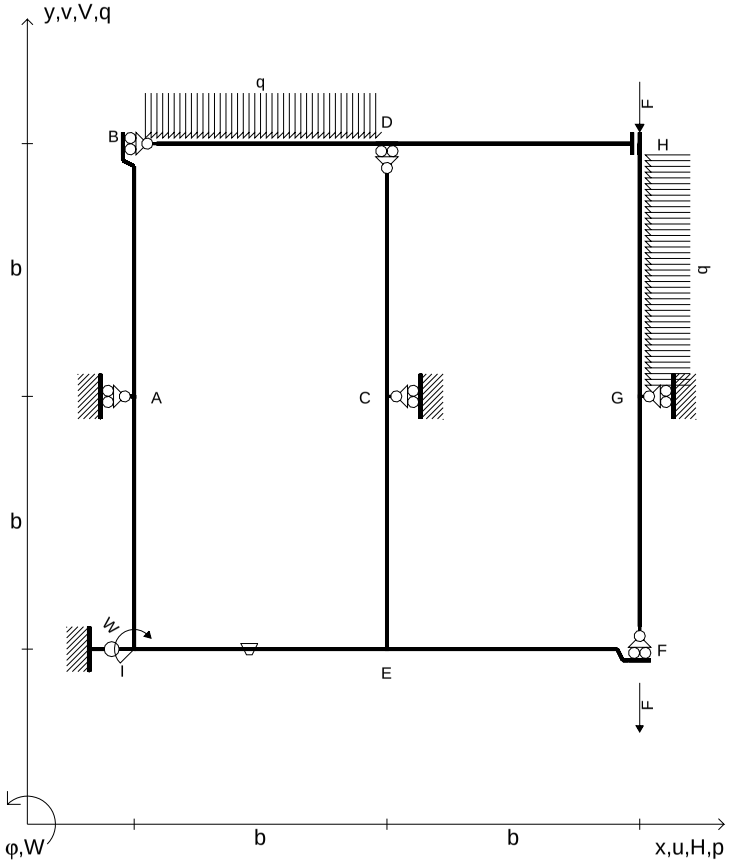
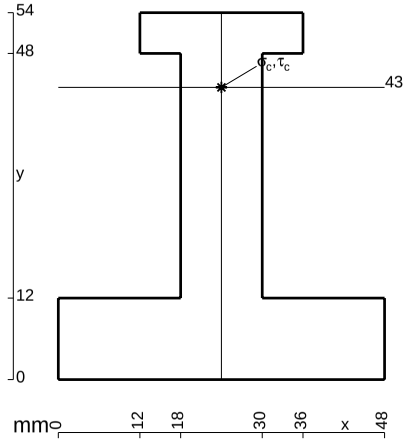
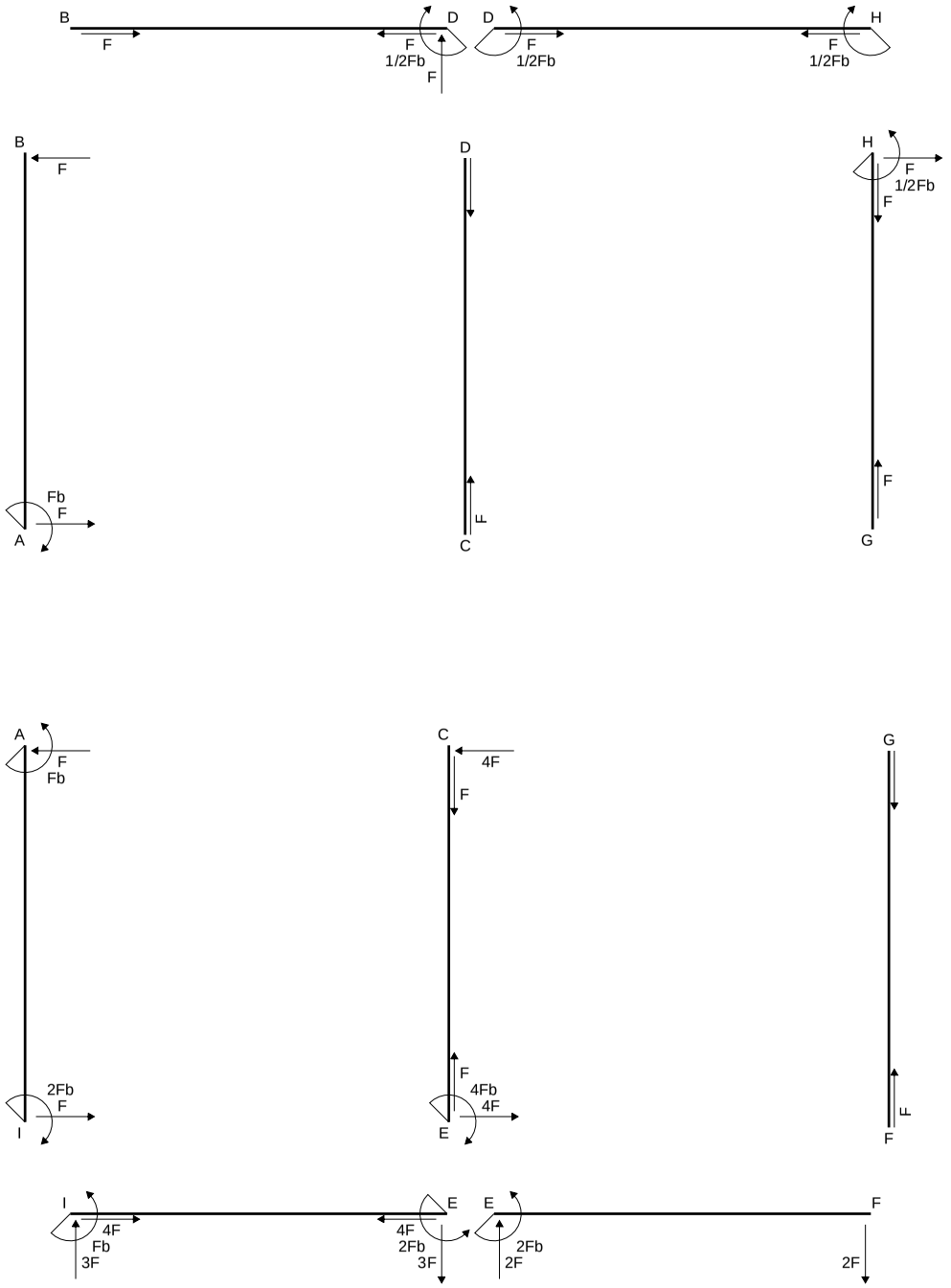
$V_{HG} = -F$
$V_{FE} = -F$
$W_I = -W = -Fb$
$p_{GH} = -q = -F/b$
$q_{DB} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



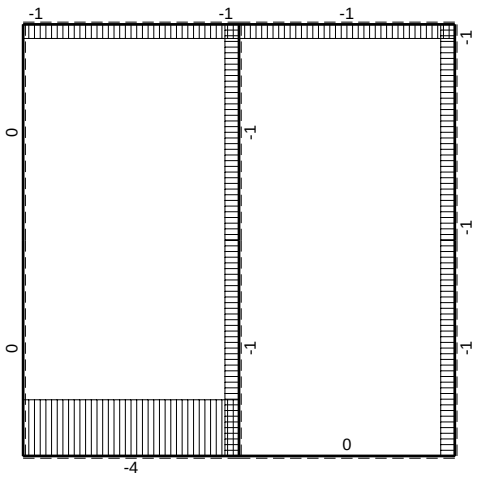


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 810 mm, F = 1280 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

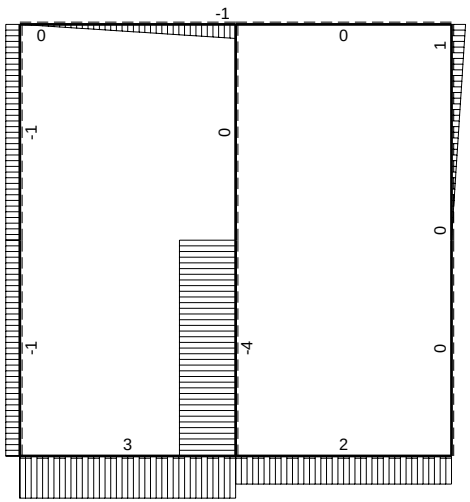
12.12.25



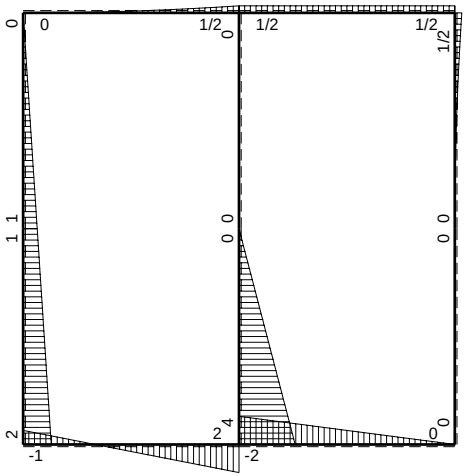
12.12.25



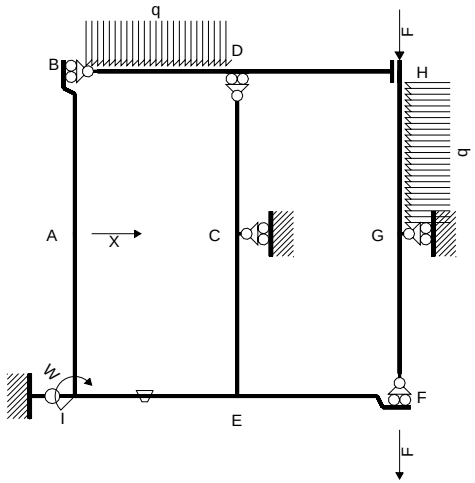
←□→ F



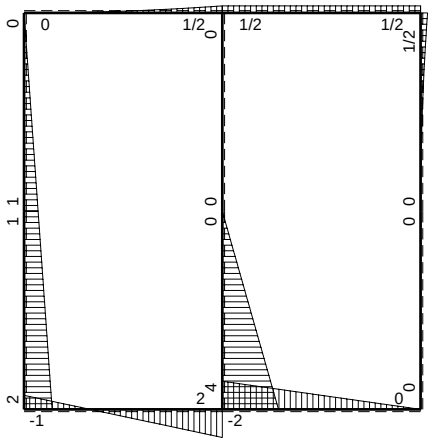
↑□↓ F



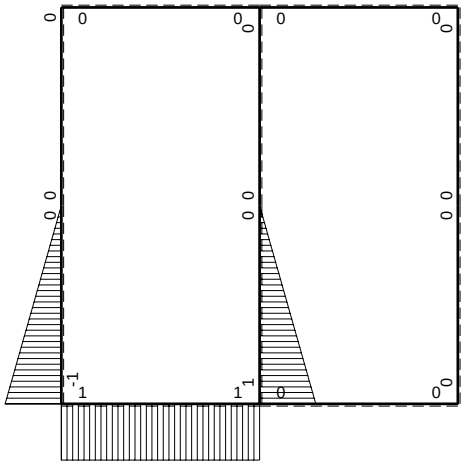
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	Fb-Fx	0	0	0	0	0+0	0
BA b	0	-Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
HD b	0	1/2Fb	0	0	0	0	0+0	0
DH b	0	-1/2Fb	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	b	-Fb+3Fx	-Fb/EJ	$-Fb^2+3Fbx$	$-Fb^2/EJ$	b^2	$(1/2-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	-2Fb+3Fx	Fb/EJ	$2Fb^2-3Fbx$	$-Fb^2/EJ$	b^2		
EC b	b-x	4Fb-4Fx	0	$4Fb^2-8Fbx+4Fx^2$	0	$b^2-2bx+x^2$	$(4/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	-4Fx	0	$4Fx^2$	0	x^2		
IA b	-b+x	2Fb-Fx	0	$-2Fb^2+3Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-5/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	-Fb-Fx	0	$-Fbx-Fx^2$	0	x^2		
	totali						0	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						0	

PROCEDIMENTO E RISULTATI 995596 ESΣ07.xxxx.015

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 12.12.25

$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (-1 + 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx = [-\, x + 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [-\, x \,]_o^b\, \theta$

$= (-\, b + 3/2\, b)\, Fb^2\, 1/EJ + (-\, b)\, \theta = -1/2\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (2 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx = [2\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (2\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = -1/2\ Fb^3/EJ$

$L_{EC}^{Xo} = \int_o^b (4 - 8\, x/b + 4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [4\, x - 4\, x^2/b + 4/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (4\, b - 4\, b + 4/3\, b \,)\, Fb^2\, 1/EJ = 4/3\ Fb^3/EJ$

$L_{CE}^{Xo} = \int_o^b (4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [4/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

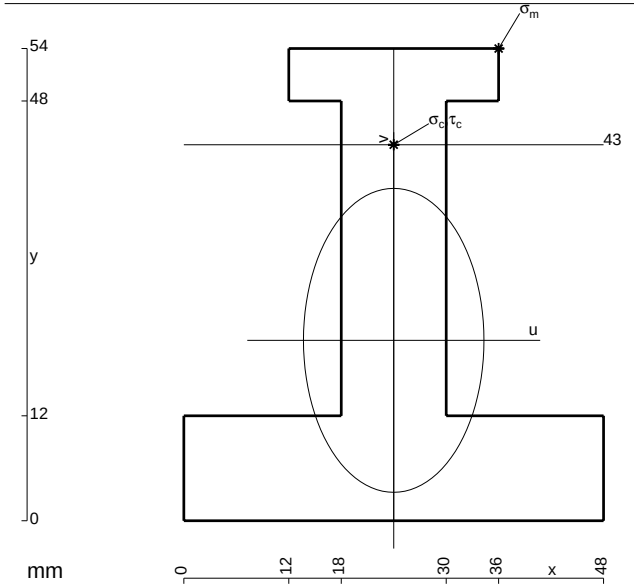
$= (4/3\, b \,)\, Fb^2\, 1/EJ = 4/3\ Fb^3/EJ$

$L_{IA}^{Xo} = \int_o^b (-2 + 3\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\, x + 3/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-2\, b + 3/2\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$

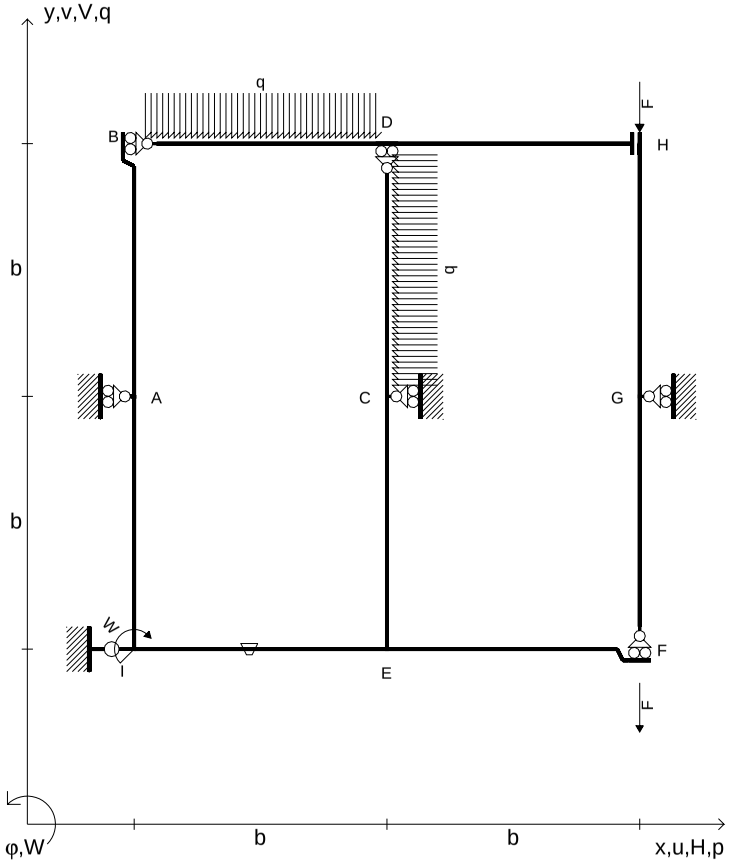
$L_{AI}^{Xo} = \int_o^b (-\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

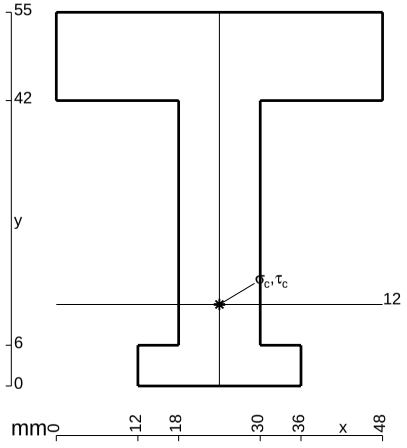
$= (-1/2\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$



A = 1152. mm^2
J_u = 348030. mm^4
J_v = 122688. mm^4
y_g = 20.63 mm
T_y = 2560. N
M_x = -2073600. Nmm
x_m = 36. mm
y_m = 54. mm
u_m = 12. mm
v_m = 33.38 mm
$\sigma_m = -Mv/J_u$ = 198.9 N/mm^2
x_c = 24. mm
y_c = 43. mm
v_c = 22.38 mm
$\sigma_c = -Mv/J_u$ = 133.3 N/mm^2
τ_c = 3.596 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 133.5 N/mm^2
S^* = 5867. mm^3

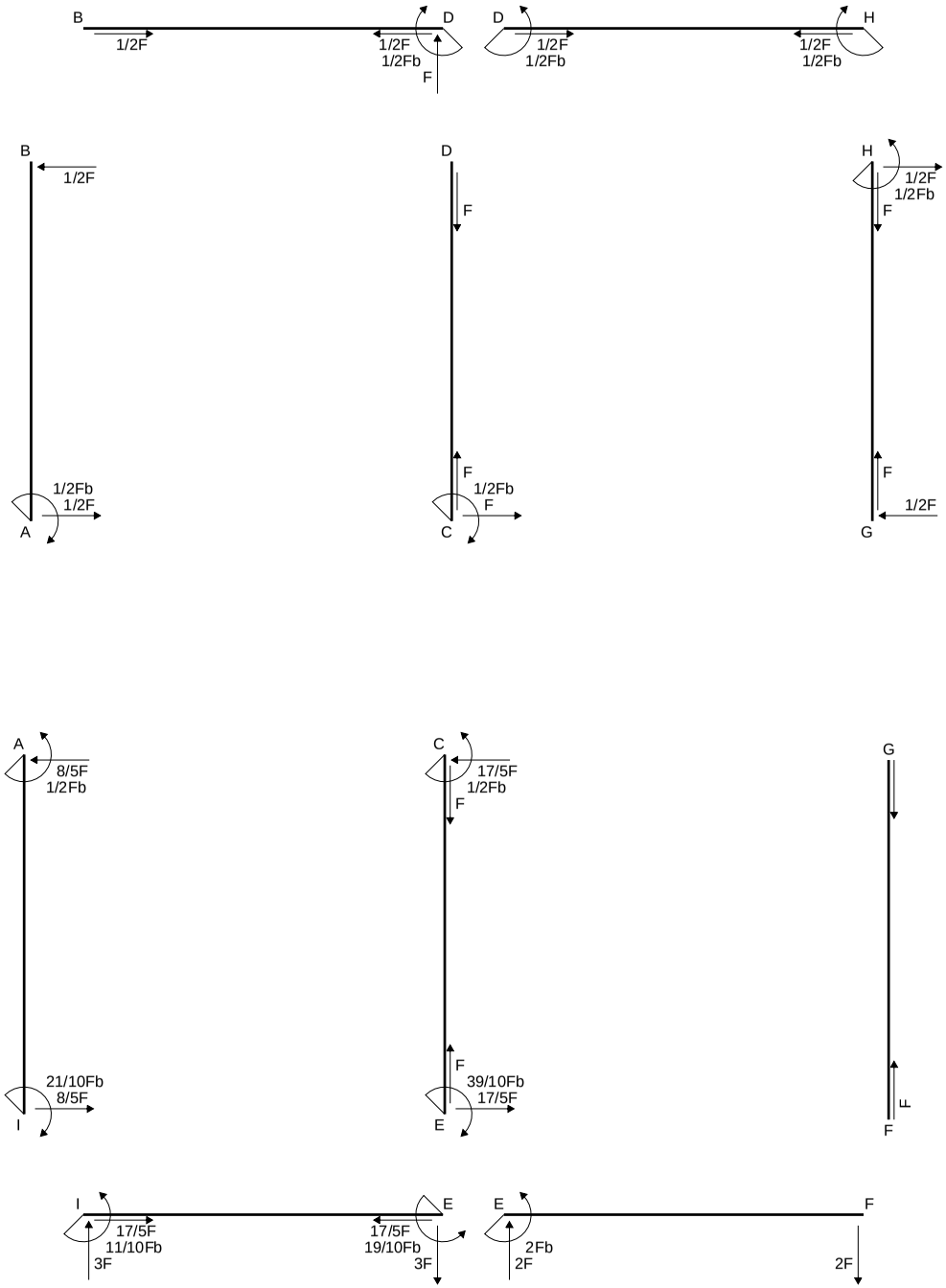
$V_{HG} = -F$
$V_{FE} = -F$
$W_I = -W = -Fb$
$p_{CD} = -q = -F/b$
$q_{DB} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



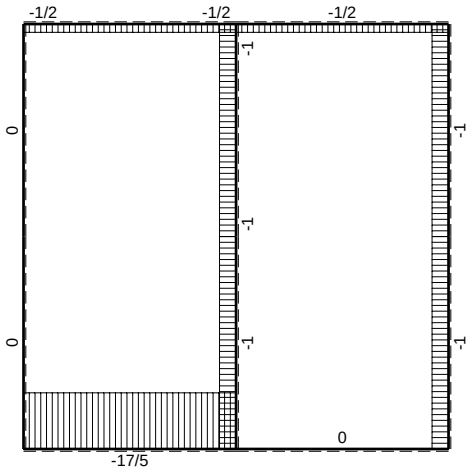


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 880 mm, F = 1280 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

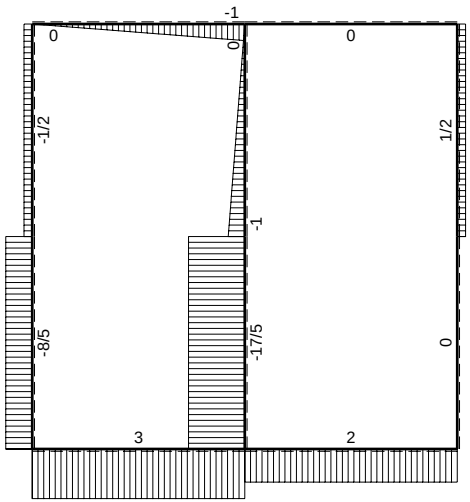
12.12.25



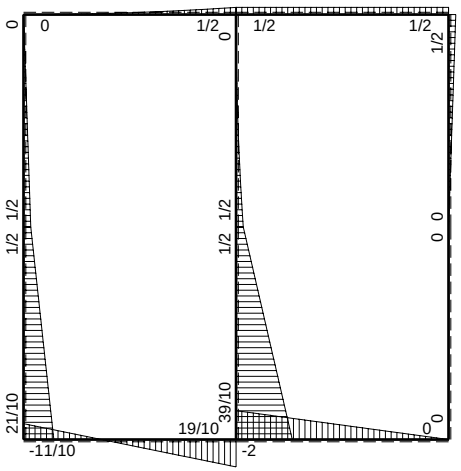
12.12.25



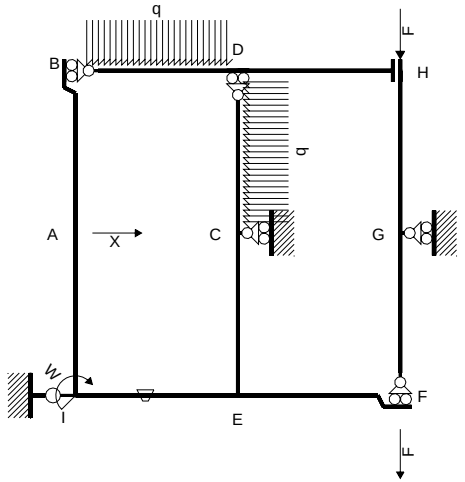
←□→ F



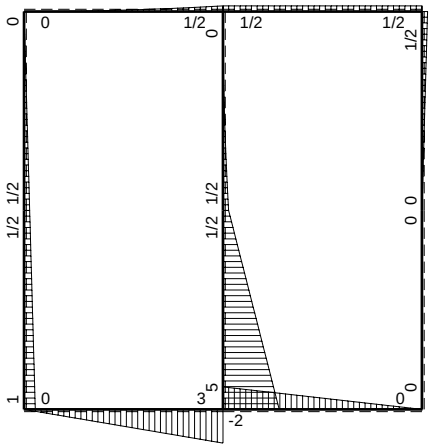
↑□↓ F



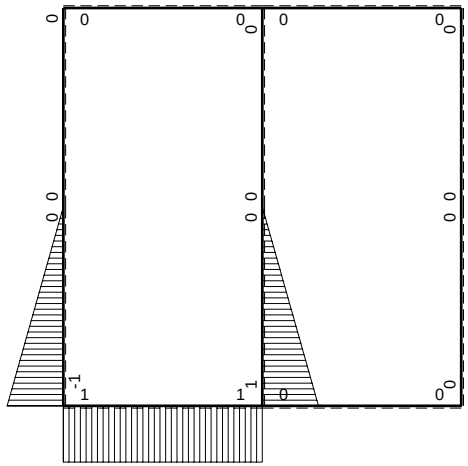
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2qx^2$	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fb+1/2Fx	0	0	0	0		
HD b	0	1/2Fb	0	0	0	0	0+0	0
DH b	0	-1/2Fb	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	b	3Fx	-Fb/EJ	3Fbx	$-Fb^2/EJ$	b^2	$(3/2-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	-3Fb+3Fx	Fb/EJ	$3Fb^2-3Fbx$	$-Fb^2/EJ$	b^2		
EC b	b-x	$5Fb-9/2Fx$	0	$5Fb^2-19/2Fbx+9/2Fx^2$	0	$b^2-2bx+x^2$	$(7/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	$-1/2Fb-9/2Fx$	0	$1/2Fbx+9/2Fx^2$	0	x^2		
IA b	-b+x	Fb-1/2Fx	0	$-Fb^2+3/2Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-5/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	$-1/2Fb-1/2Fx$	0	$-1/2Fbx-1/2Fx^2$	0	x^2		
	totali						$11/6Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-11/10F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 309819 ESΣ07.xxxx.036
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 12.12.25

$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (3/2\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = 1/2\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (3 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [3\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (3\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 1/2\ Fb^3/EJ$

$L_{EC}^{Xo} = \int_o^b (5 - 19/2\, x/b + 9/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5\, x - 19/4\, x^2/b + 3/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (5\, b - 19/4\, b + 3/2\, b)\, Fb^2\, 1/EJ = 7/4\ Fb^3/EJ$

$L_{CE}^{Xo} = \int_o^b (1/2\, x/b + 9/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b + 3/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b + 3/2\, b)\, Fb^2\, 1/EJ = 7/4\ Fb^3/EJ$

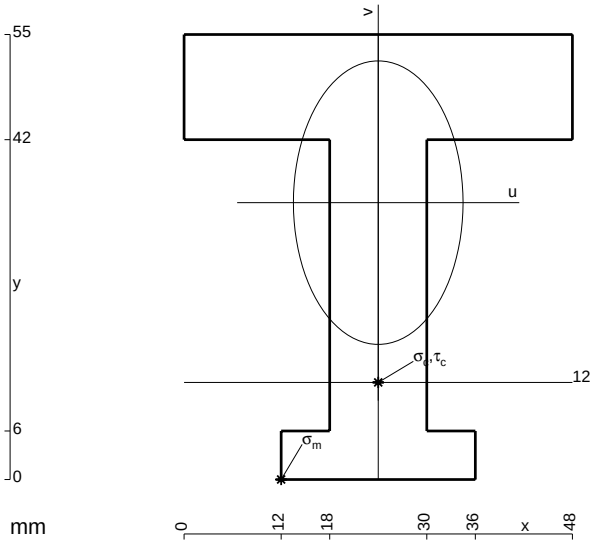
$L_{IA}^{Xo} = \int_o^b (-1 + 3/2\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x + 3/4\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (- b + 3/4\, b - 1/6\, b)\, Fb^2\, 1/EJ = -5/12\ Fb^3/EJ$

$L_{AI}^{Xo} = \int_o^b (-1/2\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

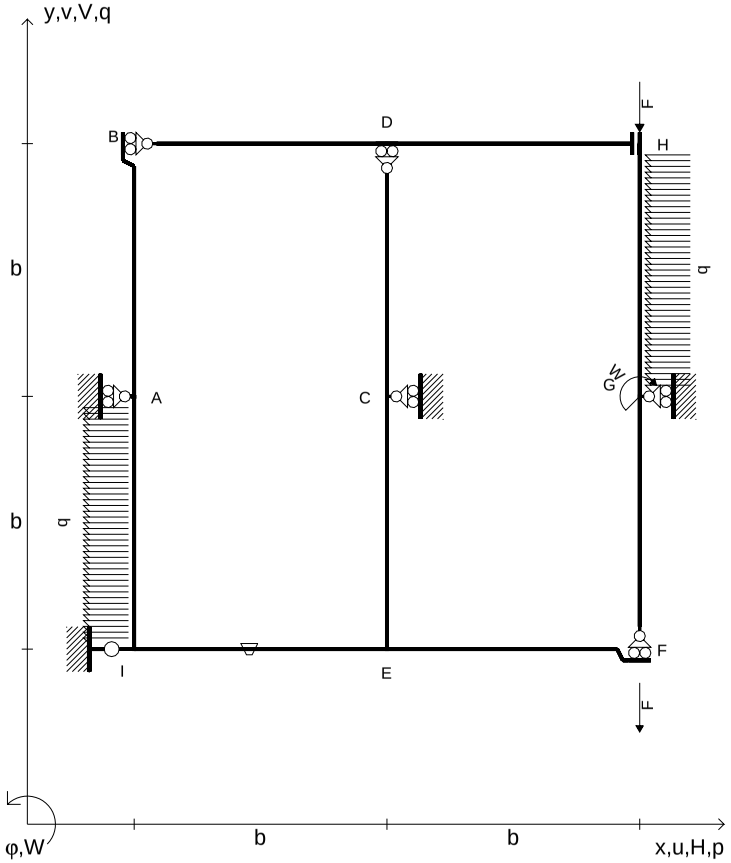
$= (-1/4\, b - 1/6\, b)\, Fb^2\, 1/EJ = -5/12\ Fb^3/EJ$

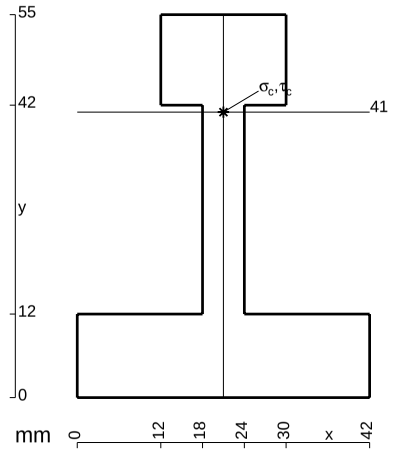
PROCEDIMENTO E RISULTATI 309819 ESΣ07.xxxx.036



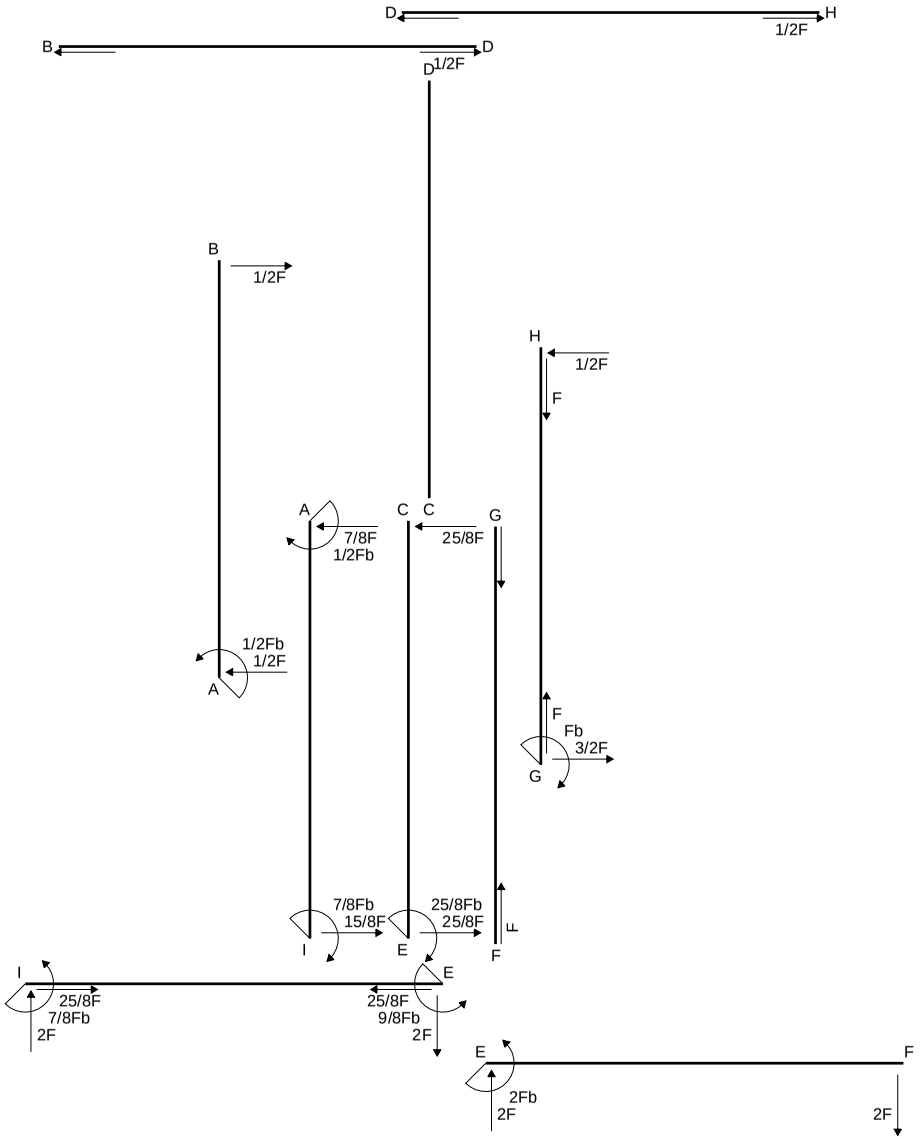
A = 1200. mm^2
J_u = 368598. mm^4
J_v = 131904. mm^4
y_g = 34.22 mm
T_y = 2560. N
M_x = -2252800. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -34.22 mm
$\sigma_m = -Mv/J_u$ = -209.1 N/mm^2
x_c = 24. mm
y_c = 12. mm
v_c = -22.22 mm
$\sigma_c = -Mv/J_u$ = -135.8 N/mm^2
τ_c = 3.653 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 136. N/mm^2
S^* = 6312. mm^3

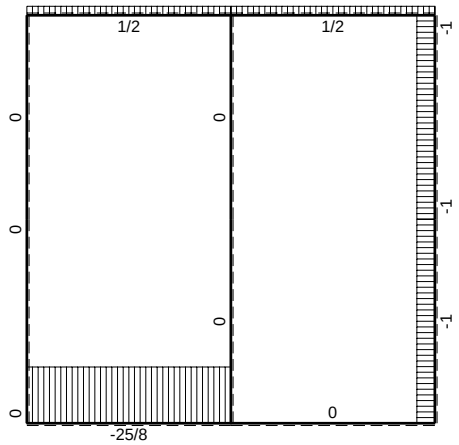
V_{HG} = -F
V_{FE} = -F
W_G = -W = -Fb
p_{GH} = -q = -F/b
p_{IA} = -q = -F/b
θ_{IE} = -θ = -αT/b = -bF/EJ
EJ_{AB} = EJ
EJ_{CD} = EJ
EJ_{EF} = EJ
EJ_{FG} = EJ
EJ_{GH} = EJ
EJ_{HD} = EJ
EJ_{DB} = EJ
EJ_{IE} = EJ
EJ_{EC} = EJ
EJ_{IA} = EJ



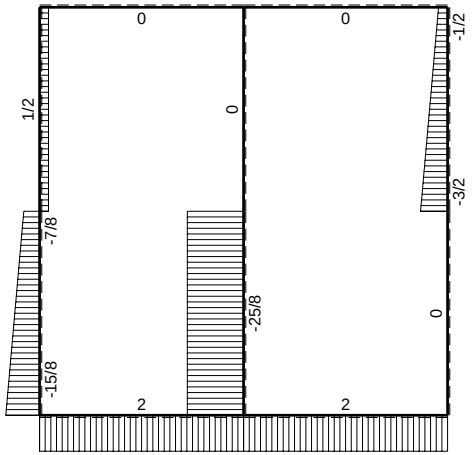


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 770 mm, F = 1460 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

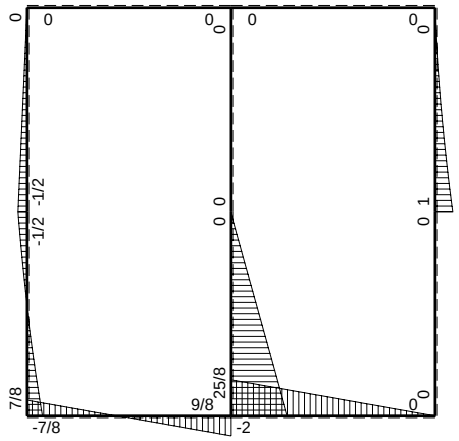




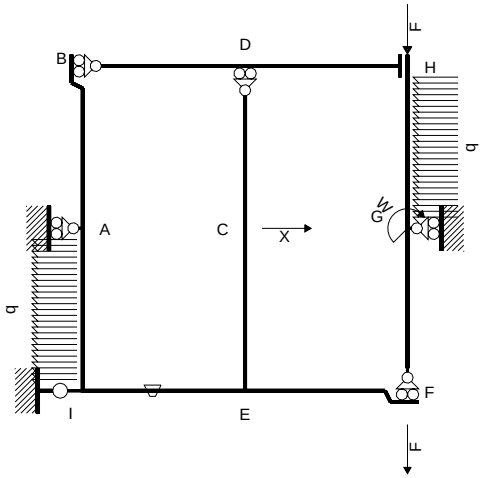
←□→ F



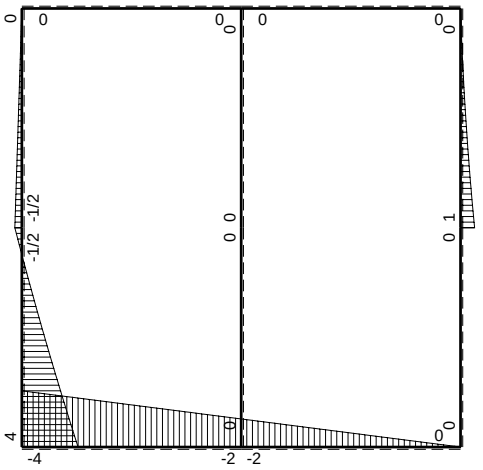
↑□↓ F



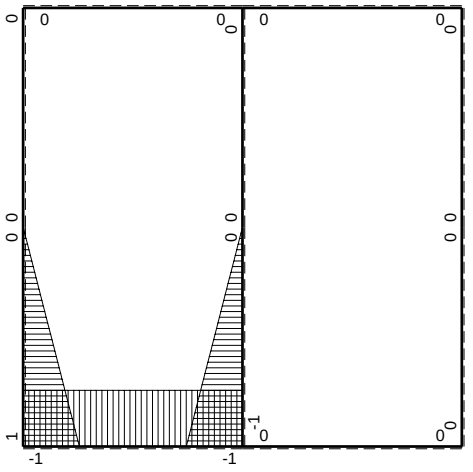
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236422 ESΣ07.xxxx.044

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$Fb-3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$-1/2Fx-1/2qx^2$	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-b	-4Fb+2Fx	-Fb/EJ	$4Fb^2-2Fbx$	Fb^2/EJ	b^2	$(3+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	2Fb+2Fx	Fb/EJ	$2Fb^2+2Fbx$	Fb^2/EJ	b^2		
EC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CE b	x	0	0	0	0	x^2		
IA b	b-x	$4Fb-5Fx+1/2qx^2$	0	$4Fb^2-9Fbx+11/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$	$(29/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	$1/2Fb-4Fx-1/2qx^2$	0	$-1/2Fbx+4Fx^2+1/2qx^3$	0	x^2		
	totali						$125/24Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-25/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 12.12.25

$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (4 - 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [4\, x - x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (4\, b - b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 4\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (2 + 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [2\, x + x^2/b \,]_o^b\, Fb^2\, 1/EJ + [-\, x \,]_o^b\, \theta$

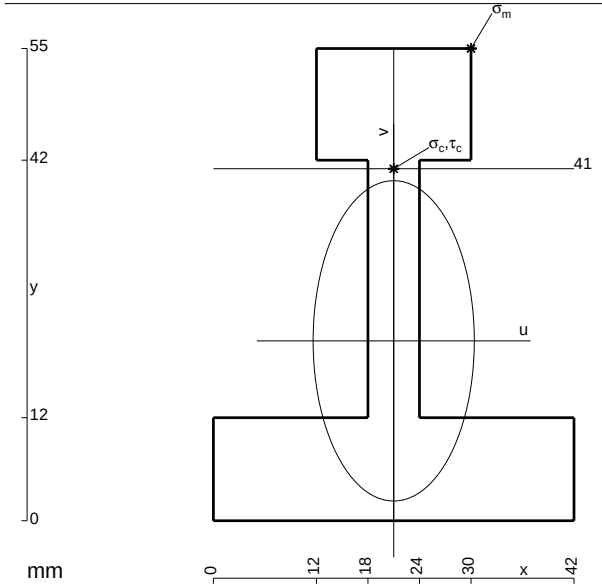
$= (2\, b + b)\, Fb^2\, 1/EJ + (-\, b)\, \theta = 4\ Fb^3/EJ$

$L_{IA}^{Xo} = \int_o^b (4 - 9\, x/b + 11/2\, x^2/b^2 - 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [4\, x - 9/2\, x^2/b + 11/6\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (4\, b - 9/2\, b + 11/6\, b - 1/8\, b)\, Fb^2\, 1/EJ = 29/24\ Fb^3/EJ$

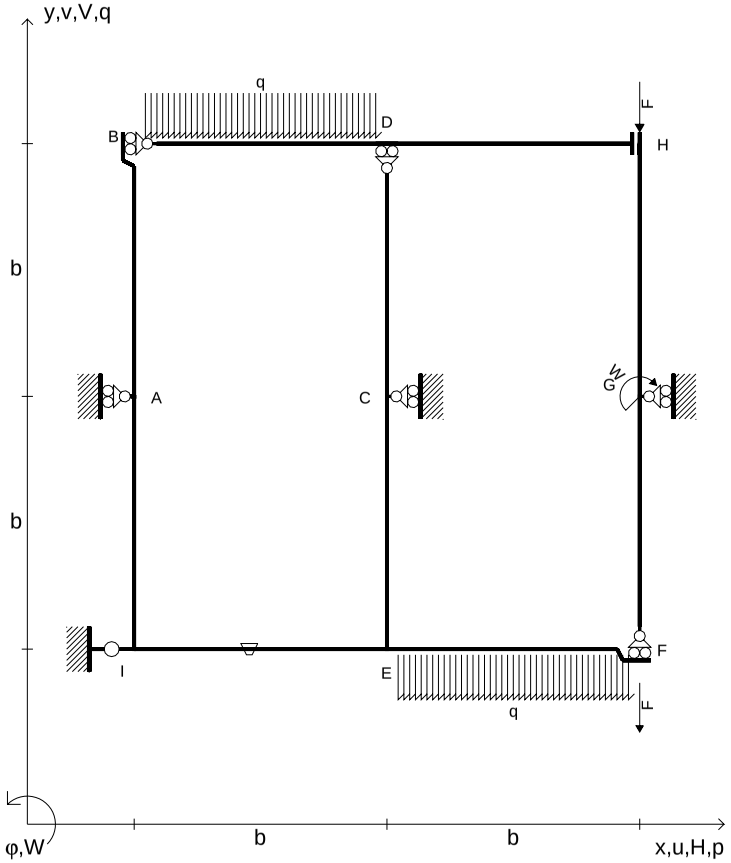
$L_{AI}^{Xo} = \int_o^b (-1/2\, x/b + 4\, x^2/b^2 + 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b + 4/3\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

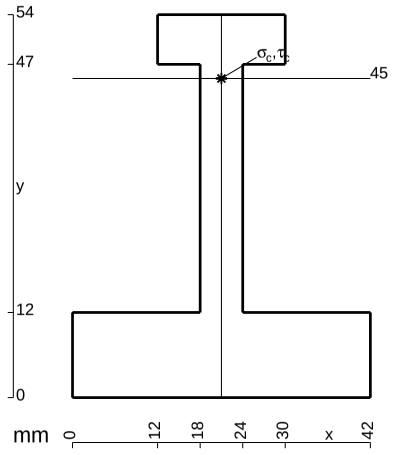
$= (-1/4\, b + 4/3\, b + 1/8\, b)\, Fb^2\, 1/EJ = 29/24\ Fb^3/EJ$



A = 918. mm^2
J_u = 319684. mm^4
J_v = 80946. mm^4
y_g = 20.95 mm
T_y = 2920. N
M_x = -2248400. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 9. mm
v_m = 34.05 mm
$\sigma_m = -Mv/J_u$ = 239.5 N/mm^2
x_c = 21. mm
y_c = 41. mm
v_c = 20.05 mm
$\sigma_c = -Mv/J_u$ = 141. N/mm^2
τ_c = 10. N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 142.1 N/mm^2
S^* = 6570. mm^3

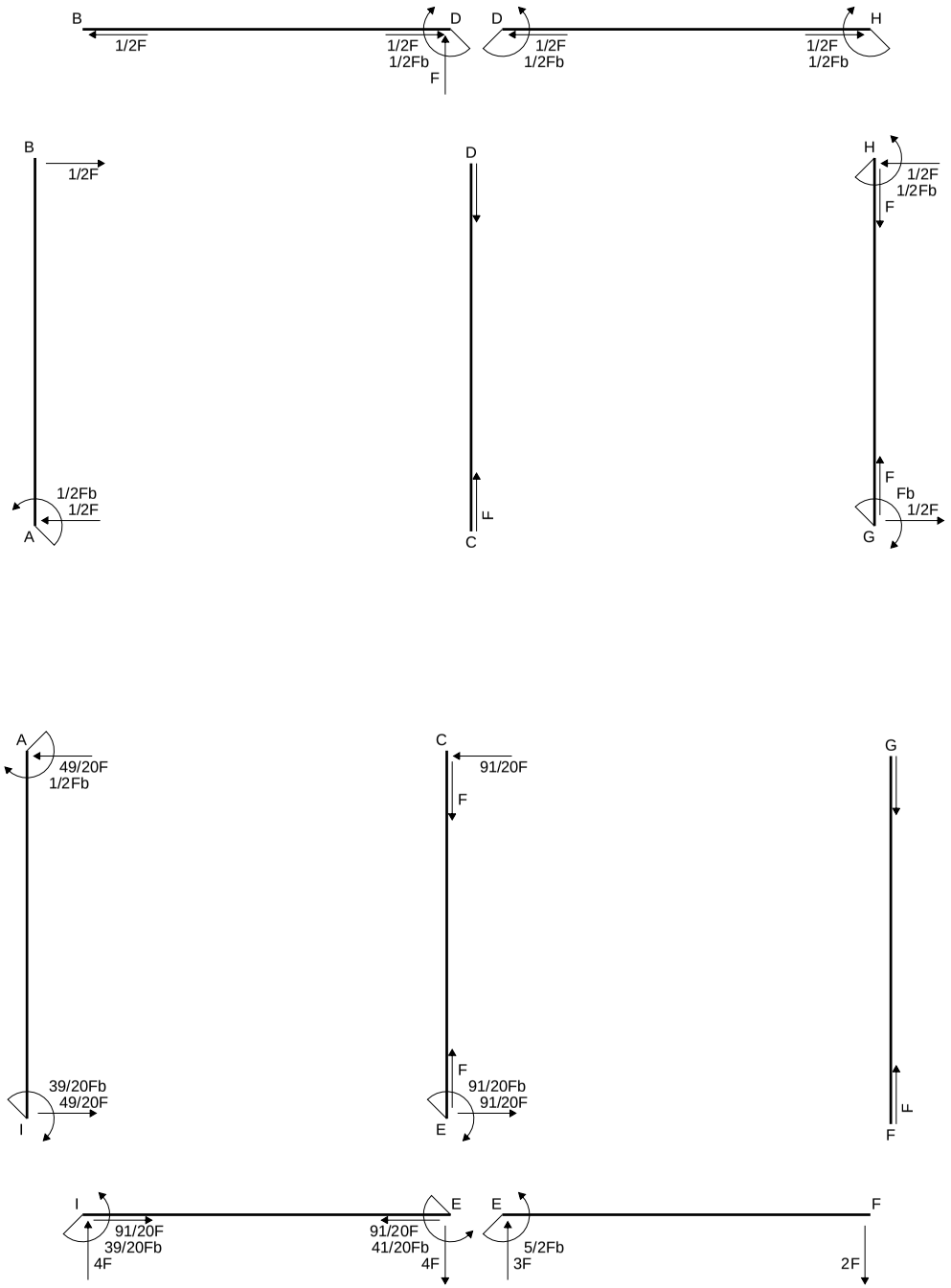
$V_{HG} = -F$
$V_{FE} = -F$
$W_G = -W = -Fb$
$q_{DB} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



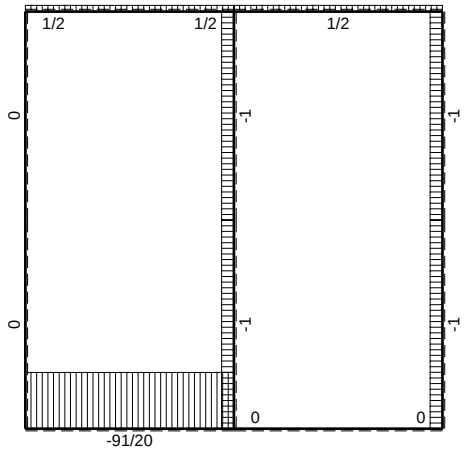


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 590 mm, F = 1040 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

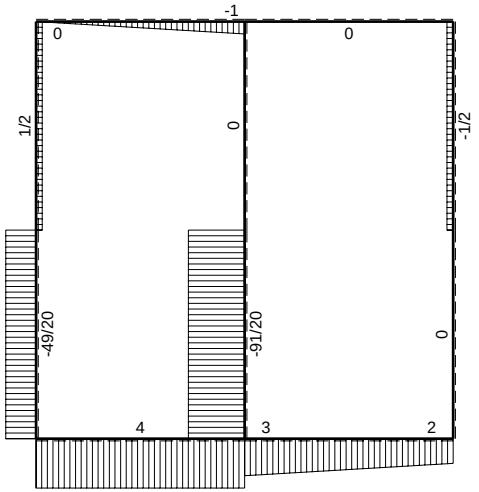
12.12.25



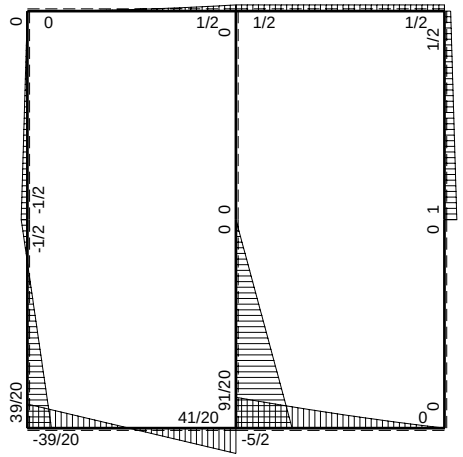
12.12.25



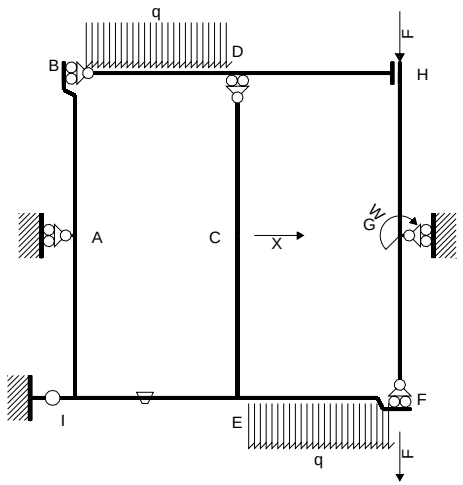
←□→ F



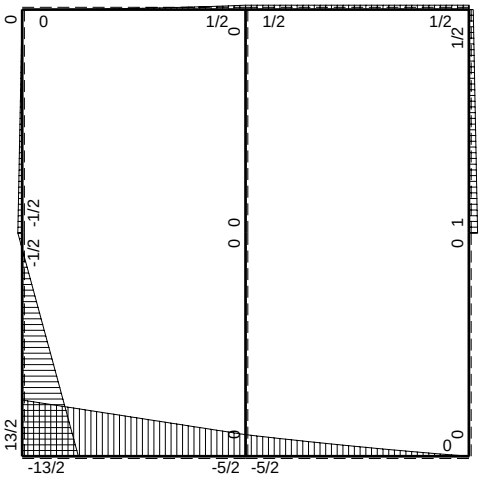
↑□↓ F



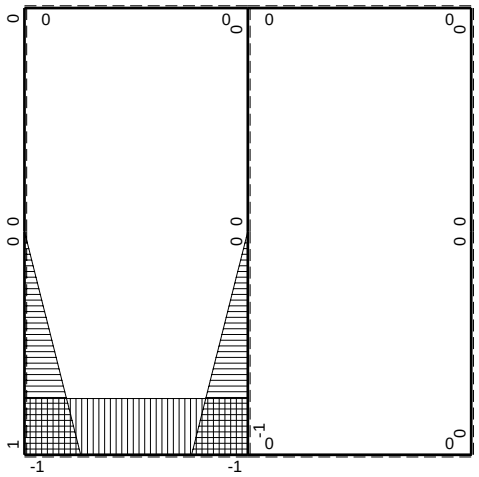
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-5/2Fb+3Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$2Fx+1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fb-1/2Fx	0	0	0	0		
HD b	0	1/2Fb	0	0	0	0	0+0	0
DH b	0	-1/2Fb	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	-b	$-13/2Fb+4Fx$	-Fb/EJ	$13/2Fb^2-4Fbx$	Fb^2/EJ	b^2	$(9/2+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$5/2Fb+4Fx$	Fb/EJ	$5/2Fb^2+4Fbx$	Fb^2/EJ	b^2		
EC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CE b	x	0	0	0	0	x^2		
IA b	b-x	13/2Fb-7Fx	0	$13/2Fb^2-27/2Fbx+7Fx^2$	0	$b^2-2bx+x^2$	$(25/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	1/2Fb-7Fx	0	$-1/2Fbx+7Fx^2$	0	x^2		
	totali						$91/12Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-91/20F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 12.12.25

$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (13/2 - 4\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [13/2\, x - 2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (13/2\, b - 2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 11/2\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (5/2 + 4\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [5/2\, x + 2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (5/2\, b + 2\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = 11/2\ Fb^3/EJ$

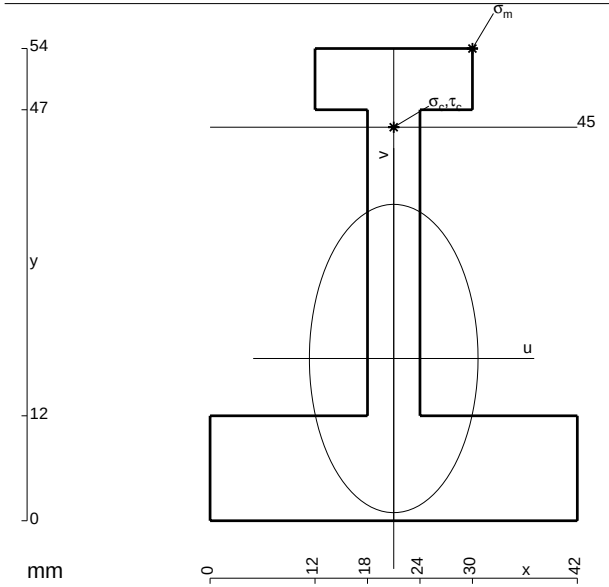
$L_{IA}^{Xo} = \int_o^b (13/2 - 27/2\, x/b + 7\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [13/2\, x - 27/4\, x^2/b + 7/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (13/2\, b - 27/4\, b + 7/3\, b)\, Fb^2\, 1/EJ = 25/12\ Fb^3/EJ$

$L_{AI}^{Xo} = \int_o^b (-1/2\, x/b + 7\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b + 7/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/4\, b + 7/3\, b)\, Fb^2\, 1/EJ = 25/12\ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 240095 ESΣ07.xxxx.061



A = 840. mm^2
J_u = 261182. mm^4
J_v = 78120. mm^4
y_g = 18.55 mm
T_y = 3120. N
M_x = -1534000. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 9. mm
v_m = 35.45 mm
$\sigma_m = -Mv/J_u$ = 208.2 N/mm^2
x_c = 21. mm
y_c = 45. mm
v_c = 26.45 mm
$\sigma_c = -Mv/J_u$ = 155.3 N/mm^2
τ_c = 8.671 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 156.1 N/mm^2
S^* = 4355. mm^3

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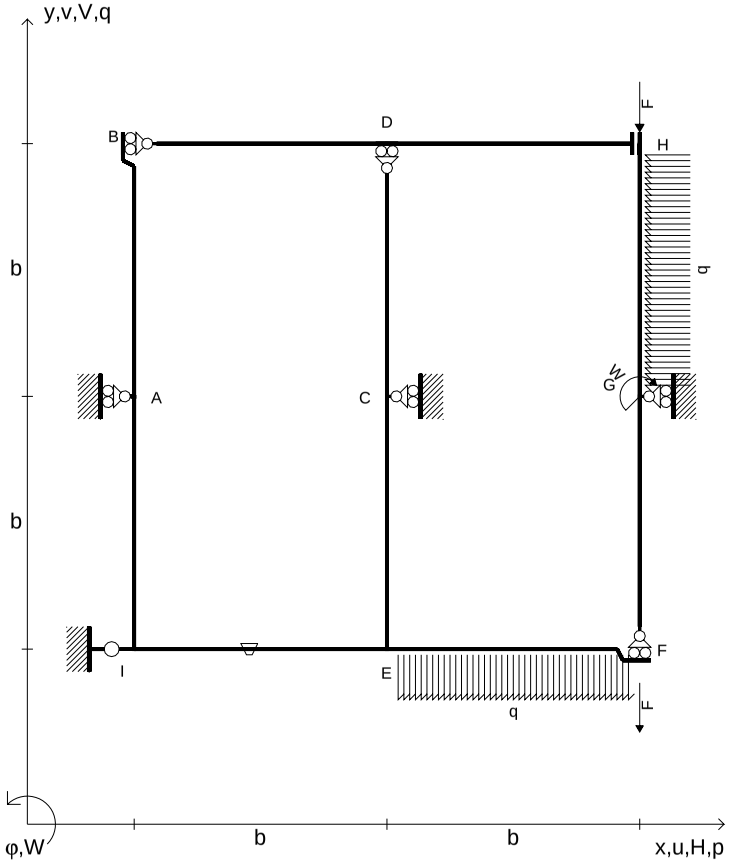
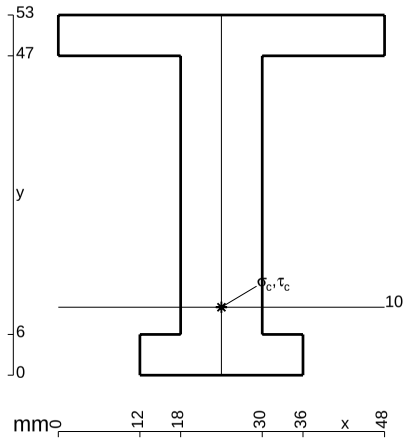
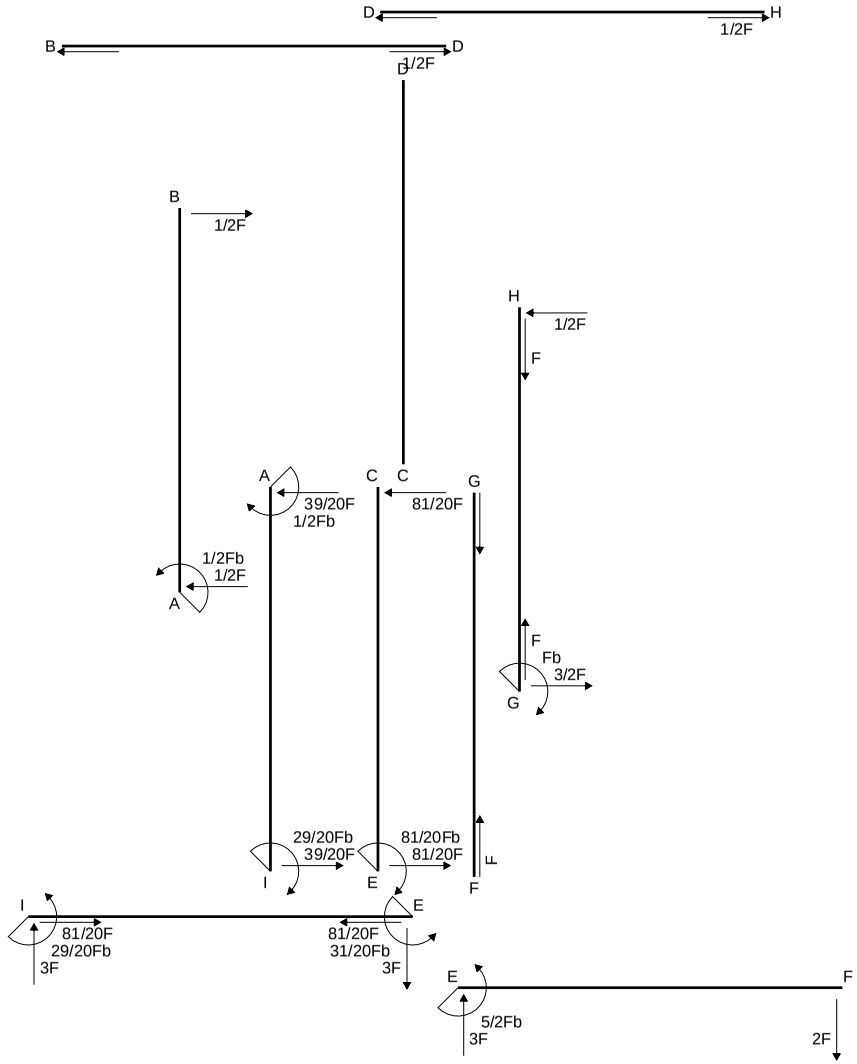
$V_{HG} = -F$
$V_{FE} = -F$
$W_G = -W = -Fb$
$p_{GH} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 900 mm, F = 960 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.



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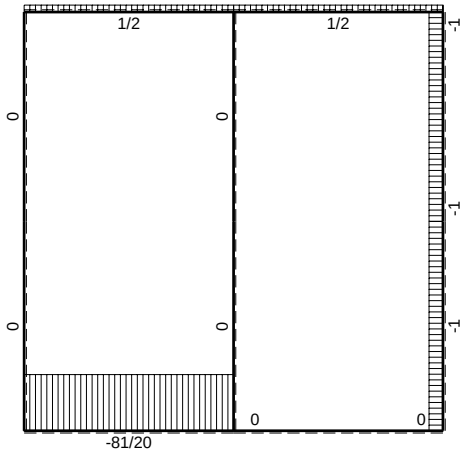
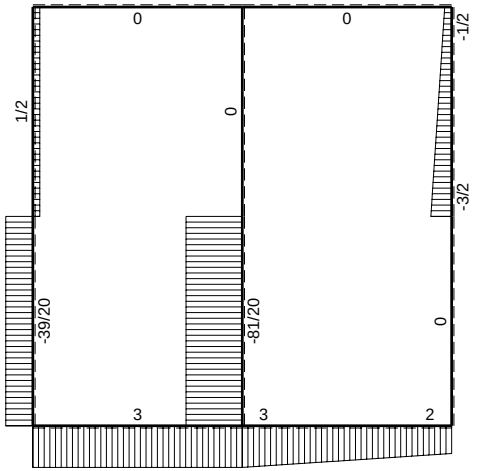
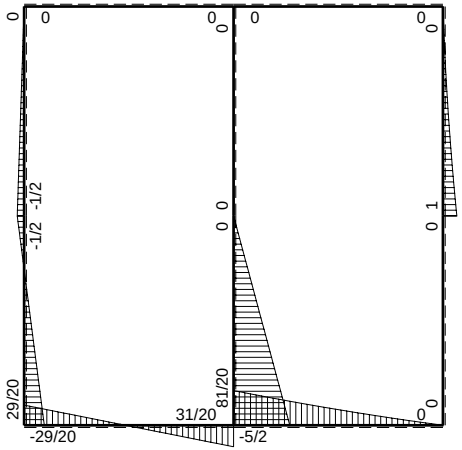


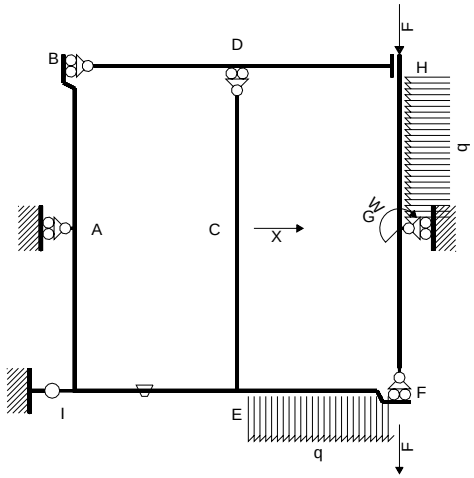
←F→



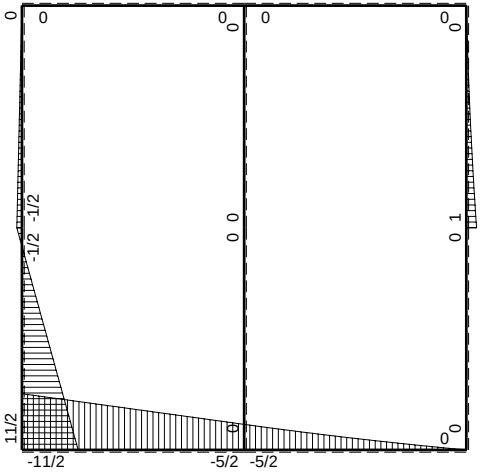
↑F↓



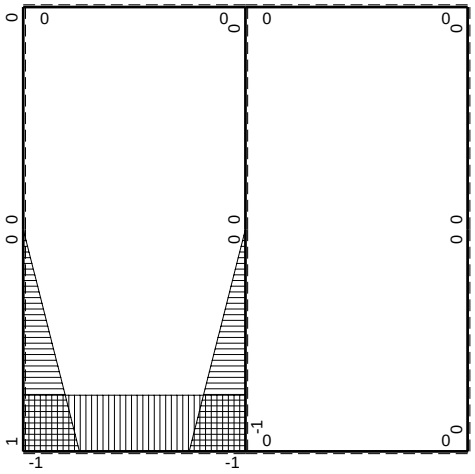
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-5/2Fb+3Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$2Fx+1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$Fb-3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$-1/2Fx-1/2qx^2$	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-b	$-11/2Fb+3Fx$	$-Fb/EJ$	$11/2Fb^2-3Fbx$	Fb^2/EJ	b^2	$(4+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$5/2Fb+3Fx$	Fb/EJ	$5/2Fb^2+3Fbx$	Fb^2/EJ	b^2		
EC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CE b	x	0	0	0	0	x^2		
IA b	b-x	$11/2Fb-6Fx$	0	$11/2Fb^2-23/2Fbx+6Fx^2$	0	$b^2-2bx+x^2$	$(7/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	$1/2Fb-6Fx$	0	$-1/2Fbx+6Fx^2$	0	x^2		
	totali						$27/4Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-81/20F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (11/2 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [11/2\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (11/2\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 5\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (5/2 + 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [5/2\, x + 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [-\, x \,]_o^b\, \theta$

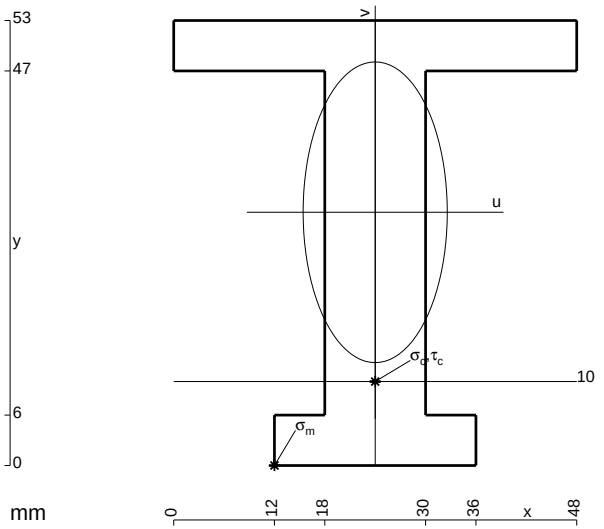
$= (5/2\, b + 3/2\, b)\, Fb^2\, 1/EJ + (-\, b)\, \theta = 5\ Fb^3/EJ$

$L^{Xo}_{IA} = \int_o^b (11/2 - 23/2\, x/b + 6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [11/2\, x - 23/4\, x^2/b + 2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (11/2\, b - 23/4\, b + 2\, b)\, Fb^2\, 1/EJ = 7/4\ Fb^3/EJ$

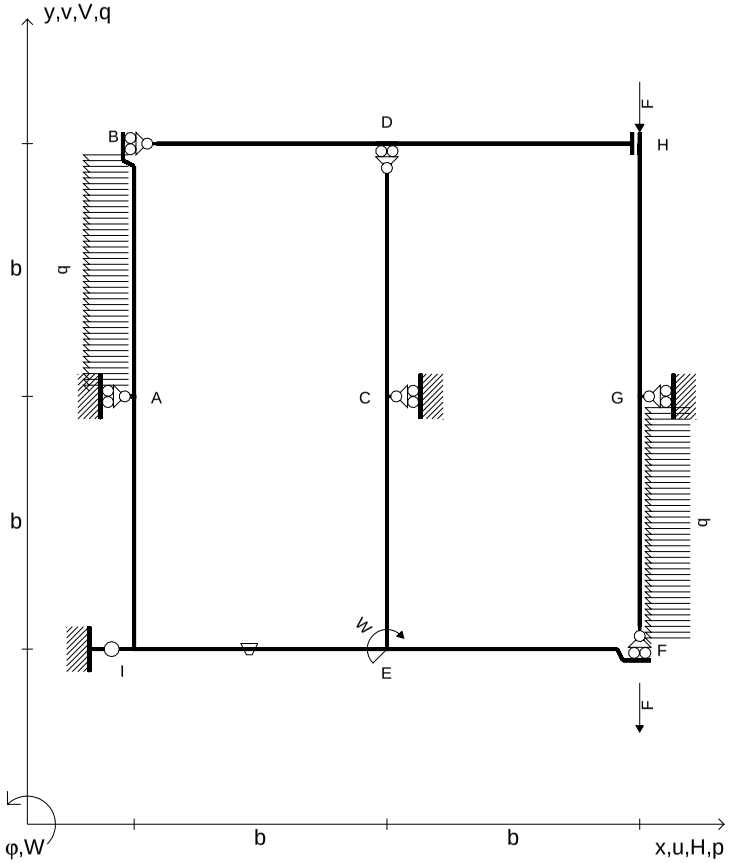
$L^{Xo}_{AI} = \int_o^b (-1/2\, x/b + 6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b + 2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

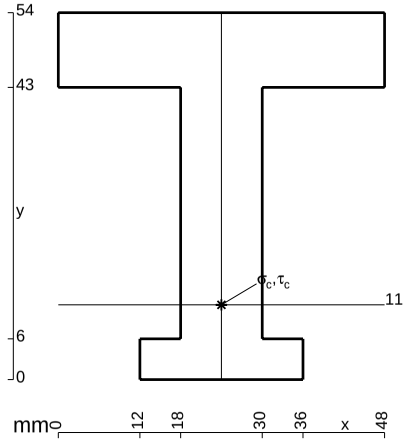
$= (-1/4\, b + 2\, b)\, Fb^2\, 1/EJ = 7/4\ Fb^3/EJ$



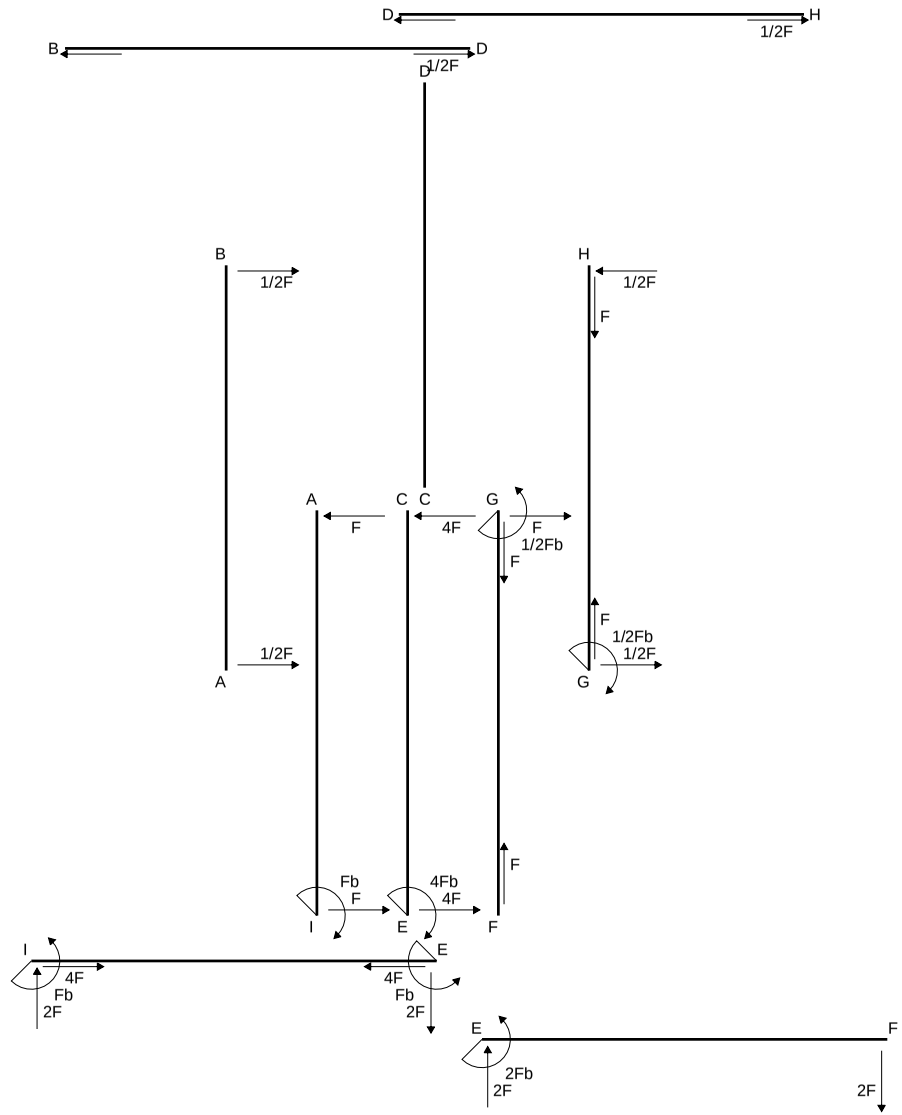
A = 924. mm^2
J_u = 296396. mm^4
J_v = 68112. mm^4
y_g = 30.16 mm
T_y = 2880. N
M_x = -2160000. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -30.16 mm
$\sigma_m = -Mv/J_u$ = -219.8 N/mm^2
x_c = 24. mm
y_c = 10. mm
v_c = -20.16 mm
$\sigma_c = -Mv/J_u$ = -146.9 N/mm^2
τ_c = 4.029 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 147.1 N/mm^2
S^* = 4975. mm^3

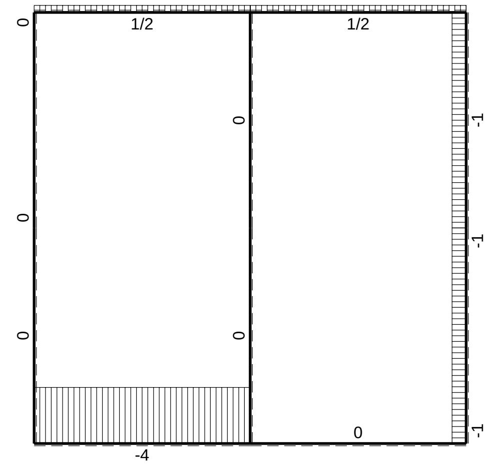
$V_{HG} = -F$
$V_{FE} = -F$
$W_E = -W = -Fb$
$p_{AB} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



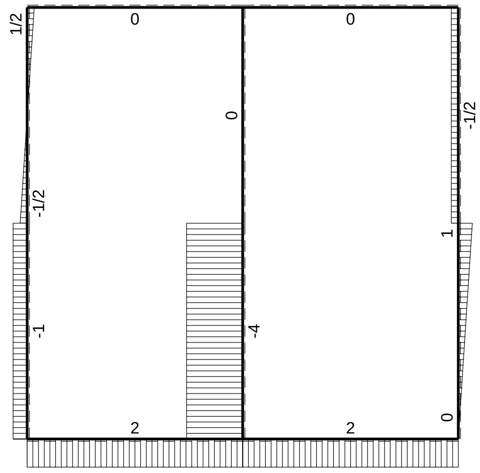


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 540 mm, F = 1930 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

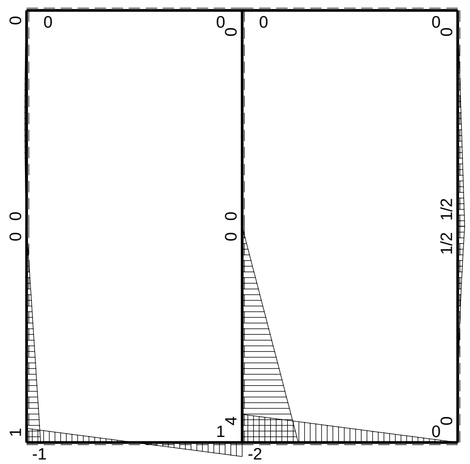




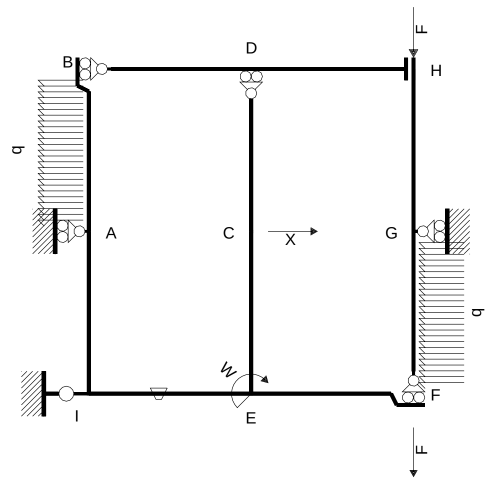
F



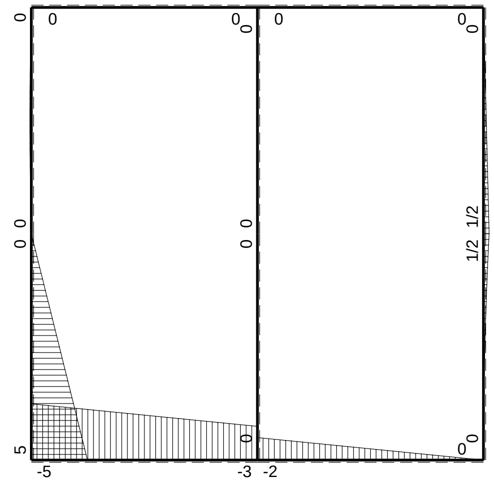
F



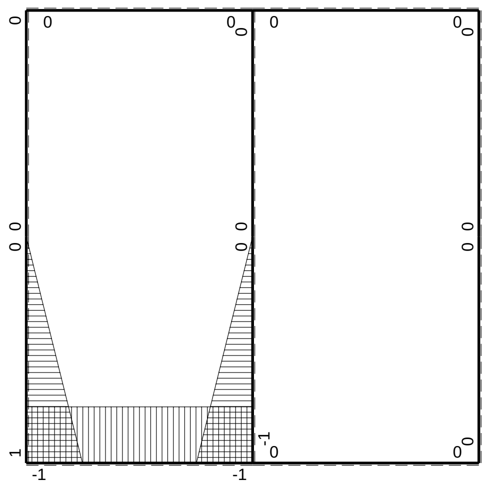
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
FE b	0	$2Fx$	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	$1/2Fb-1/2Fx$	0	0	0	0	0+0	0
HG b	0	$-1/2Fx$	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-b	$-5Fb+2Fx$	$-Fb/EJ$	$5Fb^2-2Fbx$	Fb^2/EJ	b^2	$(4+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$3Fb+2Fx$	Fb/EJ	$3Fb^2+2Fbx$	Fb^2/EJ	b^2		
EC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CE b	x	0	0	0	0	x^2		
IA b	b-x	5Fb-5Fx	0	$5Fb^2-10Fbx+5Fx^2$	0	$b^2-2bx+x^2$	$(5/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	$-5Fx$	0	$5Fx^2$	0	x^2		
	totali						$20/3Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-4F	

PROCEDIMENTO E RISULTATI 982088 ESΣ07.xxxx.100

Sviluppi di calcolo iperstatica

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$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (5 - 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [5\, x - x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (5\, b - b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 5\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (3 + 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [3\, x + x^2/b \,]_o^b\, Fb^2\, 1/EJ + [-\, x \,]_o^b\, \theta$

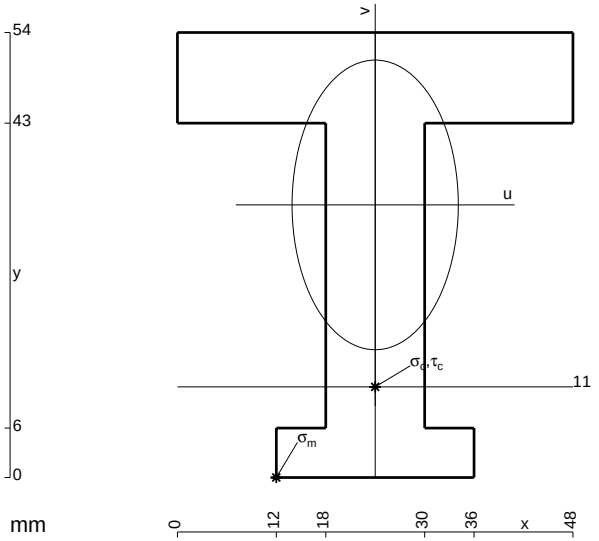
$= (3\, b + b)\, Fb^2\, 1/EJ + (-\, b \,)\, \theta = 5\ Fb^3/EJ$

$L_{IA}^{Xo} = \int_o^b (5 - 10\, x/b + 5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5\, x - 5\, x^2/b + 5/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (5\, b - 5\, b + 5/3\, b)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

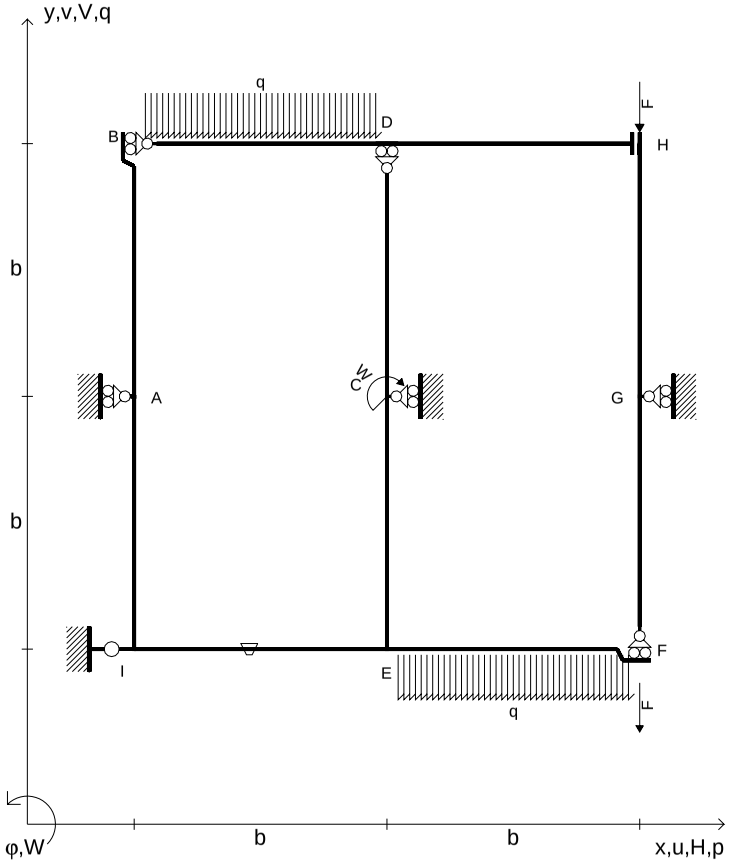
$L_{AI}^{Xo} = \int_o^b (5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

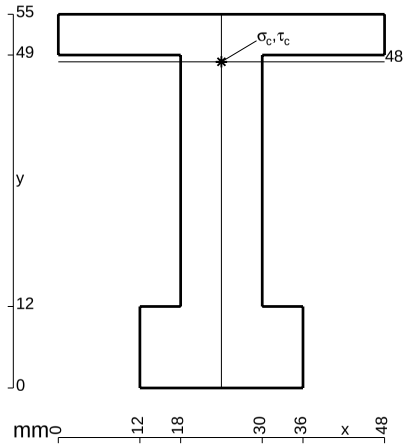
$= (5/3\, b)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$



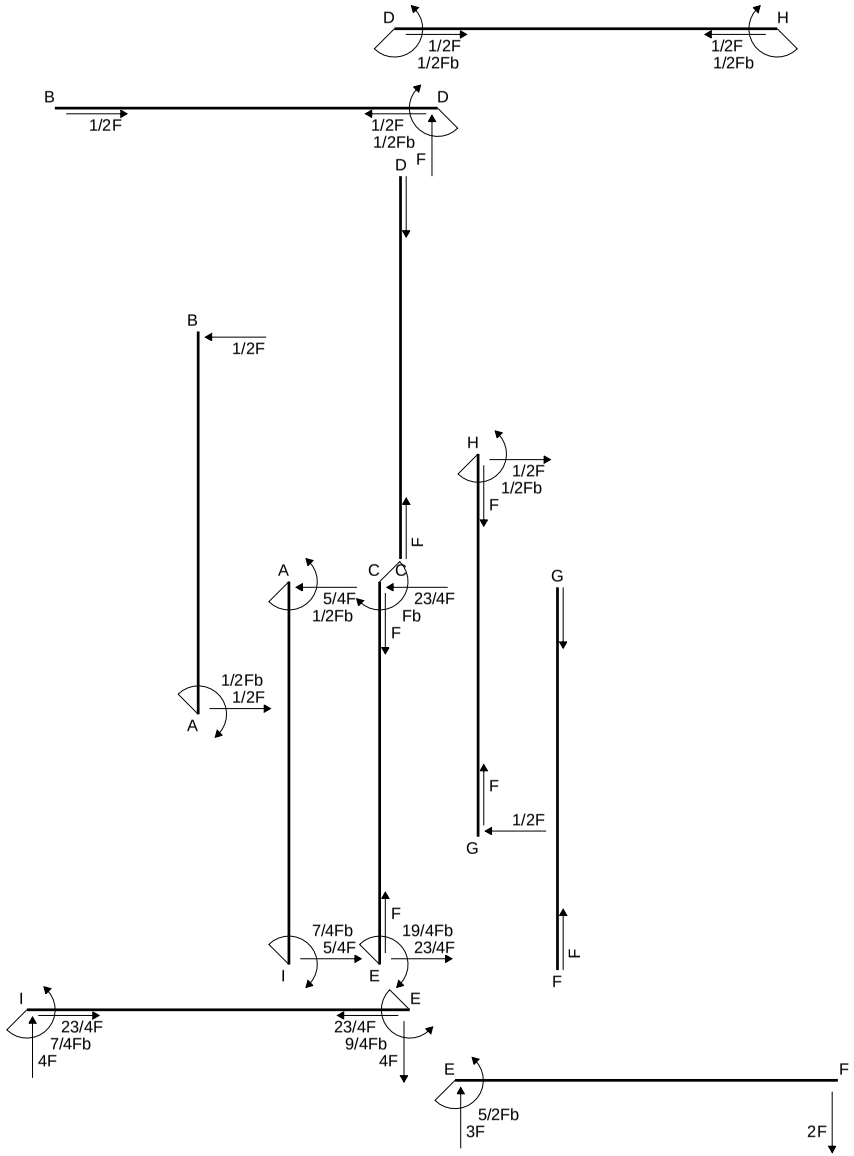
A = 1116. mm^2
J_u = 344933. mm^4
J_v = 113616. mm^4
y_g = 33.08 mm
T_y = 3860. N
M_x = -2084400. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -33.08 mm
$\sigma_m = -Mv/J_u$ = -199.9 N/mm^2
x_c = 24. mm
y_c = 11. mm
v_c = -22.08 mm
$\sigma_c = -Mv/J_u$ = -133.4 N/mm^2
τ_c = 5.415 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 133.8 N/mm^2
S^* = 5806. mm^3

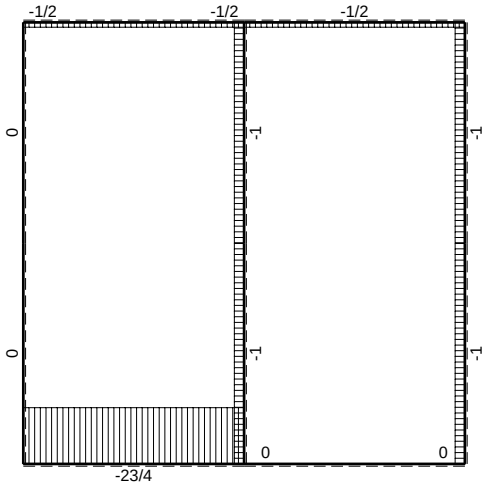
$V_{HG} = -F$
$V_{FE} = -F$
$W_C = -W = -Fb$
$q_{DB} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



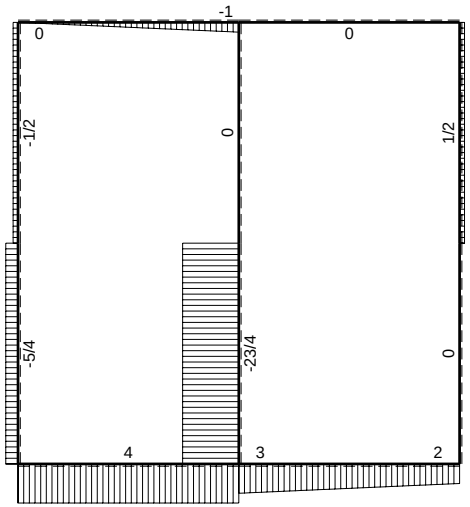


Reazioni iperstatiche in soluzione: $X=W_{EI}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 820 mm, F = 1180 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

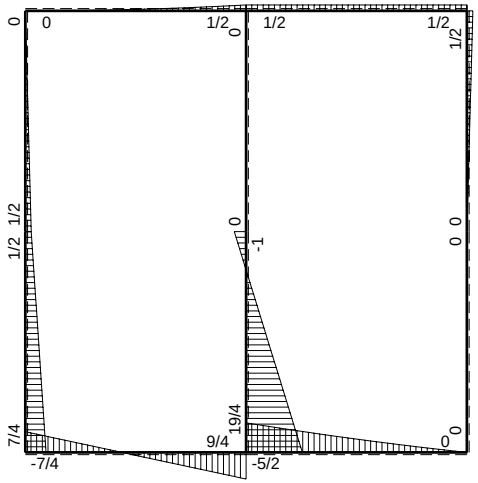




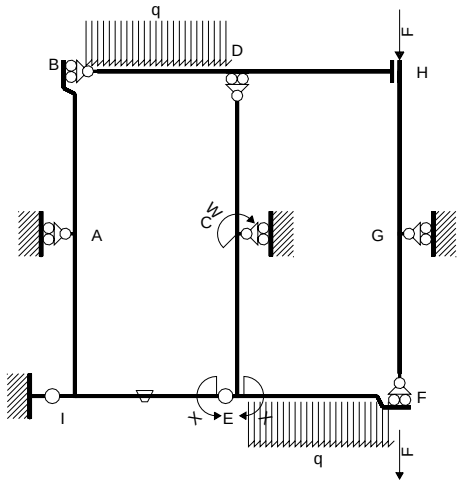
←□→ F



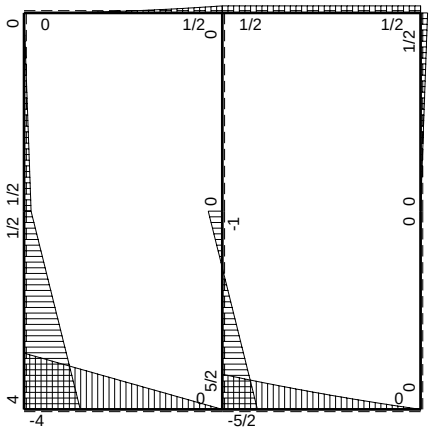
↑□↓ F



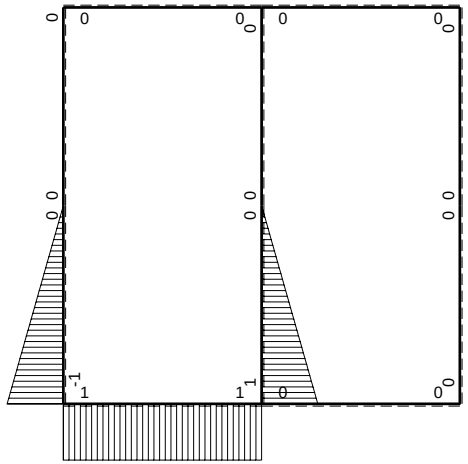
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{EI}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-5/2Fb+3Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$2Fx+1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fb+1/2Fx	0	0	0	0		
HD b	0	1/2Fb	0	0	0	0	0+0	0
DH b	0	-1/2Fb	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	1	-4Fb+4Fx	-Fb/EJ	-4Fb+4Fx	-Fb/EJ	1	$(-2-1)Fb^2/EJ$	Xb/EJ
EI b	-1	4Fx	Fb/EJ	-4Fx	-Fb/EJ	1		
EC b	1-x/b	5/2Fb-7/2Fx	0	$5/2Fb-6Fx+7/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(2/3+0)Fb^2/EJ$	1/3Xb/EJ
CE b	-x/b	Fb-7/2Fx	0	$-Fx+7/2Fx^2/b$	0	x^2/b^2		
IA b	-1+x/b	4Fb-7/2Fx	0	$-4Fb+15/2Fx-7/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-17/12+0)Fb^2/EJ$	1/3Xb/EJ
AI b	x/b	-1/2Fb-7/2Fx	0	$-1/2Fx-7/2Fx^2/b$	0	x^2/b^2		
	totali						$-15/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EI}$						9/4Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 12.12.25

$L^{XX}_{IE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{EI} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CE} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{AI} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{IE} = \int_o^b (-4 + 4\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-4\ x + 2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-4\ b + 2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -3\ Fb^2/EJ$

$L^{Xo}_{EI} = \int_o^b (-4\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3\ Fb^2/EJ$

$L^{Xo}_{EC} = \int_o^b (5/2 - 6\ x/b + 7/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x - 3\ x^2/b + 7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b - 3\ b + 7/6\ b)\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CE} = \int_o^b (-\ x/b + 7/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

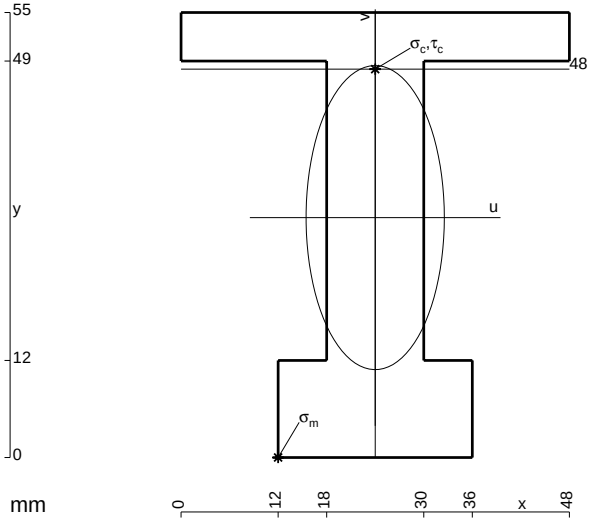
$= (-1/2\ b + 7/6\ b)\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{IA} = \int_o^b (-4 + 15/2\ x/b - 7/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-4\ x + 15/4\ x^2/b - 7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-4\ b + 15/4\ b - 7/6\ b)\ Fb\ 1/EJ\ = -17/12\ Fb^2/EJ$

$L^{Xo}_{AI} = \int_o^b (-1/2\ x/b - 7/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b - 7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b - 7/6\ b)\ Fb\ 1/EJ\ = -17/12\ Fb^2/EJ$



A = 1020. mm^2
J_u = 360241. mm^4
J_v = 74448. mm^4
y_g = 29.65 mm
T_y = 3540. N
M_x = -2419000. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -29.65 mm
$\sigma_m = -Mv/J_u$ = -199.1 N/mm^2
x_c = 24. mm
y_c = 48. mm
v_c = 18.35 mm
$\sigma_c = -Mv/J_u$ = 123.2 N/mm^2
τ_c = 5.456 N/mm^2
$\sigma_0 = \sqrt{\sigma^2 + 3\tau^2}$ = 123.6 N/mm^2
S^* = 6662. mm^3