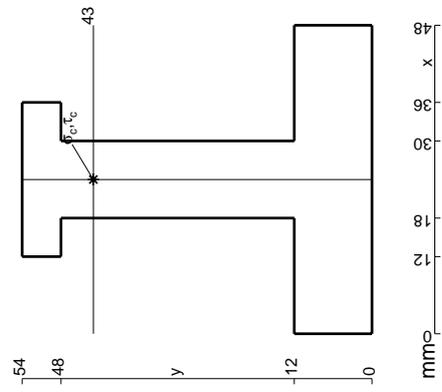
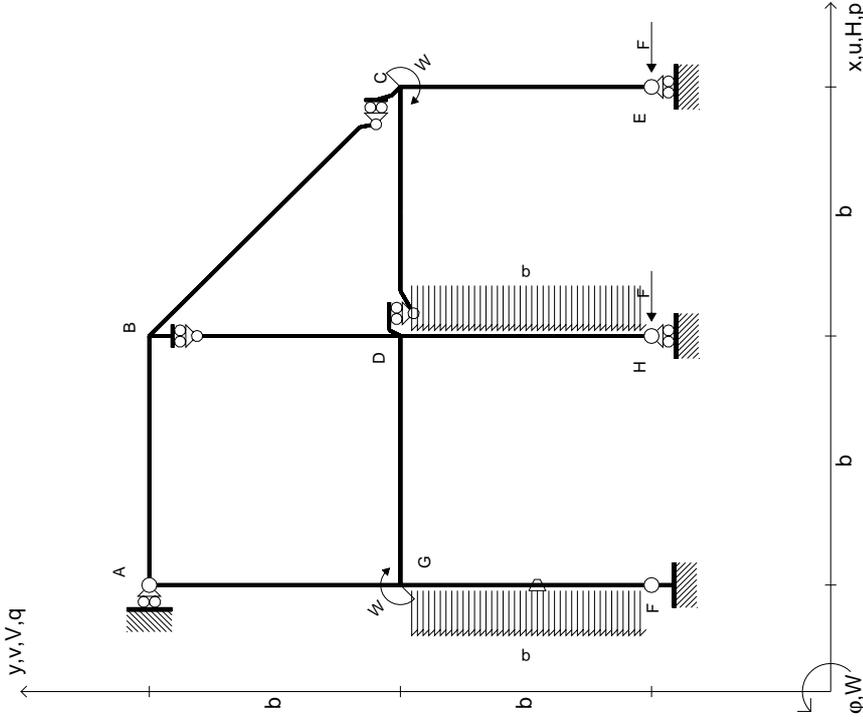
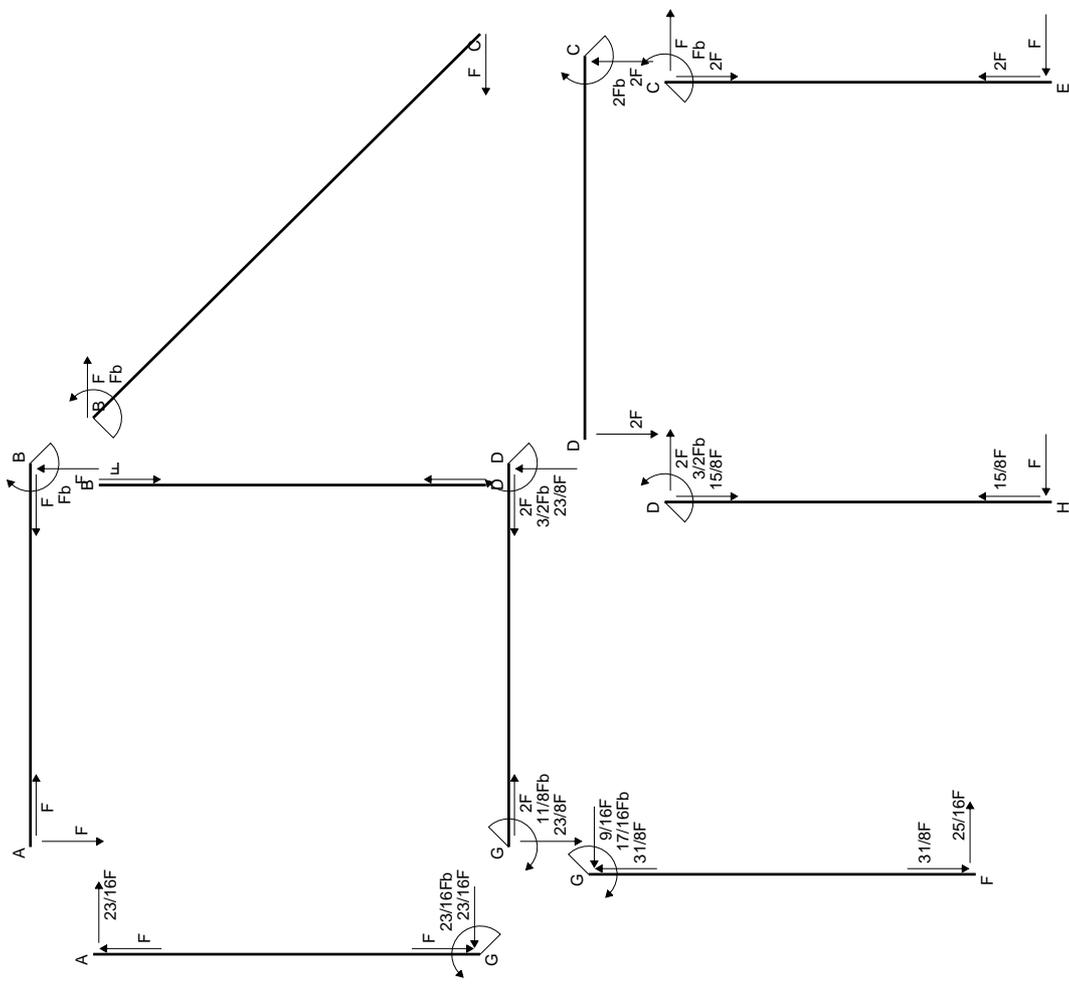


$H_E = -F$   
 $H_H = -F$   
 $W_C = -W = -Fb$   
 $W_G = -W = -Fb$   
 $P_{DH} = -q = -F/b$   
 $P_{FG} = -q = -F/b$   
 $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$   
 $EJ_{AB} = EJ$   
 $EJ_{BC} = EJ$   
 $EJ_{BD} = EJ$   
 $EJ_{DC} = EJ$   
 $EJ_{CE} = EJ$   
 $EJ_{FG} = EJ$   
 $EJ_{GD} = EJ$   
 $EJ_{DH} = EJ$   
 $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione:  $X=V_H$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste misurati in proiezione sugli assi  $x,y$ .  
 $J_{YZ} - X_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 La trave DC ha la sezione riportata e dimensioni in mm, con:  
 $b = 760 \text{ mm}, F = 1370 \text{ N}$   
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su traveggio trave, a destra da D a C  
 Curvatura  $\theta$  asta FG positiva se convessa a destra con inizio F.  
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Quadro contributi PLV per iperstatica  $X=V_H$ 

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	0	-Fx	0	0	0	0	0+0	0	
BA b	0	Fb-Fx	0	0	0	0			
BC $\sqrt{2}b$	0	-Fb+ $\sqrt{2}/2Fx$	0	0	0	0	0	0	
BD b	0	0	0	0	0	0	0+0	0	
DB b	0	0	0	0	0	0			
DC b	0	-2Fx	0	0	0	0	0+0	0	
CD b	0	2Fb-2Fx	0	0	0	0			
CE b	0	-Fb+Fx	0	0	0	0	0+0	0	
EC b	0	Fx	0	0	0	0			
FG b	1/2x	-5/2Fx+1/2qx <sup>2</sup>	-Fb/EJ	-5/4Fx <sup>2</sup> +1/4qx <sup>3</sup>	-1/2Fxb/EJ	1/4x <sup>2</sup>	(-17/48-1/4)Fb <sup>3</sup> /EJ	1/12Xb <sup>3</sup> /EJ	
GF b	-1/2b+1/2x	2Fb-3/2Fx-1/2qx <sup>2</sup>	Fb/EJ	-Fb <sup>2</sup> +7/4Fbx-1/2Fx <sup>2</sup> -1/4qx <sup>3</sup>	-1/2Fb <sup>2</sup> /EJ+1/2Fxb/EJ	1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup>			
GD b	b-x	-1/2Fb-Fx	0	-1/2Fb <sup>2</sup> -1/2Fbx+Fx <sup>2</sup>	0	b <sup>2</sup> -2bx+x <sup>2</sup>	(-5/12+0)Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ	
DG b	-x	3/2Fb-Fx	0	-3/2Fbx+Fx <sup>2</sup>	0	x <sup>2</sup>			
DH b	0	-3/2Fb+2Fx-1/2qx <sup>2</sup>	0	0	0	0	0+0	0	
HD b	0	Fx+1/2qx <sup>2</sup>	0	0	0	0			
GA b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb <sup>2</sup> -1/2Fbx+1/4Fx <sup>2</sup>	0	1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup>	(1/12+0)Fb <sup>3</sup> /EJ	1/12Xb <sup>3</sup> /EJ	
AG b	1/2x	1/2Fx	0	1/4Fx <sup>2</sup>	0	1/4x <sup>2</sup>			
	totali							-15/16Fb <sup>3</sup> /EJ	1/2Xb <sup>3</sup> /EJ
	iperstatica $X=V_H$							15/8F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-5/4 x^2/b^2 + 1/4 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx$$

$$= [-5/12 x^3/b^2 + 1/16 x^4/b^3]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (-5/12 b + 1/16 b) Fb^2 1/EJ + (-1/4 b) \theta = -29/48 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-1 + 7/4 x/b - 1/2 x^2/b^2 - 1/4 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [-x + 7/8 x^2/b - 1/6 x^3/b^2 - 1/16 x^4/b^3]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (-b + 7/8 b - 1/6 b - 1/16 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = -29/48 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-1/2 - 1/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [-1/2 x - 1/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b - 1/4 b + 1/3 b) Fb^2 1/EJ = -5/12 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-3/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [-3/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

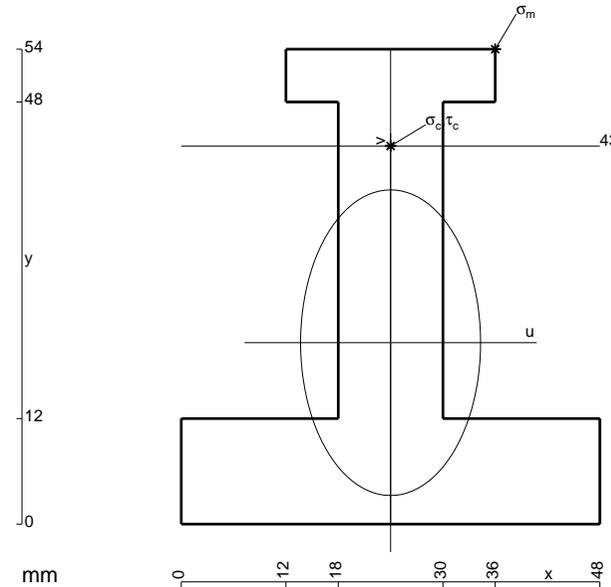
$$= (-3/4 b + 1/3 b) Fb^2 1/EJ = -5/12 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

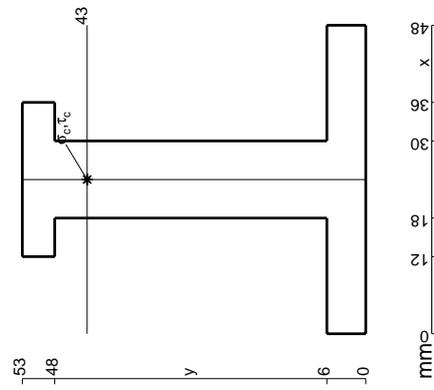
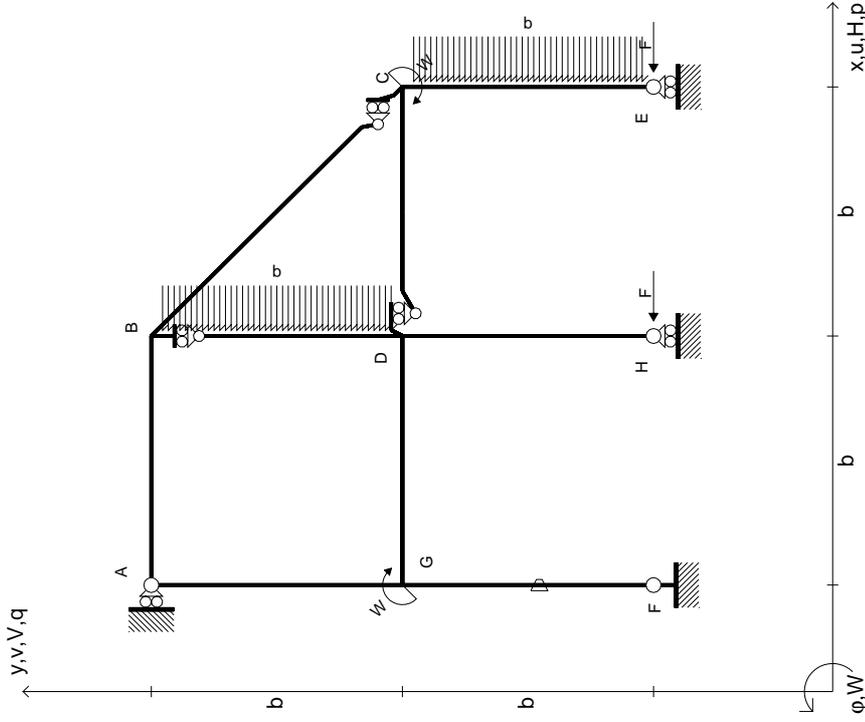
$$L_{AG}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

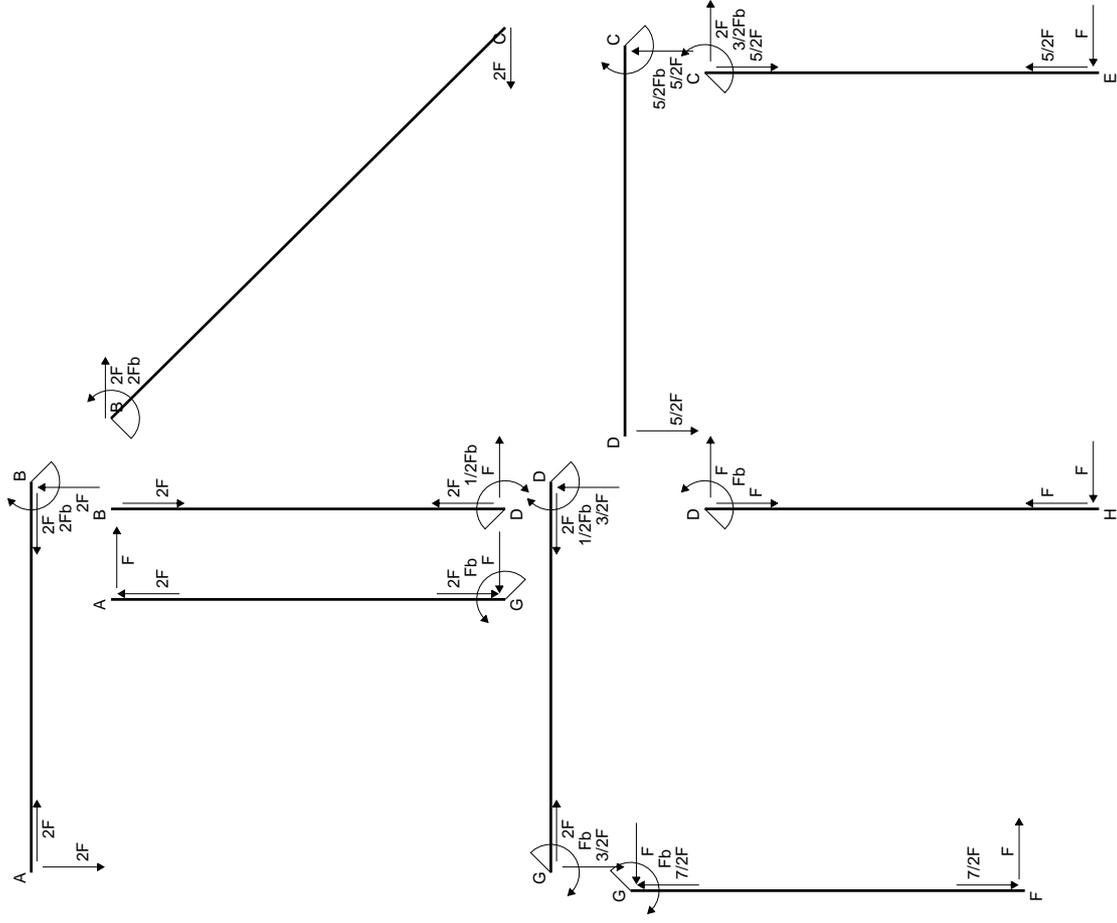


- A = 1152. mm<sup>2</sup>
- J<sub>u</sub> = 348030. mm<sup>4</sup>
- J<sub>v</sub> = 122688. mm<sup>4</sup>
- y<sub>g</sub> = 20.63 mm
- T<sub>y</sub> = -2740. N
- M<sub>x</sub> = -2082400. Nmm
- x<sub>m</sub> = 36. mm
- y<sub>m</sub> = 54. mm
- u<sub>m</sub> = 12. mm
- v<sub>m</sub> = 33.38 mm
- σ<sub>m</sub> = -Mv/J<sub>u</sub> = 199.7 N/mm<sup>2</sup>
- x<sub>c</sub> = 24. mm
- y<sub>c</sub> = 43. mm
- v<sub>c</sub> = 22.38 mm
- σ<sub>c</sub> = -Mv/J<sub>u</sub> = 133.9 N/mm<sup>2</sup>
- τ<sub>c</sub> = 3.849 N/mm<sup>2</sup>
- σ<sub>o</sub> = √σ<sup>2</sup>+3τ<sup>2</sup> = 134. N/mm<sup>2</sup>
- S = 5867. mm<sup>3</sup>

$H_E = -F$   
 $H_H = -F$   
 $W_C = -W = -Fb$   
 $W_G = -W = -Fb$   
 $P_{CE} = -q = -F/b$   
 $P_{BD} = -q = -F/b$   
 $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$   
 $EJ_{AB} = EJ$   
 $EJ_{BC} = EJ$   
 $EJ_{BD} = EJ$   
 $EJ_{DC} = EJ$   
 $EJ_{CE} = EJ$   
 $EJ_{FG} = EJ$   
 $EJ_{GD} = EJ$   
 $EJ_{DH} = EJ$   
 $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione:  $X=V_H$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi  $x,y$ .  
 $J_{yz} = x_{yz} \cdot \theta_{yz}$  riferimento locale asta YZ con origine in Y.  
 La trave DC ha la sezione riportata e dimensioni in mm, con:  
 $b = 850$  mm,  $F = 980$  N  
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da D a C  
 Curvatura  $\theta$  asta FG positiva se convessa a destra con inizio F.  
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Quadro contributi PLV per iperstatica  $X=V_H$ 

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0		
BC $\sqrt{2}b$	0	-2Fb+ $\sqrt{2}Fx$	0	0	0	0	0	0
BD b	0	-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
DB b	0	1/2Fb-Fx+1/2qx <sup>2</sup>	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	-3/2Fb+2Fx-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
EC b	0	Fx+1/2qx <sup>2</sup>	0	0	0	0		
FG b	1/2x	-3/2Fx	-Fb/EJ	-3/4Fx <sup>2</sup>	-1/2Fxb/EJ	1/4x <sup>2</sup>	(-1/4-1/4)Fb <sup>3</sup> /EJ	1/12Xb <sup>3</sup> /EJ
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	-3/4Fb <sup>2</sup> +3/2Fbx-3/4Fx <sup>2</sup>	-1/2Fb <sup>2</sup> /EJ+1/2Fxb/EJ	1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup>		
GD b	b-x	-1/2Fx	0	-1/2Fbx+1/2Fx <sup>2</sup>	0	b <sup>2</sup> -2bx+x <sup>2</sup>	(-1/12+0)Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ
DG b	-x	1/2Fb-1/2Fx	0	-1/2Fbx+1/2Fx <sup>2</sup>	0	x <sup>2</sup>		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb <sup>2</sup> -1/2Fbx+1/4Fx <sup>2</sup>	0	1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup>	(1/12+0)Fb <sup>3</sup> /EJ	1/12Xb <sup>3</sup> /EJ
AG b	1/2x	1/2Fx	0	1/4Fx <sup>2</sup>	0	1/4x <sup>2</sup>		
	totali						-1/2Fb <sup>3</sup> /EJ	1/2Xb <sup>3</sup> /EJ
	iperstatica $X=V_H$						F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [-1/4 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (-1/4 b) Fb^2 1/EJ + (-1/4 b) \theta = -1/2 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-3/4 + 3/2 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [-3/4 x + 3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (-3/4 b + 3/4 b - 1/4 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = -1/2 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-1/2 x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/4 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/4 b + 1/6 b) Fb^2 1/EJ = -1/12 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-1/2 x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/4 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

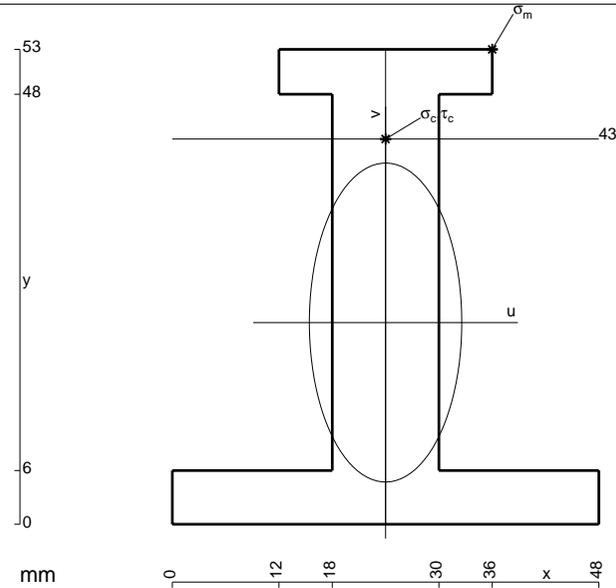
$$= (-1/4 b + 1/6 b) Fb^2 1/EJ = -1/12 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

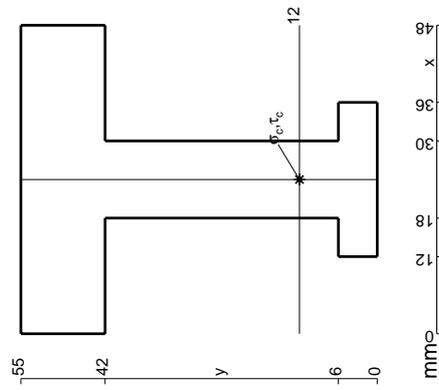
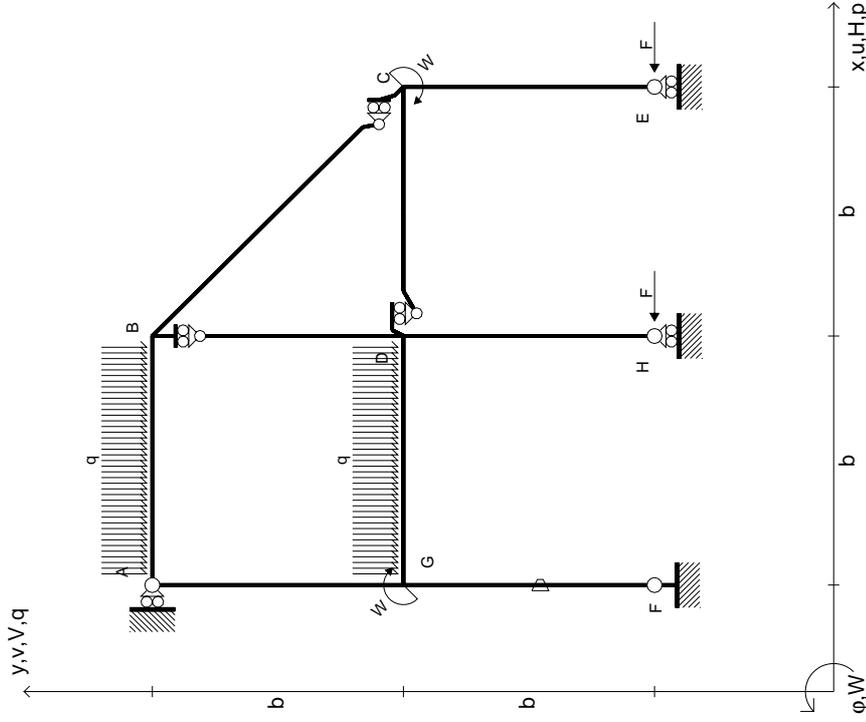
$$L_{AG}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

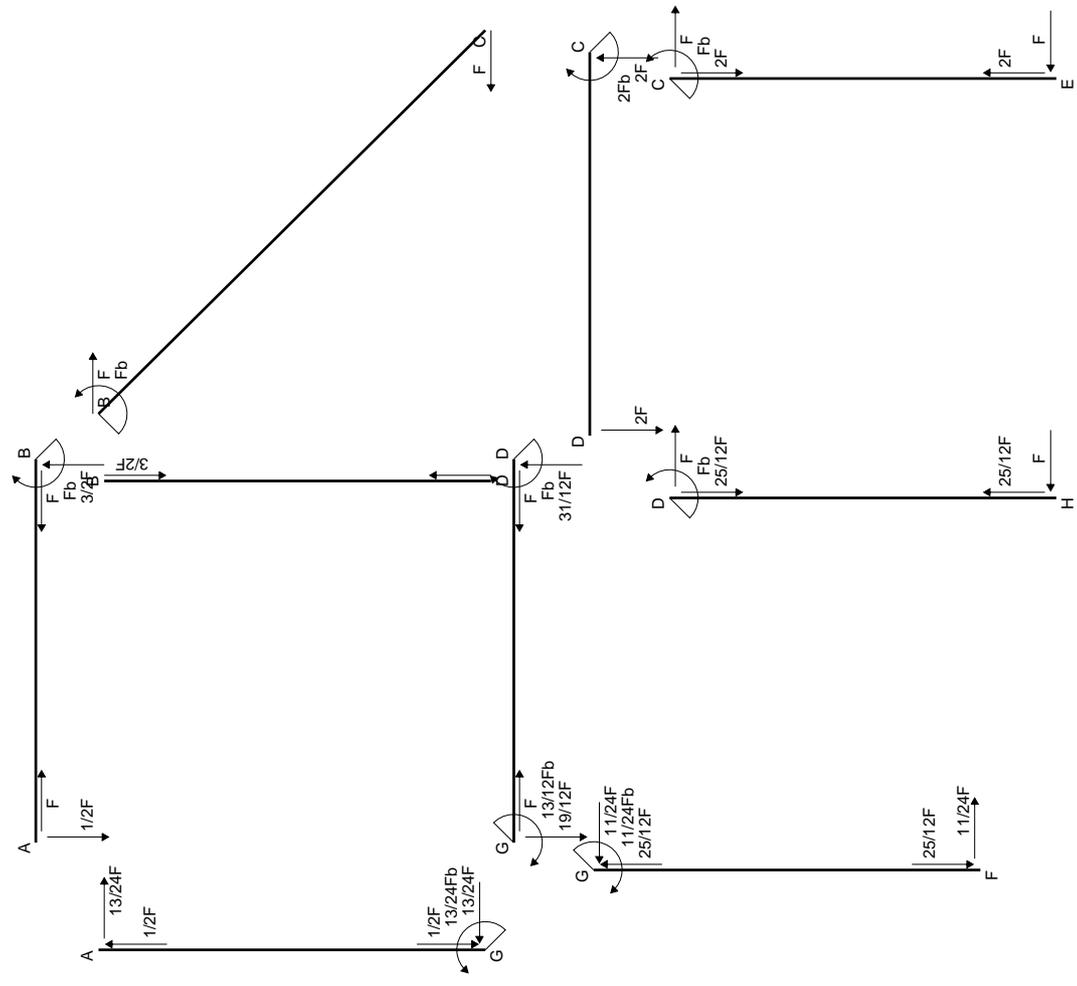


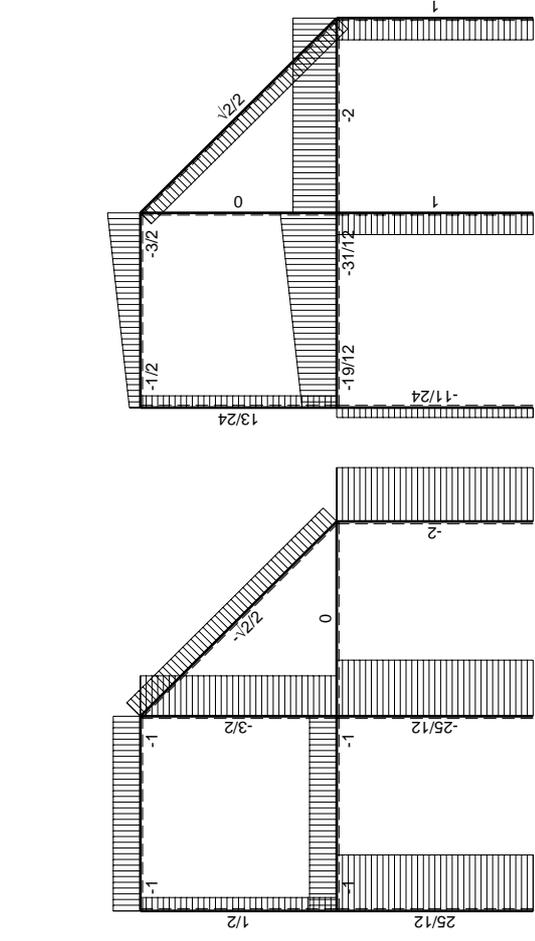
- A = 912. mm<sup>2</sup>
- J<sub>u</sub> = 289000. mm<sup>4</sup>
- J<sub>v</sub> = 67104. mm<sup>4</sup>
- y<sub>g</sub> = 22.51 mm
- T<sub>y</sub> = -2450. N
- M<sub>x</sub> = -2082500. Nmm
- x<sub>m</sub> = 36. mm
- y<sub>m</sub> = 53. mm
- u<sub>m</sub> = 12. mm
- v<sub>m</sub> = 30.49 mm
- σ<sub>m</sub> = -Mv/J<sub>u</sub> = 219.7 N/mm<sup>2</sup>
- x<sub>c</sub> = 24. mm
- y<sub>c</sub> = 43. mm
- v<sub>c</sub> = 20.49 mm
- σ<sub>c</sub> = -Mv/J<sub>u</sub> = 147.6 N/mm<sup>2</sup>
- τ<sub>c</sub> = 3.347 N/mm<sup>2</sup>
- σ<sub>q</sub> = √σ<sup>2</sup>+3τ<sup>2</sup> = 147.7 N/mm<sup>2</sup>
- S = 4738. mm<sup>3</sup>

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_G = -W = -Fb$
- $q_{AB} = -q = -F/b$
- $q_{GD} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



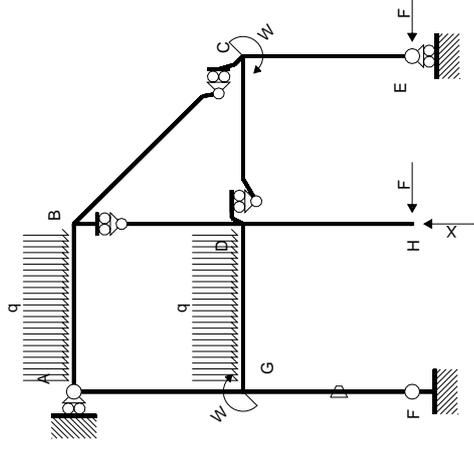
Reazioni iperstatiche in soluzione:  $X=V_H$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi  $x,y$ .  
 $J_{yz} - x_{yz} - \theta_{yz}$  riferimento locale asta YZ con origine in Y.  
 La trave DC ha la sezione riportata e dimensioni in mm, con:  
 $b = 830 \text{ mm}, F = 1360 \text{ N}$   
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da D a C  
 Curvatura  $\theta$  asta FG positiva se convessa a destra con inizio F.  
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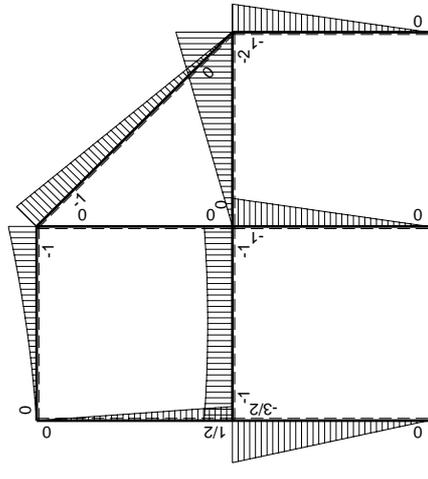


$\left[ \begin{array}{c} \square \\ \square \end{array} \right] \rightarrow F$

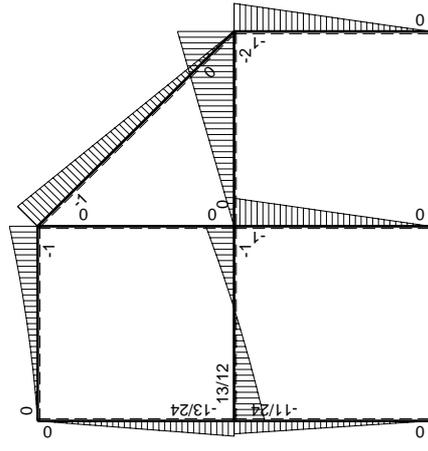
$\left[ \begin{array}{c} \square \\ \square \end{array} \right] \uparrow F$



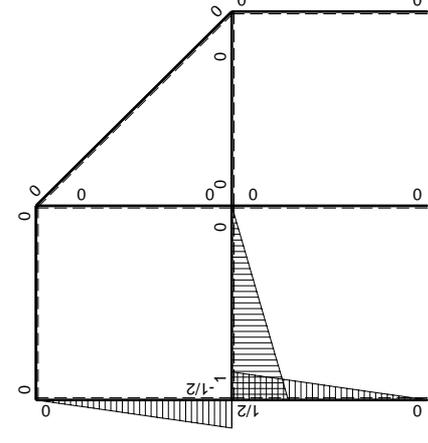
Schema di calcolo iperstatico



$\left[ \begin{array}{c} \square \\ \square \end{array} \right] M_0$  flessione da carichi assegnati



$\left[ \begin{array}{c} \square \\ \square \end{array} \right] F_b$



$\left[ \begin{array}{c} \square \\ \square \end{array} \right] M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=V_H$

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{2}b$	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	$-2Fx$	0	0	0	0	0+0	0
CD b	0	$2Fb-2Fx$	0	0	0	0		
CE b	0	$-Fb+Fx$	0	0	0	0	0+0	0
EC b	0	$Fx$	0	0	0	0		
FG b	$1/2x$	$-3/2Fx$	$-Fb/EJ$	$-3/4Fx^2$	$-1/2Fxb/EJ$	$1/4x^2$	$(-1/4-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	$-1/2b+1/2x$	$3/2Fb-3/2Fx$	$Fb/EJ$	$-3/4Fb^2+3/2Fbx-3/4Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	$b-x$	$-Fb+1/2Fx-1/2qx^2$	0	$-Fb^2+3/2Fbx-Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(-11/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	$-x$	$Fb-1/2Fx+1/2qx^2$	0	$-Fbx+1/2Fx^2-1/2qx^3$	0	$x^2$		
DH b	0	$-Fb+Fx$	0	0	0	0	0+0	0
HD b	0	$Fx$	0	0	0	0		
GA b	$-1/2b+1/2x$	$1/2Fb-1/2Fx$	0	$-1/4Fb^2+1/2Fbx-1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	$1/2x$	$-1/2Fx$	0	$-1/4Fx^2$	0	$1/4x^2$		
	totali						$-25/24Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						$25/12F$	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [-1/4 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (-1/4 b) Fb^2 1/EJ + (-1/4 b) \theta = -1/2 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-3/4 + 3/2 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [-3/4 x + 3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (-3/4 b + 3/4 b - 1/4 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = -1/2 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-1 + 3/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [-x + 3/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-b + 3/4 b - 1/3 b + 1/8 b) Fb^2 1/EJ = -11/24 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-x/b + 1/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [-1/2 x^2/b + 1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

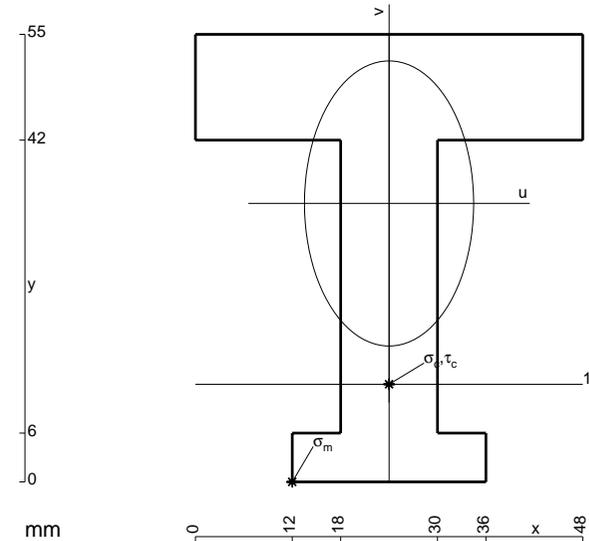
$$= (-1/2 b + 1/6 b - 1/8 b) Fb^2 1/EJ = -11/24 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (-1/4 + 1/2 x/b - 1/4 x^2/b^2) Fb^2 1/EJ dx = [-1/4 x + 1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/4 b + 1/4 b - 1/12 b) Fb^2 1/EJ = -1/12 Fb^3/EJ$$

$$L_{AG}^{xo} = \int_0^b (-1/4 x^2/b^2) Fb^2 1/EJ dx = [-1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/12 b) Fb^2 1/EJ = -1/12 Fb^3/EJ$$



$$A = 1200. \text{ mm}^2$$

$$J_u = 368598. \text{ mm}^4$$

$$J_v = 131904. \text{ mm}^4$$

$$y_g = 34.22 \text{ mm}$$

$$T_y = -2720. \text{ N}$$

$$M_x = -2257600. \text{ Nmm}$$

$$x_m = 12. \text{ mm}$$

$$u_m = -12. \text{ mm}$$

$$v_m = -34.22 \text{ mm}$$

$$\sigma_m = -Mv/J_u = -209.6 \text{ N/mm}^2$$

$$x_c = 24. \text{ mm}$$

$$y_c = 12. \text{ mm}$$

$$v_c = -22.22 \text{ mm}$$

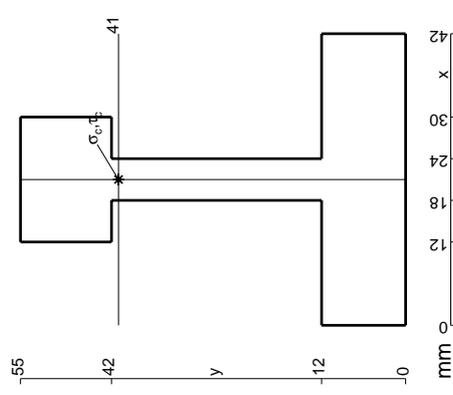
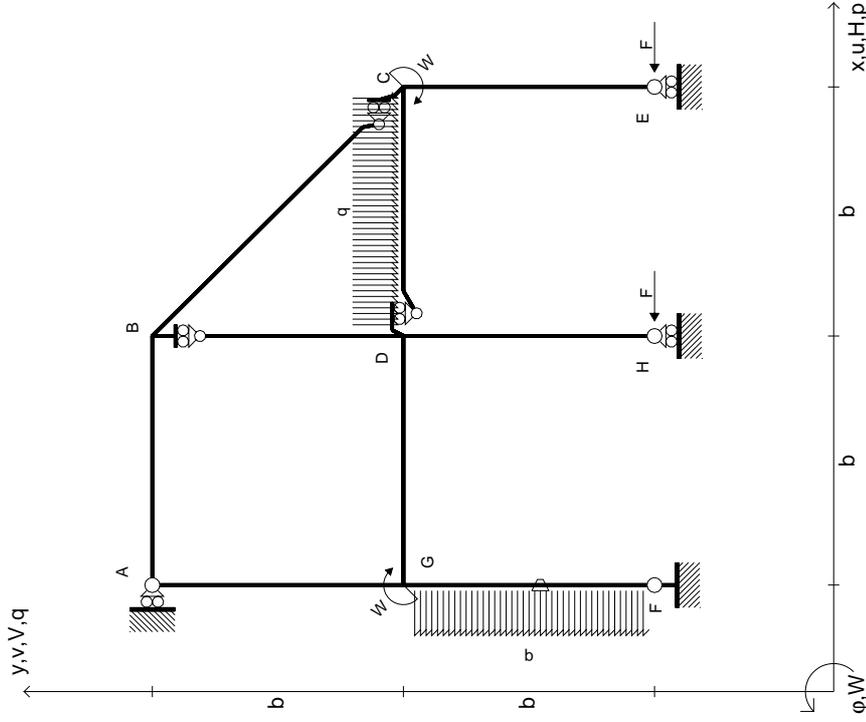
$$\sigma_c = -Mv/J_u = -136.1 \text{ N/mm}^2$$

$$\tau_c = 3.881 \text{ N/mm}^2$$

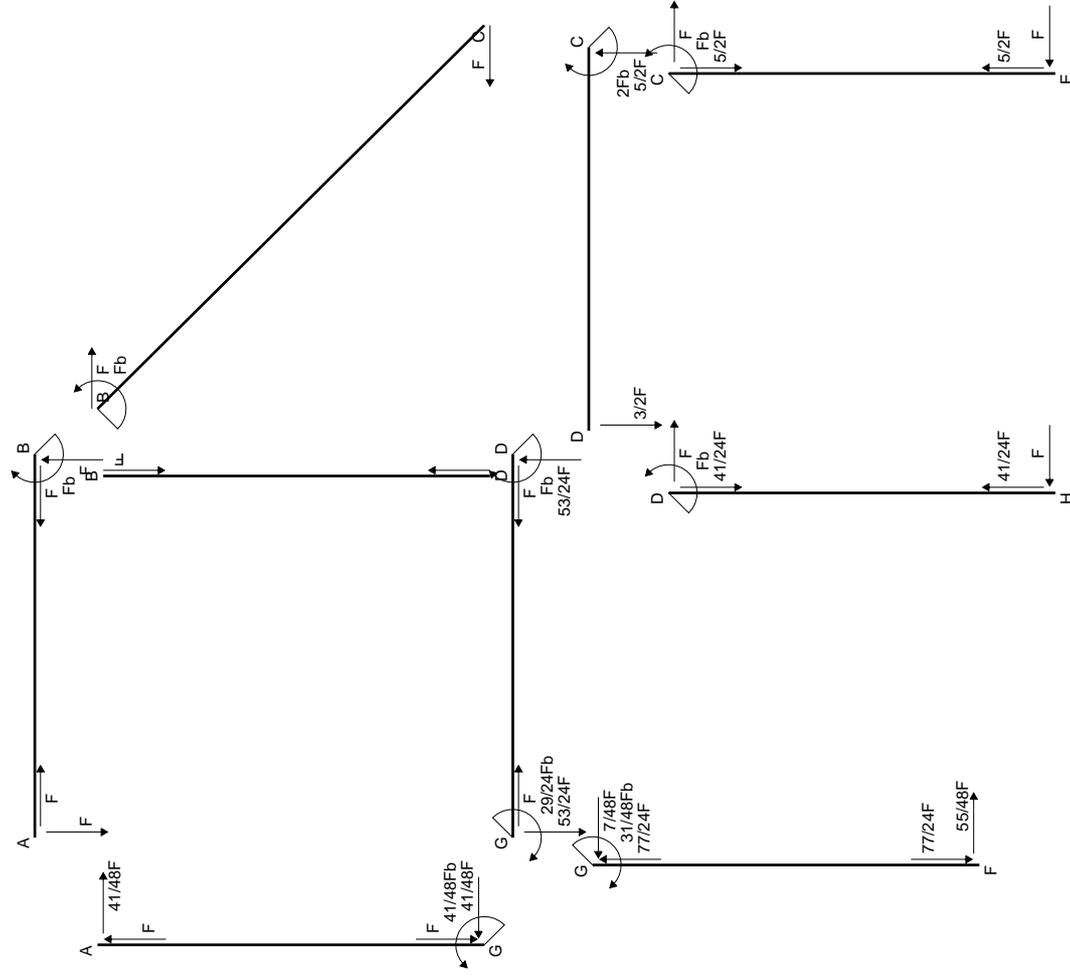
$$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 136.3 \text{ N/mm}^2$$

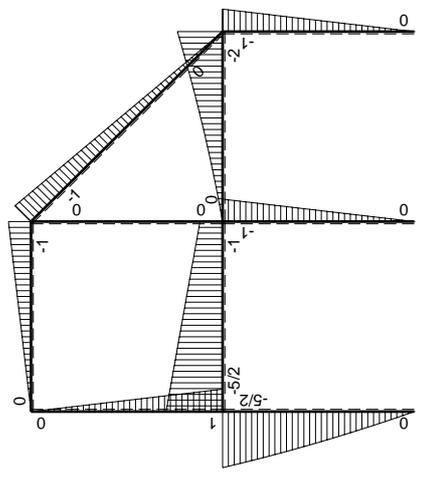
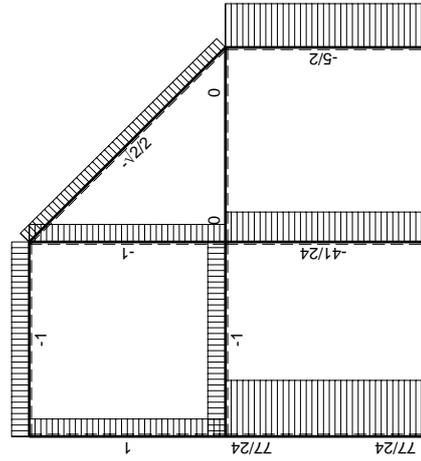
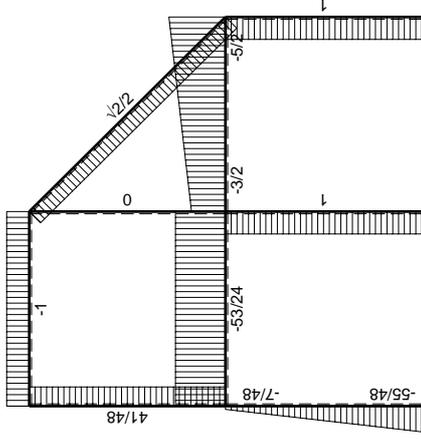
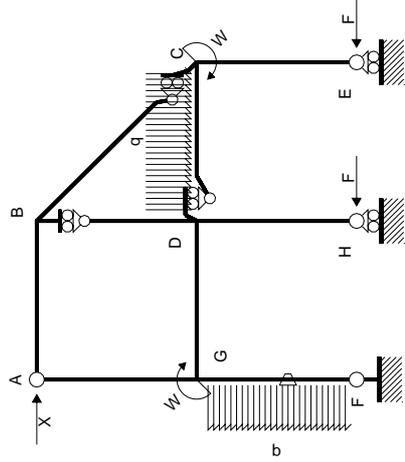
$$S = 6312. \text{ mm}^3$$

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_G = -W = -Fb$
- $q_{bc} = -q = -F/b$
- $p_{FG} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curvati in proiezione sugli assi x,y.  
 $J_{YZ} - X_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 La trave DC ha la sezione riportata e dimensioni in mm, con:  
 $b = 720 \text{ mm}, F = 1560 \text{ N}$   
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da D a C  
 Curvatura  $\theta$  asta FG positiva se convessa a destra con inizio F.  
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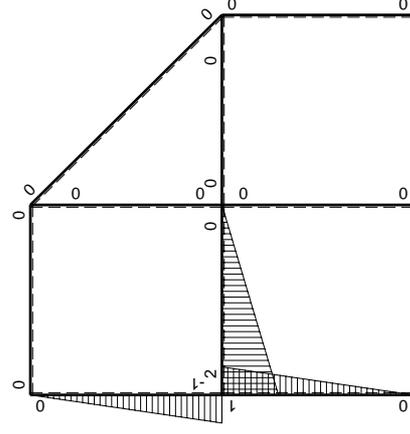
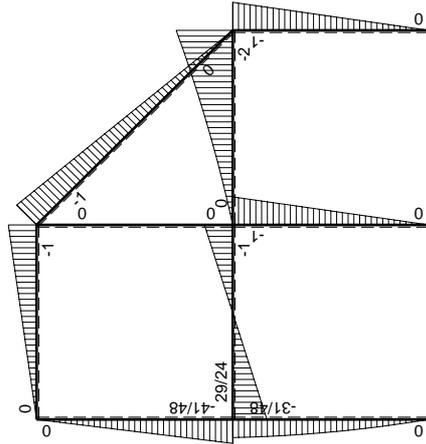


$\left[ \begin{array}{c} + \\ - \end{array} \right] \rightarrow F$

$\left[ \begin{array}{c} + \\ - \end{array} \right] \uparrow F$

Schema di calcolo iperstatico

$\left[ \begin{array}{c} + \\ - \end{array} \right] \curvearrowright M_0$  flessione da carichi assegnati



$\left[ \begin{array}{c} + \\ - \end{array} \right] F_b$

$\left[ \begin{array}{c} + \\ - \end{array} \right] M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	0	-Fx	0	0	0	0	0+0	0
BA b	0	Fb-Fx	0	0	0	0	0	0
BC $\sqrt{2}b$	0	-Fb+ $\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0	0	0
DC b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
CD b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0	0
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0	0	0
FG b	x	$-3Fx+1/2qx^2$	-Fb/EJ	$-3Fx^2+1/2qx^3$	-Fxb/EJ	$x^2$	$(-7/8-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	-b+x	$5/2Fb-2Fx-1/2qx^2$	Fb/EJ	$-5/2Fb^2+9/2Fbx-3/2Fx^2-1/2qx^3$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$		
GD b	2b-2x	$-5/2Fb+3/2Fx$	0	$-5Fb^2+8Fbx-3Fx^2$	0	$4b^2-8bx+4x^2$	$(-2+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	-2x	$Fb+3/2Fx$	0	$-2Fbx-3Fx^2$	0	$4x^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-b+x	Fb-Fx	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	-Fx	0	$-Fx^2$	0	$x^2$		
	totali						$-89/24Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_A$						89/48F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-3x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx$$

$$= [-x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [-1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/8 b) Fb^2 1/EJ + (-1/2 b) \theta = -11/8 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-5/2 + 9/2 x/b - 3/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

$$= [-5/2 x + 9/4 x^2/b - 1/2 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [x - 1/2 x^2/b]_0^b \theta$$

$$= (-5/2 b + 9/4 b - 1/2 b - 1/8 b) Fb^2 1/EJ + (b - 1/2 b) \theta = -11/8 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-5 + 8x/b - 3x^2/b^2) Fb^2 1/EJ dx = [-5x + 4x^2/b - x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-5b + 4b - b) Fb^2 1/EJ = -2 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-2x/b - 3x^2/b^2) Fb^2 1/EJ dx = [-x^2/b - x^3/b^2]_0^b Fb^2 1/EJ$$

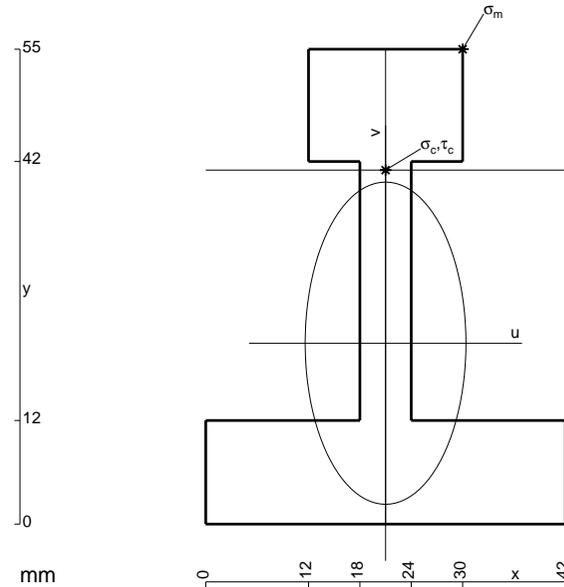
$$= (-b - b) Fb^2 1/EJ = -2 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (-1 + 2x/b - x^2/b^2) Fb^2 1/EJ dx = [-x + x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-b + b - 1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

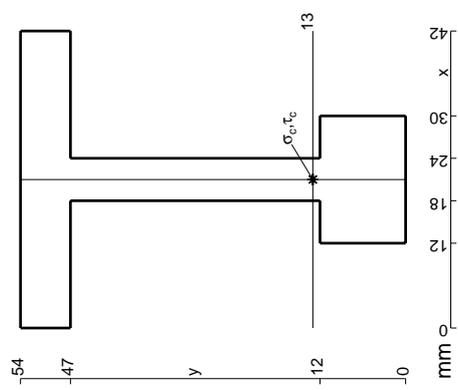
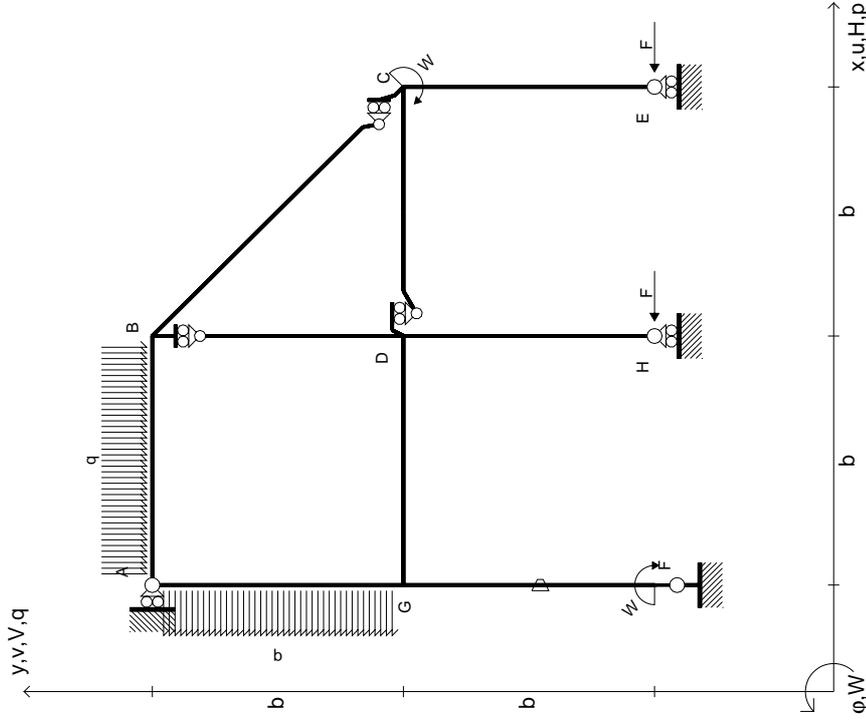
$$L_{AG}^{xo} = \int_0^b (-x^2/b^2) Fb^2 1/EJ dx = [-1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

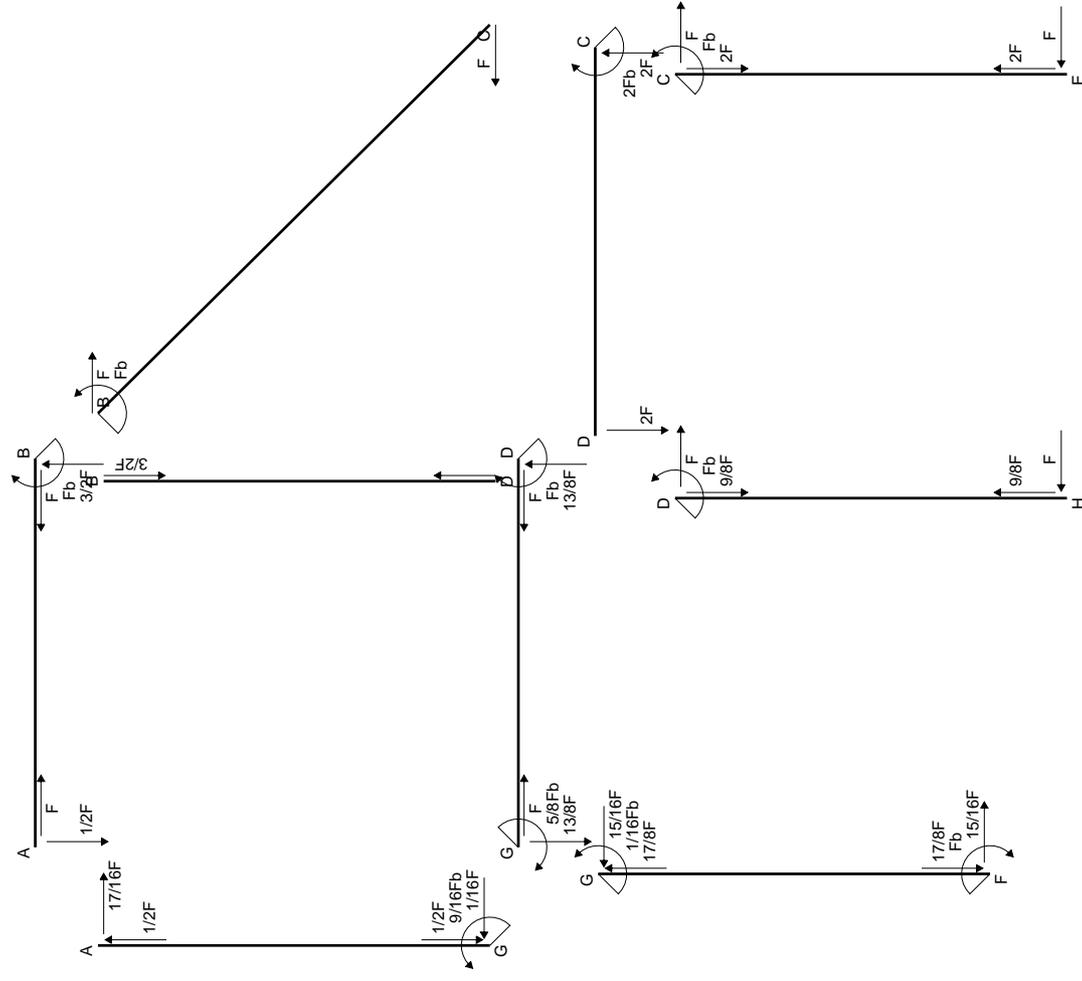


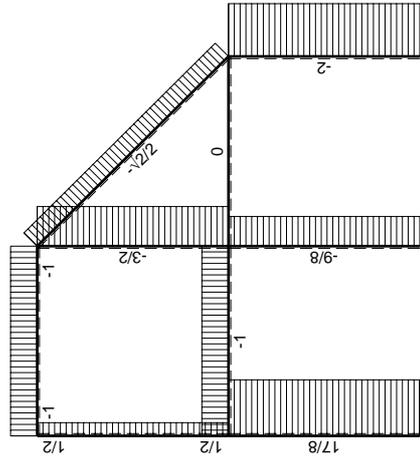
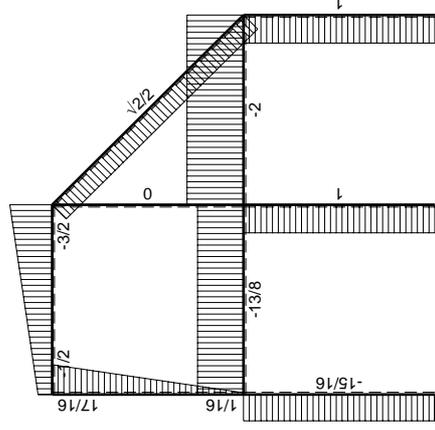
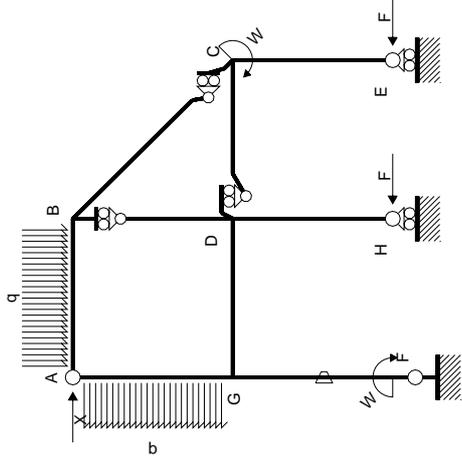
- A = 918. mm<sup>2</sup>
- J<sub>u</sub> = 319684. mm<sup>4</sup>
- J<sub>v</sub> = 80946. mm<sup>4</sup>
- y<sub>g</sub> = 20.95 mm
- T<sub>y</sub> = -3900. N
- M<sub>x</sub> = -2246400. Nmm
- x<sub>m</sub> = 30. mm
- y<sub>m</sub> = 55. mm
- u<sub>m</sub> = 9. mm
- v<sub>m</sub> = 34.05 mm
- σ<sub>m</sub> = -Mv/J<sub>u</sub> = 239.3 N/mm<sup>2</sup>
- x<sub>c</sub> = 21. mm
- y<sub>c</sub> = 41. mm
- v<sub>c</sub> = 20.05 mm
- σ<sub>c</sub> = -Mv/J<sub>v</sub> = 140.9 N/mm<sup>2</sup>
- τ<sub>c</sub> = 13.36 N/mm<sup>2</sup>
- σ<sub>o</sub> = √σ<sup>2</sup>+3τ<sup>2</sup> = 142.8 N/mm<sup>2</sup>
- S = 6570. mm<sup>3</sup>

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_F = -W = -Fb$
- $q_{AB} = -q = -F/b$
- $p_{EA} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi  $x, y$ .  
 $J_{YZ} - X_{YZ} - \theta_{YZ}$  riferimento locale asta  $YZ$  con origine in  $Y$ .  
 La trave  $DC$  ha la sezione riportata e dimensioni in mm, con:  
 $b = 660$  mm,  $F = 1530$  N  
 Calcolare sulla sezione  $C$  la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da  $D$  a  $C$   
 Curvatura  $\theta$  asta  $FG$  positiva se convessa a destra con inizio  $F$ .  
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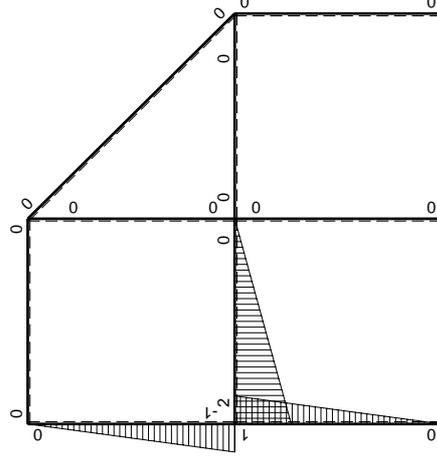




$\left[ \begin{array}{c} + \\ - \end{array} \right] \rightarrow F$

$\left[ \begin{array}{c} + \\ - \end{array} \right] \rightarrow F$

Schema di calcolo iperstatico



$\left[ \begin{array}{c} + \\ - \end{array} \right] F_b$

$\left[ \begin{array}{c} + \\ - \end{array} \right] M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=H_A$ 

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{2}b$	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	$-2Fx$	0	0	0	0	0+0	0
CD b	0	$2Fb-2Fx$	0	0	0	0		
CE b	0	$-Fb+Fx$	0	0	0	0	0+0	0
EC b	0	$Fx$	0	0	0	0		
FG b	x	$Fb-3Fx$	$-Fb/EJ$	$Fbx-3Fx^2$	$-Fxb/EJ$	$x^2$	$(-1/2-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	$-b+x$	$2Fb-3Fx$	$Fb/EJ$	$-2Fb^2+5Fbx-3Fx^2$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$		
GD b	$2b-2x$	$-7/2Fb+5/2Fx$	0	$-7Fb^2+12Fbx-5Fx^2$	0	$4b^2-8bx+4x^2$	$(-8/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	$-2x$	$Fb+5/2Fx$	0	$-2Fbx-5Fx^2$	0	$4x^2$		
DH b	0	$-Fb+Fx$	0	0	0	0	0+0	0
HD b	0	$Fx$	0	0	0	0		
GA b	$-b+x$	$3/2Fb-2Fx+1/2qx^2$	0	$-3/2Fb^2+7/2Fbx-5/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(-11/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	$-Fx-1/2qx^2$	0	$-Fx^2-1/2qx^3$	0	$x^2$		
	totali						$-33/8Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_A$						$33/16F$	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (x/b - 3x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx$$

$$= [1/2 x^2/b - x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x^2/b]_0^b \theta$$

$$= (1/2 b - b) Fb^2 1/EJ + (-1/2 b) \theta = - Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-2 + 5x/b - 3x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

$$= [-2x + 5/2 x^2/b - x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/2 x^2/b]_0^b \theta$$

$$= (-2b + 5/2 b - b) Fb^2 1/EJ + (b - 1/2 b) \theta = - Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-7 + 12x/b - 5x^2/b^2) Fb^2 1/EJ dx = [-7x + 6x^2/b - 5/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-7b + 6b - 5/3 b) Fb^2 1/EJ = -8/3 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-2x/b - 5x^2/b^2) Fb^2 1/EJ dx = [-x^2/b - 5/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-b - 5/3 b) Fb^2 1/EJ = -8/3 Fb^3/EJ$$

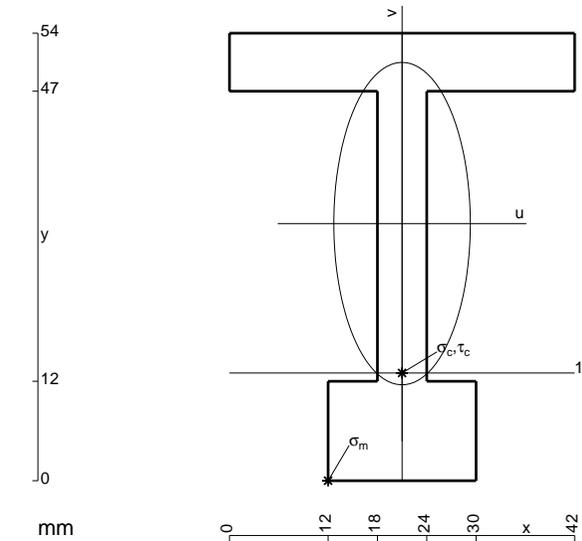
$$L_{GA}^{xo} = \int_0^b (-3/2 + 7/2 x/b - 5/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [-3/2 x + 7/4 x^2/b - 5/6 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-3/2 b + 7/4 b - 5/6 b + 1/8 b) Fb^2 1/EJ = -11/24 Fb^3/EJ$$

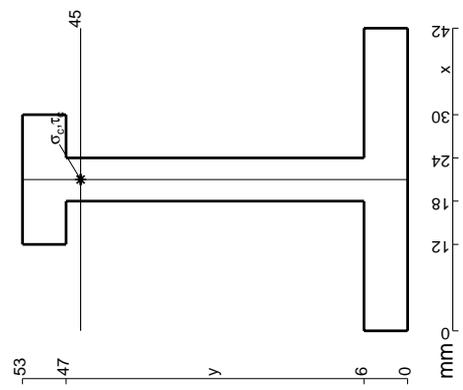
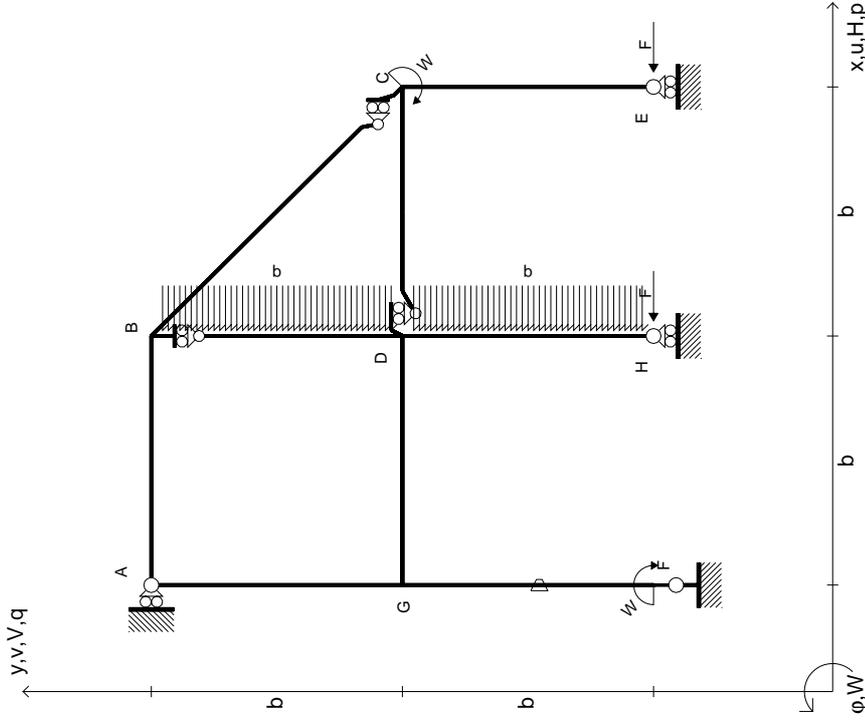
$$L_{AG}^{xo} = \int_0^b (-x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [-1/3 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/3 b - 1/8 b) Fb^2 1/EJ = -11/24 Fb^3/EJ$$

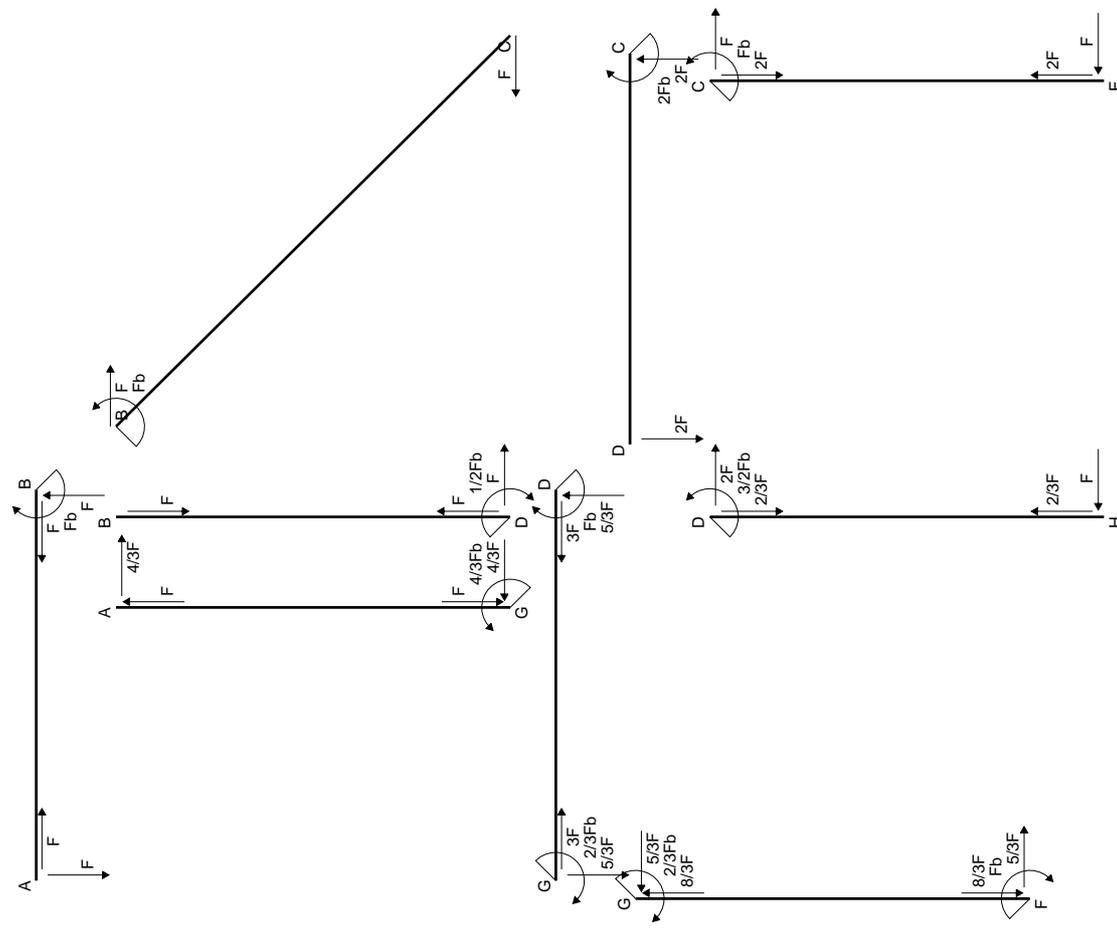


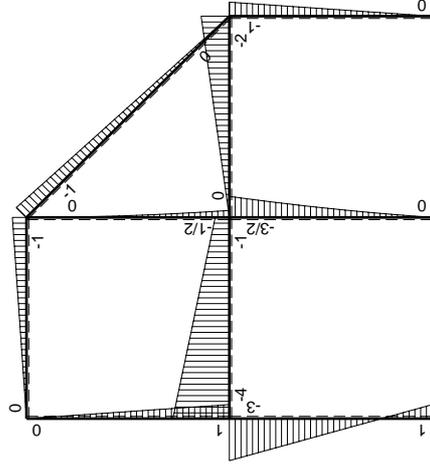
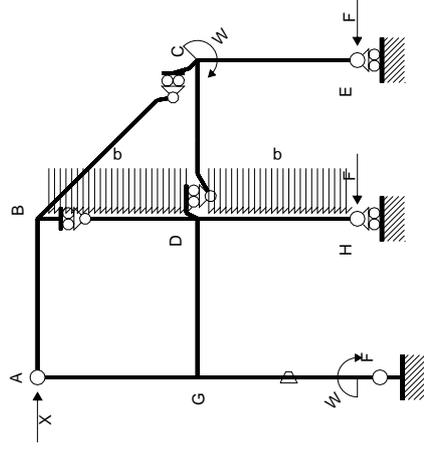
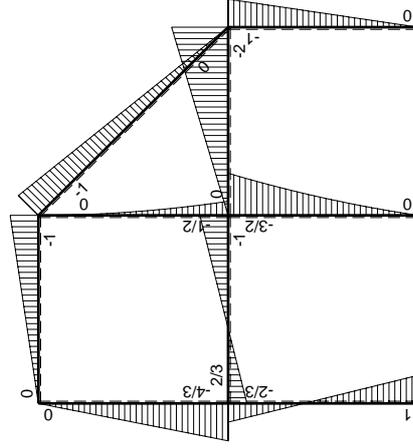
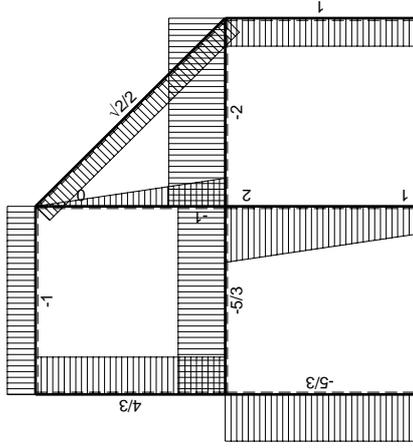
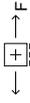
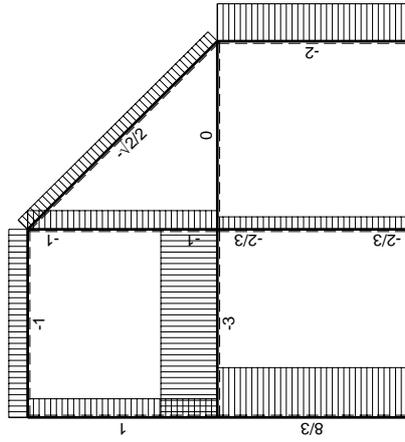
- A = 720. mm<sup>2</sup>
- J<sub>u</sub> = 272496. mm<sup>4</sup>
- J<sub>v</sub> = 49680. mm<sup>4</sup>
- y<sub>g</sub> = 31.02 mm
- T<sub>y</sub> = -3060. N
- M<sub>x</sub> = -2019600. Nmm
- x<sub>m</sub> = 12. mm
- u<sub>m</sub> = -9. mm
- v<sub>m</sub> = -31.02 mm
- σ<sub>m</sub> = -Mv/J<sub>u</sub> = -229.9 N/mm<sup>2</sup>
- x<sub>c</sub> = 21. mm
- y<sub>c</sub> = 13. mm
- v<sub>c</sub> = -18.02 mm
- σ<sub>c</sub> = -Mv/J<sub>v</sub> = -133.6 N/mm<sup>2</sup>
- τ<sub>c</sub> = 10.32 N/mm<sup>2</sup>
- σ<sub>q</sub> = √σ<sup>2</sup>+3τ<sup>2</sup> = 134.8 N/mm<sup>2</sup>
- S = 5517. mm<sup>3</sup>

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_F = -W = -Fb$
- $P_{BD} = -q = -F/b$
- $P_{DH} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



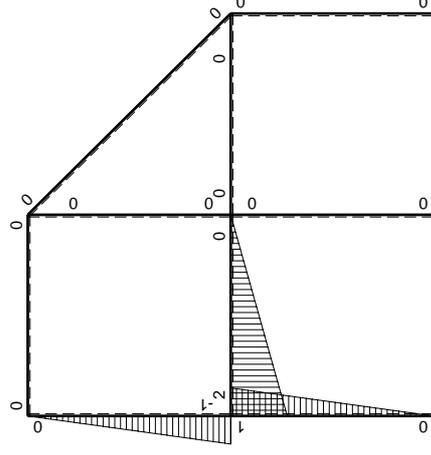
Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{yz} = x_{yz} \cdot \theta_{yz}$  riferimento locale asta YZ con origine in Y.  
 La trave DC ha la sezione riportata e dimensioni in mm, con:  
 $b = 500 \text{ mm}$ ,  $F = 1340 \text{ N}$   
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da D a C  
 Curvatura  $\theta$  asta FG positiva se convessa a destra con inizio F.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13





Schema di calcolo iperstatico

$M_y$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=H_A$ 

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	0	-Fx	0	0	0	0	0+0	0
BA b	0	Fb-Fx	0	0	0	0		
BC $\sqrt{2}b$	0	-Fb+ $\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
DB b	0	1/2Fb-Fx+1/2qx <sup>2</sup>	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	x	Fb-4Fx	-Fb/EJ	Fbx-4Fx <sup>2</sup>	-Fxb/EJ	x <sup>2</sup>	(-5/6-1/2)Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ
GF b	-b+x	3Fb-4Fx	Fb/EJ	-3Fb <sup>2</sup> +7Fbx-4Fx <sup>2</sup>	-Fb <sup>2</sup> /EJ+Fxb/EJ	b <sup>2</sup> -2bx+x <sup>2</sup>		
GD b	2b-2x	-4Fb+3Fx	0	-8Fb <sup>2</sup> +14Fbx-6Fx <sup>2</sup>	0	4b <sup>2</sup> -8bx+4x <sup>2</sup>	(-3+0)Fb <sup>3</sup> /EJ	4/3Xb <sup>3</sup> /EJ
DG b	-2x	Fb+3Fx	0	-2Fbx-6Fx <sup>2</sup>	0	4x <sup>2</sup>		
DH b	0	-3/2Fb+2Fx-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
HD b	0	Fx+1/2qx <sup>2</sup>	0	0	0	0		
GA b	-b+x	Fb-Fx	0	-Fb <sup>2</sup> +2Fbx-Fx <sup>2</sup>	0	b <sup>2</sup> -2bx+x <sup>2</sup>	(-1/3+0)Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ
AG b	x	-Fx	0	-Fx <sup>2</sup>	0	x <sup>2</sup>		
	totali						-14/3Fb <sup>3</sup> /EJ	2Xb <sup>3</sup> /EJ
	iperstatica $X=H_A$						7/3F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (x/b - 4x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx$$

$$= [1/2 x^2/b - 4/3 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x^2/b]_0^b \theta$$

$$= (1/2 b - 4/3 b) Fb^2 1/EJ + (-1/2 b) \theta = -4/3 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-3 + 7x/b - 4x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

$$= [-3x + 7/2 x^2/b - 4/3 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/2 x^2/b]_0^b \theta$$

$$= (-3b + 7/2 b - 4/3 b) Fb^2 1/EJ + (b - 1/2 b) \theta = -4/3 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-8 + 14x/b - 6x^2/b^2) Fb^2 1/EJ dx = [-8x + 7x^2/b - 2x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-8b + 7b - 2b) Fb^2 1/EJ = -3 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-2x/b - 6x^2/b^2) Fb^2 1/EJ dx = [-x^2/b - 2x^3/b^2]_0^b Fb^2 1/EJ$$

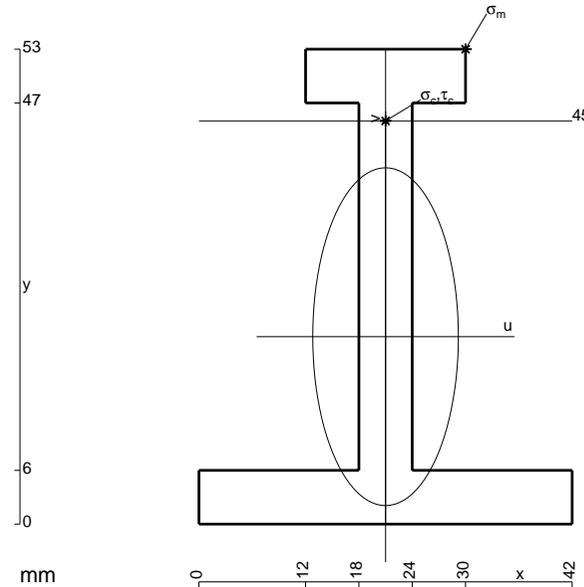
$$= (-b - 2b) Fb^2 1/EJ = -3 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (-1 + 2x/b - x^2/b^2) Fb^2 1/EJ dx = [-x + x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-b + b - 1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

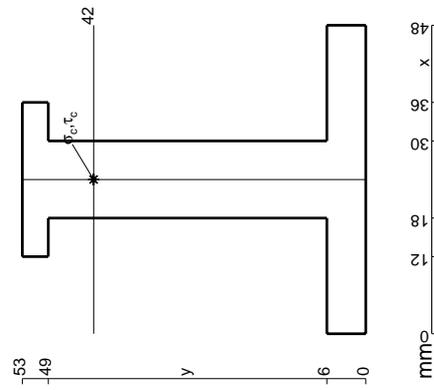
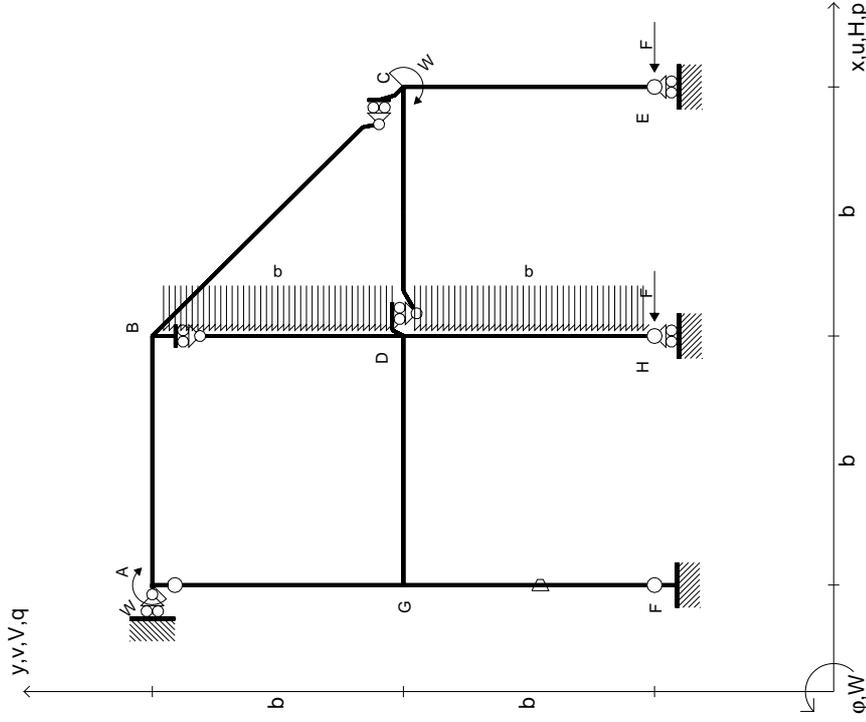
$$L_{AG}^{xo} = \int_0^b (-x^2/b^2) Fb^2 1/EJ dx = [-1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

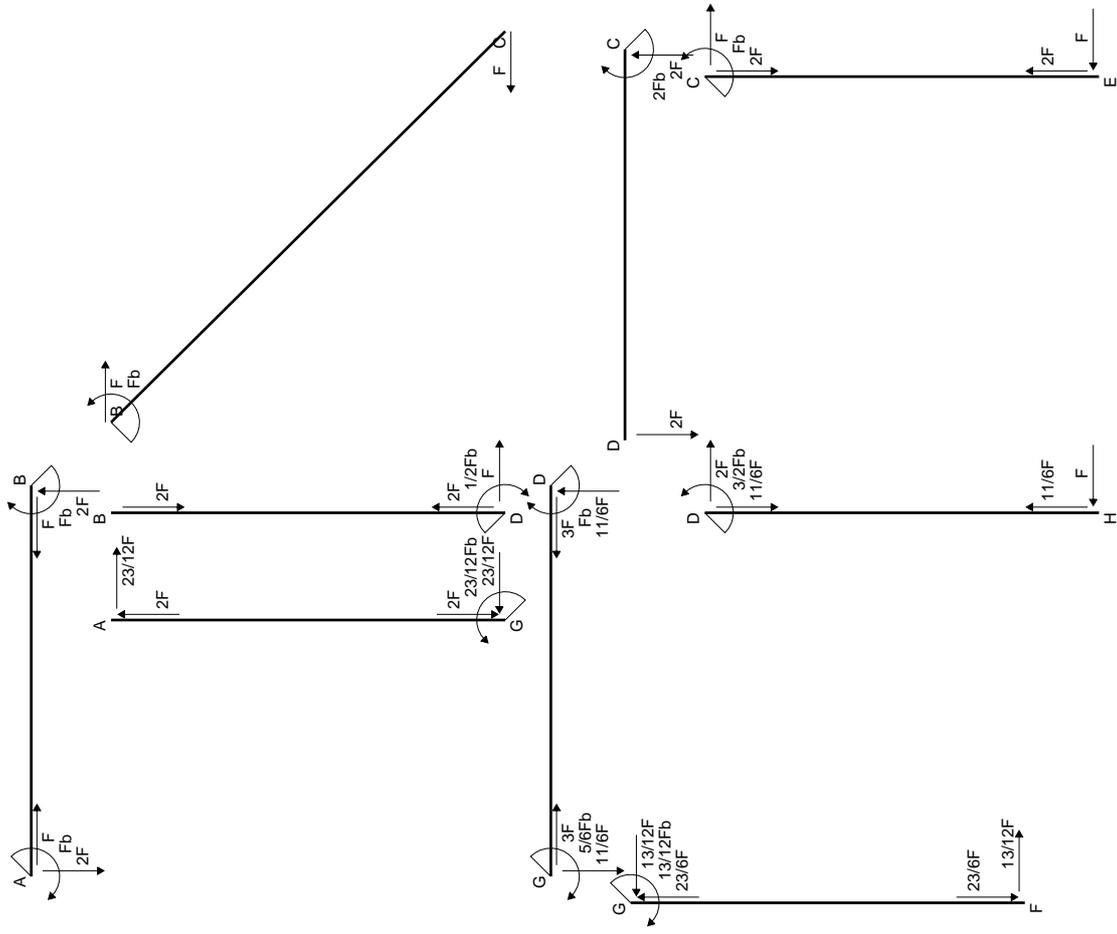


- A = 606. mm<sup>2</sup>
- J<sub>u</sub> = 215454. mm<sup>4</sup>
- J<sub>v</sub> = 40698. mm<sup>4</sup>
- y<sub>g</sub> = 20.92 mm
- T<sub>y</sub> = -2680. N
- M<sub>x</sub> = -1340000. Nmm
- x<sub>m</sub> = 30. mm
- y<sub>m</sub> = 53. mm
- u<sub>m</sub> = 9. mm
- v<sub>m</sub> = 32.08 mm
- σ<sub>m</sub> = -Mv/J<sub>u</sub> = 199.5 N/mm<sup>2</sup>
- x<sub>c</sub> = 21. mm
- y<sub>c</sub> = 45. mm
- v<sub>c</sub> = 24.08 mm
- σ<sub>c</sub> = -Mv/J<sub>u</sub> = 149.8 N/mm<sup>2</sup>
- τ<sub>c</sub> = 7.136 N/mm<sup>2</sup>
- σ<sub>o</sub> = √σ<sup>2</sup>+3τ<sup>2</sup> = 150.3 N/mm<sup>2</sup>
- S = 3442. mm<sup>3</sup>

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_A = -W = -Fb$
- $P_{BD} = -q = -F/b$
- $P_{DH} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione:  $X=V_H$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - X_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 La trave DC ha la sezione riportata e dimensioni in mm, con:  
 $b = 650 \text{ mm}, F = 1610 \text{ N}$   
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da D a C  
 Curvatura  $\theta$  asta FG positiva se convessa a destra con inizio F.  
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Quadro contributi PLV per iperstatica  $X=V_H$

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	0	Fb-2Fx	0	0	0	0	0+0	0
BA b	0	Fb-2Fx	0	0	0	0	0	0
BC $\sqrt{2}b$	0	-Fb+ $\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
DB b	0	1/2Fb-Fx+1/2qx <sup>2</sup>	0	0	0	0	0	0
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0	0	0
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0	0	0
FG b	1/2x	-2Fx	-Fb/EJ	-Fx <sup>2</sup>	-1/2Fxb/EJ	1/4x <sup>2</sup>	$(-1/3-1/4)Fb^3/EJ$	1/12Xb <sup>3</sup> /EJ
GF b	-1/2b+1/2x	2Fb-2Fx	Fb/EJ	-Fb <sup>2</sup> +2Fbx-Fx <sup>2</sup>	-1/2Fb <sup>2</sup> /EJ+1/2Fxb/EJ	1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup>		
GD b	b-x	-Fb	0	-Fb <sup>2</sup> +Fbx	0	b <sup>2</sup> -2bx+x <sup>2</sup>	$(-1/2+0)Fb^3/EJ$	1/3Xb <sup>3</sup> /EJ
DG b	-x	Fb	0	-Fbx	0	x <sup>2</sup>		
DH b	0	-3/2Fb+2Fx-1/2qx <sup>2</sup>	0	0	0	0	0+0	0
HD b	0	Fx+1/2qx <sup>2</sup>	0	0	0	0	0	0
GA b	-1/2b+1/2x	-Fb+Fx	0	1/2Fb <sup>2</sup> -Fbx+1/2Fx <sup>2</sup>	0	1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup>	$(1/6+0)Fb^3/EJ$	1/12Xb <sup>3</sup> /EJ
AG b	1/2x	Fx	0	1/2Fx <sup>2</sup>	0	1/4x <sup>2</sup>		
	totali						-11/12Fb <sup>3</sup> /EJ	1/2Xb <sup>3</sup> /EJ
	iperstatica $X=V_H$						11/6F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [-1/3 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (-1/3 b) Fb^2 1/EJ + (-1/4 b) \theta = -7/12 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-1 + 2x/b - x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [-x + x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (-b + b - 1/3 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = -7/12 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-1 + x/b) Fb^2 1/EJ dx = [-x + 1/2 x^2/b]_0^b Fb^2 1/EJ$$

$$= (-b + 1/2 b) Fb^2 1/EJ = -1/2 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-x/b) Fb^2 1/EJ dx = [-1/2 x^2/b]_0^b Fb^2 1/EJ$$

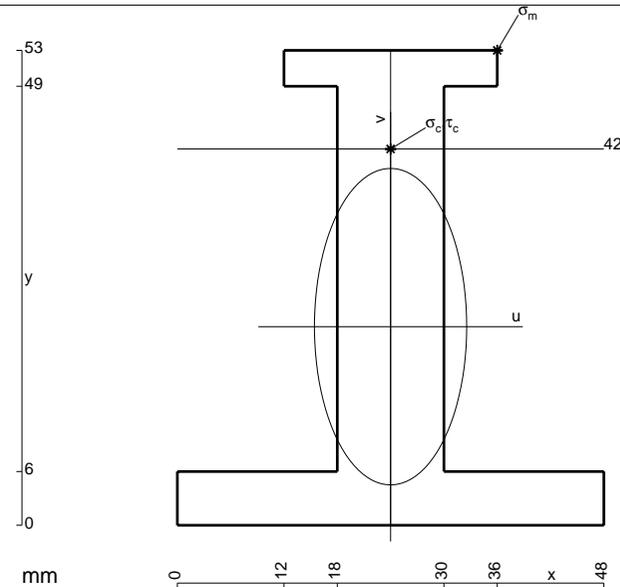
$$= (-1/2 b) Fb^2 1/EJ = -1/2 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

$$L_{AG}^{xo} = \int_0^b (1/2 x^2/b^2) Fb^2 1/EJ dx = [1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$



$$A = 900. \text{ mm}^2$$

$$J_u = 280787. \text{ mm}^4$$

$$J_v = 66096. \text{ mm}^4$$

$$y_g = 22.17 \text{ mm}$$

$$T_y = -3220. \text{ N}$$

$$M_x = -2093000. \text{ Nmm}$$

$$x_m = 36. \text{ mm}$$

$$y_m = 53. \text{ mm}$$

$$u_m = 12. \text{ mm}$$

$$v_m = 30.83 \text{ mm}$$

$$\sigma_m = -Mv/J_u = 229.8 \text{ N/mm}^2$$

$$x_c = 24. \text{ mm}$$

$$y_c = 42. \text{ mm}$$

$$v_c = 19.83 \text{ mm}$$

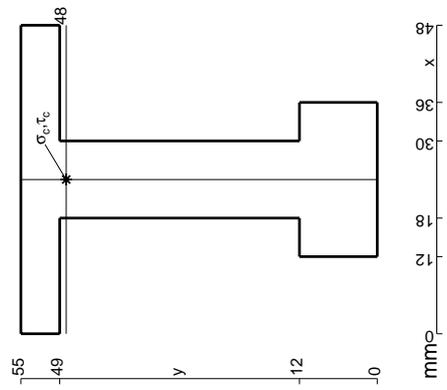
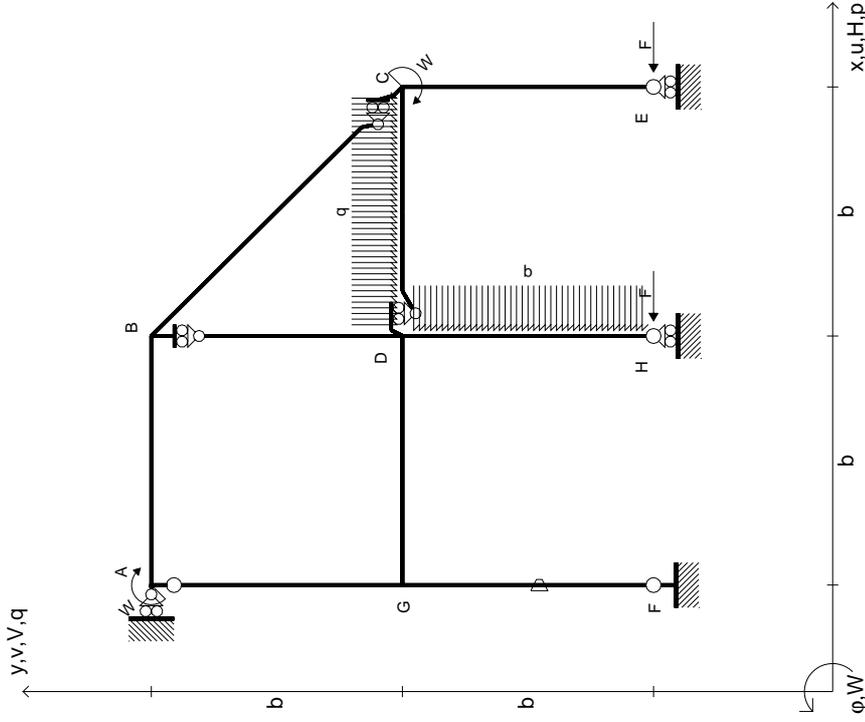
$$\sigma_c = -Mv/J_u = 147.8 \text{ N/mm}^2$$

$$\tau_c = 4.518 \text{ N/mm}^2$$

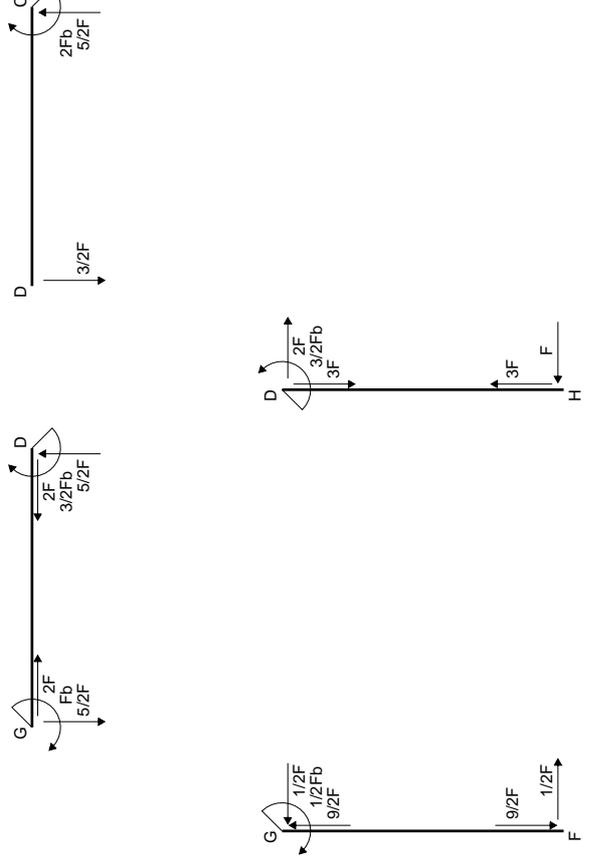
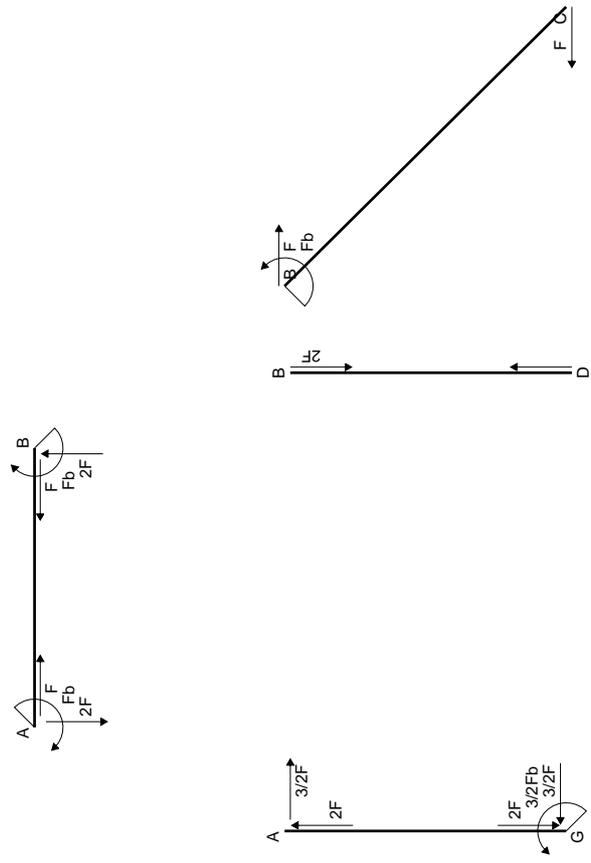
$$\sigma_\rho = \sqrt{\sigma^2 + 3\tau^2} = 148. \text{ N/mm}^2$$

$$S = 4728. \text{ mm}^3$$

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_A = -W = -Fb$
- $q_{bc} = -q = -F/b$
- $p_{DH} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione:  $X=V_H$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi  $x,y$ .  
 $J_{yz} - x_{yz} - \theta_{yz}$  riferimento locale asta YZ con origine in Y.  
 La trave DC ha la sezione riportata e dimensioni in mm, con:  
 $b = 780 \text{ mm}$ ,  $F = 1550 \text{ N}$   
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da D a C  
 Curvatura  $\theta$  asta FG positiva se convessa a destra con inizio F.  
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Quadro contributi PLV per iperstatica  $X=V_H$ 

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	0	$Fb-2Fx$	0	0	0	0	0+0	0	
BA b	0	$Fb-2Fx$	0	0	0	0	0	0	
BC $\sqrt{2}b$	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0	
BD b	0	0	0	0	0	0	0+0	0	
DB b	0	0	0	0	0	0	0	0	
DC b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0	
CD b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0	0	
CE b	0	$-Fb+Fx$	0	0	0	0	0+0	0	
EC b	0	$Fx$	0	0	0	0	0	0	
FG b	$1/2x$	$-2Fx$	$-Fb/EJ$	$-Fx^2$	$-1/2Fxb/EJ$	$1/4x^2$	$(-1/3-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$	
GF b	$-1/2b+1/2x$	$2Fb-2Fx$	$Fb/EJ$	$-Fb^2+2Fbx-Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$			
GD b	$b-x$	$-2Fb+1/2Fx$	0	$-2Fb^2+5/2Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-11/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
DG b	$-x$	$3/2Fb+1/2Fx$	0	$-3/2Fbx-1/2Fx^2$	0	$x^2$			
DH b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0	
HD b	0	$Fx+1/2qx^2$	0	0	0	0	0	0	
GA b	$-1/2b+1/2x$	0	0	0	0	$1/4b^2-1/2bx+1/4x^2$	0+0	$1/12Xb^3/EJ$	
AG b	$1/2x$	0	0	0	0	$1/4x^2$			
	totali							$-3/2Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$							3F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [-1/3 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (-1/3 b) Fb^2 1/EJ + (-1/4 b) \theta = -7/12 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-1 + 2x/b - x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [-x + x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

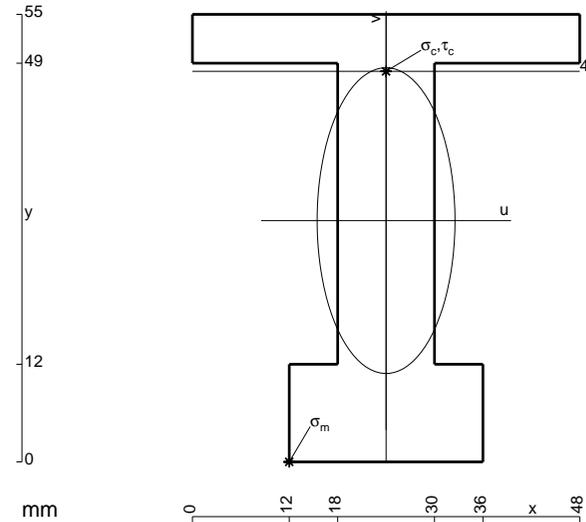
$$= (-b + b - 1/3 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = -7/12 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-2 + 5/2 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-2x + 5/4 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2b + 5/4 b - 1/6 b) Fb^2 1/EJ = -11/12 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-3/2 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-3/4 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-3/4 b - 1/6 b) Fb^2 1/EJ = -11/12 Fb^3/EJ$$



- A = 1020. mm<sup>2</sup>
- J<sub>u</sub> = 360241. mm<sup>4</sup>
- J<sub>v</sub> = 74448. mm<sup>4</sup>
- y<sub>g</sub> = 29.65 mm
- T<sub>y</sub> = -3875. N
- M<sub>x</sub> = -2418000. Nmm
- x<sub>m</sub> = 12. mm
- u<sub>m</sub> = -12. mm
- v<sub>m</sub> = -29.65 mm
- σ<sub>m</sub> = -Mv/J<sub>u</sub> = -199. N/mm<sup>2</sup>
- x<sub>c</sub> = 24. mm
- y<sub>c</sub> = 48. mm
- v<sub>c</sub> = 18.35 mm
- σ<sub>c</sub> = -Mv/J<sub>u</sub> = 123.1 N/mm<sup>2</sup>
- τ<sub>c</sub> = 5.972 N/mm<sup>2</sup>
- σ<sub>o</sub> = √σ<sup>2</sup>+3τ<sup>2</sup> = 123.6 N/mm<sup>2</sup>
- S = 6662. mm<sup>3</sup>