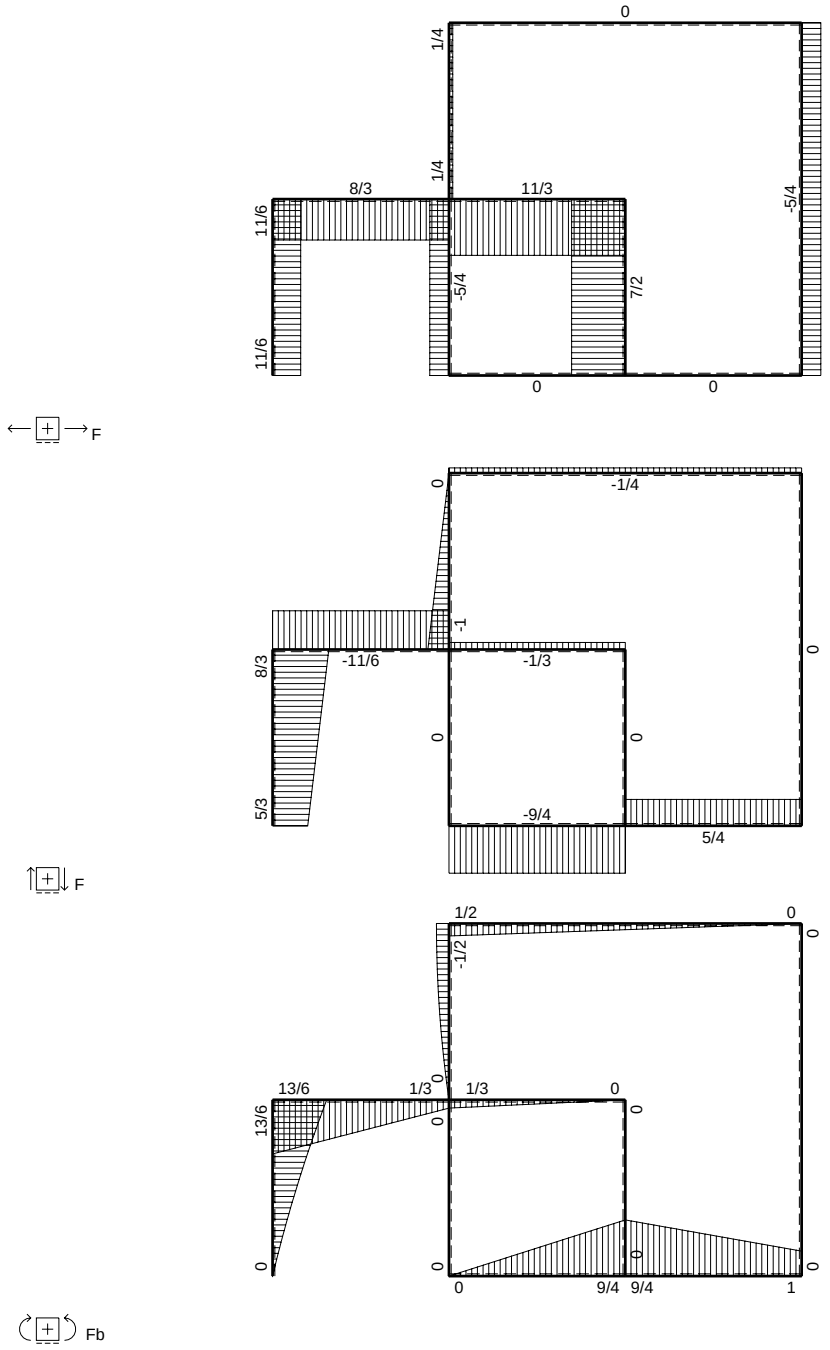

A = 672. $mm^2$
$J_u$ = 167384. $mm^4$
$J_v$ = 32256. $mm^4$
$y_g$ = 38.5 mm
N = -270. N
$T_y$ = -1890. N
$M_x$ = 869400. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -38.5 mm
$\sigma_m = N/A - Mv/J_u$ = 199.6 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 16. mm
$v_c$ = -22.5 mm
$\sigma_c = N/A - Mv/J_u$ = 116.5 $N/mm^2$
$\tau_c$ = 5.51 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.9 $N/mm^2$
$S^*$ = 2928. $mm^3$

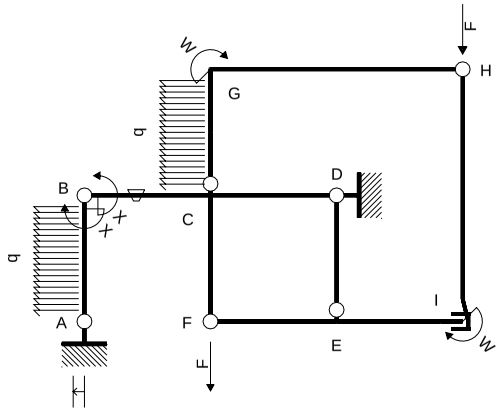
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H
I
5/4F
B
A
I
11/6F
13/6Fb
8/3F
11/6F
5/3F
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
1/3F
9/4F
9/4Fb
11/3F
1/2Fb
1/4F
G
11/3F
1/3Fb
1/3F
9/4F
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
C
11/6F
1/3Fb
8/3F
8/3F
13/6Fb
11/6F
B

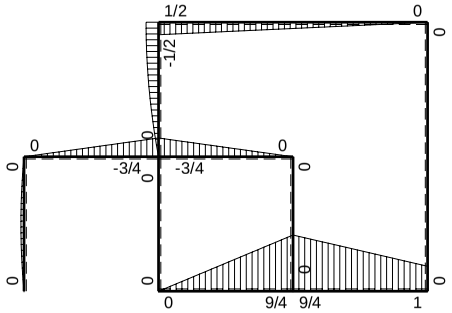




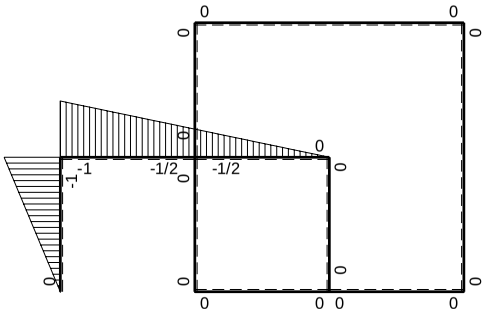

F
F
Fb
0
1/4
1/4
8/3
11/3
-5/4
11/6
-5/4
7/2
11/6
0
0
-1/4
0
-1
8/3
-11/6
-1/3
0
0
0
5/3
-9/4
5/4
1/2
0
0
-1/2
0
13/6
1/3
1/3
0
13/6
0
0
0
0
0
0
9/4
9/4
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 216391 CI01.xxxx.071

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PROCEDIMENTO E RISULTATI 216391 CI01.xxxx.071

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

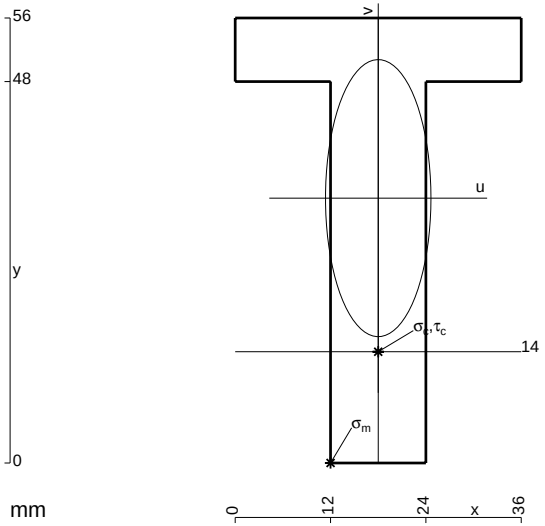
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

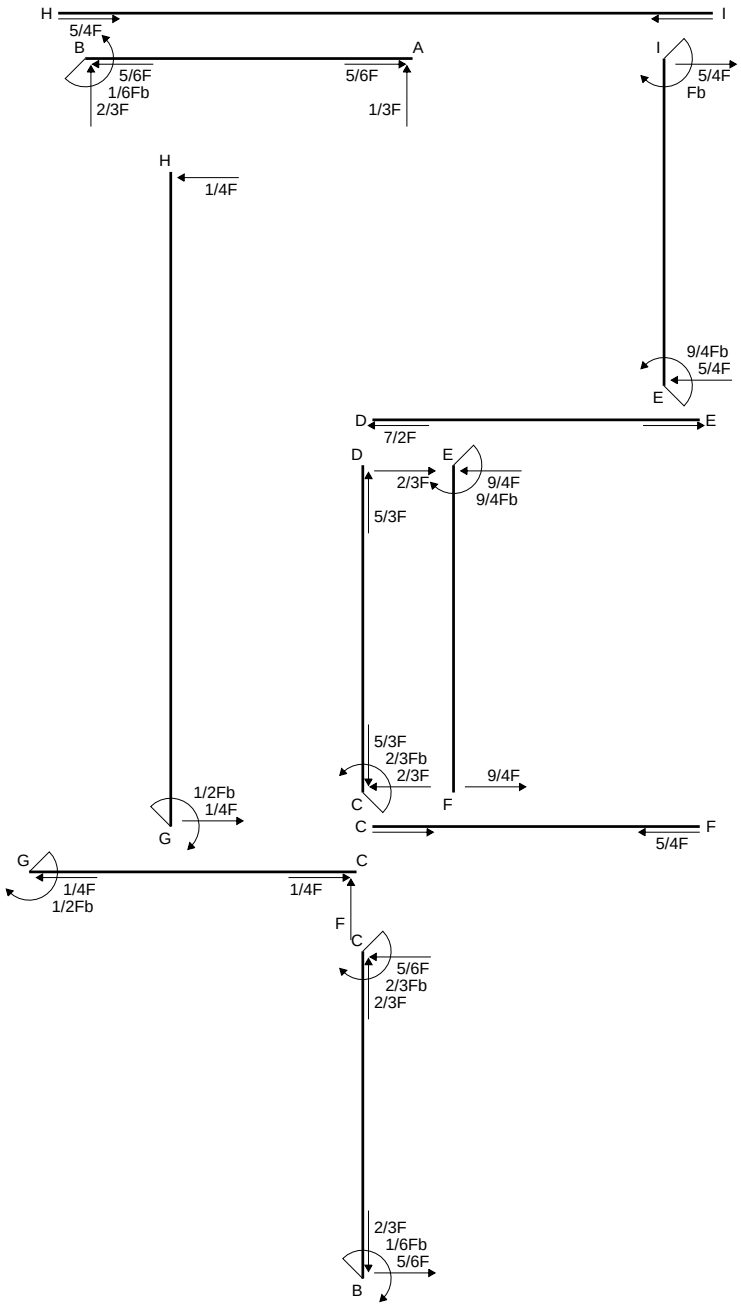


PROCEDIMENTO E RISULTATI 216391 CI01.xxxx.071



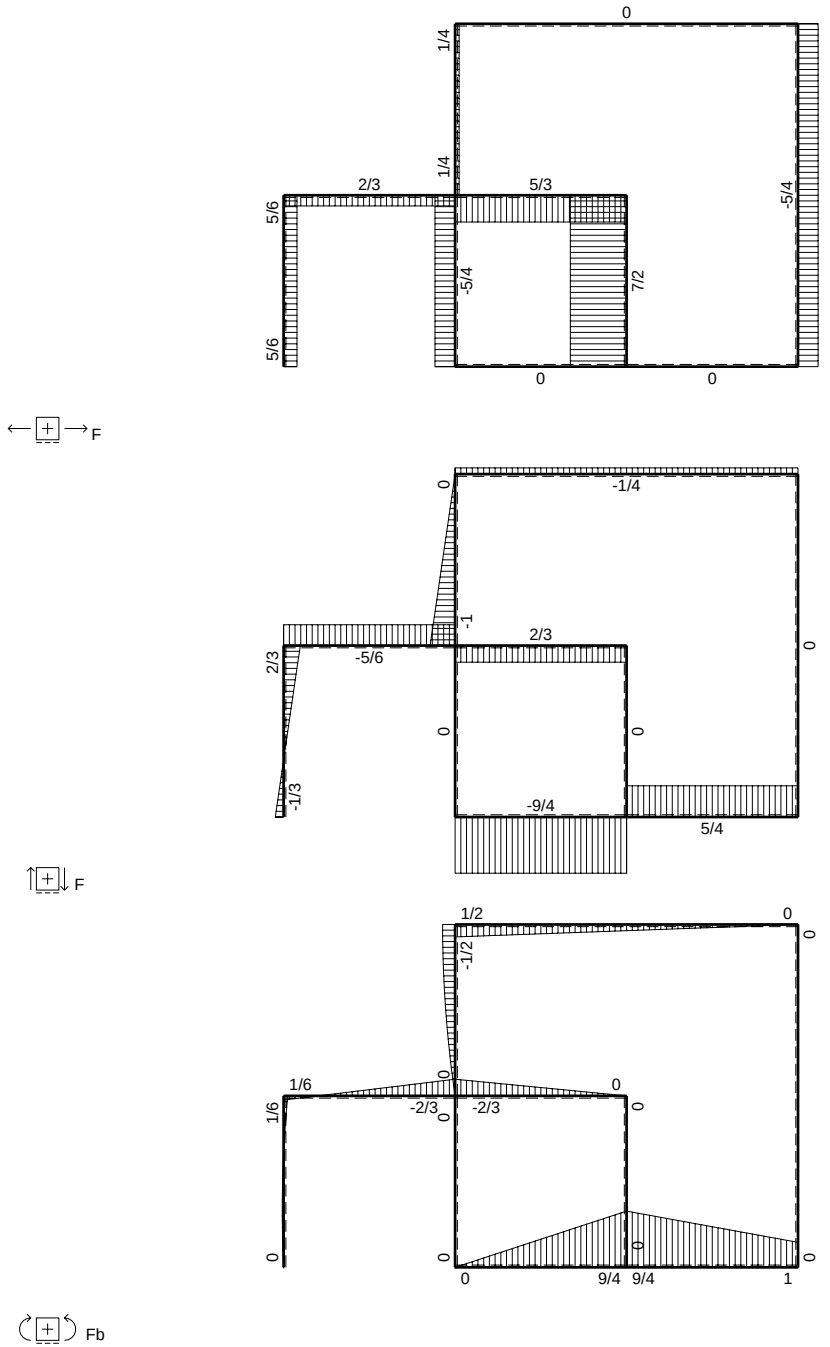

A = 864. $mm^2$
$J_u$ = 262656. $mm^4$
$J_v$ = 38016. $mm^4$
$y_g$ = 33.33 mm
$T_y$ = -3308. N
$M_x$ = 1653750. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -33.33 mm
$\sigma_m = -Mv/J_u$ = 209.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -19.33 mm
$\sigma_c = -Mv/J_u$ = 121.7 $N/mm^2$
$\tau_c$ = 4.642 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122. $N/mm^2$
$S^*$ = 4424. $mm^3$

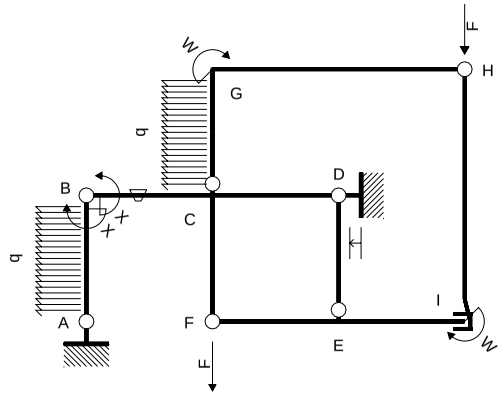
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H
I
5/4F
B
A
5/6F
1/6Fb
2/3F
5/6F
1/3F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
2/3F
9/4F
9/4Fb
5/3F
5/3F
2/3Fb
2/3F
9/4F
C
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1/2Fb
1/4F
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C
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G
C
1/4F
1/4F
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C
5/6F
2/3Fb
2/3F
2/3F
1/6Fb
5/6F
B

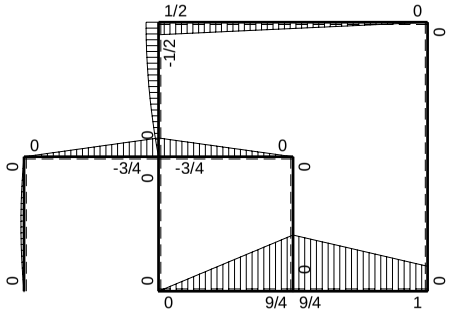




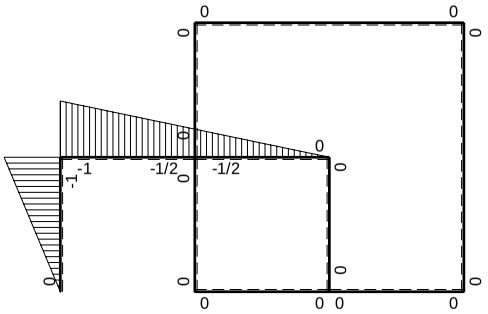

F
F
Fb
0
1/4
1/4
2/3
5/3
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7/2
0
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-1/4
0
-1
2/3
2/3
-5/6
0
0
0
-1/3
-9/4
5/4
1/2
0
0
-1/2
1/6
0
1/6
-2/3
-2/3
0
0
0
0
0
0
9/4
9/4
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 939077 CI01.xxxx.072

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PROCEDIMENTO E RISULTATI 939077 CI01.xxxx.072

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/3\ b )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2 ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/12\ b )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3 ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3 ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b ]_o^b\ \theta$

$= (3/8\ b -1/8\ b )\ Fb\ 1/EJ\ + (\ b -1/4\ b )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b ]_o^b\ \theta$

$= (3/8\ b -1/8\ b )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

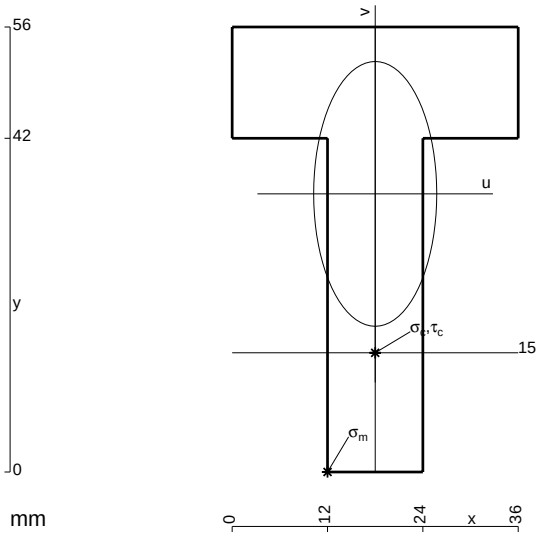
$= (3/8\ b -3/8\ b +1/8\ b )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

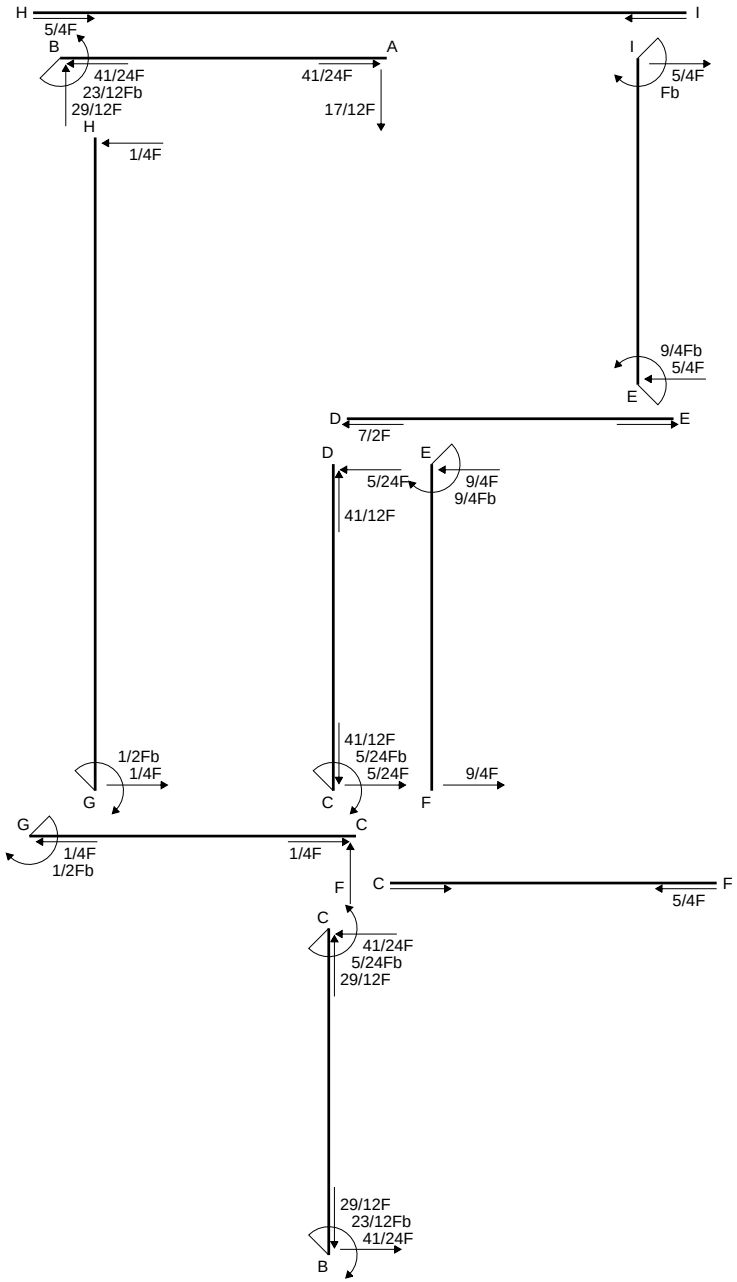
$= (1/8\ b )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





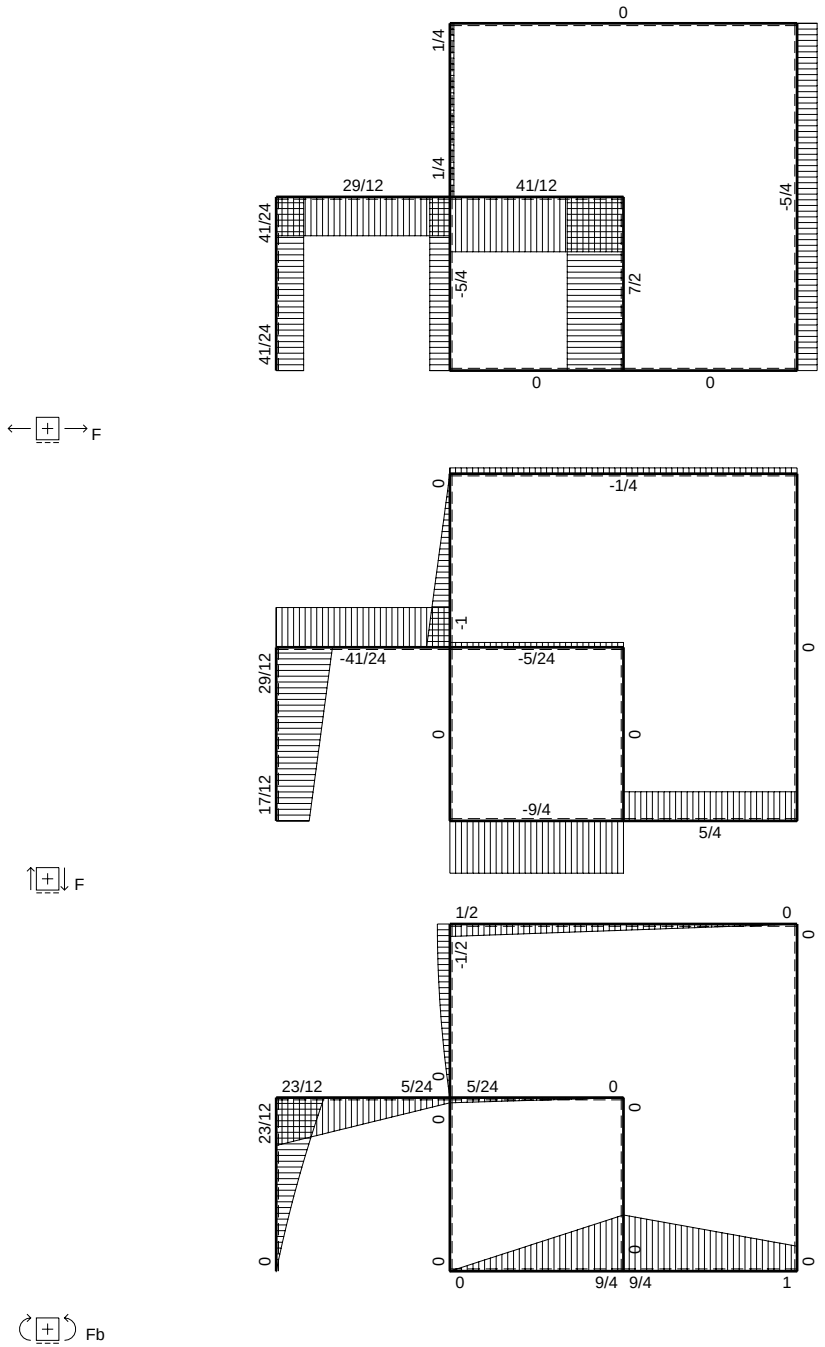


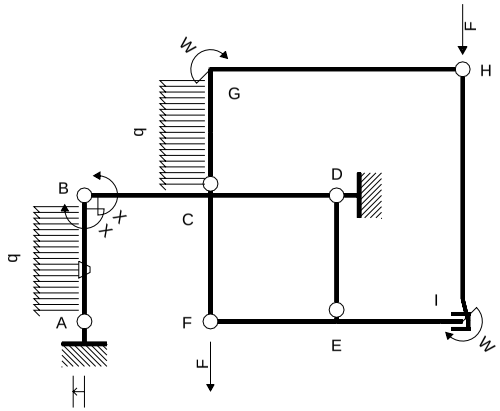

A = 1008. $mm^2$
$J_u$ = 279888. $mm^4$
$J_v$ = 60480. $mm^4$
$y_g$ = 35. mm
$T_y$ = -3195. N
$M_x$ = 1757250. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -35. mm
$\sigma_m = -Mv/J_u$ = 219.7 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 15. mm
$v_c$ = -20. mm
$\sigma_c = -Mv/J_u$ = 125.6 $N/mm^2$
$\tau_c$ = 4.709 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.8 $N/mm^2$
$S^*$ = 4950. $mm^3$


H
I
5/4F
B
A
41/24F
23/12Fb
29/12F
41/24F
17/12F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
5/24F
9/4F
9/4Fb
41/12F
1/2Fb
1/4F
41/12F
5/24Fb
5/24F
9/4F
G
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
C
41/24F
5/24Fb
29/12F
29/12F
23/12Fb
41/24F
B

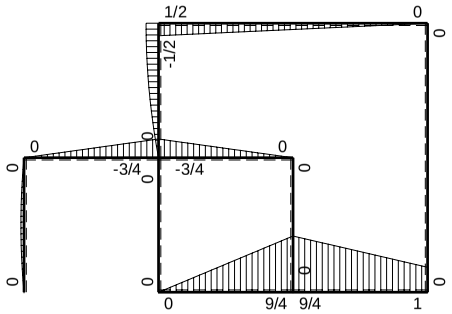




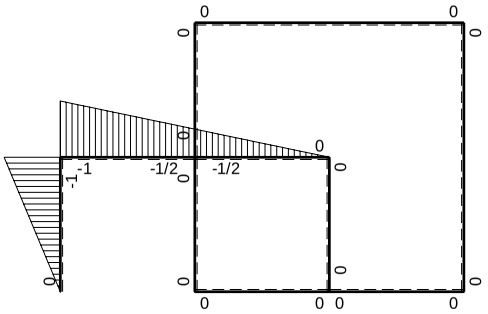

0
1/4
1/4
29/12
41/12
-5/4
41/24
41/24
-5/4
7/2
0
0
F
0
-1/4
-1
29/12
-41/24
-5/24
0
0
0
17/12
-9/4
5/4
F
1/2
0
0
-1/2
0
23/12
5/24
5/24
0
23/12
0
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264989 CI01.xxxx.073

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 264989 CI01.xxxx.073

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

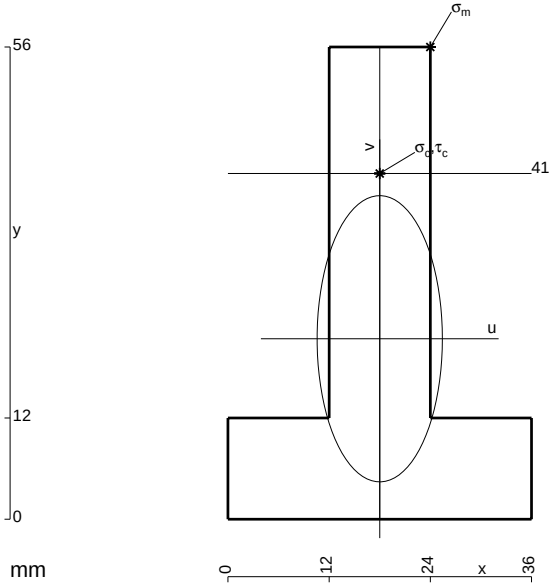
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

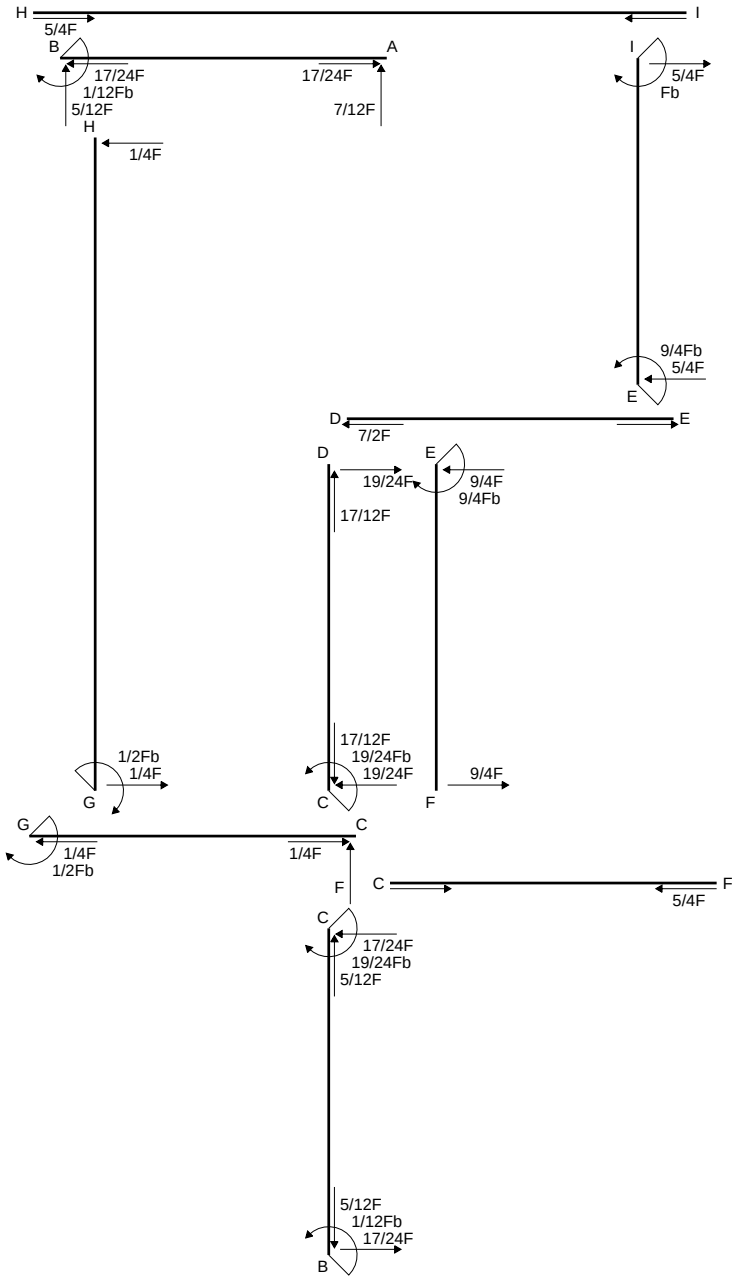
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





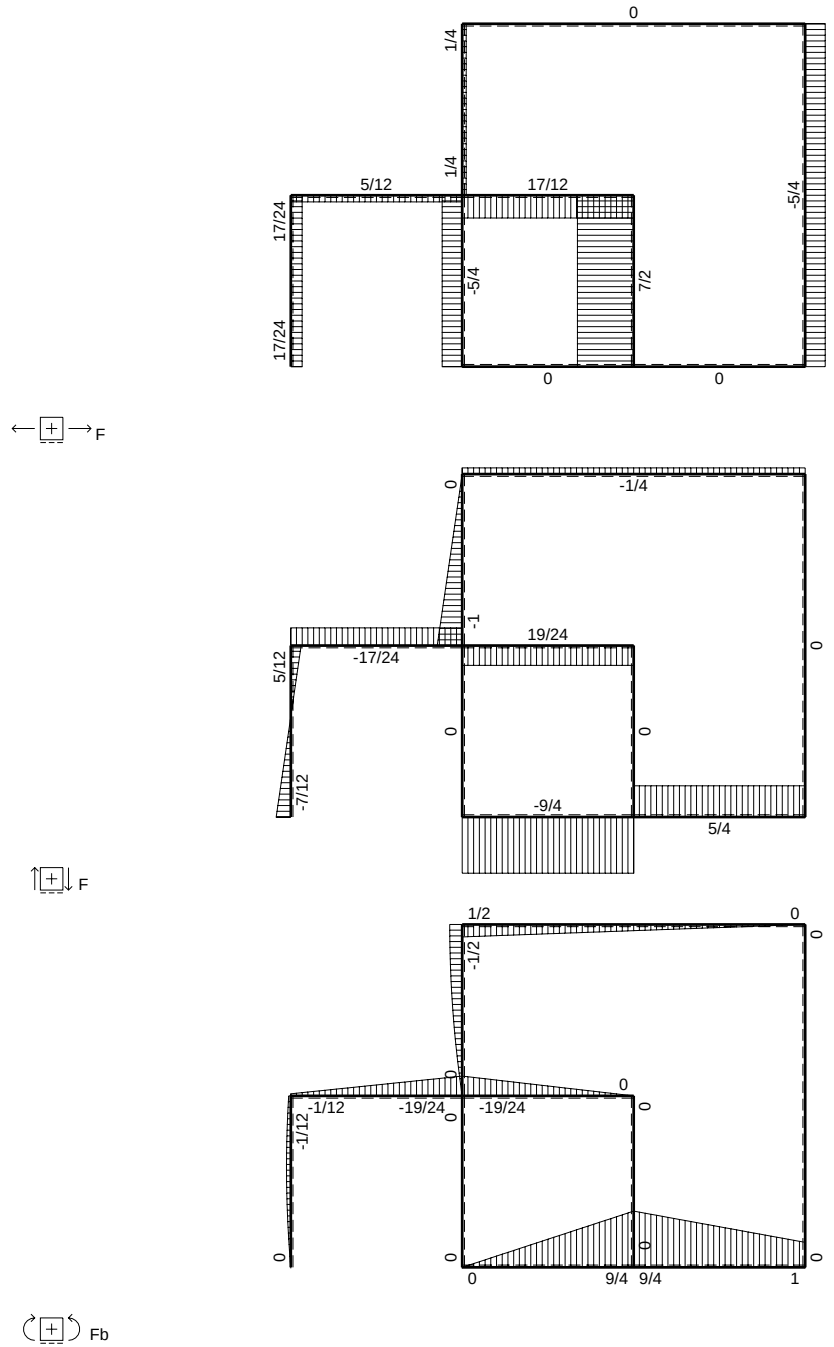


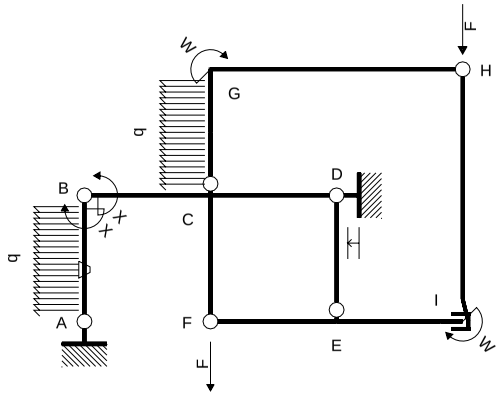

A = 960. $mm^2$
$J_u$ = 276646. $mm^4$
$J_v$ = 52992. $mm^4$
$y_g$ = 21.4 mm
$T_y$ = -3105. N
$M_x$ = 1831950. Nmm
$x_m$ = 24. mm
$y_m$ = 56. mm
$u_m$ = 6. mm
$v_m$ = 34.6 mm
$\sigma_m = -Mv/J_u$ = -229.1 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 41. mm
$v_c$ = 19.6 mm
$\sigma_c = -Mv/J_u$ = -129.8 $N/mm^2$
$\tau_c$ = 4.562 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 130. $N/mm^2$
$S^*$ = 4878. $mm^3$


H
5/4F
I
B
A
I
17/24F
17/24F
5/4F
1/12Fb
Fb
5/12F
7/12F
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
19/24F
9/4F
9/4Fb
17/12F
1/2Fb
1/4F
17/12F
19/24Fb
19/24F
9/4F
G
C
F
G
C
1/4F
1/4F
1/2Fb
F
C
F
5/4F
C
17/24F
19/24Fb
5/12F
5/12F
1/12Fb
17/24F
B

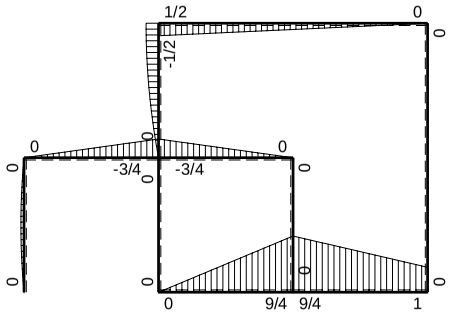




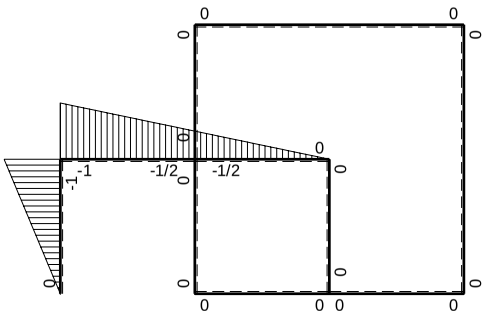

0
1/4
1/4
5/12
17/12
-5/4
17/24
-5/4
7/2
17/24
0
0
F
0
-1/4
-1
19/24
5/12
-17/24
0
0
0
-7/12
-9/4
5/4
F
1/2
0
0
-1/2
0
0
-1/12
-19/24
-19/24
-1/12
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 274704 CI01.xxxx.074

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 274704 CI01.xxxx.074

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2 ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2 ]_o^b\ 1/EJ$

$= (1/12\ b )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b ( x/b )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3 ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b ]_o^b\ \theta$

$= (1/6\ b -1/8\ b )\ Fb\ 1/EJ\ + (1/2\ b )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3 ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b )\ Fb\ 1/EJ\ + (- b +1/2\ b )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

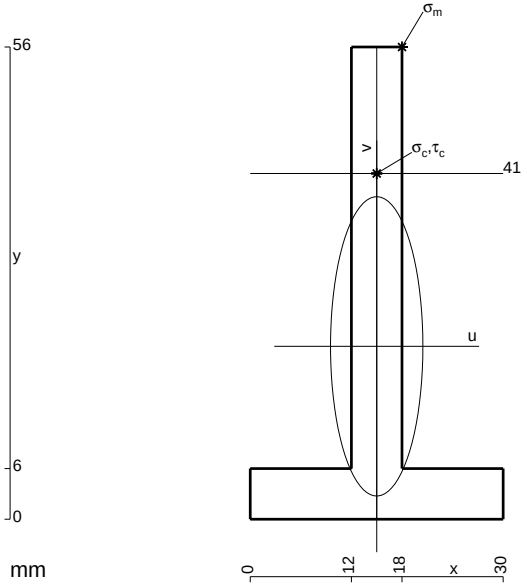
$= (3/8\ b -3/8\ b +1/8\ b )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2 ]_o^b\ Fb\ 1/EJ$

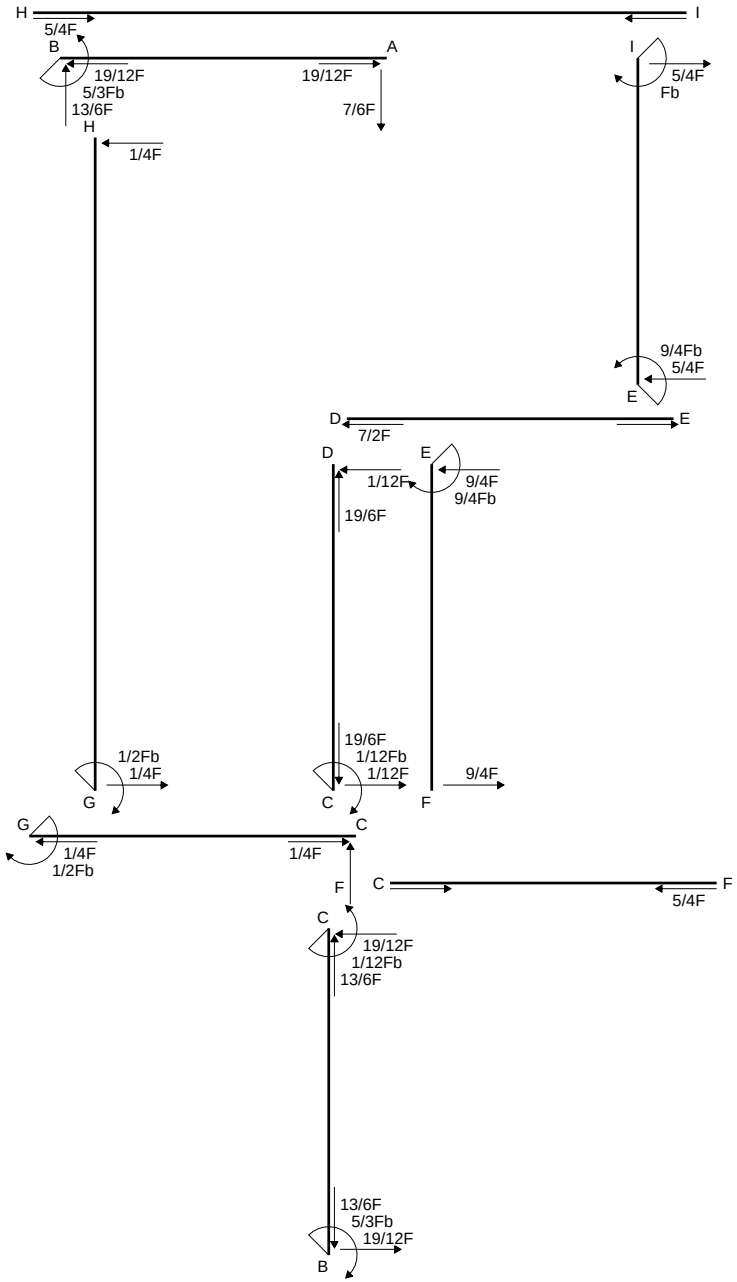
$= (1/8\ b )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





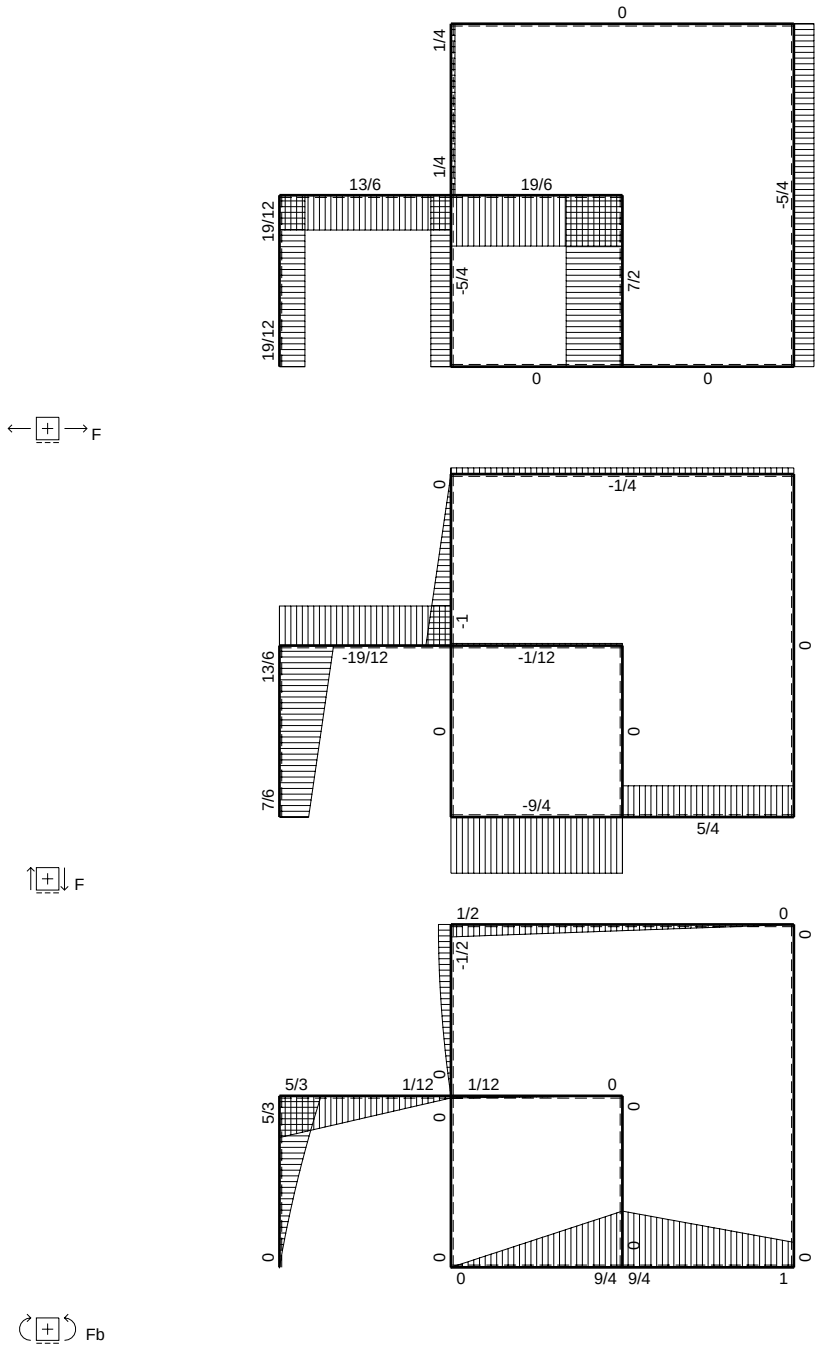


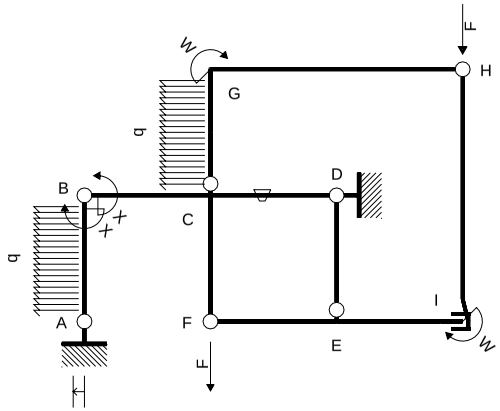

A = 480. $mm^2$
$J_u$ = 151240. $mm^4$
$J_v$ = 14400. $mm^4$
$y_g$ = 20.5 mm
$T_y$ = -1598. N
$M_x$ = 1022400. Nmm
$x_m$ = 18. mm
$y_m$ = 56. mm
$u_m$ = 3. mm
$v_m$ = 35.5 mm
$\sigma_m = -Mv/J_u$ = -240. $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 41. mm
$v_c$ = 20.5 mm
$\sigma_c = -Mv/J_u$ = -138.6 $N/mm^2$
$\tau_c$ = 4.436 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.8 $N/mm^2$
$S^*$ = 2520. $mm^3$


H
I
5/4F
B
A
I
19/12F
19/12F
5/4F
5/3Fb
Fb
13/6F
7/6F
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
1/12F
9/4F
9/4Fb
19/6F
19/6F
1/2Fb
1/12Fb
1/4F
1/12F
9/4F
G
C
F
G
C
1/4F
1/4F
1/2Fb
F
C
F
5/4F
C
19/12F
1/12Fb
13/6F
13/6F
5/3Fb
19/12F
B

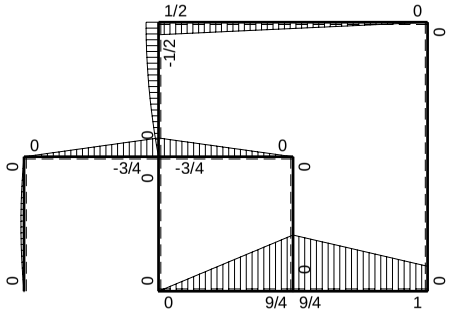




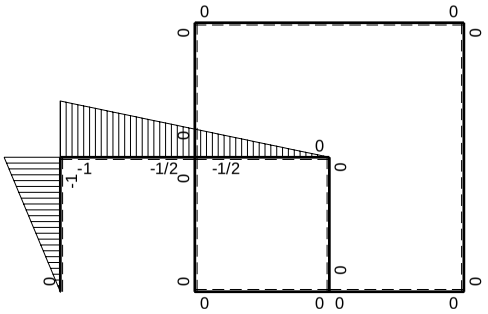

0
1/4
1/4
13/6
19/6
-5/4
19/12
-5/4
7/2
19/12
0
0
F
0
-1/4
-1
13/6
-19/12
-1/12
0
0
0
7/6
-9/4
5/4
F
1/2
0
0
-1/2
0
5/3
1/12
1/12
0
5/3
0
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 263877 CI01.xxxx.075

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $5/3Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

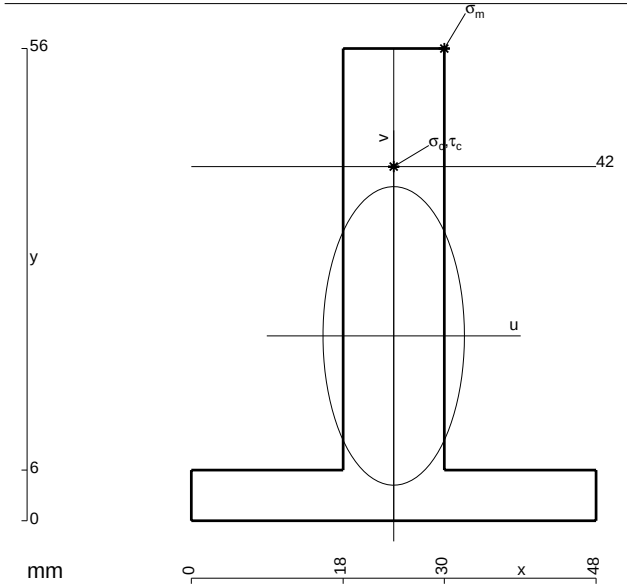
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

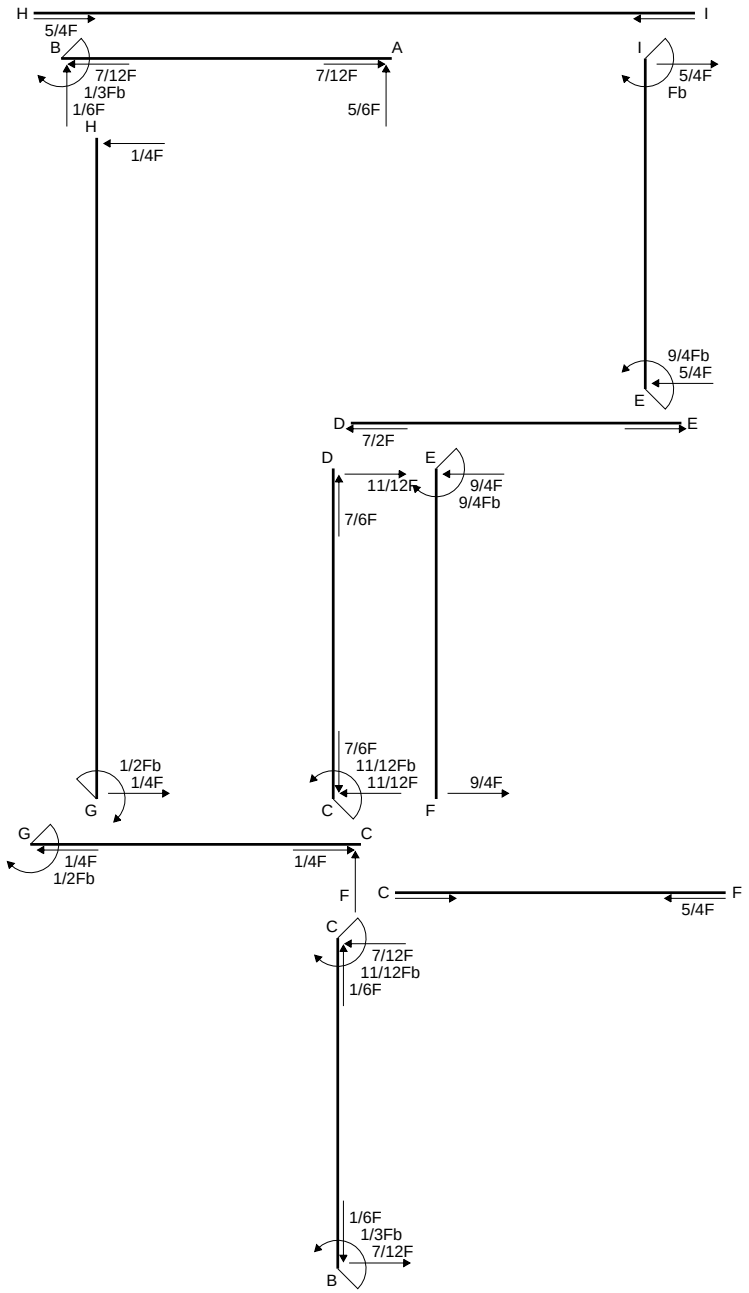


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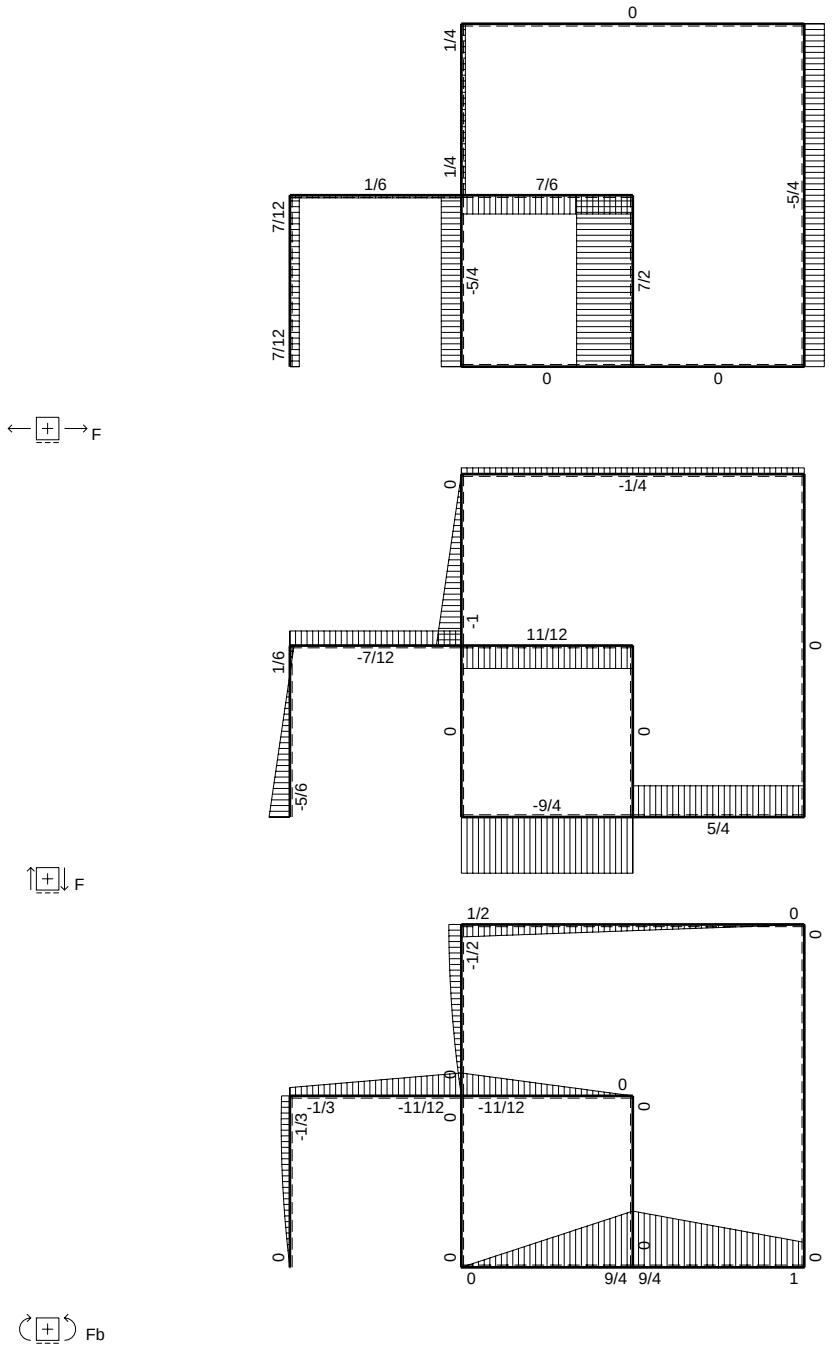

A = 888. $mm^2$
$J_u$ = 278426. $mm^4$
$J_v$ = 62496. $mm^4$
$y_g$ = 21.92 mm
$T_y$ = -2363. N
$M_x$ = 1630130. Nmm
$x_m$ = 30. mm
$y_m$ = 56. mm
$u_m$ = 6. mm
$v_m$ = 34.08 mm
$\sigma_m = -Mv/J_u$ = -199.5 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 42. mm
$v_c$ = 20.08 mm
$\sigma_c = -Mv/J_u$ = -117.6 $N/mm^2$
$\tau_c$ = 3.217 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.7 $N/mm^2$
$S^*$ = 4550. $mm^3$

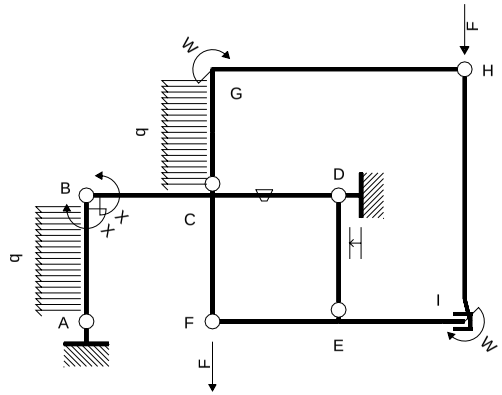
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H
I
5/4F
B
A
7/12F
1/3Fb
1/6F
7/12F
5/6F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
11/12F
9/4F
9/4Fb
7/6F
1/2Fb
1/4F
G
7/6F
11/12Fb
11/12F
C
F
9/4F
G
C
1/4F
1/4F
1/2Fb
F
C
F
5/4F
C
7/12F
11/12Fb
1/6F
1/6F
1/3Fb
7/12F
B

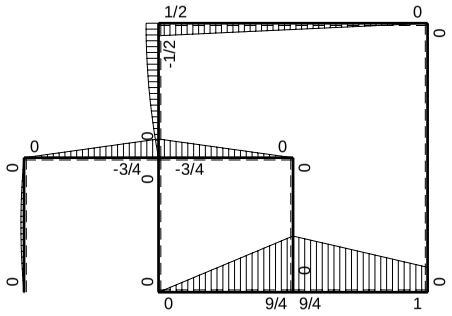




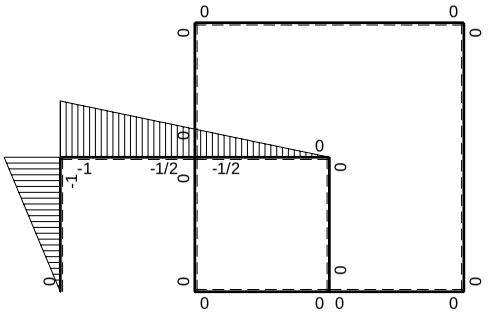

F
F
Fb
0
1/4
1/4
1/6
7/6
-5/4
7/12
-5/4
7/2
7/12
0
0
-1/4
0
-1
11/12
1/6
-7/12
0
0
0
-5/6
-9/4
5/4
1/2
0
0
-1/2
0
-1/3
-11/12
-11/12
-1/3
0
0
0
0
0
0
9/4
9/4
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265708 CI01.xxxx.076

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PROCEDIMENTO E RISULTATI 265708 CI01.xxxx.076

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | $1/3Xb/EJ$ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | $7/12Xb/EJ$ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | $1/12Xb/EJ$ |
| DC b | $1/2x/b$ | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | $1/2Fx/EJ$ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/3Fb^2/EJ$ | $Xb/EJ$ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

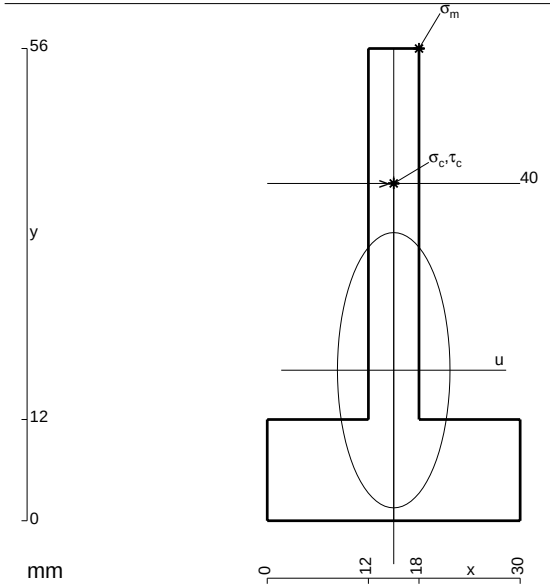
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

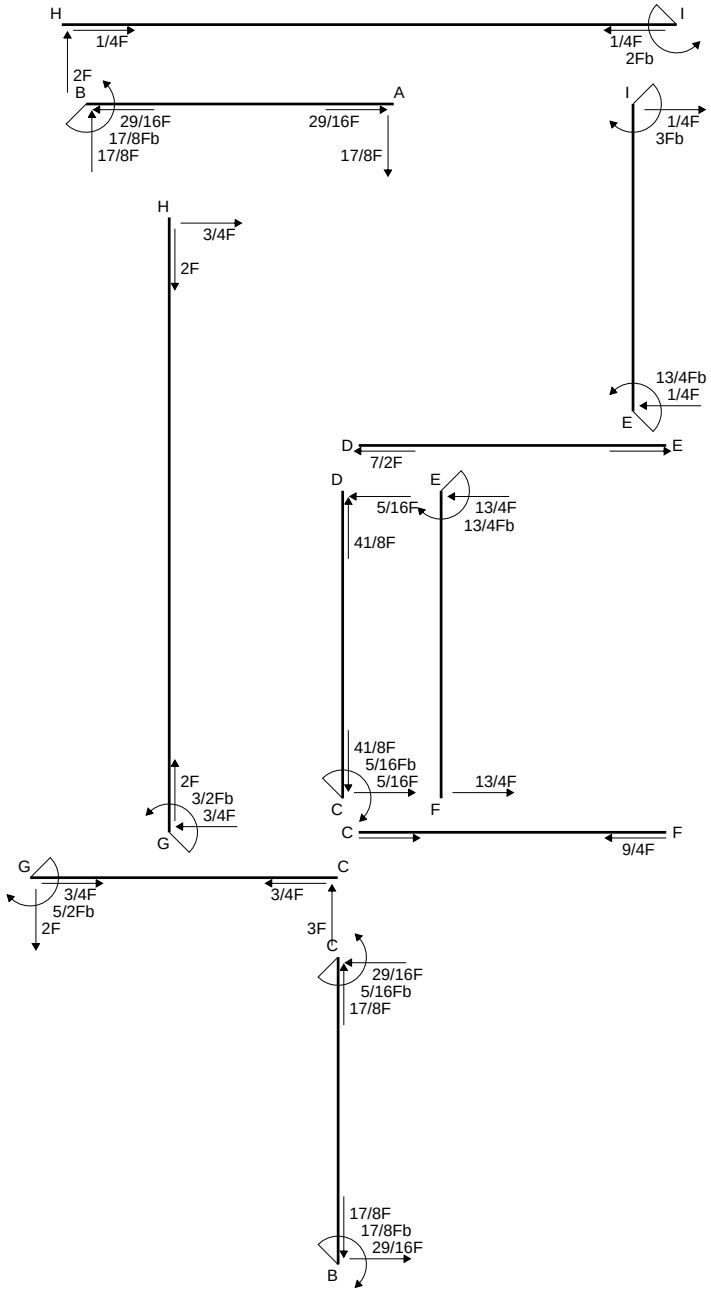


PROCEDIMENTO E RISULTATI 265708 CI01.xxxx.076

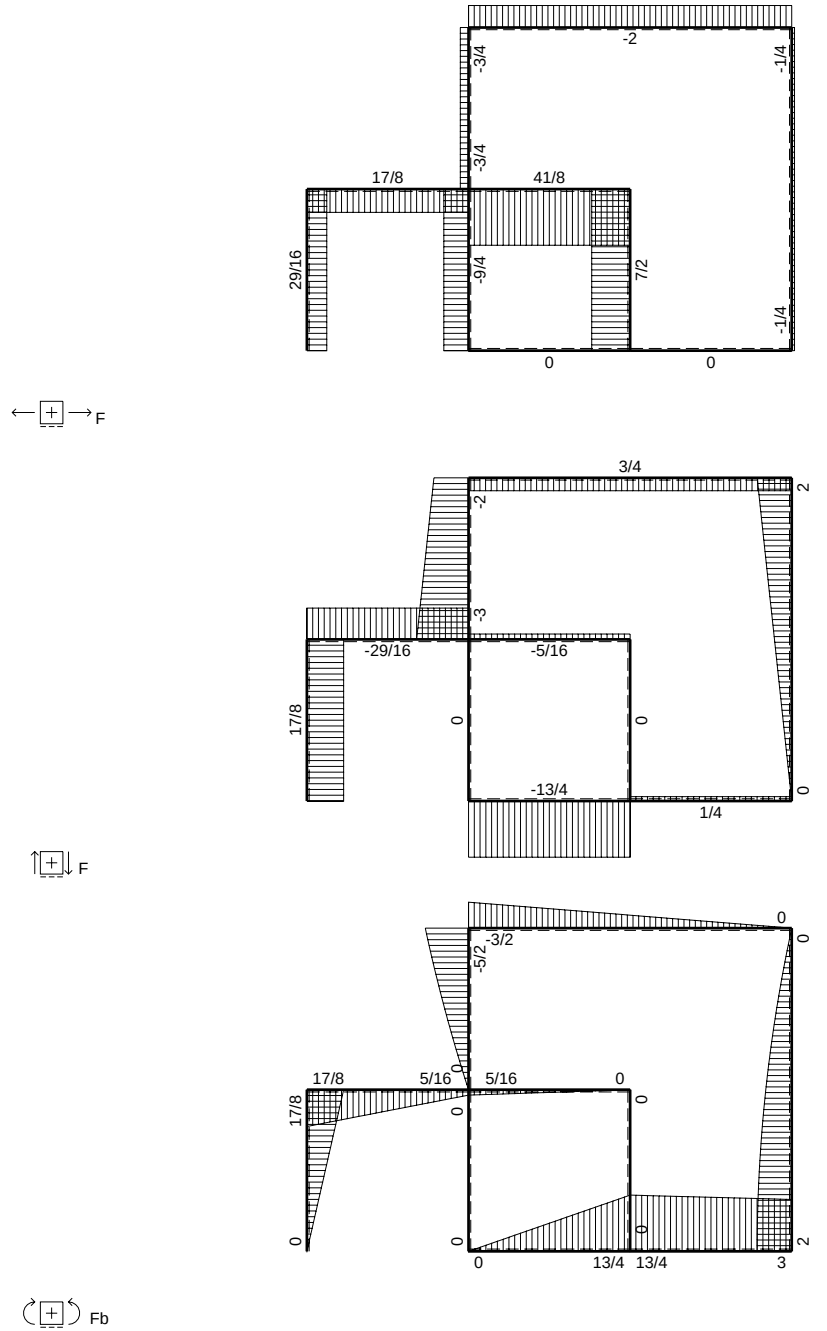



A = 624. $mm^2$
$J_u$ = 166321. $mm^4$
$J_v$ = 27792. $mm^4$
$y_g$ = 17.85 mm
$T_y$ = -1238. N
$M_x$ = 903375. Nmm
$x_m$ = 18. mm
$y_m$ = 56. mm
$u_m$ = 3. mm
$v_m$ = 38.15 mm
$\sigma_m = -Mv/J_u$ = -207.2 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 40. mm
$v_c$ = 22.15 mm
$\sigma_c = -Mv/J_u$ = -120.3 $N/mm^2$
$\tau_c$ = 3.59 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.5 $N/mm^2$
$S^*$ = 2895. $mm^3$

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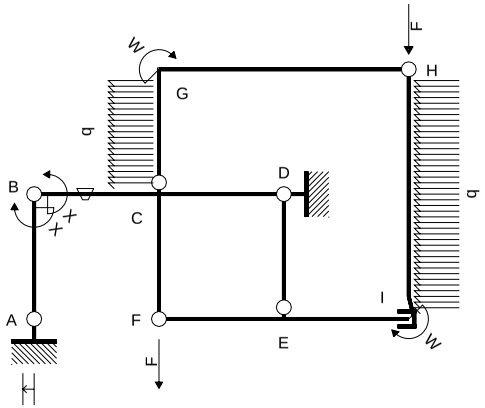




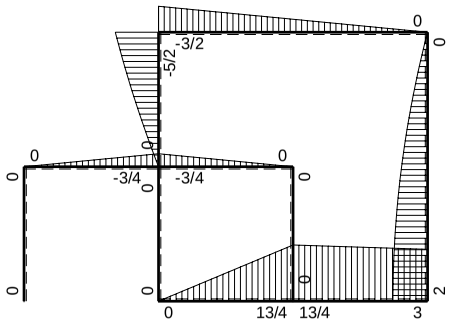





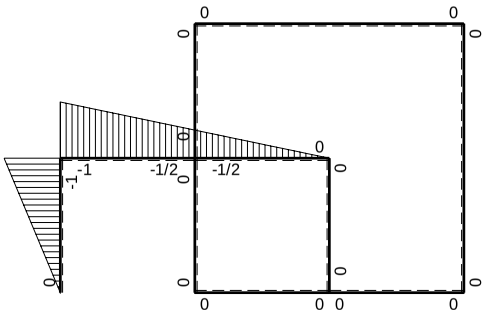

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Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265715 CI01.xxxx.077

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PROCEDIMENTO E RISULTATI 265715 CI01.xxxx.077

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $17/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -17/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

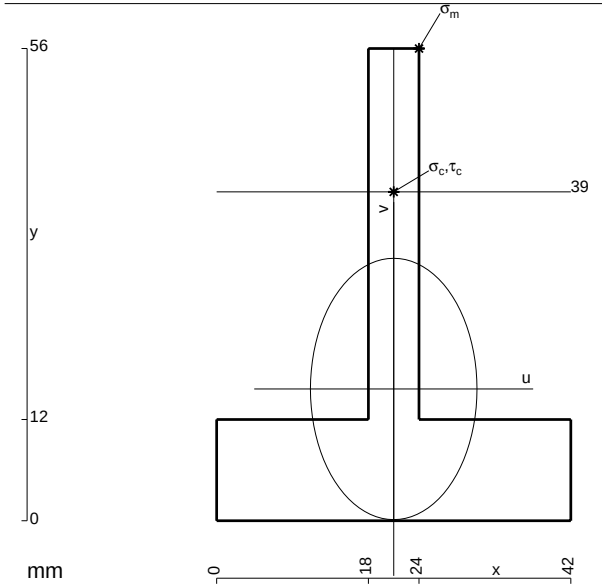
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

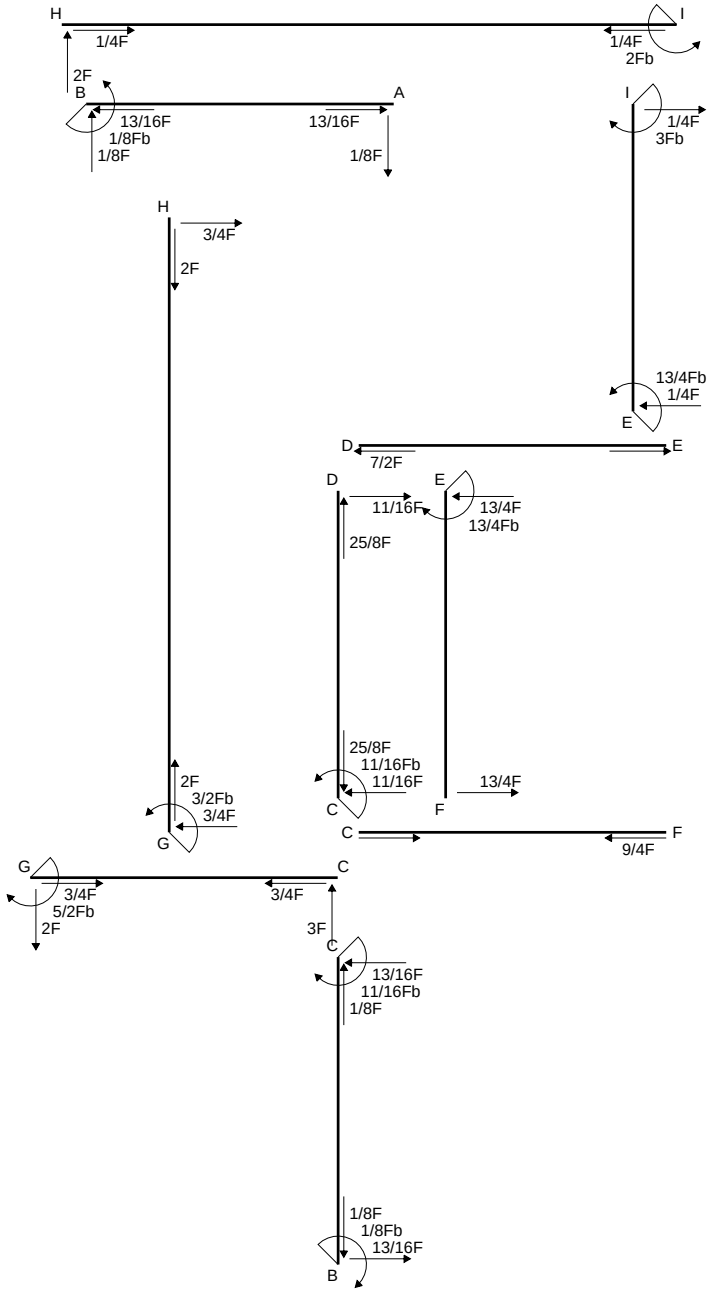


PROCEDIMENTO E RISULTATI 265715 CI01.xxxx.077



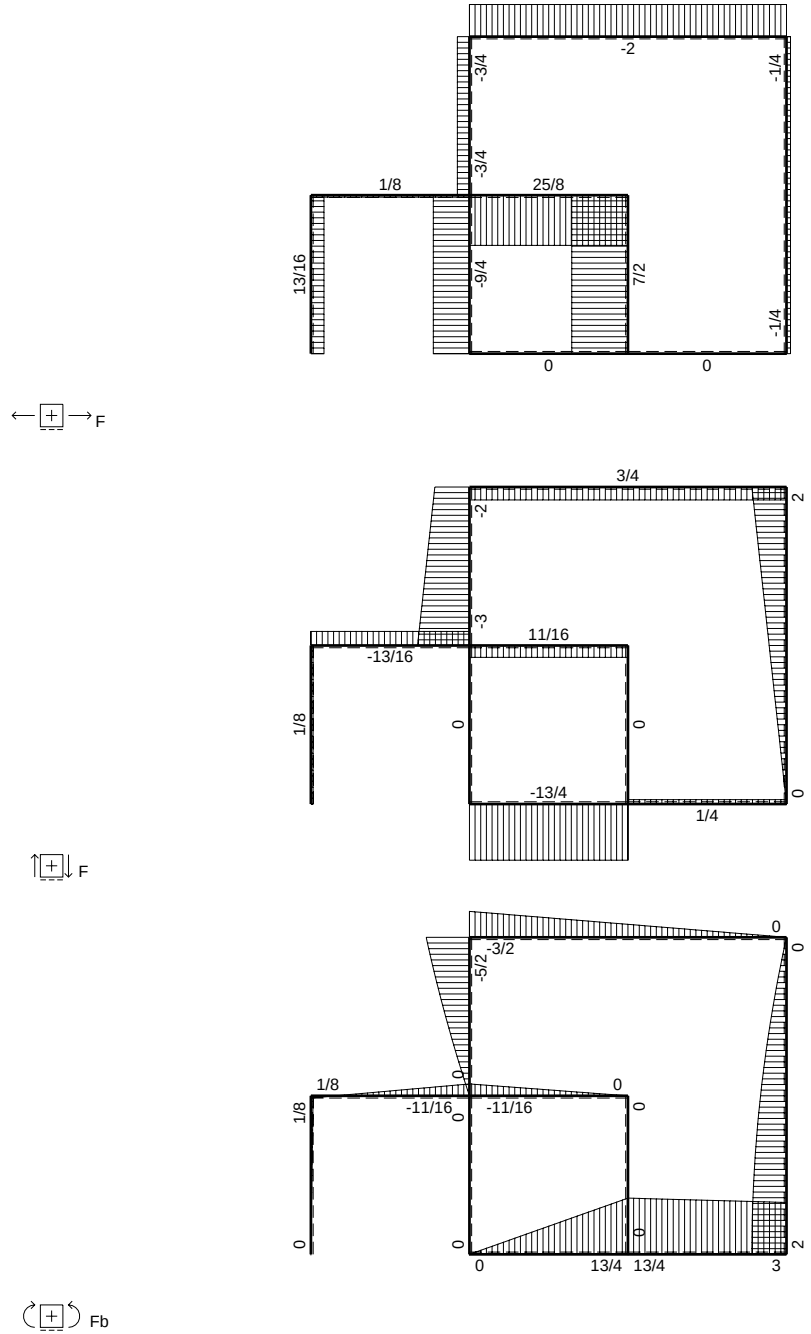

A = 768. $mm^2$
$J_u$ = 184468. $mm^4$
$J_v$ = 74880. $mm^4$
$y_g$ = 15.63 mm
$T_y$ = -1268. N
$M_x$ = 988650. Nmm
$x_m$ = 24. mm
$y_m$ = 56. mm
$u_m$ = 3. mm
$v_m$ = 40.38 mm
$\sigma_m = -Mv/J_u$ = -216.4 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 39. mm
$v_c$ = 23.38 mm
$\sigma_c = -Mv/J_u$ = -125.3 $N/mm^2$
$\tau_c$ = 3.723 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.4 $N/mm^2$
$S^*$ = 3251. $mm^3$

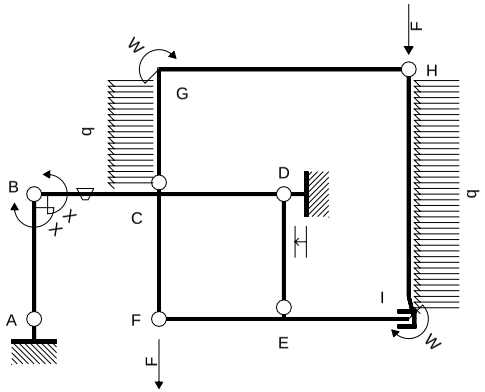
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H
I
1/4F
1/4F
2Fb
2F
B
A
13/16F
1/8Fb
1/8F
13/16F
1/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
11/16F
13/4F
13/4Fb
25/8F
25/8F
11/16Fb
11/16F
13/4F
C
F
2F
3/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
5/2Fb
2F
3F
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13/16F
11/16Fb
1/8F
1/8F
1/8Fb
13/16F
B

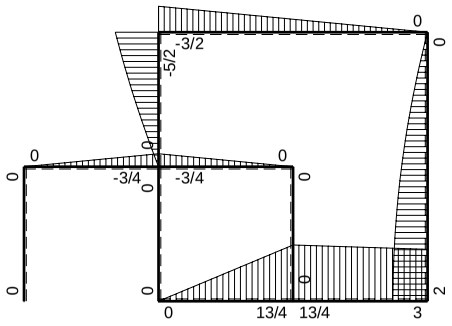




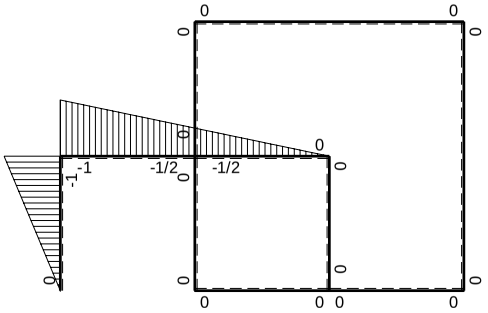

-2
-3/4
-1/4
-3/4
1/8
25/8
13/16
-9/4
7/2
-1/4
0
0
F
3/4
-2
2
-3
11/16
-13/16
1/8
0
0
-13/4
0
1/4
F
0
-3/2
0
-5/2
0
1/8
0
1/8
-11/16
-11/16
0
0
0
0
2
0
13/4
13/4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 216487 CI01.xxxx.078

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 216487 CI01.xxxx.078

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

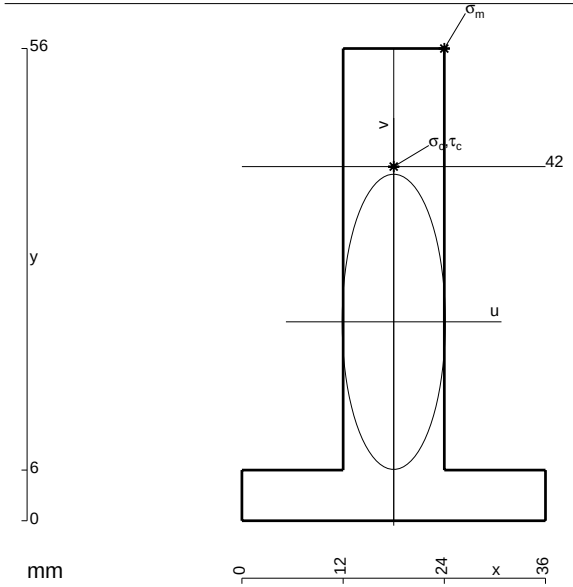
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

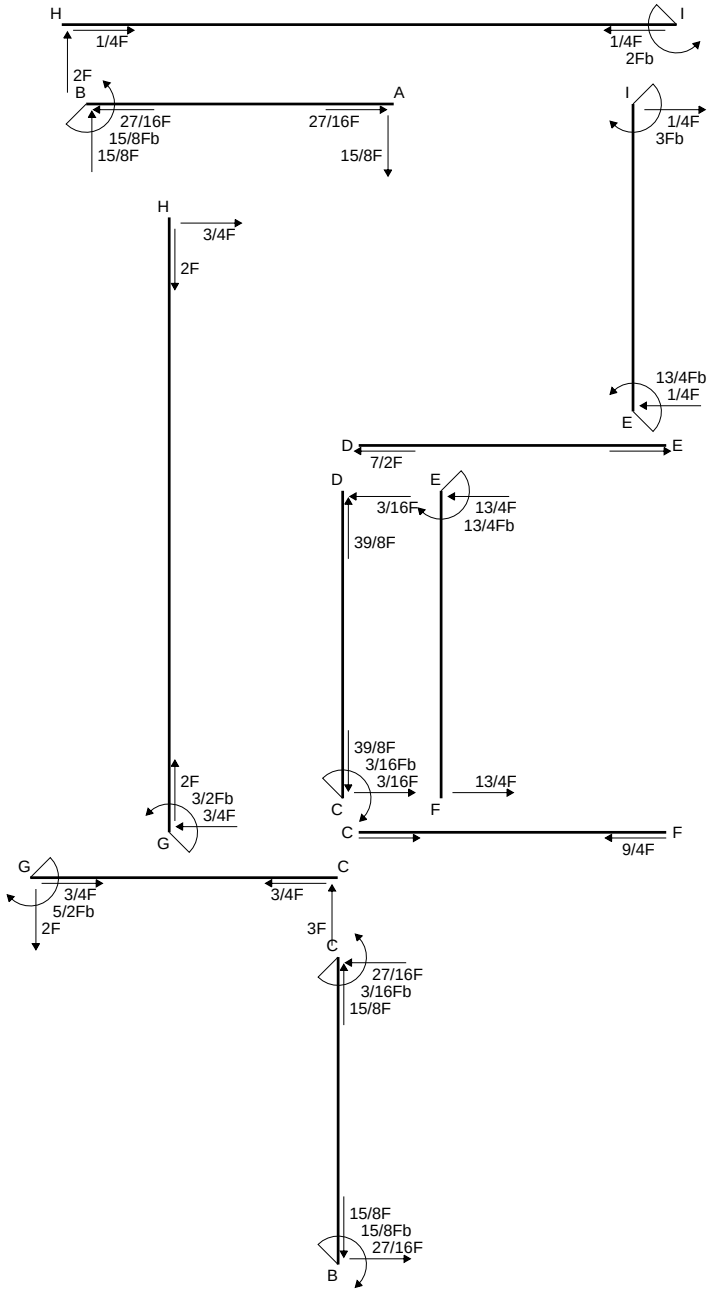
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





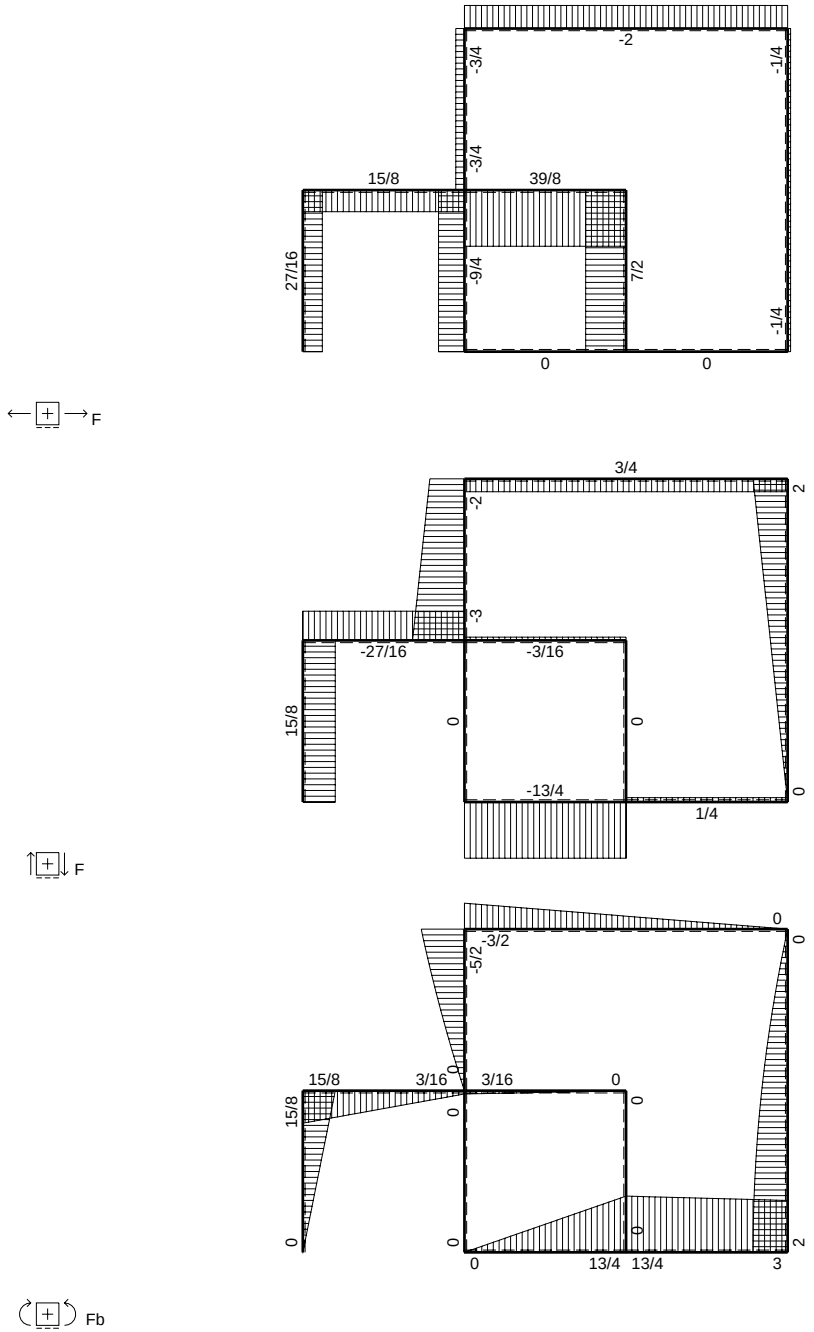


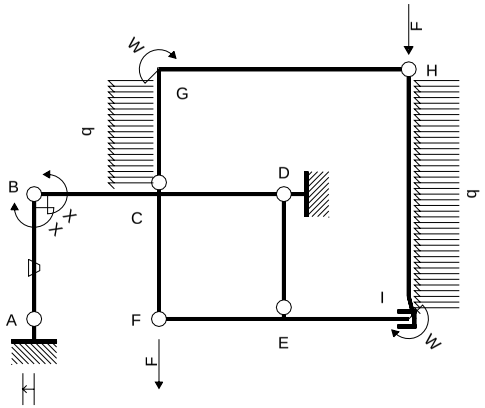

A = 816. $mm^2$
$J_u$ = 250166. $mm^4$
$J_v$ = 30528. $mm^4$
$y_g$ = 23.59 mm
$T_y$ = -2145. N
$M_x$ = 1758900. Nmm
$x_m$ = 24. mm
$y_m$ = 56. mm
$u_m$ = 6. mm
$v_m$ = 32.41 mm
$\sigma_m = -Mv/J_u$ = -227.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 42. mm
$v_c$ = 18.41 mm
$\sigma_c = -Mv/J_u$ = -129.5 $N/mm^2$
$\tau_c$ = 3.05 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.6 $N/mm^2$
$S^*$ = 4269. $mm^3$


H
I
1/4F
1/4F
2Fb
2F
B
A
27/16F
15/8Fb
15/8F
27/16F
15/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
3/16F
13/4F
13/4Fb
39/8F
39/8F
3/16Fb
3/16F
13/4F
C
F
2F
3/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
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27/16F
3/16Fb
15/8F
15/8F
15/8Fb
27/16F
B

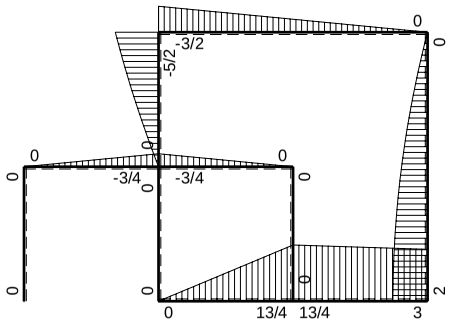




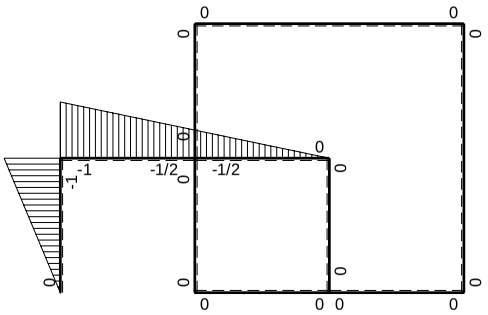

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-1/4
-3/4
15/8
39/8
27/16
-9/4
7/2
-1/4
0
0
F
3/4
-2
2
-3
-27/16
-3/16
15/8
0
0
-13/4
0
1/4
F
0
-3/2
0
-5/2
15/8
3/16
3/16
0
15/8
0
0
0
0
0
0
13/4
13/4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264526 CI01.xxxx.079

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PROCEDIMENTO E RISULTATI 264526 CI01.xxxx.079

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $15/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -15/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

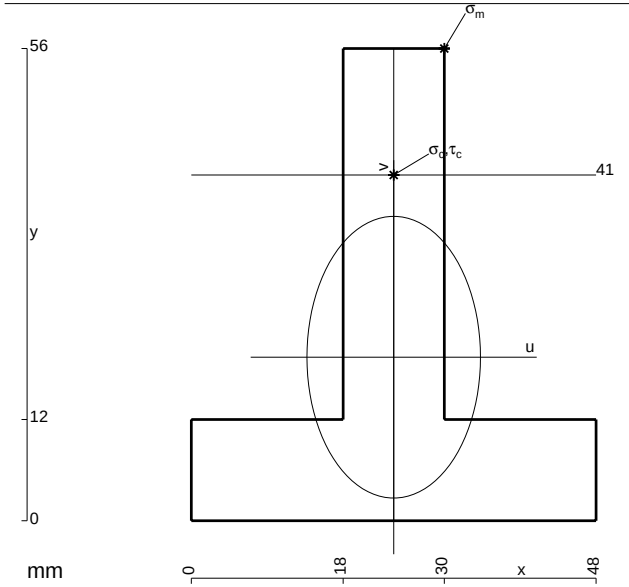
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

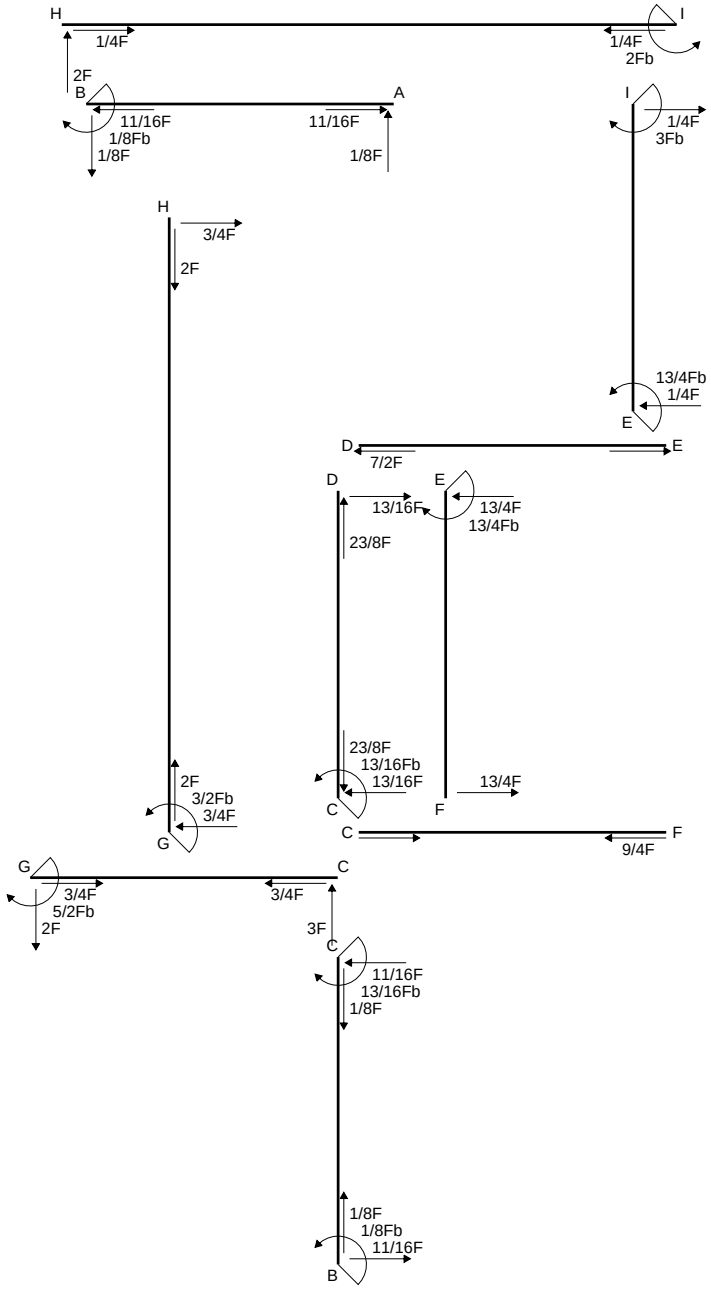
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 264526 CI01.xxxx.079

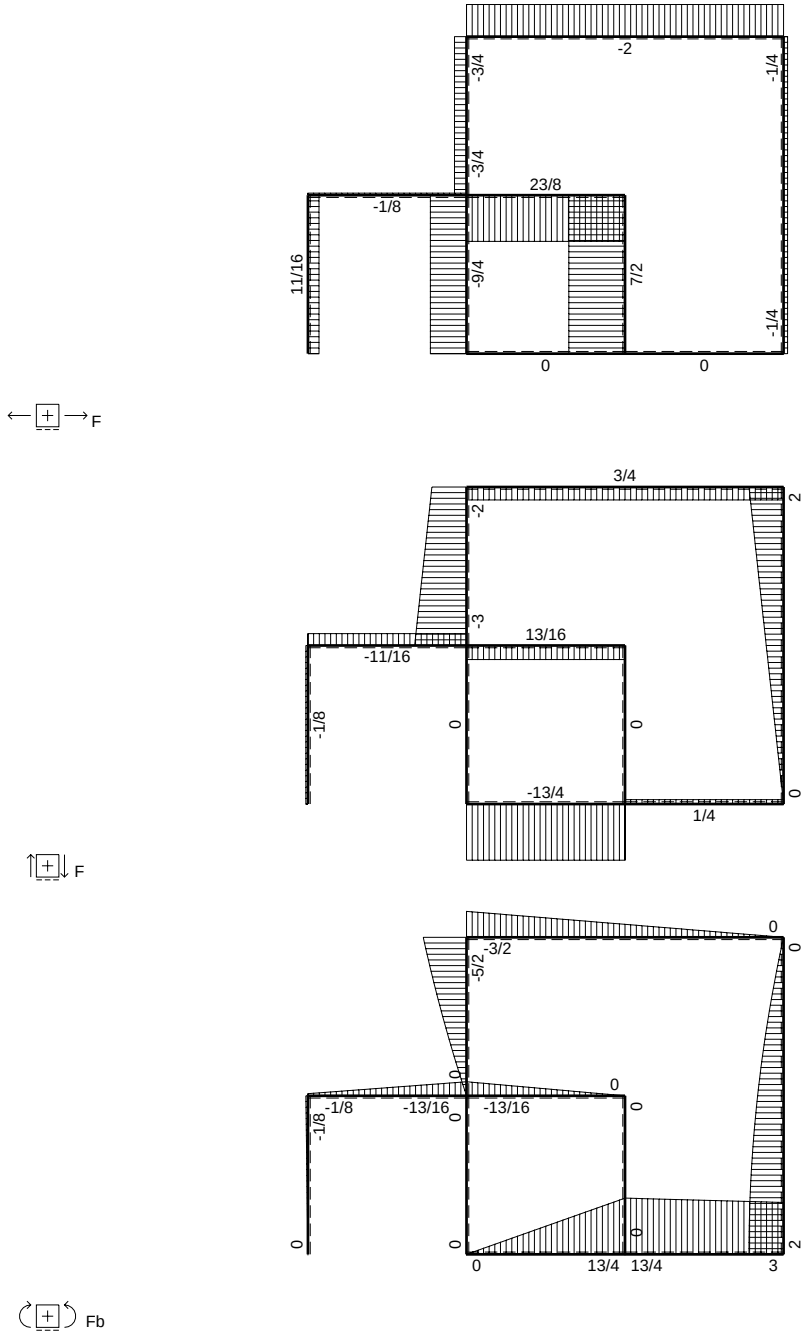


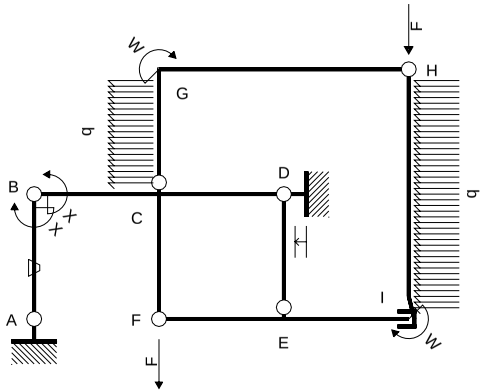

A = 1104. $mm^2$
$J_u$ = 308071. $mm^4$
$J_v$ = 116928. $mm^4$
$y_g$ = 19.39 mm
$T_y$ = -2308. N
$M_x$ = 2007530. Nmm
$x_m$ = 30. mm
$y_m$ = 56. mm
$u_m$ = 6. mm
$v_m$ = 36.61 mm
$\sigma_m = -Mv/J_u$ = -238.6 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 41. mm
$v_c$ = 21.61 mm
$\sigma_c = -Mv/J_u$ = -140.8 $N/mm^2$
$\tau_c$ = 3.27 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.9 $N/mm^2$
$S^*$ = 5240. $mm^3$


H
I
1/4F
1/4F
2Fb
2F
B
A
11/16F
1/8Fb
1/8F
11/16F
1/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
13/16F
13/4F
13/4Fb
23/8F
23/8F
13/16Fb
13/16F
13/4F
C
F
2F
3/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
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1/8F
1/8F
1/8Fb
11/16F
B

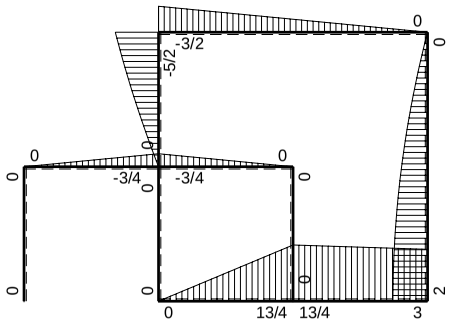




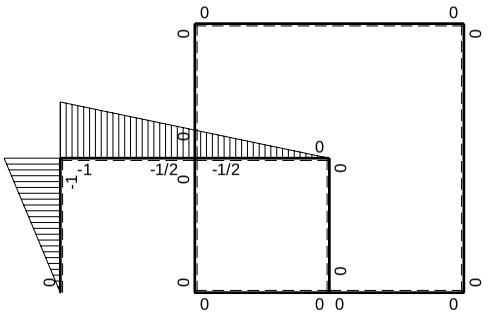

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23/8
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11/16
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7/2
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F
3/4
-2
2
-3
13/16
-11/16
-1/8
0
0
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F
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-3/2
0
-5/2
0
0
-1/8
-13/16
-13/16
0
0
0
0
0
2
0
13/4
13/4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267010 CI01.xxxx.080

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 267010 CI01.xxxx.080

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

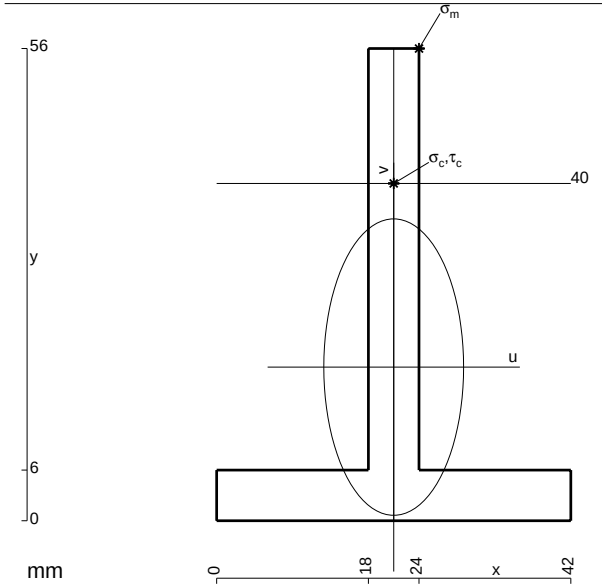
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

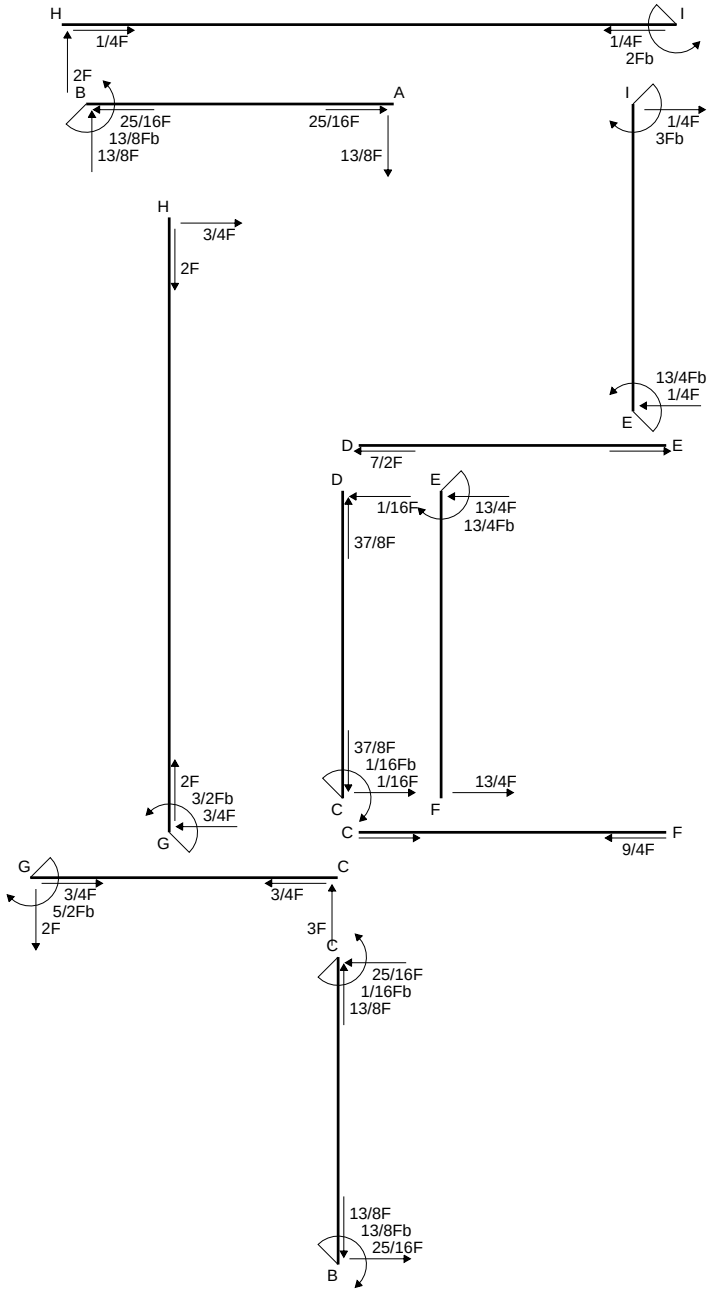
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





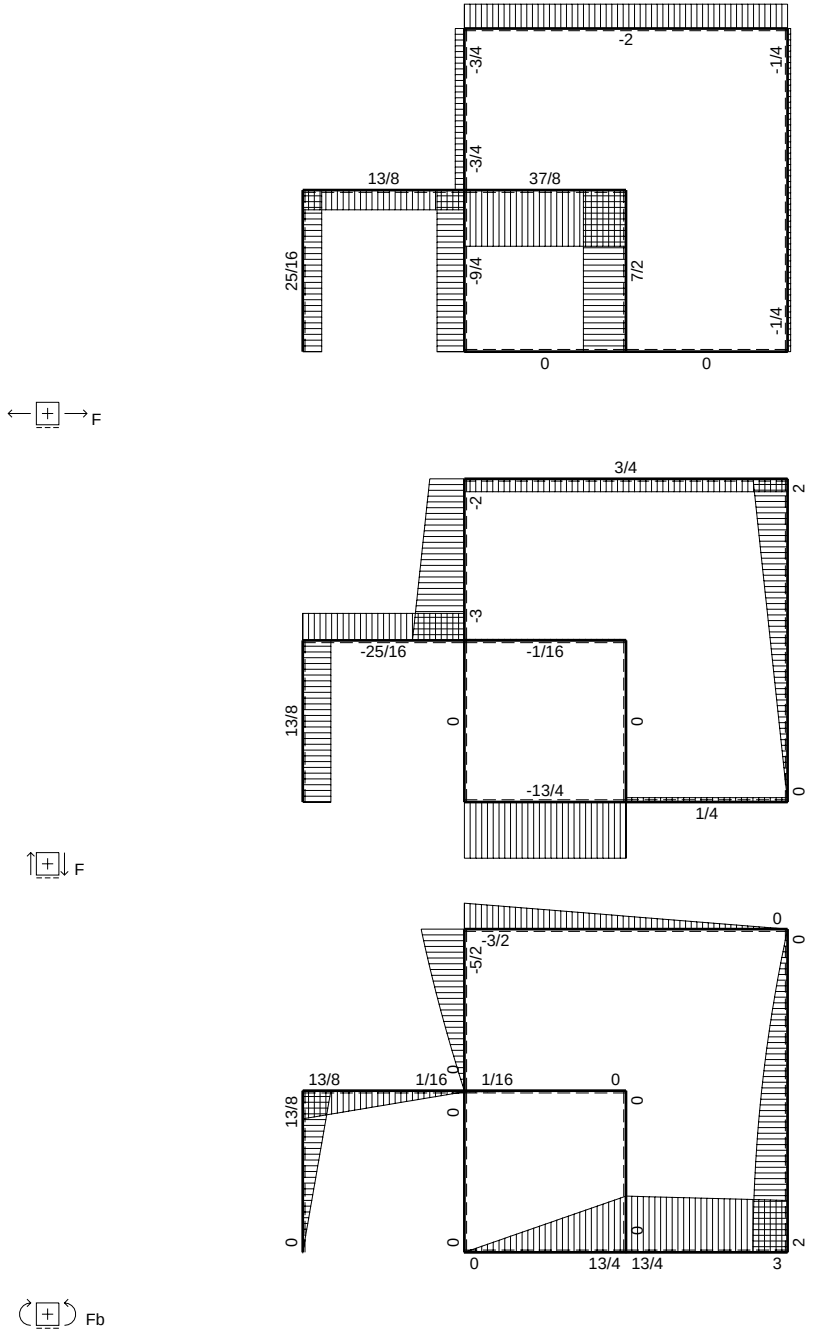


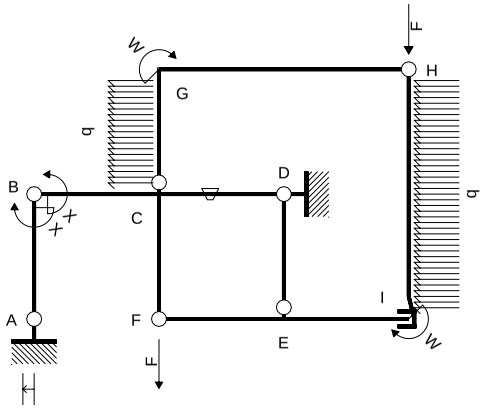

A = 552. $mm^2$
$J_u$ = 170630. $mm^4$
$J_v$ = 37944. $mm^4$
$y_g$ = 18.22 mm
$T_y$ = -1950. N
$M_x$ = 897000. Nmm
$x_m$ = 24. mm
$y_m$ = 56. mm
$u_m$ = 3. mm
$v_m$ = 37.78 mm
$\sigma_m = -Mv/J_u$ = -198.6 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 40. mm
$v_c$ = 21.78 mm
$\sigma_c = -Mv/J_u$ = -114.5 $N/mm^2$
$\tau_c$ = 5.446 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.9 $N/mm^2$
$S^*$ = 2859. $mm^3$


H
I
1/4F
1/4F
2Fb
2F
B
A
25/16F
13/8Fb
13/8F
25/16F
13/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
1/16F
13/4F
13/4Fb
37/8F
37/8F
1/16Fb
1/16F
13/4F
C
F
2F
3/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
5/2Fb
2F
3F
C
25/16F
1/16Fb
13/8F
13/8F
13/8Fb
25/16F
B

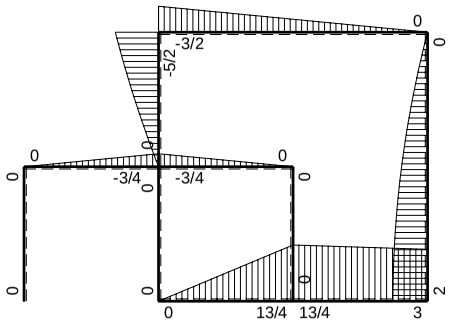




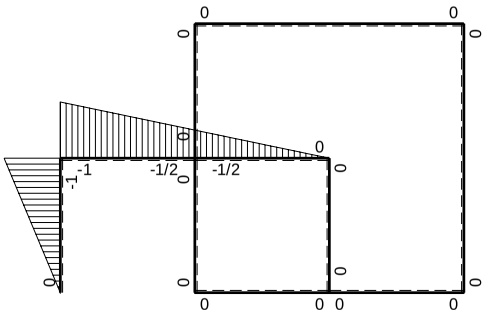

F
-2
-3/4
-1/4
-3/4
13/8
37/8
25/16
-9/4
7/2
-1/4
0
0
F
3/4
-2
2
-3
-25/16
-1/16
13/8
0
0
0
-13/4
1/4
Fb
0
-3/2
-5/2
0
0
13/8
1/16
1/16
0
13/8
0
0
0
0
0
0
13/4
13/4
3
2




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 962891 CI01.xxxx.081

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

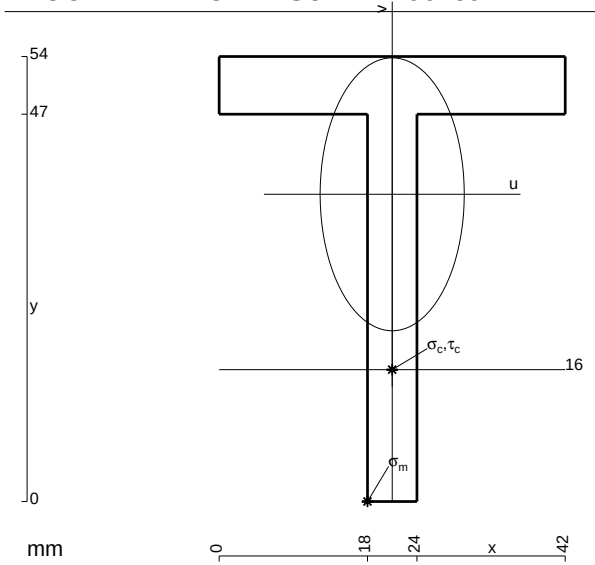
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

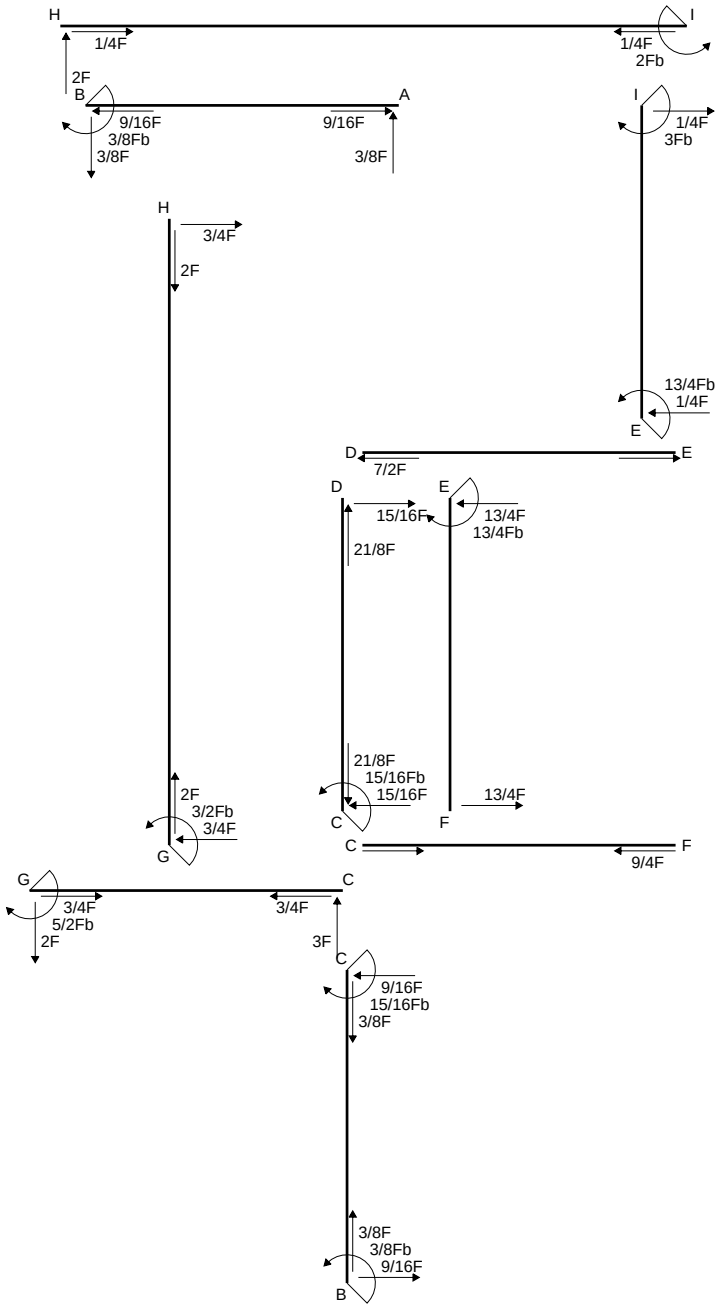


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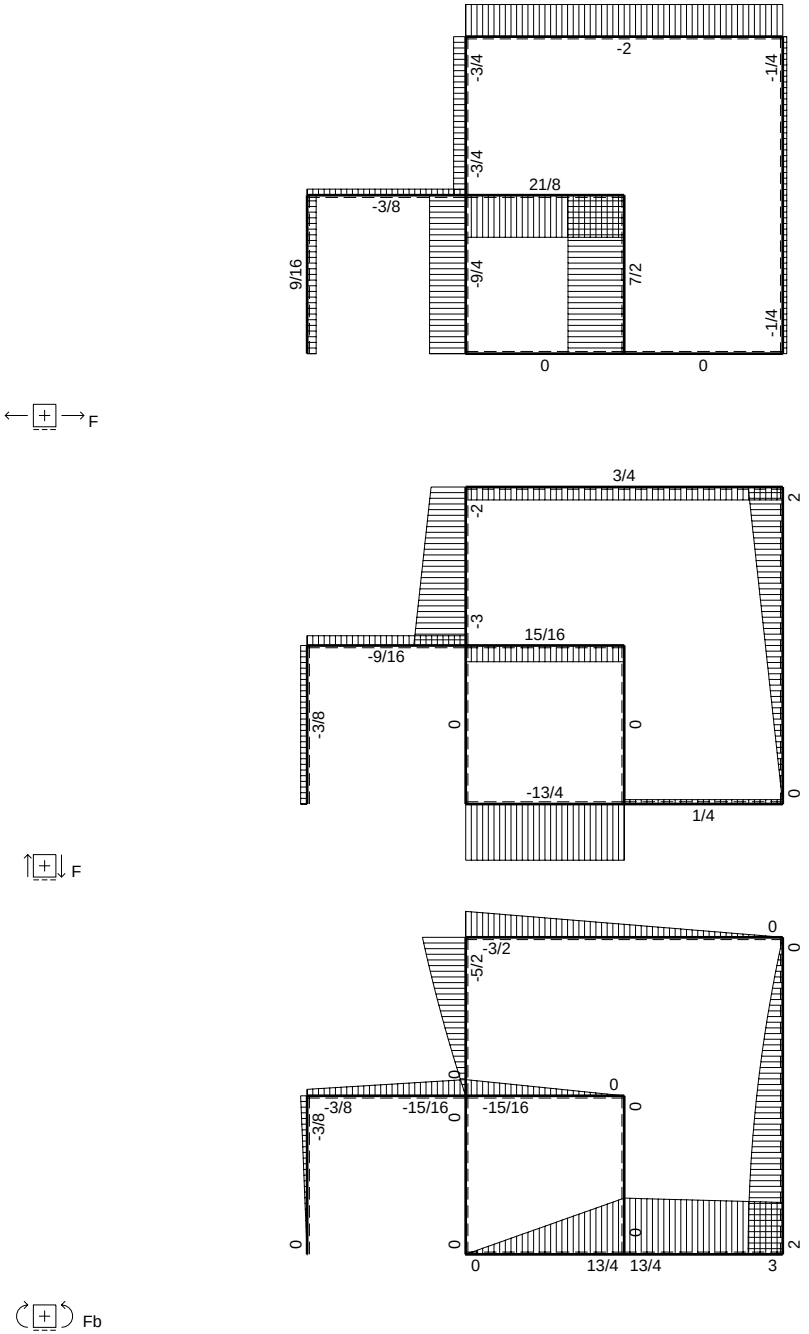

A = 576. $mm^2$
$J_u$ = 158042. $mm^4$
$J_v$ = 44064. $mm^4$
$y_g$ = 37.28 mm
$T_y$ = -1788. N
$M_x$ = 875875. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -37.28 mm
$\sigma_m = -Mv/J_u$ = 206.6 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -21.28 mm
$\sigma_c = -Mv/J_u$ = 117.9 $N/mm^2$
$\tau_c$ = 5.299 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 118.3 $N/mm^2$
$S^*$ = 2811. $mm^3$

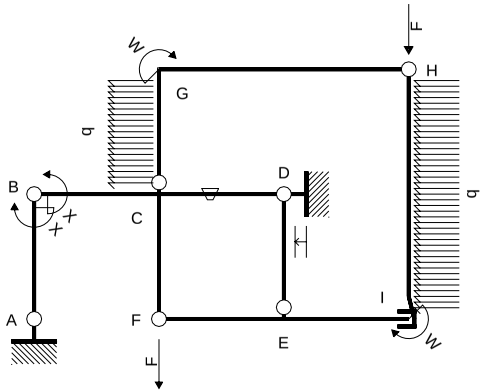
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H
1/4F
I
1/4F
2Fb
2F
B
A
9/16F
3/8Fb
3/8F
9/16F
3/8F
I
1/4F
3Fb
H
3/4F
2F
13/4Fb
1/4F
E
D
E
7/2F
D
E
15/16F
13/4F
13/4Fb
21/8F
21/8F
15/16Fb
15/16F
13/4F
C
F
2F
3/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
5/2Fb
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3F
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9/16F
15/16Fb
3/8F
3/8F
3/8Fb
9/16F
B

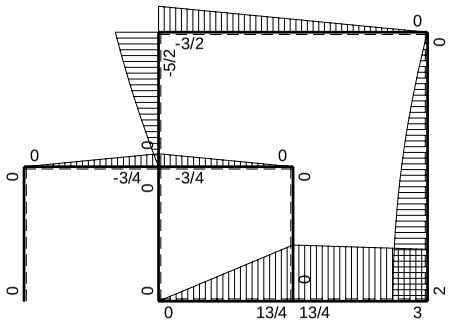




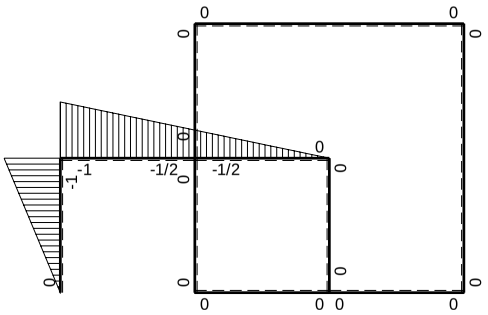

F
-2
-3/4
-1/4
-3/4
21/8
-3/8
9/16
-9/4
7/2
-1/4
0
0
F
3/4
2
-2
-3
15/16
-9/16
-3/8
0
0
-13/4
0
1/4
F
0
0
-3/2
-5/2
0
0
-3/8
-15/16
-15/16
0
0
-3/8
0
0
0
0
13/4
13/4
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 266362 CI01.xxxx.082

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PROCEDIMENTO E RISULTATI 266362 CI01.xxxx.082

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $5/2Fb-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -3/2Fb+3/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 3/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+1/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-3/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 3/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

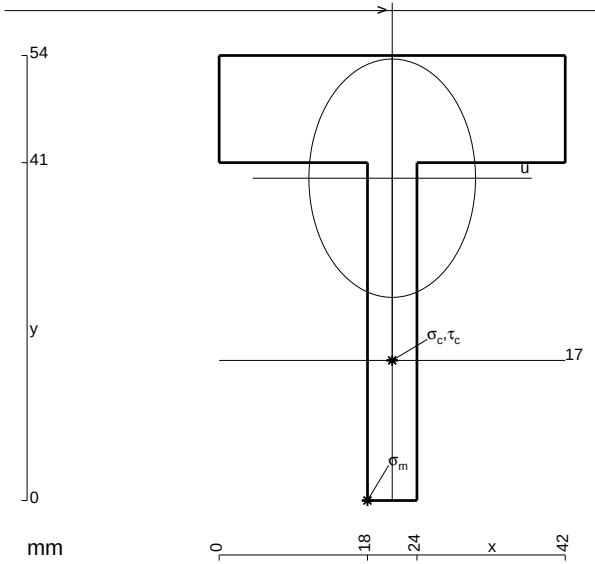
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

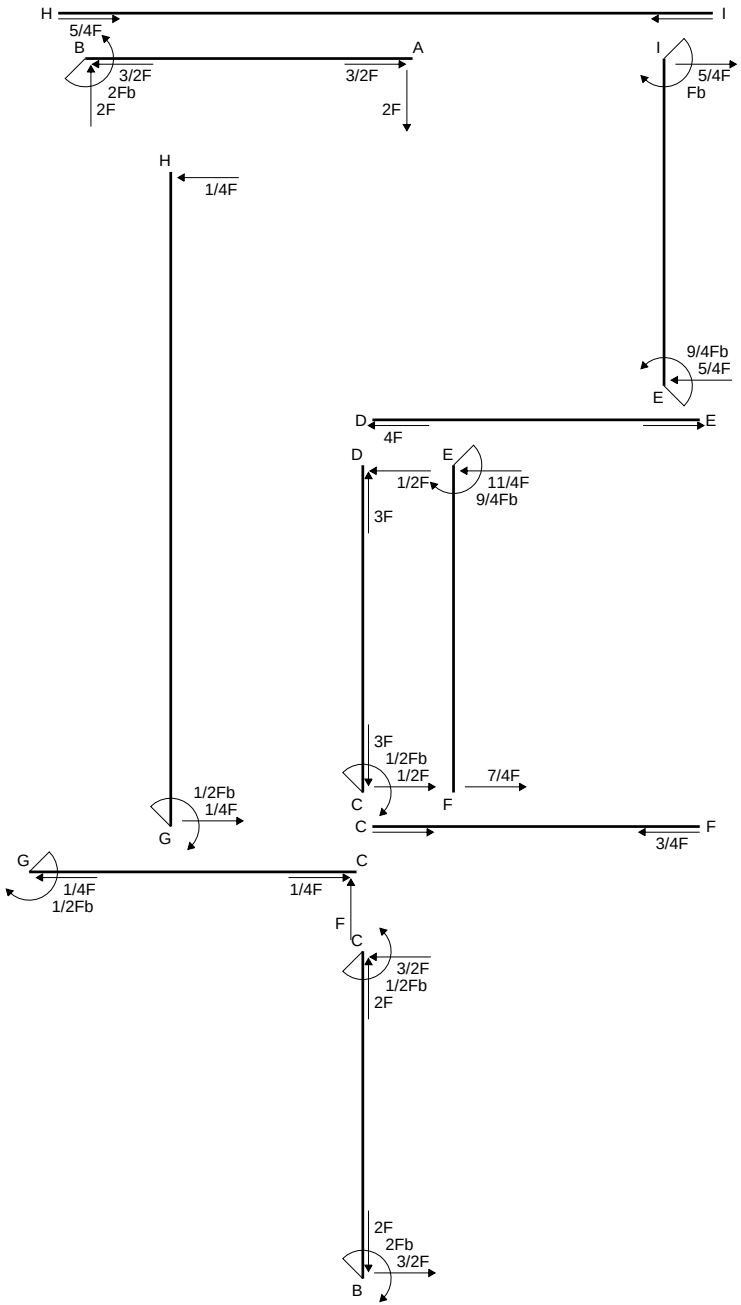


PROCEDIMENTO E RISULTATI 266362 CI01.xxxx.082



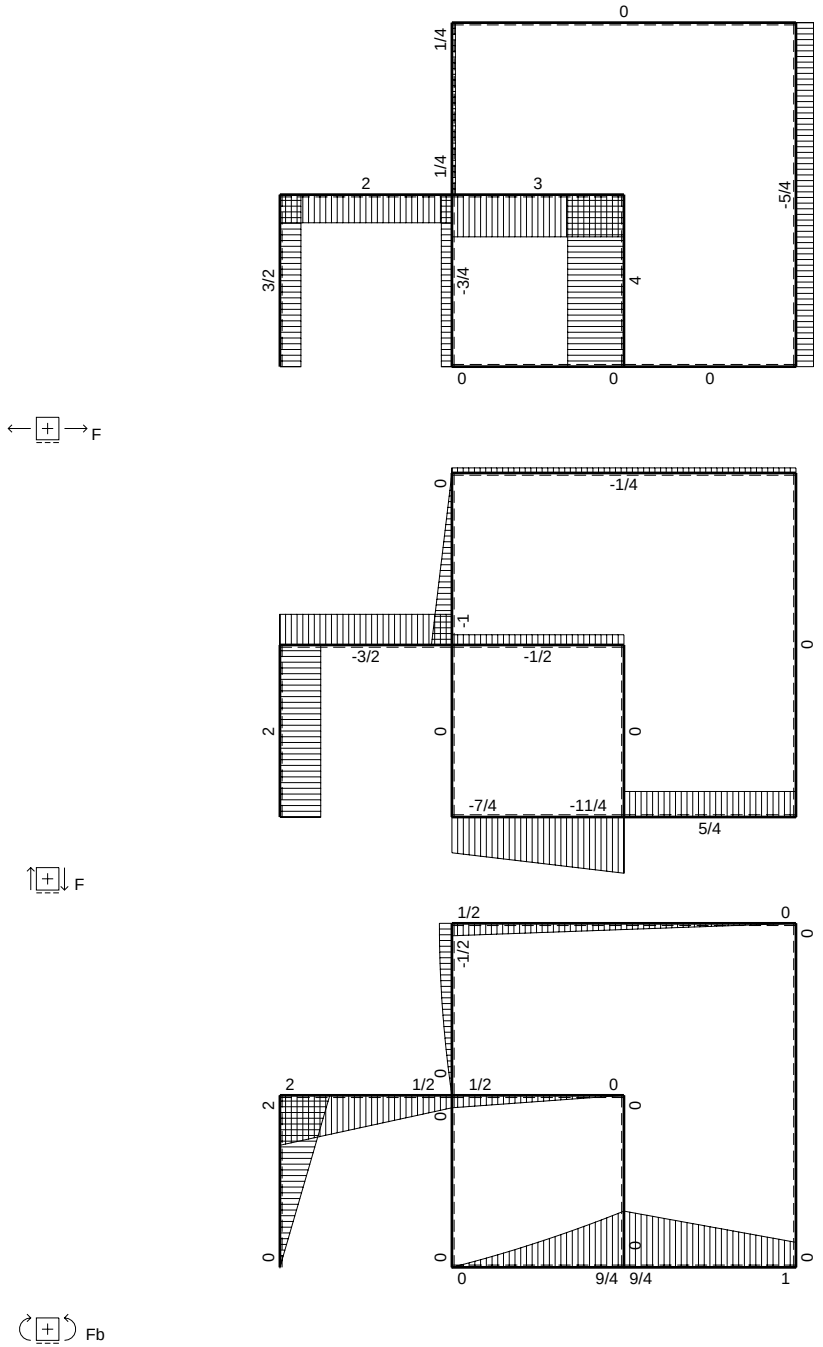

A = 792. $mm^2$
$J_u$ = 165782. $mm^4$
$J_v$ = 81000. $mm^4$
$y_g$ = 39.11 mm
$T_y$ = -1723. N
$M_x$ = 930150. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -39.11 mm
$\sigma_m = -Mv/J_u$ = 219.5 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 17. mm
$v_c$ = -22.11 mm
$\sigma_c = -Mv/J_u$ = 124.1 $N/mm^2$
$\tau_c$ = 5.407 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 124.4 $N/mm^2$
$S^*$ = 3123. $mm^3$

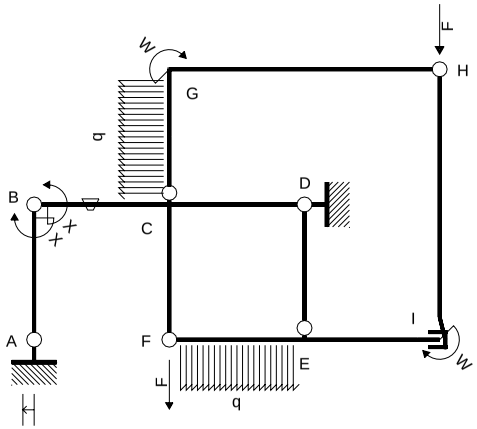
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H
I
5/4F
B
A
3/2F
2Fb
2F
3/2F
2F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
1/2F
11/4F
9/4Fb
3F
3F
1/2Fb
1/2F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/4F
1/2Fb
F
C
3/2F
1/2Fb
2F
2F
2Fb
3/2F
B

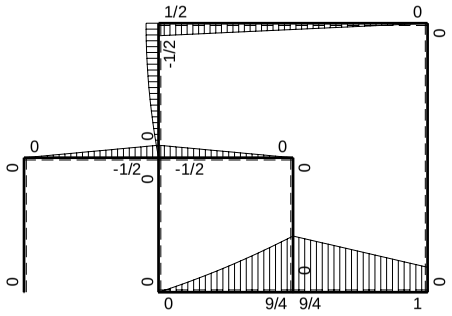




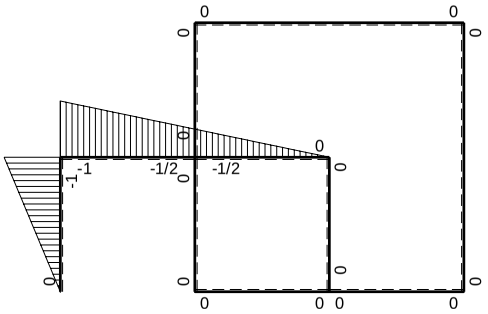

F
0
1/4
1/4
2
3
-5/4
3/2
-3/4
4
0
0
0
F
0
-1/4
-1
-3/2
-1/2
2
0
0
0
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
2
1/2
1/2
0
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264390 CI01.xxxx.083

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PROCEDIMENTO E RISULTATI 264390 CI01.xxxx.083

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -1/2Fx | -Fb/EJ | $1/2Fx-1/4Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/6+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 1/2Fb-1/2Fx | Fb/EJ | $1/4Fb-1/4Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

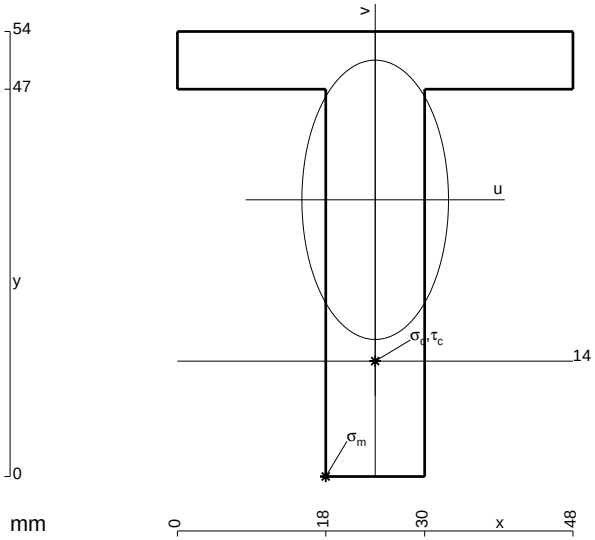
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

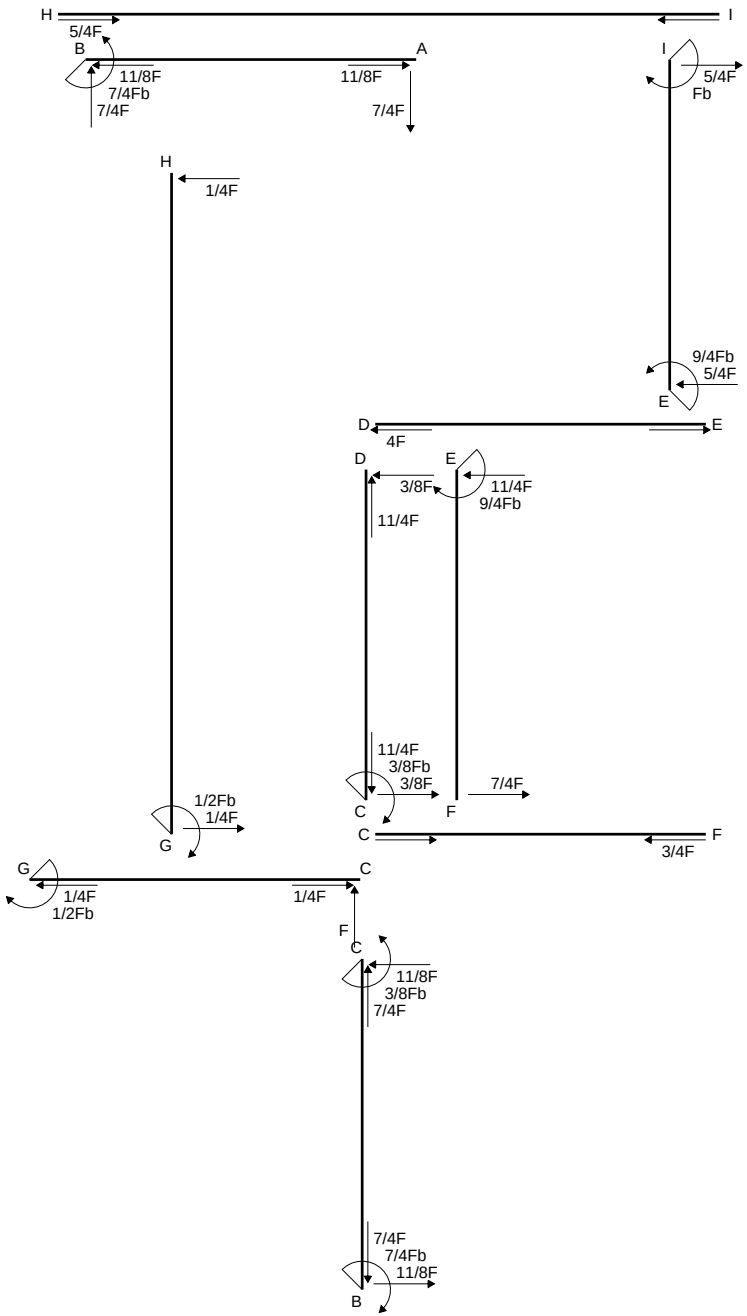
$= (1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





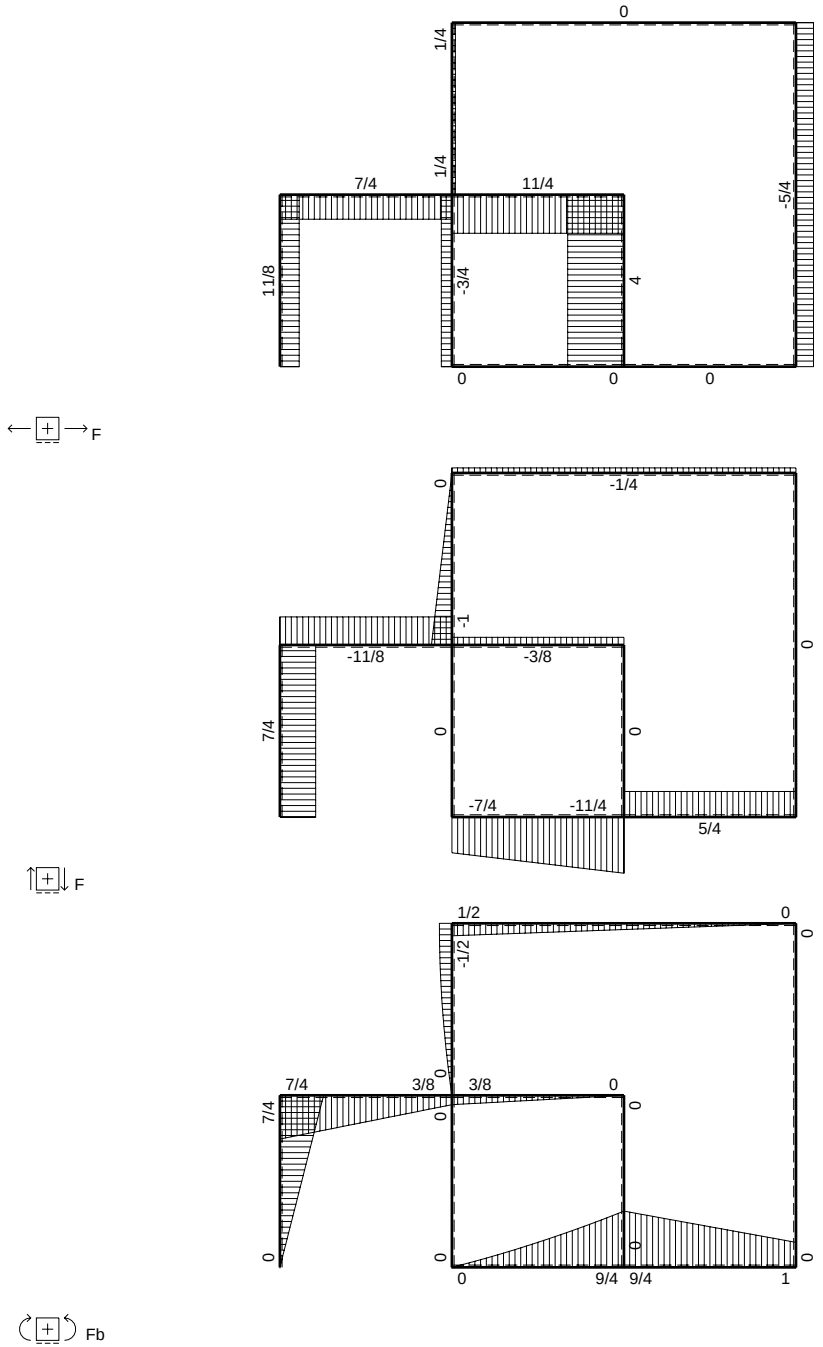


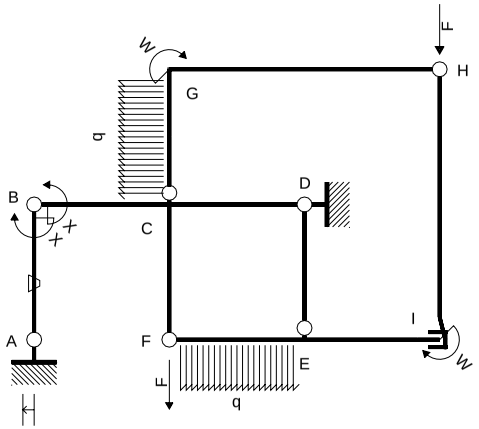

A = 900. $mm^2$
$J_u$ = 258693. $mm^4$
$J_v$ = 71280. $mm^4$
$y_g$ = 33.58 mm
$T_y$ = -3713. N
$M_x$ = 1761750. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -33.58 mm
$\sigma_m$ = -Mv/$J_u$ = 228.7 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 14. mm
$v_c$ = -19.58 mm
$\sigma_c$ = -Mv/$J_u$ = 133.3 $N/mm^2$
$\tau_c$ = 5.34 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.7 $N/mm^2$
$S^*$ = 4465. $mm^3$


H
I
5/4F
B
A
11/8F
7/4Fb
7/4F
11/8F
7/4F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
3/8F
11/4F
9/4Fb
11/4F
11/4F
3/8Fb
3/8F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/2Fb
1/4F
F
C
11/8F
3/8Fb
7/4F
7/4F
7/4Fb
11/8F
B

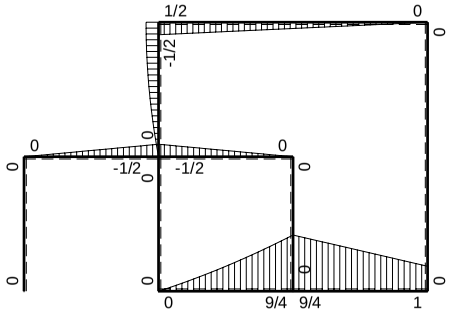




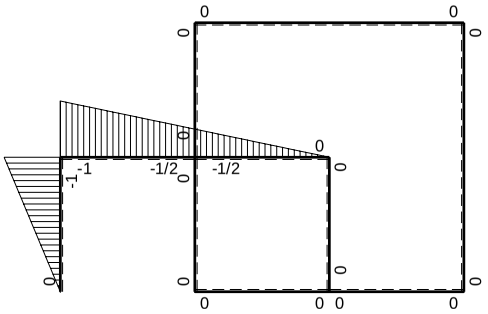

0
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F
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-1
-11/8
-3/8
0
7/4
0
0
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
0
7/4
3/8
3/8
0
7/4
0
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 302320 CI01.xxxx.084

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PROCEDIMENTO E RISULTATI 302320 CI01.xxxx.084

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-1/2Fx$ | 0 | $1/2Fx-1/4Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $1/2Fb-1/2Fx$ | 0 | $1/4Fb-1/4Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+1/2Fx$ | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $7/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

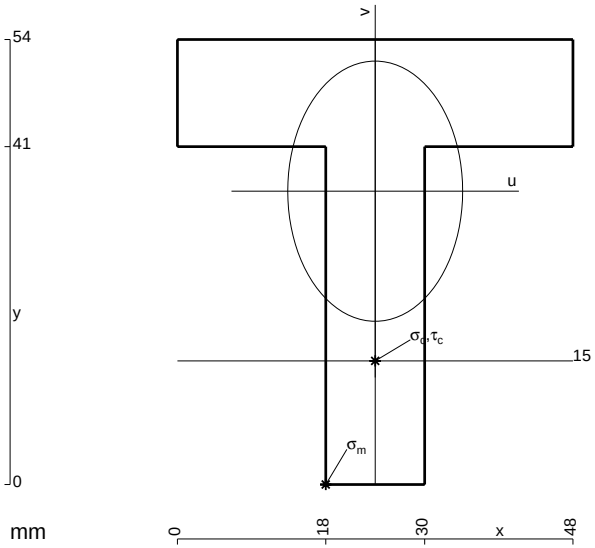
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

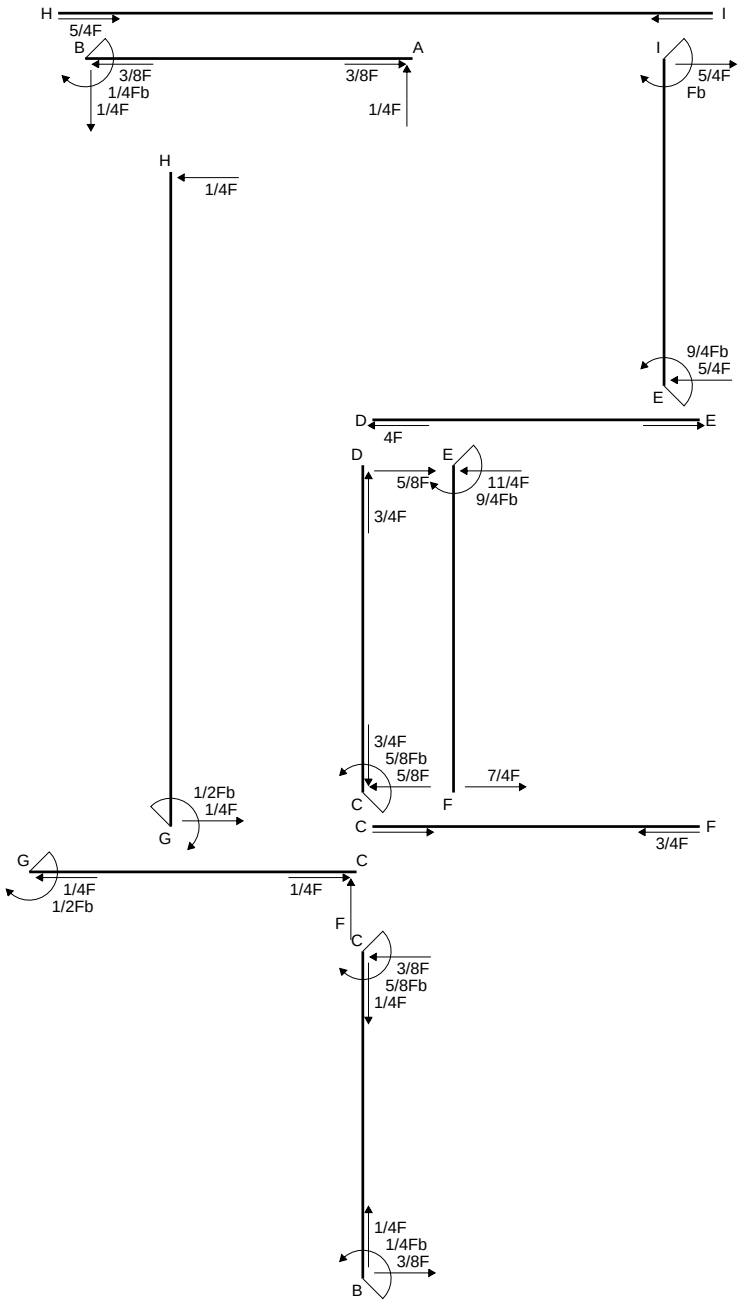
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





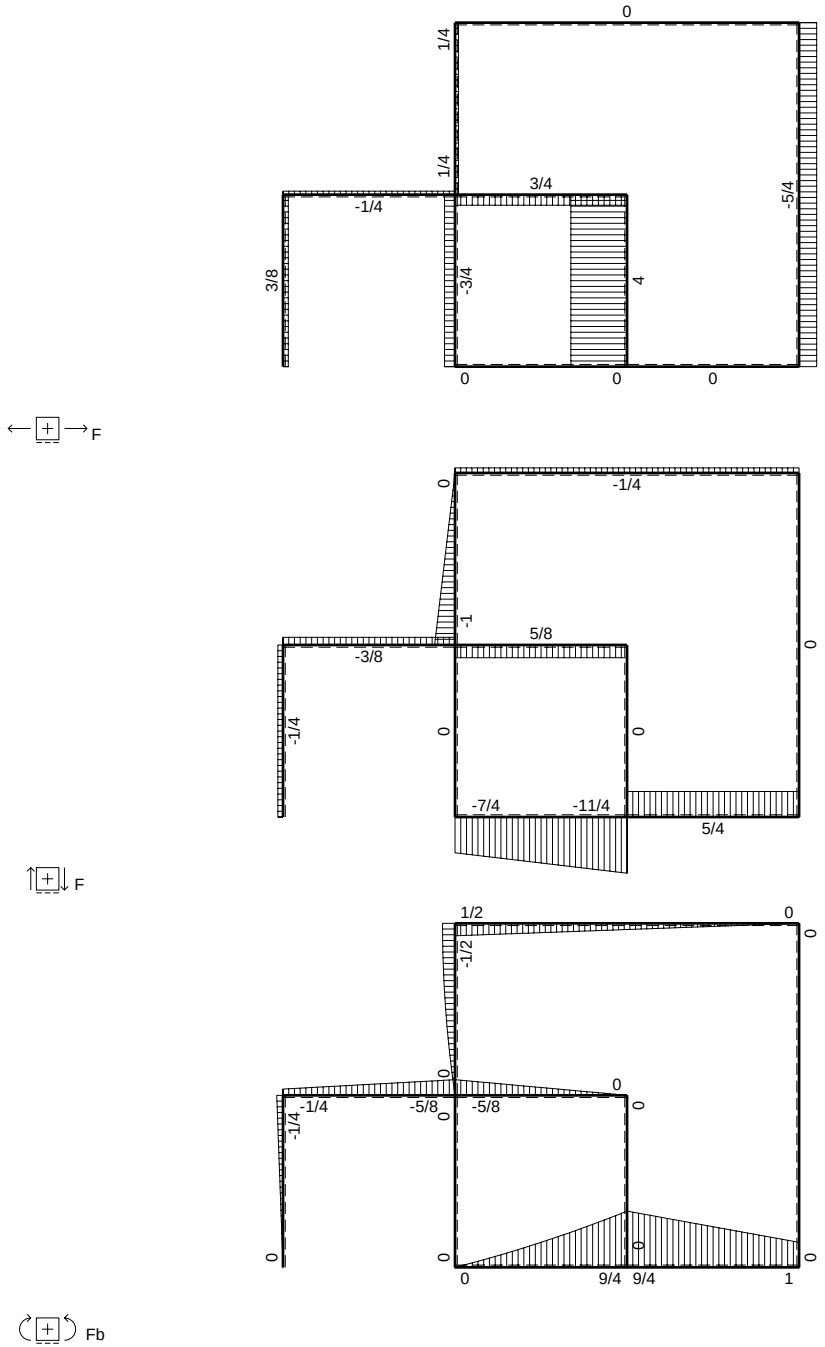


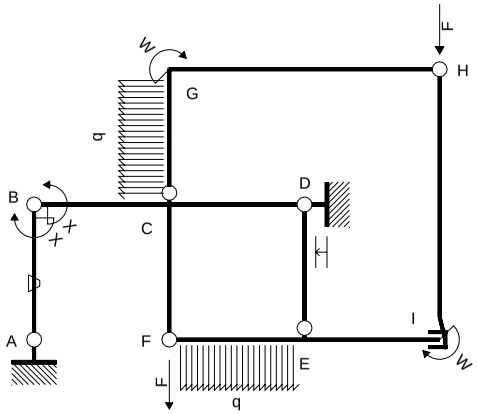

A = 1116. $mm^2$
$J_u$ = 278254. $mm^4$
$J_v$ = 125712. $mm^4$
$y_g$ = 35.6 mm
$T_y$ = -3630. N
$M_x$ = 1871100. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -35.6 mm
$\sigma_m = -Mv/J_u$ = 239.4 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -20.6 mm
$\sigma_c = -Mv/J_u$ = 138.5 $N/mm^2$
$\tau_c$ = 5.498 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.8 $N/mm^2$
$S^*$ = 5057. $mm^3$


H
I
5/4F
B
A
3/8F
1/4Fb
1/4F
3/8F
1/4F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
5/8F
11/4F
9/4Fb
3/4F
3/4F
5/8Fb
5/8F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/4F
1/2Fb
F
C
3/8F
5/8Fb
1/4F
1/4F
1/4Fb
3/8F
B

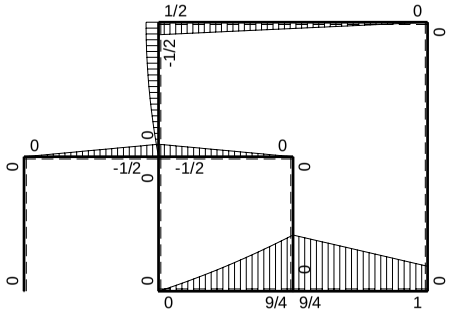




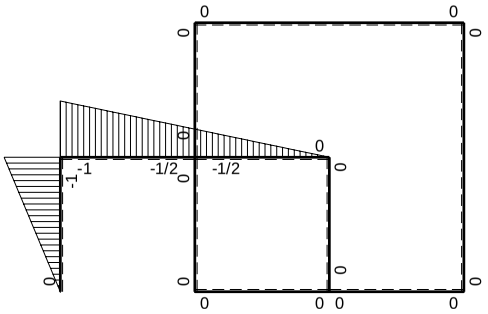

0
1/4
1/4
3/4
-1/4
-5/4
3/8
-3/4
4
0
0
0
F
-1/4
0
-1
5/8
-3/8
0
-1/4
0
0
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
0
0
-1/4
-1/4
-5/8
-5/8
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 218577 CI01.xxxx.085

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PROCEDIMENTO E RISULTATI 218577 CI01.xxxx.085

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-1/2Fx$ | 0 | $1/2Fx-1/4Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $1/2Fb-1/2Fx$ | 0 | $1/4Fb-1/4Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+1/2Fx$ | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

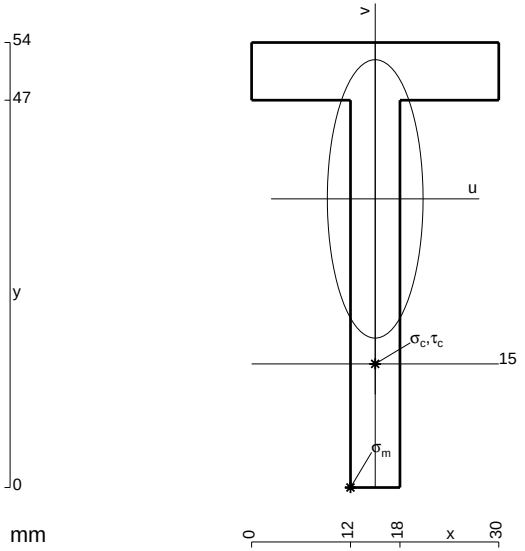
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

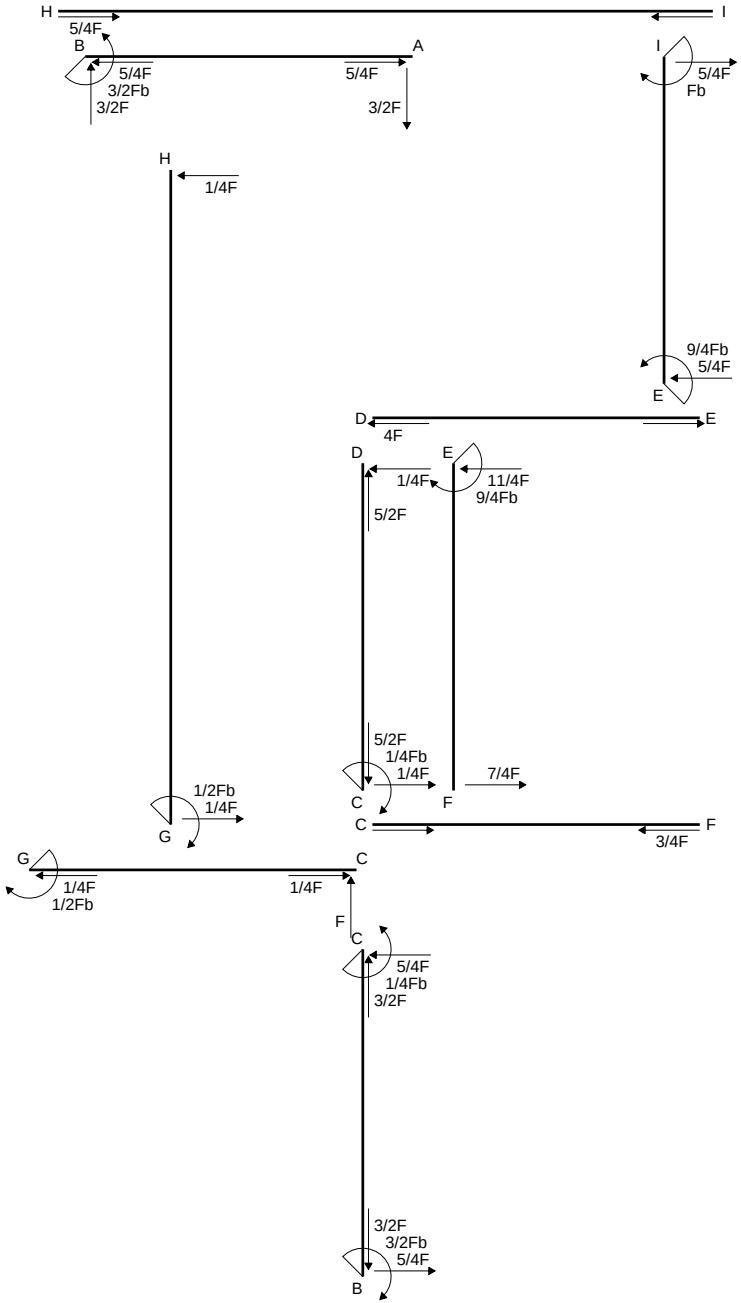
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





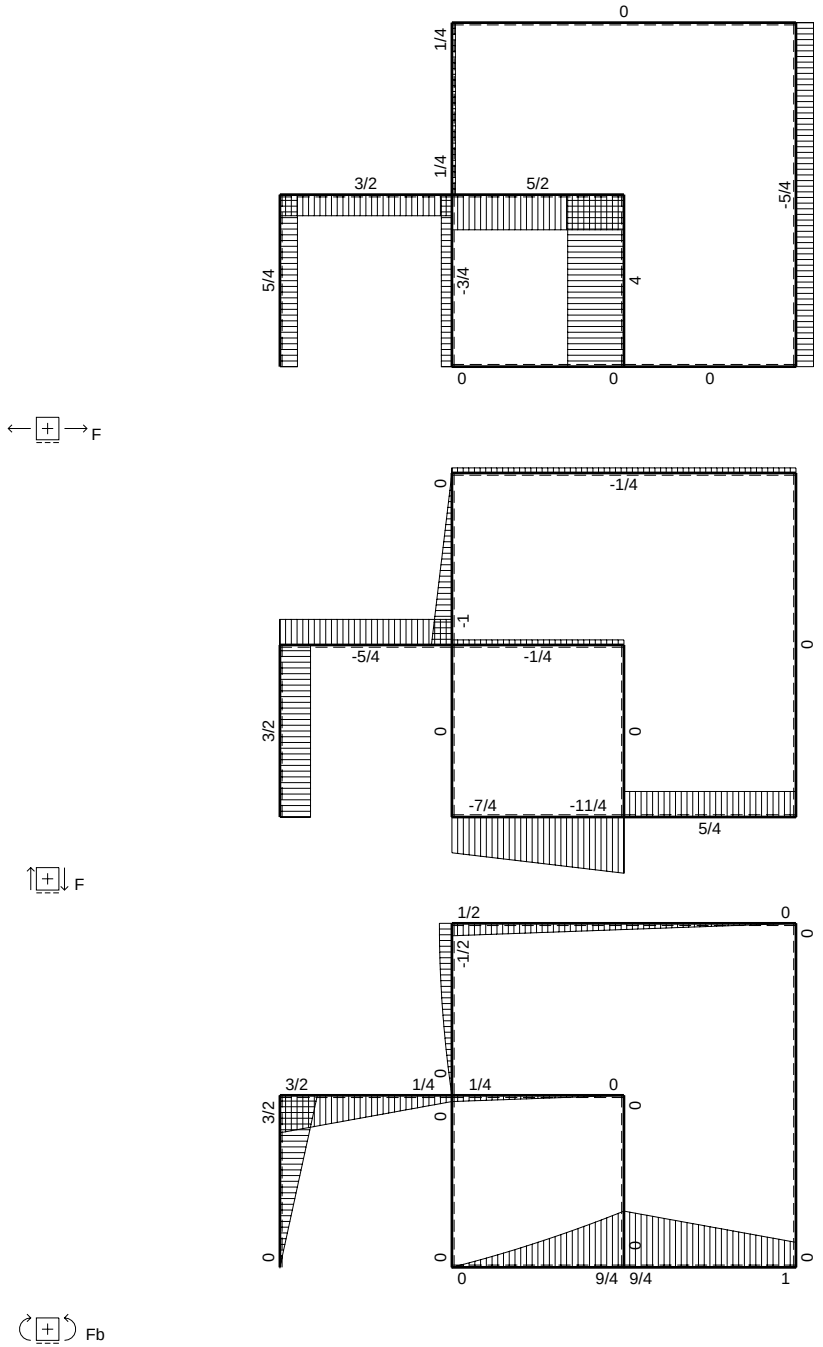


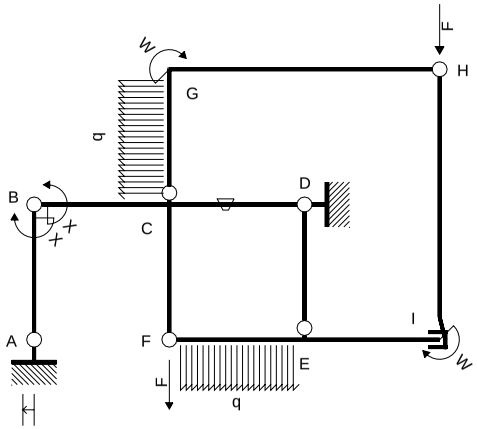

A = 492. $mm^2$
$J_u$ = 140516. $mm^4$
$J_v$ = 16596. $mm^4$
$y_g$ = 35.02 mm
$T_y$ = -1458. N
$M_x$ = 798975. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -35.02 mm
$\sigma_m = -Mv/J_u$ = 199.1 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 15. mm
$v_c$ = -20.02 mm
$\sigma_c = -Mv/J_u$ = 113.9 $N/mm^2$
$\tau_c$ = 4.282 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.1 $N/mm^2$
$S^*$ = 2477. $mm^3$


H
I
5/4F
B
A
5/4F
3/2Fb
3/2F
5/4F
3/2F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
1/4F
11/4F
9/4Fb
5/2F
5/2F
1/4Fb
1/4F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/4F
1/2Fb
F
C
5/4F
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3/2F
3/2F
3/2Fb
5/4F
B

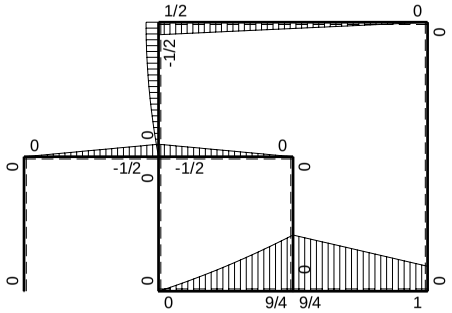




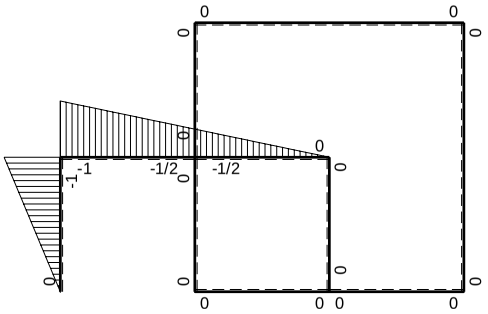

0
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0
-1
-5/4
-1/4
0
3/2
0
0
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
0
3/2
1/4
1/4
0
3/2
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265477 CI01.xxxx.086

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 265477 CI01.xxxx.086

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -1/2Fx | 0 | $1/2Fx-1/4Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 1/2Fb-1/2Fx | 0 | $1/4Fb-1/4Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $3/2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -3/2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

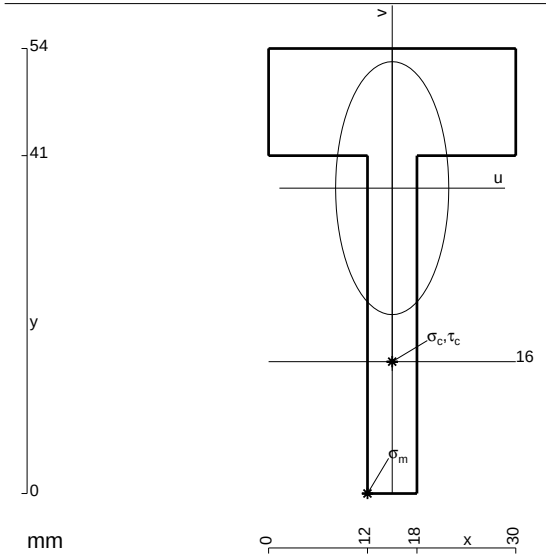
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

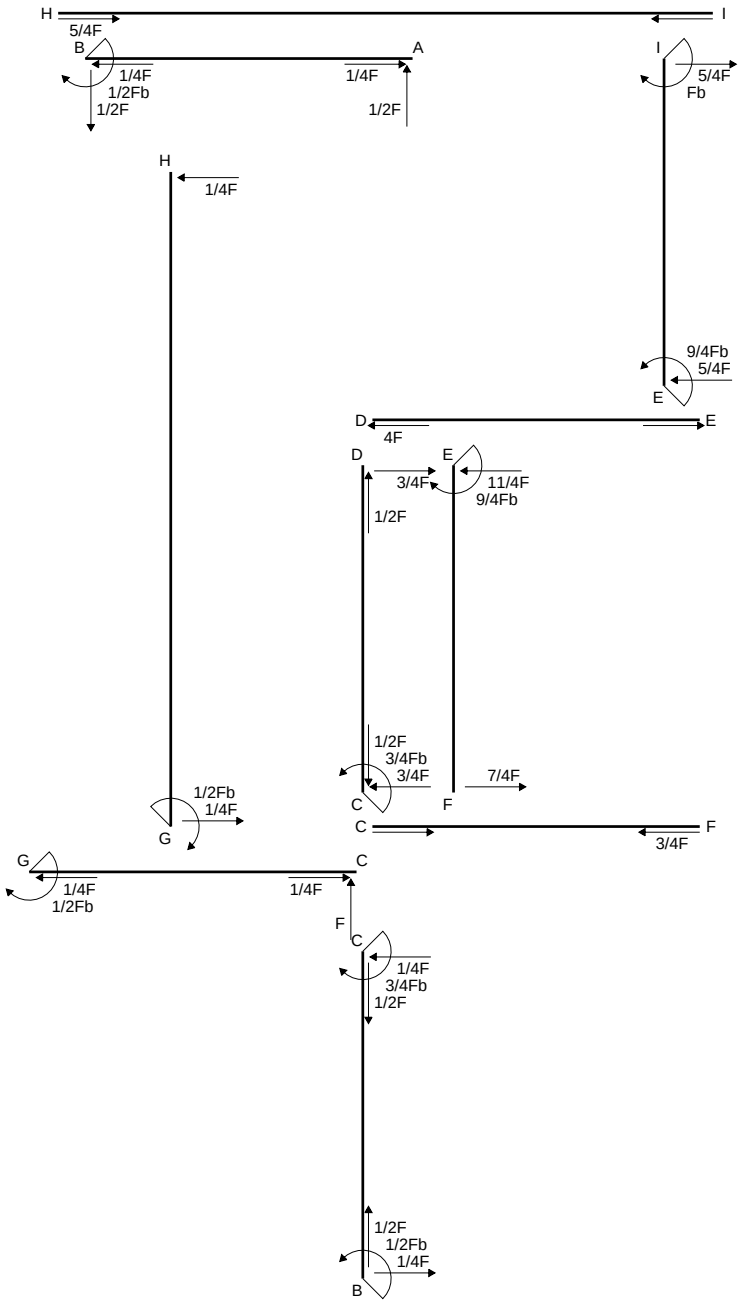
$= (1/12\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$





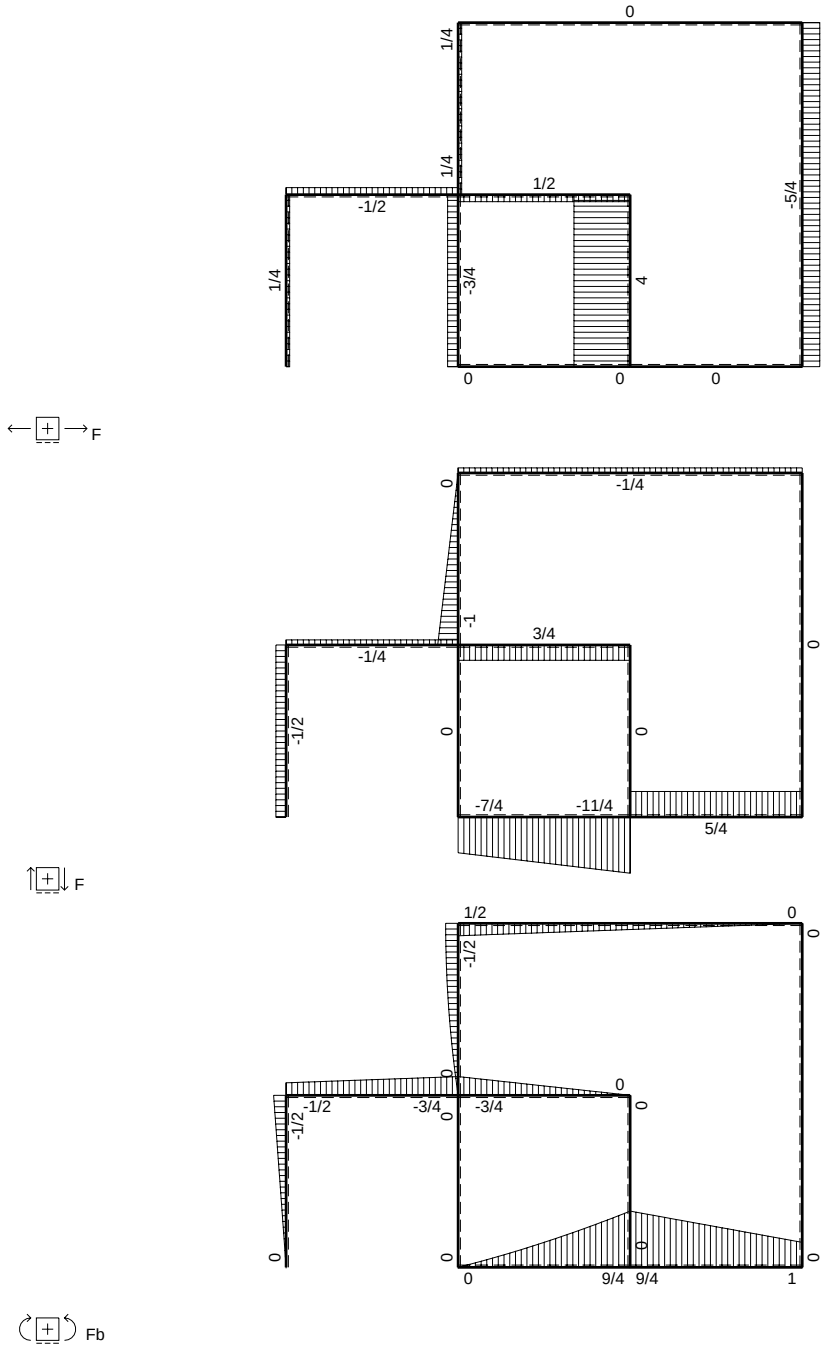


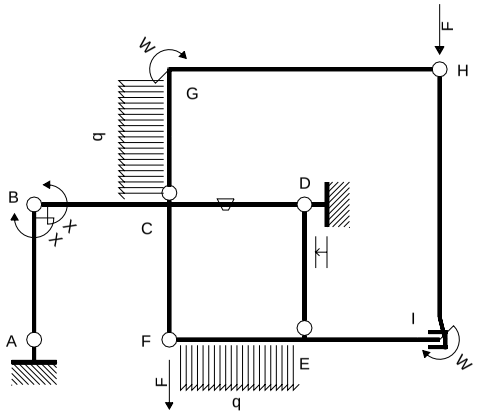

A = 636. $mm^2$
$J_u$ = 149922. $mm^4$
$J_v$ = 29988. $mm^4$
$y_g$ = 37.06 mm
$T_y$ = -1430. N
$M_x$ = 842400. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -37.06 mm
$\sigma_m = -Mv/J_u$ = 208.2 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 16. mm
$v_c$ = -21.06 mm
$\sigma_c = -Mv/J_u$ = 118.3 $N/mm^2$
$\tau_c$ = 4.434 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 118.6 $N/mm^2$
$S^*$ = 2789. $mm^3$


H
I
5/4F
B
A
1/4F
1/2Fb
1/2F
1/4F
1/2F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
3/4F
11/4F
9/4Fb
1/2F
1/2F
3/4Fb
3/4F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/2Fb
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F
C
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1/2Fb
1/4F
B

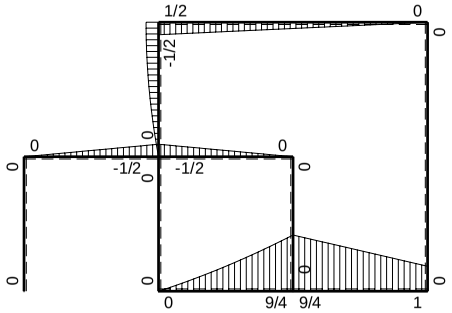




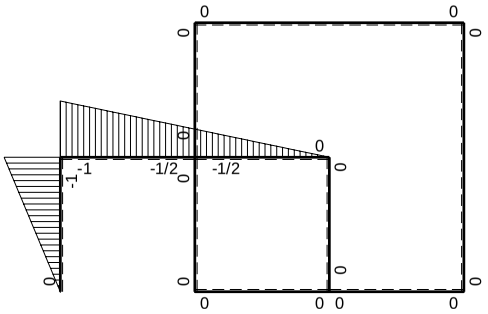

0
1/4
1/4
1/2
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1/4
-3/4
4
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0
0
F
-1/4
0
-1
3/4
-1/4
0
-1/2
0
0
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
0
0
-1/2
-1/2
-3/4
-3/4
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 986307 CI01.xxxx.087

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PROCEDIMENTO E RISULTATI 986307 CI01.xxxx.087

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -1/2Fx | 0 | $1/2Fx-1/4Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 1/2Fb-1/2Fx | 0 | $1/4Fb-1/4Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

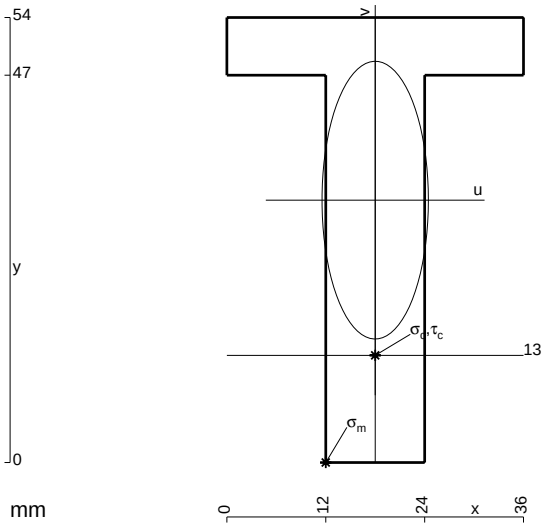
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/12\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$

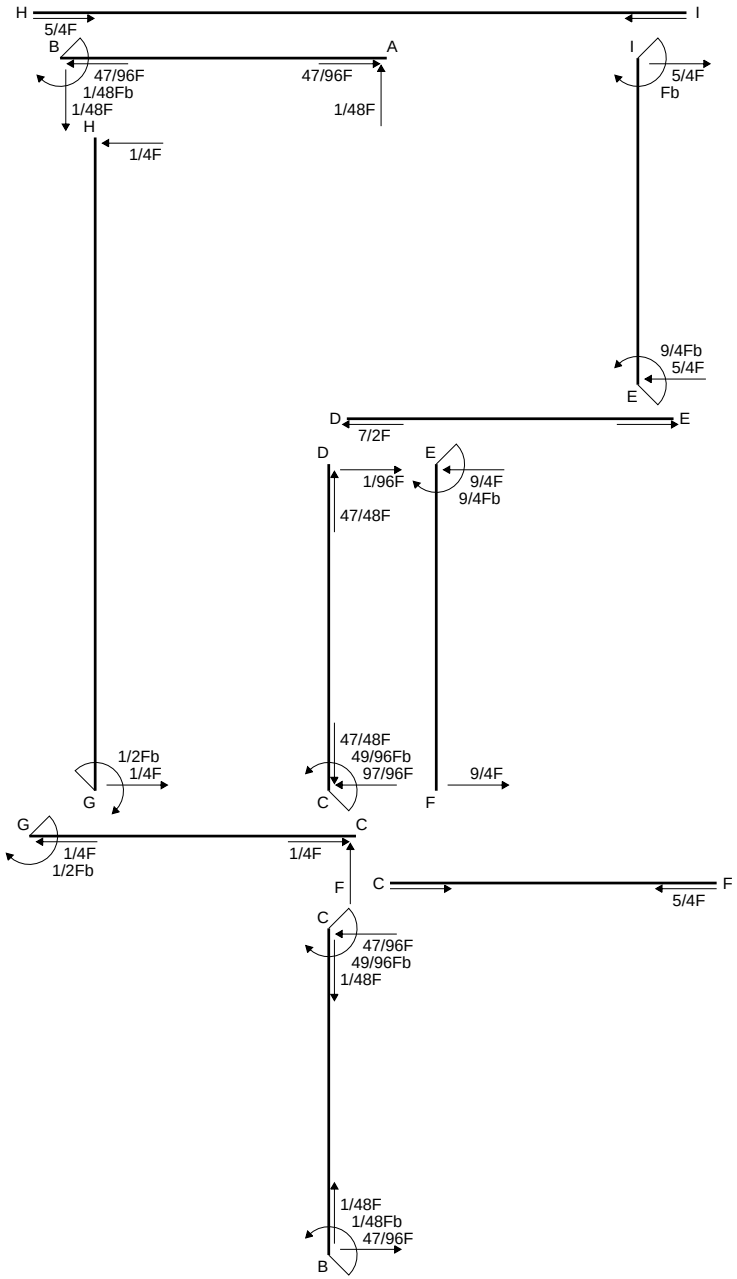


PROCEDIMENTO E RISULTATI 986307 CI01.xxxx.087



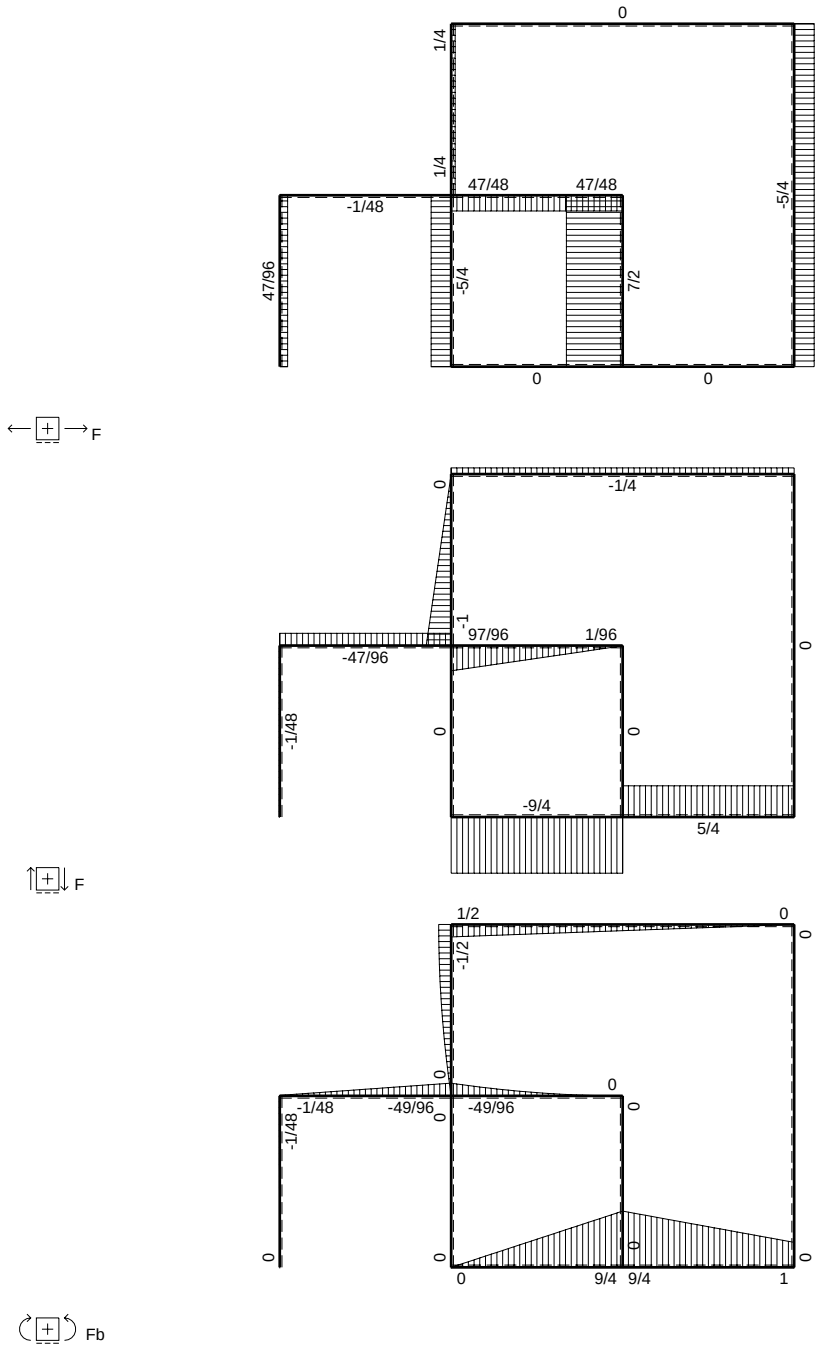

A = 816. $mm^2$
$J_u$ = 231827. $mm^4$
$J_v$ = 33984. $mm^4$
$y_g$ = 31.84 mm
$T_y$ = -2558. N
$M_x$ = 1590300. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -31.84 mm
$\sigma_m = -Mv/J_u$ = 218.4 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 13. mm
$v_c$ = -18.84 mm
$\sigma_c = -Mv/J_u$ = 129.2 $N/mm^2$
$\tau_c$ = 3.634 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 129.4 $N/mm^2$
$S^*$ = 3953. $mm^3$

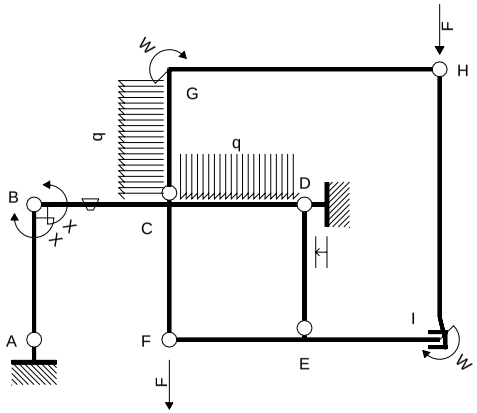
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H
5/4F
I
B
A
47/96F
1/48Fb
1/48F
47/96F
1/48F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
1/96F
9/4F
9/4Fb
47/48F
1/2Fb
1/4F
47/48F
49/96Fb
97/96F
9/4F
G
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
C
47/96F
49/96Fb
1/48F
1/48F
1/48Fb
47/96F
B

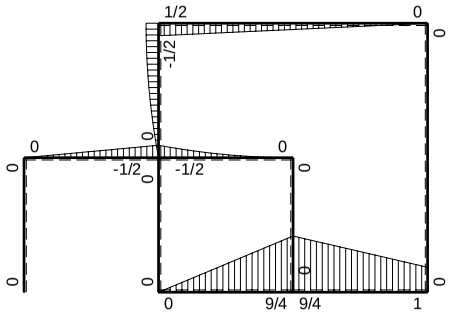




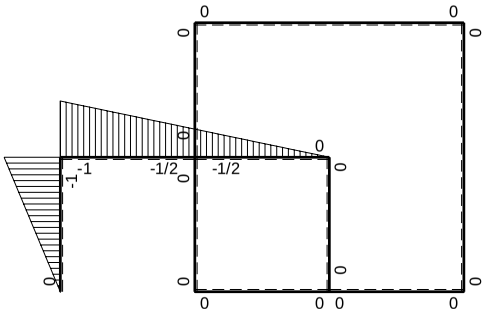

0
1/4
1/4
47/48
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47/96
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7/2
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0
F
-1/4
0
-1
97/96
1/96
-47/96
0
-1/48
0
0
-9/4
5/4
F
1/2
0
0
-1/2
0
0
-1/48
-49/96
0
-49/96
0
-1/48
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 263194 CI01.xxxx.088

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PROCEDIMENTO E RISULTATI 263194 CI01.xxxx.088

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-1/2Fx$ | -Fb/EJ | $1/2Fx-1/4Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/6+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $1/2Fb-1/2Fx$ | Fb/EJ | $1/4Fb-1/4Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+Fx-1/2qx^2$ | 0 | $1/4Fb-3/4Fx+3/4Fx^2/b-1/4qx^3/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/16+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | $1/2qx^2$ | 0 | $1/4qx^3/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/48Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/48Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -3/4\ x/b +3/4\ x^2/b^2 -1/4\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [1/4\ x -3/8\ x^2/b +1/4\ x^3/b^2 -1/16\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

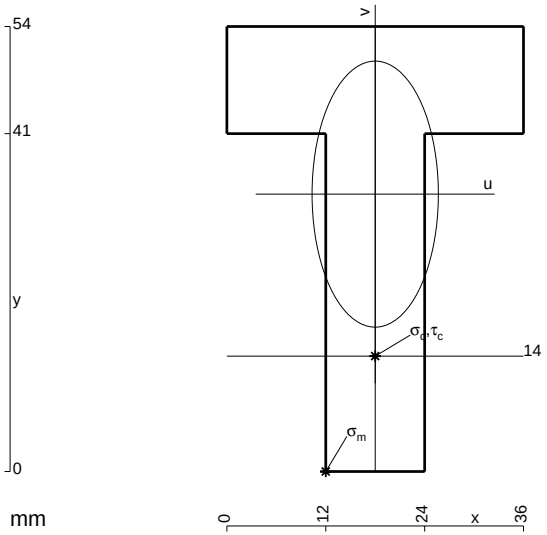
$= (1/4\ b -3/8\ b +1/4\ b -1/16\ b\ )\ Fb\ 1/EJ\ = 1/16\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/16\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

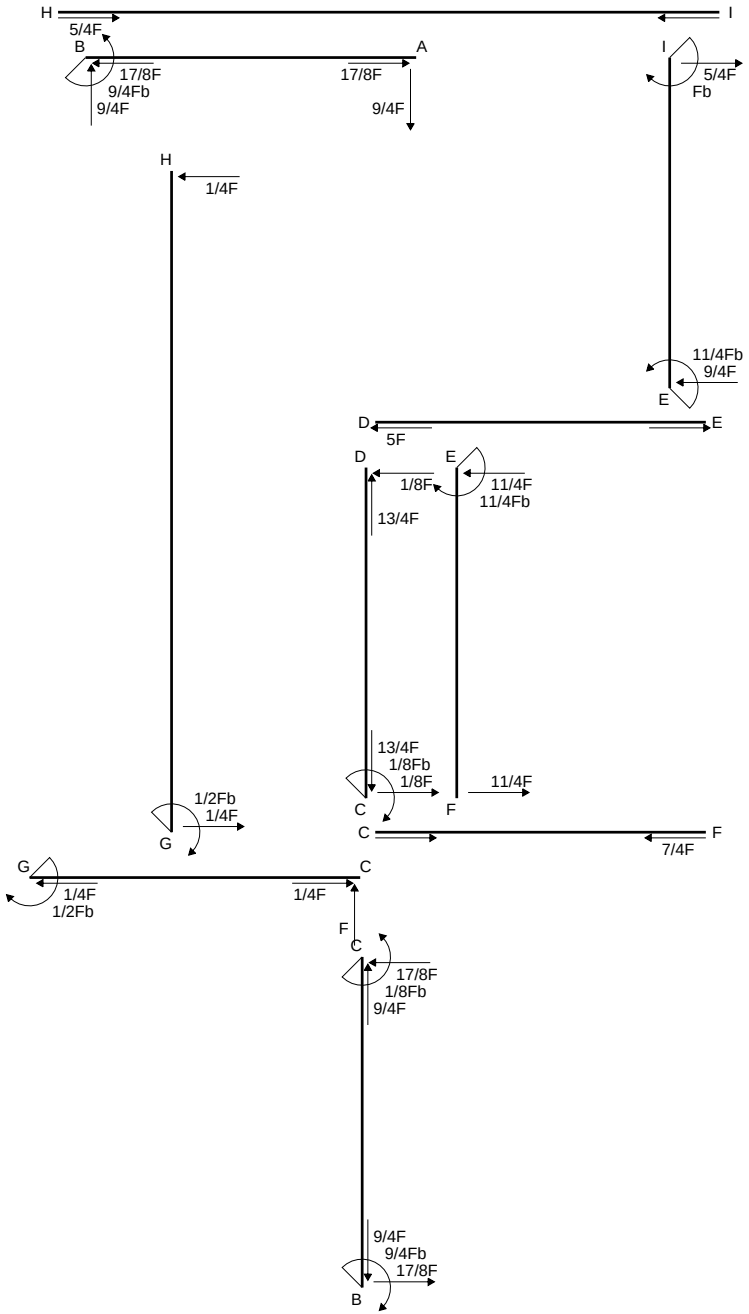
$= (1/16\ b\ )\ Fb\ 1/EJ\ = 1/16\ Fb^2/EJ$





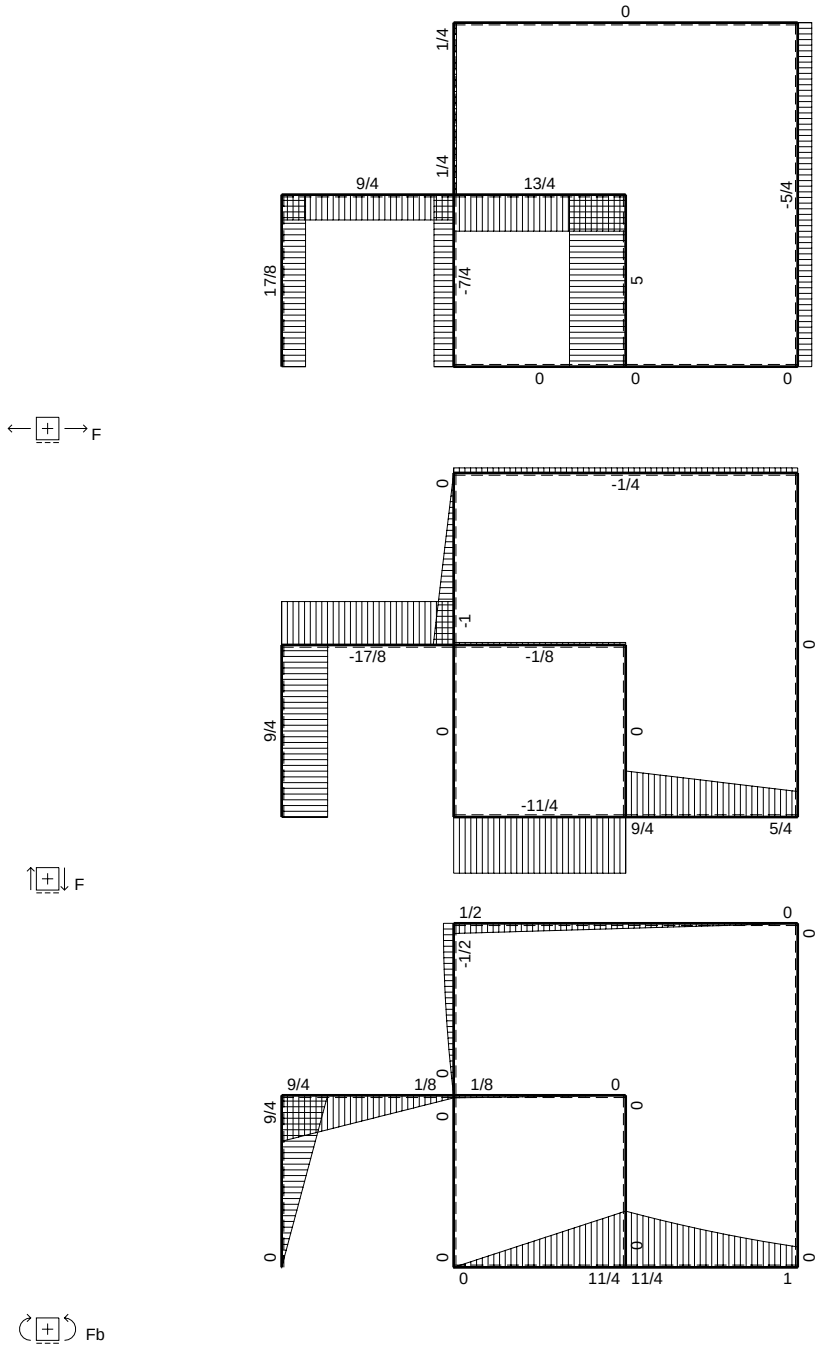


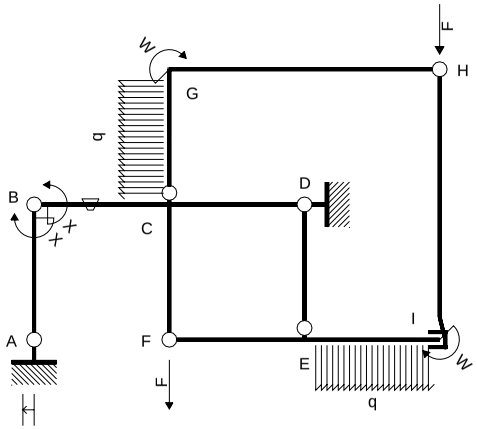

A = 960. $mm^2$
$J_u$ = 250363. $mm^4$
$J_v$ = 56448. $mm^4$
$y_g$ = 33.66 mm
$T_y$ = -2093. N
$M_x$ = 1694930. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -33.66 mm
$\sigma_m = -Mv/J_u$ = 227.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -19.66 mm
$\sigma_c = -Mv/J_u$ = 133.1 $N/mm^2$
$\tau_c$ = 3.12 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 133.2 $N/mm^2$
$S^*$ = 4479. $mm^3$


H
I
5/4F
B
A
17/8F
9/4Fb
9/4F
17/8F
9/4F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
1/8F
11/4F
11/4Fb
13/4F
13/4F
1/8Fb
1/8F
11/4F
C
F
1/2Fb
1/4F
G
C
F
7/4F
G
C
1/4F
1/2Fb
1/4F
F
C
17/8F
1/8Fb
9/4F
9/4F
9/4Fb
17/8F
B

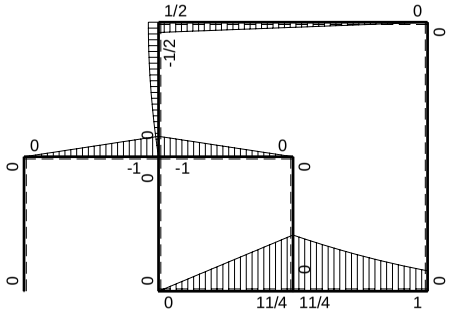




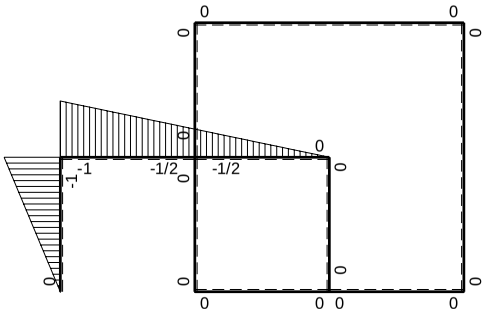

0
1/4
1/4
9/4
13/4
-5/4
17/8
-7/4
5
0
0
0
F
-1/4
0
-1
-17/8
-1/8
0
9/4
0
0
-11/4
9/4
5/4
F
1/2
0
0
-1/2
0
9/4
1/8
1/8
0
9/4
0
0
0
0
0
0
11/4
11/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 218770 CI01.xxxx.089

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $9/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -9/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

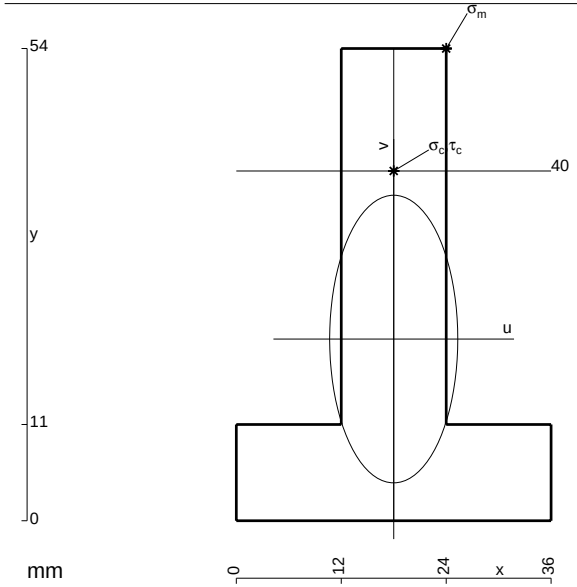
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

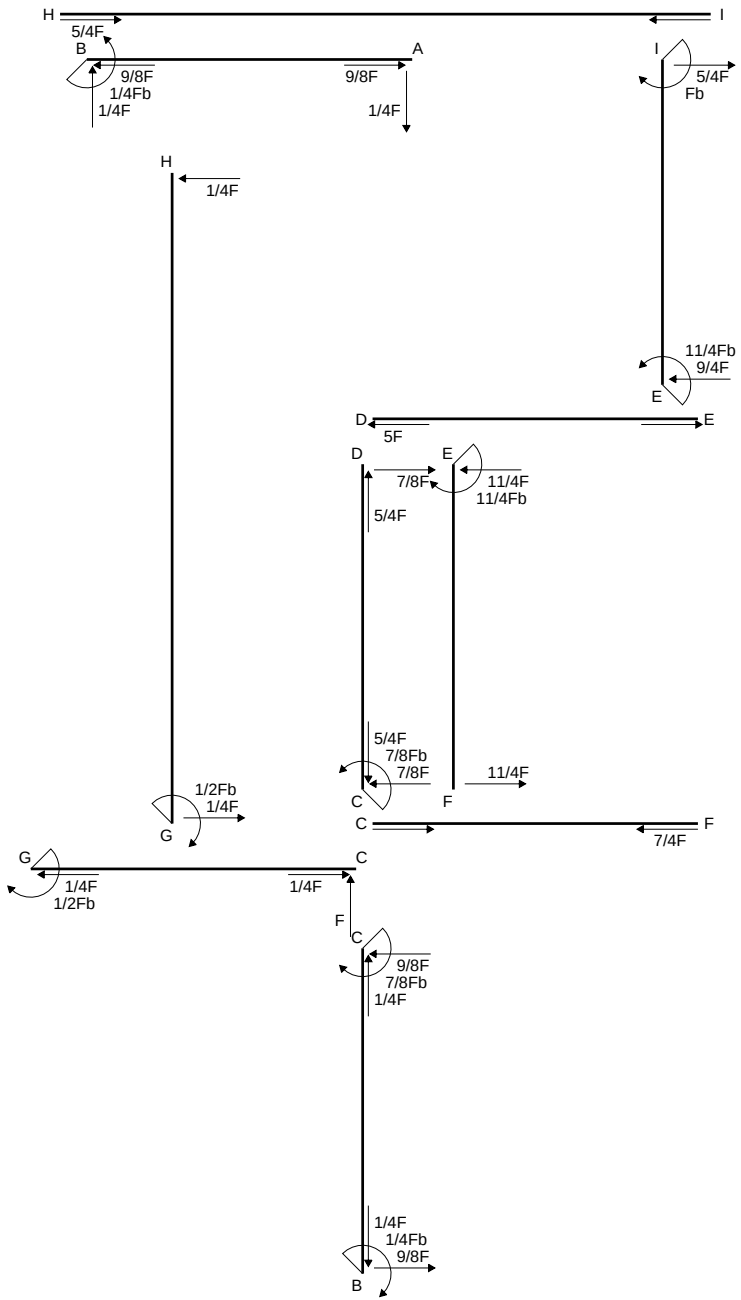
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





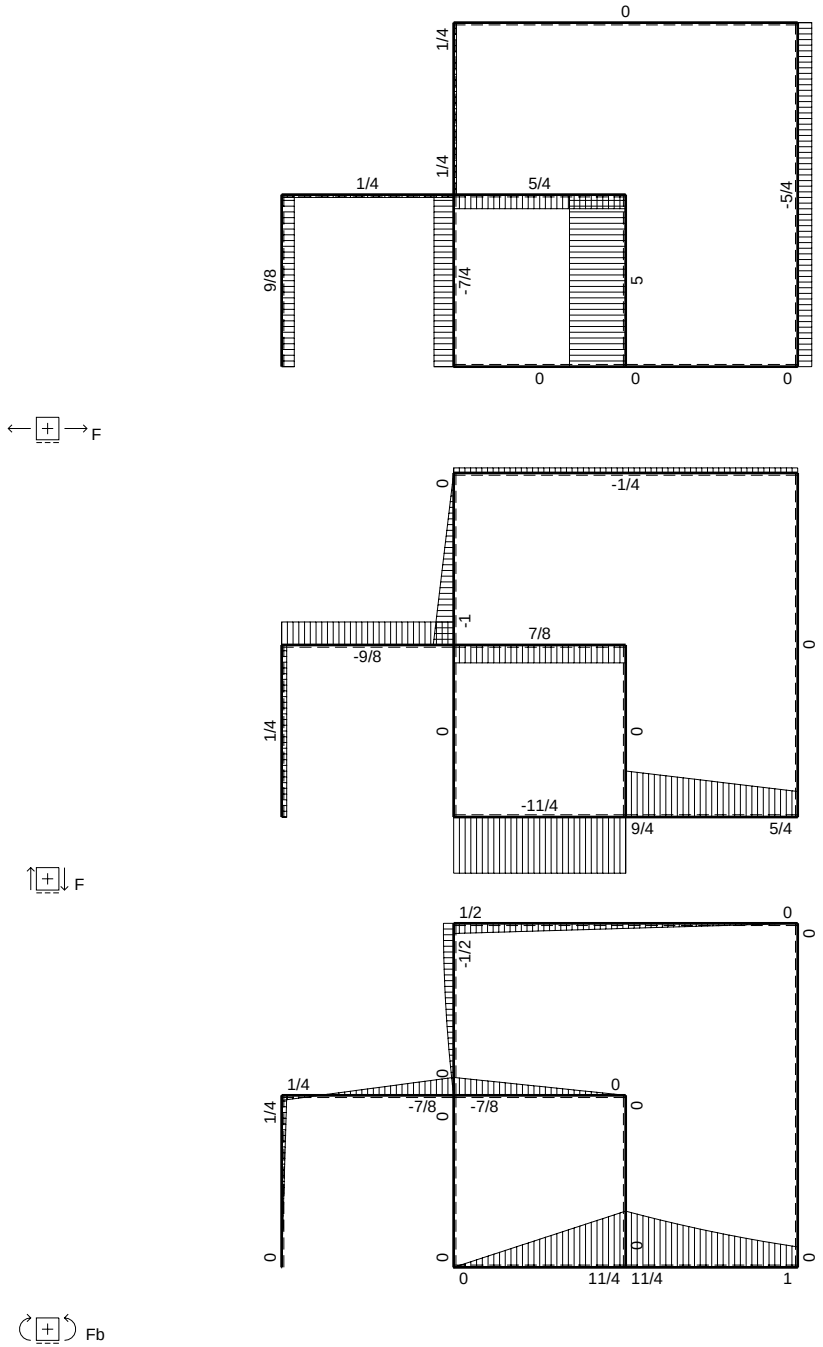


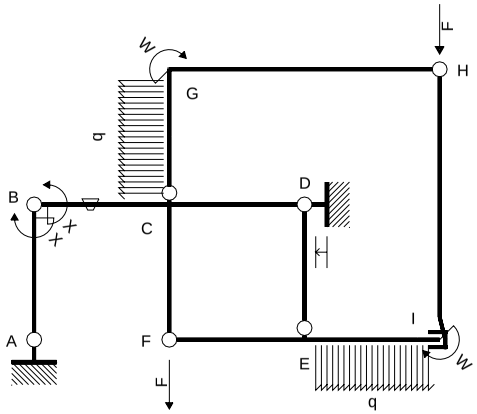

A = 912. $mm^2$
$J_u$ = 246834. $mm^4$
$J_v$ = 48960. $mm^4$
$y_g$ = 20.78 mm
$T_y$ = -2090. N
$M_x$ = 1776500. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 33.22 mm
$\sigma_m = -Mv/J_u$ = -239.1 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 40. mm
$v_c$ = 19.22 mm
$\sigma_c = -Mv/J_u$ = -138.4 $N/mm^2$
$\tau_c$ = 3.109 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.5 $N/mm^2$
$S^*$ = 4406. $mm^3$


H
I
5/4F
B
A
9/8F
1/4Fb
1/4F
9/8F
1/4F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
7/8F
11/4F
11/4Fb
5/4F
5/4F
7/8Fb
7/8F
11/4F
C
F
1/2Fb
1/4F
G
C
F
7/4F
G
C
1/4F
1/4F
1/2Fb
F
C
9/8F
7/8Fb
1/4F
1/4F
1/4Fb
9/8F
B

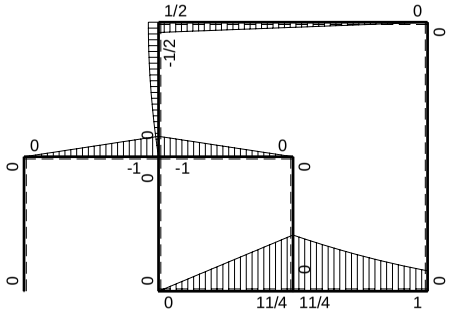




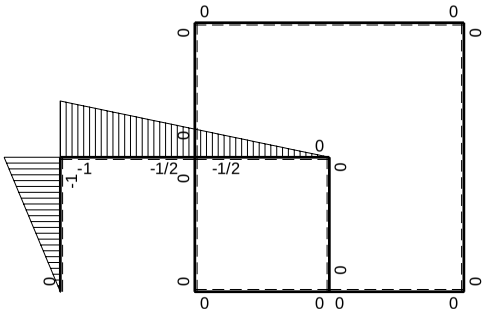

F
F
Fb
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1/4
1/4
1/4
5/4
-5/4
9/8
-7/4
5
0
0
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-1/4
0
-1
7/8
-9/8
0
1/4
0
0
-11/4
9/4
5/4
1/2
0
0
-1/2
1/4
0
0
-7/8
-7/8
1/4
0
0
0
0
0
0
11/4
11/4
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/4Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 214608 CI01.xxxx.090

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

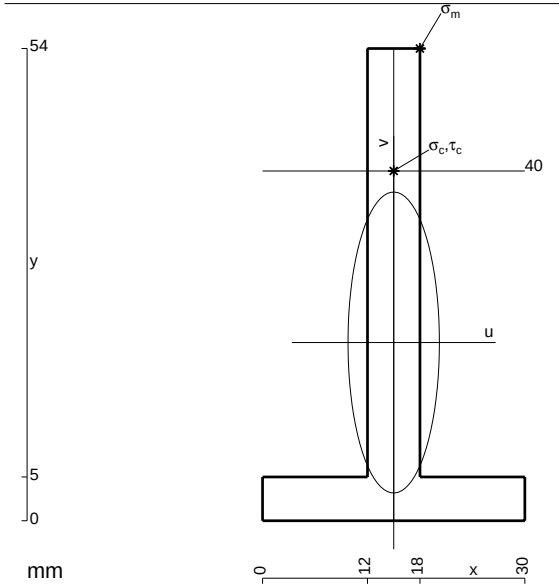
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

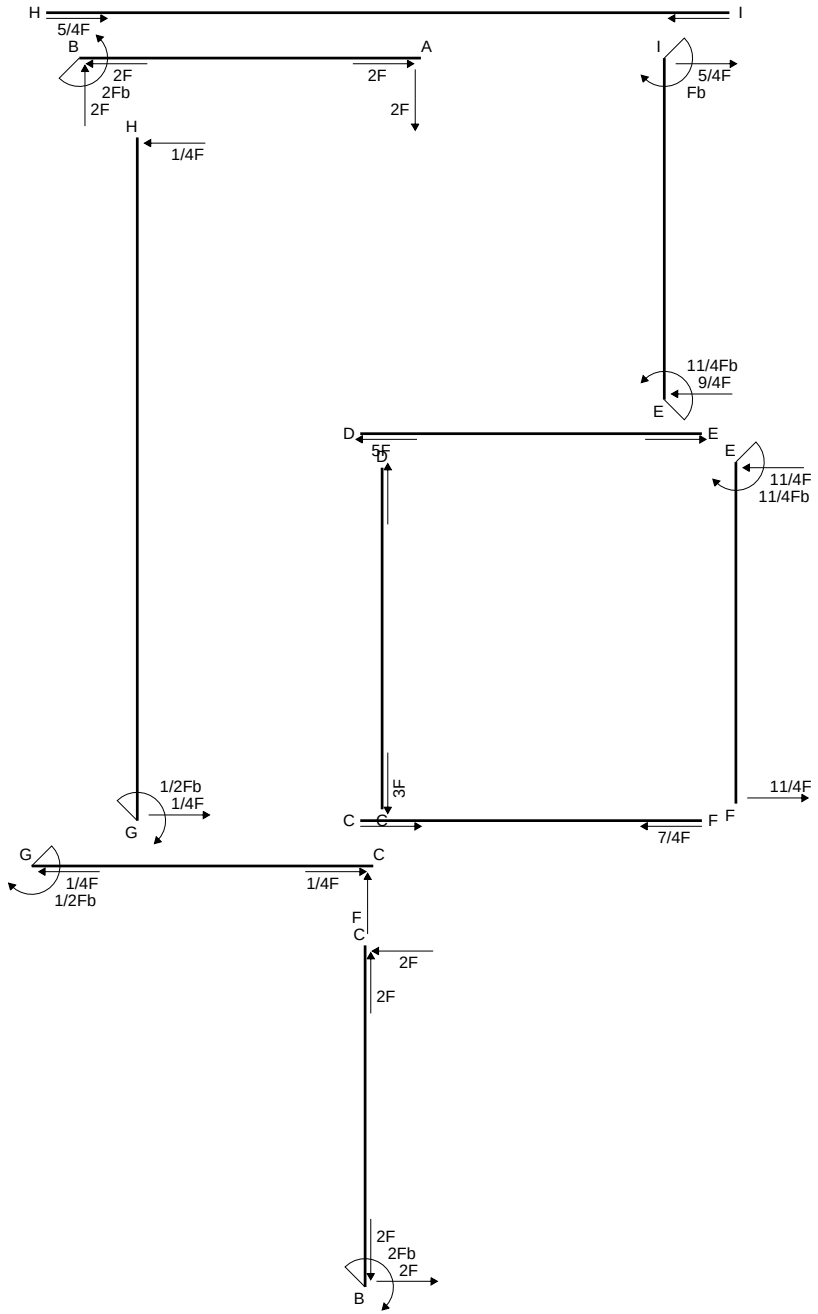
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





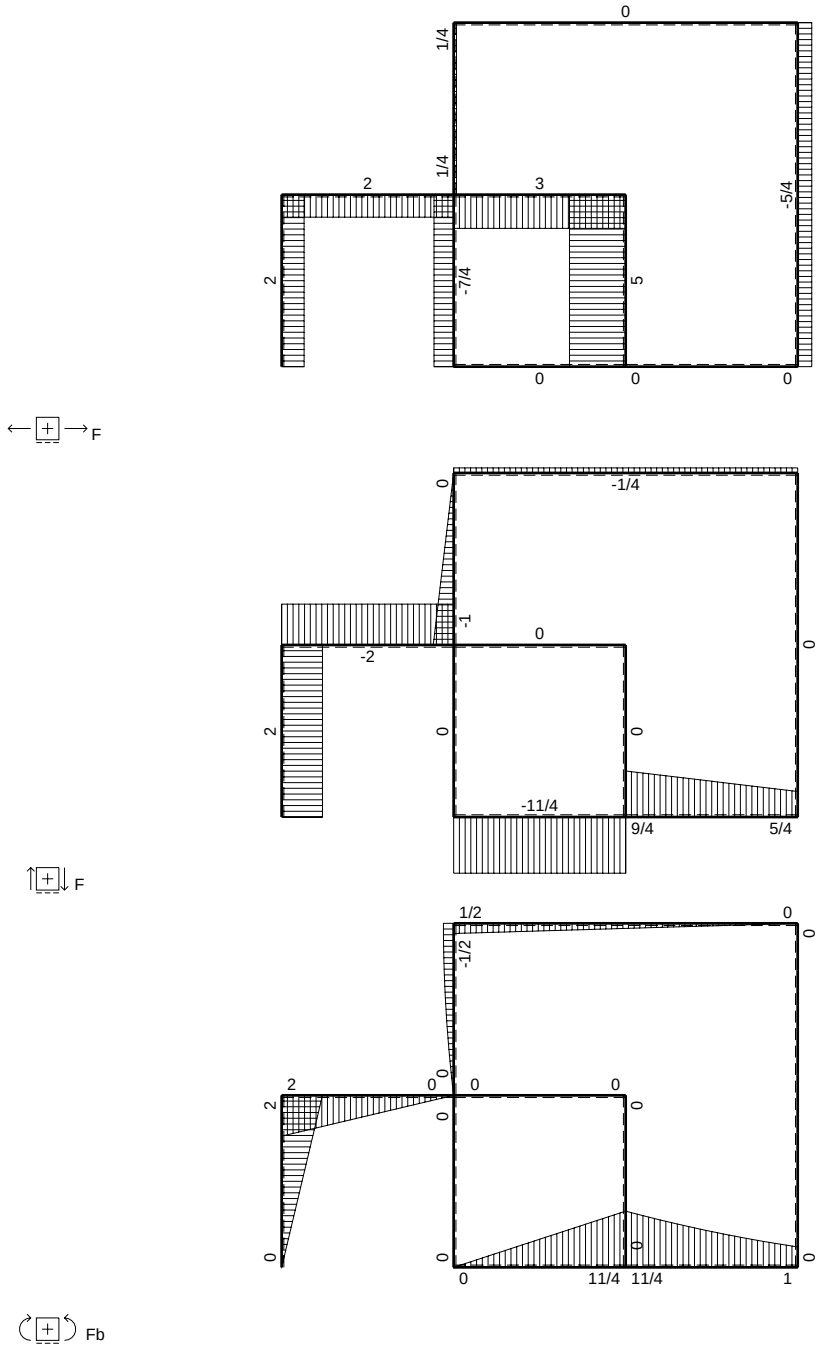


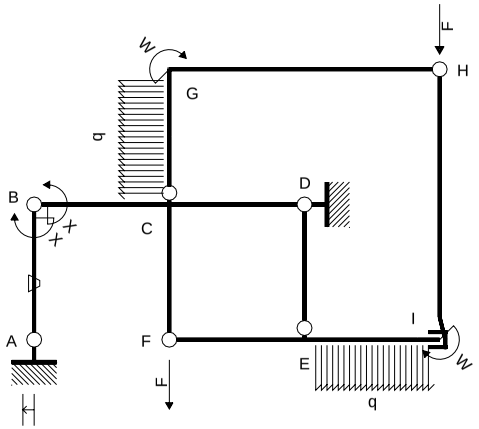

A = 444. $mm^2$
$J_u$ = 131544. $mm^4$
$J_v$ = 12132. $mm^4$
$y_g$ = 20.38 mm
$T_y$ = -1733. N
$M_x$ = 779625. Nmm
$x_m$ = 18. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 33.62 mm
$\sigma_m = -Mv/J_u$ = -199.3 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 40. mm
$v_c$ = 19.62 mm
$\sigma_c = -Mv/J_u$ = -116.3 $N/mm^2$
$\tau_c$ = 4.909 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.6 $N/mm^2$
$S^*$ = 2236. $mm^3$


H
I
5/4F
B
A
2F
2Fb
2F
2F
2F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
11/4F
11/4Fb
1/2Fb
1/4F
G
3F
C
C
F
F
11/4F
7/4F
G
C
1/4F
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1/4F
F
C
2F
2F
2F
2Fb
2F
B

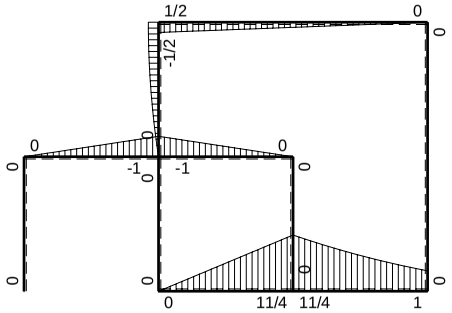




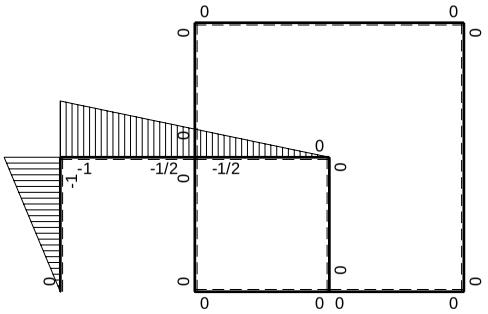

F
F
Fb
0
1/4
1/4
2
3
-5/4
2
-7/4
5
0
0
0
-1/4
0
-1
0
-2
2
0
0
0
-11/4
9/4
5/4
1/2
0
0
-1/2
2
0
0
0
0
0
0
0
0
0
11/4
11/4
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248683 CI01.xxxx.091

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b ( x/b )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

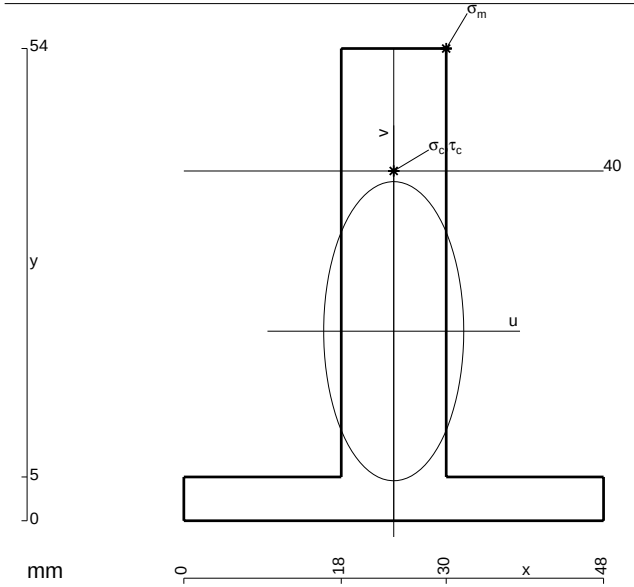
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

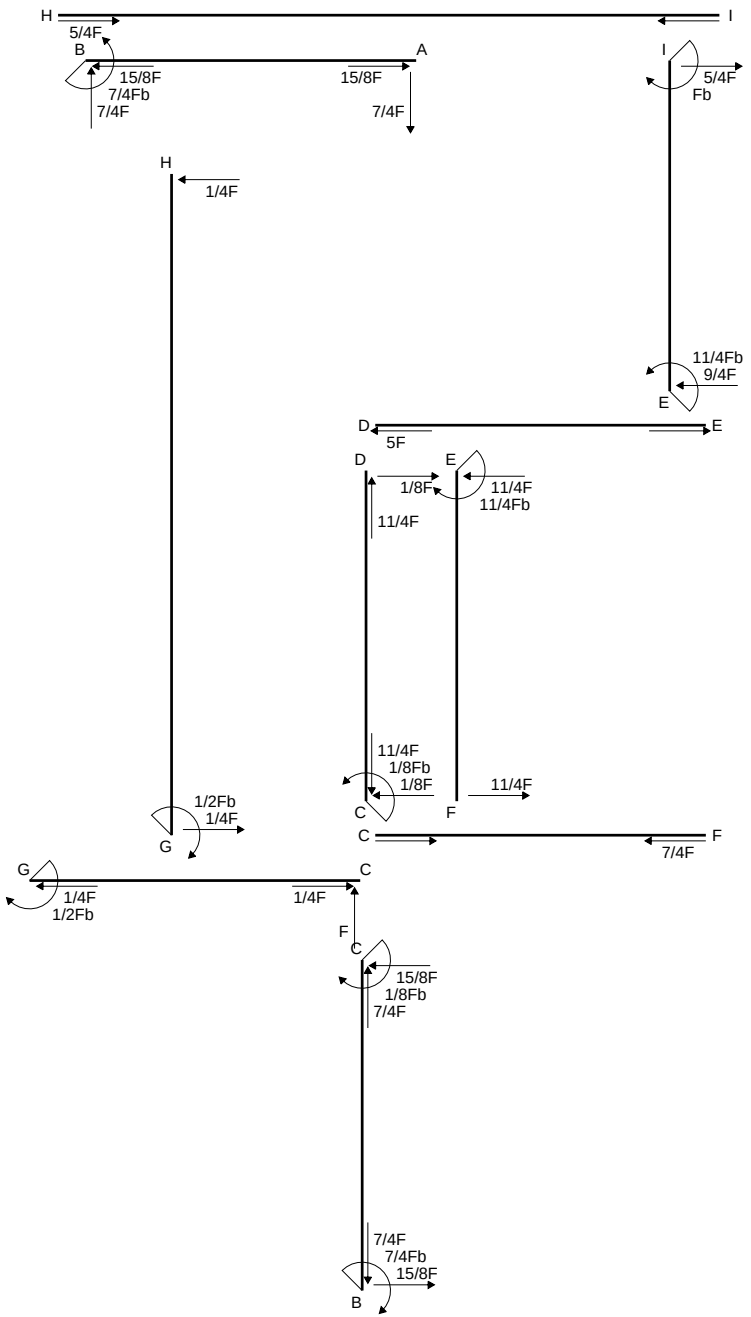


PROCEDIMENTO E RISULTATI 248683 CI01.xxxx.091



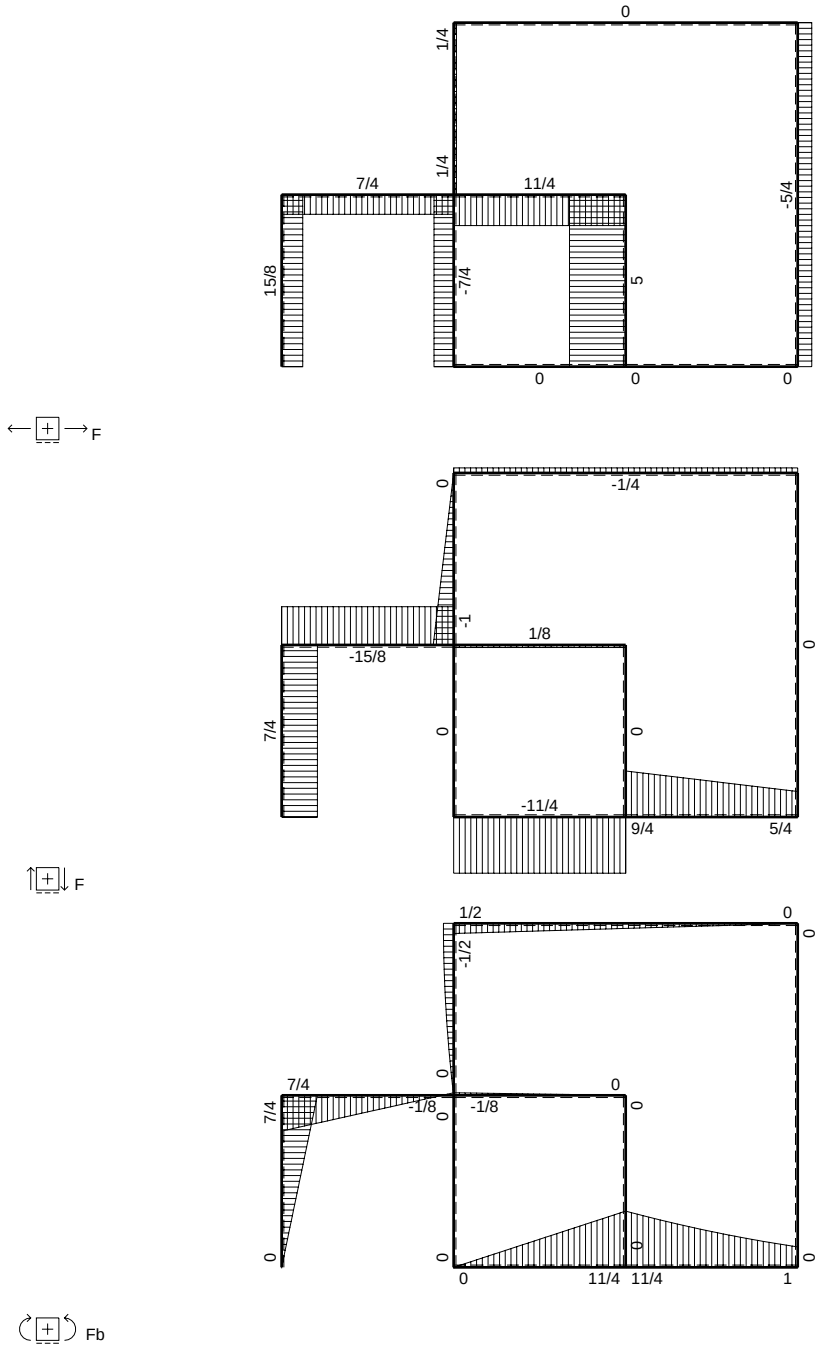

A = 828. $mm^2$
$J_u$ = 242396. $mm^4$
$J_v$ = 53136. $mm^4$
$y_g$ = 21.67 mm
$T_y$ = -3190. N
$M_x$ = 1563100. Nmm
$x_m$ = 30. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 32.33 mm
$\sigma_m = -Mv/J_u$ = -208.5 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 40. mm
$v_c$ = 18.33 mm
$\sigma_c = -Mv/J_u$ = -118.2 $N/mm^2$
$\tau_c$ = 4.666 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.5 $N/mm^2$
$S^*$ = 4255. $mm^3$

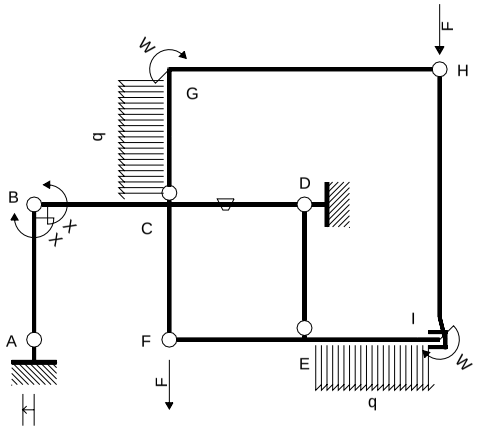
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
5/4F
B
A
15/8F
7/4Fb
7/4F
15/8F
7/4F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
1/8F
11/4F
11/4Fb
11/4F
11/4F
1/8Fb
1/8F
11/4F
C
F
1/2Fb
1/4F
G
C
F
7/4F
G
C
1/4F
1/2Fb
1/4F
F
C
15/8F
1/8Fb
7/4F
7/4F
7/4Fb
15/8F
B

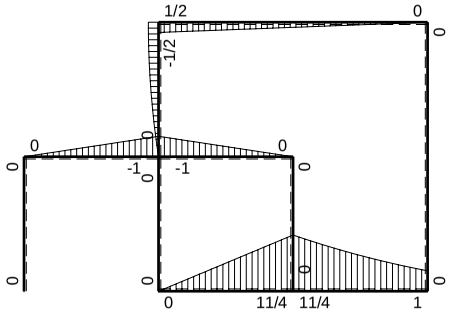




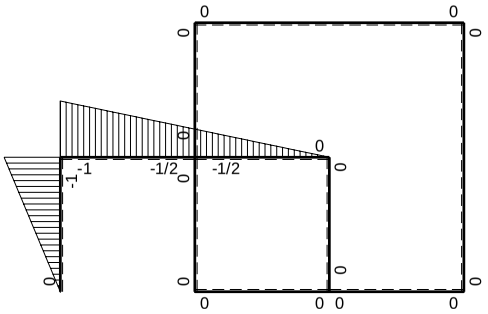

0
1/4
1/4
7/4
11/4
-5/4
15/8
-7/4
5
0
0
0
F
-1/4
0
-1
1/8
-15/8
7/4
0
0
0
-11/4
9/4
5/4
F
1/2
0
0
-1/2
0
7/4
0
7/4
-1/8
-1/8
0
0
0
0
0
0
11/4
11/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 266563 CI01.xxxx.092

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PROCEDIMENTO E RISULTATI 266563 CI01.xxxx.092

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $7/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

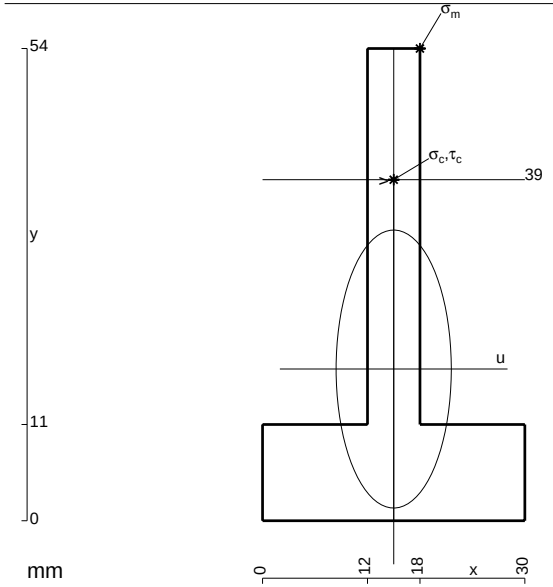
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

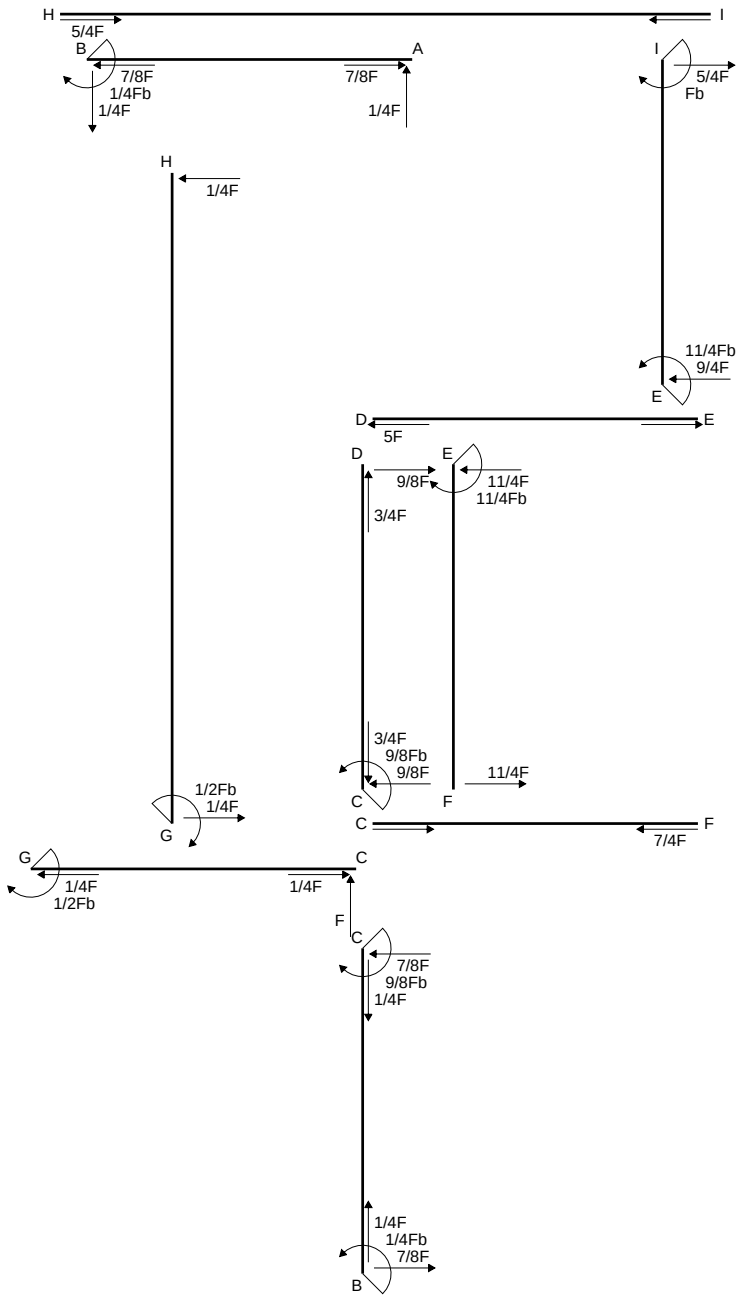


PROCEDIMENTO E RISULTATI 266563 CI01.xxxx.092



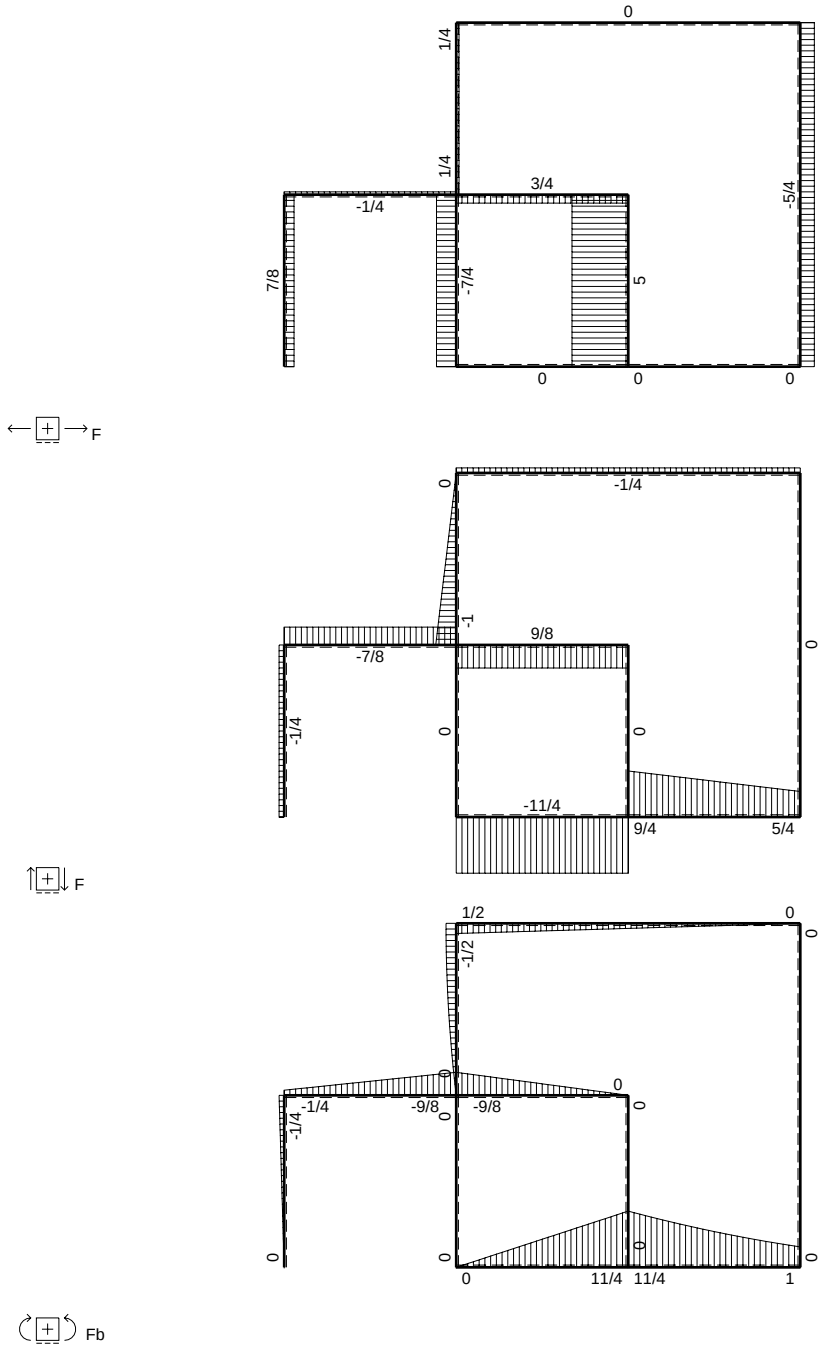

A = 588. $mm^2$
$J_u$ = 148637. $mm^4$
$J_v$ = 25524. $mm^4$
$y_g$ = 17.35 mm
$T_y$ = -1650. N
$M_x$ = 891000. Nmm
$x_m$ = 18. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 36.65 mm
$\sigma_m = -Mv/J_u$ = -219.7 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 39. mm
$v_c$ = 21.65 mm
$\sigma_c = -Mv/J_u$ = -129.8 $N/mm^2$
$\tau_c$ = 4.854 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.1 $N/mm^2$
$S^*$ = 2624. $mm^3$

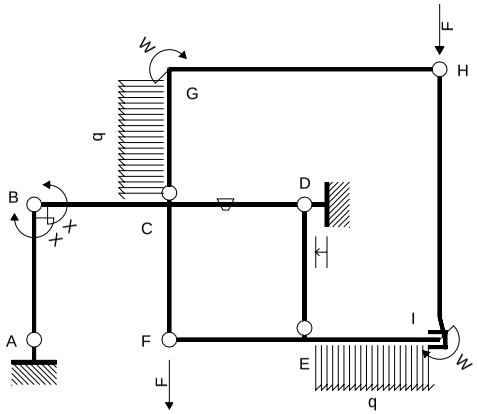
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H
I
5/4F
B
A
7/8F
1/4Fb
1/4F
7/8F
1/4F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
9/8F
11/4F
11/4Fb
3/4F
3/4F
9/8Fb
9/8F
11/4F
C
F
1/2Fb
1/4F
G
C
F
7/4F
G
C
1/4F
1/2Fb
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F
C
7/8F
9/8Fb
1/4F
1/4F
1/4Fb
7/8F
B

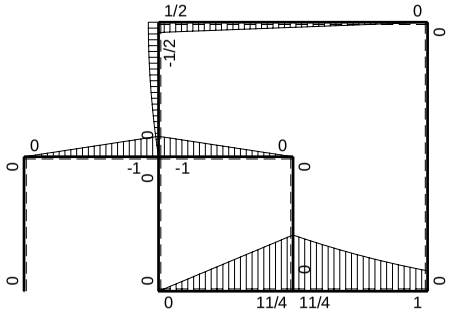




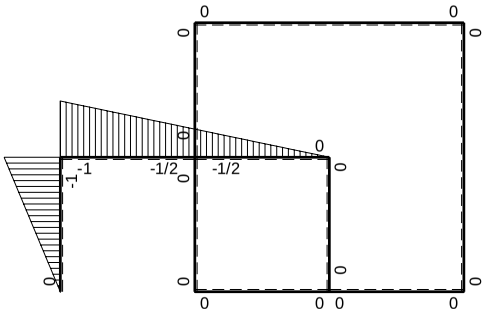

0
1/4
1/4
3/4
-1/4
-5/4
7/8
-7/4
5
0
0
0
F
-1/4
0
-1
9/8
-7/8
0
0
-1/4
0
-11/4
9/4
5/4
F
1/2
0
0
-1/2
0
0
-1/4
-1/4
-9/8
-9/8
0
0
0
0
0
0
0
11/4
11/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248282 CI01.xxxx.093

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PROCEDIMENTO E RISULTATI 248282 CI01.xxxx.093

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

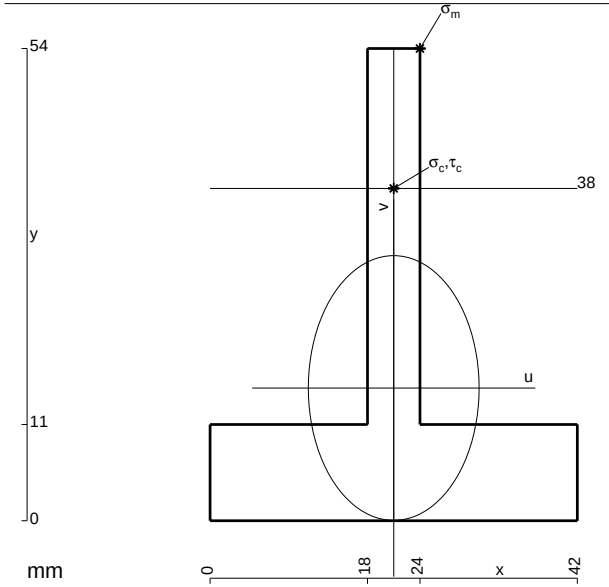
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

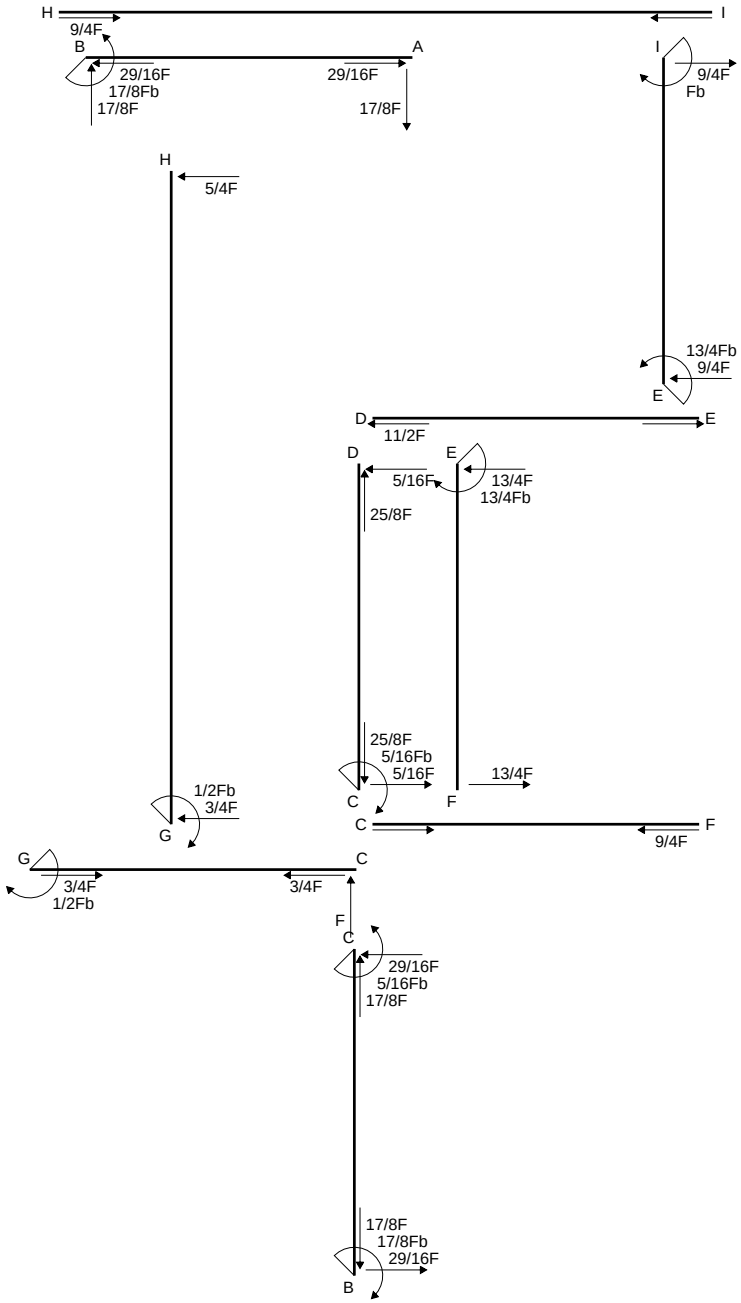


PROCEDIMENTO E RISULTATI 248282 CI01.xxxx.093



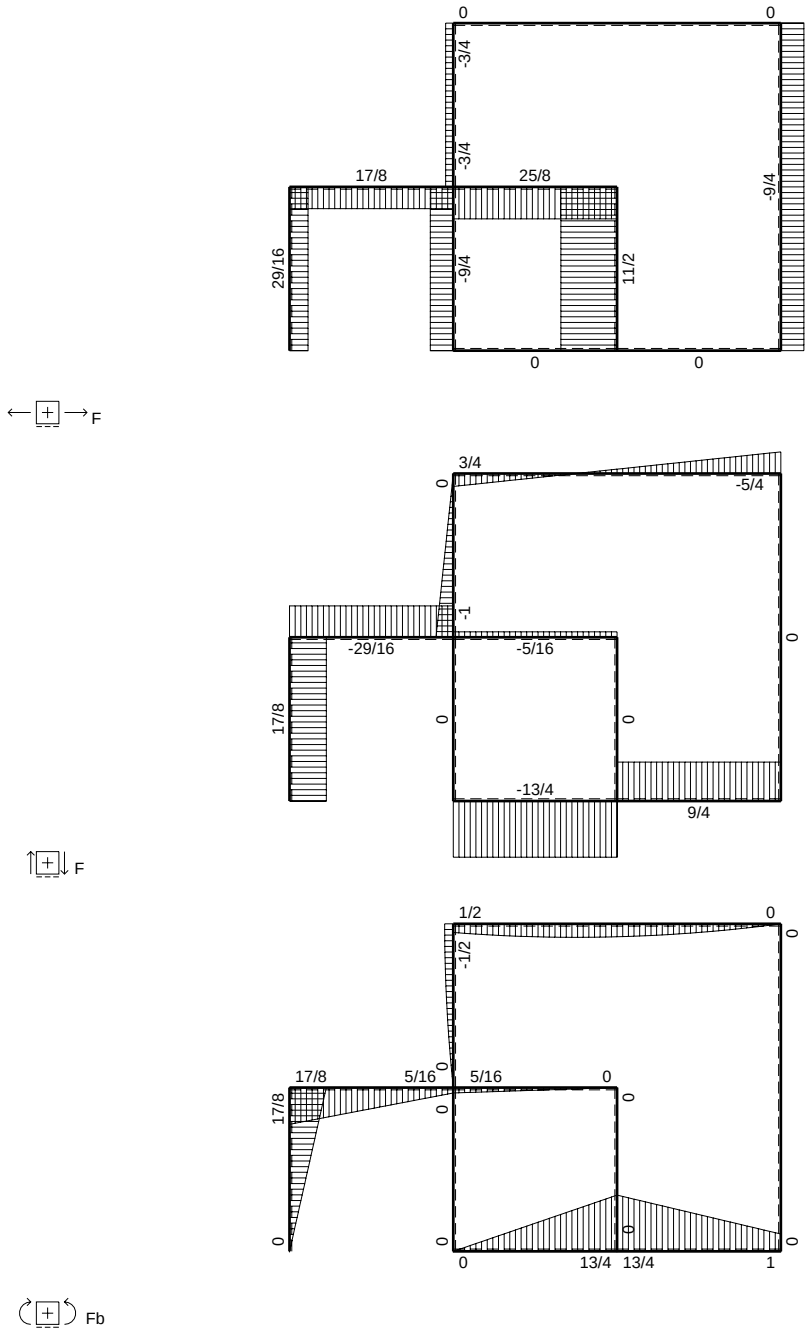

A = 720. $mm^2$
$J_u$ = 165098. $mm^4$
$J_v$ = 68688. $mm^4$
$y_g$ = 15.18 mm
$T_y$ = -1678. N
$M_x$ = 972950. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 38.83 mm
$\sigma_m = -Mv/J_u$ = -228.8 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 38. mm
$v_c$ = 22.83 mm
$\sigma_c = -Mv/J_u$ = -134.5 $N/mm^2$
$\tau_c$ = 5.011 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.8 $N/mm^2$
$S^*$ = 2959. $mm^3$

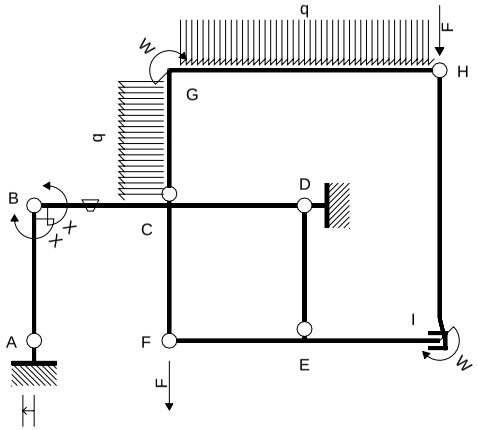
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H
I
9/4F
B
A
29/16F
17/8Fb
17/8F
29/16F
17/8F
I
9/4F
Fb
H
5/4F
13/4Fb
9/4F
E
D
E
11/2F
D
E
5/16F
13/4F
13/4Fb
25/8F
25/8F
5/16Fb
5/16F
13/4F
C
F
1/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
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F
C
29/16F
5/16Fb
17/8F
17/8F
17/8Fb
29/16F
B

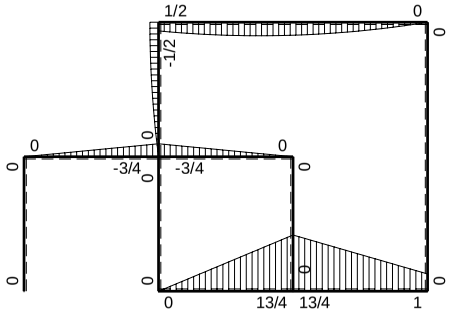




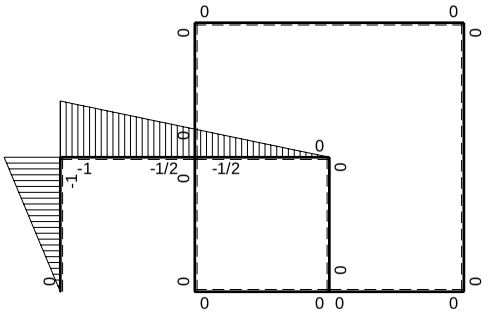

0
0
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-3/4
17/8
25/8
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29/16
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3/4
0
-5/4
-1
-29/16
-5/16
0
17/8
0
0
-13/4
9/4
F
1/2
0
0
-1/2
17/8
5/16
5/16
0
0
17/8
0
0
0
0
0
0
13/4
13/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 269001 CI01.xxxx.094

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PROCEDIMENTO E RISULTATI 269001 CI01.xxxx.094

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $1/2Fb+3/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+9/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $17/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -17/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

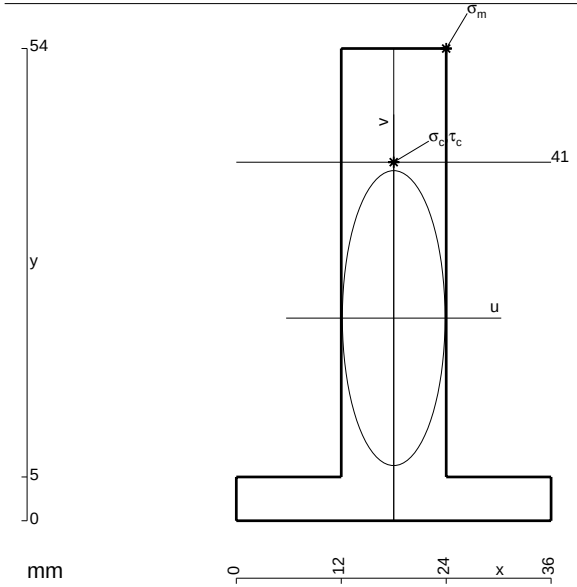
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

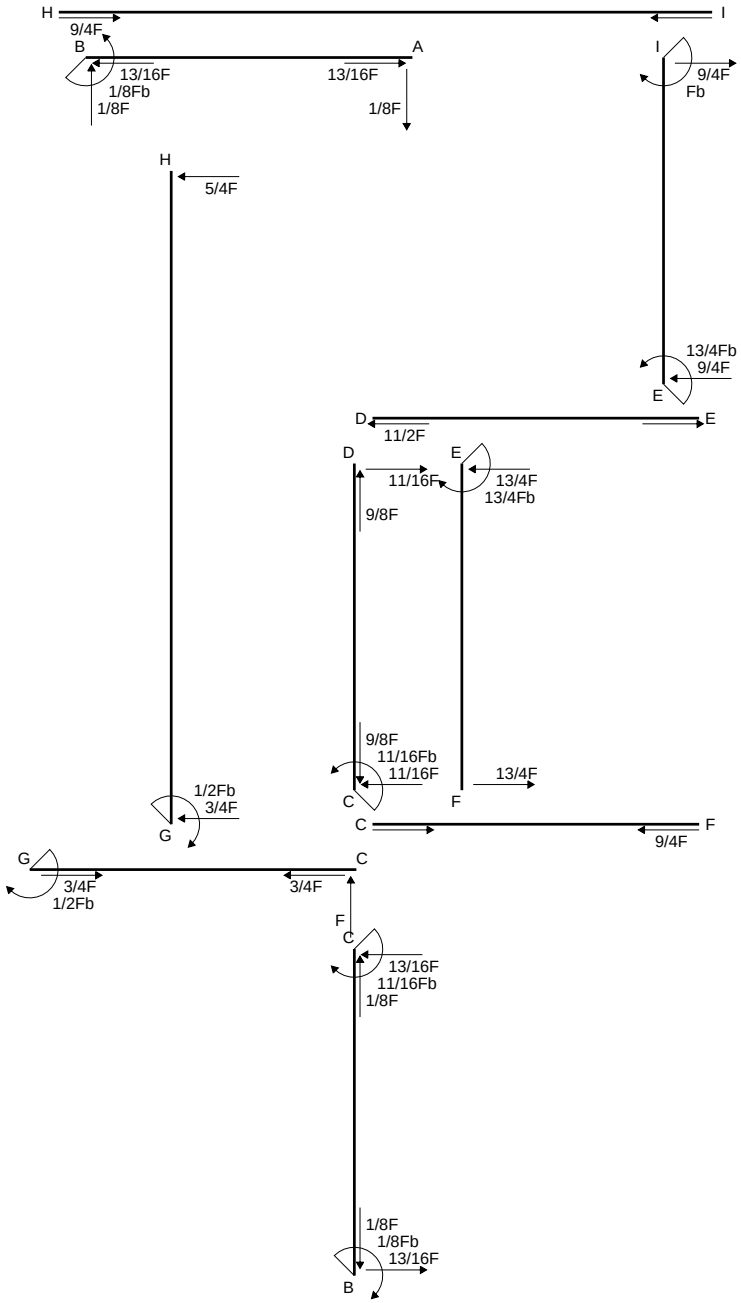
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





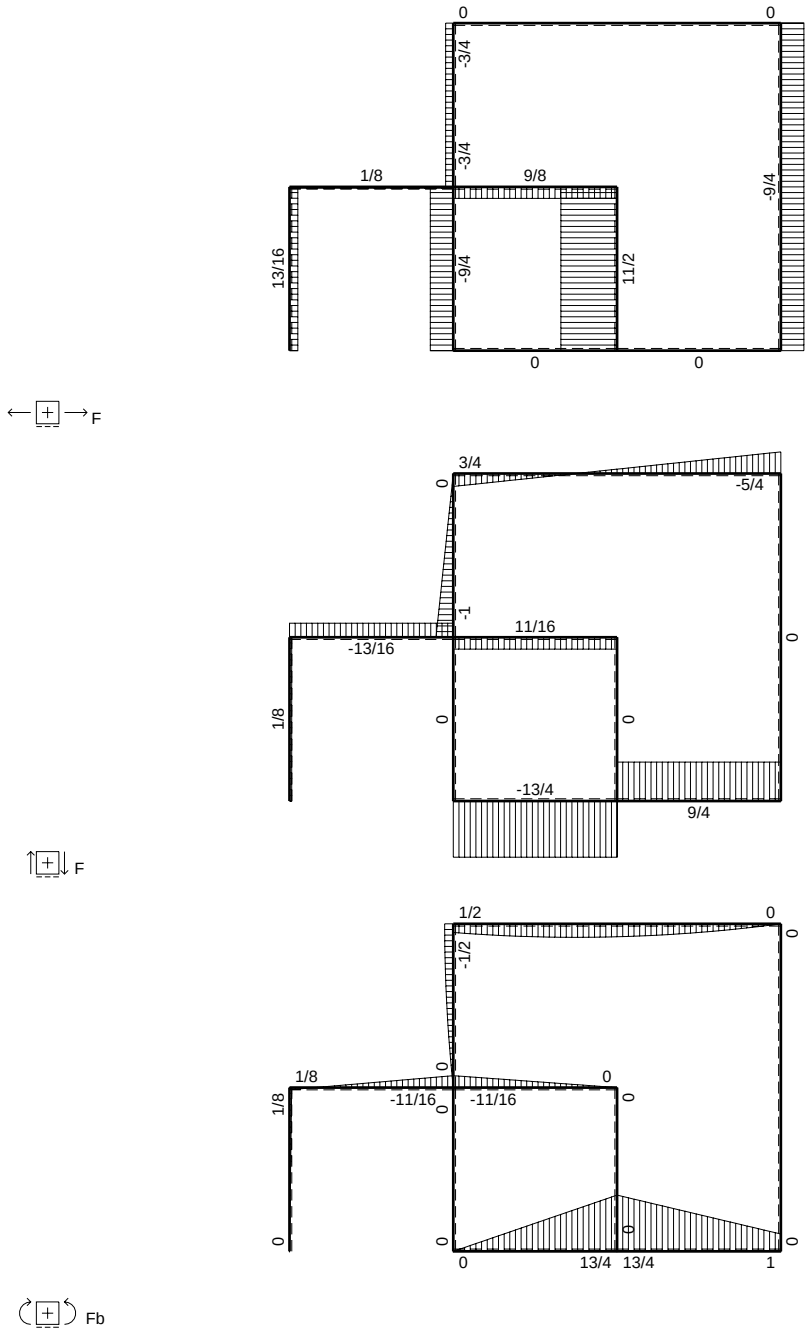


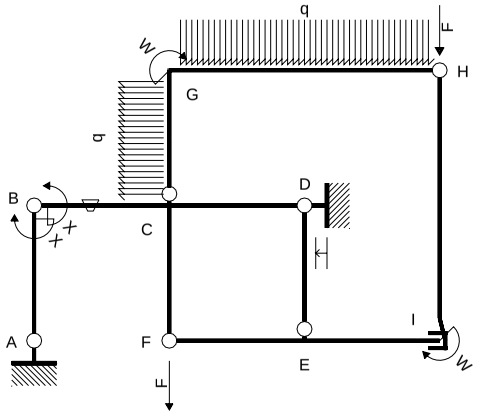

A = 768. $mm^2$
$J_u$ = 218489. $mm^4$
$J_v$ = 26496. $mm^4$
$y_g$ = 23.17 mm
$T_y$ = -2698. N
$M_x$ = 1699430. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 30.83 mm
$\sigma_m = -Mv/J_u$ = -239.8 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 41. mm
$v_c$ = 17.83 mm
$\sigma_c = -Mv/J_u$ = -138.7 $N/mm^2$
$\tau_c$ = 3.905 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.8 $N/mm^2$
$S^*$ = 3795. $mm^3$


H
I
9/4F
B
A
13/16F
1/8Fb
1/8F
13/16F
1/8F
I
9/4F
Fb
H
5/4F
13/4Fb
9/4F
E
D
E
11/2F
D
E
11/16F
13/4F
13/4Fb
9/8F
9/8F
11/16Fb
11/16F
C
F
13/4F
1/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
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F
C
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1/8F
1/8F
1/8Fb
13/16F
B

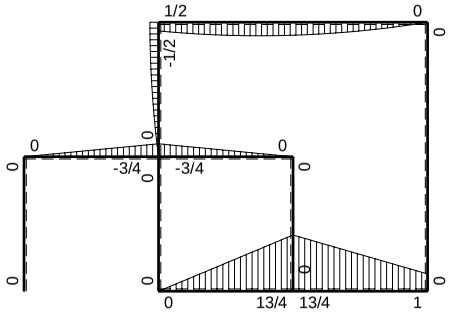




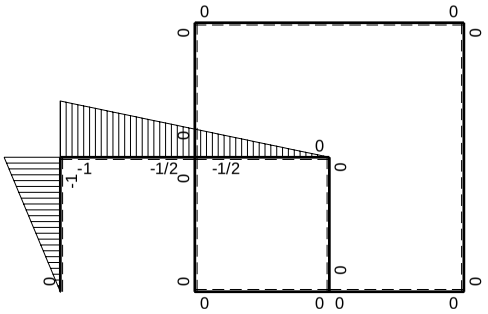

0
0
-3/4
-3/4
1/8
9/8
-9/4
13/16
-9/4
11/2
0
0
F
3/4
0
-5/4
-1
11/16
-13/16
0
1/8
0
0
-13/4
9/4
F
1/2
0
0
-1/2
0
1/8
0
1/8
-11/16
-11/16
0
0
0
0
0
0
13/4
13/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264038 CI01.xxxx.095

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $1/2Fb+3/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+9/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

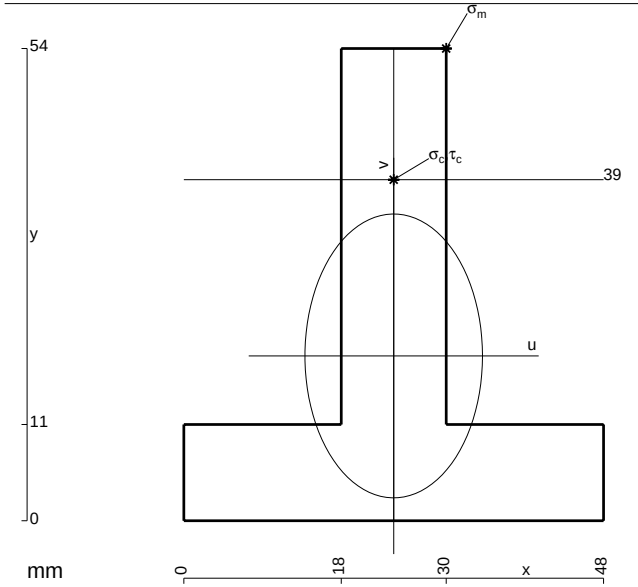
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

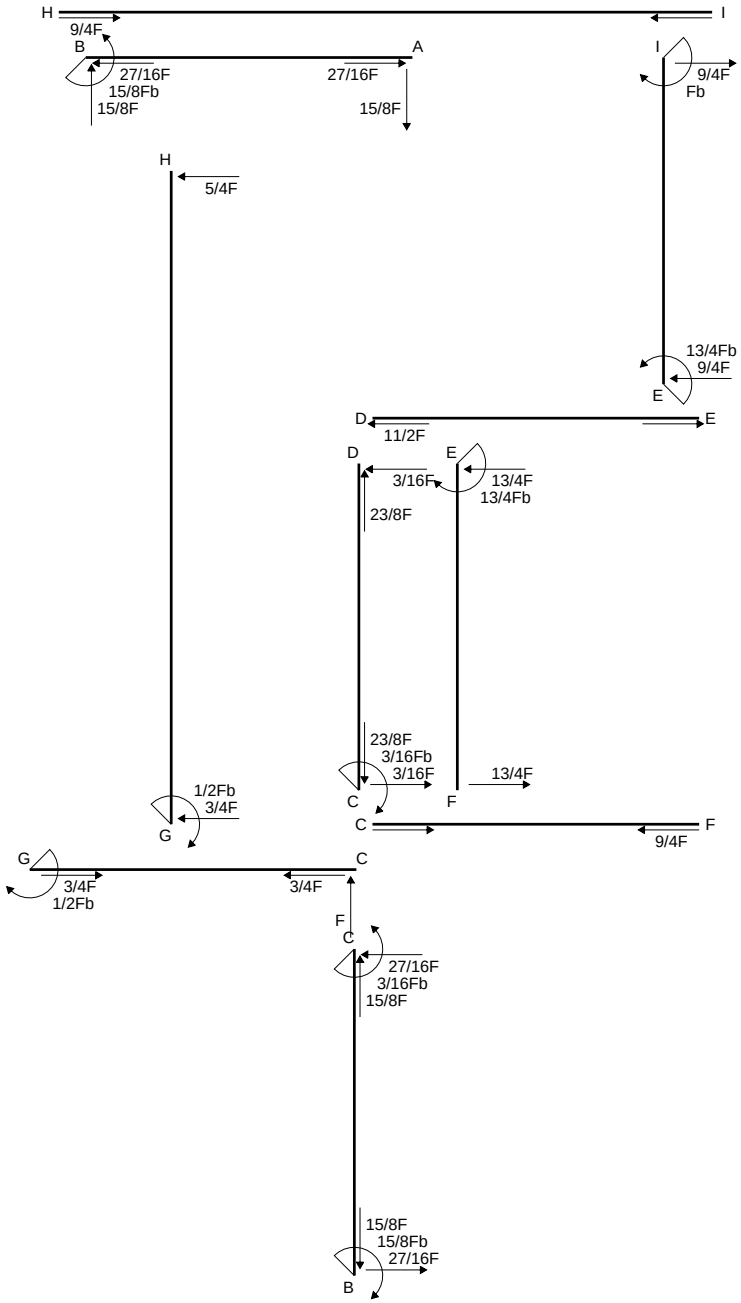


PROCEDIMENTO E RISULTATI 264038 CI01.xxxx.095



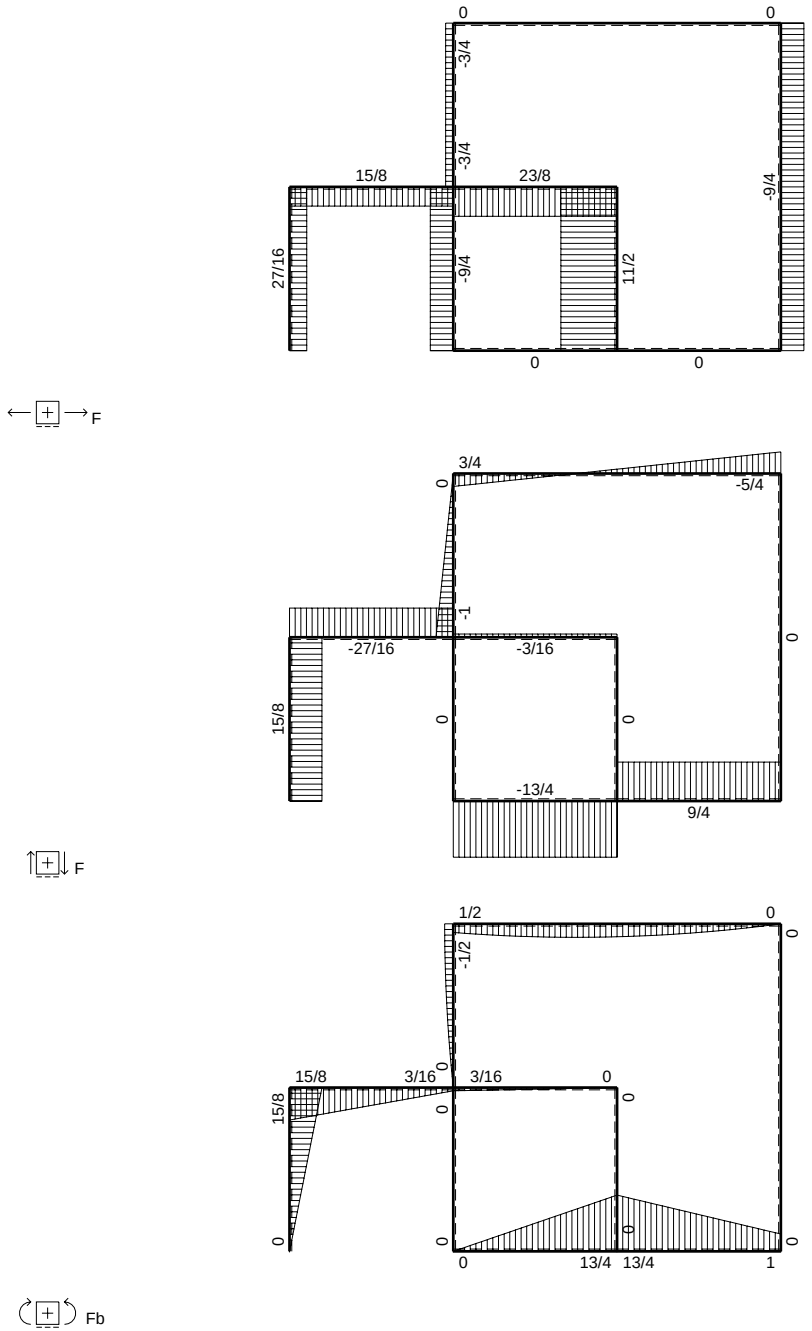

A = 1044. $mm^2$
$J_u$ = 275075. $mm^4$
$J_v$ = 107568. $mm^4$
$y_g$ = 18.84 mm
$T_y$ = -2308. N
$M_x$ = 1546030. Nmm
$x_m$ = 30. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 35.16 mm
$\sigma_m = -Mv/J_u$ = -197.6 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 39. mm
$v_c$ = 20.16 mm
$\sigma_c = -Mv/J_u$ = -113.3 $N/mm^2$
$\tau_c$ = 3.48 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.4 $N/mm^2$
$S^*$ = 4978. $mm^3$

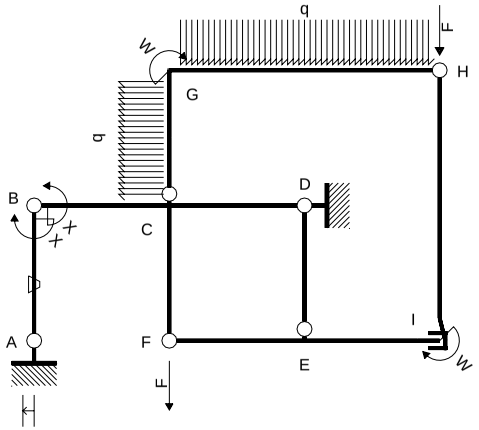
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H
I
9/4F
B
A
27/16F
15/8Fb
15/8F
27/16F
15/8F
I
9/4F
Fb
H
5/4F
13/4Fb
9/4F
E
D
E
11/2F
D
E
3/16F
13/4F
13/4Fb
23/8F
23/8F
3/16Fb
3/16F
13/4F
C
F
1/2Fb
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G
C
F
9/4F
G
C
3/4F
1/2Fb
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F
C
27/16F
3/16Fb
15/8F
15/8F
15/8Fb
27/16F
B

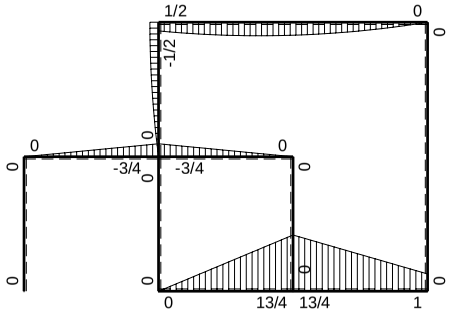




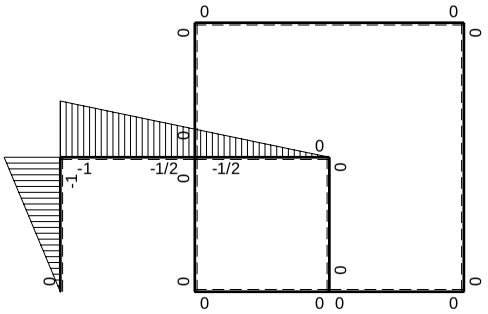

0
0
-3/4
-3/4
15/8
23/8
-9/4
27/16
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11/2
0
0
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3/4
0
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-1
-27/16
-3/16
0
15/8
0
0
-13/4
9/4
F
1/2
0
0
-1/2
0
15/8
3/16
3/16
0
15/8
0
0
0
0
0
0
13/4
13/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 266348 CI01.xxxx.096

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PROCEDIMENTO E RISULTATI 266348 CI01.xxxx.096

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-3/4Fx$ | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $3/4Fb-3/4Fx$ | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | $3/4Fx$ | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $1/2Fb+3/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+9/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $15/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -15/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

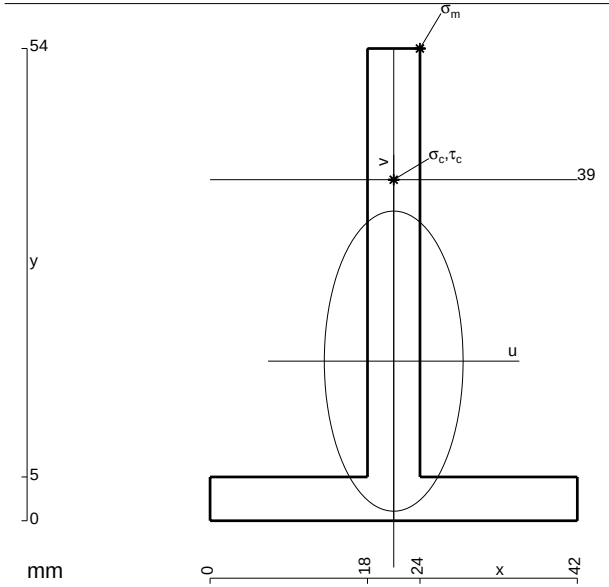
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

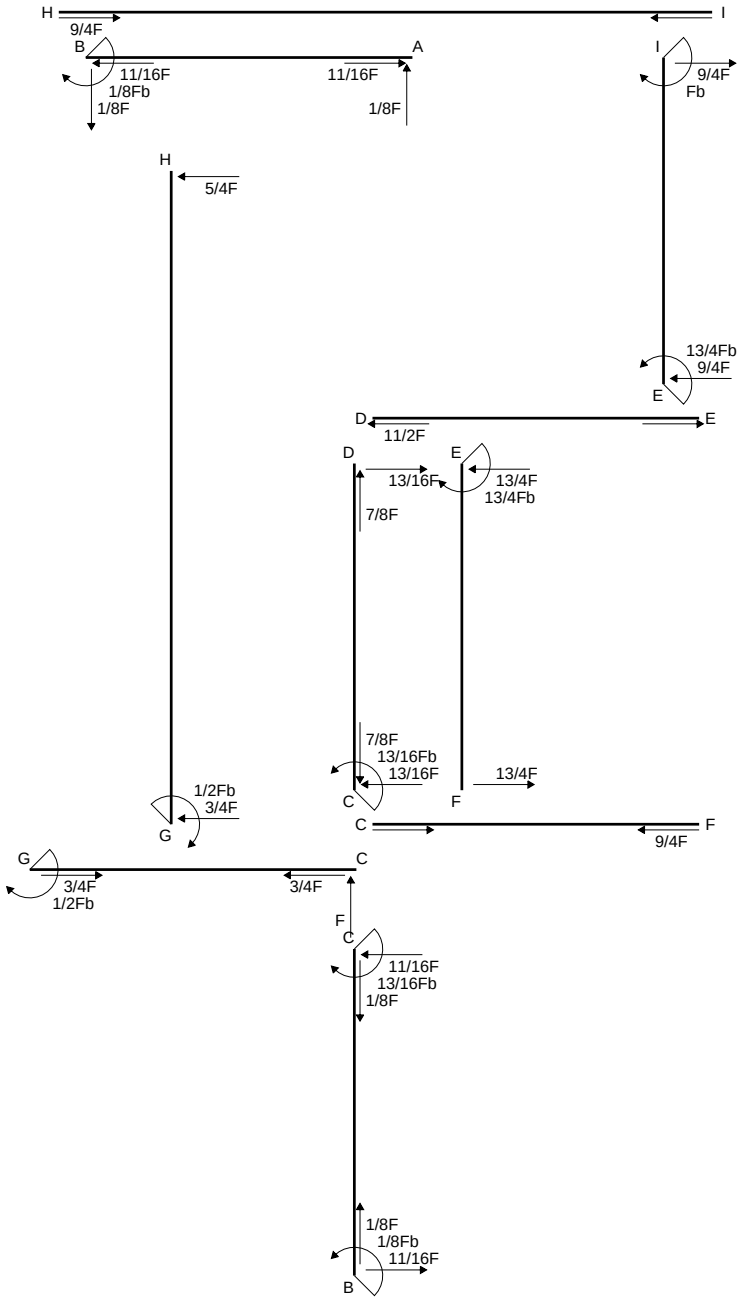
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





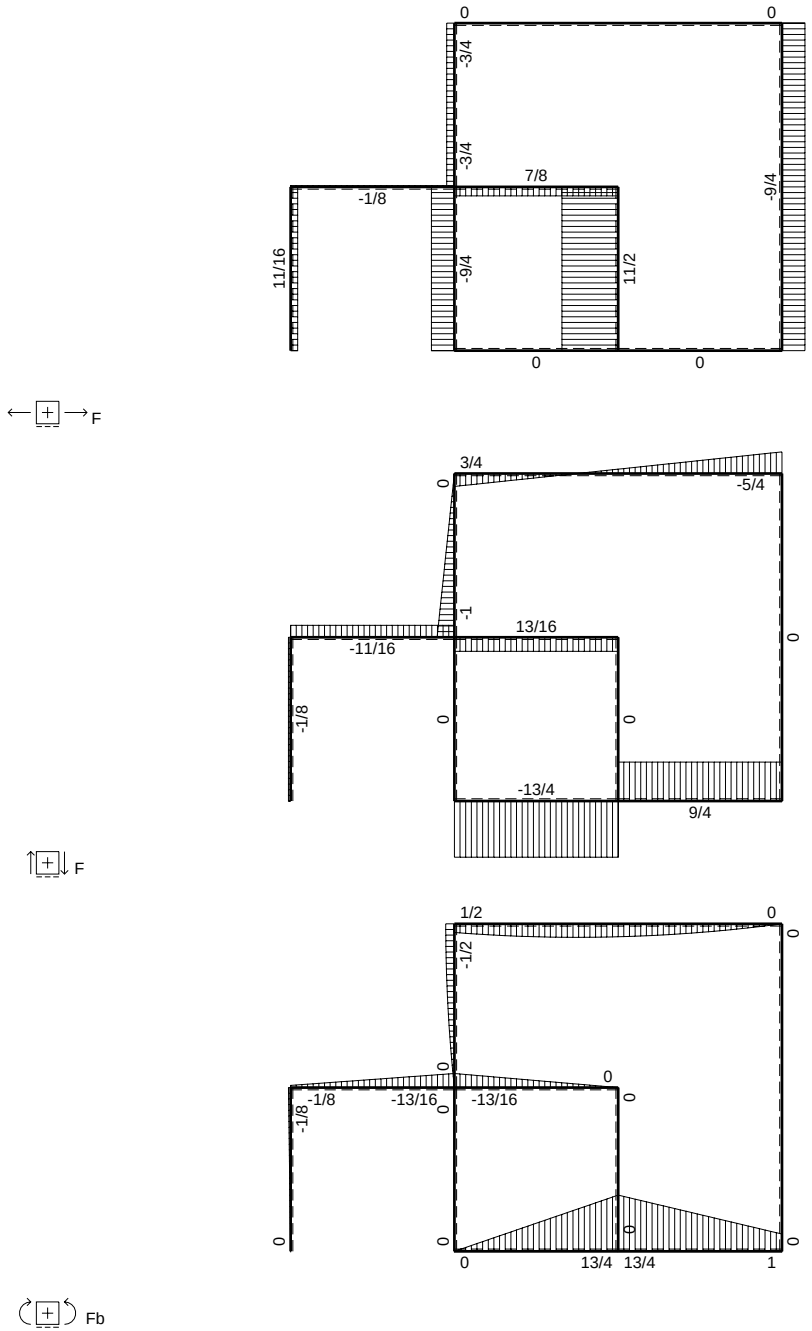


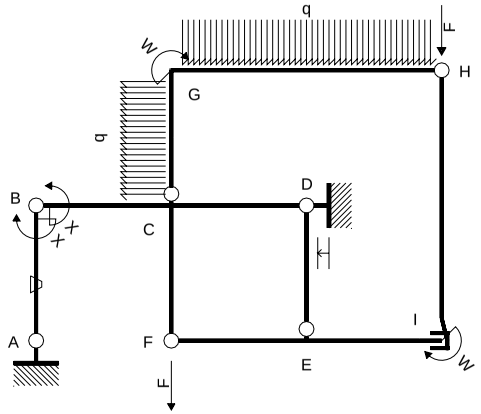

A = 504. $mm^2$
$J_u$ = 148565. $mm^4$
$J_v$ = 31752. $mm^4$
$y_g$ = 18.25 mm
$T_y$ = -1203. N
$M_x$ = 865800. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 35.75 mm
$\sigma_m = -Mv/J_u$ = -208.3 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 39. mm
$v_c$ = 20.75 mm
$\sigma_c = -Mv/J_u$ = -120.9 $N/mm^2$
$\tau_c$ = 3.43 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.1 $N/mm^2$
$S^*$ = 2543. $mm^3$


H
I
9/4F
B
A
11/16F
1/8Fb
1/8F
11/16F
1/8F
I
9/4F
Fb
H
5/4F
13/4Fb
9/4F
E
D
E
11/2F
D
E
13/16F
13/4F
13/4Fb
7/8F
7/8F
13/16Fb
13/16F
C
F
13/4F
1/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
1/2Fb
F
C
11/16F
13/16Fb
1/8F
1/8F
1/8Fb
11/16F
B

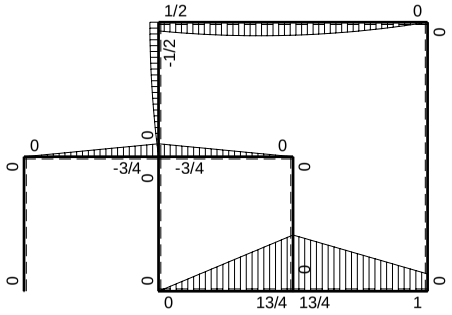




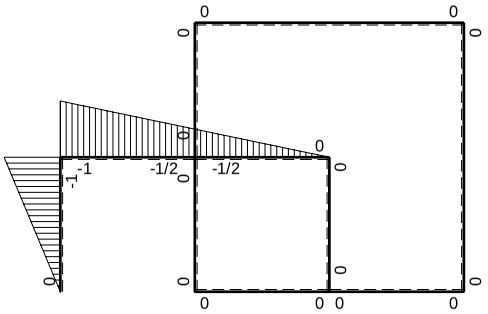

0
0
-3/4
-3/4
7/8
-1/8
-9/4
11/16
-9/4
11/2
0
0
F
3/4
0
-5/4
-1
13/16
-11/16
-1/8
0
0
-13/4
9/4
F
1/2
0
0
-1/2
0
0
-1/8
-13/16
-13/16
-1/8
0
0
0
0
0
13/4
13/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267428 CI01.xxxx.097

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | (0+1/2)$Fb^2$/EJ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | (1/4+0)$Fb^2$/EJ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | (1/8+0)$Fb^2$/EJ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $1/2Fb+3/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+9/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2$/EJ | |
| | totali | | | | | | $-1/8Fb^2$/EJ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

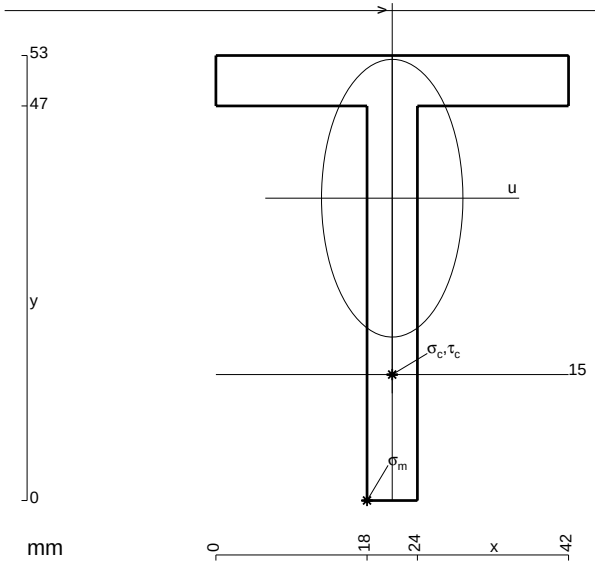
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

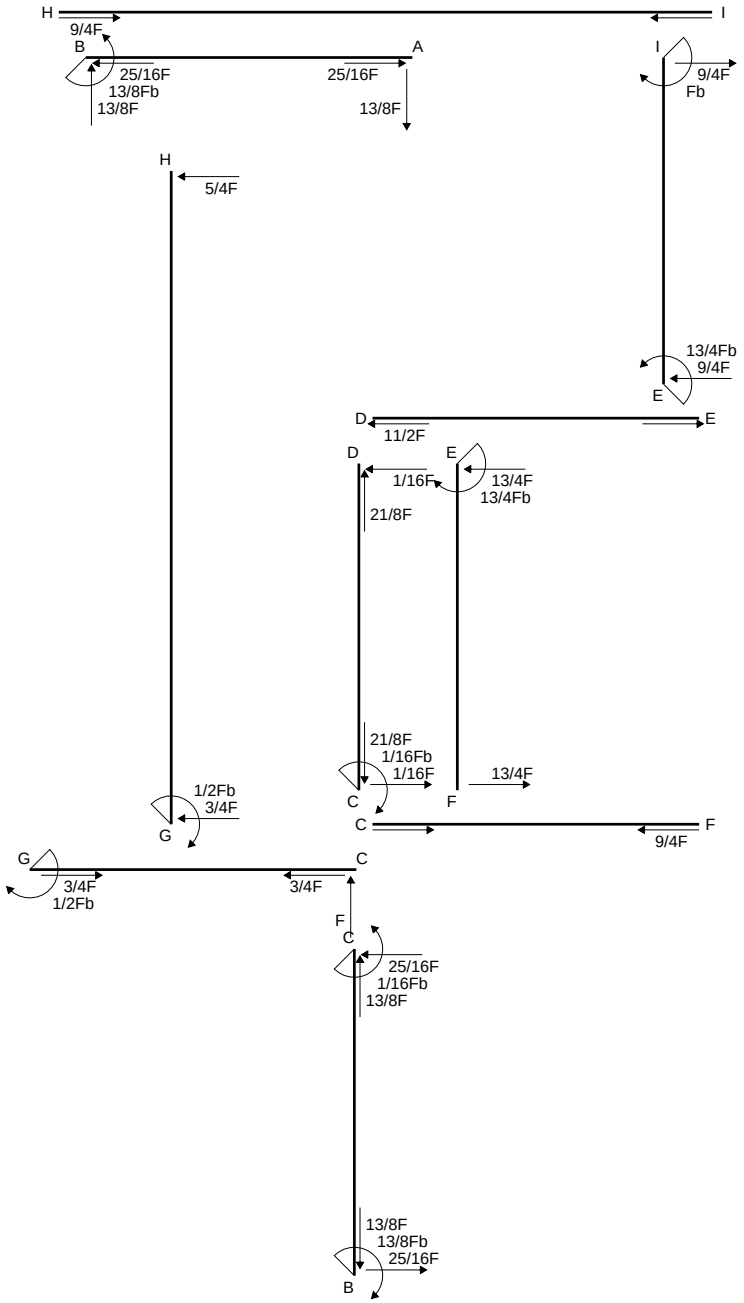
$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





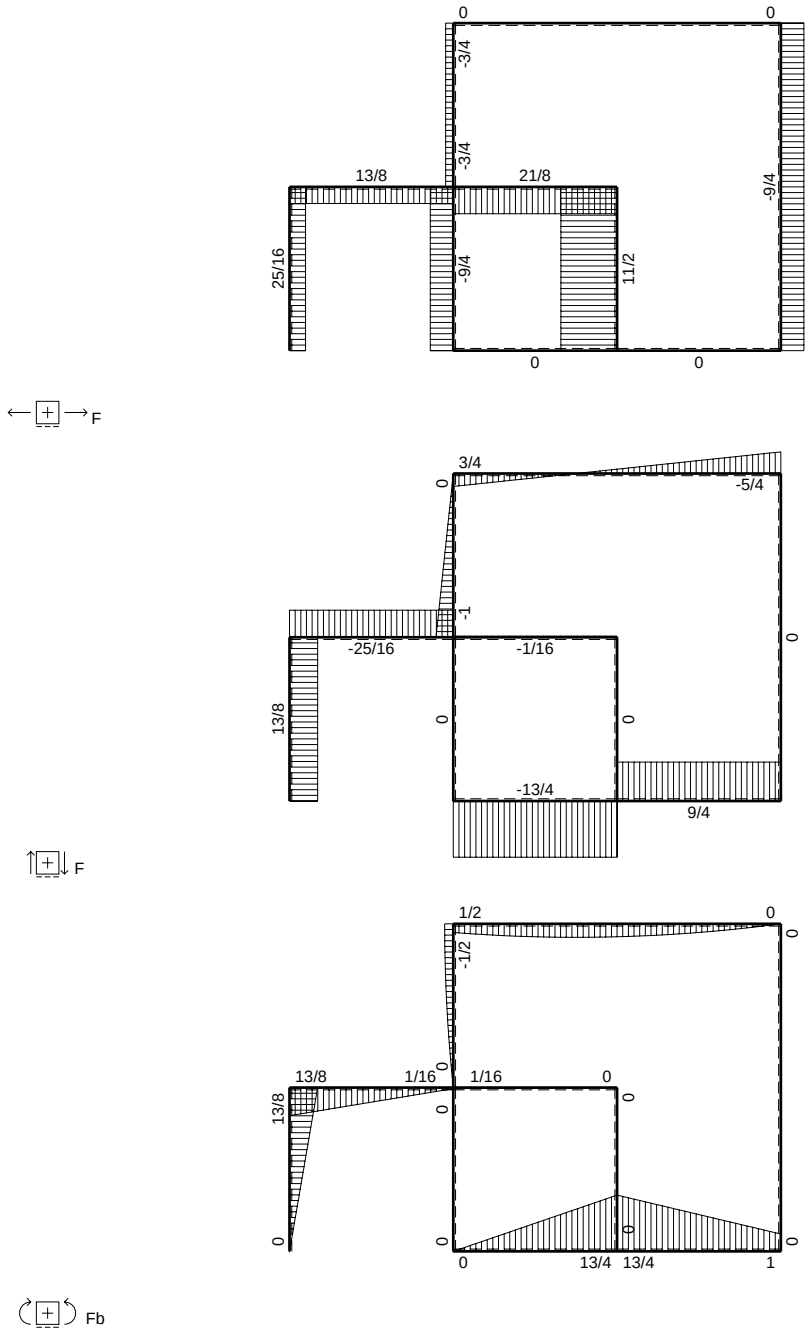


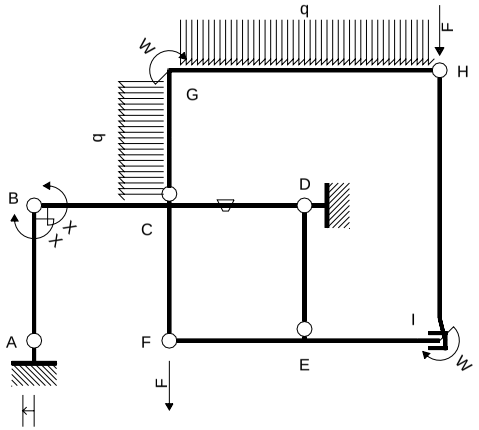

A = 534. $mm^2$
$J_u$ = 146122. $mm^4$
$J_v$ = 37890. $mm^4$
$y_g$ = 36.01 mm
$T_y$ = -1203. N
$M_x$ = 889850. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -36.01 mm
$\sigma_m = -Mv/J_u$ = 219.3 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 15. mm
$v_c$ = -21.01 mm
$\sigma_c = -Mv/J_u$ = 127.9 $N/mm^2$
$\tau_c$ = 3.519 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 128.1 $N/mm^2$
$S^*$ = 2566. $mm^3$


H
I
9/4F
B
A
25/16F
13/8Fb
13/8F
25/16F
13/8F
I
9/4F
Fb
H
5/4F
13/4Fb
9/4F
E
D
E
11/2F
D
E
1/16F
13/4F
13/4Fb
21/8F
21/8F
1/16Fb
1/16F
13/4F
C
F
1/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
1/2Fb
F
C
25/16F
1/16Fb
13/8F
13/8F
13/8Fb
25/16F
B

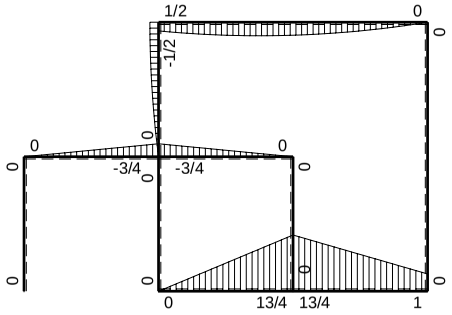




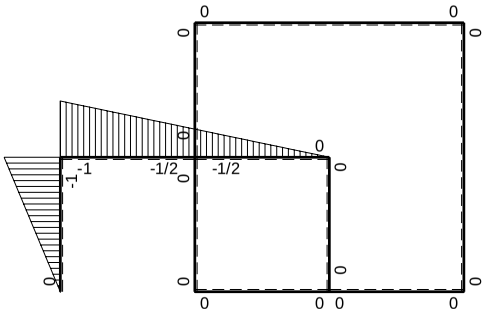

0
0
-3/4
-3/4
13/8
21/8
-9/4
25/16
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11/2
0
0
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3/4
0
-5/4
-1
-25/16
-1/16
13/8
0
0
0
-13/4
9/4
F
1/2
0
0
-1/2
13/8
1/16
1/16
0
13/8
0
0
0
0
0
0
13/4
13/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



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PROCEDIMENTO E RISULTATI 264872 CI01.xxxx.098

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $1/2Fb+3/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+9/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

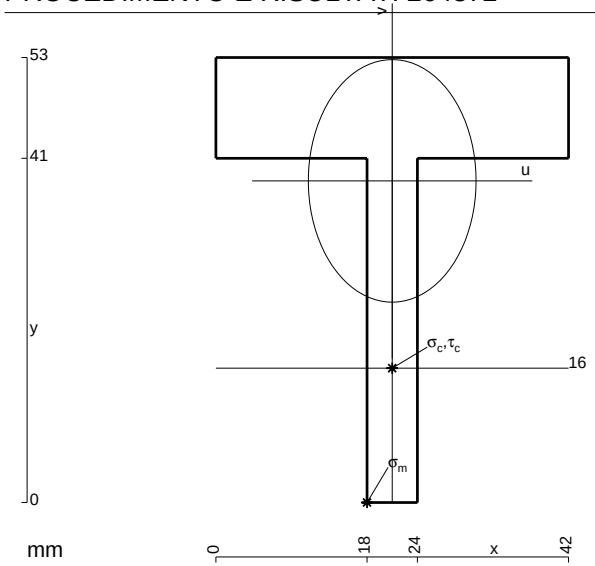
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

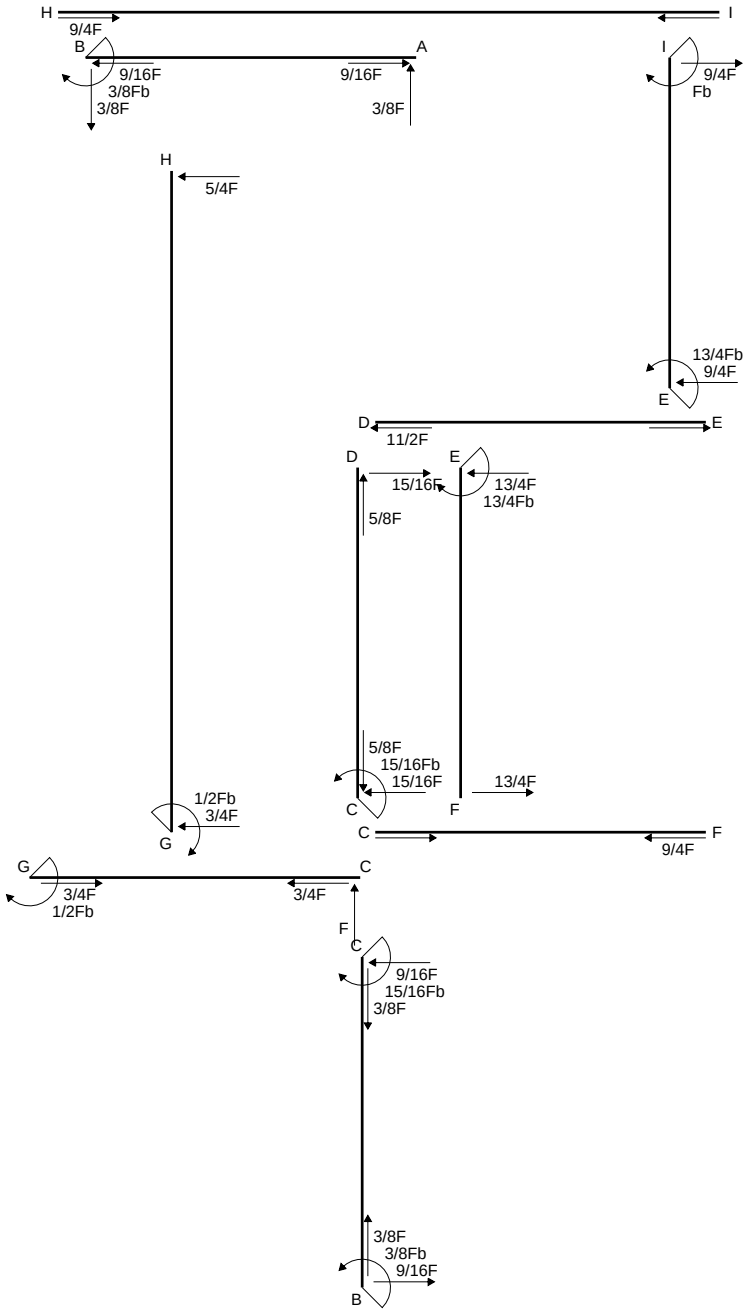


PROCEDIMENTO E RISULTATI 264872 CI01.xxxx.098



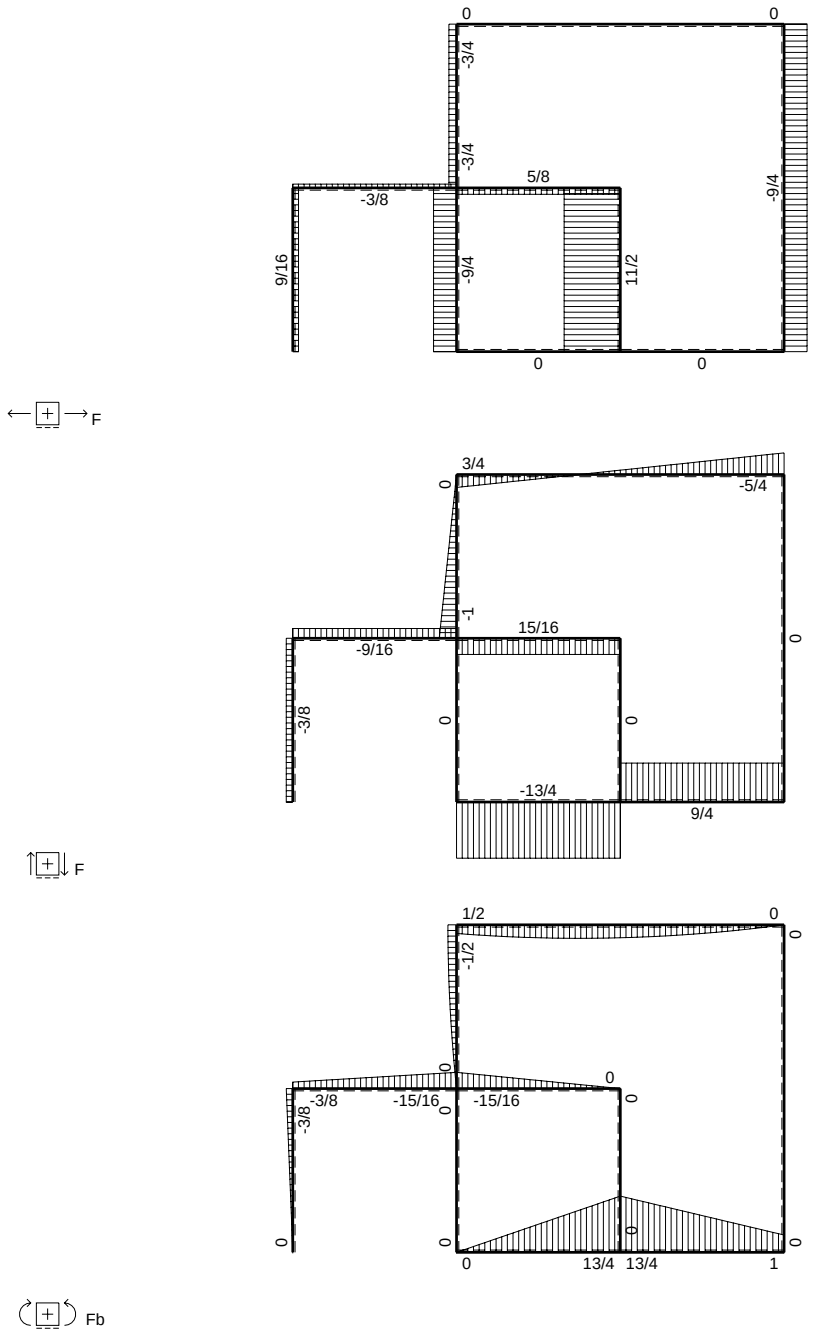

A = 750. $mm^2$
$J_u$ = 156599. $mm^4$
$J_v$ = 74826. $mm^4$
$y_g$ = 38.31 mm
$T_y$ = -1170. N
$M_x$ = 924300. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -38.31 mm
$\sigma_m = -Mv/J_u$ = 226.1 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -22.31 mm
$\sigma_c = -Mv/J_u$ = 131.7 $N/mm^2$
$\tau_c$ = 3.623 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 131.8 $N/mm^2$
$S^*$ = 2910. $mm^3$

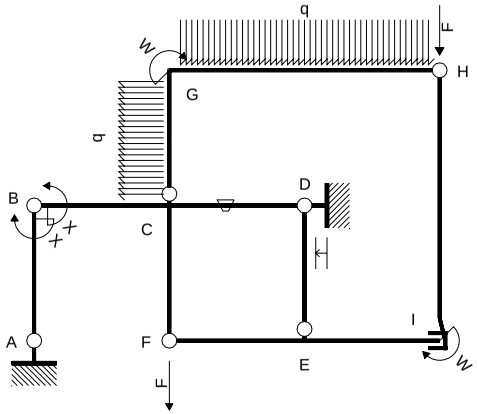
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H
I
9/4F
B
A
9/16F
3/8Fb
3/8F
9/16F
3/8F
I
9/4F
Fb
H
5/4F
13/4Fb
9/4F
E
D
E
11/2F
D
E
15/16F
13/4F
13/4Fb
5/8F
5/8F
15/16Fb
15/16F
13/4F
C
F
1/2Fb
3/4F
G
C
F
9/4F
G
C
3/4F
3/4F
1/2Fb
F
C
9/16F
15/16Fb
3/8F
3/8F
3/8Fb
9/16F
B

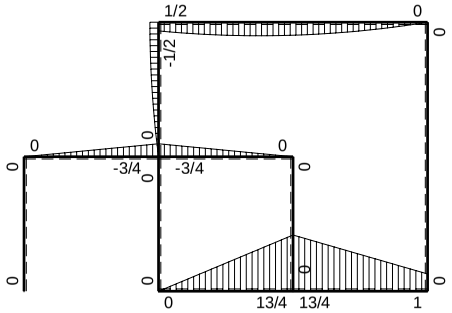




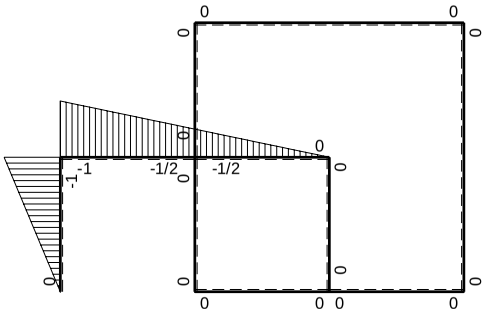

0
0
-3/4
-3/4
5/8
-3/8
-9/4
9/16
-9/4
11/2
0
0
F
3/4
0
-5/4
-1
15/16
-9/16
-3/8
0
0
-13/4
9/4
F
1/2
0
0
-1/2
0
0
-3/8
-3/8
-15/16
-15/16
0
0
0
0
0
0
13/4
13/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264047 CI01.xxxx.099

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PROCEDIMENTO E RISULTATI 264047 CI01.xxxx.099

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 13/4Fb-13/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -13/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $1/2Fb+3/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -13/4Fb+9/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-3/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 3/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

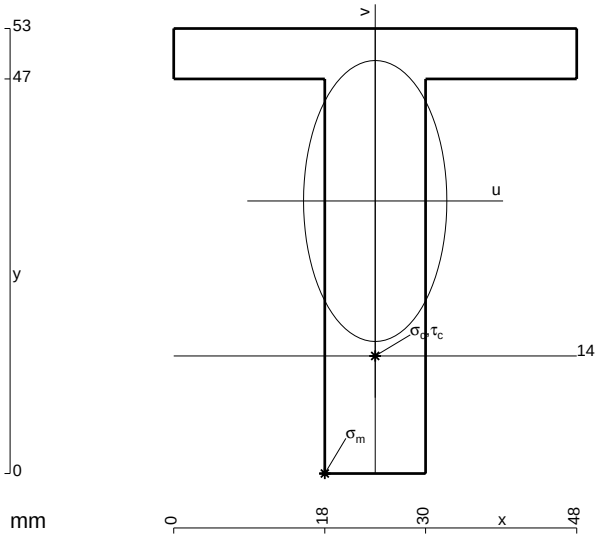
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

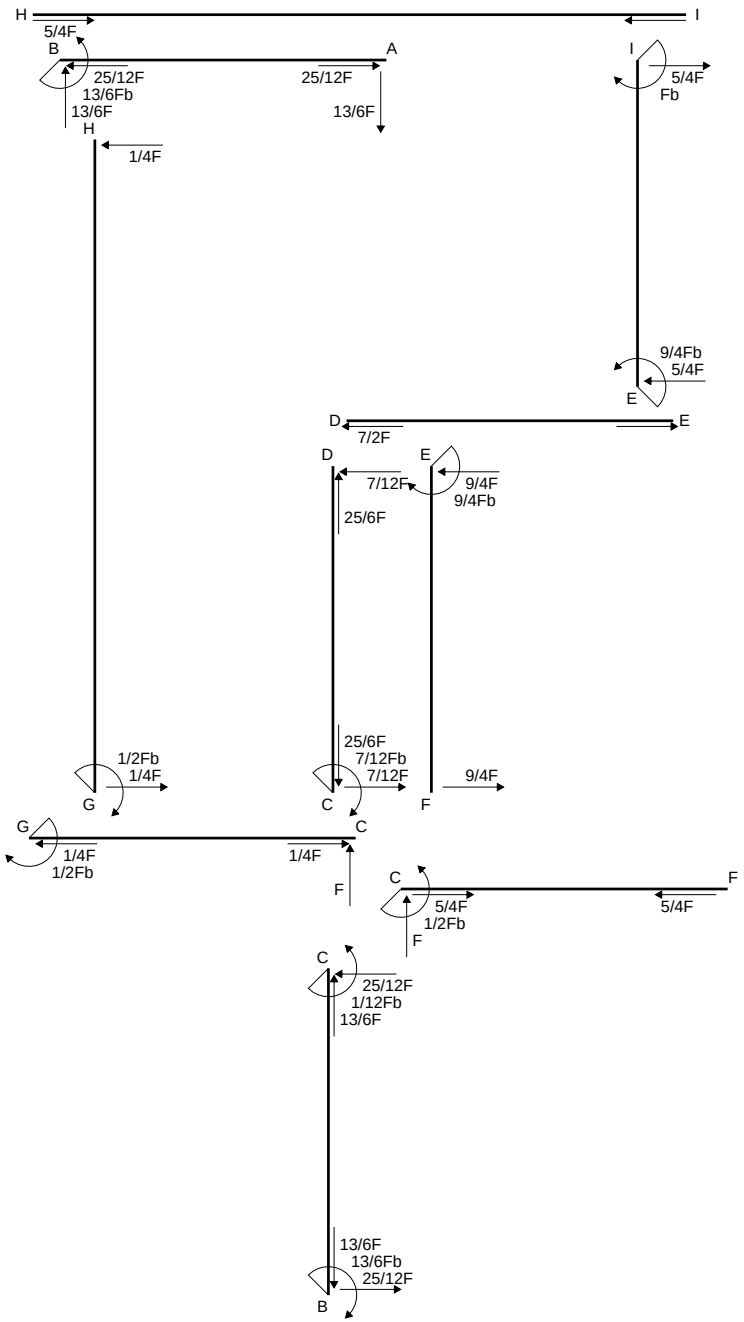
$= (1/8\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$





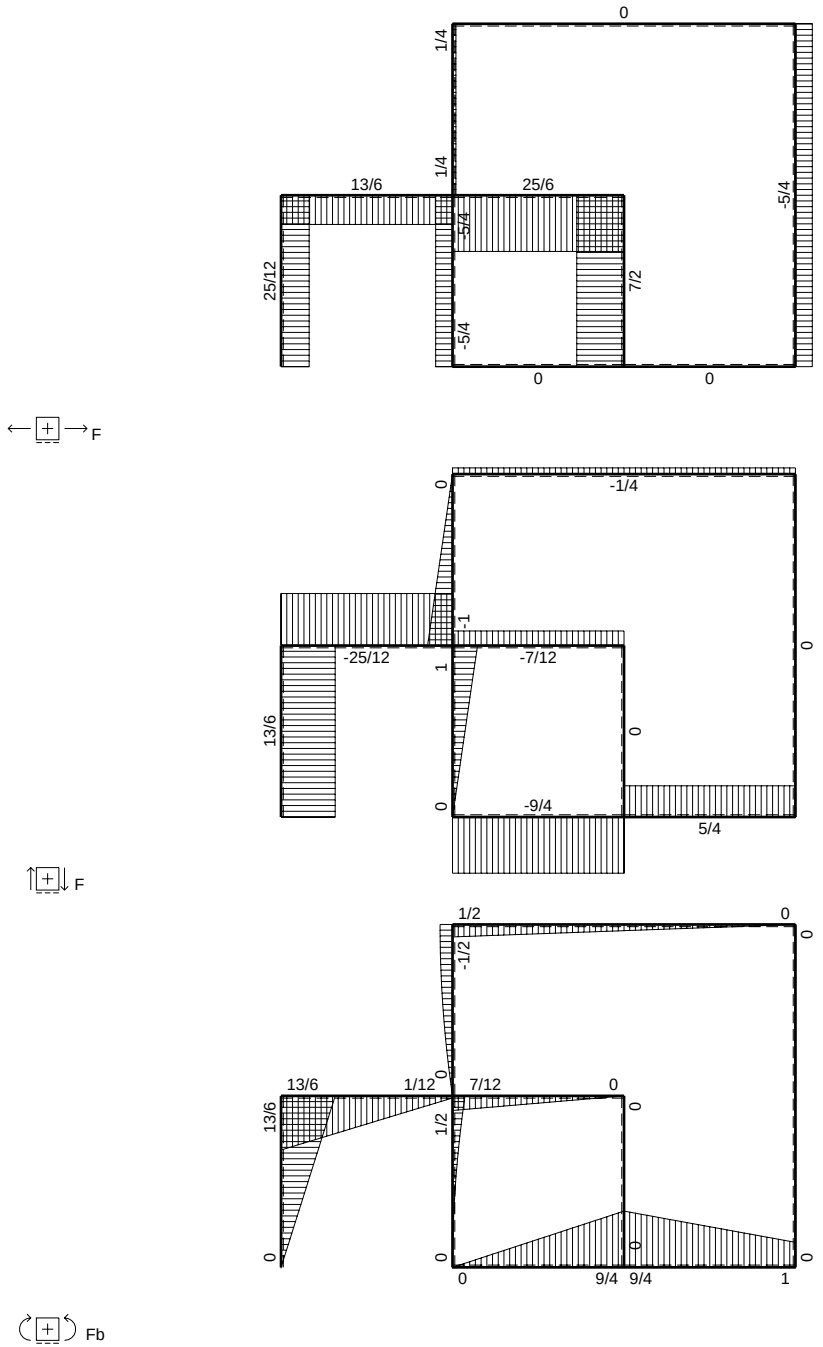


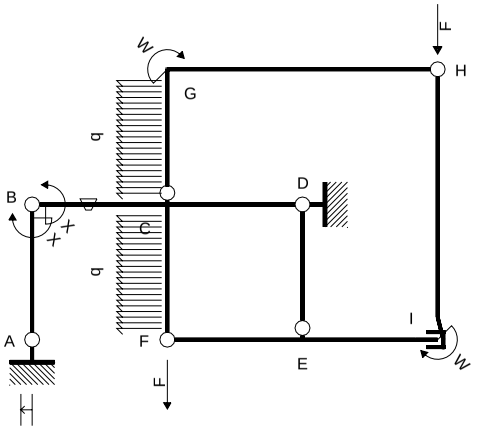

A = 852. $mm^2$
$J_u$ = 238570. $mm^4$
$J_v$ = 62064. $mm^4$
$y_g$ = 32.46 mm
$T_y$ = -2113. N
$M_x$ = 1753380. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -32.46 mm
$\sigma_m = -Mv/J_u$ = 238.5 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 14. mm
$v_c$ = -18.46 mm
$\sigma_c = -Mv/J_u$ = 135.7 $N/mm^2$
$\tau_c$ = 3.156 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.8 $N/mm^2$
$S^*$ = 4277. $mm^3$


H
I
5/4F
B
A
25/12F
13/6Fb
13/6F
25/12F
13/6F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
7/12F
9/4F
9/4Fb
25/6F
1/2Fb
1/4F
G
25/6F
7/12Fb
7/12F
C
9/4F
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
5/4F
1/2Fb
F
C
25/12F
1/12Fb
13/6F
13/6F
13/6Fb
25/12F
B

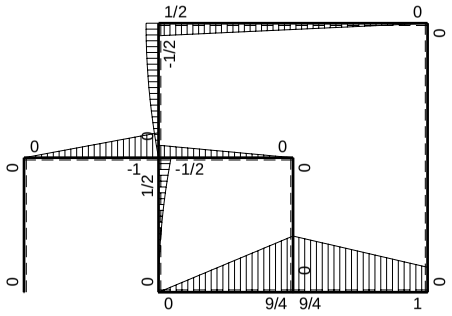




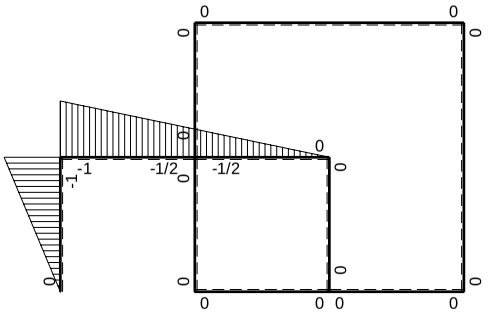

F
0
1/4
1/4
13/6
25/6
-5/4
-5/4
25/12
7/2
-5/4
0
0
F
0
-1/4
-1
-25/12
1
-7/12
13/6
0
0
-9/4
5/4
F
1/2
0
0
-1/2
0
13/6
1/12
7/12
0
13/6
1/2
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248062 CI01.xxxx.100

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 248062 CI01.xxxx.100
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

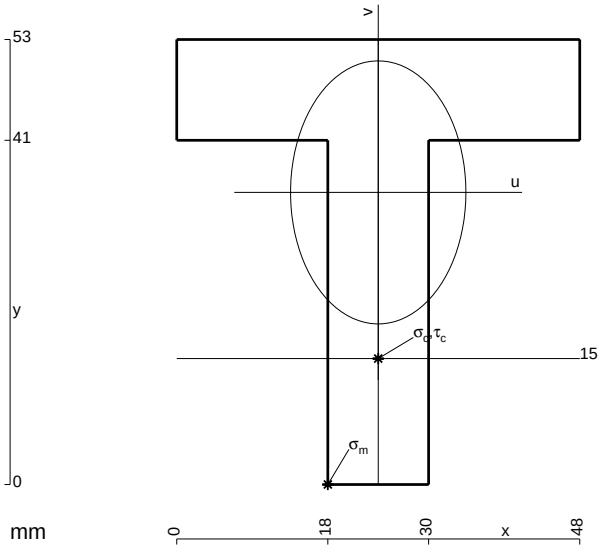
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

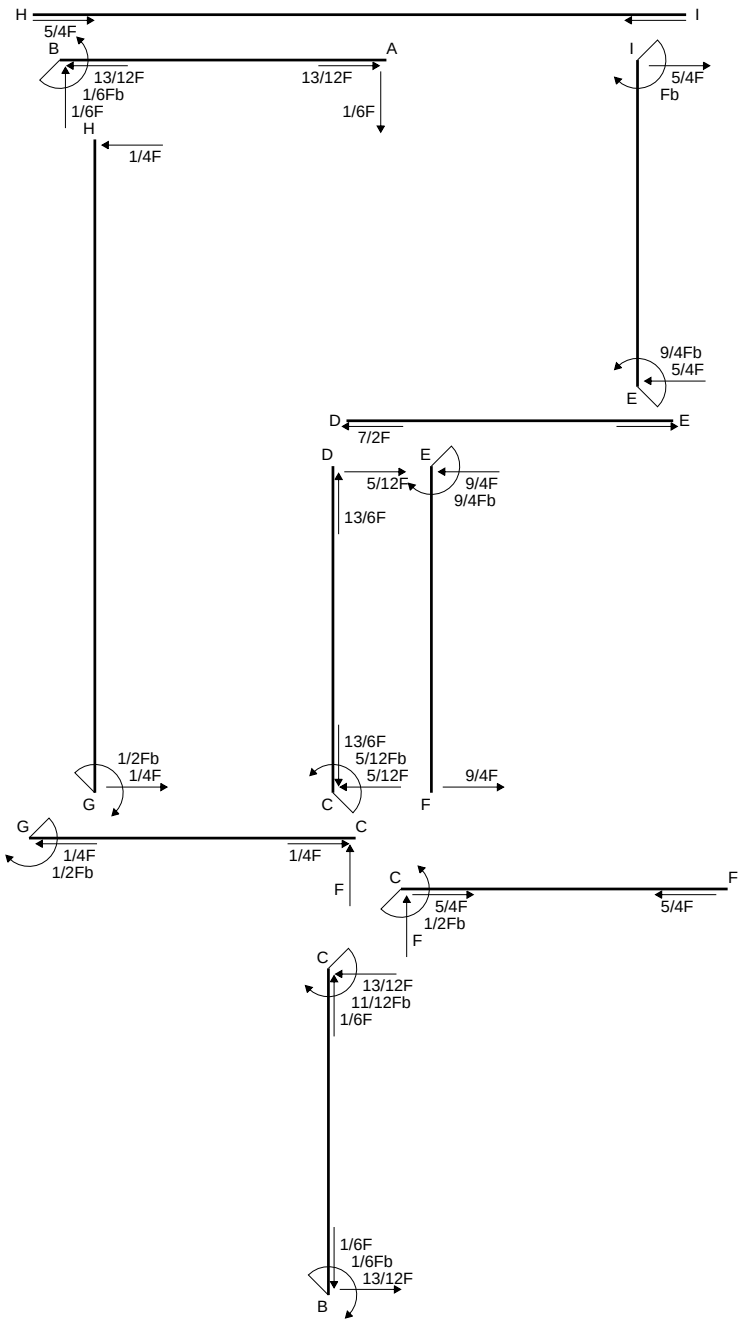
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





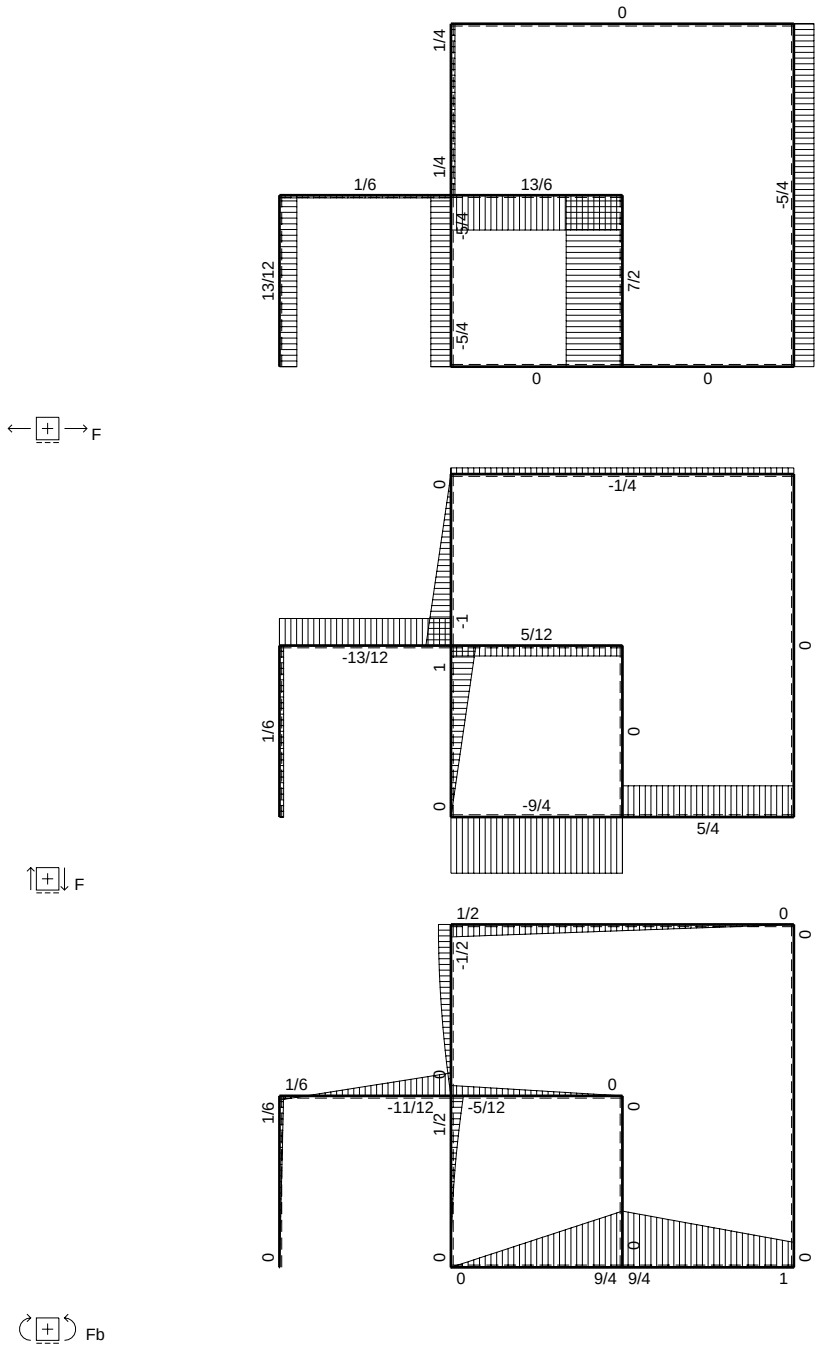


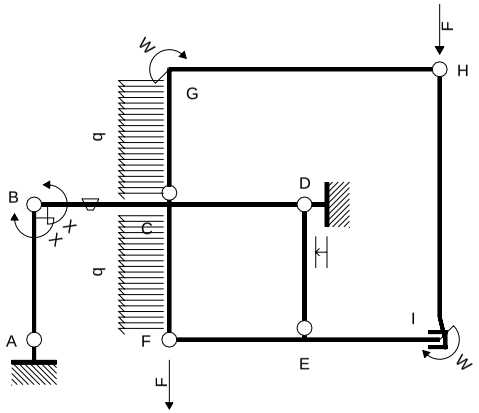

A = 1068. $mm^2$
$J_u$ = 262174. $mm^4$
$J_v$ = 116496. $mm^4$
$y_g$ = 34.79 mm
$T_y$ = -3420. N
$M_x$ = 1504800. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -34.79 mm
$\sigma_m = -Mv/J_u$ = 199.7 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -19.79 mm
$\sigma_c = -Mv/J_u$ = 113.6 $N/mm^2$
$\tau_c$ = 5.34 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114. $N/mm^2$
$S^*$ = 4913. $mm^3$


H
I
5/4F
B
A
13/12F
1/6Fb
1/6F
13/12F
1/6F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
5/12F
9/4F
9/4Fb
13/6F
1/2Fb
1/4F
G
13/6F
5/12Fb
5/12F
C
9/4F
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
5/4F
1/2Fb
F
C
13/12F
11/12Fb
1/6F
1/6F
1/6Fb
13/12F
B

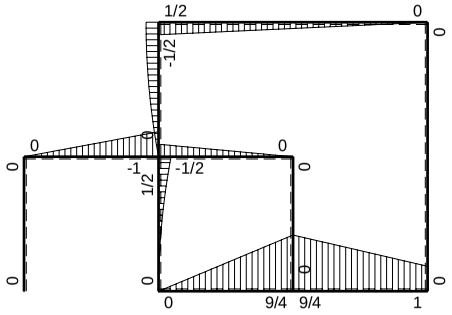




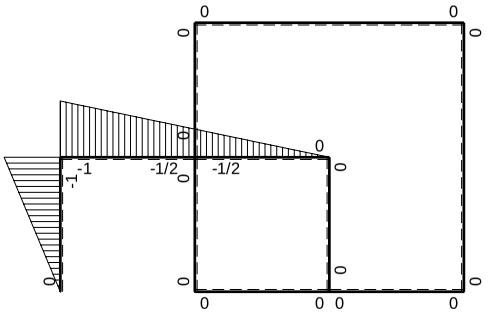

0
1/4
1/4
1/6
13/6
-5/4
-5/4
13/12
7/2
-5/4
0
0
F
0
-1/4
-1
5/12
-13/12
1
0
1/6
0
0
-9/4
5/4
F
1/2
0
0
-1/2
1/6
0
1/6
-11/12
-5/12
1/2
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264522 CI01.xxxx.101

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 264522 CI01.xxxx.101

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

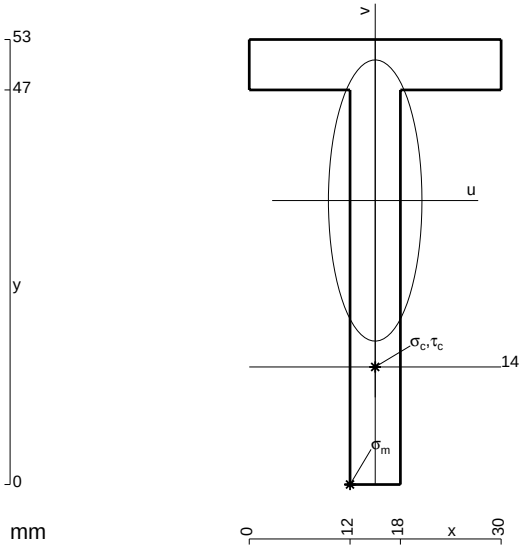
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

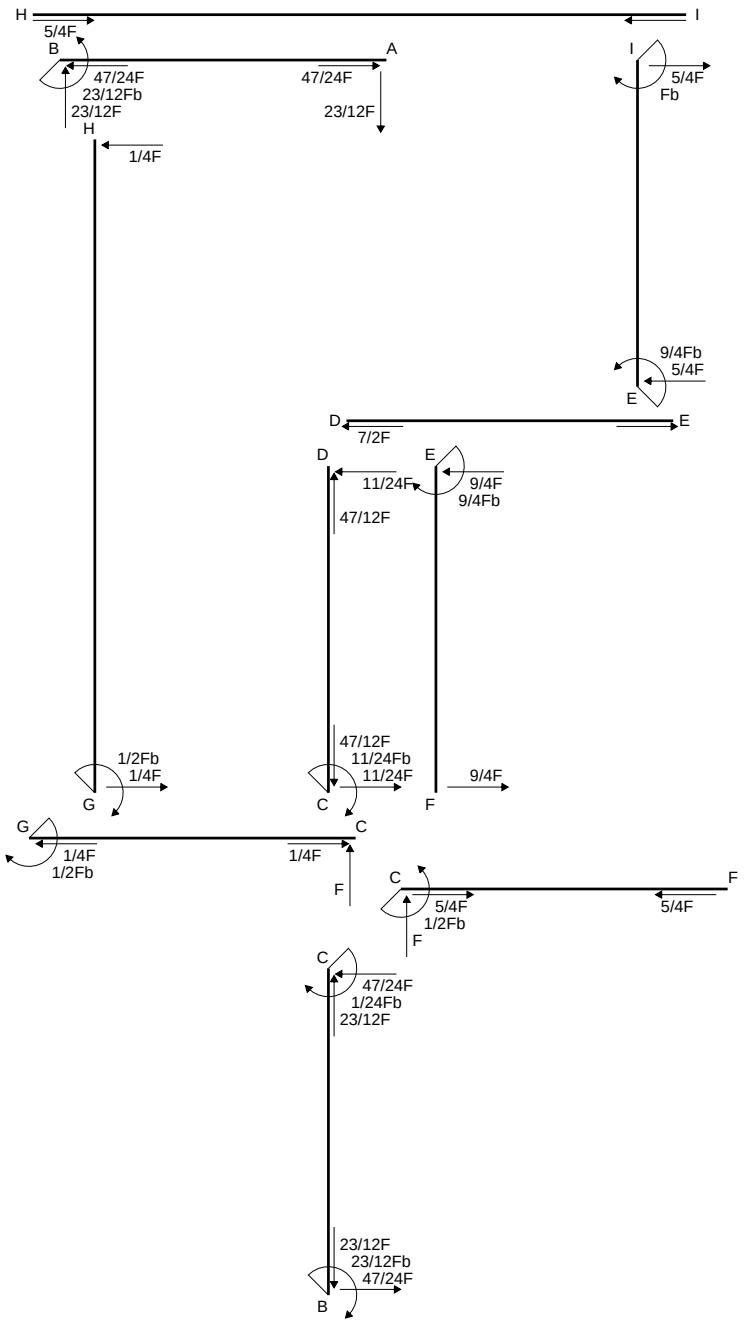
$= (1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





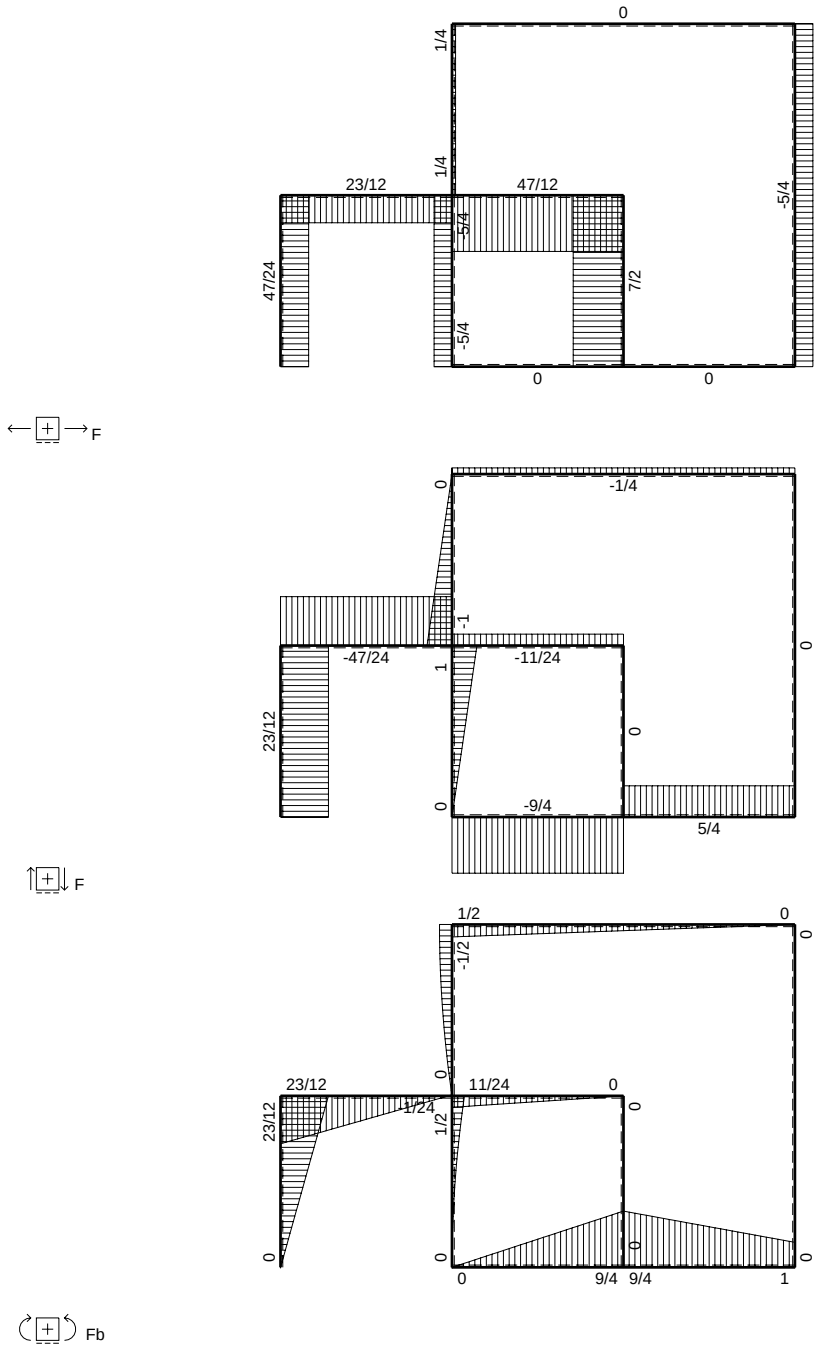


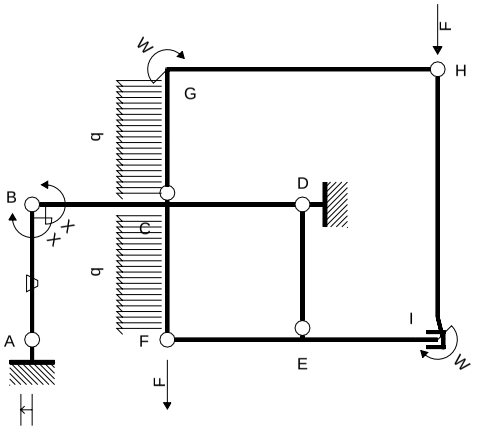

A = 462. $mm^2$
$J_u$ = 129608. $mm^4$
$J_v$ = 14346. $mm^4$
$y_g$ = 33.82 mm
$T_y$ = -1665. N
$M_x$ = 799200. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -33.82 mm
$\sigma_m = -Mv/J_u$ = 208.6 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 14. mm
$v_c$ = -19.82 mm
$\sigma_c = -Mv/J_u$ = 122.2 $N/mm^2$
$\tau_c$ = 4.824 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.5 $N/mm^2$
$S^*$ = 2253. $mm^3$


H
I
5/4F
B
A
47/24F
23/12Fb
23/12F
47/24F
23/12F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
11/24F
9/4F
9/4Fb
47/12F
1/2Fb
1/4F
G
47/12F
11/24Fb
11/24F
C
9/4F
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
5/4F
1/2Fb
F
C
47/24F
1/24Fb
23/12F
23/12F
23/12Fb
47/24F
B

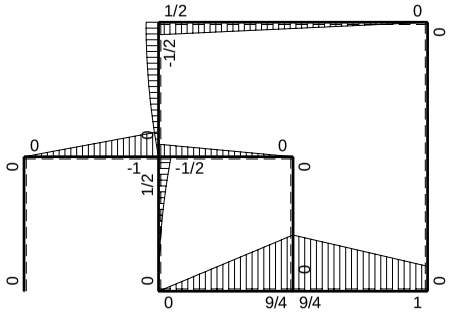




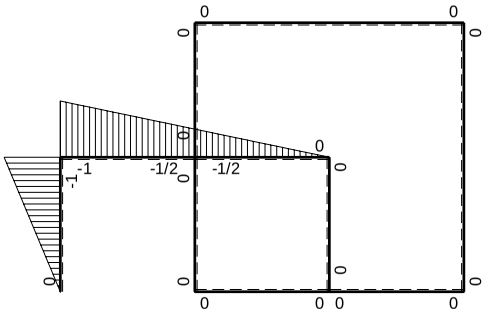

0
1/4
1/4
23/12
47/12
-5/4
-5/4
47/24
7/2
-5/4
0
0
F
0
-1/4
-1
-47/24
1
-11/24
0
23/12
0
0
-9/4
5/4
F
1/2
0
0
-1/2
0
23/12
11/24
0
-1/24
23/12
1/2
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268257 CI01.xxxx.102

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 268257 CI01.xxxx.102

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+1/2Fx$ | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b ( x/b )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

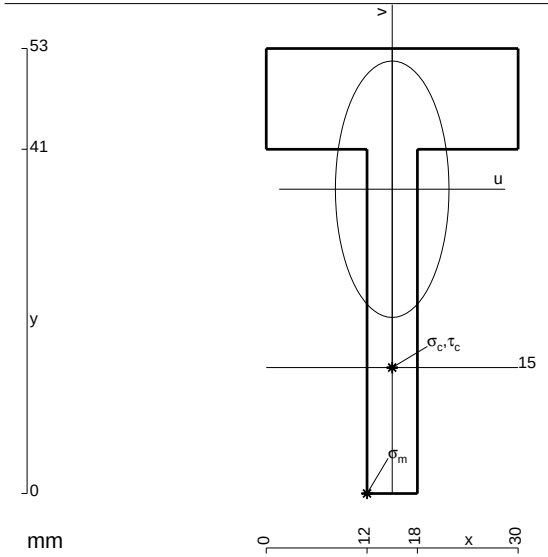
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

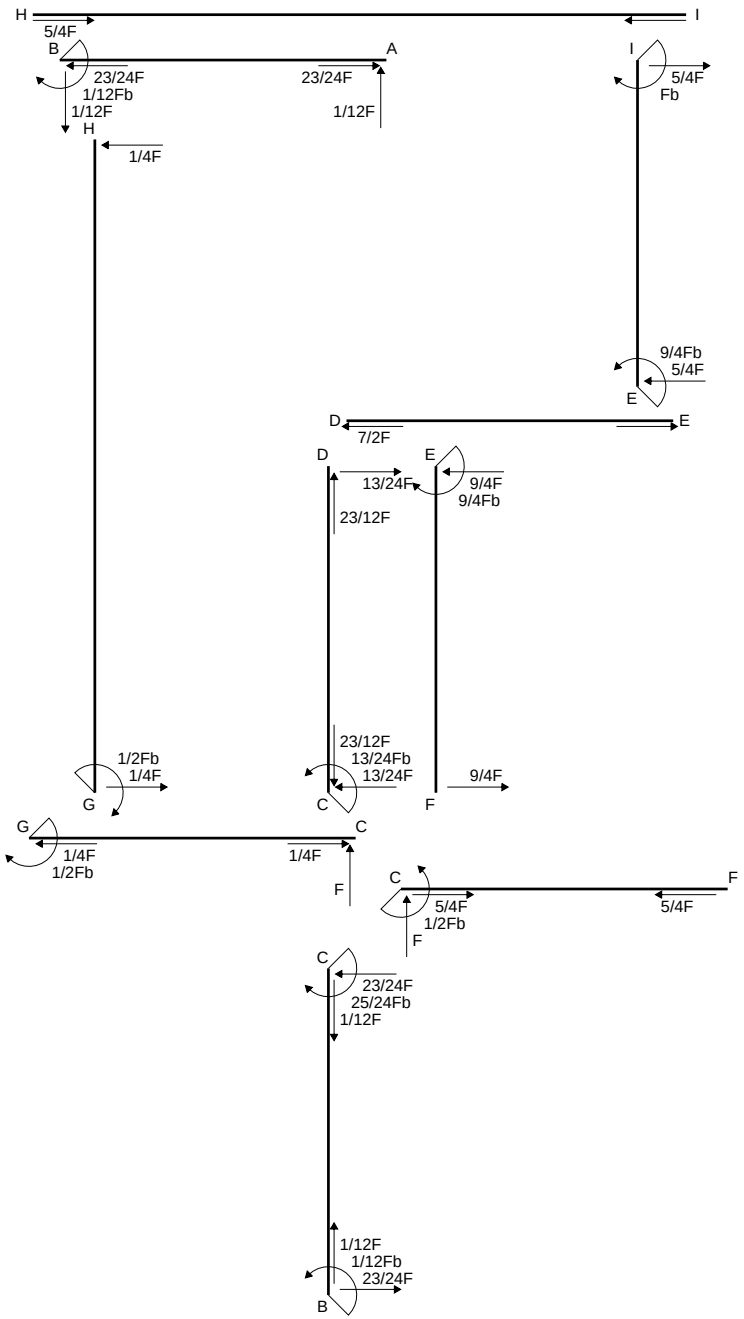
$= (1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





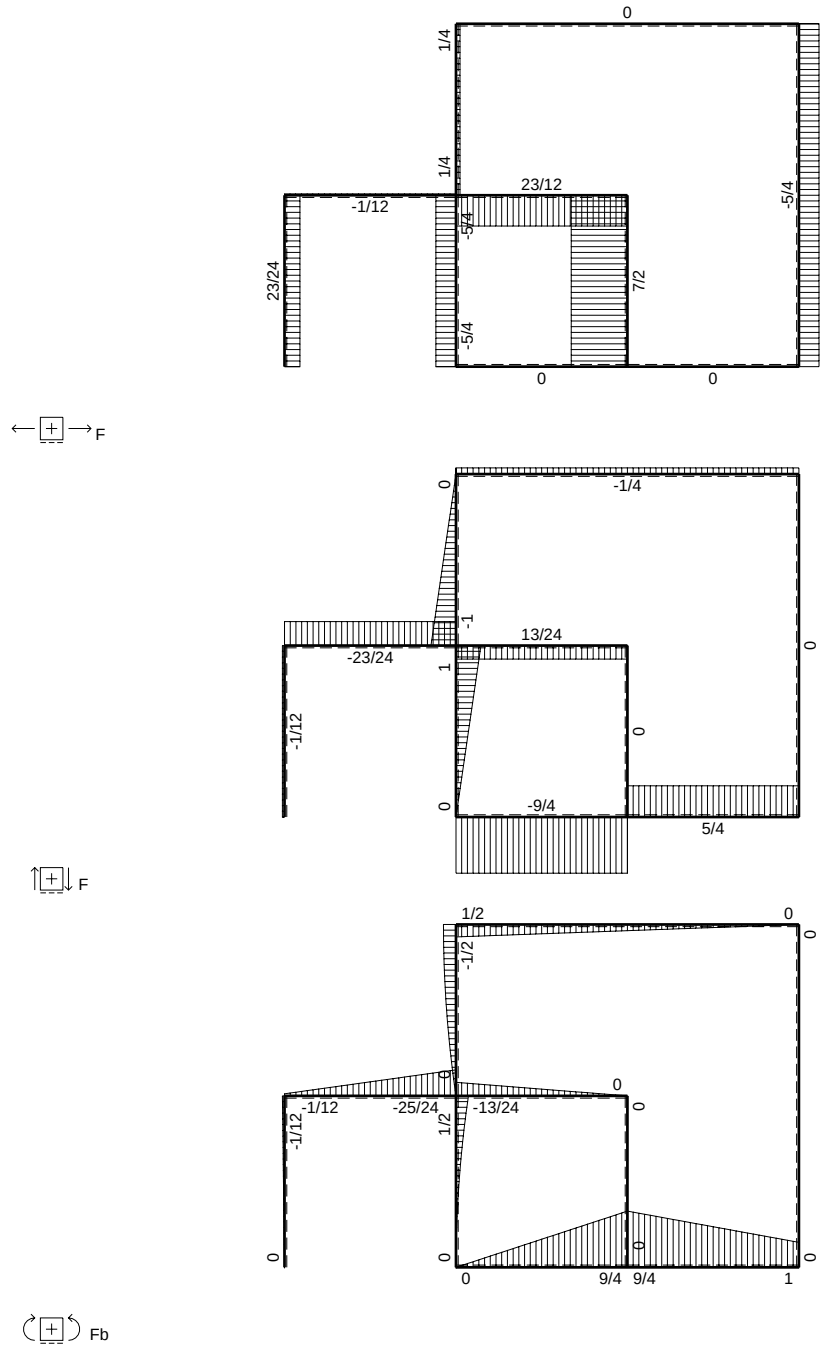


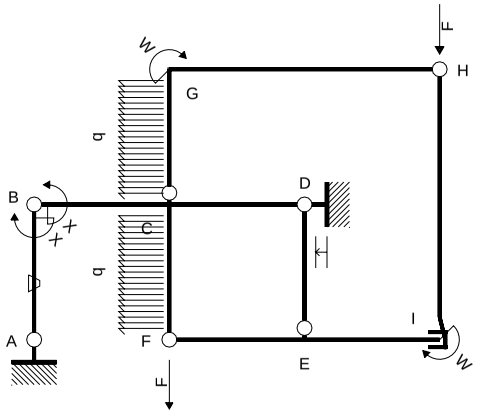

A = 606. $mm^2$
$J_u$ = 141406. $mm^4$
$J_v$ = 27738. $mm^4$
$y_g$ = 36.24 mm
$T_y$ = -1643. N
$M_x$ = 854100. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -36.24 mm
$\sigma_m = -Mv/J_u$ = 218.9 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 15. mm
$v_c$ = -21.24 mm
$\sigma_c = -Mv/J_u$ = 128.3 $N/mm^2$
$\tau_c$ = 5.008 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.6 $N/mm^2$
$S^*$ = 2587. $mm^3$


H
I
5/4F
B
A
23/24F
1/12Fb
1/12F
23/24F
1/12F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
13/24F
9/4F
9/4Fb
23/12F
1/2Fb
1/4F
23/12F
13/24Fb
13/24F
9/4F
G
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
5/4F
1/2Fb
F
C
23/24F
25/24Fb
1/12F
1/12F
1/12Fb
23/24F
B

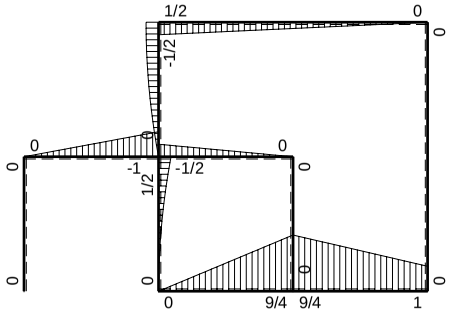




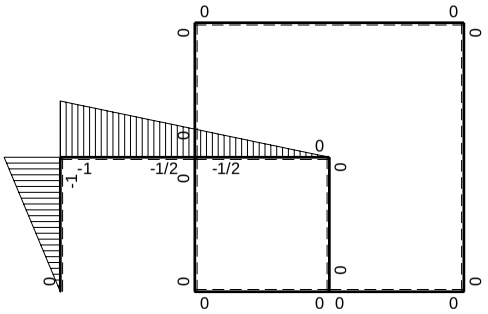

0
1/4
1/4
23/12
-1/12
-5/4
-5/4
23/24
7/2
-5/4
0
0
F
0
-1/4
-1
13/24
-23/24
1
0
-1/12
0
0
-9/4
5/4
F
1/2
0
0
-1/2
0
-1/12
-25/24
1/2
-13/24
0
-1/12
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 956343 CI01.xxxx.103

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PROCEDIMENTO E RISULTATI 956343 CI01.xxxx.103

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+1/2Fx$ | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b ( x/b )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

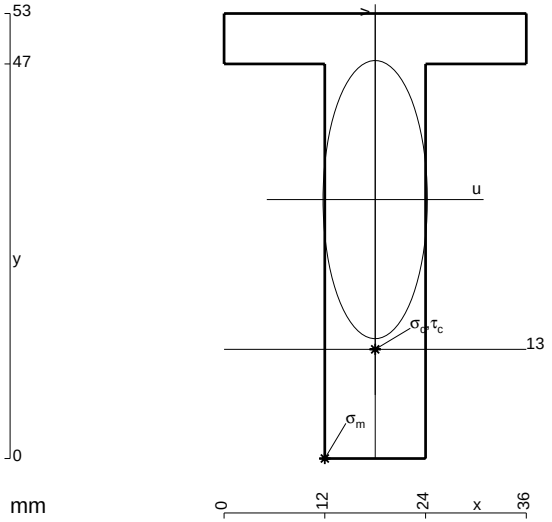
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

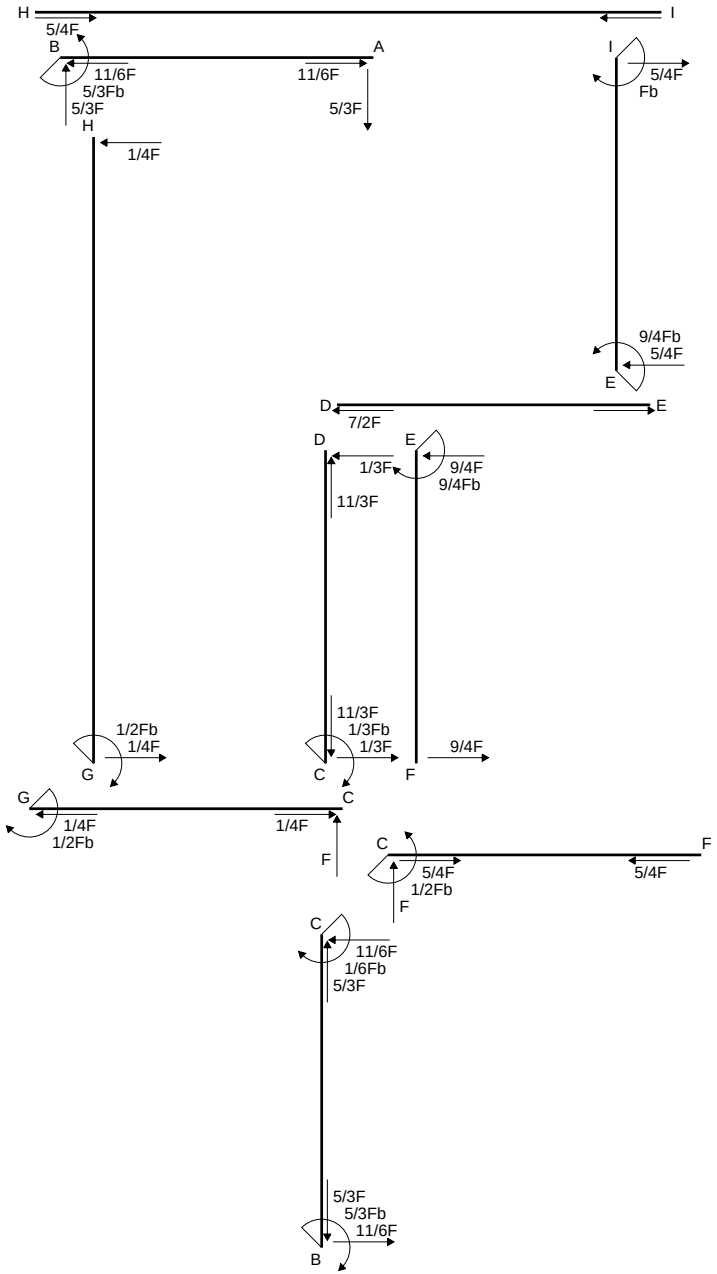
$= (1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





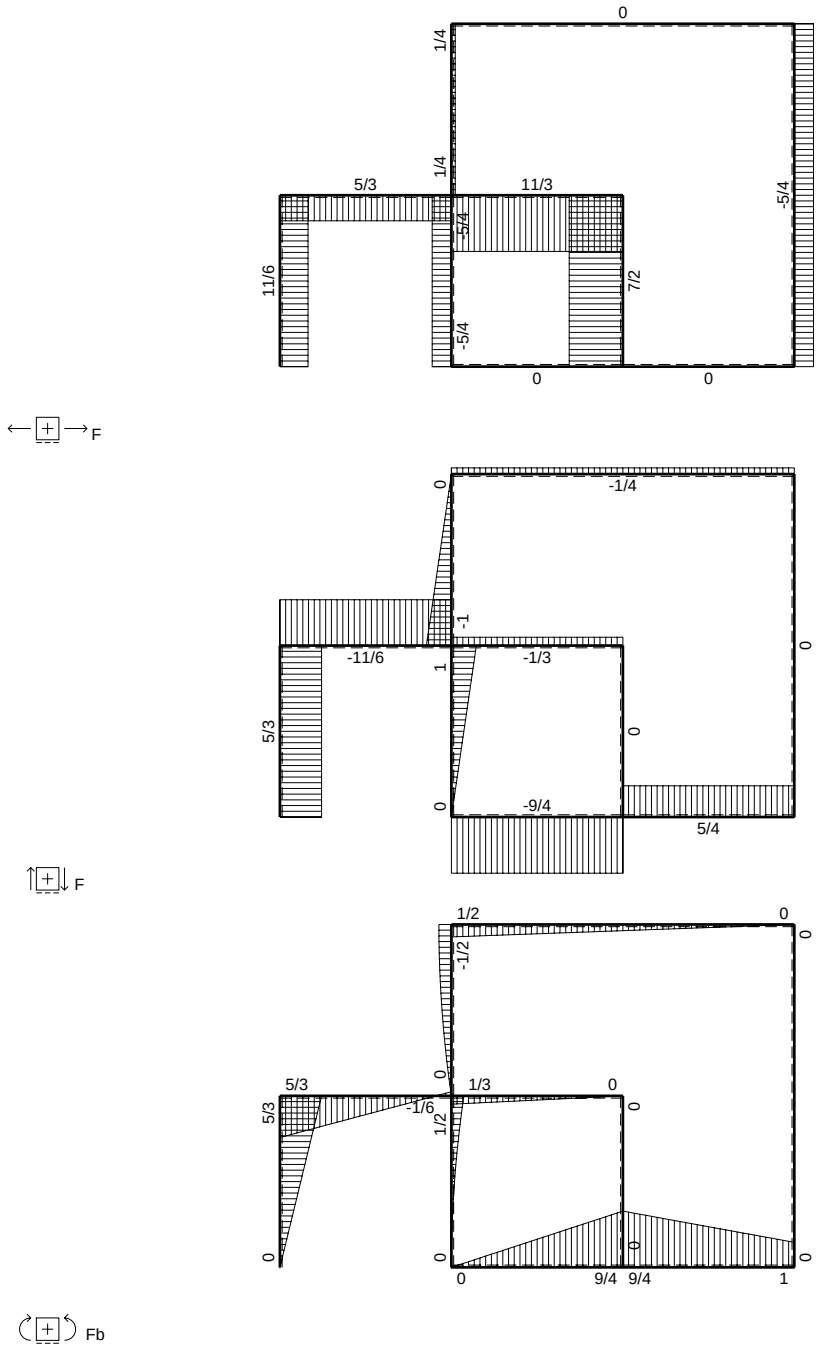


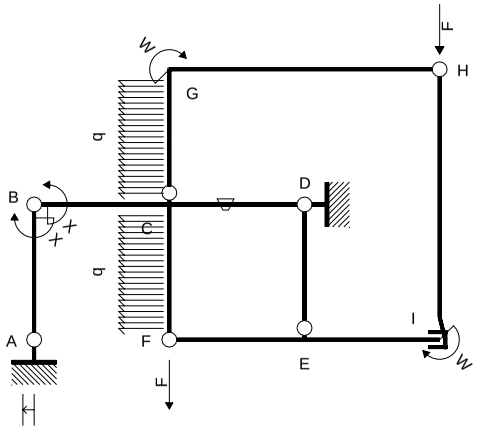

A = 780. $mm^2$
$J_u$ = 214152. $mm^4$
$J_v$ = 30096. $mm^4$
$y_g$ = 30.84 mm
$T_y$ = -2790. N
$M_x$ = 1590300. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -30.84 mm
$\sigma_m = -Mv/J_u$ = 229. $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 13. mm
$v_c$ = -17.84 mm
$\sigma_c = -Mv/J_u$ = 132.5 $N/mm^2$
$\tau_c$ = 4.122 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.7 $N/mm^2$
$S^*$ = 3797. $mm^3$


H
I
5/4F
B
A
11/6F
5/3Fb
5/3F
11/6F
5/3F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
1/3F
9/4F
9/4Fb
11/3F
1/2Fb
1/4F
G
11/3F
1/3Fb
1/3F
C
F
9/4F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
1/2Fb
5/4F
F
C
11/6F
1/6Fb
5/3F
5/3F
5/3Fb
11/6F
B

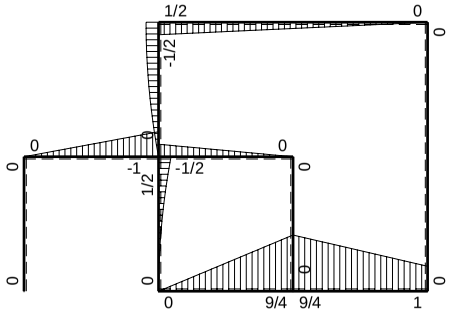




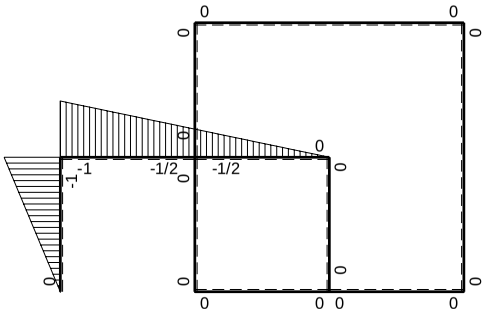

F
0
1/4
1/4
5/3
11/3
-5/4
-5/4
11/6
7/2
-5/4
0
0
F
0
-1/4
-1
-11/6
1
-1/3
0
5/3
0
0
-9/4
5/4
F
1/2
0
0
-1/2
0
5/3
1/3
0
5/3
-1/6
1/2
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 212572 CI01.xxxx.104

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 212572 CI01.xxxx.104

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $5/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

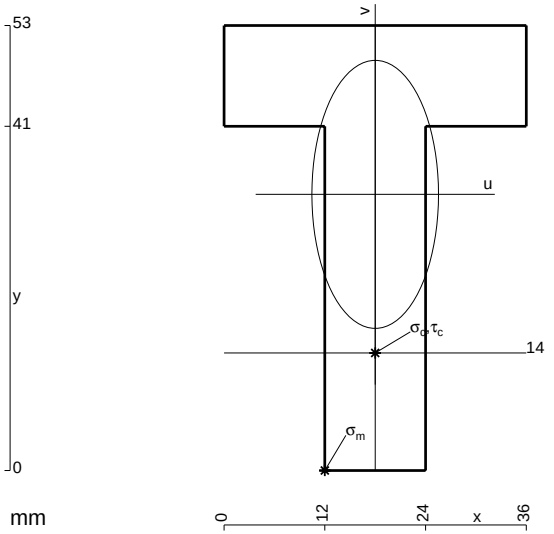
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

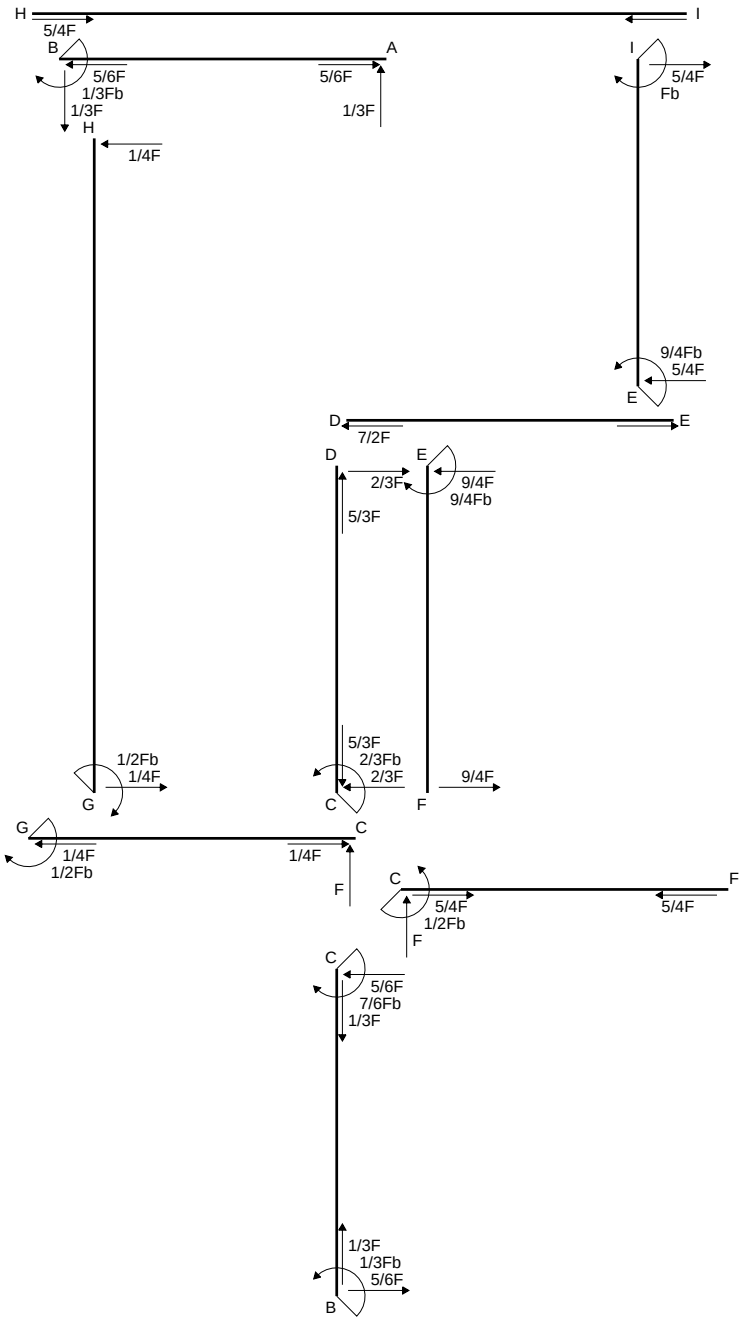
$= (1/12\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$





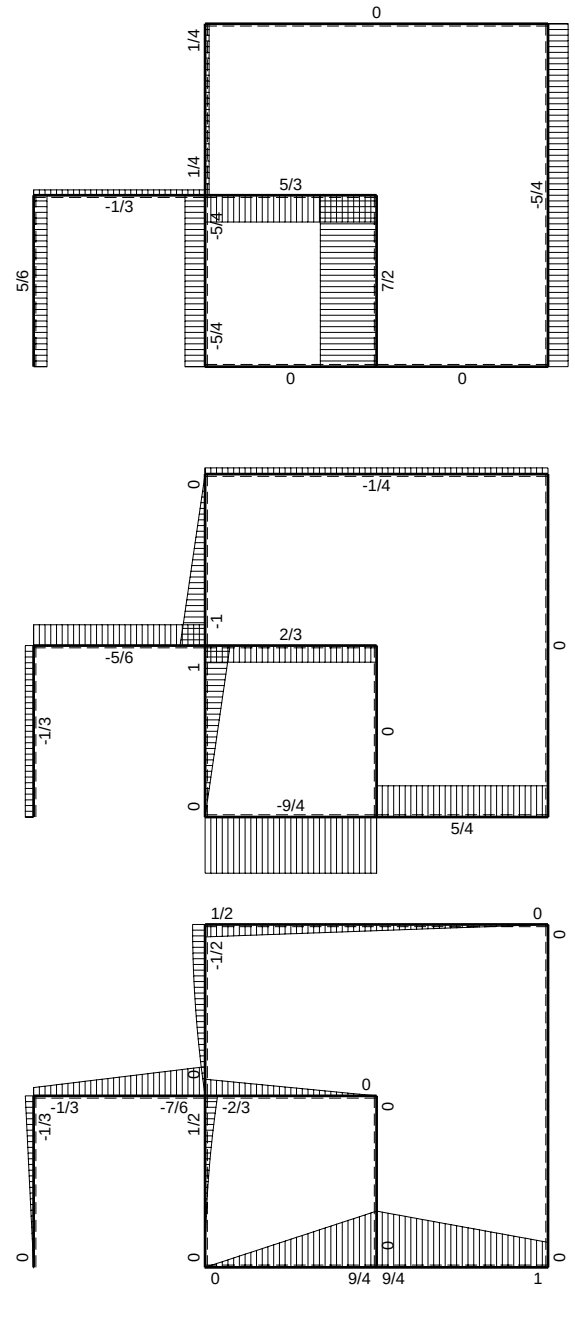


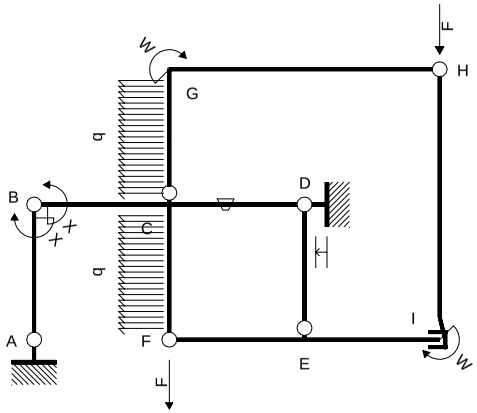

A = 924. $mm^2$
$J_u$ = 235641. $mm^4$
$J_v$ = 52560. $mm^4$
$y_g$ = 32.89 mm
$T_y$ = -2813. N
$M_x$ = 1715630. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -32.89 mm
$\sigma_m = -Mv/J_u$ = 239.5 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -18.89 mm
$\sigma_c = -Mv/J_u$ = 137.5 $N/mm^2$
$\tau_c$ = 4.326 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137.7 $N/mm^2$
$S^*$ = 4349. $mm^3$


H
I
5/4F
B
A
5/6F
1/3Fb
1/3F
5/6F
1/3F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
2/3F
9/4F
9/4Fb
5/3F
1/2Fb
1/4F
5/3F
2/3Fb
2/3F
9/4F
G
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
5/4F
1/2Fb
F
C
5/6F
7/6Fb
1/3F
1/3F
1/3Fb
5/6F
B

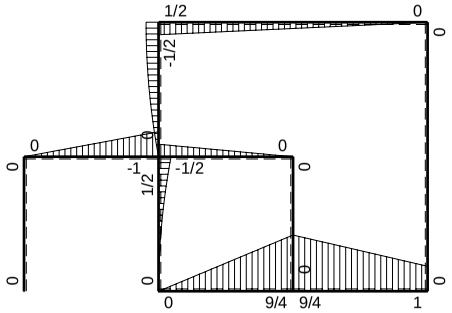




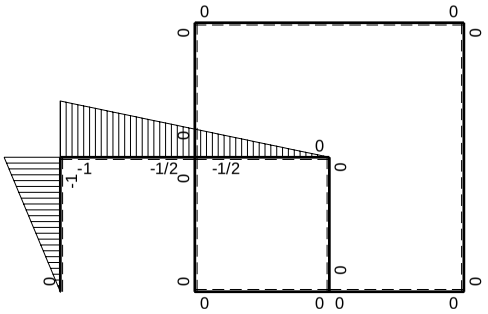

0
1/4
1/4
5/3
-1/3
-5/4
-5/4
5/6
7/2
-5/4
0
0
F
-1/4
0
-1
2/3
-5/6
1
-1/3
0
0
-9/4
5/4
F
1/2
0
0
-1/2
0
0
-1/3
-1/3
-7/6
-2/3
1/2
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268011 CI01.xxxx.105

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PROCEDIMENTO E RISULTATI 268011 CI01.xxxx.105

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

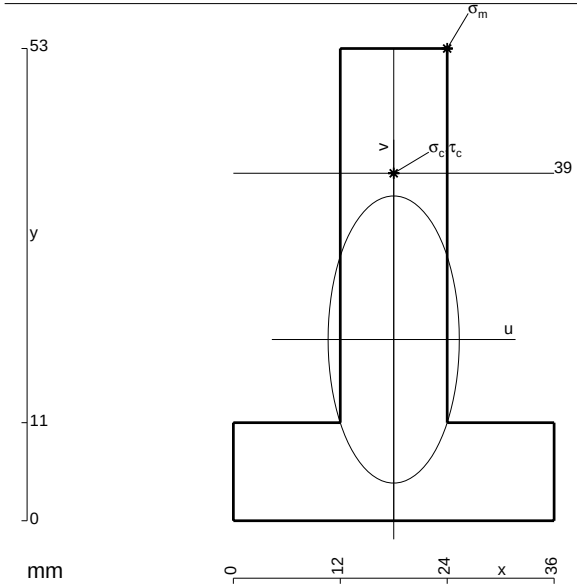
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

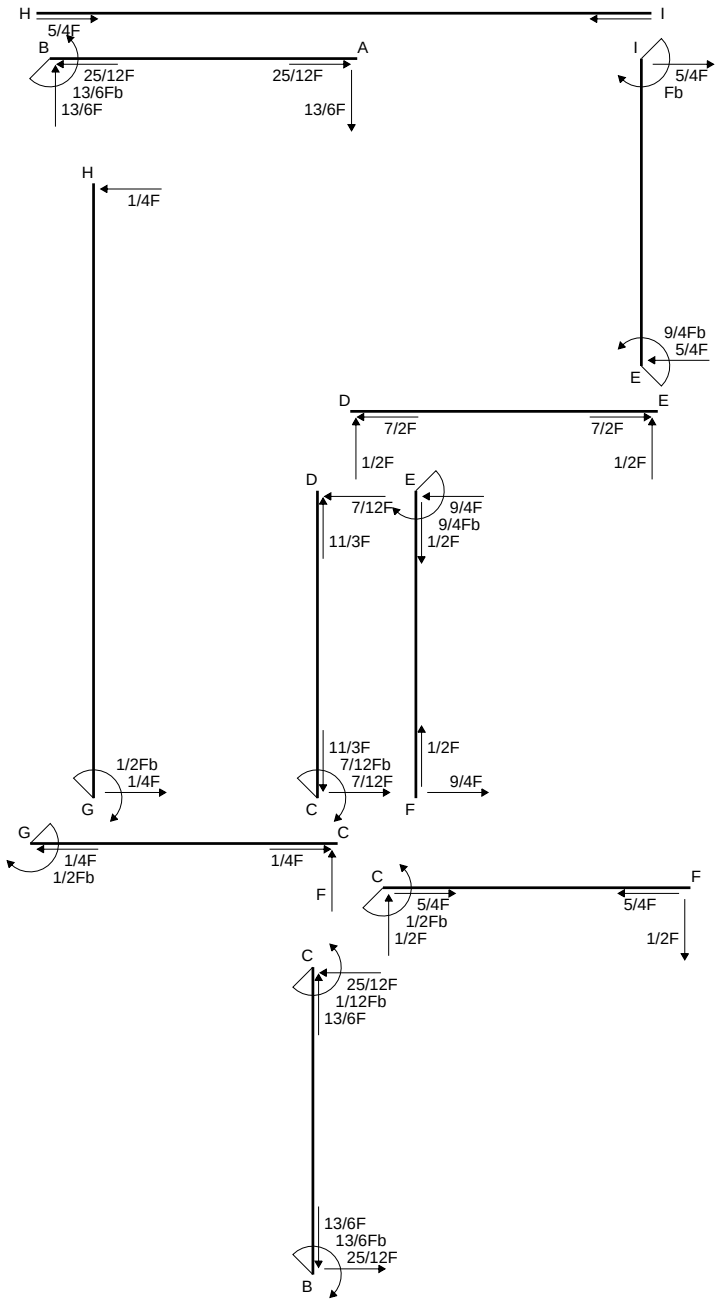
$= (1/12\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$





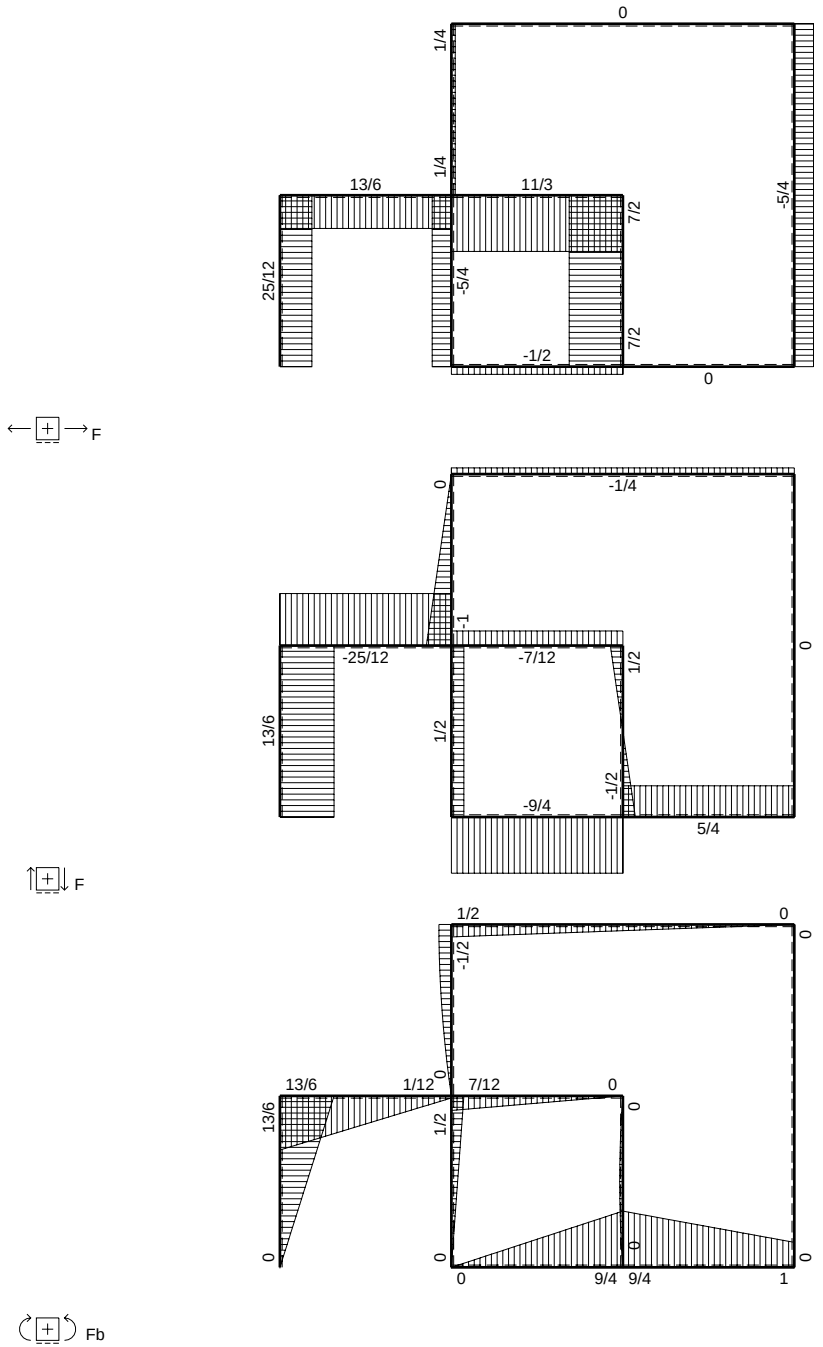


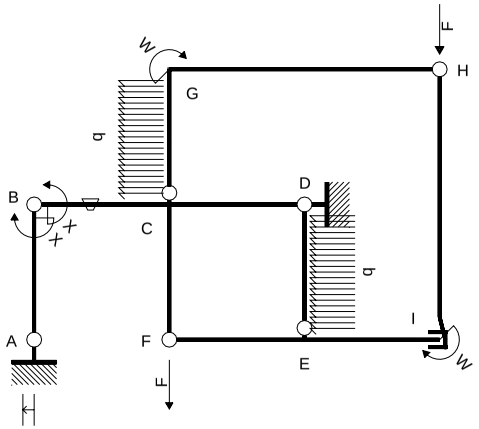

A = 900. $mm^2$
$J_u$ = 233812. $mm^4$
$J_v$ = 48816. $mm^4$
$y_g$ = 20.34 mm
$T_y$ = -2160. N
$M_x$ = 1425600. Nmm
$x_m$ = 24. mm
$y_m$ = 53. mm
$u_m$ = 6. mm
$v_m$ = 32.66 mm
$\sigma_m = -Mv/J_u$ = -199.1 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 39. mm
$v_c$ = 18.66 mm
$\sigma_c = -Mv/J_u$ = -113.8 $N/mm^2$
$\tau_c$ = 3.319 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.9 $N/mm^2$
$S^*$ = 4311. $mm^3$


H
I
5/4F
B
A
25/12F
13/6Fb
13/6F
25/12F
13/6F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
7/2F
1/2F
1/2F
D
E
7/12F
9/4F
9/4Fb
1/2F
11/3F
1/2Fb
1/4F
G
11/3F
7/12Fb
7/12F
C
1/2F
9/4F
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
1/2Fb
1/2F
5/4F
1/2F
C
25/12F
1/12Fb
13/6F
13/6F
13/6Fb
25/12F
B

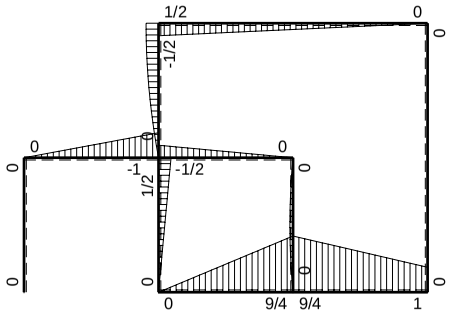




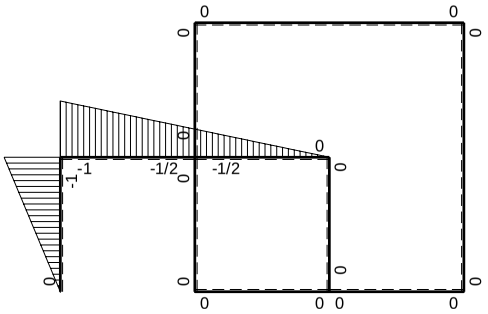

F
0
1/4
1/4
13/6
11/3
-5/4
7/2
25/12
-5/4
7/2
-1/2
0
F
0
-1/4
-1
0
-25/12
-7/12
1/2
13/6
1/2
-1/2
-9/4
5/4
F
1/2
0
0
-1/2
0
13/6
1/12
7/12
0
13/6
1/2
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 266252 CI01.xxxx.106

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 266252 CI01.xxxx.106

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

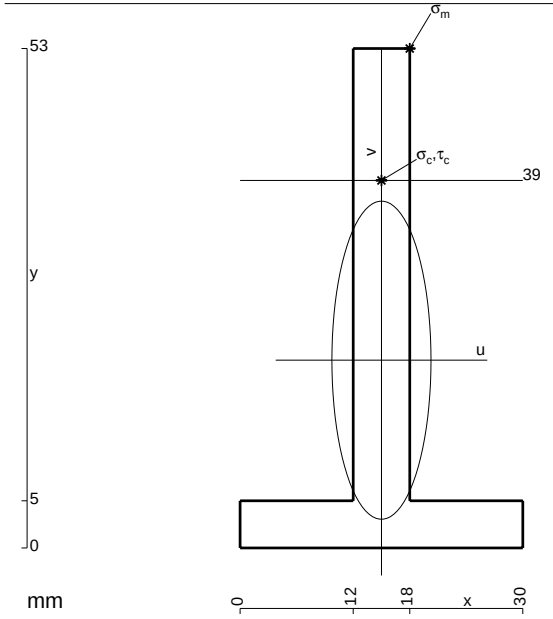
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

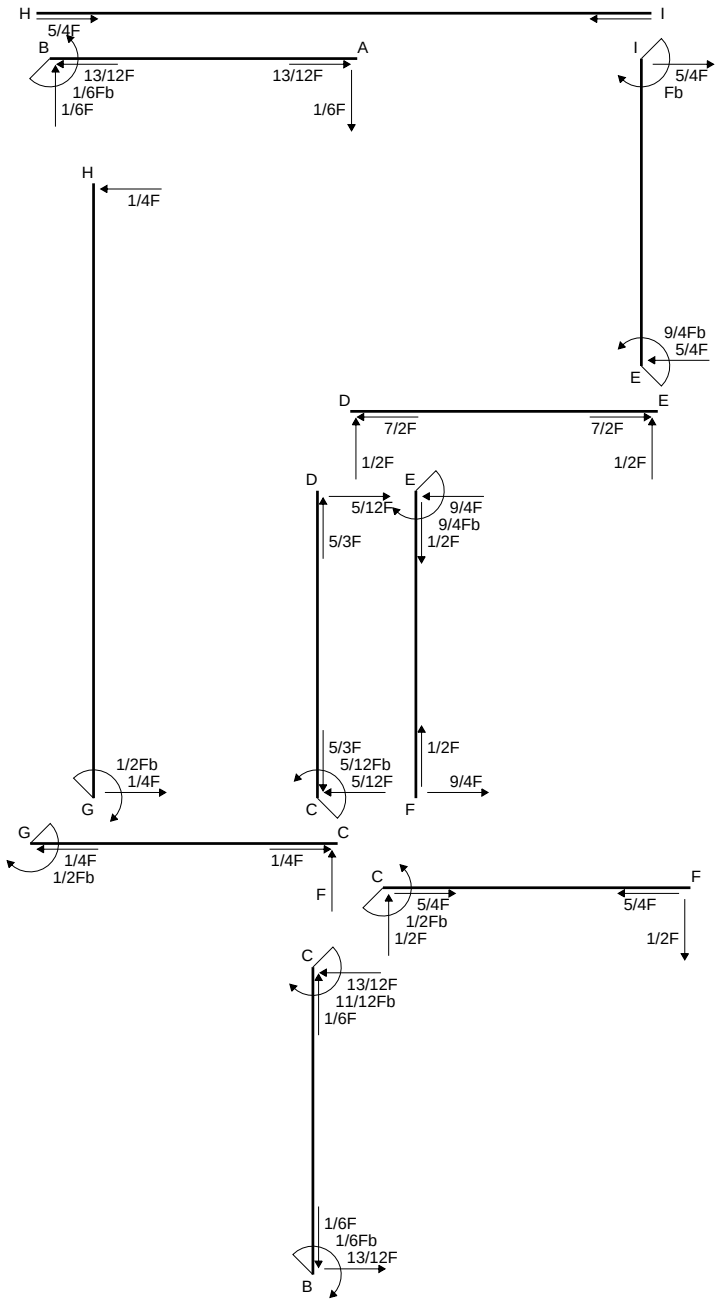
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





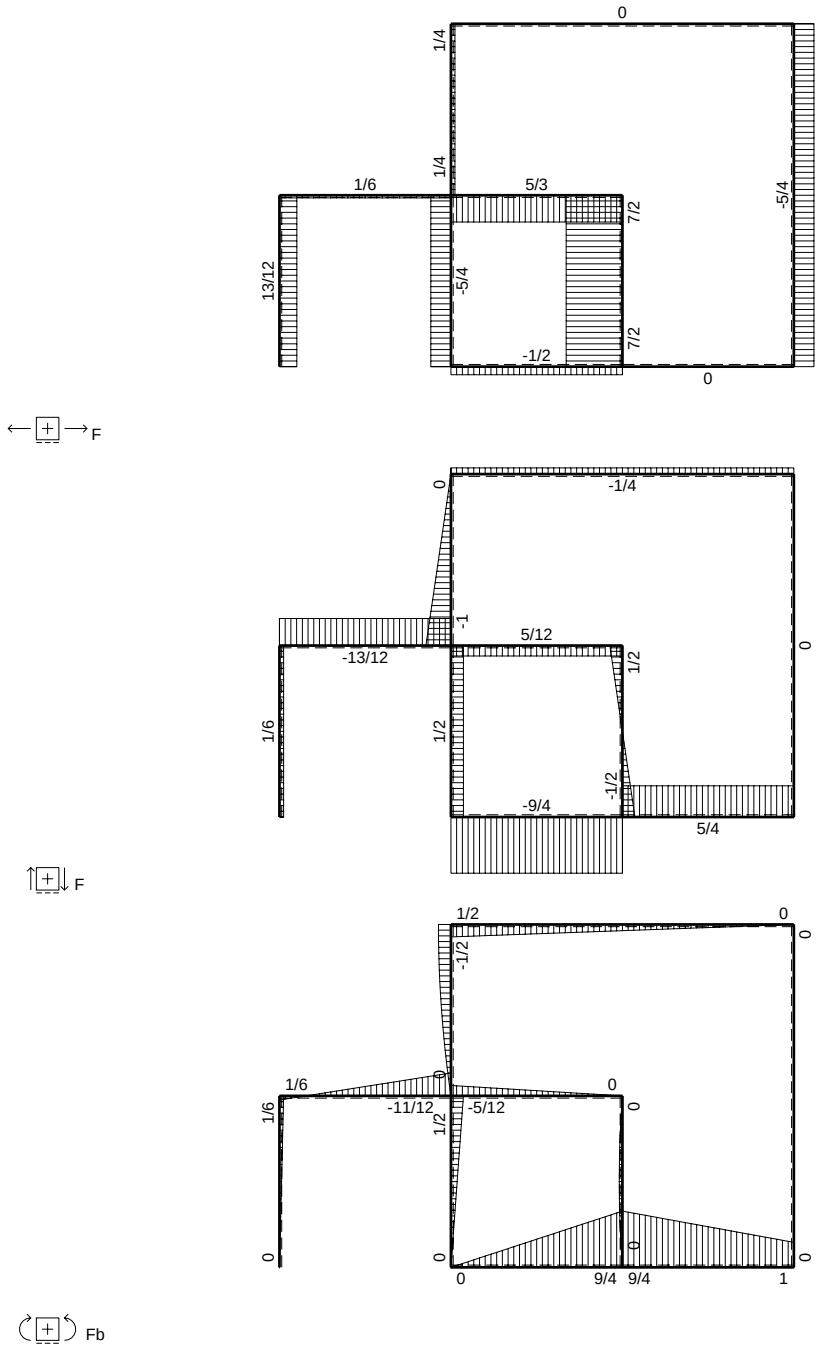


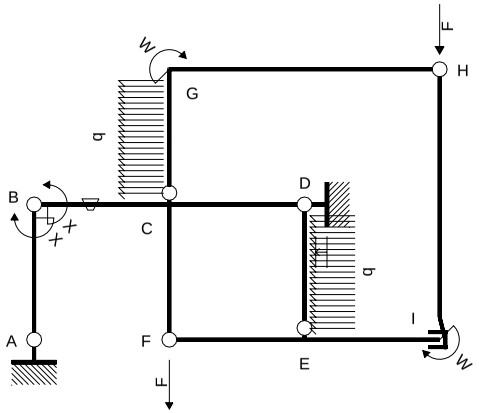

A = 438. $mm^2$
$J_u$ = 124872. $mm^4$
$J_v$ = 12114. $mm^4$
$y_g$ = 19.92 mm
N = -250. N
$T_y$ = -1125. N
$M_x$ = 787500. Nmm
$x_m$ = 18. mm
$y_m$ = 53. mm
$u_m$ = 3. mm
$v_m$ = 33.08 mm
$\sigma_m = N/A - Mv/J_u$ = -209.2 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 39. mm
$v_c$ = 19.08 mm
$\sigma_c = N/A - Mv/J_u$ = -120.9 $N/mm^2$
$\tau_c$ = 3.289 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121. $N/mm^2$
$S^*$ = 2190. $mm^3$


H
I
5/4F
B
A
13/12F
1/6Fb
1/6F
13/12F
1/6F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
7/2F
1/2F
1/2F
D
E
5/12F
9/4F
9/4Fb
1/2F
5/3F
1/2Fb
1/4F
G
5/3F
5/12Fb
5/12F
C
1/2F
9/4F
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
1/2Fb
1/2F
5/4F
1/2F
C
13/12F
11/12Fb
1/6F
1/6F
1/6Fb
13/12F
B

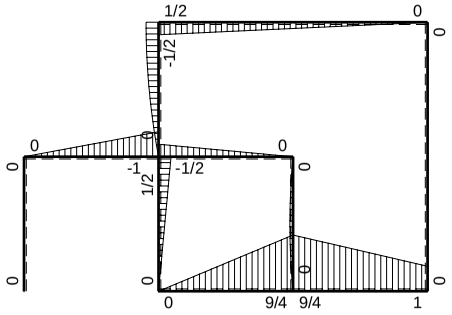




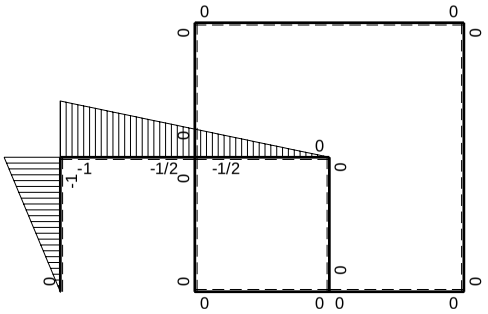

0
1/4
1/4
1/6
5/3
-5/4
7/2
13/12
-5/4
7/2
-1/2
0
F
0
-1/4
-1
5/12
-13/12
1/2
0
1/6
1/2
-1/2
-9/4
5/4
F
1/2
0
0
-1/2
1/6
0
-11/12
-5/12
1/6
1/2
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 226461 CI01.xxxx.107

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PROCEDIMENTO E RISULTATI 226461 CI01.xxxx.107

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - 1/2\ b + 1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 + 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b + 1/4\ b + 1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - 1/2\ x/b)\ \theta\ dx$

$= [1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x - 1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ + (\ b - 1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 - 1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x - 1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b - 1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

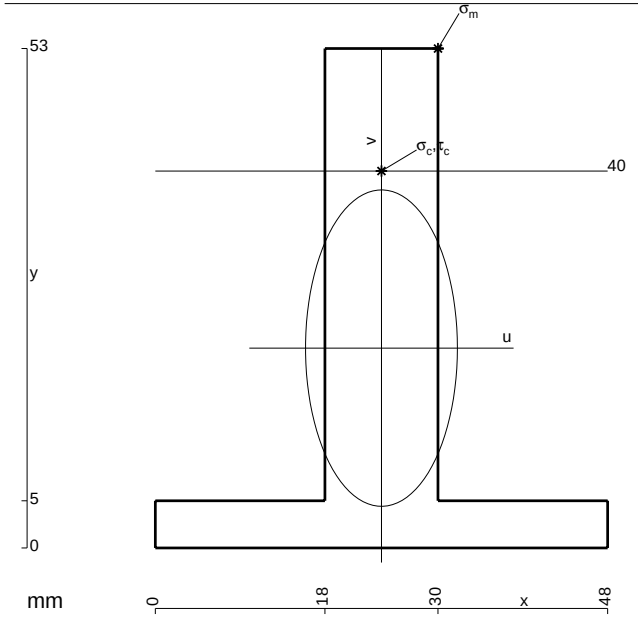
$= (1/4\ b - 1/4\ b + 1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

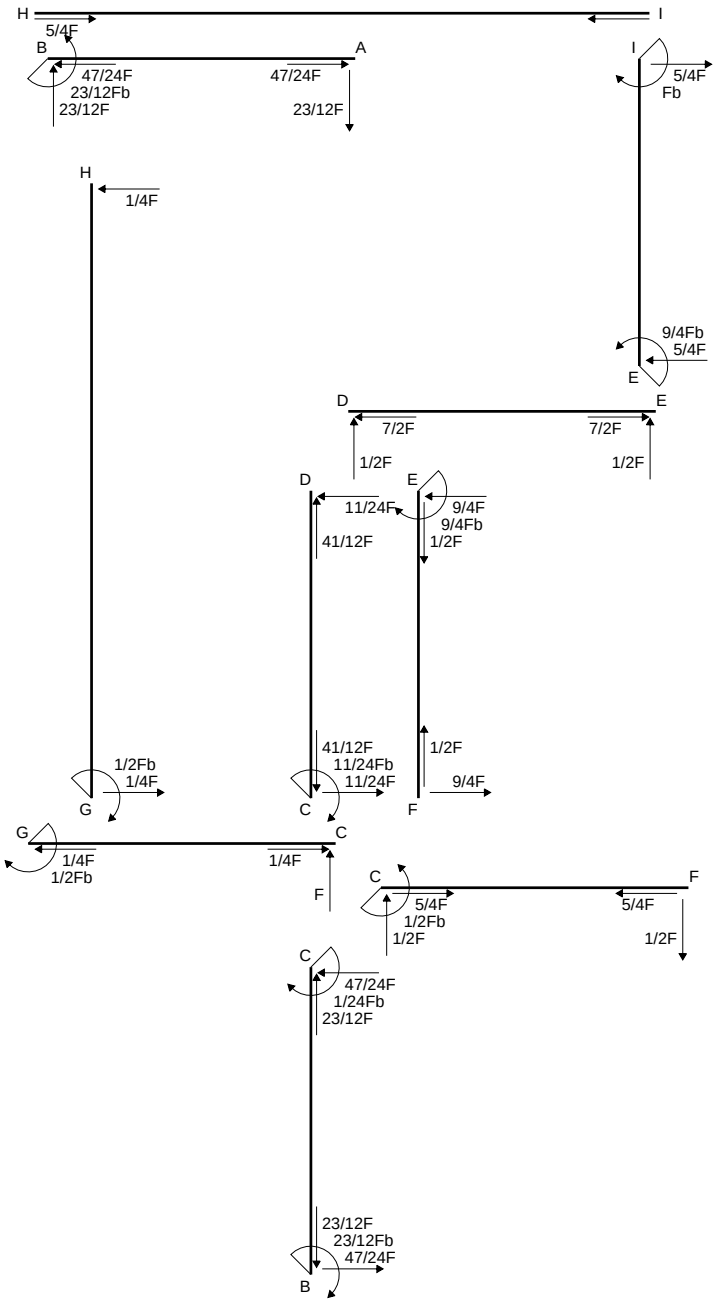
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





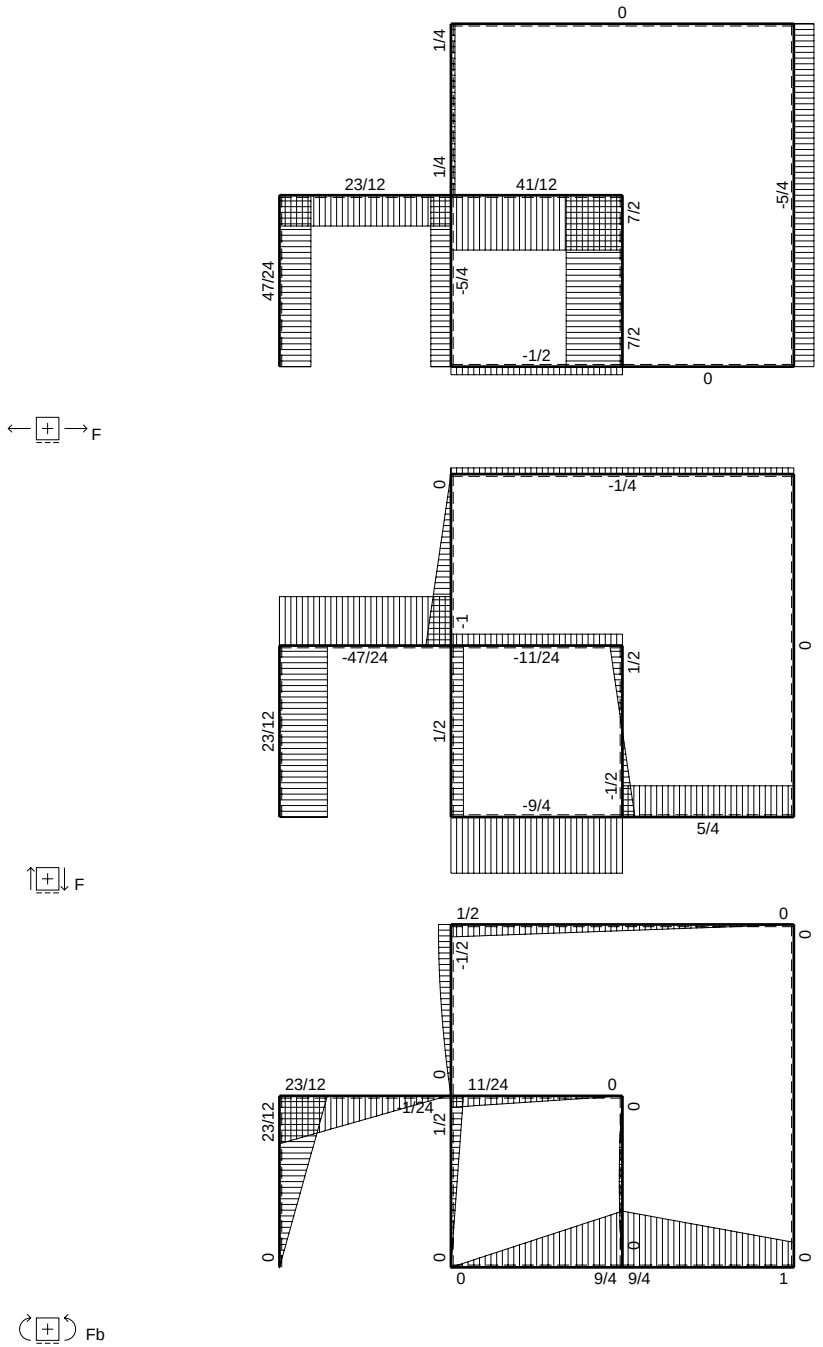


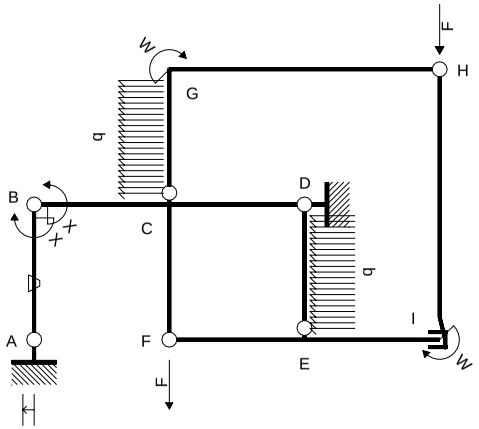

A = 816. $mm^2$
$J_u$ = 230061. $mm^4$
$J_v$ = 52992. $mm^4$
$y_g$ = 21.21 mm
N = -475. N
$T_y$ = -2138. N
$M_x$ = 1581750. Nmm
$x_m$ = 30. mm
$y_m$ = 53. mm
$u_m$ = 6. mm
$v_m$ = 31.79 mm
$\sigma_m = N/A - Mv/J_u$ = -219.2 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 40. mm
$v_c$ = 18.79 mm
$\sigma_c = N/A - Mv/J_u$ = -129.8 $N/mm^2$
$\tau_c$ = 3.055 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 129.9 $N/mm^2$
$S^*$ = 3946. $mm^3$


H
I
5/4F
B
A
47/24F
23/12Fb
23/12F
47/24F
23/12F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
7/2F
1/2F
1/2F
D
E
11/24F
9/4F
9/4Fb
41/12F
1/2F
41/12F
11/24Fb
11/24F
1/2F
9/4F
1/2Fb
1/4F
G
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
1/2Fb
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23/12F
23/12Fb
47/24F
B

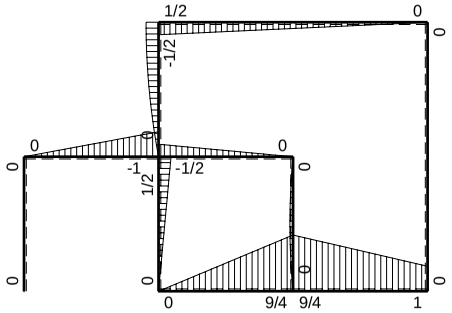




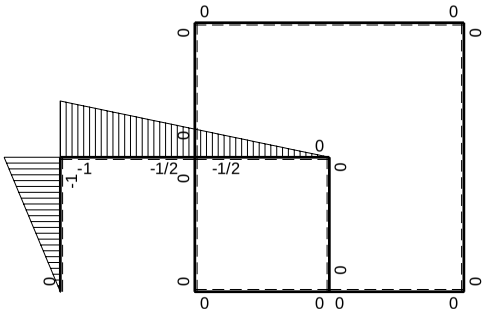

0
1/4
1/4
23/12
41/12
-5/4
7/2
47/24
-5/4
7/2
-1/2
0
F
0
-1/4
-1
0
-47/24
-11/24
1/2
23/12
1/2
-1/2
-9/4
5/4
F
1/2
0
0
-1/2
0
23/12
11/24
0
23/12
1/24
1/2
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 213451 CI01.xxxx.108

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 213451 CI01.xxxx.108

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+1/2Fx$ | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | $1/2Fx$ | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+1/2Fx$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $1/2Fb-1/4Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

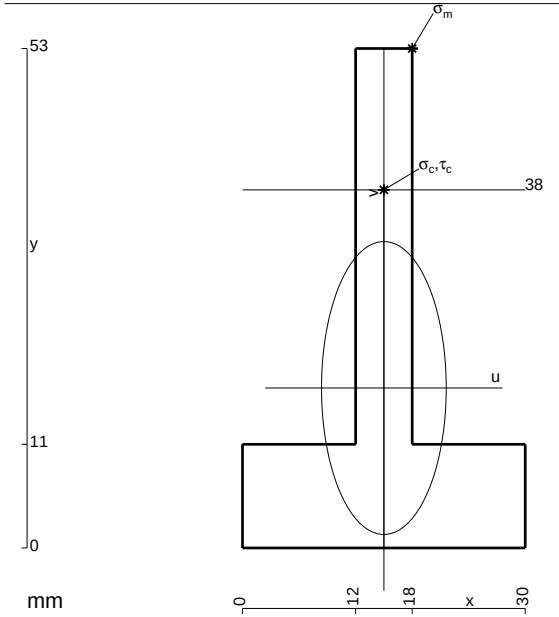
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

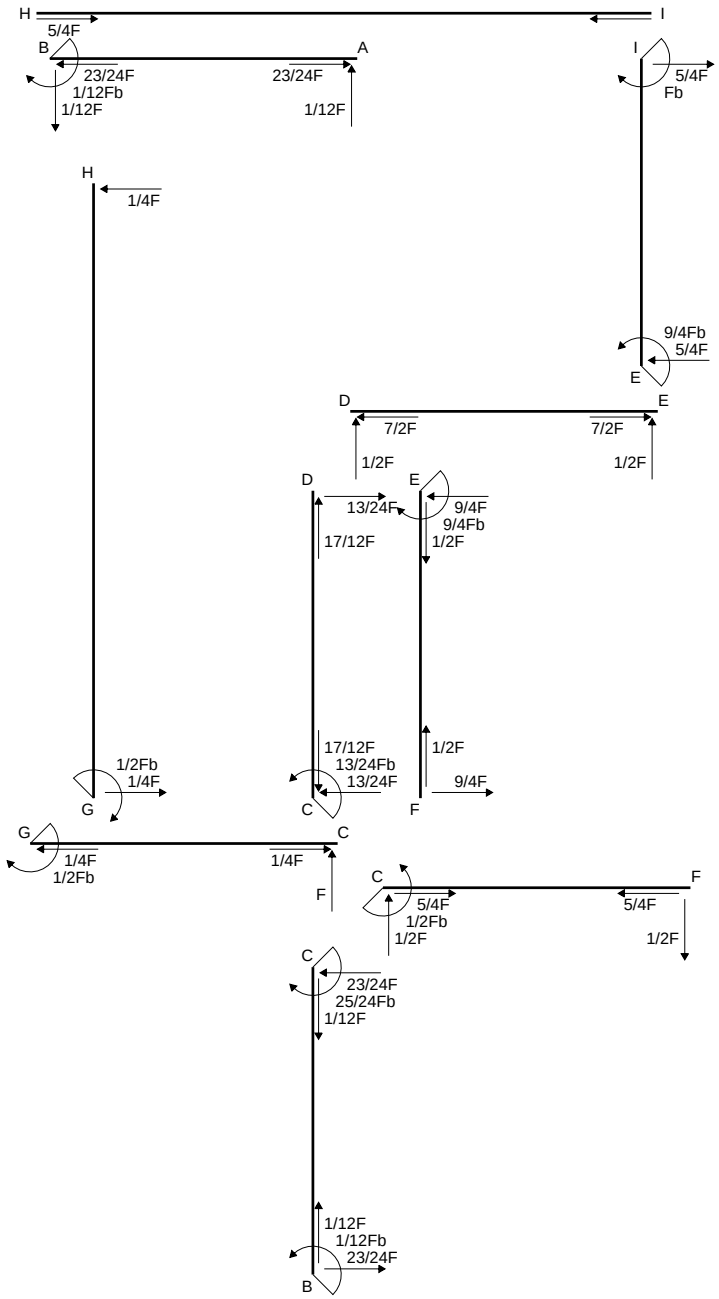


PROCEDIMENTO E RISULTATI 213451 CI01.xxxx.108



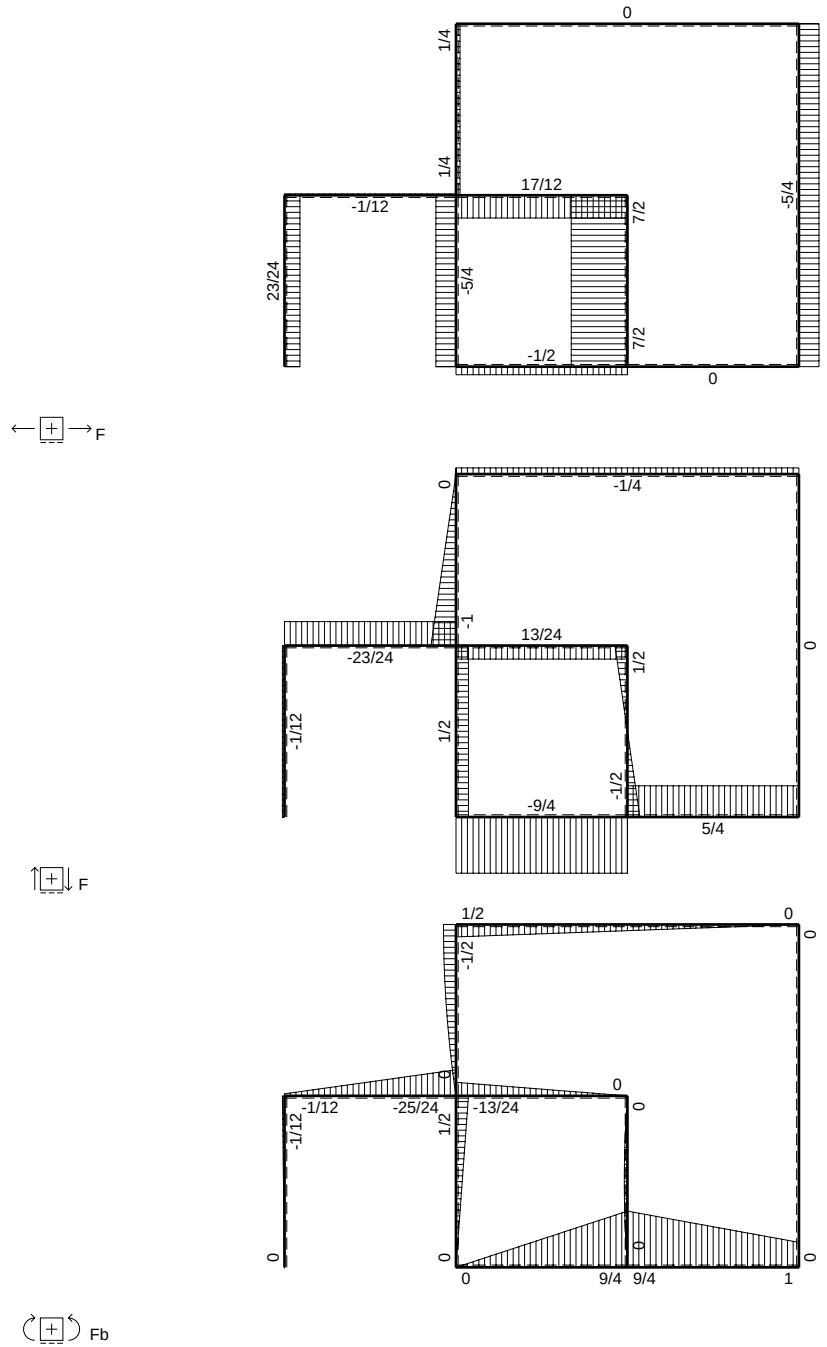

A = 582. $mm^2$
$J_u$ = 140714. $mm^4$
$J_v$ = 25506. $mm^4$
$y_g$ = 16.97 mm
N = -250. N
$T_y$ = -1125. N
$M_x$ = 888750. Nmm
$x_m$ = 18. mm
$y_m$ = 53. mm
$u_m$ = 3. mm
$v_m$ = 36.03 mm
$\sigma_m = N/A - Mv/J_u$ = -228. $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 38. mm
$v_c$ = 21.03 mm
$\sigma_c = N/A - Mv/J_u$ = -133.2 $N/mm^2$
$\tau_c$ = 3.421 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.4 $N/mm^2$
$S^*$ = 2567. $mm^3$

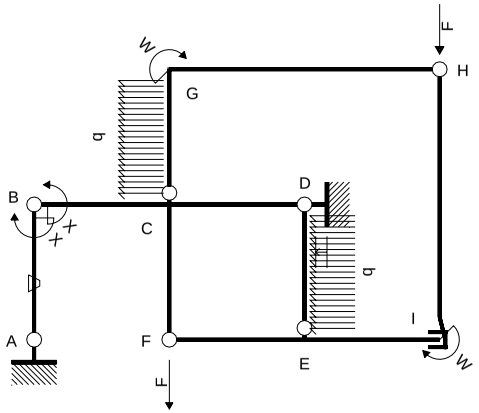
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H
I
5/4F
B
A
23/24F
1/12Fb
1/12F
23/24F
1/12F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
7/2F
1/2F
1/2F
D
E
13/24F
9/4F
9/4Fb
17/12F
1/2F
17/12F
13/24Fb
13/24F
1/2F
9/4F
1/2Fb
1/4F
G
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
5/4F
1/2Fb
1/2F
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C
23/24F
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1/12F
1/12F
1/12Fb
23/24F
B

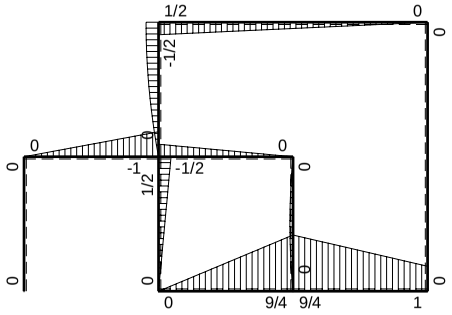




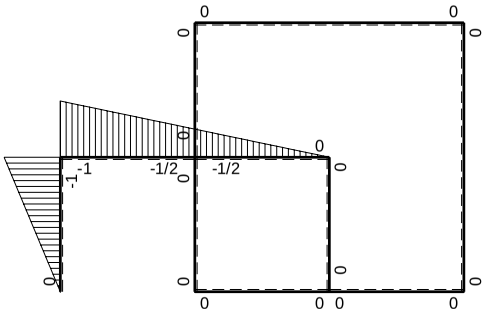

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1/4
1/4
17/12
-1/12
-5/4
7/2
23/24
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7/2
-1/2
0
F
0
-1/4
-1
13/24
-23/24
1/2
0
-1/12
1/2
-1/2
-9/4
5/4
F
1/2
0
0
-1/2
0
-1/12
-25/24
-13/24
1/2
-1/12
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248086 CI01.xxxx.109

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PROCEDIMENTO E RISULTATI 248086 CI01.xxxx.109

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+1/2Fx$ | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | $1/2Fx$ | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+1/2Fx$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $1/2Fb-1/4Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

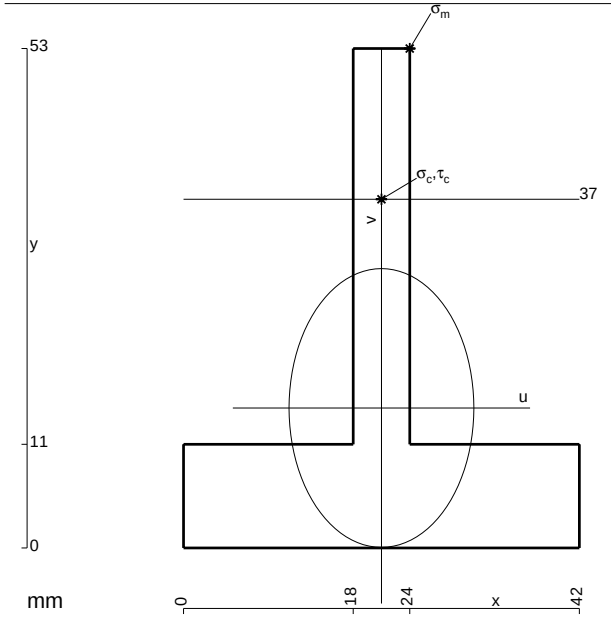
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

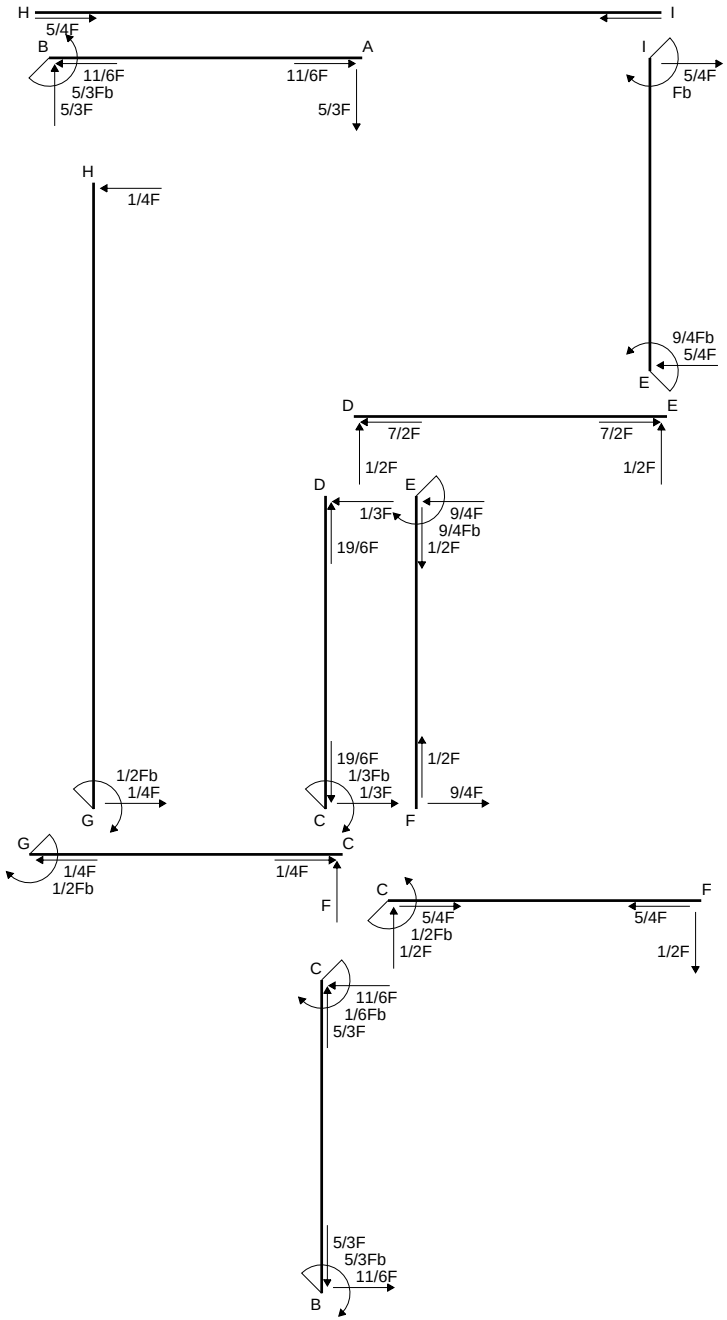
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





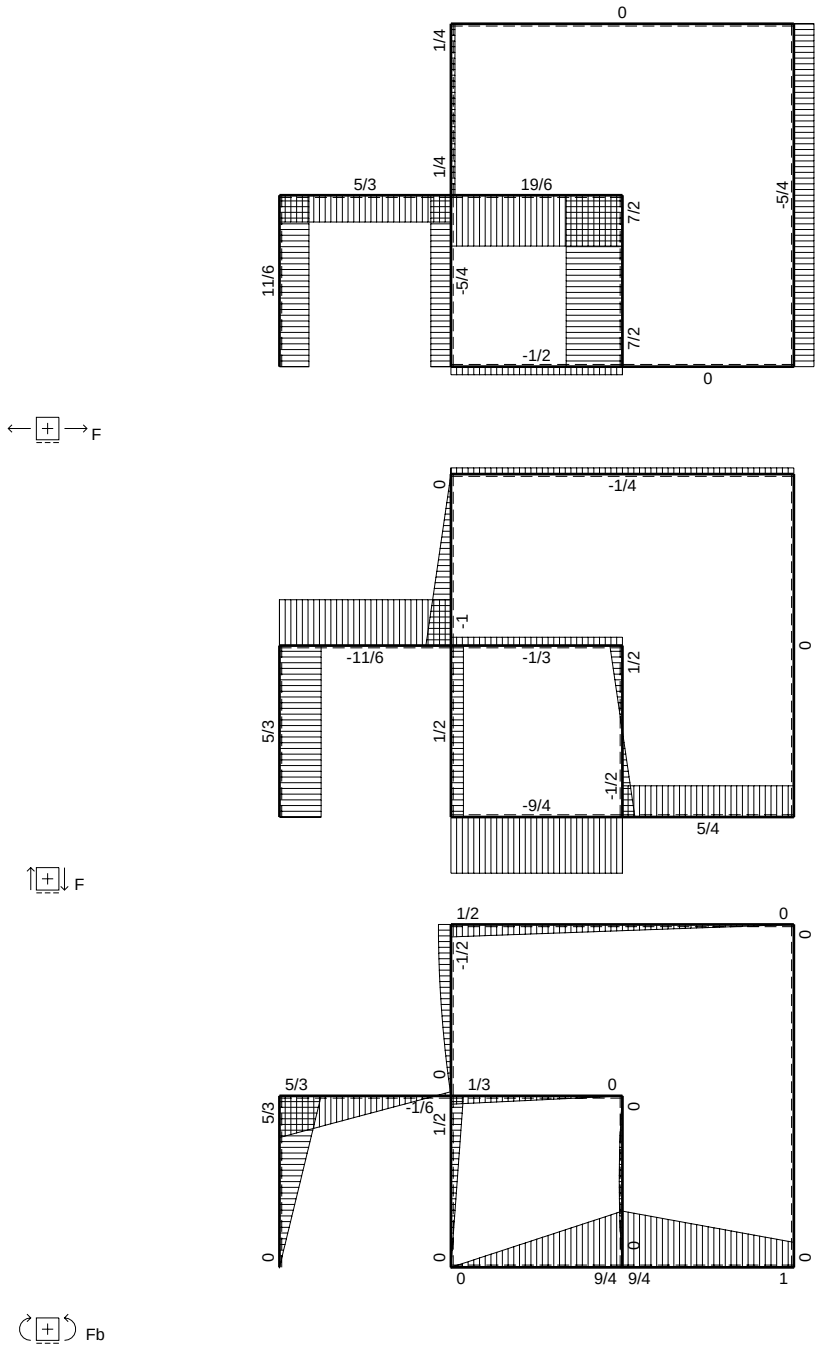


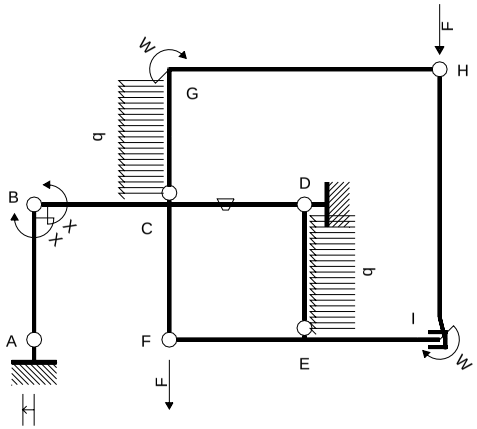

A = 714. $mm^2$
$J_u$ = 156211. $mm^4$
$J_v$ = 68670. $mm^4$
$y_g$ = 14.85 mm
N = -260. N
$T_y$ = -1170. N
$M_x$ = 971100. Nmm
$x_m$ = 24. mm
$y_m$ = 53. mm
$u_m$ = 3. mm
$v_m$ = 38.15 mm
$\sigma_m = N/A - Mv/J_u$ = -237.5 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 37. mm
$v_c$ = 22.15 mm
$\sigma_c = N/A - Mv/J_u$ = -138. $N/mm^2$
$\tau_c$ = 3.613 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.2 $N/mm^2$
$S^*$ = 2894. $mm^3$


H
I
5/4F
B
A
11/6F
5/3Fb
5/3F
11/6F
5/3F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
7/2F
1/2F
1/2F
D
E
1/3F
9/4F
9/4Fb
19/6F
1/2F
1/2Fb
1/4F
G
19/6F
1/3Fb
1/3F
C
1/2F
9/4F
F
G
C
1/4F
1/2Fb
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F
C
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1/2Fb
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11/6F
1/6Fb
5/3F
5/3F
5/3Fb
11/6F
B

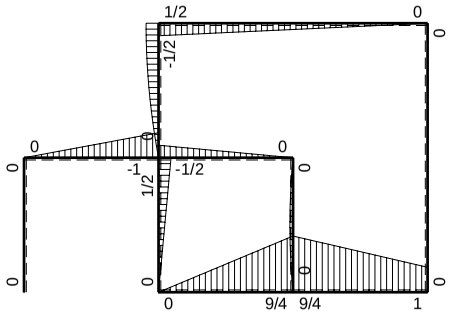




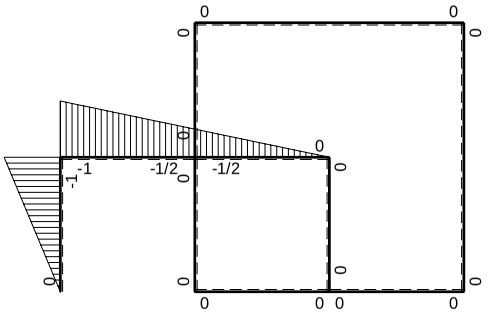

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F
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-1/4
-1
-11/6
-1/3
1/2
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5/3
1/2
-1/2
-9/4
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F
1/2
0
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0
5/3
1/3
0
5/3
-1/6
1/2
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 215080 CI01.xxxx.110

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PROCEDIMENTO E RISULTATI 215080 CI01.xxxx.110

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $5/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

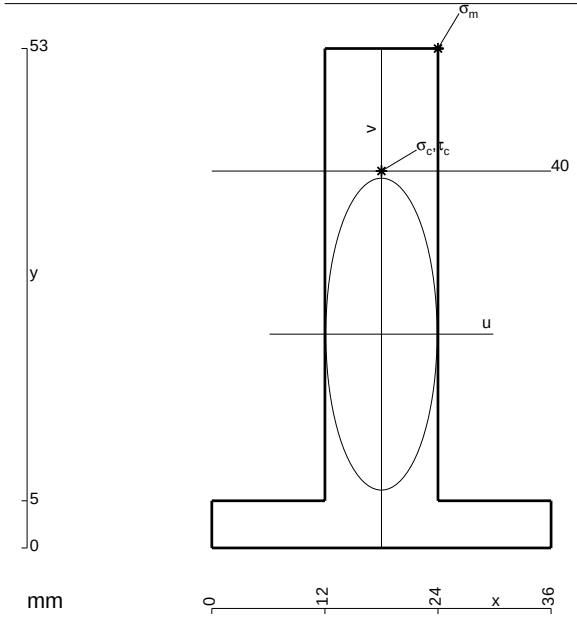
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/12\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

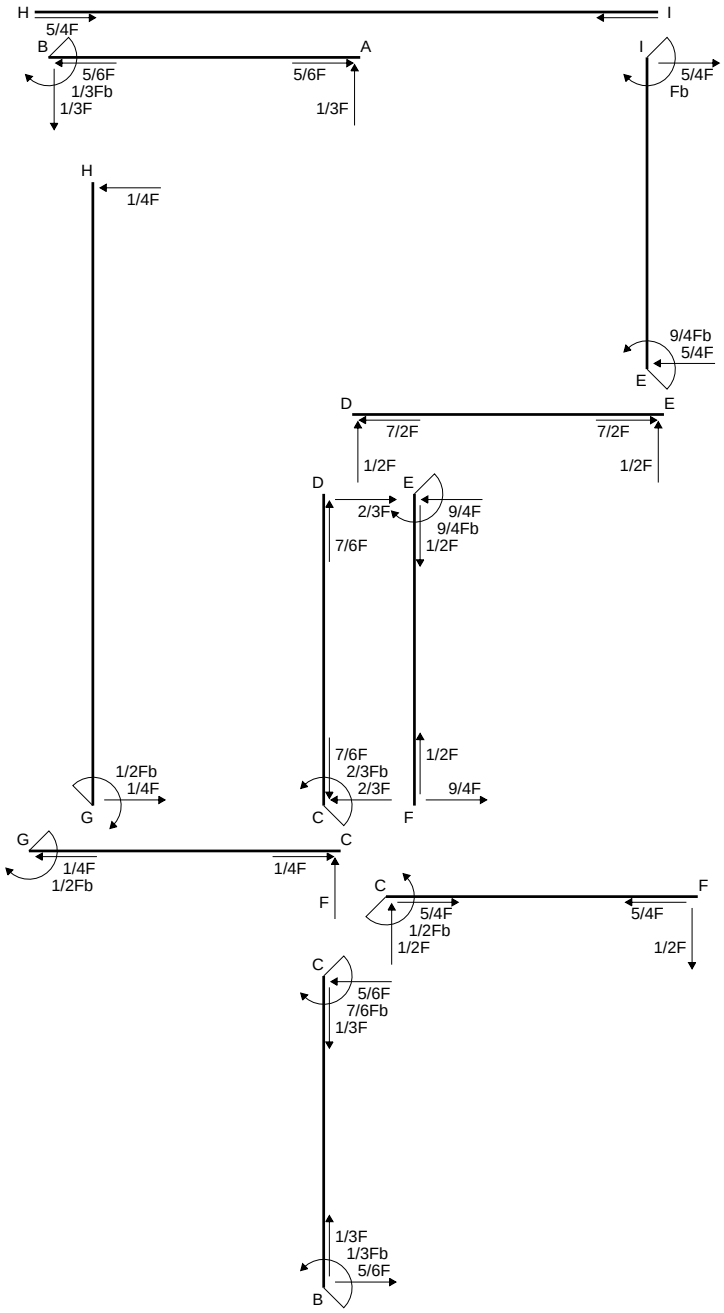


PROCEDIMENTO E RISULTATI 215080 CI01.xxxx.110



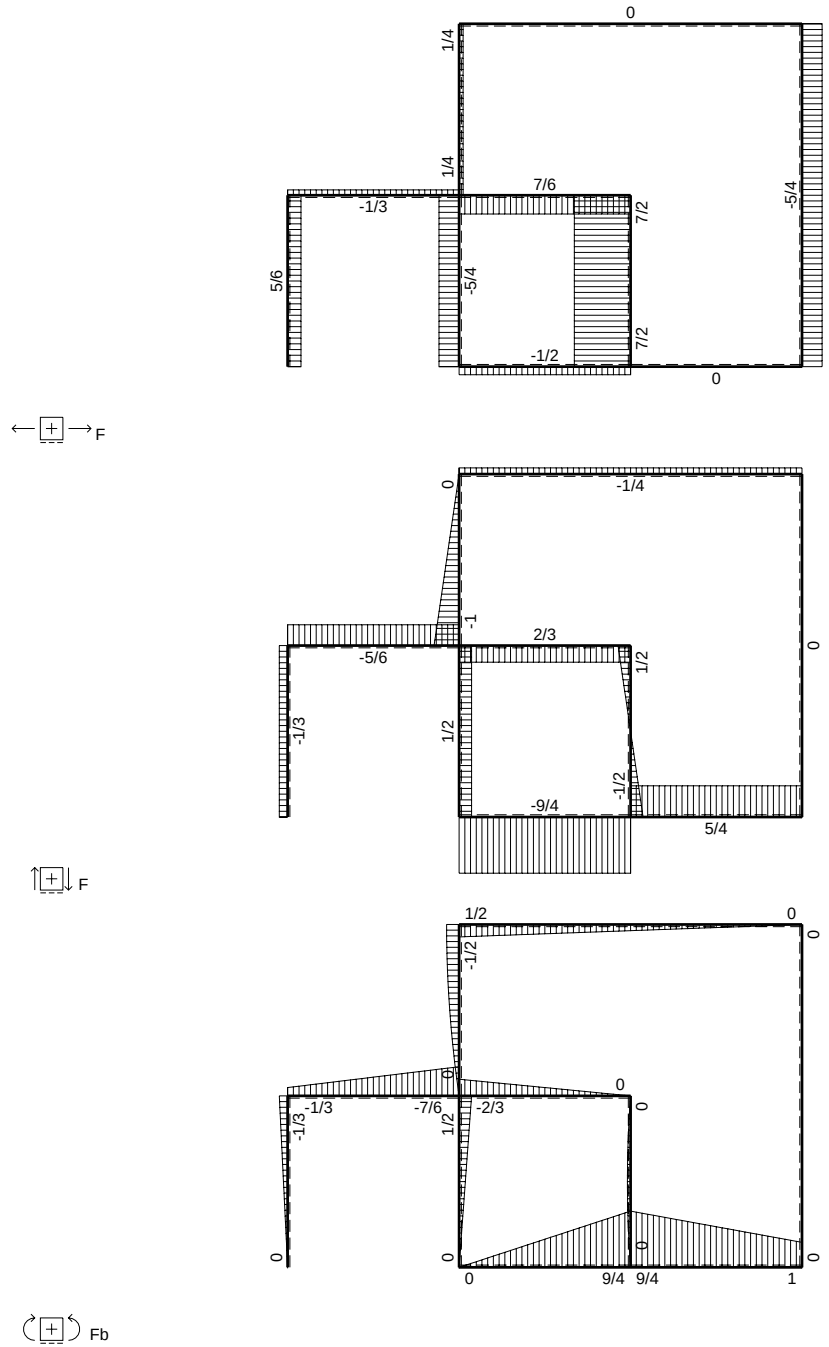

A = 756. $mm^2$
$J_u$ = 207276. $mm^4$
$J_v$ = 26352. $mm^4$
$y_g$ = 22.69 mm
N = -685. N
$T_y$ = -3083. N
$M_x$ = 1356300. Nmm
$x_m$ = 24. mm
$y_m$ = 53. mm
$u_m$ = 6. mm
$v_m$ = 30.31 mm
$\sigma_m = N/A - Mv/J_u$ = -199.2 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 40. mm
$v_c$ = 17.31 mm
$\sigma_c = N/A - Mv/J_u$ = -114.2 $N/mm^2$
$\tau_c$ = 4.603 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.4 $N/mm^2$
$S^*$ = 3714. $mm^3$

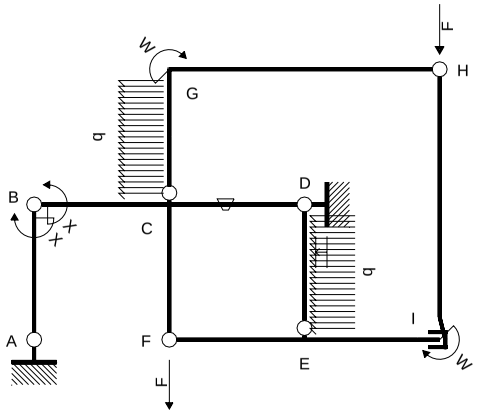
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H
I
5/4F
B
A
5/6F
1/3Fb
1/3F
5/6F
1/3F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
7/2F
1/2F
1/2F
D
E
2/3F
9/4F
9/4Fb
7/6F
1/2F
1/2Fb
1/4F
G
7/6F
2/3Fb
2/3F
C
1/2F
F
9/4F
G
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5/6F
B

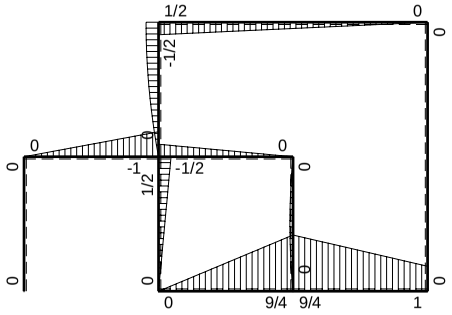




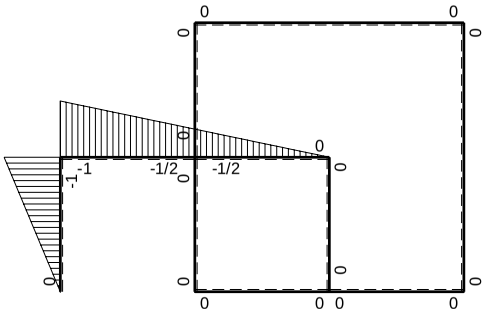

0
1/4
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7/6
-1/3
7/2
-5/4
5/6
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7/2
-1/2
0
F
0
-1/4
-1
2/3
-5/6
1/2
0
-1/3
1/2
-1/2
-9/4
5/4
F
1/2
0
0
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0
-1/3
-1/3
-7/6
-2/3
1/2
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 266284 CI01.xxxx.111

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

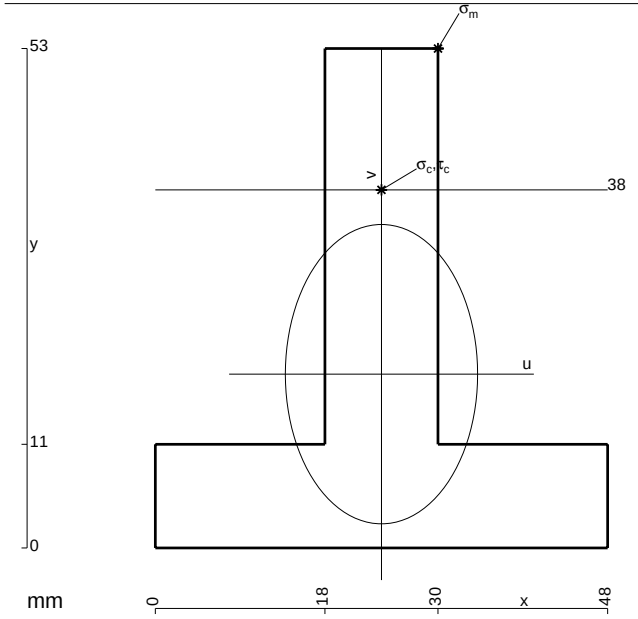
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/12\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

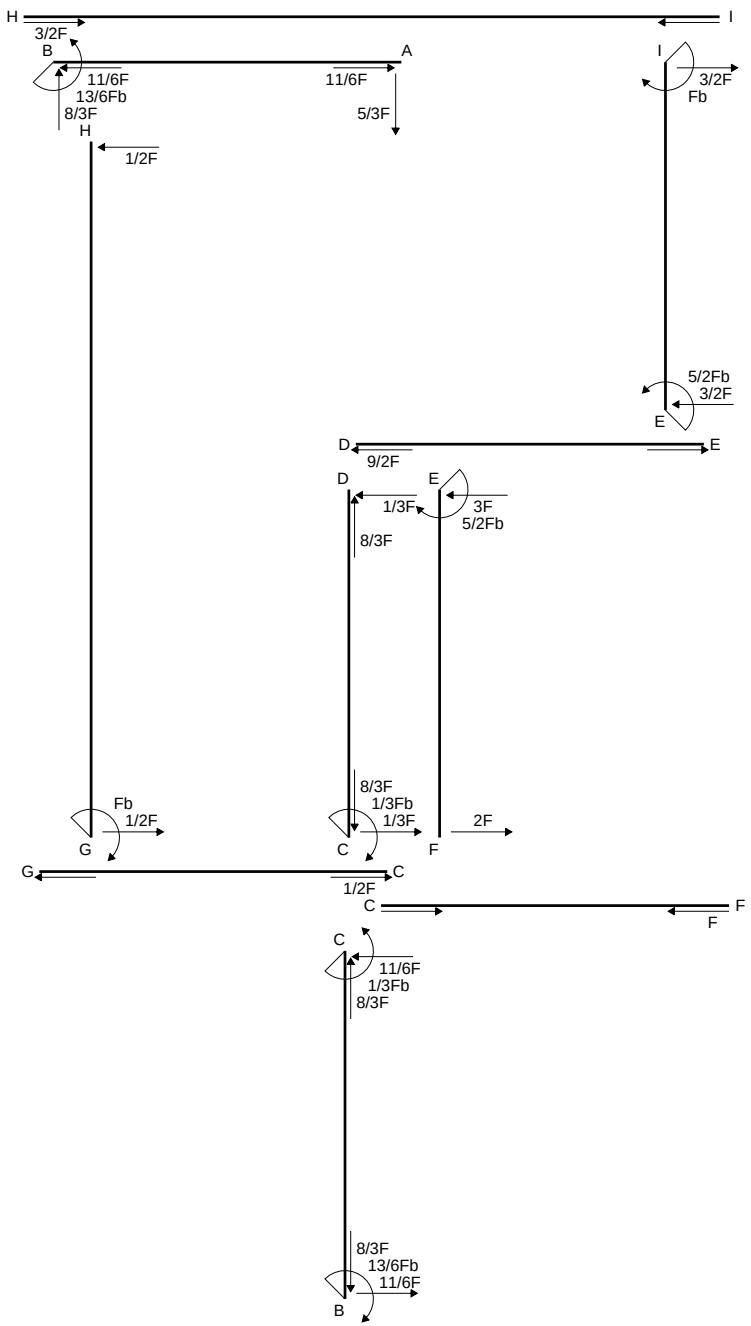


PROCEDIMENTO E RISULTATI 266284 CI01.xxxx.111



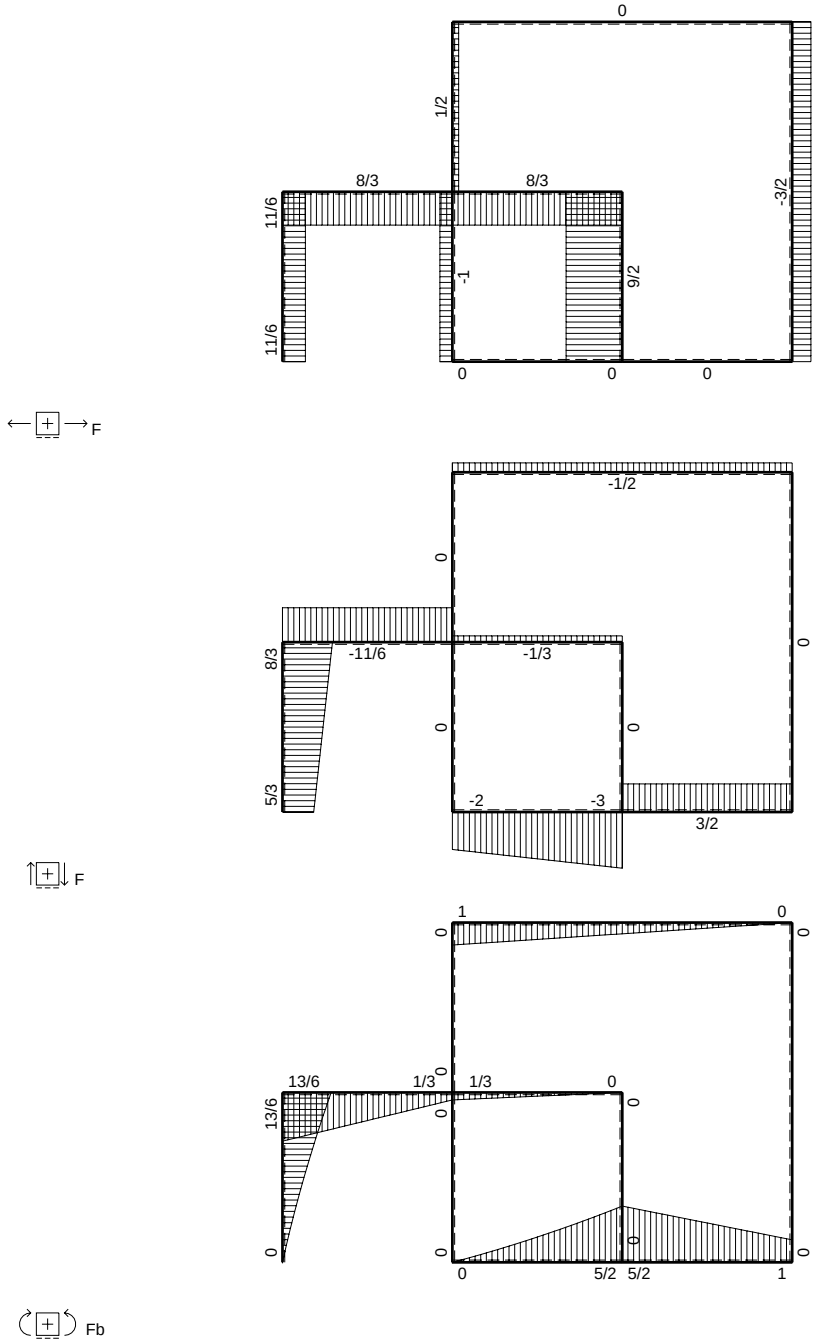

A = 1032. $mm^2$
$J_u$ = 260495. $mm^4$
$J_v$ = 107424. $mm^4$
$y_g$ = 18.44 mm
N = -730. N
$T_y$ = -3285. N
$M_x$ = 1576800. Nmm
$x_m$ = 30. mm
$y_m$ = 53. mm
$u_m$ = 6. mm
$v_m$ = 34.56 mm
$\sigma_m = N/A - Mv/J_u$ = -209.9 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 38. mm
$v_c$ = 19.56 mm
$\sigma_c = N/A - Mv/J_u$ = -119.1 $N/mm^2$
$\tau_c$ = 5.118 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.4 $N/mm^2$
$S^*$ = 4870. $mm^3$

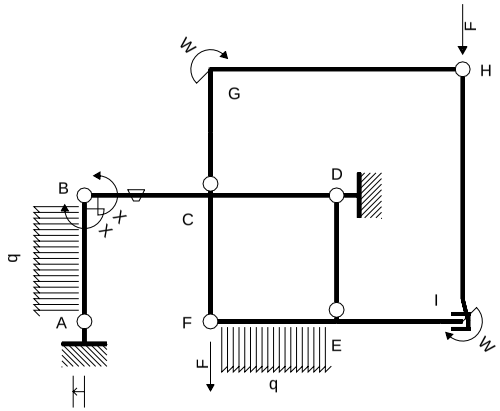
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H
I
3/2F
B
A
11/6F
13/6Fb
8/3F
11/6F
5/3F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
1/3F
3F
5/2Fb
8/3F
8/3F
1/3Fb
Fb
1/2F
1/3F
2F
G
C
F
G
C
1/2F
C
F
F
C
11/6F
1/3Fb
8/3F
8/3F
13/6Fb
11/6F
B

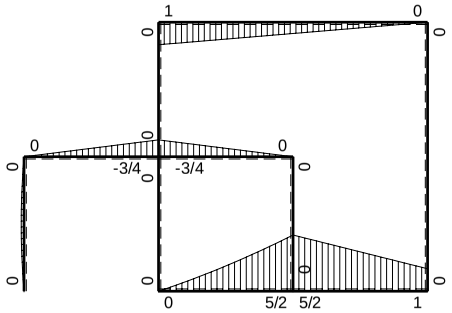




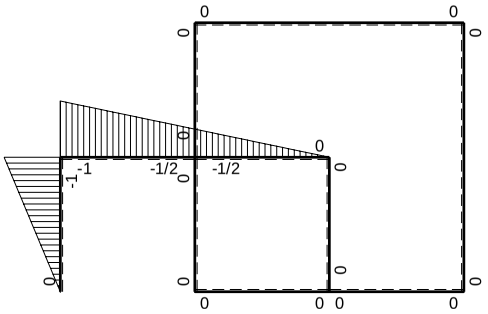

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8/3
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11/6
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0
0
0
13/6
1/3
1/3
0
13/6
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 270433 CI01.xxxx.112

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PROCEDIMENTO E RISULTATI 270433 CI01.xxxx.112

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

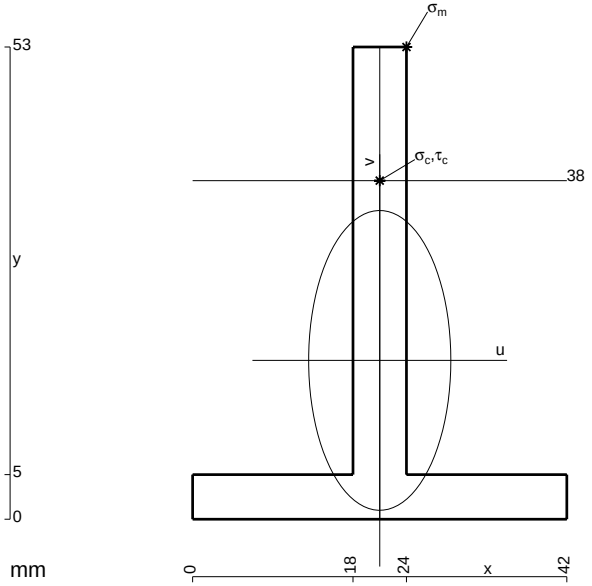
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

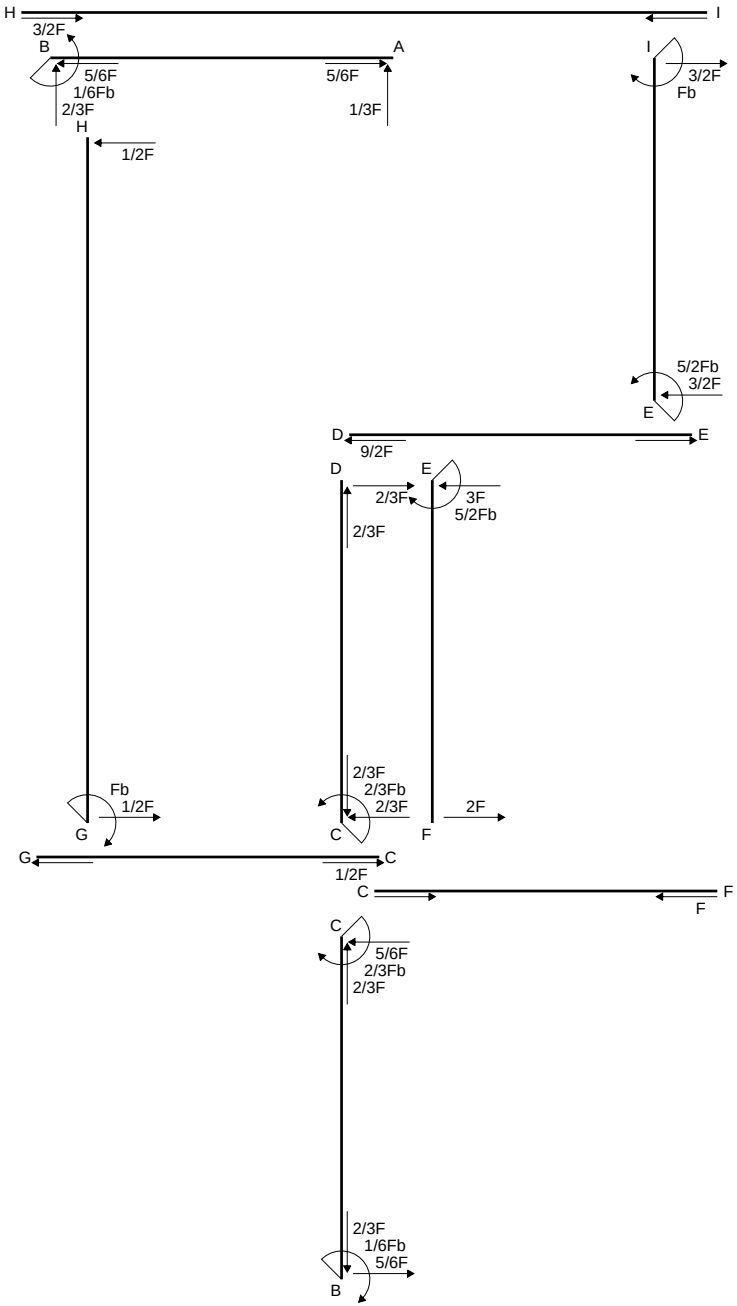


PROCEDIMENTO E RISULTATI 270433 CI01.xxxx.112



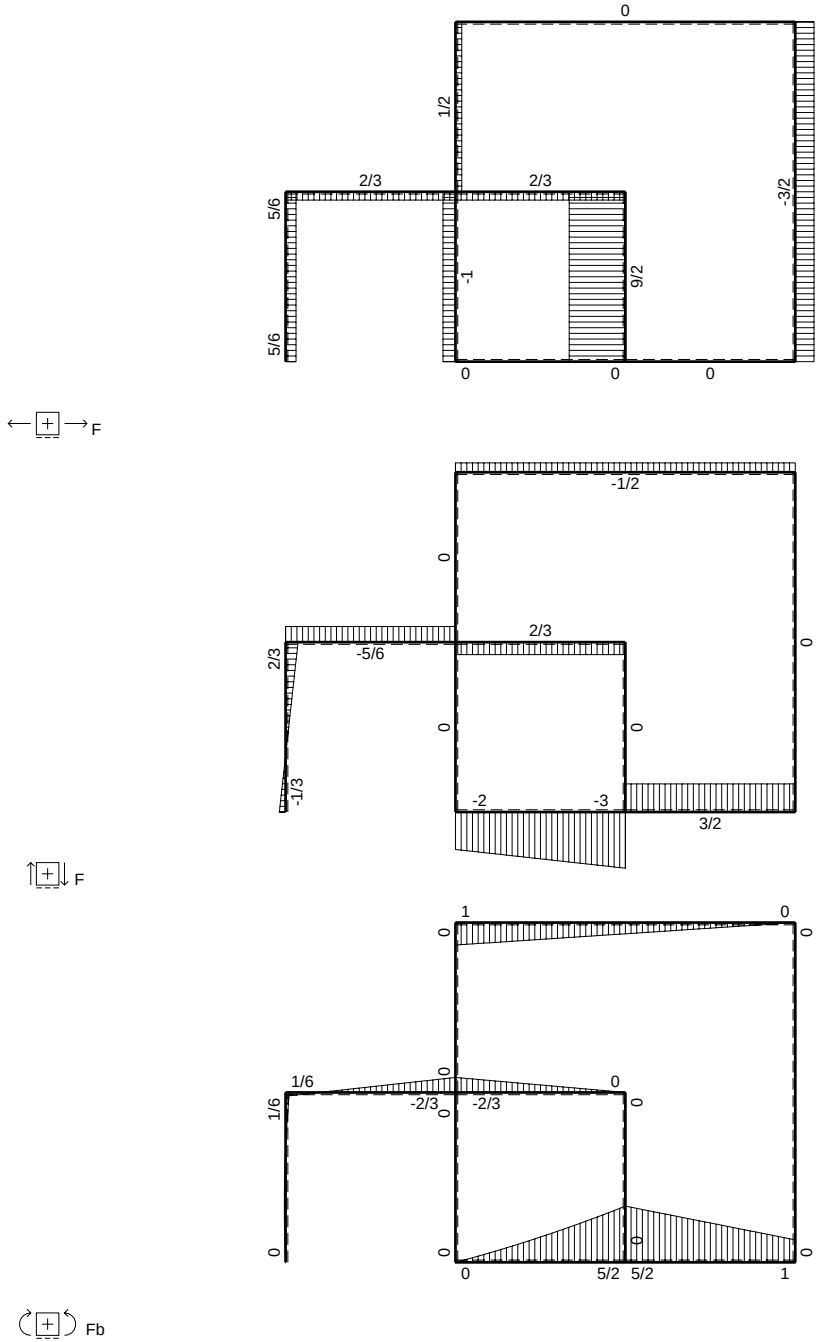

A = 498. $mm^2$
$J_u$ = 141019. $mm^4$
$J_v$ = 31734. $mm^4$
$y_g$ = 17.83 mm
$T_y$ = -2010. N
$M_x$ = 871000. Nmm
$x_m$ = 24. mm
$y_m$ = 53. mm
$u_m$ = 3. mm
$v_m$ = 35.17 mm
$\sigma_m = -Mv/J_u$ = -217.3 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 38. mm
$v_c$ = 20.17 mm
$\sigma_c = -Mv/J_u$ = -124.6 $N/mm^2$
$\tau_c$ = 5.917 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125. $N/mm^2$
$S^*$ = 2491. $mm^3$

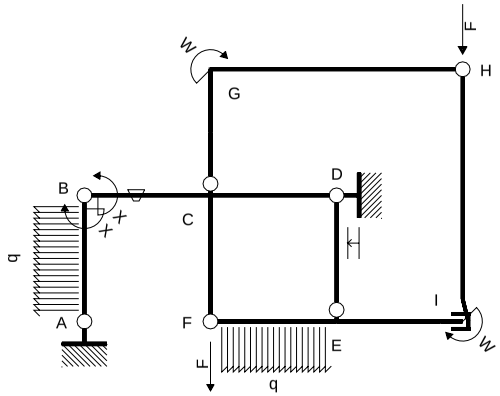
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H
3/2F
B
5/6F
1/6Fb
2/3F
A
5/6F
1/3F
I
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
9/2F
E
D
E
2/3F
3F
5/2Fb
2/3F
2/3F
2/3Fb
2/3F
Fb
1/2F
G
C
F
2F
G
C
1/2F
C
F
F
C
5/6F
2/3Fb
2/3F
2/3F
1/6Fb
5/6F
B

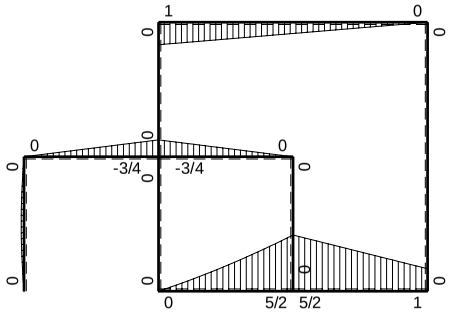




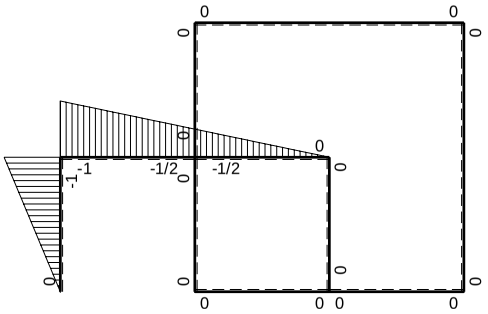

F
0
1/2
2/3
2/3
-3/2
5/6
-1
9/2
5/6
0
0
0
F
-1/2
0
2/3
-5/6
2/3
0
0
0
0
-1/3
-2
-3
3/2
F
1
0
0
0
1/6
0
-2/3
-2/3
1/6
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265736 CI01.xxxx.113

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-3/4Fb+3/4Fx$ | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 265736 CI01.xxxx.113

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

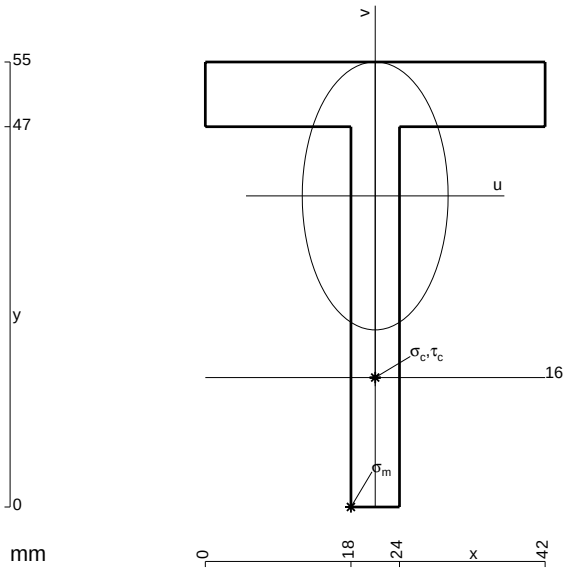
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

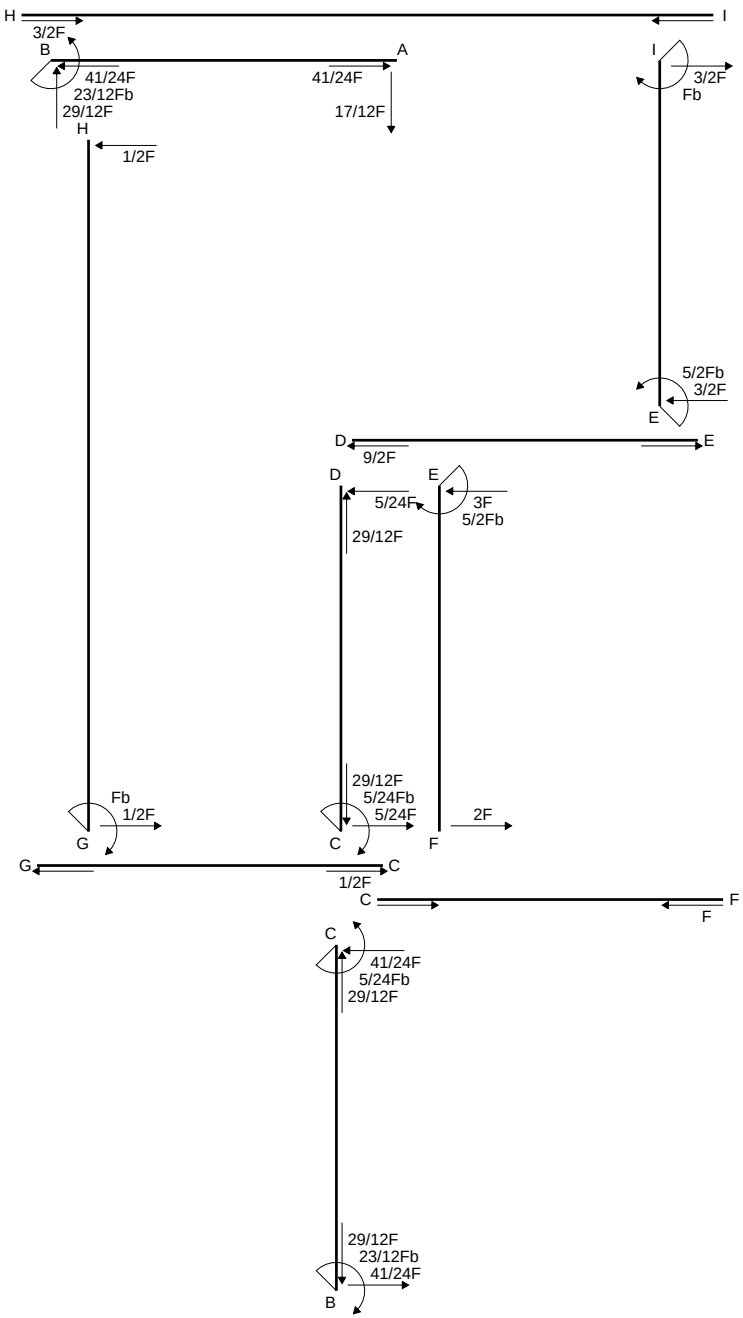
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 265736 CI01.xxxx.113

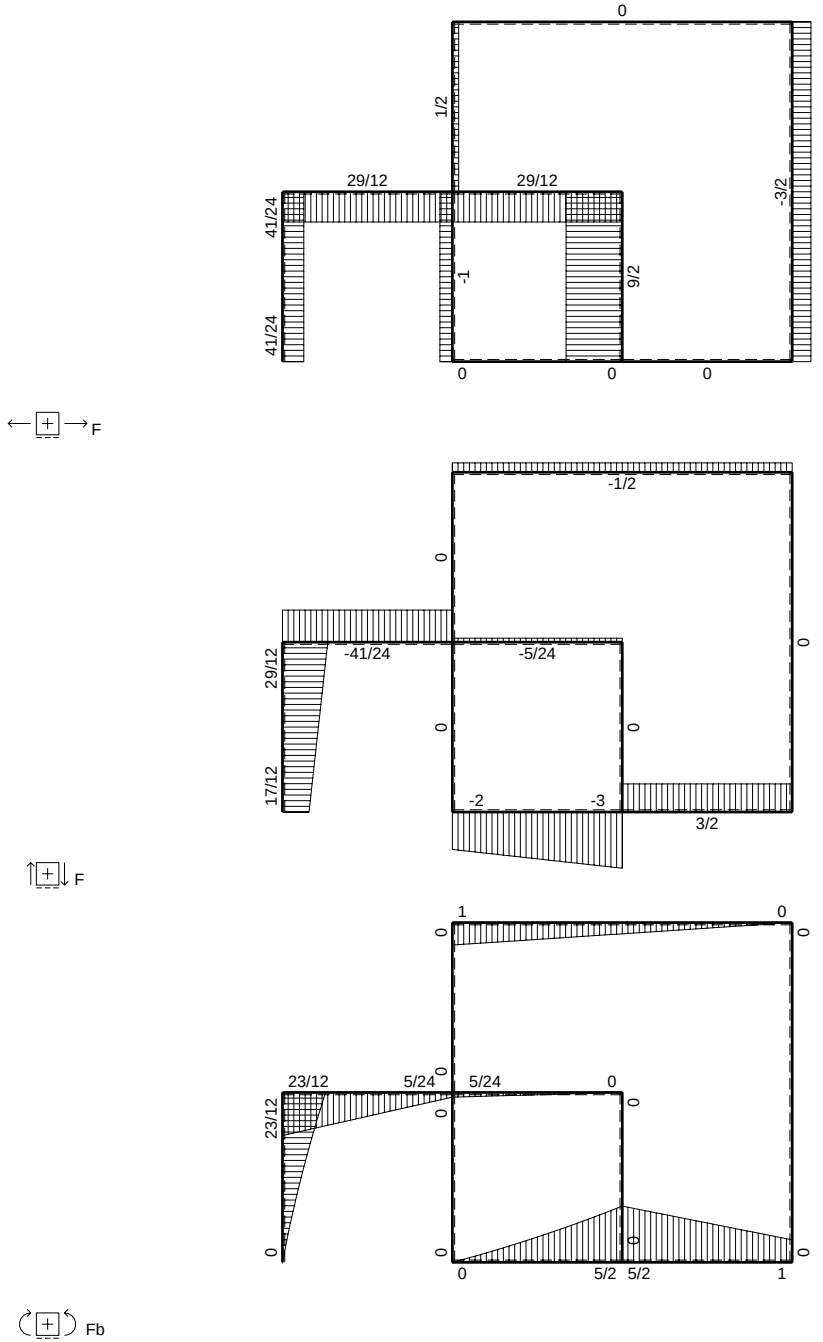


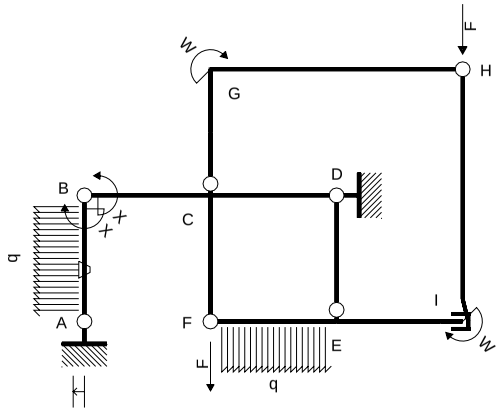

A = 618. $mm^2$
$J_u$ = 169652. $mm^4$
$J_v$ = 50238. $mm^4$
$y_g$ = 38.45 mm
$T_y$ = -2070. N
$M_x$ = 1000500. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -38.45 mm
$\sigma_m = -Mv/J_u$ = 226.8 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -22.45 mm
$\sigma_c = -Mv/J_u$ = 132.4 $N/mm^2$
$\tau_c$ = 5.945 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.8 $N/mm^2$
$S^*$ = 2923. $mm^3$


H
I
3/2F
B
A
41/24F
23/12Fb
29/12F
41/24F
17/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
5/24F
3F
5/2Fb
29/12F
Fb
1/2F
29/12F
5/24Fb
5/24F
2F
G
C
F
G
C
1/2F
C
F
F
C
41/24F
5/24Fb
29/12F
29/12F
23/12Fb
41/24F
B

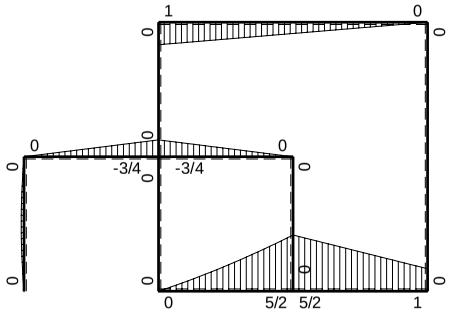




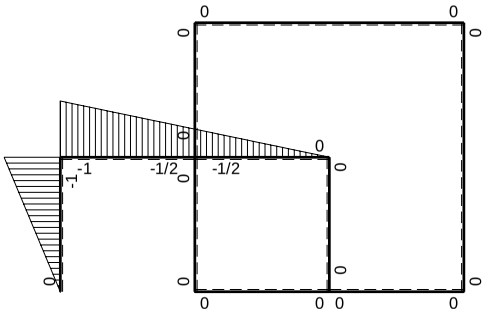

0
1/2
29/12
29/12
-3/2
41/24
-1
9/2
41/24
0
0
0
F
-1/2
0
29/12
-41/24
-5/24
0
0
0
17/12
-2
-3
3/2
F
1
0
0
0
0
23/12
5/24
5/24
0
0
23/12
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 939153 CI01.xxxx.114

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PROCEDIMENTO E RISULTATI 939153 CI01.xxxx.114

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b ( x/b )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

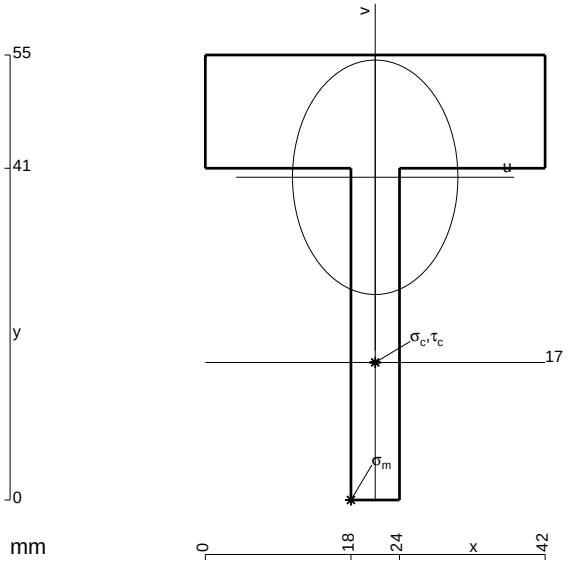
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

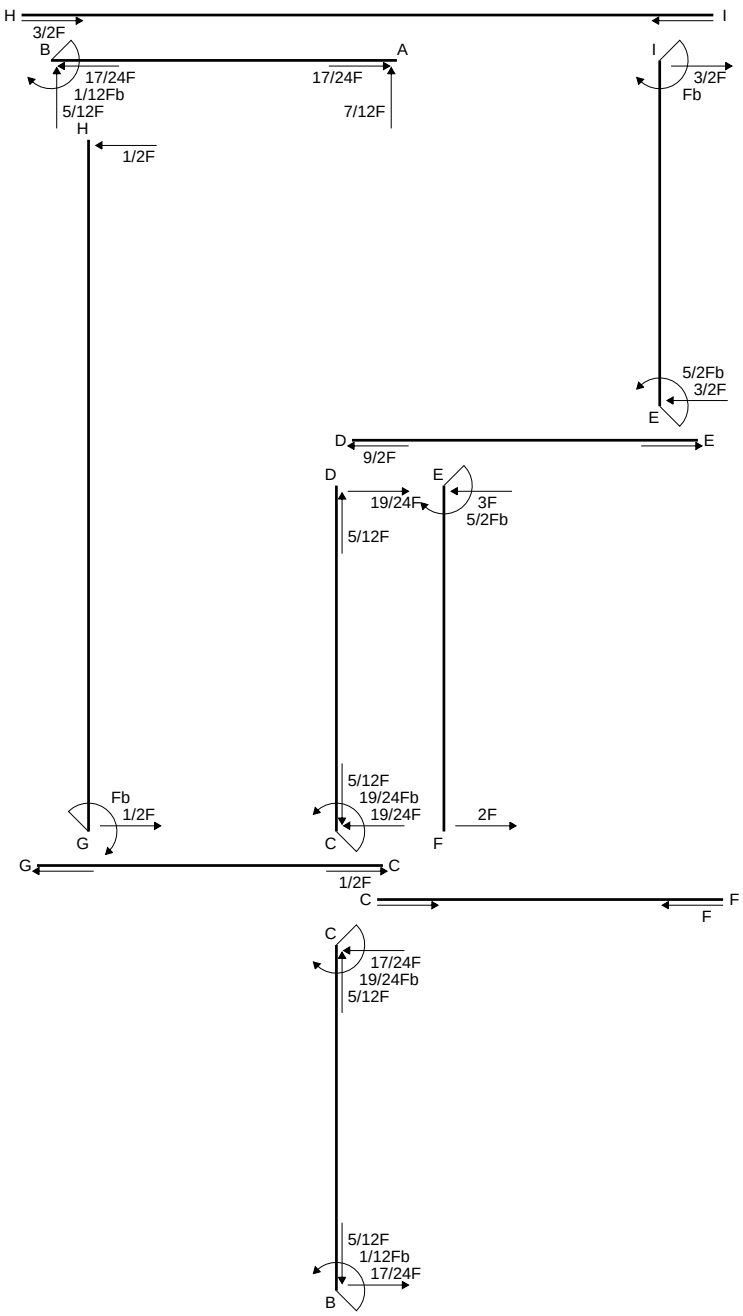


PROCEDIMENTO E RISULTATI 939153 CI01.xxxx.114



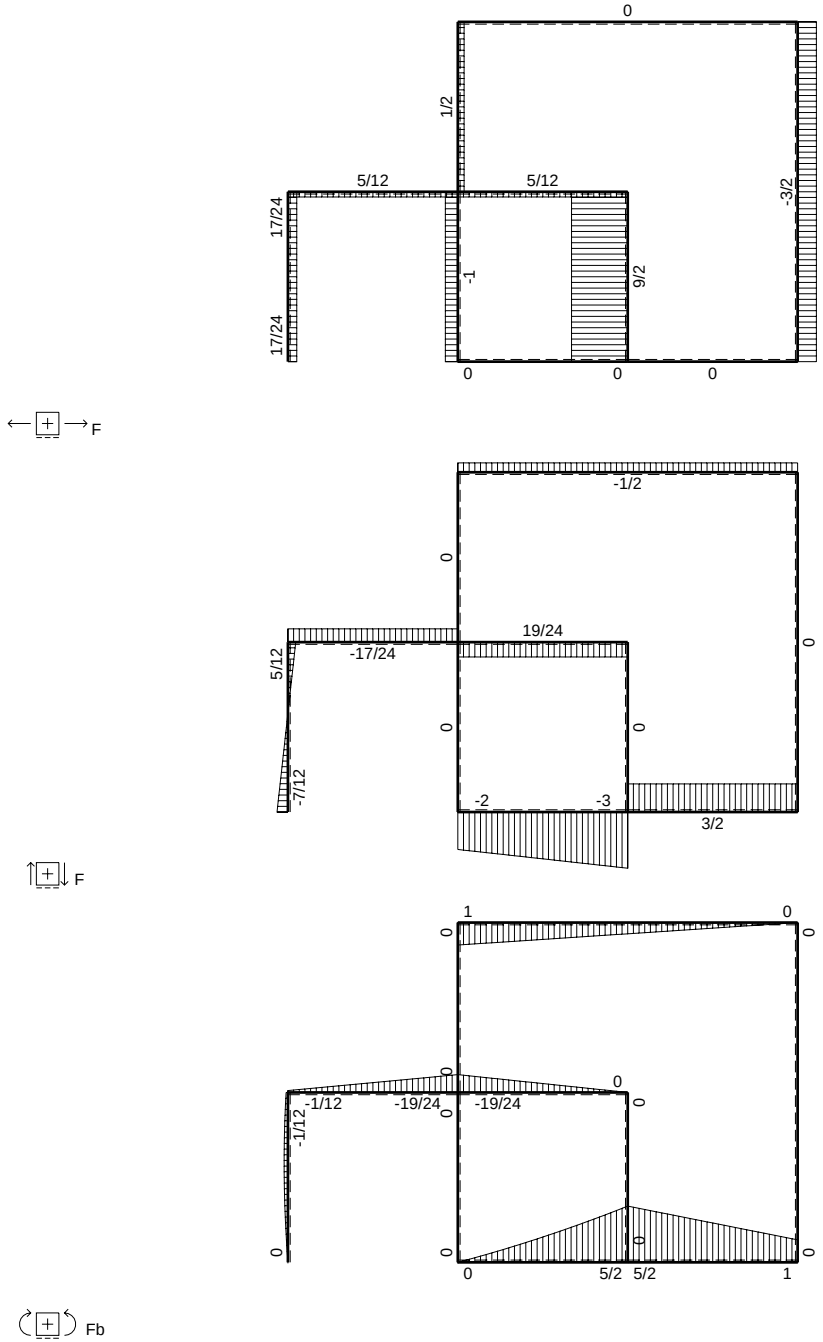

A = 834. $mm^2$
$J_u$ = 175228. $mm^4$
$J_v$ = 87174. $mm^4$
$y_g$ = 39.89 mm
$T_y$ = -1980. N
$M_x$ = 1039500. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -39.89 mm
$\sigma_m = -Mv/J_u$ = 236.6 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 17. mm
$v_c$ = -22.89 mm
$\sigma_c = -Mv/J_u$ = 135.8 $N/mm^2$
$\tau_c$ = 6.03 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.2 $N/mm^2$
$S^*$ = 3202. $mm^3$

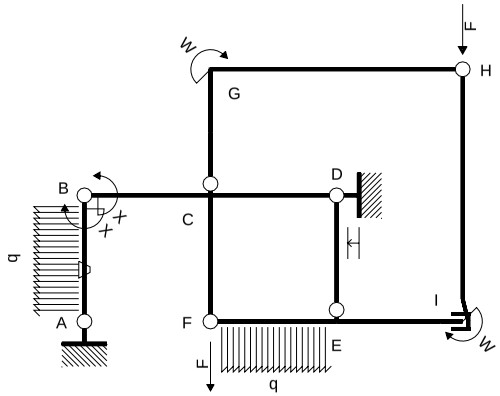
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H
I
3/2F
B
A
17/24F
1/12Fb
5/12F
17/24F
7/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
19/24F
3F
5/2Fb
5/12F
Fb
1/2F
5/12F
19/24Fb
19/24F
2F
G
C
F
G
C
1/2F
C
F
F
C
17/24F
19/24Fb
5/12F
5/12F
1/12Fb
17/24F
B

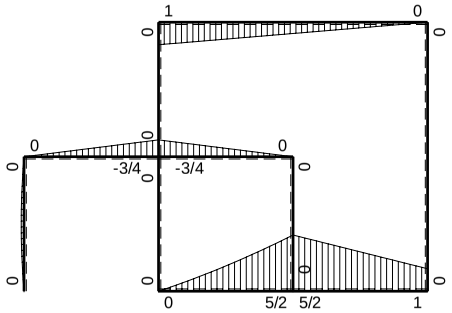




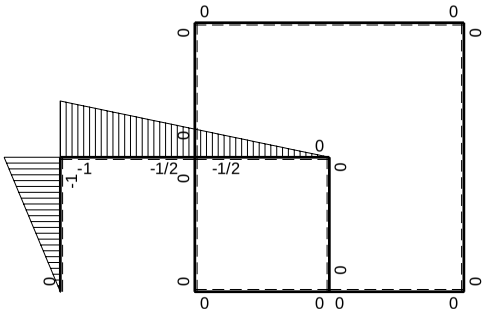

0
1/2
5/12
5/12
-3/2
17/24
-1
9/2
17/24
0
0
0
F
-1/2
0
19/24
-17/24
5/12
0
0
0
-7/12
-2
-3
3/2
F
1
0
0
0
0
0
-1/12
-19/24
-19/24
-1/12
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264484 CI01.xxxx.115

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PROCEDIMENTO E RISULTATI 264484 CI01.xxxx.115

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b ( x/b )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3 )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

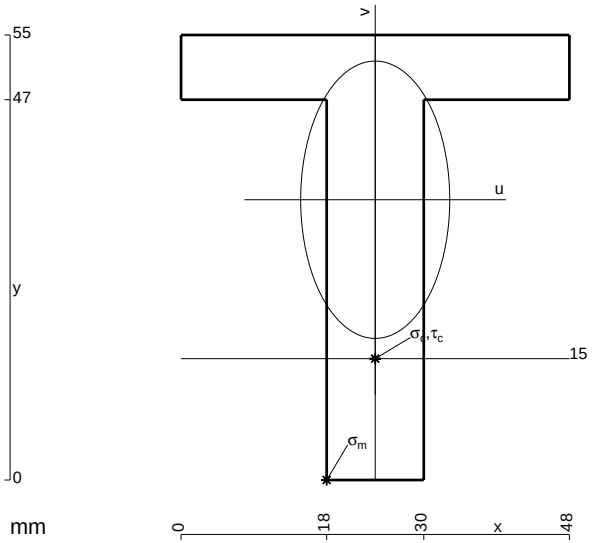
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

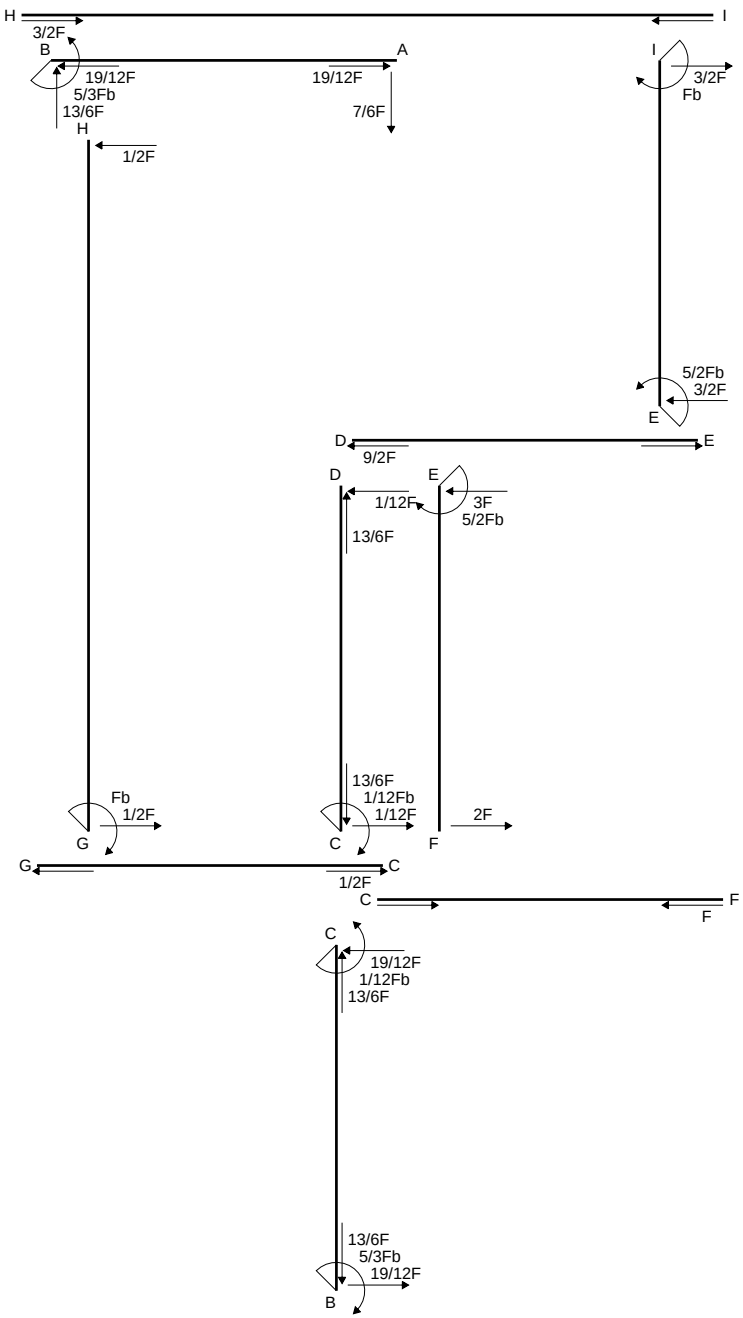
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





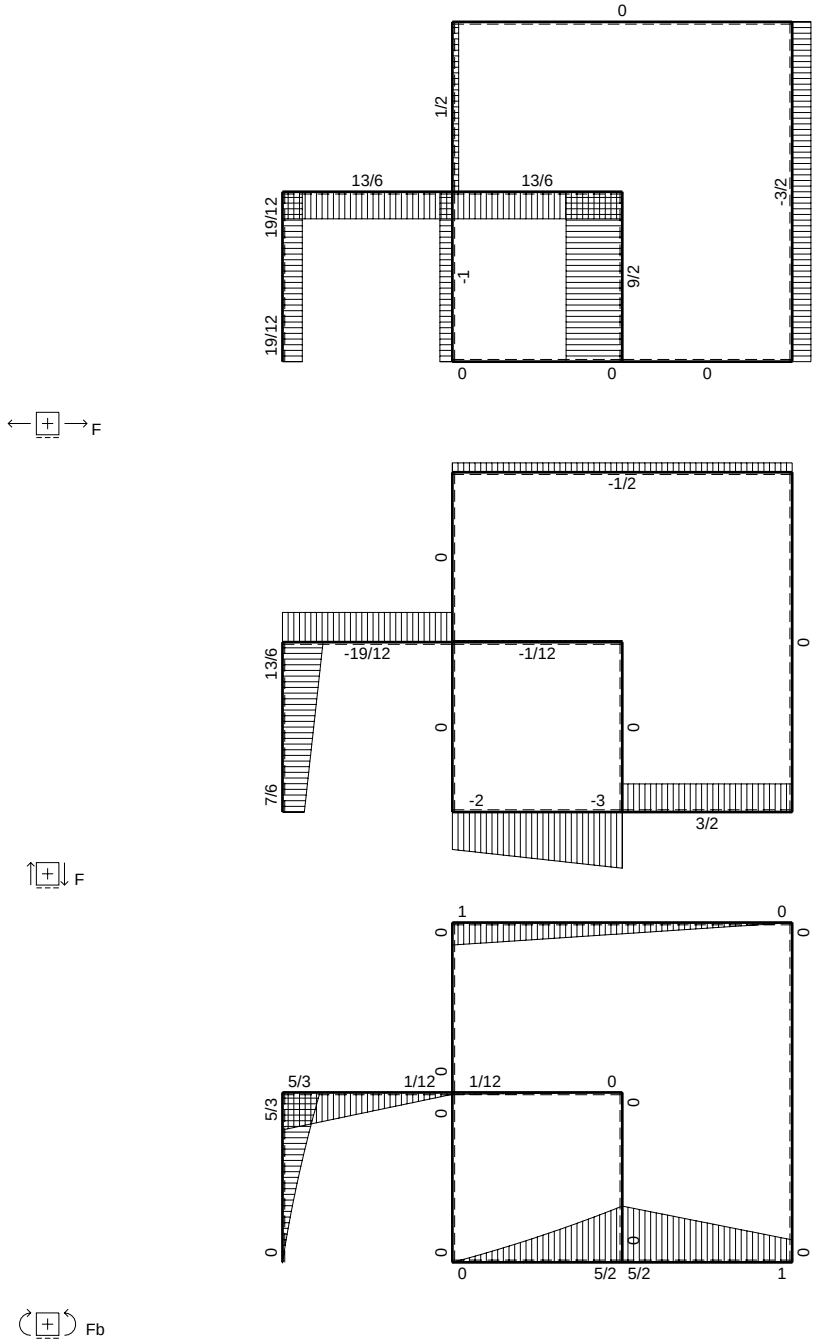


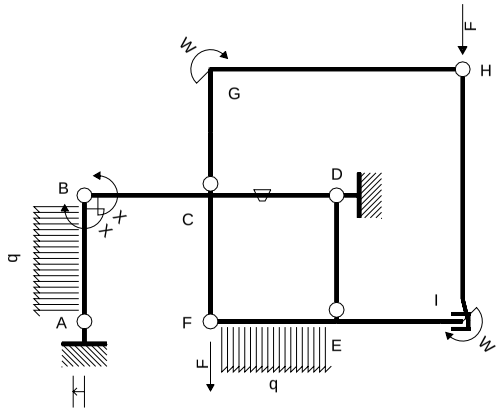

A = 948. $mm^2$
$J_u$ = 278641. $mm^4$
$J_v$ = 80496. $mm^4$
$y_g$ = 34.64 mm
$T_y$ = -2880. N
$M_x$ = 1608000. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -34.64 mm
$\sigma_m = -Mv/J_u$ = 199.9 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -19.64 mm
$\sigma_c = -Mv/J_u$ = 113.3 $N/mm^2$
$\tau_c$ = 4.208 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.6 $N/mm^2$
$S^*$ = 4885. $mm^3$


H
I
3/2F
B
A
19/12F
5/3Fb
13/6F
19/12F
7/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
1/12F
3F
5/2Fb
13/6F
13/6F
1/12Fb
1/12F
Fb
1/2F
G
C
F
2F
G
C
1/2F
C
F
F
C
19/12F
1/12Fb
13/6F
13/6F
5/3Fb
19/12F
B

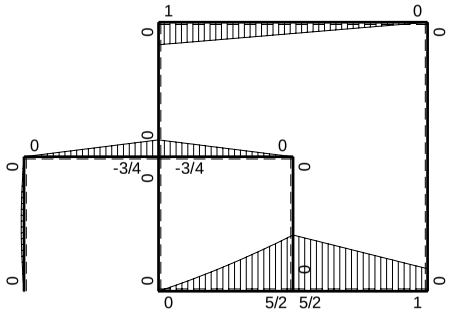




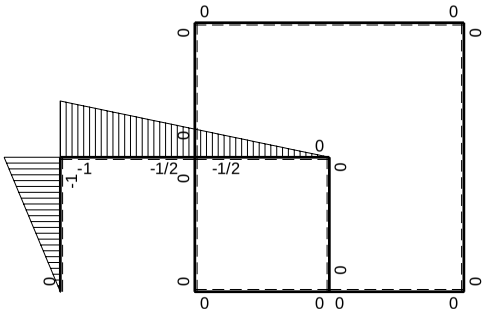

0
1/2
13/6
13/6
-3/2
19/12
-1
9/2
19/12
0
0
0
F
-1/2
0
13/6
-19/12
-1/12
0
7/6
0
0
-2
-3
3/2
F
1
0
0
0
5/3
1/12
1/12
0
5/3
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 990867 CI01.xxxx.116

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $5/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/3Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 990867 CI01.xxxx.116

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

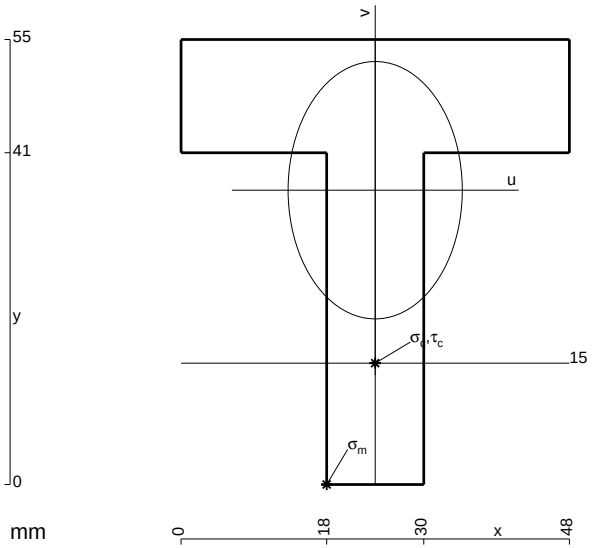
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

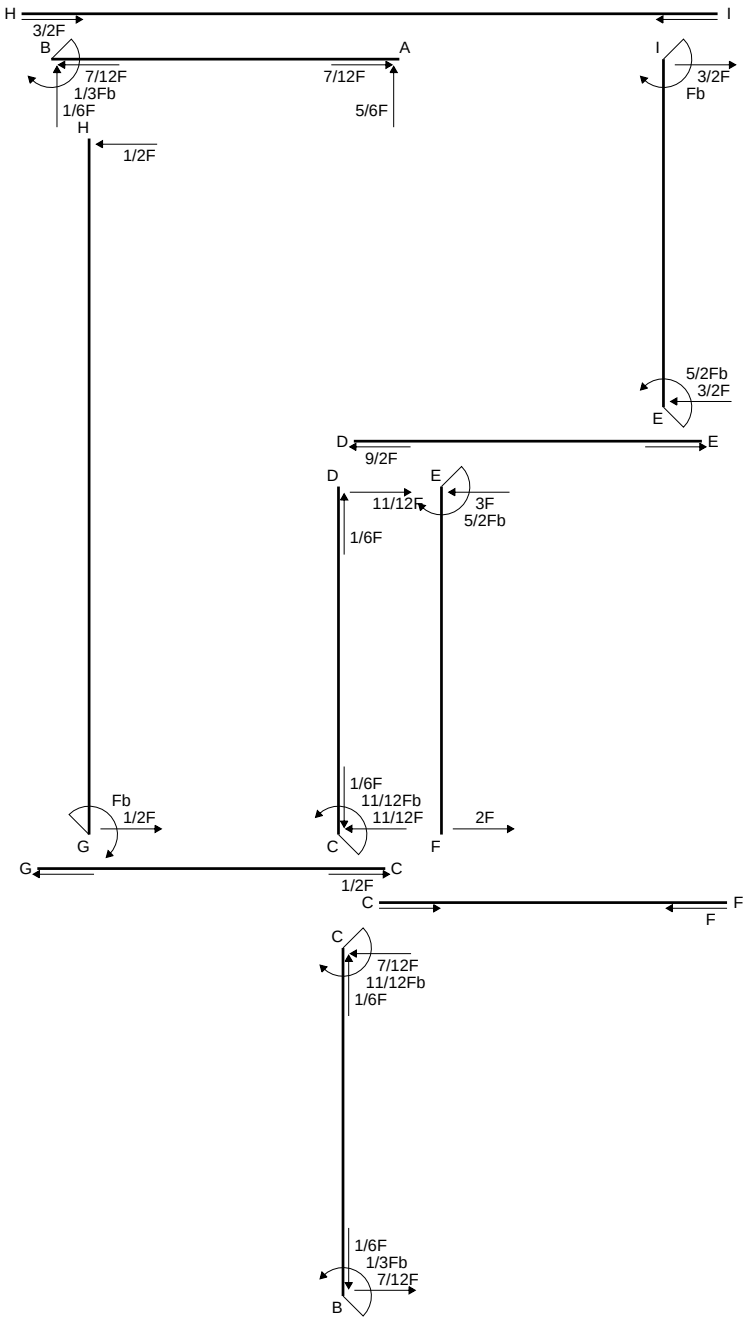


PROCEDIMENTO E RISULTATI 990867 CI01.xxxx.116



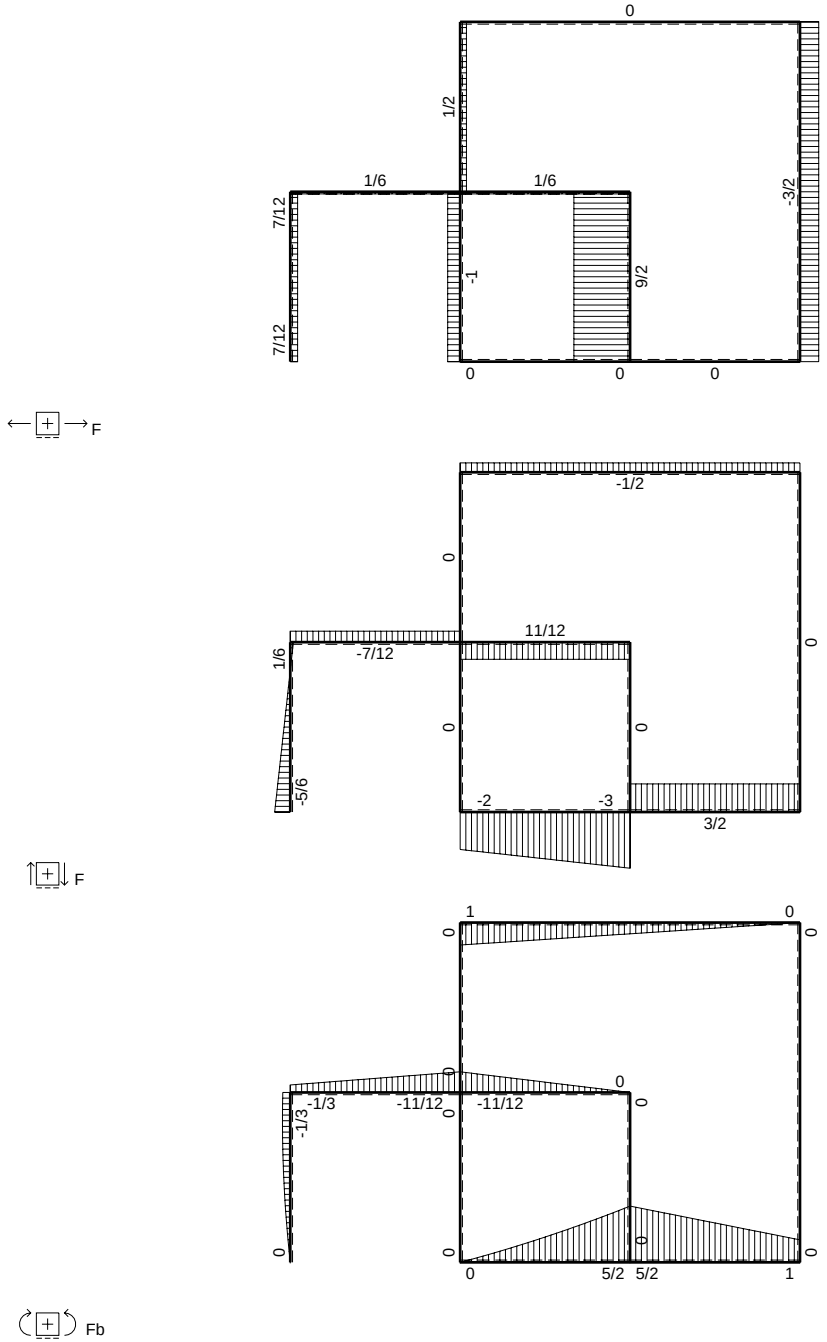

A = 1164. $mm^2$
$J_u$ = 294703. $mm^4$
$J_v$ = 134928. $mm^4$
$y_g$ = 36.38 mm
$T_y$ = -2820. N
$M_x$ = 1692000. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -36.38 mm
$\sigma_m = -Mv/J_u$ = 208.9 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -21.38 mm
$\sigma_c = -Mv/J_u$ = 122.7 $N/mm^2$
$\tau_c$ = 4.145 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 122.9 $N/mm^2$
$S^*$ = 5198. $mm^3$

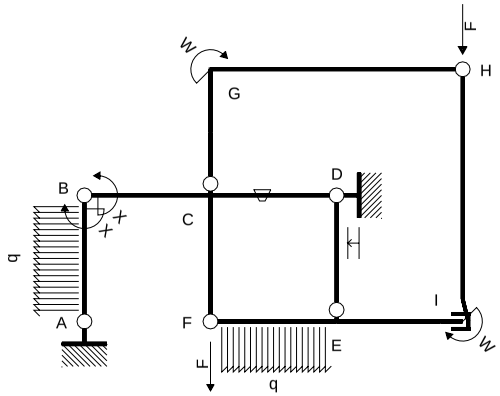
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
3/2F
B
A
7/12F
7/12F
1/3Fb
1/6F
5/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
11/12F
3F
5/2Fb
1/6F
1/6F
11/12Fb
11/12F
Fb
1/2F
2F
G
C
F
G
C
1/2F
C
F
F
C
7/12F
11/12Fb
1/6F
1/6F
1/3Fb
7/12F
B

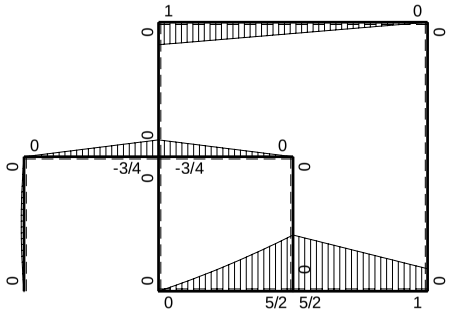




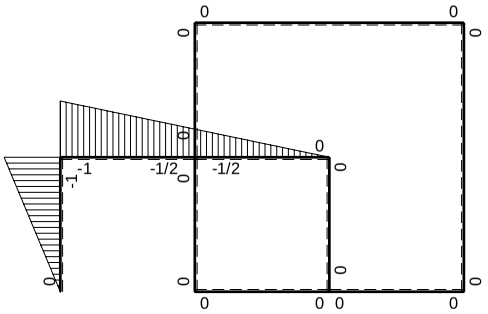

0
1/2
1/6
1/6
-3/2
7/12
-1
9/2
7/12
0
0
0
F
-1/2
0
11/12
1/6
-7/12
0
0
0
-5/6
-2
-3
3/2
F
1
0
0
0
0
0
-1/3
-1/3
-11/12
-11/12
0
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 247897 CI01.xxxx.117

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PROCEDIMENTO E RISULTATI 247897 CI01.xxxx.117

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

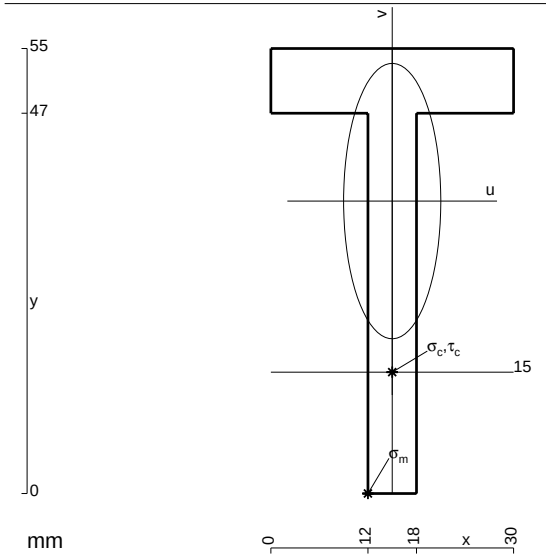
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

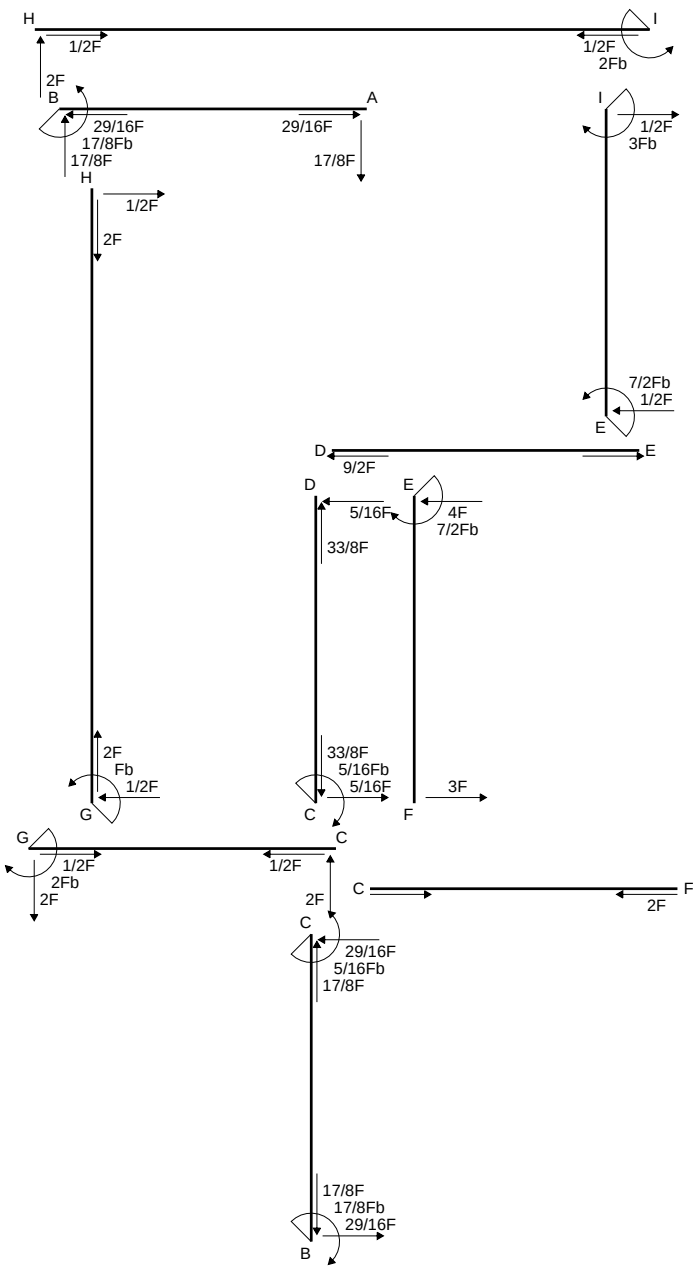


PROCEDIMENTO E RISULTATI 247897 CI01.xxxx.117



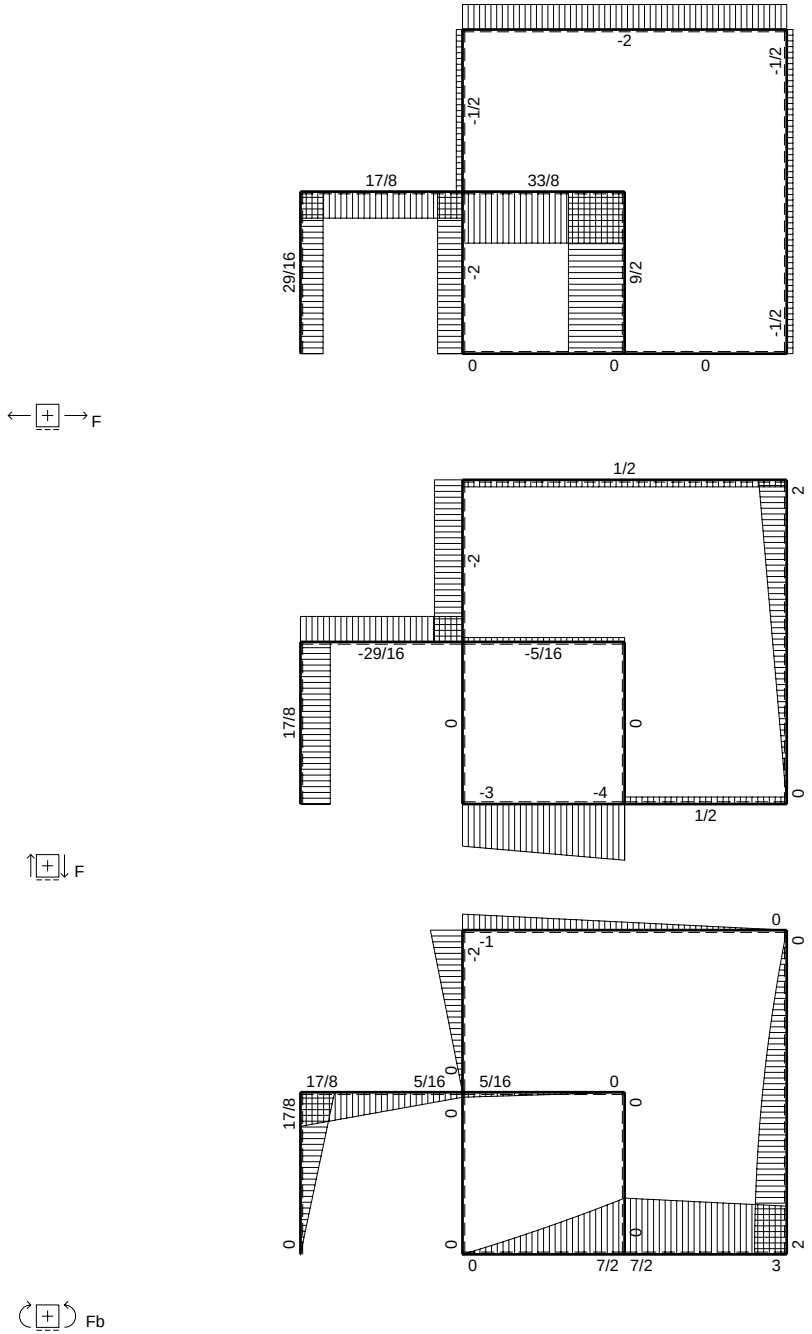

A = 522. $mm^2$
$J_u$ = 151243. $mm^4$
$J_v$ = 18846. $mm^4$
$y_g$ = 36.14 mm
$T_y$ = -1440. N
$M_x$ = 912000. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -36.14 mm
$\sigma_m = -Mv/J_u$ = 217.9 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 15. mm
$v_c$ = -21.14 mm
$\sigma_c = -Mv/J_u$ = 127.5 $N/mm^2$
$\tau_c$ = 4.091 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 127.7 $N/mm^2$
$S^*$ = 2578. $mm^3$

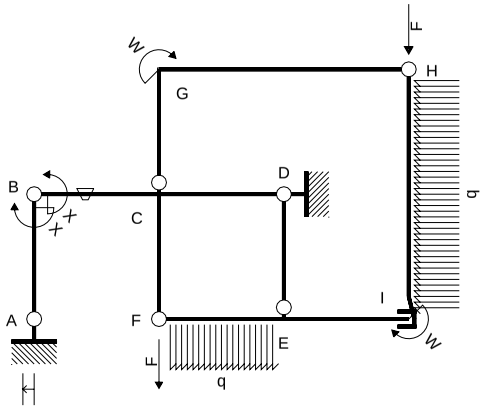
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
1/2F
1/2F
2Fb
2F
B
A
29/16F
17/8Fb
17/8F
29/16F
17/8F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
5/16F
4F
7/2Fb
33/8F
2F
Fb
1/2F
G
33/8F
5/16Fb
5/16F
C
3F
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
2F
C
29/16F
5/16Fb
17/8F
17/8F
17/8Fb
29/16F
B

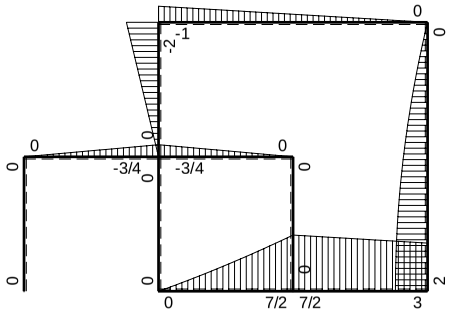




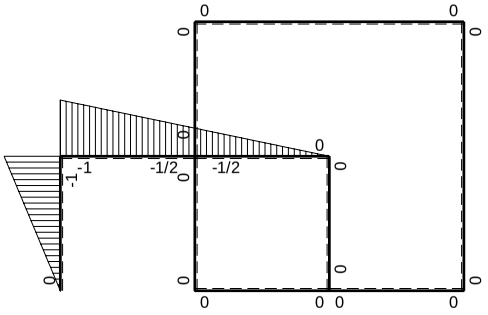

-2
-1/2
-1/2
17/8
33/8
29/16
-2
9/2
-1/2
0
0
0
F
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-2
2
-29/16
-5/16
17/8
0
0
-3
-4
0
1/2
F
0
-2
-1
0
0
17/8
5/16
5/16
0
17/8
0
0
0
0
0
2
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 247861 CI01.xxxx.118

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PROCEDIMENTO E RISULTATI 247861 CI01.xxxx.118

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $17/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -17/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

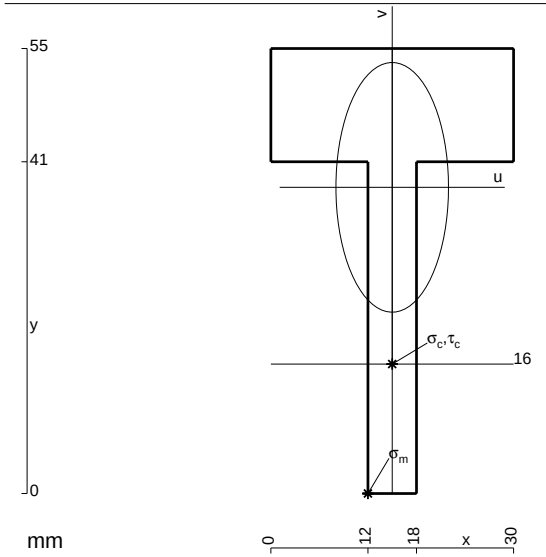
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

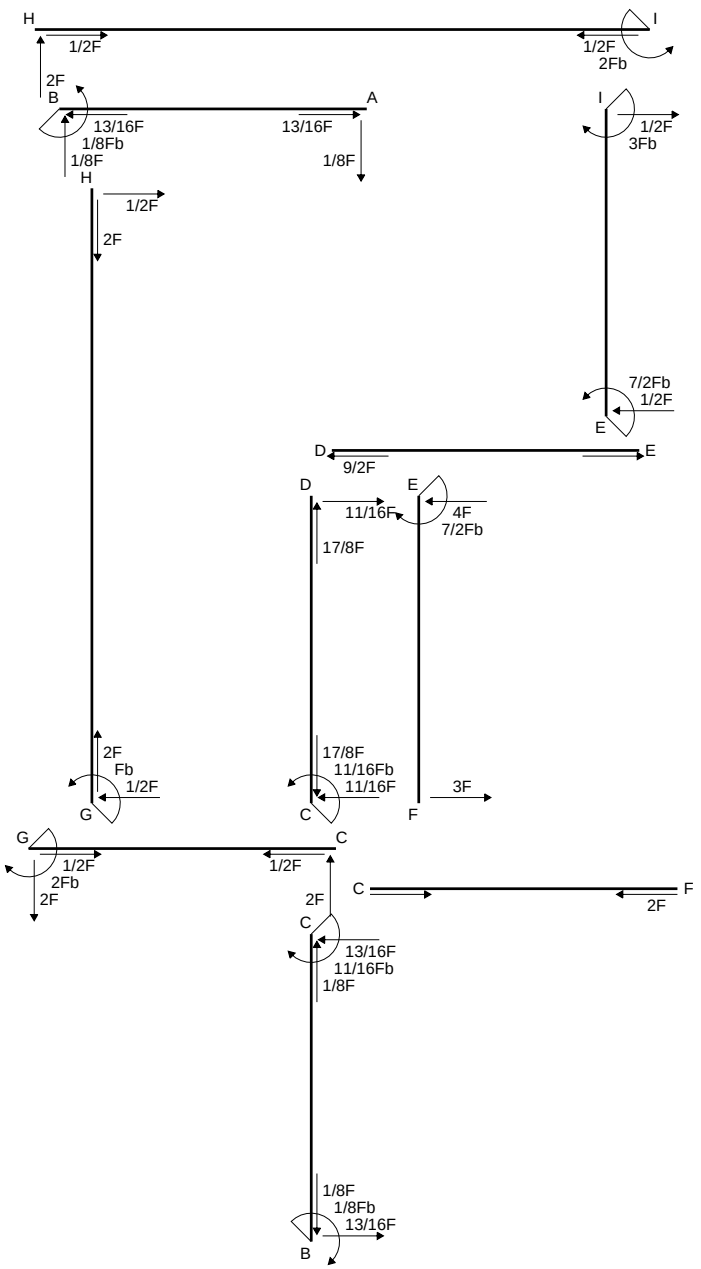
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





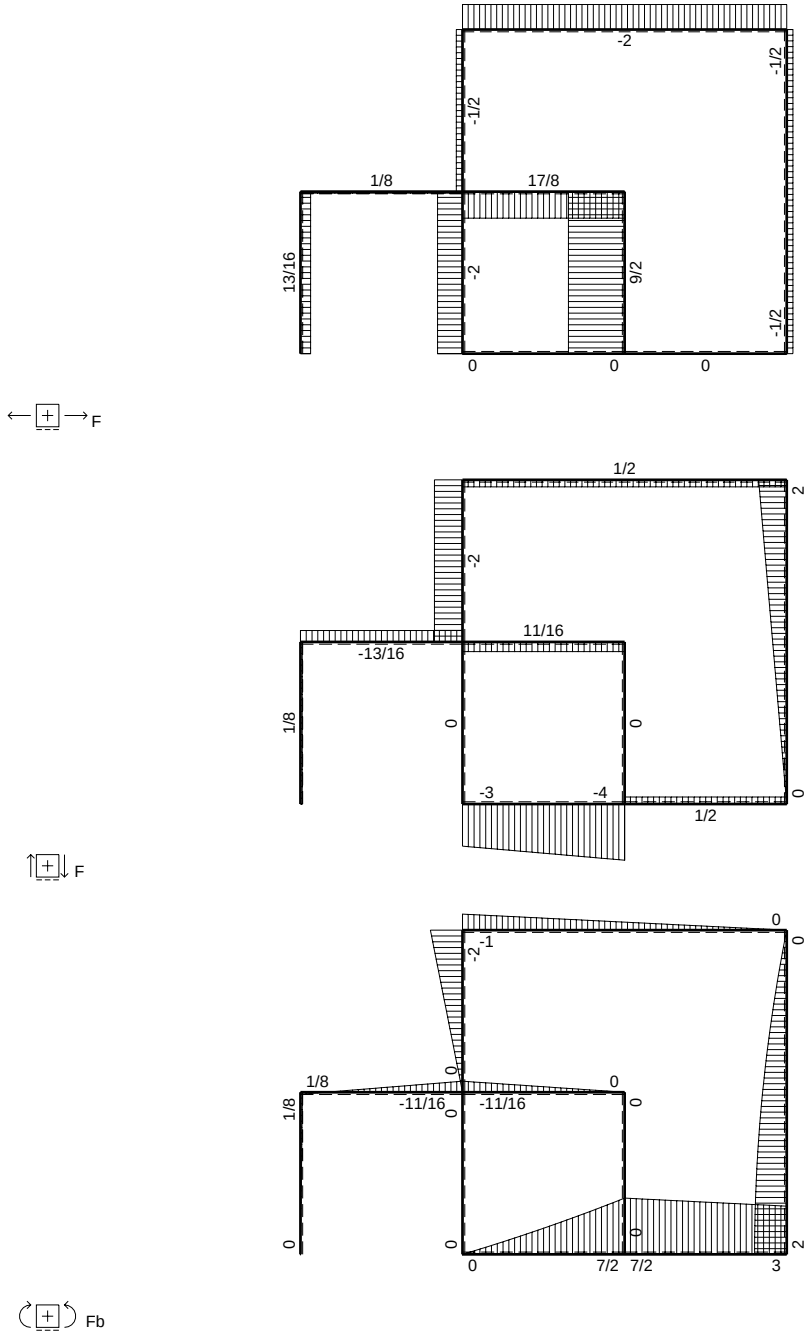


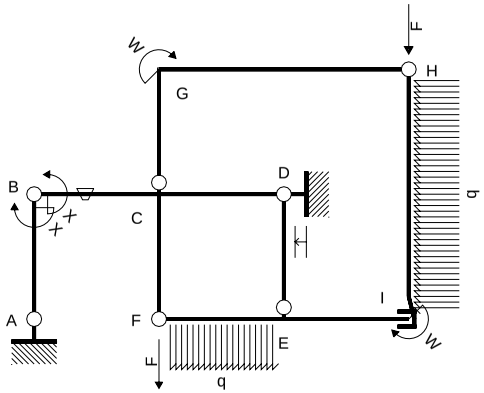

A = 666. $mm^2$
$J_u$ = 158641. $mm^4$
$J_v$ = 32238. $mm^4$
$y_g$ = 37.84 mm
$T_y$ = -1360. N
$M_x$ = 963900. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -37.84 mm
$\sigma_m = -Mv/J_u$ = 229.9 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 16. mm
$v_c$ = -21.84 mm
$\sigma_c = -Mv/J_u$ = 132.7 $N/mm^2$
$\tau_c$ = 4.093 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 132.9 $N/mm^2$
$S^*$ = 2865. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
I
13/16F
1/8Fb
1/8F
13/16F
1/8F
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
11/16F
4F
7/2Fb
17/8F
2F
Fb
1/2F
G
17/8F
11/16Fb
11/16F
C
3F
F
G
C
1/2F
2Fb
2F
1/2F
C
F
2F
C
13/16F
11/16Fb
1/8F
1/8F
1/8Fb
13/16F
B

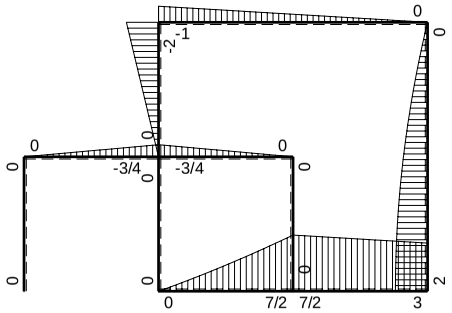




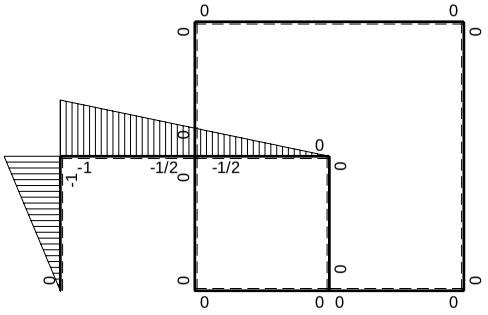

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-1/2
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1/8
17/8
13/16
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9/2
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1/2
2
-2
11/16
-13/16
1/8
0
0
-3
-4
0
1/2
F
0
0
-2
-1
1/8
0
0
-11/16
-11/16
1/8
0
0
0
0
0
2
0
7/2
7/2
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 215531 CI01.xxxx.119

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 215531 CI01.xxxx.119
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

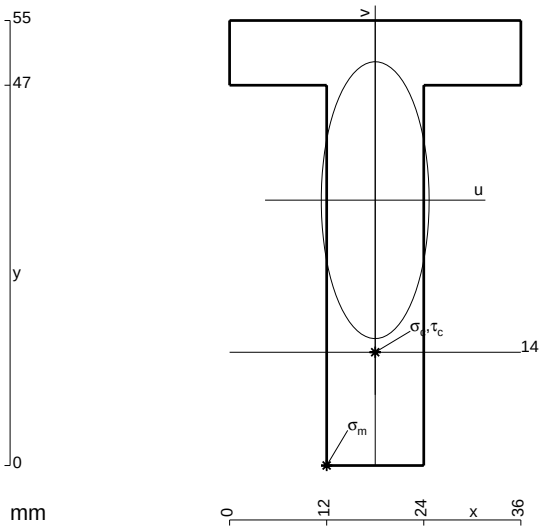
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

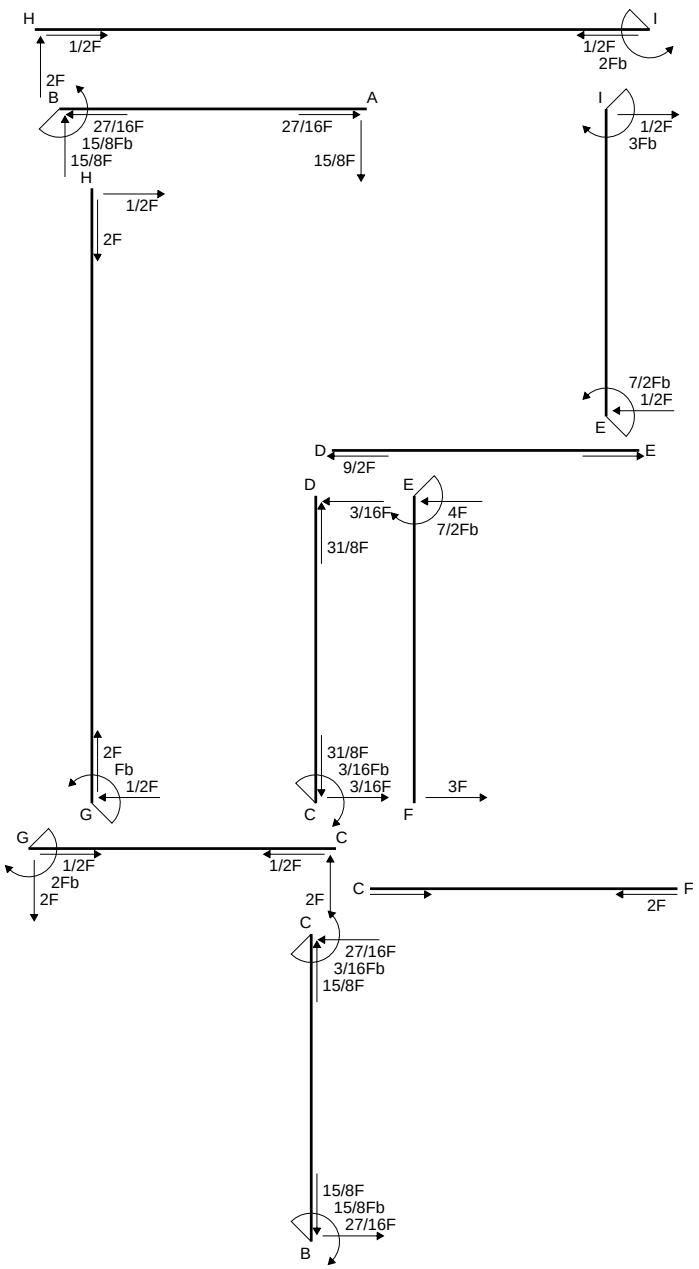


PROCEDIMENTO E RISULTATI 215531 CI01.xxxx.119



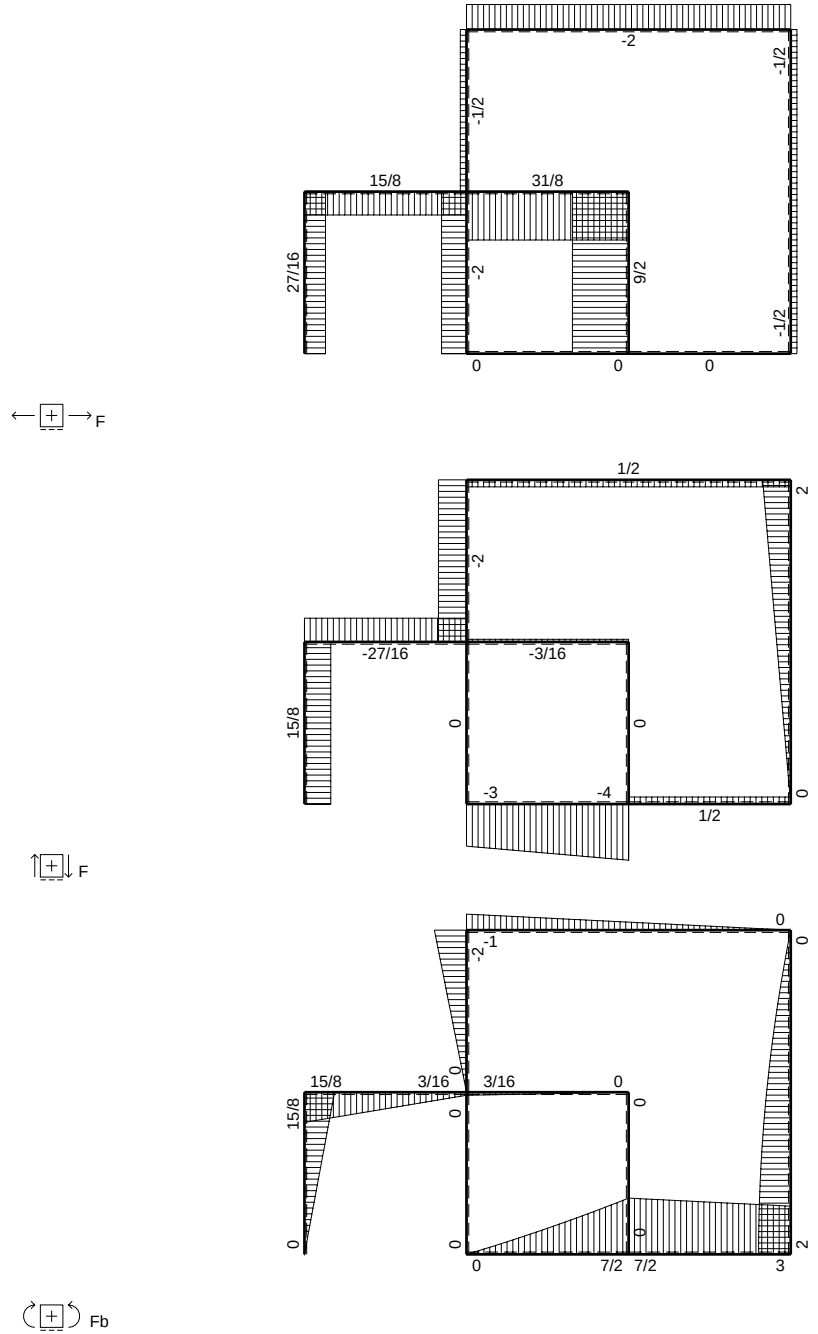

A = 852. $mm^2$
$J_u$ = 249536. $mm^4$
$J_v$ = 37872. $mm^4$
$y_g$ = 32.8 mm
$T_y$ = -2440. N
$M_x$ = 1814750. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -32.8 mm
$\sigma_m = -Mv/J_u$ = 238.5 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -18.8 mm
$\sigma_c = -Mv/J_u$ = 136.7 $N/mm^2$
$\tau_c$ = 3.531 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 136.8 $N/mm^2$
$S^*$ = 4334. $mm^3$

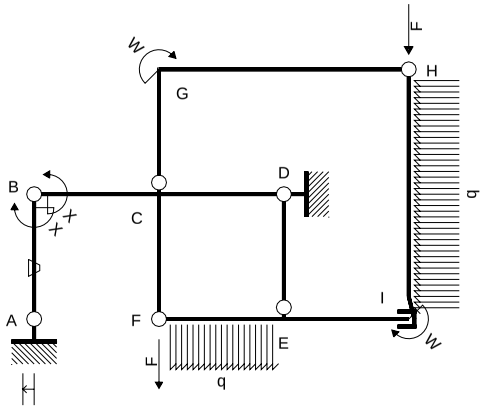
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
1/2F
1/2F
2Fb
2F
B
A
27/16F
15/8Fb
15/8F
27/16F
15/8F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
3/16F
4F
7/2Fb
31/8F
2F
Fb
1/2F
G
31/8F
3/16Fb
3/16F
C
3F
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
2F
C
27/16F
3/16Fb
15/8F
15/8F
15/8Fb
27/16F
B

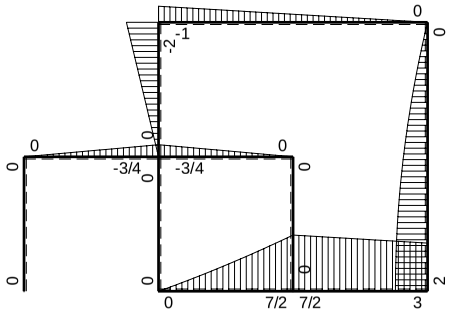




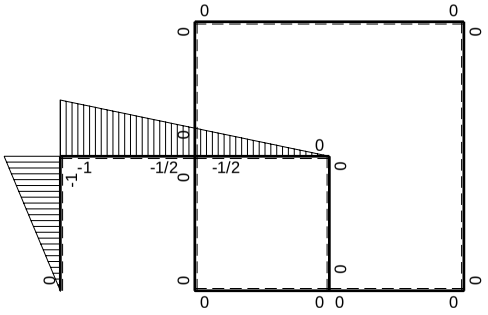

F
-2
-1/2
-1/2
15/8
31/8
27/16
-2
9/2
-1/2
0
0
0
F
1/2
2
-2
-27/16
-3/16
15/8
0
0
0
-3
-4
1/2
Fb
0
0
-2
-1
0
15/8
3/16
3/16
0
15/8
0
0
0
0
0
0
7/2
7/2
3
2




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264654 CI01.xxxx.120

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PROCEDIMENTO E RISULTATI 264654 CI01.xxxx.120

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $15/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -15/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

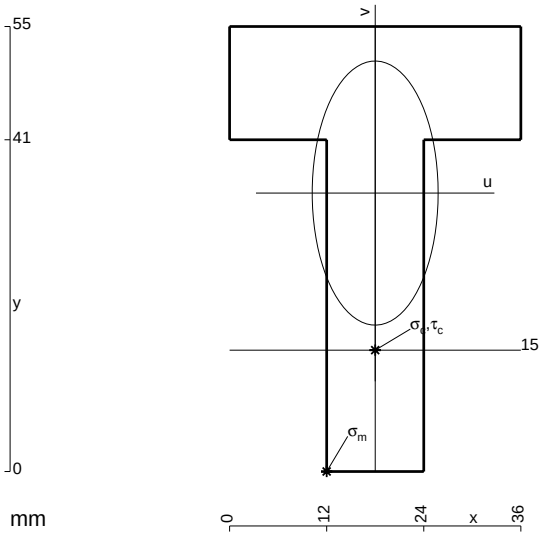
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

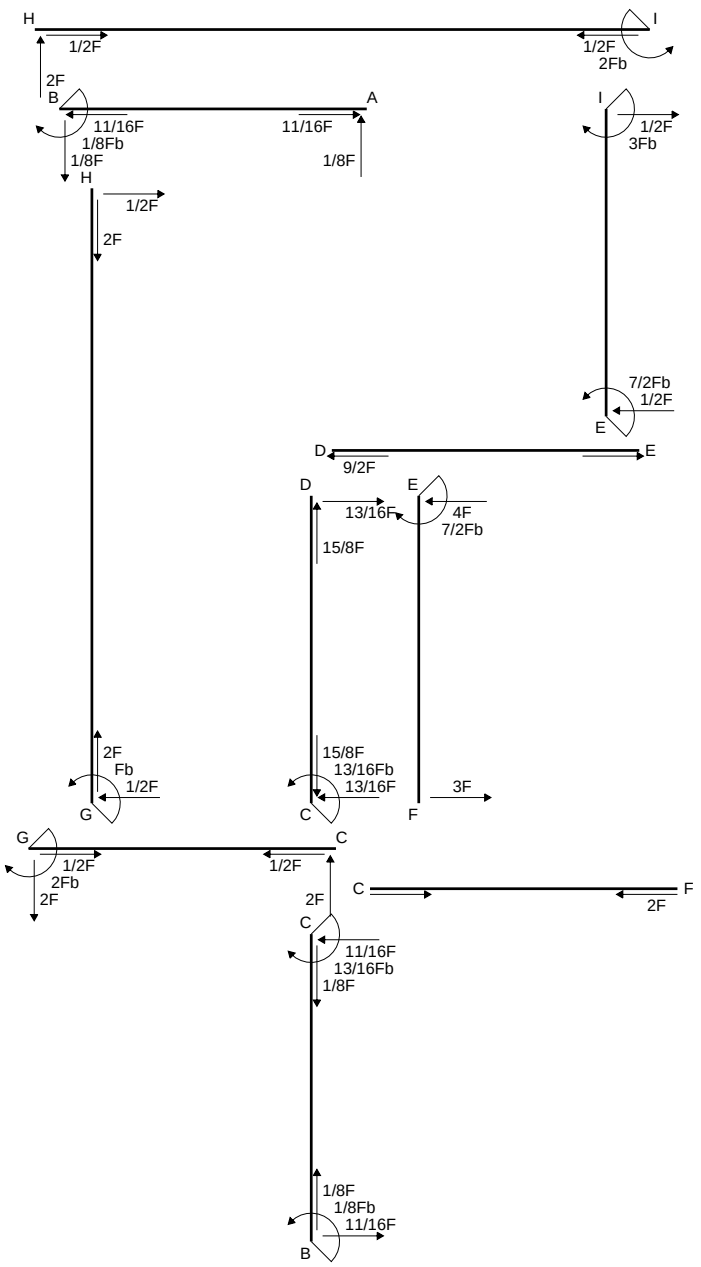
$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





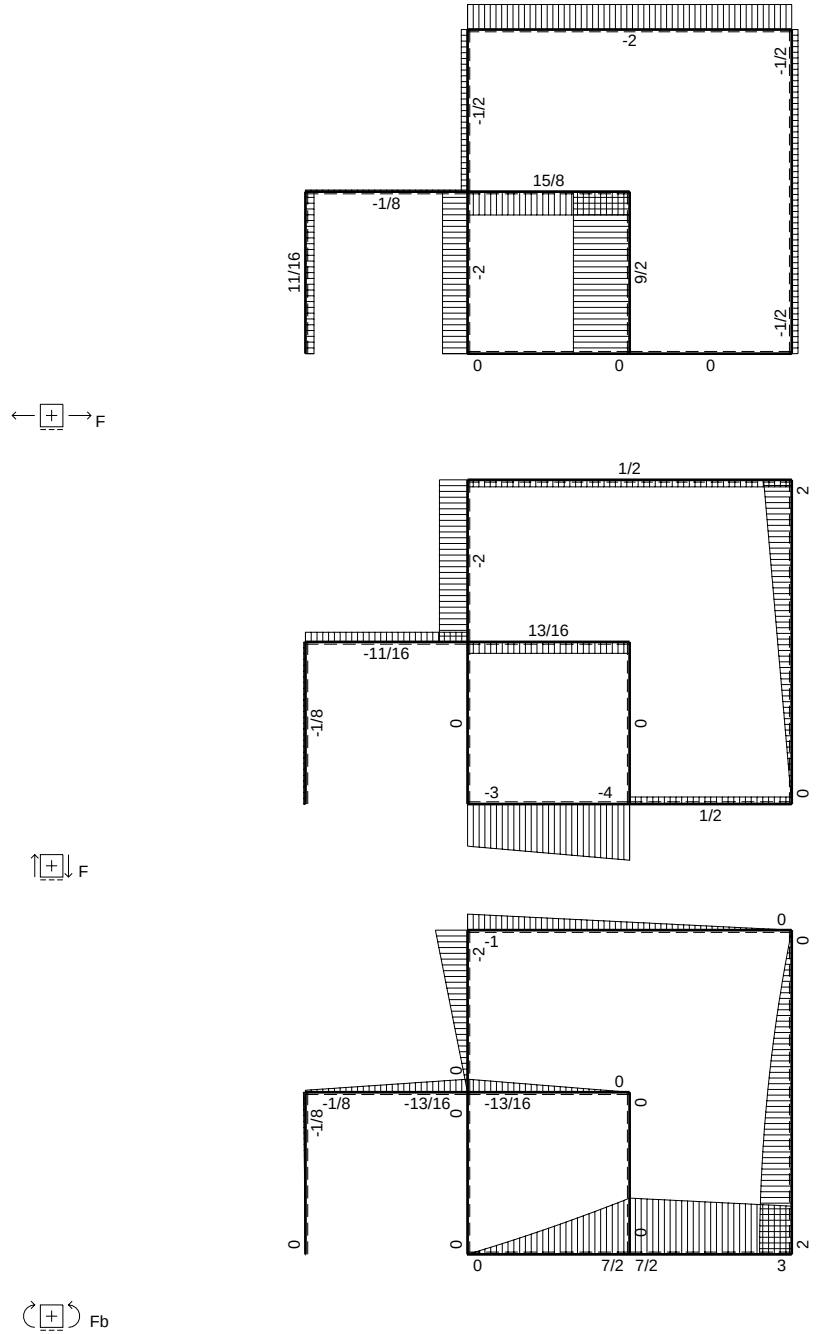


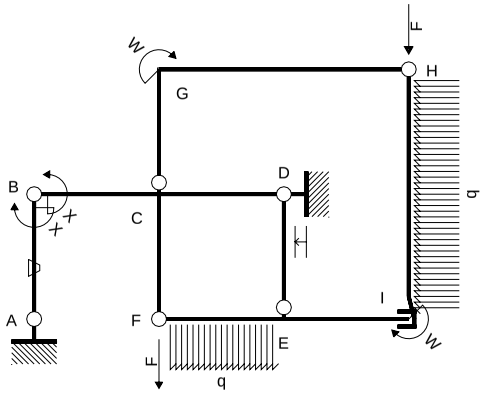

A = 996. $mm^2$
$J_u$ = 265432. $mm^4$
$J_v$ = 60336. $mm^4$
$y_g$ = 34.42 mm
$T_y$ = -3880. N
$M_x$ = 1527750. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -34.42 mm
$\sigma_m = -Mv/J_u$ = 198.1 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 15. mm
$v_c$ = -19.42 mm
$\sigma_c = -Mv/J_u$ = 111.8 $N/mm^2$
$\tau_c$ = 5.902 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 112.2 $N/mm^2$
$S^*$ = 4845. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
I
11/16F
11/16F
1/2F
1/8Fb
3Fb
1/8F
1/8F
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
13/16F
4F
7/2Fb
15/8F
2F
Fb
1/2F
15/8F
13/16Fb
13/16F
3F
G
C
F
G
C
1/2F
2Fb
1/2F
2F
C
F
2F
C
11/16F
13/16Fb
1/8F
1/8F
1/8Fb
11/16F
B

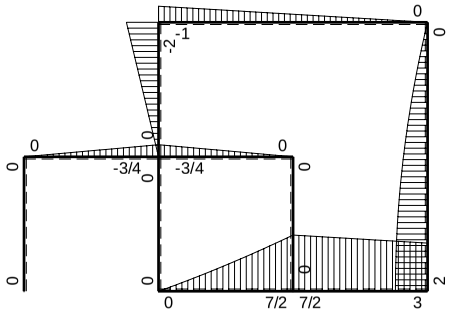




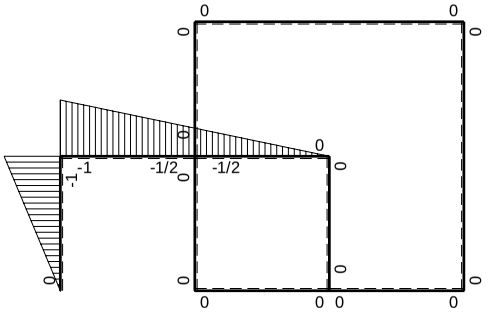

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11/16
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9/2
-1/2
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0
F
1/2
2
-2
13/16
-11/16
-1/8
0
0
-3
-4
0
1/2
F
0
0
-2
-1
0
0
-1/8
-13/16
-13/16
0
0
0
0
0
0
7/2
7/2
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 237158 CI01.xxxx.121

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PROCEDIMENTO E RISULTATI 237158 CI01.xxxx.121

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

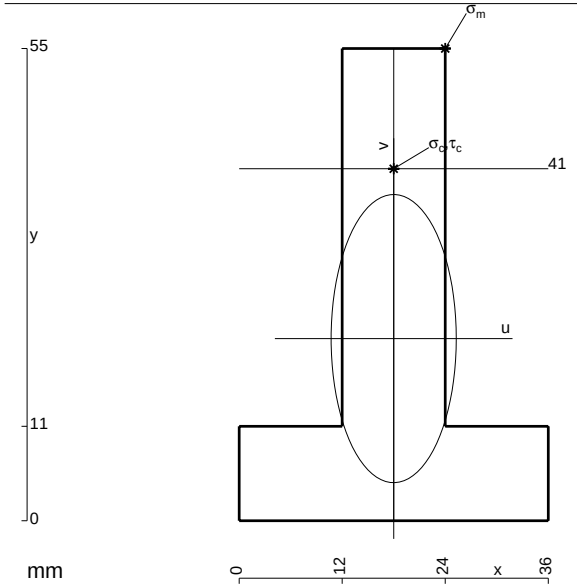
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

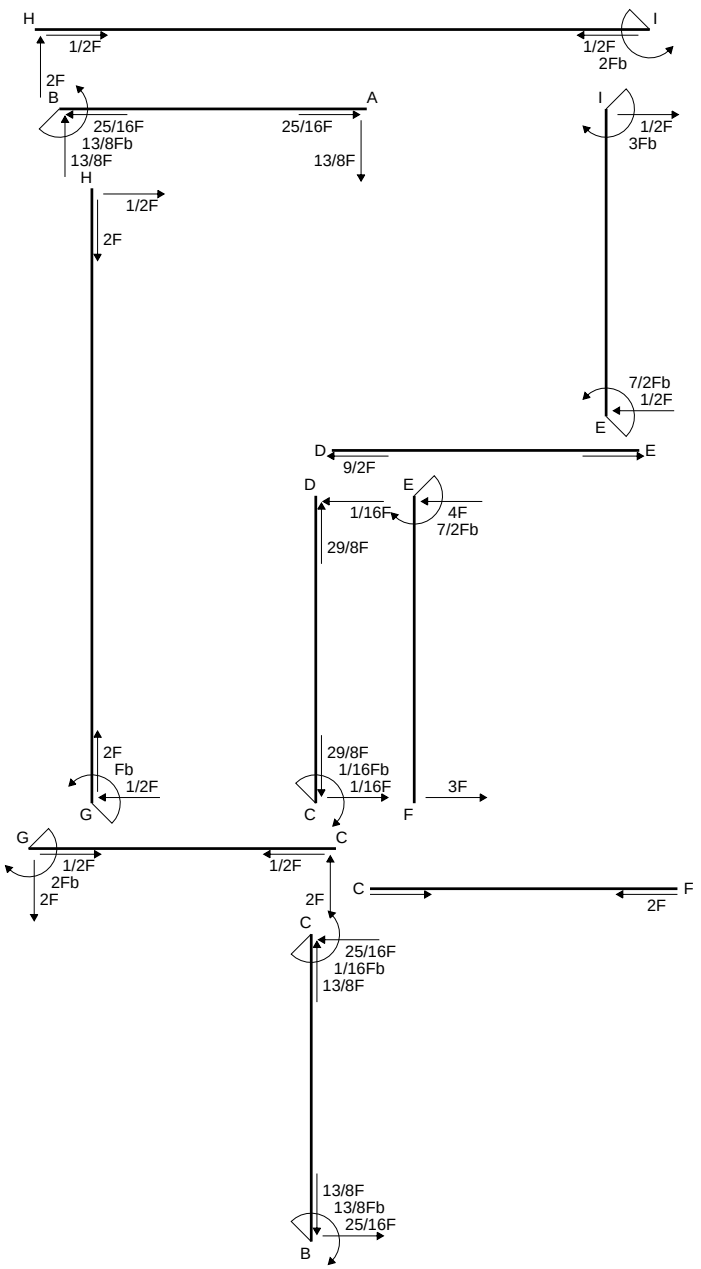


PROCEDIMENTO E RISULTATI 237158 CI01.xxxx.121



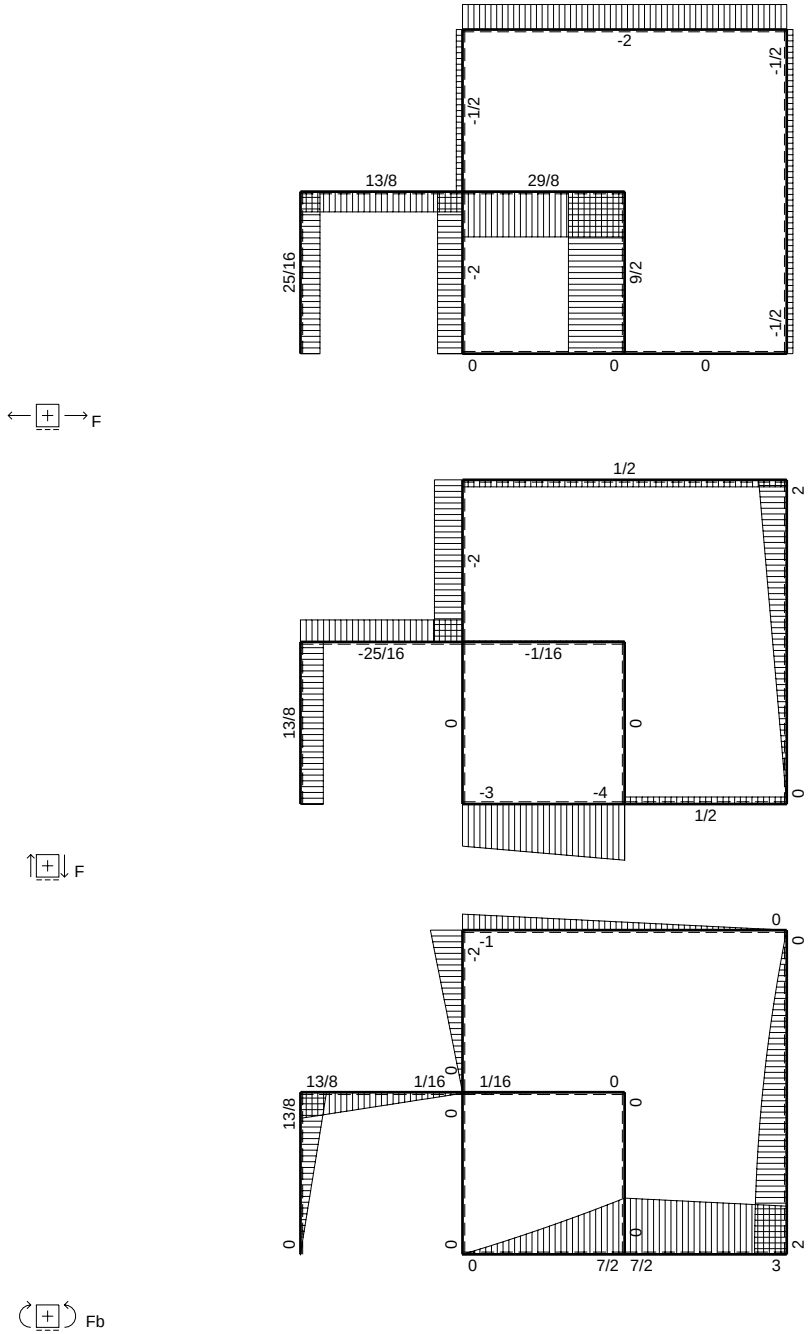

A = 924. $mm^2$
$J_u$ = 260306. $mm^4$
$J_v$ = 49104. $mm^4$
$y_g$ = 21.21 mm
$T_y$ = -3760. N
$M_x$ = 1612100. Nmm
$x_m$ = 24. mm
$y_m$ = 55. mm
$u_m$ = 6. mm
$v_m$ = 33.79 mm
$\sigma_m = -Mv/J_u$ = -209.2 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 41. mm
$v_c$ = 19.79 mm
$\sigma_c = -Mv/J_u$ = -122.5 $N/mm^2$
$\tau_c$ = 5.417 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.9 $N/mm^2$
$S^*$ = 4500. $mm^3$

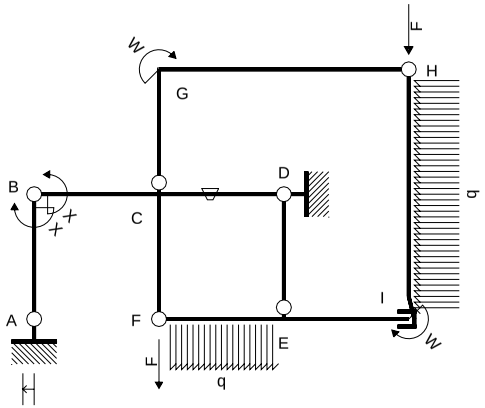
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
1/2F
1/2F
2Fb
2F
B
A
25/16F
13/8Fb
13/8F
25/16F
13/8F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
1/16F
4F
7/2Fb
29/8F
2F
Fb
1/2F
G
29/8F
1/16Fb
1/16F
C
3F
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
2F
C
25/16F
1/16Fb
13/8F
13/8F
13/8Fb
25/16F
B

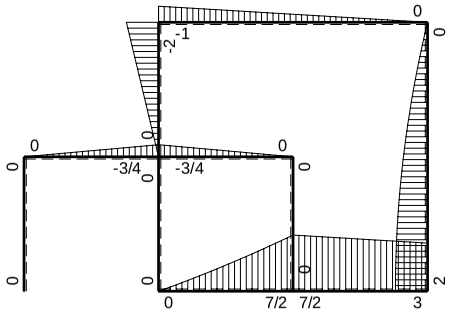




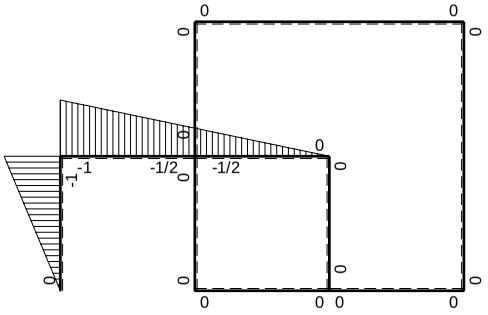

F
-2
-1/2
-1/2
13/8
29/8
25/16
-2
9/2
-1/2
0
0
0
F
1/2
2
-2
-25/16
-1/16
13/8
0
0
0
-3
-4
0
1/2
Fb
0
0
-2
-1
0
13/8
1/16
1/16
0
13/8
0
0
0
0
0
0
2
0
7/2
7/2
3




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264611 CI01.xxxx.122

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 264611 CI01.xxxx.122

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

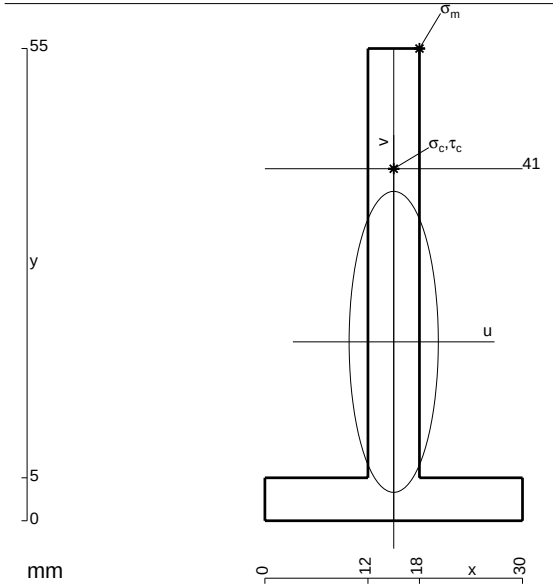
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

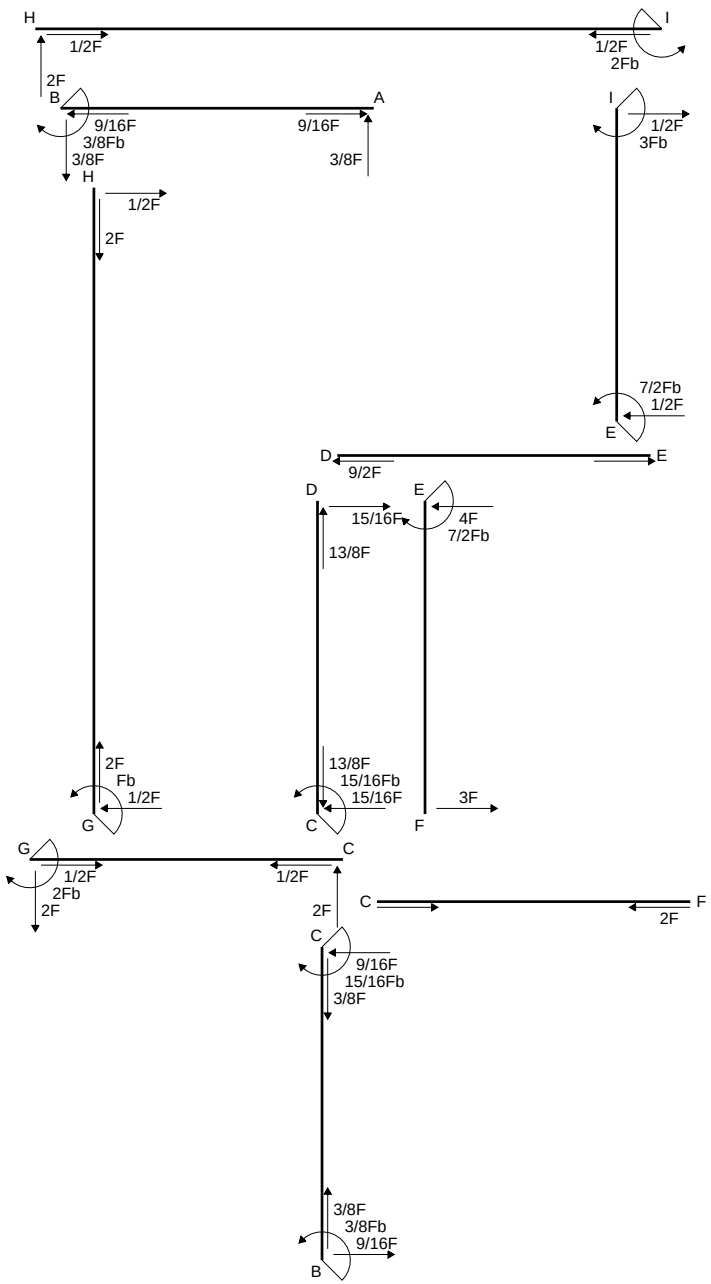
$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 264611 CI01.xxxx.122

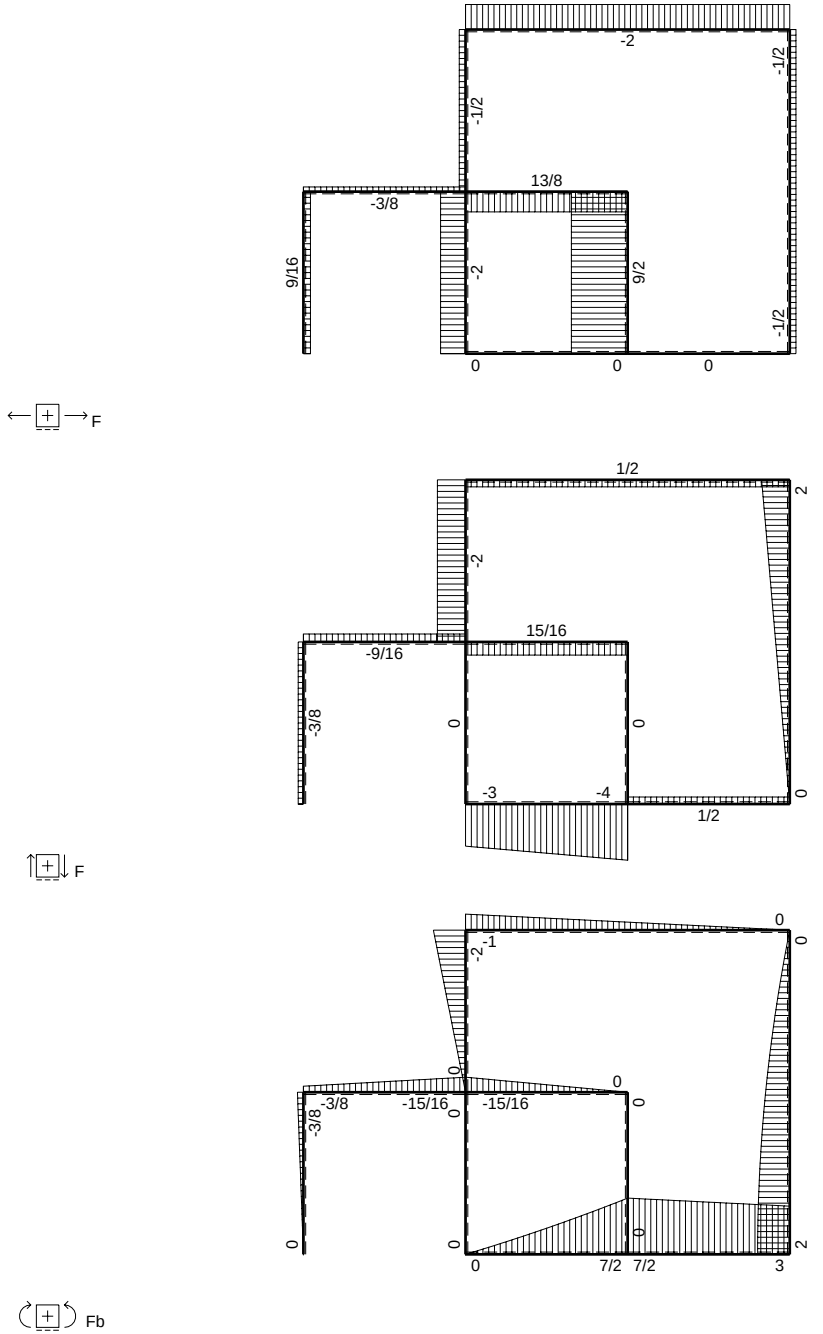


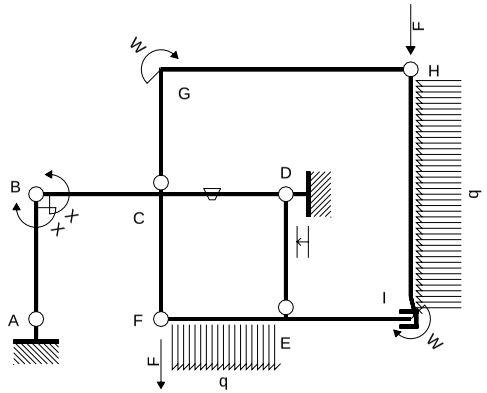

A = 450. $mm^2$
$J_u$ = 138437. $mm^4$
$J_v$ = 12150. $mm^4$
$y_g$ = 20.83 mm
$T_y$ = -1880. N
$M_x$ = 888300. Nmm
$x_m$ = 18. mm
$y_m$ = 55. mm
$u_m$ = 3. mm
$v_m$ = 34.17 mm
$\sigma_m = -Mv/J_u$ = -219.2 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 41. mm
$v_c$ = 20.17 mm
$\sigma_c = -Mv/J_u$ = -129.4 $N/mm^2$
$\tau_c$ = 5.165 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.7 $N/mm^2$
$S^*$ = 2282. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
9/16F
3/8Fb
9/16F
3/8F
3/8F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
9/2F
D
E
15/16F
4F
7/2Fb
13/8F
2F
Fb
1/2F
13/8F
15/16Fb
15/16F
3F
G
C
F
G
C
1/2F
2Fb
2F
1/2F
2F
C
F
2F
C
9/16F
15/16Fb
3/8F
3/8F
3/8Fb
9/16F
B

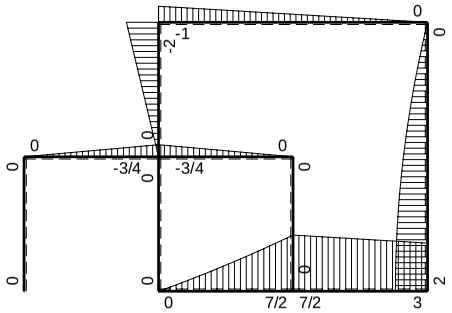




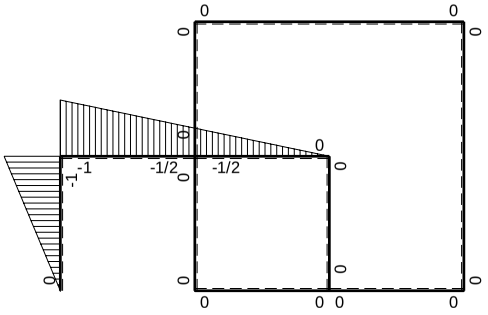

F
-2
-1/2
-1/2
13/8
-3/8
9/16
-2
9/2
-1/2
0
0
0
F
1/2
2
-2
15/16
-9/16
-3/8
0
0
-3
-4
0
1/2
F
0
0
-2
-1
0
0
-3/8
-15/16
-15/16
-3/8
0
0
0
0
0
7/2
7/2
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265166 CI01.xxxx.123

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 265166 CI01.xxxx.123

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | 3Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-3/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 3/8Fb | |

Sviluppi di calcolo iperstatica


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

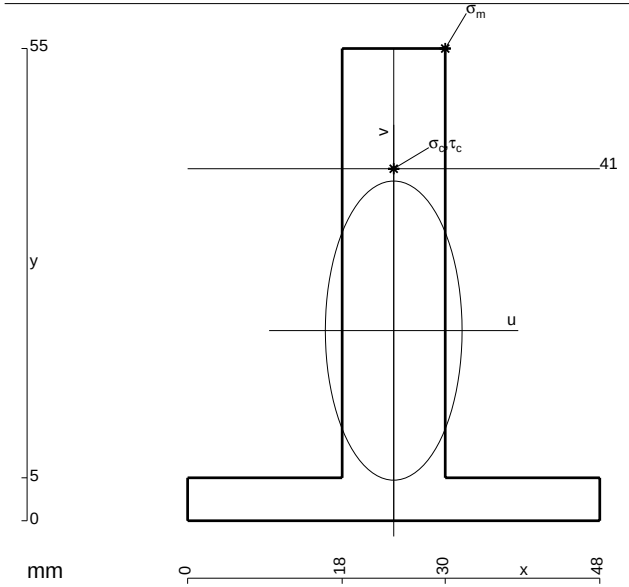
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$

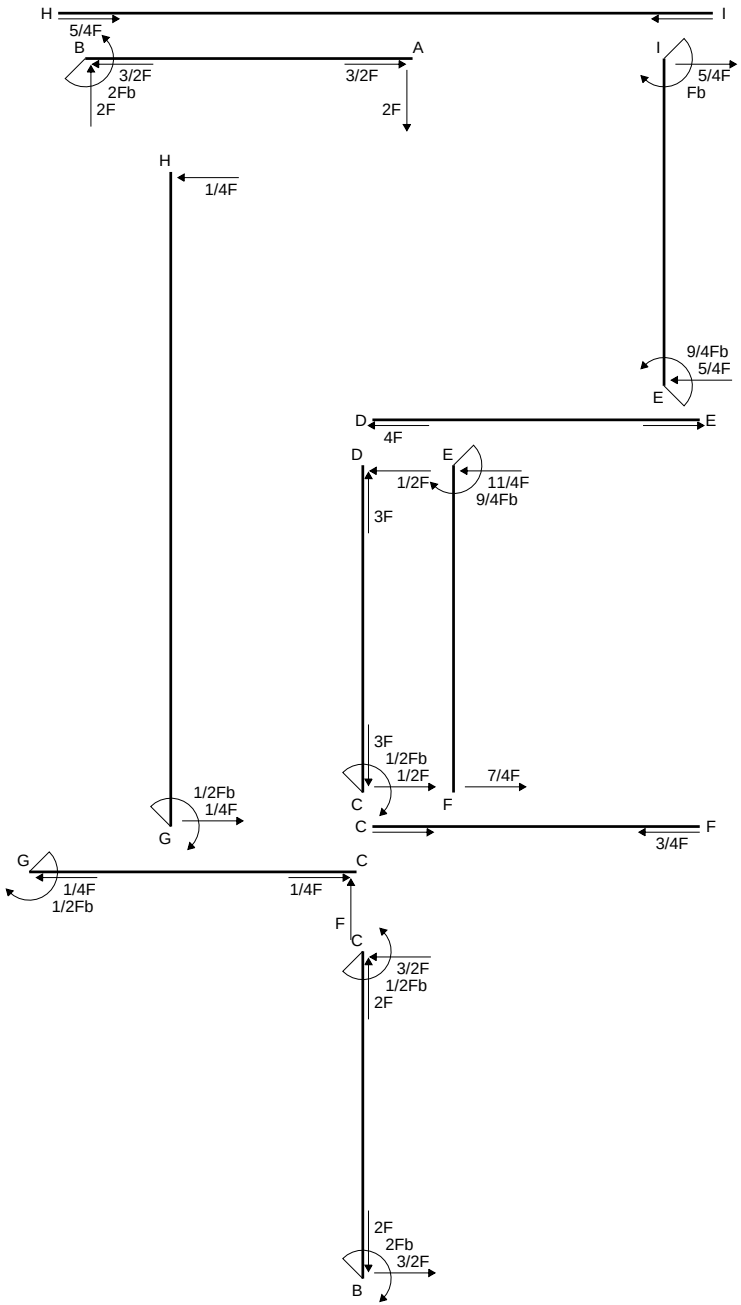


PROCEDIMENTO E RISULTATI 265166 CI01.xxxx.123



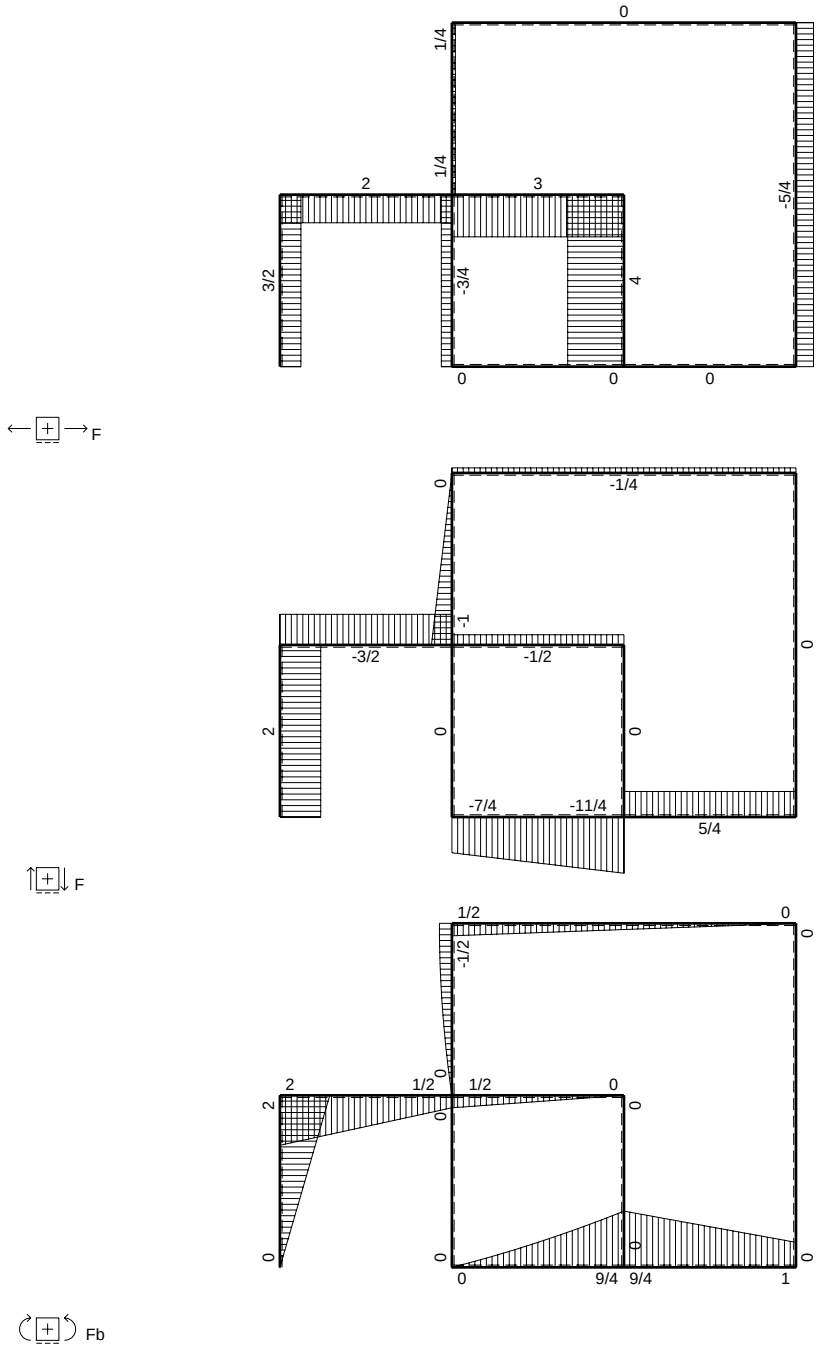

A = 840. $mm^2$
$J_u$ = 255143. $mm^4$
$J_v$ = 53280. $mm^4$
$y_g$ = 22.14 mm
$T_y$ = -3480. N
$M_x$ = 1766100. Nmm
$x_m$ = 30. mm
$y_m$ = 55. mm
$u_m$ = 6. mm
$v_m$ = 32.86 mm
$\sigma_m = -Mv/J_u$ = -227.4 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 41. mm
$v_c$ = 18.86 mm
$\sigma_c = -Mv/J_u$ = -130.5 $N/mm^2$
$\tau_c$ = 4.937 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.8 $N/mm^2$
$S^*$ = 4344. $mm^3$

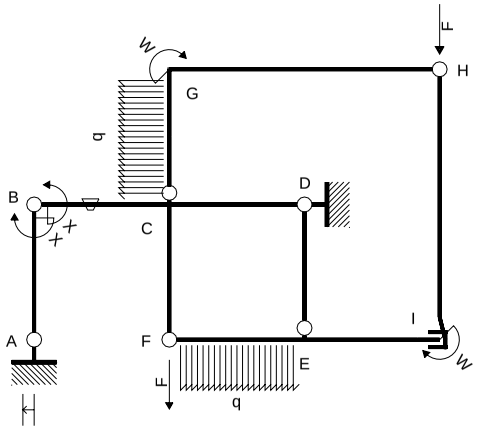
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H
I
5/4F
B
A
3/2F
2Fb
2F
3/2F
2F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
1/2F
11/4F
9/4Fb
3F
3F
1/2Fb
1/2F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/4F
1/2Fb
F
C
3/2F
1/2Fb
2F
2F
2Fb
3/2F
B

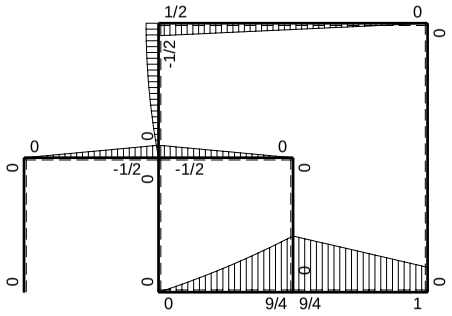




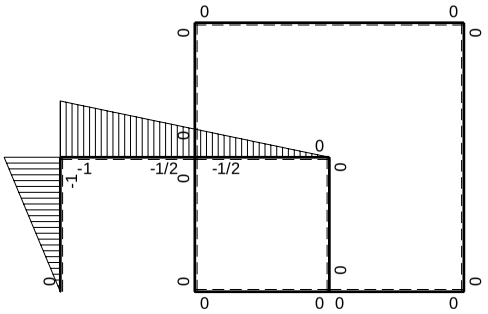

F
0
1/4
1/4
2
3
-5/4
3/2
-3/4
4
0
0
0
F
0
-1/4
-1
-3/2
-1/2
2
0
0
0
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
2
1/2
1/2
0
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264142 CI01.xxxx.124

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PROCEDIMENTO E RISULTATI 264142 CI01.xxxx.124

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -1/2Fx | -Fb/EJ | $1/2Fx-1/4Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/6+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 1/2Fb-1/2Fx | Fb/EJ | $1/4Fb-1/4Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

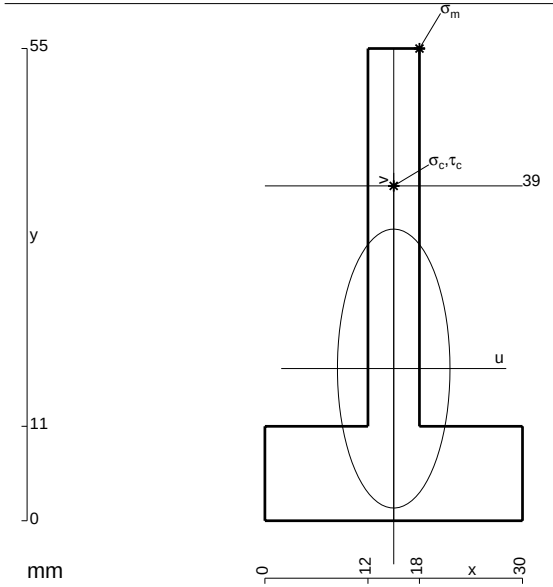
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

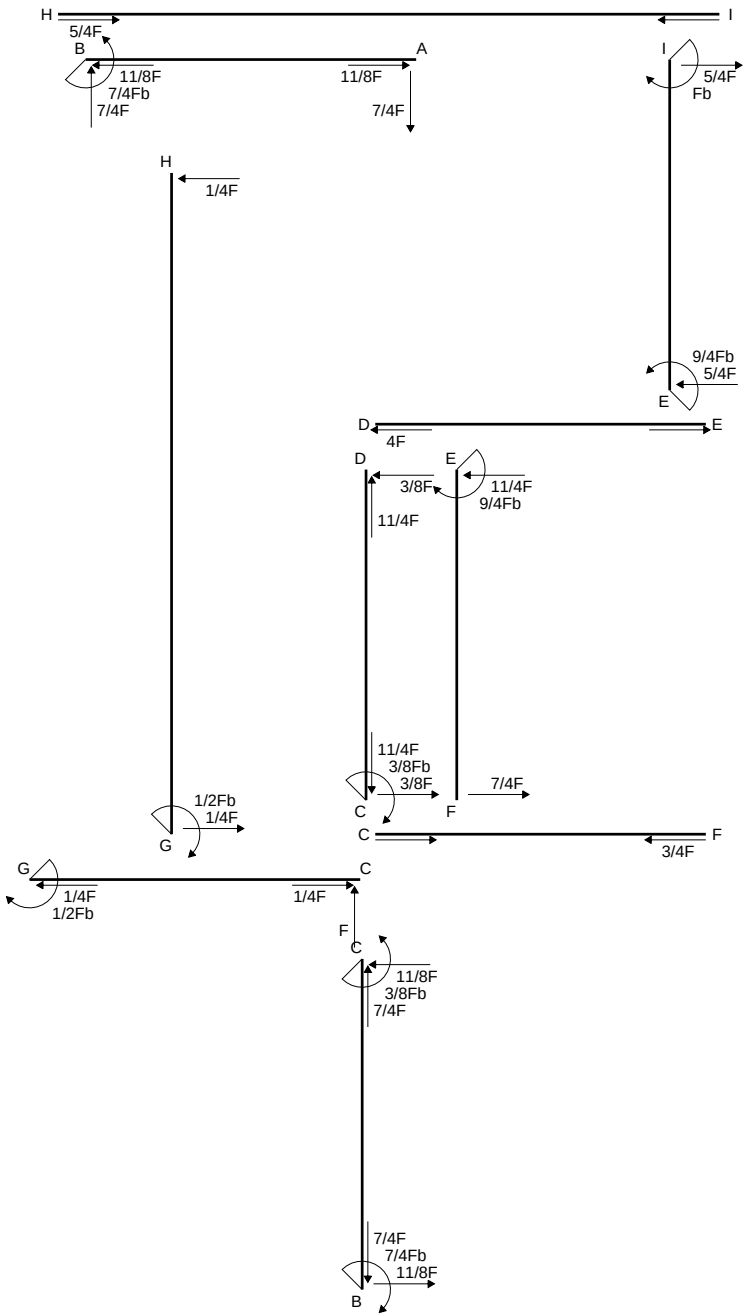


PROCEDIMENTO E RISULTATI 264142 CI01.xxxx.124



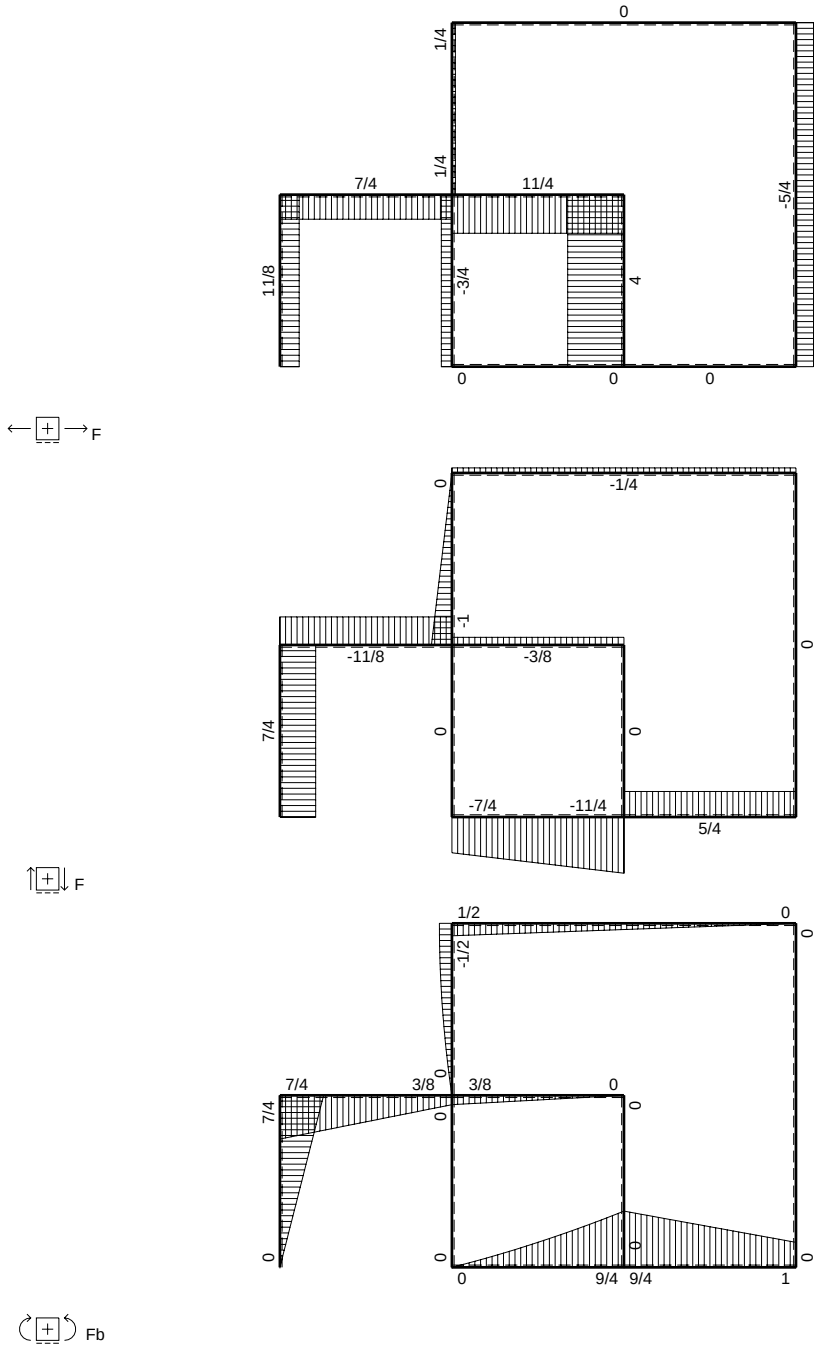

A = 594. $mm^2$
$J_u$ = 156836. $mm^4$
$J_v$ = 25542. $mm^4$
$y_g$ = 17.72 mm
$T_y$ = -1953. N
$M_x$ = 1006430. Nmm
$x_m$ = 18. mm
$y_m$ = 55. mm
$u_m$ = 3. mm
$v_m$ = 37.28 mm
$\sigma_m = -Mv/J_u$ = -239.2 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 39. mm
$v_c$ = 21.28 mm
$\sigma_c = -Mv/J_u$ = -136.5 $N/mm^2$
$\tau_c$ = 5.832 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.9 $N/mm^2$
$S^*$ = 2811. $mm^3$

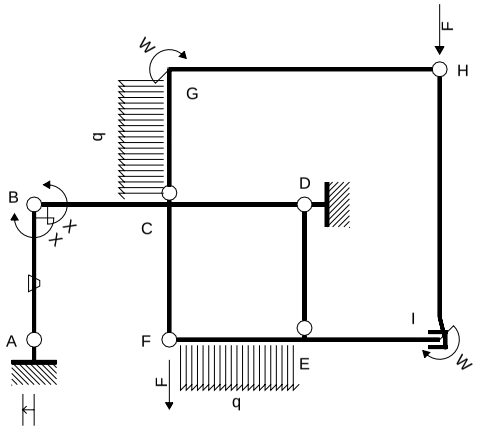
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H
I
5/4F
B
A
11/8F
7/4Fb
7/4F
11/8F
7/4F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
3/8F
11/4F
9/4Fb
11/4F
11/4F
3/8Fb
3/8F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/2Fb
1/4F
F
C
11/8F
3/8Fb
7/4F
7/4F
7/4Fb
11/8F
B

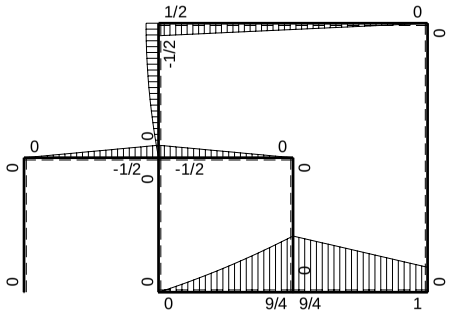




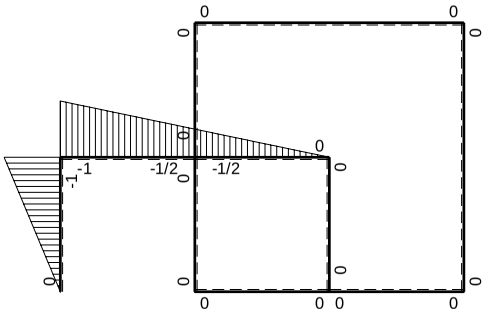

0
1/4
1/4
7/4
11/4
-5/4
11/8
-3/4
4
0
0
0
F
0
-1/4
-1
-11/8
-3/8
0
7/4
0
0
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
0
7/4
3/8
3/8
0
7/4
0
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267320 CI01.xxxx.125

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PROCEDIMENTO E RISULTATI 267320 CI01.xxxx.125

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-1/2Fx$ | 0 | $1/2Fx-1/4Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $1/2Fb-1/2Fx$ | 0 | $1/4Fb-1/4Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+1/2Fx$ | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $7/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

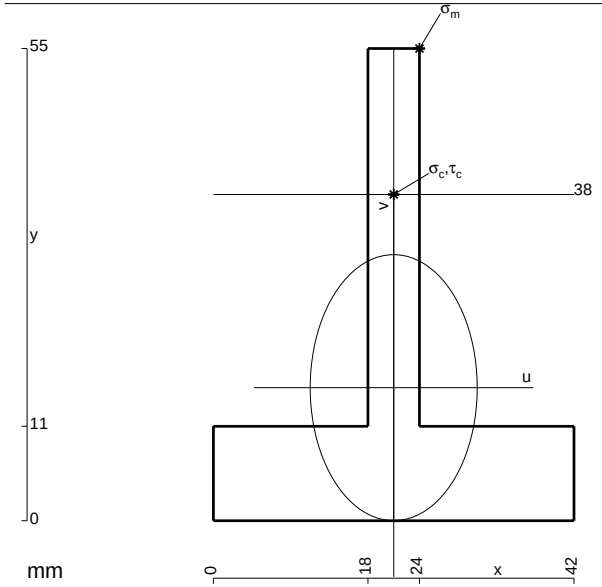
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

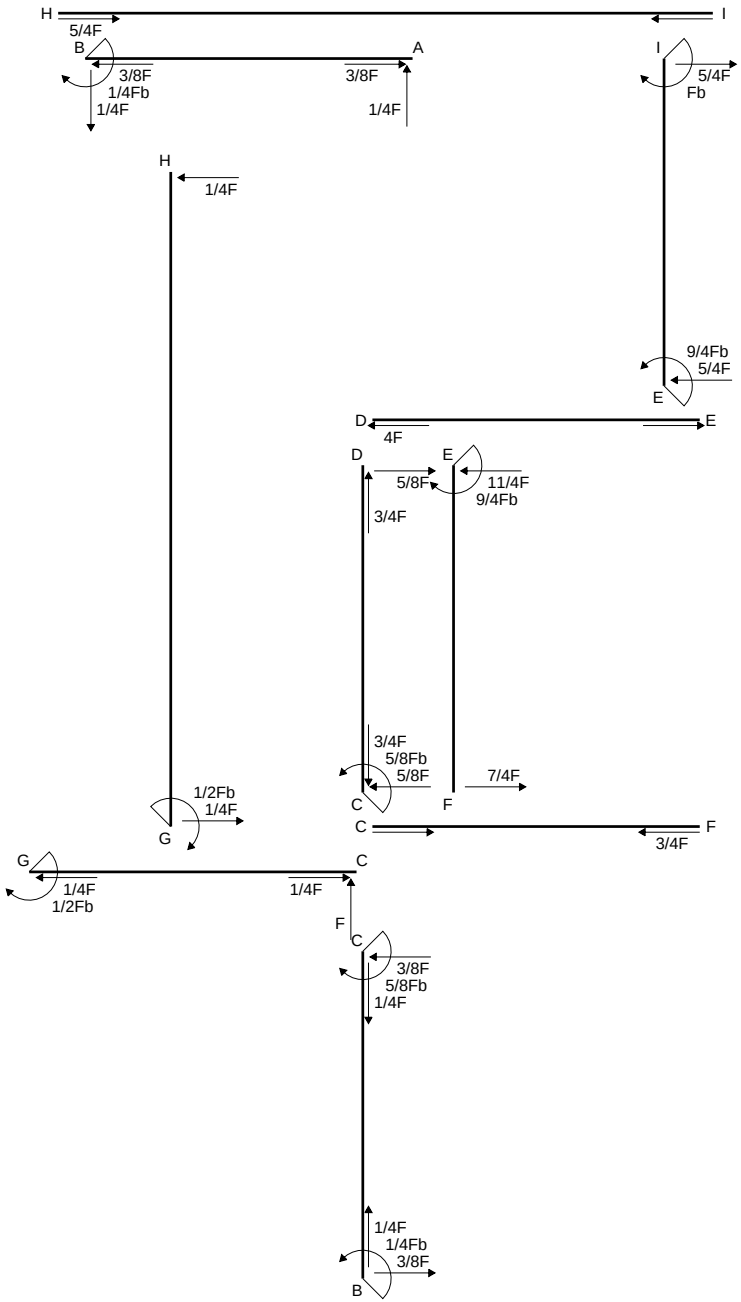
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





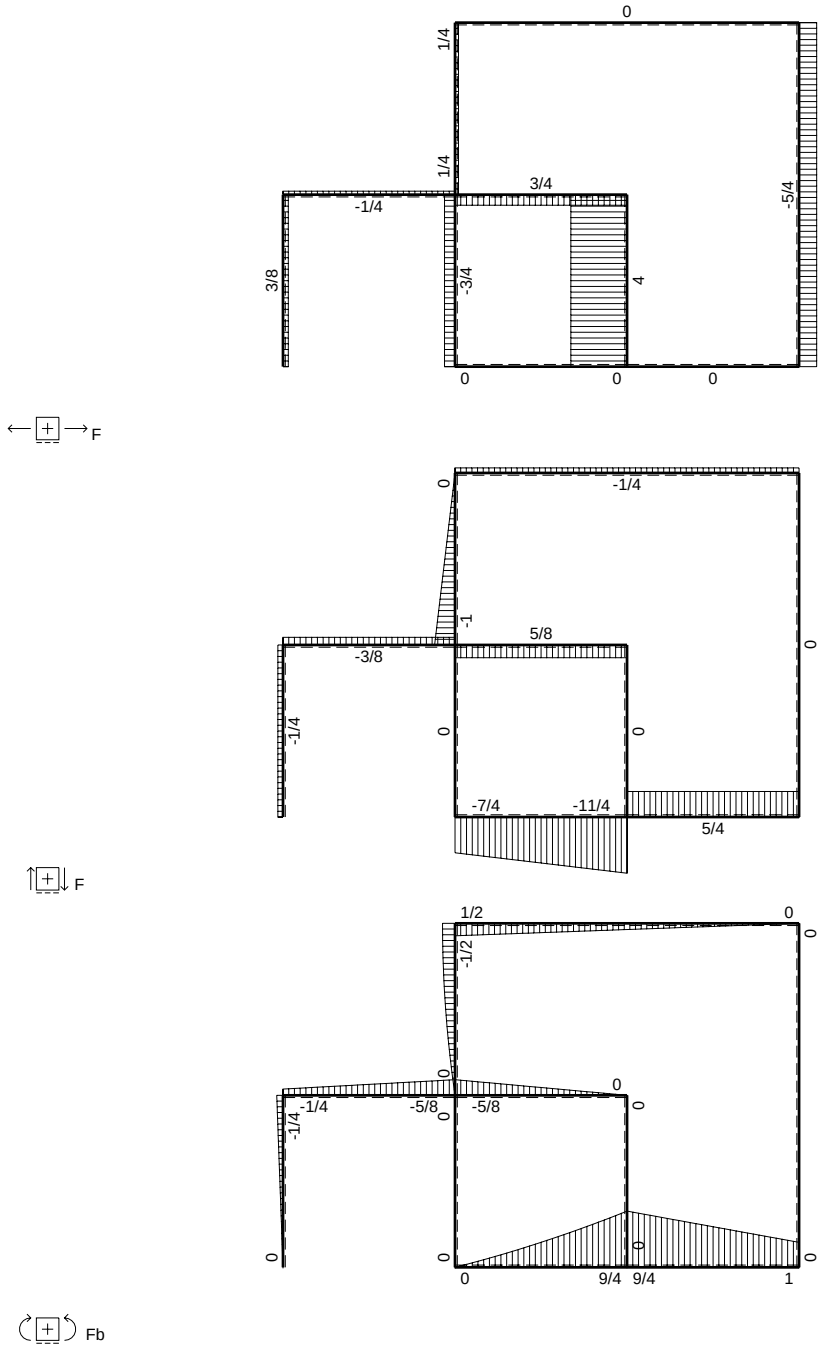


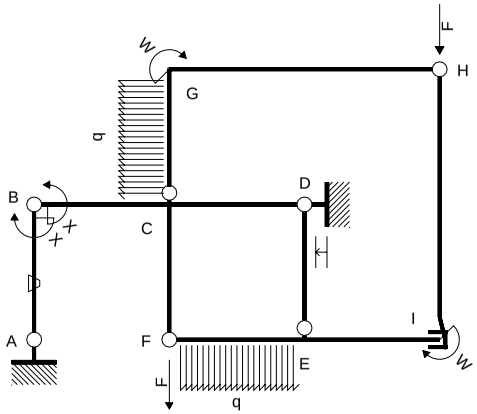

A = 726. $mm^2$
$J_u$ = 174301. $mm^4$
$J_v$ = 68706. $mm^4$
$y_g$ = 15.5 mm
$T_y$ = -1595. N
$M_x$ = 874350. Nmm
$x_m$ = 24. mm
$y_m$ = 55. mm
$u_m$ = 3. mm
$v_m$ = 39.5 mm
$\sigma_m = -Mv/J_u$ = -198.1 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 38. mm
$v_c$ = 22.5 mm
$\sigma_c = -Mv/J_u$ = -112.9 $N/mm^2$
$\tau_c$ = 4.823 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.2 $N/mm^2$
$S^*$ = 3162. $mm^3$


H
I
5/4F
B
A
3/8F
1/4Fb
1/4F
3/8F
1/4F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
5/8F
11/4F
9/4Fb
3/4F
3/4F
5/8Fb
5/8F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/4F
1/2Fb
F
C
3/8F
5/8Fb
1/4F
1/4F
1/4Fb
3/8F
B

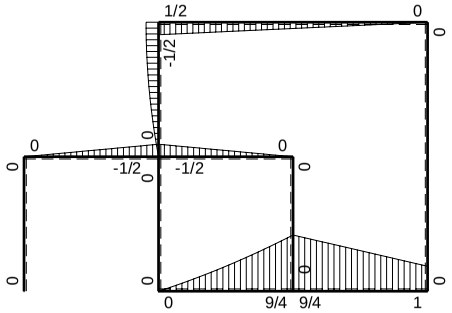




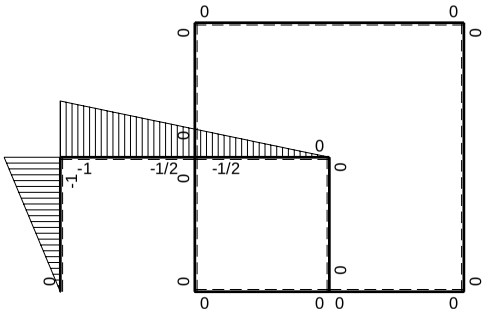

0
1/4
1/4
3/4
-1/4
-5/4
3/8
-3/4
4
0
0
0
F
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-1
5/8
-3/8
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0
-1/4
0
0
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
0
0
-1/4
-1/4
-5/8
-5/8
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 263840 CI01.xxxx.126

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PROCEDIMENTO E RISULTATI 263840 CI01.xxxx.126

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-1/2Fx$ | 0 | $1/2Fx-1/4Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $1/2Fb-1/2Fx$ | 0 | $1/4Fb-1/4Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+1/2Fx$ | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

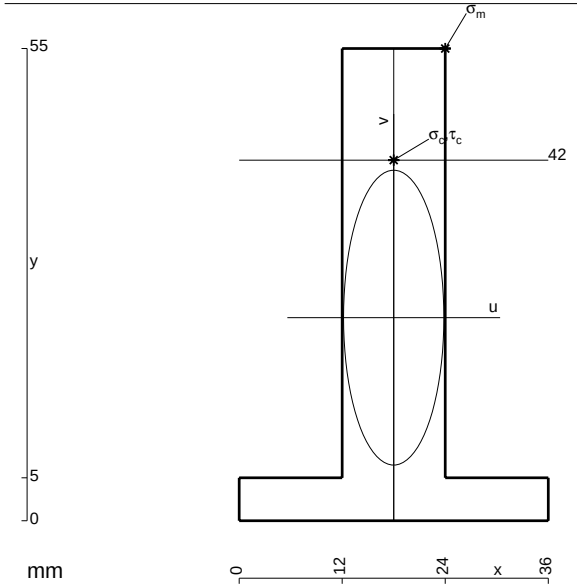
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

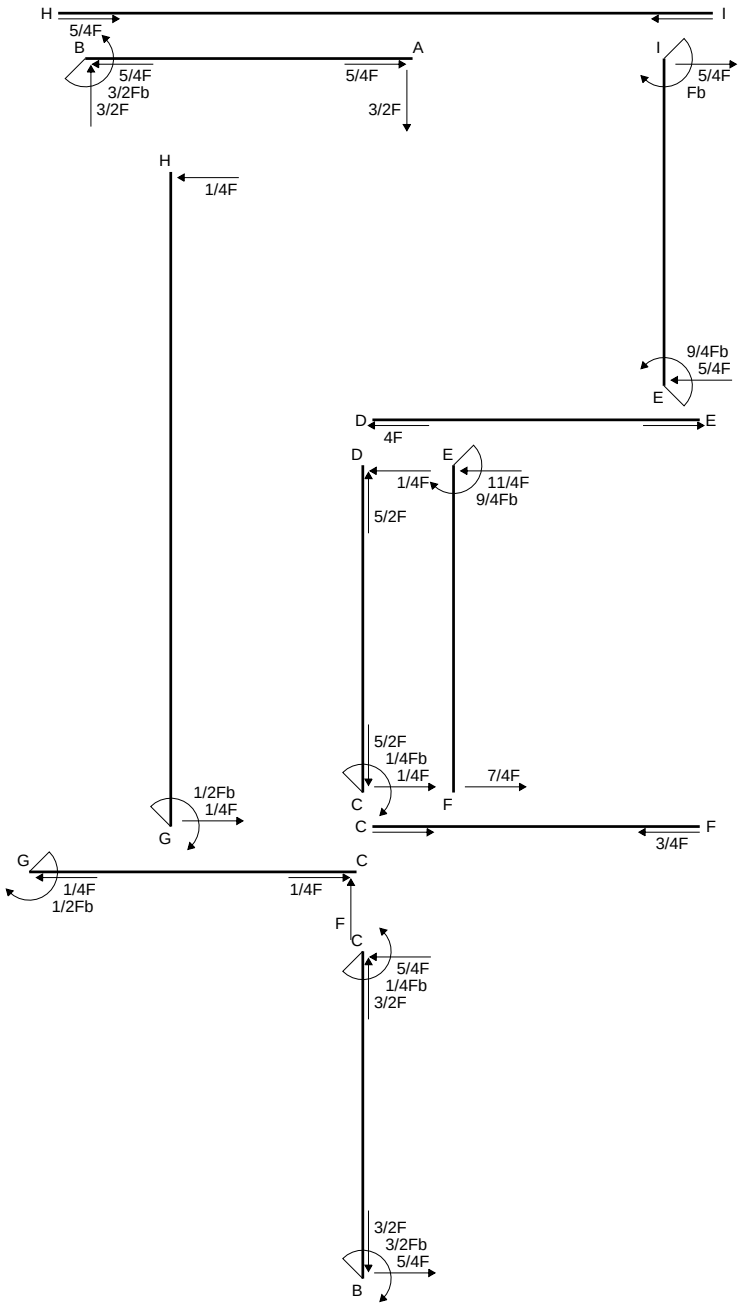


PROCEDIMENTO E RISULTATI 263840 CI01.xxxx.126



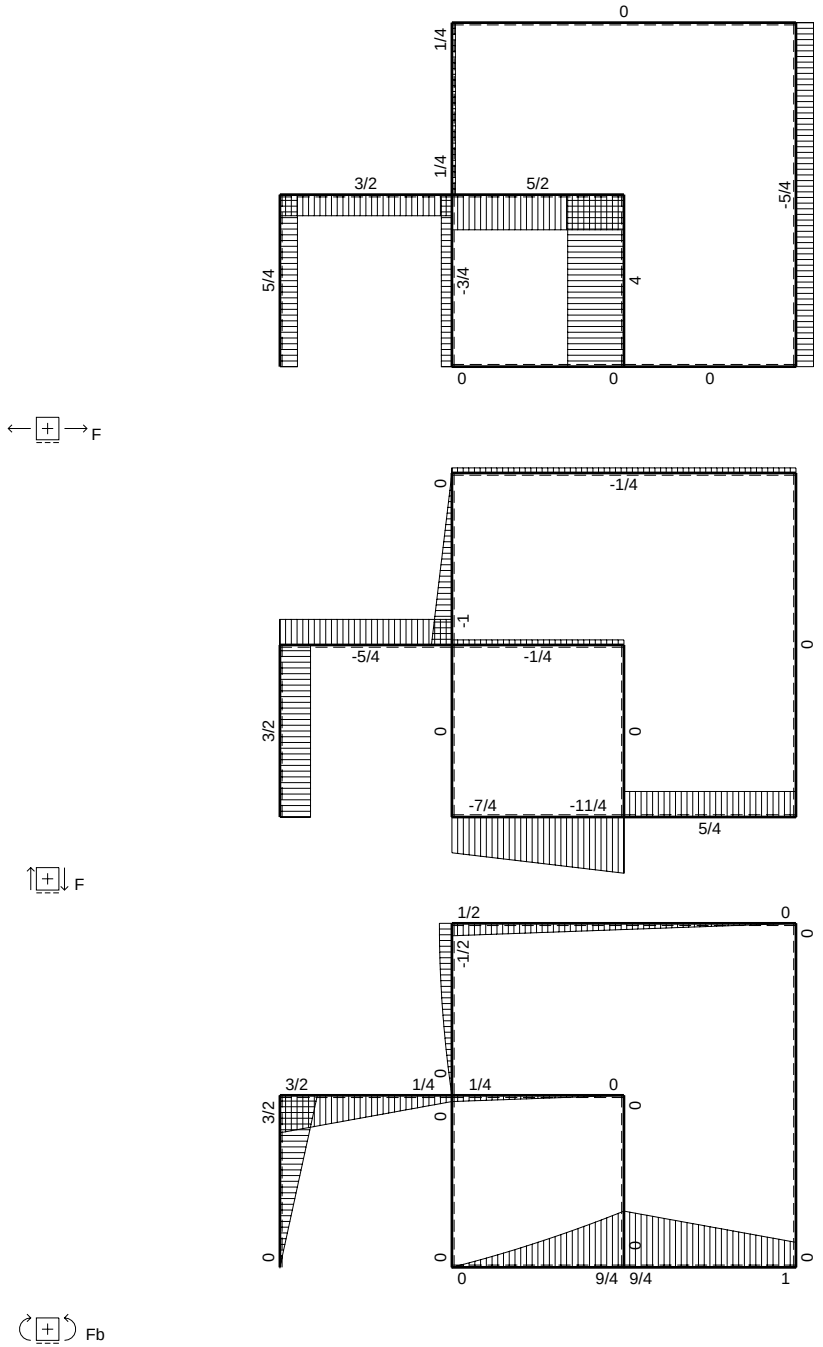

A = 780. $mm^2$
$J_u$ = 230087. $mm^4$
$J_v$ = 26640. $mm^4$
$y_g$ = 23.65 mm
$T_y$ = -2613. N
$M_x$ = 1539000. Nmm
$x_m$ = 24. mm
$y_m$ = 55. mm
$u_m$ = 6. mm
$v_m$ = 31.35 mm
$\sigma_m = -Mv/J_u$ = -209.7 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 42. mm
$v_c$ = 18.35 mm
$\sigma_c = -Mv/J_u$ = -122.7 $N/mm^2$
$\tau_c$ = 3.667 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.9 $N/mm^2$
$S^*$ = 3876. $mm^3$

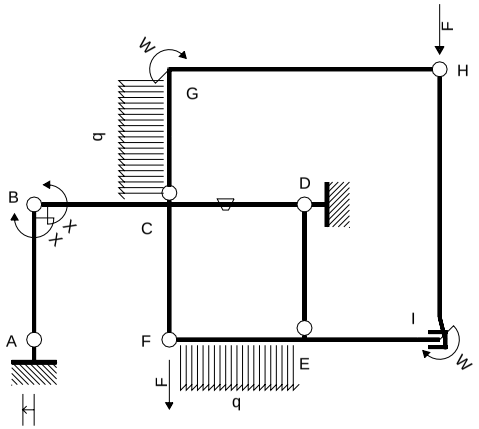
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H
I
5/4F
B
A
5/4F
3/2Fb
3/2F
5/4F
3/2F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
1/4F
11/4F
9/4Fb
5/2F
5/2F
1/4Fb
1/4F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/4F
1/2Fb
F
C
5/4F
1/4Fb
3/2F
3/2F
3/2Fb
5/4F
B

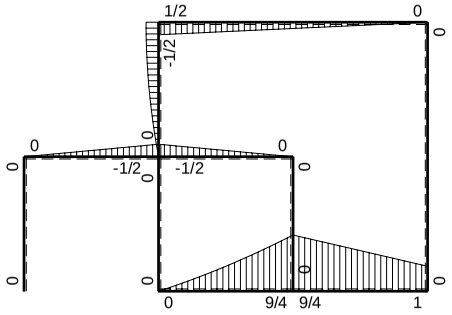




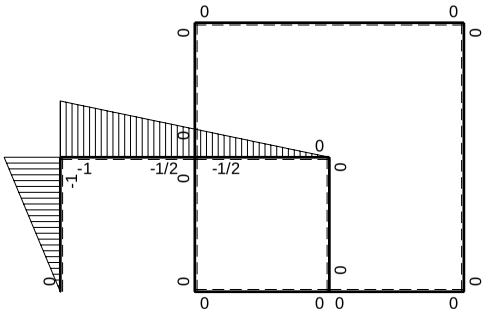

0
1/4
1/4
3/2
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-5/4
5/4
-3/4
4
0
0
0
F
0
-1/4
-1
-5/4
-1/4
0
0
0
3/2
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
0
3/2
1/4
1/4
0
0
3/2
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 240551 CI01.xxxx.127

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PROCEDIMENTO E RISULTATI 240551 CI01.xxxx.127

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -1/2Fx | 0 | $1/2Fx-1/4Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 1/2Fb-1/2Fx | 0 | $1/4Fb-1/4Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $3/2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -3/2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

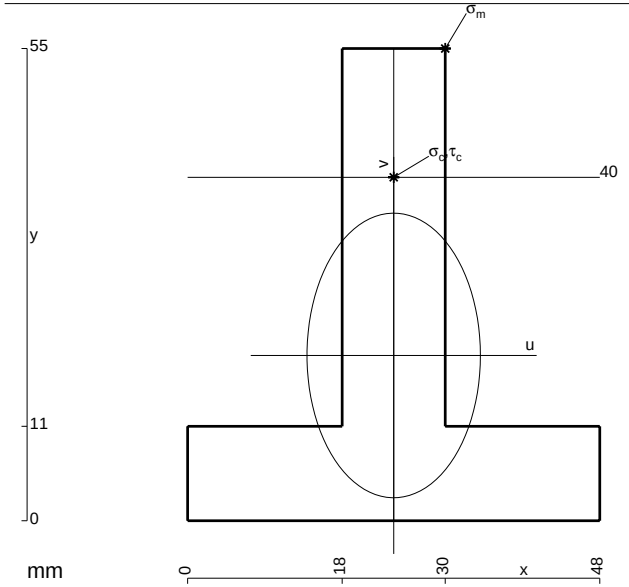
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

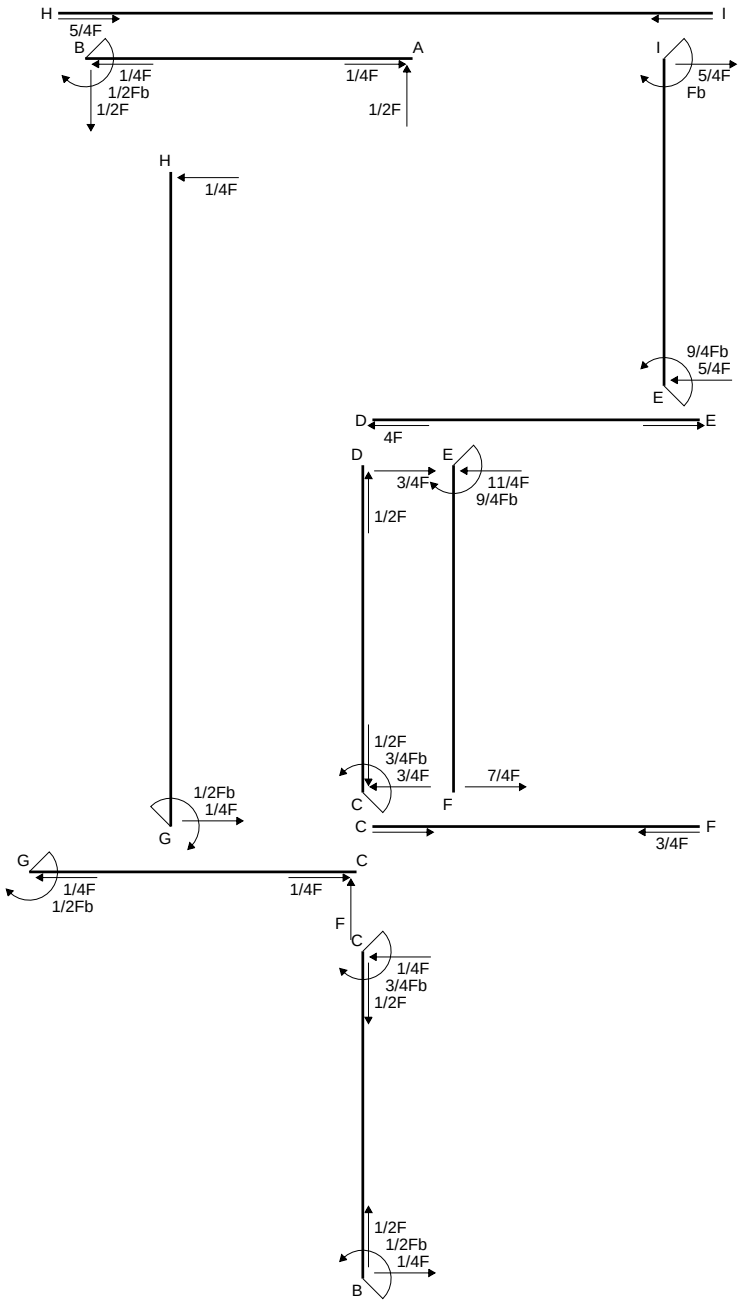
$= (1/12\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$





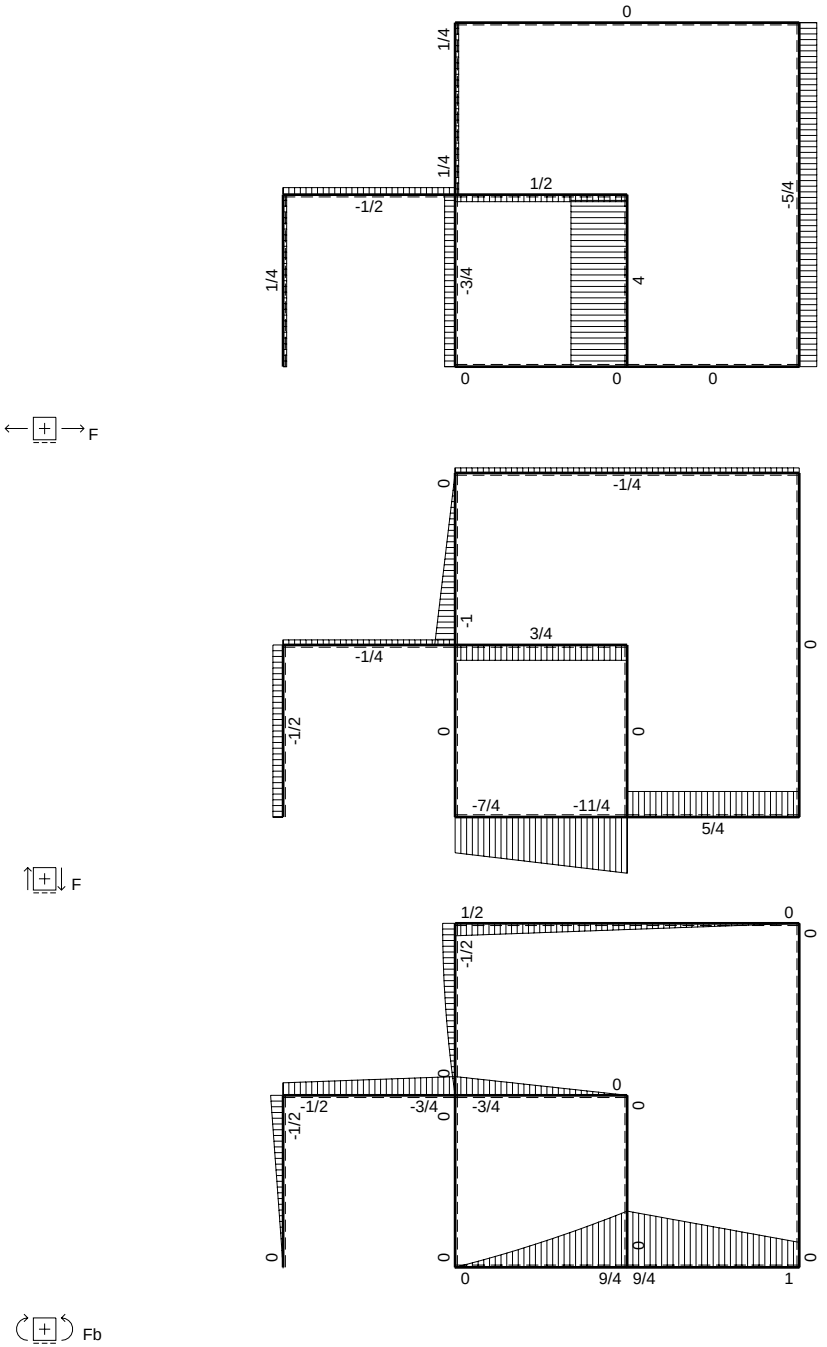


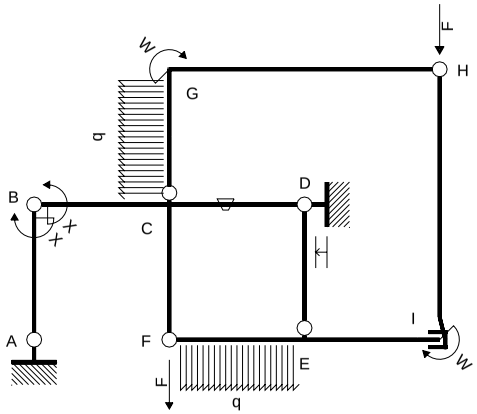

A = 1056. $mm^2$
$J_u$ = 290158. $mm^4$
$J_v$ = 107712. $mm^4$
$y_g$ = 19.25 mm
$T_y$ = -2860. N
$M_x$ = 1778400. Nmm
$x_m$ = 30. mm
$y_m$ = 55. mm
$u_m$ = 6. mm
$v_m$ = 35.75 mm
$\sigma_m = -Mv/J_u$ = -219.1 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 40. mm
$v_c$ = 20.75 mm
$\sigma_c = -Mv/J_u$ = -127.2 $N/mm^2$
$\tau_c$ = 4.177 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.4 $N/mm^2$
$S^*$ = 5085. $mm^3$


H
I
5/4F
B
A
1/4F
1/2Fb
1/2F
1/4F
1/2F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
4F
D
E
3/4F
11/4F
9/4Fb
1/2F
1/2F
3/4Fb
3/4F
7/4F
C
F
1/2Fb
1/4F
G
C
F
3/4F
G
C
1/4F
1/2Fb
1/4F
F
C
1/4F
3/4Fb
1/2F
1/2F
1/2Fb
1/4F
B

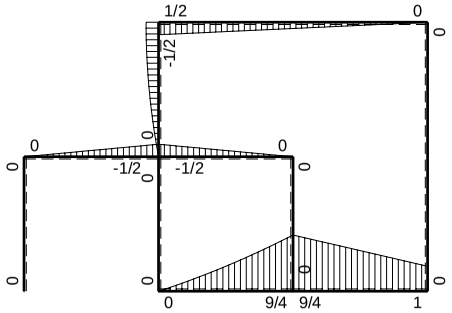




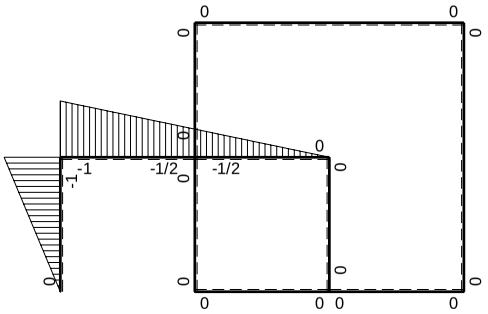

0
1/4
1/4
1/2
-1/2
-5/4
1/4
-3/4
4
0
0
0
F
-1/4
0
-1
3/4
-1/4
0
-1/2
0
0
-7/4
-11/4
5/4
F
1/2
0
0
-1/2
0
0
-1/2
-1/2
-3/4
-3/4
0
0
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 271572 CI01.xxxx.128

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PROCEDIMENTO E RISULTATI 271572 CI01.xxxx.128

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -1/2Fx | 0 | $1/2Fx-1/4Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 1/2Fb-1/2Fx | 0 | $1/4Fb-1/4Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $9/4Fb-11/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-7/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - 1/2\ b + 1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 + 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b + 1/4\ b + 1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b - 1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b - 1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b - 1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 - 1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x - 1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b - 1/12\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x - 1/4\ x^2/b\ ]_o^b\ \theta$

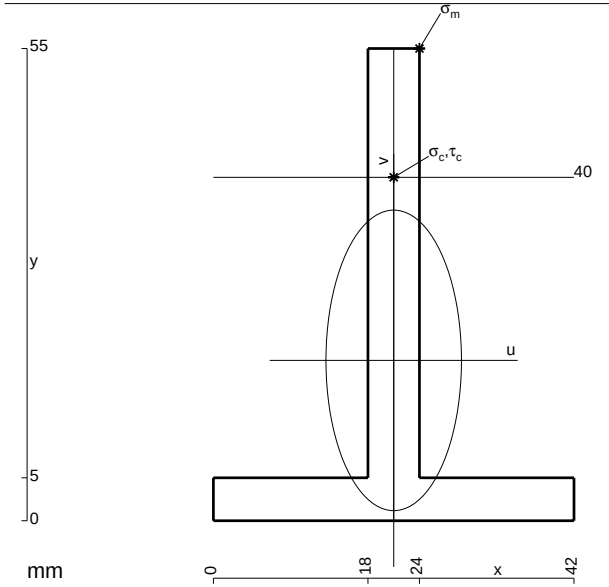
$= (1/4\ b - 1/4\ b + 1/12\ b)\ Fb\ 1/EJ\ + (1/2\ b - 1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/12\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$

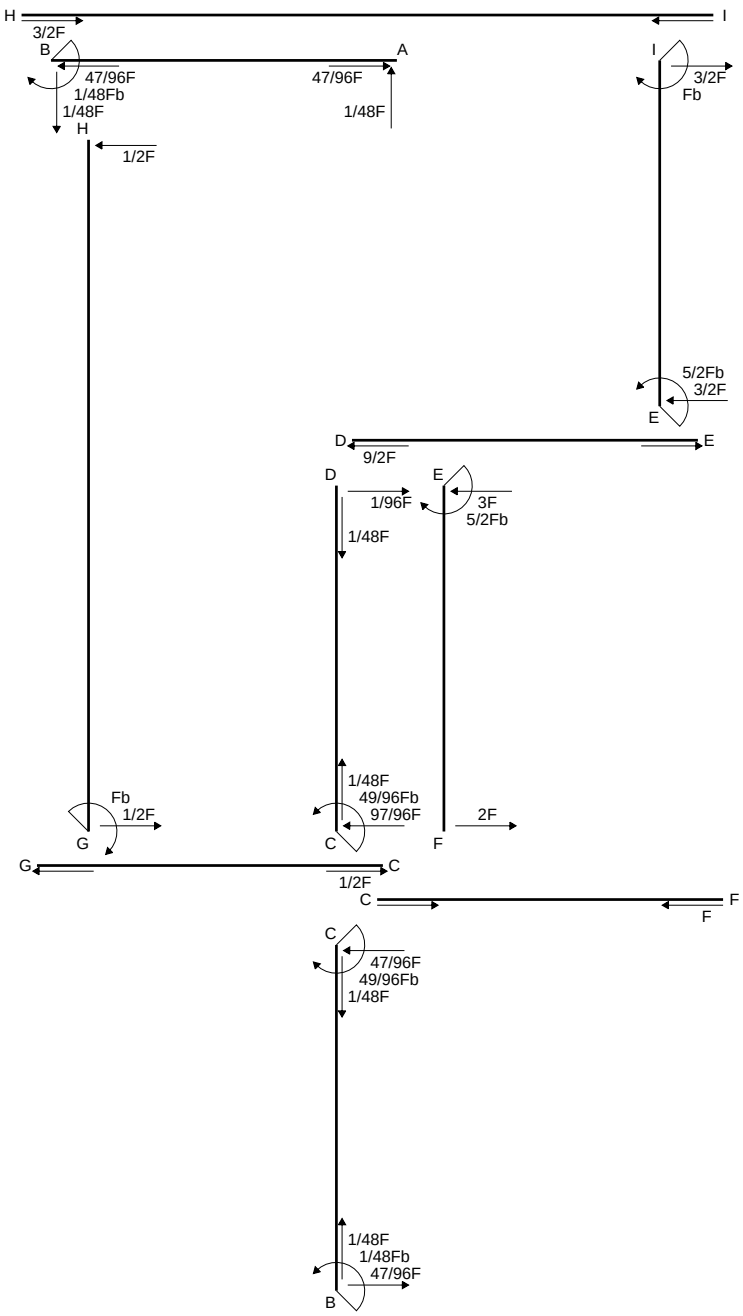


PROCEDIMENTO E RISULTATI 271572 CI01.xxxx.128



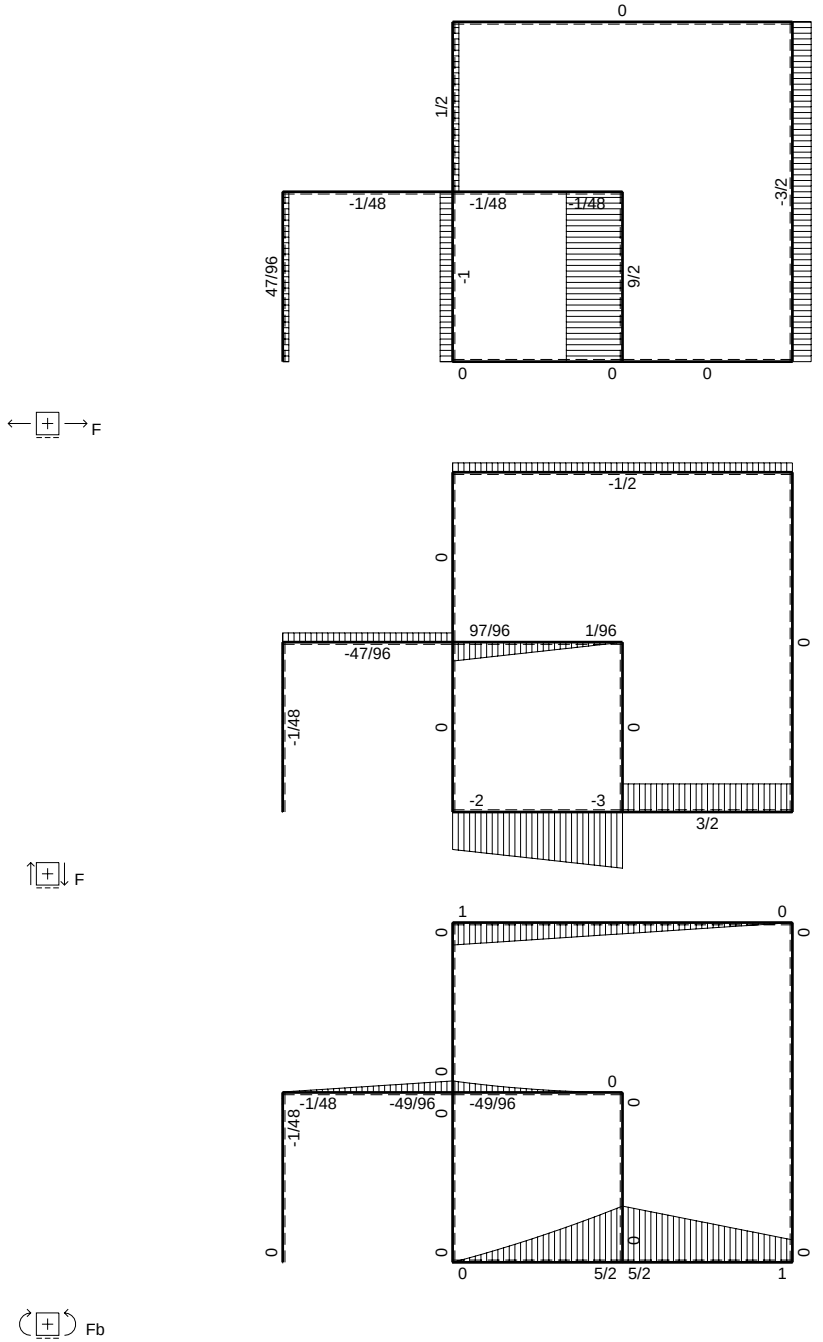

A = 510. $mm^2$
$J_u$ = 156357. $mm^4$
$J_v$ = 31770. $mm^4$
$y_g$ = 18.68 mm
$T_y$ = -1485. N
$M_x$ = 984150. Nmm
$x_m$ = 24. mm
$y_m$ = 55. mm
$u_m$ = 3. mm
$v_m$ = 36.32 mm
$\sigma_m = -Mv/J_u$ = -228.6 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 40. mm
$v_c$ = 21.32 mm
$\sigma_c = -Mv/J_u$ = -134.2 $N/mm^2$
$\tau_c$ = 4.106 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 134.4 $N/mm^2$
$S^*$ = 2594. $mm^3$

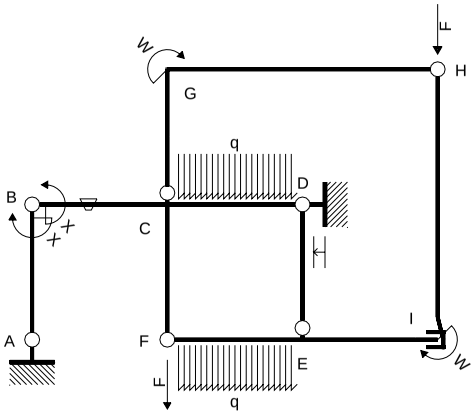
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H
I
3/2F
B
A
47/96F
1/48Fb
1/48F
47/96F
1/48F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
1/96F
3F
5/2Fb
1/48F
Fb
1/2F
G
1/48F
49/96Fb
97/96F
C
2F
F
G
C
1/2F
C
F
F
C
47/96F
49/96Fb
1/48F
1/48F
1/48Fb
47/96F
B

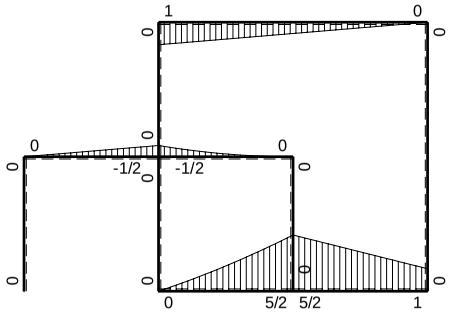




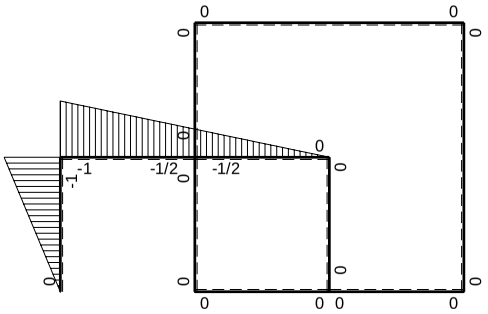

0
1/2
-3/2
-1/48
-1/48
-1/48
47/96
-1
9/2
0
0
0
F
-1/2
0
97/96
1/96
-47/96
-1/48
0
0
0
-2
-3
3/2
F
1
0
0
0
0
0
-1/48
-1/48
-49/96
-49/96
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248692 CI01.xxxx.129

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PROCEDIMENTO E RISULTATI 248692 CI01.xxxx.129

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -1/2Fx | -Fb/EJ | $1/2Fx-1/4Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/6+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 1/2Fb-1/2Fx | Fb/EJ | $1/4Fb-1/4Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+Fx-1/2qx^2$ | 0 | $1/4Fb-3/4Fx+3/4Fx^2/b-1/4qx^3/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/16+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | $1/2qx^2$ | 0 | $1/4qx^3/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/48Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/48Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -3/4\ x/b +3/4\ x^2/b^2 -1/4\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [1/4\ x -3/8\ x^2/b +1/4\ x^3/b^2 -1/16\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

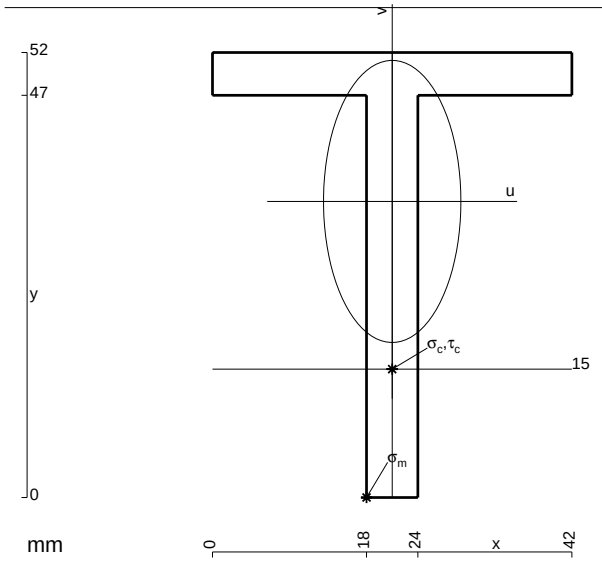
$= (1/4\ b -3/8\ b +1/4\ b -1/16\ b\ )\ Fb\ 1/EJ\ = 1/16\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/16\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/16\ b\ )\ Fb\ 1/EJ\ = 1/16\ Fb^2/EJ$

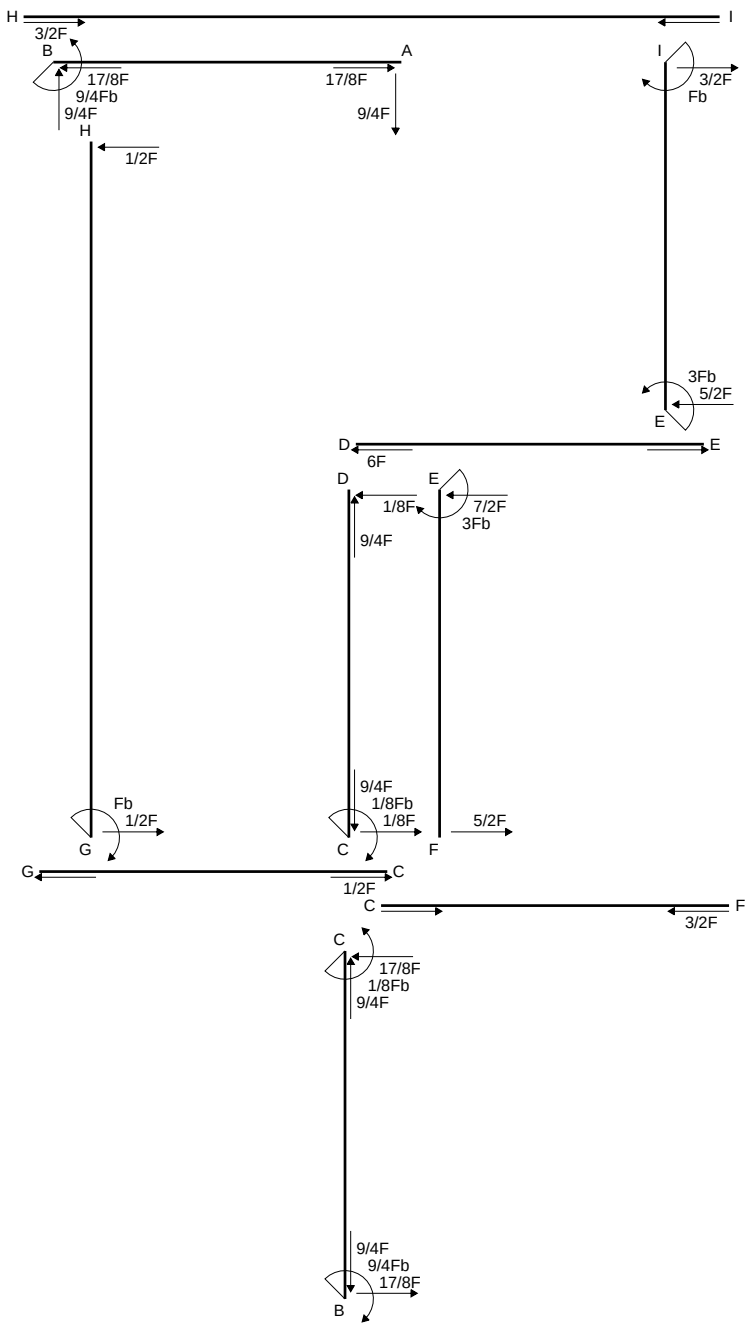


PROCEDIMENTO E RISULTATI 248692 CI01.xxxx.129



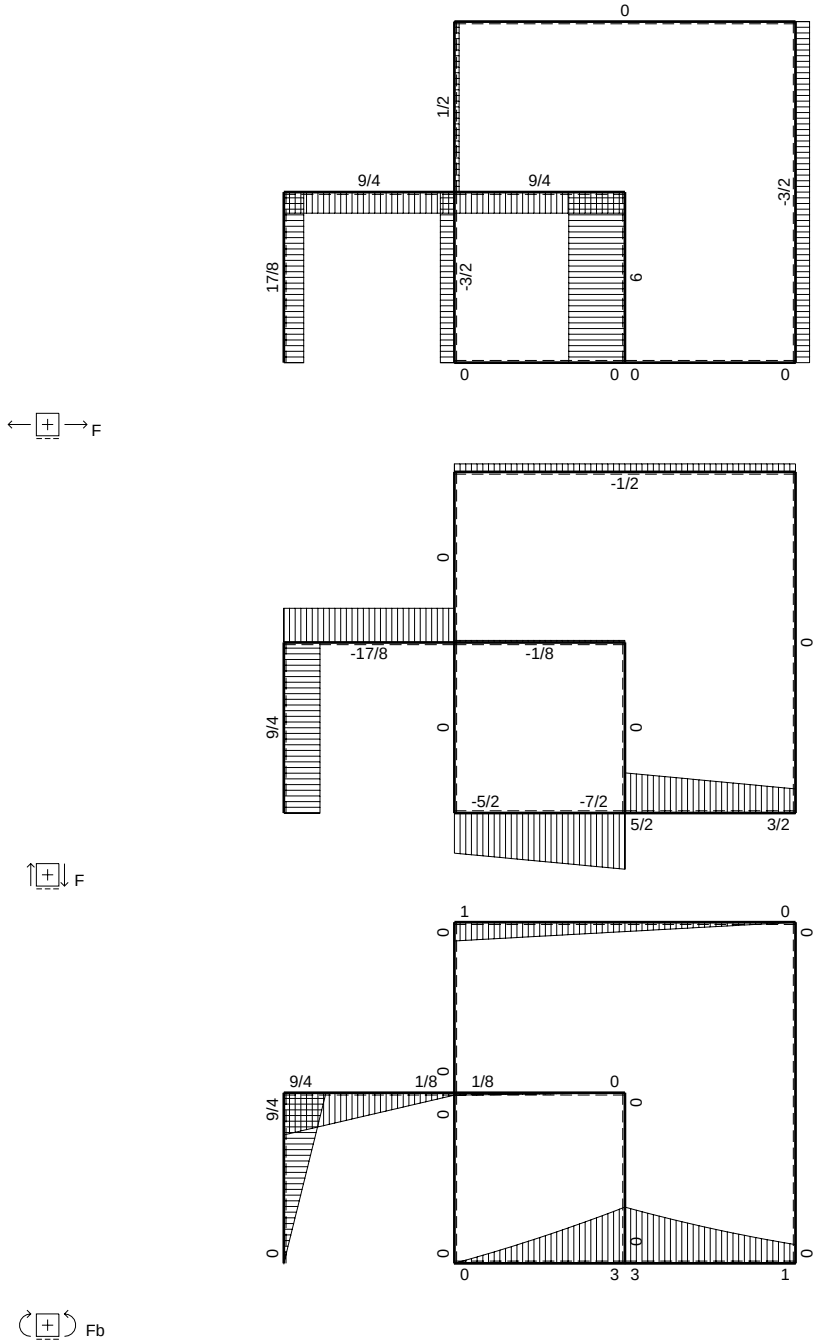

A = 492. $mm^2$
$J_u$ = 133716. $mm^4$
$J_v$ = 31716. $mm^4$
$y_g$ = 34.6 mm
$T_y$ = -1350. N
$M_x$ = 911250. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -34.6 mm
$\sigma_m = -Mv/J_u$ = 235.8 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 15. mm
$v_c$ = -19.6 mm
$\sigma_c = -Mv/J_u$ = 133.6 $N/mm^2$
$\tau_c$ = 4.104 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 133.7 $N/mm^2$
$S^*$ = 2439. $mm^3$

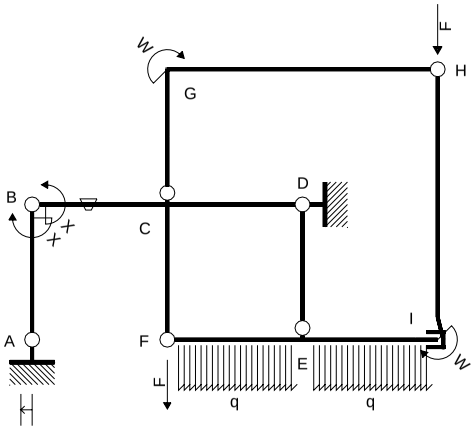
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H
I
3/2F
B
A
17/8F
17/8F
9/4Fb
9/4F
9/4F
H
1/2F
I
3/2F
Fb
3Fb
5/2F
E
D
E
6F
D
E
1/8F
7/2F
3Fb
9/4F
9/4F
1/8Fb
Fb
1/2F
1/8F
5/2F
G
C
F
G
C
1/2F
C
F
3/2F
C
17/8F
1/8Fb
9/4F
9/4F
9/4Fb
17/8F
B

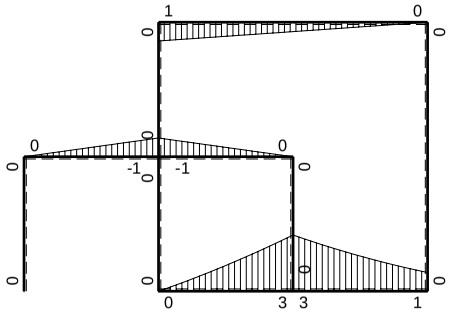




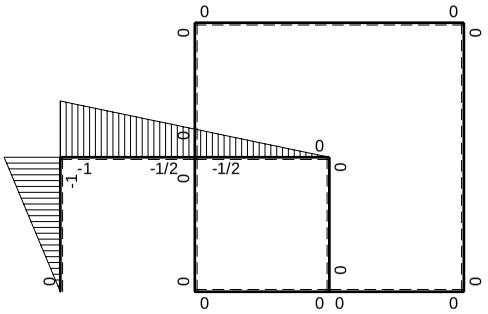

F
0
1/2
9/4
9/4
-3/2
17/8
-3/2
6
0
0 0
0
F
-1/2
0
-17/8
-1/8
0
9/4
0
0
-5/2
-7/2
5/2
3/2
Fb
1
0
0
0
9/4
1/8
1/8
0
0
0
0
0
0
3 3
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268836 CI01.xxxx.130

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PROCEDIMENTO E RISULTATI 268836 CI01.xxxx.130

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-Fb+Fx$ | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $9/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -9/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

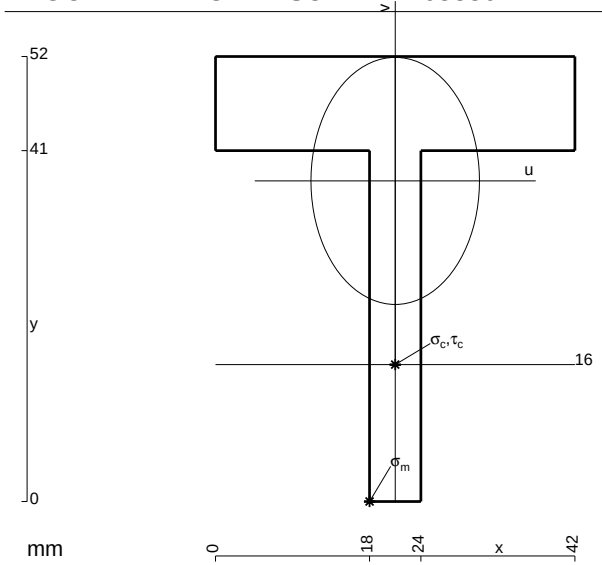
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

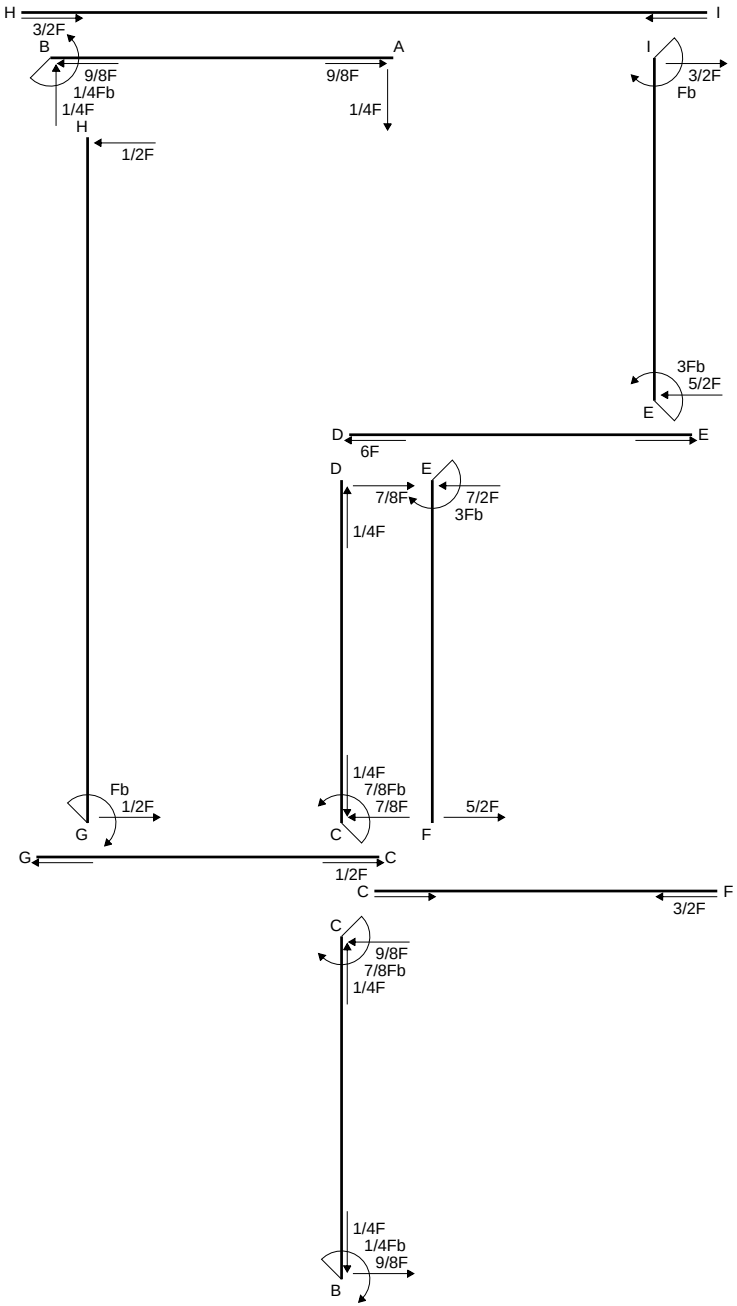
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





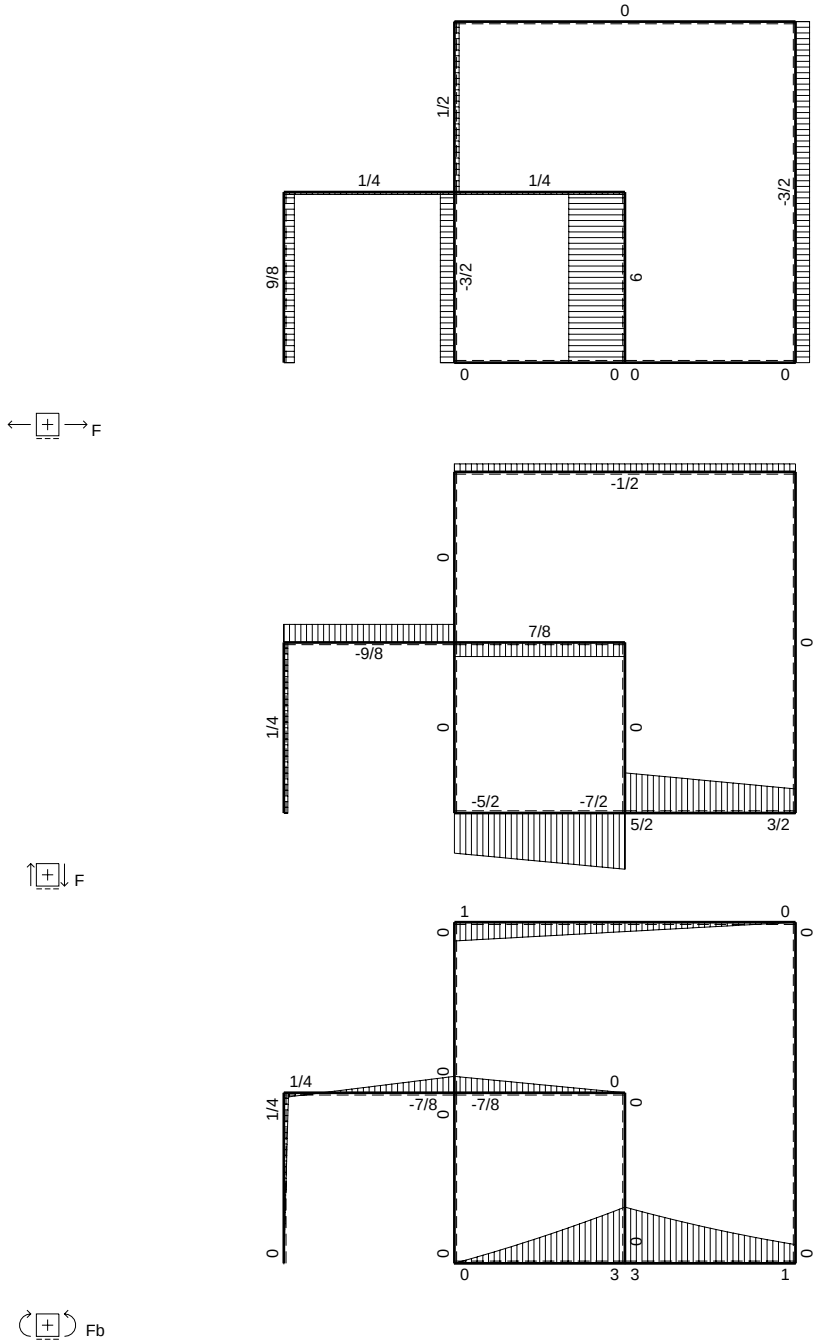


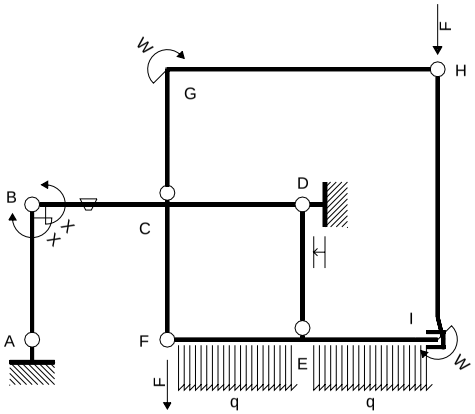

A = 708. $mm^2$
$J_u$ = 147634. $mm^4$
$J_v$ = 68652. $mm^4$
$y_g$ = 37.47 mm
$T_y$ = -2135. N
$M_x$ = 786900. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -37.47 mm
$\sigma_m = -Mv/J_u$ = 199.7 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -21.47 mm
$\sigma_c = -Mv/J_u$ = 114.4 $N/mm^2$
$\tau_c$ = 6.818 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115. $N/mm^2$
$S^*$ = 2829. $mm^3$


H
I
3/2F
B
A
I
9/8F
1/4Fb
1/4F
9/8F
1/4F
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
6F
D
E
7/8F
7/2F
3Fb
1/4F
1/4F
7/8Fb
7/8F
Fb
1/2F
5/2F
G
C
F
G
C
1/2F
C
F
3/2F
C
9/8F
7/8Fb
1/4F
1/4F
1/4Fb
9/8F
B

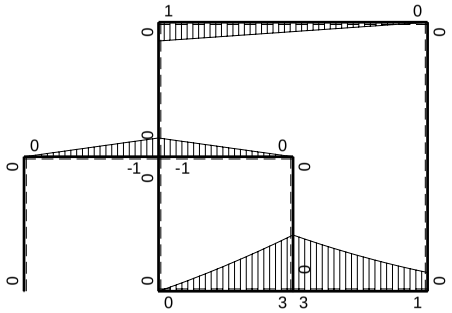




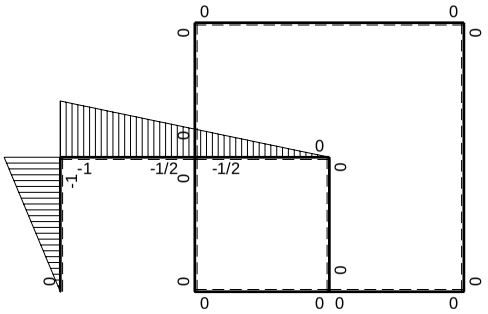

0
1/2
1/4
1/4
-3/2
9/8
-3/2
6
0
0 0
0
F
-1/2
0
7/8
-9/8
0
1/4
0
0
-5/2
-7/2
5/2
3/2
F
1
0
0
0
1/4
0
0
1/4
-7/8
-7/8
0
0
0
0
0
3 3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268787 CI01.xxxx.131

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PROCEDIMENTO E RISULTATI 268787 CI01.xxxx.131

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

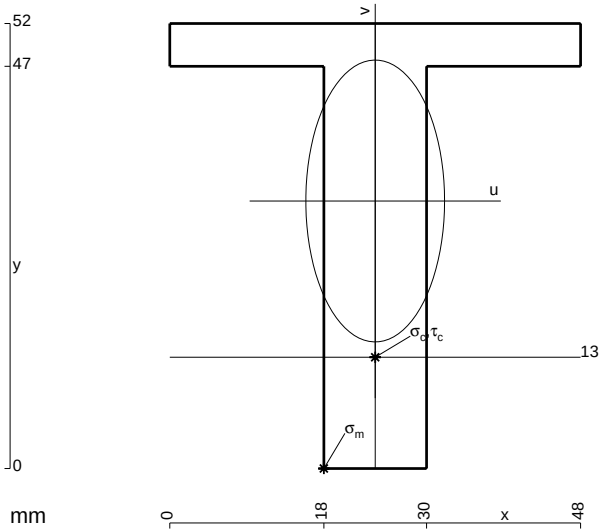
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$








$A = 804.\ mm^2$

$J_u = 218133.\ mm^4$

$J_v = 52848.\ mm^4$

$y_g = 31.26\ mm$

$T_y = -3605.\ N$

$M_x = 1452300.\ Nmm$

$x_m = 18.\ mm$

$u_m = -6.\ mm$

$v_m = -31.26\ mm$

$\sigma_m = -Mv/J_u = 208.1\ N/mm^2$

$x_c = 24.\ mm$

$y_c = 13.\ mm$

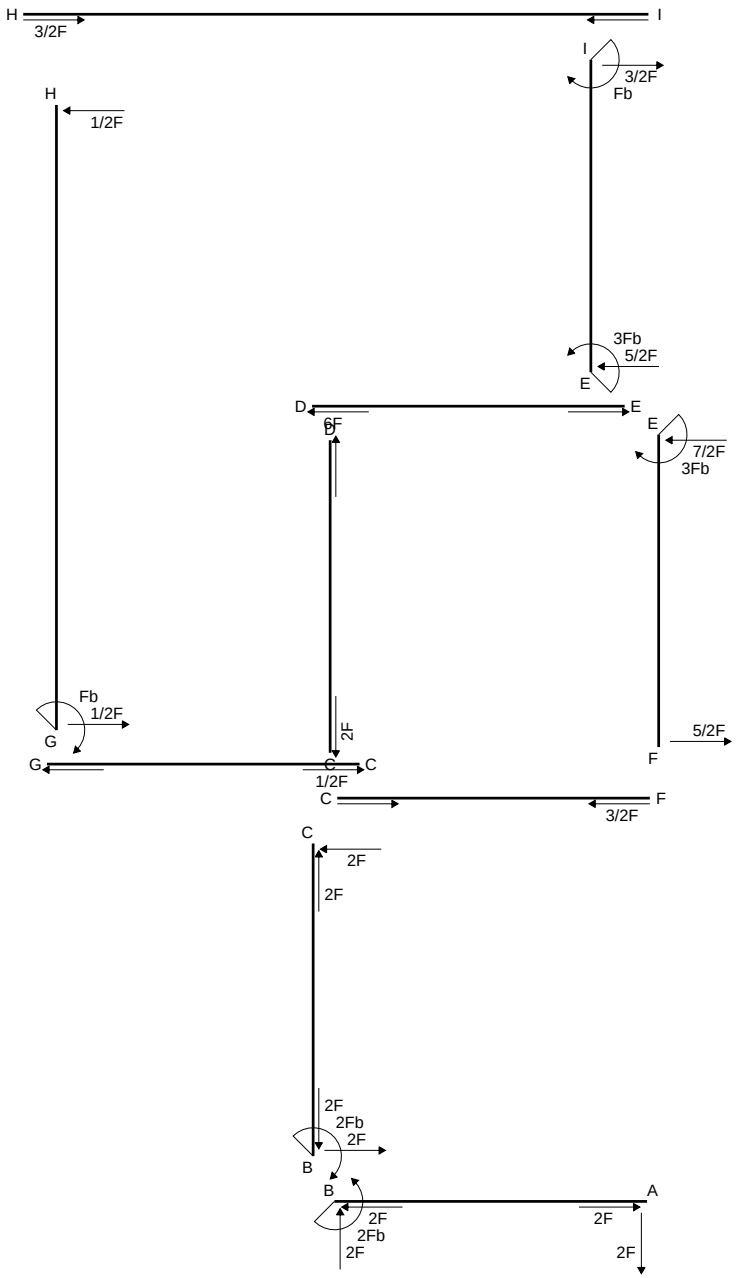
$v_c = -18.26\ mm$

$\sigma_c = -Mv/J_u = 121.6\ N/mm^2$

$\tau_c = 5.32\ N/mm^2$

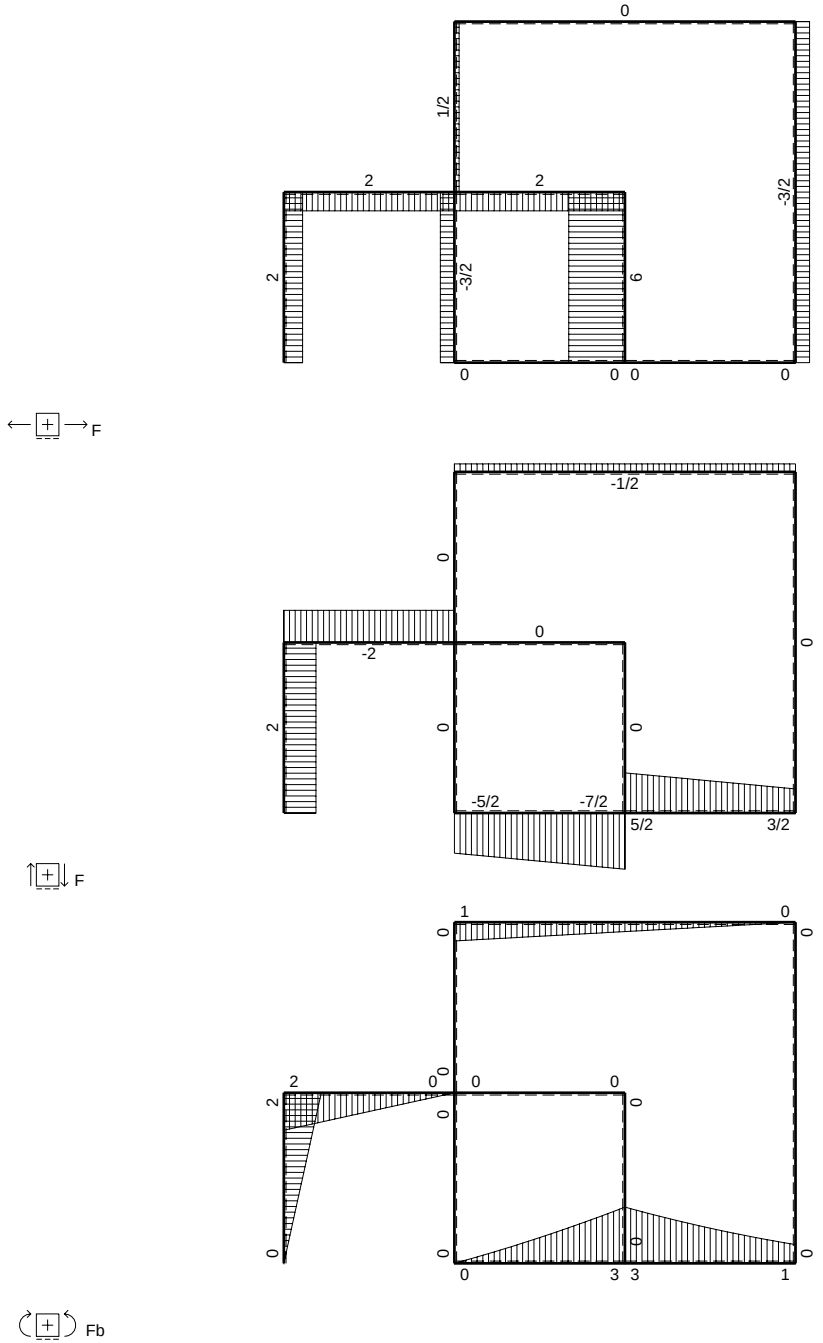
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 121.9\ N/mm^2$

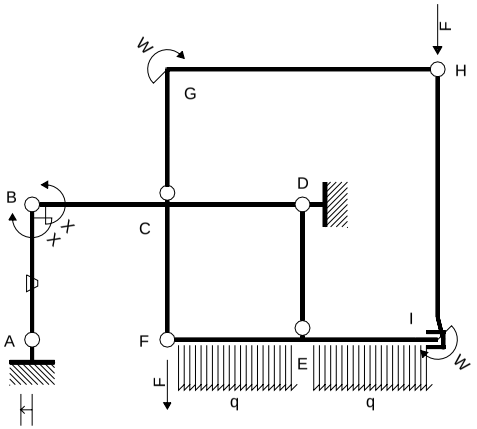
$S^* = 3863.\ mm^3$


H
I
3/2F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
6F
D
E
7/2F
3Fb
Fb
1/2F
G
2F
G
C
C
1/2F
5/2F
F
C
F
3/2F
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2Fb
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B
A
2F
2F
2Fb
2F
2F

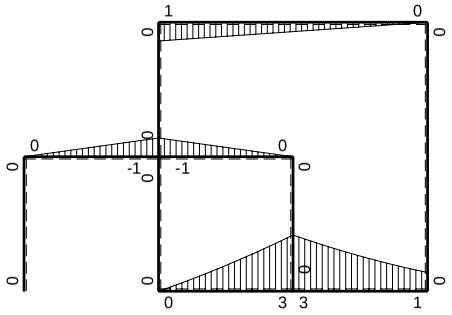




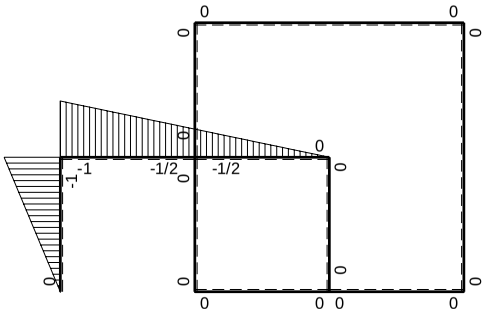

F
F
Fb
0
1/2
2
2
-3/2
2
-3/2
6
0
0 0
0
-1/2
0
0
-2
0
2
0
0
0
-5/2
-7/2
5/2
3/2
1
0
0
0
2
0 0
0
0
2
0
0
0
0
0
3 3
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 242068 CI01.xxxx.132

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 242068 CI01.xxxx.132

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-Fb+Fx$ | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

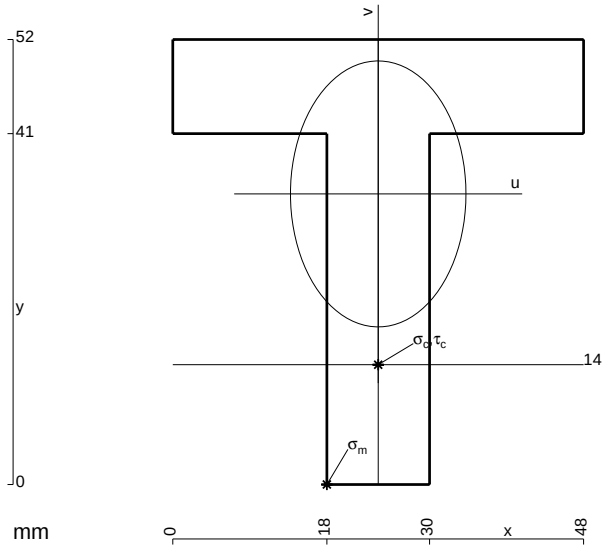
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

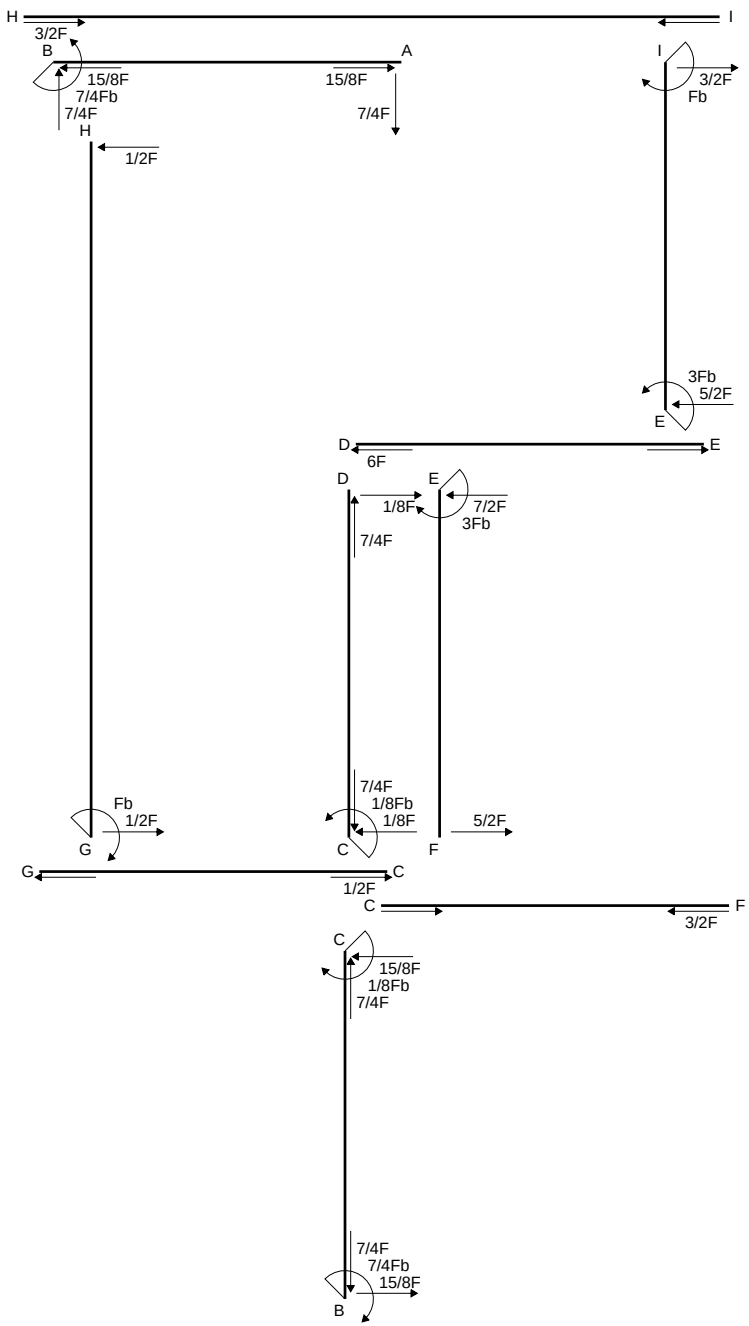
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





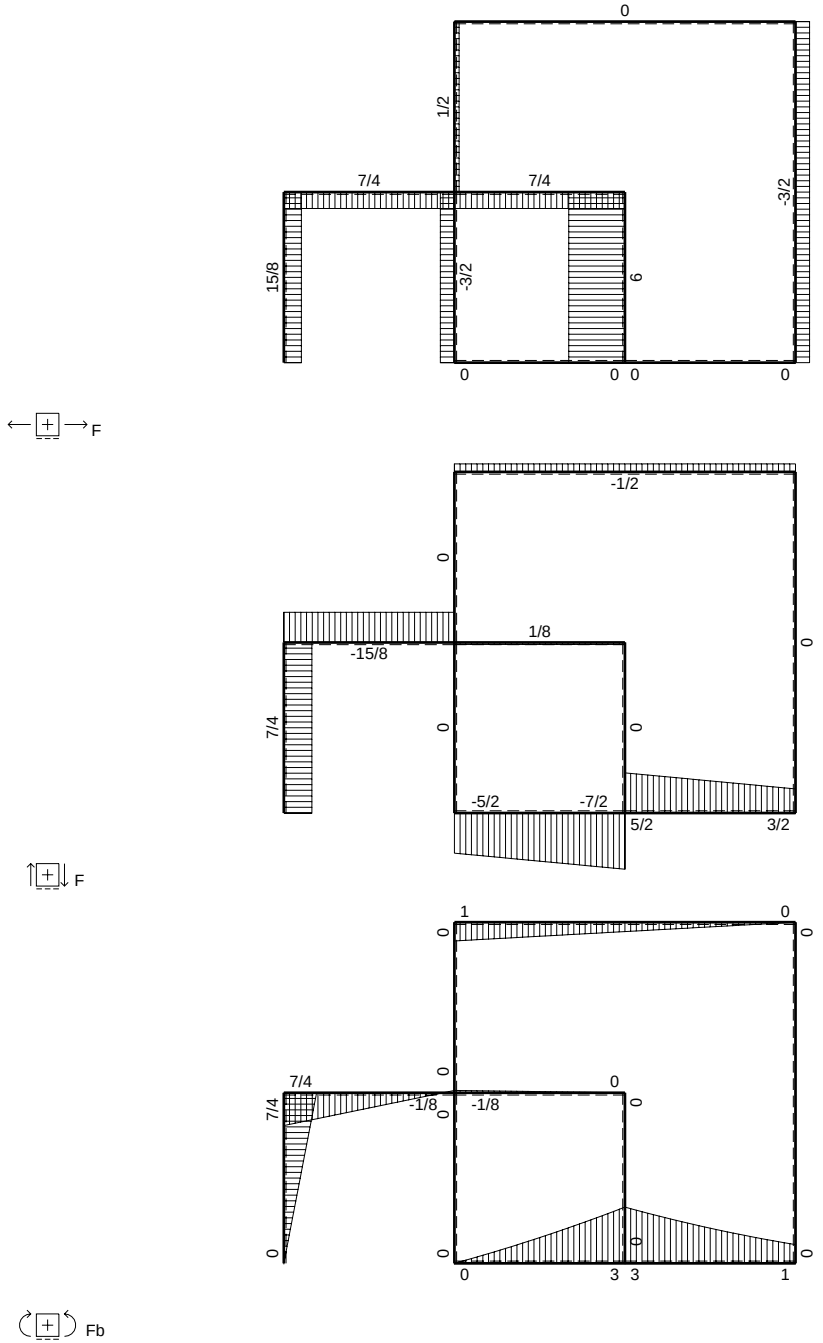


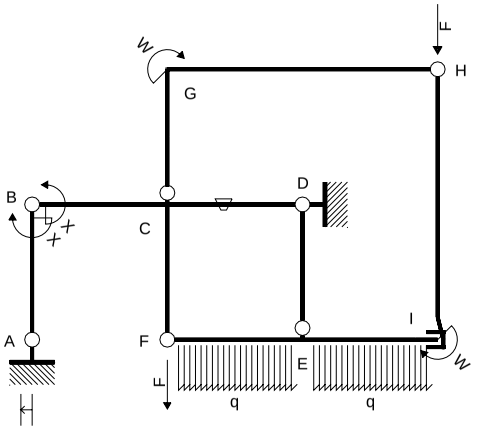

A = 1020. $mm^2$
$J_u$ = 246410. $mm^4$
$J_v$ = 107280. $mm^4$
$y_g$ = 33.96 mm
$T_y$ = -3640. N
$M_x$ = 1591200. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -33.96 mm
$\sigma_m$ = -Mv/$J_u$ = 219.3 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 14. mm
$v_c$ = -19.96 mm
$\sigma_c$ = -Mv/$J_u$ = 128.9 $N/mm^2$
$\tau_c$ = 5.575 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.2 $N/mm^2$
$S^*$ = 4529. $mm^3$


H
I
3/2F
B
A
15/8F
15/8F
7/4Fb
7/4F
7/4F
H
1/2F
I
3/2F
Fb
3Fb
5/2F
E
D
E
6F
D
E
1/8F
7/2F
3Fb
7/4F
7/4F
1/8Fb
1/8F
5/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
3/2F
C
15/8F
1/8Fb
7/4F
7/4F
7/4Fb
15/8F
B

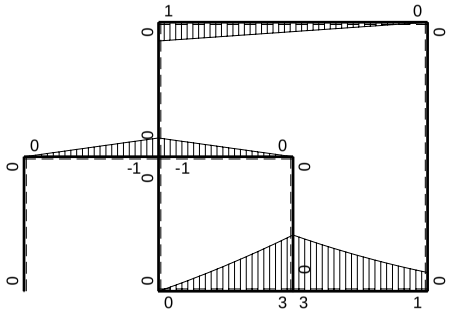




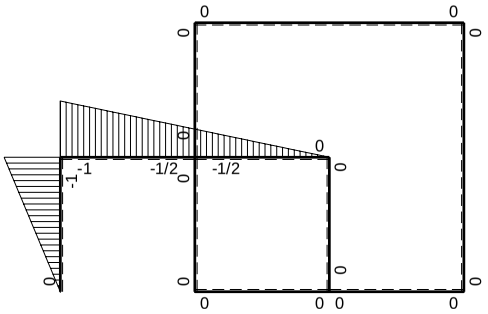

F
0
1/2
7/4
7/4
-3/2
15/8
-3/2
6
0
0 0
0
F
-1/2
0
1/8
-15/8
0
7/4
0
0
-5/2
-7/2
5/2
3/2
Fb
1
0
0
0
0
7/4
0
7/4
-1/8
-1/8
0
0
0
0
0
3 3
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 269676 CI01.xxxx.133

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 269676 CI01.xxxx.133

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $7/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

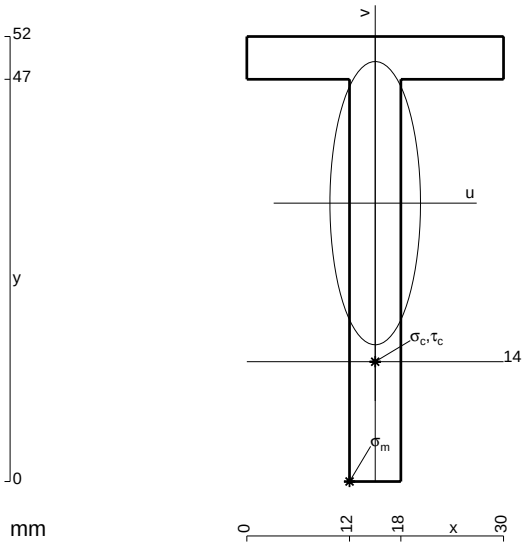
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

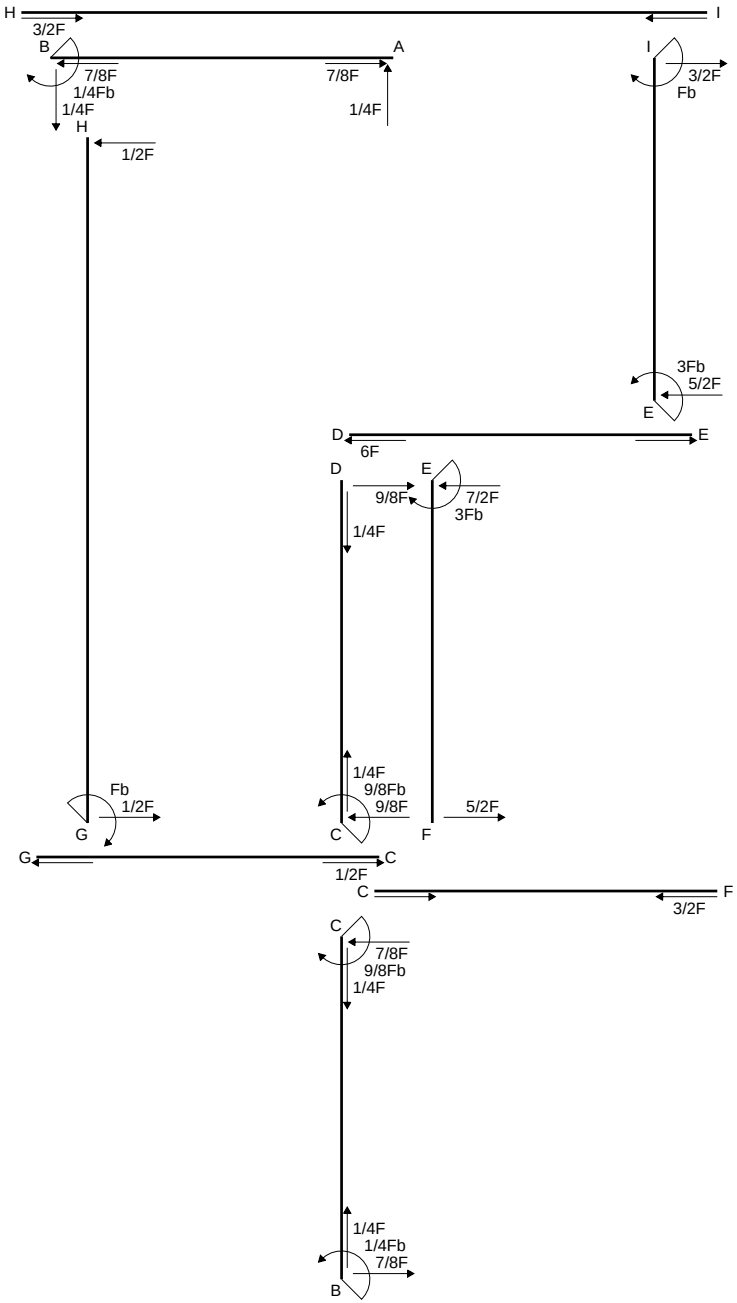
$= (1/6\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$





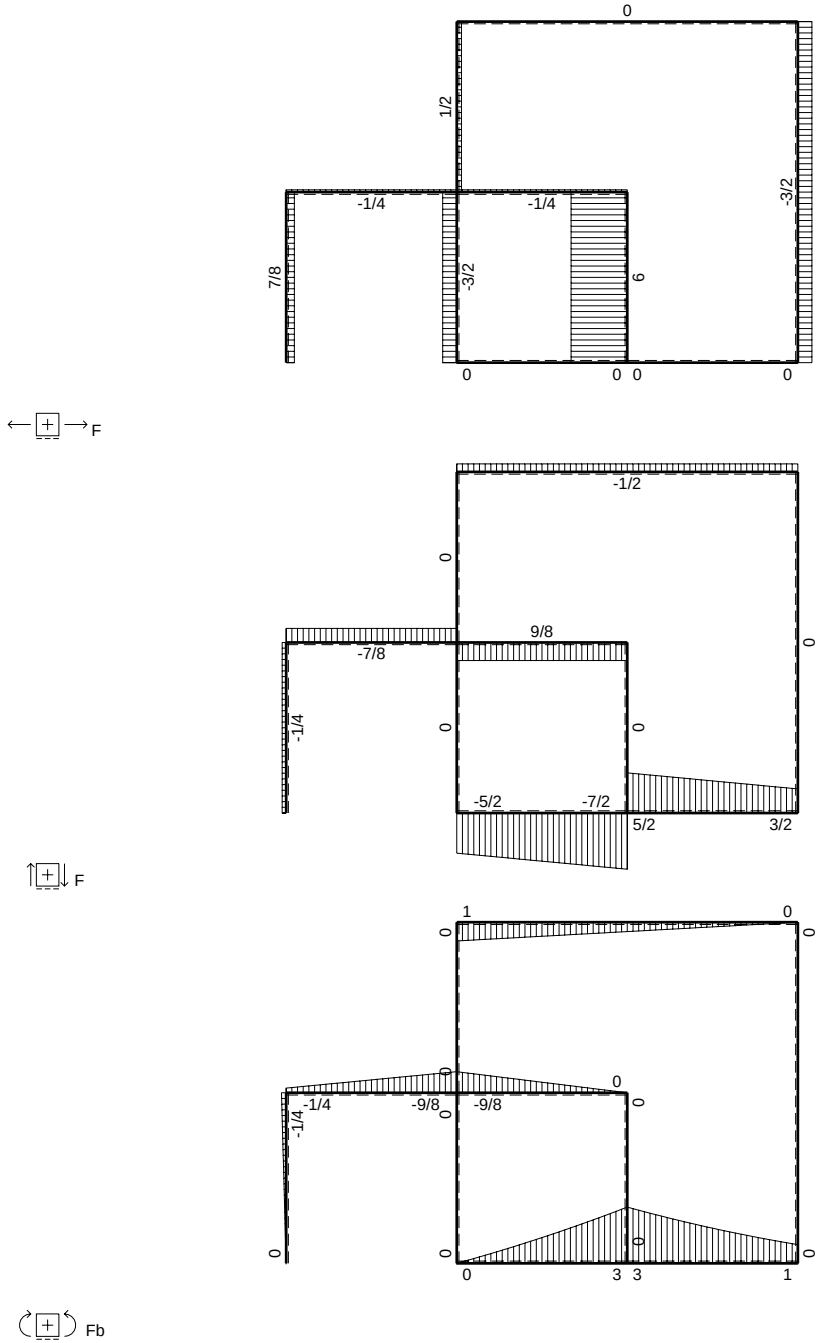


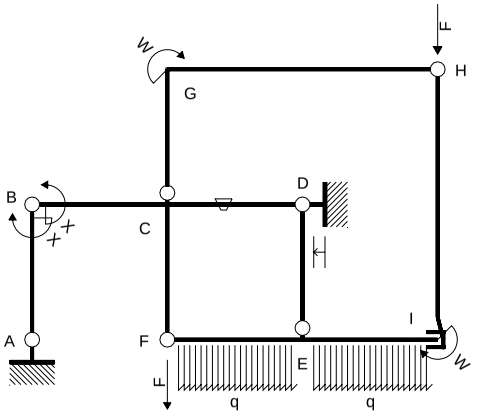

A = 432. $mm^2$
$J_u$ = 118416. $mm^4$
$J_v$ = 12096. $mm^4$
$y_g$ = 32.53 mm
$T_y$ = -1750. N
$M_x$ = 825000. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -32.53 mm
$\sigma_m = -Mv/J_u$ = 226.6 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 14. mm
$v_c$ = -18.53 mm
$\sigma_c = -Mv/J_u$ = 129.1 $N/mm^2$
$\tau_c$ = 5.282 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 129.4 $N/mm^2$
$S^*$ = 2144. $mm^3$


H
3/2F
I
B
A
7/8F
1/4Fb
1/4F
7/8F
1/4F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
6F
D
E
9/8F
7/2F
3Fb
1/4F
1/4F
9/8Fb
9/8F
Fb
1/2F
G
C
F
5/2F
G
C
1/2F
C
F
3/2F
C
7/8F
9/8Fb
1/4F
1/4F
1/4Fb
7/8F
B

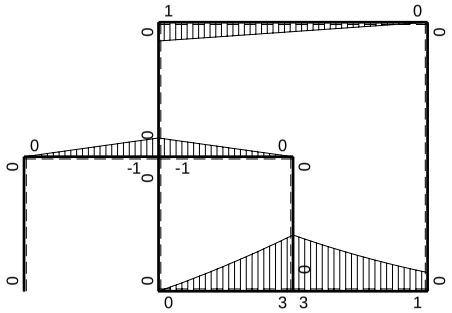




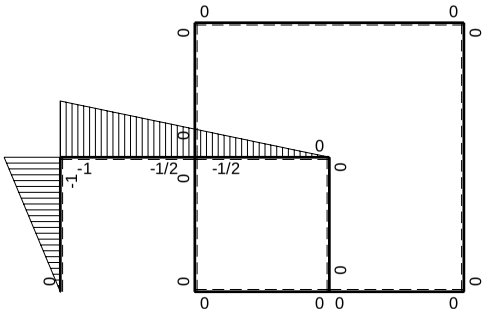

F
0
1/2
-1/4
-1/4
-3/2
7/8
-3/2
6
0
0 0
0
F
-1/2
0
9/8
-7/8
0
-1/4
0
0
-5/2
-7/2
5/2
3/2
F
1
0
0
0
0
0
-1/4
-9/8
-9/8
0
0
0
0
0
0
3 3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

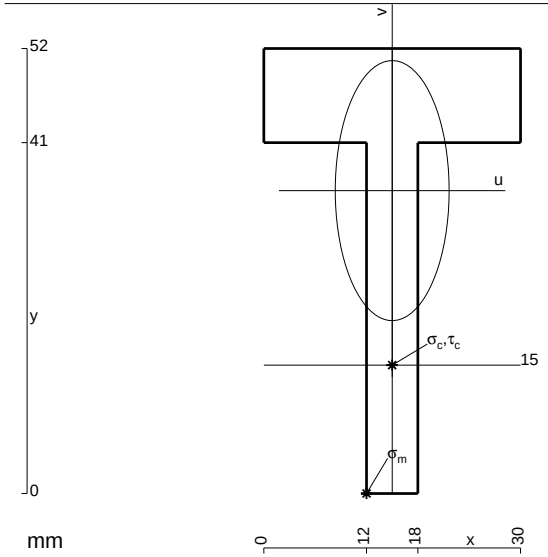
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

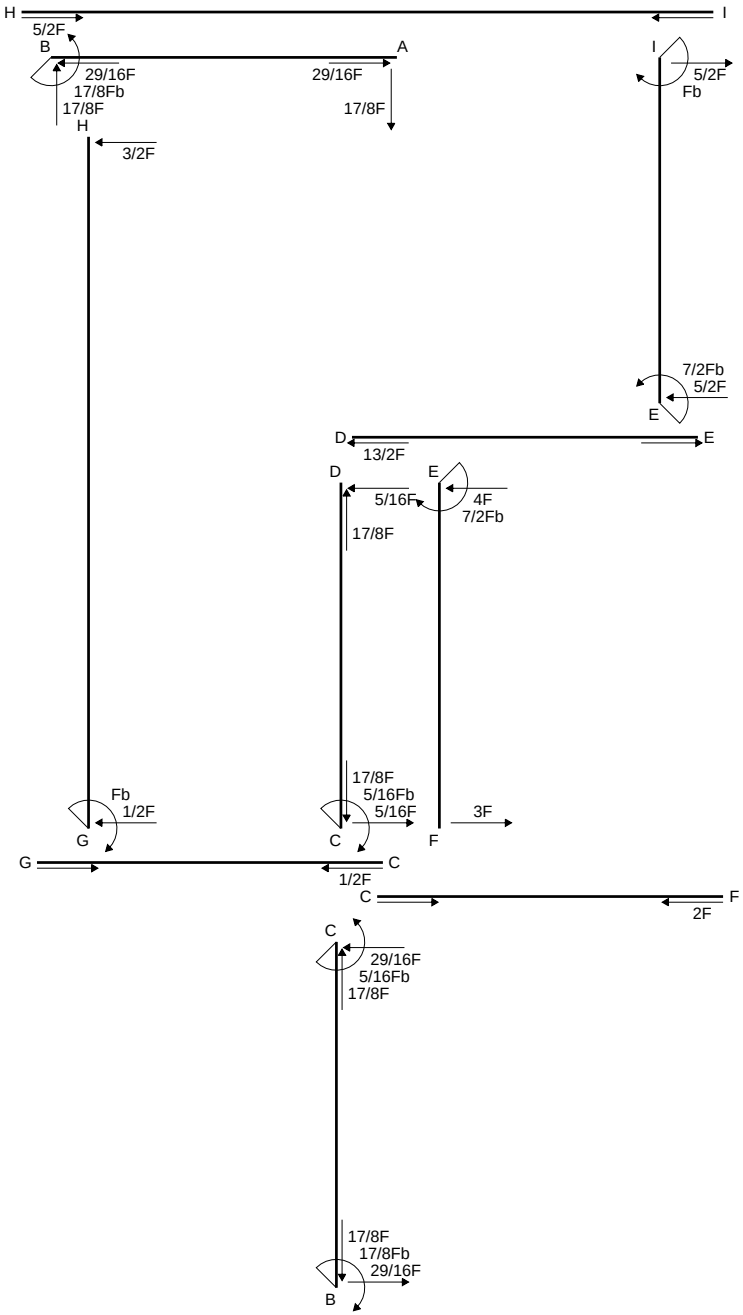


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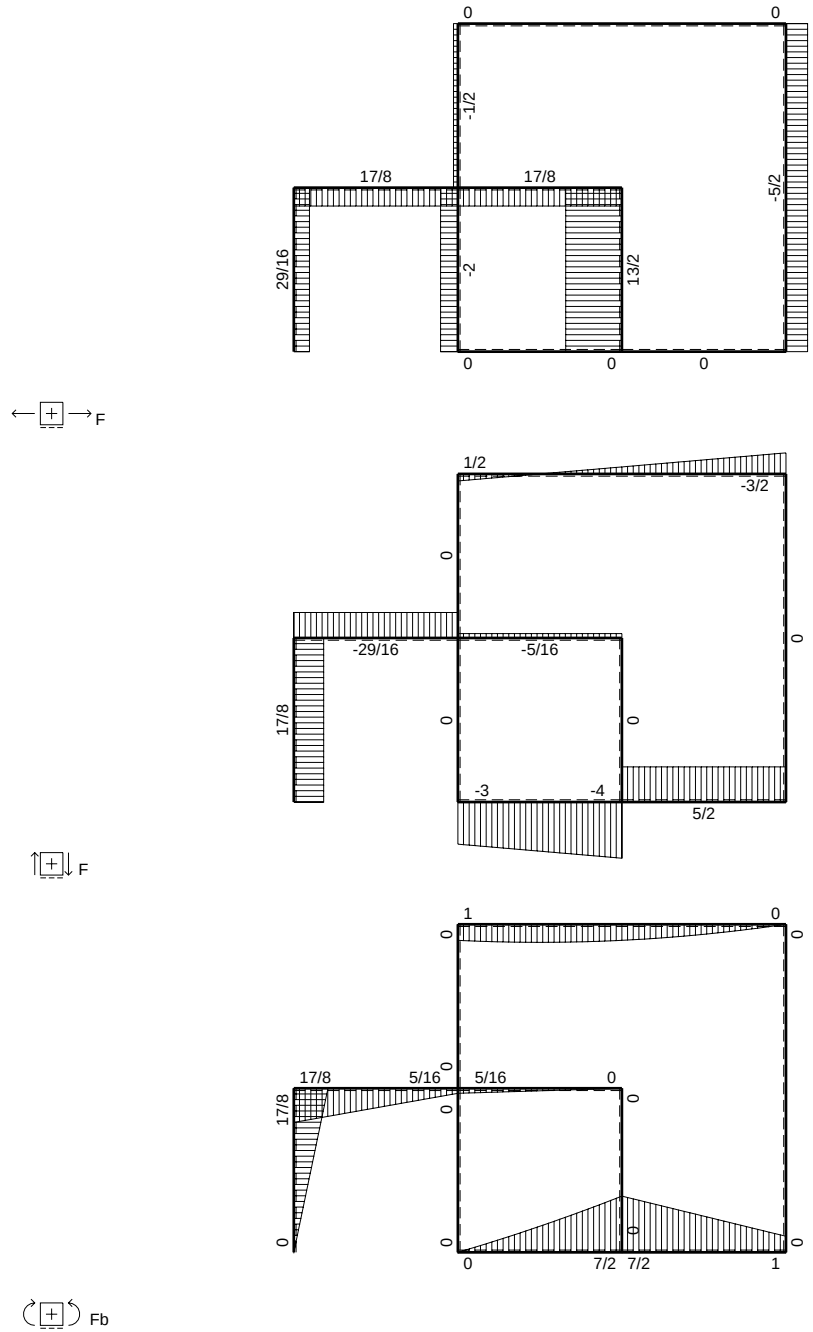

A = 576. $mm^2$
$J_u$ = 133062. $mm^4$
$J_v$ = 25488. $mm^4$
$y_g$ = 35.4 mm
$T_y$ = -1750. N
$M_x$ = 900000. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -35.4 mm
$\sigma_m = -Mv/J_u$ = 239.4 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 15. mm
$v_c$ = -20.4 mm
$\sigma_c = -Mv/J_u$ = 138. $N/mm^2$
$\tau_c$ = 5.503 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 138.3 $N/mm^2$
$S^*$ = 2511. $mm^3$

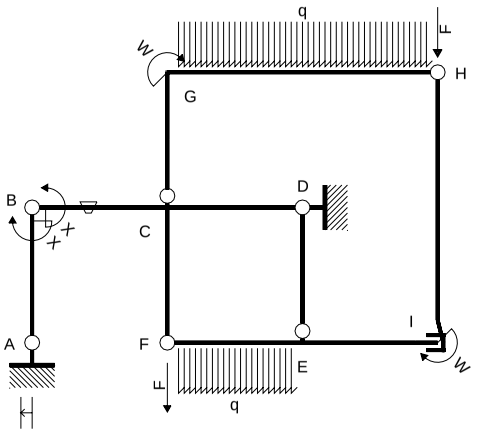
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H
I
5/2F
B
A
29/16F
17/8Fb
17/8F
29/16F
17/8F
I
5/2F
Fb
H
3/2F
7/2Fb
5/2F
E
D
E
13/2F
D
E
5/16F
4F
7/2Fb
17/8F
17/8F
5/16Fb
5/16F
3F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
2F
C
29/16F
5/16Fb
17/8F
17/8F
17/8Fb
29/16F
B

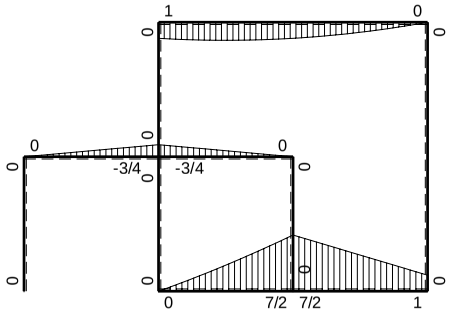




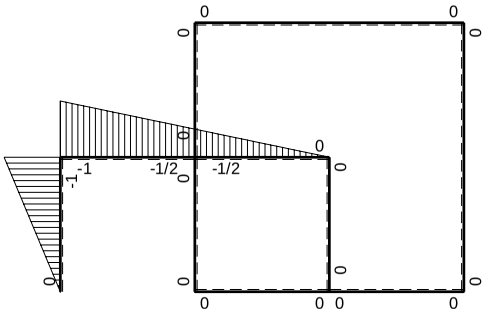

0
0
-1/2
17/8
17/8
-5/2
29/16
-2
13/2
0
0
0
F
1/2
-3/2
0
-29/16
-5/16
0
17/8
0
-3
-4
5/2
F
1
0
0
0
17/8
5/16
5/16
0
0
17/8
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $17/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -17/8Fb | |

Sviluppi di calcolo iperstatica



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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

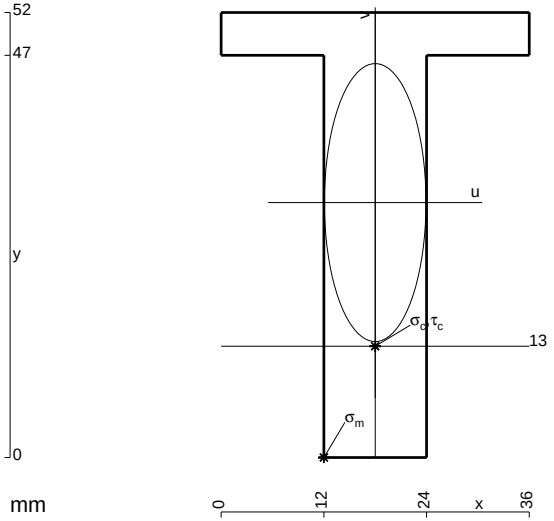
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

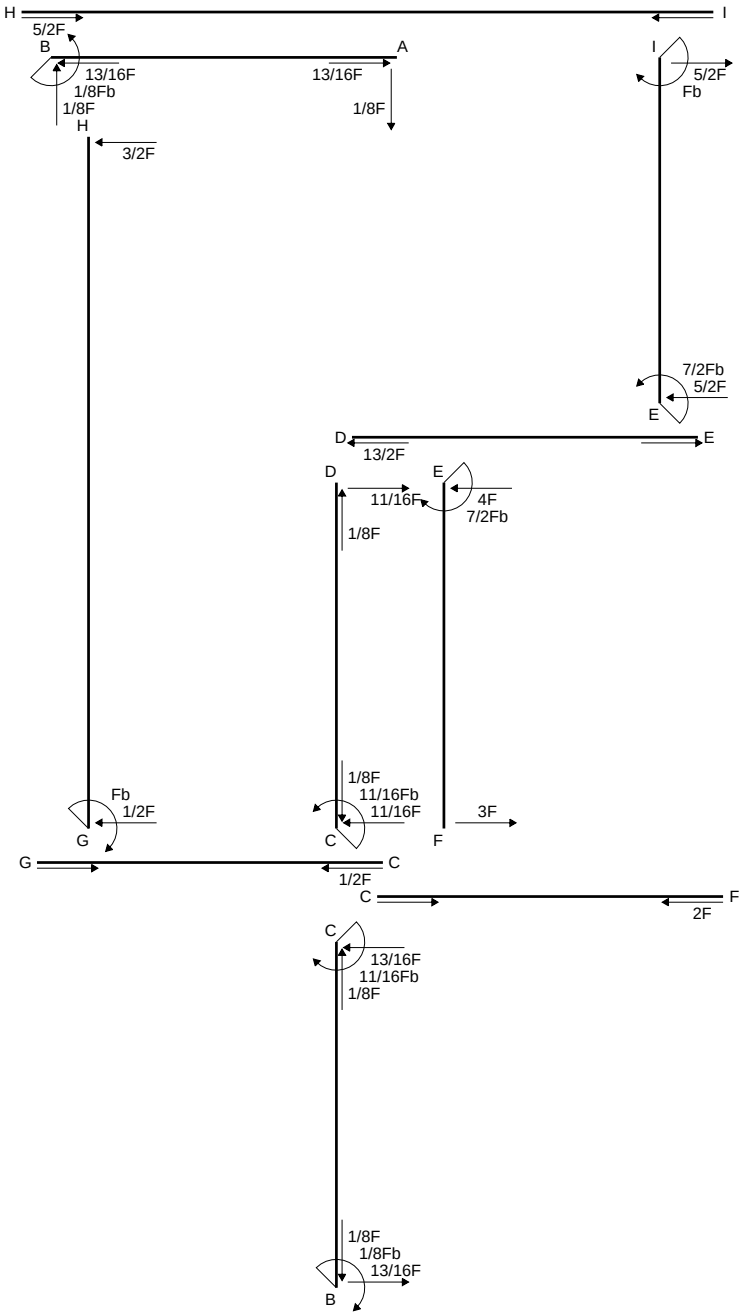


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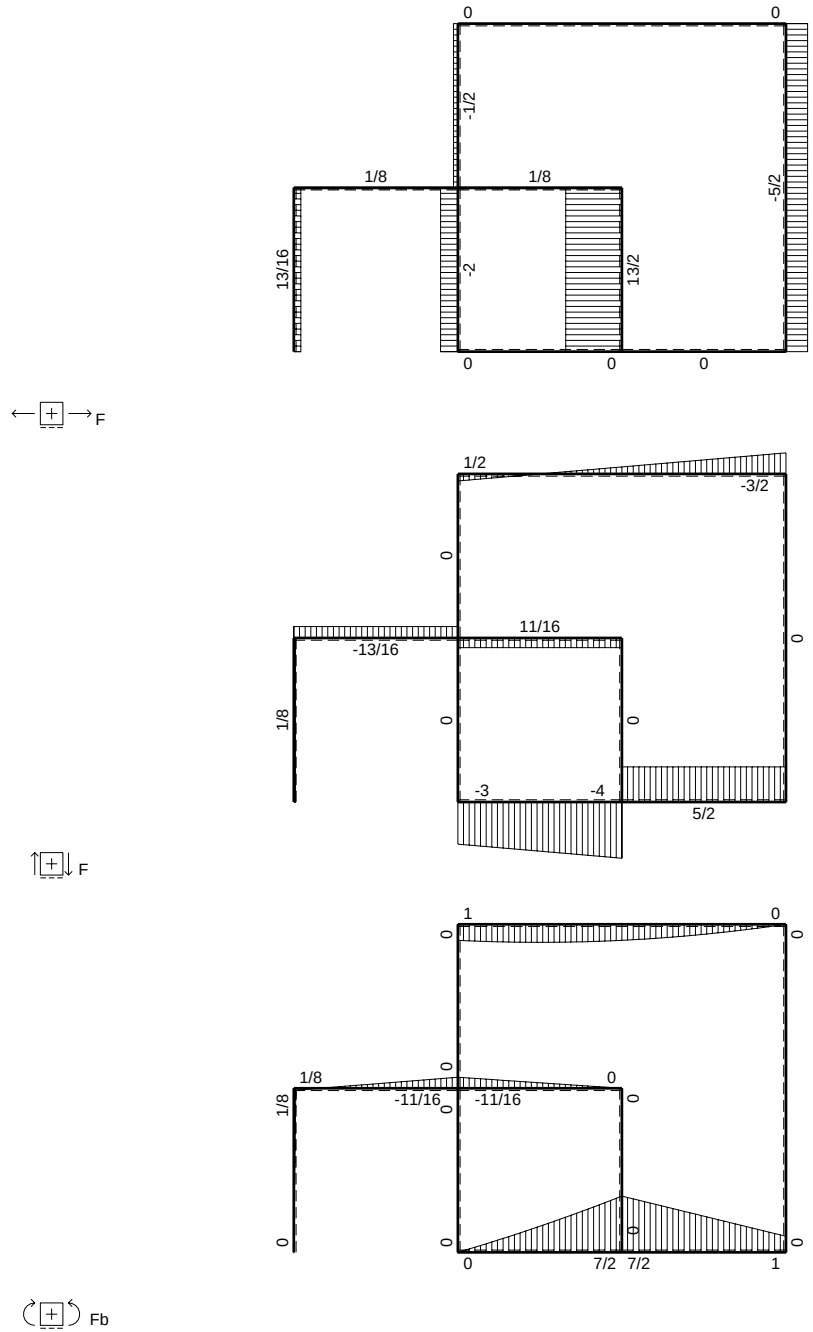

A = 744. $mm^2$
$J_u$ = 196439. $mm^4$
$J_v$ = 26208. $mm^4$
$y_g$ = 29.79 mm
$T_y$ = -2320. N
$M_x$ = 1299200. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -29.79 mm
$\sigma_m = -Mv/J_u$ = 197. $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 13. mm
$v_c$ = -16.79 mm
$\sigma_c = -Mv/J_u$ = 111. $N/mm^2$
$\tau_c$ = 3.576 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 111.2 $N/mm^2$
$S^*$ = 3633. $mm^3$

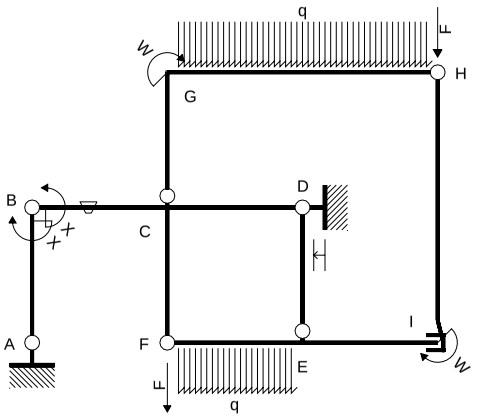
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
I
5/2F
B
A
13/16F
1/8Fb
1/8F
13/16F
1/8F
H
3/2F
I
5/2F
Fb
7/2Fb
5/2F
E
D
E
13/2F
D
E
11/16F
4F
7/2Fb
1/8F
1/8F
11/16Fb
11/16F
Fb
1/2F
G
C
F
3F
G
C
1/2F
C
F
2F
C
13/16F
11/16Fb
1/8F
1/8F
1/8Fb
13/16F
B

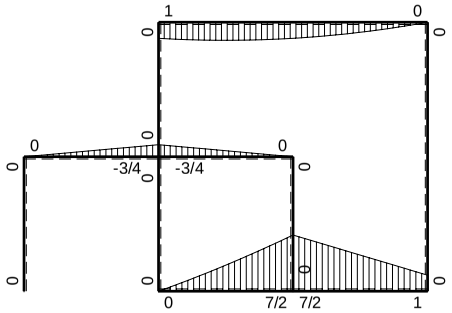




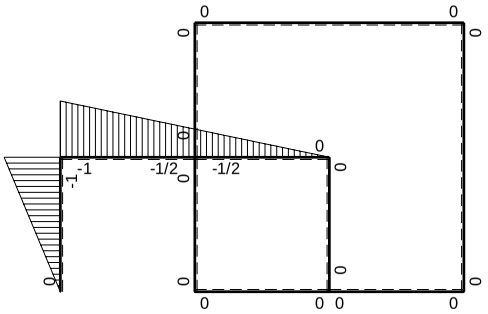

0
0
-1/2
1/8
1/8
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13/16
-2
13/2
0
0
0
F
1/2
-3/2
0
11/16
-13/16
0
1/8
0
0
-3
-4
5/2
F
1
0
0
0
0
1/8
0
-11/16
-11/16
0
1/8
0
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 274782 CI01.xxxx.136

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | -Fb/EJ | $3/4Fx-3/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/4+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | Fb/EJ | $3/8Fb-3/8Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 274782 CI01.xxxx.136

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

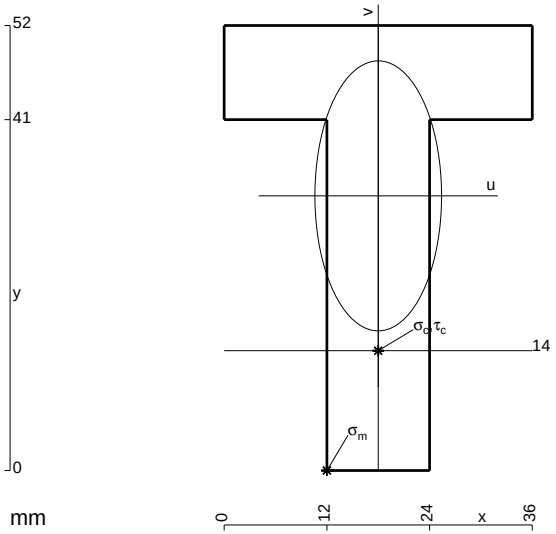
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

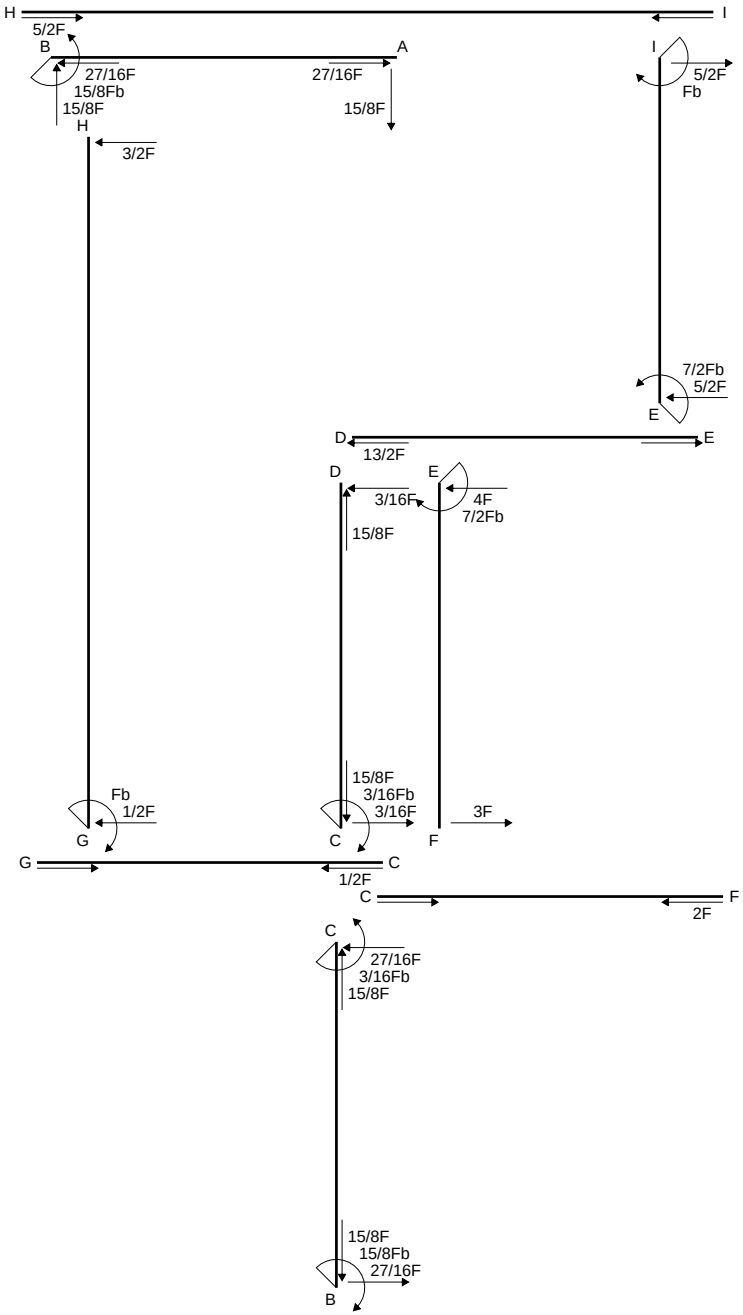
$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$





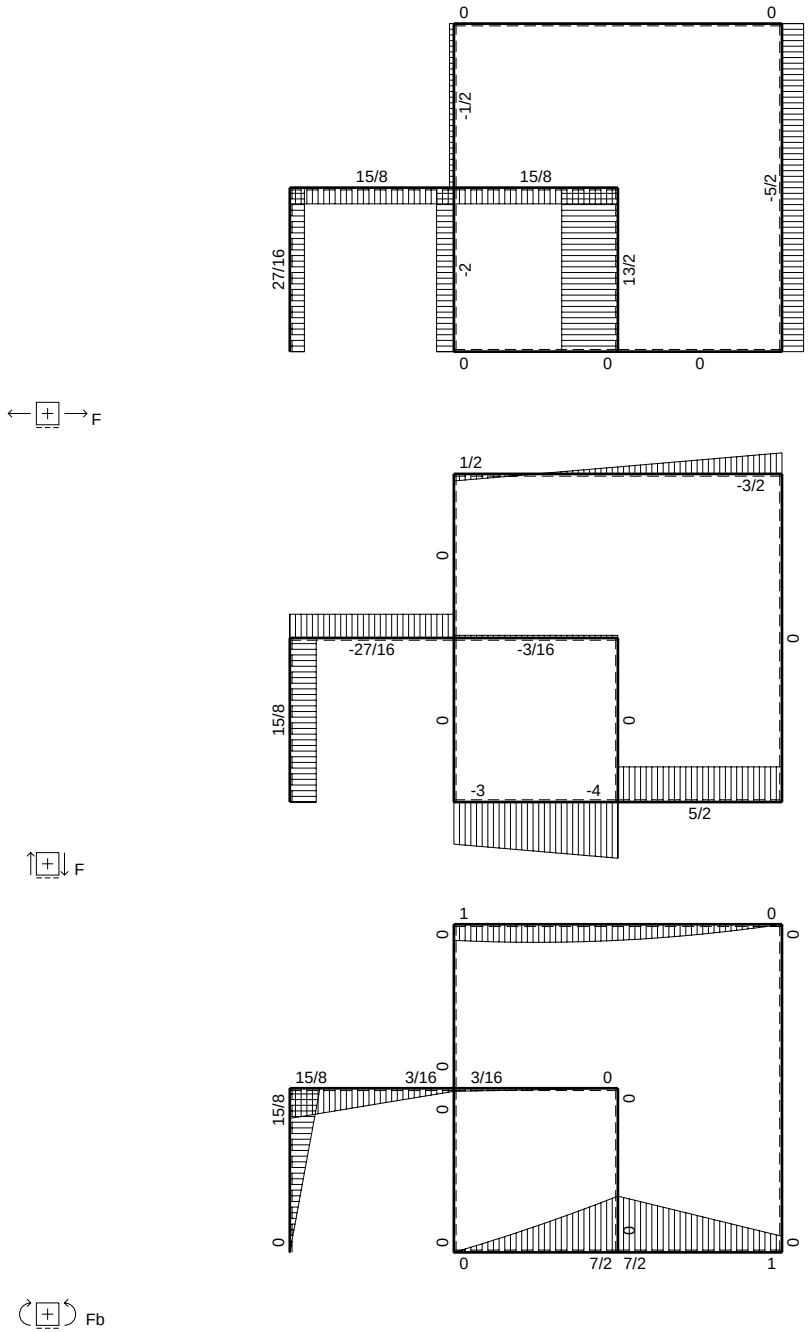


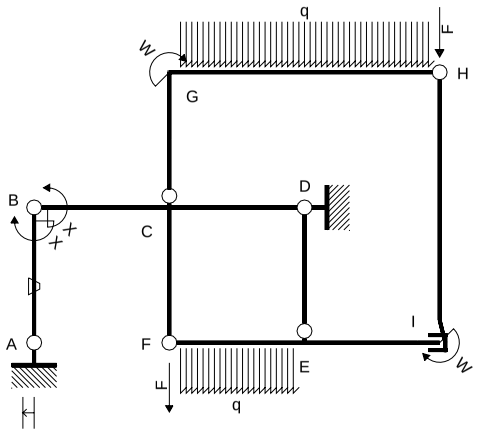

A = 888. $mm^2$
$J_u$ = 221232. $mm^4$
$J_v$ = 48672. $mm^4$
$y_g$ = 32.09 mm
$T_y$ = -2400. N
$M_x$ = 1428000. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -32.09 mm
$\sigma_m = -Mv/J_u$ = 207.2 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -18.09 mm
$\sigma_c = -Mv/J_u$ = 116.8 $N/mm^2$
$\tau_c$ = 3.811 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117. $N/mm^2$
$S^*$ = 4216. $mm^3$


H
I
5/2F
B
A
27/16F
15/8Fb
15/8F
27/16F
15/8F
H
3/2F
I
5/2F
Fb
7/2Fb
5/2F
E
D
E
13/2F
D
E
3/16F
4F
7/2Fb
15/8F
15/8F
3/16Fb
3/16F
Fb
1/2F
3F
G
C
F
G
C
1/2F
C
F
2F
C
27/16F
3/16Fb
15/8F
15/8F
15/8Fb
27/16F
B

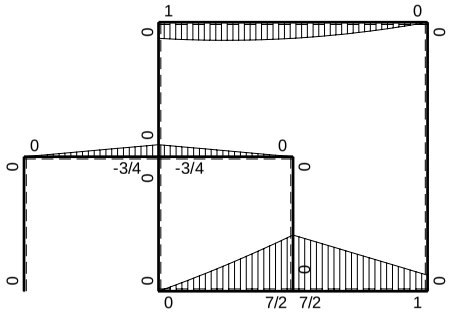




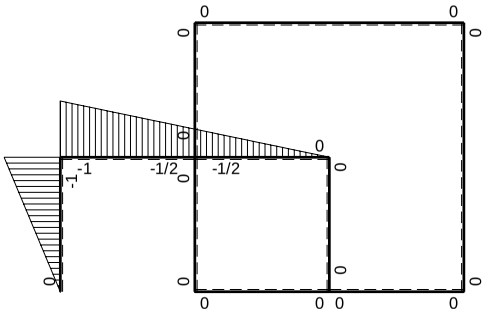

0
0
-1/2
15/8
15/8
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13/2
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0
F
1/2
-3/2
0
-27/16
-3/16
0
15/8
0
-3
-4
5/2
F
1
0
0
0
15/8
3/16
3/16
0
0
15/8
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264070 CI01.xxxx.137

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PROCEDIMENTO E RISULTATI 264070 CI01.xxxx.137

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $15/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -15/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

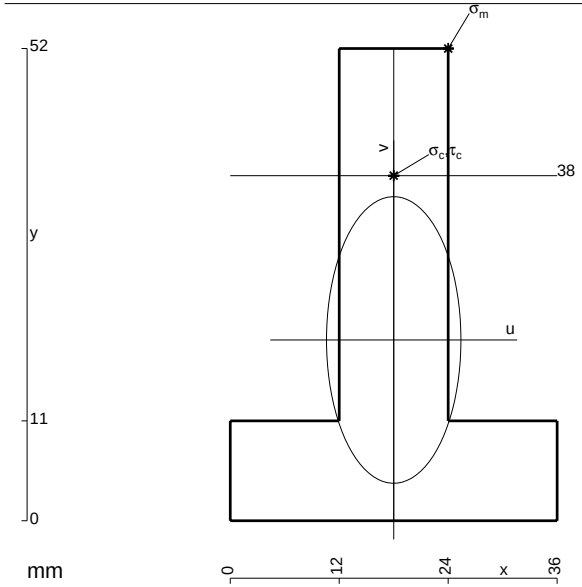
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

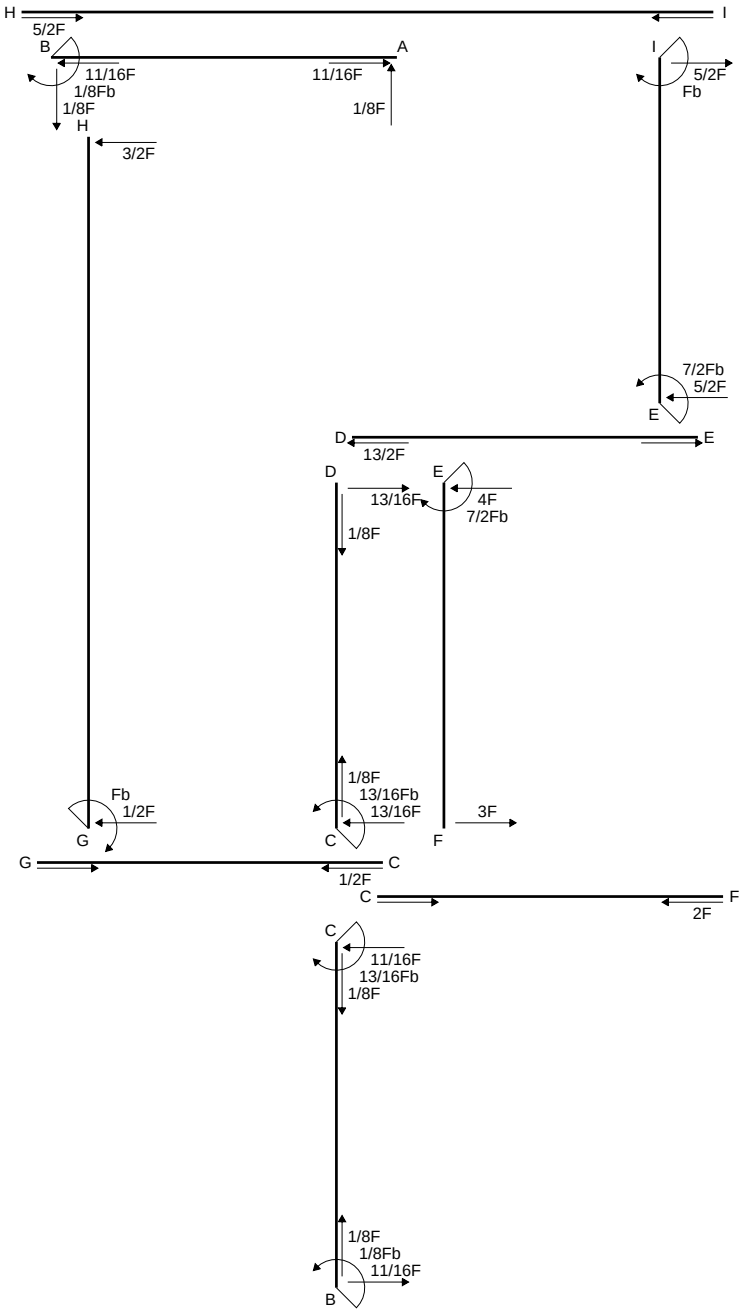


PROCEDIMENTO E RISULTATI 264070 CI01.xxxx.137



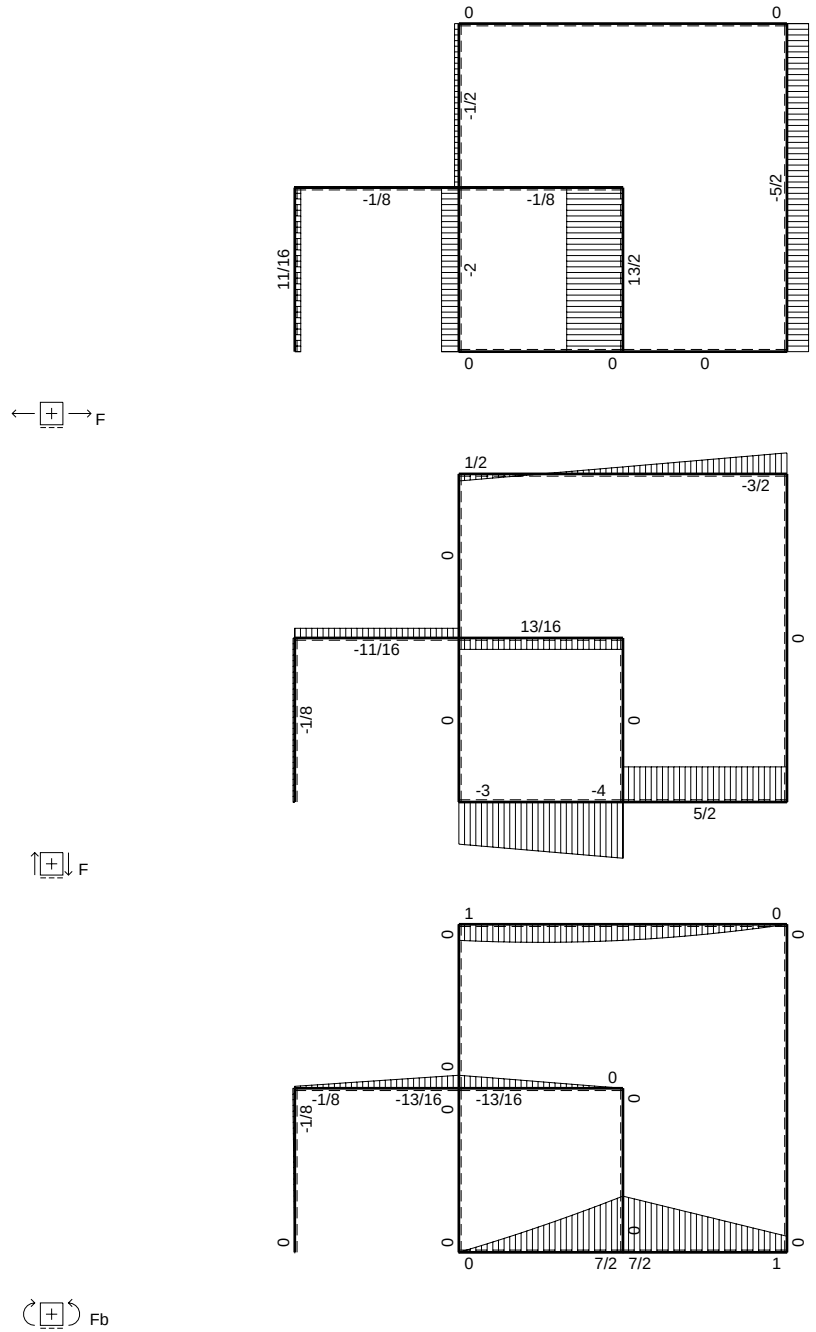

A = 888. $mm^2$
$J_u$ = 221232. $mm^4$
$J_v$ = 48672. $mm^4$
$y_g$ = 19.91 mm
$T_y$ = -2360. N
$M_x$ = 1507450. Nmm
$x_m$ = 24. mm
$y_m$ = 52. mm
$u_m$ = 6. mm
$v_m$ = 32.09 mm
$\sigma_m = -Mv/J_u$ = -218.7 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 38. mm
$v_c$ = 18.09 mm
$\sigma_c = -Mv/J_u$ = -123.3 $N/mm^2$
$\tau_c$ = 3.748 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.5 $N/mm^2$
$S^*$ = 4216. $mm^3$

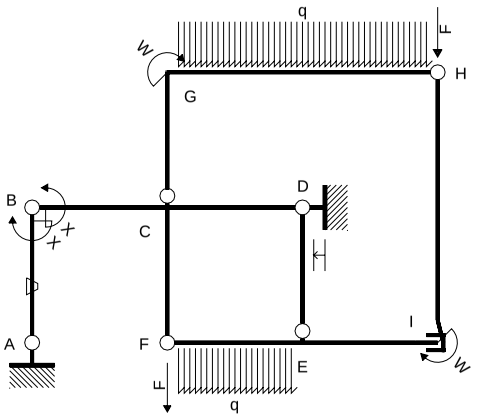
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H
I
5/2F
B
A
11/16F
1/8Fb
1/8F
11/16F
1/8F
I
5/2F
Fb
H
3/2F
7/2Fb
5/2F
E
D
E
13/2F
D
E
13/16F
4F
7/2Fb
1/8F
1/8F
13/16Fb
13/16F
Fb
1/2F
3F
G
C
F
G
C
1/2F
C
F
2F
C
11/16F
13/16Fb
1/8F
1/8F
1/8Fb
11/16F
B

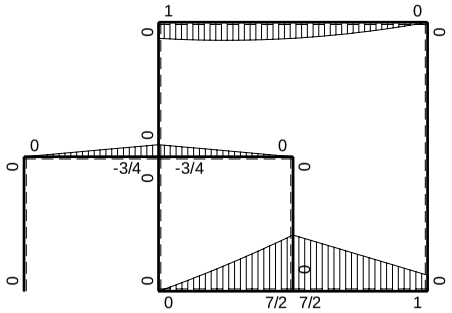




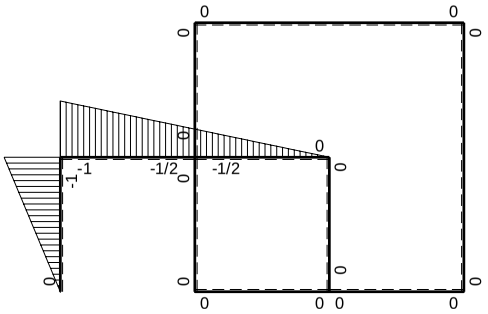

0
0
-1/2
-5/2
-1/8
-1/8
11/16
-2
13/2
0
0
0
F
1/2
-3/2
0
13/16
-11/16
0
-1/8
0
0
-3
-4
5/2
F
1
0
0
0
0
-1/8
-13/16
-13/16
0
0
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 310226 CI01.xxxx.138

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | 0 | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | 0 | $3/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

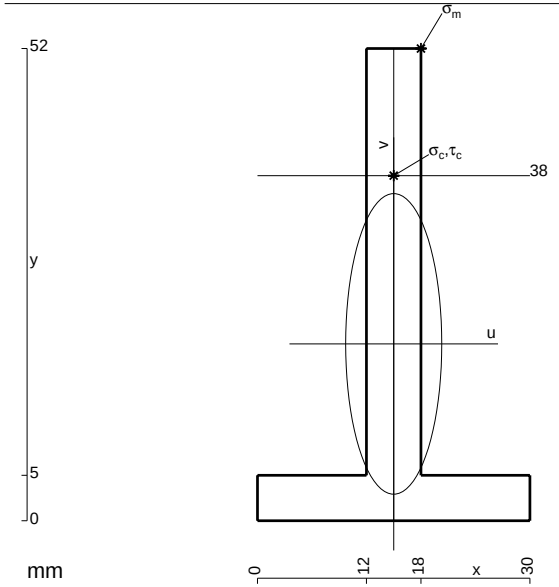
$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

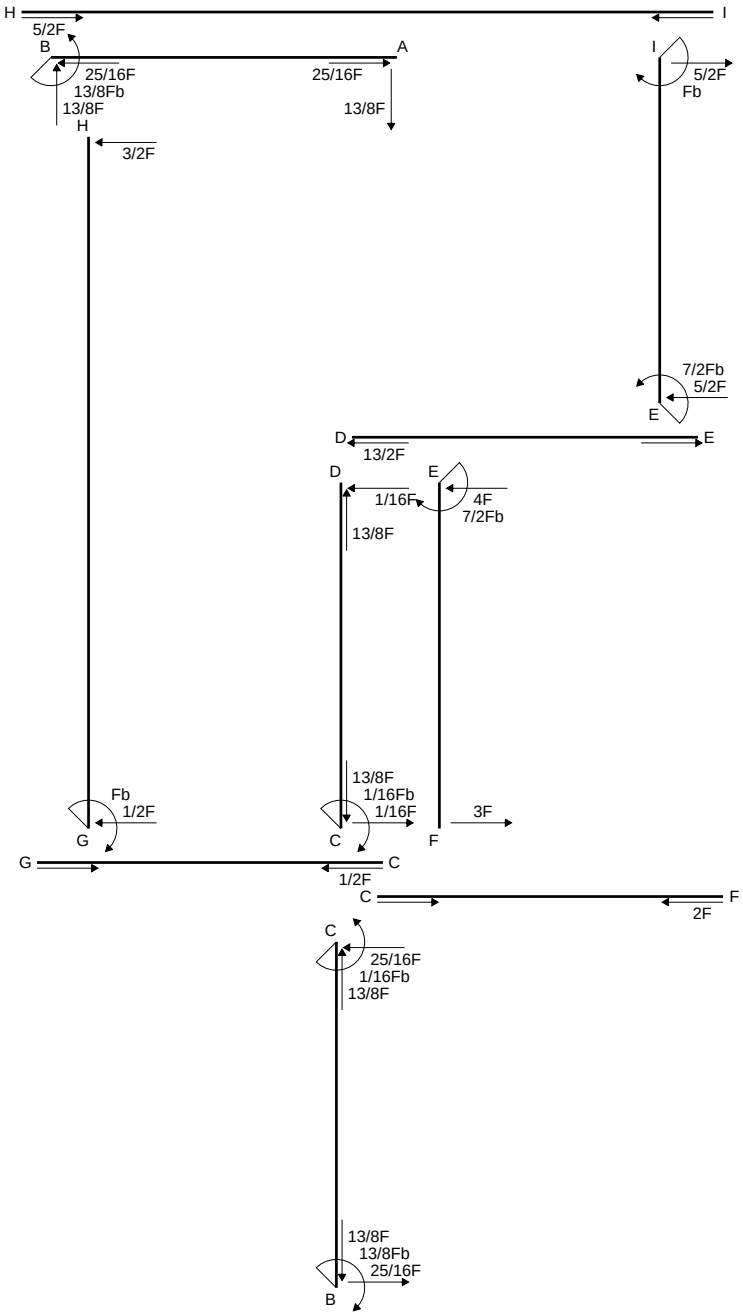


PROCEDIMENTO E RISULTATI 310226 CI01.xxxx.138



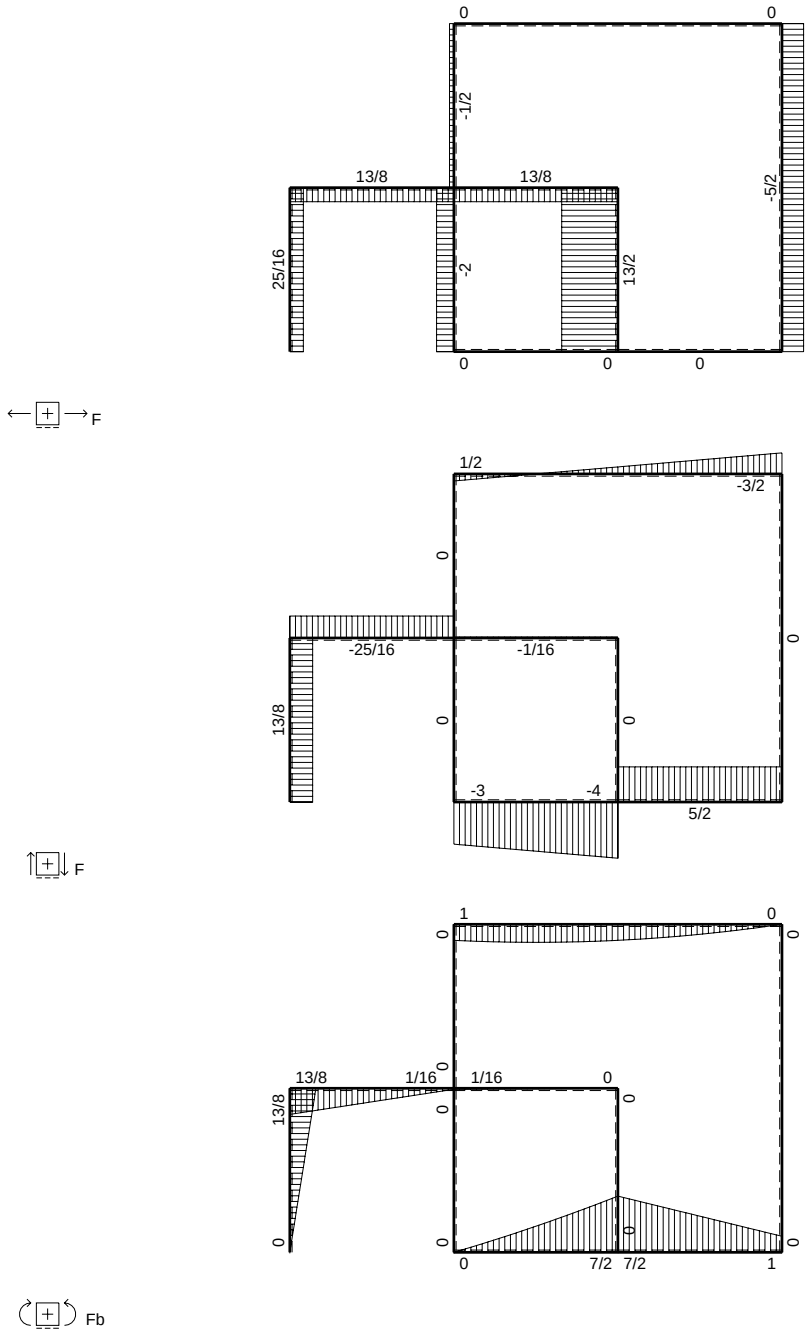

A = 432. $mm^2$
$J_u$ = 118416. $mm^4$
$J_v$ = 12096. $mm^4$
$y_g$ = 19.47 mm
$T_y$ = -1240. N
$M_x$ = 835450. Nmm
$x_m$ = 18. mm
$y_m$ = 52. mm
$u_m$ = 3. mm
$v_m$ = 32.53 mm
$\sigma_m = -Mv/J_u$ = -229.5 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 38. mm
$v_c$ = 18.53 mm
$\sigma_c = -Mv/J_u$ = -130.7 $N/mm^2$
$\tau_c$ = 3.742 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.9 $N/mm^2$
$S^*$ = 2144. $mm^3$

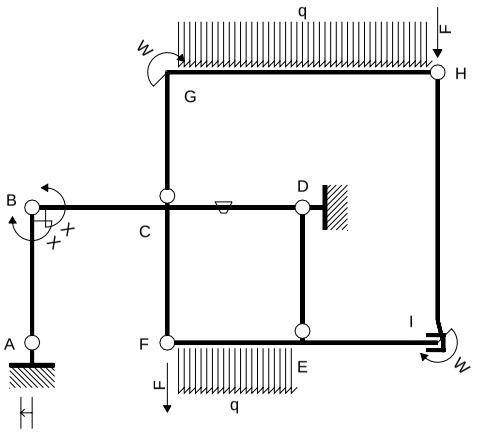
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H
I
5/2F
B
A
25/16F
13/8Fb
13/8F
25/16F
13/8F
I
5/2F
Fb
H
3/2F
7/2Fb
5/2F
E
D
E
13/2F
D
E
1/16F
4F
7/2Fb
13/8F
13/8F
1/16Fb
Fb
1/2F
1/16F
3F
G
C
F
G
C
1/2F
C
F
2F
C
25/16F
1/16Fb
13/8F
13/8F
13/8Fb
25/16F
B

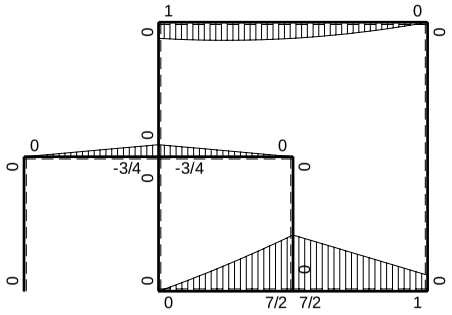




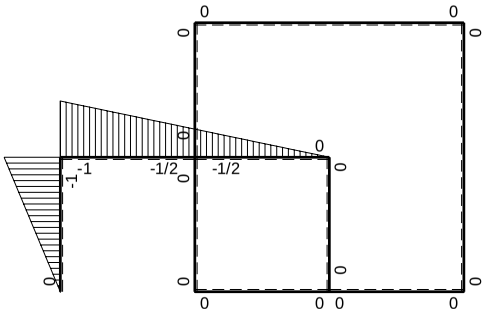

0
0
-1/2
13/8
13/8
-5/2
25/16
-2
13/2
0
0
0
F
1/2
-3/2
0
-25/16
-1/16
0
13/8
0
0
-3
-4
5/2
F
1
0
0
0
13/8
1/16
1/16
0
13/8
0
0
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 263941 CI01.xxxx.139

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 263941 CI01.xxxx.139

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

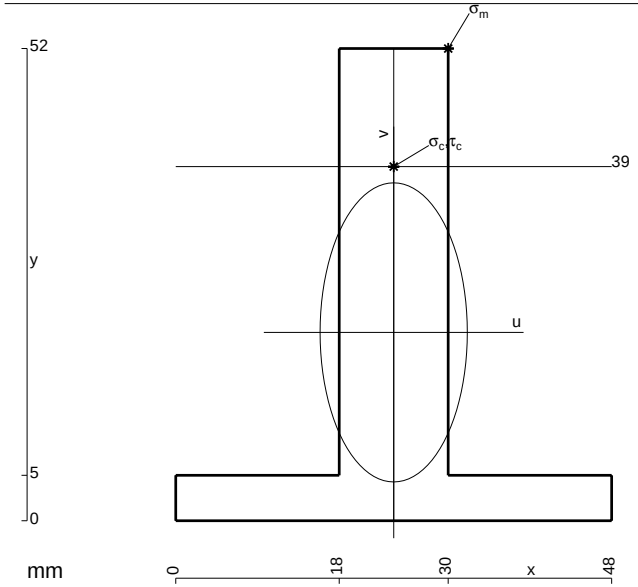
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

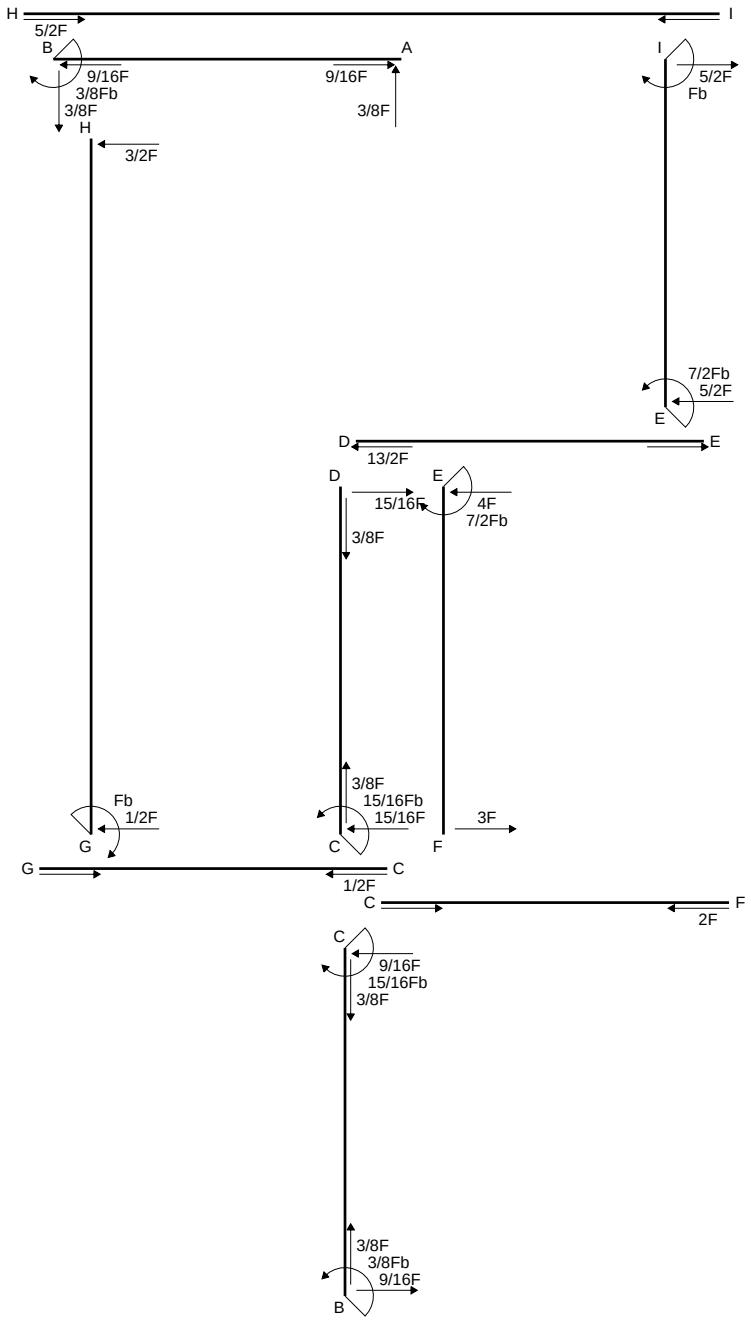
$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 263941 CI01.xxxx.139

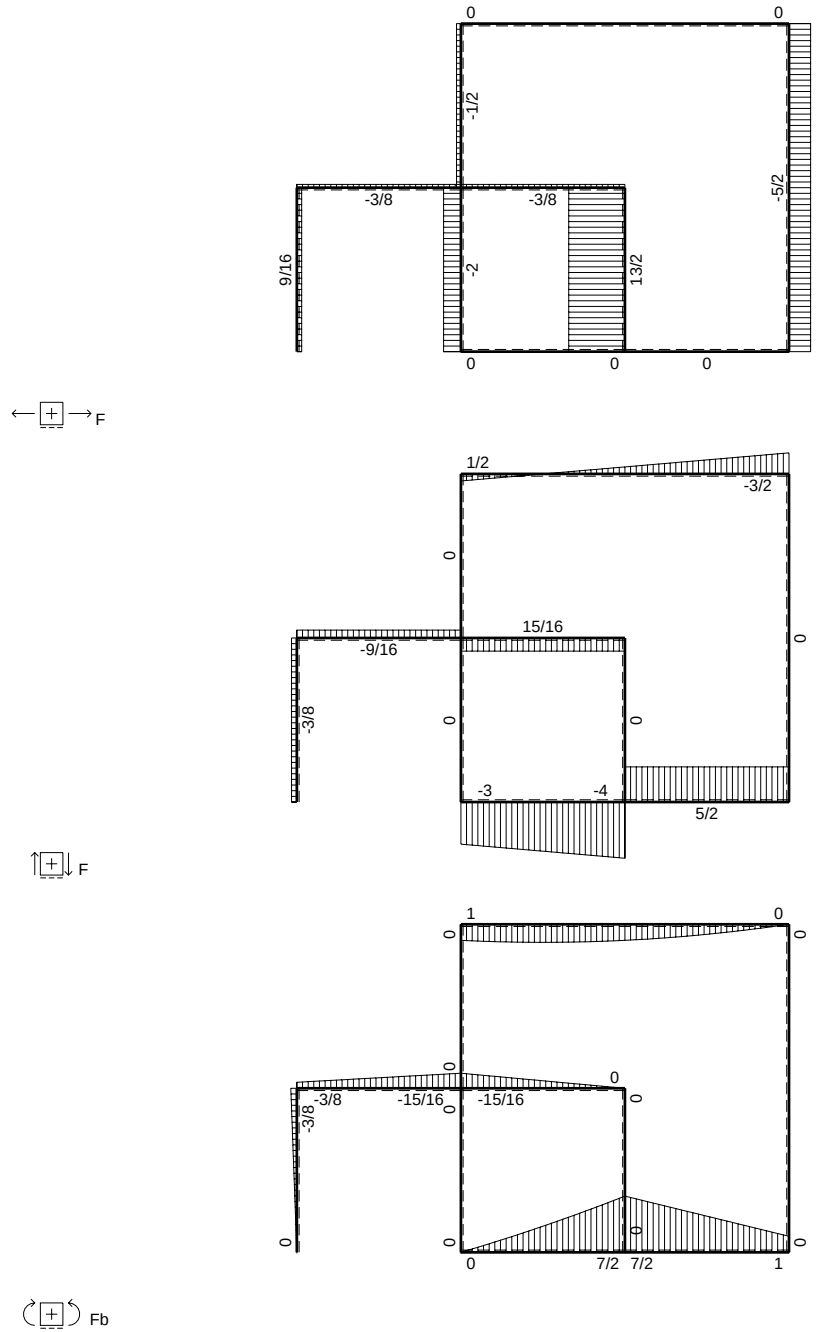


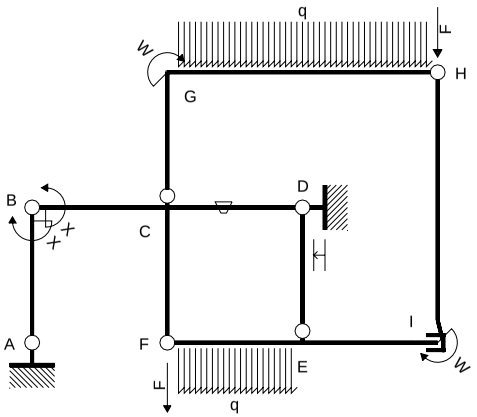

A = 804. $mm^2$
$J_u$ = 218133. $mm^4$
$J_v$ = 52848. $mm^4$
$y_g$ = 20.74 mm
$T_y$ = -2360. N
$M_x$ = 1672650. Nmm
$x_m$ = 30. mm
$y_m$ = 52. mm
$u_m$ = 6. mm
$v_m$ = 31.26 mm
$\sigma_m = -Mv/J_u$ = -239.7 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 39. mm
$v_c$ = 18.26 mm
$\sigma_c = -Mv/J_u$ = -140. $N/mm^2$
$\tau_c$ = 3.483 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.2 $N/mm^2$
$S^*$ = 3863. $mm^3$


H
I
5/2F
B
A
9/16F
9/16F
3/8Fb
3/8F
3/8F
H
3/2F
I
5/2F
Fb
7/2Fb
5/2F
E
D
E
13/2F
D
E
15/16F
4F
7/2Fb
3/8F
3/8F
15/16Fb
15/16F
Fb
1/2F
G
C
F
3F
G
C
1/2F
C
F
2F
C
9/16F
15/16Fb
3/8F
3/8F
3/8Fb
9/16F
B

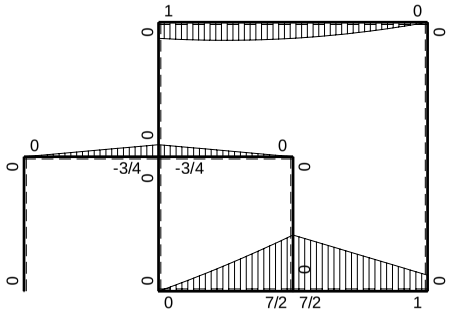




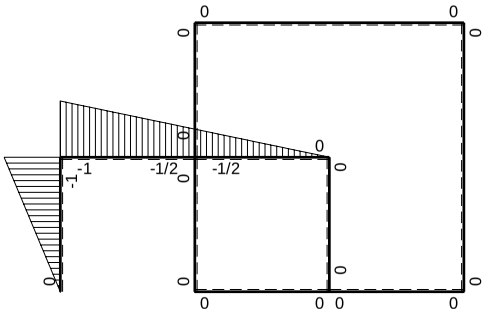

0
0
-1/2
-5/2
-3/8
-3/8
9/16
-2
13/2
0
0
0
F
1/2
-3/2
0
15/16
-9/16
-3/8
0
0
-3
-4
5/2
F
1
0
0
0
0
-3/8
-15/16
-15/16
-3/8
0
0
0
0
0
7/2
7/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 218194 CI01.xxxx.140

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PROCEDIMENTO E RISULTATI 218194 CI01.xxxx.140

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/4Fx | 0 | $3/4Fx-3/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/4+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/4Fb-3/4Fx | 0 | $3/8Fb-3/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -3/4Fb+3/4Fx | -Fb/EJ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 3/4Fx | Fb/EJ | $3/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $7/2Fb-4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-3Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -7/2Fb+5/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-3/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 3/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

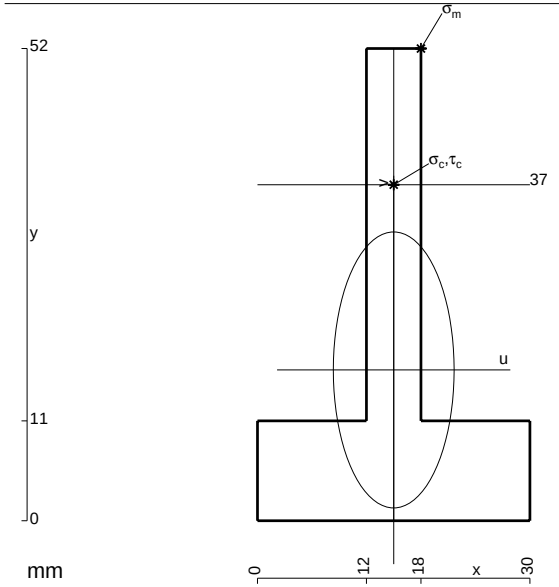
$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/8\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 3/8\ Fb^2/EJ$

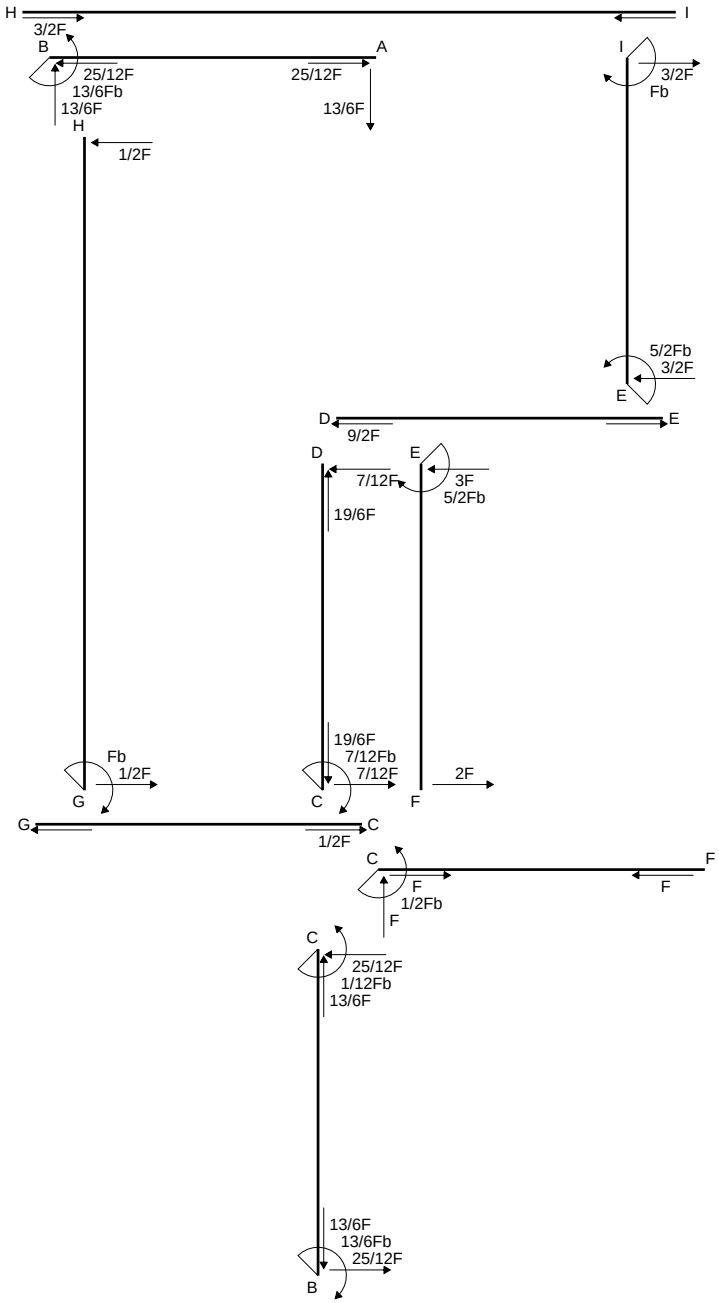


PROCEDIMENTO E RISULTATI 218194 CI01.xxxx.140



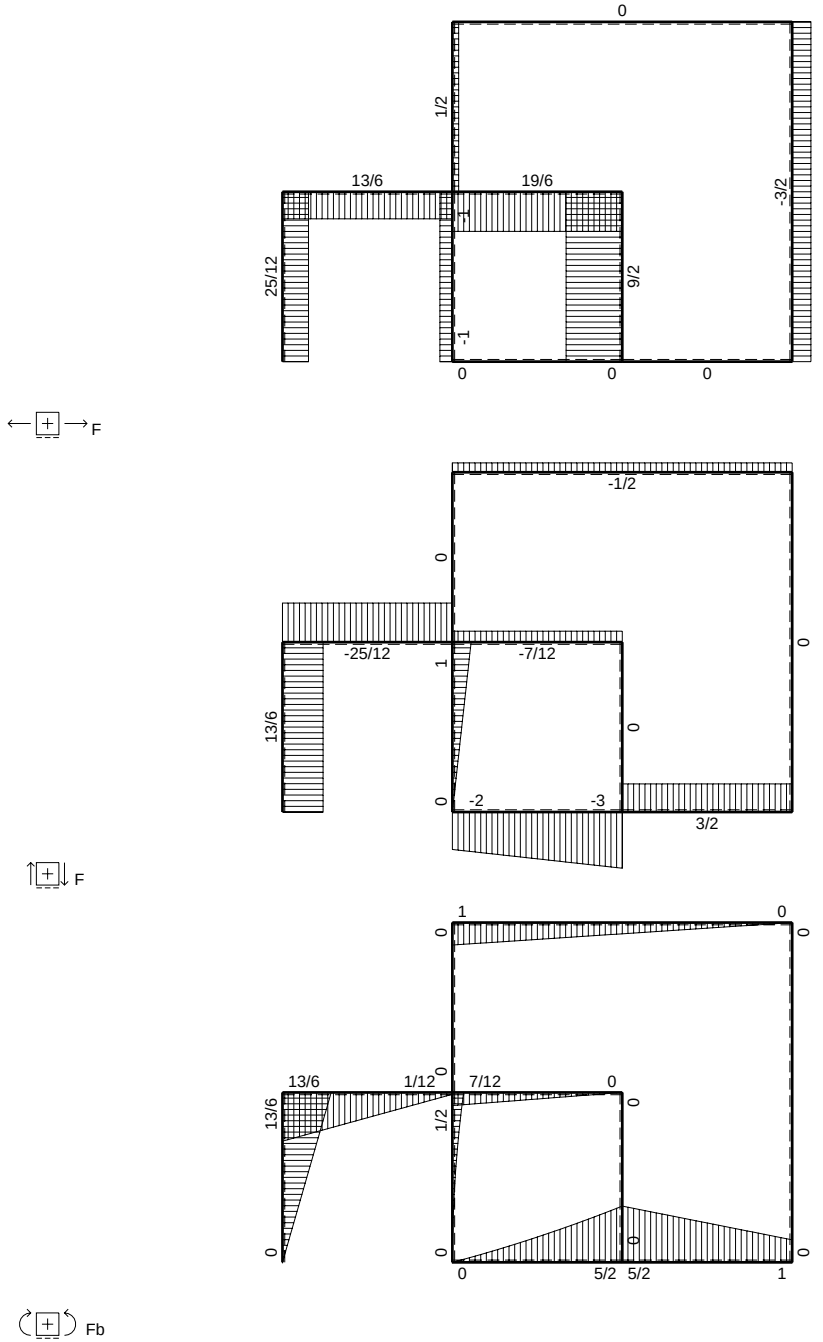

A = 576. $mm^2$
$J_u$ = 133062. $mm^4$
$J_v$ = 25488. $mm^4$
$y_g$ = 16.6 mm
$T_y$ = -1960. N
$M_x$ = 737450. Nmm
$x_m$ = 18. mm
$y_m$ = 52. mm
$u_m$ = 3. mm
$v_m$ = 35.4 mm
$\sigma_m = -Mv/J_u$ = -196.2 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 37. mm
$v_c$ = 20.4 mm
$\sigma_c = -Mv/J_u$ = -113. $N/mm^2$
$\tau_c$ = 6.164 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.5 $N/mm^2$
$S^*$ = 2511. $mm^3$

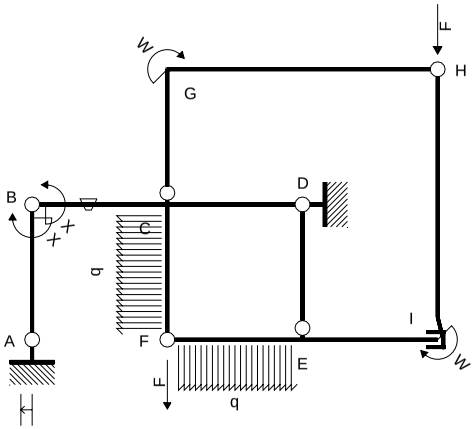
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H
I
3/2F
B
A
25/12F
13/6Fb
13/6F
25/12F
13/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
7/12F
3F
5/2Fb
19/6F
19/6F
7/12Fb
7/12F
Fb
1/2F
2F
G
C
F
G
C
1/2F
C
F
F
F
1/2Fb
F
C
25/12F
1/12Fb
13/6F
13/6F
13/6Fb
25/12F
B

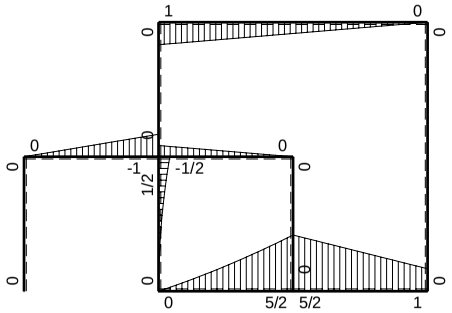




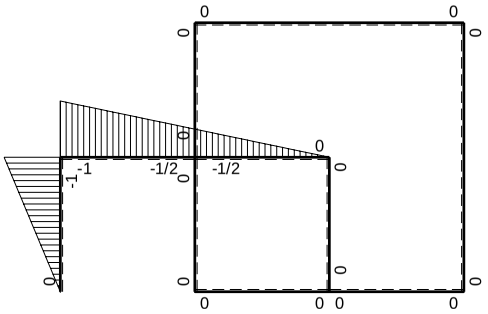

0
1/2
13/6
19/6
-3/2
-1
25/12
9/2
-1
0
0
0
F
-1/2
0
-25/12
1
-7/12
0
13/6
0
-2
-3
3/2
F
1
0
0
0
13/6
1/12
0
7/12
0
13/6
1/2
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 218622 CI01.xxxx.141

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 218622 CI01.xxxx.141

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

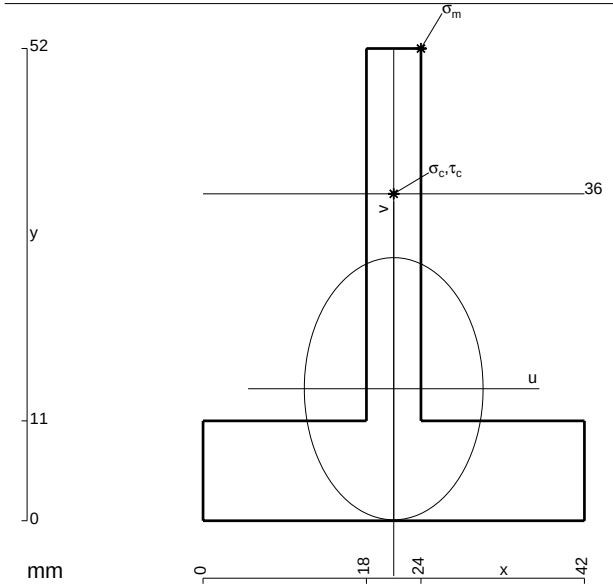
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

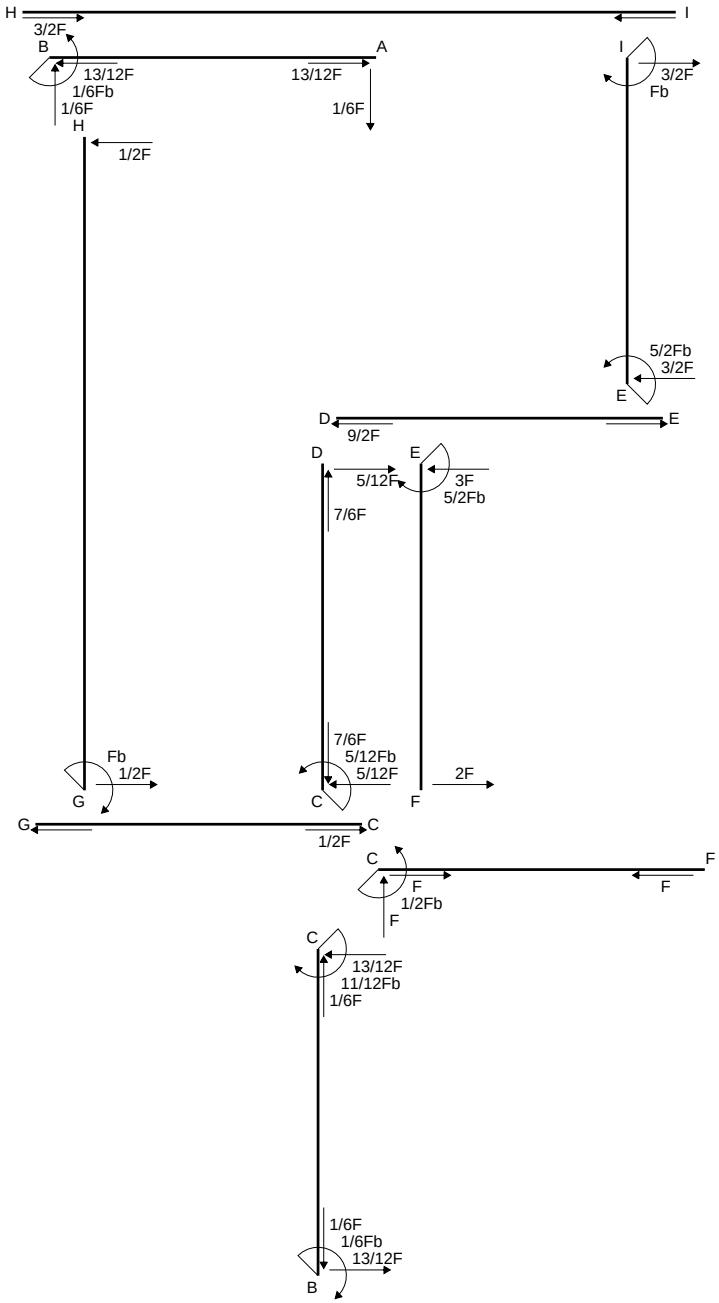


PROCEDIMENTO E RISULTATI 218622 CI01.xxxx.141



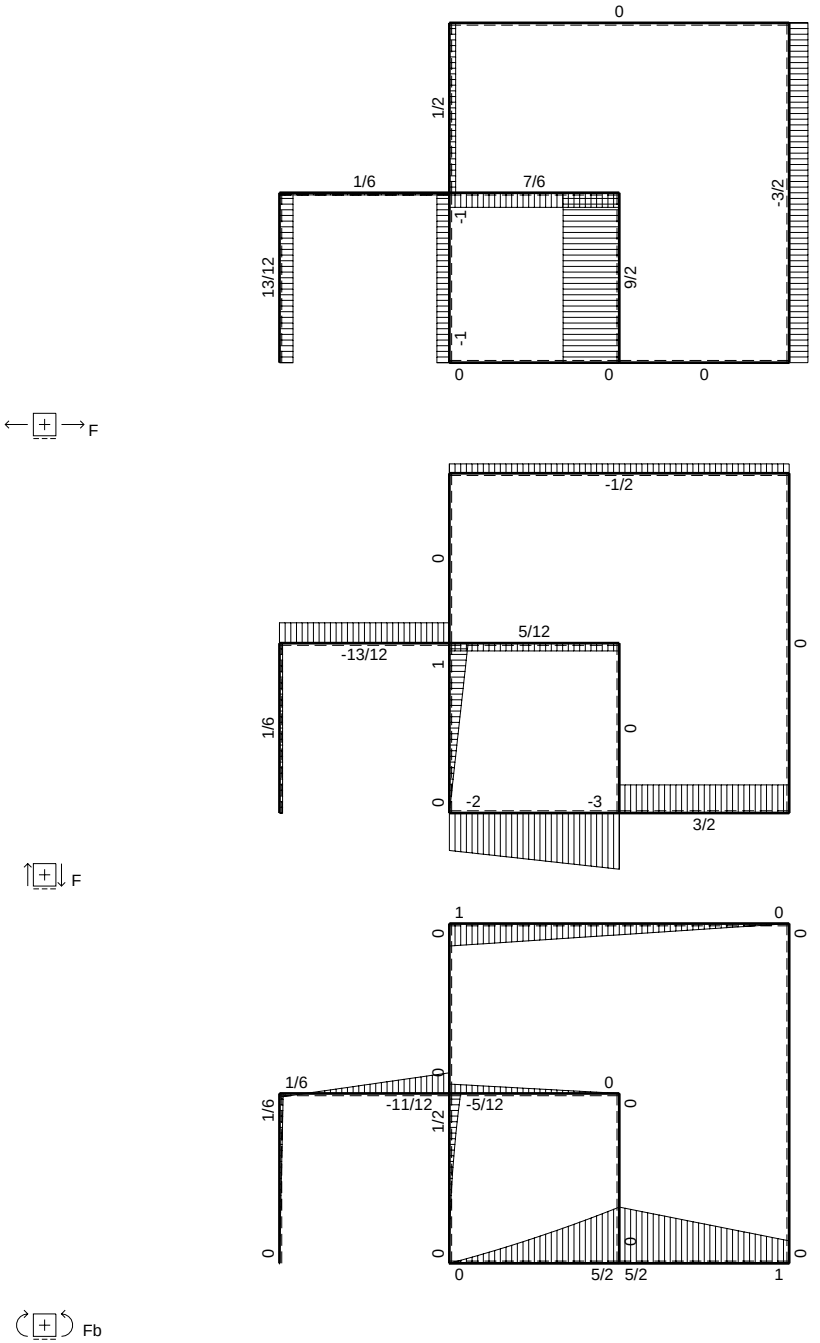

A = 708. $mm^2$
$J_u$ = 147634. $mm^4$
$J_v$ = 68652. $mm^4$
$y_g$ = 14.53 mm
$T_y$ = -2100. N
$M_x$ = 822500. Nmm
$x_m$ = 24. mm
$y_m$ = 52. mm
$u_m$ = 3. mm
$v_m$ = 37.47 mm
$\sigma_m = -Mv/J_u$ = -208.7 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 36. mm
$v_c$ = 21.47 mm
$\sigma_c = -Mv/J_u$ = -119.6 $N/mm^2$
$\tau_c$ = 6.706 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.2 $N/mm^2$
$S^*$ = 2829. $mm^3$

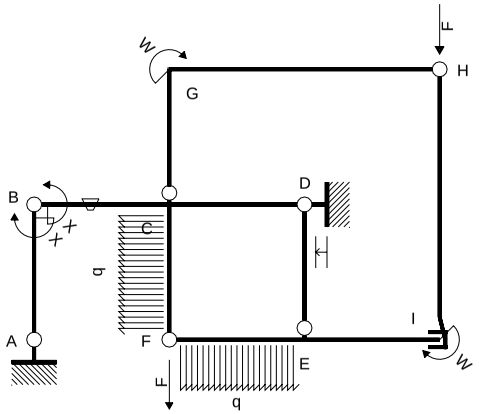
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H
I
3/2F
B
A
13/12F
1/6Fb
1/6F
13/12F
1/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
5/12F
3F
5/2Fb
7/6F
7/6F
5/12Fb
5/12F
Fb
1/2F
G
C
F
2F
G
C
1/2F
C
F
F
F
1/2Fb
C
13/12F
11/12Fb
1/6F
1/6F
1/6Fb
13/12F
B

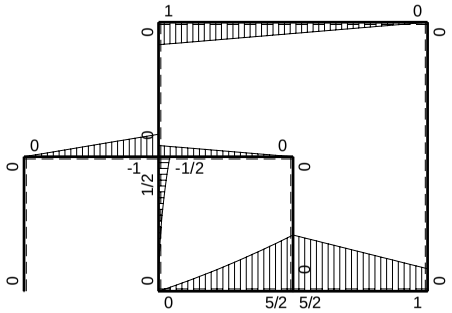




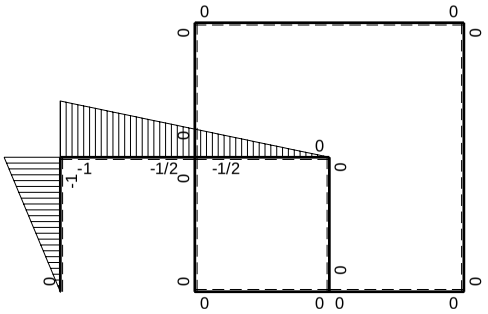

F
0
1/2
1/6
7/6
-3/2
-1
13/12
9/2
-1
0
0
0
F
-1/2
0
5/12
-13/12
1
0
1/6
0
0
-2
-3
3/2
F
1
0
0
0
1/6
0
0
-11/12
-5/12
1/2
1/6
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 218087 CI01.xxxx.142

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PROCEDIMENTO E RISULTATI 218087 CI01.xxxx.142

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

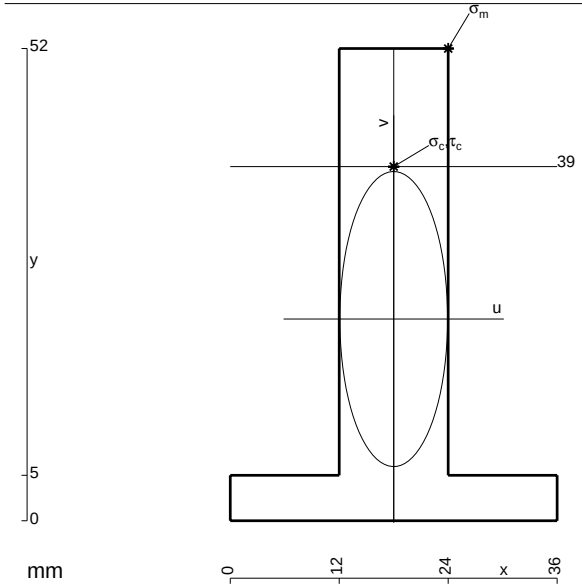
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

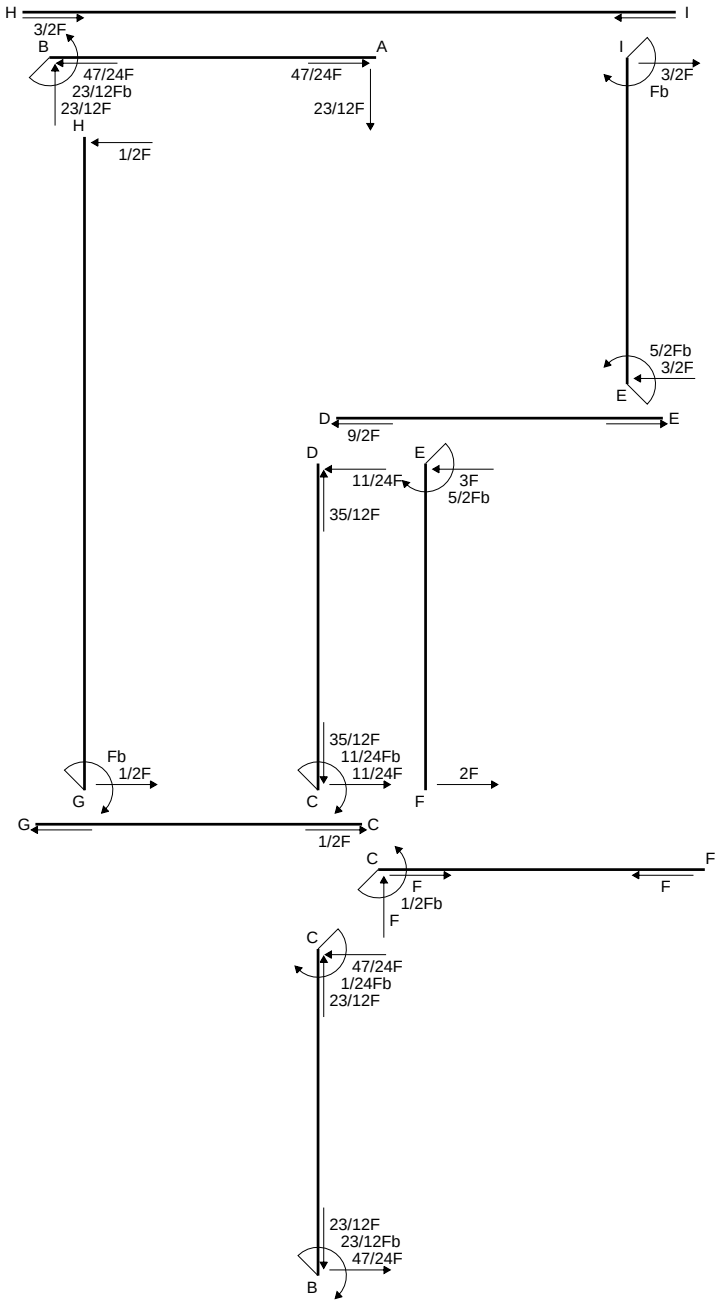


PROCEDIMENTO E RISULTATI 218087 CI01.xxxx.142



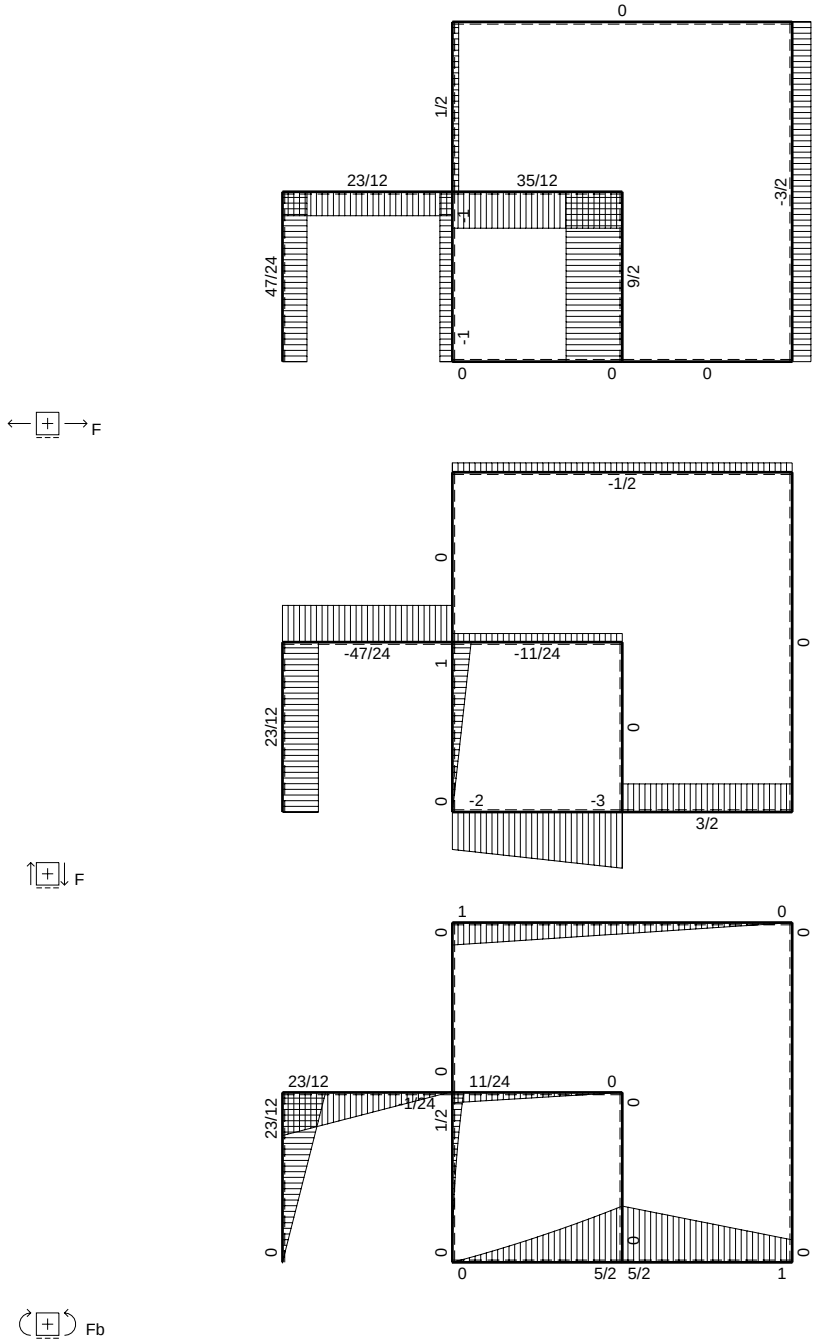

A = 744. $mm^2$
$J_u$ = 196439. $mm^4$
$J_v$ = 26208. $mm^4$
$y_g$ = 22.21 mm
$T_y$ = -3390. N
$M_x$ = 1440750. Nmm
$x_m$ = 24. mm
$y_m$ = 52. mm
$u_m$ = 6. mm
$v_m$ = 29.79 mm
$\sigma_m = -Mv/J_u$ = -218.5 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 39. mm
$v_c$ = 16.79 mm
$\sigma_c = -Mv/J_u$ = -123.1 $N/mm^2$
$\tau_c$ = 5.225 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.5 $N/mm^2$
$S^*$ = 3633. $mm^3$

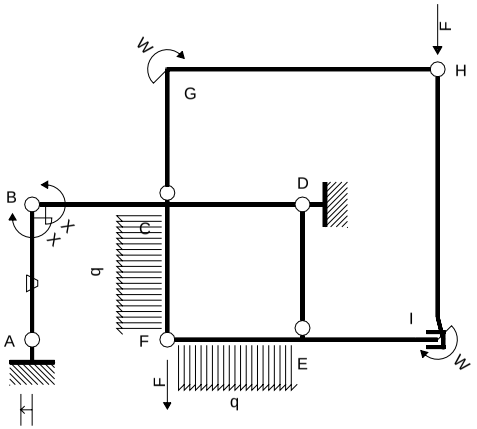
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H
I
3/2F
B
A
47/24F
23/12Fb
23/12F
47/24F
23/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
11/24F
3F
5/2Fb
35/12F
35/12F
11/24Fb
11/24F
Fb
1/2F
2F
G
C
F
G
C
1/2F
C
F
F
F
1/2Fb
F
C
47/24F
1/24Fb
23/12F
23/12F
23/12Fb
47/24F
B

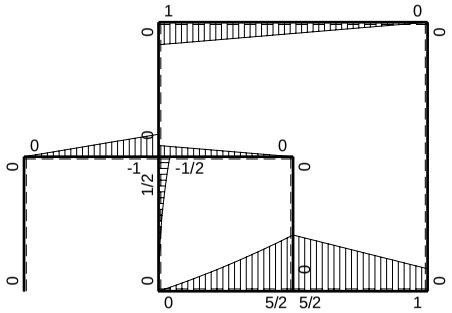




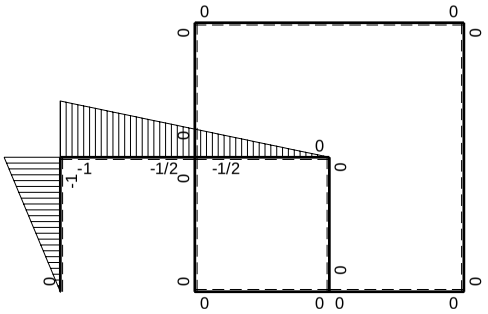

0
1/2
23/12
35/12
-3/2
47/24
9/2
-1
0
0
0
F
-1/2
0
-47/24
1
-11/24
0
23/12
0
-2
-3
3/2
F
1
0
0
0
0
23/12
11/24
0
23/12
1/24
1/2
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 249121 CI01.xxxx.143

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PROCEDIMENTO E RISULTATI 249121 CI01.xxxx.143

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

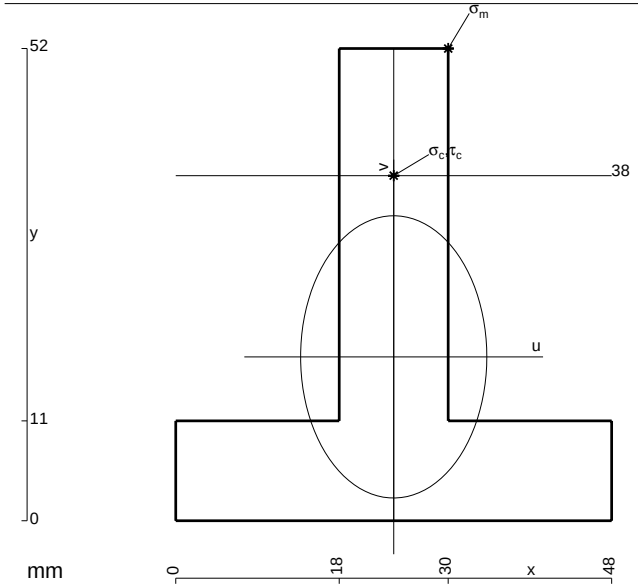
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

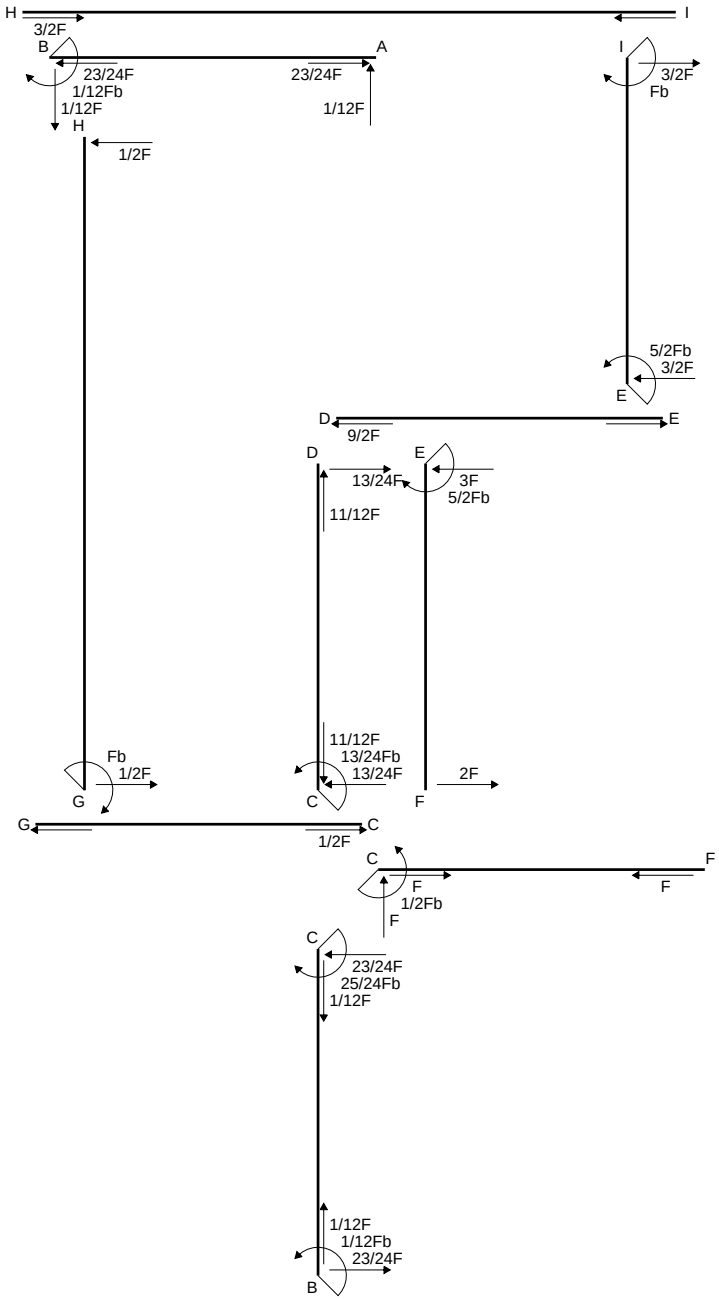


PROCEDIMENTO E RISULTATI 249121 CI01.xxxx.143



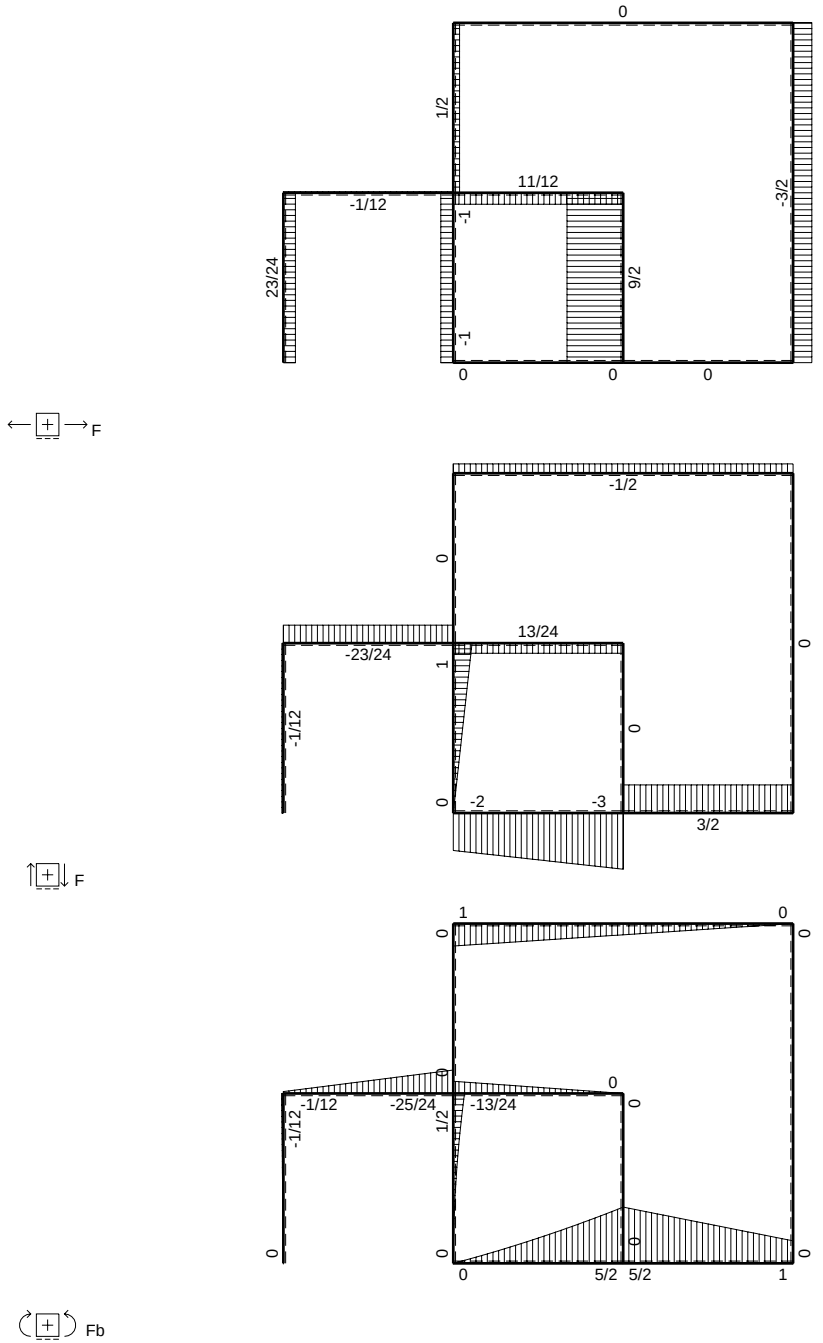

A = 1020. $mm^2$
$J_u$ = 246410. $mm^4$
$J_v$ = 107280. $mm^4$
$y_g$ = 18.04 mm
$T_y$ = -3630. N
$M_x$ = 1663750. Nmm
$x_m$ = 30. mm
$y_m$ = 52. mm
$u_m$ = 6. mm
$v_m$ = 33.96 mm
$\sigma_m = -Mv/J_u$ = -229.3 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 38. mm
$v_c$ = 19.96 mm
$\sigma_c = -Mv/J_u$ = -134.8 $N/mm^2$
$\tau_c$ = 5.56 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.1 $N/mm^2$
$S^*$ = 4529. $mm^3$

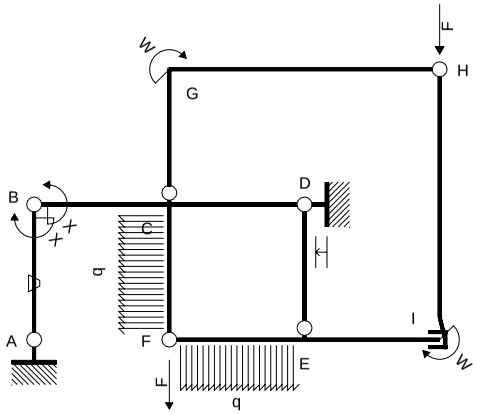
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H
I
3/2F
B
A
23/24F
1/12Fb
1/12F
23/24F
1/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
13/24F
3F
5/2Fb
11/12F
11/12F
13/24Fb
13/24F
Fb
1/2F
G
C
2F
F
G
C
1/2F
C
F
F
F
1/2Fb
F
C
23/24F
25/24Fb
1/12F
1/12F
1/12Fb
23/24F
B

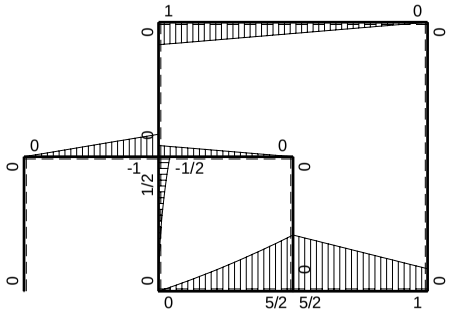




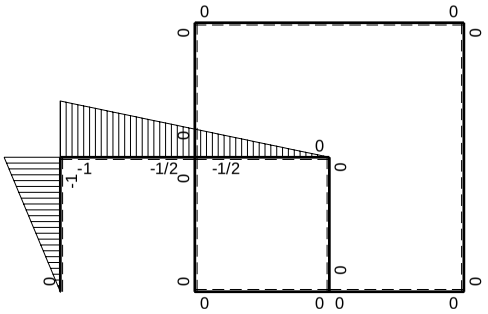

F
0
1/2
11/12
-1/12
-3/2
-1
23/24
9/2
-1
0
0
0
F
-1/2
0
13/24
-23/24
1
0
0
-1/12
0
-2
-3
3/2
F
1
0
0
0
0
-1/12
-25/24
-13/24
1/2
-1/12
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 239853 CI01.xxxx.144

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PROCEDIMENTO E RISULTATI 239853 CI01.xxxx.144

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b ( x/b )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

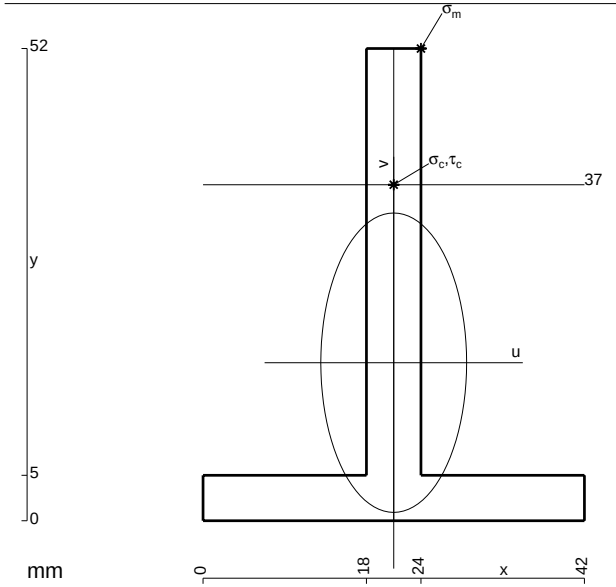
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

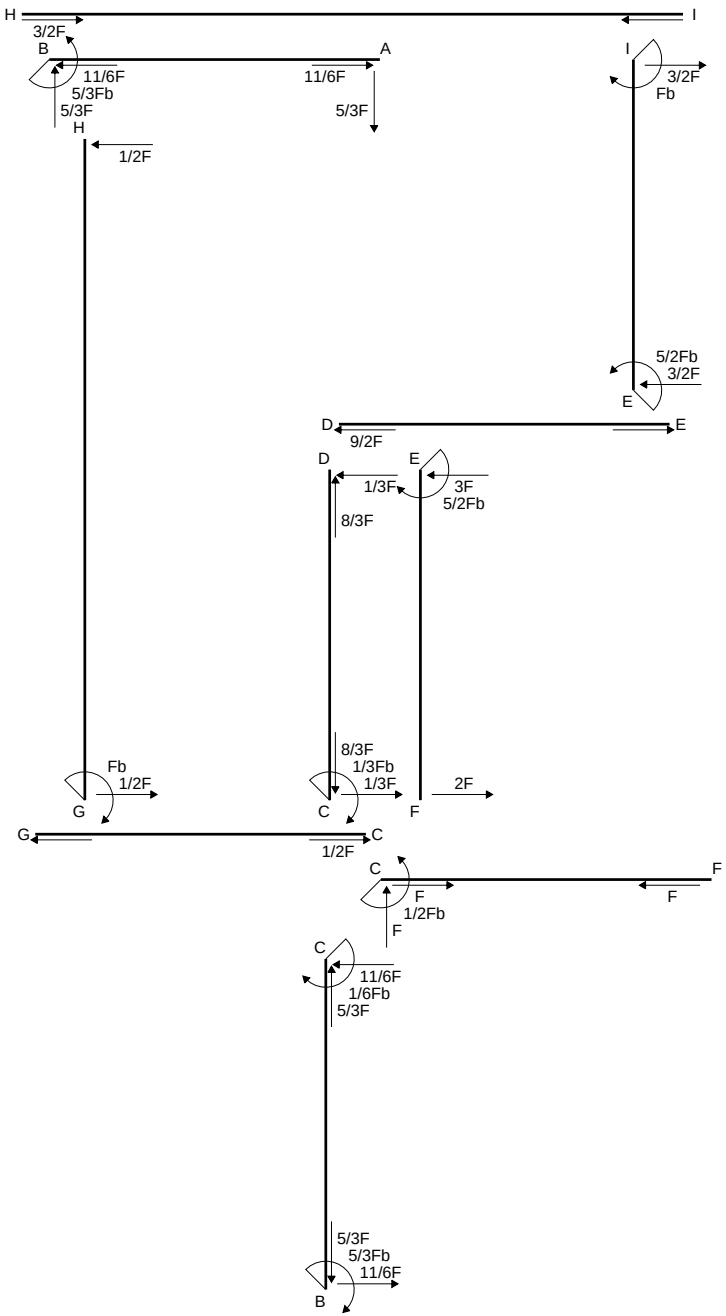
$= (1/12\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





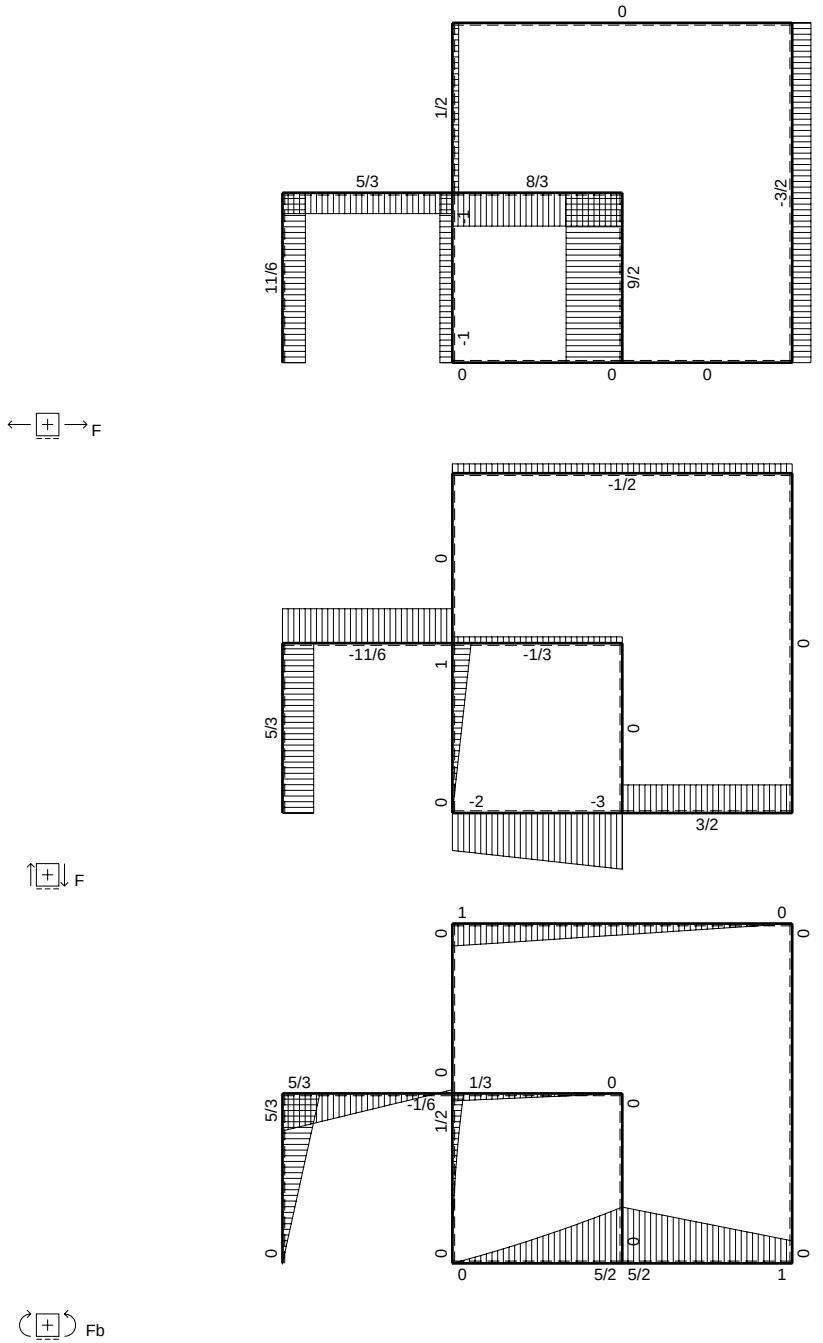


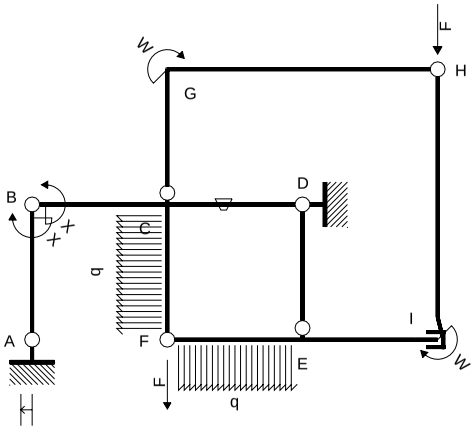

A = 492. $mm^2$
$J_u$ = 133716. $mm^4$
$J_v$ = 31716. $mm^4$
$y_g$ = 17.4 mm
$T_y$ = -1830. N
$M_x$ = 915000. Nmm
$x_m$ = 24. mm
$y_m$ = 52. mm
$u_m$ = 3. mm
$v_m$ = 34.6 mm
$\sigma_m = -Mv/J_u$ = -236.7 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 37. mm
$v_c$ = 19.6 mm
$\sigma_c = -Mv/J_u$ = -134.1 $N/mm^2$
$\tau_c$ = 5.563 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.4 $N/mm^2$
$S^*$ = 2439. $mm^3$


H
I
3/2F
B
A
11/6F
5/3Fb
5/3F
11/6F
5/3F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
1/3F
3F
5/2Fb
8/3F
8/3F
1/3Fb
Fb
1/2F
1/3F
2F
G
C
F
G
C
1/2F
C
F
F
F
1/2Fb
F
C
11/6F
1/6Fb
5/3F
5/3F
5/3Fb
11/6F
B

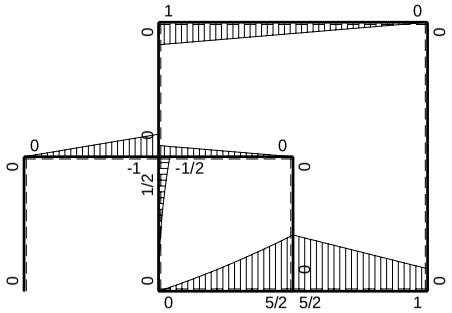




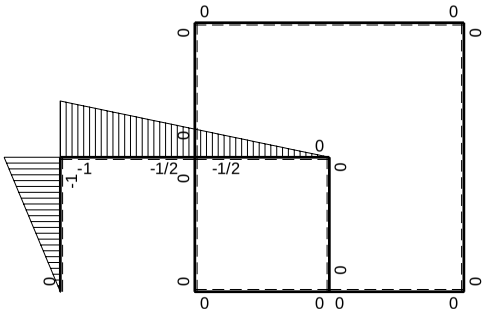

F
0
1/2
5/3
8/3
-3/2
11/6
9/2
-1
0
0
0
F
-1/2
0
-11/6
1
-1/3
0
5/3
0
-2
-3
3/2
F
1
0
0
0
5/3
1/3
0
5/3
-1/6
1/2
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 274905 CI01.xxxx.145

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PROCEDIMENTO E RISULTATI 274905 CI01.xxxx.145

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $5/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

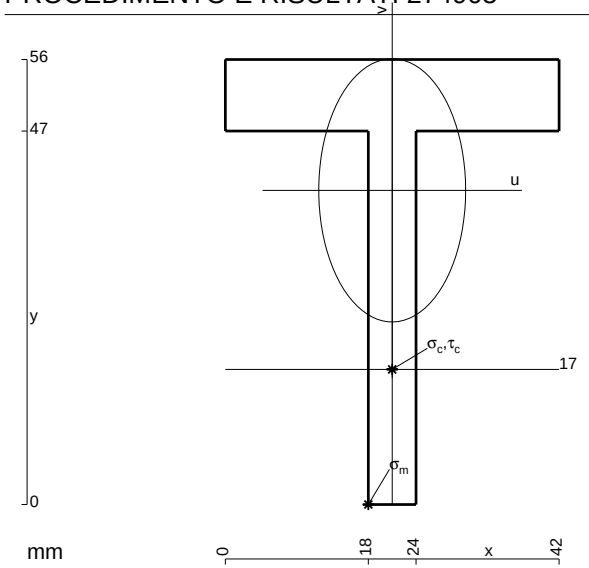
$= (1/4\ b -1/4\ b +1/12\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/12\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 1/3\ Fb^2/EJ$

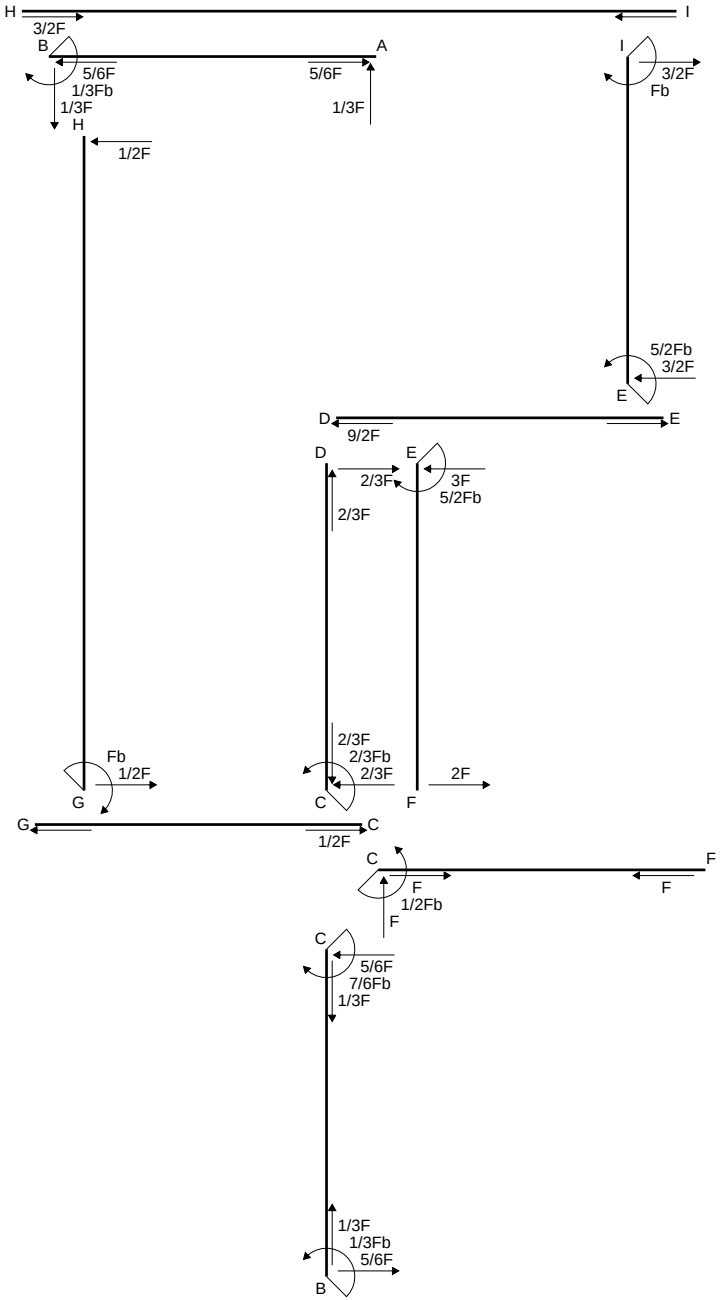


PROCEDIMENTO E RISULTATI 274905 CI01.xxxx.145



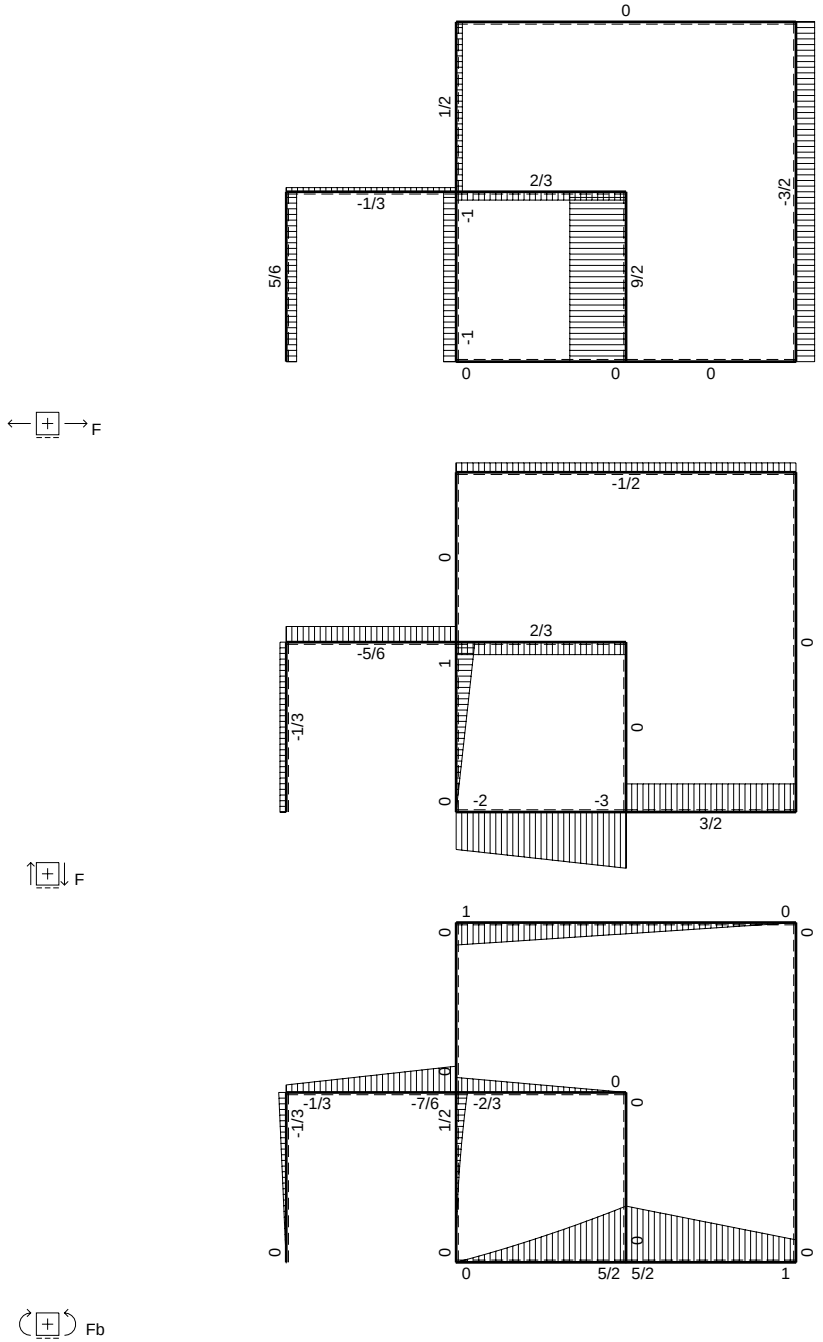

A = 660. $mm^2$
$J_u$ = 181086. $mm^4$
$J_v$ = 56412. $mm^4$
$y_g$ = 39.54 mm
$T_y$ = -1590. N
$M_x$ = 914250. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -39.54 mm
$\sigma_m = -Mv/J_u$ = 199.6 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 17. mm
$v_c$ = -22.54 mm
$\sigma_c = -Mv/J_u$ = 113.8 $N/mm^2$
$\tau_c$ = 4.633 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 114.1 $N/mm^2$
$S^*$ = 3166. $mm^3$

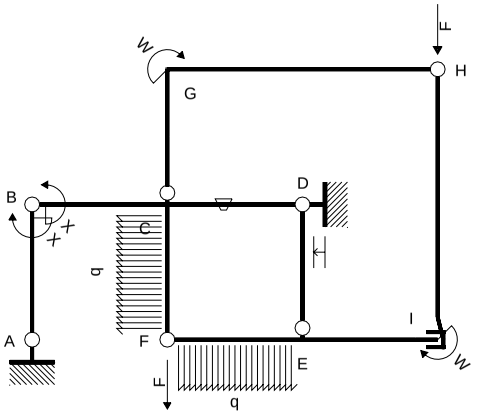
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H
3/2F
B
5/6F
1/3Fb
1/3F
H
1/2F
A
5/6F
1/3F
I
I
3/2F
Fb
5/2Fb
3/2F
E
D
9/2F
E
D
E
2/3F
3F
5/2Fb
2/3F
Fb
1/2F
G
2/3F
2/3Fb
2/3F
C
F
2F
G
C
1/2F
C
F
F
F
1/2Fb
F
C
5/6F
7/6Fb
1/3F
1/3F
1/3Fb
5/6F
B

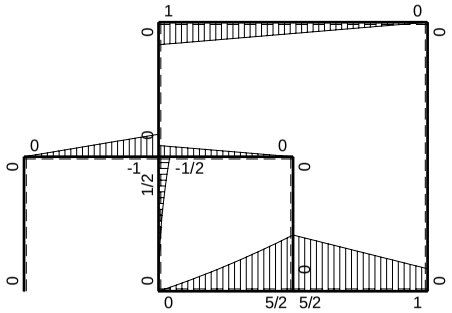




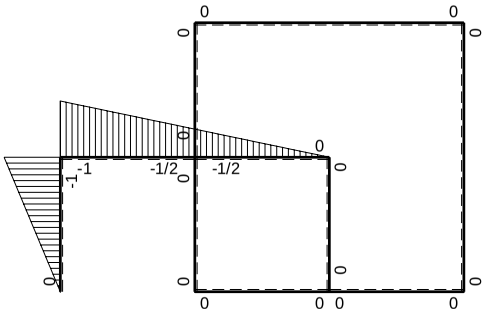

F
0
1/2
2/3
-1/3
-1
-3/2
5/6
9/2
-1
0
0
0
F
-1/2
0
2/3
-5/6
1
0
-1/3
0
0
-2
-3
3/2
F
1
0
0
0
0
-1/3
-1/3
-7/6
-2/3
1/2
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264372 CI01.xxxx.146

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/3Fb | |

Sviluppi di calcolo iperstatica



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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

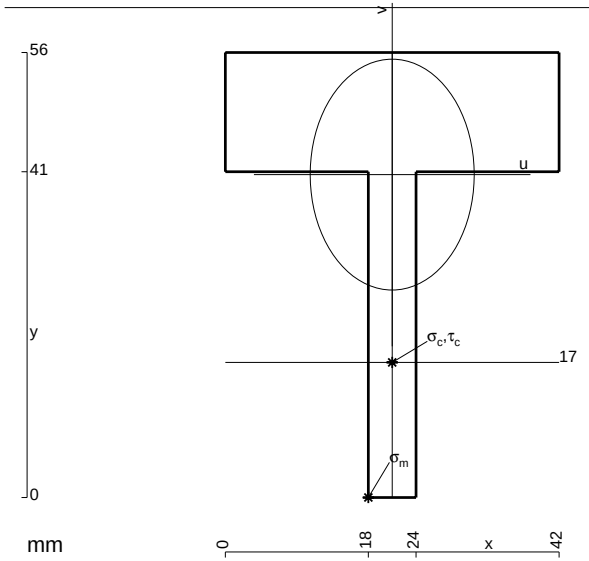
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/12\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

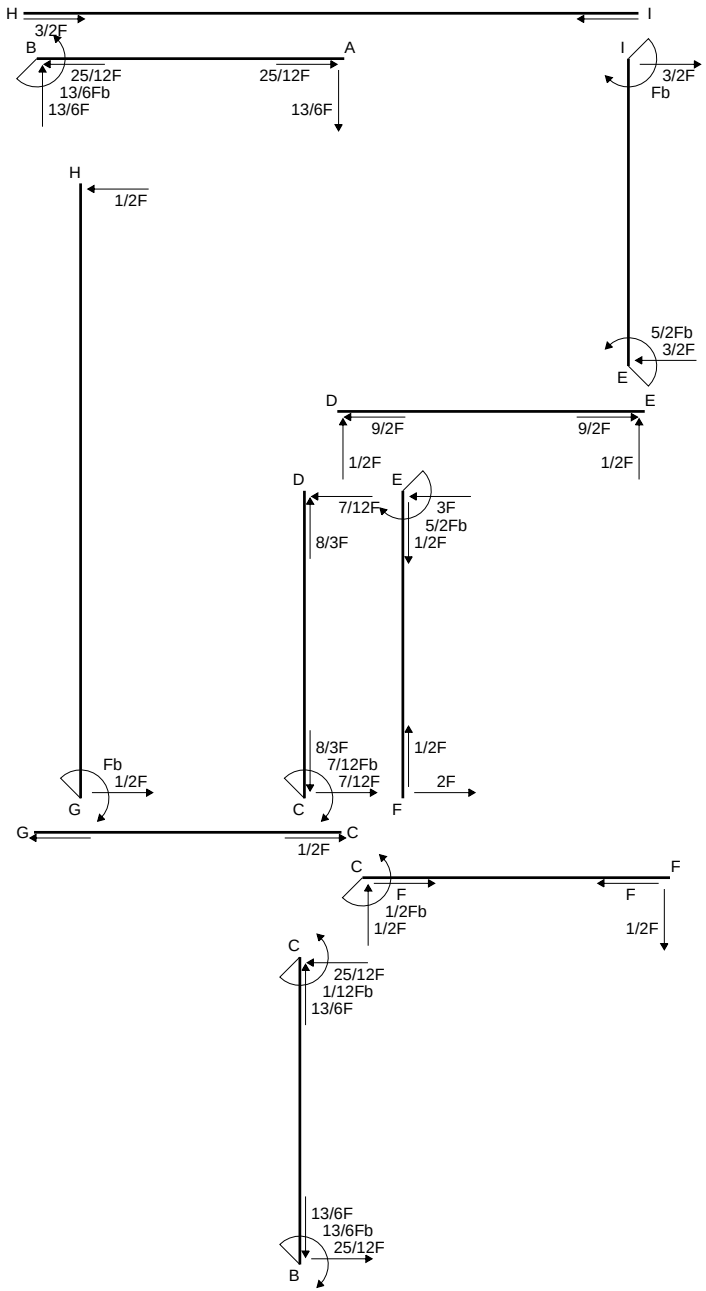


PROCEDIMENTO E RISULTATI 264372 CI01.xxxx.146



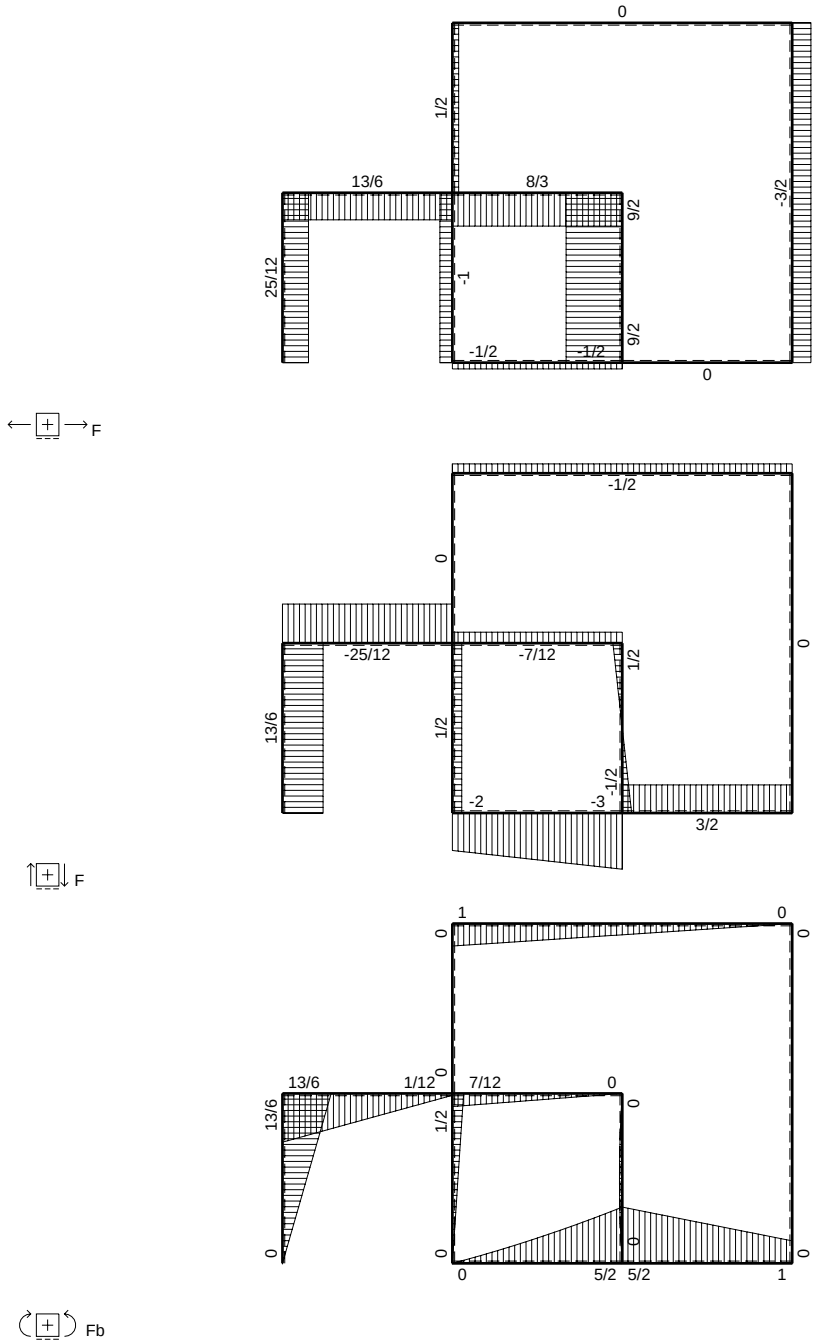

A = 876. $mm^2$
$J_u$ = 184977. $mm^4$
$J_v$ = 93348. $mm^4$
$y_g$ = 40.64 mm
$T_y$ = -1560. N
$M_x$ = 949000. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -40.64 mm
$\sigma_m = -Mv/J_u$ = 208.5 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 17. mm
$v_c$ = -23.64 mm
$\sigma_c = -Mv/J_u$ = 121.3 $N/mm^2$
$\tau_c$ = 4.607 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 121.5 $N/mm^2$
$S^*$ = 3278. $mm^3$

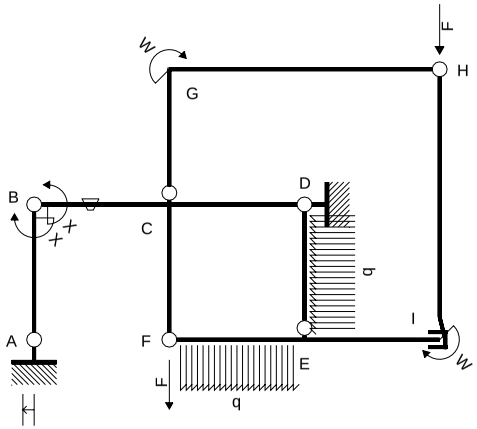
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H
I
3/2F
B
A
25/12F
13/6Fb
13/6F
25/12F
13/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
9/2F
1/2F
1/2F
D
E
7/12F
3F
5/2Fb
8/3F
1/2F
8/3F
7/12Fb
7/12F
1/2F
2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
F
1/2Fb
1/2F
1/2F
C
25/12F
1/12Fb
13/6F
13/6F
13/6Fb
25/12F
B

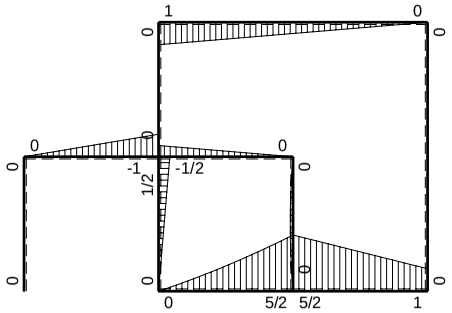




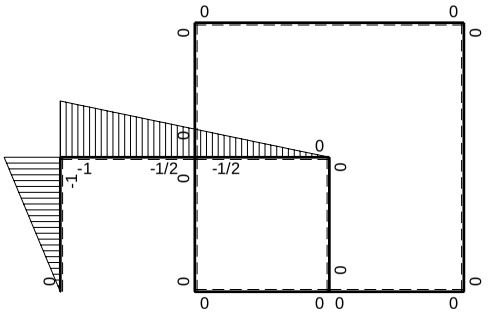

F
0
1/2
13/6
8/3
-3/2
9/2
25/12
-1
9/2
-1/2
-1/2
0
F
-1/2
0
-25/12
-7/12
1/2
0
13/6
1/2
-1/2
-2
-3
3/2
F
1
0
0
0
0
13/6
1/12
7/12
0
13/6
1/2
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264631 CI01.xxxx.147

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 264631 CI01.xxxx.147

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

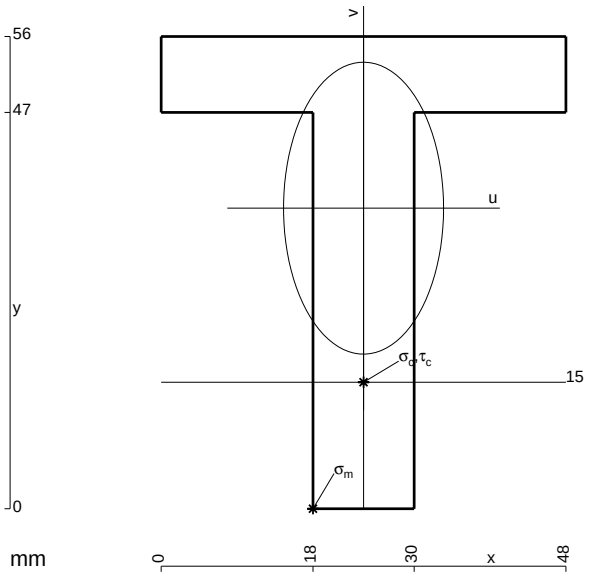
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

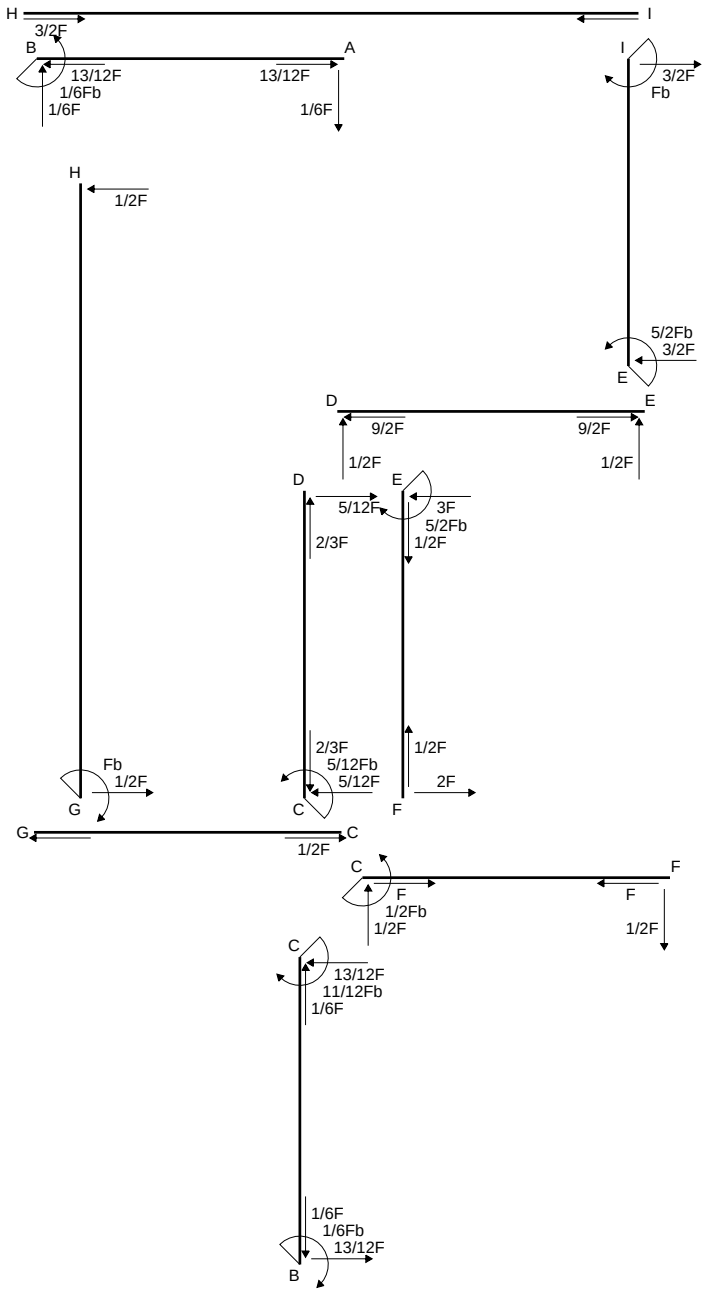
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





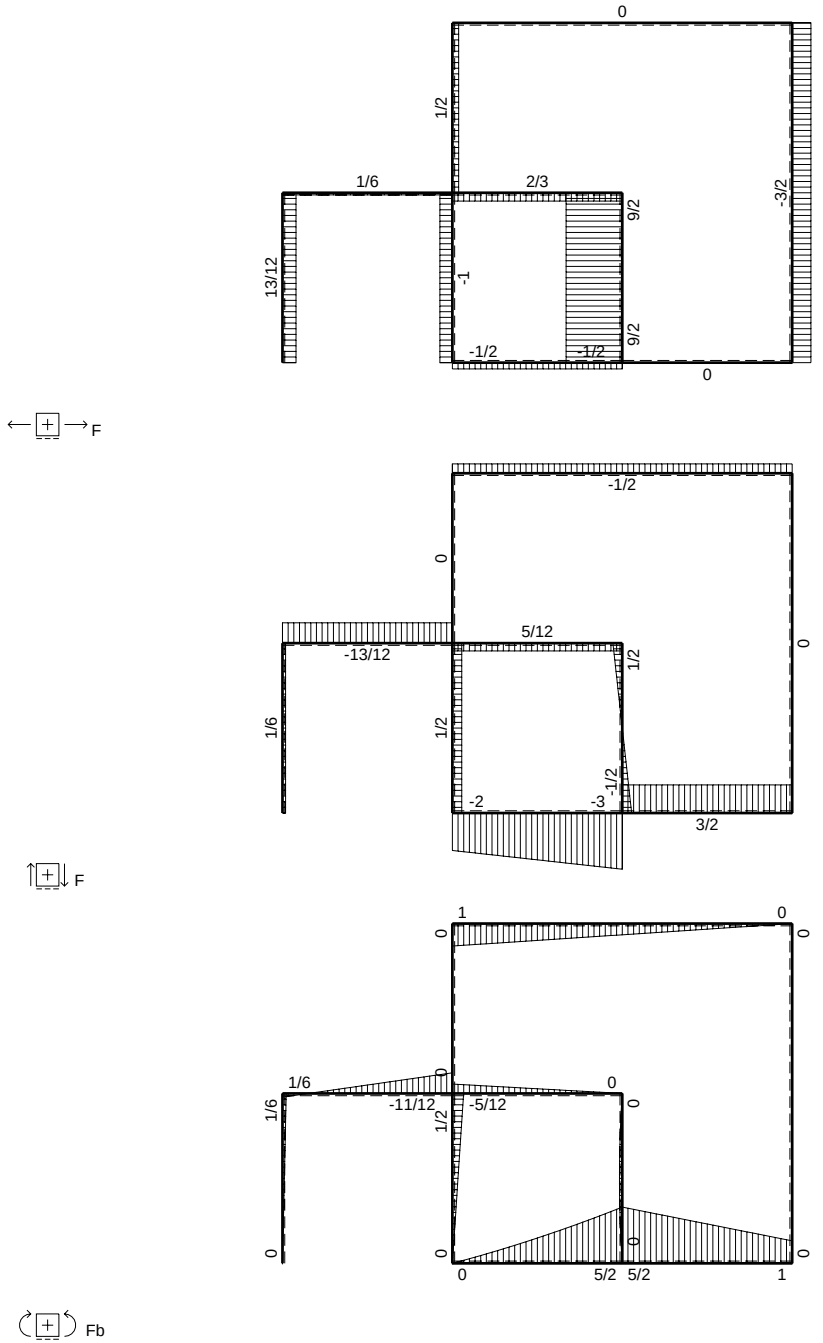


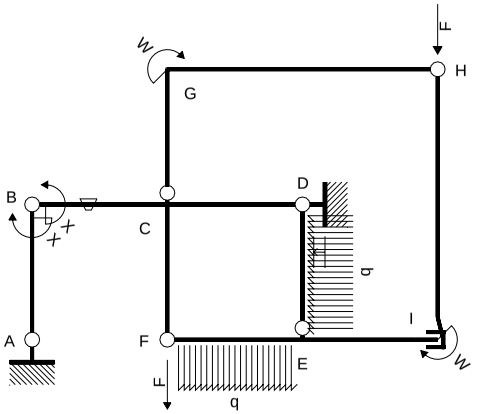

A = 996. $mm^2$
$J_u$ = 298526. $mm^4$
$J_v$ = 89712. $mm^4$
$y_g$ = 35.64 mm
N = -470. N
$T_y$ = -2820. N
$M_x$ = 1833000. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -35.64 mm
$\sigma_m$ = N/A-Mv/$J_u$ = 218.4 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -20.64 mm
$\sigma_c$ = N/A-Mv/$J_u$ = 126.3 $N/mm^2$
$\tau_c$ = 3.988 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.5 $N/mm^2$
$S^*$ = 5066. $mm^3$


H
I
3/2F
B
A
13/12F
1/6Fb
1/6F
13/12F
1/6F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
9/2F
1/2F
1/2F
D
E
5/12F
3F
5/2Fb
2/3F
1/2F
2/3F
5/12Fb
5/12F
1/2F
2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
F
1/2Fb
1/2F
1/2F
C
13/12F
11/12Fb
1/6F
1/6F
1/6Fb
13/12F
B

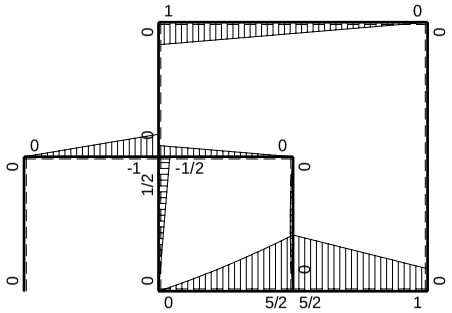




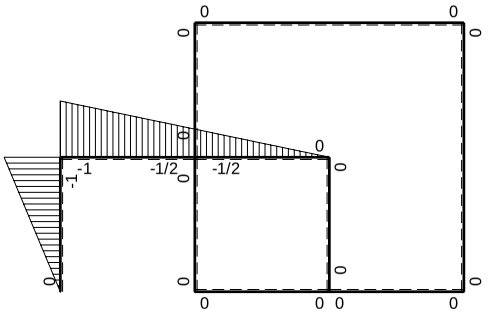

F
0
1/2
1/6
2/3
-3/2
13/12
-1
9/2
9/2
-1/2
-1/2
0
F
-1/2
0
5/12
-13/12
1/2
0
1/6
1/2
-2
-3
-1/2
3/2
F
1
0
0
0
1/6
0
-11/12
-5/12
1/6
1/2
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267798 CI01.xxxx.148

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PROCEDIMENTO E RISULTATI 267798 CI01.xxxx.148

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+1/2Fx$ | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+1/2Fx$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

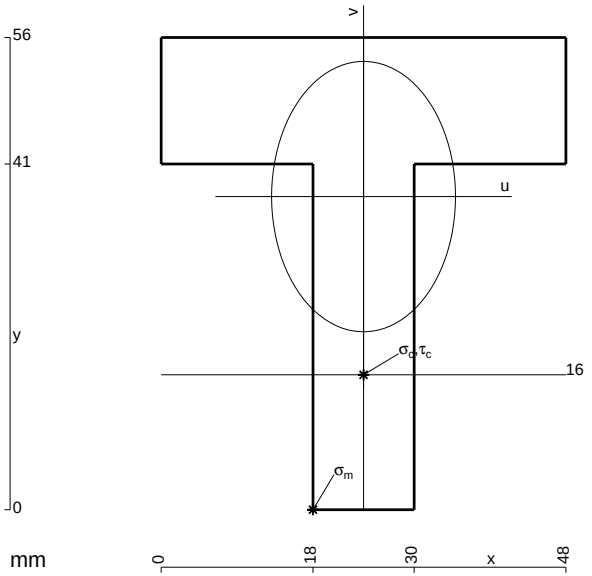
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

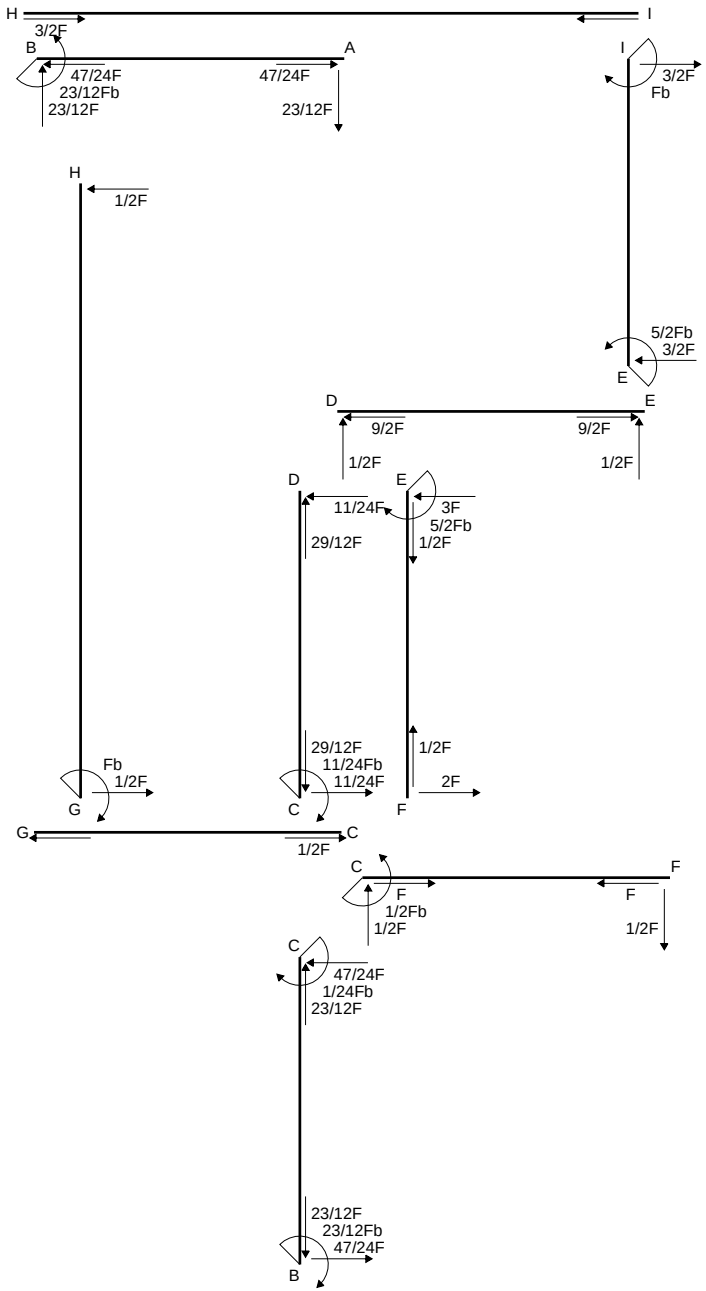
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





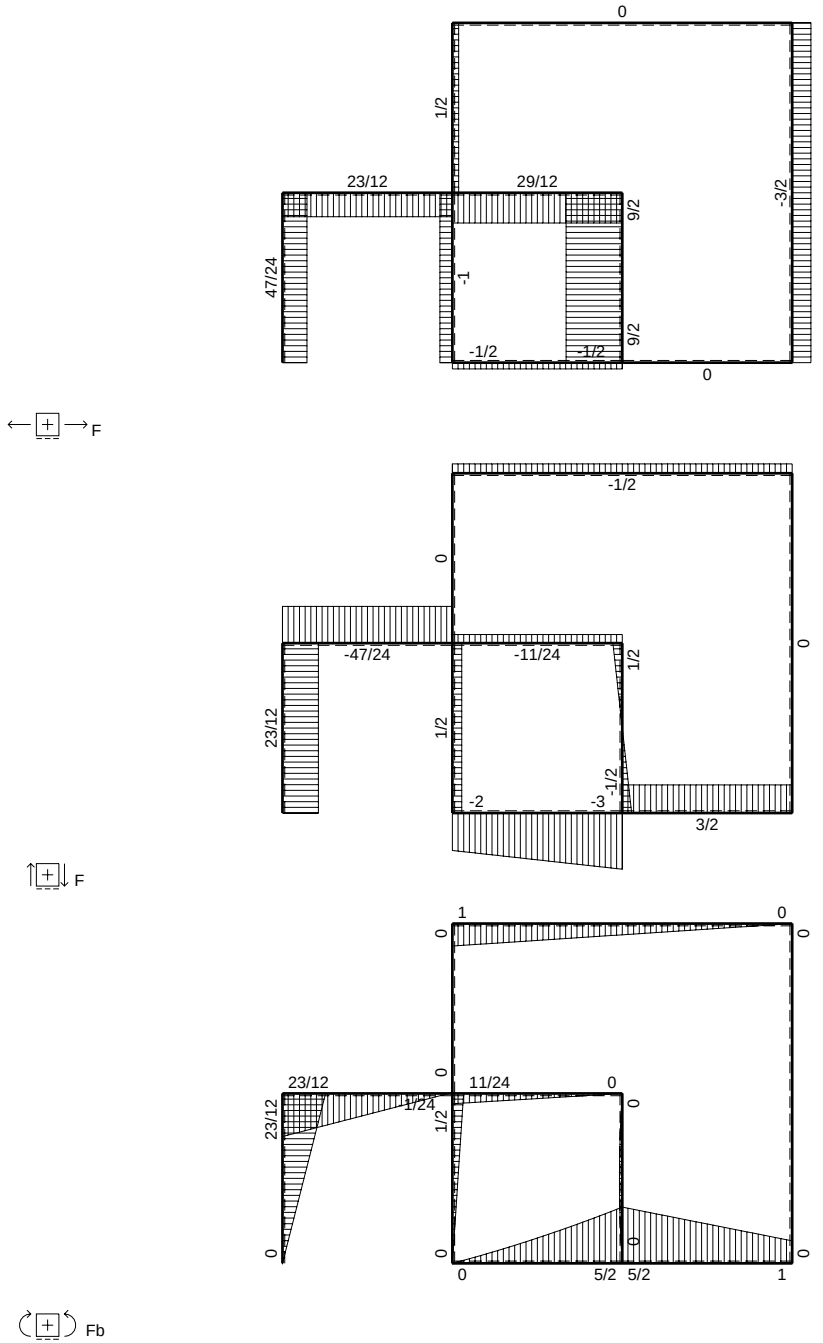


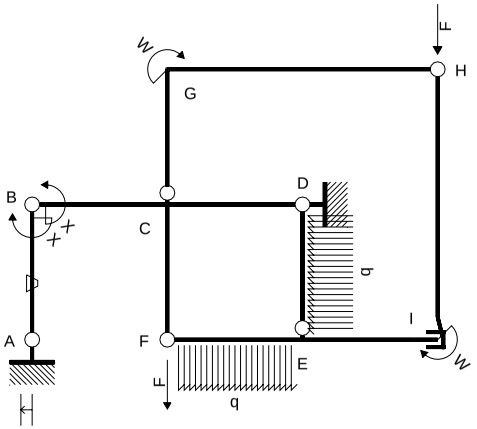

A = 1212. $mm^2$
$J_u$ = 311566. $mm^4$
$J_v$ = 144144. $mm^4$
$y_g$ = 37.13 mm
N = -470. N
$T_y$ = -2820. N
$M_x$ = 1927000. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -37.13 mm
$\sigma_m = N/A - Mv/J_u$ = 229.3 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 16. mm
$v_c$ = -21.13 mm
$\sigma_c = N/A - Mv/J_u$ = 130.3 $N/mm^2$
$\tau_c$ = 4.219 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 130.5 $N/mm^2$
$S^*$ = 5594. $mm^3$


H
3/2F
B
A
47/24F
23/12Fb
23/12F
47/24F
23/12F
I
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
9/2F
1/2F
1/2F
D
E
11/24F
3F
5/2Fb
29/12F
1/2F
29/12F
11/24Fb
11/24F
1/2F
2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
F
1/2Fb
1/2F
1/2F
C
47/24F
1/24Fb
23/12F
23/12F
23/12Fb
47/24F
B

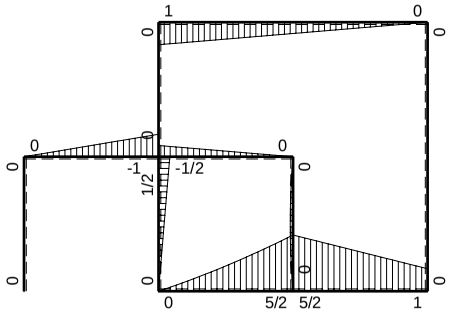




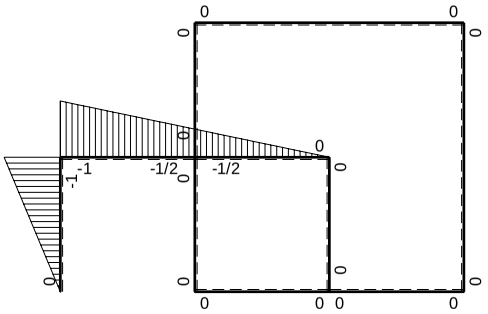

F
0
1/2
23/12
29/12
-3/2
9/2
47/24
-1
9/2
-1/2
-1/2
0
F
-1/2
0
0
-47/24
-11/24
1/2
23/12
1/2
-2
-3
-1/2
3/2
F
1
0
0
0
0
0
23/12
11/24
0
23/12
1/24
1/2
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264866 CI01.xxxx.149

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PROCEDIMENTO E RISULTATI 264866 CI01.xxxx.149

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

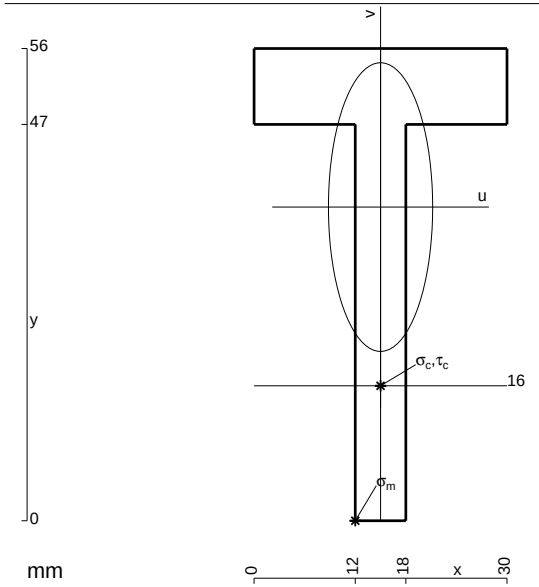
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

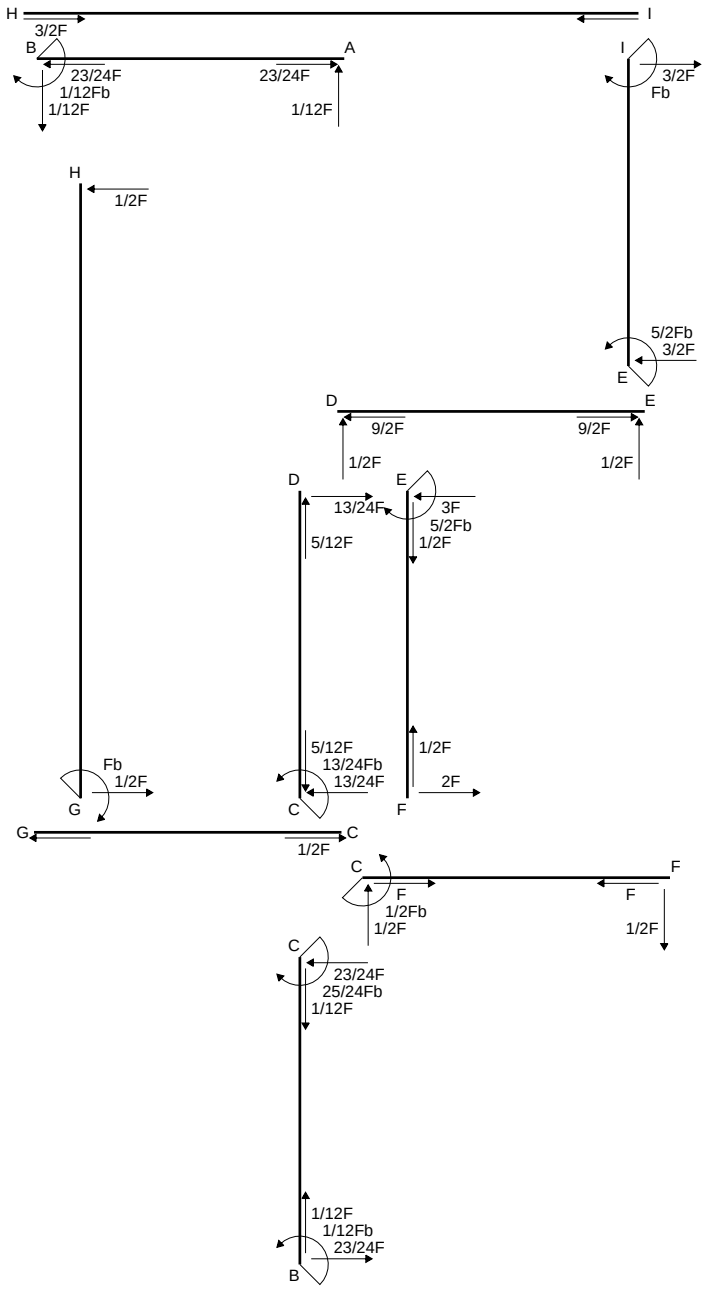


PROCEDIMENTO E RISULTATI 264866 CI01.xxxx.149



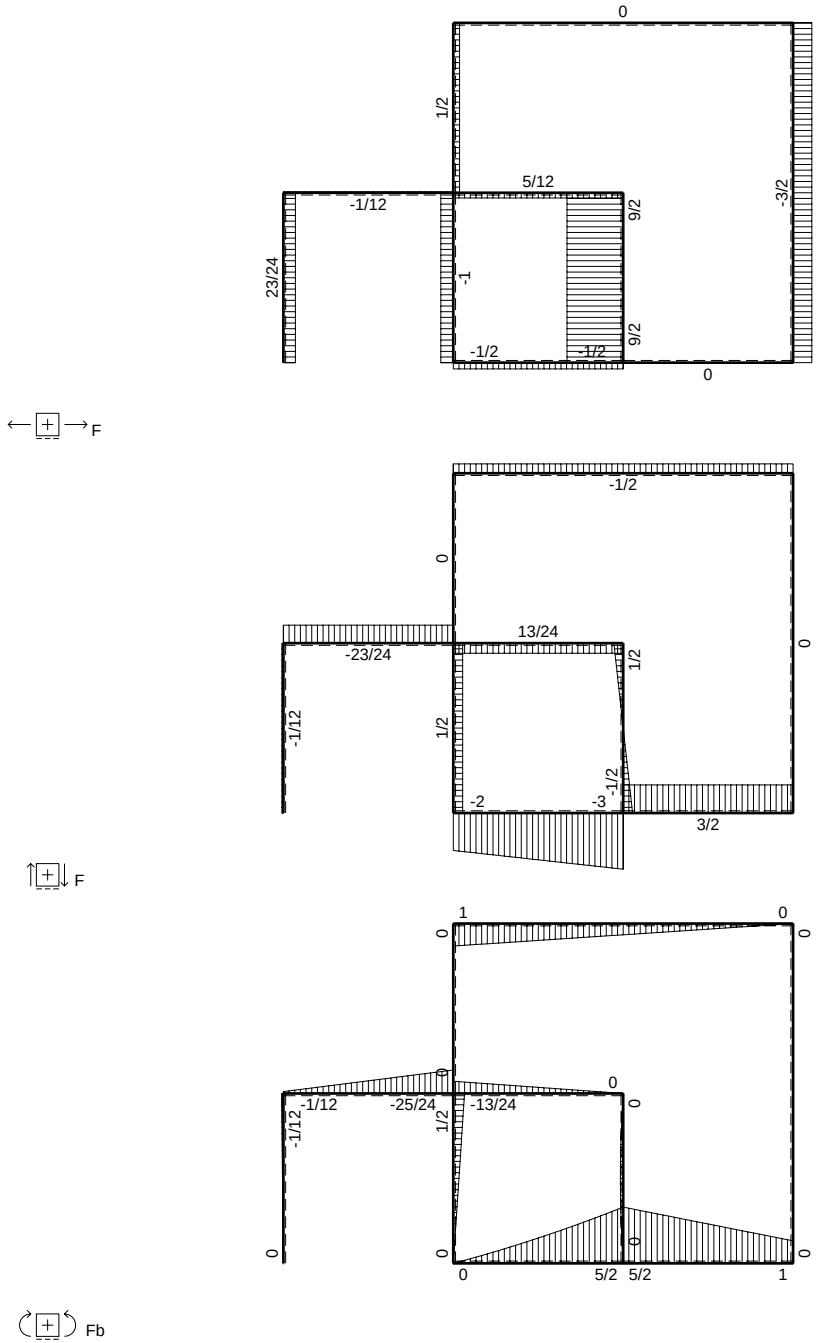

A = 552. $mm^2$
$J_u$ = 161875. $mm^4$
$J_v$ = 21096. $mm^4$
$y_g$ = 37.2 mm
N = -240. N
$T_y$ = -1440. N
$M_x$ = 1044000. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -37.2 mm
$\sigma_m = N/A - Mv/J_u$ = 239.5 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 16. mm
$v_c$ = -21.2 mm
$\sigma_c = N/A - Mv/J_u$ = 136.3 $N/mm^2$
$\tau_c$ = 4.155 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.5 $N/mm^2$
$S^*$ = 2803. $mm^3$

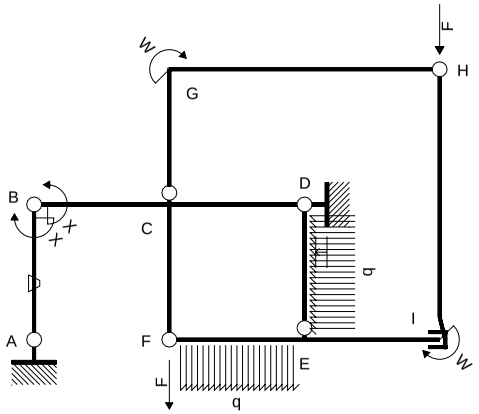
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H
3/2F
I
B
A
23/24F
1/12Fb
1/12F
23/24F
1/12F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
9/2F
1/2F
1/2F
D
E
13/24F
3F
5/2Fb
1/2F
5/12F
5/12F
13/24Fb
13/24F
1/2F
2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
F
1/2Fb
1/2F
1/2F
C
23/24F
25/24Fb
1/12F
1/12F
1/12Fb
23/24F
B

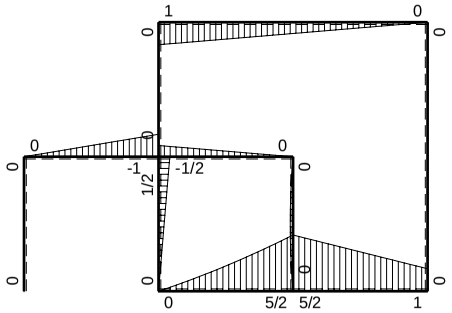




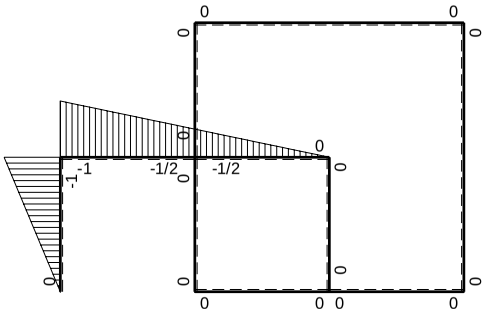

0
1/2
5/12
-1/12
-3/2
9/2
23/24
-1
9/2
-1/2
-1/2
0
F
-1/2
0
0
13/24
-23/24
1/2
-1/12
1/2
-2
-3
-1/2
3/2
F
1
0
0
0
0
-1/12
-25/24
-13/24
1/2
-1/12
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264990 CI01.xxxx.150

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 264990 CI01.xxxx.150

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | 0 | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | 0 | $1/4Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

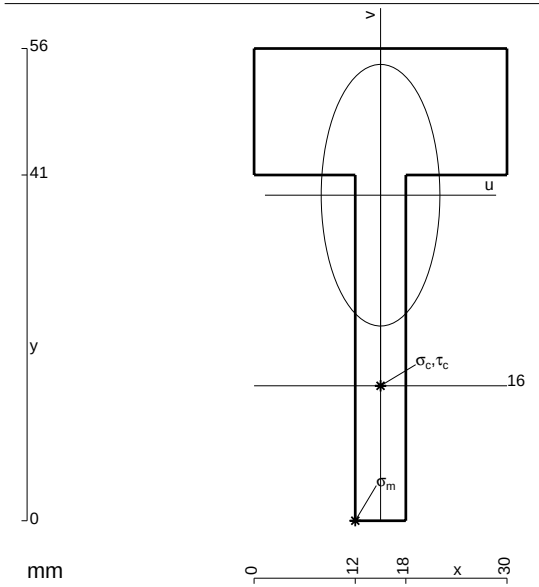
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

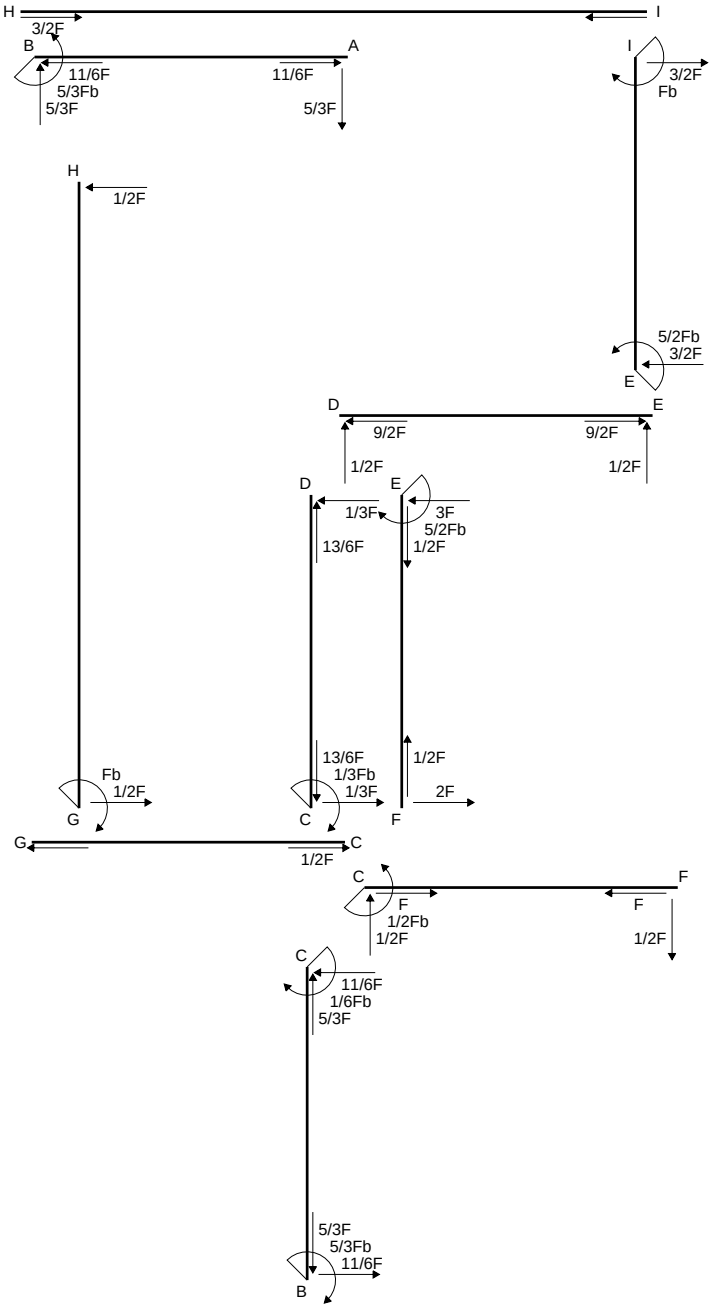
$= (1/12\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$





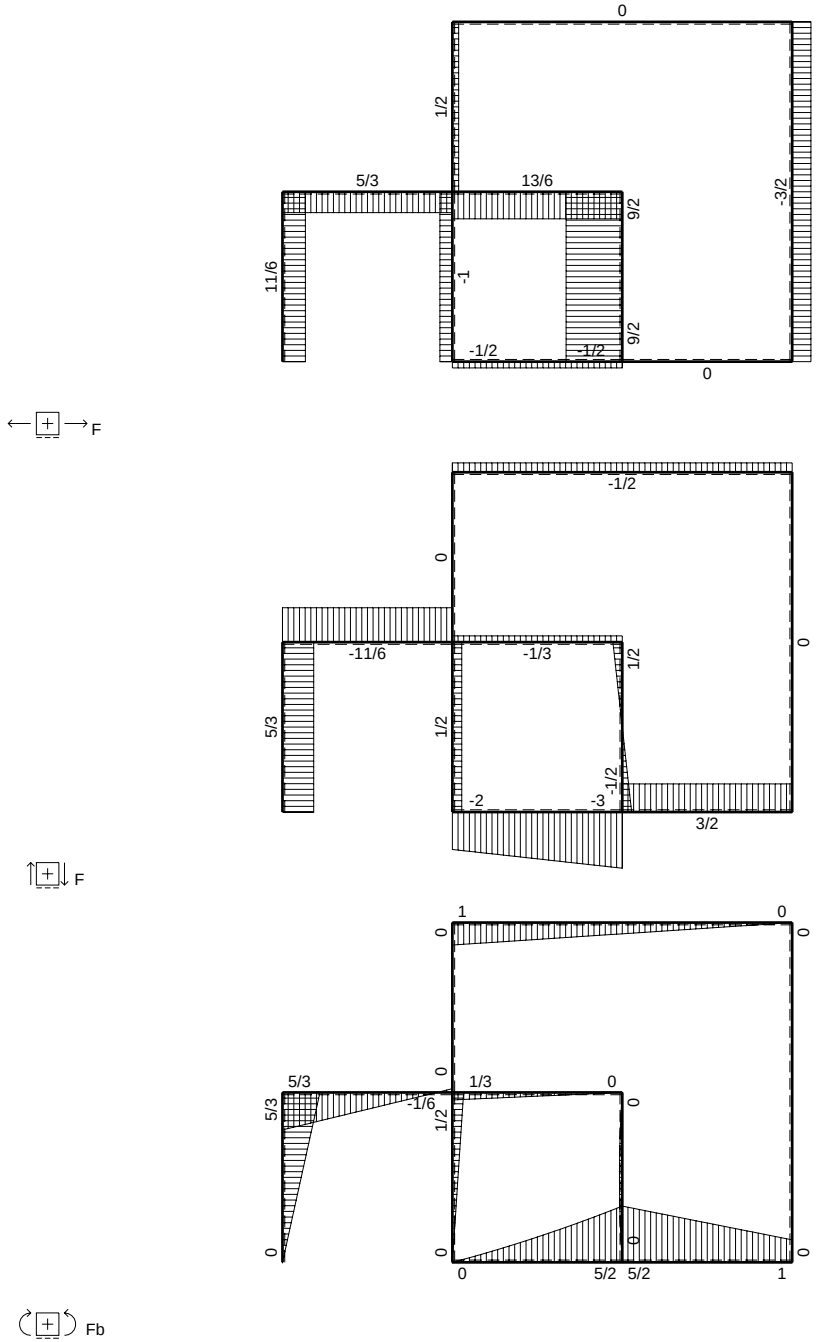


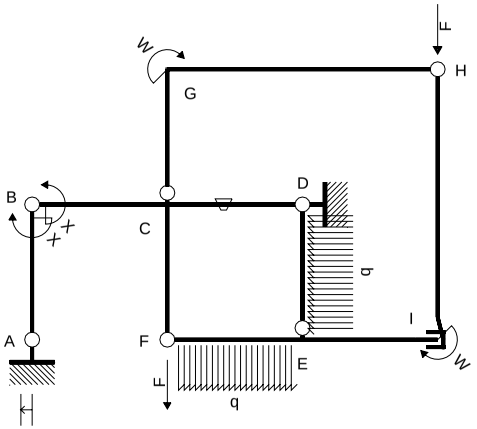

A = 696. $mm^2$
$J_u$ = 167595. $mm^4$
$J_v$ = 34488. $mm^4$
$y_g$ = 38.6 mm
N = -375. N
$T_y$ = -2250. N
$M_x$ = 862500. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -38.6 mm
$\sigma_m = N/A - Mv/J_u$ = 198.1 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 16. mm
$v_c$ = -22.6 mm
$\sigma_c = N/A - Mv/J_u$ = 115.8 $N/mm^2$
$\tau_c$ = 6.574 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.3 $N/mm^2$
$S^*$ = 2938. $mm^3$


H
I
3/2F
B
A
11/6F
5/3Fb
5/3F
11/6F
5/3F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
9/2F
1/2F
1/2F
D
E
1/3F
3F
5/2Fb
13/6F
1/2F
13/6F
1/3Fb
1/3F
1/2F
2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
F
F
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C
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5/3Fb
11/6F
B

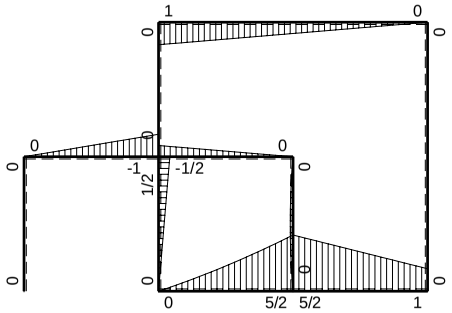




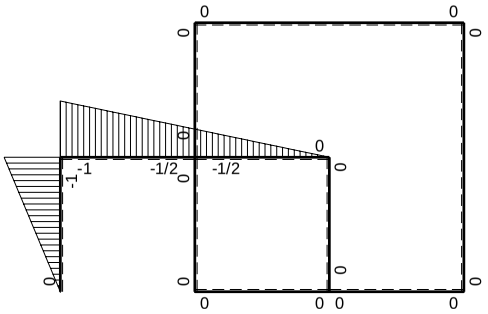

0
1/2
5/3
13/6
-3/2
9/2
11/6
-1
9/2
-1/2
-1/2
0
F
-1/2
0
-11/6
-1/3
1/2
0
5/3
1/2
-2
-3
-1/2
3/2
F
1
0
0
0
0
5/3
1/3
0
-1/6
5/3
1/2
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 263702 CI01.xxxx.151

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $5/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/3Fb | |

Sviluppi di calcolo iperstatica



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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

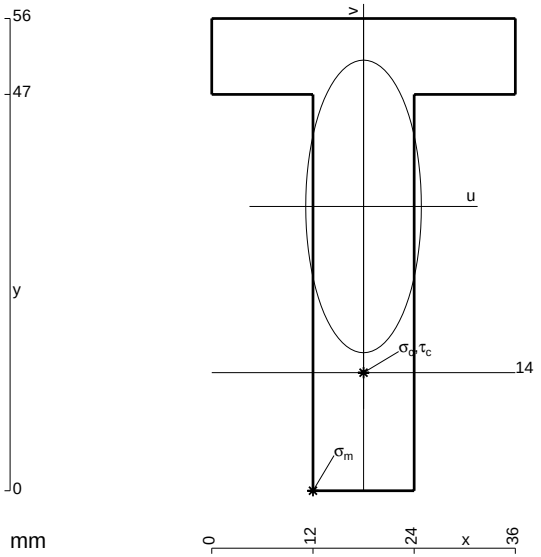
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/12\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

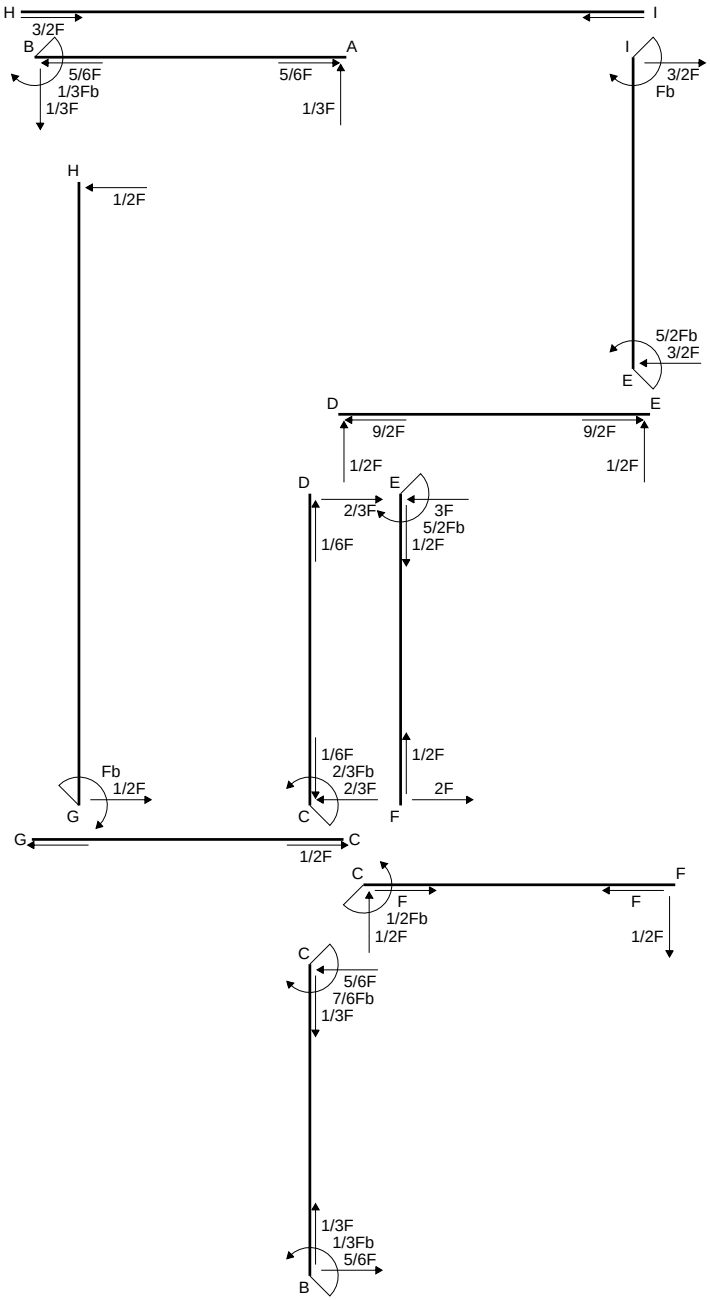


PROCEDIMENTO E RISULTATI 263702 CI01.xxxx.151



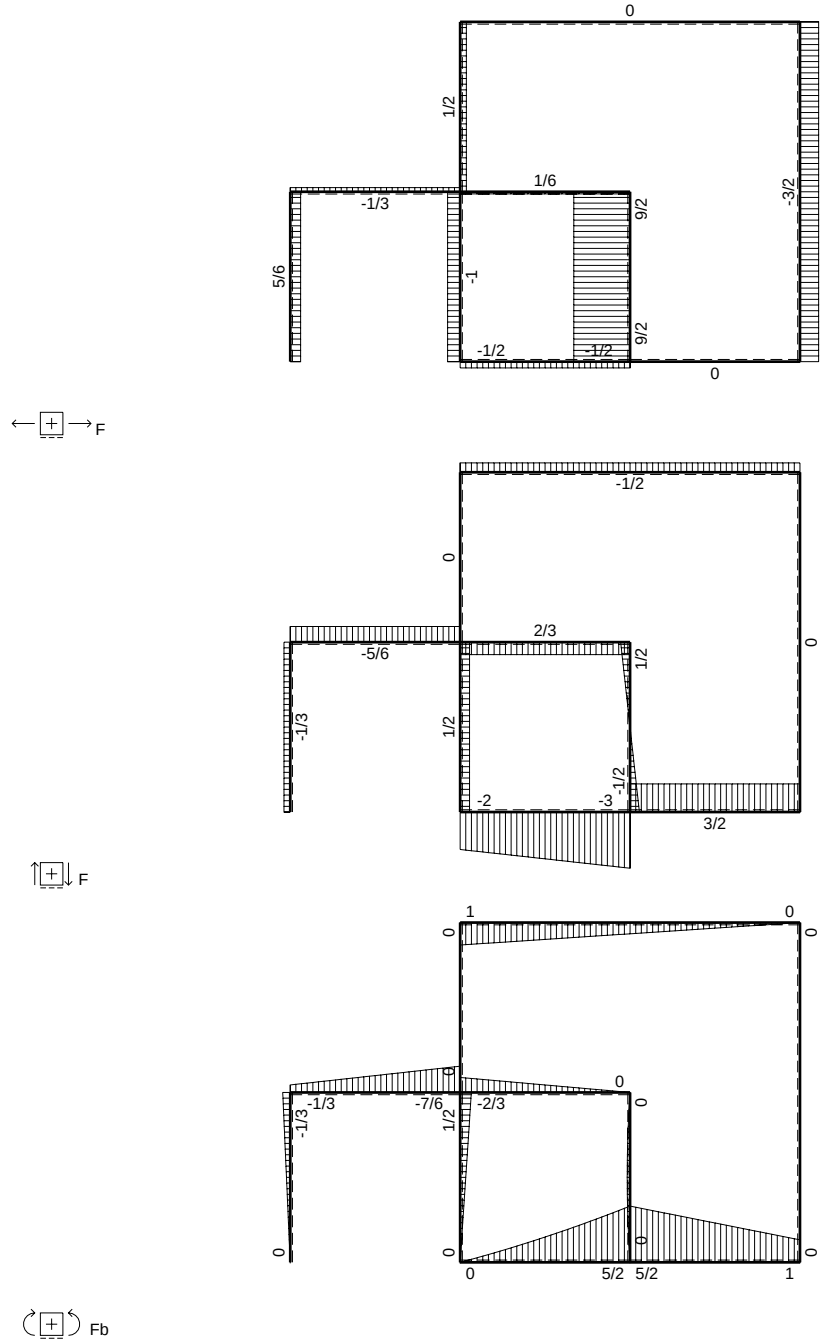

A = 888. $mm^2$
$J_u$ = 267344. $mm^4$
$J_v$ = 41760. $mm^4$
$y_g$ = 33.72 mm
N = -665. N
$T_y$ = -3990. N
$M_x$ = 1662500. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -33.72 mm
$\sigma_m$ = N/A-Mv/$J_u$ = 208.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -19.72 mm
$\sigma_c$ = N/A-Mv/$J_u$ = 121.9 $N/mm^2$
$\tau_c$ = 5.582 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.2 $N/mm^2$
$S^*$ = 4488. $mm^3$

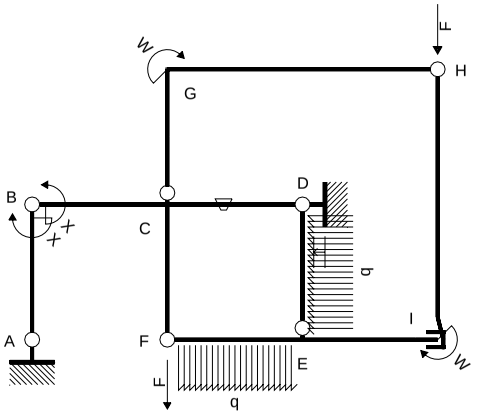
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H
I
3/2F
B
A
5/6F
1/3Fb
1/3F
5/6F
1/3F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
9/2F
1/2F
1/2F
D
E
2/3F
3F
5/2Fb
1/6F
1/2F
1/6F
2/3Fb
2/3F
1/2F
2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
F
1/2Fb
1/2F
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C
5/6F
7/6Fb
1/3F
1/3F
1/3Fb
5/6F
B

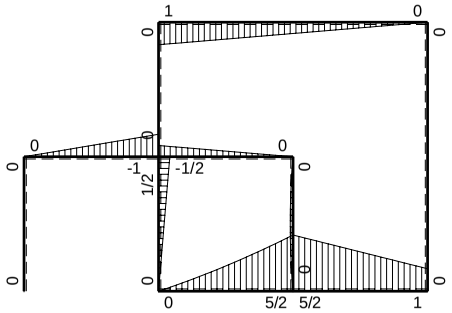




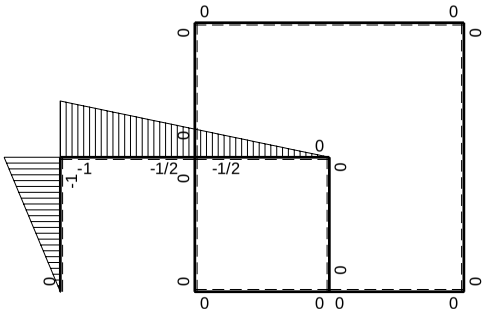

F
0
1/2
1/6
-1/3
-3/2
9/2
5/6
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9/2
-1/2
-1/2
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2/3
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1/2
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-1/2
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0
-1/3
-1/3
-7/6
-2/3
1/2
0
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 271087 CI01.xxxx.152

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PROCEDIMENTO E RISULTATI 271087 CI01.xxxx.152

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -1/2Fb+1/2Fx | -Fb/EJ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 1/2Fx | Fb/EJ | $1/4Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/3Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/3Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

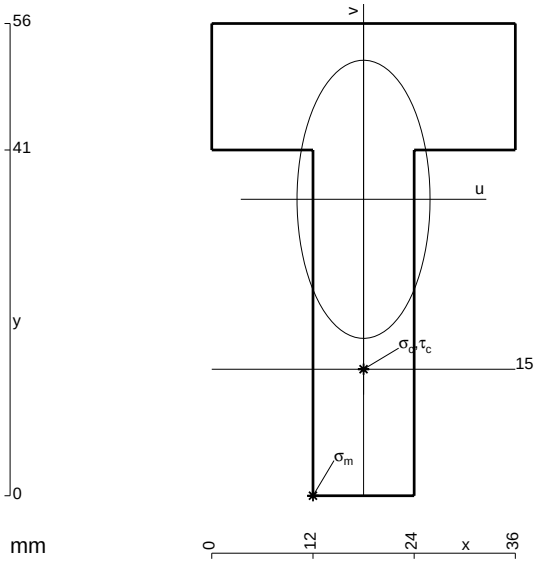
$= (1/4\ b -1/4\ b +1/12\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

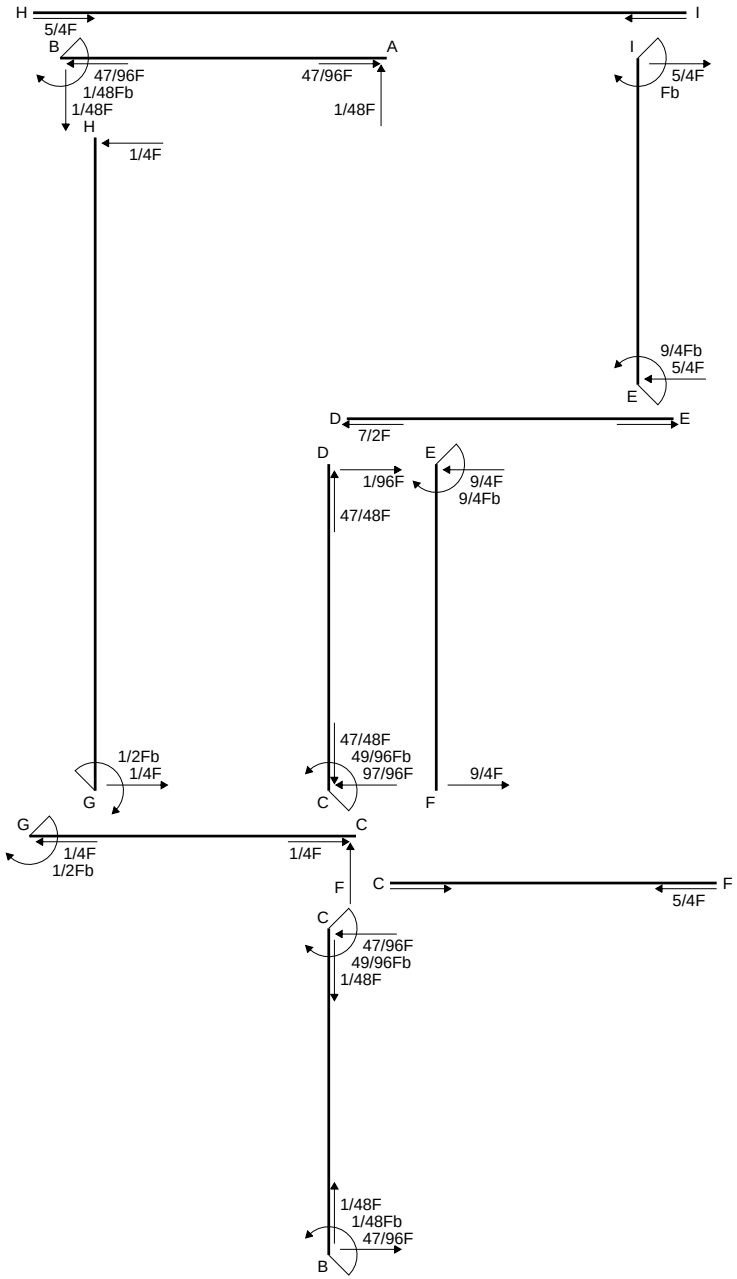
$= (1/12\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 1/3\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 271087 CI01.xxxx.152

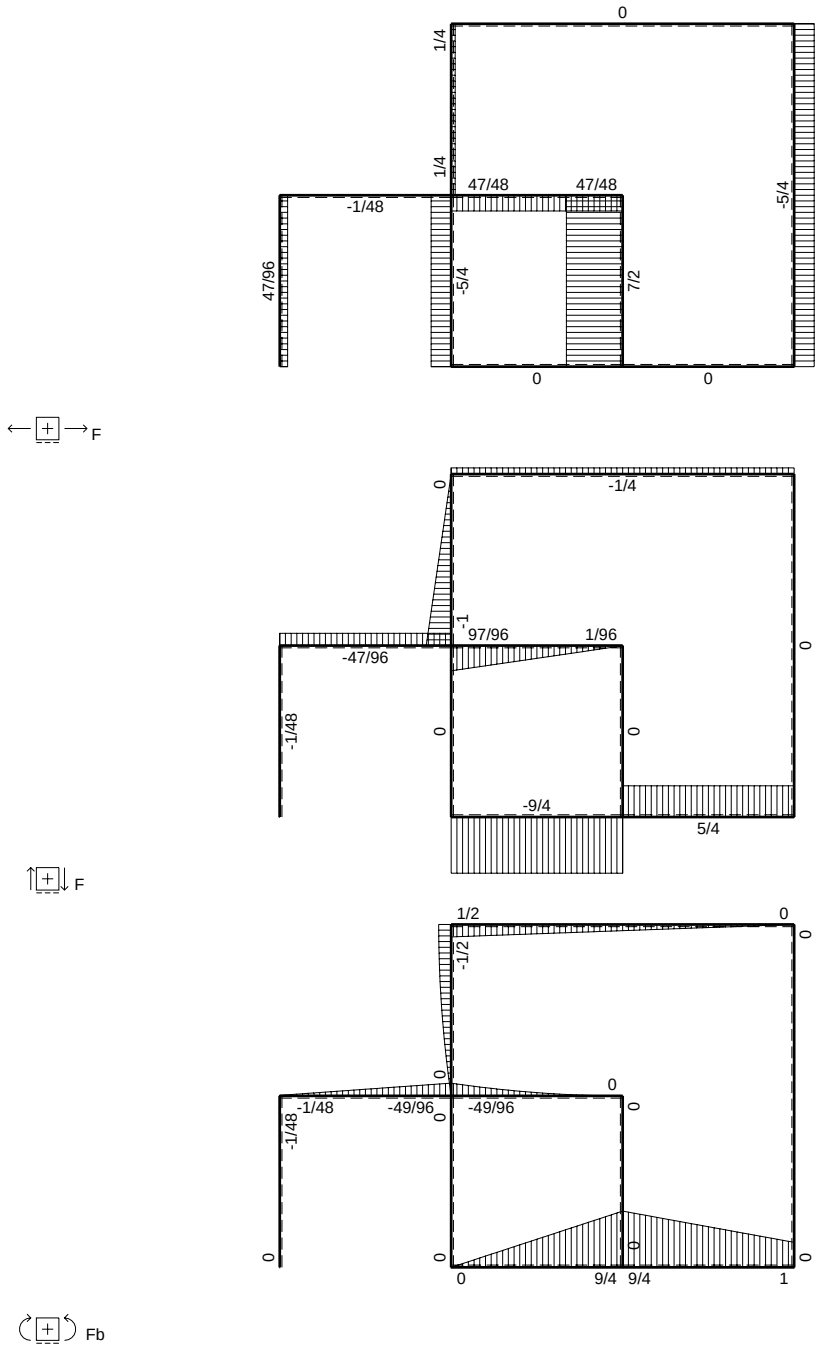


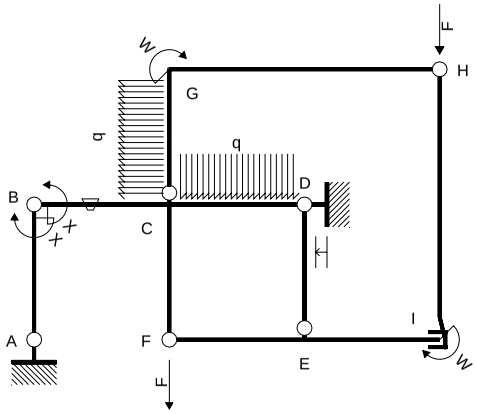

A = 1032. $mm^2$
$J_u$ = 280881. $mm^4$
$J_v$ = 64224. $mm^4$
$y_g$ = 35.15 mm
N = -640. N
$T_y$ = -3840. N
$M_x$ = 1760000. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -35.15 mm
$\sigma_m$ = N/A-Mv/$J_u$ = 219.6 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 15. mm
$v_c$ = -20.15 mm
$\sigma_c$ = N/A-Mv/$J_u$ = 125.6 $N/mm^2$
$\tau_c$ = 5.67 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126. $N/mm^2$
$S^*$ = 4977. $mm^3$


H
5/4F
I
B
A
47/96F
1/48Fb
1/48F
47/96F
1/48F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
1/96F
9/4F
9/4Fb
47/48F
1/2Fb
1/4F
47/48F
49/96Fb
97/96F
9/4F
G
C
F
G
C
1/4F
1/2Fb
1/4F
F
C
F
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47/96F
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1/48F
1/48Fb
47/96F
B

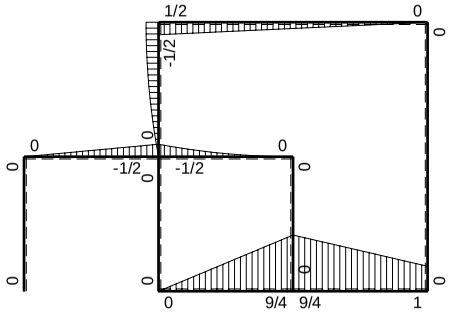




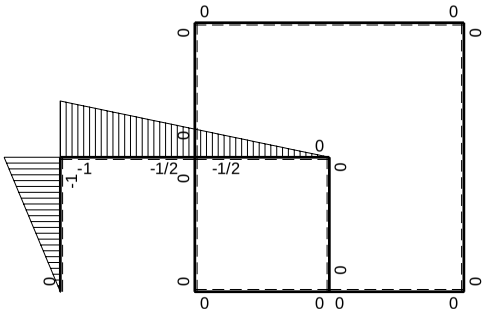

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1/4
47/48
47/48
-1/48
-5/4
47/96
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7/2
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97/96
1/96
-47/96
0
-1/48
0
0
-9/4
5/4
F
1/2
0
0
-1/2
0
0
-1/48
-49/96
0
-49/96
0
-1/48
0
0
0
0
9/4
9/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 249262 CI01.xxxx.153

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PROCEDIMENTO E RISULTATI 249262 CI01.xxxx.153

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-1/2Fx$ | -Fb/EJ | $1/2Fx-1/4Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/6+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $1/2Fb-1/2Fx$ | Fb/EJ | $1/4Fb-1/4Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+Fx-1/2qx^2$ | 0 | $1/4Fb-3/4Fx+3/4Fx^2/b-1/4qx^3/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/16+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | $1/2qx^2$ | 0 | $1/4qx^3/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 9/4Fb-9/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -9/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+5/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -9/4Fb+5/4Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/48Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/48Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -3/4\ x/b +3/4\ x^2/b^2 -1/4\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [1/4\ x -3/8\ x^2/b +1/4\ x^3/b^2 -1/16\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

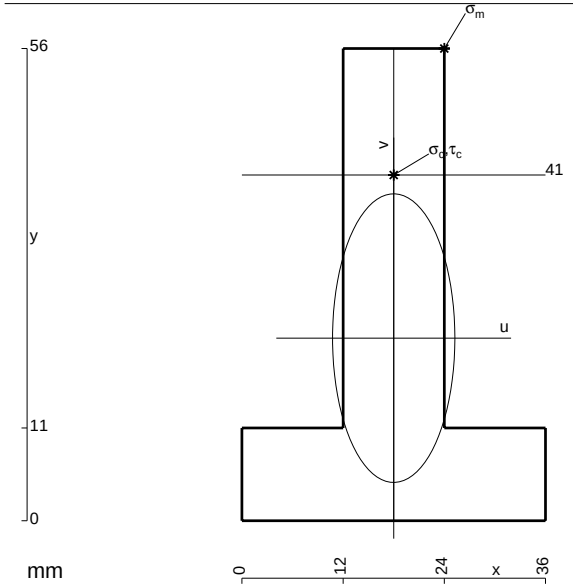
$= (1/4\ b -3/8\ b +1/4\ b -1/16\ b\ )\ Fb\ 1/EJ\ = 1/16\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/16\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/16\ b\ )\ Fb\ 1/EJ\ = 1/16\ Fb^2/EJ$

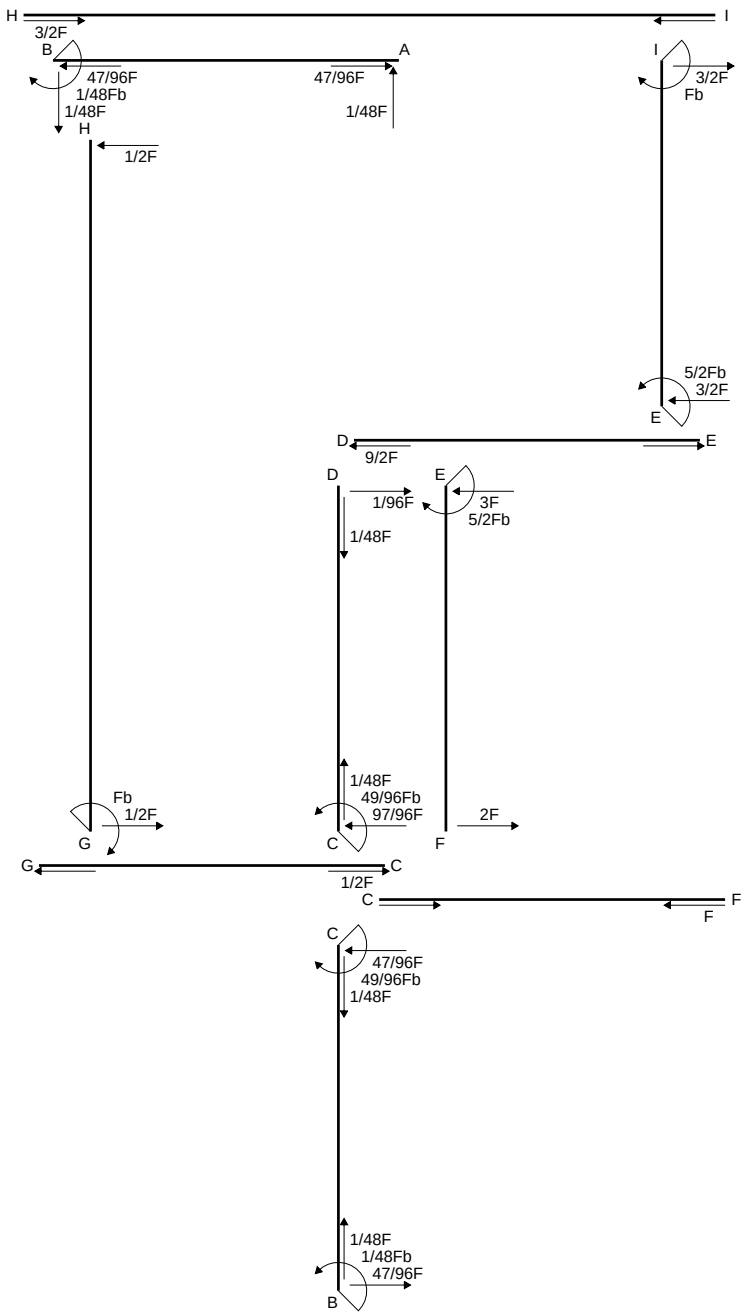


PROCEDIMENTO E RISULTATI 249262 CI01.xxxx.153



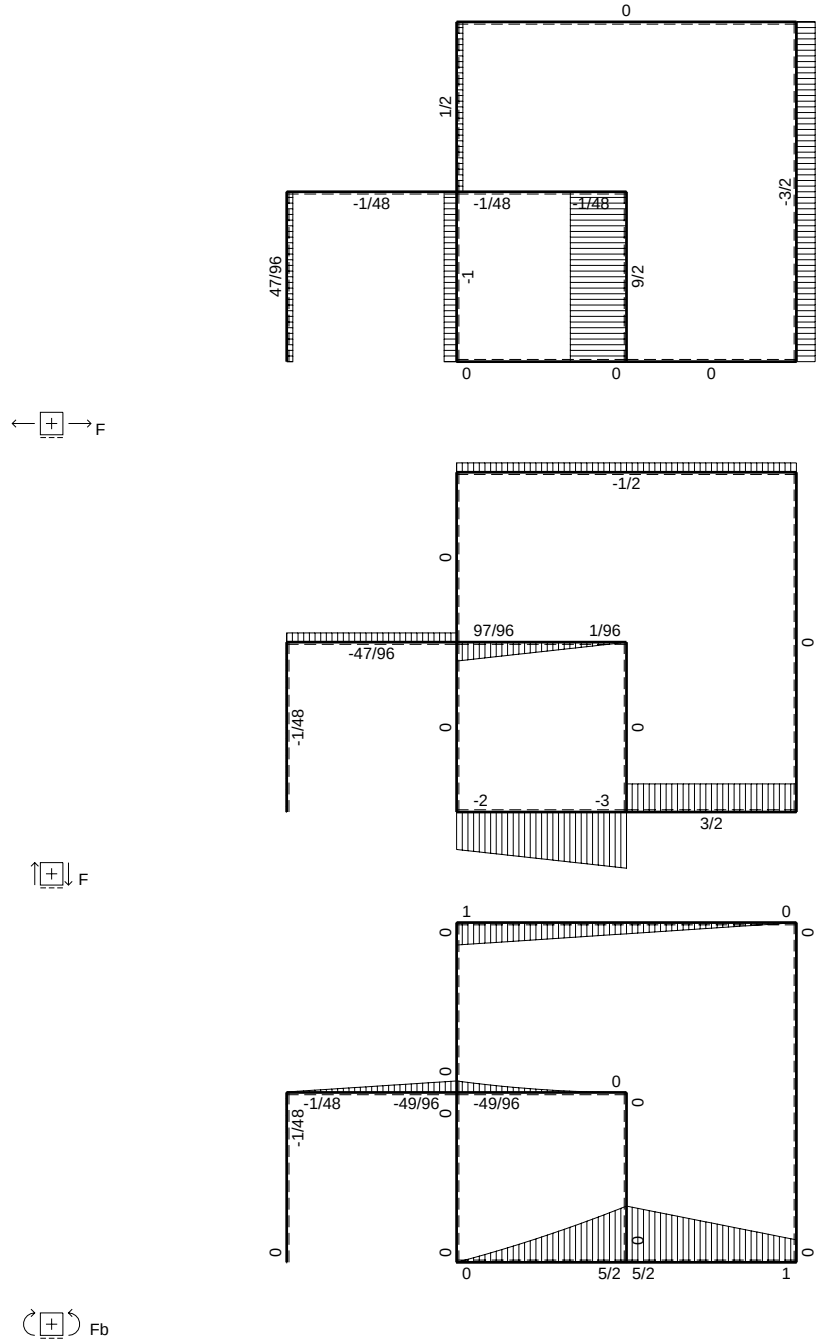

A = 936. $mm^2$
$J_u$ = 274232. $mm^4$
$J_v$ = 49248. $mm^4$
$y_g$ = 21.65 mm
$T_y$ = -3105. N
$M_x$ = 1831950. Nmm
$x_m$ = 24. mm
$y_m$ = 56. mm
$u_m$ = 6. mm
$v_m$ = 34.35 mm
$\sigma_m = -Mv/J_u$ = -229.4 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 41. mm
$v_c$ = 19.35 mm
$\sigma_c = -Mv/J_u$ = -129.2 $N/mm^2$
$\tau_c$ = 4.56 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.5 $N/mm^2$
$S^*$ = 4832. $mm^3$

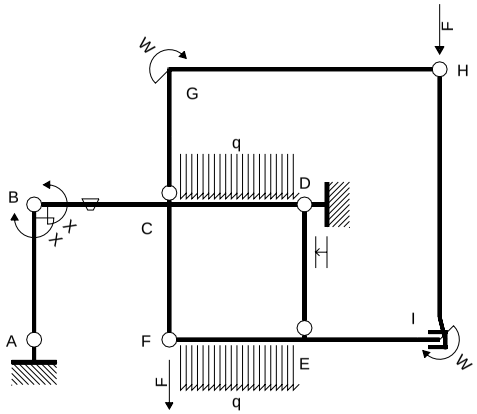
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H
I
3/2F
B
A
47/96F
1/48Fb
1/48F
47/96F
1/48F
I
3/2F
Fb
H
1/2F
5/2Fb
3/2F
E
D
E
9/2F
D
E
1/96F
3F
5/2Fb
1/48F
Fb
1/2F
G
1/48F
49/96Fb
97/96F
C
2F
F
G
C
1/2F
C
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F
C
47/96F
49/96Fb
1/48F
1/48F
1/48Fb
47/96F
B

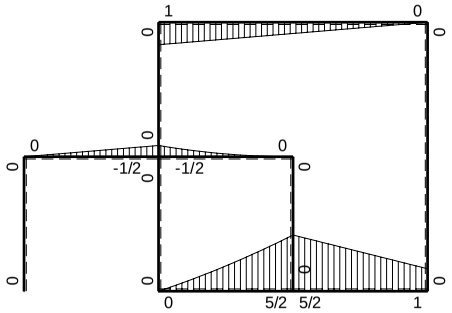




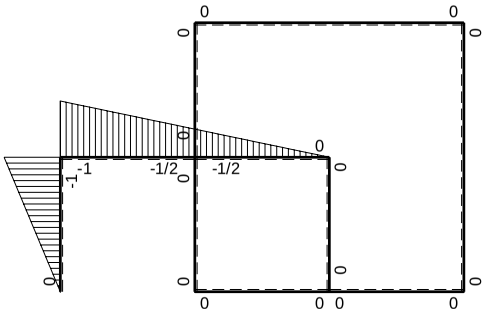

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1/2
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-1/48
-1/48
-1/48
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9/2
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0
97/96
1/96
-47/96
-1/48
0
0
0
-2
-3
3/2
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1
0
0
0
0
0
-1/48
-1/48
-49/96
-49/96
0
0
0
0
0
5/2
5/2
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 309525 CI01.xxxx.154

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PROCEDIMENTO E RISULTATI 309525 CI01.xxxx.154

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -1/2Fx | -Fb/EJ | $1/2Fx-1/4Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/6+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 1/2Fb-1/2Fx | Fb/EJ | $1/4Fb-1/4Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-1/2Fb+Fx-1/2qx^2$ | 0 | $1/4Fb-3/4Fx+3/4Fx^2/b-1/4qx^3/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/16+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | $1/2qx^2$ | 0 | $1/4qx^3/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | Fb+3/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | -5/2Fb+3/2Fx | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/48Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/48Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/4\ x -1/12\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/12\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -3/4\ x/b +3/4\ x^2/b^2 -1/4\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [1/4\ x -3/8\ x^2/b +1/4\ x^3/b^2 -1/16\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

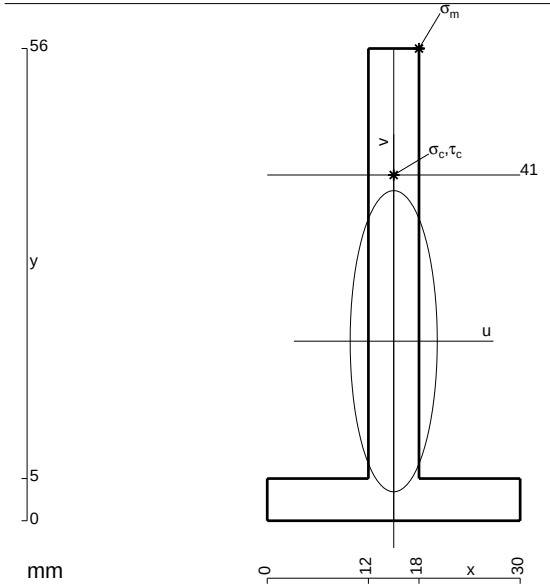
$= (1/4\ b -3/8\ b +1/4\ b -1/16\ b\ )\ Fb\ 1/EJ\ = 1/16\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/16\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/16\ b\ )\ Fb\ 1/EJ\ = 1/16\ Fb^2/EJ$

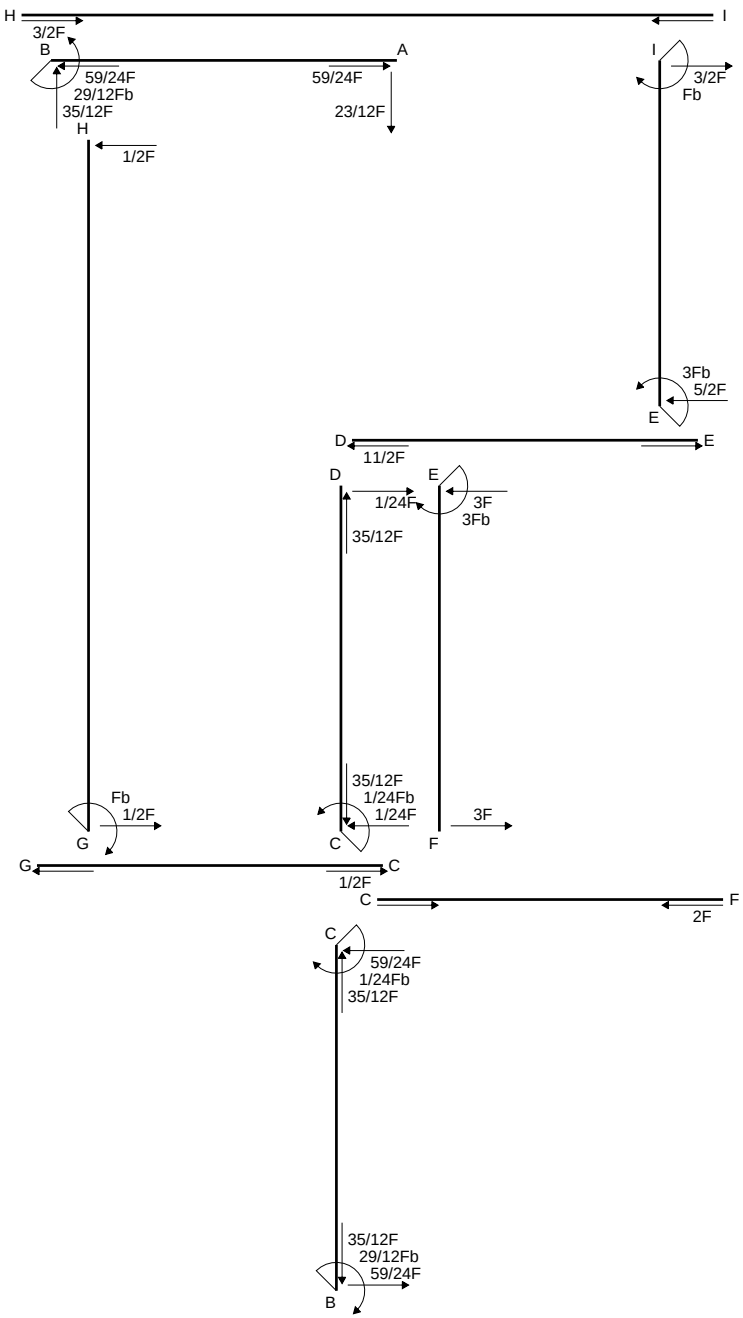


PROCEDIMENTO E RISULTATI 309525 CI01.xxxx.154



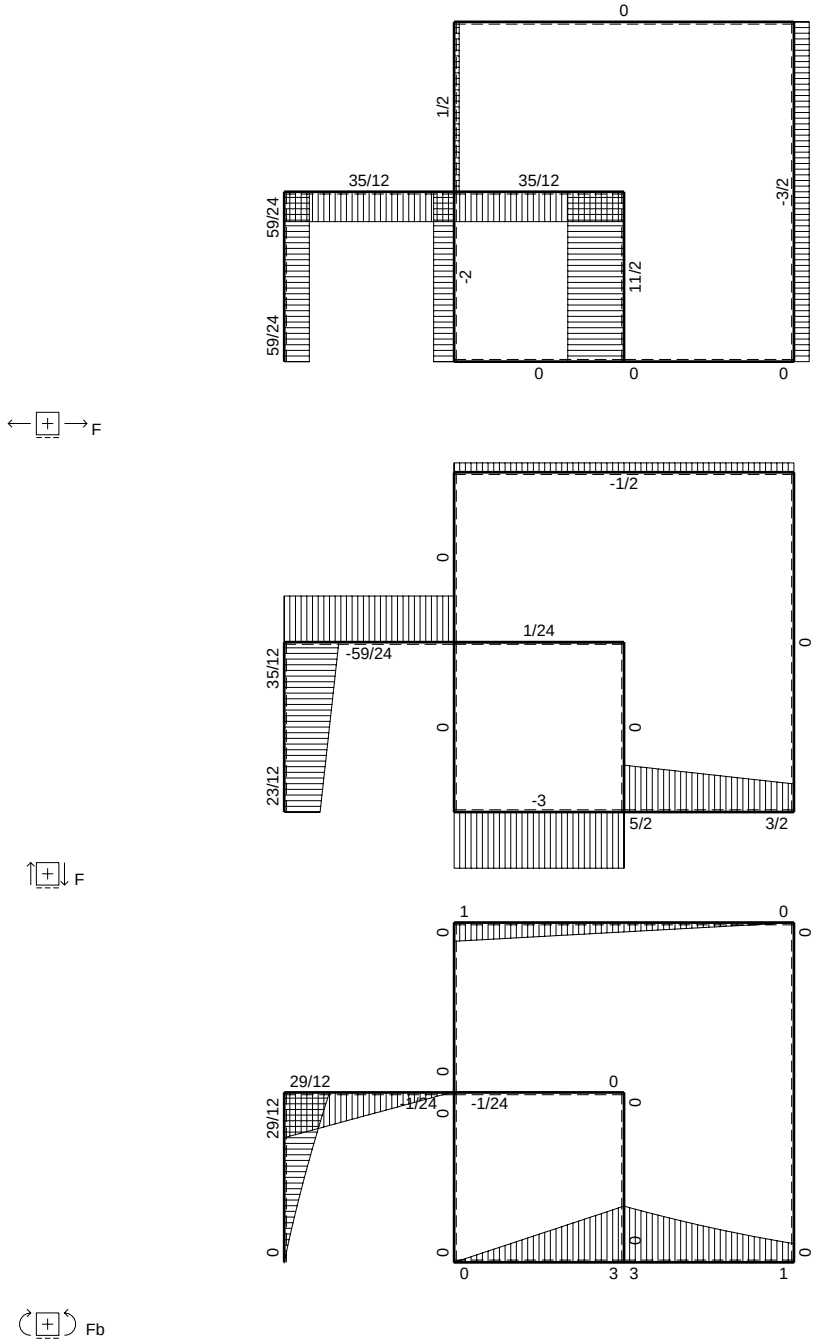

A = 456. $mm^2$
$J_u$ = 145554. $mm^4$
$J_v$ = 12168. $mm^4$
$y_g$ = 21.29 mm
$T_y$ = -1860. N
$M_x$ = 992000. Nmm
$x_m$ = 18. mm
$y_m$ = 56. mm
$u_m$ = 3. mm
$v_m$ = 34.71 mm
$\sigma_m = -Mv/J_u$ = -236.6 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 41. mm
$v_c$ = 19.71 mm
$\sigma_c = -Mv/J_u$ = -134.3 $N/mm^2$
$\tau_c$ = 5.216 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.6 $N/mm^2$
$S^*$ = 2449. $mm^3$

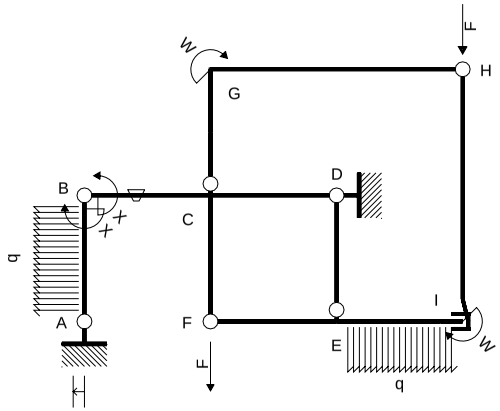
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H
I
3/2F
B
A
59/24F
29/12Fb
35/12F
59/24F
23/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
1/24F
3F
3Fb
35/12F
35/12F
1/24Fb
1/24F
Fb
1/2F
3F
G
C
F
G
C
1/2F
C
F
2F
C
59/24F
1/24Fb
35/12F
35/12F
29/12Fb
59/24F
B

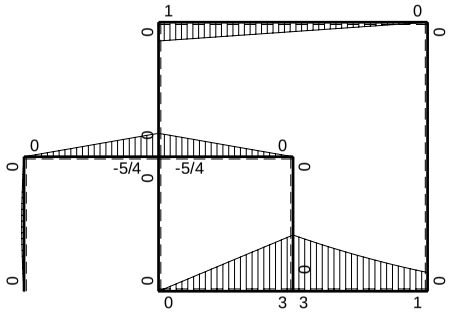




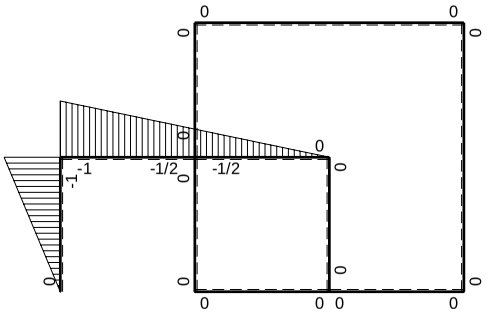

0
1/2
35/12
35/12
-3/2
59/24
-2
11/2
59/24
0
0
0
F
-1/2
0
1/24
35/12
-59/24
0
0
0
23/12
-3
5/2
3/2
F
1
0
0
0
0
29/12
0
29/12
1/24
-1/24
0
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 274656 CI01.xxxx.155

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PROCEDIMENTO E RISULTATI 274656 CI01.xxxx.155

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $29/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -29/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

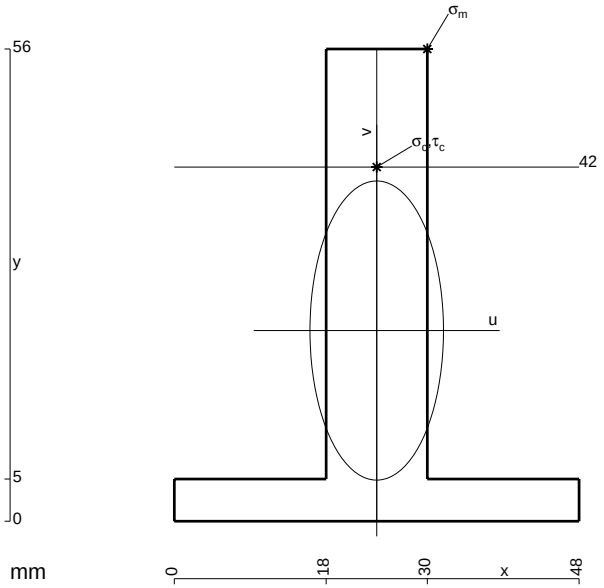
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

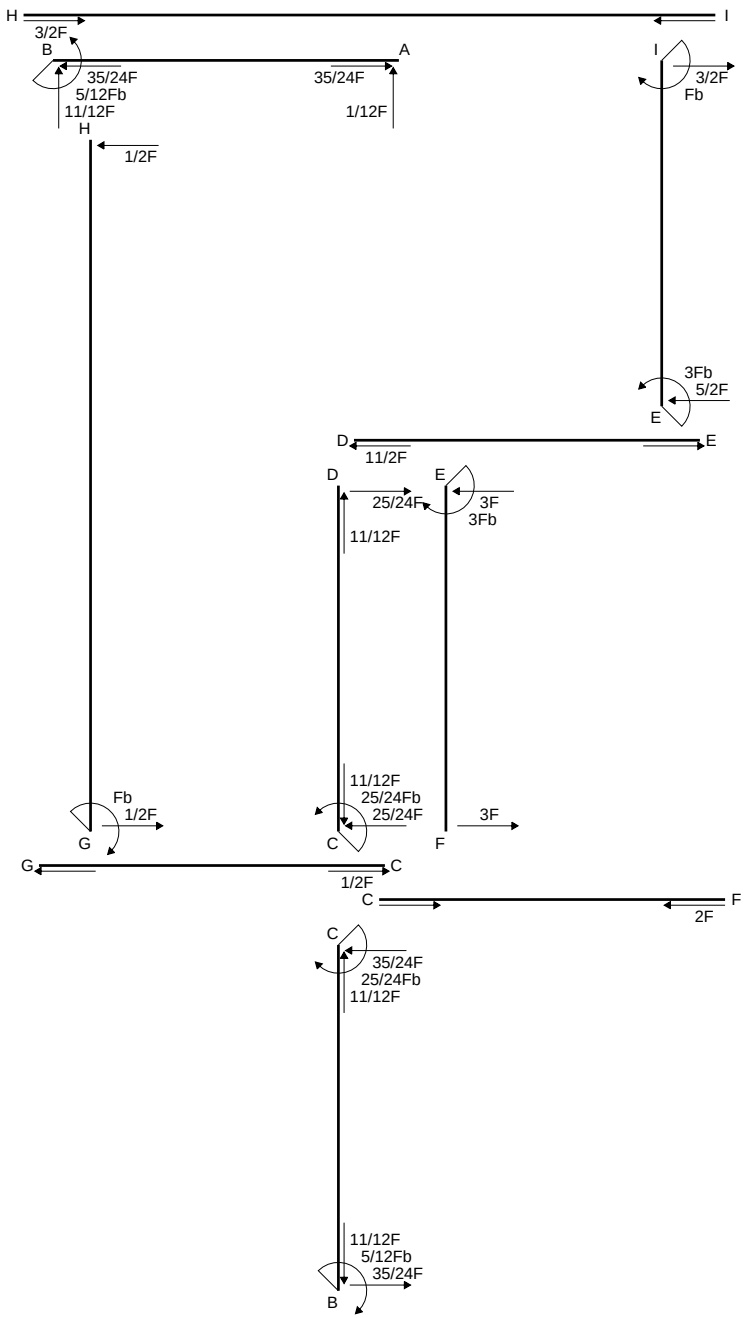
$= (5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





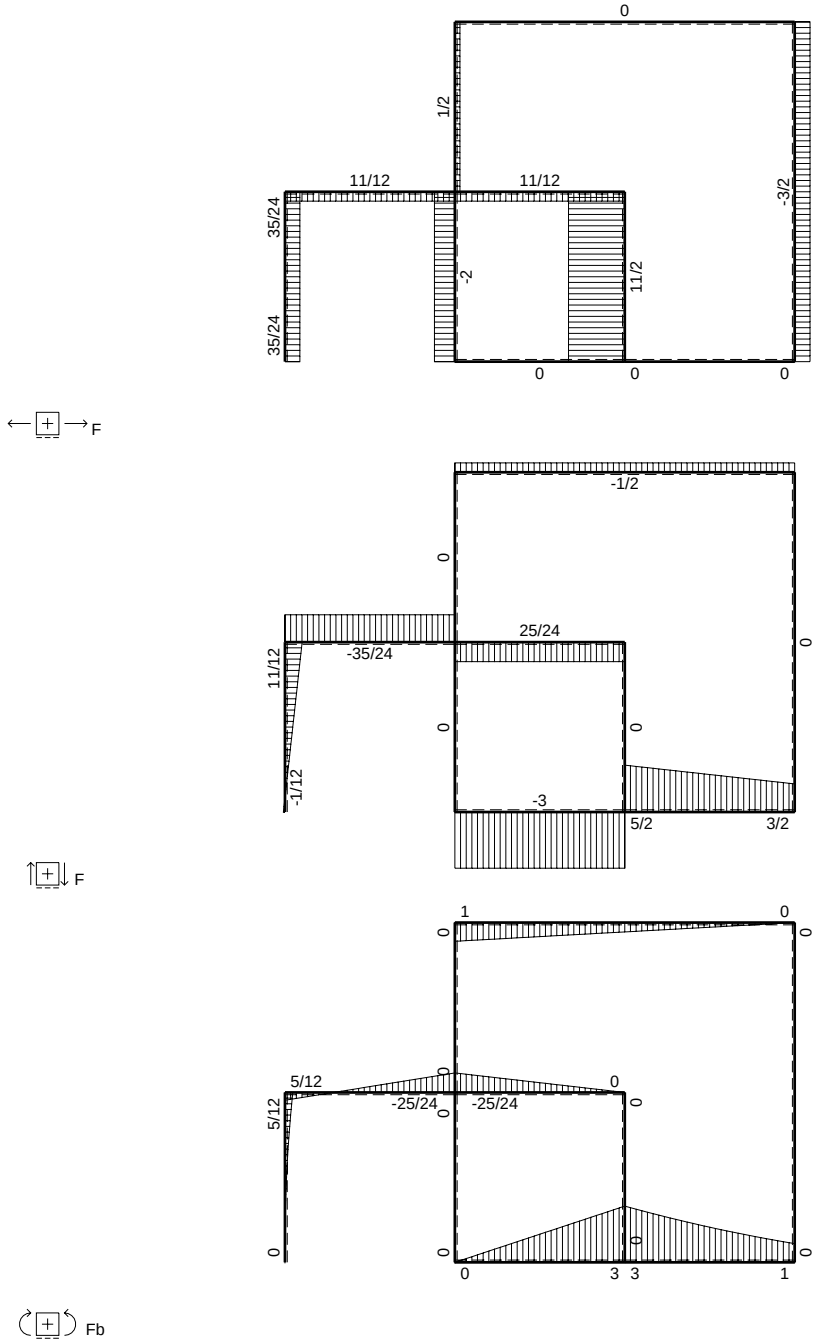


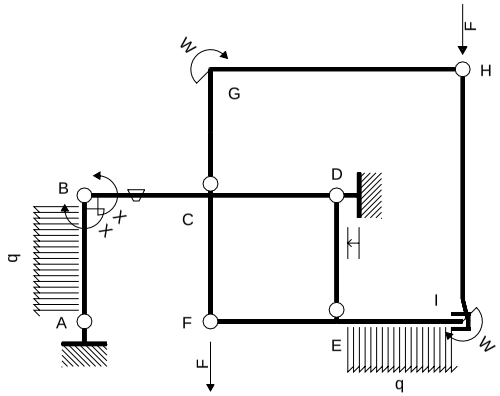

A = 852. $mm^2$
$J_u$ = 268308. $mm^4$
$J_v$ = 53424. $mm^4$
$y_g$ = 22.61 mm
$T_y$ = -2310. N
$M_x$ = 1593900. Nmm
$x_m$ = 30. mm
$y_m$ = 56. mm
$u_m$ = 6. mm
$v_m$ = 33.39 mm
$\sigma_m = -Mv/J_u$ = -198.3 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 42. mm
$v_c$ = 19.39 mm
$\sigma_c = -Mv/J_u$ = -115.2 $N/mm^2$
$\tau_c$ = 3.181 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.3 $N/mm^2$
$S^*$ = 4433. $mm^3$


H
I
3/2F
B
A
35/24F
5/12Fb
11/12F
35/24F
1/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
25/24F
3F
3Fb
11/12F
Fb
1/2F
11/12F
25/24Fb
25/24F
3F
G
C
F
G
C
1/2F
C
F
2F
C
35/24F
25/24Fb
11/12F
11/12F
5/12Fb
35/24F
B

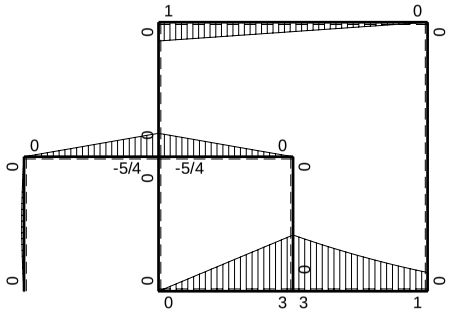




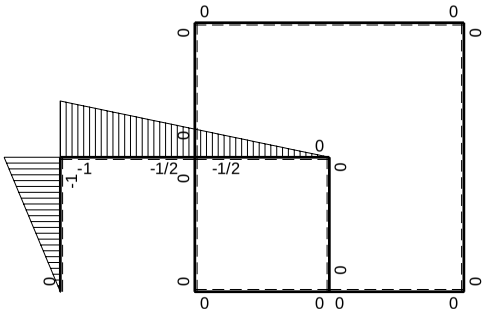

F
0
1/2
11/12
11/12
-3/2
35/24
-2
11/2
35/24
0
0
0
F
-1/2
0
25/24
-35/24
11/12
0
0
0
-1/12
-3
5/2
3/2
Fb
1
0
0
0
5/12
0
-25/24
-25/24
5/12
0
0
0
0
0
0
3
3
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 986254 CI01.xxxx.156

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $5/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/12Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 986254 CI01.xxxx.156

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

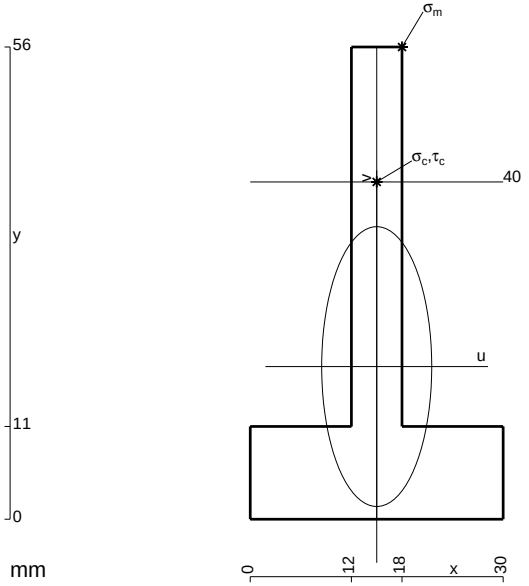
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

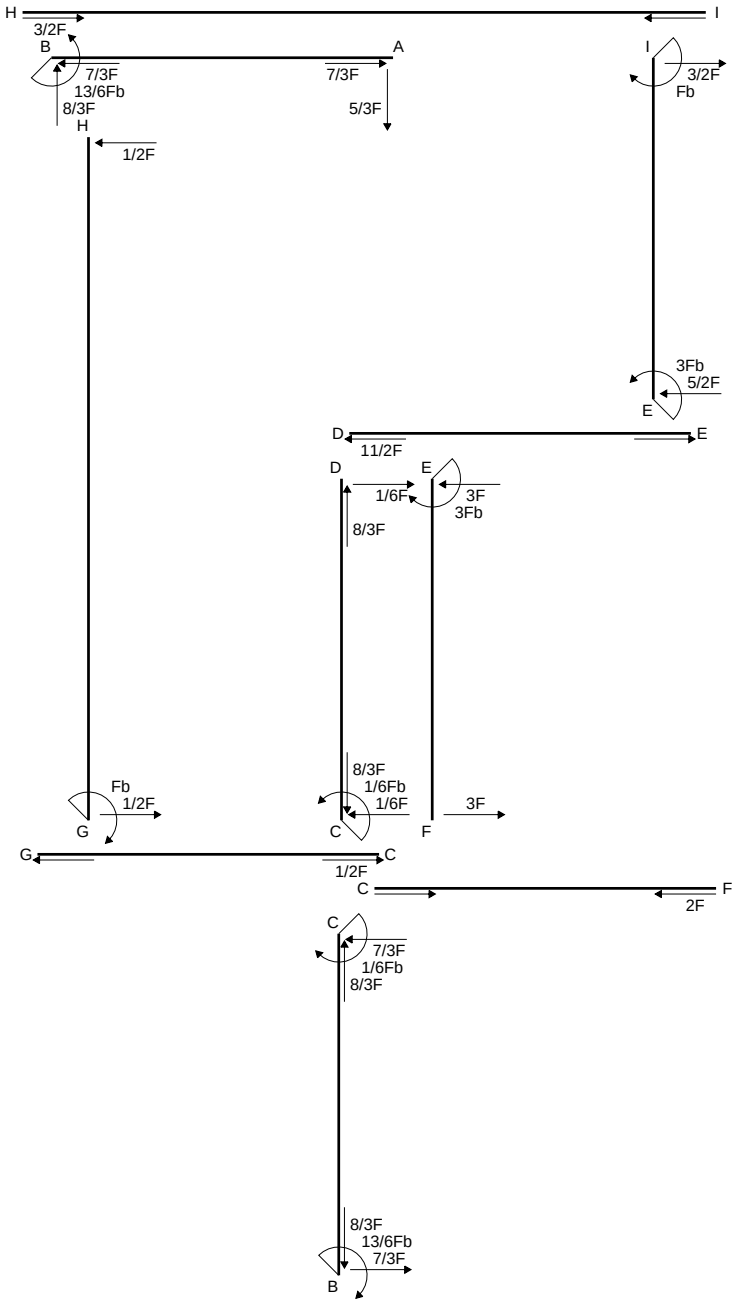
$= (5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





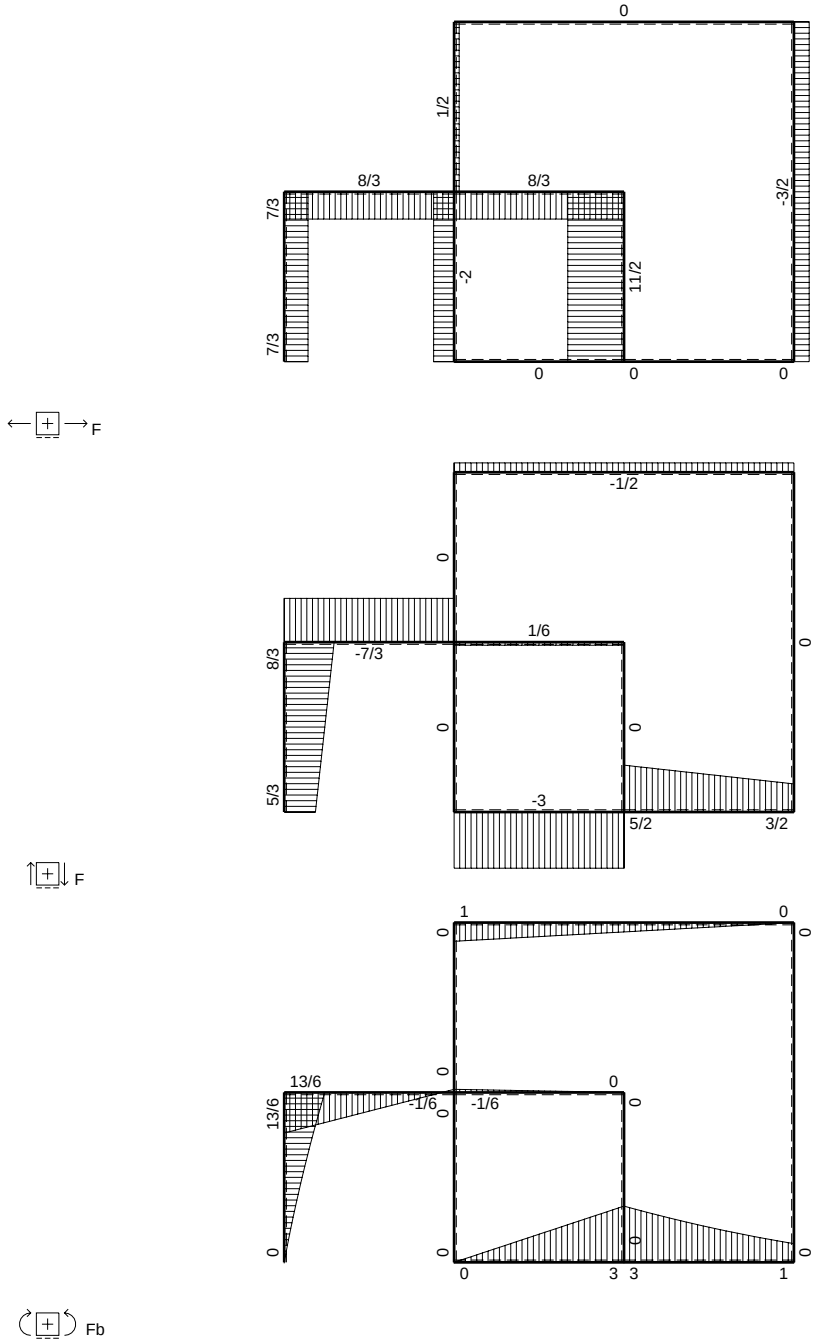


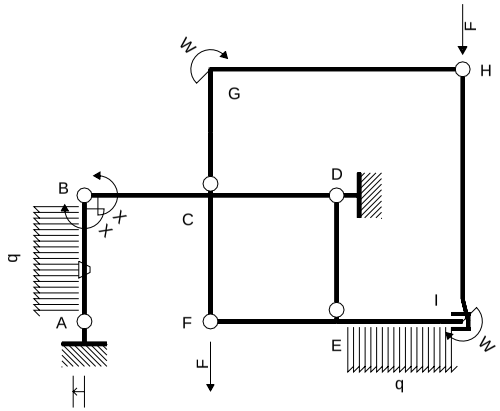

A = 600. $mm^2$
$J_u$ = 165314. $mm^4$
$J_v$ = 25560. $mm^4$
$y_g$ = 18.1 mm
$T_y$ = -1230. N
$M_x$ = 897900. Nmm
$x_m$ = 18. mm
$y_m$ = 56. mm
$u_m$ = 3. mm
$v_m$ = 37.9 mm
$\sigma_m = -Mv/J_u$ = -205.9 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 40. mm
$v_c$ = 21.9 mm
$\sigma_c = -Mv/J_u$ = -118.9 $N/mm^2$
$\tau_c$ = 3.559 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.1 $N/mm^2$
$S^*$ = 2870. $mm^3$


H
I
3/2F
B
A
7/3F
13/6Fb
8/3F
7/3F
5/3F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
1/6F
3F
3Fb
8/3F
Fb
1/2F
G
8/3F
1/6Fb
1/6F
C
F
3F
G
C
1/2F
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F
2F
C
7/3F
1/6Fb
8/3F
8/3F
13/6Fb
7/3F
B

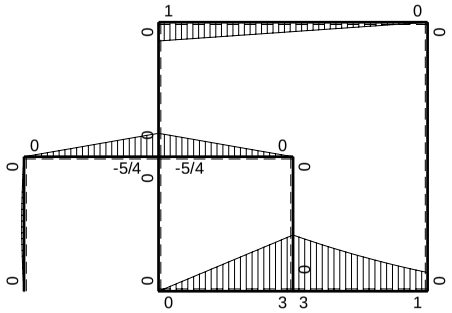




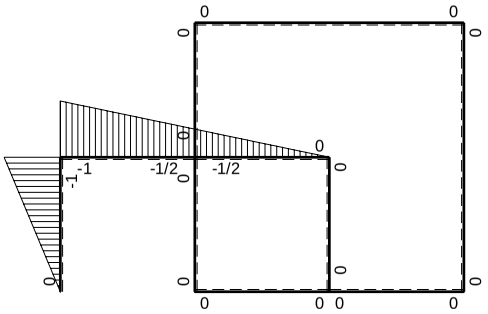

0
1/2
8/3
8/3
-3/2
7/3
-2
11/2
7/3
0
0
0
F
-1/2
0
1/6
8/3
-7/3
0
0
0
5/3
-3
5/2
3/2
F
1
0
0
0
0
13/6
0
13/6
-1/6
-1/6
0
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 218803 CI01.xxxx.157

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PROCEDIMENTO E RISULTATI 218803 CI01.xxxx.157

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $13/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -13/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

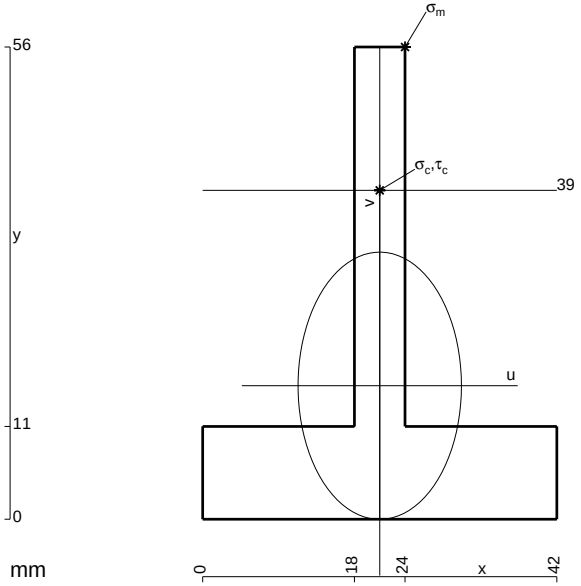
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

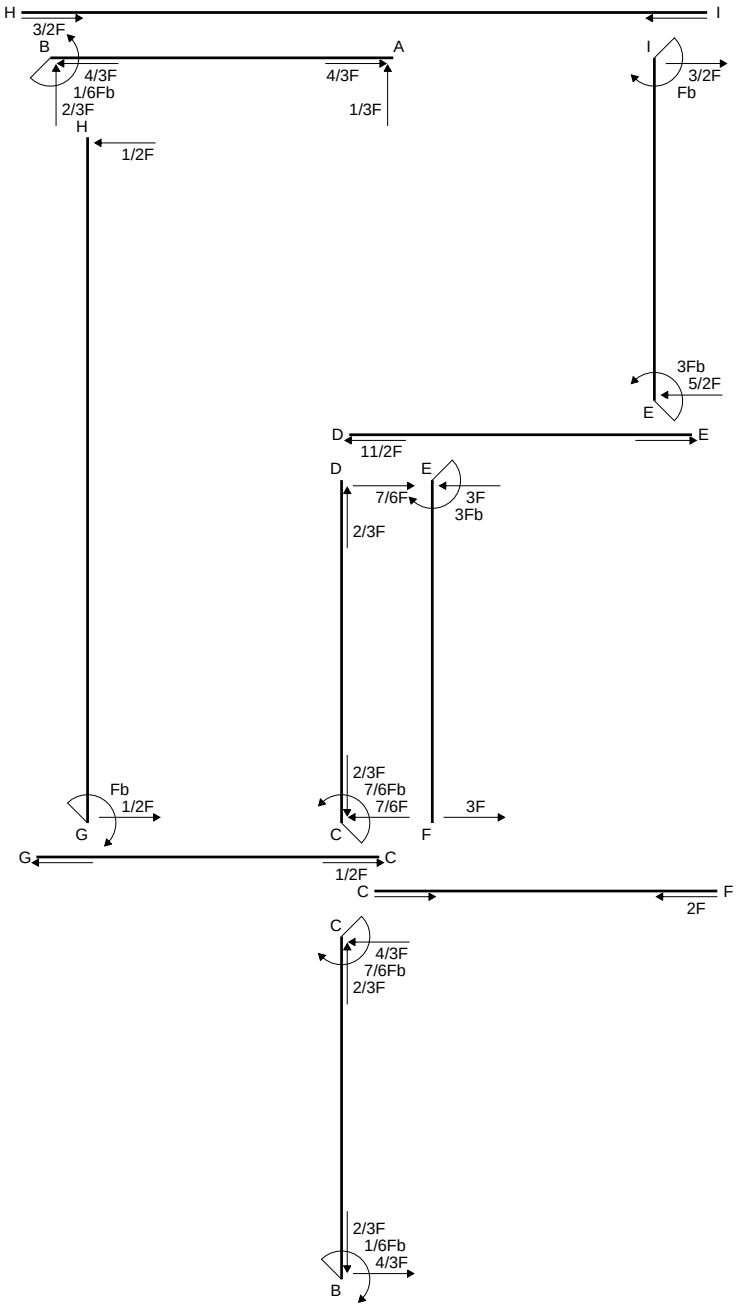


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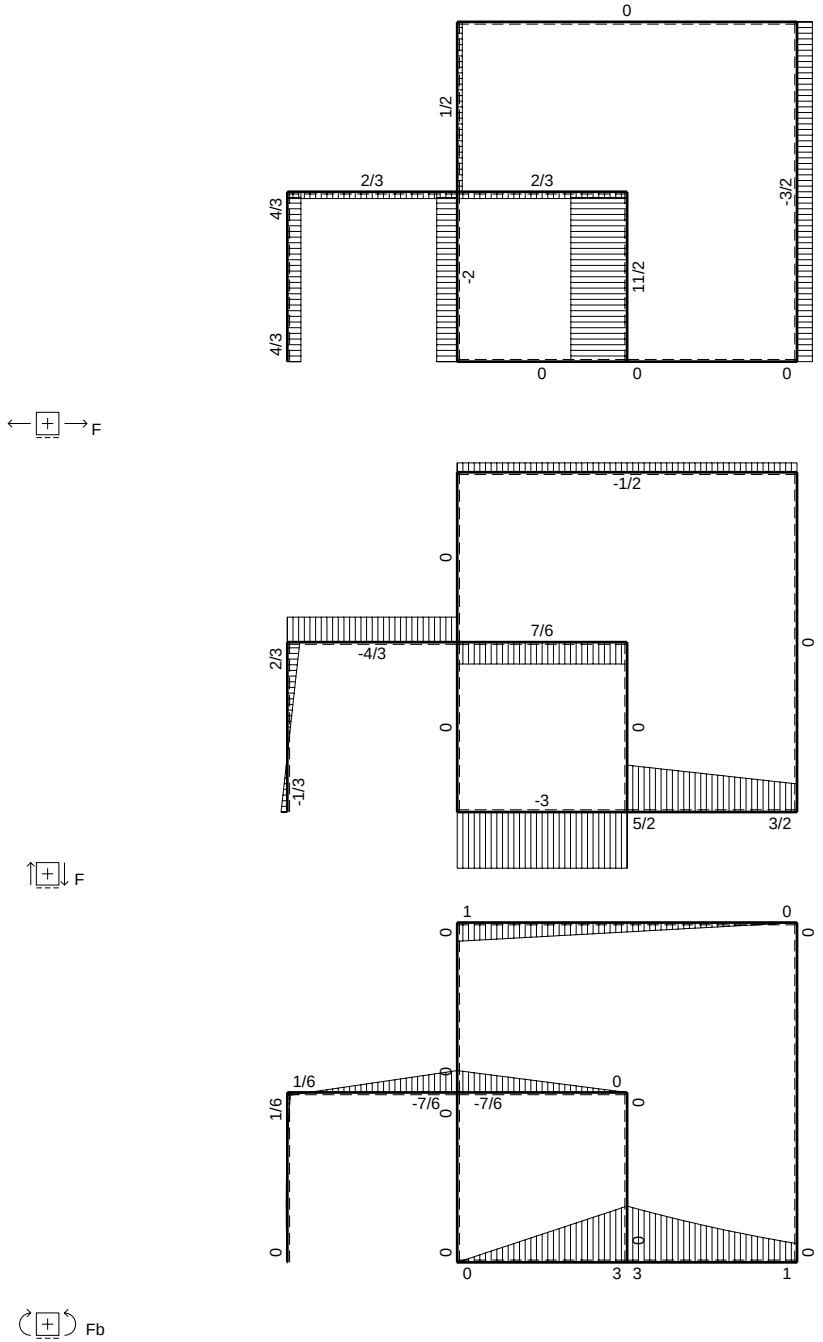

A = 732. $mm^2$
$J_u$ = 183822. $mm^4$
$J_v$ = 68724. $mm^4$
$y_g$ = 15.83 mm
$T_y$ = -1290. N
$M_x$ = 1006200. Nmm
$x_m$ = 24. mm
$y_m$ = 56. mm
$u_m$ = 3. mm
$v_m$ = 40.17 mm
$\sigma_m = -Mv/J_u$ = -219.9 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 39. mm
$v_c$ = 23.17 mm
$\sigma_c = -Mv/J_u$ = -126.8 $N/mm^2$
$\tau_c$ = 3.778 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 127. $N/mm^2$
$S^*$ = 3231. $mm^3$

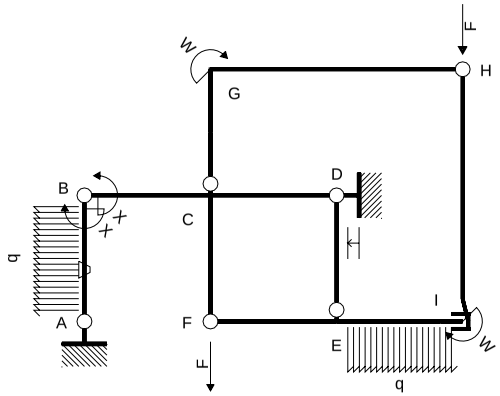
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H
3/2F
B
4/3F
1/6Fb
2/3F
A
4/3F
1/3F
I
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
7/6F
3F
3Fb
2/3F
Fb
1/2F
G
2/3F
7/6Fb
7/6F
C
F
3F
G
C
1/2F
C
F
2F
C
4/3F
7/6Fb
2/3F
2/3F
1/6Fb
4/3F
B

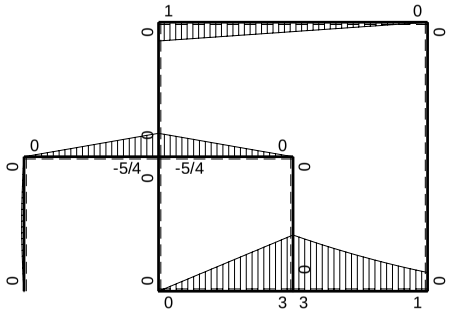




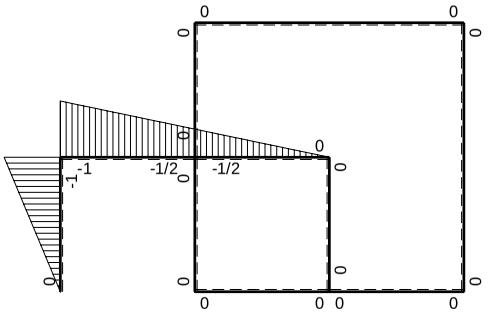

F
0
1/2
2/3
2/3
-3/2
4/3
-2
11/2
4/3
0
0
0
F
-1/2
0
7/6
2/3
-4/3
0
0
0
-1/3
-3
5/2
3/2
Fb
1
0
0
0
1/6
0
1/6
-7/6
-7/6
0
0
0
0
3
3
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 214984 CI01.xxxx.158

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | -Fb/EJ | $1/2Fx^2/b-1/2qx^3/b$ | Fx/EJ | $x^2/b^2$ | $(1/24+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | Fb/EJ | $1/2Fx-Fx^2/b+1/2qx^3/b$ | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/6Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/6Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

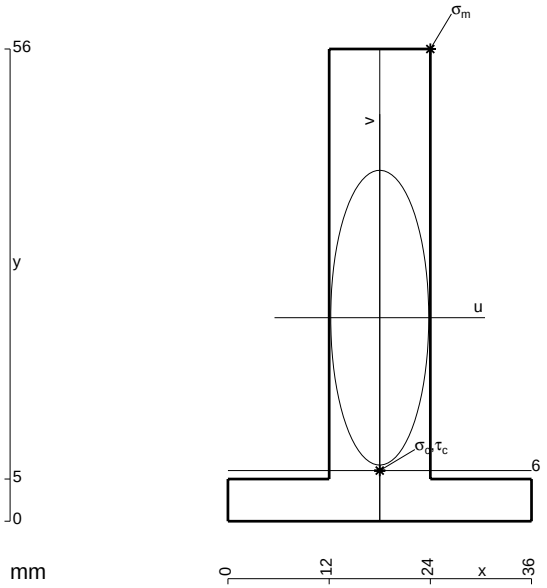
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

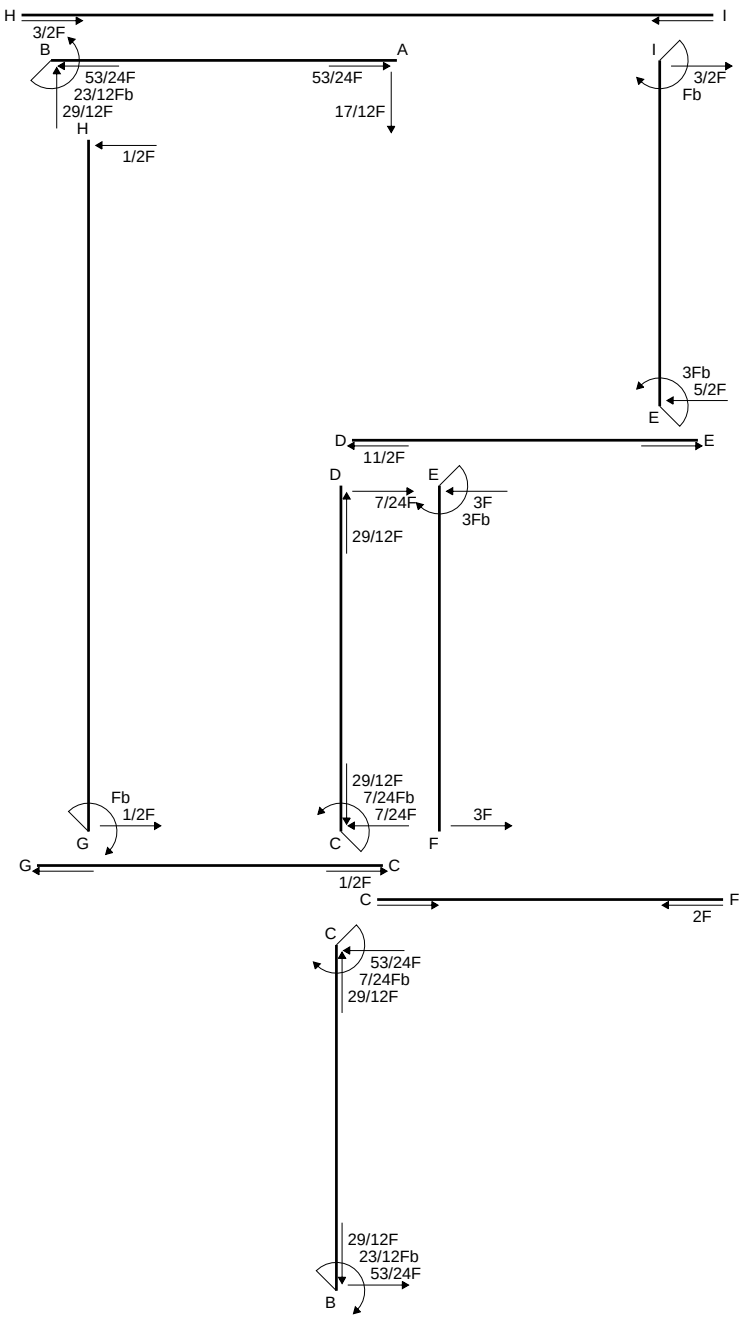
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 214984 CI01.xxxx.158

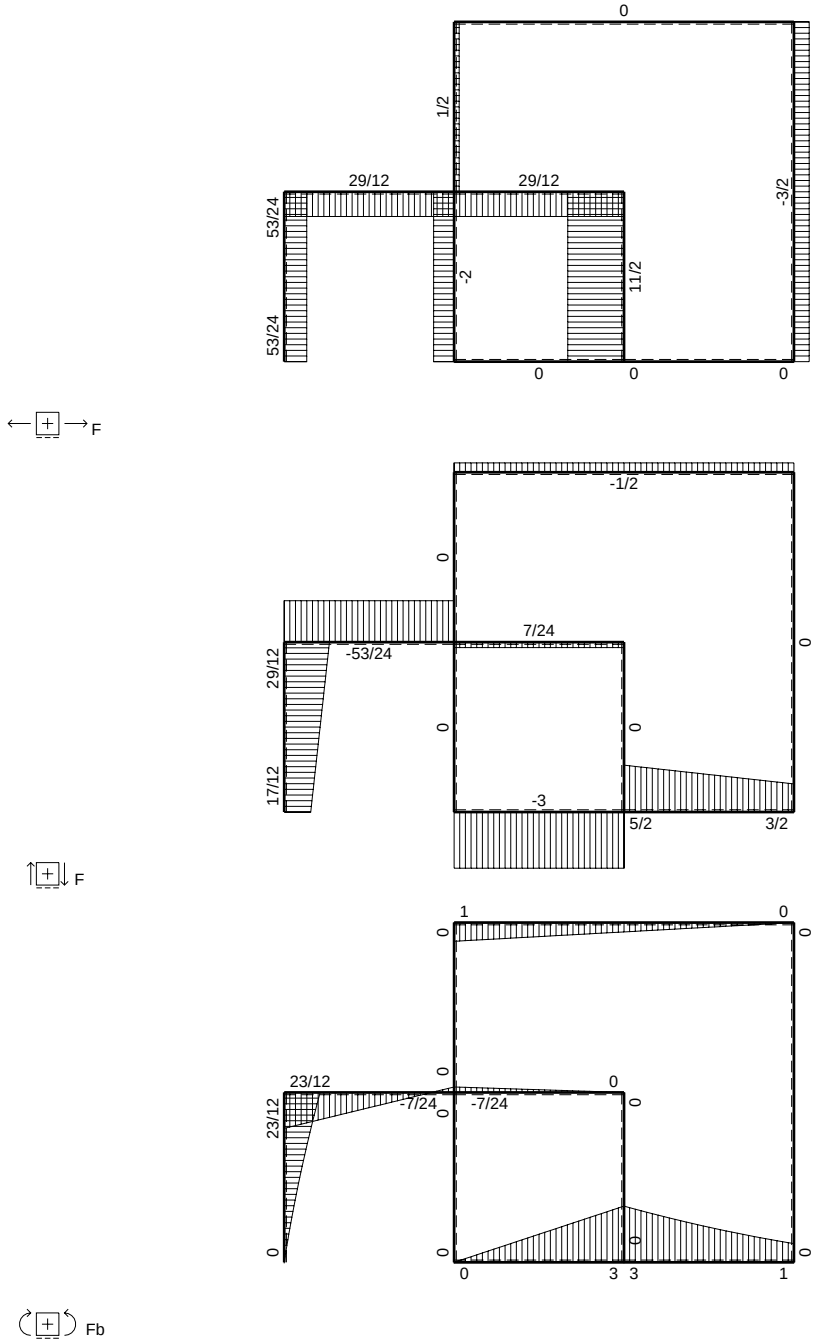


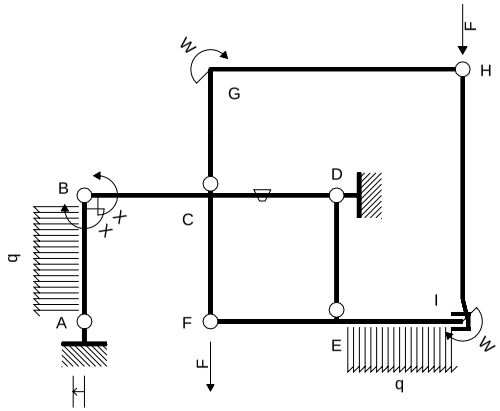

A = 792. $mm^2$
$J_u$ = 242073. $mm^4$
$J_v$ = 26784. $mm^4$
$y_g$ = 24.14 mm
$T_y$ = -2130. N
$M_x$ = 1746600. Nmm
$x_m$ = 24. mm
$y_m$ = 56. mm
$u_m$ = 6. mm
$v_m$ = 31.86 mm
$\sigma_m = -Mv/J_u$ = -229.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 6. mm
$v_c$ = -18.14 mm
$\sigma_c = -Mv/J_u$ = 130.9 $N/mm^2$
$\tau_c$ = 3.02 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 131. $N/mm^2$
$S^*$ = 4118. $mm^3$


H
I
3/2F
B
A
53/24F
23/12Fb
29/12F
53/24F
17/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
7/24F
3F
3Fb
29/12F
29/12F
7/24Fb
7/24F
Fb
1/2F
G
C
F
3F
G
C
1/2F
C
F
2F
C
53/24F
7/24Fb
29/12F
29/12F
23/12Fb
53/24F
B

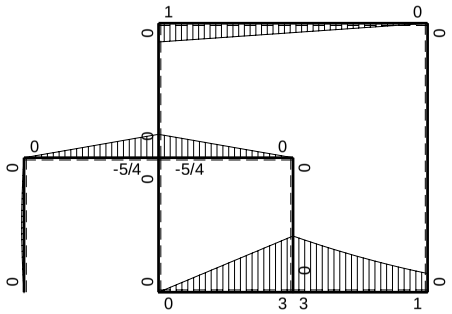




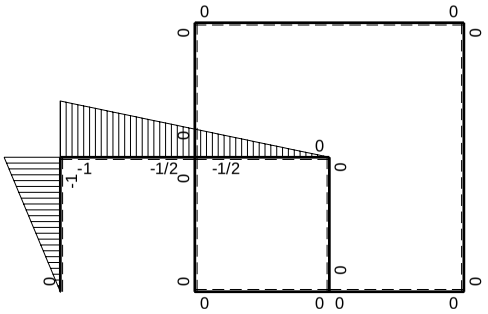

0
1/2
29/12
29/12
-3/2
53/24
-2
11/2
53/24
0
0
0
F
-1/2
0
7/24
29/12
-53/24
0
0
0
17/12
-3
5/2
3/2
F
1
0
0
0
0
23/12
0
23/12
-7/24
-7/24
0
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 269143 CI01.xxxx.159

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PROCEDIMENTO E RISULTATI 269143 CI01.xxxx.159

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $23/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -23/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

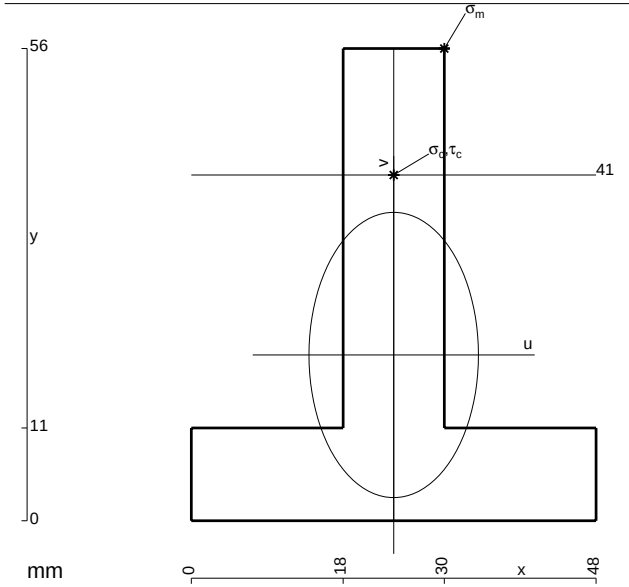
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/24\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 11/24\ Fb^2/EJ$

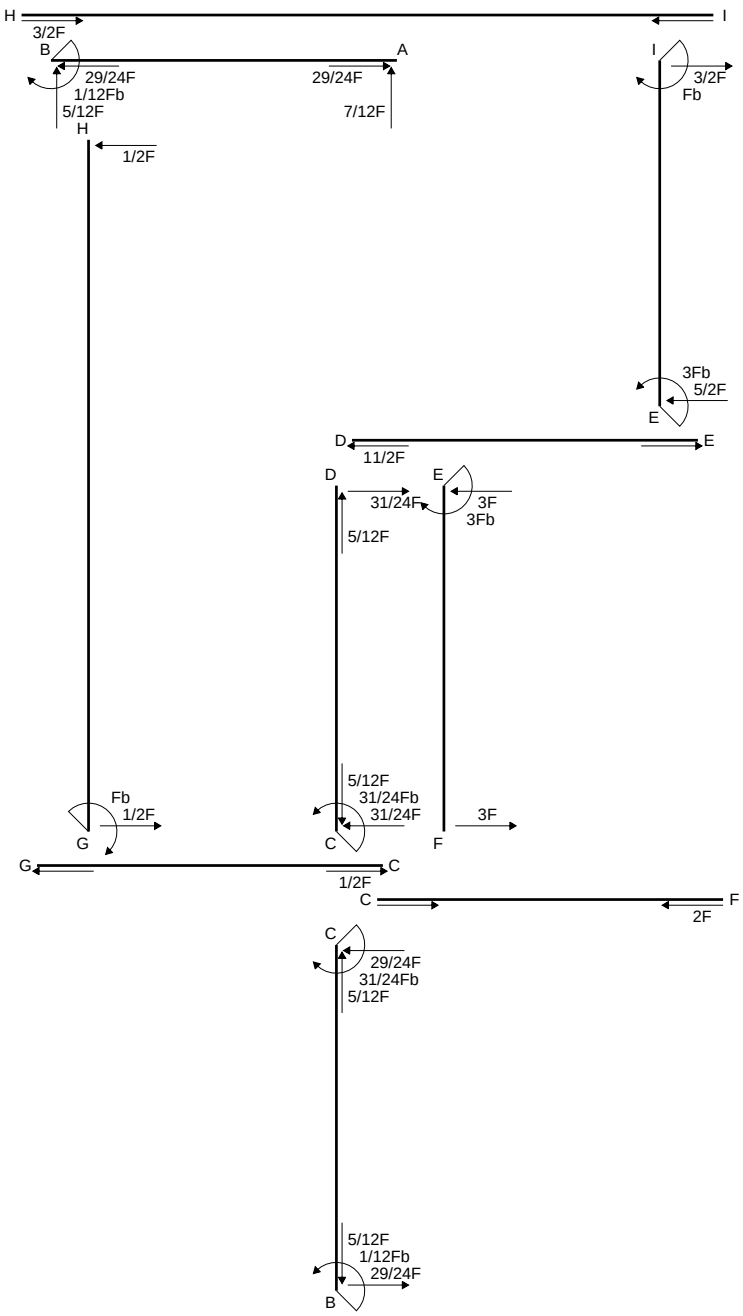


PROCEDIMENTO E RISULTATI 269143 CI01.xxxx.159



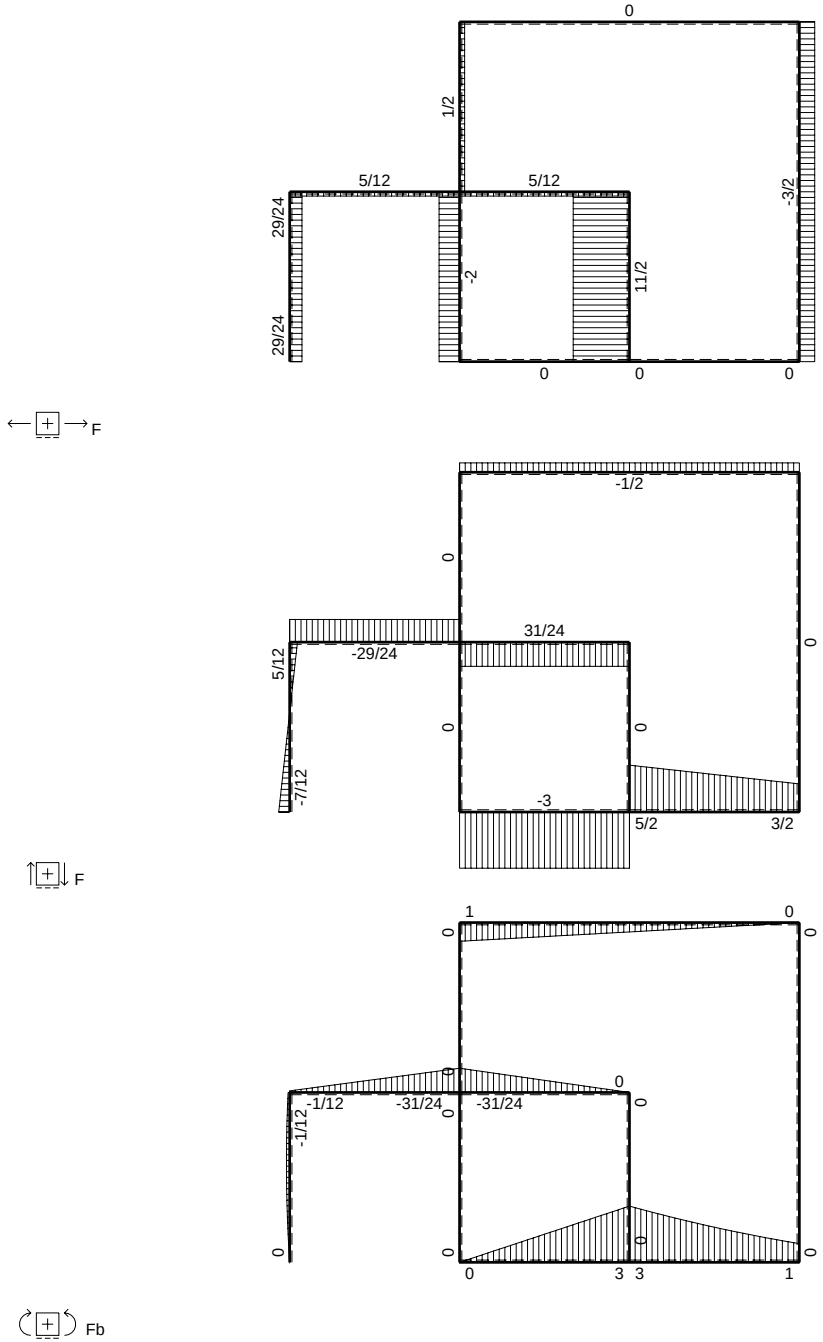

A = 1068. $mm^2$
$J_u$ = 305751. $mm^4$
$J_v$ = 107856. $mm^4$
$y_g$ = 19.66 mm
$T_y$ = -2310. N
$M_x$ = 2009700. Nmm
$x_m$ = 30. mm
$y_m$ = 56. mm
$u_m$ = 6. mm
$v_m$ = 36.34 mm
$\sigma_m = -Mv/J_u$ = -238.9 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 41. mm
$v_c$ = 21.34 mm
$\sigma_c = -Mv/J_u$ = -140.3 $N/mm^2$
$\tau_c$ = 3.269 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.4 $N/mm^2$
$S^*$ = 5192. $mm^3$

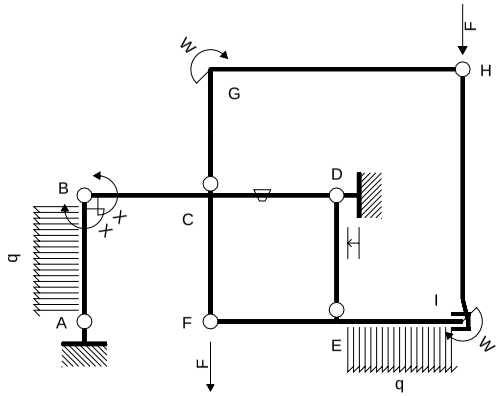
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H
I
3/2F
B
A
29/24F
1/12Fb
5/12F
29/24F
7/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
31/24F
3F
3Fb
5/12F
5/12F
31/24Fb
31/24F
Fb
1/2F
G
C
F
3F
G
C
1/2F
C
F
2F
C
29/24F
31/24Fb
5/12F
5/12F
1/12Fb
29/24F
B

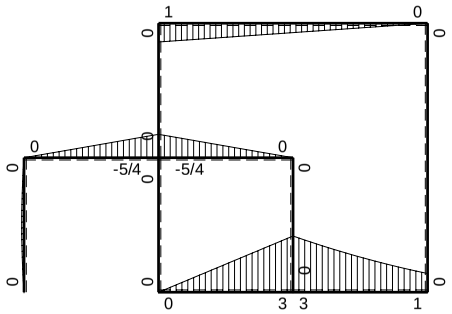




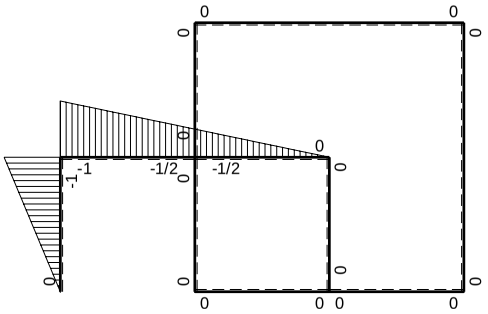

0
1/2
5/12
5/12
-3/2
29/24
-2
11/2
29/24
0
0
0
F
-1/2
0
31/24
-29/24
5/12
0
0
0
-3
-7/12
5/2
3/2
F
1
0
0
0
0
-1/12
-1/12
-31/24
-31/24
0
0
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 236686 CI01.xxxx.160

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | $-1/2Fx+1/2qx^2$ | 0 | $1/2Fx^2/b-1/2qx^3/b$ | 0 | $x^2/b^2$ | $(1/24+0)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | $1/2Fx-1/2qx^2$ | 0 | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-5/4Fb+5/4Fx$ | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | $1/2Fx/EJ$ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/12Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 236686 CI01.xxxx.160

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

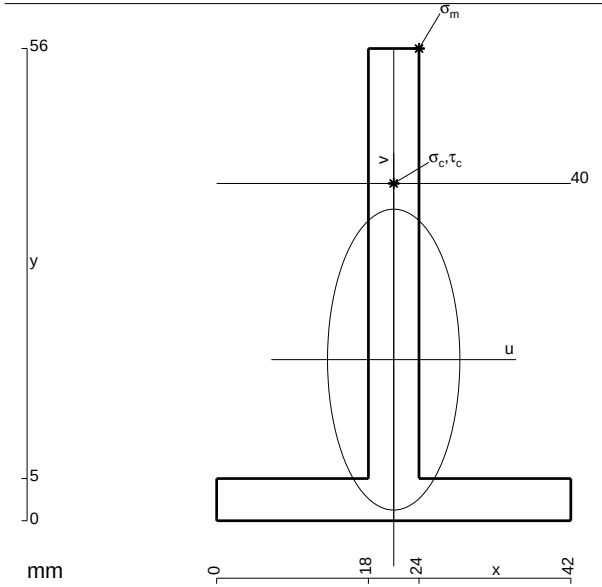
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/24\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 11/24\ Fb^2/EJ$

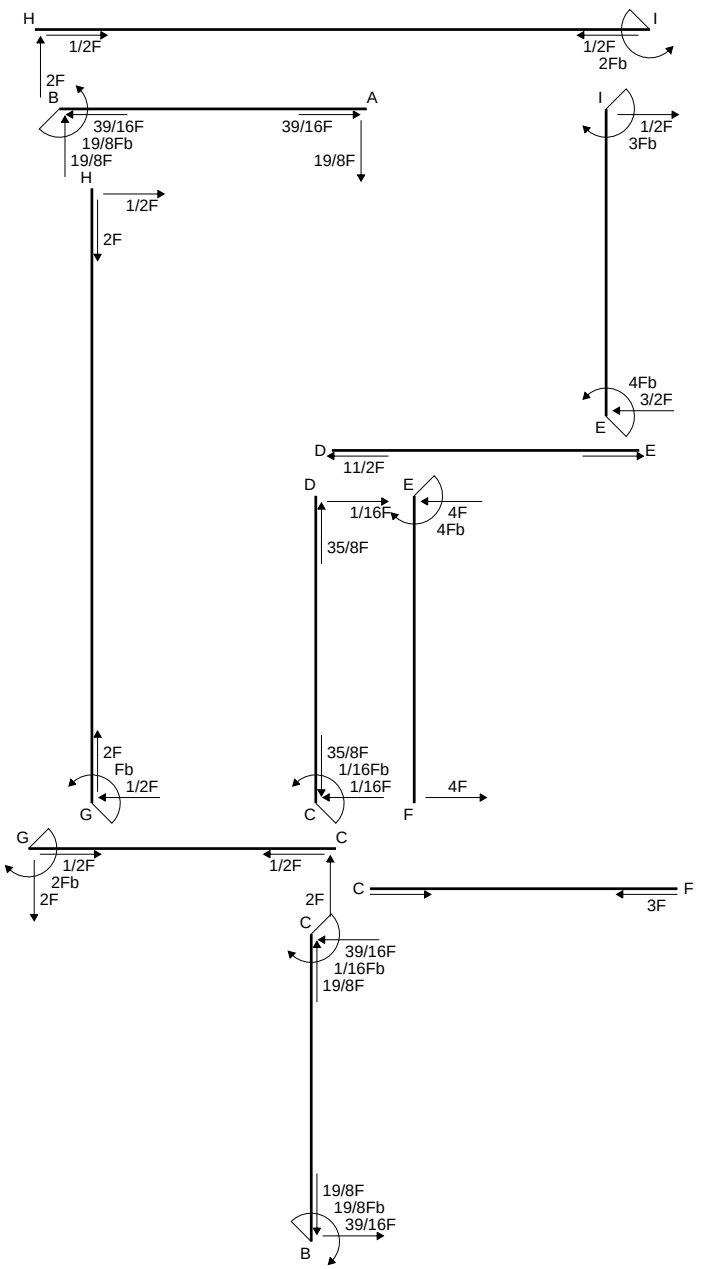


PROCEDIMENTO E RISULTATI 236686 CI01.xxxx.160



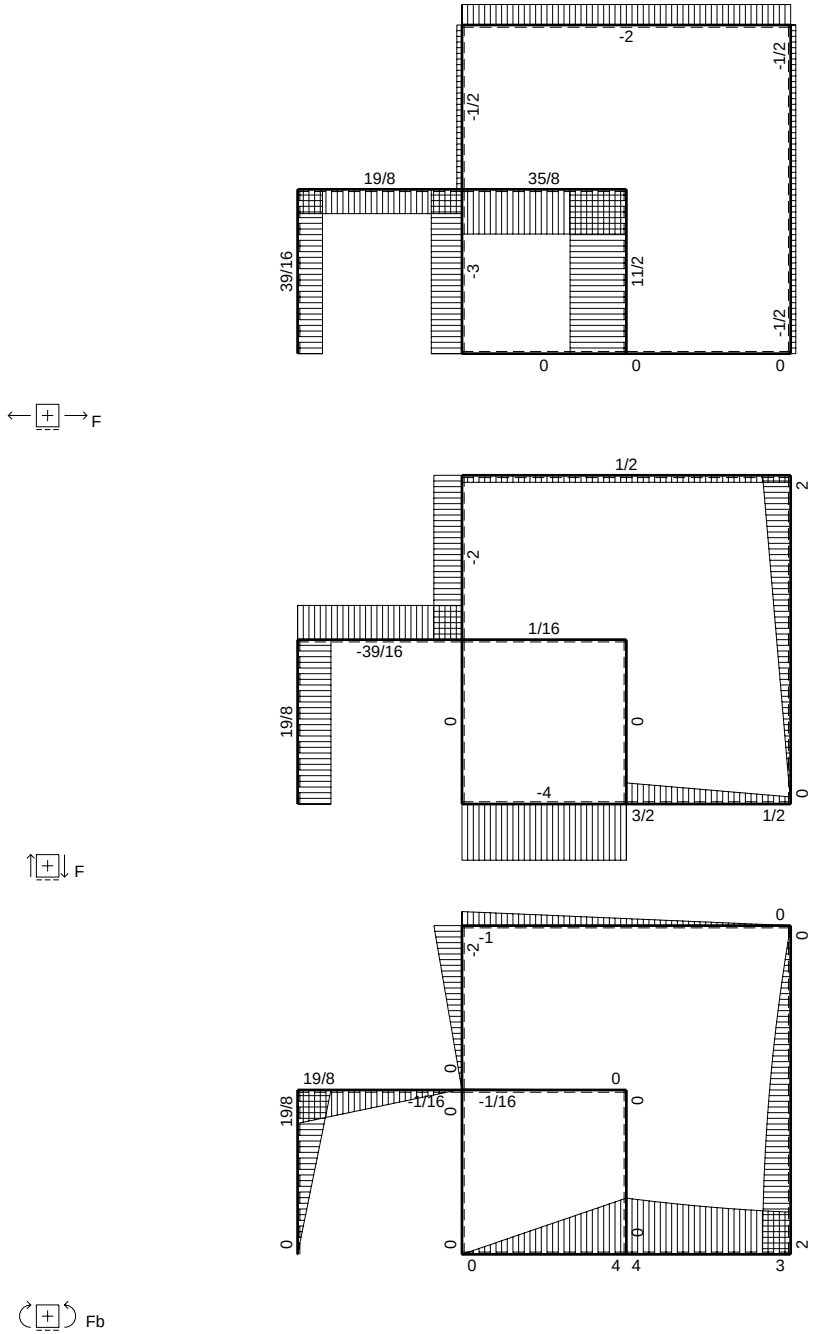

A = 516. $mm^2$
$J_u$ = 164398. $mm^4$
$J_v$ = 31788. $mm^4$
$y_g$ = 19.1 mm
$T_y$ = -1920. N
$M_x$ = 883200. Nmm
$x_m$ = 24. mm
$y_m$ = 56. mm
$u_m$ = 3. mm
$v_m$ = 36.9 mm
$\sigma_m = -Mv/J_u$ = -198.2 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 40. mm
$v_c$ = 20.9 mm
$\sigma_c = -Mv/J_u$ = -112.3 $N/mm^2$
$\tau_c$ = 5.399 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 112.6 $N/mm^2$
$S^*$ = 2774. $mm^3$

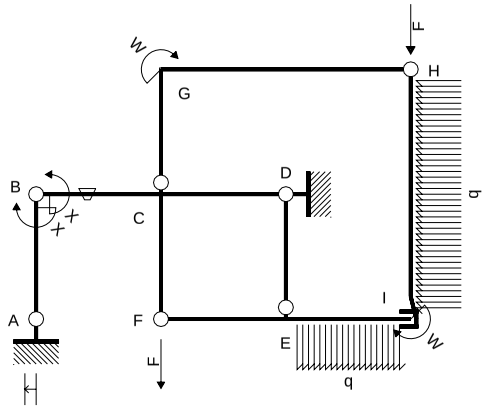
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H
I
1/2F
1/2F
2Fb
2F
B
A
39/16F
39/16F
19/8Fb
19/8F
19/8F
I
1/2F
3Fb
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
1/16F
4F
4Fb
35/8F
2F
Fb
1/2F
G
35/8F
1/16Fb
1/16F
C
4F
F
G
C
1/2F
1/2F
2Fb
2F
C
F
3F
2F
C
39/16F
1/16Fb
19/8F
19/8F
19/8Fb
39/16F
B

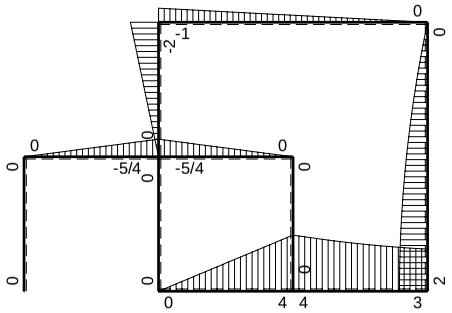




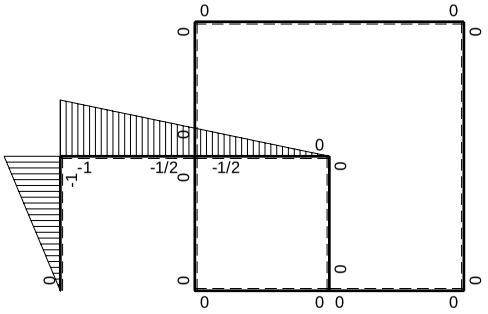

F
-2
-1/2
-1/2
19/8
35/8
39/16
-3
11/2
-1/2
0
0
0
F
1/2
2
-2
1/16
-39/16
19/8
0
0
0
-4
3/2
1/2
Fb
0
0
-2
-1
0
19/8
0
19/8
1/16
-1/16
0
0
0
0
0
0
4
4
3
2




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265627 CI01.xxxx.161

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $19/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -19/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 265627 CI01.xxxx.161

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

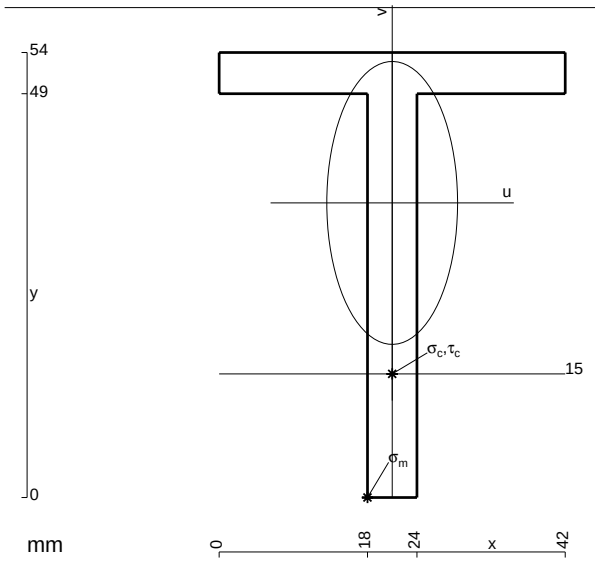
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

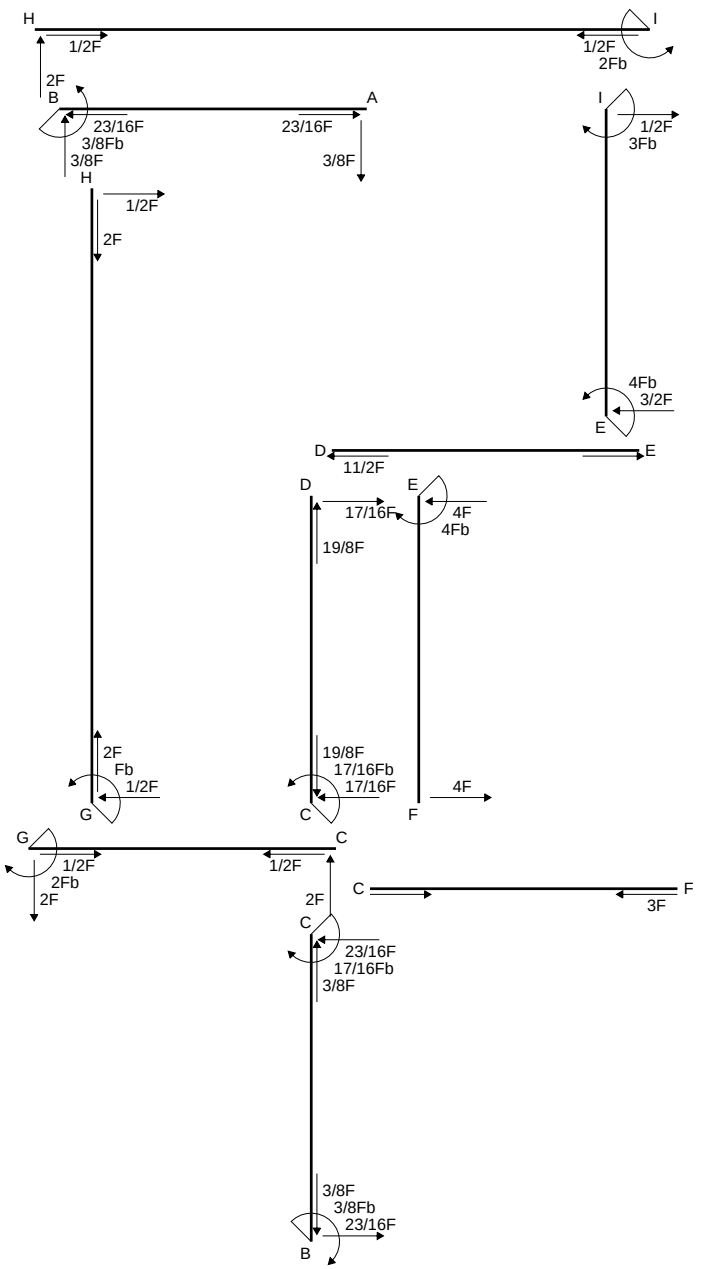
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$



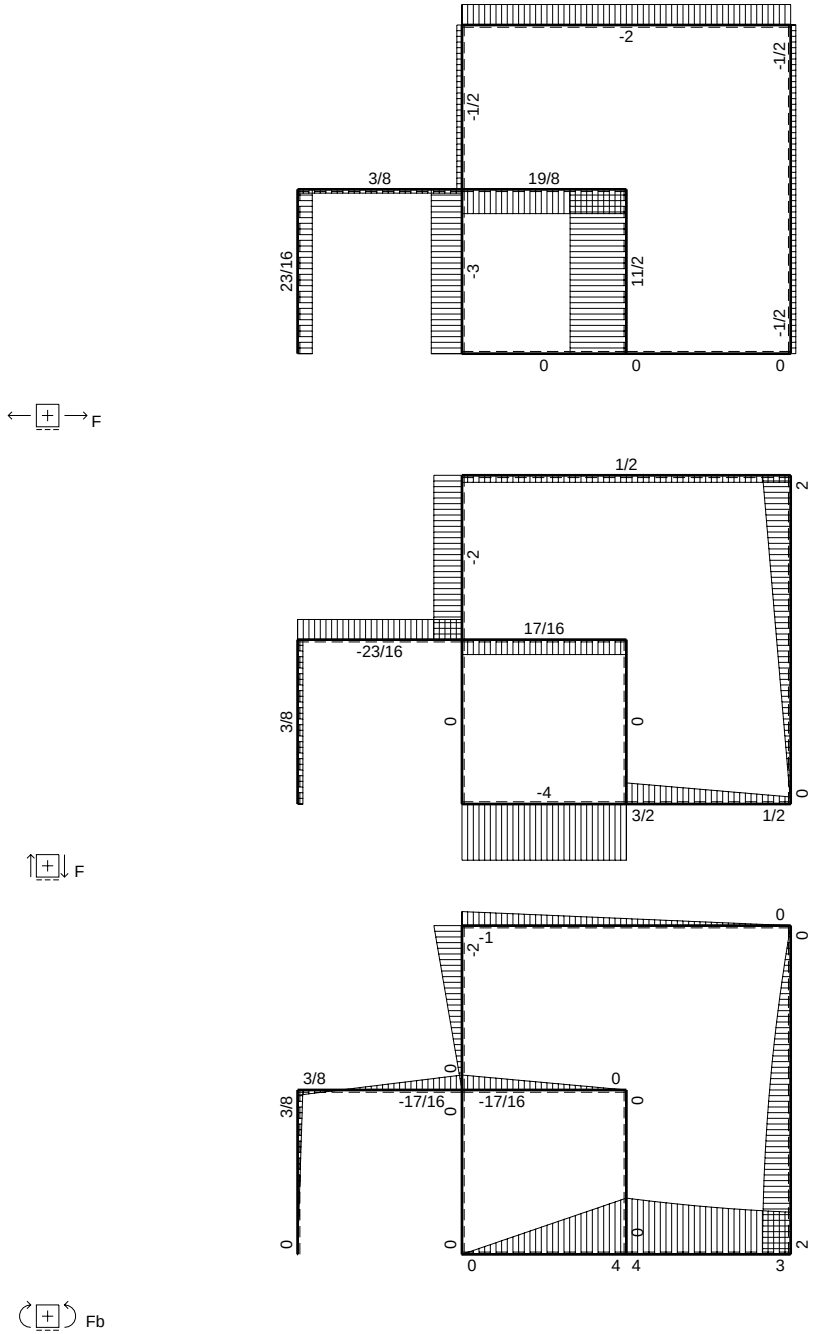





A = 504. $mm^2$
$J_u$ = 148565. $mm^4$
$J_v$ = 31752. $mm^4$
$y_g$ = 35.75 mm
$T_y$ = -1760. N
$M_x$ = 862400. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -35.75 mm
$\sigma_m = -Mv/J_u$ = 207.5 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 15. mm
$v_c$ = -20.75 mm
$\sigma_c = -Mv/J_u$ = 120.5 $N/mm^2$
$\tau_c$ = 5.02 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 120.8 $N/mm^2$
$S^*$ = 2543. $mm^3$

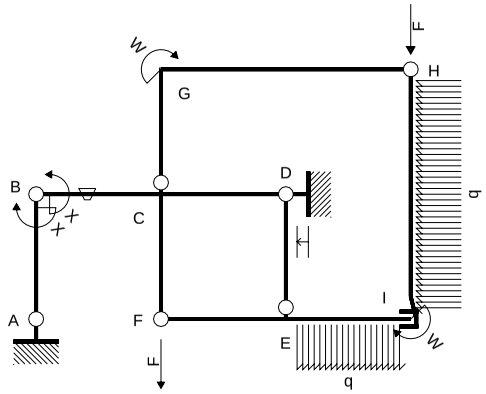




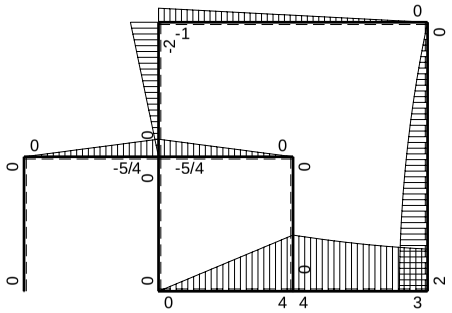





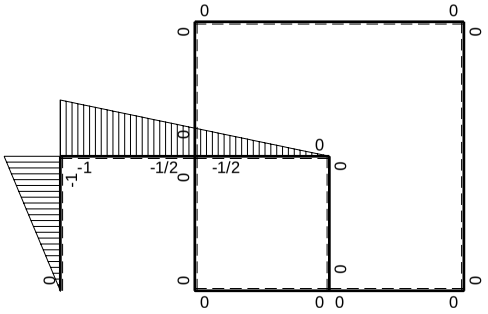

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 212930 CI01.xxxx.162

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $3/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -3/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 212930 CI01.xxxx.162

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

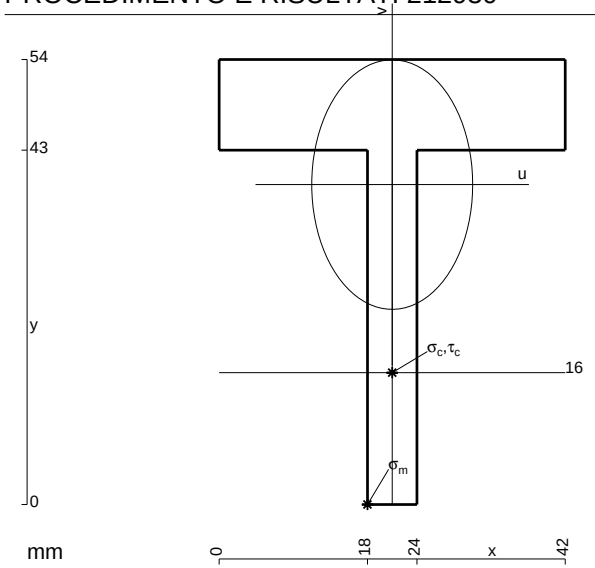
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

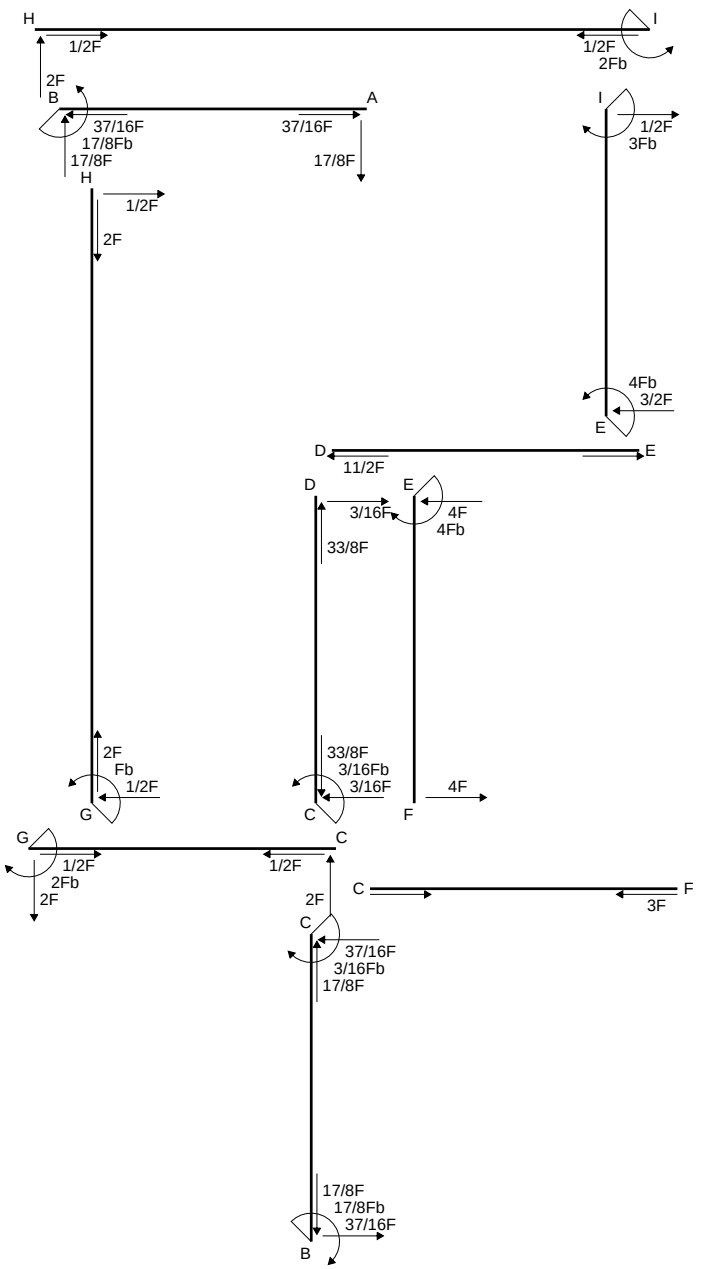
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





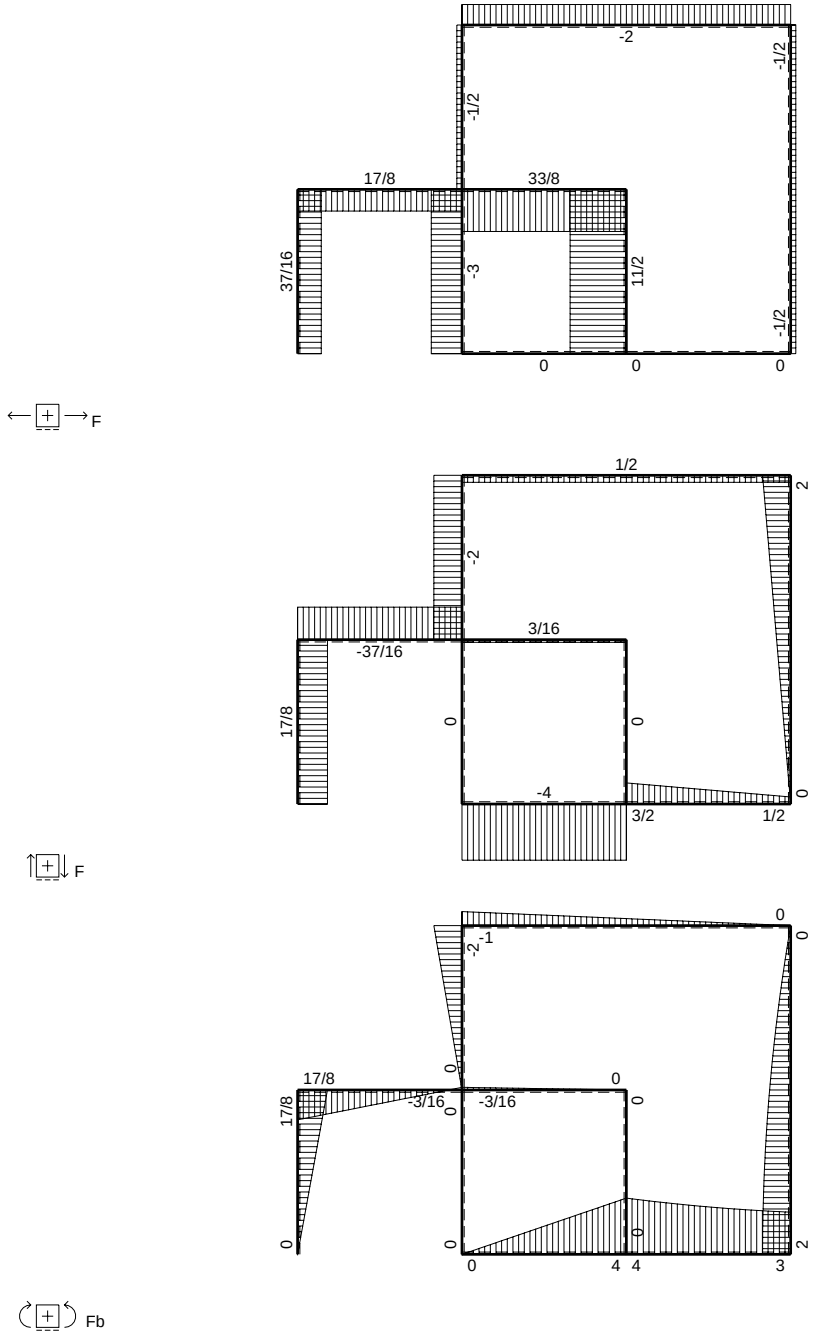


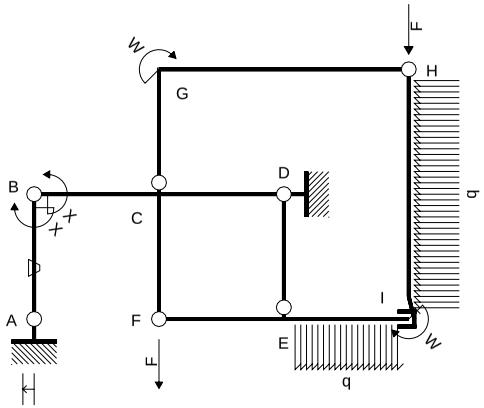

A = 720. $mm^2$
$J_u$ = 165098. $mm^4$
$J_v$ = 68688. $mm^4$
$y_g$ = 38.83 mm
$T_y$ = -1720. N
$M_x$ = 928800. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -38.83 mm
$\sigma_m = -Mv/J_u$ = 218.4 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -22.83 mm
$\sigma_c = -Mv/J_u$ = 128.4 $N/mm^2$
$\tau_c$ = 5.138 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 128.7 $N/mm^2$
$S^*$ = 2959. $mm^3$


H
1/2F
I
1/2F
2Fb
2F
B
A
37/16F
17/8Fb
17/8F
37/16F
17/8F
I
1/2F
3Fb
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
3/16F
4F
4Fb
33/8F
2F
Fb
1/2F
G
33/8F
3/16Fb
3/16F
C
4F
F
G
C
1/2F
2Fb
2F
1/2F
C
F
3F
2F
C
37/16F
3/16Fb
17/8F
17/8F
17/8Fb
37/16F
B

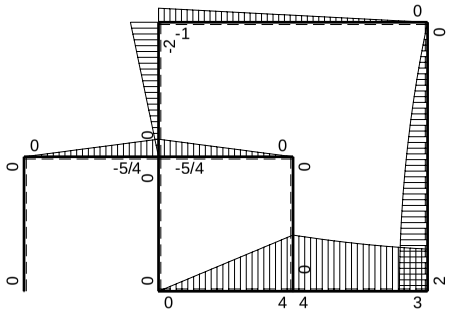




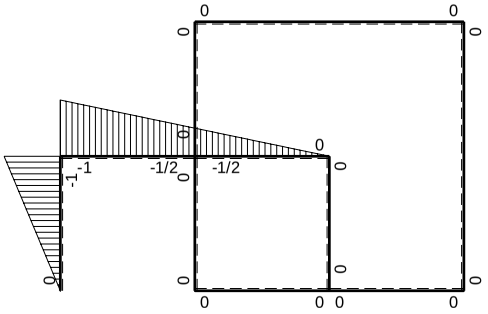

-2
-1/2
-1/2
17/8
33/8
37/16
-3
11/2
-1/2
0
0
0
F
1/2
2
-2
3/16
-37/16
17/8
0
0
0
-4
3/2
1/2
F
0
0
-2
-1
0
17/8
0
-3/16
-3/16
17/8
0
0
0
0
0
0
4
4
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264603 CI01.xxxx.163

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 264603 CI01.xxxx.163

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-5/4Fx$ | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-5/4Fb+5/4Fx$ | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | $5/4Fx$ | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+1/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $17/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -17/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

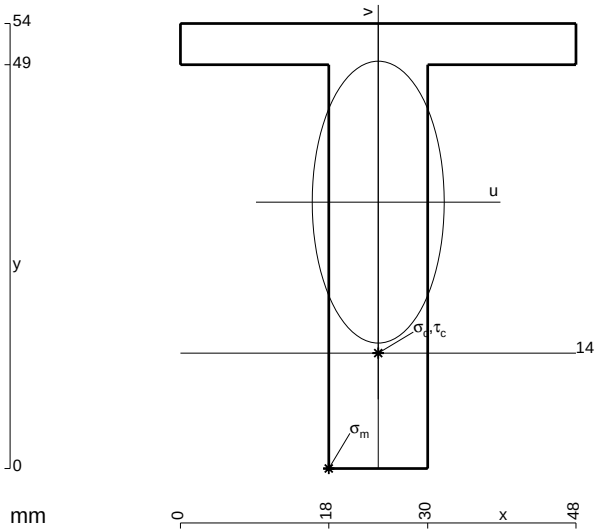
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

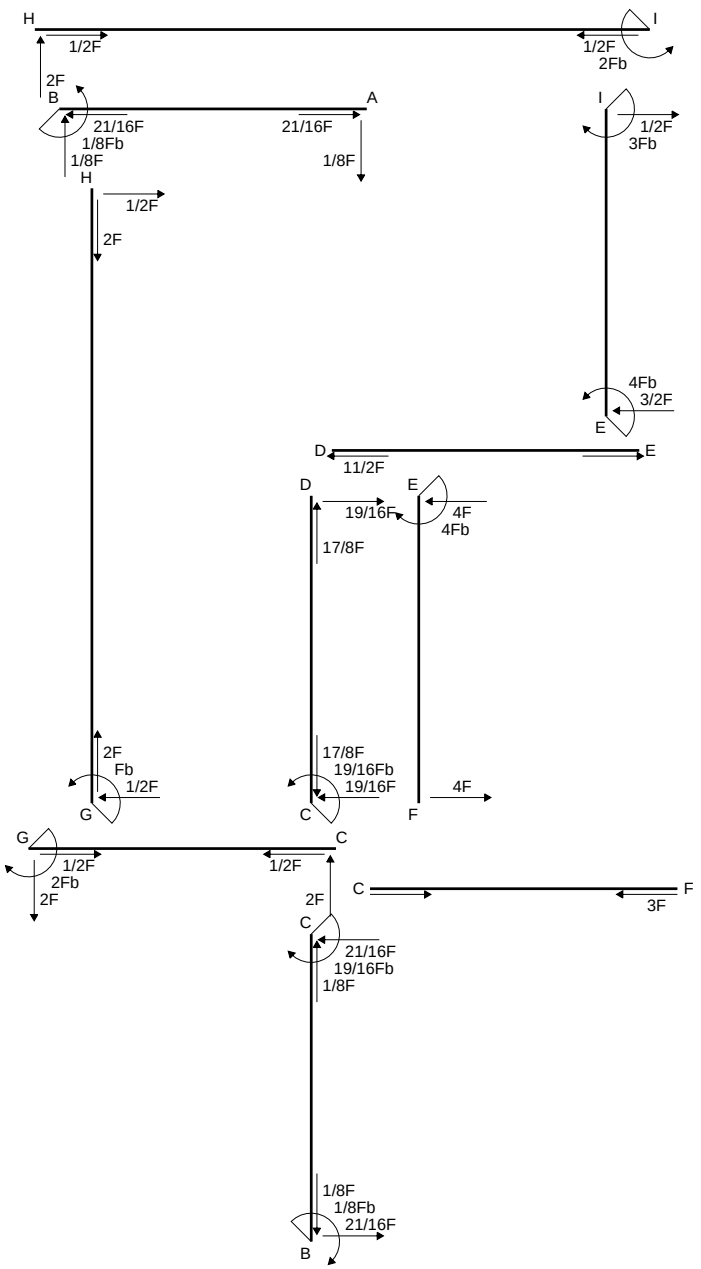
$= (5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





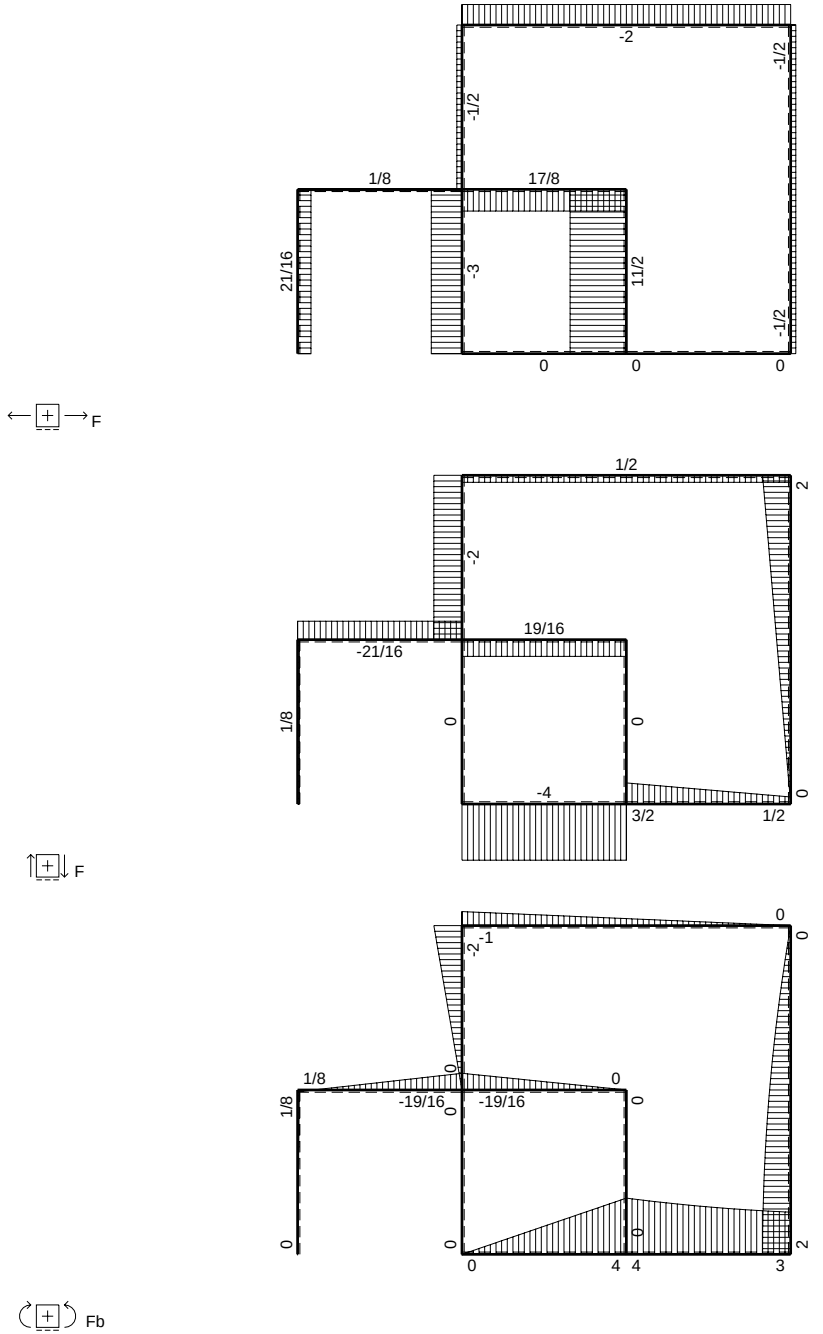


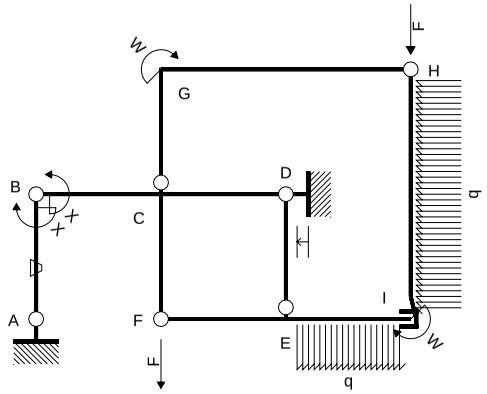

A = 828. $mm^2$
$J_u$ = 242396. $mm^4$
$J_v$ = 53136. $mm^4$
$y_g$ = 32.33 mm
$T_y$ = -2960. N
$M_x$ = 1716800. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -32.33 mm
$\sigma_m = -Mv/J_u$ = 229. $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 14. mm
$v_c$ = -18.33 mm
$\sigma_c = -Mv/J_u$ = 129.8 $N/mm^2$
$\tau_c$ = 4.33 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130. $N/mm^2$
$S^*$ = 4255. $mm^3$


H
1/2F
I
1/2F
2Fb
2F
B
A
21/16F
1/8Fb
1/8F
21/16F
1/8F
I
1/2F
3Fb
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
19/16F
4F
4Fb
17/8F
2F
Fb
1/2F
G
17/8F
19/16Fb
19/16F
C
4F
F
G
C
1/2F
2Fb
2F
1/2F
C
F
3F
2F
C
21/16F
19/16Fb
1/8F
1/8F
1/8Fb
21/16F
B

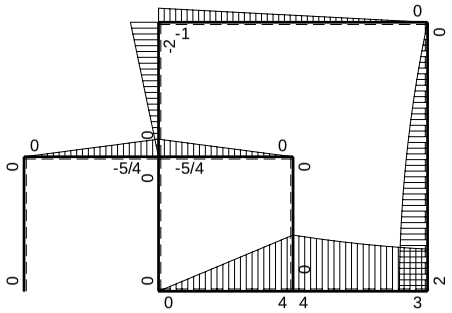




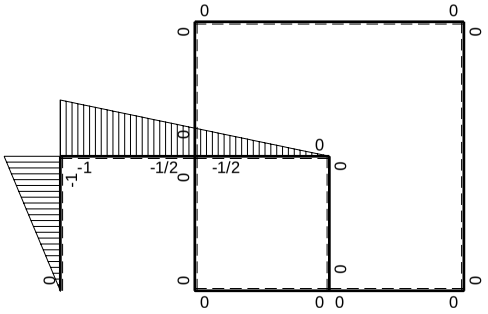

F
-2
-1/2
-1/2
1/8
17/8
21/16
-3
11/2
-1/2
0
0
0
F
1/2
2
-2
19/16
-21/16
1/8
0
0
0
-4
3/2
1/2
F
0
0
-2
-1
1/8
0
0
-19/16
-19/16
1/8
0
0
0
0
0
0
4
4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264698 CI01.xxxx.164

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 264698 CI01.xxxx.164

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-5/4Fx$ | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-5/4Fb+5/4Fx$ | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $-Fb+1/2Fx$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

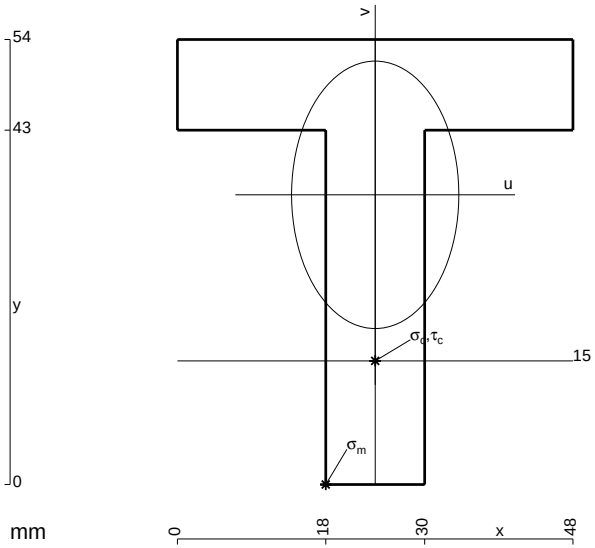
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

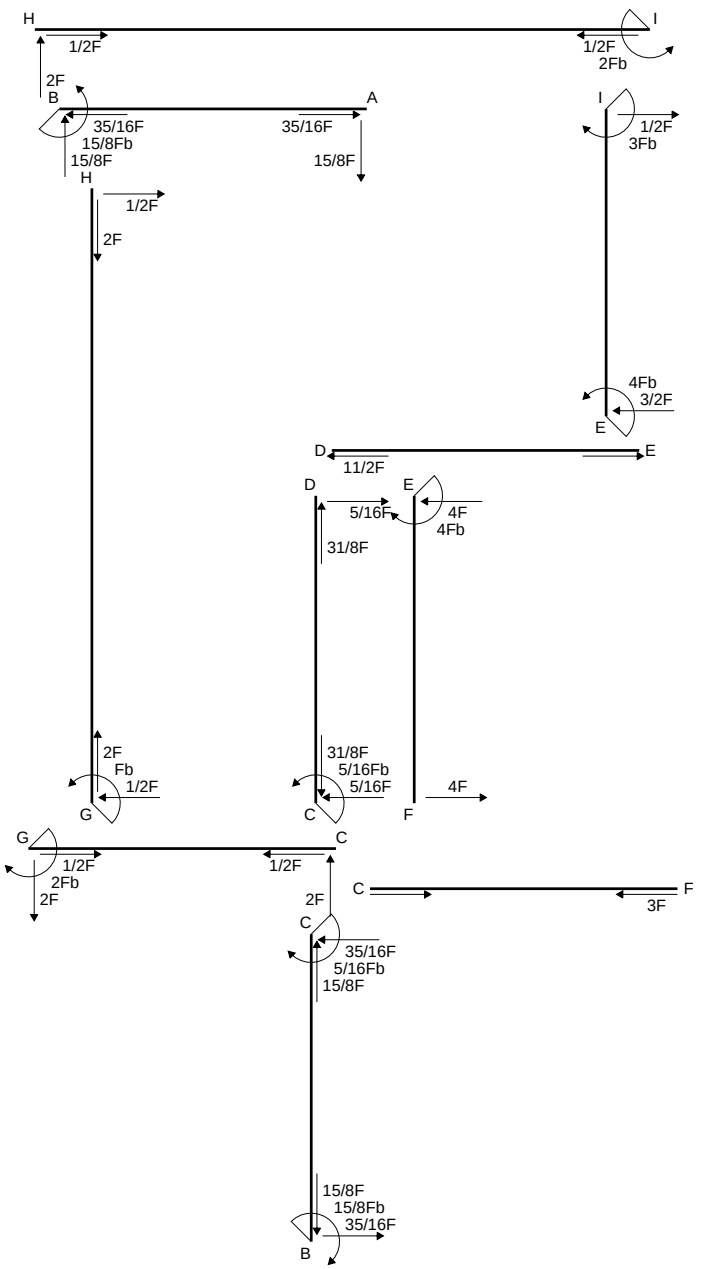
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





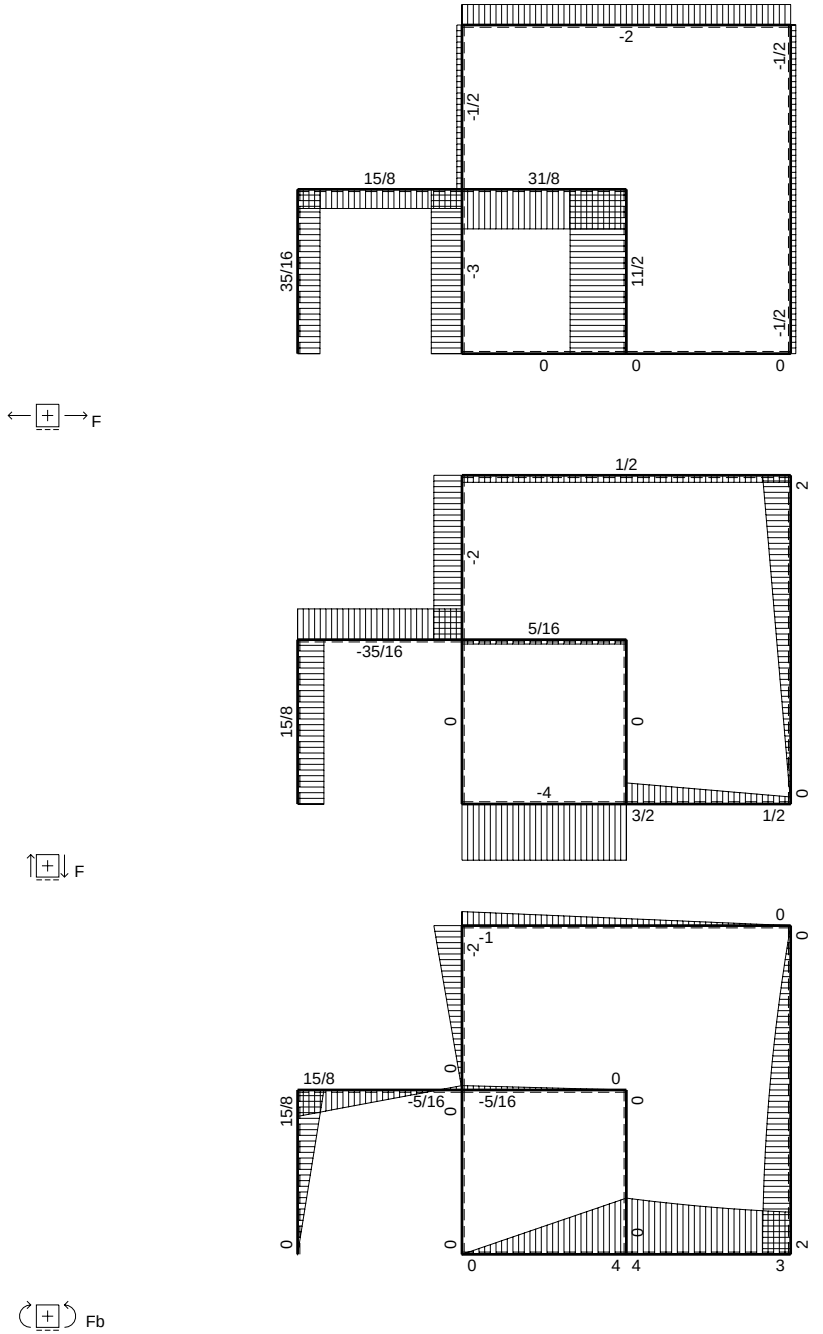


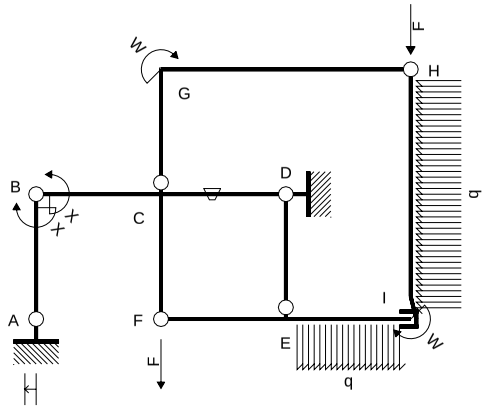

A = 1044. $mm^2$
$J_u$ = 275075. $mm^4$
$J_v$ = 107568. $mm^4$
$y_g$ = 35.16 mm
$T_y$ = -2960. N
$M_x$ = 1864800. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -35.16 mm
$\sigma_m = -Mv/J_u$ = 238.3 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 15. mm
$v_c$ = -20.16 mm
$\sigma_c = -Mv/J_u$ = 136.6 $N/mm^2$
$\tau_c$ = 4.464 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 136.9 $N/mm^2$
$S^*$ = 4978. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
I
35/16F
15/8Fb
15/8F
35/16F
15/8F
1/2F
3Fb
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
5/16F
4F
4Fb
31/8F
2F
Fb
1/2F
G
31/8F
5/16Fb
5/16F
C
4F
F
G
C
1/2F
2Fb
2F
1/2F
C
F
3F
2F
C
35/16F
5/16Fb
15/8F
15/8F
15/8Fb
35/16F
B

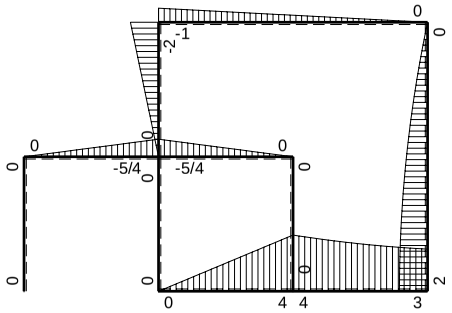




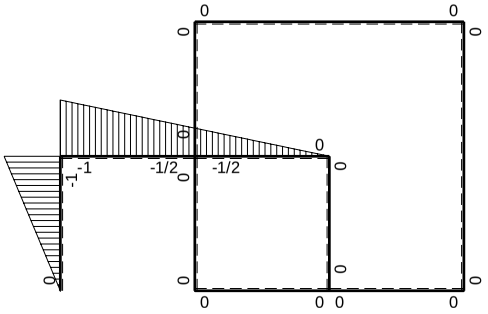

F
-2
-1/2
-1/2
15/8
31/8
35/16
-3
11/2
-1/2
0
0
0
F
1/2
2
-2
5/16
-35/16
15/8
0
0
0
-4
3/2
1/2
F
0
0
-2
-1
0
15/8
0
-5/16
-5/16
15/8
0
0
0
0
0
2
0
4
4
3
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265269 CI01.xxxx.165

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $15/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -15/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 265269 CI01.xxxx.165

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

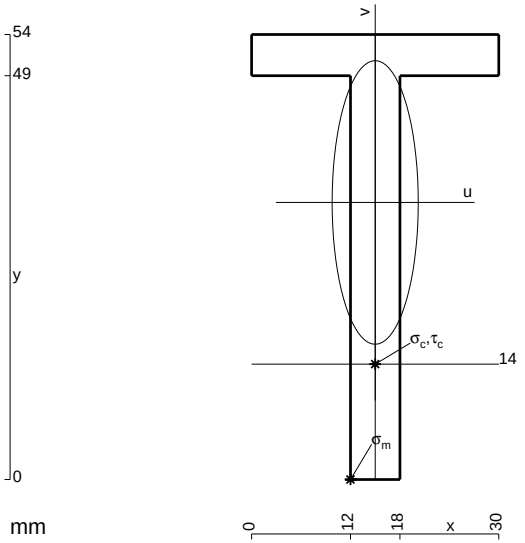
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

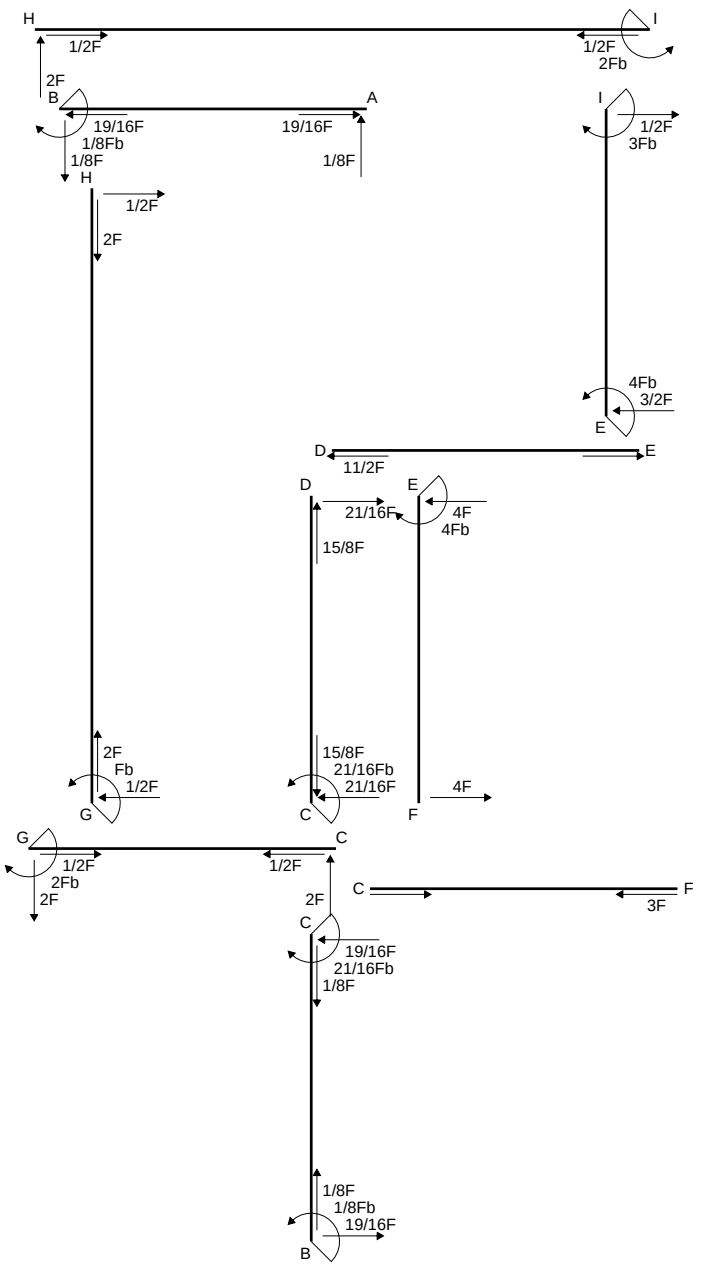
$= (5/24\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$





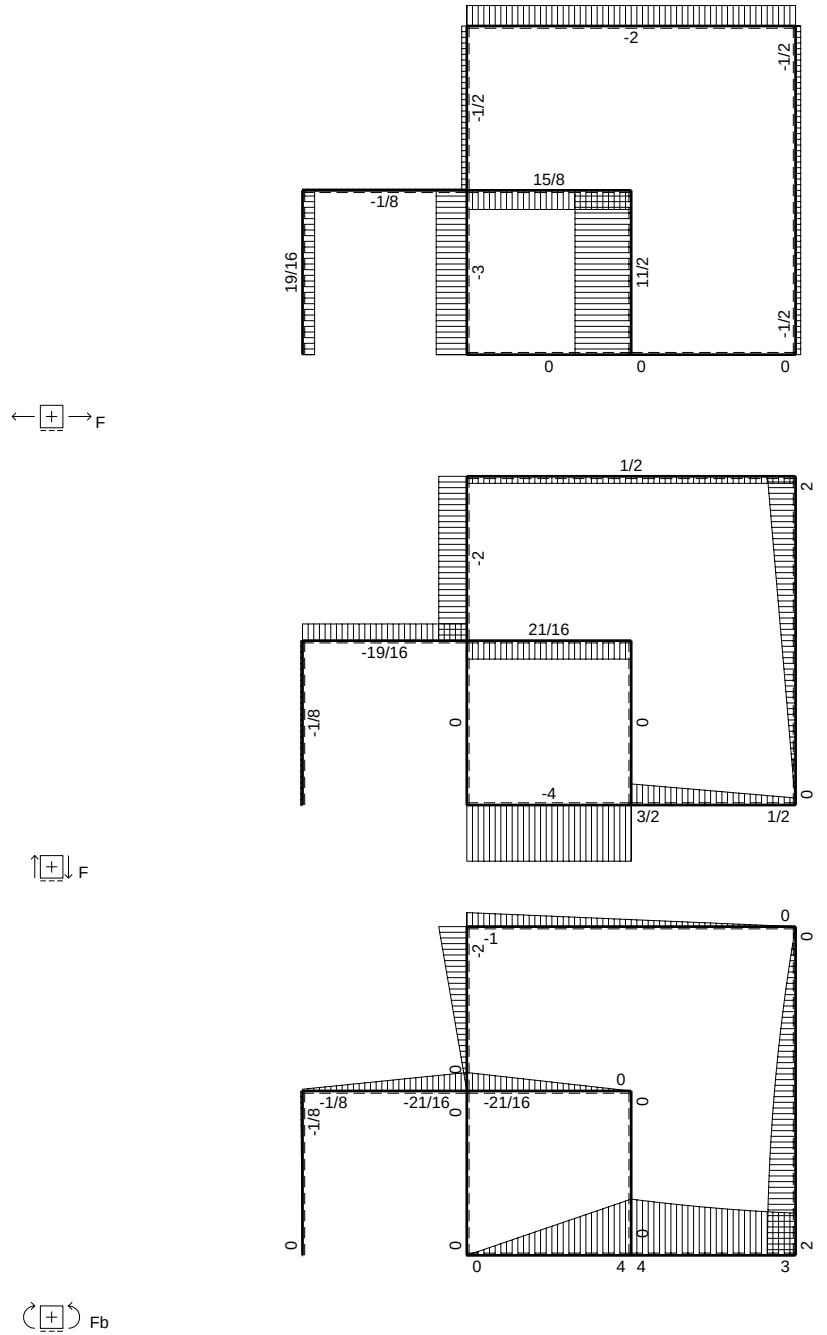


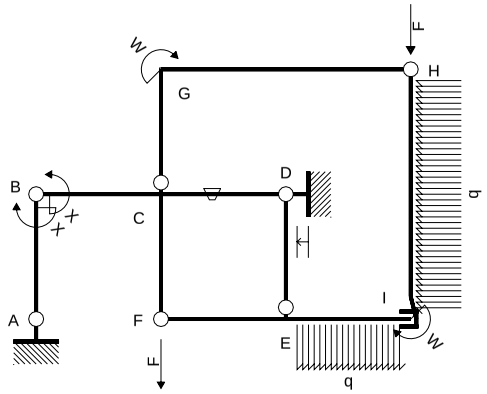

A = 444. $mm^2$
$J_u$ = 131544. $mm^4$
$J_v$ = 12132. $mm^4$
$y_g$ = 33.62 mm
$T_y$ = -1160. N
$M_x$ = 777200. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -33.62 mm
$\sigma_m = -Mv/J_u$ = 198.6 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 14. mm
$v_c$ = -19.62 mm
$\sigma_c = -Mv/J_u$ = 115.9 $N/mm^2$
$\tau_c$ = 3.287 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 116.1 $N/mm^2$
$S^*$ = 2236. $mm^3$


H
I
1/2F
1/2F
2Fb
2F
B
A
19/16F
19/16F
1/8Fb
1/8F
1/8F
I
1/2F
3Fb
H
1/2F
2F
4Fb
3/2F
E
D
E
11/2F
D
E
21/16F
4F
4Fb
15/8F
2F
Fb
1/2F
G
15/8F
21/16Fb
21/16F
C
4F
F
G
C
1/2F
1/2F
2Fb
2F
C
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C
19/16F
21/16Fb
1/8F
1/8F
1/8Fb
19/16F
B

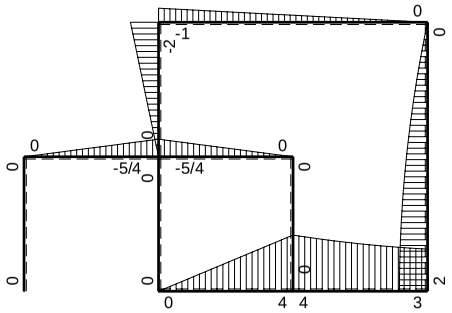




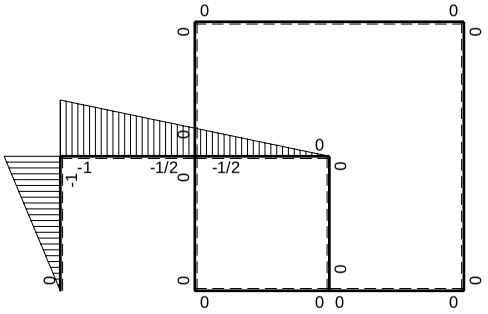

-2
-1/2
-1/2
15/8
-1/8
19/16
-3
11/2
-1/2
0
0
0
F
1/2
2
-2
21/16
-19/16
-1/8
0
0
0
-4
3/2
1/2
F
0
0
-2
-1
0
0
-1/8
-21/16
-21/16
0
0
0
0
0
0
4
4
3
2
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265852 CI01.xxxx.166

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | -2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 2Fb-2Fx | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | -Fb+1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | 1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | $2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | $-2Fb+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $3Fb+1/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 265852 CI01.xxxx.166
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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

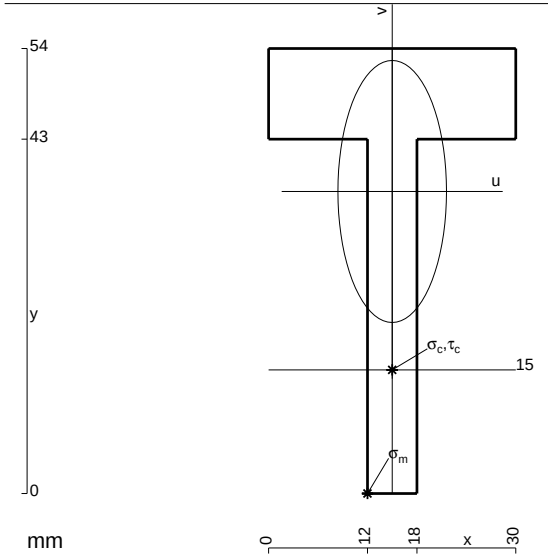
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/24\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$








$A = 588.\ mm^2$

$J_u = 148637.\ mm^4$

$J_v = 25524.\ mm^4$

$y_g = 36.65\ mm$

$T_y = -1160.\ N$

$M_x = 835200.\ Nmm$

$x_m = 12.\ mm$

$u_m = -3.\ mm$

$v_m = -36.65\ mm$

$\sigma_m = -Mv/J_u = 206.\ N/mm^2$

$x_c = 15.\ mm$

$y_c = 15.\ mm$

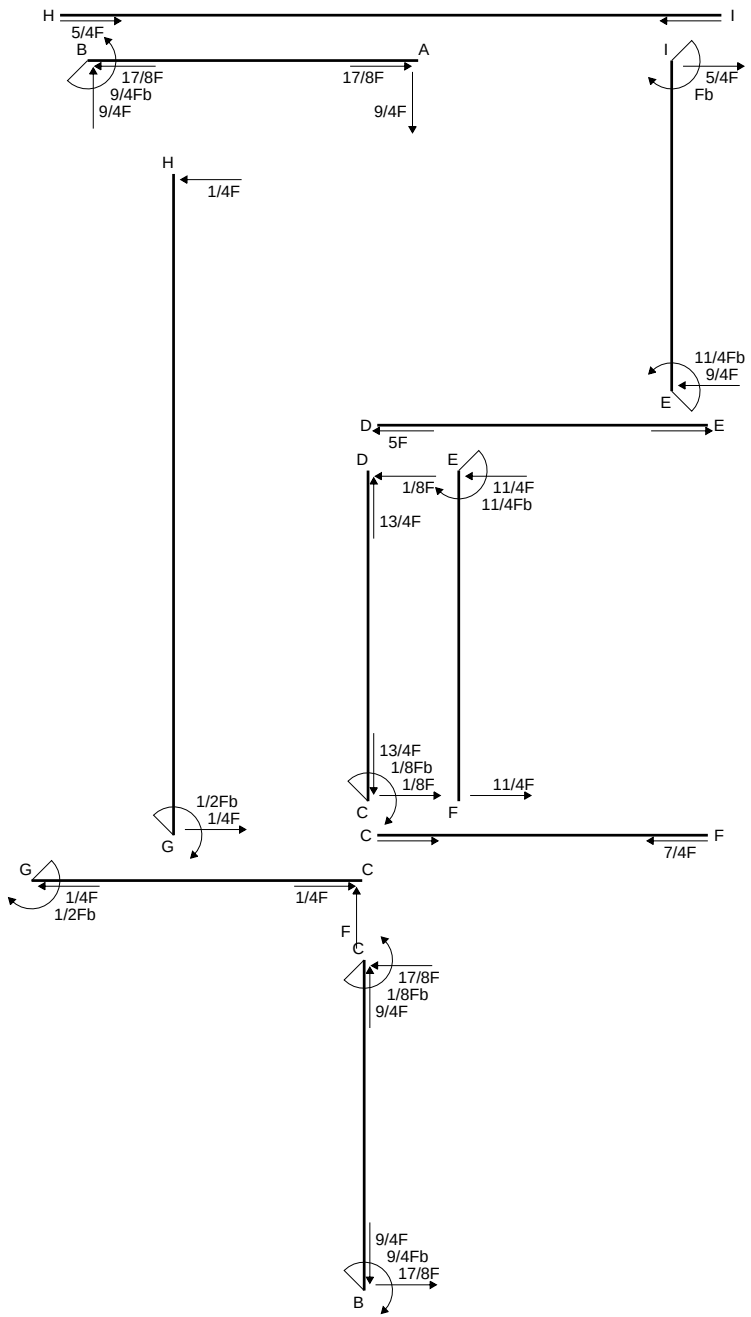
$v_c = -21.65\ mm$

$\sigma_c = -Mv/J_u = 121.7\ N/mm^2$

$\tau_c = 3.413\ N/mm^2$

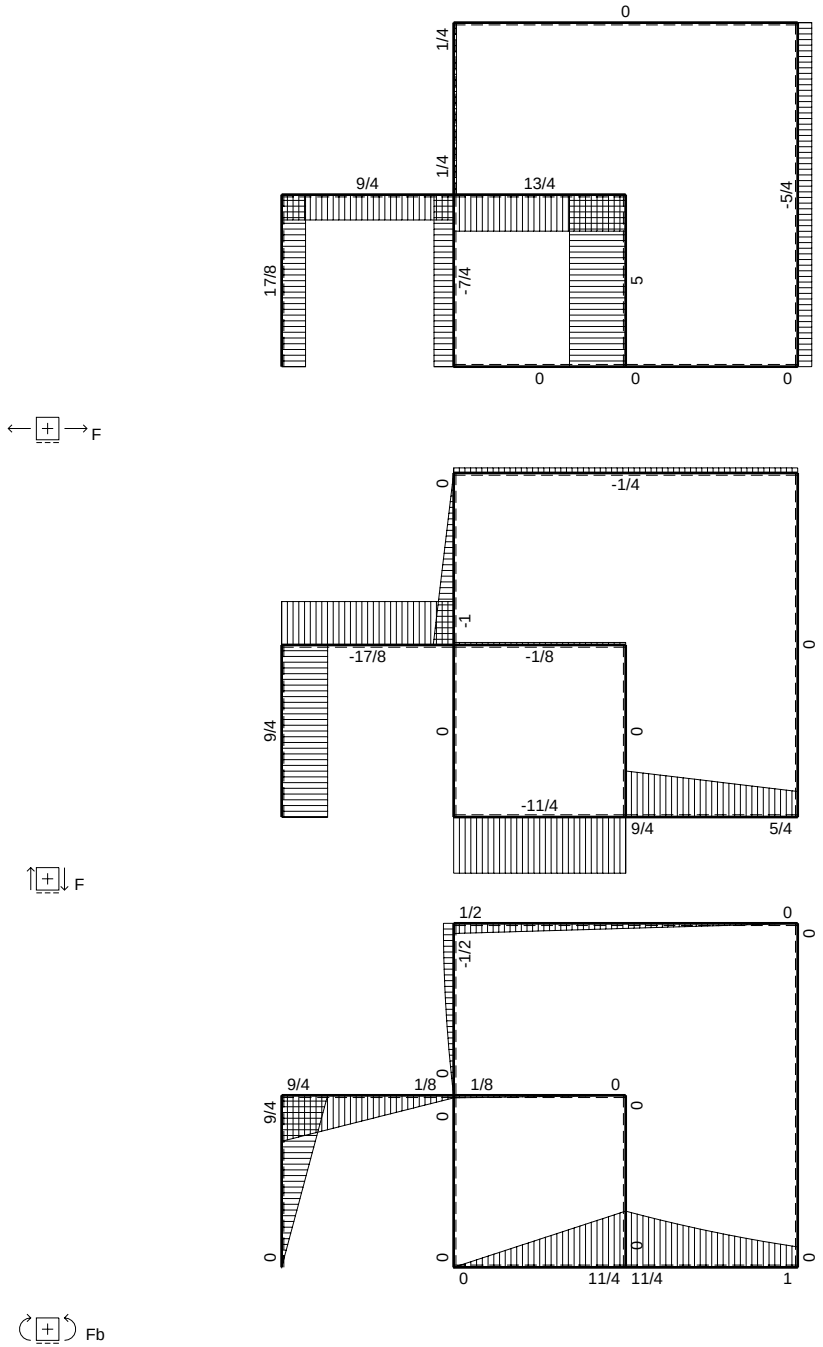
$\sigma_g = \sqrt{\sigma^2+3\tau^2} = 121.8\ N/mm^2$

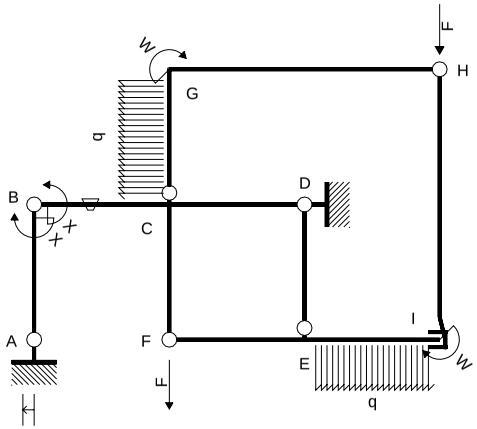
$S^* = 2624.\ mm^3$


H
I
5/4F
B
A
17/8F
9/4Fb
9/4F
17/8F
9/4F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
1/8F
11/4F
11/4Fb
13/4F
13/4F
1/8Fb
1/8F
11/4F
C
F
1/2Fb
1/4F
G
C
F
7/4F
G
C
1/4F
1/2Fb
1/4F
F
C
17/8F
1/8Fb
9/4F
9/4F
9/4Fb
17/8F
B

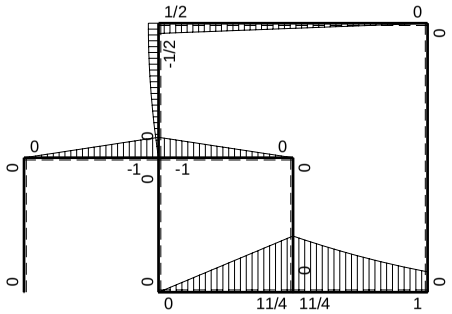




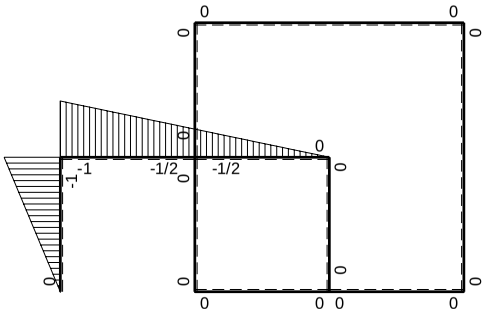

0
1/4
1/4
9/4
13/4
-5/4
17/8
-7/4
5
0
0
0
F
-1/4
0
-1
-17/8
-1/8
0
9/4
0
0
-11/4
9/4
5/4
F
1/2
0
0
-1/2
0
9/4
1/8
1/8
0
9/4
0
0
0
0
0
0
11/4
11/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267041 CI01.xxxx.167

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PROCEDIMENTO E RISULTATI 267041 CI01.xxxx.167

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $9/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -9/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

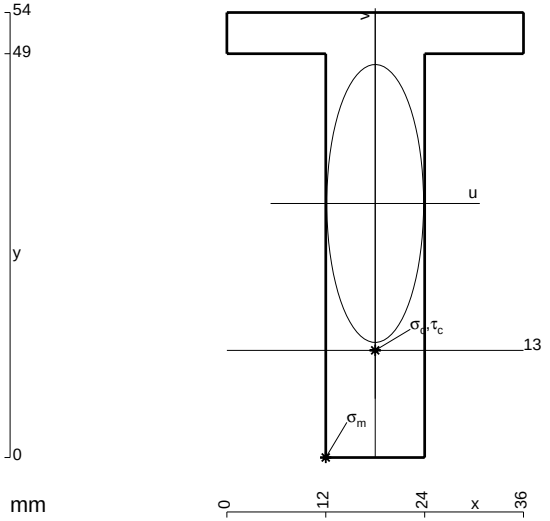
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

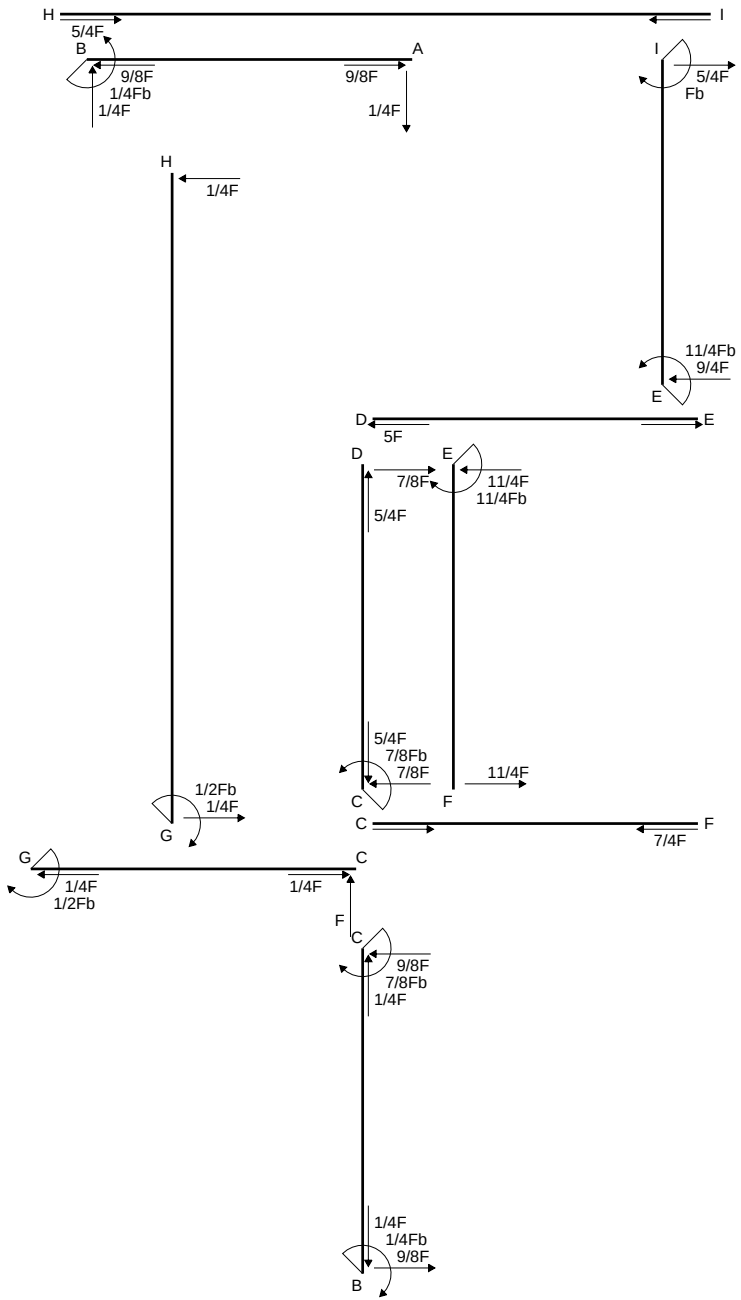


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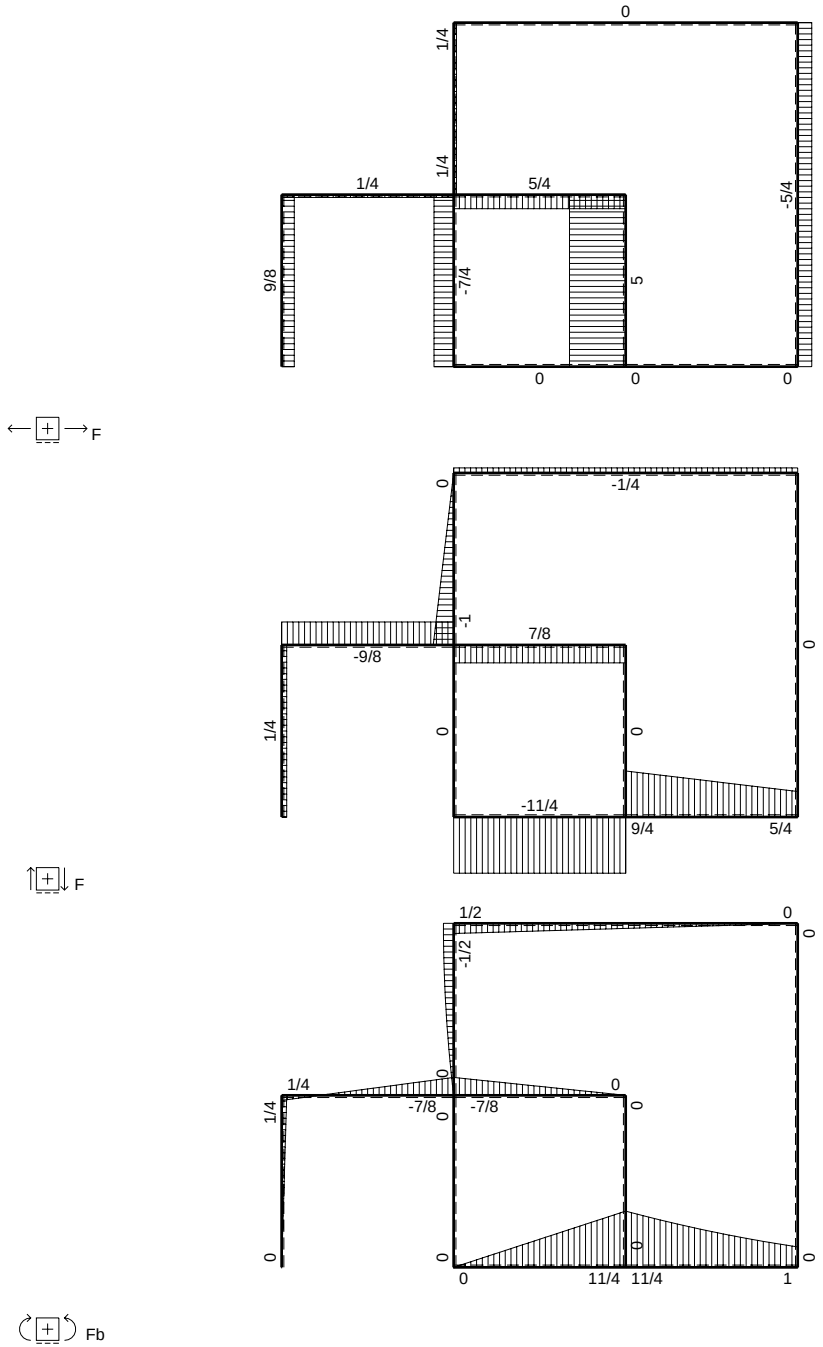

A = 768. $mm^2$
$J_u$ = 218489. $mm^4$
$J_v$ = 26496. $mm^4$
$y_g$ = 30.83 mm
$T_y$ = -2035. N
$M_x$ = 1546600. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -30.83 mm
$\sigma_m = -Mv/J_u$ = 218.2 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 13. mm
$v_c$ = -17.83 mm
$\sigma_c = -Mv/J_u$ = 126.2 $N/mm^2$
$\tau_c$ = 2.946 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 126.3 $N/mm^2$
$S^*$ = 3795. $mm^3$

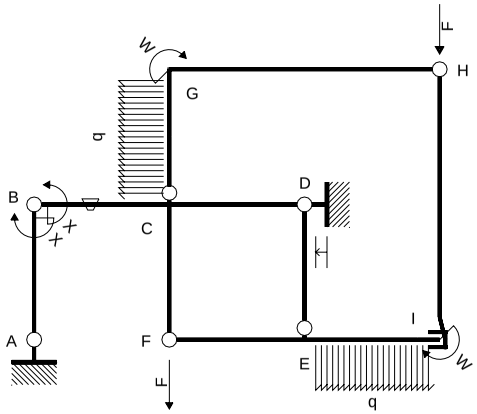
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H
I
5/4F
B
A
9/8F
1/4Fb
1/4F
9/8F
1/4F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
7/8F
11/4F
11/4Fb
5/4F
5/4F
7/8Fb
7/8F
11/4F
C
F
1/2Fb
1/4F
G
C
F
7/4F
G
C
1/4F
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1/4Fb
9/8F
B

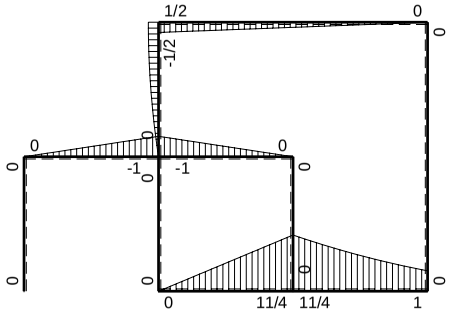




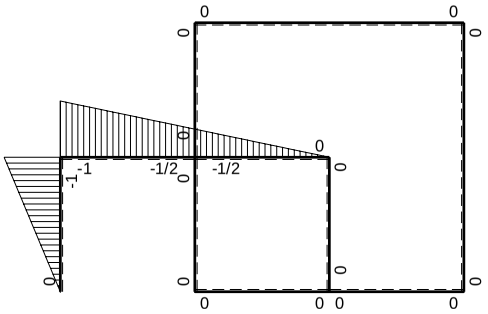

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1/4
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-5/4
9/8
-7/4
5
0
0
0
F
-1/4
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-1
7/8
-9/8
0
1/4
0
0
-11/4
9/4
5/4
F
1/2
0
0
-1/2
1/4
0
0
-7/8
-7/8
1/4
0
0
0
0
0
11/4
11/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264626 CI01.xxxx.168

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/4Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 264626 CI01.xxxx.168

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

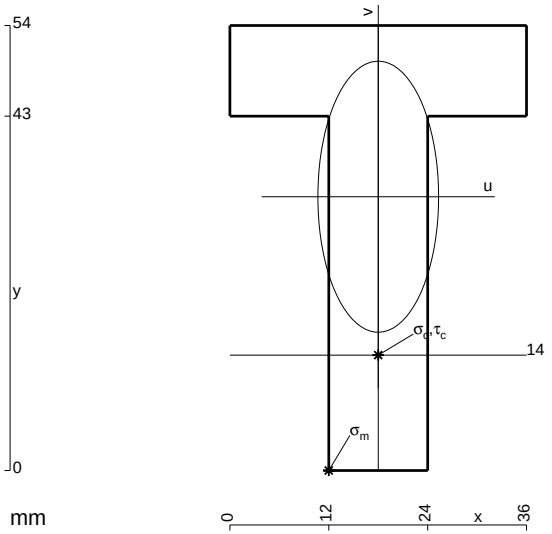
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

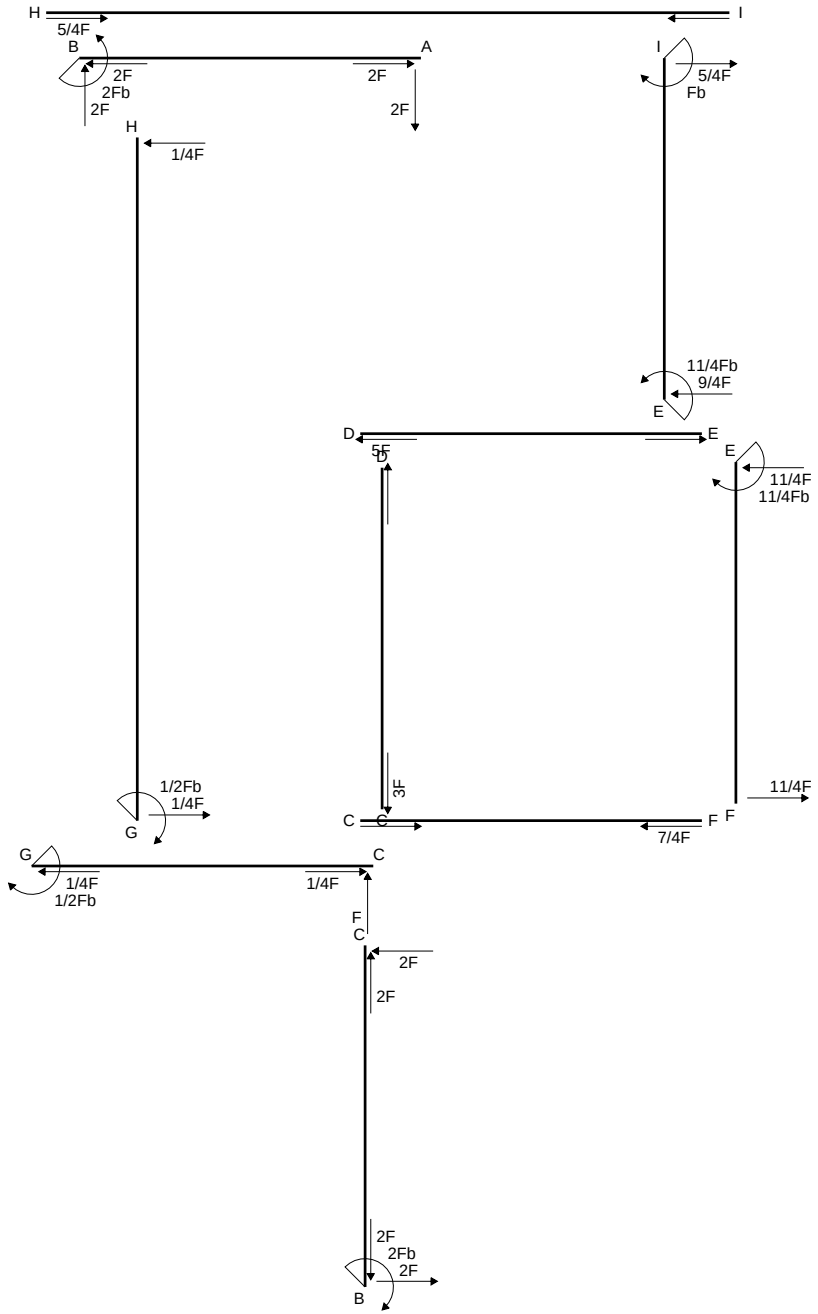
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





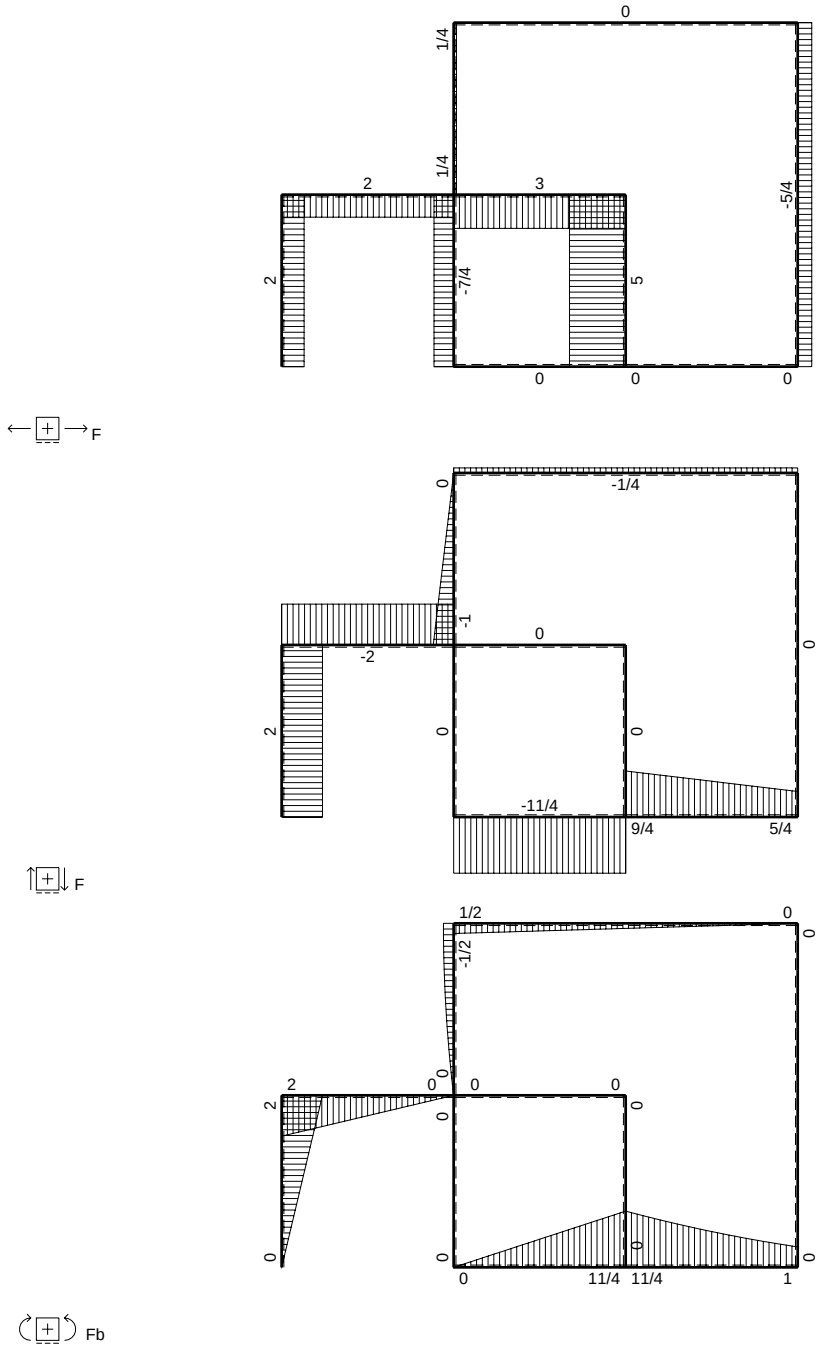


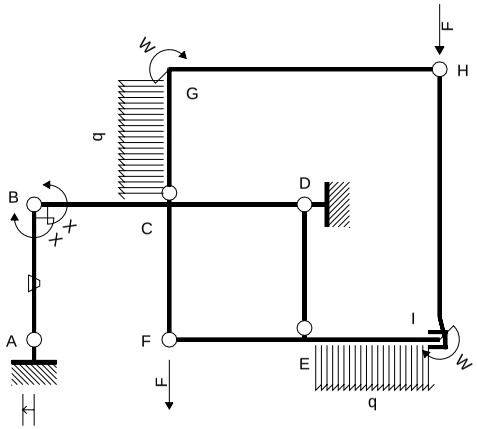

A = 912. $mm^2$
$J_u$ = 246834. $mm^4$
$J_v$ = 48960. $mm^4$
$y_g$ = 33.22 mm
$T_y$ = -2090. N
$M_x$ = 1692900. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -33.22 mm
$\sigma_m = -Mv/J_u$ = 227.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -19.22 mm
$\sigma_c = -Mv/J_u$ = 131.8 $N/mm^2$
$\tau_c$ = 3.109 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132. $N/mm^2$
$S^*$ = 4406. $mm^3$


H
I
5/4F
B
A
2F
2Fb
2F
2F
2F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
11/4F
11/4Fb
1/2Fb
1/4F
G
3F
C
C
F
F
11/4F
7/4F
G
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1/4F
F
C
2F
2F
2F
2Fb
2F
B

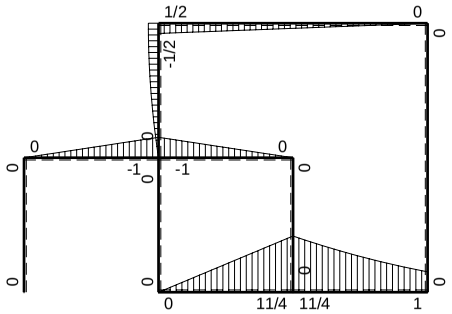




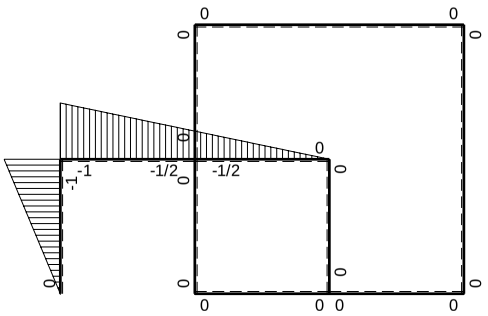

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F
Fb
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-1
0
-2
2
0
0
0
-11/4
9/4
5/4
1/2
0
0
-1/2
2
0
0
0
0
0
0
0
0
0
11/4
11/4
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268284 CI01.xxxx.169

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 268284 CI01.xxxx.169

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -2Fb | |

Sviluppi di calcolo iperstatica


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b ( x/b )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

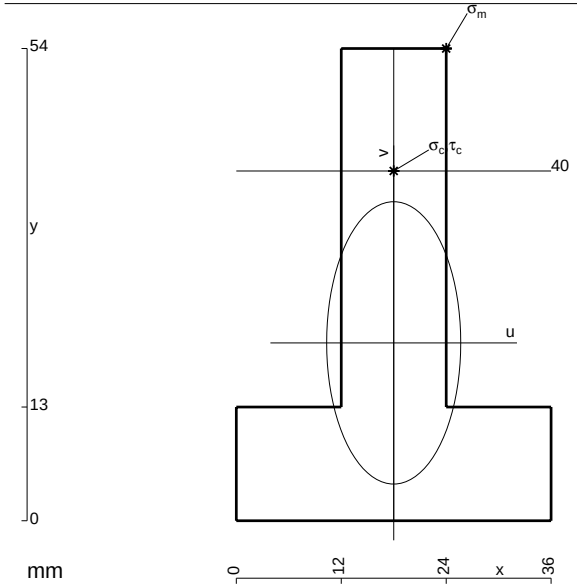
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

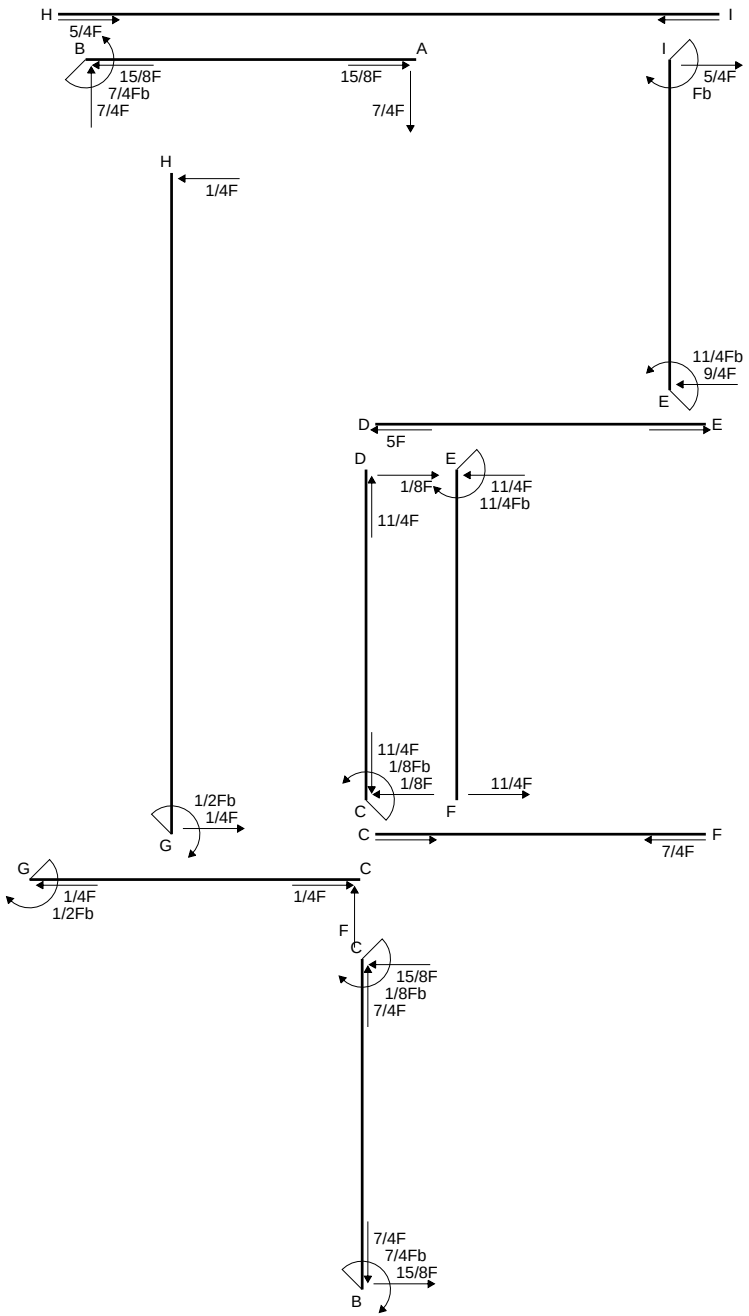
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





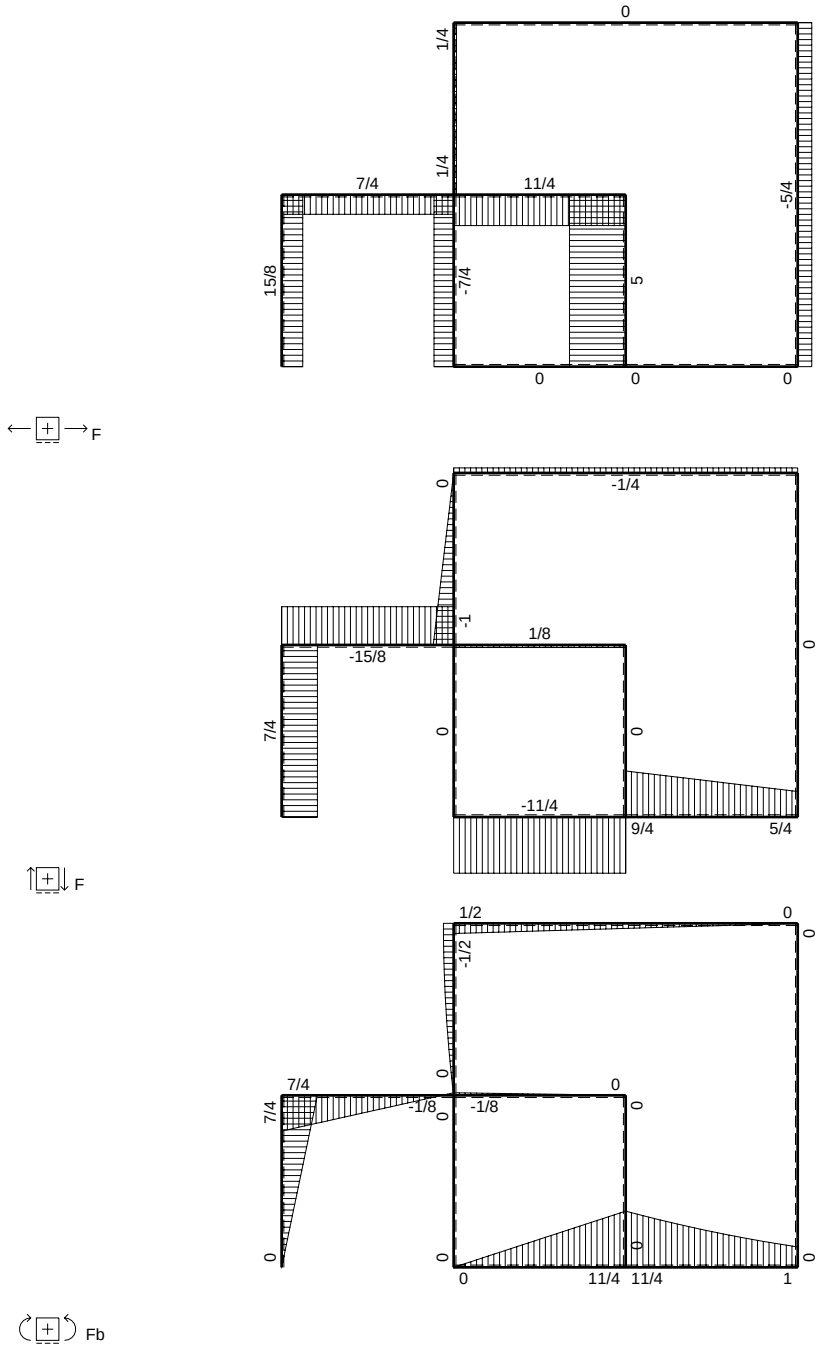


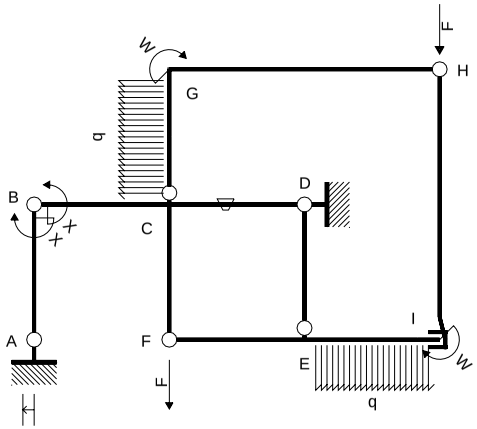

A = 960. $mm^2$
$J_u$ = 250363. $mm^4$
$J_v$ = 56448. $mm^4$
$y_g$ = 20.34 mm
$T_y$ = -2090. N
$M_x$ = 1776500. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 33.66 mm
$\sigma_m = -Mv/J_u$ = -238.9 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 40. mm
$v_c$ = 19.66 mm
$\sigma_c = -Mv/J_u$ = -139.5 $N/mm^2$
$\tau_c$ = 3.116 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.6 $N/mm^2$
$S^*$ = 4479. $mm^3$


H
I
5/4F
B
A
15/8F
7/4Fb
7/4F
15/8F
7/4F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
1/8F
11/4F
11/4Fb
11/4F
11/4F
1/8Fb
1/8F
11/4F
C
F
1/2Fb
1/4F
G
C
F
7/4F
G
C
1/4F
1/2Fb
1/4F
F
C
15/8F
1/8Fb
7/4F
7/4F
7/4Fb
15/8F
B

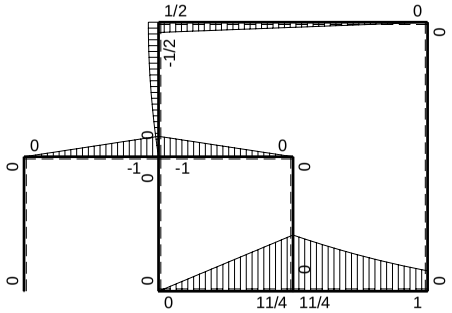




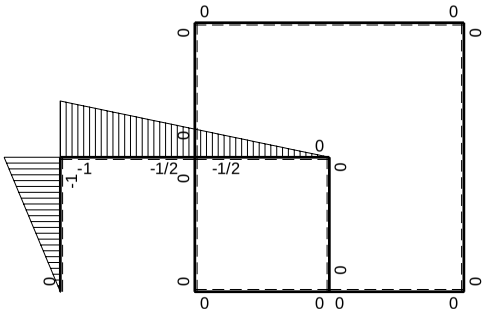

0
1/4
1/4
7/4
11/4
-5/4
15/8
-7/4
5
0
0
0
F
-1/4
0
-1
1/8
-15/8
7/4
0
0
0
-11/4
9/4
5/4
F
1/2
0
0
-1/2
0
7/4
0
7/4
-1/8
-1/8
0
0
0
0
0
0
11/4
11/4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 266409 CI01.xxxx.170

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $7/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/4Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 266409 CI01.xxxx.170

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

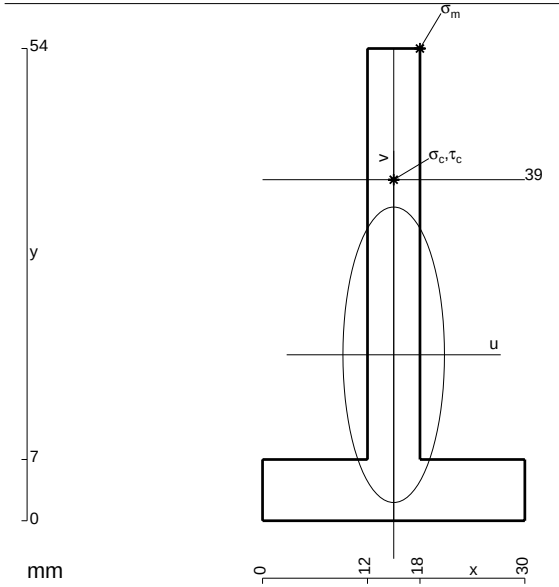
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

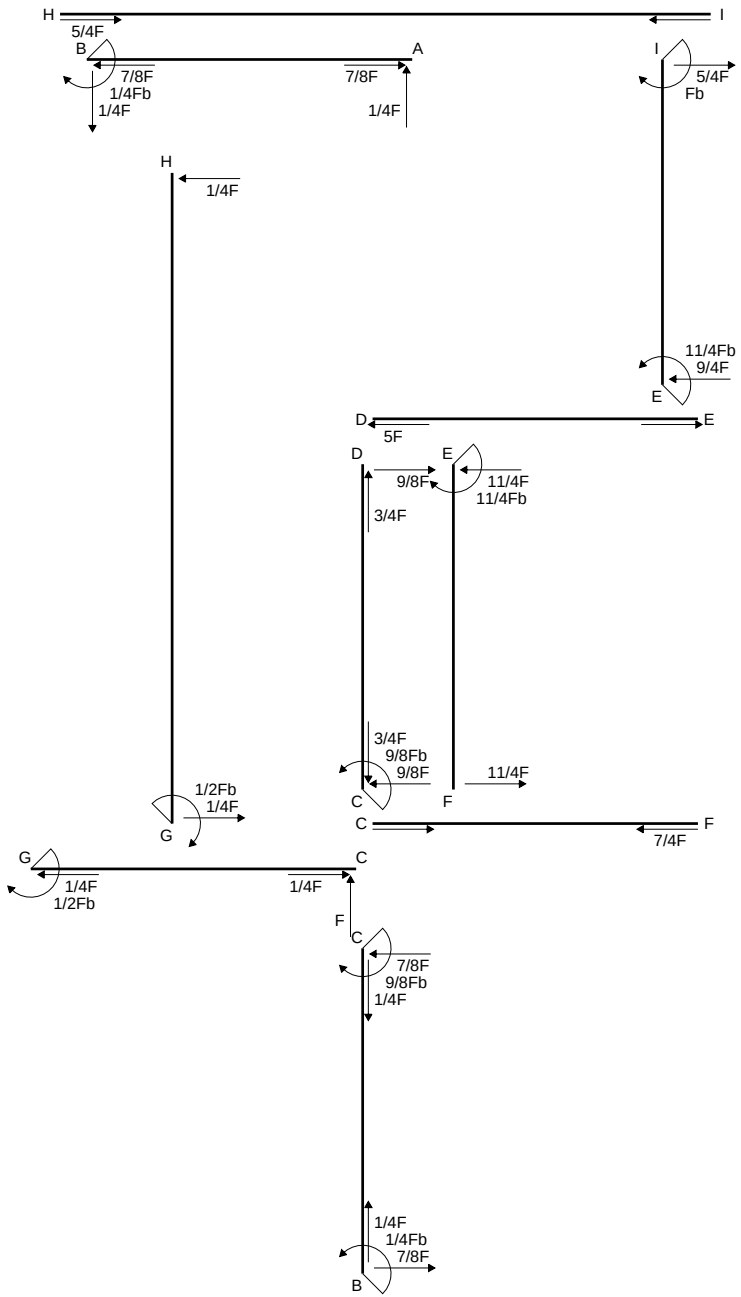
$= (1/6\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$



PROCEDIMENTO E RISULTATI 266409 CI01.xxxx.170

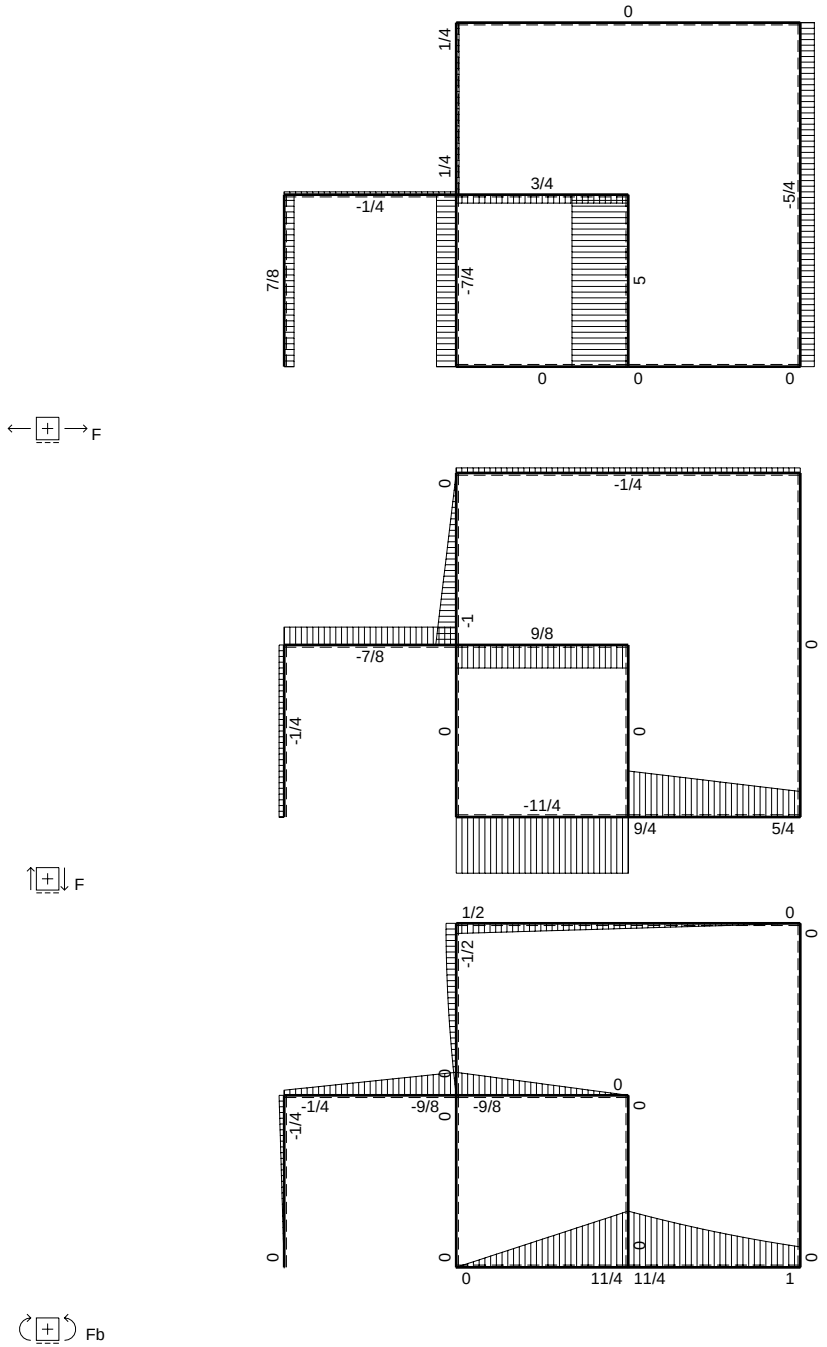


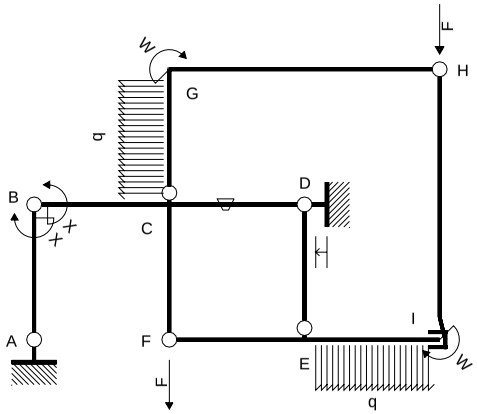

A = 492. $mm^2$
$J_u$ = 140516. $mm^4$
$J_v$ = 16596. $mm^4$
$y_g$ = 18.98 mm
$T_y$ = -1760. N
$M_x$ = 792000. Nmm
$x_m$ = 18. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 35.02 mm
$\sigma_m = -Mv/J_u$ = -197.4 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 39. mm
$v_c$ = 20.02 mm
$\sigma_c = -Mv/J_u$ = -112.9 $N/mm^2$
$\tau_c$ = 5.171 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.2 $N/mm^2$
$S^*$ = 2477. $mm^3$


H
I
5/4F
B
A
7/8F
1/4Fb
1/4F
7/8F
1/4F
I
5/4F
Fb
H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
9/8F
11/4F
11/4Fb
3/4F
3/4F
9/8Fb
9/8F
11/4F
C
F
1/2Fb
1/4F
G
C
F
7/4F
G
C
1/4F
1/2Fb
1/4F
F
C
7/8F
9/8Fb
1/4F
1/4F
1/4Fb
7/8F
B

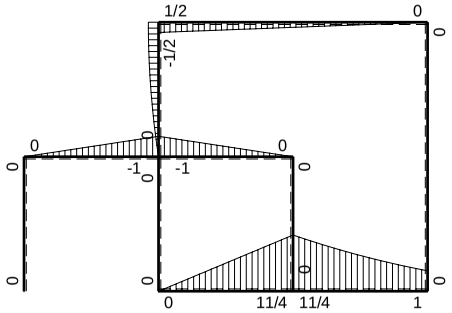




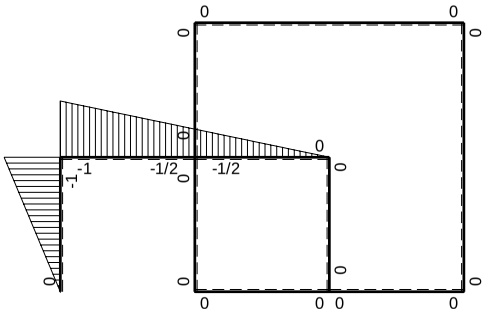

F
F
Fb
0
1/4
1/4
3/4
-1/4
-5/4
7/8
-7/4
5
0
0
0
-1/4
-1
9/8
-7/8
-1/4
0
0
0
-11/4
9/4
5/4
1/2
0
0
-1/2
0
0
-1/4
-1/4
-9/8
-9/8
0
0
0
0
0
0
0
11/4
11/4
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 213022 CI01.xxxx.171

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | 1/2Fb/EJ-1/2Fx/EJ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 11/4Fb-11/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -11/4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | $1/2Fb-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | 1/2Fb-1/4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/4Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/4Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-11/4Fb+9/4Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/4Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 213022 CI01.xxxx.171

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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - 1/2\ b + 1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 + 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b + 1/4\ b + 1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x - 1/4\ x^2/b\ ]_o^b\ \theta$

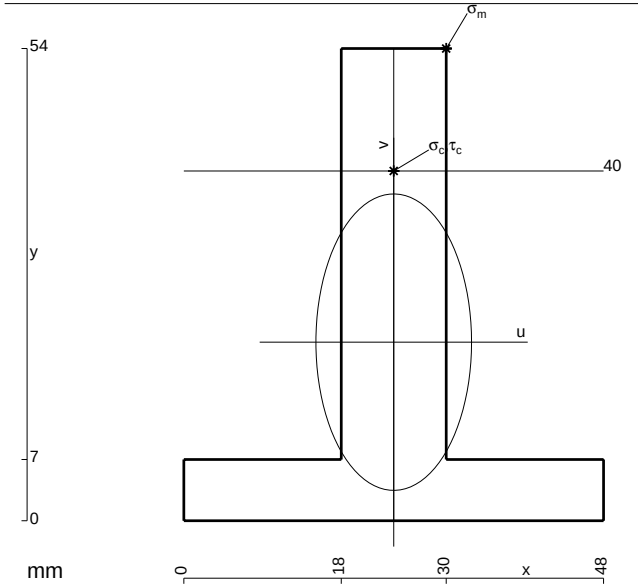
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b - 1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

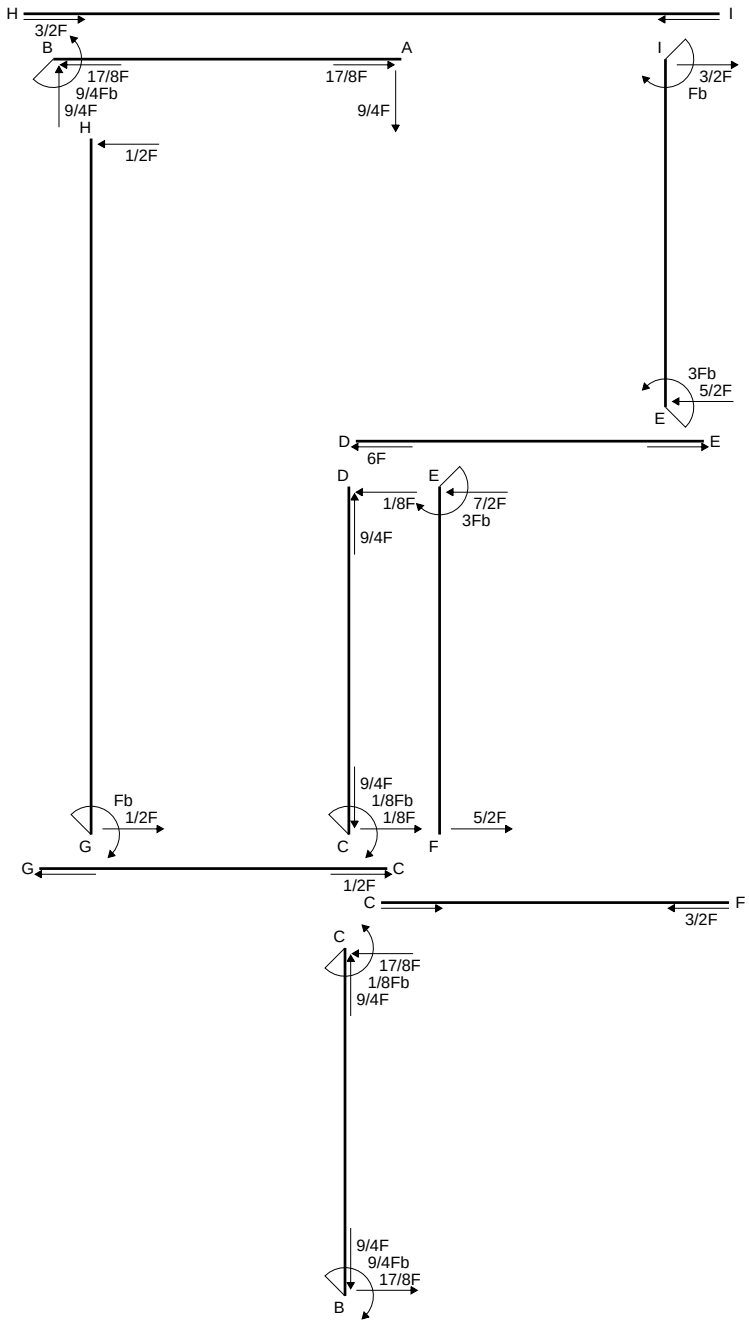


PROCEDIMENTO E RISULTATI 213022 CI01.xxxx.171



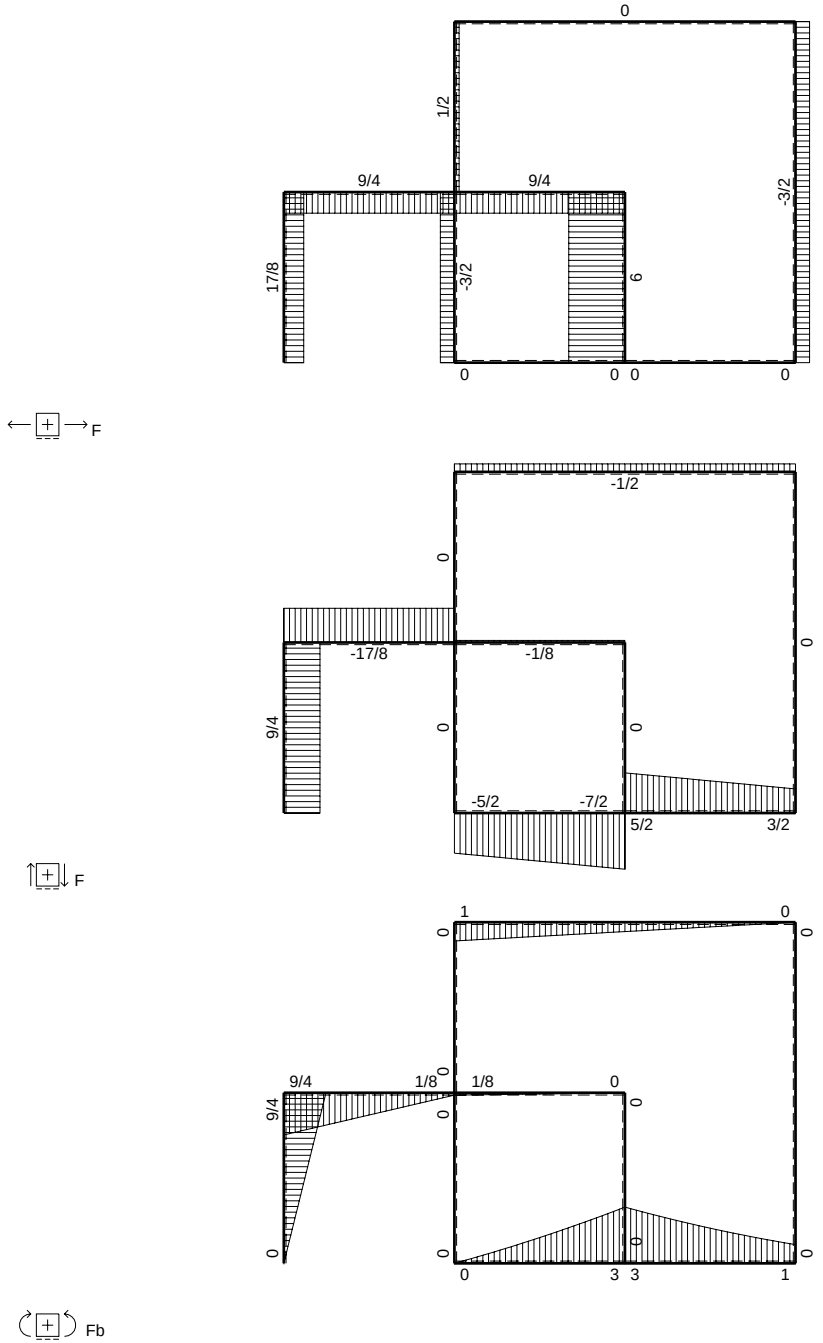

A = 900. $mm^2$
$J_u$ = 258693. $mm^4$
$J_v$ = 71280. $mm^4$
$y_g$ = 20.42 mm
$T_y$ = -3300. N
$M_x$ = 1617000. Nmm
$x_m$ = 30. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 33.58 mm
$\sigma_m = -Mv/J_u$ = -209.9 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 40. mm
$v_c$ = 19.58 mm
$\sigma_c = -Mv/J_u$ = -122.4 $N/mm^2$
$\tau_c$ = 4.747 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 122.7 $N/mm^2$
$S^*$ = 4465. $mm^3$

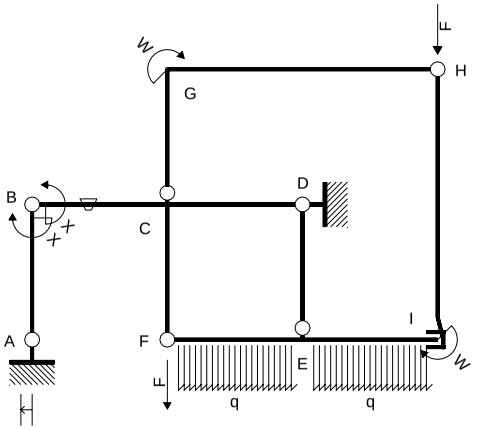
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H
I
3/2F
B
A
17/8F
17/8F
9/4Fb
9/4F
9/4F
H
1/2F
I
3/2F
Fb
3Fb
5/2F
E
D
E
6F
D
E
1/8F
7/2F
3Fb
9/4F
9/4F
1/8Fb
Fb
1/2F
1/8F
5/2F
G
C
F
G
C
1/2F
C
F
3/2F
C
17/8F
1/8Fb
9/4F
9/4F
9/4Fb
17/8F
B

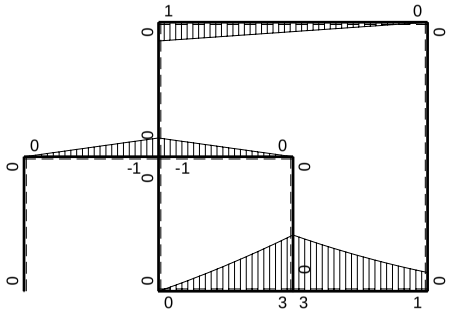




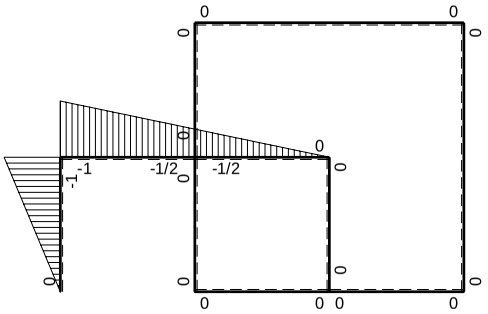

F
0
1/2
9/4
9/4
-3/2
17/8
-3/2
6
0
0 0
0
F
-1/2
0
-17/8
-1/8
9/4
0
0
0
-5/2
-7/2
5/2
3/2
F
1
0
0
0
0
9/4
1/8
1/8
0
9/4
0
0
0
0
0
0
3 3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 309547 CI01.xxxx.172

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PROCEDIMENTO E RISULTATI 309547 CI01.xxxx.172

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $9/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -9/4Fb | |

Sviluppi di calcolo iperstatica


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

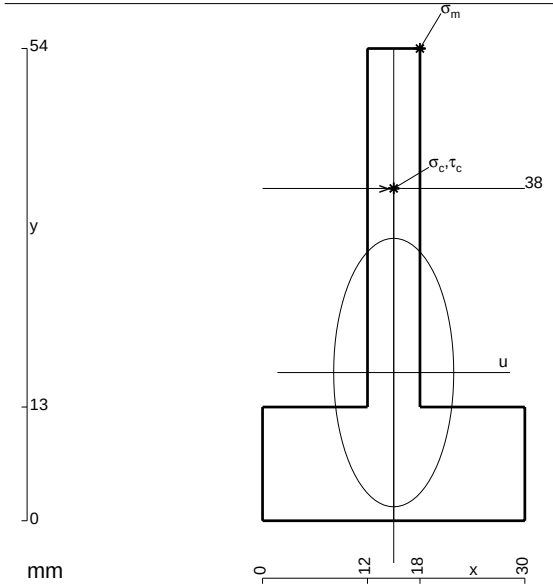
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

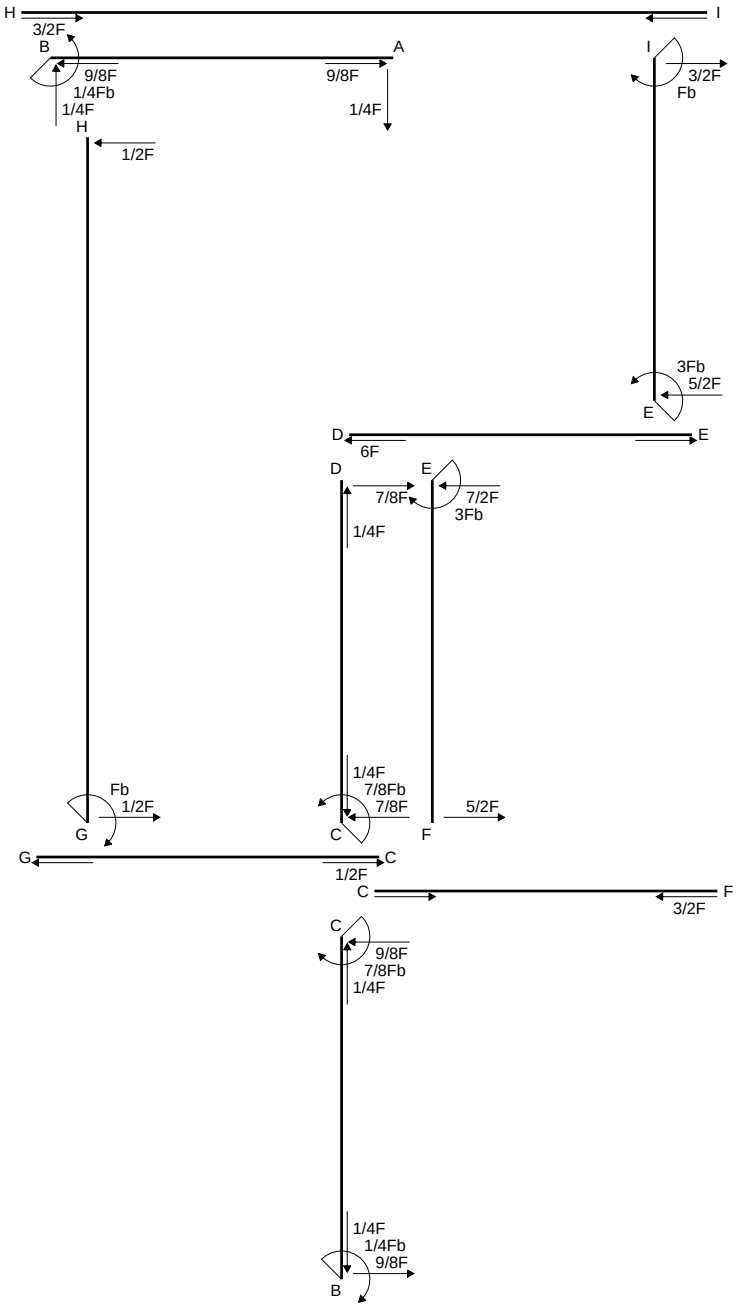
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





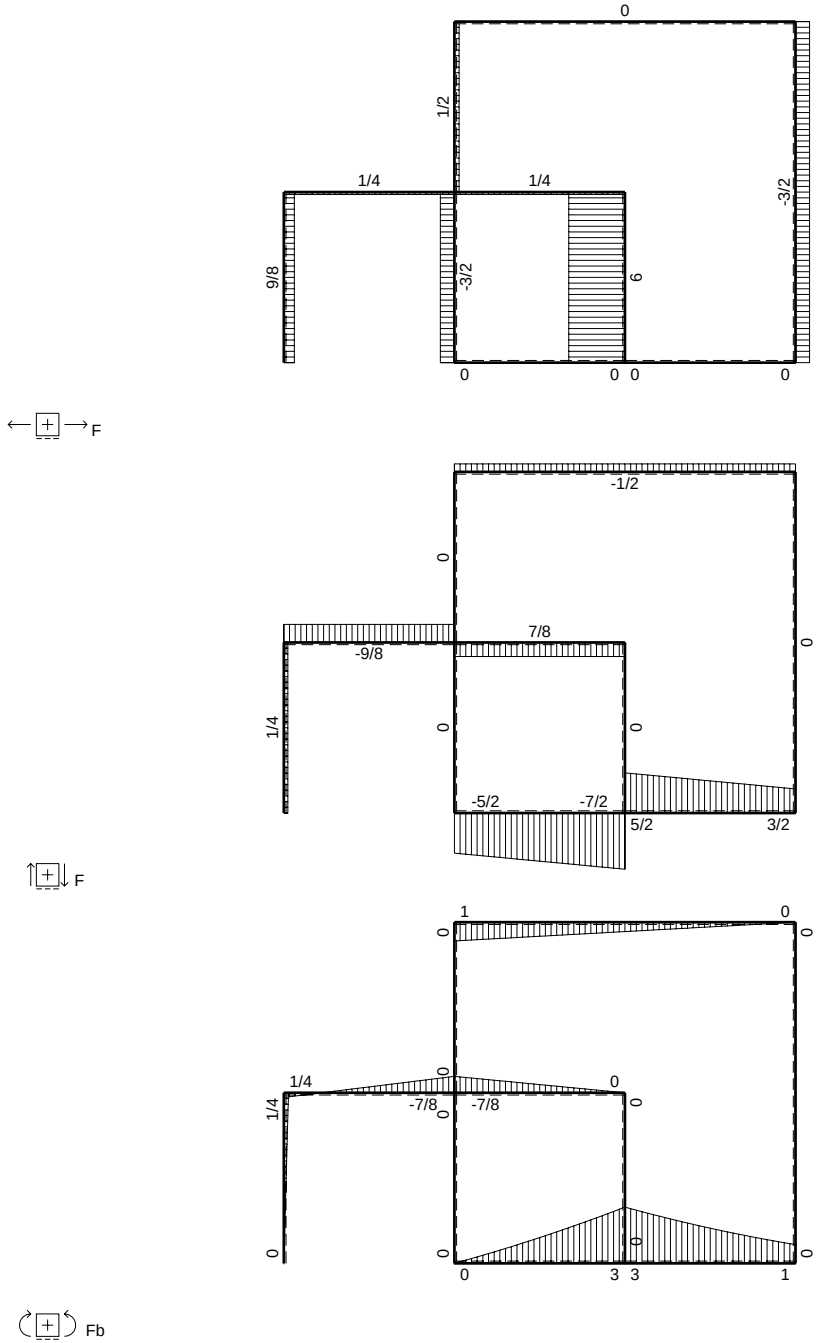


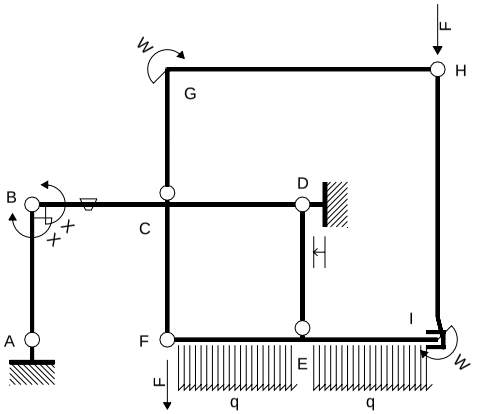

A = 636. $mm^2$
$J_u$ = 149922. $mm^4$
$J_v$ = 29988. $mm^4$
$y_g$ = 16.94 mm
$T_y$ = -1890. N
$M_x$ = 874800. Nmm
$x_m$ = 18. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 37.06 mm
$\sigma_m = -Mv/J_u$ = -216.2 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 38. mm
$v_c$ = 21.06 mm
$\sigma_c = -Mv/J_u$ = -122.9 $N/mm^2$
$\tau_c$ = 5.861 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.3 $N/mm^2$
$S^*$ = 2789. $mm^3$


H
I
3/2F
B
A
I
9/8F
9/8F
3/2F
1/4Fb
Fb
1/4F
1/4F
H
1/2F
3Fb
5/2F
E
D
E
6F
D
E
7/8F
7/2F
3Fb
1/4F
1/4F
7/8Fb
Fb
7/8F
5/2F
1/2F
G
C
F
G
C
1/2F
C
F
3/2F
C
9/8F
7/8Fb
1/4F
1/4F
1/4Fb
9/8F
B

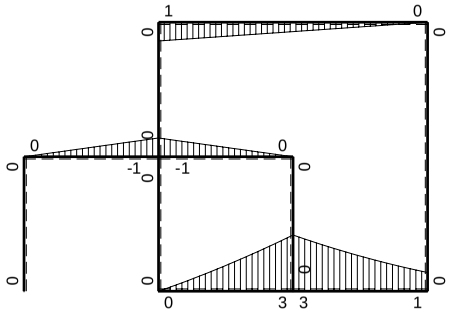




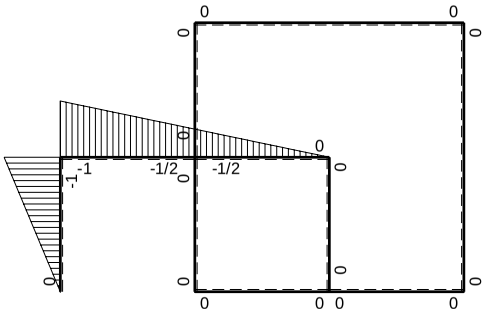

0
1/2
1/4
1/4
-3/2
9/8
-3/2
6
0
0 0
0
F
-1/2
0
7/8
-9/8
0
1/4
0
0
-5/2
-7/2
5/2
3/2
F
1
0
0
0
1/4
0
0
1/4
-7/8
-7/8
0
0
0
0
0
3 3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 275579 CI01.xxxx.173

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PROCEDIMENTO E RISULTATI 275579 CI01.xxxx.173

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | -Fb/EJ | $Fx-1/2Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/3+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | Fb/EJ | $1/2Fb-1/2Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-Fb+Fx$ | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/4Fb | |

Sviluppi di calcolo iperstatica


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

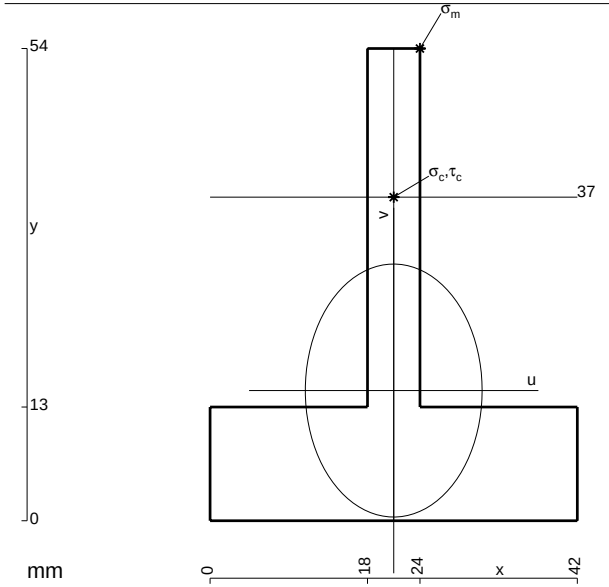
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

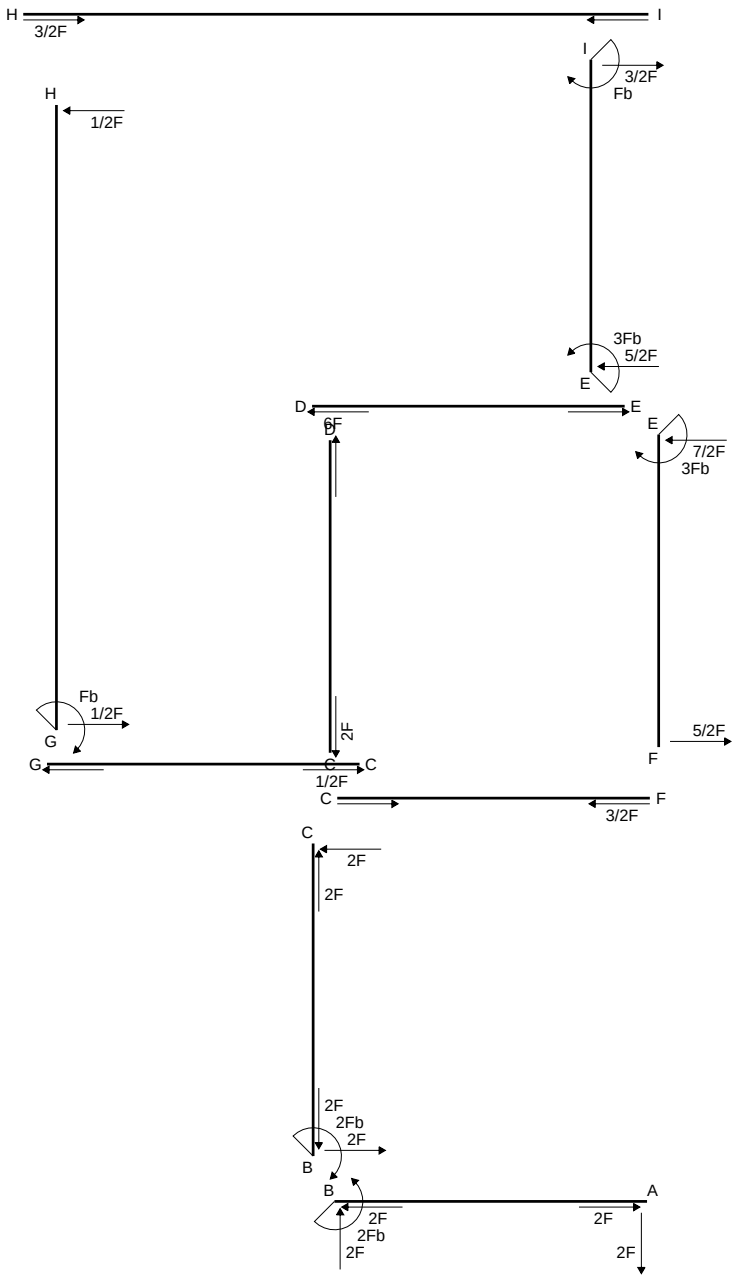
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





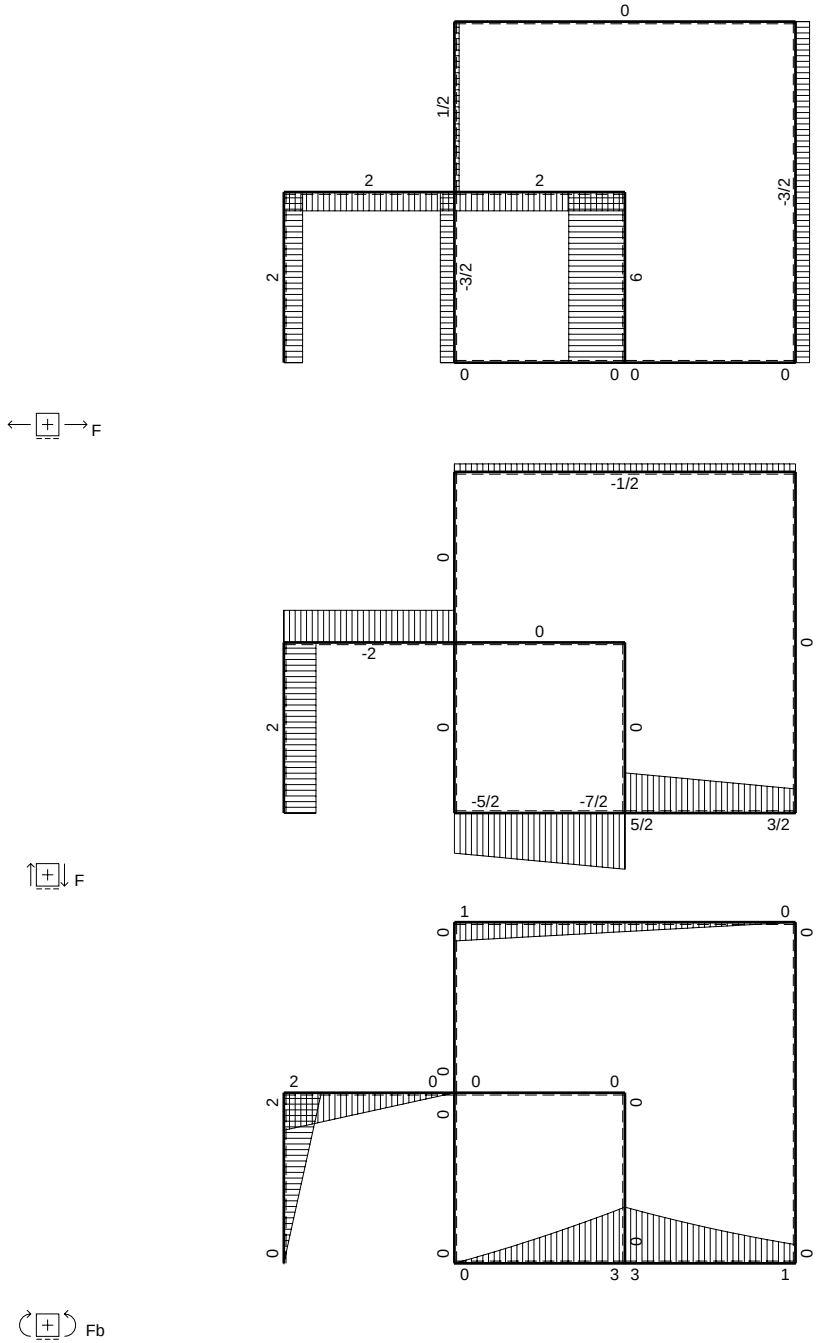


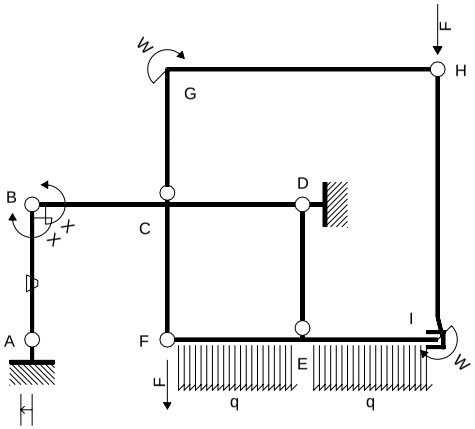

A = 792. $mm^2$
$J_u$ = 165782. $mm^4$
$J_v$ = 81000. $mm^4$
$y_g$ = 14.89 mm
$T_y$ = -1960. N
$M_x$ = 974400. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 39.11 mm
$\sigma_m = -Mv/J_u$ = -229.9 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 37. mm
$v_c$ = 22.11 mm
$\sigma_c = -Mv/J_u$ = -130. $N/mm^2$
$\tau_c$ = 6.153 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.4 $N/mm^2$
$S^*$ = 3123. $mm^3$


H
3/2F
I
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
6F
D
E
7/2F
3Fb
Fb
1/2F
G
2F
G
C
C
1/2F
5/2F
F
C
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2F
2Fb
2F
2F

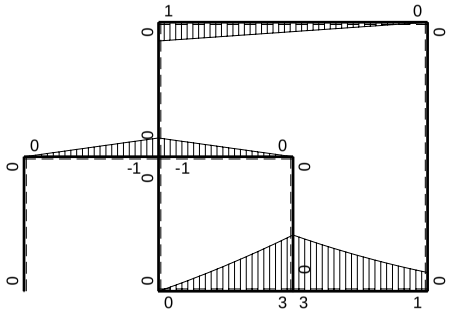




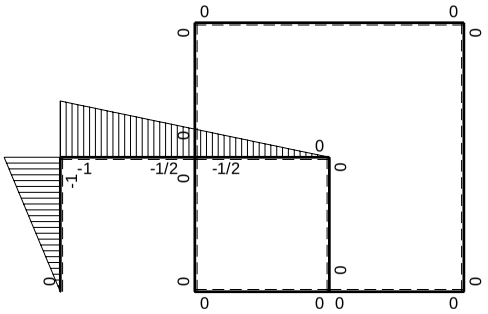

F
F
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 215204 CI01.xxxx.174

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

PROCEDIMENTO E RISULTATI 215204 CI01.xxxx.174

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-Fb+Fx$ | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $2Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -2Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b ( x/b )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

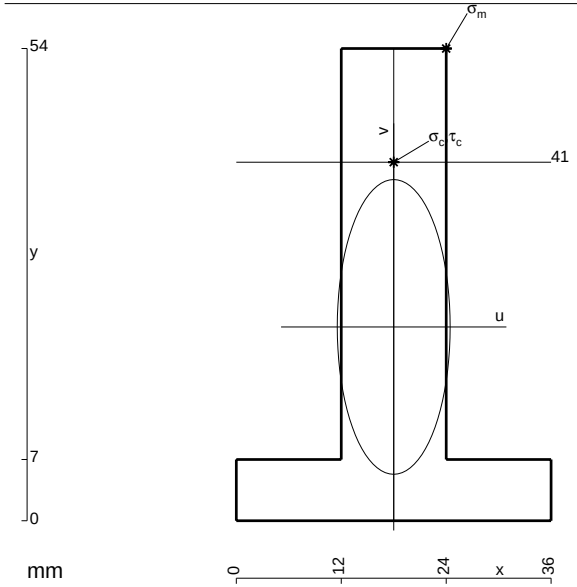
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

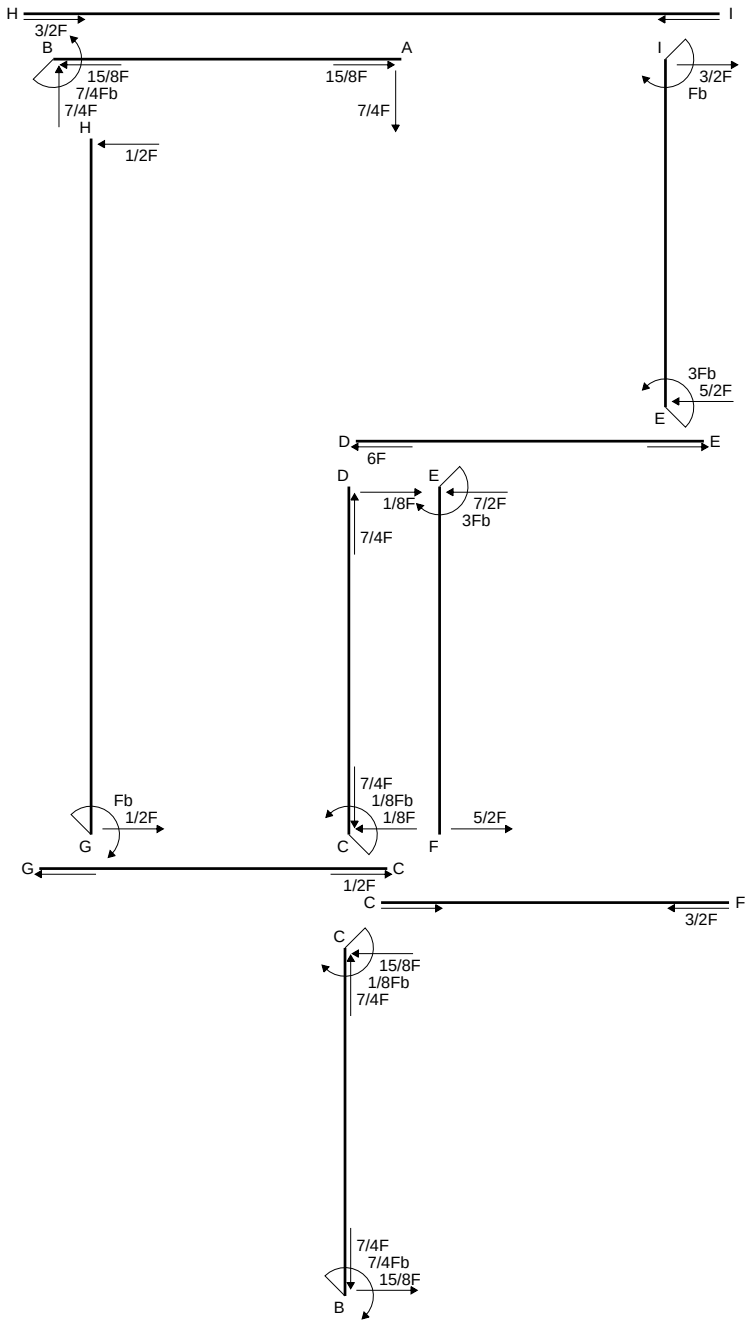


PROCEDIMENTO E RISULTATI 215204 CI01.xxxx.174



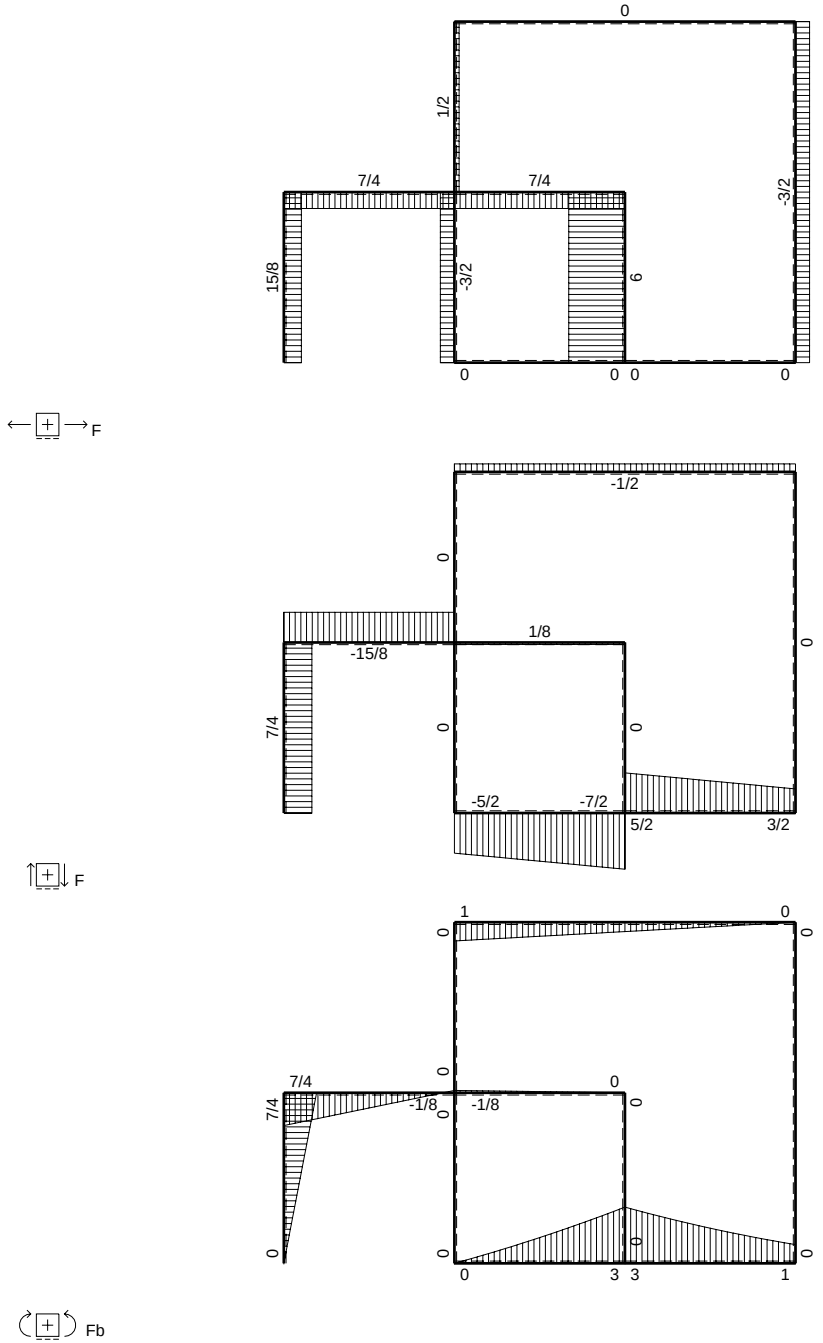

A = 816. $mm^2$
$J_u$ = 231827. $mm^4$
$J_v$ = 33984. $mm^4$
$y_g$ = 22.16 mm
$T_y$ = -3220. N
$M_x$ = 1738800. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 31.84 mm
$\sigma_m = -Mv/J_u$ = -238.8 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 41. mm
$v_c$ = 18.84 mm
$\sigma_c = -Mv/J_u$ = -141.3 $N/mm^2$
$\tau_c$ = 4.575 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 141.5 $N/mm^2$
$S^*$ = 3953. $mm^3$

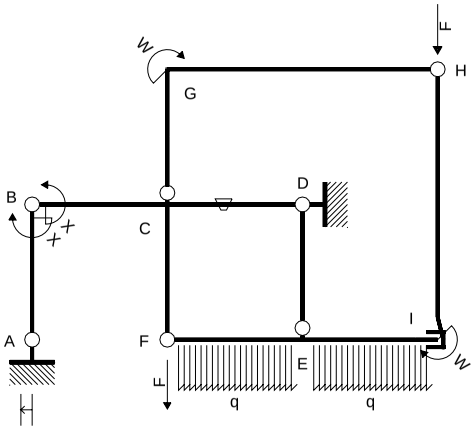
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H
I
3/2F
B
A
15/8F
15/8F
7/4Fb
7/4F
7/4F
H
1/2F
I
3/2F
Fb
3Fb
5/2F
E
D
E
6F
D
E
1/8F
7/2F
3Fb
7/4F
7/4F
1/8Fb
1/8F
5/2F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
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C
15/8F
1/8Fb
7/4F
7/4F
7/4Fb
15/8F
B

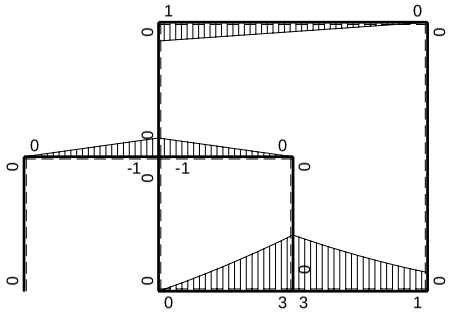




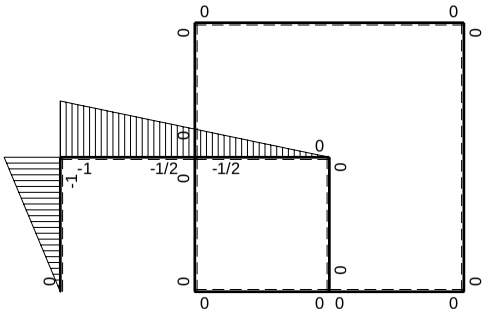

0
1/2
7/4
7/4
-3/2
15/8
-3/2
6
0
0 0
0
F
-1/2
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1/8
-15/8
0
7/4
0
0
-5/2
-7/2
5/2
3/2
F
1
0
0
0
0
7/4
0
7/4
-1/8
-1/8
0
0
0
0
0
3 3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 265726 CI01.xxxx.175

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PROCEDIMENTO E RISULTATI 265726 CI01.xxxx.175

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $7/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -7/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

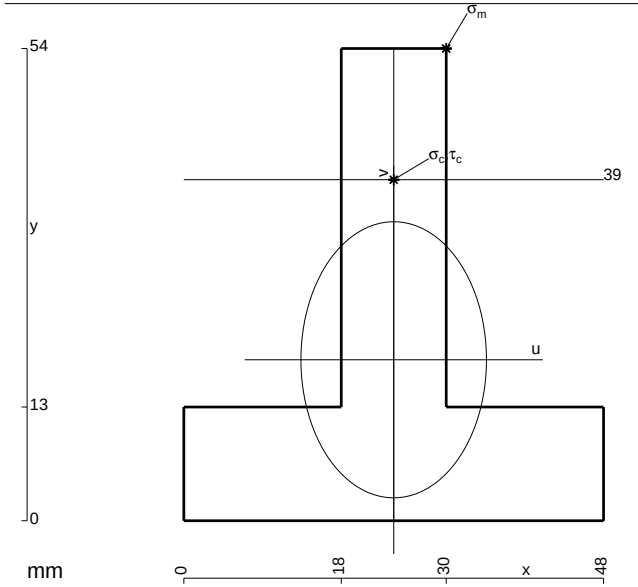
$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

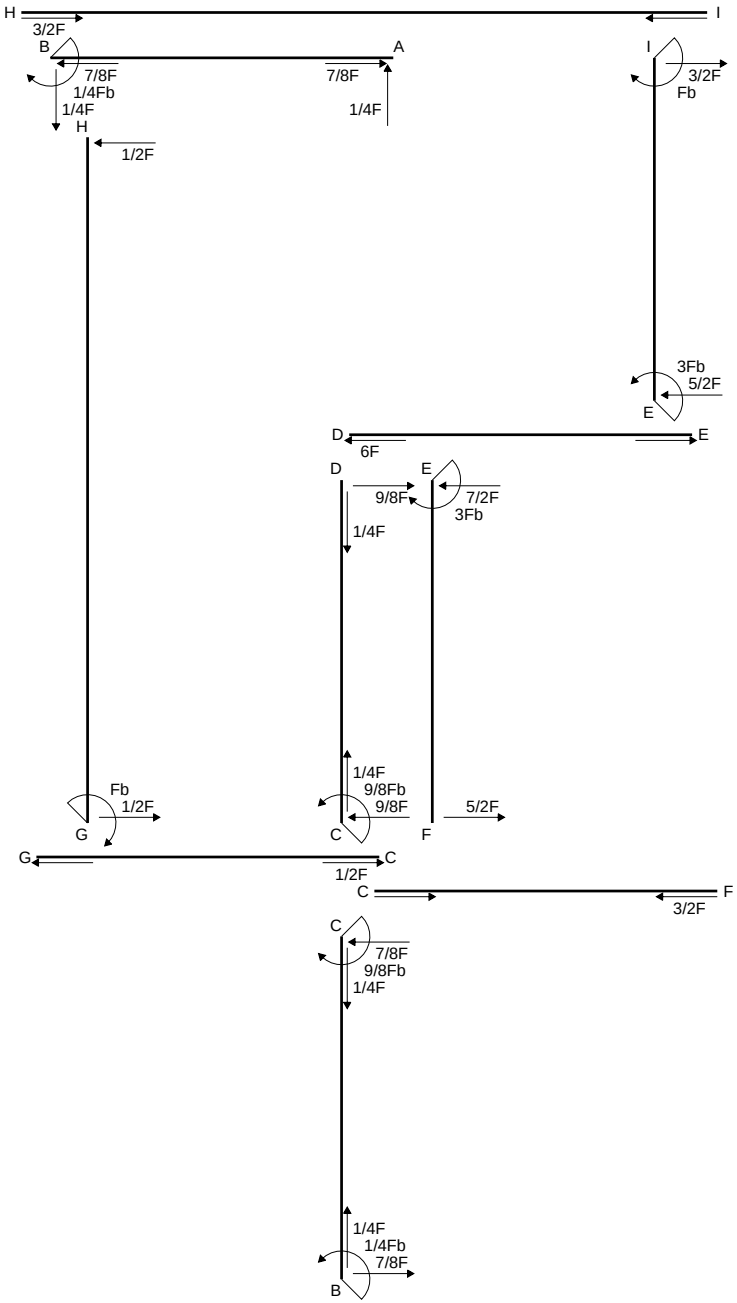


PROCEDIMENTO E RISULTATI 265726 CI01.xxxx.175



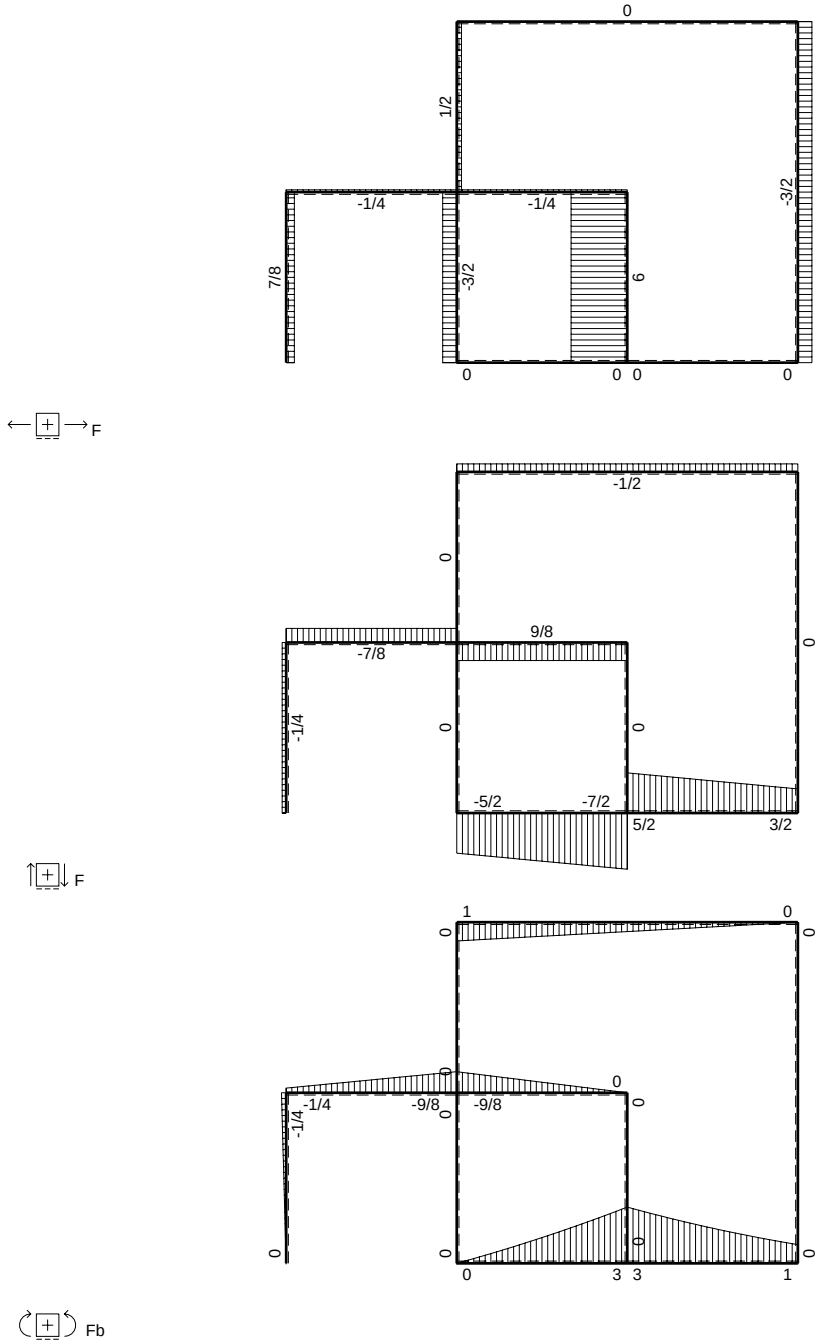

A = 1116. $mm^2$
$J_u$ = 278255. $mm^4$
$J_v$ = 125712. $mm^4$
$y_g$ = 18.4 mm
$T_y$ = -2695. N
$M_x$ = 1547700. Nmm
$x_m$ = 30. mm
$y_m$ = 54. mm
$u_m$ = 6. mm
$v_m$ = 35.6 mm
$\sigma_m = -Mv/J_u$ = -198. $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 39. mm
$v_c$ = 20.6 mm
$\sigma_c = -Mv/J_u$ = -114.6 $N/mm^2$
$\tau_c$ = 4.082 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.8 $N/mm^2$
$S^*$ = 5057. $mm^3$

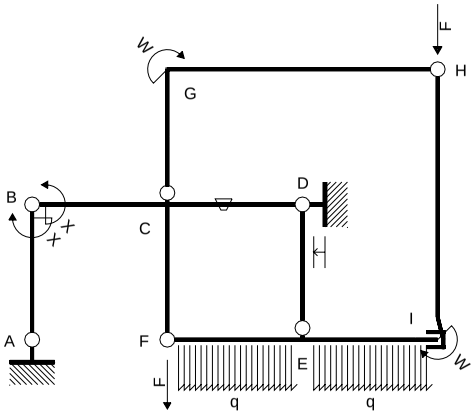
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26


H
3/2F
B
7/8F
1/4Fb
1/4F
H
1/2F
A
7/8F
1/4F
I
I
3/2F
Fb
3Fb
5/2F
E
D
6F
E
D
E
9/8F
7/2F
3Fb
1/4F
1/4F
9/8Fb
9/8F
C
F
5/2F
Fb
1/2F
G
G
C
1/2F
C
F
3/2F
C
7/8F
9/8Fb
1/4F
1/4F
1/4Fb
7/8F
B

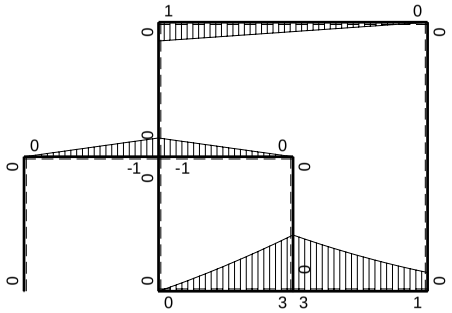




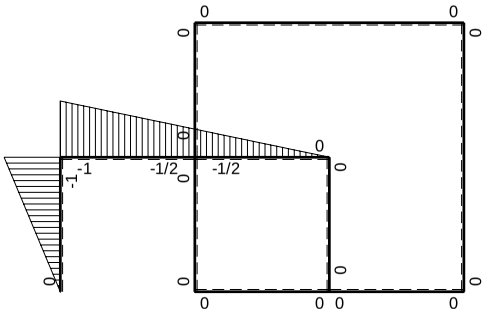

F
F
Fb
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1/2
-1/4
-1/4
-3/2
7/8
-3/2
6
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0
-1/2
0
9/8
-7/8
0
0
-1/4
0
-5/2
-7/2
5/2
3/2
1
0
0
0
0
-1/4
-9/8
-9/8
0
0
0
0
0
3 3
1




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 270371 CI01.xxxx.176

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PROCEDIMENTO E RISULTATI 270371 CI01.xxxx.176

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -Fx | 0 | $Fx-1/2Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(1/3+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | Fb-Fx | 0 | $1/2Fb-1/2Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | -Fb/EJ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | Fb/EJ | $1/2Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | $3Fb-7/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | $-5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/4Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/4Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b ( x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

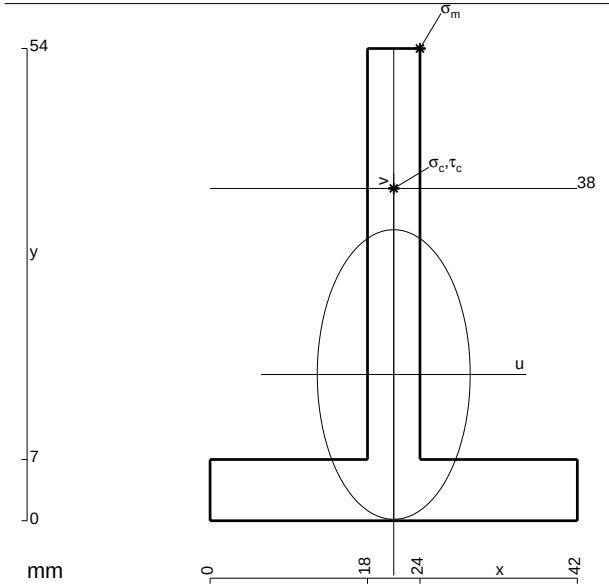
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$

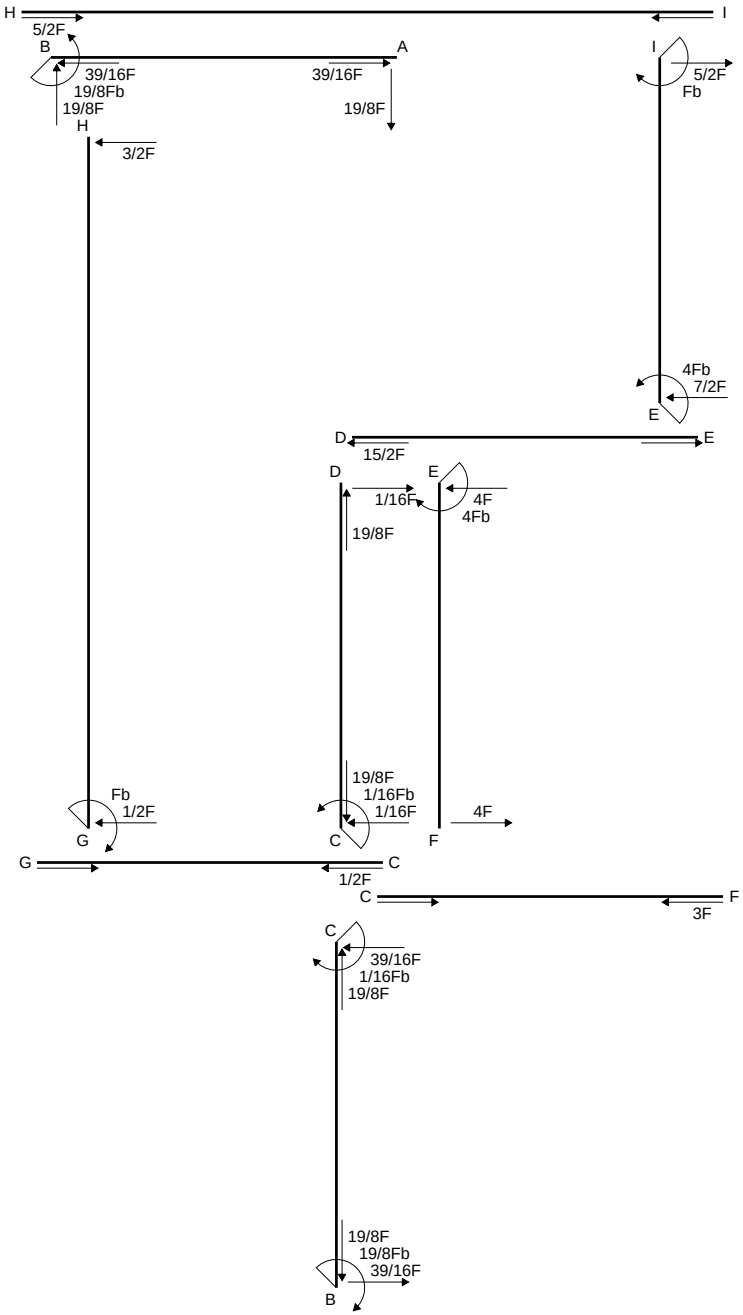


PROCEDIMENTO E RISULTATI 270371 CI01.xxxx.176



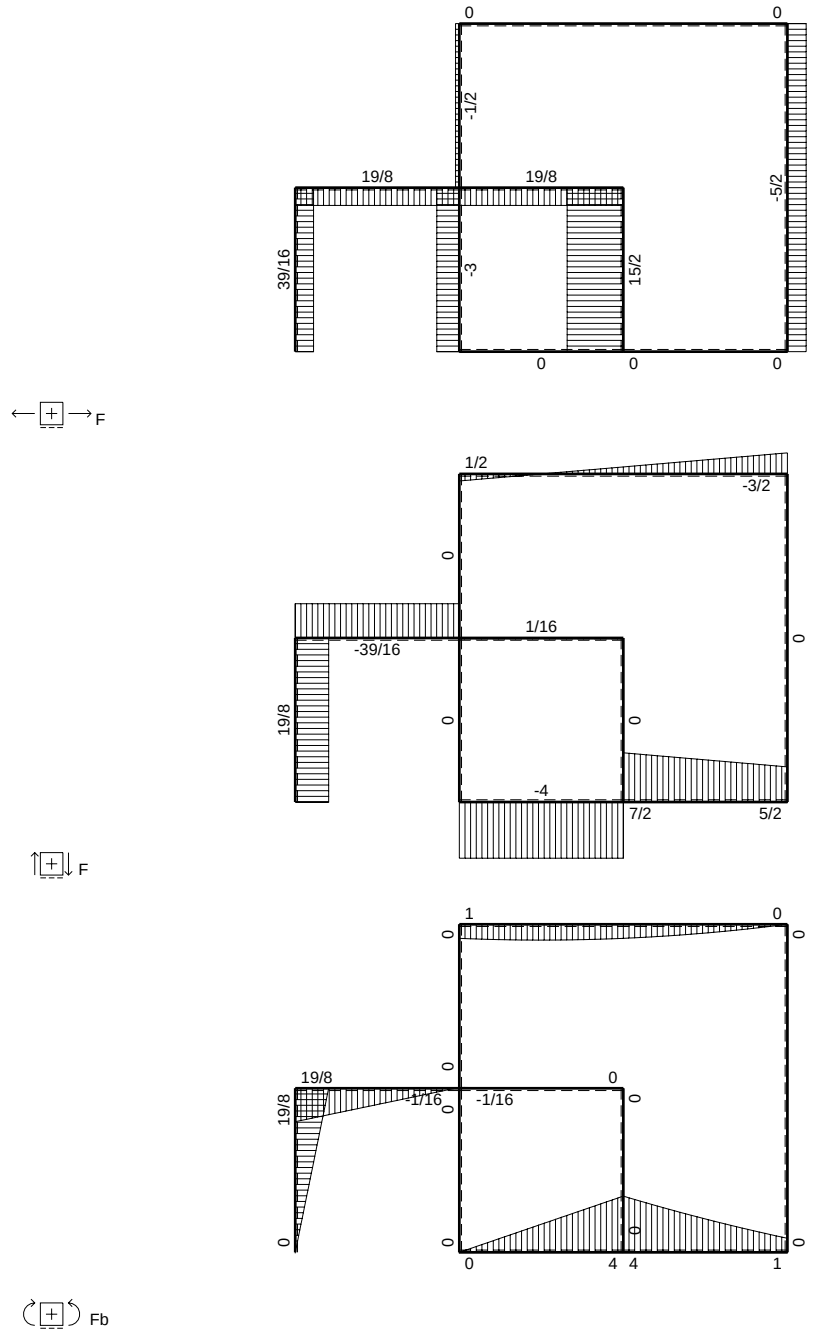

A = 576. $mm^2$
$J_u$ = 158042. $mm^4$
$J_v$ = 44064. $mm^4$
$y_g$ = 16.72 mm
$T_y$ = -1435. N
$M_x$ = 885600. Nmm
$x_m$ = 24. mm
$y_m$ = 54. mm
$u_m$ = 3. mm
$v_m$ = 37.28 mm
$\sigma_m = -Mv/J_u$ = -208.9 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 38. mm
$v_c$ = 21.28 mm
$\sigma_c = -Mv/J_u$ = -119.3 $N/mm^2$
$\tau_c$ = 4.254 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.5 $N/mm^2$
$S^*$ = 2811. $mm^3$

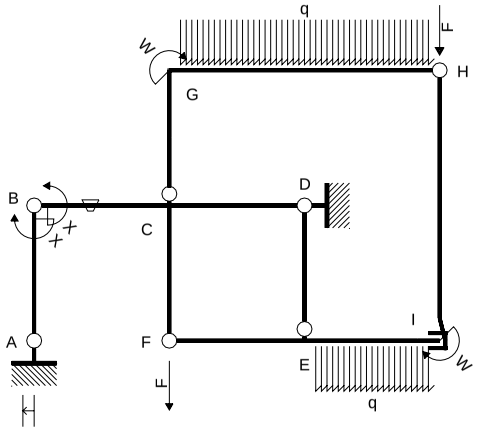
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H
I
5/2F
B
A
39/16F
19/8Fb
19/8F
39/16F
19/8F
I
5/2F
Fb
H
3/2F
4Fb
7/2F
E
D
E
15/2F
D
E
1/16F
4F
4Fb
19/8F
19/8F
1/16Fb
1/16F
Fb
1/2F
G
C
F
4F
G
C
1/2F
C
F
3F
C
39/16F
1/16Fb
19/8F
19/8F
19/8Fb
39/16F
B

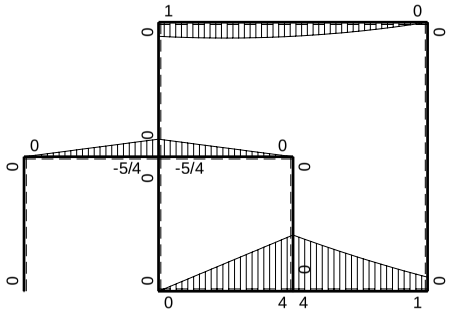




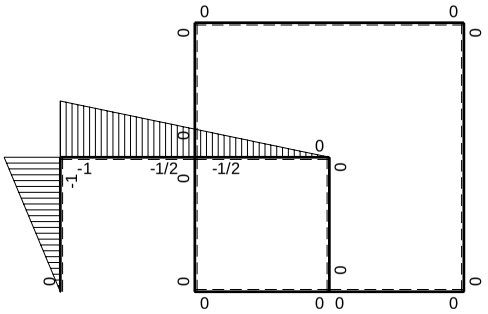

0
0
-1/2
19/8
19/8
-5/2
39/16
-3
15/2
0
0
0
F
1/2
-3/2
0
1/16
-39/16
19/8
0
0
-4
7/2
5/2
F
1
0
0
0
19/8
0
19/8
-1/16
-1/16
0
0
0
0
0
0
4
4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 306451 CI01.xxxx.177

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PROCEDIMENTO E RISULTATI 306451 CI01.xxxx.177

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+7/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $19/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -19/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

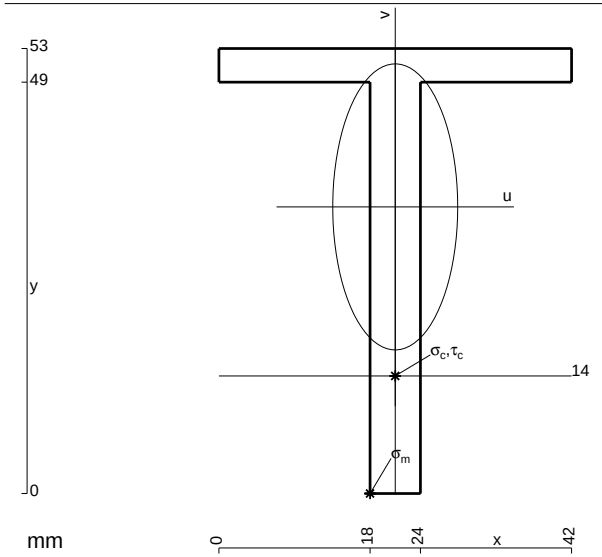
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/24\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

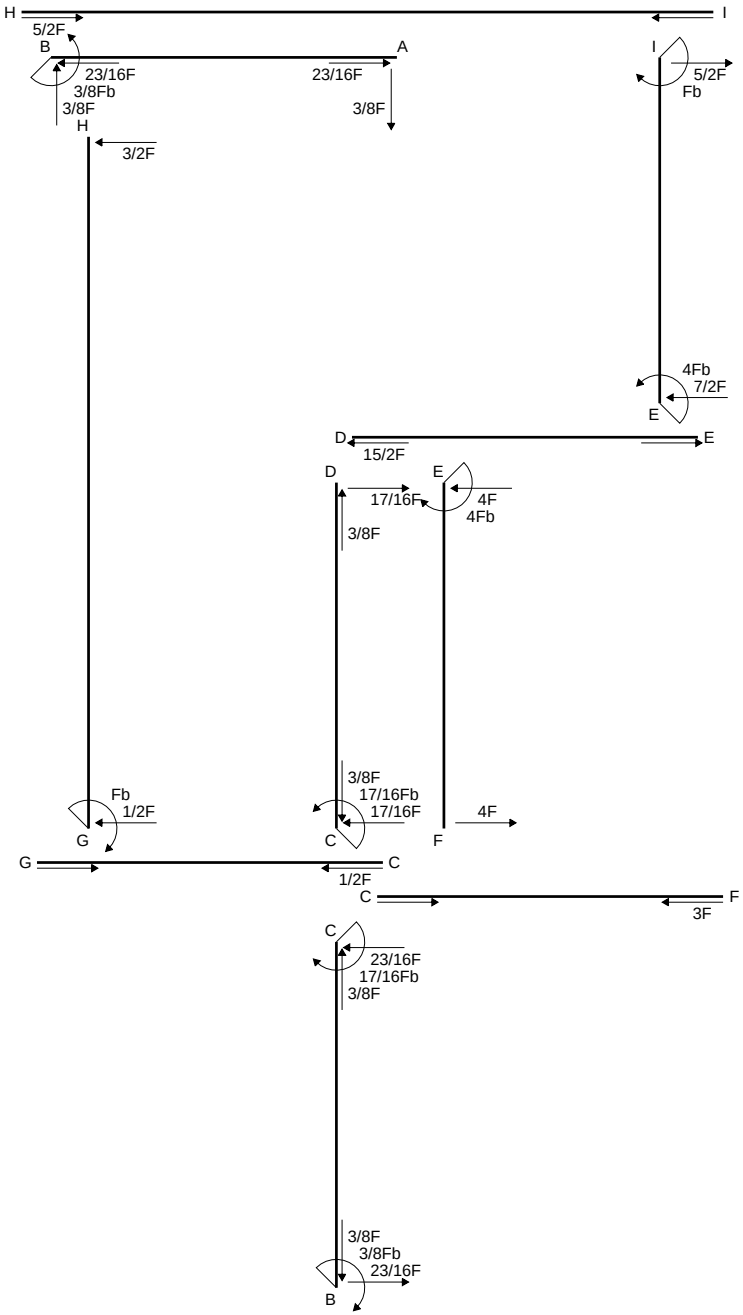


PROCEDIMENTO E RISULTATI 306451 CI01.xxxx.177



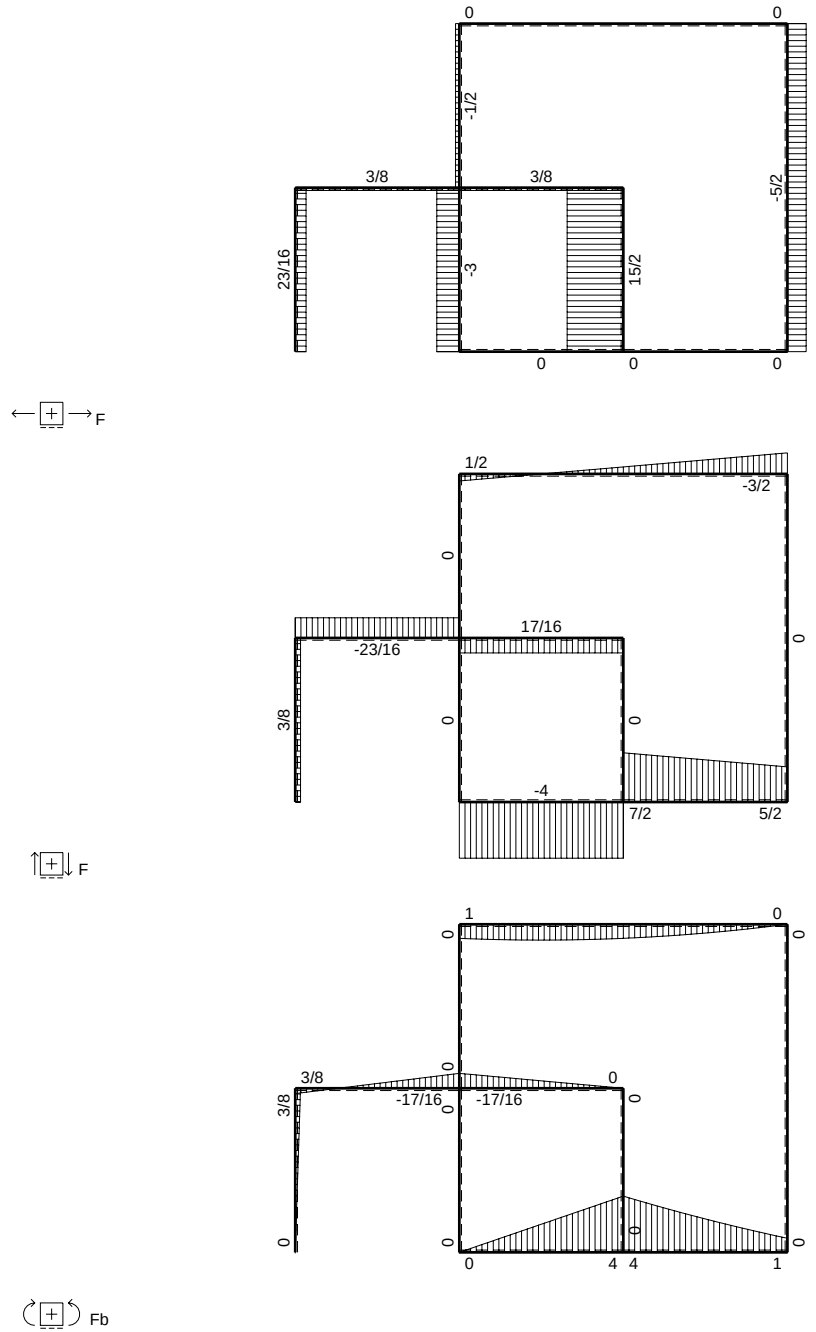

A = 462. $mm^2$
$J_u$ = 134125. $mm^4$
$J_v$ = 25578. $mm^4$
$y_g$ = 34.14 mm
$T_y$ = -1160. N
$M_x$ = 858400. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -34.14 mm
$\sigma_m = -Mv/J_u$ = 218.5 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 14. mm
$v_c$ = -20.14 mm
$\sigma_c = -Mv/J_u$ = 128.9 $N/mm^2$
$\tau_c$ = 3.286 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129. $N/mm^2$
$S^*$ = 2279. $mm^3$

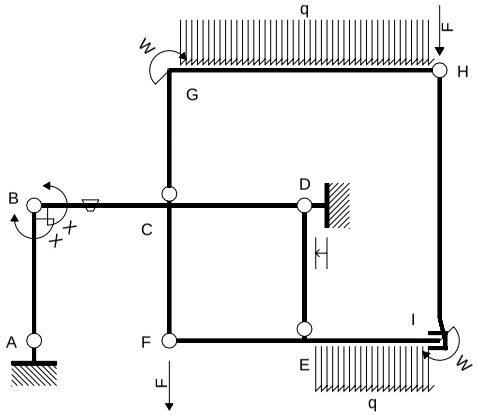
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H
I
5/2F
B
A
23/16F
3/8Fb
3/8F
23/16F
3/8F
I
5/2F
Fb
H
3/2F
4Fb
7/2F
E
D
E
15/2F
D
E
17/16F
4F
4Fb
3/8F
3/8F
17/16Fb
17/16F
Fb
1/2F
4F
G
C
F
G
C
1/2F
C
F
3F
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23/16F
17/16Fb
3/8F
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3/8Fb
23/16F
B

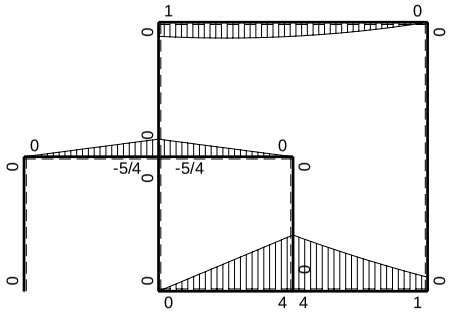




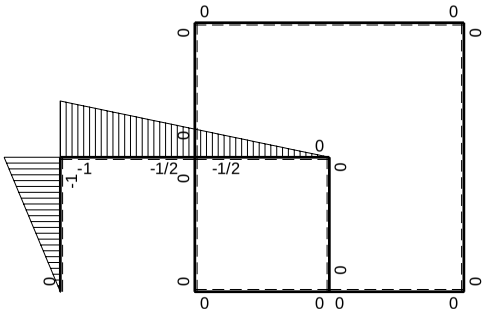

0
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-5/2
23/16
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0
17/16
-23/16
0
3/8
0
0
-4
7/2
5/2
F
1
0
0
0
3/8
0
-17/16
-17/16
0
3/8
0
0
0
0
0
4
4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 268272 CI01.xxxx.178

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PROCEDIMENTO E RISULTATI 268272 CI01.xxxx.178

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | -Fb/EJ | $5/4Fx-5/8Fx^2/b$ | Fb/EJ-1/2Fx/EJ | $1-x/b+1/4x^2/b^2$ | $(5/12+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | Fb/EJ | $5/8Fb-5/8Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+7/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $3/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -3/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

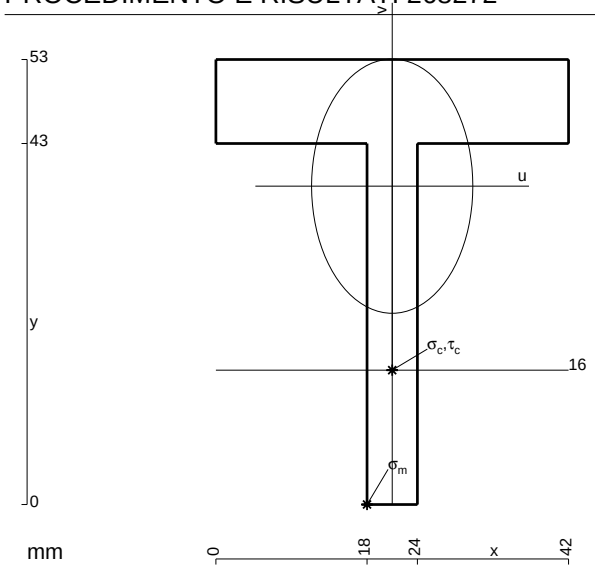
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

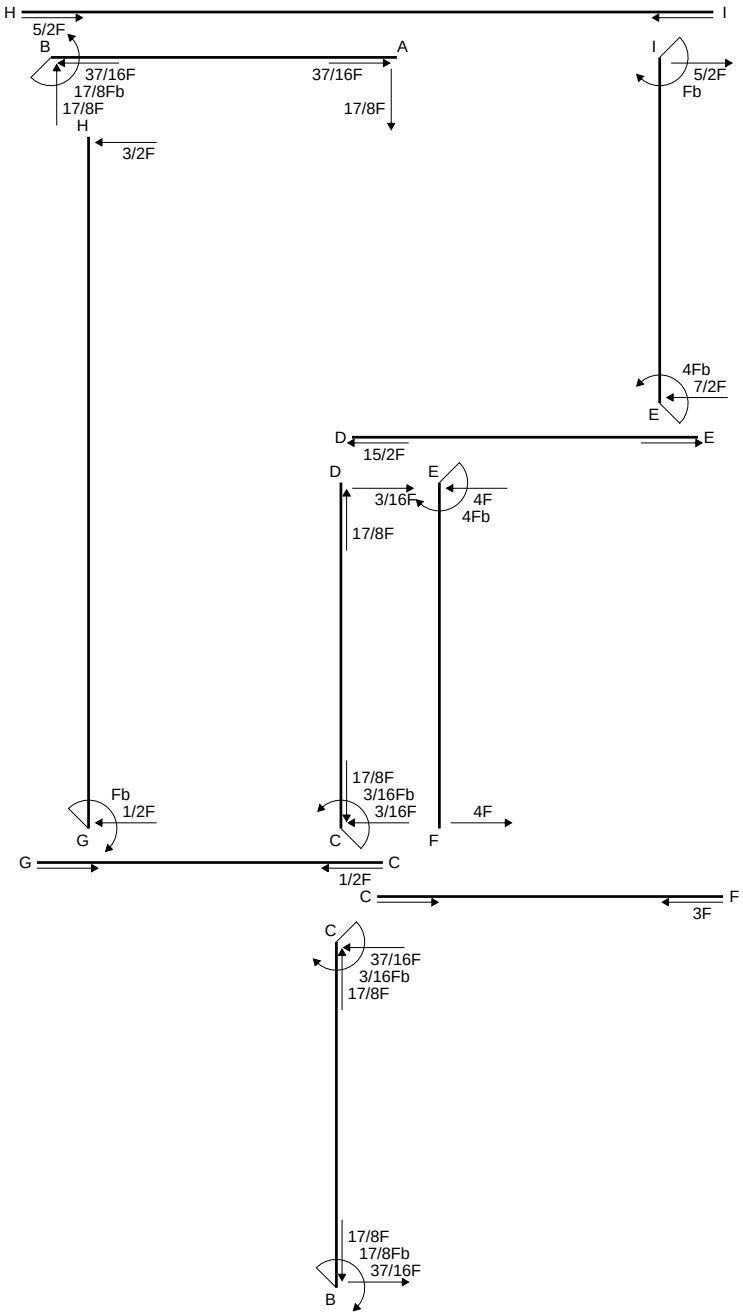
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





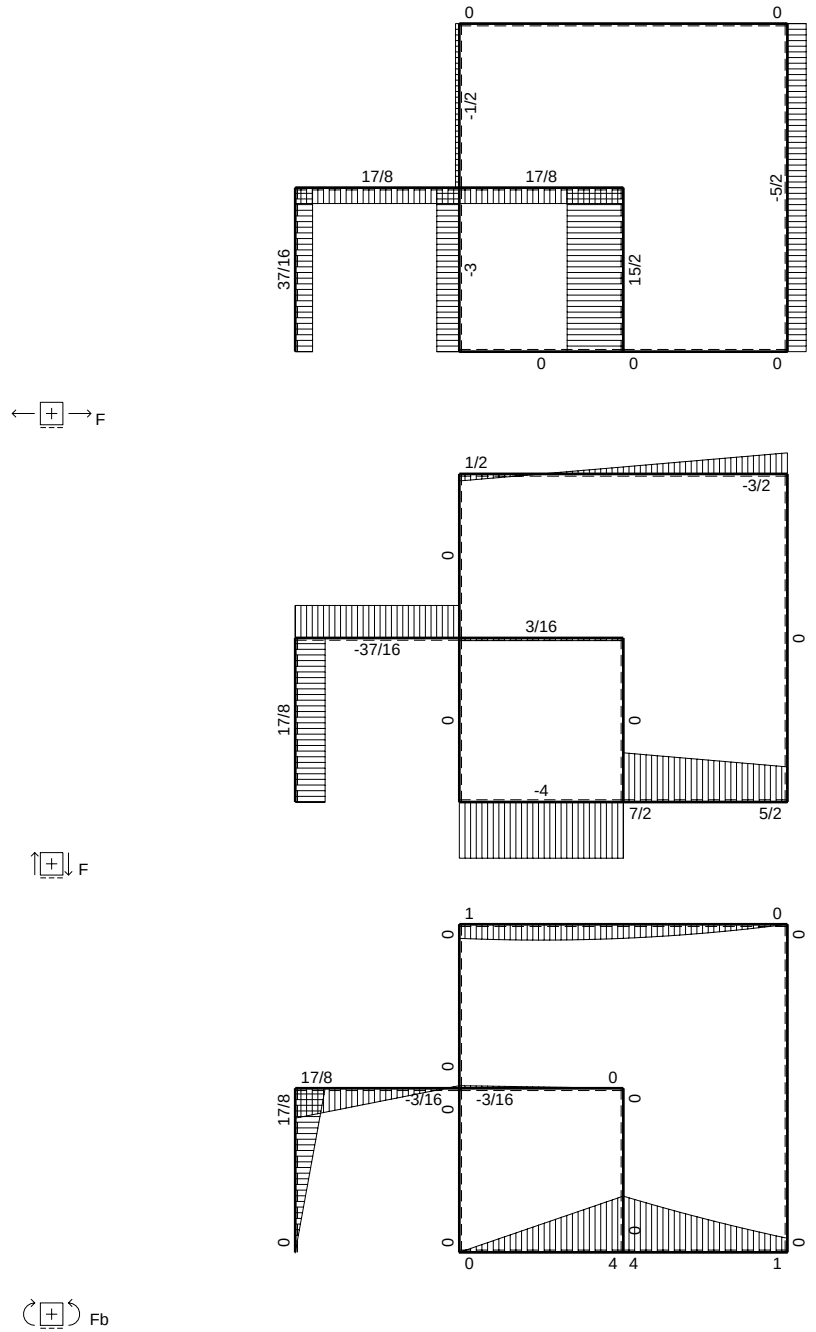


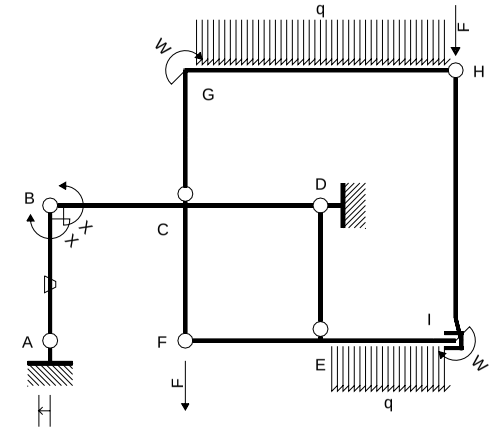

A = 678. $mm^2$
$J_u$ = 155489. $mm^4$
$J_v$ = 62514. $mm^4$
$y_g$ = 37.92 mm
$T_y$ = -1160. N
$M_x$ = 916400. Nmm
$x_m$ = 18. mm
$u_m$ = -3. mm
$v_m$ = -37.92 mm
$\sigma_m = -Mv/J_u$ = 223.5 $N/mm^2$
$x_c$ = 21. mm
$y_c$ = 16. mm
$v_c$ = -21.92 mm
$\sigma_c = -Mv/J_u$ = 129.2 $N/mm^2$
$\tau_c$ = 3.571 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 129.3 $N/mm^2$
$S^*$ = 2872. $mm^3$


H
I
5/2F
B
A
37/16F
17/8Fb
17/8F
37/16F
17/8F
I
5/2F
Fb
H
3/2F
4Fb
7/2F
E
D
E
15/2F
D
E
3/16F
4F
4Fb
17/8F
17/8F
3/16Fb
3/16F
Fb
1/2F
4F
G
C
F
G
C
1/2F
C
F
3F
C
37/16F
3/16Fb
17/8F
17/8F
17/8Fb
37/16F
B

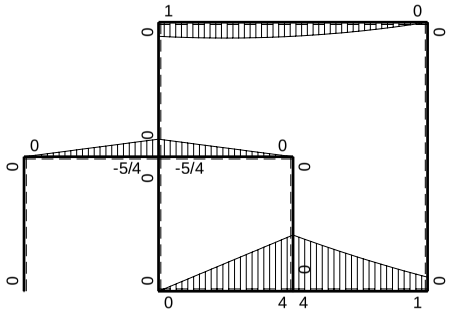




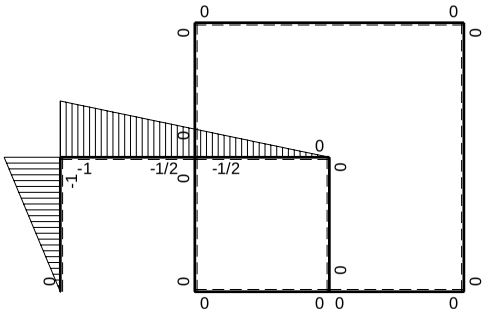

0
0
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17/8
17/8
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37/16
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15/2
0
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0
F
1/2
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0
3/16
-37/16
17/8
0
0
-4
7/2
5/2
F
1
0
0
0
17/8
0
17/8
-3/16
-3/16
0
0
0
0
0
0
4
4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 267819 CI01.xxxx.179

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+7/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $17/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -17/8Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

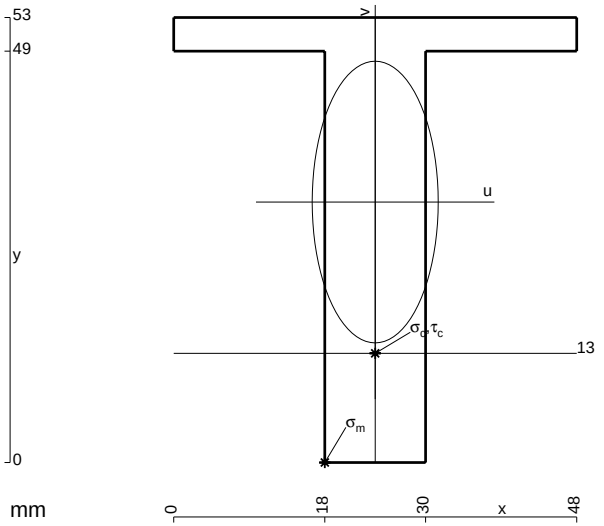
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

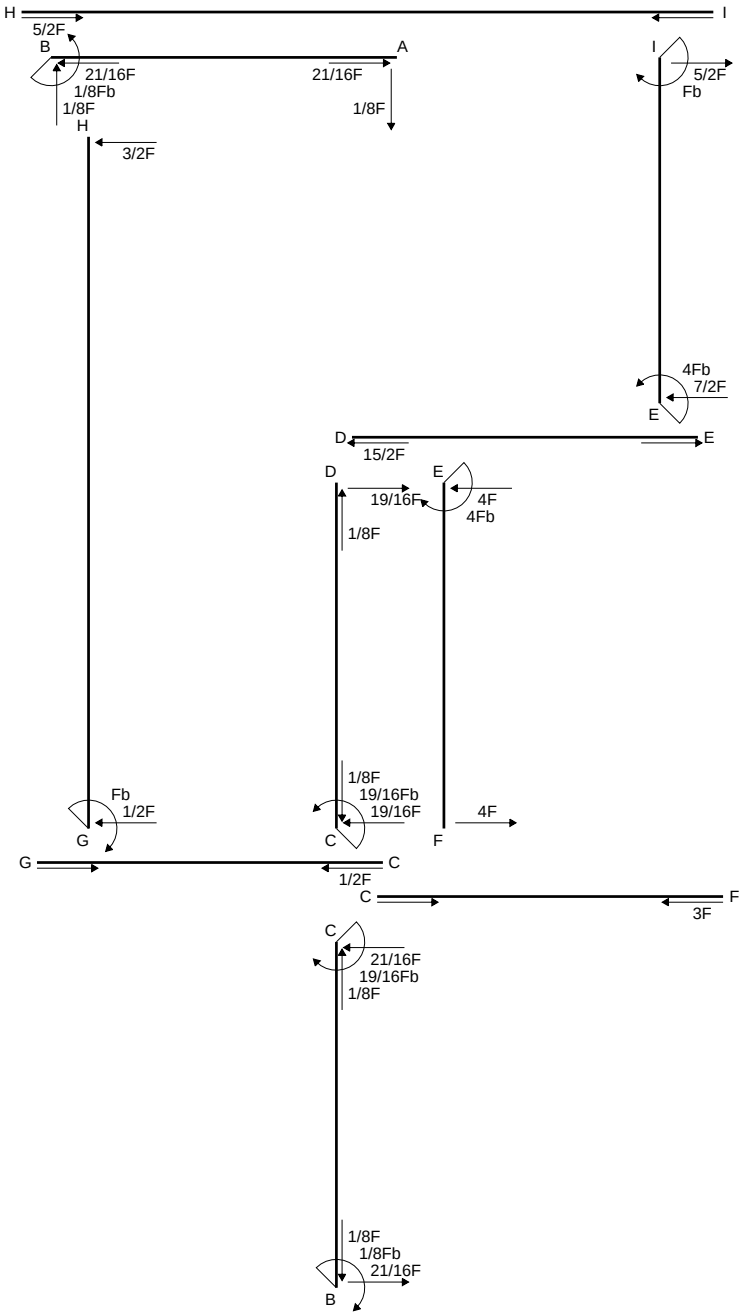
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





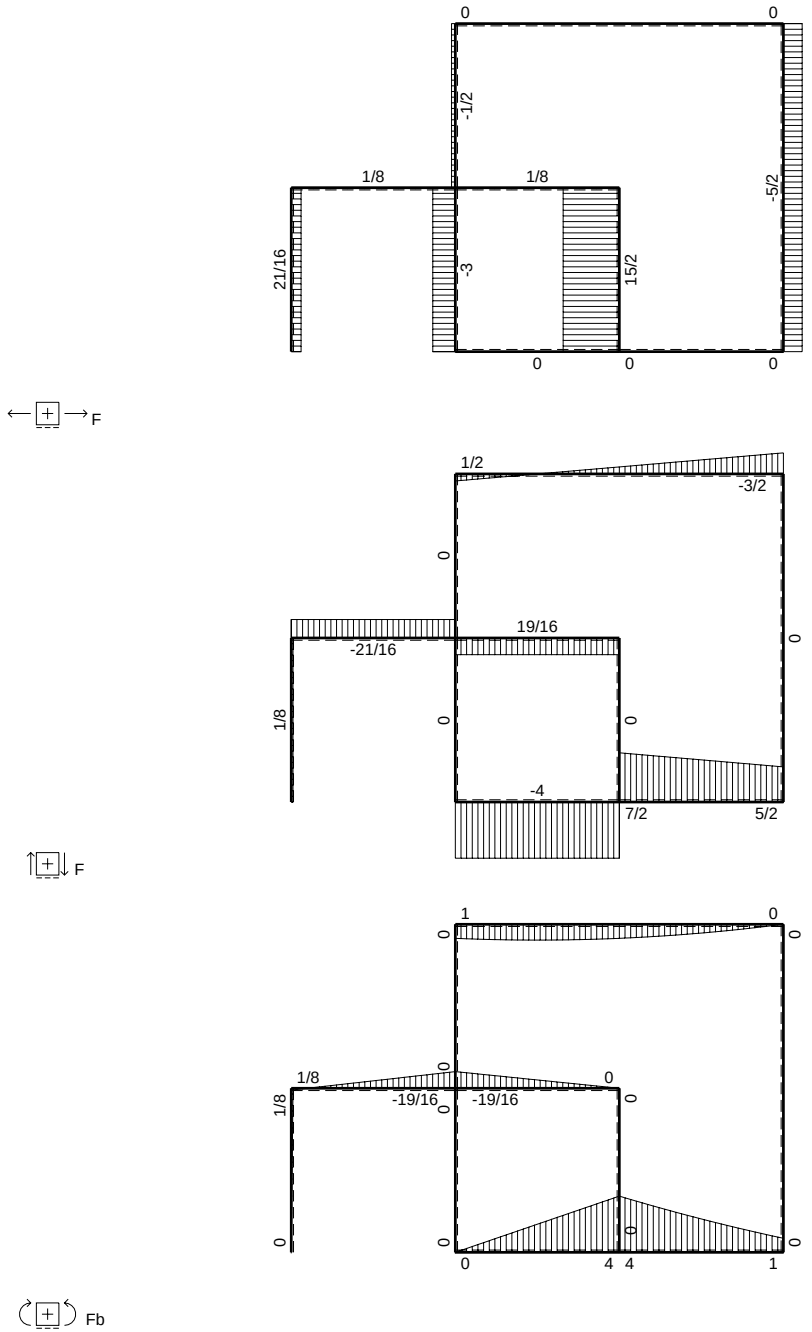


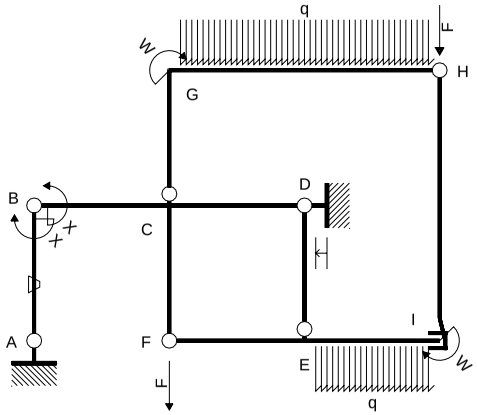

A = 780. $mm^2$
$J_u$ = 219548. $mm^4$
$J_v$ = 43920. $mm^4$
$y_g$ = 31.02 mm
$T_y$ = -2040. N
$M_x$ = 1693200. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -31.02 mm
$\sigma_m = -Mv/J_u$ = 239.3 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 13. mm
$v_c$ = -18.02 mm
$\sigma_c = -Mv/J_u$ = 139. $N/mm^2$
$\tau_c$ = 2.962 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 139.1 $N/mm^2$
$S^*$ = 3826. $mm^3$


H
I
5/2F
B
A
21/16F
1/8Fb
1/8F
21/16F
1/8F
I
5/2F
Fb
H
3/2F
4Fb
7/2F
E
D
E
15/2F
D
E
19/16F
4F
4Fb
1/8F
1/8F
19/16Fb
19/16F
4F
Fb
1/2F
G
C
F
G
C
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3F
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21/16F
19/16Fb
1/8F
1/8F
1/8Fb
21/16F
B

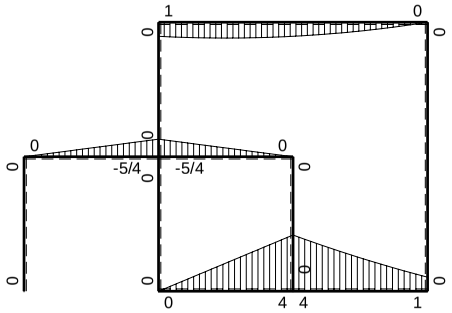




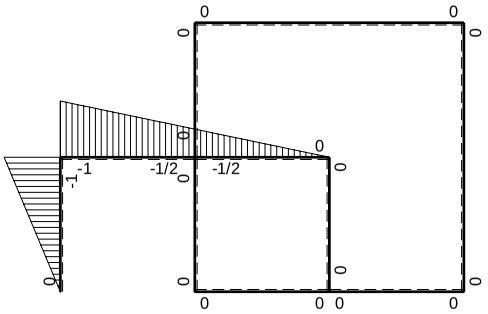

0
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19/16
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0
1/8
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0
-4
7/2
5/2
F
1
0
0
0
1/8
0
0
-19/16
-19/16
1/8
0
0
0
0
0
0
4
4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 264071 CI01.xxxx.180

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PROCEDIMENTO E RISULTATI 264071 CI01.xxxx.180

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | -Fb/EJ | 0 | Fx/EJ | $x^2/b^2$ | $(0+1/2)Fb^2/EJ$ | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | Fb/EJ | 0 | Fb/EJ-Fx/EJ | $1-2x/b+x^2/b^2$ | | |
| BC b | $-1+1/2x/b$ | $-5/4Fx$ | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | $1/2+1/2x/b$ | $5/4Fb-5/4Fx$ | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | $-1/2+1/2x/b$ | $-5/4Fb+5/4Fx$ | 0 | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | $1/2x/b$ | $5/4Fx$ | 0 | $5/8Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+7/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -1/8Fb | |

Sviluppi di calcolo iperstatica


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2 )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2 )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

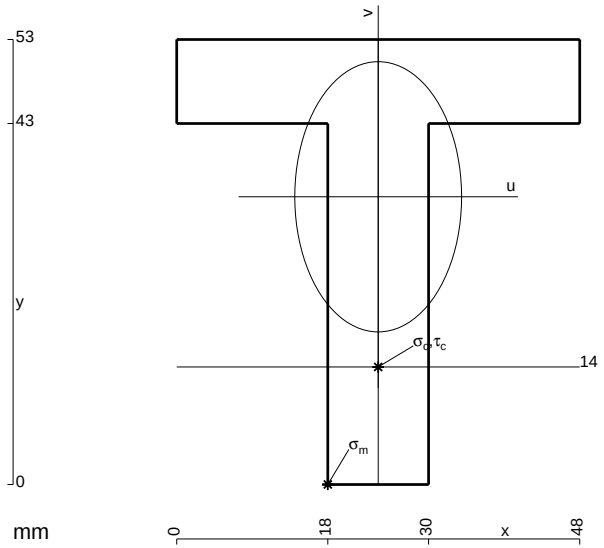
$= (5/8\ b -5/8\ b +5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2 )\ Fb\ 1/EJ\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

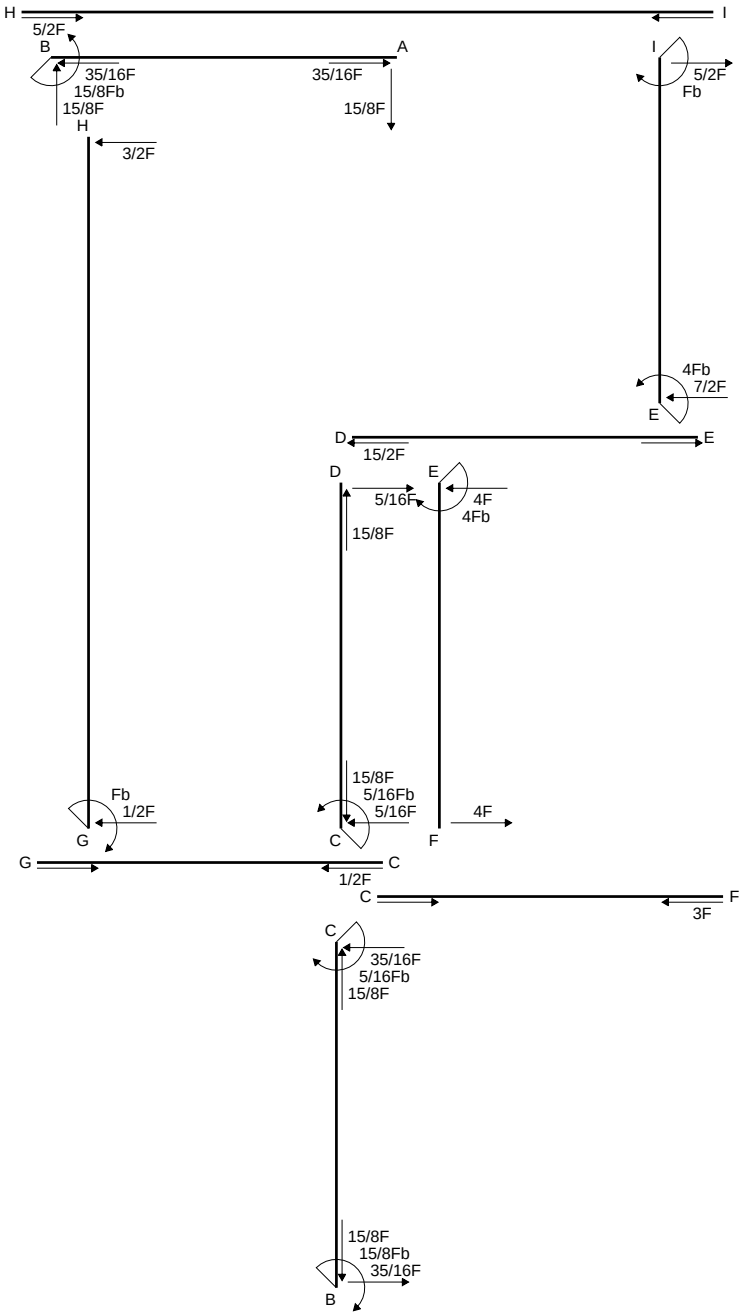
$= (5/24\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$





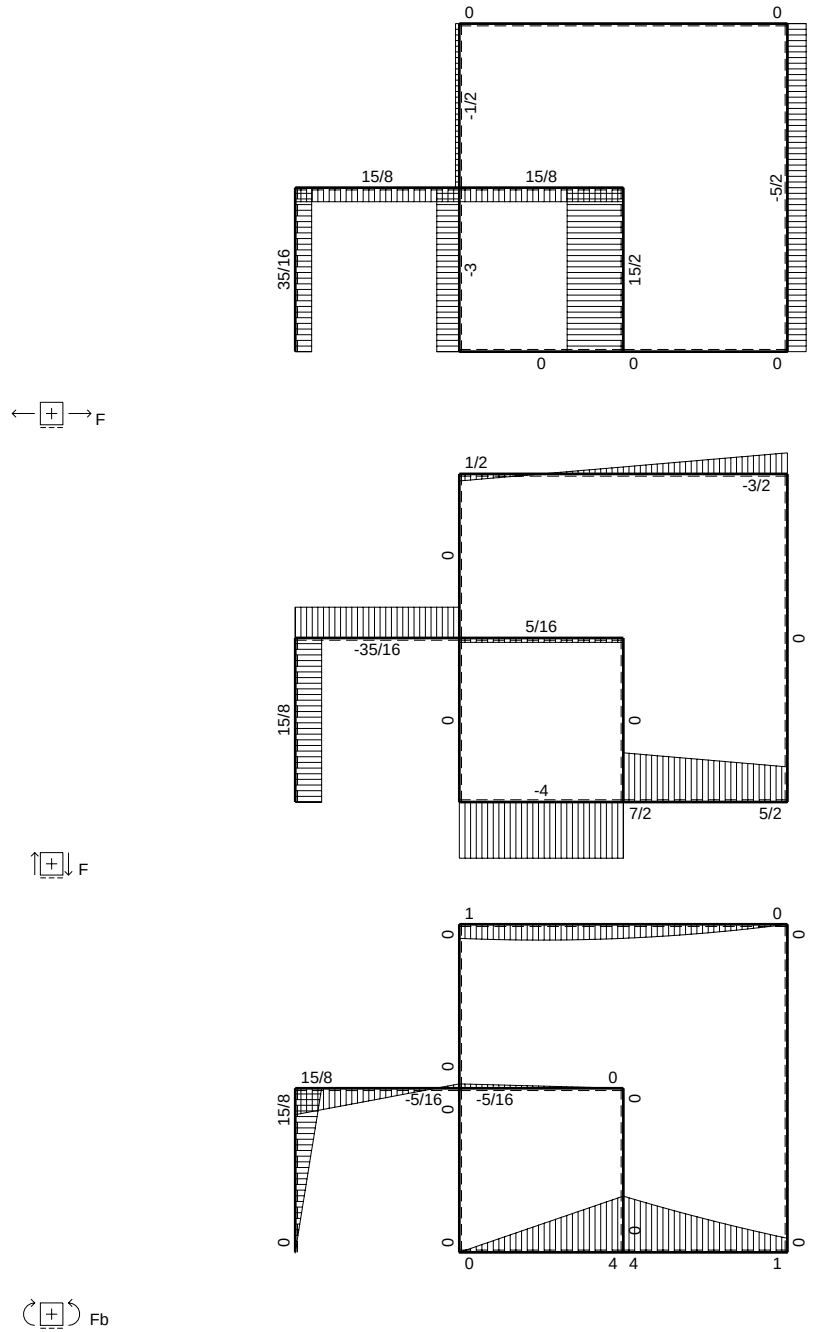


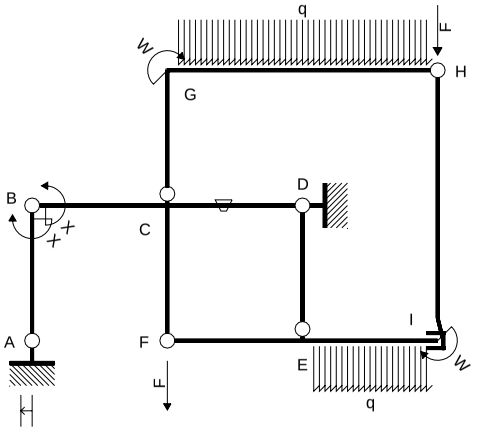

A = 996. $mm^2$
$J_u$ = 258139. $mm^4$
$J_v$ = 98352. $mm^4$
$y_g$ = 34.27 mm
$T_y$ = -3400. N
$M_x$ = 1496000. Nmm
$x_m$ = 18. mm
$u_m$ = -6. mm
$v_m$ = -34.27 mm
$\sigma_m = -Mv/J_u$ = 198.6 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 14. mm
$v_c$ = -20.27 mm
$\sigma_c = -Mv/J_u$ = 117.5 $N/mm^2$
$\tau_c$ = 5.029 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.8 $N/mm^2$
$S^*$ = 4582. $mm^3$


H
I
5/2F
B
A
35/16F
15/8Fb
15/8F
35/16F
15/8F
I
5/2F
Fb
H
3/2F
4Fb
7/2F
E
D
E
15/2F
D
E
5/16F
4F
4Fb
15/8F
15/8F
5/16Fb
5/16F
Fb
1/2F
G
C
F
4F
G
C
1/2F
C
F
3F
C
35/16F
5/16Fb
15/8F
15/8F
15/8Fb
35/16F
B

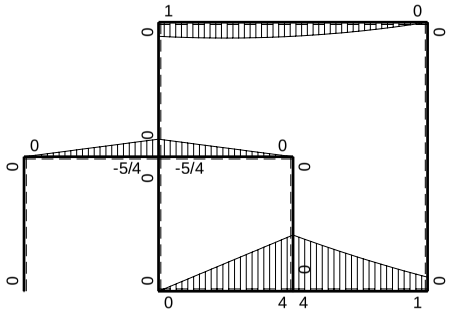




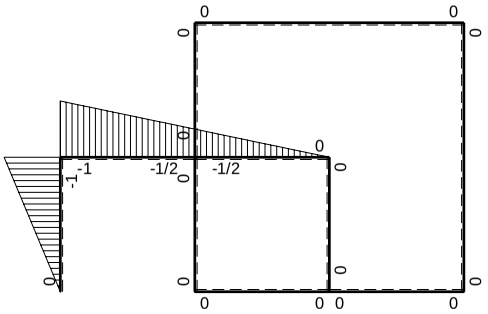

0
0
-1/2
15/8
15/8
-5/2
35/16
-3
15/2
0
0
0
F
1/2
-3/2
0
5/16
-35/16
15/8
0
0
-4
7/2
5/2
F
1
0
0
0
15/8
0
15/8
-5/16
-5/16
0
0
0
0
0
4
4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 274715 CI01.xxxx.181

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+7/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $15/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -15/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 274715 CI01.xxxx.181

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

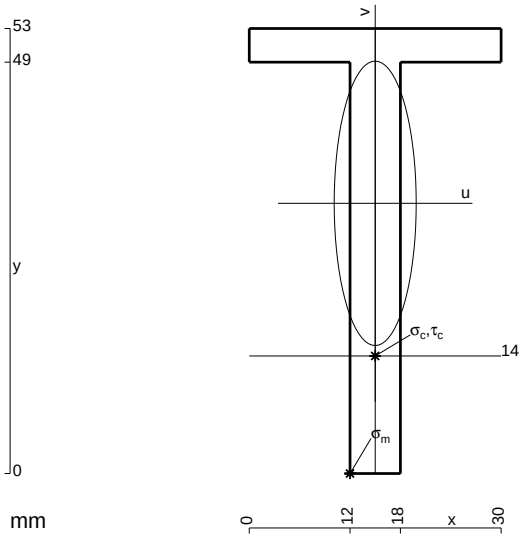
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

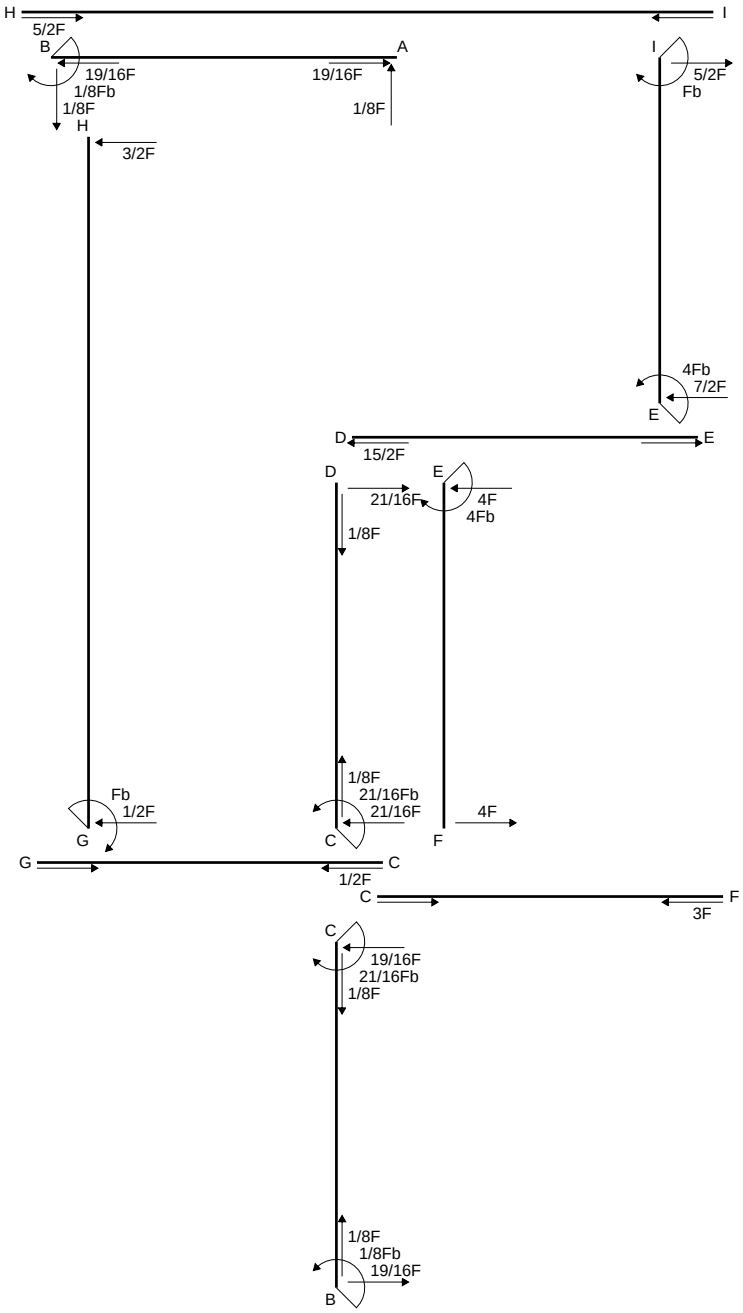
$= (5/24\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$





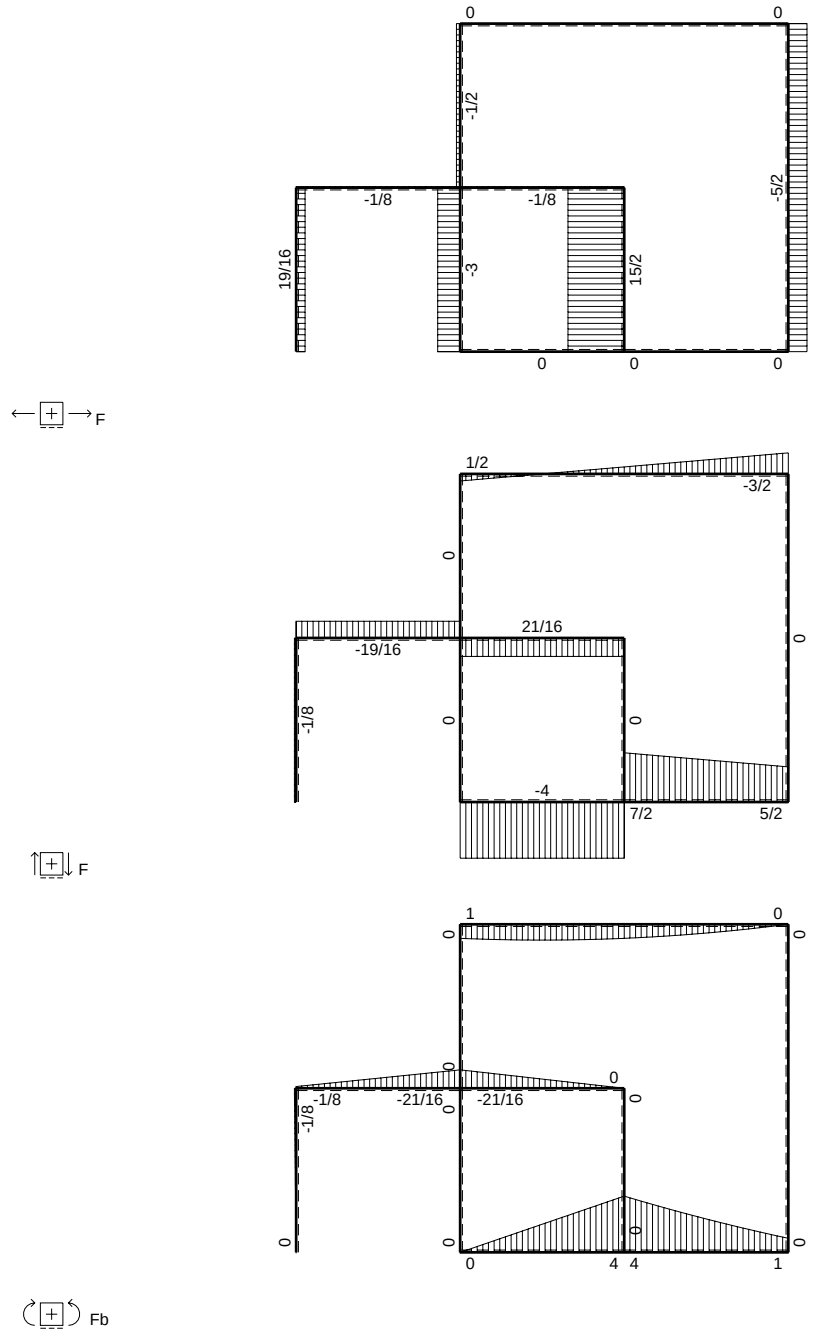


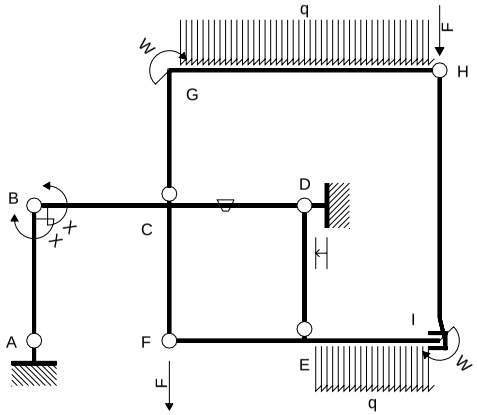

A = 414. $mm^2$
$J_u$ = 118828. $mm^4$
$J_v$ = 9882. $mm^4$
$y_g$ = 32.18 mm
$T_y$ = -1600. N
$M_x$ = 768000. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -32.18 mm
$\sigma_m = -Mv/J_u$ = 208. $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 14. mm
$v_c$ = -18.18 mm
$\sigma_c = -Mv/J_u$ = 117.5 $N/mm^2$
$\tau_c$ = 4.747 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 117.8 $N/mm^2$
$S^*$ = 2115. $mm^3$


H
I
5/2F
B
A
19/16F
1/8Fb
1/8F
19/16F
1/8F
I
5/2F
Fb
H
3/2F
4Fb
7/2F
E
D
E
15/2F
D
E
21/16F
4F
4Fb
1/8F
1/8F
21/16Fb
21/16F
Fb
1/2F
4F
G
C
F
G
C
1/2F
C
F
3F
C
19/16F
21/16Fb
1/8F
1/8F
1/8Fb
19/16F
B

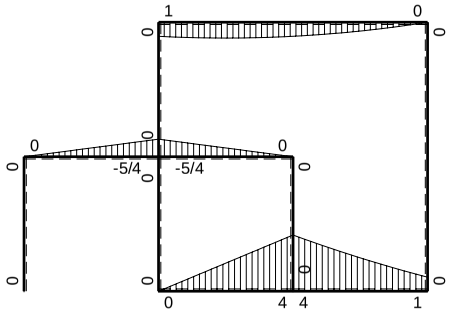




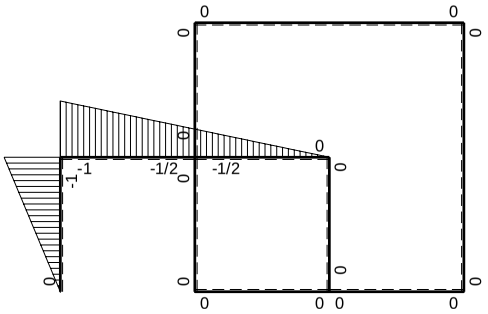

0
0
-1/2
-5/2
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-1/8
19/16
-3
15/2
0
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0
F
1/2
-3/2
0
21/16
-19/16
-1/8
0
0
-4
7/2
5/2
F
1
0
0
0
0
0
-1/8
-21/16
-21/16
0
0
0
0
0
0
4
4
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 247954 CI01.xxxx.182

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -5/4Fx | 0 | $5/4Fx-5/8Fx^2/b$ | 0 | $1-x/b+1/4x^2/b^2$ | $(5/12+0)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 5/4Fb-5/4Fx | 0 | $5/8Fb-5/8Fx^2/b$ | 0 | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -5/4Fb+5/4Fx | -Fb/EJ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | 5/4Fx | Fb/EJ | $5/8Fx^2/b$ | 1/2Fx/EJ | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 4Fb-4Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -4Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | $-3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+5/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-4Fb+7/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $-1/8Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | 1/8Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 247954 CI01.xxxx.182

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 16.04.26

$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

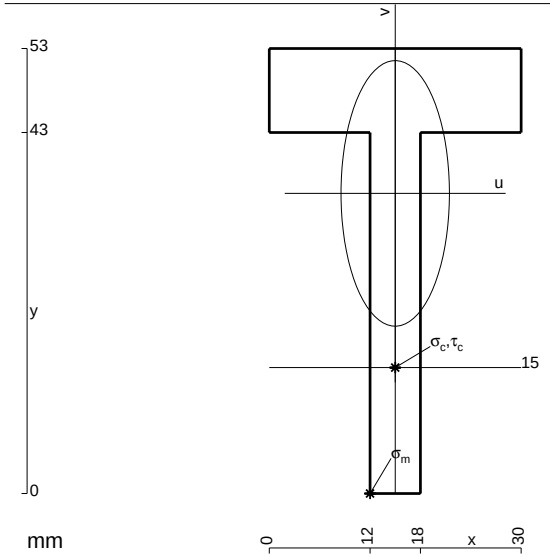
$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [5/24\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

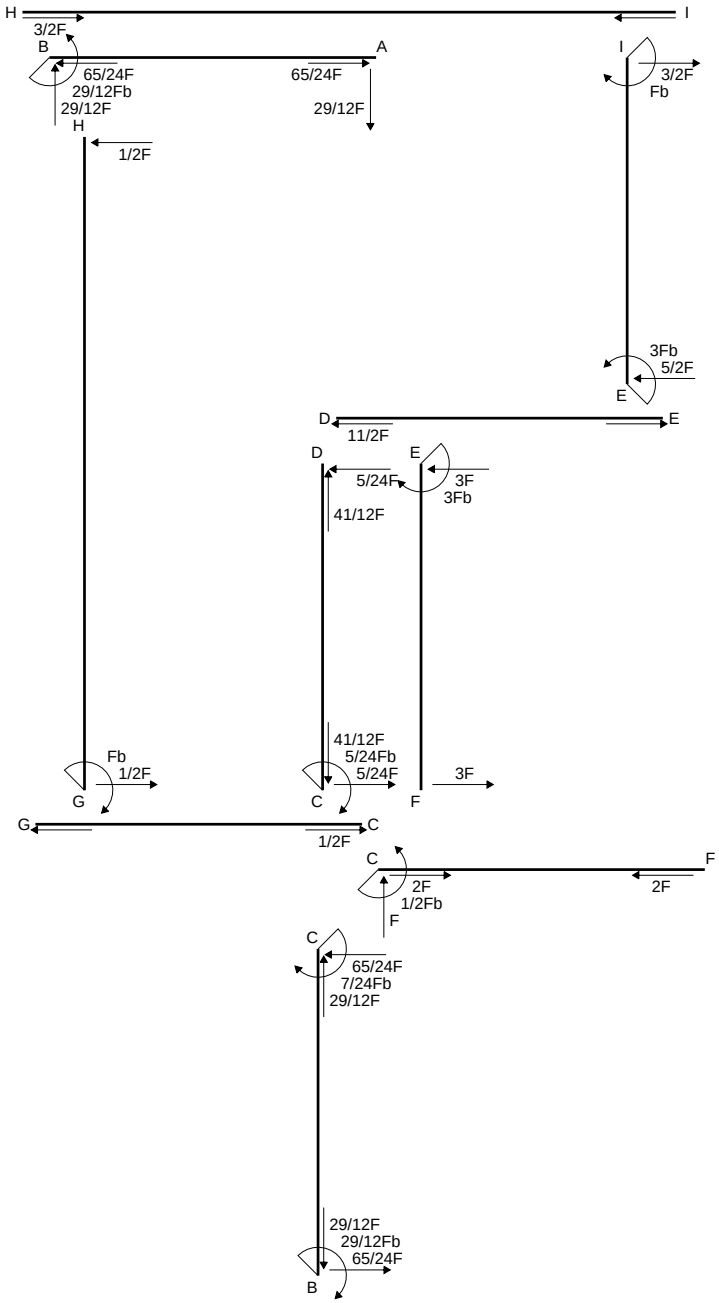
$= (5/24\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$





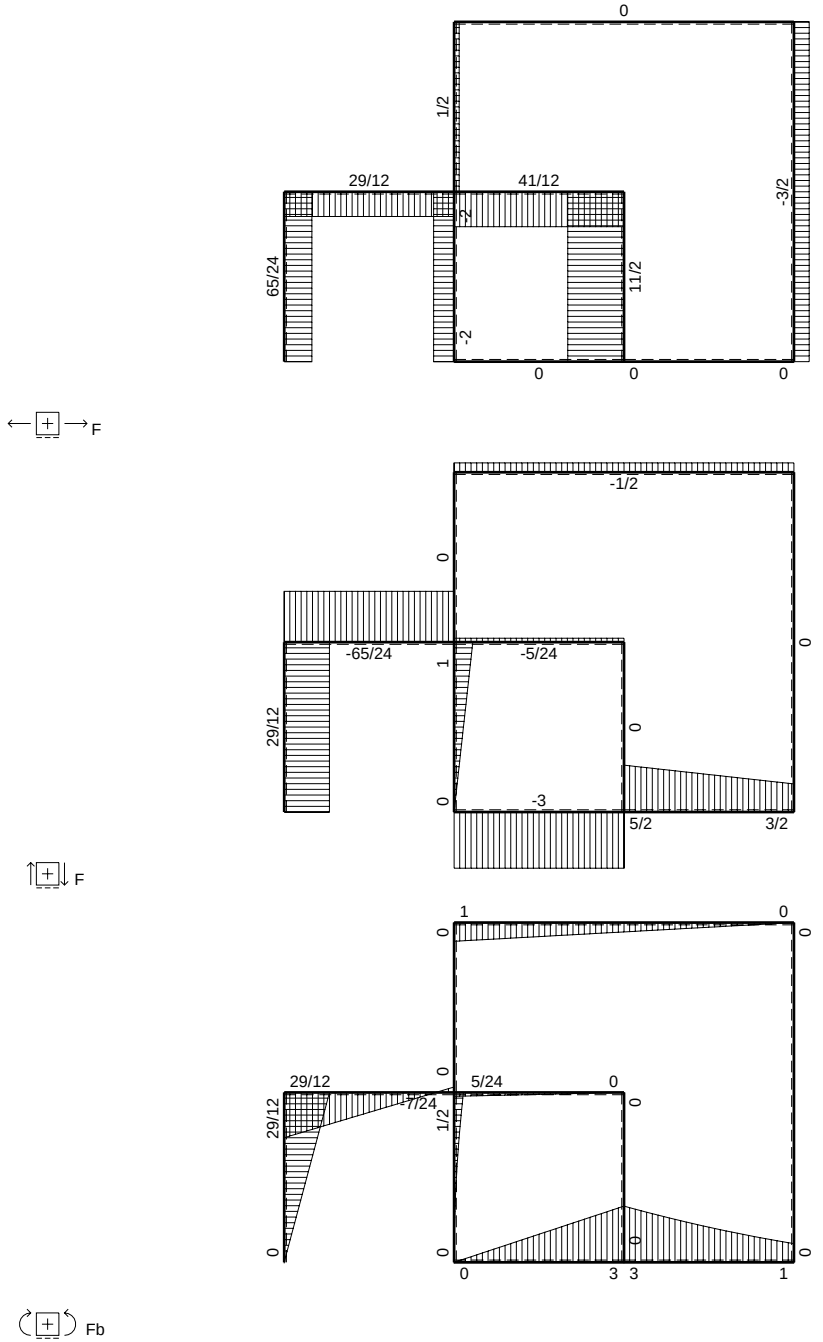


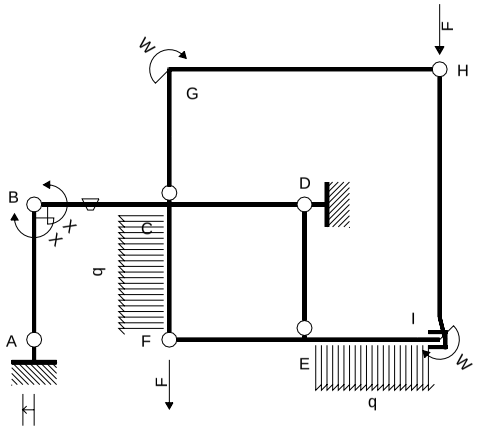

A = 558. $mm^2$
$J_u$ = 139662. $mm^4$
$J_v$ = 23274. $mm^4$
$y_g$ = 35.75 mm
$T_y$ = -1640. N
$M_x$ = 852800. Nmm
$x_m$ = 12. mm
$u_m$ = -3. mm
$v_m$ = -35.75 mm
$\sigma_m = -Mv/J_u$ = 218.3 $N/mm^2$
$x_c$ = 15. mm
$y_c$ = 15. mm
$v_c$ = -20.75 mm
$\sigma_c = -Mv/J_u$ = 126.7 $N/mm^2$
$\tau_c$ = 4.975 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127. $N/mm^2$
$S^*$ = 2542. $mm^3$


H
I
3/2F
B
A
65/24F
29/12Fb
29/12F
65/24F
29/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
5/24F
3F
3Fb
41/12F
41/12F
5/24Fb
5/24F
3F
Fb
1/2F
G
C
F
G
C
1/2F
C
F
2F
2F
1/2Fb
F
C
65/24F
7/24Fb
29/12F
29/12F
29/12Fb
65/24F
B

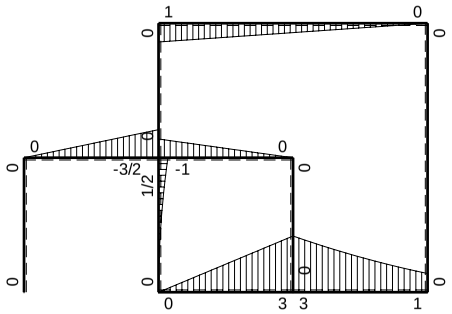




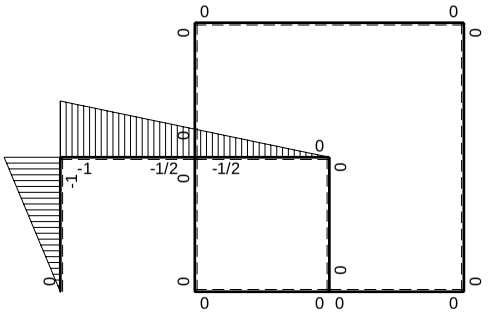

F
0
1/2
29/12
41/12
-3/2
65/24
-2
11/2
-2
0
0
0
F
-1/2
0
-65/24
1
-5/24
0
29/12
0
-3
5/2
3/2
F
1
0
0
0
0
29/12
5/24
0
7/24
29/12
1/2
0
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248666 CI01.xxxx.183

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/2Fx | -Fb/EJ | $3/2Fx-3/4Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/2+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/2Fb-3/2Fx | Fb/EJ | $3/4Fb-3/4Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| A | cedimento nodo $-H_{1A}u_A$ | | | | | | $Fb^2/EJ$ | |
| | totali | | | | | | $29/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -29/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -3/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/4\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/4\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/4 -3/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/4\ x -1/4\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/4\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

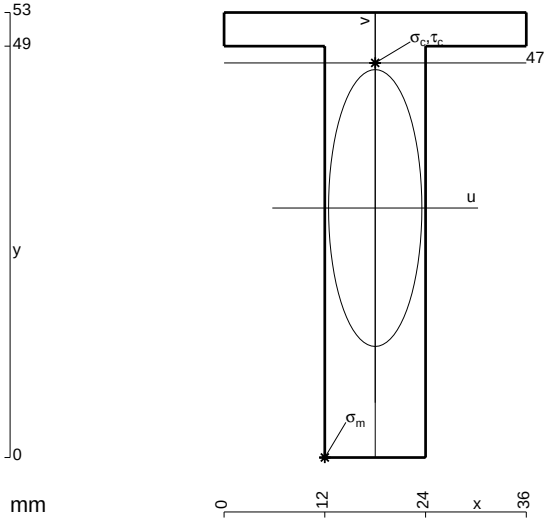
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

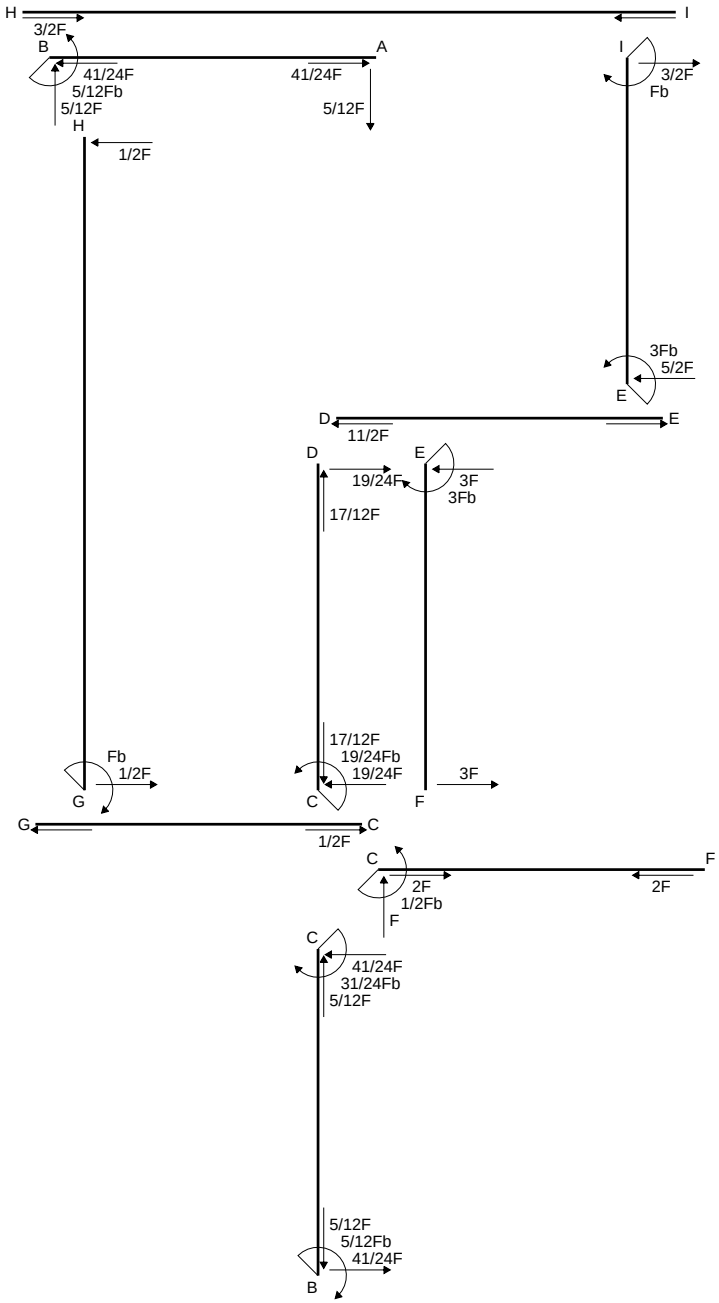
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$





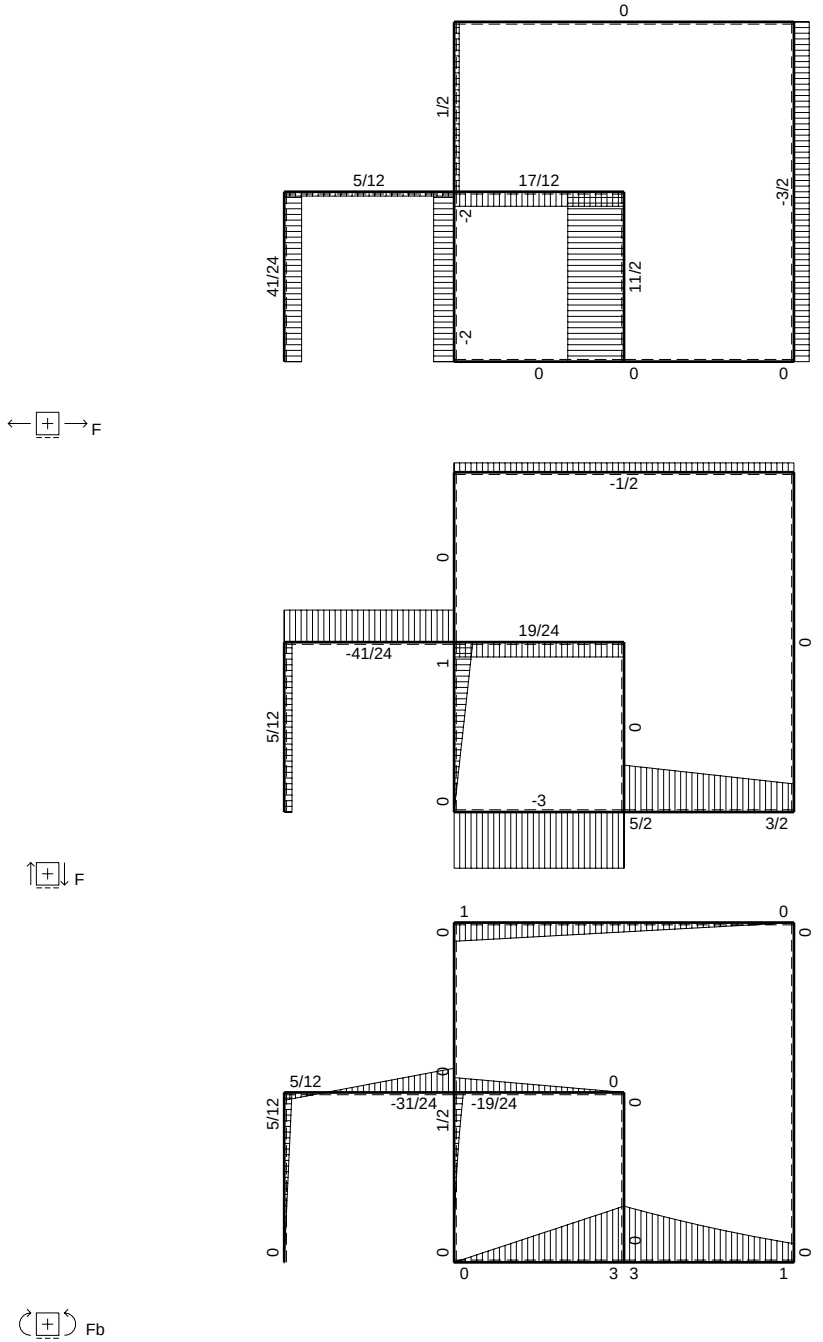


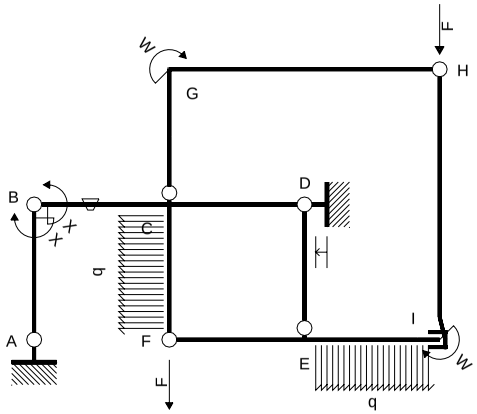

A = 732. $mm^2$
$J_u$ = 199072. $mm^4$
$J_v$ = 22608. $mm^4$
$y_g$ = 29.71 mm
$T_y$ = -2700. N
$M_x$ = 1539000. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -29.71 mm
$\sigma_m = -Mv/J_u$ = 229.7 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 47. mm
$v_c$ = 17.29 mm
$\sigma_c = -Mv/J_u$ = -133.6 $N/mm^2$
$\tau_c$ = 3.961 $N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 133.8 $N/mm^2$
$S^*$ = 3504. $mm^3$


H
I
3/2F
B
A
41/24F
5/12Fb
5/12F
41/24F
5/12F
I
3/2F
Fb
H
1/2F
3Fb
5/2F
E
D
E
11/2F
D
E
19/24F
3F
3Fb
17/12F
17/12F
19/24Fb
19/24F
Fb
1/2F
G
C
F
3F
G
C
1/2F
C
F
2F
2F
1/2Fb
F
C
41/24F
31/24Fb
5/12F
5/12F
5/12Fb
41/24F
B

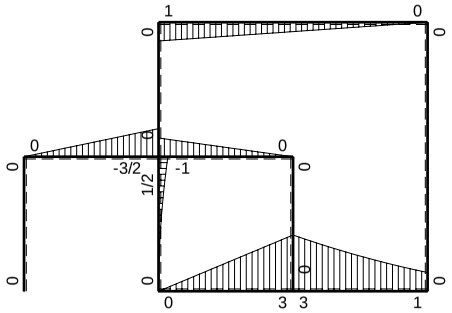




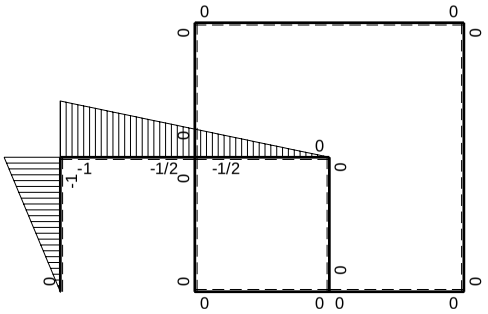

0
1/2
5/12
17/12
-3/2
-2
41/24
11/2
-2
0
0
0
F
-1/2
0
19/24
-41/24
1
0
5/12
0
-3
5/2
3/2
F
1
0
0
0
5/12
0
-31/24
-19/24
1/2
5/12
0
0
0
0
0
3
3
1
Fb




Schema di calcolo iperstatico




$M_0$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1



PROCEDIMENTO E RISULTATI 248460 CI01.xxxx.184

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Quadro contributi PLV per iperstatica $X=W_{BC}$

| → | $M_x(x)$ | $M_o(x)$ | $\theta$ | $M_xM_o$ | $M_x\theta$ | $M_xM_x$ | $\int M_x(M_o/EJ+\theta)dx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|---|---|
| AB b | -x/b | 0 | 0 | 0 | 0 | $x^2/b^2$ | 0+0 | 1/3Xb/EJ |
| BA b | 1-x/b | 0 | 0 | 0 | 0 | $1-2x/b+x^2/b^2$ | | |
| BC b | -1+1/2x/b | -3/2Fx | -Fb/EJ | $3/2Fx-3/4Fx^2/b$ | $Fb/EJ-1/2Fx/EJ$ | $1-x/b+1/4x^2/b^2$ | $(1/2+3/4)Fb^2/EJ$ | 7/12Xb/EJ |
| CB b | 1/2+1/2x/b | 3/2Fb-3/2Fx | Fb/EJ | $3/4Fb-3/4Fx^2/b$ | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ | | |
| CD b | -1/2+1/2x/b | -Fb+Fx | 0 | $1/2Fb-Fx+1/2Fx^2/b$ | 0 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$ | 1/12Xb/EJ |
| DC b | 1/2x/b | Fx | 0 | $1/2Fx^2/b$ | 0 | $1/4x^2/b^2$ | | |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| ED b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| EF b | 0 | 3Fb-3Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| FE b | 0 | -3Fx | 0 | 0 | 0 | 0 | | |
| FC b | 0 | $1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| CF b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| CG b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| GC b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| GH 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 | 0+0 | 0 |
| HG 2b | 0 | -1/2Fx | 0 | 0 | 0 | 0 | | |
| HI 2b | 0 | 0 | 0 | 0 | 0 | 0 | 0+0 | 0 |
| IH 2b | 0 | 0 | 0 | 0 | 0 | 0 | | |
| IE b | 0 | $Fb+3/2Fx+1/2qx^2$ | 0 | 0 | 0 | 0 | 0+0 | 0 |
| EI b | 0 | $-3Fb+5/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 | | |
| D | cedimento nodo $-H_{1D}u_D$ | | | | | | $-Fb^2/EJ$ | |
| | totali | | | | | | $5/12Fb^2/EJ$ | Xb/EJ |
| | iperstatica $X=W_{BC}$ | | | | | | -5/12Fb | |

Sviluppi di calcolo iperstatica


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$L^{XX}_{AB} = \int_o^b ( x^2/b^2 )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -3/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/4\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/4\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/4 -3/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/4\ x -1/4\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/4\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

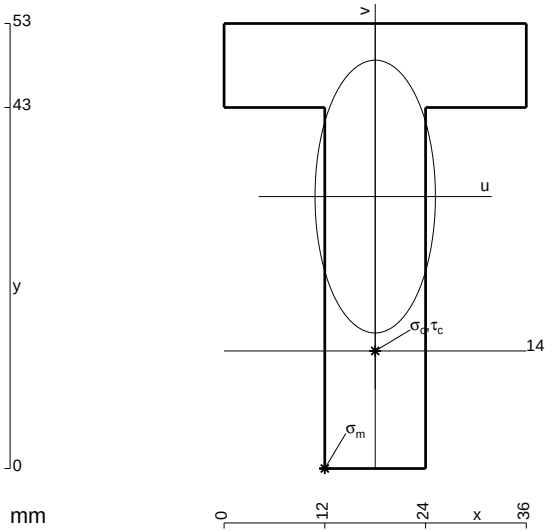
$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$








A = 876. $mm^2$
$J_u$ = 231422. $mm^4$
$J_v$ = 45072. $mm^4$
$y_g$ = 32.39 mm
$T_y$ = -2790. N
$M_x$ = 1701900. Nmm
$x_m$ = 12. mm
$u_m$ = -6. mm
$v_m$ = -32.39 mm
$\sigma_m = -Mv/J_u$ = 238.2 $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 14. mm
$v_c$ = -18.39 mm
$\sigma_c = -Mv/J_u$ = 135.2 $N/mm^2$
$\tau_c$ = 4.285 $N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.4 $N/mm^2$
$S^*$ = 4266. $mm^3$