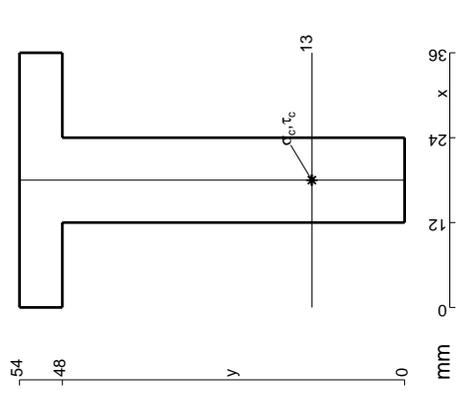
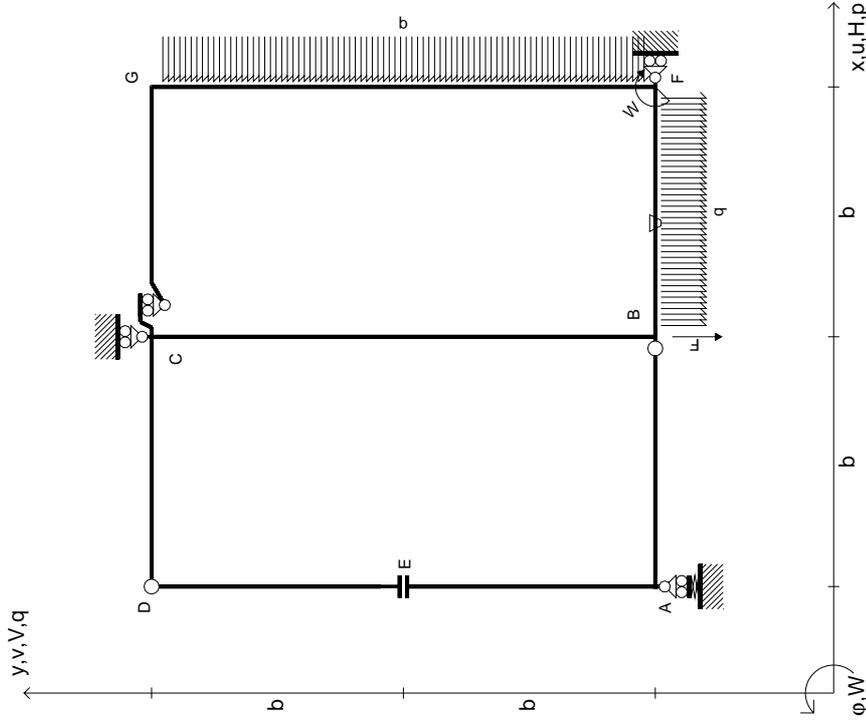
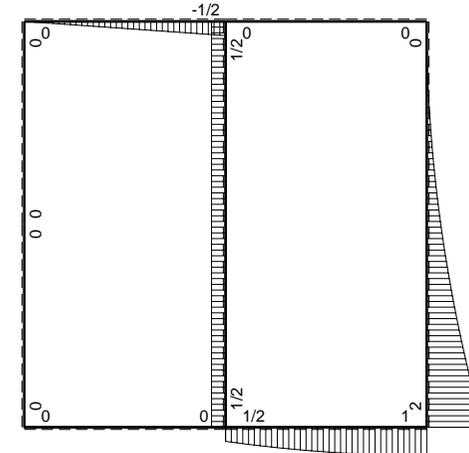
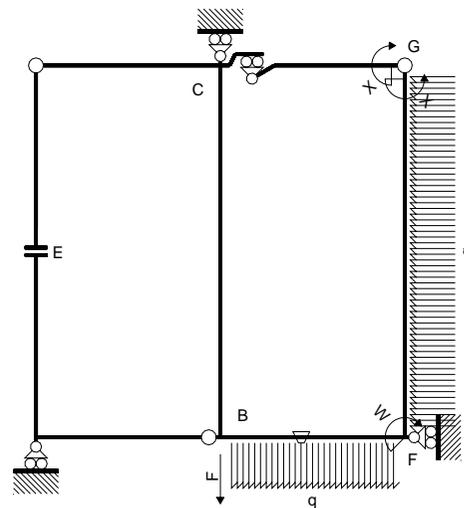
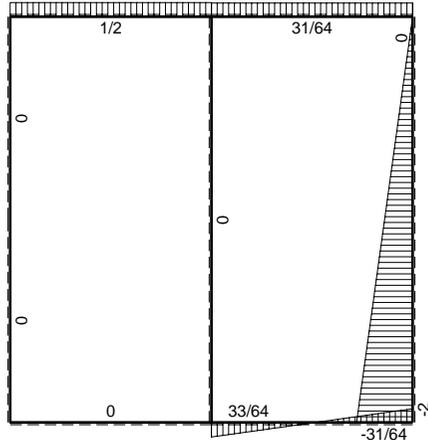
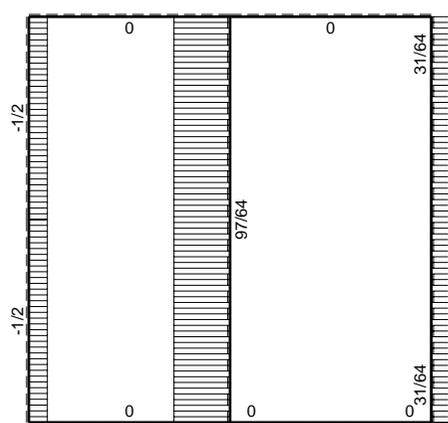


$V_b = -F$
 $W_F = -W = -Fb$
 $q_{BF} = -q = -F/b$
 $P_{FG} = -q = -F/b$
 $\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
 $K_A = 4EJ/b^3$
 $EJ_{AB} = EJ$
 $EJ_{CD} = EJ$
 $EJ_{DE} = EJ$
 $EJ_{EA} = EJ$
 $EJ_{BF} = EJ$
 $EJ_{GC} = EJ$
 $EJ_{FG} = EJ$
 $EJ_{CB} = EJ$



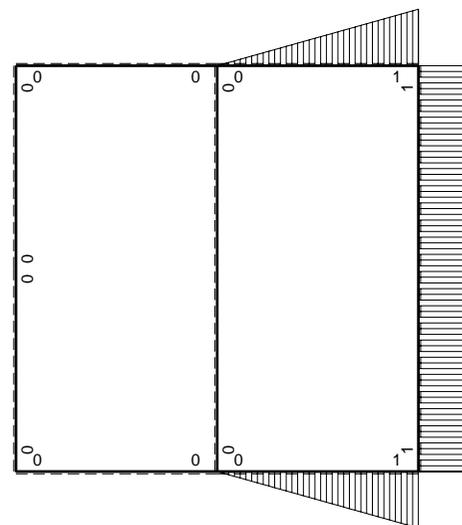
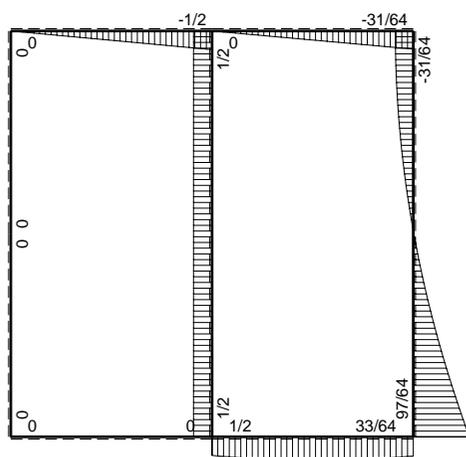
Reazioni iperstatiche in soluzione: $X=W_{GF}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{Y-Z} - X_{Y-Z} - \theta_{Y-Z}$ riferimento locale asta YZ con origine in Y.
 La trave CD ha la sezione riportata e dimensioni in mm, con:
 $b = 730 \text{ mm}$, $F = 4330 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su traveggio trave, a destra da C a D
 Curvatura θ asta BF positiva se convessa a destra con inizio B.
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← ⊕ → F

↑ ⊕ ↓ F

⊕ M_o flessione da carichi assegnati



⊕ F_b

⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GF}

→	M _x (x)	M ₀ (x)	θ	M _x M ₀	M _x θ	M _x M _x	∫M _x (M ₀ /E ₀ +θ)dx	∫XM _x M _x /E ₀ Jdx
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0	0+0	0
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0	0+0	0
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0	0+0	0
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0	0+0	0
BF b	x/b	1/2Fb+Fx-1/2qx ²	-Fb/EJ	1/2Fx+Fx ² /b-1/2qx ³ /b	-Fx/EJ	x ² /b ²	(1/24-1/2)Fb ² /EJ	1/3Xb/EJ
FB b	-1+x/b	-Fb+1/2qx ²	Fb/EJ	Fb-Fx-1/2Fx ² /b+1/2qx ³ /b	-Fb/EJ+Fx/EJ	1-2x/b+x ² /b ²	0+0	1/3Xb/EJ
GC b	1-x/b	0	0	0	0	1-2x/b+x ² /b ²	0+0	1/3Xb/EJ
CG b	-x/b	0	0	0	0	x ² /b ²	0+0	2Xb/EJ
FG 2b	1	2Fb-2Fx+1/2qx ²	0	2Fb-2Fx+1/2Fx ² /b	0	1	(4/3+0)Fb ² /EJ	0
GF 2b	-1	-1/2qx ²	0	1/2Fx ² /b	0	1	0+0	0
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0	0+0	0
	totali						31/24Fb ² /EJ	8/3Xb/EJ
	iperstatica X=W _{GF}						-31/64Fb	

Svilupi di calcolo iperstatica

$$L_{BF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FB}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{GC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FG}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2b) 1/EJ = 2 b/EJ$$

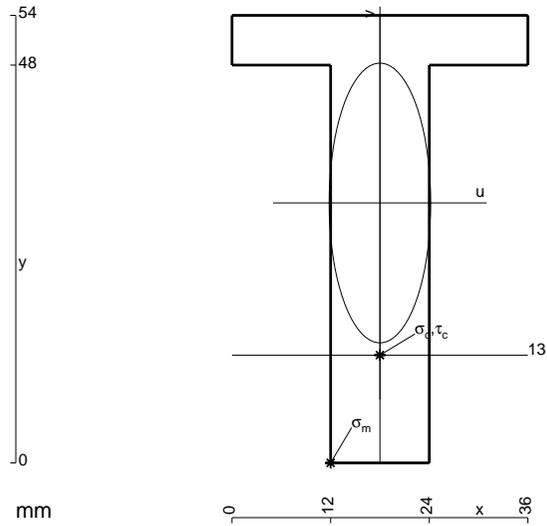
$$L_{GF}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2b) 1/EJ = 2 b/EJ$$

$$L_{BF}^{X0} = \int_0^b (1/2 x/b + x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-x/b) \theta dx = [1/4 x^2/b + 1/3 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [-1/2 x^2/b]_0^b \theta = (1/4 b + 1/3 b - 1/8 b) Fb 1/EJ + (-1/2 b) \theta = -1/24 Fb^2/EJ$$

$$L_{FB}^{X0} = \int_0^b (1 - x/b - 1/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (1 - x/b) \theta dx = [x - 1/2 x^2/b - 1/6 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [x - 1/2 x^2/b]_0^b \theta = (b - 1/2 b - 1/6 b + 1/8 b) Fb 1/EJ + (b - 1/2 b) \theta = -1/24 Fb^2/EJ$$

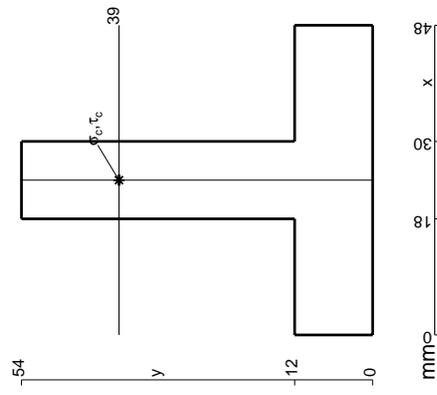
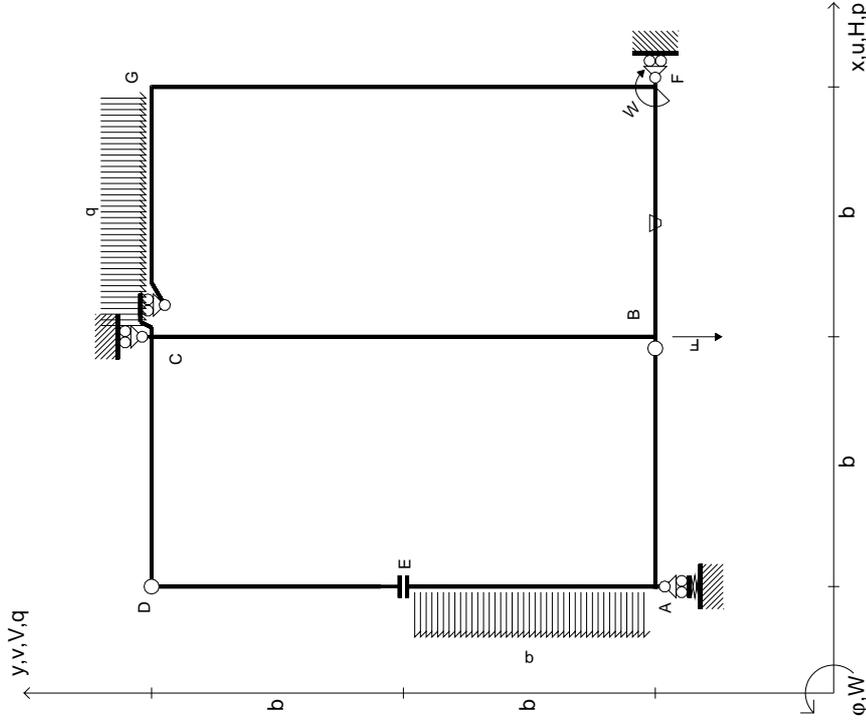
$$L_{FG}^{X0} = \int_0^{2b} (2 - 2x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [2x - x^2/b + 1/6 x^3/b^2]_0^{2b} Fb 1/EJ = (4b - 4b + 4/3 b) Fb 1/EJ = 4/3 Fb^2/EJ$$

$$L_{GF}^{X0} = \int_0^{2b} (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^{2b} Fb 1/EJ = (4/3 b) Fb 1/EJ = 4/3 Fb^2/EJ$$

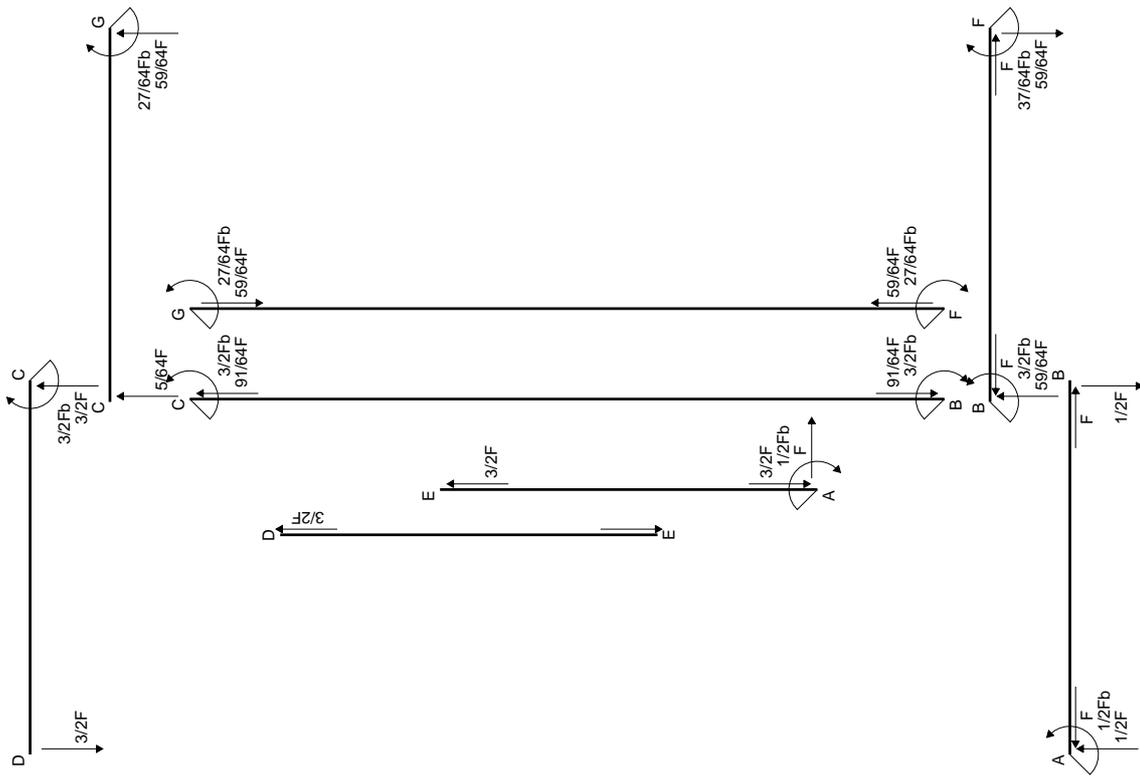


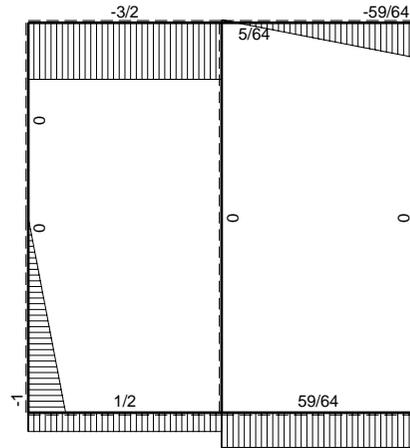
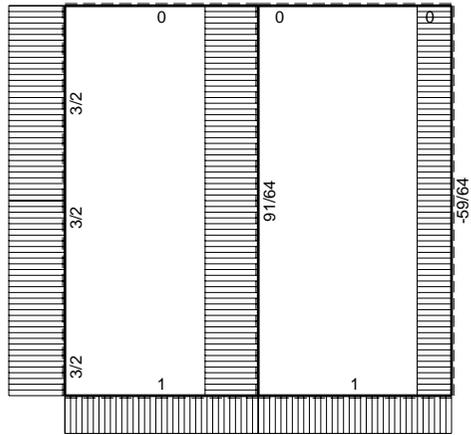
$$\begin{aligned}
 A &= 792. \text{ mm}^2 \\
 J_u &= 225759. \text{ mm}^4 \\
 J_v &= 30240. \text{ mm}^4 \\
 y_g &= 31.36 \text{ mm} \\
 T_y &= 2165. \text{ N} \\
 M_x &= -1580450. \text{ Nmm} \\
 x_m &= 12. \text{ mm} \\
 u_m &= -6. \text{ mm} \\
 v_m &= -31.36 \text{ mm} \\
 \sigma_m &= -Mv/J_u = -219.6 \text{ N/mm}^2 \\
 x_c &= 18. \text{ mm} \\
 y_c &= 13. \text{ mm} \\
 v_c &= -18.36 \text{ mm} \\
 \sigma_c &= -Mv/J_u = -128.6 \text{ N/mm}^2 \\
 \tau_c &= 3.1 \text{ N/mm}^2 \\
 \sigma_\rho &= \sqrt{\sigma^2 + 3\tau^2} = 128.7 \text{ N/mm}^2 \\
 S &= 3879. \text{ mm}^3
 \end{aligned}$$

$V_b = -F$
 $W_F = -W = -Fb$
 $P_{EA} = -q = -F/b$
 $q_{GC} = -q = -F/b$
 $\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
 $K_A = 4EJ/b^3$
 $EJ_{AB} = EJ$
 $EJ_{CD} = EJ$
 $EJ_{DE} = EJ$
 $EJ_{EA} = EJ$
 $EJ_{BF} = EJ$
 $EJ_{GC} = EJ$
 $EJ_{FG} = EJ$
 $EJ_{CB} = EJ$



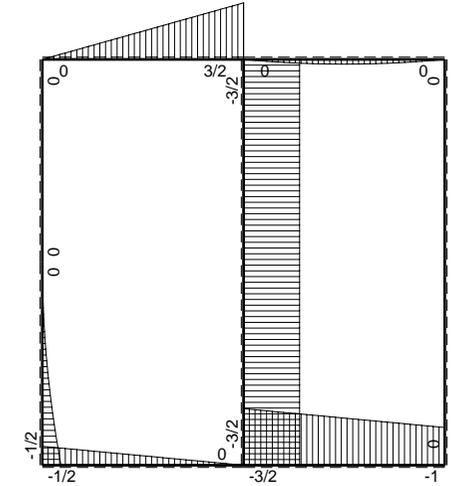
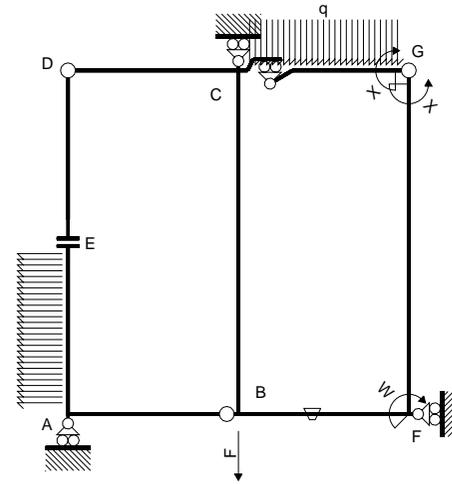
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 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave CD ha la sezione riportata e dimensioni in mm, con:
 $b = 640 \text{ mm}$, $F = 1620 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su traveggio trave, a destra da C a D
 Curvatura θ asta BF positiva se convessa a destra con inizio B.
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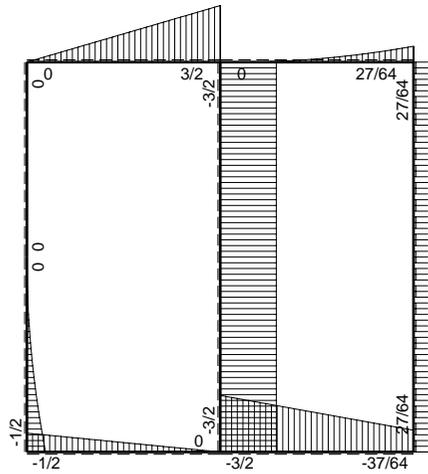


← (+) → F

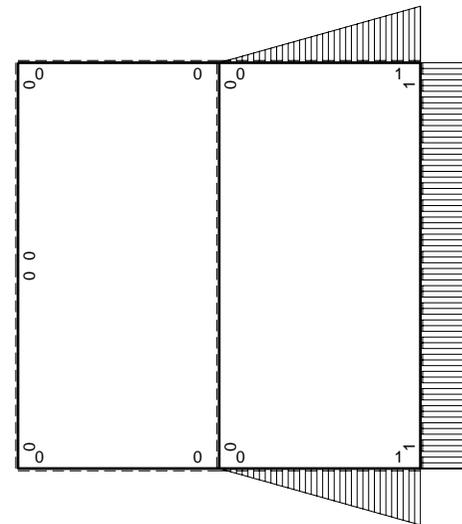
↑ (+) ↓ F



⊕ M_o flessione da carichi assegnati



⊕ F_b



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W _{GF}									
→	M _x (x)	M ₀ (x)	θ	M _x M ₀	M _x θ	M _x M _x	∫M _x (M ₀ /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0	
BA b	0	1/2Fx	0	0	0	0	0+0	0	
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0	
DC b	0	-3/2Fx	0	0	0	0	0+0	0	
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0	0+0	0	
EA b	0	-1/2qx ²	0	0	0	0	0+0	0	
AE b	0	1/2Fb-Fx+1/2qx ²	0	0	0	0	0+0	0	
BF b	x/b	-3/2Fb+1/2Fx	-Fb/EJ	-3/2Fx+1/2Fx ² /b	-Fx/EJ	x ² /b ²	(-7/12-1/2)Fb ² /EJ	1/3Xb/EJ	
FB b	-1+x/b	Fb+1/2Fx	Fb/EJ	-Fb+1/2Fx+1/2Fx ² /b	-Fb/EJ+Fx/EJ	1-2x/b+x ² /b ²	(-1/24+0)Fb ² /EJ	1/3Xb/EJ	
GC b	1-x/b	-1/2Fx+1/2qx ²	0	-1/2Fx+Fx ² /b-1/2qx ³ /b	0	1-2x/b+x ² /b ²			
CG b	-x/b	1/2Fx-1/2qx ²	0	-1/2Fx ² /b+1/2qx ³ /b	0	x ² /b ²			
FG 2b	1	0	0	0	0	1	0+0	2Xb/EJ	
GF 2b	-1	0	0	0	0	1	0+0	0	
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0	
BC 2b	0	3/2Fb	0	0	0	0	0+0	0	
	totali						-9/8Fb ² /EJ	8/3Xb/EJ	
	iperstatica X=W _{GF}						27/64Fb		

Svilupi di calcolo iperstatica

$$L_{BF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FB}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{GC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FG}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2b) 1/EJ = 2 b/EJ$$

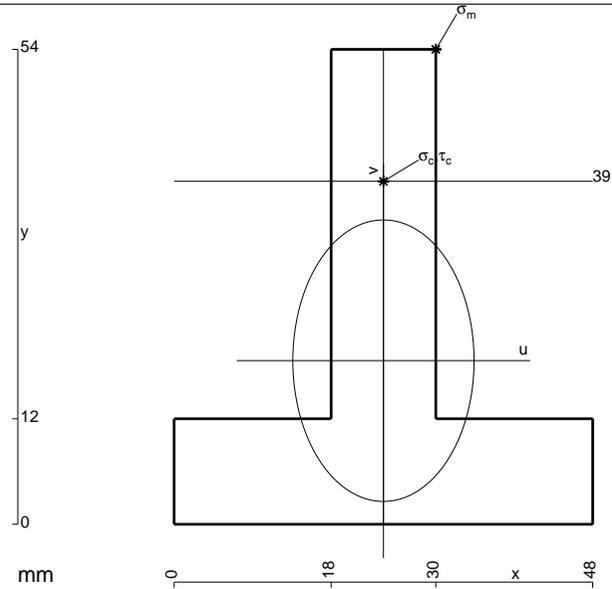
$$L_{GF}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2b) 1/EJ = 2 b/EJ$$

$$L_{BF}^{X0} = \int_0^b (-3/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-x/b) \theta dx = [-3/4 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x^2/b]_0^b \theta = (-3/4 b + 1/6 b) Fb 1/EJ + (-1/2 b) \theta = -13/12 Fb^2/EJ$$

$$L_{FB}^{X0} = \int_0^b (-1 + 1/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - x/b) \theta dx = [-x + 1/4 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/2 x^2/b]_0^b \theta = (-b + 1/4 b + 1/6 b) Fb 1/EJ + (b - 1/2 b) \theta = -13/12 Fb^2/EJ$$

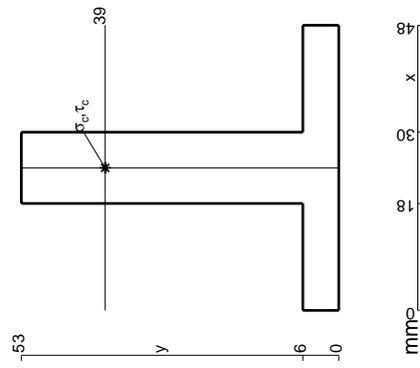
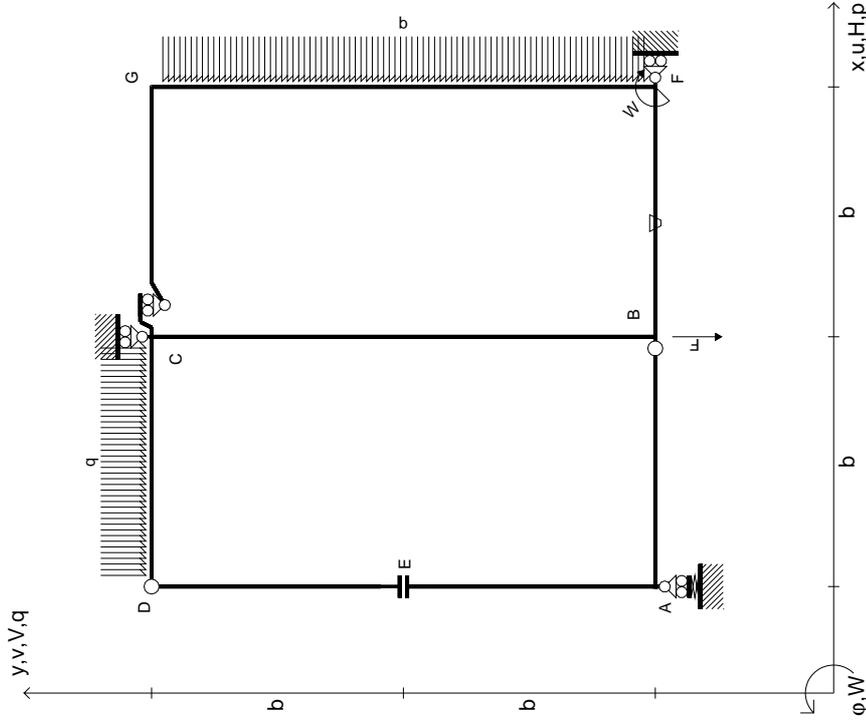
$$L_{GC}^{X0} = \int_0^b (-1/2 x/b + x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [-1/4 x^2/b + 1/3 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ = (-1/4 b + 1/3 b - 1/8 b) Fb 1/EJ = -1/24 Fb^2/EJ$$

$$L_{CG}^{X0} = \int_0^b (-1/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [-1/6 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ = (-1/6 b + 1/8 b) Fb 1/EJ = -1/24 Fb^2/EJ$$

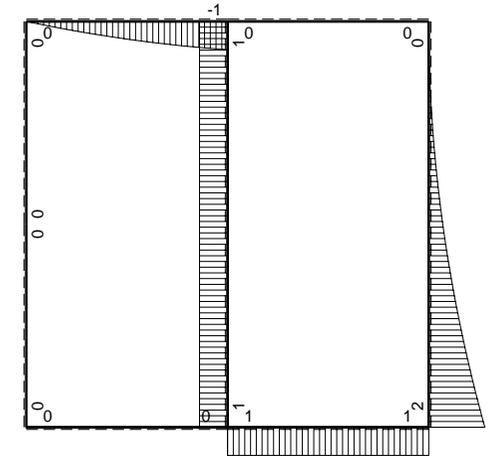
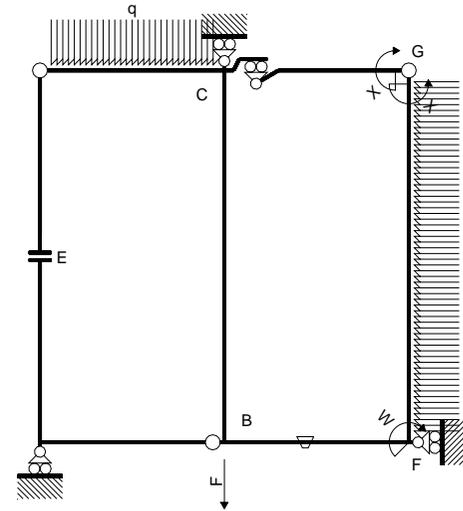
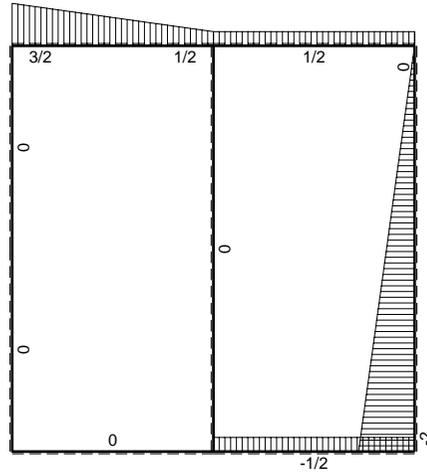
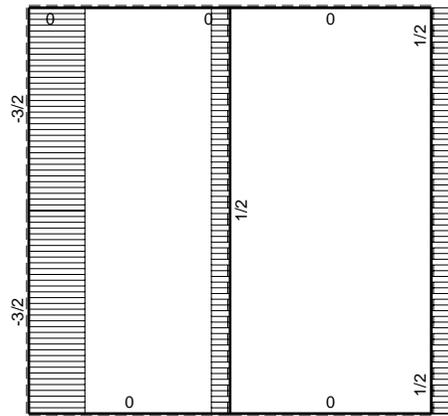


- $A = 1080. \text{ mm}^2$
- $J_u = 276955. \text{ mm}^4$
- $J_v = 116640. \text{ mm}^4$
- $y_g = 18.6 \text{ mm}$
- $T_y = -2430. \text{ N}$
- $M_x = 1555200. \text{ Nmm}$
- $x_m = 30. \text{ mm}$
- $y_m = 54. \text{ mm}$
- $u_m = 6. \text{ mm}$
- $v_m = 35.4 \text{ mm}$
- $\sigma_m = -Mv/J_u = -198.8 \text{ N/mm}^2$
- $x_c = 24. \text{ mm}$
- $y_c = 39. \text{ mm}$
- $v_c = 20.4 \text{ mm}$
- $\sigma_c = -Mv/J_u = -114.6 \text{ N/mm}^2$
- $\tau_c = 3.672 \text{ N/mm}^2$
- $\sigma_\varphi = \sqrt{\sigma^2 + 3\tau^2} = 114.7 \text{ N/mm}^2$
- $S = 5022. \text{ mm}^3$

$V_b = -F$
 $W_F = -W = -Fb$
 $q_{CD} = -q = -F/b$
 $P_{FG} = -q = -F/b$
 $\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
 $K_A = 4EJ/b^3$
 $EJ_{AB} = EJ$
 $EJ_{CD} = EJ$
 $EJ_{BE} = EJ$
 $EJ_{EA} = EJ$
 $EJ_{BF} = EJ$
 $EJ_{GC} = EJ$
 $EJ_{FG} = EJ$
 $EJ_{CB} = EJ$



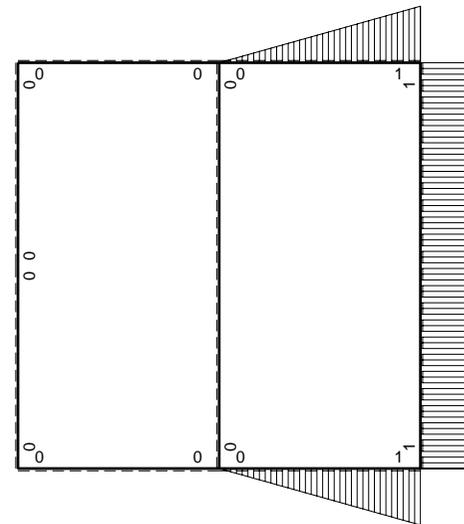
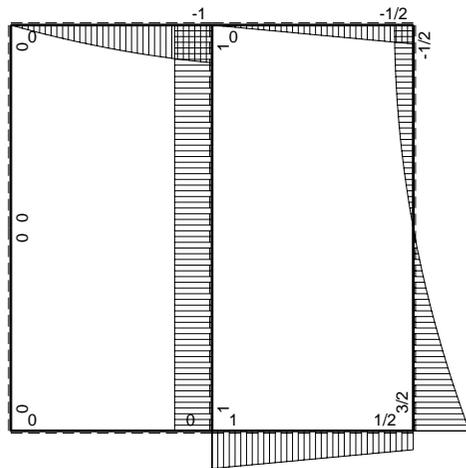
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 La trave CD ha la sezione riportata e dimensioni in mm, con:
 $b = 710$ mm, $F = 3640$ N
 Calcolare sulla sezione mediana la massima tensione normale σ_m
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da C a D
 Curvatura θ asta BF positiva se convessa a destra con inizio B.
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← ⊕ → F

⊕ ↓ F

⊕ ↻ M_o flessione da carichi assegnati



⊕ ↻ F_b

⊕ ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W _{GF}									
→	M _x (x)	M ₀ (x)	θ	M _x M ₀	M _x θ	M _x M _x	∫M _x (M ₀ /EJ+θ)dx	∫XM _x M ₀ EJdx	
AB b	0	0	0	0	0	0	0+0	0	
BA b	0	0	0	0	0	0	0+0	0	
CD b	0	-Fb+1/2Fx+1/2qx ²	0	0	0	0	0+0	0	
DC b	0	3/2Fx-1/2qx ²	0	0	0	0	0+0	0	
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0	0+0	0	
EA b	0	0	0	0	0	0	0+0	0	
AE b	0	0	0	0	0	0	0+0	0	
BF b	x/b	Fb	-Fb/EJ	Fx	-Fx/EJ	x ² /b ²	(1/2-1/2)Fb ² /EJ	1/3Xb/EJ	
FB b	-1+x/b	-Fb	Fb/EJ	Fb-Fx	-Fb/EJ+Fx/EJ	1-2x/b+x ² /b ²		1/3Xb/EJ	
GC b	1-x/b	0	0	0	0	1-2x/b+x ² /b ²	0+0	1/3Xb/EJ	
CG b	-x/b	0	0	0	0	x ² /b ²	0+0	1/3Xb/EJ	
FG 2b	1	2Fb-2Fx+1/2qx ²	0	2Fb-2Fx+1/2Fx ² /b	0	1	(4/3+0)Fb ² /EJ	2Xb/EJ	
GF 2b	-1	-1/2qx ²	0	1/2Fx ² /b	0	1		0	
CB 2b	0	Fb	0	0	0	0	0+0	0	
BC 2b	0	-Fb	0	0	0	0	0+0	0	
	totali						4/3Fb ² /EJ	8/3Xb/EJ	
	iperstatica X=W _{GF}								

Svilupi di calcolo iperstatica

$$L_{BF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FB}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{GC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FG}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2 b) 1/EJ = 2 b/EJ$$

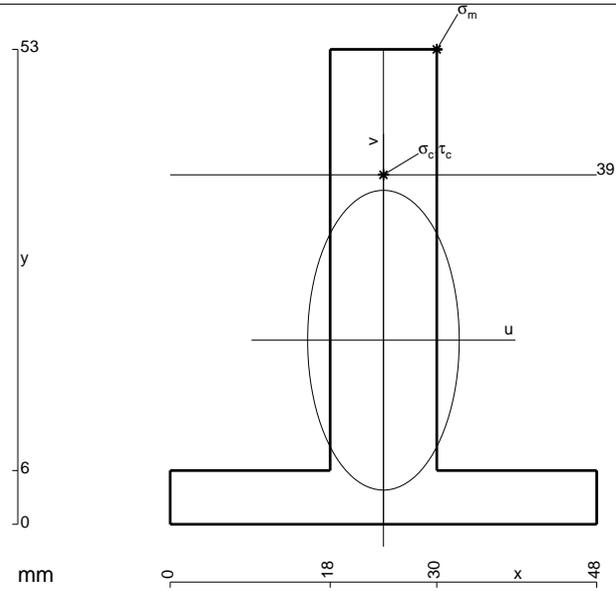
$$L_{GF}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2 b) 1/EJ = 2 b/EJ$$

$$L_{BF}^{X0} = \int_0^b (x/b) Fb 1/EJ dx + \int_0^b (-x/b) \theta dx = [1/2 x^2/b]_0^b Fb 1/EJ + [-1/2 x^2/b]_0^b \theta = (1/2 b) Fb 1/EJ + (-1/2 b) \theta = 0$$

$$L_{FB}^{X0} = \int_0^b (1 - x/b) Fb 1/EJ dx + \int_0^b (1 - x/b) \theta dx = [x - 1/2 x^2/b]_0^b Fb 1/EJ + [x - 1/2 x^2/b]_0^b \theta = (b - 1/2 b) Fb 1/EJ + (b - 1/2 b) \theta = 0$$

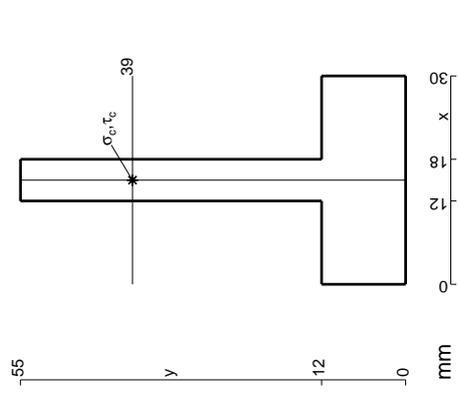
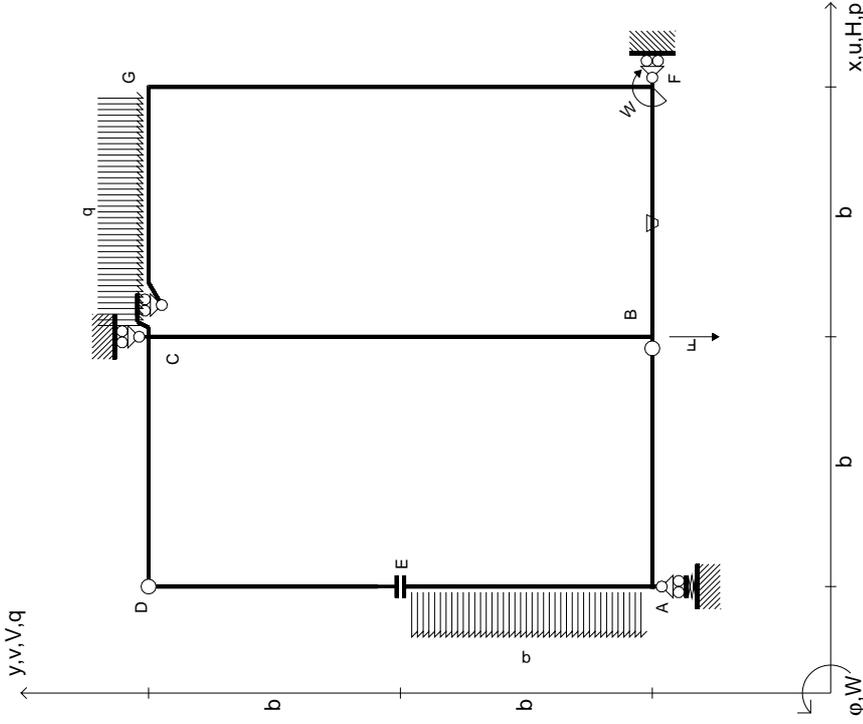
$$L_{FG}^{X0} = \int_0^{2b} (2 - 2x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [2x - x^2/b + 1/6 x^3/b^2]_0^{2b} Fb 1/EJ = (4b - 4b + 4/3 b) Fb 1/EJ = 4/3 Fb²/EJ$$

$$L_{GF}^{X0} = \int_0^{2b} (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^{2b} Fb 1/EJ = (4/3 b) Fb 1/EJ = 4/3 Fb²/EJ$$

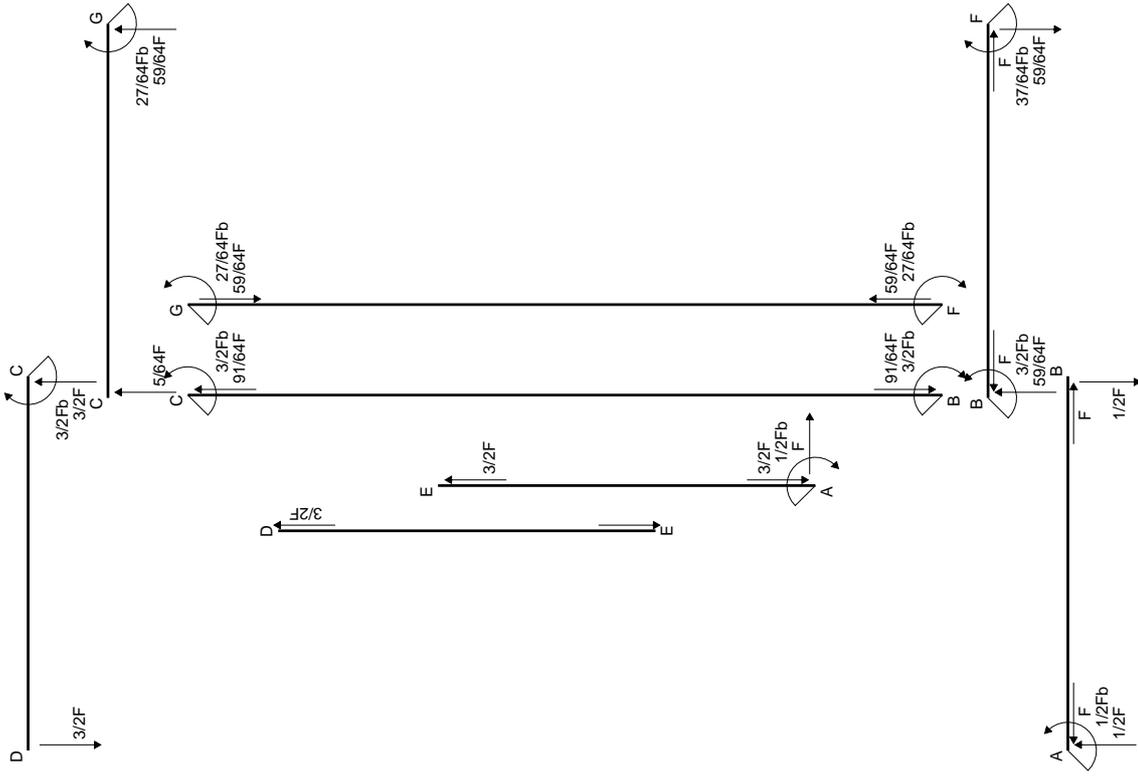


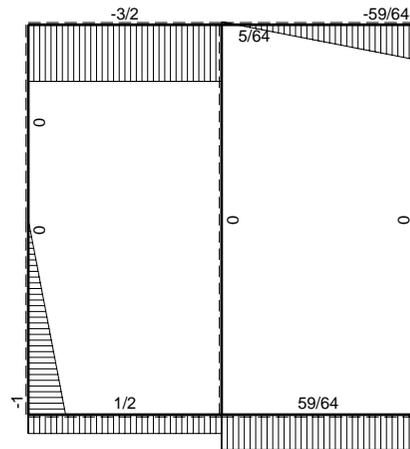
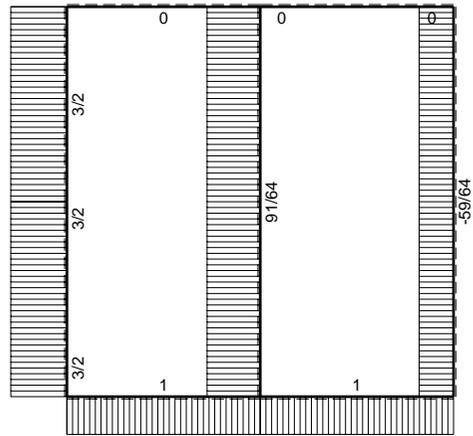
- $A = 852. \text{ mm}^2$
- $J_u = 238569. \text{ mm}^4$
- $J_v = 62064. \text{ mm}^4$
- $y_g = 20.54 \text{ mm}$
- $T_y = 3640. \text{ N}$
- $M_x = -1615250. \text{ Nmm}$
- $x_m = 30. \text{ mm}$
- $y_m = 53. \text{ mm}$
- $u_m = 6. \text{ mm}$
- $v_m = 32.46 \text{ mm}$
- $\sigma_m = -Mv/J_u = 219.8 \text{ N/mm}^2$
- $x_c = 24. \text{ mm}$
- $y_c = 39. \text{ mm}$
- $v_c = 18.46 \text{ mm}$
- $\sigma_c = -Mv/J_u = 125. \text{ N/mm}^2$
- $\tau_c = 5.438 \text{ N/mm}^2$
- $\sigma_q = \sqrt{\sigma^2 + 3\tau^2} = 125.3 \text{ N/mm}^2$
- $S = 4277. \text{ mm}^3$

$V_b = -F$
 $W_F = -W = -Fb$
 $P_{EA} = -q = -F/b$
 $q_{GC} = -q = -F/b$
 $\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
 $K_A = 3EJ/b^3$
 $EJ_{AB} = EJ$
 $EJ_{CD} = EJ$
 $EJ_{DE} = EJ$
 $EJ_{EA} = EJ$
 $EJ_{BF} = EJ$
 $EJ_{GC} = EJ$
 $EJ_{FG} = EJ$
 $EJ_{CB} = EJ$



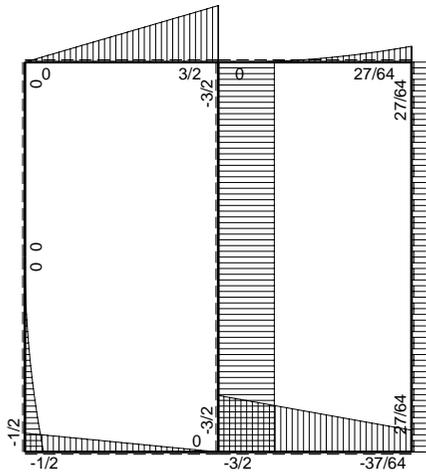
Reazioni iperstatiche in soluzione: $X=W_{GC}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} = X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave CD ha la sezione riportata e dimensioni in mm, con:
 $b = 610$ mm, $F = 1100$ N
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su traveggio trave, a destra da C a D
 Curvatura θ asta BF positiva se convessa a destra con inizio B.
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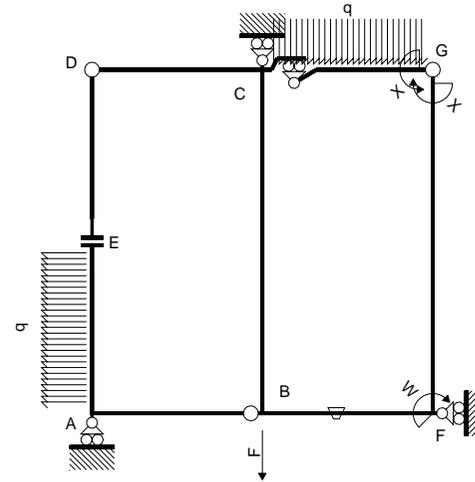


← (+) → F

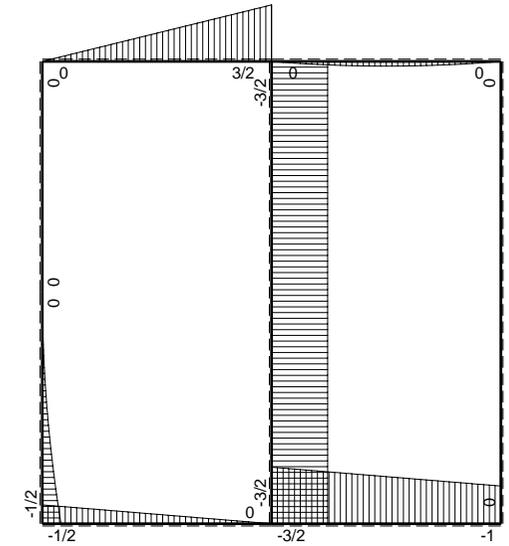
↑ (+) ↓ F



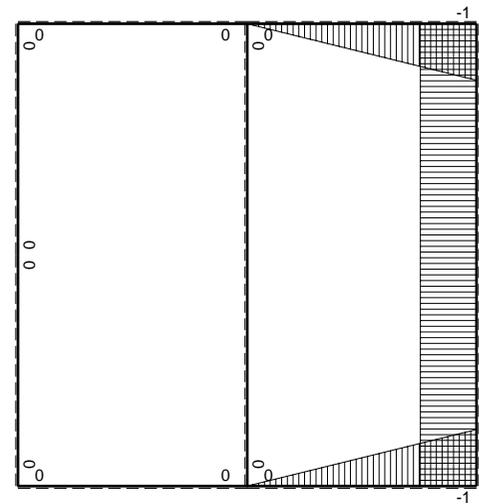
↺ (+) ↻ F_b



Schema di calcolo iperstatico



↺ (+) ↻ M₀ flessione da carichi assegnati



↺ (+) ↻ M_x flessione da iperstatica X=1

→	$M_x(x)$	$M_0(x)$	θ	M_{M_0}	$M_{x\theta}$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int x M_x M_x/EJ dx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0	0+0	0
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0	0+0	0
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0	0+0	0
EA b	0	-1/2qx ²	0	0	0	0	0+0	0
AE b	0	1/2Fb-Fx+1/2qx ²	0	0	0	0	0+0	0
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx ² /b	Fx/EJ	x ² /b ²	(7/12+1/2)Fb ² /EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx ² /b	Fb/EJ-Fx/EJ	1-2x/b+x ² /b ²	(1/24+0)Fb ² /EJ	1/3Xb/EJ
GC b	-1+x/b	-1/2Fx+1/2qx ²	0	1/2Fx-Fx ² /b+1/2qx ³ /b	0	1-2x/b+x ² /b ²	0	2Xb/EJ
CG b	x/b	1/2Fx-1/2qx ²	0	1/2Fx ² /b-1/2qx ³ /b	0	x ² /b ²	(1/24+0)Fb ² /EJ	0
FG 2b	-1	0	0	0	0	1	0+0	0
GF 2b	1	0	0	0	0	1	0+0	0
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0	0+0	0
totali								
iperstatica X=W _{GC}								

Sviluppi di calcolo iperstatica

$$L_{BF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FB}^{XX} = \int_0^b (1-2x/b+x^2/b^2) 1/EJ dx = [x-x^2/b+1/3 x^3/b^2]_0^b 1/EJ = (b-b+1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{GC}^{XX} = \int_0^b (1-2x/b+x^2/b^2) 1/EJ dx = [x-x^2/b+1/3 x^3/b^2]_0^b 1/EJ = (b-b+1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FG}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2b) 1/EJ = 2 b/EJ$$

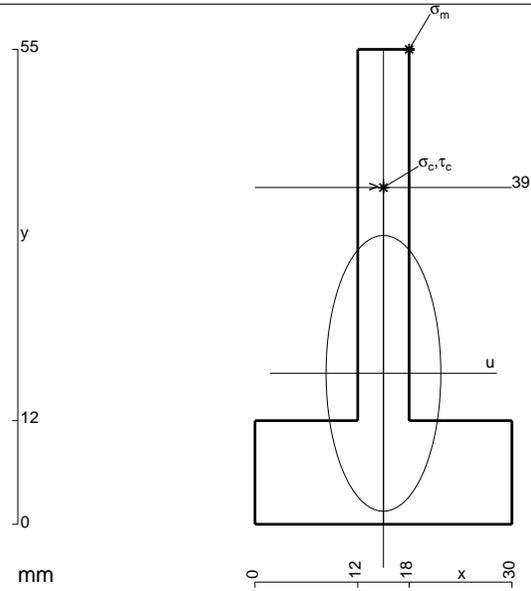
$$L_{GF}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2b) 1/EJ = 2 b/EJ$$

$$L_{BF}^{X\theta} = \int_0^b (3/2 x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (x/b) \theta dx = [3/4 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta = (3/4 b - 1/6 b) Fb 1/EJ + (1/2 b) \theta = 13/12 Fb^2/EJ$$

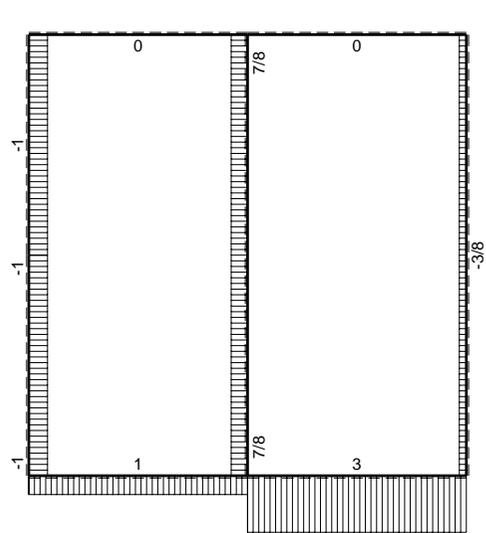
$$L_{FB}^{X\theta} = \int_0^b (1-1/2 x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1+x/b) \theta dx = [x-1/4 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-x+1/2 x^2/b]_0^b \theta = (b-1/4 b - 1/6 b) Fb 1/EJ + (-b+1/2 b) \theta = 13/12 Fb^2/EJ$$

$$L_{GC}^{X\theta} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ = (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

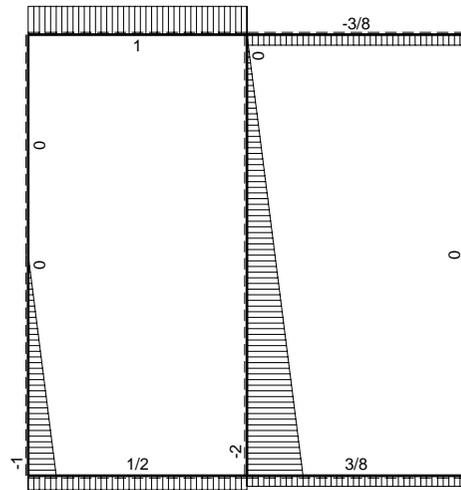
$$L_{CG}^{X\theta} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ = (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$



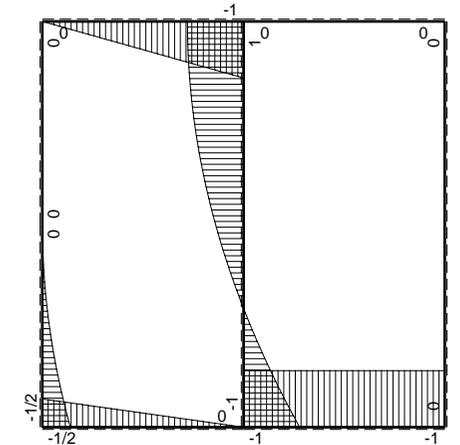
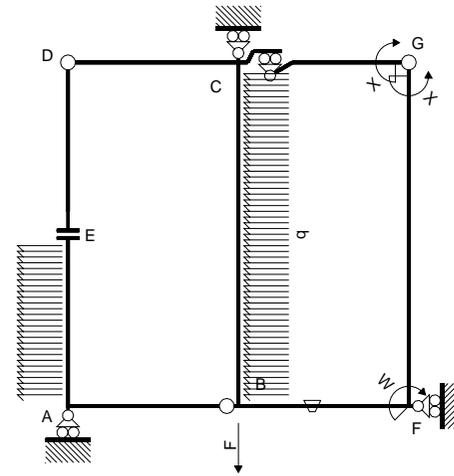
- $A = 618. \text{ mm}^2$
- $J_u = 157731. \text{ mm}^4$
- $J_v = 27774. \text{ mm}^4$
- $y_g = 17.48 \text{ mm}$
- $T_y = -1650. \text{ N}$
- $M_x = 1006500. \text{ Nmm}$
- $x_m = 18. \text{ mm}$
- $y_m = 55. \text{ mm}$
- $u_m = 3. \text{ mm}$
- $v_m = 37.52 \text{ mm}$
- $\sigma_m = -Mv/J_u = -239.4 \text{ N/mm}^2$
- $x_c = 15. \text{ mm}$
- $y_c = 39. \text{ mm}$
- $v_c = 21.52 \text{ mm}$
- $\sigma_c = -Mv/J_u = -137.3 \text{ N/mm}^2$
- $\tau_c = 4.941 \text{ N/mm}^2$
- $\sigma_\varphi = \sqrt{\sigma^2 + 3\tau^2} = 137.6 \text{ N/mm}^2$
- $S = 2834. \text{ mm}^3$



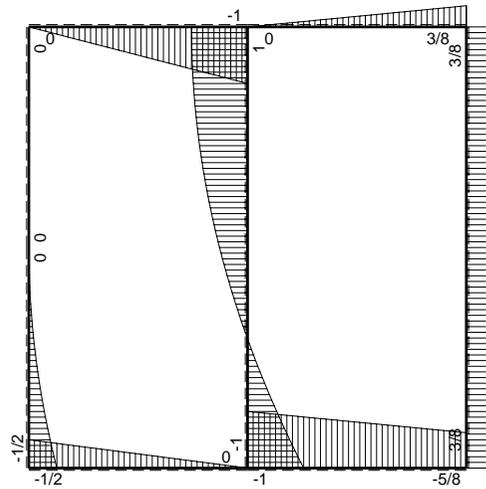
← (+) → F



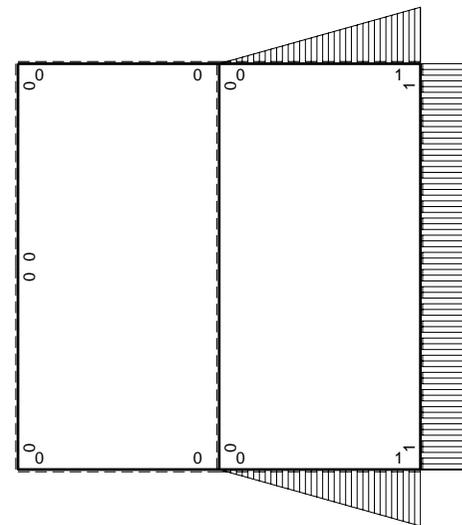
↑ (+) ↓ F



⌚ (+) ↻ M₀ flessione da carichi assegnati



⌚ (+) ↻ F_b



⌚ (+) ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W _{GF}									
→	M _x (x)	M ₀ (x)	θ	M _x M ₀	M _x θ	M _x M _x	∫M _x (M ₀ /EJ+θ)dx	∫XM _x M _x EJdx	
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0	
BA b	0	1/2Fx	0	0	0	0	0+0	0	
CD b	0	-Fb+Fx	0	0	0	0	0+0	0	
DC b	0	Fx	0	0	0	0	0+0	0	
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0	0+0	0	
EA b	0	-1/2qx ²	0	0	0	0	0+0	0	
AE b	0	1/2Fb-Fx+1/2qx ²	0	0	0	0	0+0	0	
BF b	x/b	-Fb	-Fb/EJ	-Fx	-Fx/EJ	x ² /b ²	(-1/2-1/2)Fb ² /EJ	1/3Xb/EJ	
FB b	-1+x/b	Fb	Fb/EJ	-Fb+Fx	-Fb/EJ+Fx/EJ	1-2x/b+x ² /b ²		1/3Xb/EJ	
GC b	1-x/b	0	0	0	0	1-2x/b+x ² /b ²	0+0	1/3Xb/EJ	
CG b	-x/b	0	0	0	0	x ² /b ²	0+0	2Xb/EJ	
FG 2b	1	0	0	0	0	1	0+0	0	
GF 2b	-1	0	0	0	0	1	0+0	0	
CB 2b	0	Fb-1/2qx ²	0	0	0	0	0+0	0	
BC 2b	0	Fb-2Fx+1/2qx ²	0	0	0	0	0+0	0	
	totali						-Fb ² /EJ	8/3Xb/EJ	
	iperstatica X=W _{GF}								

Sviluppi di calcolo iperstatica

$$L_{BF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FB}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{GC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FG}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ$$

$$= (2 b) 1/EJ = 2 b/EJ$$

$$L_{GF}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ$$

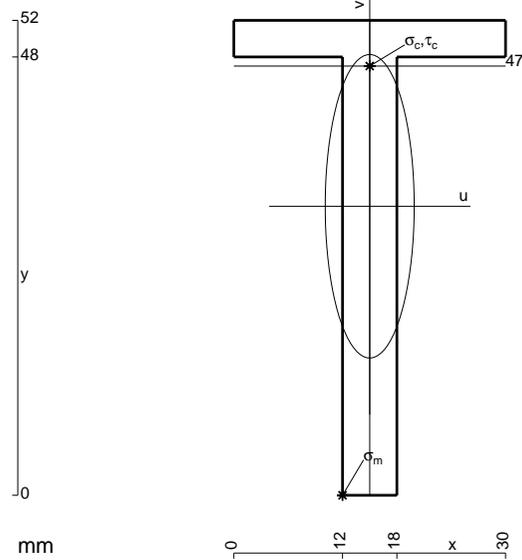
$$= (2 b) 1/EJ = 2 b/EJ$$

$$L_{BF}^{Xθ} = \int_0^b (-x/b) Fb 1/EJ dx + \int_0^b (-x/b) θ dx = [-1/2 x^2/b]_0^b Fb 1/EJ + [-1/2 x^2/b]_0^b θ$$

$$= (-1/2 b) Fb 1/EJ + (-1/2 b) θ = - Fb^2/EJ$$

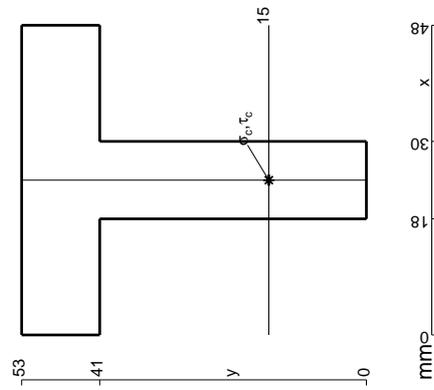
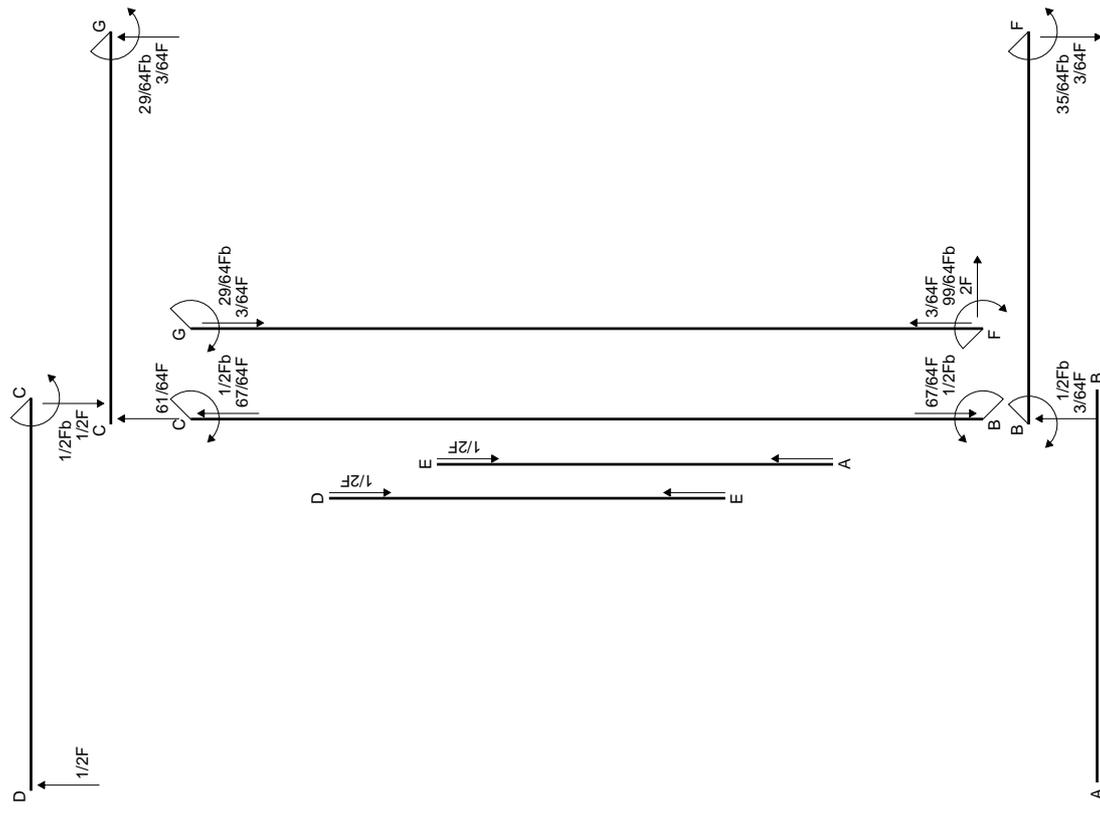
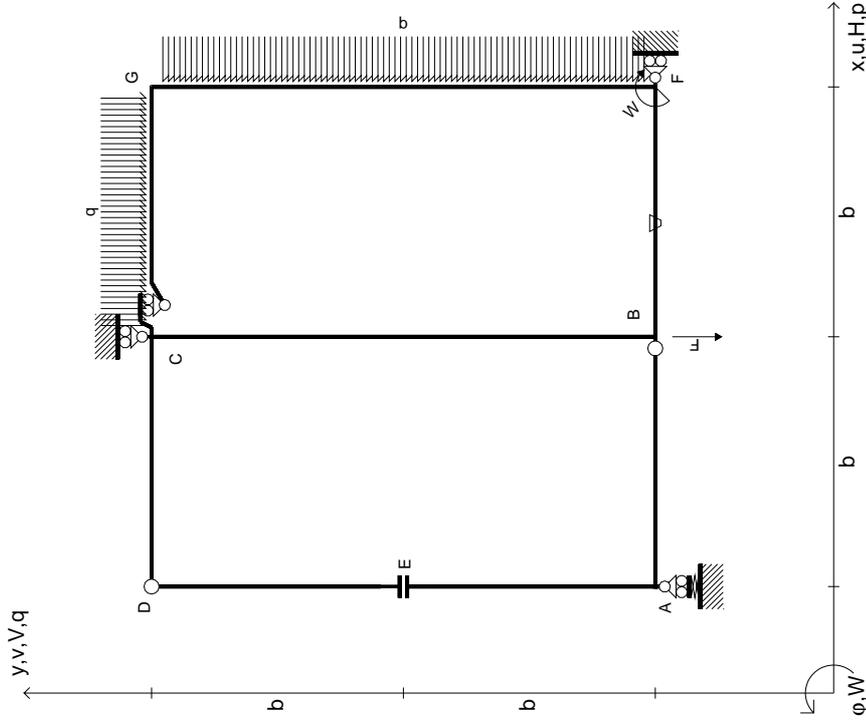
$$L_{FB}^{Xθ} = \int_0^b (-1 + x/b) Fb 1/EJ dx + \int_0^b (1 - x/b) θ dx = [-x + 1/2 x^2/b]_0^b Fb 1/EJ + [x - 1/2 x^2/b]_0^b θ$$

$$= (-b + 1/2 b) Fb 1/EJ + (b - 1/2 b) θ = - Fb^2/EJ$$

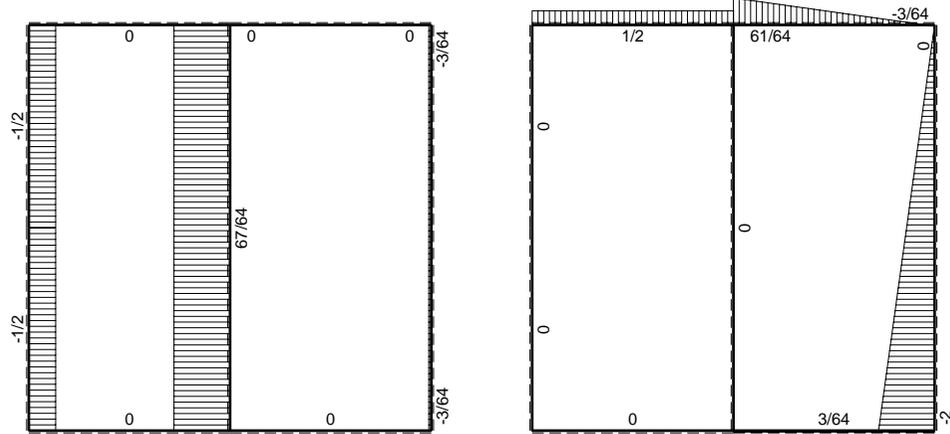


- $A = 408. \text{ mm}^2$
- $J_u = 112717. \text{ mm}^4$
- $J_v = 9864. \text{ mm}^4$
- $y_g = 31.65 \text{ mm}$
- $N = 1365. \text{ N}$
- $T_y = -3120. \text{ N}$
- $M_x = -826800. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $u_m = -3. \text{ mm}$
- $v_m = -31.65 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = -228.8 \text{ N/mm}^2$
- $x_c = 15. \text{ mm}$
- $y_c = 47. \text{ mm}$
- $v_c = 15.35 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = 116. \text{ N/mm}^2$
- $\tau_c = 10.6 \text{ N/mm}^2$
- $\sigma_g = \sqrt{\sigma^2 + 3\tau^2} = 117.4 \text{ N/mm}^2$
- $S = 2297. \text{ mm}^3$

$V_b = -F$
 $W_F = -W = -Fb$
 $q_{GC} = -q = -F/b$
 $P_{FG} = -q = -F/b$
 $\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
 $K_A = EJ/b^3$
 $EJ_{AB} = EJ$
 $EJ_{CD} = EJ$
 $EJ_{DE} = EJ$
 $EJ_{EA} = EJ$
 $EJ_{BF} = EJ$
 $EJ_{GC} = EJ$
 $EJ_{FG} = EJ$
 $EJ_{CB} = EJ$

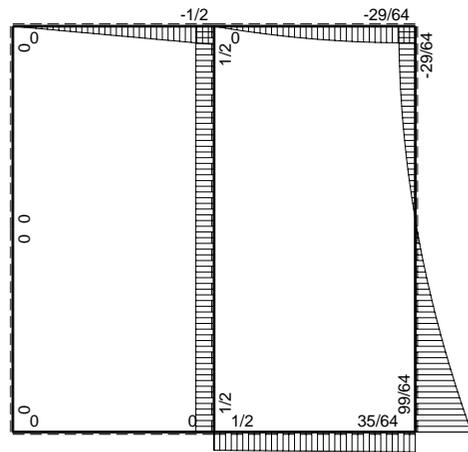


Reazioni iperstatiche in soluzione: $X=W_{FG}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave CD ha la sezione riportata e dimensioni in mm, con:
 $b = 420 \text{ mm}$, $F = 7170 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su traveggio trave, a destra da C a D
 Curvatura θ asta BF positiva se convessa a destra con inizio B.
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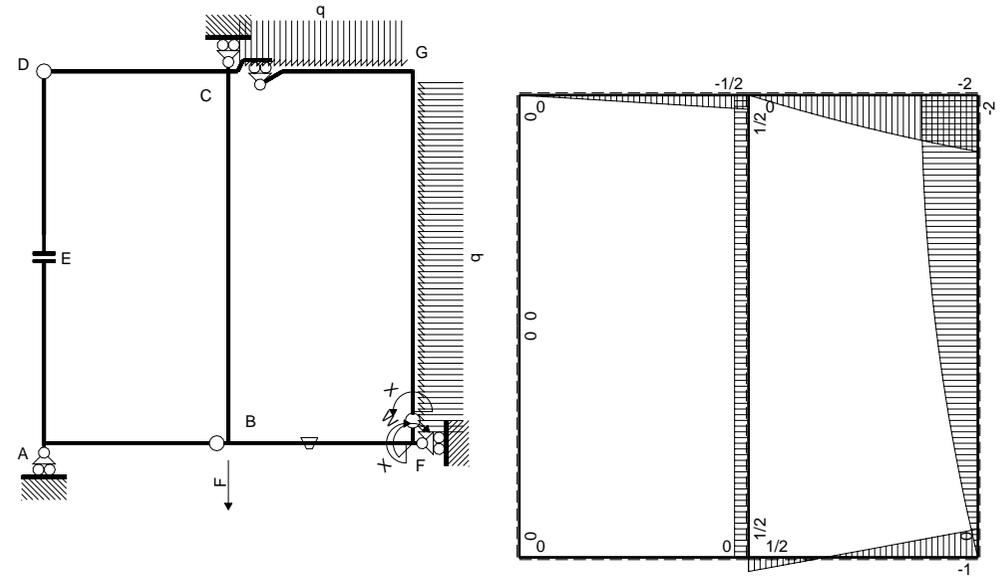


← (+) → F

↑ (+) ↓ Mb

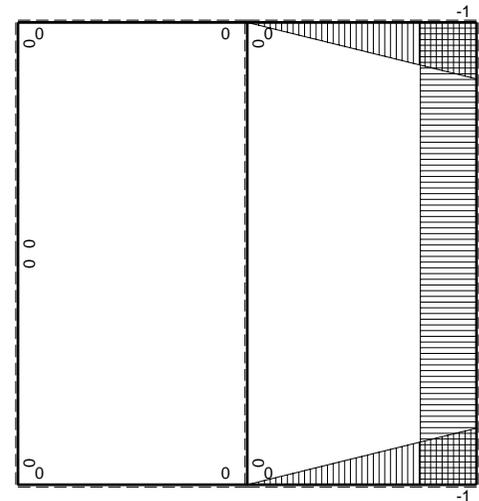


↺ (+) ↻ Mb

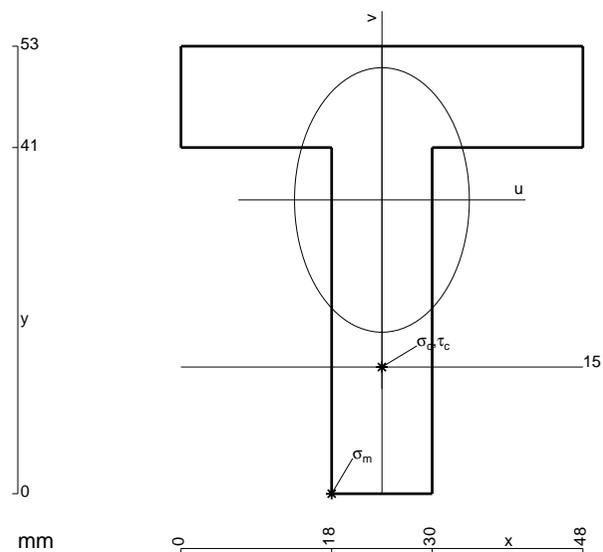


Schema di calcolo iperstatico

↺ (+) ↻ Mo flessione da carichi assegnati

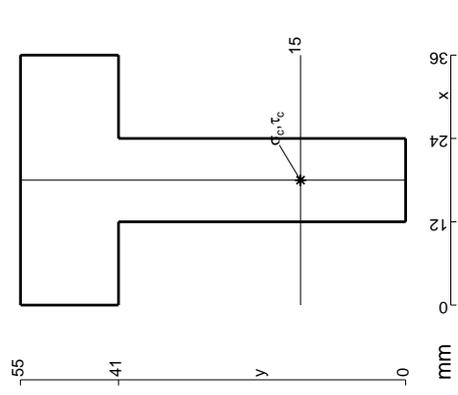
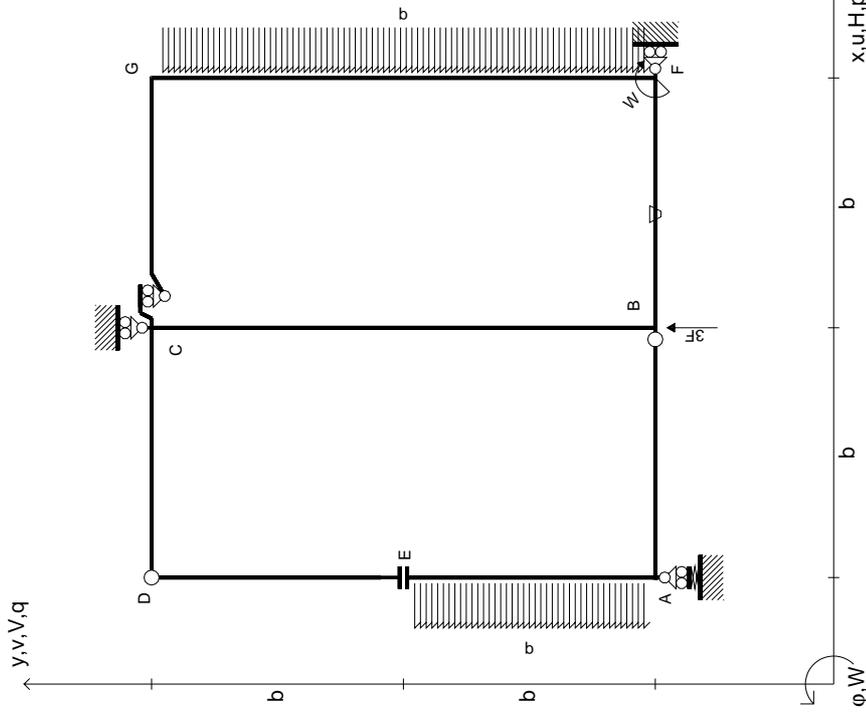


↺ (+) ↻ Mx flessione da iperstatica X=1

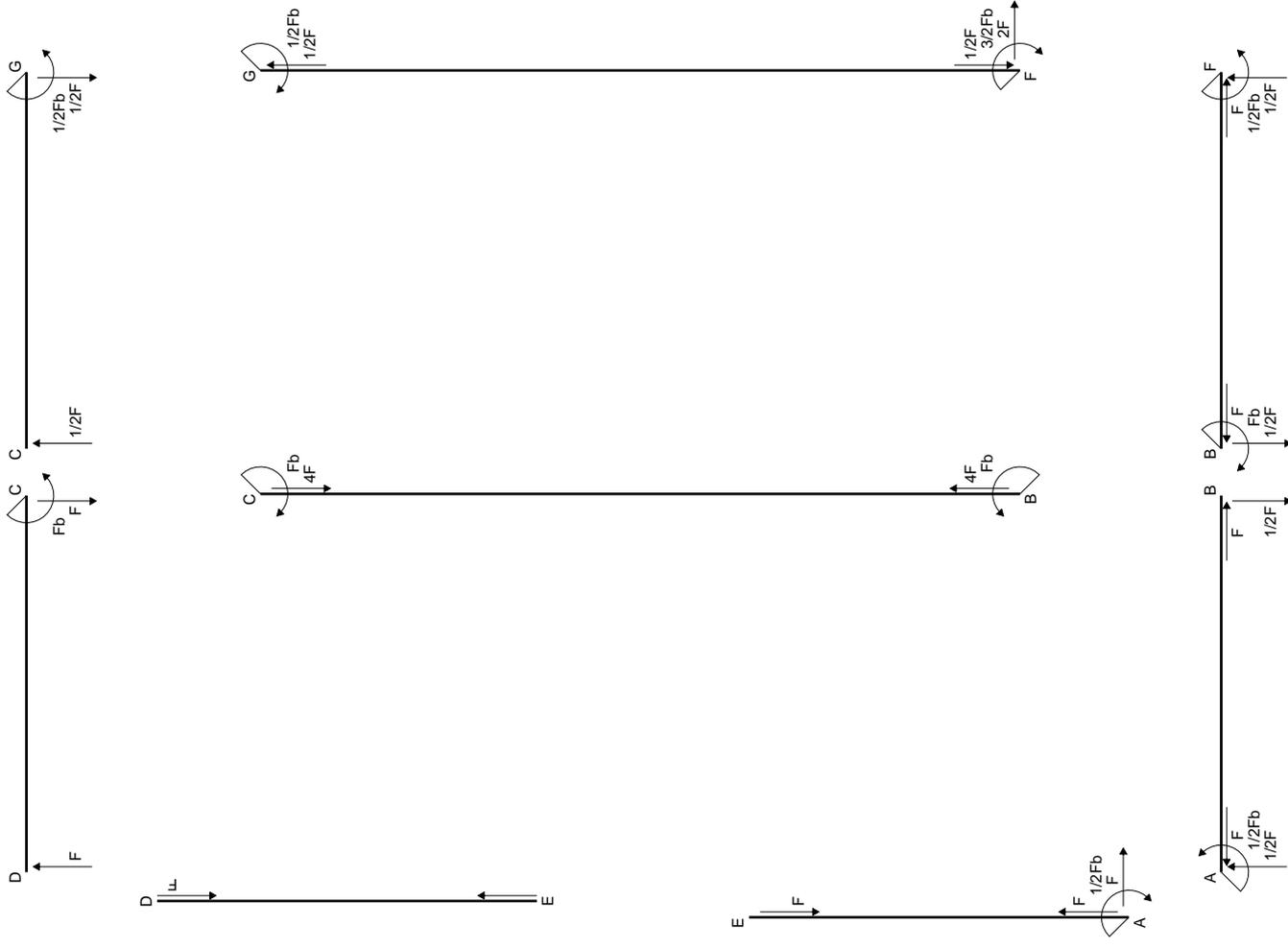


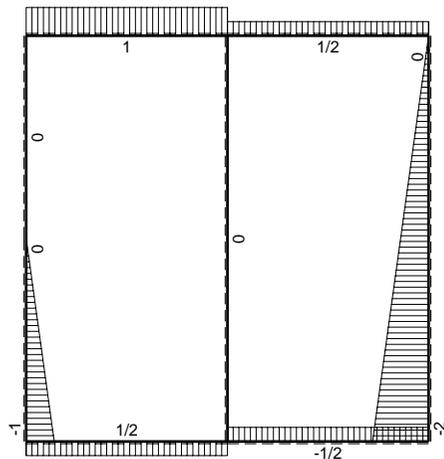
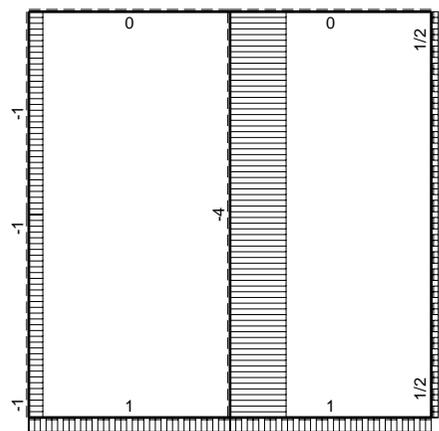
$$\begin{aligned}
 A &= 1068. \text{ mm}^2 \\
 J_u &= 262174. \text{ mm}^4 \\
 J_v &= 116496. \text{ mm}^4 \\
 y_g &= 34.79 \text{ mm} \\
 T_y &= 3585. \text{ N} \\
 M_x &= -1505700. \text{ Nmm} \\
 x_m &= 18. \text{ mm} \\
 u_m &= -6. \text{ mm} \\
 v_m &= -34.79 \text{ mm} \\
 \sigma_m &= -Mv/J_u = -199.8 \text{ N/mm}^2 \\
 x_c &= 24. \text{ mm} \\
 y_c &= 15. \text{ mm} \\
 v_c &= -19.79 \text{ mm} \\
 \sigma_c &= -Mv/J_u = -113.7 \text{ N/mm}^2 \\
 \tau_c &= 5.598 \text{ N/mm}^2 \\
 \sigma_\varphi &= \sqrt{\sigma^2 + 3\tau^2} = 114.1 \text{ N/mm}^2 \\
 S &= 4913. \text{ mm}^3
 \end{aligned}$$

- $V_b = 3F$
- $W_F = -W = -Fb$
- $P_{EA} = -q = -F/b$
- $P_{FG} = -q = -F/b$
- $\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
- $K_A = 4EJ/b^3$
- $EJ_{AB} = EJ$
- $EJ_{CD} = EJ$
- $EJ_{DE} = EJ$
- $EJ_{EA} = EJ$
- $EJ_{BF} = EJ$
- $EJ_{GC} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{CB} = EJ$



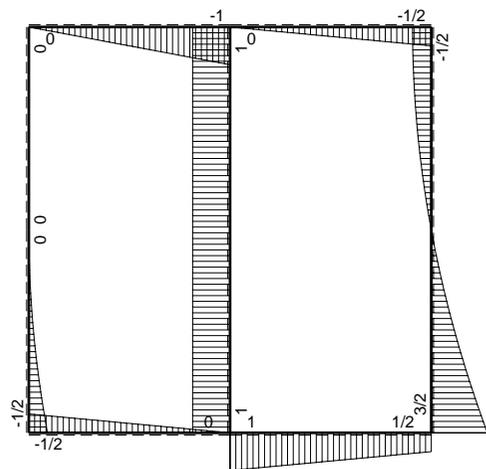
Reazioni iperstatiche in soluzione: $X=W_{FG}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave CD ha la sezione riportata e dimensioni in mm, con:
 $b = 440 \text{ mm}, F = 3500 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su traveggio trave, a destra da C a D
 Curvatura θ asta BF positiva se convessa a destra con inizio B.
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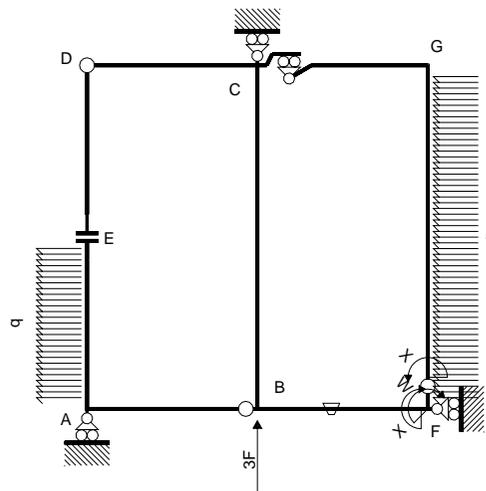


← (+) → F

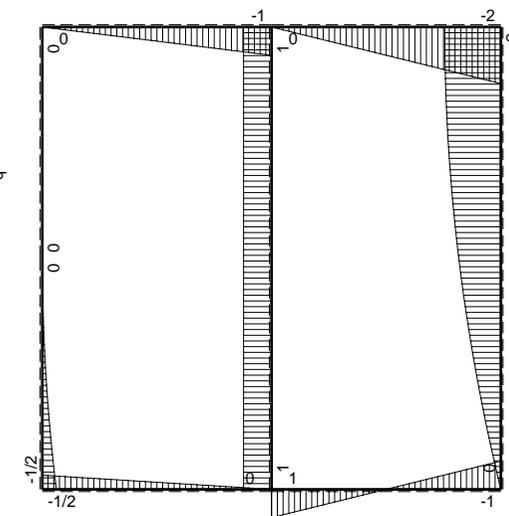
↑ (+) ↓ F



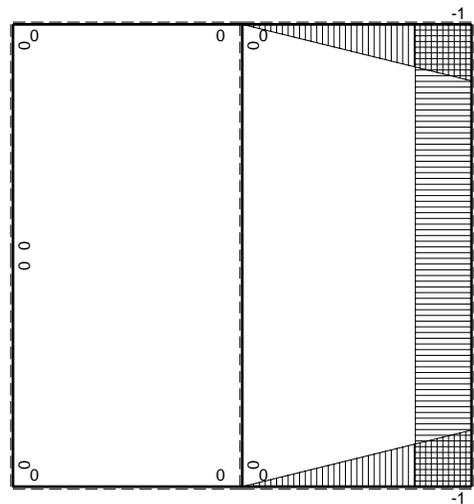
⊞ (+) ⊞ F_b



Schema di calcolo iperstatico



⊞ (+) ⊞ M₀ flessione da carichi assegnati



⊞ (+) ⊞ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{FG}

→	M _x (x)	M ₀ (x)	θ	M _x M ₀	M _x θ	M _x M _x	∫M _x (M ₀ /EJ+θ)dx	∫XM _x M ₀ /EJdx
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0	0+0	0
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0	0+0	0
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0	0+0	0
EA b	0	-1/2qx ²	0	0	0	0	0+0	0
AE b	0	1/2Fb-Fx+1/2qx ²	0	0	0	0	0+0	0
BF b	-x/b	Fb-2Fx	-Fb/EJ	-Fx+2Fx ² /b	Fx/EJ	x ² /b ²	(1/6+1/2)Fb ² /EJ	1/3Xb/EJ
FB b	1-x/b	Fb-2Fx	Fb/EJ	Fb-3Fx+2Fx ² /b	Fb/EJ-Fx/EJ	1-2x/b+x ² /b ²	(2/3+0)Fb ² /EJ	1/3Xb/EJ
GC b	-1+x/b	-2Fb+2Fx	0	2Fb-4Fx+2Fx ² /b	0	1-2x/b+x ² /b ²	(2/3+0)Fb ² /EJ	1/3Xb/EJ
CG b	x/b	2Fx	0	2Fx ² /b	0	x ² /b ²	(8/3+0)Fb ² /EJ	2Xb/EJ
FG 2b	-1	-2Fx+1/2qx ²	0	2Fx-1/2Fx ² /b	0	1	0+0	0
GF 2b	1	2Fb-1/2qx ²	0	2Fb-1/2Fx ² /b	0	1	0+0	0
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0	0+0	0
totali								
iperstatica X=W _{FG}								

Sviluppi di calcolo iperstatica

$$L_{BF}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FB}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{GC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FG}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2b) 1/EJ = 2 b/EJ$$

$$L_{GF}^{XX} = \int_0^{2b} (1) 1/EJ dx = [x]_0^{2b} 1/EJ = (2b) 1/EJ = 2 b/EJ$$

$$L_{BF}^{X\theta} = \int_0^b (-x/b + 2x^2/b^2) Fb 1/EJ dx + \int_0^b (x/b) \theta dx = [-1/2 x^2/b + 2/3 x^3/b^2]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta = (-1/2 b + 2/3 b) Fb 1/EJ + (1/2 b) \theta = 2/3 Fb^2/EJ$$

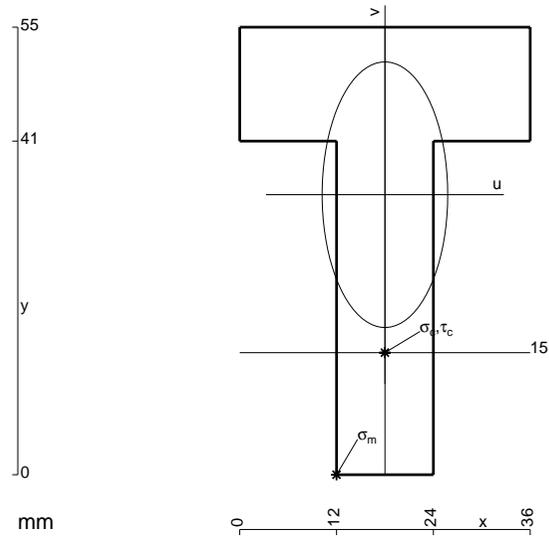
$$L_{FB}^{X\theta} = \int_0^b (1 - 3x/b + 2x^2/b^2) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx = [x - 3/2 x^2/b + 2/3 x^3/b^2]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta = (b - 3/2 b + 2/3 b) Fb 1/EJ + (-b + 1/2 b) \theta = 2/3 Fb^2/EJ$$

$$L_{GC}^{X\theta} = \int_0^b (2 - 4x/b + 2x^2/b^2) Fb 1/EJ dx = [2x - 2x^2/b + 2/3 x^3/b^2]_0^b Fb 1/EJ = (2b - 2b + 2/3 b) Fb 1/EJ = 2/3 Fb^2/EJ$$

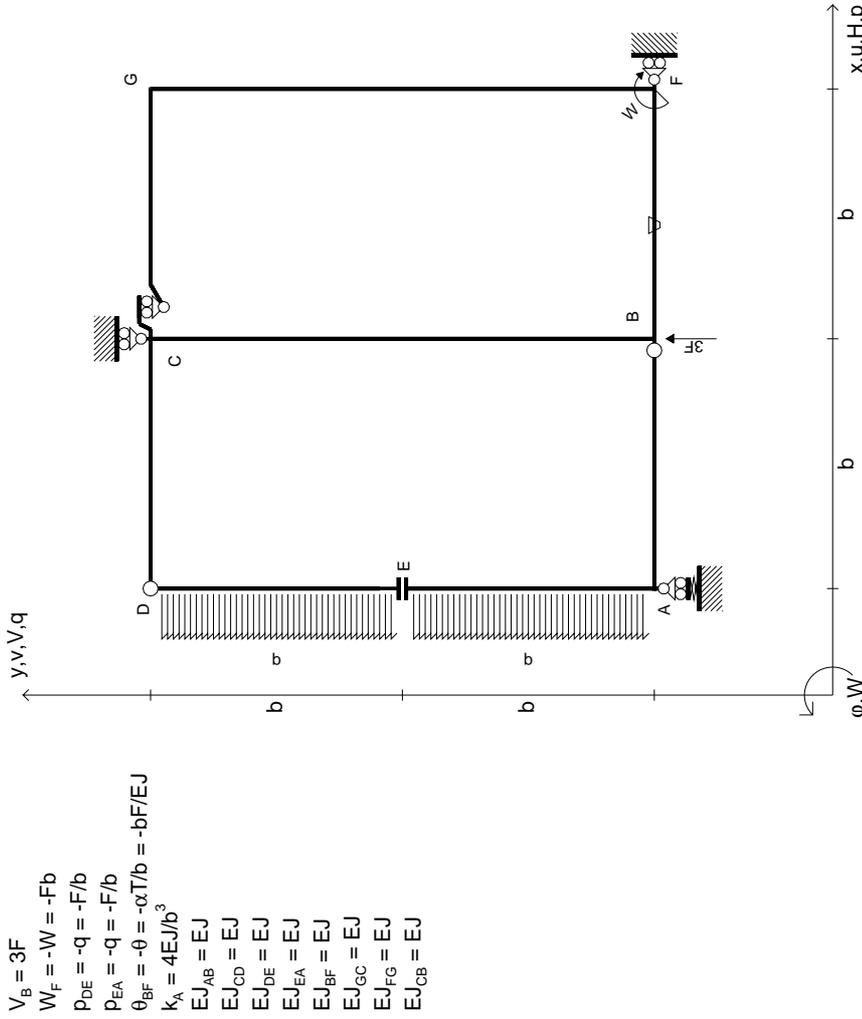
$$L_{CG}^{X\theta} = \int_0^b (2x^2/b^2) Fb 1/EJ dx = [2/3 x^3/b^2]_0^b Fb 1/EJ = (2/3 b) Fb 1/EJ = 2/3 Fb^2/EJ$$

$$L_{FG}^{X\theta} = \int_0^{2b} (2x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [x^2/b - 1/6 x^3/b^2]_0^{2b} Fb 1/EJ = (4b - 4/3 b) Fb 1/EJ = 8/3 Fb^2/EJ$$

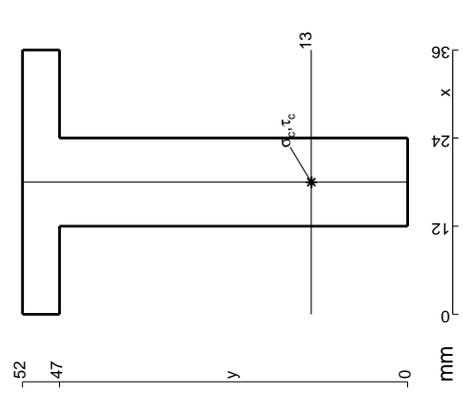
$$L_{GF}^{X\theta} = \int_0^{2b} (2 - 1/2 x^2/b^2) Fb 1/EJ dx = [2x - 1/6 x^3/b^2]_0^{2b} Fb 1/EJ = (4b - 4/3 b) Fb 1/EJ = 8/3 Fb^2/EJ$$



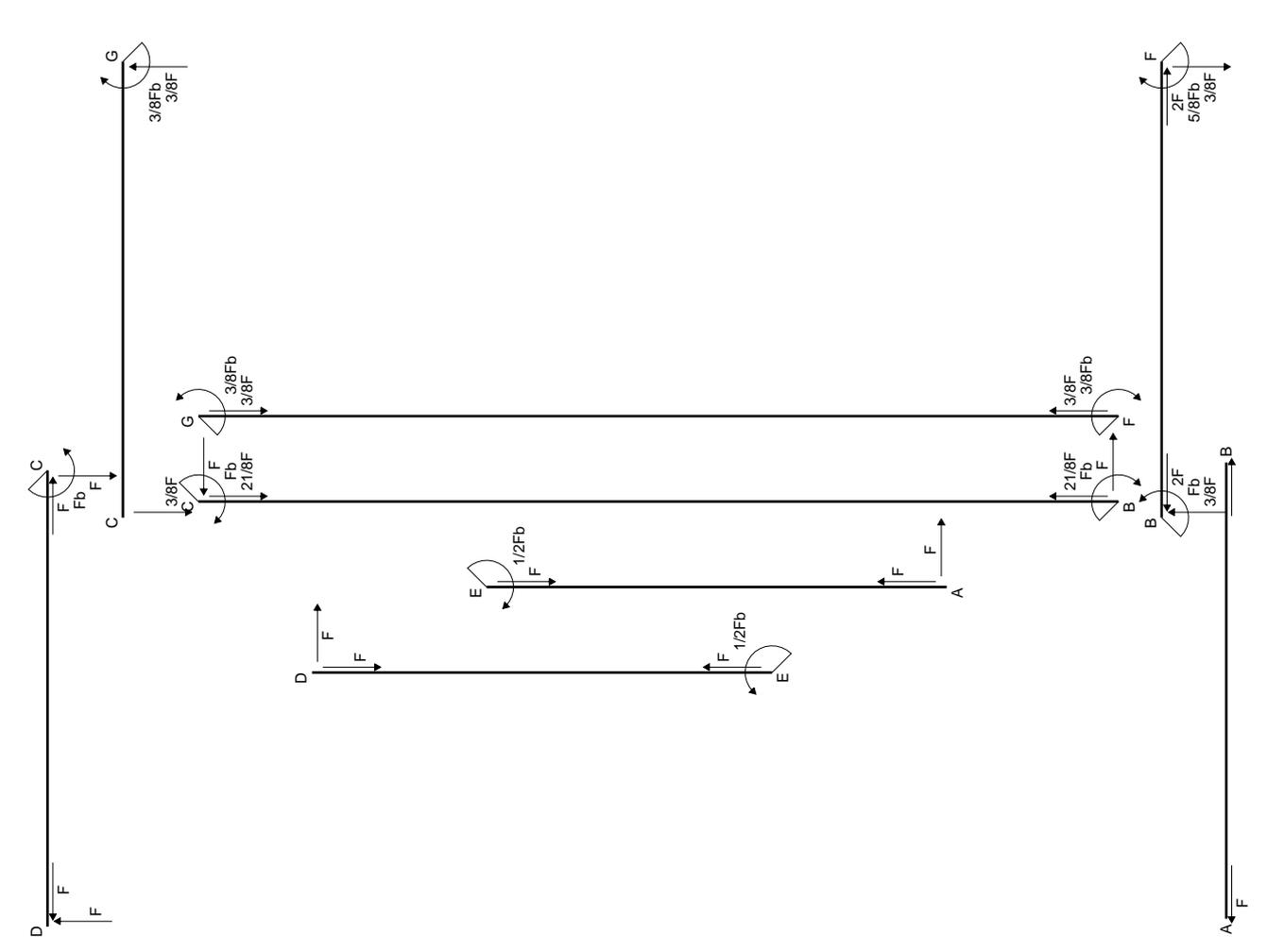
$A = 996. \text{ mm}^2$
 $J_u = 265432. \text{ mm}^4$
 $J_v = 60336. \text{ mm}^4$
 $y_g = 34.42 \text{ mm}$
 $T_y = 3500. \text{ N}$
 $M_x = -1540000. \text{ Nmm}$
 $x_m = 12. \text{ mm}$
 $u_m = -6. \text{ mm}$
 $v_m = -34.42 \text{ mm}$
 $\sigma_m = -Mv/J_u = -199.7 \text{ N/mm}^2$
 $x_c = 18. \text{ mm}$
 $y_c = 15. \text{ mm}$
 $v_c = -19.42 \text{ mm}$
 $\sigma_c = -Mv/J_u = -112.6 \text{ N/mm}^2$
 $\tau_c = 5.324 \text{ N/mm}^2$
 $\sigma_\varphi = \sqrt{\sigma^2 + 3\tau^2} = 113. \text{ N/mm}^2$
 $S = 4845. \text{ mm}^3$

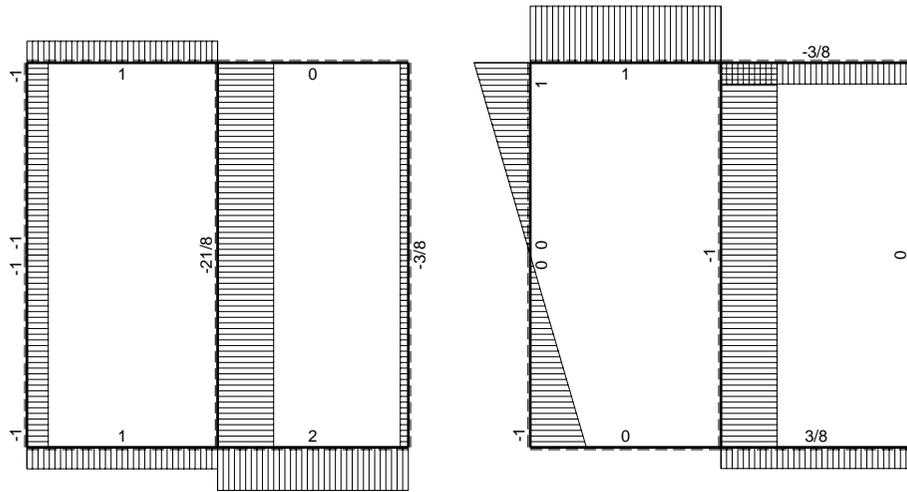


$V_b = 3F$
 $W_F = -W = -Fb$
 $P_{DE} = -q = -F/b$
 $P_{EA} = -q = -F/b$
 $\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
 $K_A = 4EJ/b^3$
 $EJ_{AB} = EJ$
 $EJ_{CD} = EJ$
 $EJ_{DE} = EJ$
 $EJ_{EA} = EJ$
 $EJ_{BF} = EJ$
 $EJ_{GC} = EJ$
 $EJ_{FG} = EJ$
 $EJ_{CB} = EJ$



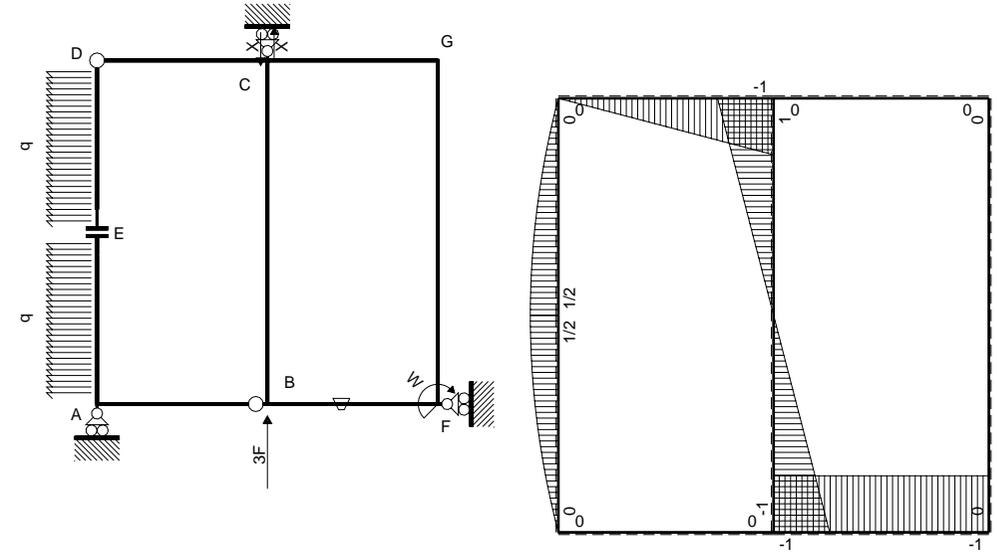
Reazioni iperstatiche in soluzione: $X = V_{CG}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave CB ha la sezione riportata e dimensioni in mm, con:
 $b = 610 \text{ mm}$, $F = 2080 \text{ N}$
 Calcolare sulla sezione B la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su traveggio trave, a destra da C a B
 Curvatura θ asta BF positiva se convessa a destra con inizio B.
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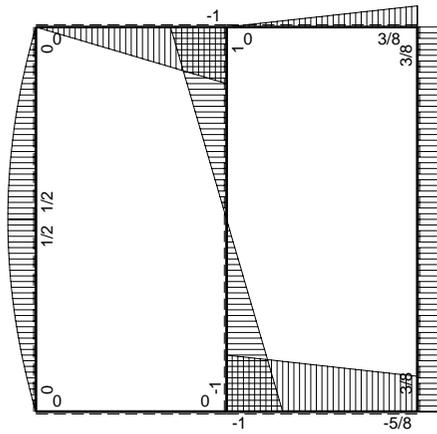
← (+) → F

↑ (+) ↓ F

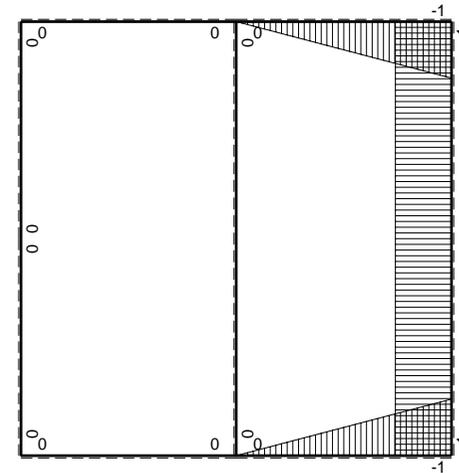


Schema di calcolo iperstatico

(+) Mo flessione da carichi assegnati



(+) Mb



(+) Mx flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=V _{CG}									
→	M _x (x)	M ₀ (x)	θ	M _x M ₀	M _x θ	M _x M _x	∫M _x (M ₀ /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	0	0	0	0	0	0	0+0	0	
BA b	0	0	0	0	0	0	0+0	0	
CD b	0	-Fb+Fx	0	0	0	0	0+0	0	
DC b	0	Fx	0	0	0	0	0+0	0	
DE b	0	Fx-1/2qx ²	0	0	0	0	0+0	0	
ED b	0	-1/2Fb+1/2qx ²	0	0	0	0	0+0	0	
EA b	0	1/2Fb-1/2qx ²	0	0	0	0	0+0	0	
AE b	0	-Fx+1/2qx ²	0	0	0	0	0+0	0	
BF b	-x	-Fb	-Fb/EJ	Fbx	Fxb/EJ	x ²	(1/2+1/2)Fb ³ /EJ	1/3Xb ³ /EJ	
FB b	b-x	Fb	Fb/EJ	Fb ² -Fbx	Fb ² /EJ-Fxb/EJ	b ² -2bx+x ²	0+0	1/3Xb ³ /EJ	
GC b	-b+x	0	0	0	0	b ² -2bx+x ²	0+0	0+0	
CG b	x	0	0	0	0	x ²	0+0	2Xb ³ /EJ	
FG 2b	-b	0	0	0	0	b ²	0+0	0	
GF 2b	b	0	0	0	0	b ²	0+0	0	
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0	
BC 2b	0	Fb-Fx	0	0	0	0	0+0	0	
	totali						Fb ³ /EJ	8/3Xb ³ /EJ	
	iperstatica X=V _{CG}								-3/8F

Sviluppi di calcolo iperstatica

$$L_{BF}^{XX} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{FB}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CG}^{XX} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{FG}^{XX} = \int_0^{2b} (1) b^2 1/EJ dx = [x]_0^{2b} b^2 1/EJ$$

$$= (2 b) b^2 1/EJ = 2 b^3/EJ$$

$$L_{GF}^{XX} = \int_0^{2b} (1) b^2 1/EJ dx = [x]_0^{2b} b^2 1/EJ$$

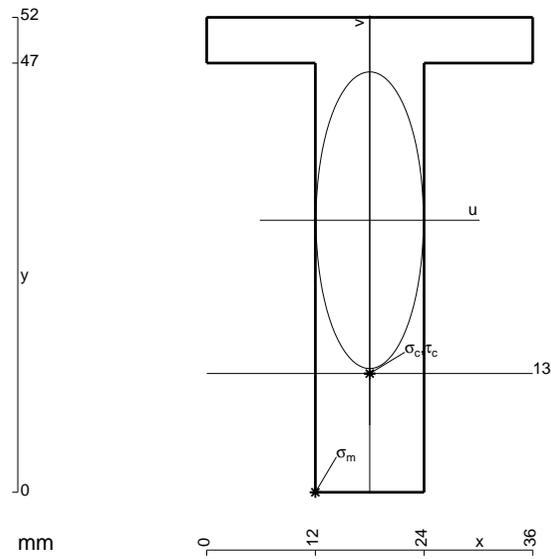
$$= (2 b) b^2 1/EJ = 2 b^3/EJ$$

$$L_{BF}^{X0} = \int_0^b (x/b) Fb^2 1/EJ dx + \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b Fb^2 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) Fb^2 1/EJ + (1/2 b) \theta = Fb^3/EJ$$

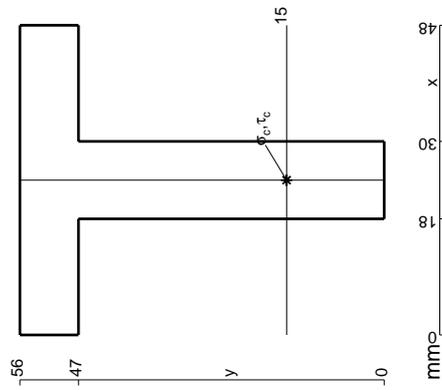
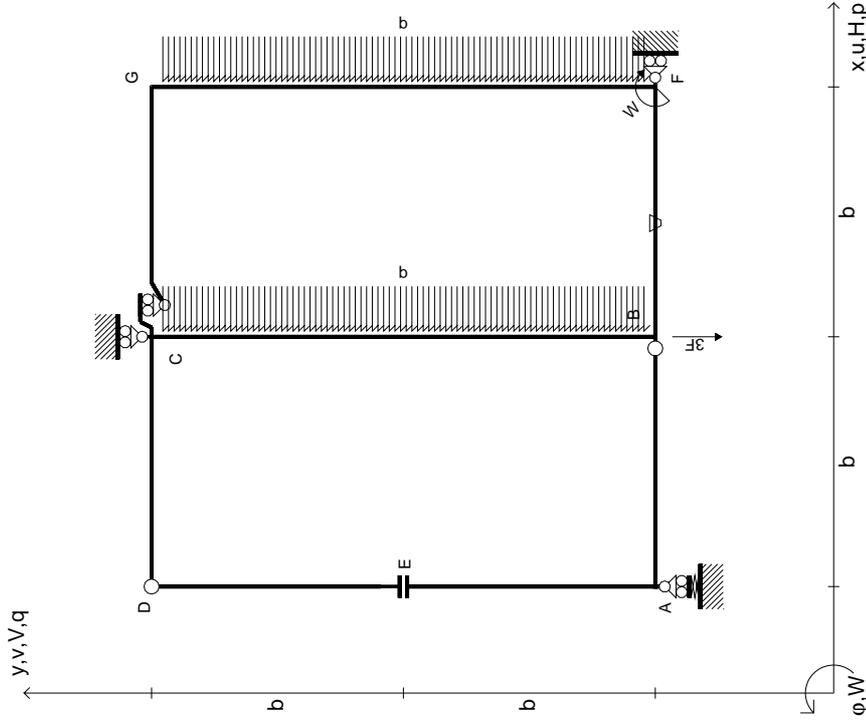
$$L_{FB}^{X0} = \int_0^b (1 - x/b) Fb^2 1/EJ dx + \int_0^b (-1 + x/b) \theta dx = [x - 1/2 x^2/b]_0^b Fb^2 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (b - 1/2 b) Fb^2 1/EJ + (-b + 1/2 b) \theta = Fb^3/EJ$$

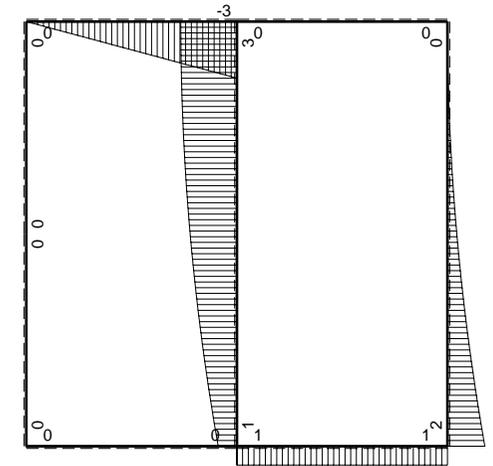
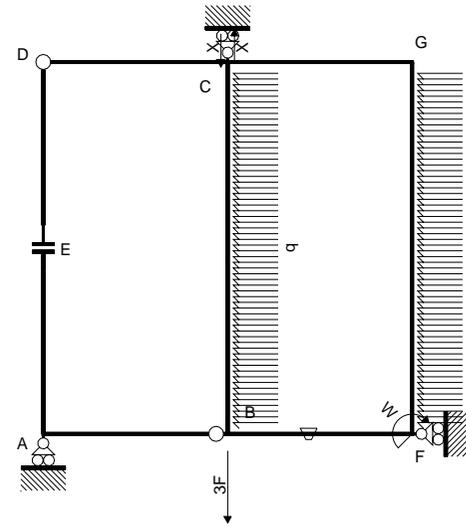
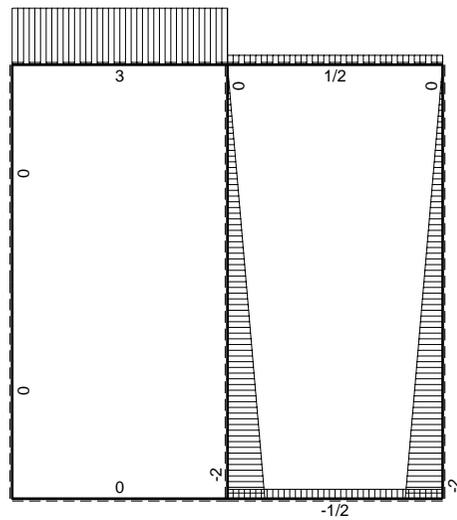
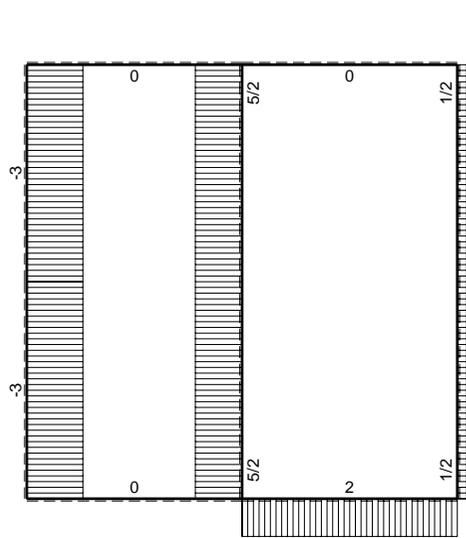


- $A = 744. \text{ mm}^2$
- $J_u = 196439. \text{ mm}^4$
- $J_v = 26208. \text{ mm}^4$
- $y_g = 29.79 \text{ mm}$
- $N = -5460. \text{ N}$
- $T_y = -2080. \text{ N}$
- $M_x = -1268800. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $u_m = -6. \text{ mm}$
- $v_m = -29.79 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = -199.8 \text{ N/mm}^2$
- $x_c = 18. \text{ mm}$
- $y_c = 13. \text{ mm}$
- $v_c = -16.79 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = -115.8 \text{ N/mm}^2$
- $\tau_c = 3.206 \text{ N/mm}^2$
- $\sigma_g = \sqrt{\sigma^2 + 3\tau^2} = 115.9 \text{ N/mm}^2$
- $S = 3633. \text{ mm}^3$

$V_b = -3F$
 $W_F = -W = -Fb$
 $P_{CB} = -q = -F/b$
 $P_{FG} = -q = -F/b$
 $\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
 $K_A = 4EJ/b^3$
 $EJ_{AB} = EJ$
 $EJ_{CD} = EJ$
 $EJ_{DE} = EJ$
 $EJ_{EA} = EJ$
 $EJ_{BF} = EJ$
 $EJ_{GC} = EJ$
 $EJ_{FG} = EJ$
 $EJ_{CB} = EJ$



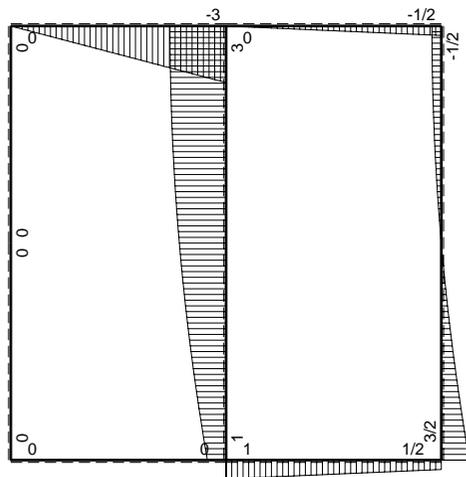
Reazioni iperstatiche in soluzione: $X = V_{CG}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} = X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave CD ha la sezione riportata e dimensioni in mm, con:
 $b = 740 \text{ mm}$, $F = 820 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su traveggio trave, a destra da C a D
 Curvatura θ asta BF positiva se convessa a destra con inizio B.
 © Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



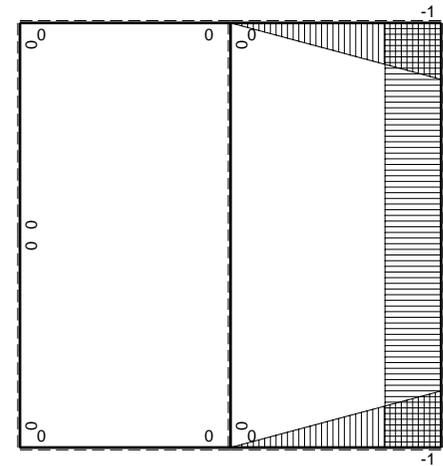
← (+) → F

↑ (+) ↓ F

⌚ (+) ⌚ M_o flessione da carichi assegnati



⌚ (+) ⌚ F_b



⌚ (+) ⌚ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=V_{CG}

→	M _x (x)	M ₀ (x)	θ	M _x M ₀	M _x θ	M _x M _x	∫M _x (M ₀ /EJ+θ)dx	∫XM _x M _x /EJdx
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0	0+0	0
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0	0+0	0
DE b	0	0	0	0	0	0	0+0	0
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0	0+0	0
BF b	-x	Fb	-Fb/EJ	-Fbx	Fxb/EJ	x ²	(-1/2+1/2)Fb ³ /EJ	1/3Xb ³ /EJ
FB b	b-x	-Fb	Fb/EJ	-Fb ² +Fbx	Fb ² /EJ-Fxb/EJ	b ² -2bx+x ²	0+0	1/3Xb ³ /EJ
GC b	-b+x	0	0	0	0	b ² -2bx+x ²	0+0	1/3Xb ³ /EJ
CG b	x	0	0	0	0	x ²	0+0	2Xb ³ /EJ
FG 2b	-b	2Fb-2Fx+1/2qx ²	0	-2Fb ² +2Fbx-1/2Fx ²	0	b ²	(-4/3+0)Fb ³ /EJ	0
GF 2b	b	-1/2qx ²	0	-1/2Fx ²	0	b ²	0+0	0
CB 2b	0	3Fb-1/2qx ²	0	0	0	0	0+0	0
BC 2b	0	-Fb-2Fx+1/2qx ²	0	0	0	0	-4/3Fb ³ /EJ	8/3Xb ³ /EJ
	totali						1/2F	
	iperstatica X=V _{CG}							

Sviluppi di calcolo iperstatica

$$L_{BF}^{XX} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{FB}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CG}^{XX} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{FG}^{XX} = \int_0^{2b} (1) b^2 1/EJ dx = [x]_0^{2b} b^2 1/EJ$$

$$= (2 b) b^2 1/EJ = 2 b^3/EJ$$

$$L_{GF}^{XX} = \int_0^{2b} (1) b^2 1/EJ dx = [x]_0^{2b} b^2 1/EJ$$

$$= (2 b) b^2 1/EJ = 2 b^3/EJ$$

$$L_{BF}^{Xθ} = \int_0^b (-x/b) Fb^2 1/EJ dx + \int_0^b (x/b) θ dx = [-1/2 x^2/b]_0^b Fb^2 1/EJ + [1/2 x^2/b]_0^b θ$$

$$= (-1/2 b) Fb^2 1/EJ + (1/2 b) θ = 0$$

$$L_{FB}^{Xθ} = \int_0^b (-1 + x/b) Fb^2 1/EJ dx + \int_0^b (-1 + x/b) θ dx$$

$$= [-x + 1/2 x^2/b]_0^b Fb^2 1/EJ + [-x + 1/2 x^2/b]_0^b θ$$

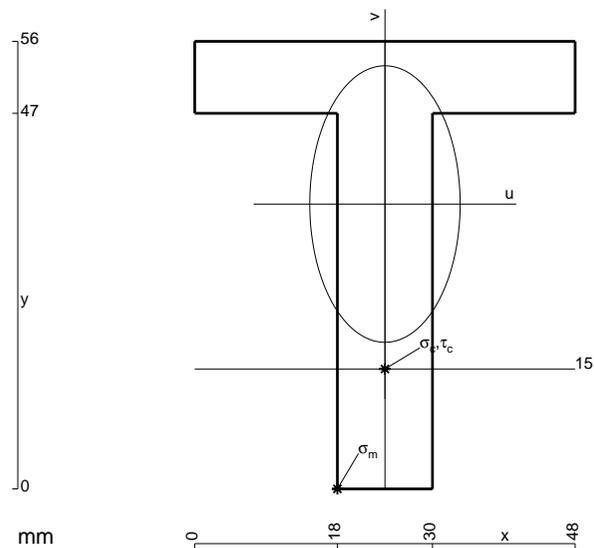
$$= (-b + 1/2 b) Fb^2 1/EJ + (-b + 1/2 b) θ = 0$$

$$L_{FG}^{Xθ} = \int_0^{2b} (-2 + 2x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-2x + x^2/b - 1/6 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (-4b + 4b - 4/3 b) Fb^2 1/EJ = -4/3 Fb^3/EJ$$

$$L_{GF}^{Xθ} = \int_0^{2b} (-1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/6 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (-4/3 b) Fb^2 1/EJ = -4/3 Fb^3/EJ$$



- $A = 996. \text{ mm}^2$
- $J_u = 298526. \text{ mm}^4$
- $J_v = 89712. \text{ mm}^4$
- $y_g = 35.64 \text{ mm}$
- $T_y = 2460. \text{ N}$
- $M_x = -1820400. \text{ Nmm}$
- $x_m = 18. \text{ mm}$
- $u_m = -6. \text{ mm}$
- $v_m = -35.64 \text{ mm}$
- $\sigma_m = -Mv/J_u = -217.4 \text{ N/mm}^2$
- $x_c = 24. \text{ mm}$
- $y_c = 15. \text{ mm}$
- $v_c = -20.64 \text{ mm}$
- $\sigma_c = -Mv/J_u = -125.9 \text{ N/mm}^2$
- $\tau_c = 3.479 \text{ N/mm}^2$
- $\sigma_\rho = \sqrt{\sigma^2 + 3\tau^2} = 126. \text{ N/mm}^2$
- $S = 5066. \text{ mm}^3$