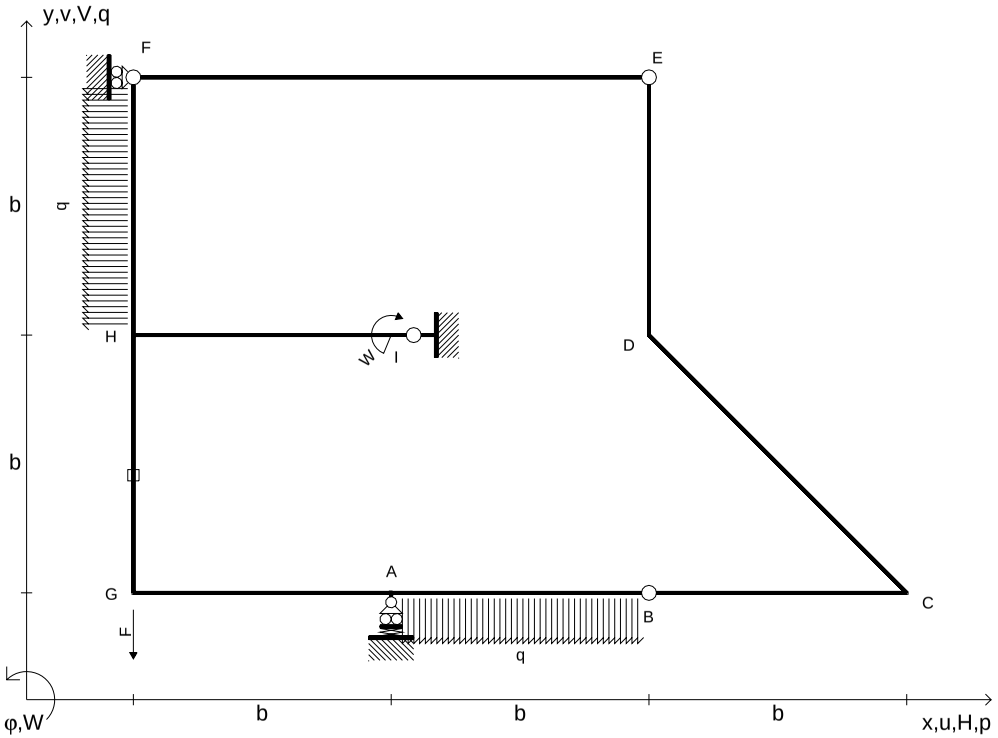
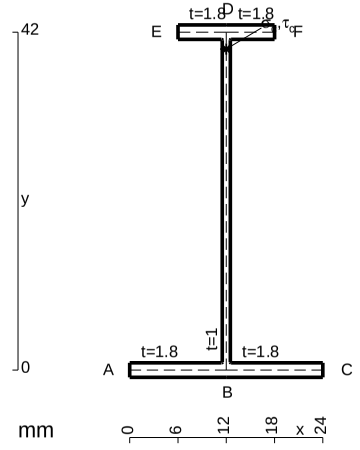


$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

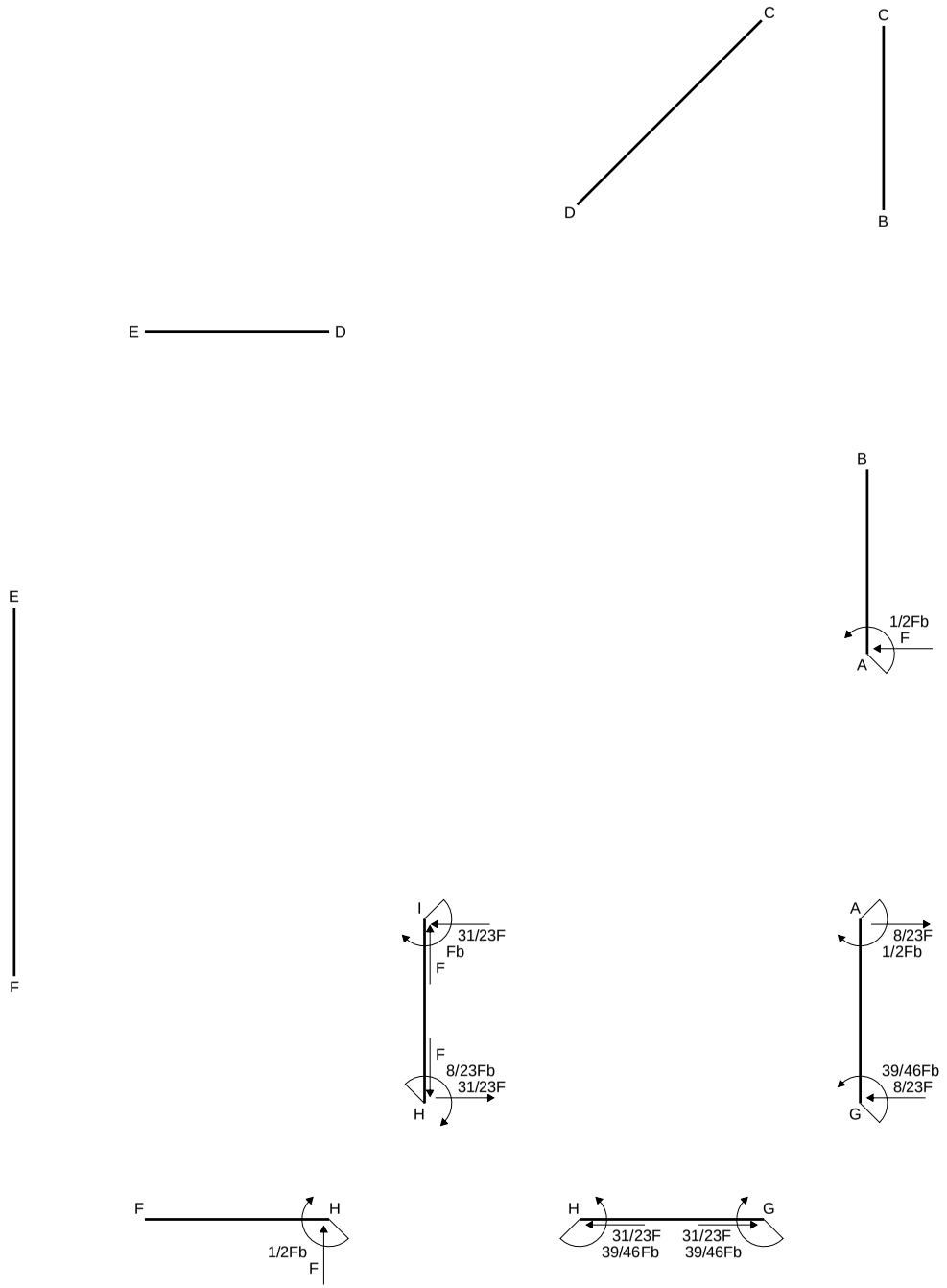
$W_I = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

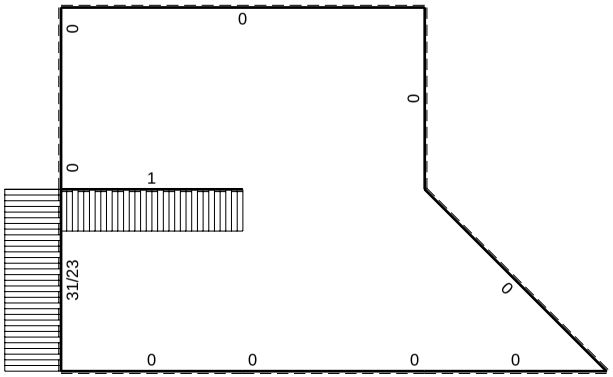
$q_{AB} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

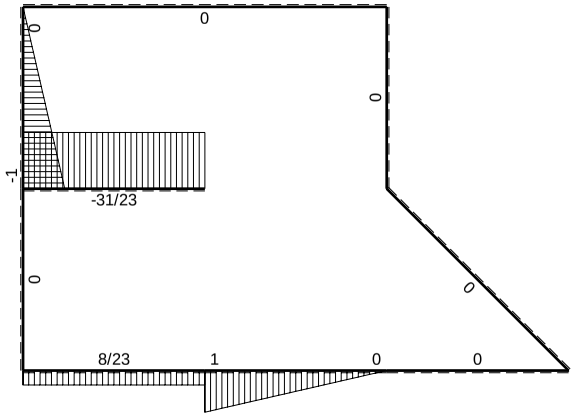


Reazioni iperstatiche in soluzione: $X=V_I$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 610 mm, F = 930 N
Calcolare sulla sezione A la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.

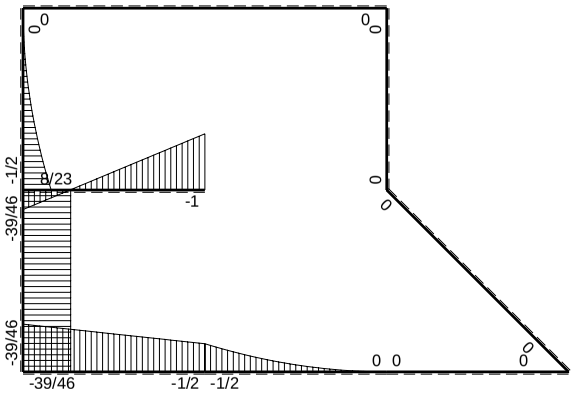




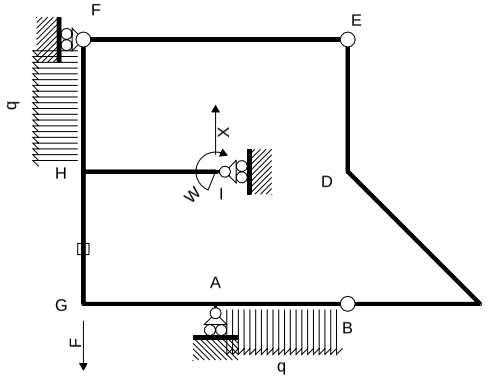
F



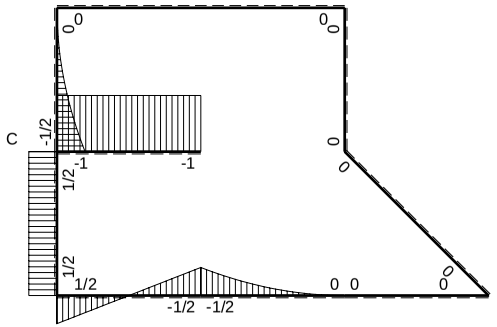
F



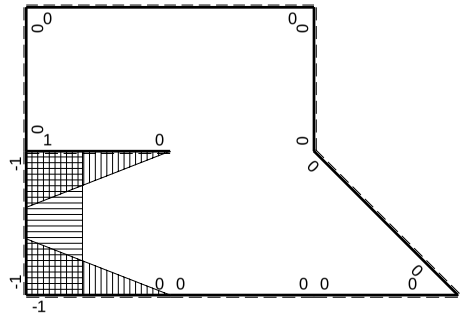
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0
BA b	0	$1/2qx^2$	0	0		
BC b	0	0	0	0	0	0
CB b	0	0	0	0		
CD $\sqrt{2}b$	0	0	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0		
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0		
GA b	-b+x	$1/2Fb-Fx$	$-1/2Fb^2+3/2Fbx-Fx^2$	$b^2-2bx+x^2$	$-1/12Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	$1/2Fb-Fx$	$1/2Fbx-Fx^2$	x^2		
FH b	0	$-1/2qx^2$	0	0	0	0
HF b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
HI b	b-x	-Fb	$-Fb^2+Fbx$	$b^2-2bx+x^2$	$-1/2Fb^3/EJ$	$1/3Xb^3/EJ$
IH b	-x	Fb	-Fbx	x^2		
HG b	-b	$1/2Fb$	$-1/2Fb^2$	b^2	$-1/2Fb^3/EJ$	Xb^3/EJ
GH b	b	$-1/2Fb$	$-1/2Fb^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				$-Fb^3/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-1/2Fb^3/EJ$	$1/4Xb^3/EJ$
	totali				$-31/12Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_I$				31/23F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 236761 ESΣ05.xxxx.007

$L^{XX}_{GA} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [\, 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [\, 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1/2 + 3/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x + 3/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b + 3/4\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -1/12\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (1/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [\, 1/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -1/12\ Fb^3/EJ$

$L^{Xo}_{HI} = \int_o^b (-1 + x/b \,)\, Fb^2\, 1/EJ\, dx = [\, - x + 1/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ$

$= (\, - b + 1/2\, b \,)\, Fb^2\, 1/EJ = -1/2\ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b (\, - x/b \,)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b \,)\, Fb^2\, 1/EJ = -1/2\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (-1/2 \,)\, Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ = [-1/2\, x \,]_o^b\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= (-1/2\, b \,)\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -3/2\ Fb^3/EJ$

$L^{Xo}_{GH} = \int_o^b (-1/2 \,)\, Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ = [-1/2\, x \,]_o^b\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= (-1/2\, b \,)\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -3/2\ Fb^3/EJ$

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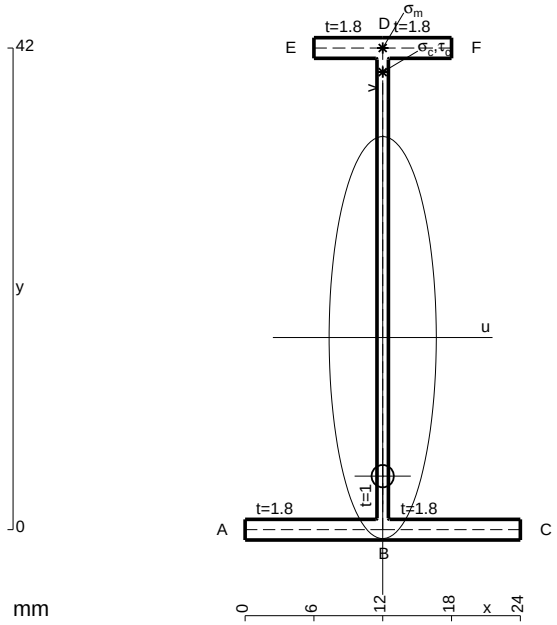

A = 106.8 mm^2
J_u = 32824. mm^4
J_v = 2333. mm^4
J_t = 83.98 mm^4
y_o = -12.09 mm
y_g = 16.75 mm
T_y = 930. N
M_x = -283650. Nmm
x_m = 12. mm
y_m = 42. mm
v_m = 25.25 mm
$\sigma_m = -Mv/J_u$ = 218.2 N/mm^2
y_c = 2. mm
u_c = -12. mm
v_c = -14.75 mm
$\sigma_c = -Mv/J_u$ = 218.2 N/mm^2
$\tau_c = TS^*/tJ_u$ = 15.45 N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.45 N/mm^2
t_c = 930. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 219.8 N/mm^2

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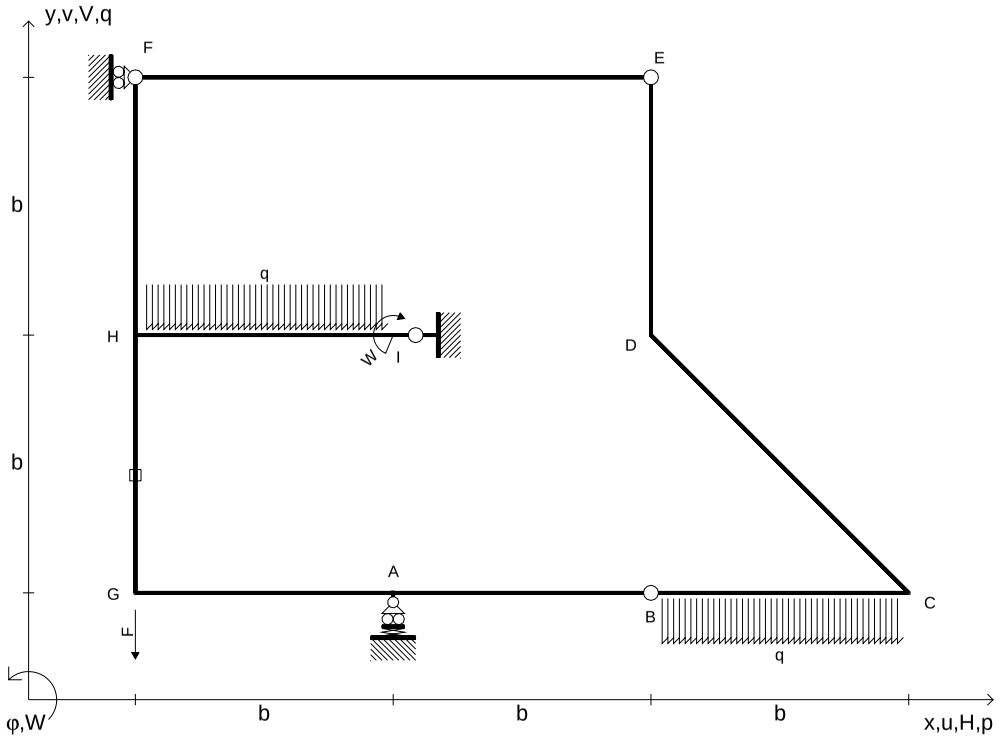
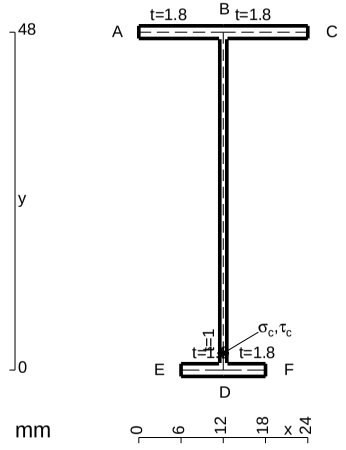


$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_I = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{BC} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

$q_{HI} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione: $X=V_I$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

La trave AB ha la sezione riportata e dimensioni in mm, con:

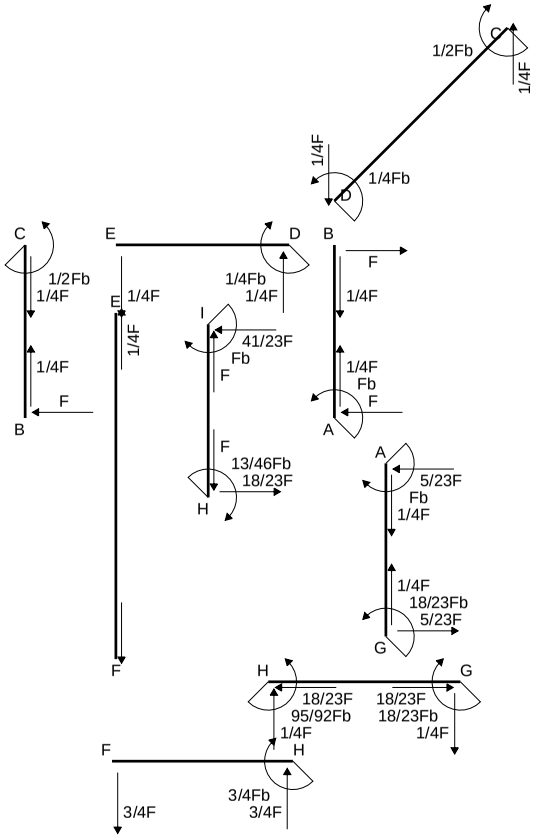
b = 610 mm, F = 500 N

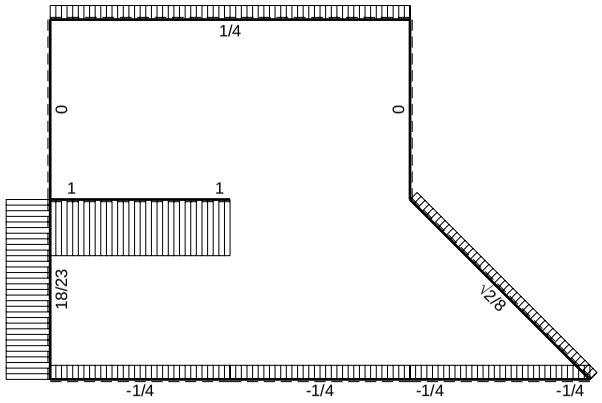
Calcolare sulla sezione A la massima tensione normale σ_m.

Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.

Lembo inferiore sezione su tratteggio trave, a destra da A a B

Elongazione termica specifica ε assegnata su asta HG.





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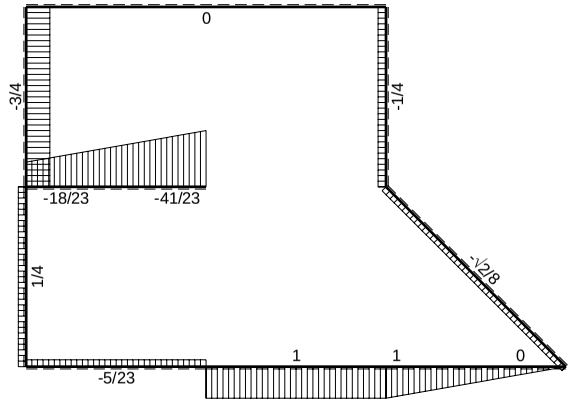
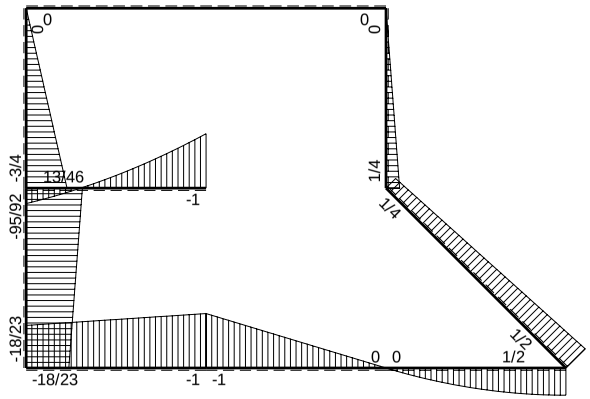
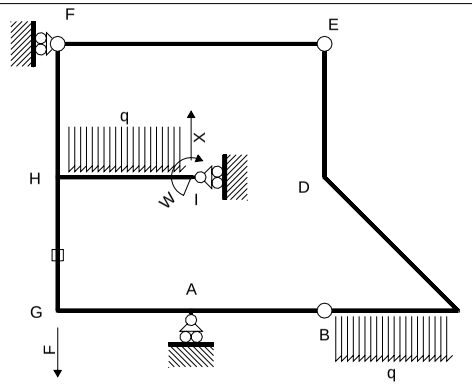


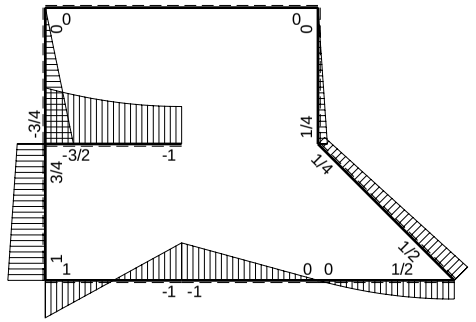
F



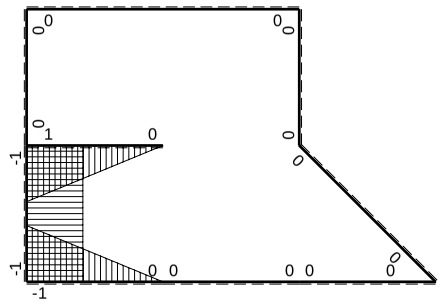
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fb+Fx	0	0	0	0
BA b	0	Fx	0	0		
BC b	0	$Fx-1/2qx^2$	0	0	0	0
CB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD √2b	0	1/2Fb-√2/8Fx	0	0	0	0
DE b	0	1/4Fb-1/4Fx	0	0	0	0
ED b	0	-1/4Fx	0	0		
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0		
GA b	-b+x	Fb-2Fx	$-Fb^2+3Fbx-2Fx^2$	$b^2-2bx+x^2$	$-1/6Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	Fb-2Fx	$Fbx-2Fx^2$	x^2		
FH b	0	-3/4Fx	0	0	0	0
HF b	0	3/4Fb-3/4Fx	0	0		
HI b	b-x	$-3/2Fb+Fx-1/2qx^2$	$-3/2Fb^2+5/2Fbx-3/2Fx^2+1/2qx^3$	$b^2-2bx+x^2$	$-5/8Fb^3/EJ$	$1/3Xb^3/EJ$
IH b	-x	$Fb+1/2qx^2$	$-Fbx-1/2qx^3$	x^2		
HG b	-b	3/4Fb+1/4Fx	$-3/4Fb^2-1/4Fbx$	b^2	$-7/8Fb^3/EJ$	Xb^3/EJ
GH b	b	-Fb+1/4Fx	$-Fb^2+1/4Fbx$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				$-Fb^3/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-3/4Fb^3/EJ$	$1/4Xb^3/EJ$
	totali				$-41/12Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_I$				41/23F	

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Sviluppi di calcolo iperstatica

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$L^{XX}_{GA} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1 + 3\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x + 3/2\, x^2/b - 2/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (- b + 3/2\, b - 2/3\, b \,)\, Fb^2\, 1/EJ = -1/6\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (\, x/b - 2\,x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b - 2/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b - 2/3\, b \,)\, Fb^2\, 1/EJ = -1/6\ Fb^3/EJ$

$L^{Xo}_{HI} = \int_o^b (-3/2 + 5/2\, x/b - 3/2\, x^2/b^2 + 1/2\, x^3/b^3 \,)\, Fb^2\, 1/EJ\, dx$

$= [-3/2\, x + 5/4\, x^2/b - 1/2\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 5/4\, b - 1/2\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = -5/8\ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b (- x/b - 1/2\, x^3/b^3 \,)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = -5/8\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (-3/4 - 1/4\, x/b \,)\, Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

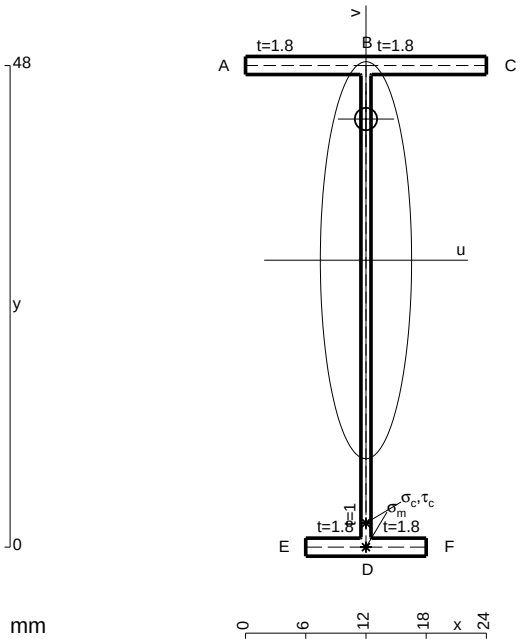
$= [-3/4\, x - 1/8\, x^2/b \,]_o^b\, Fb^2\, 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-3/4\, b - 1/8\, b \,)\, Fb^2\, 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -15/8\ Fb^3/EJ$

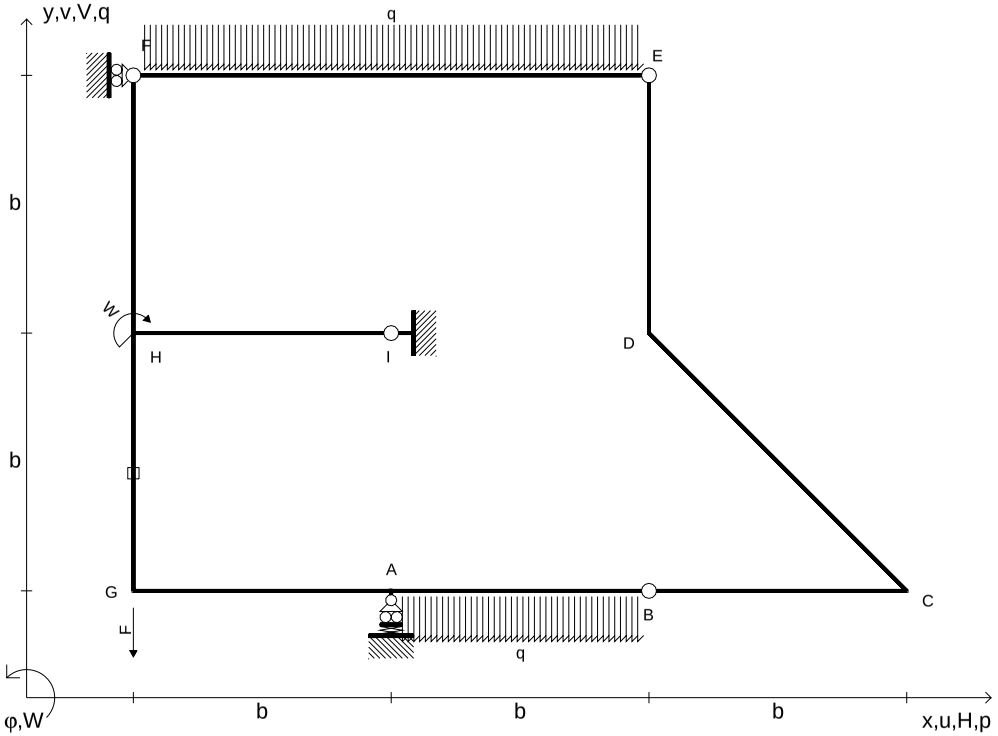
$L^{Xo}_{GH} = \int_o^b (-1 + 1/4\, x/b \,)\, Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [- x + 1/8\, x^2/b \,]_o^b\, Fb^2\, 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (- b + 1/8\, b \,)\, Fb^2\, 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -15/8\ Fb^3/EJ$



A = 112.8 mm^2
J_u = 44158. mm^4
J_v = 2333. mm^4
J_t = 85.98 mm^4
y_o = 14.07 mm
y_g = 28.6 mm
N = -125. N
T_y = 500. N
M_x = -305000. Nmm
x_m = 12. mm
v_m = -28.6 mm
$\sigma_m = N/A - Mv/J_u$ = -198.6 N/mm^2
y_c = 3. mm
u_c = -12. mm
v_c = -25.6 mm
$\sigma_c = N/A - Mv/J_u$ = -198.6 N/mm^2
$\tau_c = TS^*/tJ_u$ = 6.994 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.994 N/mm^2
t_c = 500. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 199. N/mm^2

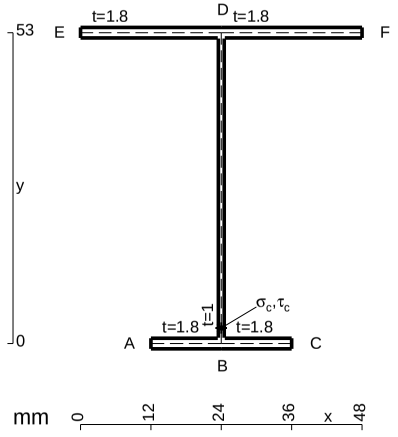


$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_H = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

$q_{EF} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione: $X=V_I$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

La trave AB ha la sezione riportata e dimensioni in mm, con:

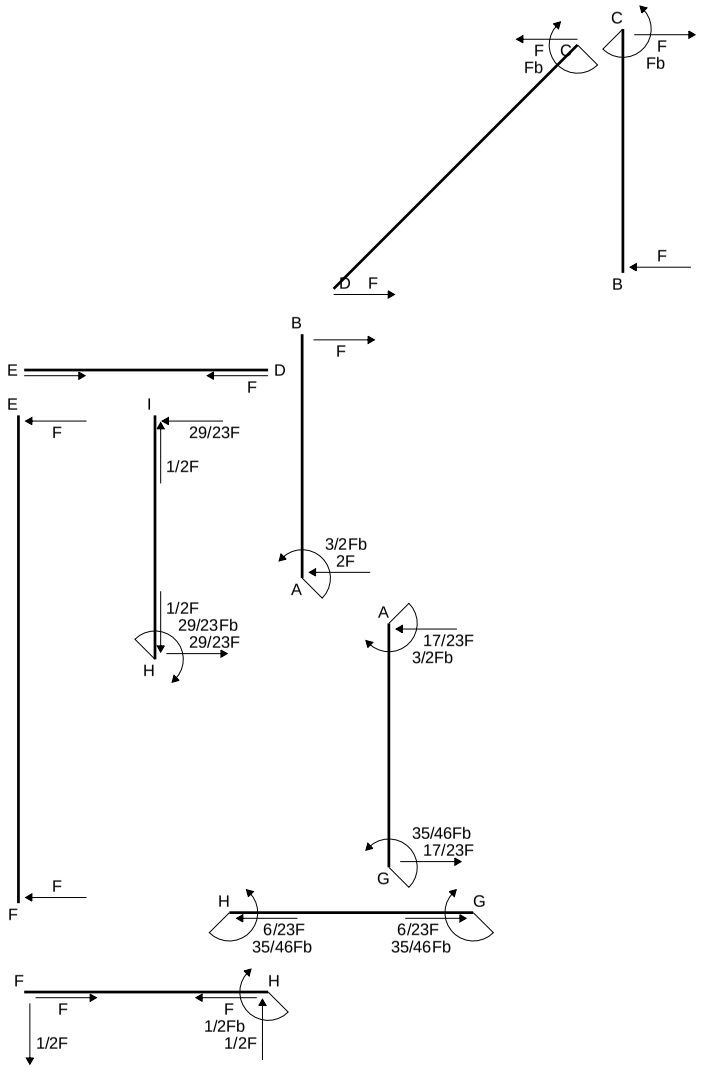
b = 760 mm, F = 560 N

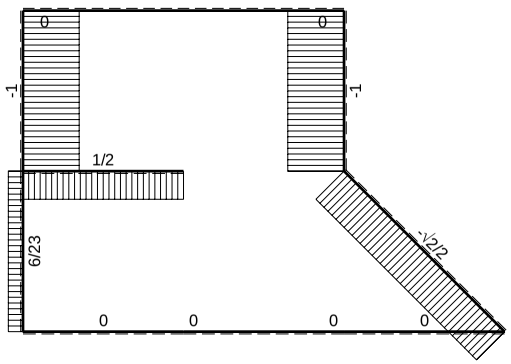
Calcolare sulla sezione A la massima tensione normale σ_m.

Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.

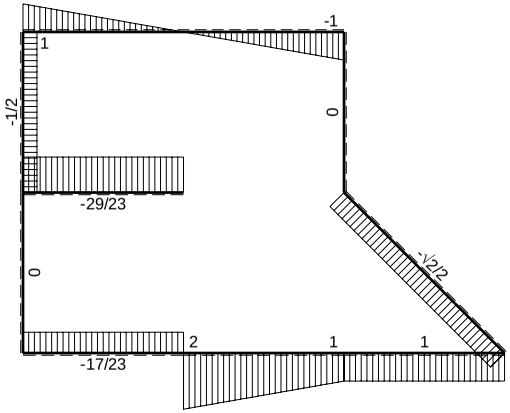
Lembo inferiore sezione su tratteggio trave, a destra da A a B

Elongazione termica specifica ε assegnata su asta HG.

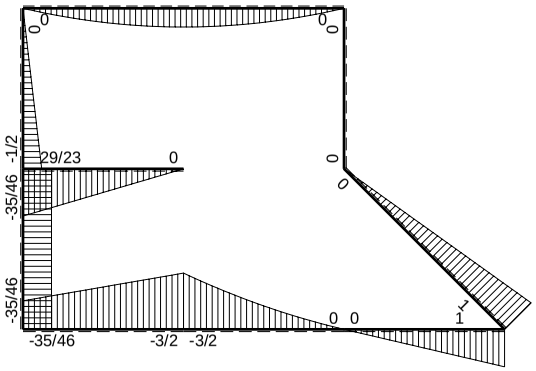




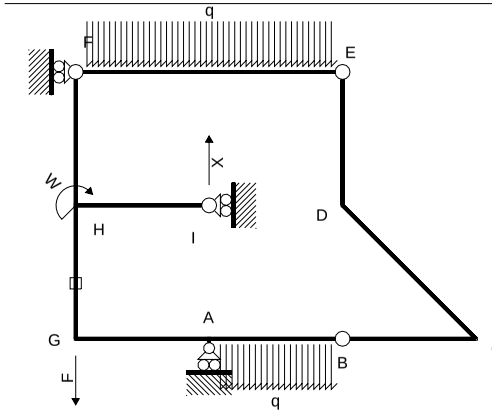
←□→ F



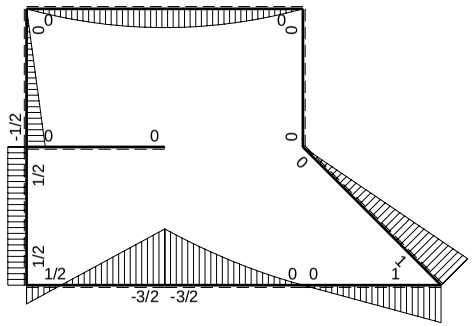
↑□↓ F



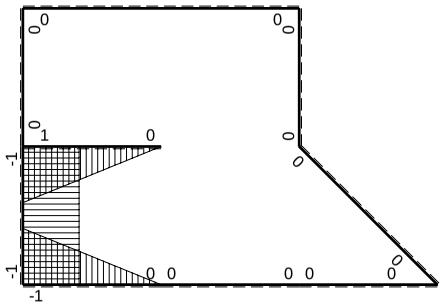
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_0 flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fx+1/2qx^2$	0	0		
BC b	0	Fx	0	0	0	0
CB b	0	-Fb+Fx	0	0		
CD $\sqrt{2}b$	0	$Fb-\sqrt{2}/2Fx$	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0		
EF 2b	0	$-Fx+1/2qx^2$	0	0	0	0
FE 2b	0	$Fx-1/2qx^2$	0	0		
GA b	-b+x	1/2Fb-2Fx	$-1/2Fb^2+5/2Fbx-2Fx^2$	$b^2-2bx+x^2$	$1/12Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	3/2Fb-2Fx	$3/2Fbx-2Fx^2$	x^2		
FH b	0	-1/2Fx	0	0	0	0
HF b	0	1/2Fb-1/2Fx	0	0		
HI b	b-x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
IH b	-x	0	0	x^2		
HG b	-b	1/2Fb	$-1/2Fb^2$	b^2	$-1/2Fb^3/EJ$	Xb^3/EJ
GH b	b	-1/2Fb	$-1/2Fb^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				$-Fb^3/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-Fb^3/EJ$	$1/4Xb^3/EJ$
	totali				$-29/12Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_I$				29/23F	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-1/2 +5/2\, x/b -2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/2\, x +5/4\, x^2/b -2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/2\, b +5/4\, b -2/3\, b \,\right) Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(3/2\, x/b -2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[3/4\, x^2/b -2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

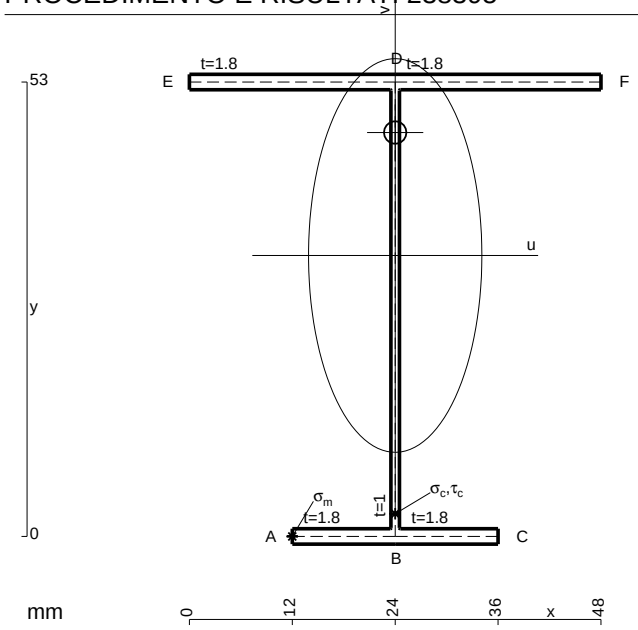
$$= \left(3/4\, b -2/3\, b \,\right) Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-1/2\,\right) Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = \left[-1/2\, x \,\right]_o^b Fb^2\, 1/EJ\ -1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-1/2\, b \,\right) Fb^2\, 1/EJ\ -1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -3/2\ \ Fb^3/EJ$$

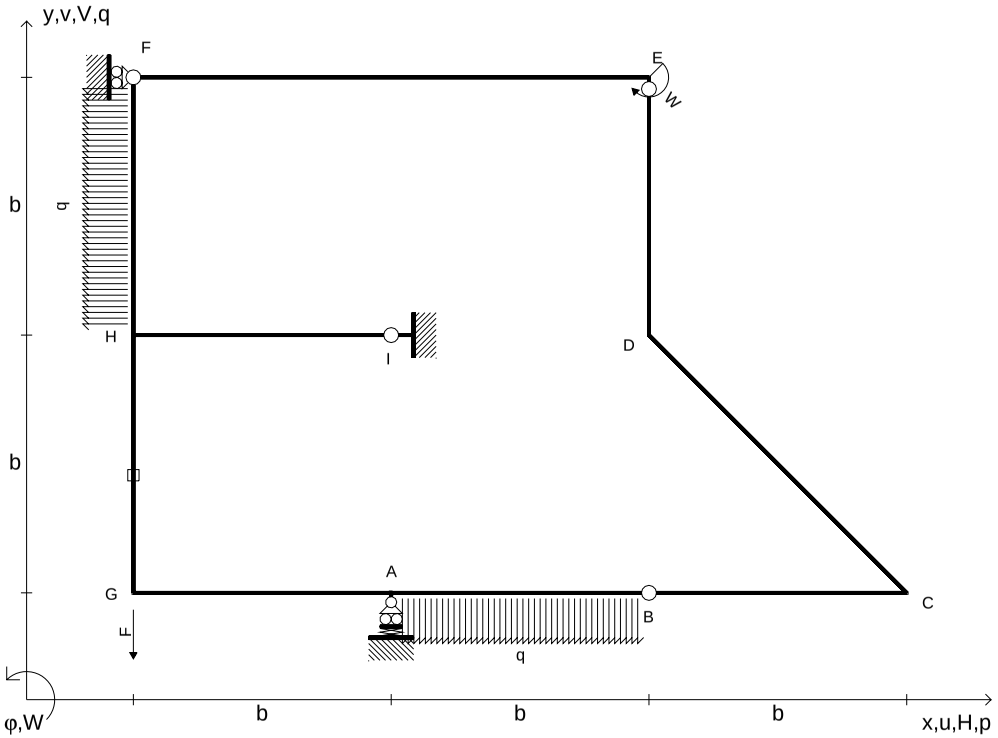
$$L_{GH}^{Xo} = \int_o^b \left(-1/2\,\right) Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = \left[-1/2\, x \,\right]_o^b Fb^2\, 1/EJ\ -1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-1/2\, b \,\right) Fb^2\, 1/EJ\ -1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -3/2\ \ Fb^3/EJ$$



A = 182.6 mm^2
J_u = 96241. mm^4
J_v = 18662. mm^4
J_t = 157.6 mm^4
y_o = 14.34 mm
y_g = 32.77 mm
T_y = 1120. N
M_x = -638400. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -32.77 mm
$\sigma_m = -Mv/J_u$ = -217.4 N/mm^2
x_c = 24. mm
v_c = -32.77 mm
$\sigma_c = -Mv/J_u$ = -217.4 N/mm^2
$\tau_c = TS^*/tJ_u$ = 16.47 N/mm^2
$\tau_g = TS^*/tJ_u$ = 16.47 N/mm^2
t_c = 560. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 219.2 N/mm^2

ESΣ05.xxxx.027
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 12.12.25



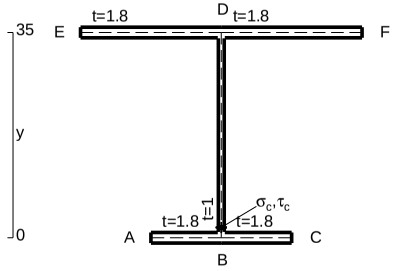
$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

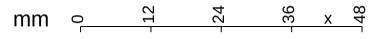
$W_E = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

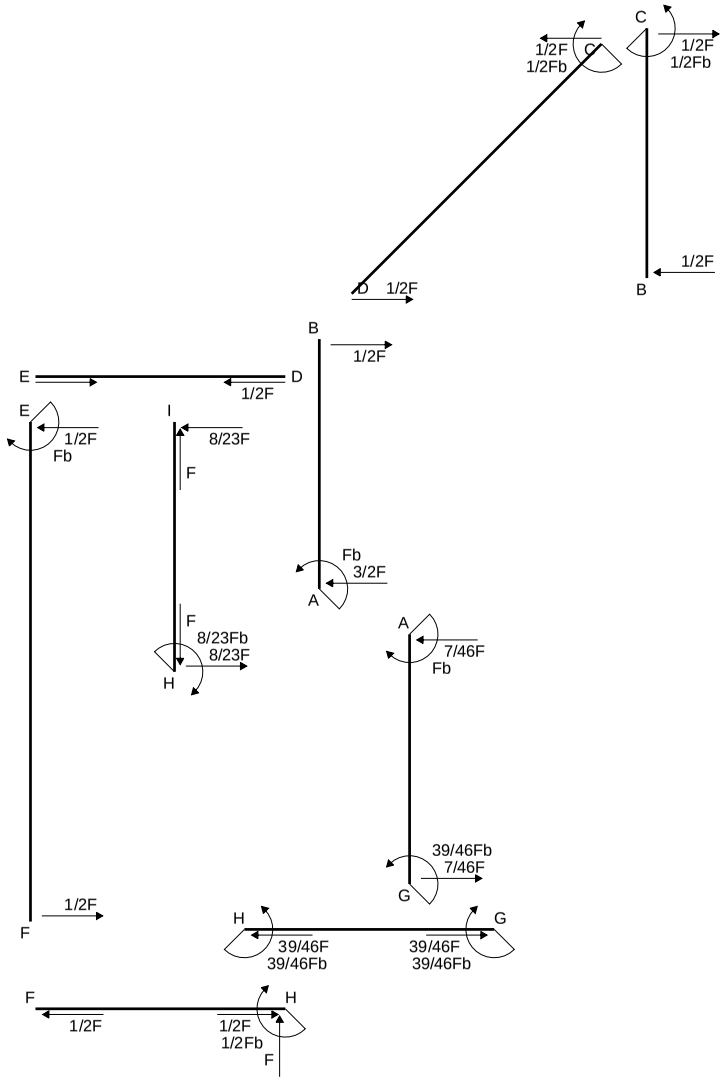
$q_{AB} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

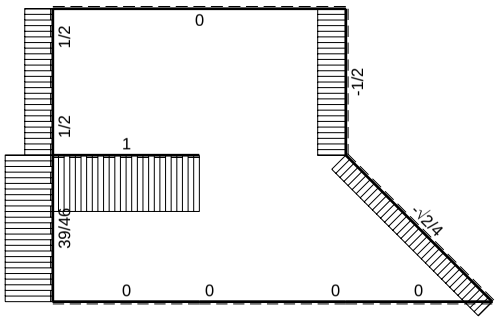
$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 480 mm, F = 780 N
Calcolare sulla sezione A la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.

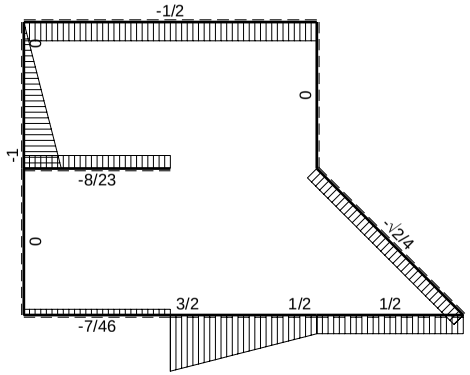




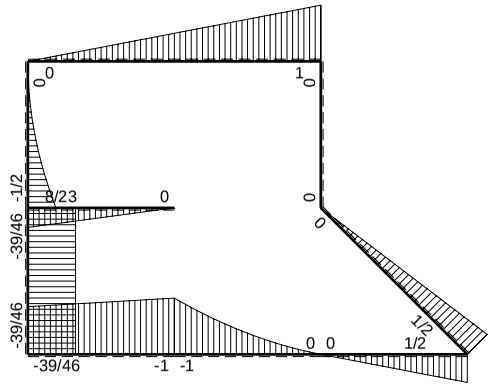




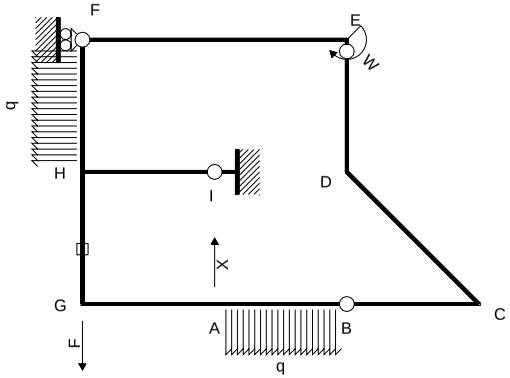
←□→ F



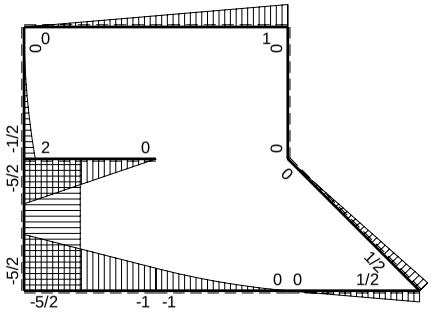
↑□↓ F



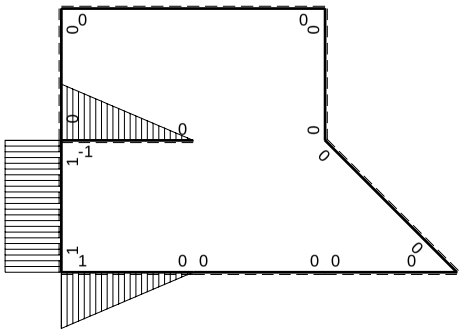
□ Fb



Schema di calcolo iperstatico



□ M_0 flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fb+3/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fx+1/2qx^2$	0	0		
BC b	0	1/2Fx	0	0	0	0
CB b	0	-1/2Fb+1/2Fx	0	0		
CD √2b	0	1/2Fb-√2/4Fx	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0		
EF 2b	0	Fb-1/2Fx	0	0	0	0
FE 2b	0	-1/2Fx	0	0		
GA b	b-x	$-5/2Fb+3/2Fx$	$-5/2Fb^2+4Fbx-3/2Fx^2$	$b^2-2bx+x^2$	$-Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	$Fb+3/2Fx$	$-Fbx-3/2Fx^2$	x^2		
FH b	0	$-1/2qx^2$	0	0	0	0
HF b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
HI b	-b+x	2Fb-2Fx	$-2Fb^2+4Fbx-2Fx^2$	$b^2-2bx+x^2$	$-2/3Fb^3/EJ$	$1/3Xb^3/EJ$
IH b	x	-2Fx	$-2Fx^2$	x^2		
HG b	b	-5/2Fb	$-5/2Fb^2$	b^2	$-5/2Fb^3/EJ$	Xb^3/EJ
GH b	-b	5/2Fb	$-5/2Fb^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				Fb^3/EJ	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$					$1/4Xb^3/EJ$
	totali				$-19/6Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_A$				38/23F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 309819 ESΣ05.xxxx.036

$$L_{GA}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$
$$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$
$$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$
$$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$
$$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b (1 \,)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$$
$$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b (1 \,)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$$
$$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b (-5/2 + 4\,x/b - 3/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-5/2\, x + 2\, x^2/b - 1/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$
$$= (-5/2\, b + 2\, b - 1/2\, b \,)\, Fb^2\, 1/EJ = -\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b (-\,x/b - 3/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b - 1/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$
$$= (-1/2\, b - 1/2\, b \,)\, Fb^2\, 1/EJ = -\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b (-2 + 4\,x/b - 2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-2\, x + 2\, x^2/b - 2/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$
$$= (-2\, b + 2\, b - 2/3\, b \,)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b (-2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$
$$= (-2/3\, b \,)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b (-5/2 \,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = [-5/2\, x \,]_o^b\, Fb^2\, 1/EJ\ +\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= (-5/2\, b \,)\, Fb^2\, 1/EJ\ +\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -3/2\ \ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b (-5/2 \,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = [-5/2\, x \,]_o^b\, Fb^2\, 1/EJ\ +\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= (-5/2\, b \,)\, Fb^2\, 1/EJ\ +\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -3/2\ \ Fb^3/EJ$$

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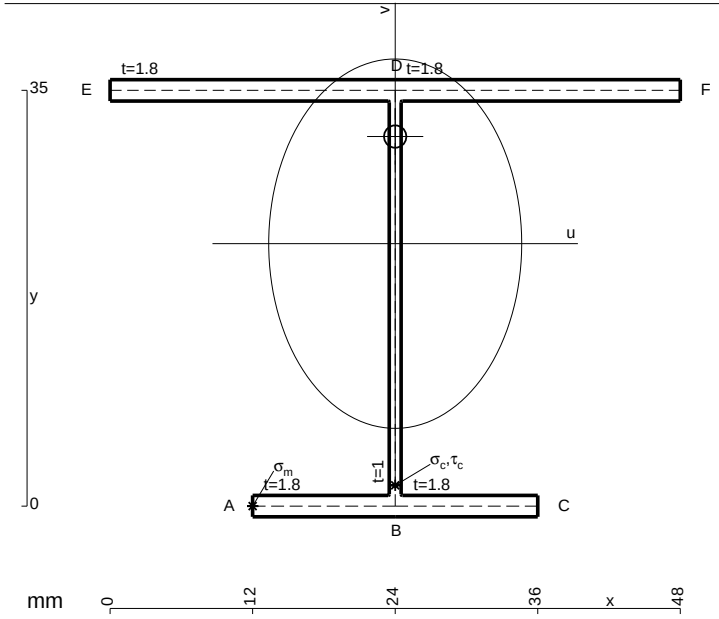

$A = 164.6\ mm^2$
$J_u = 39791.\ mm^4$
$J_v = 18662.\ mm^4$
$J_t = 151.6\ mm^4$
$y_o = 9.018$ mm
$y_g = 22.09$ mm
$T_y = 1170.$ N
$M_x = -374400.$ Nmm
$x_m = 12.$ mm
$u_m = -12.$ mm
$v_m = -22.09$ mm
$\sigma_m = -Mv/J_u = -207.9\ N/mm^2$
$x_c = 24.$ mm
$v_c = -22.09$ mm
$\sigma_c = -Mv/J_u = -207.9\ N/mm^2$
$\tau_c = TS^*/tJ_u = 28.06\ N/mm^2$
$\tau_g = TS^*/tJ_u = 28.06\ N/mm^2$
$t_c = 780.$ mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 213.5\ N/mm^2$

ESΣ05.xxxx.036

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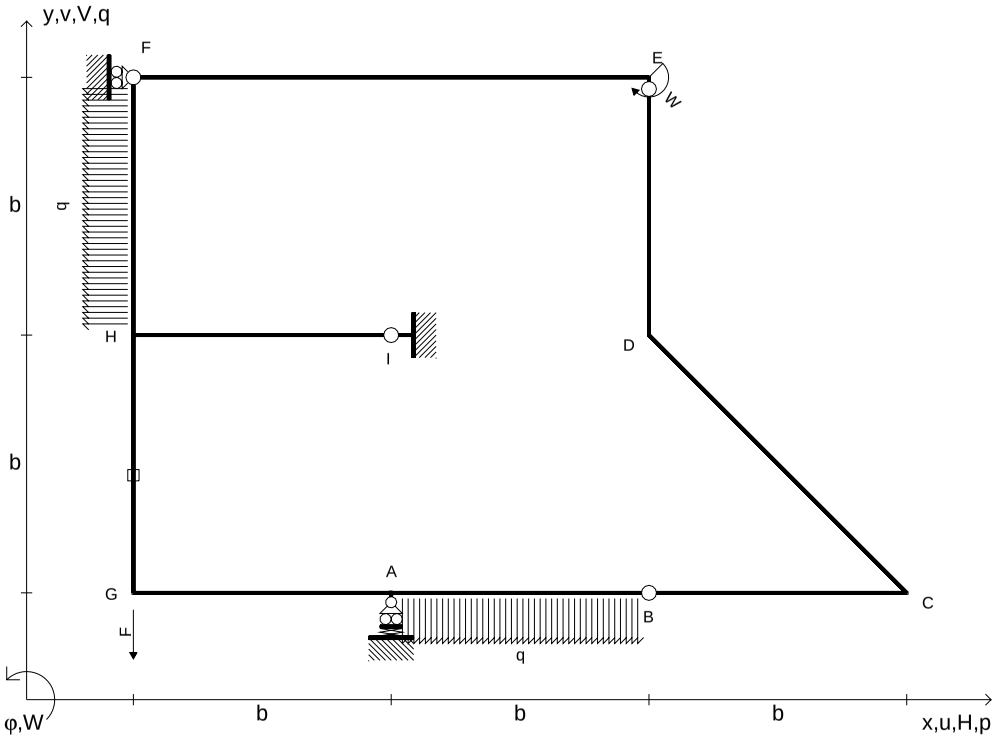
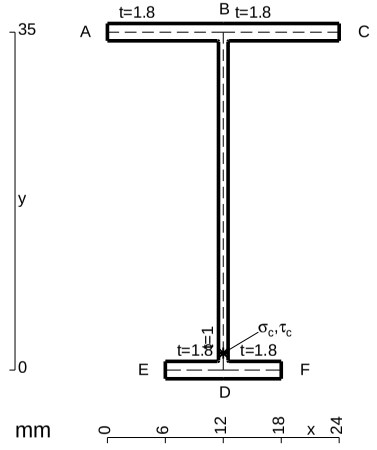


$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

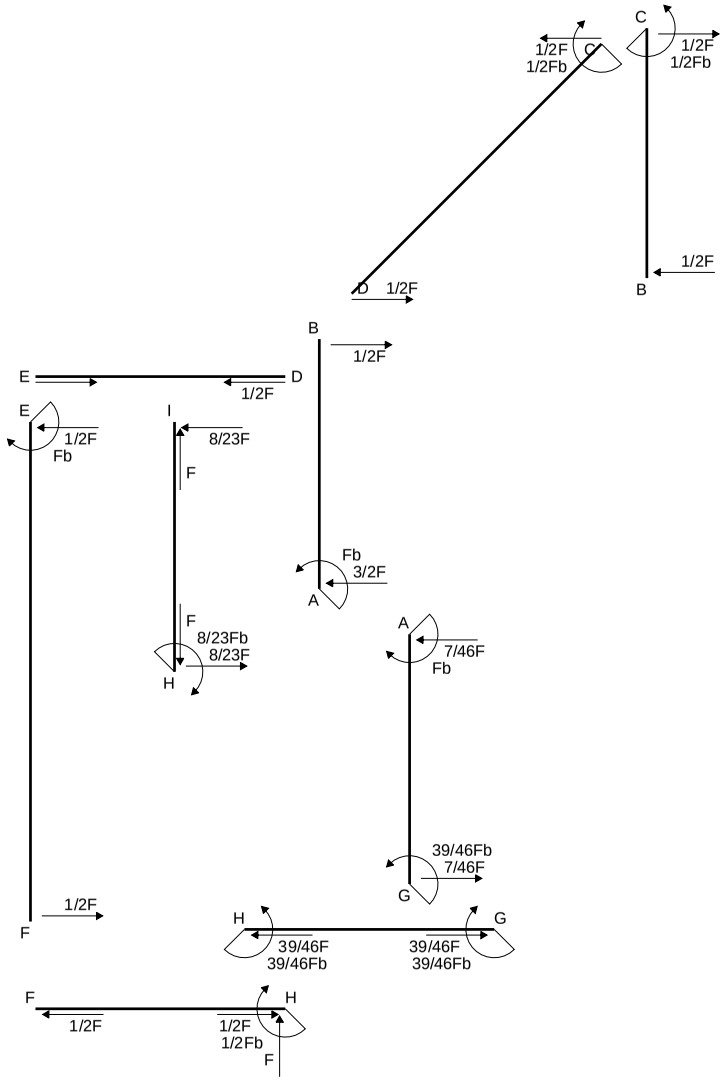
$W_E = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

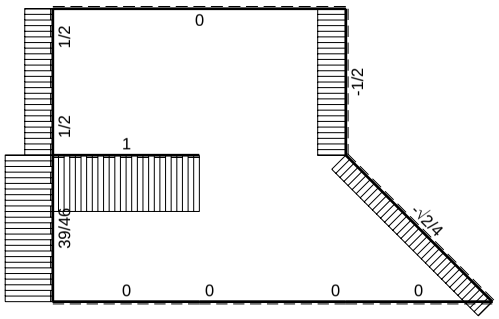
$q_{AB} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

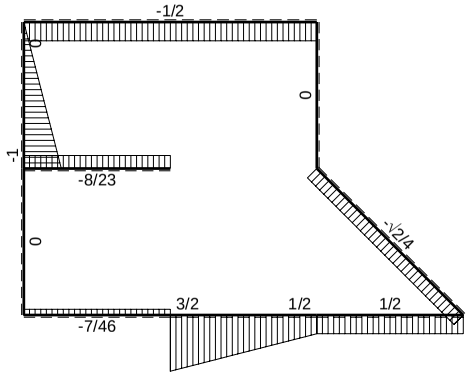


Reazioni iperstatiche in soluzione: $X=V_I$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 420 mm, F = 590 N
Calcolare sulla sezione A la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.

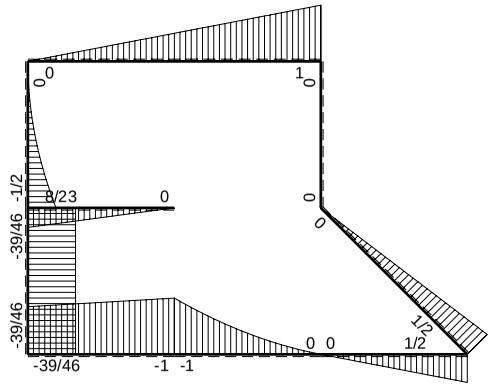




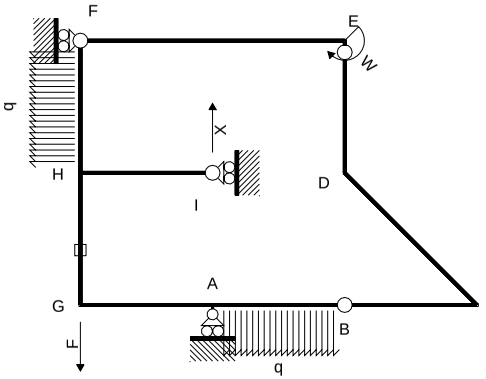
←□→ F

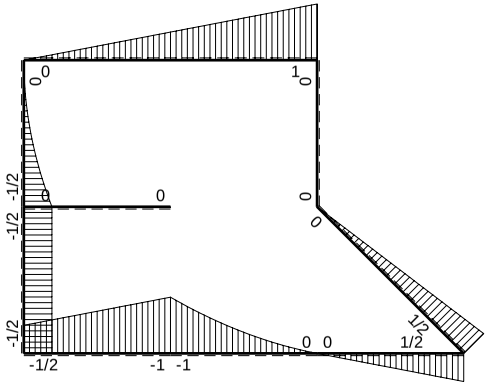


↑□↓ F

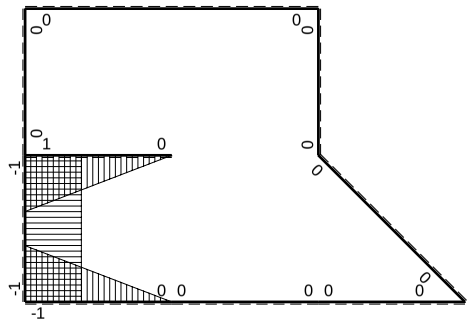


↺□↻ Fb





↺□↻ M_0 flessione da carichi assegnati



↺□↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fb+3/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fx+1/2qx^2$	0	0		
BC b	0	1/2Fx	0	0	0	0
CB b	0	-1/2Fb+1/2Fx	0	0		
CD $\sqrt{2}$b	0	$1/2Fb-\sqrt{2}/4Fx$	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0		
EF 2b	0	Fb-1/2Fx	0	0	0	0
FE 2b	0	-1/2Fx	0	0		
GA b	-b+x	-1/2Fb-1/2Fx	$1/2Fb^2-1/2Fx^2$	$b^2-2bx+x^2$	$1/3Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	Fb-1/2Fx	$Fbx-1/2Fx^2$	x^2		
FH b	0	$-1/2qx^2$	0	0	0	0
HF b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
HI b	b-x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
IH b	-x	0	0	x^2		
HG b	-b	-1/2Fb	$1/2Fb^2$	b^2	$1/2Fb^3/EJ$	Xb^3/EJ
GH b	b	1/2Fb	$1/2Fb^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				$-Fb^3/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-1/2Fb^3/EJ$	$1/4Xb^3/EJ$
	totali				$-2/3Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_I$				8/23F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 236422 ESΣ05.xxxx.044

$$L_{GA}^{XX} = \int_o^b \left(1 - 2\,x/b + x^2/b^2\right) b^2\,1/EJ\,dx = \left[\, x - x^2/b + 1/3\,x^3/b^2 \,\right]_o^b b^2\,1/EJ$$
$$= \left(b - b + 1/3\,b \right) b^2\,1/EJ = 1/3\;b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(x^2/b^2 \right) b^2\,1/EJ\,dx = \left[1/3\,x^3/b^2\right]_o^b b^2\,1/EJ$$
$$= \left(1/3\,b\right) b^2\,1/EJ = 1/3\;b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 - 2\,x/b + x^2/b^2\right) b^2\,1/EJ\,dx = \left[\, x - x^2/b + 1/3\,x^3/b^2 \,\right]_o^b b^2\,1/EJ$$
$$= \left(b - b + 1/3\,b \right) b^2\,1/EJ = 1/3\;b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(x^2/b^2 \right) b^2\,1/EJ\,dx = \left[1/3\,x^3/b^2\right]_o^b b^2\,1/EJ$$
$$= \left(1/3\,b\right) b^2\,1/EJ = 1/3\;b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\,1/EJ\,dx = \left[\, x \,\right]_o^b b^2\,1/EJ$$
$$= \left(b \right) b^2\,1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\,1/EJ\,dx = \left[\, x \,\right]_o^b b^2\,1/EJ$$
$$= \left(b \right) b^2\,1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(1/2 - 1/2\,x^2/b^2\right) Fb^2\,1/EJ\,dx = \left[1/2\,x - 1/6\,x^3/b^2\,\right]_o^b Fb^2\,1/EJ$$
$$= \left(1/2\,b - 1/6\,b\right) Fb^2\,1/EJ = 1/3\;Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(x/b - 1/2\,x^2/b^2 \right) Fb^2\,1/EJ\,dx = \left[1/2\,x^2/b - 1/6\,x^3/b^2\,\right]_o^b Fb^2\,1/EJ$$
$$= \left(1/2\,b - 1/6\,b\right) Fb^2\,1/EJ = 1/3\;Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(1/2\right) Fb^2\,1/EJ\,dx - 1\;\;(-1)\;\;(-1)\;\;Fb^3/EJ = \left[1/2\,x\,\right]_o^b Fb^2\,1/EJ\;-1\;\;(-1)\;\;(-1)\;\;Fb^3/EJ$$
$$= \left(1/2\,b\right) Fb^2\,1/EJ\;-1\;\;(-1)\;\;(-1)\;\;Fb^3/EJ = -1/2\;\;Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(1/2\right) Fb^2\,1/EJ\,dx - 1\;\;(-1)\;\;(-1)\;\;Fb^3/EJ = \left[1/2\,x\,\right]_o^b Fb^2\,1/EJ\;-1\;\;(-1)\;\;(-1)\;\;Fb^3/EJ$$
$$= \left(1/2\,b\right) Fb^2\,1/EJ\;-1\;\;(-1)\;\;(-1)\;\;Fb^3/EJ = -1/2\;\;Fb^3/EJ$$

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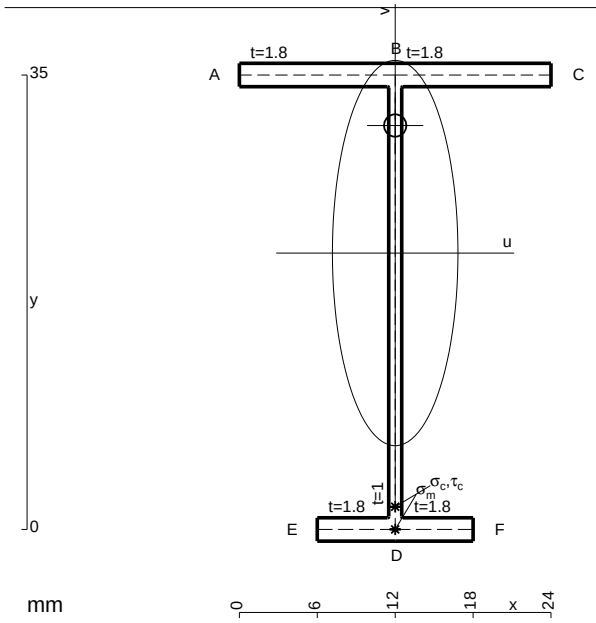

A = 99.8 mm^2
J_u = 21986. mm^4
J_v = 2333. mm^4
J_t = 81.65 mm^4
y_o = 9.824 mm
y_g = 21.29 mm
T_y = 885. N
M_x = -247800. Nmm
x_m = 12. mm
v_m = -21.29 mm
$\sigma_m = -Mv/J_u$ = -239.9 N/mm^2
y_c = 3. mm
u_c = -12. mm
v_c = -18.29 mm
$\sigma_c = -Mv/J_u$ = -239.9 N/mm^2
$\tau_c = TS^*/tJ_u$ = 18.51 N/mm^2
$\tau_g = TS^*/tJ_u$ = 18.51 N/mm^2
t_c = 590. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 242.1 N/mm^2

ESΣ05.xxxx.044
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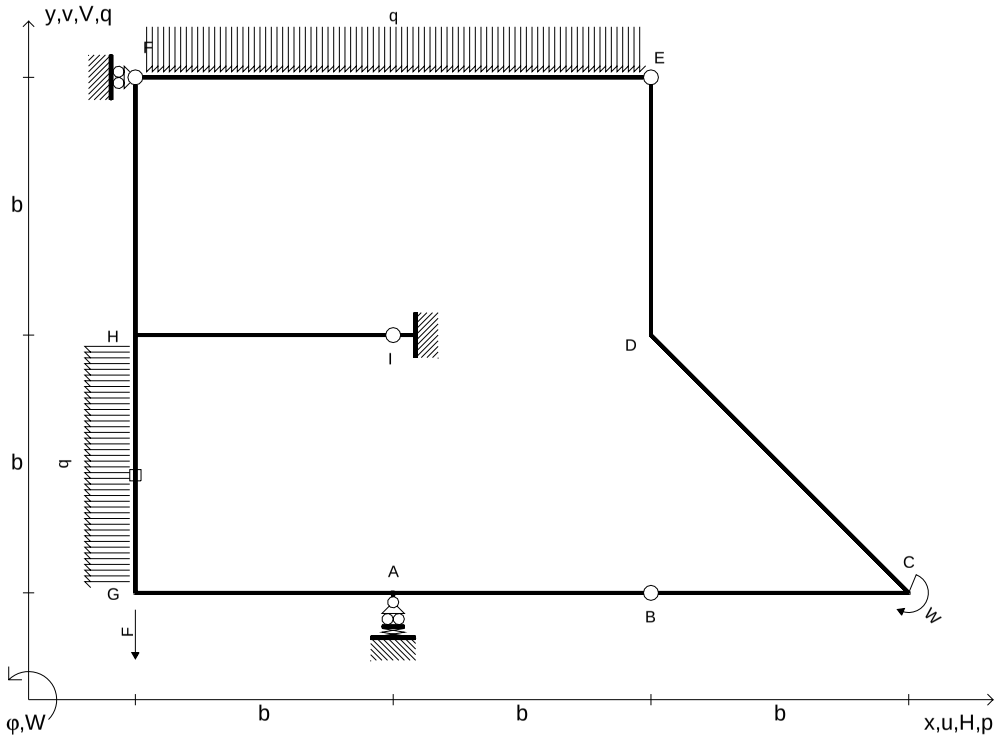
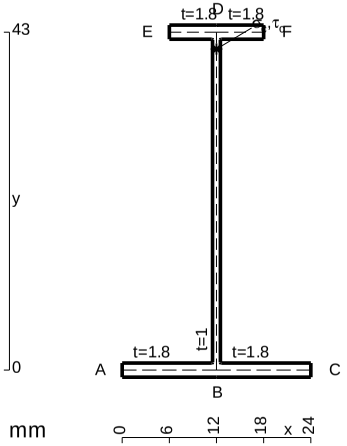


$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_C = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{HG} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione: $X=V_I$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

La trave CD ha la sezione riportata e dimensioni in mm, con:

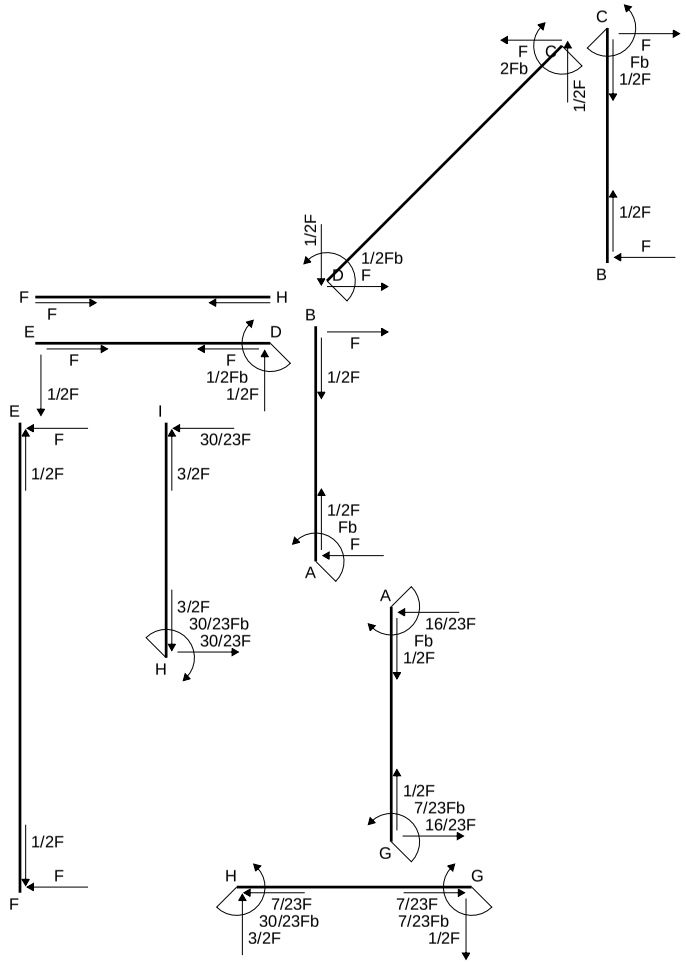
b = 550 mm, F = 240 N

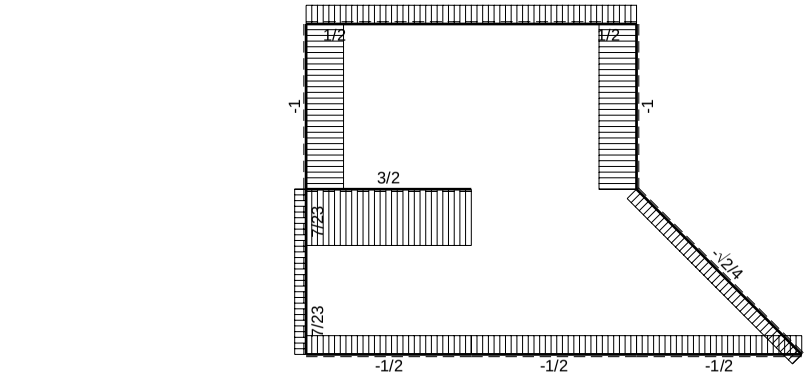
Calcolare sulla sezione C la massima tensione normale σ_m.

Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.

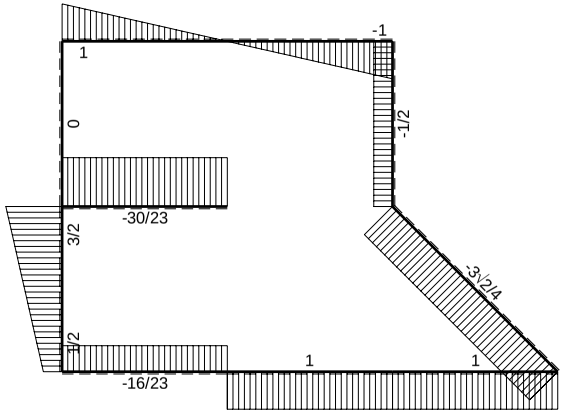
Lembo inferiore sezione su tratteggio trave, a destra da C a D

Elongazione termica specifica ε assegnata su asta HG.

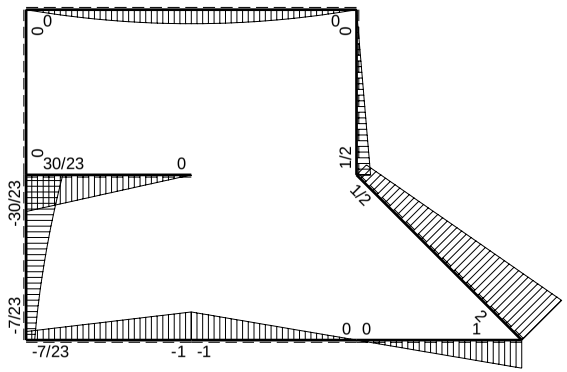




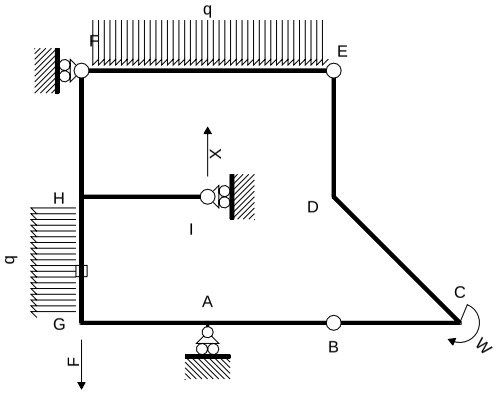
F



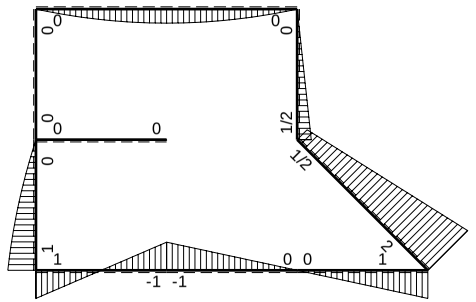
F



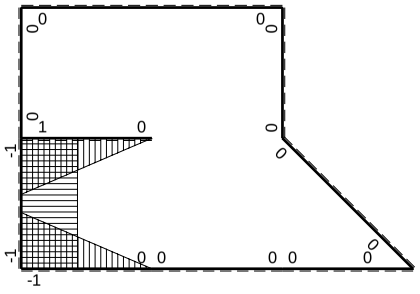
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fb+Fx	0	0	0	0
BA b	0	Fx	0	0		
BC b	0	Fx	0	0	0	0
CB b	0	-Fb+Fx	0	0		
CD √2b	0	2Fb-3√2/4Fx	0	0	0	0
DE b	0	1/2Fb-1/2Fx	0	0	0	0
ED b	0	-1/2Fx	0	0		
EF 2b	0	$-Fx+1/2qx^2$	0	0	0	0
FE 2b	0	$Fx-1/2qx^2$	0	0		
GA b	-b+x	Fb-2Fx	$-Fb^2+3Fbx-2Fx^2$	$b^2-2bx+x^2$	$-1/6Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	Fb-2Fx	$Fbx-2Fx^2$	x^2		
FH b	0	0	0	0	0	0
HF b	0	0	0	0		
HI b	b-x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
IH b	-x	0	0	x^2		
HG b	-b	$3/2Fx-1/2qx^2$	$-3/2Fbx+1/2Fx^2$	b^2	$-7/12Fb^3/EJ$	Xb^3/EJ
GH b	b	$-Fb+1/2Fx+1/2qx^2$	$-Fb^2+1/2Fbx+1/2Fx^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				$-Fb^3/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-3/4Fb^3/EJ$	$1/4Xb^3/EJ$
	totali				$-5/2Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_I$				30/23F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 237351 ESΣ05.xxxx.055

$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-1 +3\, x/b -2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[- x +3/2\, x^2/b -2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(- b +3/2\, b -2/3\, b \,\right) Fb^2\, 1/EJ = -1/6\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(\, x/b -2\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[1/2\, x^2/b -2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(1/2\, b -2/3\, b \,\right) Fb^2\, 1/EJ = -1/6\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-3/2\, x/b +1/2\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[-3/4\, x^2/b +1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(-3/4\, b +1/6\, b \,\right) Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -19/12\ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-1 +1/2\, x/b +1/2\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[- x +1/4\, x^2/b +1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(- b +1/4\, b +1/6\, b \,\right) Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -19/12\ Fb^3/EJ$$

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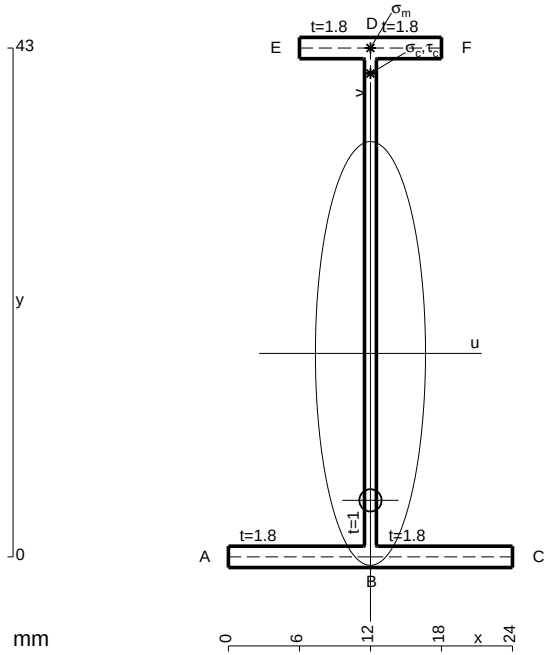

A = 107.8 mm^2
J_u = 34579. mm^4
J_v = 2333. mm^4
J_t = 84.32 mm^4
y_o = -12.41 mm
y_g = 17.19 mm
N = -84.85 N
T_y = -254.6 N
M_x = 264000. Nmm
x_m = 12. mm
y_m = 43. mm
v_m = 25.81 mm
$\sigma_m = N/A - Mv/J_u$ = -197.8 N/mm^2
y_c = 2. mm
u_c = -12. mm
v_c = -15.19 mm
$\sigma_c = N/A - Mv/J_u$ = -197.8 N/mm^2
$\tau_c = TS^*/tJ_u$ = 4.104 N/mm^2
$\tau_g = TS^*/tJ_u$ = 4.104 N/mm^2
t_c = 240. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 198. N/mm^2

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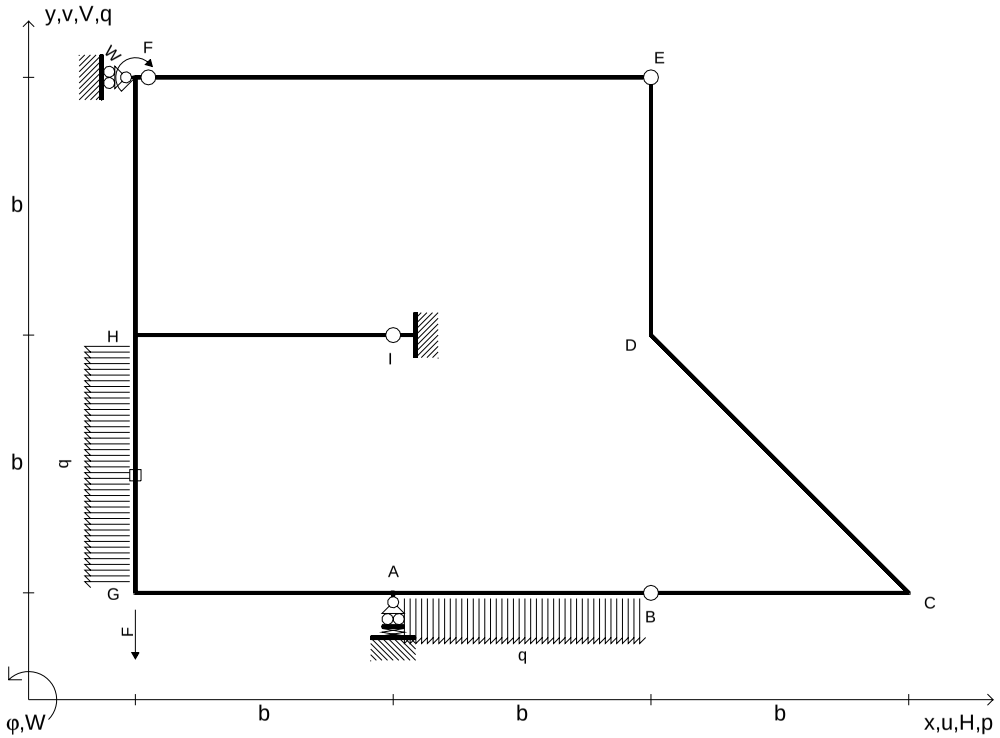
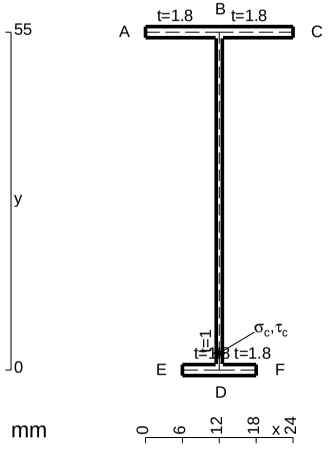


$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_F = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{HG} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione: $X=V_I$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

La trave FH ha la sezione riportata e dimensioni in mm, con:

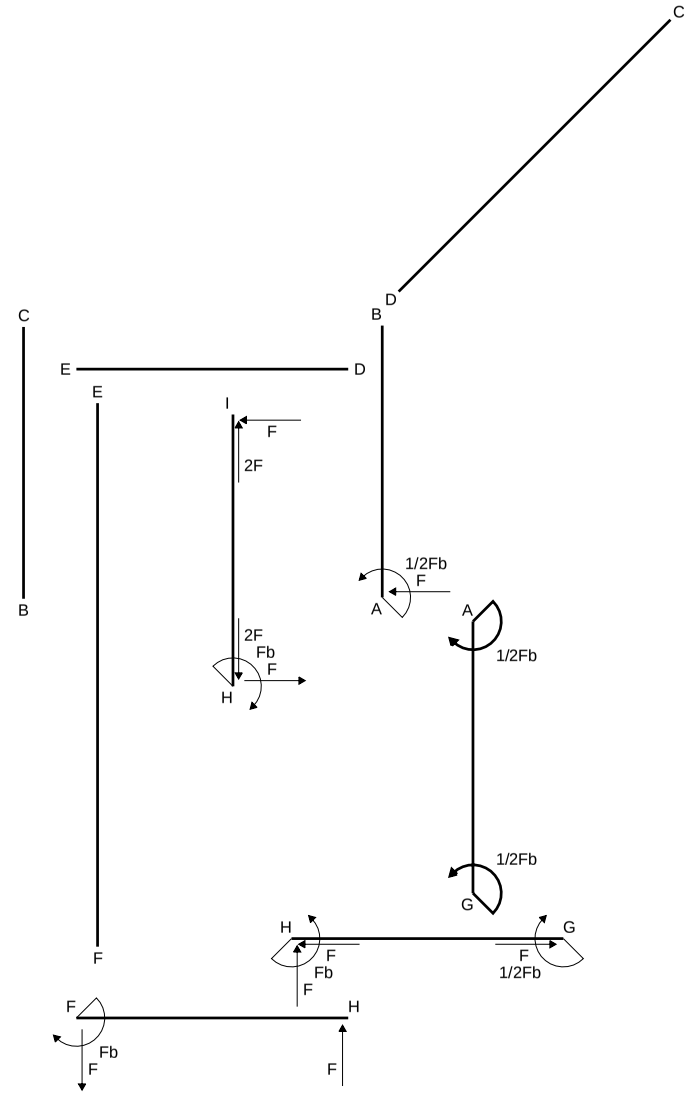
b = 700 mm, F = 520 N

Calcolare sulla sezione F la massima tensione normale σ_m.

Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.

Lembo inferiore sezione su tratteggio trave, a destra da F a H

Elongazione termica specifica ε assegnata su asta HG.



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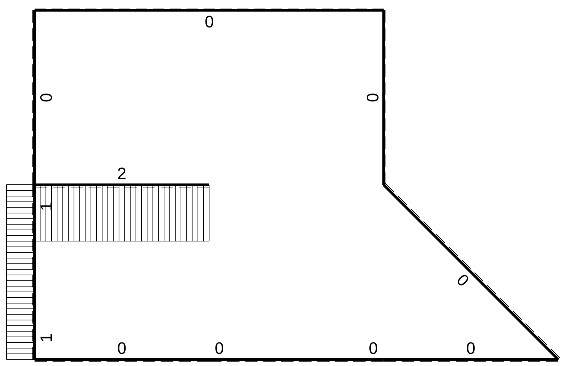
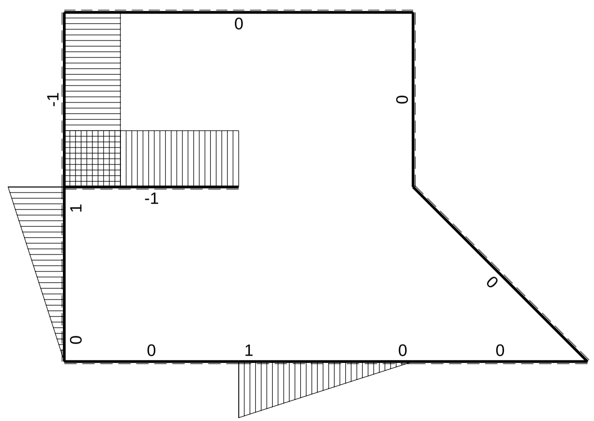
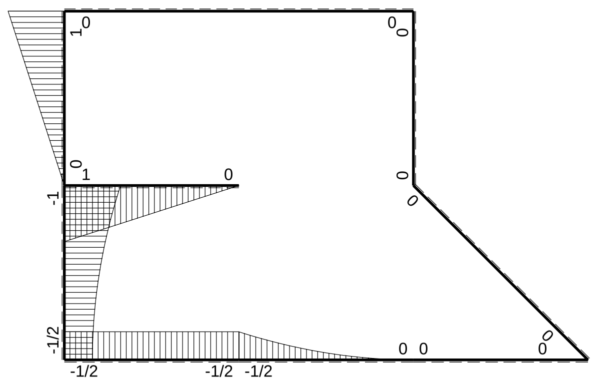


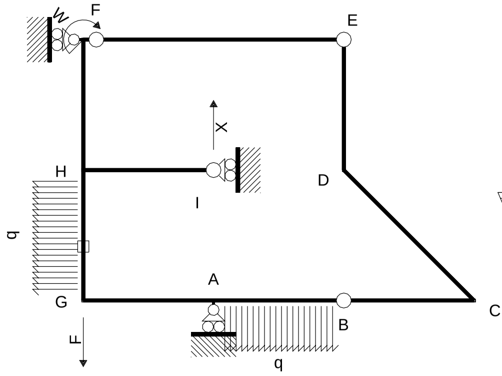
F



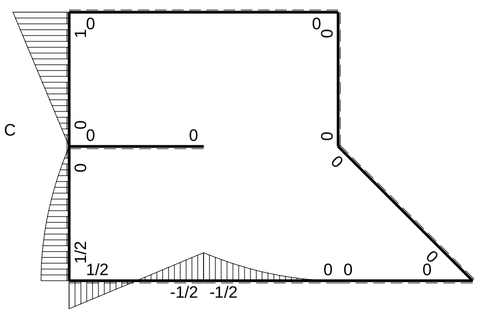
F



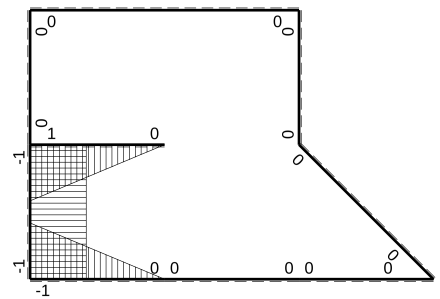
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0
BA b	0	$1/2qx^2$	0	0		
BC b	0	0	0	0	0	0
CB b	0	0	0	0		
CD $\sqrt{2}b$	0	0	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0		
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0		
GA b	-b+x	1/2Fb-Fx	$-1/2Fb^2+3/2Fbx-Fx^2$	$b^2-2bx+x^2$	$-1/12Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	1/2Fb-Fx	$1/2Fbx-Fx^2$	x^2		
FH b	0	Fb-Fx	0	0	0	0
HF b	0	-Fx	0	0		
HI b	b-x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
IH b	-x	0	0	x^2		
HG b	-b	$Fx-1/2qx^2$	$-Fbx+1/2Fx^2$	b^2	$-1/3Fb^3/EJ$	Xb^3/EJ
GH b	b	$-1/2Fb+1/2qx^2$	$-1/2Fb^2+1/2Fx^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				$-Fb^3/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-1/2Fb^3/EJ$	$1/4Xb^3/EJ$
	totali				$-23/12Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_I$				F	

Sviluppi di calcolo iperstatica

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$L_{GA}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AG}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HI}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IH}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HG}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{GH}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{GA}^{Xo} = \int_o^b (-1/2 + 3/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x + 3/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b + 3/4\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -1/12\ Fb^3/EJ$

$L_{AG}^{Xo} = \int_o^b (1/2\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -1/12\ Fb^3/EJ$

$L_{HG}^{Xo} = \int_o^b (-\, x/b + 1/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= [-1/2\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= (-1/2\, b + 1/6\, b \,)\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -4/3\ Fb^3/EJ$

$L_{GH}^{Xo} = \int_o^b (-1/2 + 1/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= [-1/2\, x + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= (-1/2\, b + 1/6\, b \,)\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -4/3\ Fb^3/EJ$

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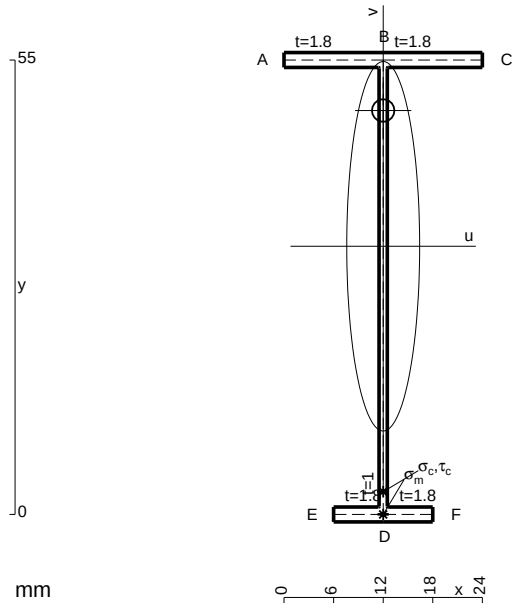

$A = 119.8\ mm^2$
$J_u = 59924.\ mm^4$
$J_v = 2333.\ mm^4$
$J_t = 88.32\ mm^4$
$y_o = 16.43\ mm$
$y_g = 32.46\ mm$
$T_y = -520.\ N$
$M_x = 364000.\ Nmm$
$x_m = 12.\ mm$
$v_m = -32.46\ mm$
$\sigma_m = -Mv/J_u = 197.2\ N/mm^2$
$y_c = 3.\ mm$
$u_c = -12.\ mm$
$v_c = -29.46\ mm$
$\sigma_c = -Mv/J_u = 197.2\ N/mm^2$
$\tau_c = TS^*/tJ_u = 6.084\ N/mm^2$
$\tau_g = TS^*/tJ_u = 6.084\ N/mm^2$
$t_c = 520.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 197.4\ N/mm^2$

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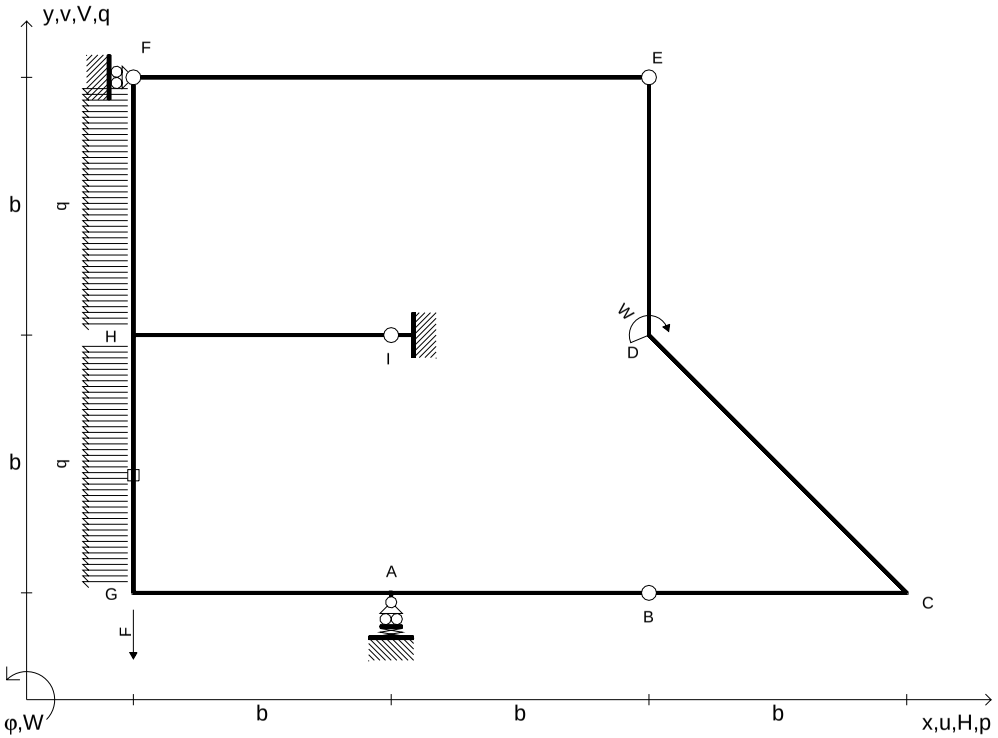
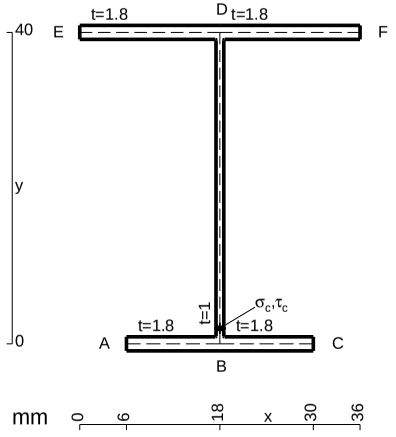


$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

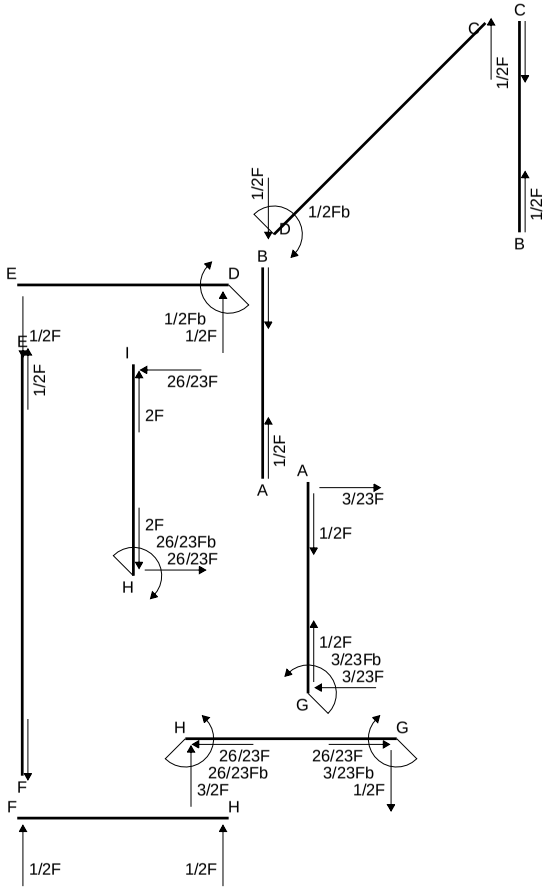
$W_D = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

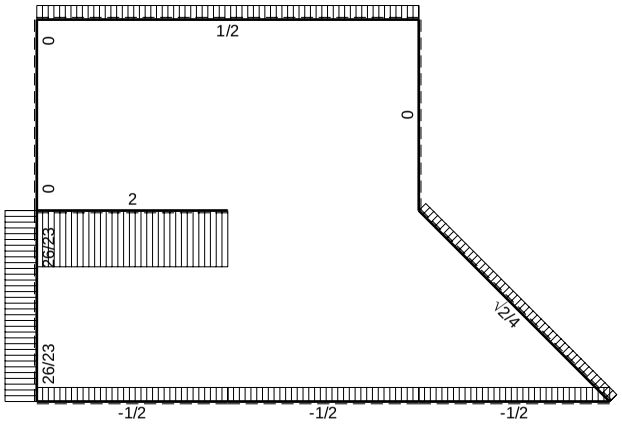
$p_{FH} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{HG} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione: $X=V_I$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DE ha la sezione riportata e dimensioni in mm, con:
b = 340 mm, F = 2420 N
Calcolare sulla sezione D la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a E
Elongazione termica specifica ε assegnata su asta HG.





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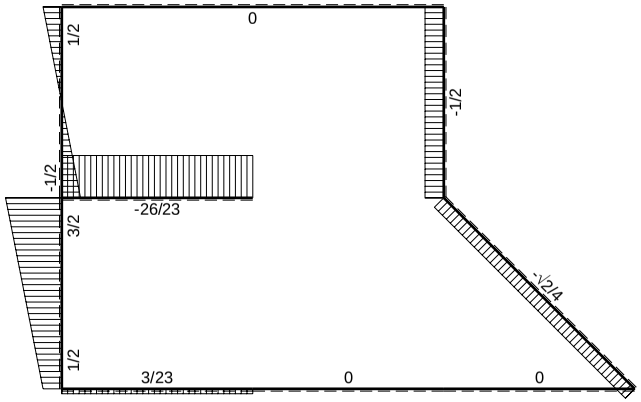

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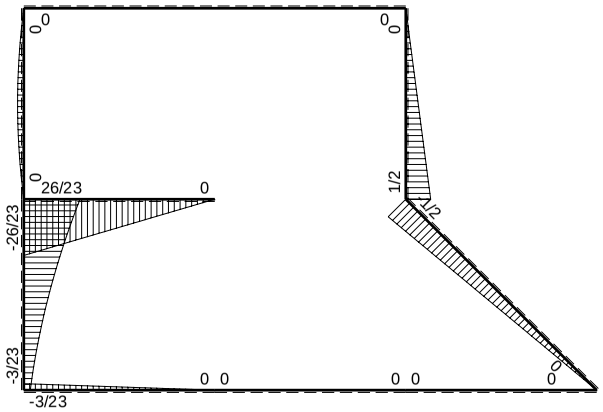
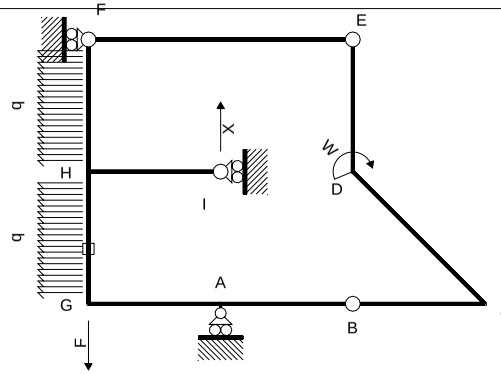
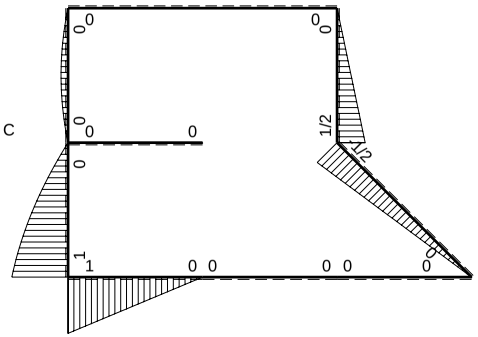


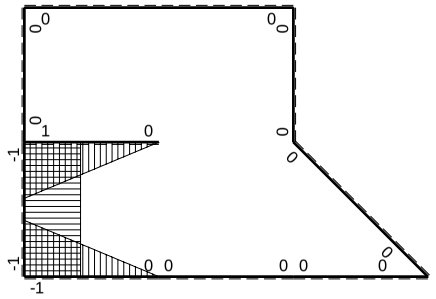
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	0	0	0	0	0	0
CB b	0	0	0	0		
CD √2b	0	$-\sqrt{2}/4Fx$	0	0	0	0
DE b	0	1/2Fb-1/2Fx	0	0	0	0
ED b	0	-1/2Fx	0	0		
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0		
GA b	-b+x	Fb-Fx	$-Fb^2+2Fbx-Fx^2$	$b^2-2bx+x^2$	$-1/3Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	-Fx	$-Fx^2$	x^2		
FH b	0	$1/2Fx-1/2qx^2$	0	0	0	0
HF b	0	$-1/2Fx+1/2qx^2$	0	0		
HI b	b-x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
IH b	-x	0	0	x^2		
HG b	-b	$3/2Fx-1/2qx^2$	$-3/2Fbx+1/2Fx^2$	b^2	$-7/12Fb^3/EJ$	Xb^3/EJ
GH b	b	$-Fb+1/2Fx+1/2qx^2$	$-Fb^2+1/2Fbx+1/2Fx^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				$-Fb^3/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-1/4Fb^3/EJ$	$1/4Xb^3/EJ$
	totali				$-13/6Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_I$				26/23F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238991 ESΣ05.xxxx.080

$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-1 +2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[- x + x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(- b + b - 1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(- x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[-1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-3/2\, x/b + 1/2\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[-3/4\, x^2/b + 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(-3/4\, b + 1/6\, b \,\right) Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -19/12\ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-1 + 1/2\, x/b + 1/2\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[- x + 1/4\, x^2/b + 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(- b + 1/4\, b + 1/6\, b \,\right) Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -19/12\ Fb^3/EJ$$

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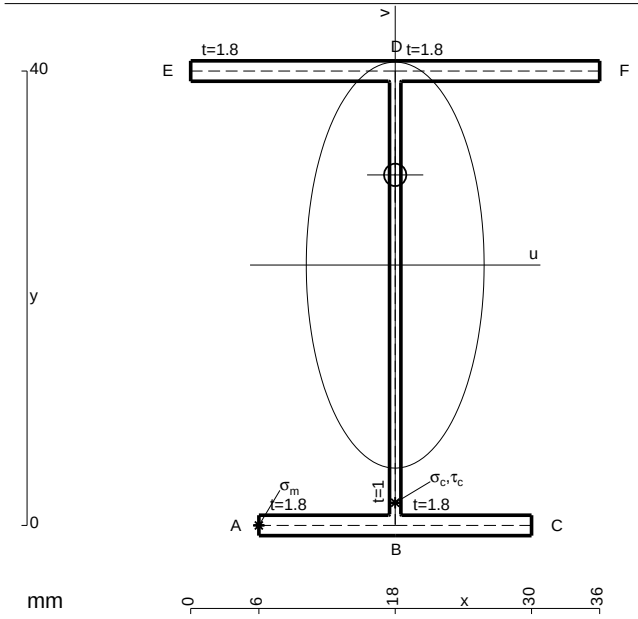

A = 148. mm^2
J_u = 47272. mm^4
J_v = 9072. mm^4
J_t = 130. mm^4
y_o = 7.938 mm
y_g = 22.92 mm
T_y = -1210. N
M_x = 411400. Nmm
x_m = 6. mm
u_m = -12. mm
v_m = -22.92 mm
$\sigma_m = -Mv/J_u$ = 199.5 N/mm^2
x_c = 18. mm
v_c = -22.92 mm
$\sigma_c = -Mv/J_u$ = 199.5 N/mm^2
$\tau_c = TS^*/tJ_u$ = 25.34 N/mm^2
$\tau_g = TS^*/tJ_u$ = 25.34 N/mm^2
t_c = 2420. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 204.2 N/mm^2

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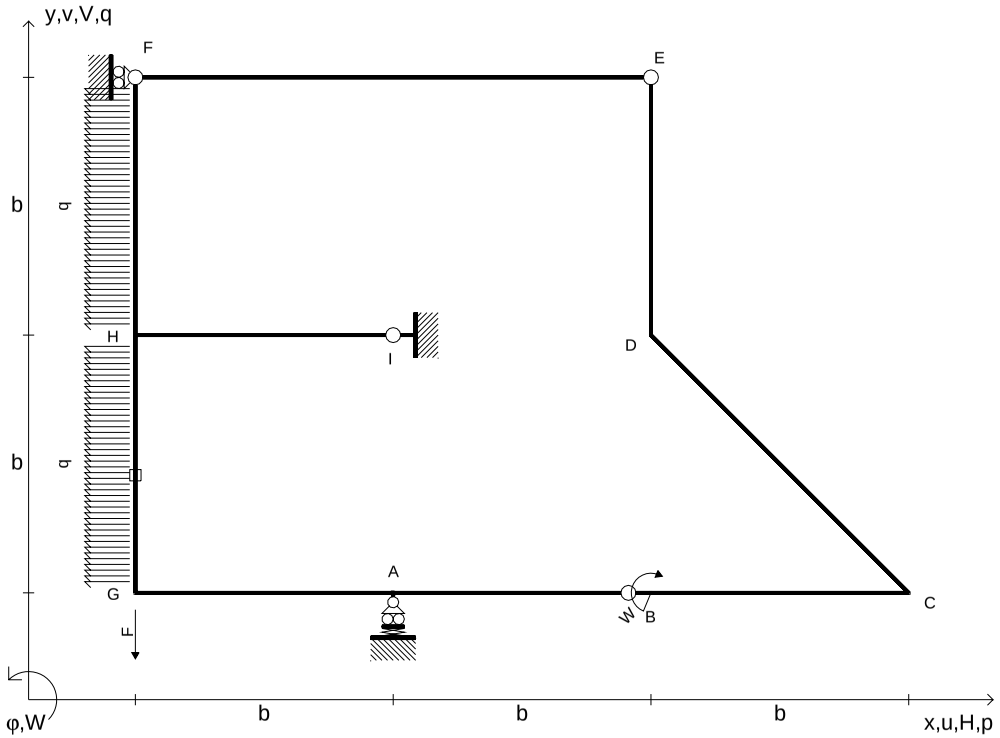
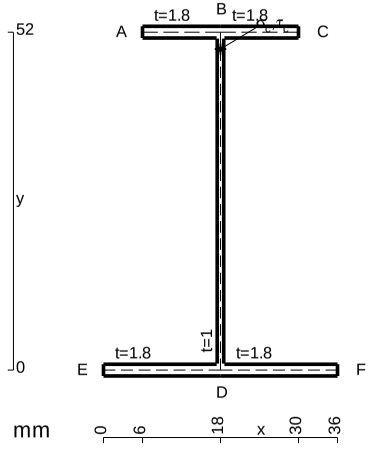


$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

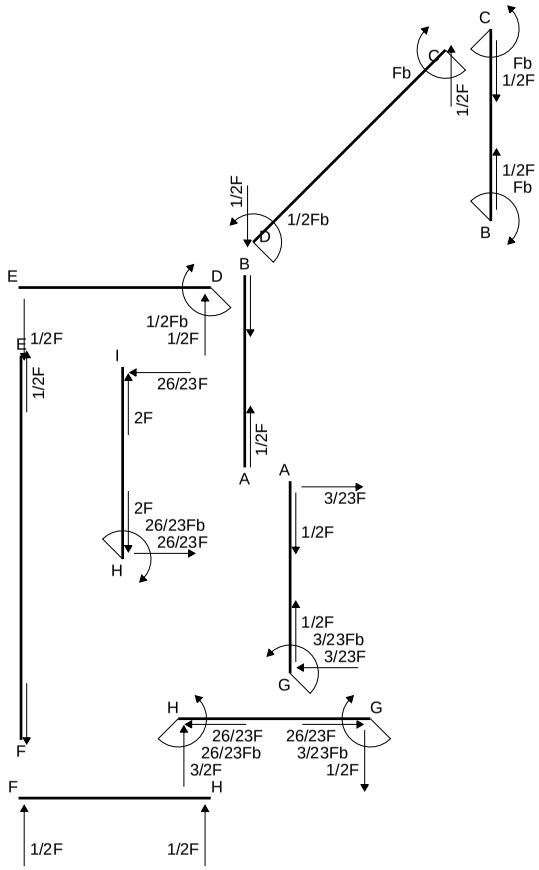
$W_B = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

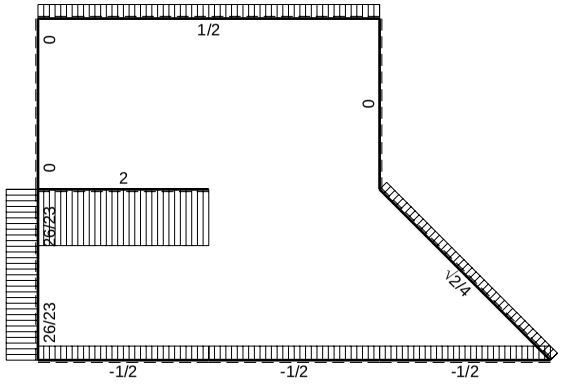
$p_{FH} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

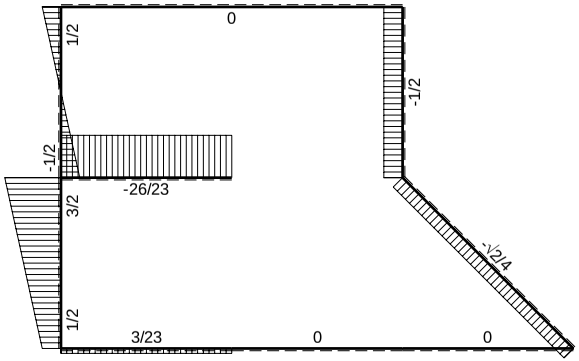


Reazioni iperstatiche in soluzione: $X=V_I$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 700 mm, F = 840 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Elongazione termica specifica ε assegnata su asta HG.

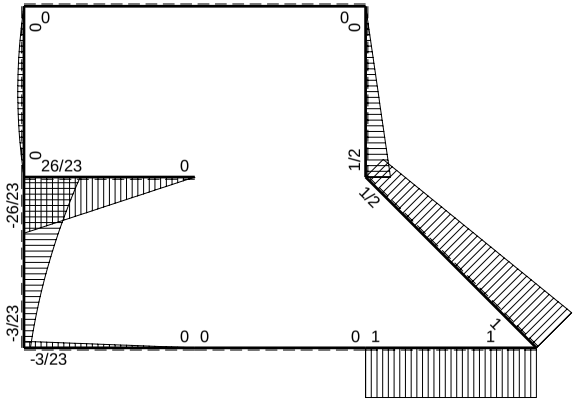




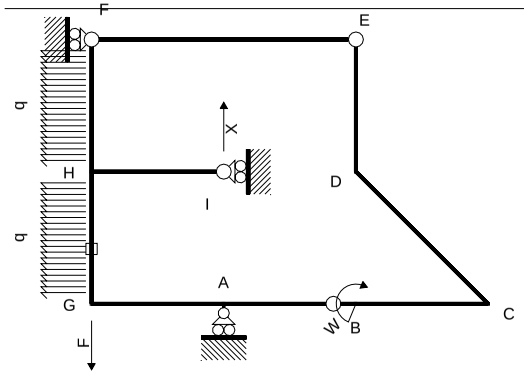
←□→ F



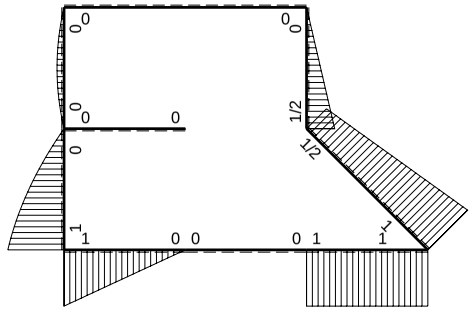
↑□↓ F



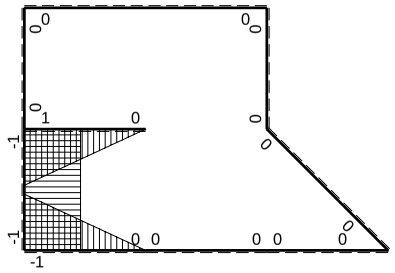
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_0 flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	0	Fb	0	0	0	0
CB b	0	-Fb	0	0		
CD $\sqrt{2}$b	0	Fb-$\sqrt{2}$/4Fx	0	0	0	0
DE b	0	1/2Fb-1/2Fx	0	0	0	0
ED b	0	-1/2Fx	0	0		
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0		
GA b	-b+x	Fb-Fx	$-Fb^2+2Fbx-Fx^2$	$b^2-2bx+x^2$	$-1/3Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	-Fx	$-Fx^2$	x^2		
FH b	0	$1/2Fx-1/2qx^2$	0	0	0	0
HF b	0	$-1/2Fx+1/2qx^2$	0	0		
HI b	b-x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
IH b	-x	0	0	x^2		
HG b	-b	$3/2Fx-1/2qx^2$	$-3/2Fbx+1/2Fx^2$	b^2	$-7/12Fb^3/EJ$	Xb^3/EJ
GH b	b	$-Fb+1/2Fx+1/2qx^2$	$-Fb^2+1/2Fbx+1/2Fx^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				$-Fb^3/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-1/4Fb^3/EJ$	$1/4Xb^3/EJ$
	totali				$-13/6Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_I$				26/23F	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$

$$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$

$$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$

$$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$

$$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$$

$$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$$

$$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b (-1 + 2\,x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x + x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$

$$= (- b + b - 1/3\, b \,)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b (- x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$

$$= (-1/3\, b \,)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b (-3/2\,x/b + 1/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

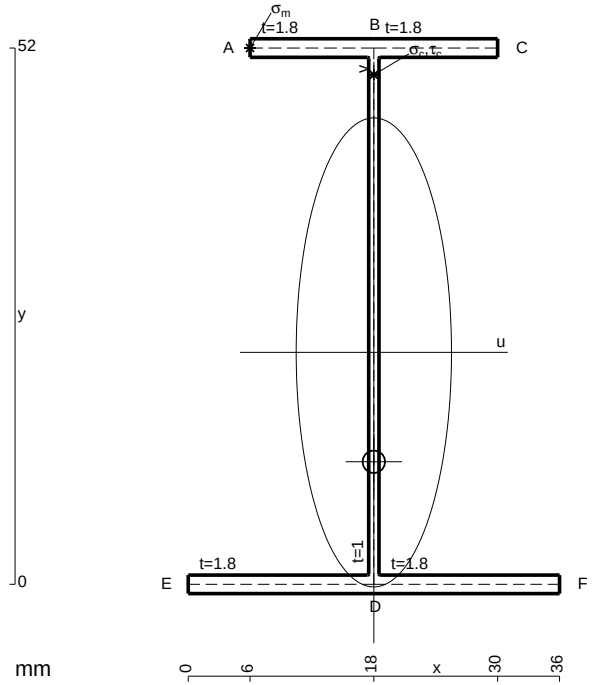
$$= [-3/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= (-3/4\, b + 1/6\, b \,)\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -19/12\ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b (-1 + 1/2\,x/b + 1/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= [- x + 1/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= (- b + 1/4\, b + 1/6\, b \,)\, Fb^2\, 1/EJ \ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -19/12\ Fb^3/EJ$$



$A = 160.\ mm^2$
$J_u = 82754.\ mm^4$
$J_v = 9072.\ mm^4$
$J_t = 134.\ mm^4$
$y_o = -10.6\ mm$
$y_g = 22.49\ mm$
$N = 297.\ N$
$T_y = -297.\ N$
$M_x = 588000.\ Nmm$
$x_m = 6.\ mm$
$y_m = 52.\ mm$
$u_m = -12.\ mm$
$v_m = 29.51\ mm$
$\sigma_m = N/A - Mv/J_u = -207.8\ N/mm^2$
$x_c = 18.\ mm$
$y_c = 52.\ mm$
$v_c = 29.51\ mm$
$\sigma_c = N/A - Mv/J_u = -207.8\ N/mm^2$
$\tau_c = TS^*/tJ_u = 4.575\ N/mm^2$
$\tau_g = TS^*/tJ_u = 4.575\ N/mm^2$
$t_c = 840.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 208.\ N/mm^2$

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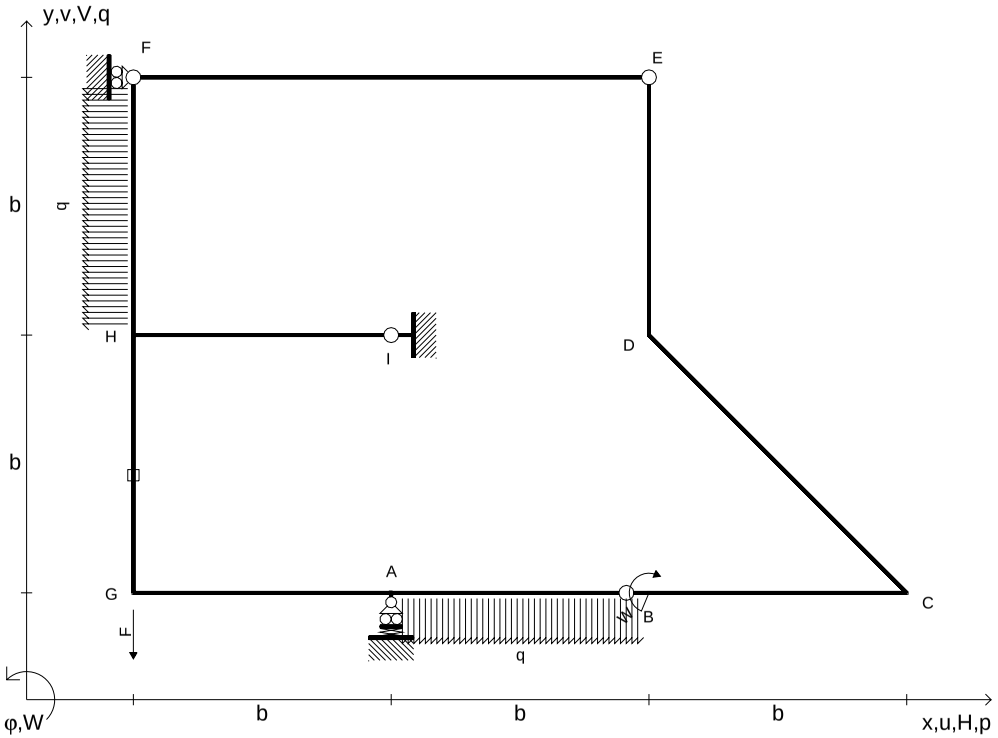
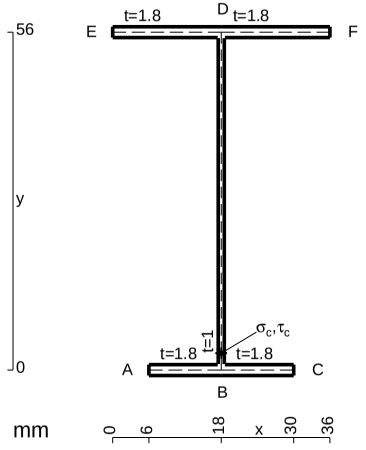


$V_G = -F$ $\quad \varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $\quad EJ_{CD} = EJ$ $\quad EJ_{FH} = EJ$

$W_B = -W = -Fb$ $\quad k_A = 4EJ/b^3$ $\quad EJ_{DE} = EJ$ $\quad EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ $\quad EJ_{AB} = EJ$ $\quad EJ_{EF} = EJ$ $\quad EJ_{HG} = EJ$

$p_{FH} = -q = -F/b$ $\quad EJ_{BC} = EJ$ $\quad EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione: $X=V_A$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

La trave AB ha la sezione riportata e dimensioni in mm, con:

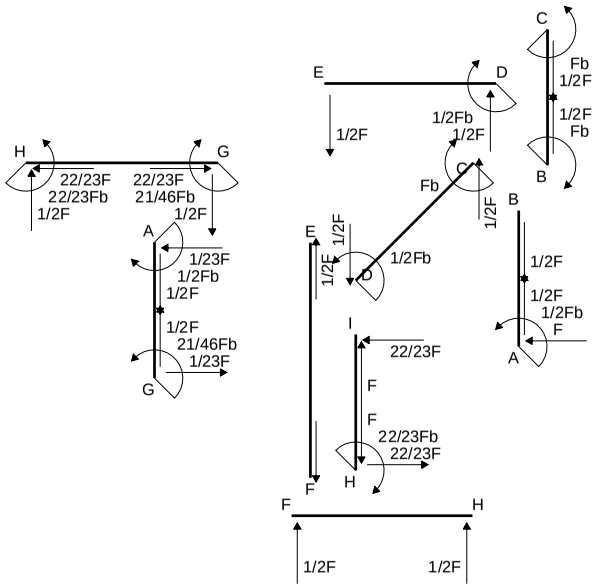
b = 860 mm, F = 1600 N

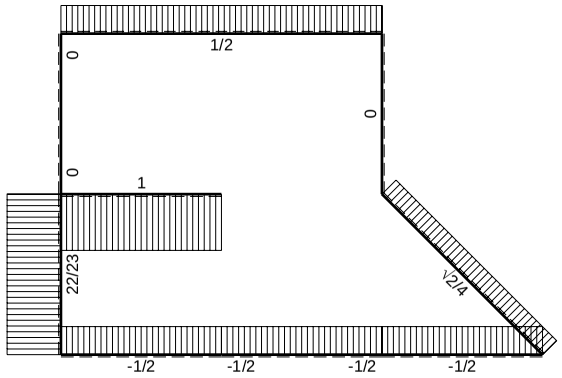
Calcolare sulla sezione A la massima tensione normale σ_m.

Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.

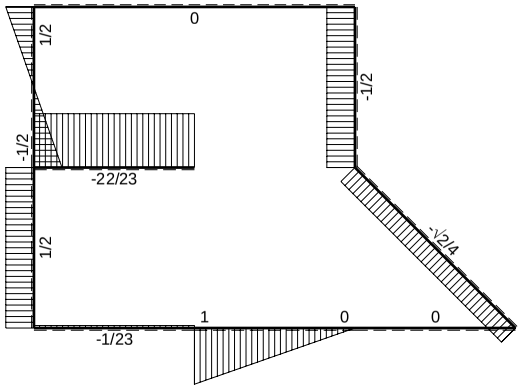
Lembo inferiore sezione su tratteggio trave, a destra da A a B

Elongazione termica specifica ε assegnata su asta HG.

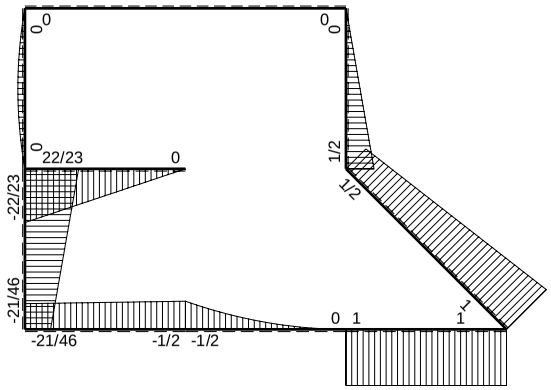




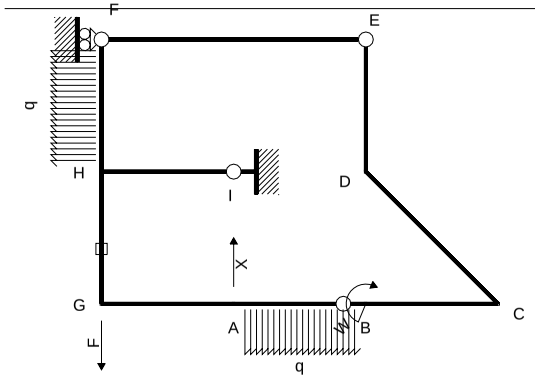
←□→ F



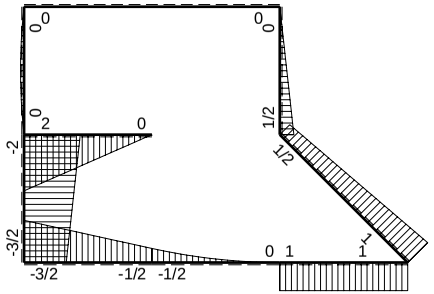
↑□↓ F



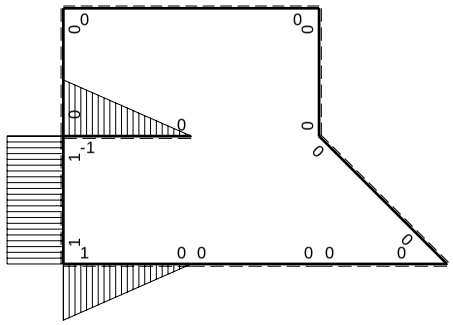
□ Fb



Schema di calcolo iperstatico



□ M_0 flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0
BA b	0	$1/2qx^2$	0	0		
BC b	0	Fb	0	0	0	0
CB b	0	-Fb	0	0		
CD $\sqrt{2}b$	0	$Fb-\sqrt{2}/4Fx$	0	0	0	0
DE b	0	1/2Fb-1/2Fx	0	0	0	0
ED b	0	-1/2Fx	0	0		
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0		
GA b	b-x	$-3/2Fb+Fx$	$-3/2Fb^2+5/2Fbx-Fx^2$	$b^2-2bx+x^2$	$-7/12Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	$1/2Fb+Fx$	$-1/2Fbx-Fx^2$	x^2		
FH b	0	$1/2Fx-1/2qx^2$	0	0	0	0
HF b	0	$-1/2Fx+1/2qx^2$	0	0		
HI b	-b+x	2Fb-2Fx	$-2Fb^2+4Fbx-2Fx^2$	$b^2-2bx+x^2$	$-2/3Fb^3/EJ$	$1/3Xb^3/EJ$
IH b	x	-2Fx	$-2Fx^2$	x^2		
HG b	b	-2Fb+1/2Fx	$-2Fb^2+1/2Fbx$	b^2	$-7/4Fb^3/EJ$	Xb^3/EJ
GH b	-b	3/2Fb+1/2Fx	$-3/2Fb^2-1/2Fbx$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				Fb^3/EJ	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$					$1/4Xb^3/EJ$
	totali				$-2Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_A$				24/23F	

Sviluppi di calcolo iperstatica

$L^{XX}_{GA} = \int_o^b (1 -2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b\,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\, x^2/b^2\,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b\,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 -2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b\,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\, x^2/b^2\,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b\,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x\,]_o^b\, b^2\, 1/EJ$

$= (\, b\,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x\,]_o^b\, b^2\, 1/EJ$

$= (\, b\,)\, b^2\, 1/EJ = b^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-3/2 +5/2\,x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/2\, x +5/4\, x^2/b -1/3\, x^3/b^2\,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b +5/4\, b -1/3\, b\,)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-1/2\,x/b - x^2/b^2\,)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b -1/3\, x^3/b^2\,]_o^b\, Fb^2\, 1/EJ$

$= (-1/4\, b -1/3\, b\,)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L^{Xo}_{HI} = \int_o^b (-2 +4\,x/b -2\,x^2/b^2\,)\, Fb^2\, 1/EJ\, dx = [-2\, x +2\, x^2/b -2/3\, x^3/b^2\,]_o^b\, Fb^2\, 1/EJ$

$= (-2\, b +2\, b -2/3\, b\,)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b (-2\, x^2/b^2\,)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2\,]_o^b\, Fb^2\, 1/EJ$

$= (-2/3\, b\,)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (-2 +1/2\,x/b\,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [-2\, x +1/4\, x^2/b\,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-2\, b +1/4\, b\,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -3/4\ \ Fb^3/EJ$

$L^{Xo}_{GH} = \int_o^b (-3/2 -1/2\,x/b\,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [-3/2\, x -1/4\, x^2/b\,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-3/2\, b -1/4\, b\,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -3/4\ \ Fb^3/EJ$

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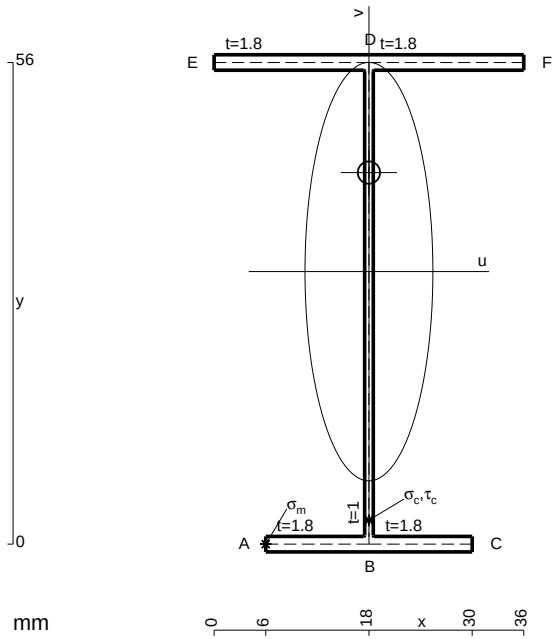

A = 164. mm^2
J_u = 97076. mm^4
J_v = 9072. mm^4
J_t = 135.3 mm^4
y_o = 11.51 mm
y_g = 31.69 mm
N = -800. N
T_y = 1600. N
M_x = -688000. Nmm
x_m = 6. mm
u_m = -12. mm
v_m = -31.69 mm
$\sigma_m = N/A - Mv/J_u$ = -229.5 N/mm^2
x_c = 18. mm
v_c = -31.69 mm
$\sigma_c = N/A - Mv/J_u$ = -229.5 N/mm^2
$\tau_c = TS^*/tJ_u$ = 22.56 N/mm^2
$\tau_g = TS^*/tJ_u$ = 22.56 N/mm^2
t_c = 1600. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 232.8 N/mm^2

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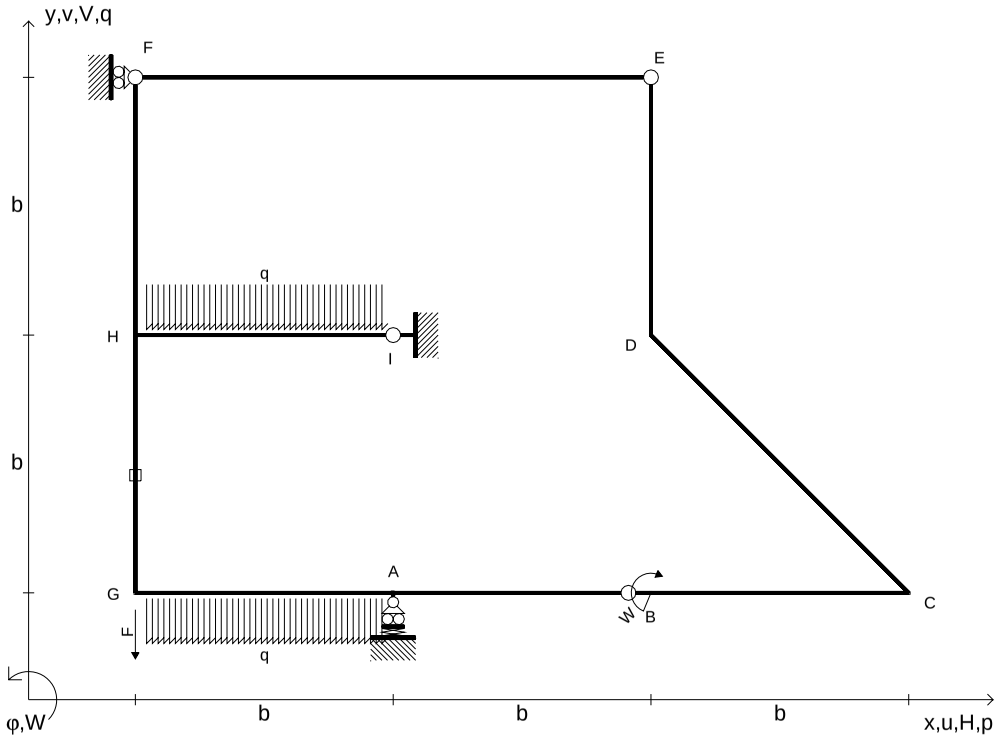
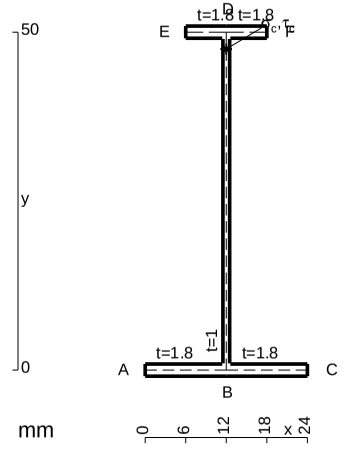


$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

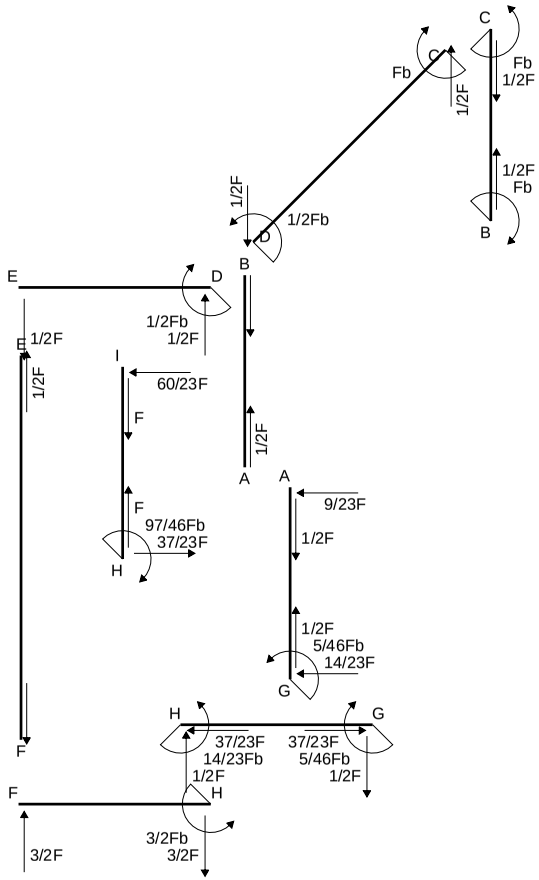
$W_B = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

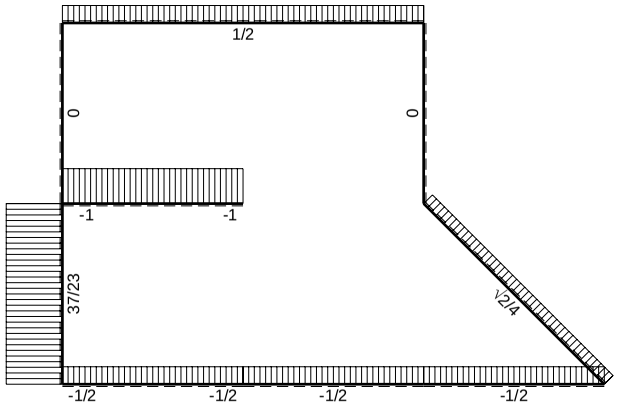
$q_{GA} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$q_{HI} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

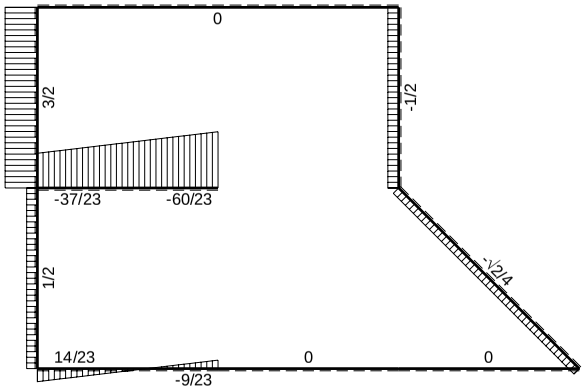


Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 430 mm, F = 500 N
Calcolare sulla sezione H la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

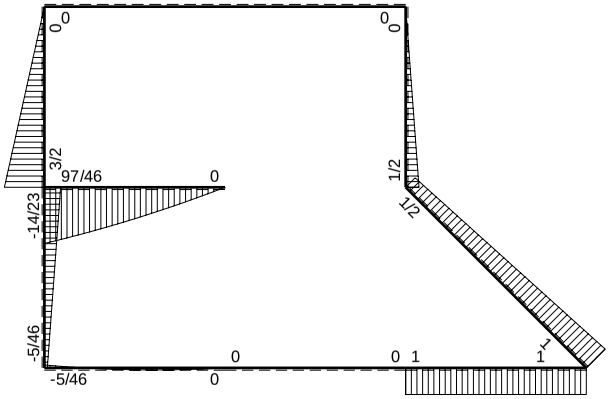




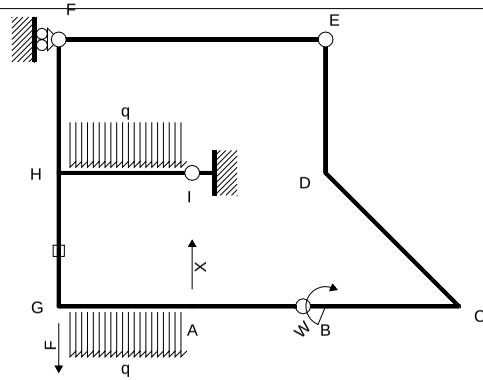
←□→ F



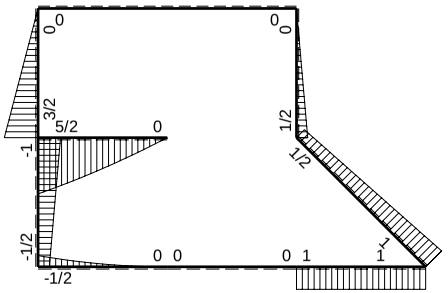
□↑↓ F



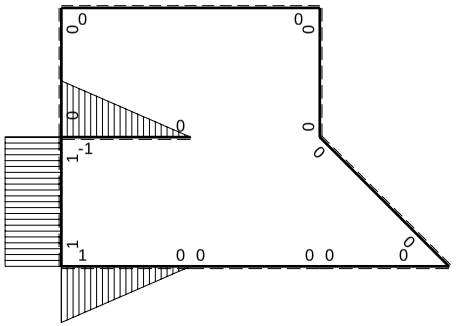
□ Fb



Schema di calcolo iperstatico



□ M_0 flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	0	Fb	0	0	0	0
CB b	0	-Fb	0	0		
CD $\sqrt{2}b$	0	$Fb-\sqrt{2}/4Fx$	0	0	0	0
DE b	0	1/2Fb-1/2Fx	0	0	0	0
ED b	0	-1/2Fx	0	0		
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0		
GA b	b-x	$-1/2Fb+Fx-1/2qx^2$	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	$b^2-2bx+x^2$	$-1/8Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	$1/2qx^2$	$-1/2qx^3$	x^2		
FH b	0	3/2Fx	0	0	0	0
HF b	0	-3/2Fb+3/2Fx	0	0		
HI b	-b+x	$5/2Fb-2Fx-1/2qx^2$	$-5/2Fb^2+9/2Fbx-3/2Fx^2-1/2qx^3$	$b^2-2bx+x^2$	$-7/8Fb^3/EJ$	$1/3Xb^3/EJ$
IH b	x	$-3Fx+1/2qx^2$	$-3Fx^2+1/2qx^3$	x^2		
HG b	b	-Fb+1/2Fx	$-Fb^2+1/2Fbx$	b^2	$-3/4Fb^3/EJ$	Xb^3/EJ
GH b	-b	1/2Fb+1/2Fx	$-1/2Fb^2-1/2Fbx$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				Fb^3/EJ	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$					$1/4Xb^3/EJ$
	totali				$-3/4Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_A$				9/23F	

PROCEDIMENTO E RISULTATI 982088 ESΣ05.xxxx.100

Sviluppi di calcolo iperstatica

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$L^{XX}_{GA} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AG} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{HI} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{IH} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{HG} = \int_o^b \left(1\ \right) b^2\ 1/EJ\ dx = \left[\ x\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b\ \right) b^2\ 1/EJ\ = \ b^3/EJ$

$L^{XX}_{GH} = \int_o^b \left(1\ \right) b^2\ 1/EJ\ dx = \left[\ x\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b\ \right) b^2\ 1/EJ\ = \ b^3/EJ$

$L^{Xo}_{GA} = \int_o^b \left(-1/2 +3/2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx$

$= \left[-1/2\ x +3/4\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-1/2\ b +3/4\ b -1/2\ b +1/8\ b\ \right) Fb^2\ 1/EJ\ = -1/8\ \ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b \left(-1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx = \left[-1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-1/8\ b\ \right) Fb^2\ 1/EJ\ = -1/8\ \ Fb^3/EJ$

$L^{Xo}_{HI} = \int_o^b \left(-5/2 +9/2\ x/b -3/2\ x^2/b^2 -1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx$

$= \left[-5/2\ x +9/4\ x^2/b -1/2\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-5/2\ b +9/4\ b -1/2\ b -1/8\ b\ \right) Fb^2\ 1/EJ\ = -7/8\ \ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b \left(-3\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx = \left[-\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-\ b +1/8\ b\ \right) Fb^2\ 1/EJ\ = -7/8\ \ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b \left(-1 +1/2\ x/b\ \right) Fb^2\ 1/EJ\ dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

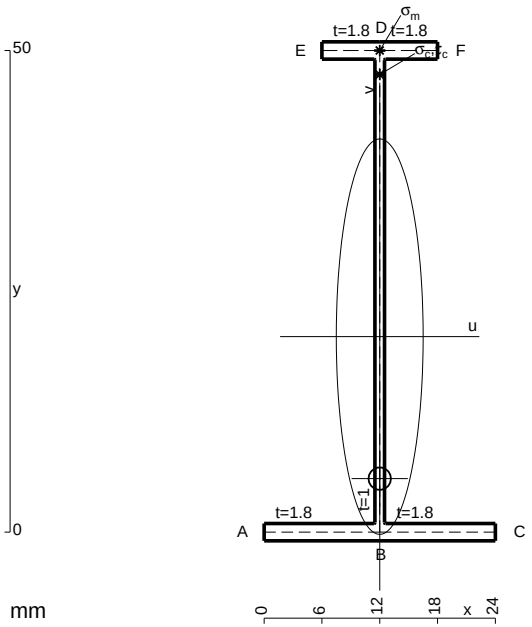
$= \left[-\ x +1/4\ x^2/b\ \right]_o^b Fb^2\ 1/EJ\ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= \left(-\ b +1/4\ b\ \right) Fb^2\ 1/EJ\ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = 1/4\ \ Fb^3/EJ$

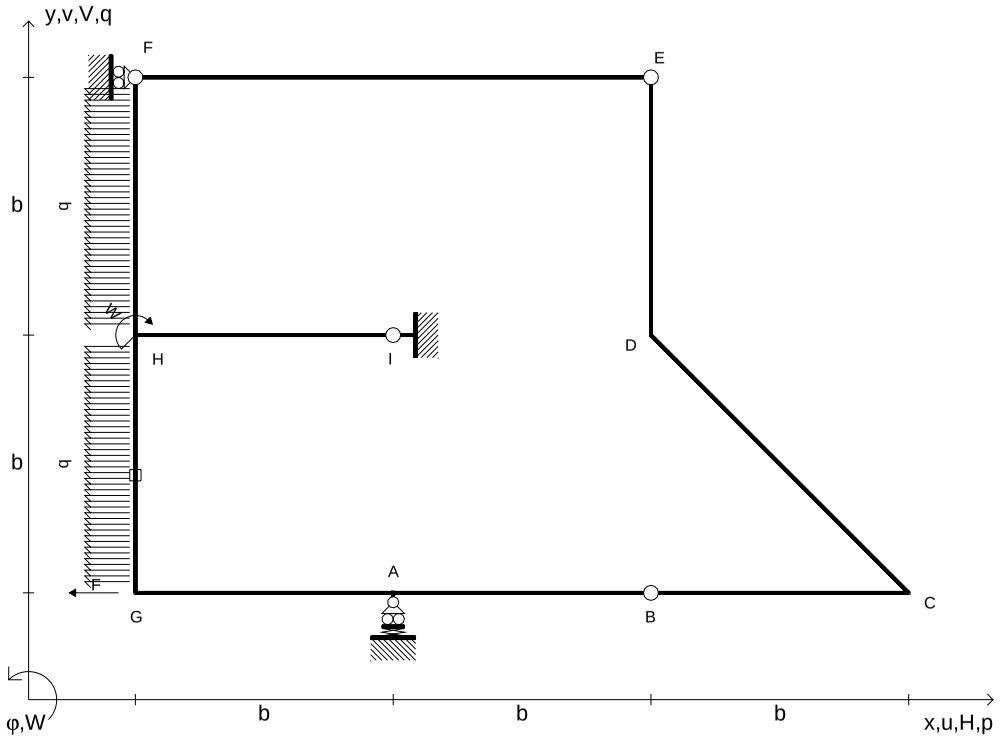
$L^{Xo}_{GH} = \int_o^b \left(-1/2 -1/2\ x/b\ \right) Fb^2\ 1/EJ\ dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= \left[-1/2\ x -1/4\ x^2/b\ \right]_o^b Fb^2\ 1/EJ\ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

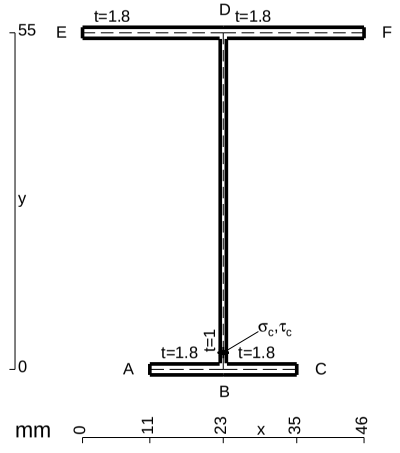
$= \left(-1/2\ b -1/4\ b\ \right) Fb^2\ 1/EJ\ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = 1/4\ \ Fb^3/EJ$



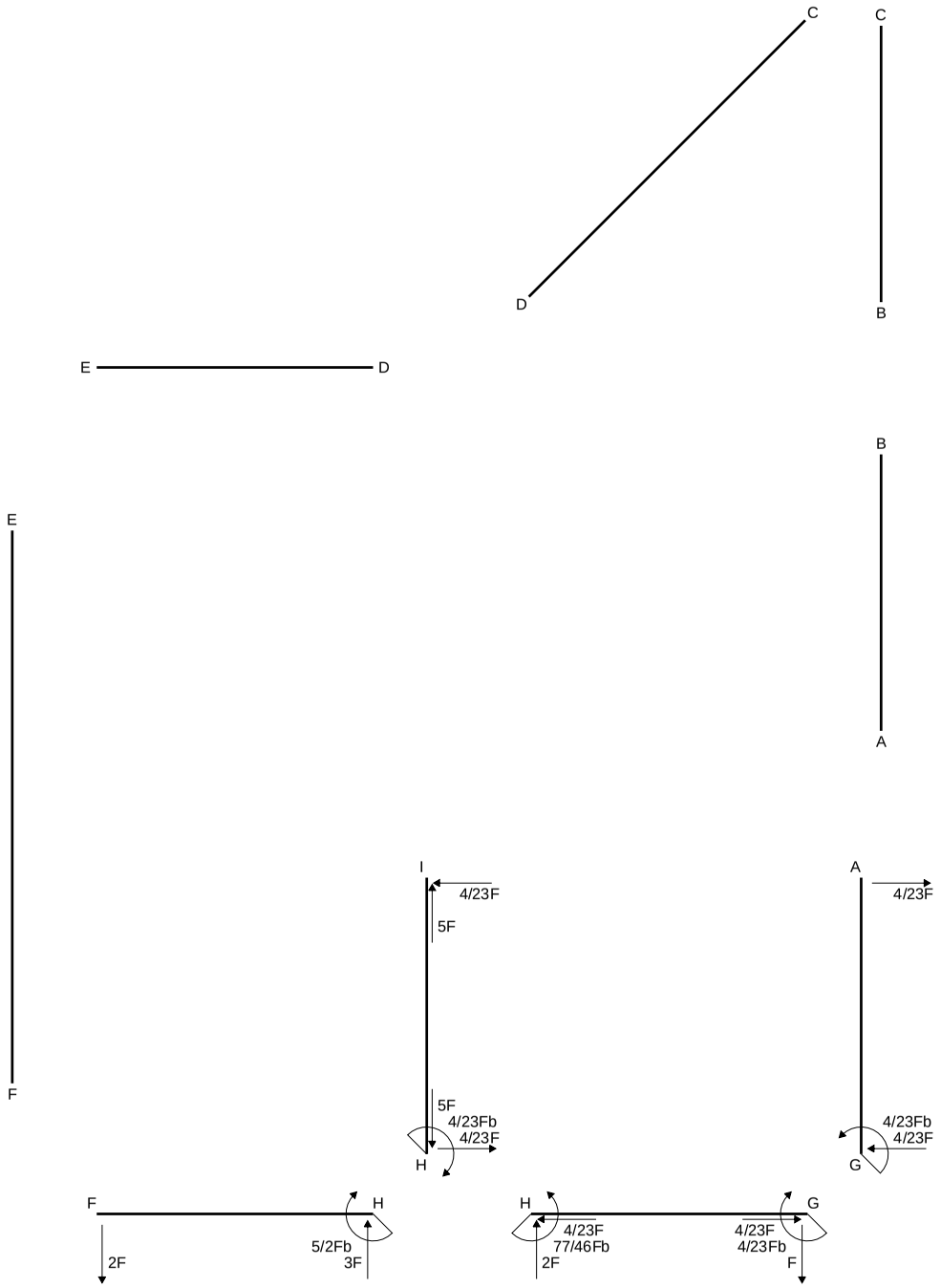
A = 114.8 mm^2
J_u = 48377. mm^4
J_v = 2333. mm^4
J_t = 86.65 mm^4
y_o = -14.74 mm
y_g = 20.3 mm
T_y = 750. N
M_x = 322500. Nmm
x_m = 12. mm
y_m = 50. mm
v_m = 29.7 mm
$\sigma_m = -Mv/J_u$ = -198. N/mm^2
y_c = 2. mm
u_c = -12. mm
v_c = -18.3 mm
$\sigma_c = -Mv/J_u$ = -198. N/mm^2
$\tau_c = TS^*/tJ_u$ = 9.947 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.947 N/mm^2
t_c = 500. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 198.8 N/mm^2

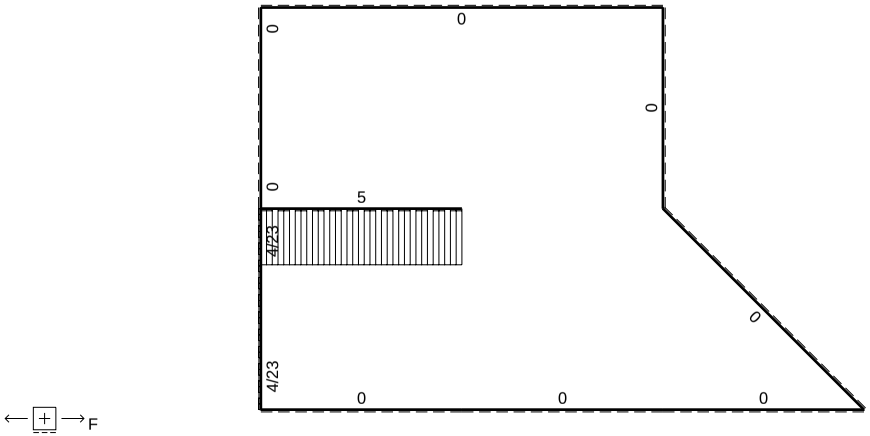


$H_G = -F$	$\varepsilon_{HG} = -\alpha T = -b^2F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_H = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$p_{HG} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$p_{FH} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

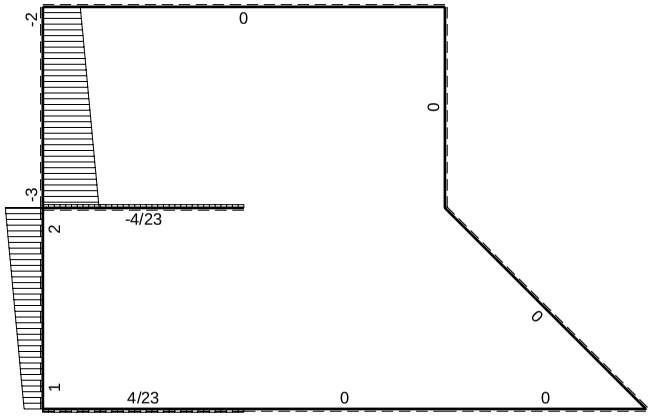


Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 610 mm, F = 460 N
Calcolare sulla sezione H la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

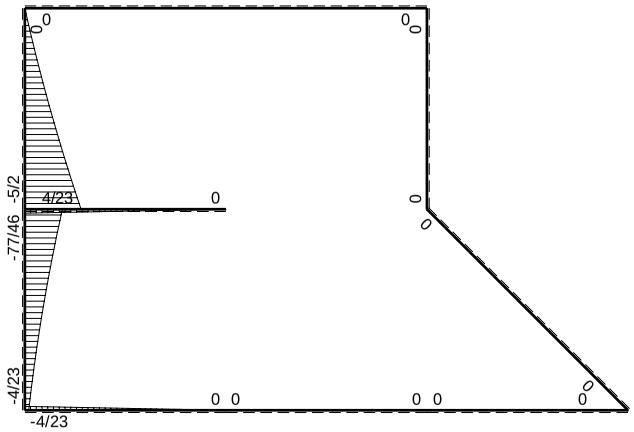




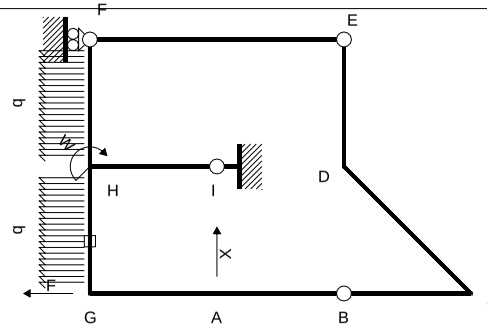
F



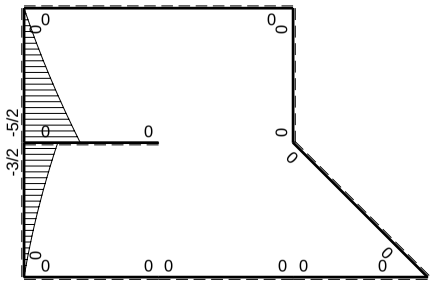
F



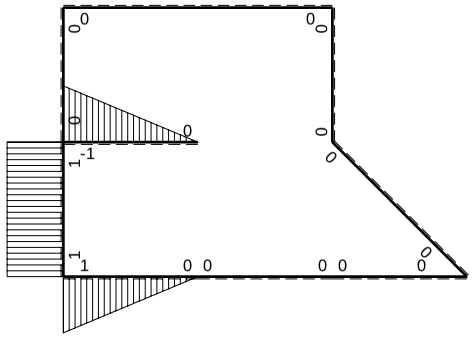
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	0	0	0	0	0	0
CB b	0	0	0	0		
CD √2b	0	0	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0		
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0		
GA b	b-x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
AG b	-x	0	0	x^2		
FH b	0	$-2Fx-1/2qx^2$	0	0	0	0
HF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0		
HI b	-b+x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
IH b	x	0	0	x^2		
HG b	b	$-3/2Fb+2Fx-1/2qx^2$	$-3/2Fb^2+2Fbx-1/2Fx^2$	b^2	$-2/3Fb^3/EJ$	Xb^3/EJ
GH b	-b	$Fx+1/2qx^2$	$-Fbx-1/2Fx^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				Fb^3/EJ	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$					$1/4Xb^3/EJ$
	totali				$1/3Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_A$				-4/23F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 237187 ESΣ05.xxxx.123

$$L^{XX}_{GA} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{AG} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{HI} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{IH} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{HG} = \int_o^b \left(1\,\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L^{XX}_{GH} = \int_o^b \left(1\,\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L^{Xo}_{HG} = \int_o^b \left(-3/2 + 2\, x/b - 1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[-3/2\, x + x^2/b - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(-3/2\, b + b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = 1/3\ Fb^3/EJ$$

$$L^{Xo}_{GH} = \int_o^b \left(-\, x/b - 1/2\, x^2/b^2\,\right) Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[-1/2\, x^2/b - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(-1/2\, b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = 1/3\ Fb^3/EJ$$

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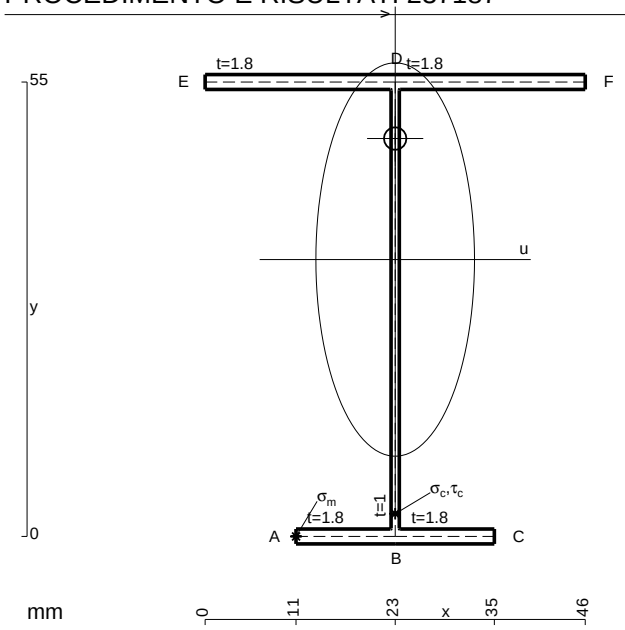

A = 181. mm^2
J_u = 102600. mm^4
J_v = 16674. mm^4
J_t = 154.4 mm^4
y_o = 14.64 mm
y_g = 33.52 mm
T_y = -1380. N
M_x = -701500. Nmm
x_m = 11. mm
u_m = -12. mm
v_m = -33.52 mm
$\sigma_m = -Mv/J_u$ = -229.2 N/mm^2
x_c = 23. mm
v_c = -33.52 mm
$\sigma_c = -Mv/J_u$ = -229.2 N/mm^2
$\tau_c = TS^*/tJ_u$ = 19.47 N/mm^2
$\tau_g = TS^*/tJ_u$ = 19.47 N/mm^2
t_c = 460. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 231.6 N/mm^2

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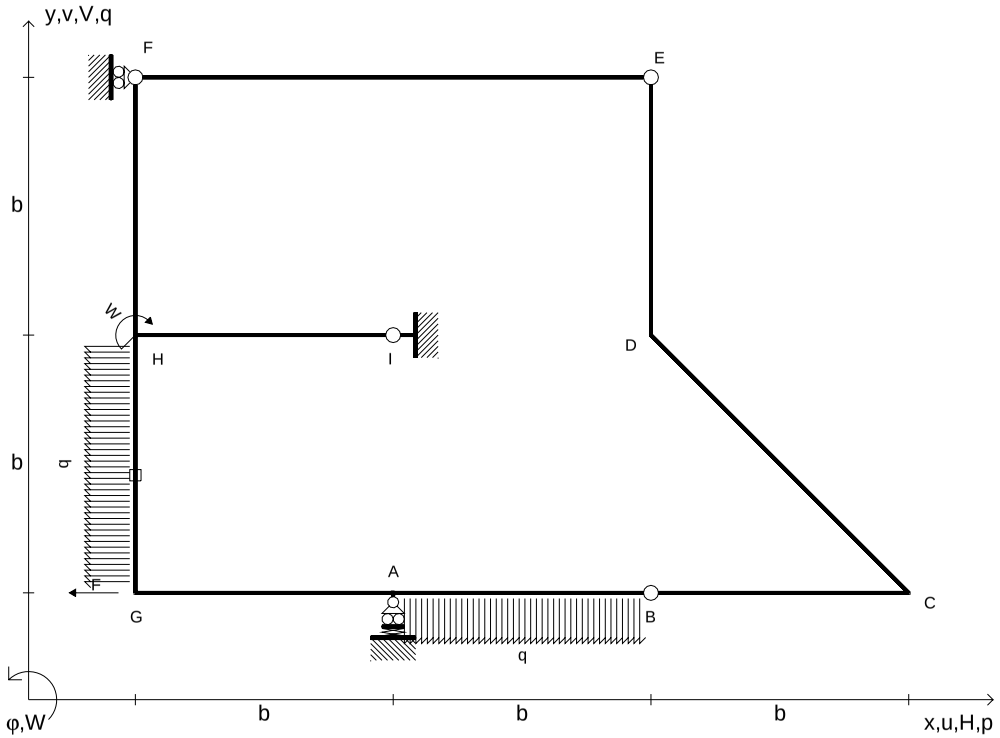
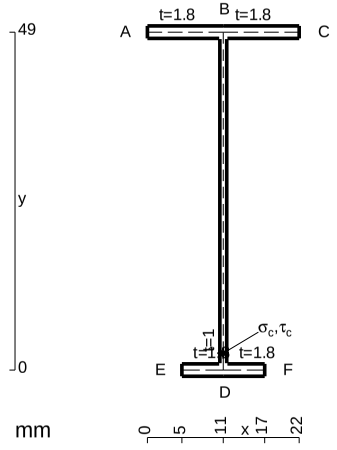


$H_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

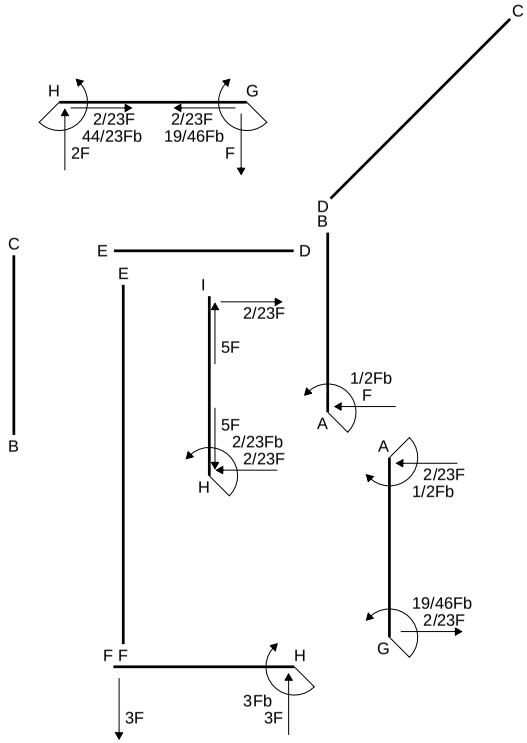
$W_H = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

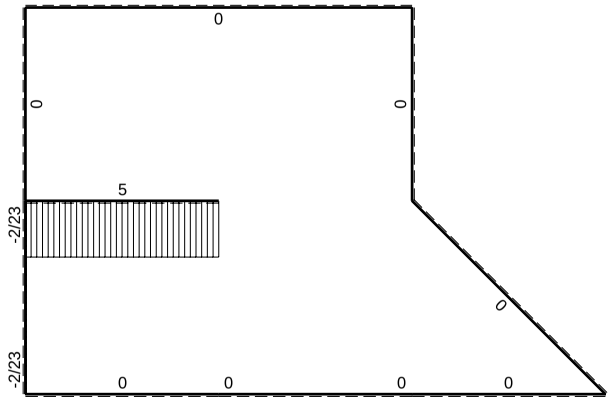
$q_{AB} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

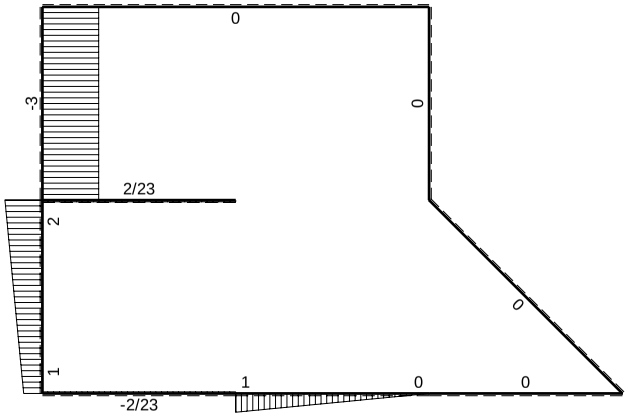


Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 630 mm, F = 160 N
Calcolare sulla sezione H la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

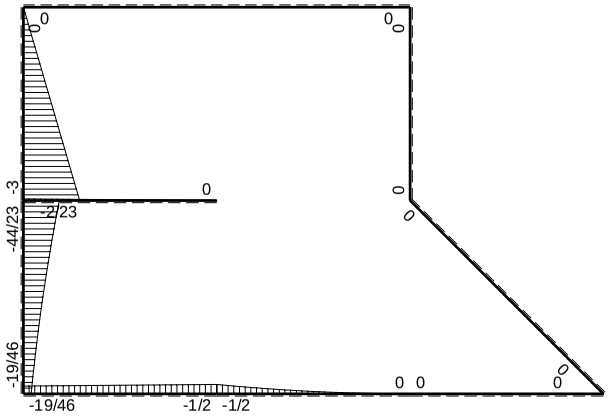




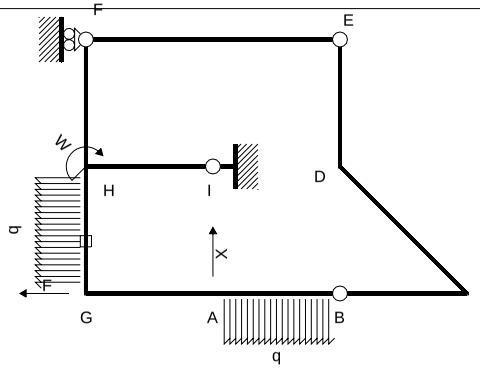
←□→ F



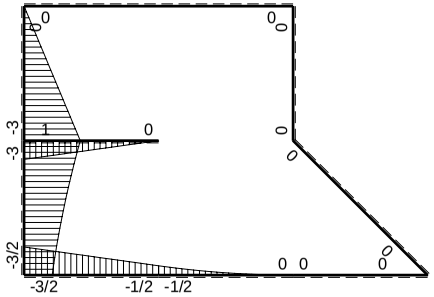
↑□↓ F



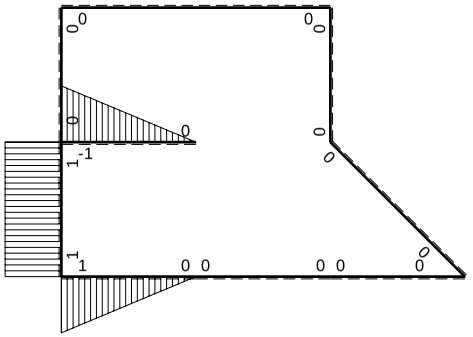
Fb



Schema di calcolo iperstatico



□ M_0 flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0
BA b	0	$1/2qx^2$	0	0		
BC b	0	0	0	0	0	0
CB b	0	0	0	0		
CD $\sqrt{2}b$	0	0	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0		
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0		
GA b	b-x	$-3/2Fb+Fx$	$-3/2Fb^2+5/2Fbx-Fx^2$	$b^2-2bx+x^2$	$-7/12Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	$1/2Fb+Fx$	$-1/2Fbx-Fx^2$	x^2		
FH b	0	$-3Fx$	0	0	0	0
HF b	0	$3Fb-3Fx$	0	0		
HI b	$-b+x$	$Fb-Fx$	$-Fb^2+2Fbx-Fx^2$	$b^2-2bx+x^2$	$-1/3Fb^3/EJ$	$1/3Xb^3/EJ$
IH b	x	$-Fx$	$-Fx^2$	x^2		
HG b	b	$-3Fb+2Fx-1/2qx^2$	$-3Fb^2+2Fbx-1/2Fx^2$	b^2	$-13/6Fb^3/EJ$	Xb^3/EJ
GH b	-b	$3/2Fb+Fx+1/2qx^2$	$-3/2Fb^2-Fbx-1/2Fx^2$	b^2		
HG	elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$				Fb^3/EJ	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$					$1/4Xb^3/EJ$
	totali				$-25/12Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_A$				25/23F	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-3/2 +5/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-3/2\, x +5/4\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-3/2\, b +5/4\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(-1/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/4\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/4\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-1 +2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[- x + x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-\, b + b -1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(-\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-3 +2\, x/b -1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

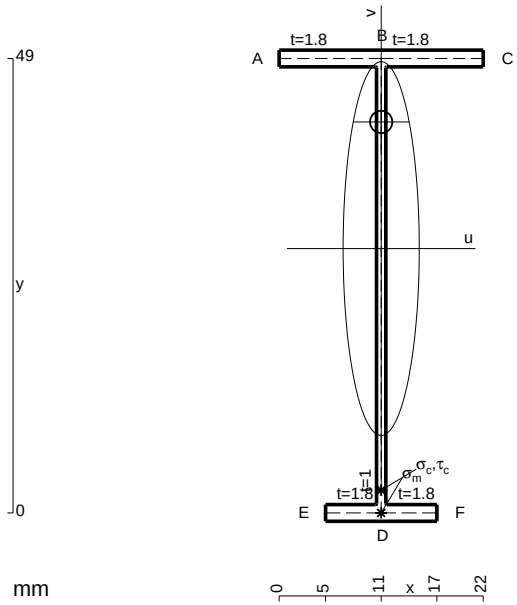
$$= \left[-3\, x + x^2/b -1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-3\, b + b -1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -7/6\ \ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-3/2 - x/b -1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left[-3/2\, x -1/2\, x^2/b -1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-3/2\, b -1/2\, b -1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -7/6\ \ Fb^3/EJ$$



A = 110.2 mm^2
J_u = 44775. mm^4
J_v = 1856. mm^4
J_t = 82.43 mm^4
y_o = 13.66 mm
y_g = 28.5 mm
T_y = -480. N
M_x = -302400. Nmm
x_m = 11. mm
v_m = -28.5 mm
$\sigma_m = -Mv/J_u$ = -192.5 N/mm^2
y_c = 3. mm
u_c = -11. mm
v_c = -25.5 mm
$\sigma_c = -Mv/J_u$ = -192.5 N/mm^2
$\tau_c = TS^*/tJ_u$ = 6.6 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.6 N/mm^2
t_c = 160. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 192.8 N/mm^2

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