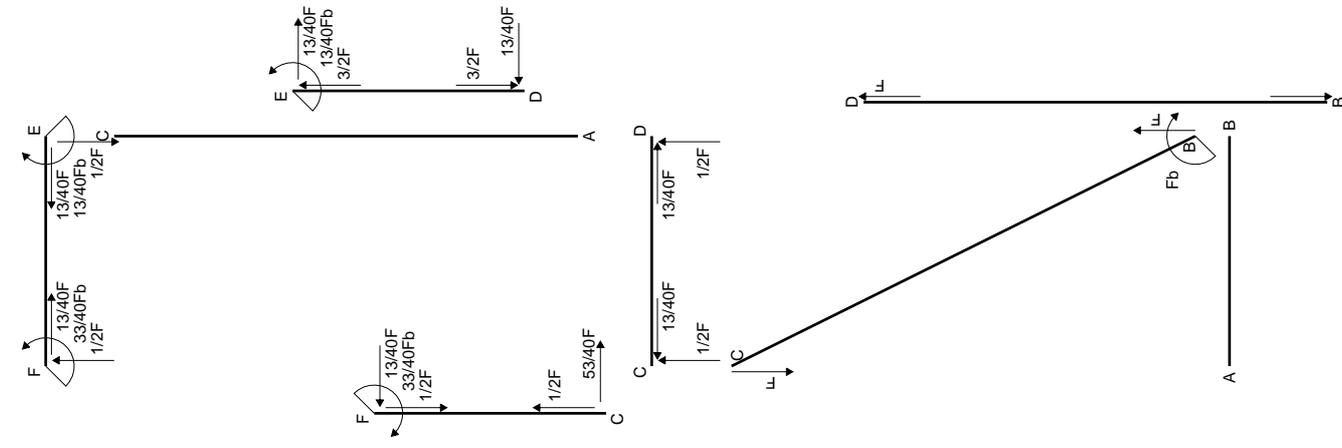
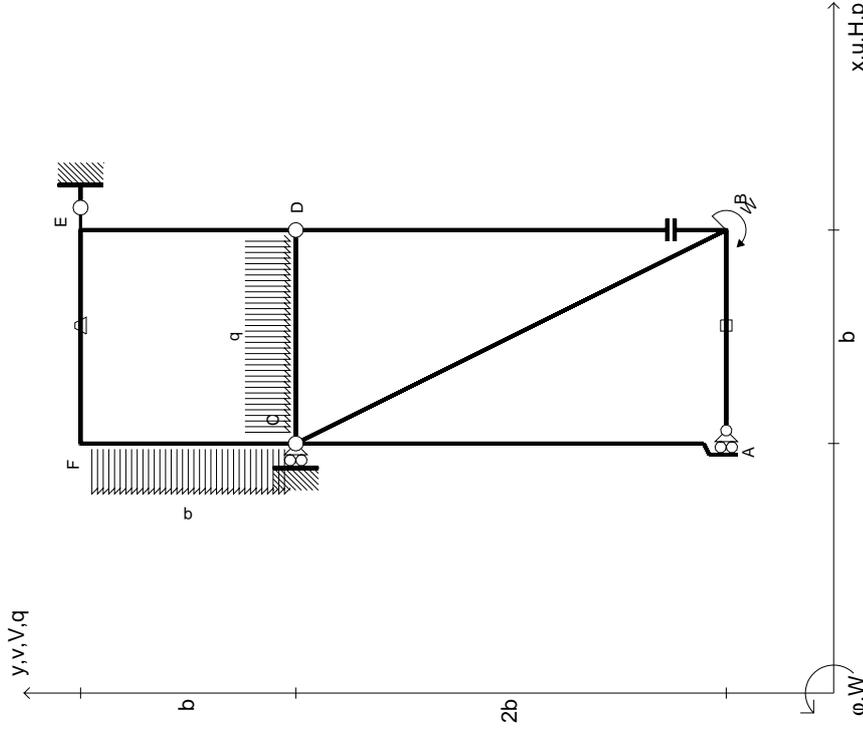
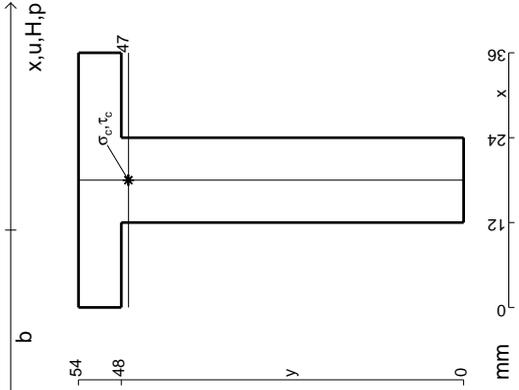
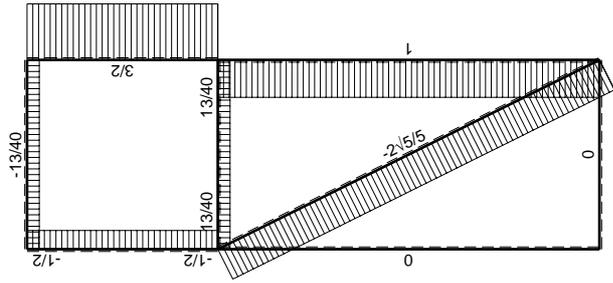


$$\begin{aligned}
 W_B &= -W = -Fb \\
 P_{FC} &= -q = -F/b \\
 q_{CD} &= -q = -F/b \\
 \varepsilon_{AB} &= -\alpha T = -b^2 F/EJ \\
 \theta_{EF} &= -\theta = -\alpha T/b = -bF/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{AC}} &= EJ \\
 E_{J_{DB}} &= EJ \\
 E_{J_{DE}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ
 \end{aligned}$$

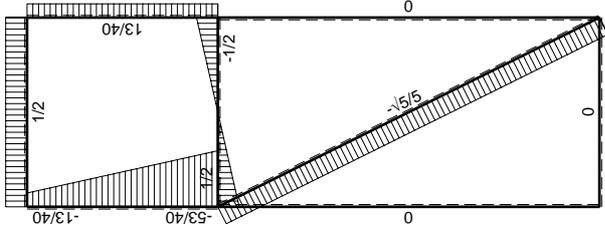


Reazioni iperstatiche in soluzione:  $X=W_{FC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{yz} - x_{yz} - \theta_{yz}$  riferimento locale asta YZ con origine in Y.  
 La trave BC ha la sezione riportata e dimensioni in mm, con:  
 $b = 640 \text{ mm}, F = 2500 \text{ N}$   
 Calcolare sulla sezione B la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da B a C  
 Elongazione termica specifica  $\varepsilon$  assegnata su asta AB.  
 Curvatura  $\theta$  asta EF positiva se convessa a destra con inizio E.  
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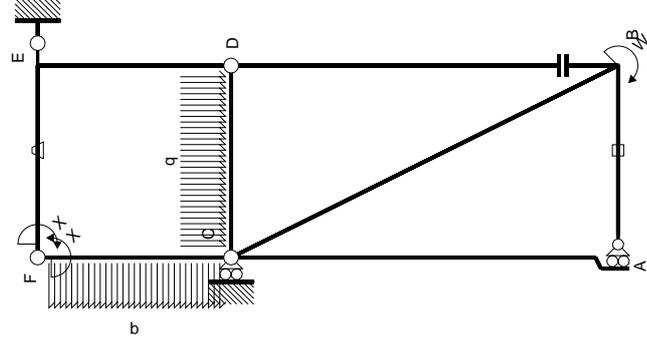




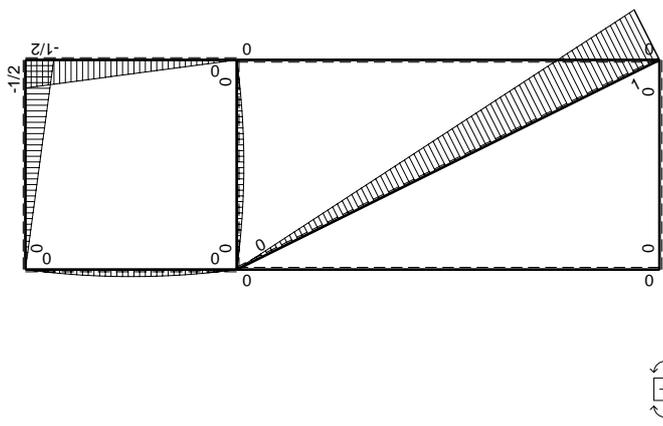
← ⊕ → F



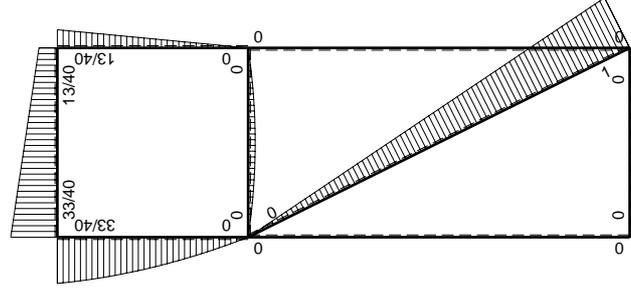
↑ ⊕ ↓ F



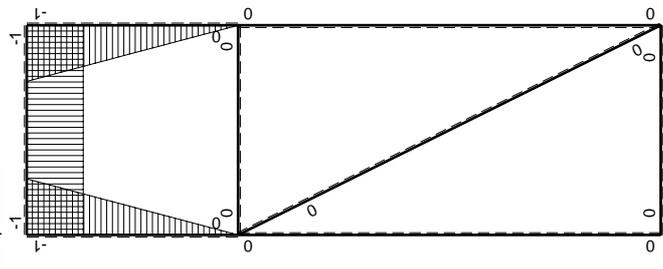
Schema di calcolo iperstatico



⊕ ↺ M<sub>0</sub> flessione da carichi assegnati



⊕ ↺ F<sub>b</sub>



⊕ ↺ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W<sub>FC</sub>

→	M <sub>x</sub> (x)	M <sub>0</sub> (x)	θ	M <sub>x</sub> M <sub>0</sub>	M <sub>x</sub> θ	M <sub>x</sub> M <sub>x</sub>	∫M <sub>x</sub> (M <sub>0</sub> /EJ+θ)dx	∫XM <sub>x</sub> M <sub>x</sub> /EJdx
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0	0	0
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0	0+0	0
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0	0+0	0
DE b	-x/b	-1/2Fx	0	1/2Fx <sup>2</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>	(1/6+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fb-1/2Fx	0	1/2Fb-Fx+1/2Fx <sup>2</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>		
CD b	0	1/2Fx-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx <sup>2</sup>	0	0	0	0	0+0	0
EF b	-1	-1/2Fb+1/2Fx	-Fb/EJ	1/2Fb-1/2Fx	Fb/EJ	1	(1/4+1)Fb <sup>2</sup> /EJ	Xb/EJ
FE b	1	1/2Fx	Fb/EJ	1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fx-1/2qx <sup>2</sup>	0	-1/2Fx+Fx <sup>2</sup> /b-1/2qx <sup>3</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	(-1/24+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx+1/2qx <sup>2</sup>	0	-1/2Fx <sup>2</sup> /b+1/2qx <sup>3</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>		
totali								
iperstatica X=W <sub>FC</sub>								

Sviluppi di calcolo iperstatica

$$L_{DE}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FE}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DE}^{X0} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ = (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

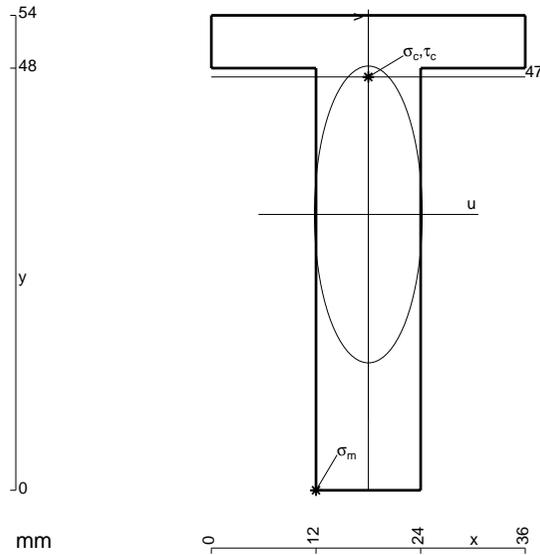
$$L_{ED}^{X0} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ = (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{EF}^{X0} = \int_0^b (1/2 - 1/2 x/b) Fb 1/EJ dx + \int_0^b (1) \theta dx = [1/2 x - 1/4 x^2/b]_0^b Fb 1/EJ + [x]_0^b \theta = (1/2 b - 1/4 b) Fb 1/EJ + (b) \theta = 5/4 Fb^2/EJ$$

$$L_{FE}^{X0} = \int_0^b (1/2 x/b) Fb 1/EJ dx + \int_0^b (-1) \theta dx = [1/4 x^2/b]_0^b Fb 1/EJ + [-x]_0^b \theta = (1/4 b) Fb 1/EJ + (-b) \theta = 5/4 Fb^2/EJ$$

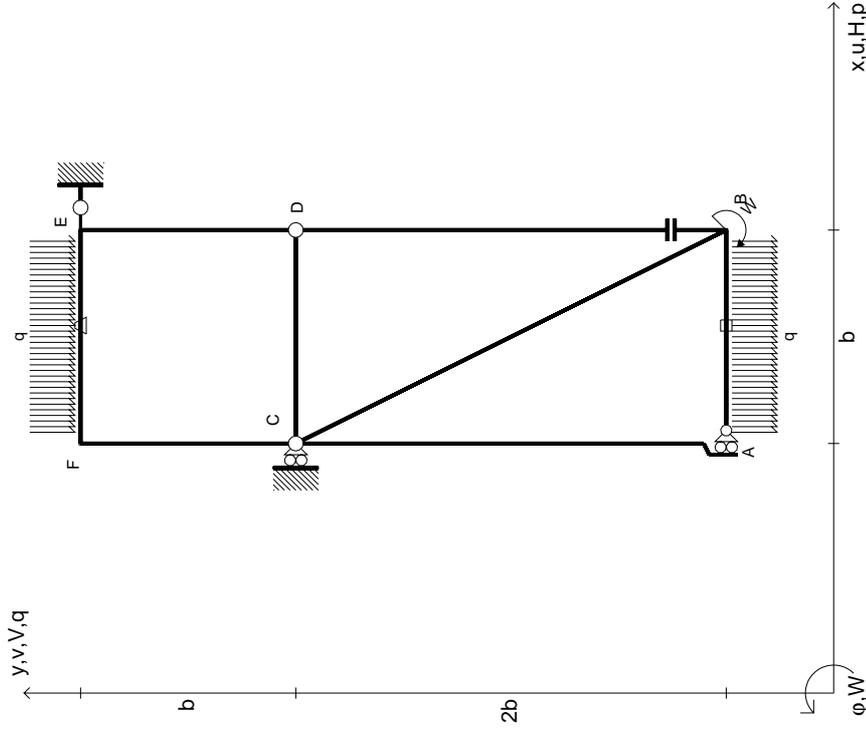
$$L_{FC}^{X0} = \int_0^b (-1/2 x/b + x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [-1/4 x^2/b + 1/3 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ = (-1/4 b + 1/3 b - 1/8 b) Fb 1/EJ = -1/24 Fb^2/EJ$$

$$L_{CF}^{X0} = \int_0^b (-1/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [-1/6 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ = (-1/6 b + 1/8 b) Fb 1/EJ = -1/24 Fb^2/EJ$$

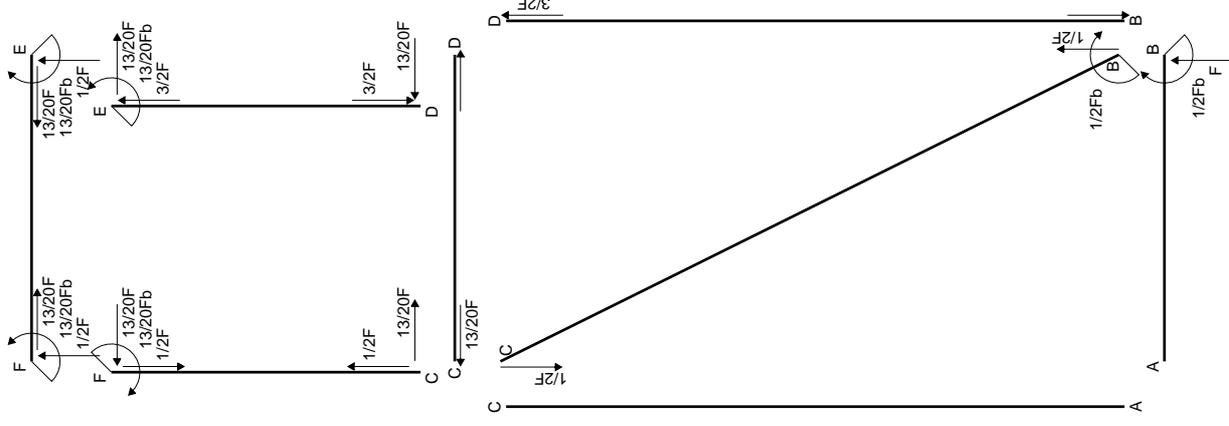
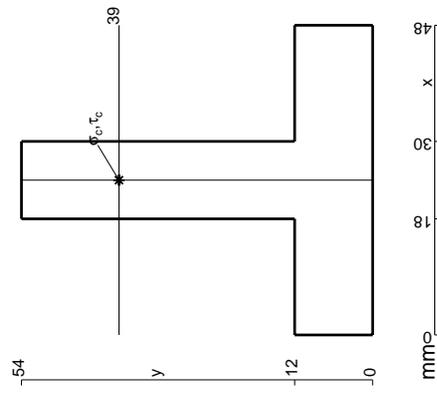


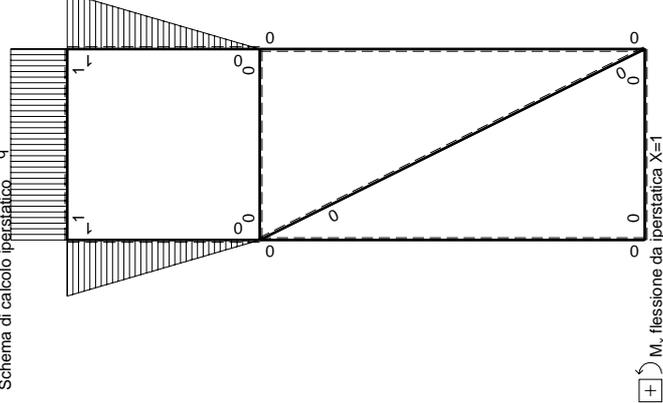
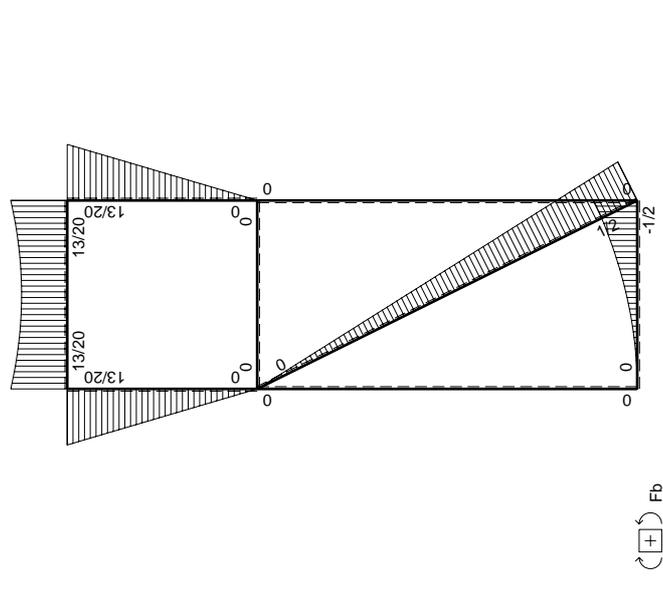
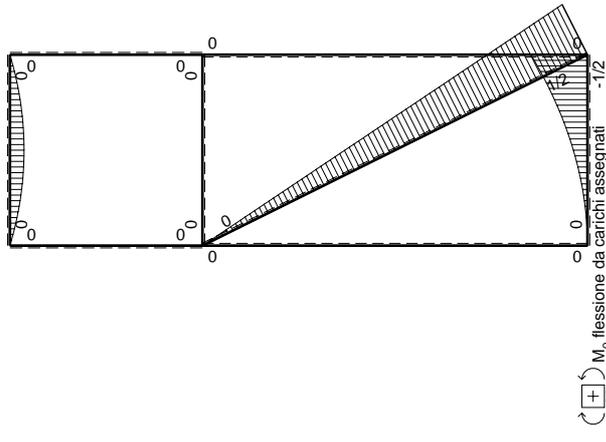
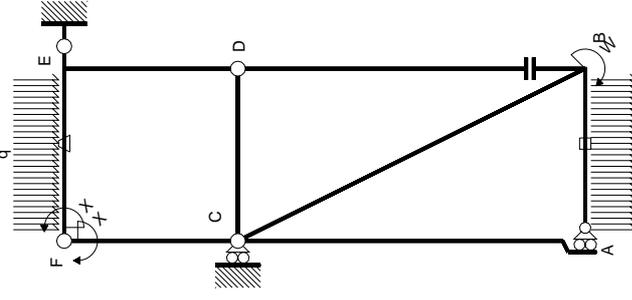
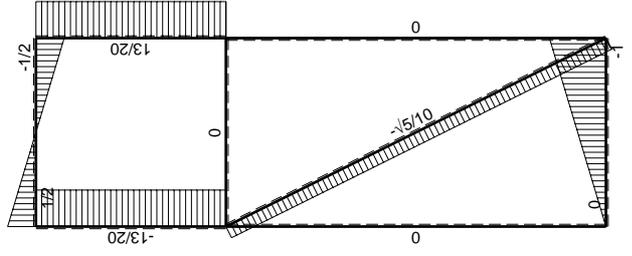
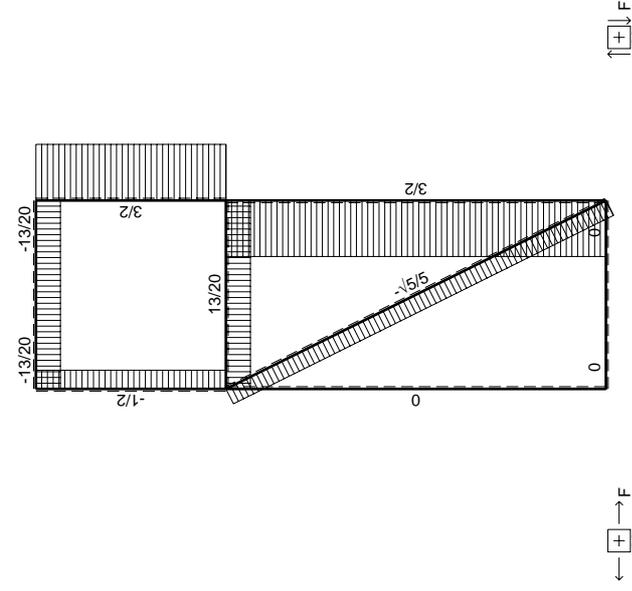
- A = 792. mm<sup>2</sup>
- J<sub>u</sub> = 225759. mm<sup>4</sup>
- J<sub>v</sub> = 30240. mm<sup>4</sup>
- y<sub>g</sub> = 31.36 mm
- N = -2236. N
- T<sub>y</sub> = -1118. N
- M<sub>x</sub> = 1600000. Nmm
- x<sub>m</sub> = 12. mm
- u<sub>m</sub> = -6. mm
- v<sub>m</sub> = -31.36 mm
- σ<sub>m</sub> = N/A-Mv/J<sub>u</sub> = 219.5 N/mm<sup>2</sup>
- x<sub>c</sub> = 18. mm
- y<sub>c</sub> = 47. mm
- v<sub>c</sub> = 15.64 mm
- σ<sub>c</sub> = N/A-Mv/J<sub>u</sub> = -113.6 N/mm<sup>2</sup>
- τ<sub>c</sub> = 1.83 N/mm<sup>2</sup>
- σ<sub>q</sub> = √σ<sup>2</sup>+3τ<sup>2</sup> = 113.7 N/mm<sup>2</sup>
- S = 4435. mm<sup>3</sup>

$$\begin{aligned}
 W_B &= -W = -Fb \\
 q_{AB} &= -q = -F/b \\
 q_{EF} &= -q = -F/b \\
 \varepsilon_{AB} &= -\alpha T = -b^2 F/EJ \\
 \theta_{EF} &= -\theta = -\alpha T/b = -bF/EJ \\
 E J_{AB} &= EJ \\
 E J_{BC} &= EJ \\
 E J_{AC} &= EJ \\
 E J_{DB} &= EJ \\
 E J_{DE} &= EJ \\
 E J_{CD} &= EJ \\
 E J_{EF} &= EJ \\
 E J_{FC} &= EJ
 \end{aligned}$$



Reazioni iperstatiche in soluzione:  $X=W_{FE}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{yz} - x_{yz} - \theta_{yz}$  riferimento locale asta YZ con origine in Y.  
 La trave AB ha la sezione riportata e dimensioni in mm, con:  
 $b = 570 \text{ mm}$ ,  $F = 5490 \text{ N}$   
 Calcolare sulla sezione B la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su traveggio trave, a destra da A a B  
 Elongazione termica specifica  $\varepsilon$  assegnata su asta AB.  
 Curvatura  $\theta$  asta EF positiva se convessa a destra con inizio E.  
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Schema di calcolo iperstatico.

Quadro contributi PLV per iperstatica X=W<sub>FE</sub>

→	M <sub>x</sub> (x)	M <sub>0</sub> (x)	θ	M <sub>x</sub> M <sub>0</sub>	M <sub>x</sub> θ	M <sub>x</sub> M <sub>x</sub>	∫M <sub>x</sub> (M <sub>0</sub> /EJ+θ)dx	∫xM <sub>x</sub> M <sub>x</sub> /EJdx
AB b	0	-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
BA b	0	1/2Fb-Fx+1/2qx <sup>2</sup>	0	0	0	0	0+0	0
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0+0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0	0+0	0
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0	0+0	0
DE b	x/b	0	0	0	0	x <sup>2</sup> /b <sup>2</sup>	0+0	1/3Xb/EJ
ED b	-1+x/b	0	0	0	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0+0	0
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0	0+0	0
EF b	1	-1/2Fx+1/2qx <sup>2</sup>	-Fb/EJ	-1/2Fx+1/2Fx <sup>2</sup> /b	-Fb/EJ	1	(-1/12-1)Fb <sup>2</sup> /EJ	Xb/EJ
FE b	-1	1/2Fx-1/2qx <sup>2</sup>	Fb/EJ	-1/2Fx+1/2Fx <sup>2</sup> /b	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x <sup>2</sup> /b <sup>2</sup>		
	totali						-13/12Fb <sup>2</sup> /EJ	5/3Xb/EJ
	iperstatica X=W <sub>FE</sub>						13/20Fb	

Sviluppi di calcolo iperstatica

$$L_{DE}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{FE}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{FC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FE}^{X0} = \int_0^b (-1/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1) \theta dx$$

$$= [-1/4 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [-x]_0^b \theta$$

$$= (-1/4 b + 1/6 b) Fb 1/EJ + (-b) \theta = -13/12 Fb^2/EJ$$

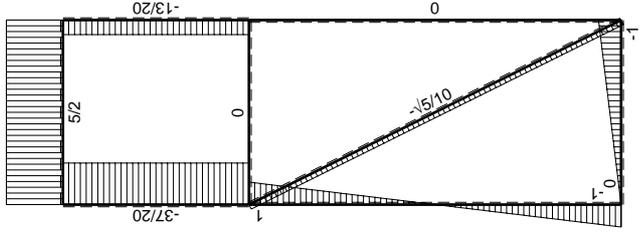
$$L_{FE}^{X0} = \int_0^b (-1/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1) \theta dx$$

$$= [-1/4 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [x]_0^b \theta$$

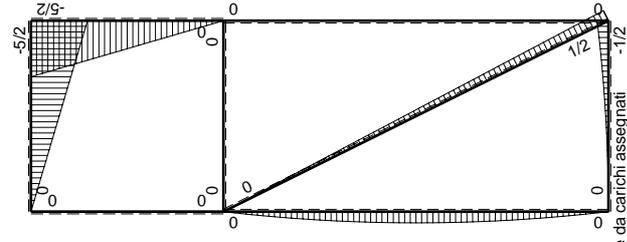
$$= (-1/4 b + 1/6 b) Fb 1/EJ + (b) \theta = -13/12 Fb^2/EJ$$



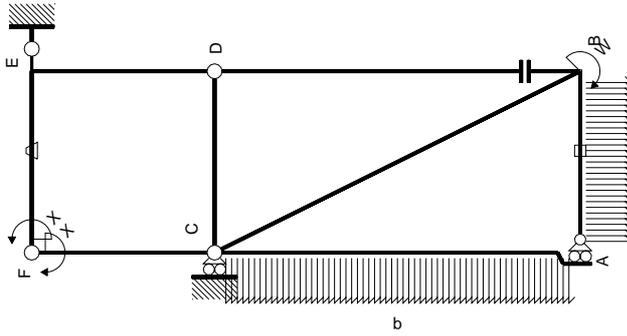




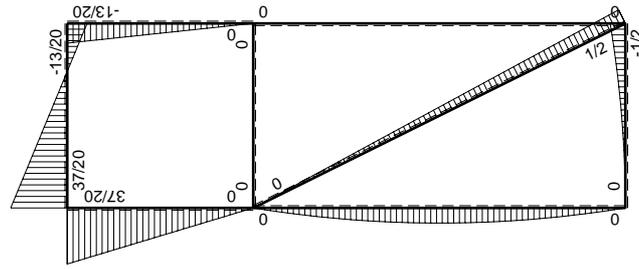
$\uparrow \oplus \rightarrow F$



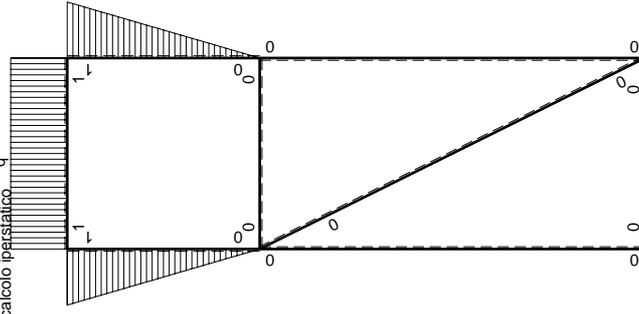
$\curvearrowright \oplus M_0$  flessione da carichi assegnati



Schema di calcolo iperstatico q



$\uparrow \oplus \rightarrow F_b$



$\curvearrowright \oplus M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W<sub>FE</sub>

→	M <sub>x</sub> (x)	M <sub>0</sub> (x)	θ	M <sub>x</sub> M <sub>0</sub>	M <sub>x</sub> θ	M <sub>x</sub> M <sub>x</sub>	∫M <sub>x</sub> (M <sub>0</sub> /EJ+θ)dx	∫XM <sub>x</sub> M <sub>x</sub> /EJdx
AB b	0	-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
BA b	0	1/2Fb-Fx+1/2qx <sup>2</sup>	0	0	0	0	0+0	0
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx <sup>2</sup>	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0	0+0	0
DE b	x/b	-5/2Fx	0	-5/2Fx <sup>2</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>	(-5/6+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ
ED b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx <sup>2</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0	0+0	0
EF b	1	-5/2Fb+5/2Fx	-Fb/EJ	-5/2Fb+5/2Fx	-Fb/EJ	1	(-5/4-1)Fb <sup>2</sup> /EJ	Xb/EJ
FE b	-1	5/2Fx	Fb/EJ	-5/2Fx	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x <sup>2</sup> /b <sup>2</sup>		
totali							-37/12Fb <sup>2</sup> /EJ	5/3Xb/EJ
iperstatica X=W <sub>FE</sub>							37/20Fb	

Sviluppi di calcolo iperstatica

$$L_{DE}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FE}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

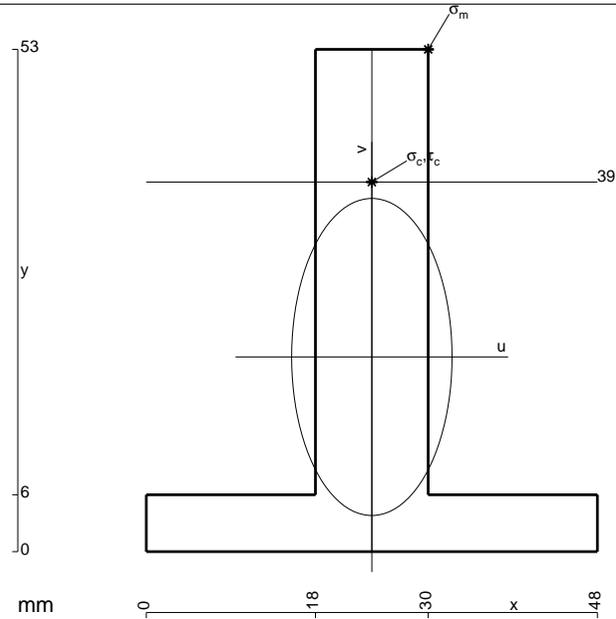
$$L_{CF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DE}^{X0} = \int_0^b (-5/2 x^2/b^2) Fb 1/EJ dx = [-5/6 x^3/b^2]_0^b Fb 1/EJ = (-5/6 b) Fb 1/EJ = -5/6 Fb^2/EJ$$

$$L_{ED}^{X0} = \int_0^b (-5/2 + 5x/b - 5/2 x^2/b^2) Fb 1/EJ dx = [-5/2 x + 5/2 x^2/b - 5/6 x^3/b^2]_0^b Fb 1/EJ = (-5/2 b + 5/2 b - 5/6 b) Fb 1/EJ = -5/6 Fb^2/EJ$$

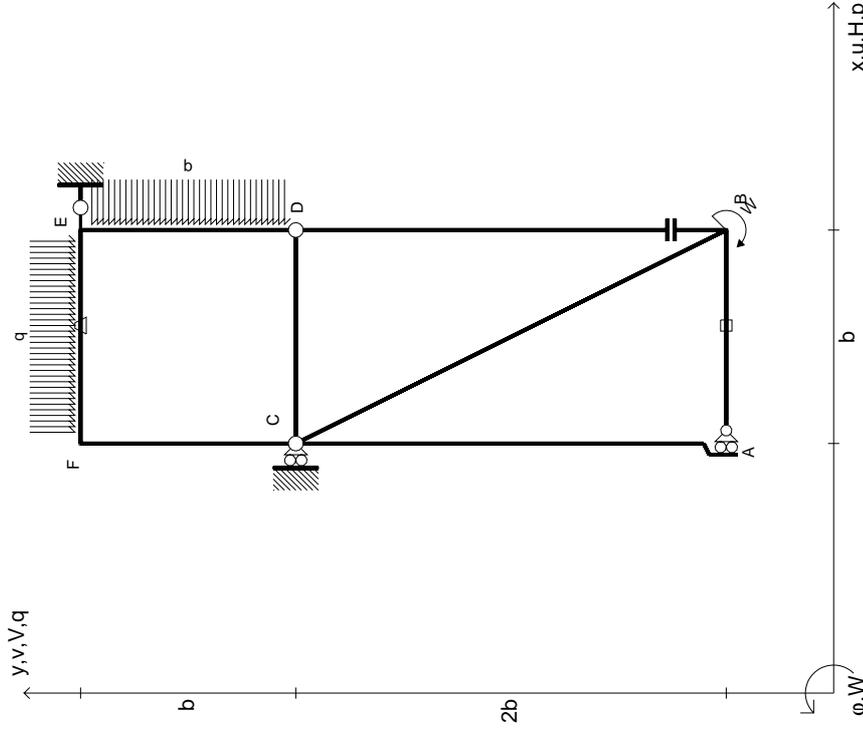
$$L_{EF}^{X0} = \int_0^b (-5/2 + 5/2 x/b) Fb 1/EJ dx + \int_0^b (-1) \theta dx = [-5/2 x + 5/4 x^2/b]_0^b Fb 1/EJ + [-x]_0^b \theta = (-5/2 b + 5/4 b) Fb 1/EJ + (-b) \theta = -9/4 Fb^2/EJ$$

$$L_{FE}^{X0} = \int_0^b (-5/2 x/b) Fb 1/EJ dx + \int_0^b (1) \theta dx = [-5/4 x^2/b]_0^b Fb 1/EJ + [x]_0^b \theta = (-5/4 b) Fb 1/EJ + (b) \theta = -9/4 Fb^2/EJ$$

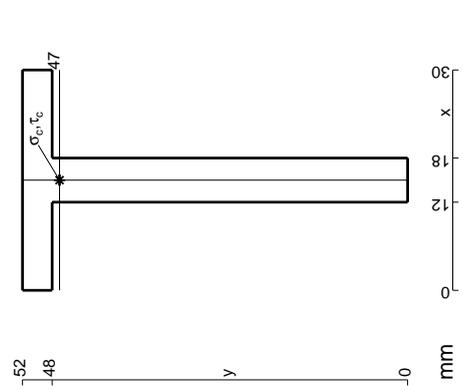


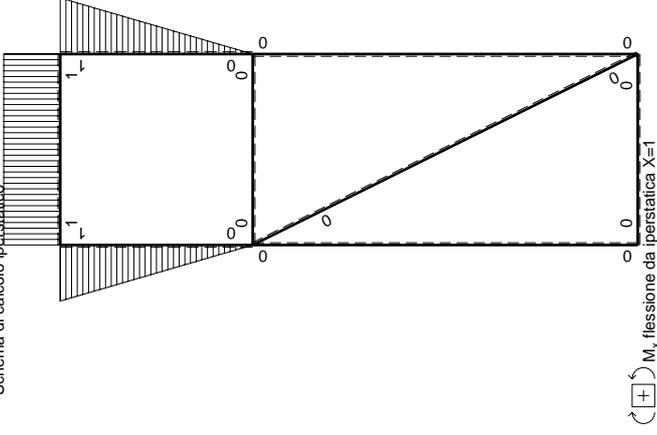
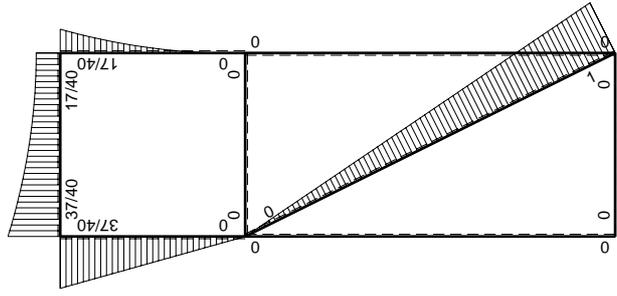
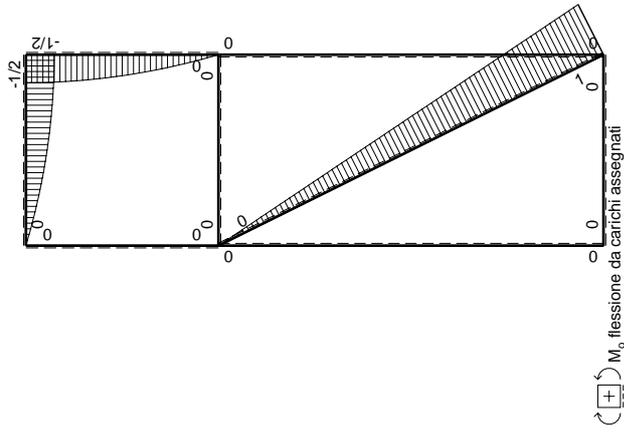
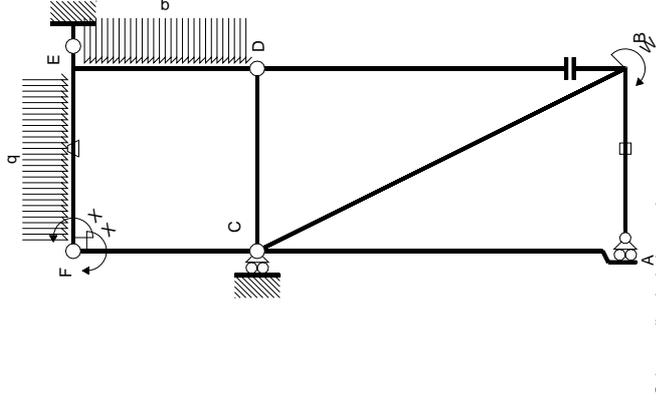
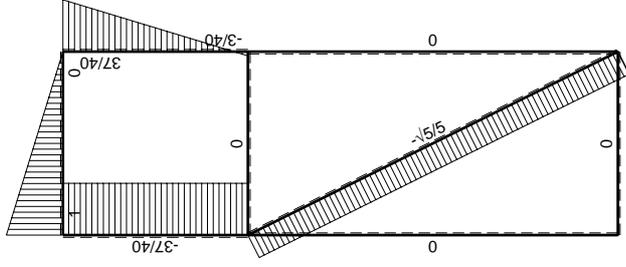
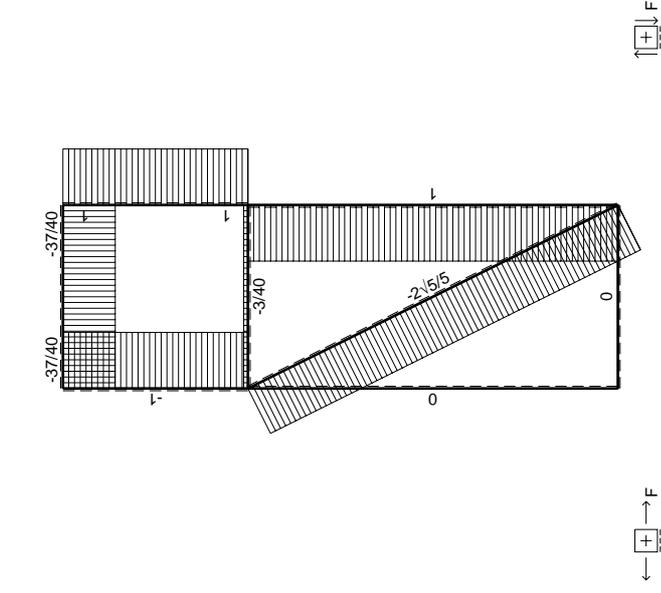
- $A = 852. \text{ mm}^2$
- $J_u = 238569. \text{ mm}^4$
- $J_v = 62064. \text{ mm}^4$
- $y_g = 20.54 \text{ mm}$
- $N = 5070. \text{ N}$
- $T_y = -5070. \text{ N}$
- $M_x = -1571700. \text{ Nmm}$
- $x_m = 30. \text{ mm}$
- $y_m = 53. \text{ mm}$
- $u_m = 6. \text{ mm}$
- $v_m = 32.46 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = 219.8 \text{ N/mm}^2$
- $x_c = 24. \text{ mm}$
- $y_c = 39. \text{ mm}$
- $v_c = 18.46 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = 127.6 \text{ N/mm}^2$
- $\tau_c = 7.574 \text{ N/mm}^2$
- $\sigma_\varrho = \sqrt{\sigma^2 + 3\tau^2} = 128.2 \text{ N/mm}^2$
- $S = 4277. \text{ mm}^3$

$$\begin{aligned}
 W_B &= -W = -Fb \\
 P_{DE} &= -q = -F/b \\
 q_{EF} &= -q = -F/b \\
 \varepsilon_{AB} &= 4\alpha T = 4b^2 F/EJ \\
 \theta_{EF} &= -\theta = -\alpha T/b = -bF/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{AC}} &= EJ \\
 E_{J_{DB}} &= EJ \\
 E_{J_{DE}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ
 \end{aligned}$$



Reazioni iperstatiche in soluzione:  $X=W_{FE}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{y,z} - x_{y,z} - \theta_{y,z}$  riferimento locale asta YZ con origine in Y.  
 La trave BC ha la sezione riportata e dimensioni in mm, con:  
 $b = 480 \text{ mm}$ ,  $F = 1730 \text{ N}$   
 Calcolare sulla sezione B la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su traveggio trave, a destra da B a C  
 Elongazione termica specifica  $\varepsilon$  assegnata su asta AB.  
 Curvatura  $\theta$  asta EF positiva se convessa a destra con inizio E.  
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→	$M_x(x)$	$M_0(x)$	$\theta$	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int x M_x M_x/EJ dx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0	0	0
BC $\sqrt{5}b$	0	$Fb-\sqrt{5}Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0	0+0	0
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0	0+0	0
DE b	$x/b$	$-Fx+1/2qx^2$	0	$-F^2/b+1/2qx^2/b$	0	$x^2/b^2$	$(-5/24+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0	0+0	0
EF b	1	$-1/2Fb+1/2qx^2$	$-Fb/EJ$	$-1/2Fb+1/2Fx^2/b$	$-Fb/EJ$	1	$(-1/3-1)Fb^2/EJ$	$Xb/EJ$
FE b	-1	$Fx-1/2qx^2$	$Fb/EJ$	$-Fx+1/2Fx^2/b$	$-Fb/EJ$	1		
FC b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CF b	$-x/b$	0	0	0	0	$x^2/b^2$		
totali								
iperstatica $X=W_{FE}$								

Sviluppi di calcolo iperstatica

$$L_{DE}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{XX} = \int_0^b (1-2x/b+x^2/b^2) 1/EJ dx = [x-x^2/b+1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b-b+1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{FE}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{FC}^{XX} = \int_0^b (1-2x/b+x^2/b^2) 1/EJ dx = [x-x^2/b+1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b-b+1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DE}^{X_0} = \int_0^b (-x^2/b^2+1/2 x^3/b^3) Fb 1/EJ dx = [-1/3 x^3/b^2+1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (-1/3 b+1/8 b) Fb 1/EJ = -5/24 Fb^2/EJ$$

$$L_{ED}^{X_0} = \int_0^b (-1/2+1/2 x/b+1/2 x^2/b^2-1/2 x^3/b^3) Fb 1/EJ dx$$

$$= [-1/2 x+1/4 x^2/b+1/6 x^3/b^2-1/8 x^4/b^3]_0^b Fb 1/EJ$$

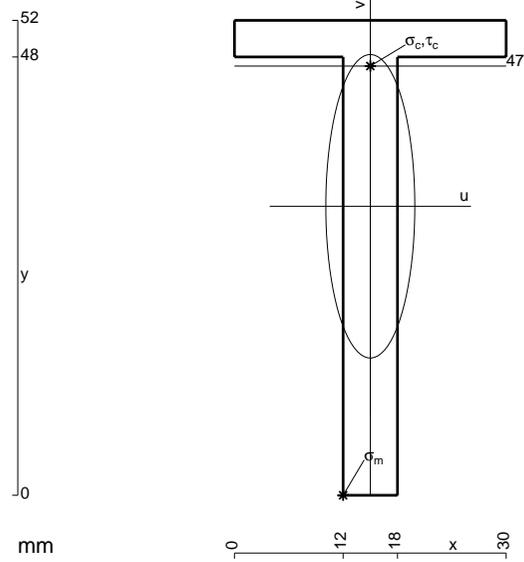
$$= (-1/2 b+1/4 b+1/6 b-1/8 b) Fb 1/EJ = -5/24 Fb^2/EJ$$

$$L_{EF}^{X_0} = \int_0^b (-1/2+1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1) \theta dx = [-1/2 x+1/6 x^3/b^2]_0^b Fb 1/EJ + [-x]_0^b \theta$$

$$= (-1/2 b+1/6 b) Fb 1/EJ + (-b) \theta = -4/3 Fb^2/EJ$$

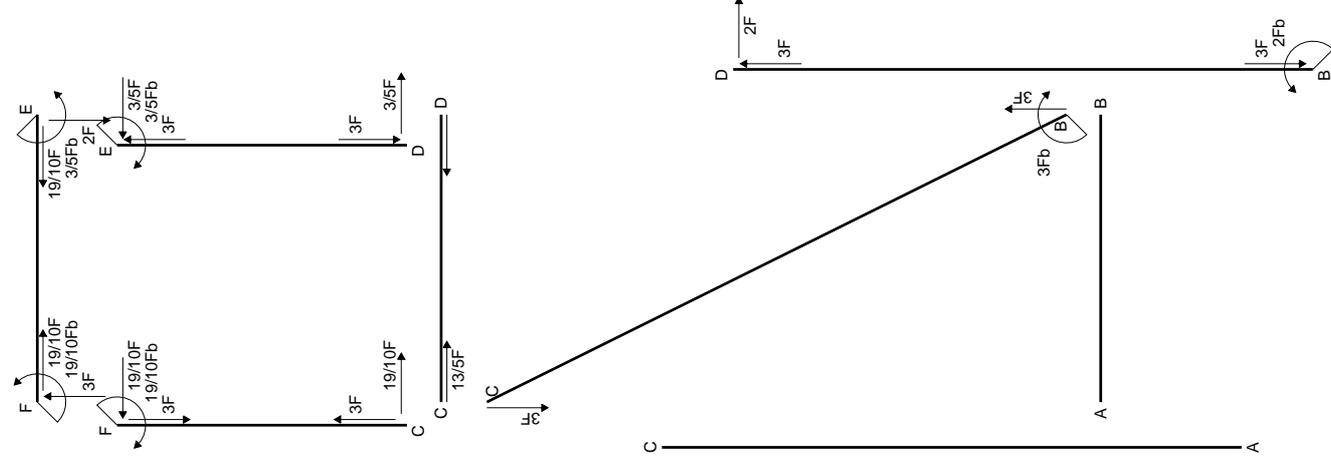
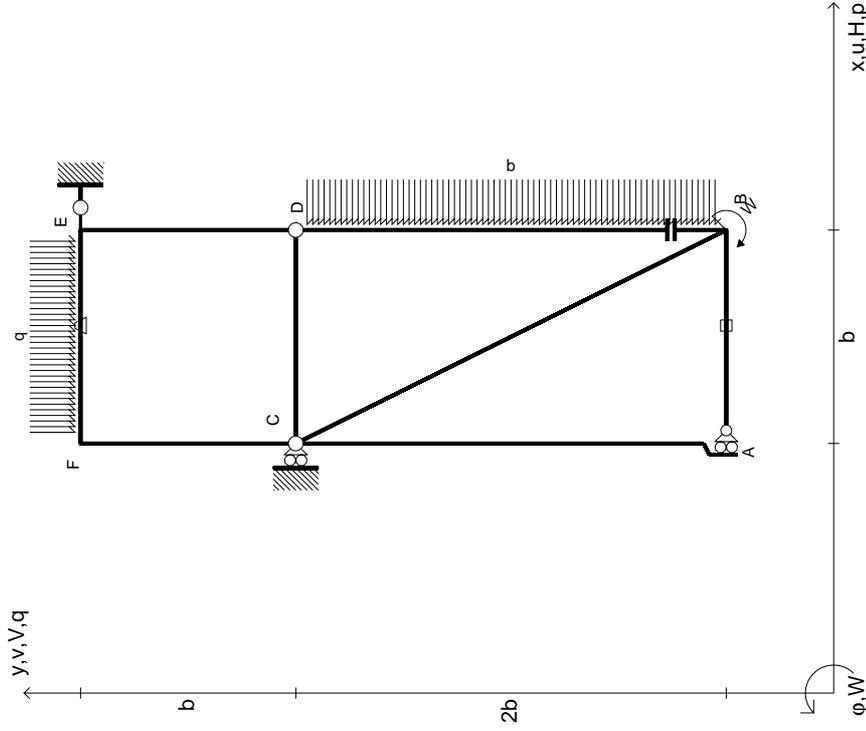
$$L_{FE}^{X_0} = \int_0^b (-x/b+1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1) \theta dx = [-1/2 x^2/b+1/6 x^3/b^2]_0^b Fb 1/EJ + [x]_0^b \theta$$

$$= (-1/2 b+1/6 b) Fb 1/EJ + (b) \theta = -4/3 Fb^2/EJ$$

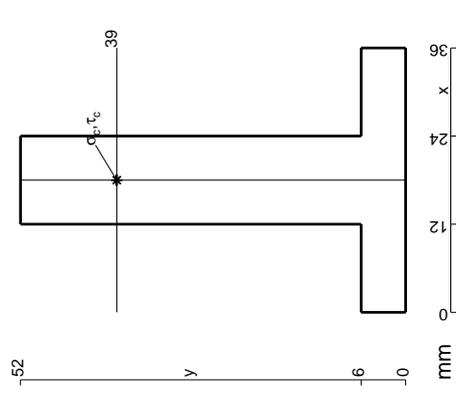


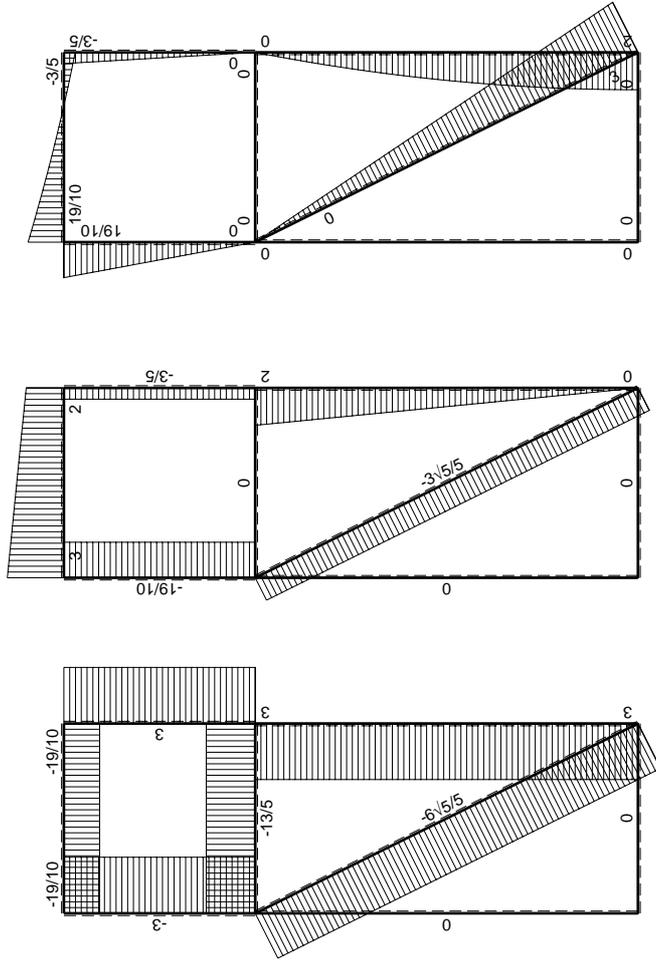
- $A = 408. \text{ mm}^2$
- $J_u = 112717. \text{ mm}^4$
- $J_v = 9864. \text{ mm}^4$
- $y_g = 31.65 \text{ mm}$
- $N = -1547. \text{ N}$
- $T_y = -773.7 \text{ N}$
- $M_x = 830400. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $u_m = -3. \text{ mm}$
- $v_m = -31.65 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = 229.4 \text{ N/mm}^2$
- $x_c = 15. \text{ mm}$
- $y_c = 47. \text{ mm}$
- $v_c = 15.35 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = -116.9 \text{ N/mm}^2$
- $\tau_c = 2.628 \text{ N/mm}^2$
- $\sigma_q = \sqrt{\sigma^2 + 3\tau^2} = 117. \text{ N/mm}^2$
- $S = 2297. \text{ mm}^3$

$$\begin{aligned}
 W_B &= -W = -Fb \\
 P_{DB} &= -q = -F/b \\
 q_{EF} &= -q = -F/b \\
 \varepsilon_{AB} &= 4\alpha T = 4b^2 F/EJ \\
 \theta_{EF} &= -\theta = -\alpha T/b = -bF/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{AC}} &= EJ \\
 E_{J_{DB}} &= EJ \\
 E_{J_{DE}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ
 \end{aligned}$$



Reazioni iperstatiche in soluzione:  $X=W_{FC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{yz} - x_{yz} - \theta_{yz}$  riferimento locale asta YZ con origine in Y.  
 La trave BC ha la sezione riportata e dimensioni in mm, con:  
 $b = 440 \text{ mm}, F = 1090 \text{ N}$   
 Calcolare sulla sezione B la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da B a C  
 Elongazione termica specifica  $\varepsilon$  assegnata su asta AB.  
 Curvatura  $\theta$  asta EF positiva se convessa a destra con inizio E.  
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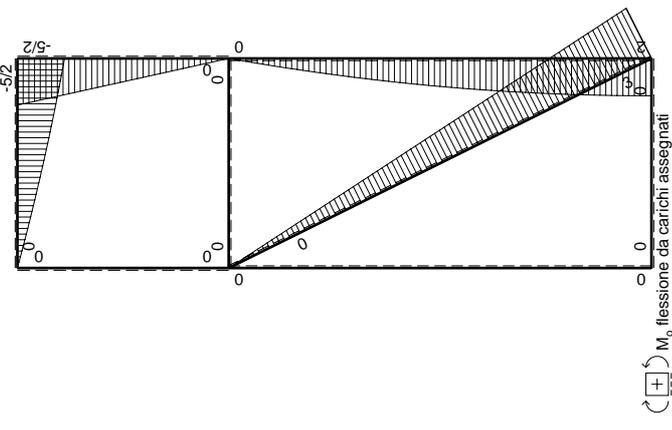




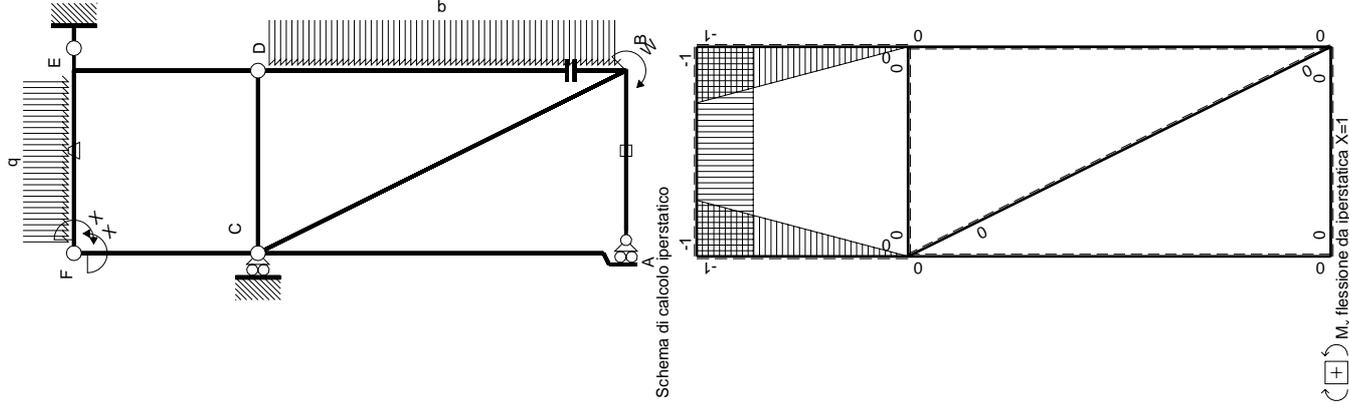
←  $\boxed{+}$  →  $F$

↑  $\boxed{+}$  ↓  $F_b$

↺  $\boxed{+}$  ↻  $F_b$



↺  $\boxed{+}$  ↻  $M_0$  flessione da carichi assegnati



↺  $\boxed{+}$  ↻  $M_x$  flessione da iperstatica  $X=1$

Schema di calcolo iperstatico

Quadro contributi PLV per iperstatica X=W<sub>FC</sub>

→	M <sub>x</sub> (x)	M <sub>0</sub> (x)	θ	M <sub>x</sub> M <sub>0</sub>	M <sub>x</sub> θ	M <sub>x</sub> M <sub>x</sub>	∫M <sub>x</sub> (M <sub>0</sub> /EJ+θ)dx	∫XM <sub>x</sub> M <sub>x</sub> /EJdx
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0	0	0
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0	0+0	0
DB 2b	0	2Fx-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx <sup>2</sup>	0	0	0	0	0+0	0
DE b	-x/b	-5/2Fx	0	5/2Fx <sup>2</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>	(5/6+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ
ED b	1-x/b	5/2Fb-5/2Fx	0	5/2Fb-5Fx+5/2Fx <sup>2</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0	0
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0	0+0	0
EF b	-1	-5/2Fb+2Ex+1/2qx <sup>2</sup>	-Fb/EJ	5/2Fb-2Fx-1/2Fx <sup>2</sup> /b	Fb/EJ	1	(4/3+1)Fb <sup>2</sup> /EJ	Xb/EJ
FE b	1	3Fx-1/2qx <sup>2</sup>	Fb/EJ	3Fx-1/2Fx <sup>2</sup> /b	Fb/EJ	1	0+0	1/3Xb/EJ
FC b	-1+x/b	0	0	0	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0	0
CF b	x/b	0	0	0	0	x <sup>2</sup> /b <sup>2</sup>	0	0
	totali						19/6Fb <sup>2</sup> /EJ	5/3Xb/EJ
	iperstatica X=W <sub>FC</sub>						-19/10Fb	

Sviluppi di calcolo iperstatica

$$L_{DE}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{XX} = \int_0^b (1-2x/b+x^2/b^2) 1/EJ dx = [x-x^2/b+1/3 x^3/b^2]_0^b 1/EJ = (b-b+1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FE}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FC}^{XX} = \int_0^b (1-2x/b+x^2/b^2) 1/EJ dx = [x-x^2/b+1/3 x^3/b^2]_0^b 1/EJ = (b-b+1/3 b) 1/EJ = 1/3 b/EJ$$

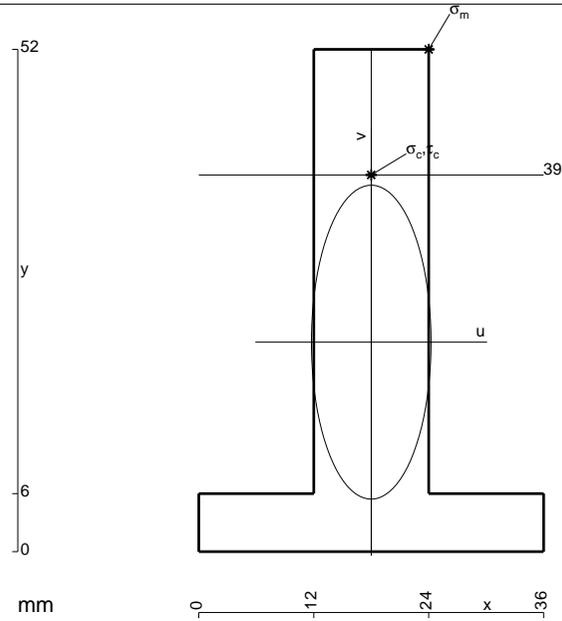
$$L_{CF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DE}^{X0} = \int_0^b (5/2 x^2/b^2) Fb 1/EJ dx = [5/6 x^3/b^2]_0^b Fb 1/EJ = (5/6 b) Fb 1/EJ = 5/6 Fb^2/EJ$$

$$L_{ED}^{X0} = \int_0^b (5/2 -5x/b+5/2 x^2/b^2) Fb 1/EJ dx = [5/2 x -5/2 x^2/b+5/6 x^3/b^2]_0^b Fb 1/EJ = (5/2 b -5/2 b +5/6 b) Fb 1/EJ = 5/6 Fb^2/EJ$$

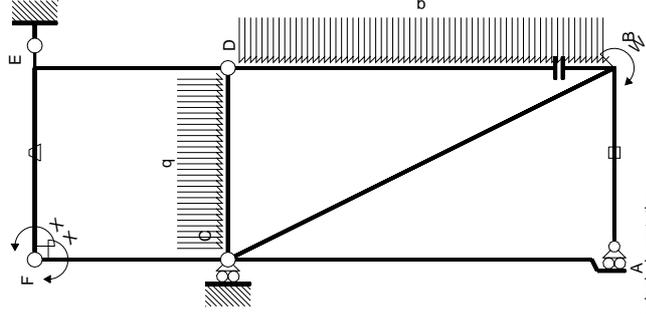
$$L_{EF}^{X0} = \int_0^b (5/2 -2x/b -1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1) \theta dx = [5/2 x -x^2/b -1/6 x^3/b^2]_0^b Fb 1/EJ + [x]_0^b \theta = (5/2 b -b -1/6 b) Fb 1/EJ + (b) \theta = 7/3 Fb^2/EJ$$

$$L_{FE}^{X0} = \int_0^b (3x/b -1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1) \theta dx = [3/2 x^2/b -1/6 x^3/b^2]_0^b Fb 1/EJ + [-x]_0^b \theta = (3/2 b -1/6 b) Fb 1/EJ + (-b) \theta = 7/3 Fb^2/EJ$$

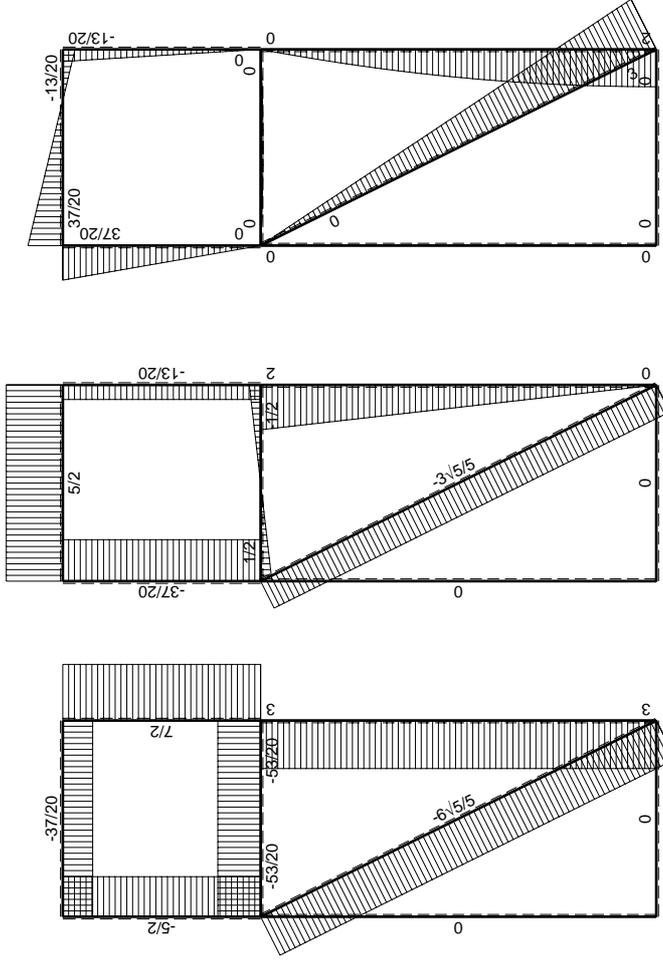


$A = 768. \text{ mm}^2$   
 $J_u = 202933. \text{ mm}^4$   
 $J_v = 29952. \text{ mm}^4$   
 $y_g = 21.69 \text{ mm}$   
 $N = -2925. \text{ N}$   
 $T_y = -1462. \text{ N}$   
 $M_x = 1438800. \text{ Nmm}$   
 $x_m = 24. \text{ mm}$   
 $y_m = 52. \text{ mm}$   
 $u_m = 6. \text{ mm}$   
 $v_m = 30.31 \text{ mm}$   
 $\sigma_m = N/A - Mv/J_u = -218.7 \text{ N/mm}^2$   
 $x_c = 18. \text{ mm}$   
 $y_c = 39. \text{ mm}$   
 $v_c = 17.31 \text{ mm}$   
 $\sigma_c = N/A - Mv/J_u = -126.6 \text{ N/mm}^2$   
 $\tau_c = 2.231 \text{ N/mm}^2$   
 $\sigma_g = \sqrt{\sigma^2 + 3\tau^2} = 126.6 \text{ N/mm}^2$   
 $S = 3715. \text{ mm}^3$





Schema di calcolo iperstatico



$\leftarrow \rightarrow$  N

$\uparrow \downarrow$  V

$\curvearrowright$  M

$\curvearrowright$  Fb

$\leftarrow \rightarrow$  F

$\leftarrow \rightarrow$  F

$\curvearrowright$  M<sub>x</sub> flessione da iperstatica X=1

$\curvearrowright$  M<sub>y</sub> flessione da carichi assegnati

Quadro contributi PLV per iperstatica X=W<sub>FE</sub>

→	M <sub>x</sub> (x)	M <sub>0</sub> (x)	θ	M <sub>x</sub> M <sub>0</sub>	M <sub>x</sub> θ	M <sub>x</sub> M <sub>x</sub>	∫M <sub>x</sub> (M <sub>0</sub> /EJ+θ)dx	∫XM <sub>x</sub> M <sub>x</sub> /EJdx
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0	0+0	0
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0	0+0	0
DB 2b	0	2Fx-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx <sup>2</sup>	0	0	0	0	0+0	0
DE b	x/b	-5/2Fx	0	-5/2Fx <sup>2</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>	(-5/6+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ
ED b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx <sup>2</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0+0	0
CD b	0	1/2Fx-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx <sup>2</sup>	0	0	0	0	0+0	0
EF b	1	-5/2Fb+5/2Fx	-Fb/EJ	-5/2Fb+5/2Fx	-Fb/EJ	1	(-5/4-1)Fb <sup>2</sup> /EJ	Xb/EJ
FE b	-1	5/2Fx	Fb/EJ	-5/2Fx	-Fb/EJ	1	0+0	1/3Xb/EJ
FC b	1-x/b	0	0	0	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x <sup>2</sup> /b <sup>2</sup>	-37/12Fb <sup>2</sup> /EJ	5/3Xb/EJ
totali								
	iperstatica X=W <sub>FE</sub>							

Sviluppi di calcolo iperstatica

$$L_{DE}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FE}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

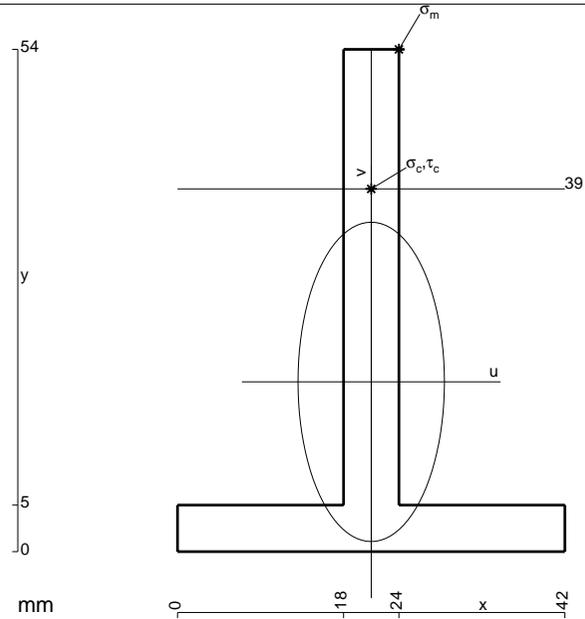
$$L_{CF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DE}^{X0} = \int_0^b (-5/2 x^2/b^2) Fb 1/EJ dx = [-5/6 x^3/b^2]_0^b Fb 1/EJ = (-5/6 b) Fb 1/EJ = -5/6 Fb^2/EJ$$

$$L_{ED}^{X0} = \int_0^b (-5/2 + 5x/b - 5/2 x^2/b^2) Fb 1/EJ dx = [-5/2 x + 5/2 x^2/b - 5/6 x^3/b^2]_0^b Fb 1/EJ = (-5/2 b + 5/2 b - 5/6 b) Fb 1/EJ = -5/6 Fb^2/EJ$$

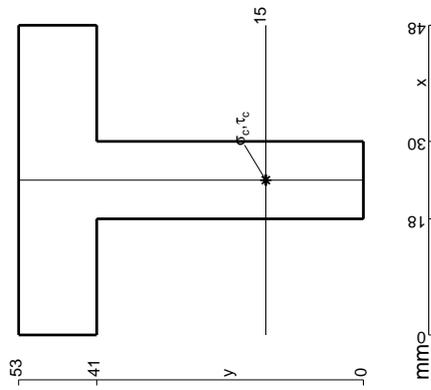
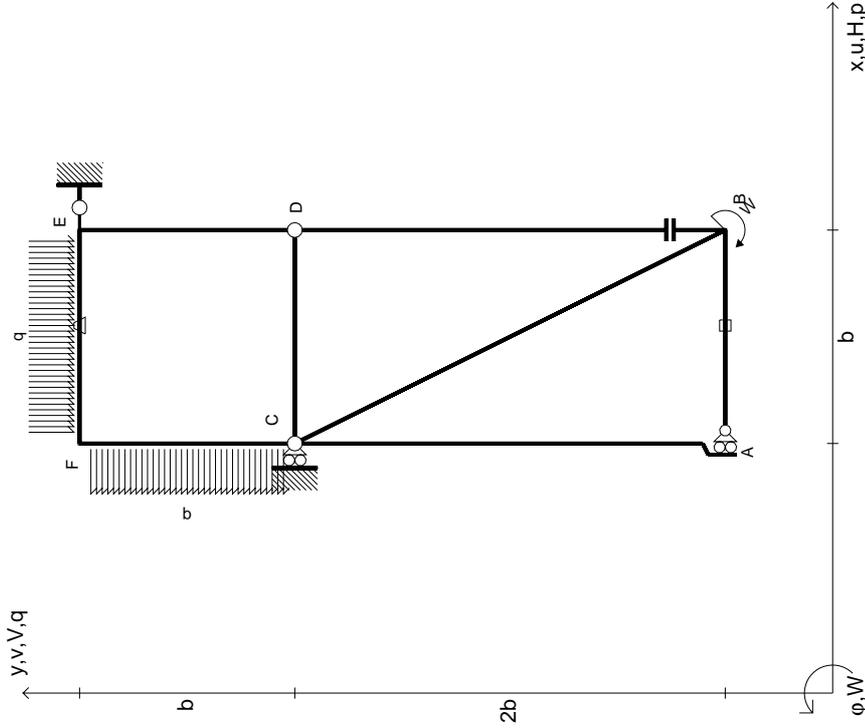
$$L_{EF}^{X0} = \int_0^b (-5/2 + 5/2 x/b) Fb 1/EJ dx + \int_0^b (-1) \theta dx = [-5/2 x + 5/4 x^2/b]_0^b Fb 1/EJ + [-x]_0^b \theta = (-5/2 b + 5/4 b) Fb 1/EJ + (-b) \theta = -9/4 Fb^2/EJ$$

$$L_{FE}^{X0} = \int_0^b (-5/2 x/b) Fb 1/EJ dx + \int_0^b (1) \theta dx = [-5/4 x^2/b]_0^b Fb 1/EJ + [x]_0^b \theta = (-5/4 b) Fb 1/EJ + (b) \theta = -9/4 Fb^2/EJ$$

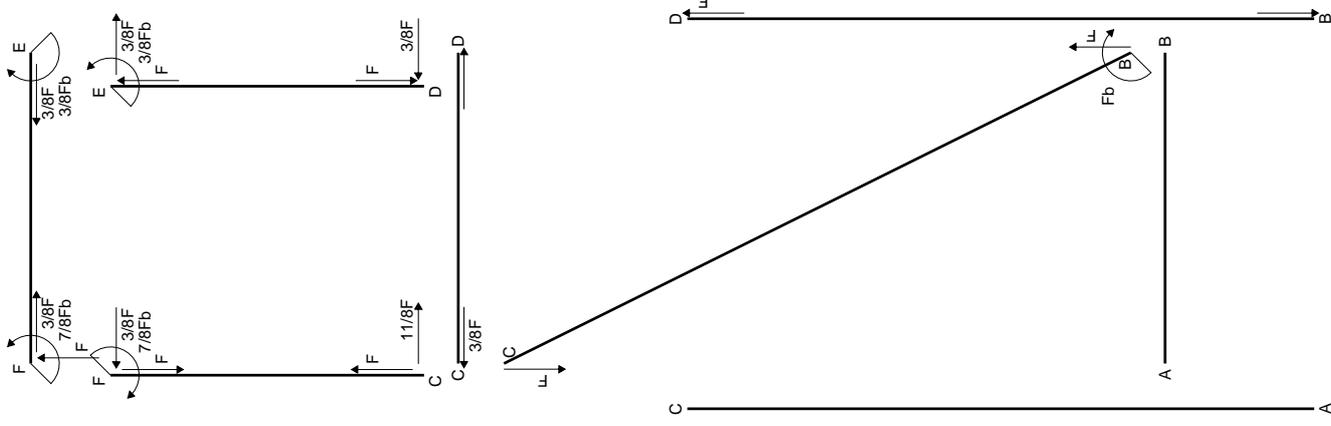


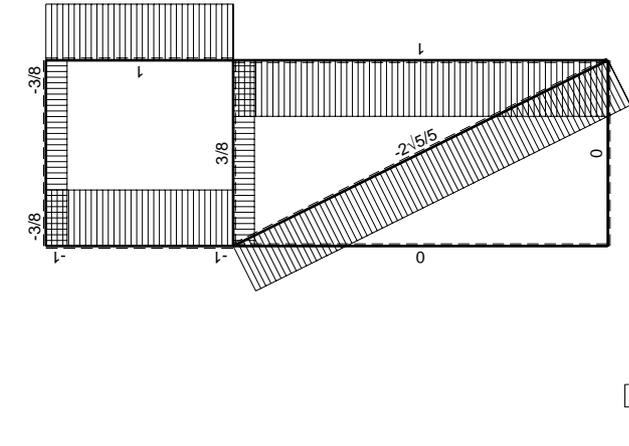
$A = 504. \text{ mm}^2$   
 $J_u = 148565. \text{ mm}^4$   
 $J_v = 31752. \text{ mm}^4$   
 $y_g = 18.25 \text{ mm}$   
 $N = -1261. \text{ N}$   
 $T_y = -630.6 \text{ N}$   
 $M_x = 846000. \text{ Nmm}$   
 $x_m = 24. \text{ mm}$   
 $y_m = 54. \text{ mm}$   
 $u_m = 3. \text{ mm}$   
 $v_m = 35.75 \text{ mm}$   
 $\sigma_m = N/A - Mv/J_u = -206.1 \text{ N/mm}^2$   
 $x_c = 21. \text{ mm}$   
 $y_c = 39. \text{ mm}$   
 $v_c = 20.75 \text{ mm}$   
 $\sigma_c = N/A - Mv/J_u = -120.7 \text{ N/mm}^2$   
 $\tau_c = 1.799 \text{ N/mm}^2$   
 $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 120.7 \text{ N/mm}^2$   
 $S = 2543. \text{ mm}^3$

$$\begin{aligned}
 W_B &= -W = -Fb \\
 q_{EF} &= -q = -F/b \\
 P_{FC} &= -q = -F/b \\
 \varepsilon_{AB} &= 3\alpha T = 3b^2 F/EJ \\
 \theta_{EF} &= -\theta = -\alpha T/b = -bF/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{AC}} &= EJ \\
 E_{J_{DB}} &= EJ \\
 E_{J_{DE}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ
 \end{aligned}$$



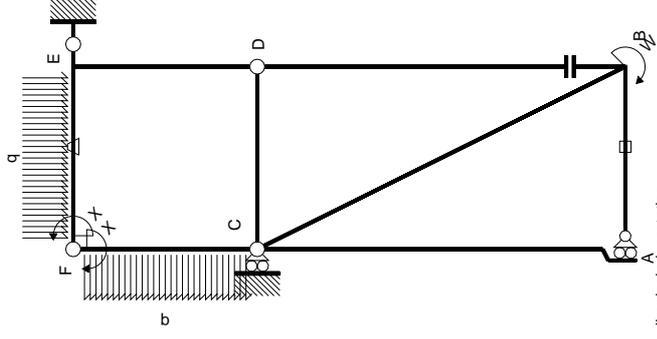
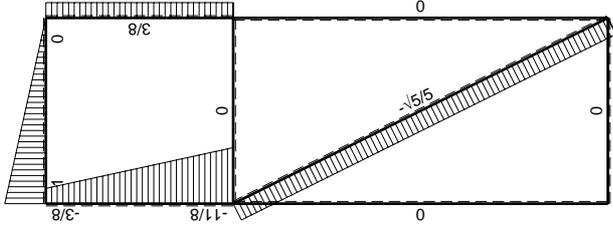
Reazioni iperstatiche in soluzione:  $X=W_{FE}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{yz} = X_{yz} - \theta_{yz}$  riferimento locale asta YZ con origine in Y.  
 La trave BC ha la sezione riportata e dimensioni in mm, con:  
 $b = 370$  mm,  $F = 4140$  N  
 Calcolare sulla sezione B la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da B a C  
 Elongazione termica specifica  $\varepsilon$  assegnata su asta AB.  
 Curvatura  $\theta$  asta EF positiva se convessa a destra con inizio E.  
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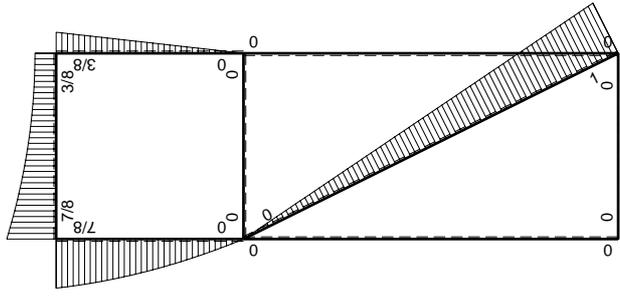


$\left[ \begin{matrix} + \\ - \end{matrix} \right] \rightarrow F$

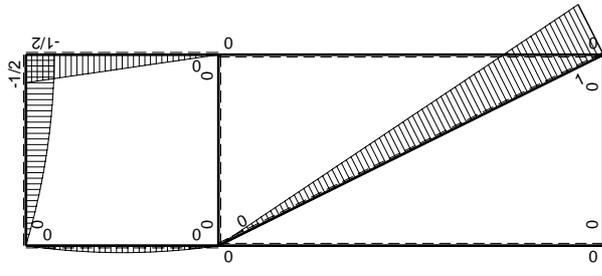
$\uparrow \left[ \begin{matrix} + \\ - \end{matrix} \right] F$



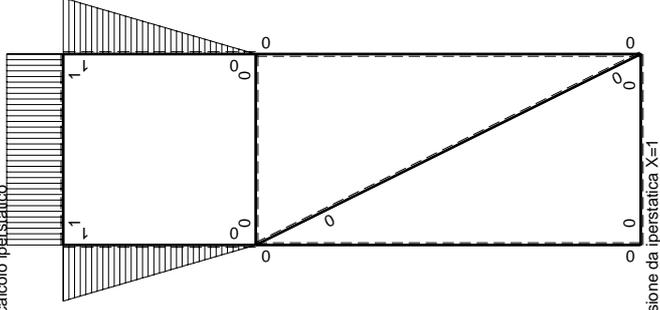
Schema di calcolo iperstatico



$\left[ \begin{matrix} + \\ - \end{matrix} \right] F_b$



$\left[ \begin{matrix} + \\ - \end{matrix} \right] M_0$  flessione da carichi assegnati



$\left[ \begin{matrix} + \\ - \end{matrix} \right] M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica X=W<sub>FE</sub>

→	M <sub>x</sub> (x)	M <sub>0</sub> (x)	θ	M <sub>x</sub> M <sub>0</sub>	M <sub>x</sub> θ	M <sub>x</sub> M <sub>x</sub>	∫M <sub>x</sub> (M <sub>0</sub> /EJ+θ)dx	∫xM <sub>x</sub> M <sub>0</sub> /EJdx
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0	0	0
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0	0+0	0
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0	0+0	0
DE b	x/b	-1/2Fx	0	-1/2Fx <sup>2</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>	(-1/6+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ
ED b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx <sup>2</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0	0+0	0
EF b	1	-1/2Fb+1/2qx <sup>2</sup>	-Fb/EJ	-1/2Fb+1/2Fx <sup>2</sup> /b	-Fb/EJ	1	(-1/3-1)Fb <sup>2</sup> /EJ	Xb/EJ
FE b	-1	Fx-1/2qx <sup>2</sup>	Fb/EJ	-Fx+1/2Fx <sup>2</sup> /b	-Fb/EJ	1	(1/24+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ
FC b	1-x/b	1/2Fx-1/2qx <sup>2</sup>	0	1/2Fx-Fx <sup>2</sup> /b+1/2qx <sup>3</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>		
CF b	-x/b	-1/2Fx+1/2qx <sup>2</sup>	0	1/2Fx <sup>2</sup> /b-1/2qx <sup>3</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>		
totali								
iperstatica X=W <sub>FE</sub>								

Sviluppi di calcolo iperstatica

$$L_{DE}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FE}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DE}^{X0} = \int_0^b (-1/2 x^2/b^2) Fb 1/EJ dx = [-1/6 x^3/b^2]_0^b Fb 1/EJ = (-1/6 b) Fb 1/EJ = -1/6 Fb<sup>2</sup>/EJ$$

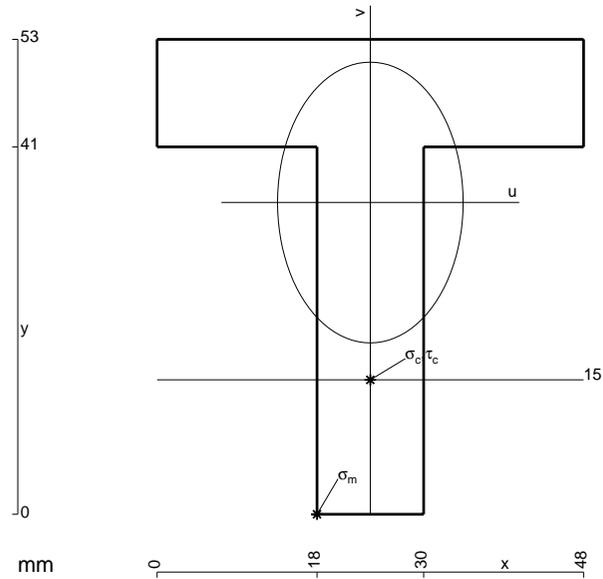
$$L_{ED}^{X0} = \int_0^b (-1/2 + x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [-1/2 x + 1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ = (-1/2 b + 1/2 b - 1/6 b) Fb 1/EJ = -1/6 Fb<sup>2</sup>/EJ$$

$$L_{EF}^{X0} = \int_0^b (-1/2 + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1) \theta dx = [-1/2 x + 1/6 x^3/b^2]_0^b Fb 1/EJ + [-x]_0^b \theta = (-1/2 b + 1/6 b) Fb 1/EJ + (-b) \theta = -4/3 Fb<sup>2</sup>/EJ$$

$$L_{FE}^{X0} = \int_0^b (-x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1) \theta dx = [-1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [x]_0^b \theta = (-1/2 b + 1/6 b) Fb 1/EJ + (b) \theta = -4/3 Fb<sup>2</sup>/EJ$$

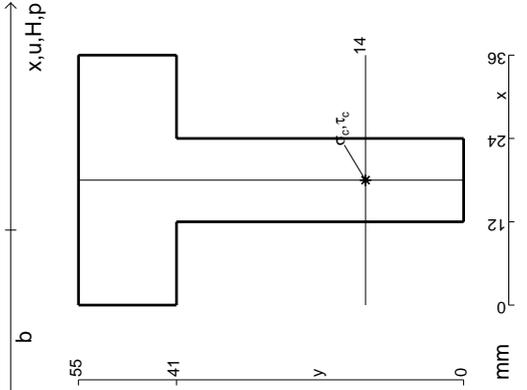
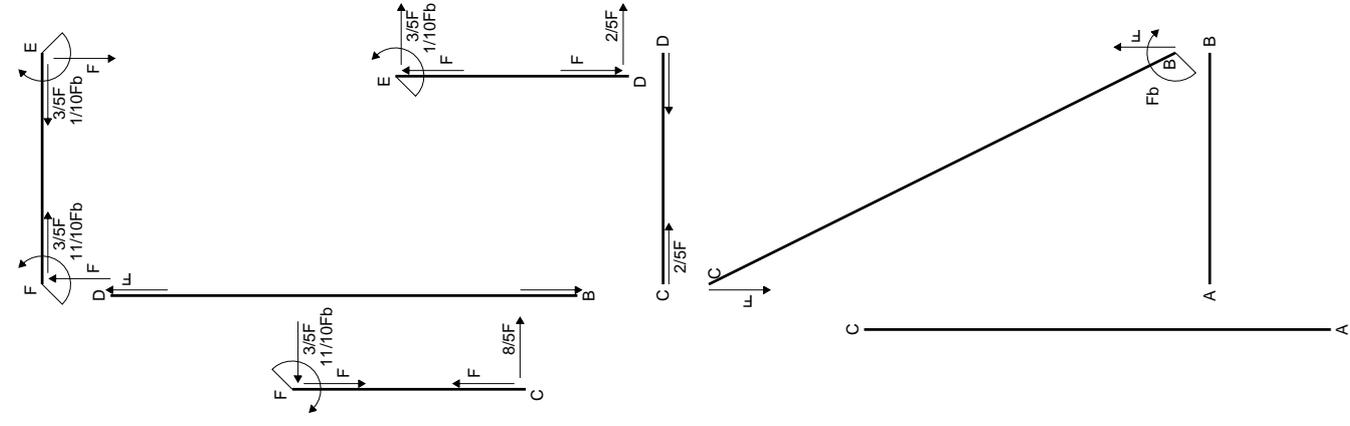
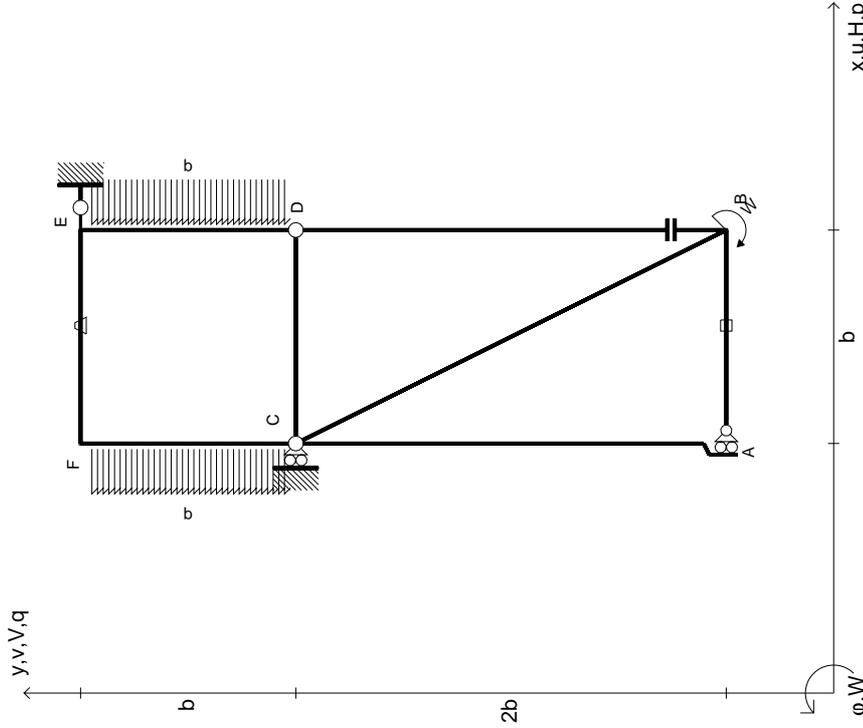
$$L_{FC}^{X0} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ = (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb<sup>2</sup>/EJ$$

$$L_{CF}^{X0} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ = (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb<sup>2</sup>/EJ$$

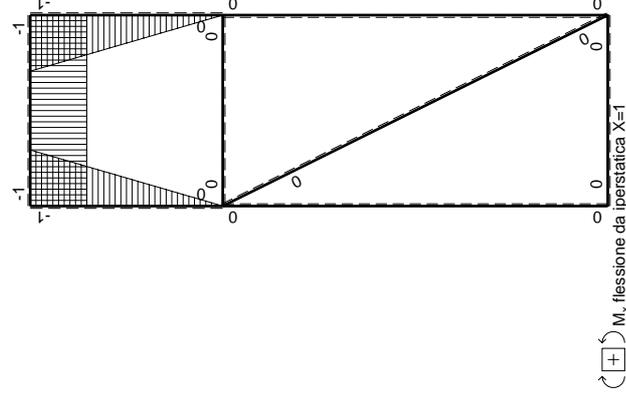
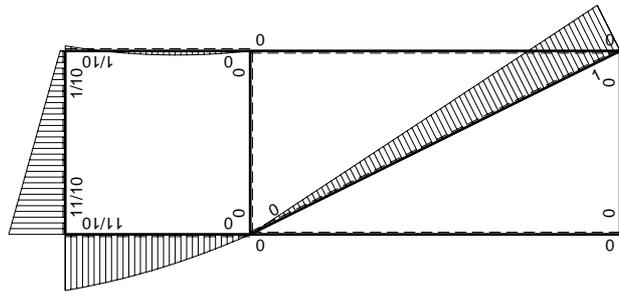
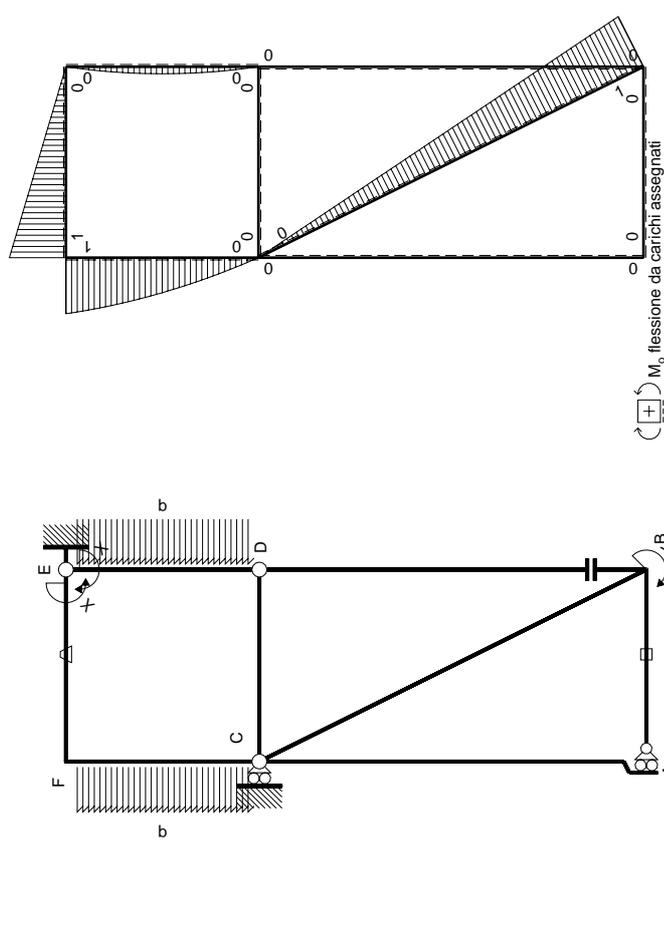
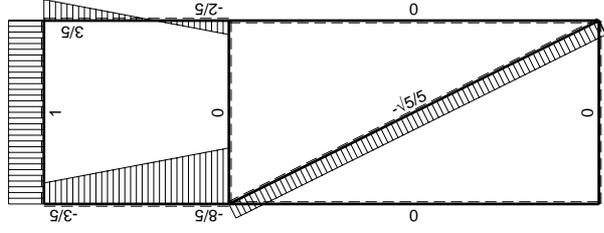
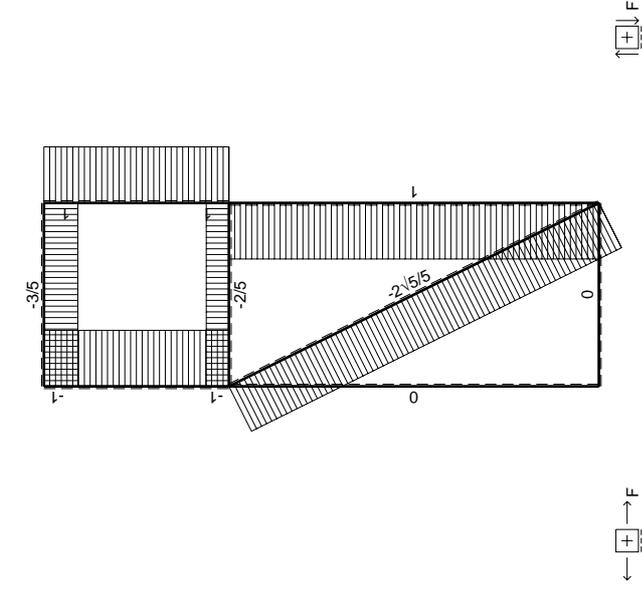


$A = 1068. \text{ mm}^2$   
 $J_u = 262174. \text{ mm}^4$   
 $J_v = 116496. \text{ mm}^4$   
 $y_g = 34.79 \text{ mm}$   
 $N = -3703. \text{ N}$   
 $T_y = -1851. \text{ N}$   
 $M_x = 1531800. \text{ Nmm}$   
 $x_m = 18. \text{ mm}$   
 $u_m = -6. \text{ mm}$   
 $v_m = -34.79 \text{ mm}$   
 $\sigma_m = N/A - Mv/J_u = 199.8 \text{ N/mm}^2$   
 $x_c = 24. \text{ mm}$   
 $y_c = 15. \text{ mm}$   
 $v_c = -19.79 \text{ mm}$   
 $\sigma_c = N/A - Mv/J_u = 112.2 \text{ N/mm}^2$   
 $\tau_c = 2.891 \text{ N/mm}^2$   
 $\sigma_\varrho = \sqrt{\sigma^2 + 3\tau^2} = 112.3 \text{ N/mm}^2$   
 $S = 4913. \text{ mm}^3$

$$\begin{aligned}
 W_B &= -W = -Fb \\
 P_{DE} &= -q = -F/b \\
 P_{FC} &= -q = -F/b \\
 \varepsilon_{AB} &= 3\alpha T = 3b^2 F/EJ \\
 \theta_{EF} &= -\theta = -\alpha T/b = -bF/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{AC}} &= EJ \\
 E_{J_{DB}} &= EJ \\
 E_{J_{DE}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ
 \end{aligned}$$



Reazioni iperstatiche in soluzione:  $X=W_{EF}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{y,z} - x_{y,z} - \theta_{y,z}$  riferimento locale asta YZ con origine in Y.  
 La trave BC ha la sezione riportata e dimensioni in mm, con:  
 $b = 390 \text{ mm}, F = 4020 \text{ N}$   
 Calcolare sulla sezione B la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su traveggio trave, a destra da B a C  
 Elongazione termica specifica  $\varepsilon$  assegnata su asta AB.  
 Curvatura  $\theta$  asta EF positiva se convessa a destra con inizio E.  
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$\left[ \begin{array}{c} + \\ - \end{array} \right] F_b$

Quadro contributi PLV per iperstatica X=W <sub>EF</sub>		M <sub>x</sub> (x)	M <sub>0</sub> (x)	θ	M <sub>x</sub> M <sub>0</sub>	M <sub>x</sub> θ	M <sub>x</sub> M <sub>x</sub>	∫M <sub>x</sub> (M <sub>0</sub> /EJ+θ)dx	∫XM <sub>x</sub> M <sub>0</sub> /EJdx
→									
AB b	0	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0	0	0	0
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0	0	0+0	0
DB 2b	0	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0	0	0+0	0
DE b	-x/b	-1/2Fx+1/2qx <sup>2</sup>	0	0	1/2Fx <sup>2</sup> /b-1/2qx <sup>3</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>	(1/24+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx <sup>2</sup>	0	0	1/2Fx-Fx <sup>2</sup> /b+1/2qx <sup>3</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>		
CD b	0	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0	0	0+0	0
EF b	-1	Fx	-Fb/EJ	0	-Fx	Fb/EJ	1	(-1/2+1)Fb <sup>2</sup> /EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	0	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx <sup>2</sup>	0	0	-Fb+3/2Fx-1/2qx <sup>3</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	(-3/8+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ
CF b	x/b	-3/2Fx+1/2qx <sup>2</sup>	0	0	-3/2Fx <sup>2</sup> /b+1/2qx <sup>3</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>		
		totali						1/6Fb <sup>2</sup> /EJ	5/3Xb/EJ
		iperstatica X=W <sub>EF</sub>						-1/10Fb	

Sviluppi di calcolo iperstatica

$$L_{DE}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FE}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DE}^{X0} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ = (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

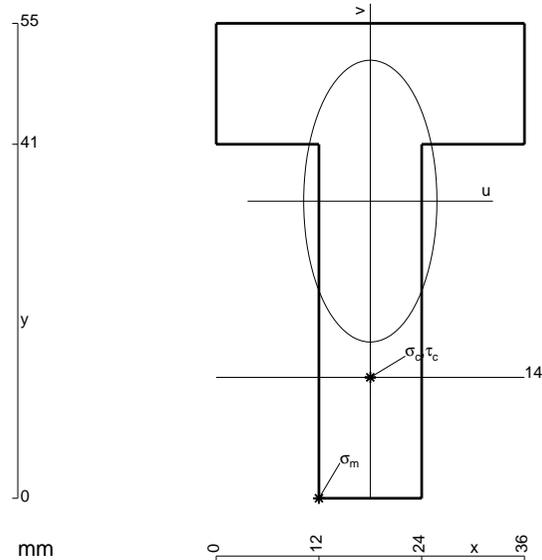
$$L_{ED}^{X0} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ = (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{EF}^{X0} = \int_0^b (-x/b) Fb 1/EJ dx + \int_0^b (1) \theta dx = [-1/2 x^2/b]_0^b Fb 1/EJ + [x]_0^b \theta = (-1/2 b) Fb 1/EJ + (b) \theta = 1/2 Fb^2/EJ$$

$$L_{FE}^{X0} = \int_0^b (-1 + x/b) Fb 1/EJ dx + \int_0^b (-1) \theta dx = [-x + 1/2 x^2/b]_0^b Fb 1/EJ + [-x]_0^b \theta = (-b + 1/2 b) Fb 1/EJ + (-b) \theta = 1/2 Fb^2/EJ$$

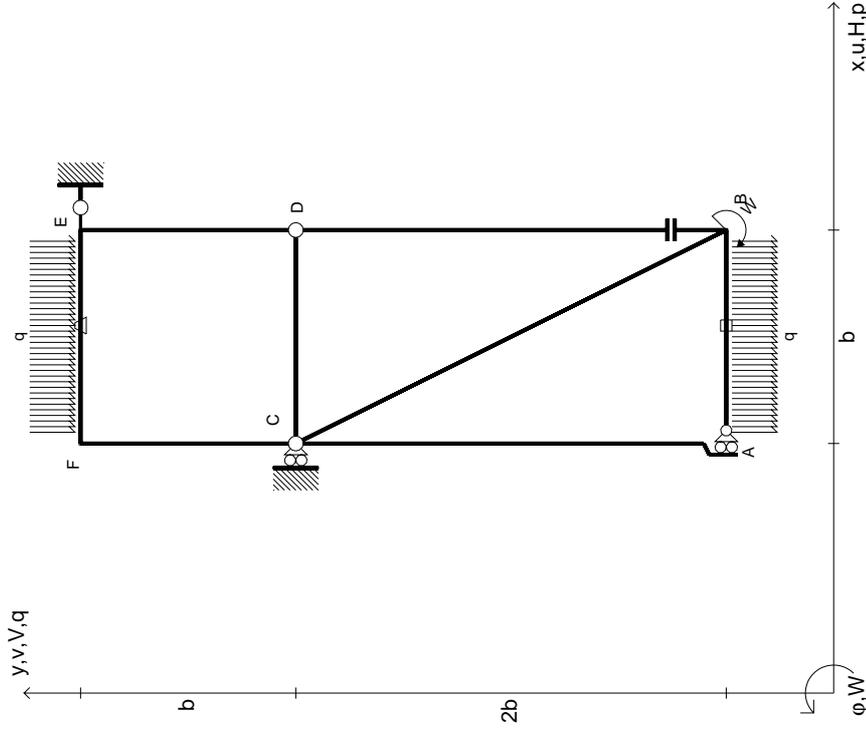
$$L_{FC}^{X0} = \int_0^b (-1 + 3/2 x/b - 1/2 x^3/b^3) Fb 1/EJ dx = [-x + 3/4 x^2/b - 1/8 x^4/b^3]_0^b Fb 1/EJ = (-b + 3/4 b - 1/8 b) Fb 1/EJ = -3/8 Fb^2/EJ$$

$$L_{CF}^{X0} = \int_0^b (-3/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [-1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ = (-1/2 b + 1/8 b) Fb 1/EJ = -3/8 Fb^2/EJ$$

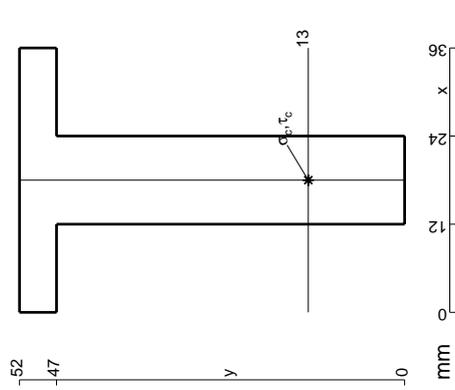
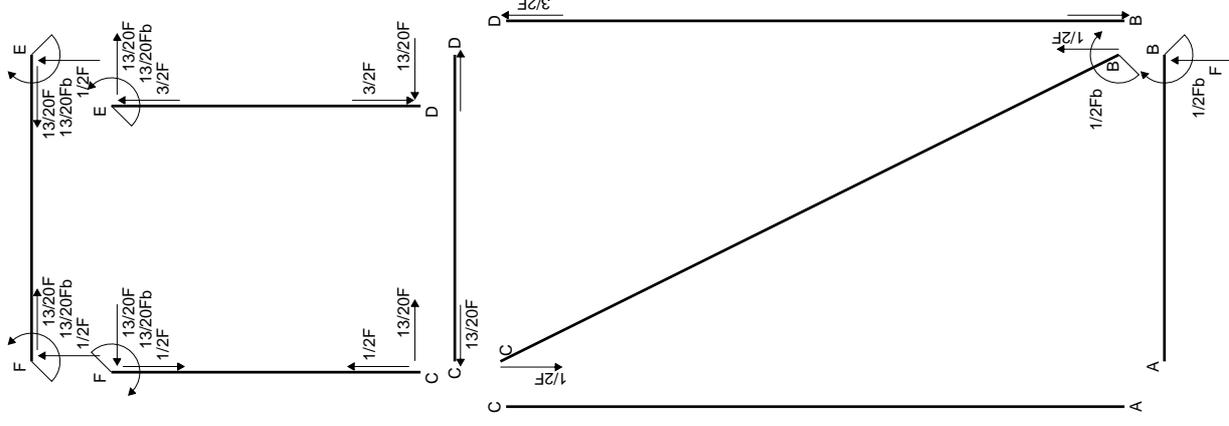


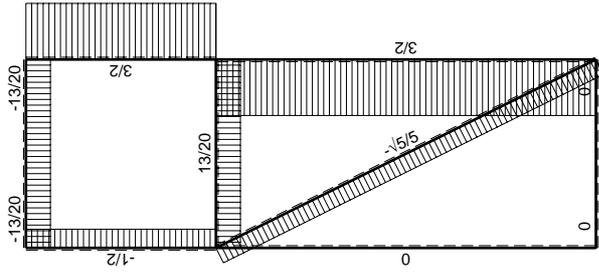
- $A = 996. \text{ mm}^2$
- $J_u = 265432. \text{ mm}^4$
- $J_v = 60336. \text{ mm}^4$
- $y_g = 34.42 \text{ mm}$
- $N = -3596. \text{ N}$
- $T_y = -1798. \text{ N}$
- $M_x = 1567800. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $u_m = -6. \text{ mm}$
- $v_m = -34.42 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = 199.7 \text{ N/mm}^2$
- $x_c = 18. \text{ mm}$
- $y_c = 14. \text{ mm}$
- $v_c = -20.42 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = 117. \text{ N/mm}^2$
- $\tau_c = 2.6 \text{ N/mm}^2$
- $\sigma_\varrho = \sqrt{\sigma^2 + 3\tau^2} = 117.1 \text{ N/mm}^2$
- $S = 4606. \text{ mm}^3$

$$\begin{aligned}
 W_B &= -W = -Fb \\
 q_{AB} &= -q = -F/b \\
 q_{EF} &= -q = -F/b \\
 \varepsilon_{AB} &= -3\alpha T = -3b^2 F/EJ \\
 \theta_{EF} &= -\theta = -\alpha T/b = -bF/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{AC}} &= EJ \\
 E_{J_{DB}} &= EJ \\
 E_{J_{DE}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ
 \end{aligned}$$

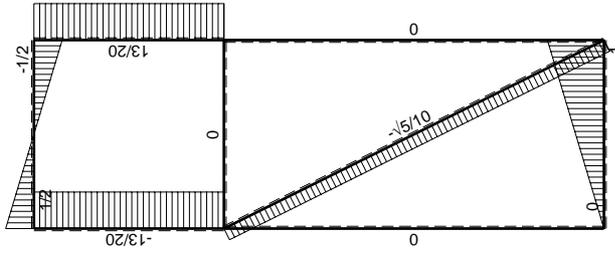


Reazioni iperstatiche in soluzione:  $X=W_{FC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{y,z} - x_{y,z} - \theta_{y,z}$  riferimento locale asta YZ con origine in Y.  
 La trave AB ha la sezione riportata e dimensioni in mm, con:  
 $b = 550 \text{ mm}$ ,  $F = 4790 \text{ N}$   
 Calcolare sulla sezione B la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da A a B  
 Elongazione termica specifica  $\varepsilon$  assegnata su asta AB.  
 Curvatura  $\theta$  asta EF positiva se convessa a destra con inizio E.  
 © Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

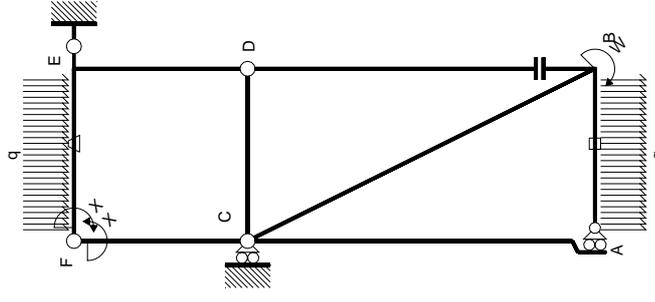




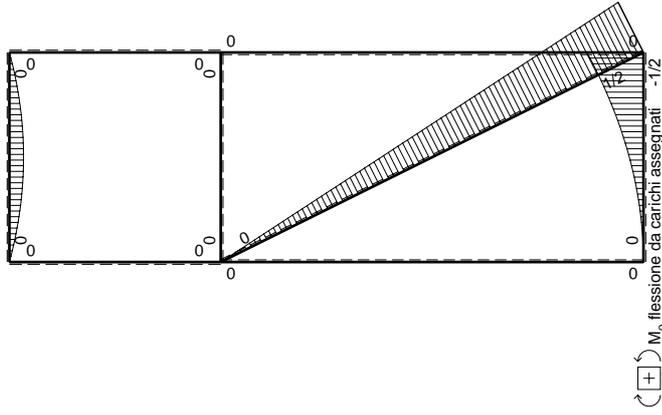
$\left[ \begin{array}{c} \leftarrow \\ \oplus \\ \rightarrow \end{array} \right] F_v$



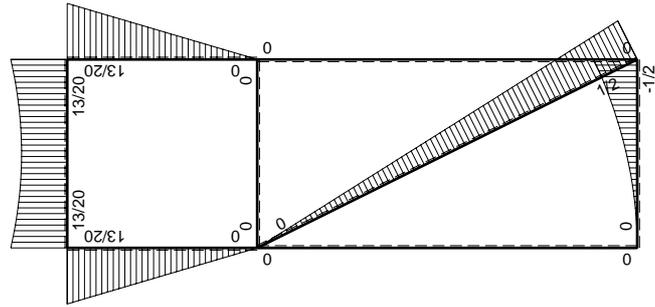
$\left[ \begin{array}{c} \oplus \\ \rightarrow \\ \leftarrow \end{array} \right] F_v$



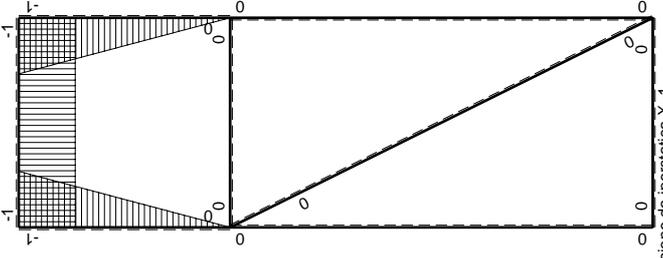
Schema di calcolo iperstatico



$\left[ \begin{array}{c} \oplus \\ \curvearrowright \end{array} \right] M_0$  flessione da carichi assegnati -1/2



$\left[ \begin{array}{c} \oplus \\ \curvearrowright \end{array} \right] F_b$



$\left[ \begin{array}{c} \oplus \\ \curvearrowright \end{array} \right] M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W<sub>FC</sub>

→	M <sub>x</sub> (x)	M <sub>0</sub> (x)	θ	M <sub>x</sub> M <sub>0</sub>	M <sub>x</sub> θ	M <sub>x</sub> M <sub>x</sub>	∫M <sub>x</sub> (M <sub>0</sub> (EJ+θ)dx	∫XM <sub>x</sub> M <sub>x</sub> /EJdx
AB b	0	-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
BA b	0	1/2Fb-Fx+1/2qx <sup>2</sup>	0	0	0	0	0+0	0
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0+0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0	0+0	0
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0	0+0	0
DE b	-x/b	0	0	0	0	x <sup>2</sup> /b <sup>2</sup>	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0+0	0
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0	0+0	0
EF b	-1	-1/2Fx+1/2qx <sup>2</sup>	-Fb/EJ	1/2Fx-1/2Fx <sup>2</sup> /b	Fb/EJ	1	(1/12+1)Fb <sup>2</sup> /EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx <sup>2</sup>	Fb/EJ	1/2Fx-1/2Fx <sup>2</sup> /b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x <sup>2</sup> /b <sup>2</sup>		
totali								
iperstatica X=W <sub>FC</sub>								

Sviluppi di calcolo iperstatica

$$L_{DE}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

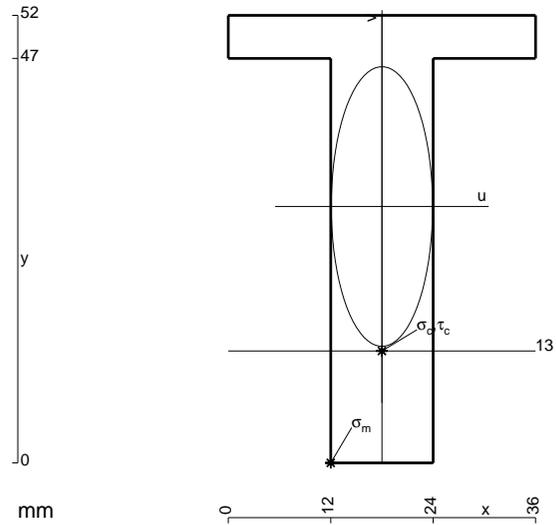
$$L_{FE}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{FC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{X0} = \int_0^b (1/2 x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1) \theta dx = [1/4 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x]_0^b \theta = (1/4 b - 1/6 b) Fb 1/EJ + (b) \theta = 13/12 Fb^2/EJ$$

$$L_{FE}^{X0} = \int_0^b (1/2 x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1) \theta dx = [1/4 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-x]_0^b \theta = (1/4 b - 1/6 b) Fb 1/EJ + (-b) \theta = 13/12 Fb^2/EJ$$



- $A = 744. \text{ mm}^2$
- $J_u = 196439. \text{ mm}^4$
- $J_v = 26208. \text{ mm}^4$
- $y_g = 29.79 \text{ mm}$
- $T_y = -4790. \text{ N}$
- $M_x = -1317250. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $u_m = -6. \text{ mm}$
- $v_m = -29.79 \text{ mm}$
- $\sigma_m = -Mv/J_u = -199.8 \text{ N/mm}^2$
- $x_c = 18. \text{ mm}$
- $y_c = 13. \text{ mm}$
- $v_c = -16.79 \text{ mm}$
- $\sigma_c = -Mv/J_u = -112.6 \text{ N/mm}^2$
- $\tau_c = 7.383 \text{ N/mm}^2$
- $\sigma_\rho = \sqrt{\sigma^2 + 3\tau^2} = 113.3 \text{ N/mm}^2$
- $S = 3633. \text{ mm}^3$