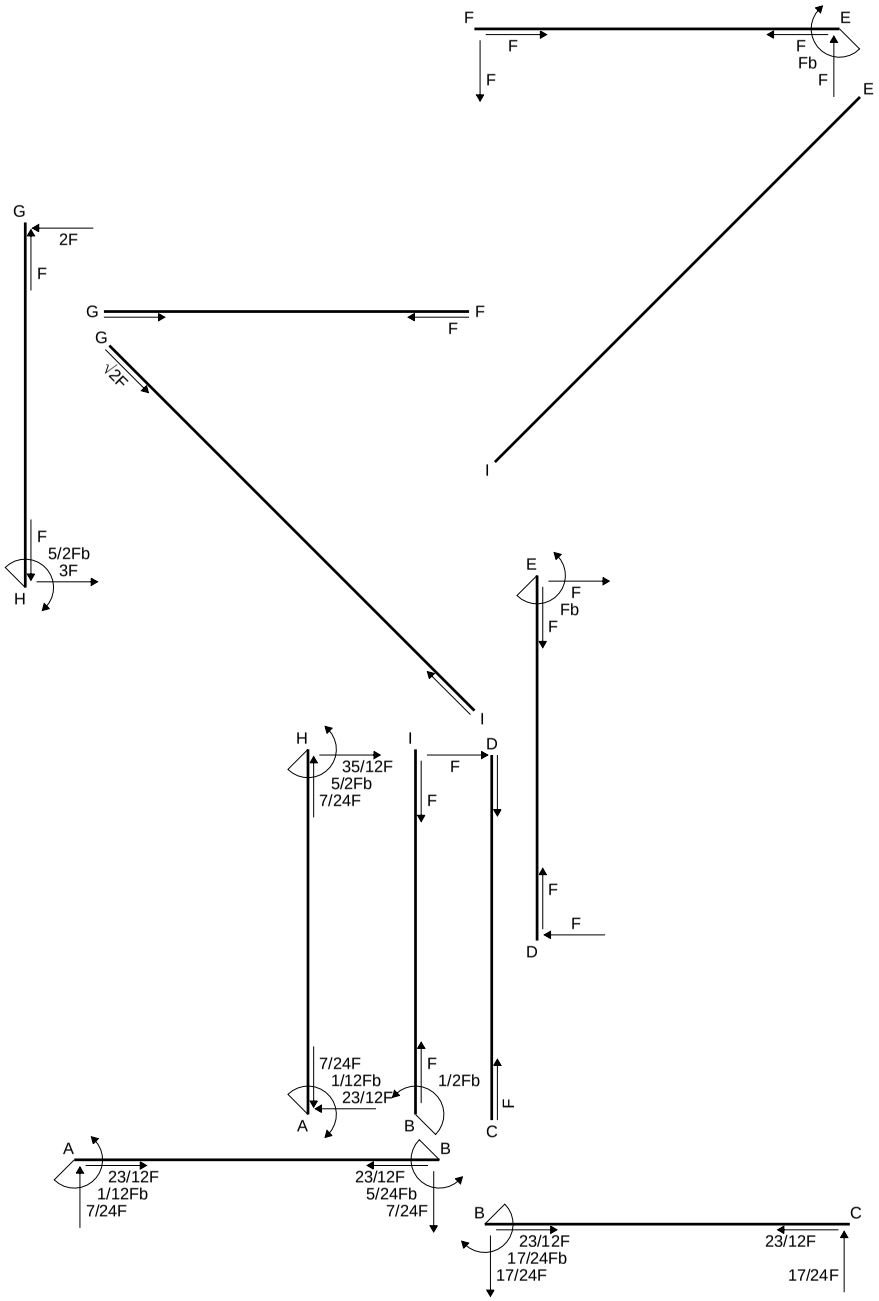
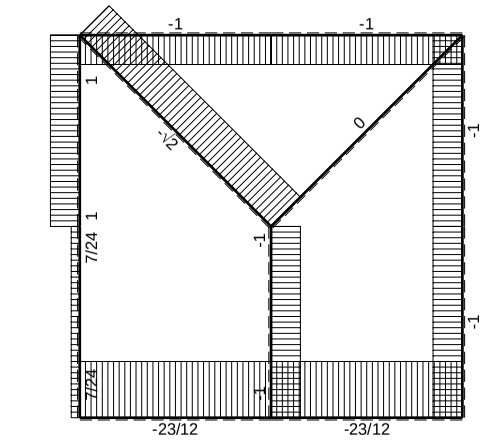
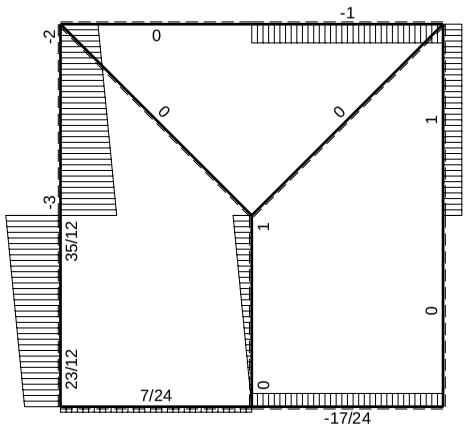
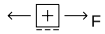
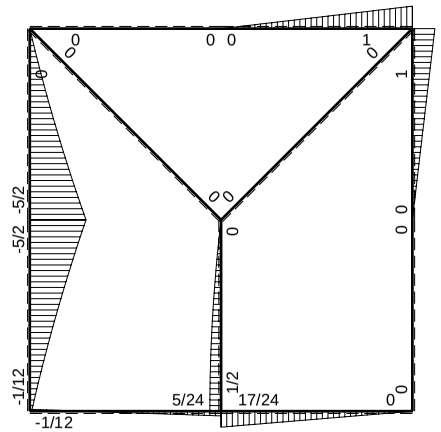

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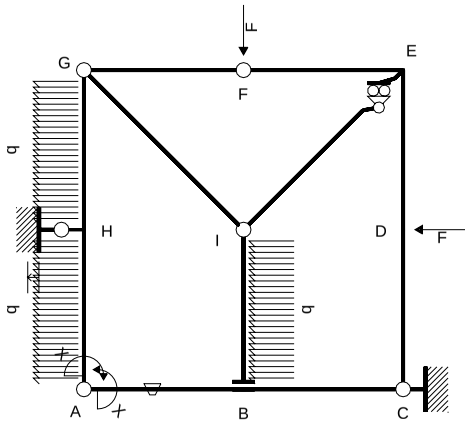

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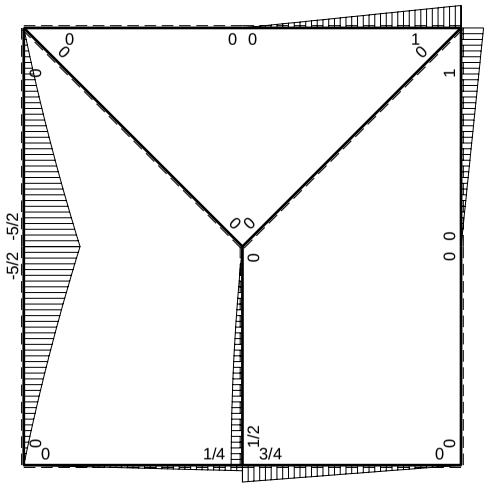

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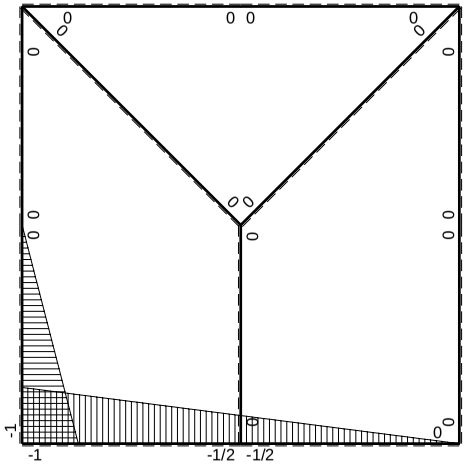

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1/2
17/24
0
0
-1/12


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	-Fb/EJ	$-1/4Fx+1/8Fx^2/b$	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(-1/12+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	Fb/EJ	$-1/8Fb+1/8Fx^2/b$	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

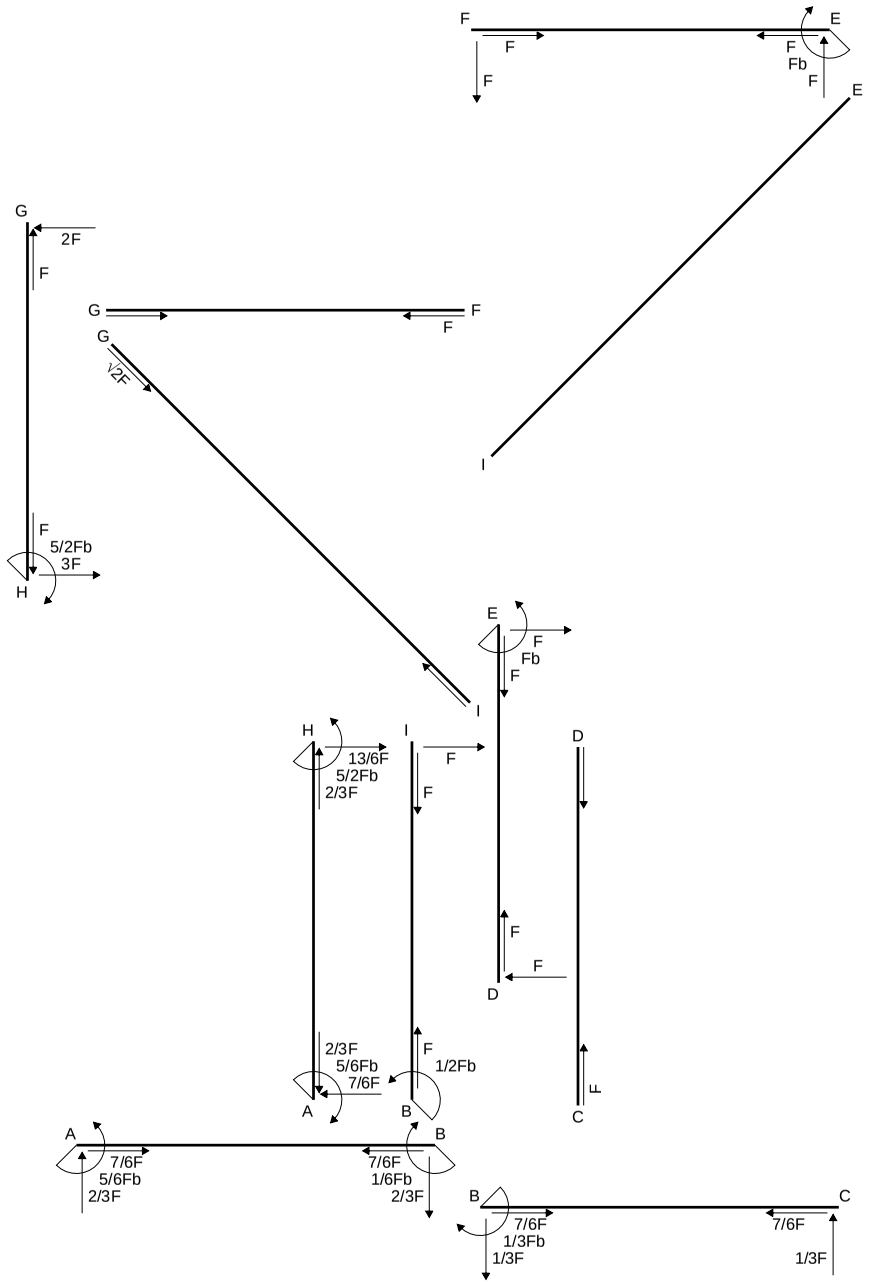
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

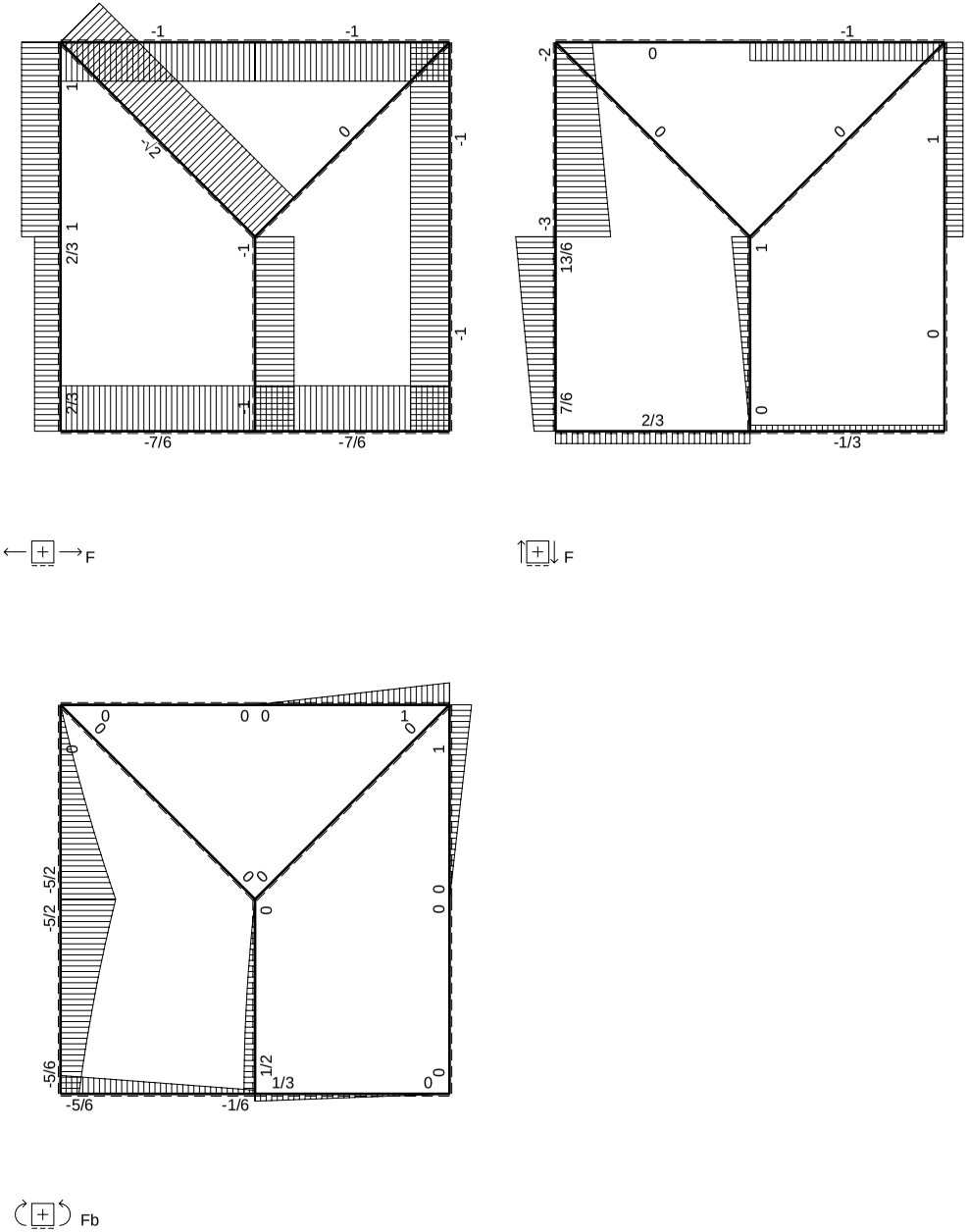
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

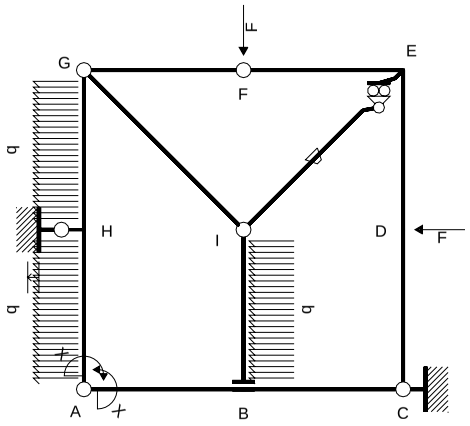
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

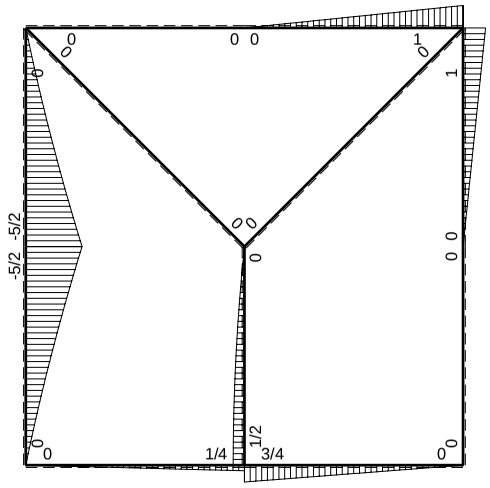
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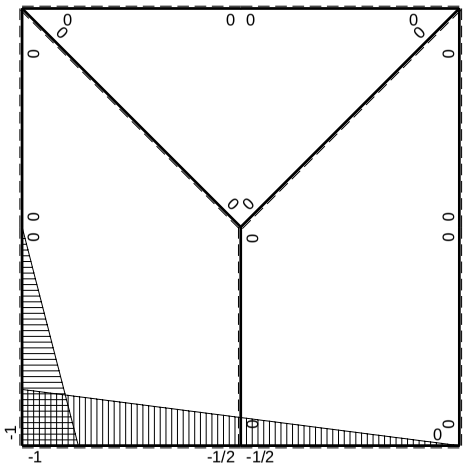

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-5/6
-1/6
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-3/4Fx$	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

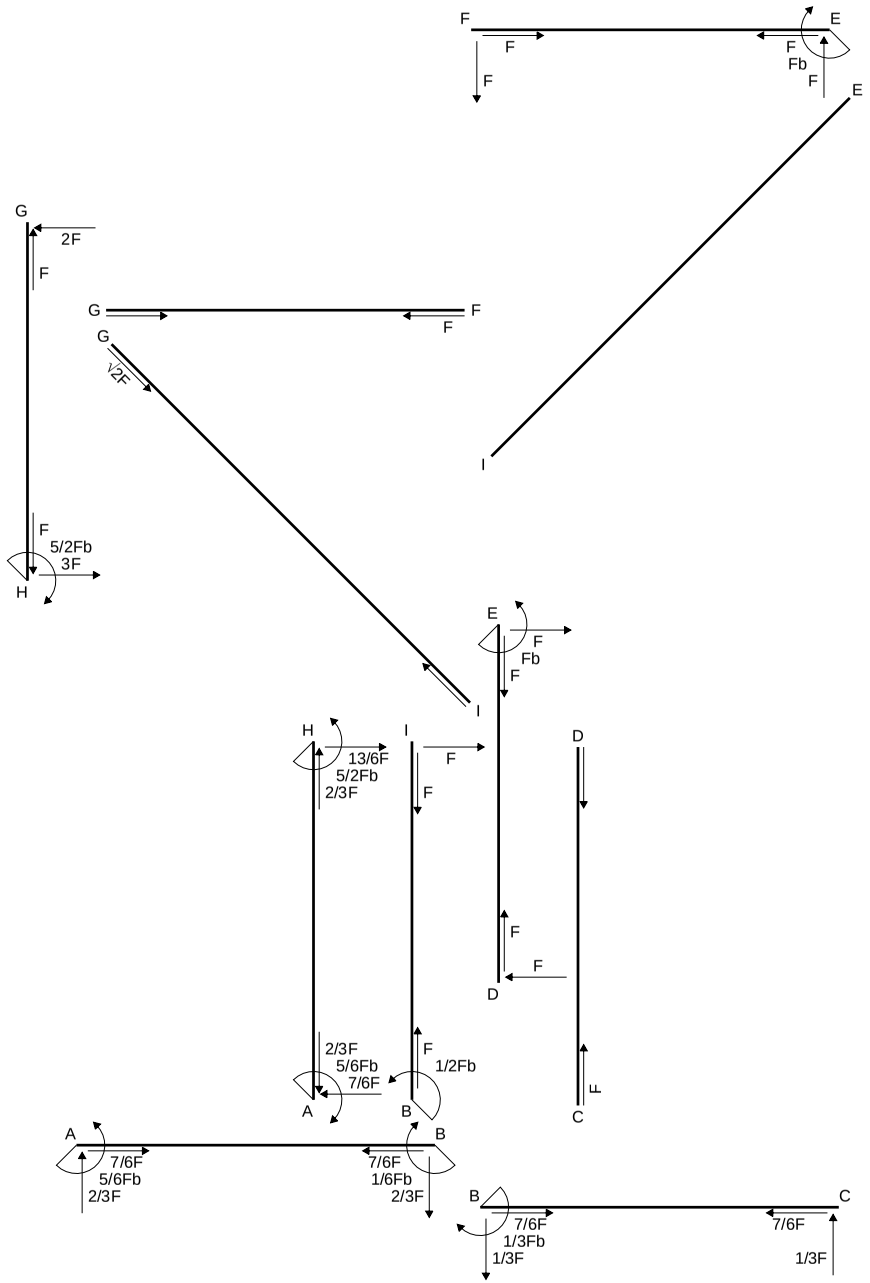
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

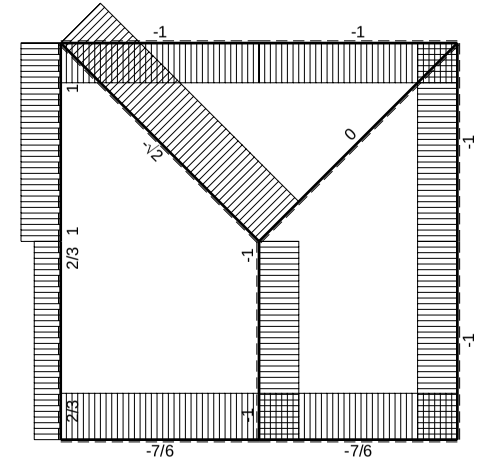
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

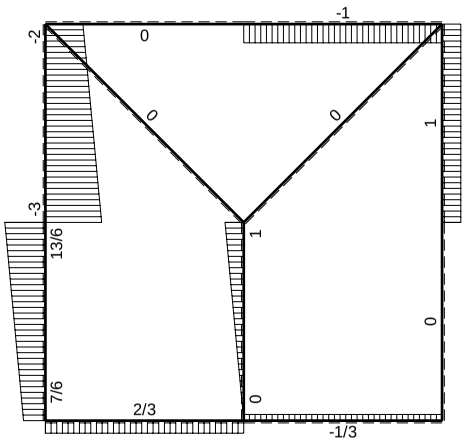
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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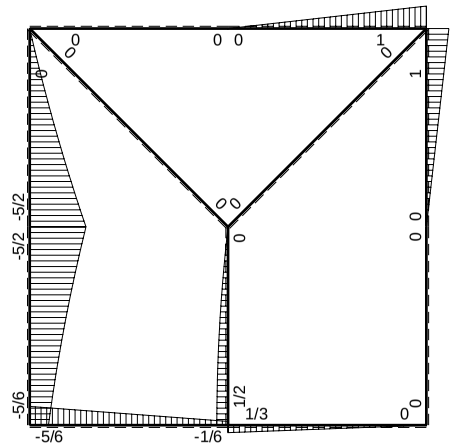

F
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Fb
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2F
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F
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½F
I
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5/2Fb
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E
F
Fb
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I
H
13/6F
5/2Fb
2/3F
I
F
F
D
F
F
D
2/3F
5/6Fb
7/6F
F
1/2Fb
A
B
F
C
A
B
7/6F
5/6Fb
2/3F
7/6F
1/6Fb
2/3F
B
C
7/6F
1/3Fb
1/3F
7/6F
1/3F


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-1
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-1
-1
2/3
1
-7/6
-7/6

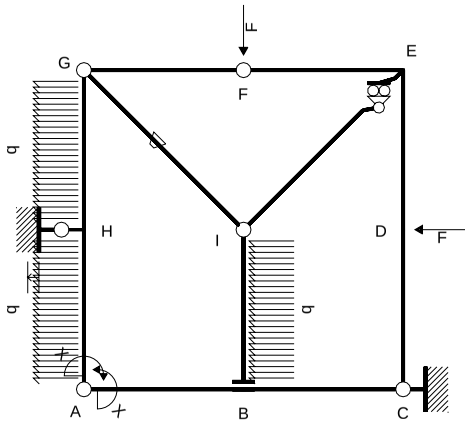

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7/6
2/3
0
-1/3

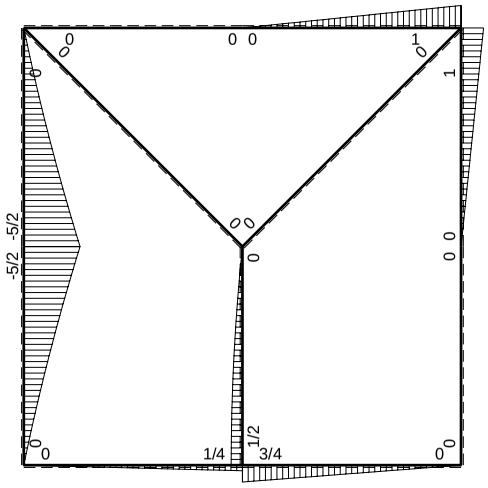
←□→ F

↑□↓ F

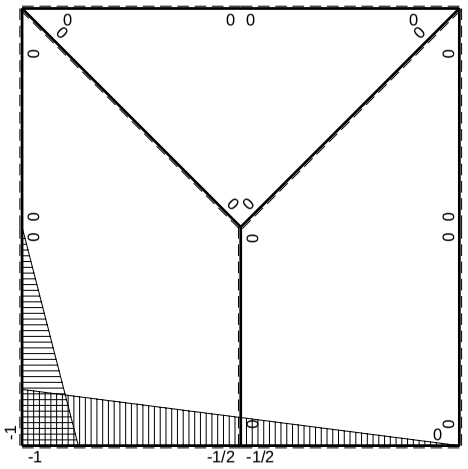

0
0 0
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0
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1
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0
0
-5/2
-5/2
0 0
-5/6
1/2
1/3
0
-5/6
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

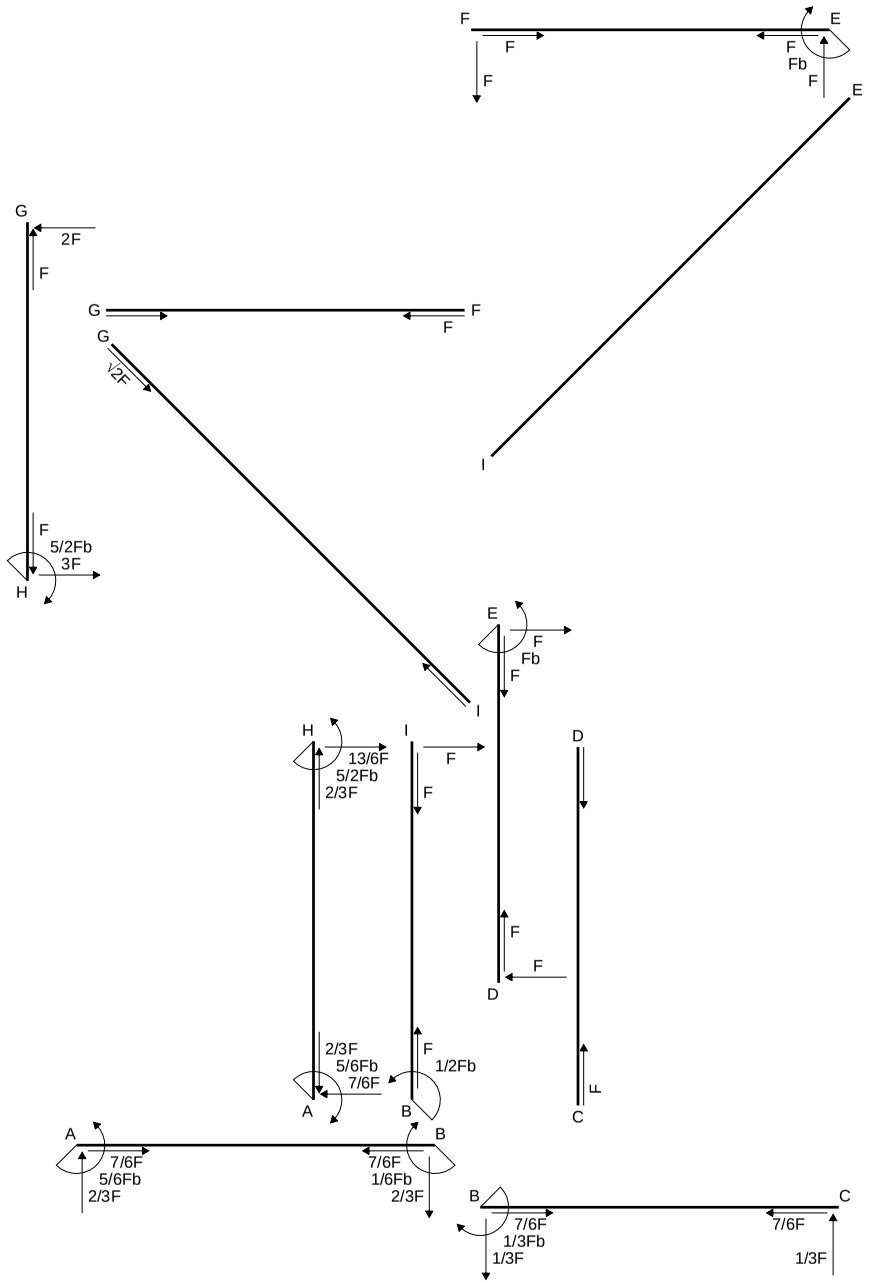
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

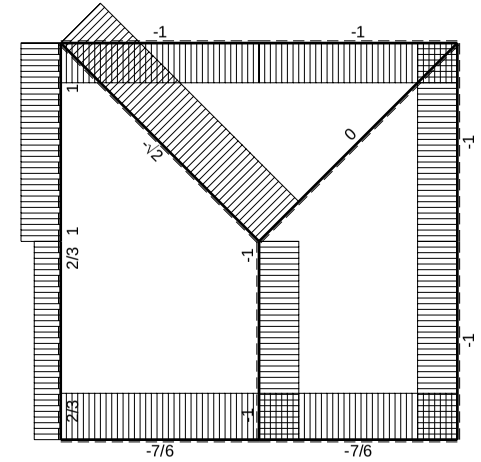
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

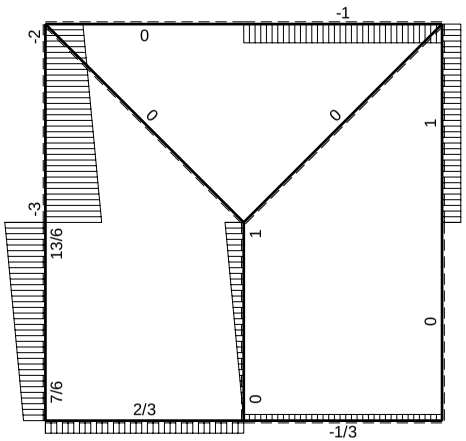
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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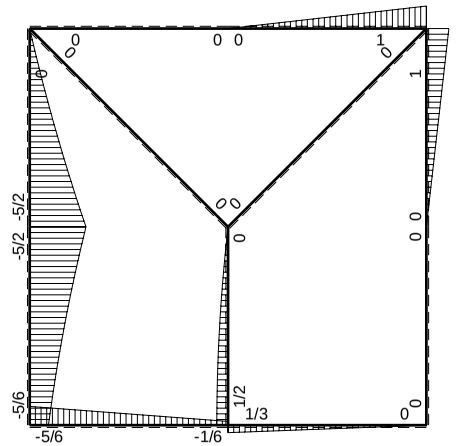

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5/2Fb
3F
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E
F
Fb
F
H
I
13/6F
5/2Fb
2/3F
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2/3F
B
C
7/6F
1/3Fb
1/3F
7/6F
1/3F


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-7/6
-7/6

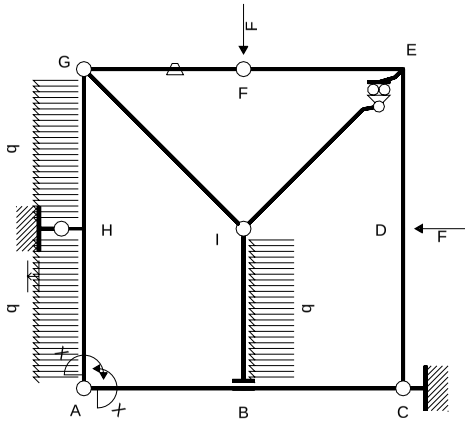

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0
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1
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13/6
1
0
7/6
2/3
0
-1/3

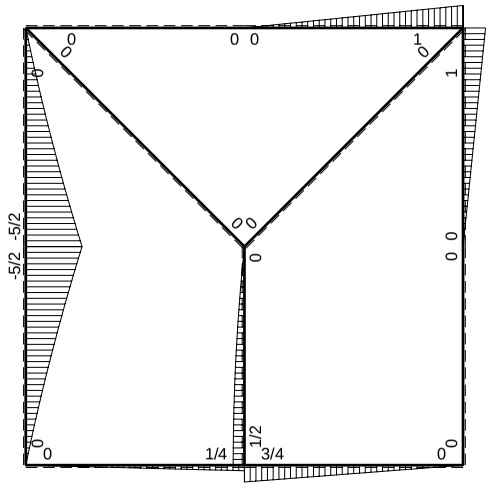
←□→ F

↑□↓ F

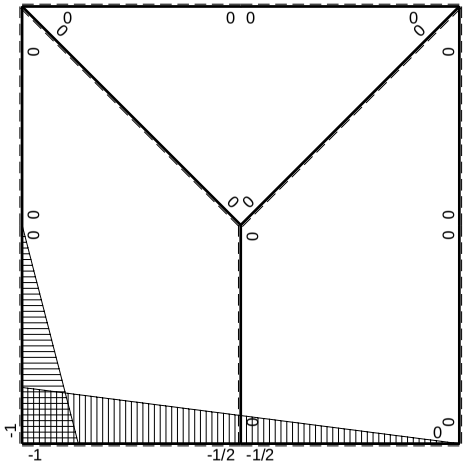

0
0 0
1
0
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1
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0
0
-5/2 -5/2
0 0
-5/6
1/2
1/3
0
0
-5/6
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238207 ESΣ04.xxxx.004

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PROCEDIMENTO E RISULTATI 238207 ESΣ04.xxxx.004

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

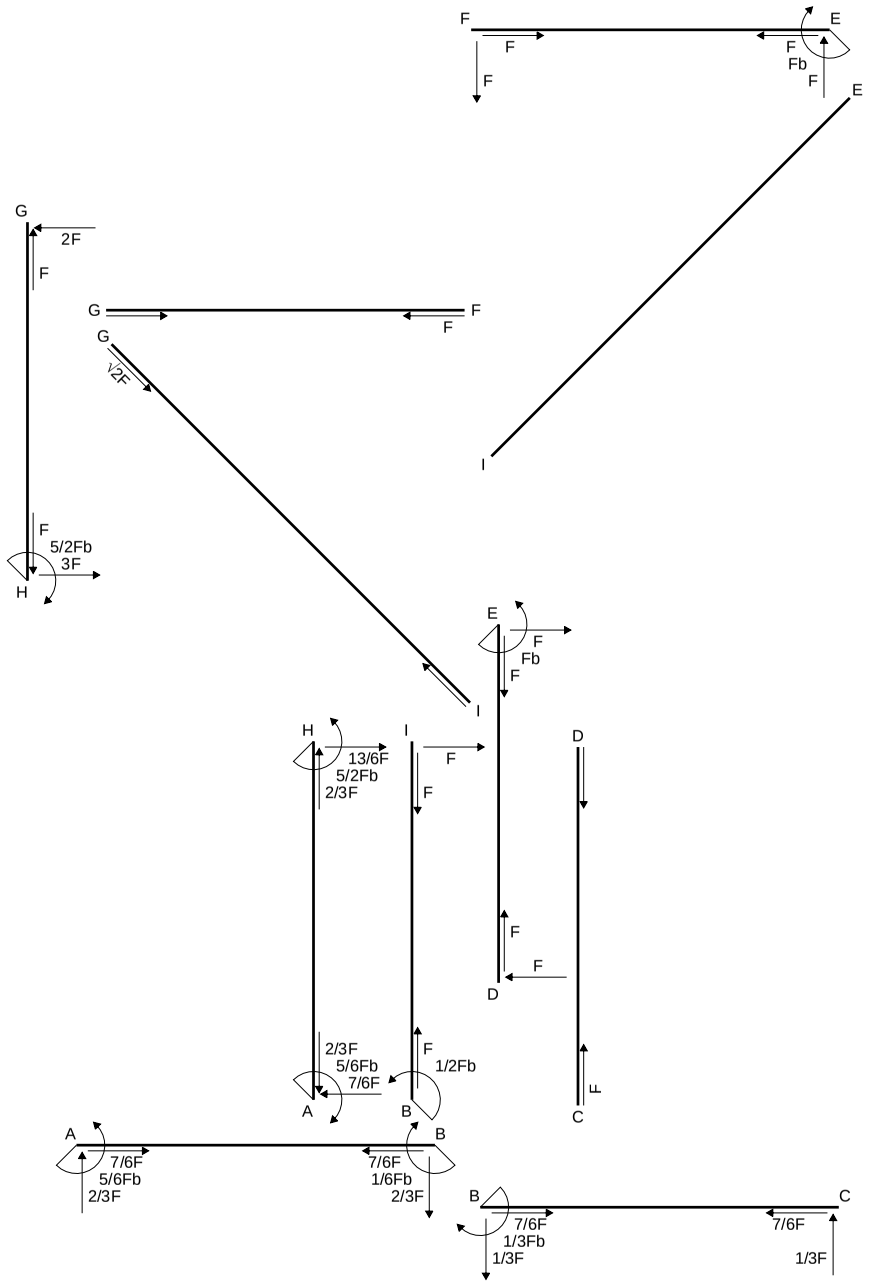
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

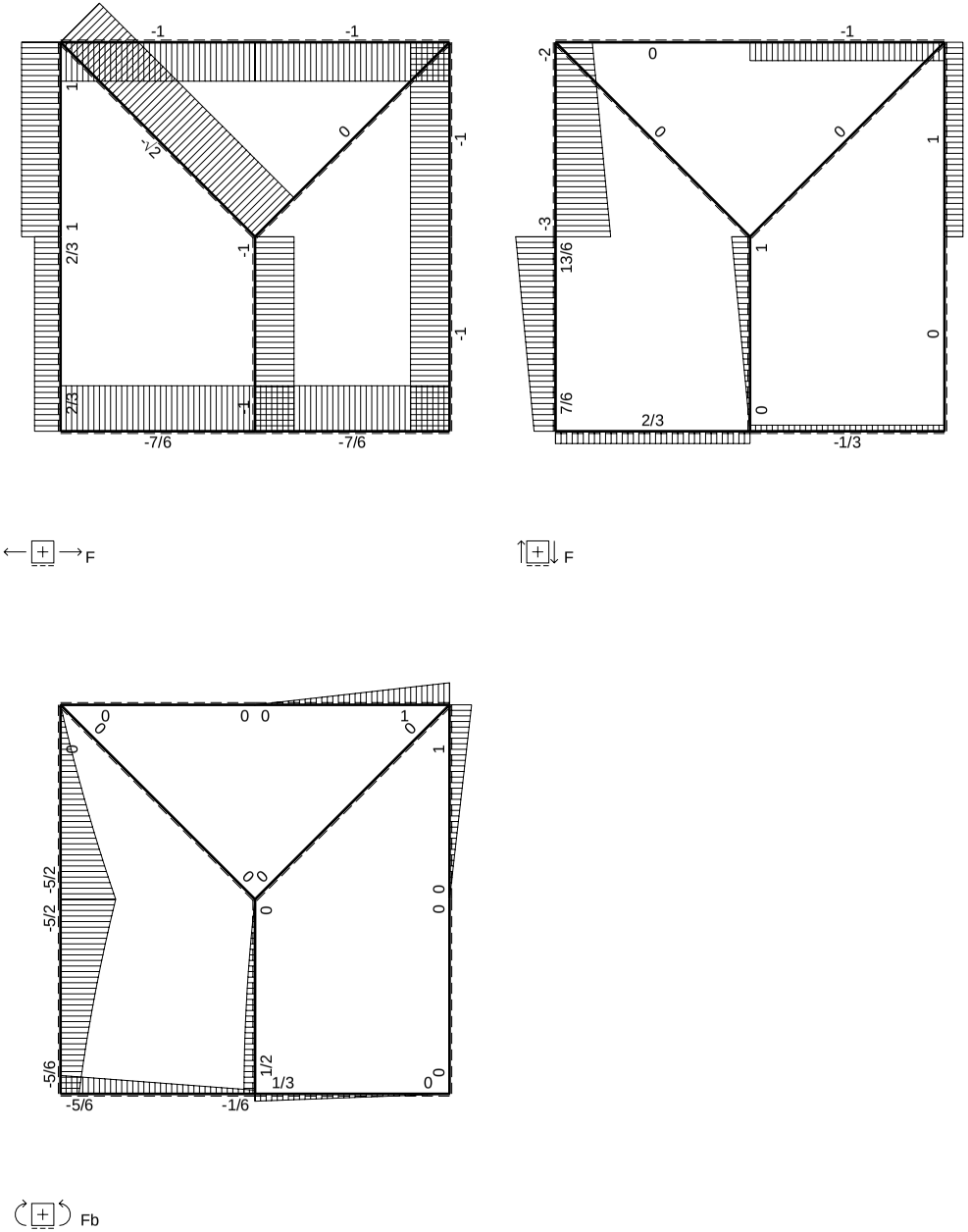
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

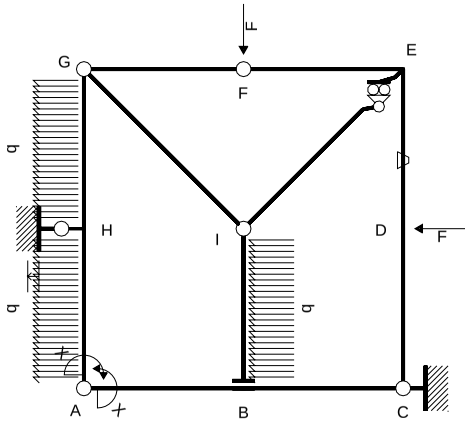
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

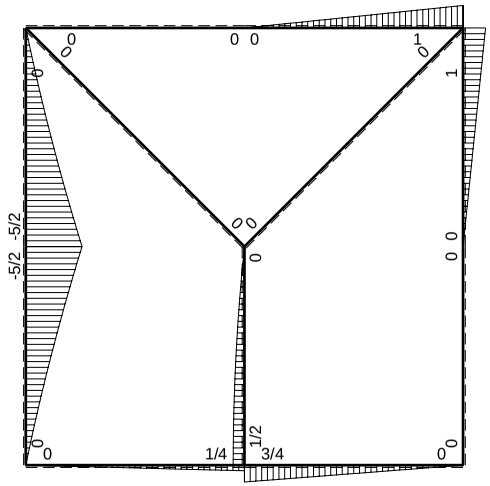
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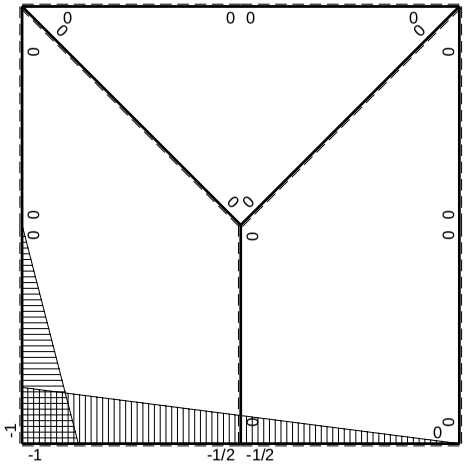

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7/6F
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0
-5/6
-1/6
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238304 ESΣ04.xxxx.005

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

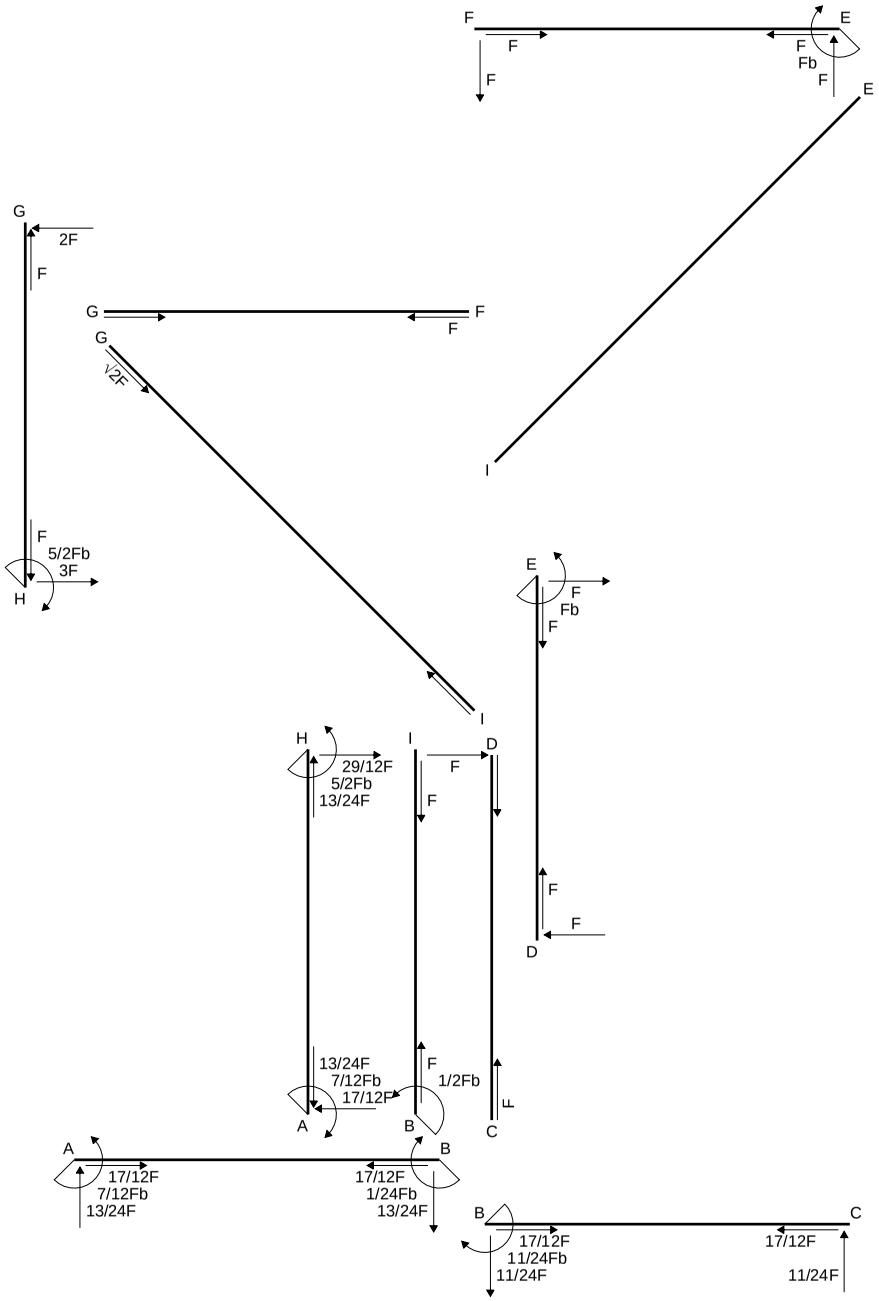
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

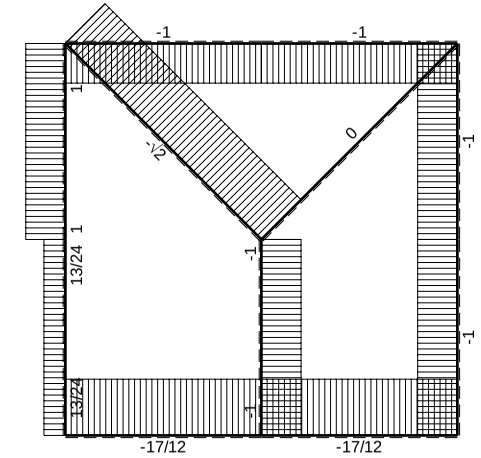
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

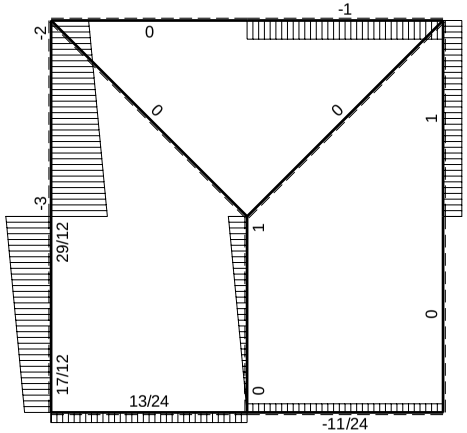
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

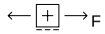
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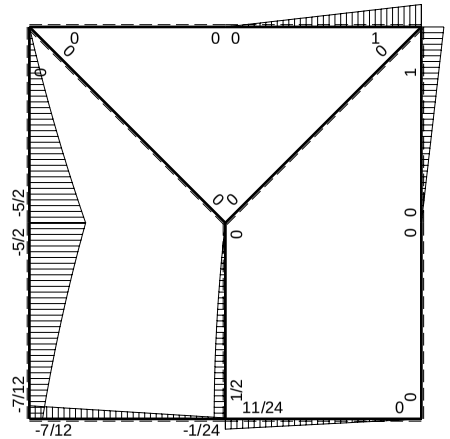

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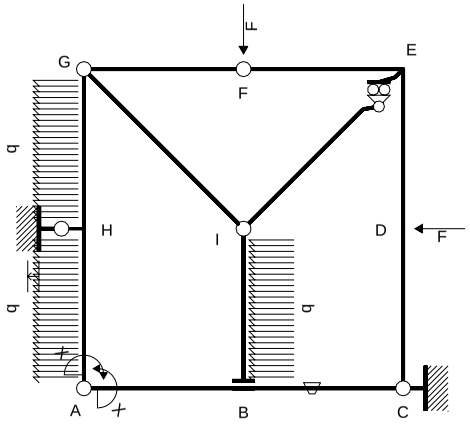

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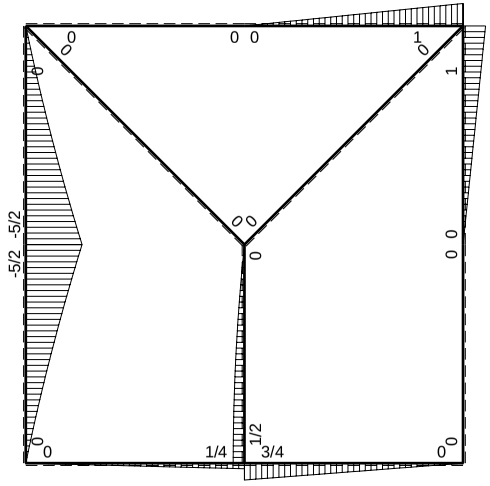

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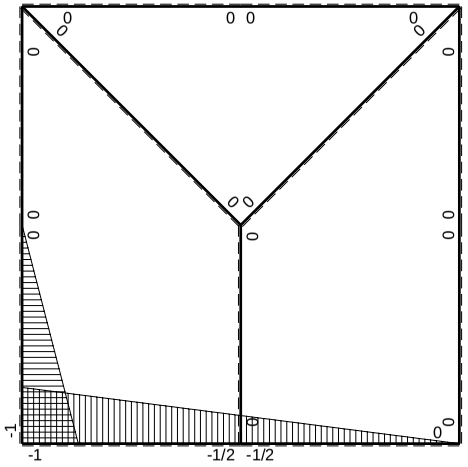

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 310182 ESΣ04.xxxx.006

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	-Fb/EJ	$-3/8Fb+3/4Fx-3/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-3/4Fx$	Fb/EJ	$-3/8Fx^2/b$	$1/2Fx/EJ$	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/12Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 310182 ESΣ04.xxxx.006

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\ \right)\ \theta\ \ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

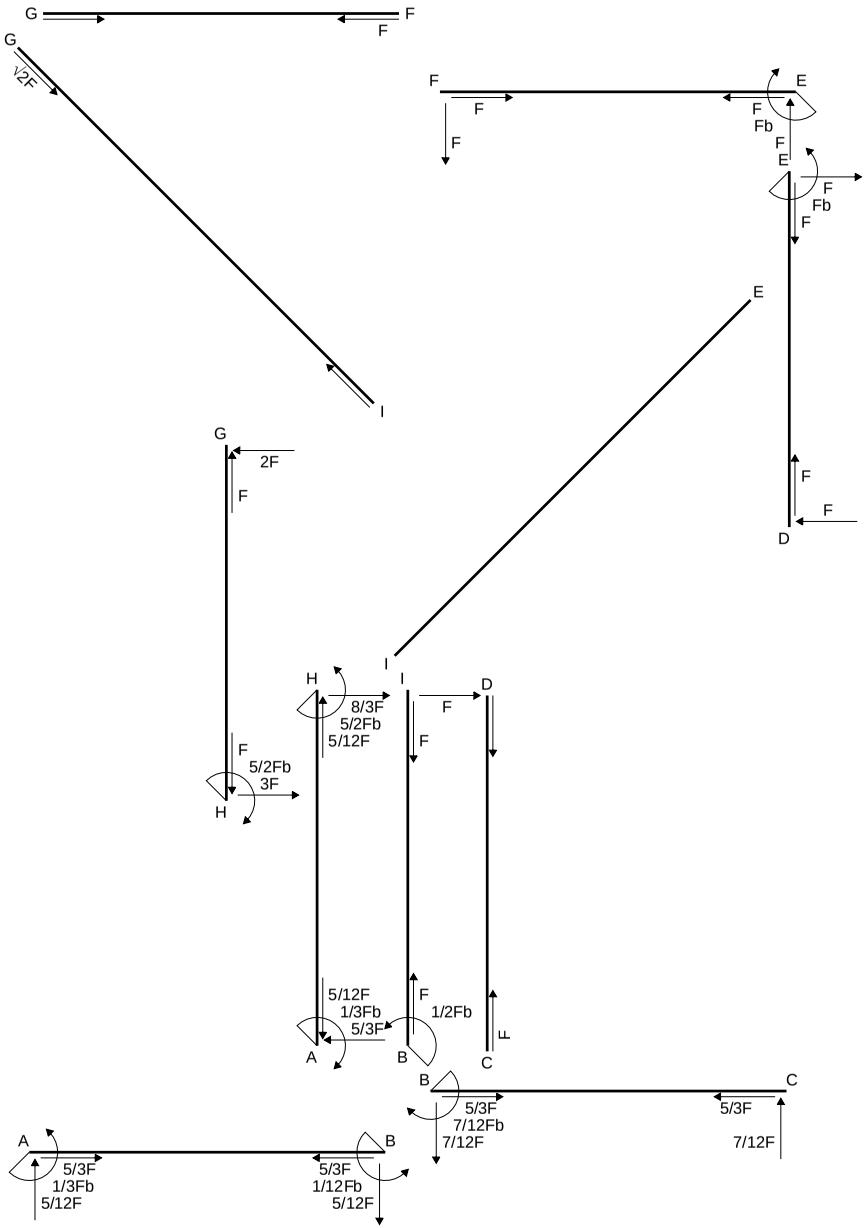
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

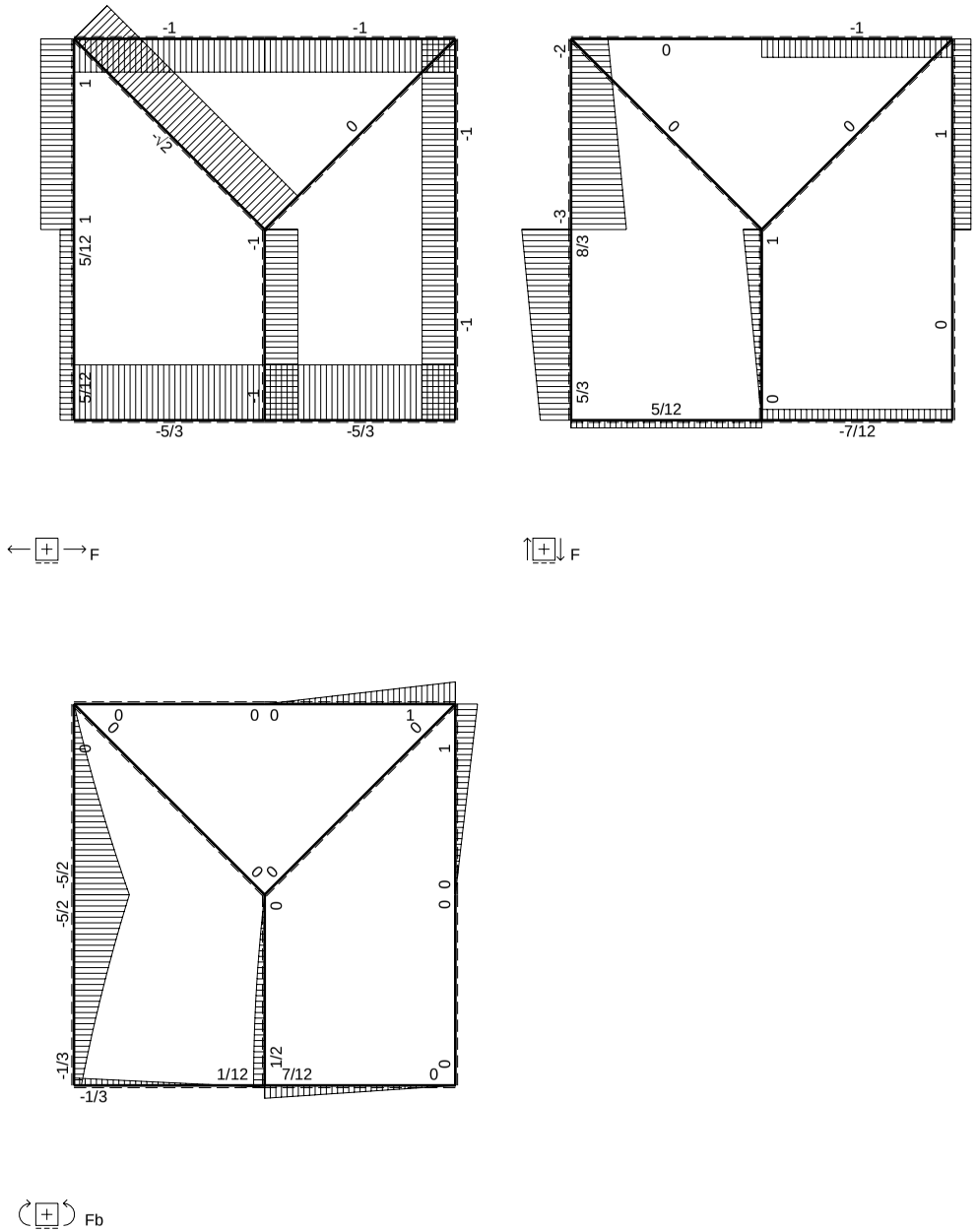
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

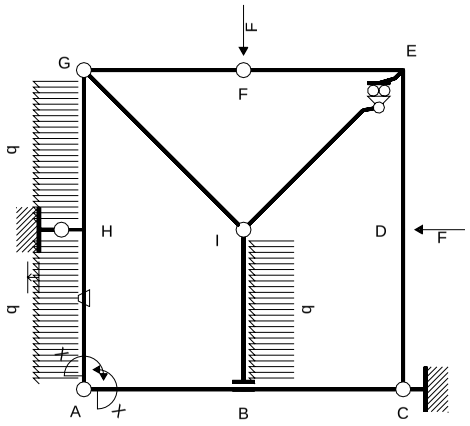
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

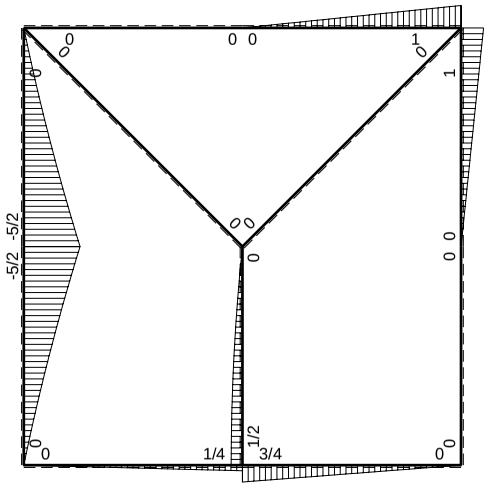
ESΣ04.xxxx.006

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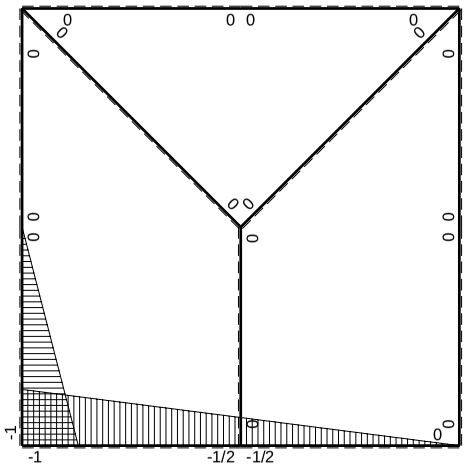

G
F
√2F
I
E
Fb
D
2F
H
8/3F
5/2Fb
5/12F
5/2Fb
3F
5/12F
1/3Fb
5/3F
1/2Fb
A
B
C
5/3F
7/12Fb
7/12F
1/3Fb
1/12Fb


-1
-√2
5/12
-5/3
F
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	-Fb/EJ	$5/2Fx-3Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(3/8+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	Fb/EJ	$2Fx-3/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/3Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/3Fb	

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ dx$

$= \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = 7/8\ \ Fb^2/EJ$

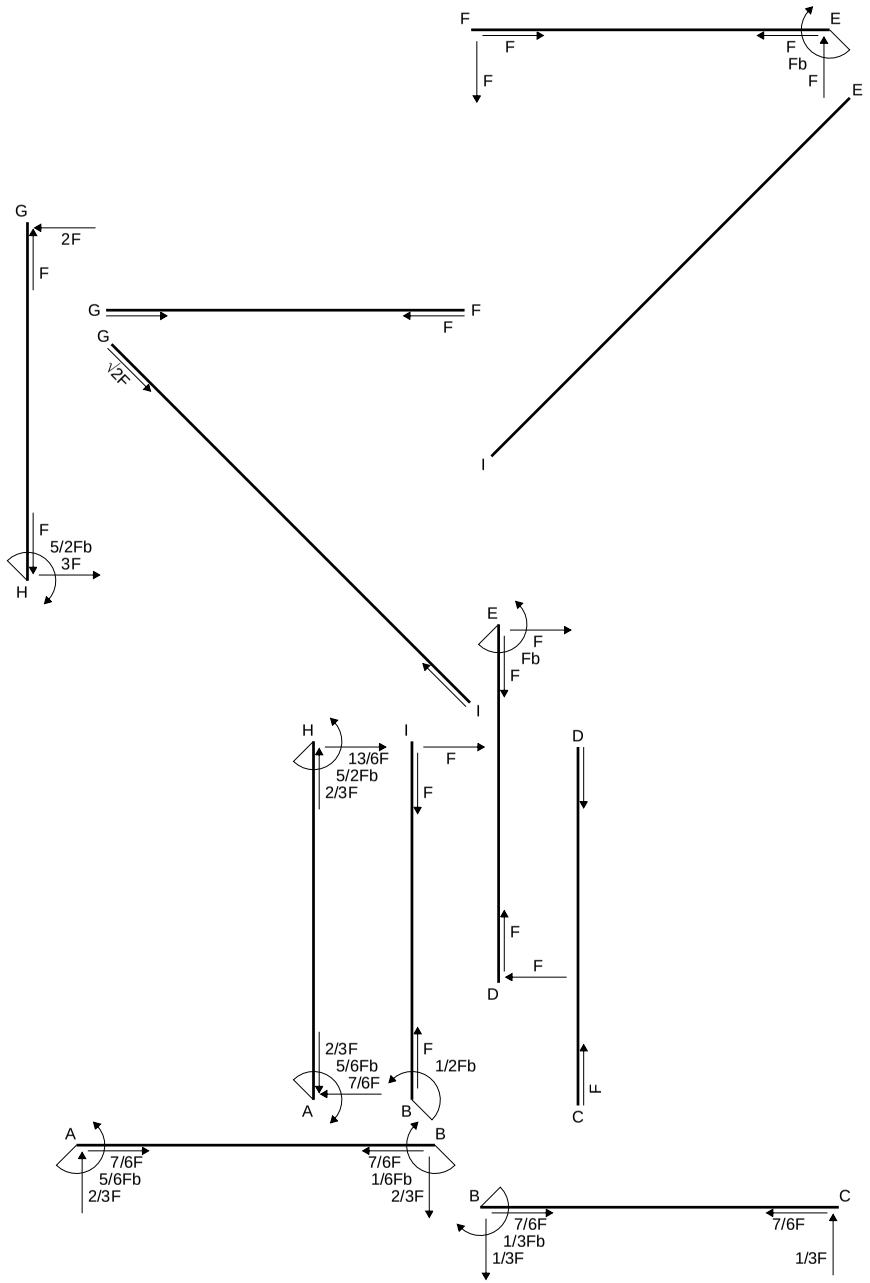
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ dx$

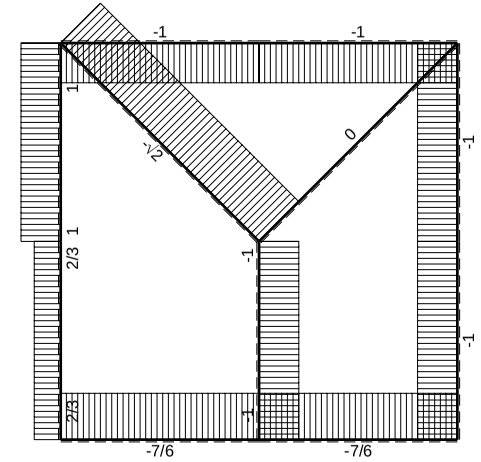
$= \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

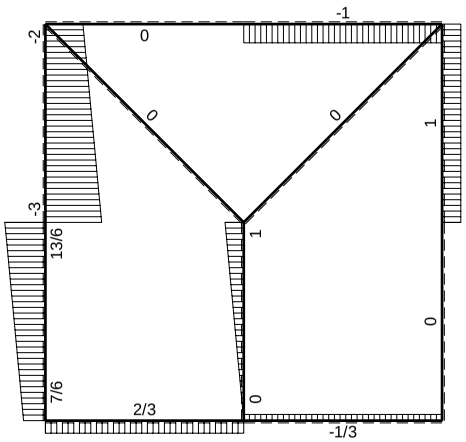
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\ \theta\ \ = 7/8\ \ Fb^2/EJ$

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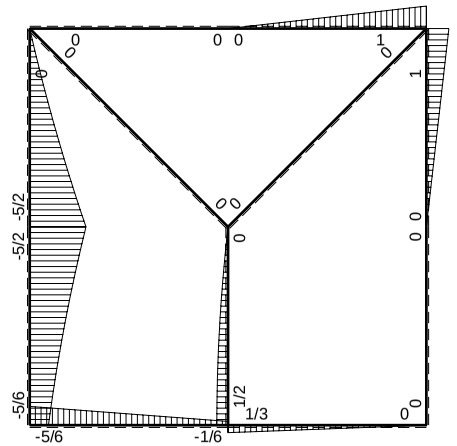

F
E
F
F
Fb
F
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G
2F
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G
F
F
G
½F
I
F
5/2Fb
3F
H
E
F
Fb
F
I
H
13/6F
5/2Fb
2/3F
I
F
F
D
F
F
D
2/3F
5/6Fb
7/6F
F
1/2Fb
A
B
F
C
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B
7/6F
5/6Fb
2/3F
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1/6Fb
2/3F
B
C
7/6F
1/3Fb
1/3F
7/6F
1/3F


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½
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-1
-1
2/3
1
-7/6
-7/6

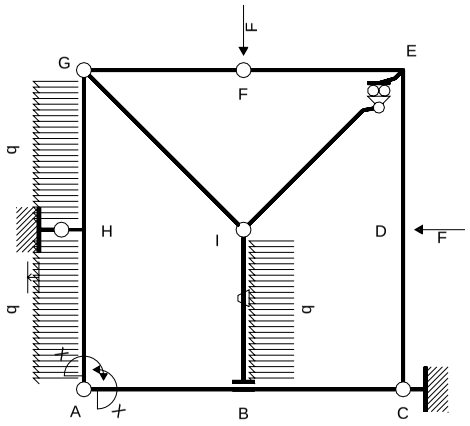

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1
-3
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1
0
7/6
2/3
0
-1/3

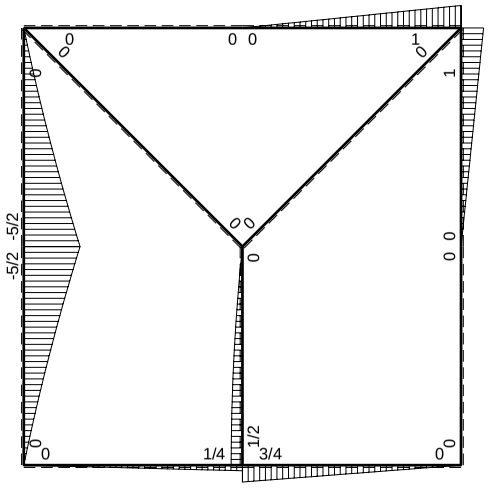
←□→ F

↑□↓ F

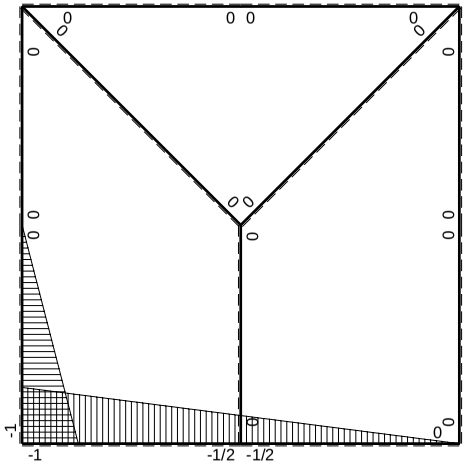

0
0 0
1
0
0
1
0
0
0
0 0
-5/2
-5/2
0
0 0
-5/6
1/2
1/3
0
0
-5/6
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 957348 ESΣ04.xxxx.008

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-3/4Fx$	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

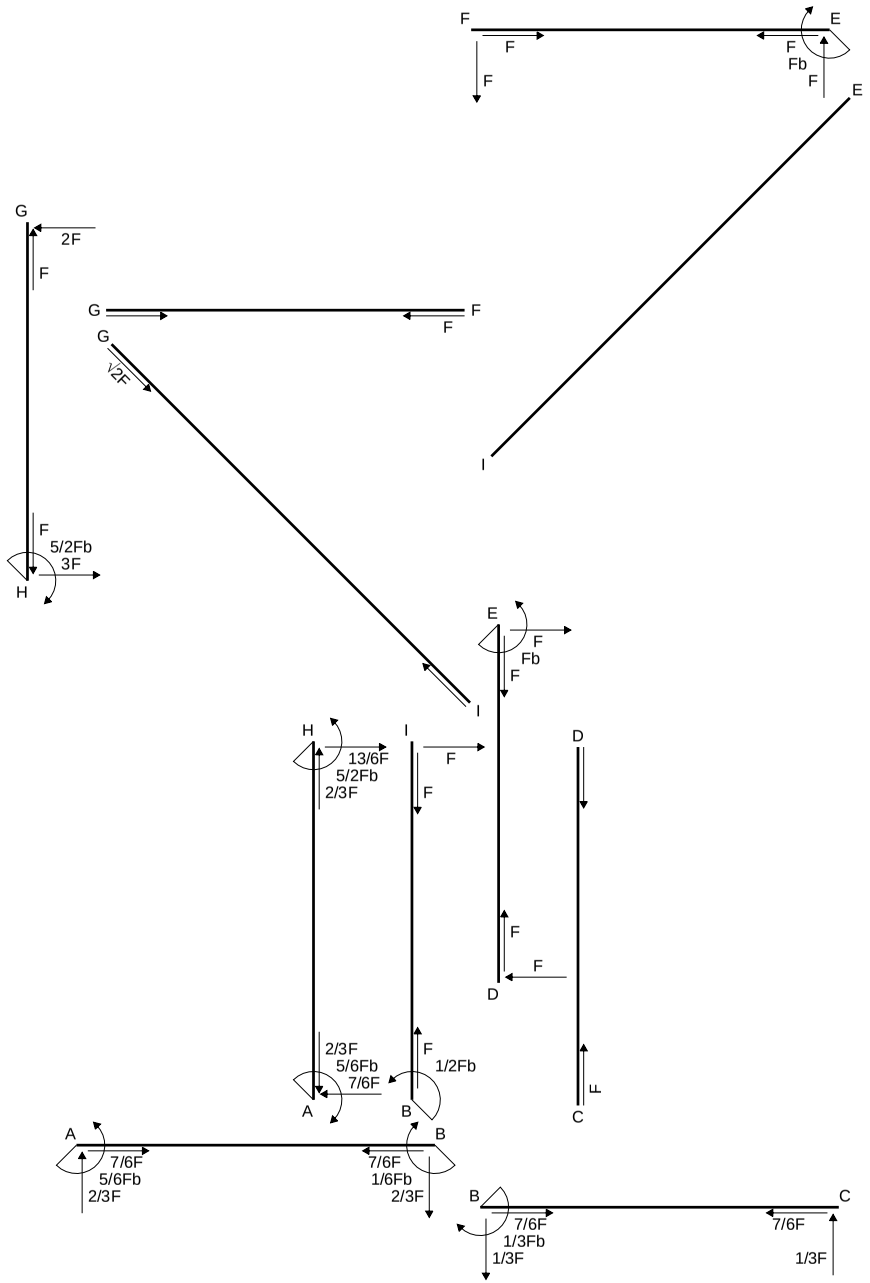
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

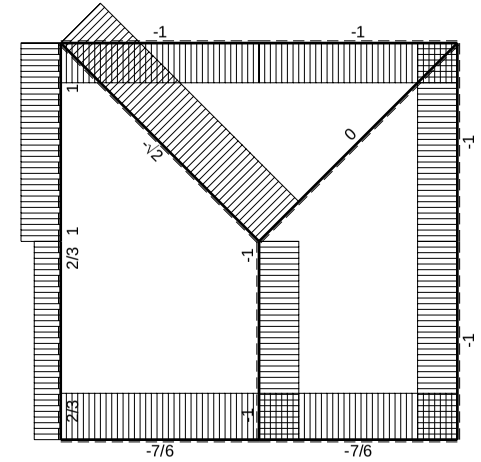
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

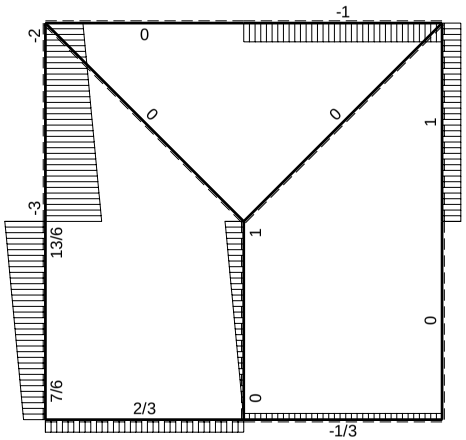
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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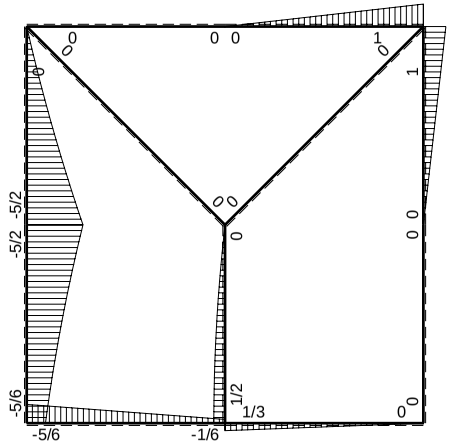

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5/2Fb
2/3F
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2/3F
B
C
7/6F
1/3Fb
1/3F
7/6F
1/3F


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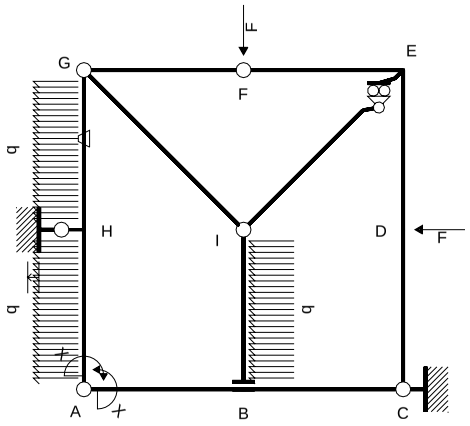

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2/3
0
-1/3

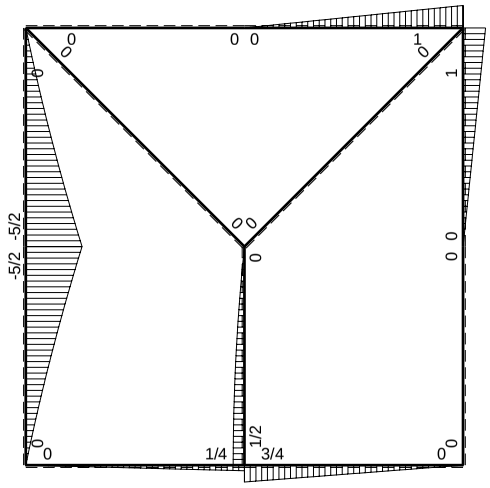
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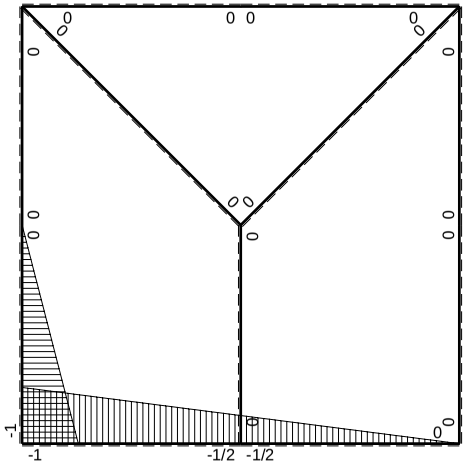

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-5/2
0 0
-5/6
1/2
1/3
0
0
-5/6
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

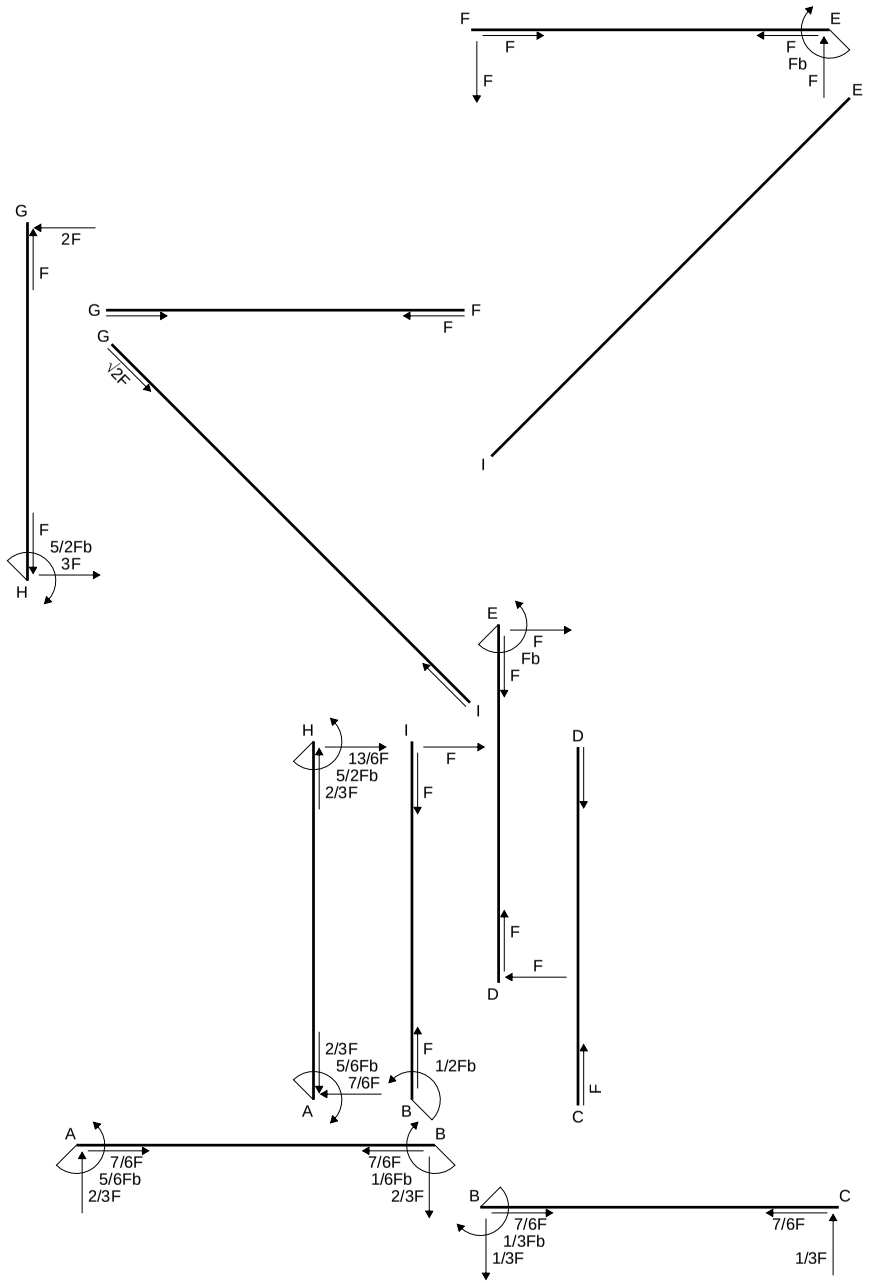
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

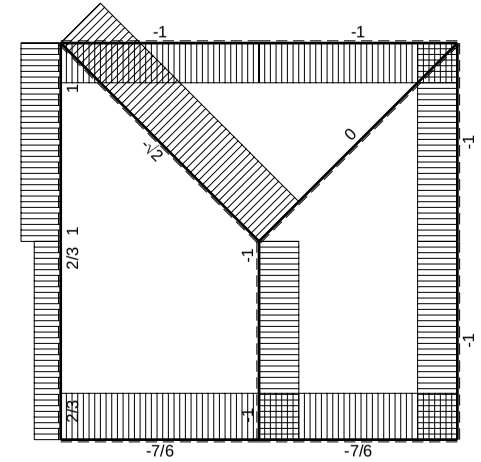
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

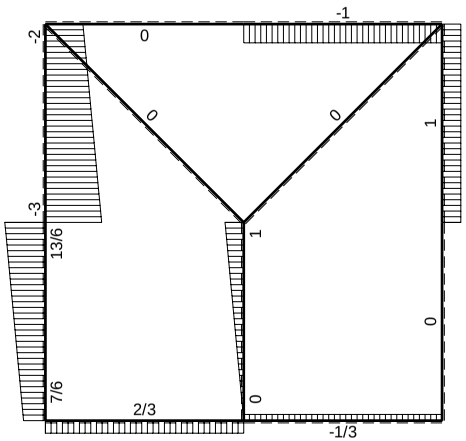
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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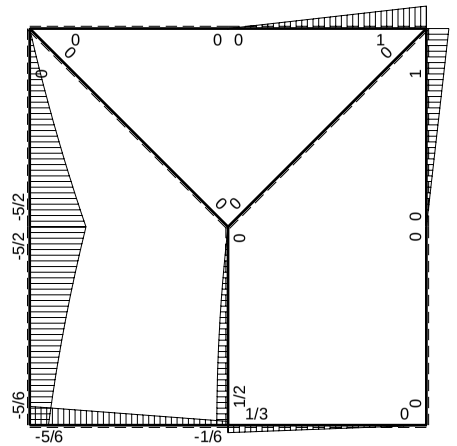

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D
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2/3F
5/6Fb
7/6F
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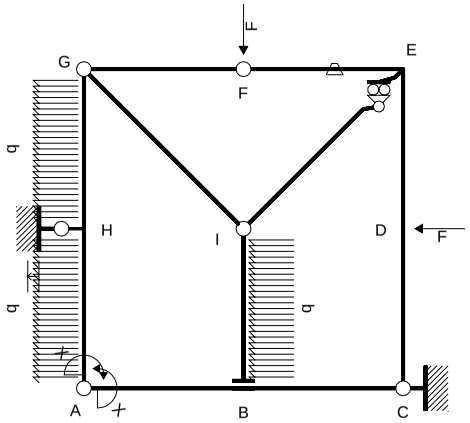

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-1/3

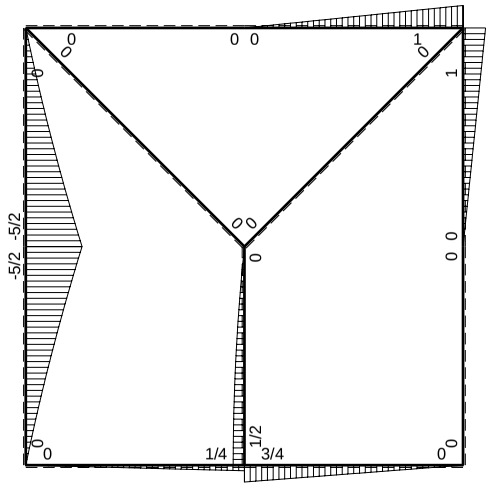
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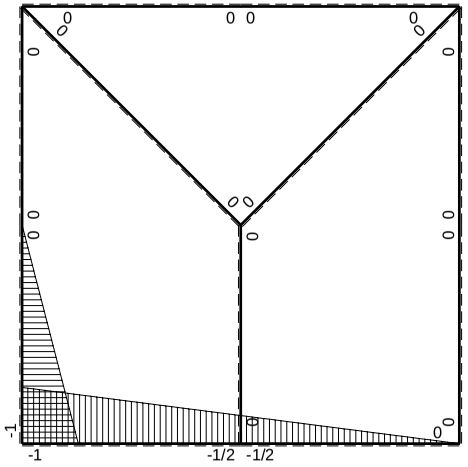

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-5/6
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

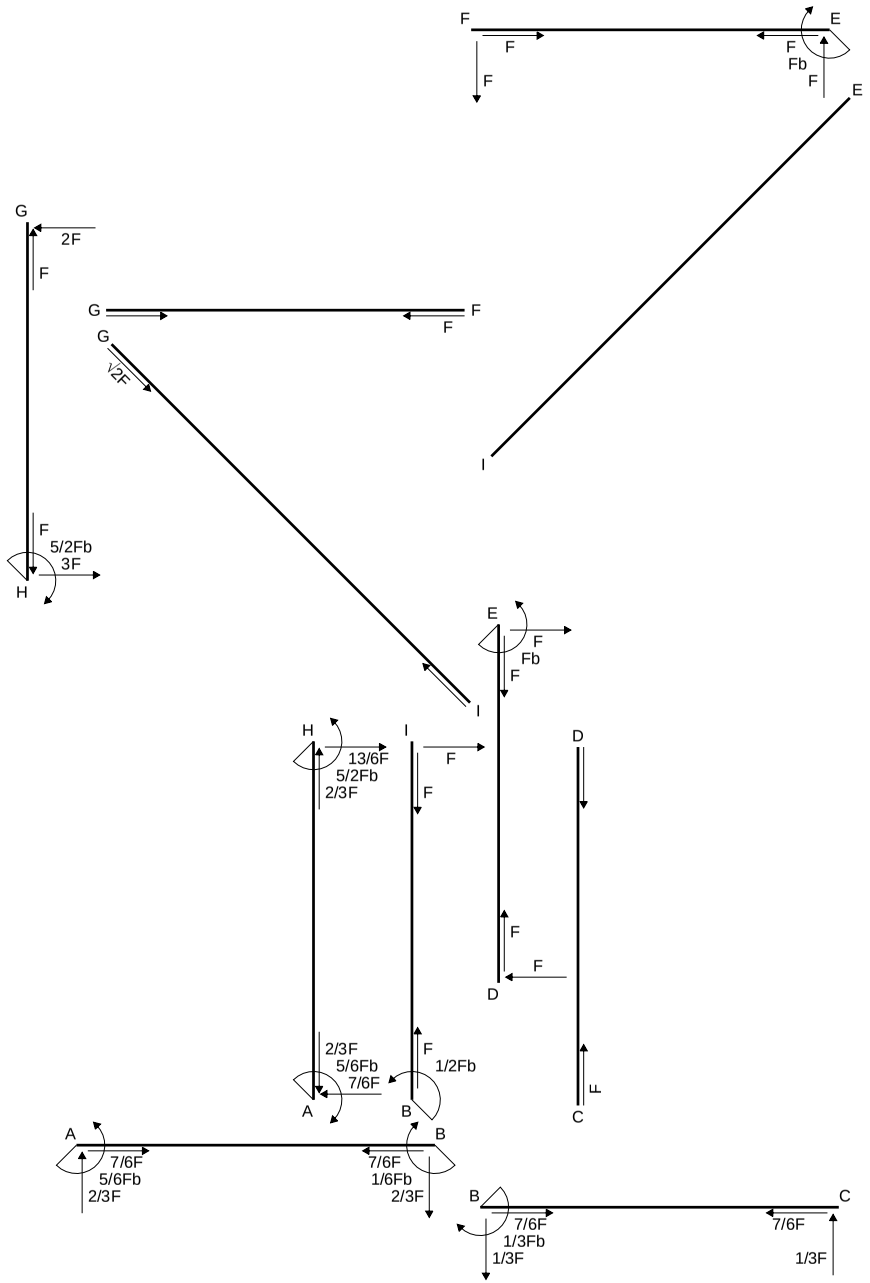
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

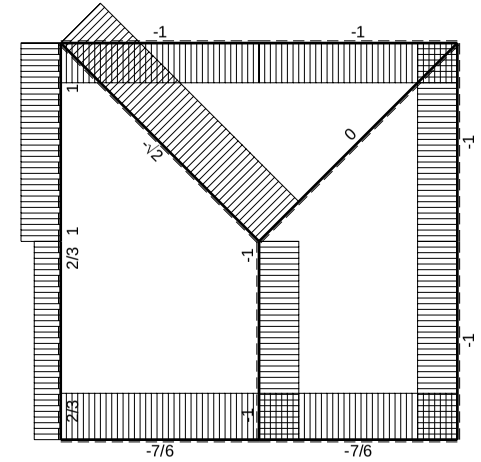
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

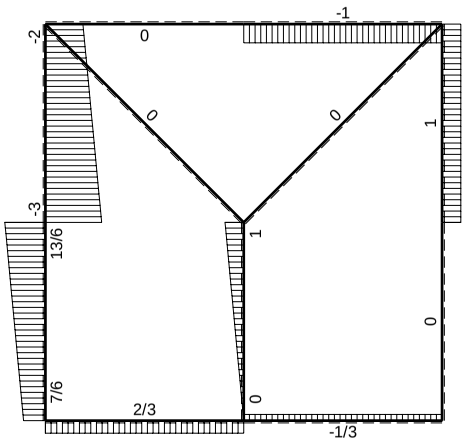
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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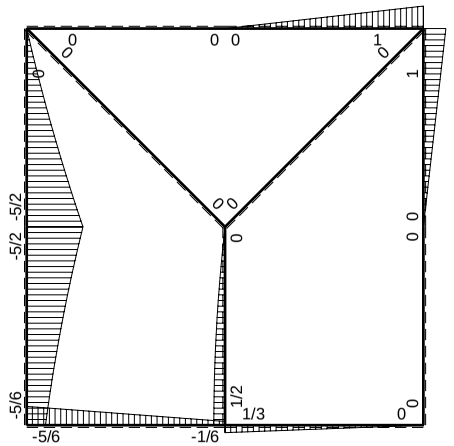

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B
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7/6F
1/3Fb
1/3F
7/6F
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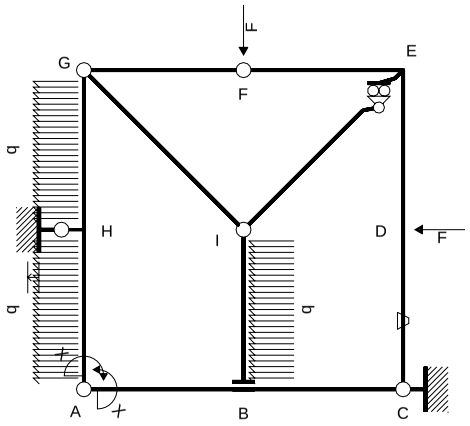

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7/6
2/3
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-1/3

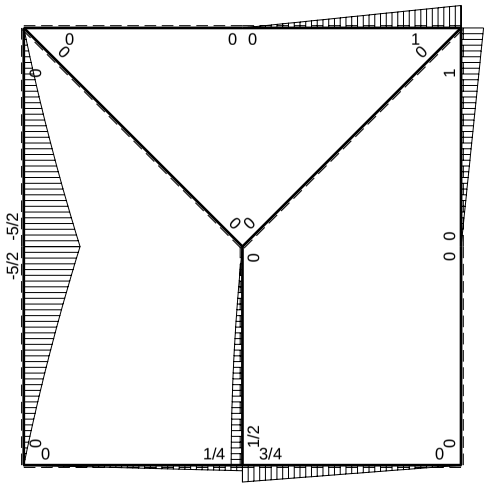
←□→ F

↑□↓ F

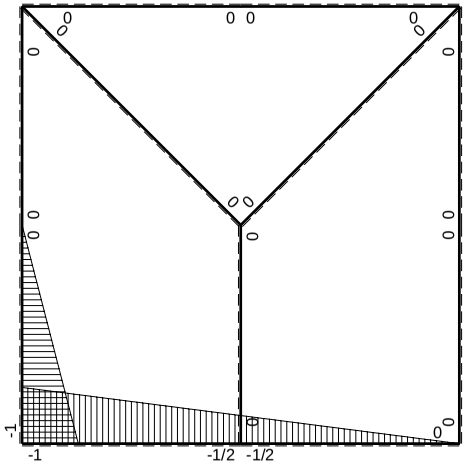

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0 0
-5/6
1/2
1/3
0
0
-5/6
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	-Fb/EJ	0	0	0	0+0	0
DC b	0	0	Fb/EJ	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

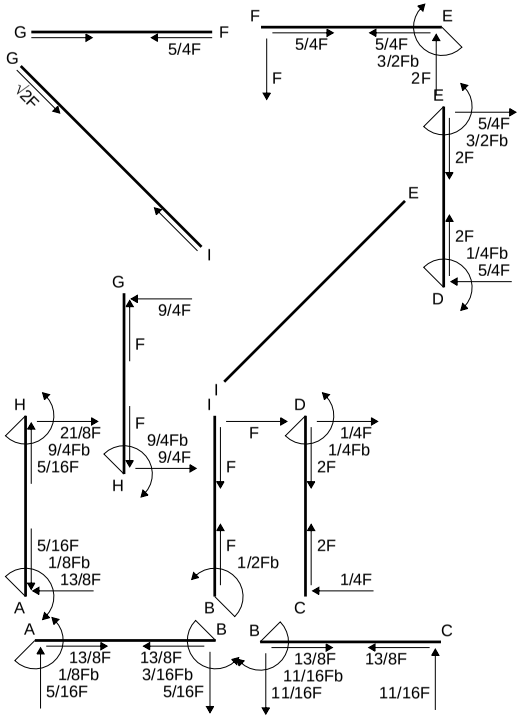
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

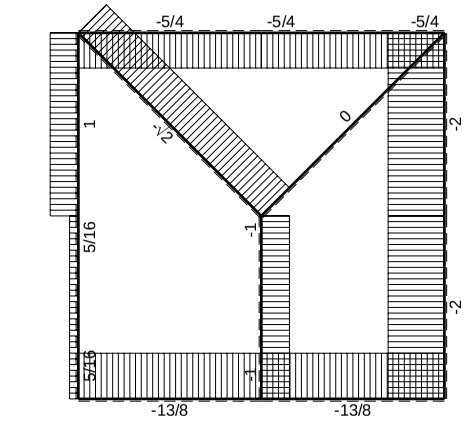
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

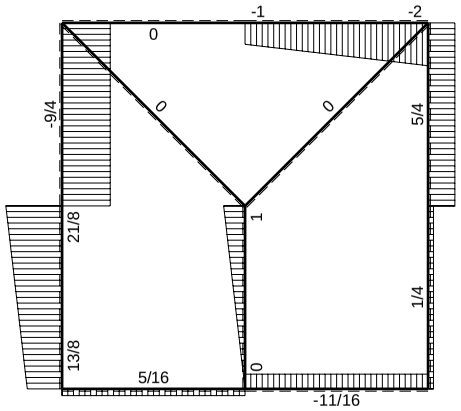
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

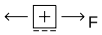
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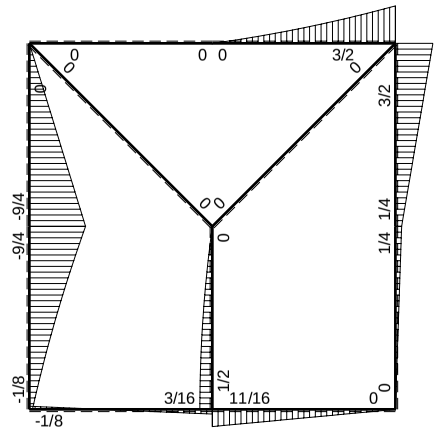

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3/2Fb
2F
E
2F
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G
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F
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-13/8

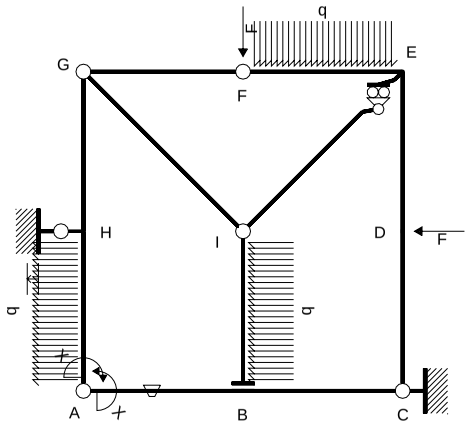

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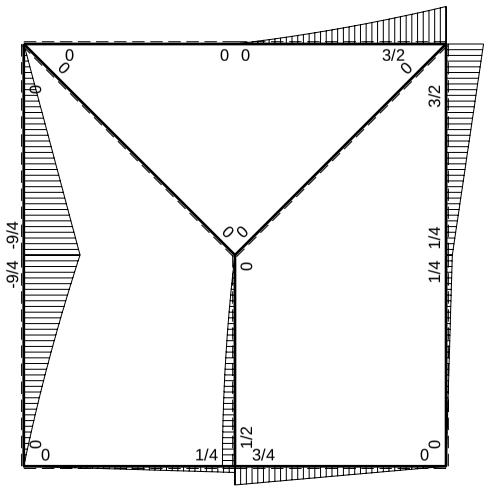

F


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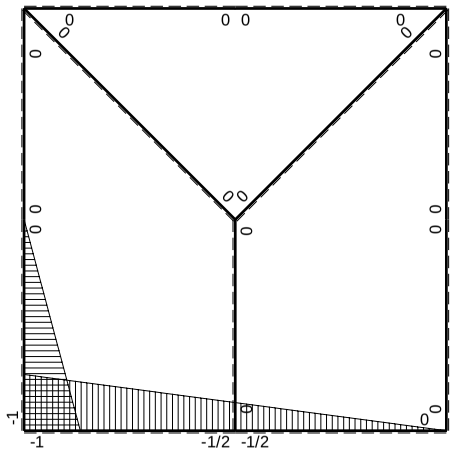

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3/2
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-9/4
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1/4
1/4
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3/16
-1/2
11/16
0
0
-1/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238503 ESΣ04.xxxx.012

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	-Fb/EJ	-1/4Fx+1/8Fx^2/b	Fb/EJ-1/2Fx/EJ	1-x/b+1/4x^2/b^2	(-1/12+3/4)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	Fb/EJ	-1/8Fb+1/8Fx^2/b	1/2Fb/EJ+1/2Fx/EJ	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	-3/8Fb+3/4Fx-3/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-1/8+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	-3/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-2Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-Fx-1/2qx^2	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
BI b	0	-1/2Fb+1/2qx^2	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	-9/4Fb+11/4Fx-1/2qx^2	0	9/4Fx-11/4Fx^2/b+1/2qx^3/b	0	x^2/b^2	(1/3+0)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	7/4Fx+1/2qx^2	0	7/4Fx-5/4Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-1/8Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						1/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

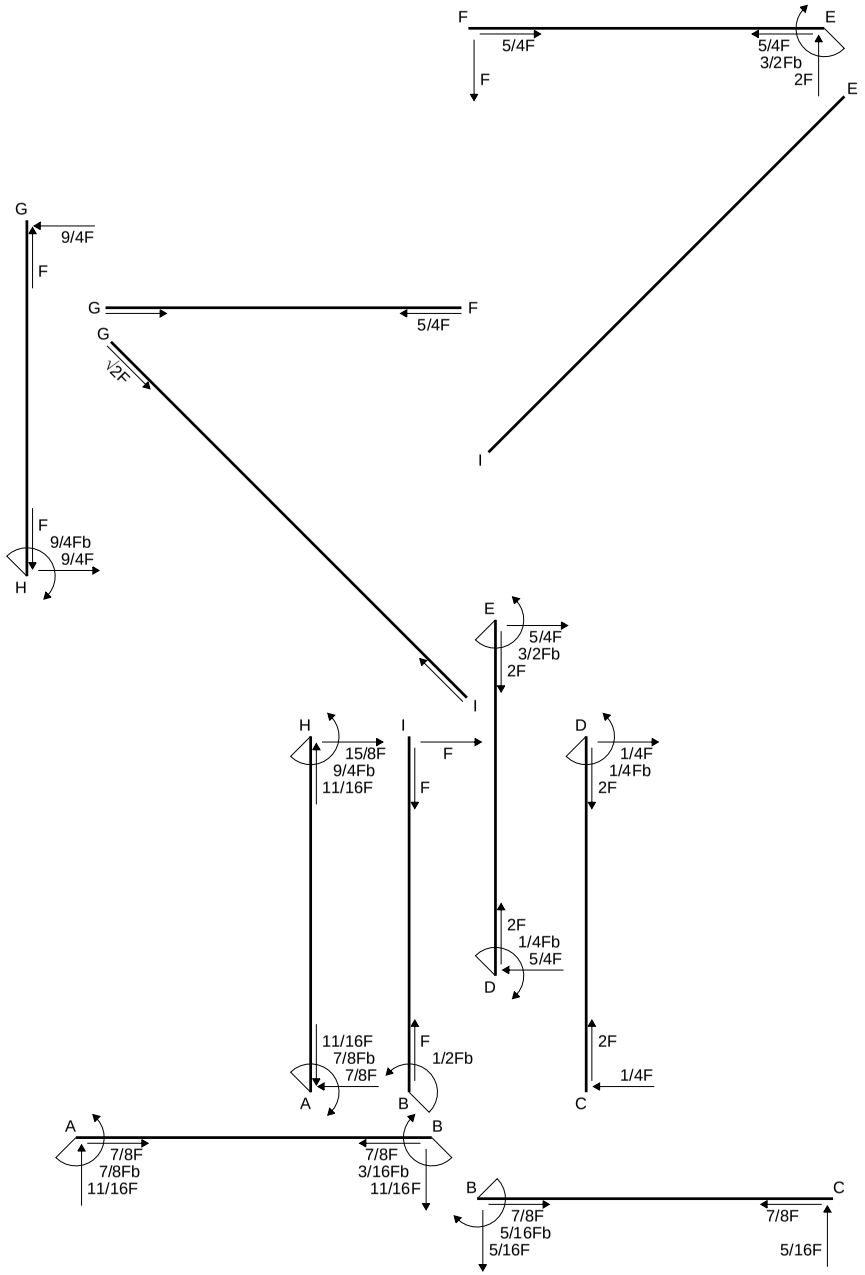
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

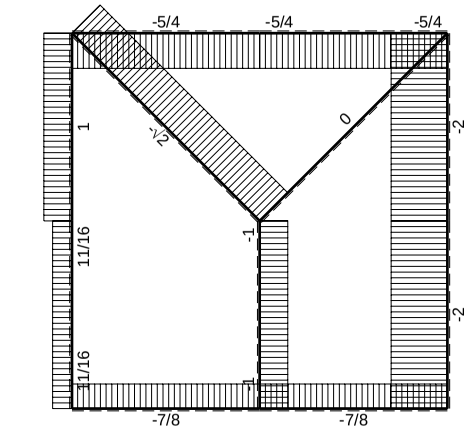
$L^{Xo}_{AH} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

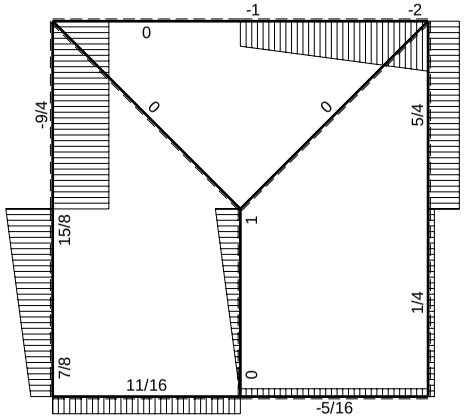
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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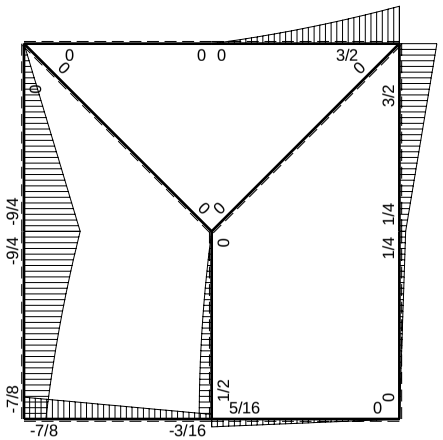

F
5/4F
E
5/4F
3/2Fb
2F
F
E
G
9/4F
F
G
F
5/4F
G
½F
I
F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
2F
I
H
15/8F
9/4Fb
11/16F
I
F
F
D
1/4F
1/4Fb
2F
2F
1/4Fb
5/4F
D
11/16F
7/8Fb
7/8F
F
1/2Fb
A
B
2F
1/4F
C
A
B
7/8F
7/8Fb
11/16F
7/8F
3/16Fb
11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
-5/4
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1
½
0
-2
-1
11/16
-2
11/16
-1
-7/8
-7/8

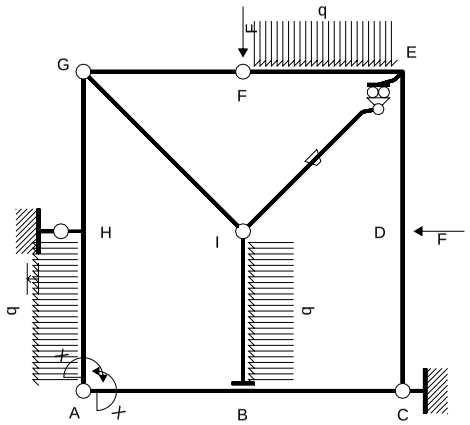

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-9/4
0
0
5/4
15/8
1
1/4
7/8
0
11/16
-5/16

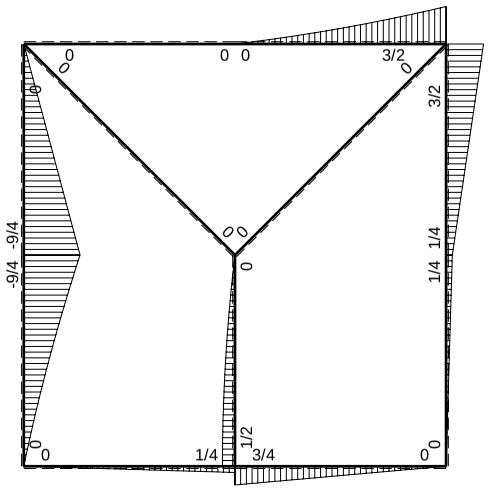
←+→ F

↑+↓ F

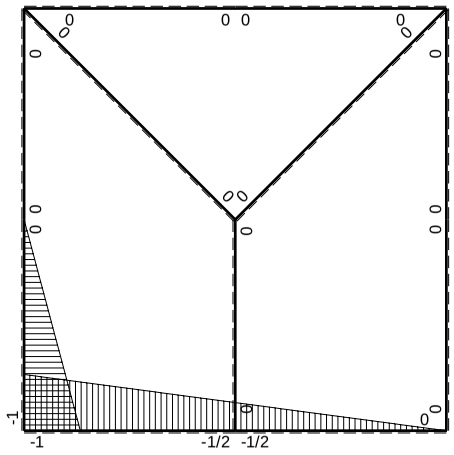

0
0 0
3/2
0
0
3/2
0
0
-9/4
-9/4
0
1/4
1/4
-9/4
0
1/2
5/16
0
0
-7/8
-7/8
-3/16

↻+↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

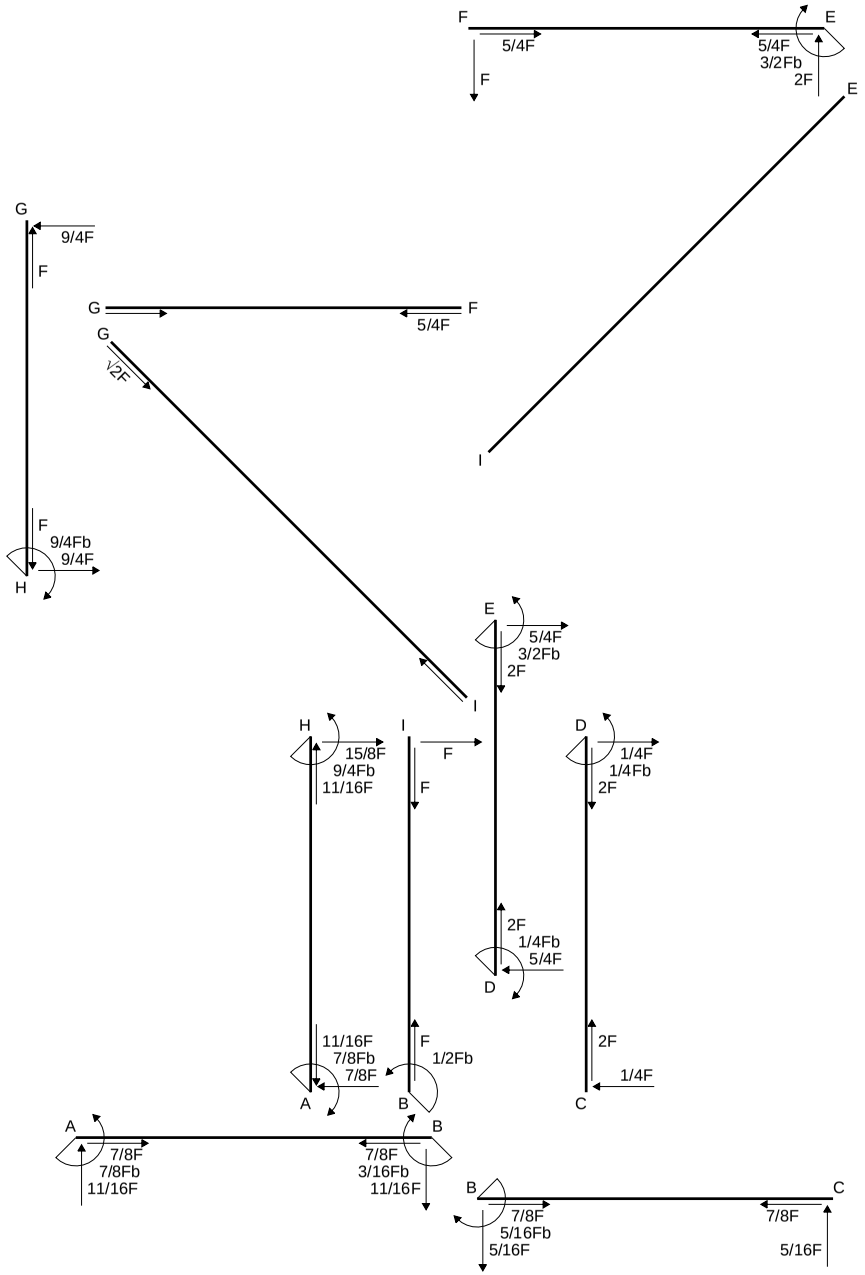
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

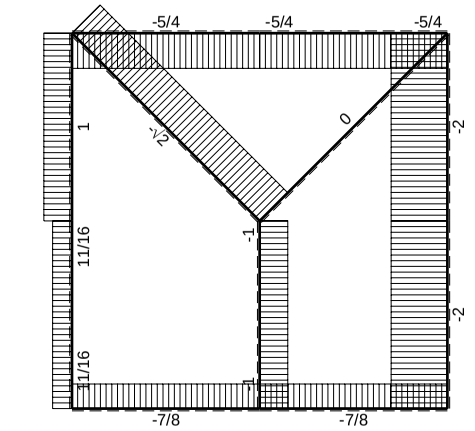
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

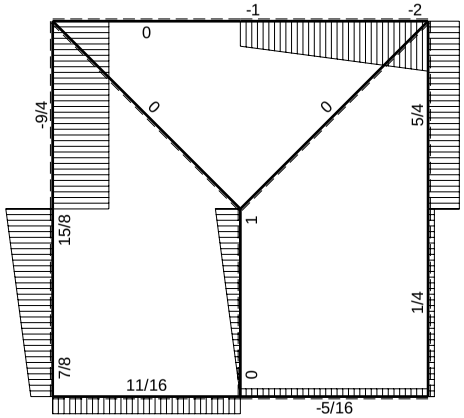
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

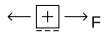
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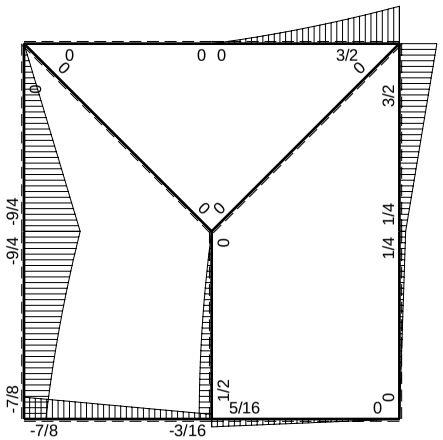






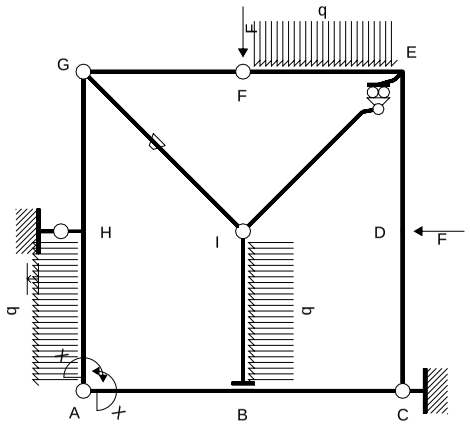


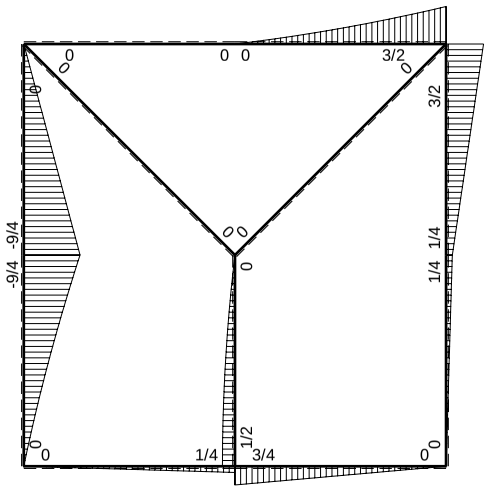




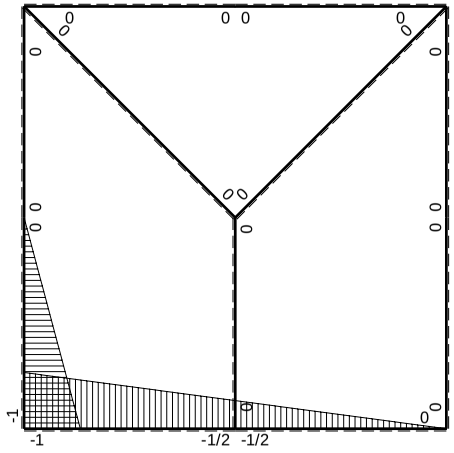


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

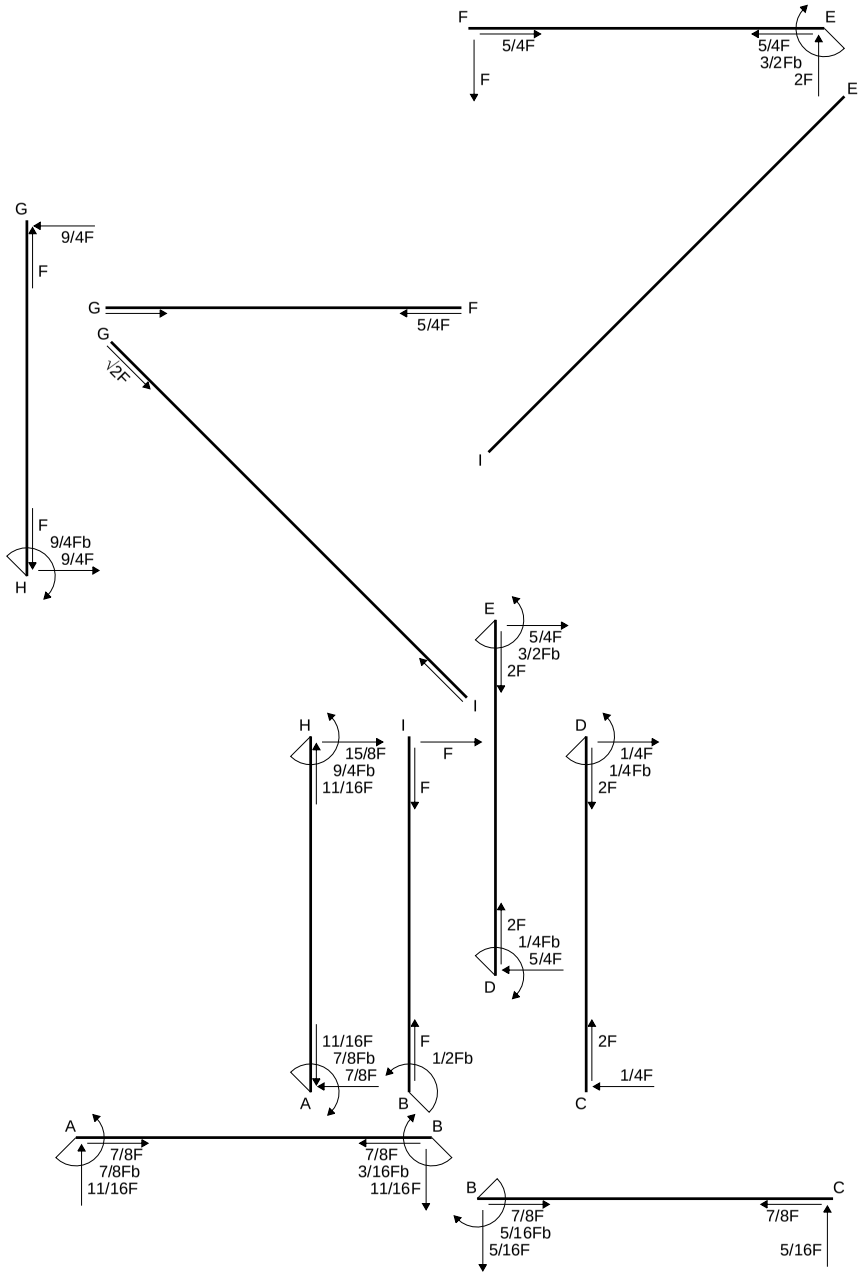
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

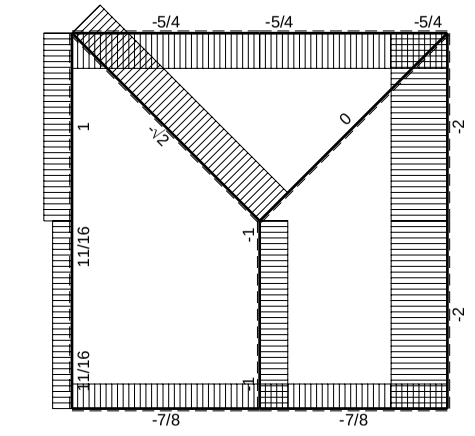
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

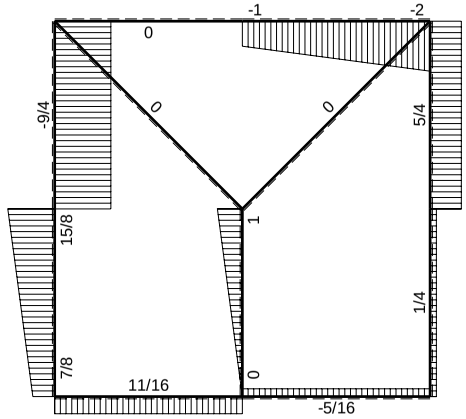
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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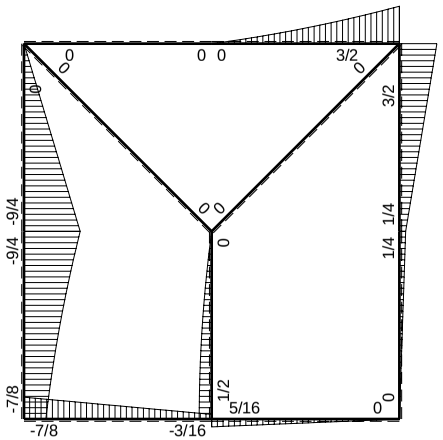

F
5/4F
E
5/4F
3/2Fb
2F
F
E
G
9/4F
F
G
F
5/4F
G
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I
F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
2F
I
H
15/8F
9/4Fb
11/16F
I
F
F
D
1/4F
1/4Fb
2F
2F
1/4Fb
5/4F
D
11/16F
7/8Fb
7/8F
F
1/2Fb
A
B
2F
1/4F
C
A
B
7/8F
7/8Fb
11/16F
7/8F
3/16Fb
11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
-5/4
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1
½
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-2
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11/16
-2
11/16
-1
-7/8
-7/8

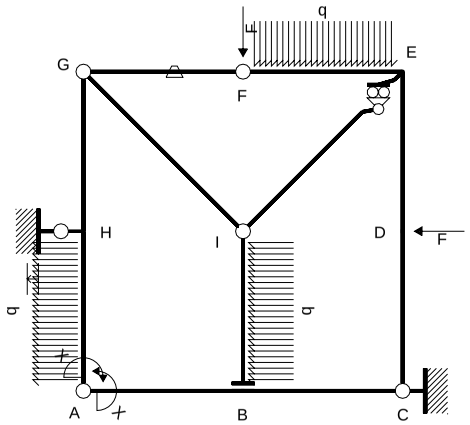

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0
5/4
15/8
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1/4
7/8
0
11/16
-5/16

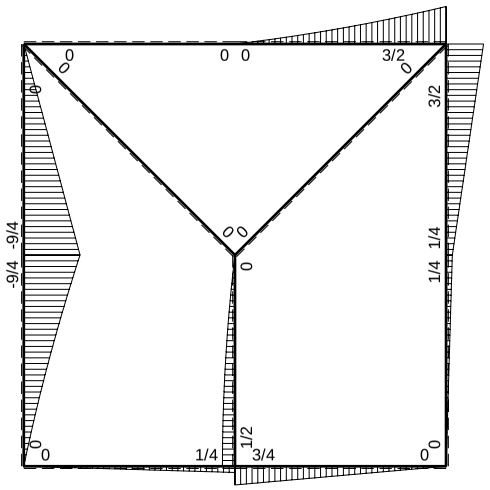
←□→ F

↑□↓ F

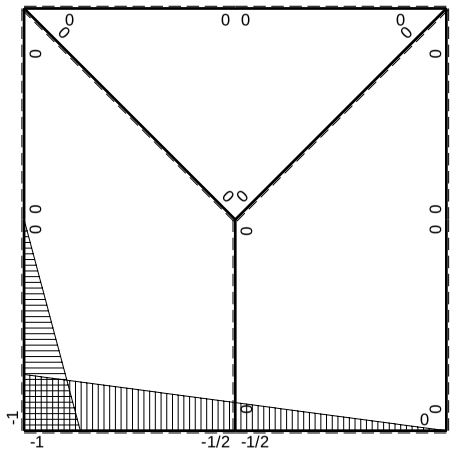

0
0 0
3/2
0
0
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-9/4
0
0
-9/4
1/4
1/4
0
-7/8
1/2
5/16
0
0
-7/8
-3/16

□ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

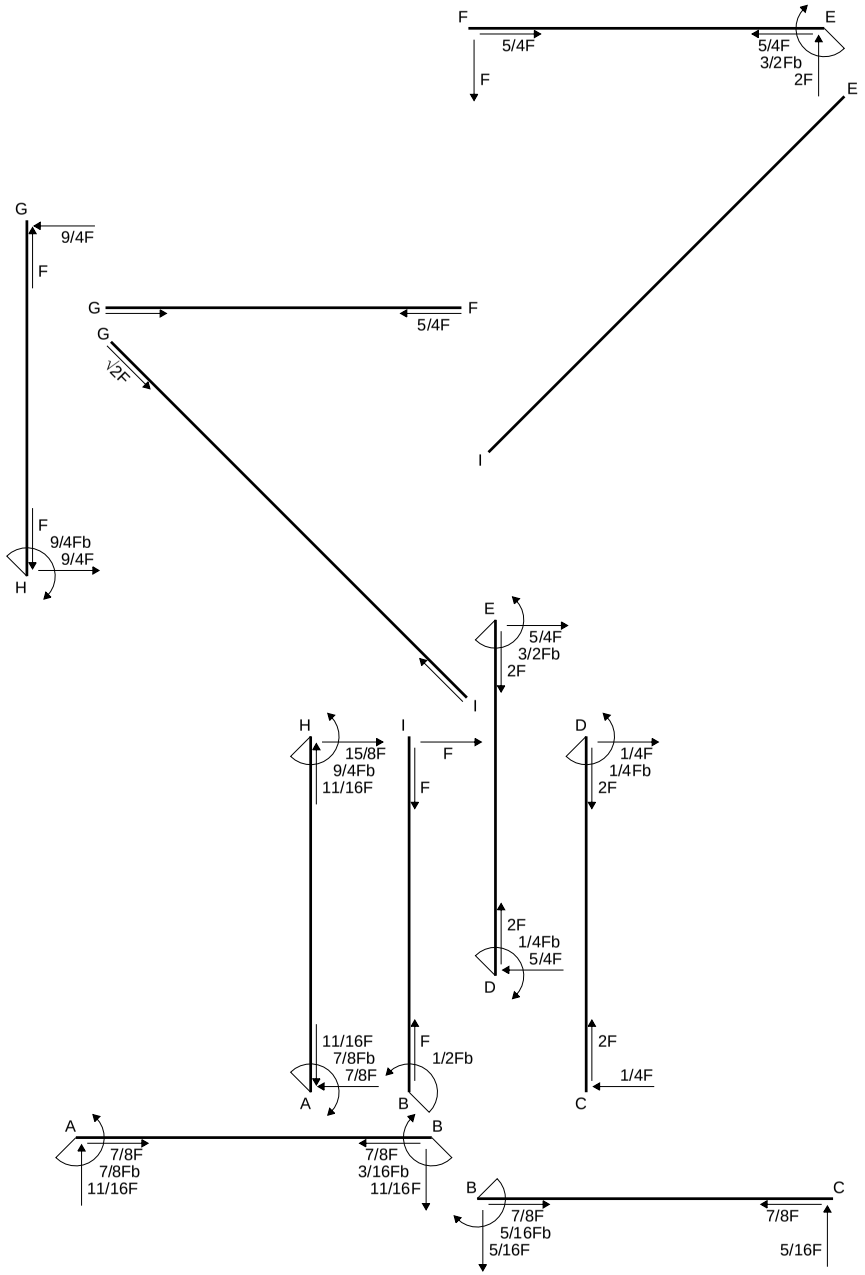
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

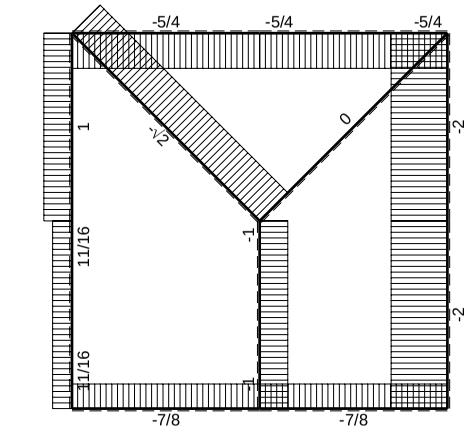
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

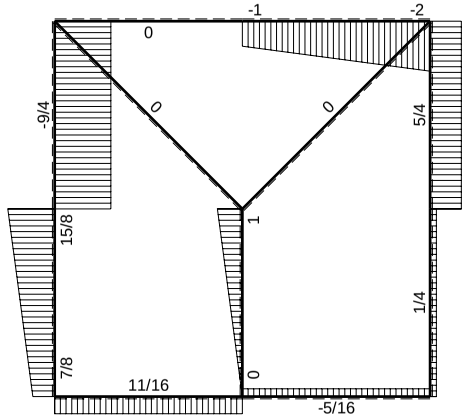
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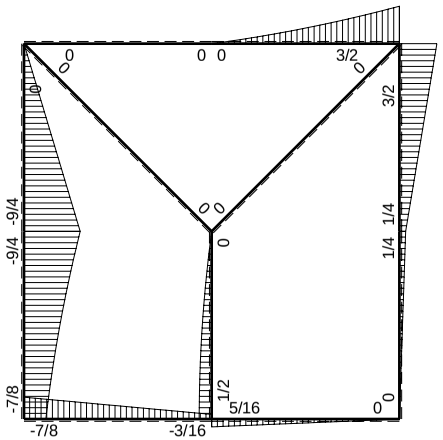




F

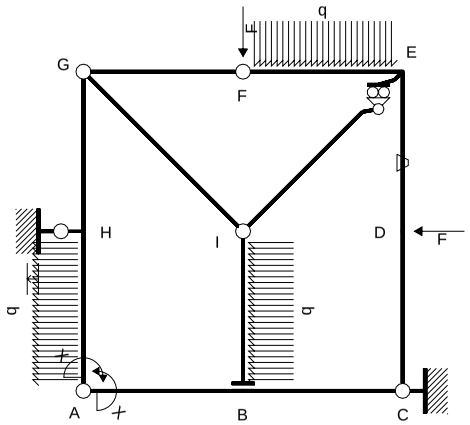


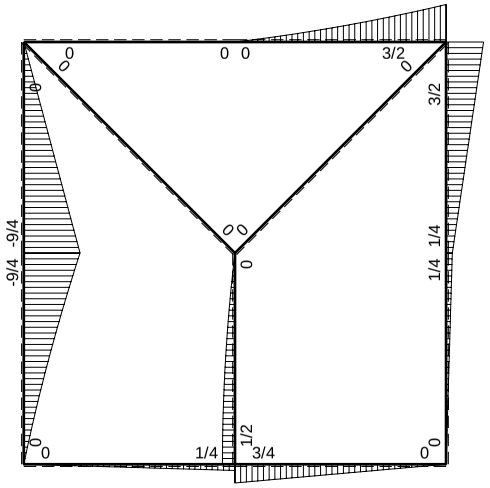
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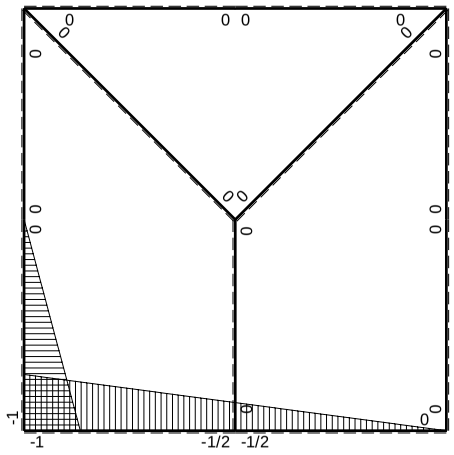
Fb

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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

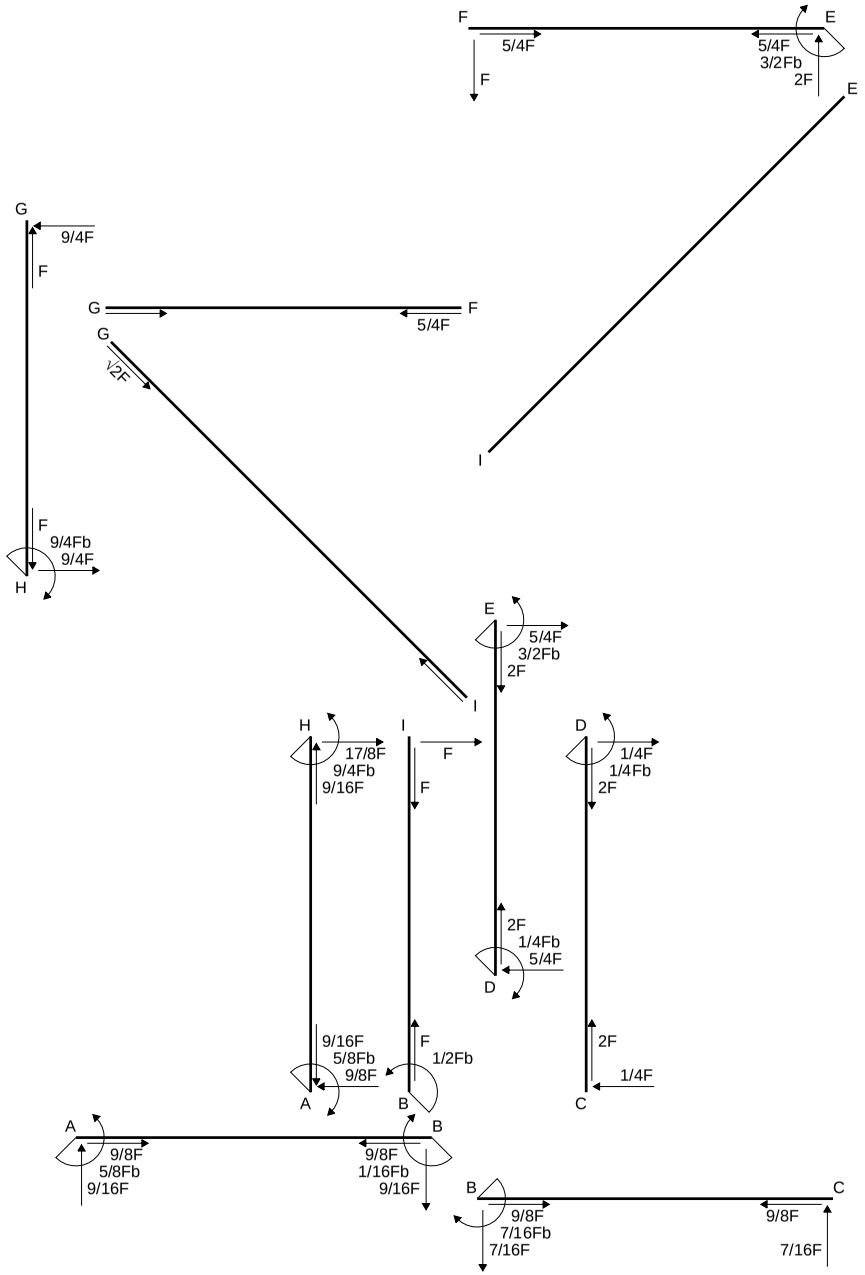
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

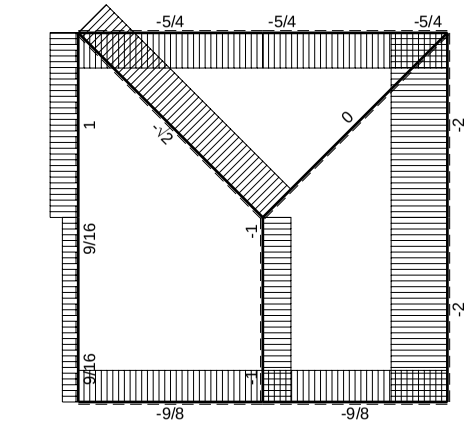
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

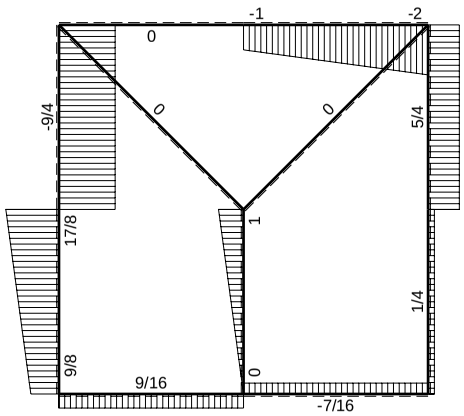
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

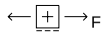
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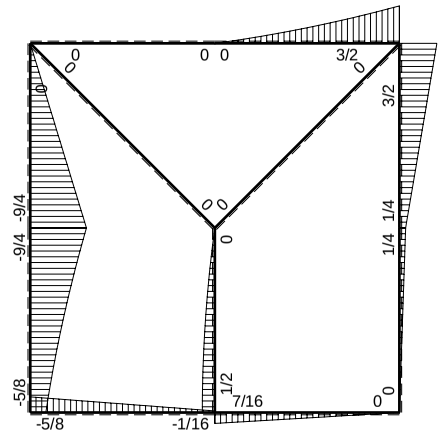

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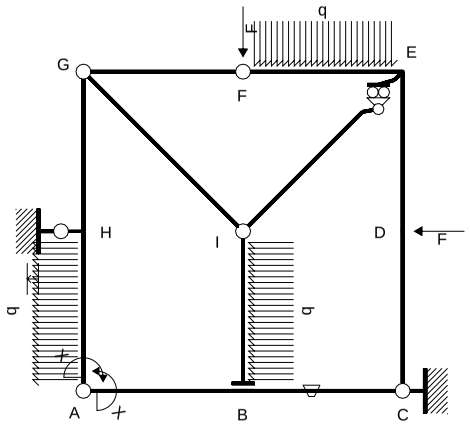

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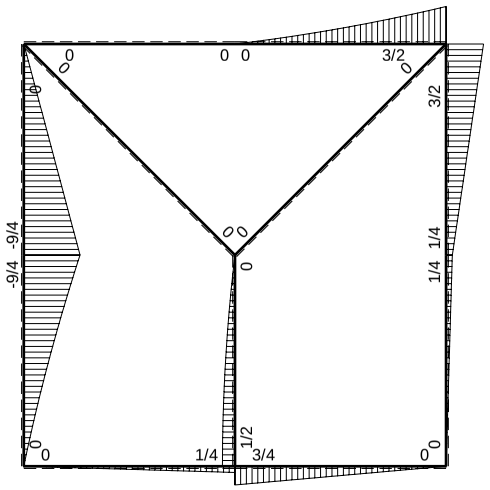

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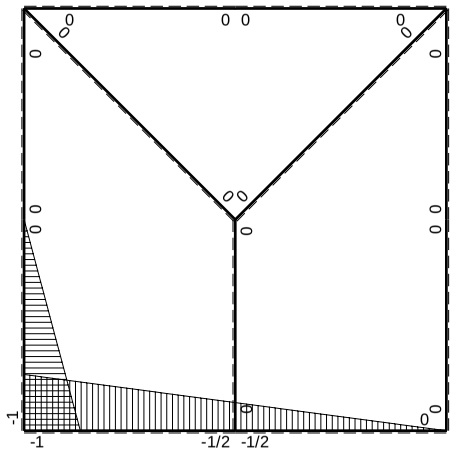

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7/16
0
0
-5/8
-1/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	-Fb/EJ	$-3/8Fb+3/4Fx-3/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	Fb/EJ	$-3/8Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/8Fb	

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Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\ \right)\ \theta\ \ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

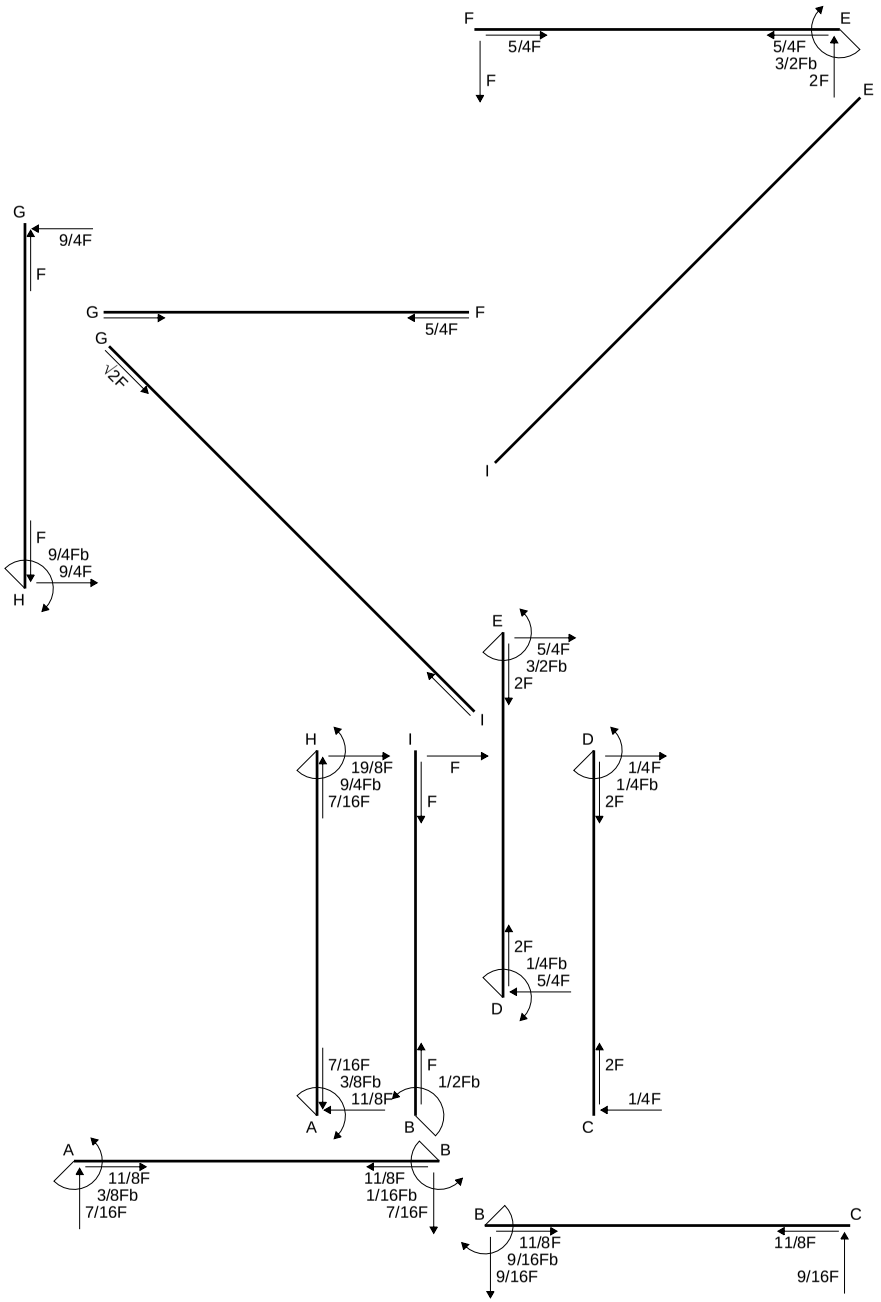
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

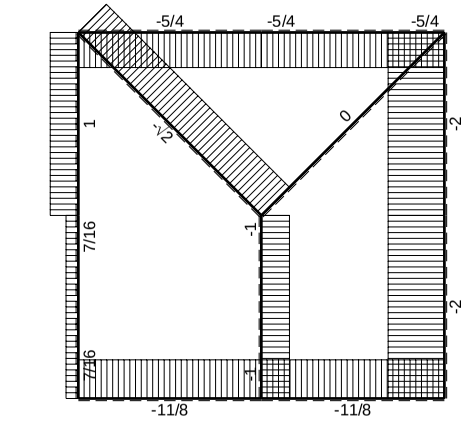
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

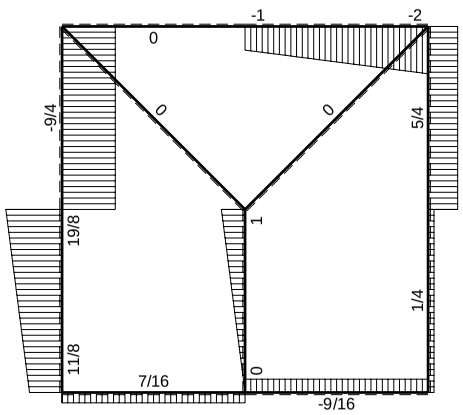
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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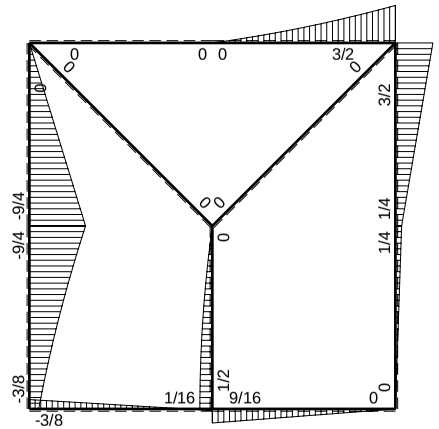

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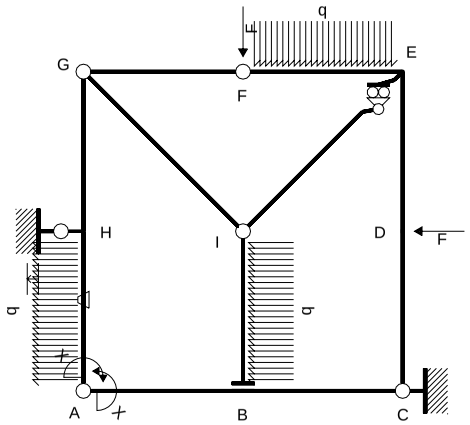

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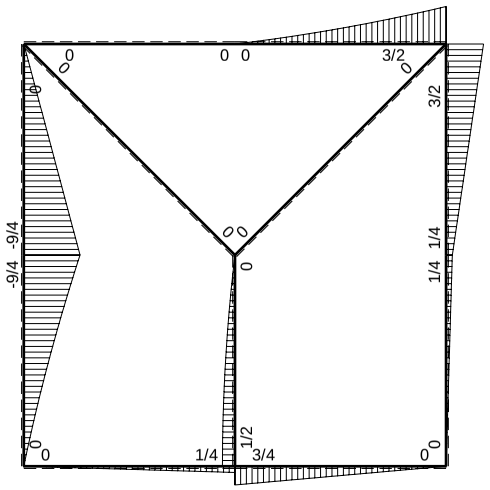
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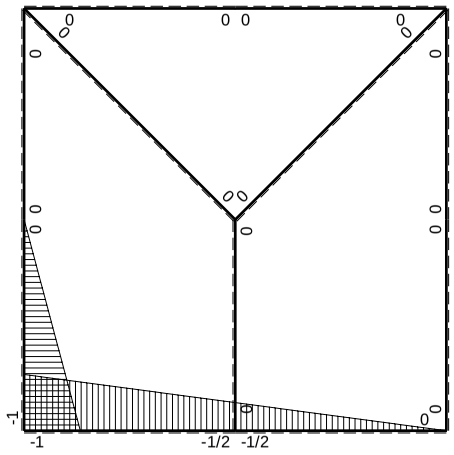

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-1/2
9/16
0
0
-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	-Fb/EJ	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/3+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	Fb/EJ	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx$

$= \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

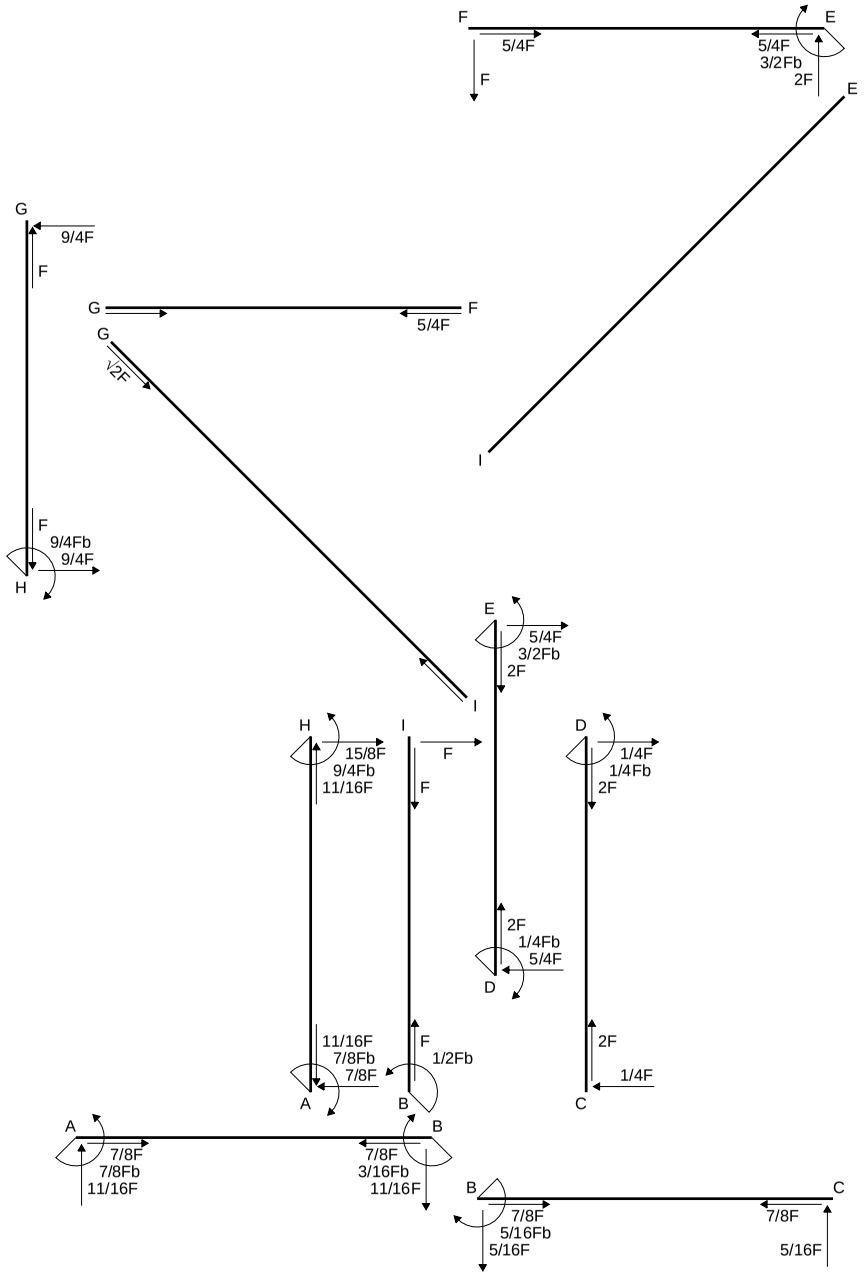
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

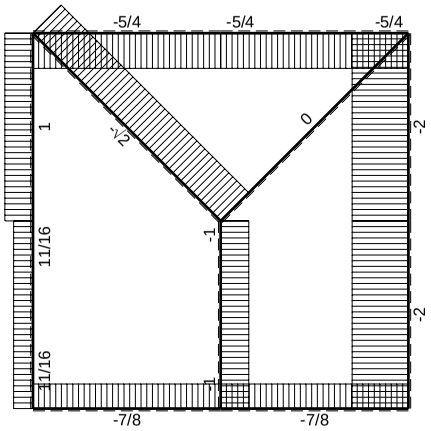
$= \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

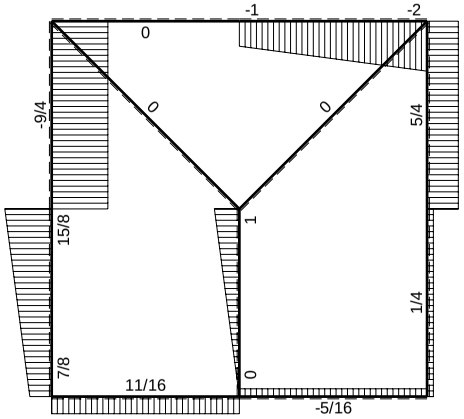
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

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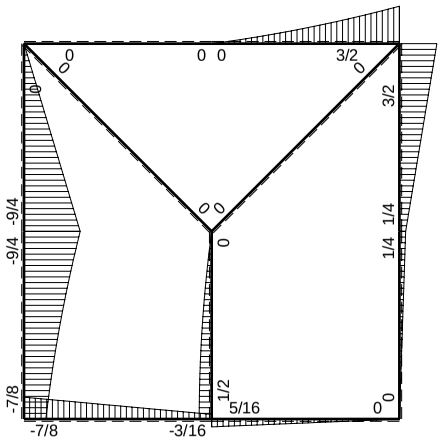

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5/16Fb
5/16F
7/8F
5/16F


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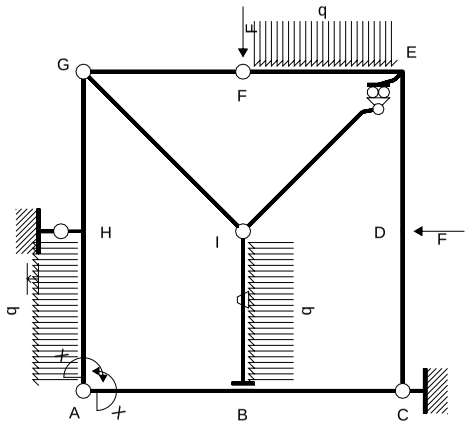

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-5/16

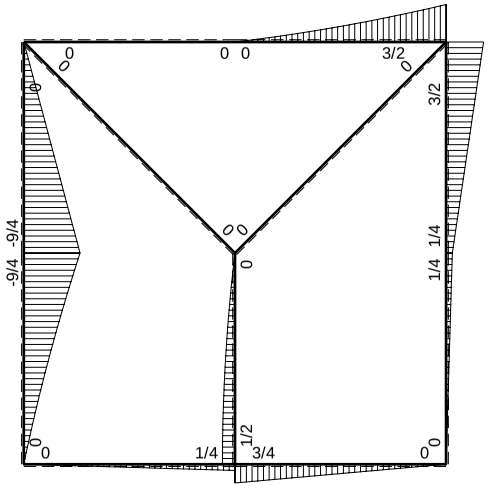
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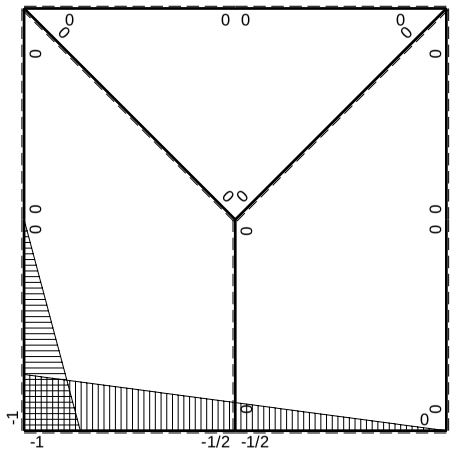

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-9/4
1/4
1/4
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-7/8
1/2
5/16
0
0
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

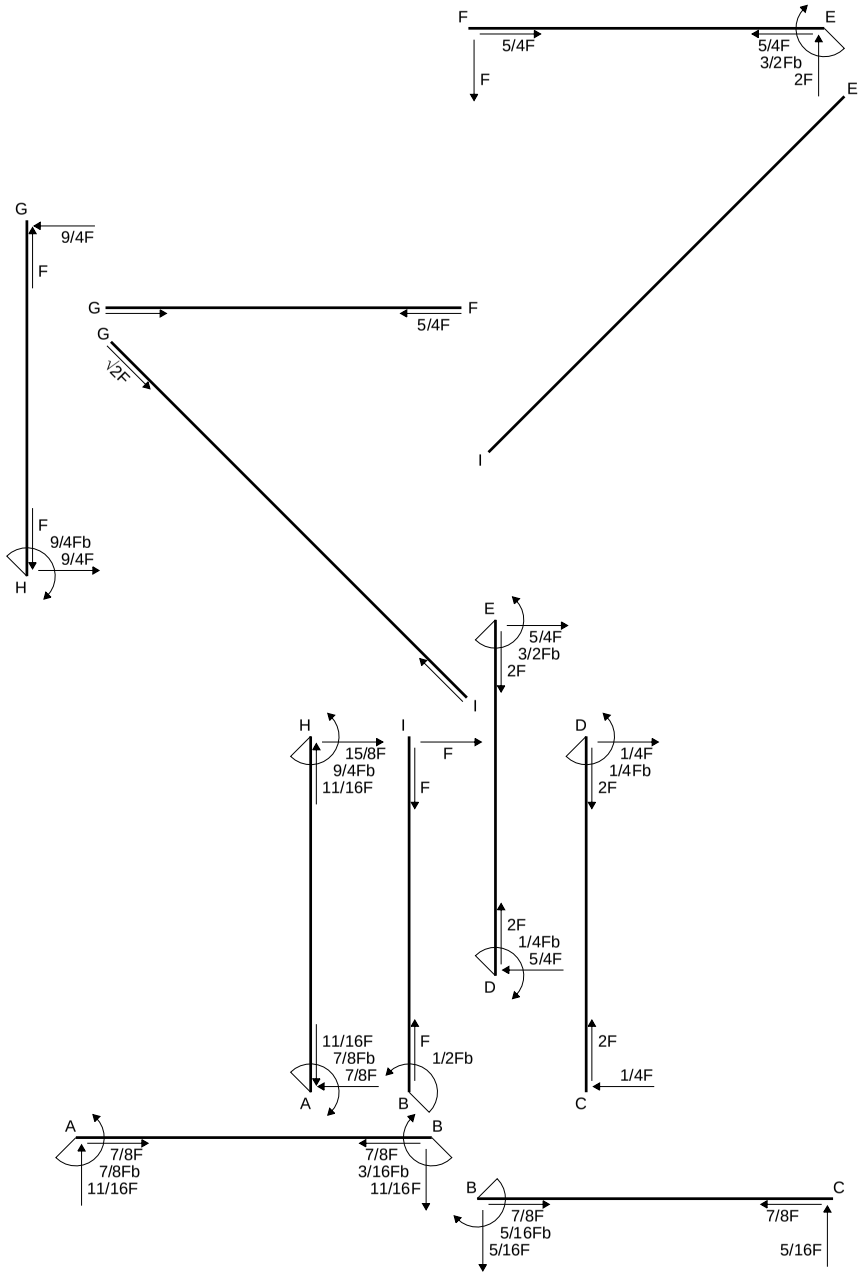
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

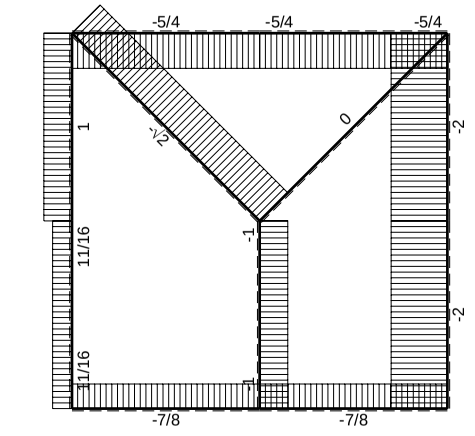
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

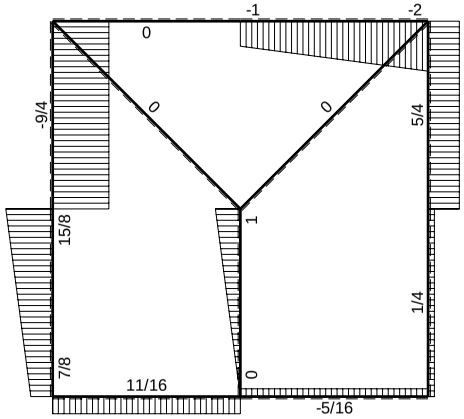
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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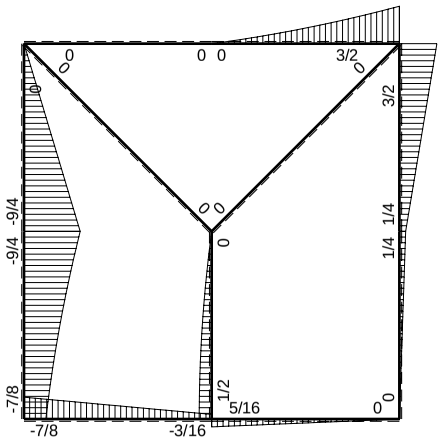

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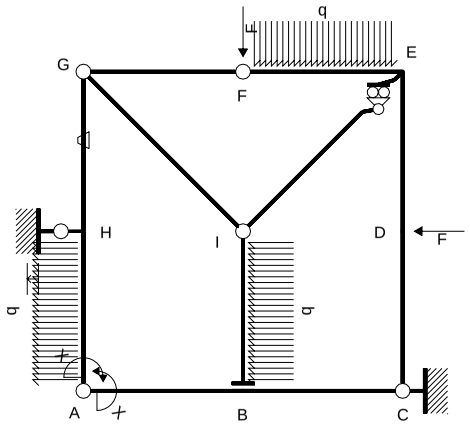

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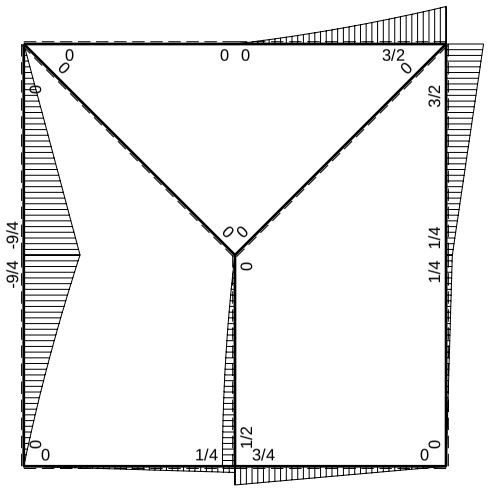
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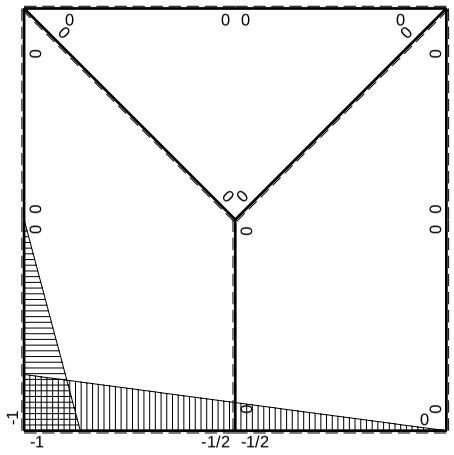

0
0 0
3/2
0
0
3/2
0
0
-9/4
-9/4
0
1/4
1/4
-9/4
0
1/2
5/16
0
0
-7/8
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	Fb/EJ	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

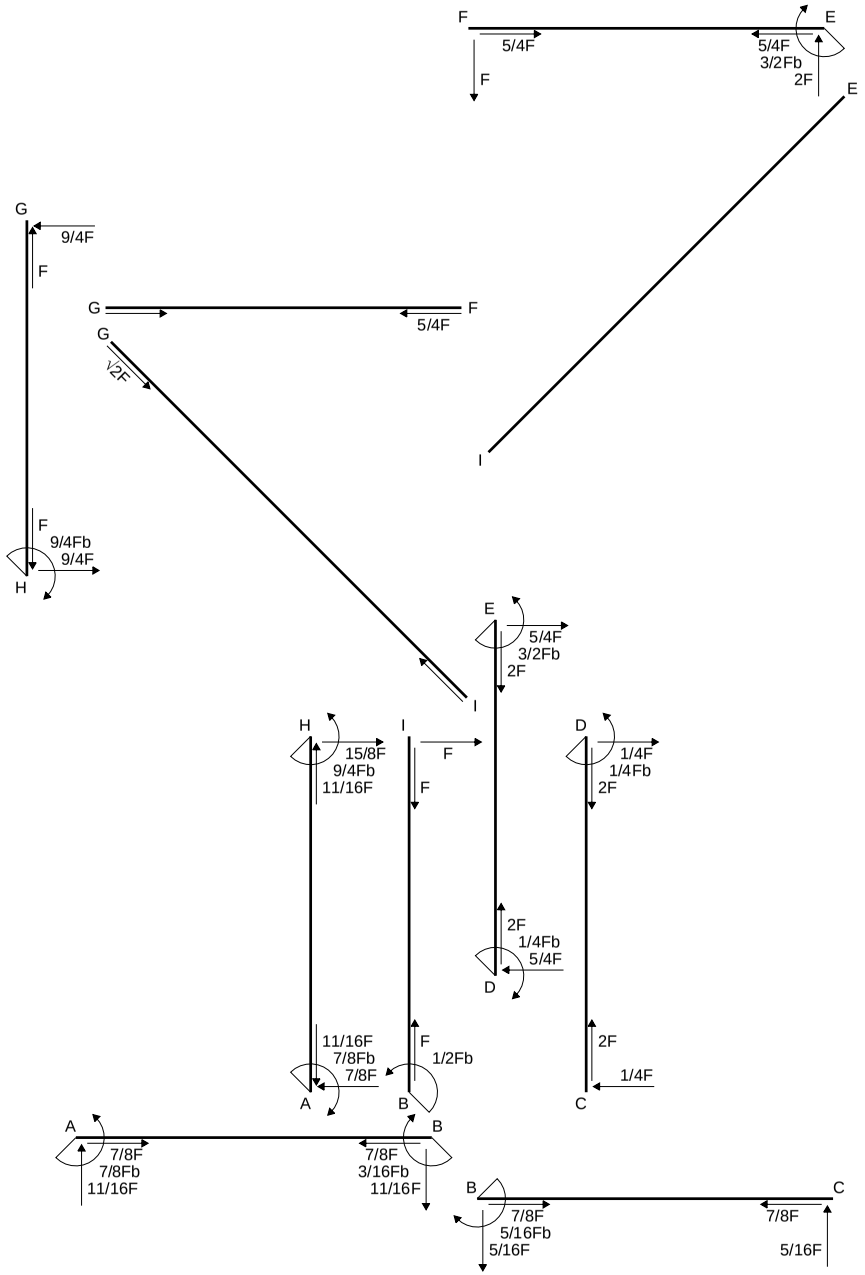
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

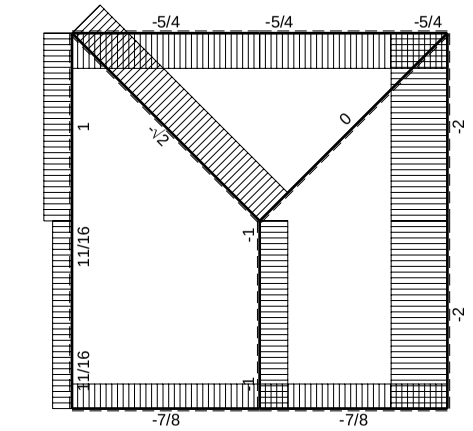
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

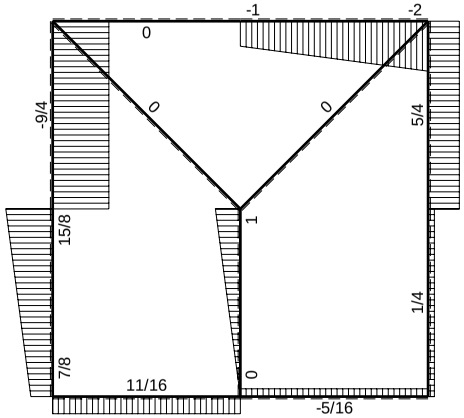
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

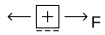
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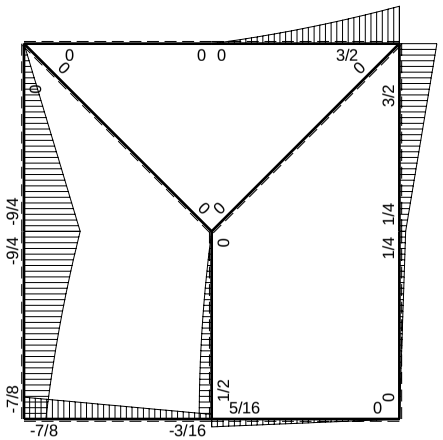






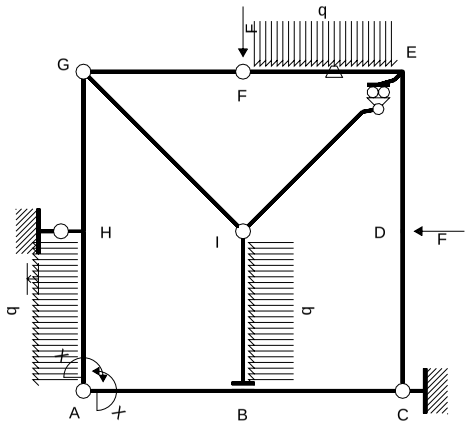


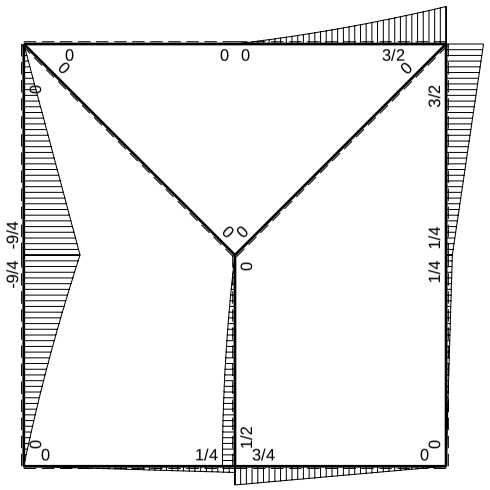




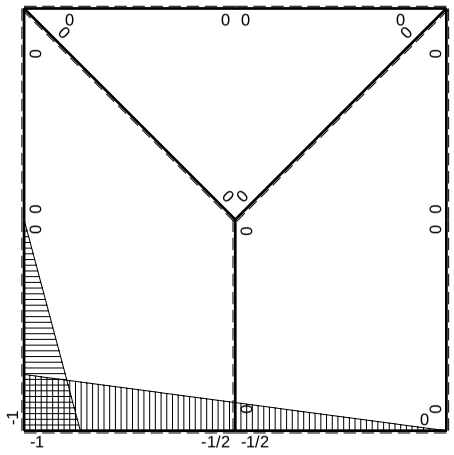


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

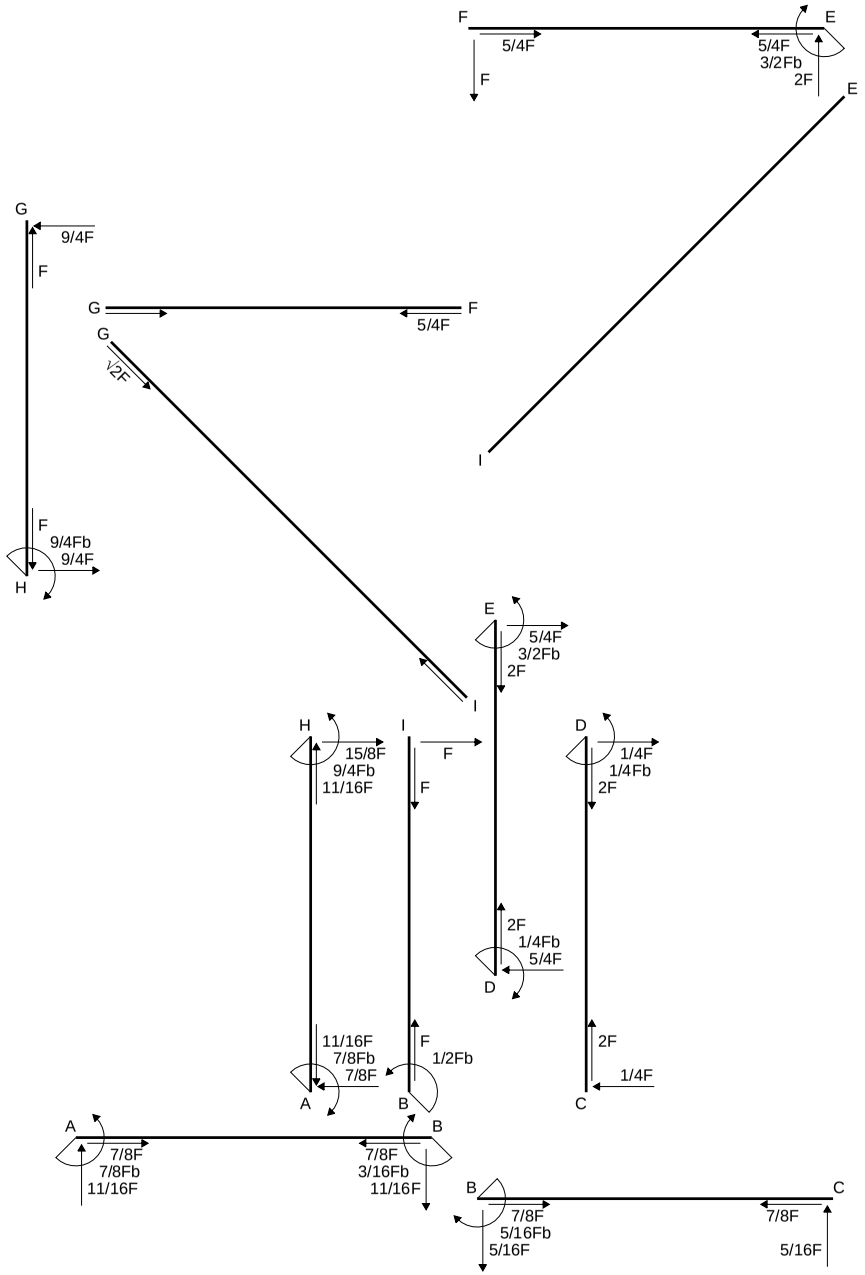
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

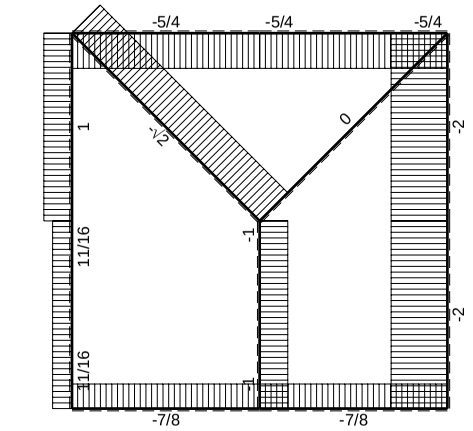
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

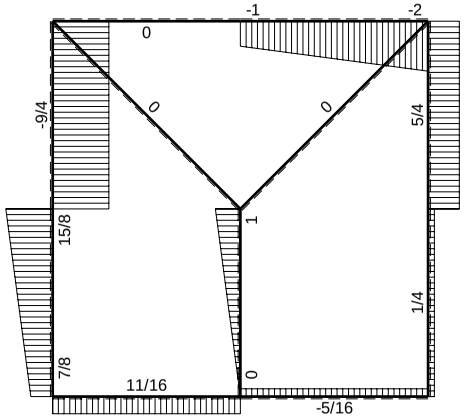
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

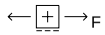
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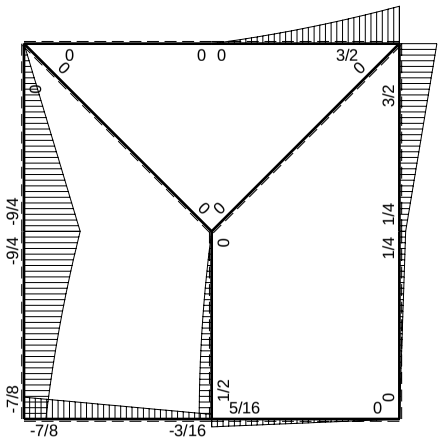






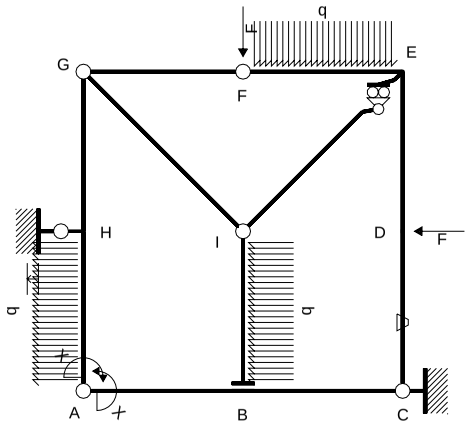


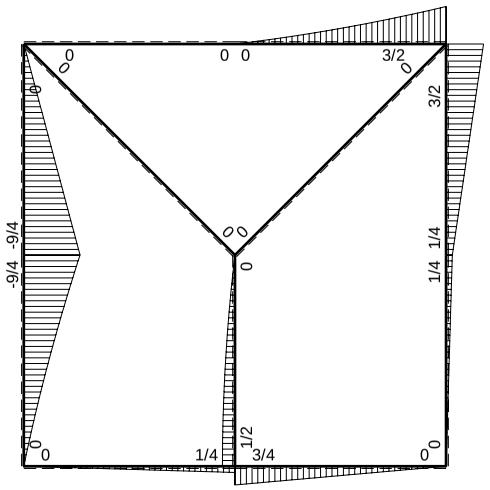




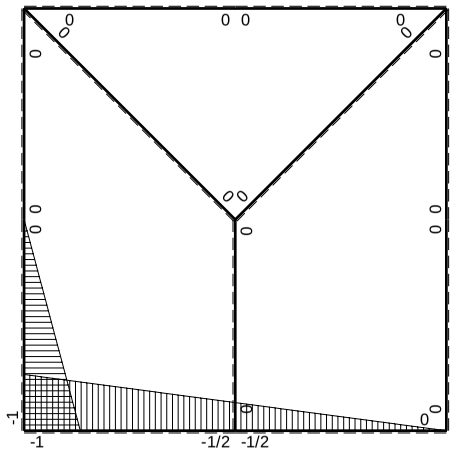


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

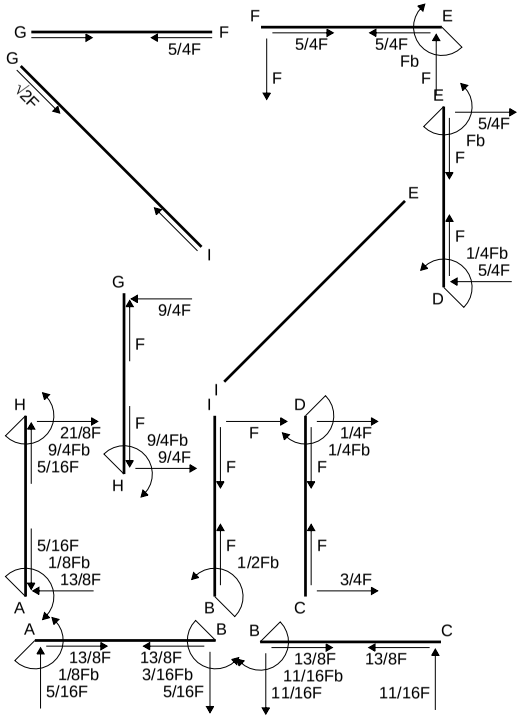
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

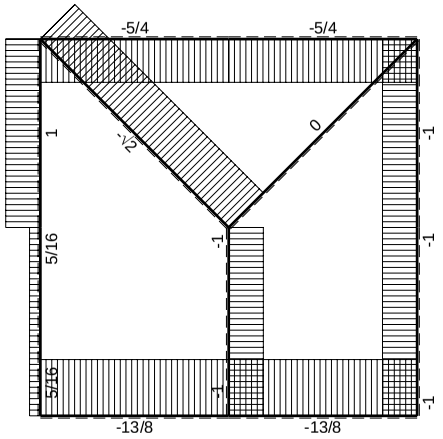
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

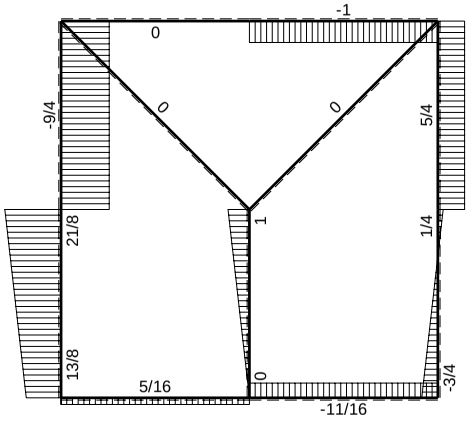
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

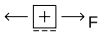
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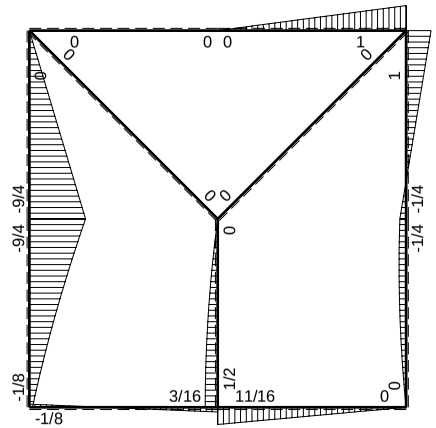

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11/16F


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-13/8

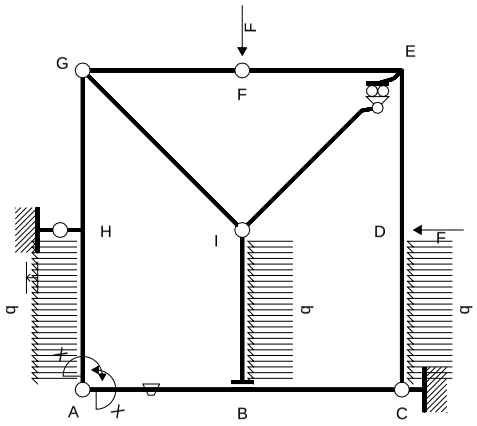

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21/8
1/4
13/8
5/16
0
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-11/16

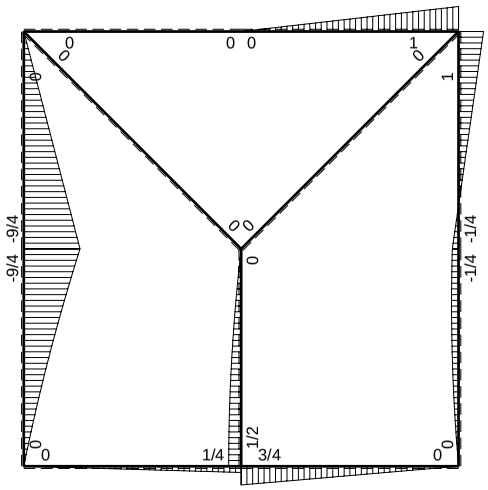

F


F

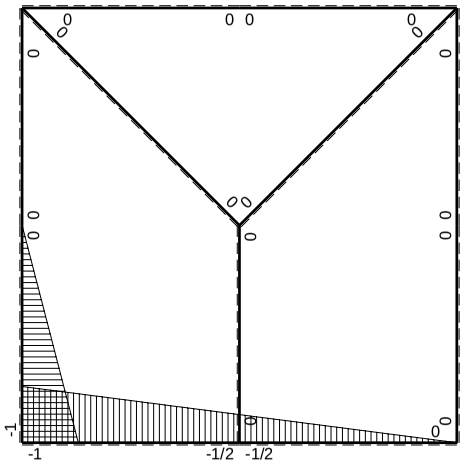

0
0 0
1
0
0
1
0 0
-9/4
-9/4
0
-1/4
-1/4
-1/8
3/16
1/2
11/16
0
-1/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	-Fb/EJ	$-1/4Fx+1/8Fx^2/b$	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(-1/12+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	Fb/EJ	$-1/8Fb+1/8Fx^2/b$	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

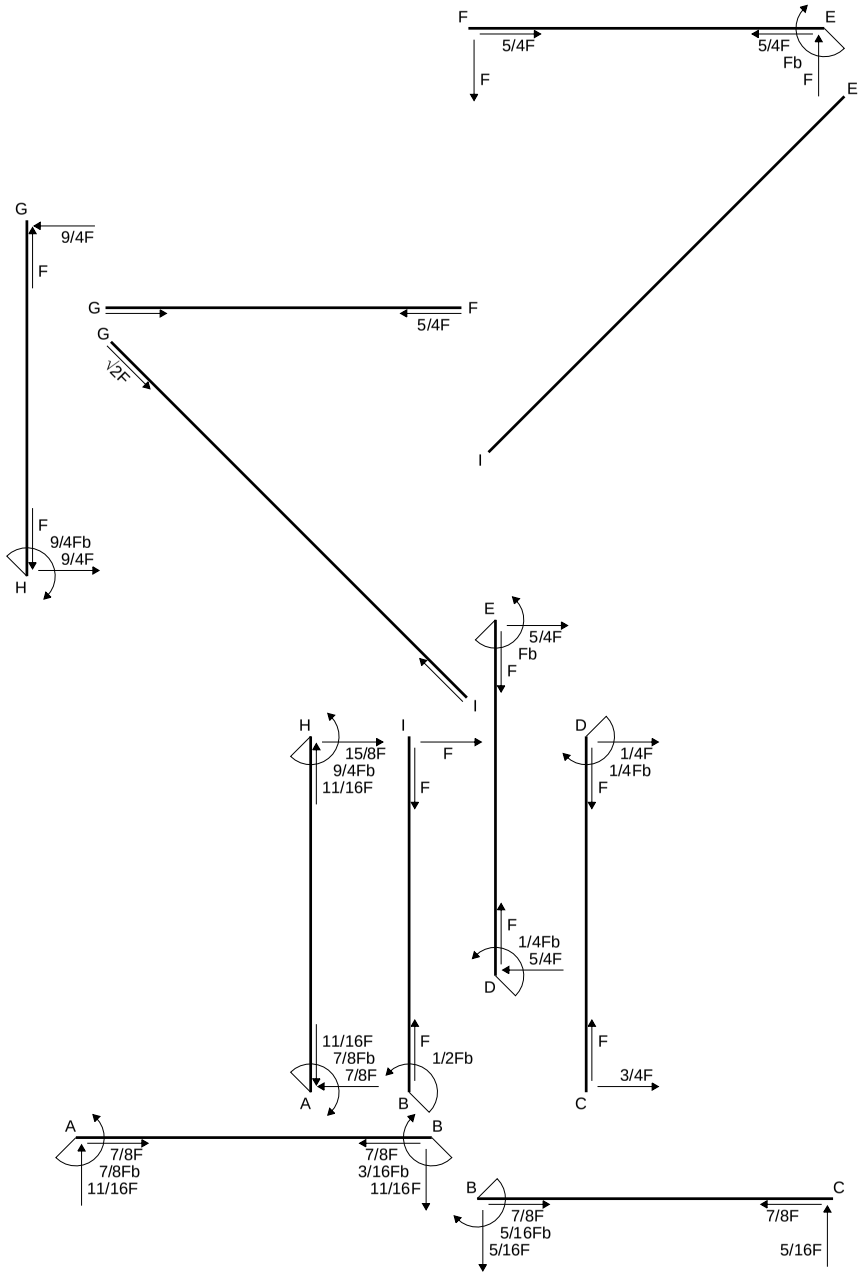
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

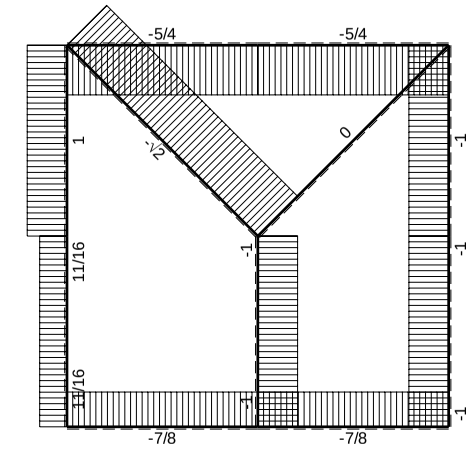
$L^{Xo}_{AH} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

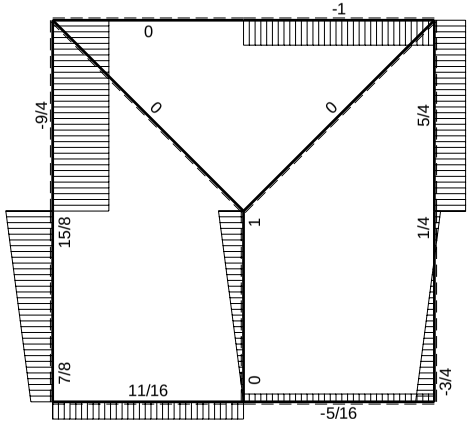
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

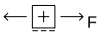
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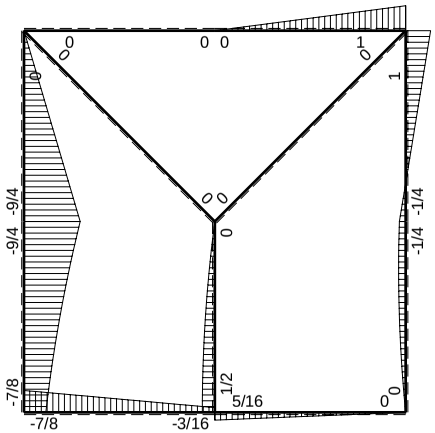

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9/4F
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E
5/4F
Fb
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H
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B
7/8F
7/8Fb
11/16F
7/8F
3/16Fb
11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
-5/4
1
-1/2
0
-1
11/16
-1
-1
11/16
-1
-1
-7/8
-7/8

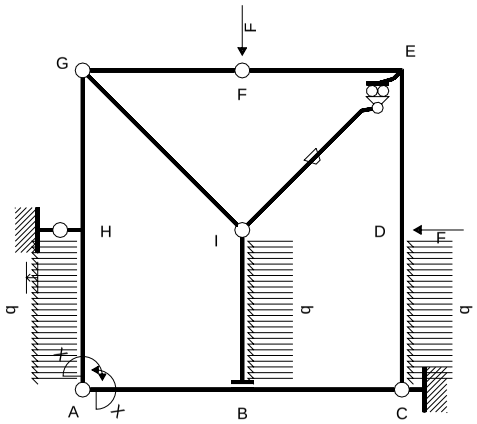

-1
0
-9/4
0
0
5/4
15/8
1
1/4
7/8
0
-3/4
11/16
-5/16

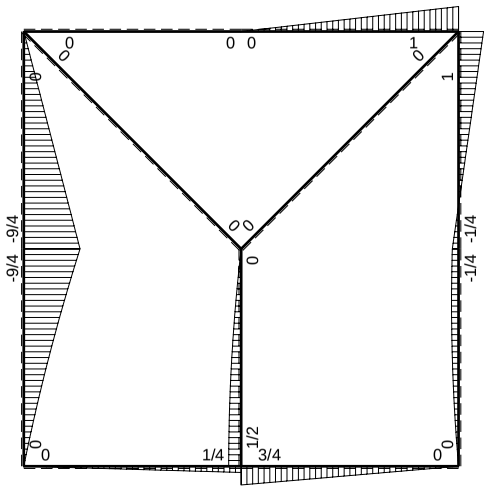

F


F

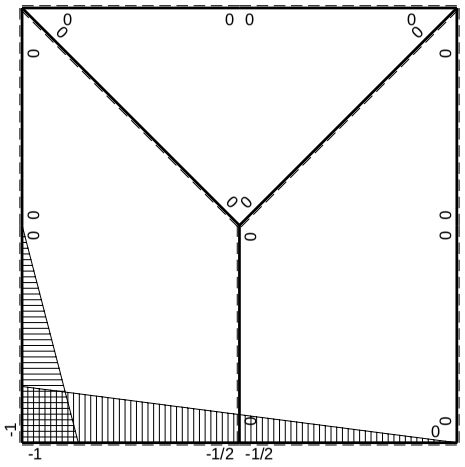

0
0 0
1
0
0
1
0 0
0
-9/4
-9/4
-1/4
-1/4
0
1/2
5/16
0
-7/8
-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237225 ESΣ04.xxxx.024

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

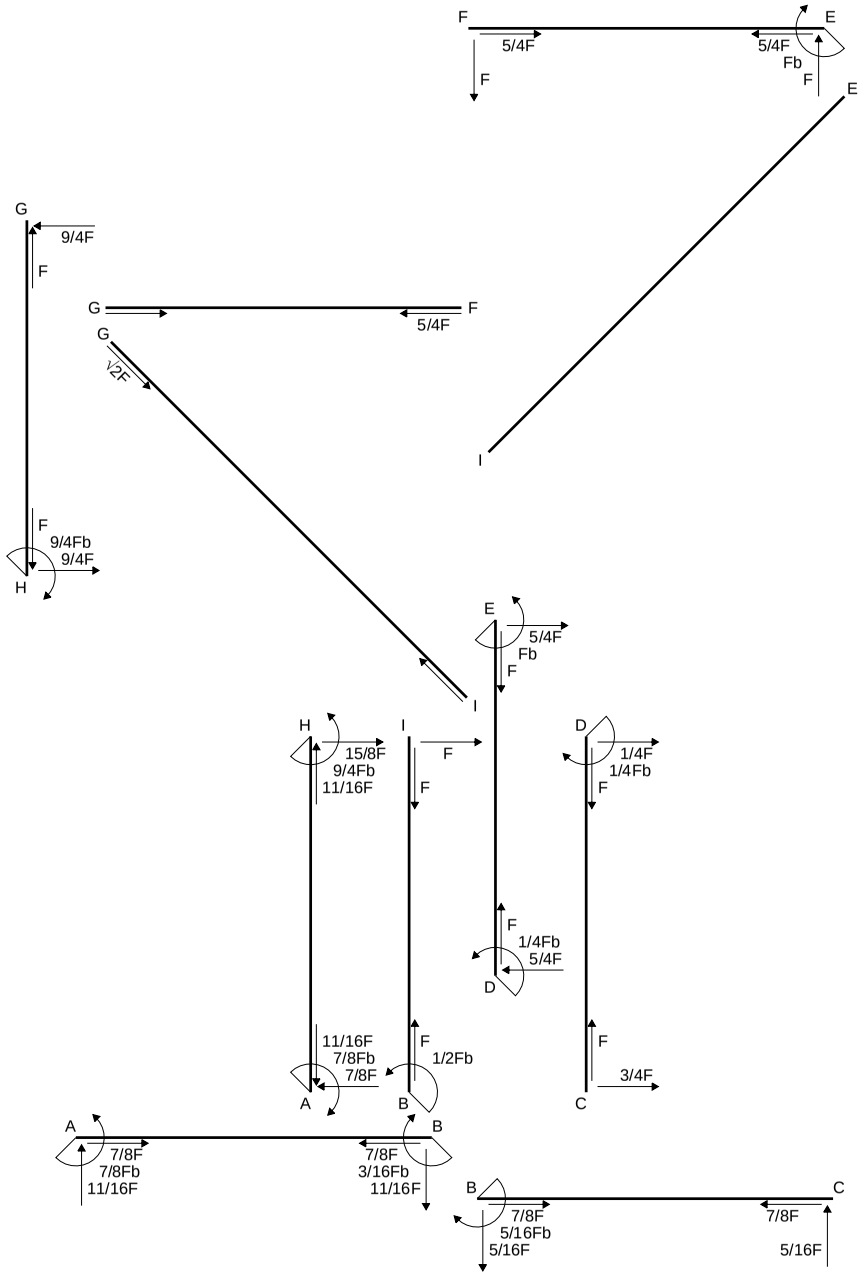
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

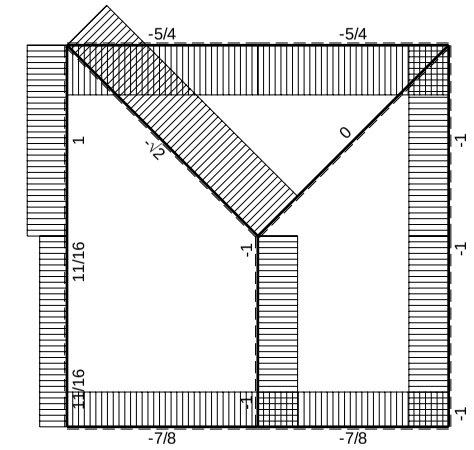
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

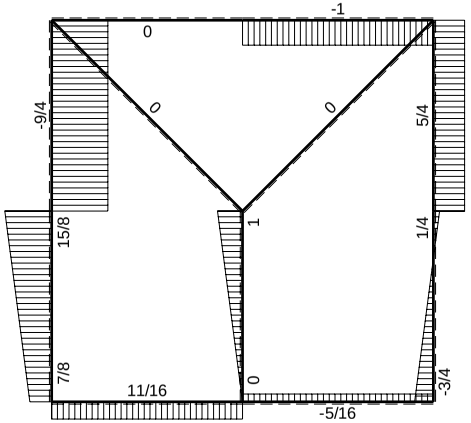
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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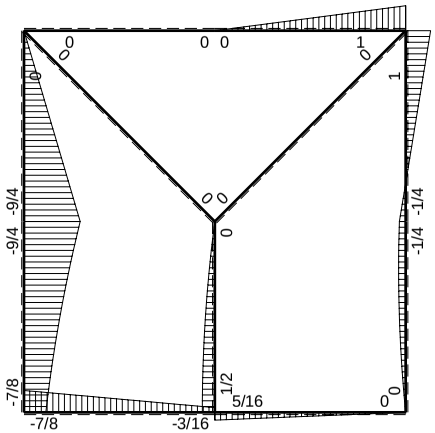

F
5/4F
E
5/4F
Fb
F
F
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G
9/4F
F
G
F
5/4F
G
½F
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F
9/4Fb
9/4F
H
E
5/4F
Fb
F
I
H
15/8F
9/4Fb
11/16F
I
F
F
D
1/4F
1/4Fb
F
F
1/4Fb
5/4F
D
11/16F
7/8Fb
7/8F
A
F
1/2Fb
B
F
3/4F
C
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11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
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11/16
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11/16
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-1
-7/8
-7/8

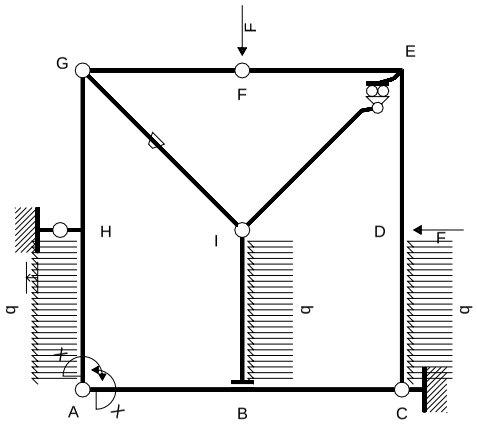

-1
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0
0
5/4
15/8
1
1/4
7/8
0
-3/4
11/16
-5/16

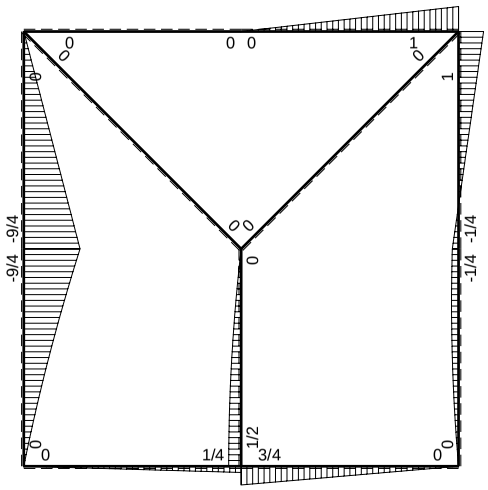
←□→ F

↑□↓ F

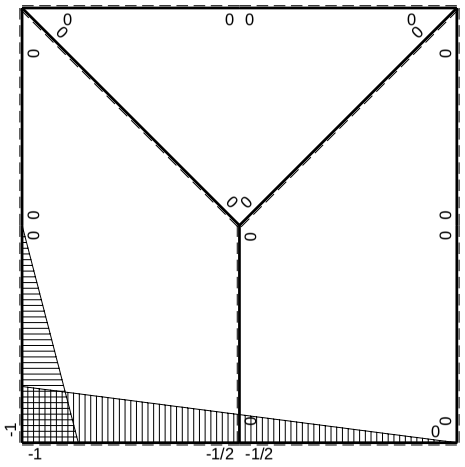

0
0 0
1
0
0
1
0 0
-9/4
-9/4
-1/4
-1/4
0
-7/8
1/2
5/16
0
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237330 ESΣ04.xxxx.025

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PROCEDIMENTO E RISULTATI 237330 ESΣ04.xxxx.025

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

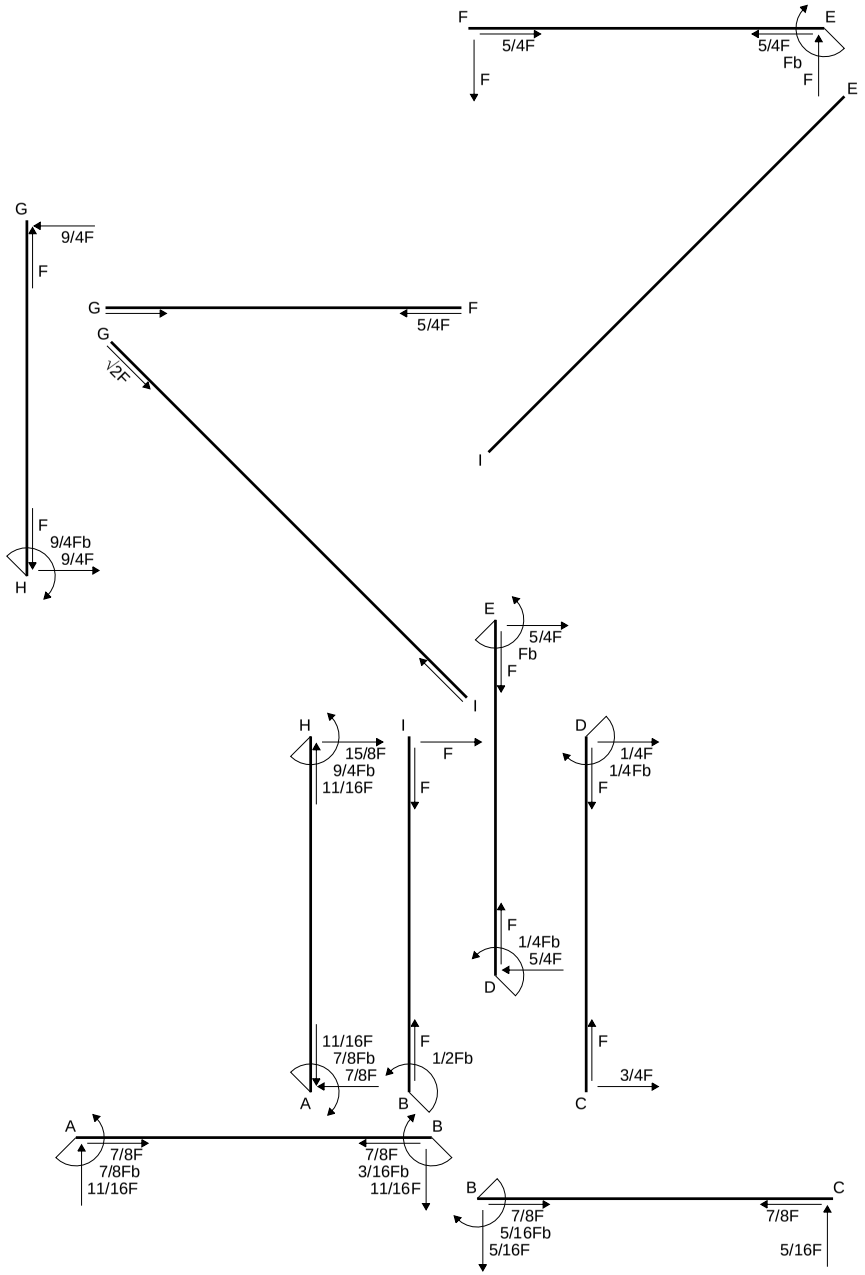
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

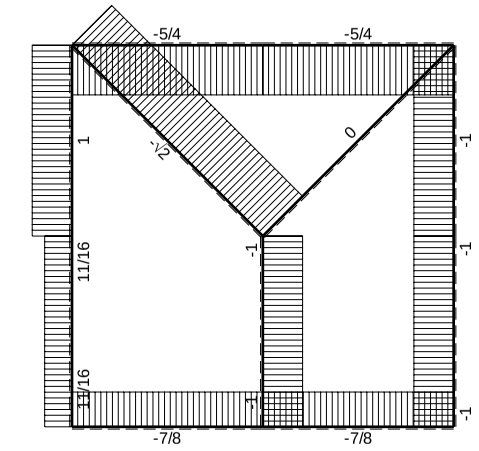
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

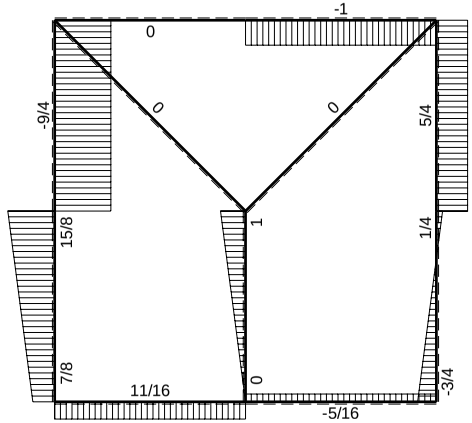
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

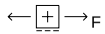
ESΣ04.xxxx.025

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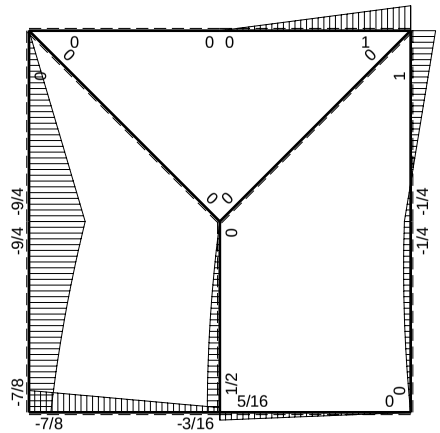

F
5/4F
E
5/4F
Fb
F
F
E
G
9/4F
F
G
F
5/4F
G
½F
I
F
9/4Fb
9/4F
H
E
5/4F
Fb
F
I
H
15/8F
9/4Fb
11/16F
I
F
D
1/4F
1/4Fb
F
F
1/4Fb
5/4F
D
11/16F
7/8Fb
7/8F
A
F
1/2Fb
B
F
3/4F
C
A
B
7/8F
7/8Fb
11/16F
7/8F
3/16Fb
11/16F
B
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7/8F
5/16Fb
5/16F
7/8F
5/16F


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-1
-1
-7/8
-7/8

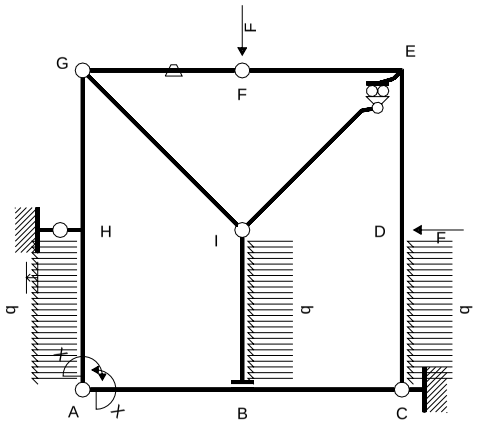

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1/4
7/8
0
-3/4
11/16
-5/16

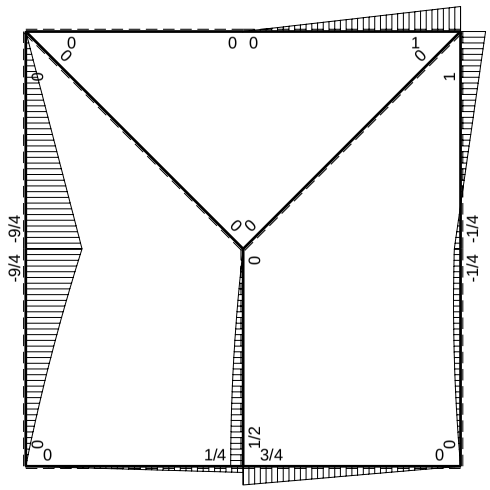

F


F

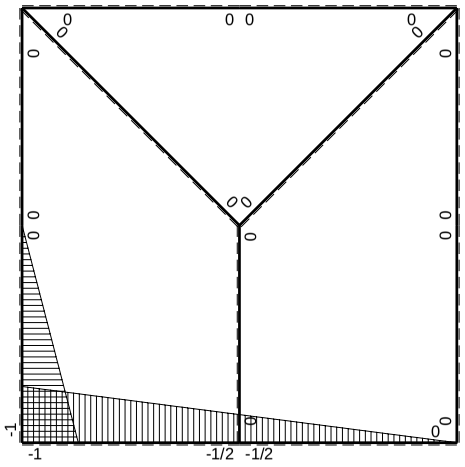

0
0 0
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0
0
1
0 0
-9/4
-9/4
-1/4
-1/4
0
1/2
5/16
0
-7/8
-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212625 ESΣ04.xxxx.026

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

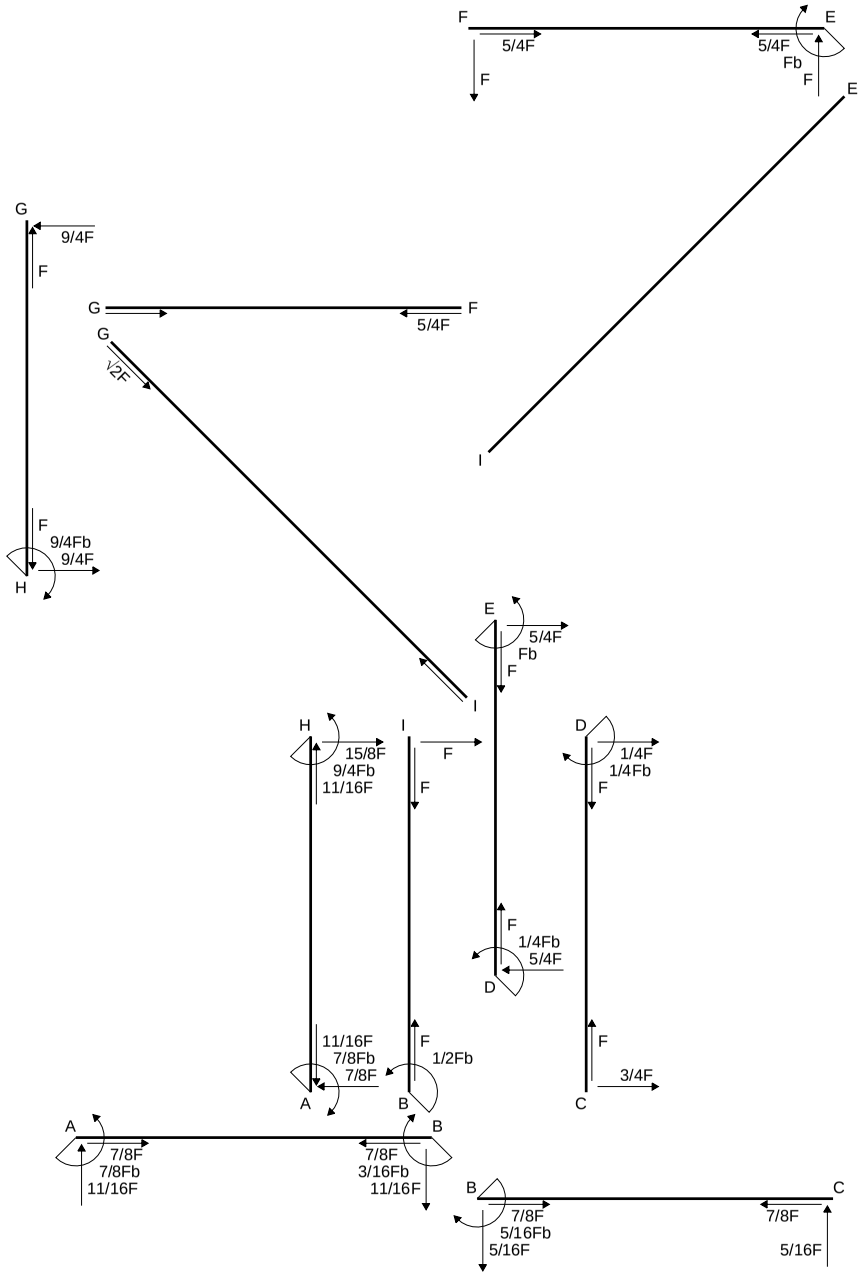
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

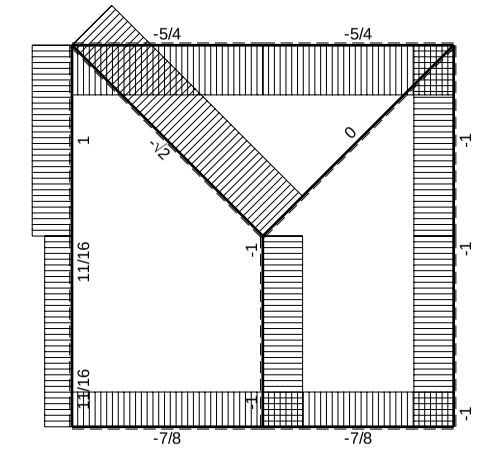
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

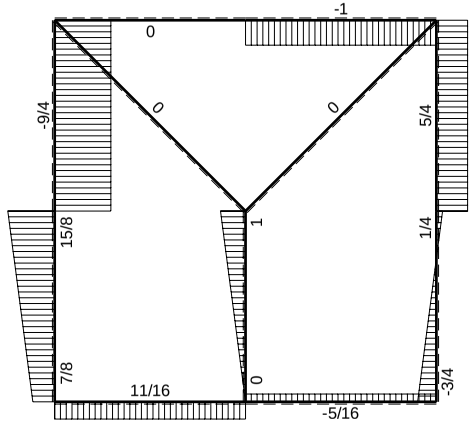
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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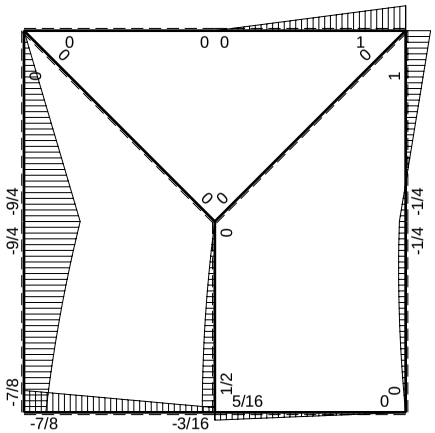

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F
9/4Fb
9/4F
H
E
5/4F
Fb
F
I
H
15/8F
9/4Fb
11/16F
I
F
F
D
1/4F
1/4Fb
F
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7/8F
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11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


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11/16
-1
-1
-7/8
-7/8

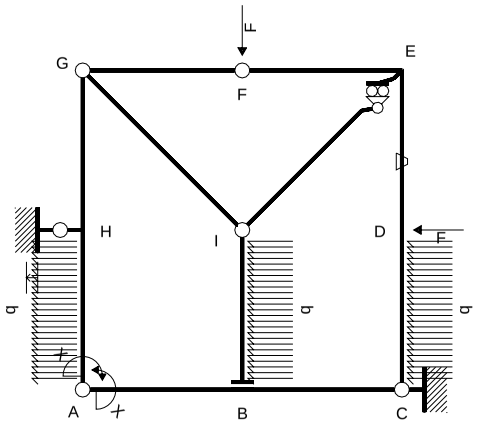

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1/4
7/8
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11/16
-5/16

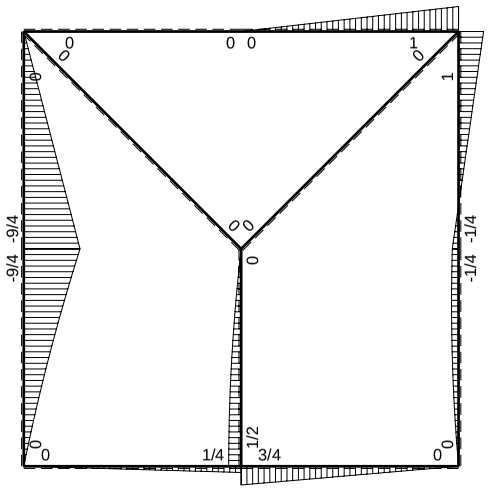
←□→ F

↑□↓ F

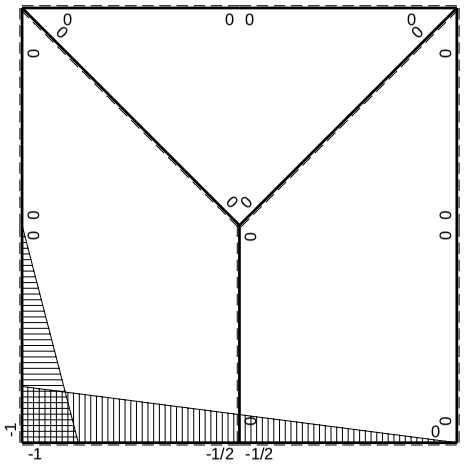

0
0 0
1
0
0
1
0 0
-9/4
-9/4
-1/4
-1/4
0
-7/8
1/2
5/16
0
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238395 ESΣ04.xxxx.027

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PROCEDIMENTO E RISULTATI 238395 ESΣ04.xxxx.027

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

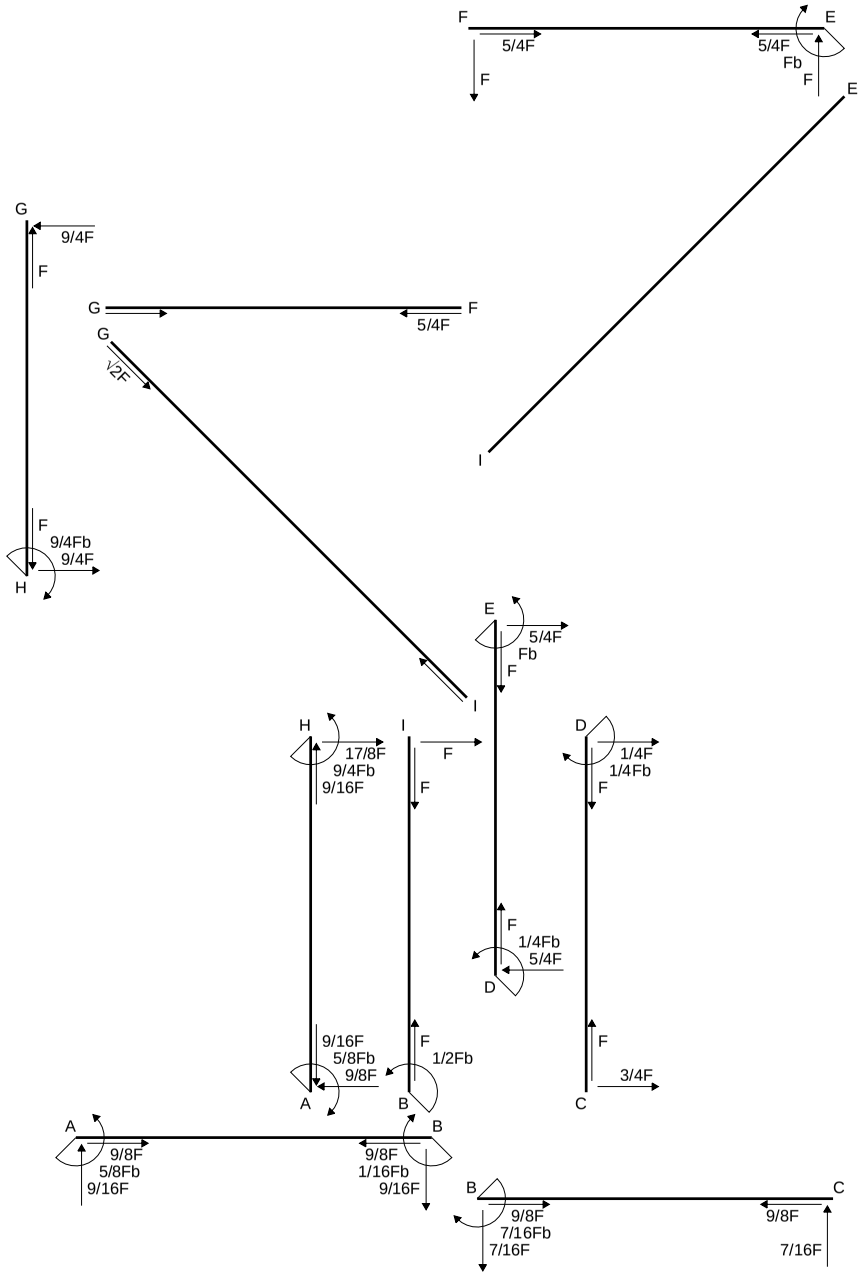
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

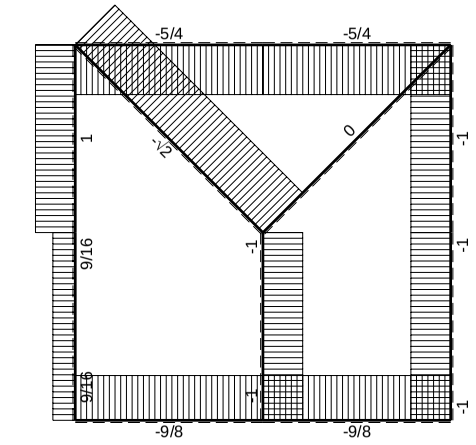
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

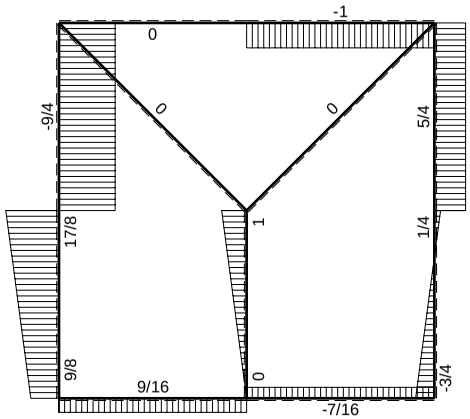
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

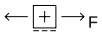
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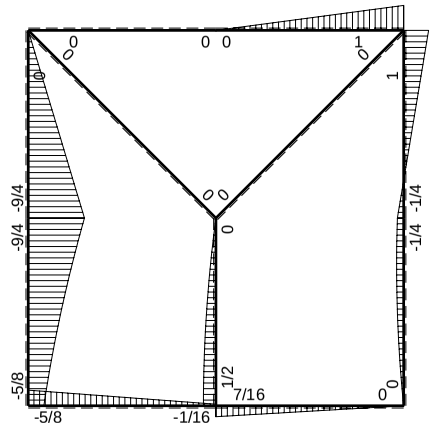

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G
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H
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5/4F
Fb
F
I
H
17/8F
9/4Fb
9/16F
I
F
F
D
1/4F
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F
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9/8F
7/16F


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½
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-1
9/16
-1
-1
-9/8
-9/8

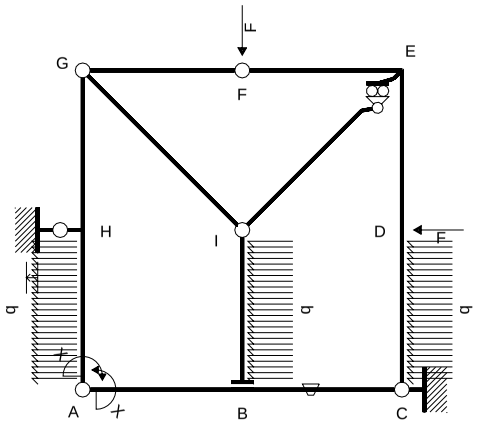

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9/8
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-3/4
-7/16

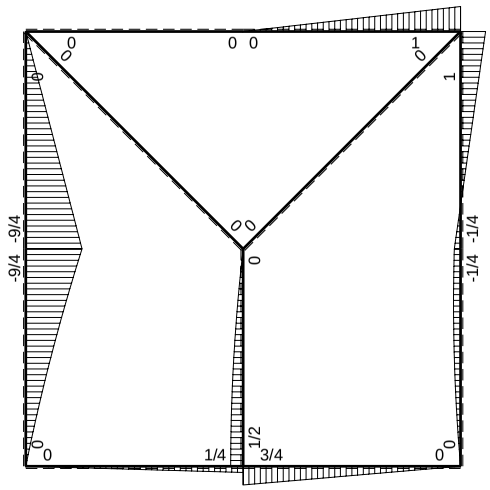

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F

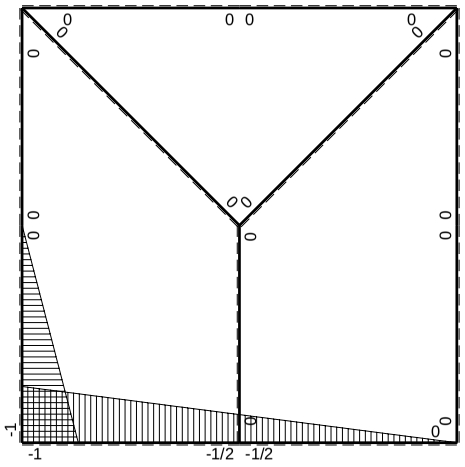

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0 0
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-9/4
-9/4
-1/4
-1/4
-5/8
1/2
7/16
0
0
-5/8
-1/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	-Fb/EJ	$-3/8Fb+3/4Fx-3/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	Fb/EJ	$-3/8Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/8Fb	

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Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\right)\ \theta\ \ dx$

$= \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\right)\ \theta\ \ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ + \left(-1/4\ b\right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

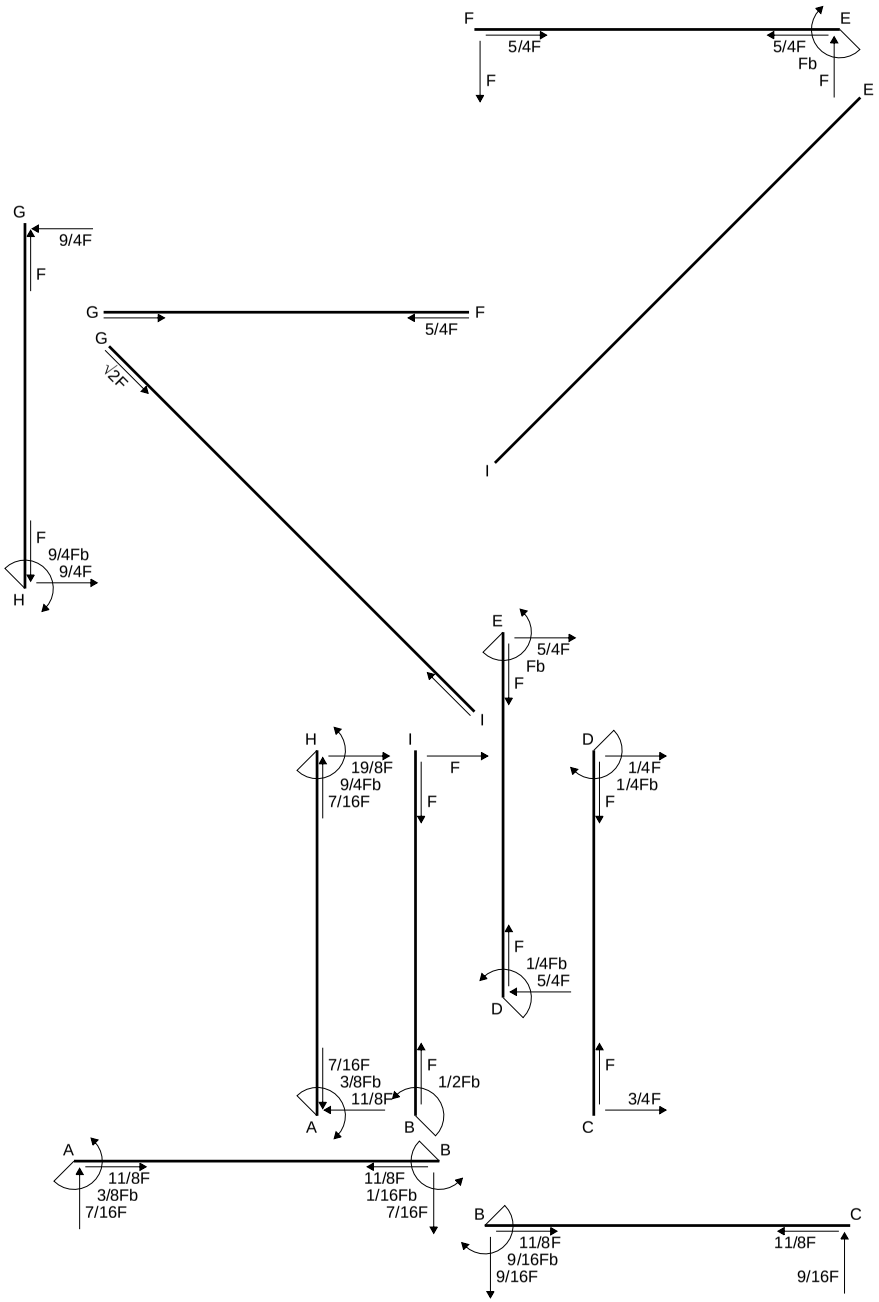
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

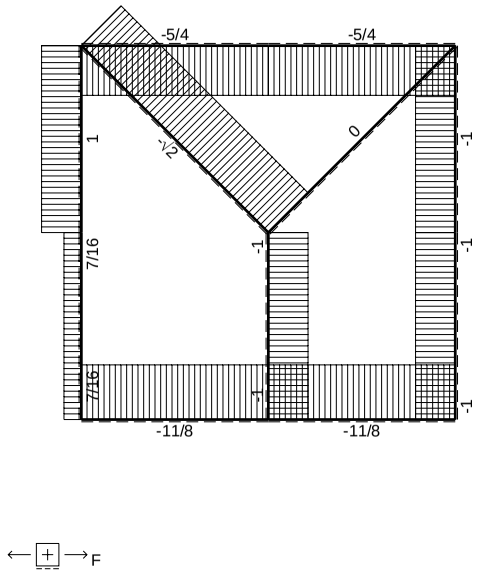
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

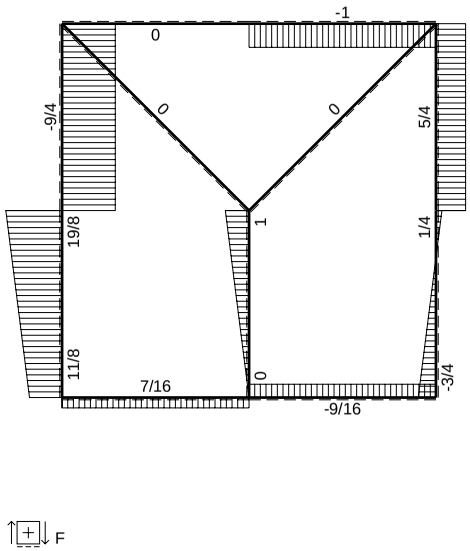
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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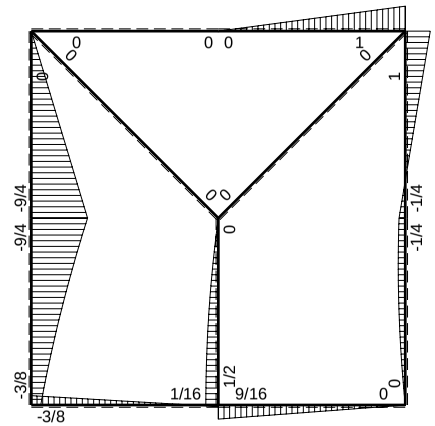

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9/16Fb
9/16F
C
11/8F
9/16F


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7/16
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-11/8

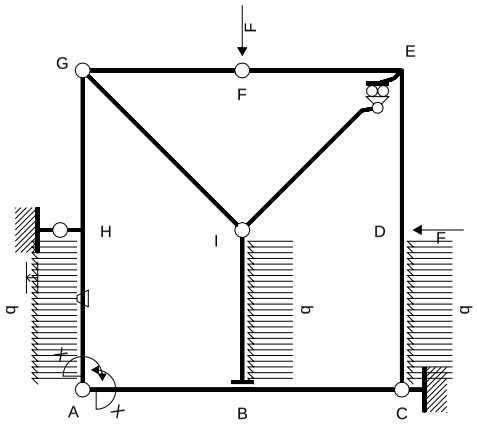

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-9/16

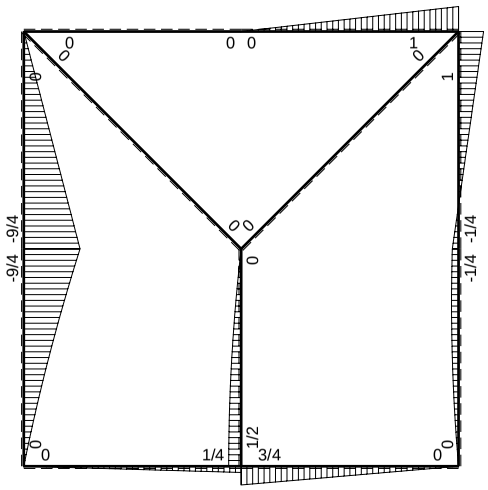
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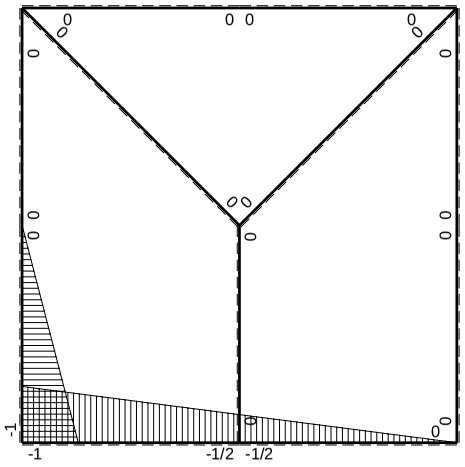

0
0 0
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-9/4
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-1/4
-1/4
0
-3/8
1/16
1/2
9/16
0
0
-3/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	-Fb/EJ	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/3+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	Fb/EJ	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\right)\ \theta\ \ dx$

$= \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

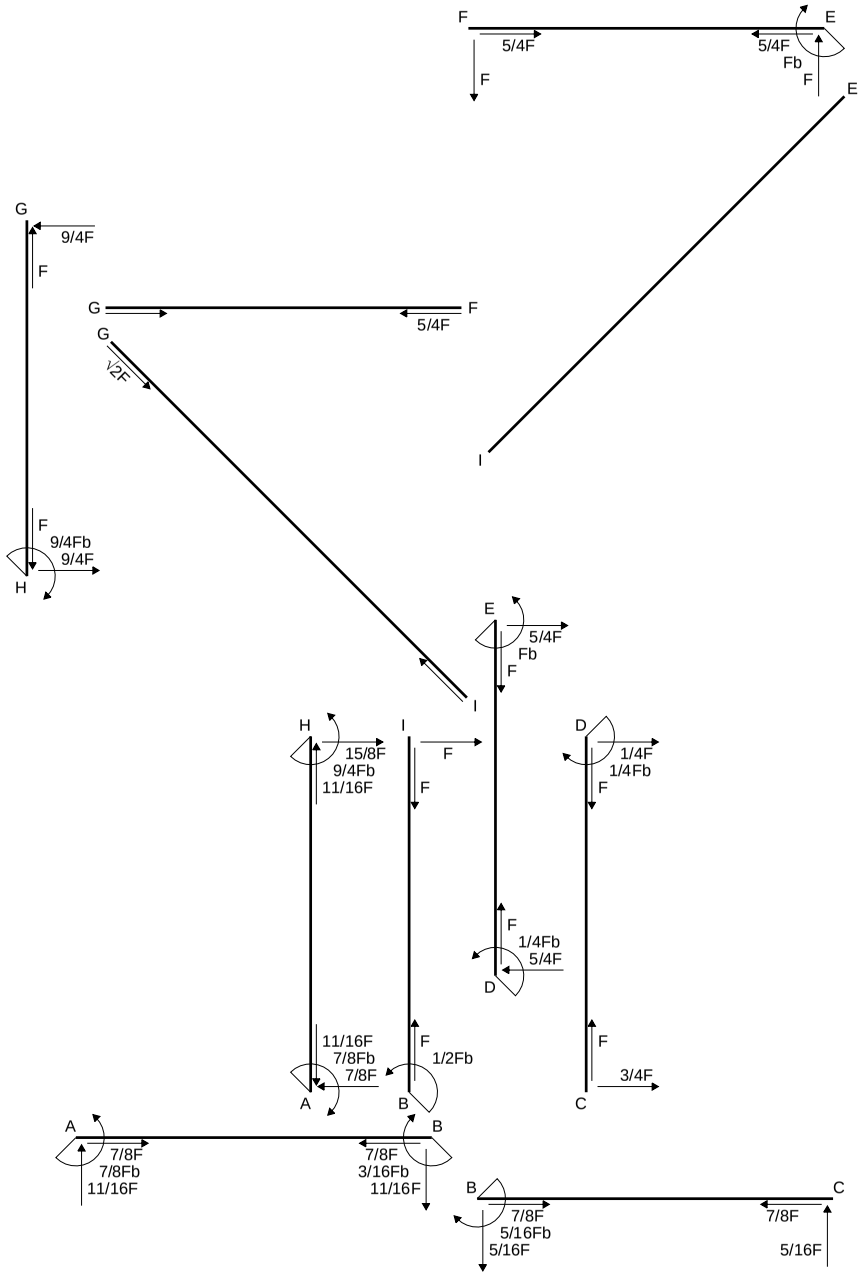
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\right)\ \theta\ \ dx$

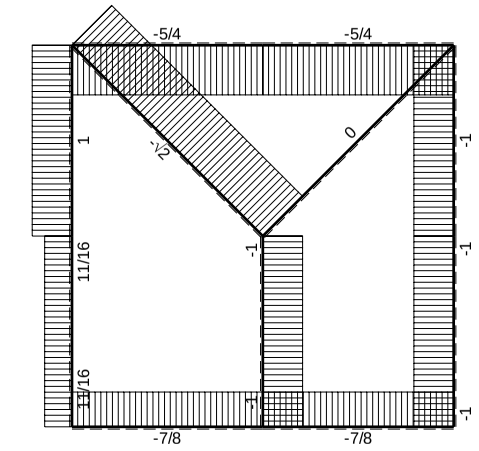
$= \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

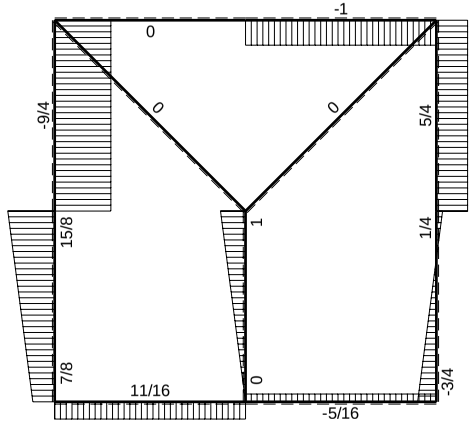
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

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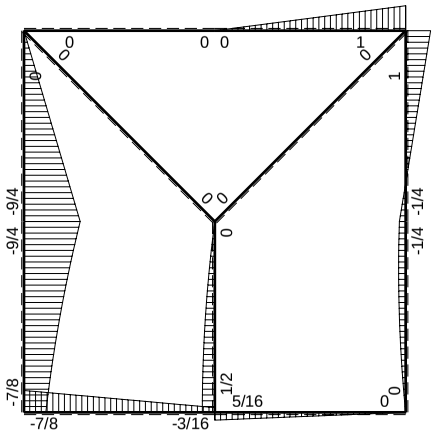

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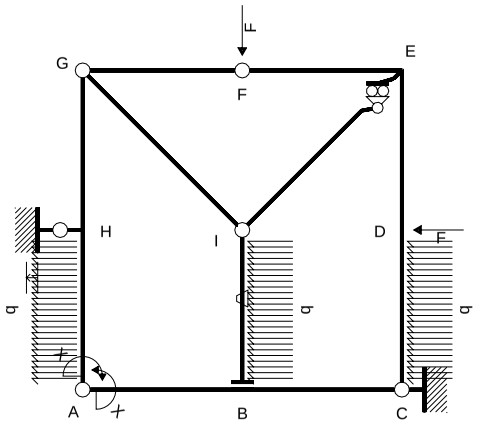

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-5/16

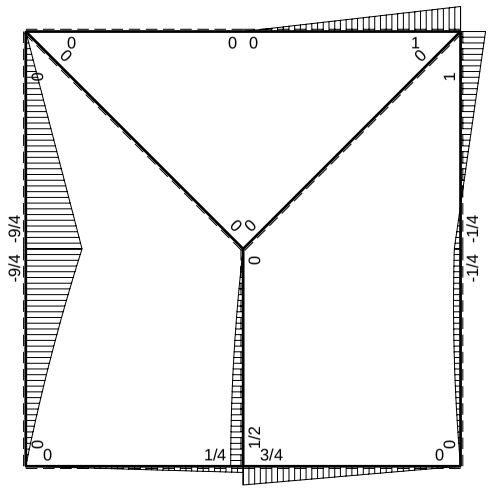
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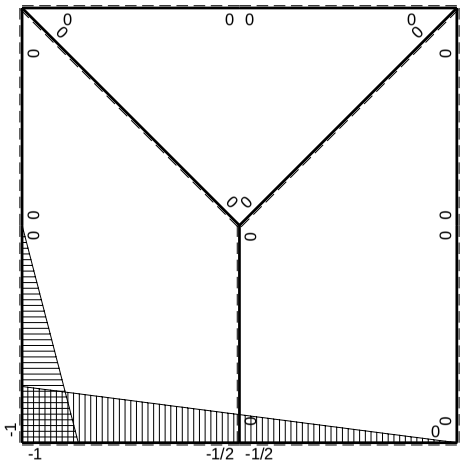

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-1/4
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5/16
0
-7/8
-3/16

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211617 ESΣ04.xxxx.030

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

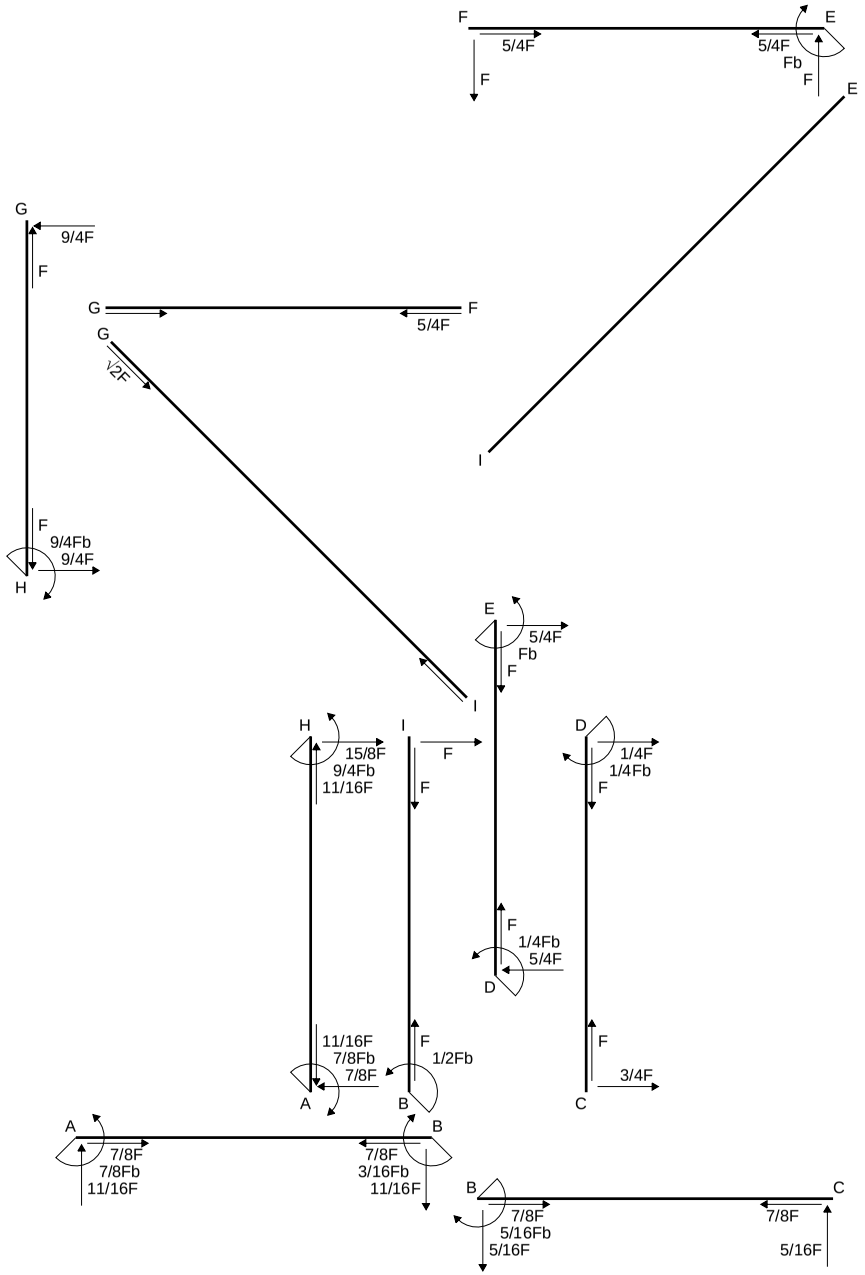
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

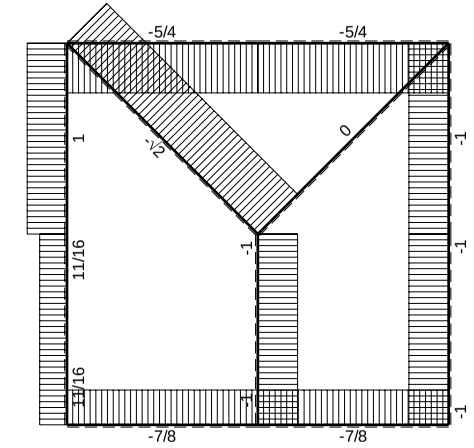
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

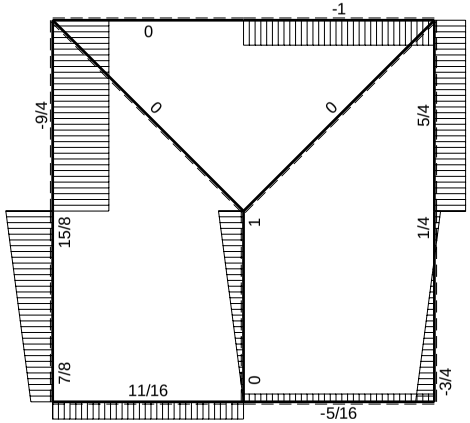
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

ESΣ04.xxxx.030

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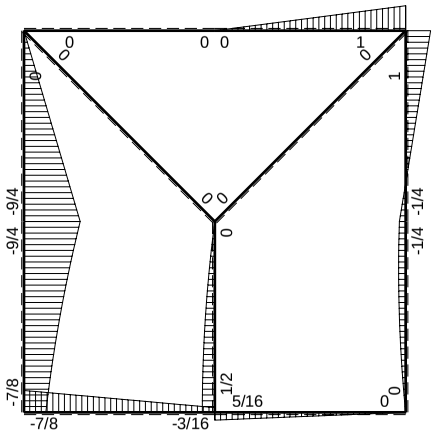

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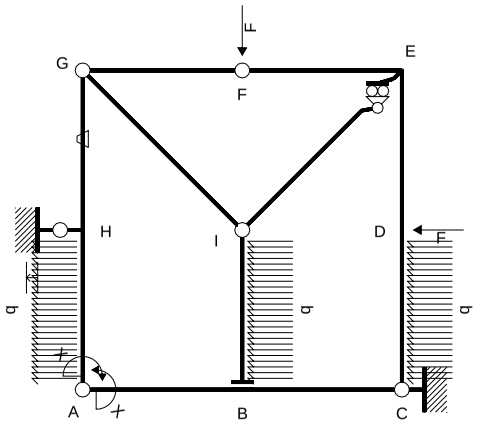

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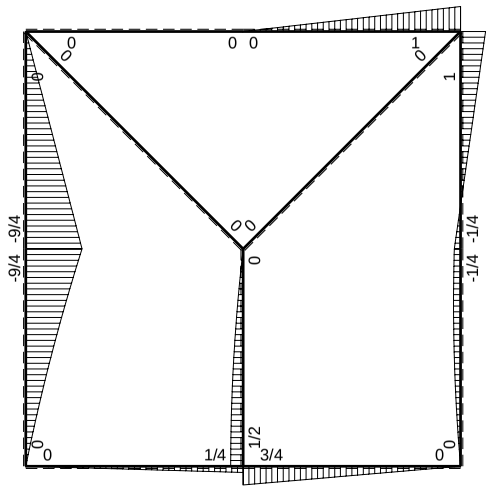
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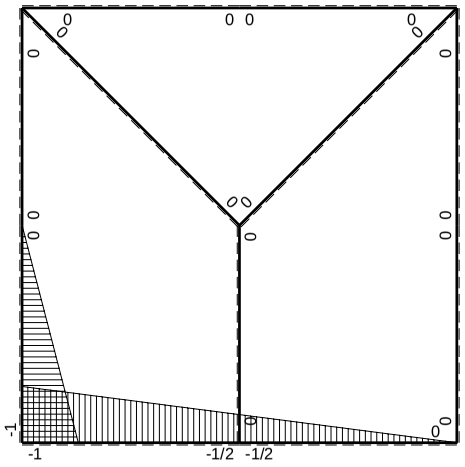

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↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 984894 ESΣ04.xxxx.031

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PROCEDIMENTO E RISULTATI 984894 ESΣ04.xxxx.031

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

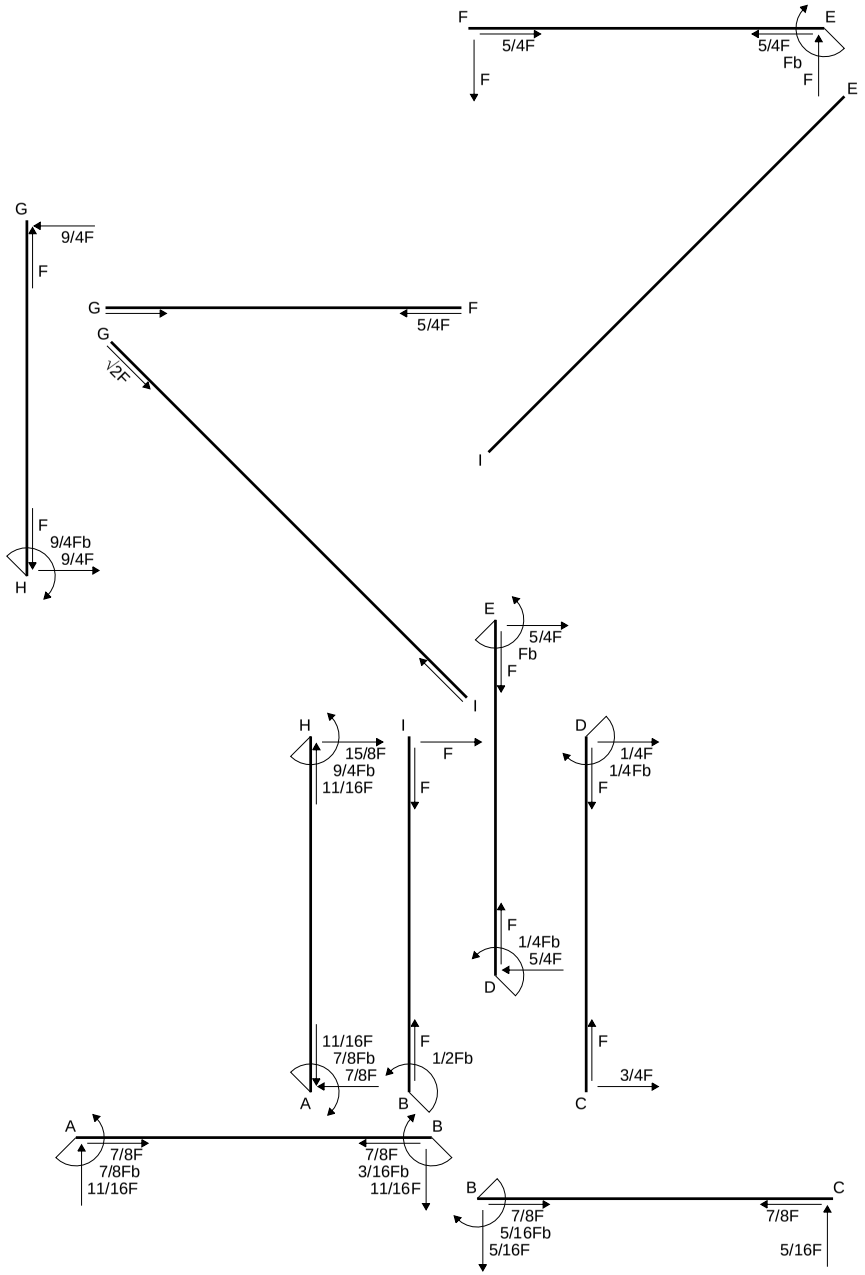
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

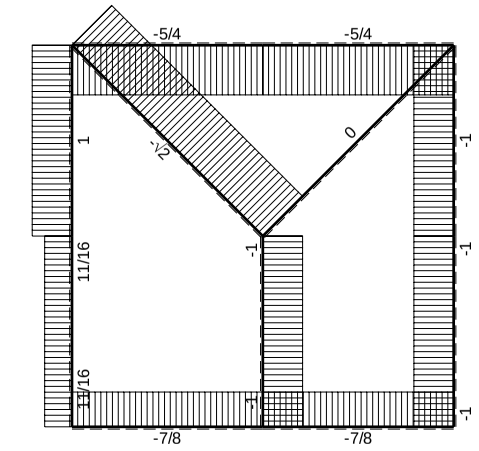
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

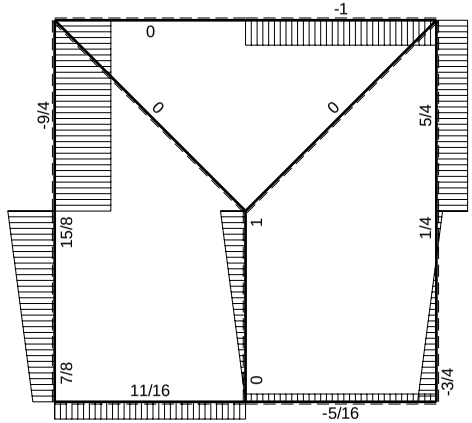
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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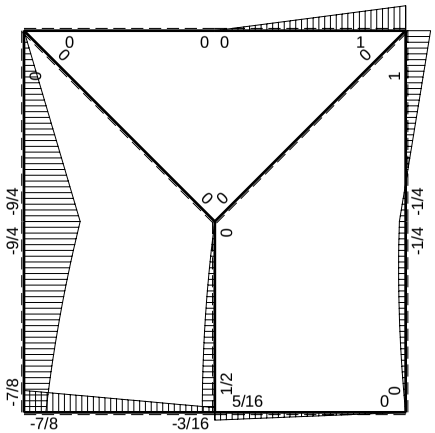

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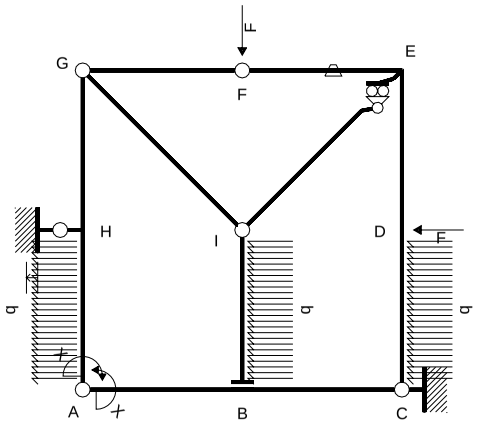

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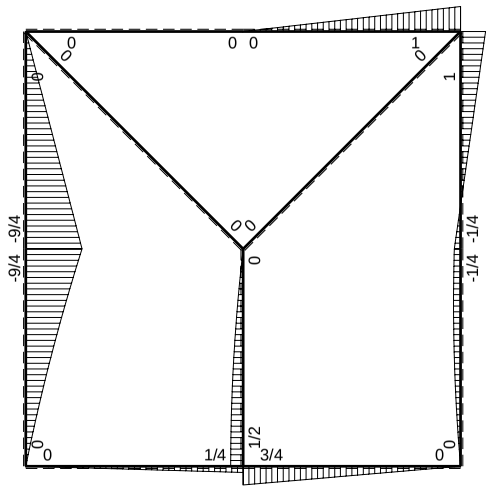
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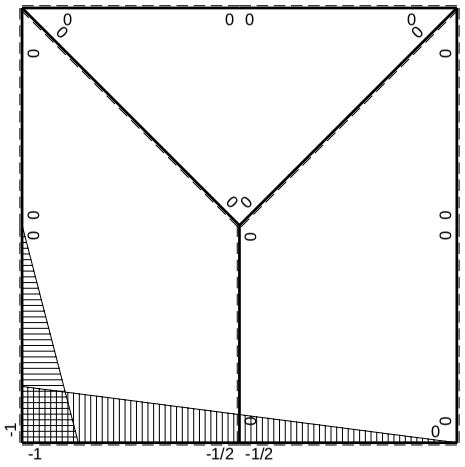

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-1/4
0
-7/8
1/2
5/16
0
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

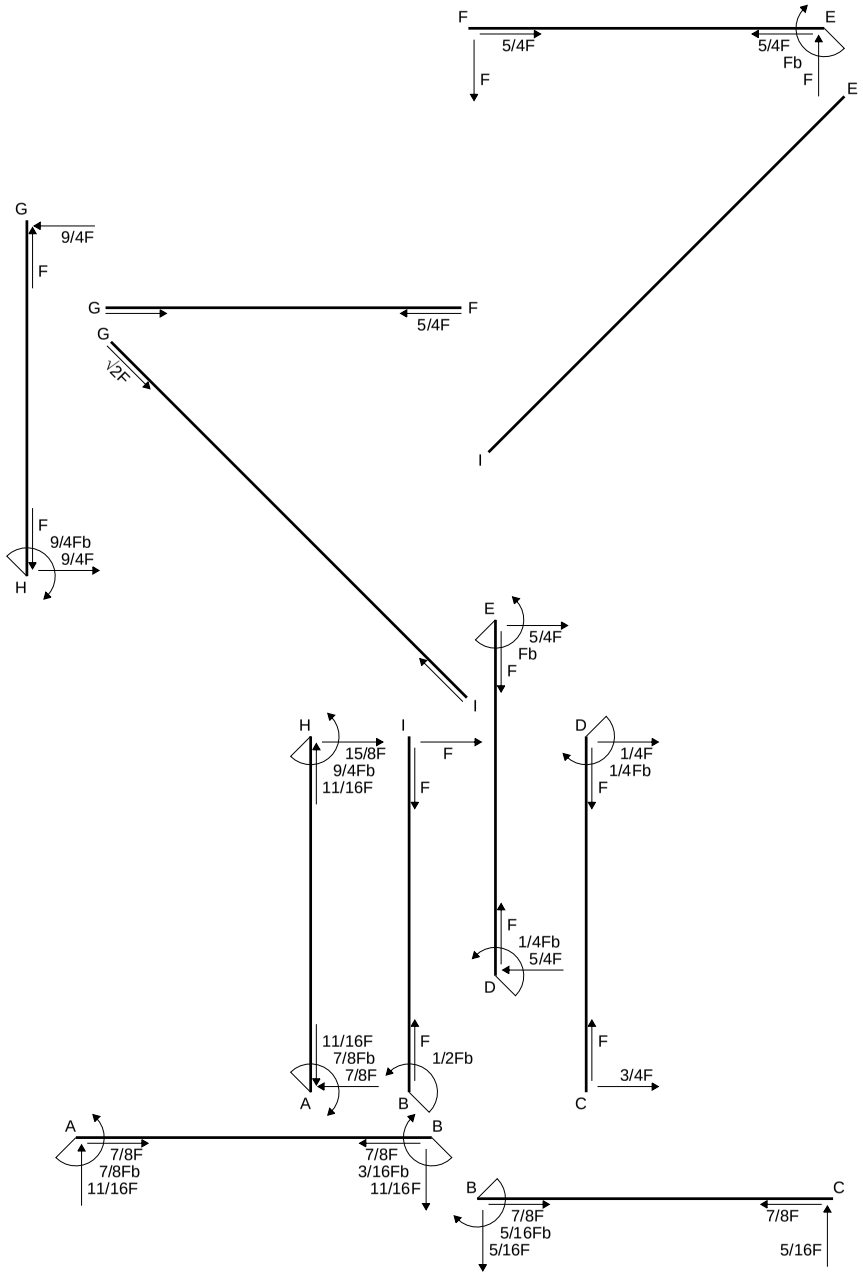
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

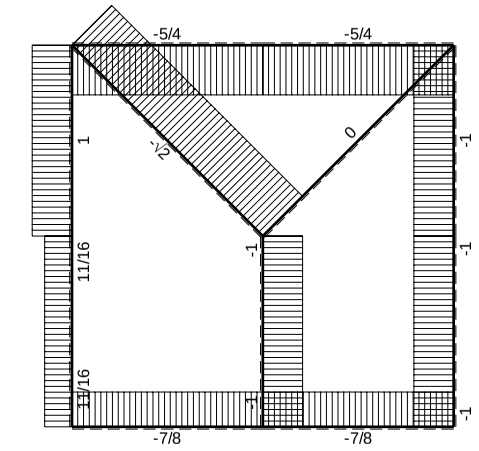
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

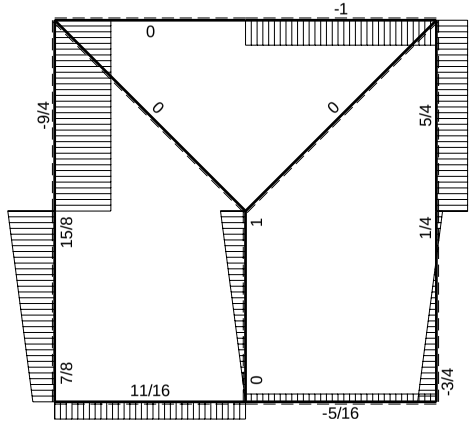
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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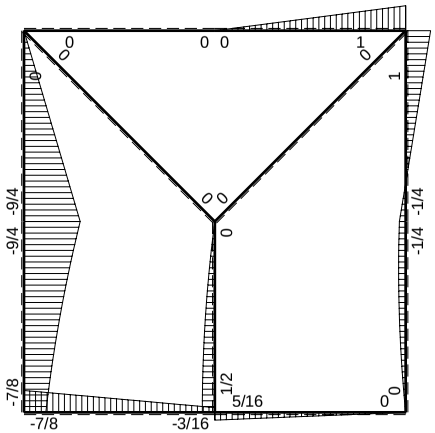

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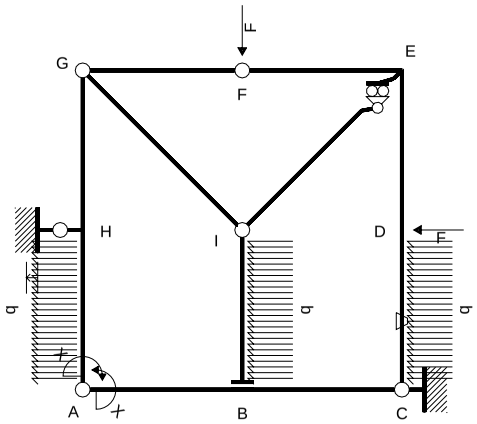

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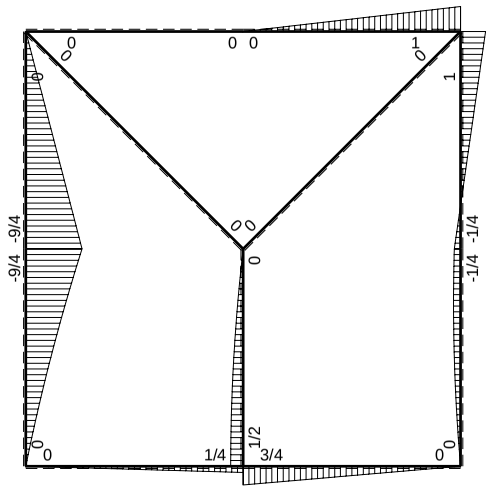
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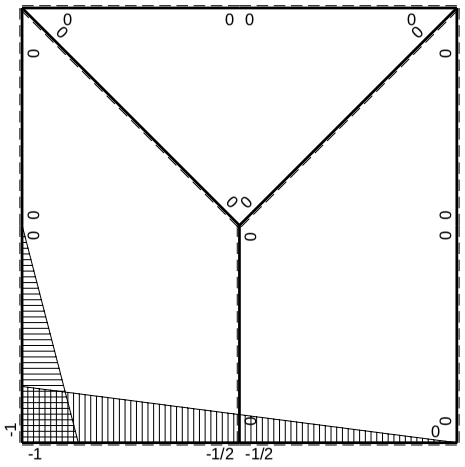

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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238530 ESΣ04.xxxx.033

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PROCEDIMENTO E RISULTATI 238530 ESΣ04.xxxx.033

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	Fb/EJ	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

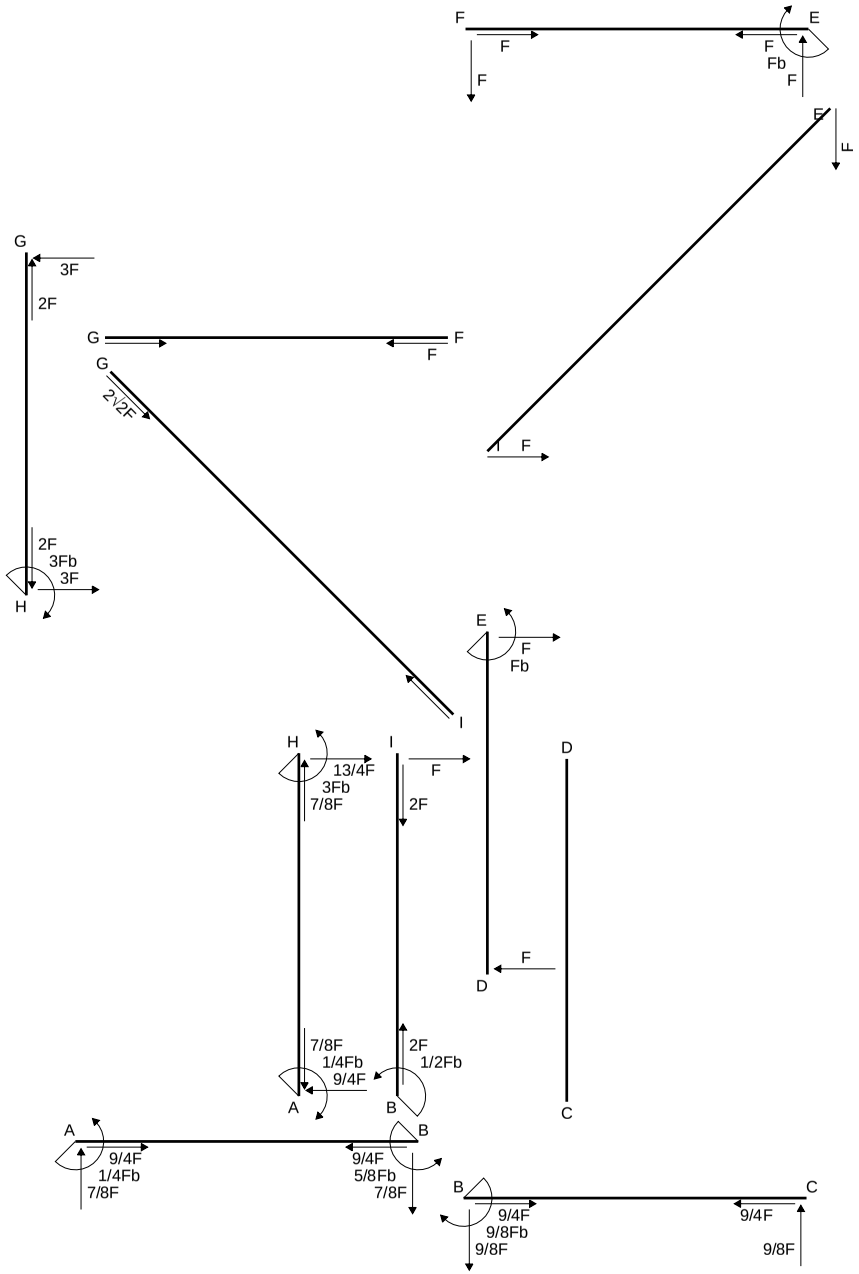
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

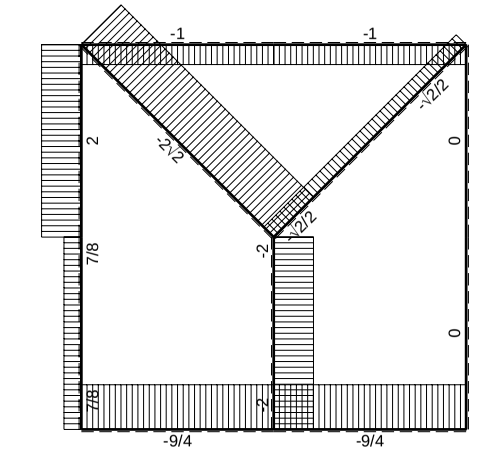
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

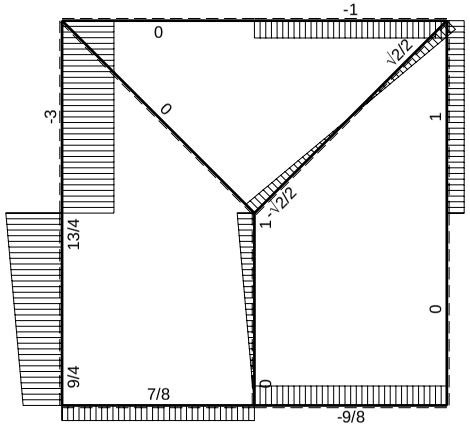
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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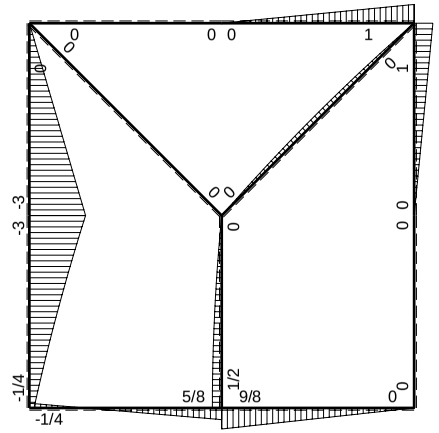

F
E
F
Fb
F
F
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F
G
3F
2F
G
F
F
G
2√2F
F
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3F
H
I
E
F
Fb
H
I
13/4F
3Fb
7/8F
F
2F
D
F
D
C
7/8F
1/4Fb
9/4F
2F
1/2Fb
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B
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B
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C
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9/4F
9/8F


-1
-1
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-√2/2
0
7/8
-2
-9/4
-9/4

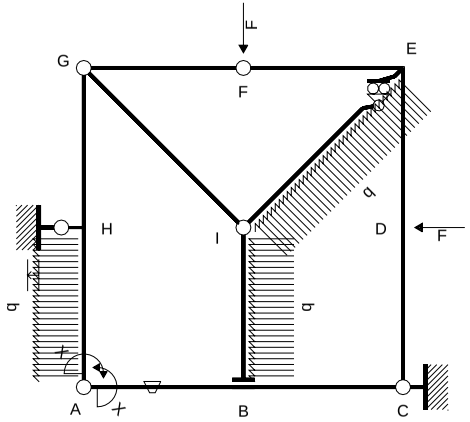

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√2/2
-3
0
1
13/4
1-√2/2
0
9/4
7/8
-9/8

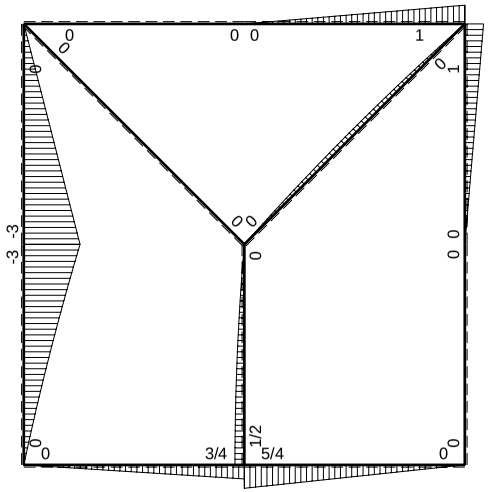
←□→ F

↑□↓ F

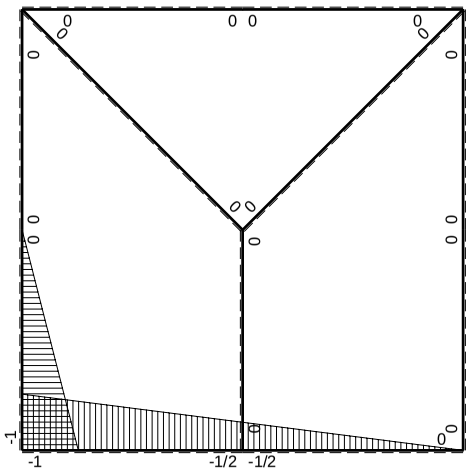

0
0 0
1
0
0
1
0 0
-3 -3
0
0 0
-1/4
5/8
1/2
9/8
0 0
-1/4

↺□↻ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	-Fb/EJ	-3/4Fx+3/8Fx^2/b	Fb/EJ-1/2Fx/EJ	1-x/b+1/4x^2/b^2	(-1/4+3/4)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-3/4Fb+3/4Fx	Fb/EJ	-3/8Fb+3/8Fx^2/b	1/2Fb/EJ+1/2Fx/EJ	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	-5/8Fb+5/4Fx-5/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-5/24+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-5/4Fx	0	-5/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
BI b	0	-1/2Fb+1/2qx^2	0	0	0	0		
IE $\sqrt{2}$b	0	-$\sqrt{2}$/2Fx+1/2qx^2	0	0	0	0	0	0
HA b	-x/b	-3Fb+7/2Fx-1/2qx^2	0	3Fx-7/2Fx^2/b+1/2qx^3/b	0	x^2/b^2	(11/24+0)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	5/2Fx+1/2qx^2	0	5/2Fx-2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-1/4Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						1/4Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\ \right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

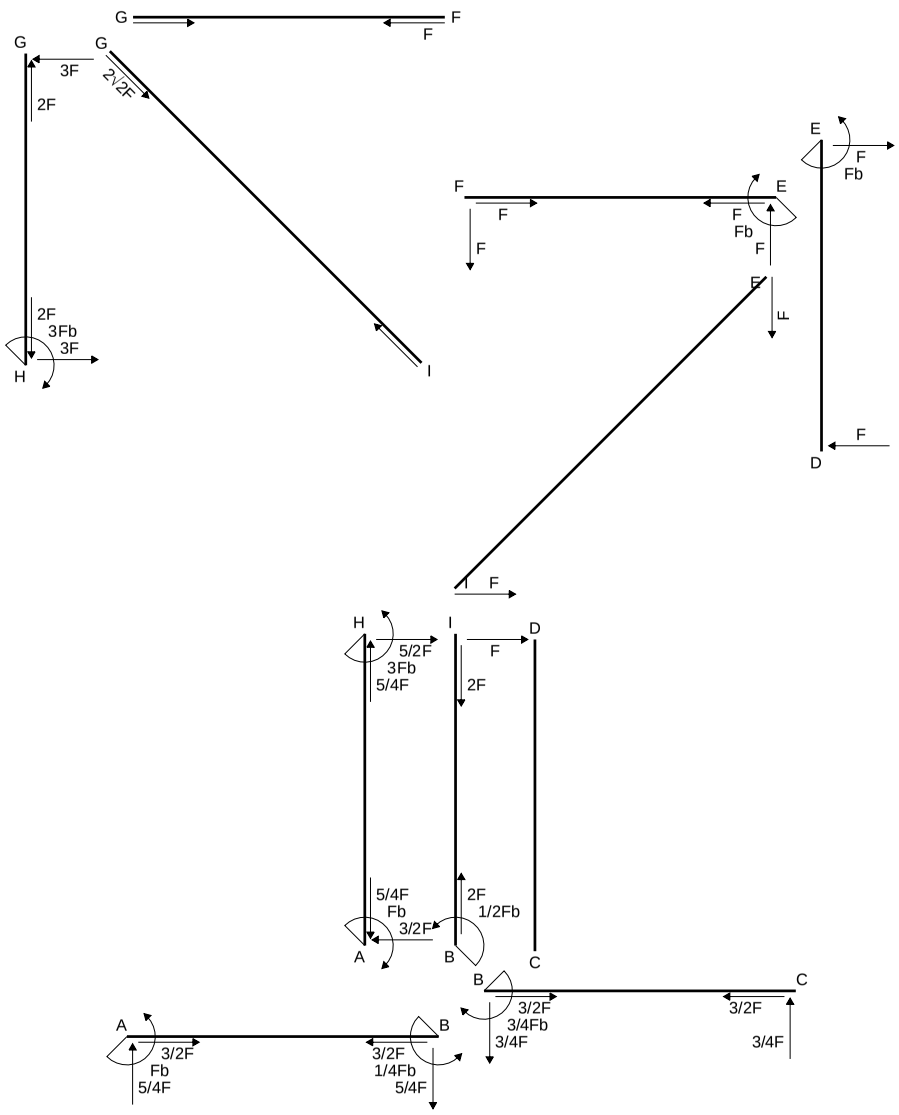
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

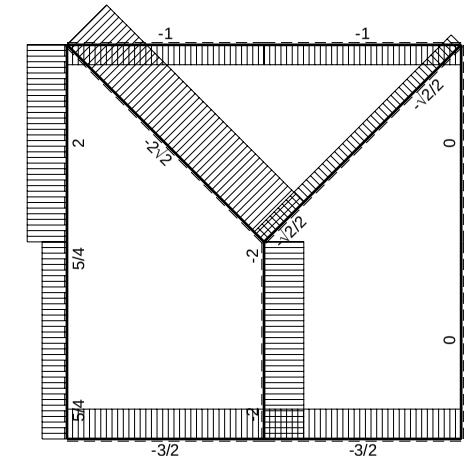
$L^{Xo}_{AH} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\ \right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

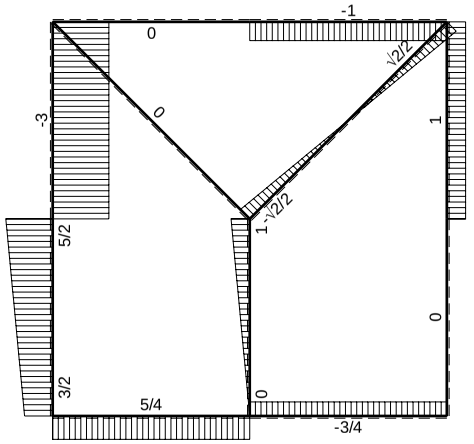
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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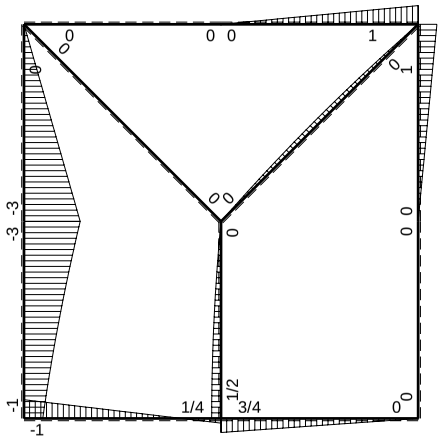

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3F
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3/2F
1/4Fb
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-3/2
-3/2

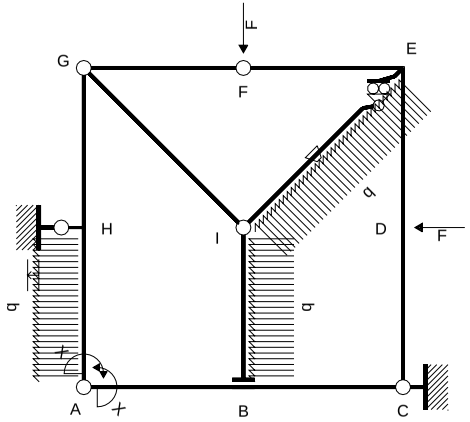

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5/4
0
-3/4

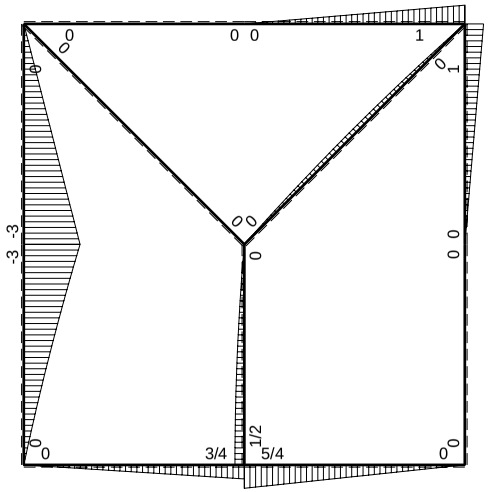
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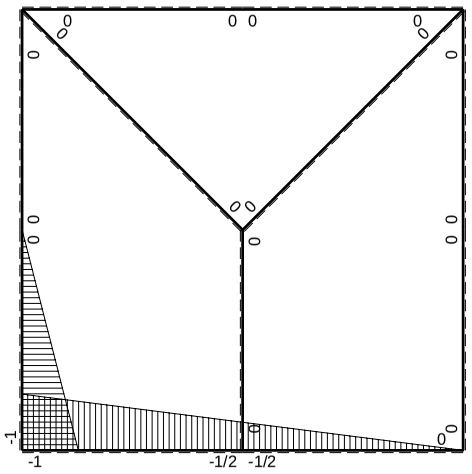

0
0 0
1
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0
1
0 0
-3
-3
0
0 0
-1
1/4
1/2
3/4
0
0
-1

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210192 ESΣ04.xxxx.035

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PROCEDIMENTO E RISULTATI 210192 ESΣ04.xxxx.035

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

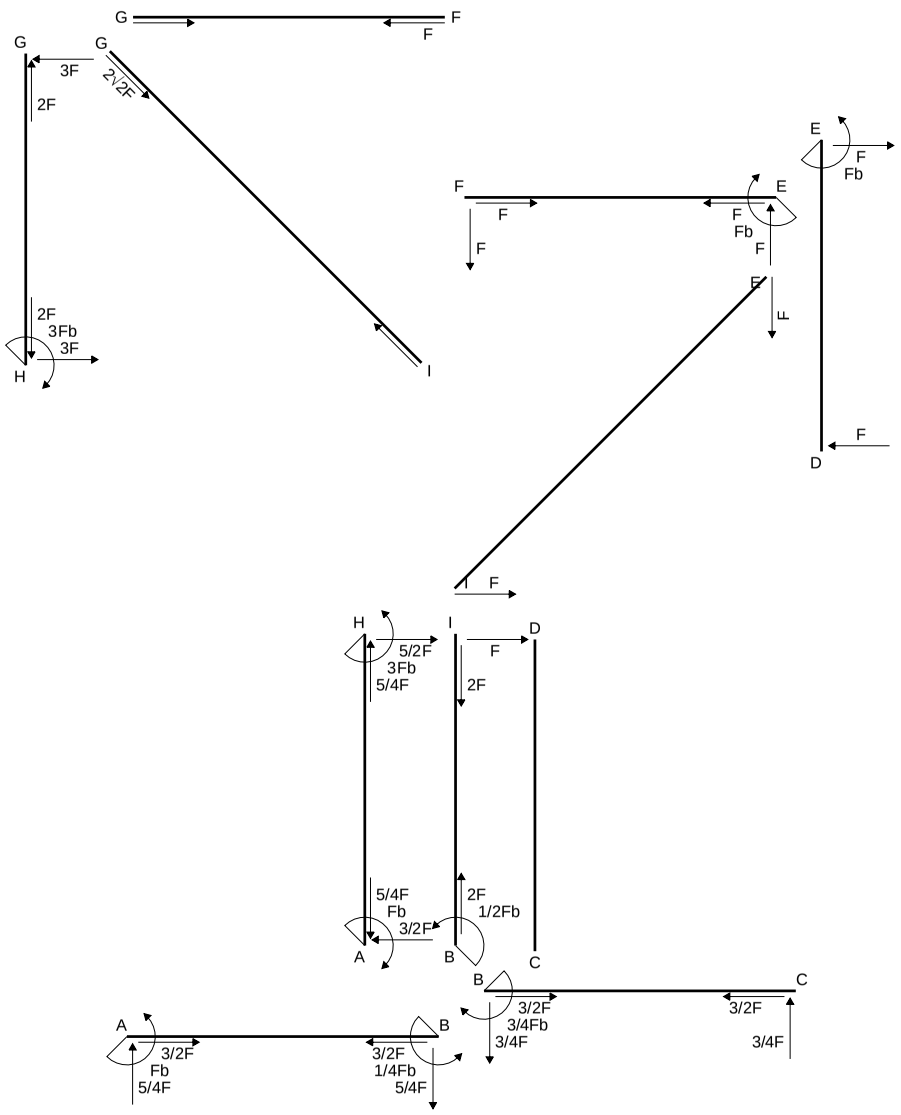
$$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$$

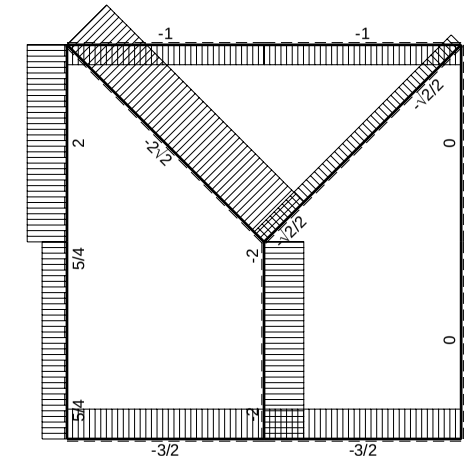
$$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

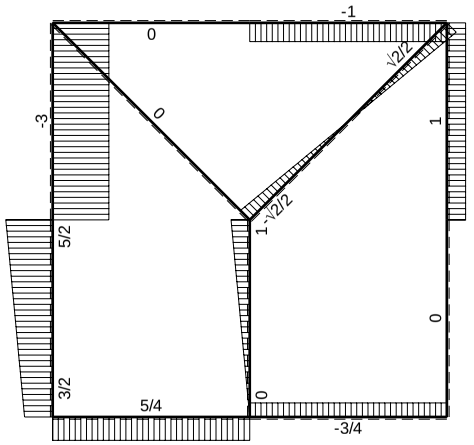
$$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$$

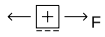
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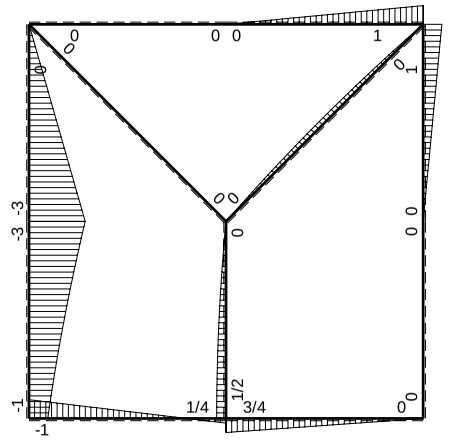

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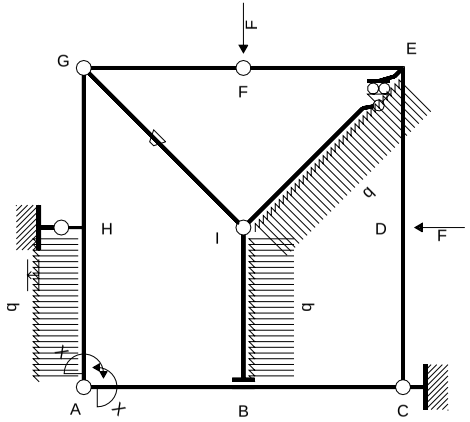

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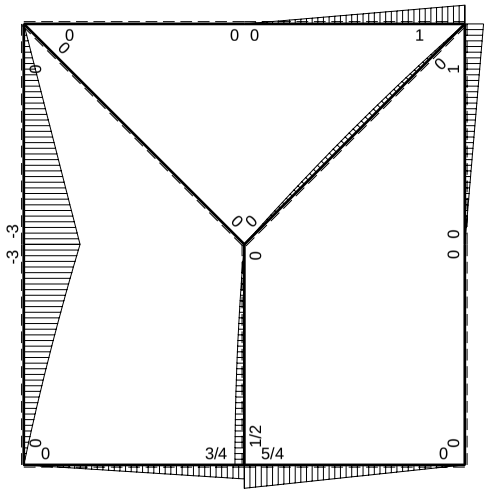

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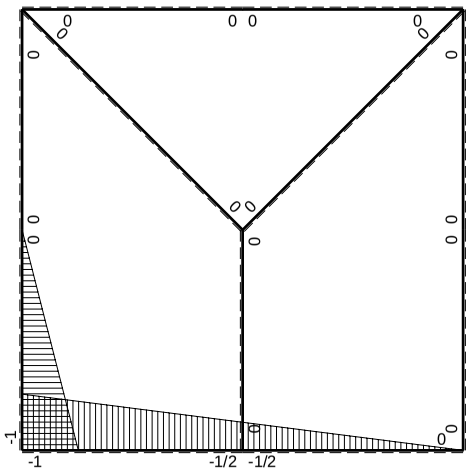

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0


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

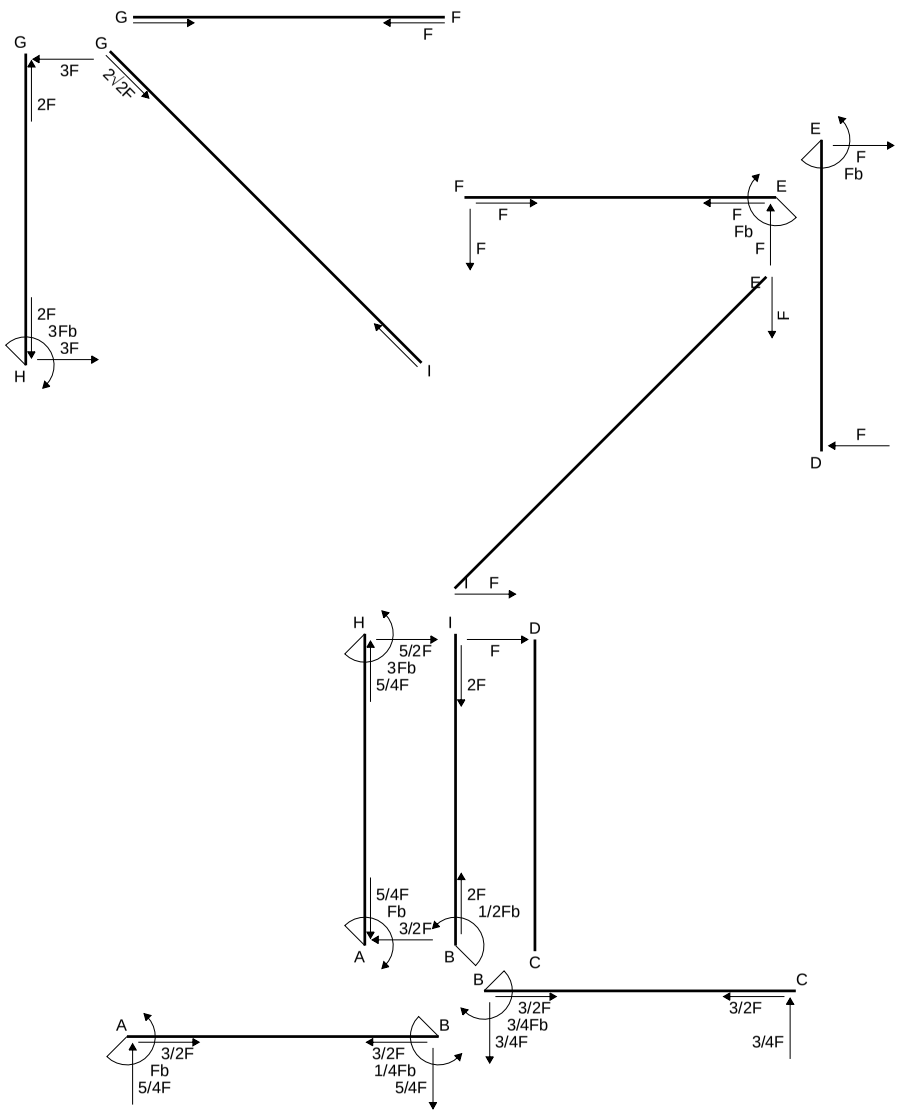
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

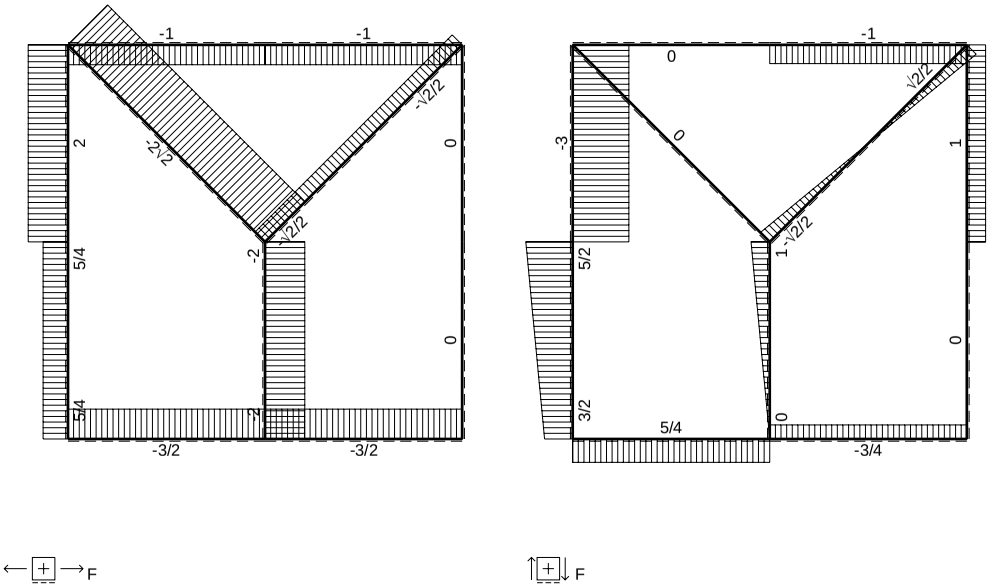
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

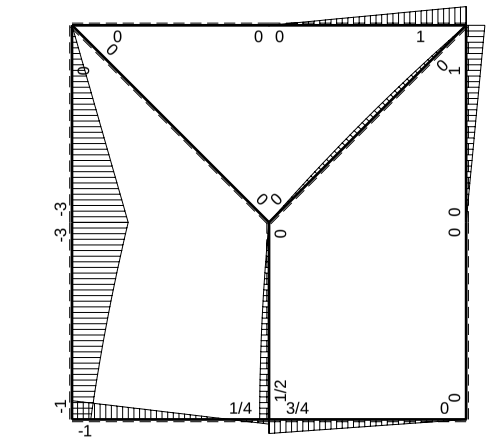
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

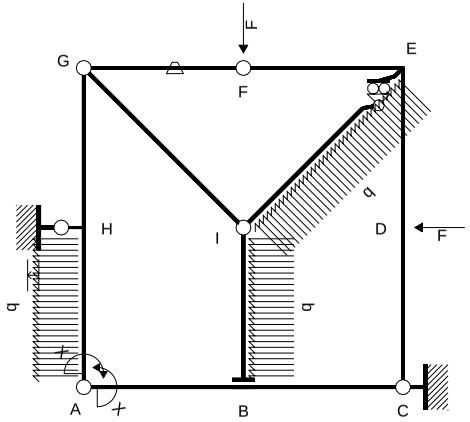
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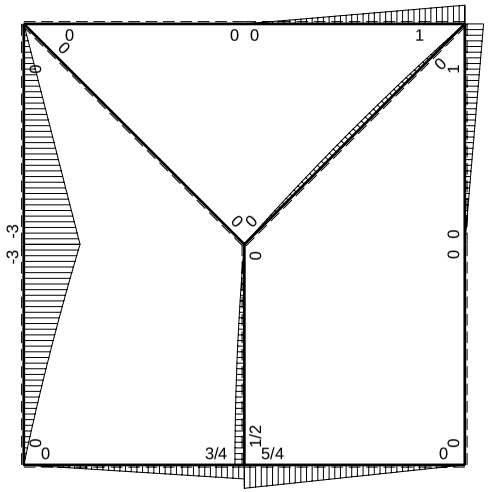
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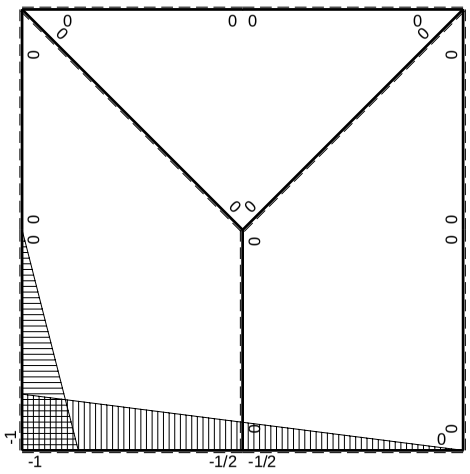

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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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PROCEDIMENTO E RISULTATI 239217 ESΣ04.xxxx.037

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

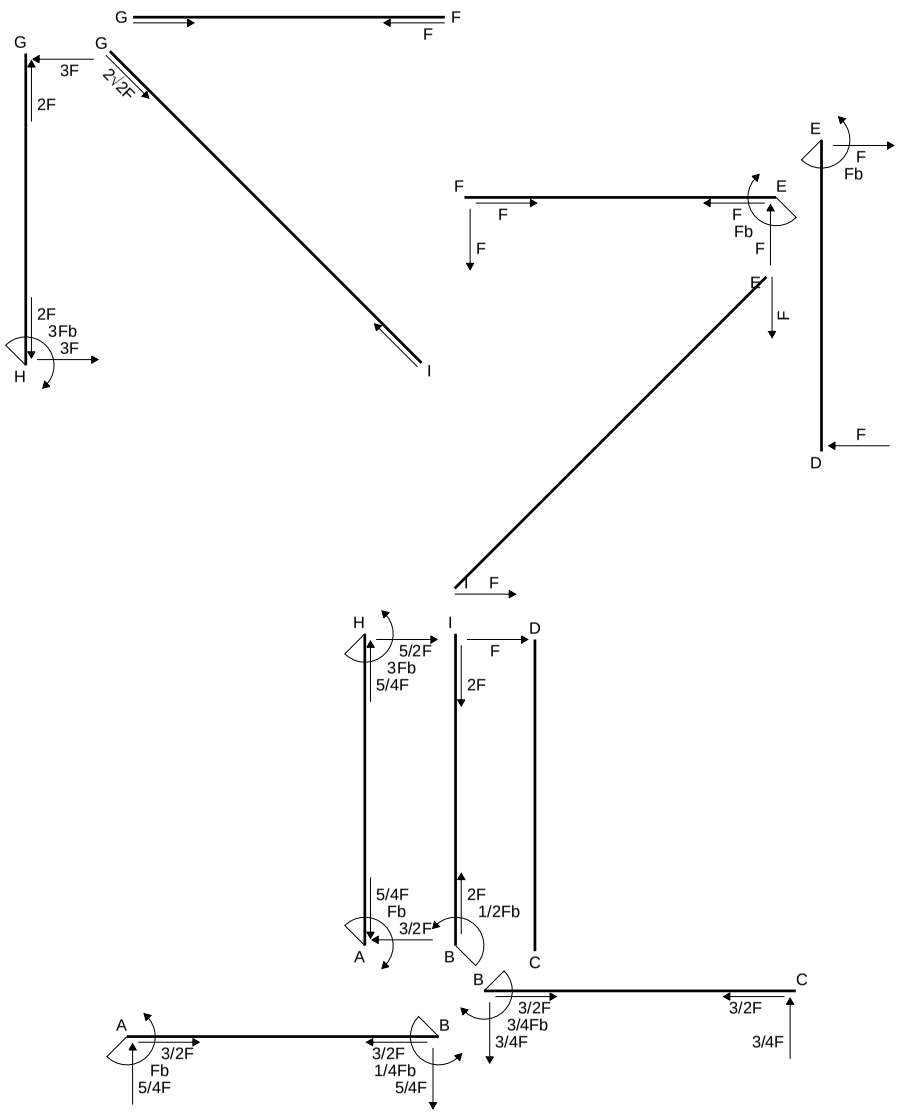
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

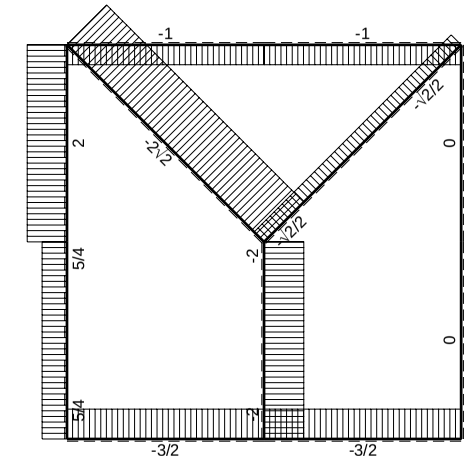
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

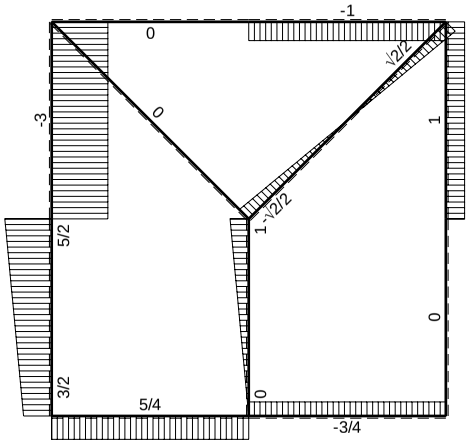
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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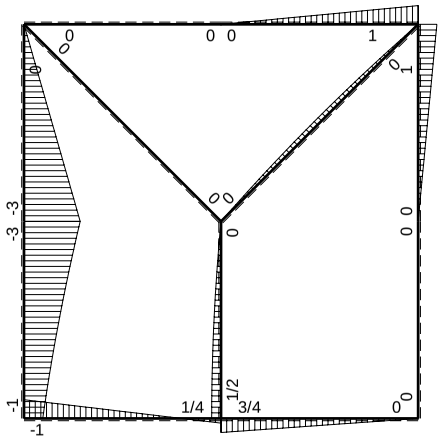

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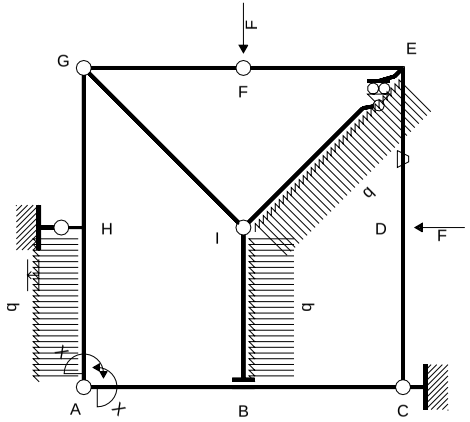

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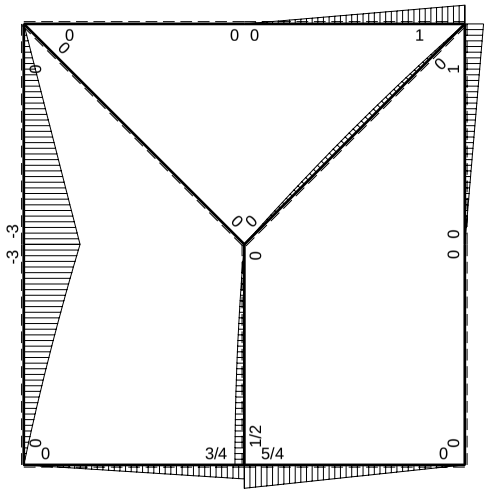
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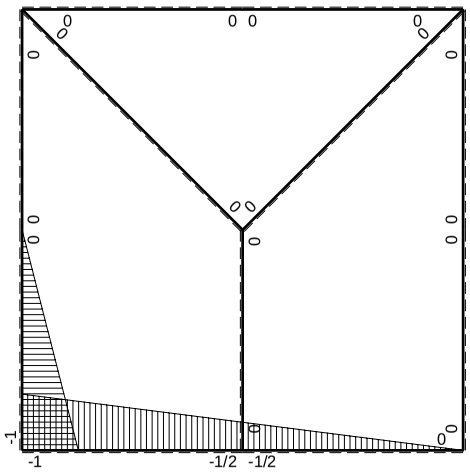

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3/4
0
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⟲□⟳ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-5/4Fx	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

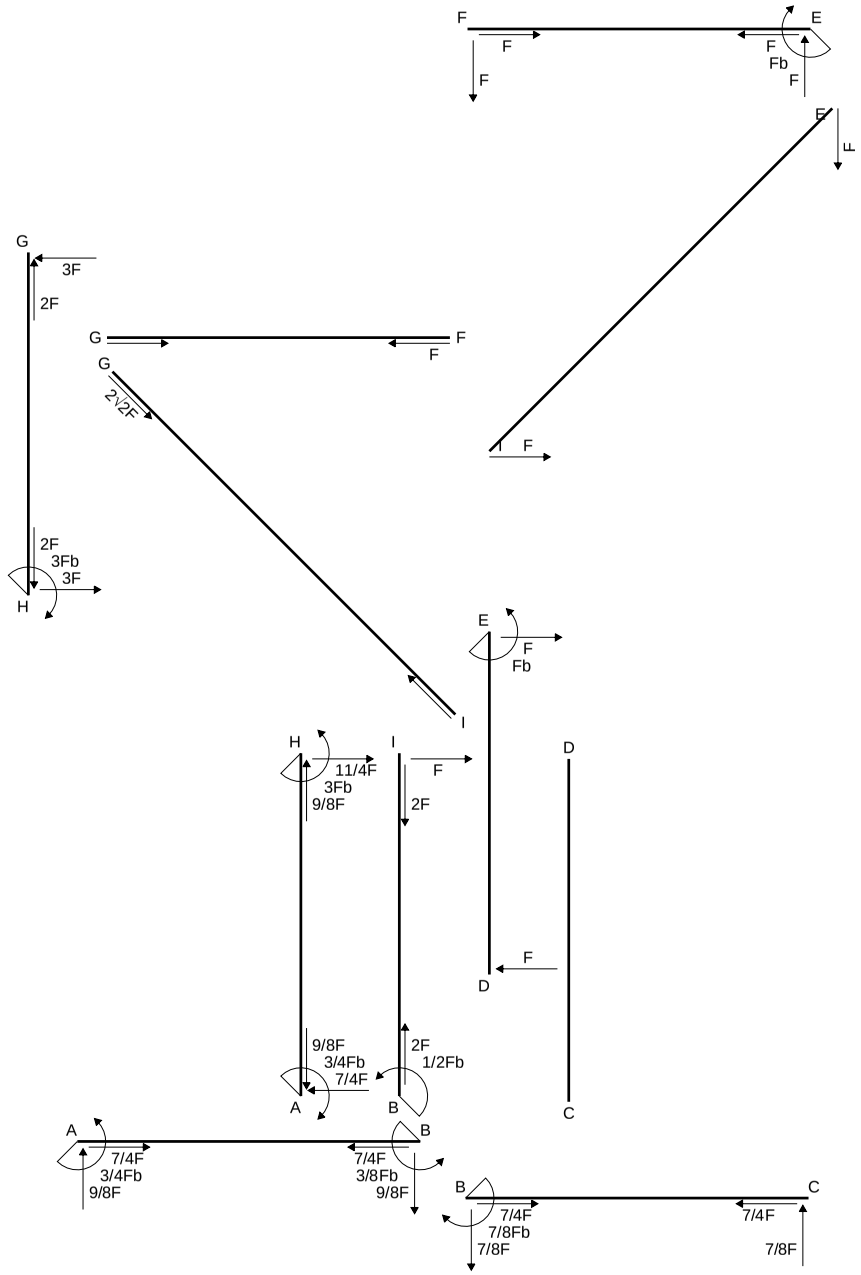
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

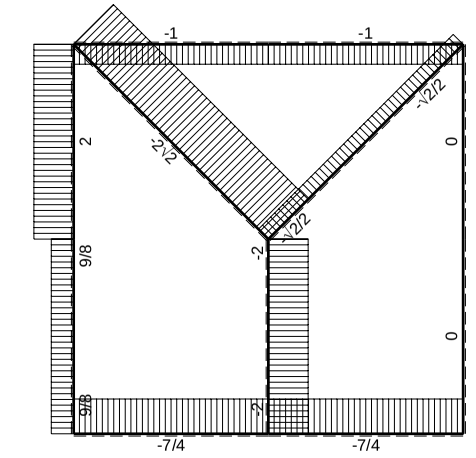
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

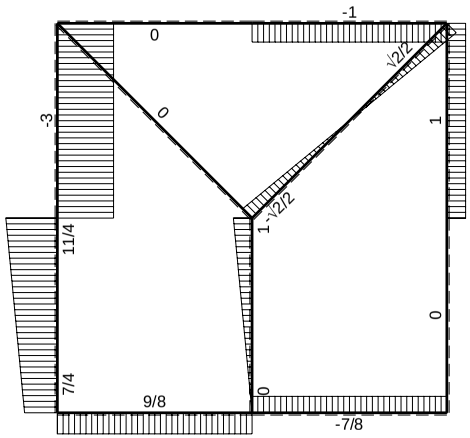
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

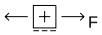
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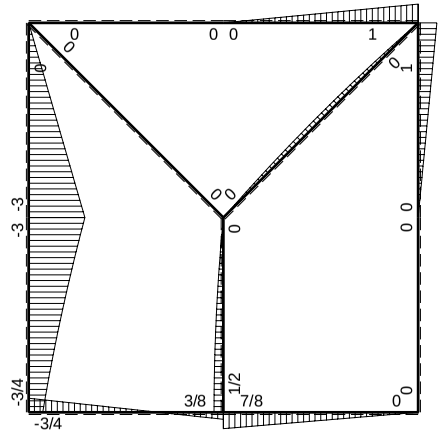

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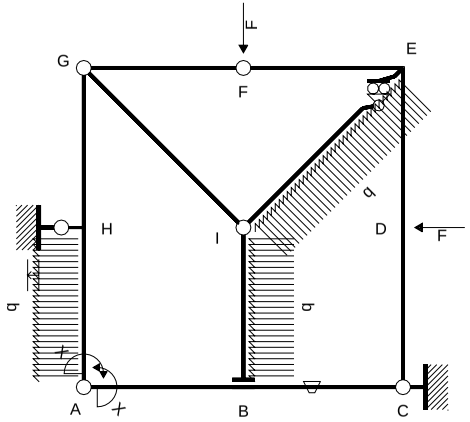

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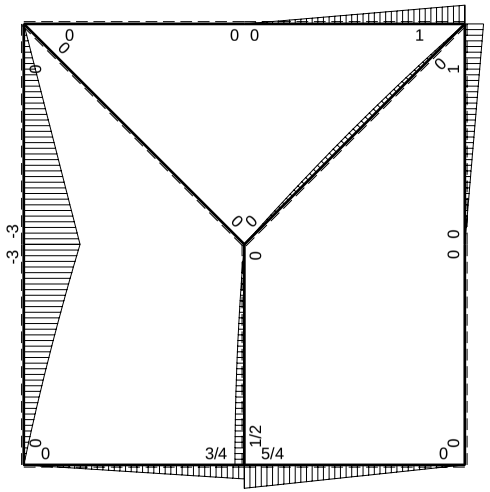

F


F

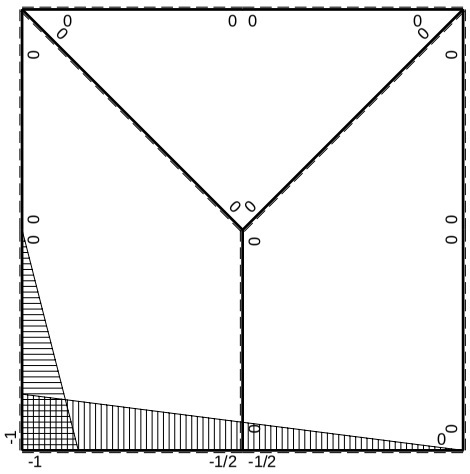

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0
0
1
0 0
-3 -3
0
0 0
-3/4
3/8
1/2
7/8
0
0
-3/4


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	-Fb/EJ	$-5/8Fb+5/4Fx-5/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-5/4Fx	Fb/EJ	$-5/8Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\right)\ \theta\ \ = 1/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\right)\ \theta\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-5/24\ b\right)\ Fb\ 1/EJ\ + \left(-1/4\ b\right)\ \theta\ \ = 1/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

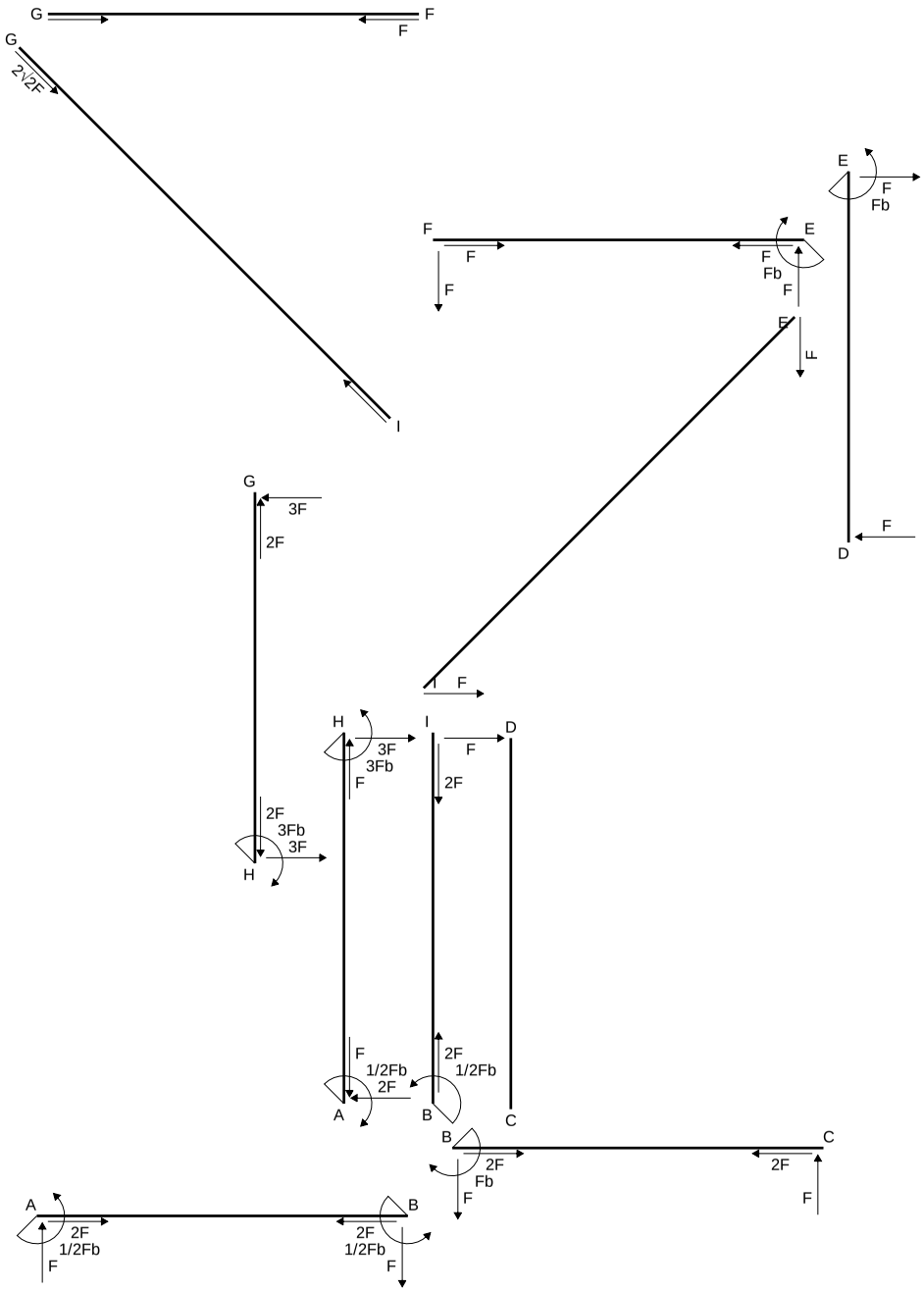
$= \left(3/2\ b - 7/6\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

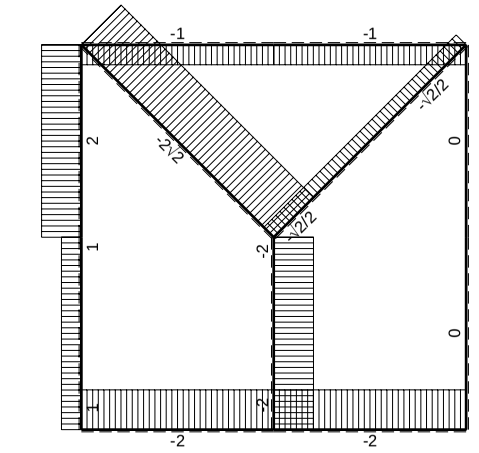
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(5/4\ b - 2/3\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

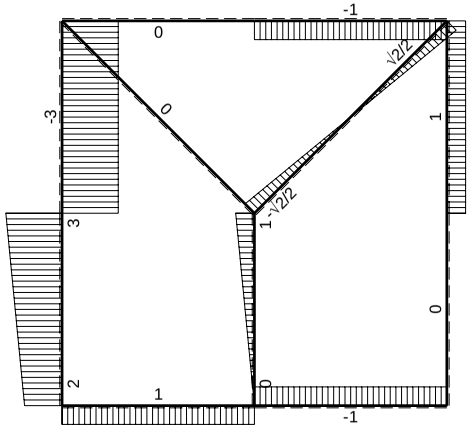
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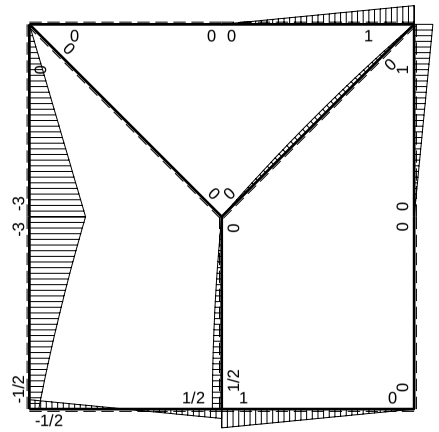

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F


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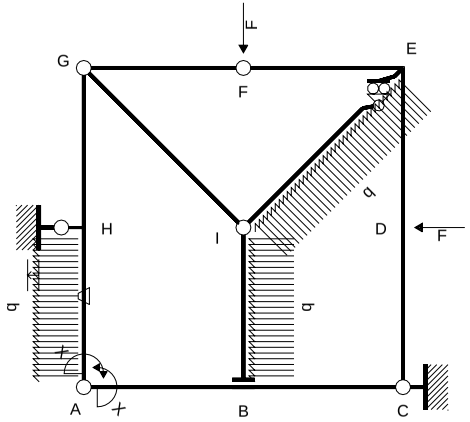
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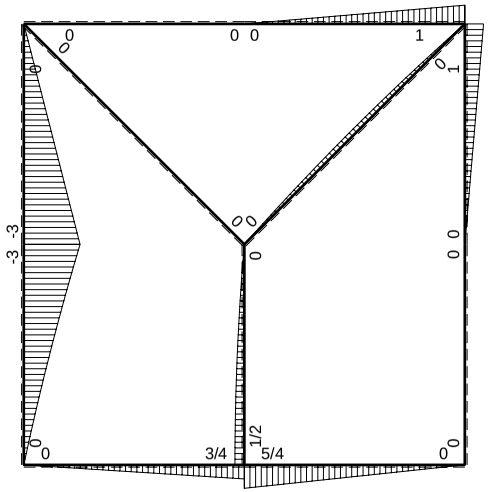

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√2/2
-3
0
1
3
1
-√2/2
0
2
1
D
-1

F

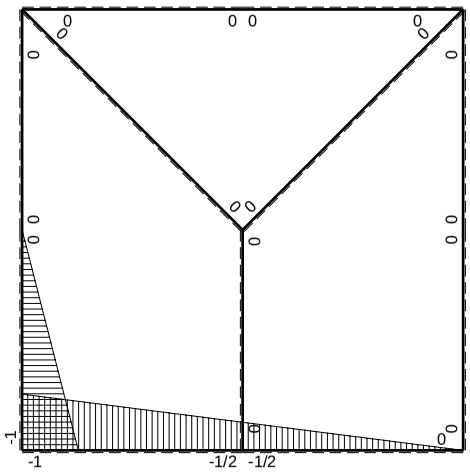

0
0 0
1
0
1
0 0
-3 -3
0
0 0
-1/2
1/2
1/2
1
0 0
-1/2

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238922 ESΣ04.xxxx.040

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	-Fb/EJ	$3Fx-7/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	Fb/EJ	$5/2Fx-2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/2Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx$

$= \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = 23/24\ \ Fb^2/EJ$

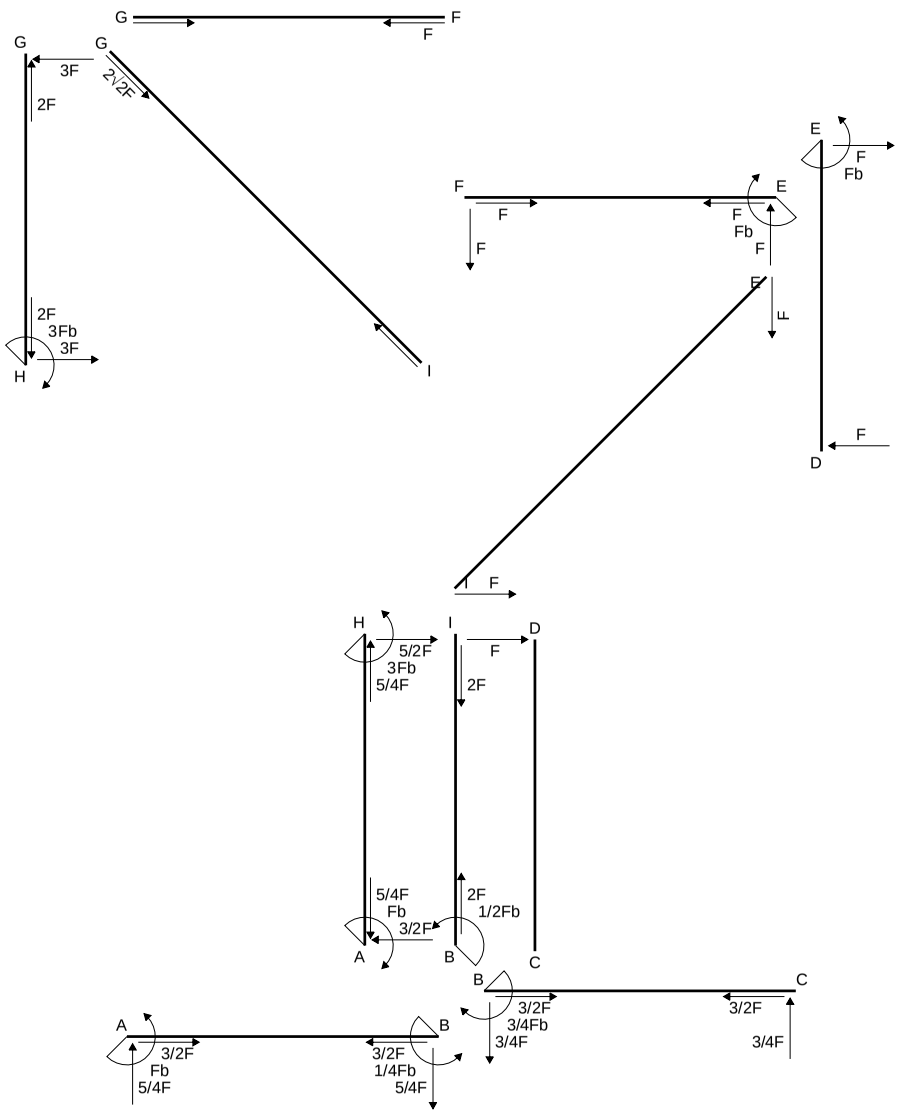
$L^{Xo}_{AH} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

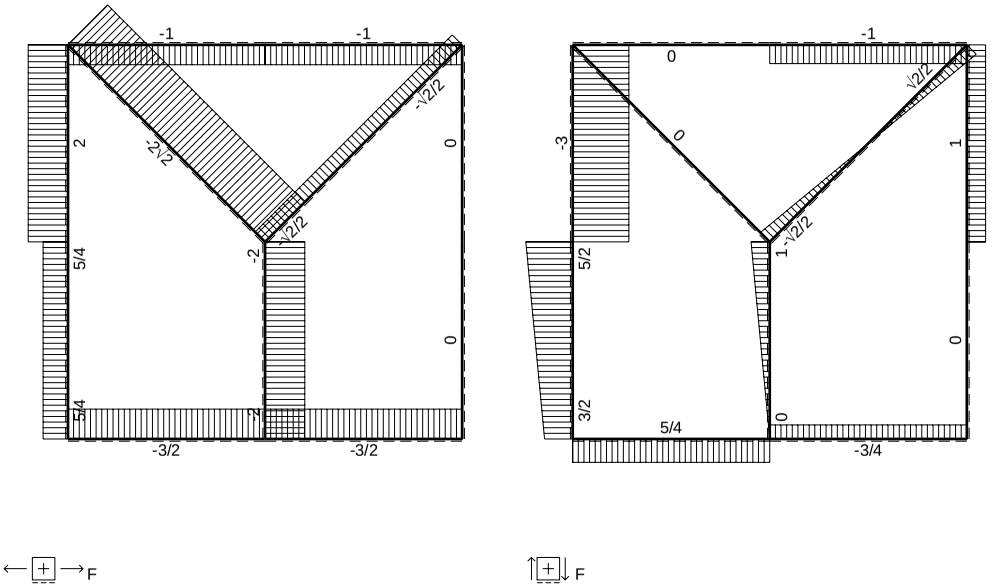
$= \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\ \theta\ \ = 23/24\ \ Fb^2/EJ$

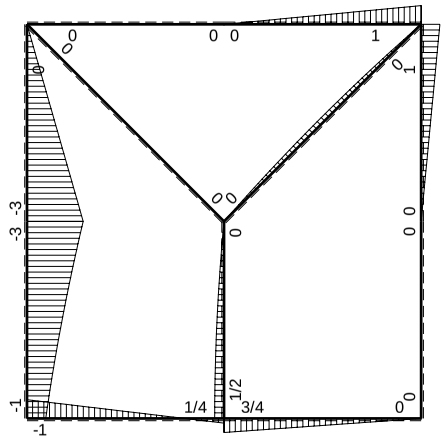
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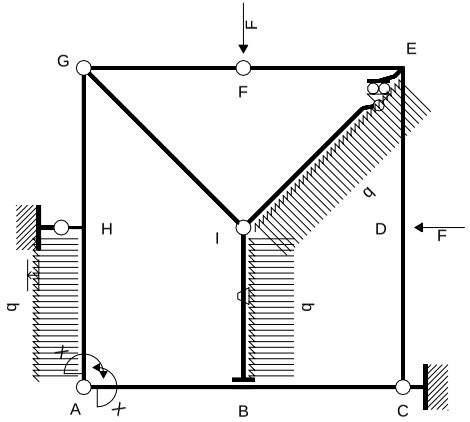

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5/4F

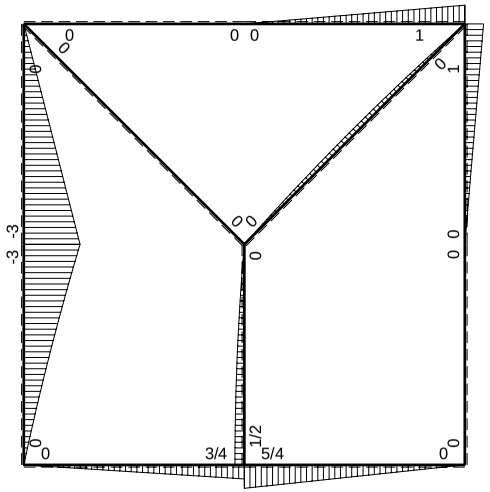

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5/4
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-1
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F F

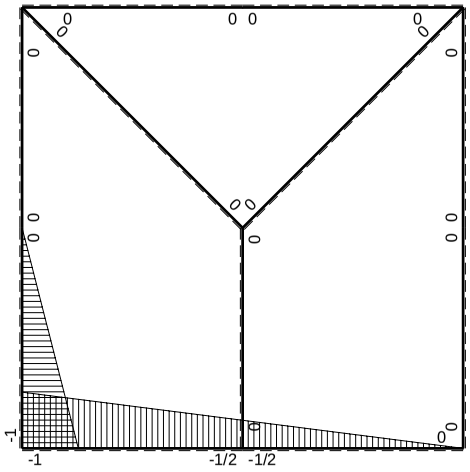

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 956982 ESΣ04.xxxx.041

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

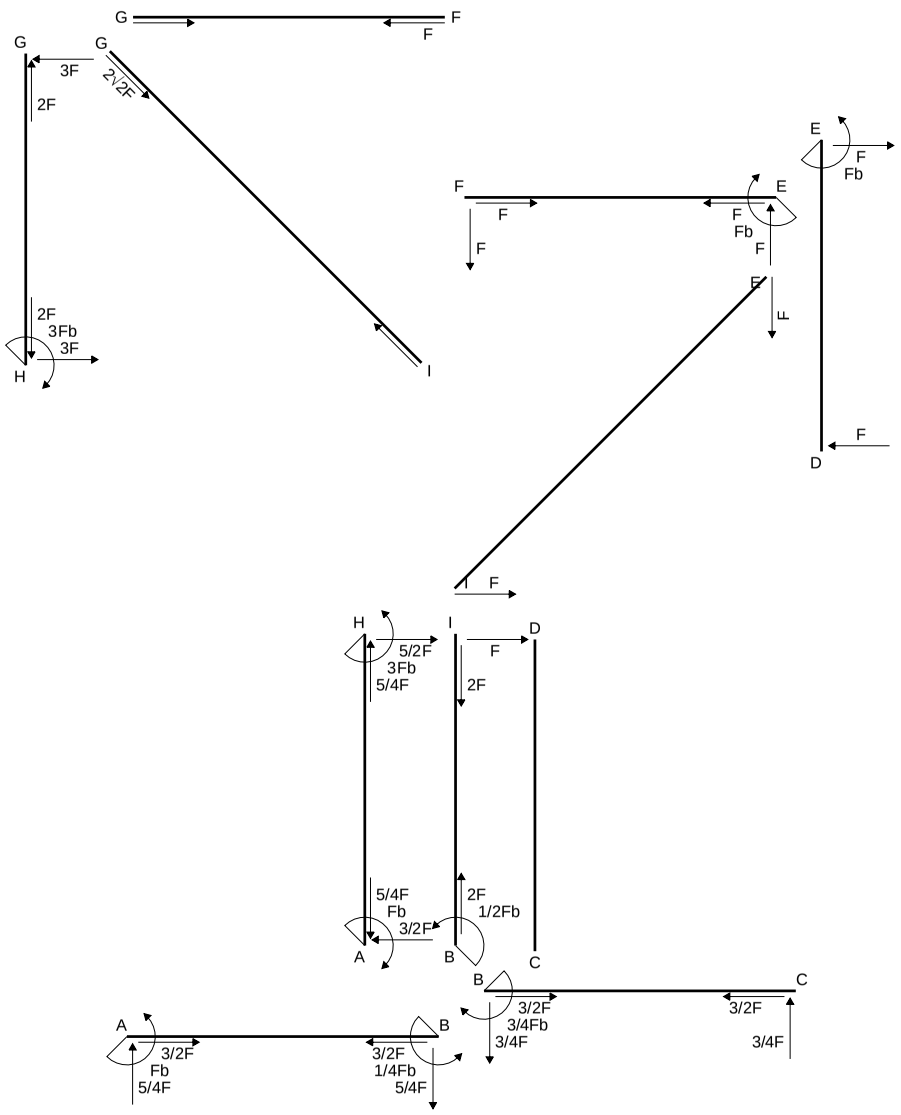
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

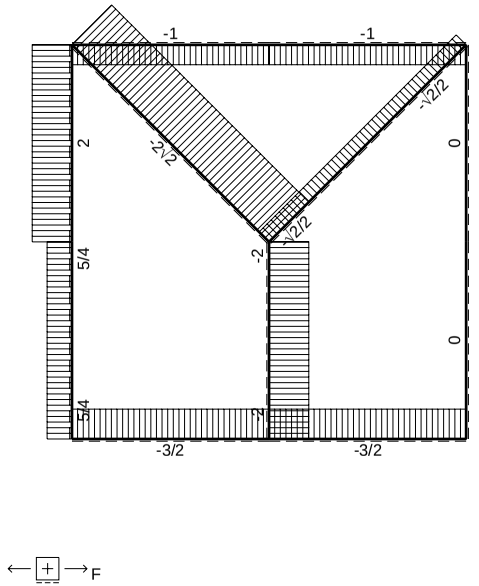
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

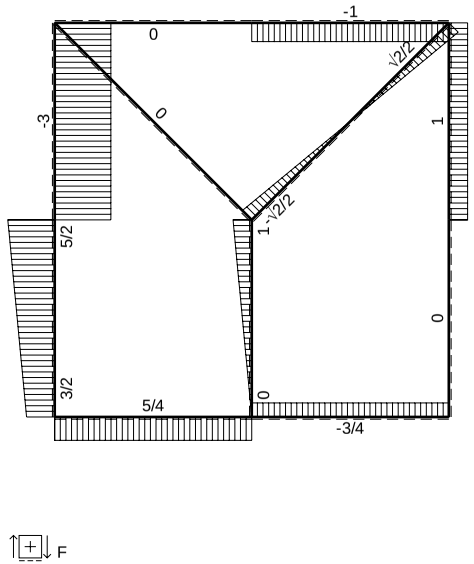
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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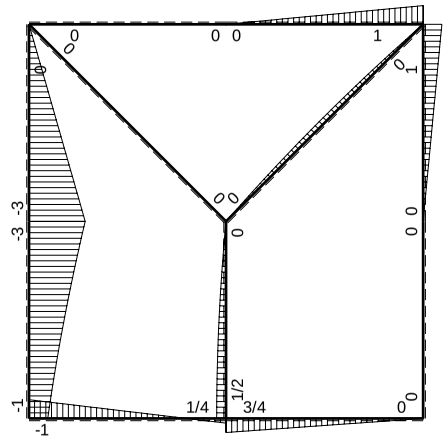
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2F
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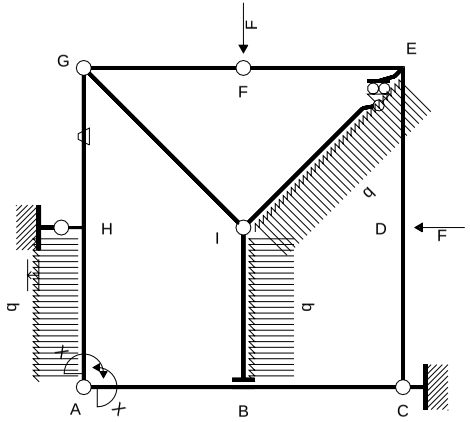

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-√2/2
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5/4
-2
-3/2
-3/2

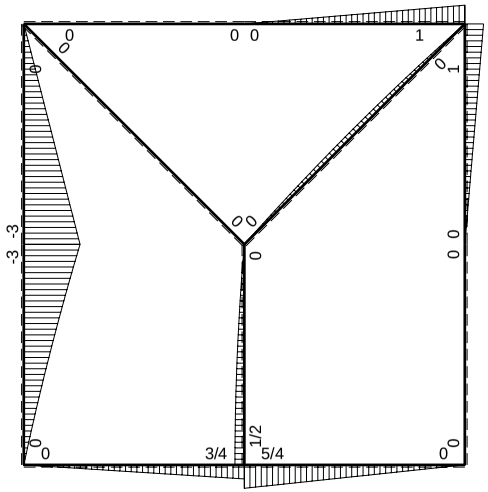

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√2/2
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-√2/2
5/2
0
3/2
5/4
0
-3/4

F F

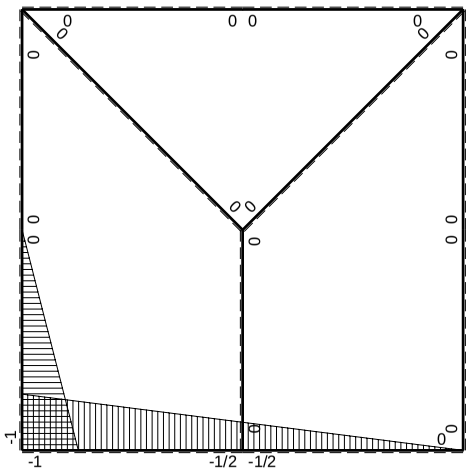

0
0 0
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0
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1
0 0
-3 -3
0
0 0
-1
1/4
1/2
3/4
0
0
-1

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211619 ESΣ04.xxxx.042

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-5/4Fx	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	3Fb-3Fx	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

PROCEDIMENTO E RISULTATI 211619 ESΣ04.xxxx.042

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

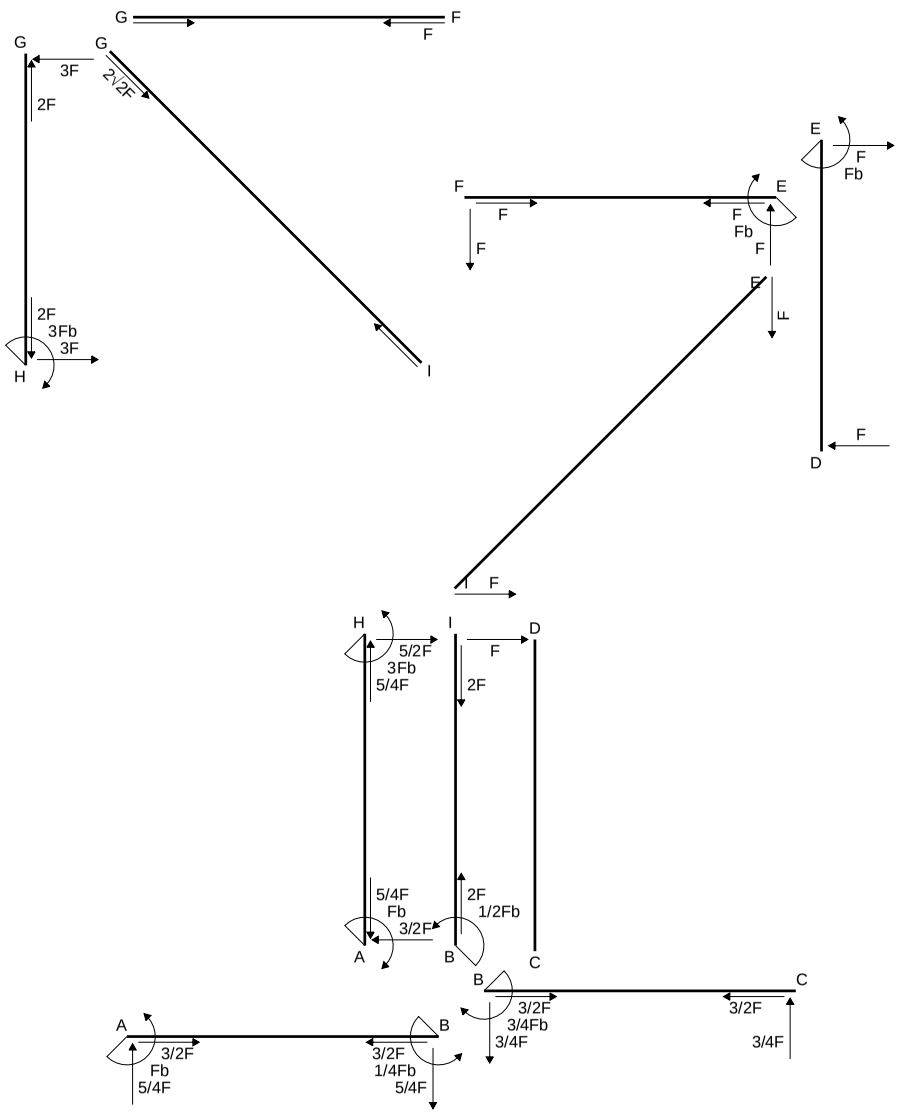
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

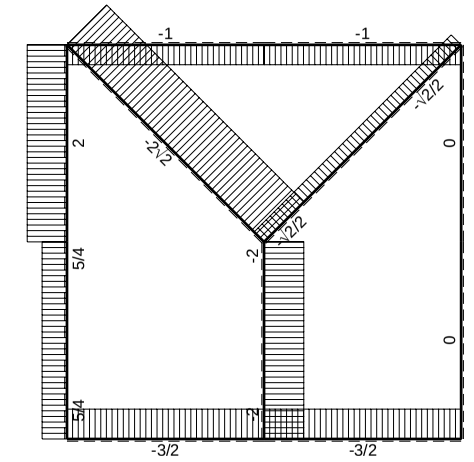
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

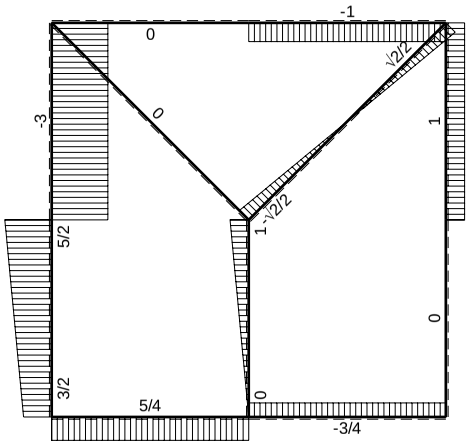
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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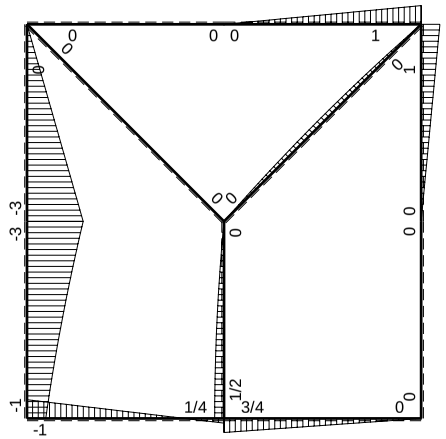

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-3/2
-3/2

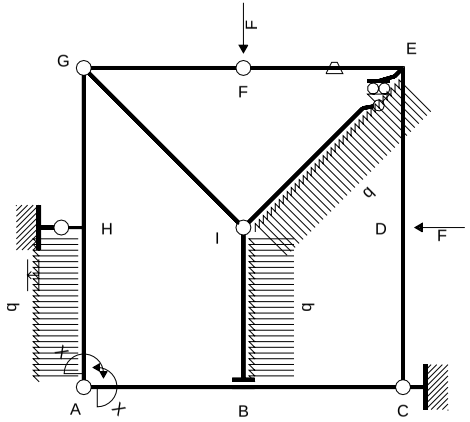

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0
-3/4

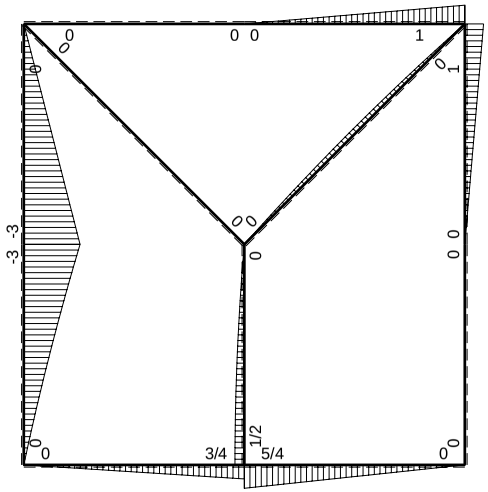
←□→ F

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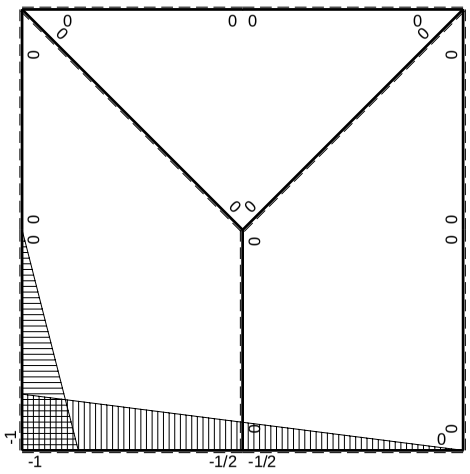

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-1

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238581 ESΣ04.xxxx.043

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

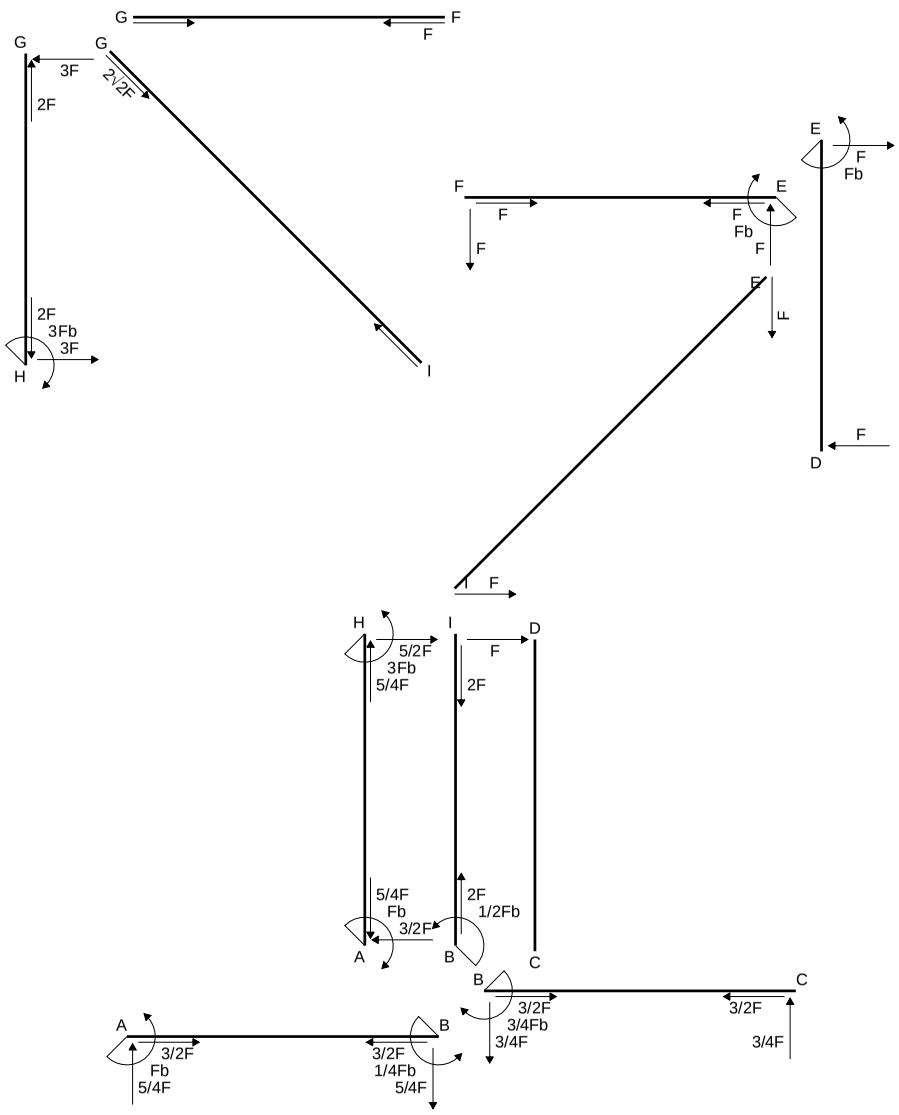
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

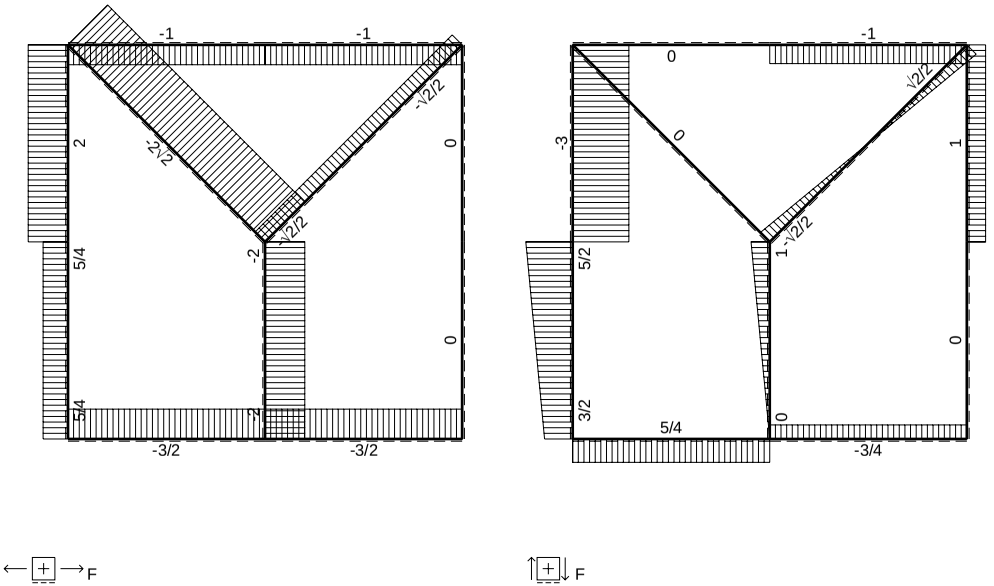
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

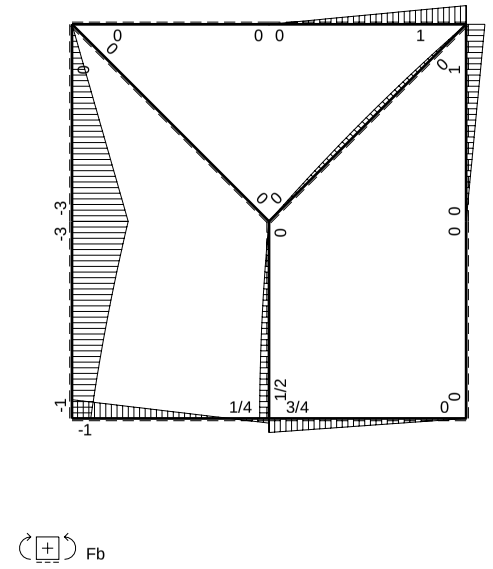
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

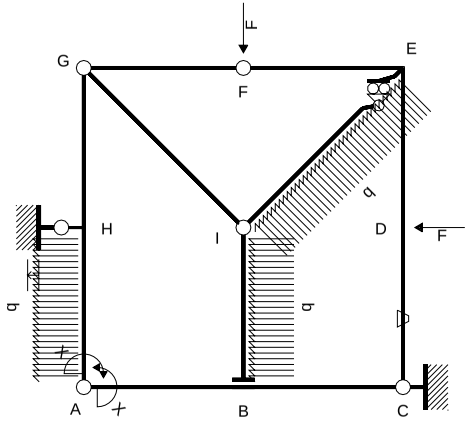
ESΣ04.xxxx.043

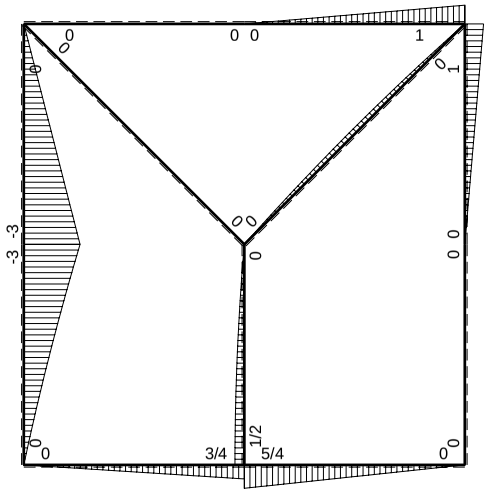
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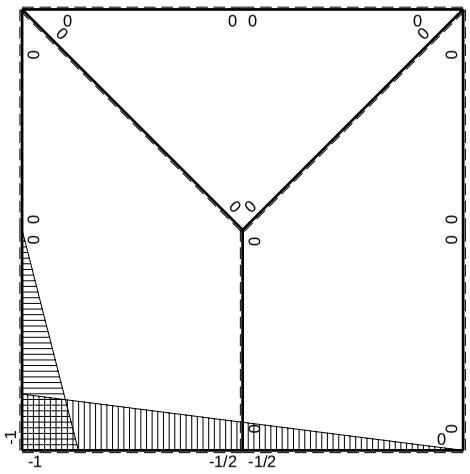

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-3/4
F


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1/2
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-1
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236422 ESΣ04.xxxx.044

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PROCEDIMENTO E RISULTATI 236422 ESΣ04.xxxx.044

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-5/4Fx	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	-Fb/EJ	0	0	0	0+0	0
DC b	0	0	Fb/EJ	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

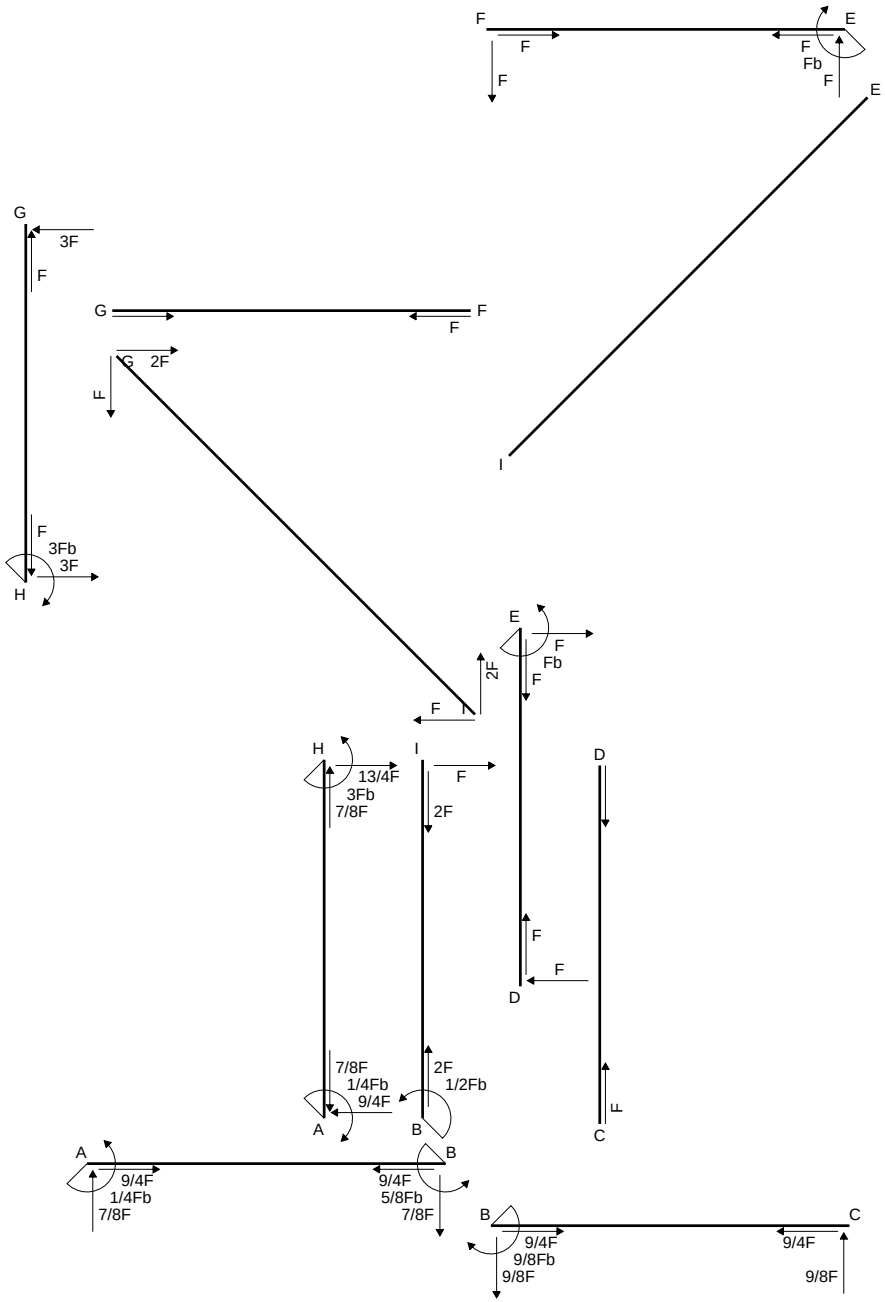
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

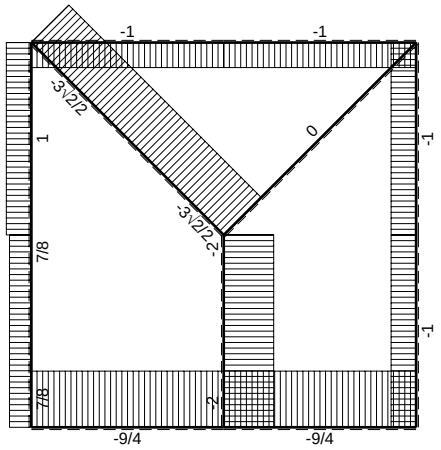
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

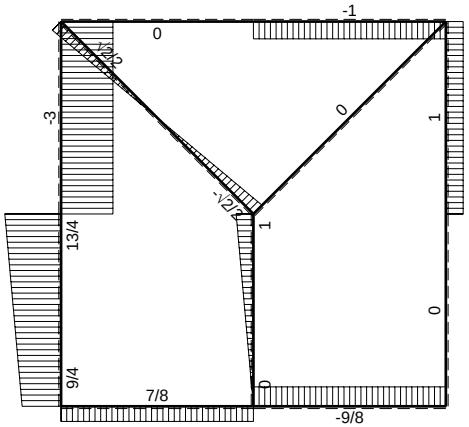
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

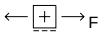
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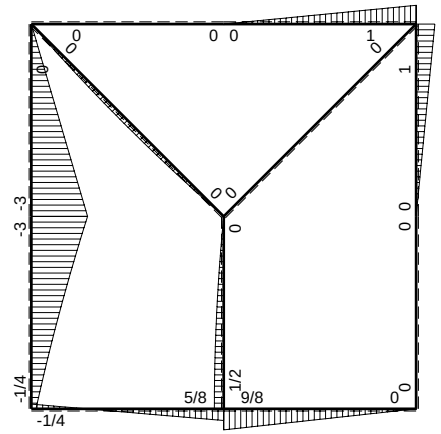

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7/8F
B
9/4F
9/8Fb
9/8F
C
9/4F
9/8F


-1
-1
-3√2/2
1
0
-1
-3√2/2
-2
-1
7/8
7/8
-2
-9/4
-9/4

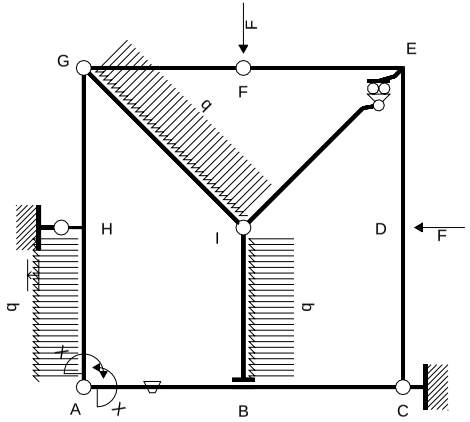

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√2/2
-3
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1
-√2/2
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13/4
0
9/4
7/8
0
-9/8

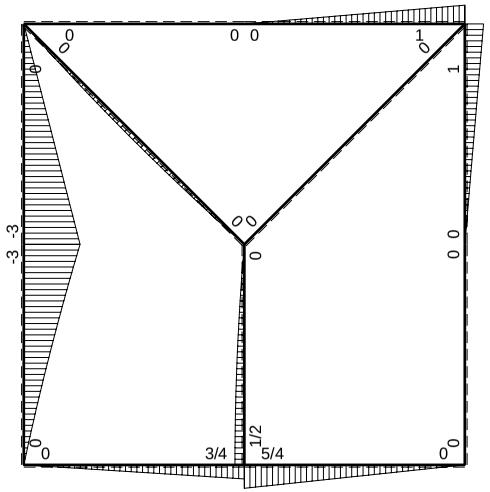

F


F

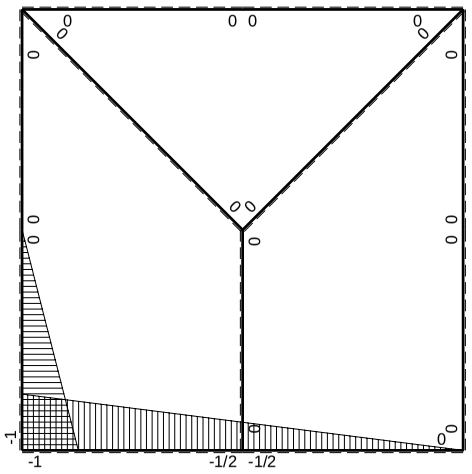

0
0 0
1
0
0
1
0 0
-3 -3
0
0 0
-1/4
5/8
1/2
9/8
0
0
-1/4


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	-Fb/EJ	$-3/4Fx+3/8Fx^2/b$	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(-1/4+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-3/4Fb+3/4Fx	Fb/EJ	$-3/8Fb+3/8Fx^2/b$	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-5/4Fx	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 1/2\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 1/2\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\ \right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

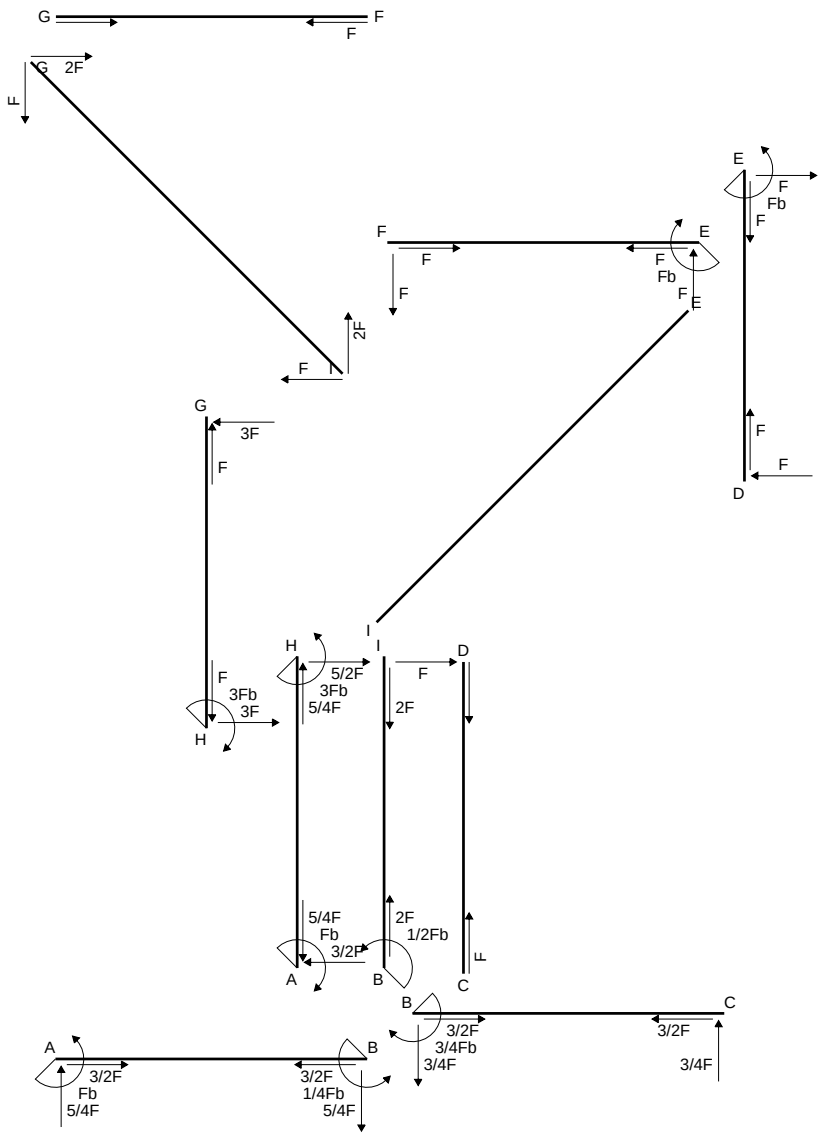
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

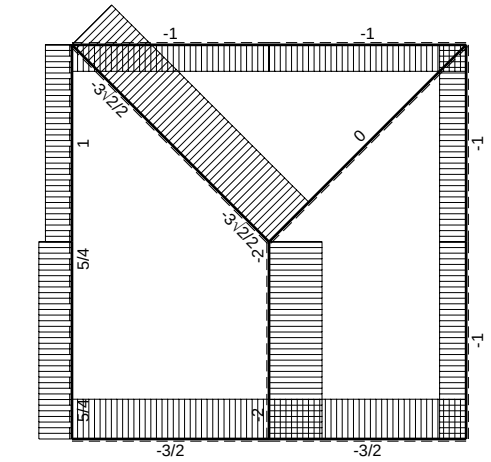
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\ \right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

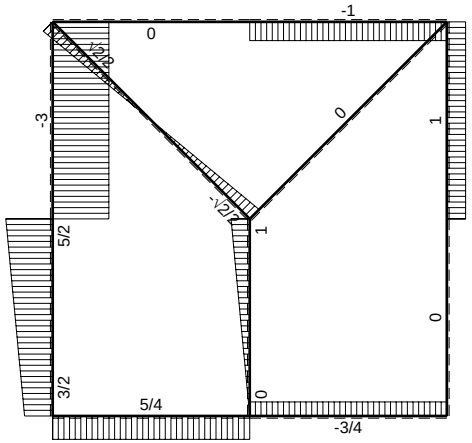
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

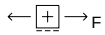
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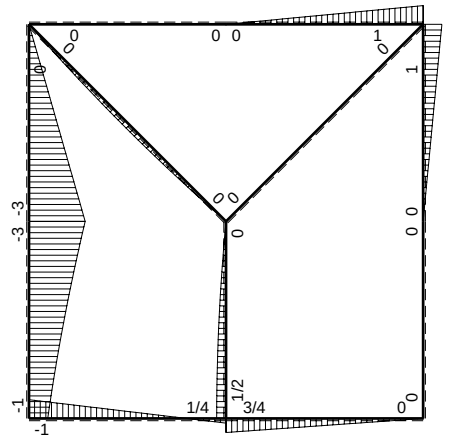

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Fb
3/2F
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1/2Fb
B
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C
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3/4Fb
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3/2F
3/4F
A
3/2F
Fb
5/4F
B
3/2F
1/4Fb
5/4F


-1
-1
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-3/2

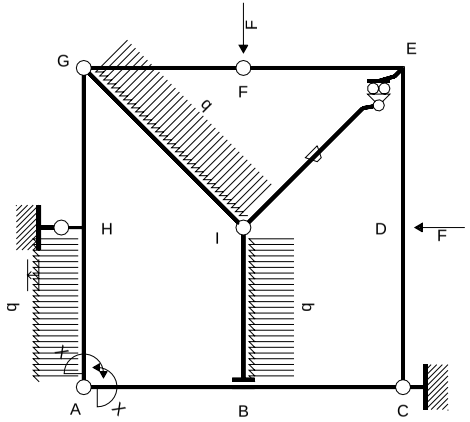

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√2/2
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5/2
0
3/2
5/4
0
-3/4

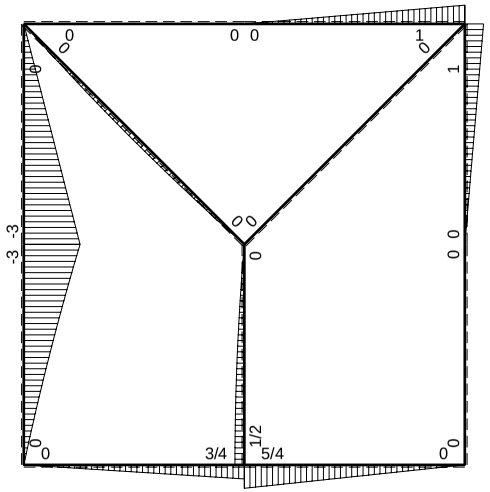

F


F

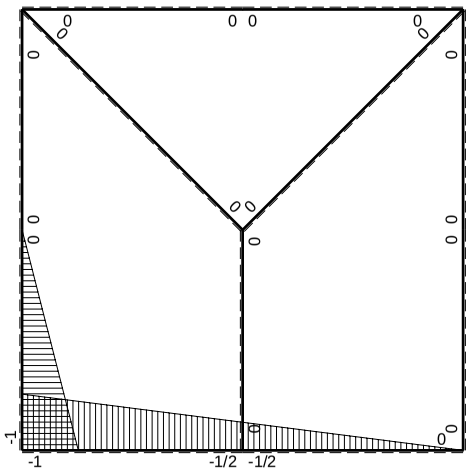

0
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1
0
0
1
0 0
-3 -3
0
0 0
-1
1/4
1/2
3/4
0
0
-1


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237363 ESΣ04.xxxx.046

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PROCEDIMENTO E RISULTATI 237363 ESΣ04.xxxx.046

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

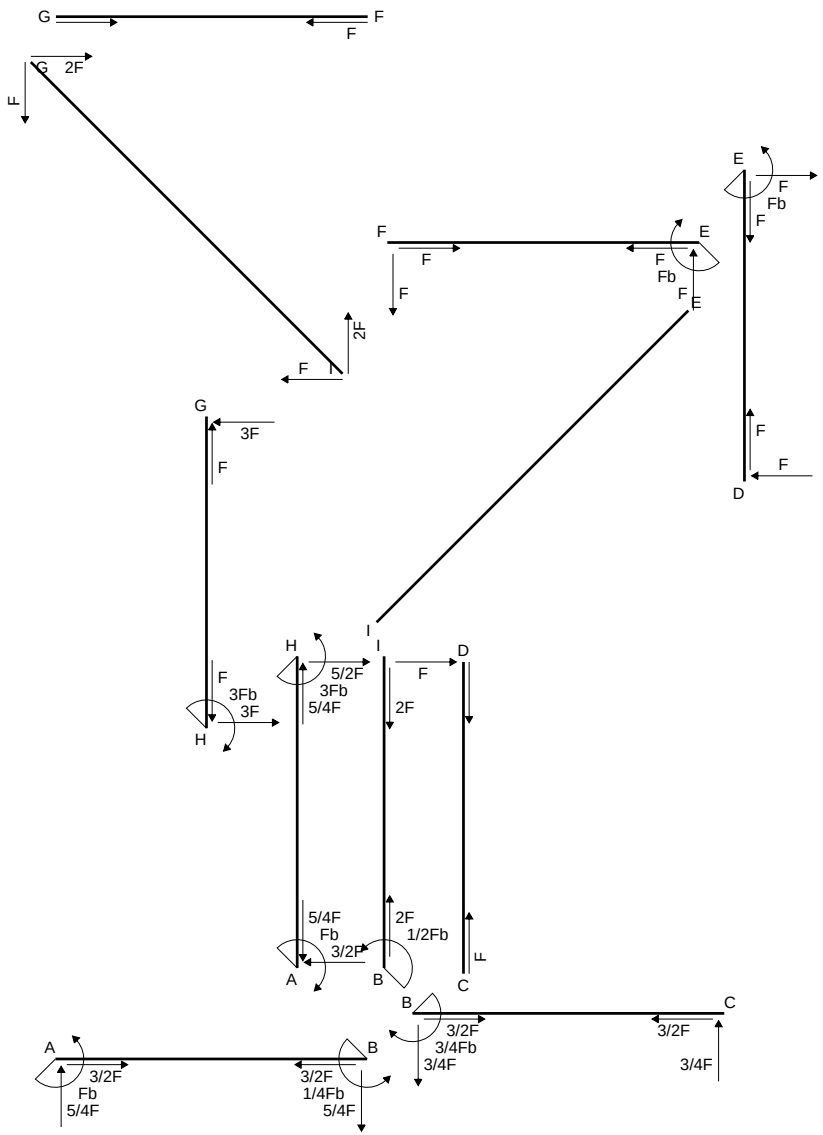
$= \left(3/2\ b - 7/6\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

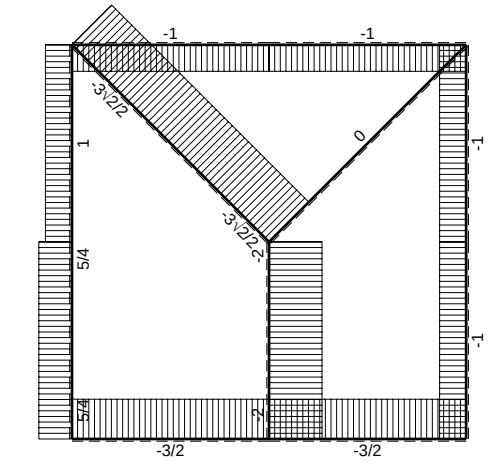
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

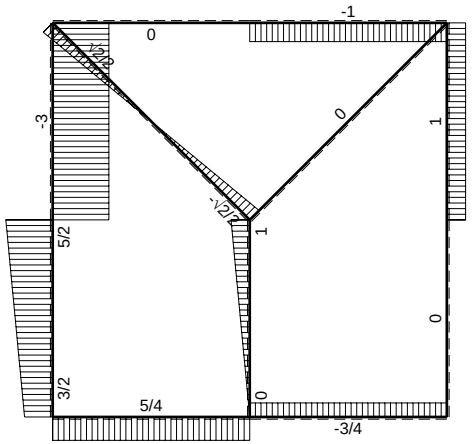
$= \left(5/4\ b - 2/3\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

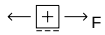
ESΣ04.xxxx.046

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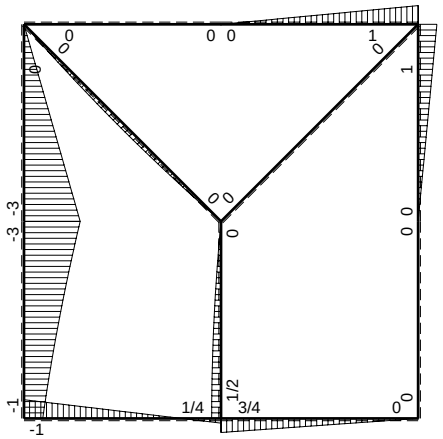

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3Fb
3F
2F
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1/2Fb
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1/4Fb
5/4F


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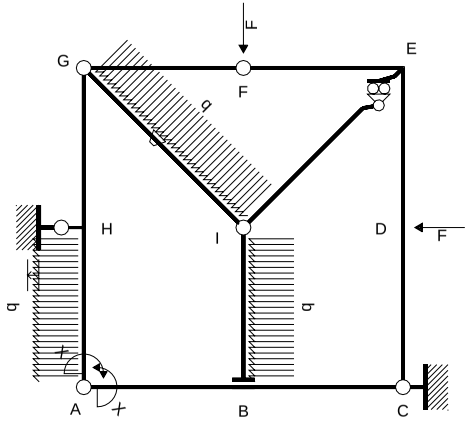

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-3/4

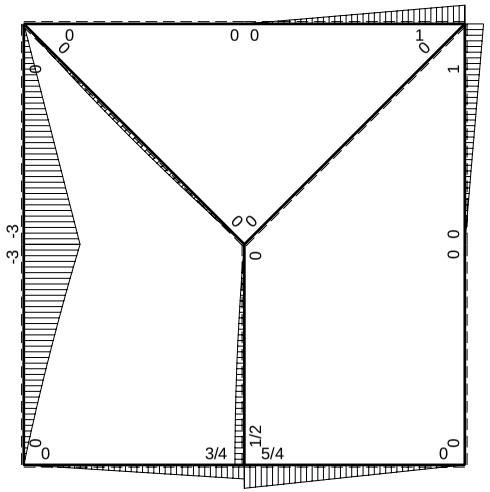

F


F

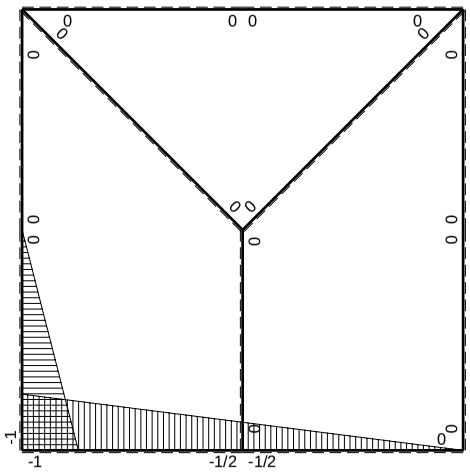

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1/2
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0
-1


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238341 ESΣ04.xxxx.047

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

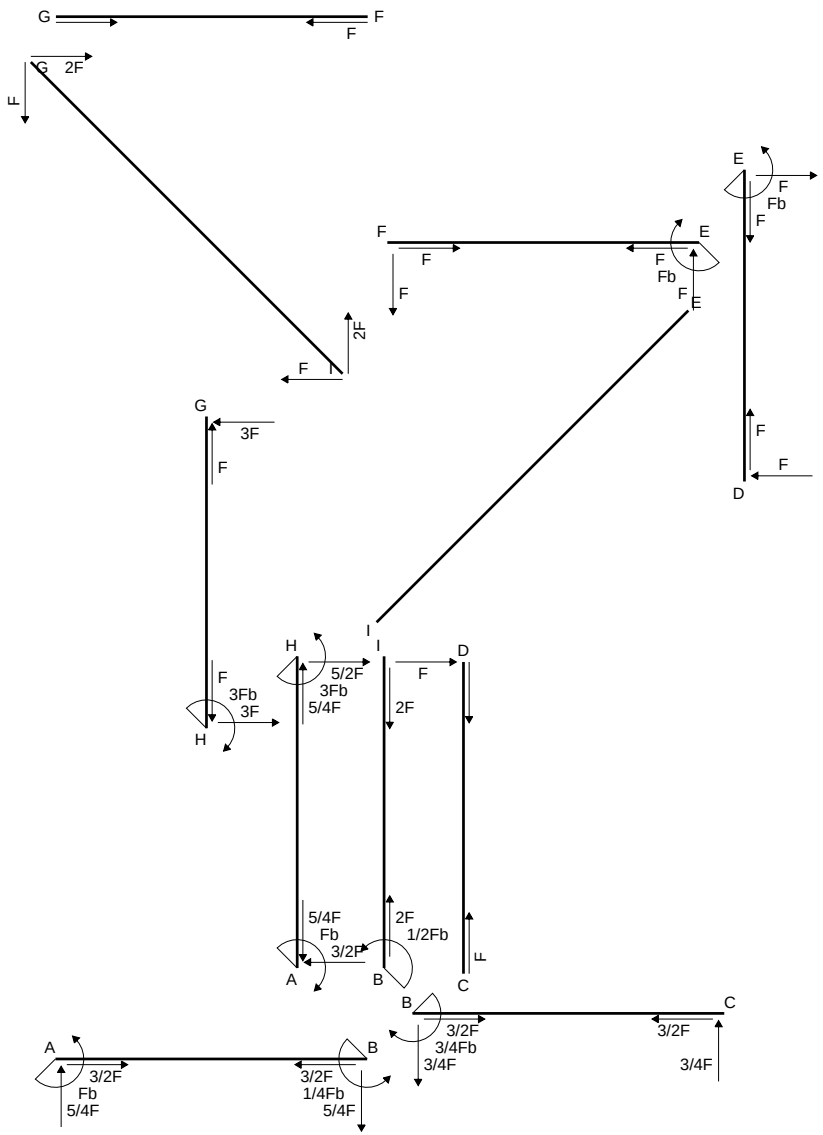
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

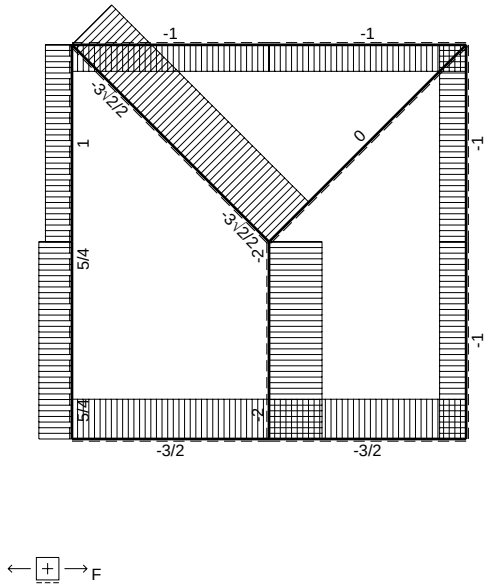
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

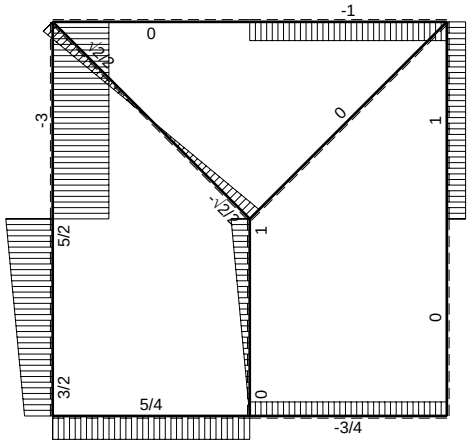
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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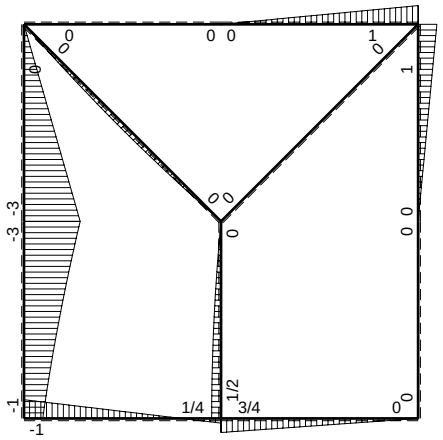

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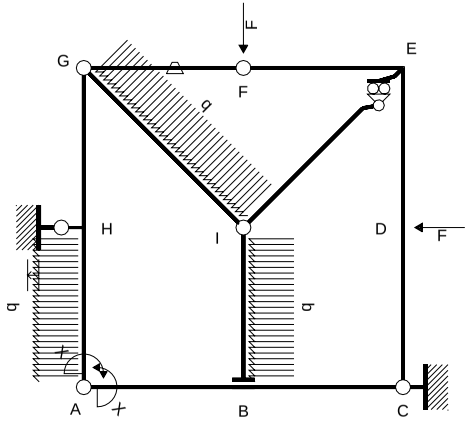

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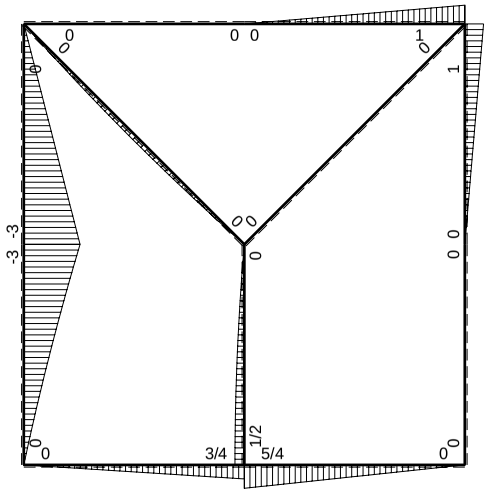
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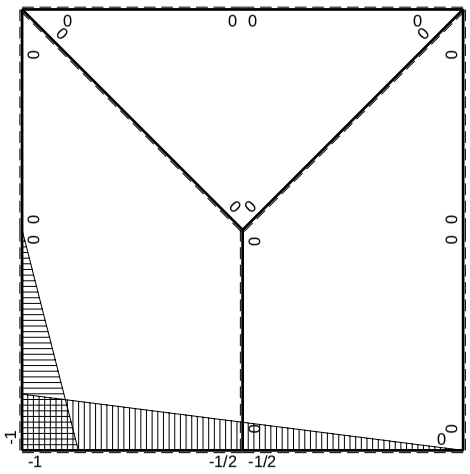

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 934507 ESΣ04.xxxx.048

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

PROCEDIMENTO E RISULTATI 934507 ESΣ04.xxxx.048

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

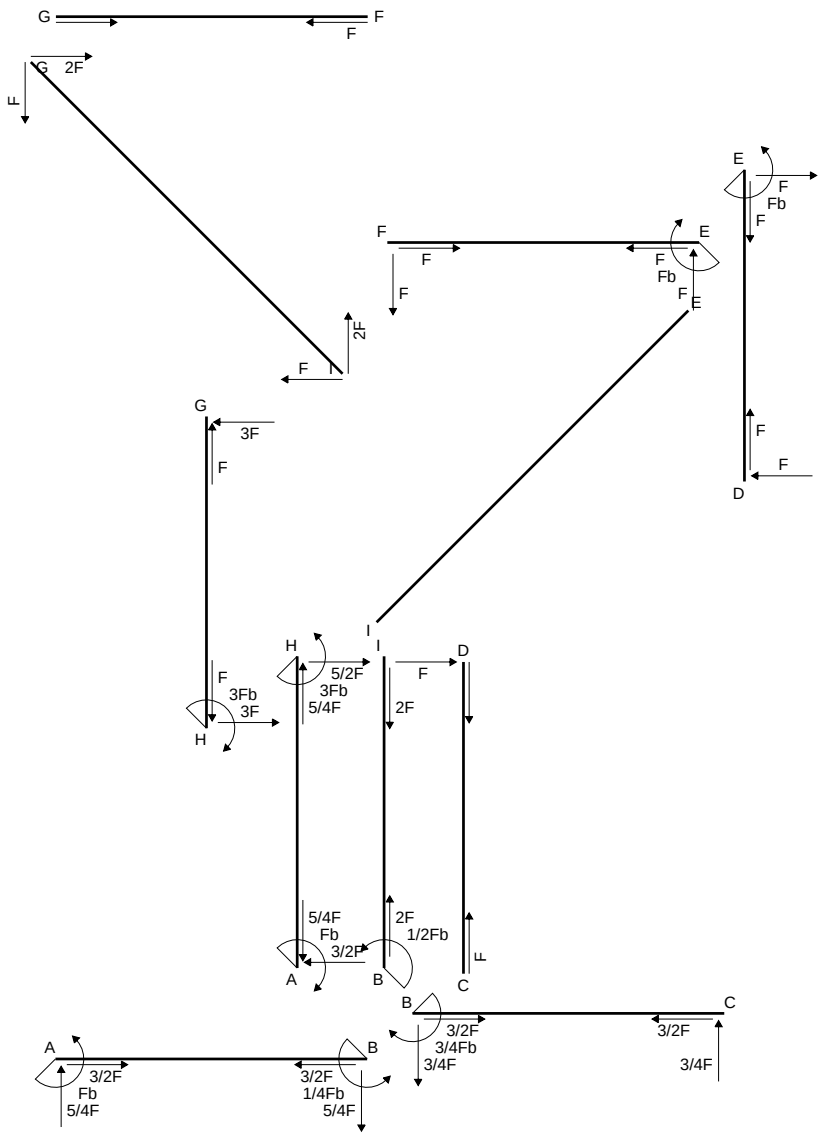
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

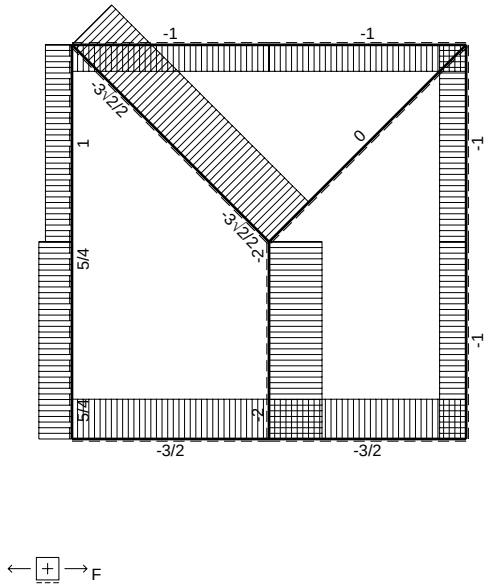
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

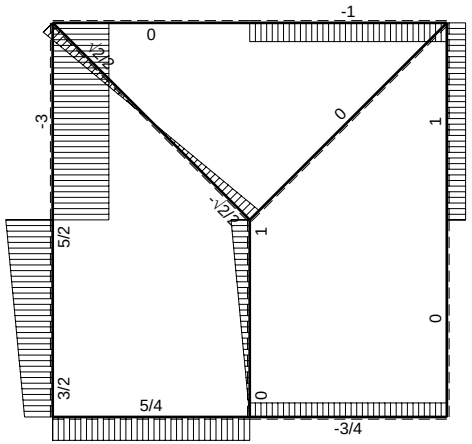
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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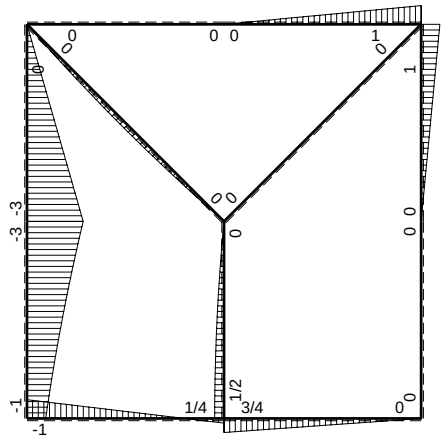

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1/4Fb
5/4F


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-2
-3/2
-3/2

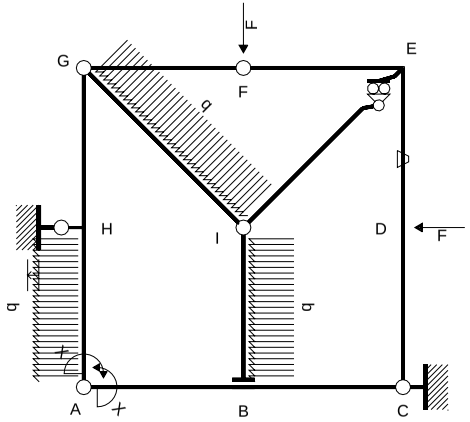

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-3/4

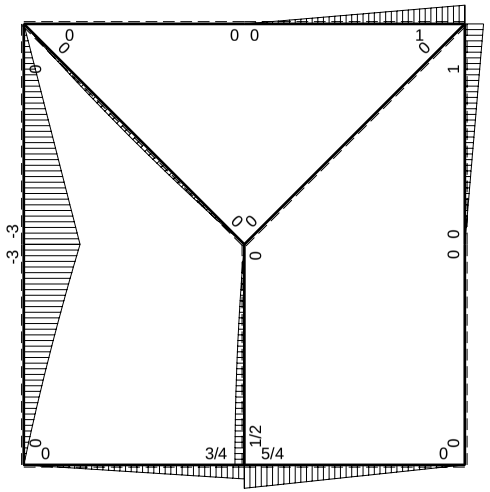
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F

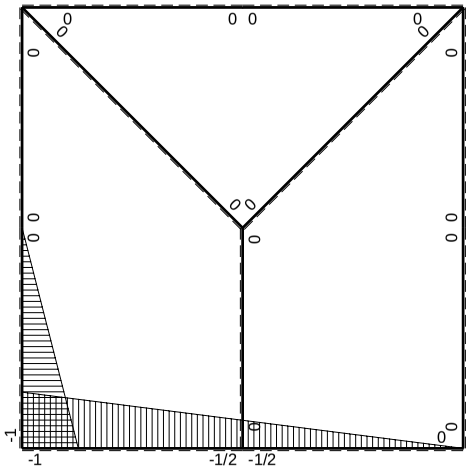

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0
0 0
-1
1/4
1/2
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0
0
-1


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 280338 ESΣ04.xxxx.049

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-5/4Fx	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

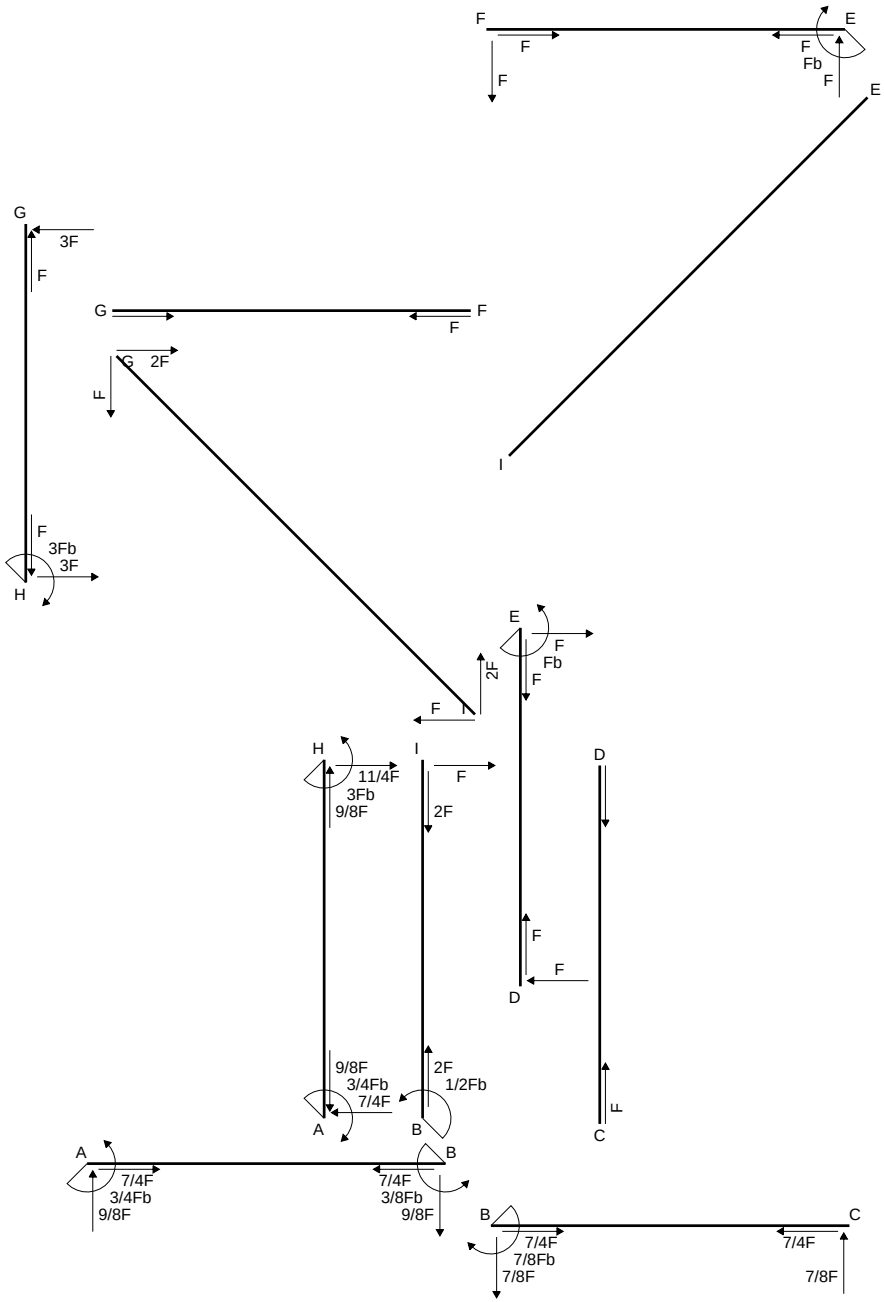
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

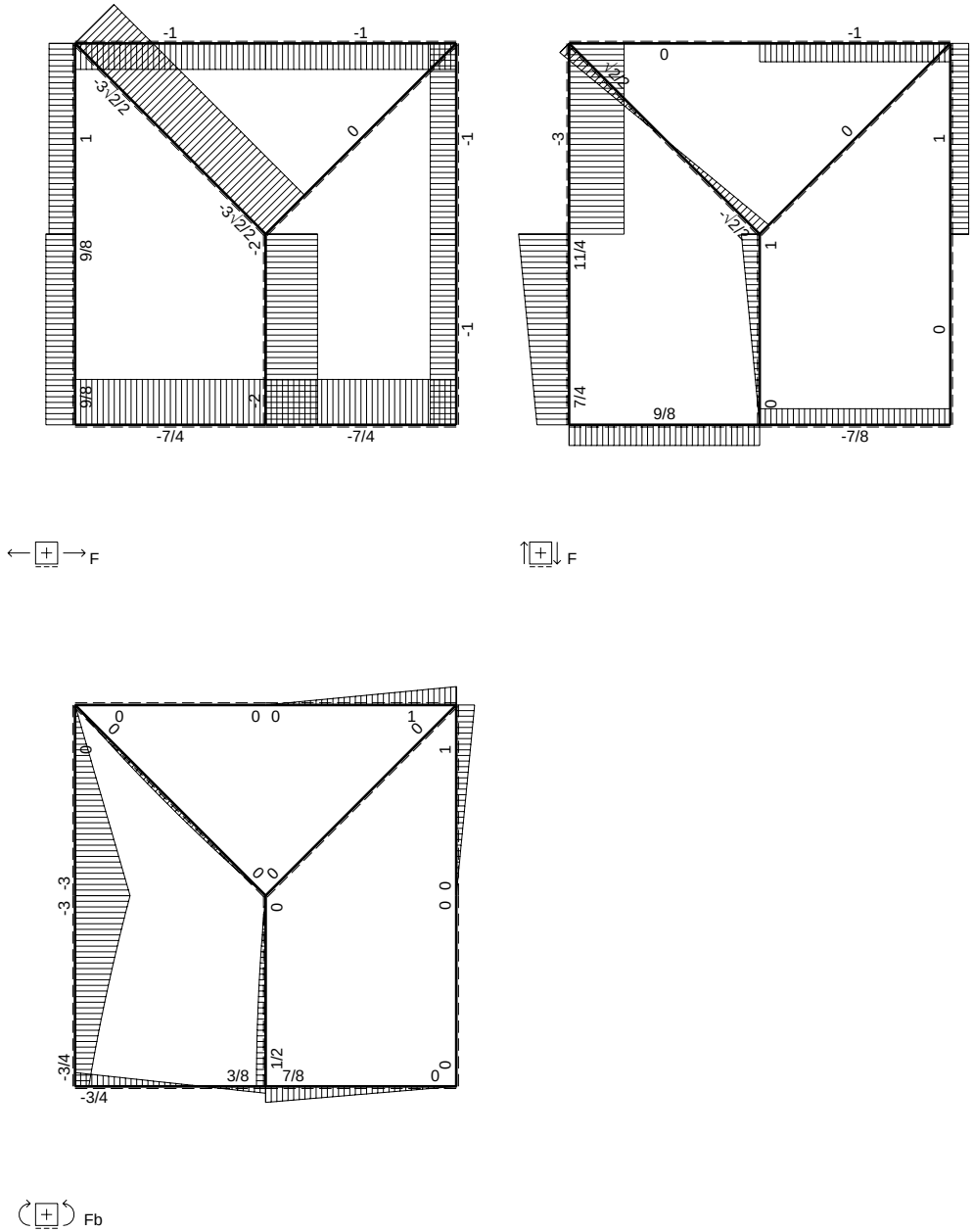
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

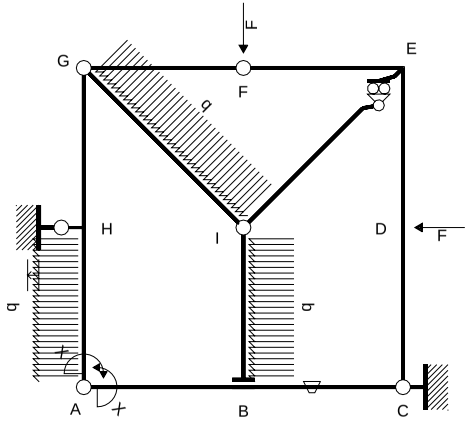
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

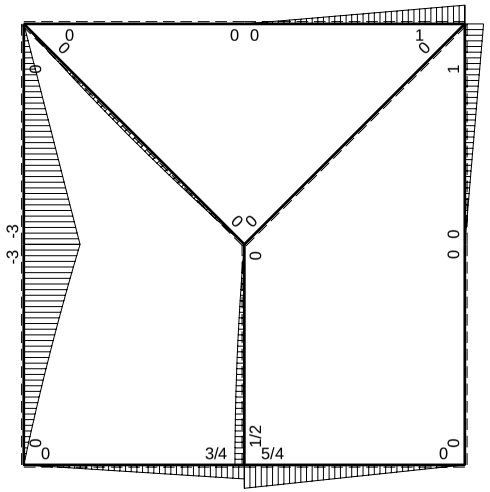
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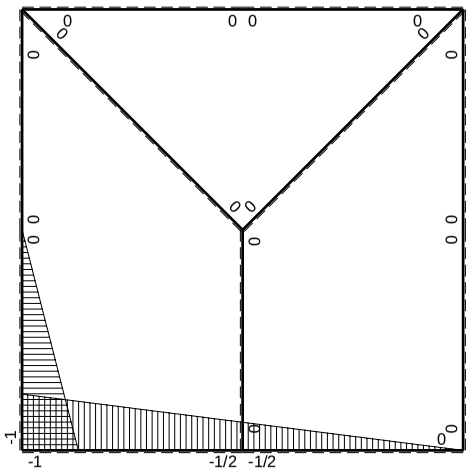

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7/8F


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-7/4
-1
0
√2/2
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1
-√2/2
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7/4
9/8
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-7/8
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7/8
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0
-3/4
-3/4
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 242960 ESΣ04.xxxx.050

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	-Fb/EJ	$-5/8Fb+5/4Fx-5/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-5/4Fx	Fb/EJ	$-5/8Fx^2/b$	$1/2Fx/EJ$	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

PROCEDIMENTO E RISULTATI 242960 ESΣ04.xxxx.050

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\right)\ \theta\ \ = 1/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\right)\ \theta\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-5/24\ b\right)\ Fb\ 1/EJ\ + \left(-1/4\ b\right)\ \theta\ \ = 1/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

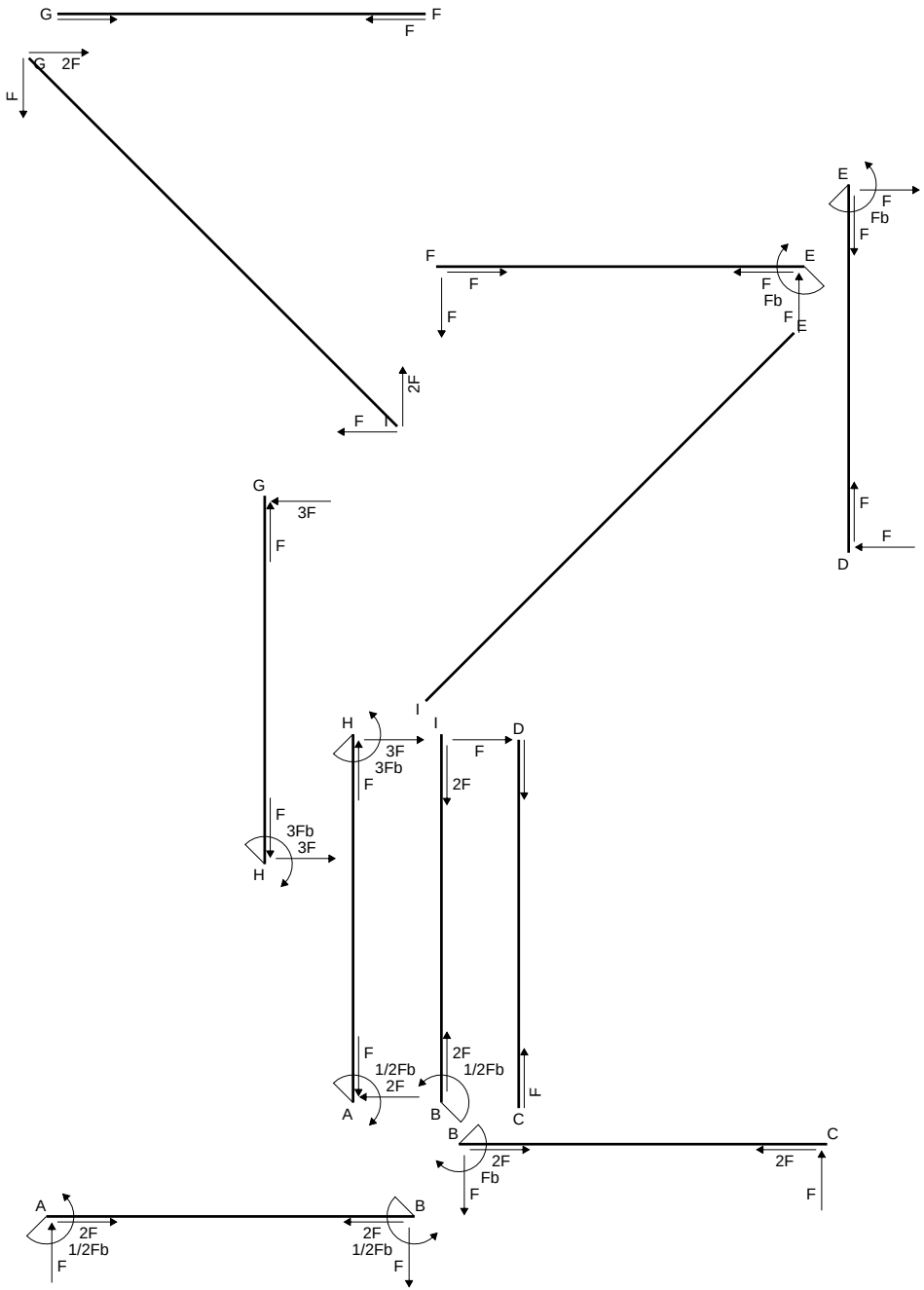
$= \left(3/2\ b - 7/6\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

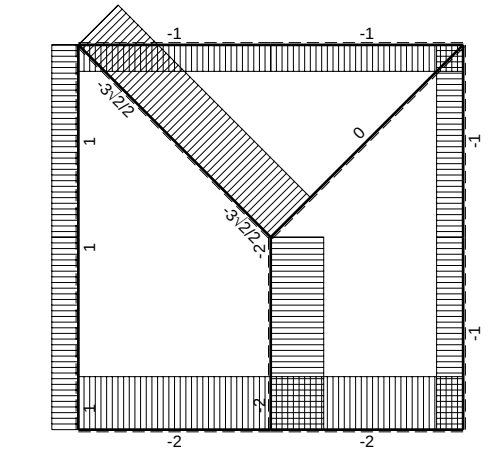
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

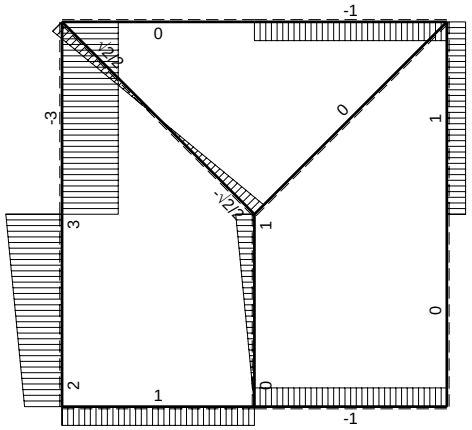
$= \left(5/4\ b - 2/3\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

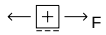
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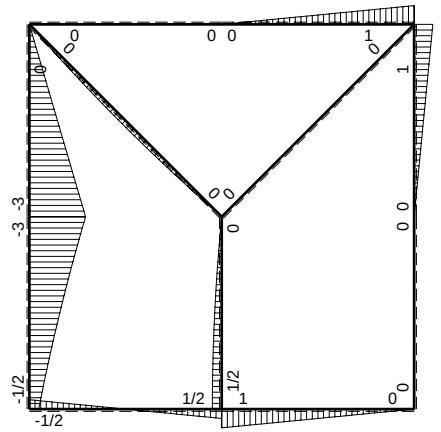

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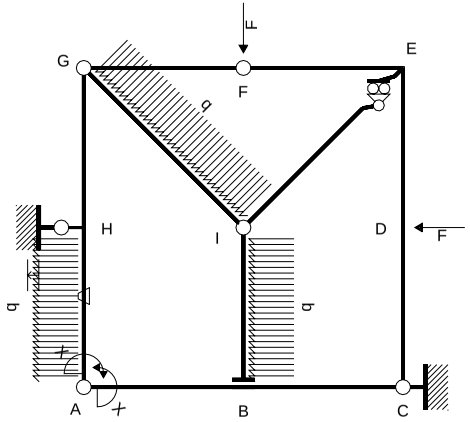

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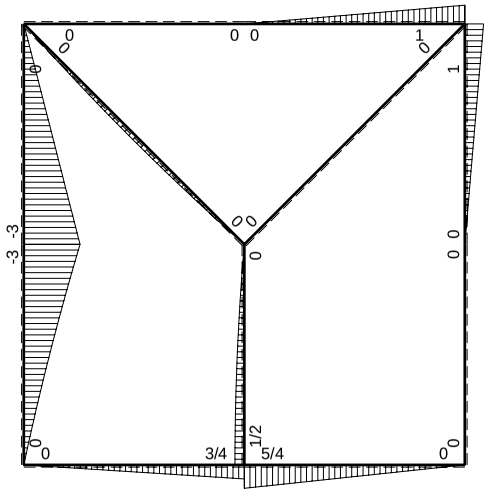

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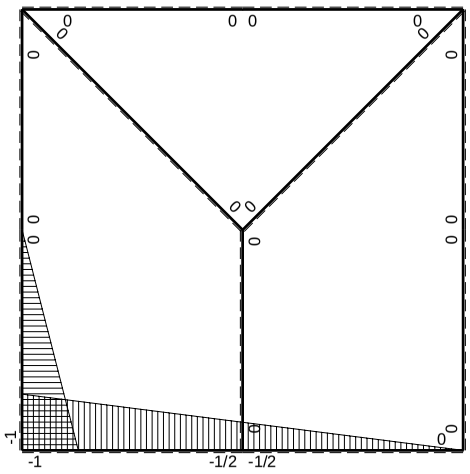

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	-Fb/EJ	$3Fx-7/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	Fb/EJ	$5/2Fx-2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/2Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right) Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right) Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\right) Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-5/24\ b\right) Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\right)\ \theta\ dx$

$= \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b \theta$

$= \left(3/2\ b - 7/6\ b + 1/8\ b\right) Fb\ 1/EJ\ + \left(1/2\ b\right)\ \theta\ \ = 23/24\ \ Fb^2/EJ$

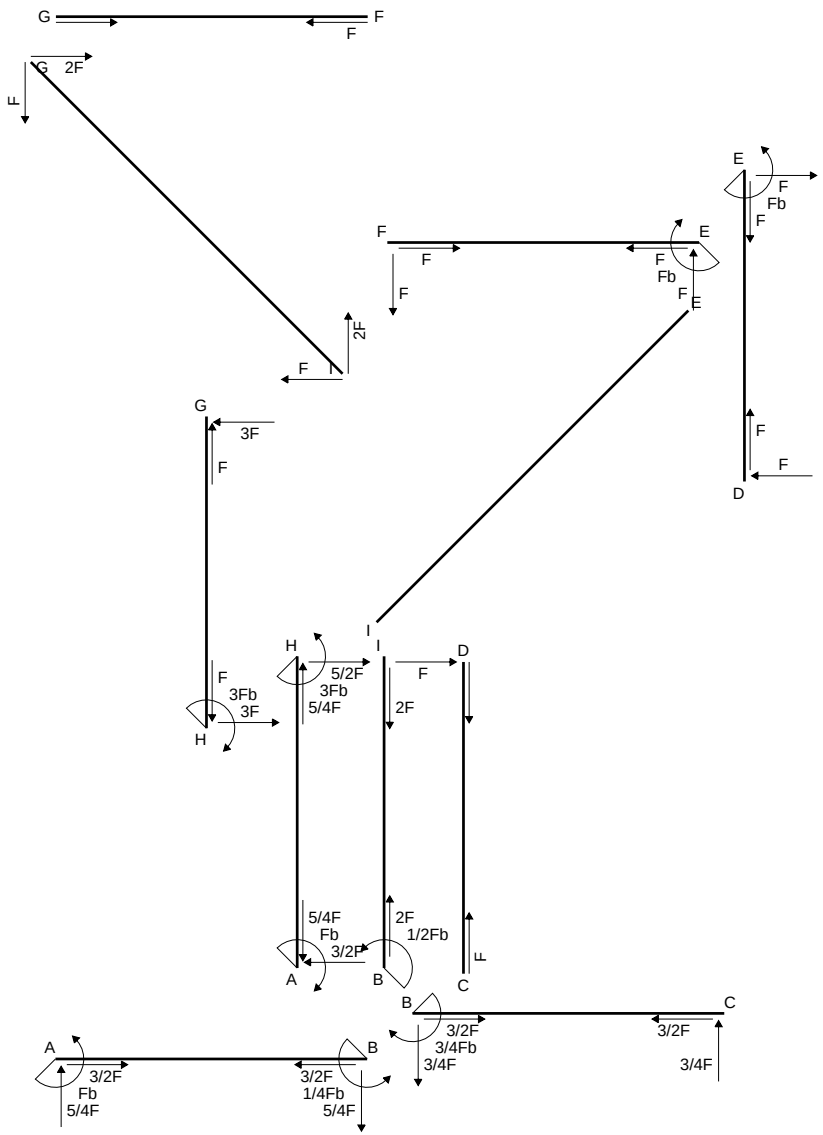
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\right)\ \theta\ dx$

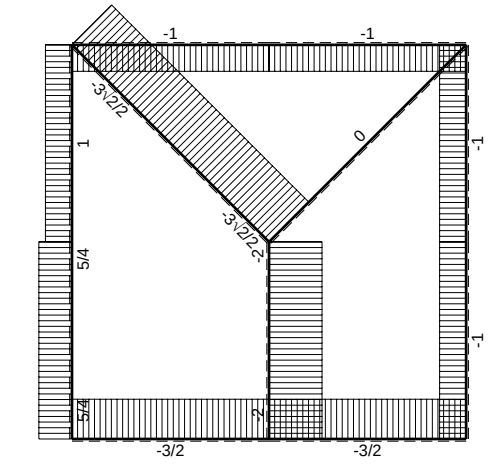
$= \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b \theta$

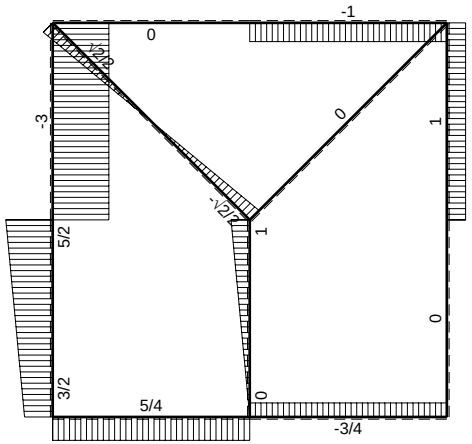
$= \left(5/4\ b - 2/3\ b - 1/8\ b\right) Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\right)\ \theta\ \ = 23/24\ \ Fb^2/EJ$

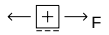
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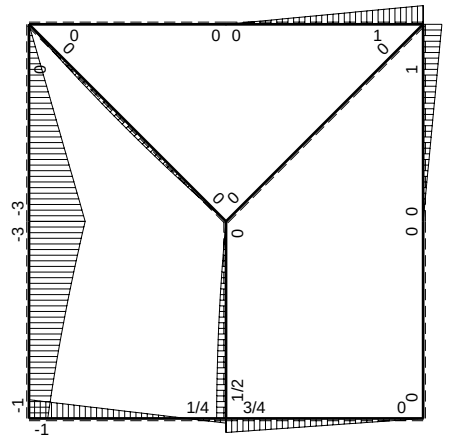

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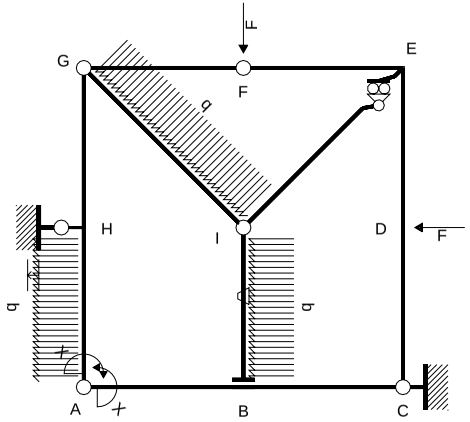

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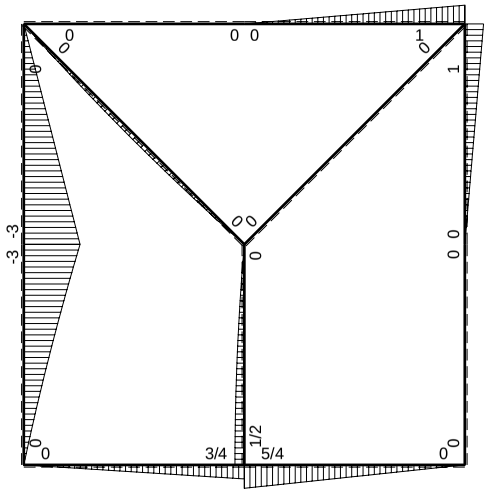

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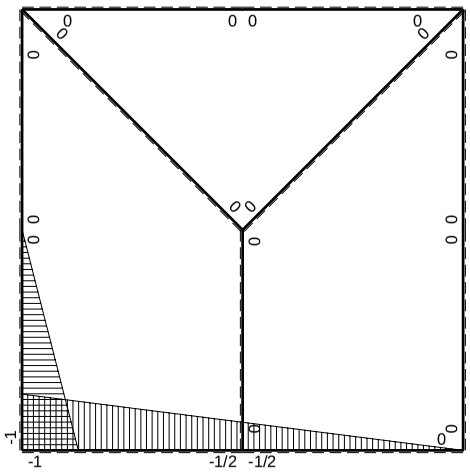

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238387 ESΣ04.xxxx.052

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

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Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

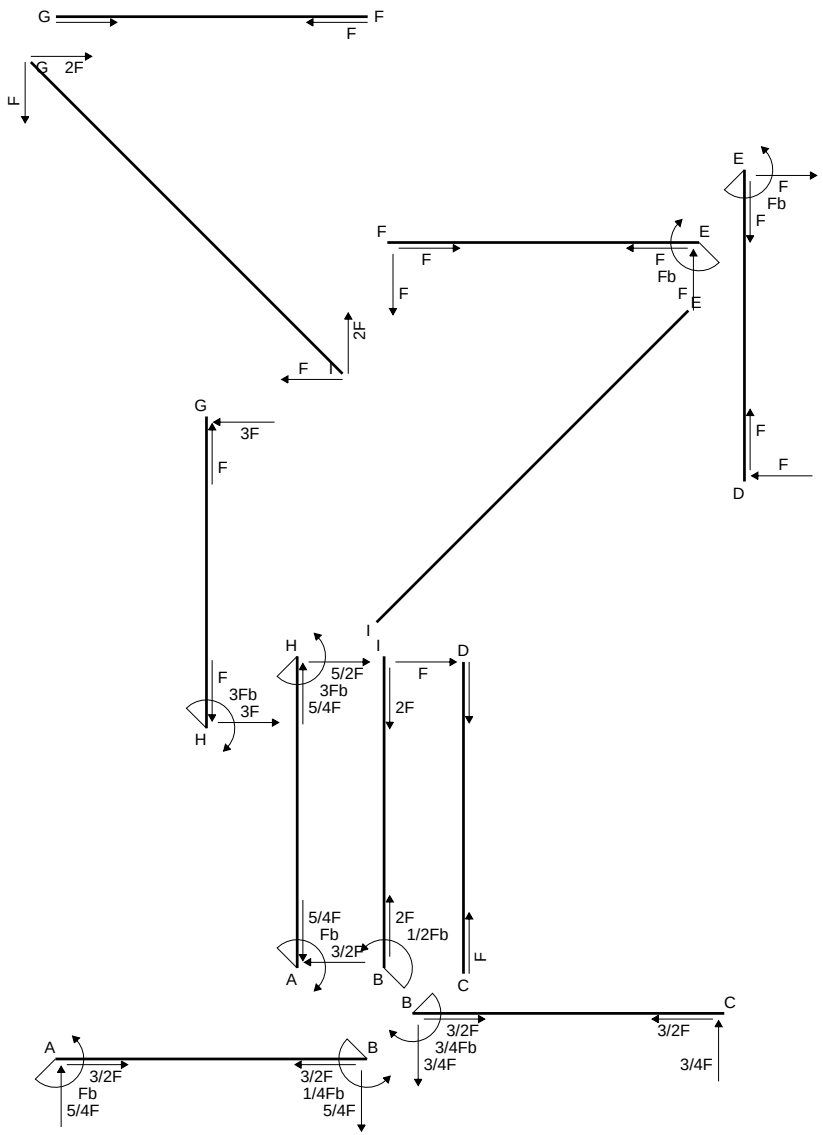
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

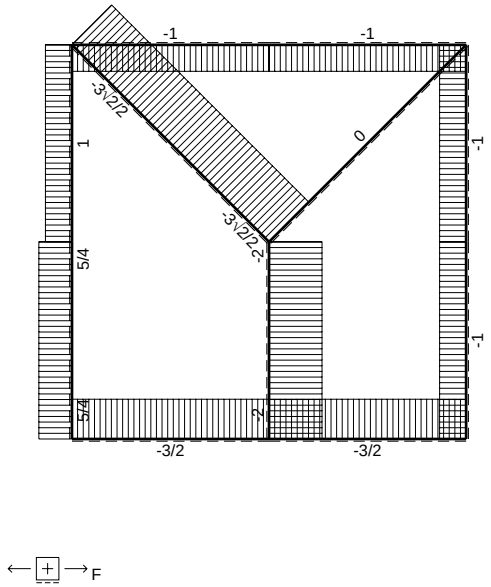
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

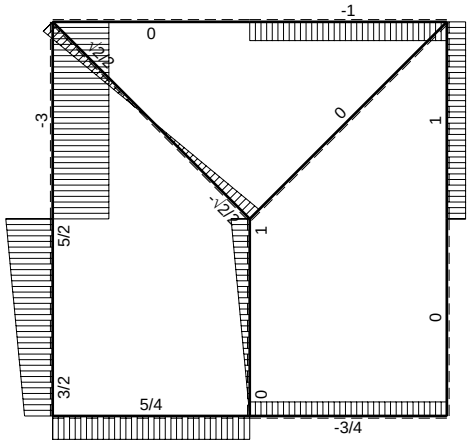
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

ESΣ04.xxxx.052

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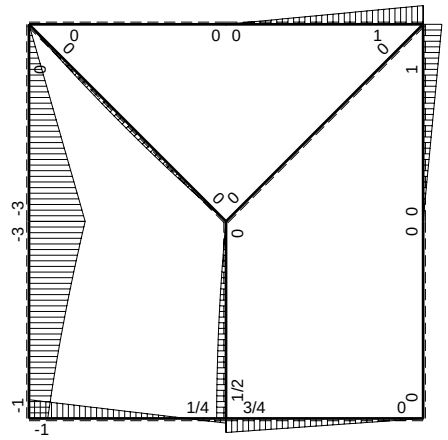

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3/2F
1/4Fb
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-3/2

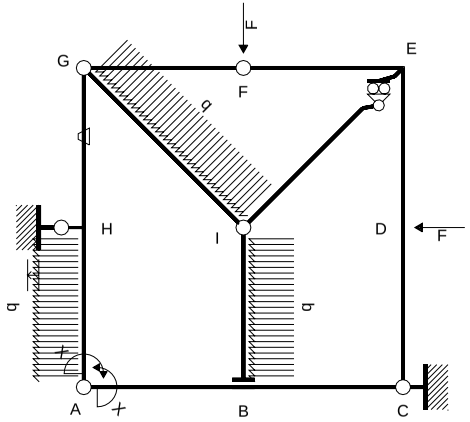

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-3/4

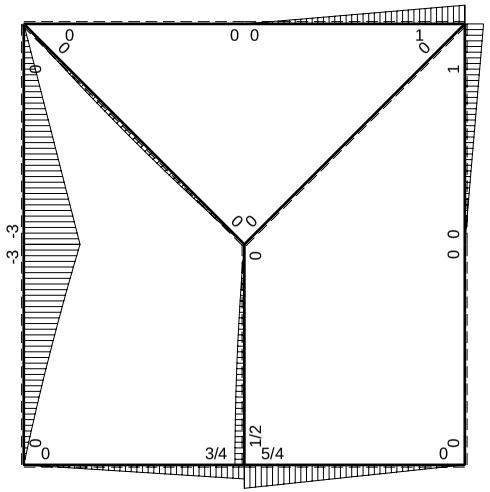
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F

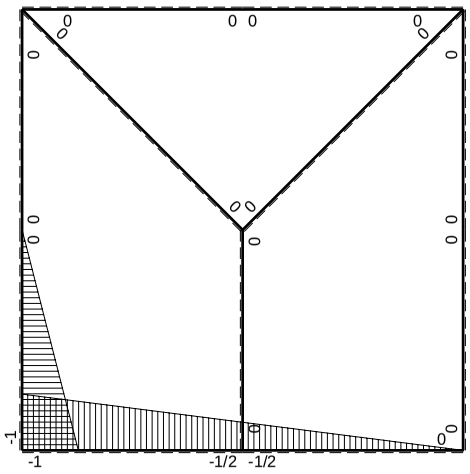

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-3 -3
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0 0
-1
1/4
1/2
3/4
0
0
-1


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238485 ESΣ04.xxxx.053

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	3Fb-3Fx	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

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Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

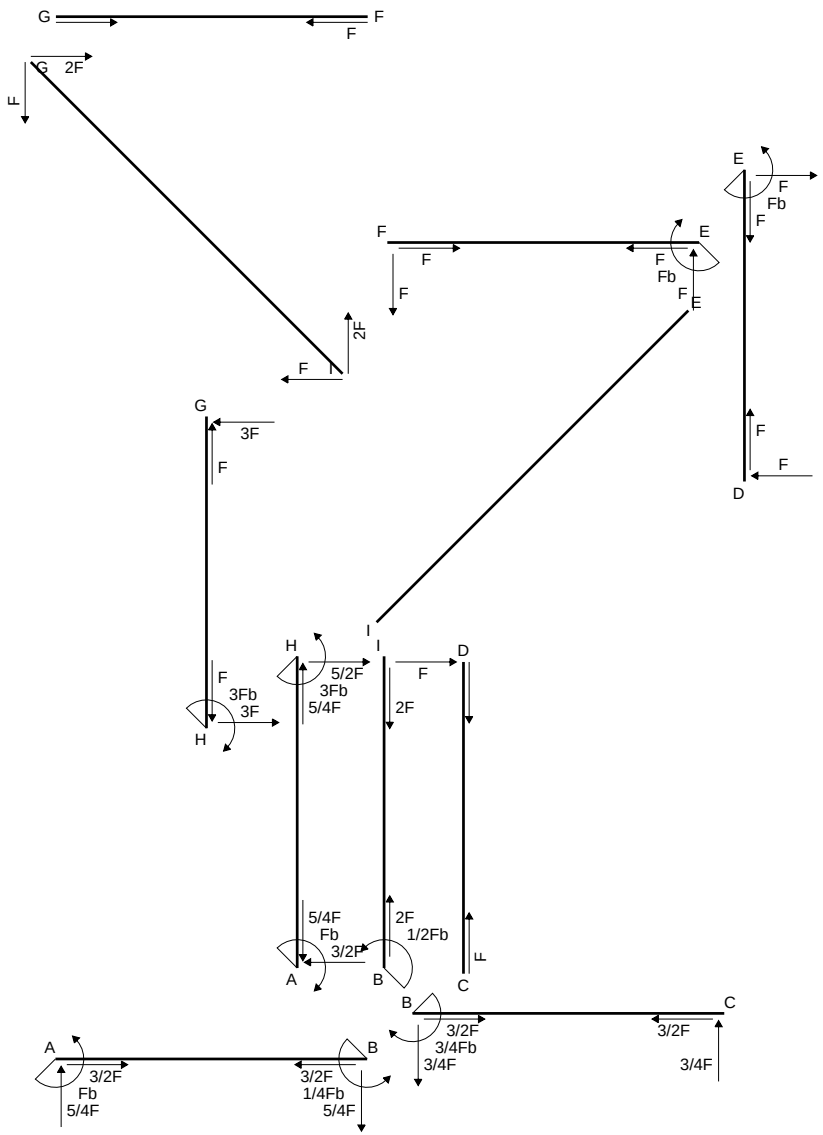
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

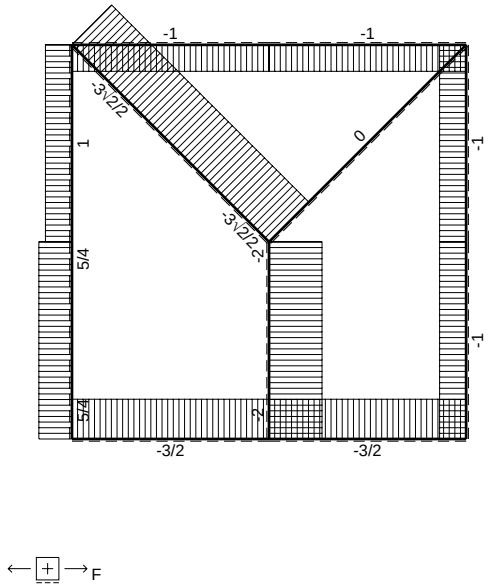
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

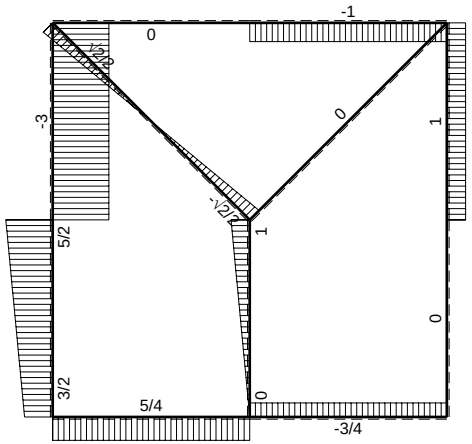
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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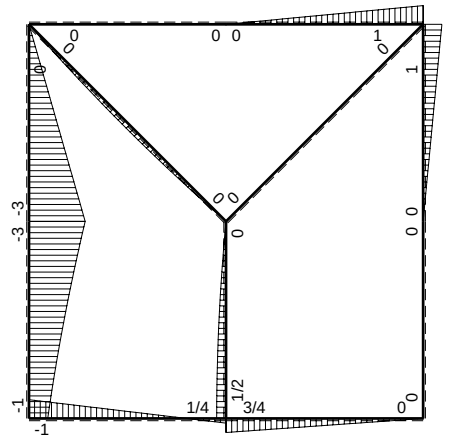

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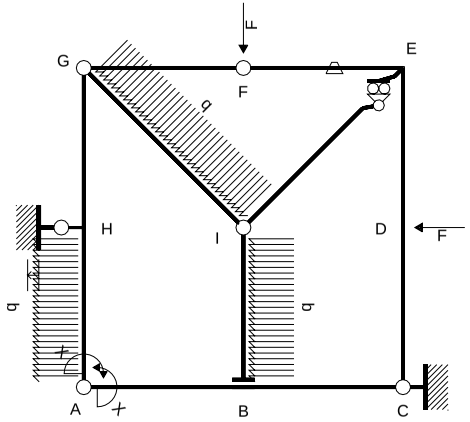

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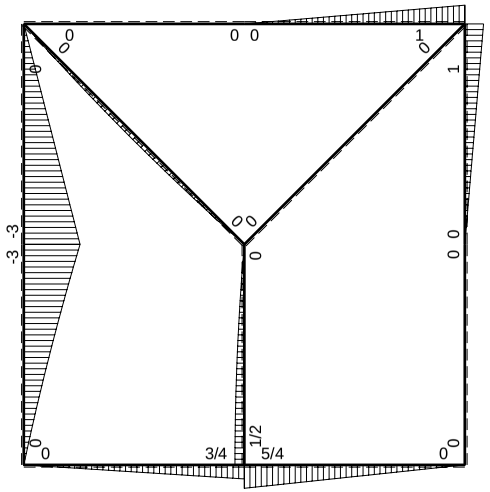
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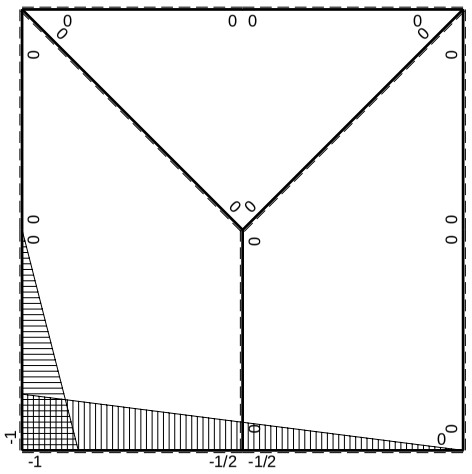

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1/4
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3/4
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0
-1


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 214395 ESΣ04.xxxx.054

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PROCEDIMENTO E RISULTATI 214395 ESΣ04.xxxx.054

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

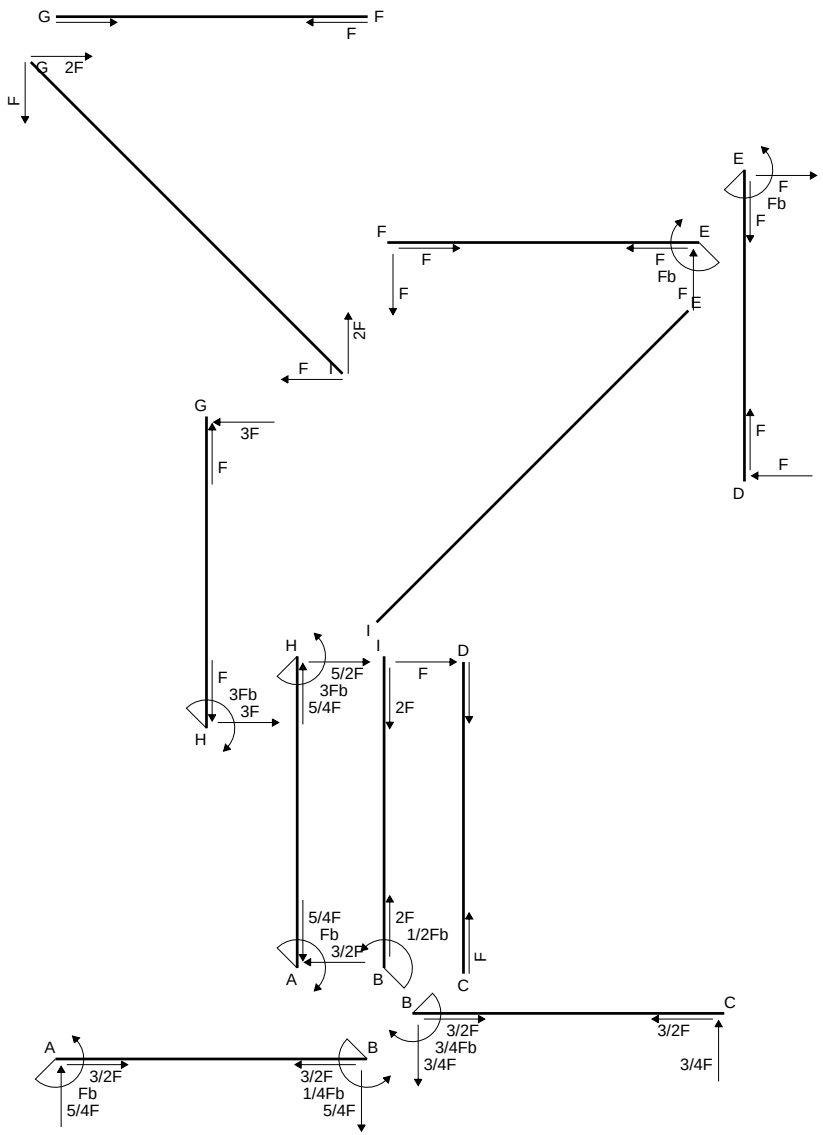
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

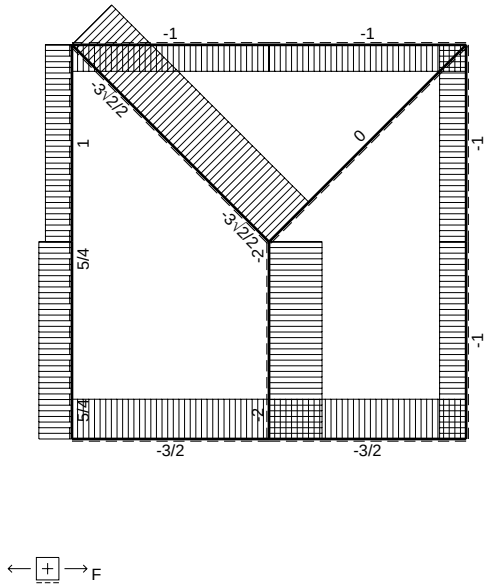
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

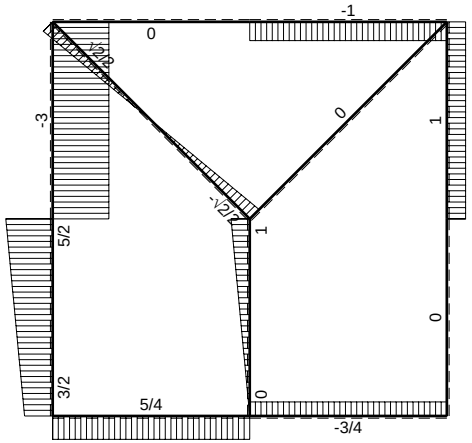
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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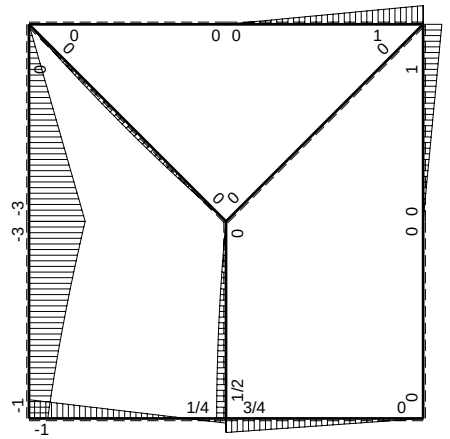

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3/4F


-1
-1
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-2
-3/2
-3/2

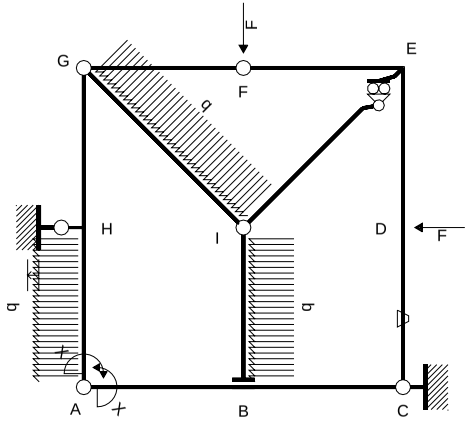

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0
-3/4

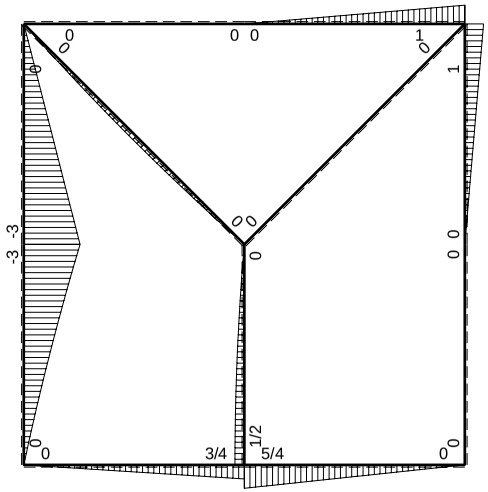
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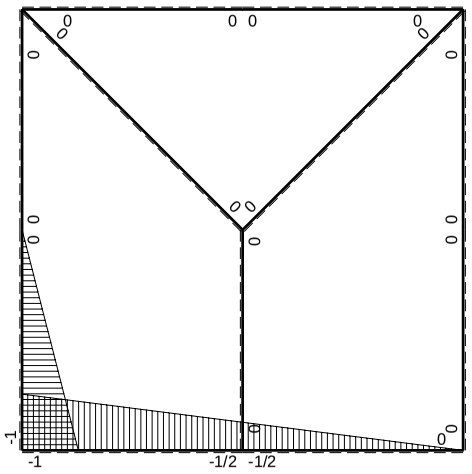

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-3 -3
0
0 0
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1/4
1/2
3/4
0
0
-1


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-5/4Fx	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	-Fb/EJ	0	0	0	0+0	0
DC b	0	0	Fb/EJ	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI √2b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	0	$3Fx-7/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(11/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	0	$5/2Fx-2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-5/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-5/24\ b\ \right)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

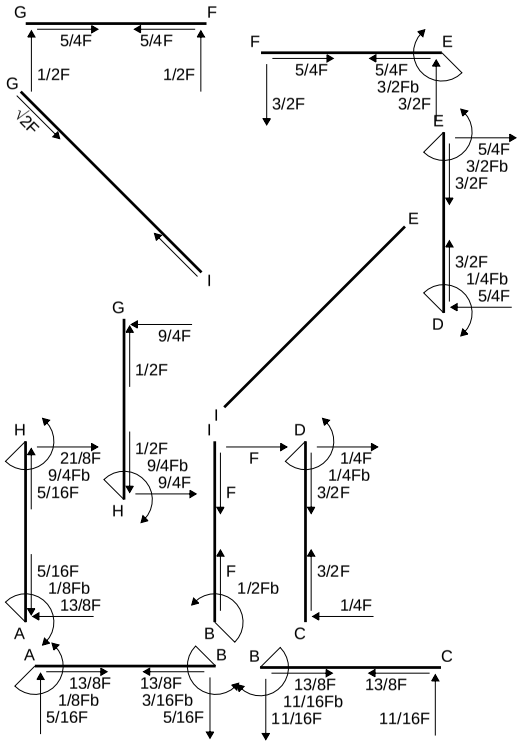
$= \left(3/2\ b - 7/6\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

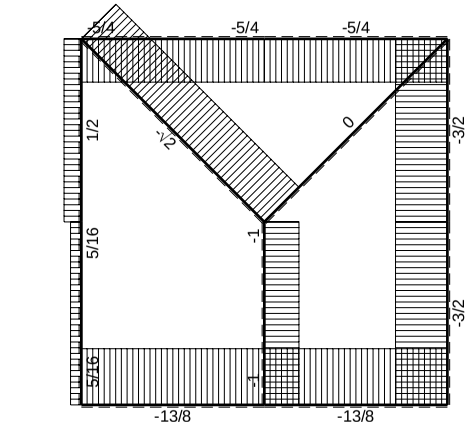
$L_{AH}^{Xo} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

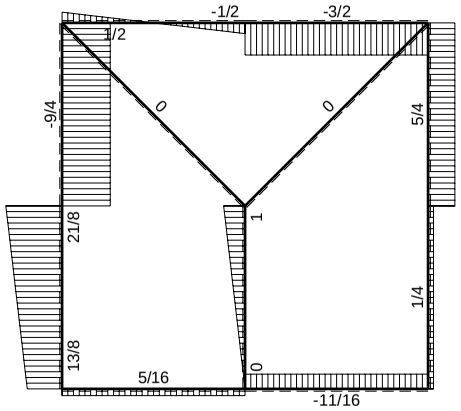
$= \left(5/4\ b - 2/3\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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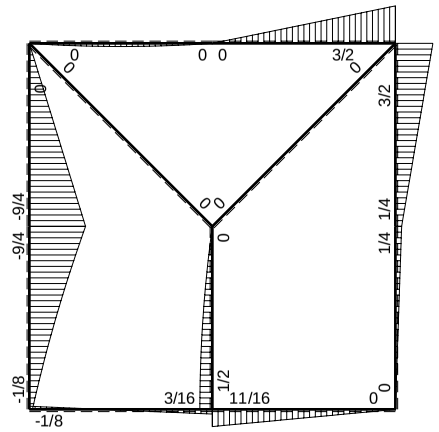

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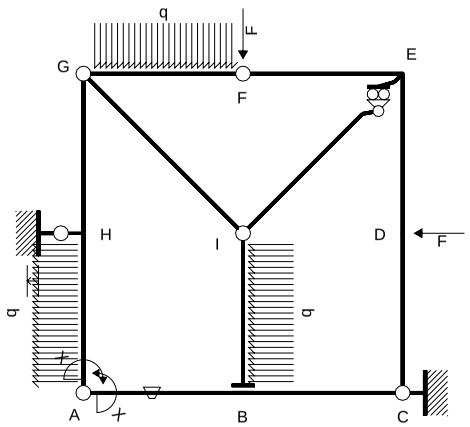

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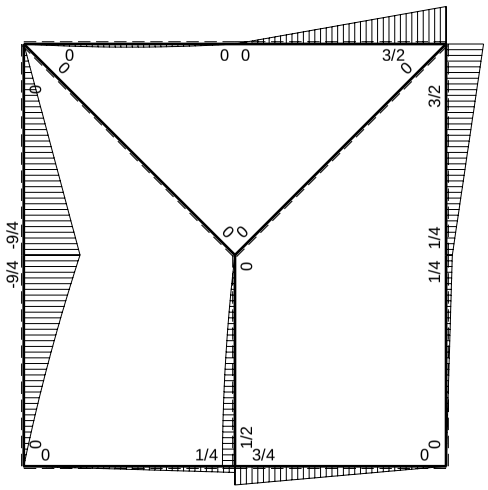
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↑□↓ F

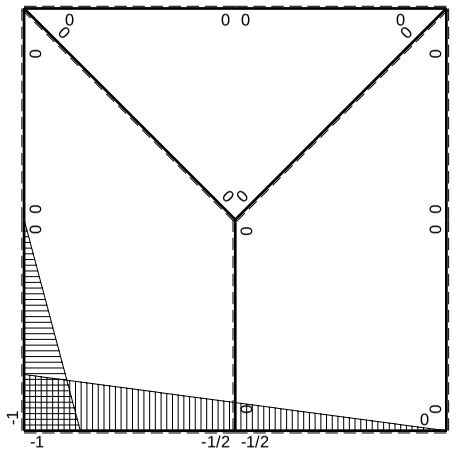

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3/16
-1/2
11/16
0
0
-1/8

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	-Fb/EJ	-1/4Fx+1/8Fx^2/b	Fb/EJ-1/2Fx/EJ	1-x/b+1/4x^2/b^2	(-1/12+3/4)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	Fb/EJ	-1/8Fb+1/8Fx^2/b	1/2Fb/EJ+1/2Fx/EJ	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	-3/8Fb+3/4Fx-3/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-1/8+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	-3/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	-1/2Fx+1/2qx^2	0	0	0	0	0+0	0
GF b	0	1/2Fx-1/2qx^2	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
BI b	0	-1/2Fb+1/2qx^2	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	-9/4Fb+11/4Fx-1/2qx^2	0	9/4Fx-11/4Fx^2/b+1/2qx^3/b	0	x^2/b^2	(1/3+0)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	7/4Fx+1/2qx^2	0	7/4Fx-5/4Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-1/8Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						1/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

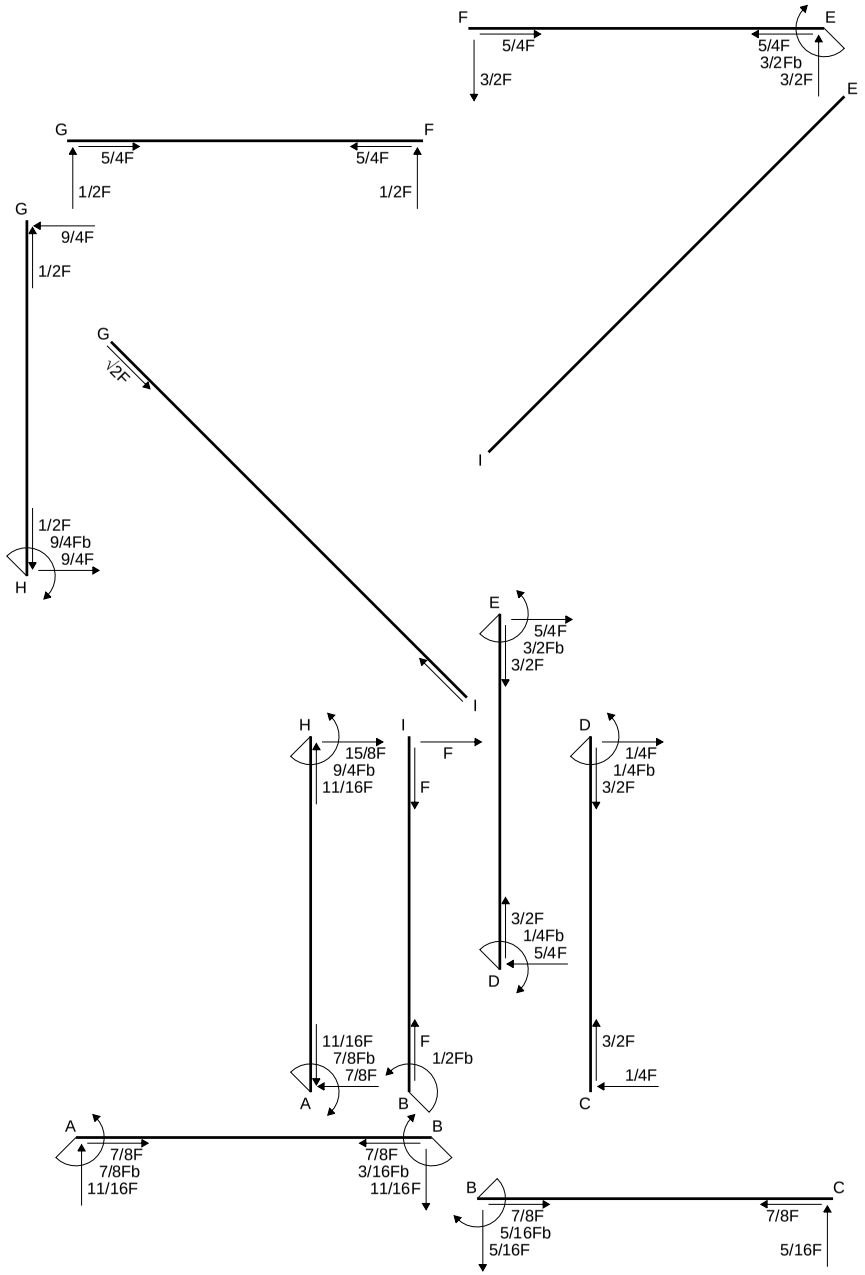
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

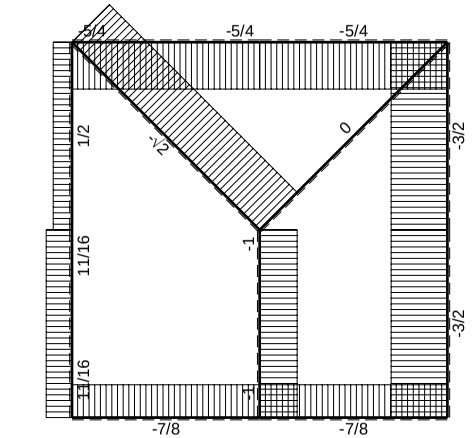
$L^{Xo}_{AH} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

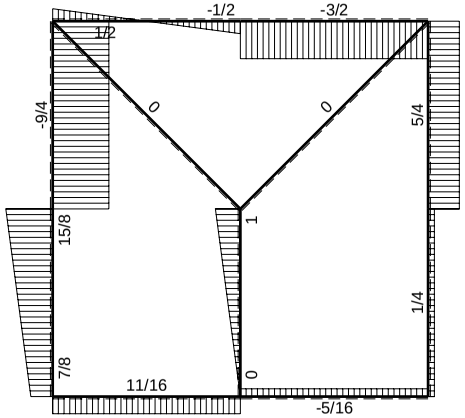
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

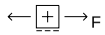
PROCEDIMENTO E RISULTATI 238123 ESΣ04.xxxx.056

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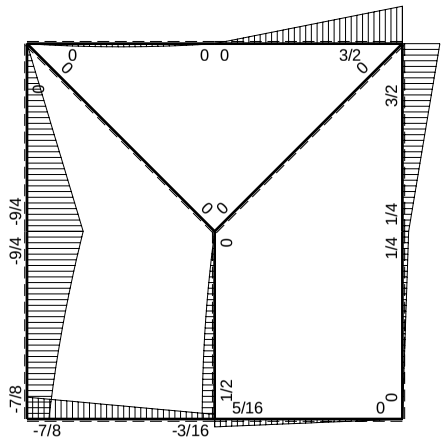

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5/4F
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3/2F
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E
G
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1/2F
G
9/4F
1/2F
G
½F
I
1/2F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
3/2F
I
H
I
D
15/8F
9/4Fb
11/16F
F
F
1/4F
1/4Fb
3/2F
3/2F
1/4Fb
5/4F
D
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C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
-5/4
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-7/8
-7/8

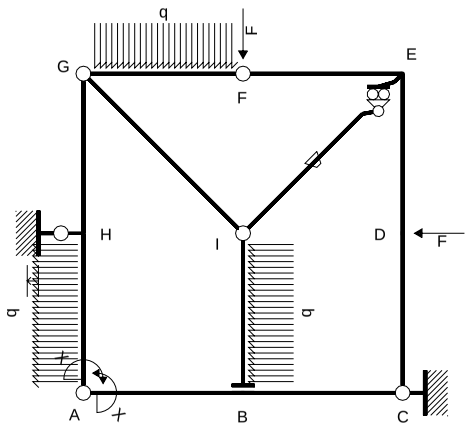

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5/4
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1
1/4
7/8
11/16
0
-5/16

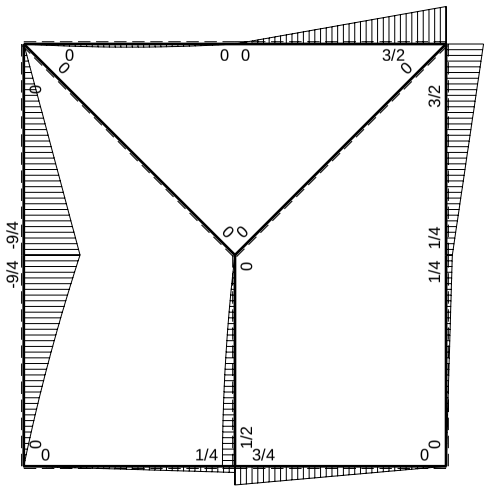

F


F

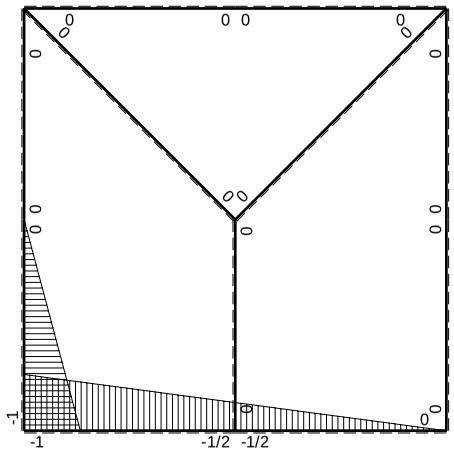

0
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3/2
0
0
3/2
0
0
-9/4
-9/4
0
1/4
1/4
0
-7/8
1/2
5/16
0
0
-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

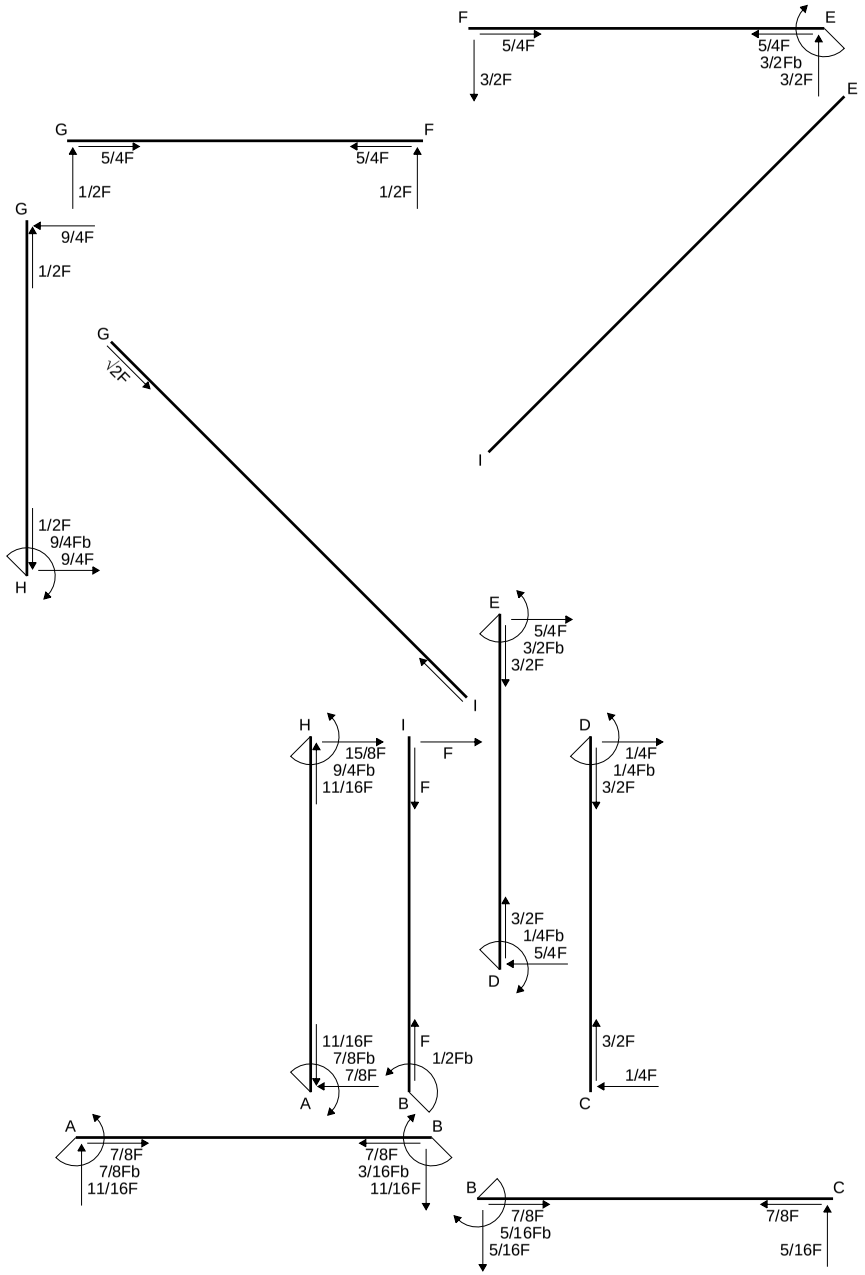
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

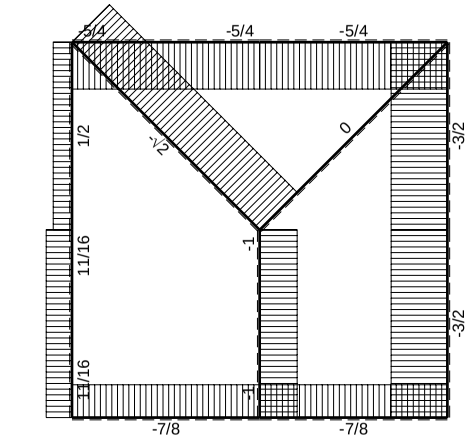
$L^{Xo}_{AH} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

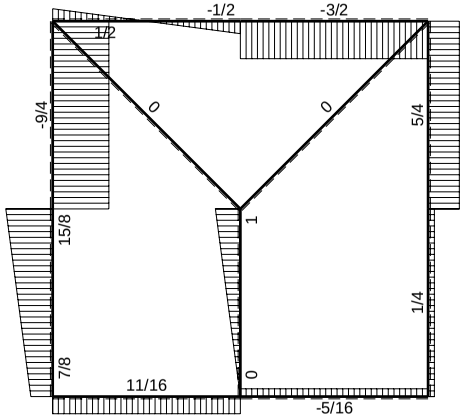
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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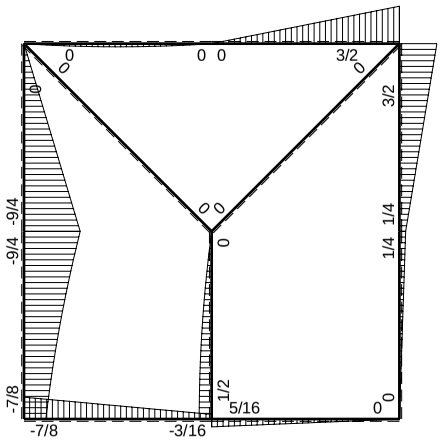

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G
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G
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G
½F
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1/2F
9/4Fb
9/4F
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H
I
D
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D
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B
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C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
-5/4
-5/4
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-7/8

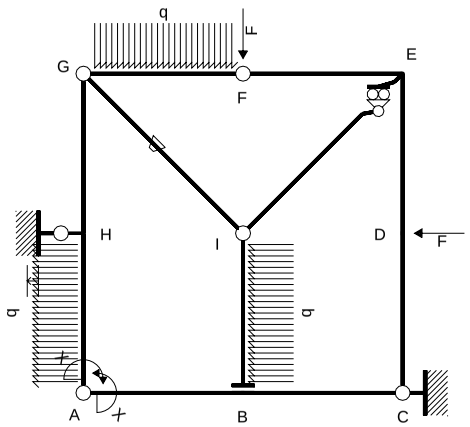

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-5/16

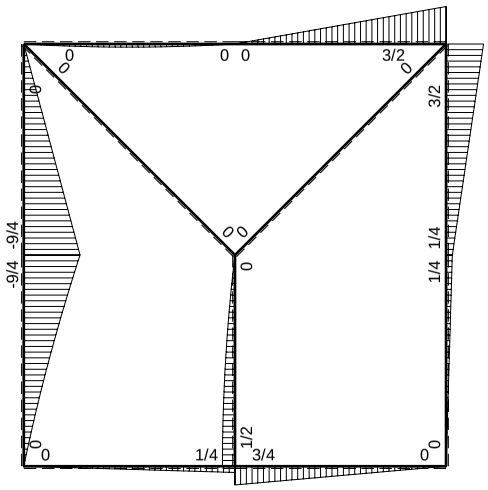
←□→ F

↑□↓ F

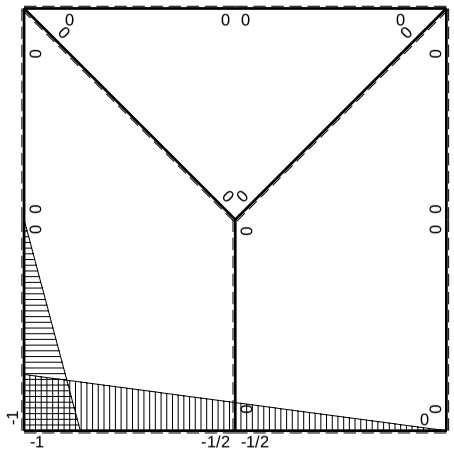

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1/4
0
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1/2
5/16
0
0
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240378 ESΣ04.xxxx.058

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PROCEDIMENTO E RISULTATI 240378 ESΣ04.xxxx.058

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

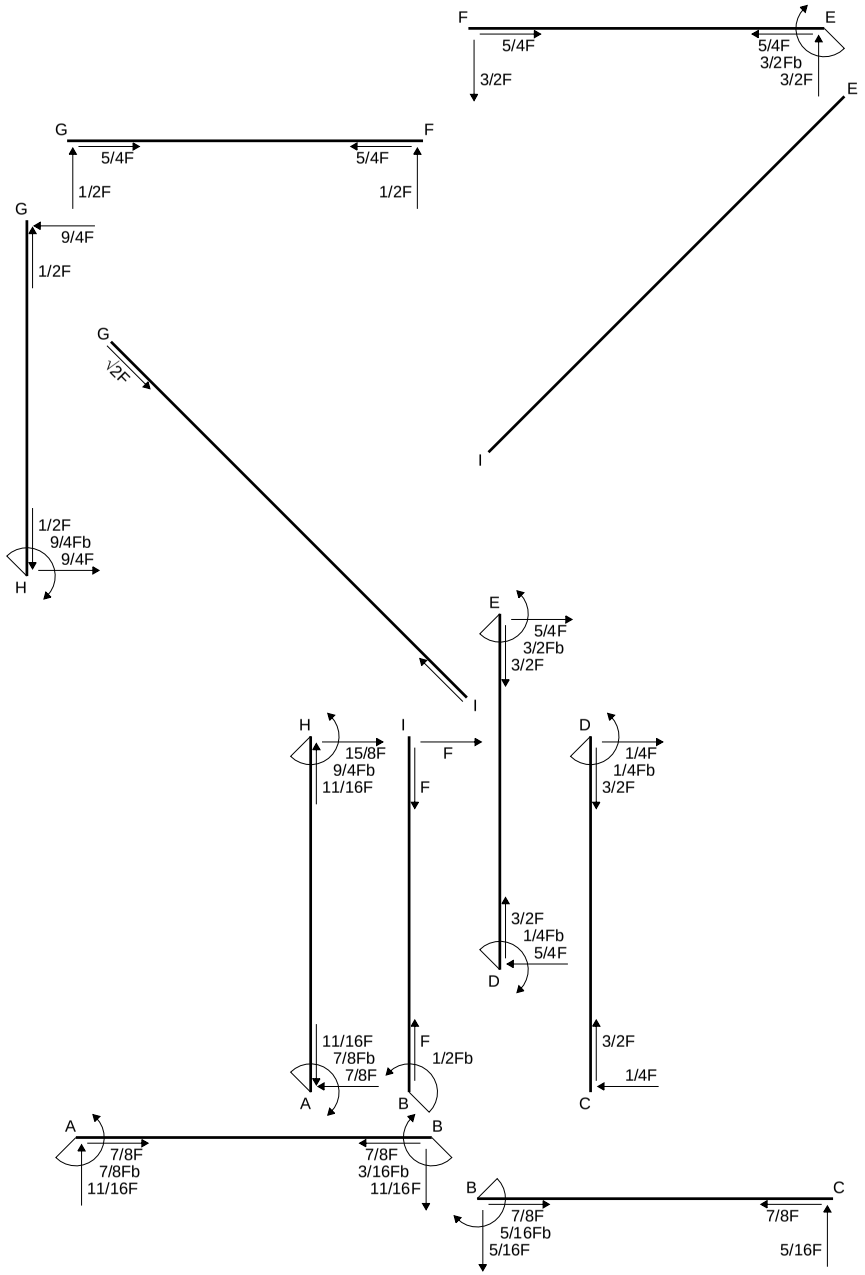
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

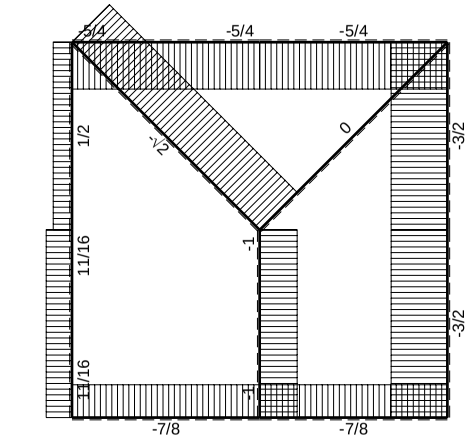
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

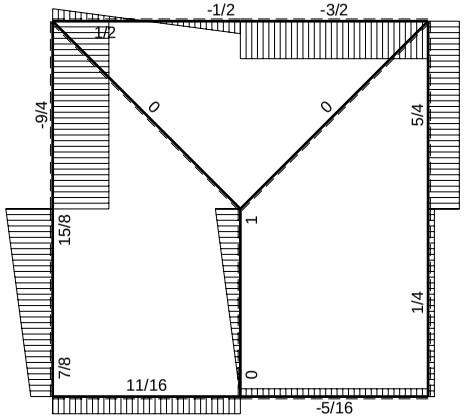
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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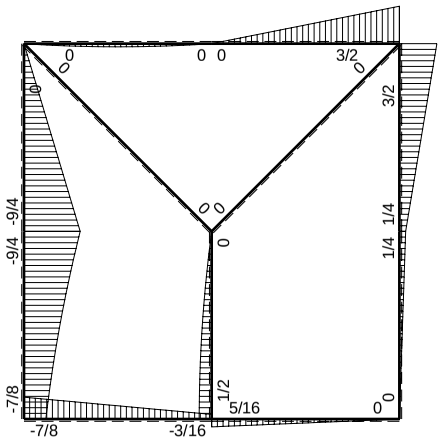

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G
9/4F
1/2F
G
½F
I
1/2F
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9/4F
H
E
5/4F
3/2Fb
3/2F
I
H
I
D
15/8F
9/4Fb
11/16F
F
F
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F
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B
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3/16Fb
11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
-5/4
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-7/8

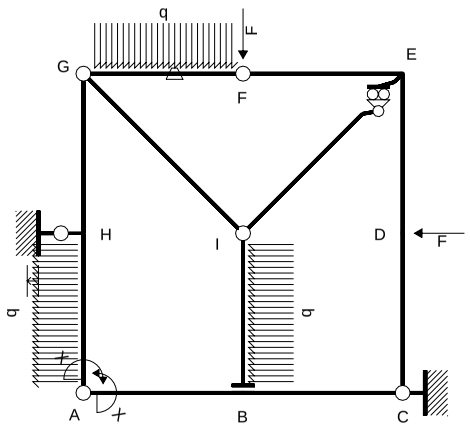

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5/4
15/8
1
1/4
7/8
11/16
0
-5/16

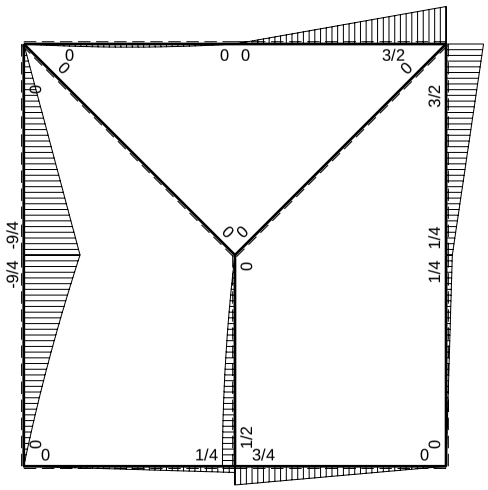
←□→ F

↑□↓ F

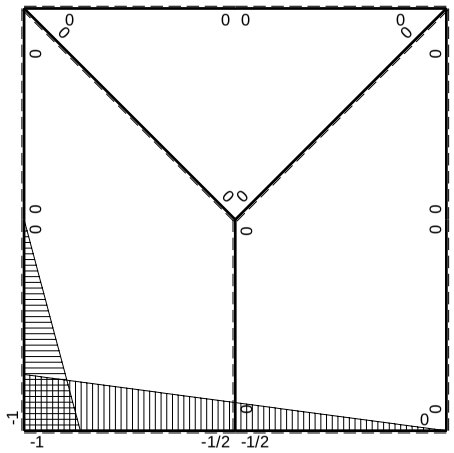

0
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0
0
3/2
-9/4
0 0
-9/4
1/4
1/4
0
-7/8
1/2
5/16
0
0
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

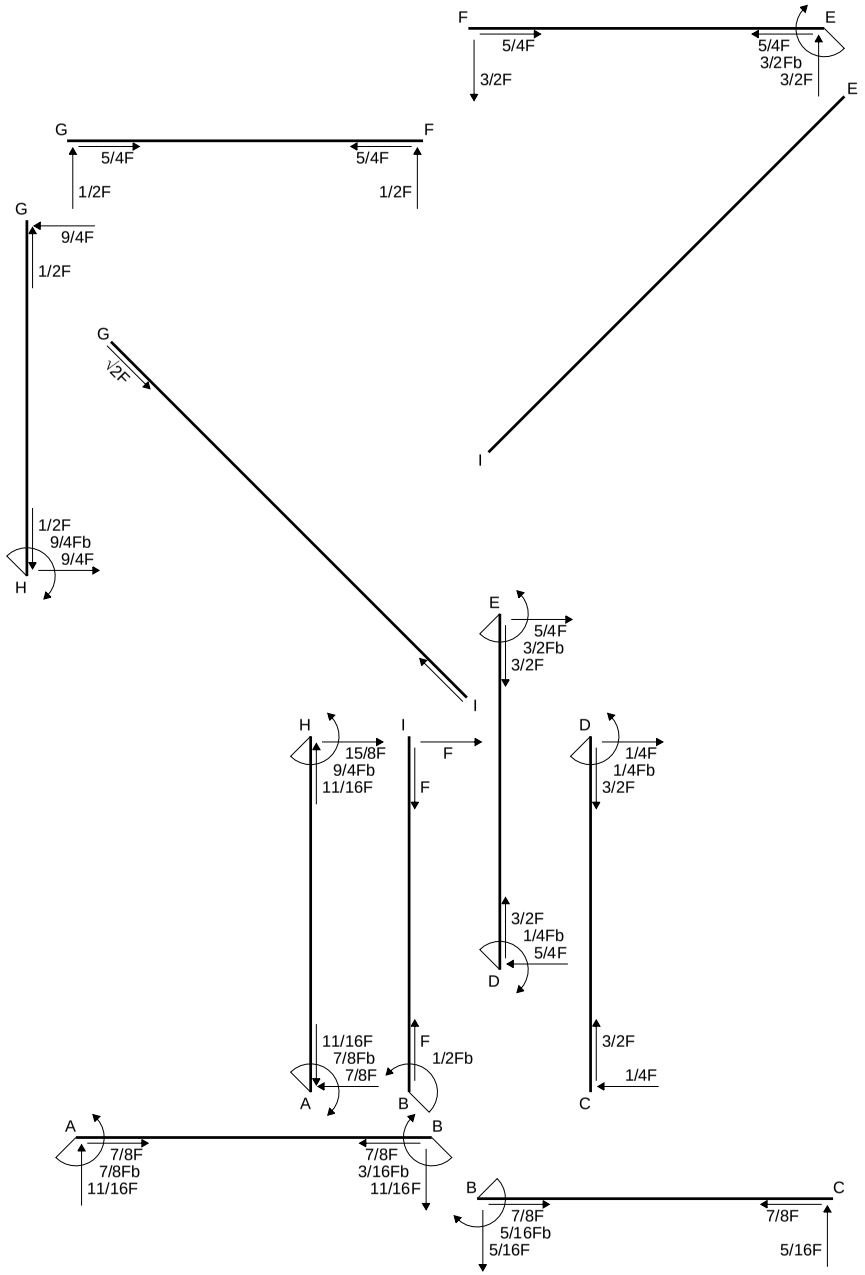
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

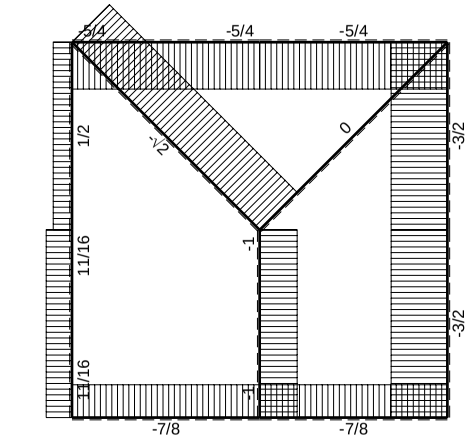
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

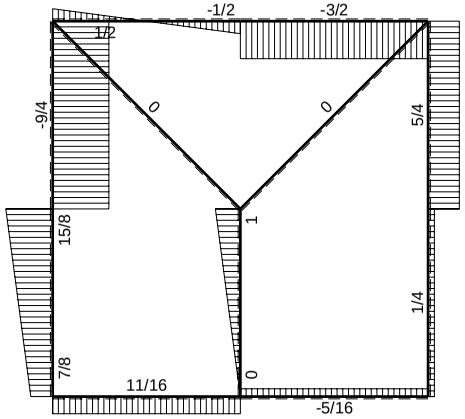
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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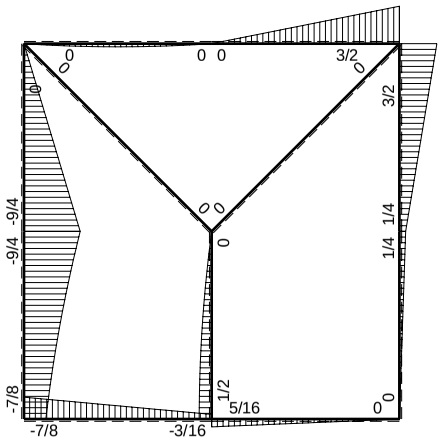

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G
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½F
I
1/2F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
3/2F
I
H
I
D
15/8F
9/4Fb
11/16F
F
F
1/4F
1/4Fb
3/2F
3/2F
1/4Fb
5/4F
D
11/16F
7/8Fb
7/8F
F
1/2Fb
3/2F
1/4F
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B
C
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B
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7/8F
5/16Fb
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7/8F
5/16F


-5/4
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-7/8

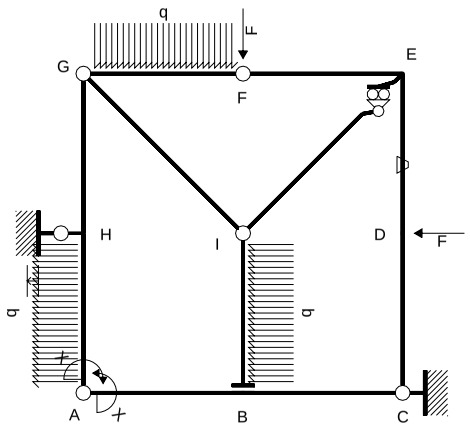

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-3/2
1/2
-9/4
0
0
5/4
15/8
1
1/4
7/8
11/16
0
-5/16

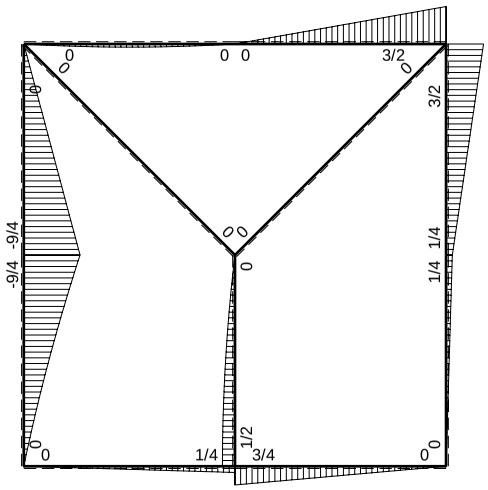
←□→ F

↑□↓ F

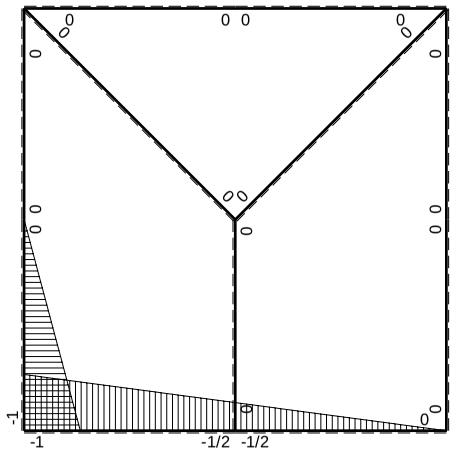

0
0 0
3/2
0
0
3/2
-9/4
0 0
-9/4
1/4
1/4
0
-7/8
1/2
5/16
0
0
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237067 ESΣ04.xxxx.060

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

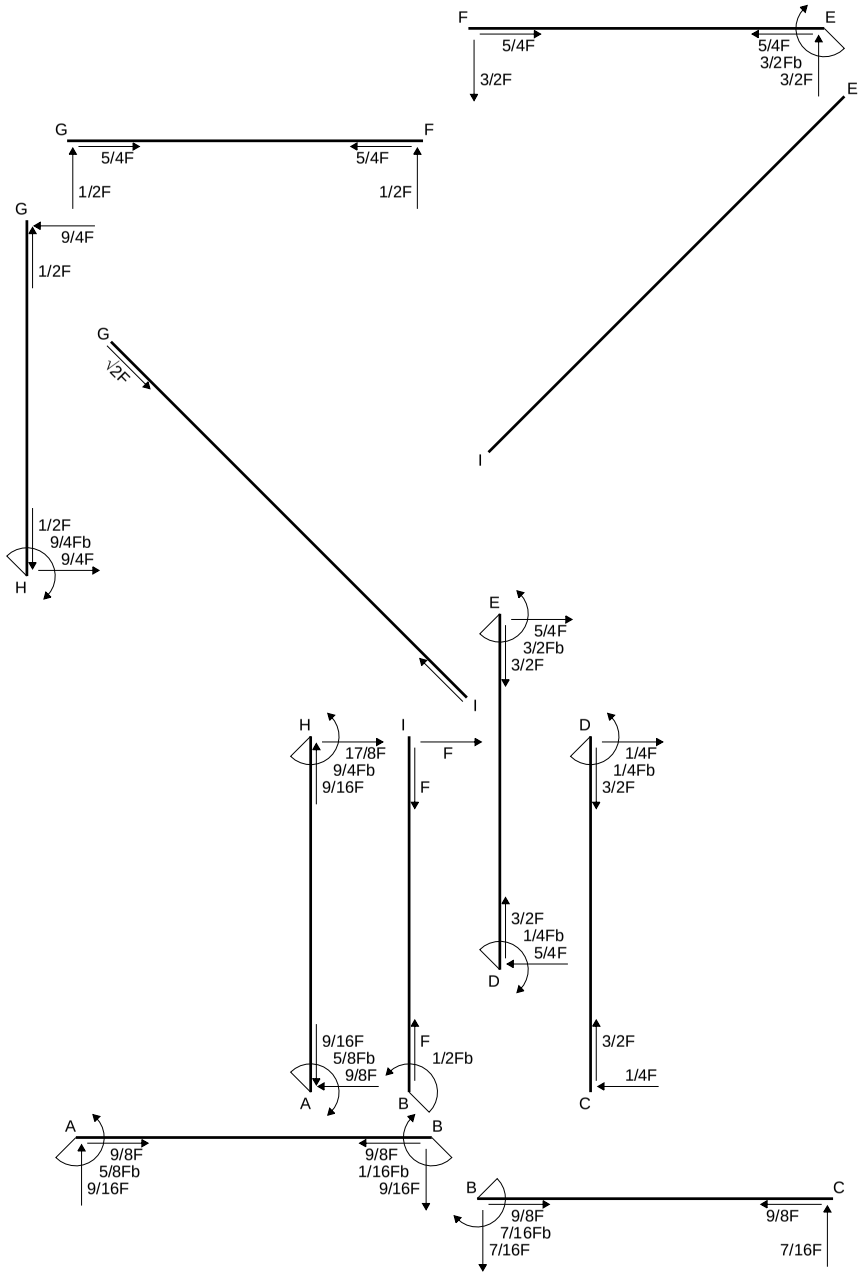
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

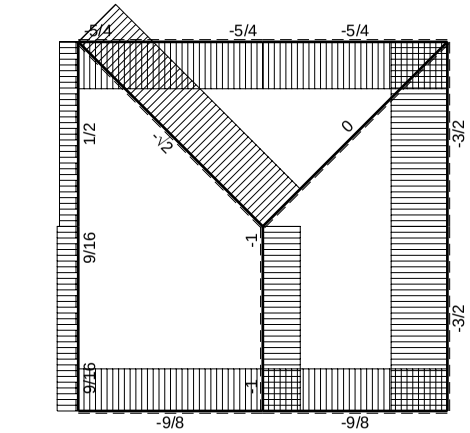
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

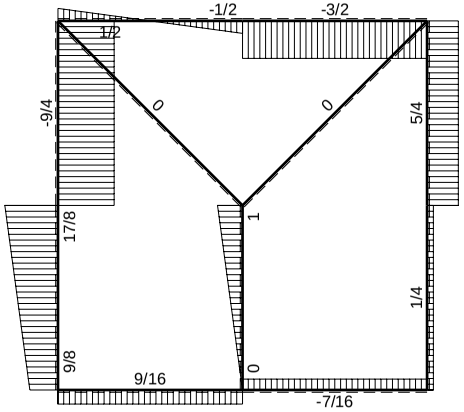
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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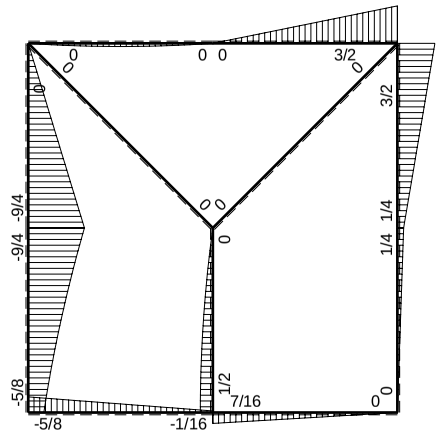

F
E
5/4F
5/4F
3/2Fb
3/2F
3/2F
E
G
F
5/4F
5/4F
1/2F
1/2F
G
9/4F
1/2F
G
½F
I
1/2F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
3/2F
I
H
I
D
17/8F
9/4Fb
9/16F
F
F
1/4F
1/4Fb
3/2F
3/2F
1/4Fb
5/4F
D
9/16F
5/8Fb
9/8F
F
1/2Fb
3/2F
1/4F
A
B
C
A
B
9/8F
5/8Fb
9/16F
9/8F
1/16Fb
9/16F
B
C
9/8F
7/16Fb
7/16F
9/8F
7/16F


-5/4
-5/4
-5/4
1/2
-½
0
-3/2
9/16
-1
-3/2
9/16
-1
-9/8
-9/8

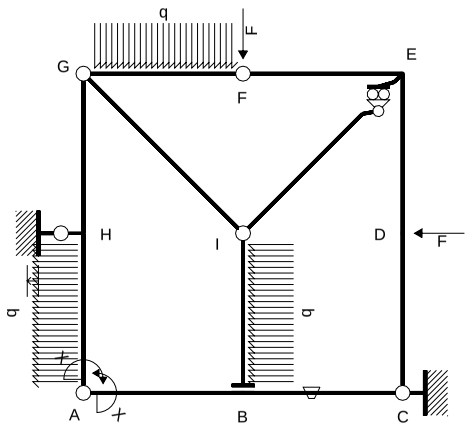

-1/2
-3/2
½
-9/4
0
0
5/4
17/8
1
9/8
9/16
0
1/4
-7/16

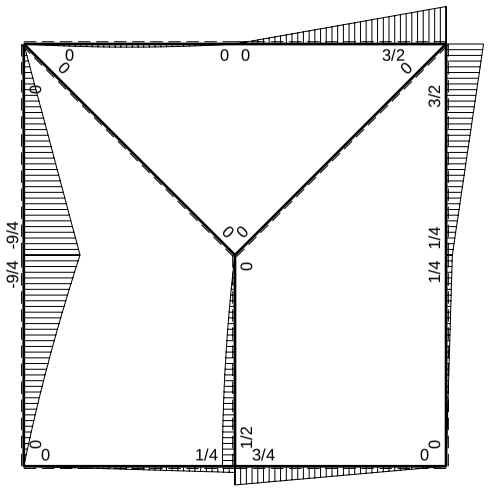
←□→ F

↑□↓ F

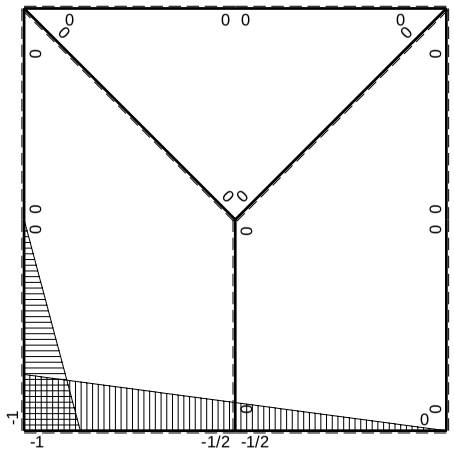

0
0 0
3/2
0
0
3/2
-9/4
0 0
-9/4
1/4
1/4
0
-5/8
1/2
7/16
0
0
-5/8
-1/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240095 ESΣ04.xxxx.061

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	-Fb/EJ	$-3/8Fb+3/4Fx-3/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	Fb/EJ	$-3/8Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/8Fb	

PROCEDIMENTO E RISULTATI 240095 ESΣ04.xxxx.061

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\ \right)\ \theta\ \ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

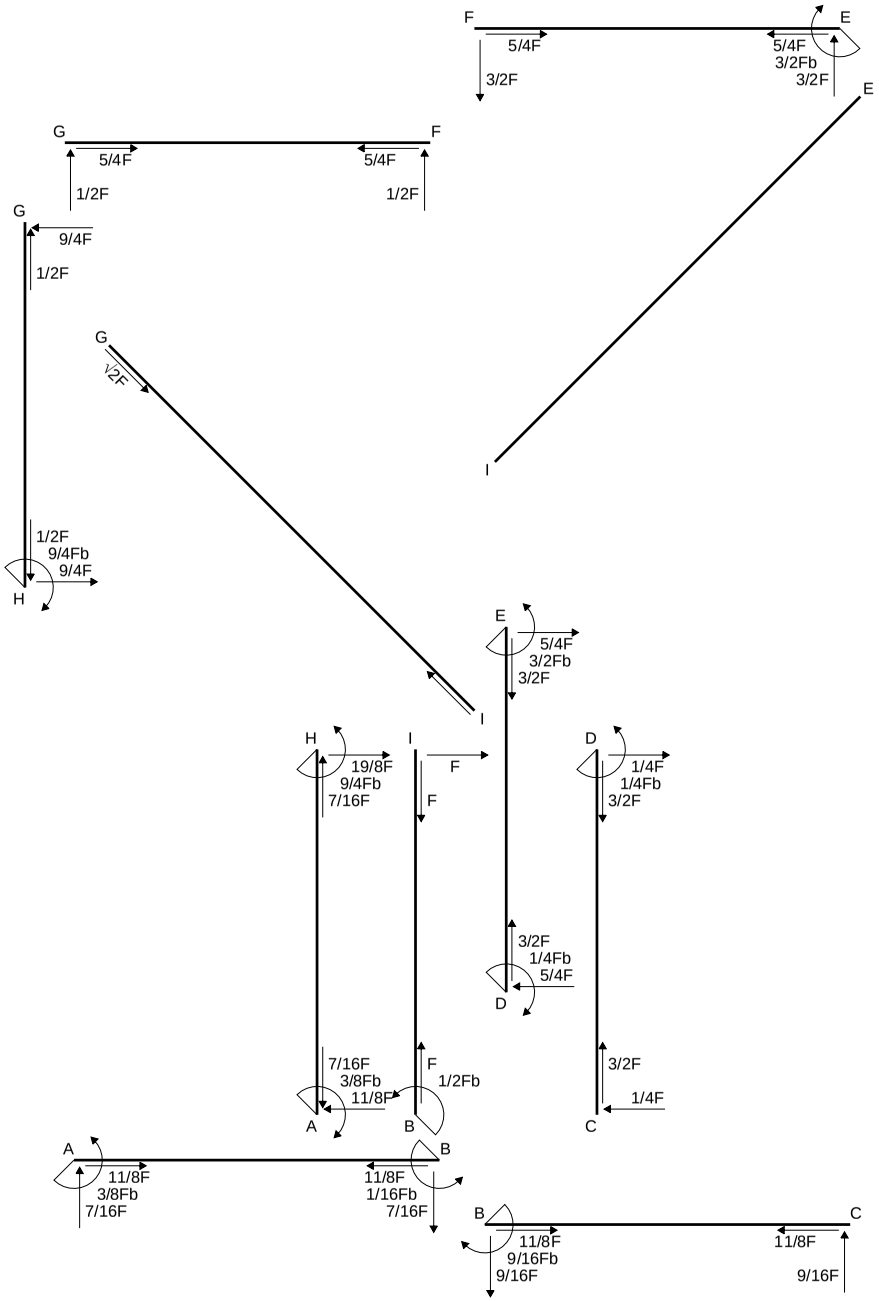
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

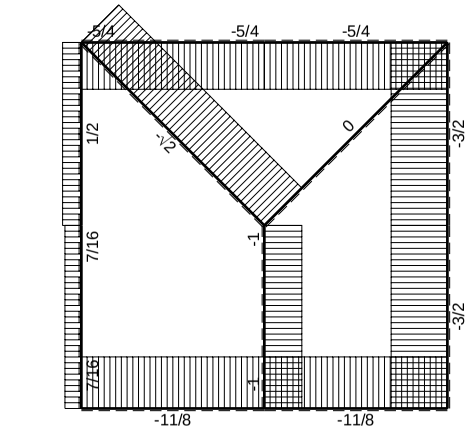
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

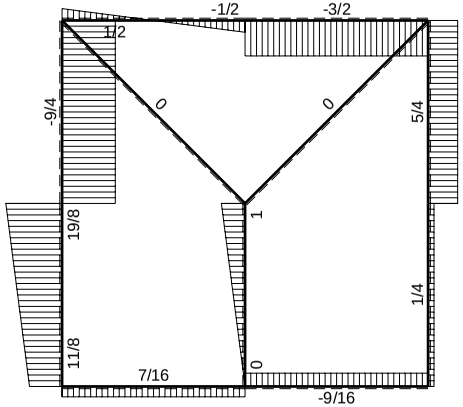
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

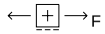
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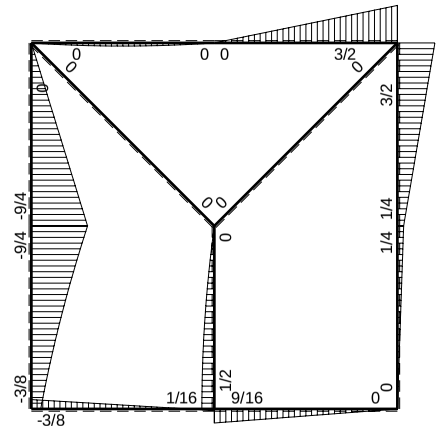

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5/4F
3/2Fb
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3/2F
G
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F
5/4F
1/2F
1/2F
G
9/4F
1/2F
G
√2F
I
E
1/2F
9/4Fb
9/4F
H
I
E
5/4F
3/2Fb
3/2F
H
19/8F
9/4Fb
7/16F
I
F
F
D
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1/4Fb
3/2F
3/2F
1/4Fb
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C
A
11/8F
3/8Fb
7/16F
B
11/8F
1/16Fb
7/16F
B
11/8F
9/16Fb
9/16F
C
11/8F
9/16F


5/4
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-11/8

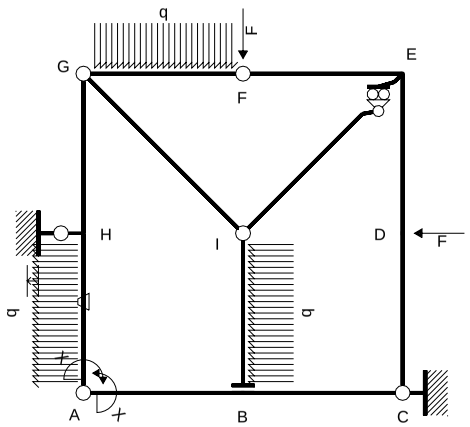

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1/4
11/8
7/16
0
-9/16

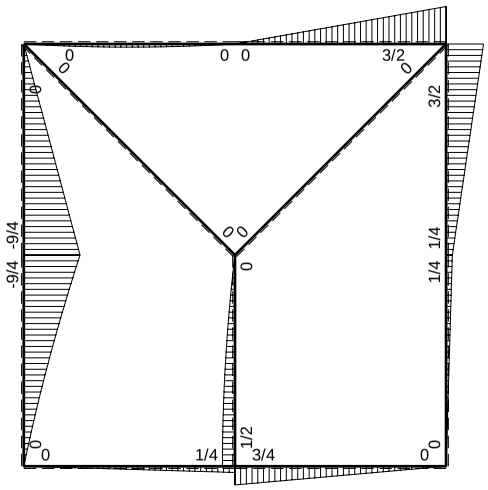

F


F

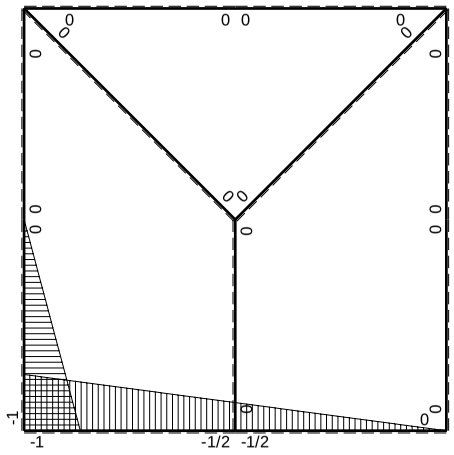

0
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0
3/2
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-9/4
1/4
1/4
0
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1/16
-1/2
9/16
0
0
-3/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 241068 ESΣ04.xxxx.062

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	-Fb/EJ	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/3+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	Fb/EJ	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\right)\ \theta\ \ dx$

$= \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

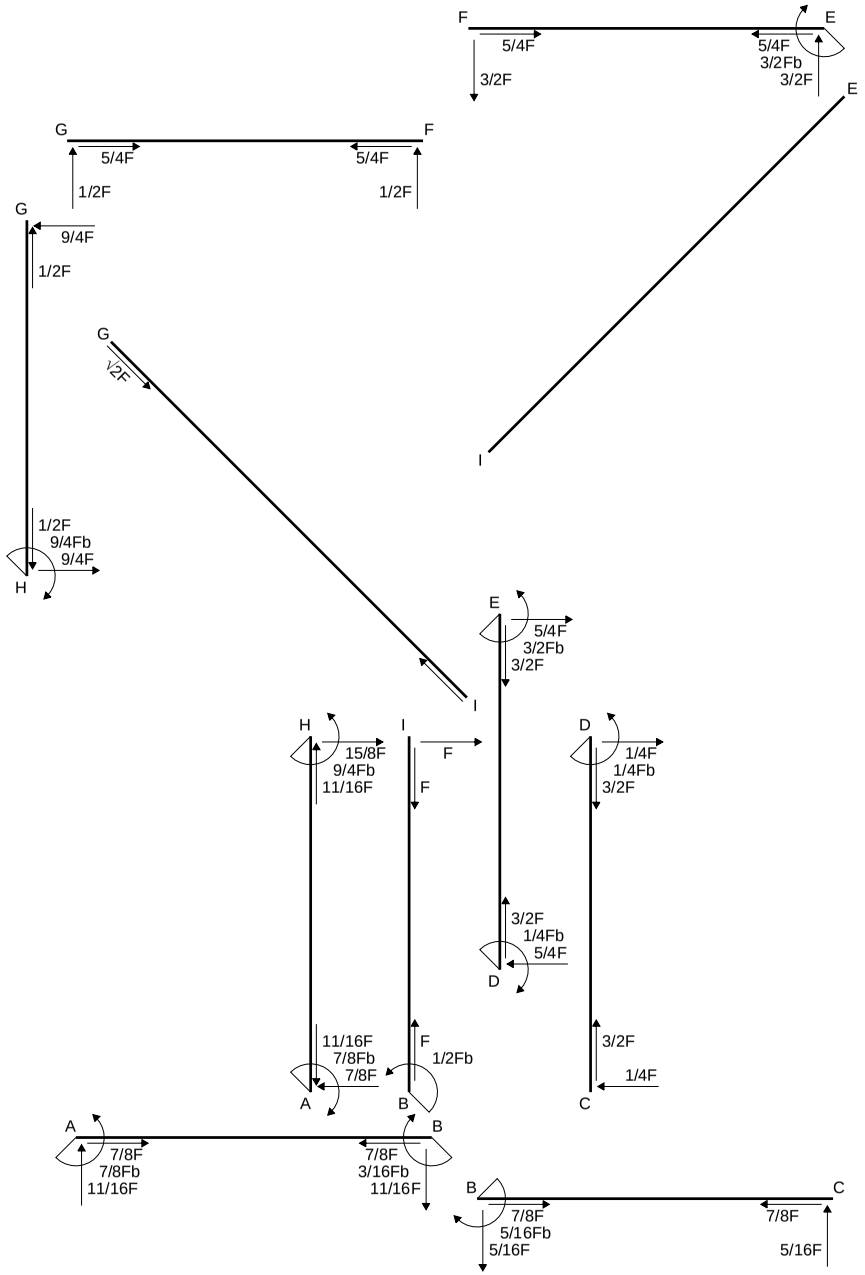
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\right)\ \theta\ \ dx$

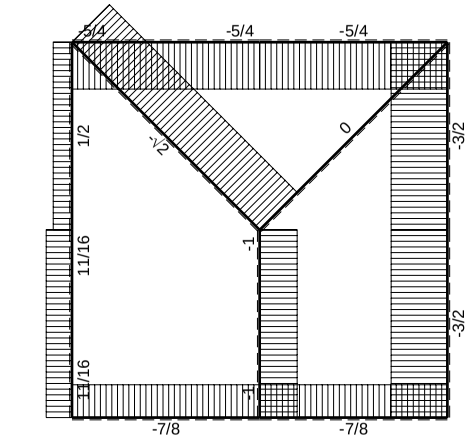
$= \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

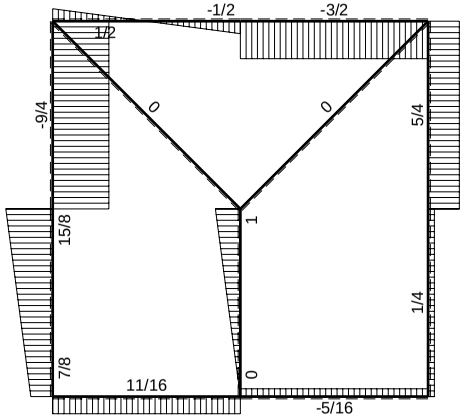
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

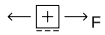
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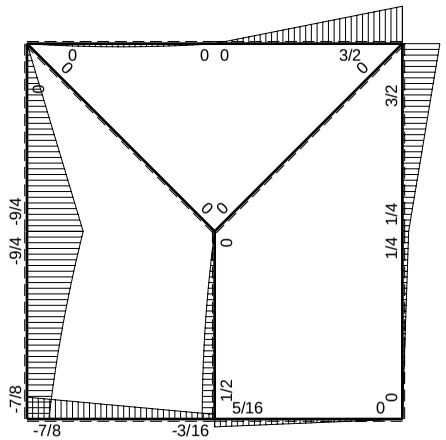

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5/4F
3/2Fb
3/2F
3/2F
E
G
F
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5/4F
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1/2F
G
9/4F
1/2F
G
½F
I
1/2F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
3/2F
I
H
I
D
15/8F
9/4Fb
11/16F
F
F
1/4F
1/4Fb
3/2F
3/2F
1/4Fb
5/4F
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5/16F


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-1
-7/8
-7/8

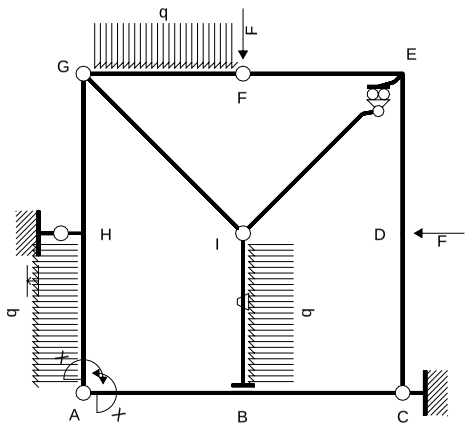

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5/4
15/8
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1/4
7/8
11/16
0
-5/16

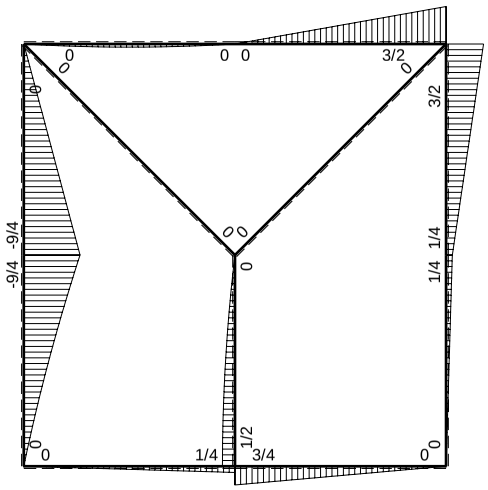

F


F

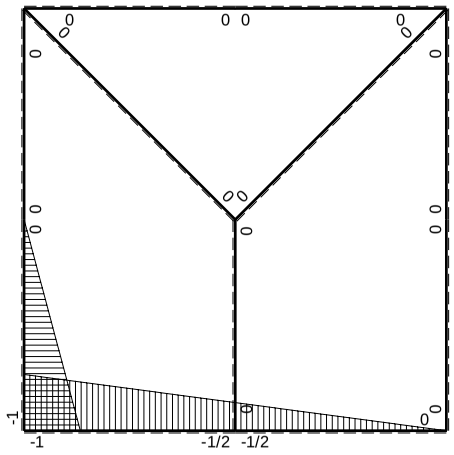

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1/4
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1/2
5/16
0
0
-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 955133 ESΣ04.xxxx.063

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PROCEDIMENTO E RISULTATI 955133 ESΣ04.xxxx.063

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

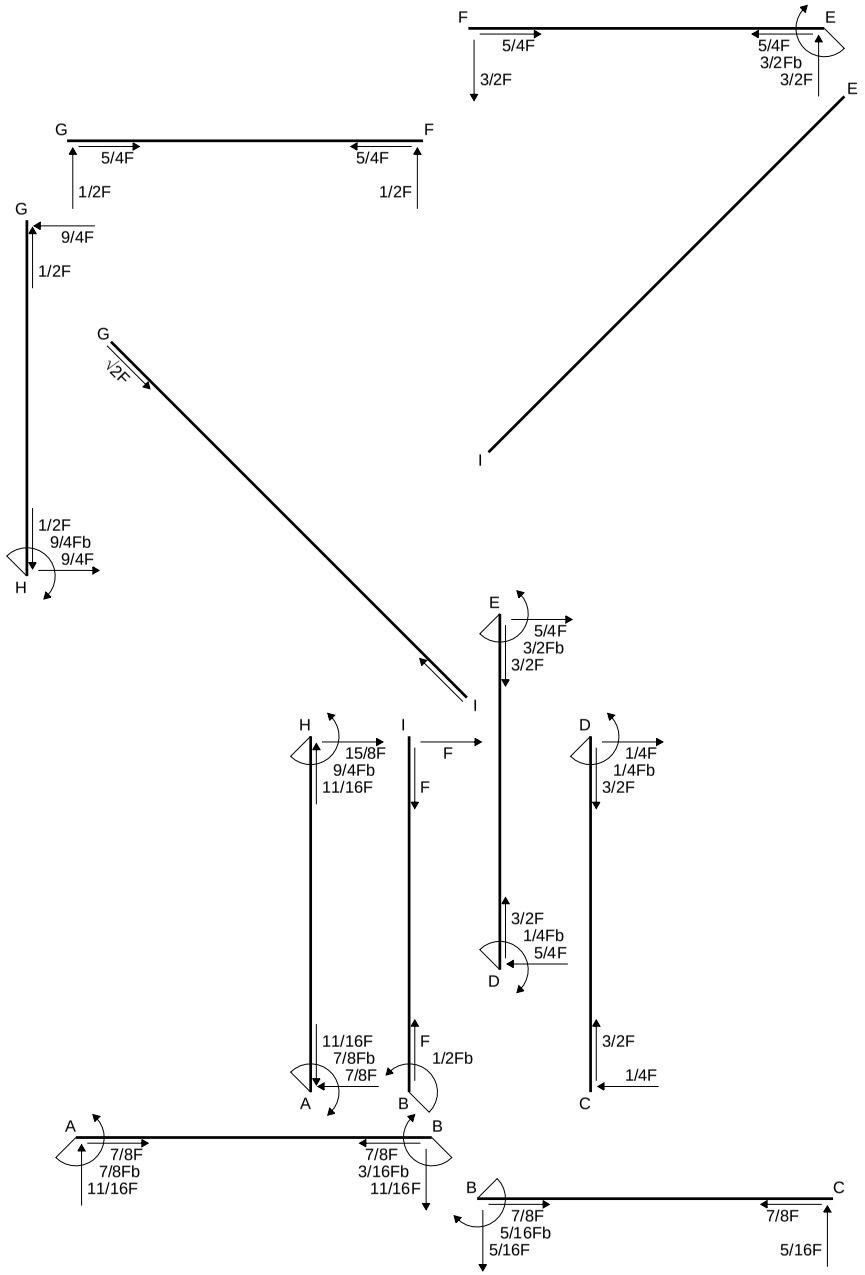
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

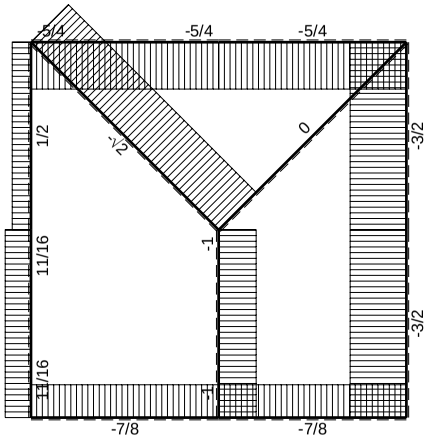
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

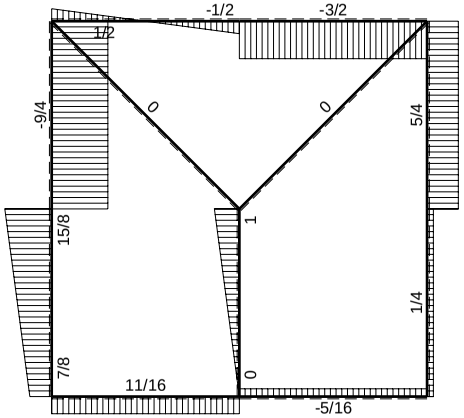
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

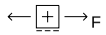
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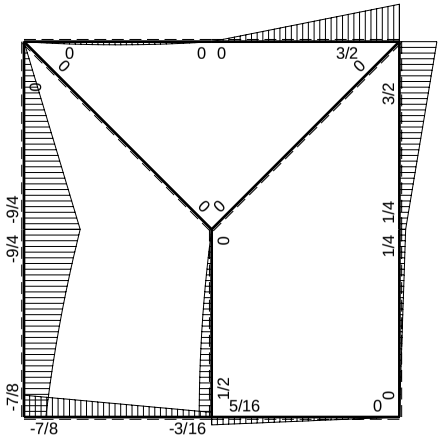

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G
½F
I
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9/4F
H
E
5/4F
3/2Fb
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I
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D
15/8F
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F
F
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-7/8

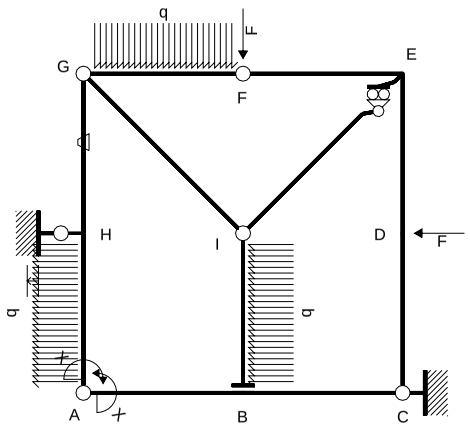

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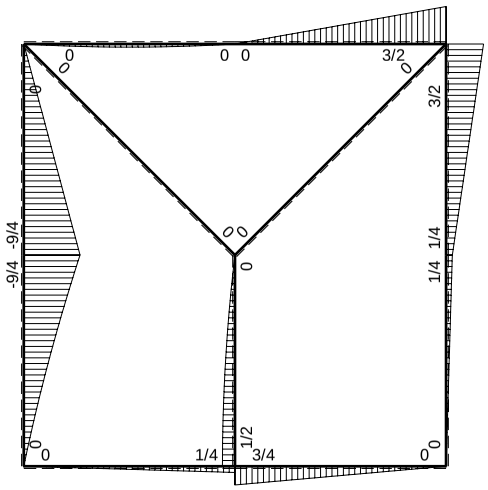

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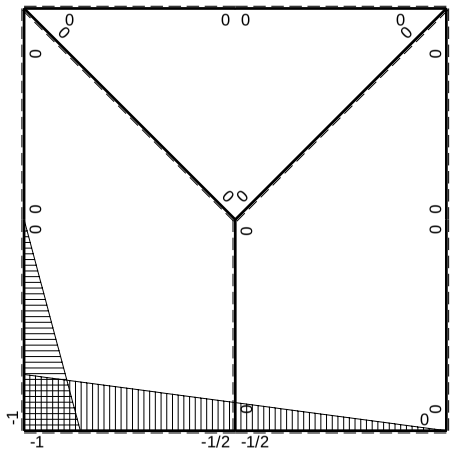

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1/2
5/16
0
0
-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

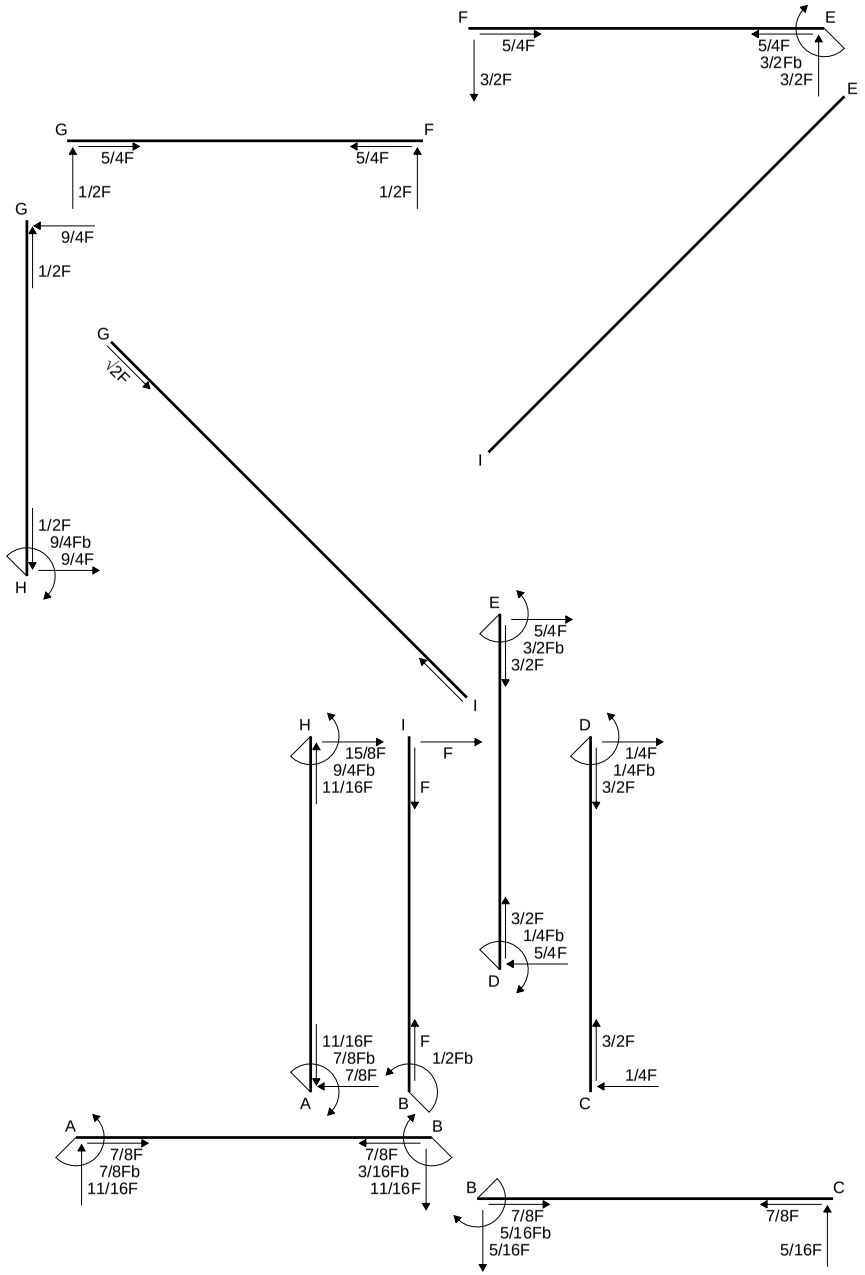
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

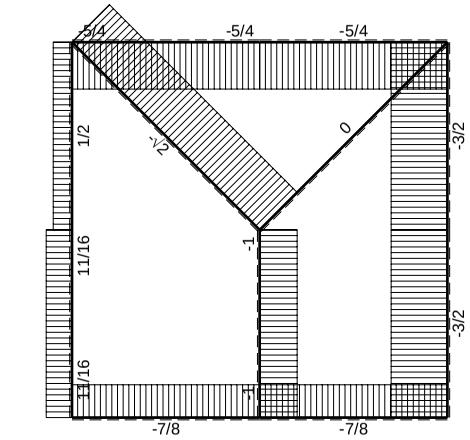
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

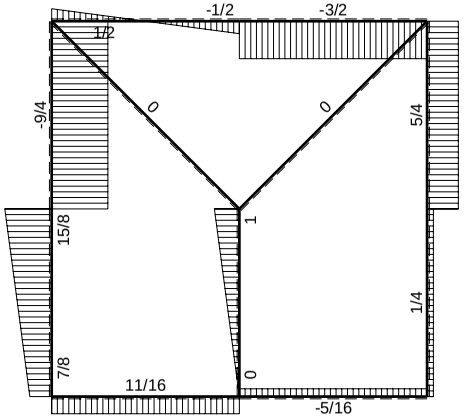
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

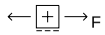
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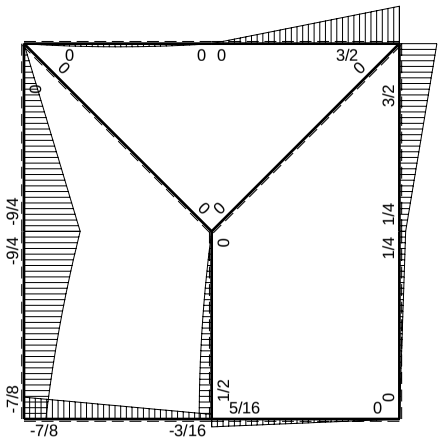

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C
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5/16F


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-7/8

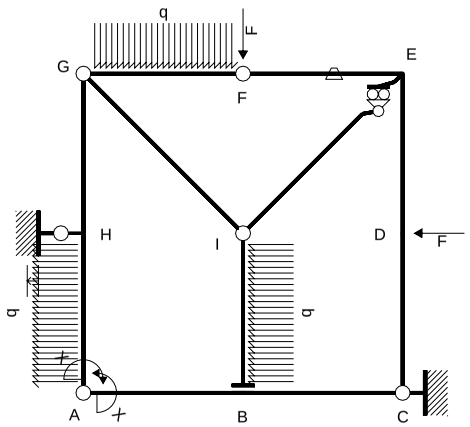

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-5/16

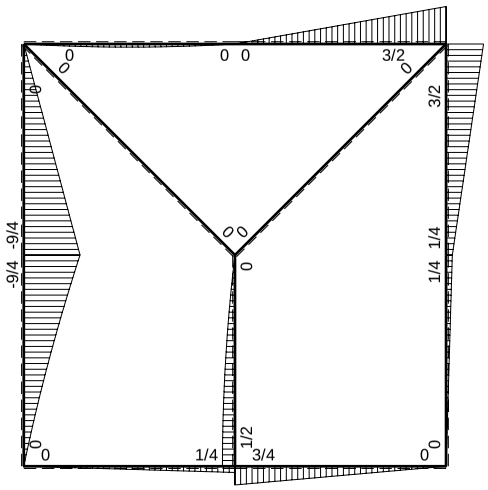

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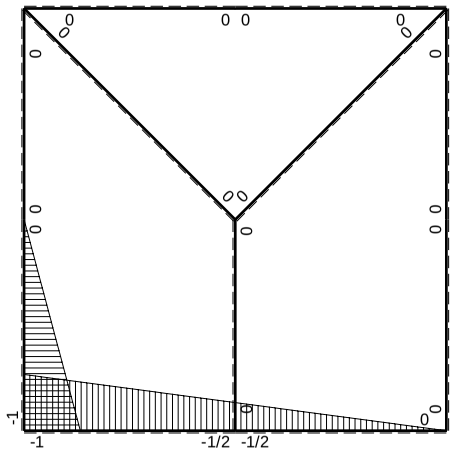

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5/16
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-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-3/2Fx	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

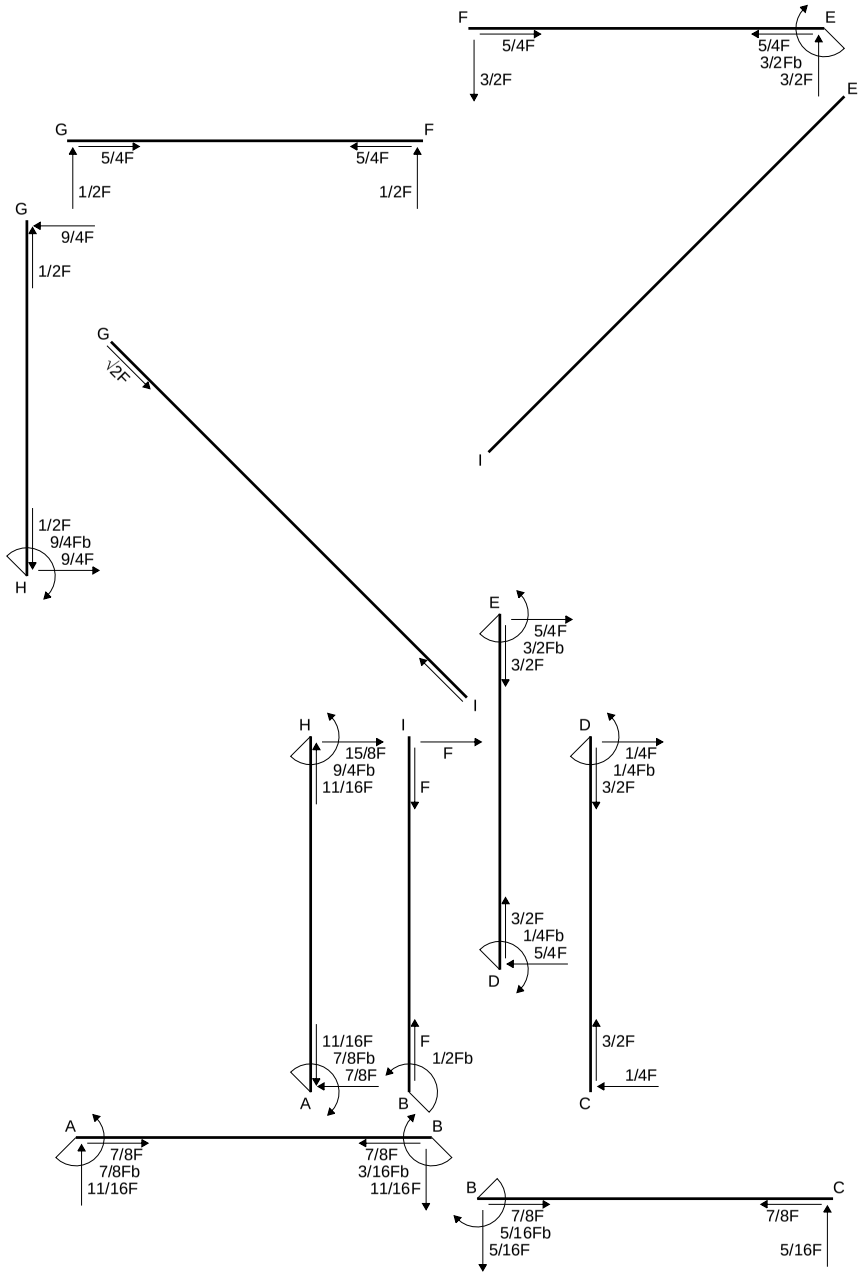
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

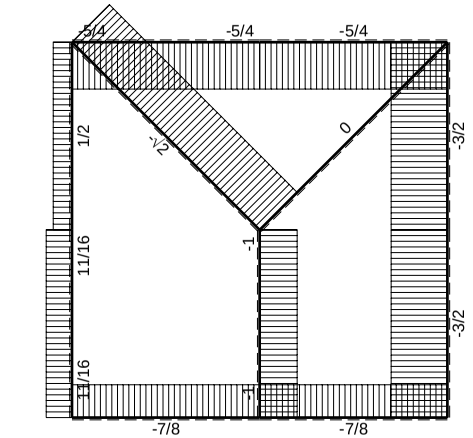
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

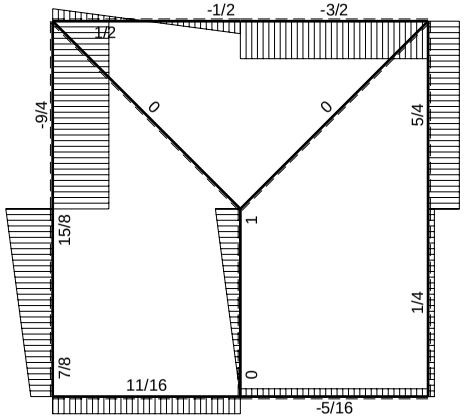
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

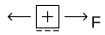
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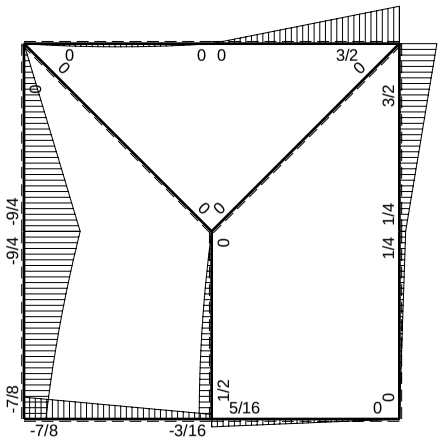

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3/2Fb
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E
G
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1/2F
G
9/4F
1/2F
G
½F
I
1/2F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
3/2F
I
H
I
D
15/8F
9/4Fb
11/16F
F
F
1/4F
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7/8F
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B
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B
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7/8Fb
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3/16Fb
11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


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-7/8
-7/8

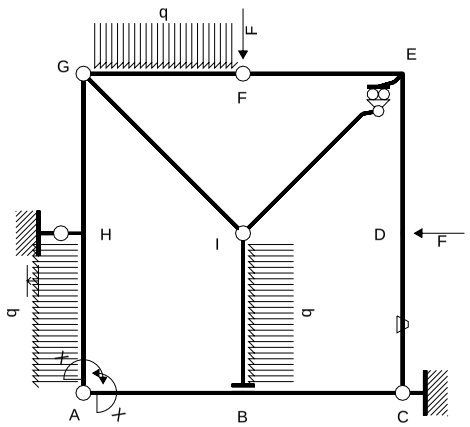

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11/16
0
-5/16

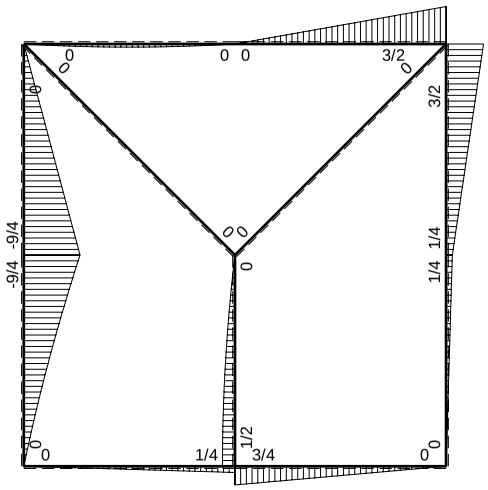

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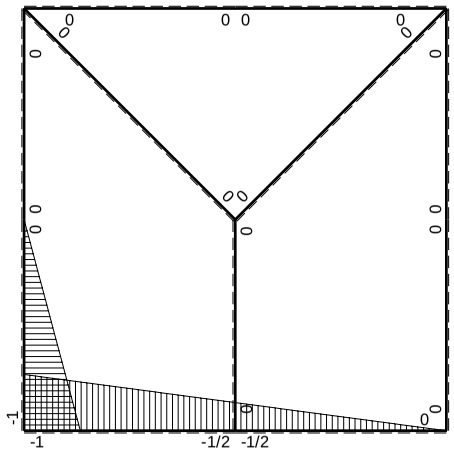

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3/2
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-9/4
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0
1/4
1/4
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-7/8
1/2
5/16
0
0
-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

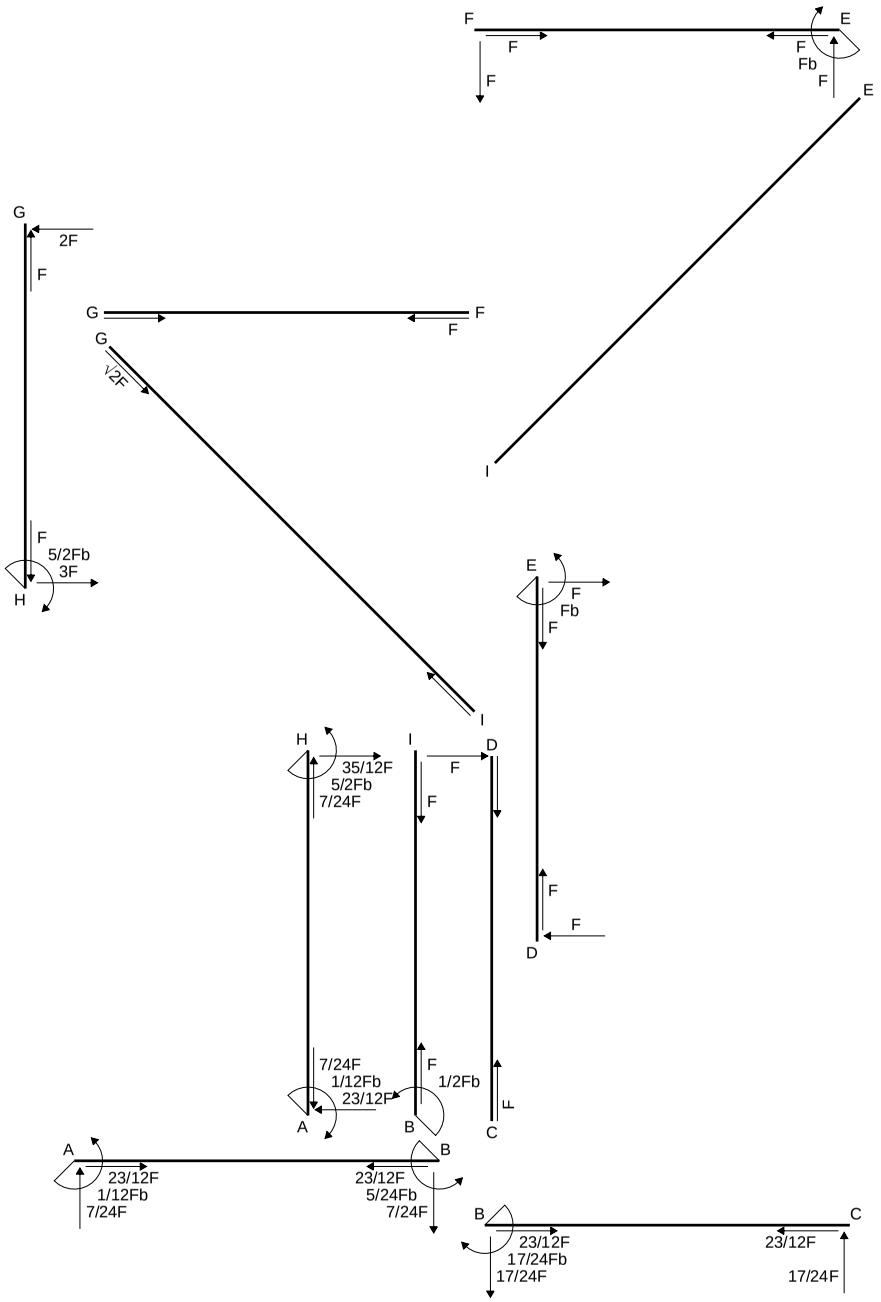
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

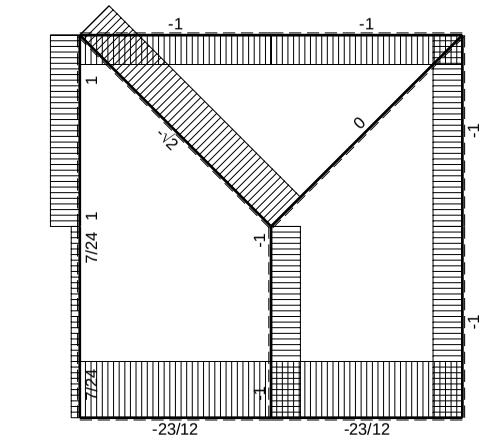
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

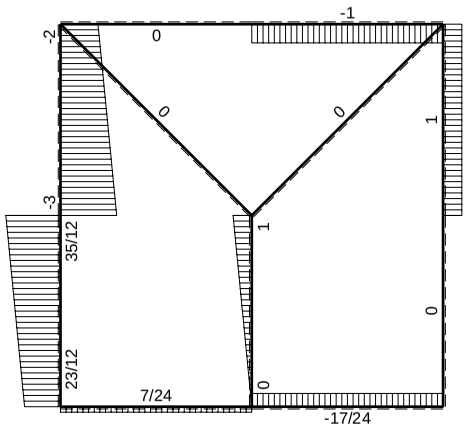
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

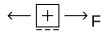
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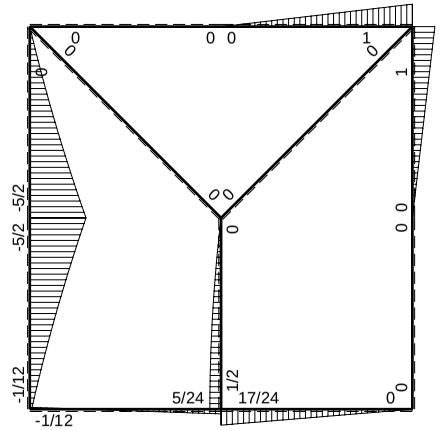

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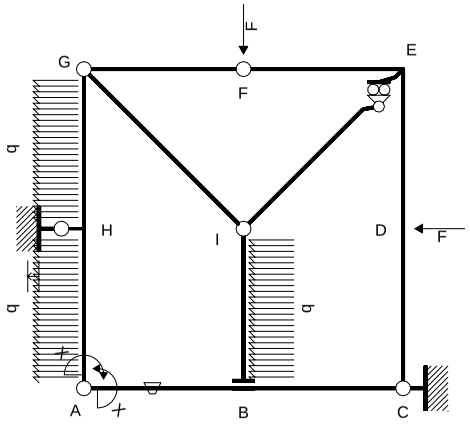

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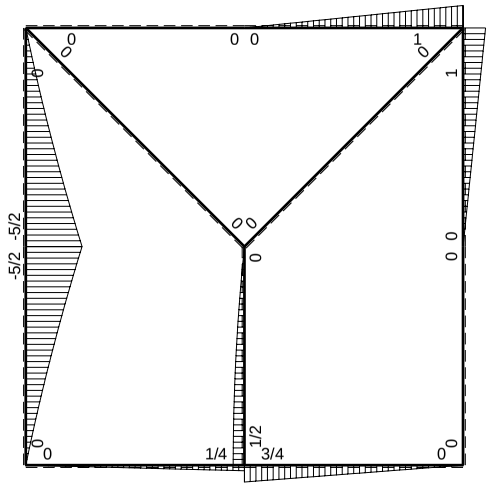

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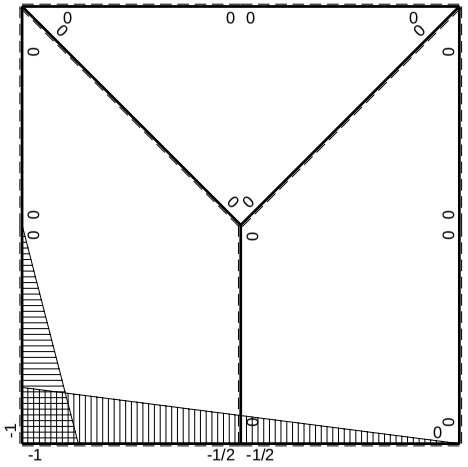

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5/24
1/2
17/24
0
0
-1/12


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 280947 ESΣ04.xxxx.067

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	-Fb/EJ	$-1/4Fx+1/8Fx^2/b$	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(-1/12+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	Fb/EJ	$-1/8Fb+1/8Fx^2/b$	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/12Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

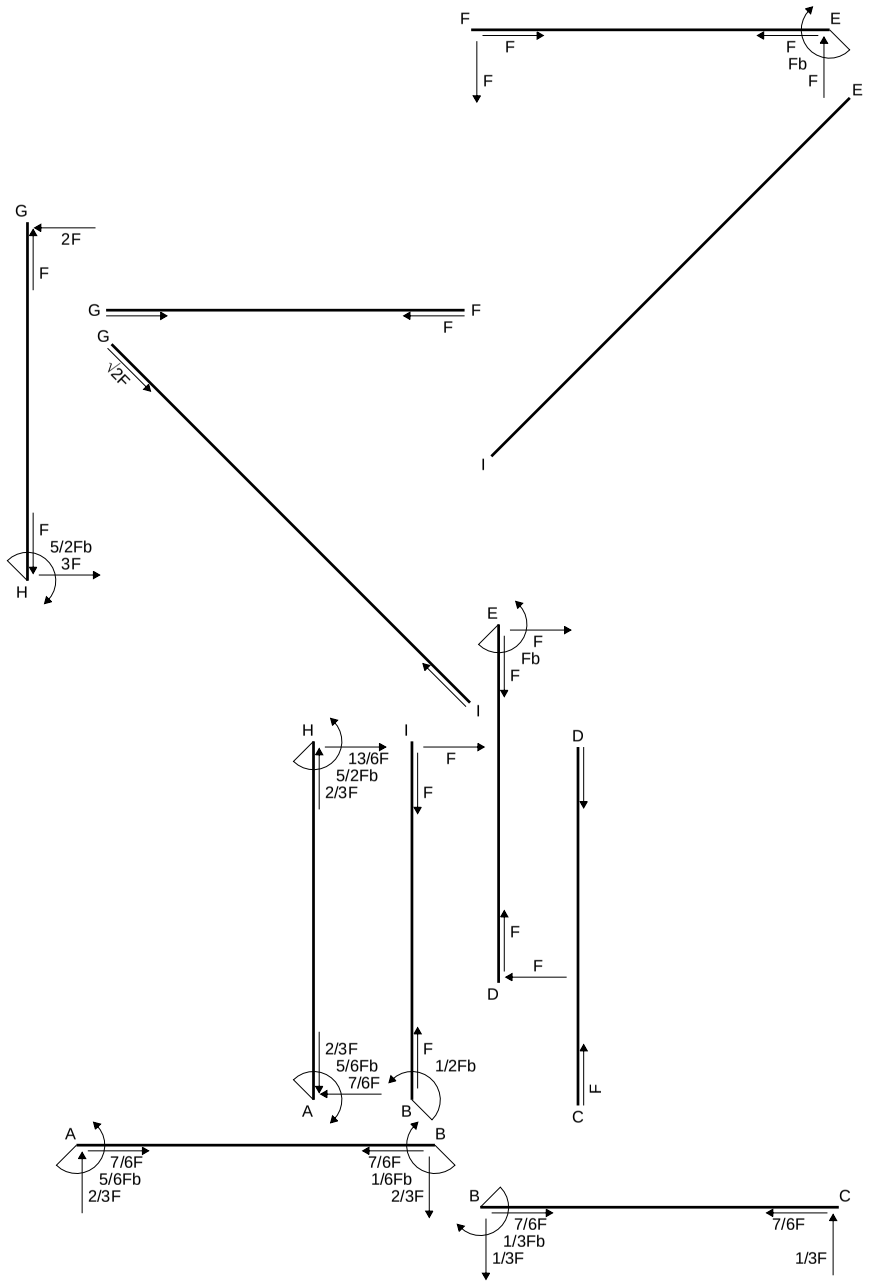
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

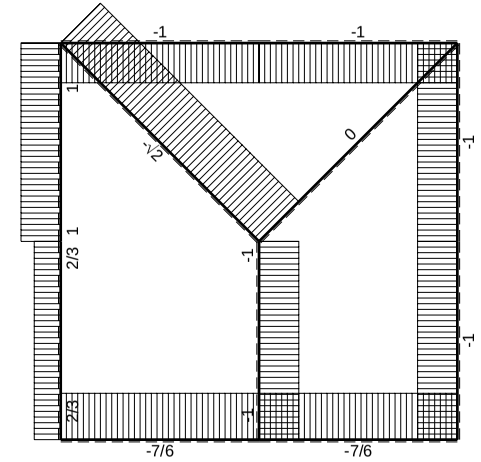
$L^{Xo}_{AH} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

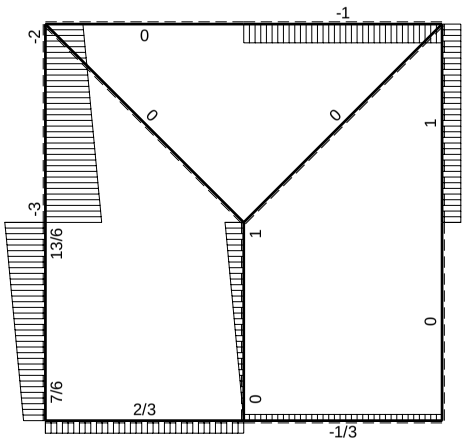
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

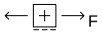
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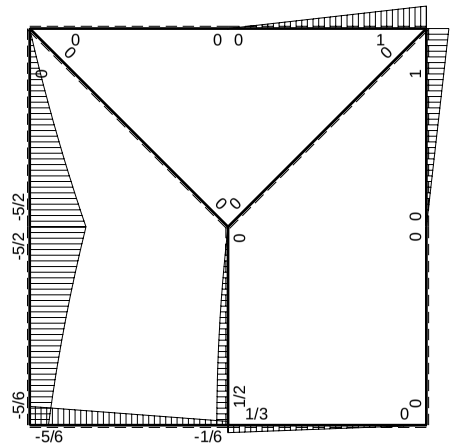

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A
B
C
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B
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B
C
7/6F
1/3Fb
1/3F
7/6F
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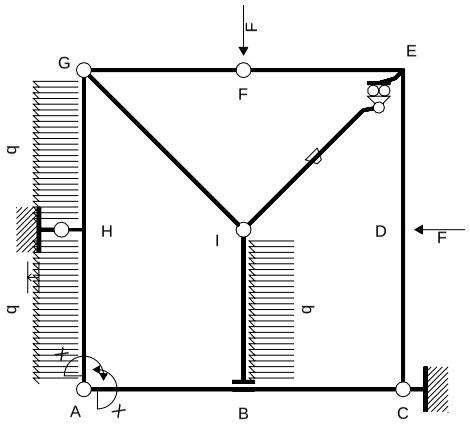

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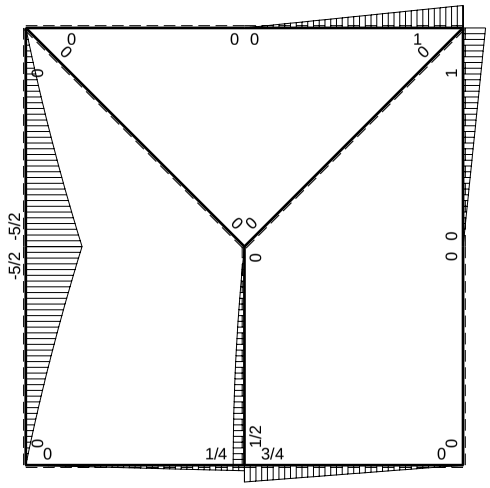

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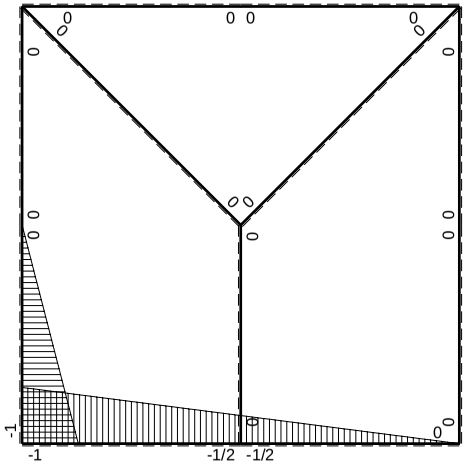

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-5/2
0 0
-5/6
1/2
1/3
0
0
-5/6
-1/6


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	-1/4Fx+1/8Fx^2/b	0	1-x/b+1/4x^2/b^2	(-1/12+0)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	-1/8Fb+1/8Fx^2/b	0	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	-3/8Fb+3/4Fx-3/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-1/8+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	-3/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-2Fx-1/2qx^2	0	0	0	0	0+0	0
HG b	0	5/2Fb-3Fx+1/2qx^2	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
BI b	0	-1/2Fb+1/2qx^2	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	-5/2Fb+3Fx-1/2qx^2	0	5/2Fx-3Fx^2/b+1/2qx^3/b	0	x^2/b^2	(3/8+0)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	2Fx+1/2qx^2	0	2Fx-3/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-5/6Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

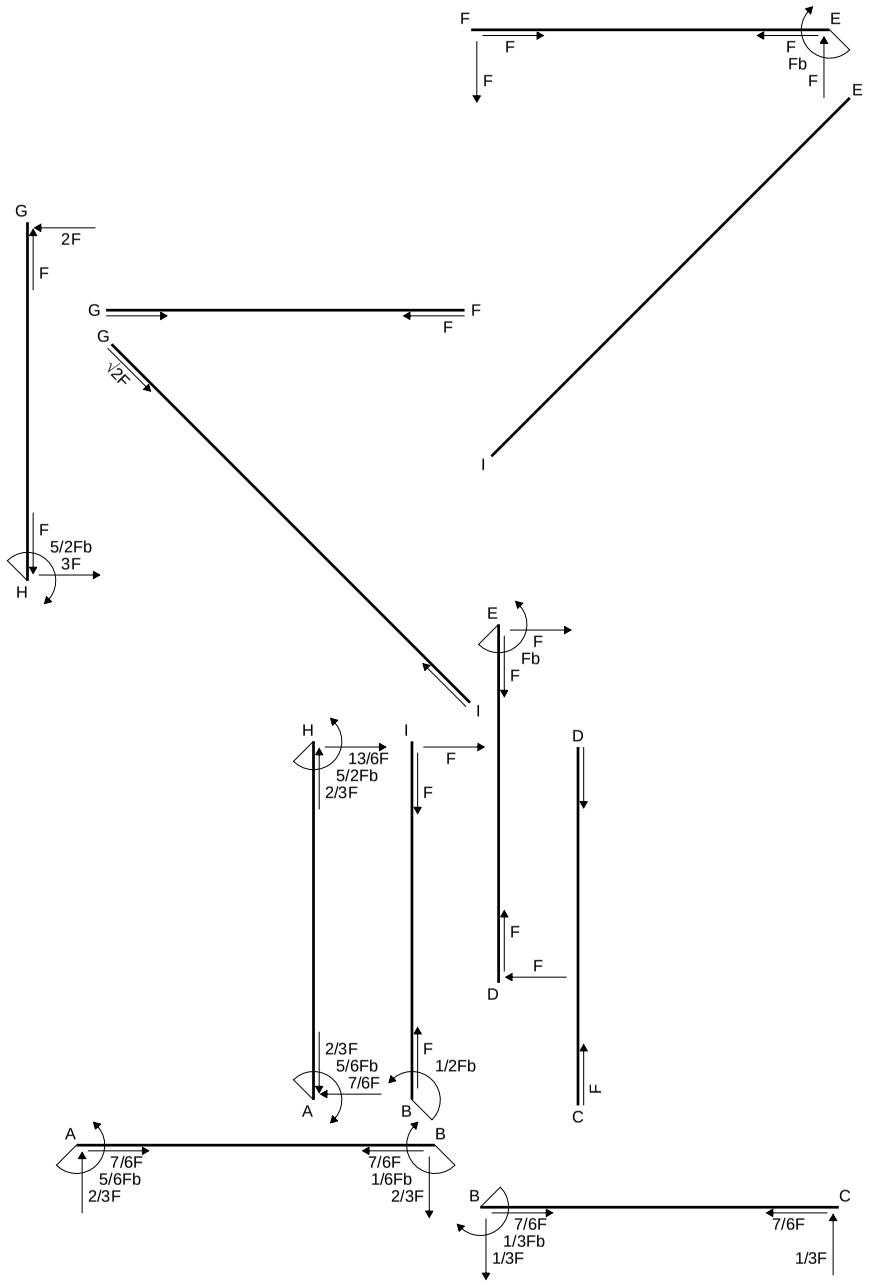
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

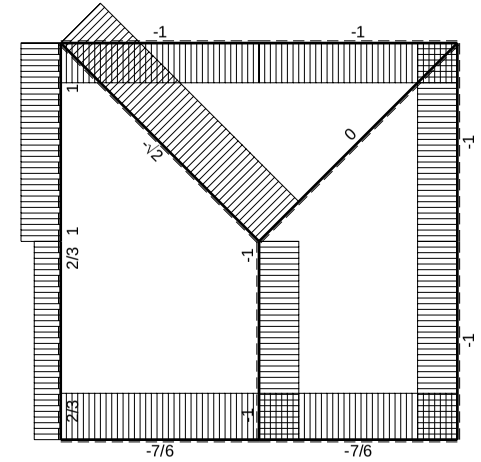
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

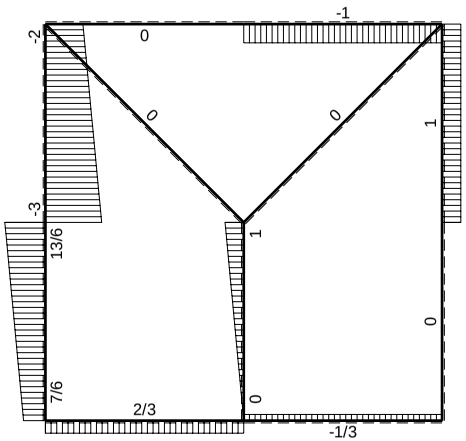
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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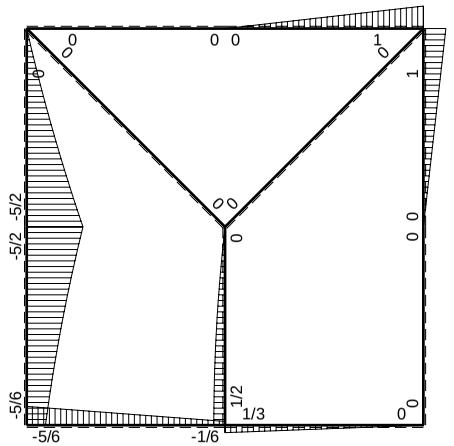

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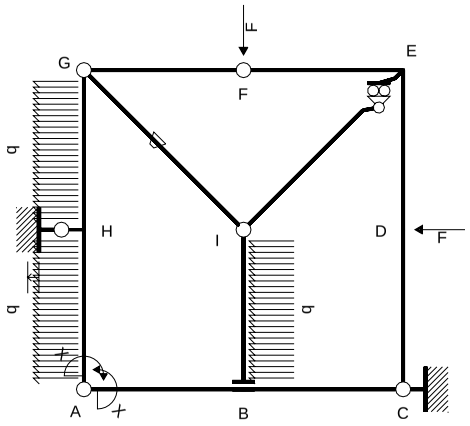

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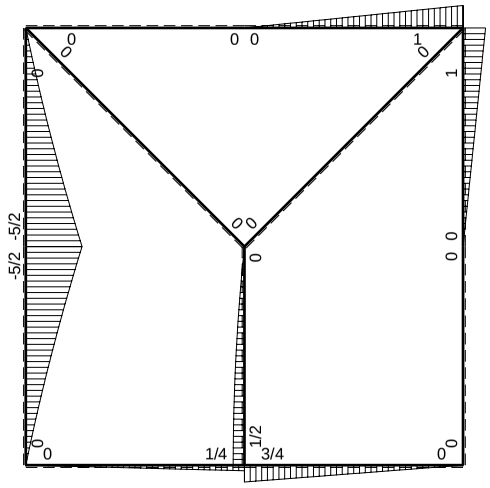
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↑□↓ F

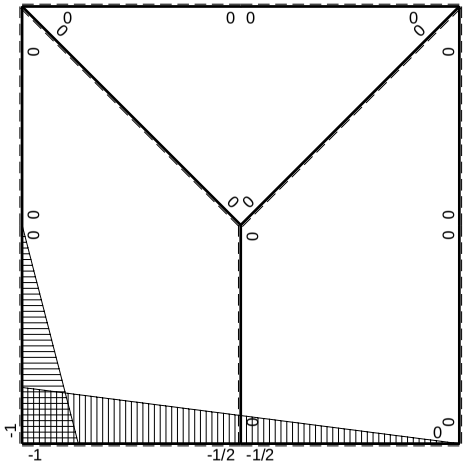

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0
0
-5/6
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	-1/4Fx+1/8Fx^2/b	0	1-x/b+1/4x^2/b^2	(-1/12+0)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	-1/8Fb+1/8Fx^2/b	0	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	-3/8Fb+3/4Fx-3/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-1/8+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	-3/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-2Fx-1/2qx^2	0	0	0	0	0+0	0
HG b	0	5/2Fb-3Fx+1/2qx^2	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
BI b	0	-1/2Fb+1/2qx^2	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	-5/2Fb+3Fx-1/2qx^2	0	5/2Fx-3Fx^2/b+1/2qx^3/b	0	x^2/b^2	(3/8+0)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	2Fx+1/2qx^2	0	2Fx-3/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-5/6Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

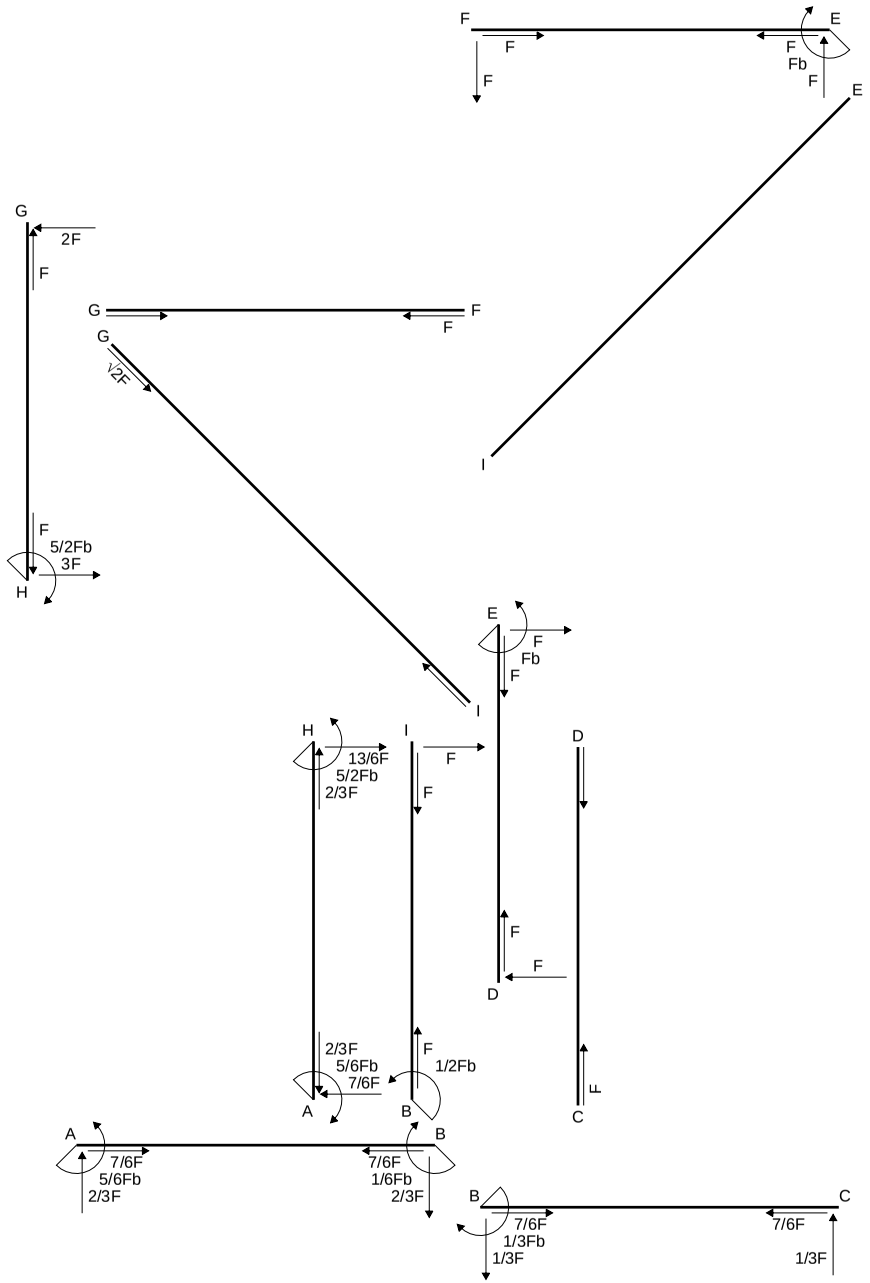
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

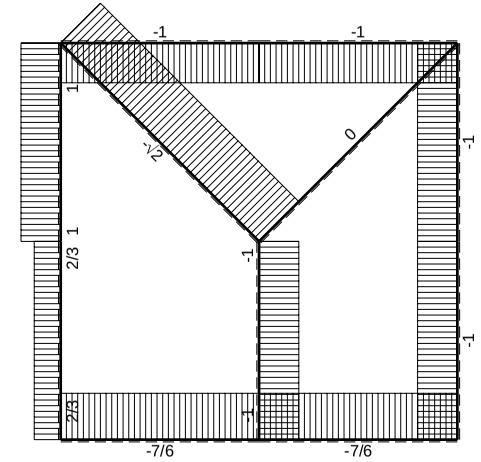
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

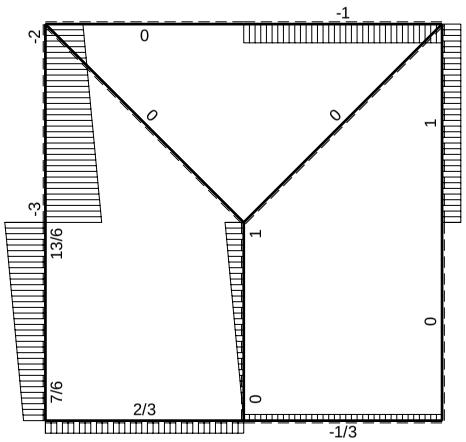
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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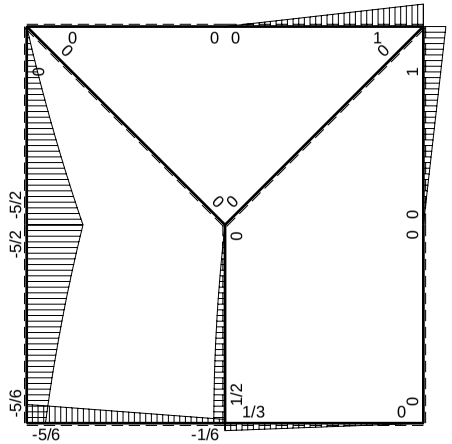

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Fb
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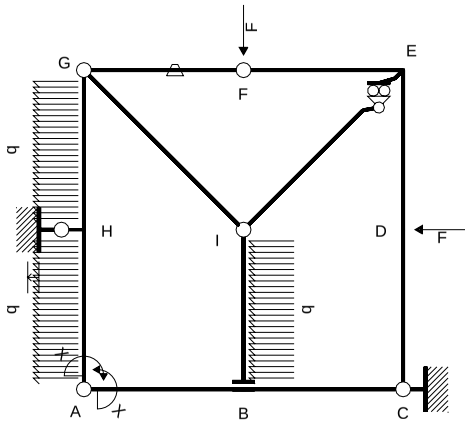

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-1/3

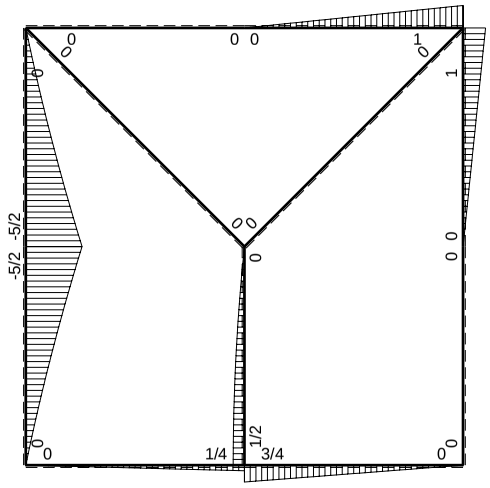
←□→ F

↑□↓ F

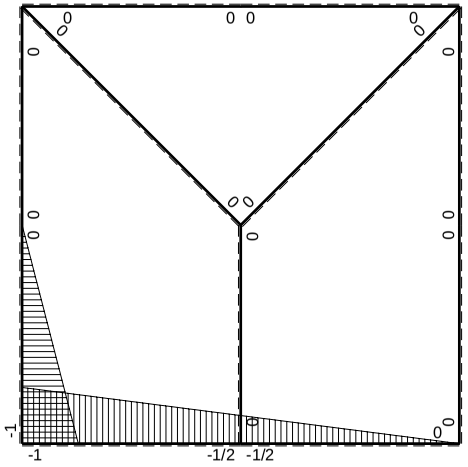

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-5/2
-5/2
0 0
-5/6
1/2
1/3
0
0
-5/6
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

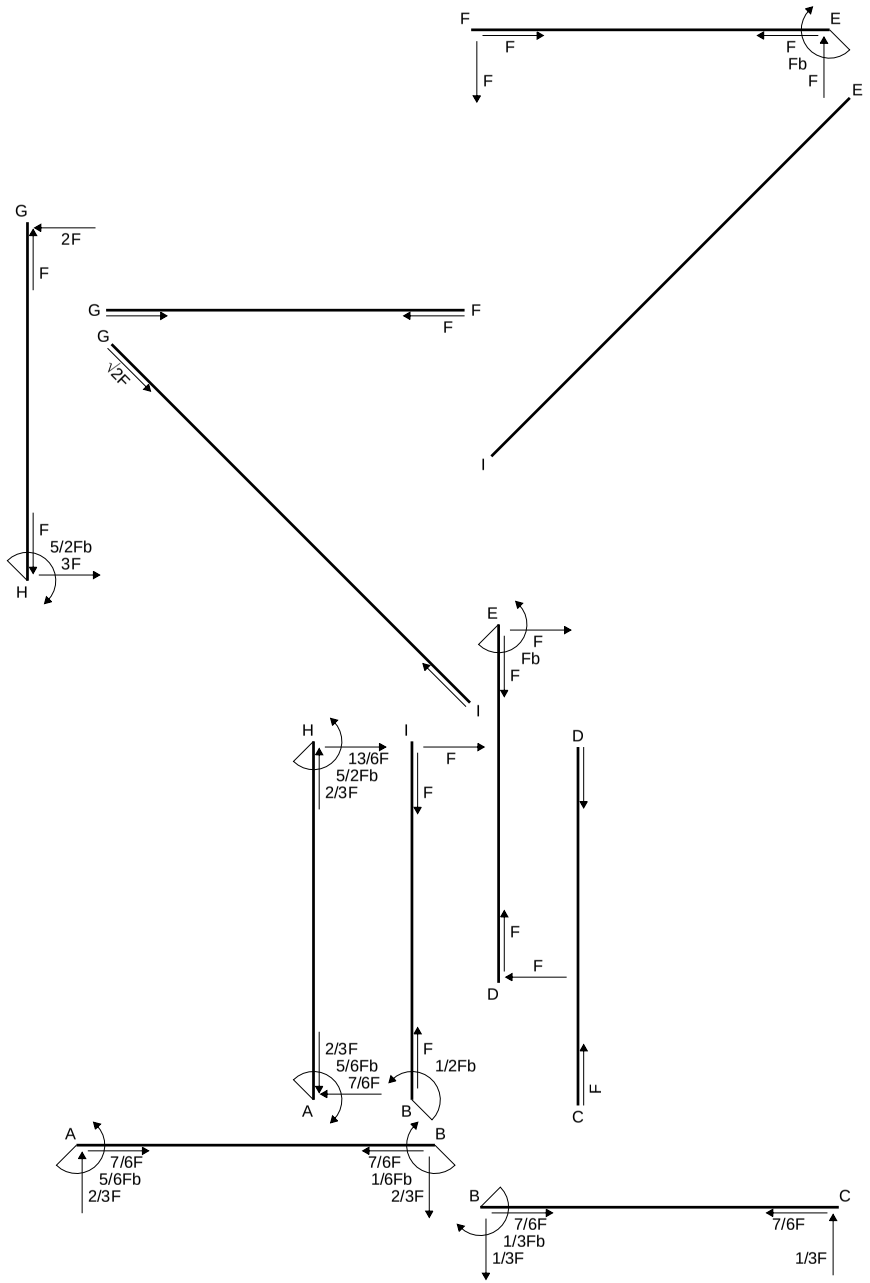
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

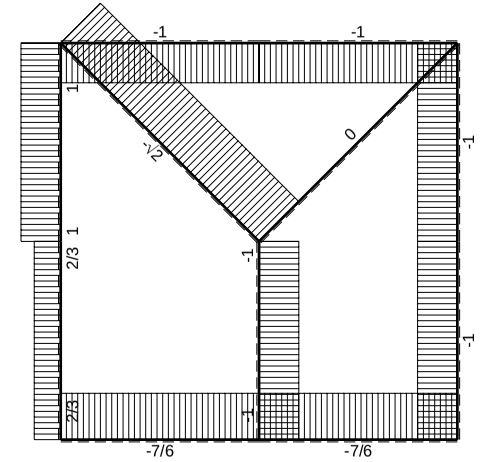
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

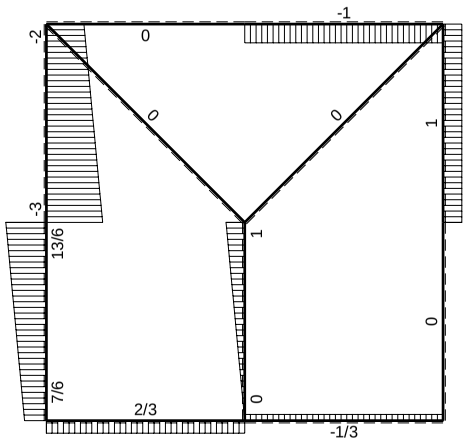
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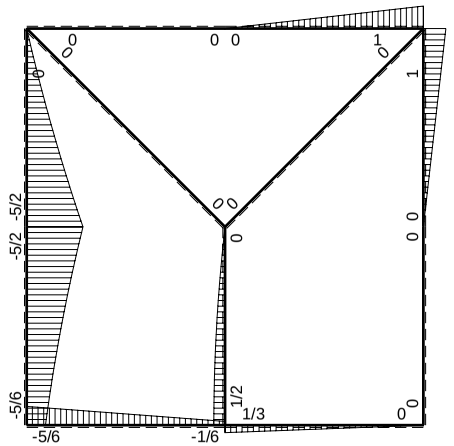

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B
C
7/6F
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1/3F
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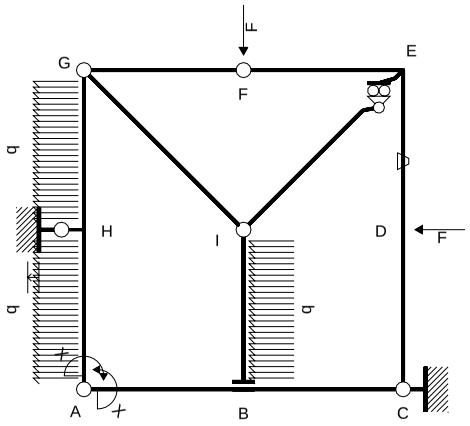
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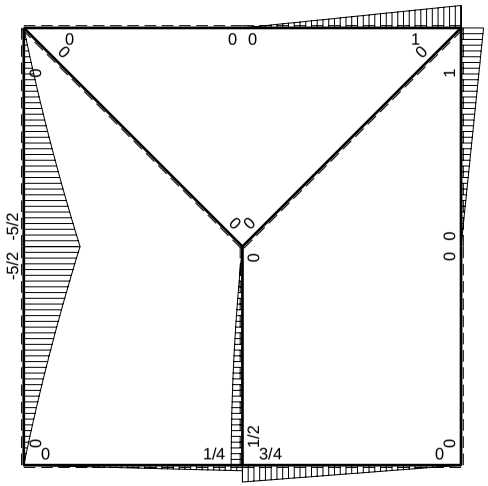

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-1/3

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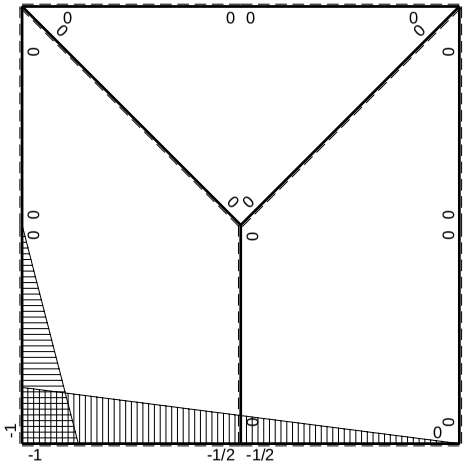

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-5/6
1/2
1/3
0
0
-5/6
-1/6

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 961051 ESΣ04.xxxx.071

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PROCEDIMENTO E RISULTATI 961051 ESΣ04.xxxx.071

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

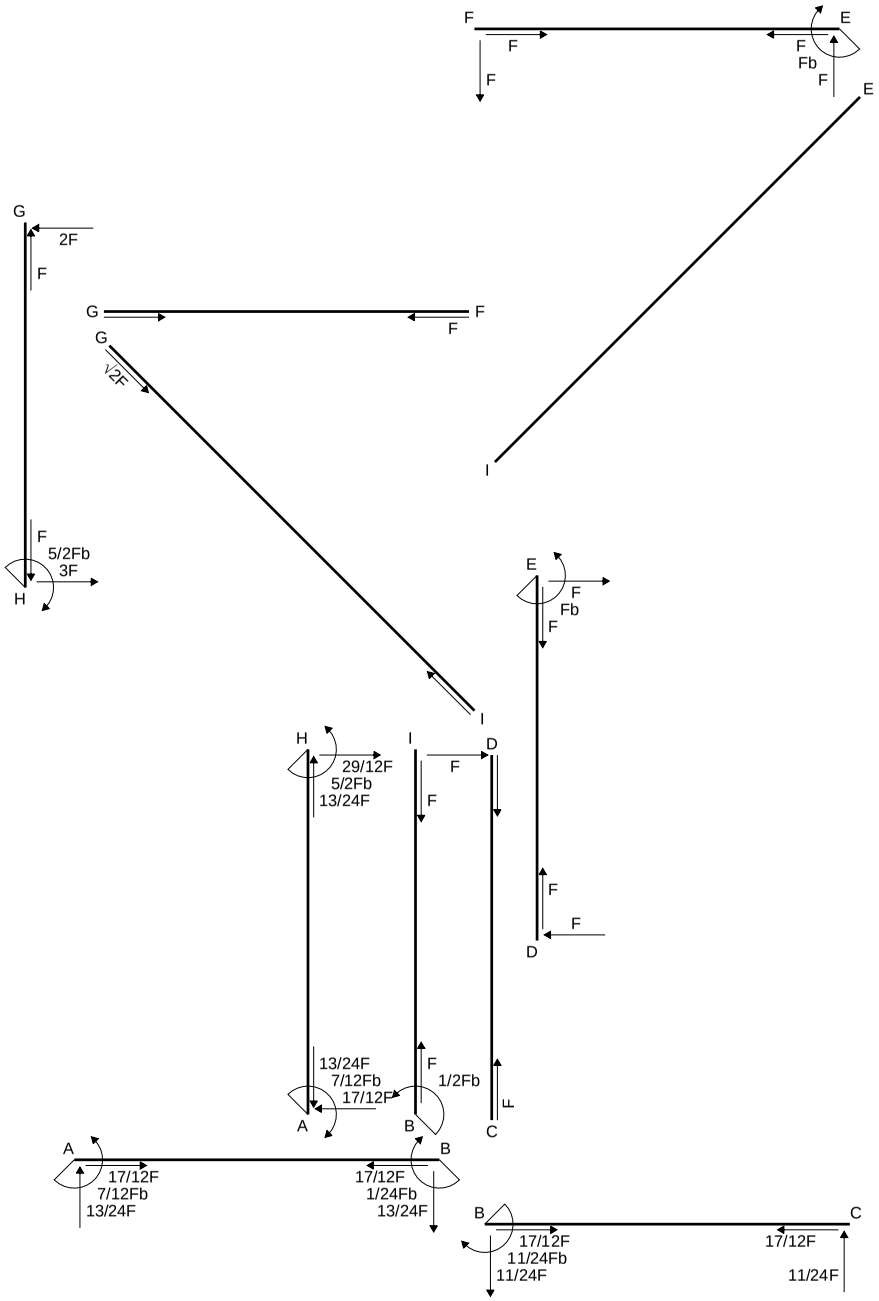
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

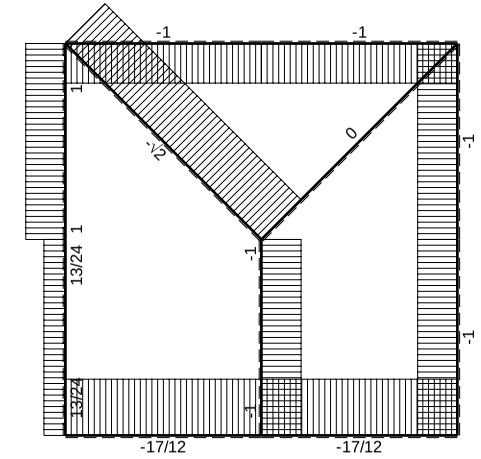
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

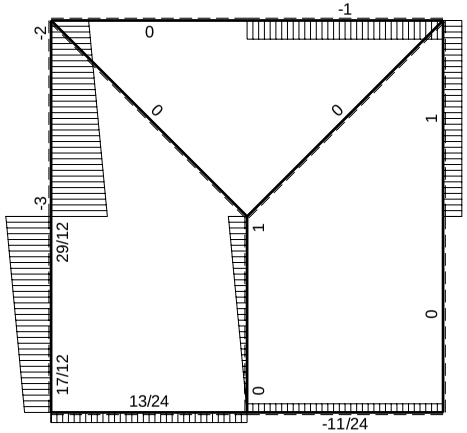
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

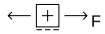
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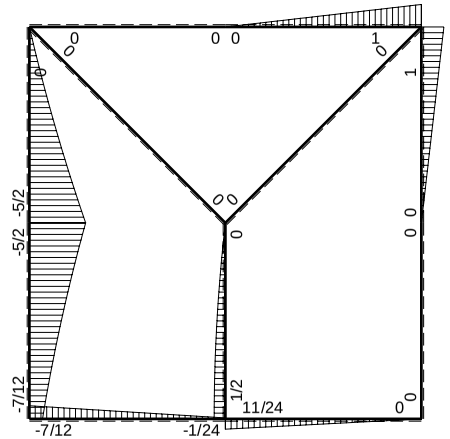

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11/24Fb
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11/24F


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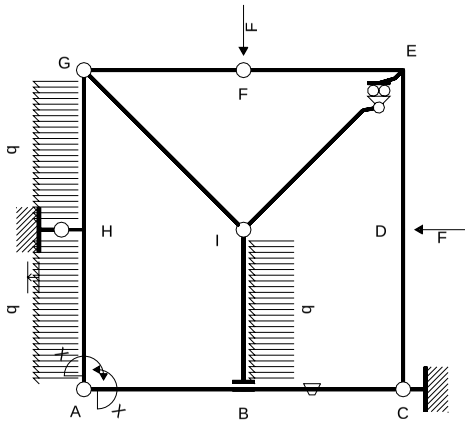

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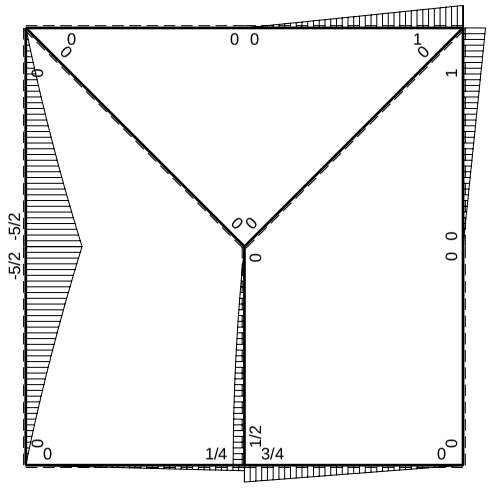

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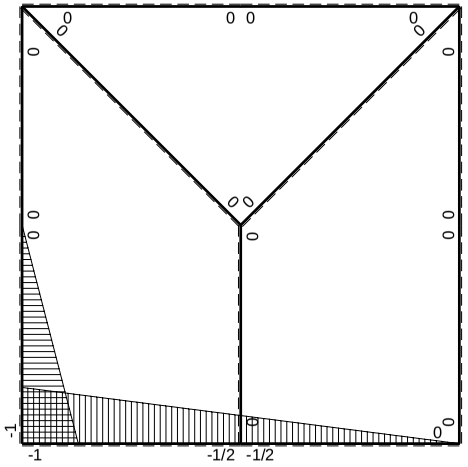

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-7/12
1/2
11/24
0
0
-7/12
-1/24


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	-Fb/EJ	$-3/8Fb+3/4Fx-3/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-3/4Fx$	Fb/EJ	$-3/8Fx^2/b$	$1/2Fx/EJ$	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\ \right)\ \theta\ \ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

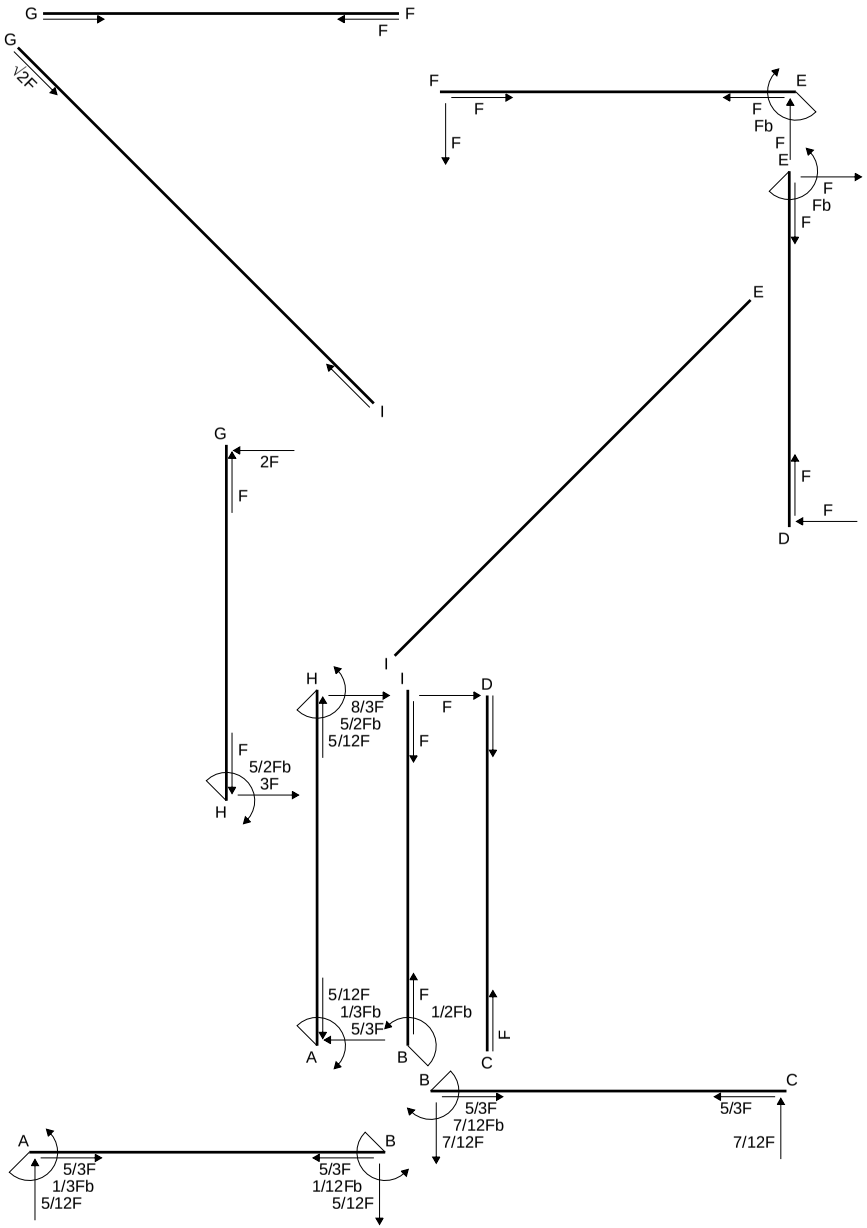
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

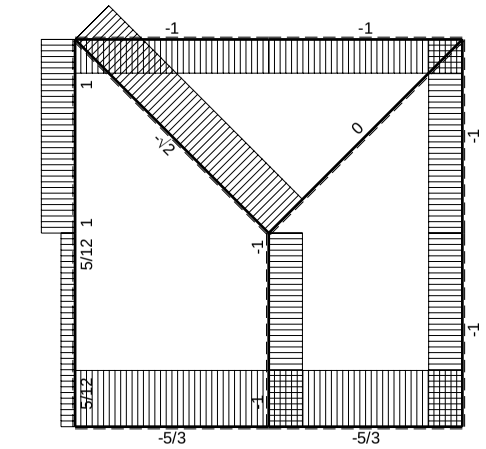
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

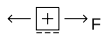
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

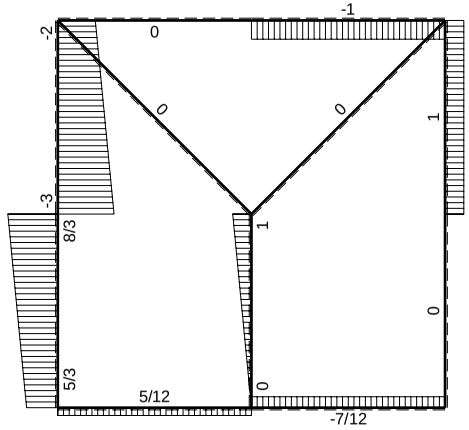
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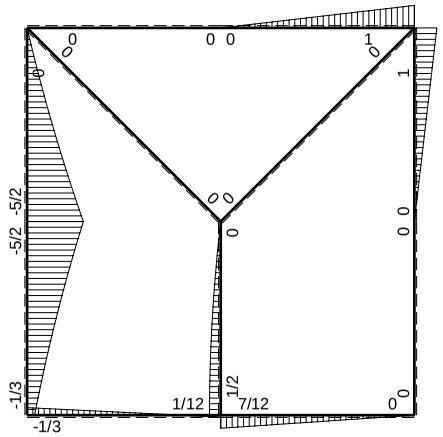

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-5/3
-5/3

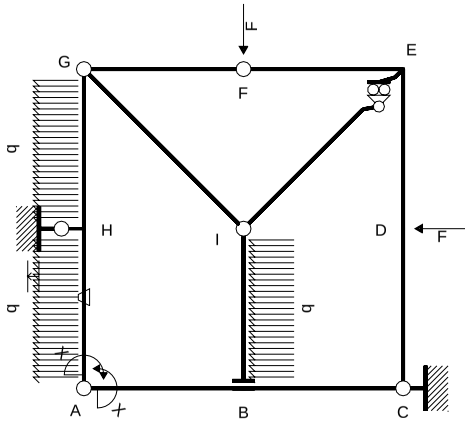

F

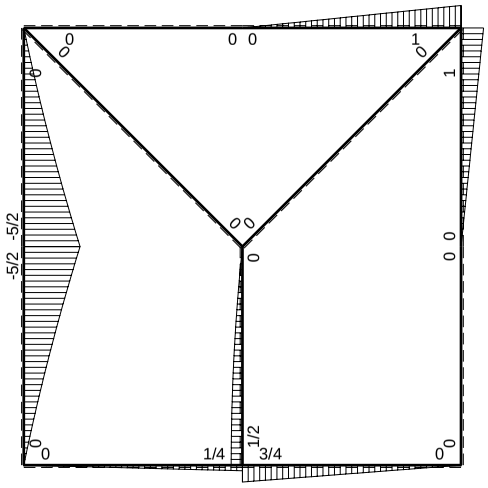

-2
0
-1
0
0
1
-3
8/3
1
5/3
0
5/12
0
-7/12


F

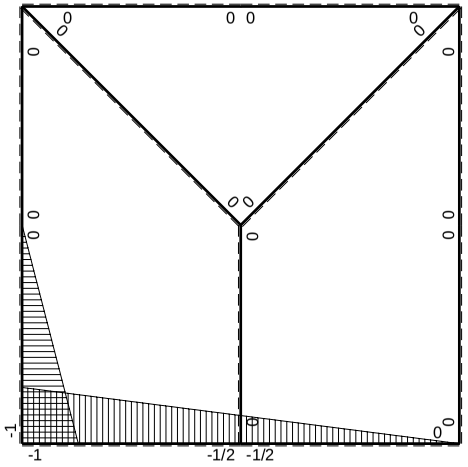

0
0 0
1
0
0
1
0 0
-5/2 -5/2
0
0 0
-1/3
1/12
1/2
7/12
0
0
-1/3


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 984792 ESΣ04.xxxx.073

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	-Fb/EJ	$5/2Fx-3Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(3/8+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	Fb/EJ	$2Fx-3/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/3Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/3Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx$

$= \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = 7/8\ \ Fb^2/EJ$

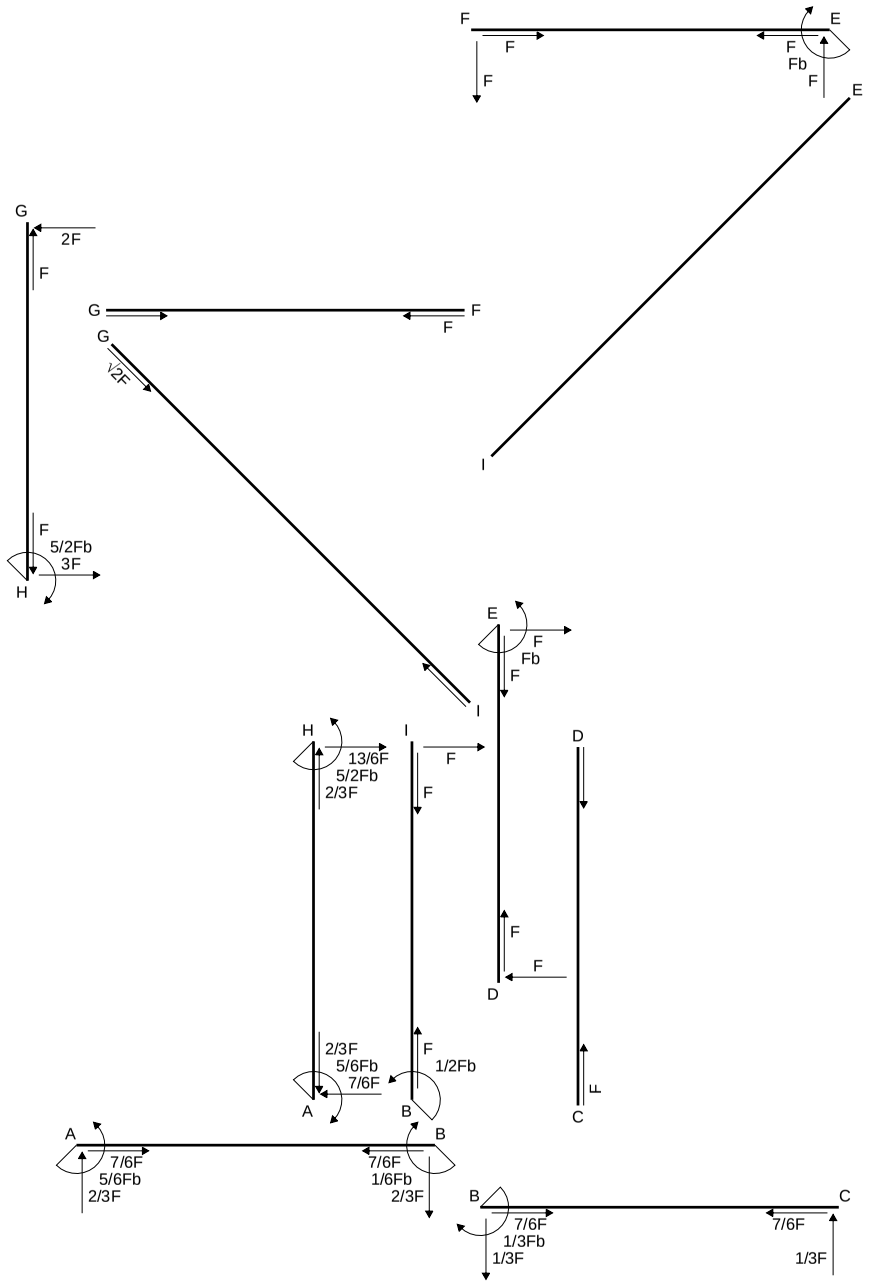
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

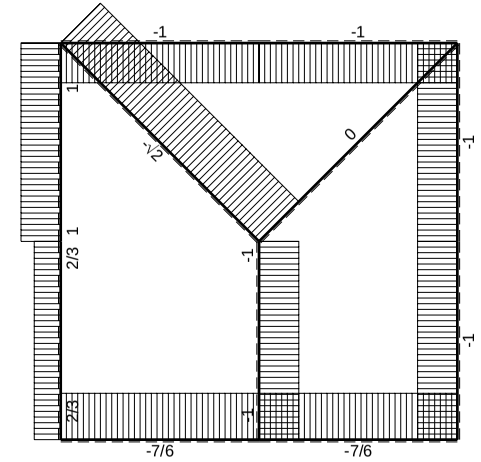
$= \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

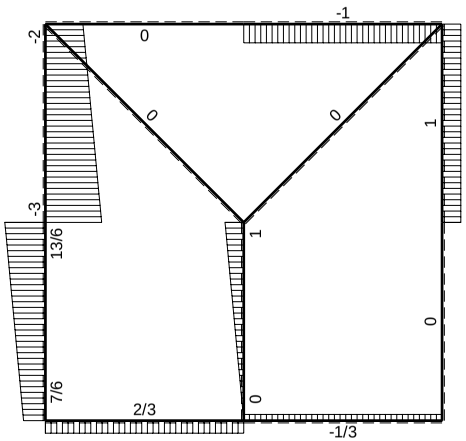
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\ \theta\ \ = 7/8\ \ Fb^2/EJ$

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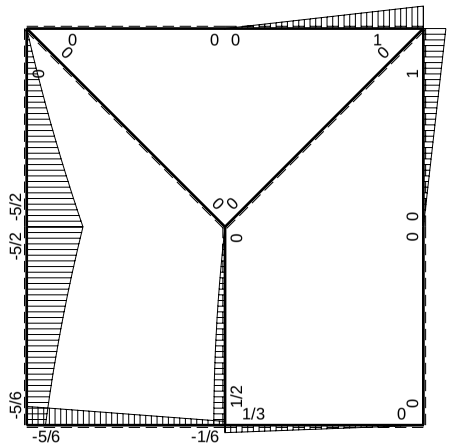

F
E
F
F
Fb
F
E
G
2F
F
G
F
F
G
½F
I
F
5/2Fb
3F
H
E
F
Fb
F
I
H
13/6F
5/2Fb
2/3F
I
F
F
D
F
F
D
2/3F
5/6Fb
7/6F
F
1/2Fb
A
B
C
F
A
B
7/6F
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1/6Fb
2/3F
B
C
7/6F
1/3Fb
1/3F
7/6F
1/3F


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½
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-1
2/3 1
-1
-1
2/3
1
-7/6
-7/6

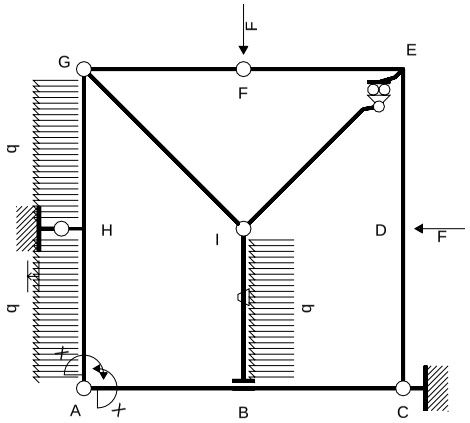

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0
0
1
-3
13/6
1
0
7/6
2/3
0
-1/3

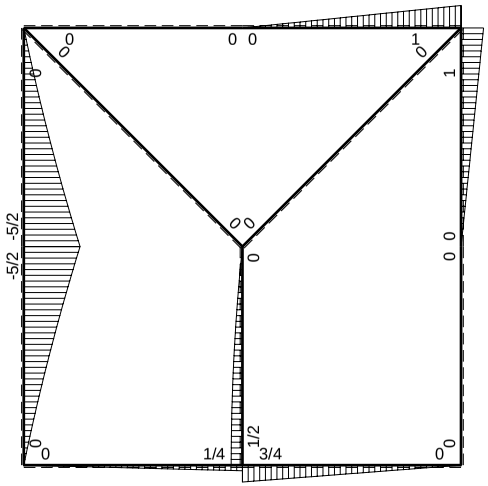
←□→ F

↑□↓ F

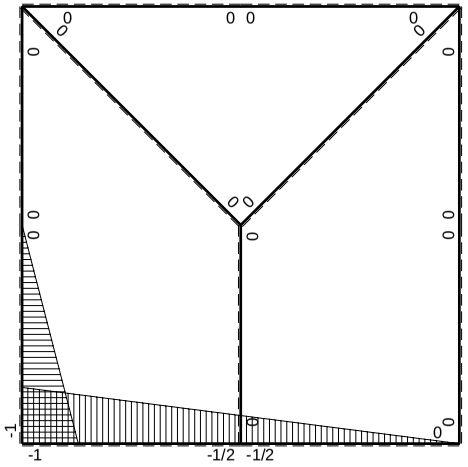

0
0 0
1
0
0
1
0
0
0
-5/2
-5/2
0 0
-5/6
1/2
1/3
0
0
-5/6
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237122 ESΣ04.xxxx.074

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PROCEDIMENTO E RISULTATI 237122 ESΣ04.xxxx.074

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-3/4Fx$	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

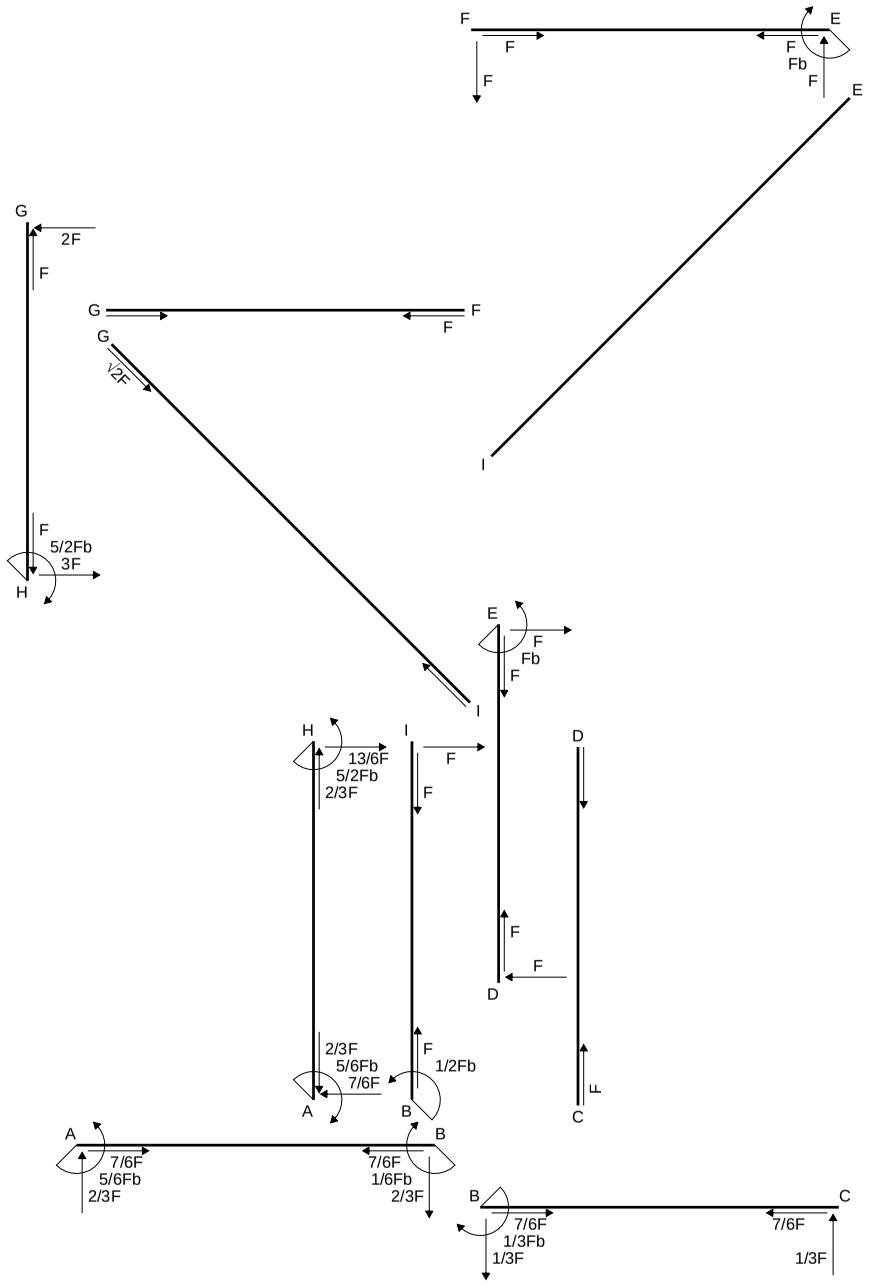
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

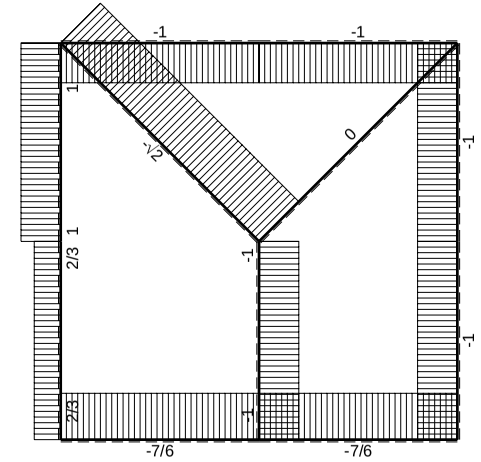
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

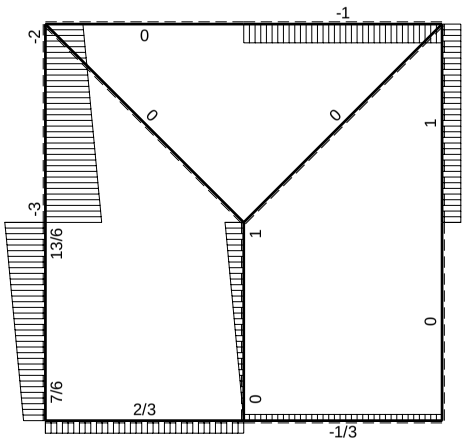
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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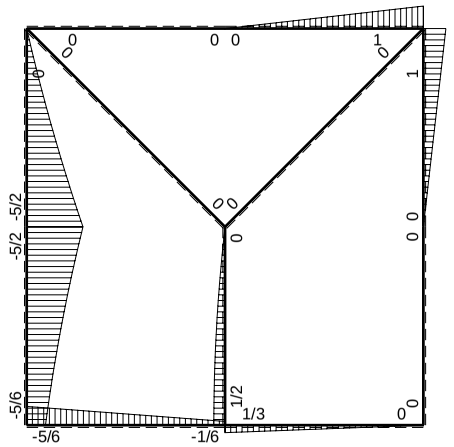

F
E
F
F
Fb
F
E
G
2F
F
G
F
F
G
½F
I
F
5/2Fb
3F
H
E
F
Fb
F
H
I
D
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5/2Fb
2/3F
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F
D
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5/6Fb
7/6F
F
1/2Fb
A
B
C
F
A
B
7/6F
5/6Fb
2/3F
7/6F
1/6Fb
2/3F
B
C
7/6F
1/3Fb
1/3F
7/6F
1/3F


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-1
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-7/6

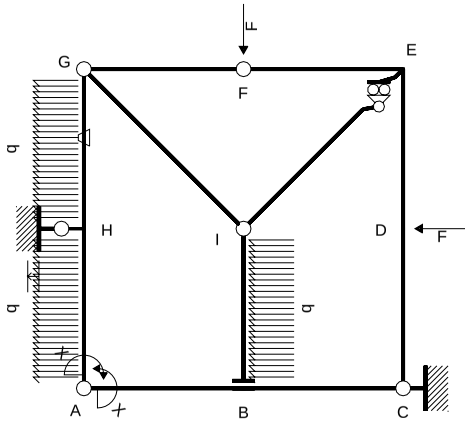

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7/6
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0
-1/3

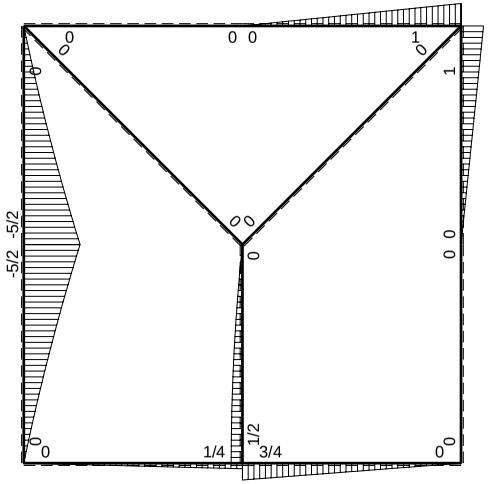
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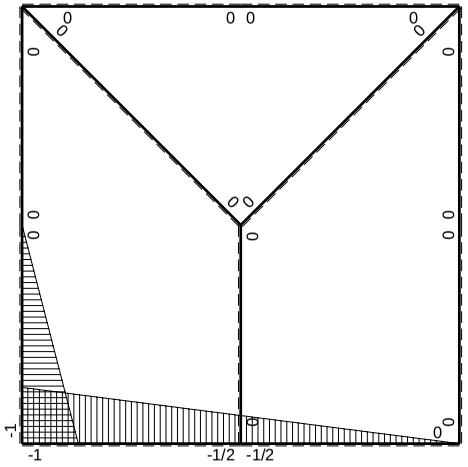

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-5/2
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-5/6
1/2
1/3
0
0
-5/6
-1/6

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238160 ESΣ04.xxxx.075

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

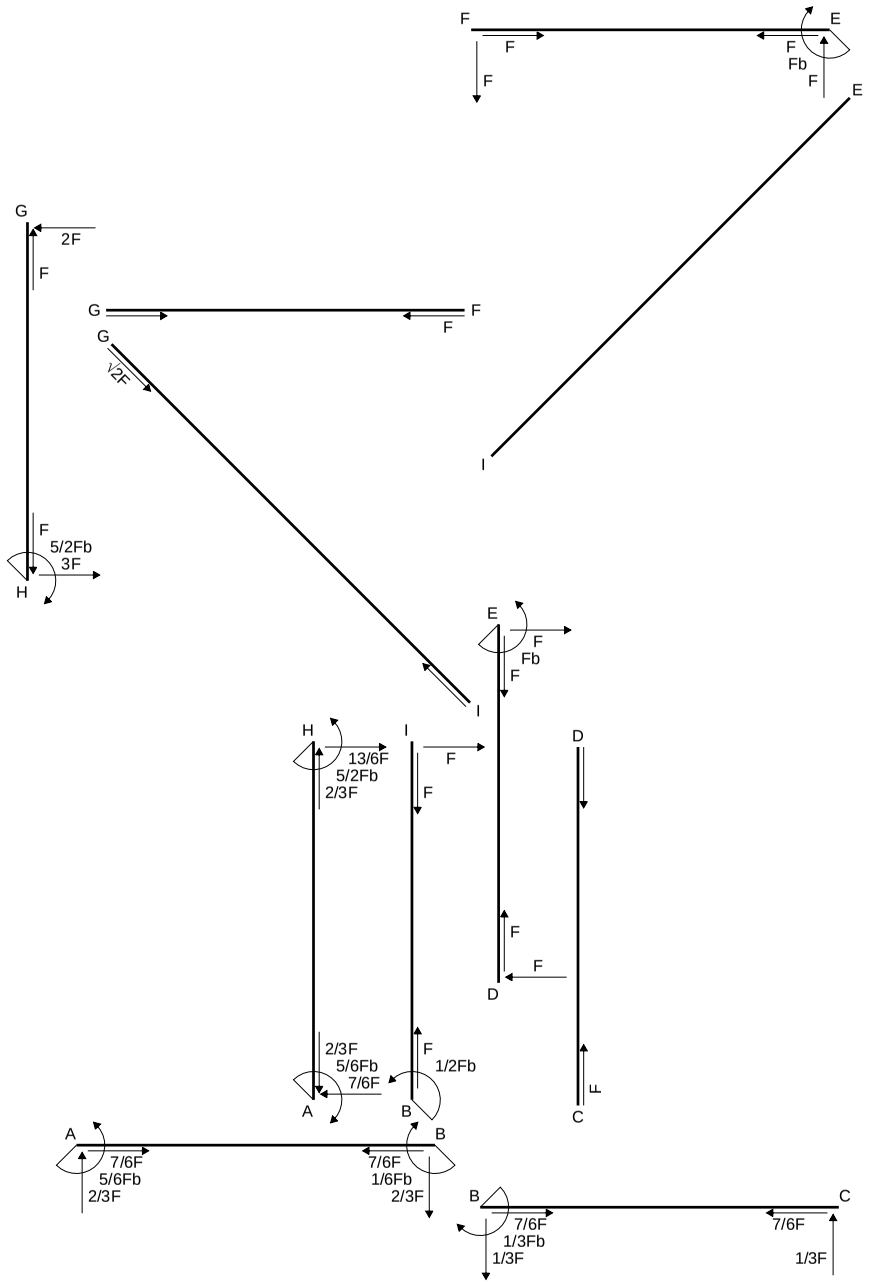
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

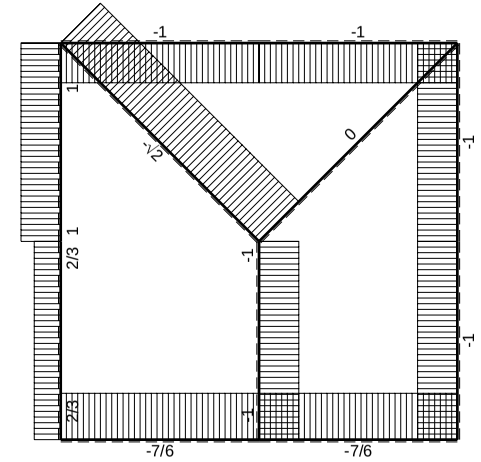
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

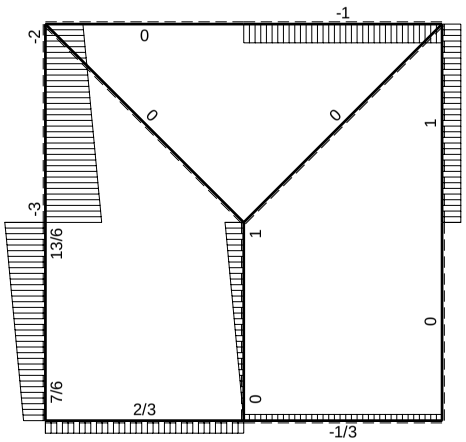
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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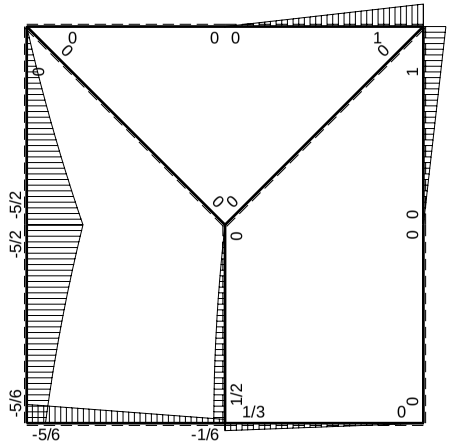

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1/3Fb
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7/6F
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-7/6

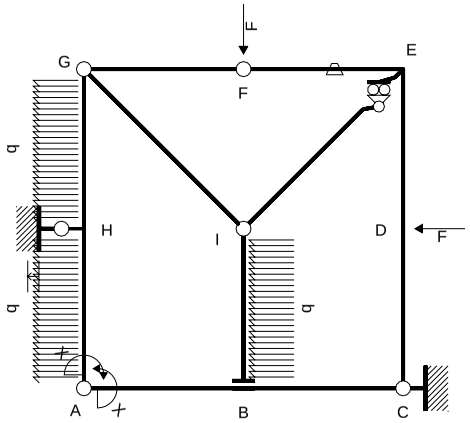

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13/6
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7/6
2/3
0
-1/3

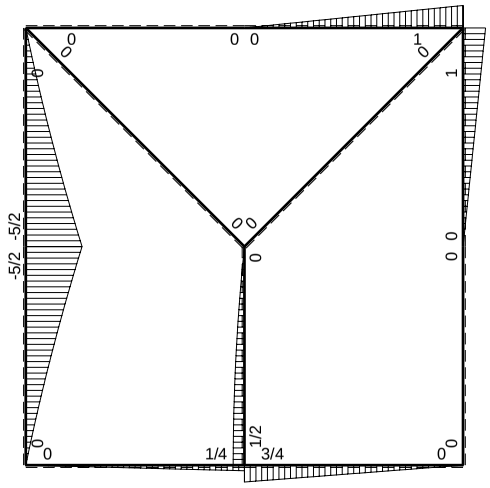
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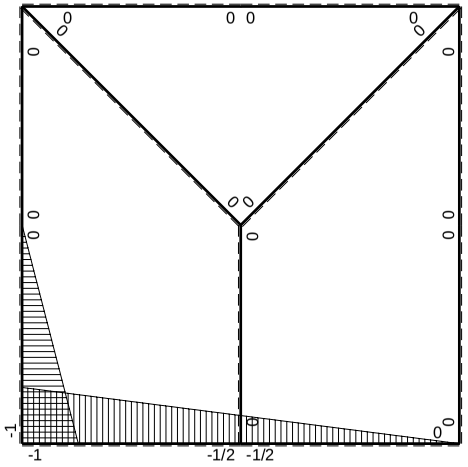

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-5/2
-5/2
0 0
-5/6
1/2
1/3
0
0
-5/6
-1/6

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212411 ESΣ04.xxxx.076

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PROCEDIMENTO E RISULTATI 212411 ESΣ04.xxxx.076

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

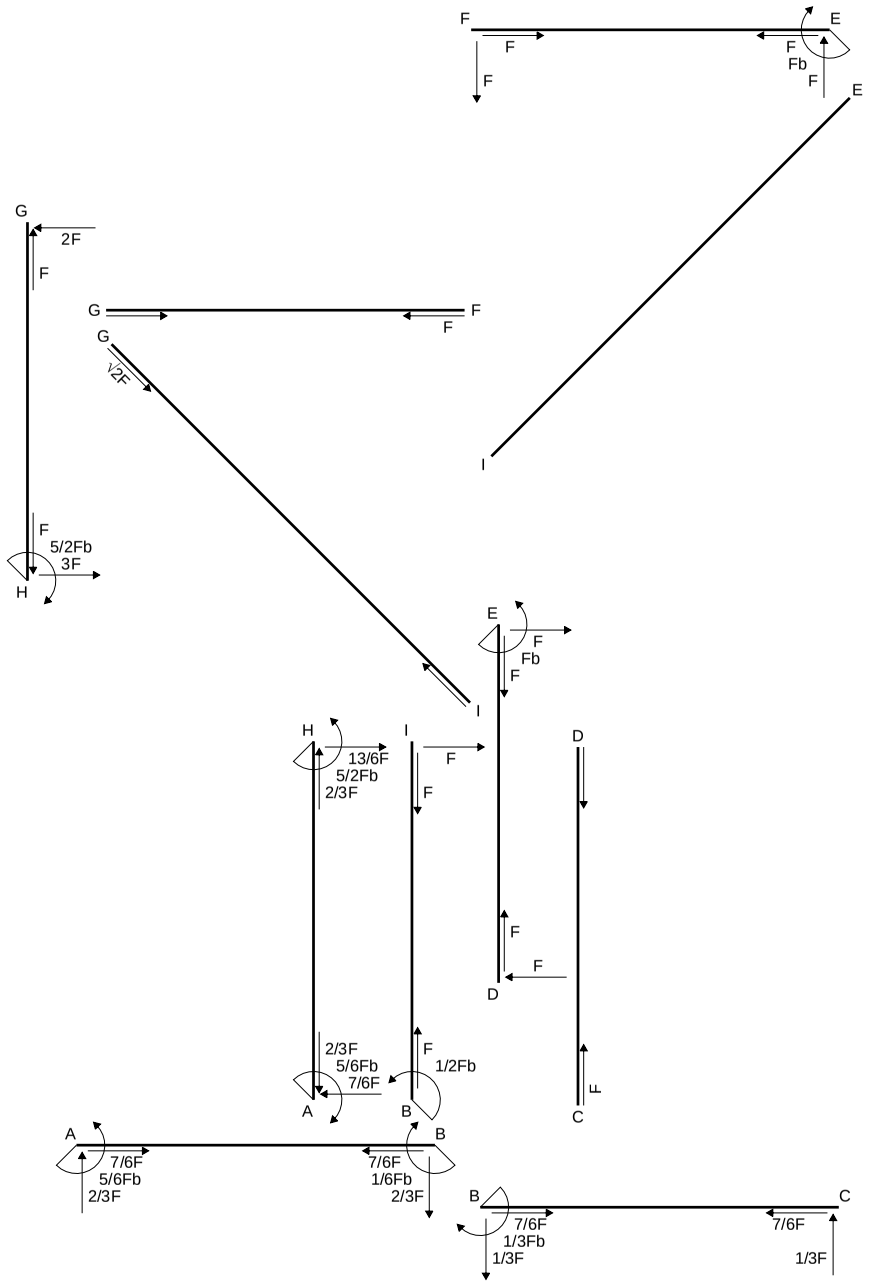
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

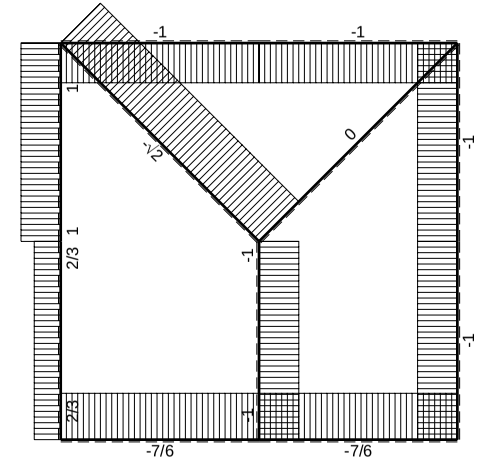
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

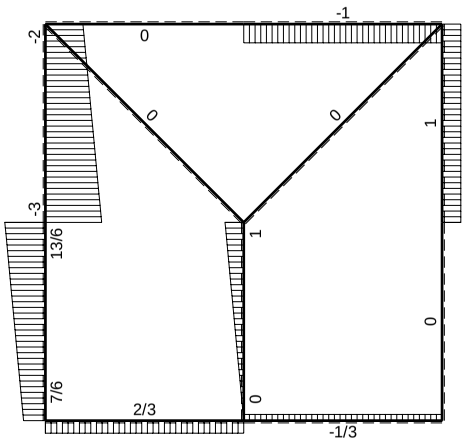
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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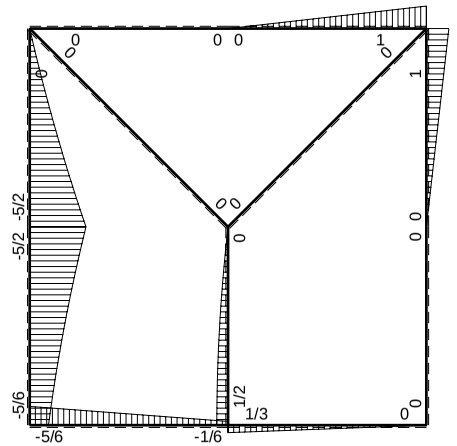

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G
½F
I
F
5/2Fb
3F
H
E
F
Fb
F
I
H
13/6F
5/2Fb
2/3F
I
F
F
D
F
F
D
2/3F
5/6Fb
7/6F
F
1/2Fb
A
B
F
C
A
B
7/6F
5/6Fb
2/3F
7/6F
1/6Fb
2/3F
B
C
7/6F
1/3Fb
1/3F
7/6F
1/3F


-1
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1
½
0
-1
2/3 1
-1
-1
2/3
1
-7/6
-7/6

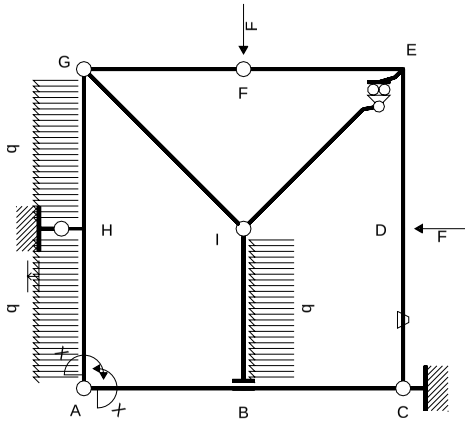

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0
0
1
-3
13/6
1
0
7/6
2/3
0
-1/3

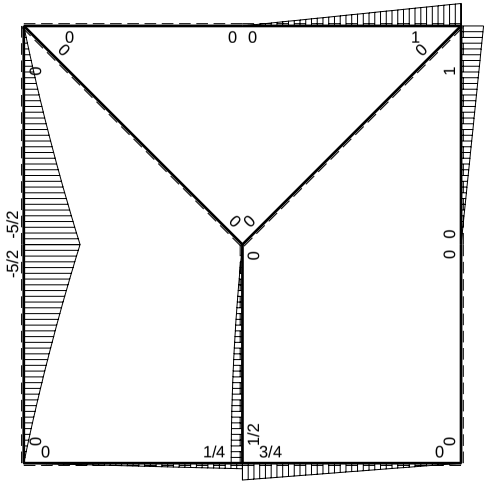
F

F

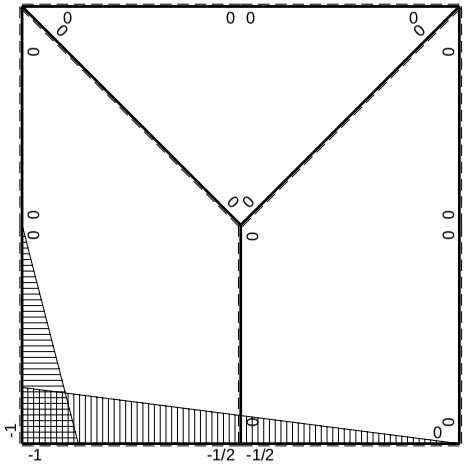

0
0 0
1
0
0
1
0
0
0
-5/2
-5/2
0 0
-5/6
1/2
1/3
0
0
-5/6
-1/6

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-3/4Fx$	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	-Fb/EJ	0	0	0	0+0	0
DC b	0	0	Fb/EJ	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/6Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

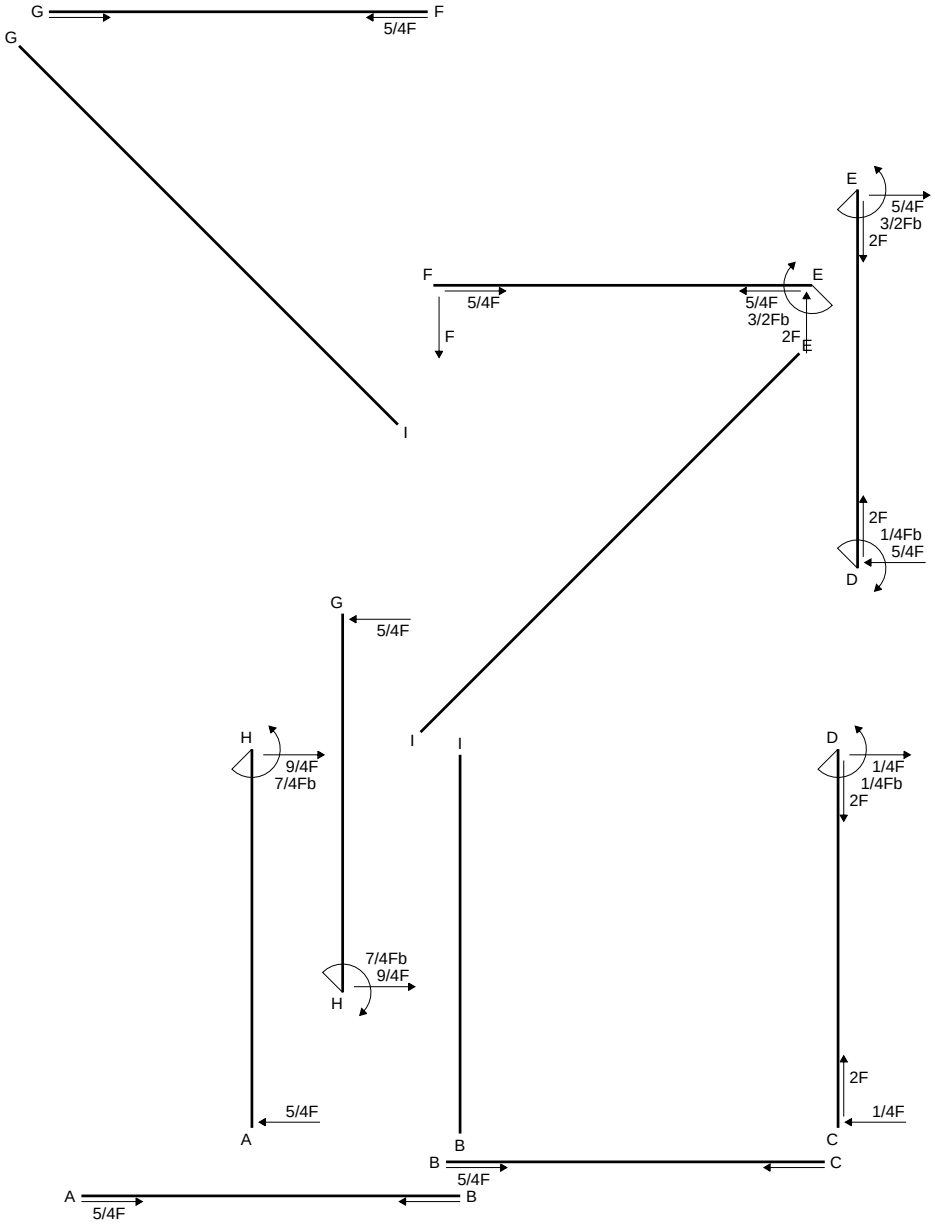
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

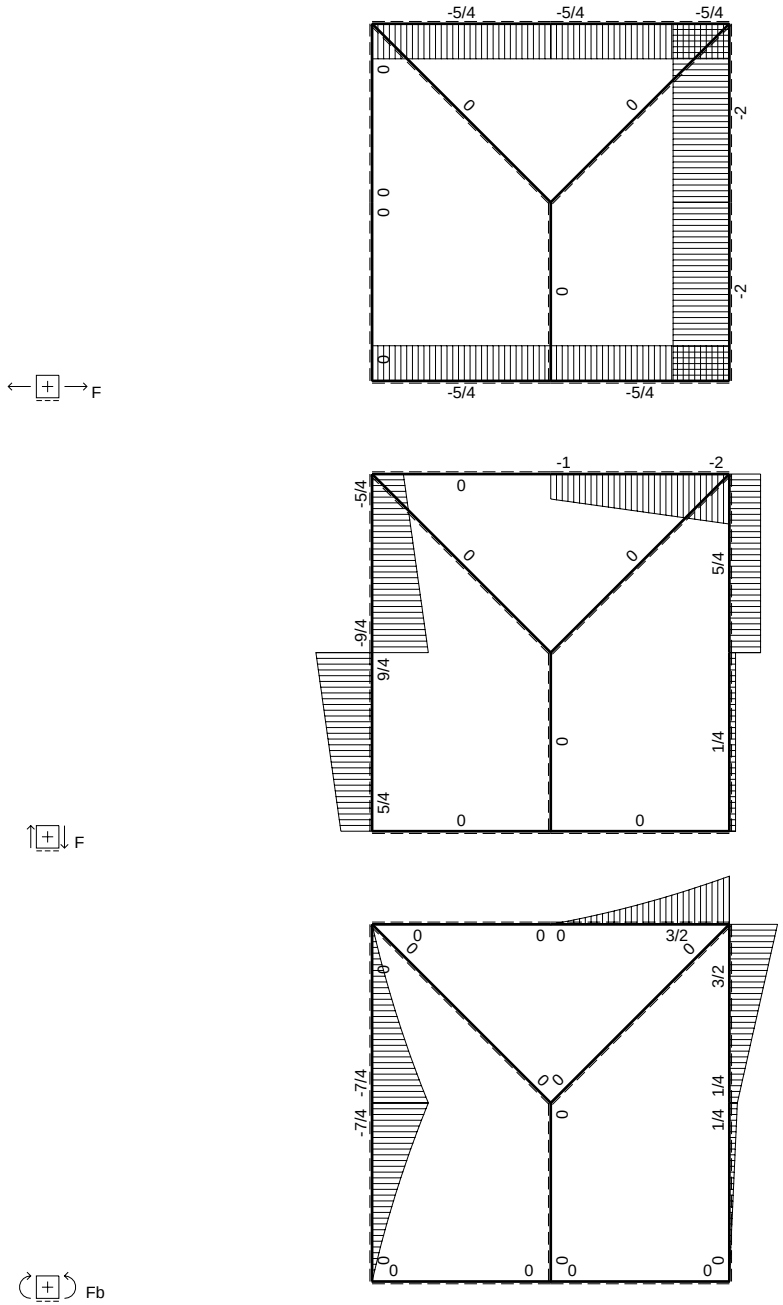
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

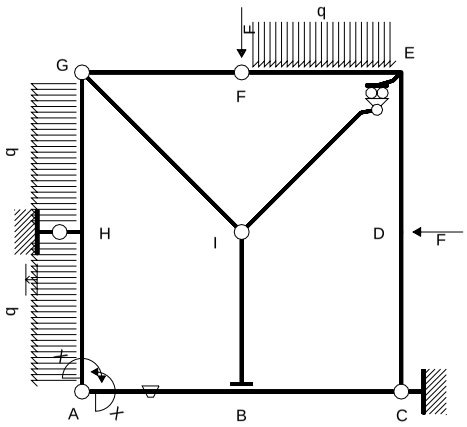
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

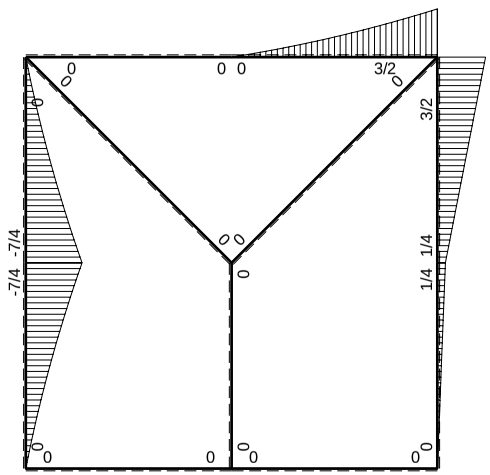
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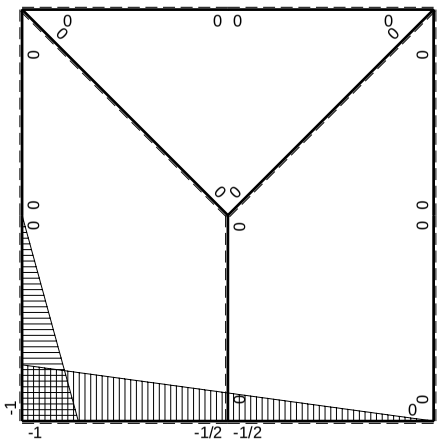

G
F
5/4F
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E
5/4F
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3/2Fb
2F
F
2F
1/4Fb
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G
5/4F
H
9/4F
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H
2F
1/4F
5/4F
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B
C
B
C
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A
B
5/4F


F
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F
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5/4
-9/4
9/4
1/4
5/4
F
3/2
3/2
-7/4
-7/4
1/4
1/4
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981024 ESΣ04.xxxx.078

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	-Fb/EJ	0	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(0+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	0	Fb/EJ	0	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						0	Xb/EJ
	iperstatica $X=W_{AB}$						0	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(1 - 1/2\ x/b\ \right)\theta\ \ dx = \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/2\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

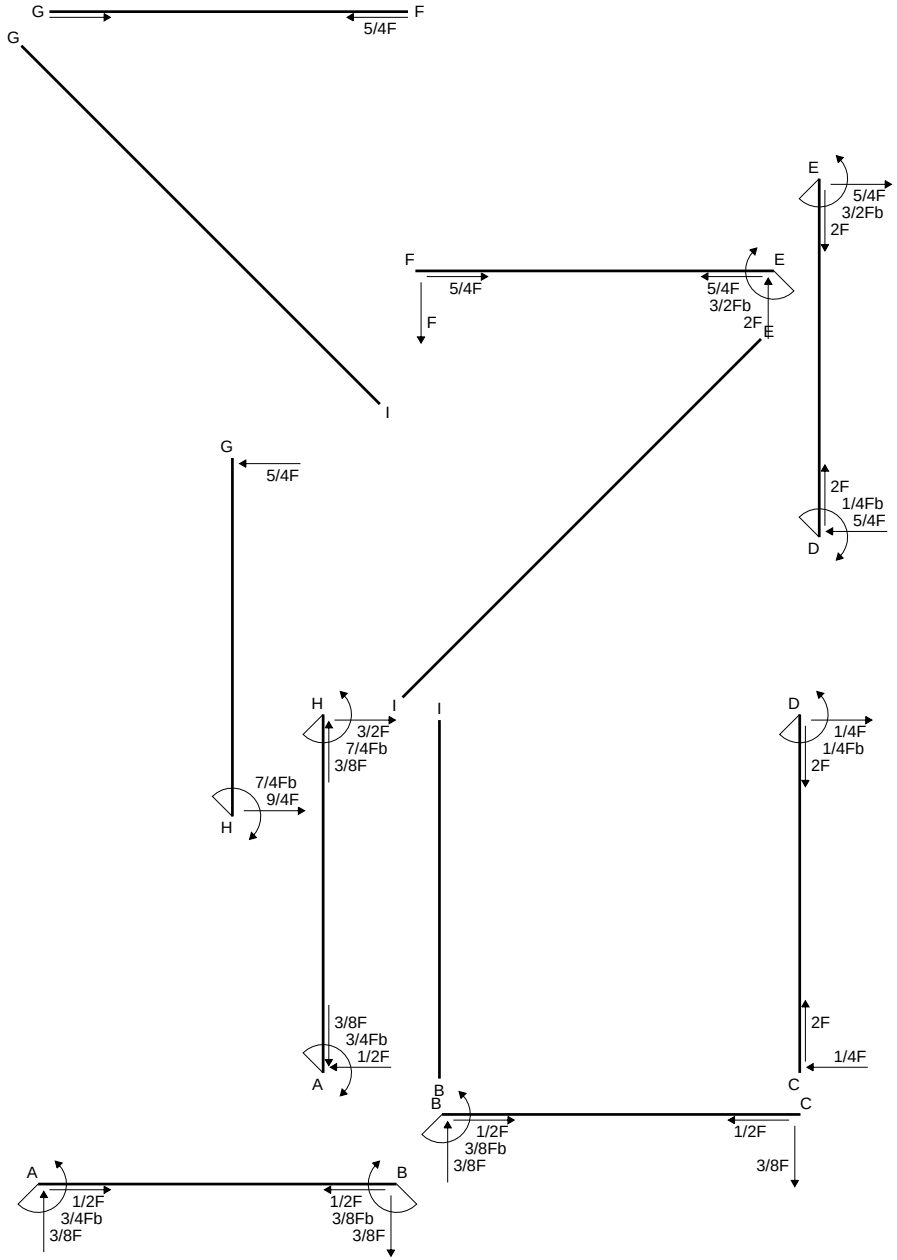
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

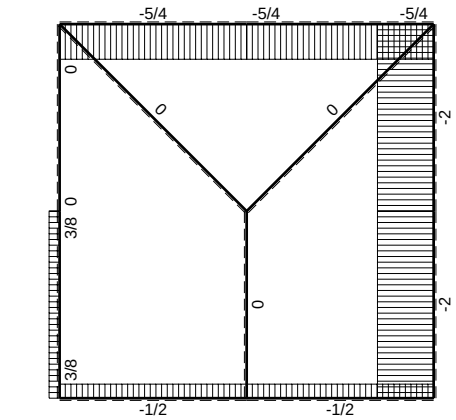
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

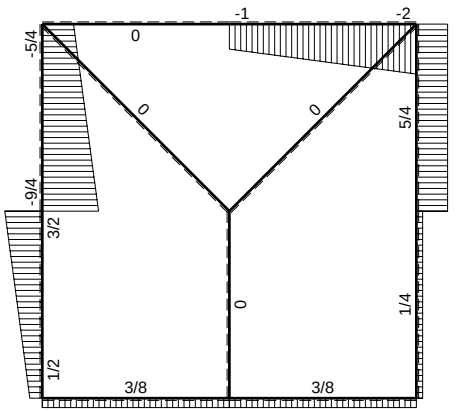
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

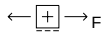
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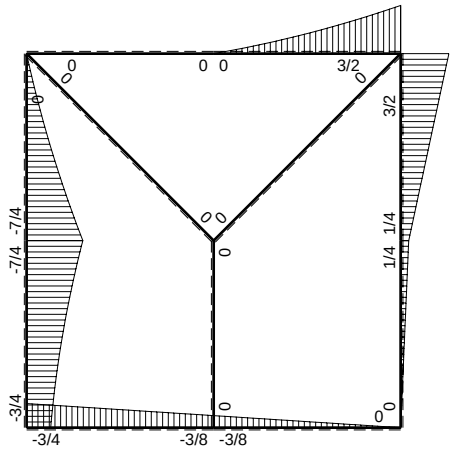






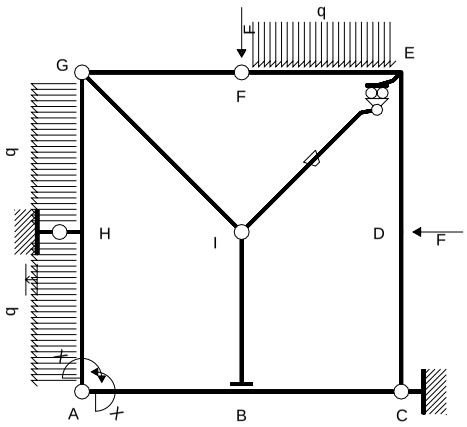


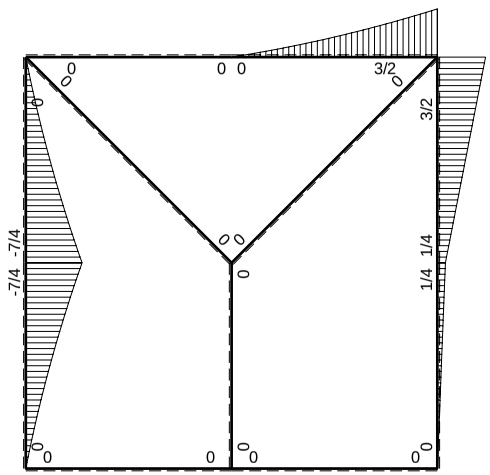




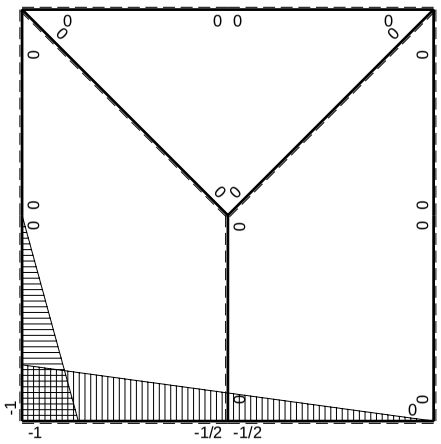


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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PROCEDIMENTO E RISULTATI 211257 ESΣ04.xxxx.079

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

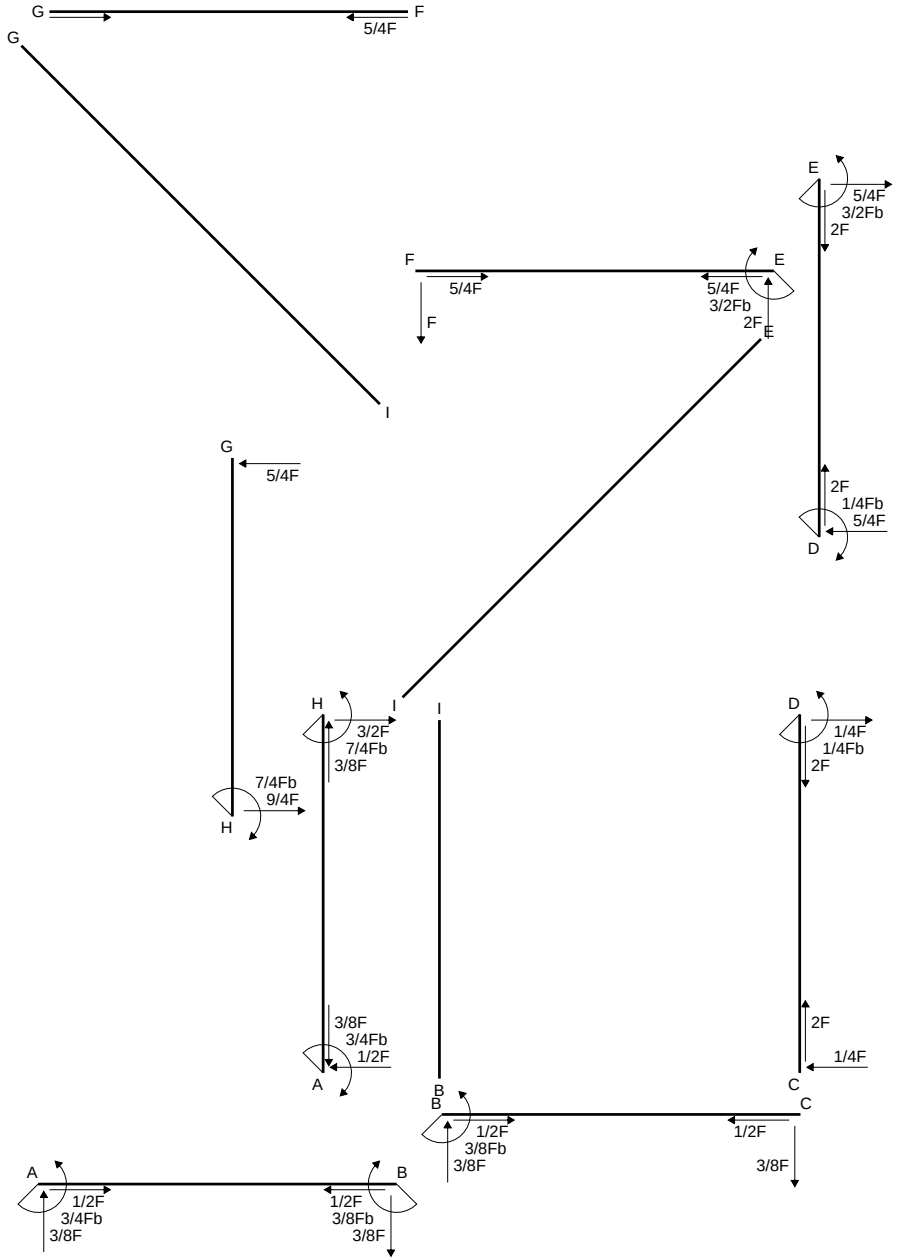
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

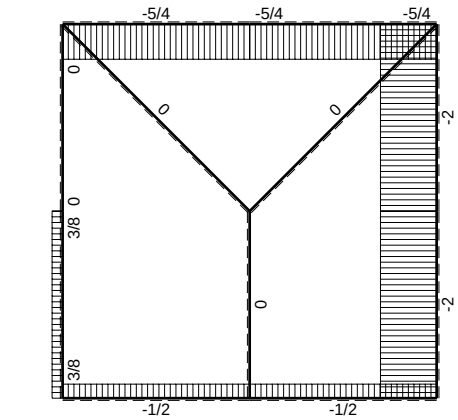
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

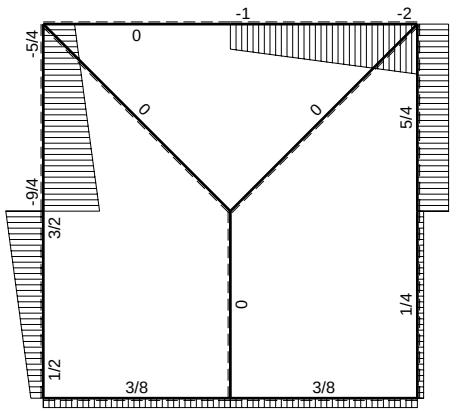
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

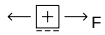
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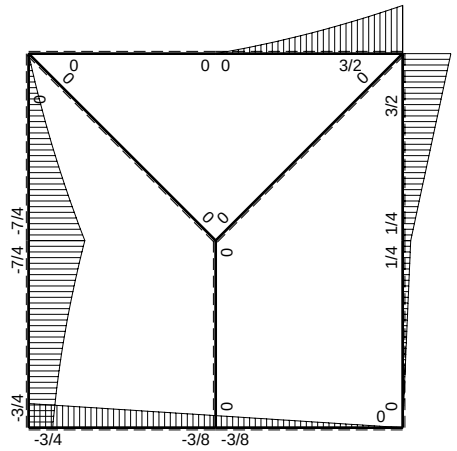






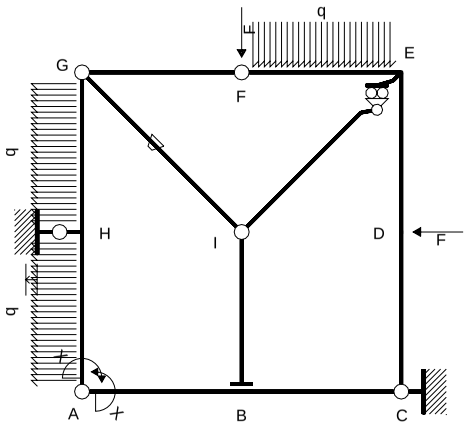


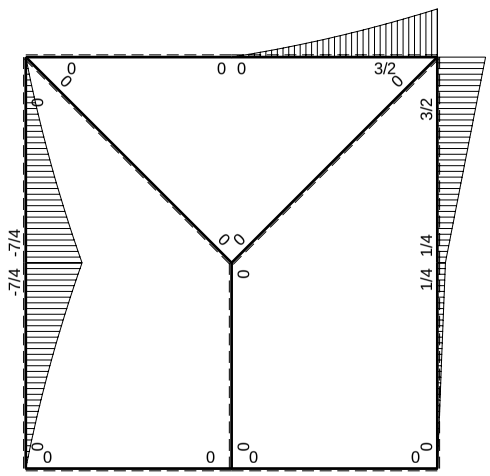




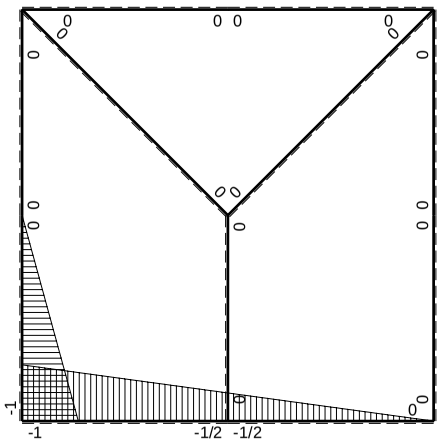


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238991 ESΣ04.xxxx.080

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

PROCEDIMENTO E RISULTATI 238991 ESΣ04.xxxx.080

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

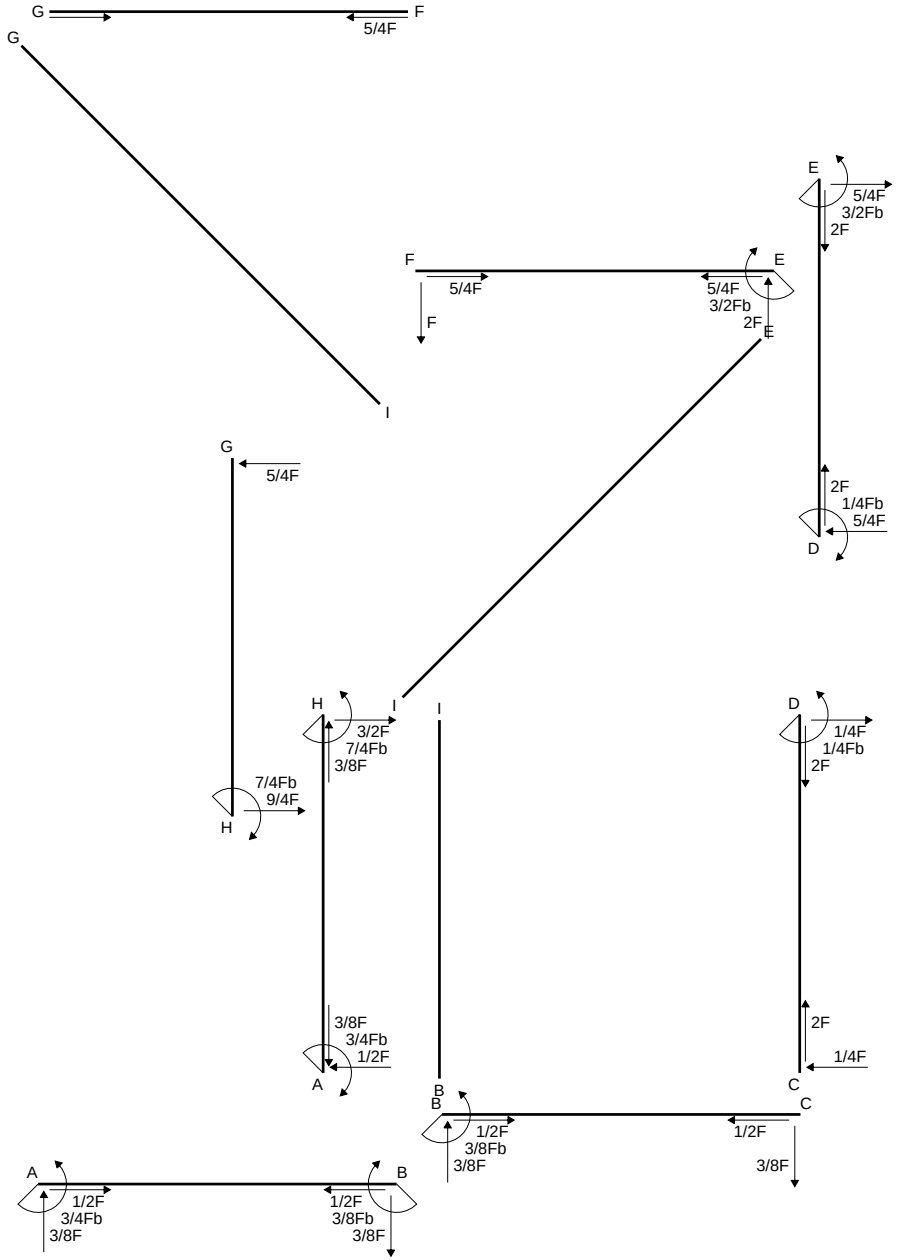
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

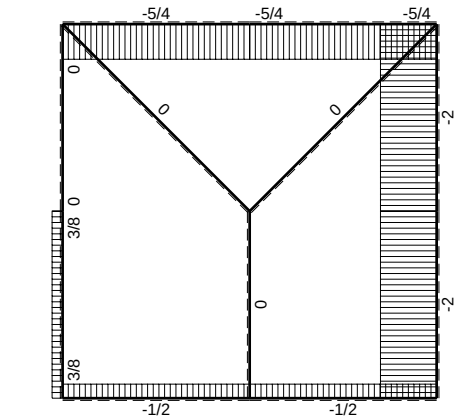
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

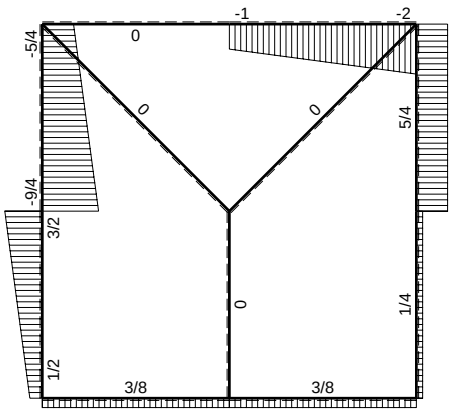
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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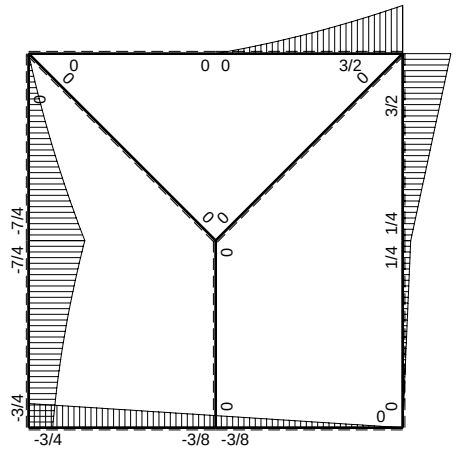

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F
5/4F
G
I
E
5/4F
3/2Fb
2F
F
5/4F
F
5/4F
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2F
E
E
2F
1/4Fb
5/4F
D
G
5/4F
H
I
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3/8F
7/4Fb
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1/2F
3/4Fb
3/8F
1/2F
3/8Fb
3/8F


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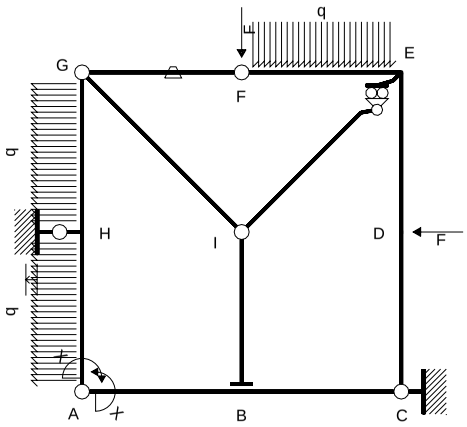

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1/4
1/2
3/8
3/8

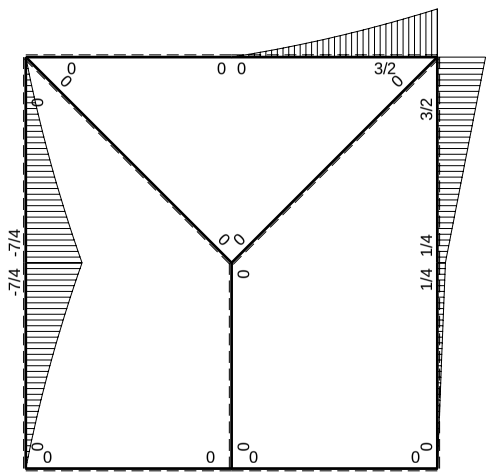
F

F

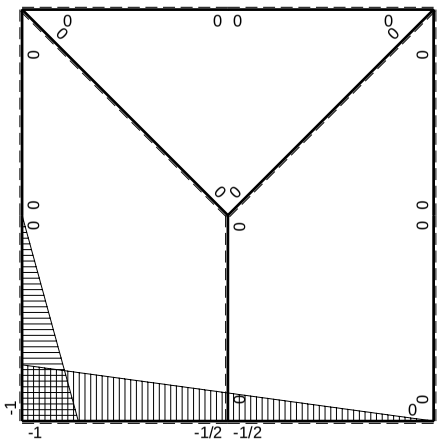

0
0 0
3/2
0
0
3/2
-7/4
-7/4
0
0
1/4
1/4
-3/4
0
0
0
-3/4
-3/8
-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

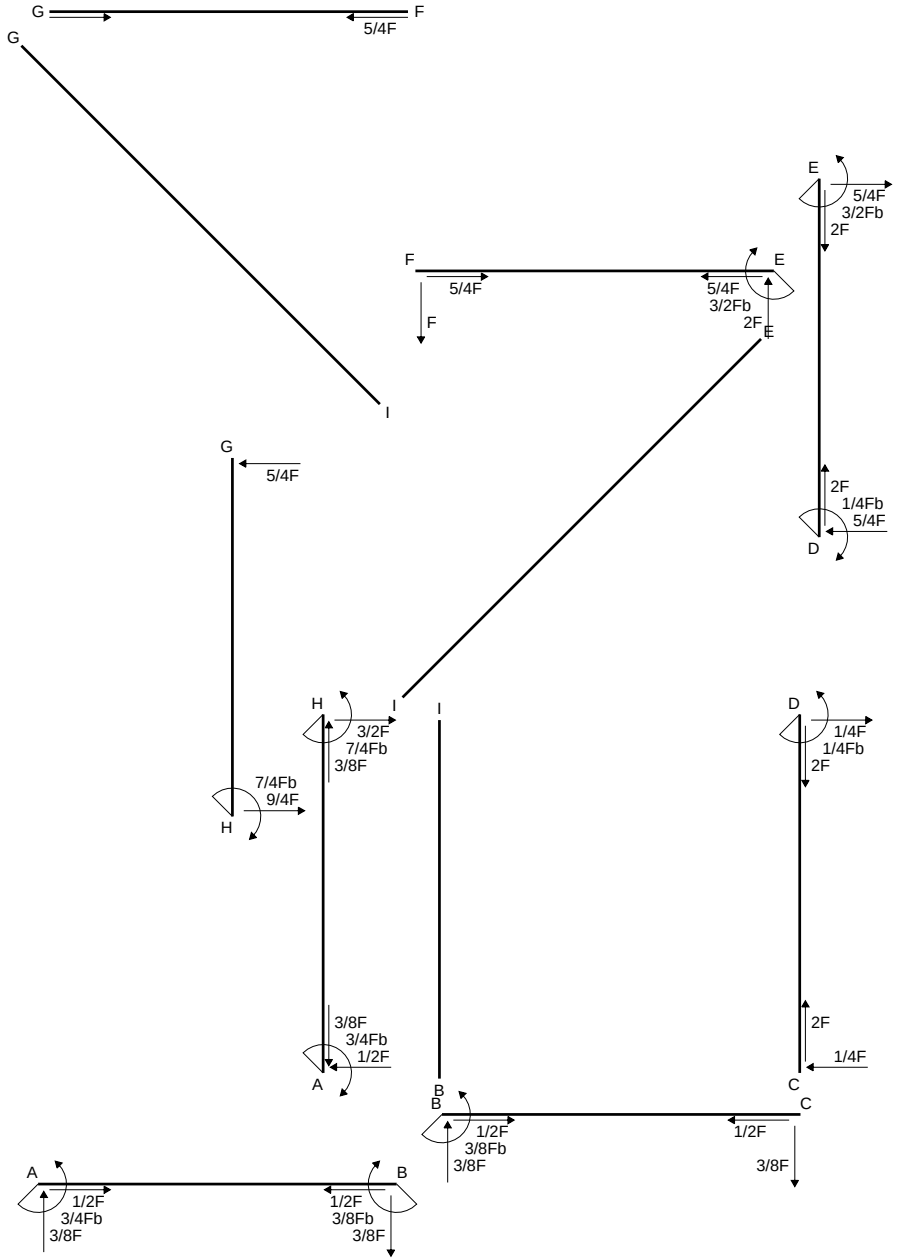
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

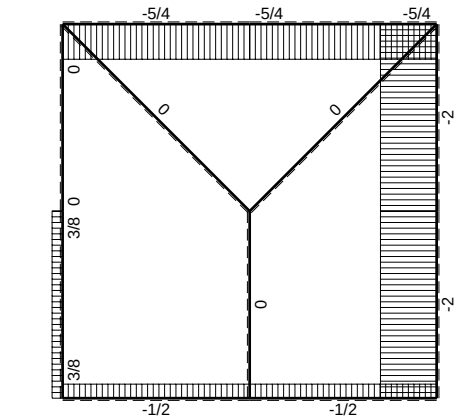
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

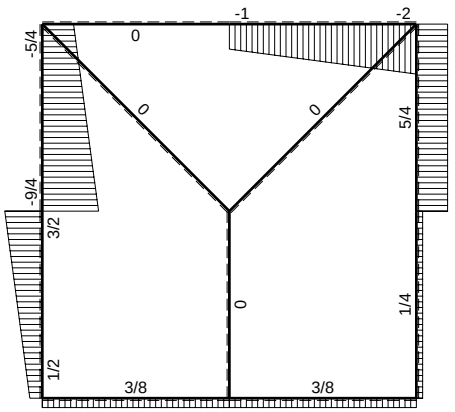
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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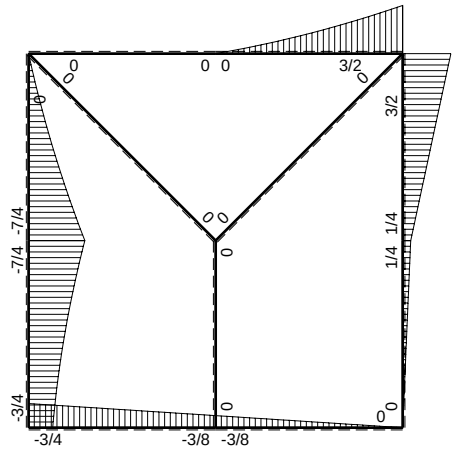

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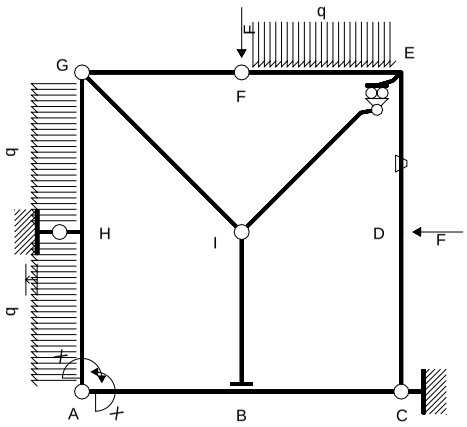

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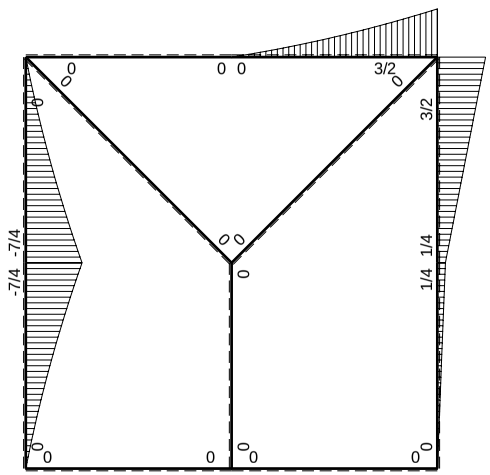
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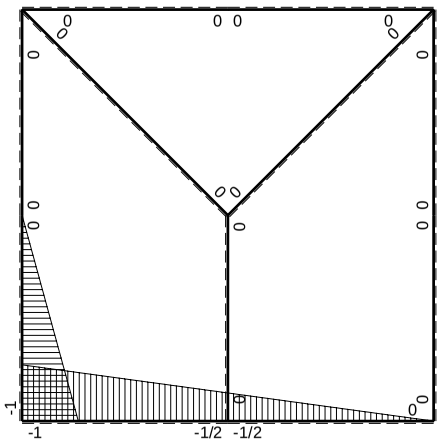

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-3/4
-3/8 -3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 252216 ESΣ04.xxxx.082

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

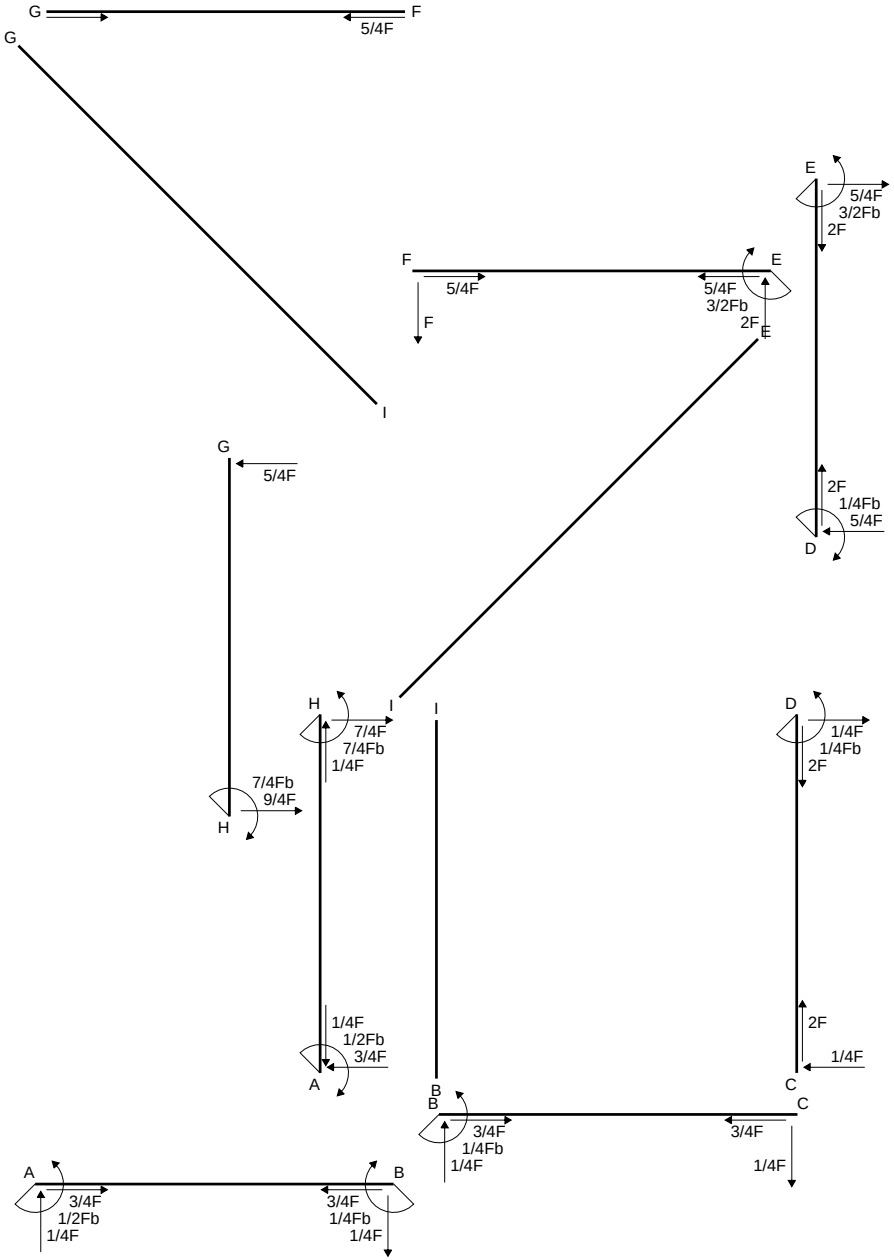
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

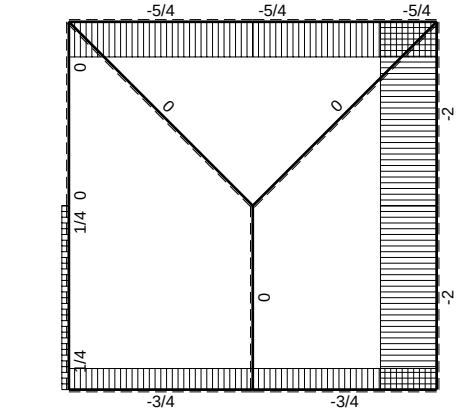
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

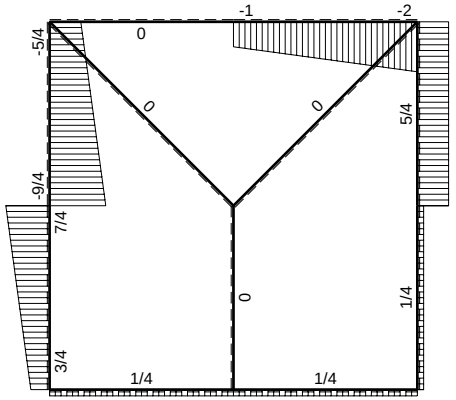
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

ESΣ04.xxxx.082

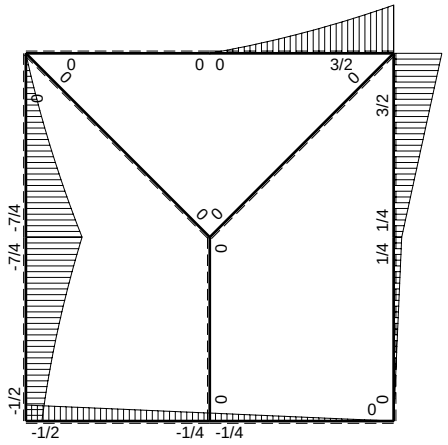
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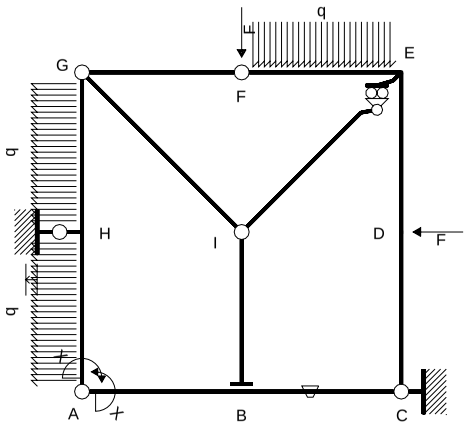

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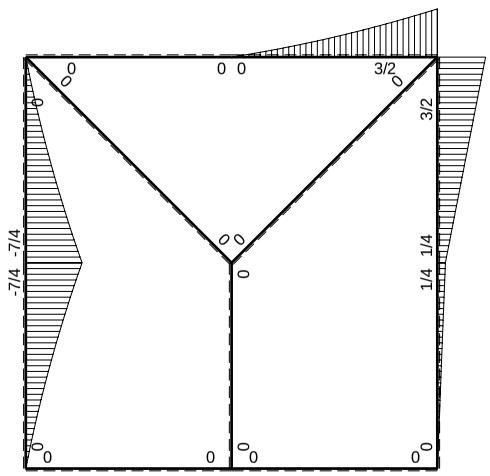

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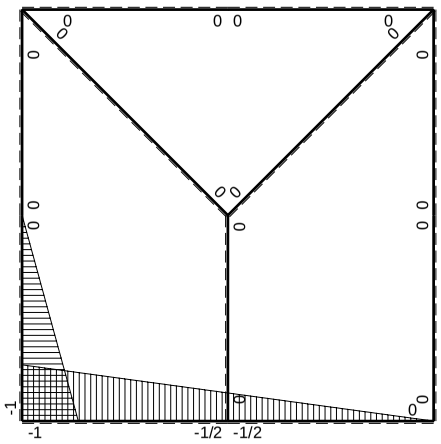

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236511 ESΣ04.xxxx.083

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	-Fb/EJ	0	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(0+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	0	Fb/EJ	0	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/2Fb	

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(1/2\ b - 1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/2\ x/b\ \right)\theta\ \ dx = \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

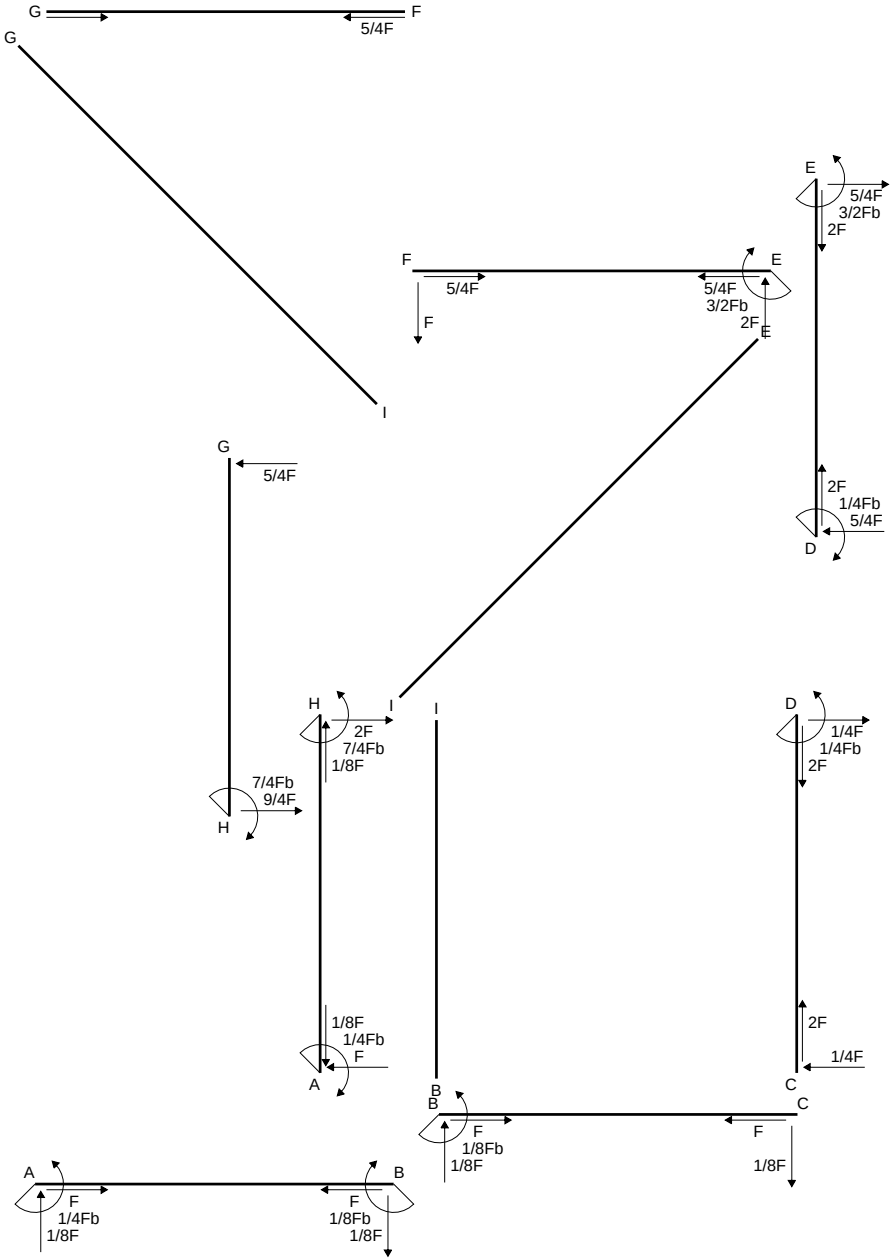
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

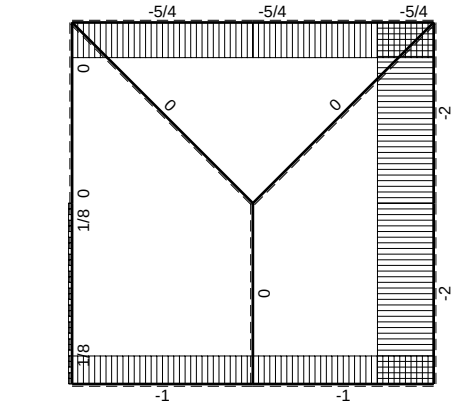
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

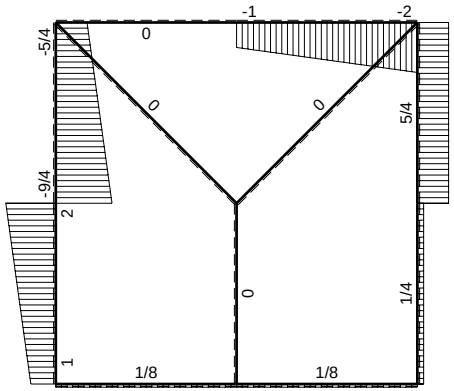
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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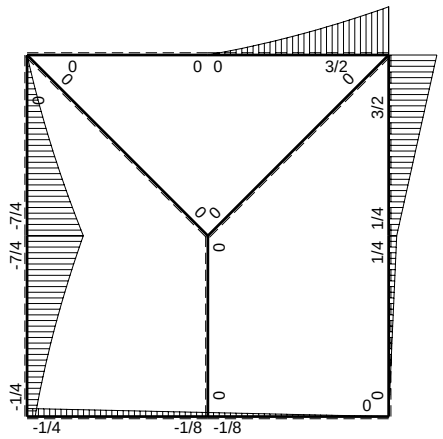

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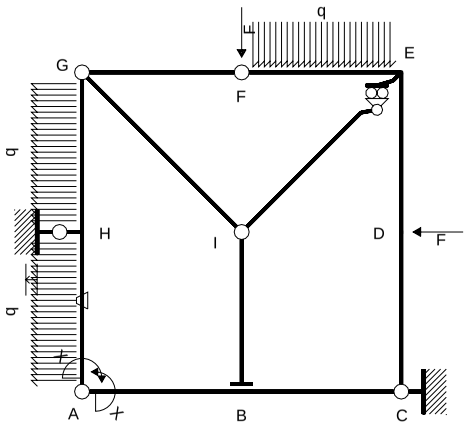

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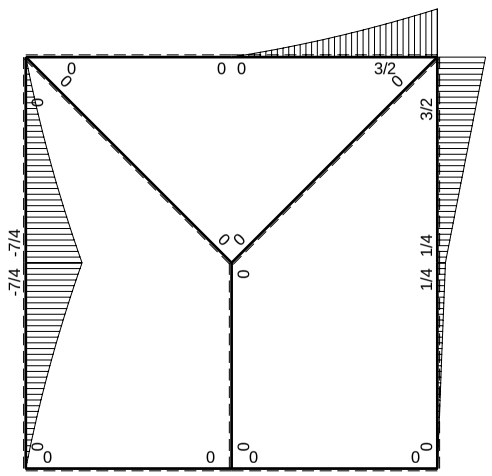
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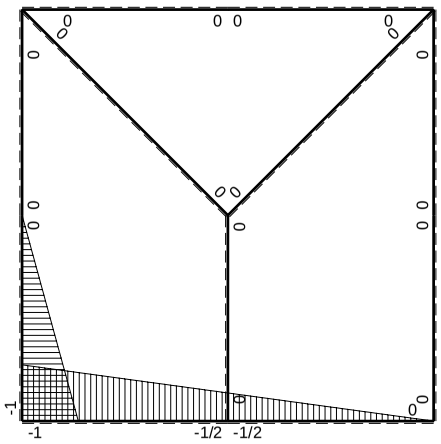

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236427 ESΣ04.xxxx.084

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	-Fb/EJ	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/4+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	Fb/EJ	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/4Fb	

PROCEDIMENTO E RISULTATI 236427 ESΣ04.xxxx.084

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\right)\theta\ \ dx$

$= \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b \theta$

$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ + \left(1/2\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

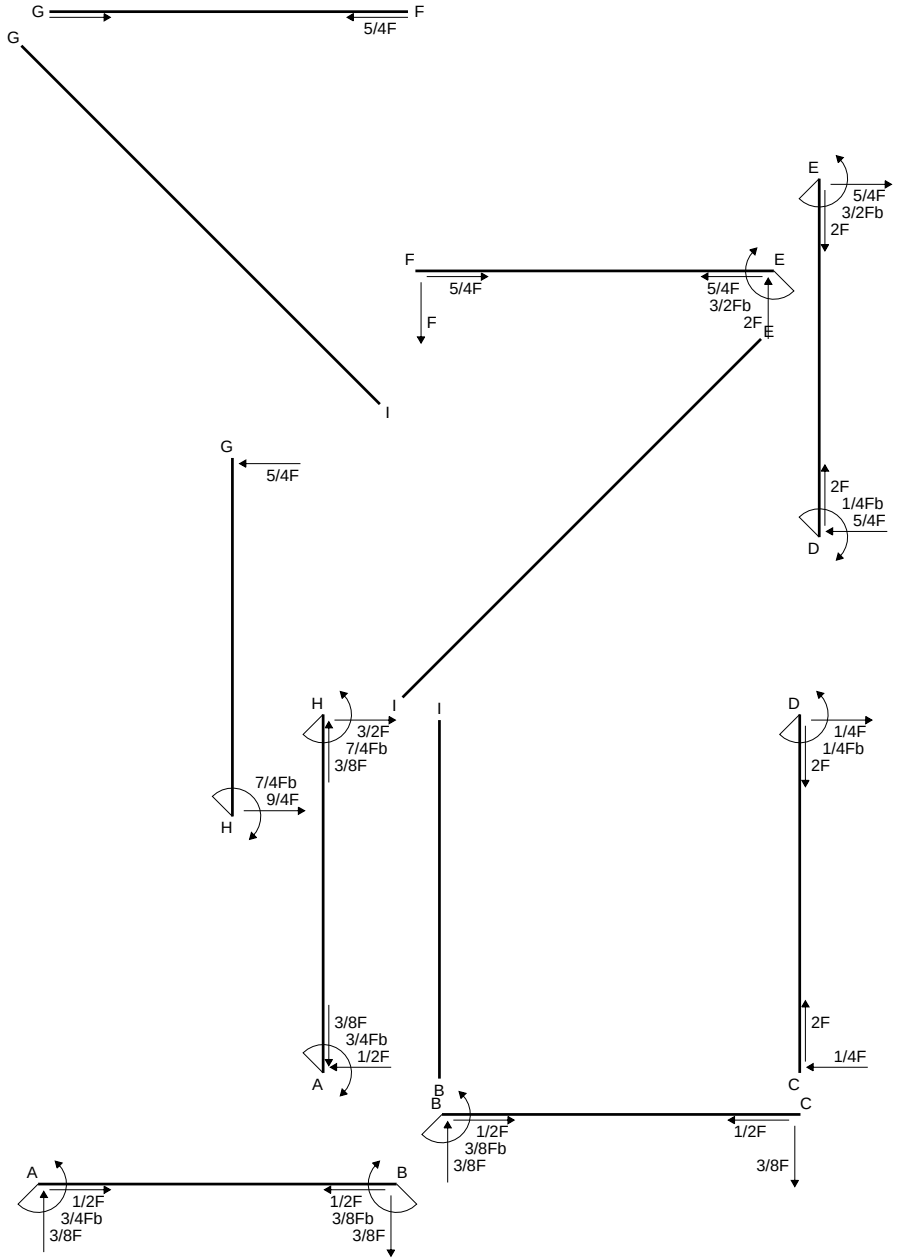
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\theta\ \ dx$

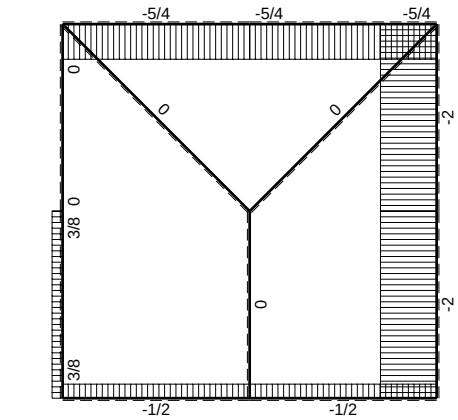
$= \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b \theta$

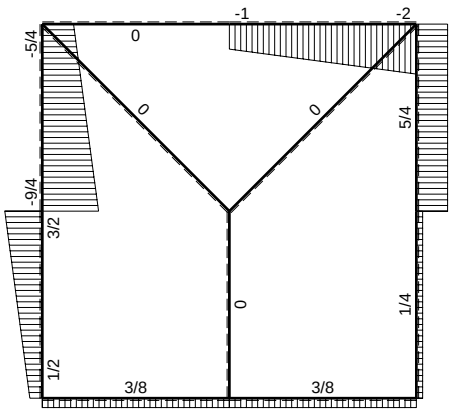
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

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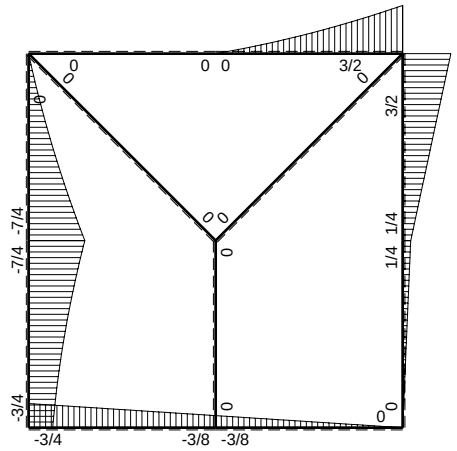

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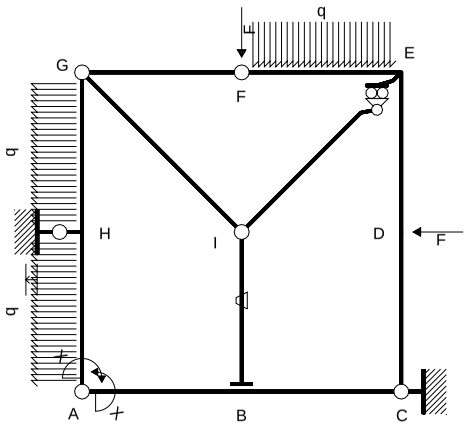

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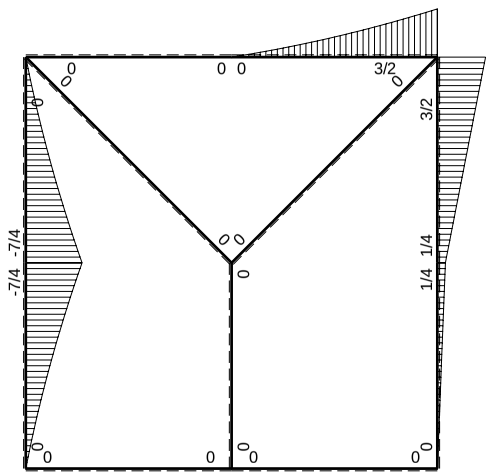
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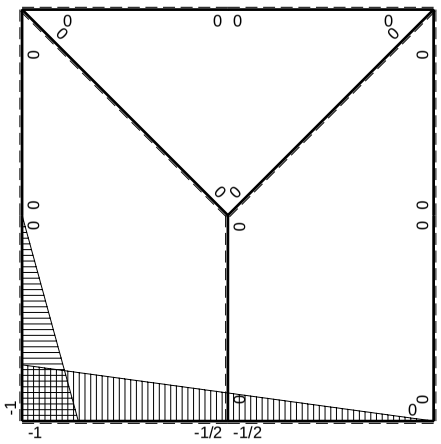

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237813 ESΣ04.xxxx.085

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PROCEDIMENTO E RISULTATI 237813 ESΣ04.xxxx.085

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

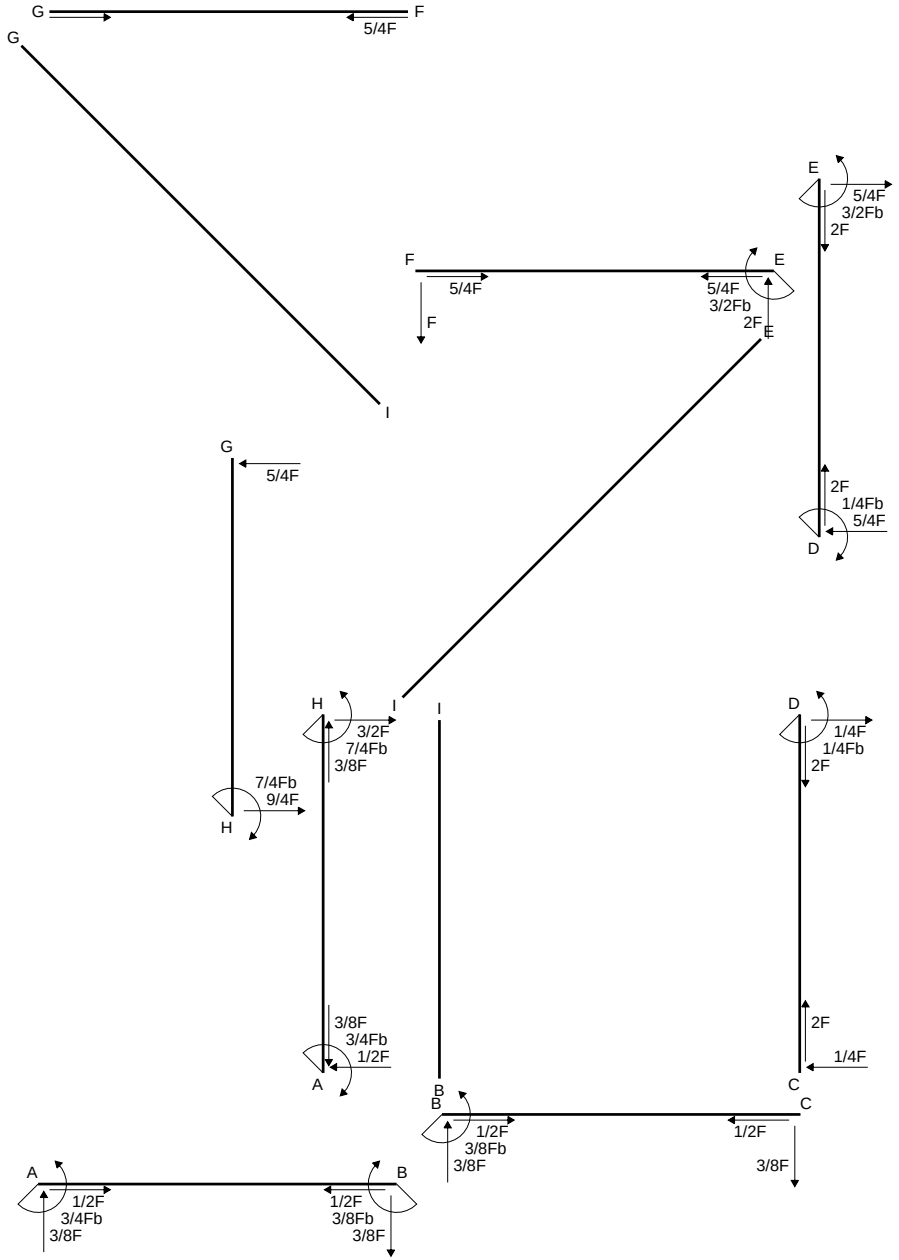
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

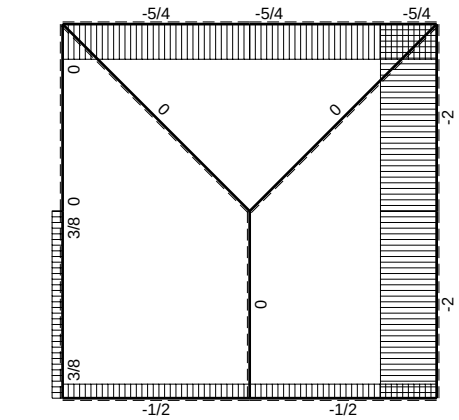
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

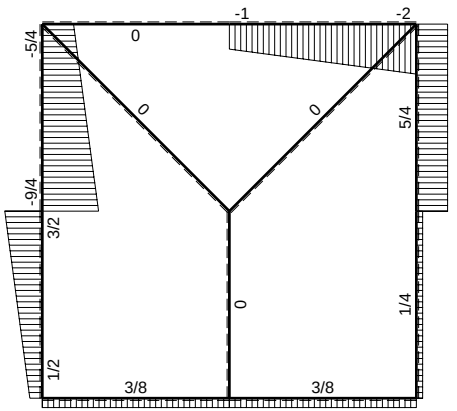
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

ESΣ04.xxxx.085

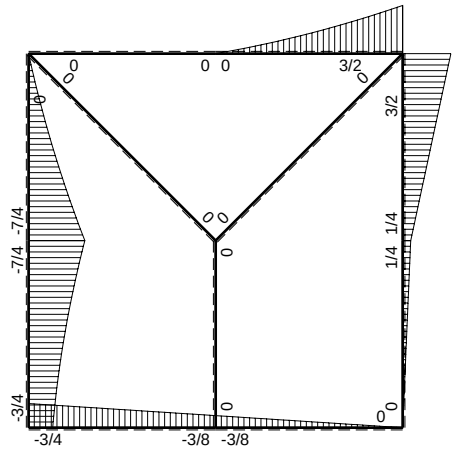
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9/4F
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2F
3/8F
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2F
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3/8Fb
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3/8Fb
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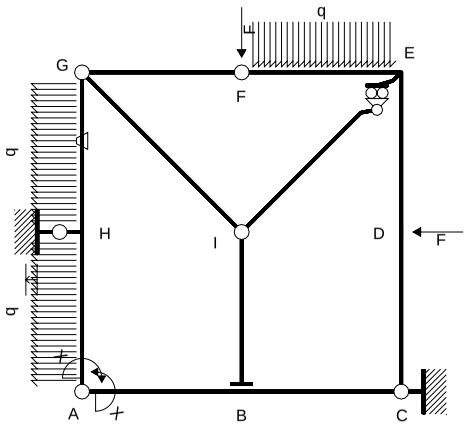

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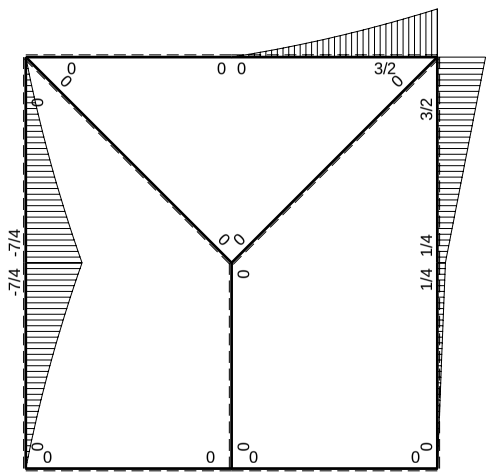

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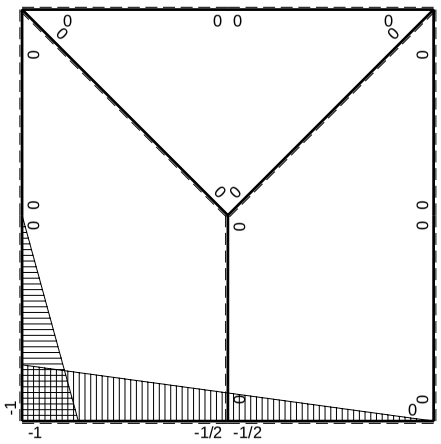

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237504 ESΣ04.xxxx.086

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

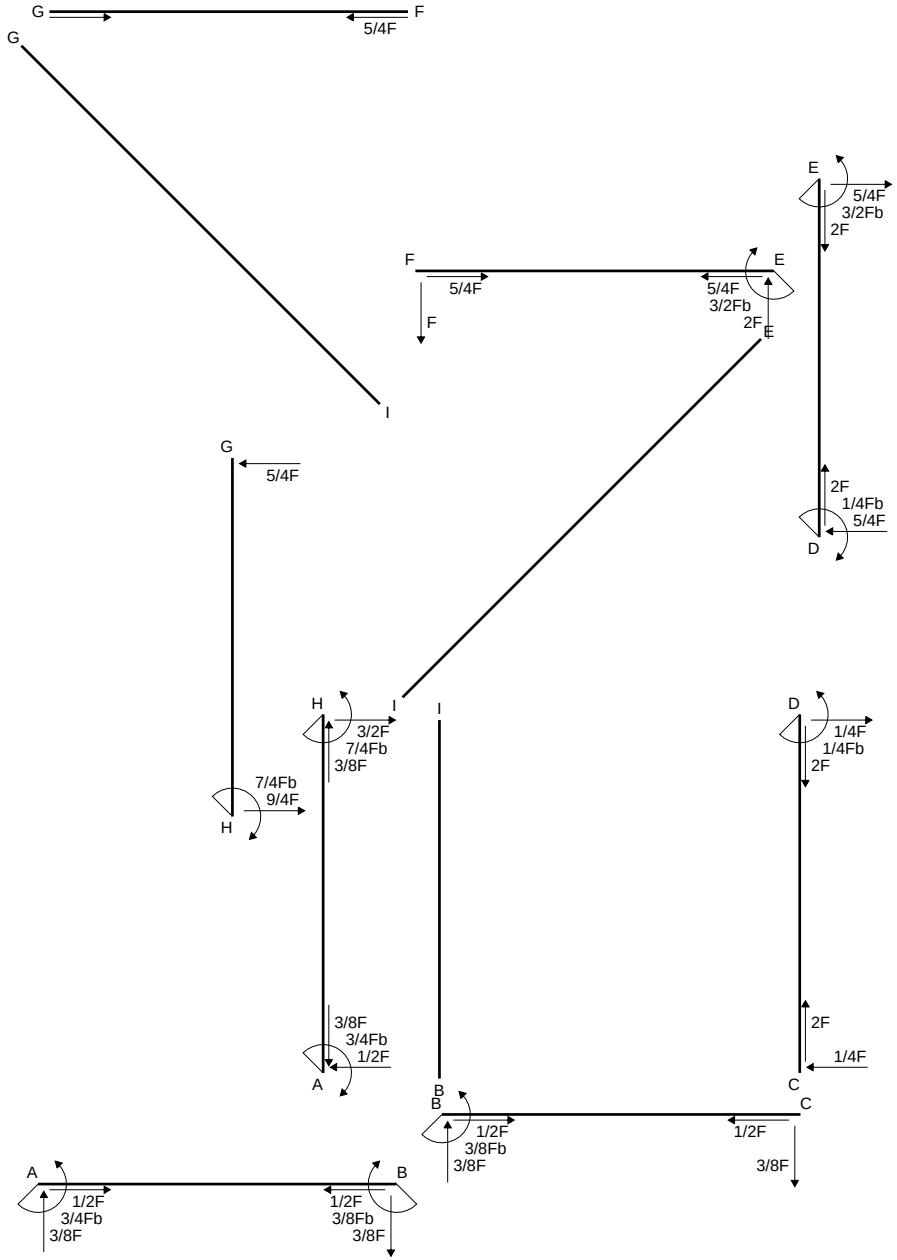
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

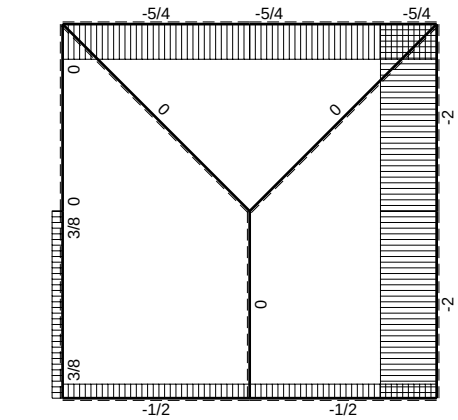
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

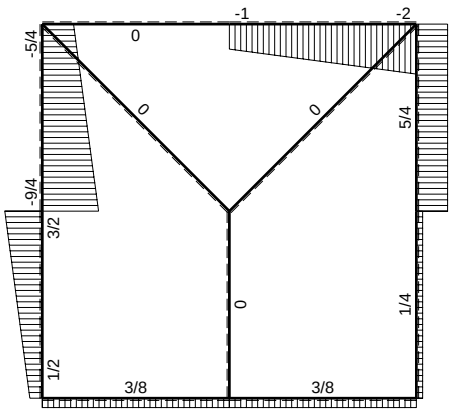
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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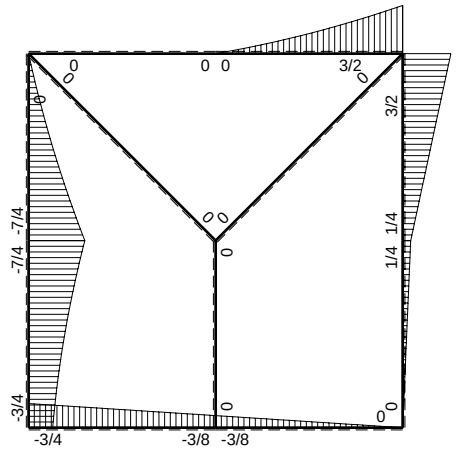

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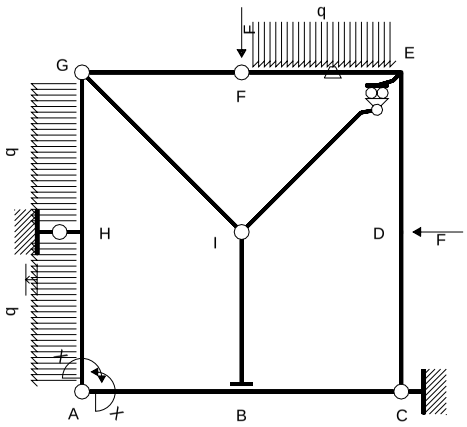

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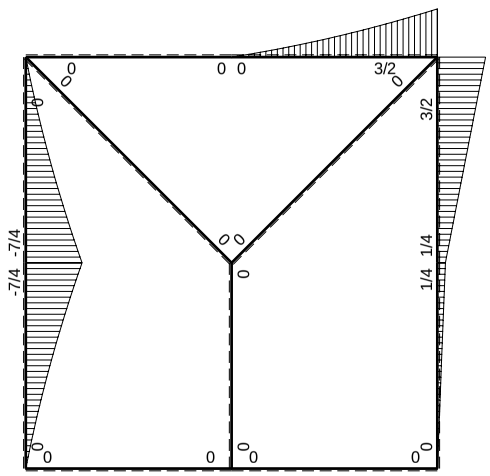
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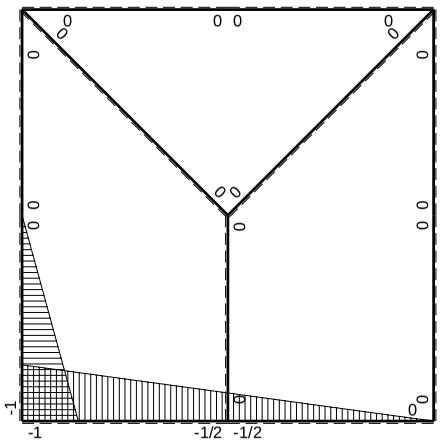

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-3/4
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-3/8

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237785 ESΣ04.xxxx.087

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PROCEDIMENTO E RISULTATI 237785 ESΣ04.xxxx.087

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

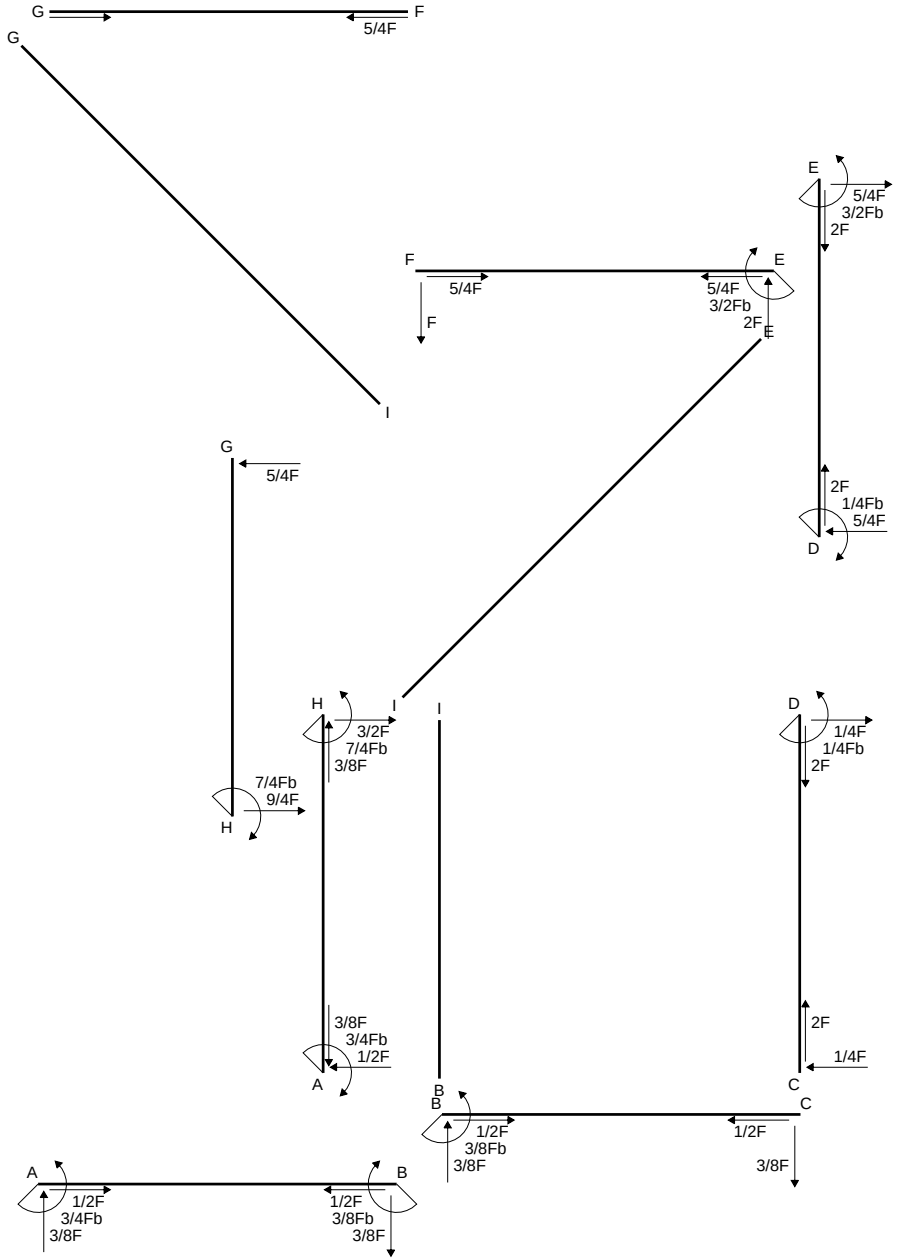
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

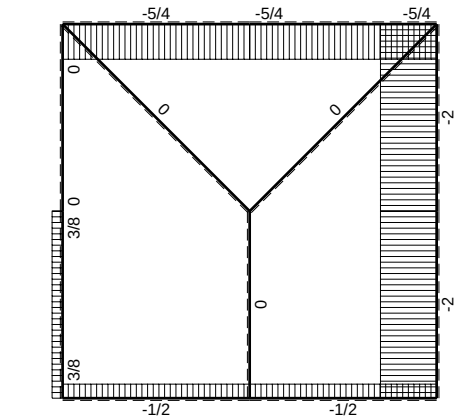
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

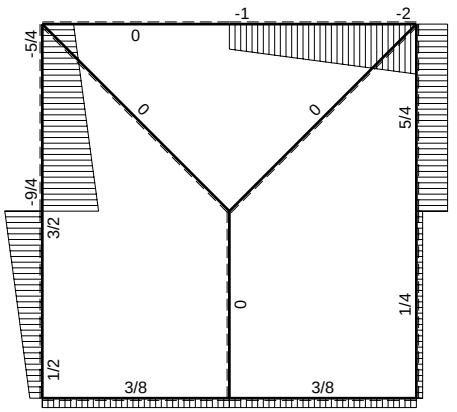
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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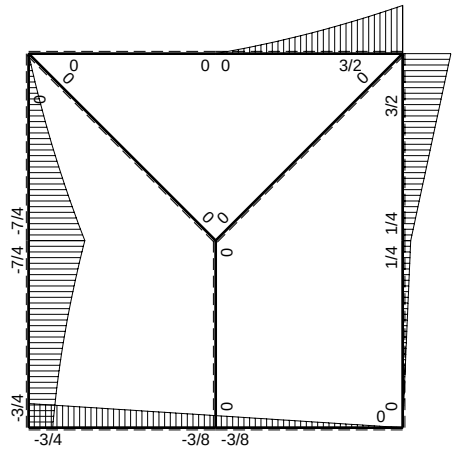

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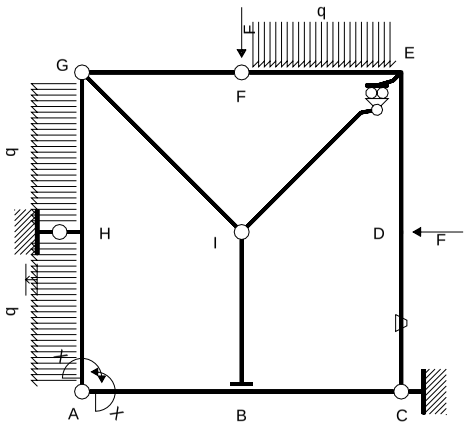

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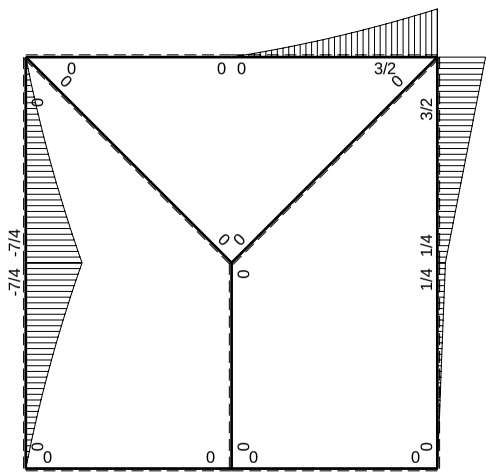
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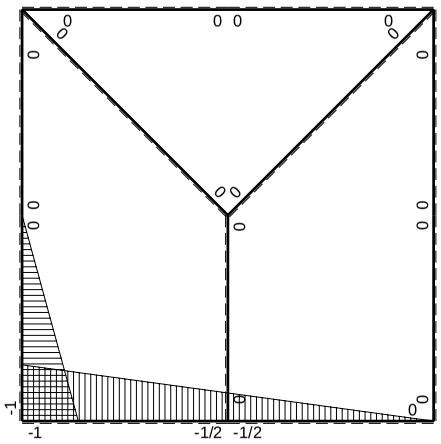

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↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236384 ESΣ04.xxxx.088

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PROCEDIMENTO E RISULTATI 236384 ESΣ04.xxxx.088

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

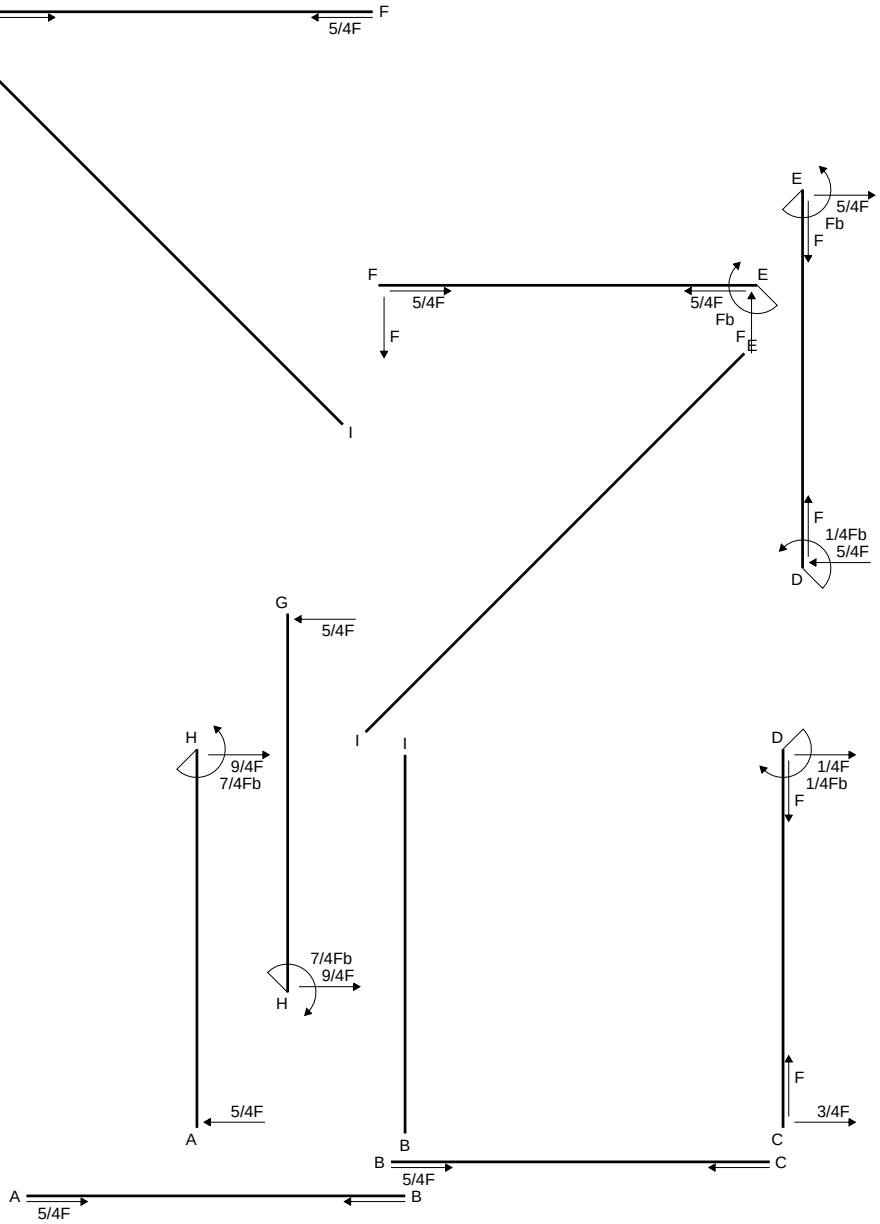
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

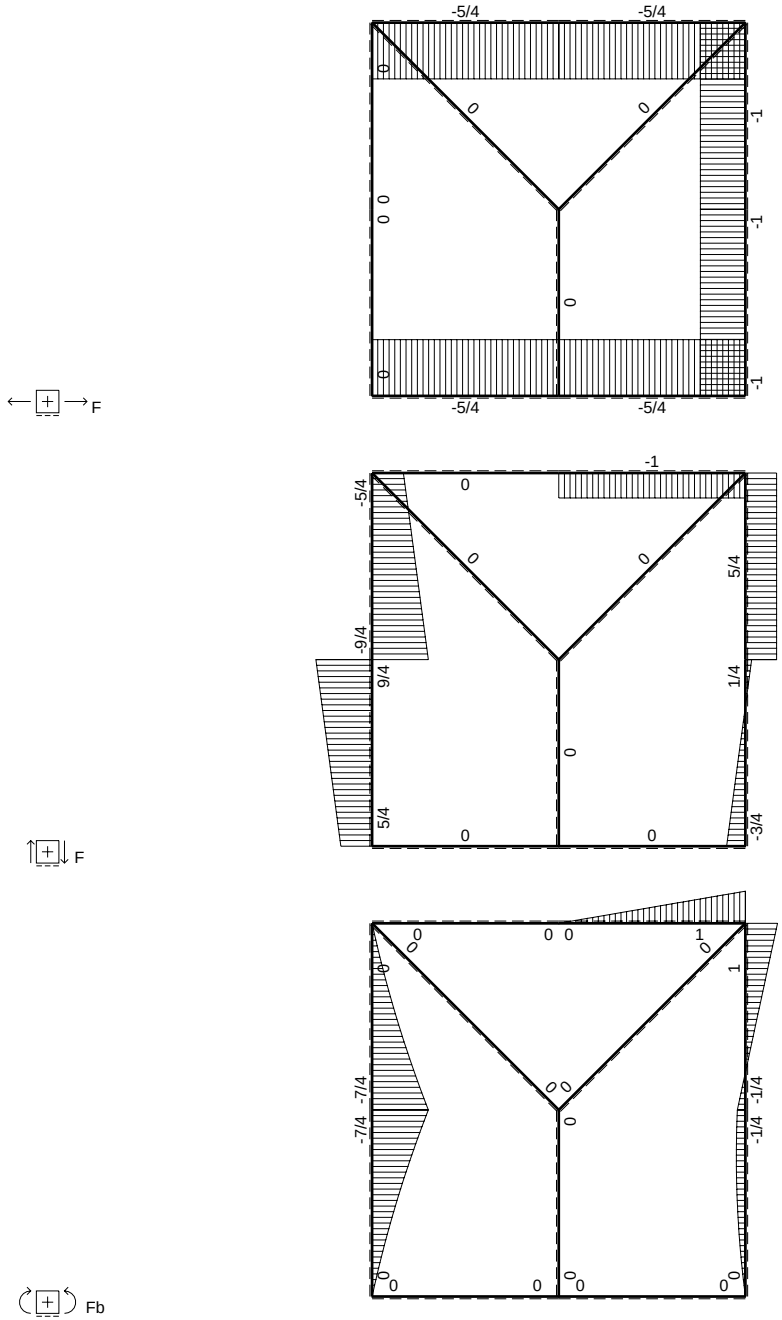
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

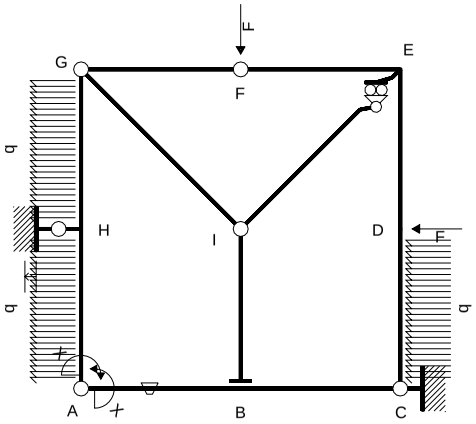
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

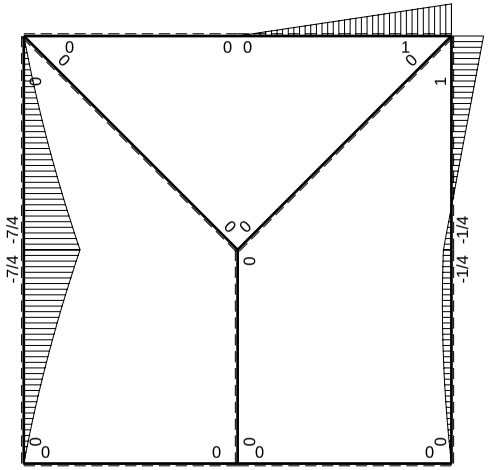
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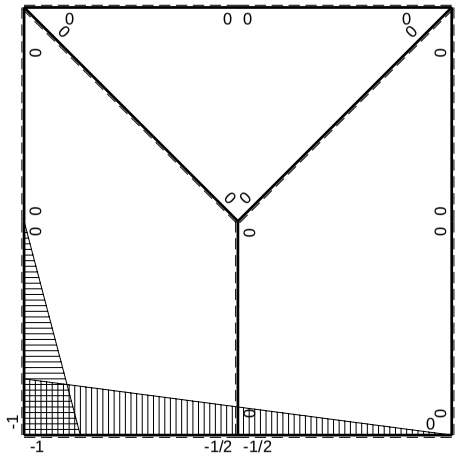

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-1/4
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	-Fb/EJ	0	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(0+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	0	Fb/EJ	0	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						0	Xb/EJ
	iperstatica $X=W_{AB}$						0	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(1 - 1/2\ x/b\ \right)\theta\ \ dx = \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/2\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

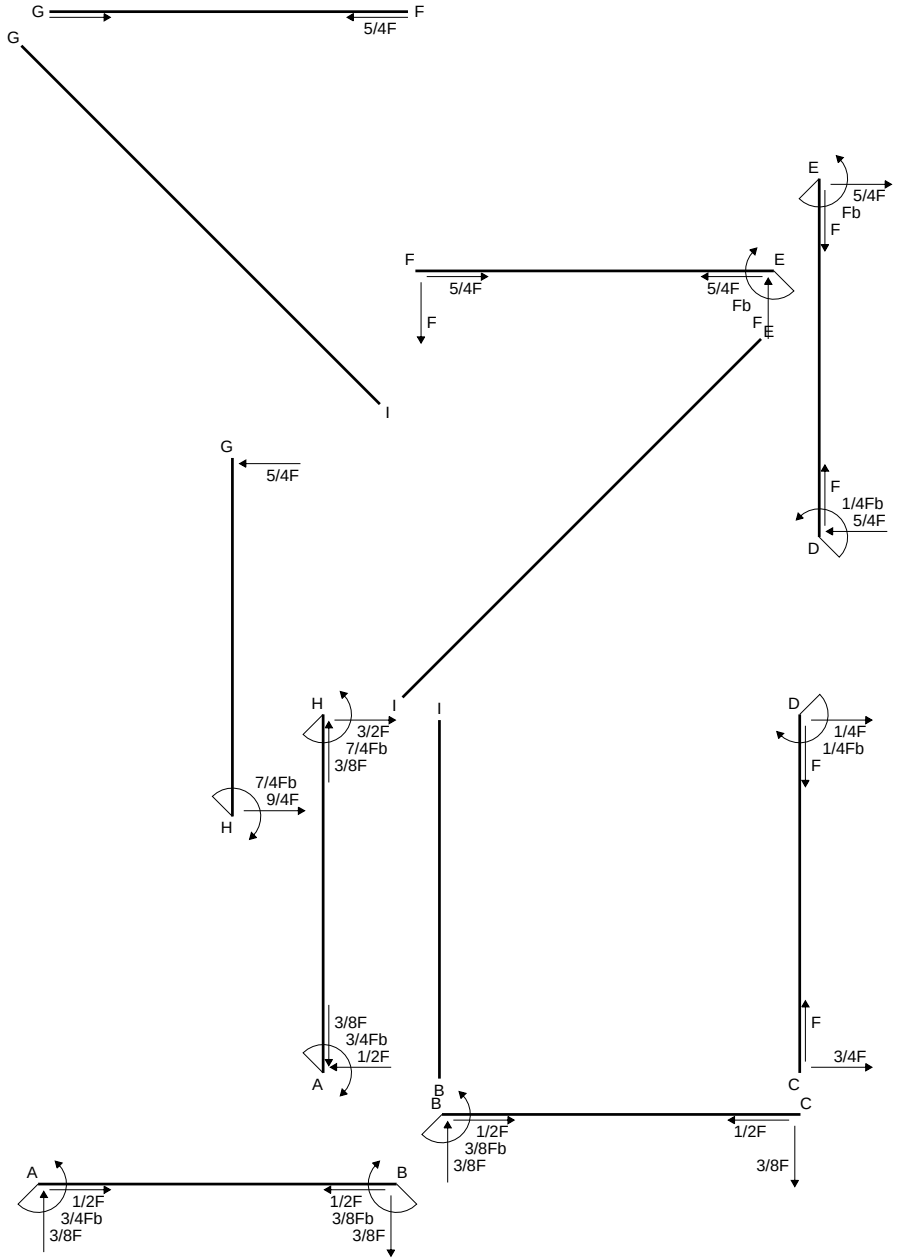
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

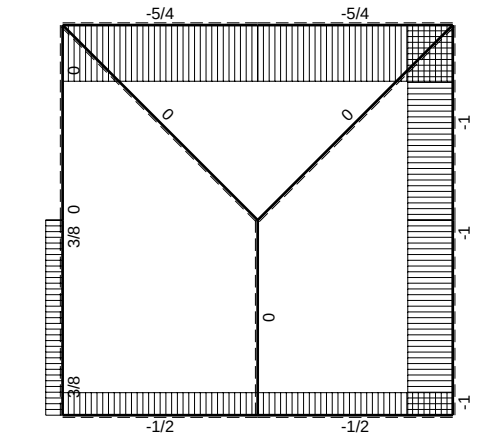
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

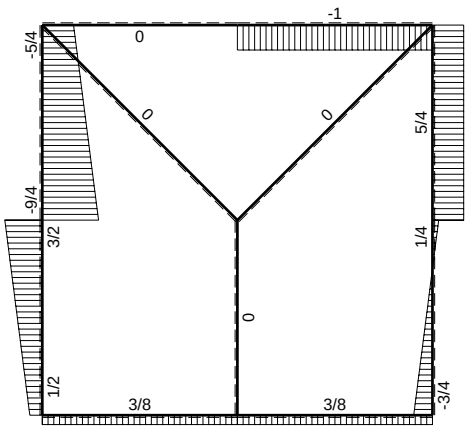
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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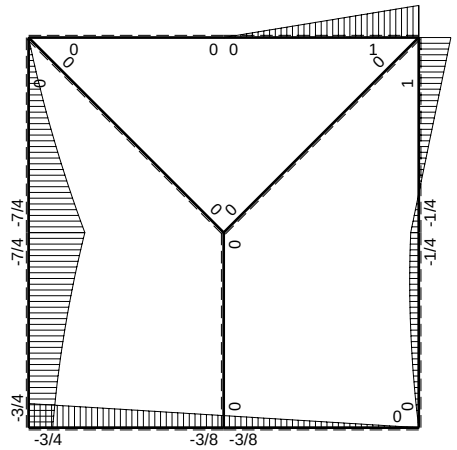

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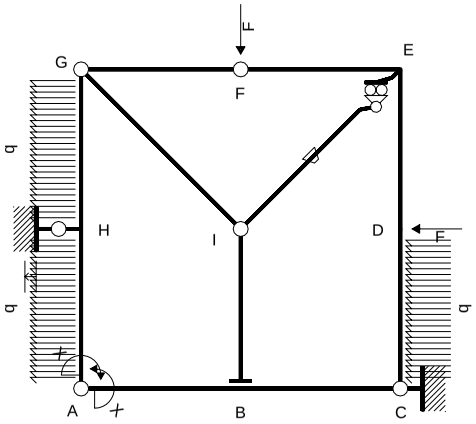

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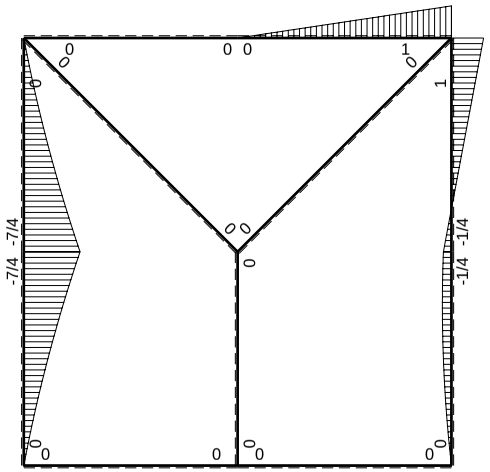
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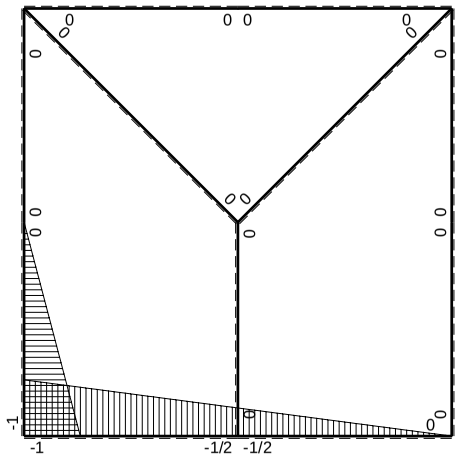

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-3/4
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-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238705 ESΣ04.xxxx.090
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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

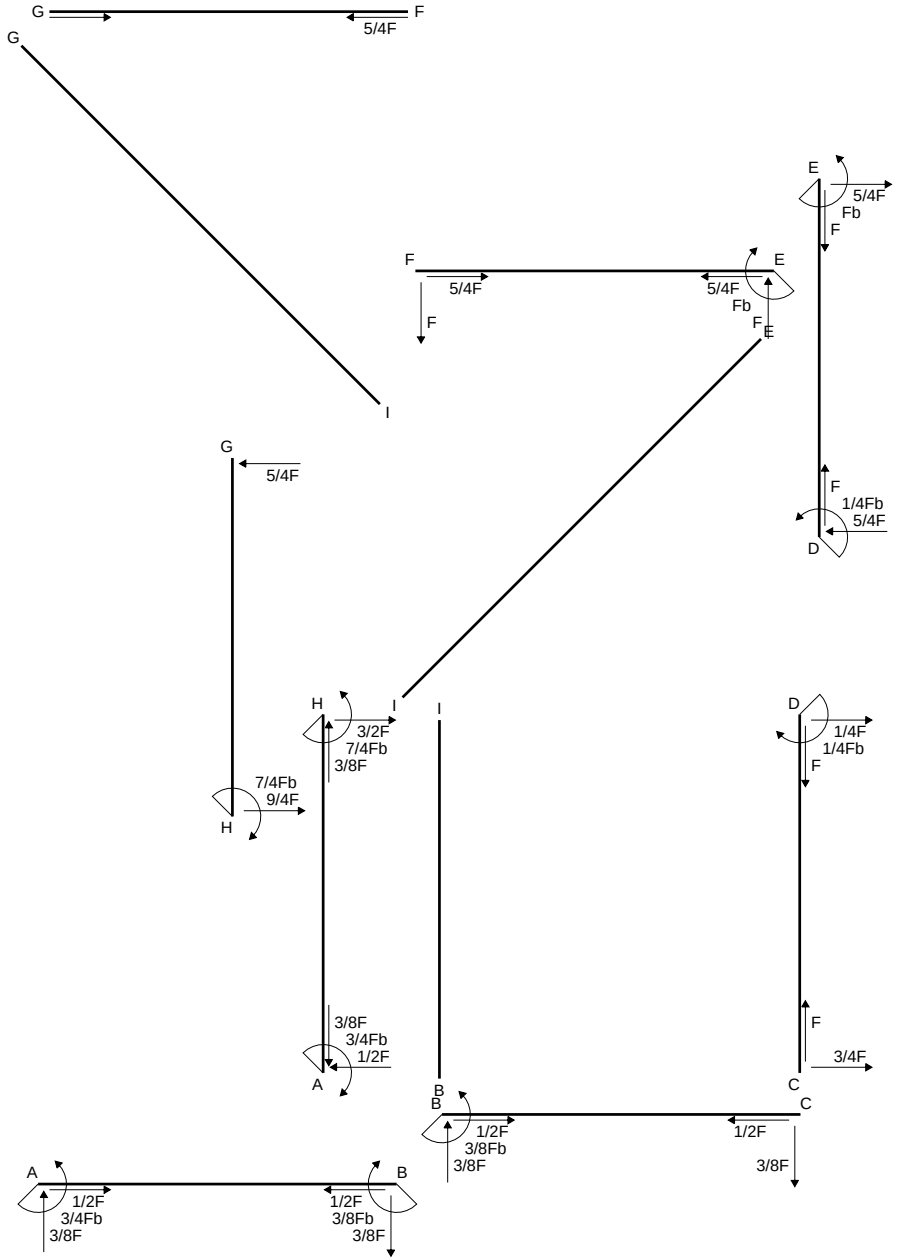
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

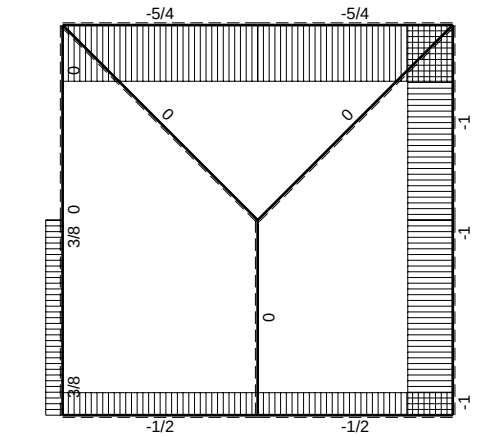
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

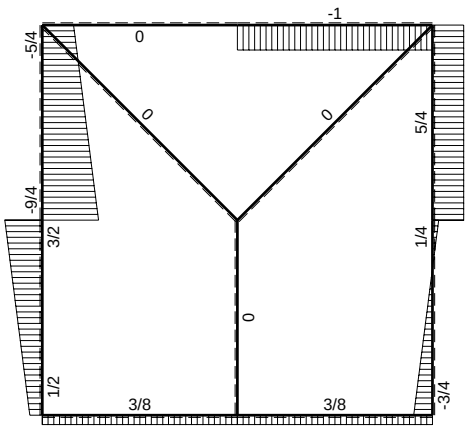
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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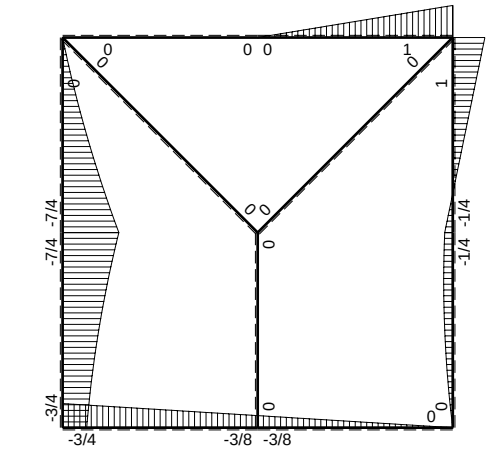

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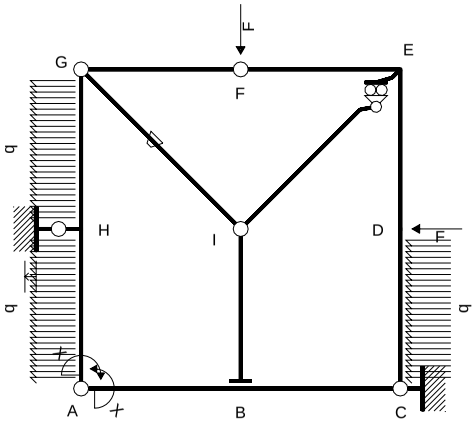

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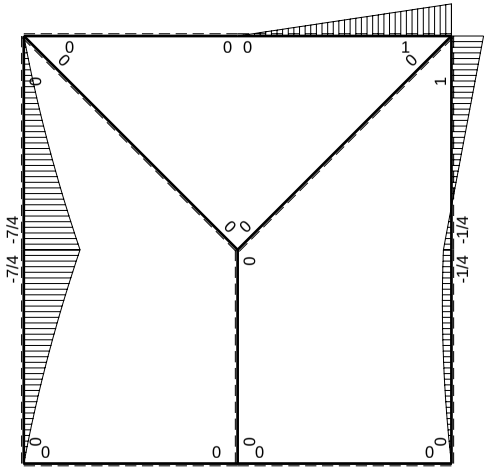
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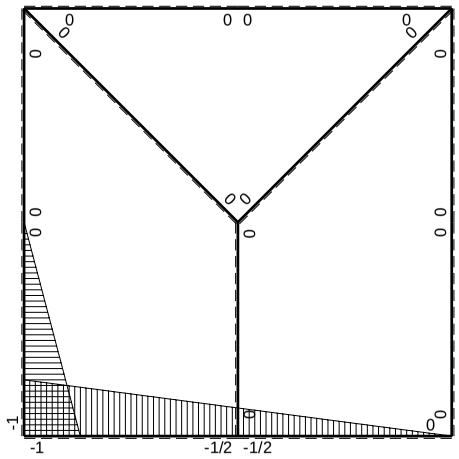

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-3/4
-3/8
-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

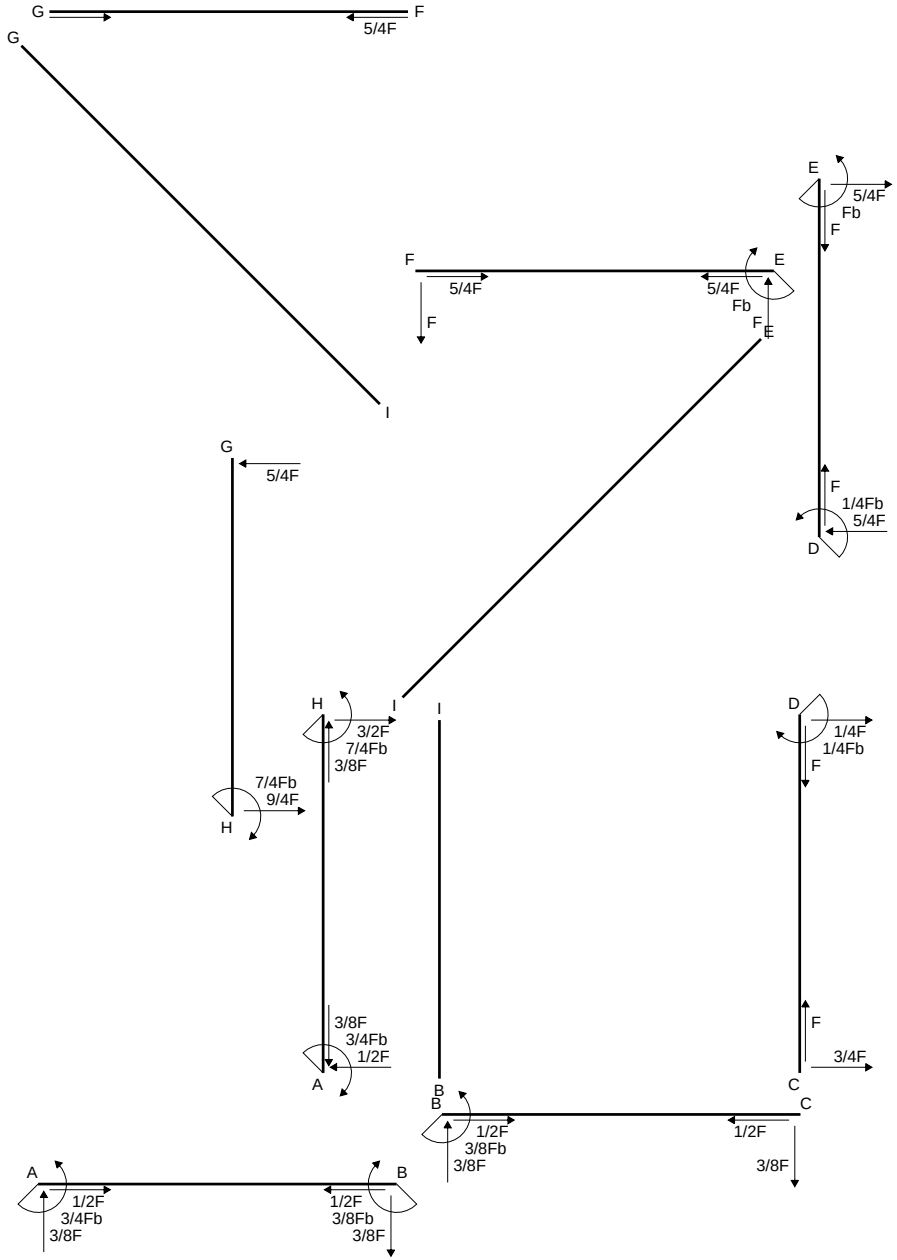
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

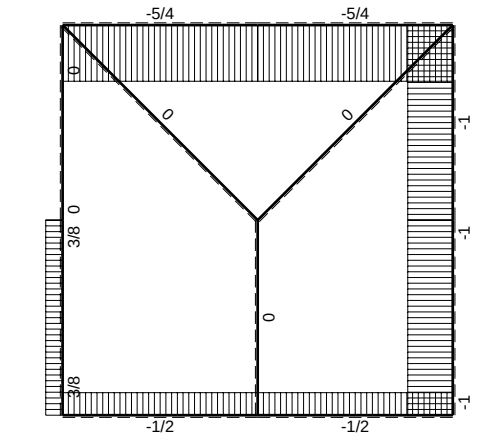
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

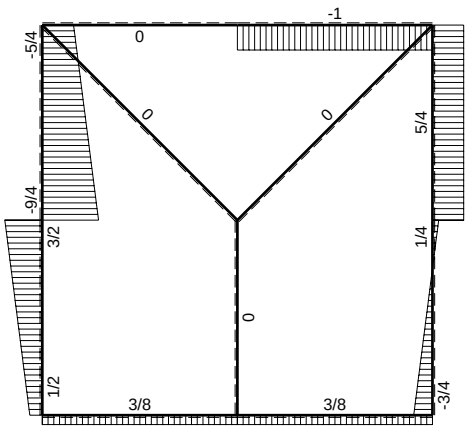
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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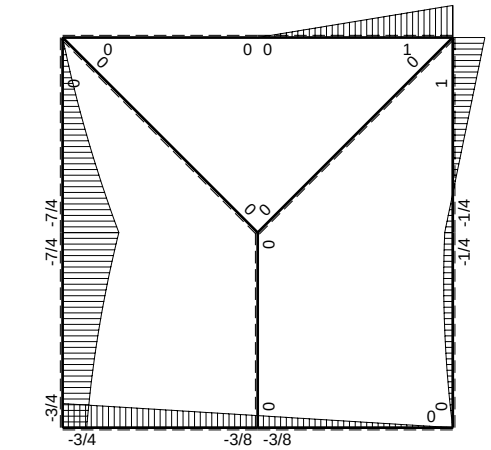

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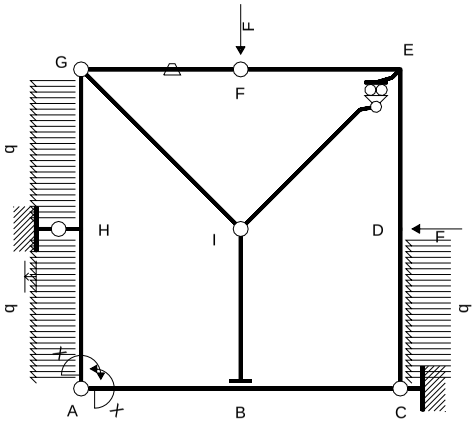

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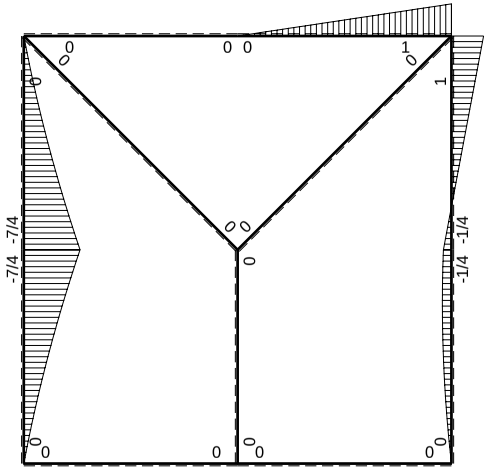
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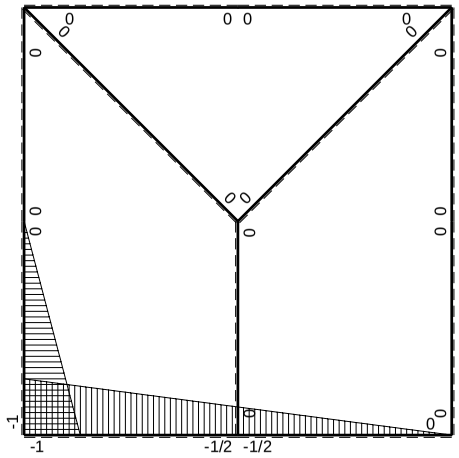

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-3/4
-3/4
-3/8
-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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PROCEDIMENTO E RISULTATI 239114 ESΣ04.xxxx.092

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

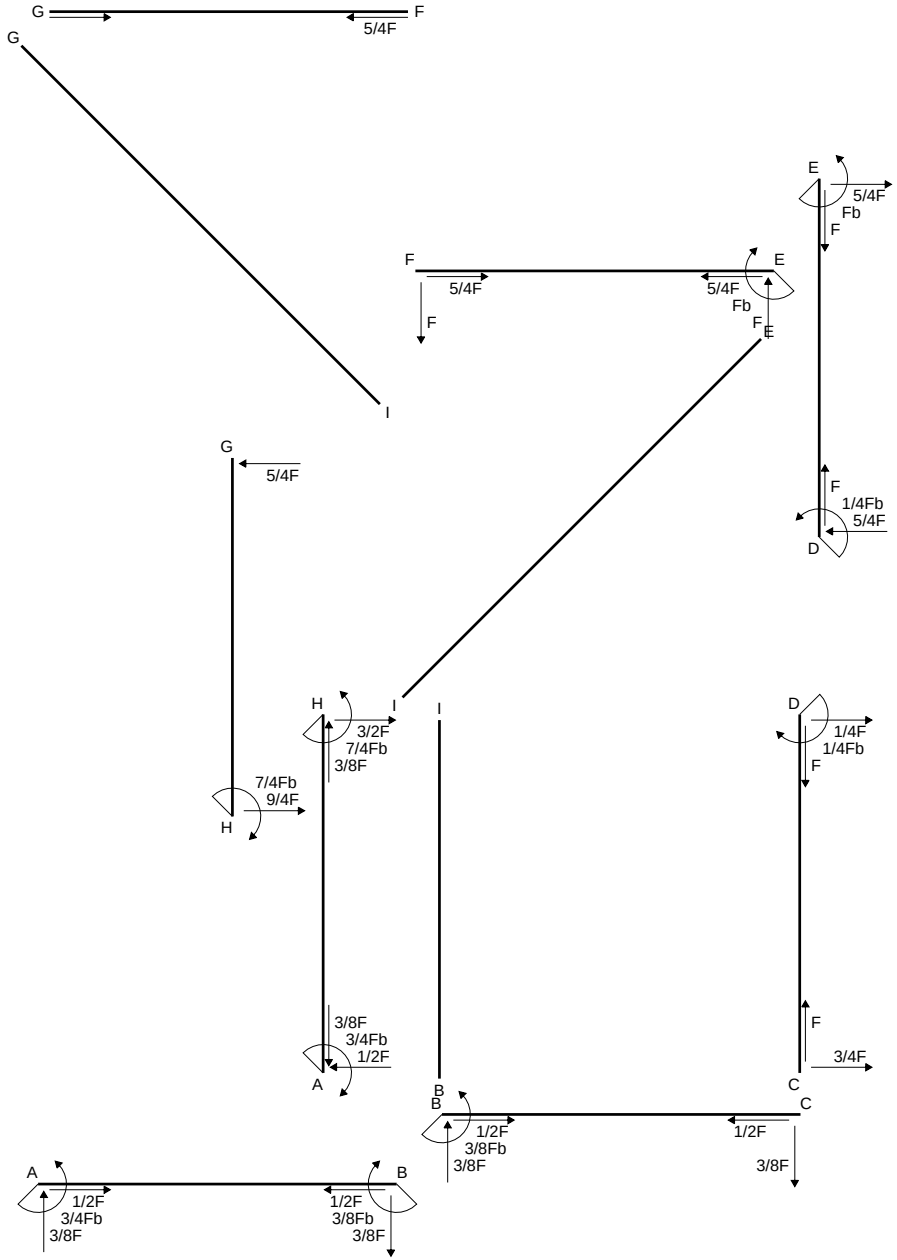
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

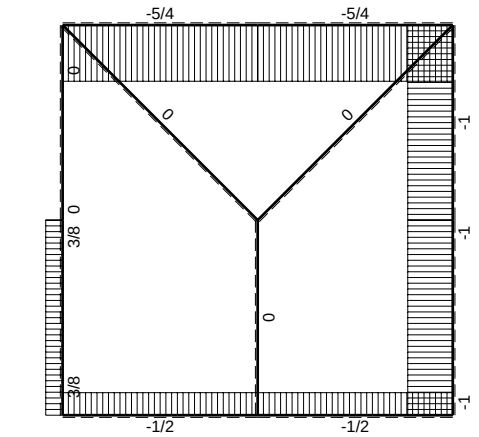
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

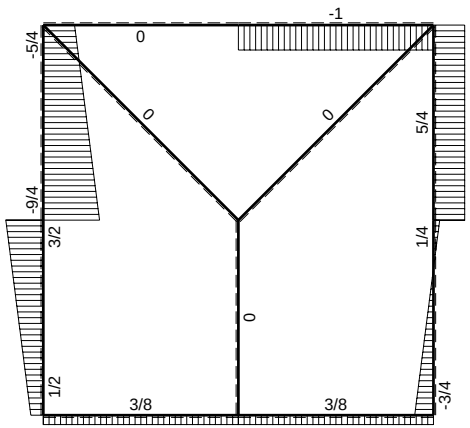
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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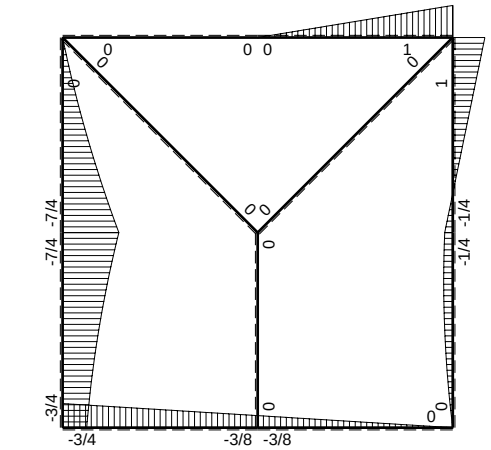

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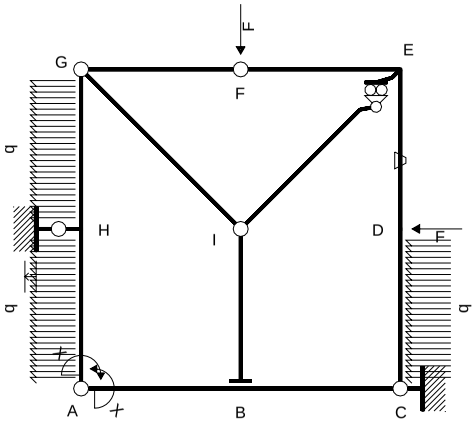

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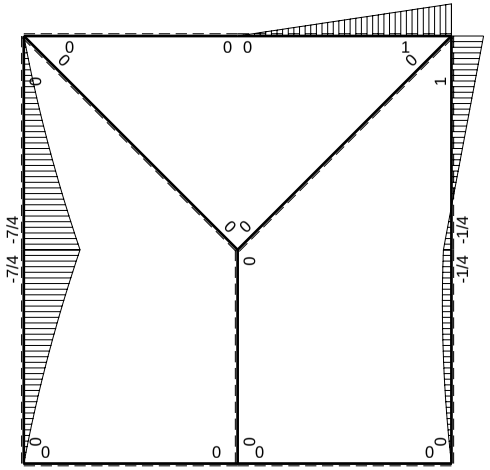
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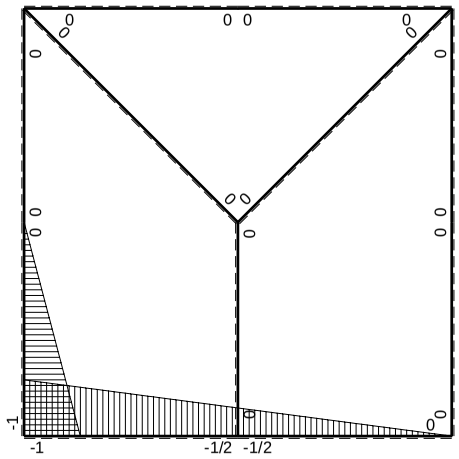

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210483 ESΣ04.xxxx.093

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

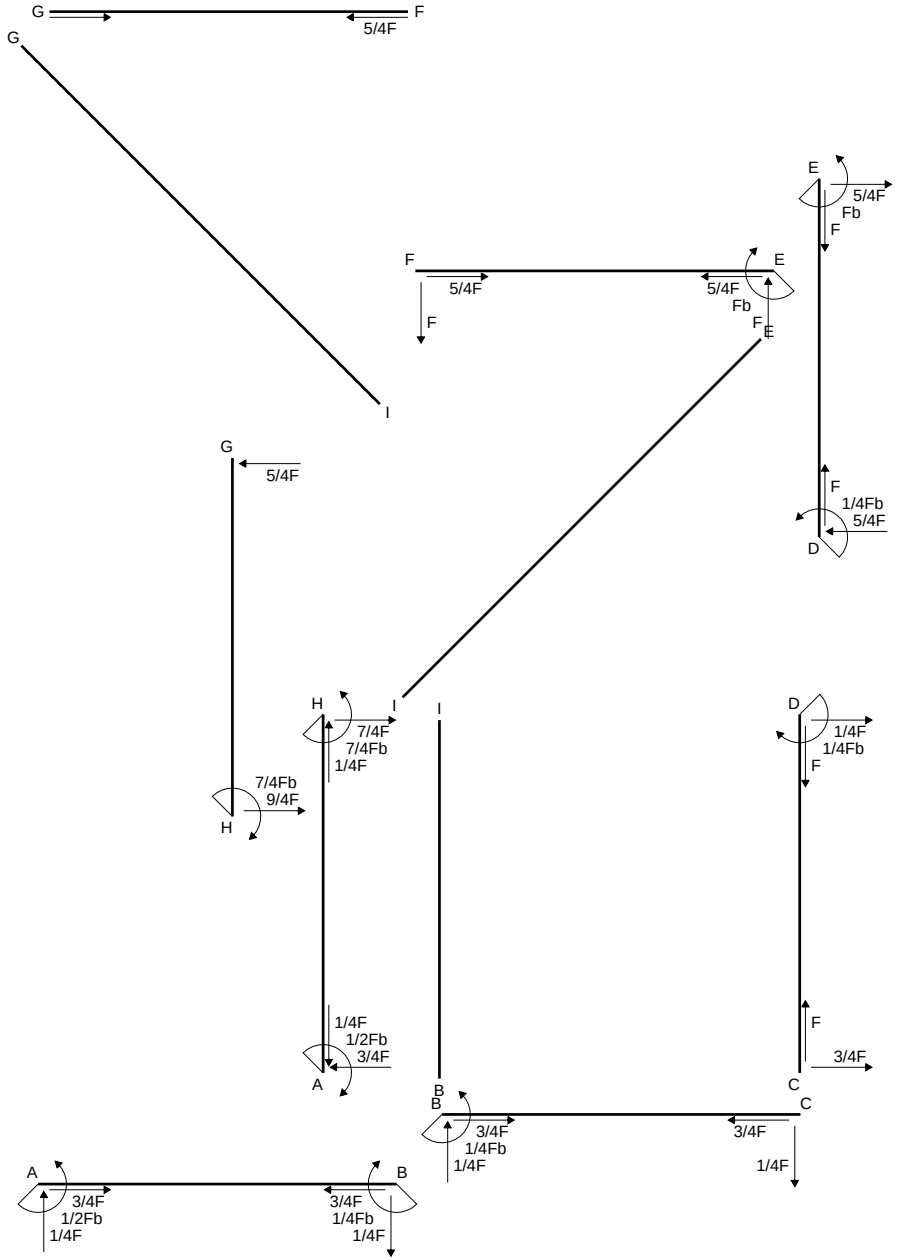
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

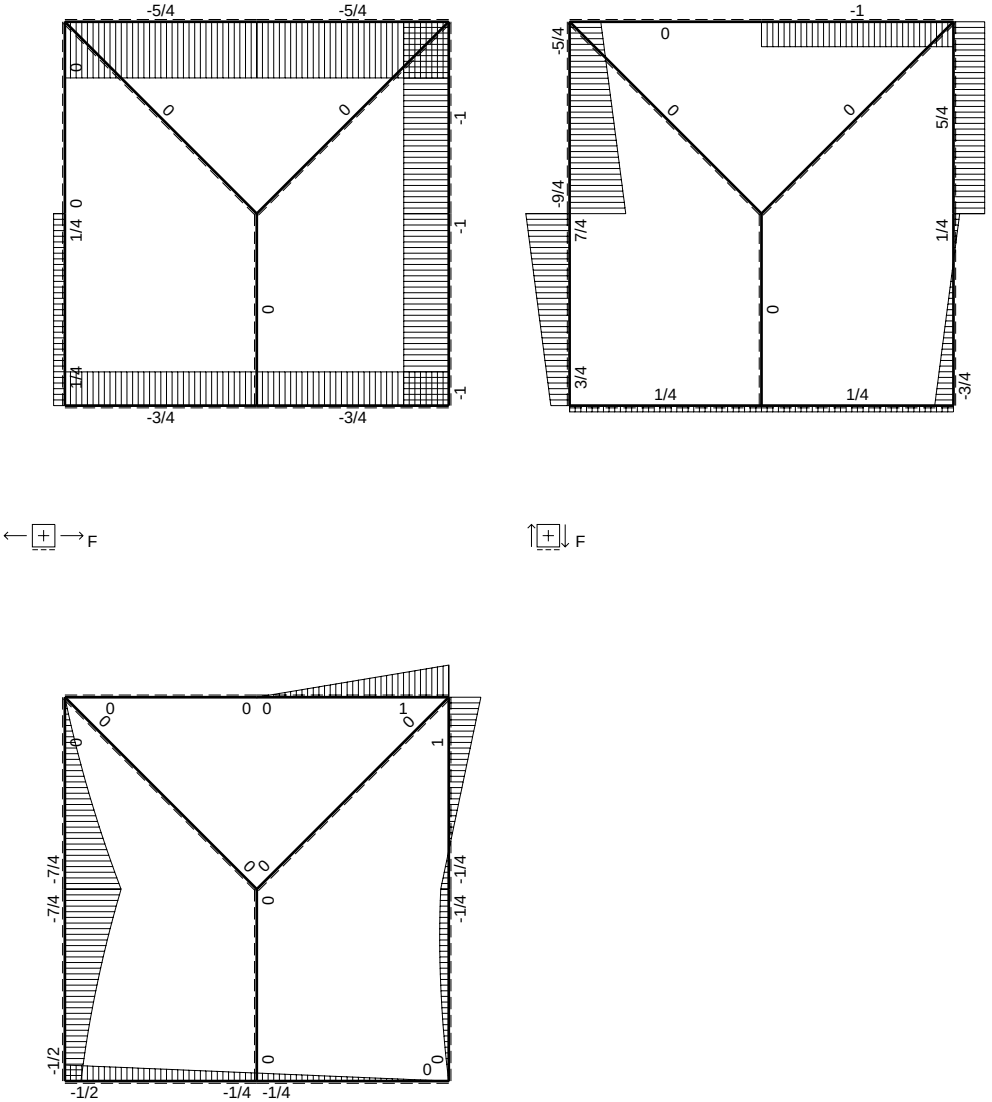
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

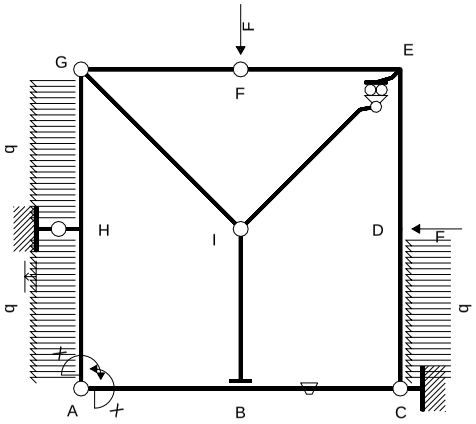
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

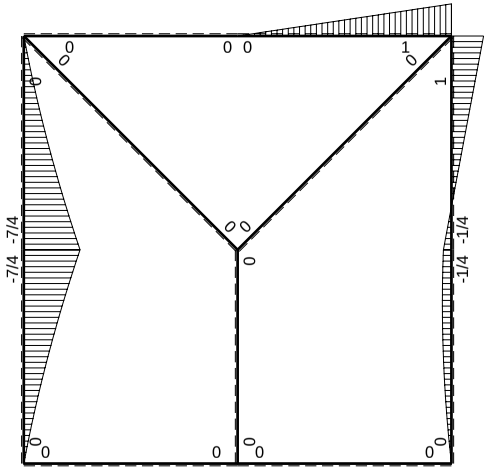
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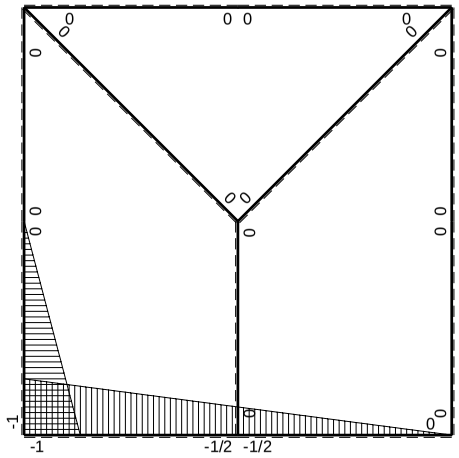

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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	-Fb/EJ	0	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(0+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	0	Fb/EJ	0	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/2Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(1/2\ b - 1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/2\ x/b\ \right)\theta\ \ dx = \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

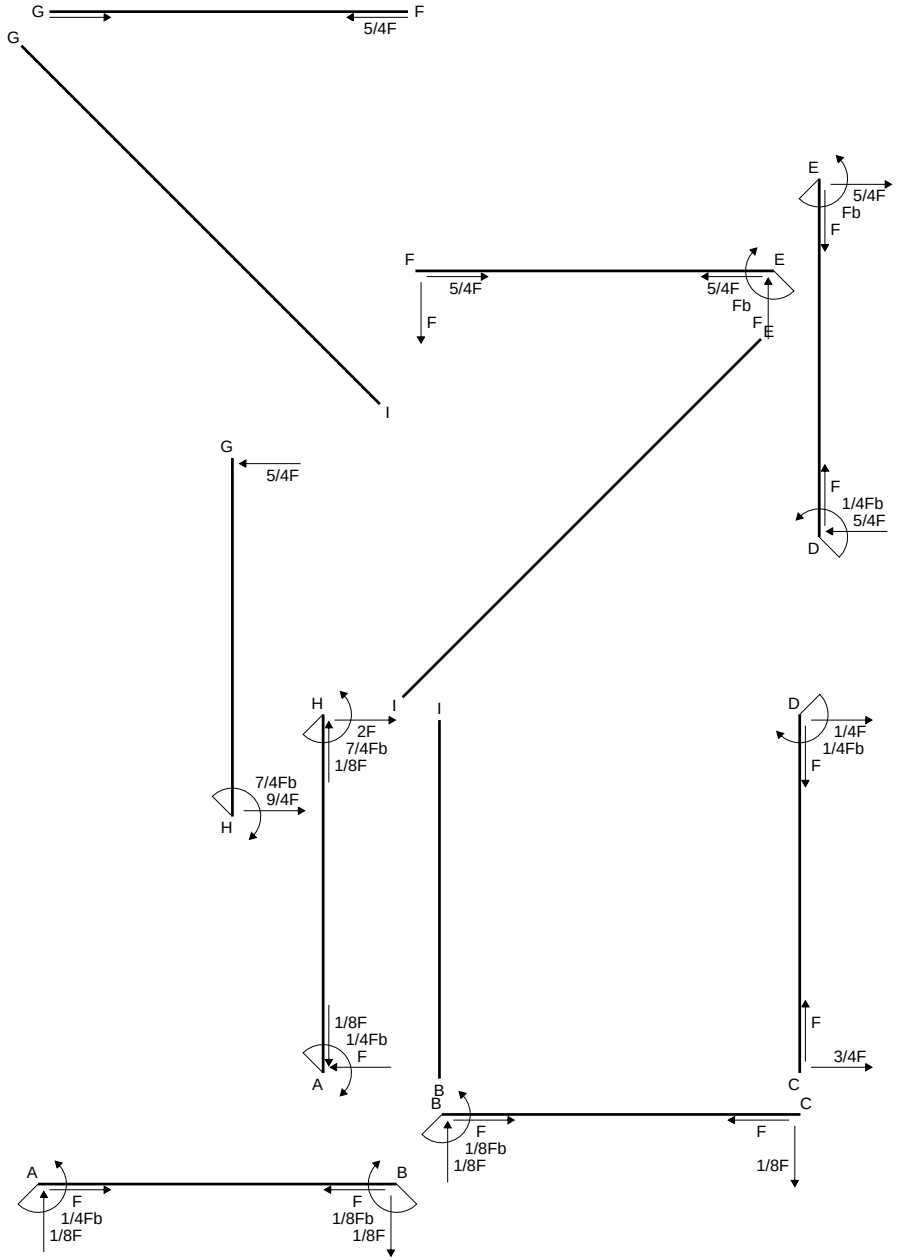
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

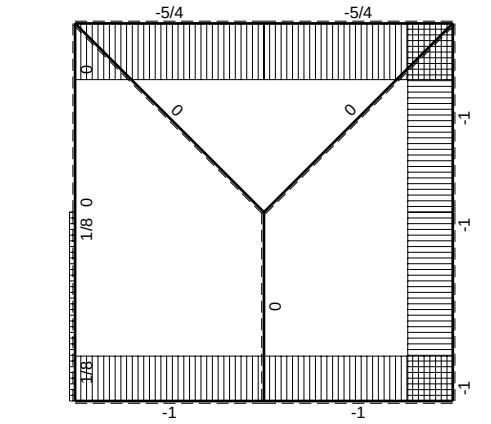
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

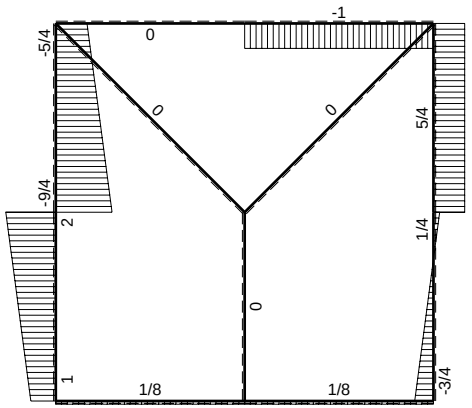
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

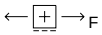
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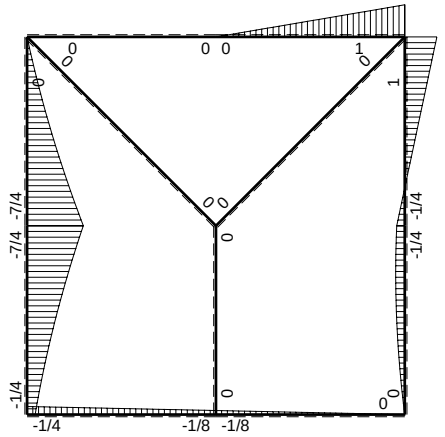

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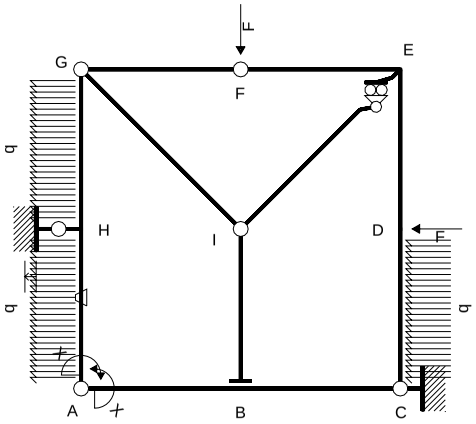

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-3/4

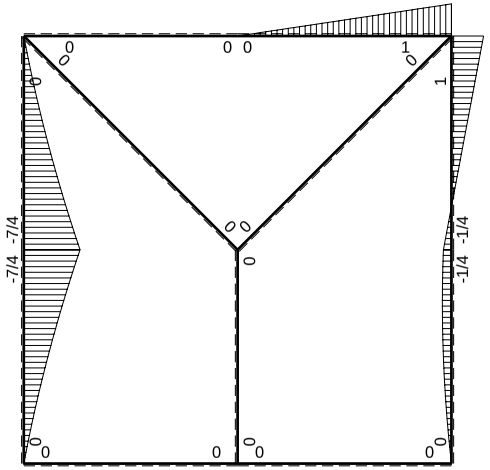

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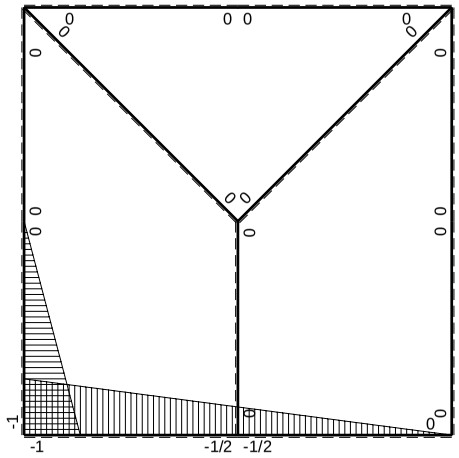

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238443 ESΣ04.xxxx.095

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	-Fb/EJ	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/4+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	Fb/EJ	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/4Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238443 ESΣ04.xxxx.095
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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\right)\theta\ \ dx$$

$$= \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$$

$$= \left(7/8\ b - 3/4\ b + 1/8\ b\right) Fb\ 1/EJ\ + \left(1/2\ b\right)\theta\ \ = 3/4\ \ Fb^2/EJ$$

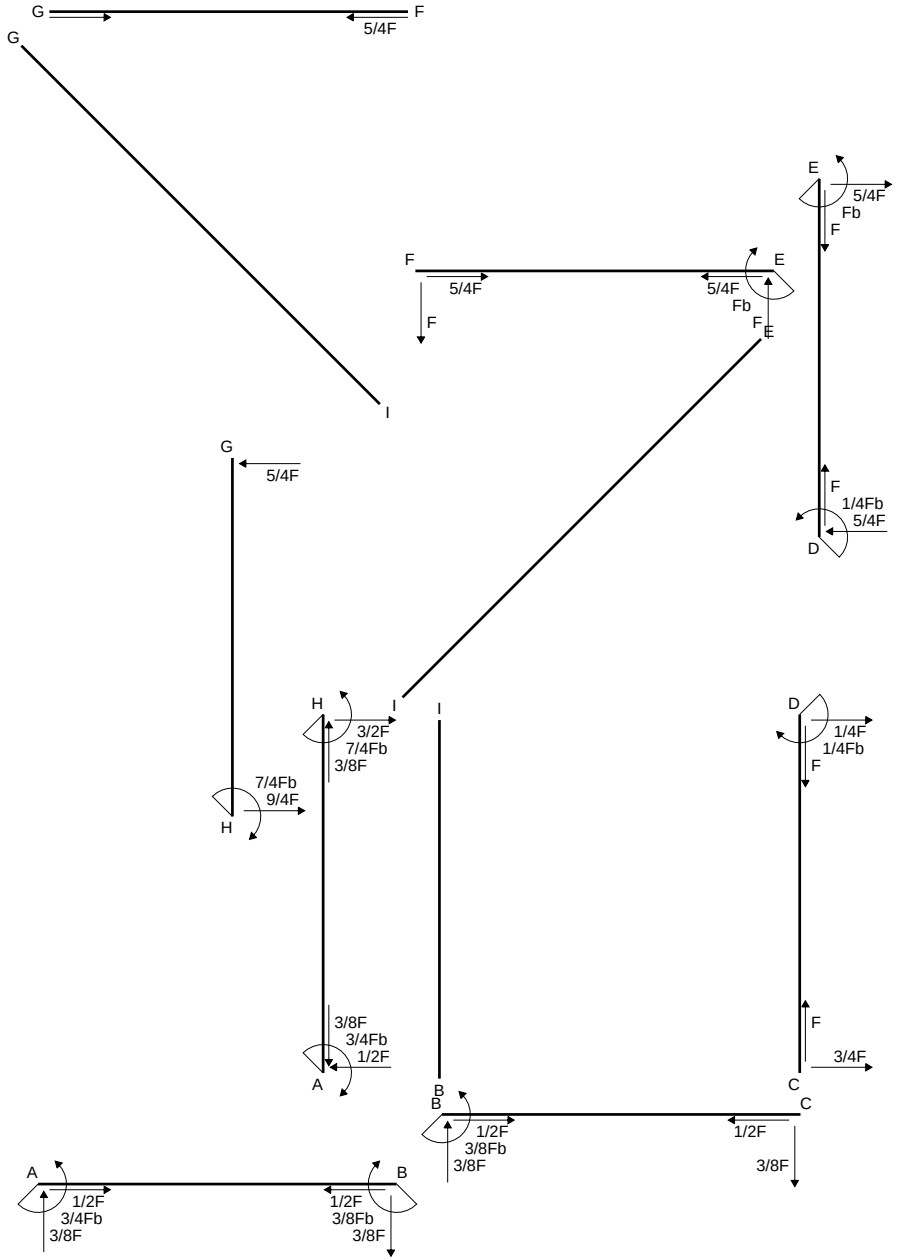
$$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\right)\theta\ \ dx$$

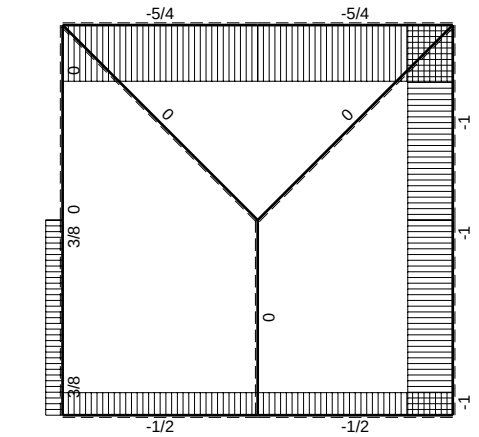
$$= \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$$

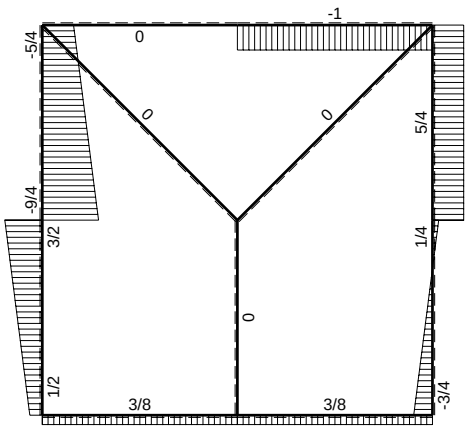
$$= \left(5/8\ b - 1/4\ b - 1/8\ b\right) Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\right)\theta\ \ = 3/4\ \ Fb^2/EJ$$

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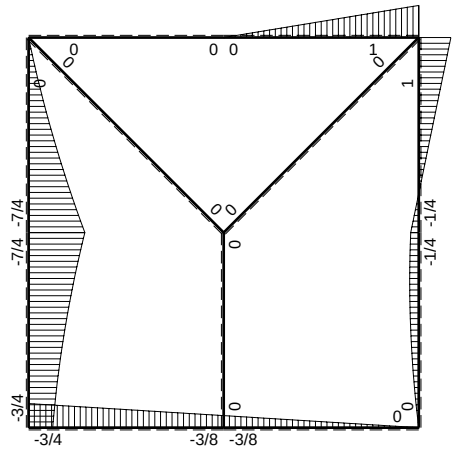

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3/8F


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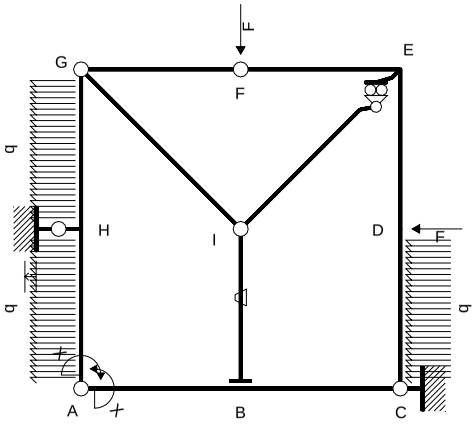

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3/8
-3/4

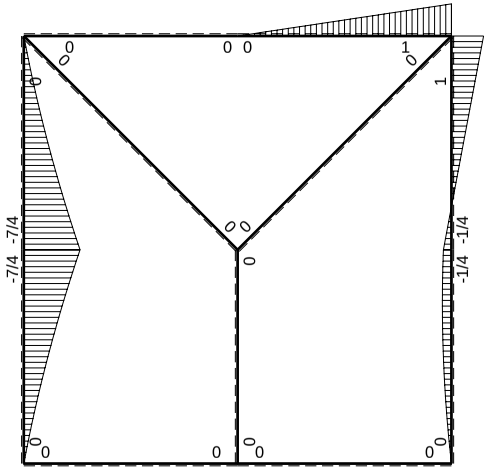
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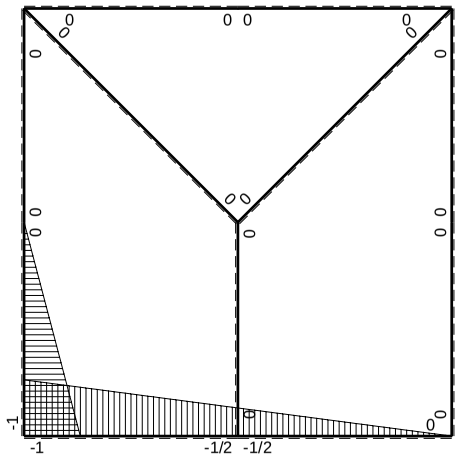

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-7/4
-7/4
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-1/4
0
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0
0
0
-3/4
-3/4
-3/8
-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 955615 ESΣ04.xxxx.096

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PROCEDIMENTO E RISULTATI 955615 ESΣ04.xxxx.096

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{HA} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

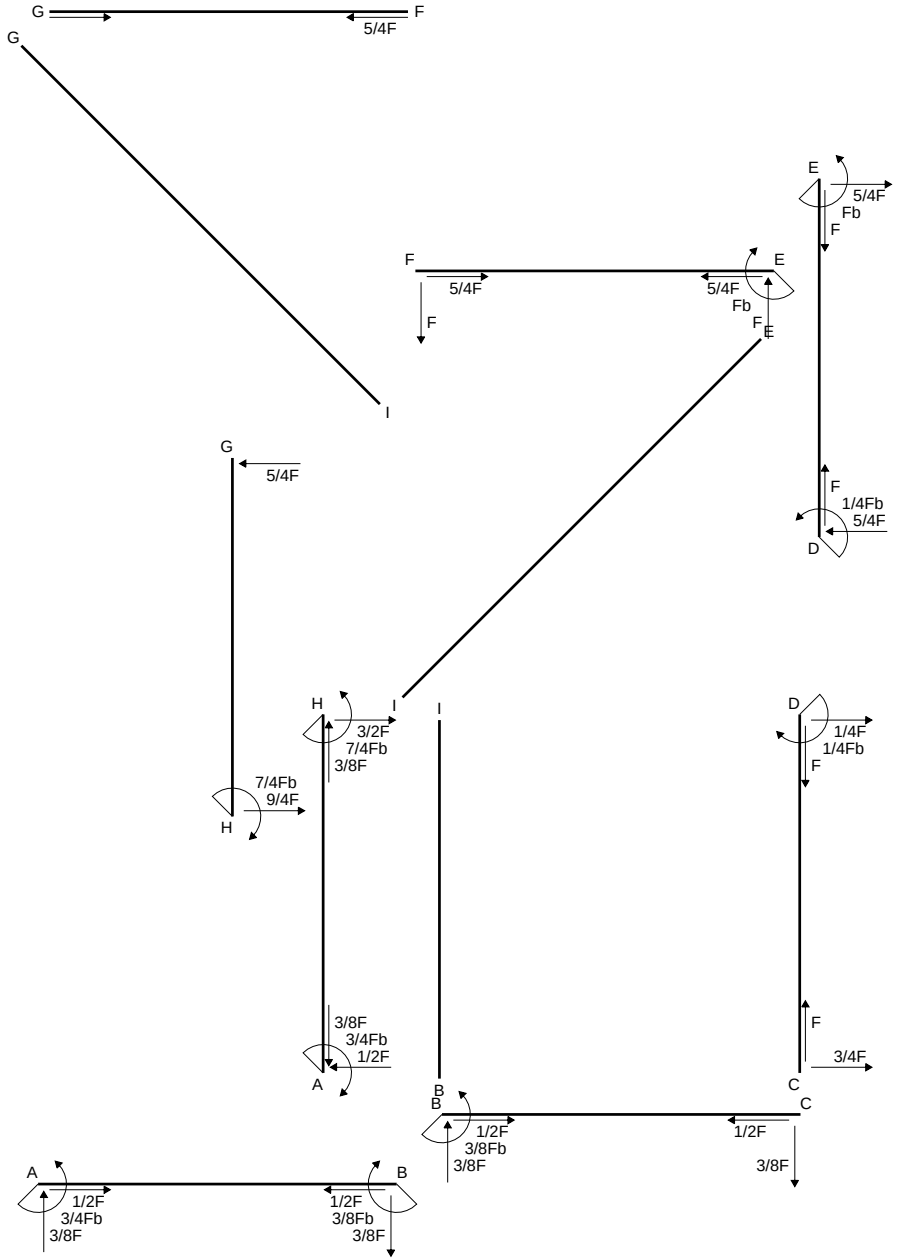
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

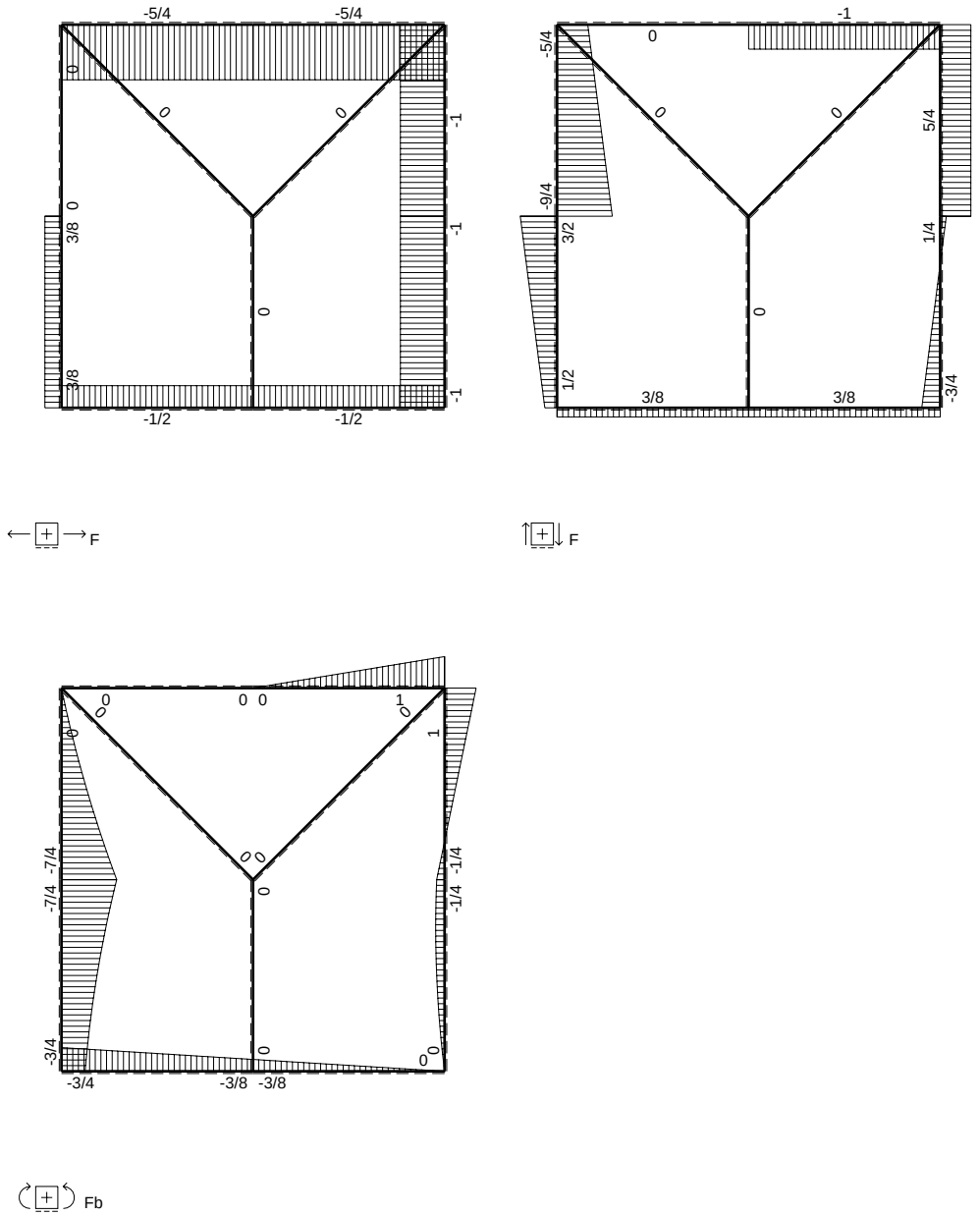
$L^{Xo}_{AH} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

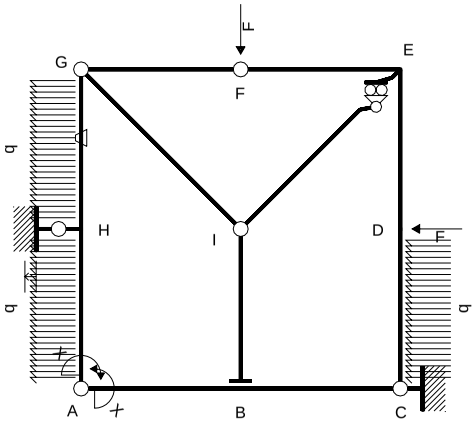
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

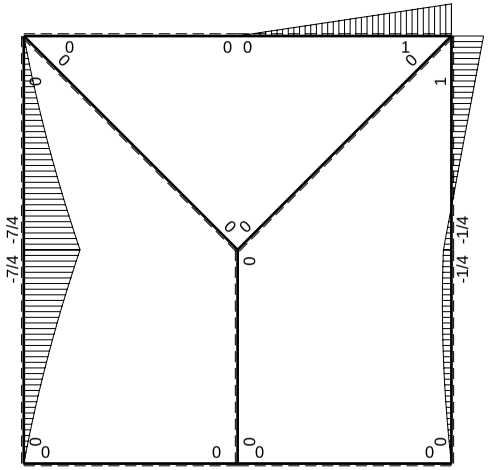
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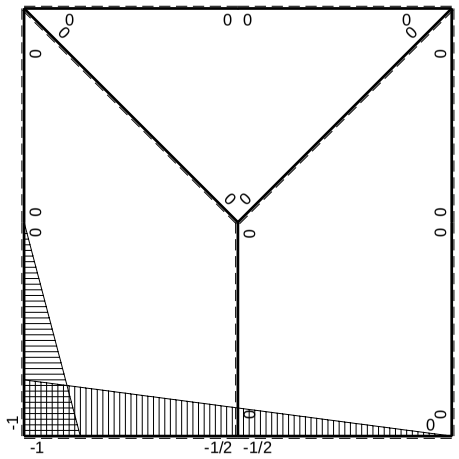

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-3/8
-3/8
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236762 ESΣ04.xxxx.097

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PROCEDIMENTO E RISULTATI 236762 ESΣ04.xxxx.097

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

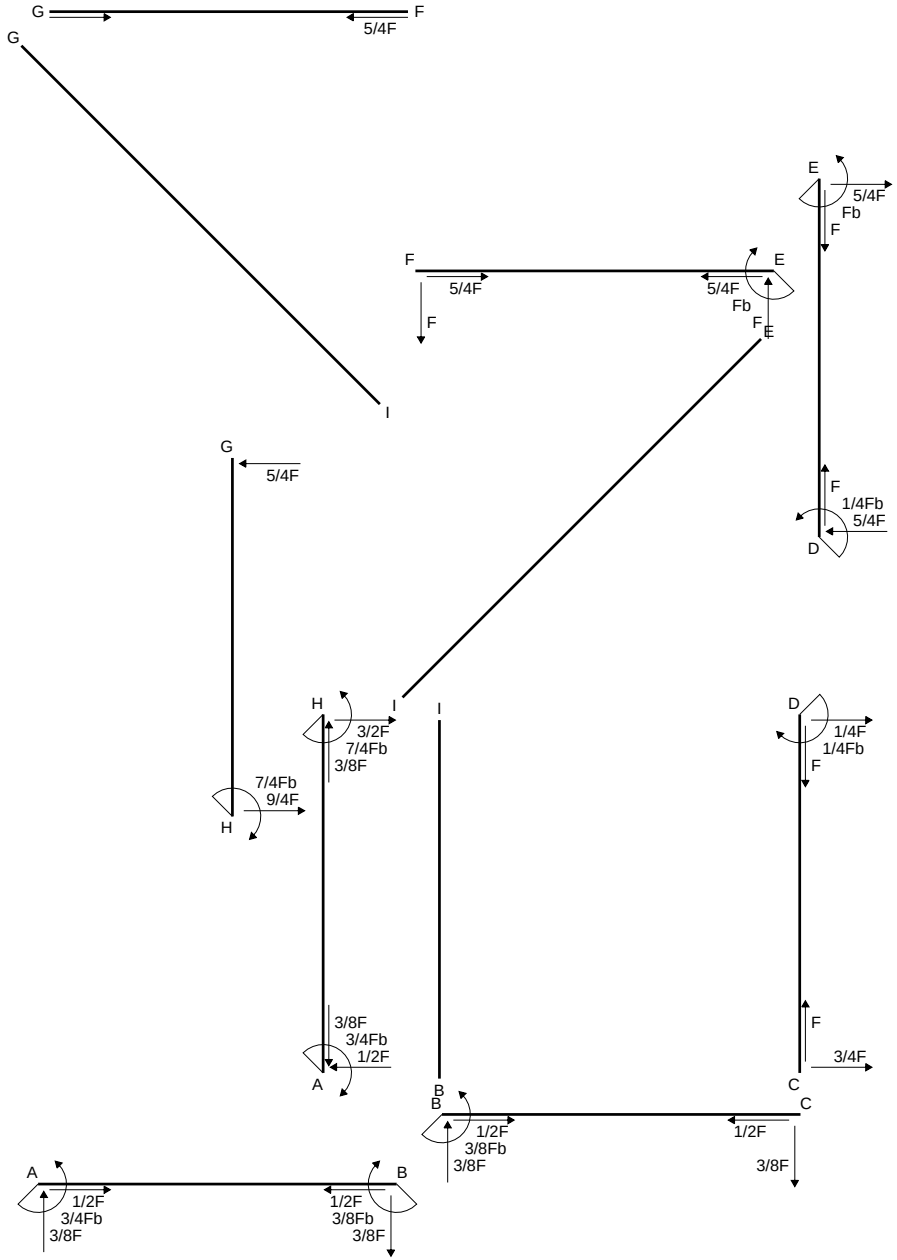
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

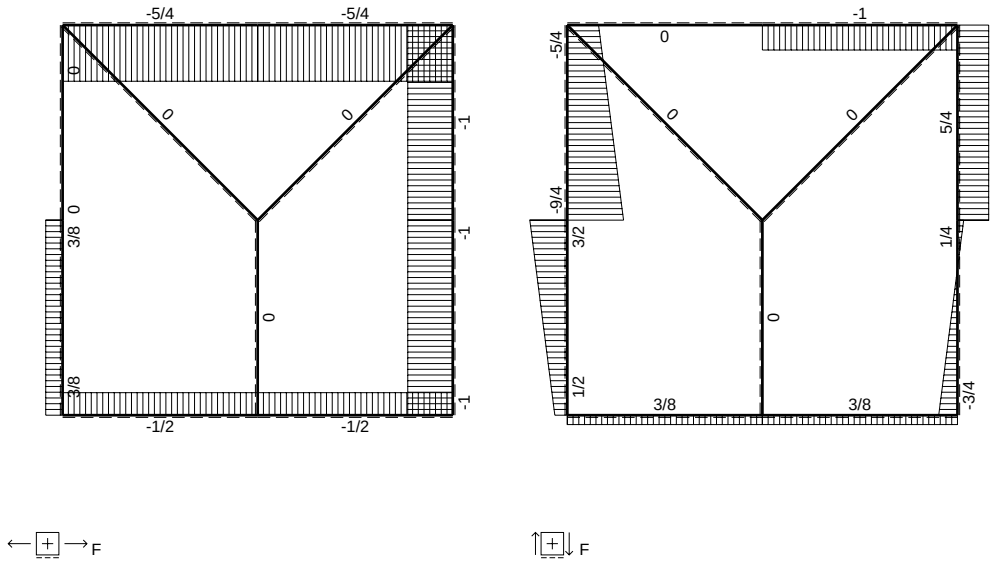
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

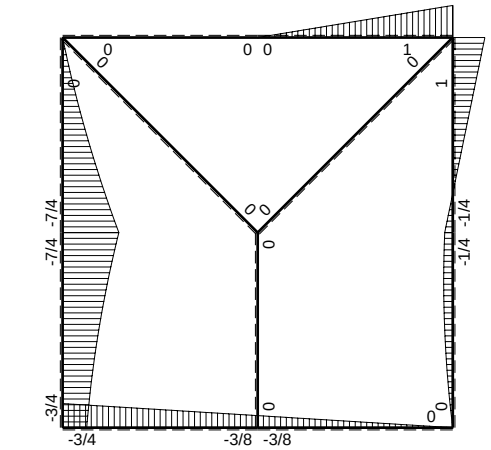
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

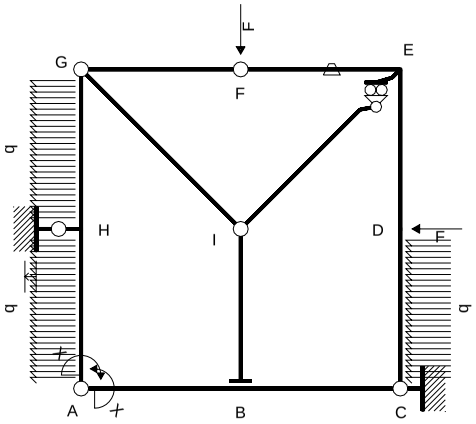
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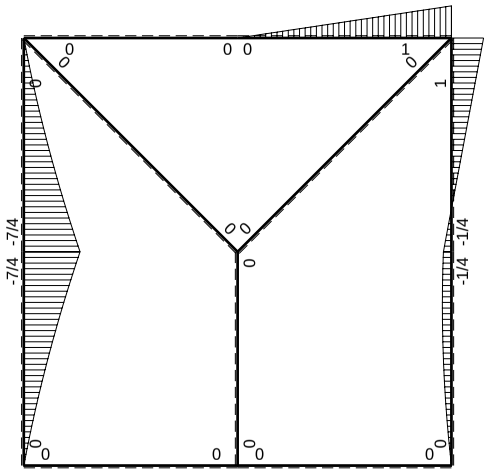
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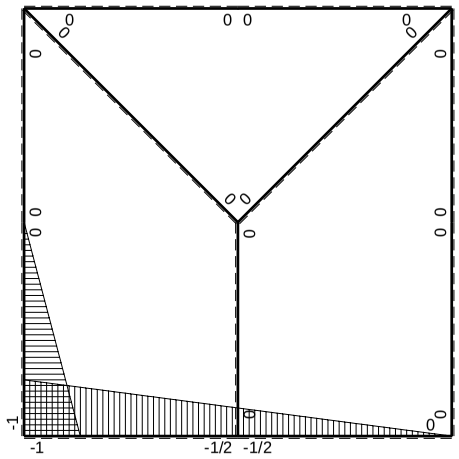

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F


-7/4
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-1/4
-3/4
-3/4
-3/8
-3/8
1
1
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

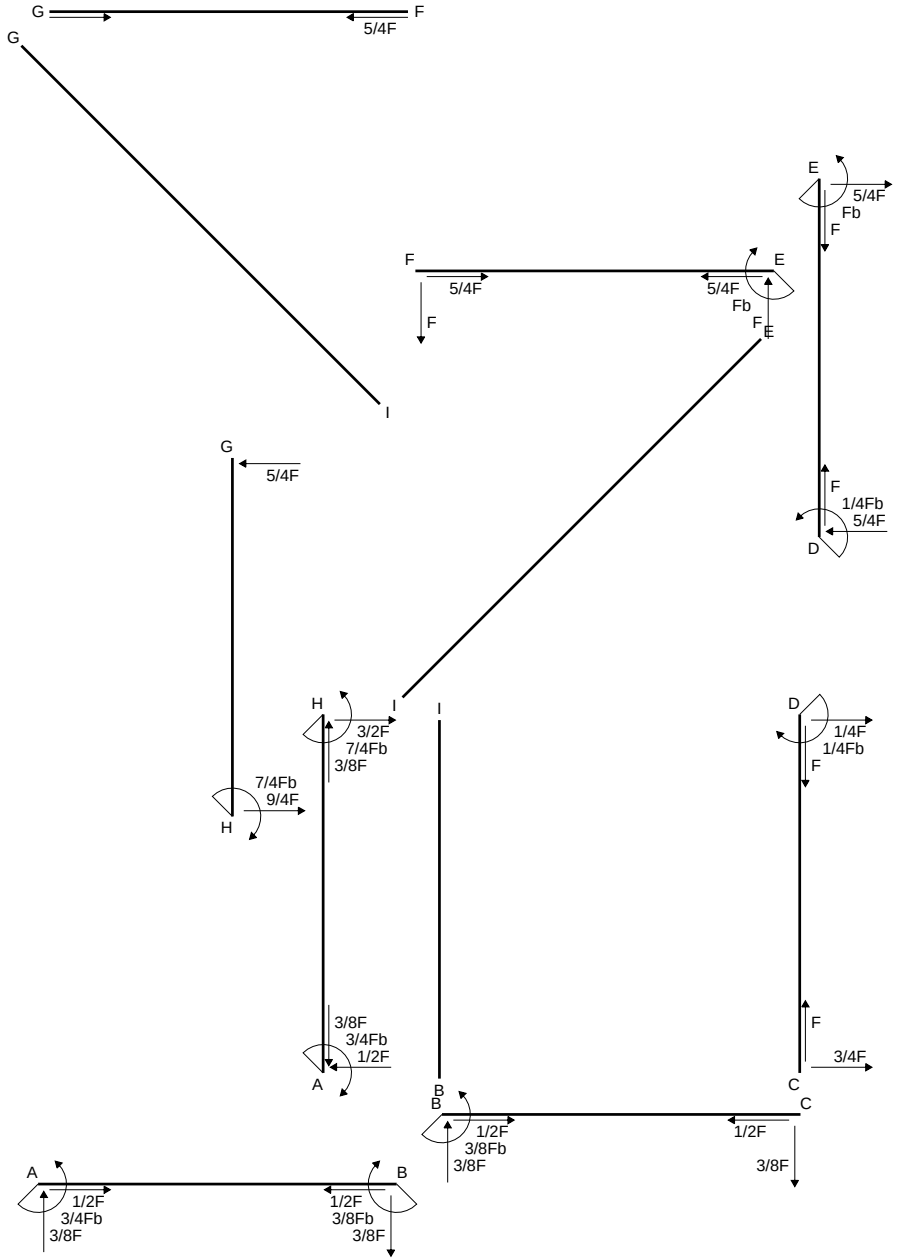
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

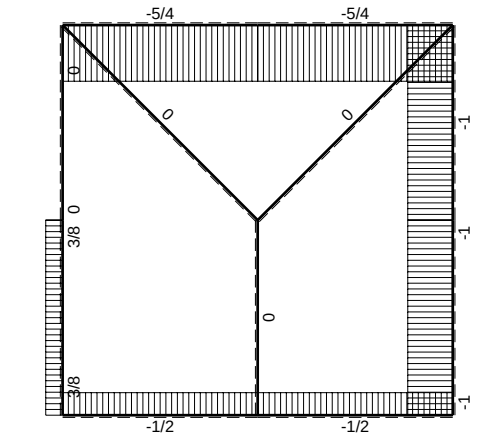
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

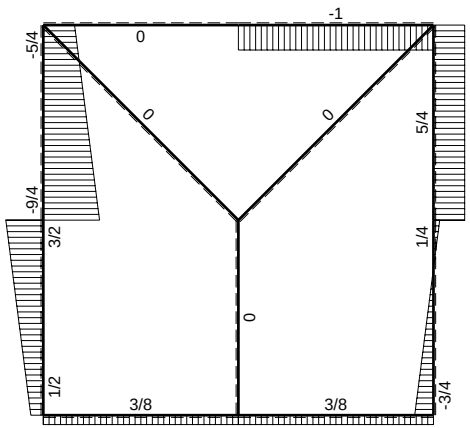
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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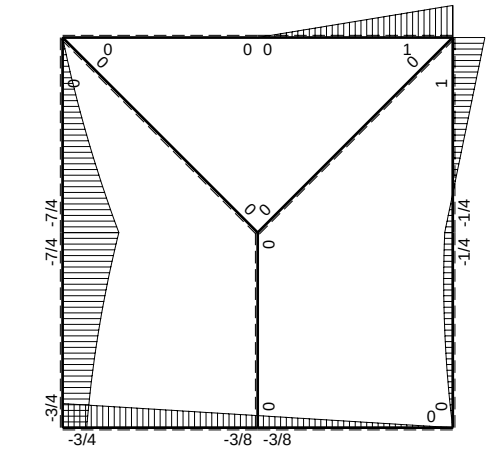

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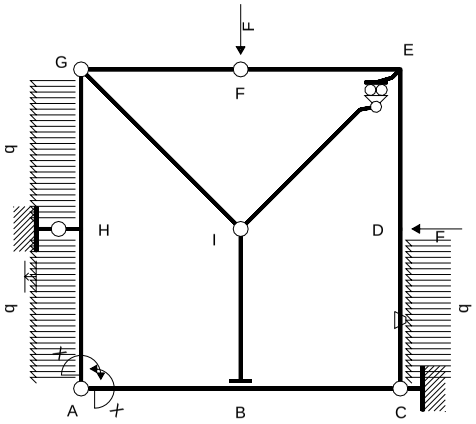

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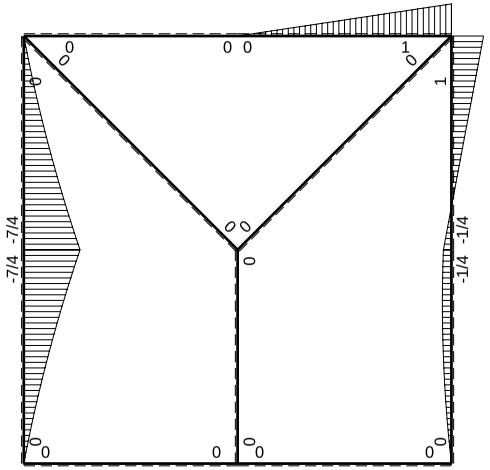
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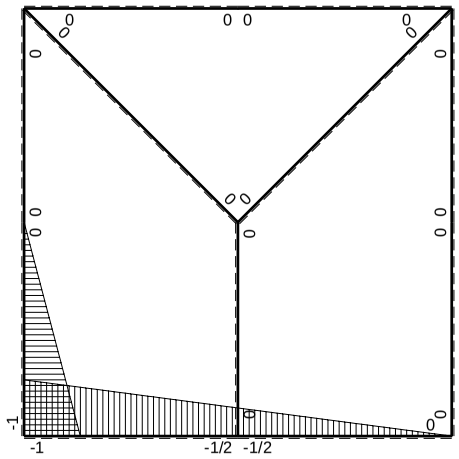

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-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	Fb/EJ	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

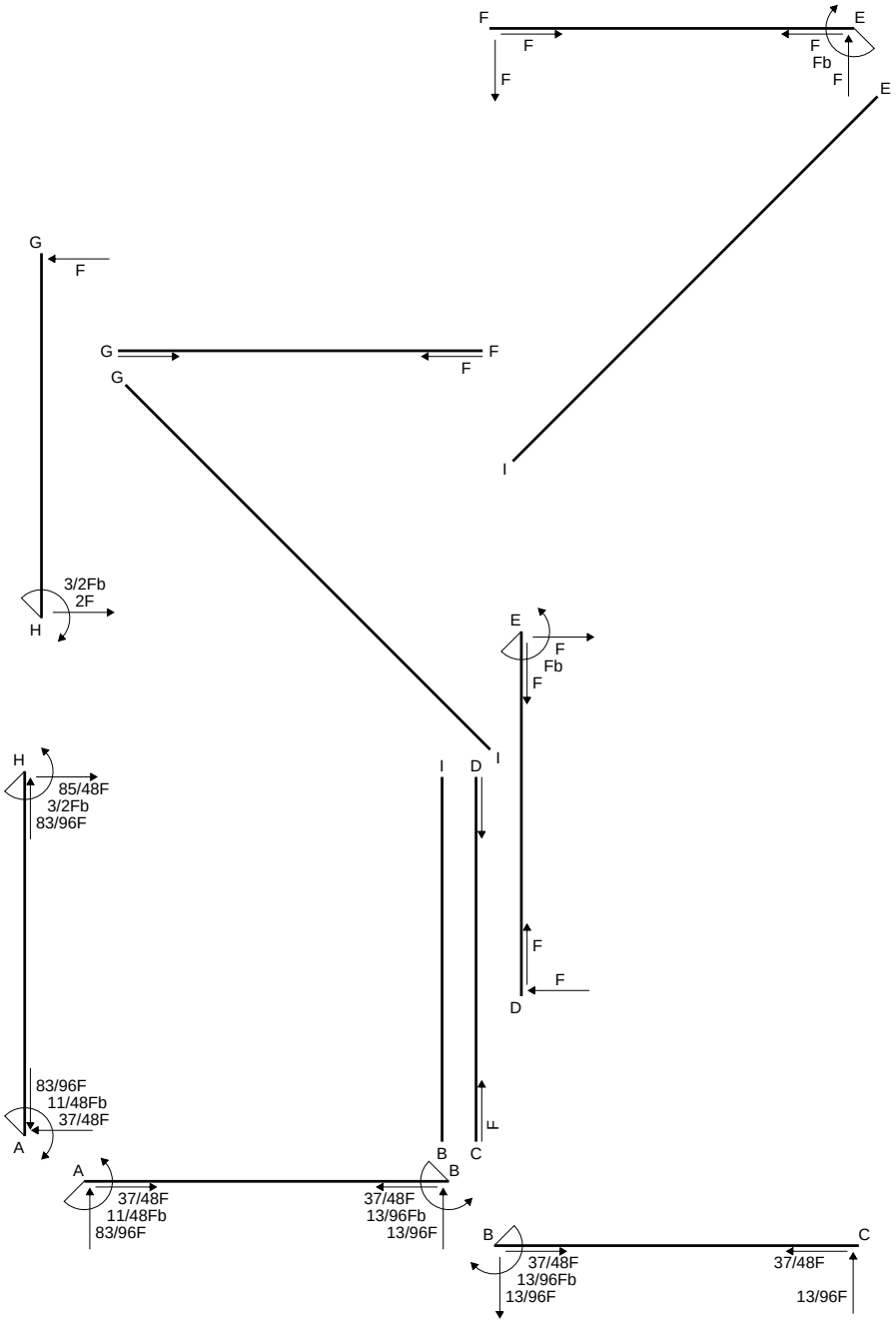
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

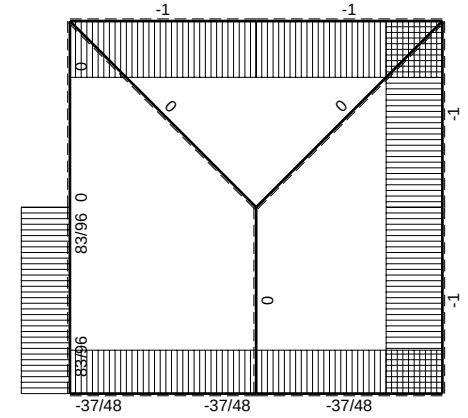
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

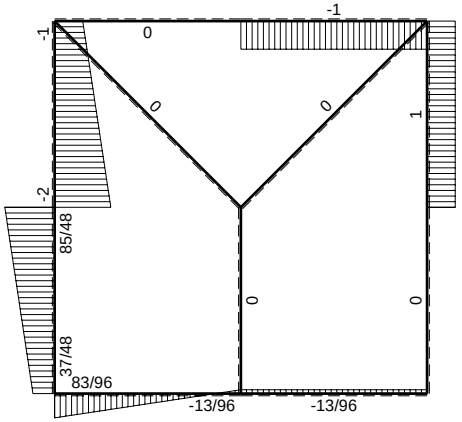
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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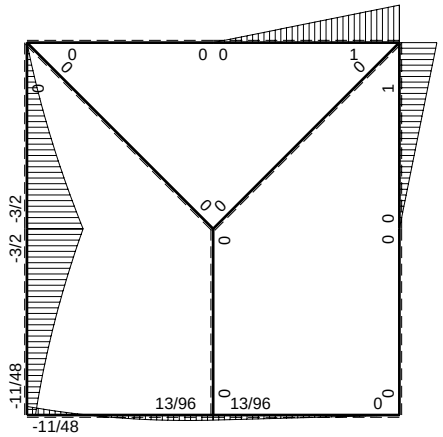

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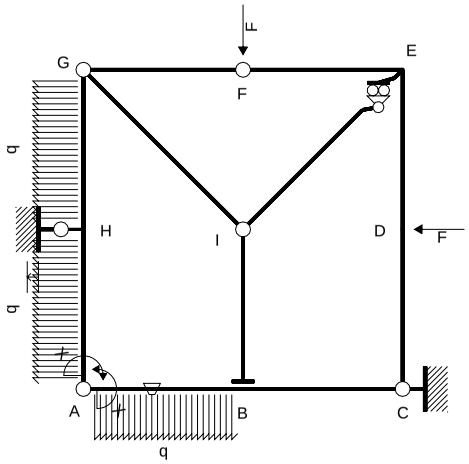

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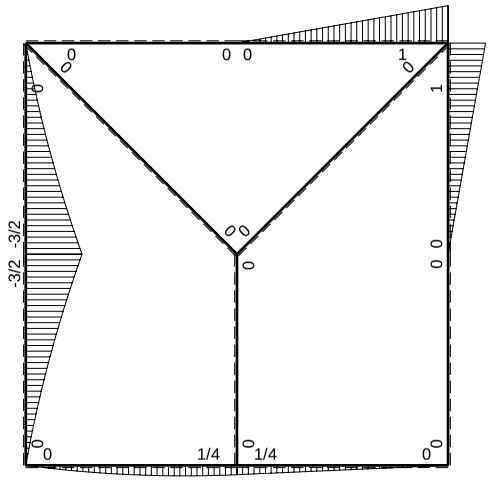
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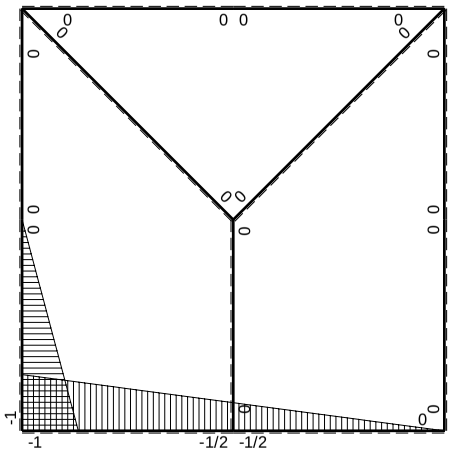

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-11/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx-1/2qx^2	-Fb/EJ	-3/4Fx+7/8Fx^2/b-1/4qx^3/b	Fb/EJ-1/2Fx/EJ	1-x/b+1/4x^2/b^2	(-7/48+3/4)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb-1/4Fx+1/2qx^2	Fb/EJ	-1/8Fb-1/4Fx+1/8Fx^2/b+1/4qx^3/b	1/2Fb/EJ+1/2Fx/EJ	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	1/4Fb-1/4Fx	0	-1/8Fb+1/4Fx-1/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-1/24+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-1/4Fx	0	-1/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-Fx-1/2qx^2	0	0	0	0	0+0	0
HG b	0	3/2Fb-2Fx+1/2qx^2	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	-3/2Fb+2Fx-1/2qx^2	0	3/2Fx-2Fx^2/b+1/2qx^3/b	0	x^2/b^2	(5/24+0)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	Fx+1/2qx^2	0	Fx-1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-11/48Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						11/48Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-3/4\ x/b + 7/8\ x^2/b^2 - 1/4\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\theta\ dx$

$= \left[-3/8\ x^2/b + 7/24\ x^3/b^2 - 1/16\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(-3/8\ b + 7/24\ b - 1/16\ b\right) Fb\ 1/EJ\ + \left(\ b - 1/4\ b\right)\theta\ \ = 29/48\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/8 - 1/4\ x/b + 1/8\ x^2/b^2 + 1/4\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\theta\ dx$

$= \left[-1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2 + 1/16\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(-1/8\ b - 1/8\ b + 1/24\ b + 1/16\ b\right) Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\right)\theta\ \ = 29/48\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-1/8 + 1/4\ x/b - 1/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/8\ x^2/b - 1/24\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/8\ b + 1/8\ b - 1/24\ b\right) Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-1/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/24\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/24\ b\right) Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

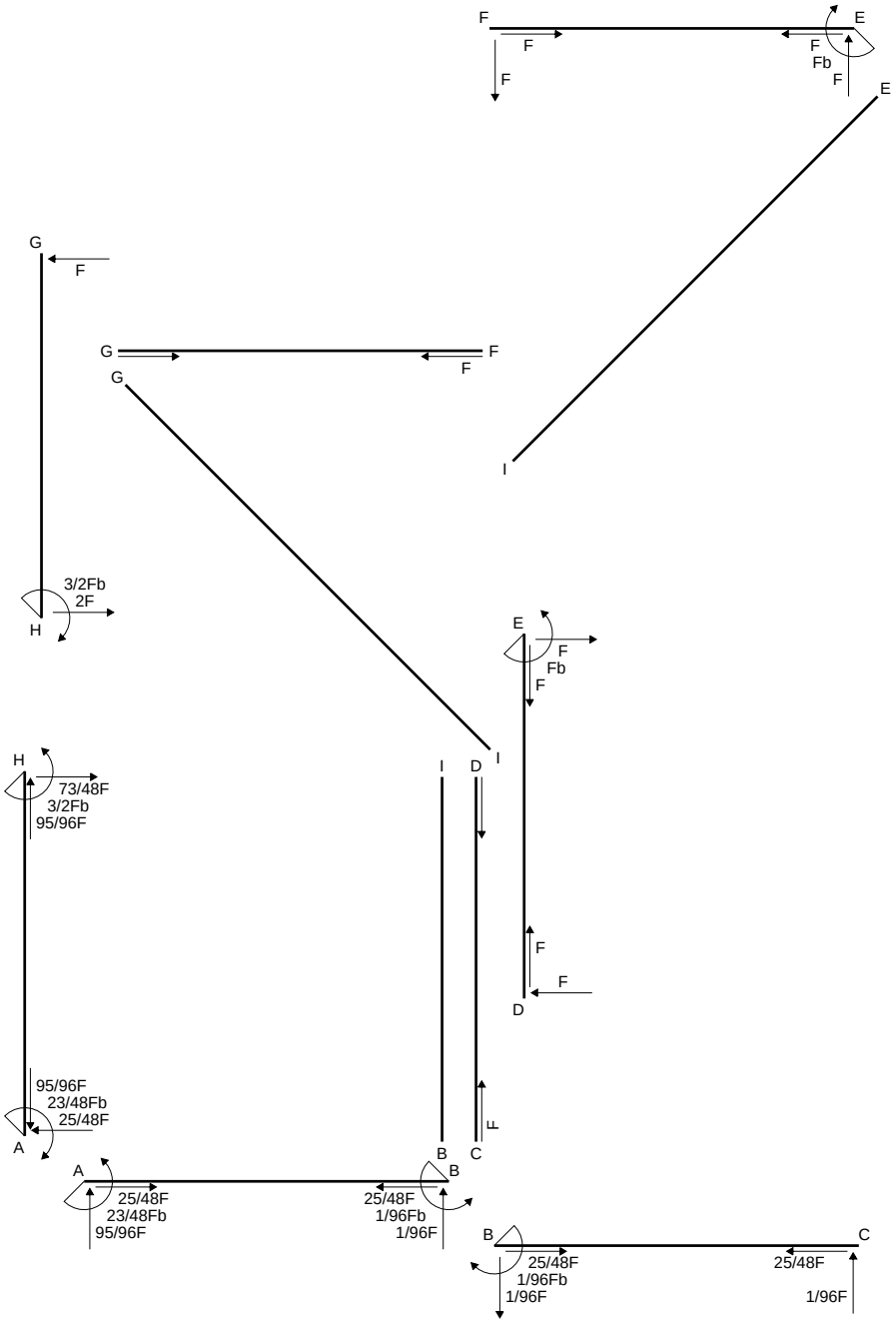
$= \left(3/4\ b - 2/3\ b + 1/8\ b\right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

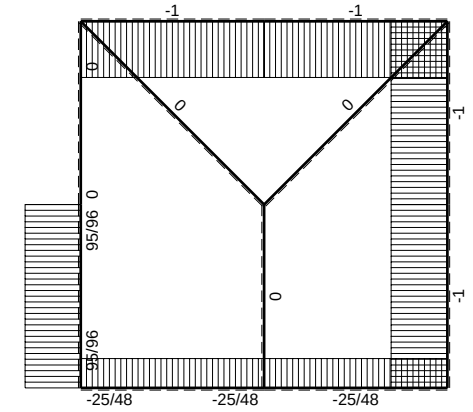
$L^{Xo}_{AH} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

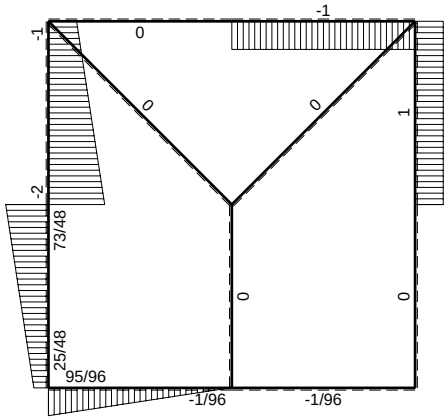
$= \left(1/2\ b - 1/6\ b - 1/8\ b\right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

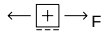
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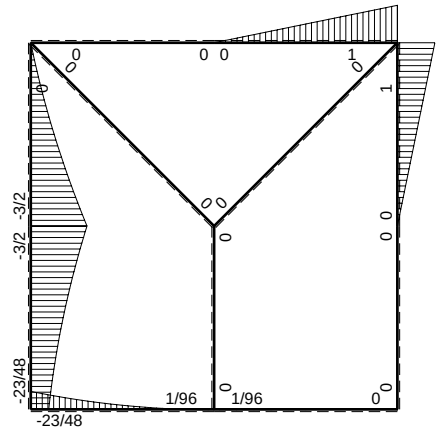

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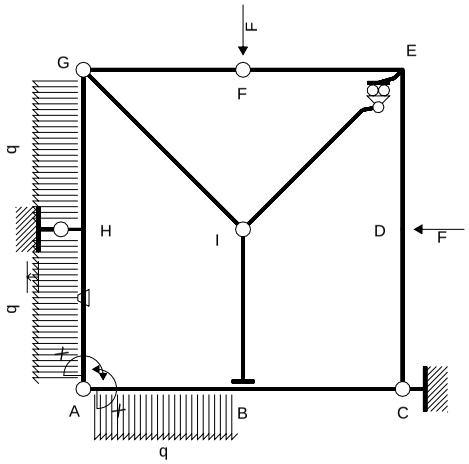

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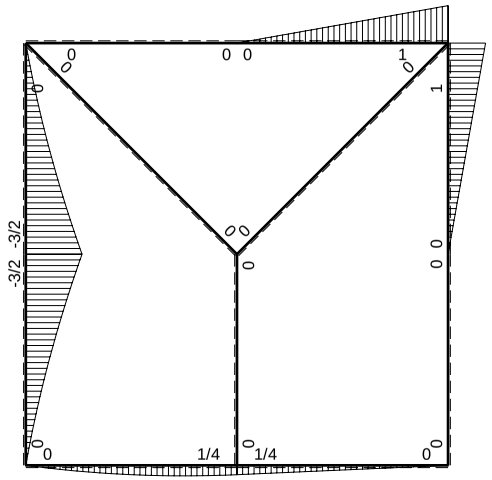

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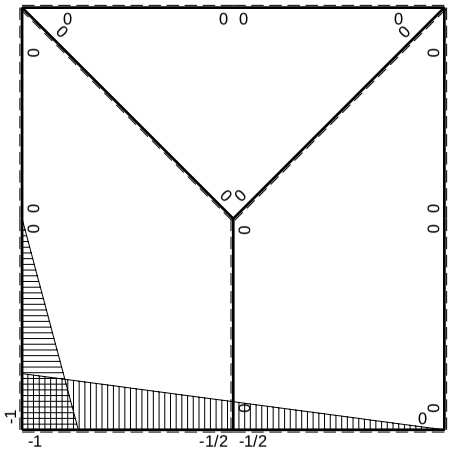

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-23/48


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx-1/2qx^2	0	-3/4Fx+7/8Fx^2/b-1/4qx^3/b	0	1-x/b+1/4x^2/b^2	(-7/48+0)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb-1/4Fx+1/2qx^2	0	-1/8Fb-1/4Fx+1/8Fx^2/b+1/4qx^3/b	0	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	1/4Fb-1/4Fx	0	-1/8Fb+1/4Fx-1/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-1/24+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-1/4Fx	0	-1/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-Fx-1/2qx^2	0	0	0	0	0+0	0
HG b	0	3/2Fb-2Fx+1/2qx^2	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	-3/2Fb+2Fx-1/2qx^2	-Fb/EJ	3/2Fx-2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/24+1/2)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	Fx+1/2qx^2	Fb/EJ	Fx-1/2Fx^2/b-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-23/48Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						23/48Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 7/8\ x^2/b^2 - 1/4\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 7/24\ x^3/b^2 - 1/16\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 7/24\ b - 1/16\ b\ \right)\ Fb\ 1/EJ\ = -7/48\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 - 1/4\ x/b + 1/8\ x^2/b^2 + 1/4\ x^3/b^3\right)\ Fb\ 1/EJ\ dx$

$= \left[-1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2 + 1/16\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b - 1/8\ b + 1/24\ b + 1/16\ b\ \right)\ Fb\ 1/EJ\ = -7/48\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/8 + 1/4\ x/b - 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/8\ x^2/b - 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/8\ b - 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ dx$

$= \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = 17/24\ \ Fb^2/EJ$

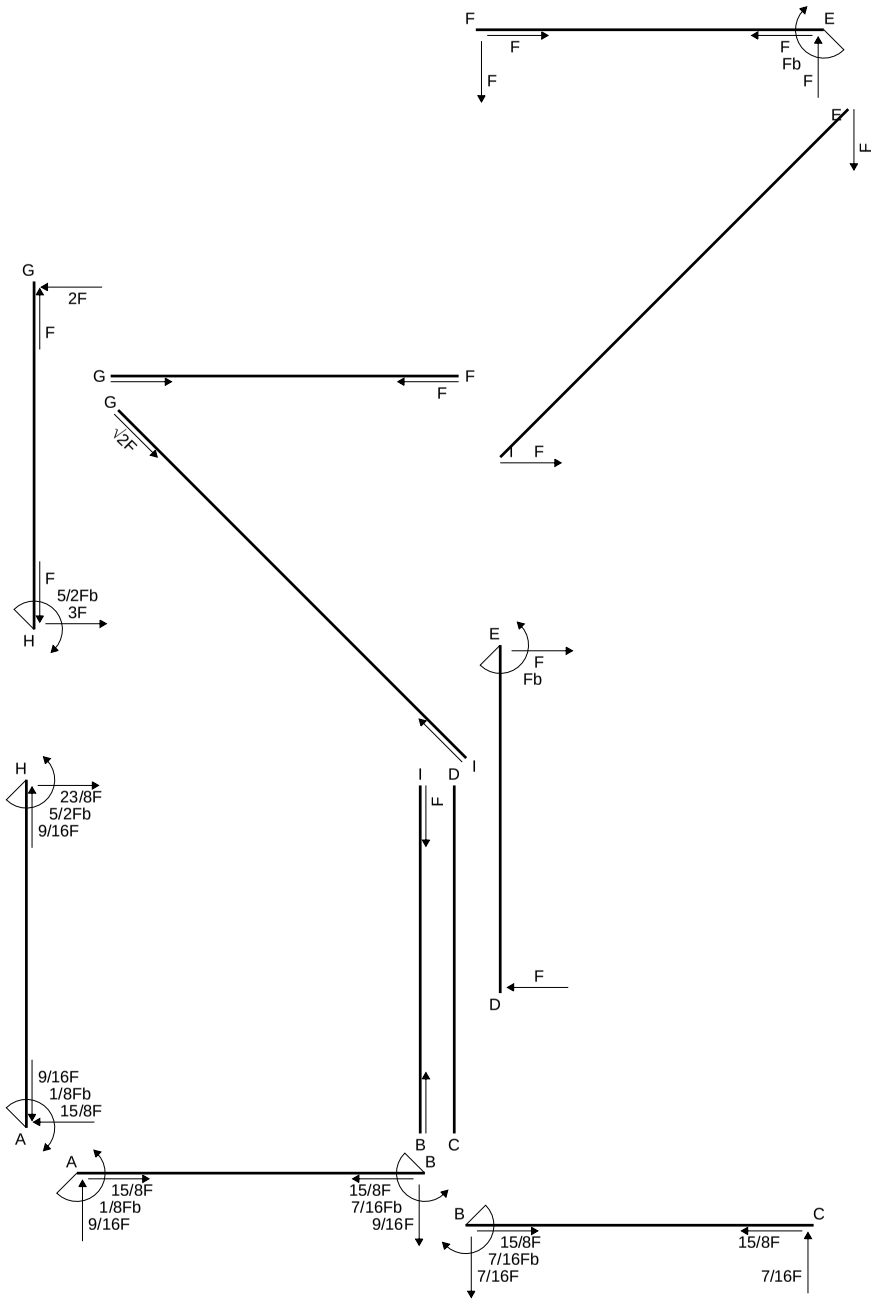
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ dx$

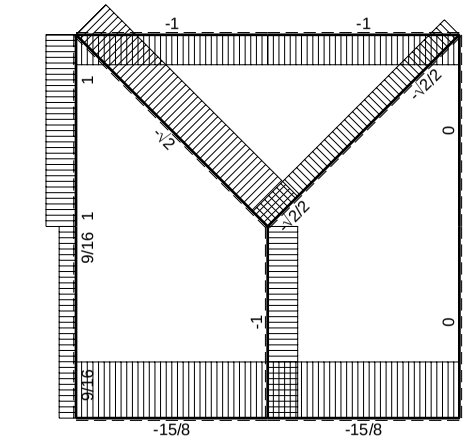
$= \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

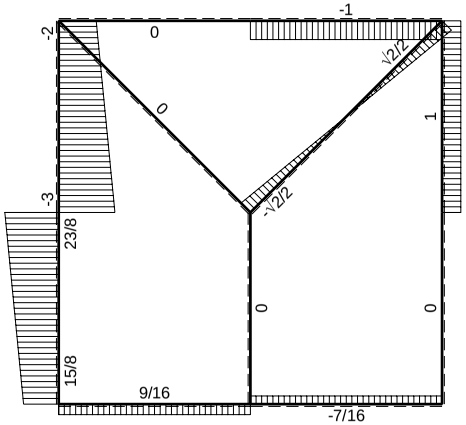
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\ \theta\ \ = 17/24\ \ Fb^2/EJ$

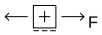
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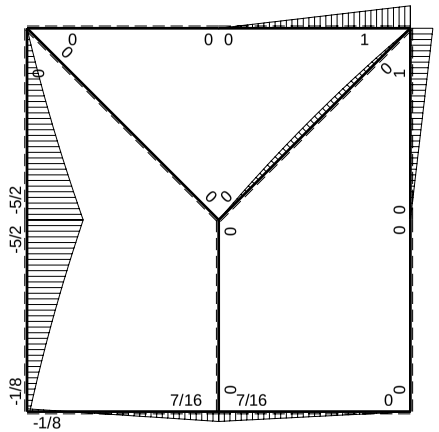

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7/16F
15/8F
7/16F


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9/16
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-15/8

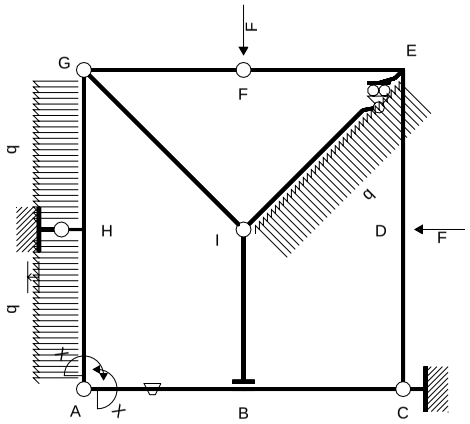

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-√2/2
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15/8
9/16
-7/16

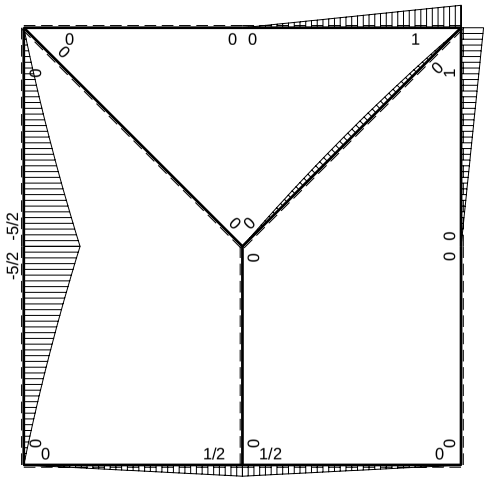

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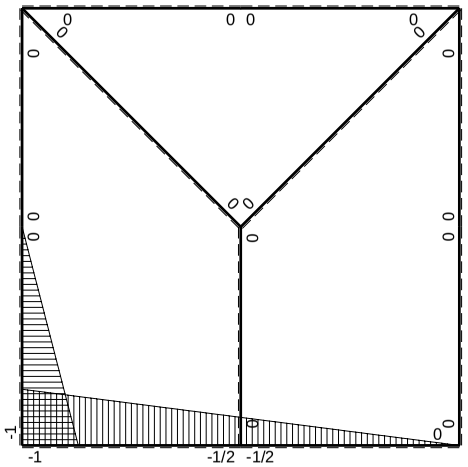

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0 0
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0 0
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-5/2
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0 0
-1/8
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0
7/16
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0
-1/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	-Fb/EJ	$-1/2Fx+1/4Fx^2/b$	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(-1/6+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/2Fb+1/2Fx	Fb/EJ	$-1/4Fb+1/4Fx^2/b$	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 7/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 7/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

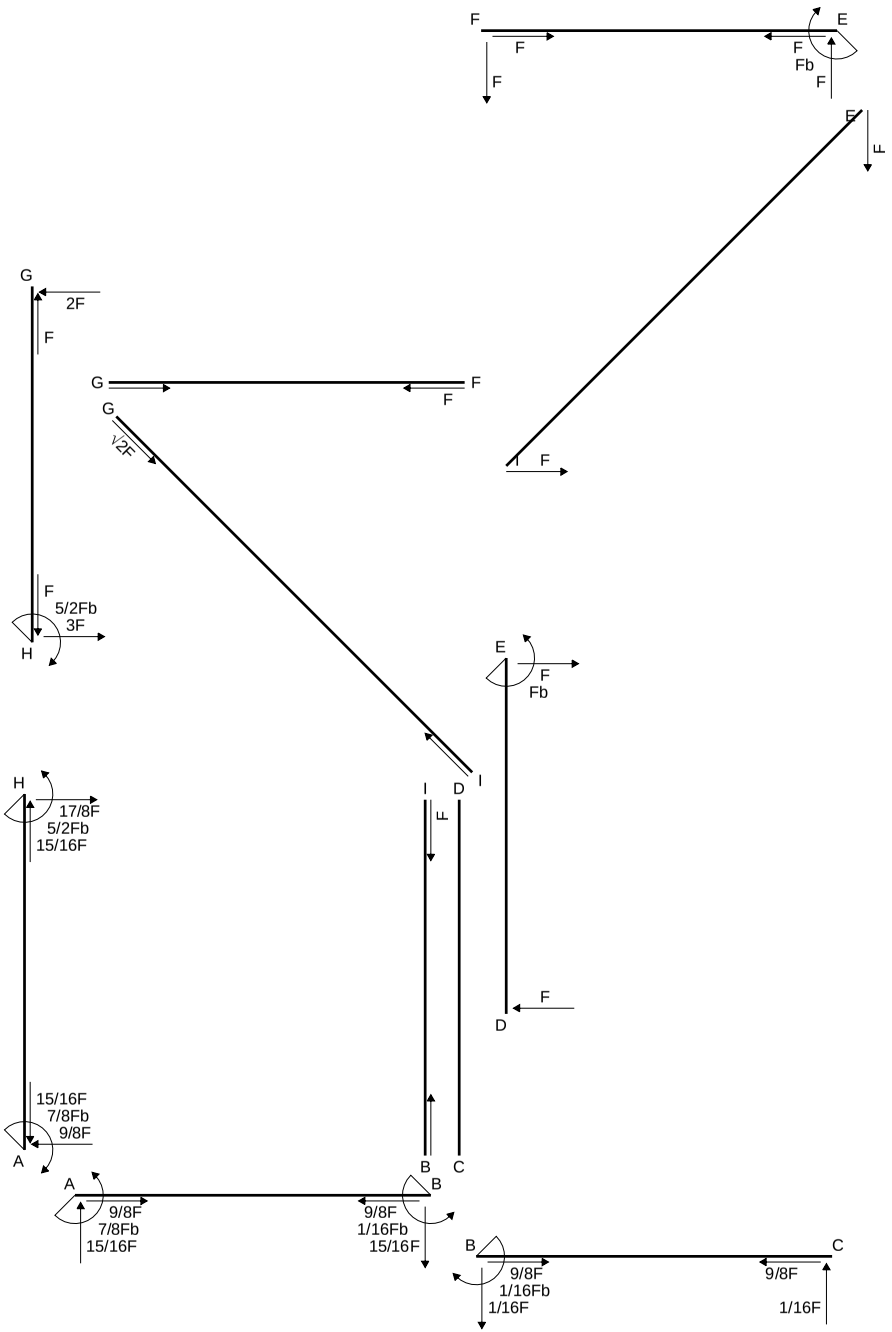
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

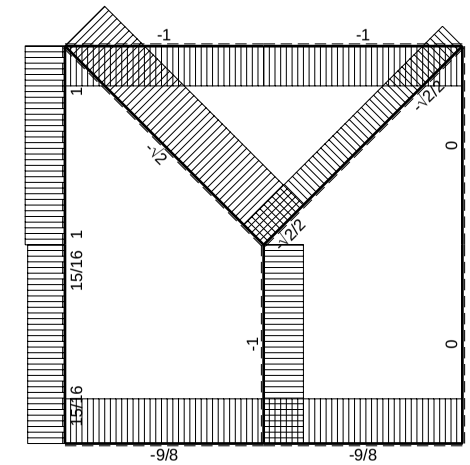
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

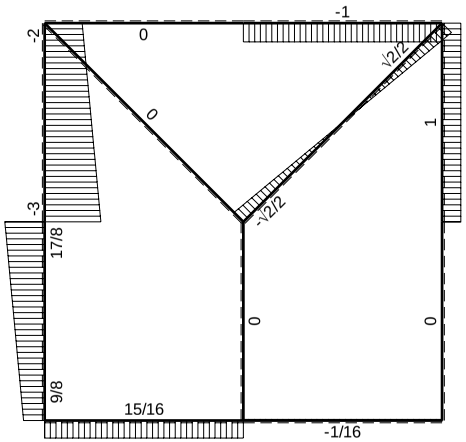
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

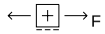
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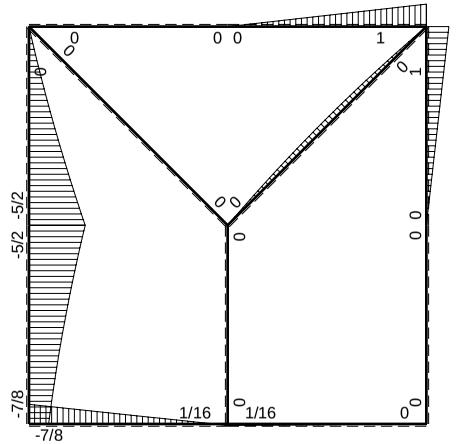

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1/16F


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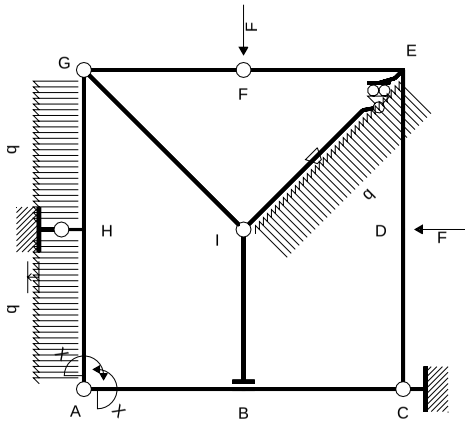

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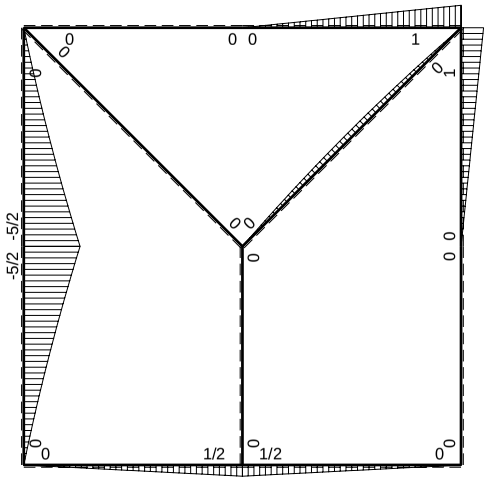

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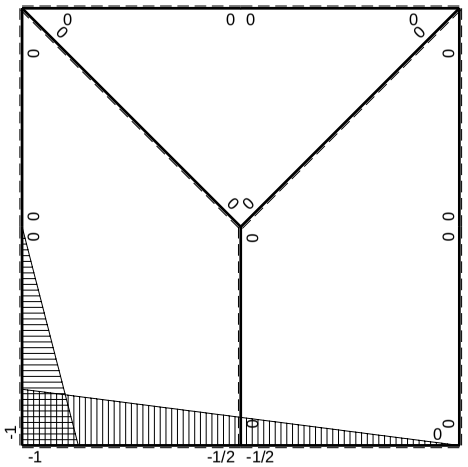

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-7/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236538 ESΣ04.xxxx.103

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

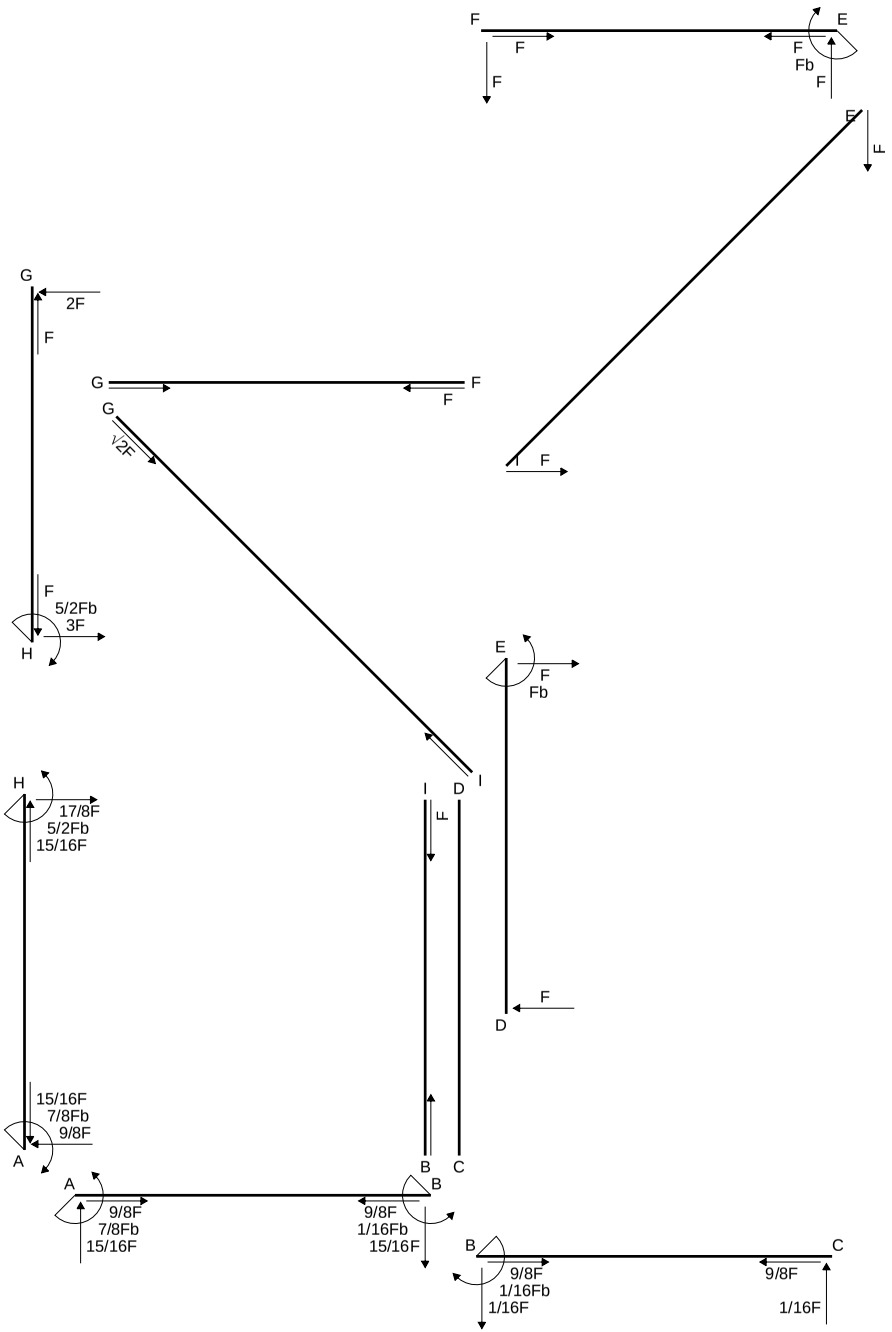
$$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$$

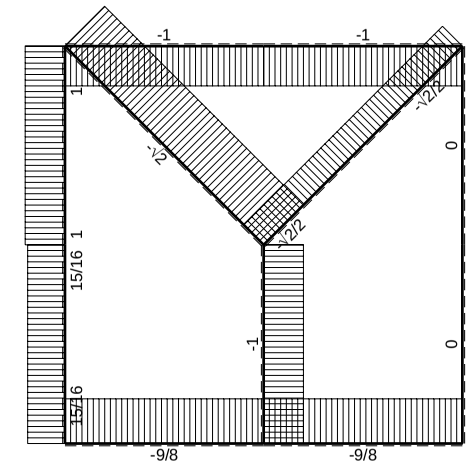
$$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

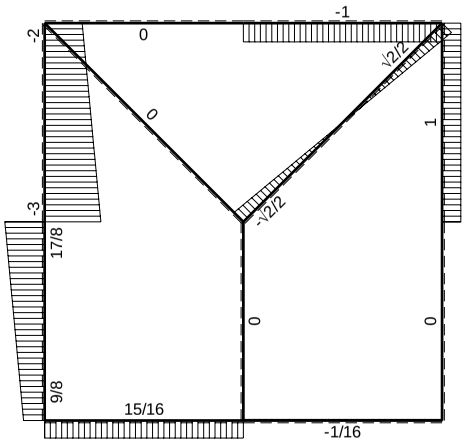
$$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$$

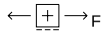
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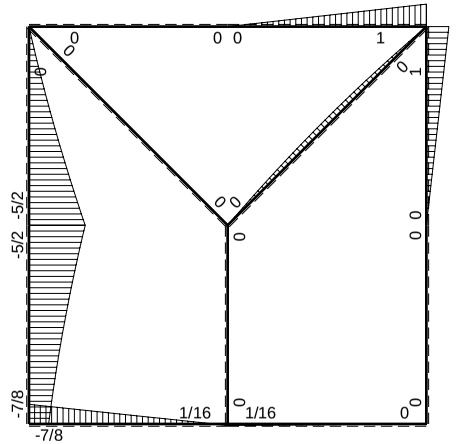

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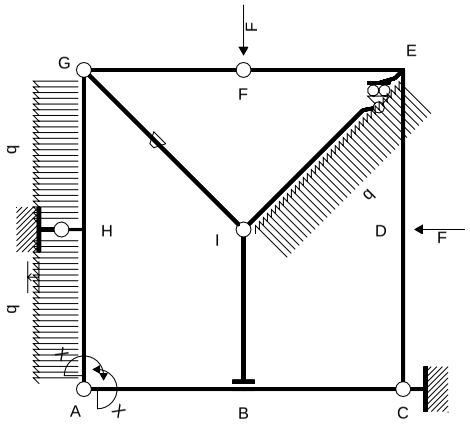

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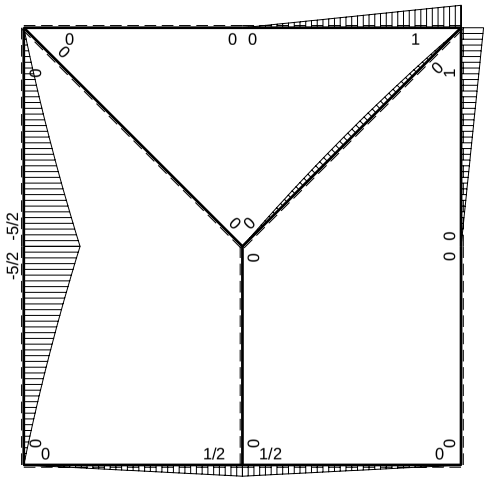

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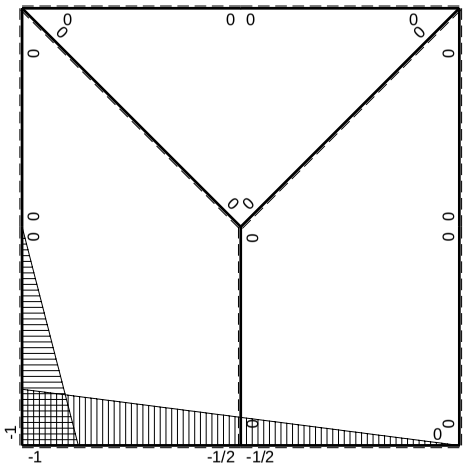

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-5/2
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0
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0 0
-7/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right) Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/12\ b\right) Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

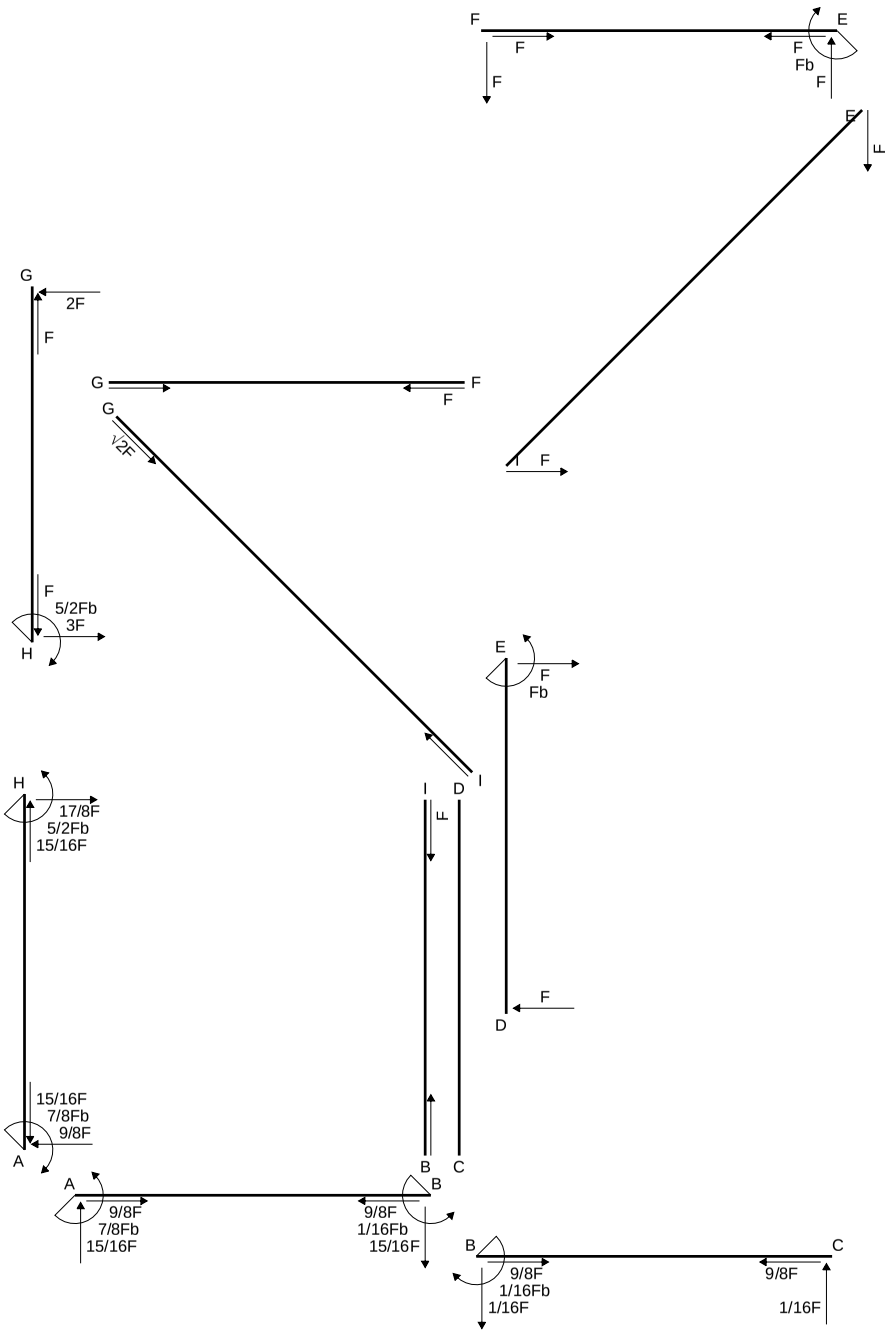
$= \left(5/4\ b - b + 1/8\ b\right) Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

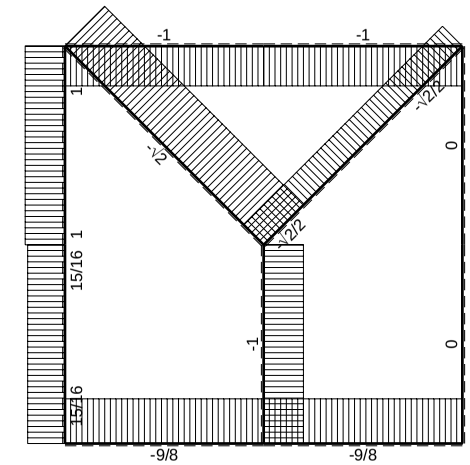
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

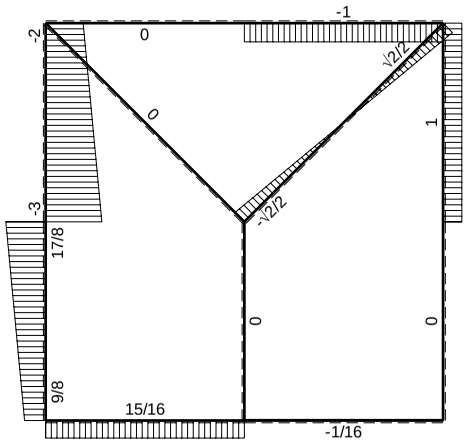
$= \left(\ b - 1/2\ b - 1/8\ b\right) Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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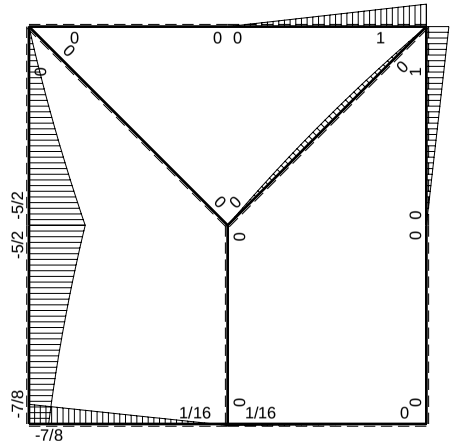

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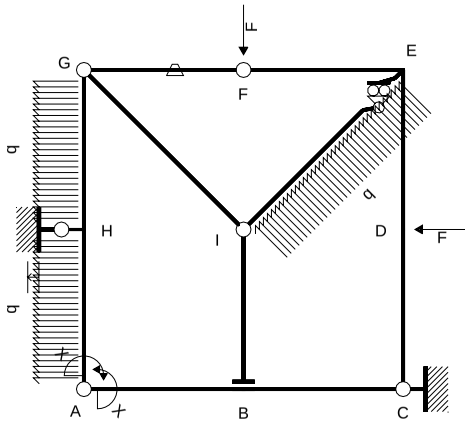

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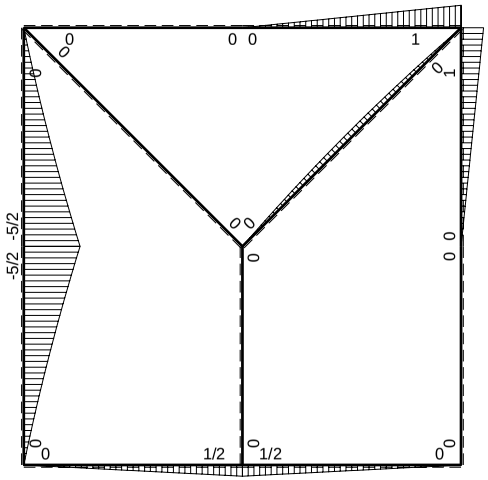
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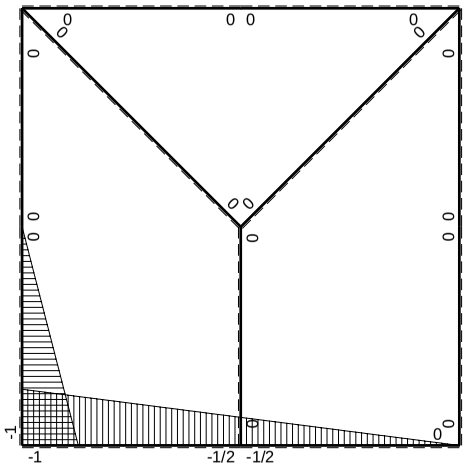

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0
1/16
0
-7/8

□ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237131 ESΣ04.xxxx.105

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

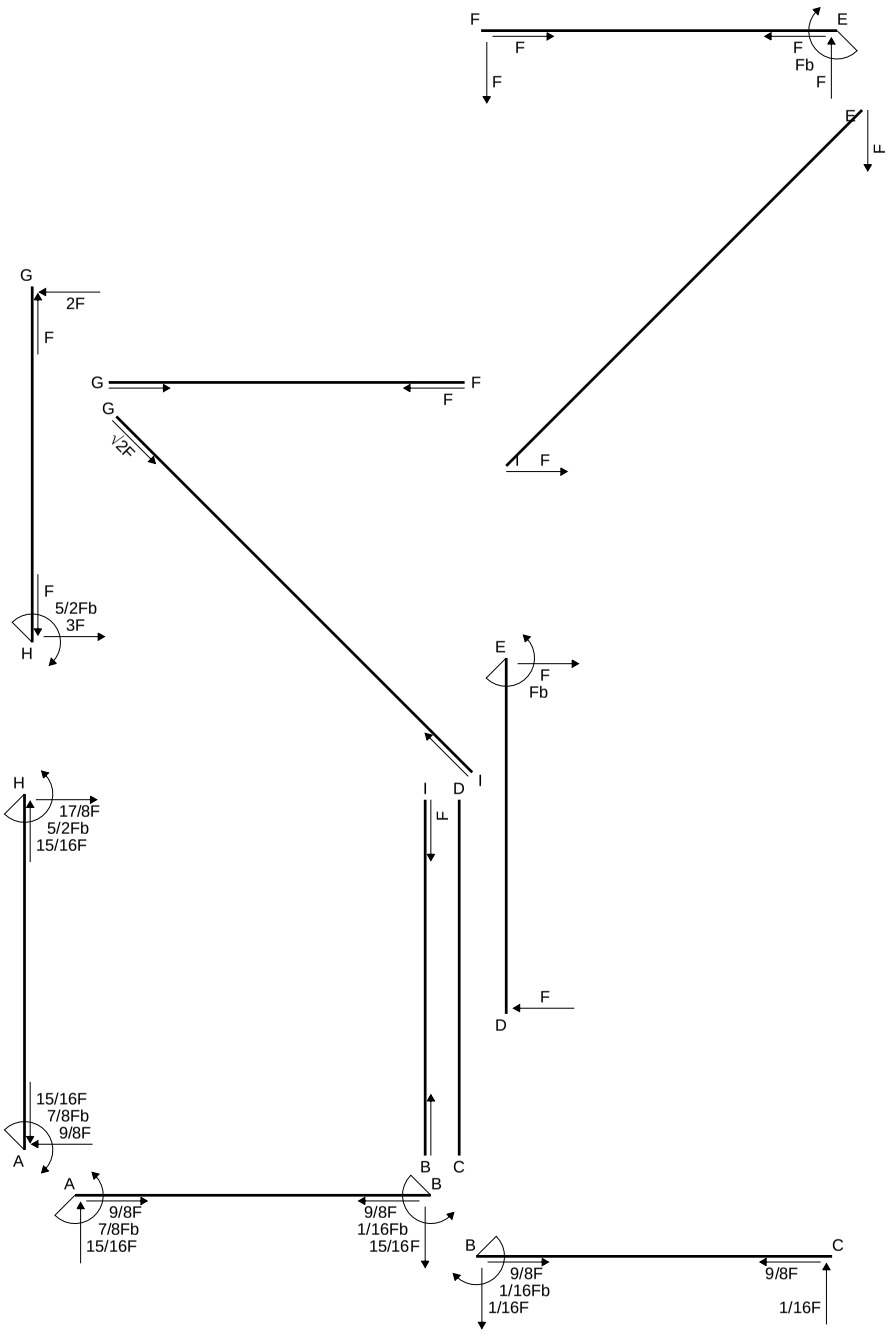
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

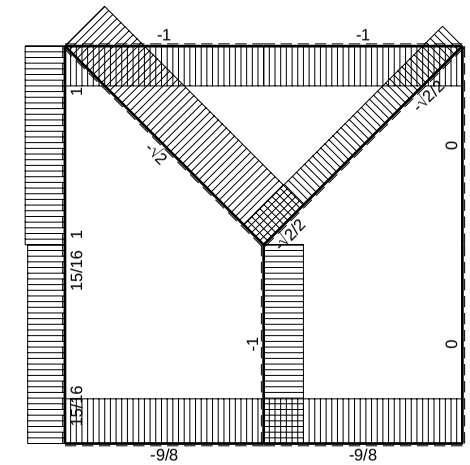
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

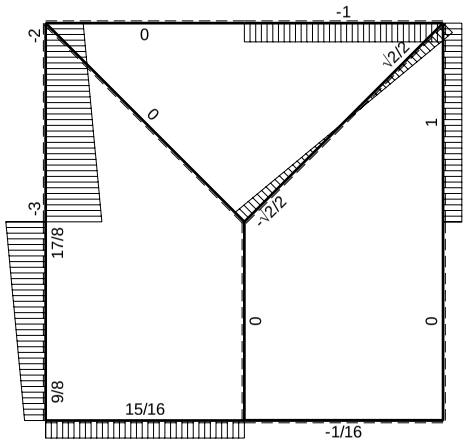
ESΣ04.xxxx.105

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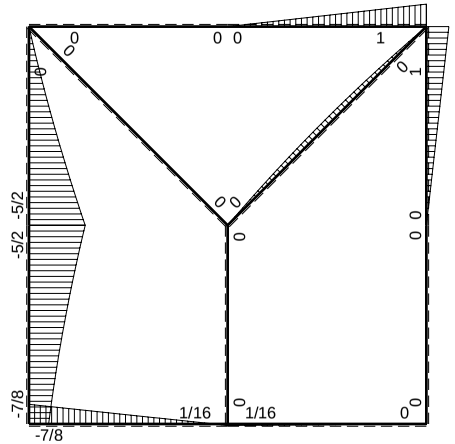

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1/16F
9/8F
1/16F


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-9/8
-9/8

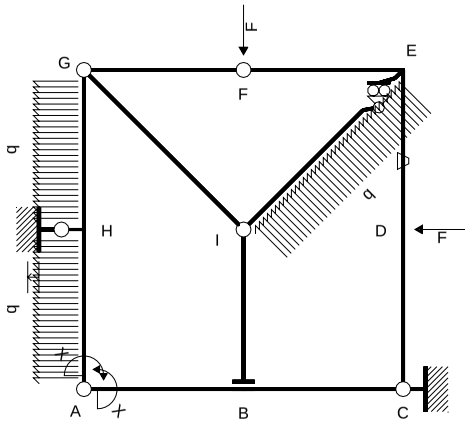
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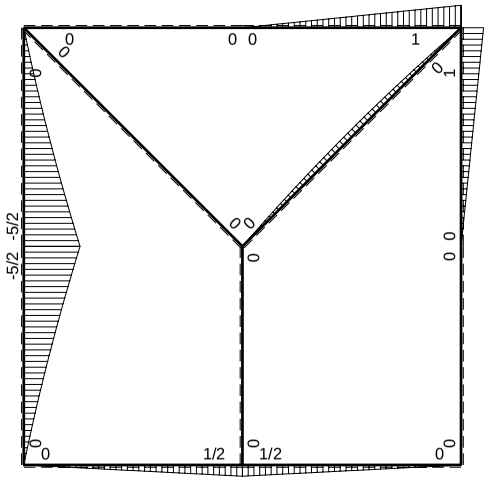

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-√2/2
17/8
0
0
9/8
15/16
-1/16

F

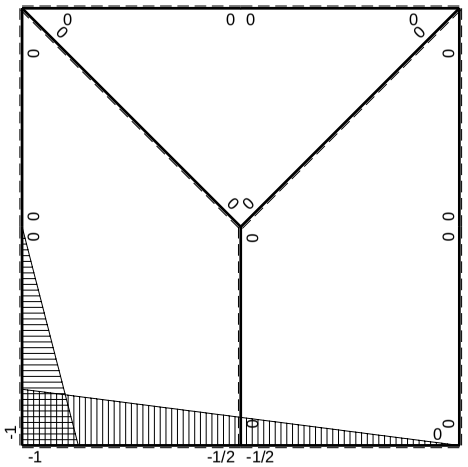

0
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1
0
0
1
0 0
-5/2
-5/2
0
0 0
-7/8
1/16
1/16
0
0
-7/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239106 ESΣ04.xxxx.106

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

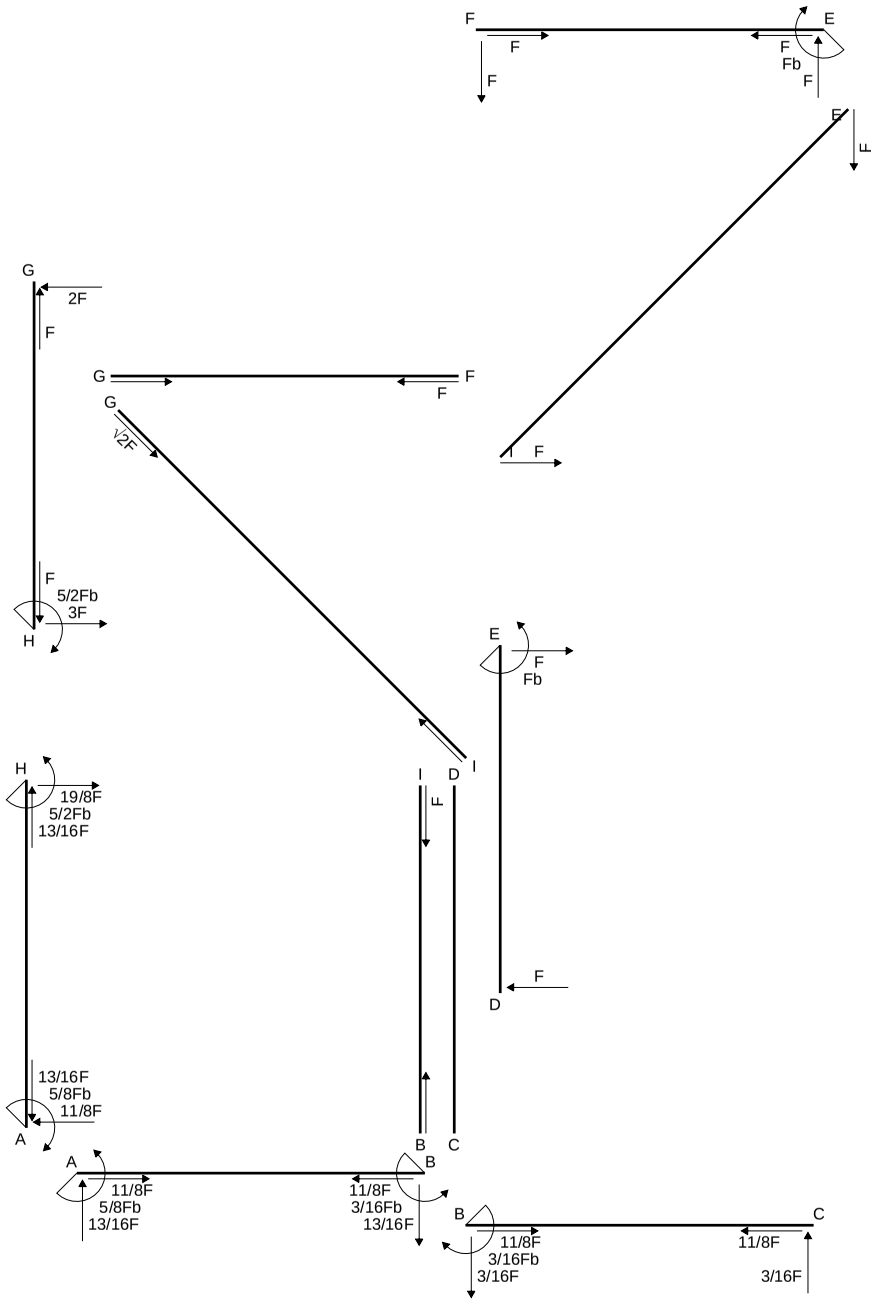
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

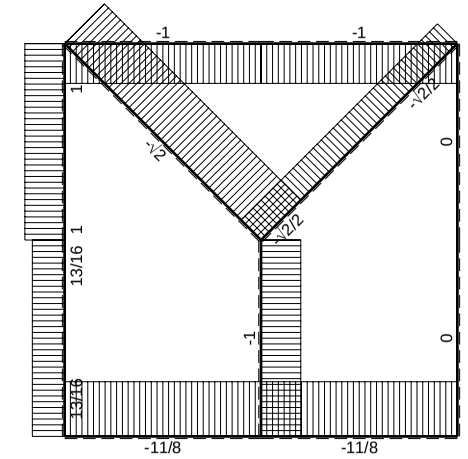
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

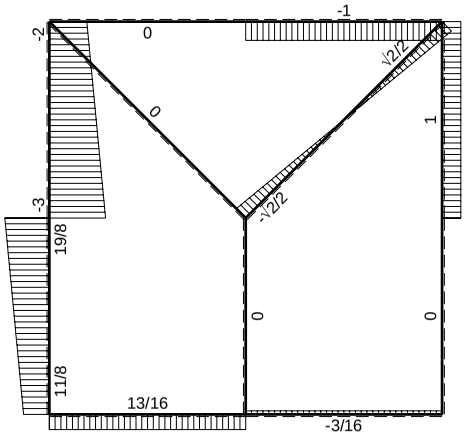
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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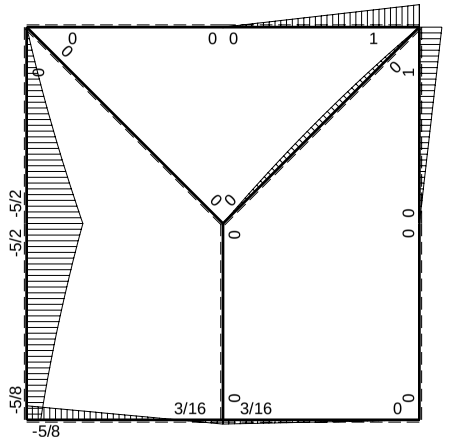

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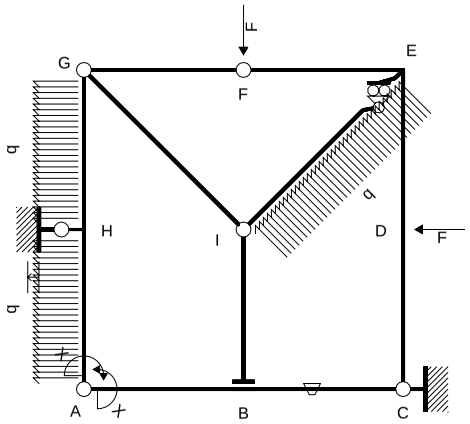

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13/16
-3/16

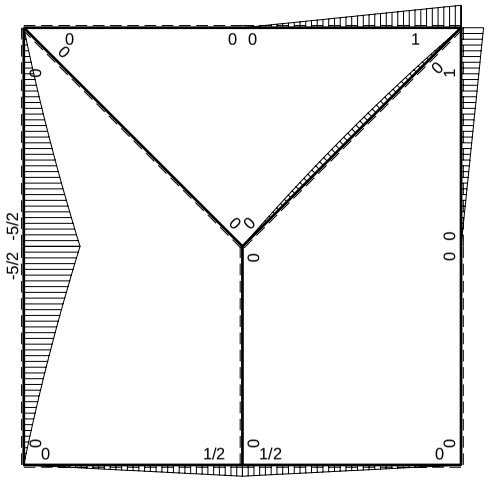
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↑□↓ F

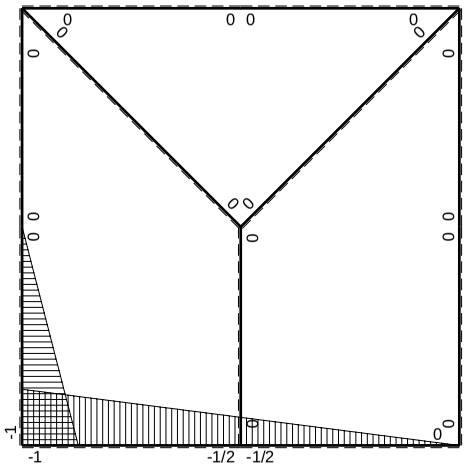

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-5/8
3/16
3/16
0
0
-5/8

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 983430 ESΣ04.xxxx.107

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	-Fb/EJ	$-1/4Fb+1/2Fx-1/4Fx^2/b$	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	Fb/EJ	$-1/4Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\right)\ \theta\ \ = 1/6\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\right)\ \theta\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ + \left(-1/4\ b\right)\ \theta\ \ = 1/6\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

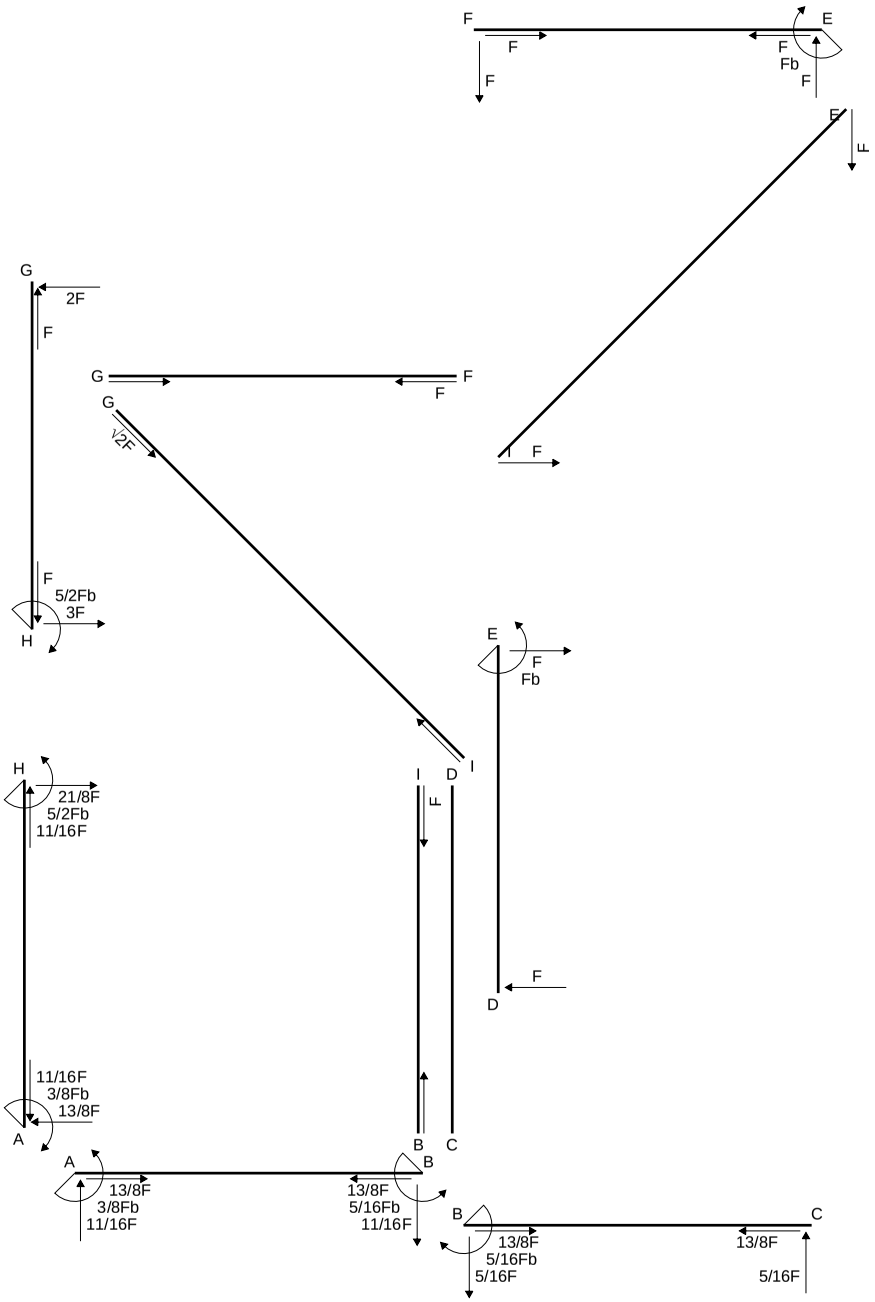
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

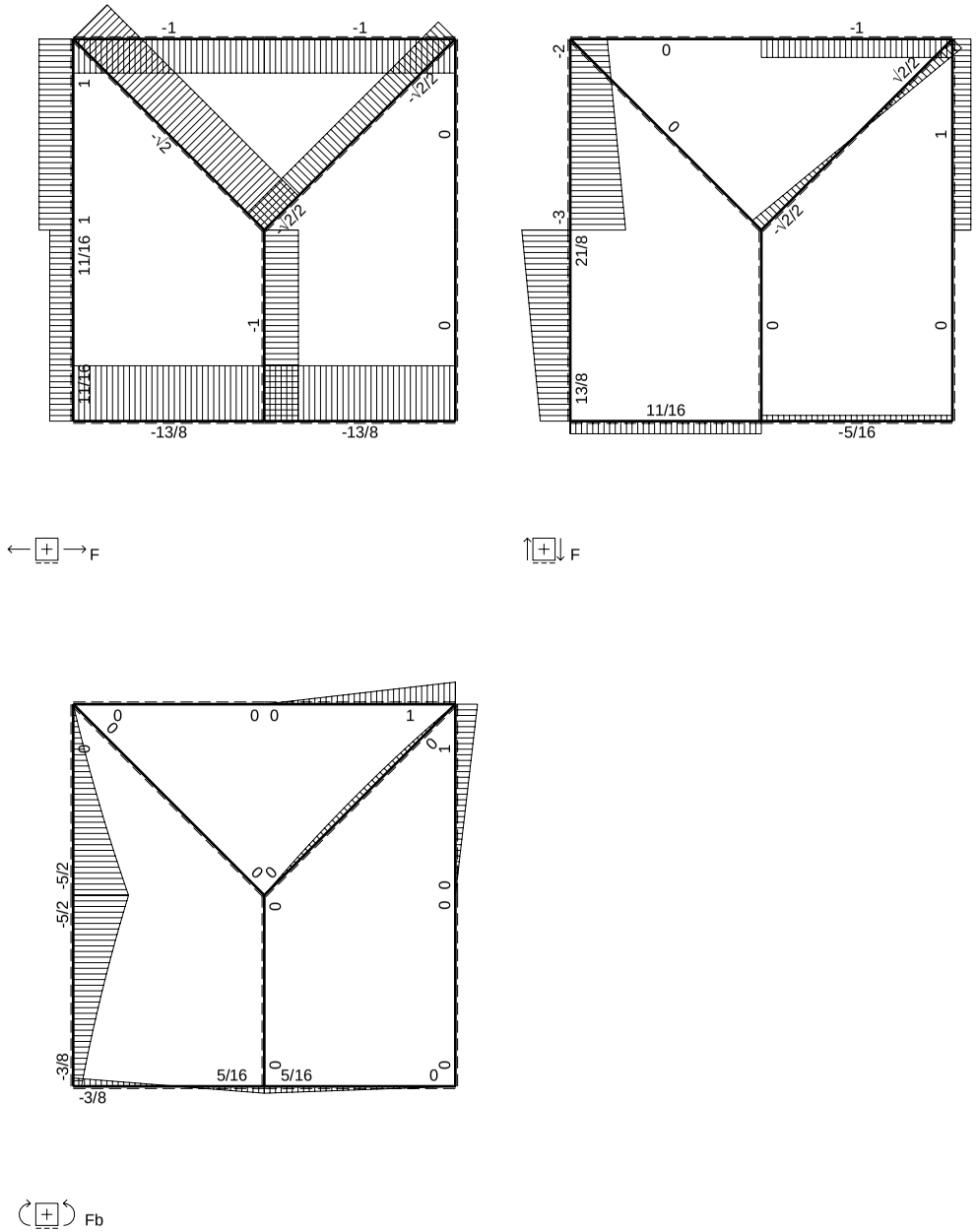
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

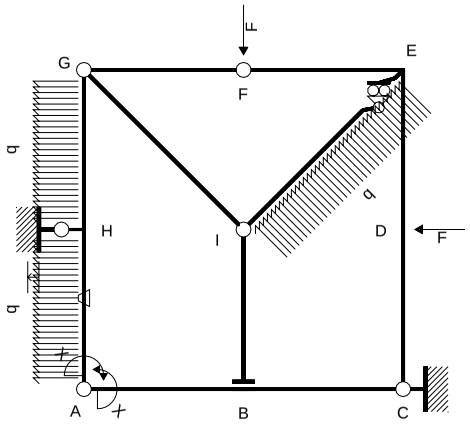
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

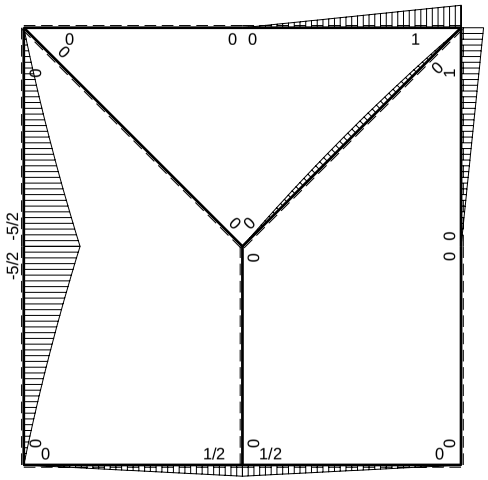
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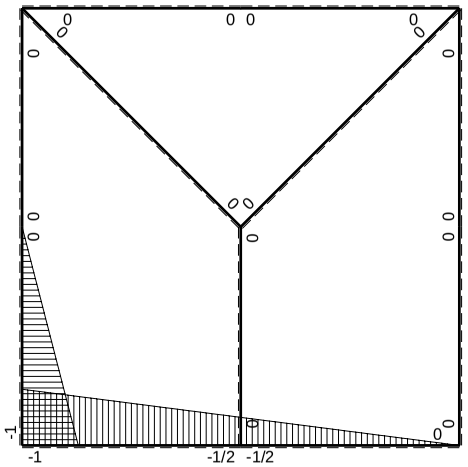

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-√2/2
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1
0 0
0
-5/2 -5/2
0 0
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0 0
-3/8
5/16
5/16
0
0 0
-3/8
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	-Fb/EJ	$5/2Fx-3Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(3/8+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	Fb/EJ	$2Fx-3/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx$

$= \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = 7/8\ \ Fb^2/EJ$

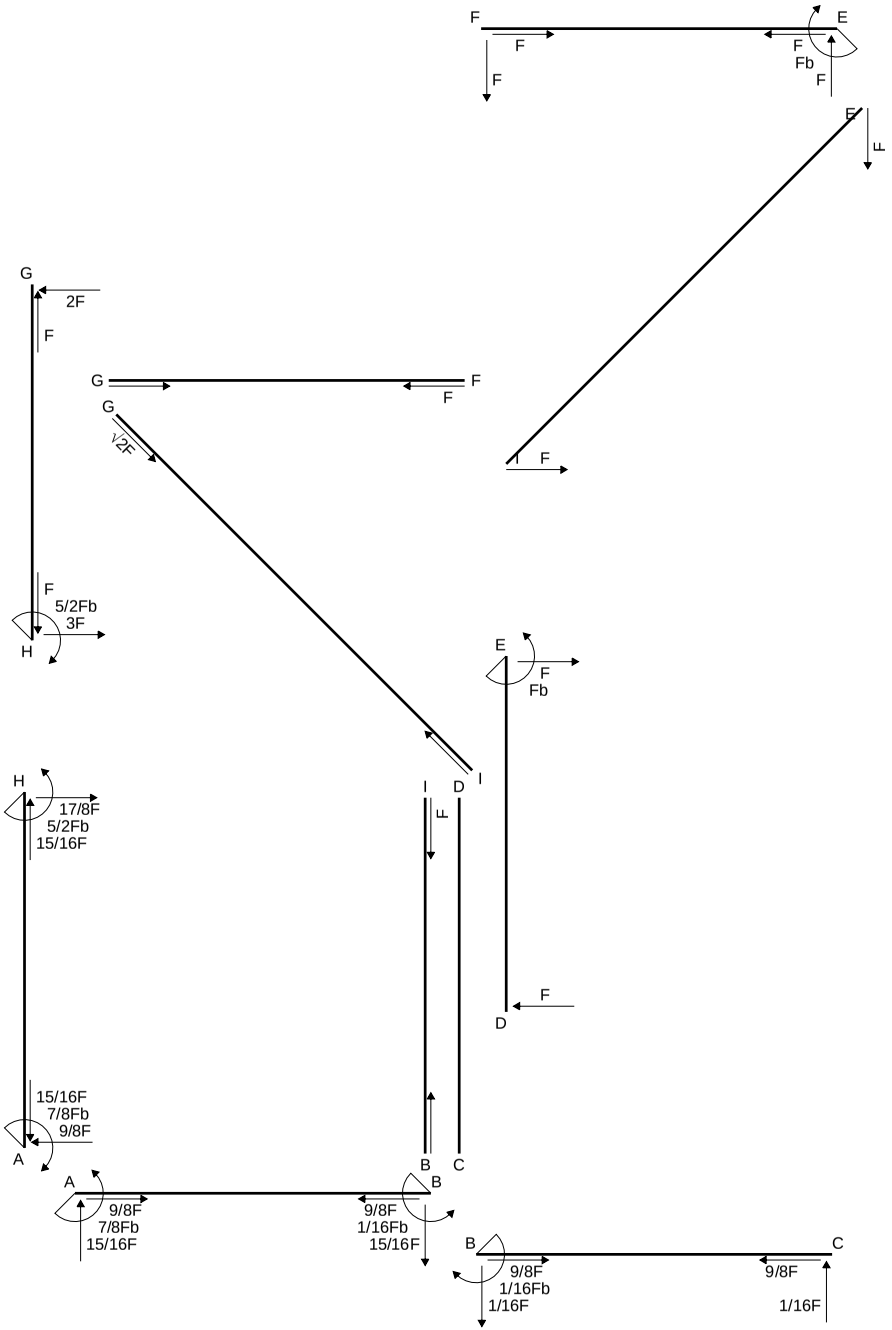
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

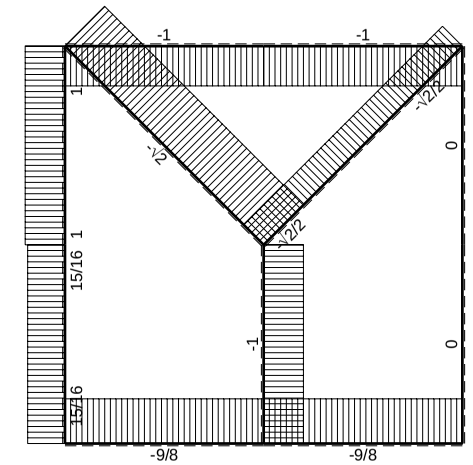
$= \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

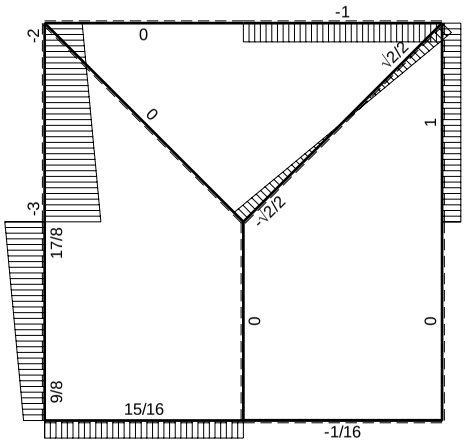
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\ \theta\ \ = 7/8\ \ Fb^2/EJ$

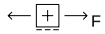
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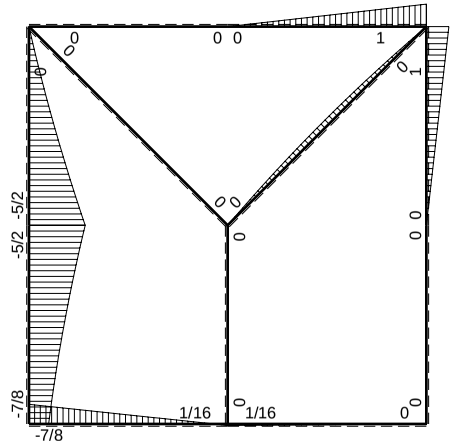

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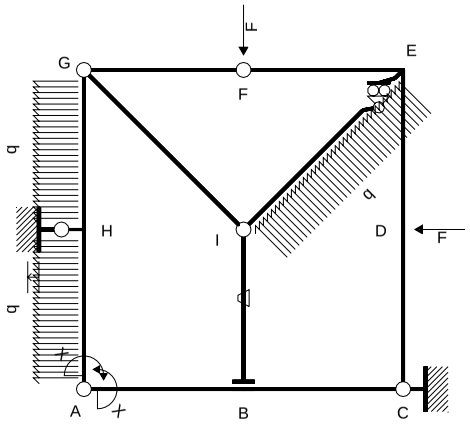

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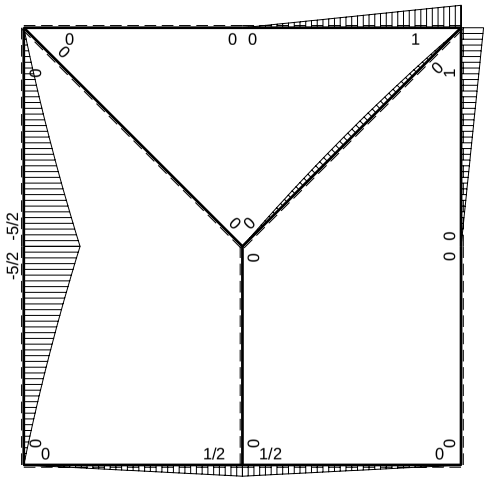

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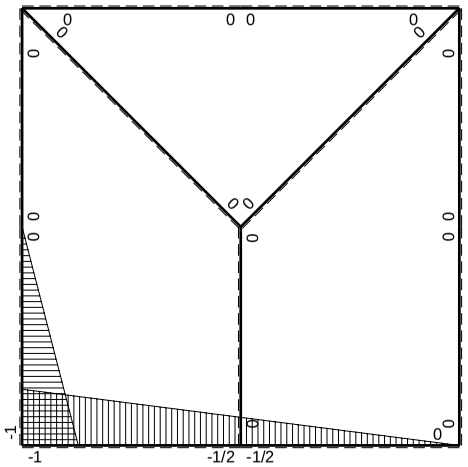

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1/16
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-7/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 309494 ESΣ04.xxxx.109

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PROCEDIMENTO E RISULTATI 309494 ESΣ04.xxxx.109

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

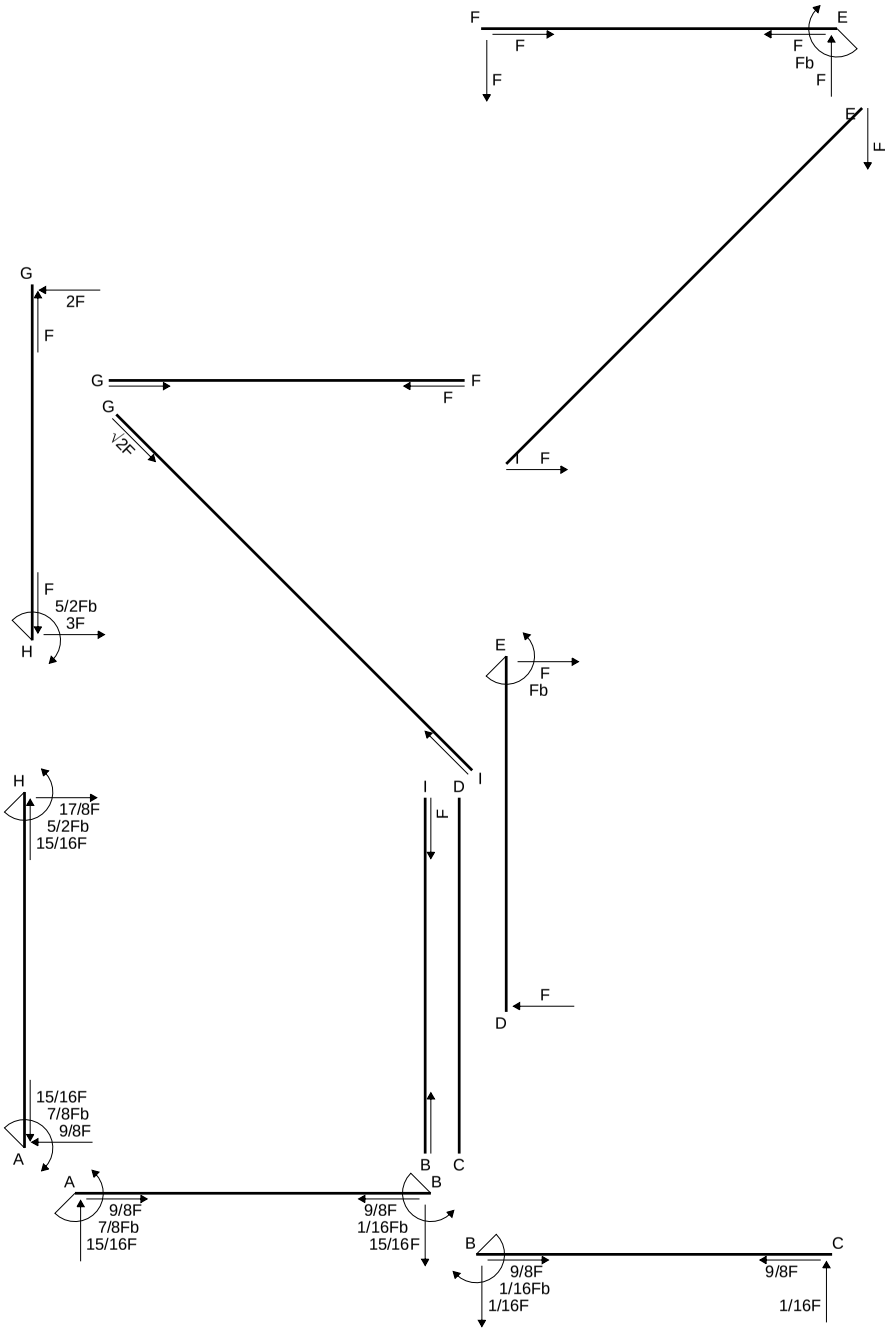
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

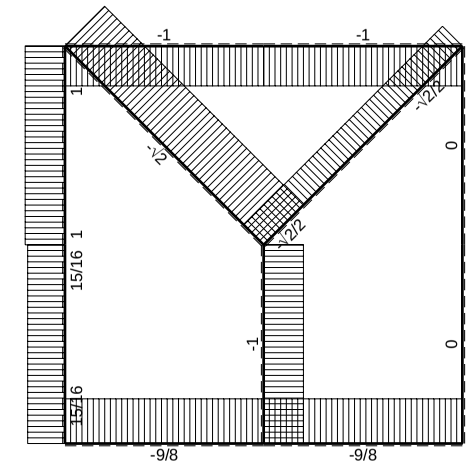
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

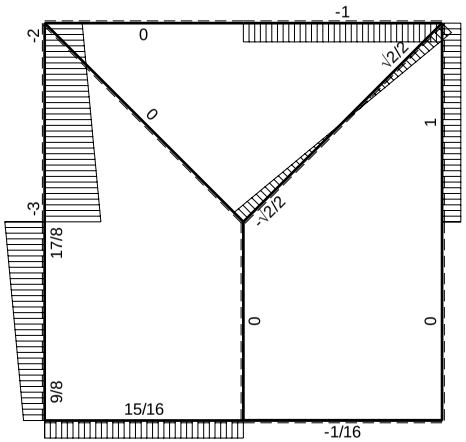
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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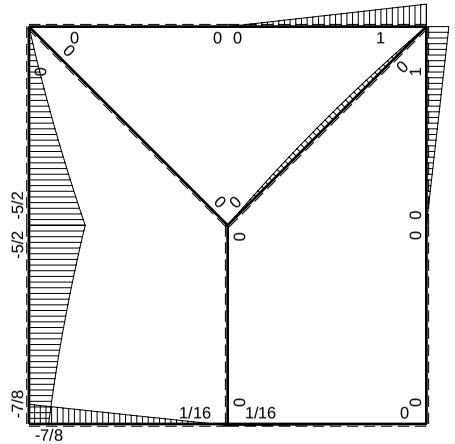

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1/16F


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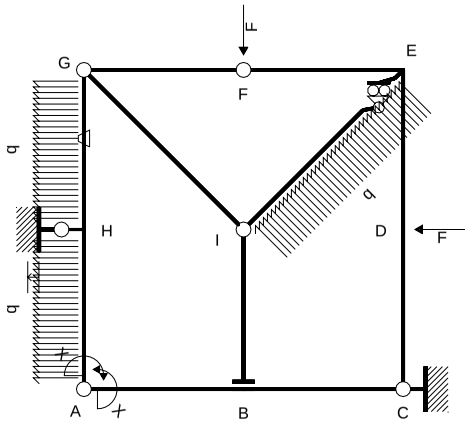

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9/8
15/16
-1/16

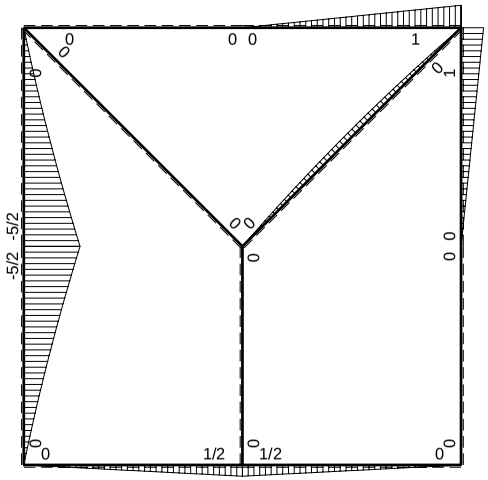
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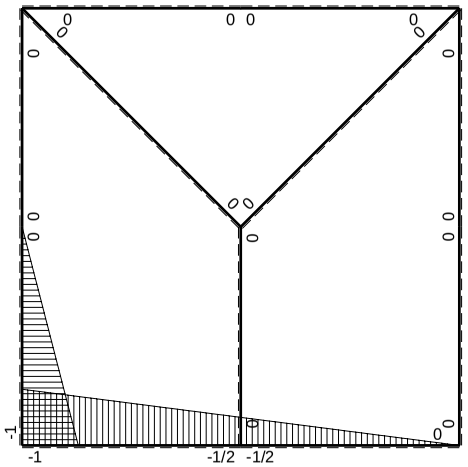

0
0 0
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0
1
0
0
-5/2
-5/2
0
0 0
0
0 0
-7/8
1/16
1/16
0
0
-7/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

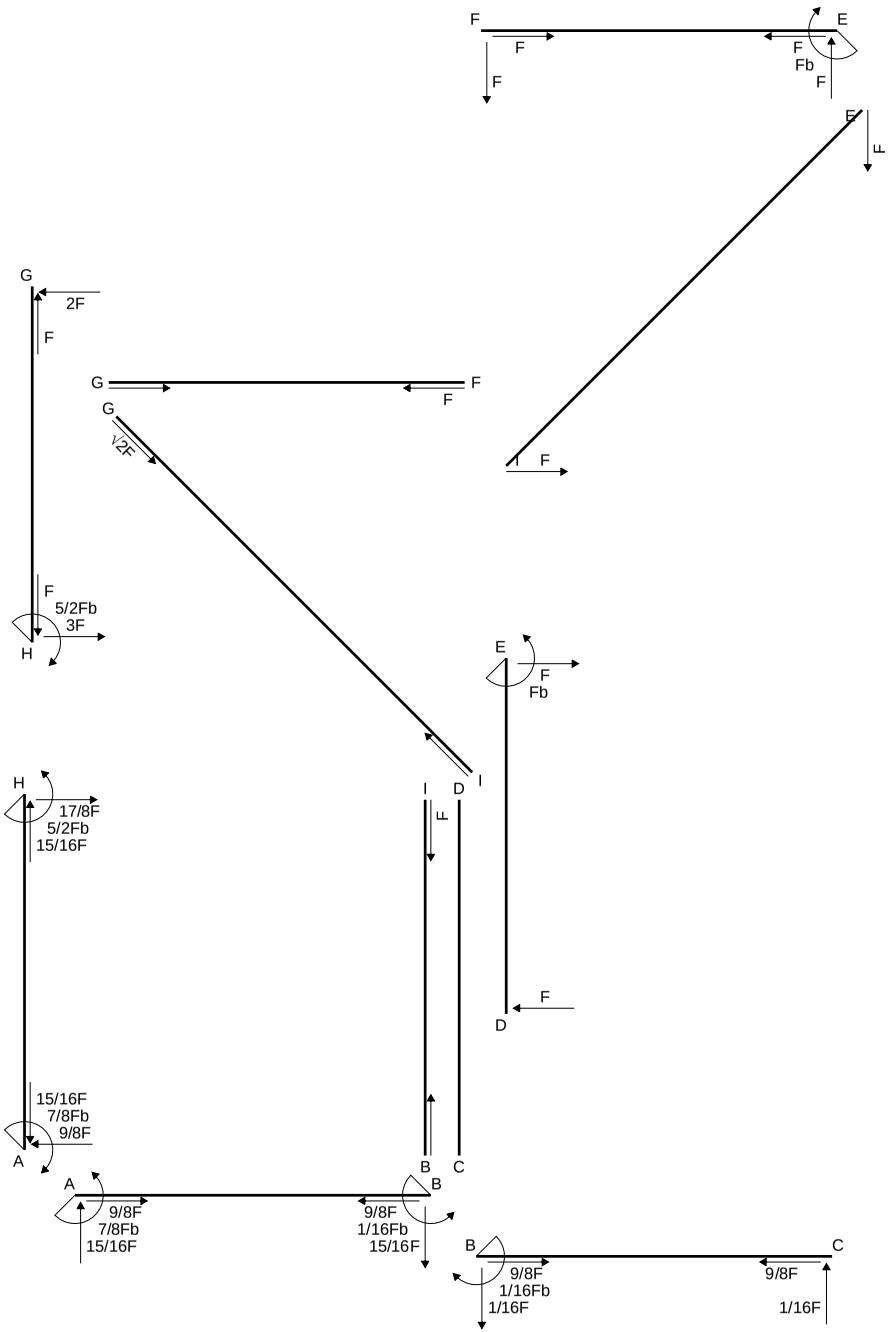
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

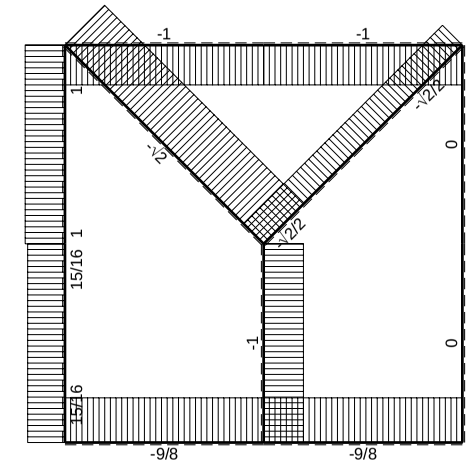
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

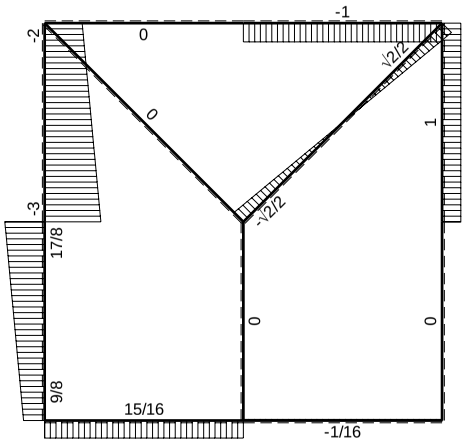
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

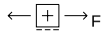
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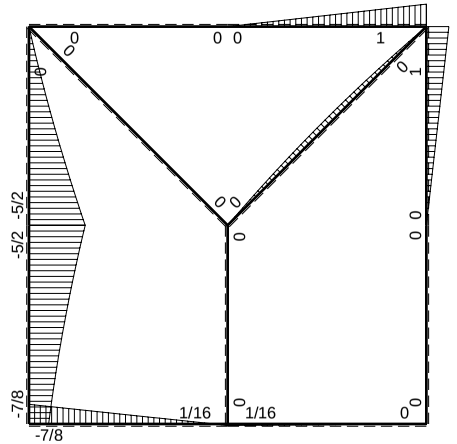

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-9/8

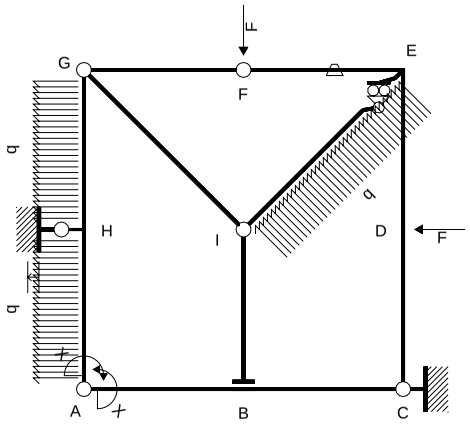

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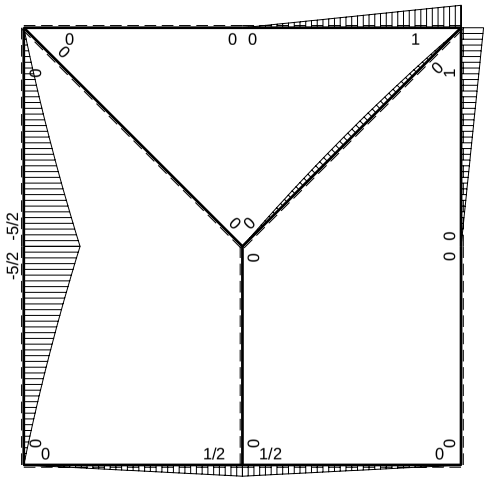

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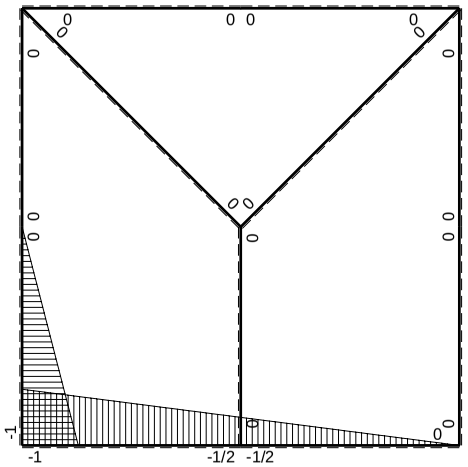

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-5/2
0
0 0
-7/8
1/16
0
1/16
0 0
-7/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237870 ESΣ04.xxxx.111

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PROCEDIMENTO E RISULTATI 237870 ESΣ04.xxxx.111

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

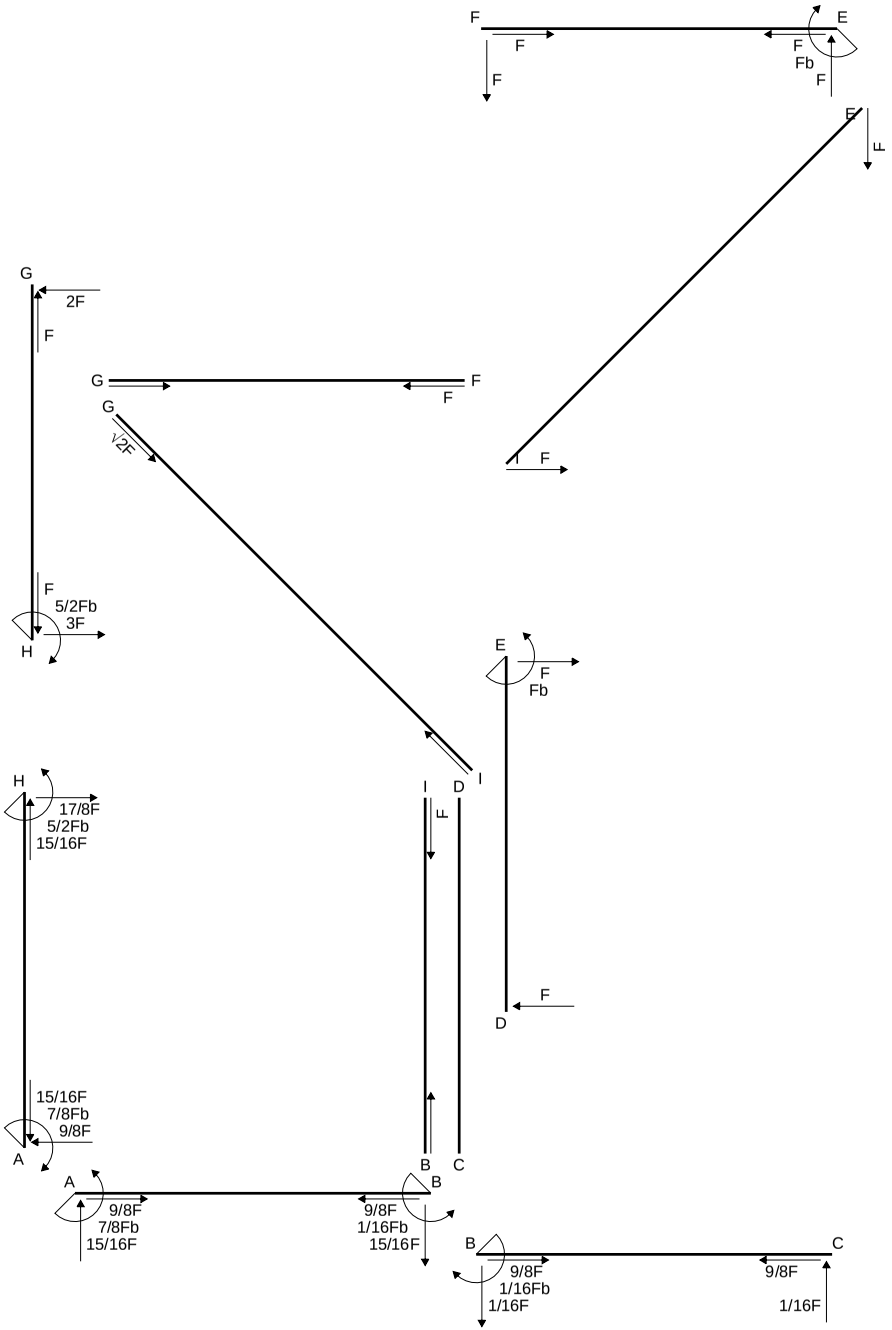
$$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$$

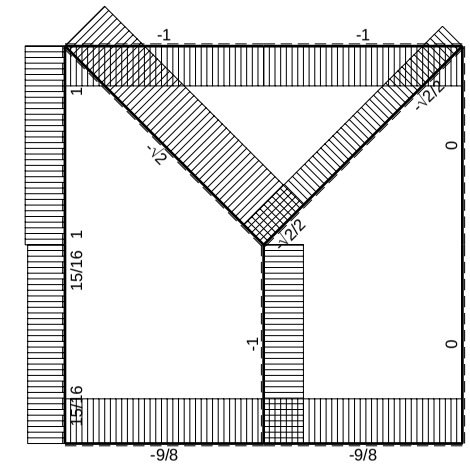
$$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

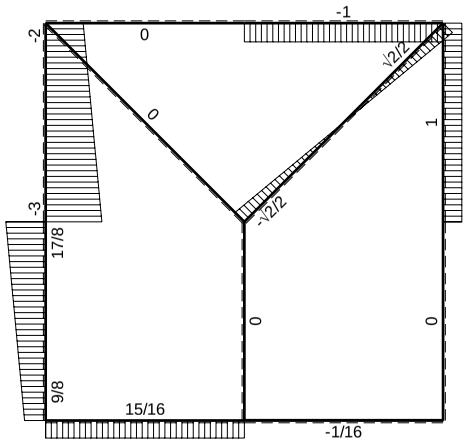
$$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$$

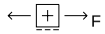
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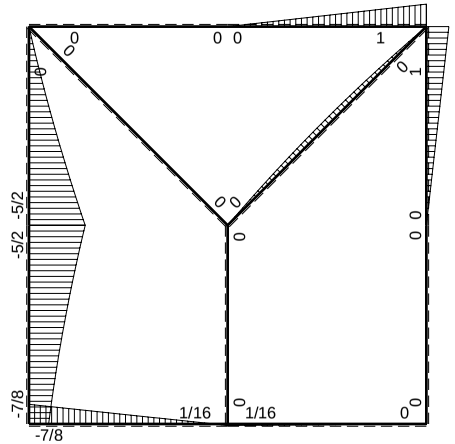

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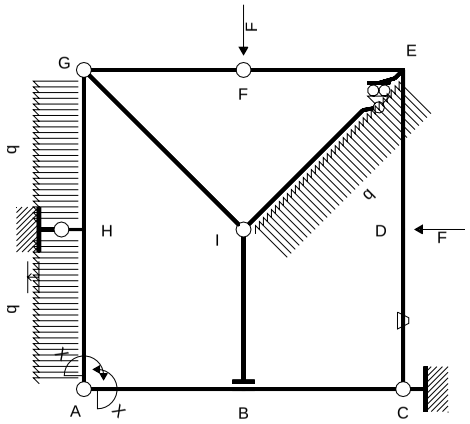

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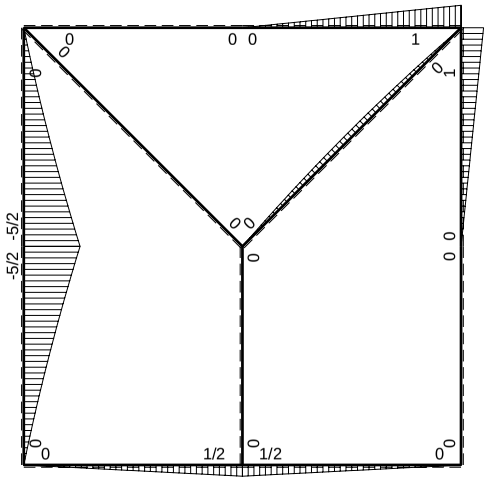

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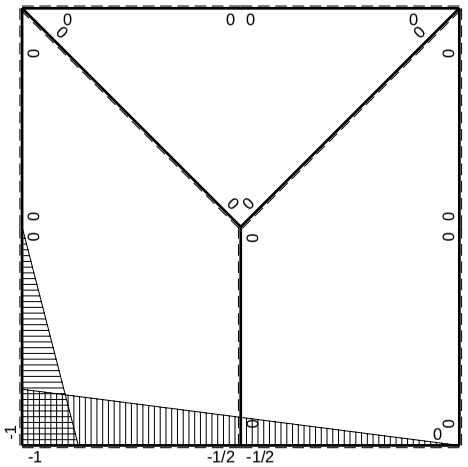

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211786 ESΣ04.xxxx.112

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	-Fb/EJ	0	0	0	0+0	0
DC b	0	0	Fb/EJ	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

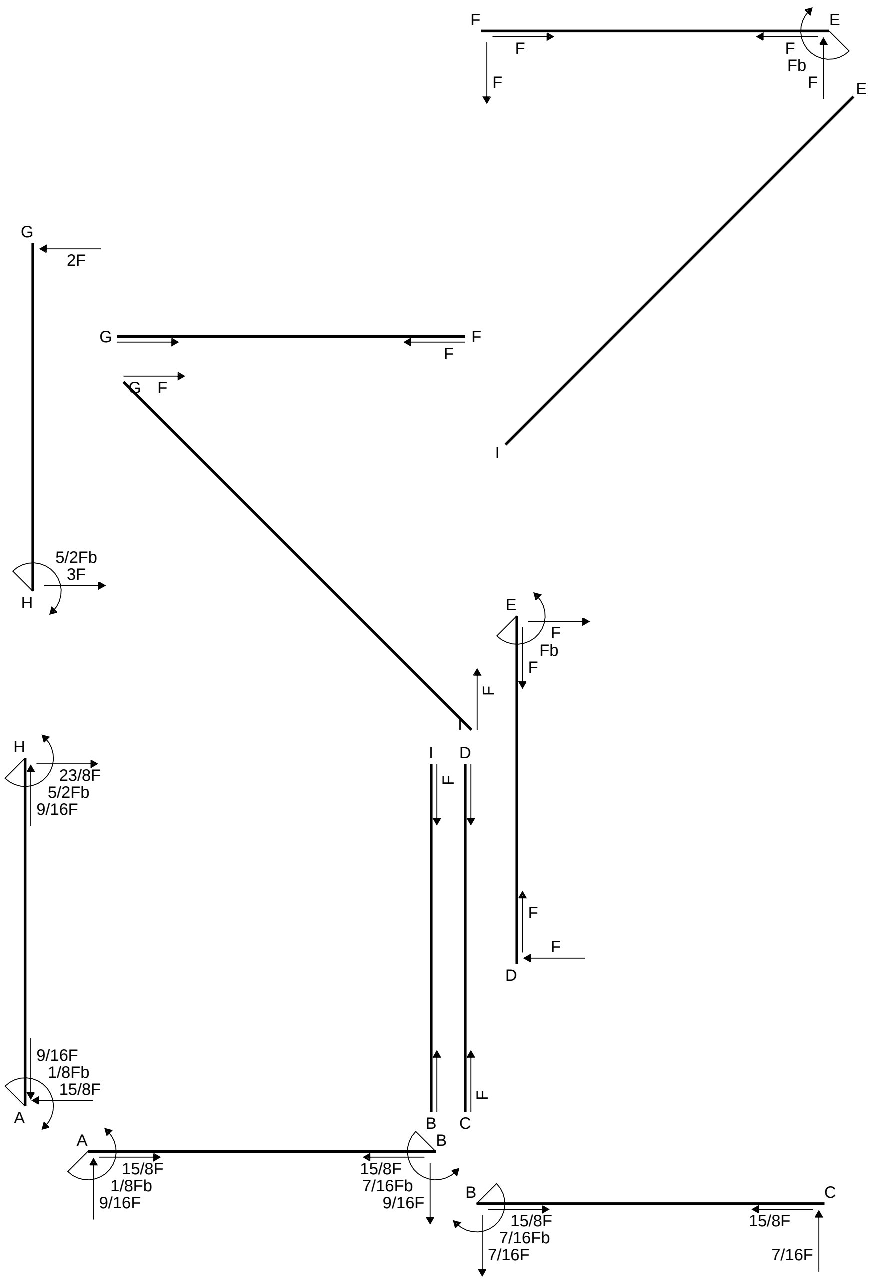
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

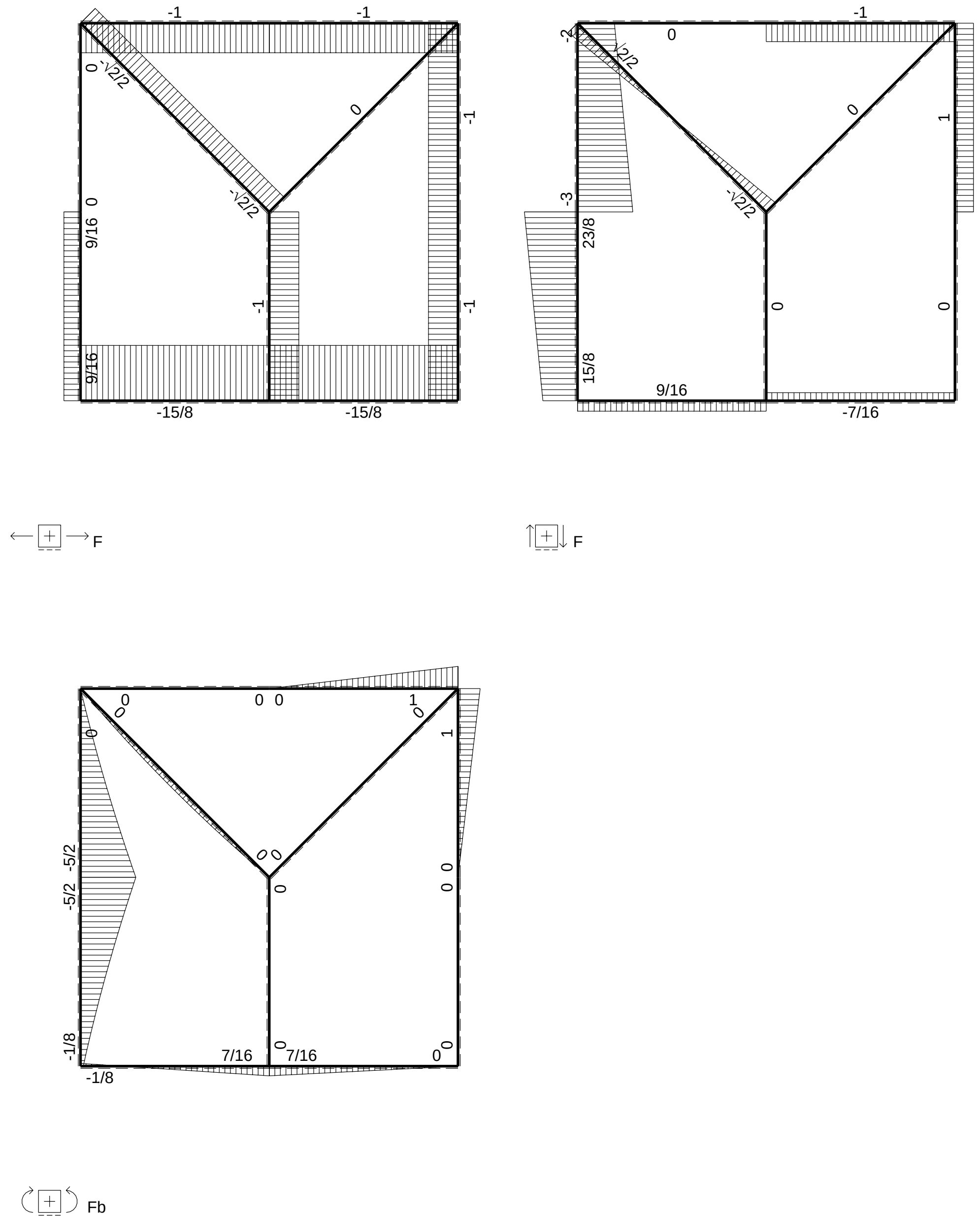
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

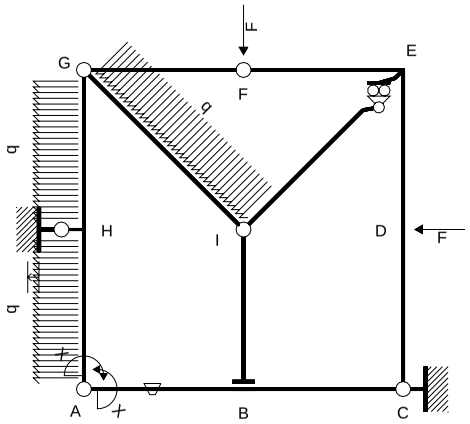
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

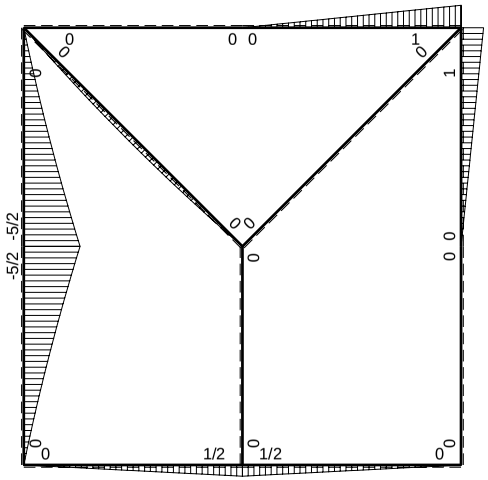
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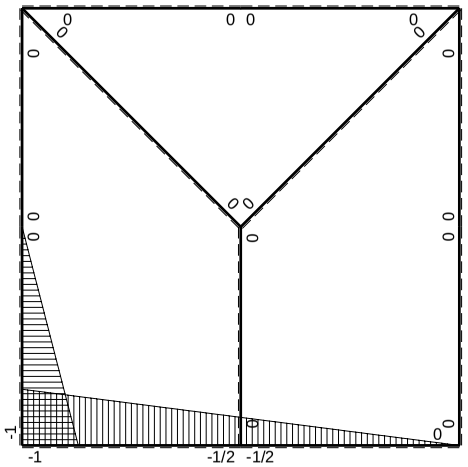

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210306 ESΣ04.xxxx.113

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	-Fb/EJ	$-1/2Fx+1/4Fx^2/b$	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(-1/6+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/2Fb+1/2Fx	Fb/EJ	$-1/4Fb+1/4Fx^2/b$	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 7/12\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 7/12\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

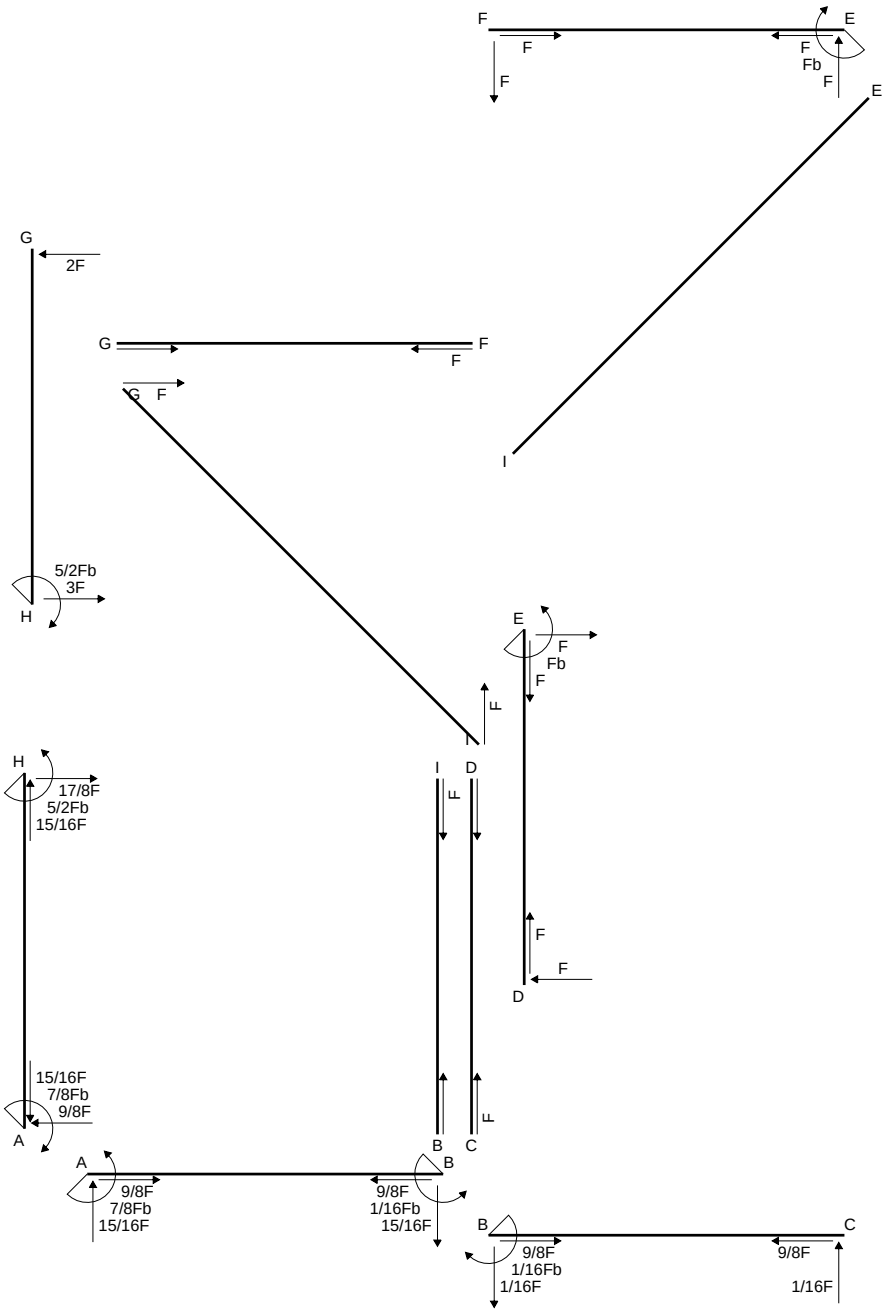
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

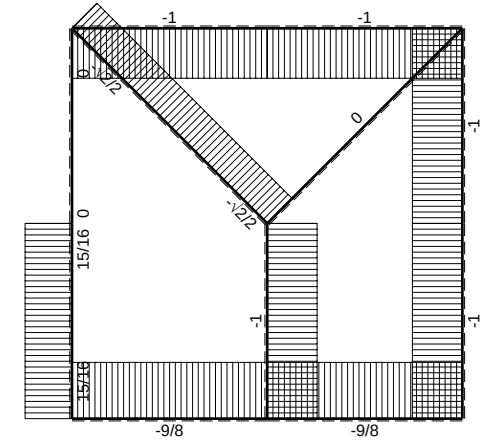
$L^{Xo}_{AH} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

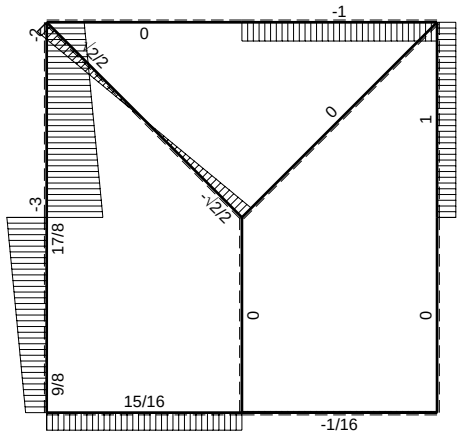
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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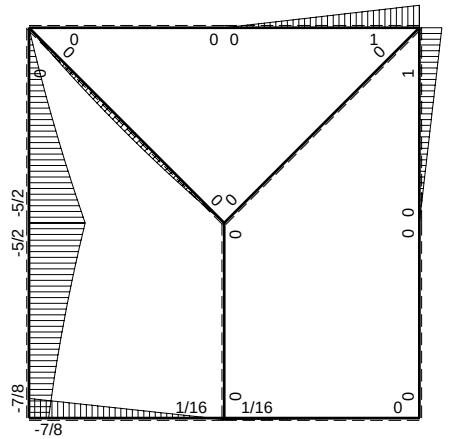

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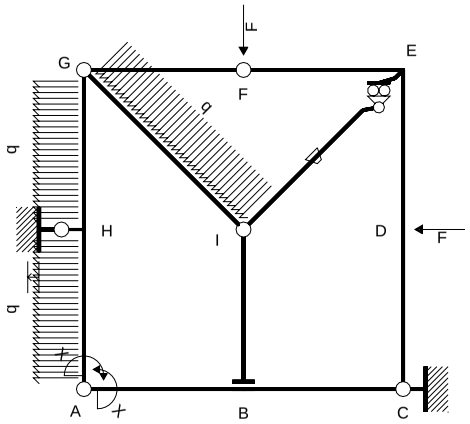

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9/8
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-1/16

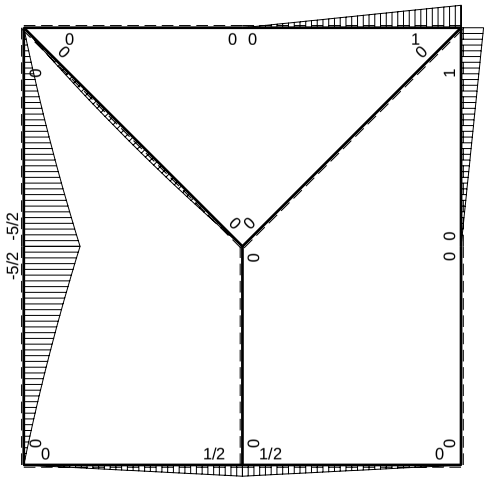
←□→ F

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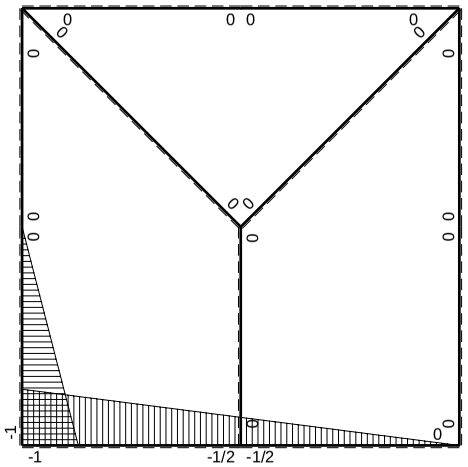

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0
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↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

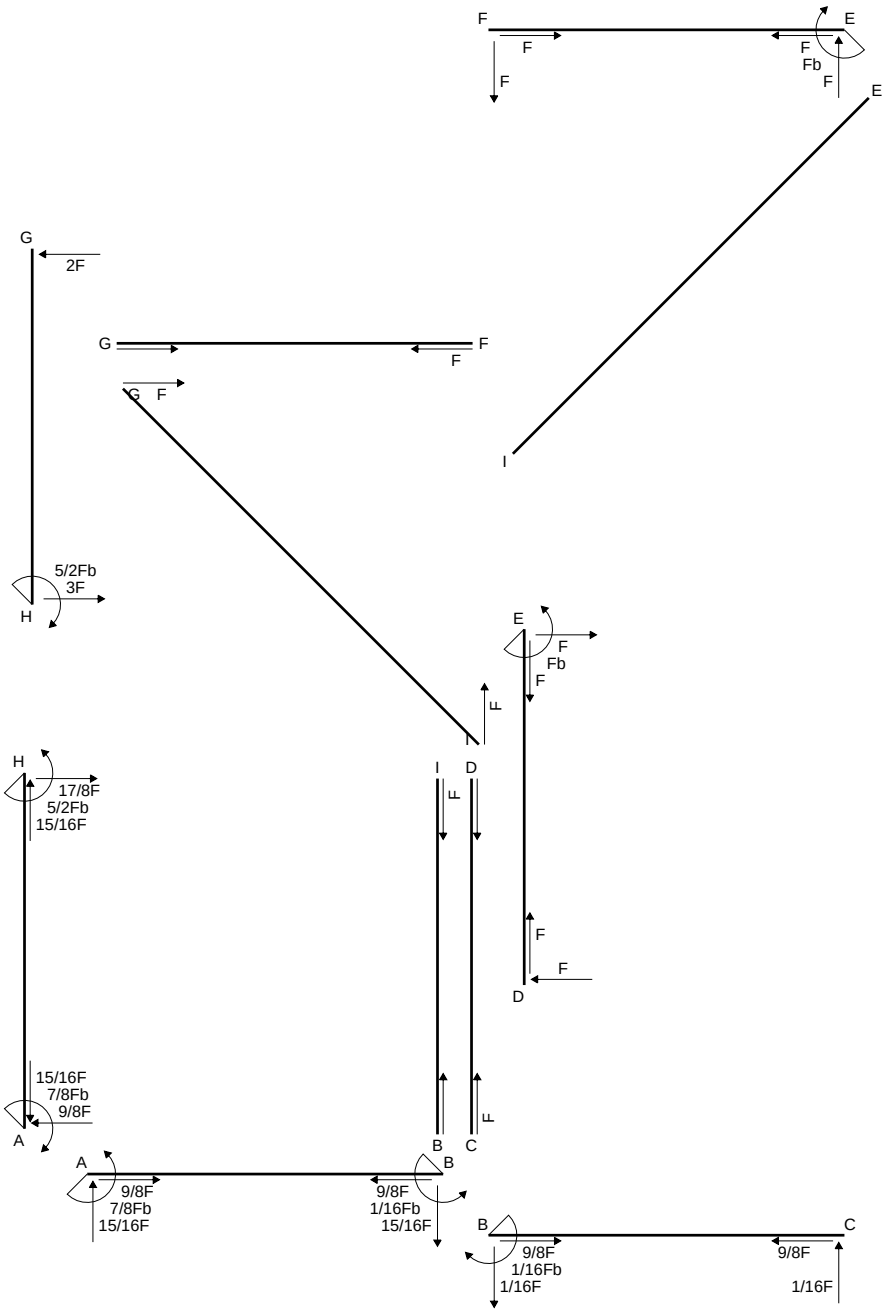
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

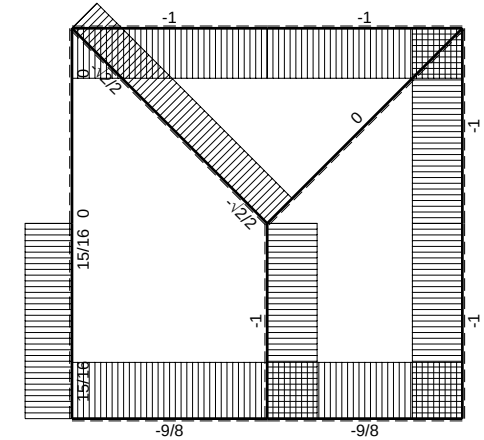
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

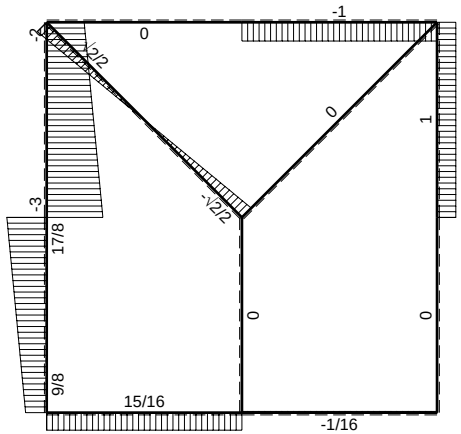
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

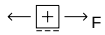
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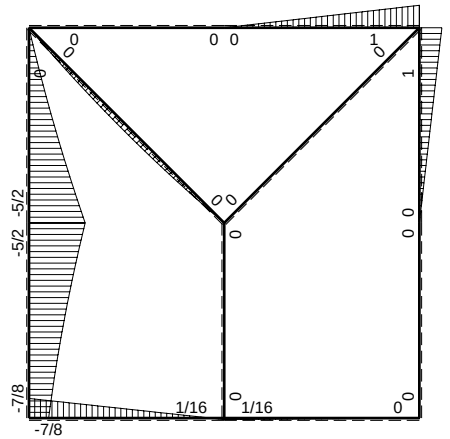

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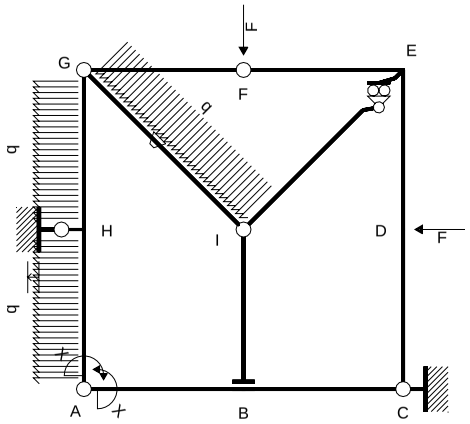

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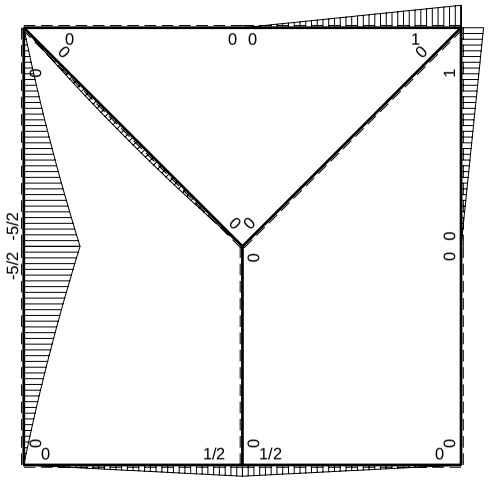

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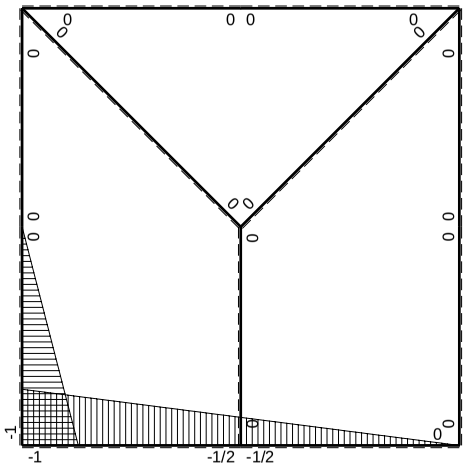

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

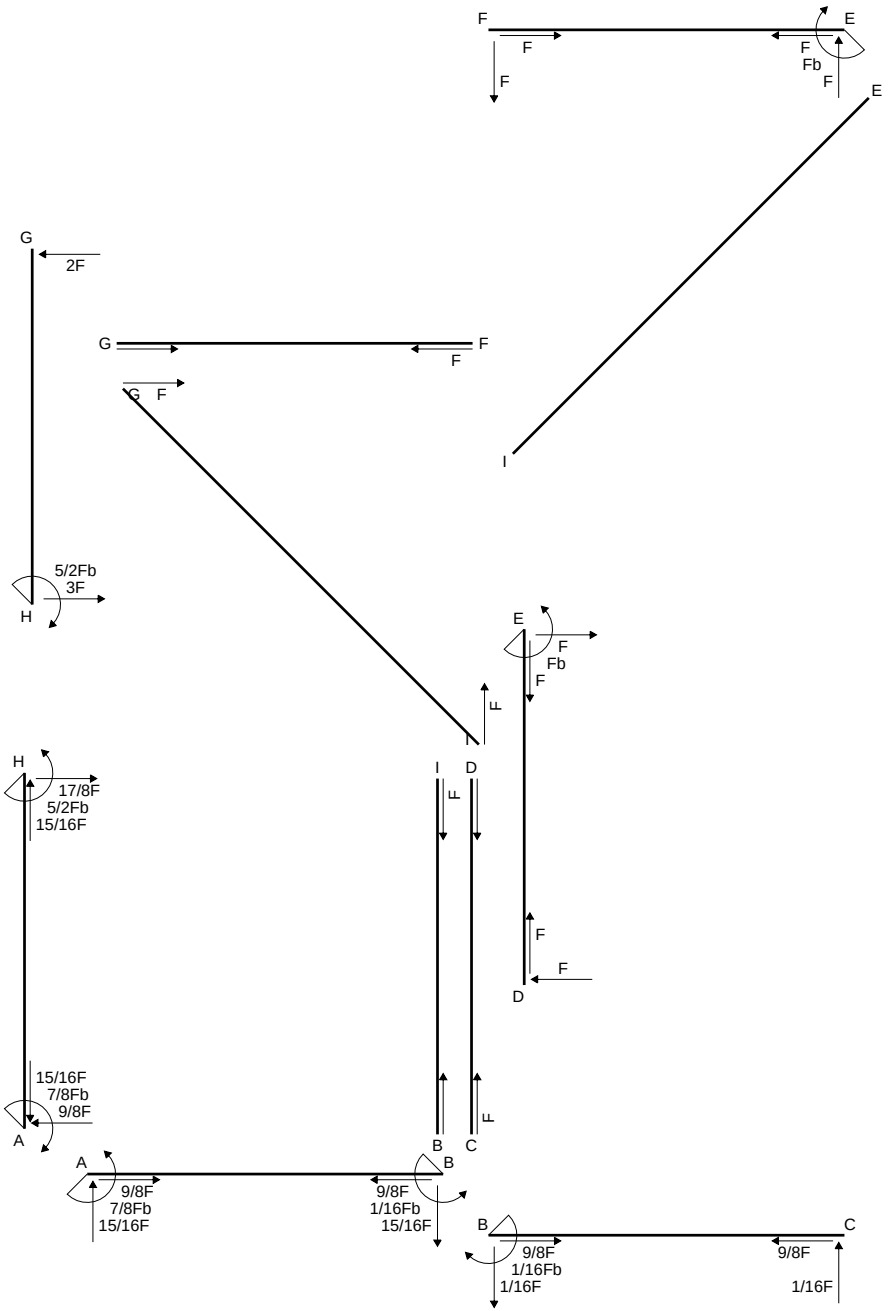
$$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$$

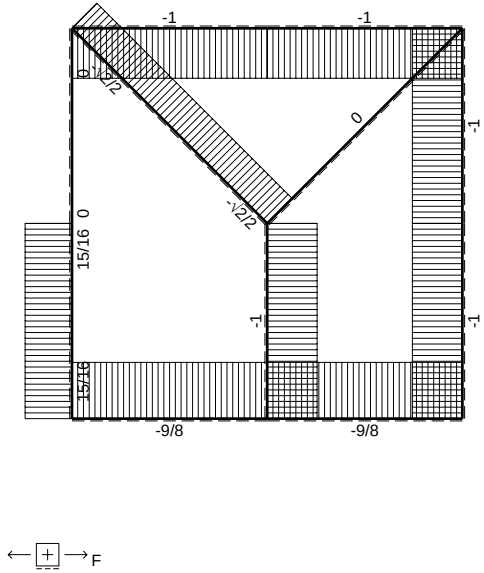
$$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

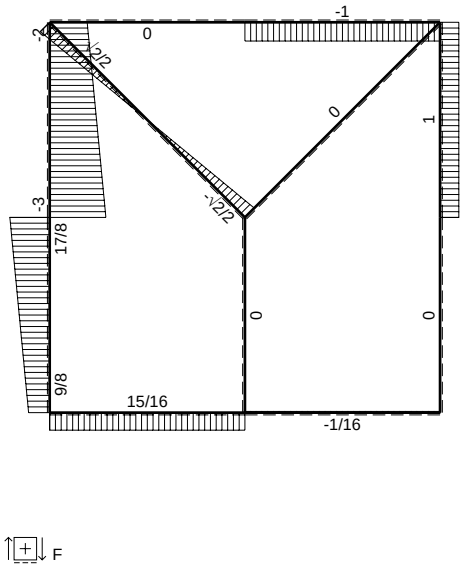
$$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$$

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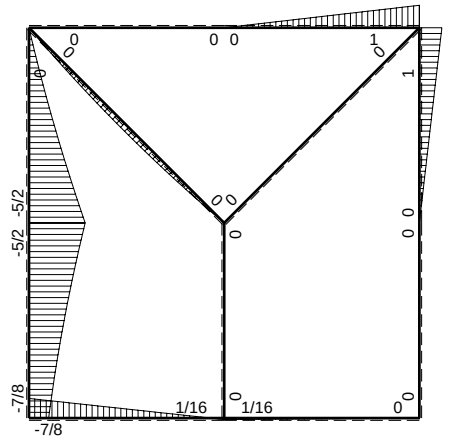

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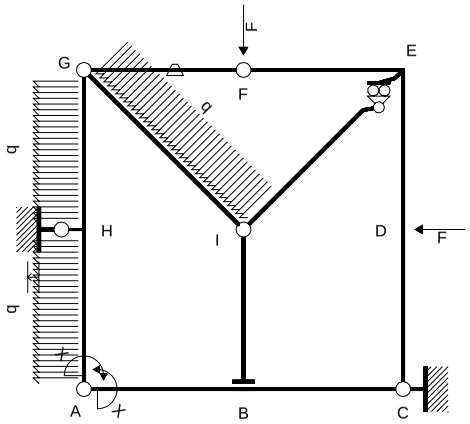

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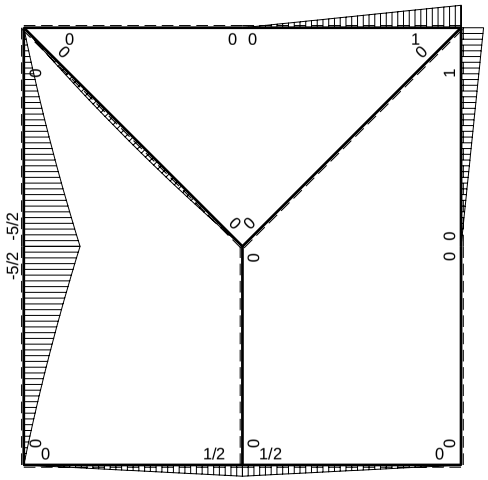
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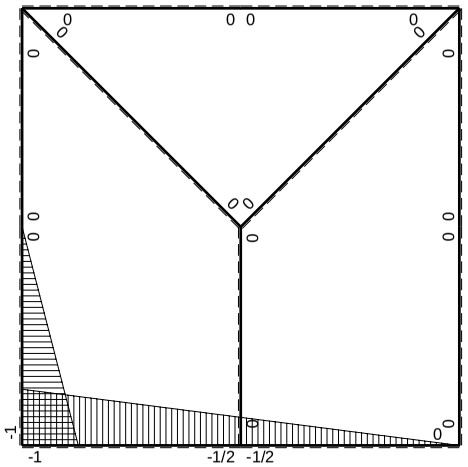

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0
-7/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

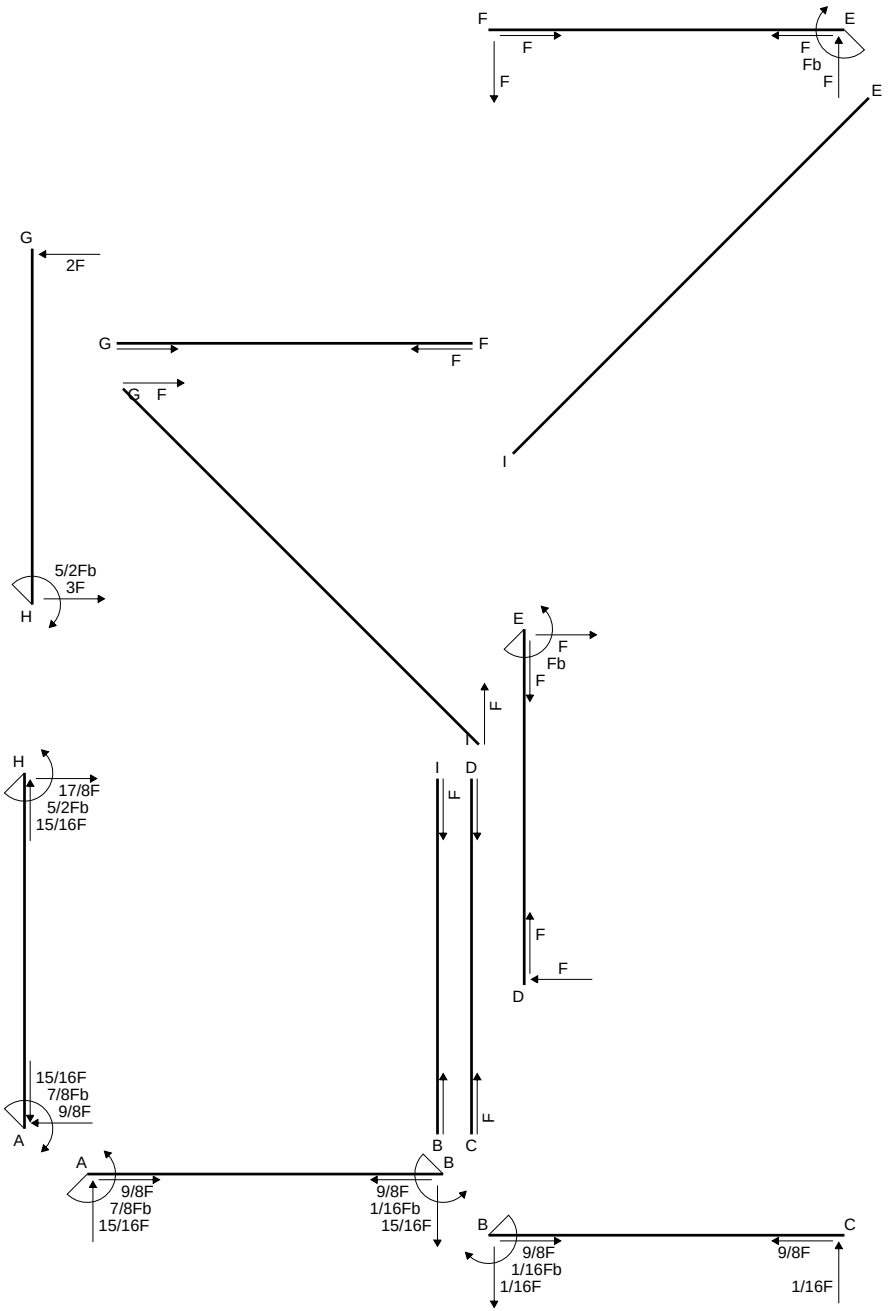
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

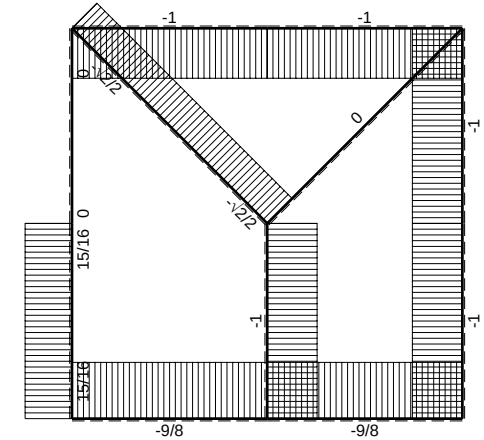
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

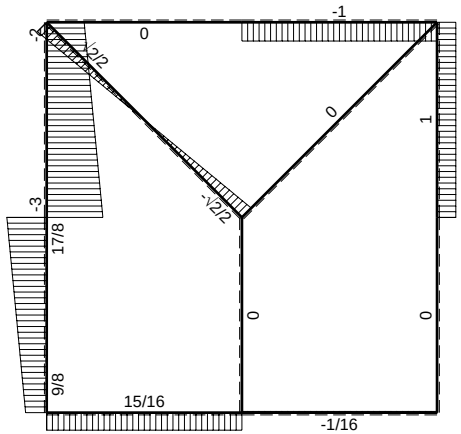
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

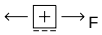
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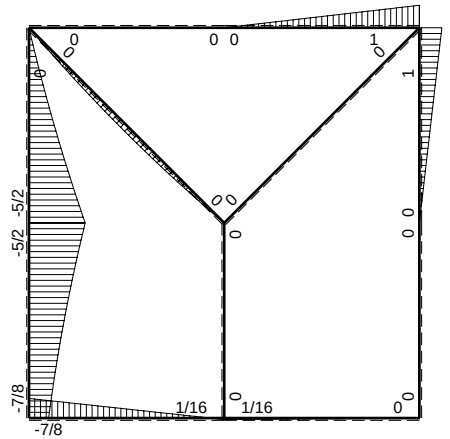

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-9/8

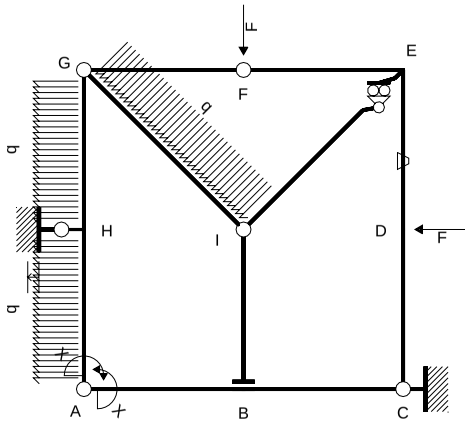

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9/8
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-1/16

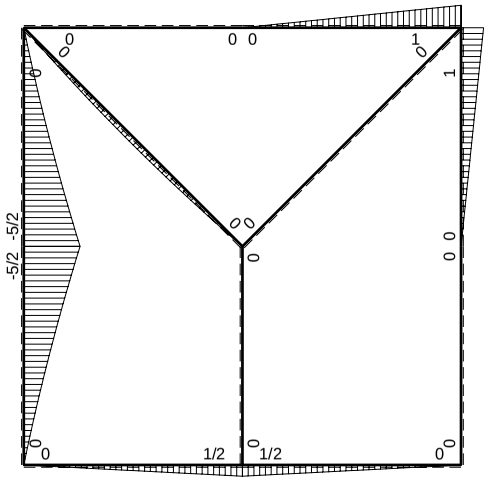

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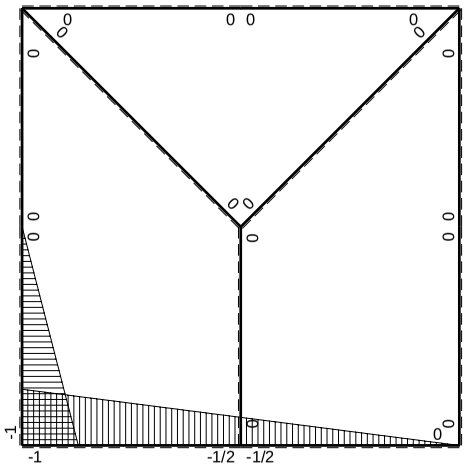

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0
-7/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237413 ESΣ04.xxxx.117

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

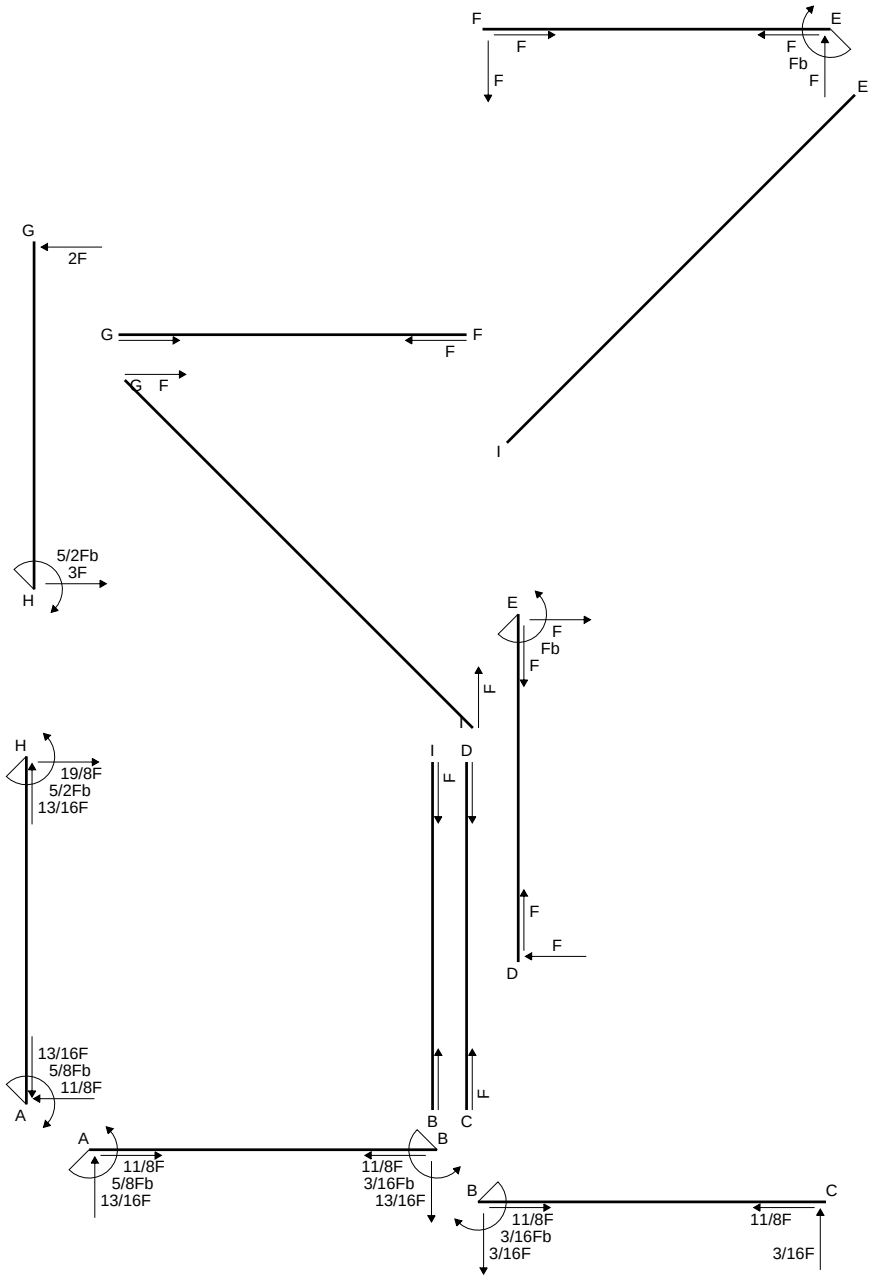
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

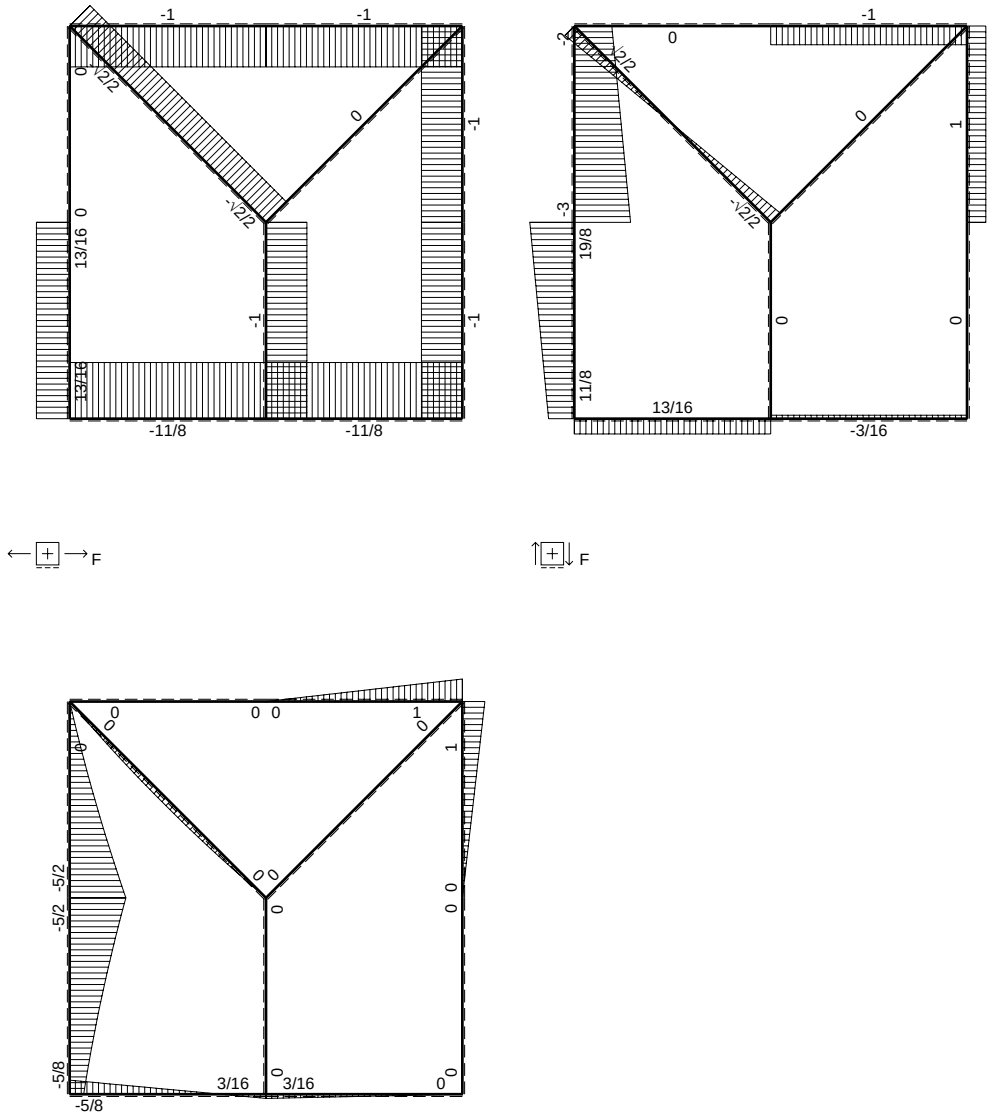
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

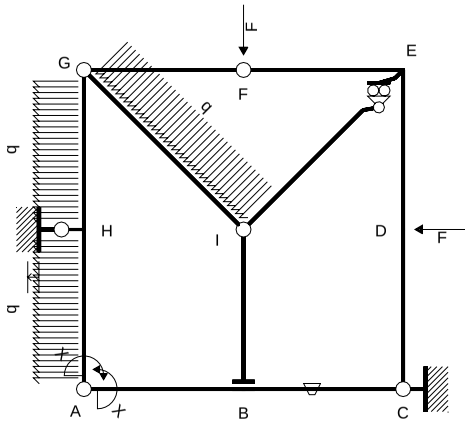
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

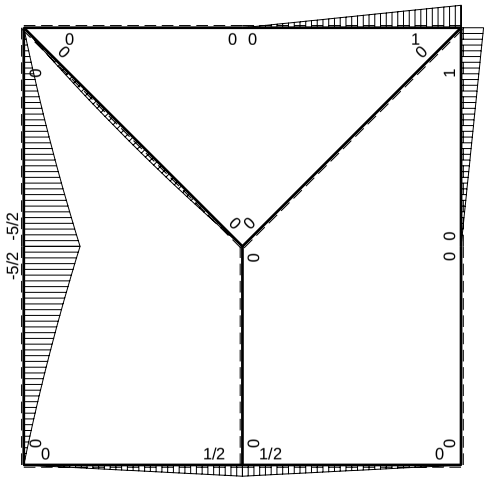
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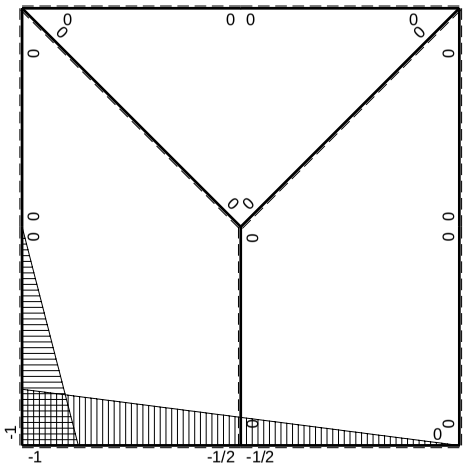

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3/16F
11/8F
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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236903 ESΣ04.xxxx.118

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	-Fb/EJ	$-1/4Fb+1/2Fx-1/4Fx^2/b$	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	Fb/EJ	$-1/4Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\right)\ \theta\ \ = 1/6\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\right)\ \theta\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ + \left(-1/4\ b\right)\ \theta\ \ = 1/6\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

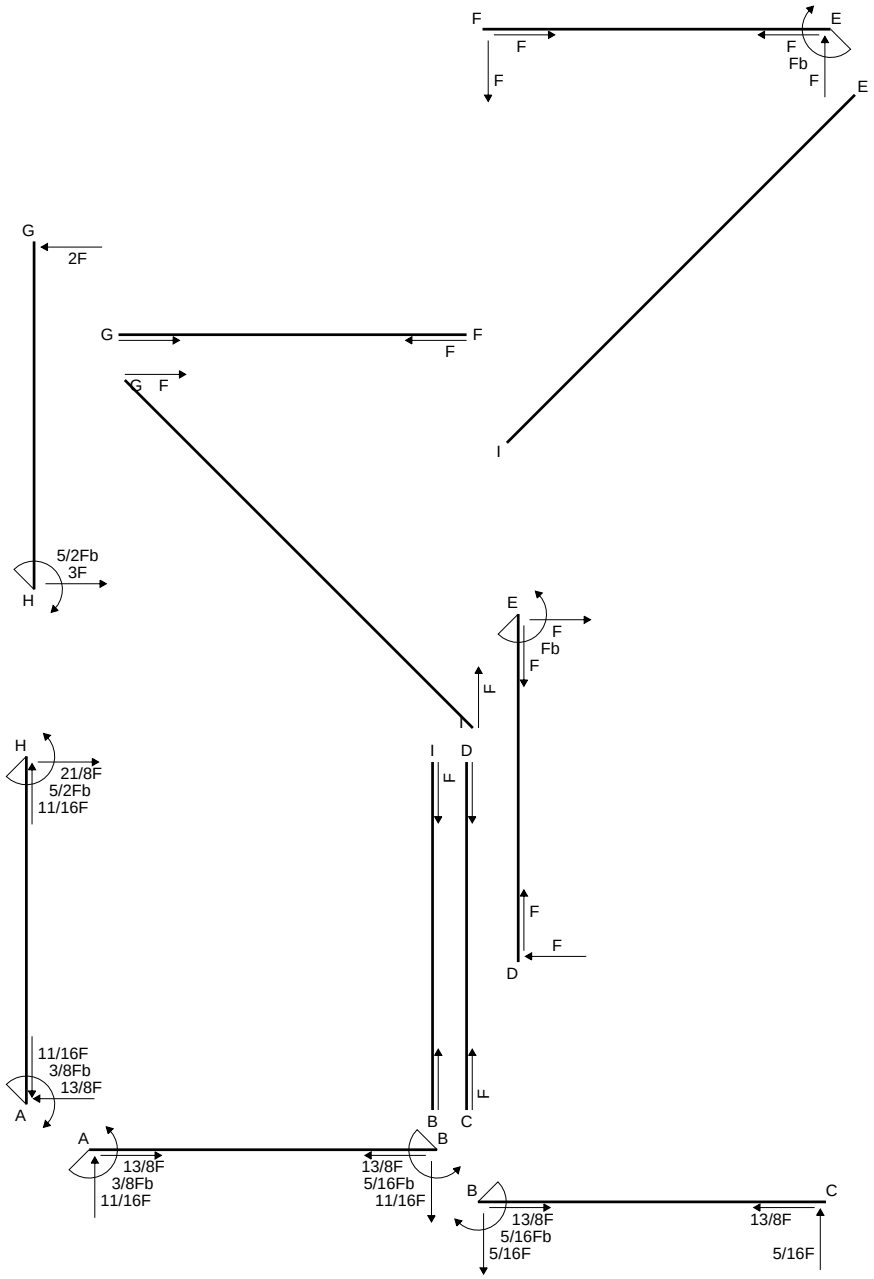
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

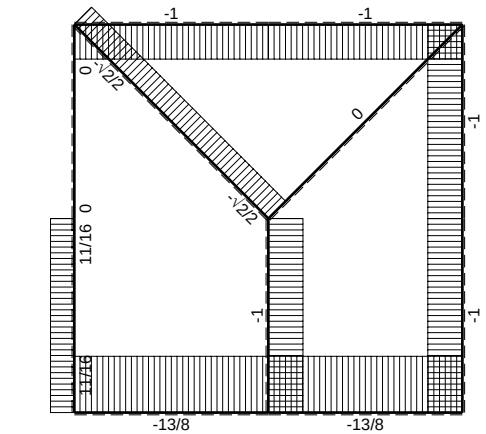
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

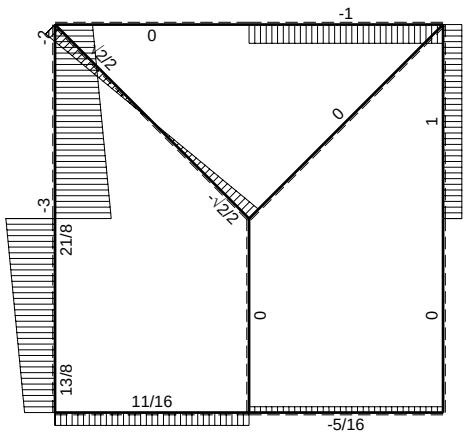
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

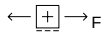
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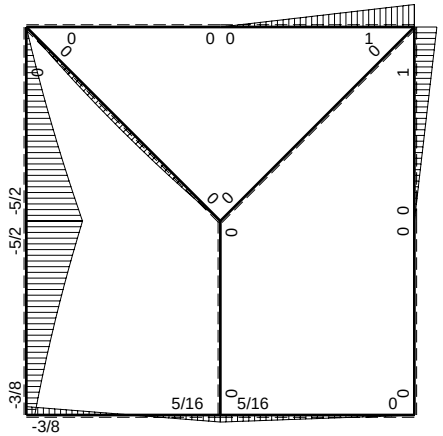

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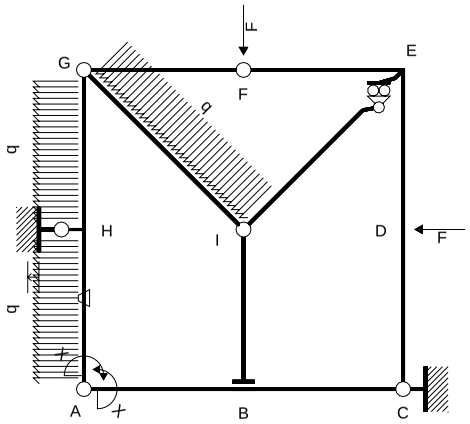

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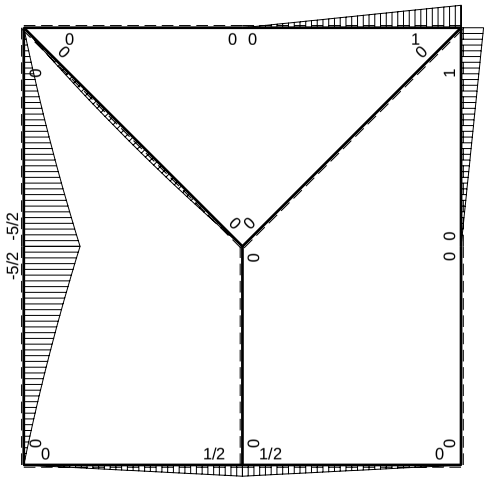

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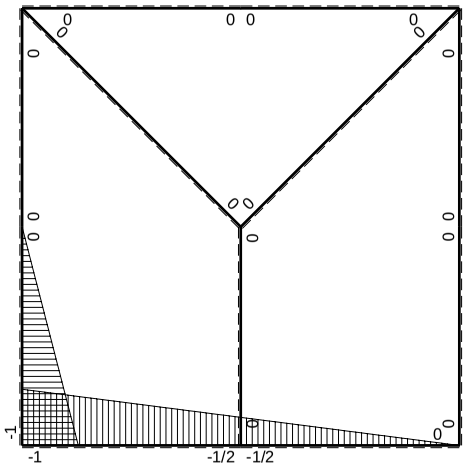

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0
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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	-Fb/EJ	$5/2Fx-3Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(3/8+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	Fb/EJ	$2Fx-3/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx$

$= \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = 7/8\ \ Fb^2/EJ$

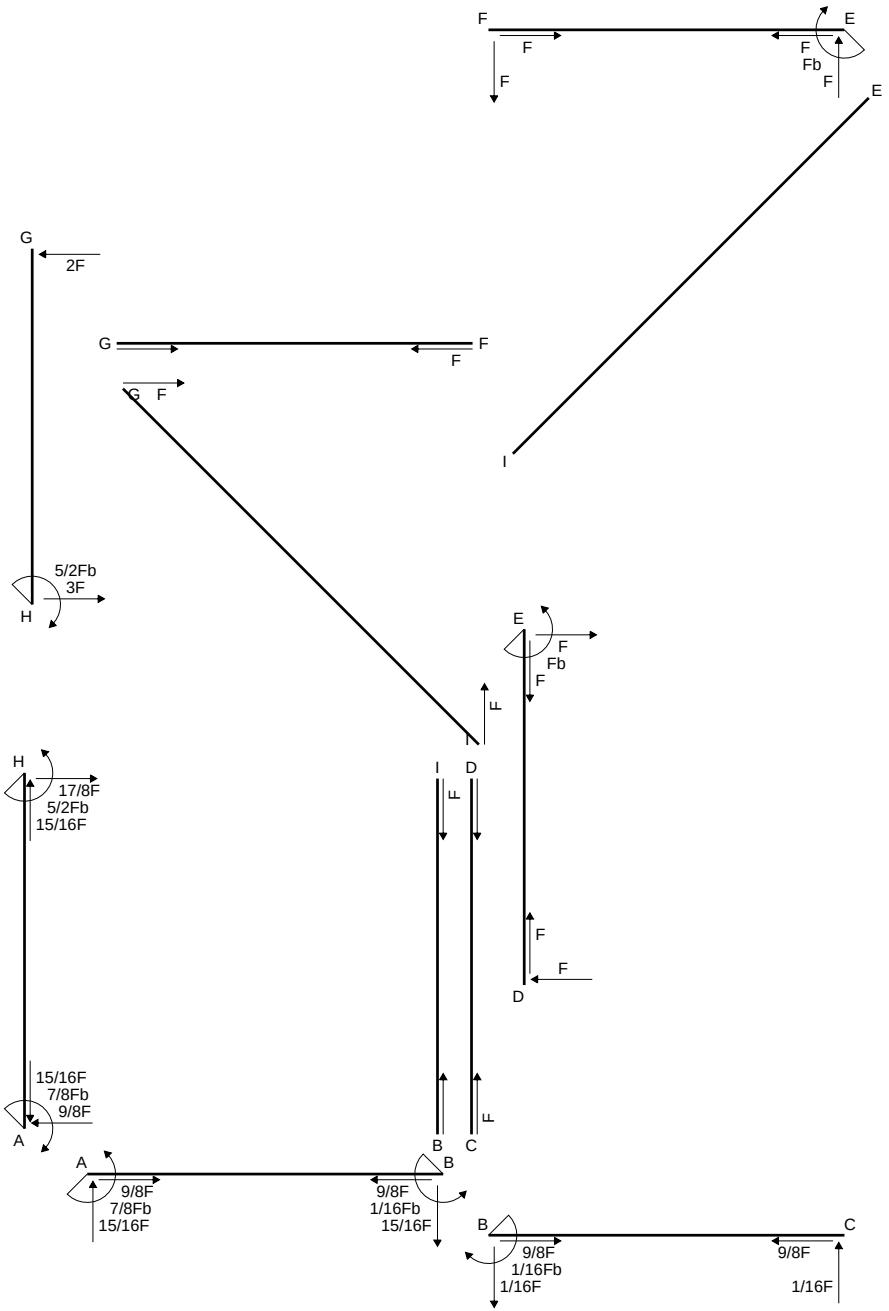
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

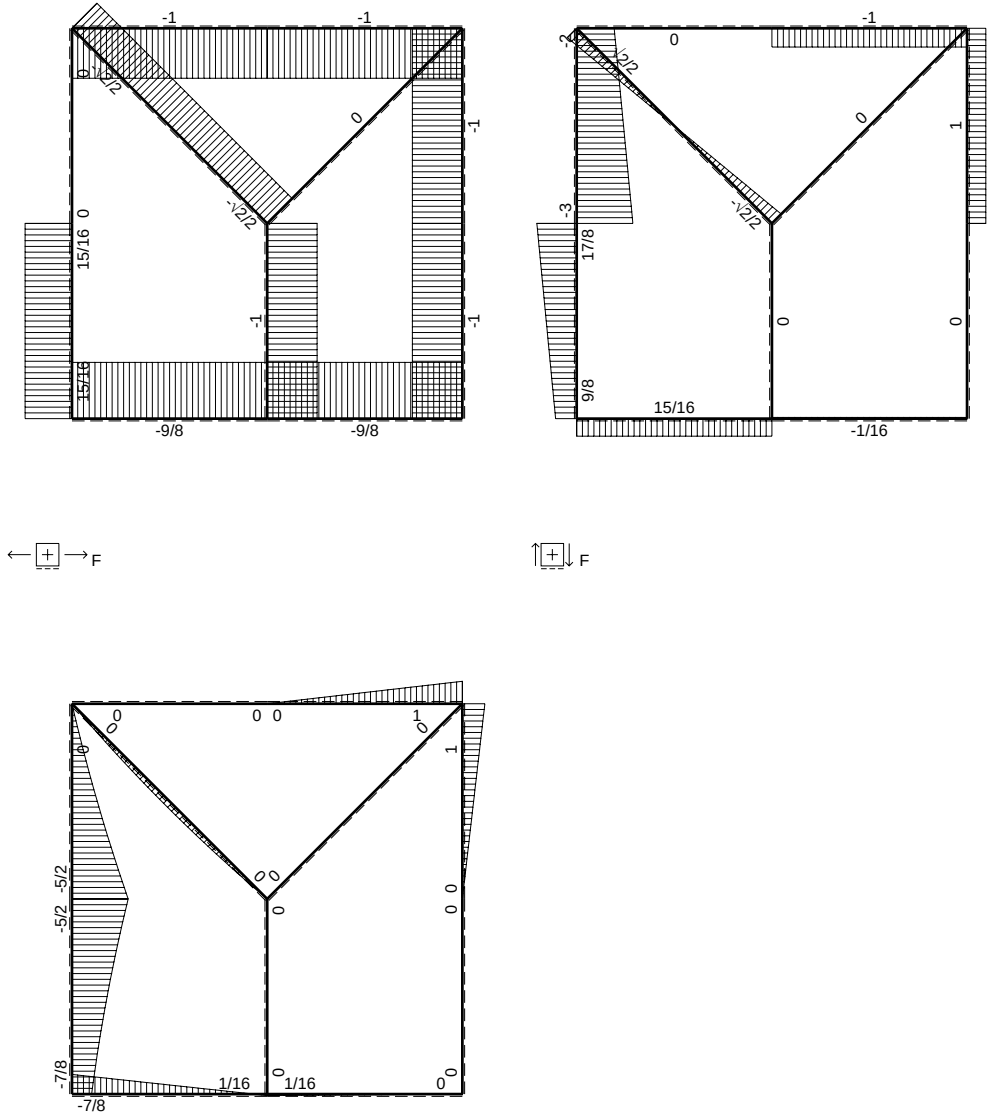
$= \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

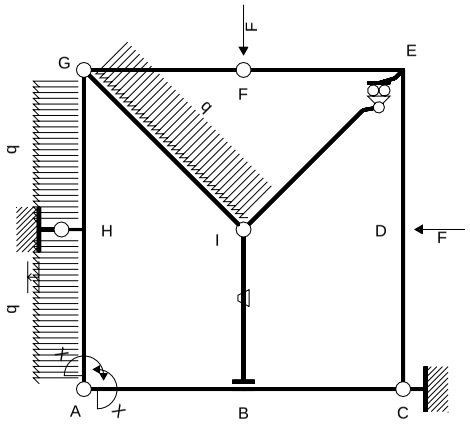
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\ \theta\ \ = 7/8\ \ Fb^2/EJ$

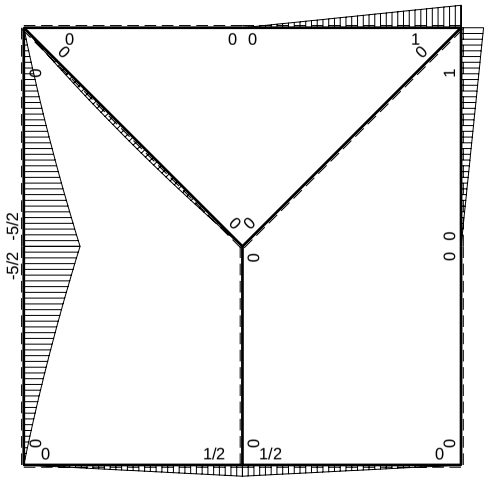
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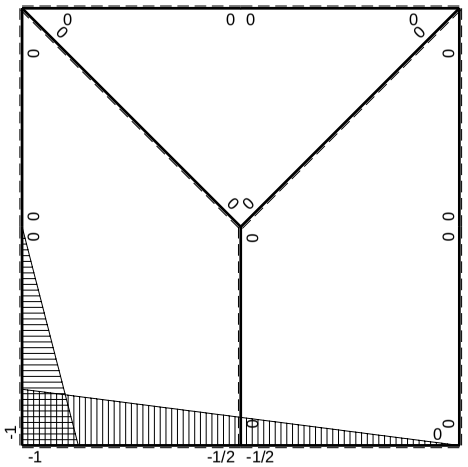

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1/16F


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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

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Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

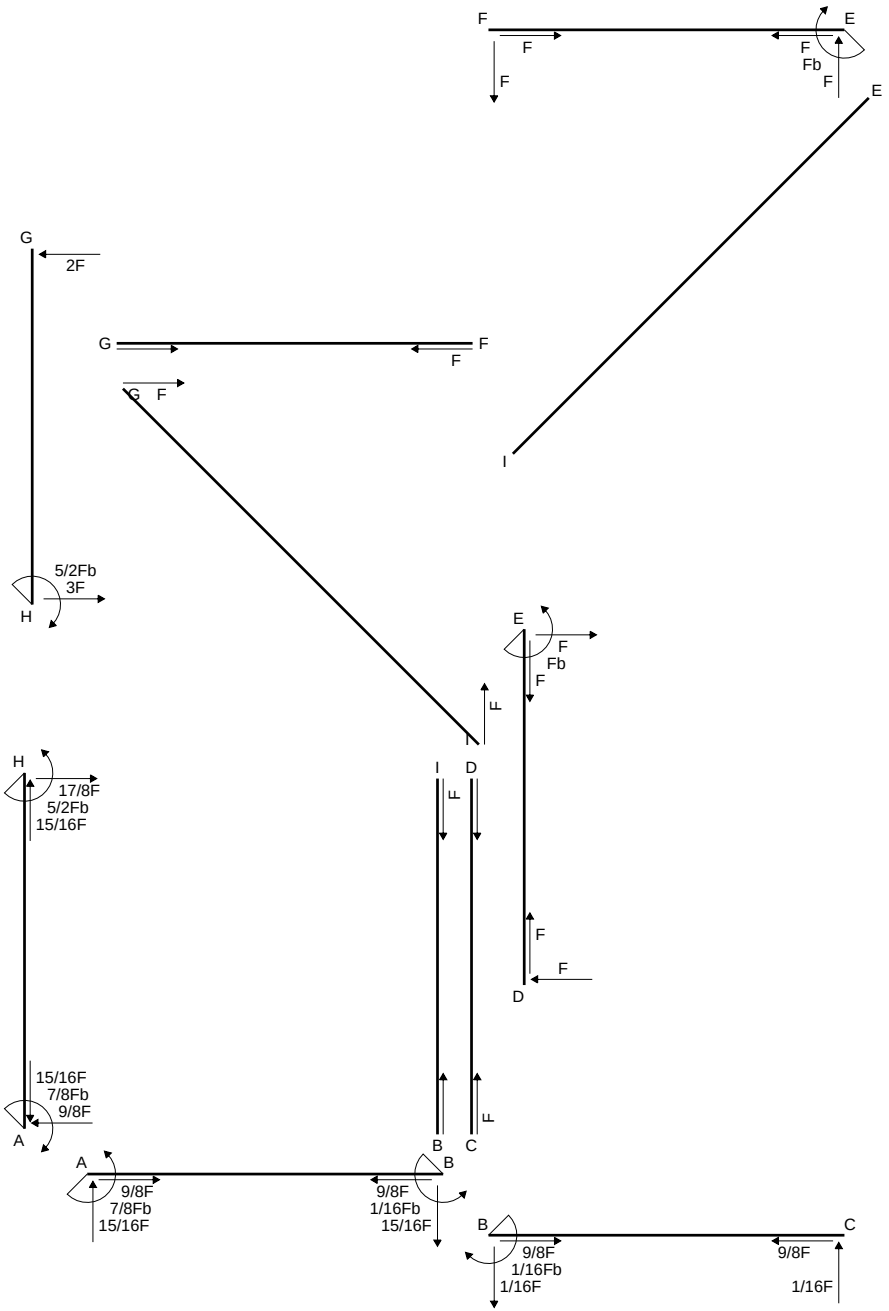
$$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$$

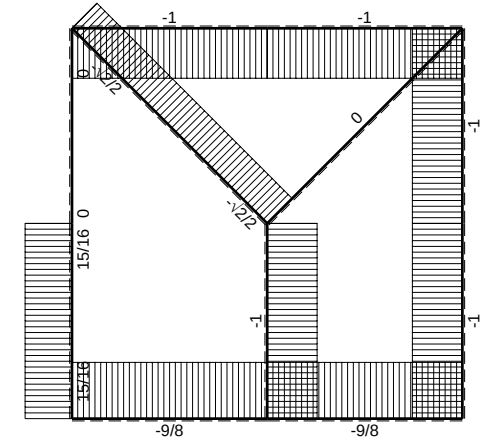
$$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

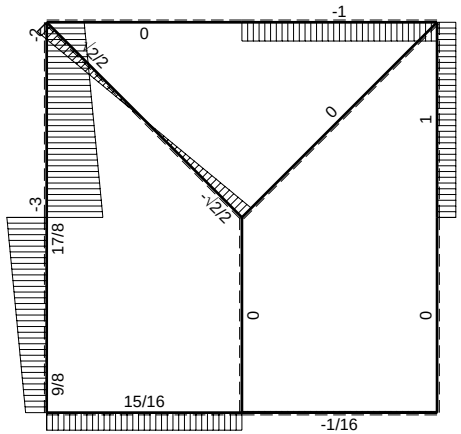
$$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$$

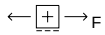
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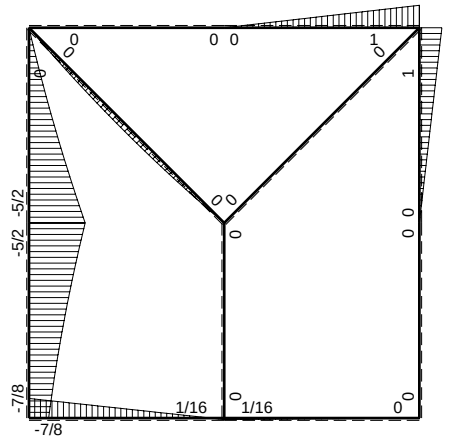

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9/8F
1/16Fb
15/16F
B
C
9/8F
1/16Fb
1/16F
9/8F
1/16F


-1
-1
-√2/2
0
-1
-√2/2
0
15/16
-1
-1
15/16
-9/8
-9/8

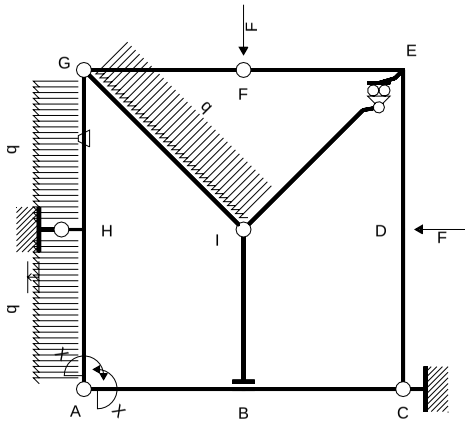

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-3
-√2/2
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0
9/8
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-1/16

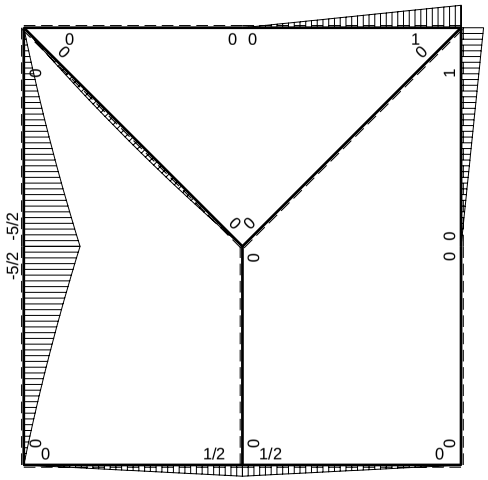

F


F

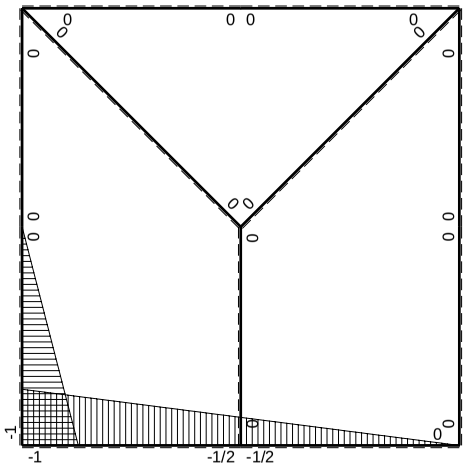

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-5/2
-5/2
0 0
-7/8
1/16
0
1/16
0
0
-7/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

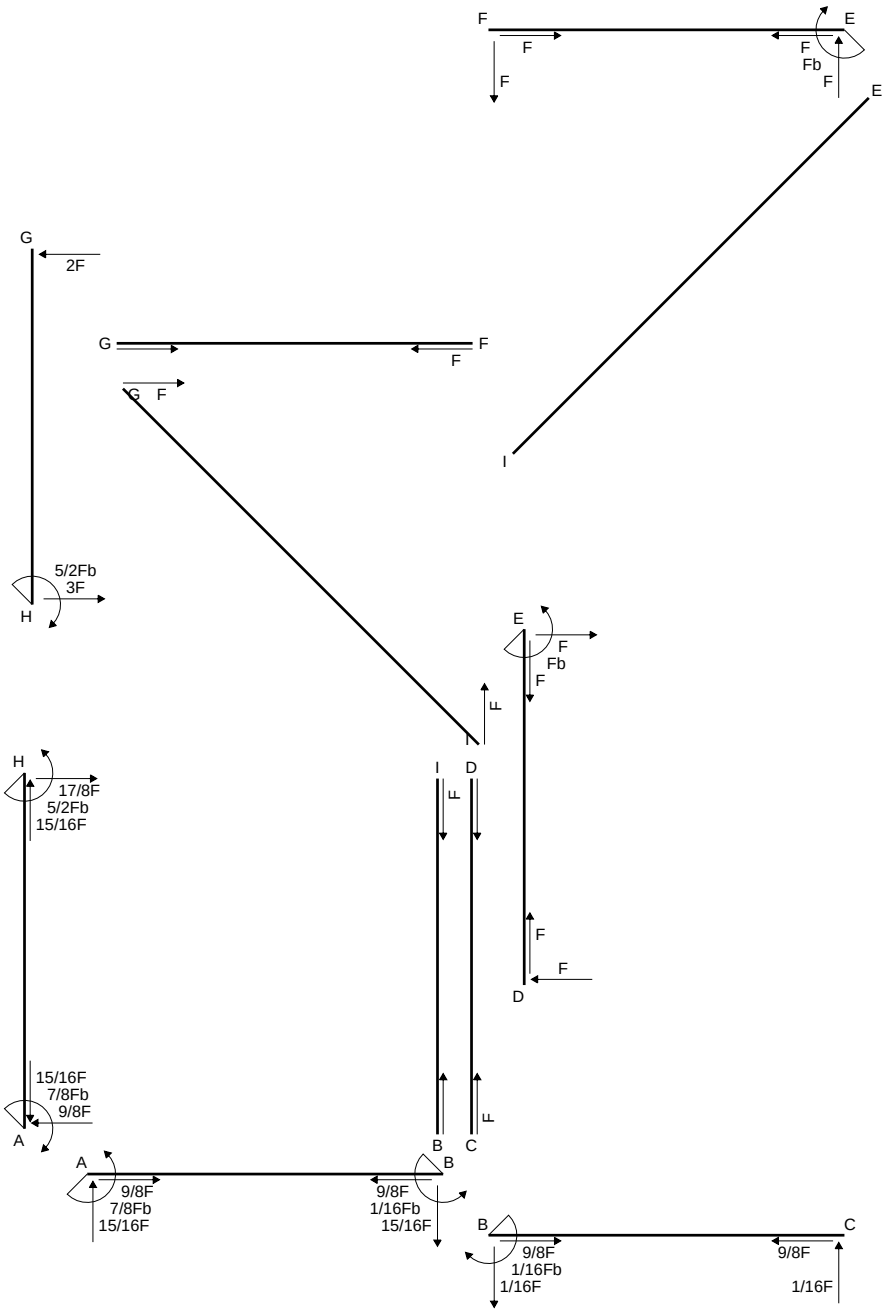
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

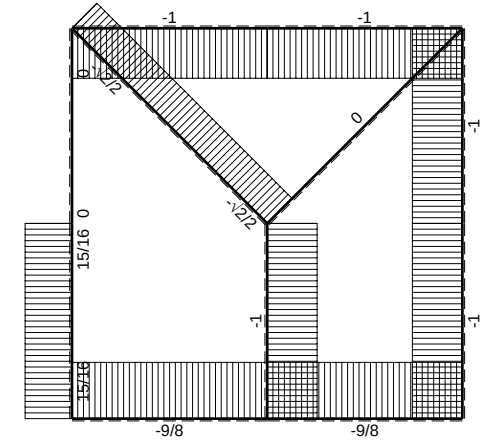
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

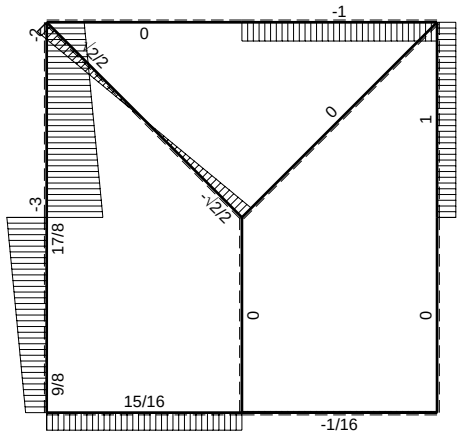
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

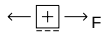
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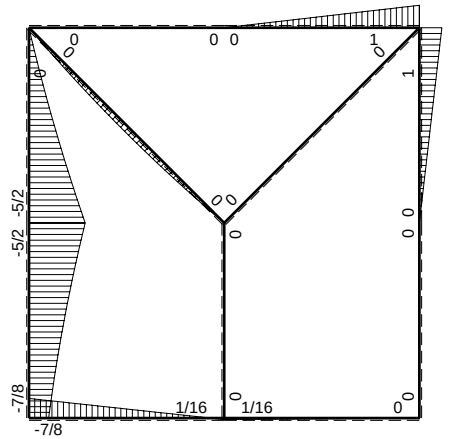

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Fb
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H
17/8F
5/2Fb
15/16F
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C
9/8F
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1/16F


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-9/8

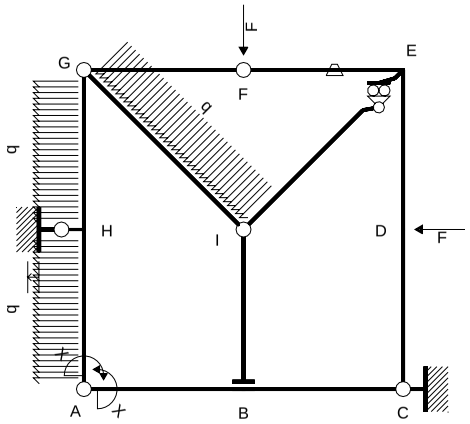

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9/8
15/16
-1/16

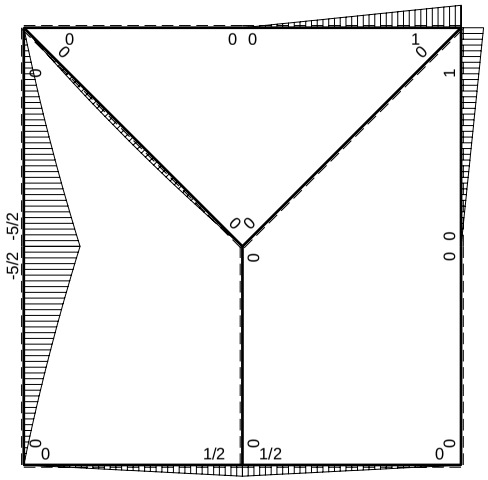

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F

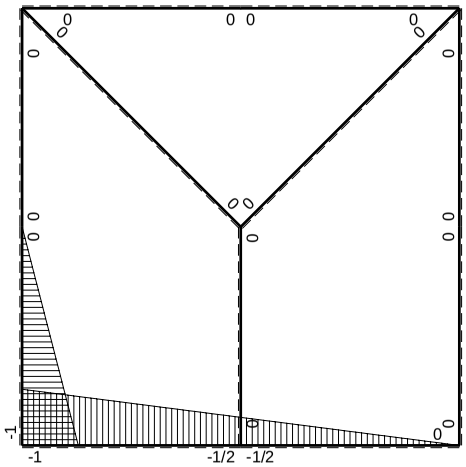

0
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0
-5/2
-5/2
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0 0
-7/8
1/16
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1/16
0
0
-7/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

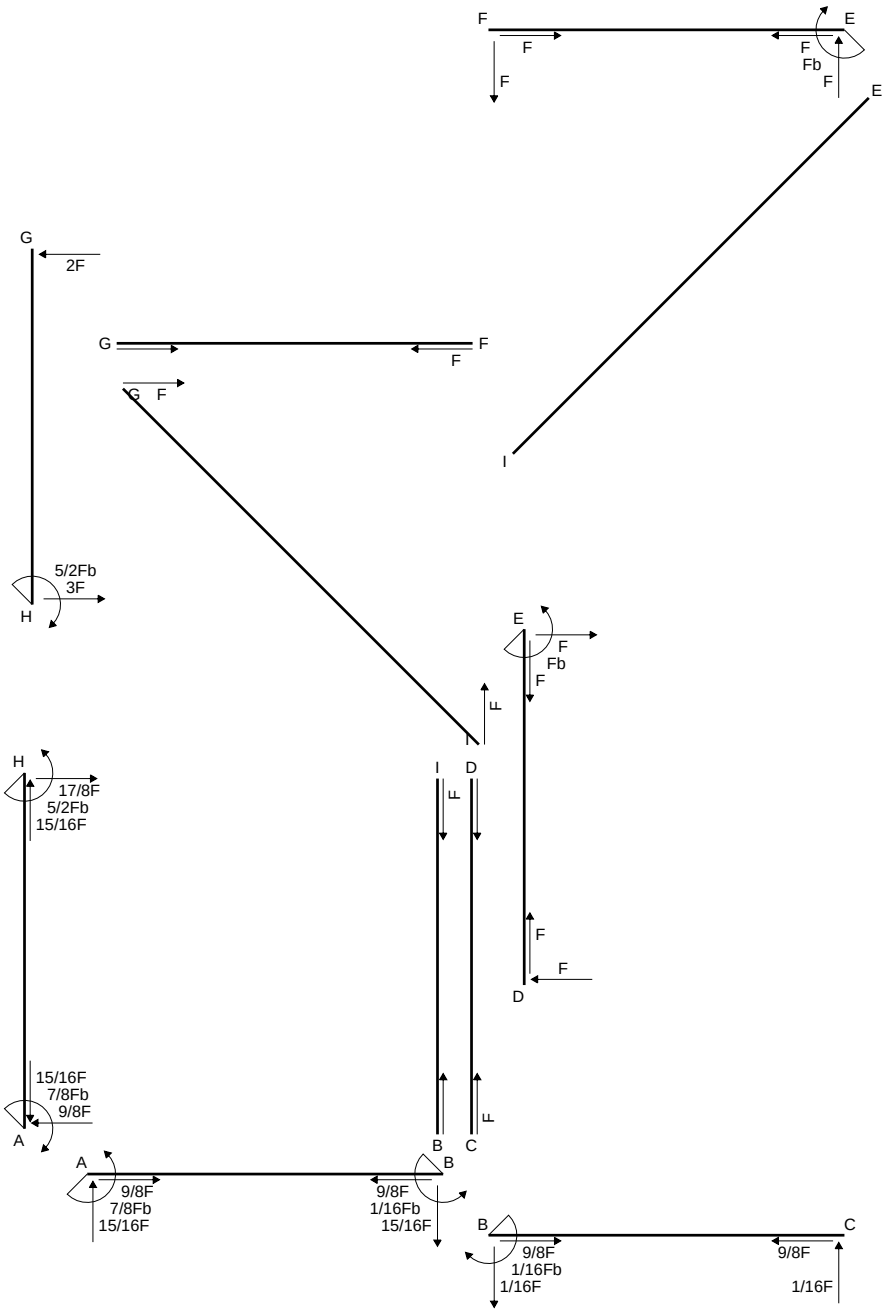
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

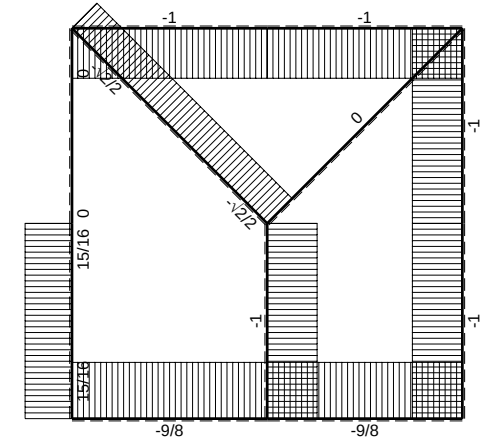
$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

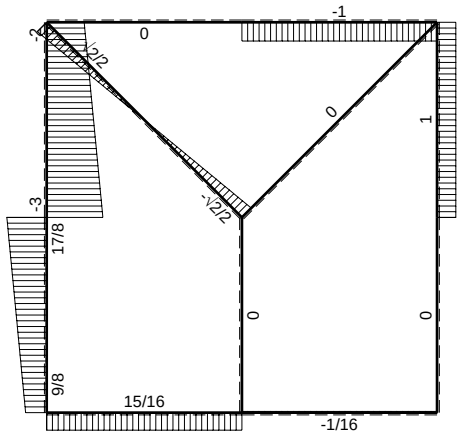
$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

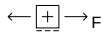
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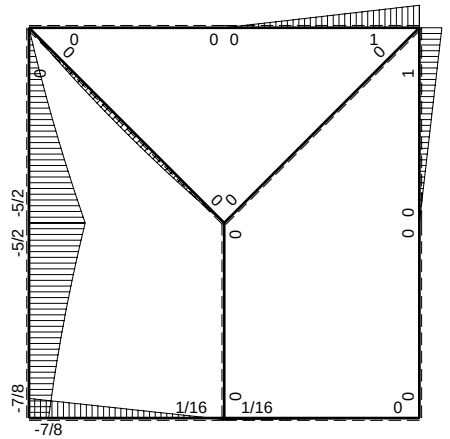

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H
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5/2Fb
15/16F
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D
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7/8Fb
9/8F
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1/16Fb
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9/8F
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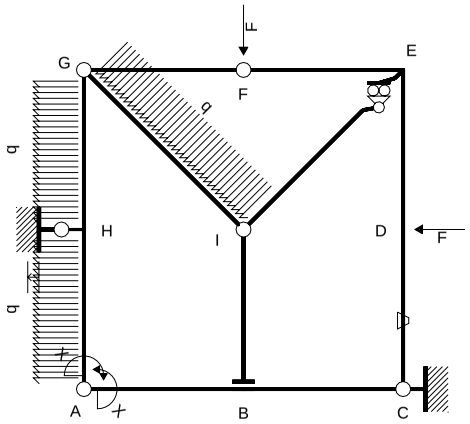

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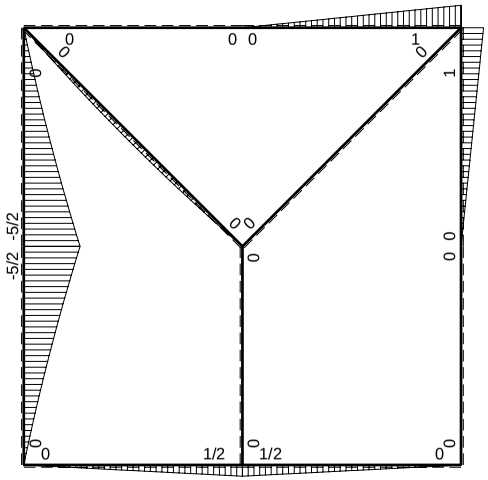

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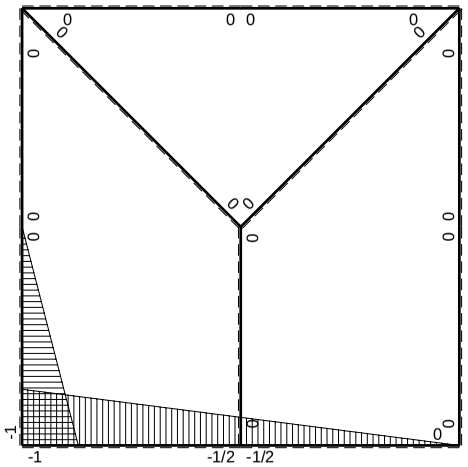

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-7/8
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0
-7/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	-Fb/EJ	0	0	0	0+0	0
DC b	0	0	Fb/EJ	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

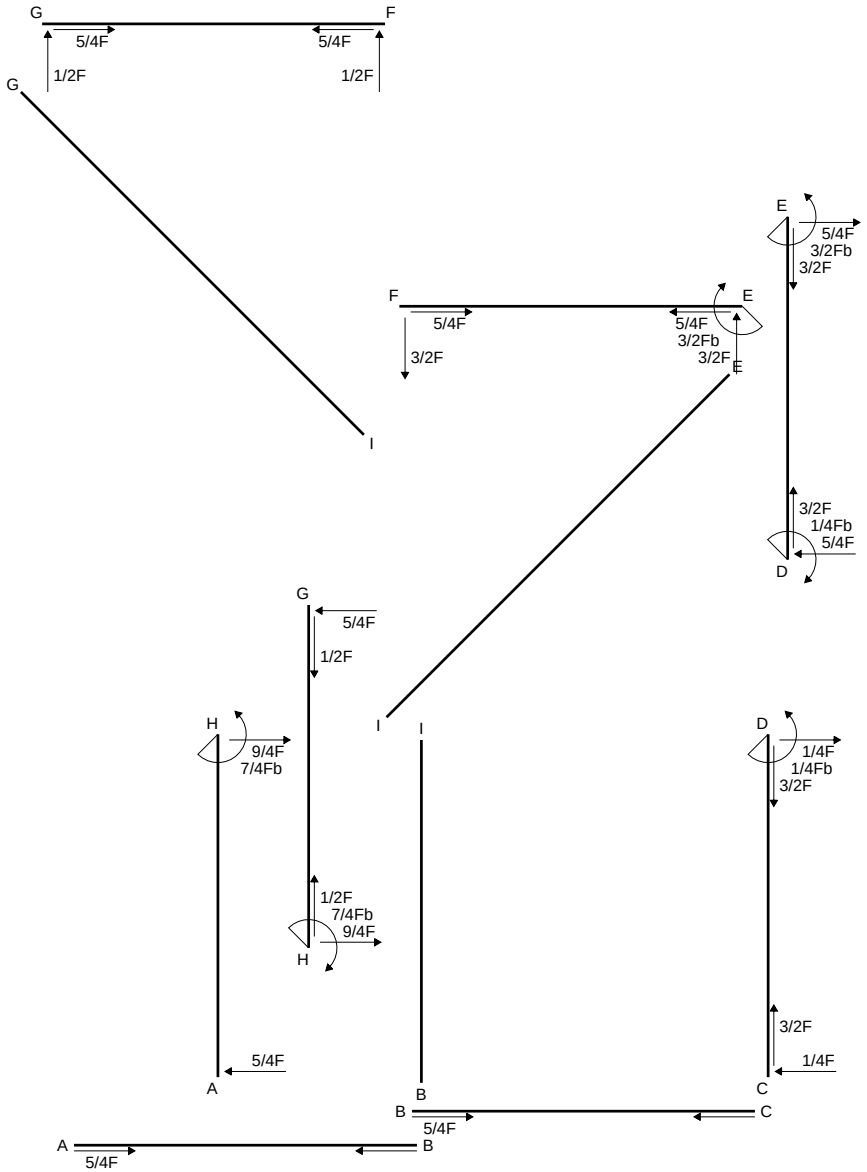
$= \left(5/4\ b - b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

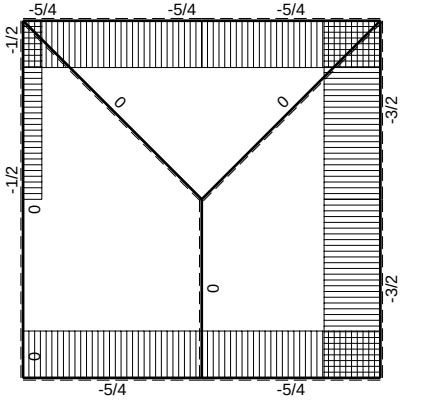
$= \left(\ b - 1/2\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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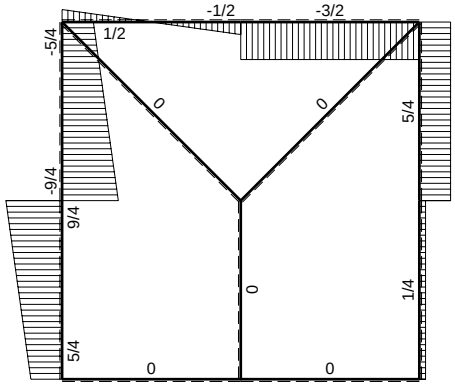
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G
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5/4F
1/2F
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G
E
5/4F
3/2Fb
3/2F
F
5/4F
E
5/4F
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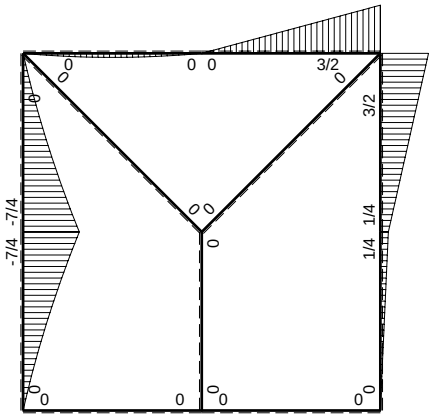
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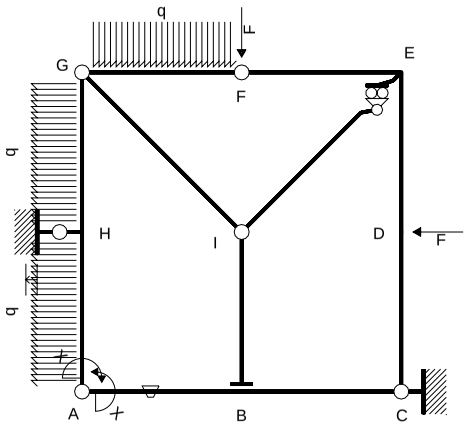

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-5/4
-5/4

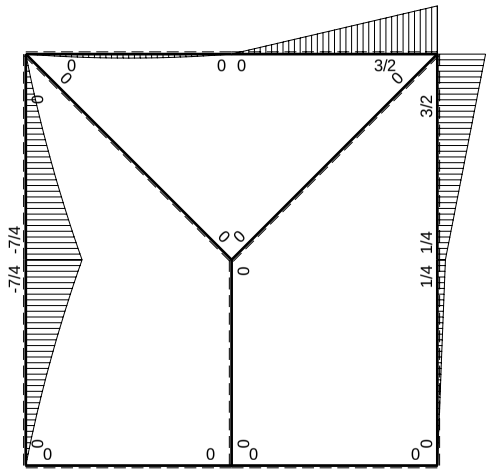
F


-1/2
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1/4
5/4

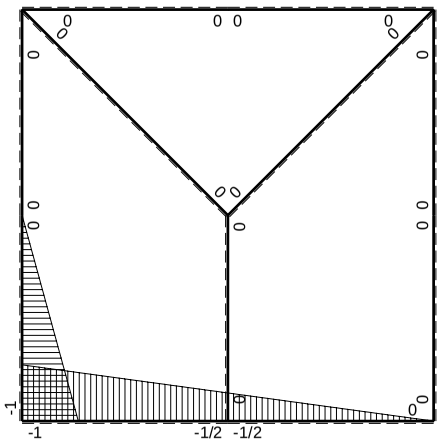
Fb


3/2
3/2
-7/4
-7/4
1/4
1/4





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	-Fb/EJ	0	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(0+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	0	Fb/EJ	0	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						0	Xb/EJ
	iperstatica $X=W_{AB}$						0	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(1 - 1/2\ x/b\ \right)\theta\ \ dx = \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/2\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

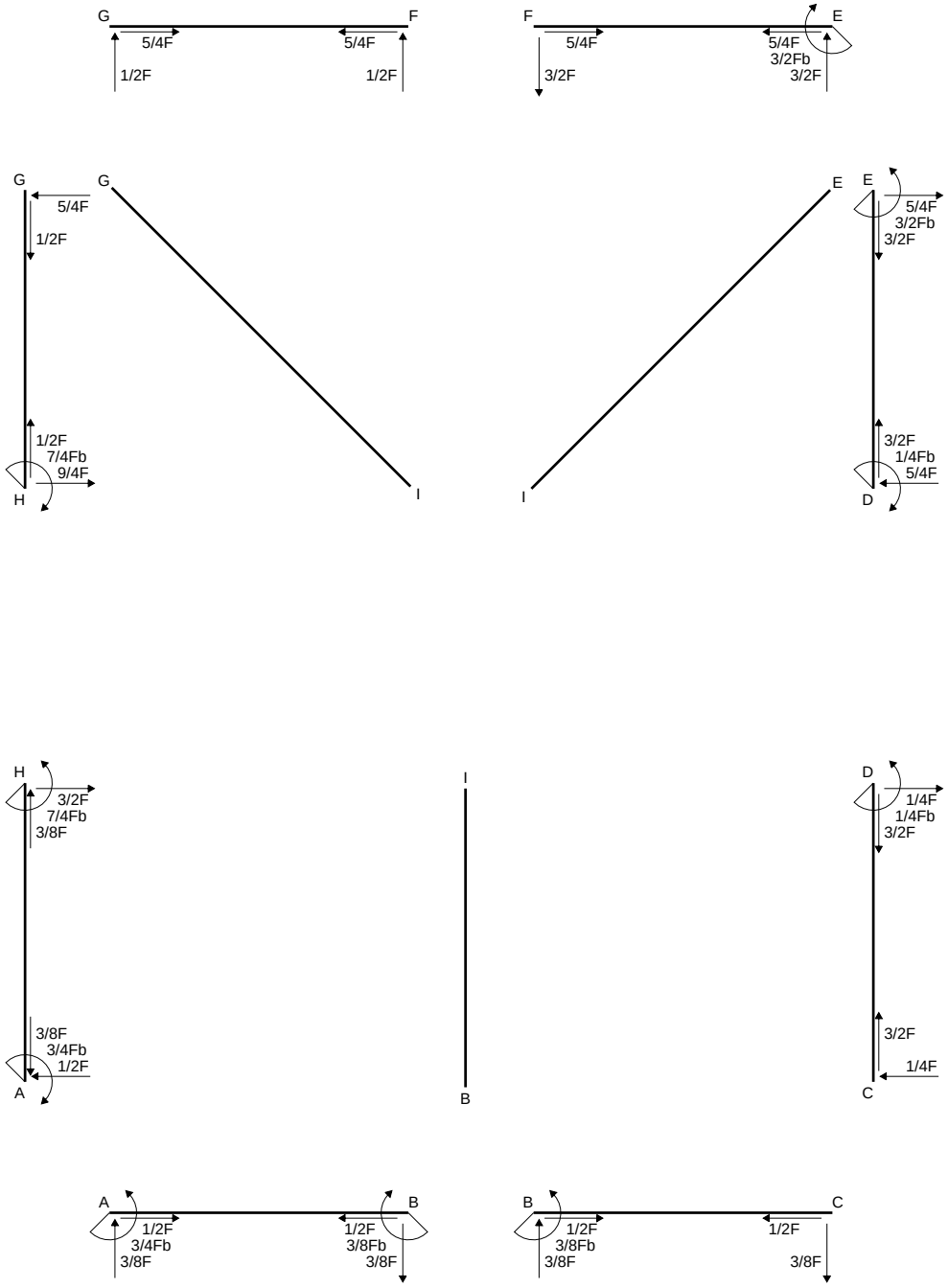
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

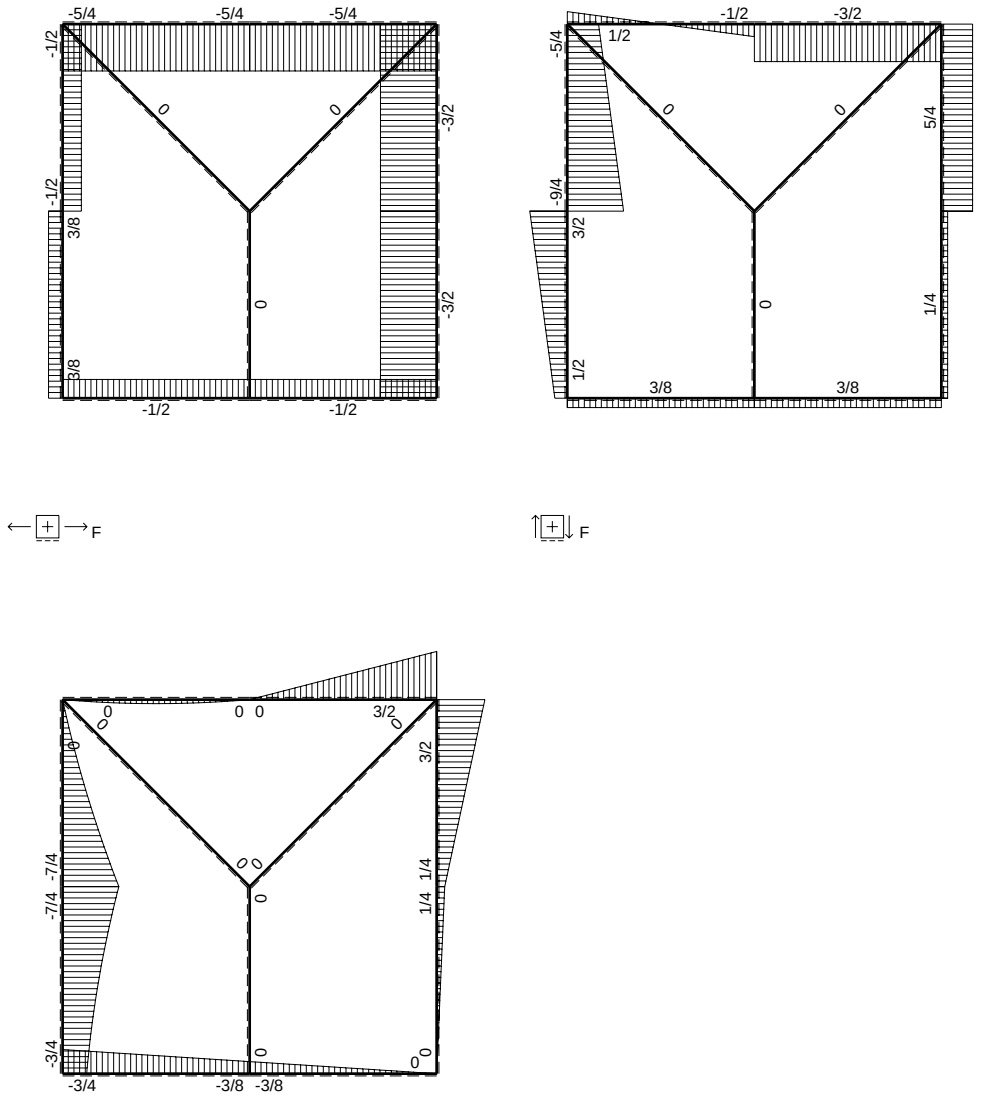
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

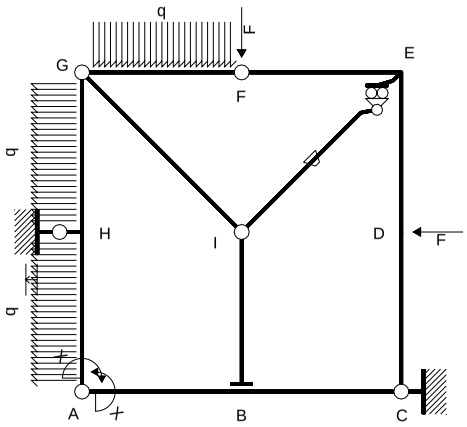
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

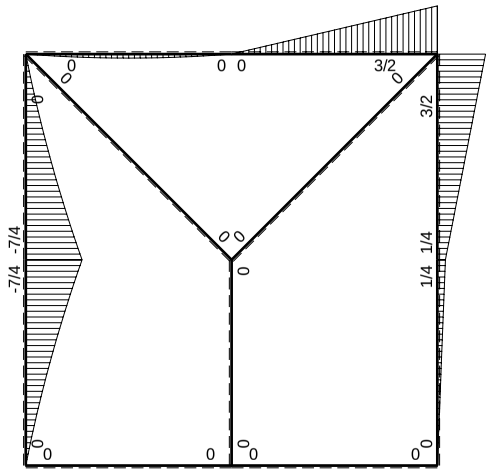
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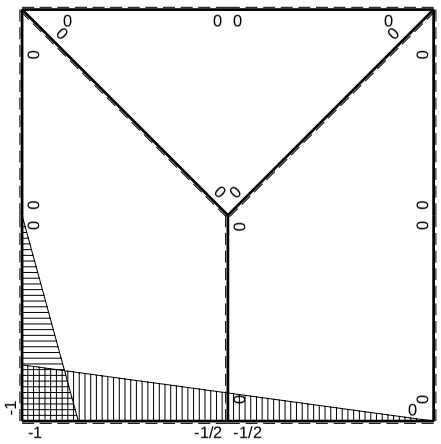

G F
5/4F 5/4F
1/2F 1/2F
F E
5/4F 5/4F
3/2Fb
3/2F 3/2F
G G
5/4F
1/2F
1/2F
7/4Fb
9/4F
H I
E E
5/4F
3/2Fb
3/2F
3/2F
1/4Fb
5/4F
I D
H
3/2F
7/4Fb
3/8F
3/8F
3/4Fb
1/2F
A
I
B
D
1/4F
1/4Fb
3/2F
3/2F
1/4F
C
A B
1/2F 1/2F
3/4Fb 3/8Fb
3/8F 3/8F
B C
1/2F 1/2F
3/8Fb
3/8F 3/8F


-5/4 -5/4 -5/4
-1/2
-1/2
-3/2
3/8
0
-3/2
3/8
-1/2 -1/2
F
-1/2 -3/2
1/2
-5/4
5/4
-9/4
3/2
1/4
1/2
3/8 3/8
F
0 0 3/2
3/2
-7/4 -7/4
1/4 1/4
-3/4
-3/4 -3/8 -3/8
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211461 ESΣ04.xxxx.125

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

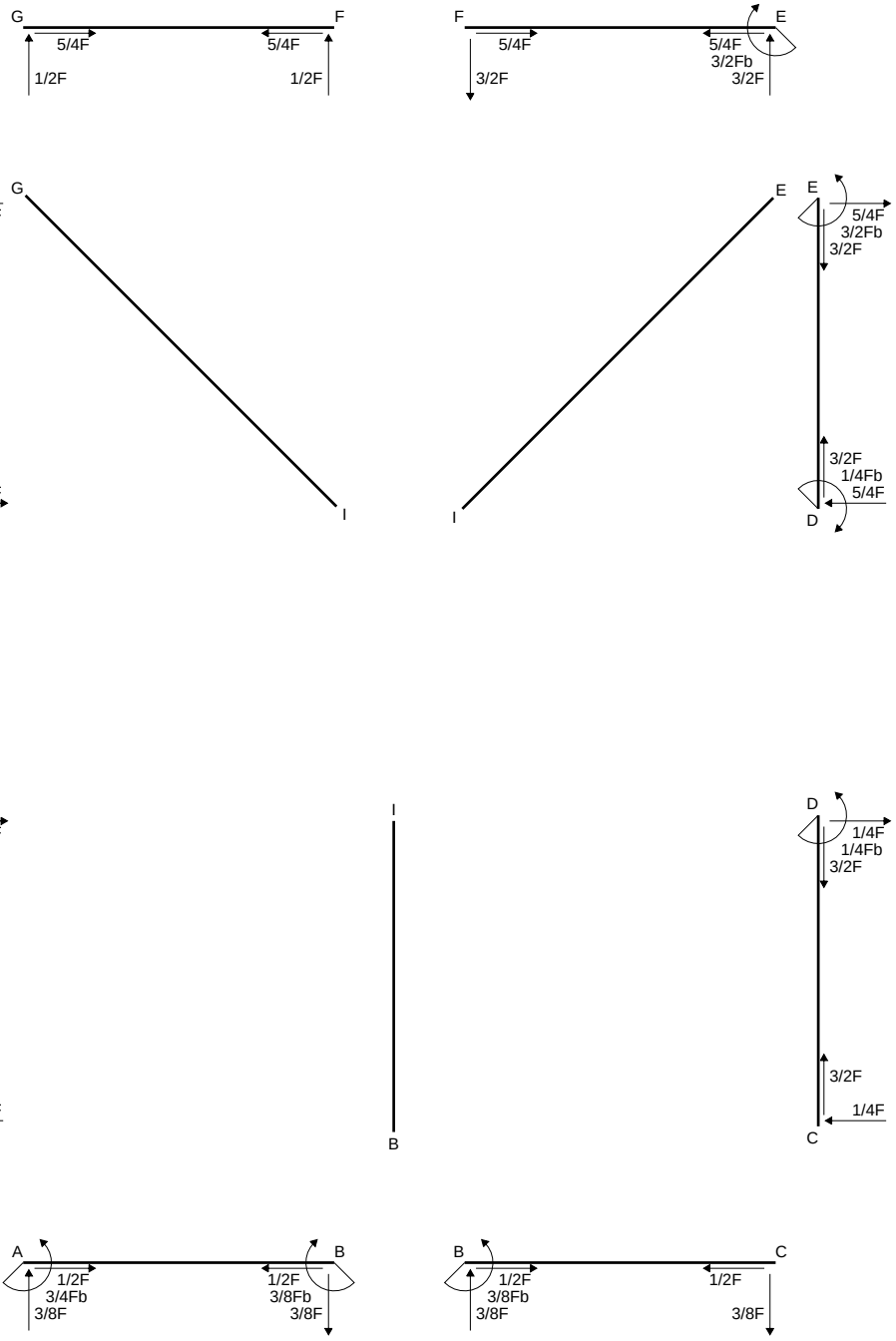
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

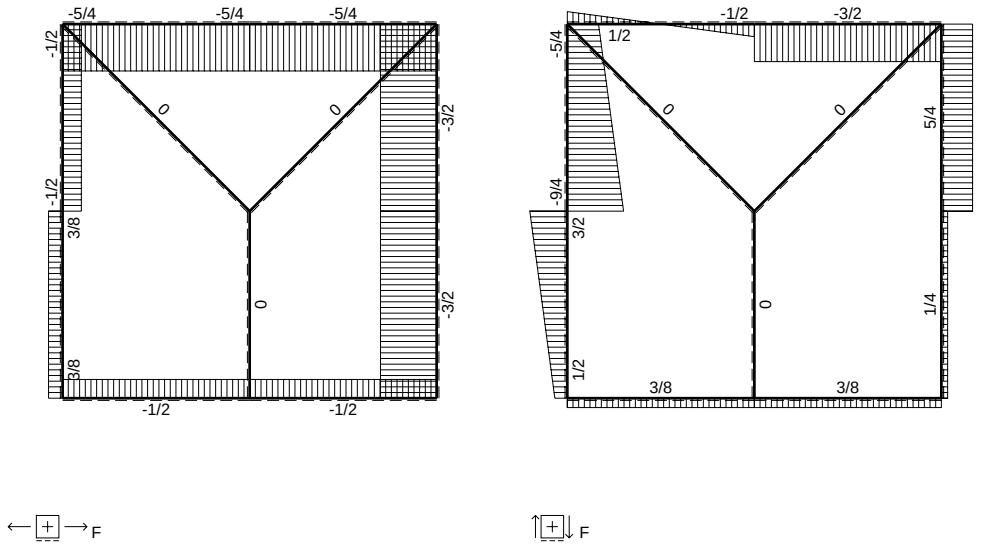
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

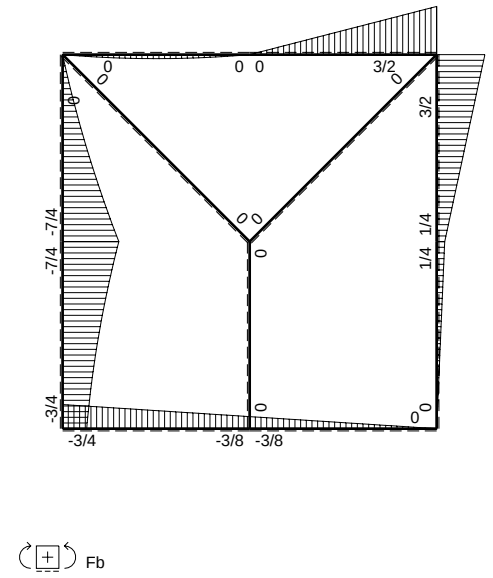
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

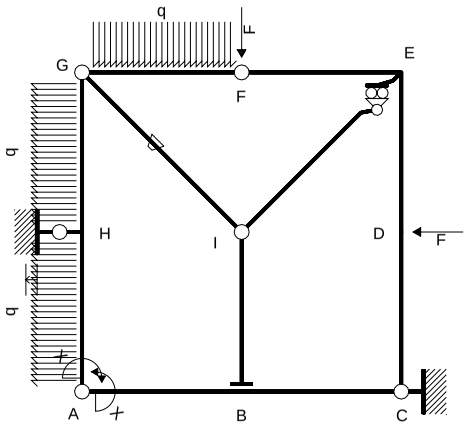
ESΣ04.xxxx.125

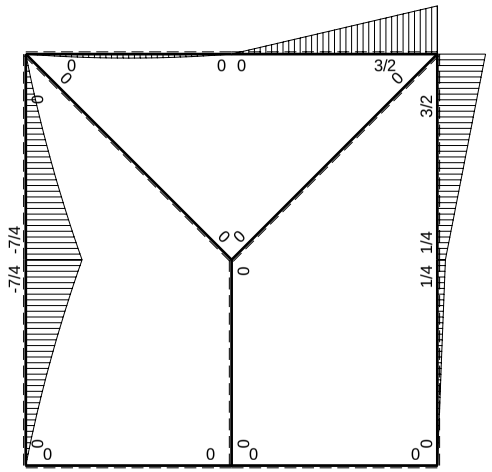
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G
F
5/4F
5/4F
1/2F
1/2F
F
E
5/4F
5/4F
3/2Fb
3/2F
3/2F
G
G
5/4F
1/2F
1/2F
7/4Fb
9/4F
H
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3/8F

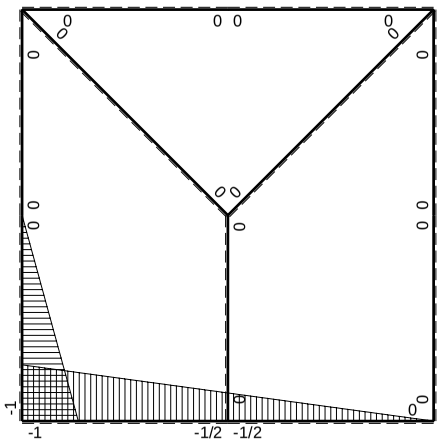

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3/8
3/8
F
F


3/2
-7/4
-7/4
1/4
1/4
-3/4
-3/8
-3/8
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240146 ESΣ04.xxxx.126

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PROCEDIMENTO E RISULTATI 240146 ESΣ04.xxxx.126

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

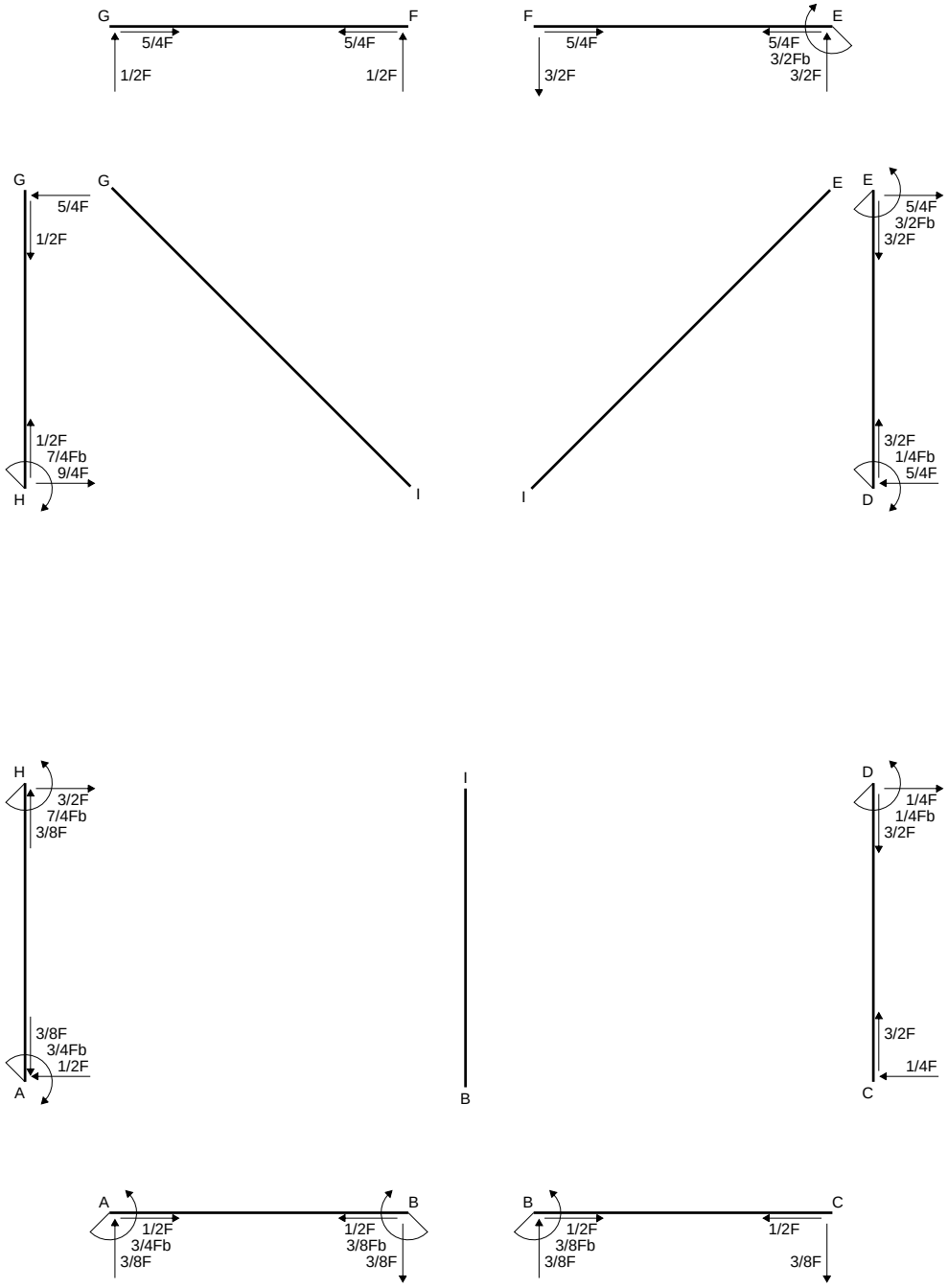
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

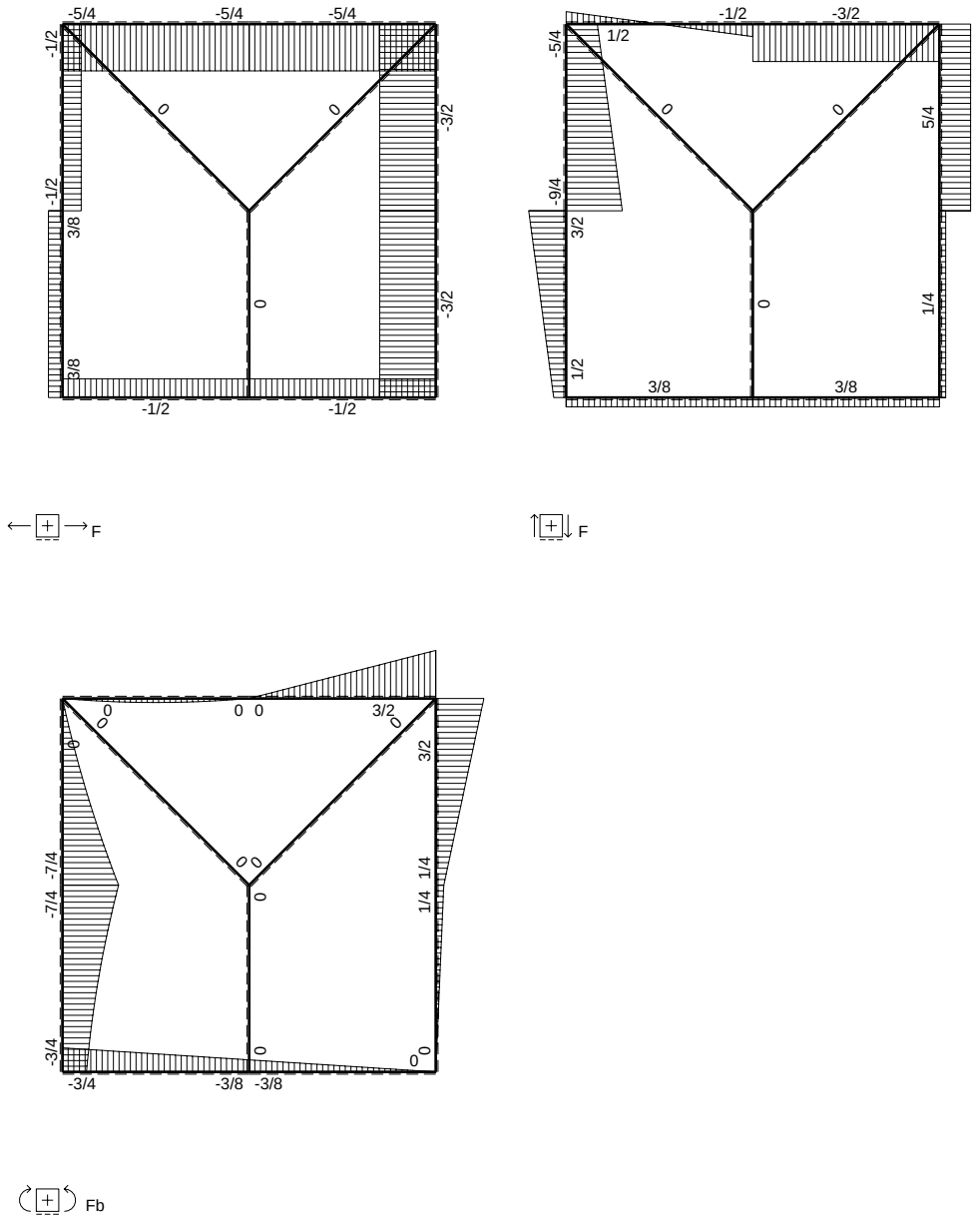
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

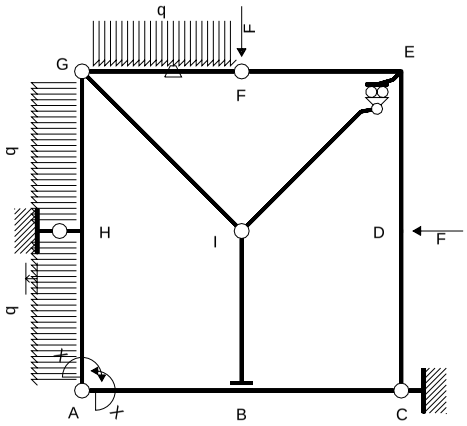
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

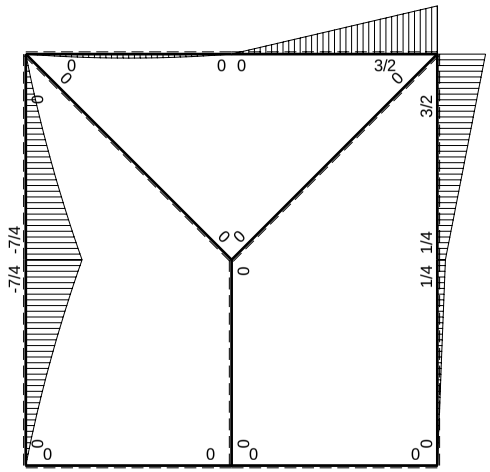
ESΣ04.xxxx.126

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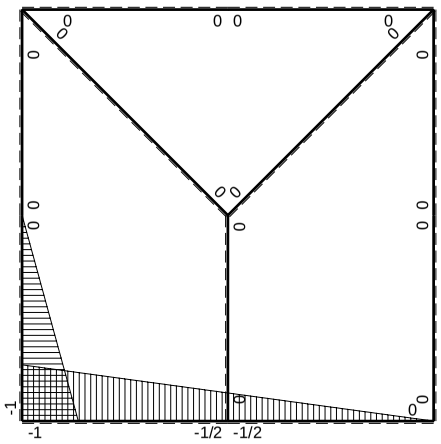

G
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-3/4
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-3/4
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239766 ESΣ04.xxxx.127

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PROCEDIMENTO E RISULTATI 239766 ESΣ04.xxxx.127

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

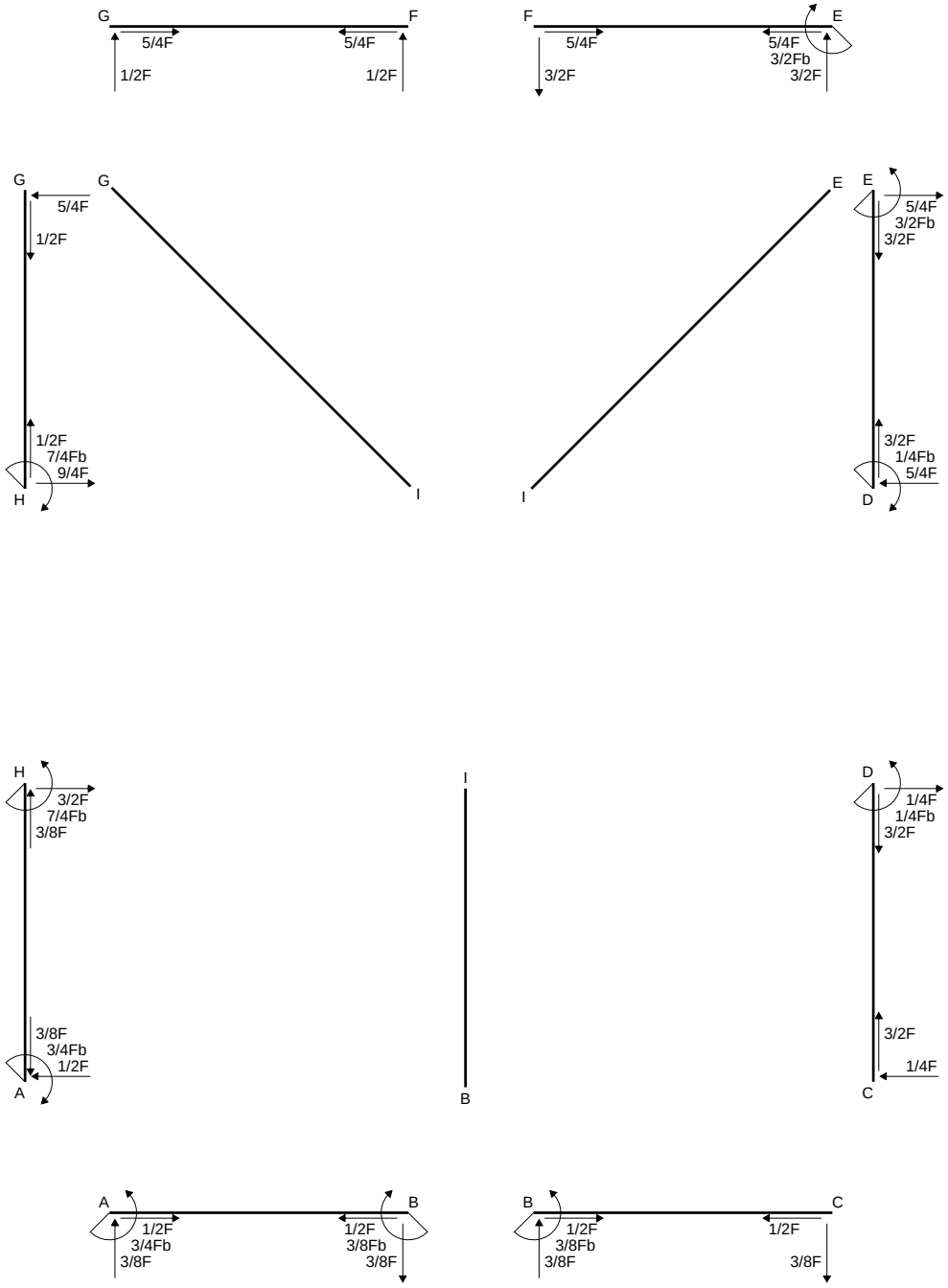
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

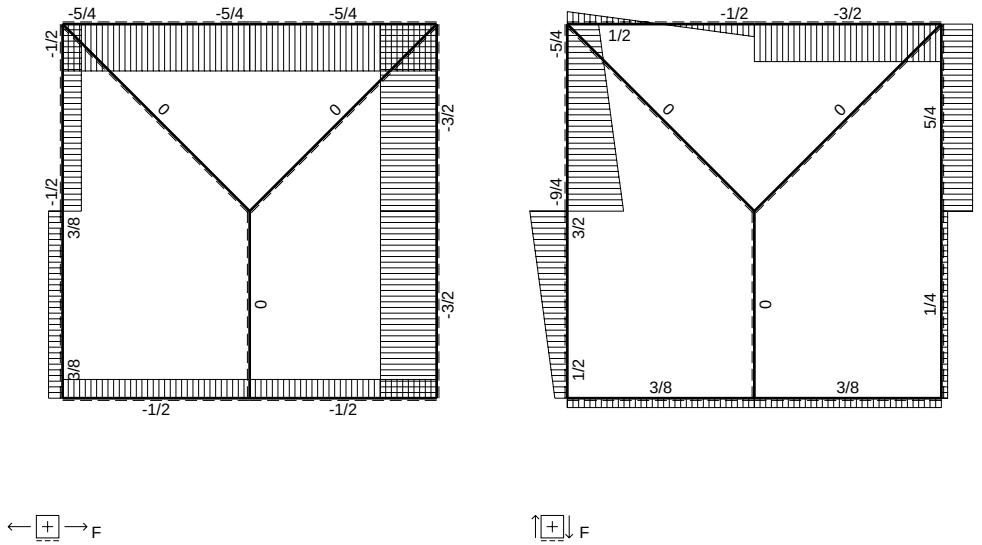
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

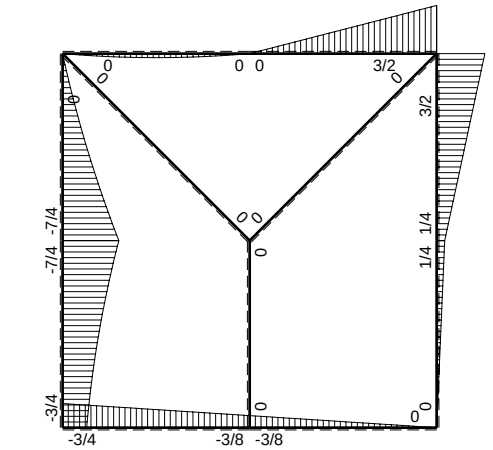
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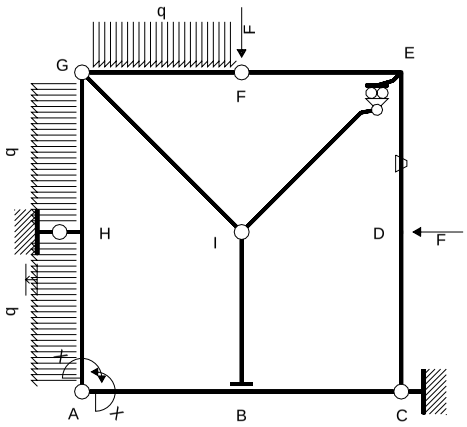

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5/4F
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F
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5/4F
5/4F
3/2Fb
3/2F
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G
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1/2F
1/2F
7/4Fb
9/4F
H
I
E
E
5/4F
3/2Fb
3/2F
I
3/2F
1/4Fb
5/4F
D
H
3/2F
7/4Fb
3/8F
3/8F
3/4Fb
1/2F
A
I
B
D
1/4F
1/4Fb
3/2F
3/2F
1/4F
C
A
B
1/2F
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3/8F

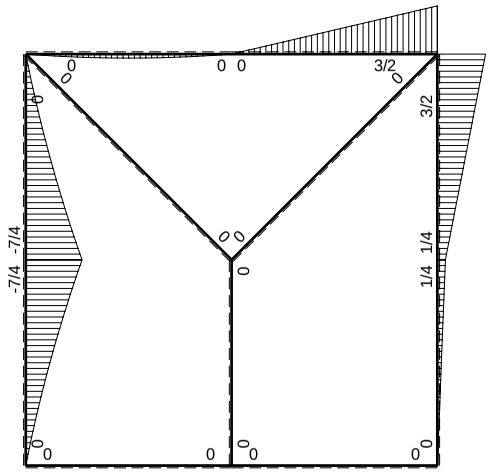

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1/2
3/8
3/8

F F

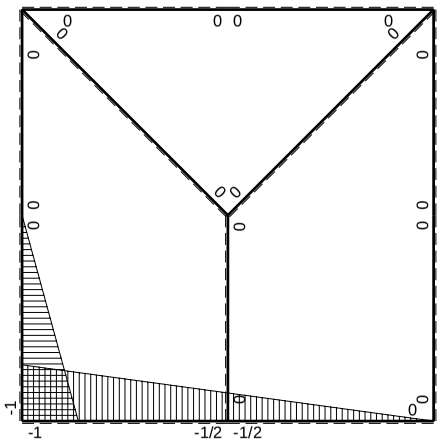

-7/4
-7/4
-3/4
-3/8
-3/8
3/2
1/4
1/4

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211653 ESΣ04.xxxx.128

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

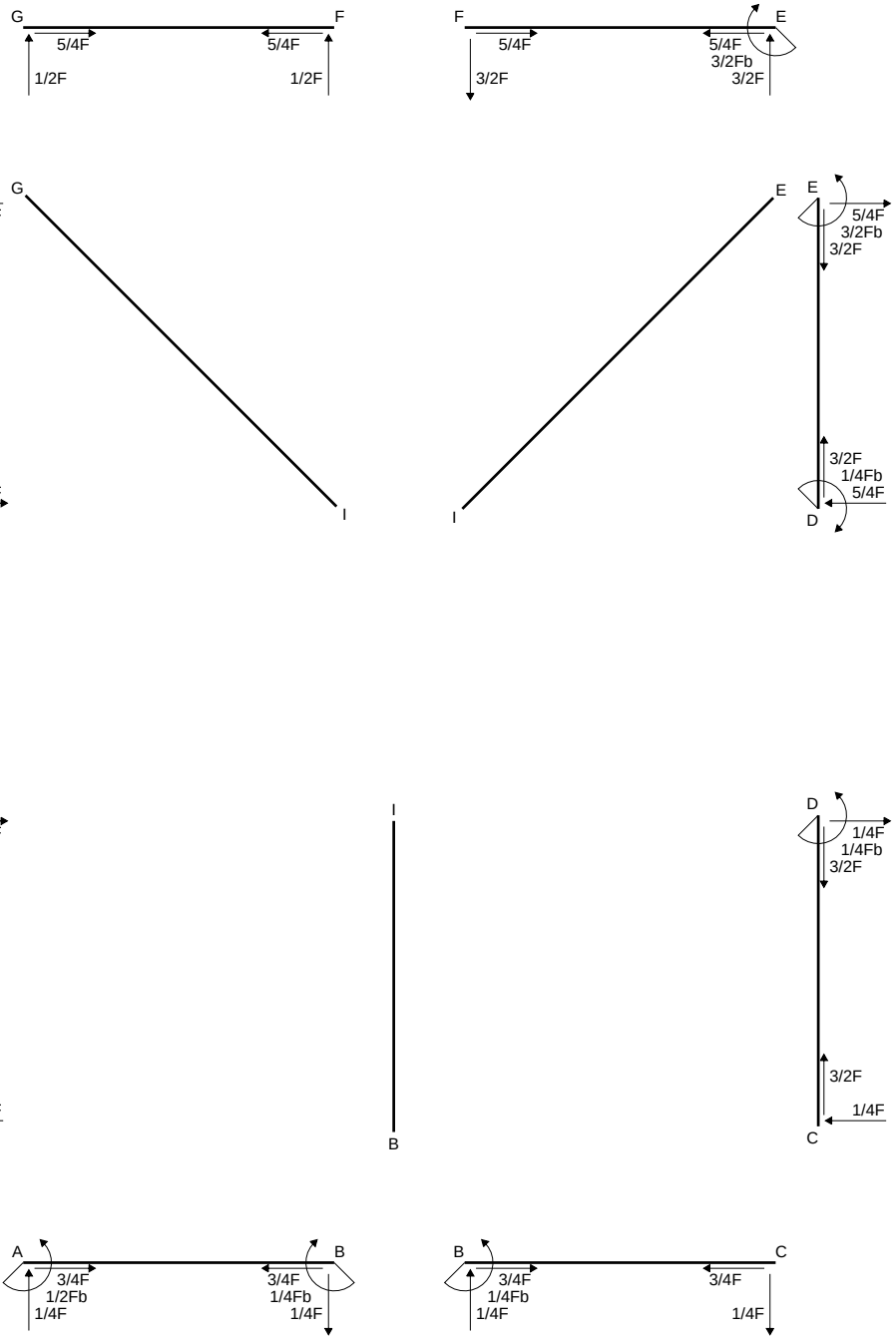
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

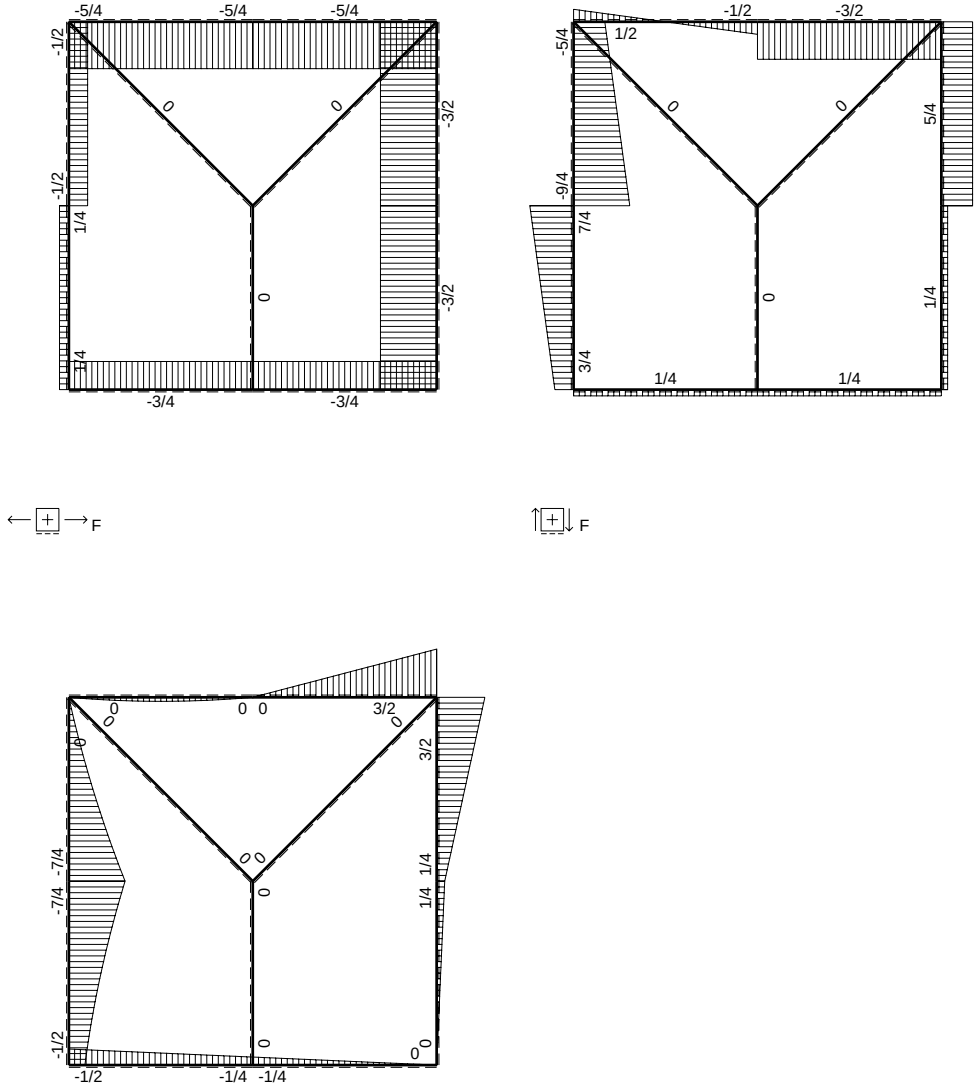
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

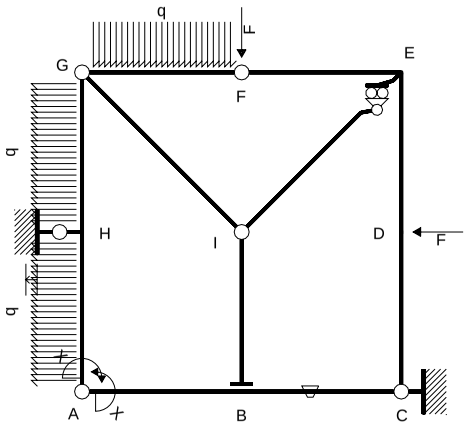
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

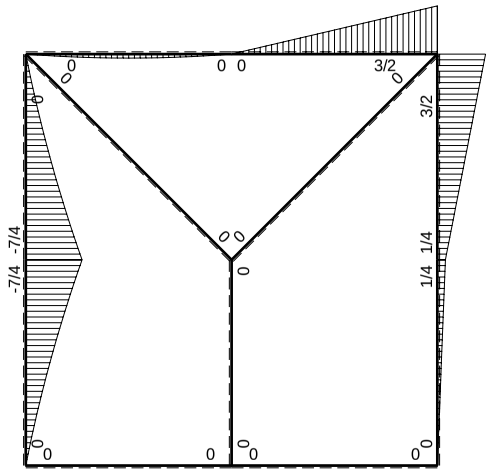
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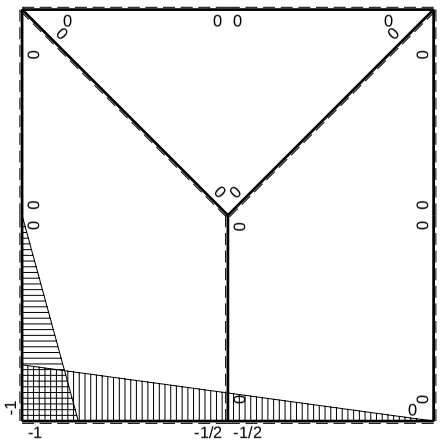

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3/2Fb
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5/4F
1/2F
1/2F
7/4Fb
9/4F
H I
E E
5/4F
3/2Fb
3/2F
I
3/2F
1/4Fb
5/4F
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1/4
3/4
1/4 1/4
F
3/2
-7/4 -7/4
1/4 1/4
-1/2
-1/2 -1/4 -1/4
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238810 ESΣ04.xxxx.129

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	-Fb/EJ	0	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(0+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	0	Fb/EJ	0	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/2Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(1/2\ b - 1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/2\ x/b\ \right)\theta\ \ dx = \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

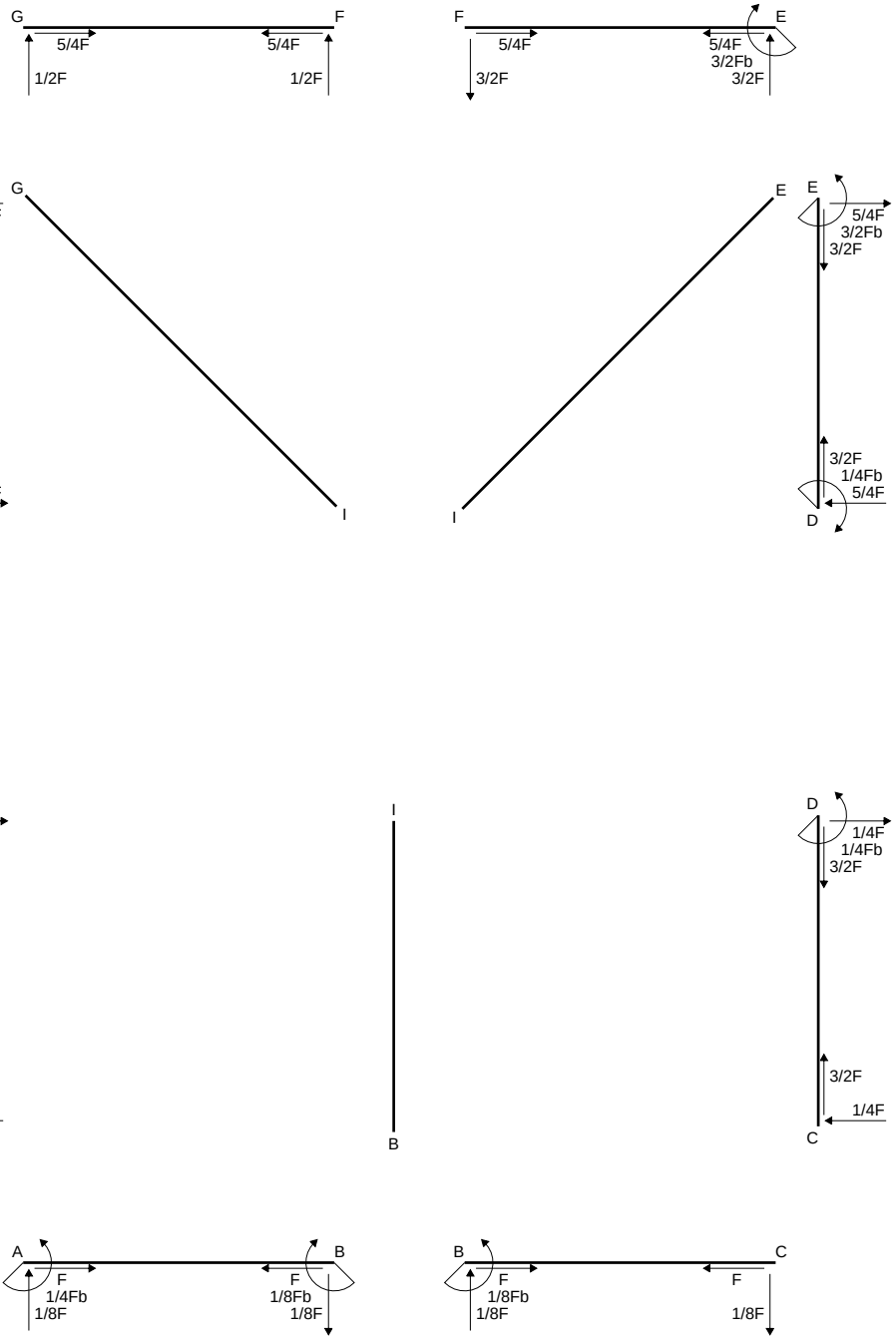
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

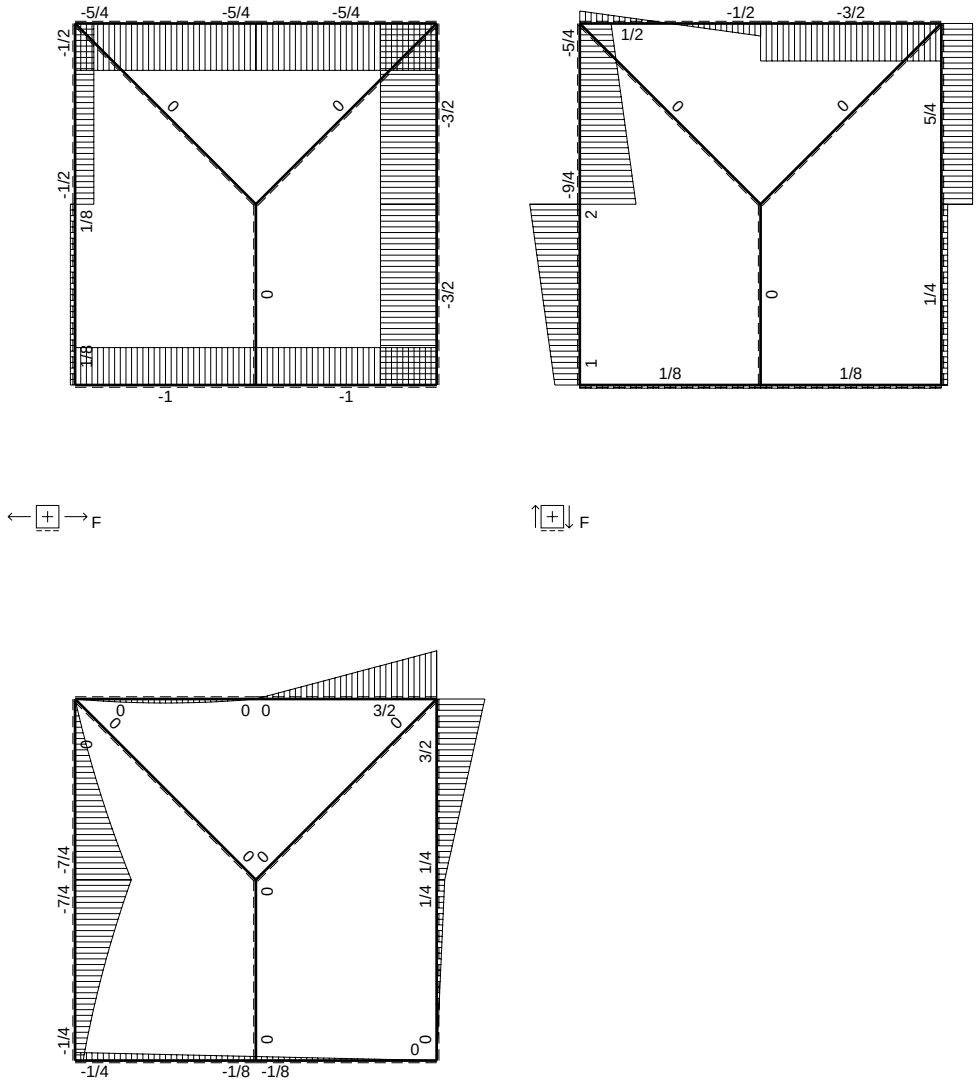
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

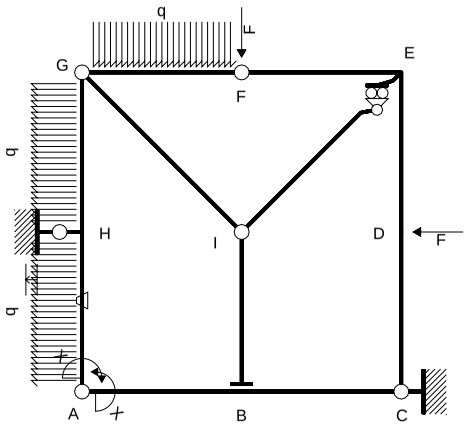
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

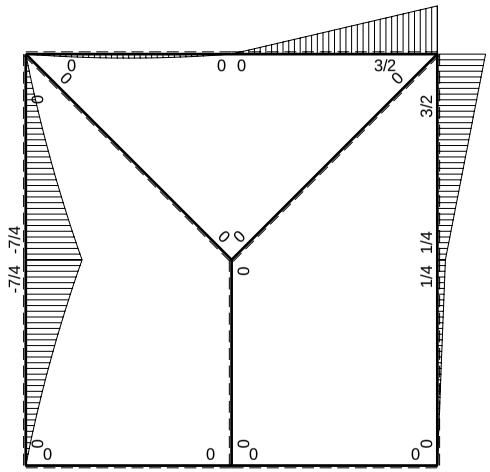
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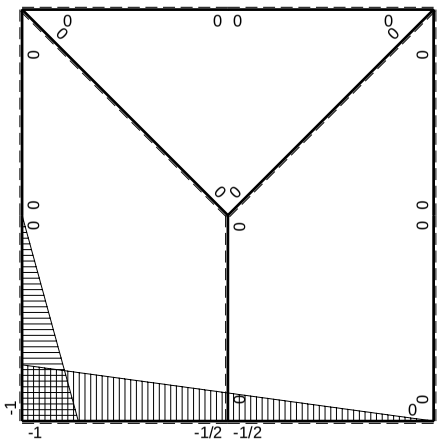

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	-Fb/EJ	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/4+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	Fb/EJ	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\theta\ \ dx$

$= \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ + \left(1/2\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

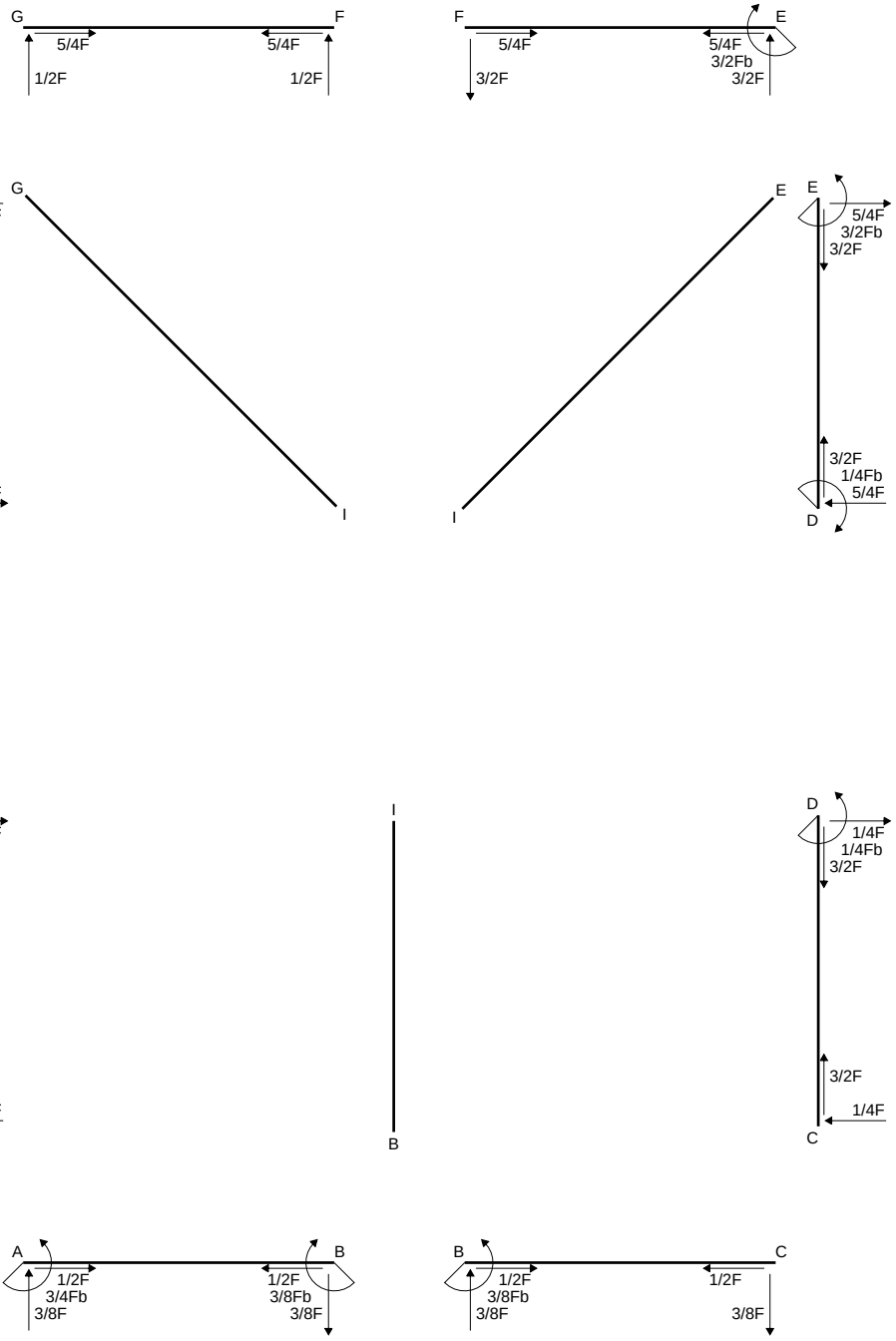
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\theta\ \ dx$

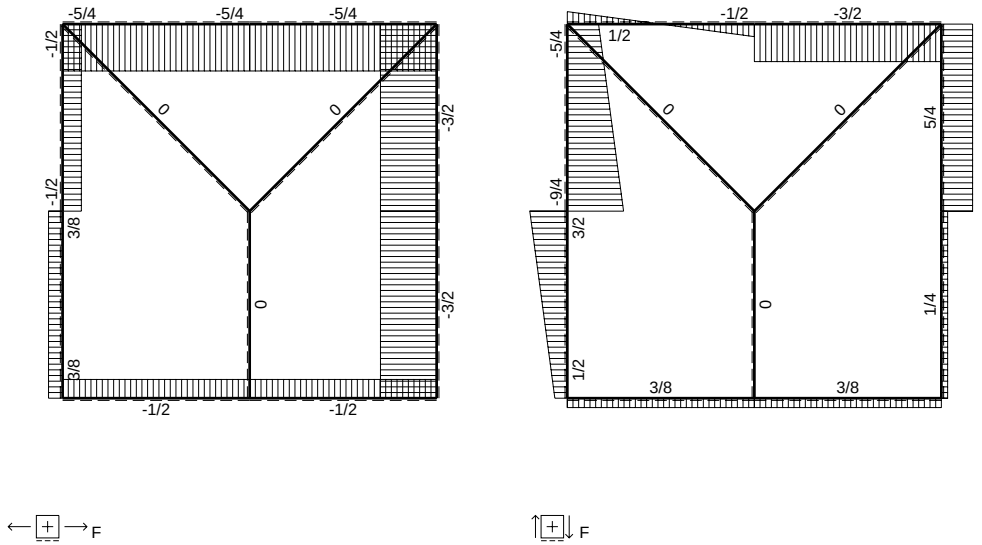
$= \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

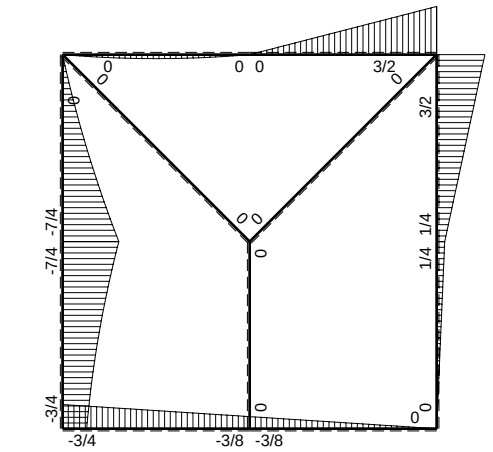
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

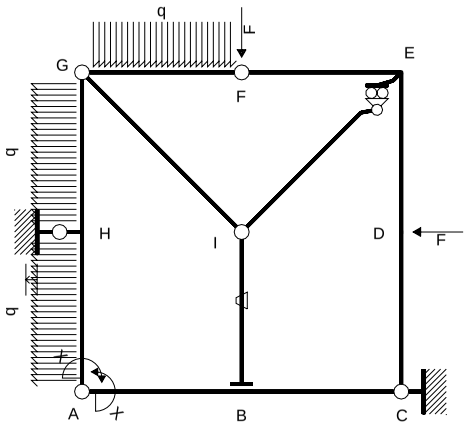
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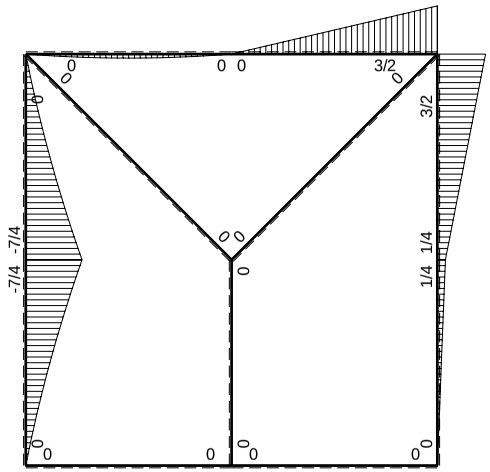
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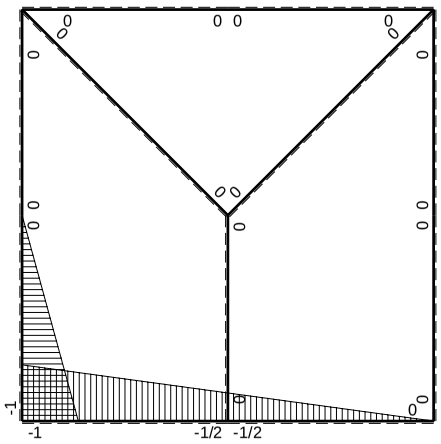

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-3/8
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 280952 ESΣ04.xxxx.131

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PROCEDIMENTO E RISULTATI 280952 ESΣ04.xxxx.131

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

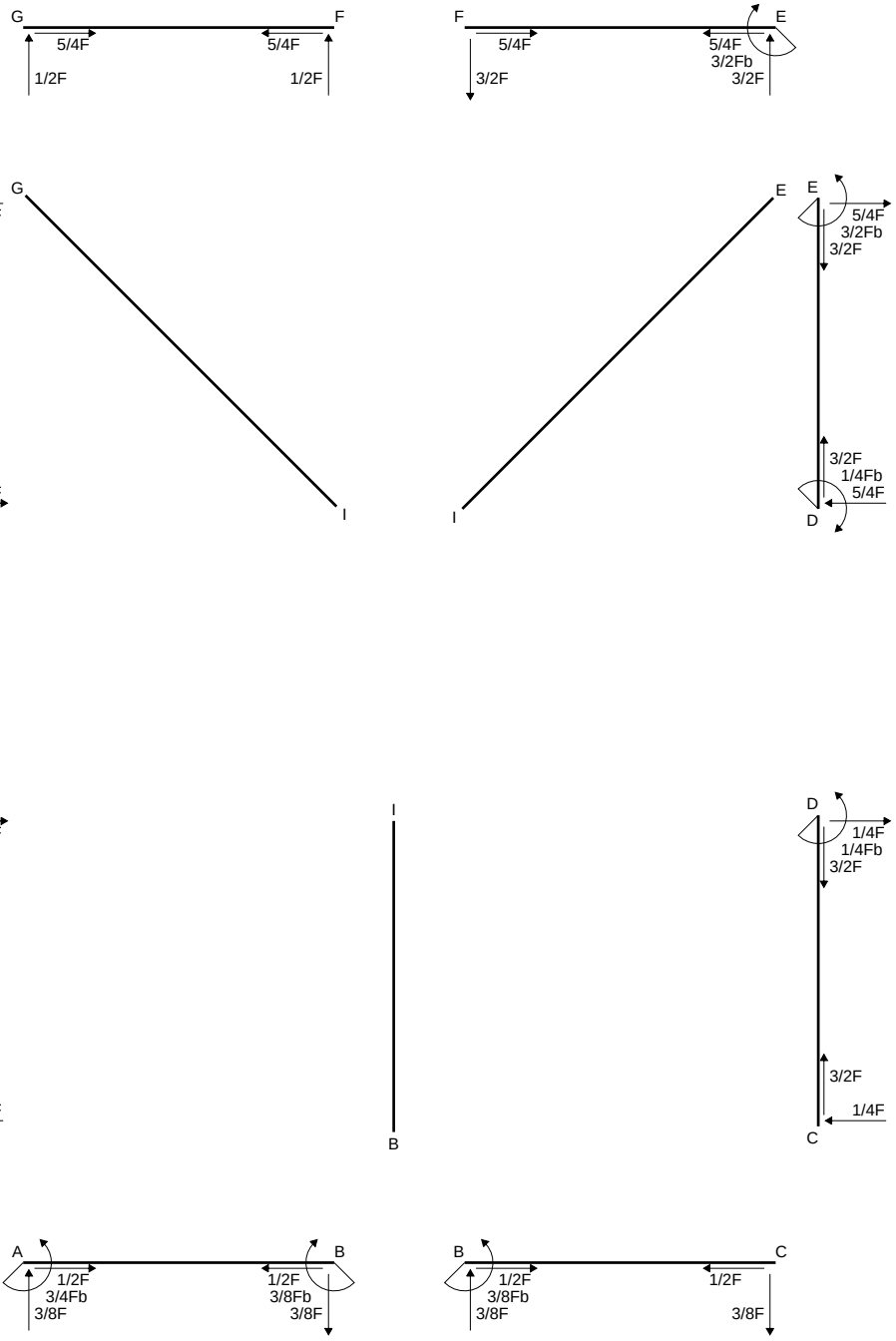
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

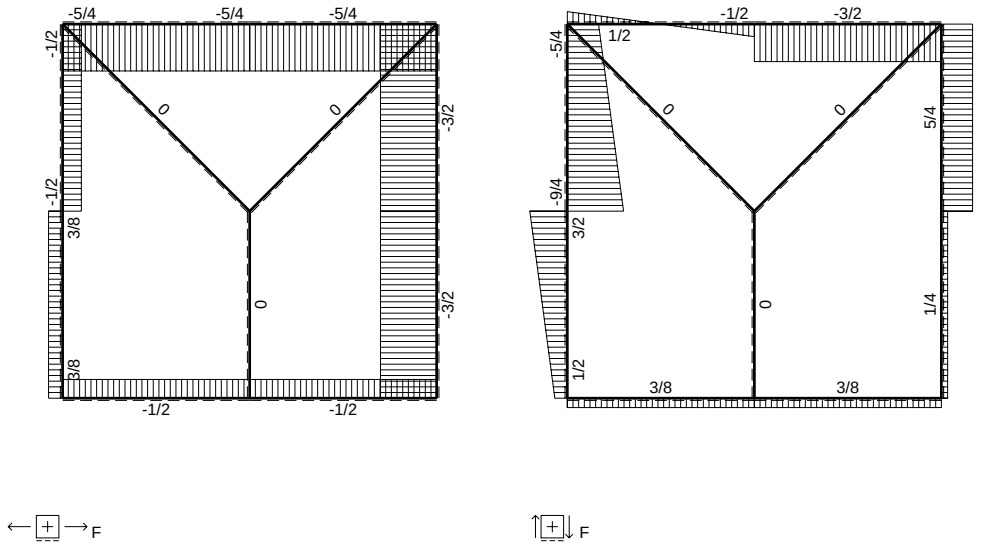
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

$= \left(5/8\ b - 1/4\ b - 1/8\ b\right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

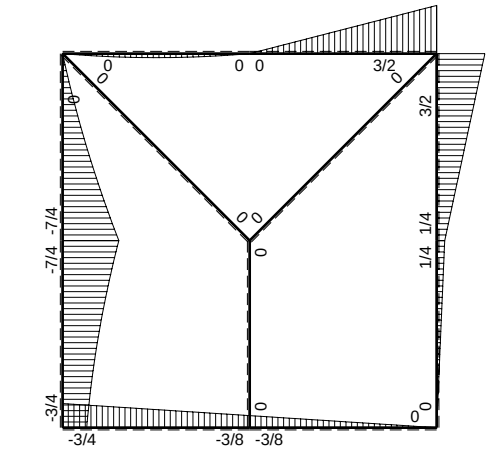
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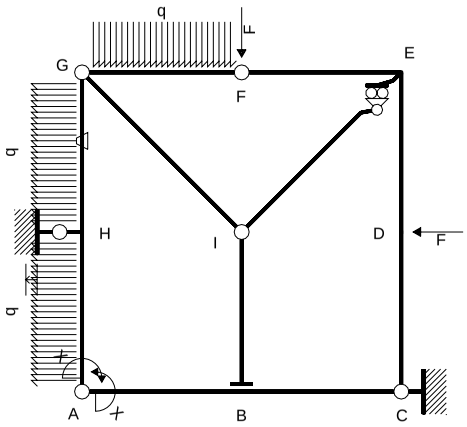

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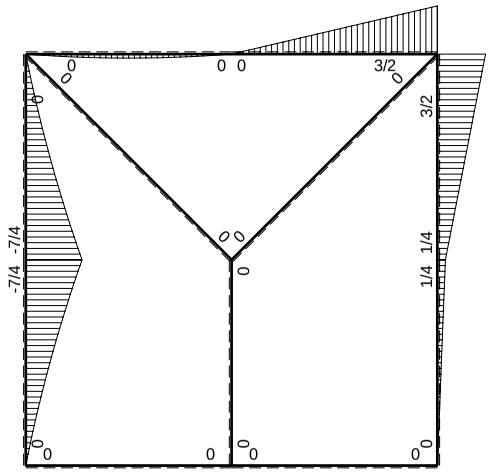

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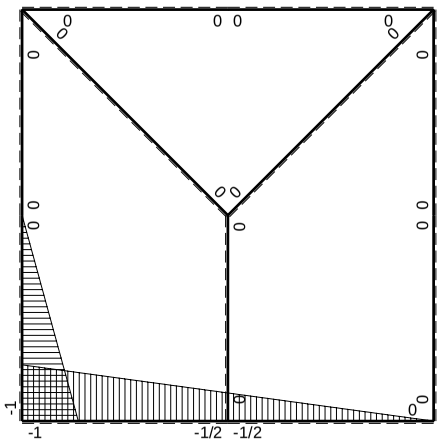

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238699 ESΣ04.xxxx.132

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

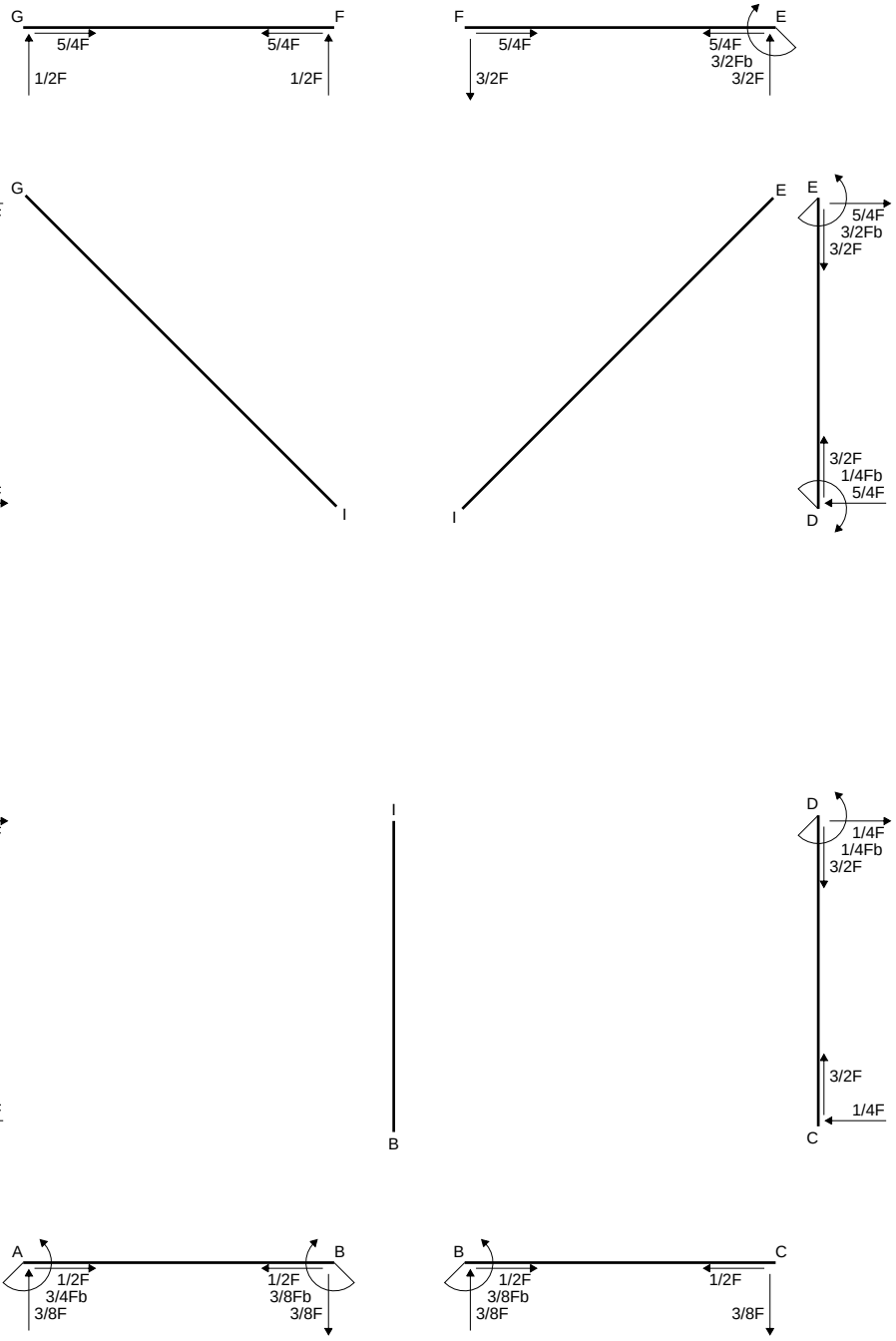
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

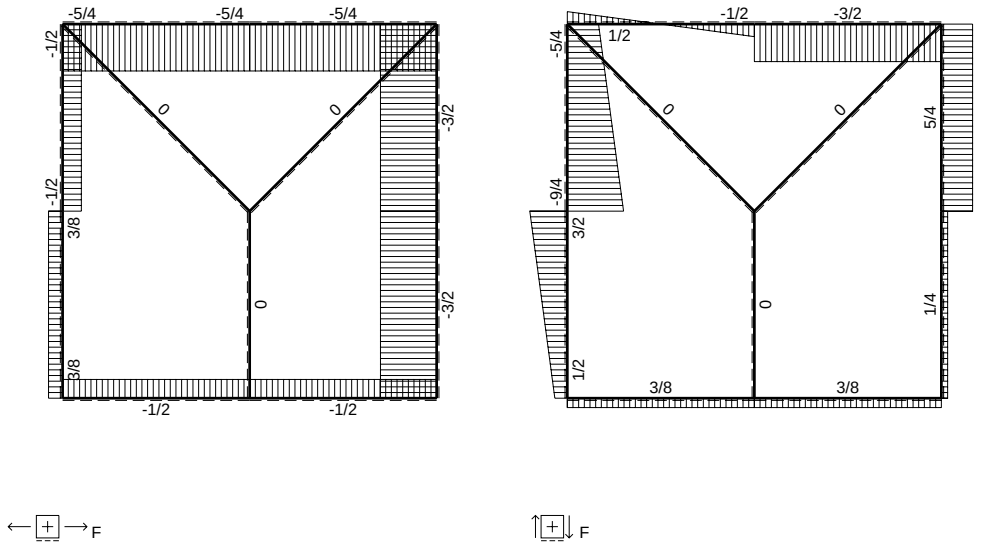
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

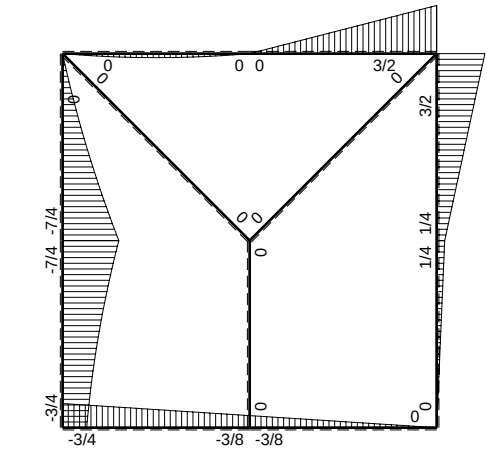
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

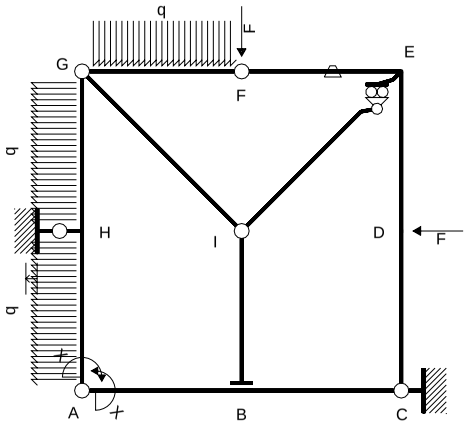
ESΣ04.xxxx.132

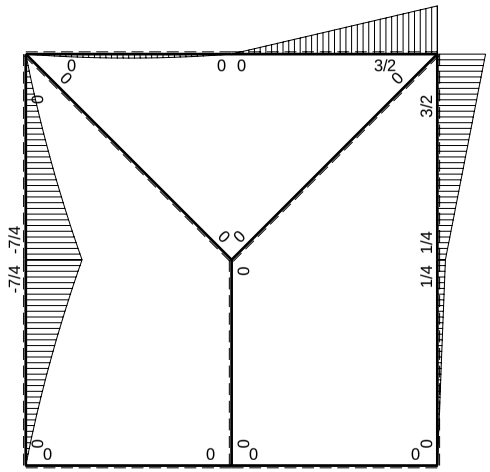
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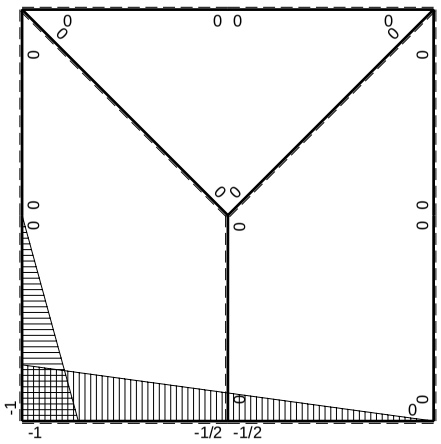

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-3/8
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236693 ESΣ04.xxxx.133

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-3/2Fx	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

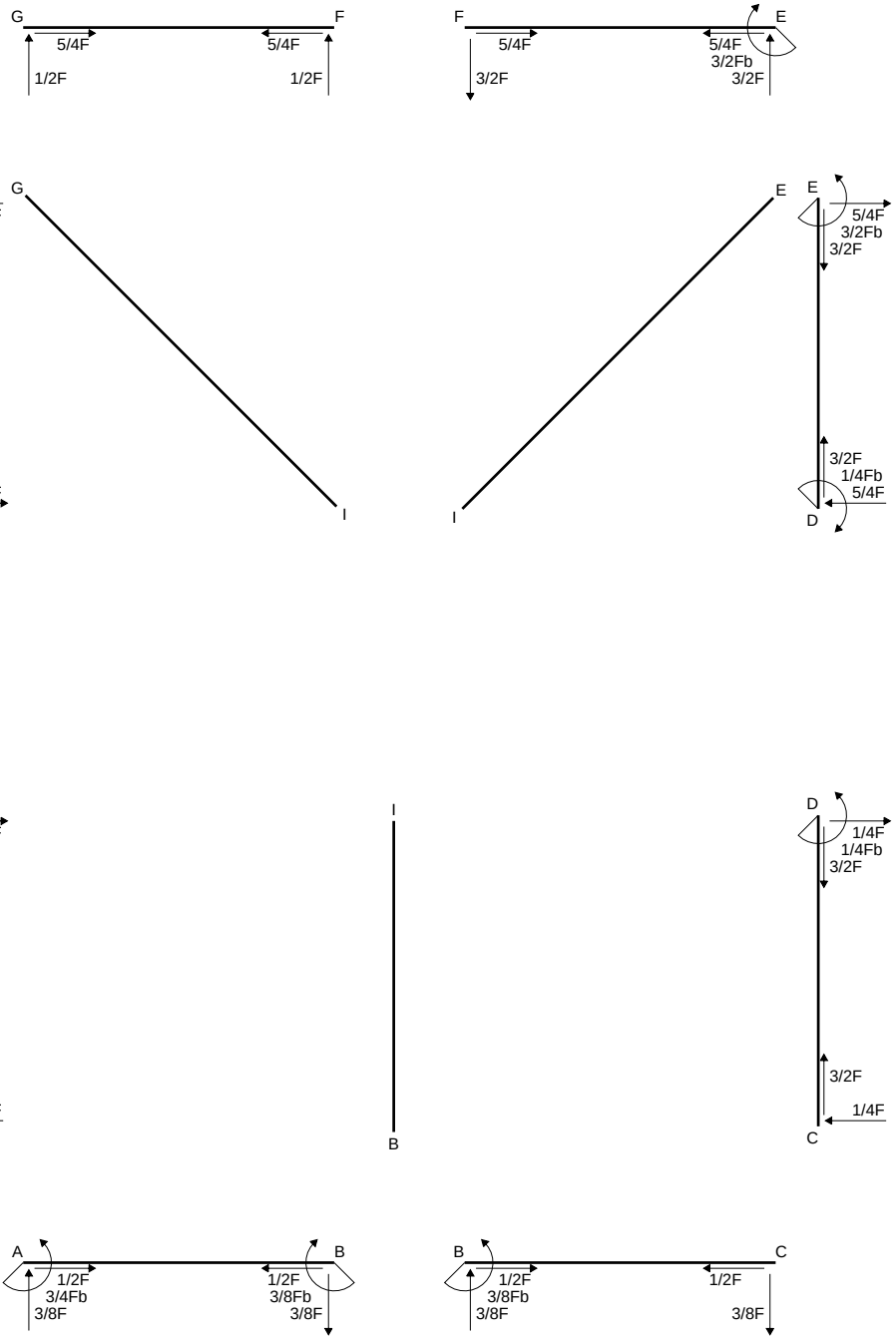
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

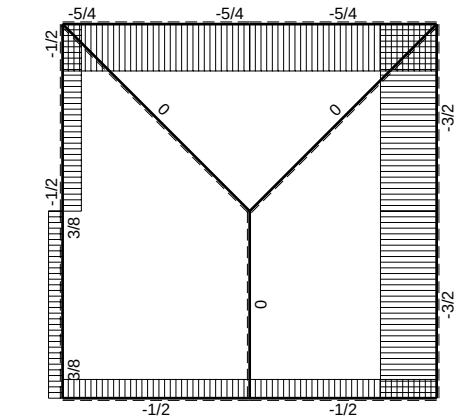
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

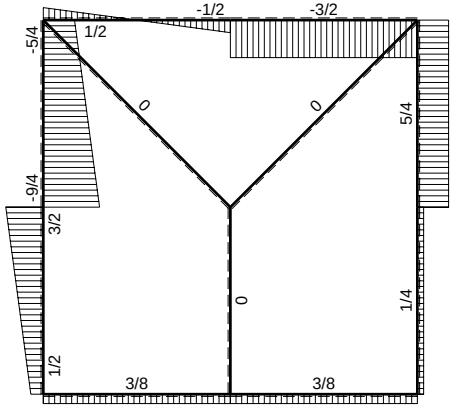
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

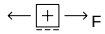
ESΣ04.xxxx.133

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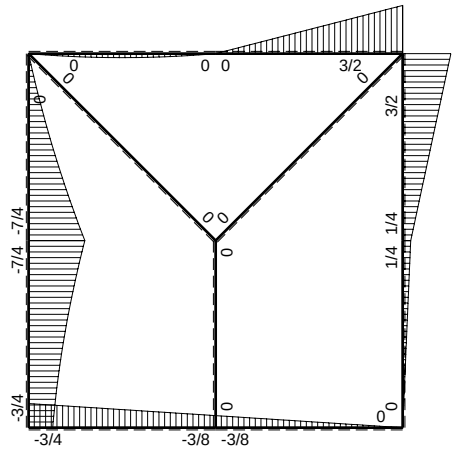

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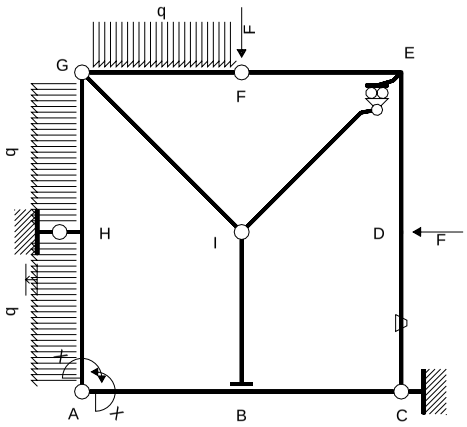

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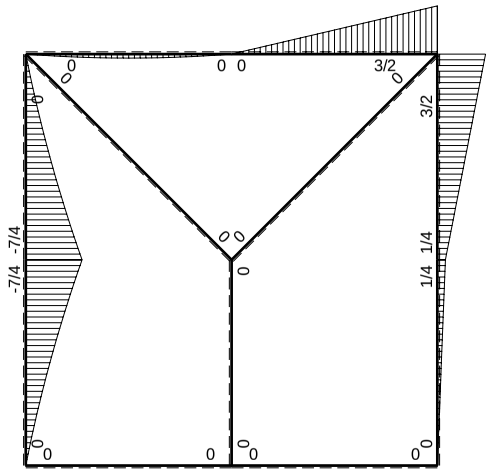

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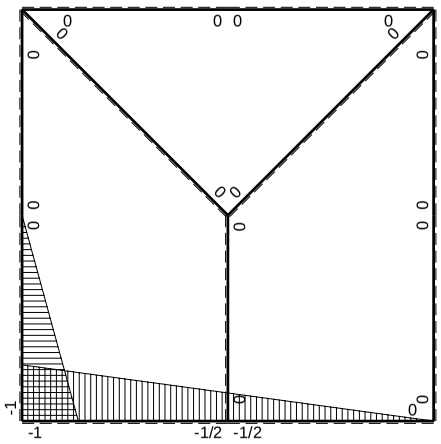

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981992 ESΣ04.xxxx.134

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PROCEDIMENTO E RISULTATI 981992 ESΣ04.xxxx.134

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

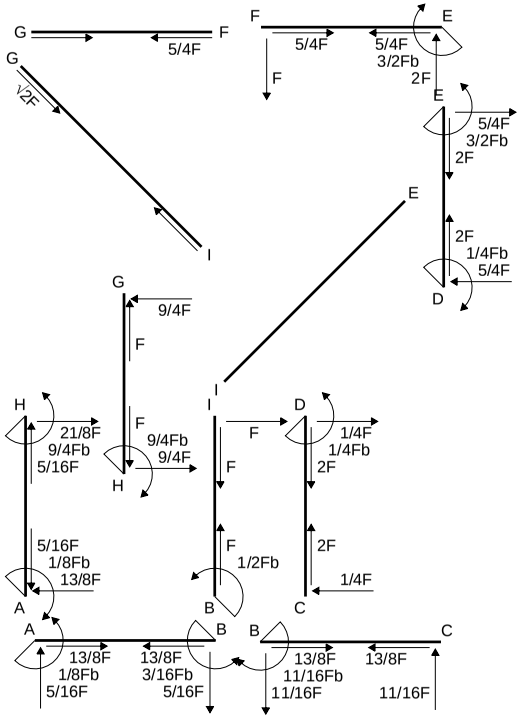
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

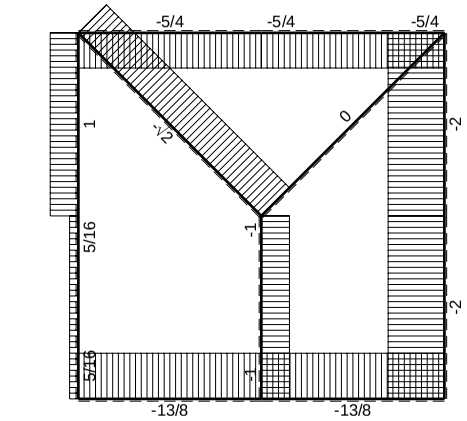
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

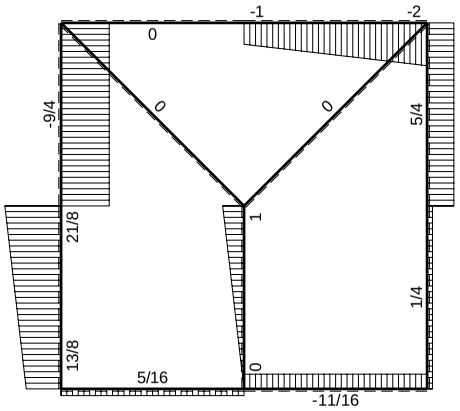
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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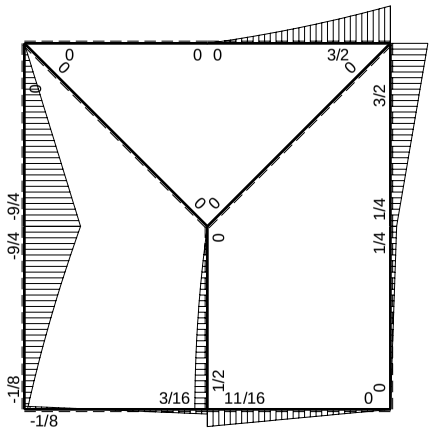

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F
5/4F
G
√2F
I
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5/4F
5/4F
3/2Fb
2F
F
E
E
5/4F
3/2Fb
2F
E
2F
1/4Fb
5/4F
D
G
9/4F
F
H
21/8F
9/4Fb
5/16F
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9/4Fb
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H
I
I
F
F
D
1/4F
1/4Fb
2F
5/16F
1/8Fb
13/8F
F
1/2Fb
2F
1/4F
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B
C
A
B
B
C
13/8F
1/8Fb
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13/8F
11/16Fb
11/16F
13/8F
11/16F


-5/4
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-1
-2
5/16
-1
-13/8
-13/8

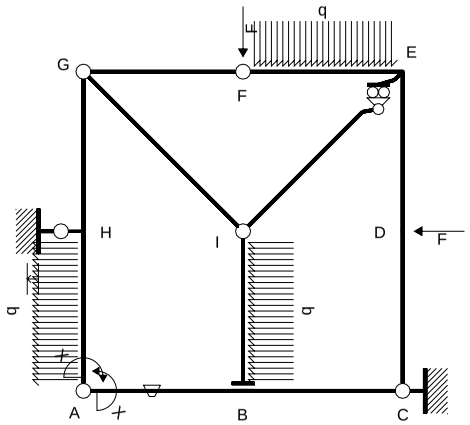

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0
5/4
1
21/8
1/4
13/8
5/16
0
-11/16

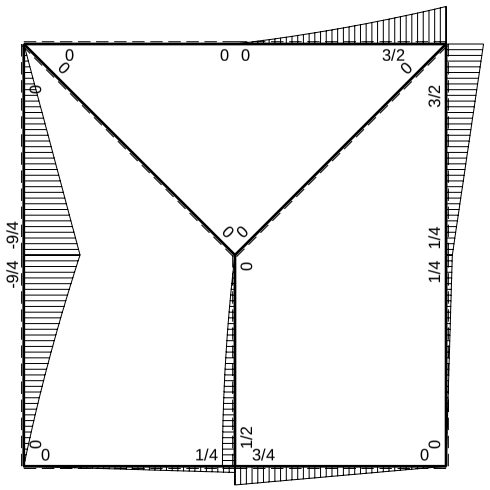
F

F

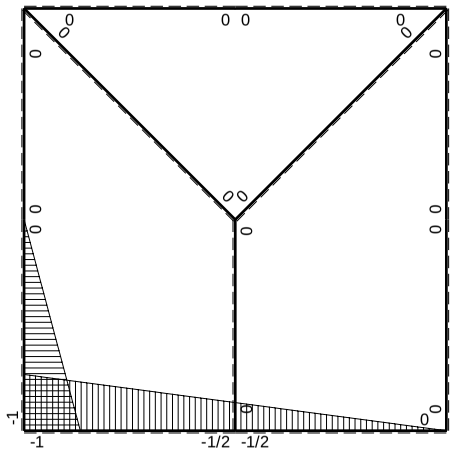

0
0 0
3/2
0
0
3/2
-9/4
0 0
0
-9/4
1/4
1/4
-1/8
3/16
-1/2
11/16
0
0
-1/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	-Fb/EJ	-1/4Fx+1/8Fx^2/b	Fb/EJ-1/2Fx/EJ	1-x/b+1/4x^2/b^2	(-1/12+3/4)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	Fb/EJ	-1/8Fb+1/8Fx^2/b	1/2Fb/EJ+1/2Fx/EJ	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	-3/8Fb+3/4Fx-3/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-1/8+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	-3/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-2Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-Fx-1/2qx^2	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
BI b	0	-1/2Fb+1/2qx^2	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	-9/4Fb+11/4Fx-1/2qx^2	0	9/4Fx-11/4Fx^2/b+1/2qx^3/b	0	x^2/b^2	(1/3+0)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	7/4Fx+1/2qx^2	0	7/4Fx-5/4Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-1/8Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						1/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

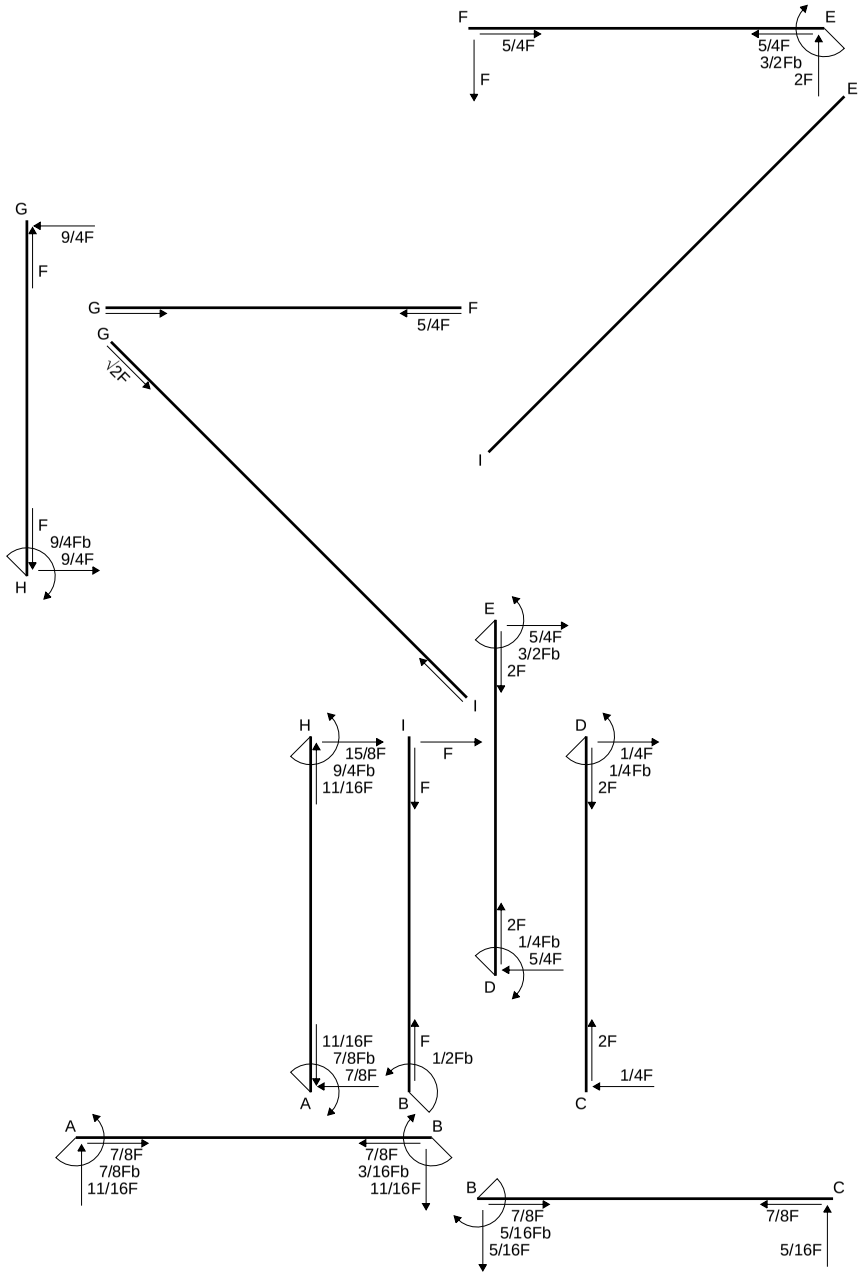
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

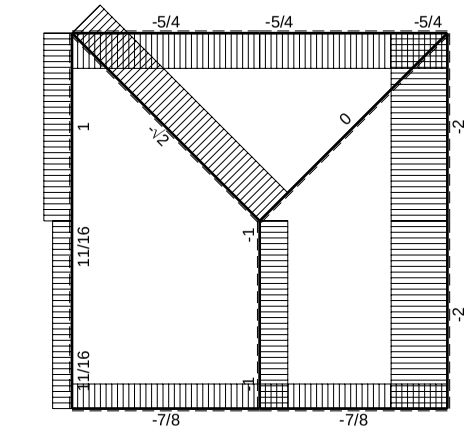
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

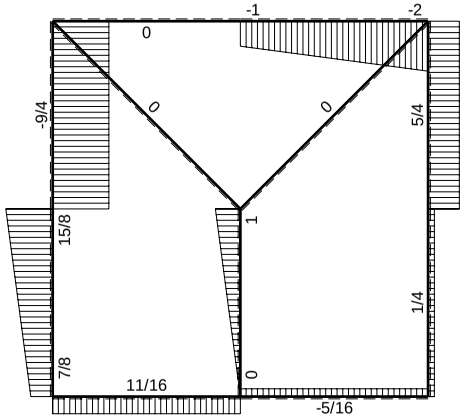
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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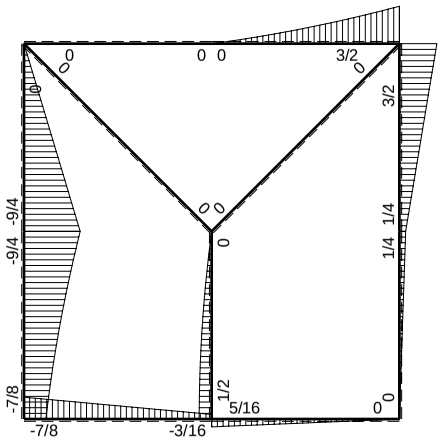






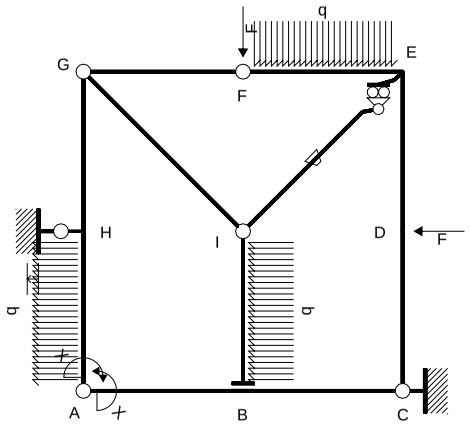
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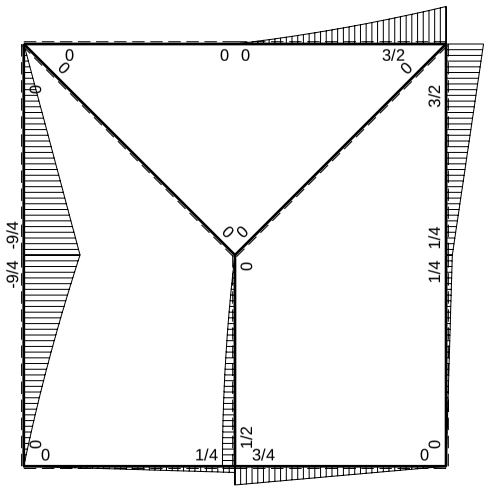
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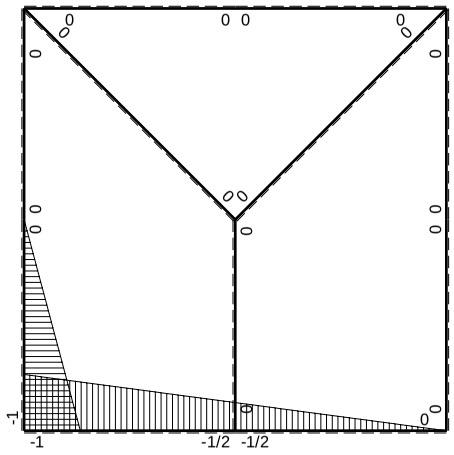
Fb

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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 991200 ESΣ04.xxxx.136

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PROCEDIMENTO E RISULTATI 991200 ESΣ04.xxxx.136

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

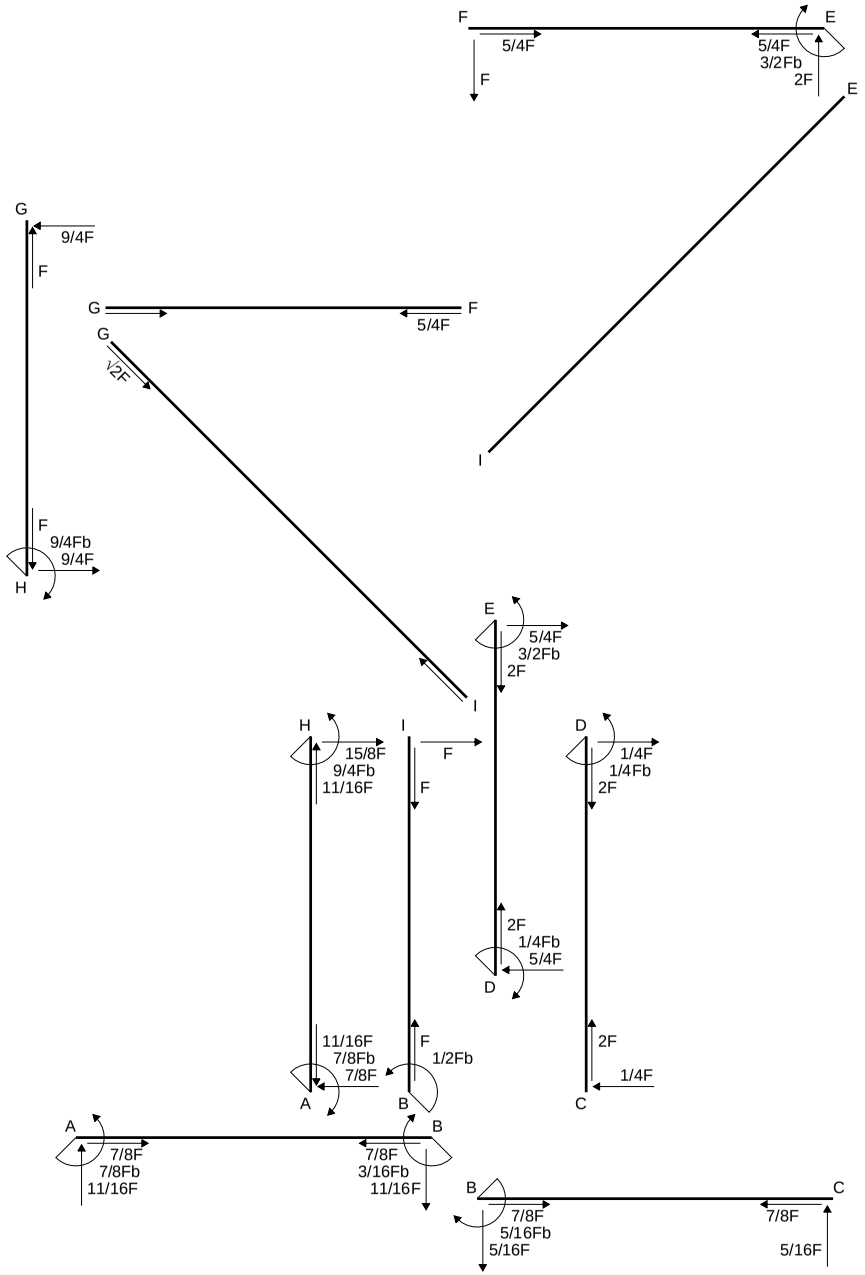
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

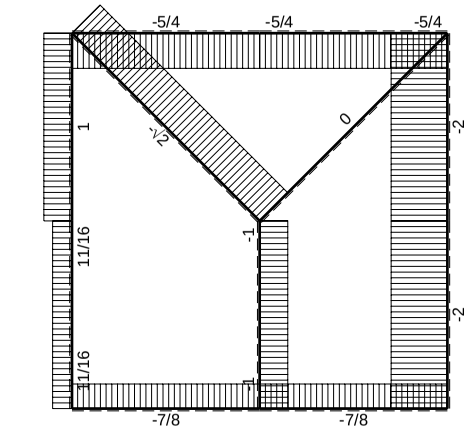
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

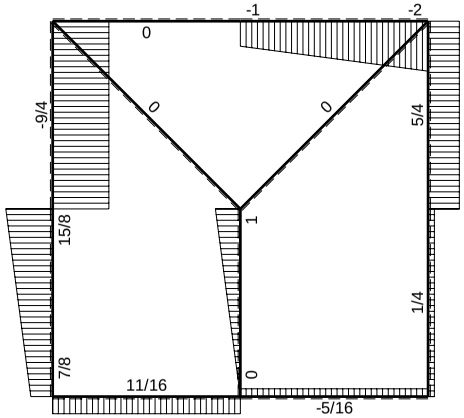
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

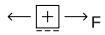
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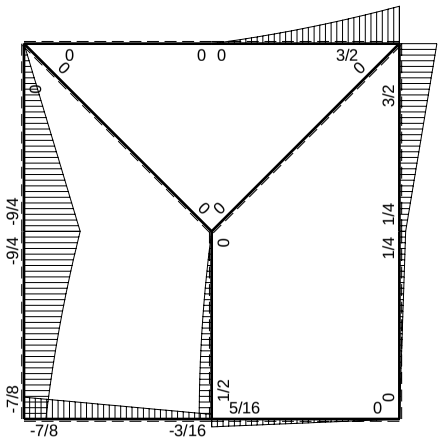






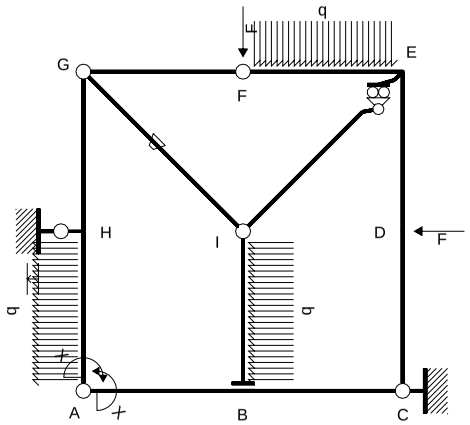


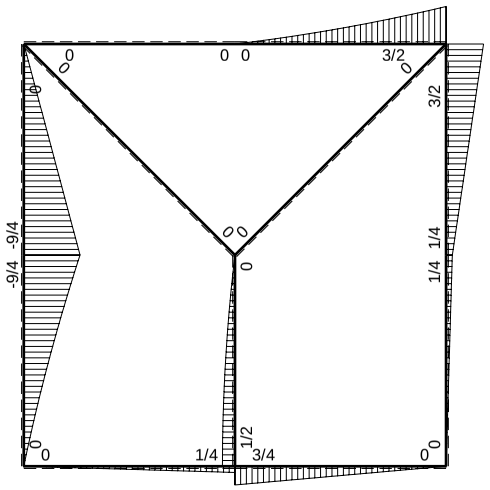




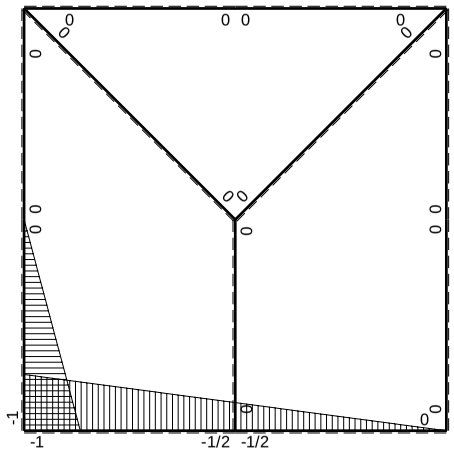


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

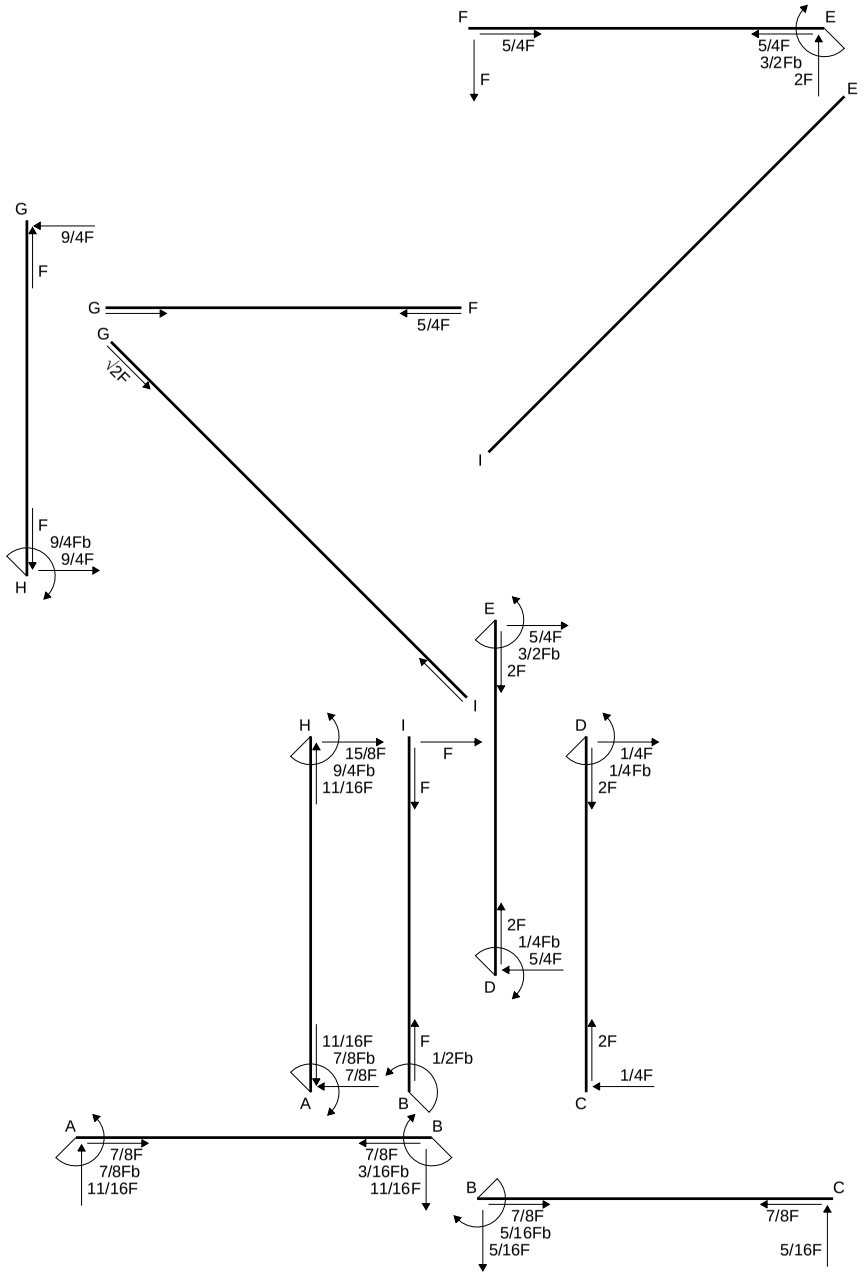
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

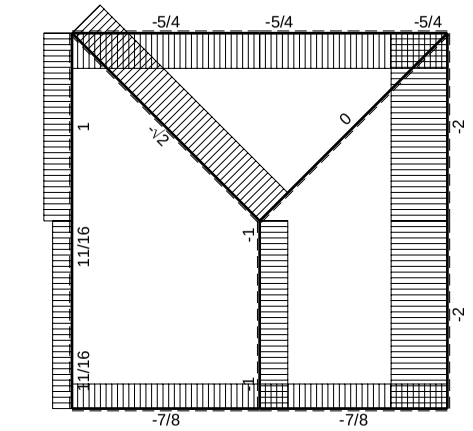
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

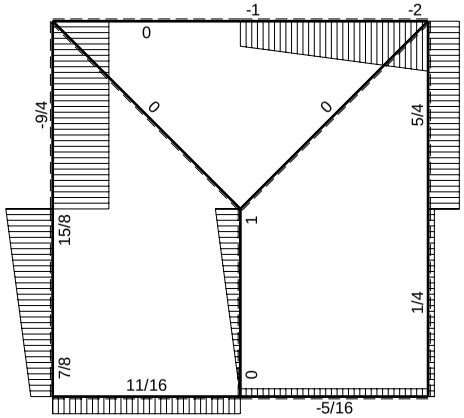
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

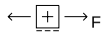
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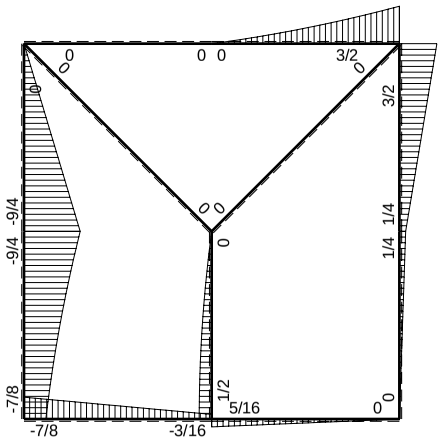






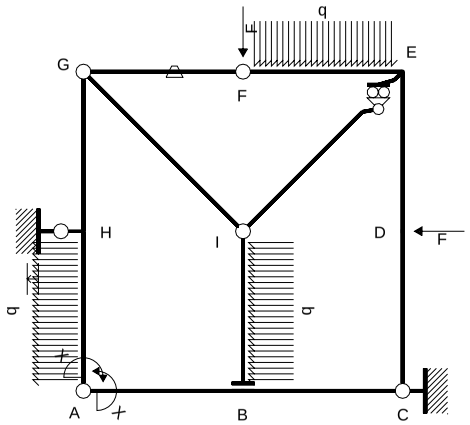


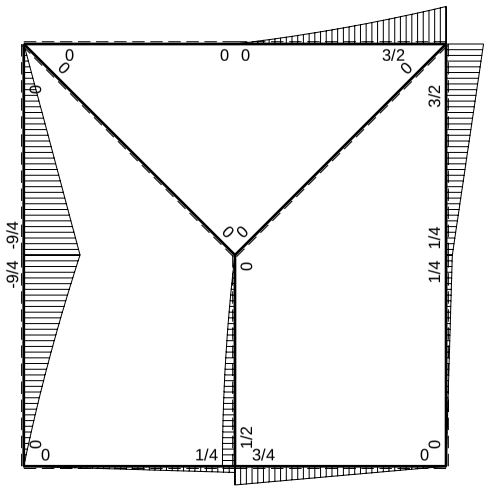




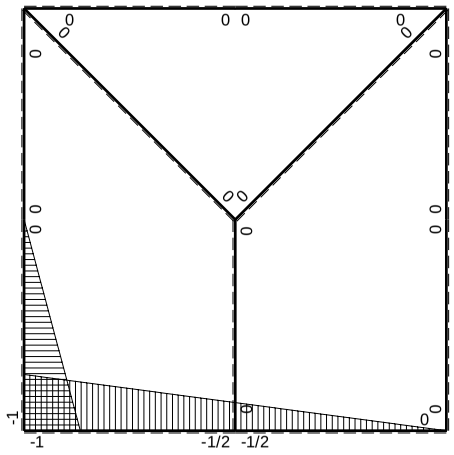


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

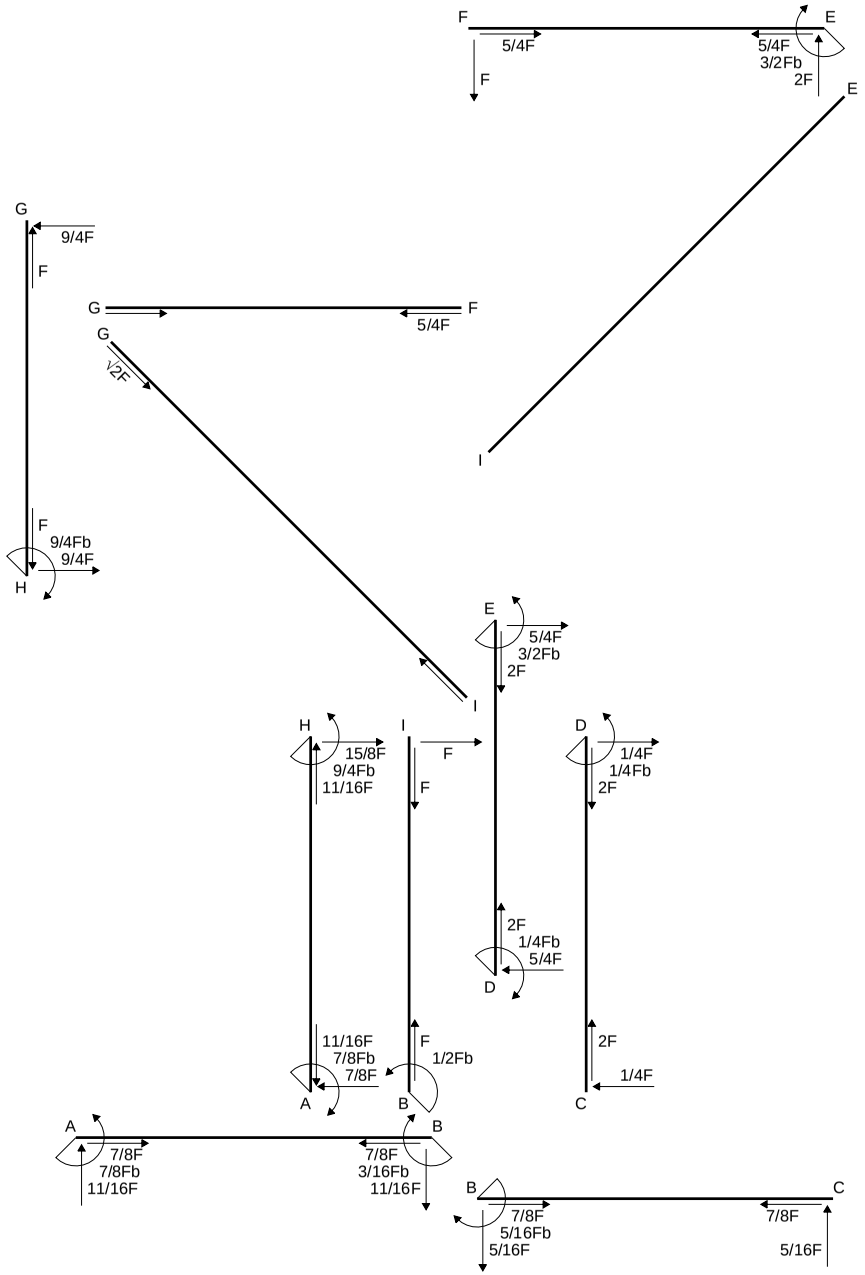
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

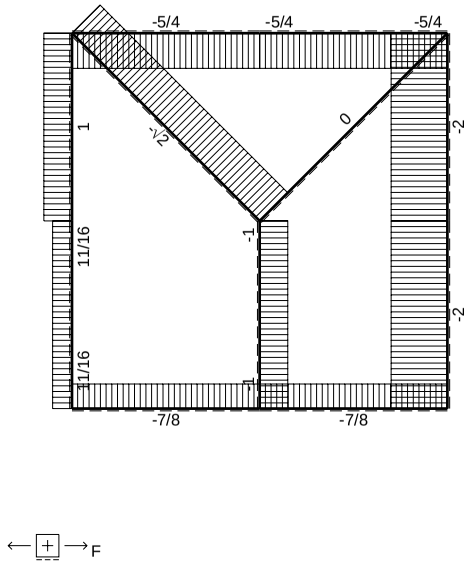
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

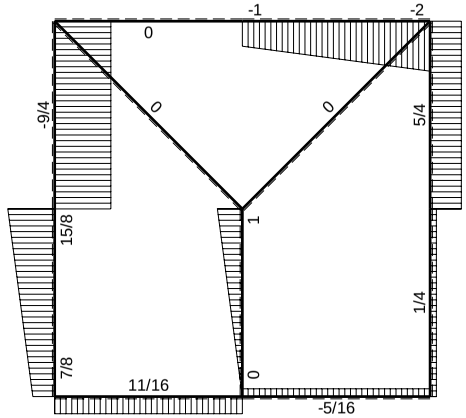
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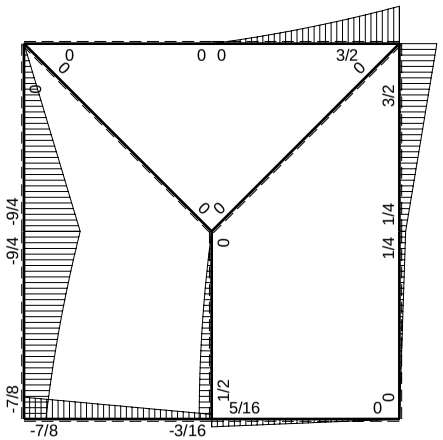




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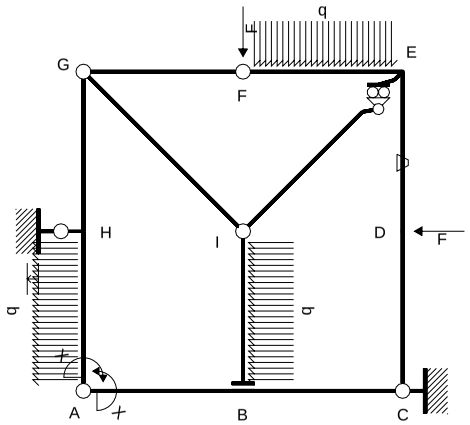


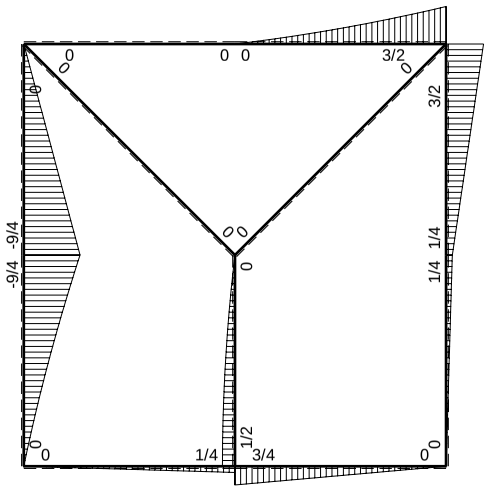
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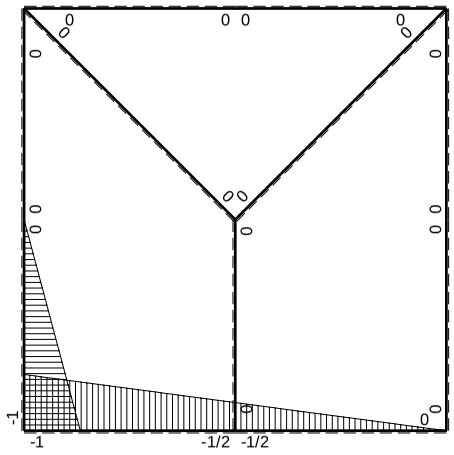
Fb

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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

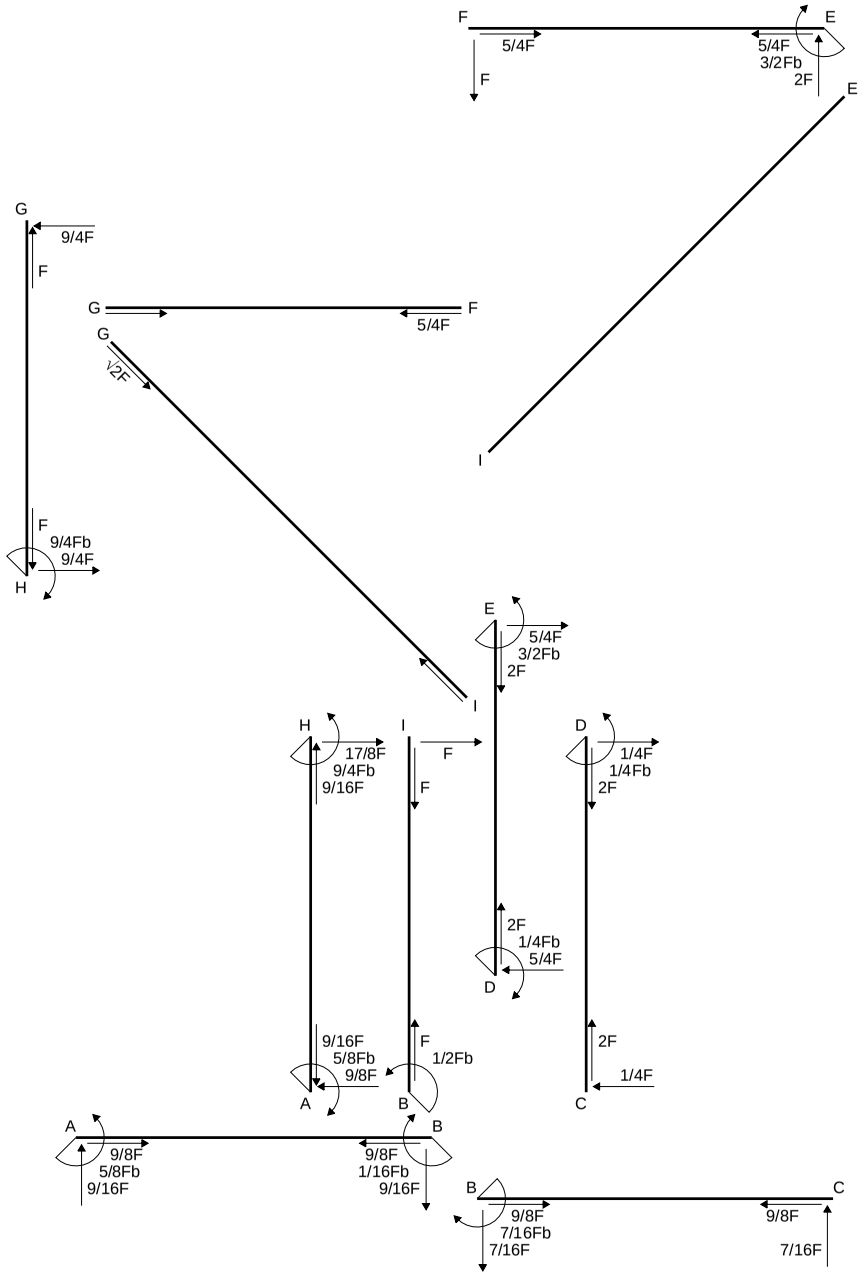
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

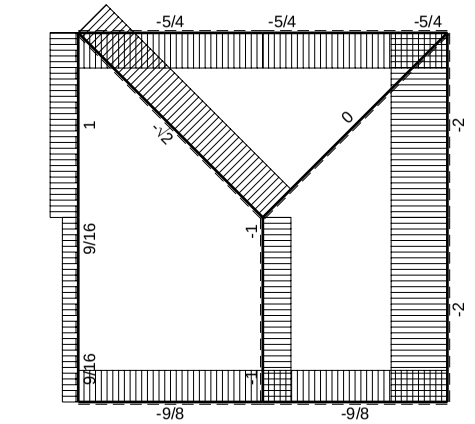
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

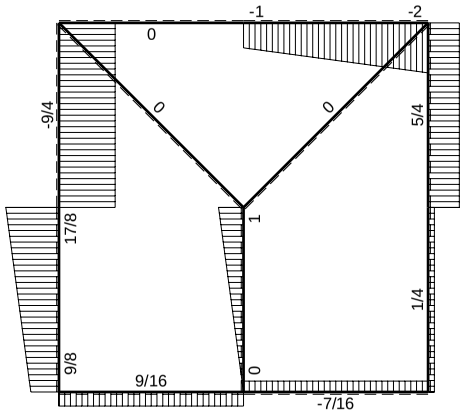
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

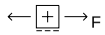
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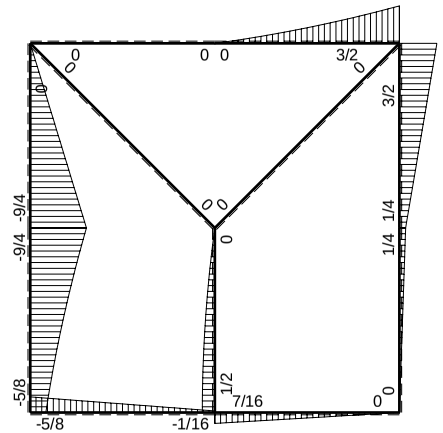

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3/2Fb
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G
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I
H
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B
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C
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7/16F


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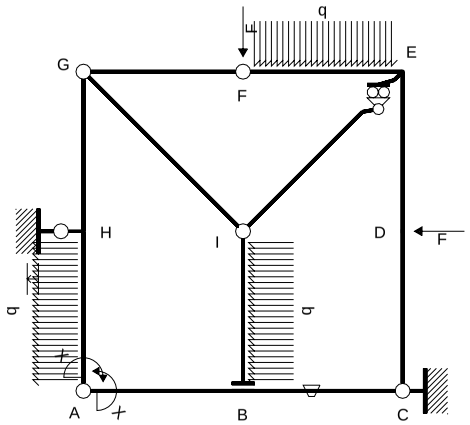

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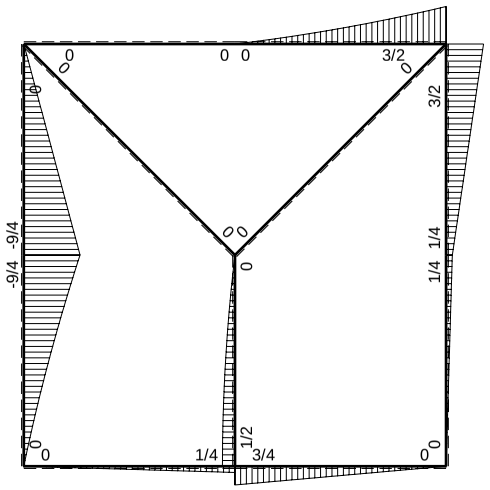

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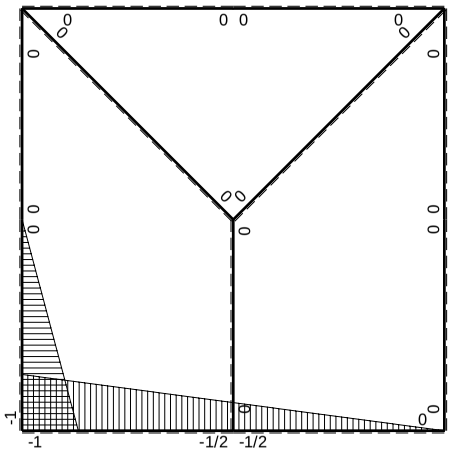

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1/2
7/16
0
0
-5/8
-1/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	-Fb/EJ	$-3/8Fb+3/4Fx-3/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	Fb/EJ	$-3/8Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/8Fb	

PROCEDIMENTO E RISULTATI 982534 ESΣ04.xxxx.140

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\right)\ \theta\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ + \left(-1/4\ b\right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

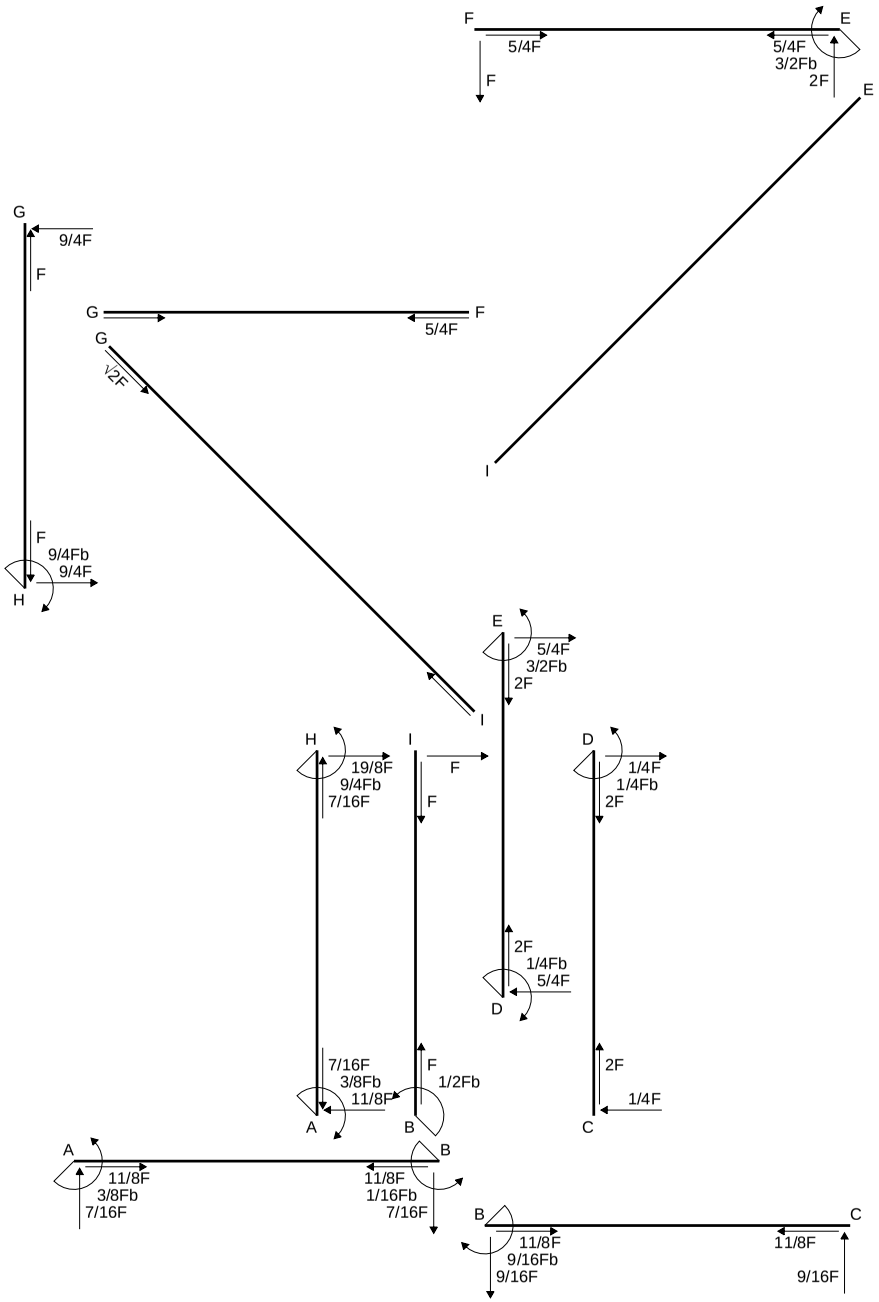
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

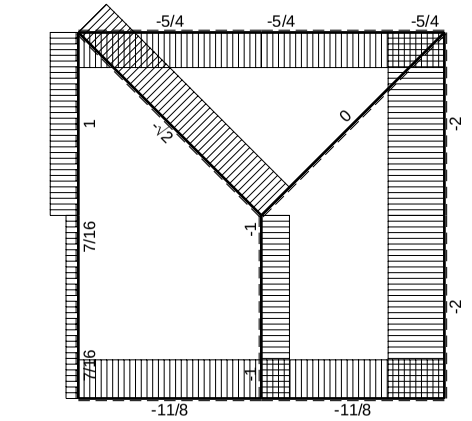
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

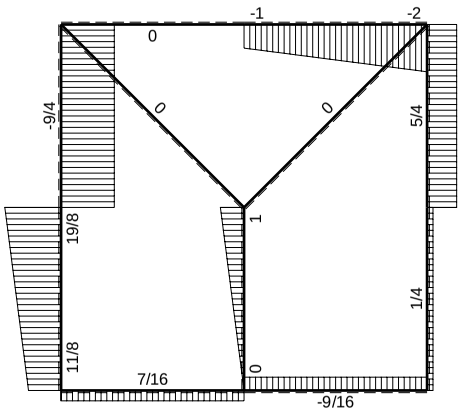
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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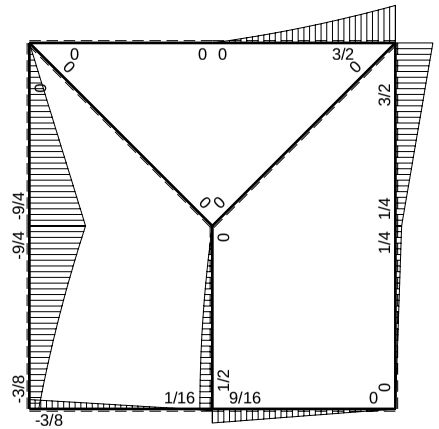

F
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E
5/4F
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2F
F
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G
9/4F
F
G
F
5/4F
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I
F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
2F
I
H
19/8F
9/4Fb
7/16F
I
F
F
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1/4Fb
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C
11/8F
9/16Fb
9/16F
11/8F
9/16F


-5/4
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7/16
-1
-11/8
-11/8

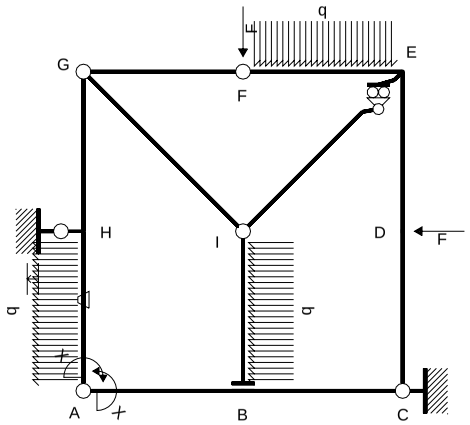

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-9/16

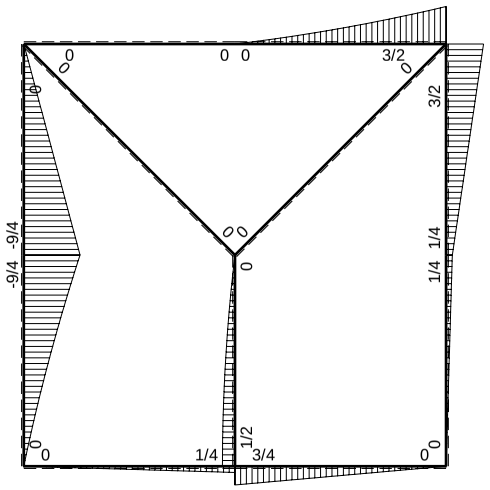
F

F

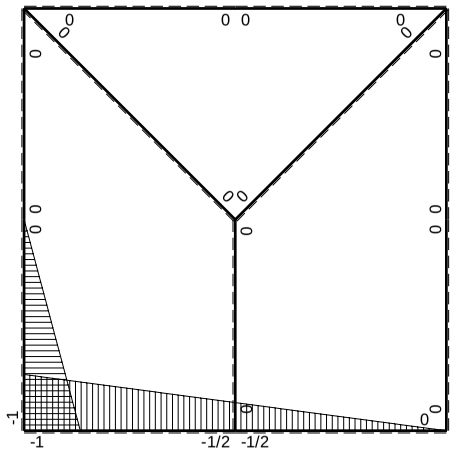

0
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3/2
0
0
3/2
-9/4
0 0
-9/4
1/4
1/4
0
-3/8
1/16
-√2
9/16
0
0
-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-3/4Fx$	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	-Fb/EJ	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/3+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	Fb/EJ	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/8Fb	

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Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\right)\ \theta\ dx$

$= \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

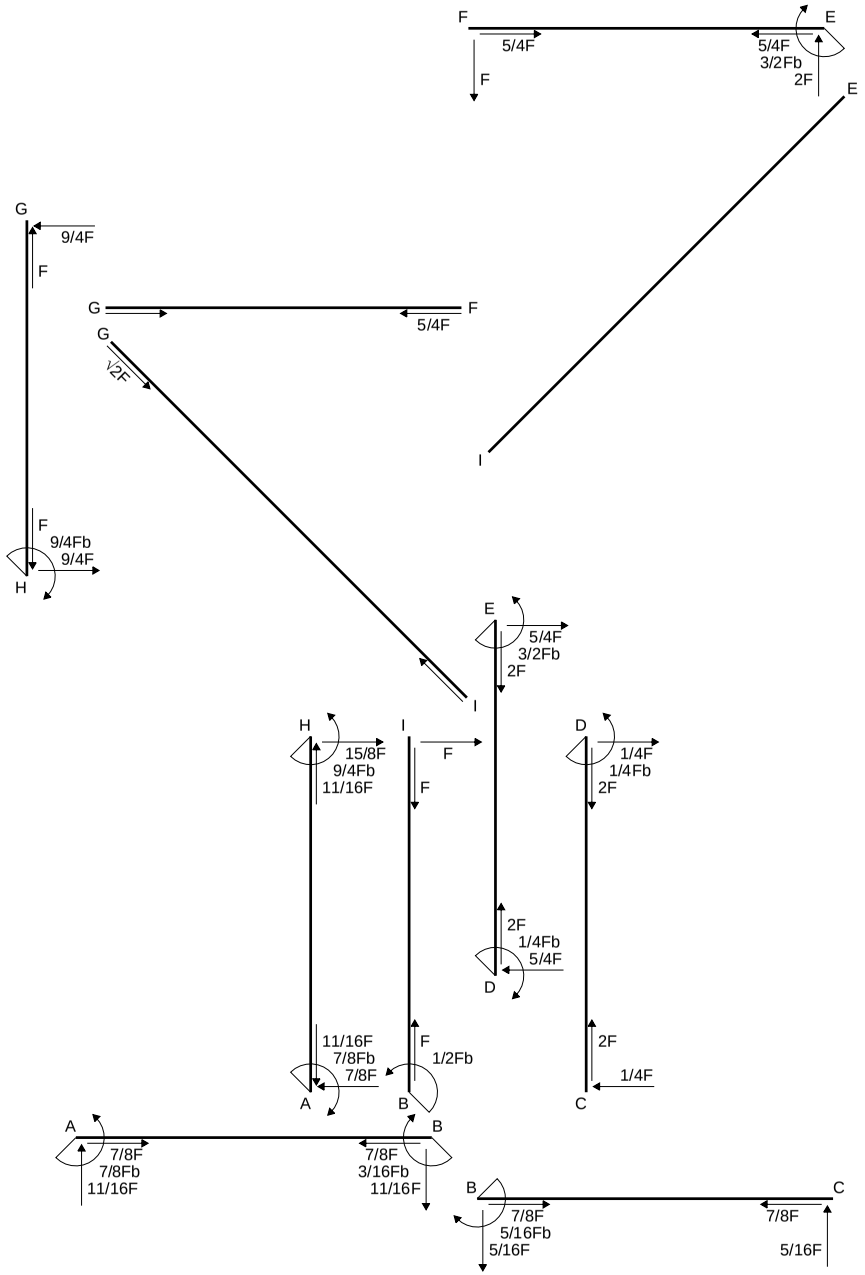
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\right)\ \theta\ dx$

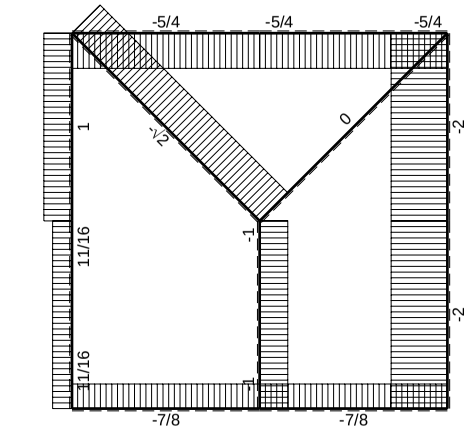
$= \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

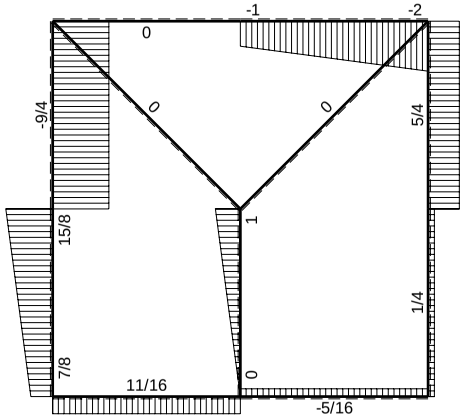
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

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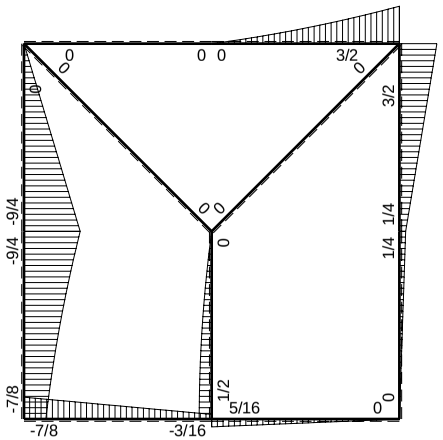

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E
5/4F
3/2Fb
2F
I
H
15/8F
9/4Fb
11/16F
I
F
F
D
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1/4Fb
2F
2F
1/4Fb
5/4F
D
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B
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B
C
7/8F
5/16Fb
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7/8F
5/16F


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-7/8

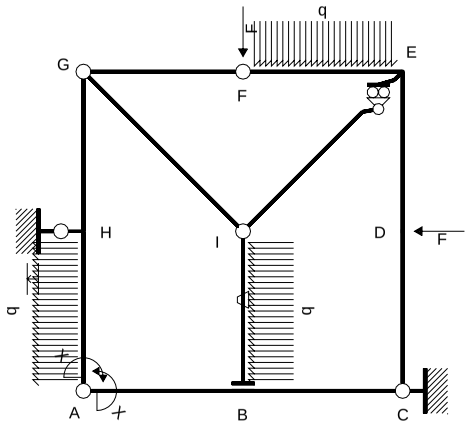

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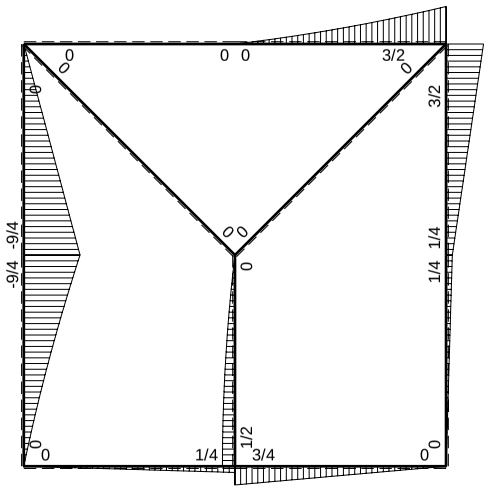
←⊞→ F

↑⊞↓ F

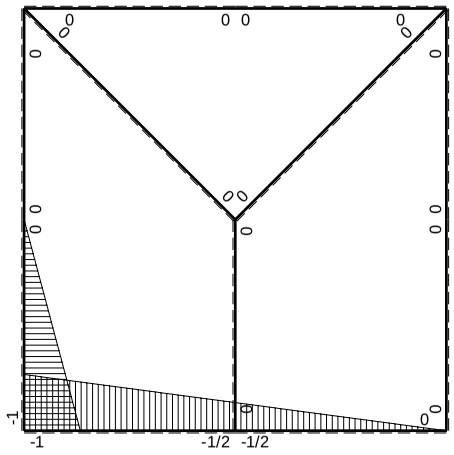

0
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-9/4
1/4
1/4
0
-7/8
1/2
5/16
0
0
-7/8
-3/16

↻⊞↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238186 ESΣ04.xxxx.142

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PROCEDIMENTO E RISULTATI 238186 ESΣ04.xxxx.142

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x\theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

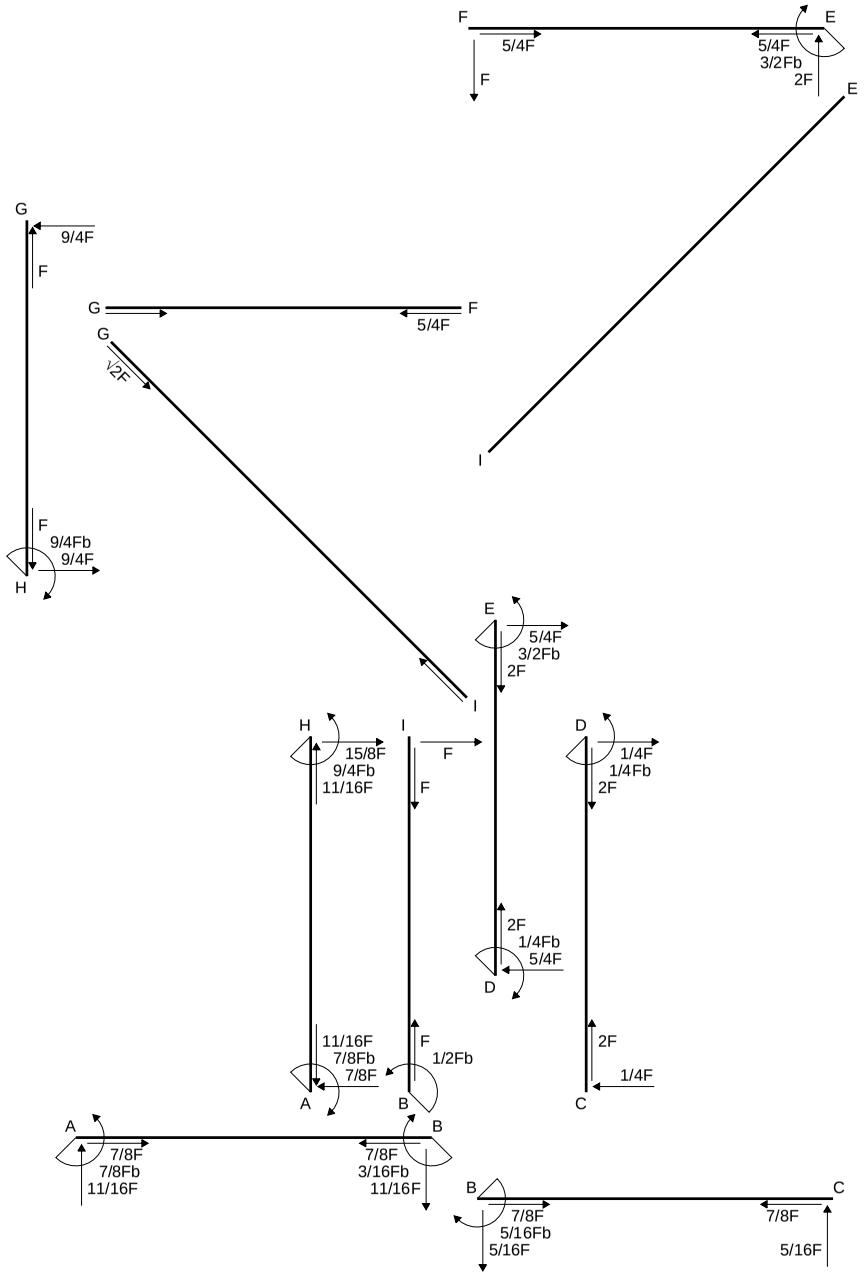
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

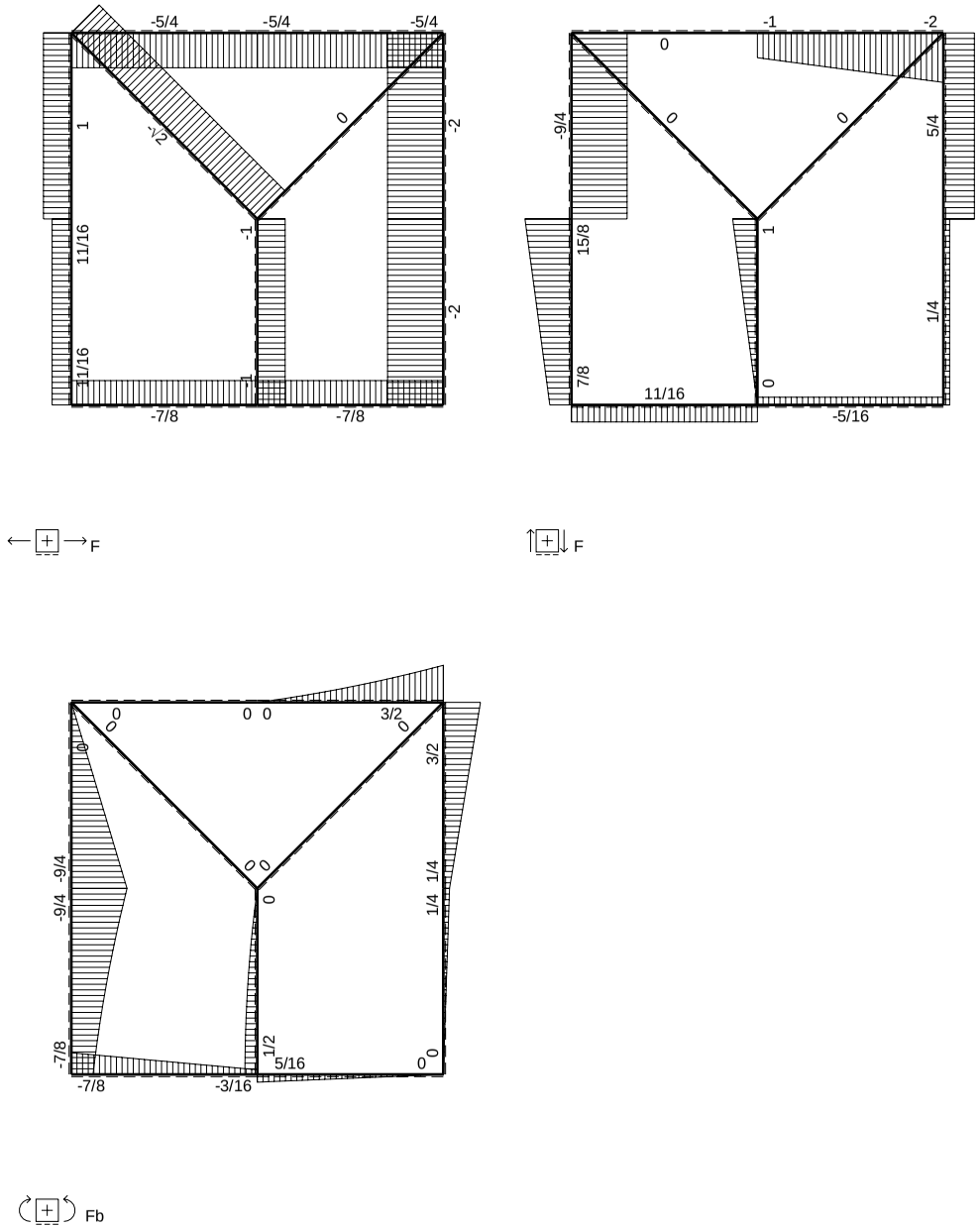
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

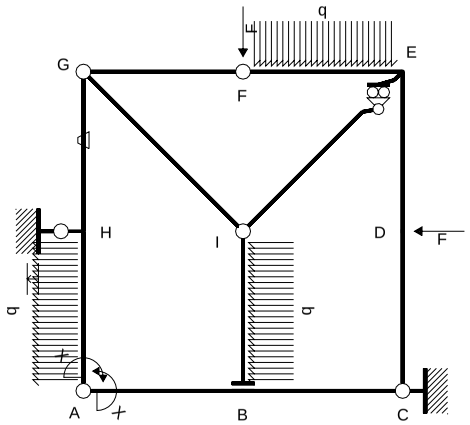
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

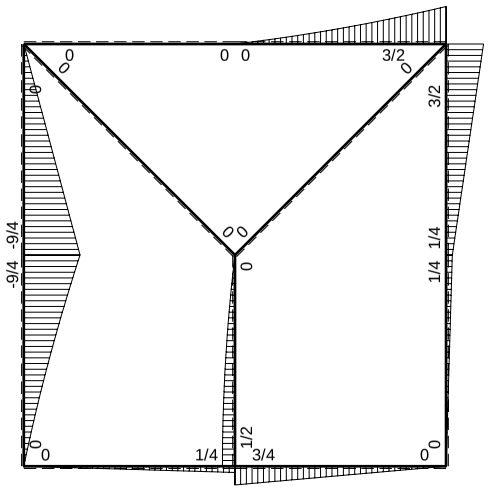
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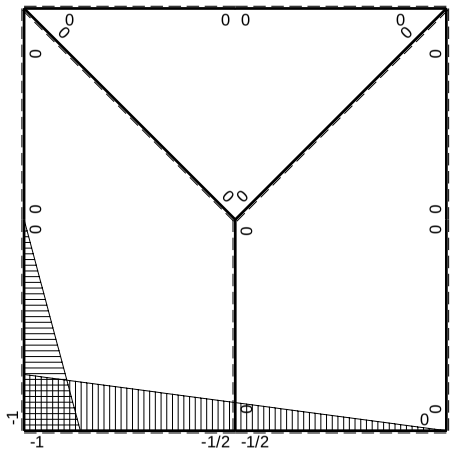

F
5/4F
E
5/4F
3/2Fb
2F
F
E
G
9/4F
F
G
F
5/4F
G
½F
I
F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
2F
I
H
15/8F
9/4Fb
11/16F
I
F
F
D
1/4F
1/4Fb
2F
2F
1/4Fb
5/4F
D
11/16F
7/8Fb
7/8F
F
1/2Fb
A
B
2F
1/4F
C
A
B
7/8F
7/8Fb
11/16F
7/8F
3/16Fb
11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
-5/4
-5/4
1
½
0
-2
-1
11/16
-2
11/16
-1
-7/8
-7/8
F
-1
-2
0
-9/4
0
0
5/4
15/8
1
1/4
7/8
11/16
0
-5/16
F
0
0 0
3/2
0
0
3/2
-9/4
0
-9/4
1/4
1/4
0
-7/8
1/2
5/16
0
-7/8
-3/16
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237118 ESΣ04.xxxx.143

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

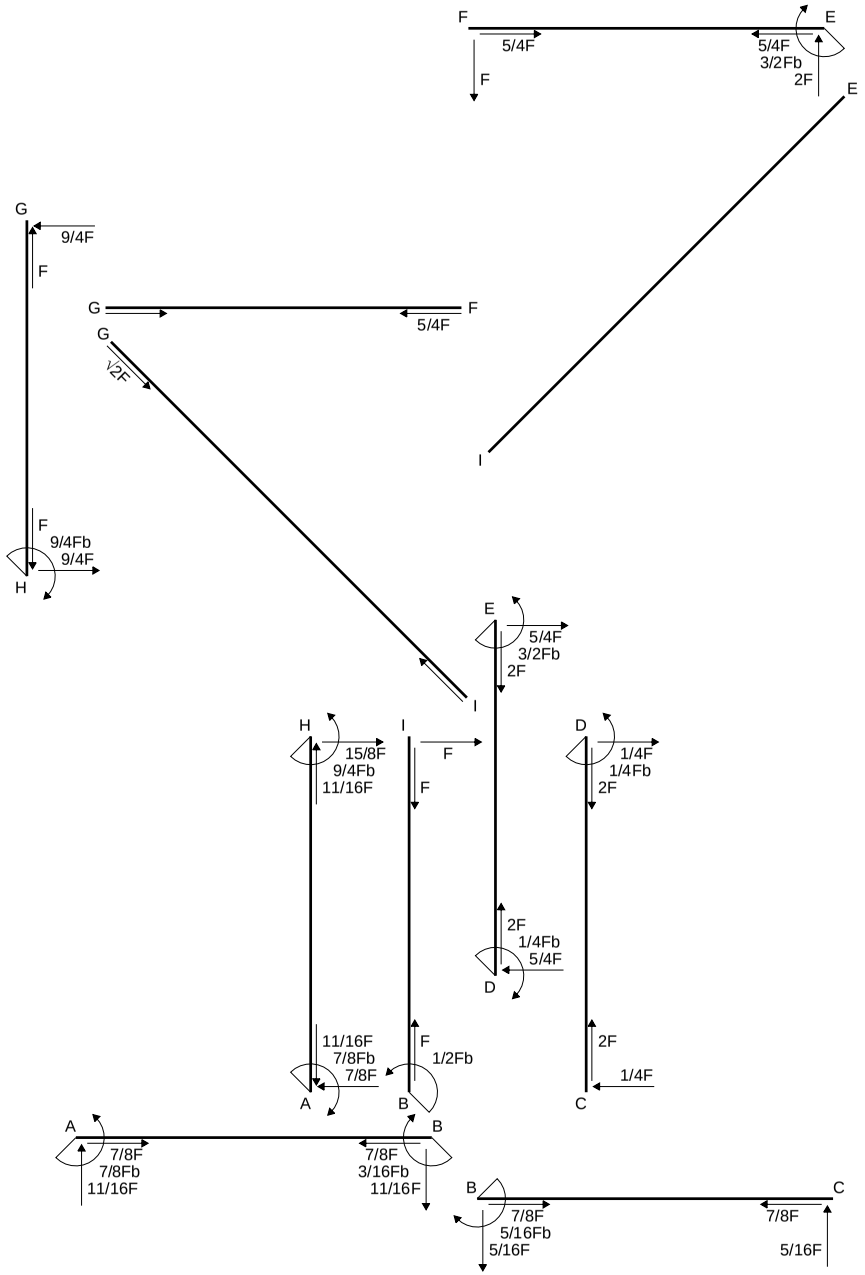
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

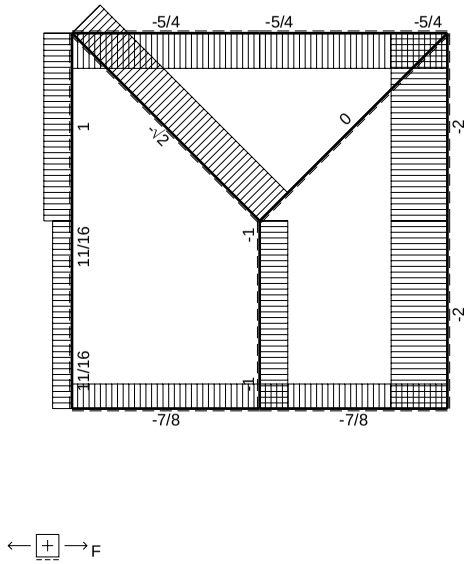
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

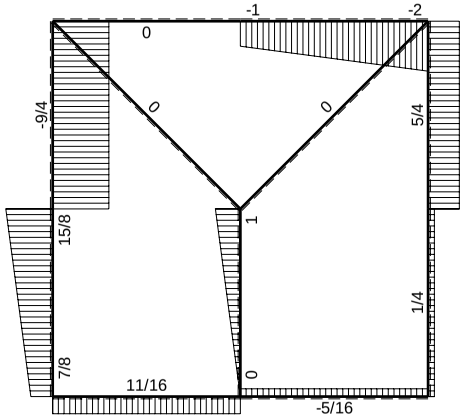
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

ESΣ04.xxxx.143

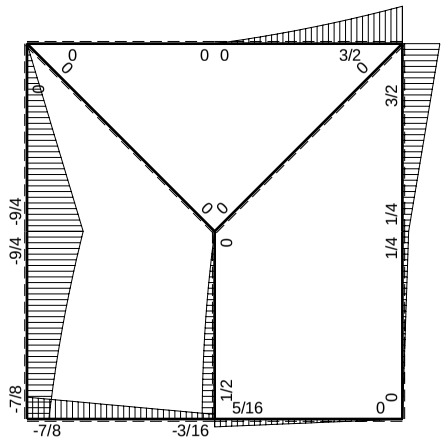
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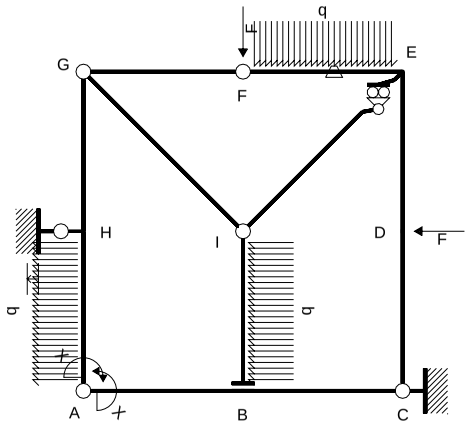


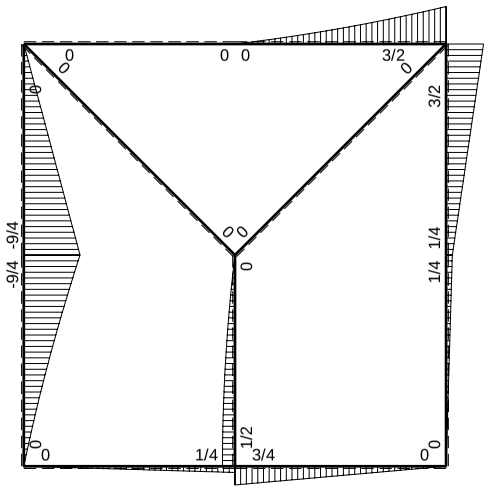




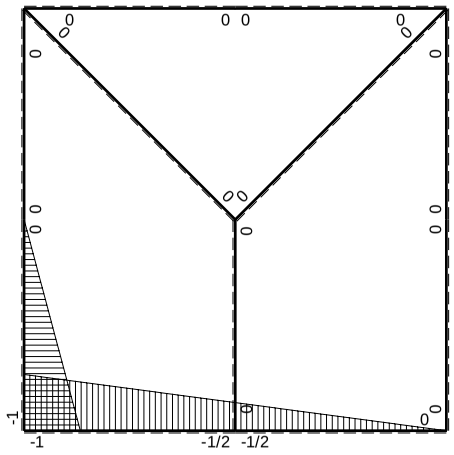


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 241258 ESΣ04.xxxx.144

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

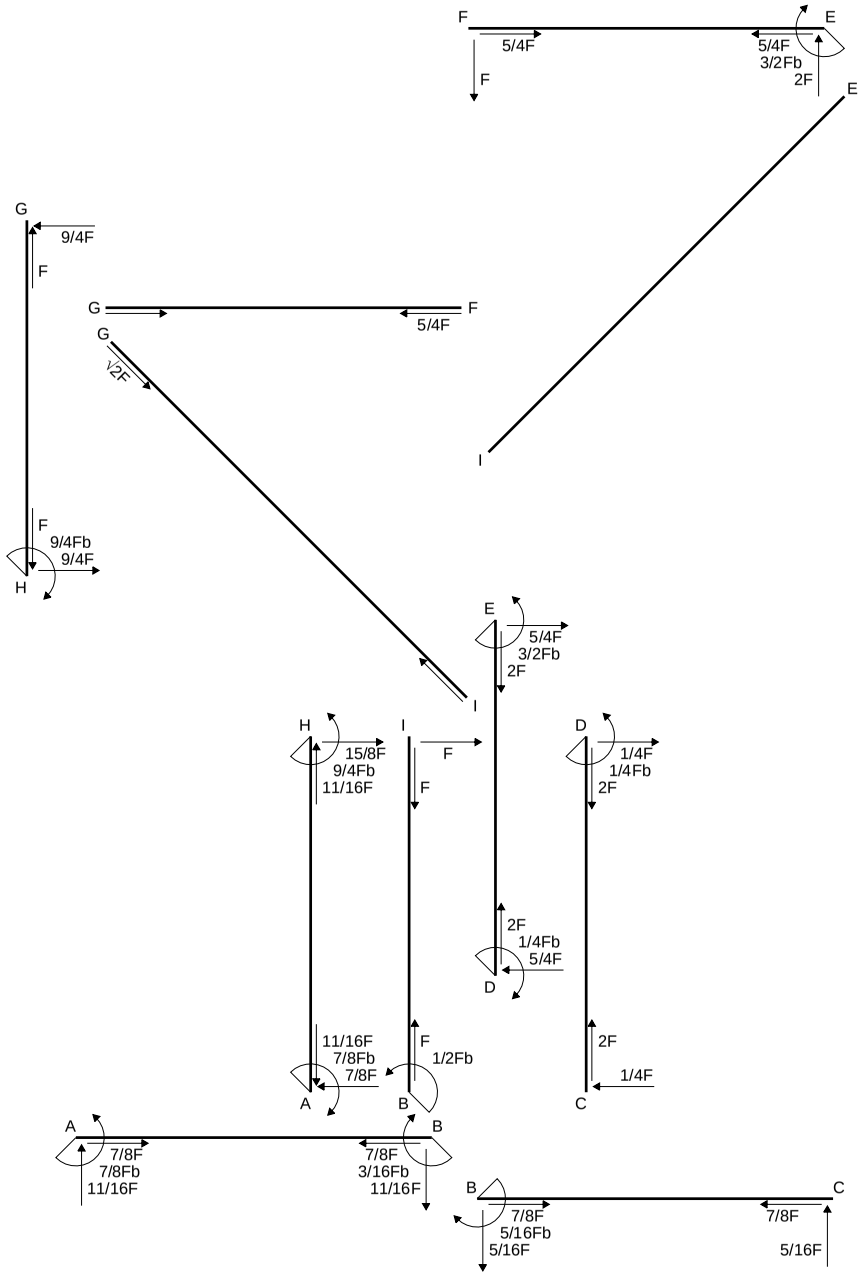
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

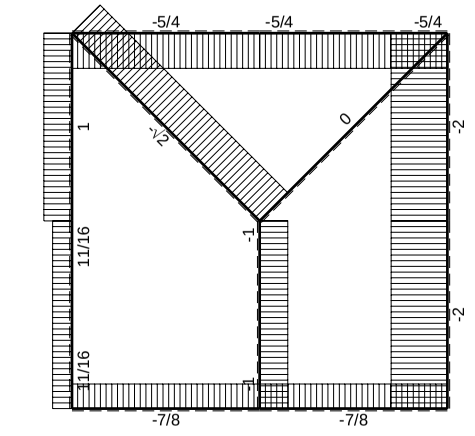
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

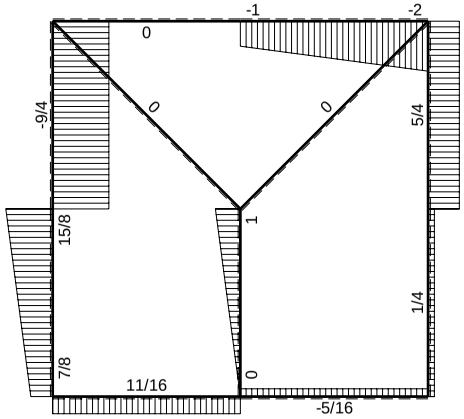
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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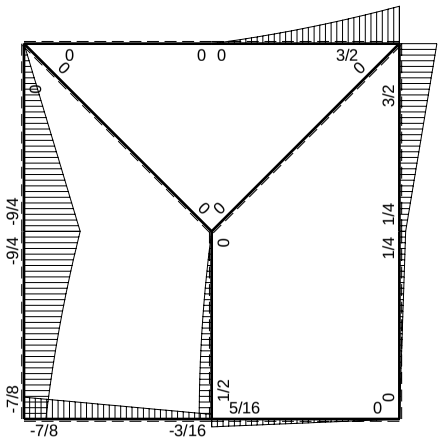

F
5/4F
E
5/4F
3/2Fb
2F
F
E
G
9/4F
F
G
F
5/4F
G
½F
I
F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
2F
I
H
15/8F
9/4Fb
11/16F
I
F
F
D
1/4F
1/4Fb
2F
2F
1/4Fb
5/4F
D
11/16F
7/8Fb
7/8F
F
1/2Fb
2F
1/4F
A
B
C
A
B
7/8F
7/8Fb
11/16F
7/8F
3/16Fb
11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
-5/4
-5/4
1
½
0
-2
-1
11/16
-2
11/16
-1
-7/8
-7/8

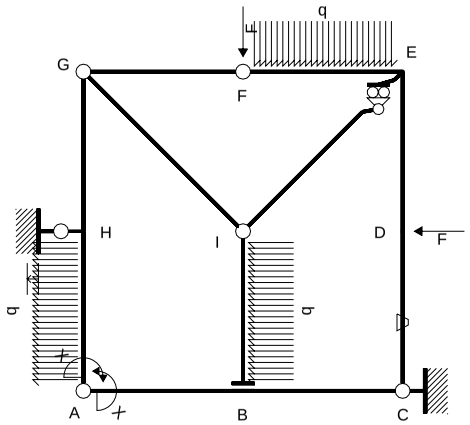

-1
-2
0
-9/4
0
0
5/4
15/8
1
1/4
7/8
0
11/16
-5/16

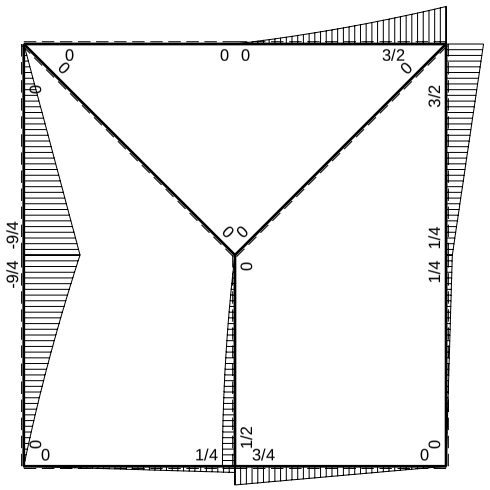
←+→ F

↑+↓ F

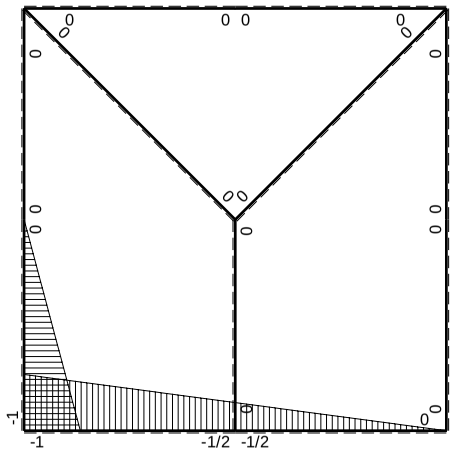

0
0 0
3/2
0
0
3/2
0
0
-9/4
-9/4
0
1/4
1/4
-9/4
0
-7/8
1/2
5/16
0
0
-7/8
-3/16

↻+↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

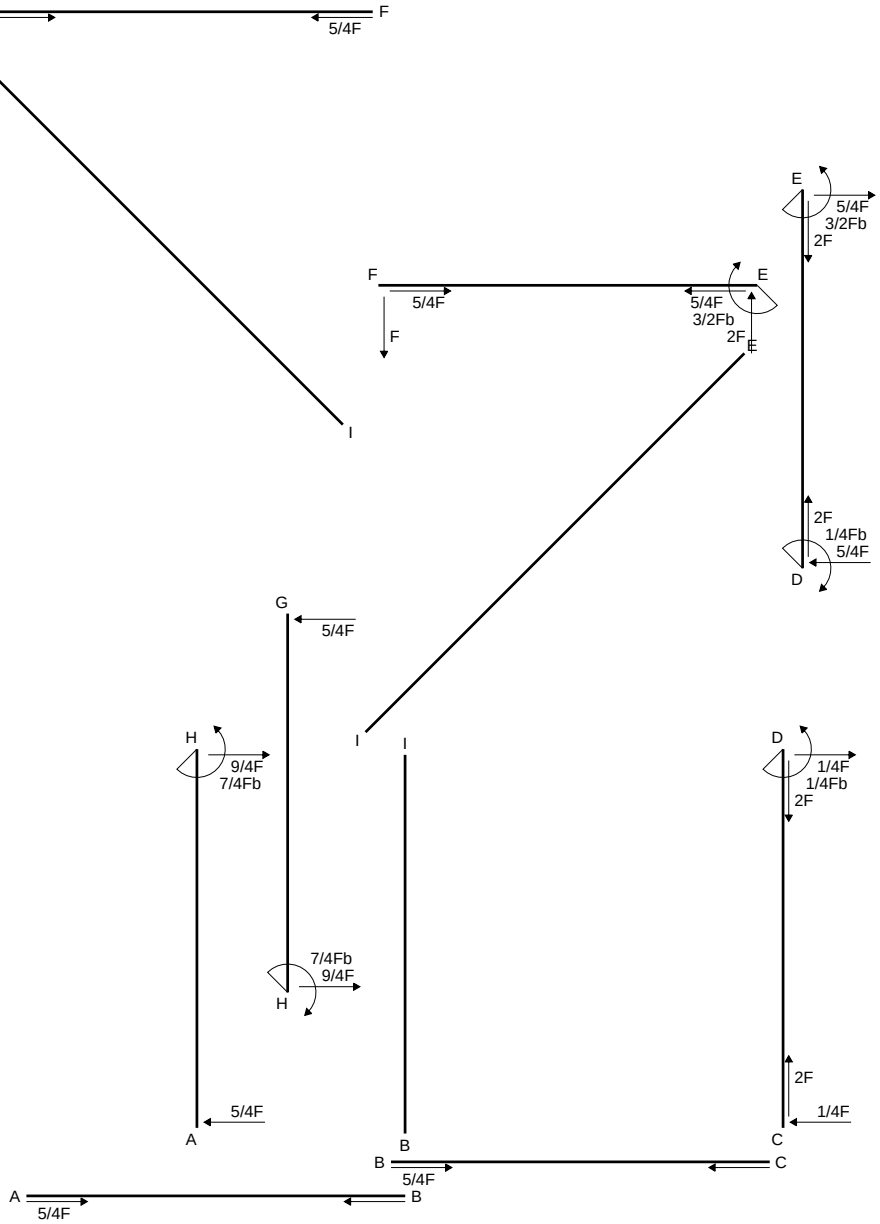
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

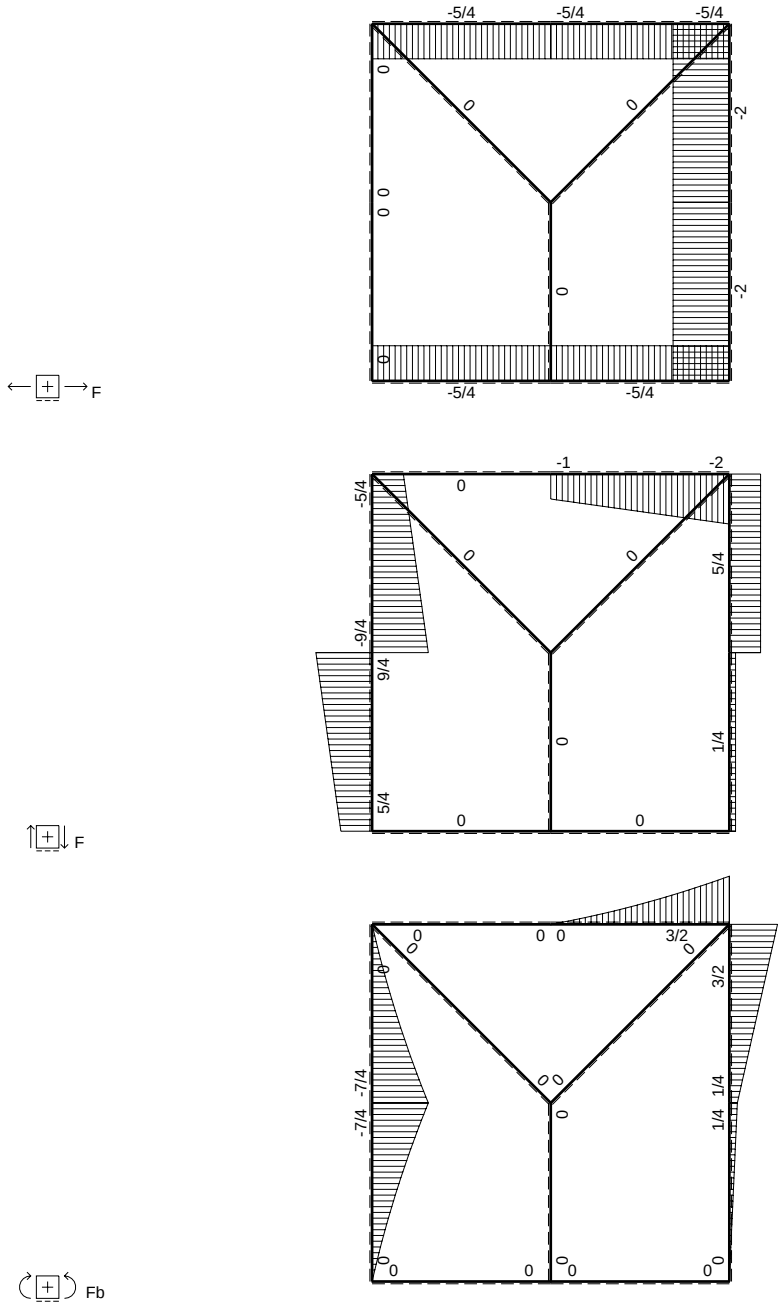
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

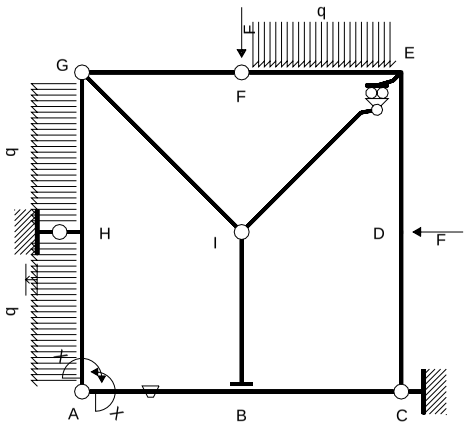
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

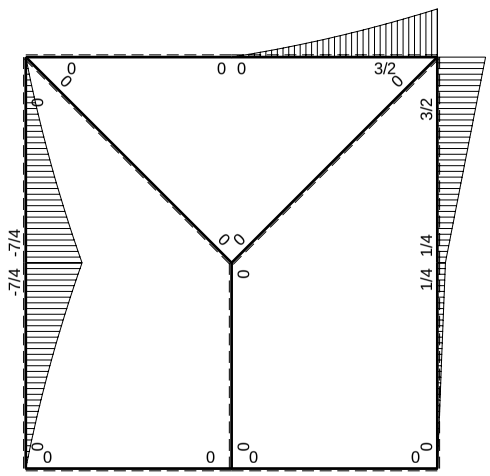
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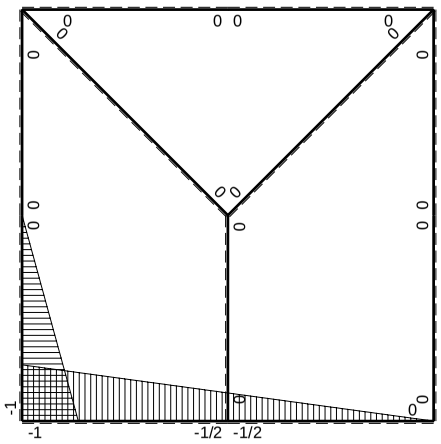

G
F
5/4F
G
I
E
5/4F
3/2Fb
2F
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D
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2F
7/4Fb
9/4F
H
2F
1/4F
5/4F
A
B
C
B
C
5/4F
A
B
5/4F


F
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-5/4
-5/4
-2
-2
-5/4
-5/4
F
-1
-2
-5/4
5/4
-9/4
9/4
1/4
5/4
Fb
3/2
3/2
-7/4
-7/4
1/4
1/4





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	-Fb/EJ	0	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(0+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	0	Fb/EJ	0	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						0	Xb/EJ
	iperstatica $X=W_{AB}$						0	

Sviluppi di calcolo iperstatica

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ESΣ04.xxxx.146

$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\ \right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(1 - 1/2\ x/b\ \right)\theta\ \ dx = \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$$

$$= \left(\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$$

$$= \left(-1/2\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$$

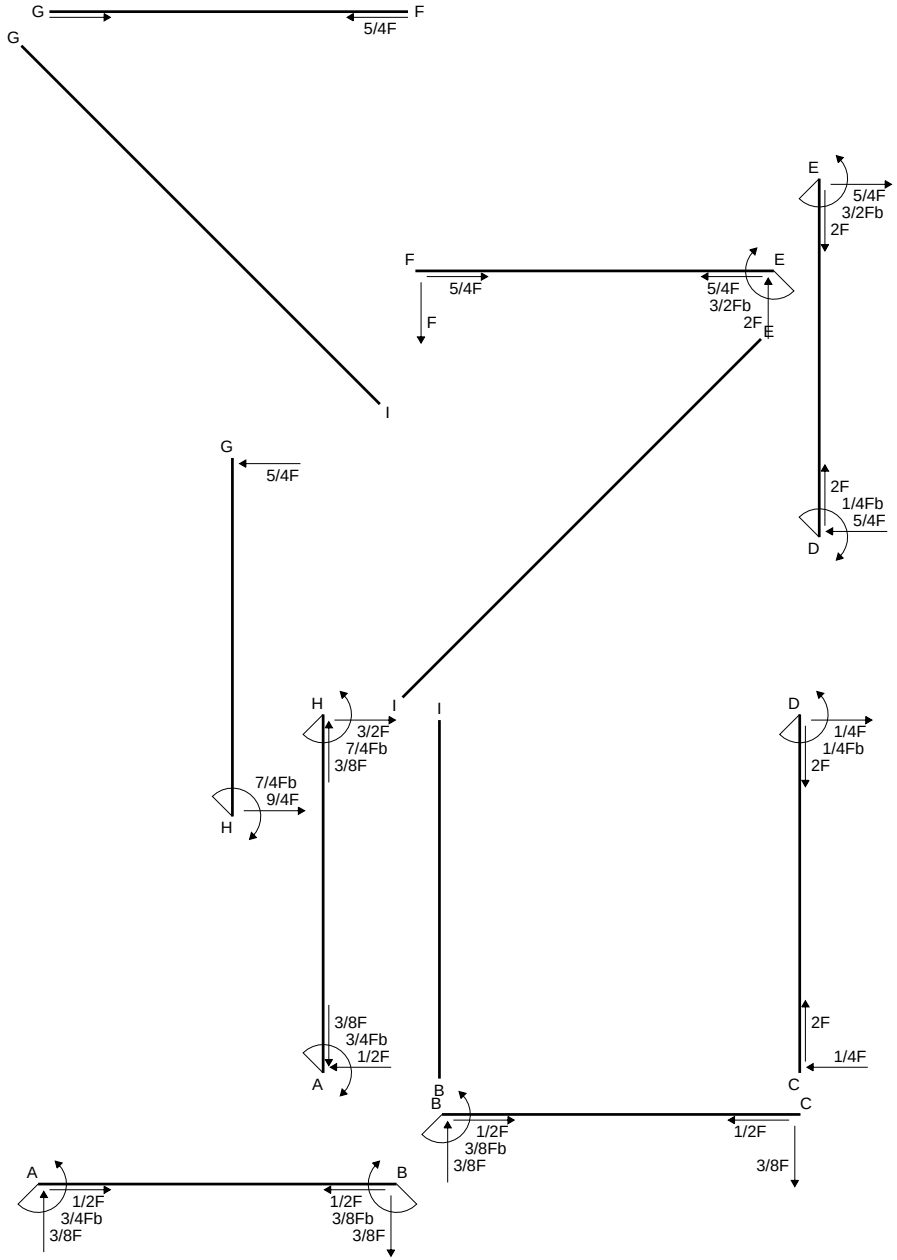
$$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

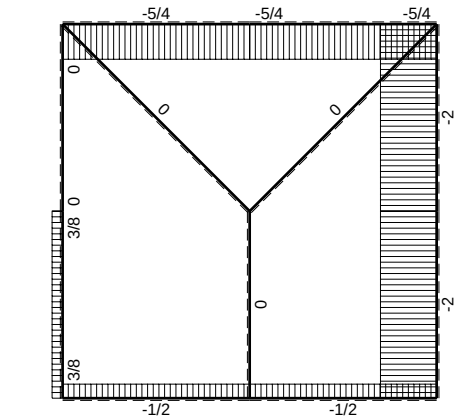
$$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$$

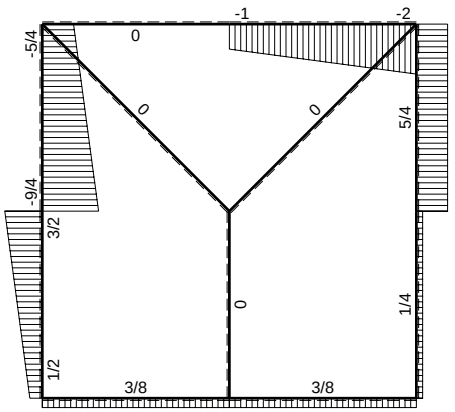
$$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$$

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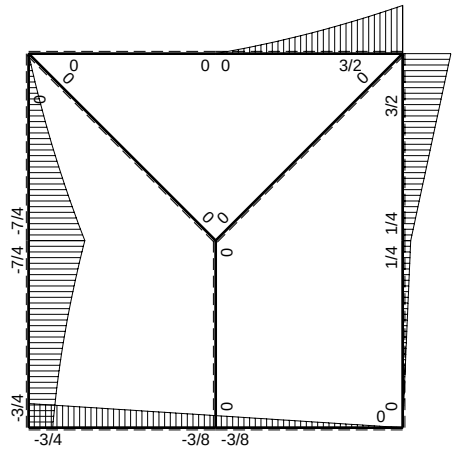

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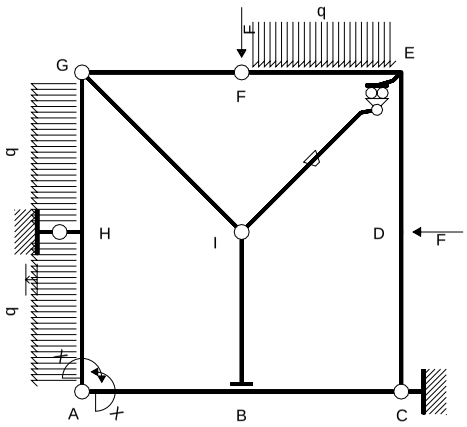

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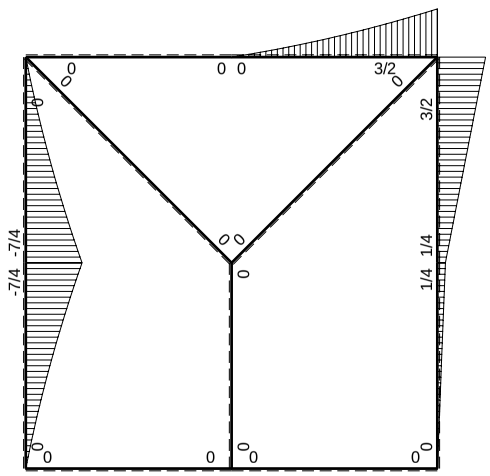
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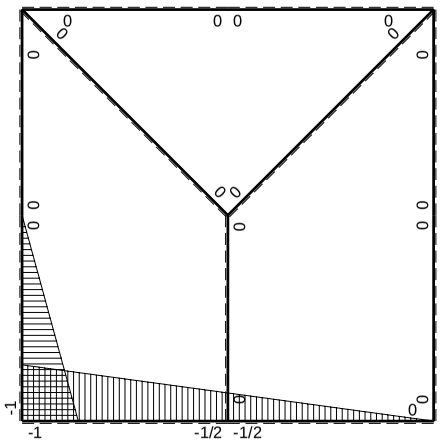

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212317 ESΣ04.xxxx.147

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

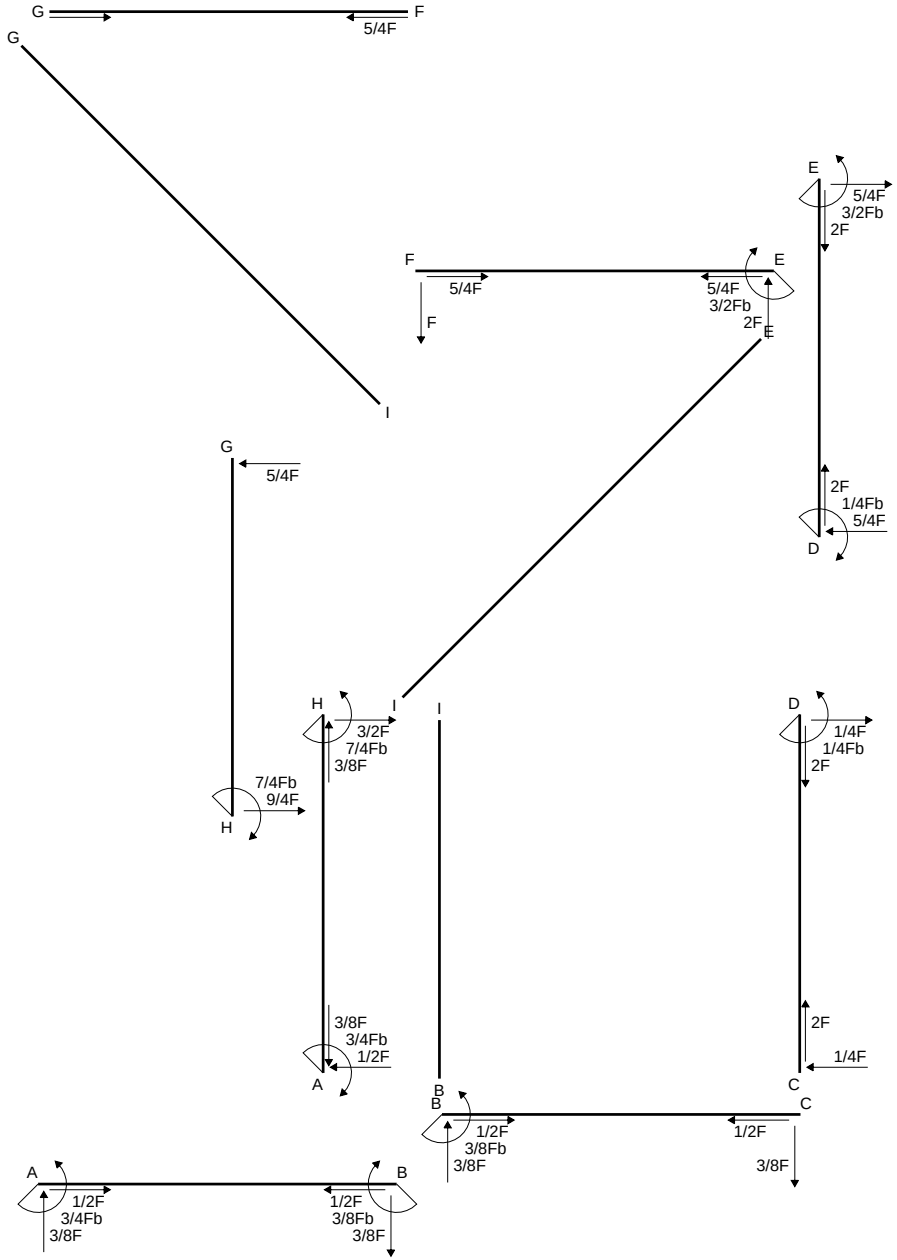
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

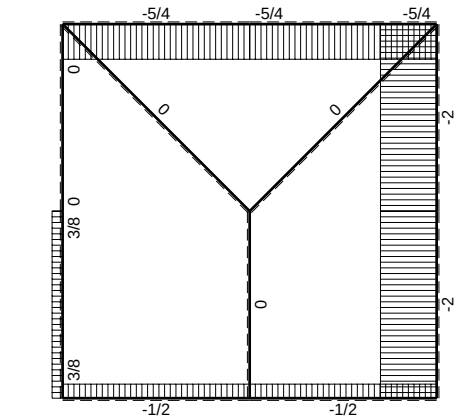
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

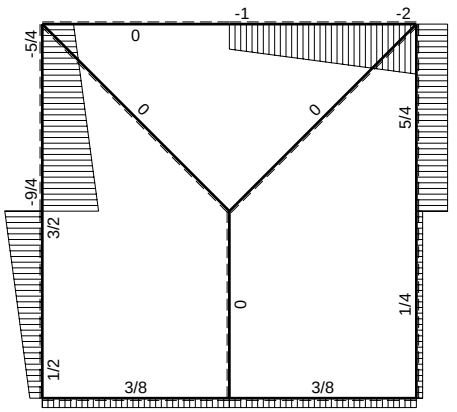
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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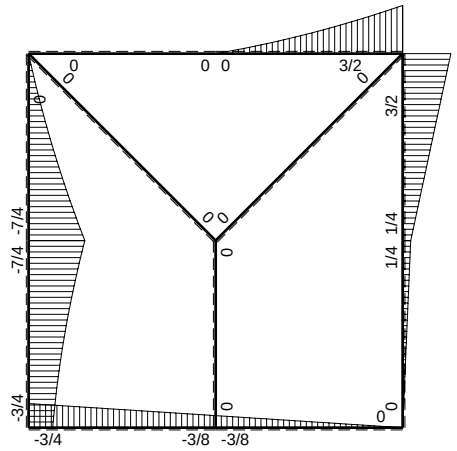

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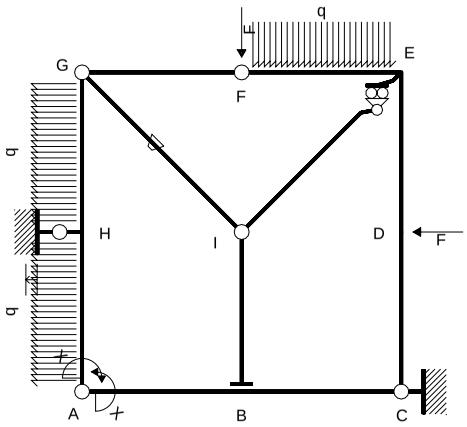

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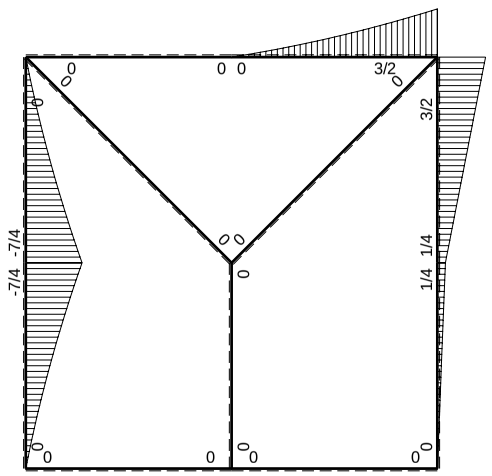
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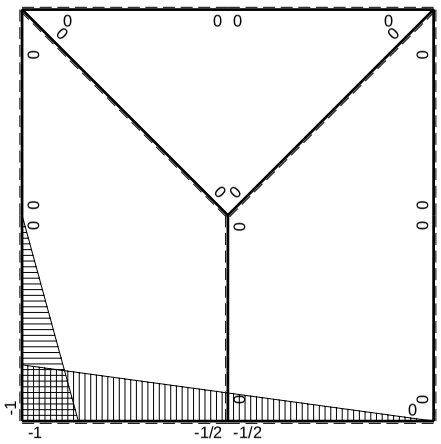

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236719 ESΣ04.xxxx.148

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

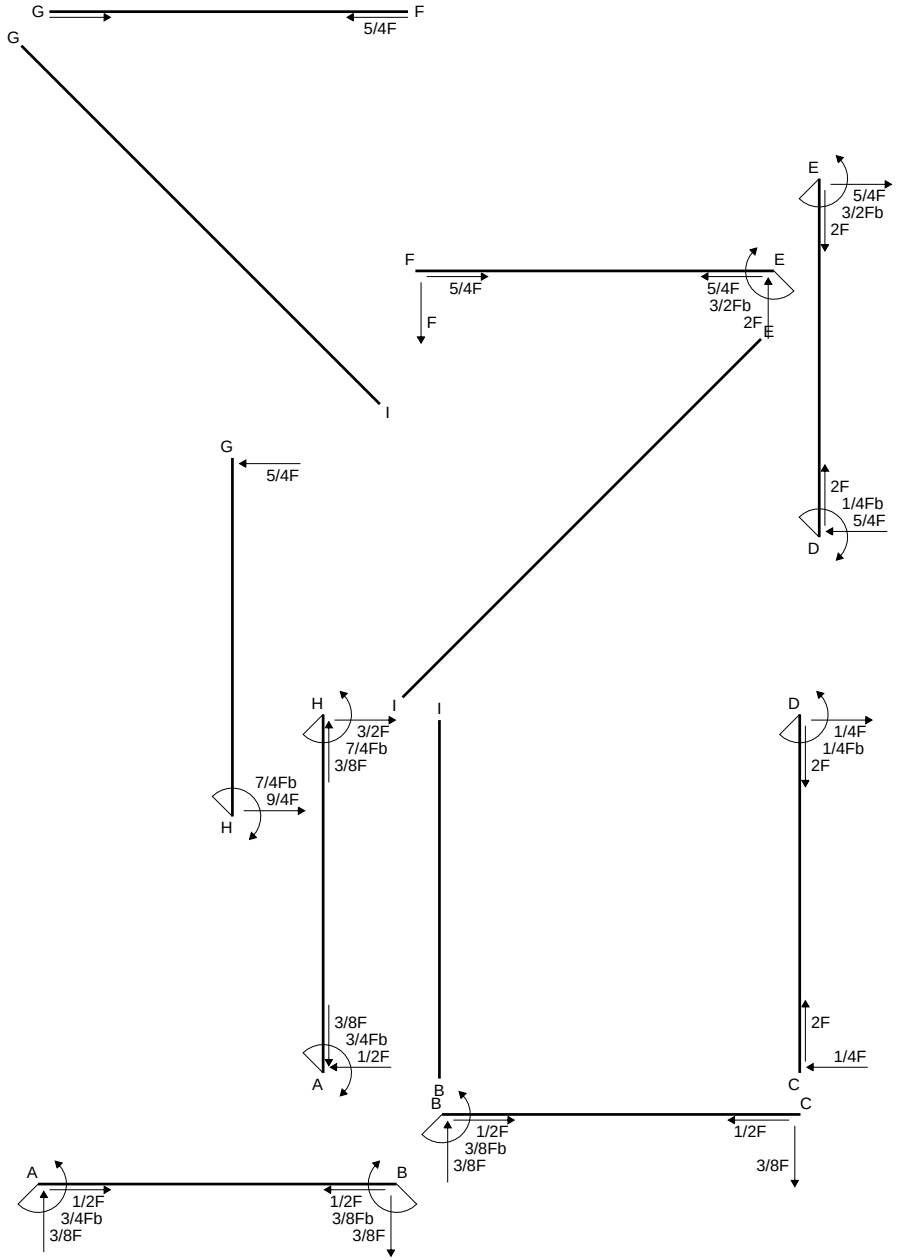
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

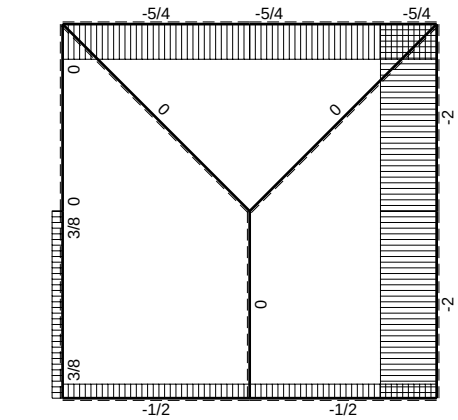
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

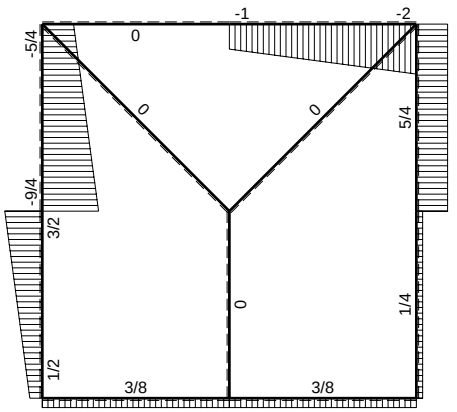
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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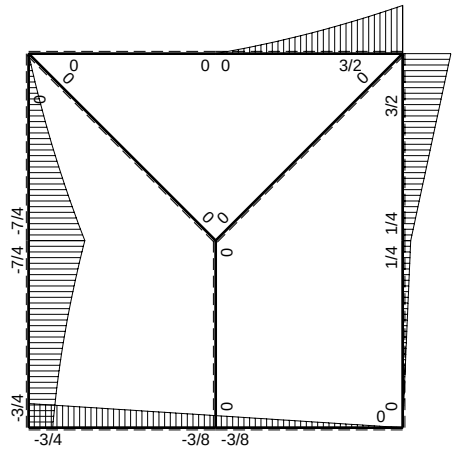

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2F
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7/4Fb
3/8F
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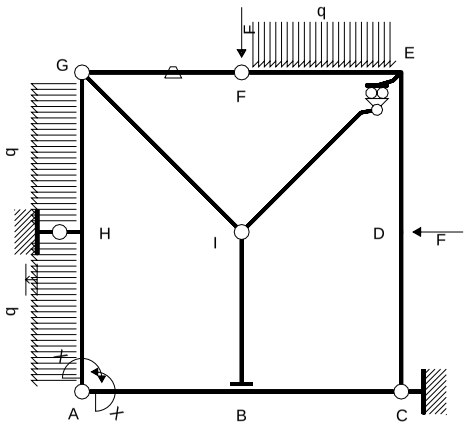

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3/8

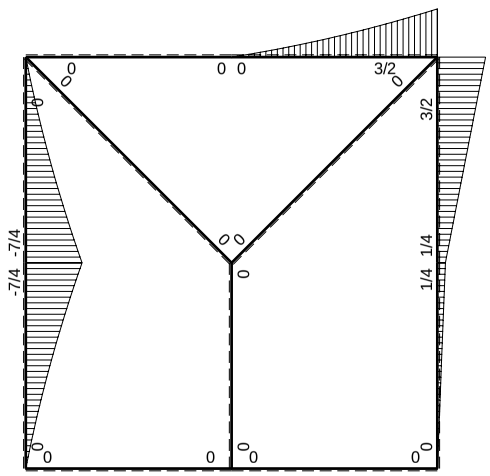
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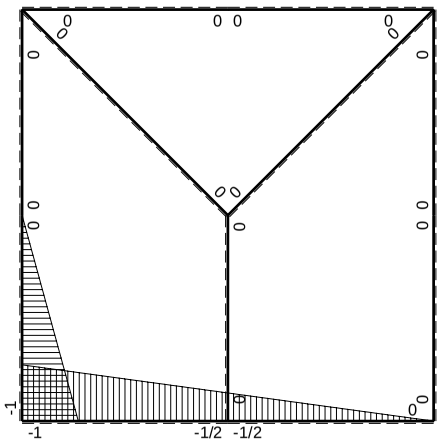

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0
0
3/2
0
0
-7/4 -7/4
0
1/4 1/4
-3/4
0
0
0
-3/4
-3/8 -3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237735 ESΣ04.xxxx.149

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

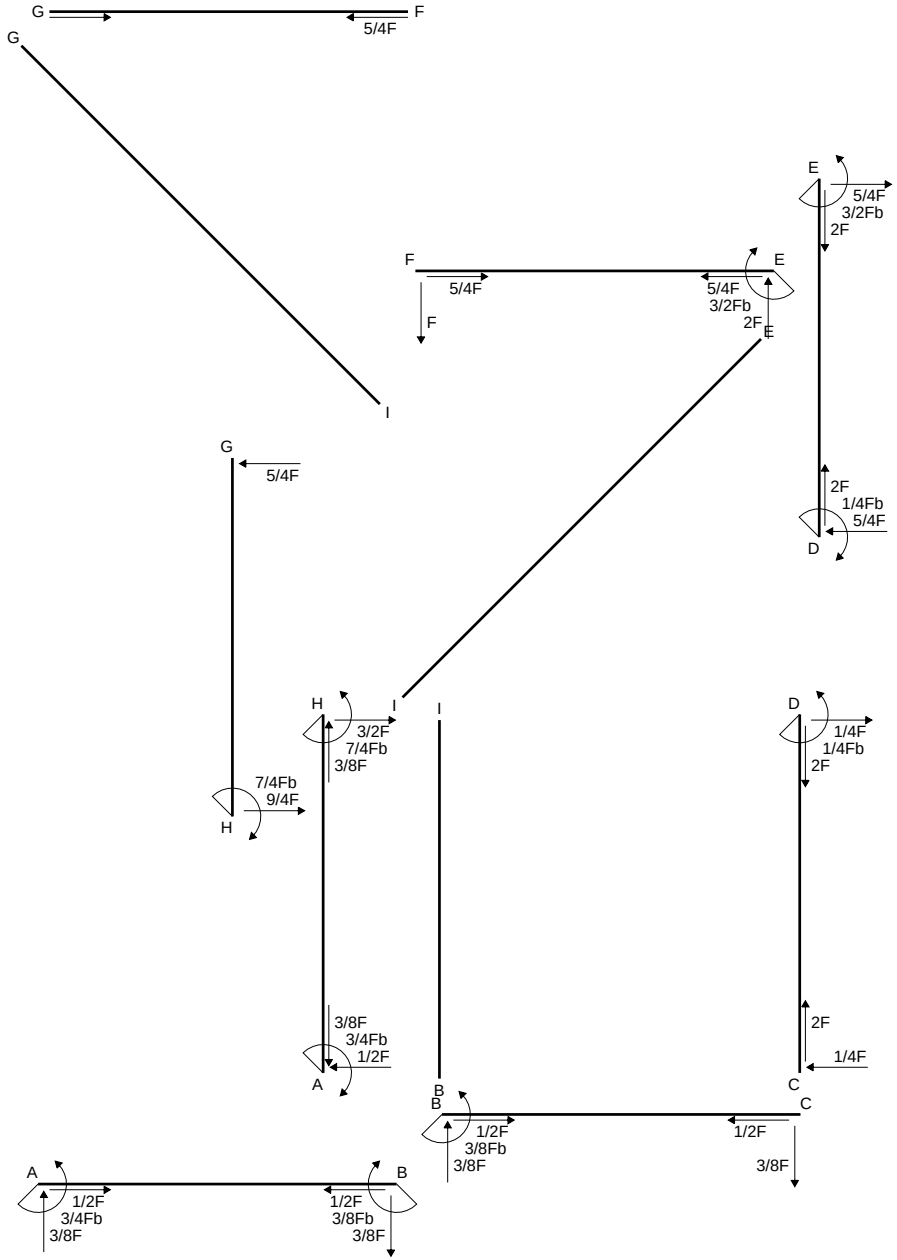
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

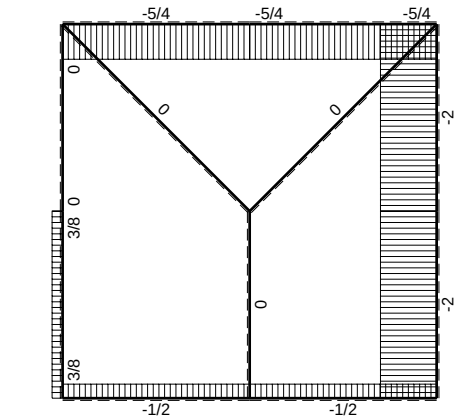
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

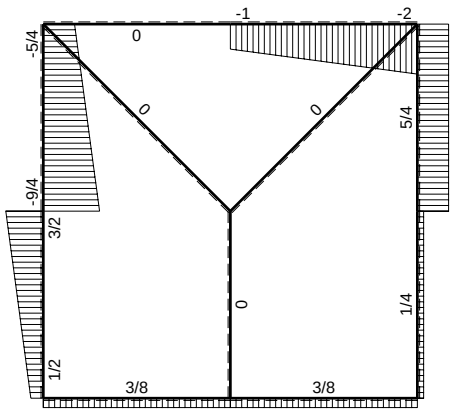
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

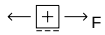
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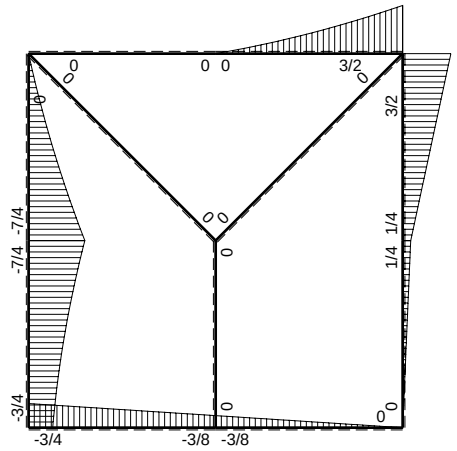






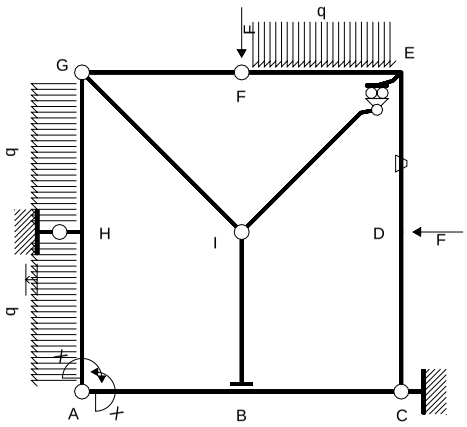


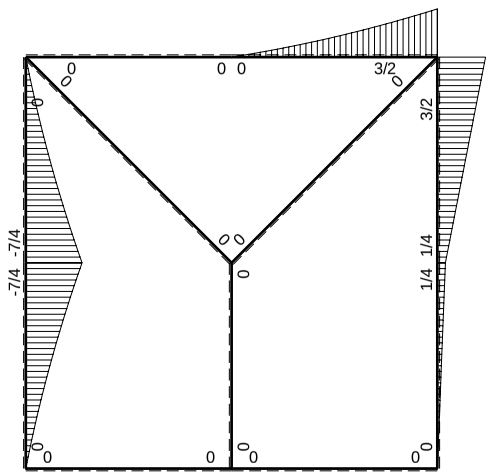




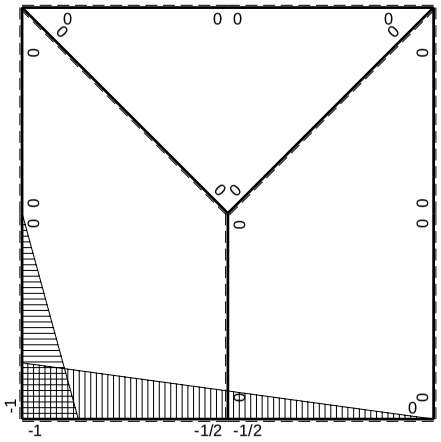


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

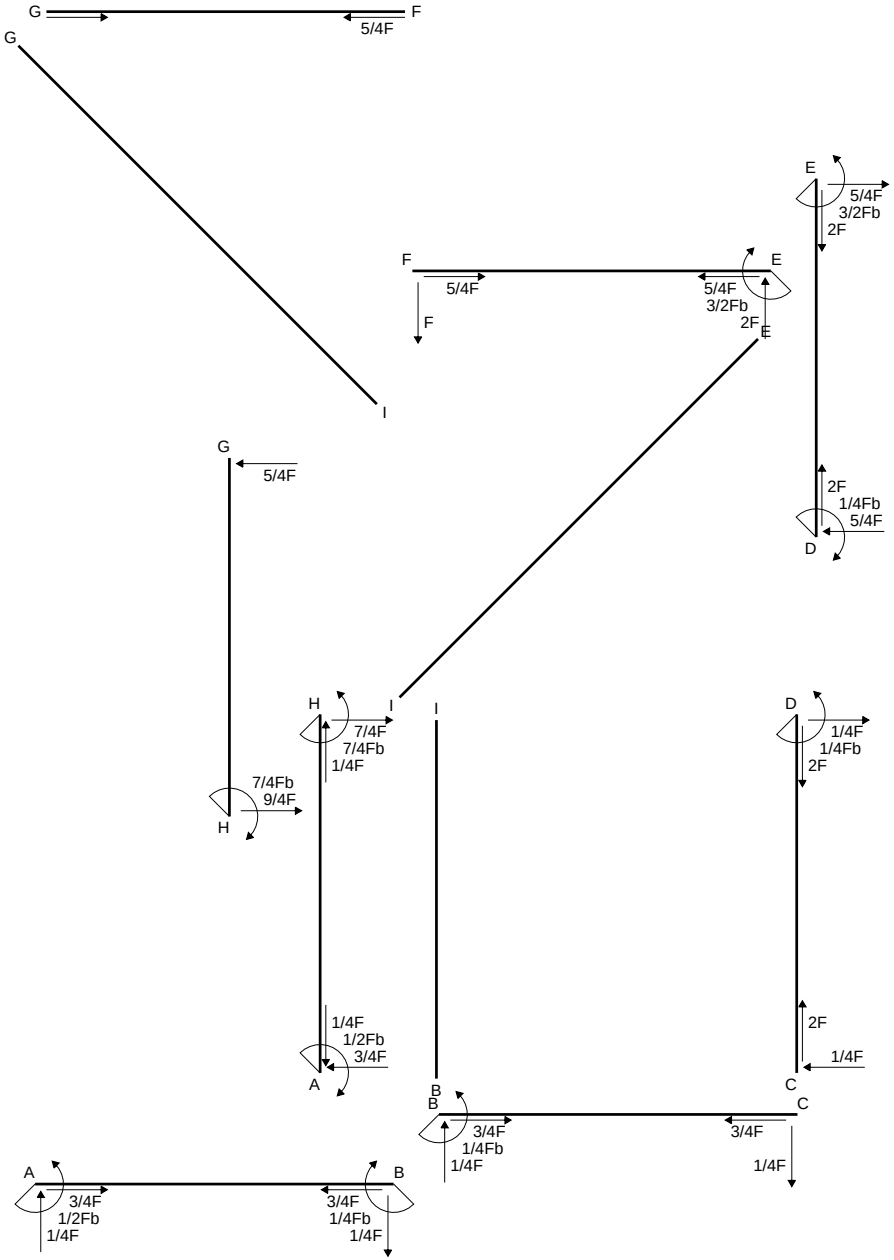
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

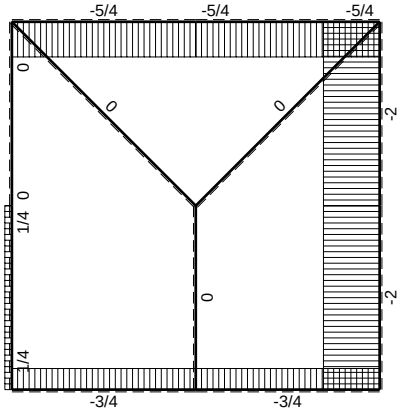
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

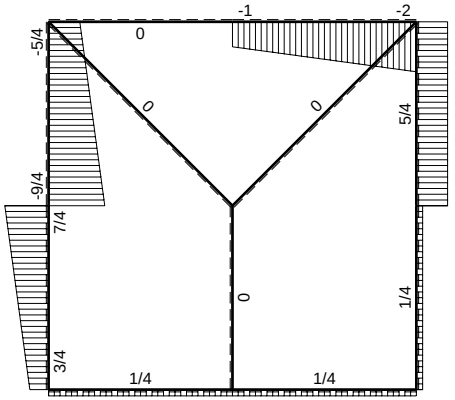
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

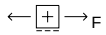
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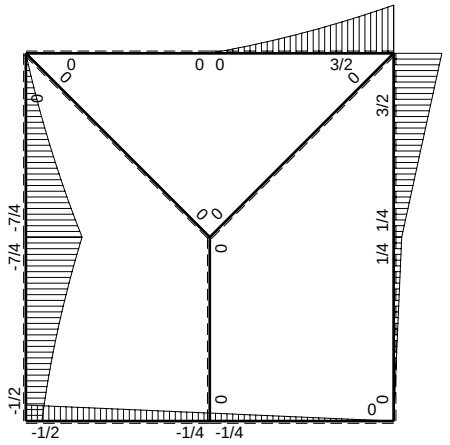






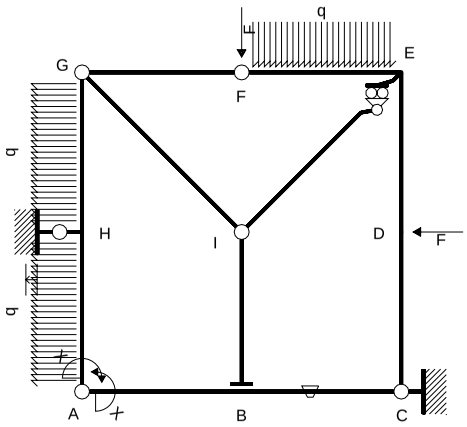


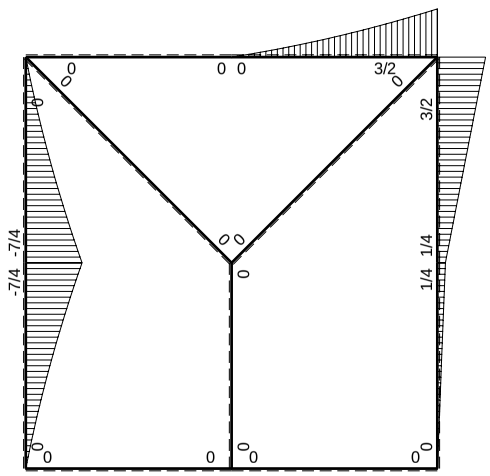




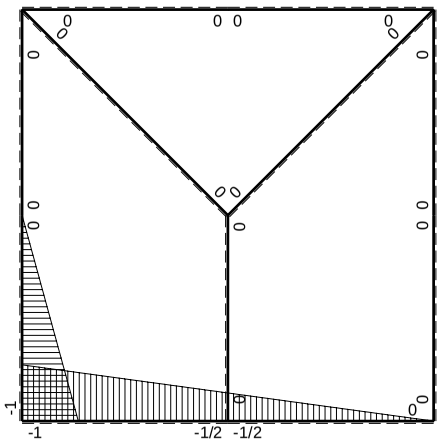


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237895 ESΣ04.xxxx.151

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	-Fb/EJ	0	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(0+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	0	Fb/EJ	0	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/2Fb	

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(1/2\ b - 1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/2\ x/b\ \right)\theta\ \ dx = \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

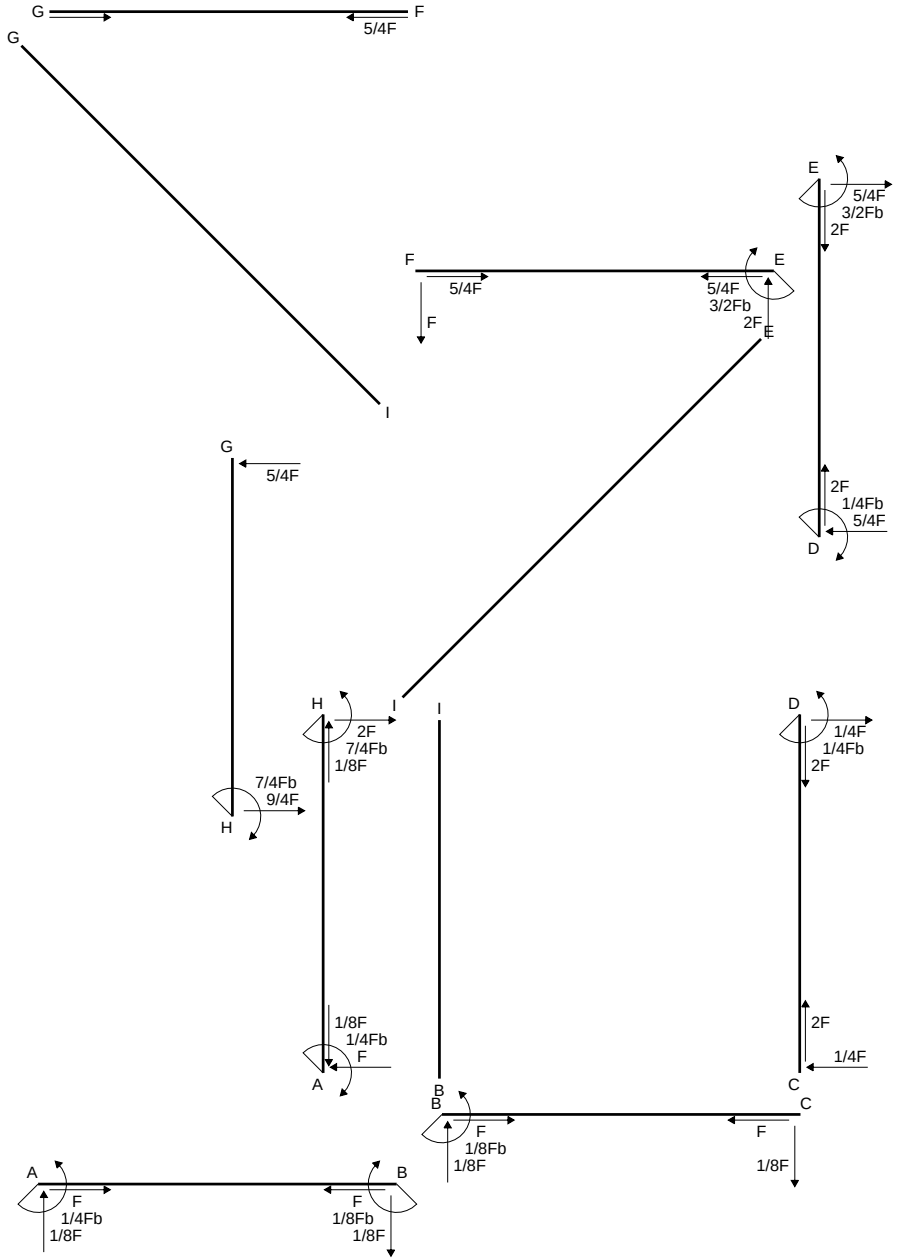
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

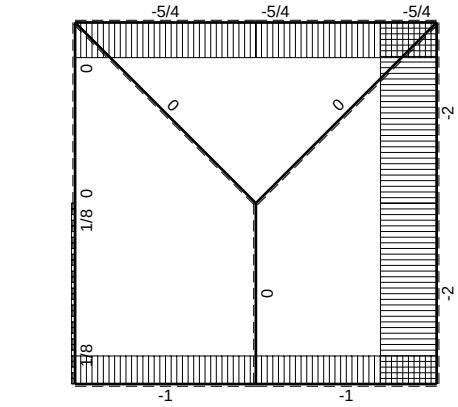
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

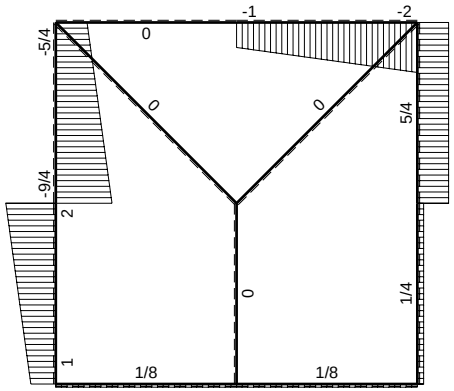
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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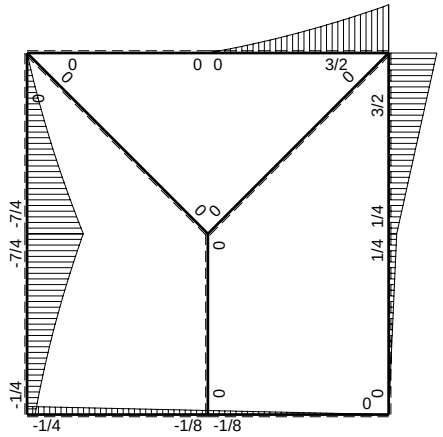

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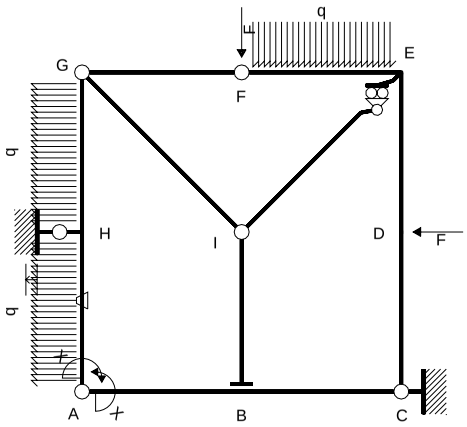

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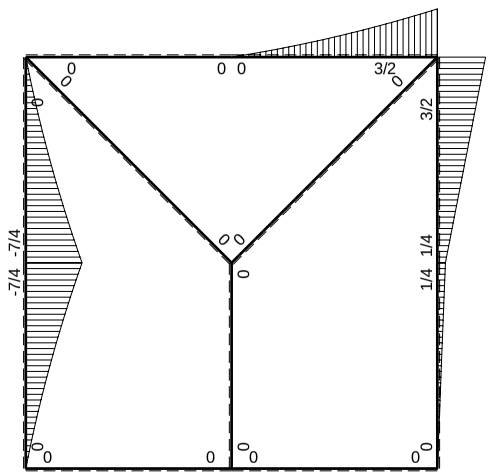
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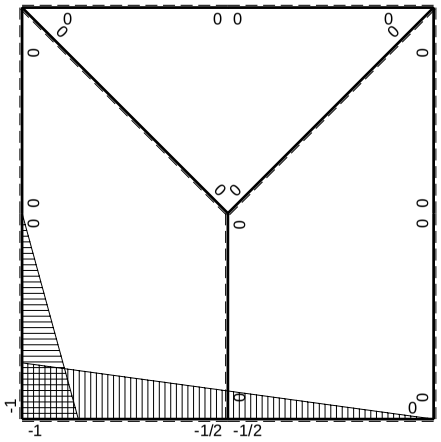

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↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	-Fb/EJ	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/4+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	Fb/EJ	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/4Fb	

Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\ \right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\theta\ \ dx$$

$$= \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$$

$$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ + \left(1/2\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$$

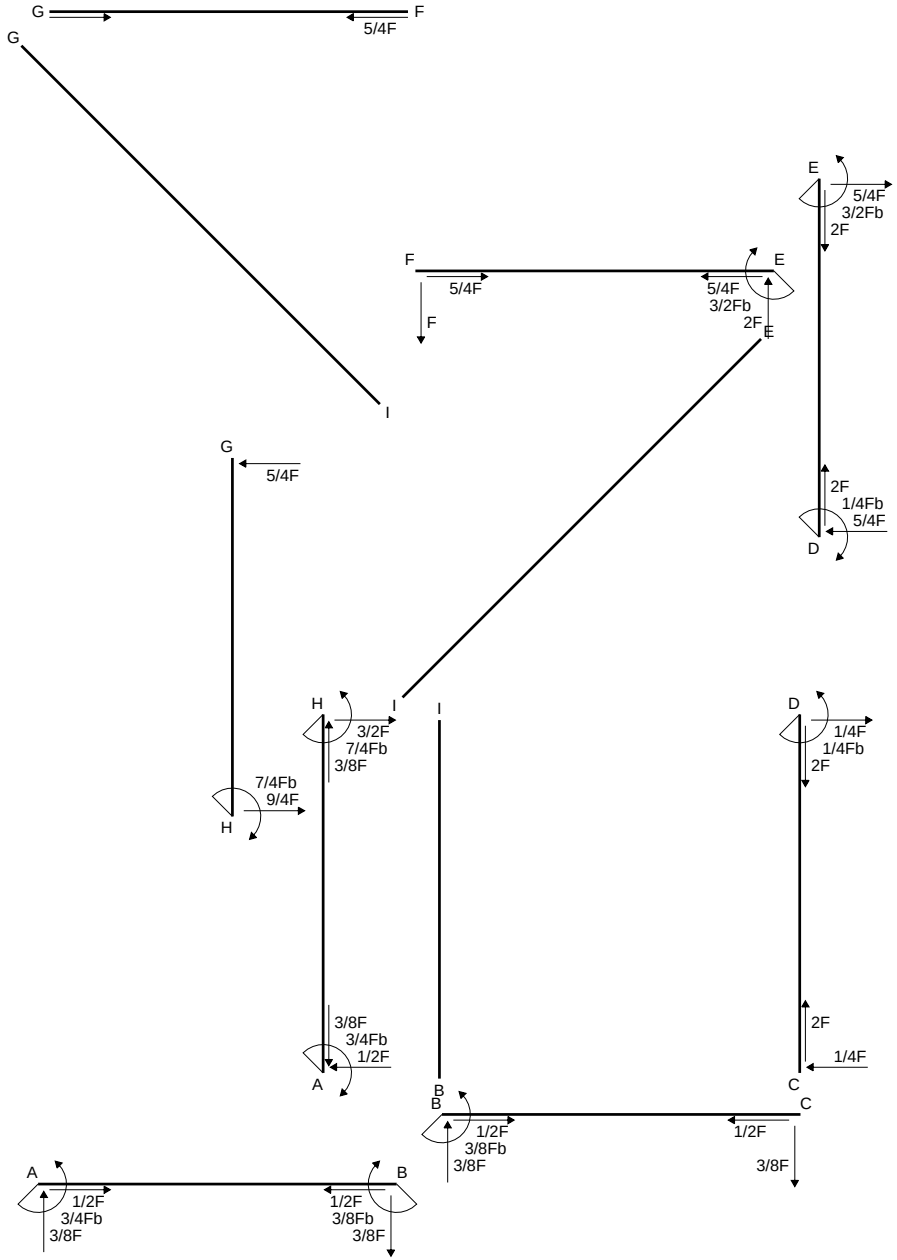
$$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\theta\ \ dx$$

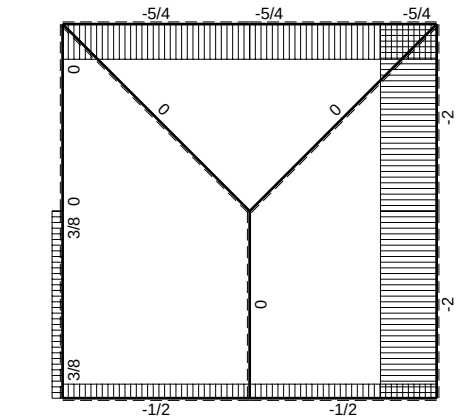
$$= \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$$

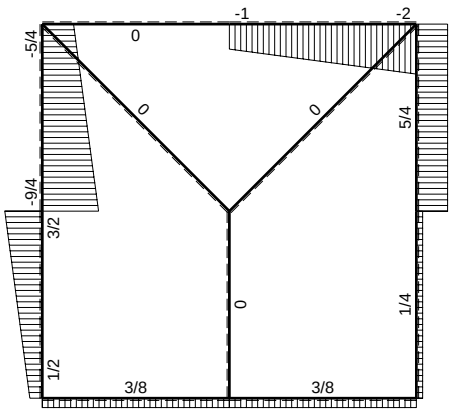
$$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$$

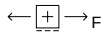
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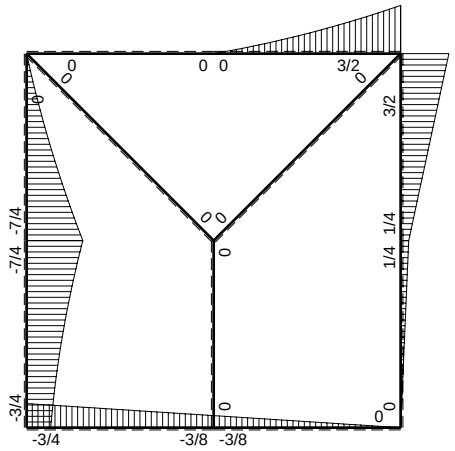

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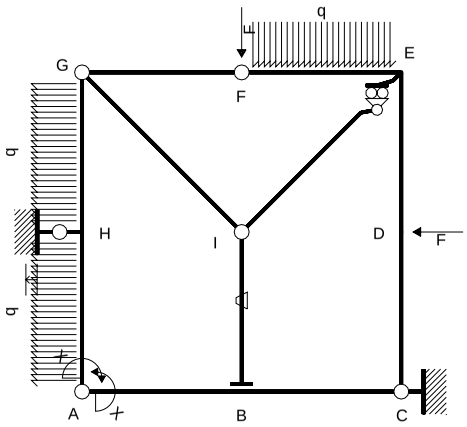

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1/4
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3/8
3/8

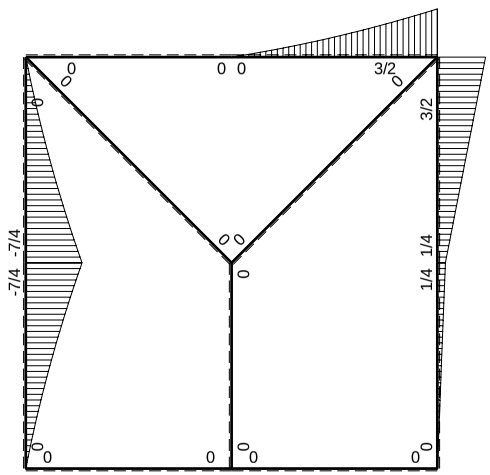

F


F

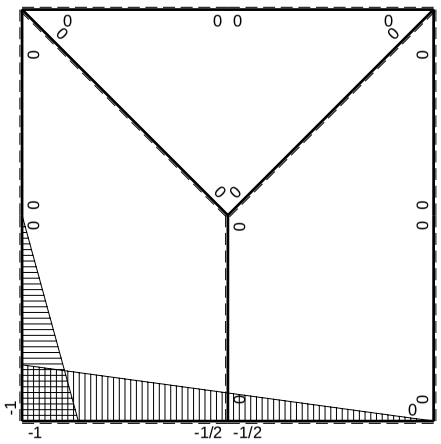

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-3/4
-3/4
-3/8
-3/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238254 ESΣ04.xxxx.153

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

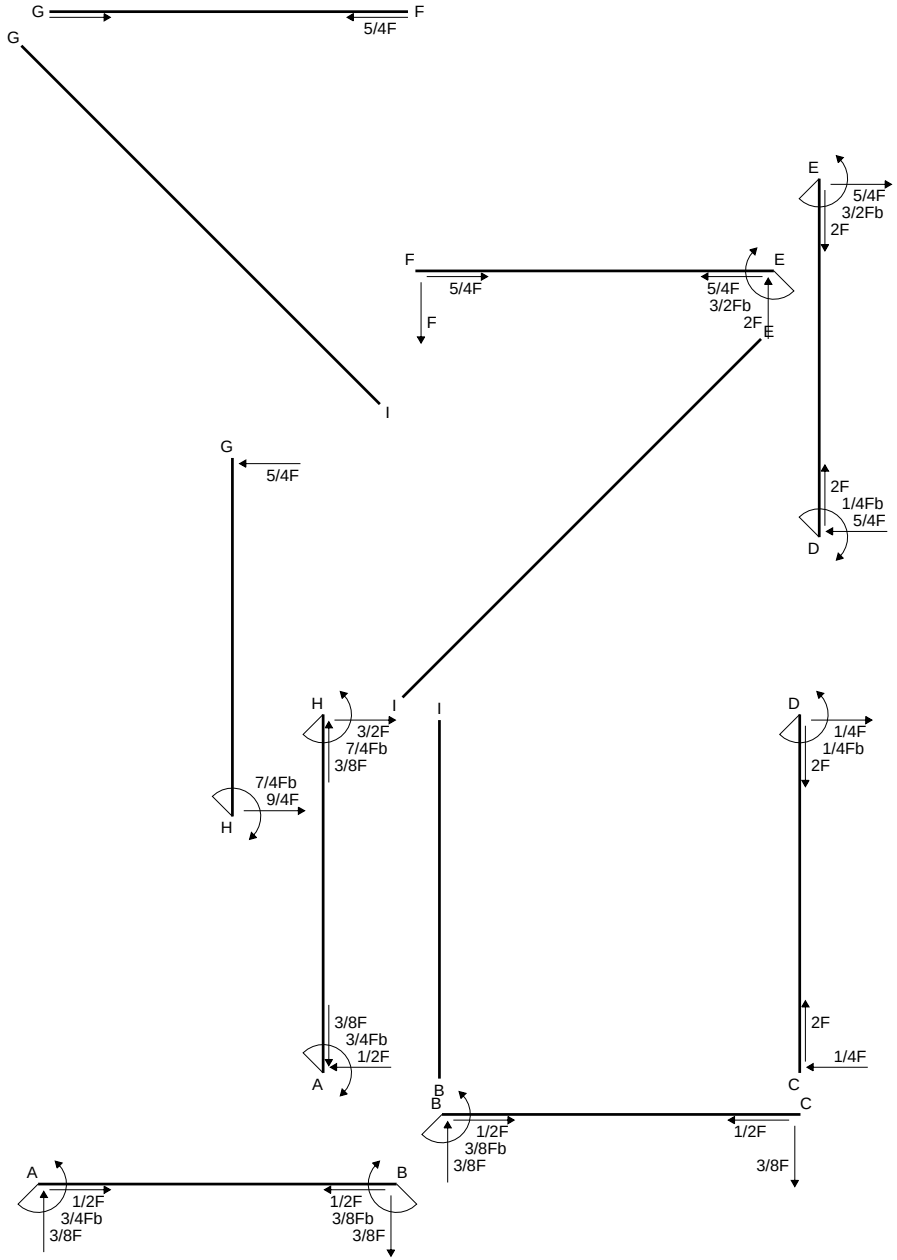
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

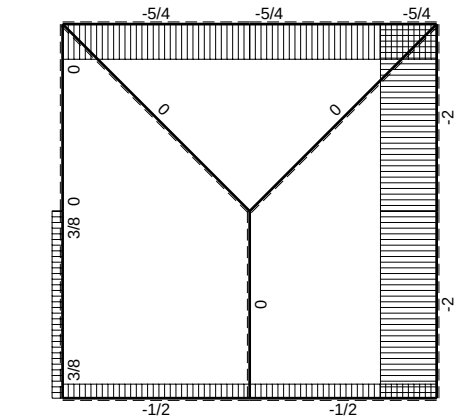
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

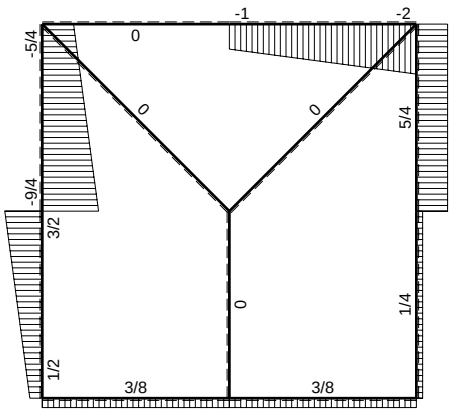
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

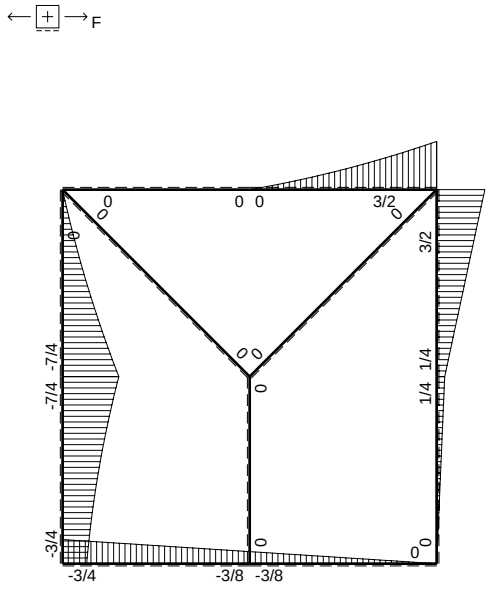
ESΣ04.xxxx.153

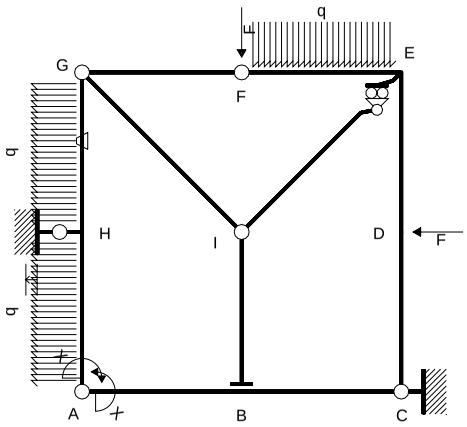
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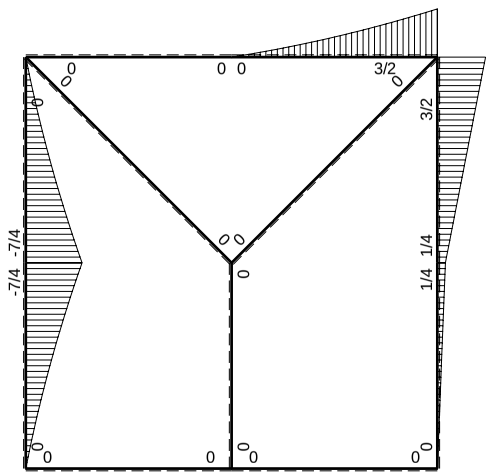

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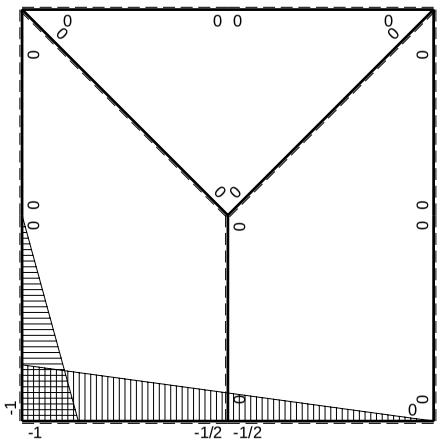

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-3/4
-3/8
-3/8





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238210 ESΣ04.xxxx.154

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x\theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

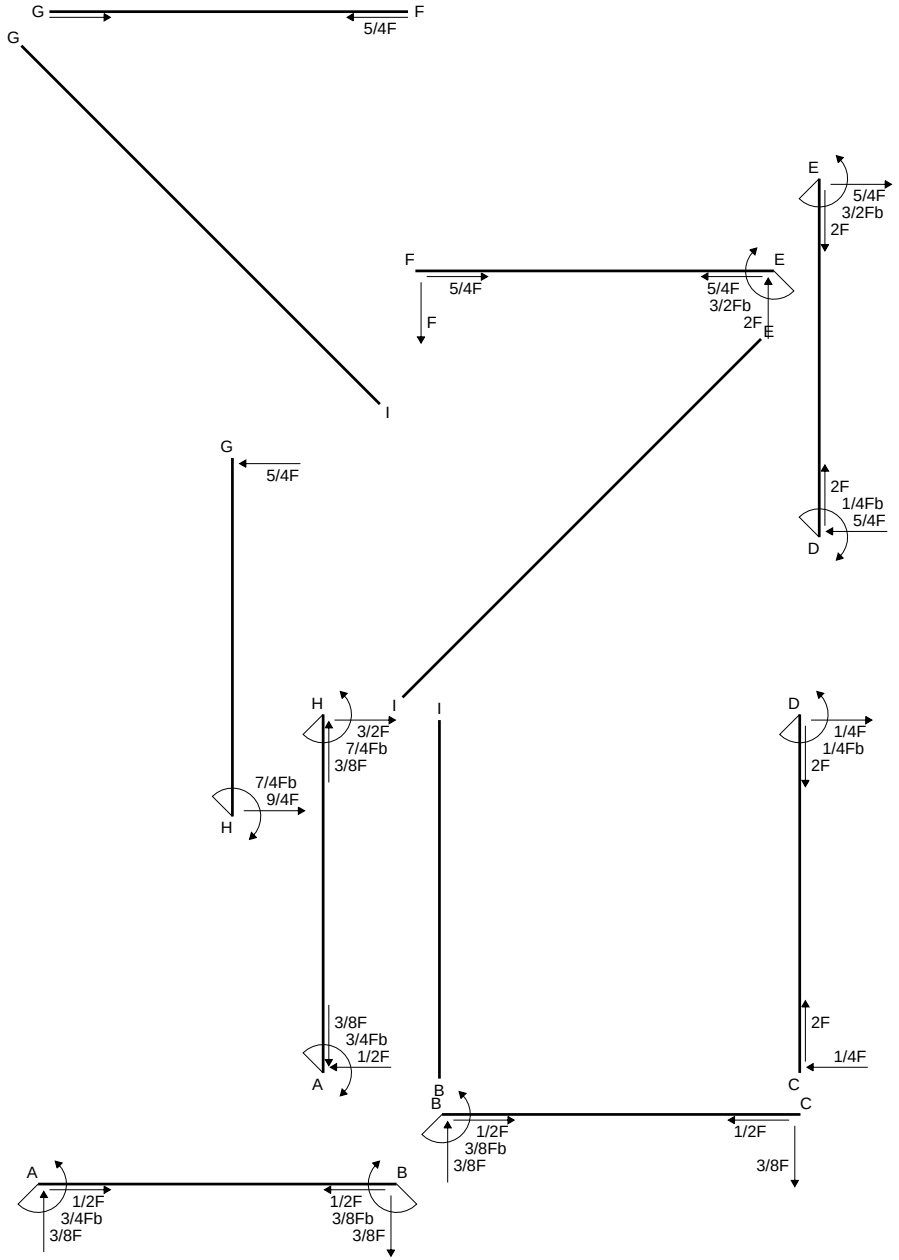
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

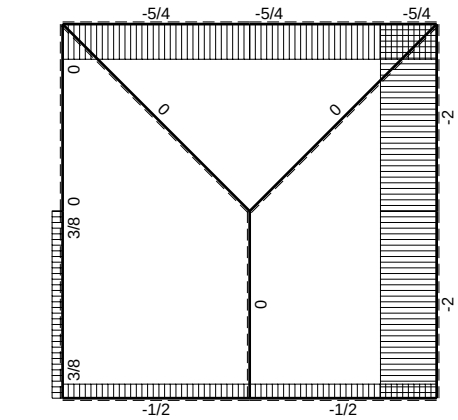
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

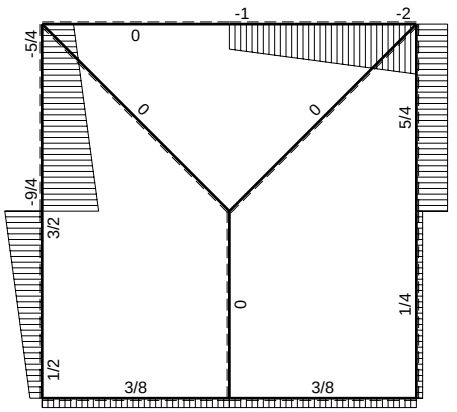
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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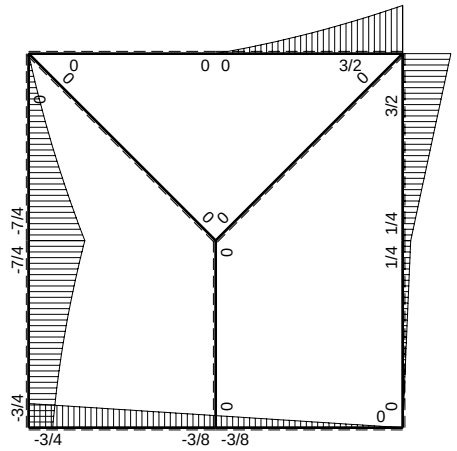

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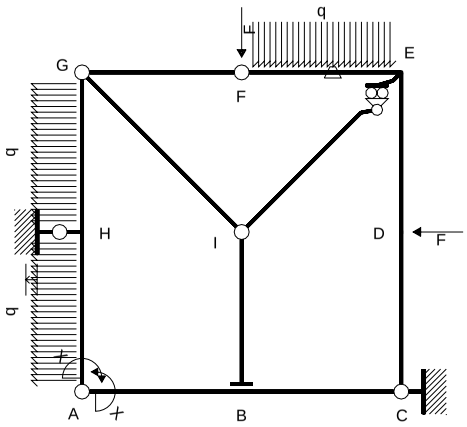

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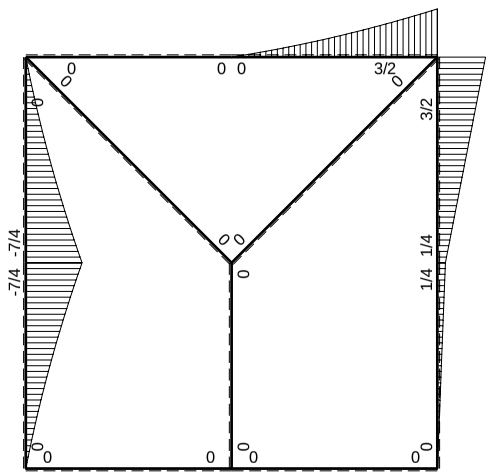
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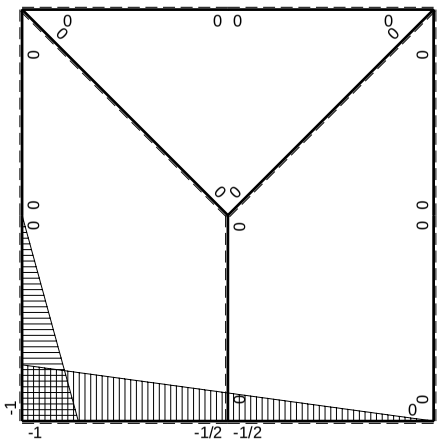

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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236938 ESΣ04.xxxx.155

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PROCEDIMENTO E RISULTATI 236938 ESΣ04.xxxx.155

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x\theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

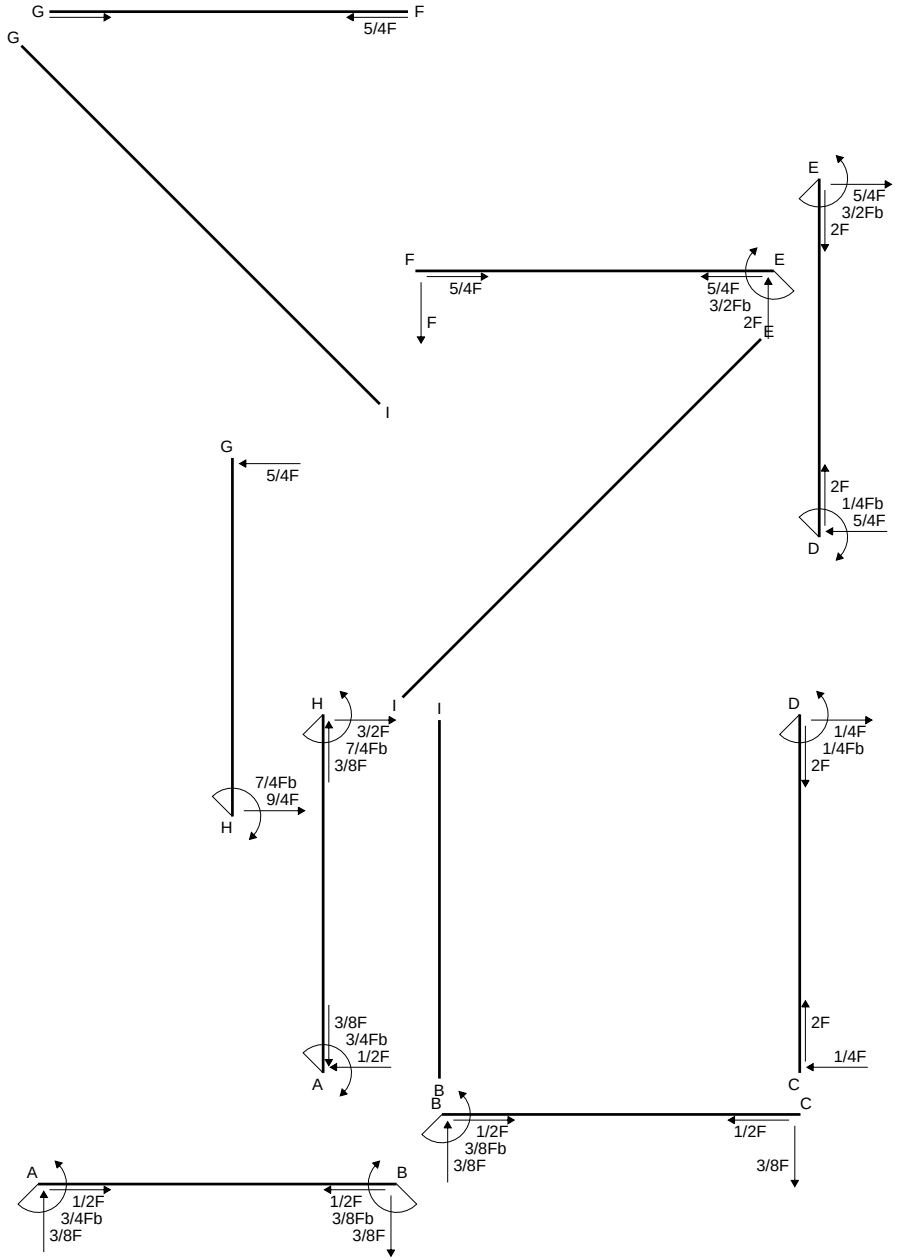
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

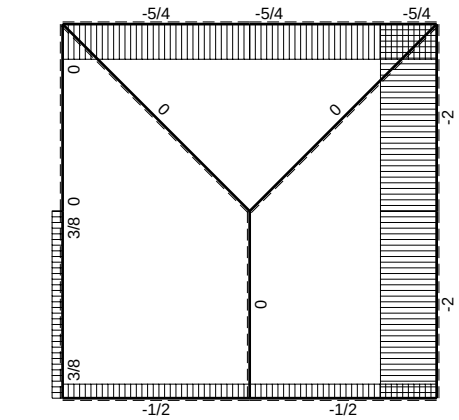
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

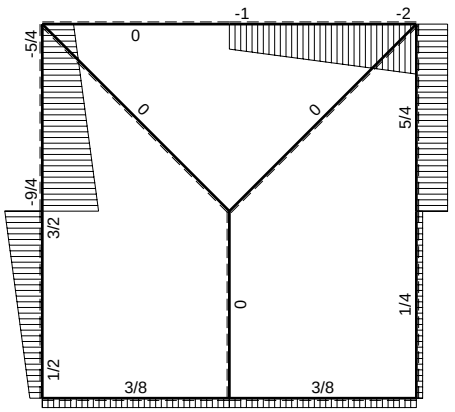
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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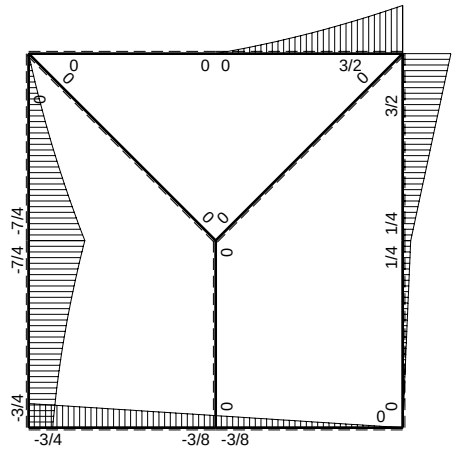

G
F
5/4F
G
I
E
5/4F
3/2Fb
2F
F
5/4F
F
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2F
E
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2F
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5/4F
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5/4F
H
I
3/2F
7/4Fb
3/8F
7/4Fb
9/4F
H
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D
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3/4Fb
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C
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3/8Fb
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3/8
-1/2
-1/2

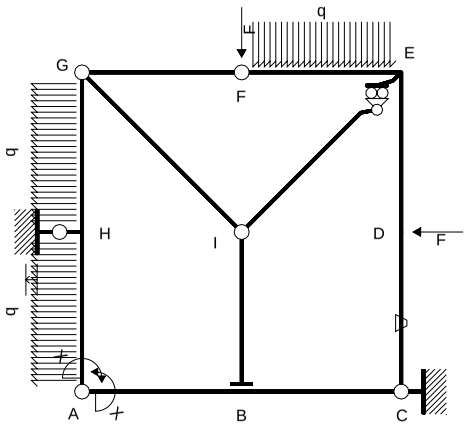

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-2
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5/4
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3/2
0
1/4
1/2
3/8
3/8

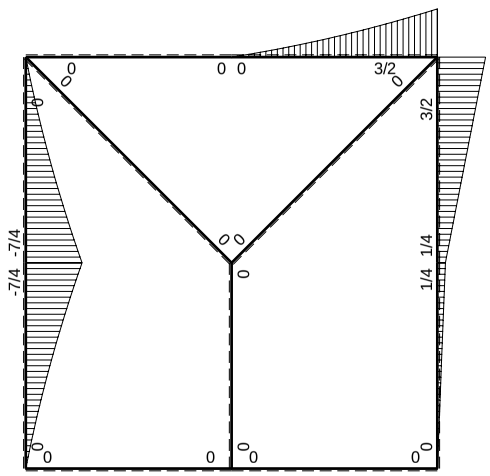
←□→ F

↑□↓ F

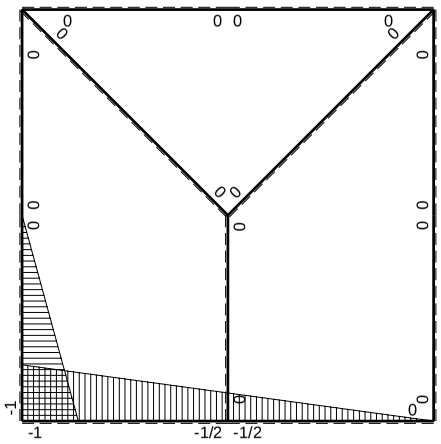

0
0 0
3/2
0
0
3/2
0
0
-7/4
-7/4
0
1/4
1/4
-3/4
0
0
0
-3/4
-3/8
-3/8

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211694 ESΣ04.xxxx.156

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

PROCEDIMENTO E RISULTATI 211694 ESΣ04.xxxx.156

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

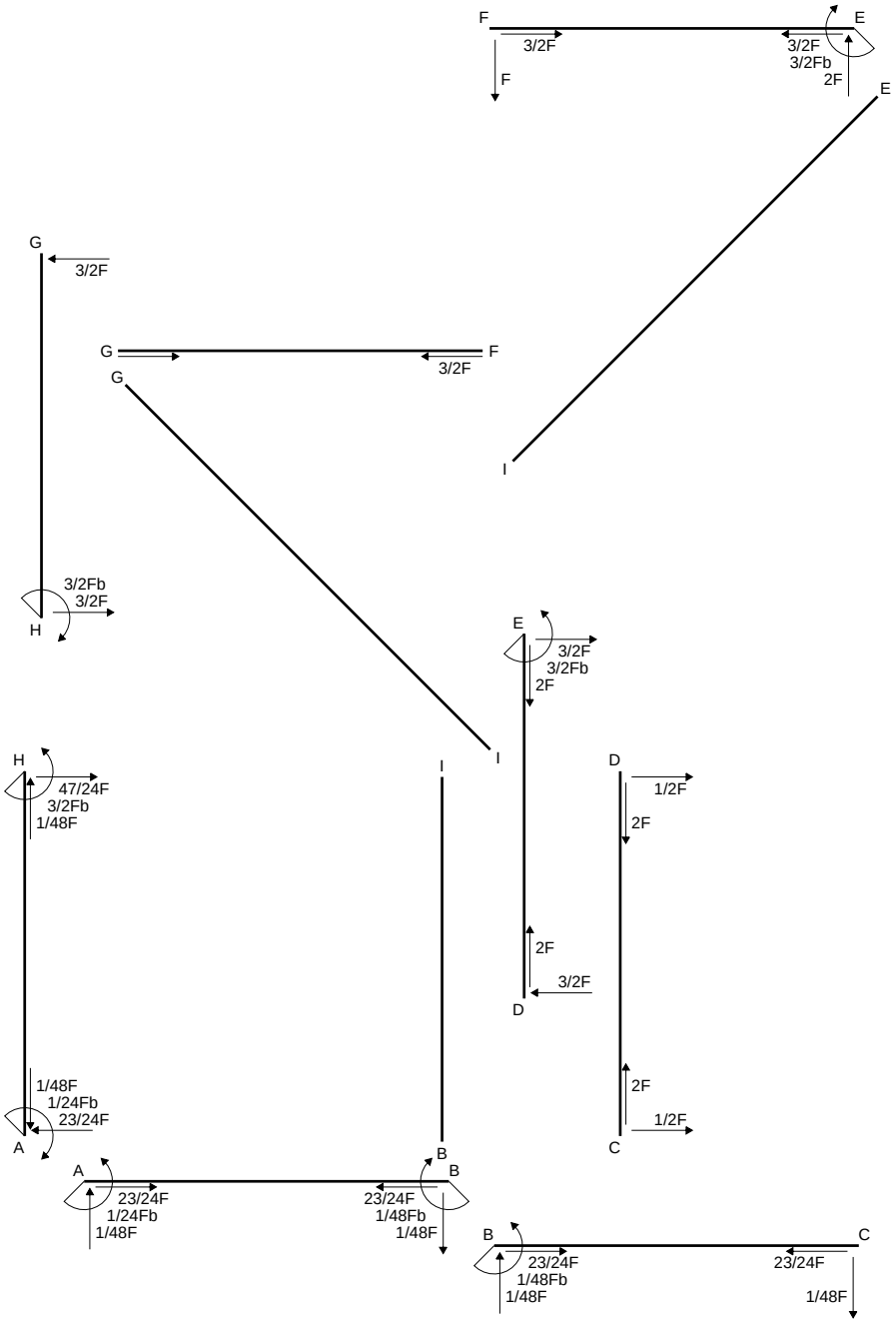
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

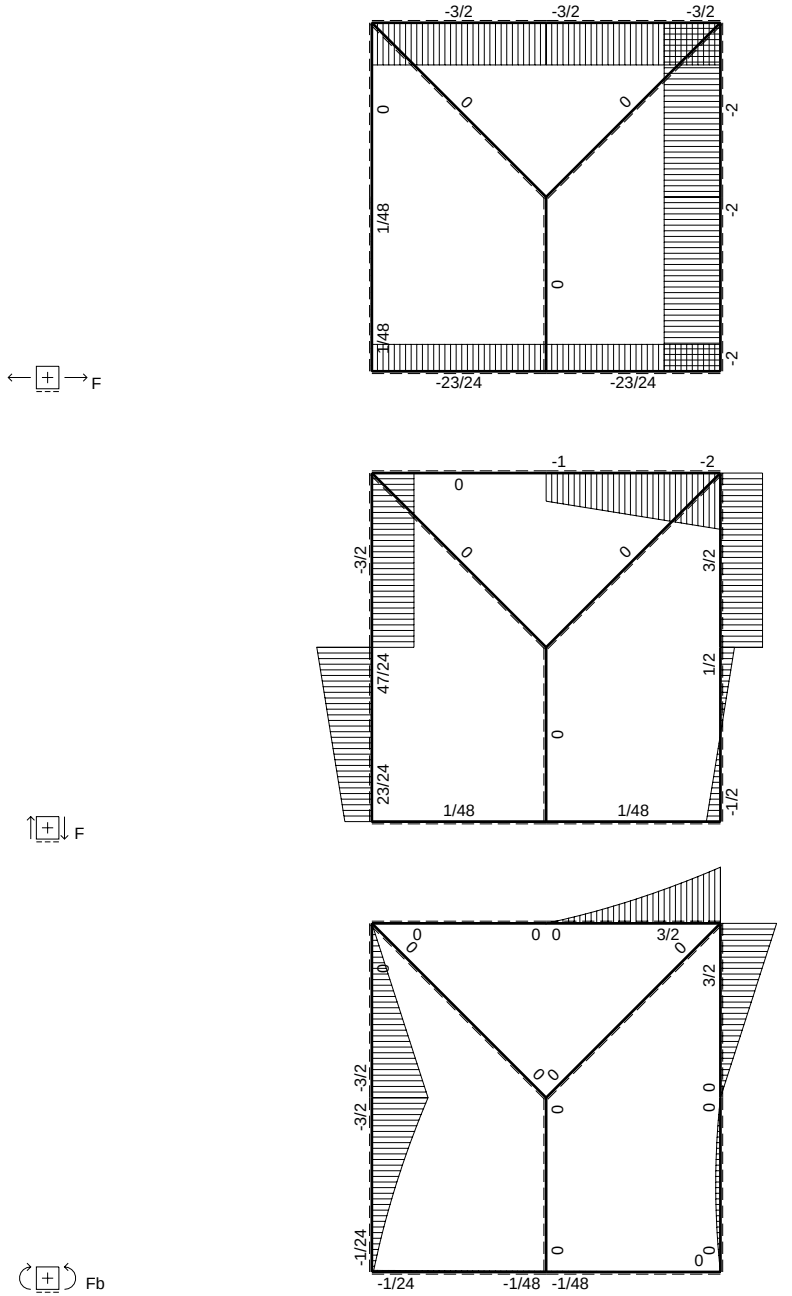
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

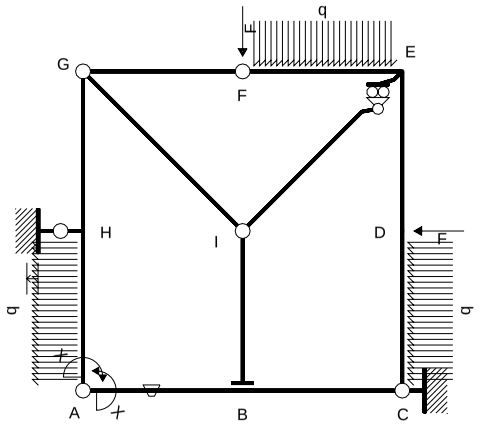
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

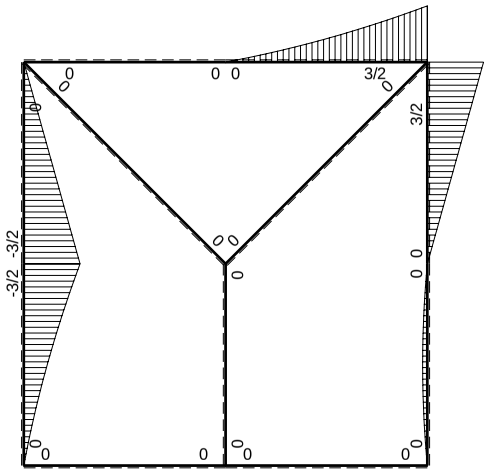
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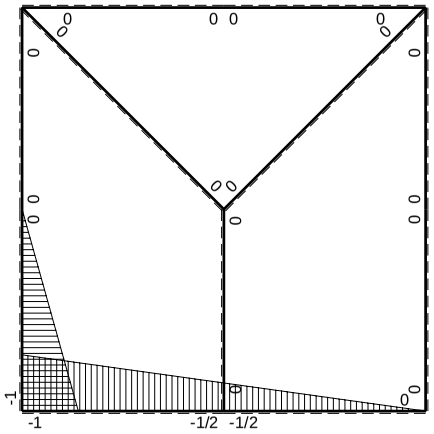

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3/2F
3/2Fb
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G
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F
3/2F
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I
3/2Fb
3/2F
H
E
3/2F
3/2Fb
2F
H
I
I
D
47/24F
3/2Fb
1/48F
1/2F
2F
2F
3/2F
D
1/48F
1/24Fb
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2F
1/2F
C
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C
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1/48Fb
1/48F
23/24F
1/48F


F
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1/48
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-3/2
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0
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1/48
1/48
-1/2
Fb
0
0 0
3/2
0
0
3/2
-3/2
-3/2
0 0
0
0
0
-1/24
-1/48 -1/48





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	-Fb/EJ	0	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(0+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	0	Fb/EJ	0	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/24Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238079 ESΣ04.xxxx.157
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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(1 - 1/2\ x/b\ \right)\theta\ \ dx = \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/2\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

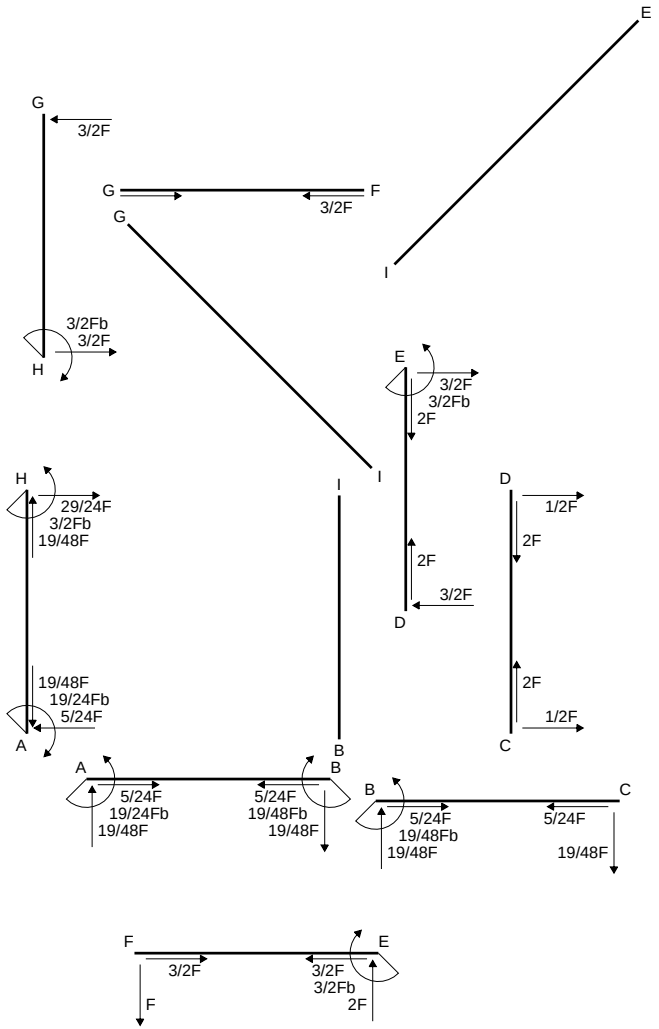
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

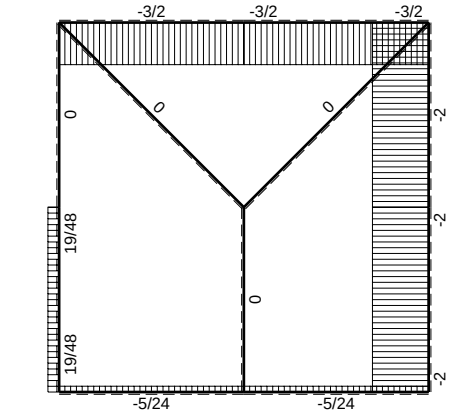
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

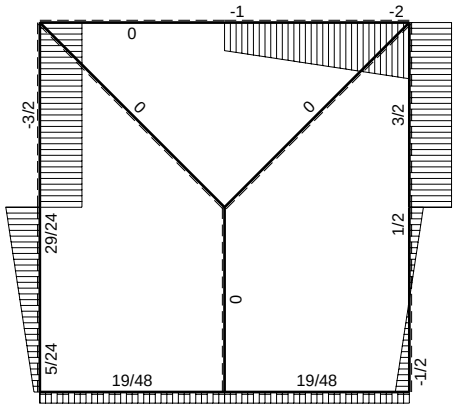
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

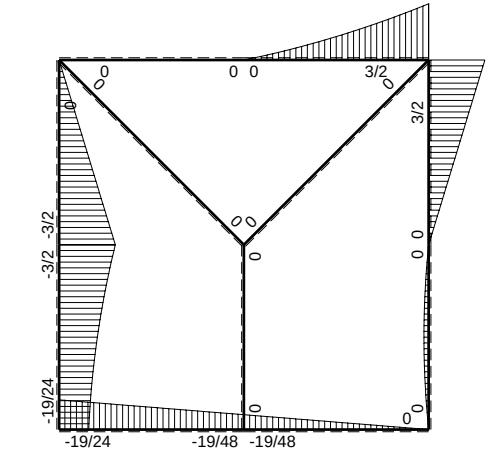
ESΣ04.xxxx.157

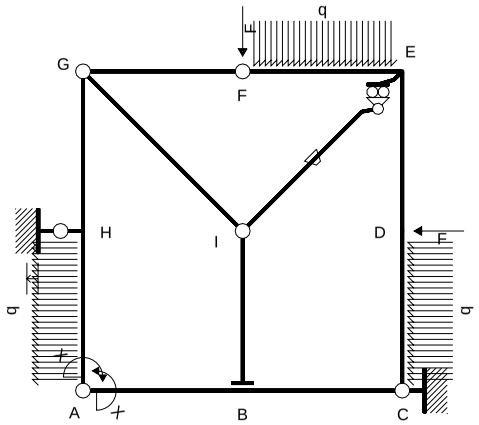
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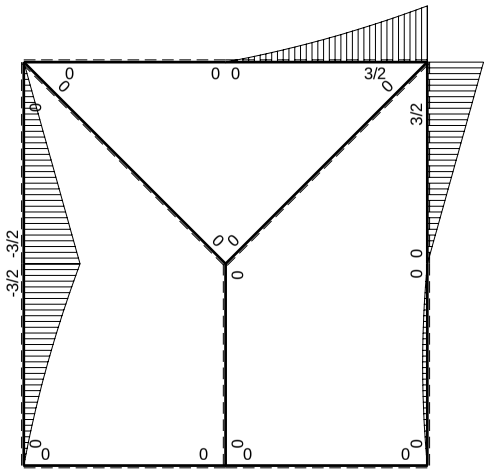

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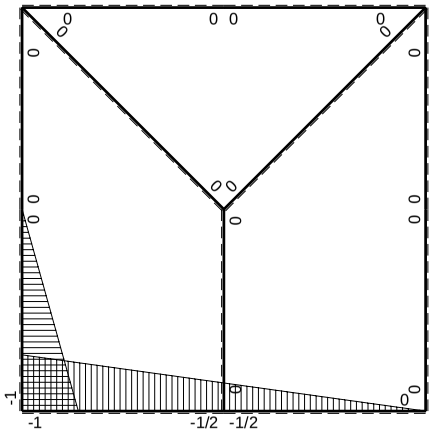

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-19/48
-19/48





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237078 ESΣ04.xxxx.158

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

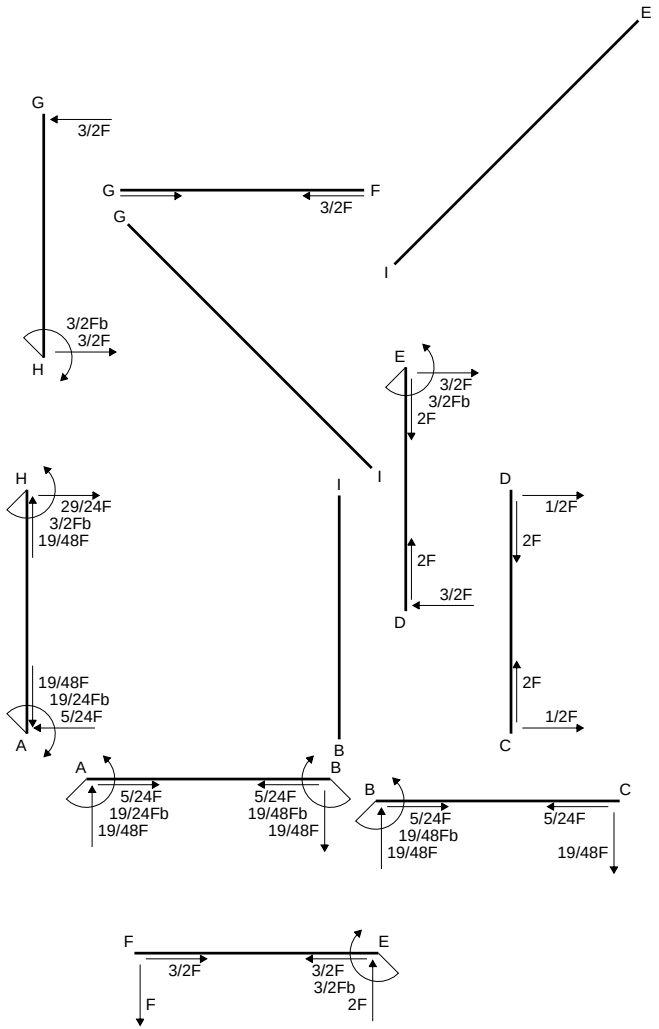
$= \left(3/4\ b - 2/3\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

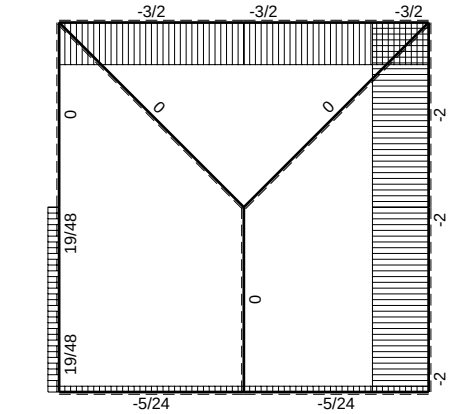
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

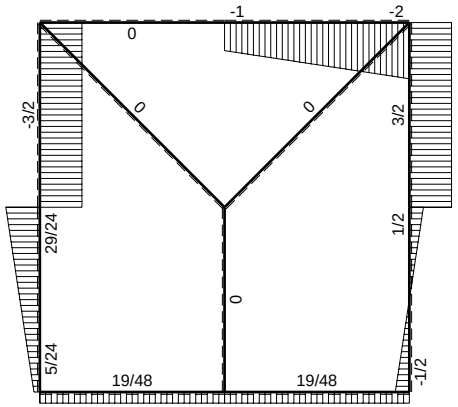
$= \left(1/2\ b - 1/6\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

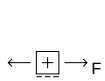
ESΣ04.xxxx.158

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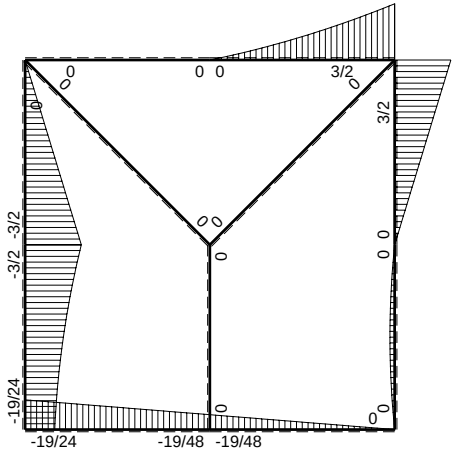

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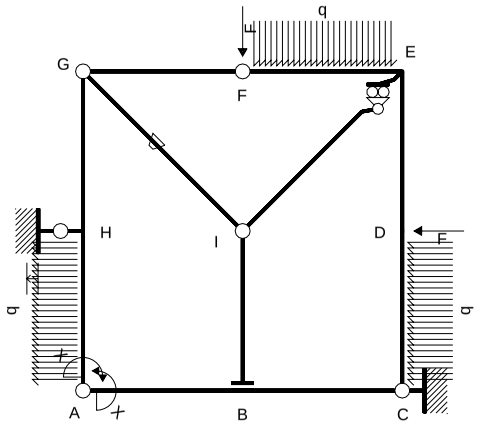

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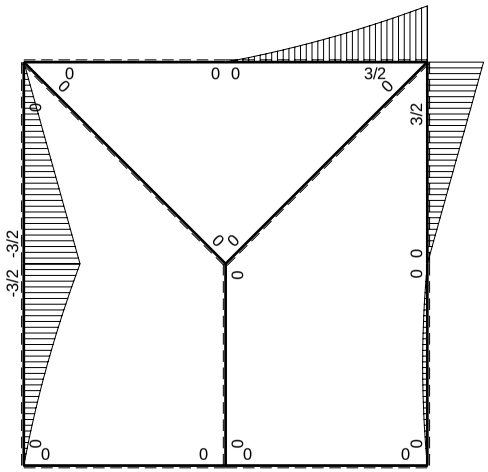

F


F

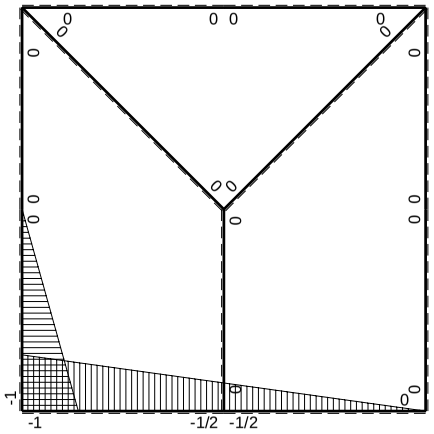

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-19/48


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982585 ESΣ04.xxxx.159

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI √2b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

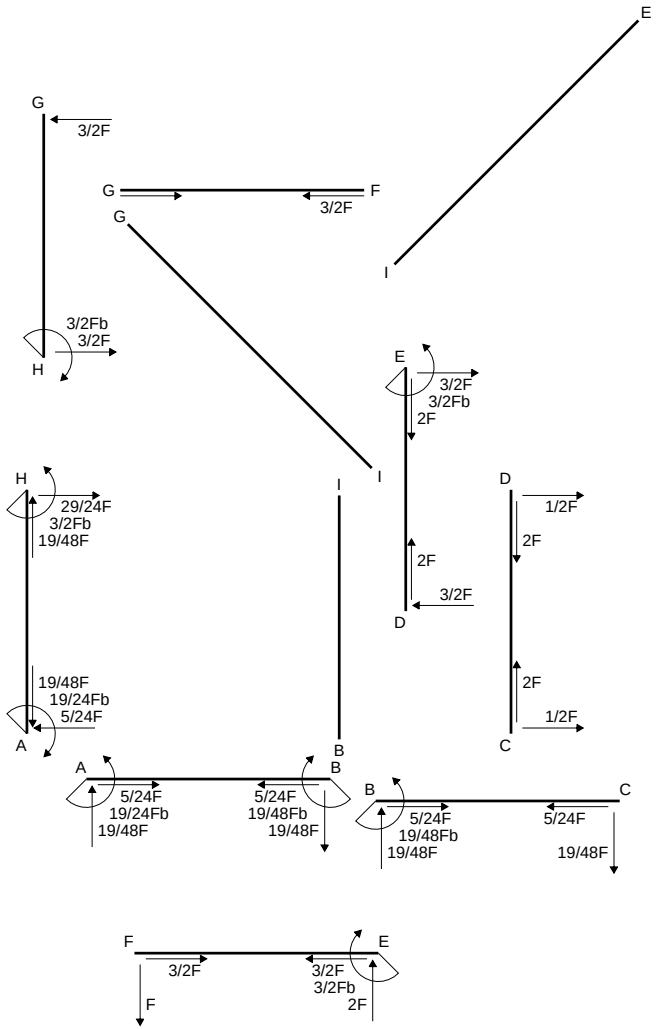
$= \left(3/4\ b - 2/3\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

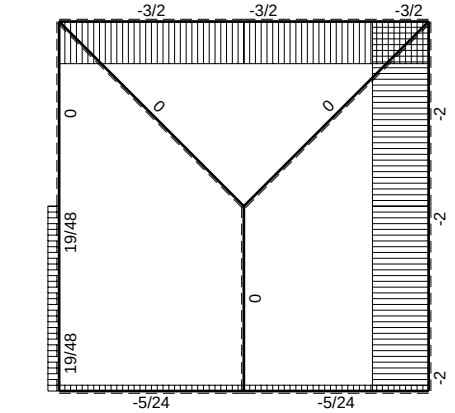
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

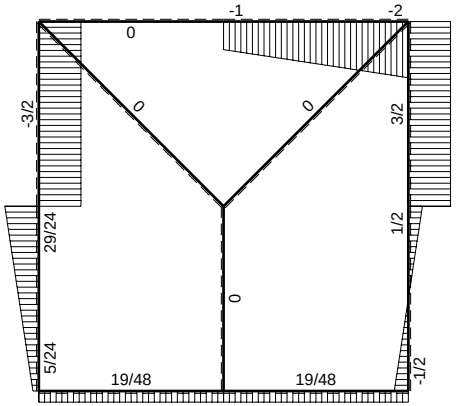
$= \left(1/2\ b - 1/6\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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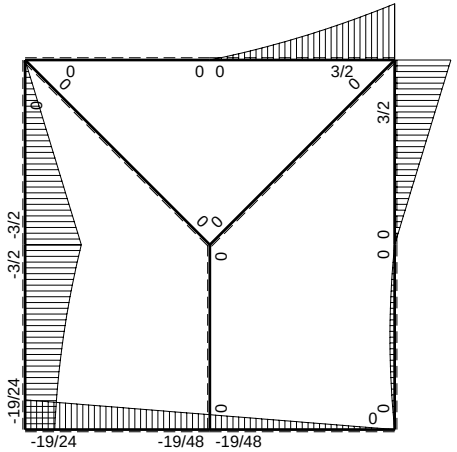

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F
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H
E
3/2F
3/2Fb
2F
I
I
H
29/24F
3/2Fb
19/48F
D
1/2F
2F
2F
3/2F
D
19/48F
19/24Fb
5/24F
2F
1/2F
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B
C
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B
5/24F
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19/48F
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C
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2F
F


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-5/24

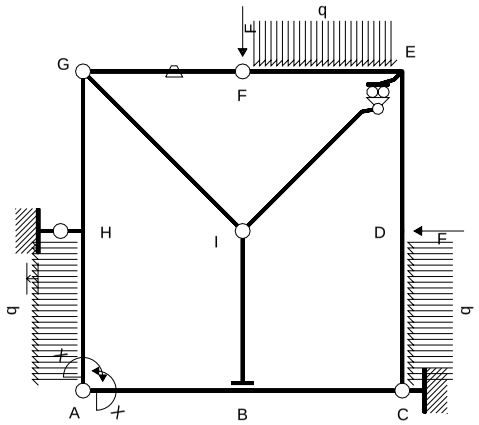

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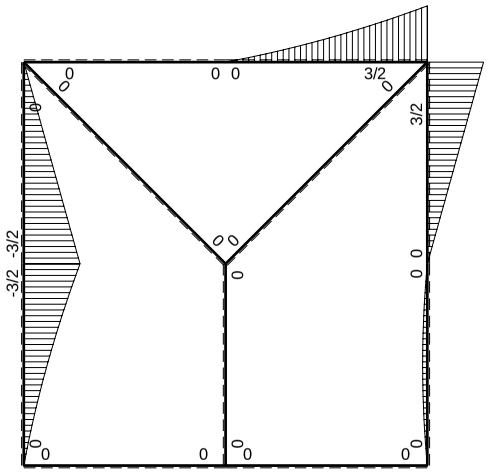
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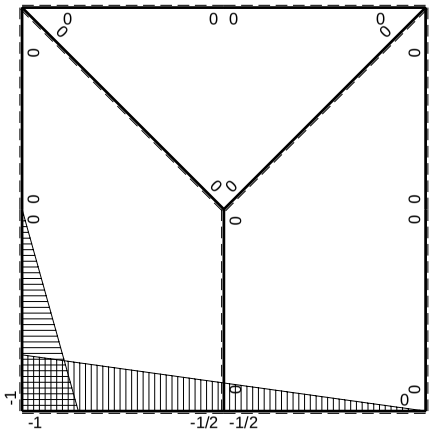

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-19/24
-19/24
-19/48
-19/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210489 ESΣ04.xxxx.160

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

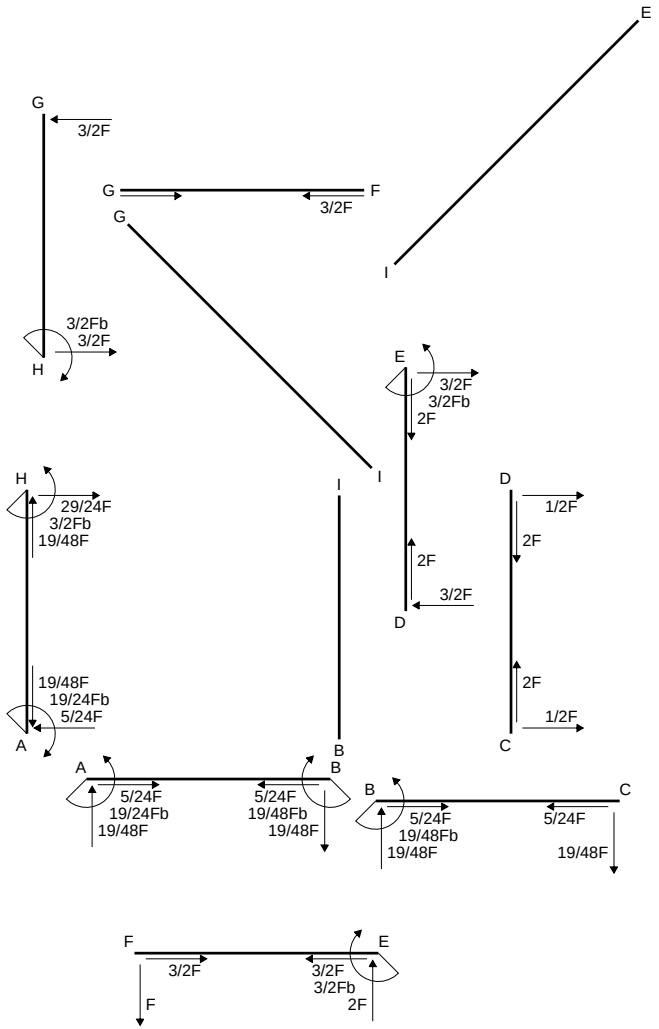
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

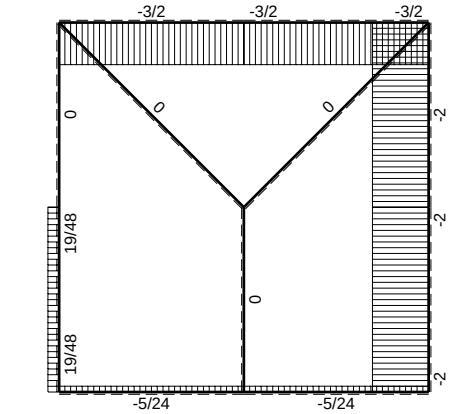
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

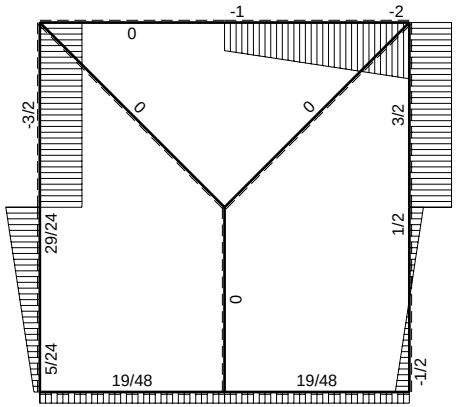
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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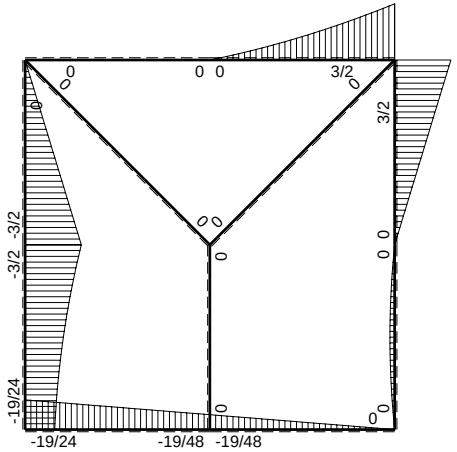

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2F
2F
3/2F
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19/48F
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2F
F


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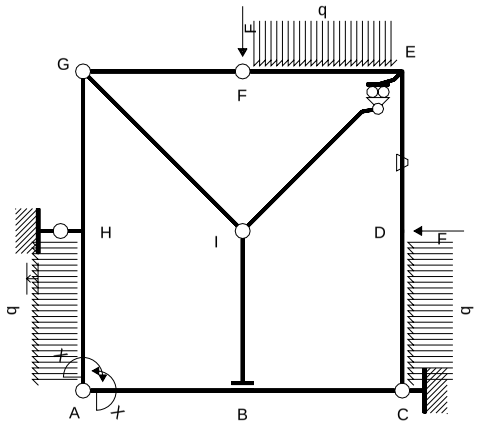

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19/48
-1/2

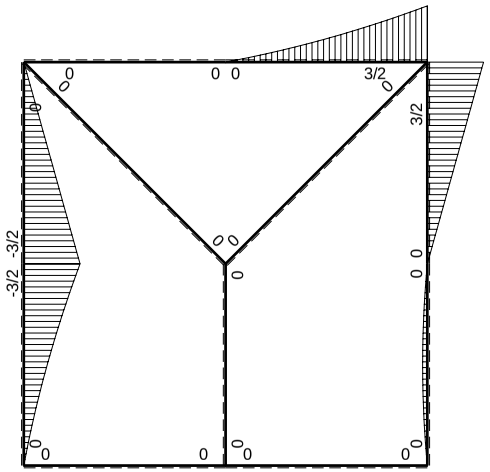
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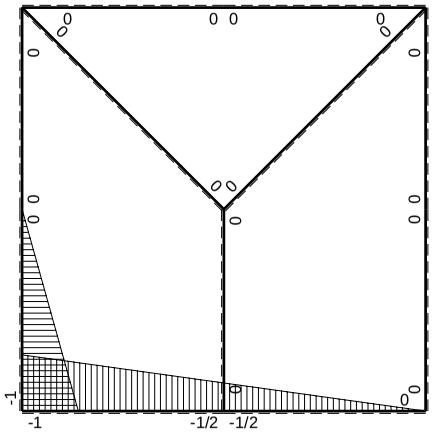

0
0 0
3/2
0
0
3/2
-3/2
-3/2
0
0 0
0
0
-19/24
-19/24
-19/48
-19/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237549 ESΣ04.xxxx.161

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PROCEDIMENTO E RISULTATI 237549 ESΣ04.xxxx.161

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	Fb/EJ	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

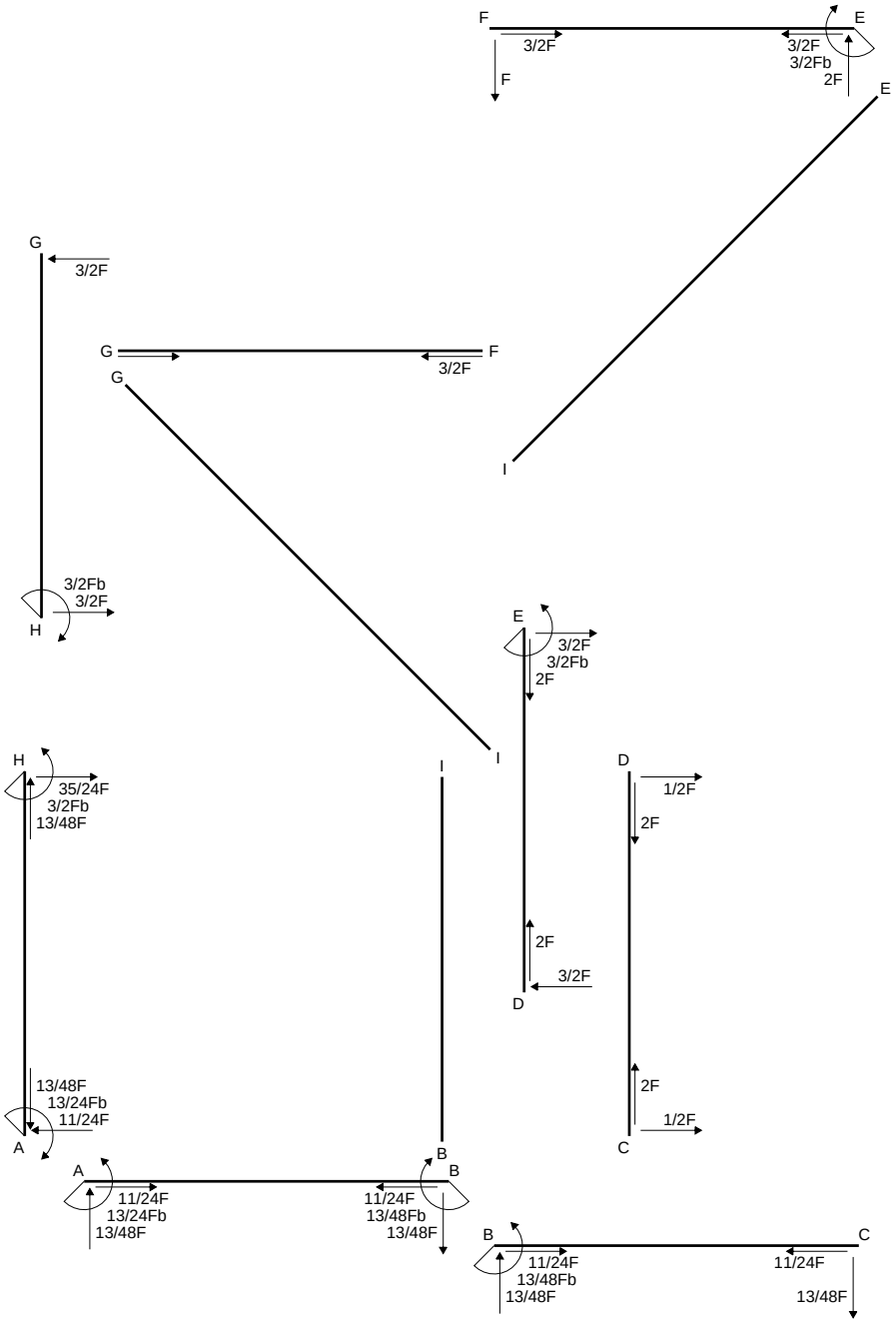
$= \left(3/4\ b - 2/3\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

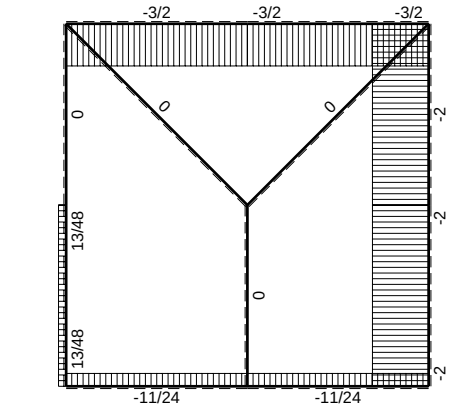
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

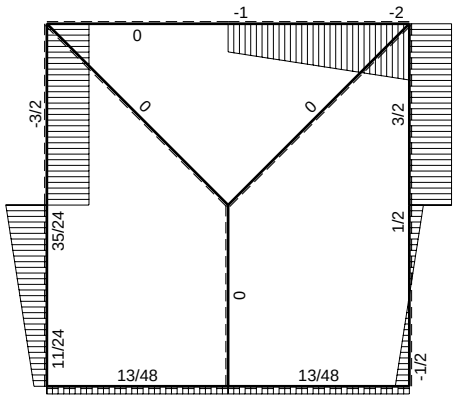
$= \left(1/2\ b - 1/6\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

ESΣ04.xxxx.161

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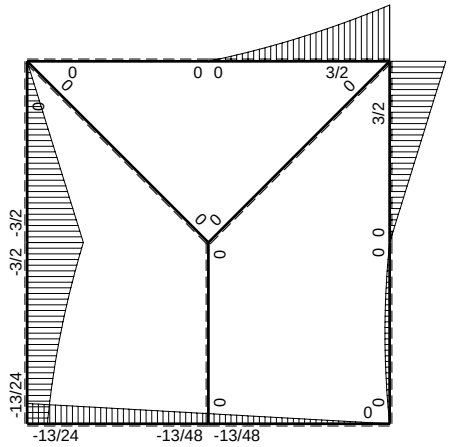

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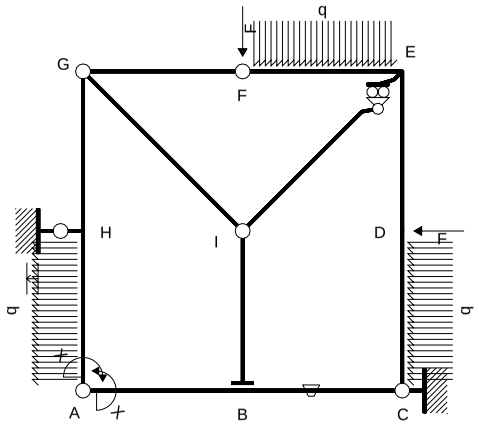

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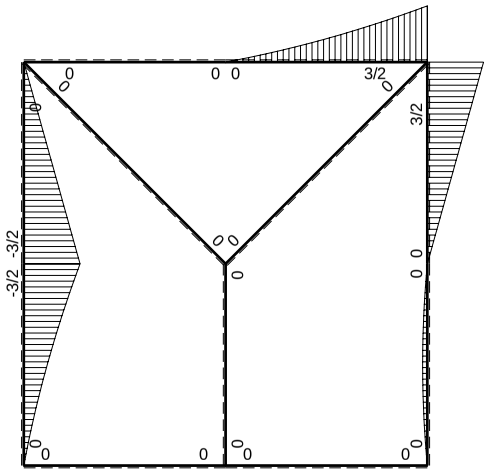
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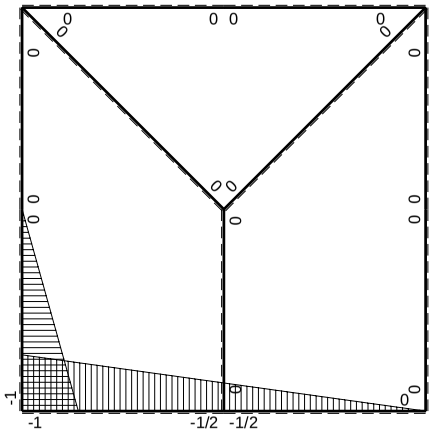

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 966407 ESΣ04.xxxx.162

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	-Fb/EJ	0	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(0+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	0	Fb/EJ	0	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-13/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						13/24Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 966407 ESΣ04.xxxx.162
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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(1/2\ b - 1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/2\ x/b\ \right)\theta\ \ dx = \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

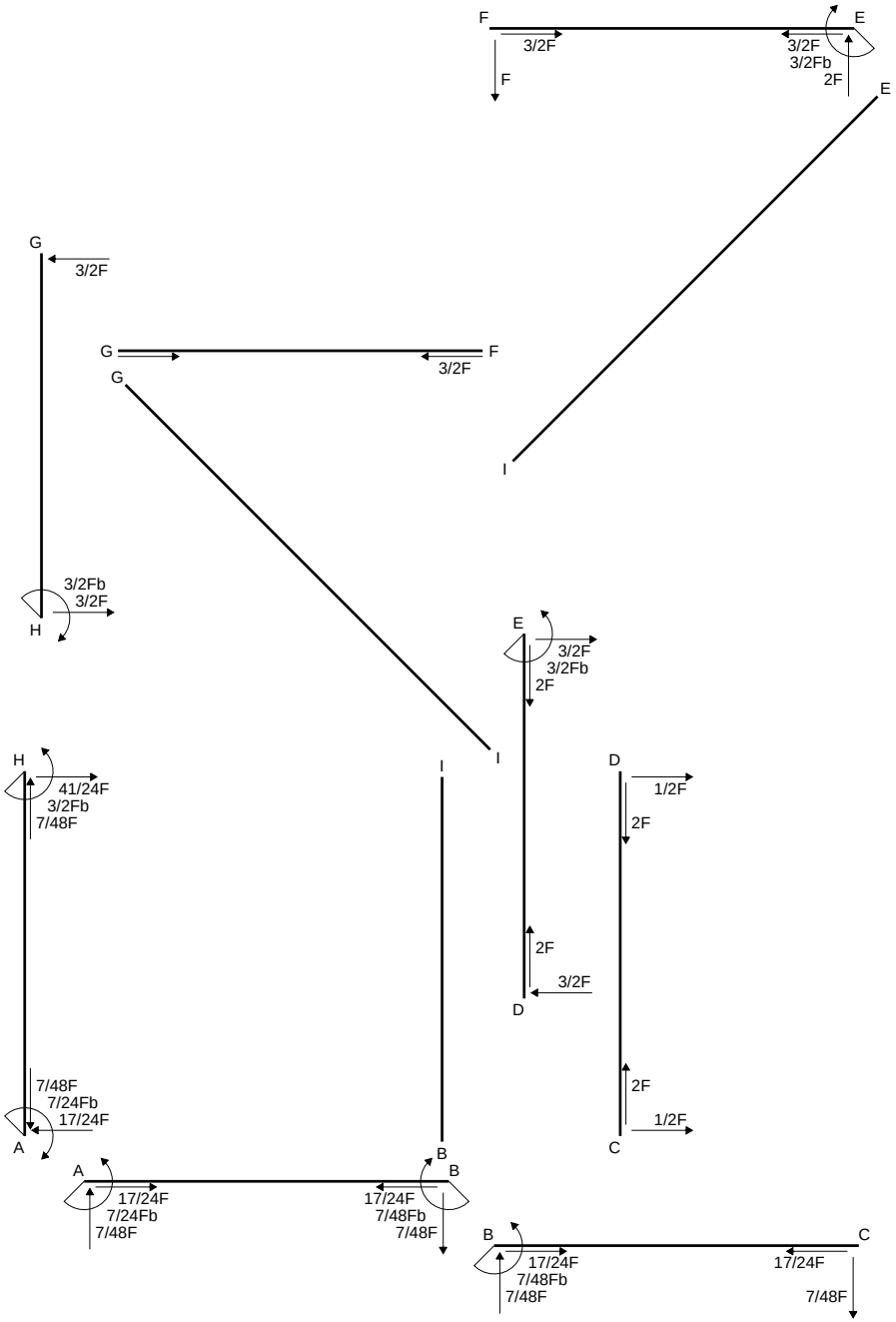
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

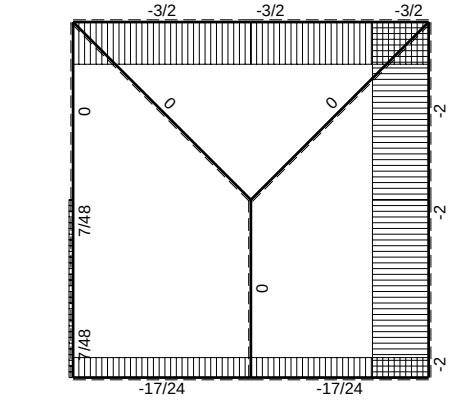
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

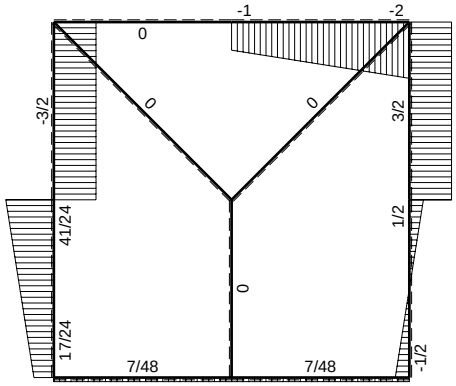
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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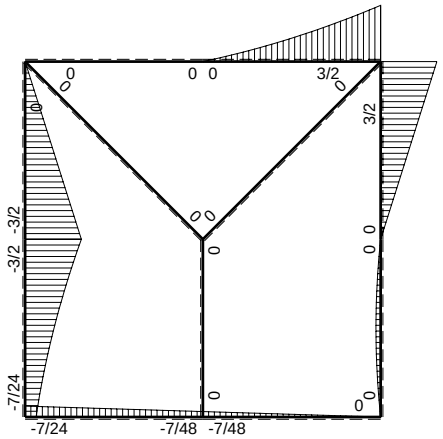

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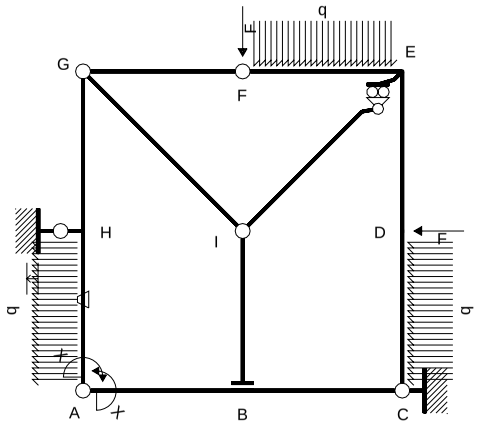

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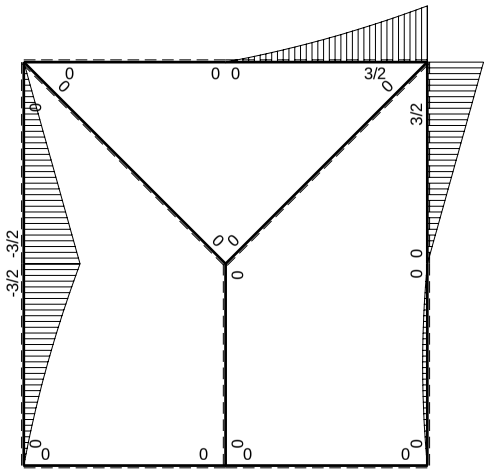
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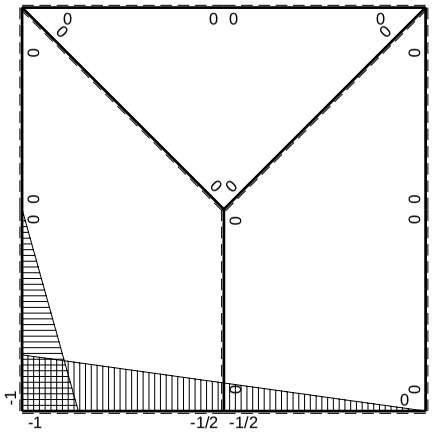

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 213077 ESΣ04.xxxx.163

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	-Fb/EJ	$3/2Fx-2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/24+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	Fb/EJ	$Fx-1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/24Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213077 ESΣ04.xxxx.163
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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\theta\ \ dx$

$= \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\theta\ \ = 17/24\ \ Fb^2/EJ$

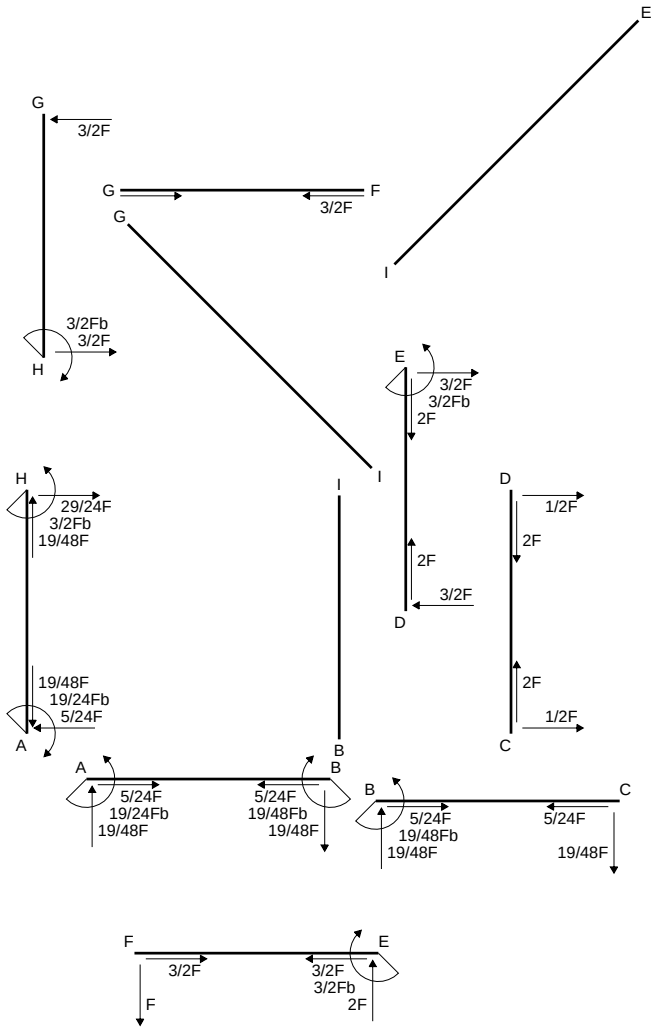
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\theta\ \ dx$

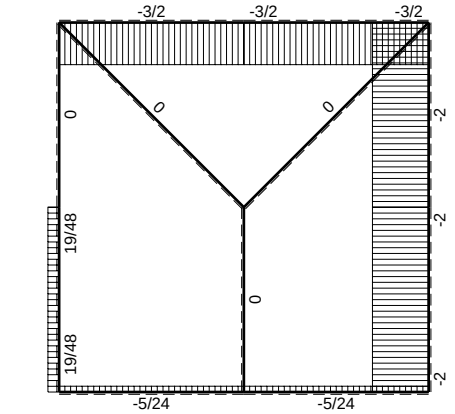
$= \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

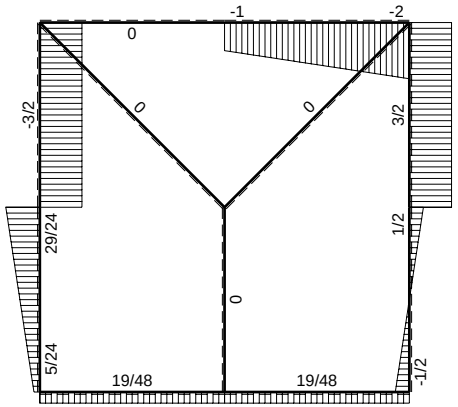
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\theta\ \ = 17/24\ \ Fb^2/EJ$

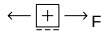
ESΣ04.xxxx.163

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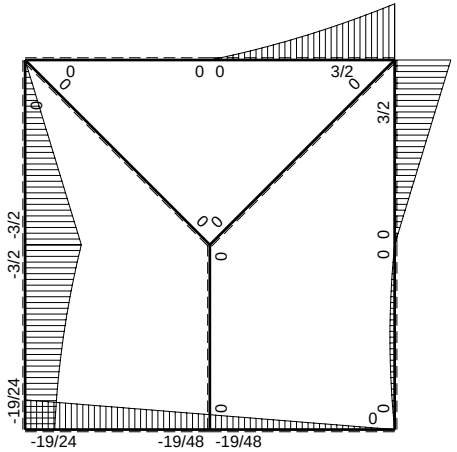

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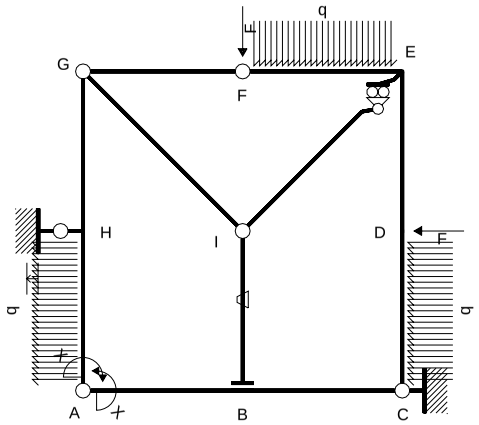

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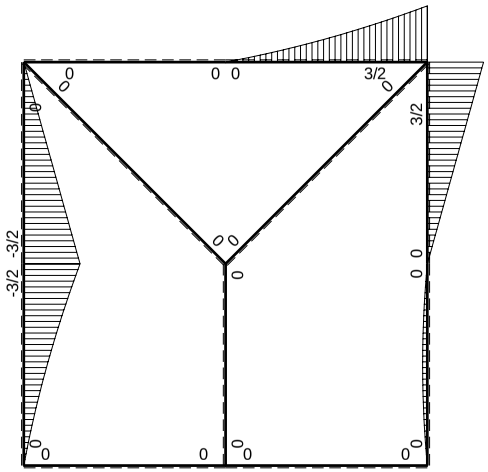

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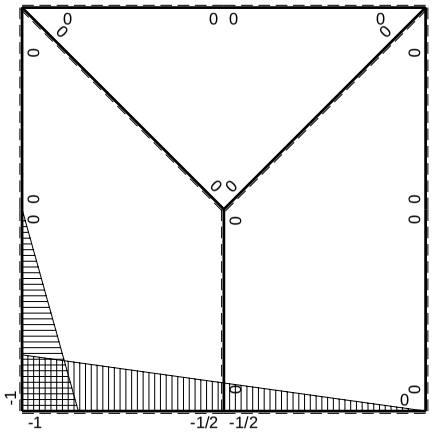

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239868 ESΣ04.xxxx.164

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PROCEDIMENTO E RISULTATI 239868 ESΣ04.xxxx.164

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

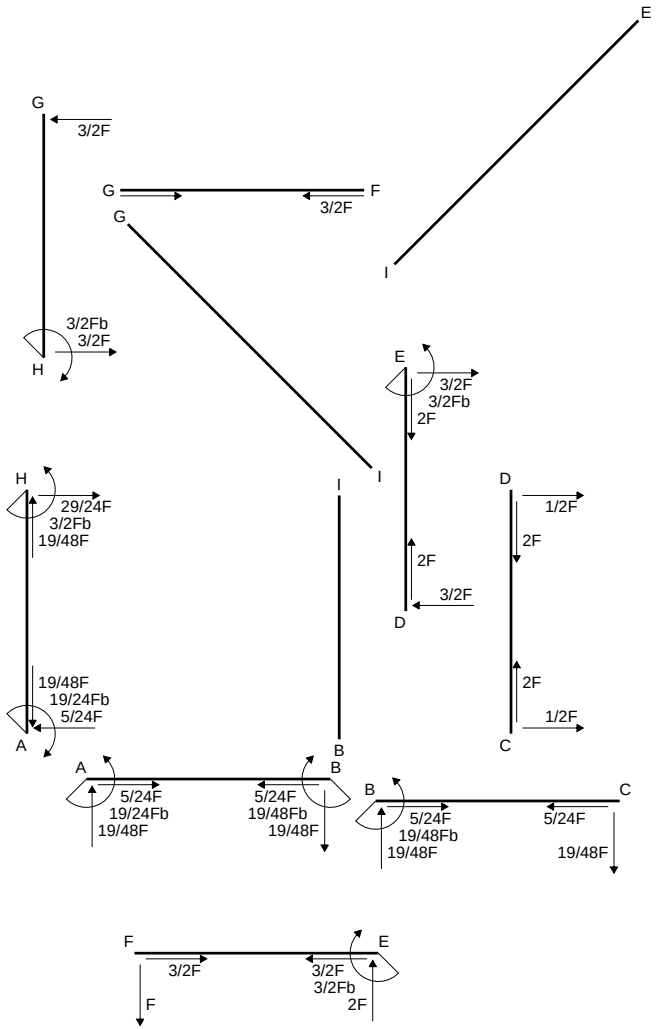
$= \left(3/4\ b - 2/3\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

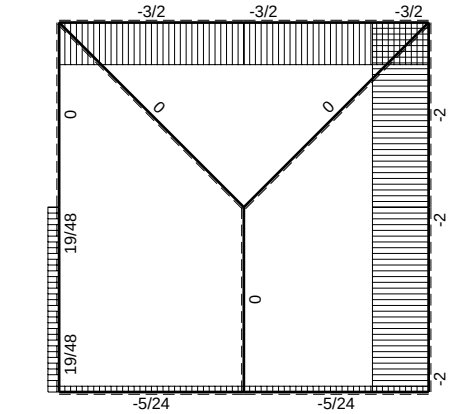
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

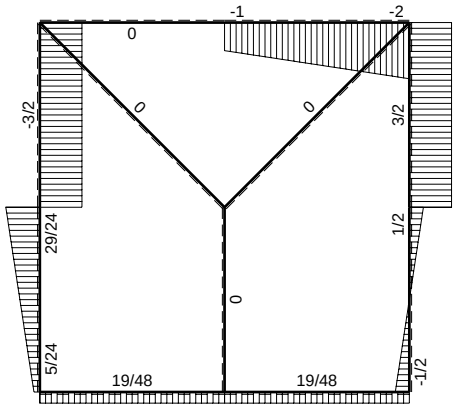
$= \left(1/2\ b - 1/6\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

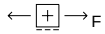
ESΣ04.xxxx.164

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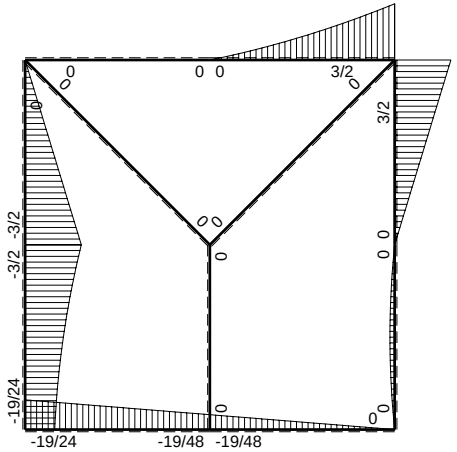

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2F
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2F
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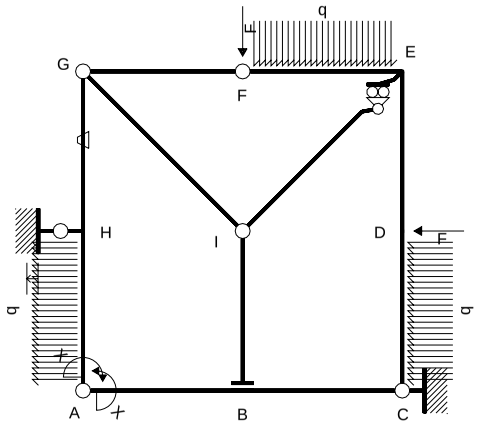

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-1/2

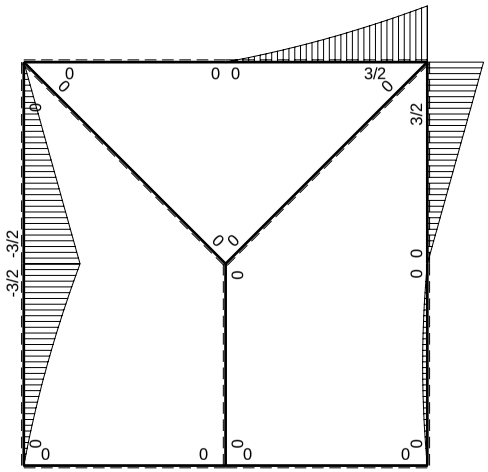

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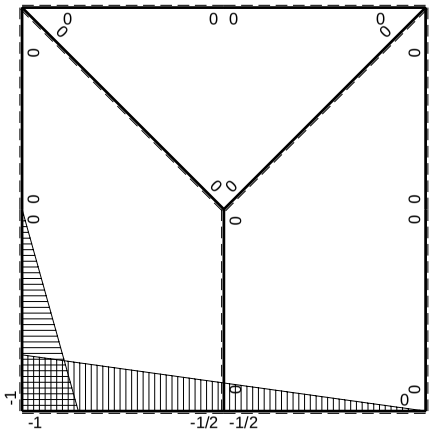

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-3/2
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0 0
0
0
-19/24
-19/24
-19/48
-19/48


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 983188 ESΣ04.xxxx.165

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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ESΣ04.xxxx.165

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

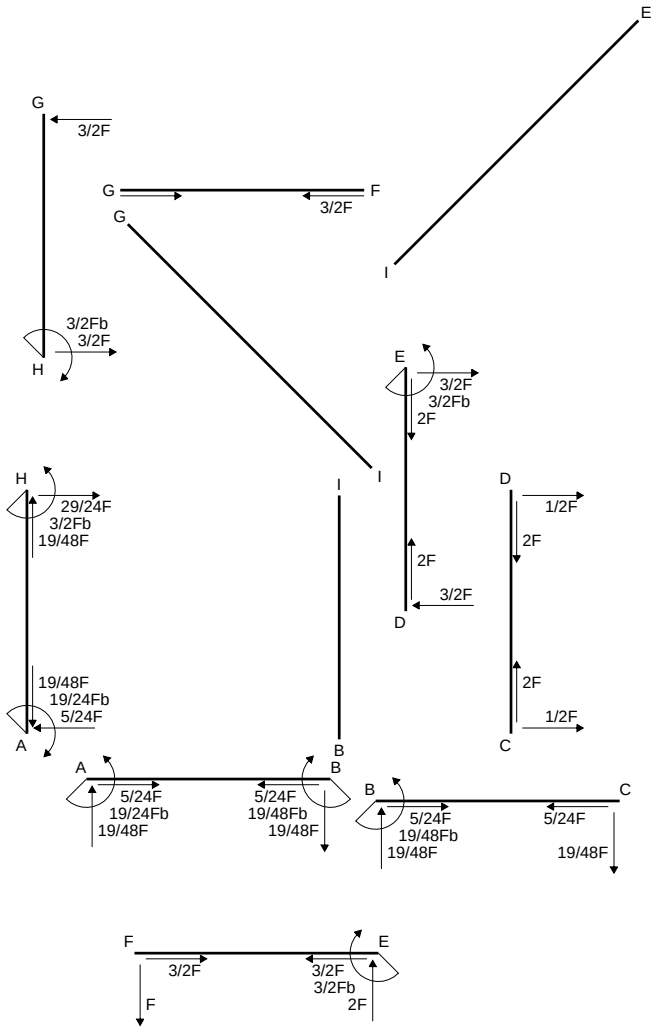
$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

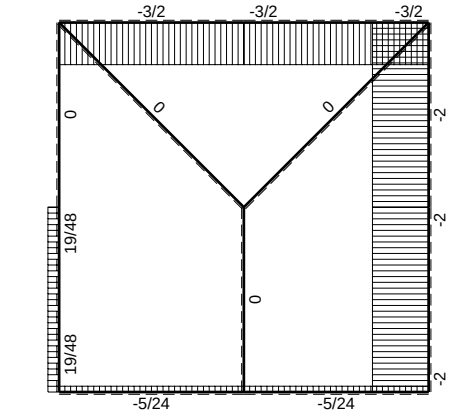
$= \left(3/4\ b - 2/3\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

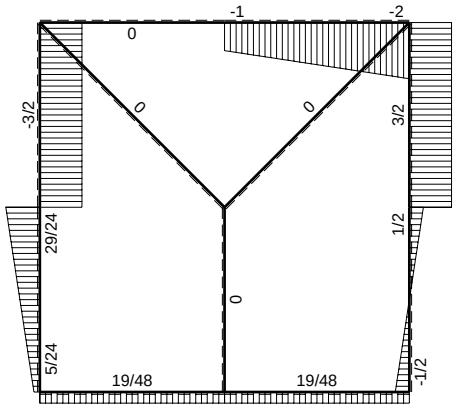
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

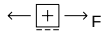
$= \left(1/2\ b - 1/6\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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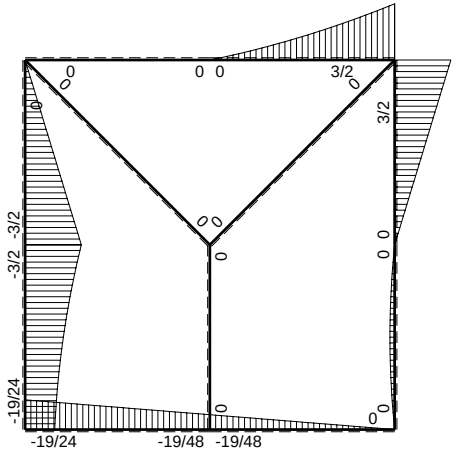

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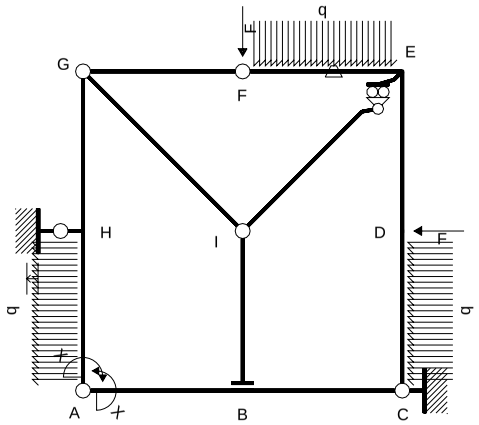

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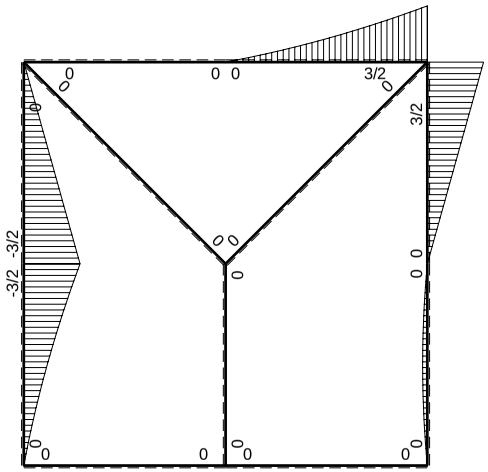

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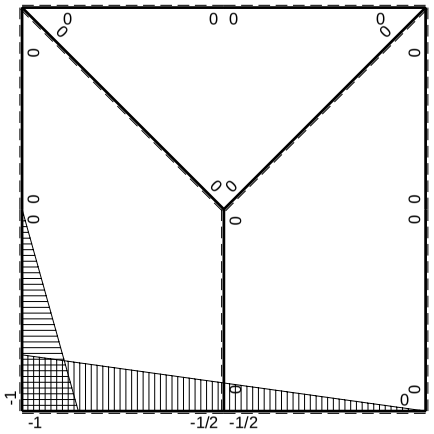

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-19/48
-19/48


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237215 ESΣ04.xxxx.166

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

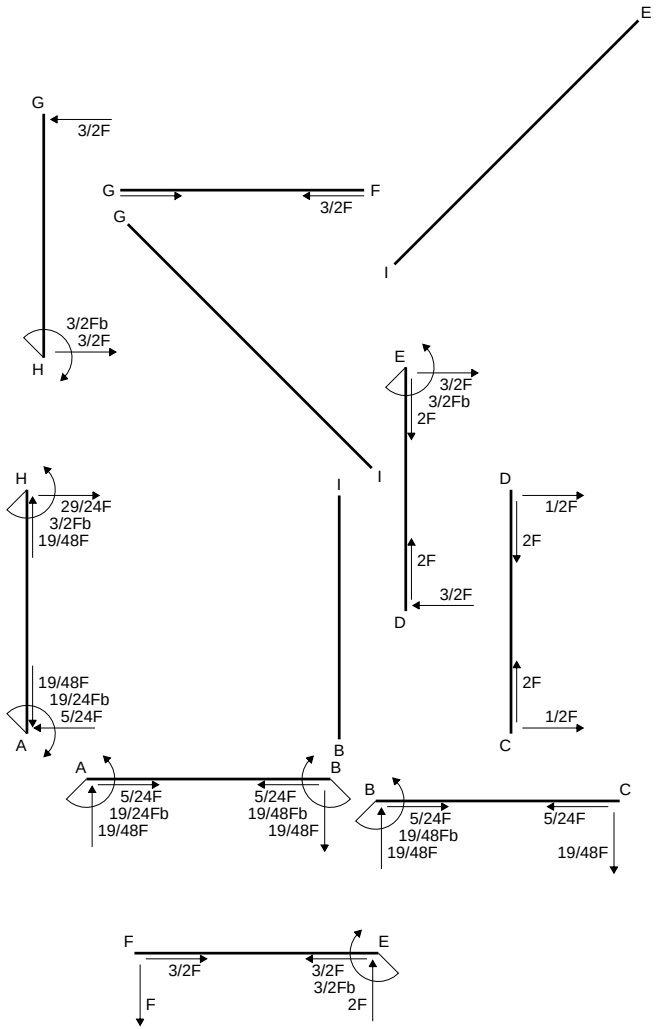
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

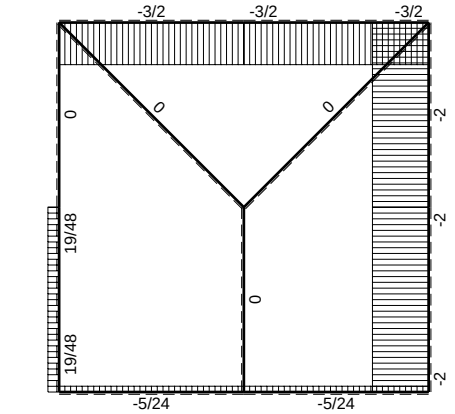
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

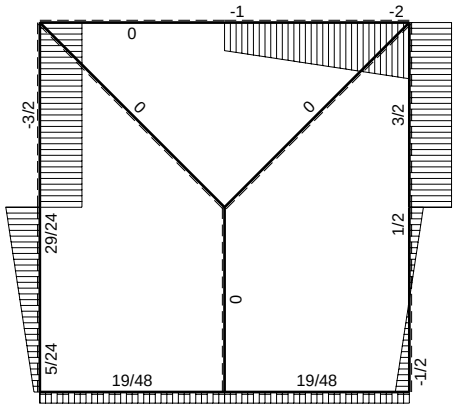
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

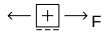
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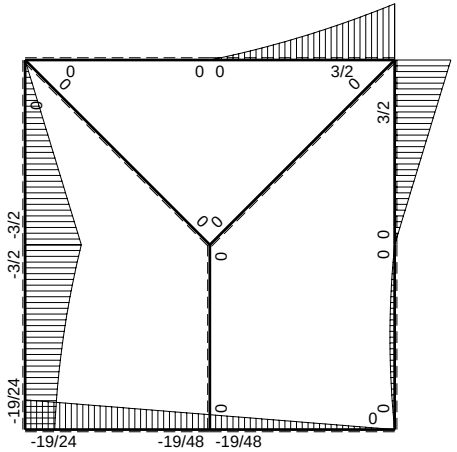

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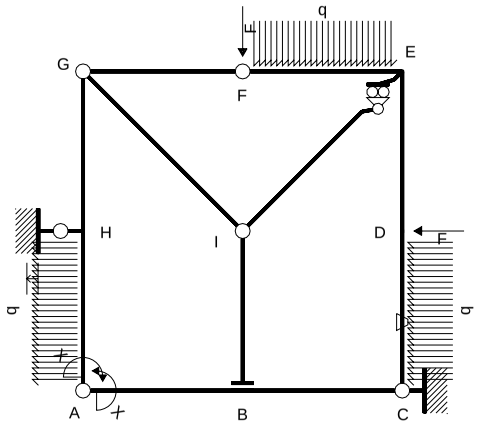

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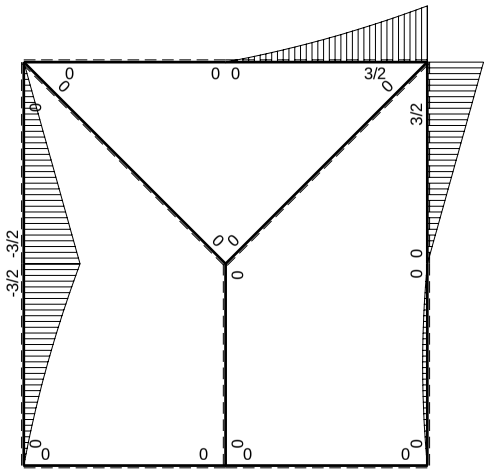

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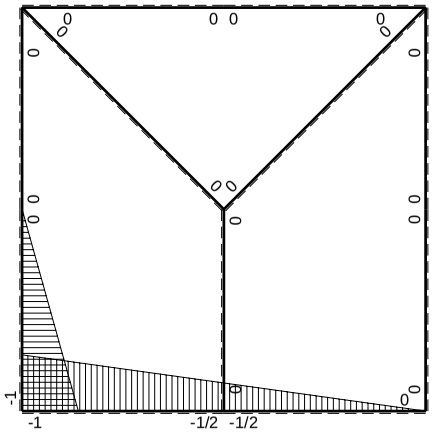

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-19/24
-19/24
-19/48
-19/48


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-1/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

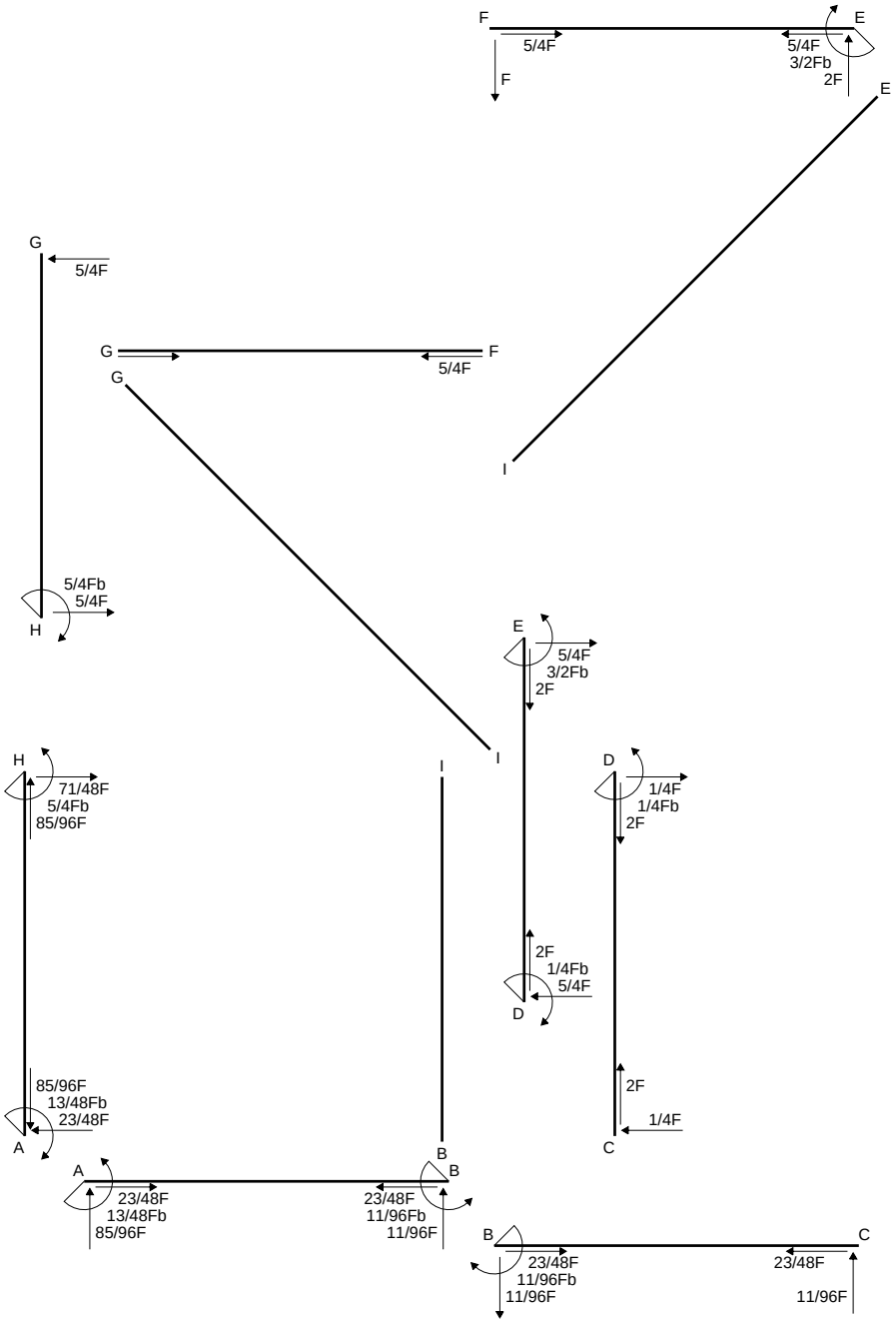
$= \left(3/4\ b - 2/3\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

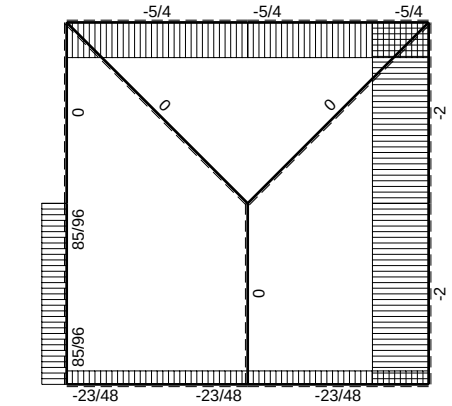
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

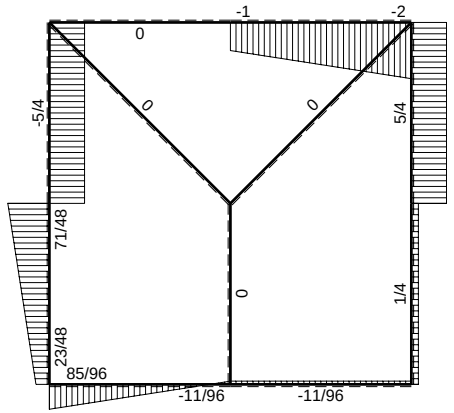
$= \left(1/2\ b - 1/6\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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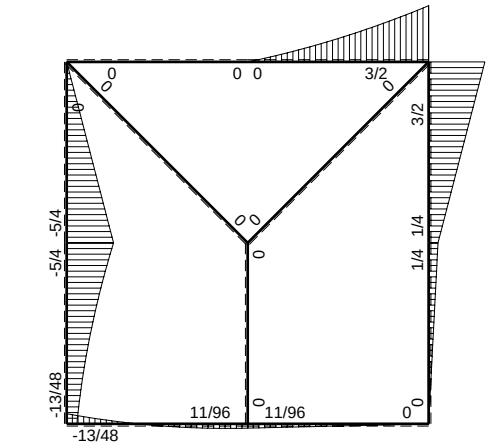

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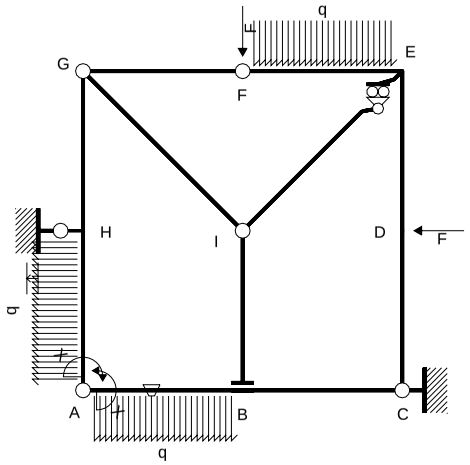

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-11/96

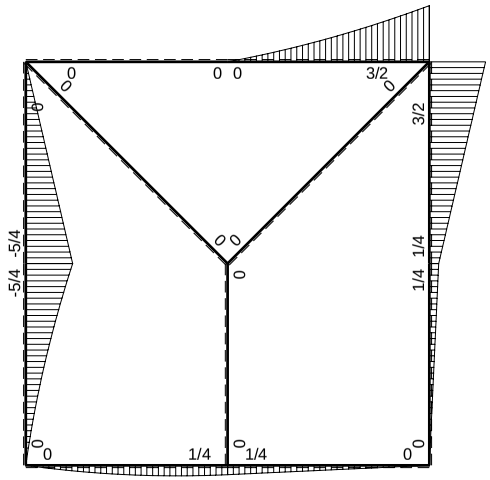
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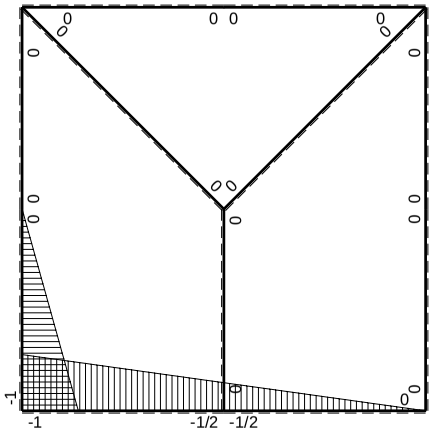

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-13/48
11/96
11/96
0
0
-13/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x\theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-1+1/2x/b	3/4Fx-1/2qx^2	-Fb/EJ	-3/4Fx+7/8Fx^2/b-1/4qx^3/b	Fb/EJ-1/2Fx/EJ	1-x/b+1/4x^2/b^2	(-7/48+3/4)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb-1/4Fx+1/2qx^2	Fb/EJ	-1/8Fb-1/4Fx+1/8Fx^2/b+1/4qx^3/b	1/2Fb/EJ+1/2Fx/EJ	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	1/4Fb-1/4Fx	0	-1/8Fb+1/4Fx-1/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-1/24+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-1/4Fx	0	-1/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-2Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-Fx-1/2qx^2	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-5/4Fx	0	0	0	0	0+0	0
HG b	0	5/4Fb-5/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	-5/4Fb+7/4Fx-1/2qx^2	0	5/4Fx-7/4Fx^2/b+1/2qx^3/b	0	x^2/b^2	(1/6+0)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	3/4Fx+1/2qx^2	0	3/4Fx-1/4Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-13/48Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						13/48Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 7/8\ x^2/b^2 - 1/4\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\theta\ dx$

$= \left[-3/8\ x^2/b + 7/24\ x^3/b^2 - 1/16\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(-3/8\ b + 7/24\ b - 1/16\ b\right) Fb\ 1/EJ\ + \left(\ b - 1/4\ b\right)\theta\ \ = 29/48\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 - 1/4\ x/b + 1/8\ x^2/b^2 + 1/4\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\theta\ dx$

$= \left[-1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2 + 1/16\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(-1/8\ b - 1/8\ b + 1/24\ b + 1/16\ b\right) Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\right)\theta\ \ = 29/48\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/8 + 1/4\ x/b - 1/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/8\ x^2/b - 1/24\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/8\ b + 1/8\ b - 1/24\ b\right) Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/24\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/24\ b\right) Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/4\ x/b - 7/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 7/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

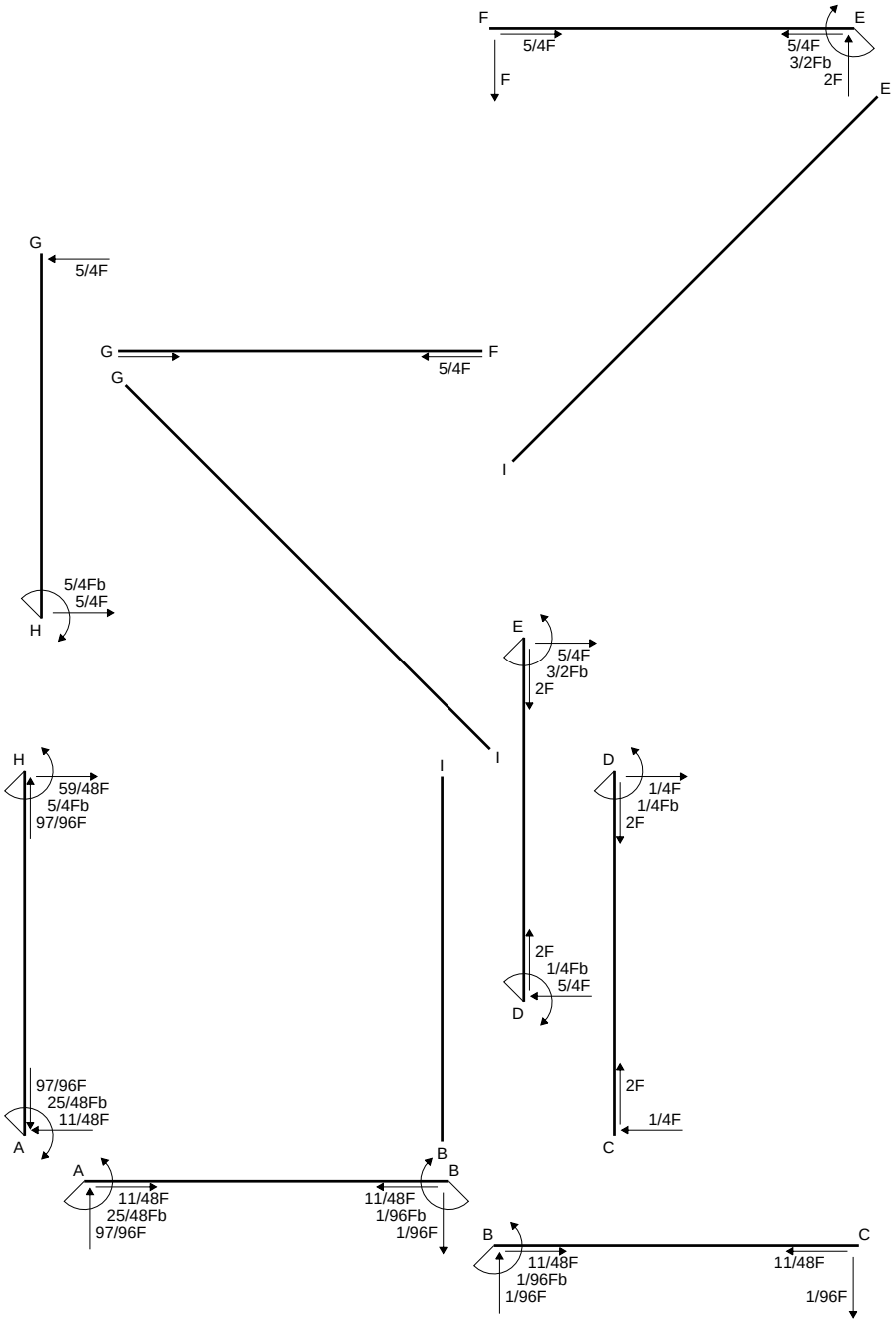
$= \left(5/8\ b - 7/12\ b + 1/8\ b\right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

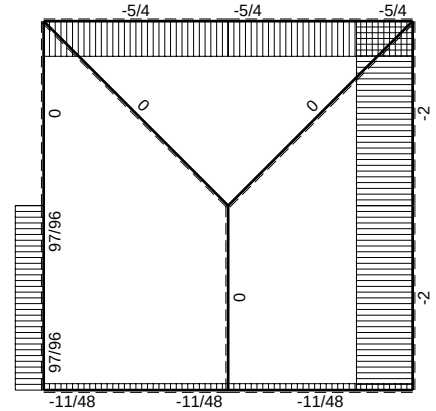
$L_{AH}^{Xo} = \int_o^b \left(3/4\ x/b - 1/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[3/8\ x^2/b - 1/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

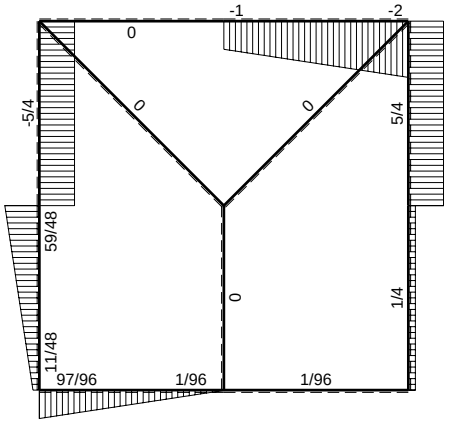
$= \left(3/8\ b - 1/12\ b - 1/8\ b\right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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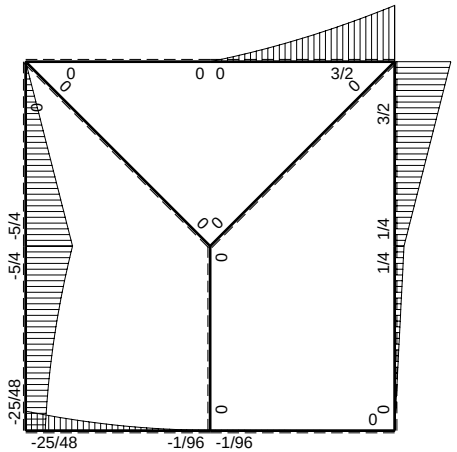

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G
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3/2Fb
2F
I
H
59/48F
5/4Fb
97/96F
I
D
1/4F
1/4Fb
2F
2F
1/4Fb
5/4F
D
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B
11/48F
1/96Fb
1/96F
11/48F
1/96F
C


-5/4
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97/96
-11/48
-11/48
-11/48

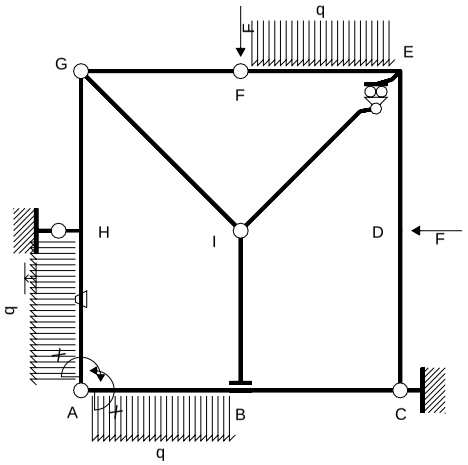

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5/4
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1/4
11/48
97/96
1/96
1/96

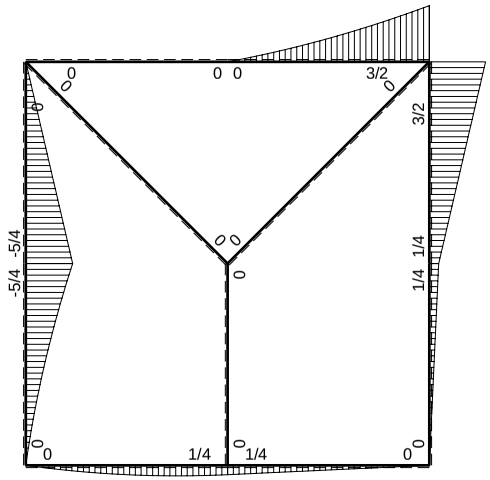
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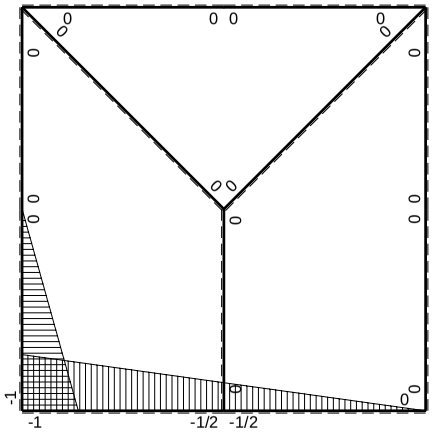

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0
3/2
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-5/4
-5/4
0
1/4
1/4
0
0
0
-25/48
-25/48
-1/96
-1/96

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	$3/4Fx-1/2qx^2$	0	$-3/4Fx+7/8Fx^2/b-1/4qx^3/b$	0	$1-x/b+1/4x^2/b^2$	$(-7/48+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb-1/4Fx+1/2qx^2$	0	$-1/8Fb-1/4Fx+1/8Fx^2/b+1/4qx^3/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/4Fb-1/4Fx	0	$-1/8Fb+1/4Fx-1/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/4Fx	0	$-1/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-5/4Fx	0	0	0	0	0+0	0
HG b	0	5/4Fb-5/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-5/4Fb+7/4Fx-1/2qx^2$	-Fb/EJ	$5/4Fx-7/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/6+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$3/4Fx+1/2qx^2$	Fb/EJ	$3/4Fx-1/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-25/48Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						25/48Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-3/4\ x/b + 7/8\ x^2/b^2 - 1/4\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 7/24\ x^3/b^2 - 1/16\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 7/24\ b - 1/16\ b\ \right)\ Fb\ 1/EJ\ = -7/48\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 - 1/4\ x/b + 1/8\ x^2/b^2 + 1/4\ x^3/b^3\right)\ Fb\ 1/EJ\ dx$

$= \left[-1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2 + 1/16\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b - 1/8\ b + 1/24\ b + 1/16\ b\ \right)\ Fb\ 1/EJ\ = -7/48\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/8 + 1/4\ x/b - 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/8\ x^2/b - 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/8\ b - 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/4\ x/b - 7/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx$

$= \left[5/8\ x^2/b - 7/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(5/8\ b - 7/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

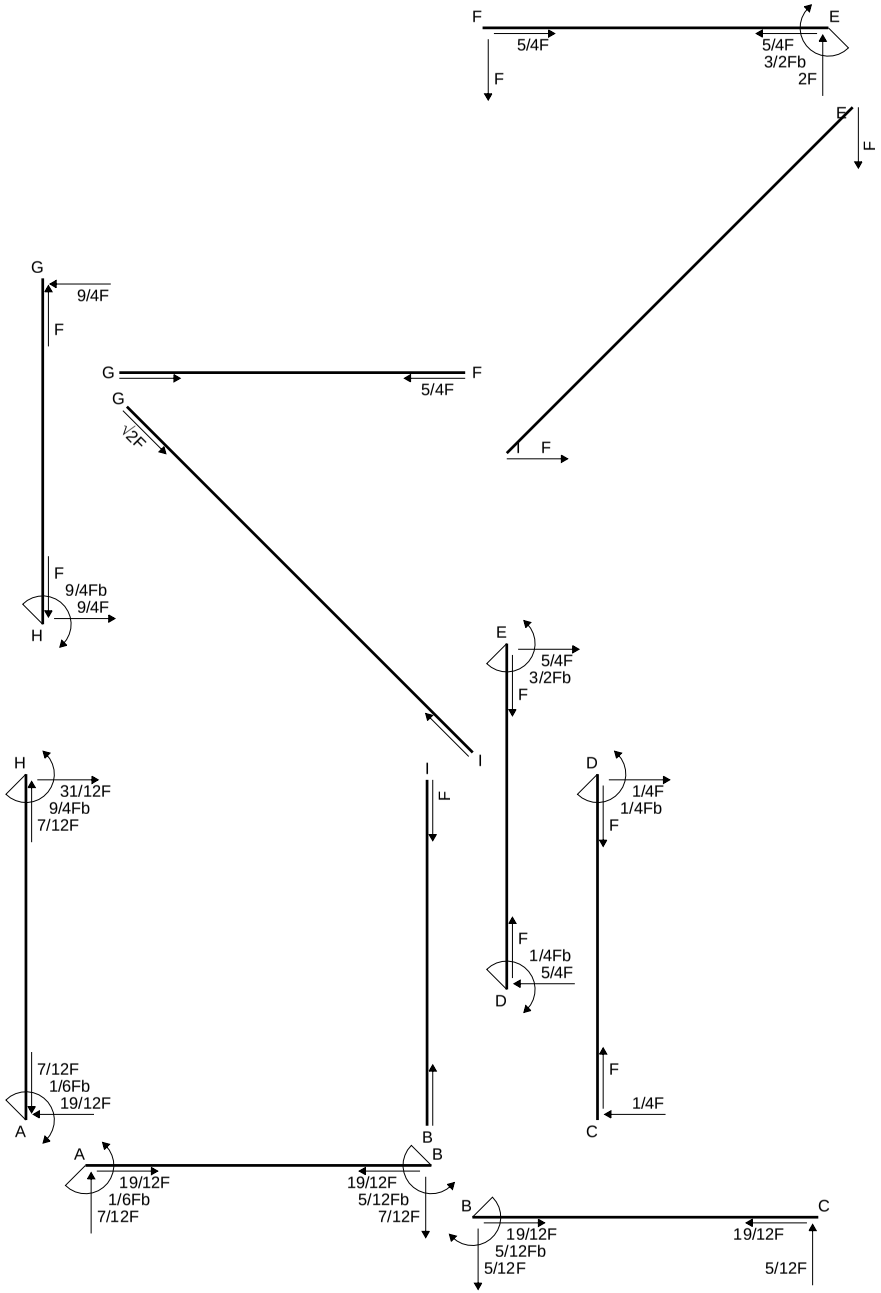
$L_{AH}^{Xo} = \int_o^b \left(3/4\ x/b - 1/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

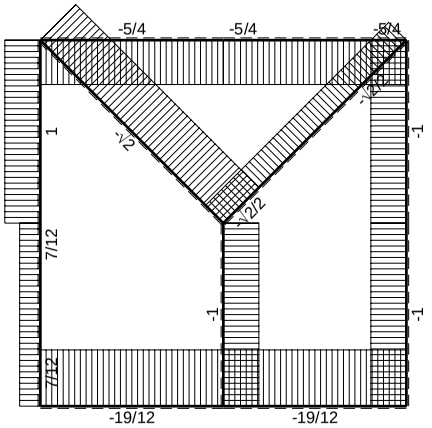
$= \left[3/8\ x^2/b - 1/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

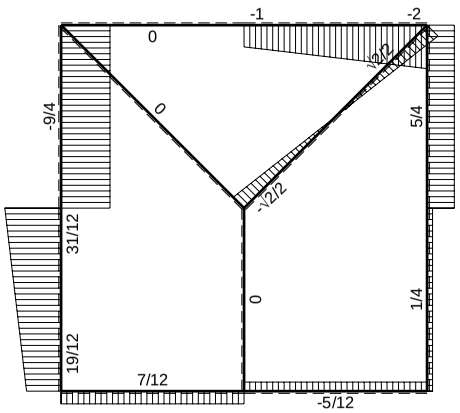
$= \left(3/8\ b - 1/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

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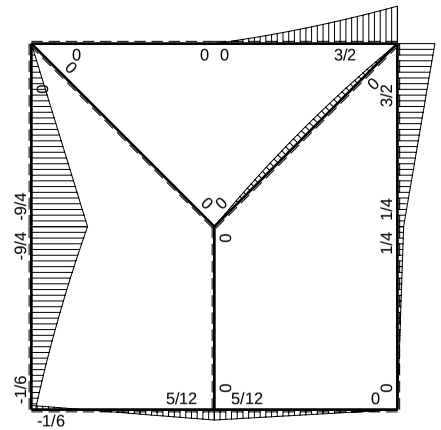

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3/2Fb
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A
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B
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-19/12

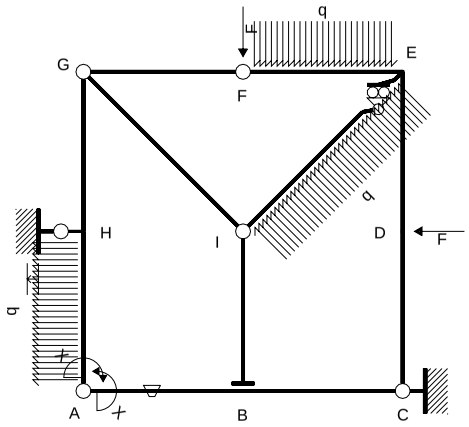

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7/12
-5/12

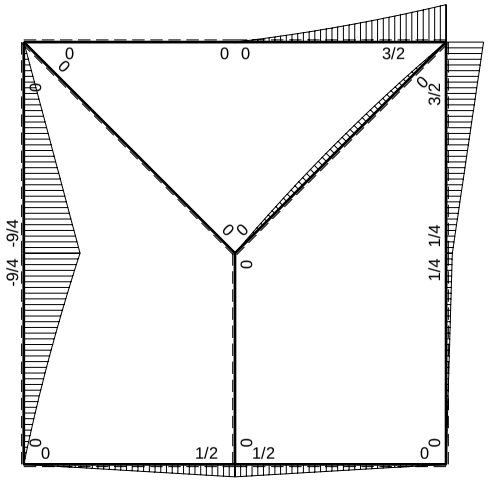
←□→ F

↑□↓ F

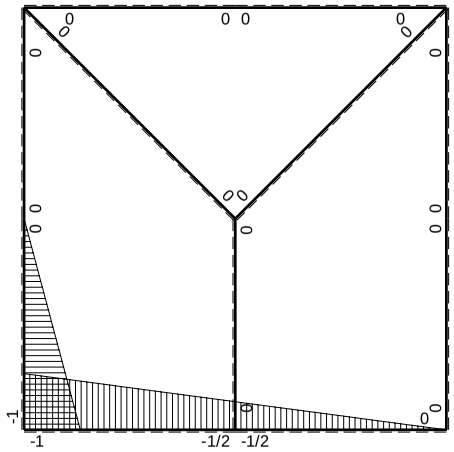

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3/2
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-9/4
0
1/4
1/4
0
-1/6
5/12
0
5/12
0 0
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 955642 ESΣ04.xxxx.170

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PROCEDIMENTO E RISULTATI 955642 ESΣ04.xxxx.170

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	-Fb/EJ	$-1/2Fx+1/4Fx^2/b$	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(-1/6+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/2Fb+1/2Fx	Fb/EJ	$-1/4Fb+1/4Fx^2/b$	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/6Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 7/12\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\right)\ \theta\ \ = 7/12\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

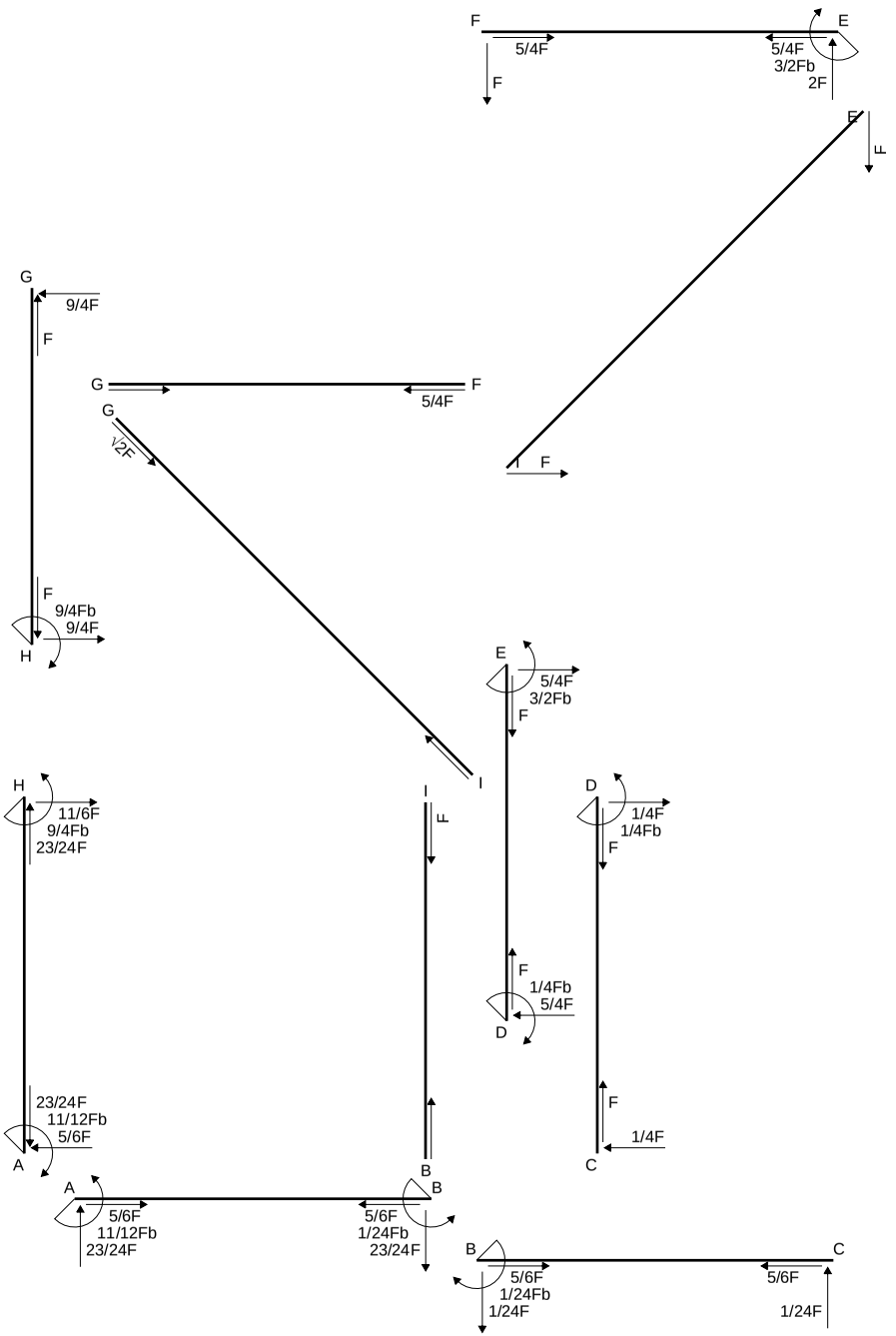
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

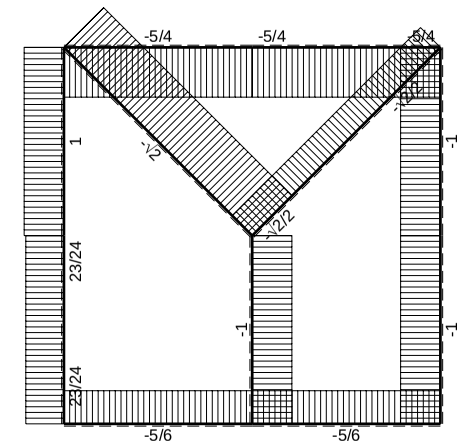
$L^{Xo}_{AH} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

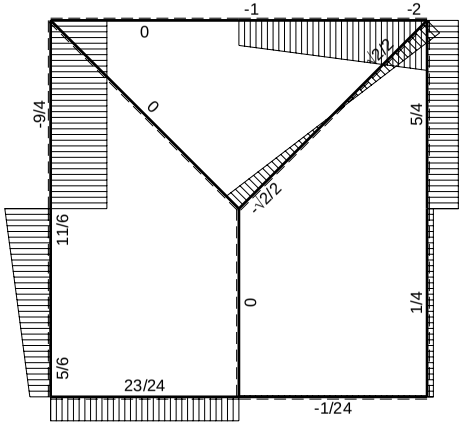
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 955642 ESΣ04.xxxx.170

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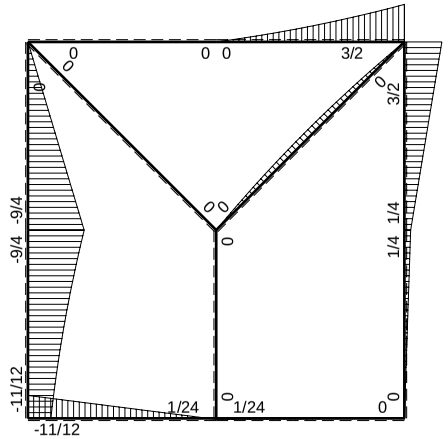

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5/4F
3/2Fb
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G
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G
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F
F
9/4Fb
9/4F
H
I
E
5/4F
3/2Fb
F
H
11/6F
9/4Fb
23/24F
I
F
D
1/4F
1/4Fb
F
F
1/4Fb
5/4F
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F
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C
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B
A
B
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B
C
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23/24
-5/6
-5/6

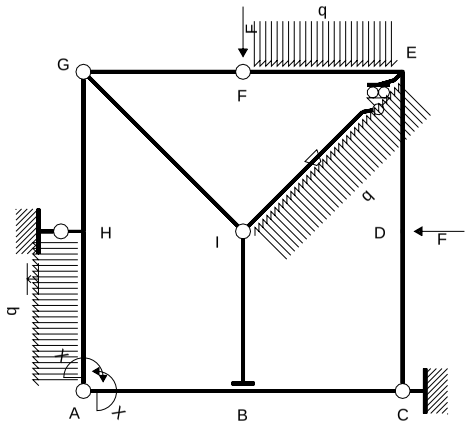

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5/6
23/24
-1/24

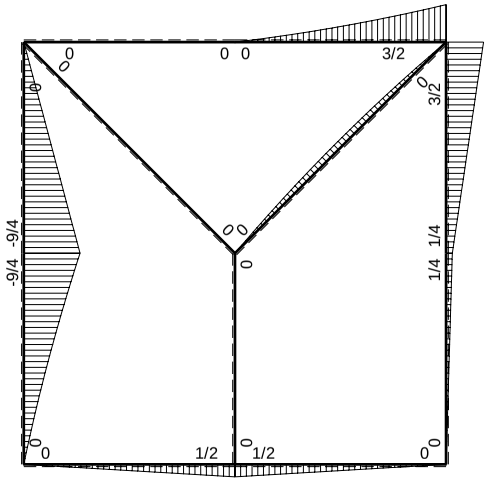
←□→ F

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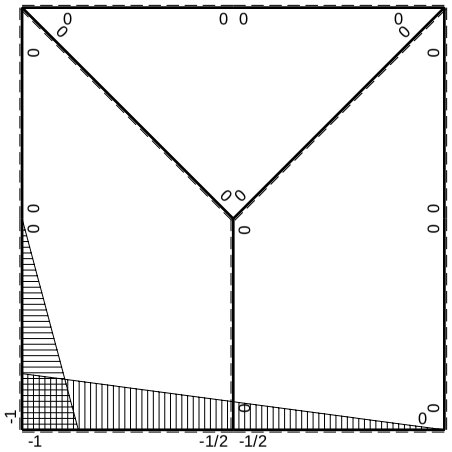

0
0 0
3/2
0
0
3/2
0 0
-9/4
-9/4
0
1/4
1/4
-11/12
1/24
0
1/24
0 0
-11/12

□ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238383 ESΣ04.xxxx.171

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	$-\sqrt{2}/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right) Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-1/12\ b\right) Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

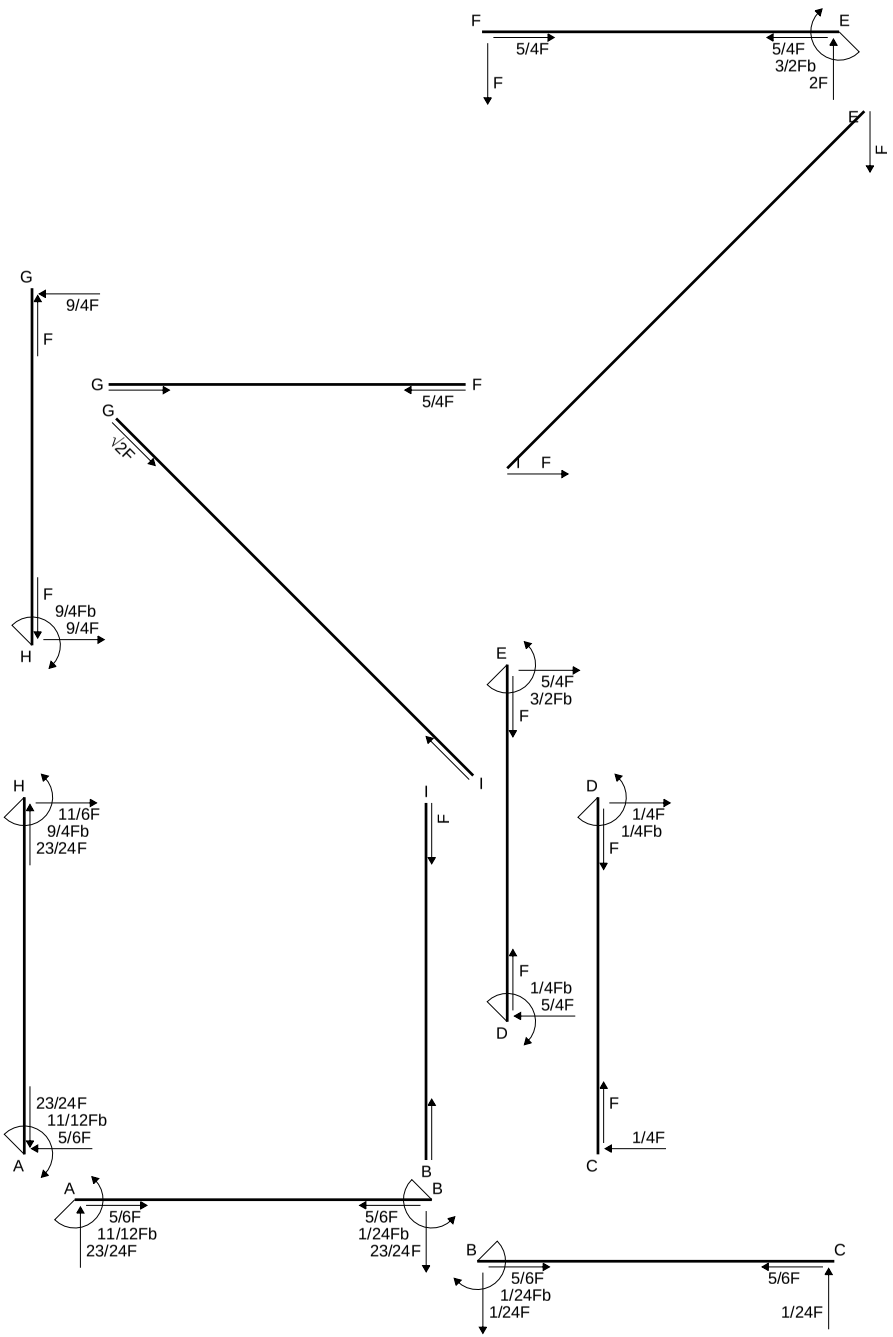
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right) Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

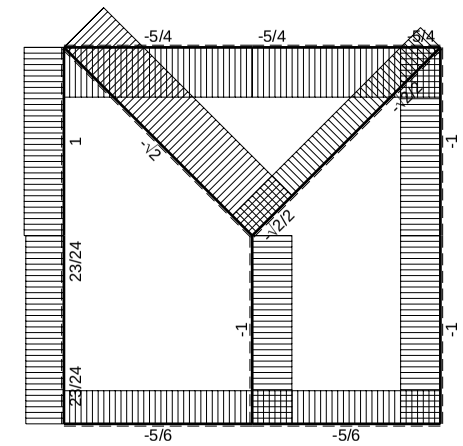
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

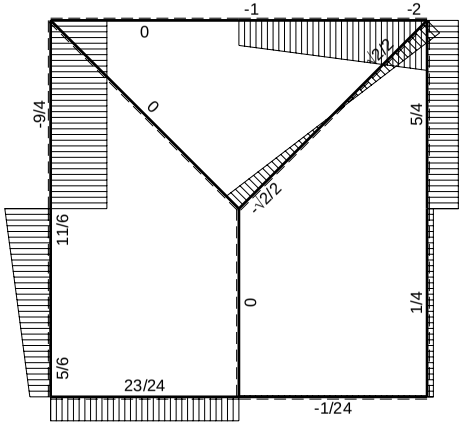
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right) Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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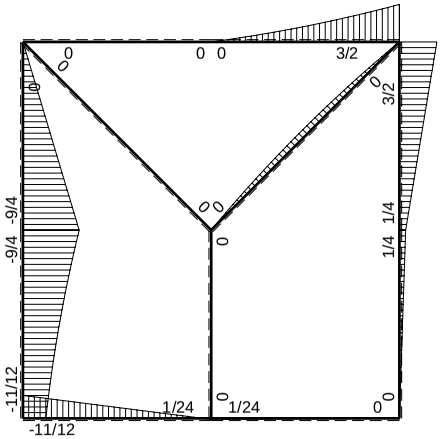

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23/24
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-5/6

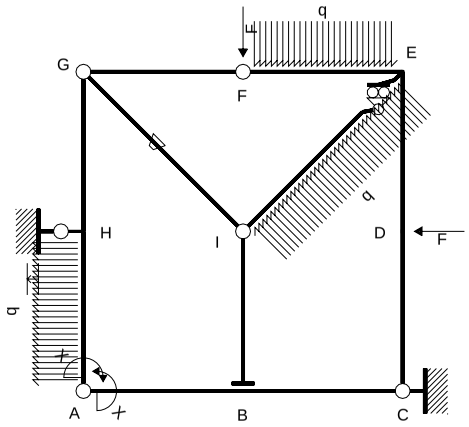

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-1/24

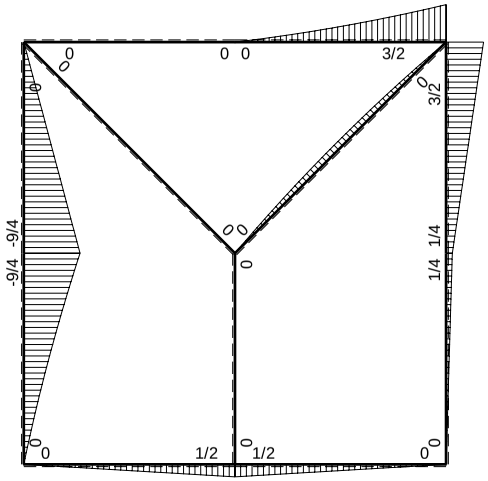
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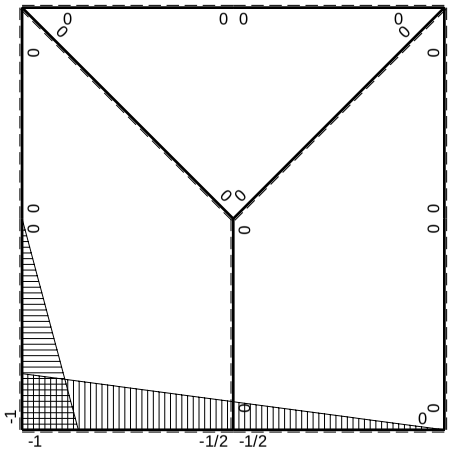

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0
3/2
0 0
0
-9/4
-9/4
1/4
1/4
0
1/24
1/24
0 0
-11/12
-11/12

□ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237138 ESΣ04.xxxx.172

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PROCEDIMENTO E RISULTATI 237138 ESΣ04.xxxx.172

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

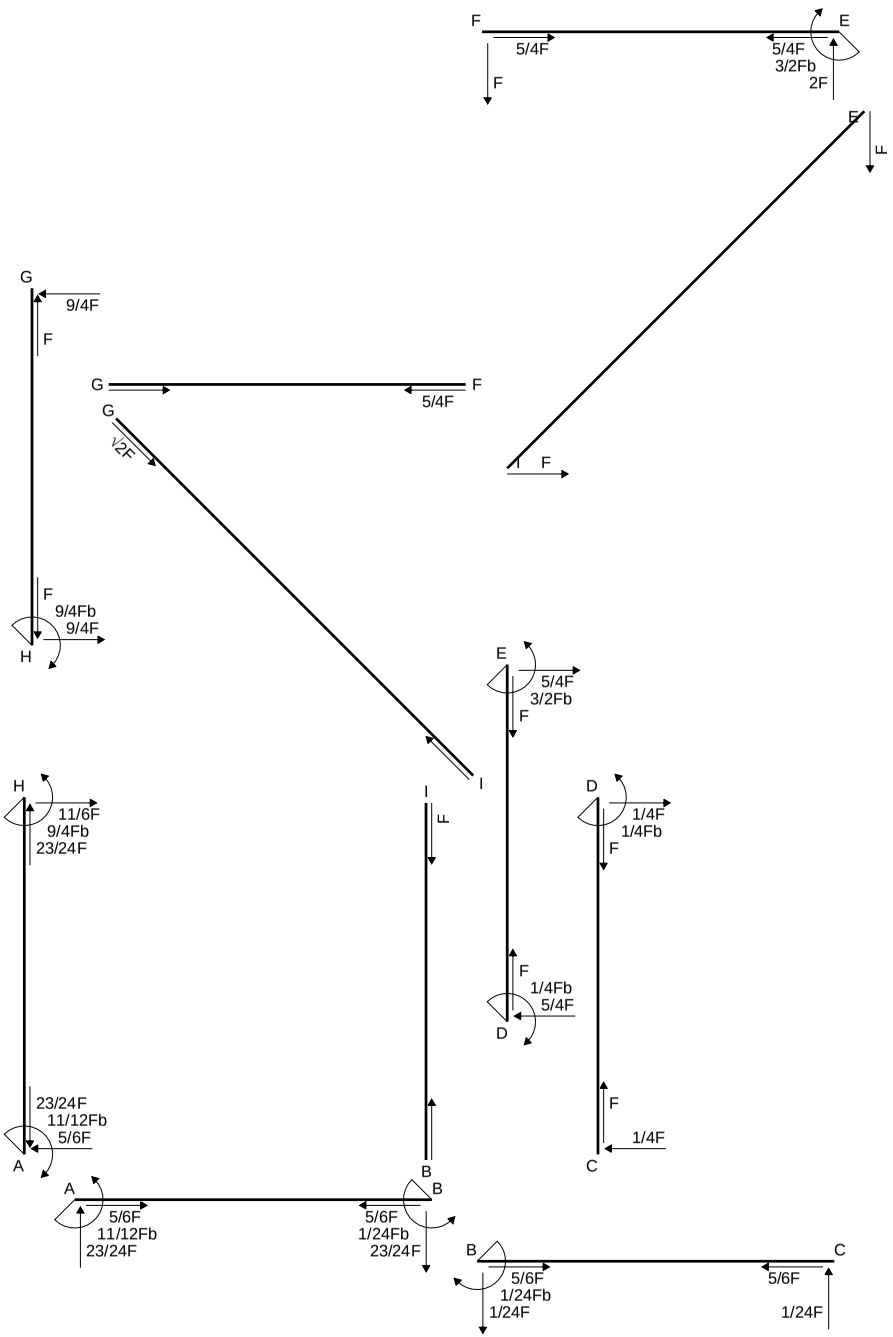
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

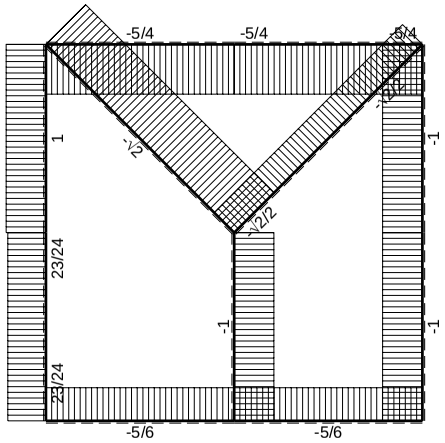
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

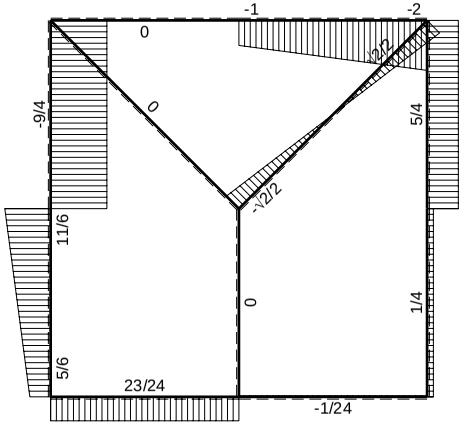
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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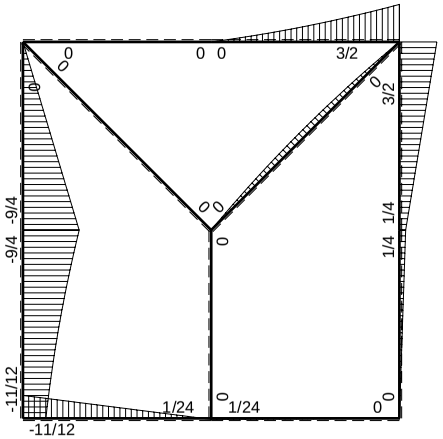

F
E
5/4F
5/4F
3/2Fb
2F
F
F
G
9/4F
F
G
F
5/4F
G
½F
F
F
9/4Fb
9/4F
H
I
E
5/4F
3/2Fb
F
H
11/6F
9/4Fb
23/24F
I
F
D
1/4F
1/4Fb
F
F
1/4Fb
5/4F
D
F
1/4F
C
23/24F
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5/6F
A
B
A
B
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5/6F
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B
C
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1/24Fb
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23/24
-5/6
-5/6

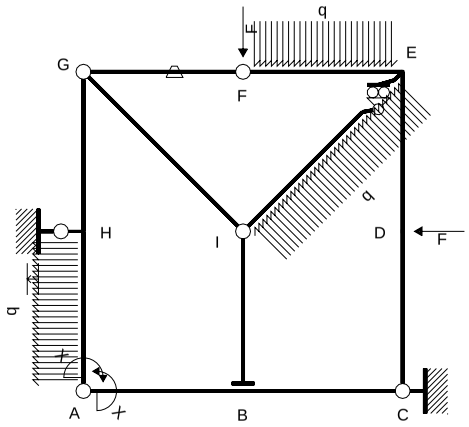

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-√2/2
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0
5/4
11/6
0
1/4
5/6
23/24
-1/24

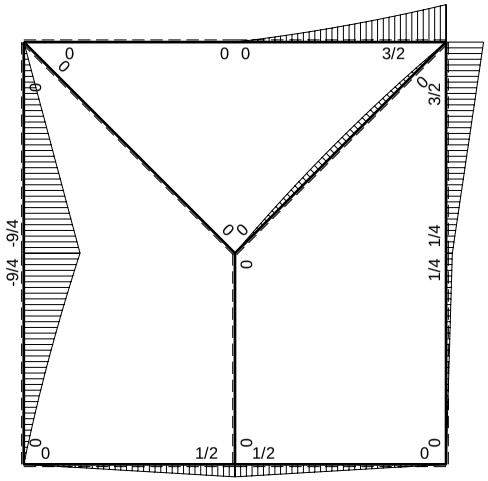
←□→ F

↑□↓ F

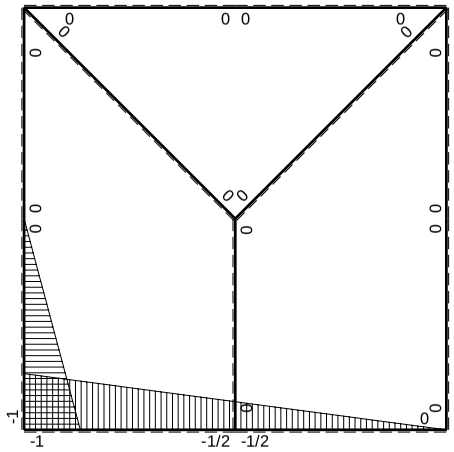

0
0 0
3/2
0
0
3/2
0
0
-9/4
-9/4
0
1/4
1/4
-11/12
1/24
0
1/24
0
0
-11/12

□ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

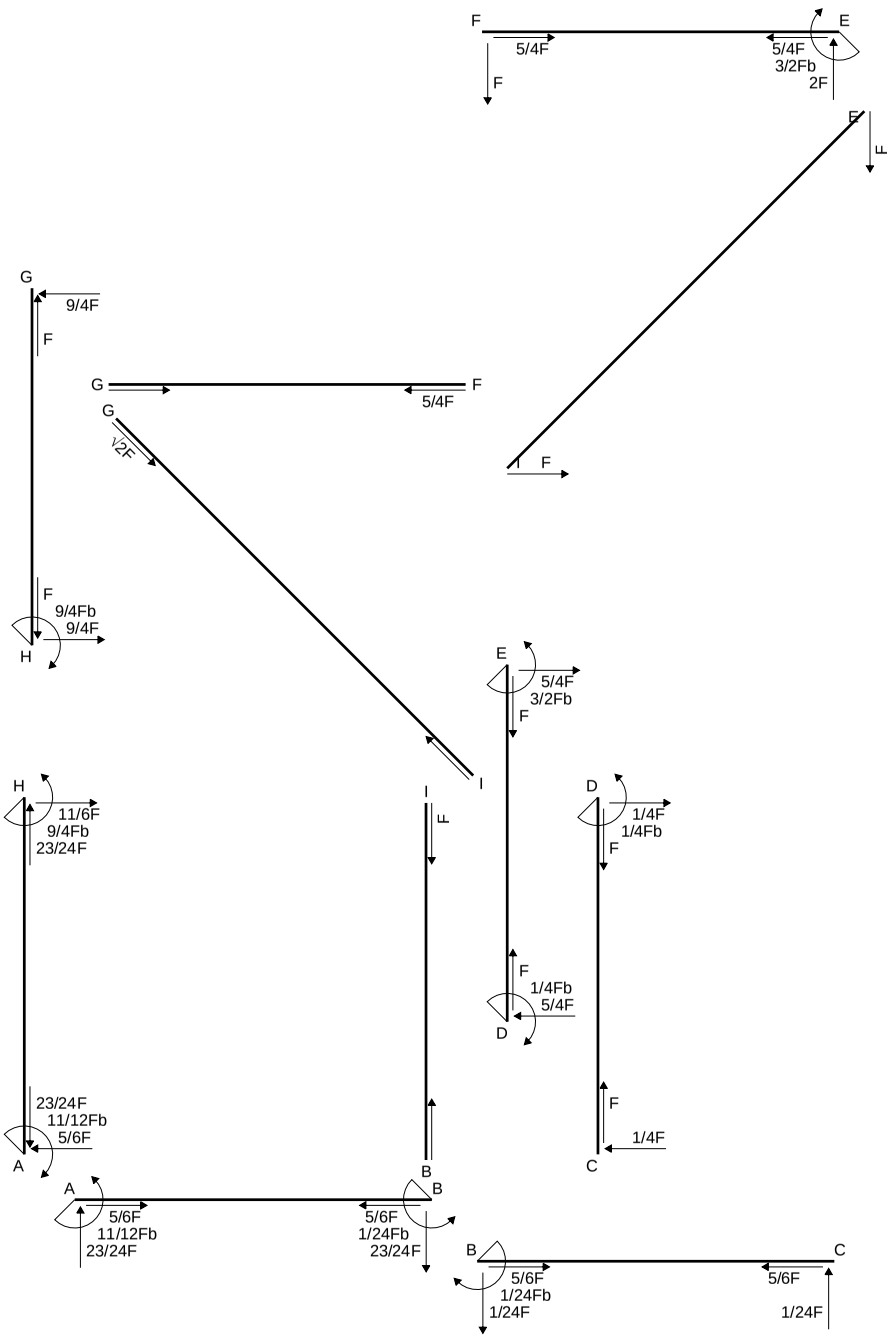
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

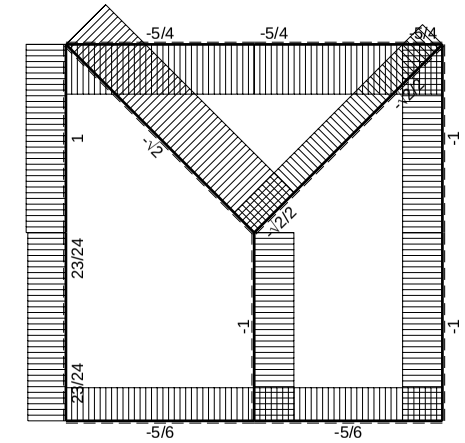
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

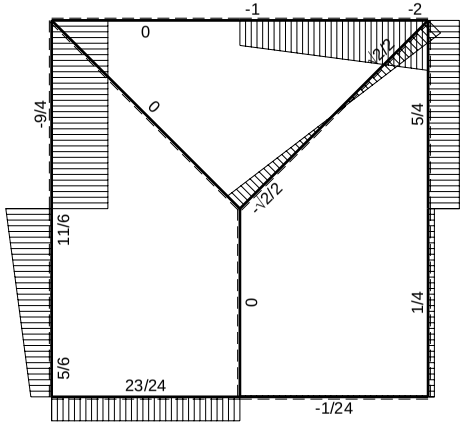
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

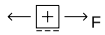
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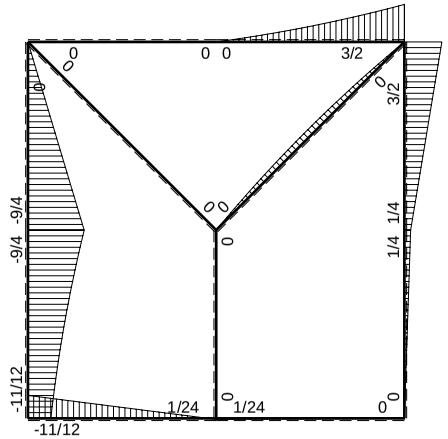






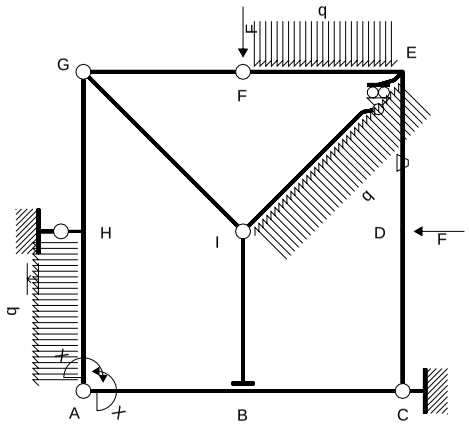


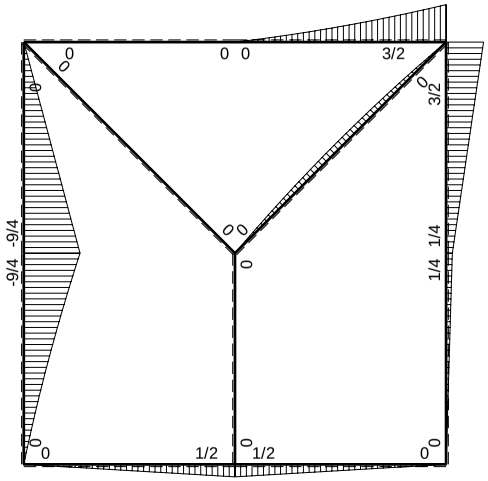




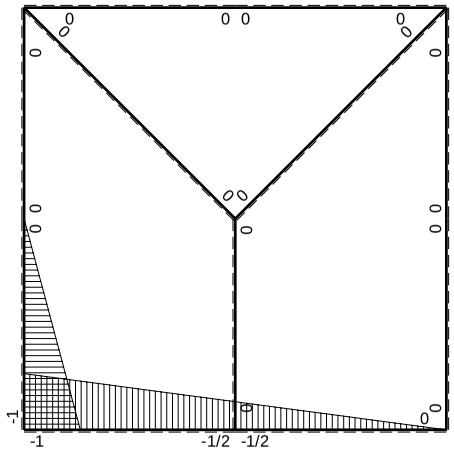


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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

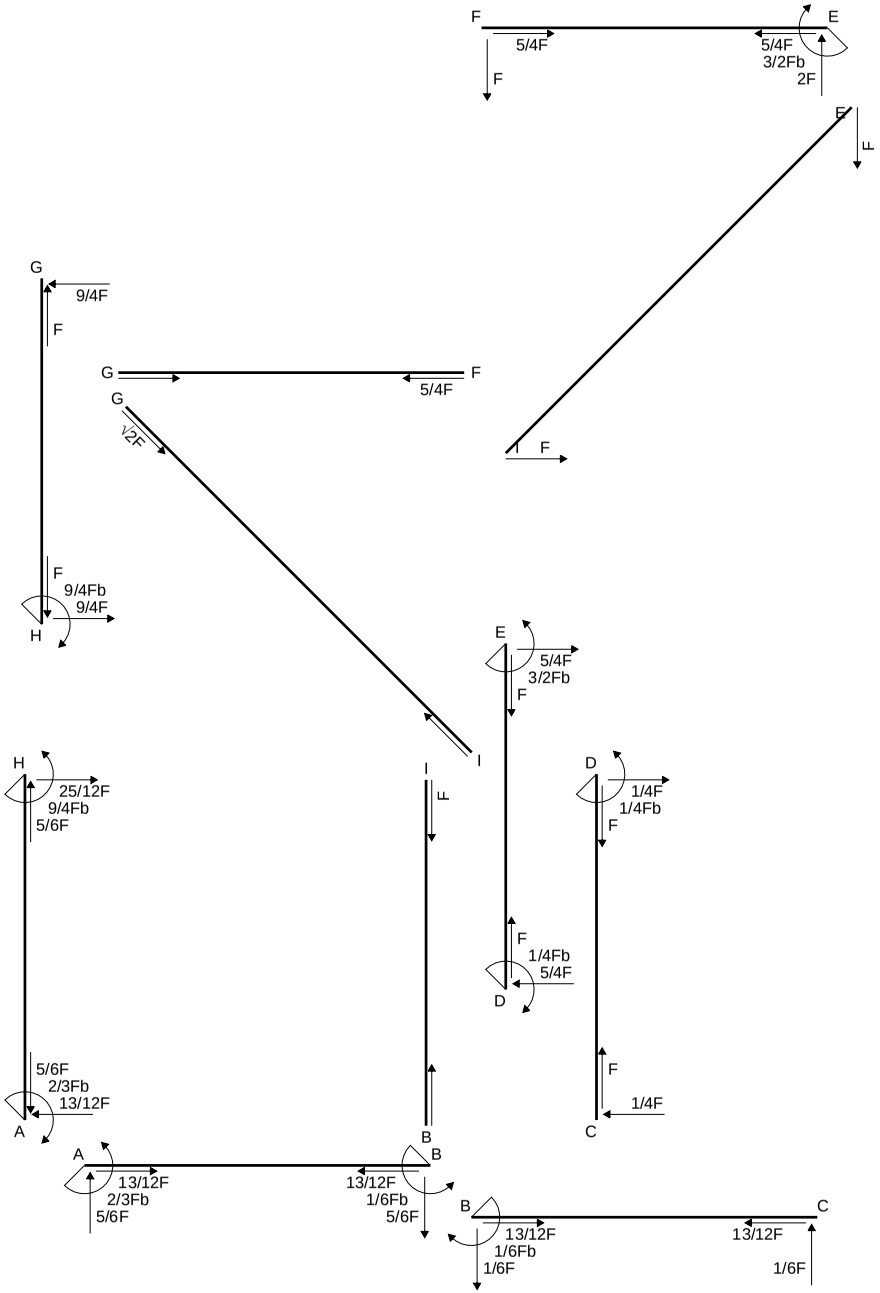
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

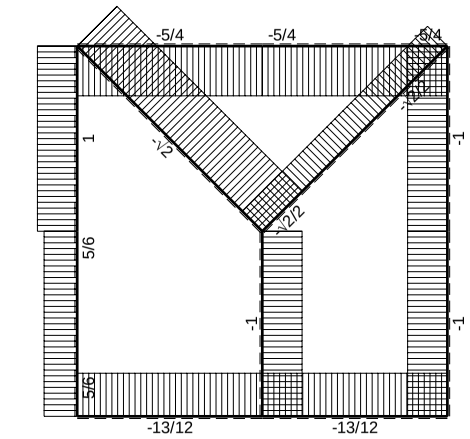
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

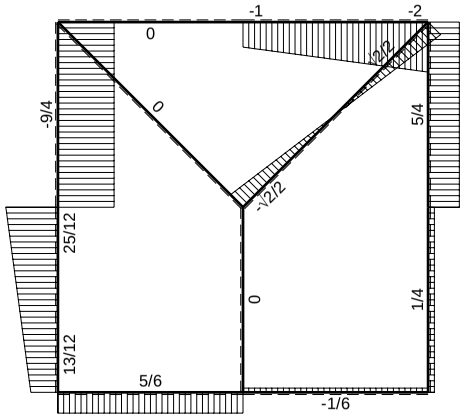
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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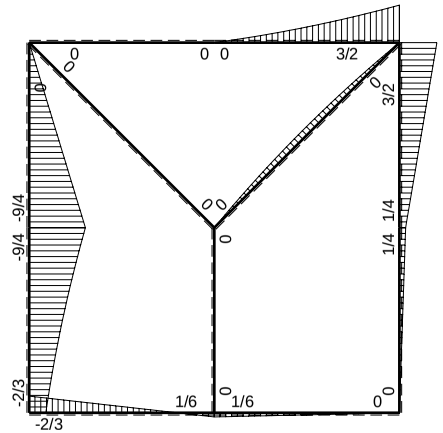

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G
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F
F
9/4Fb
9/4F
H
I
E
5/4F
3/2Fb
F
H
25/12F
9/4Fb
5/6F
I
F
D
1/4F
1/4Fb
F
F
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B
13/12F
1/6Fb
5/6F
B
13/12F
1/6Fb
1/6F
C
13/12F
1/6F


-5/4
-5/4
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-√2
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-1
-1
5/6
-13/12
-13/12

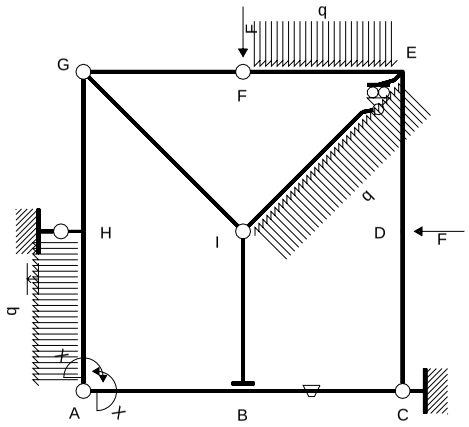

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-2
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-9/4
0
5/4
-√2/2
25/12
0
1/4
13/12
5/6
-1/6

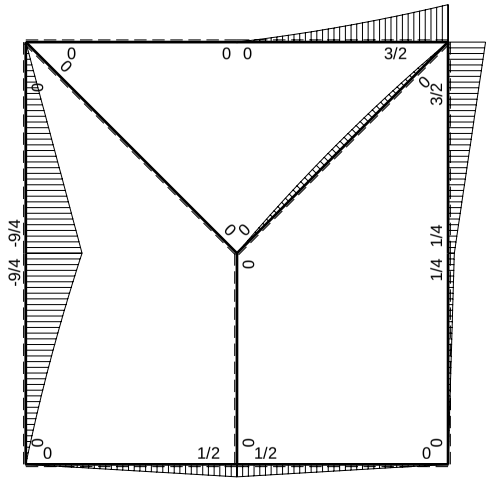
←□→ F

↑□↓ F

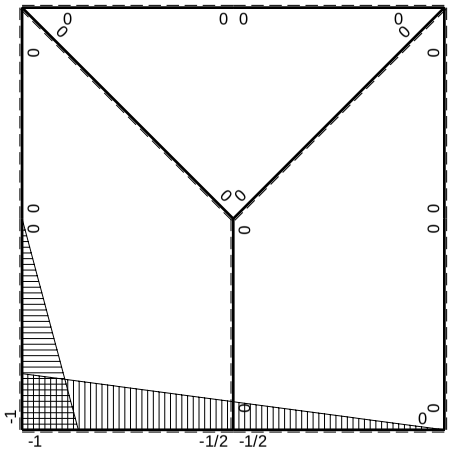

0
0 0
3/2
0
0
3/2
0
0
-9/4
-9/4
0
1/4
1/4
-2/3
1/6
0
1/6
0
0
-2/3

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	-Fb/EJ	$-1/4Fb+1/2Fx-1/4Fx^2/b$	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	Fb/EJ	$-1/4Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-2/3Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						2/3Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\right)\ \theta\ dx$

$= \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\right)\ \theta\ \ = 1/6\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\right)\ \theta\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ + \left(-1/4\ b\right)\ \theta\ \ = 1/6\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

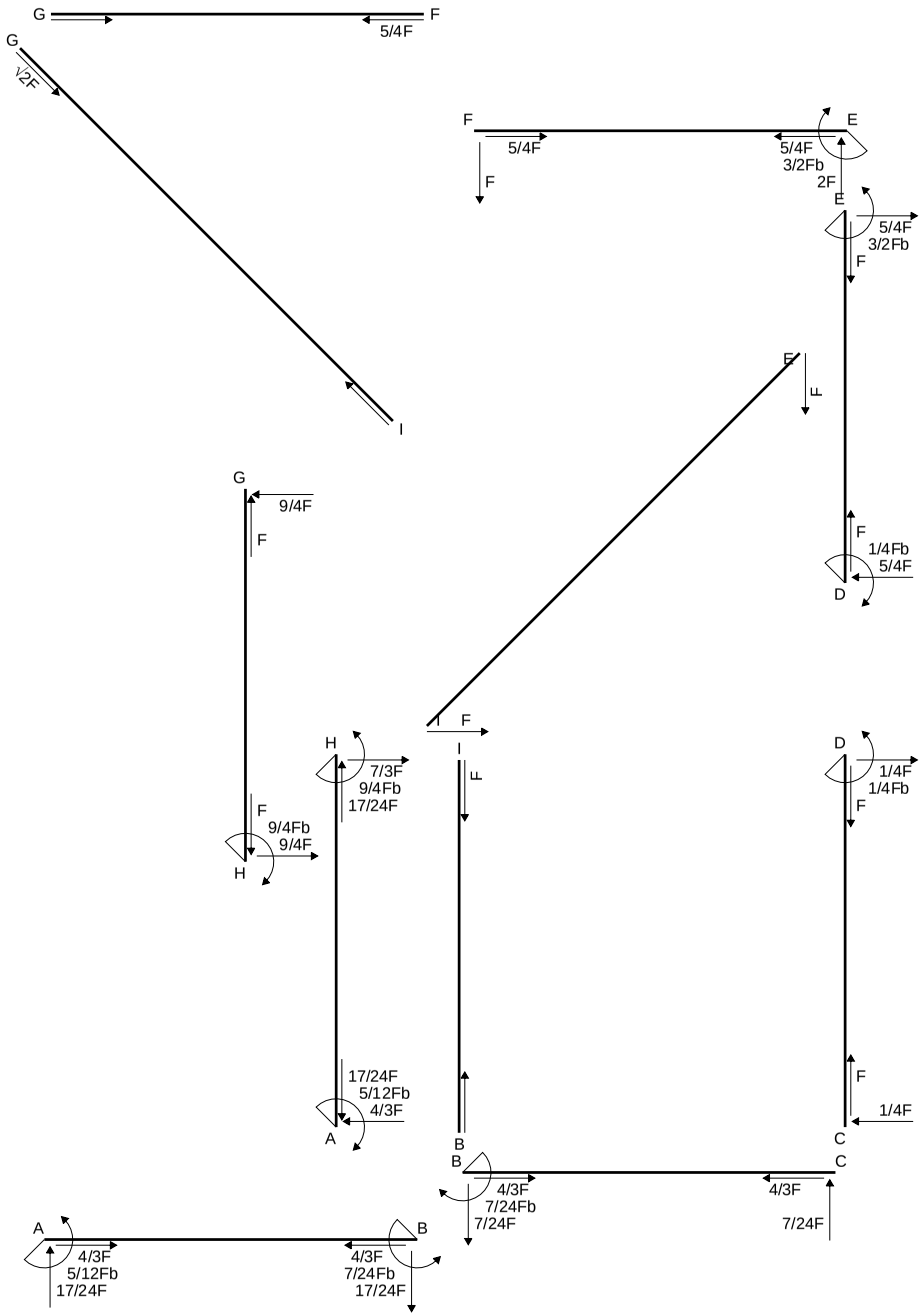
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

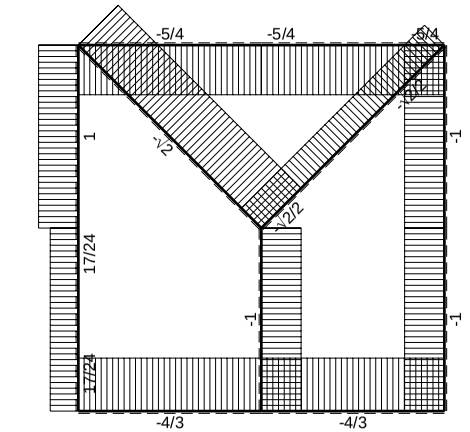
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

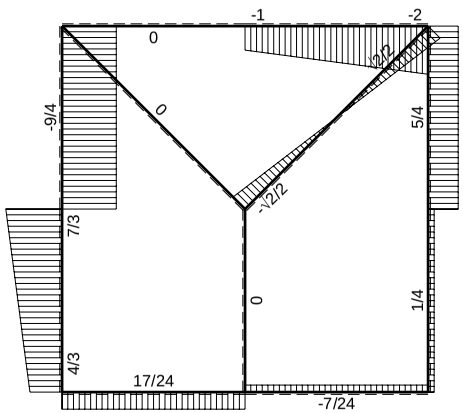
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

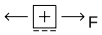
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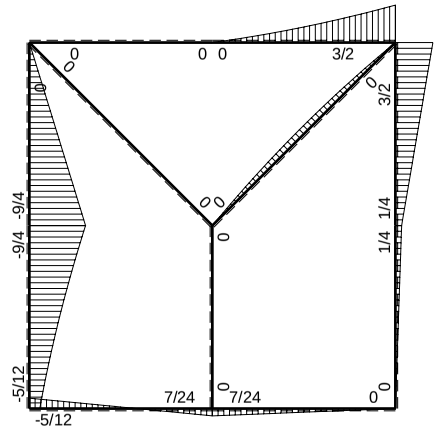

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3/2Fb
F
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F
1/4Fb
5/4F
D
G
9/4F
F
F
9/4Fb
9/4F
H
H
7/3F
9/4Fb
17/24F
F
I
F
D
1/4F
1/4Fb
F
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F
1/4F
B
B
C
C
4/3F
7/24Fb
7/24F
4/3F
7/24F
A
B
4/3F
5/12Fb
17/24F
4/3F
7/24Fb
17/24F


-5/4
-5/4
-5/4
1
-√2
-√2/2
-1
17/24
-1
-1
17/24
-4/3
-4/3

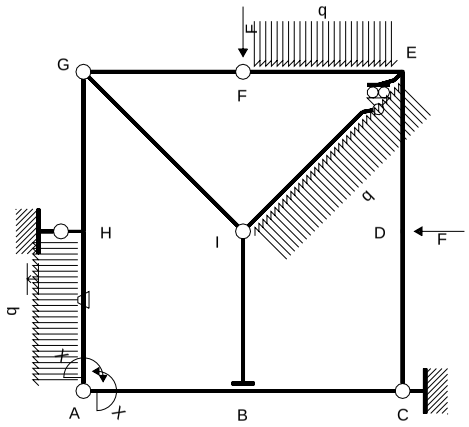

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-1
-2
√2/2
-9/4
0
5/4
-√2/2
7/3
0
1/4
4/3
17/24
-7/24

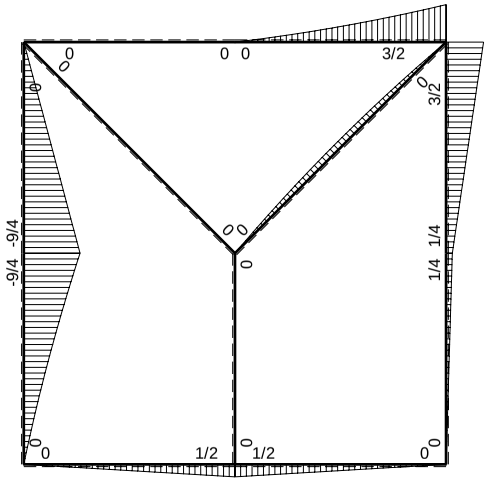

F


F

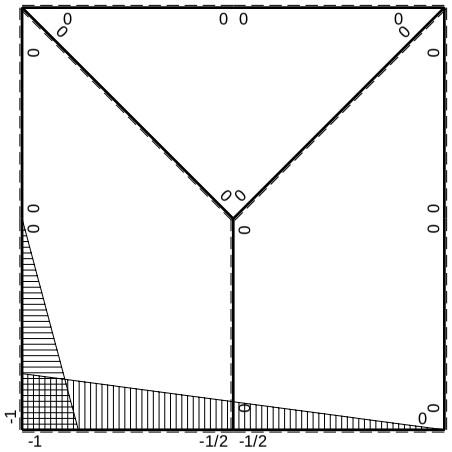

0
0 0
3/2
0
0
3/2
0 0
0
-9/4
-9/4
1/4
1/4
0
-5/12
7/24
0
7/24
0 0
-5/12


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.176

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	-Fb/EJ	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/3+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	Fb/EJ	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/12Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.176

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\right)\ \theta\ dx$

$= \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

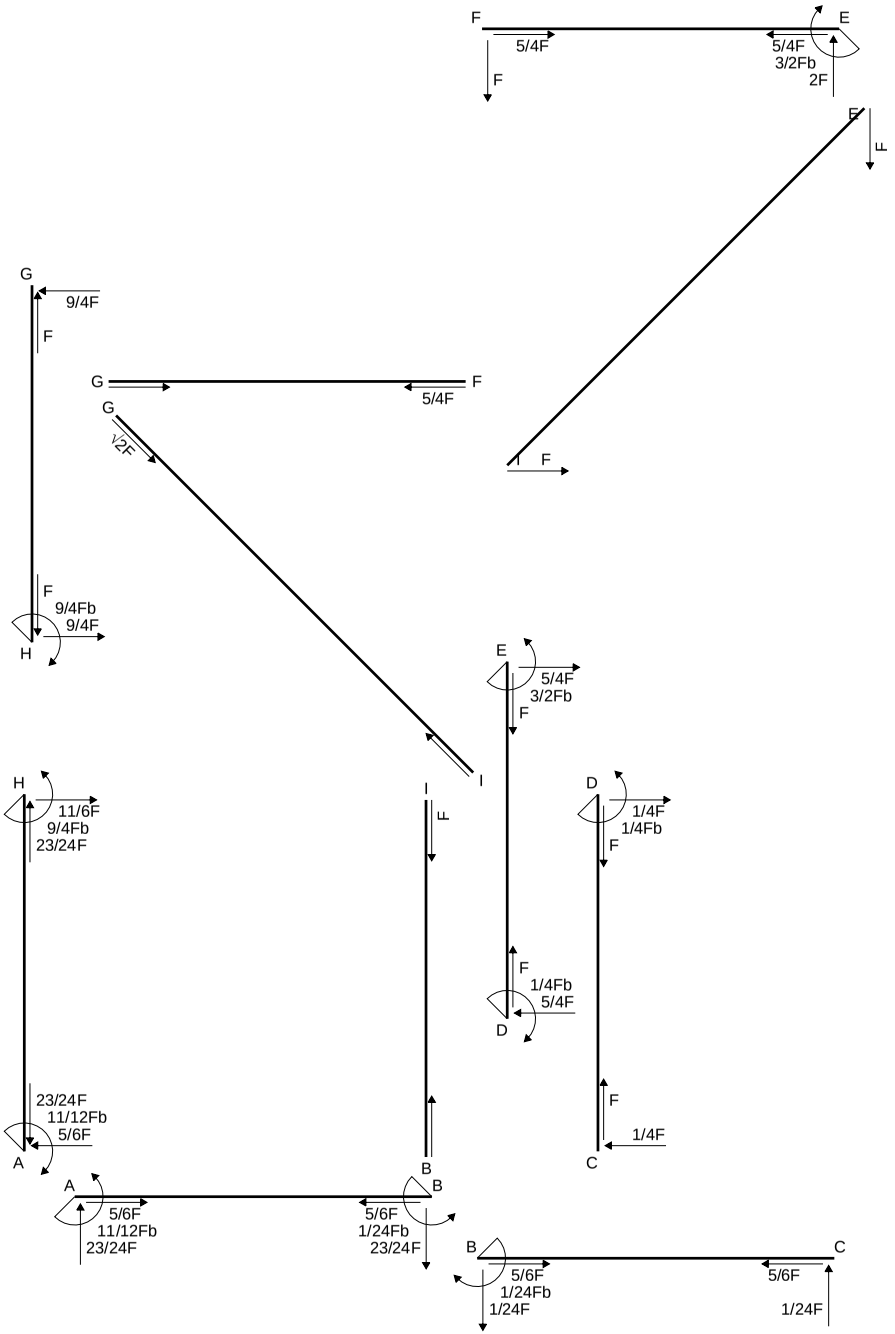
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\right)\ \theta\ dx$

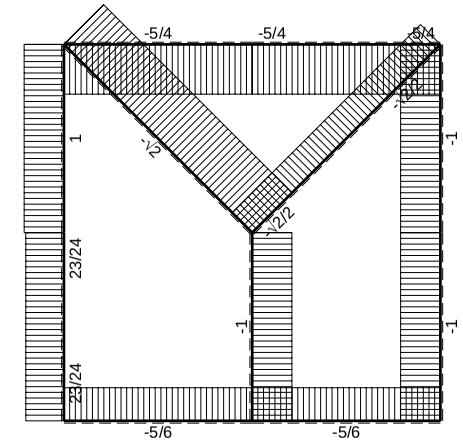
$= \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

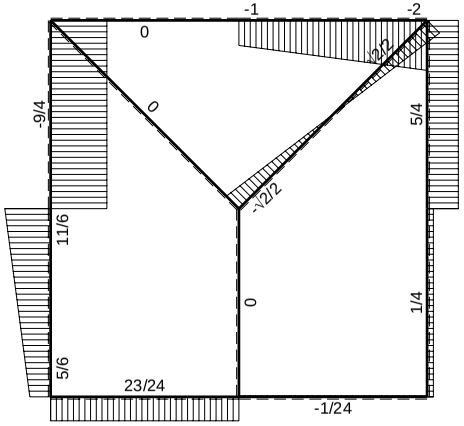
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

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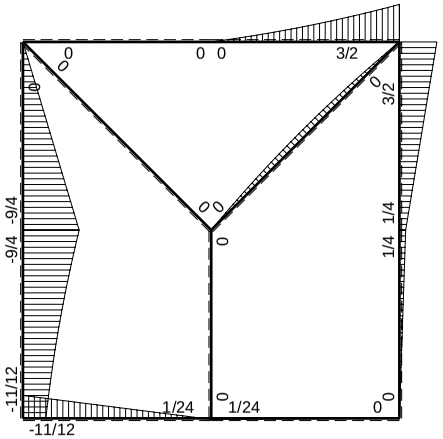

F
E
5/4F
5/4F
3/2Fb
2F
F
F
G
9/4F
F
G
F
5/4F
G
½F
F
F
9/4Fb
9/4F
H
I
E
5/4F
3/2Fb
F
H
11/6F
9/4Fb
23/24F
I
F
D
1/4F
1/4Fb
F
F
1/4Fb
5/4F
D
F
1/4F
C
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11/12Fb
5/6F
A
B
A
B
5/6F
11/12Fb
23/24F
5/6F
1/24Fb
23/24F
B
C
5/6F
1/24Fb
1/24F
5/6F
1/24F


-5/4
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-1/2
23/24
-1
-1
23/24
-5/6
-5/6

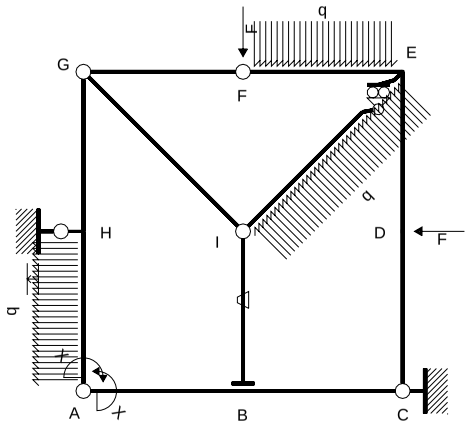

-1
-2
0
3/2
-9/4
0
5/4
-1/2
11/6
0
1/4
5/6
23/24
-1/24

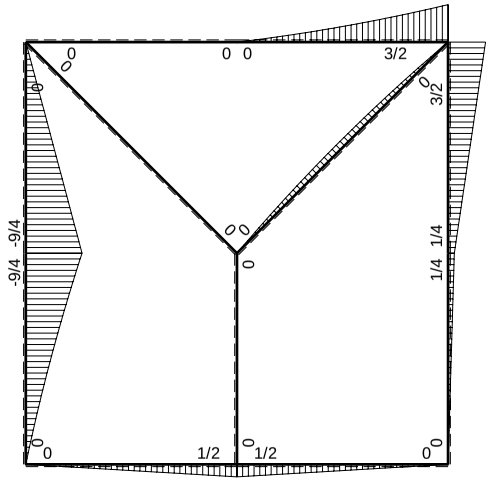
←□→ F

↑□↓ F

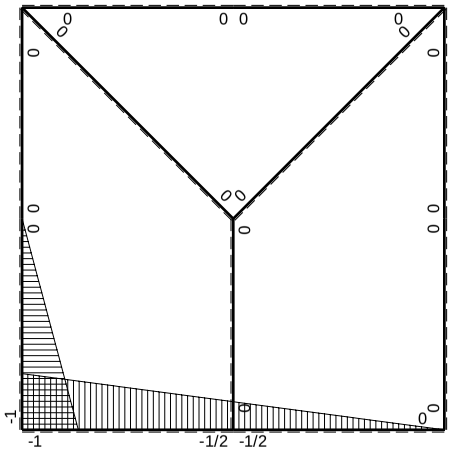

0
0 0
3/2
0
0
3/2
0 0
-9/4
-9/4
0
1/4 1/4
-11/12
1/24
0
1/24
0 0
-11/12

⟲□⟳ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.177

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

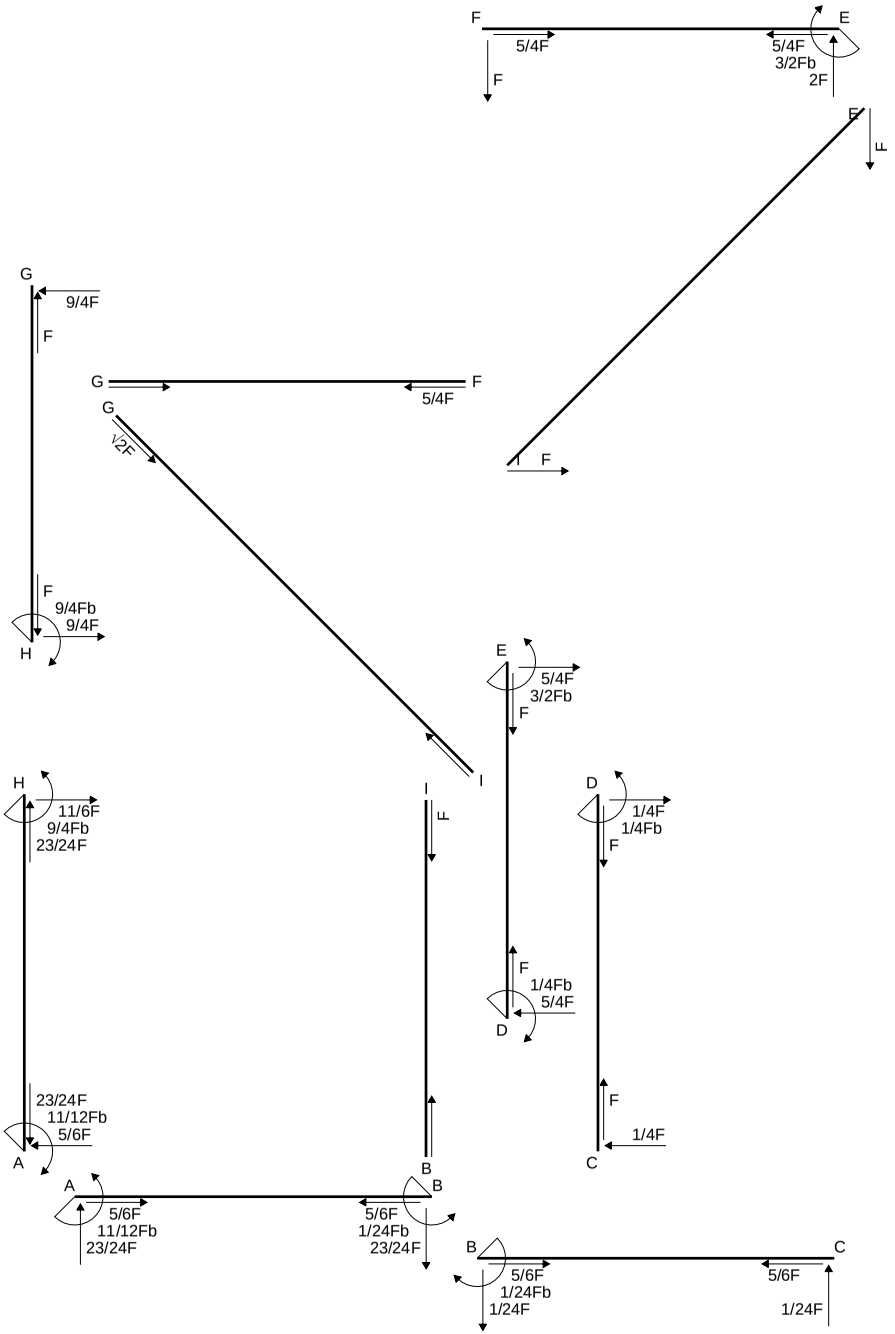
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

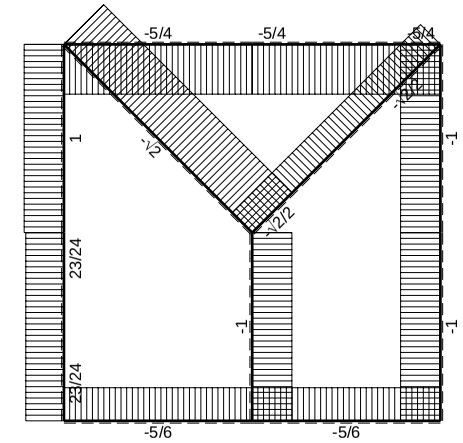
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

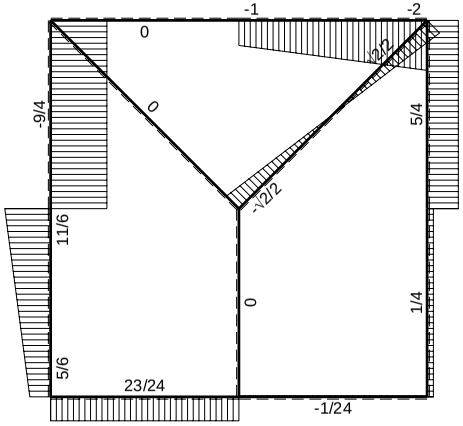
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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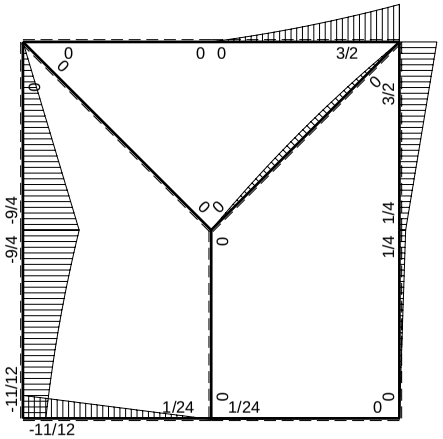

F
E
5/4F
5/4F
3/2Fb
2F
F
F
G
9/4F
F
G
F
5/4F
G
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F
F
9/4Fb
9/4F
H
I
E
5/4F
3/2Fb
F
H
11/6F
9/4Fb
23/24F
I
F
D
1/4F
1/4Fb
F
F
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5/4F
D
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5/6F
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1/4F
C
B
A
B
5/6F
11/12Fb
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-5/4
-5/4
-5/4
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-1
23/24
-5/6
-5/6

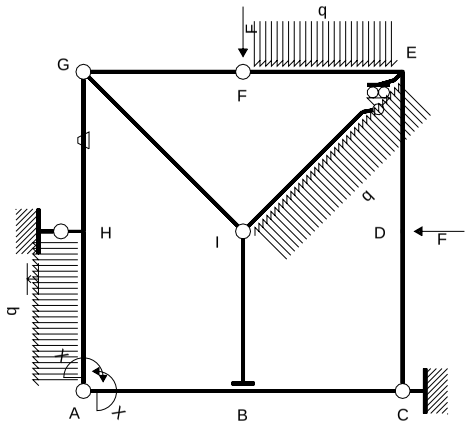

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11/6
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5/6
23/24
-1/24

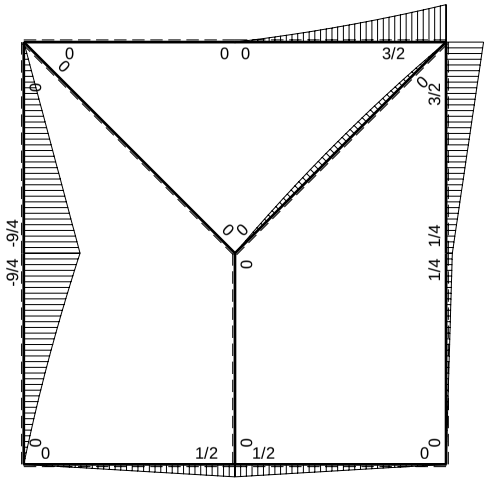
←□→ F

↑□↓ F

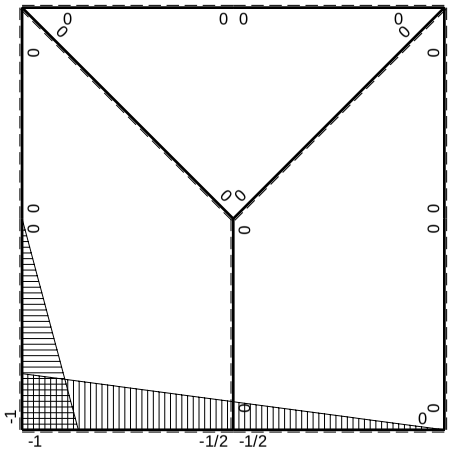

0
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3/2
0
0
3/2
0 0
-9/4
-9/4
0
1/4
1/4
-11/12
1/24
0
1/24
0 0
-11/12

□ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.178

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/2Fb+1/2Fx	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.178

Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

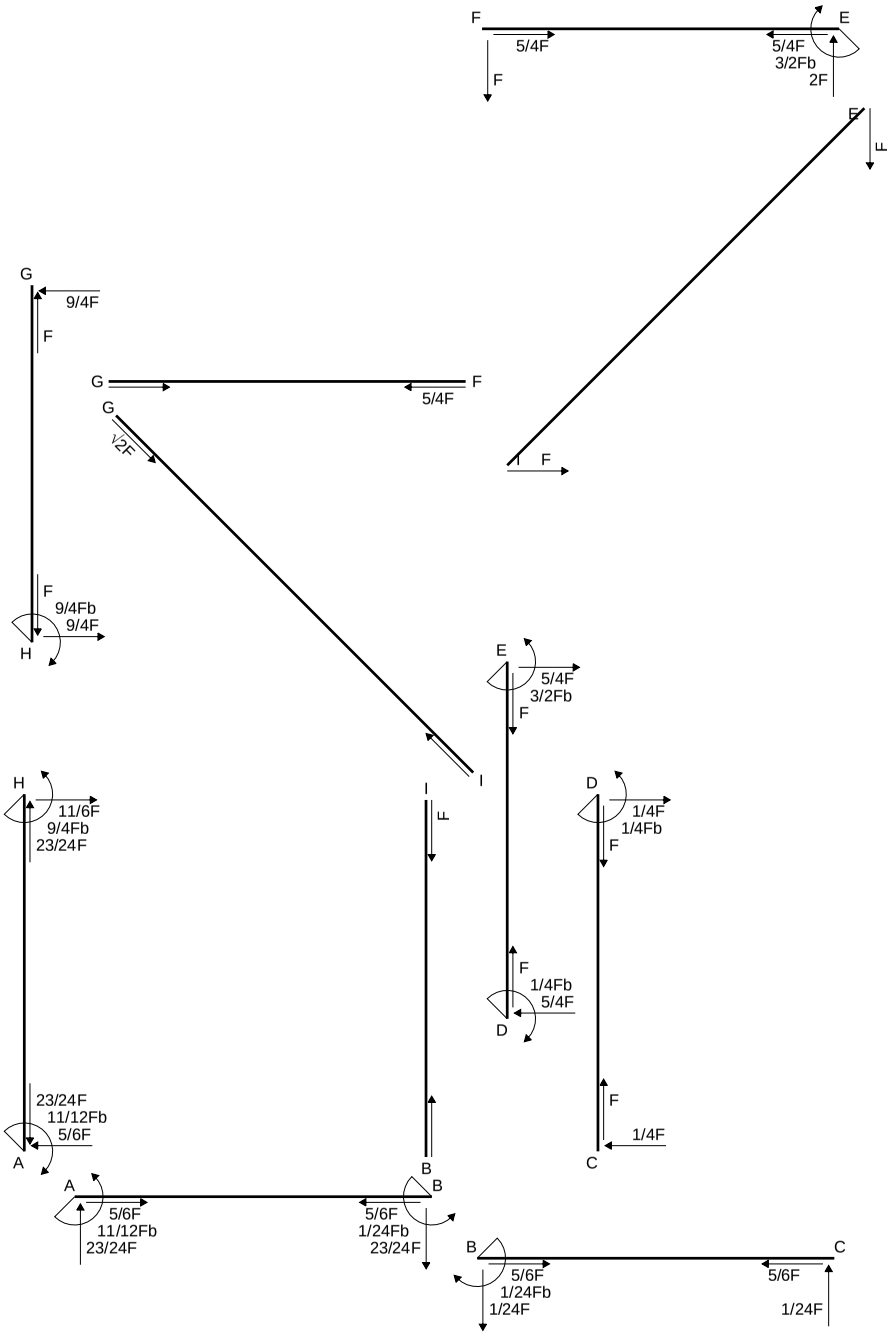
$$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$$

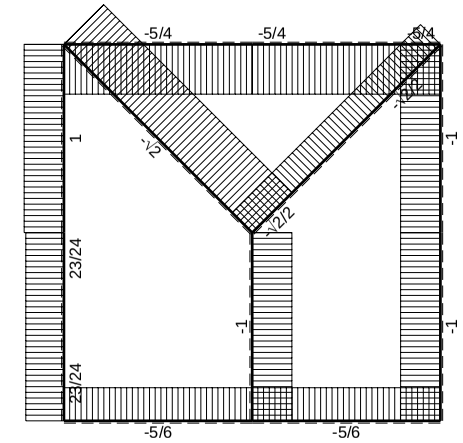
$$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

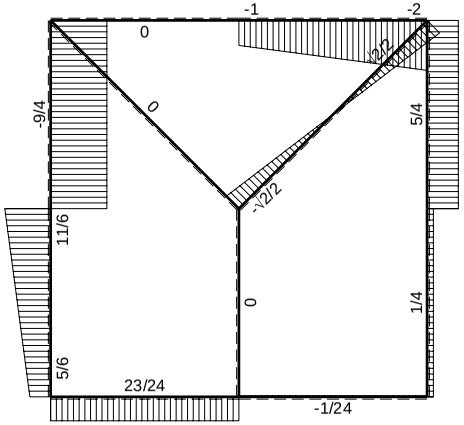
$$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$$

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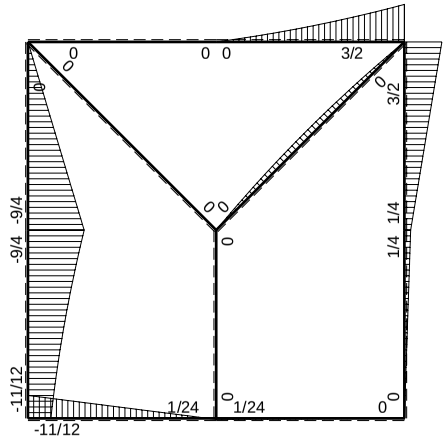

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G
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G
5/4F
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F
F
9/4Fb
9/4F
H
I
E
5/4F
3/2Fb
F
H
11/6F
9/4Fb
23/24F
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F
D
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1/4Fb
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1/24F


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-5/6

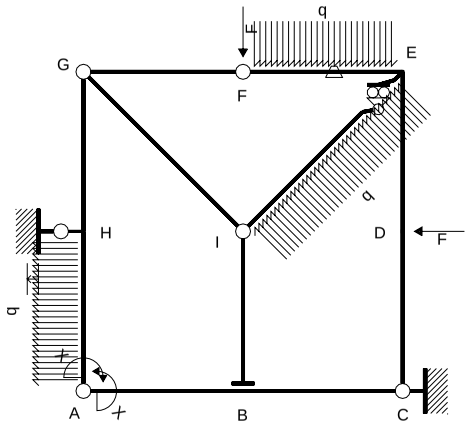

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5/4
-1/2
11/6
0
1/4
5/6
23/24
-1/24

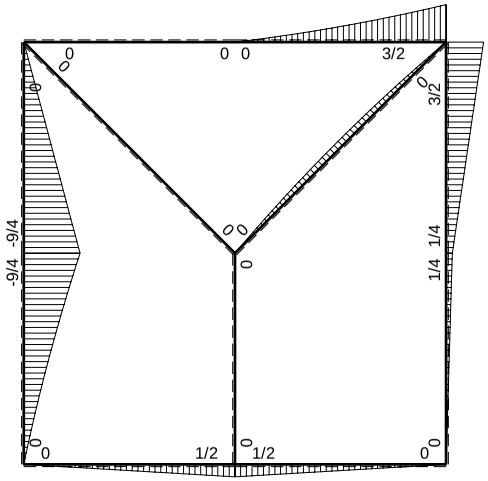
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F

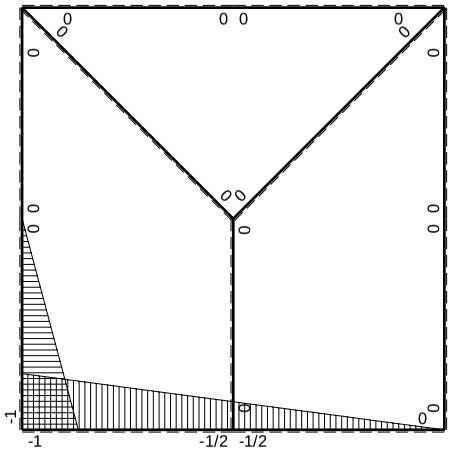

0
0 0
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0
0
3/2
0 0
-9/4
-9/4
0
1/4 1/4
-11/12
1/24
0
1/24
0 0
-11/12

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.179

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.179

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

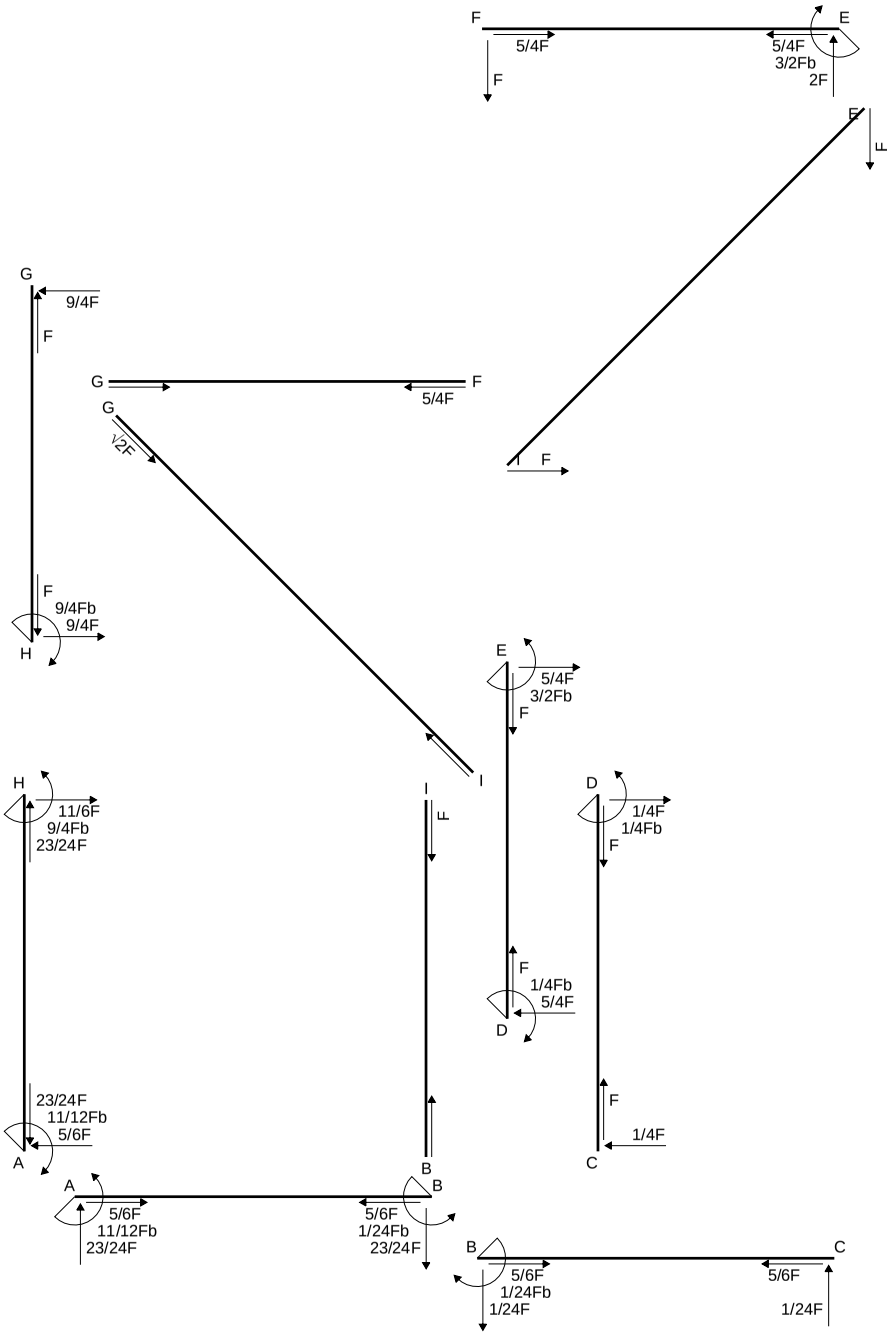
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

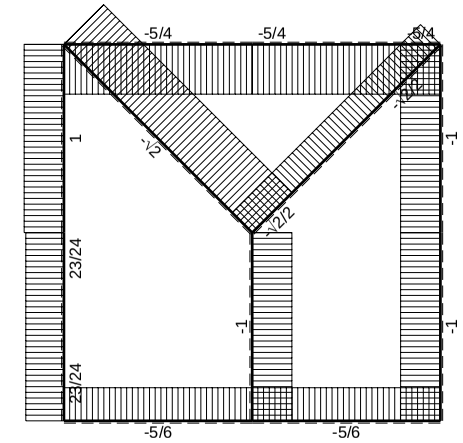
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

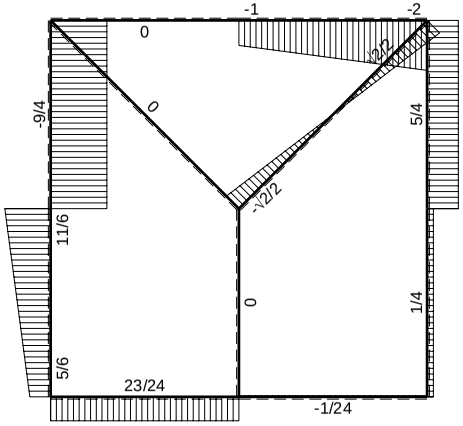
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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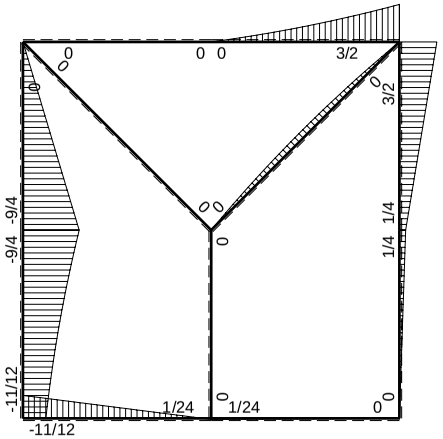

F
5/4F
E
5/4F
3/2Fb
2F
F
F
G
9/4F
F
G
5/4F
F
G
√2F
F
F
9/4Fb
9/4F
H
E
5/4F
3/2Fb
F
I
H
11/6F
9/4Fb
23/24F
I
F
D
1/4F
1/4Fb
F
F
1/4Fb
5/4F
D
F
1/4F
C
23/24F
11/12Fb
5/6F
A
B
A
5/6F
11/12Fb
23/24F
B
5/6F
1/24Fb
23/24F
B
5/6F
1/24Fb
1/24F
C
5/6F
1/24F


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23/24
-5/6
-5/6

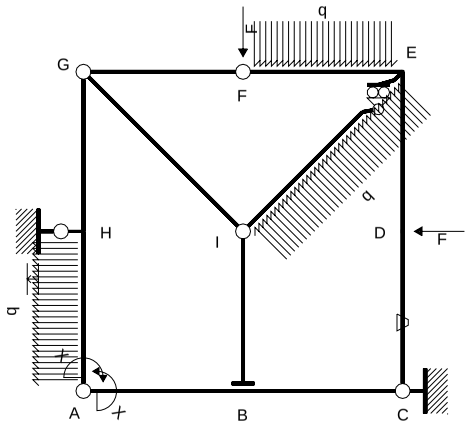

-1
-2
0
-1/2
-9/4
0
5/4
-√2/2
11/6
0
1/4
5/6
23/24
-1/24

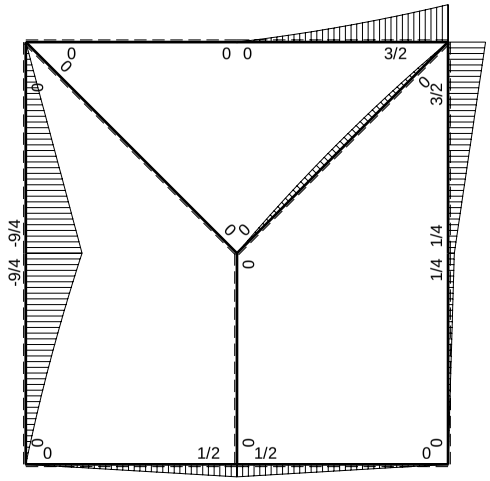
←⊞→ F

↑⊞↓ F

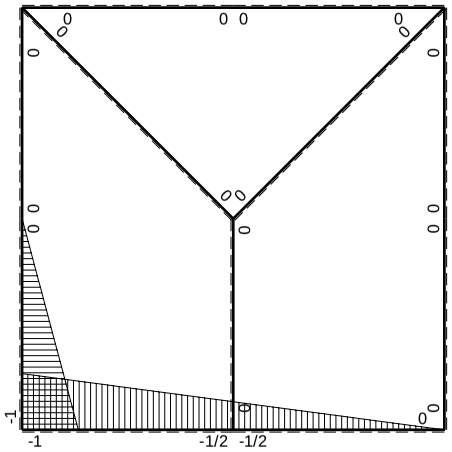

0
0 0
3/2
0
0
3/2
0 0
0
-9/4
-9/4
1/4
1/4
-11/12
1/24
0
1/24
0 0
-11/12

⊞ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.180

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.180

Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

$$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

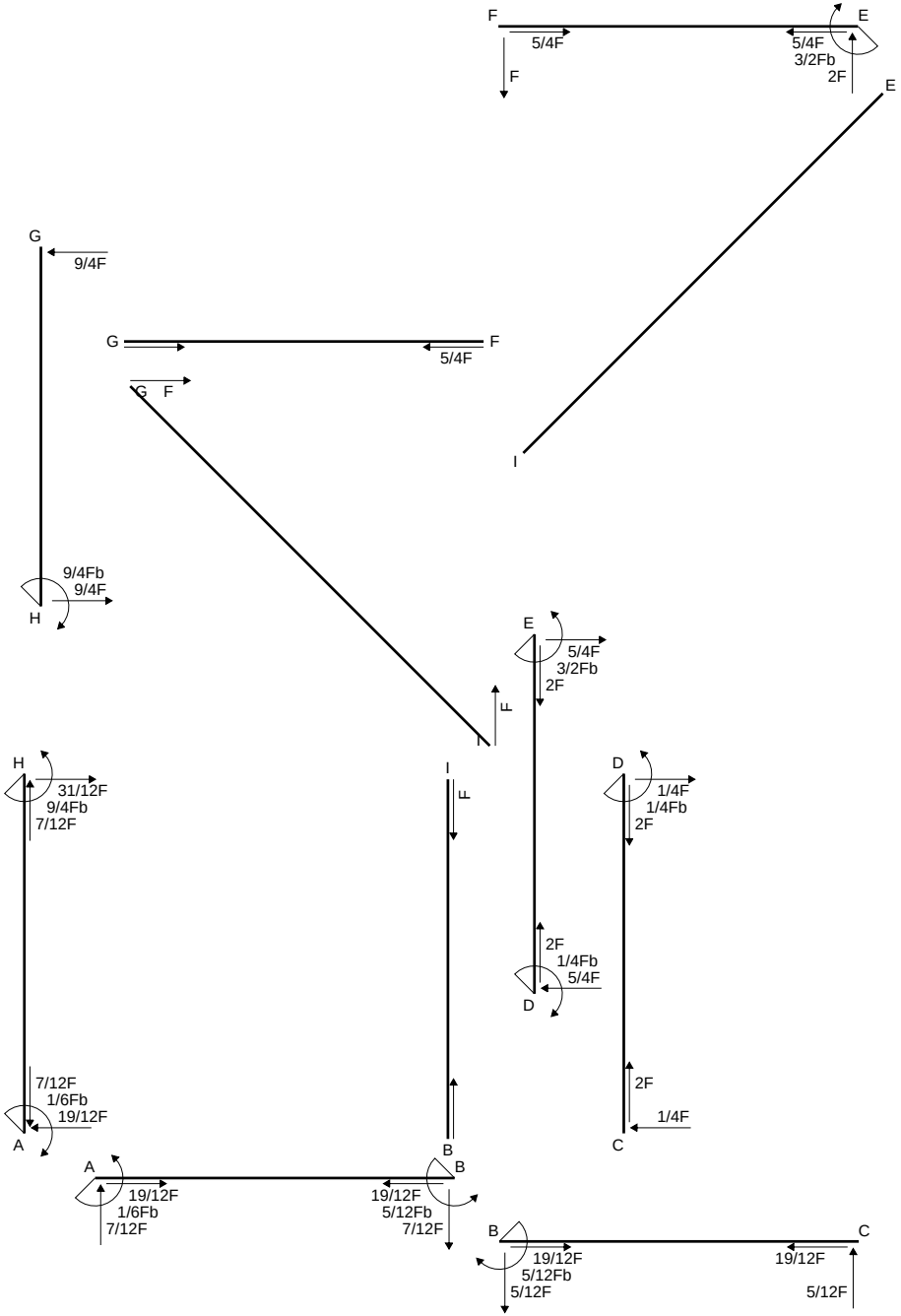
$$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$$

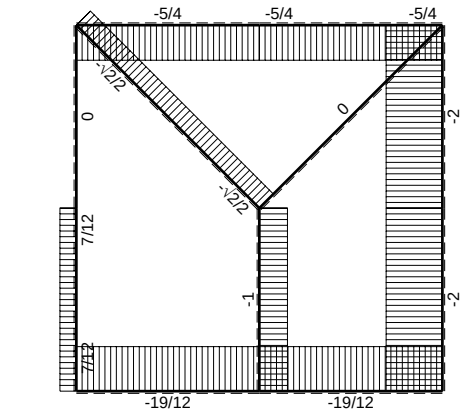
$$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$$

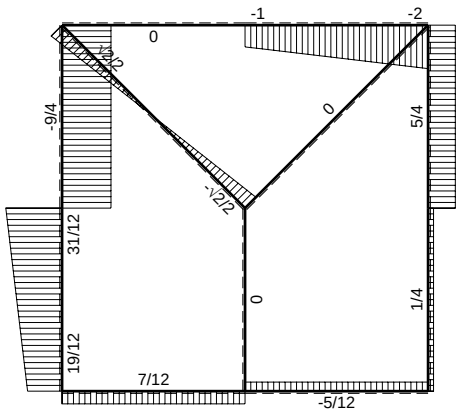
$$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$$

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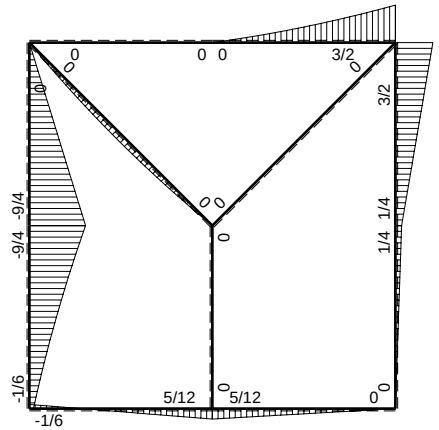

F
5/4F
E
5/4F
3/2Fb
2F
F
E
G
9/4F
G
F
5/4F
G
F
I
9/4Fb
9/4F
H
E
5/4F
3/2Fb
2F
F
H
31/12F
9/4Fb
7/12F
I
F
D
1/4F
1/4Fb
2F
2F
1/4Fb
5/4F
D
7/12F
1/6Fb
19/12F
A
2F
1/4F
C
B
A
B
19/12F
1/6Fb
7/12F
19/12F
5/12Fb
7/12F
B
C
19/12F
5/12Fb
5/12F
19/12F
5/12F


-5/4
-5/4
-5/4
-√2/2
0
0
-2
-√2/2
7/12
-1
-2
7/12
-19/12
-19/12

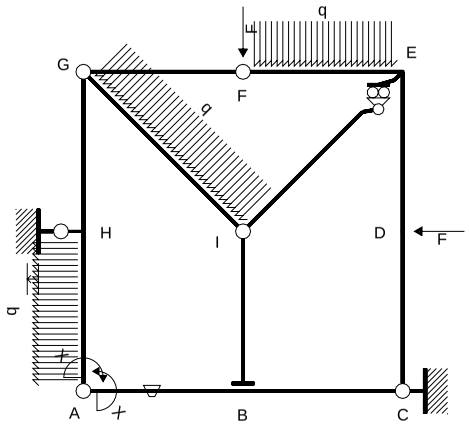

-1
-2
0
-√2/2
-9/4
0
5/4
-√2/2
31/12
0
1/4
19/12
7/12
-5/12

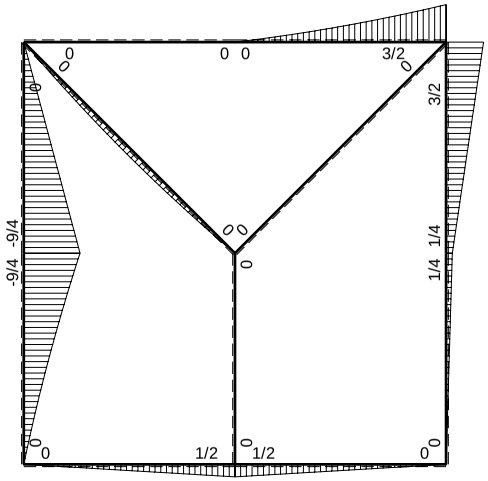
←□→ F

↑□↓ F

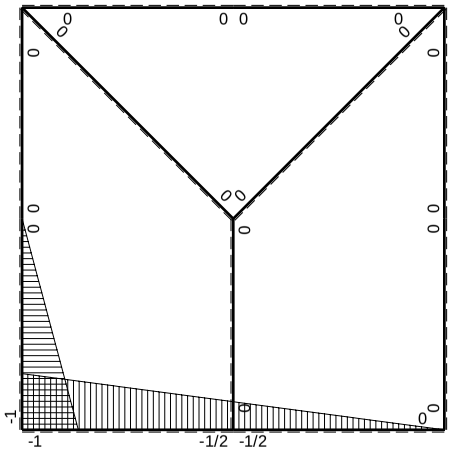

0
0 0
3/2
0
0
3/2
0
0
-9/4
-9/4
0
1/4
1/4
-1/6
5/12
5/12
0
0
-1/6

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.181

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	-Fb/EJ	$-1/2Fx+1/4Fx^2/b$	$Fb/EJ-1/2Fx/EJ$	$1-x/b+1/4x^2/b^2$	$(-1/6+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/2Fb+1/2Fx	Fb/EJ	$-1/4Fb+1/4Fx^2/b$	$1/2Fb/EJ+1/2Fx/EJ$	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/6Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\right)\ \theta\ \ dx$

$= \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 7/12\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\right)\ \theta\ \ dx$

$= \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 7/12\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

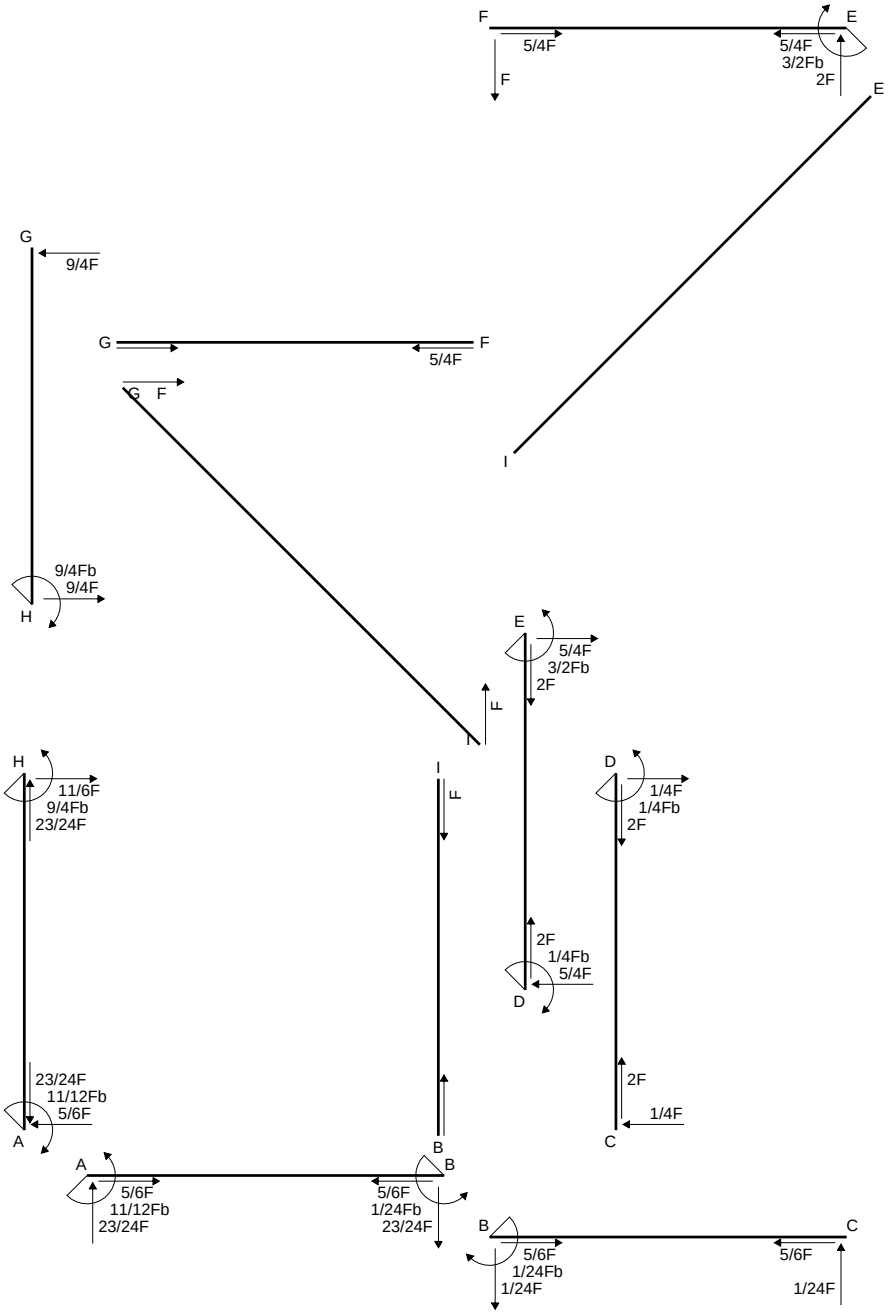
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

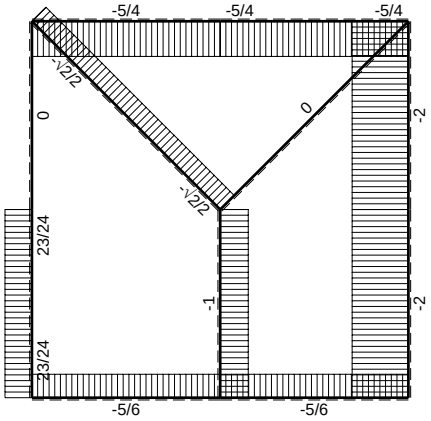
$L^{Xo}_{AH} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

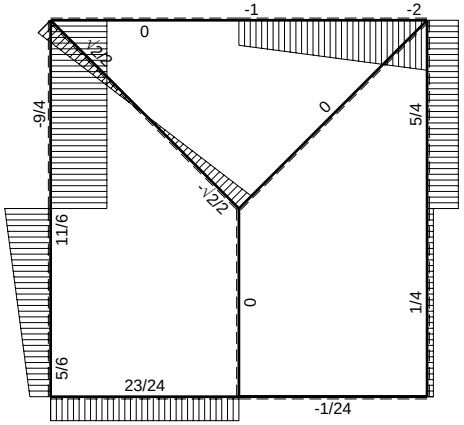
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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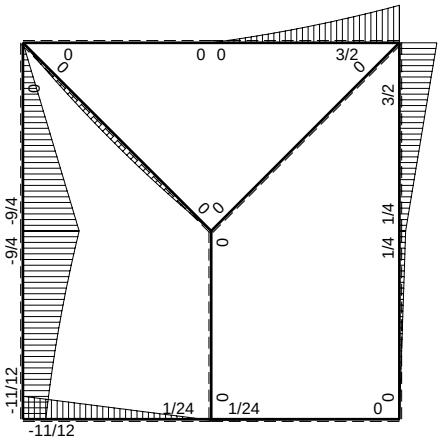

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E
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5/4F
3/2Fb
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F
E
G
9/4F
G
F
5/4F
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9/4Fb
9/4F
H
E
5/4F
3/2Fb
2F
F
H
11/6F
9/4Fb
23/24F
I
F
D
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1/4Fb
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1/4Fb
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D
2F
1/4F
C
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11/12Fb
5/6F
A
B
A
B
5/6F
11/12Fb
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C
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1/24Fb
1/24F
5/6F
1/24F


-5/4
-5/4
-5/4
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-2
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-1
-2
23/24
-5/6
-5/6

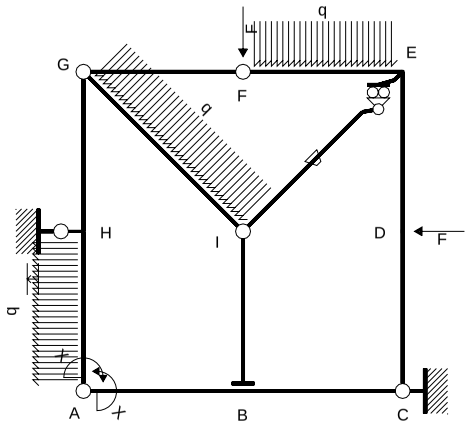

-1
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-√2/2
-√2/2
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5/4
11/6
0
1/4
5/6
23/24
-1/24

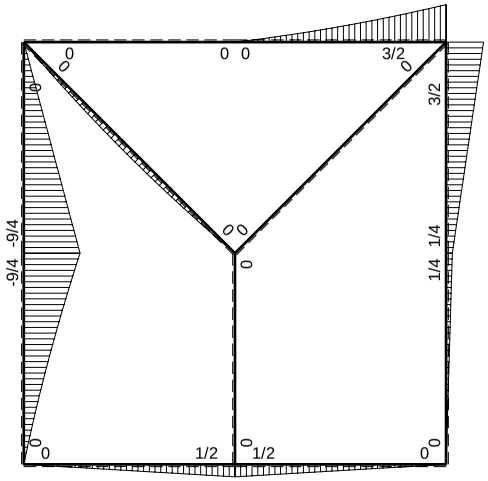
←□→ F

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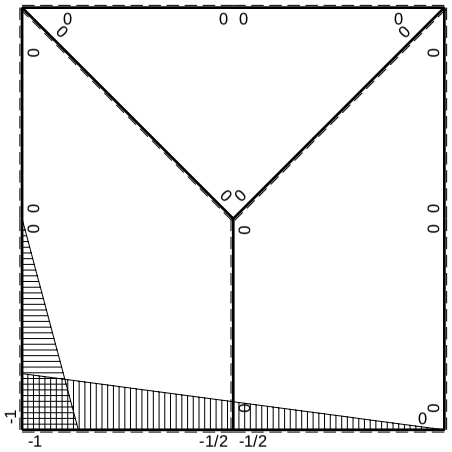

0
0 0
3/2
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3/2
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-9/4
0
1/4
1/4
0
-11/12
1/24
1/24
0
0
-11/12

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.182

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

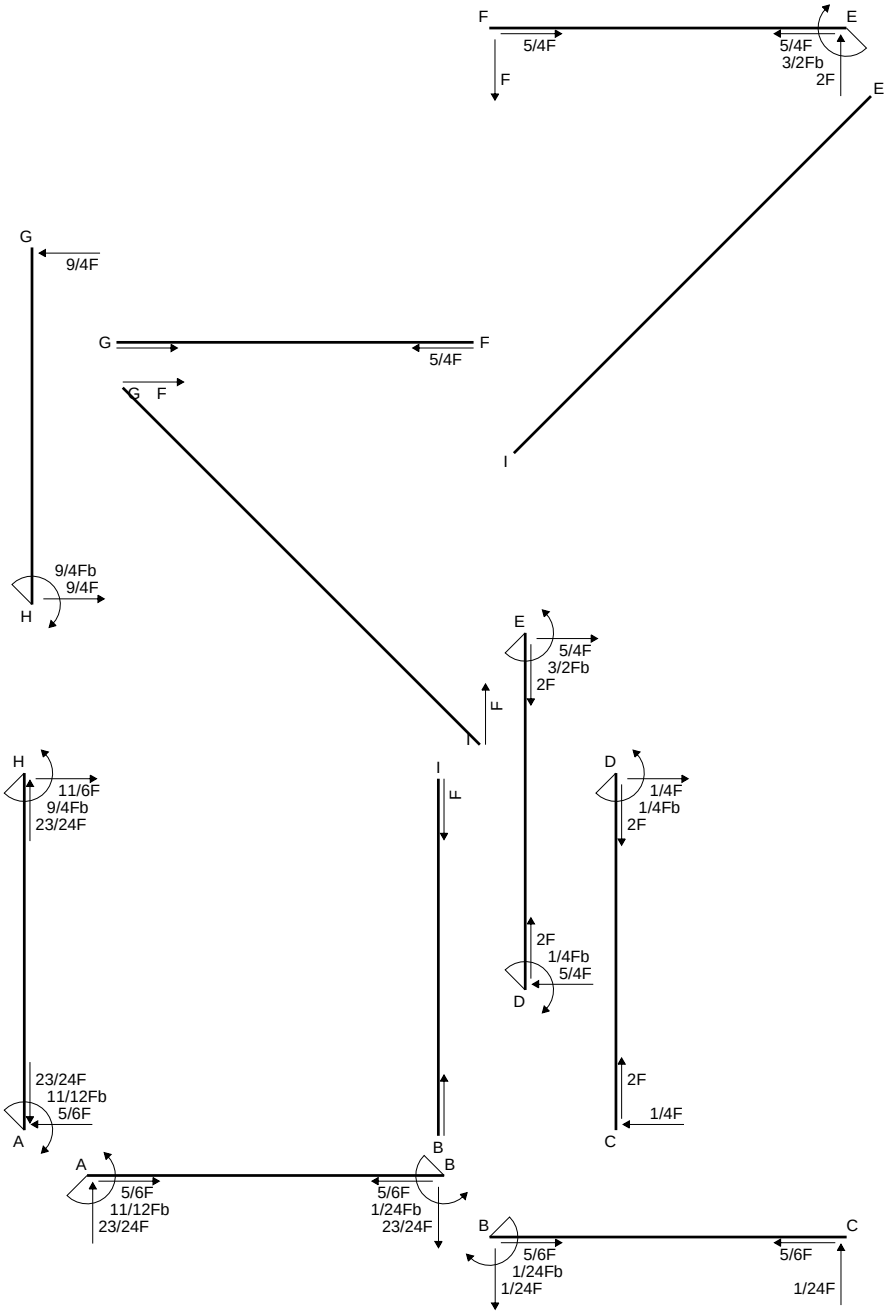
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

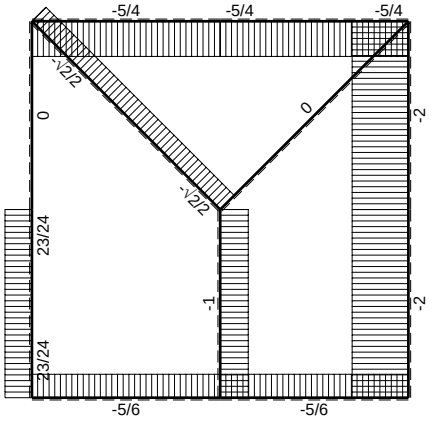
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

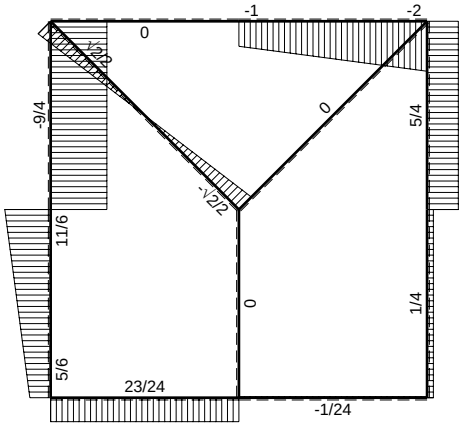
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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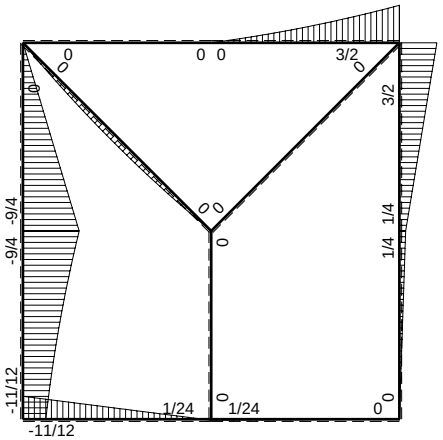

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9/4Fb
9/4F
H
E
5/4F
3/2Fb
2F
F
H
11/6F
9/4Fb
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5/6F
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1/24F
5/6F
1/24F


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-5/6

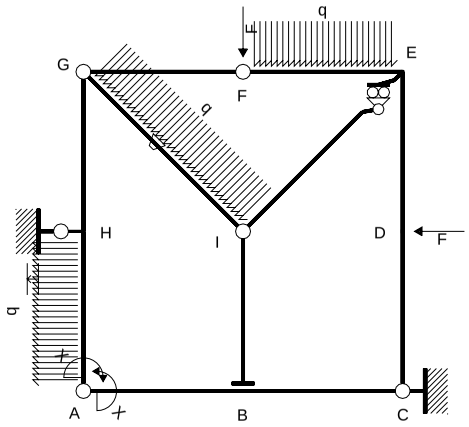

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1/4
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23/24
-1/24

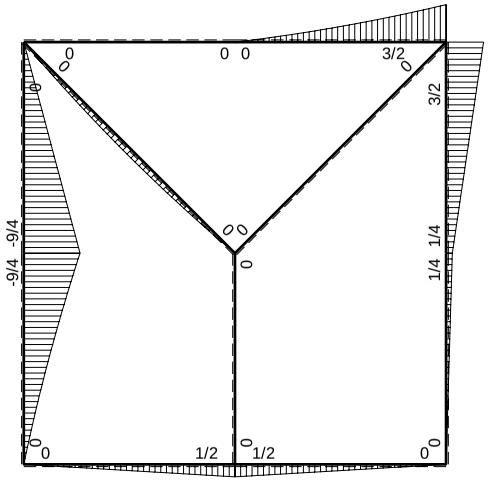
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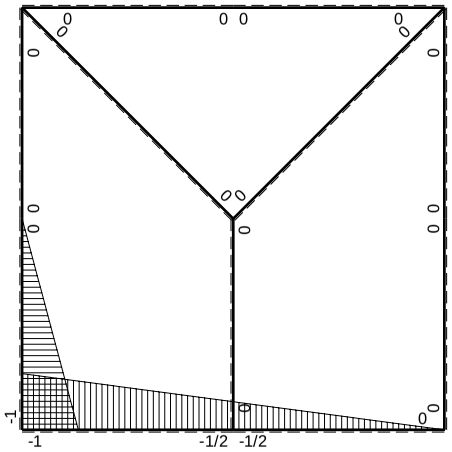

0
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3/2
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-11/12
1/24
1/24
0
0
-11/12

□ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.183

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.183

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

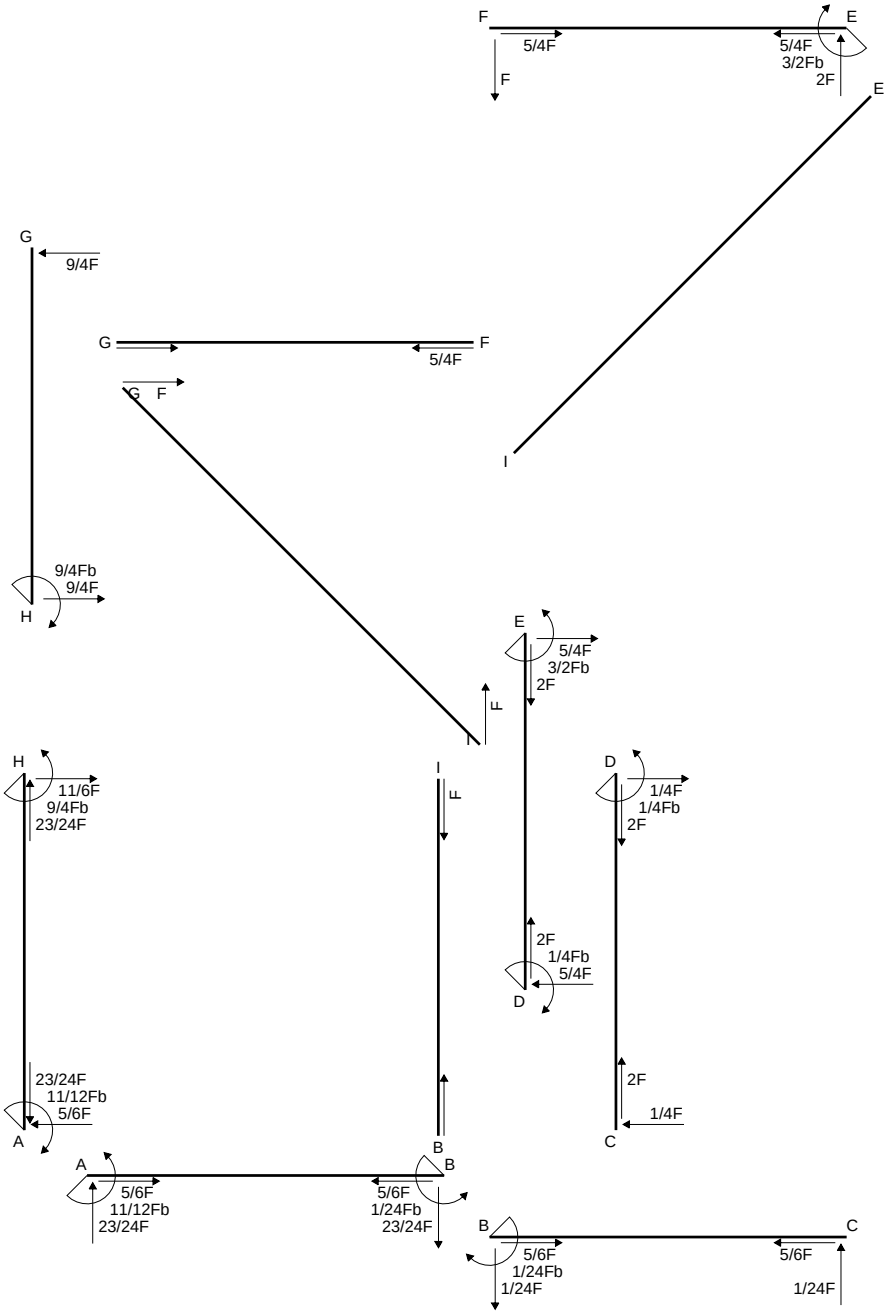
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

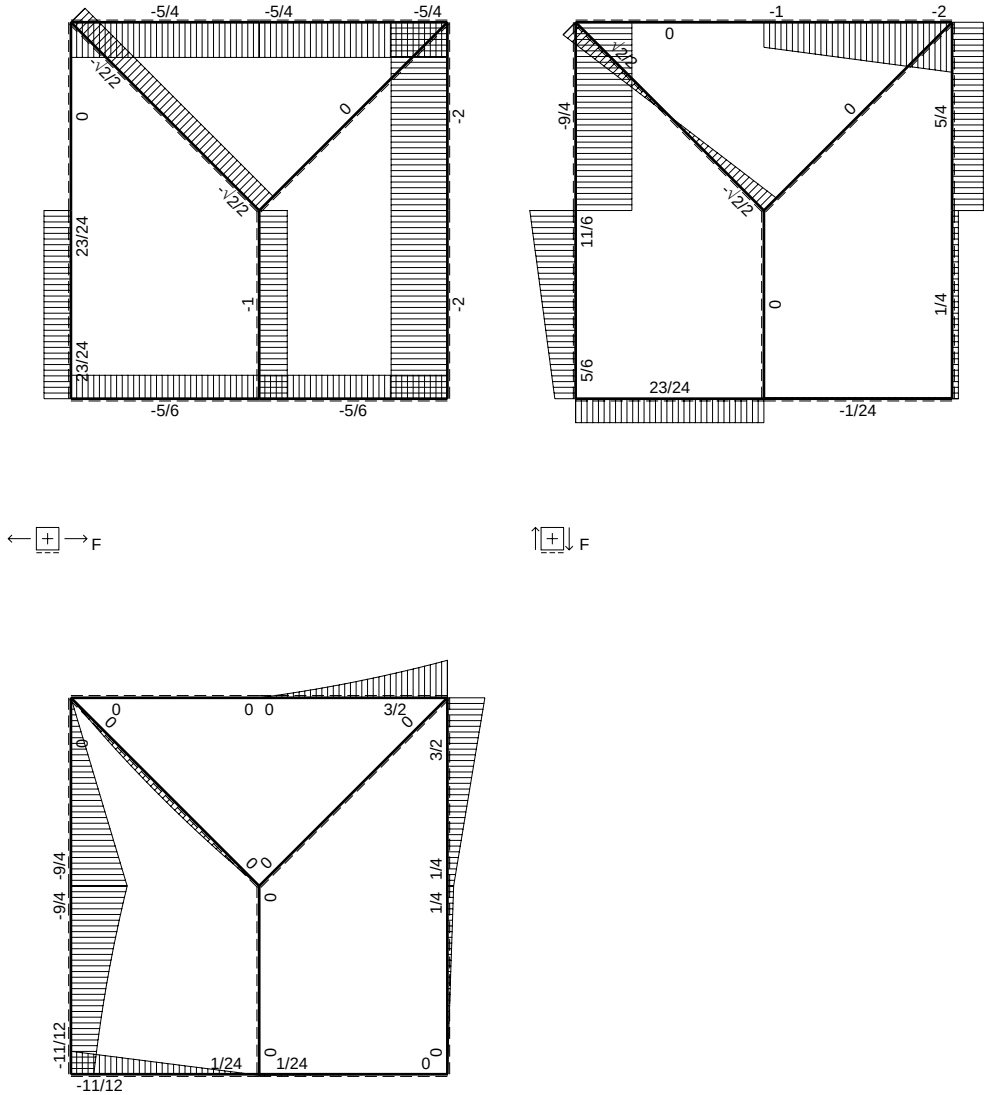
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

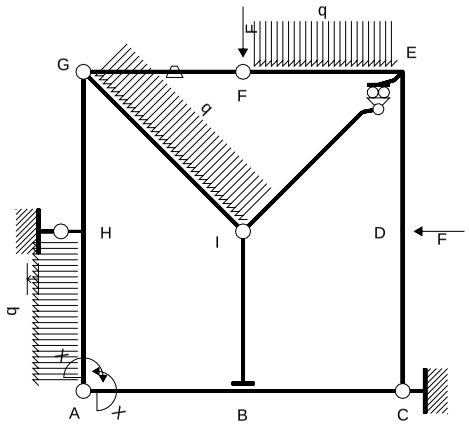
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

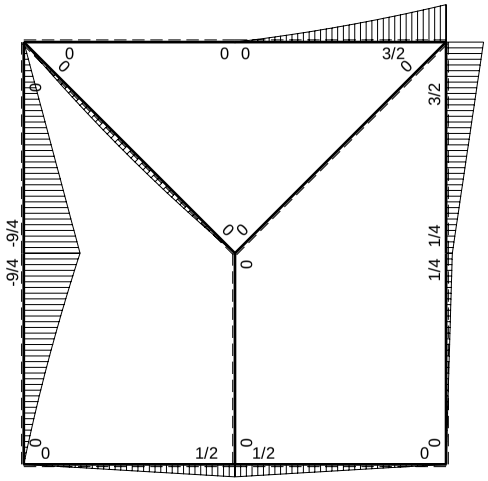
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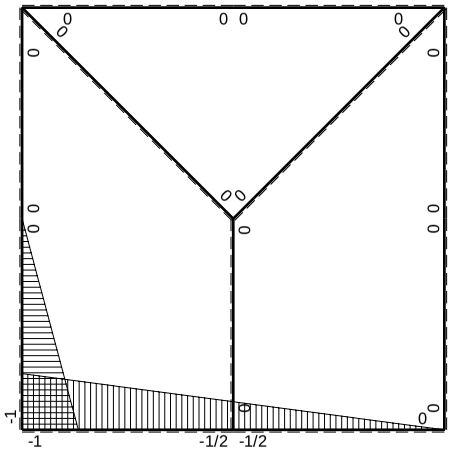

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0
-11/12
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.184

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

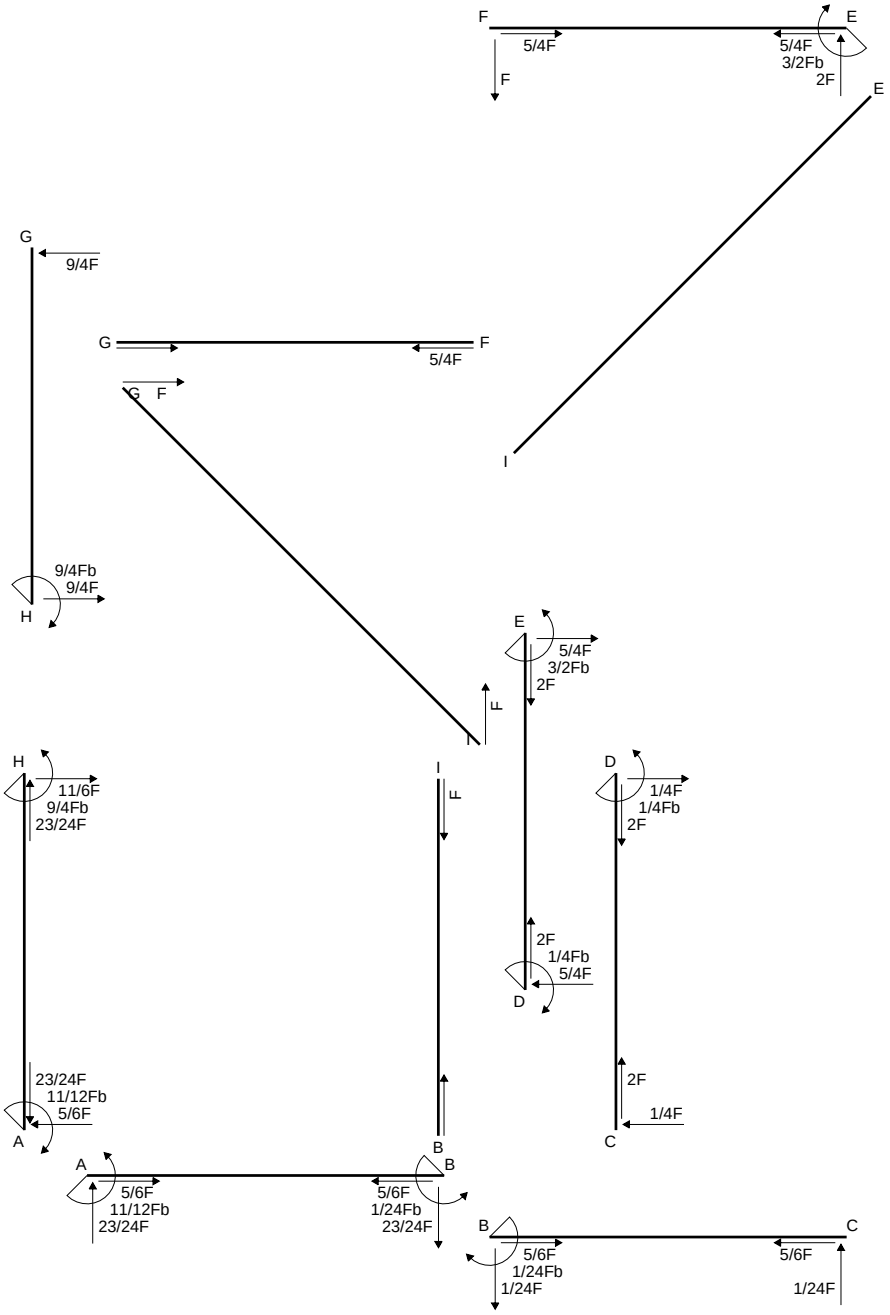
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

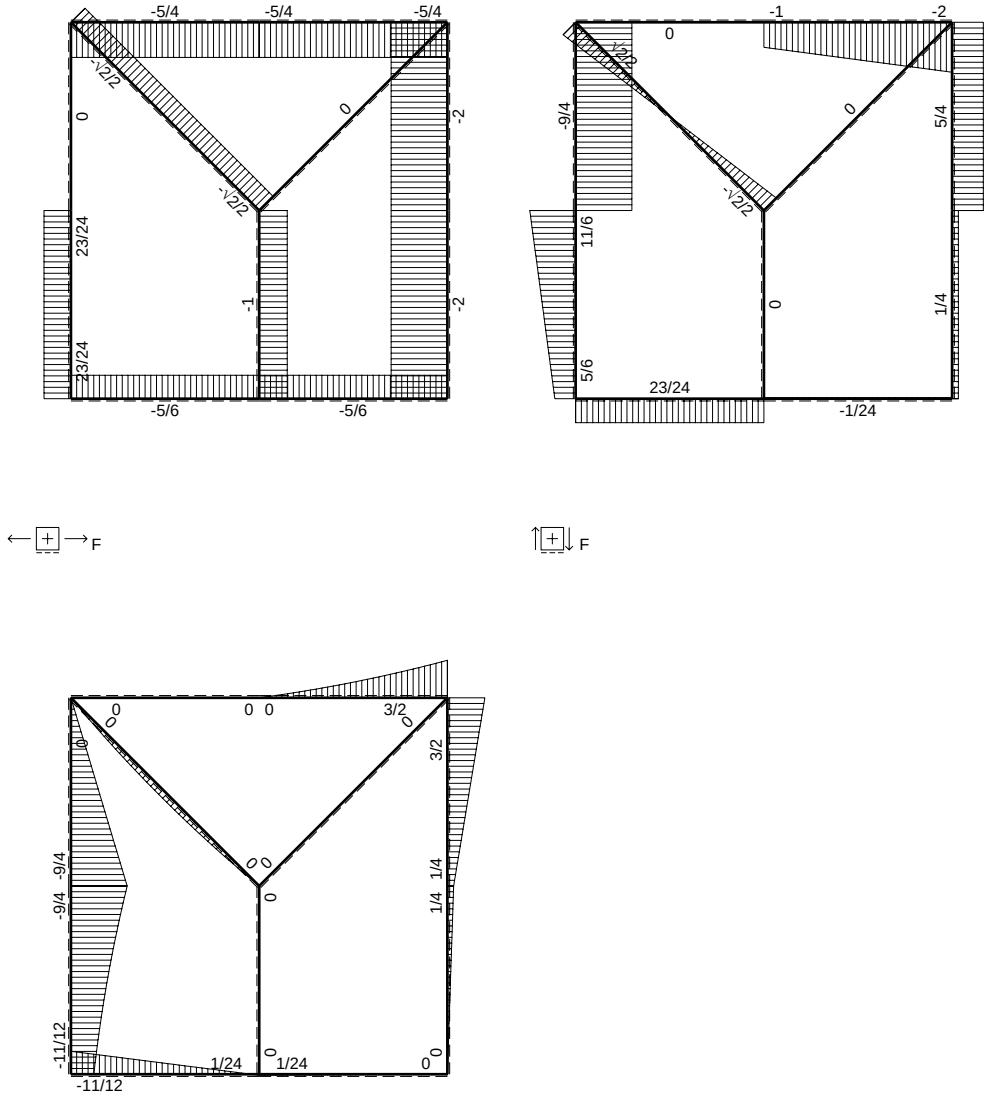
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

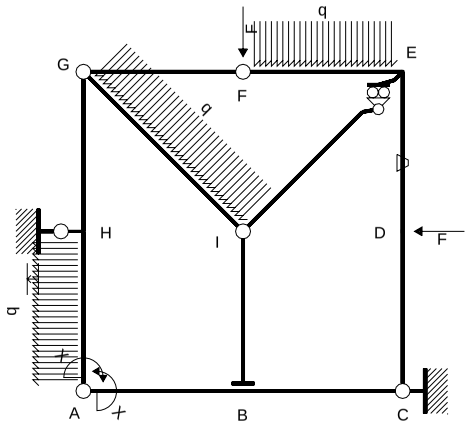
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

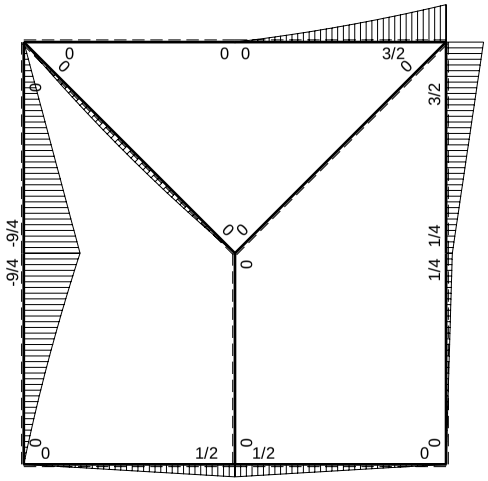
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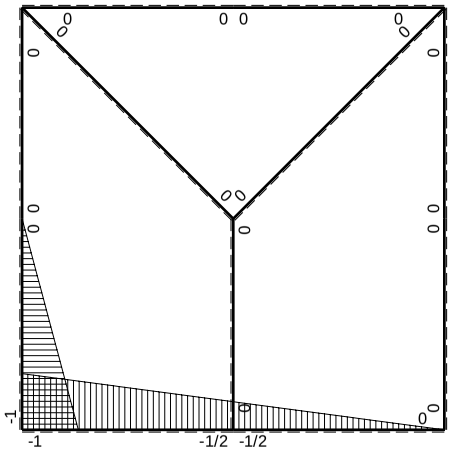

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-11/12
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1/24
0
0
-11/12
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.185

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.185

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

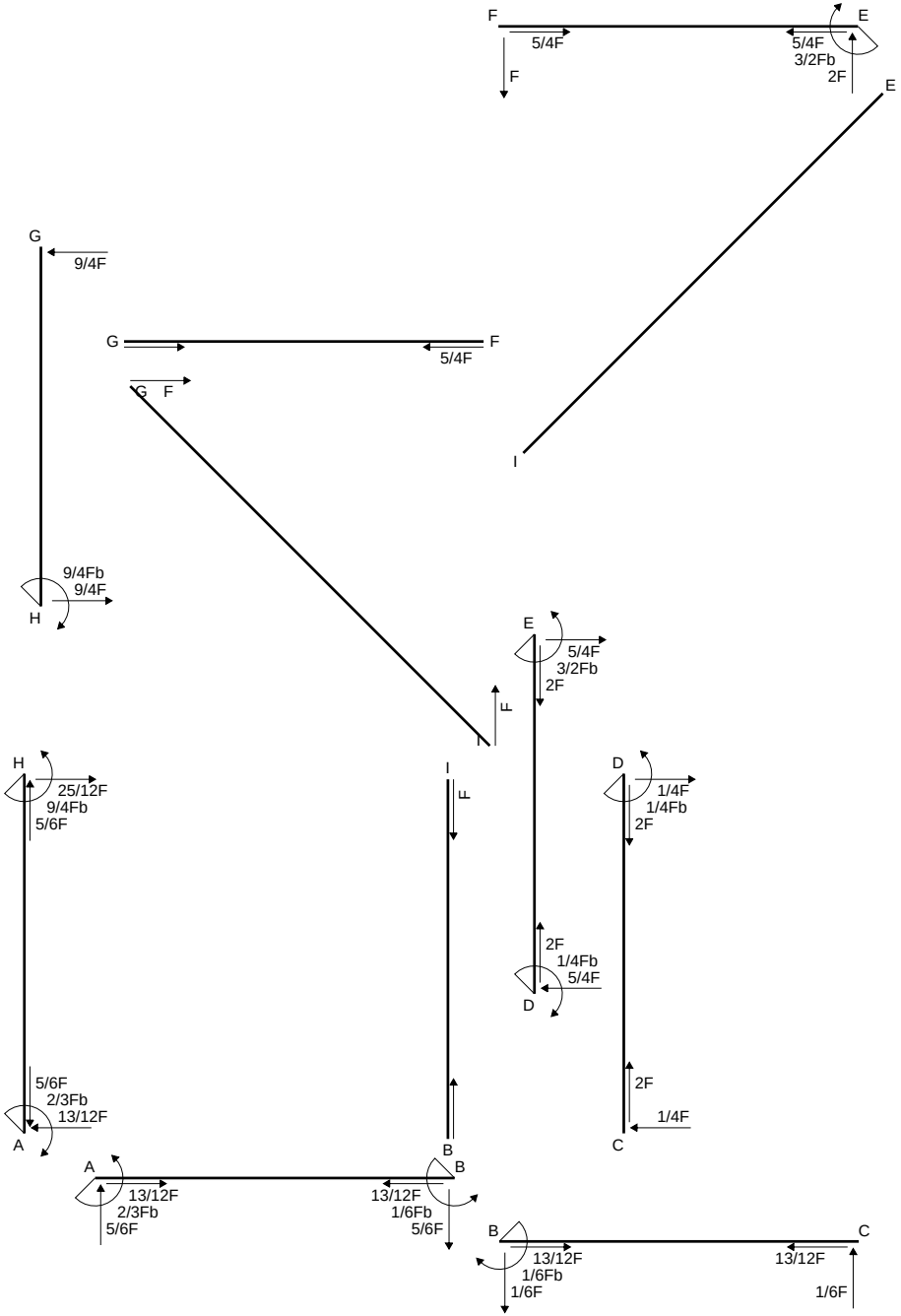
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

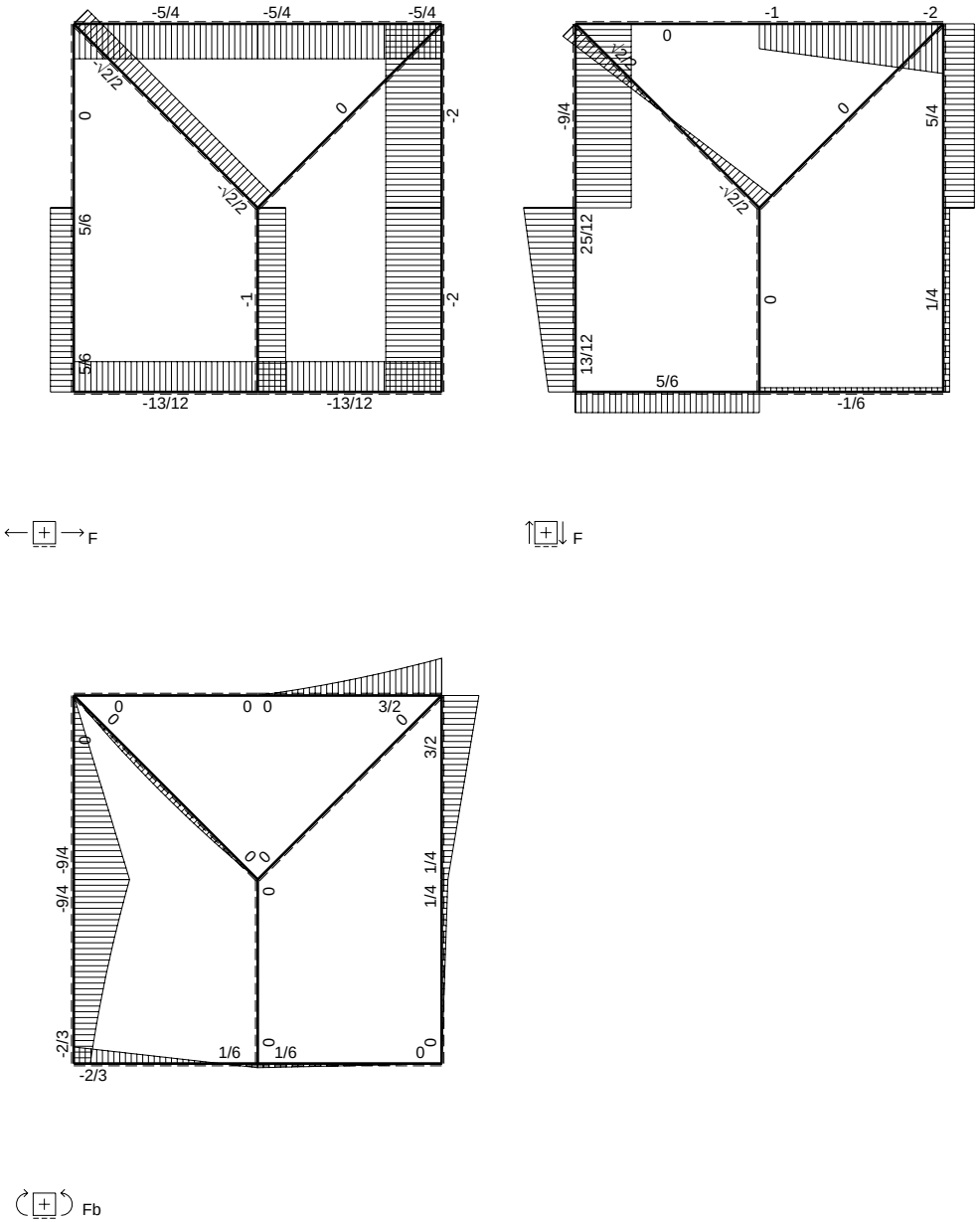
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

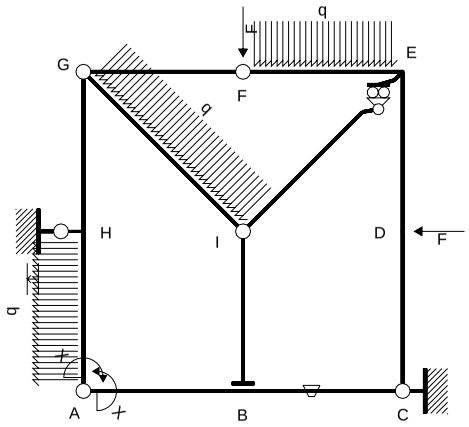
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

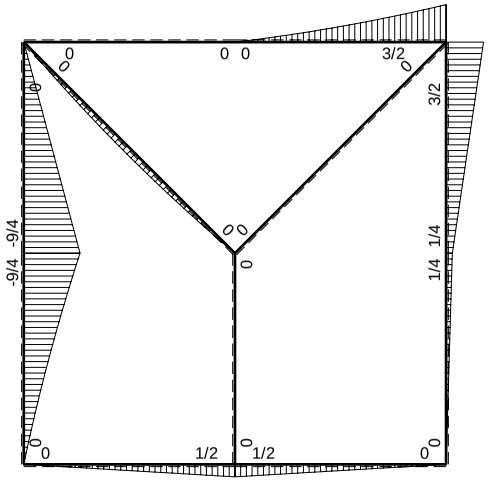
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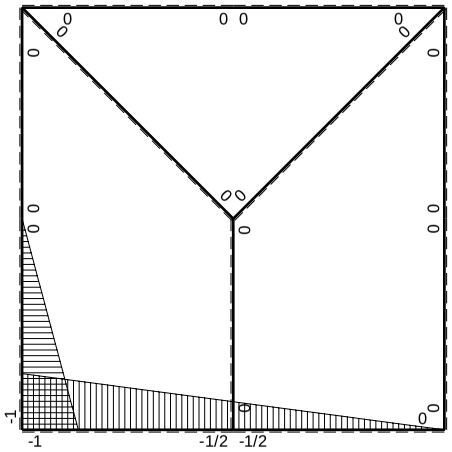

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H
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D
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2F
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A
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13/12F
1/6Fb
1/6F
13/12F
1/6F


-5/4
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-5/4
-√2/2
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-2
5/6
-13/12
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-9/4
1/4
1/4
0
-2/3
1/6
1/6
0
0
-2/3
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	-Fb/EJ	$-1/4Fb+1/2Fx-1/4Fx^2/b$	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	Fb/EJ	$-1/4Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-2/3Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						2/3Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 1/6\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\ \right)\ \theta\ \ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/12\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/4\ b\ \right)\ \theta\ \ = 1/6\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

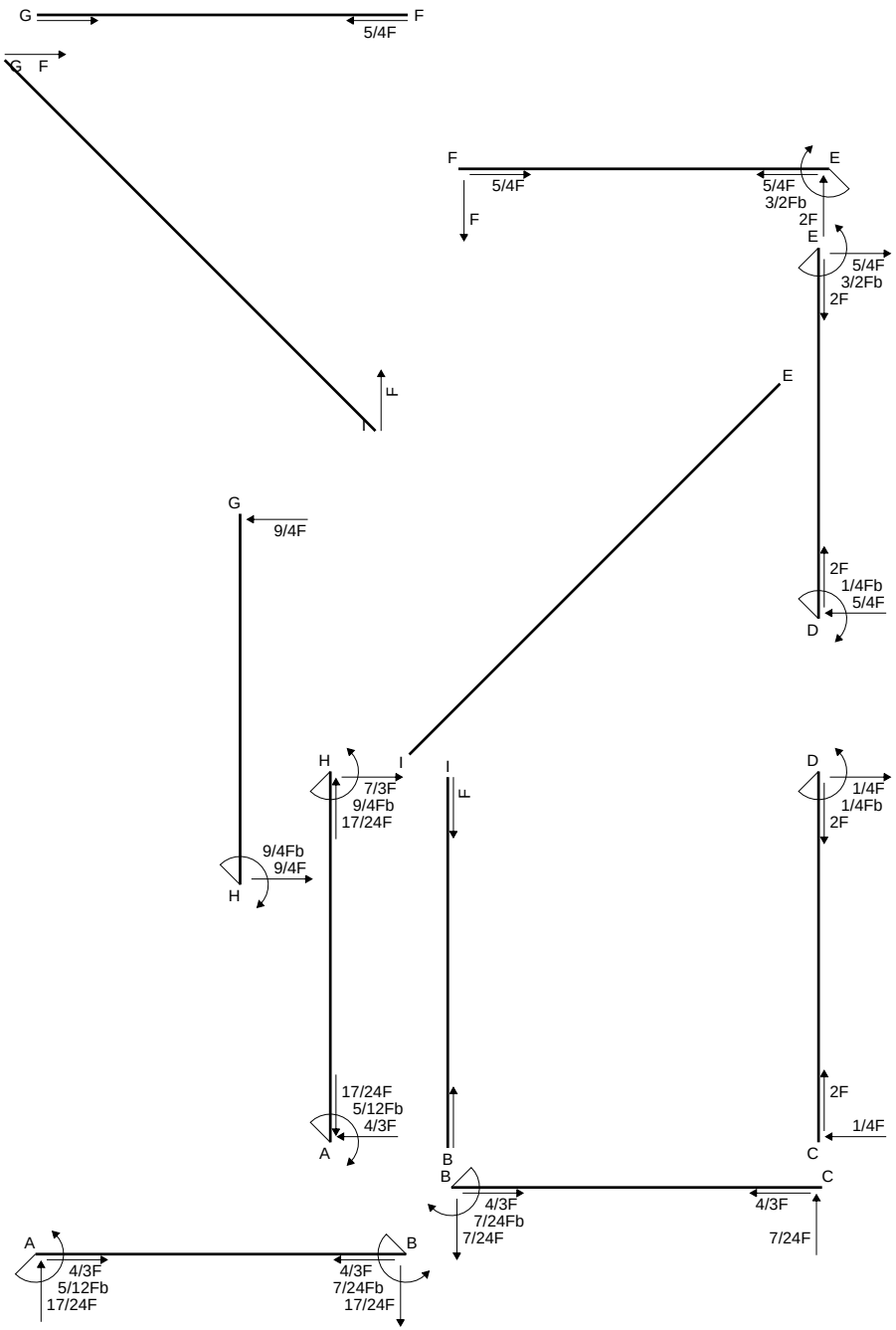
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

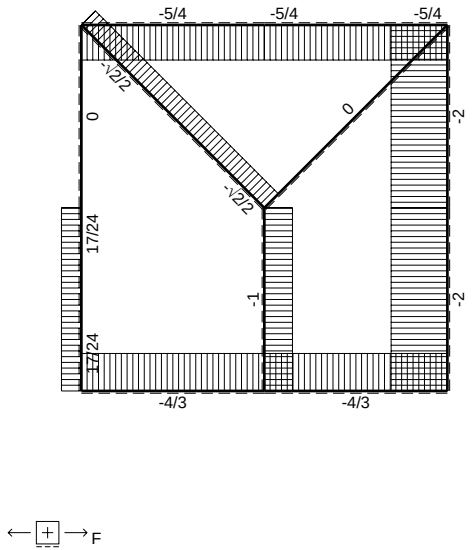
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

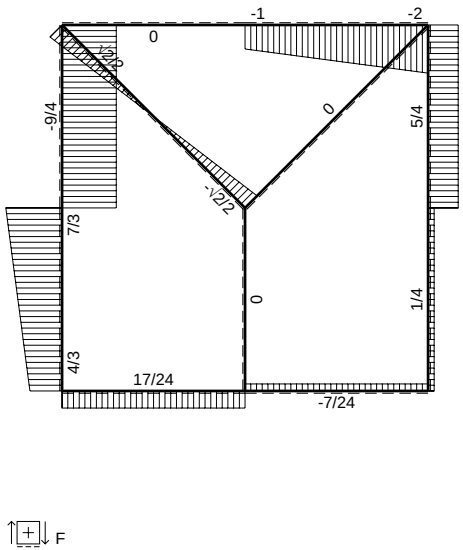
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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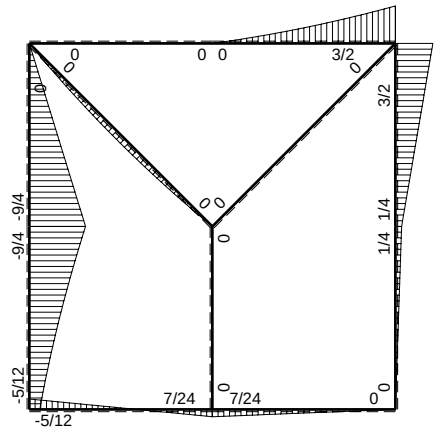

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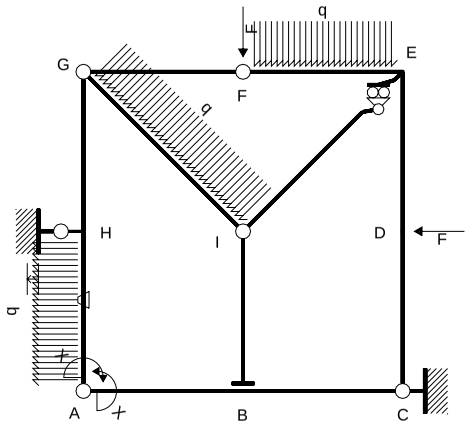

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-7/24

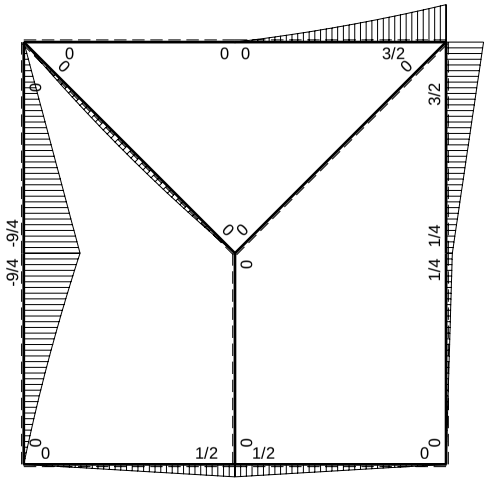
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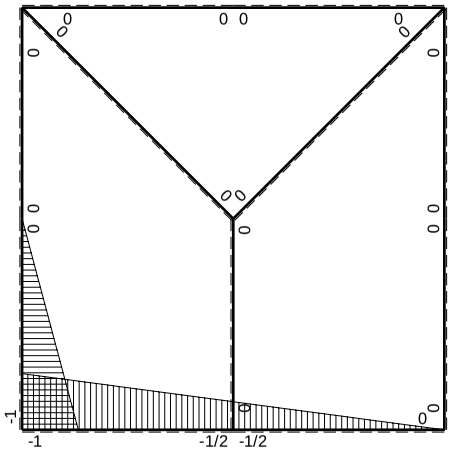

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3/2
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0
1/4
1/4
-5/12
7/24
7/24
0
0
-5/12


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.187

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	-Fb/EJ	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/3+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	Fb/EJ	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\right)\ \theta\ \ dx$

$= \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ + \left(1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

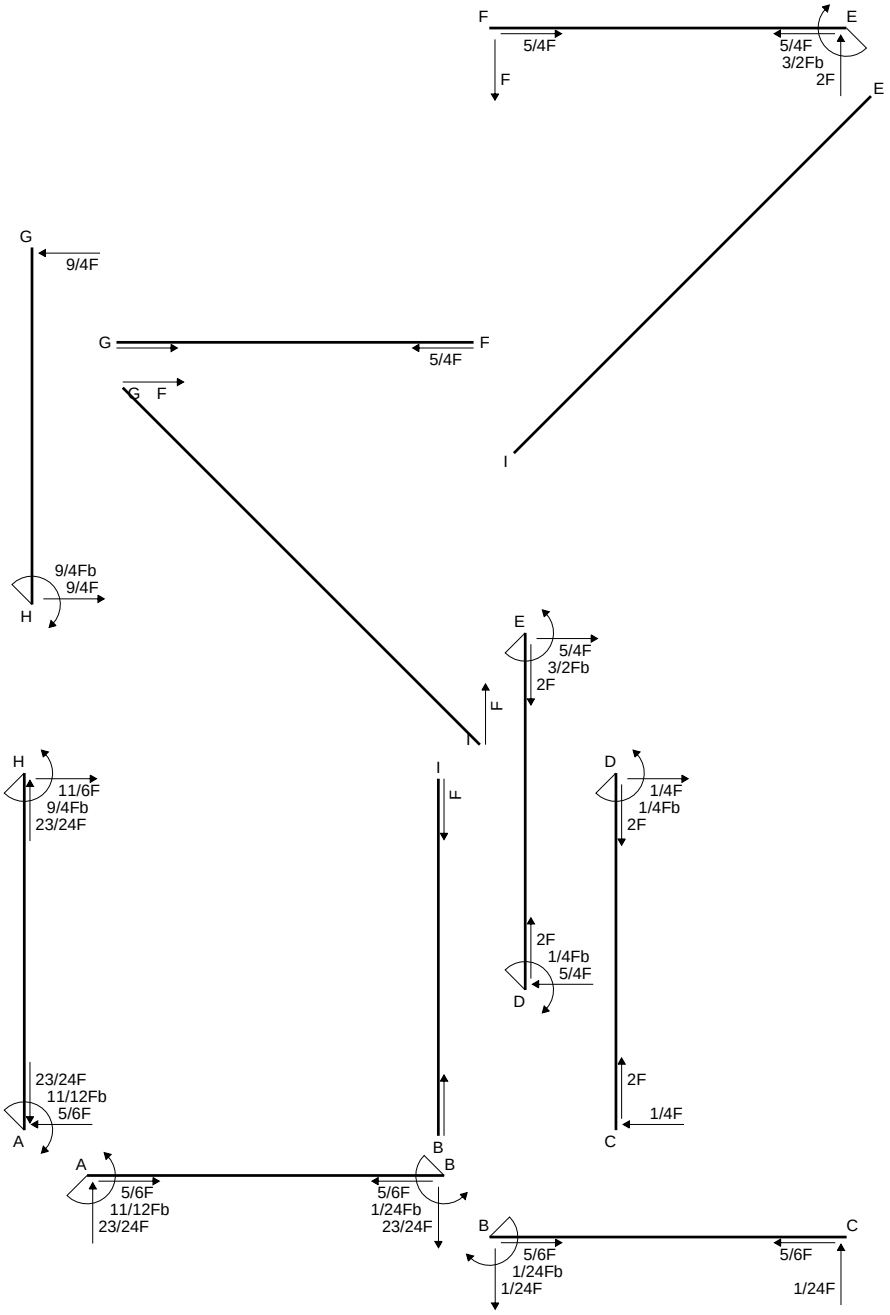
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\right)\ \theta\ \ dx$

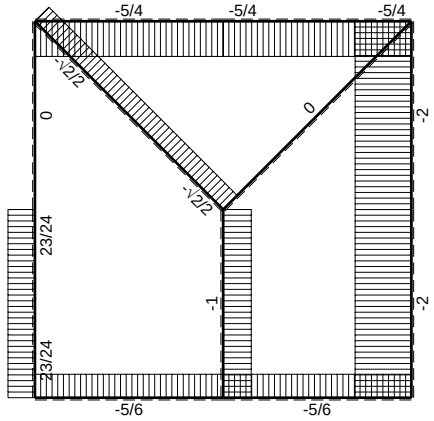
$= \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

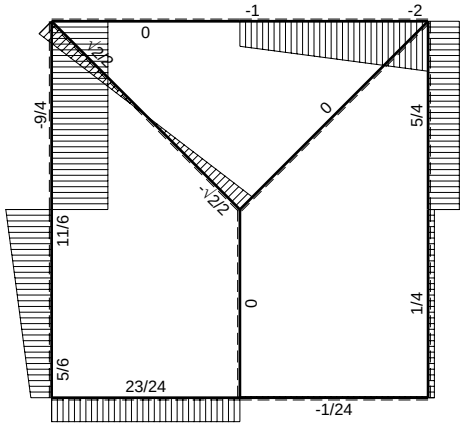
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

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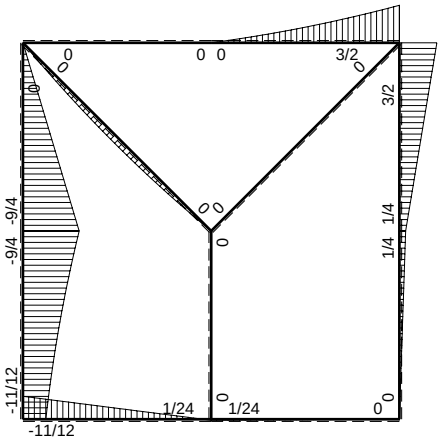

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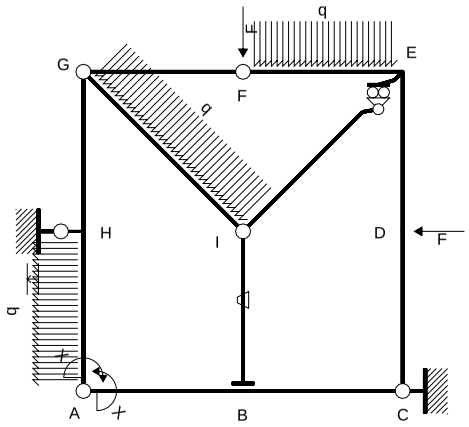

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23/24
-1/24

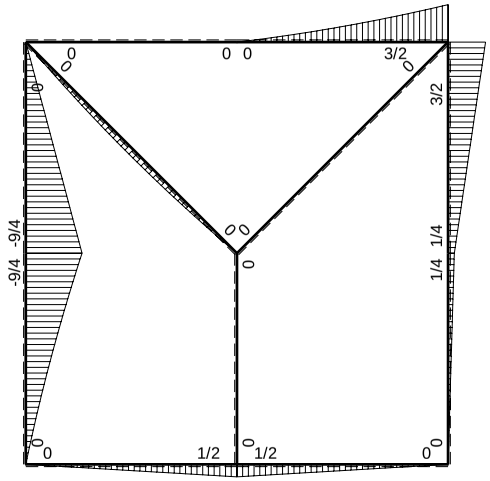
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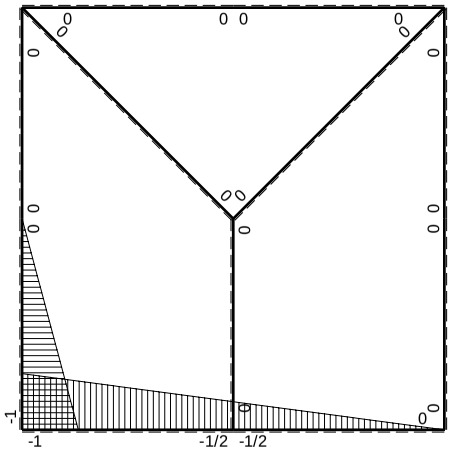

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1/4
-11/12
1/24
1/24
0
0
-11/12

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.188

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

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Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

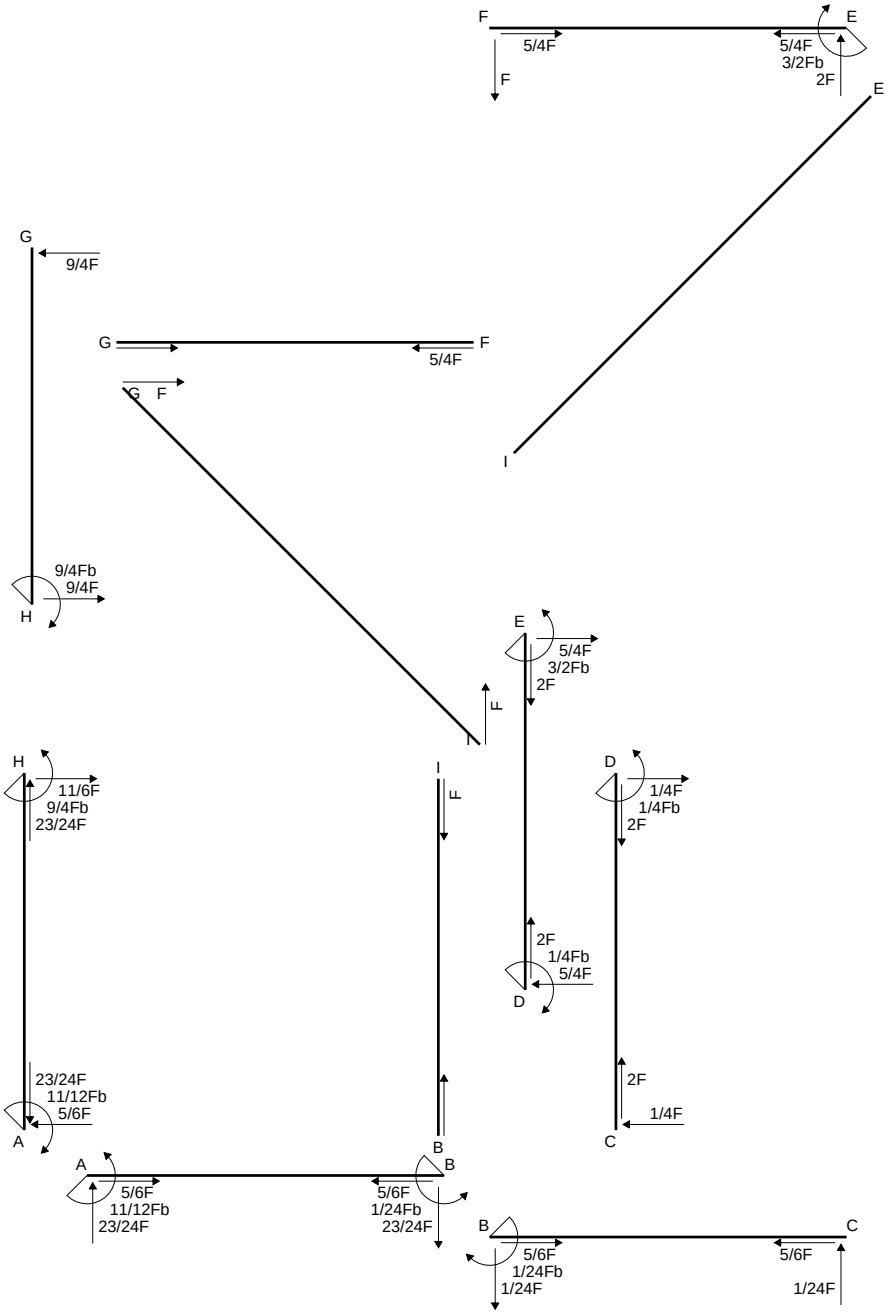
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

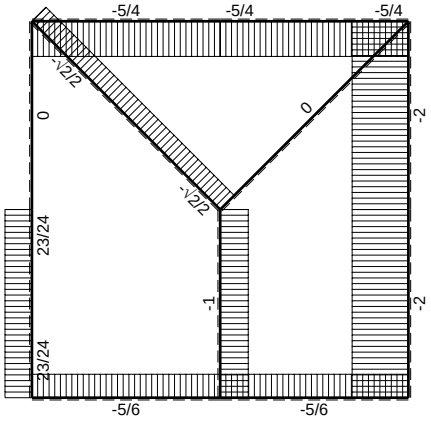
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

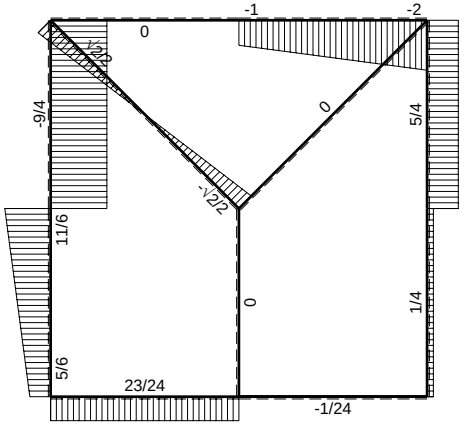
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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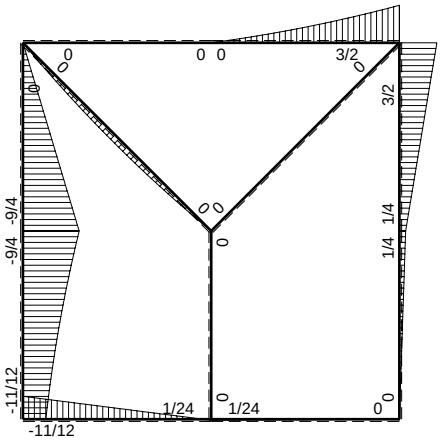

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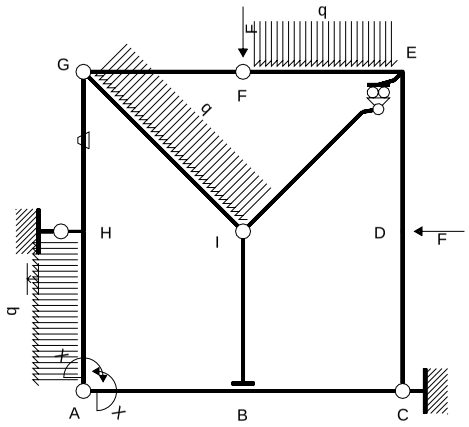

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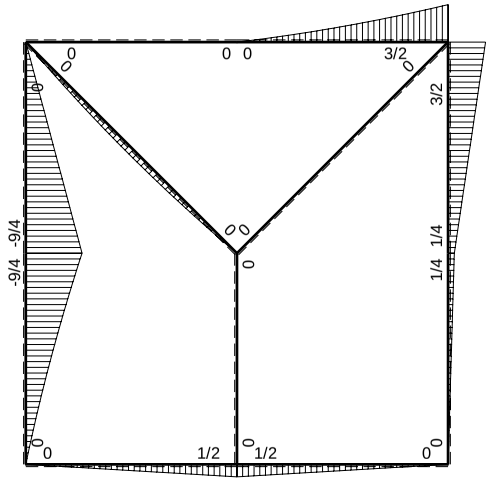
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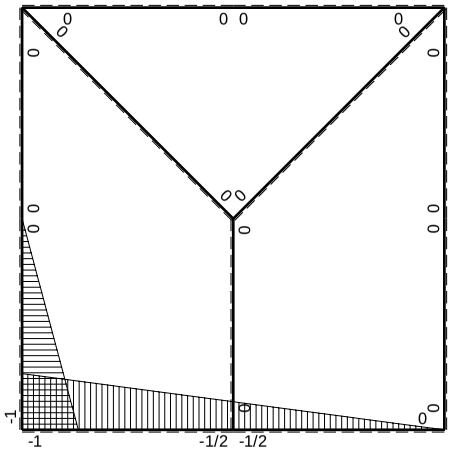

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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.189

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PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.189

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

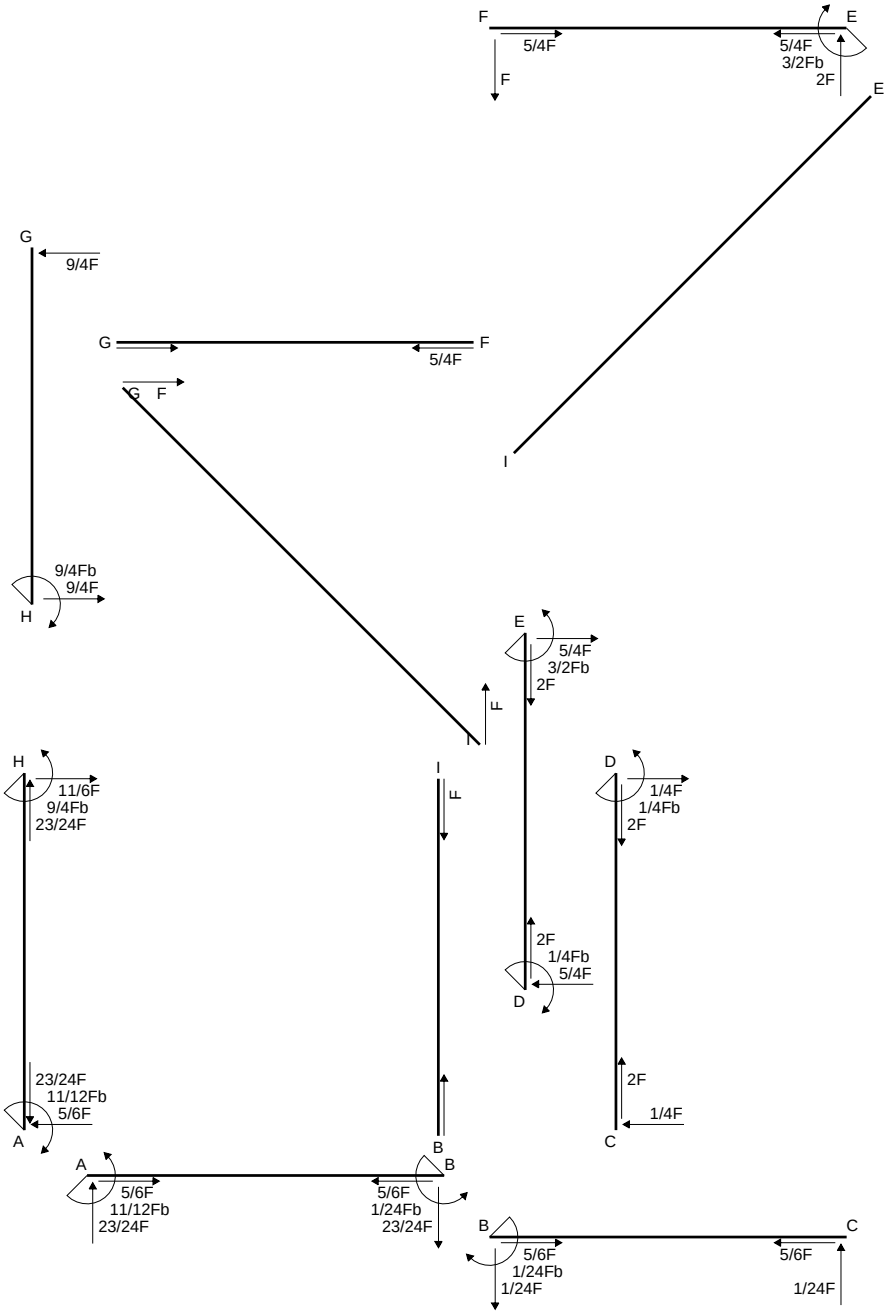
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

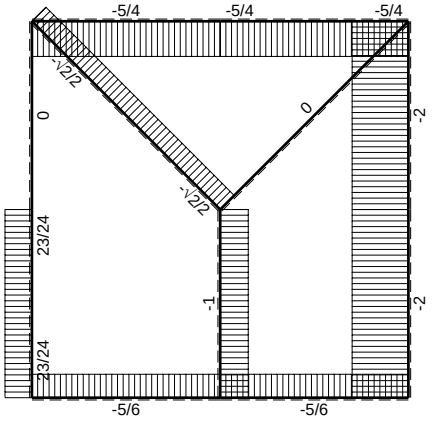
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

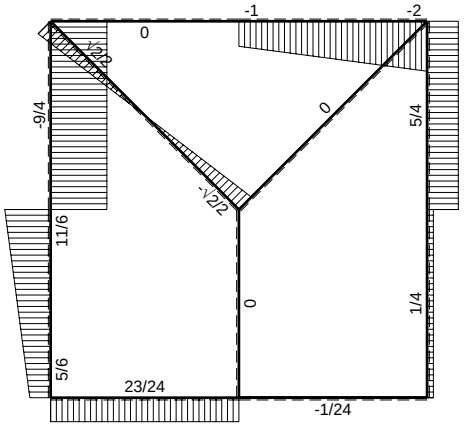
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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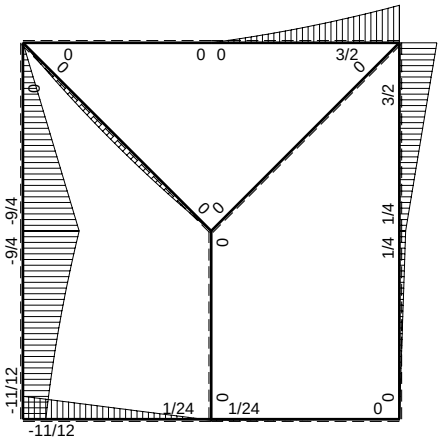

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-5/6

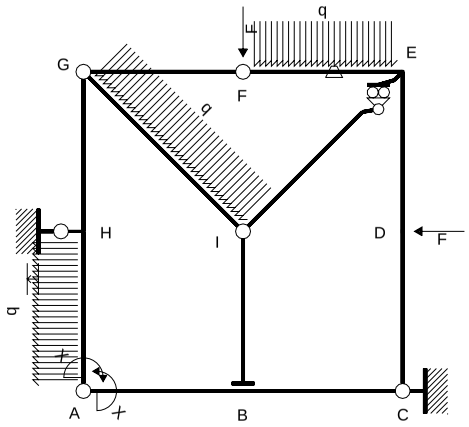

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23/24
-1/24

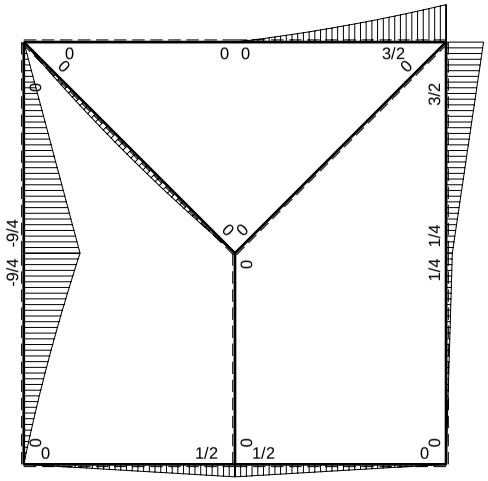
←□→ F

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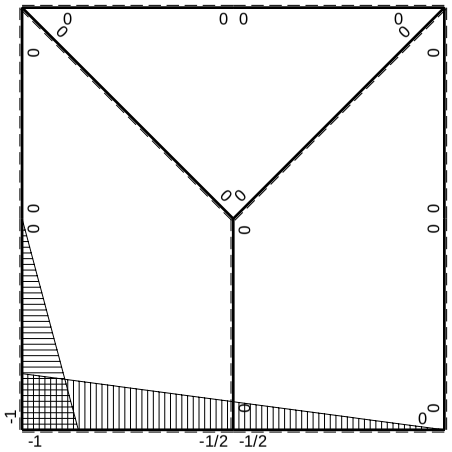

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1/4
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-11/12
1/24
1/24
0
0
-11/12

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

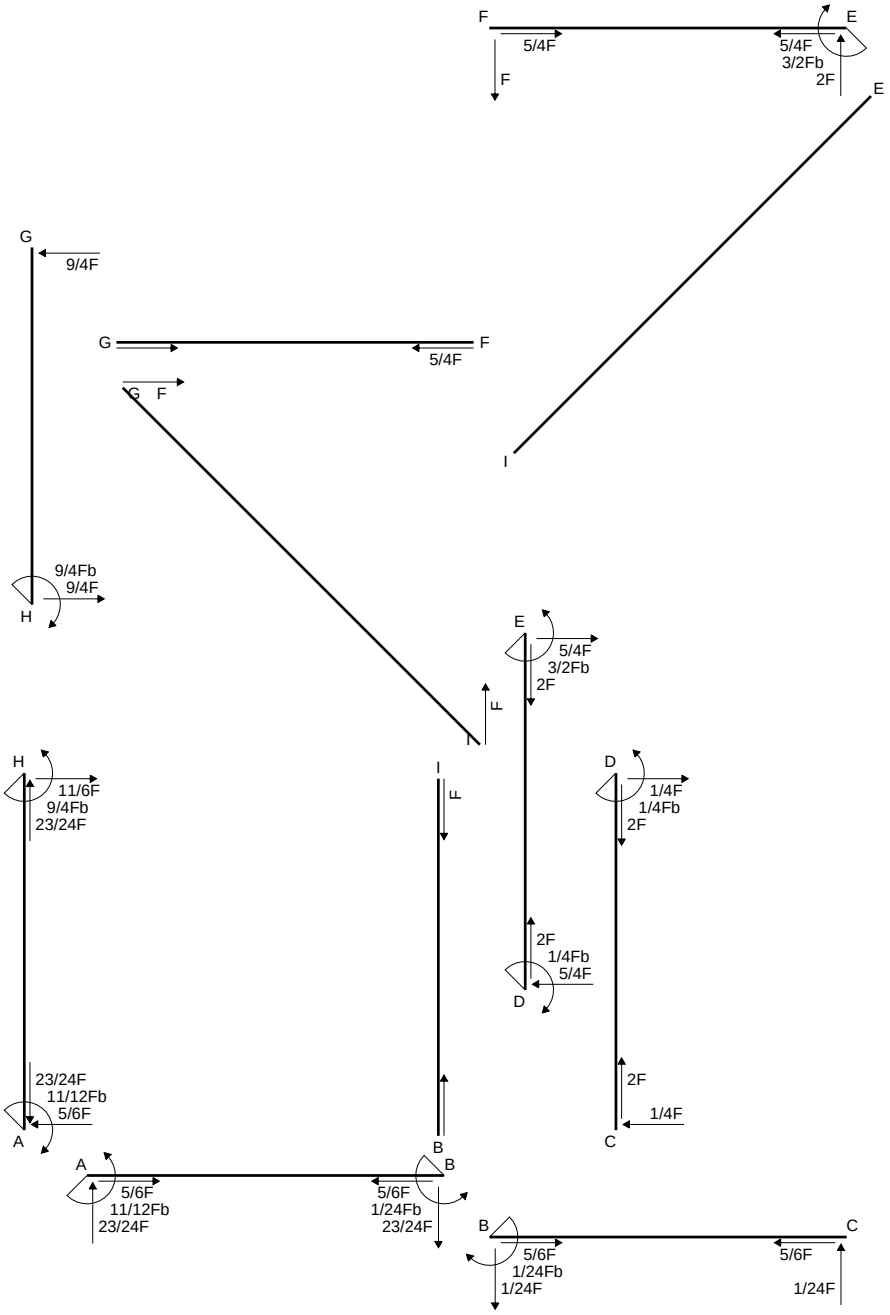
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

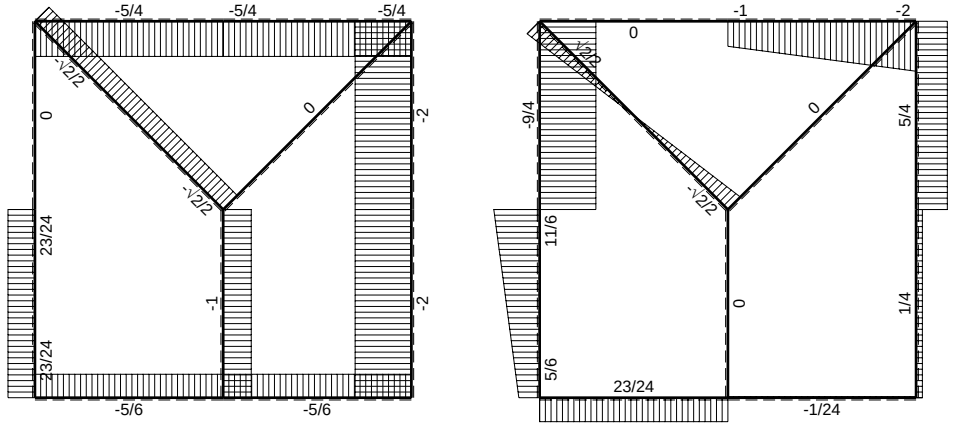
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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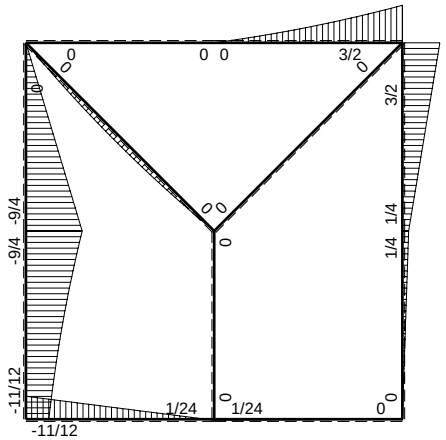
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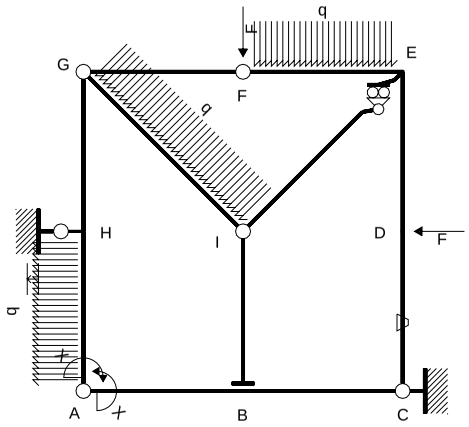

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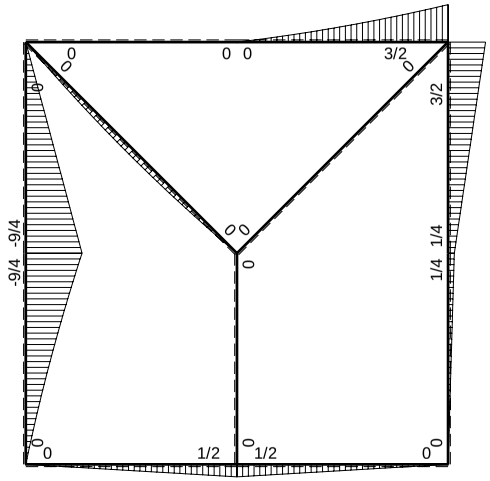
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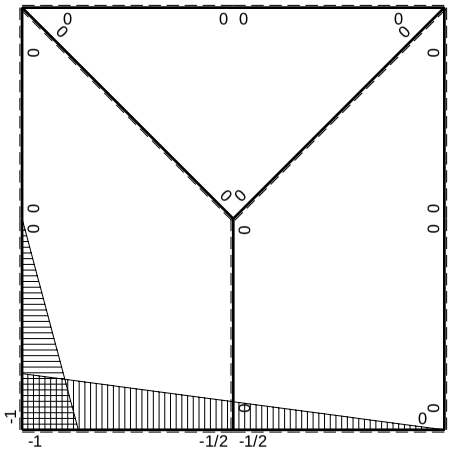

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1/4
-11/12
1/24
1/24
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0
-11/12

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.191

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PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.191

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-11/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						11/12Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

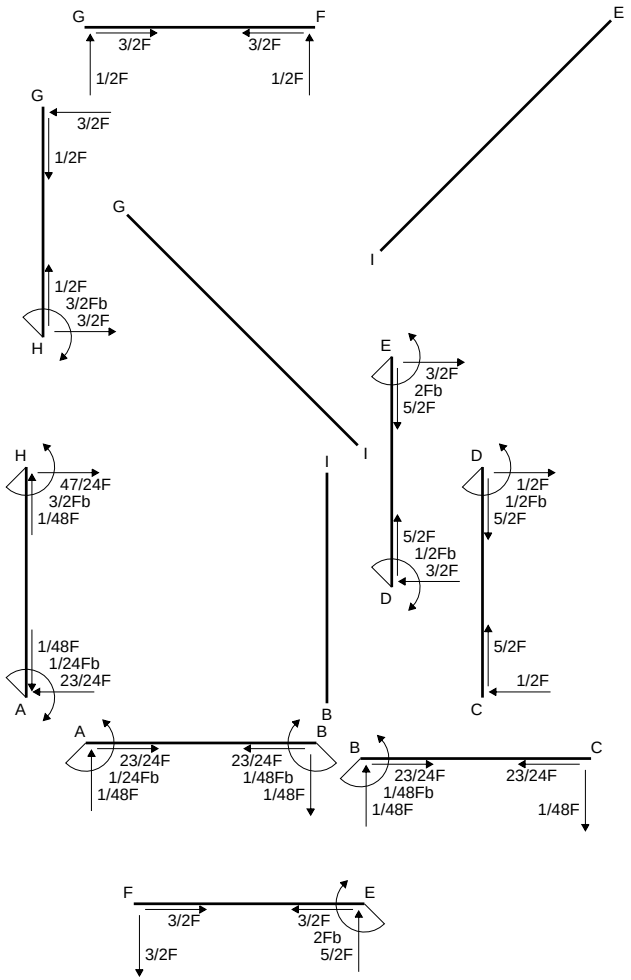
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

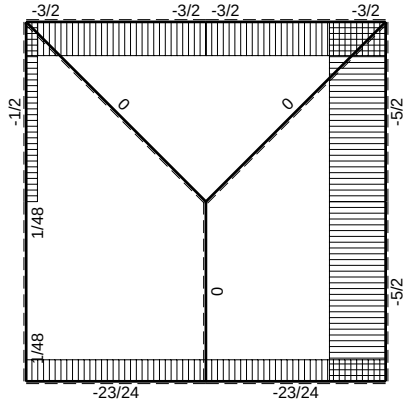
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

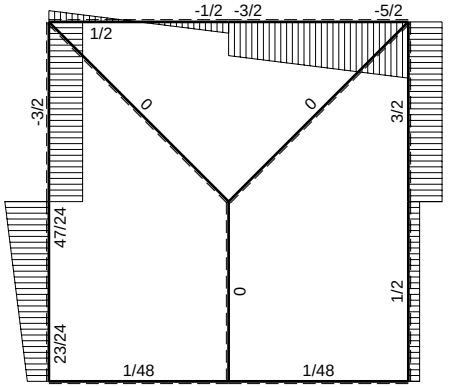
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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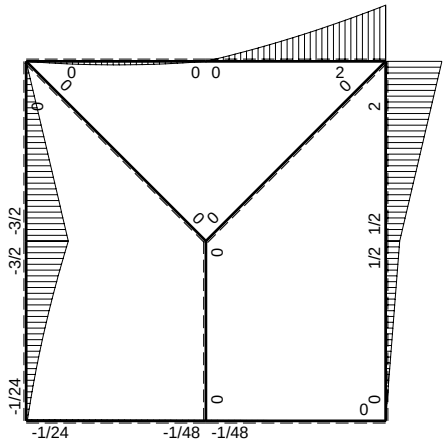

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-23/24

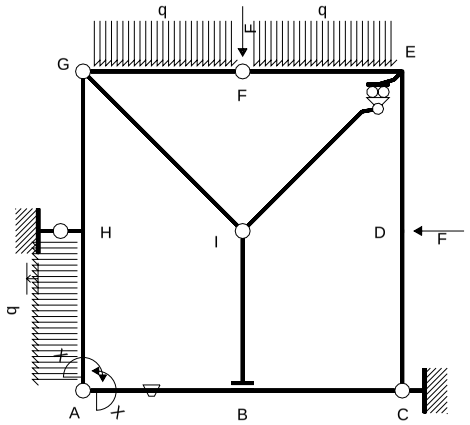

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1/48
1/48

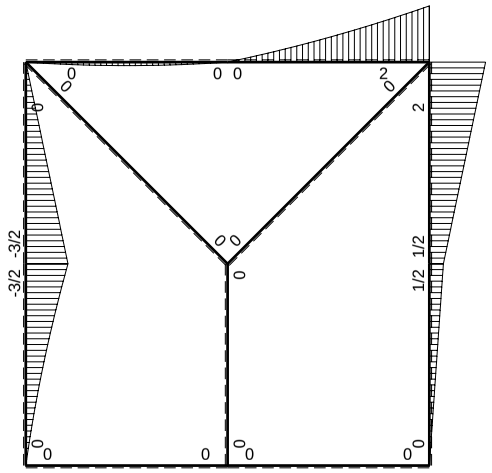
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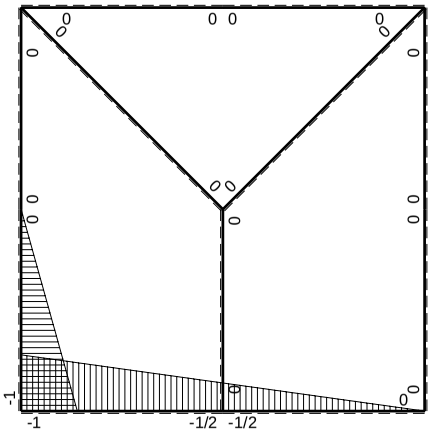

-3/2
-3/2
1/2
1/2
-1/24
-1/24
-1/48
-1/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.192

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	-Fb/EJ	0	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(0+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	0	Fb/EJ	0	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/24Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.192

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(1 - 1/2\ x/b\ \right)\theta\ \ dx = \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/2\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

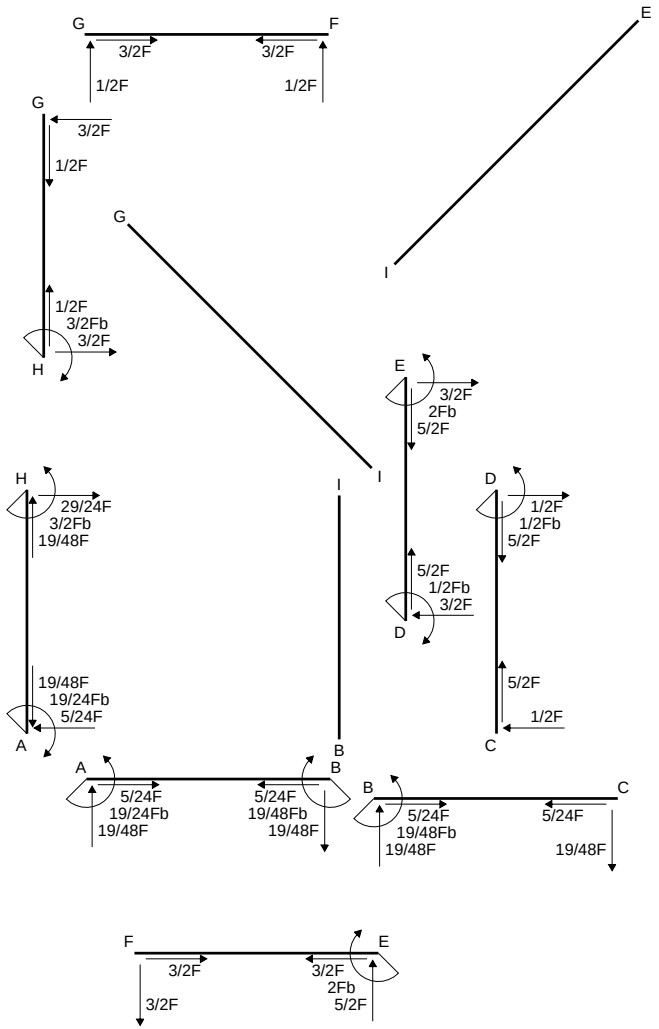
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

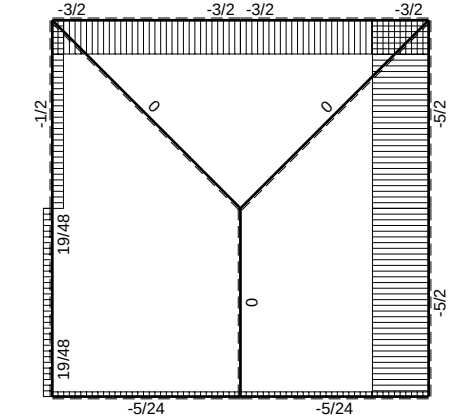
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

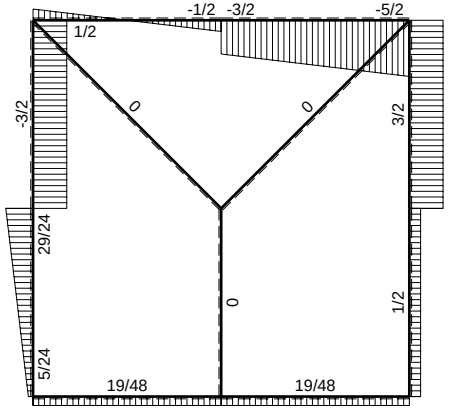
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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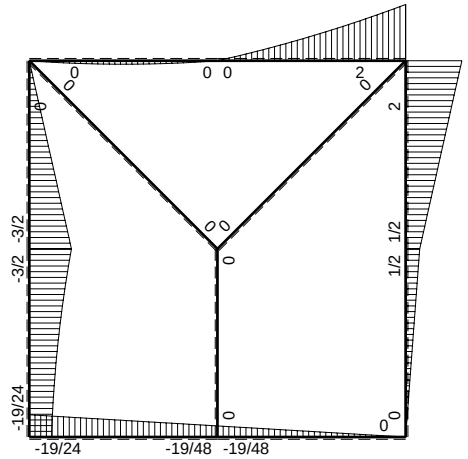

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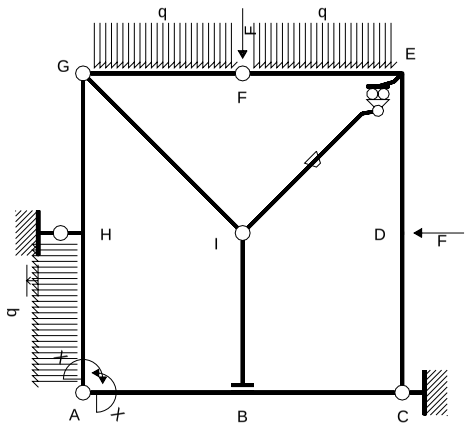

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19/48

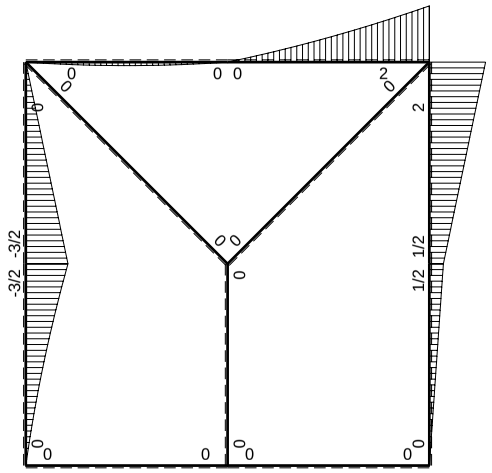
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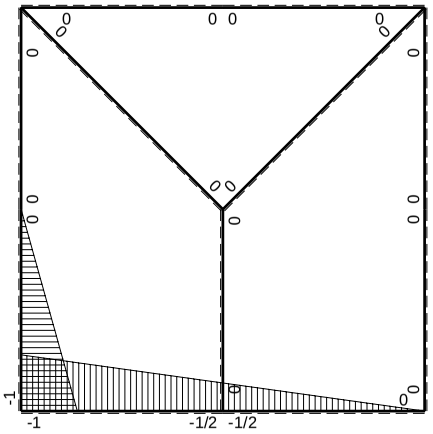

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1/2
-19/24
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-19/48 -19/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.193

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.193

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

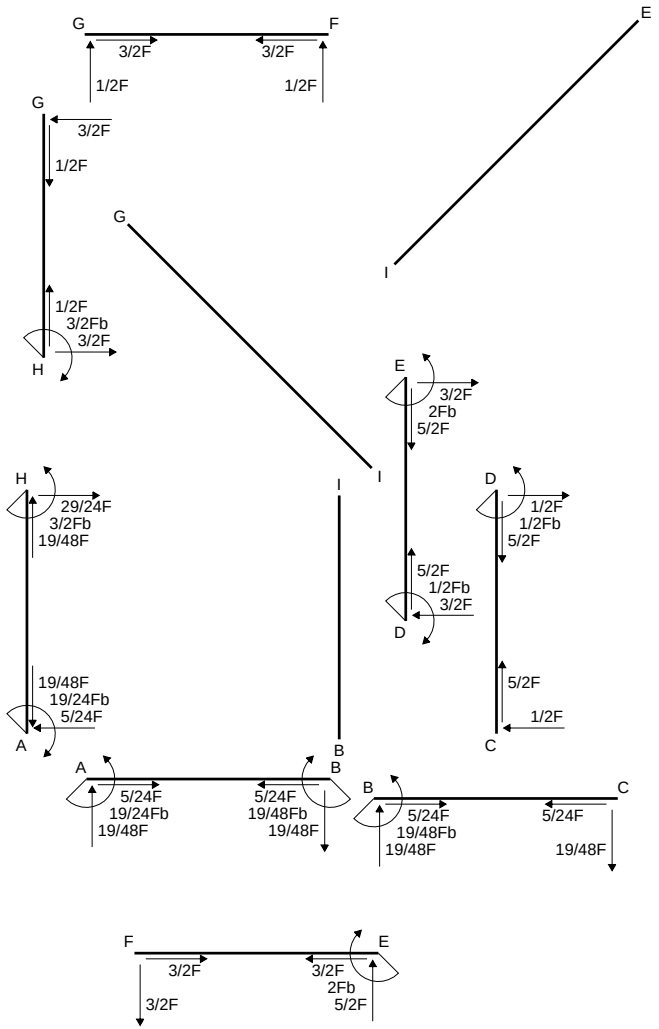
$= \left(3/4\ b - 2/3\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

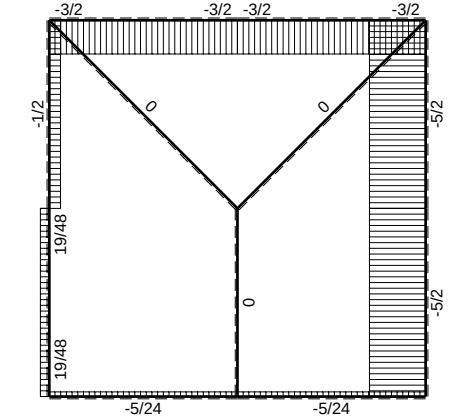
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

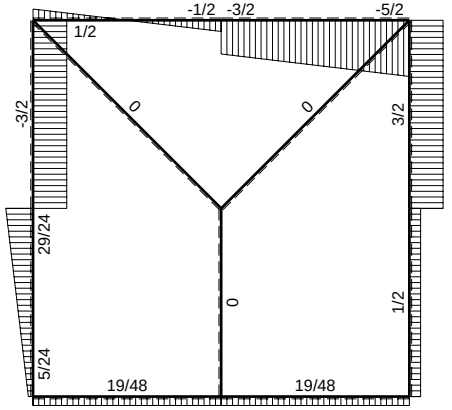
$= \left(1/2\ b - 1/6\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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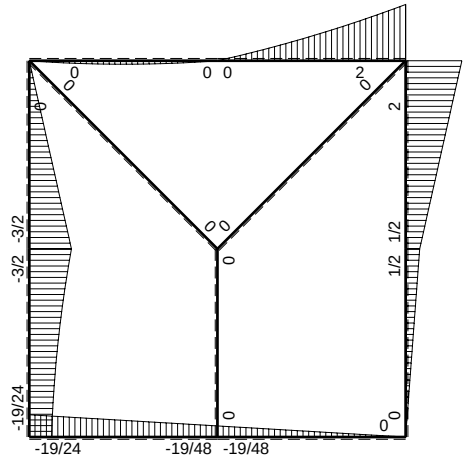

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I
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3/2F
2Fb
5/2F
H
29/24F
3/2Fb
19/48F
I
D
1/2F
1/2Fb
5/2F
5/2F
1/2Fb
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D
19/48F
19/24Fb
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1/2F
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C
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2Fb
5/2F


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-5/24

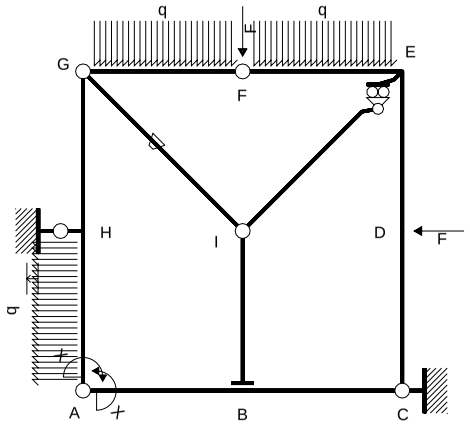

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29/24
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19/48

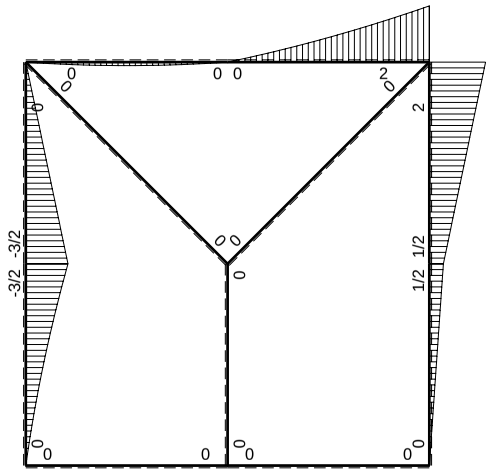
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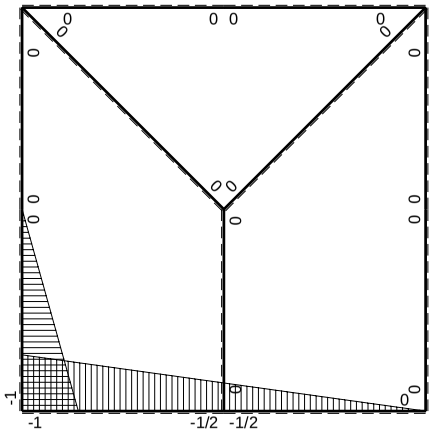

0
0 0
2
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-3/2
1/2
1/2
-19/24
-19/24
-19/48 -19/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.194

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PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.194

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

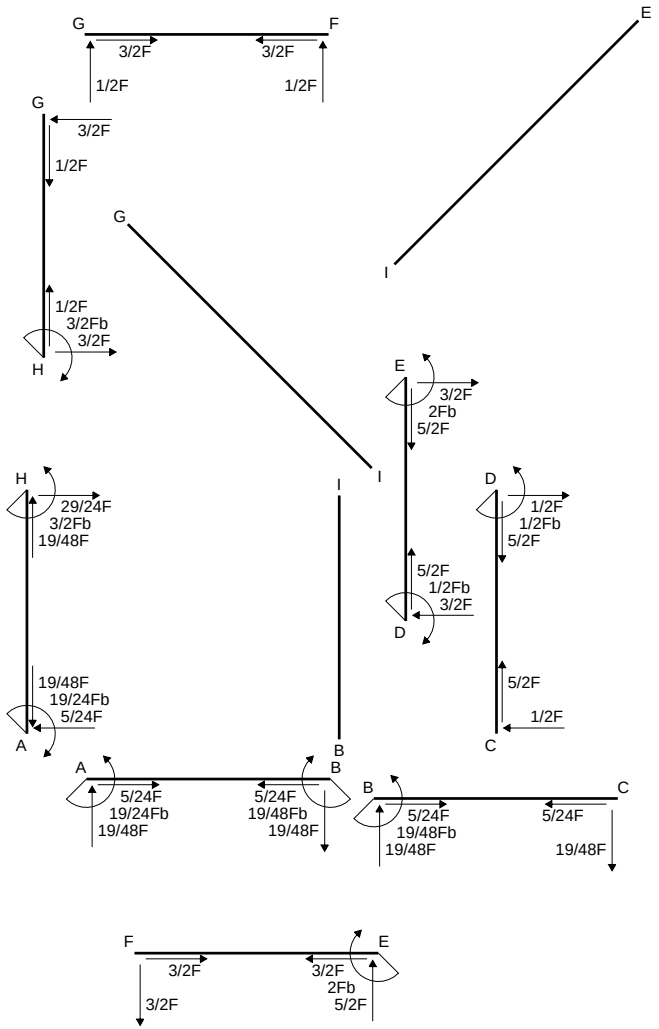
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

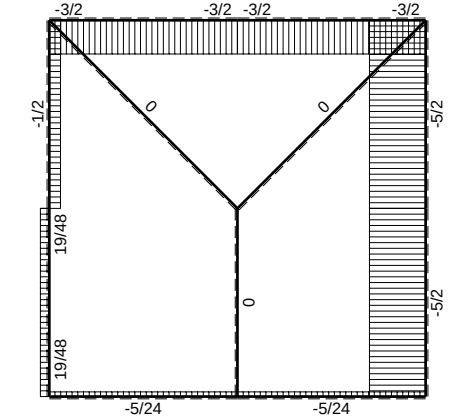
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

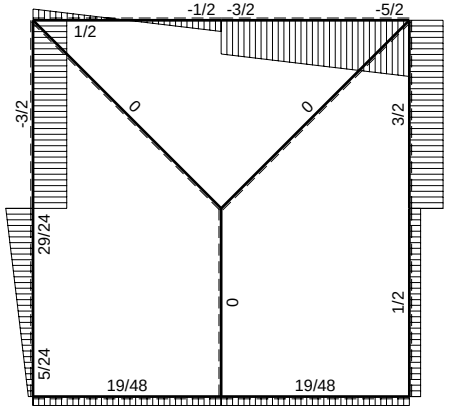
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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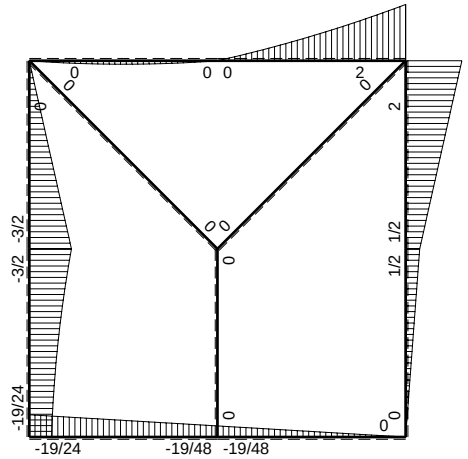

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2Fb
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29/24F
3/2Fb
19/48F
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1/2Fb
5/2F
5/2F
1/2Fb
3/2F
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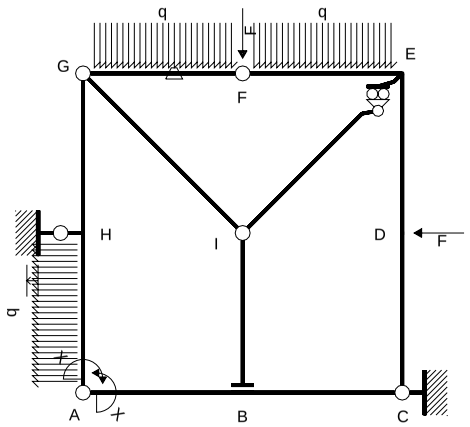

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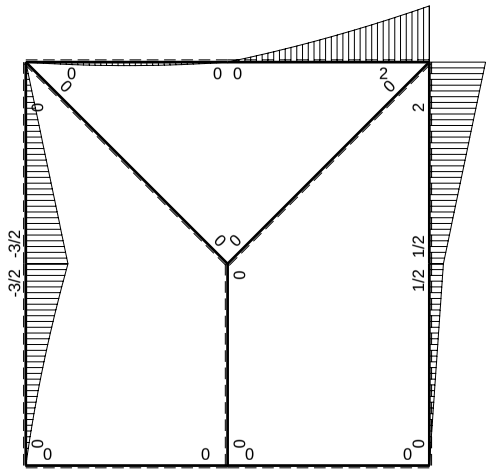
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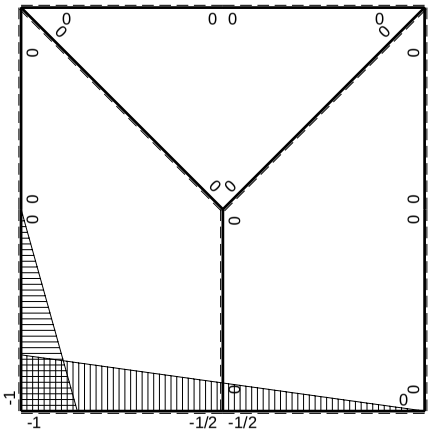

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1/2
-19/24
-19/24
-19/48 -19/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.195

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.195

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

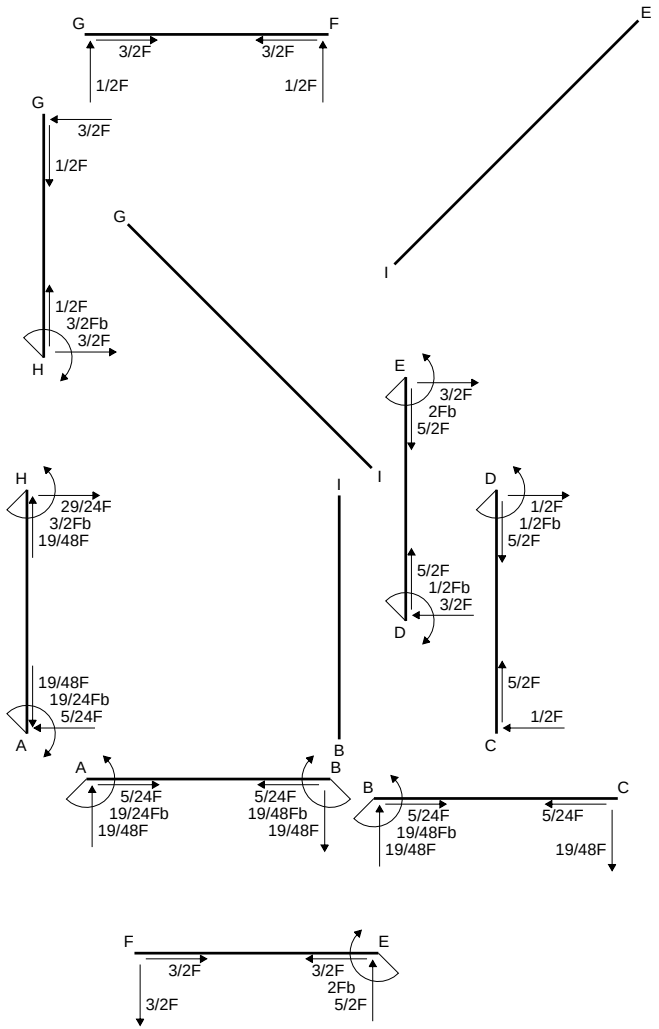
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

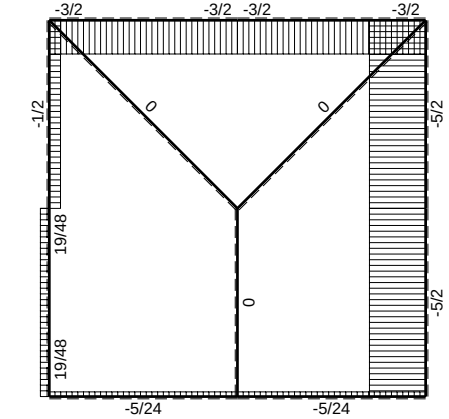
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

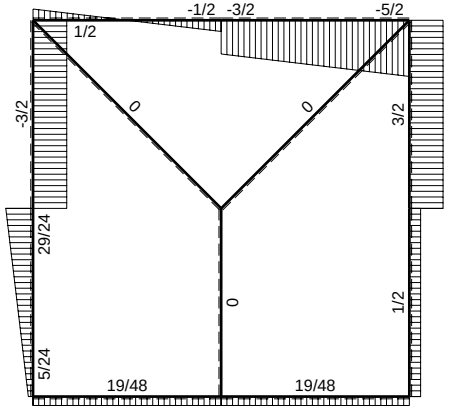
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

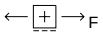
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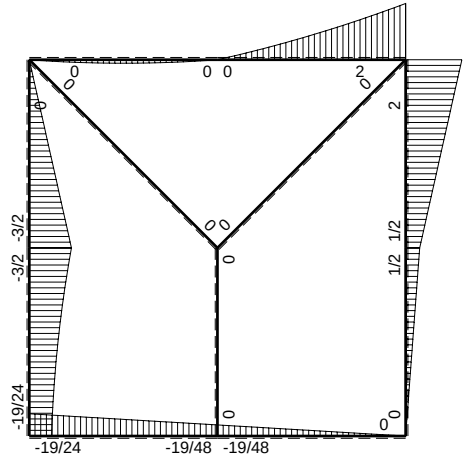

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E
3/2F
2Fb
5/2F
H
29/24F
3/2Fb
19/48F
I
D
1/2F
1/2Fb
5/2F
5/2F
1/2Fb
3/2F
D
19/48F
19/24Fb
5/24F
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A
B
C
A
B
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19/48F
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3/2F
2Fb
5/2F


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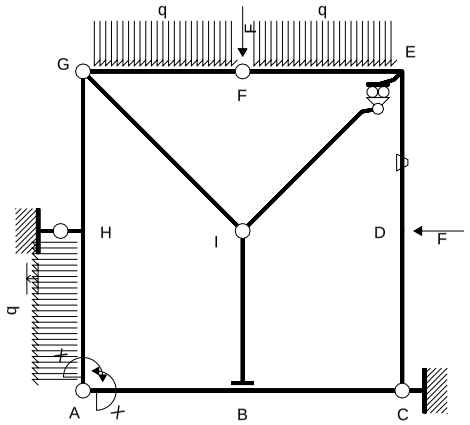

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19/48

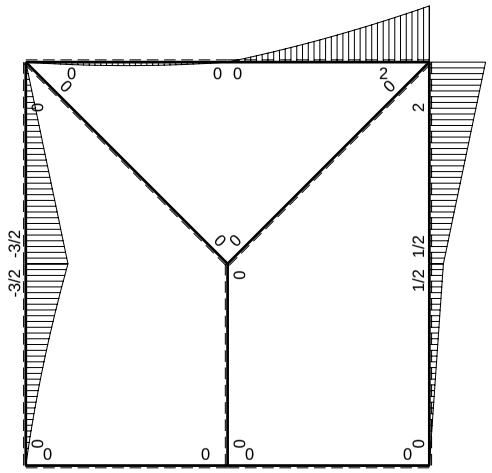

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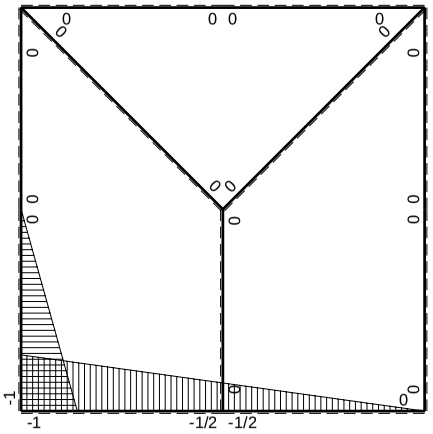

0 0 2
2
-3/2 -3/2
1/2 1/2
-19/24
-19/24 -19/48 -19/48


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.196

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	Fb/EJ	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.196

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

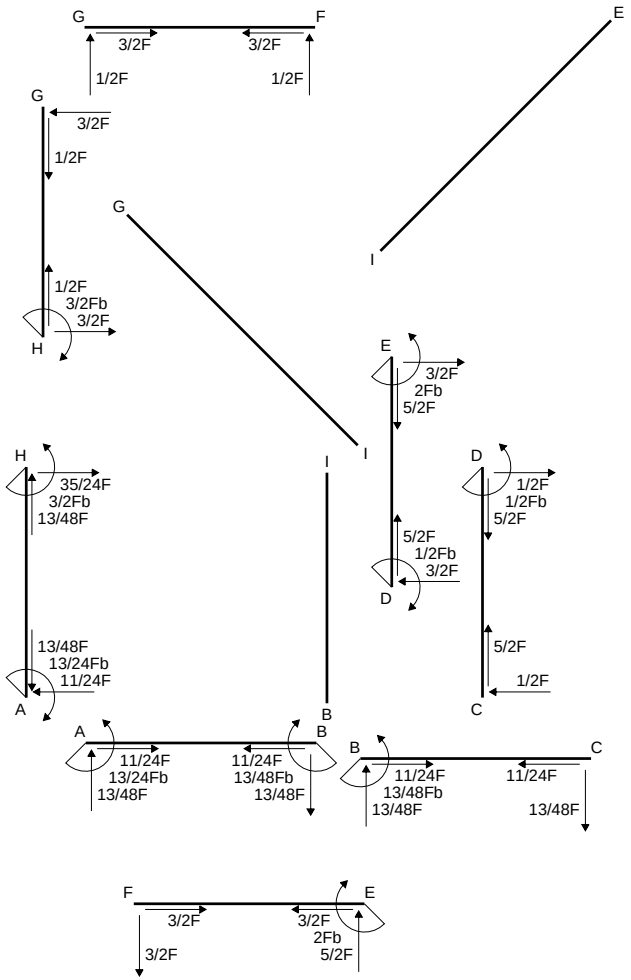
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

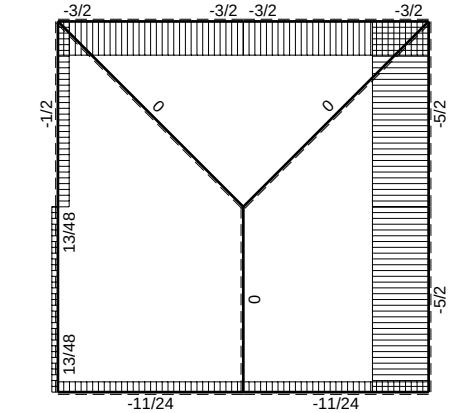
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

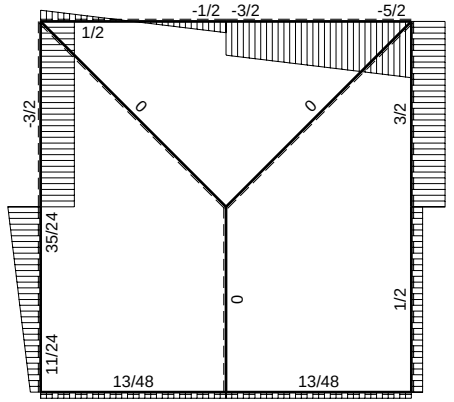
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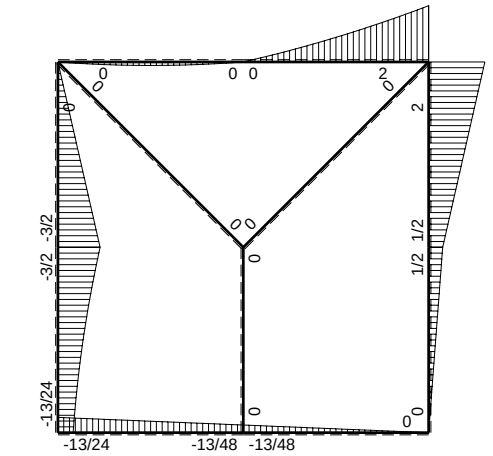




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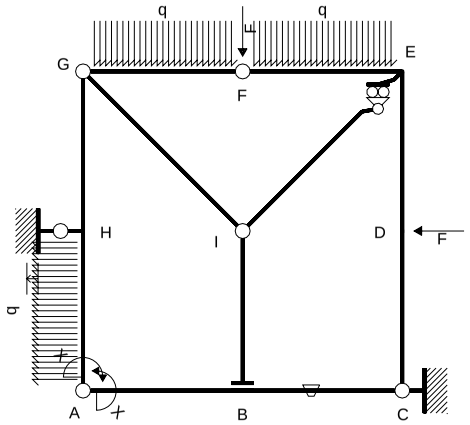


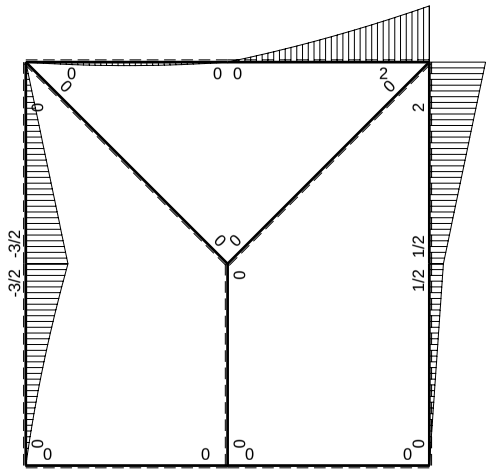
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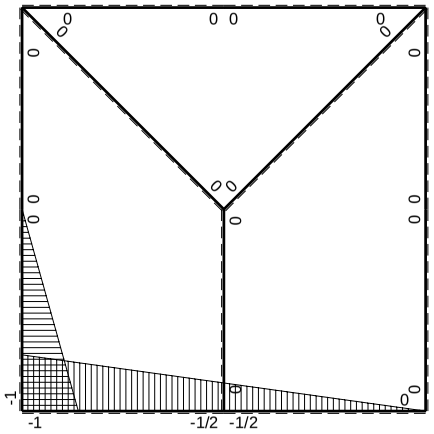
Fb

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M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.197

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	-Fb/EJ	0	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(0+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	0	Fb/EJ	0	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-13/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						13/24Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.197

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(1/2\ b - 1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/2\ x/b\ \right)\theta\ \ dx = \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

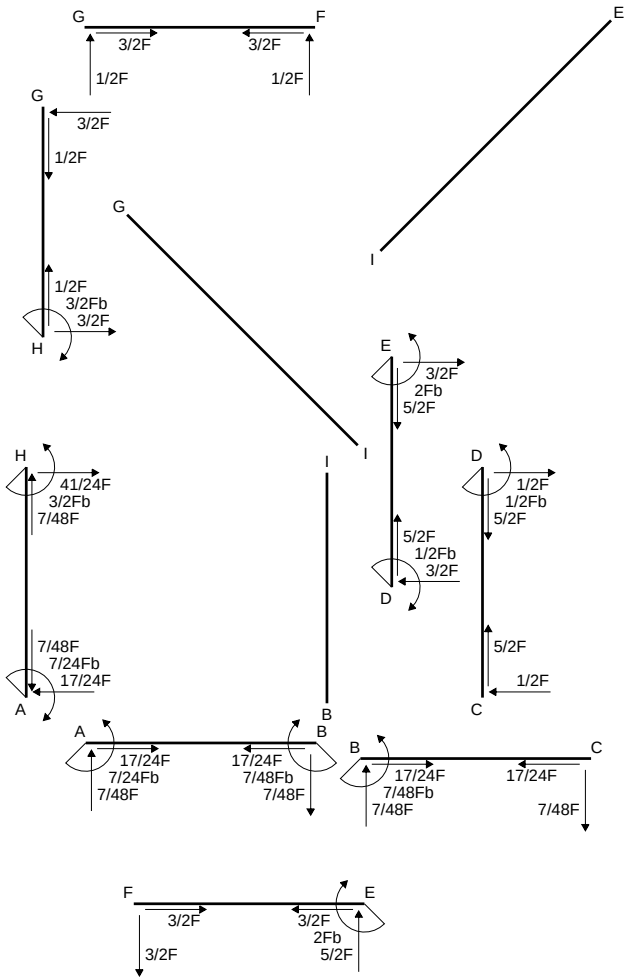
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

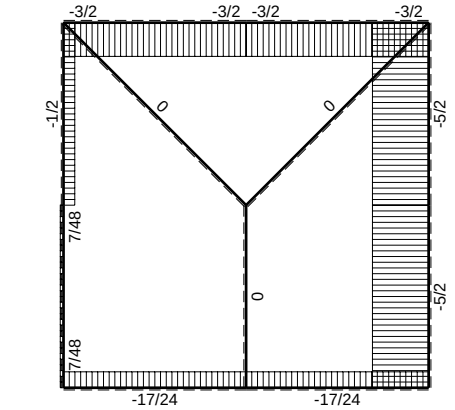
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

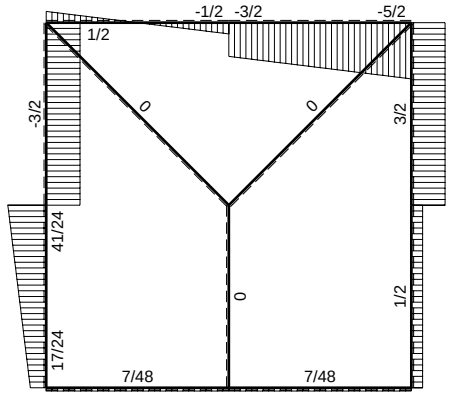
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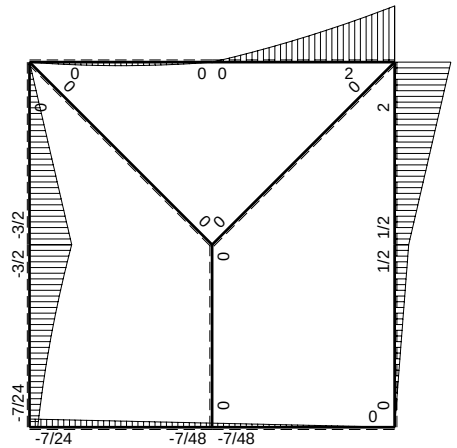

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H
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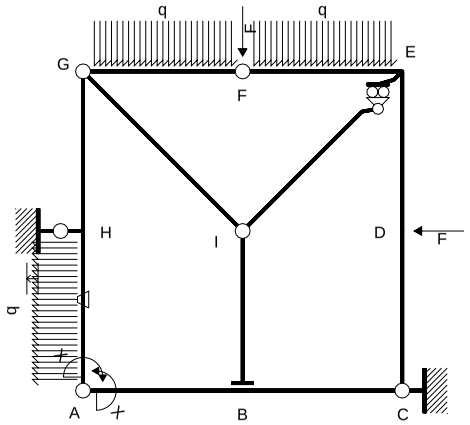
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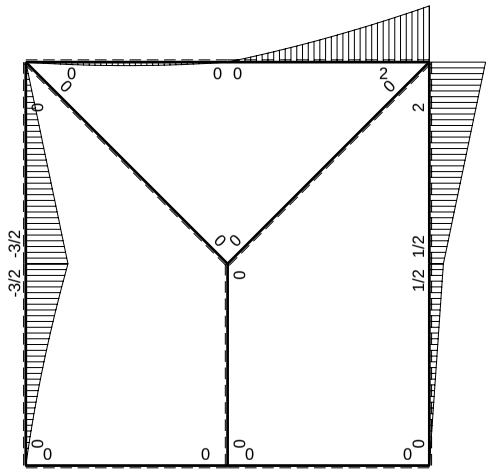

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7/48

F

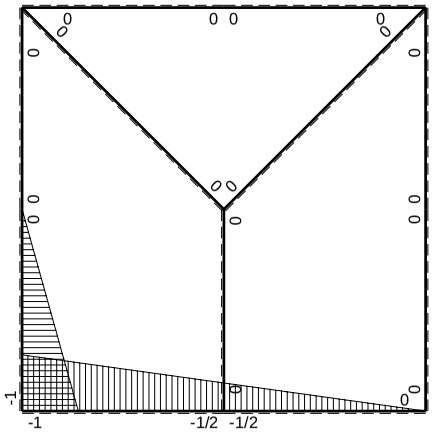

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-7/24
-7/24
-7/48
-7/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.198

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	-Fb/EJ	$3/2Fx-2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/24+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	Fb/EJ	$Fx-1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/24Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.198

Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\theta\ \ dx$$

$$= \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$$

$$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right) Fb\ 1/EJ\ + \left(1/2\ b\ \right)\theta\ \ = 17/24\ \ Fb^2/EJ$$

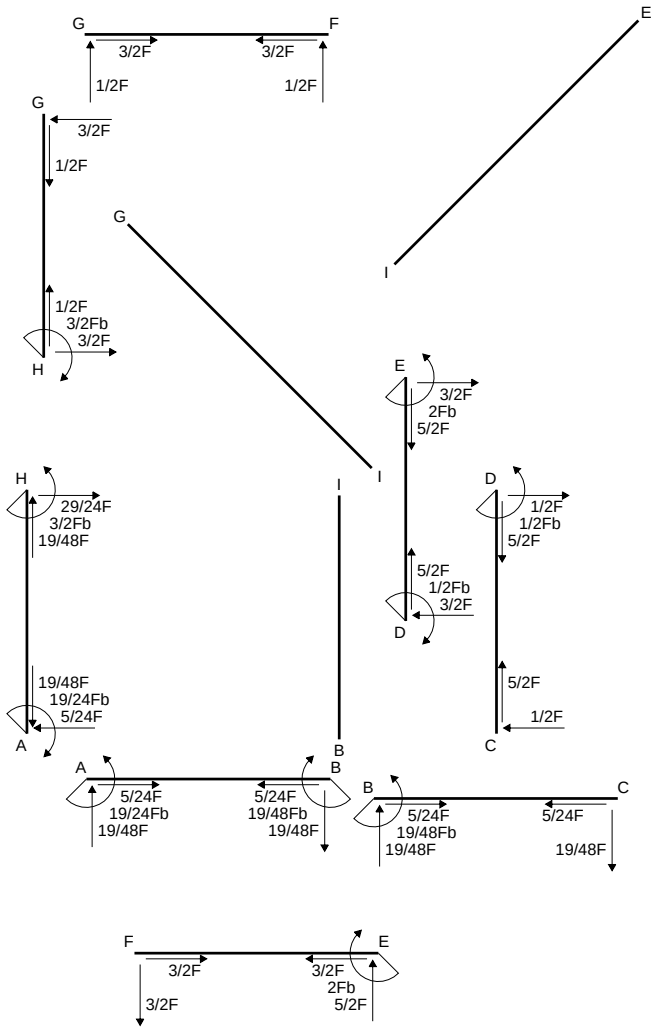
$$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\theta\ \ dx$$

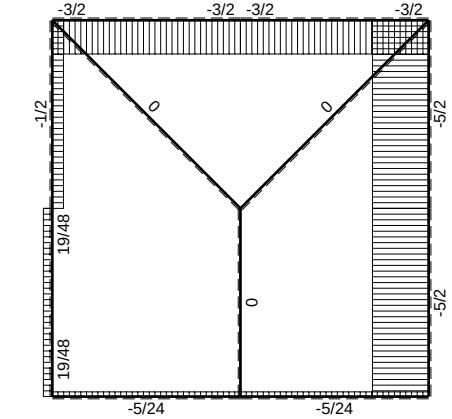
$$= \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$$

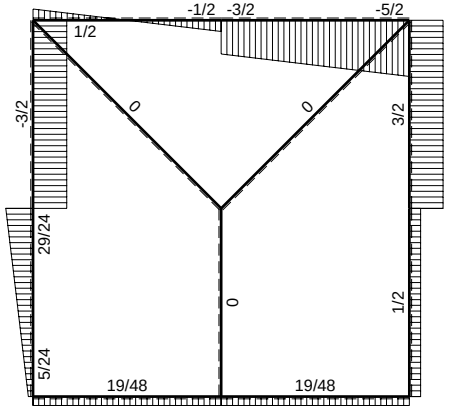
$$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right) Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\theta\ \ = 17/24\ \ Fb^2/EJ$$

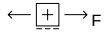
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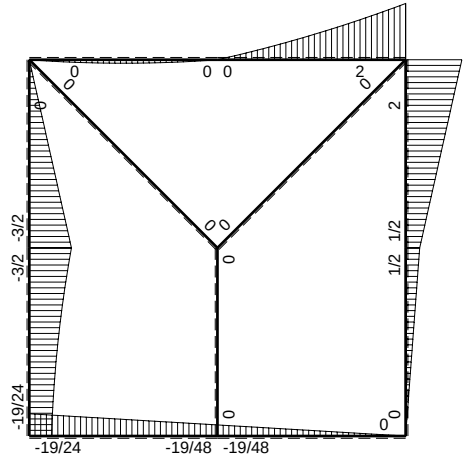

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H
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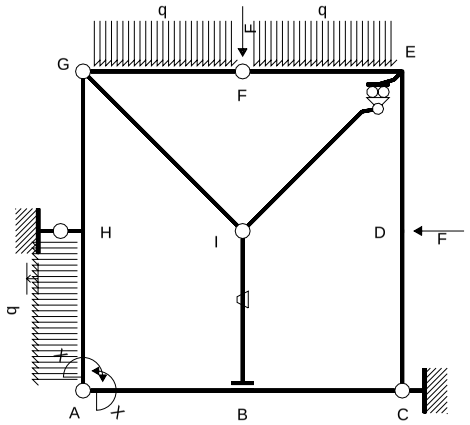

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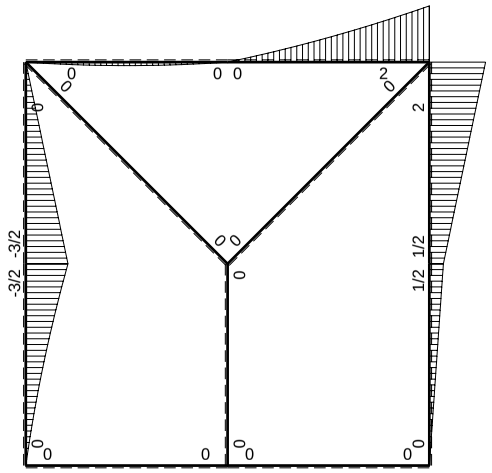

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F

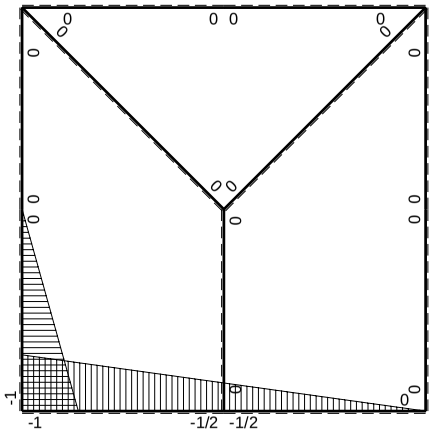

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-19/24
-19/24
-19/48 -19/48


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.199

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

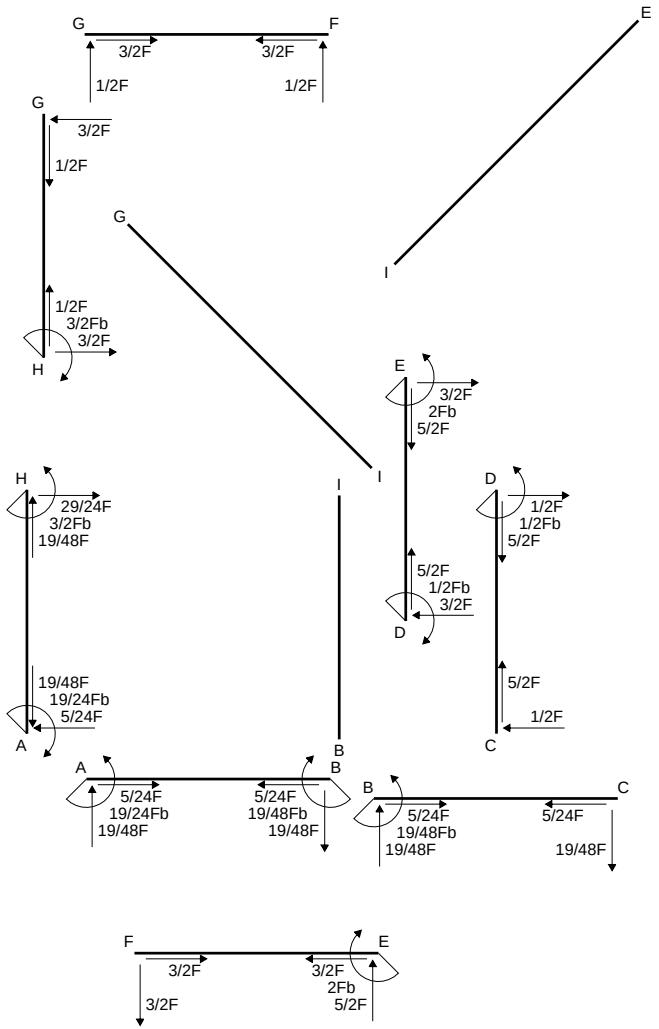
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

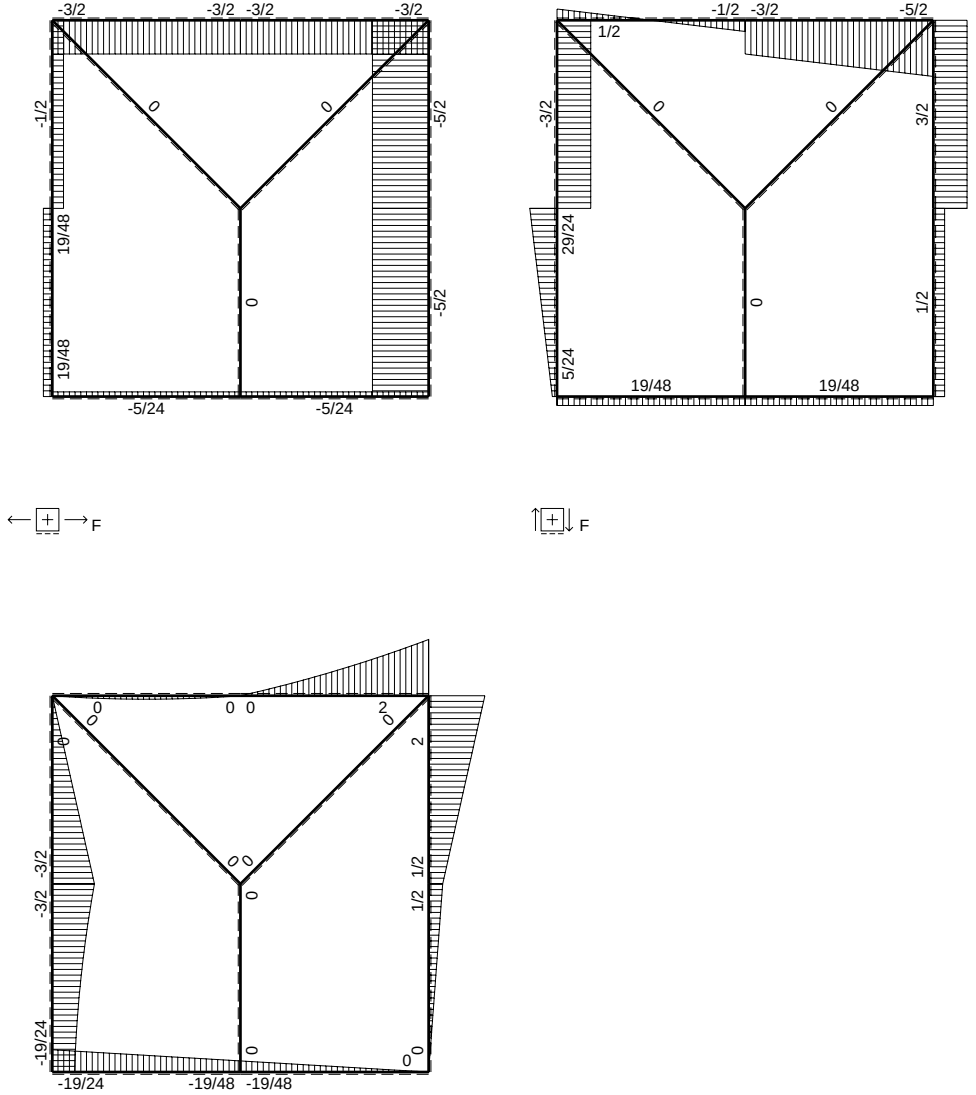
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

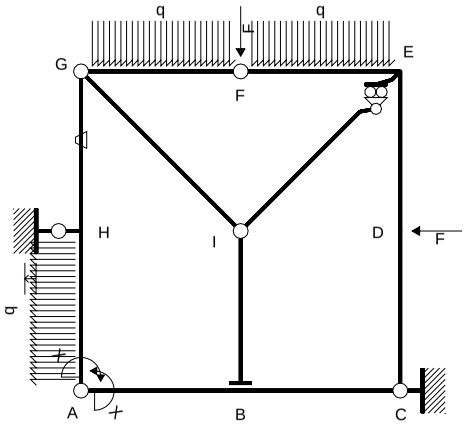
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

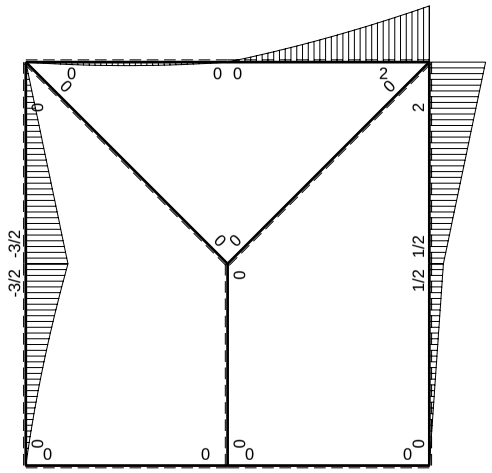
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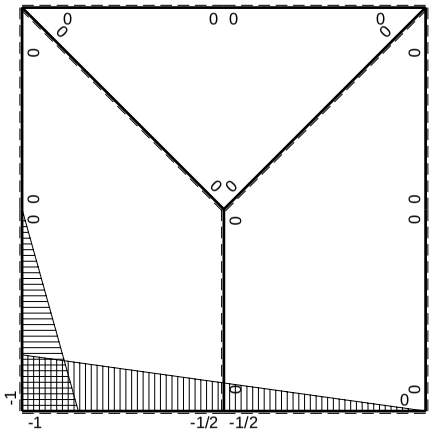

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.200

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	Fb/EJ	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

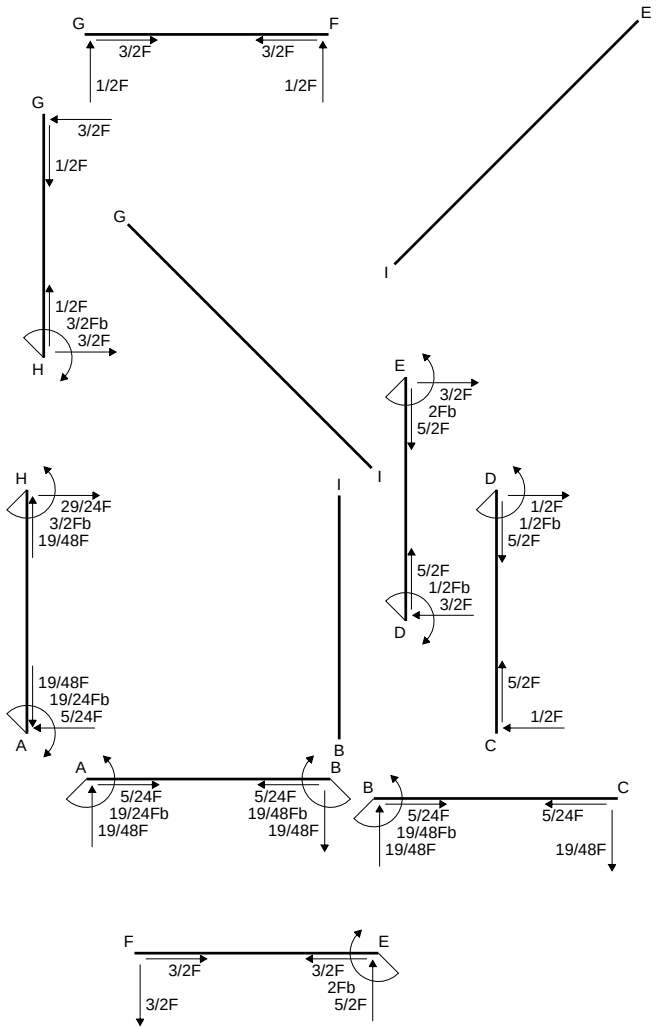
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

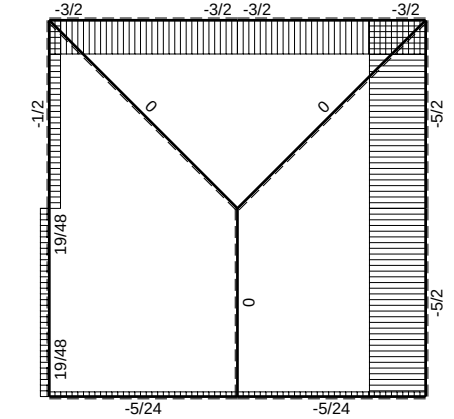
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

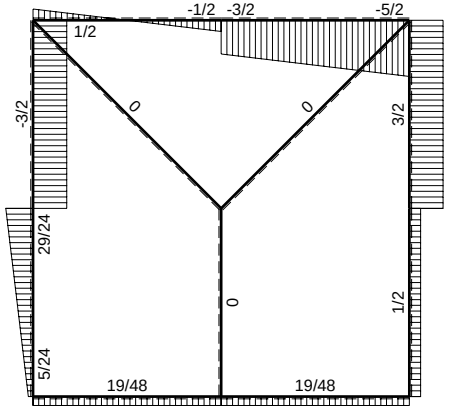
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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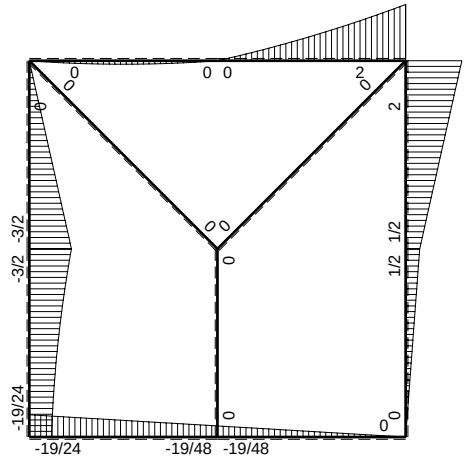

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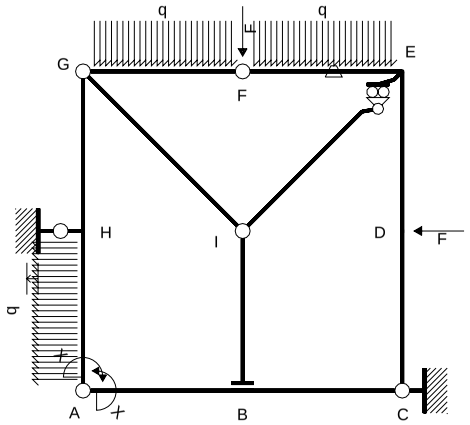

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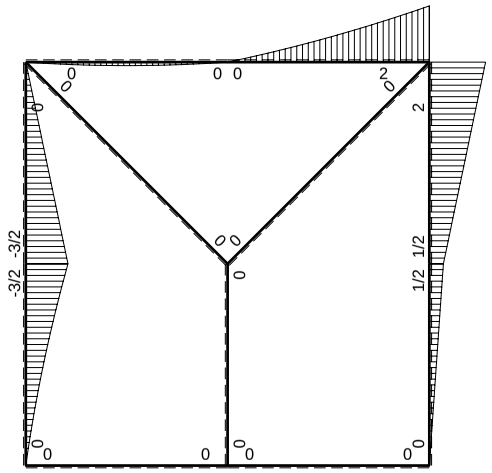
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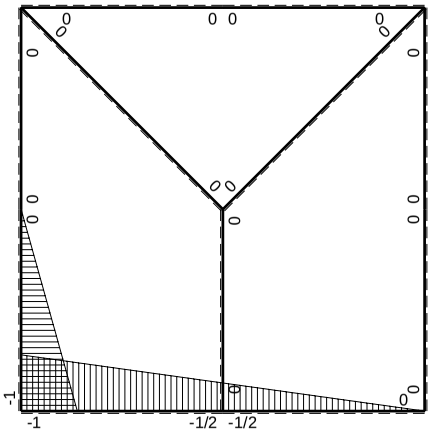

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-19/24
-19/48 -19/48

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.201

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PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.201

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

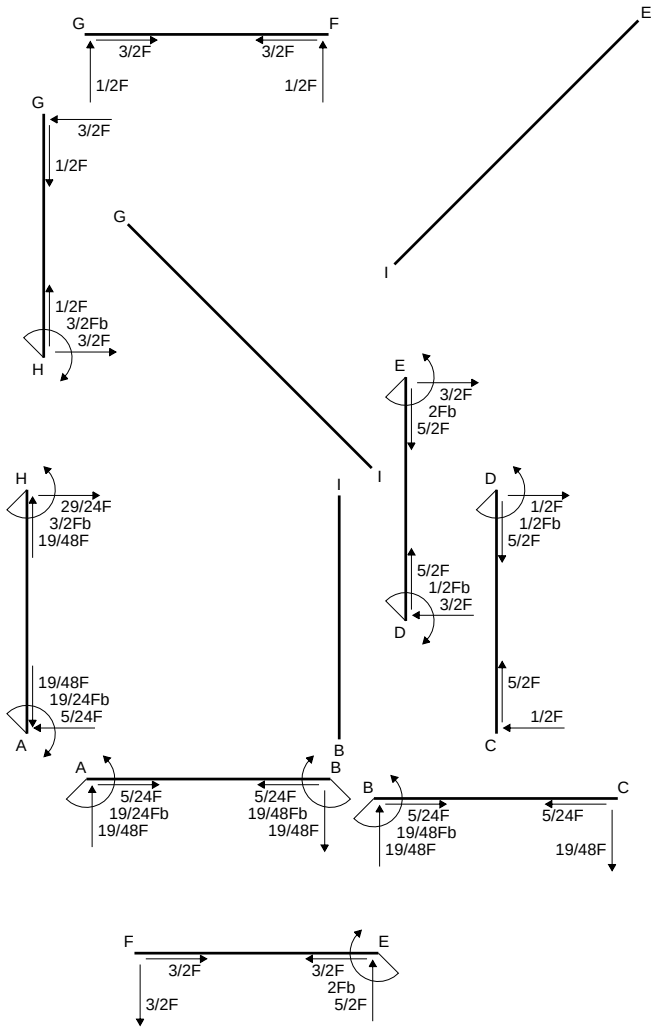
$= \left(3/4\ b - 2/3\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

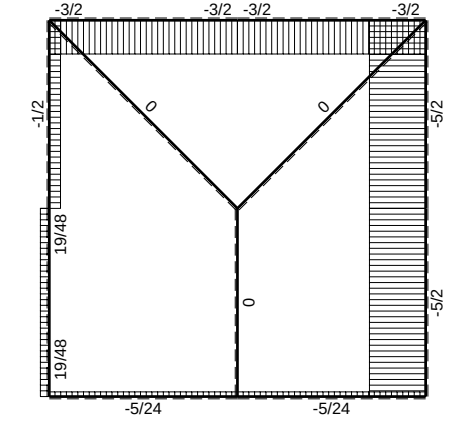
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

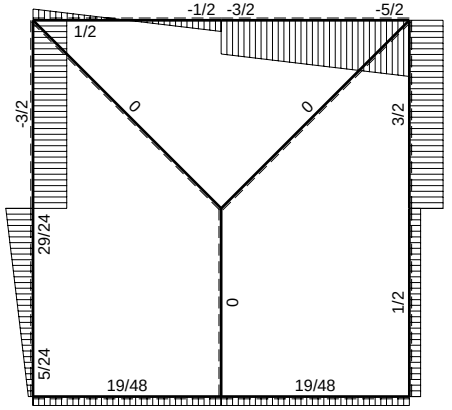
$= \left(1/2\ b - 1/6\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

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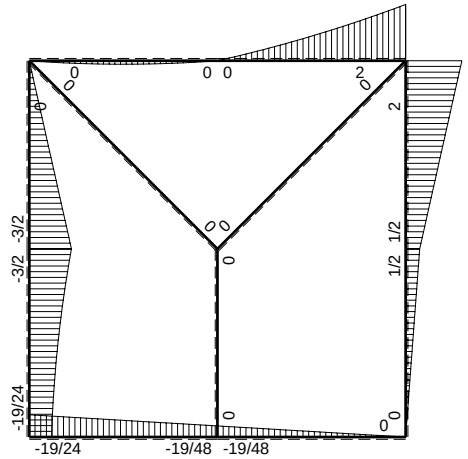

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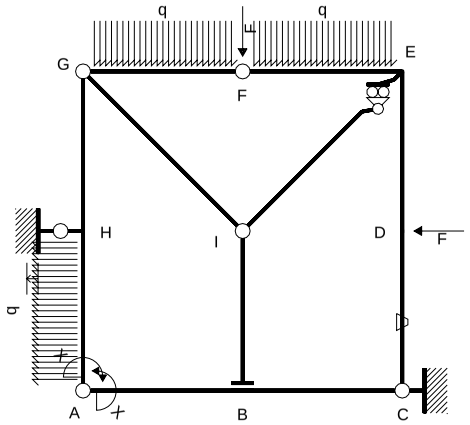

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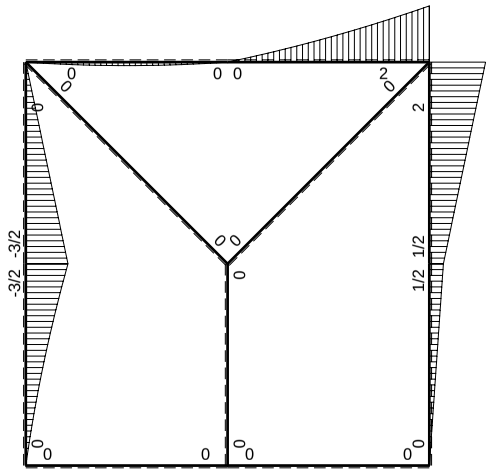
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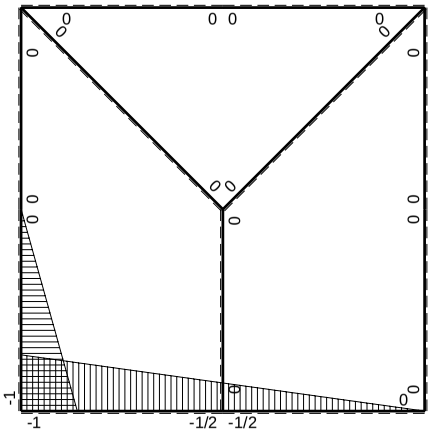

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.202

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/2Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	Fb/EJ	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3/2Fb+2Fx-1/2qx^2$	0	$3/2Fx-2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$Fx+1/2qx^2$	0	$Fx-1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-19/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						19/24Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.202

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[3/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

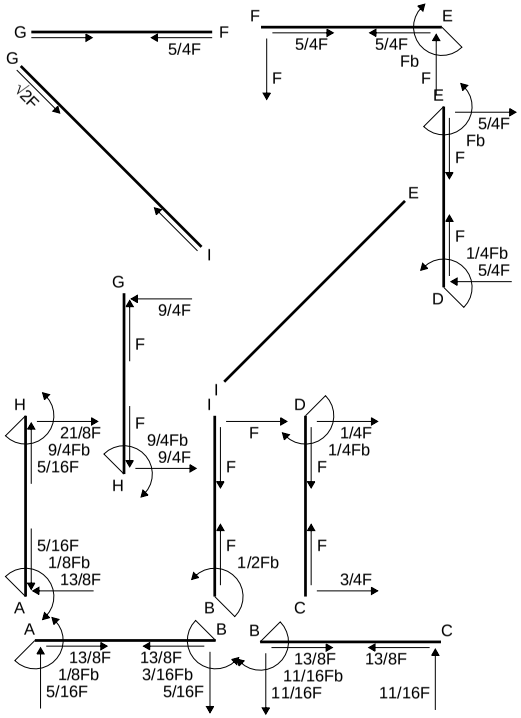
$= \left(3/4\ b - 2/3\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

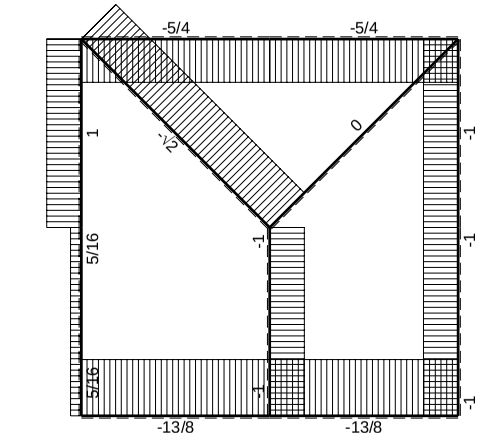
$L_{AH}^{Xo} = \int_o^b \left(\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

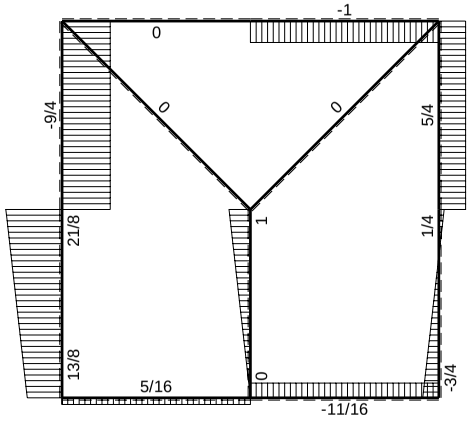
$= \left(1/2\ b - 1/6\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 5/24\ \ Fb^2/EJ$

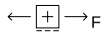
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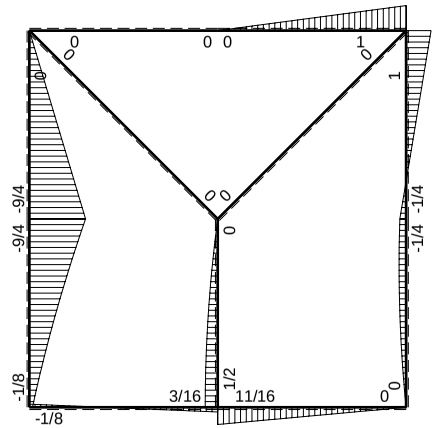

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11/16F


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-13/8

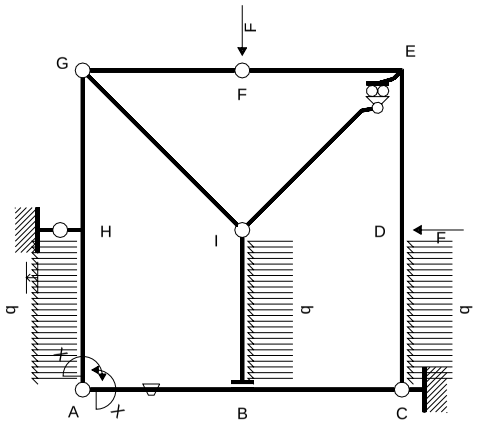

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-11/16

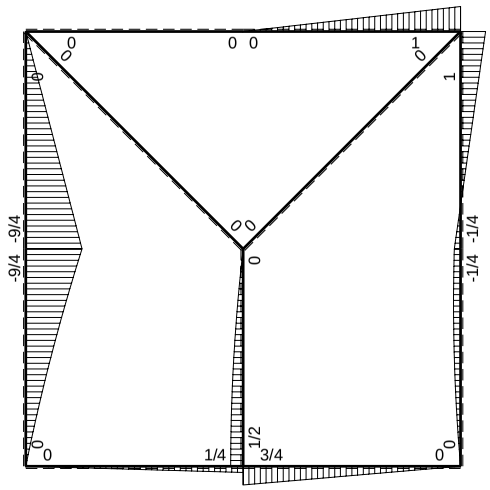

F


F

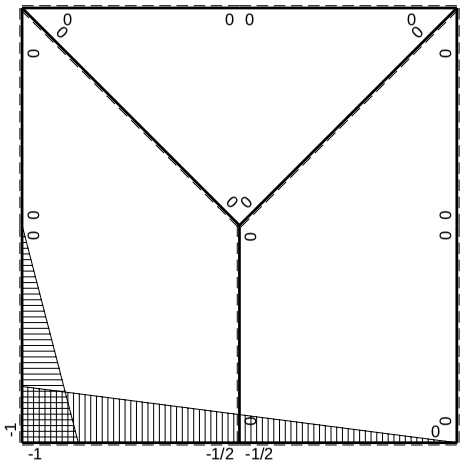

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0
0
1
0 0
-9/4
-9/4
0
-1/4
-1/4
-1/8
3/16
1/2
11/16
0
-1/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.203

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	-Fb/EJ	-1/4Fx+1/8Fx^2/b	Fb/EJ-1/2Fx/EJ	1-x/b+1/4x^2/b^2	(-1/12+3/4)Fb^2/EJ	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	Fb/EJ	-1/8Fb+1/8Fx^2/b	1/2Fb/EJ+1/2Fx/EJ	1/4+1/2x/b+1/4x^2/b^2		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	-3/8Fb+3/4Fx-3/8Fx^2/b	0	1/4-1/2x/b+1/4x^2/b^2	(-1/8+0)Fb^2/EJ	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	-3/8Fx^2/b	0	1/4x^2/b^2		
CD b	0	-3/4Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	1/4Fb+1/4Fx-1/2qx^2	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
BI b	0	-1/2Fb+1/2qx^2	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	-9/4Fb+11/4Fx-1/2qx^2	0	9/4Fx-11/4Fx^2/b+1/2qx^3/b	0	x^2/b^2	(1/3+0)Fb^2/EJ	1/3Xb/EJ
AH b	1-x/b	7/4Fx+1/2qx^2	0	7/4Fx-5/4Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2		
H	cedimento nodo $-H_{1H}u_H$						-Fb^2/EJ	
	totali						-1/8Fb^2/EJ	Xb/EJ
	iperstatica $X=W_{AB}$						1/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ + \left(\ b - 1/4\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 2/3\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

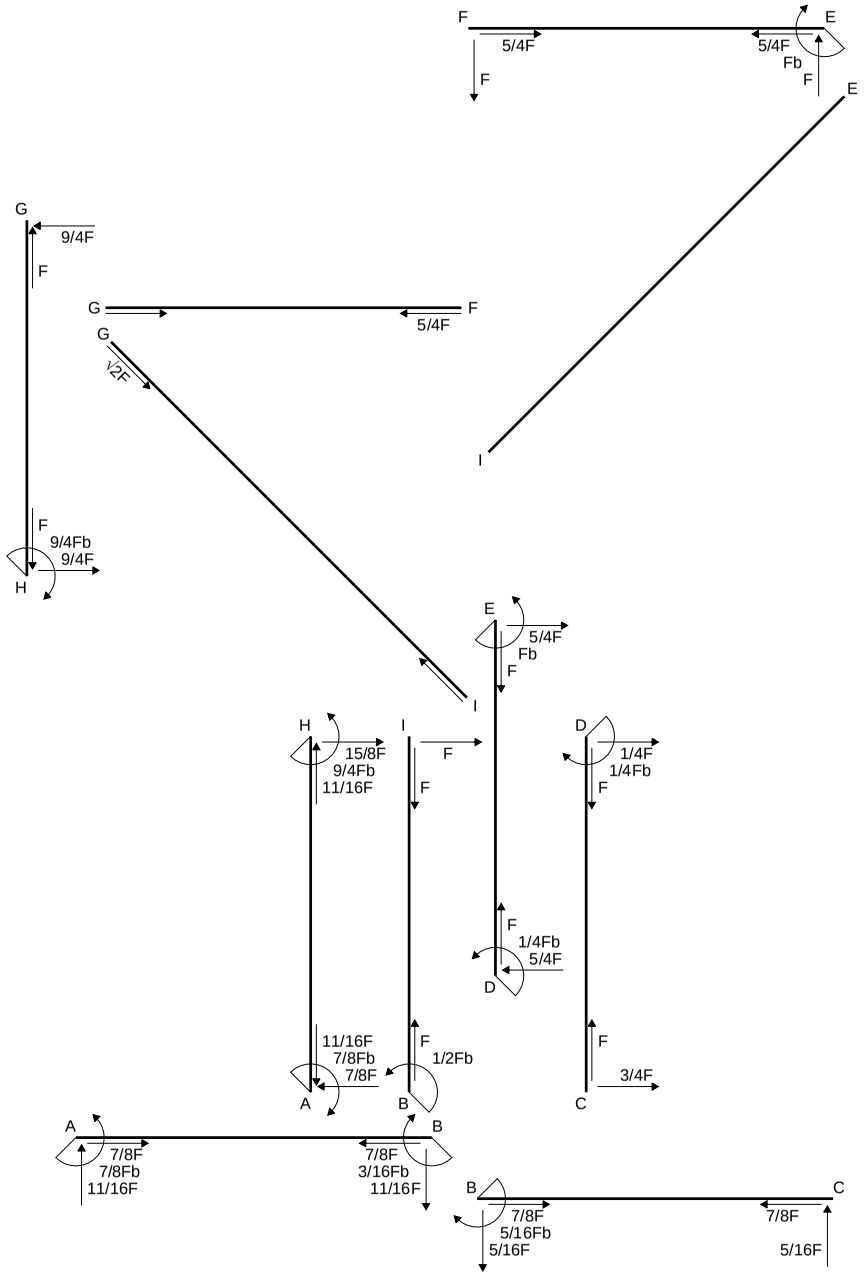
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

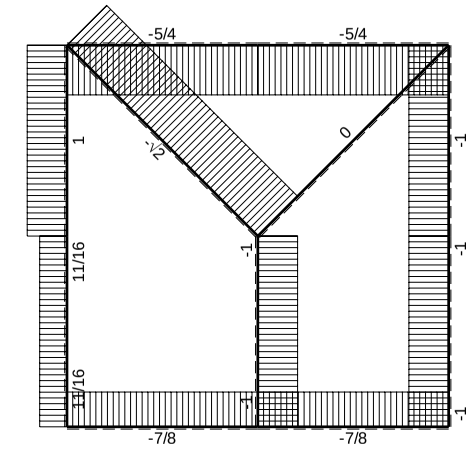
$L^{Xo}_{AH} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

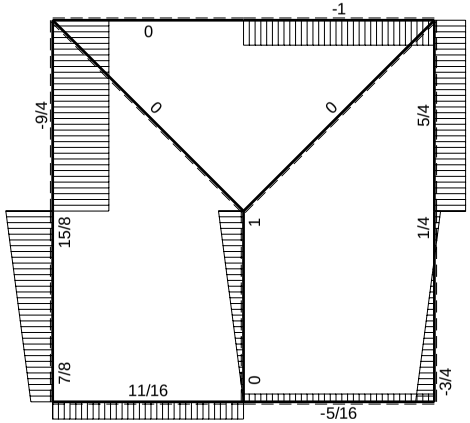
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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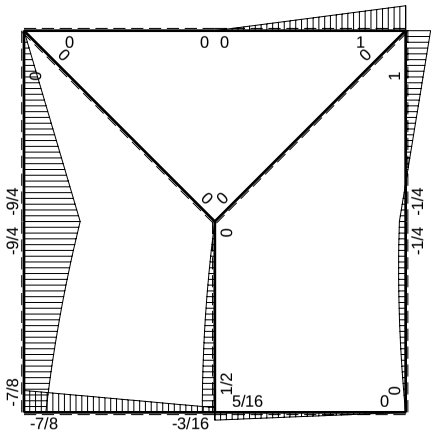

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F
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-7/8

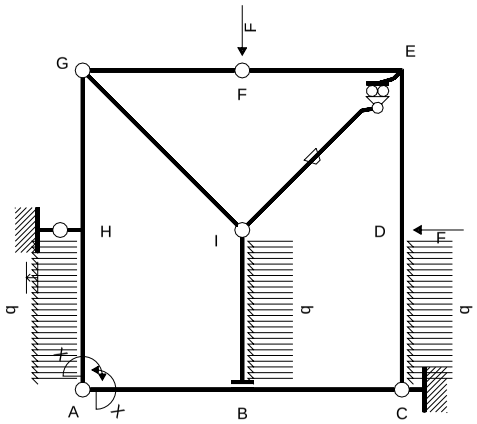

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-5/16

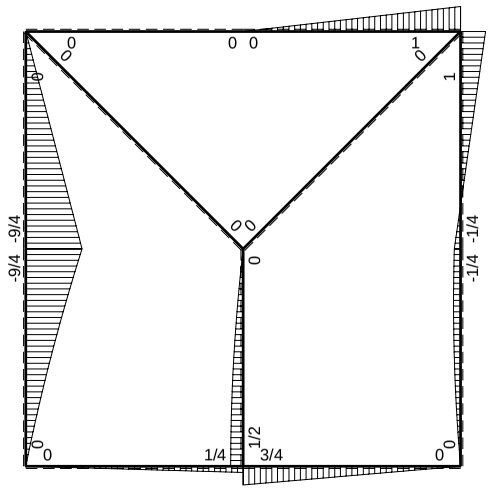
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↑□↓ F

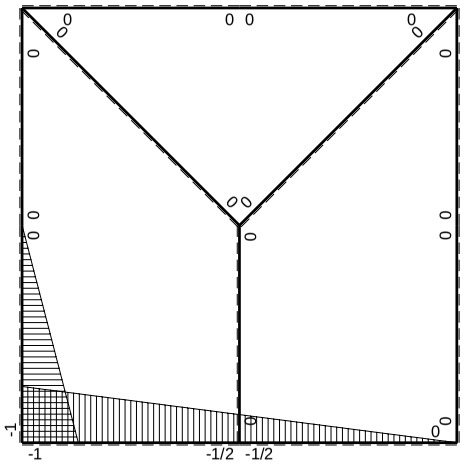

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0
-7/8
1/2
5/16
0
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.204

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PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.204

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

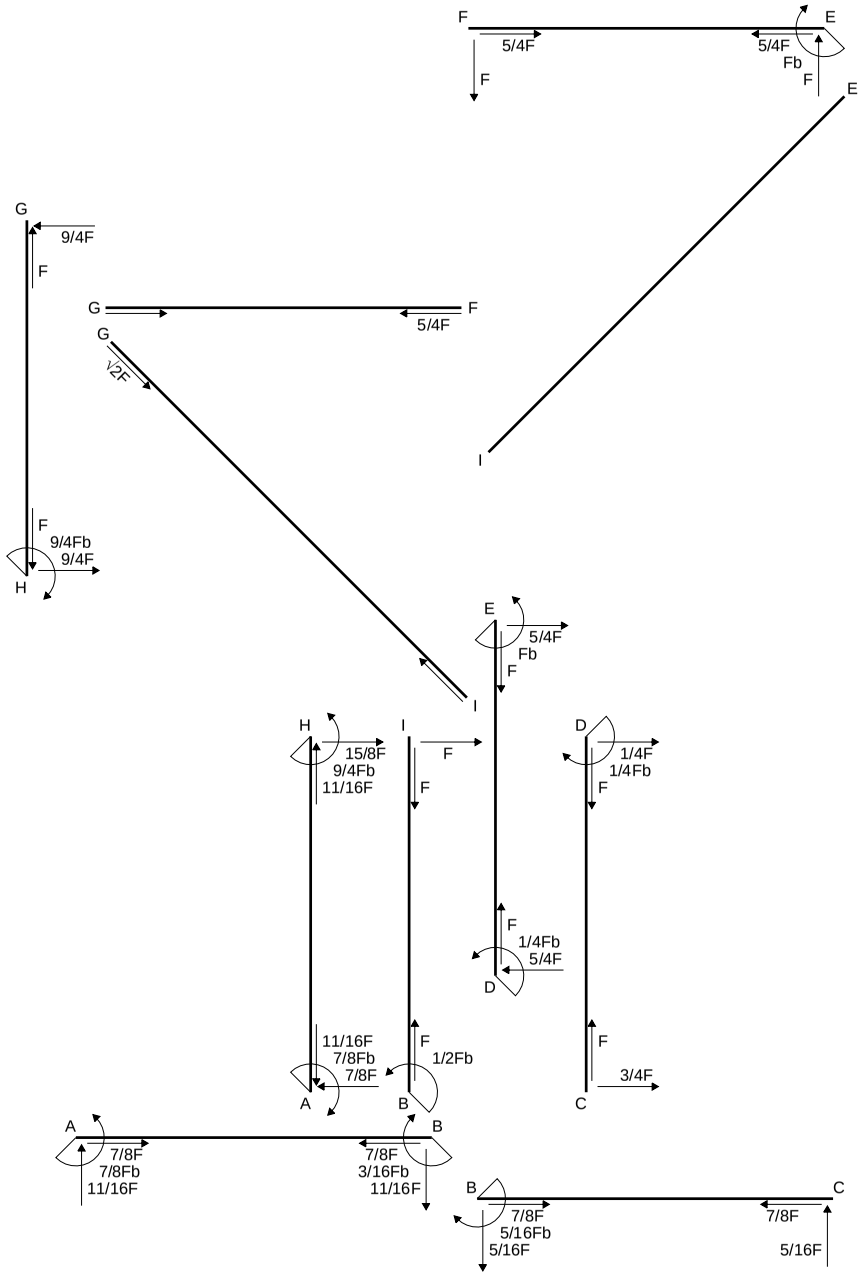
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

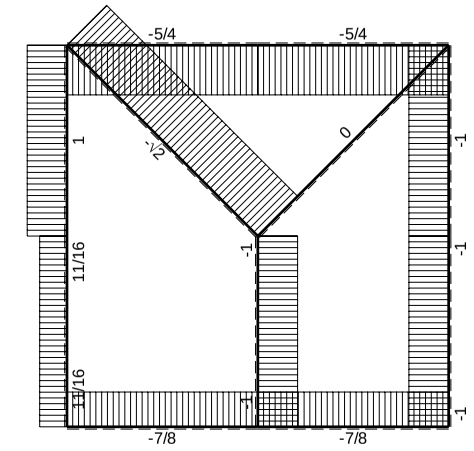
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

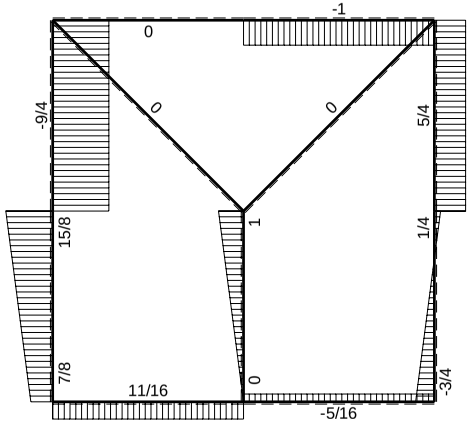
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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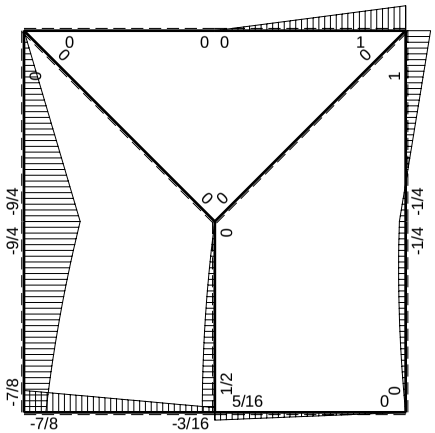

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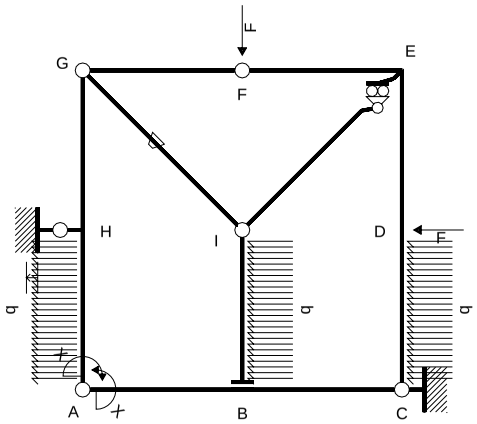

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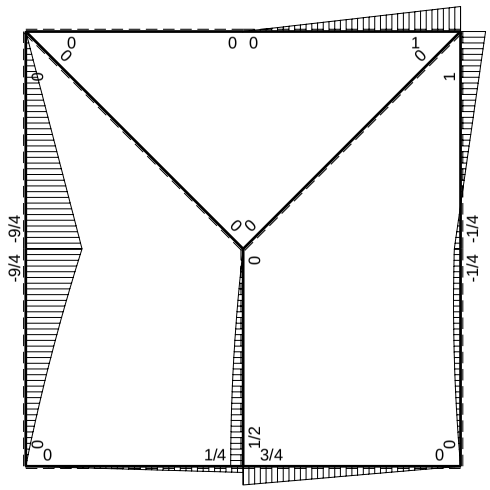
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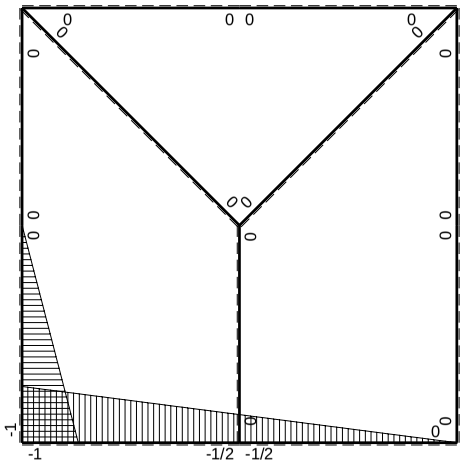

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-1/4
-1/4
0
-7/8
1/2
5/16
0
-7/8
-3/16

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.205

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

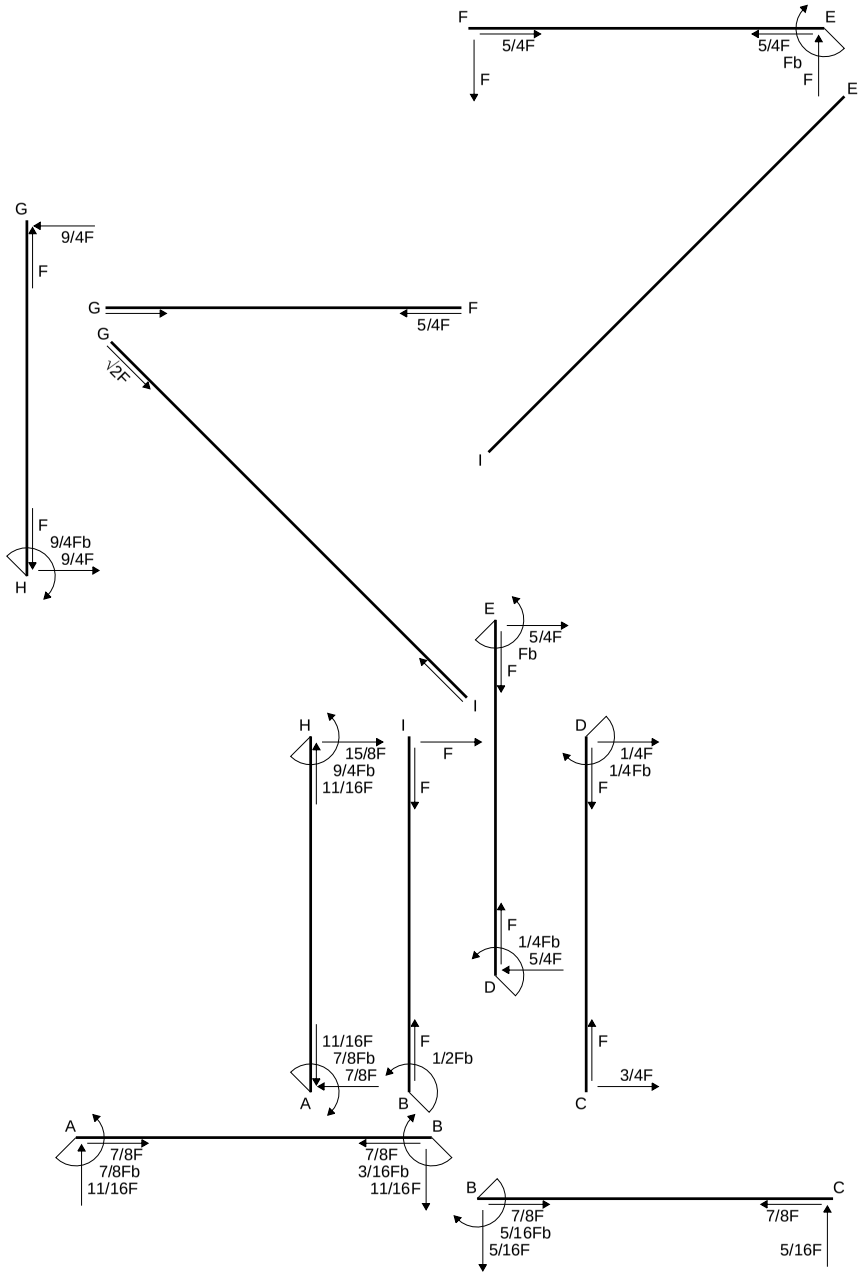
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

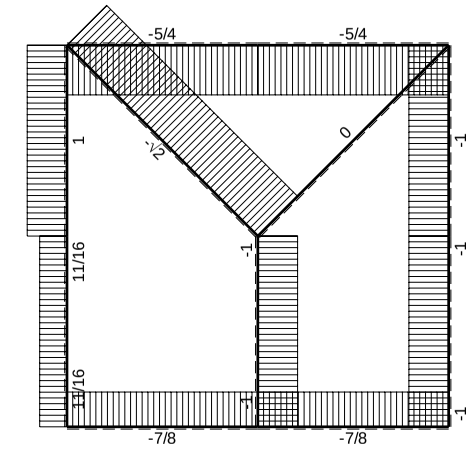
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

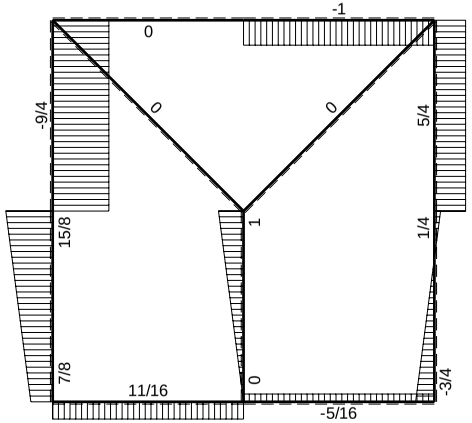
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

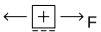
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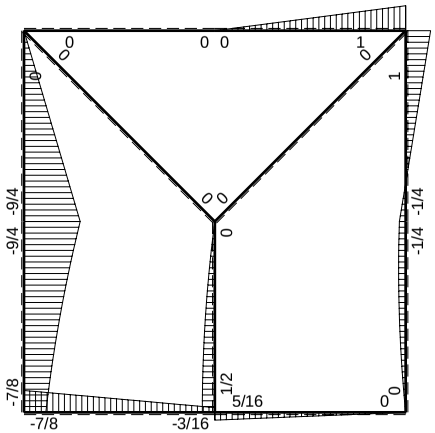

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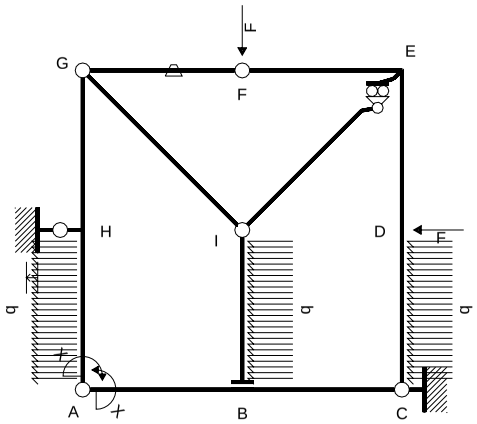

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11/16
-5/16

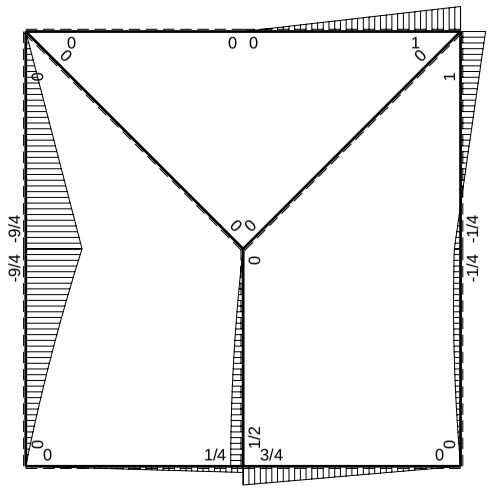

F


F

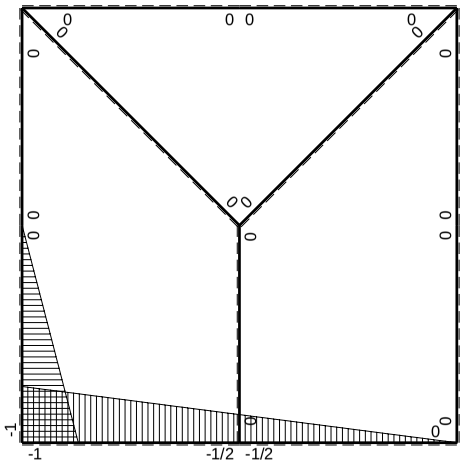

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0 0
0
-9/4
-9/4
-1/4
-1/4
0
-7/8
1/2
5/16
0
-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.206

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

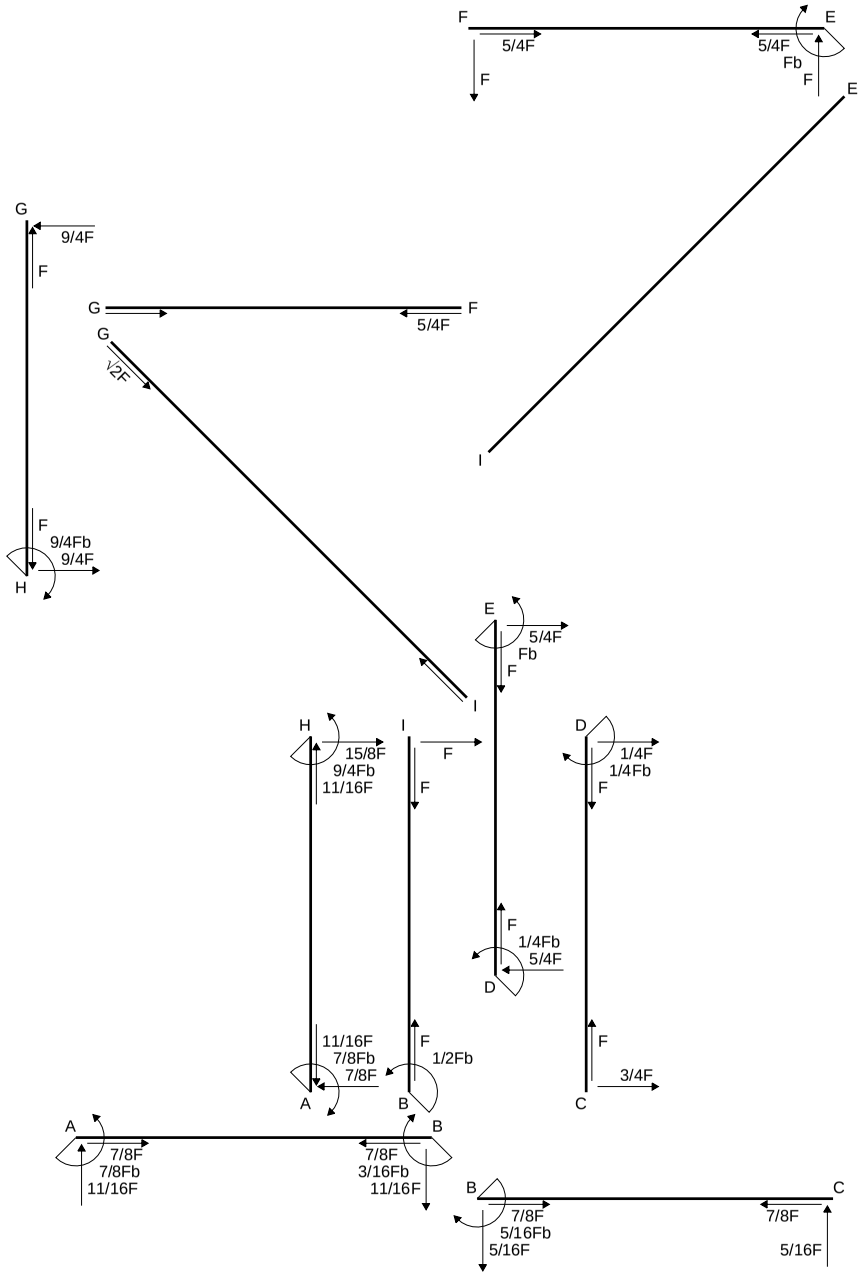
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

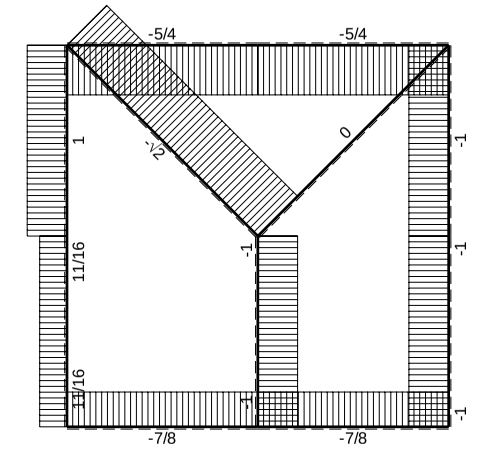
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

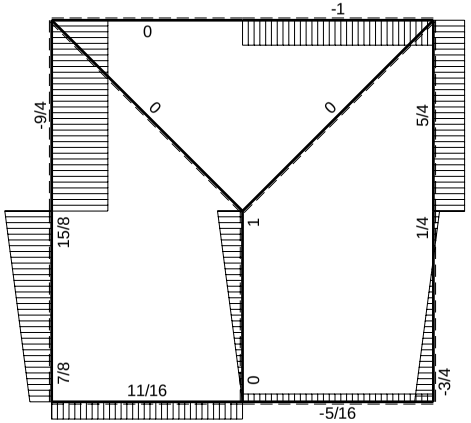
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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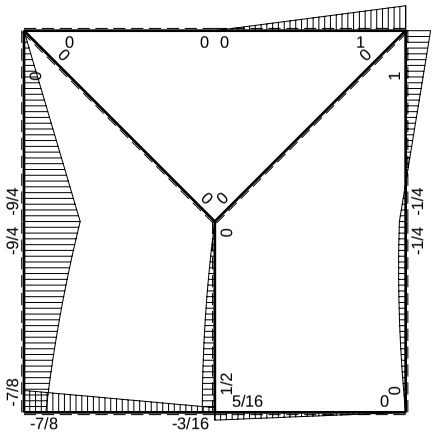

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9/4Fb
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B
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5/16F


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-7/8

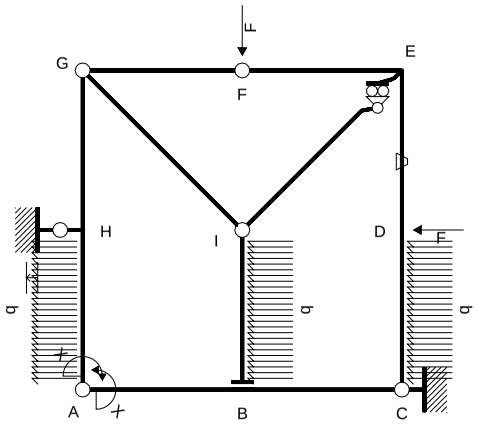

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7/8
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-3/4
11/16
-5/16

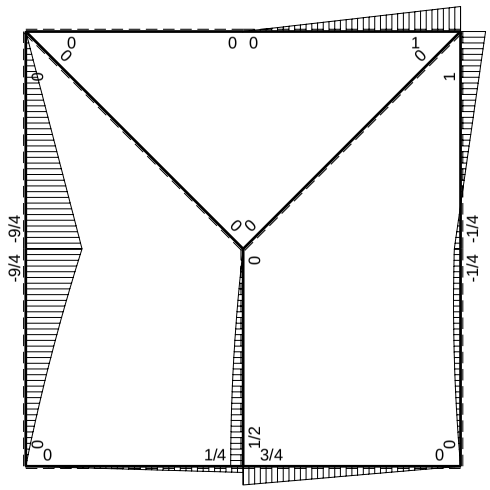
F

F

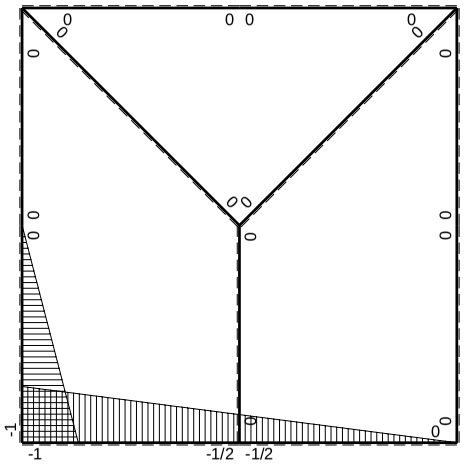

0
0 0
1
0
0
1
0
0
0
-9/4
-9/4
-1/4
-1/4
0
1/2
5/16
0
-7/8
-7/8
-3/16

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.207

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PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.207

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

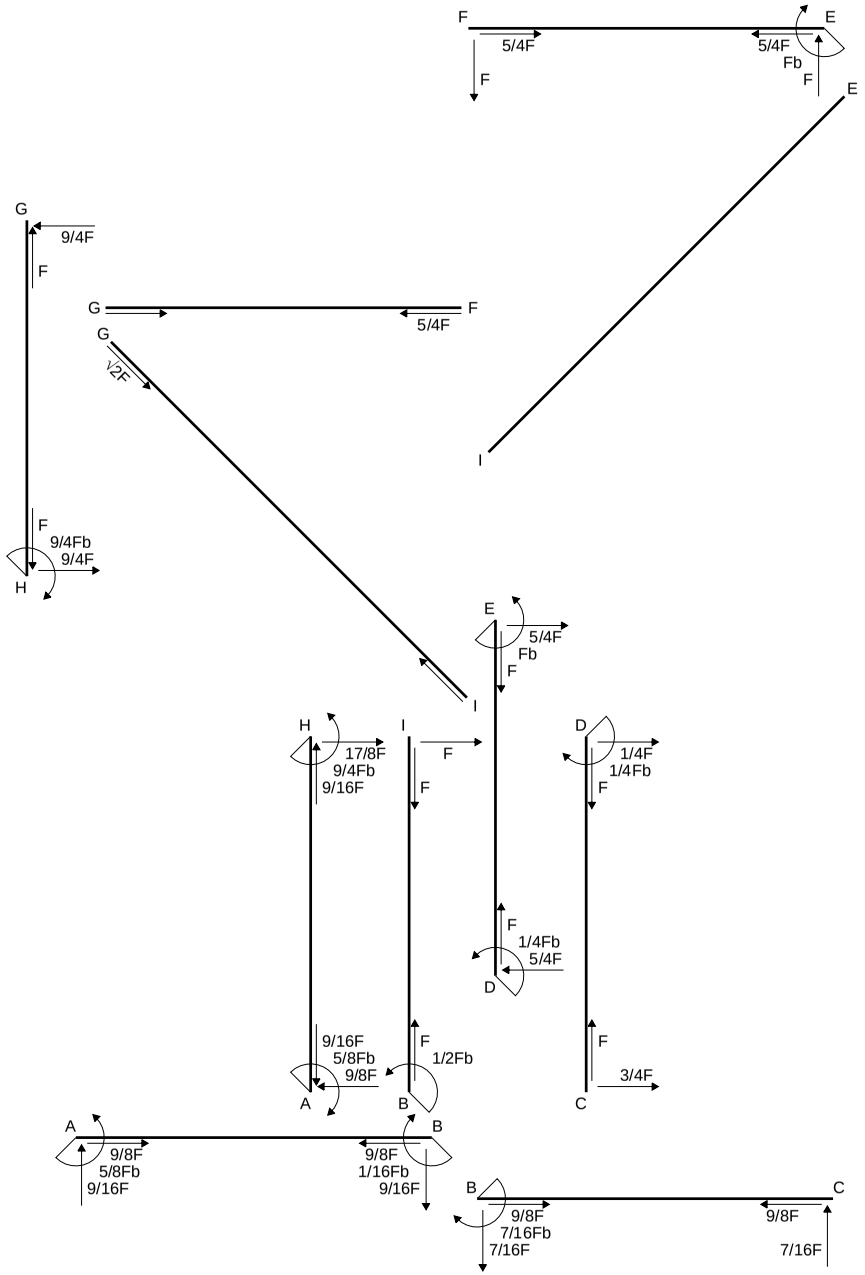
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

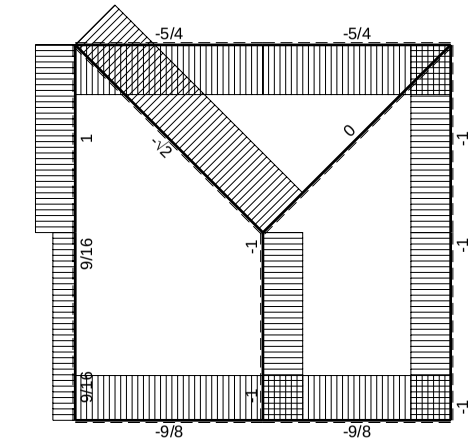
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

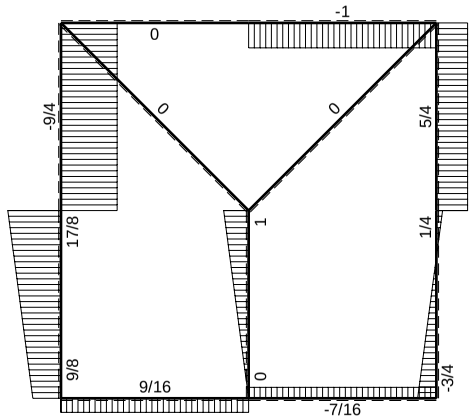
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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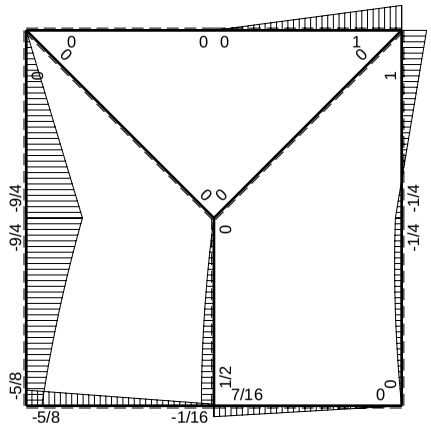

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9/4F
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Fb
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I
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9/4Fb
9/16F
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B
F
3/4F
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A
B
9/8F
5/8Fb
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9/8F
1/16Fb
9/16F
B
C
9/8F
7/16Fb
7/16F
9/8F
7/16F


-5/4
-5/4
1
½
0
-1
9/16
-1
-1
9/16
-1
-1
-9/8
-9/8

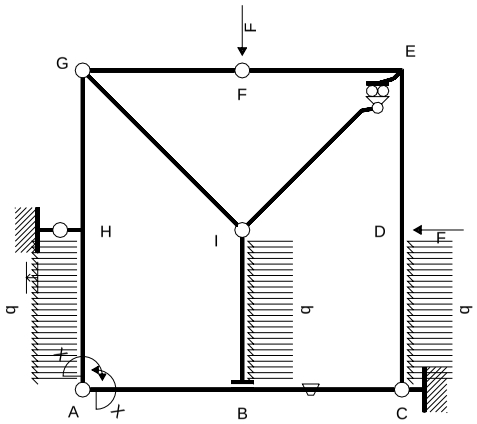

-1
0
-9/4
0
0
5/4
17/8
1
1/4
9/8
9/16
0
-3/4
-7/16

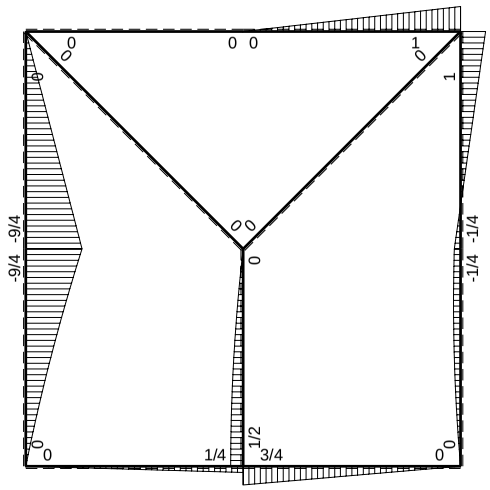
←□→ F

↑□↓ F

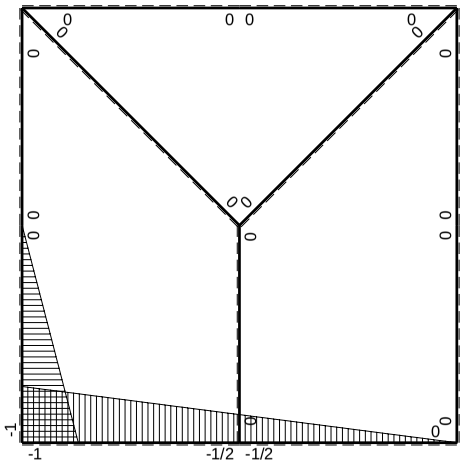

0
0 0
1
0
0
1
0
0 0
0
-9/4
-9/4
-1/4
-1/4
1/2
7/16
0
0
-5/8
-5/8
-1/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.208

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	-Fb/EJ	$-3/8Fb+3/4Fx-3/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	Fb/EJ	$-3/8Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-5/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						5/8Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.208

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\ \right)\ \theta\ \ dx$

$= \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b - 1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\ \right)\ \theta\ \ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-1/4\ b\ \right)\ \theta\ \ = 1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

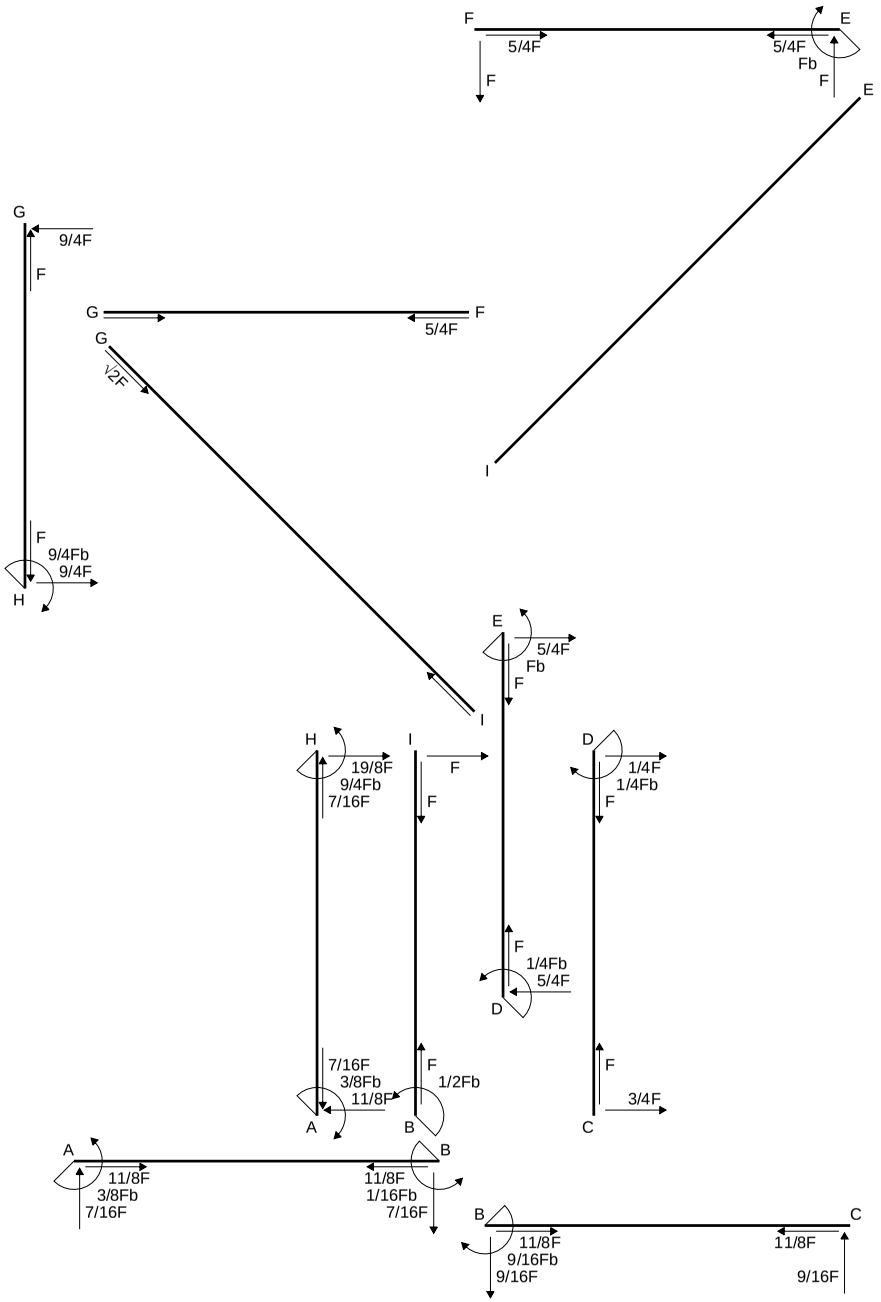
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

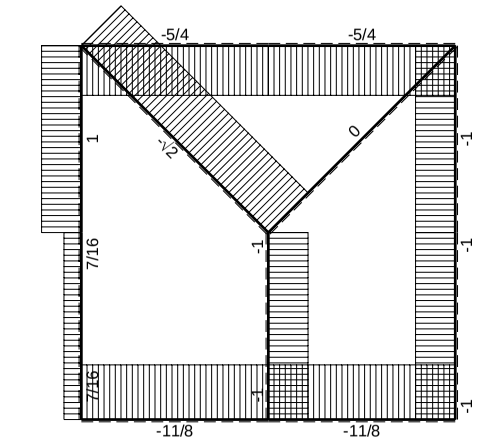
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

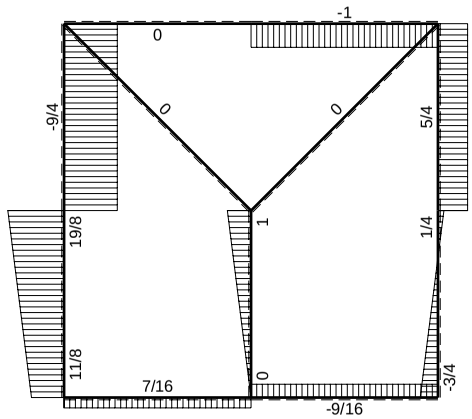
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

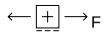
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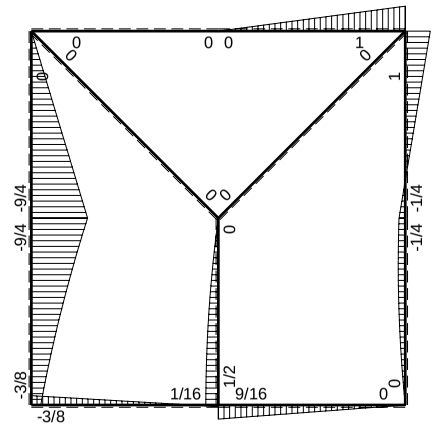

F
5/4F
E
5/4F
Fb
F
F
E
G
9/4F
F
G
F
5/4F
G
√2F
I
F
9/4Fb
9/4F
H
E
5/4F
Fb
F
I
H
19/8F
9/4Fb
7/16F
I
F
F
D
1/4F
1/4Fb
F
F
1/4Fb
5/4F
D
7/16F
3/8Fb
11/8F
F
1/2Fb
A
B
F
3/4F
C
A
11/8F
3/8Fb
7/16F
B
11/8F
1/16Fb
7/16F
B
11/8F
9/16Fb
9/16F
C
11/8F
9/16F


-5/4
-5/4
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0
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7/16
-1
-1
7/16
-1
-1
-11/8
-11/8

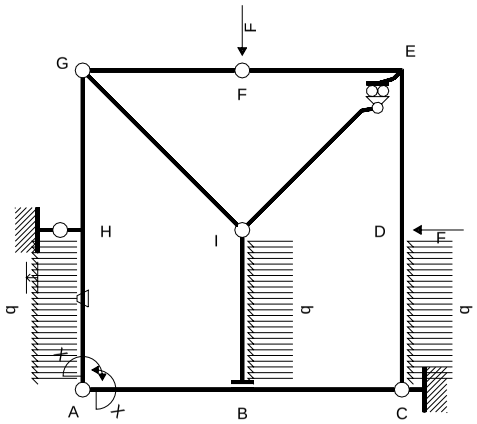

-1
0
-9/4
0
0
5/4
19/8
1
1/4
11/8
7/16
0
-3/4
-9/16

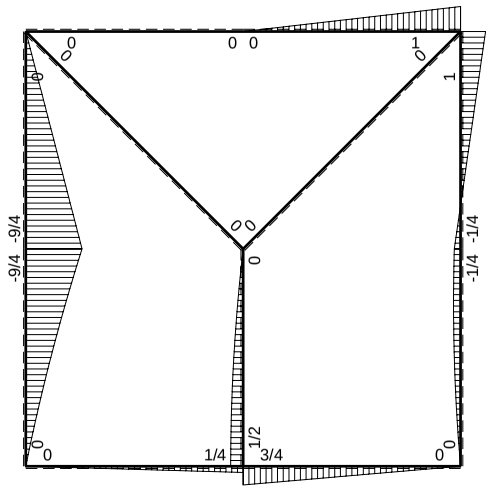

F


F

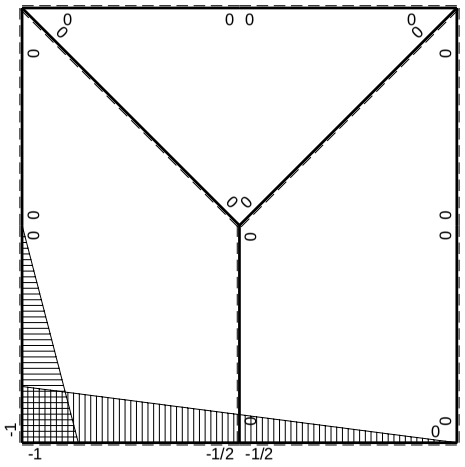

0
0 0
1
0
0
1
0
0
0
-9/4
-9/4
-1/4
-1/4
-3/8
1/16
1/2
9/16
0
0
-3/8


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.209

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-3/4Fx$	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-Fb+5/4Fx$	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	-Fb/EJ	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/3+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	Fb/EJ	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.209

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx$

$= \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

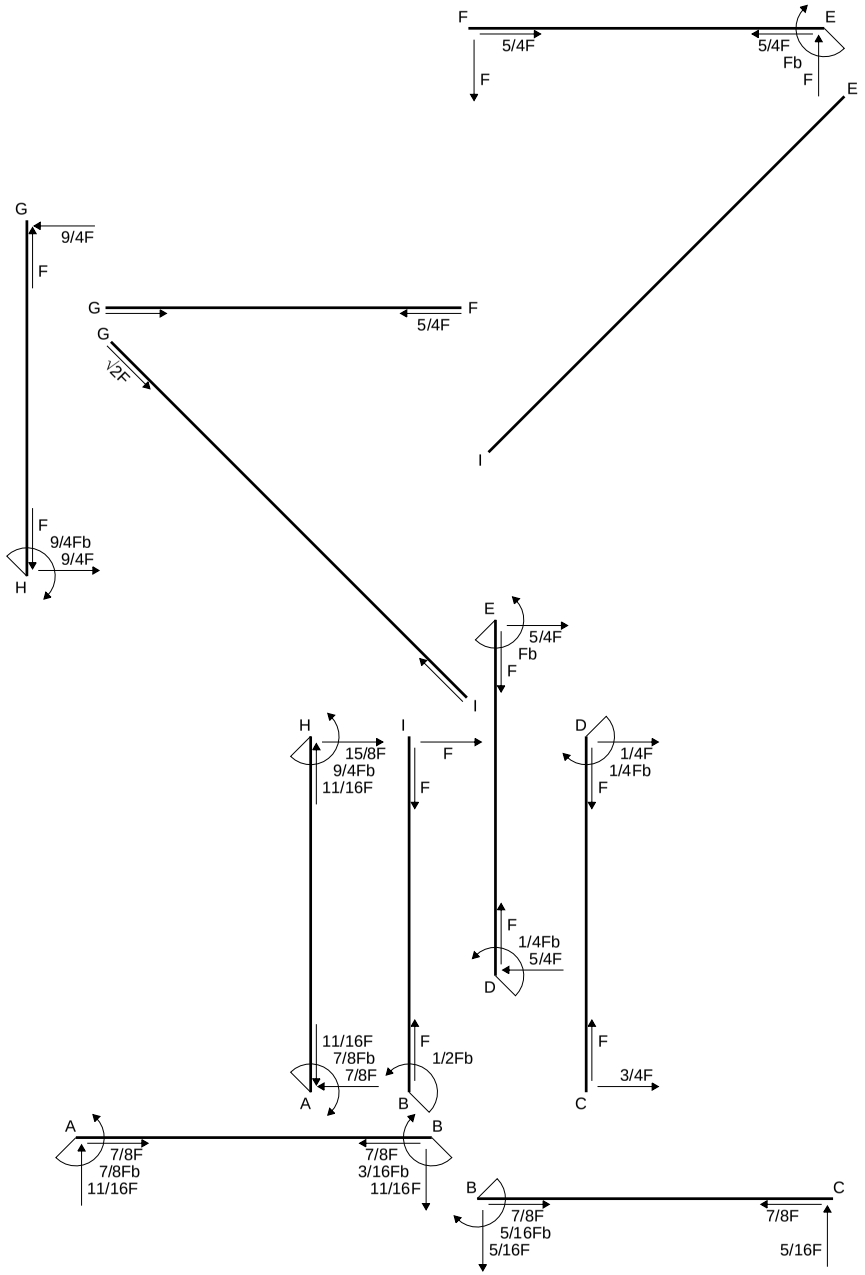
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

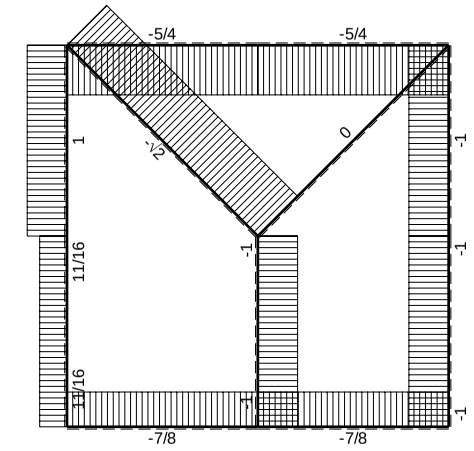
$= \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b\ \theta$

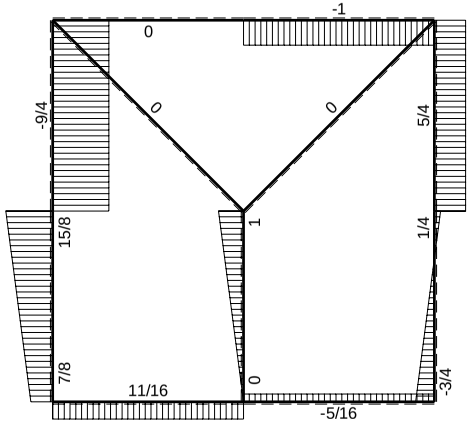
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right)\ \theta\ \ = 5/6\ \ Fb^2/EJ$

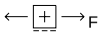
PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.209

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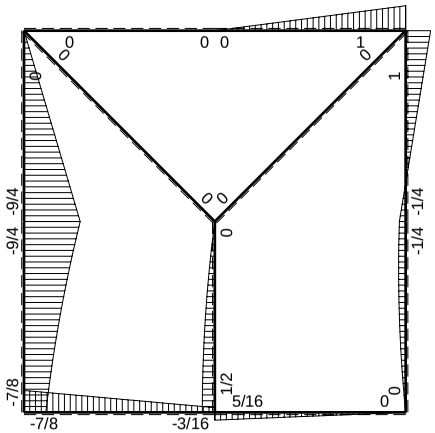

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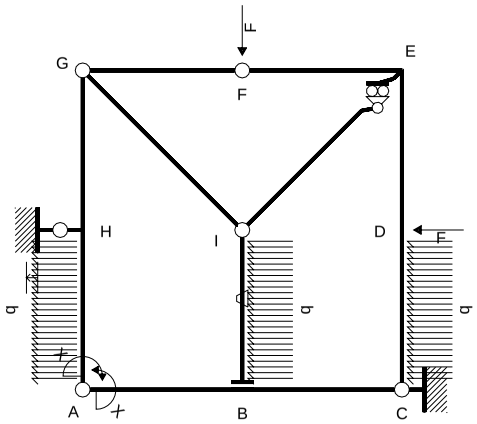

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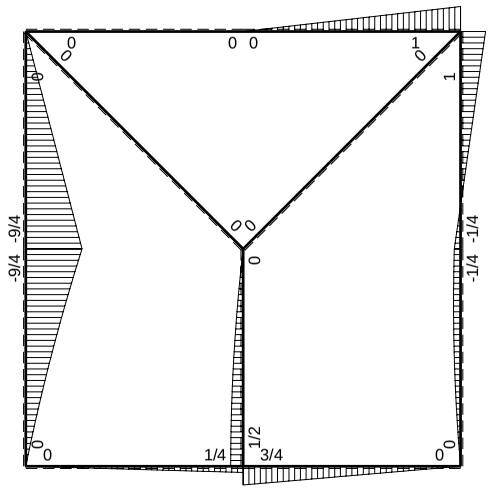

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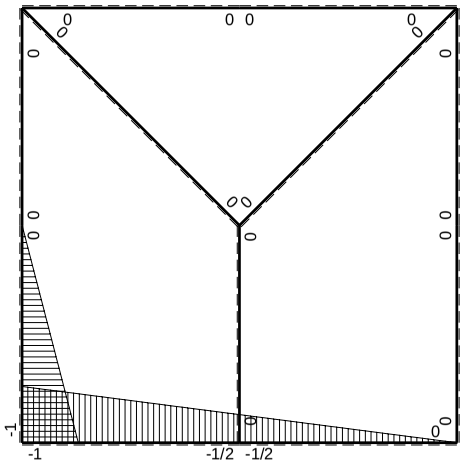

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5/16
0
-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.210

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

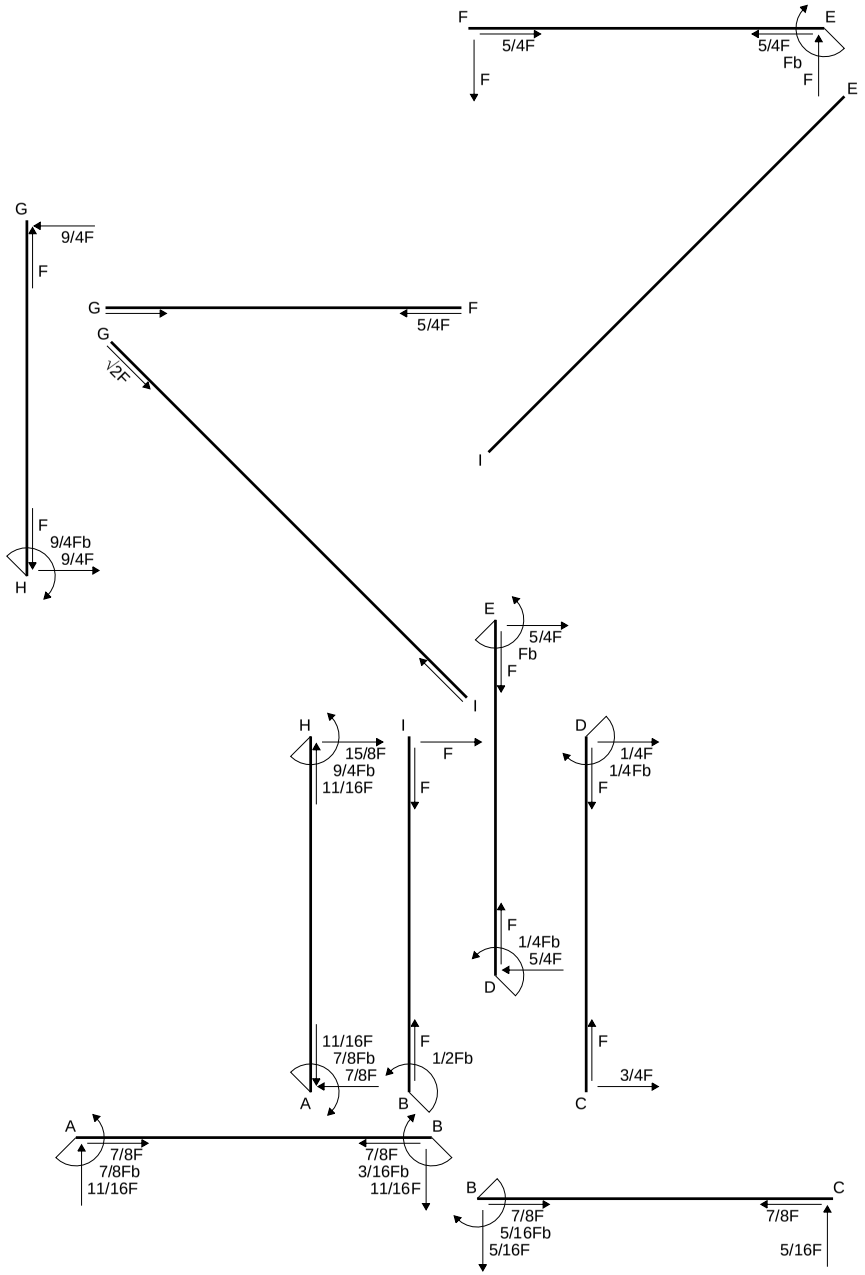
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

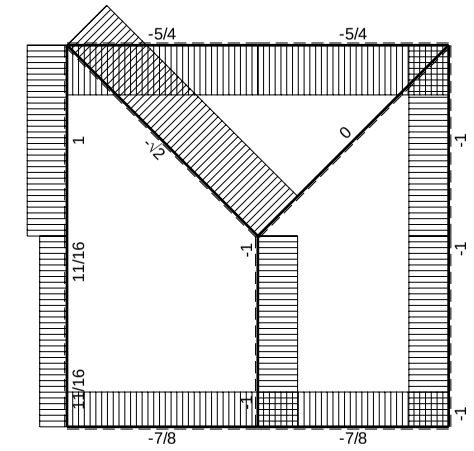
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

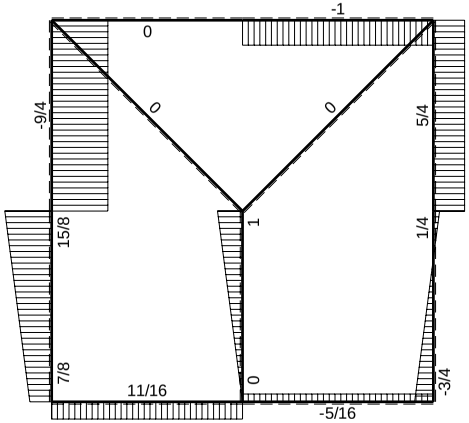
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

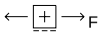
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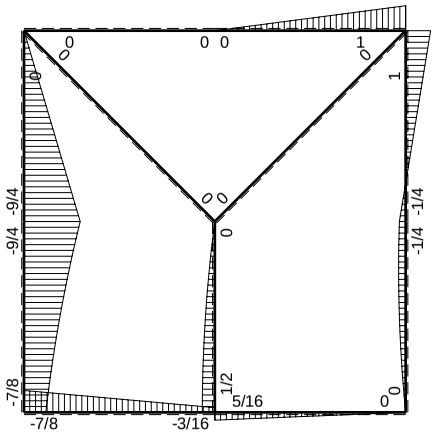

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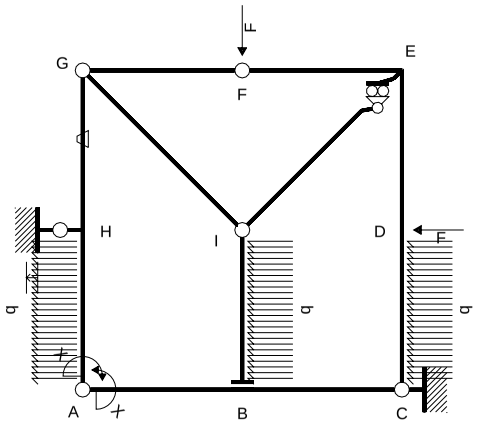

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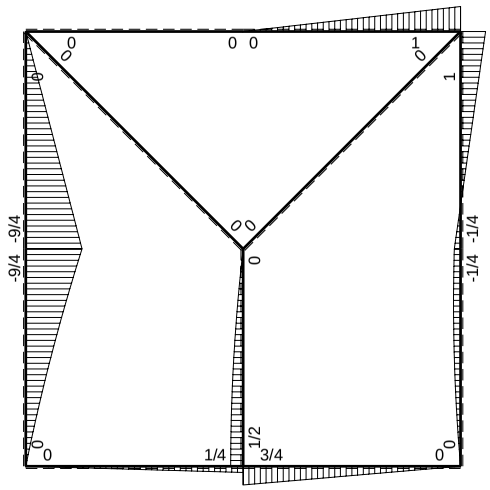

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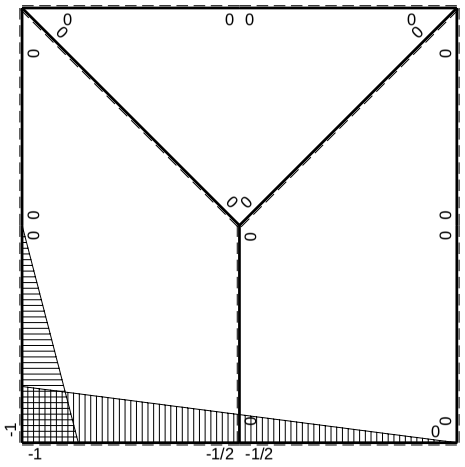

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-1/4
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-7/8
-7/8
-3/16


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.211

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PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.211

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	-Fb/EJ	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

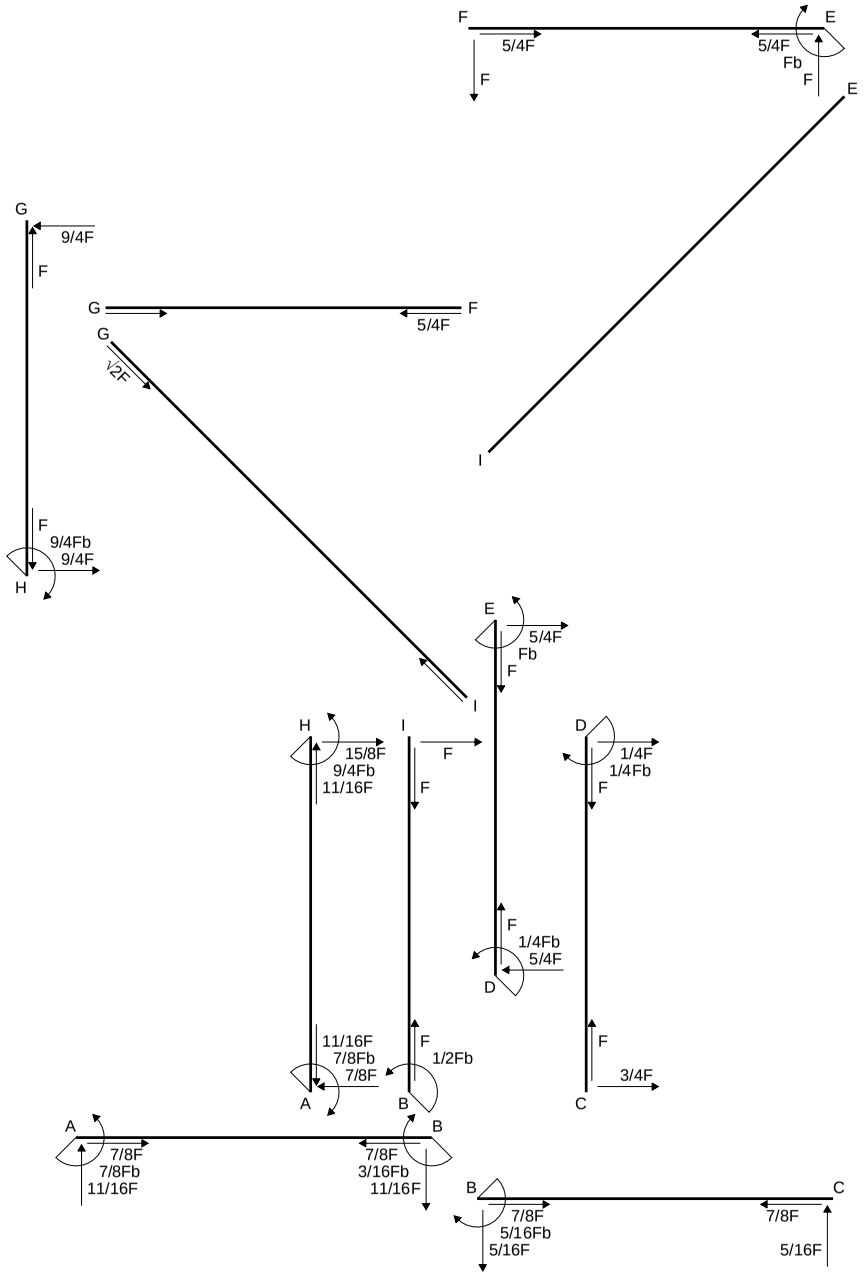
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

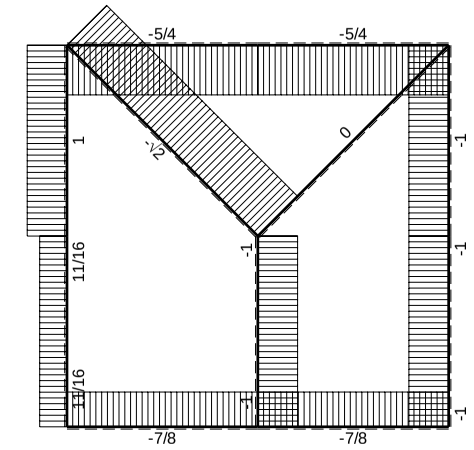
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

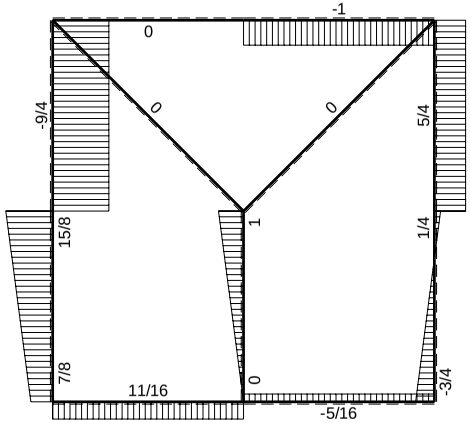
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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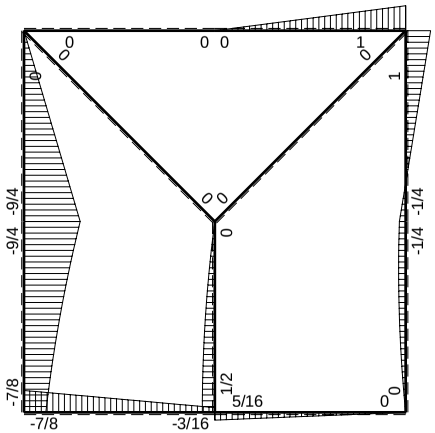

F
5/4F
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Fb
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E
G
9/4F
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G
F
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G
½F
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F
9/4Fb
9/4F
H
E
5/4F
Fb
F
I
H
15/8F
9/4Fb
11/16F
I
F
F
D
1/4F
1/4Fb
F
F
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D
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-1
-1
-7/8
-7/8

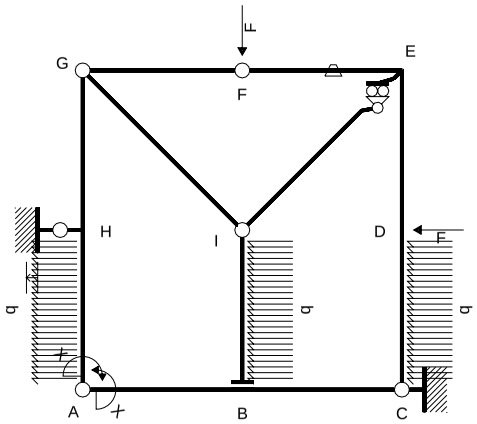

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7/8
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-3/4
11/16
-5/16

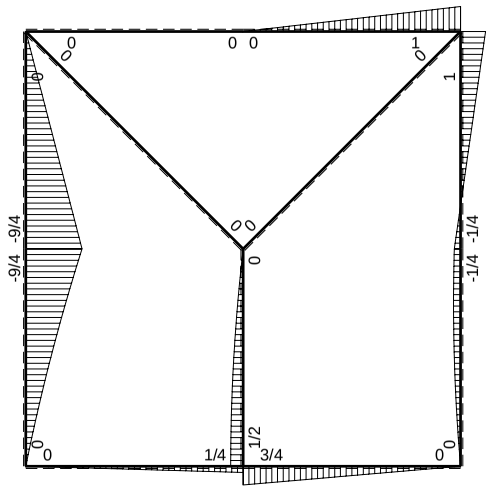
←□→ F

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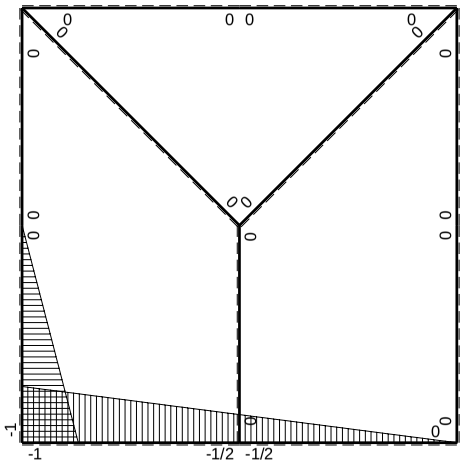

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0 0
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-9/4
-1/4
-1/4
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1/2
5/16
0
-7/8
-3/16

↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.212

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\ \right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\ \right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

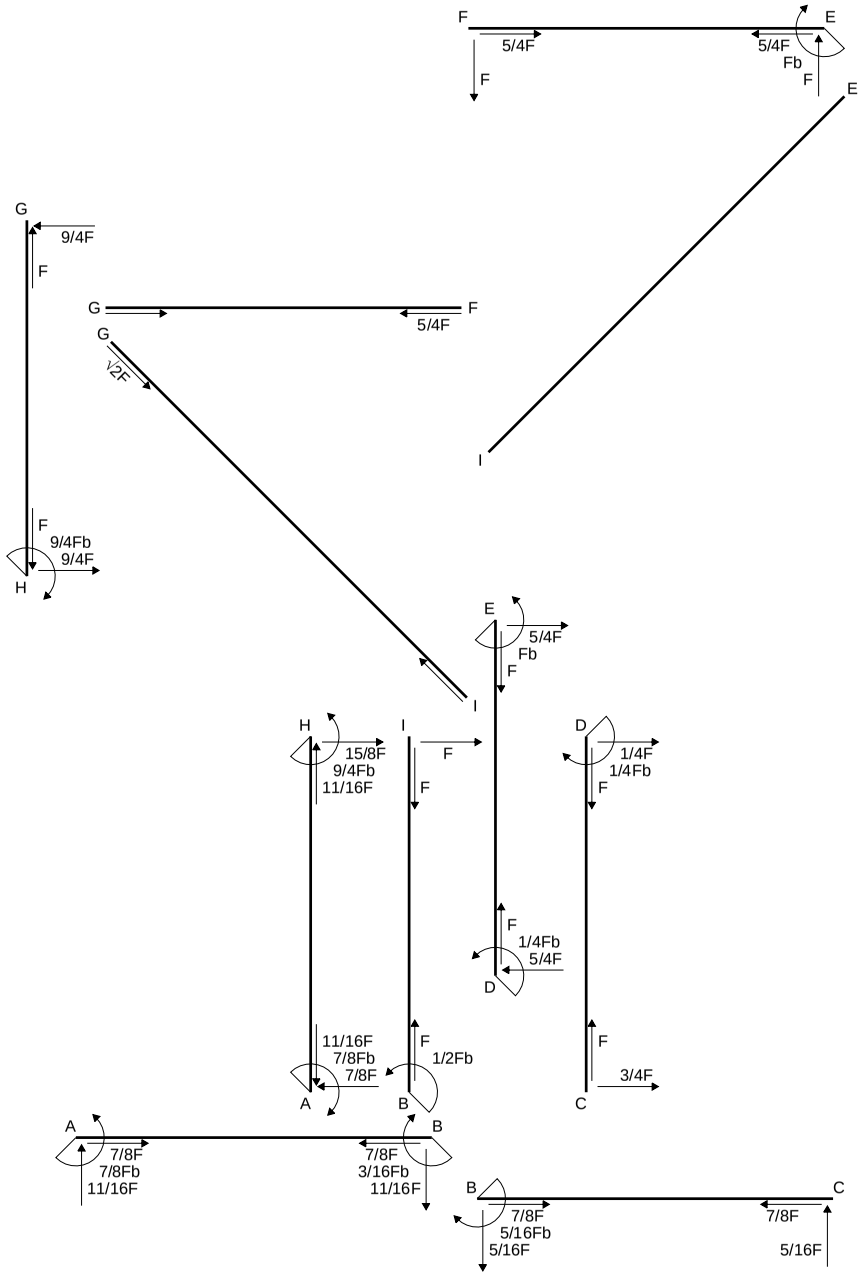
$= \left(9/8\ b - 11/12\ b + 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

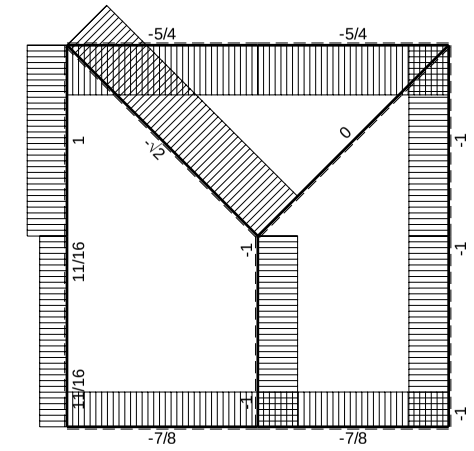
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

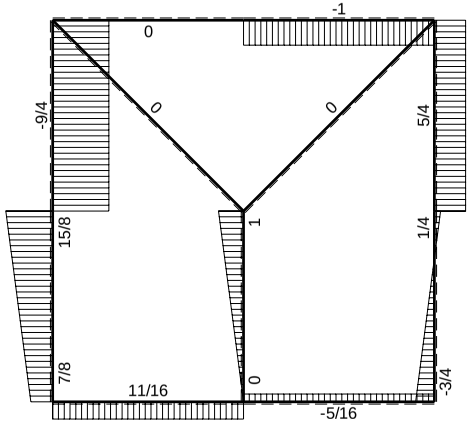
$= \left(7/8\ b - 5/12\ b - 1/8\ b\ \right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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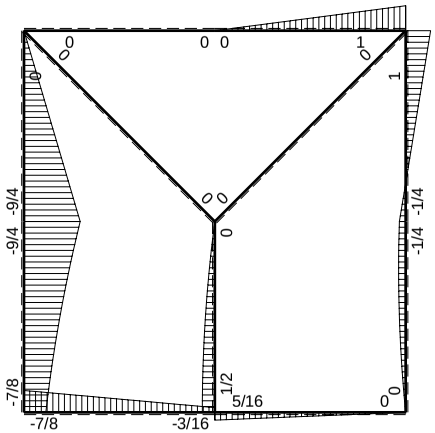

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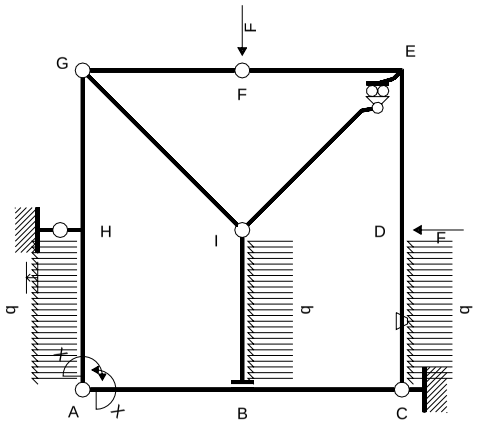

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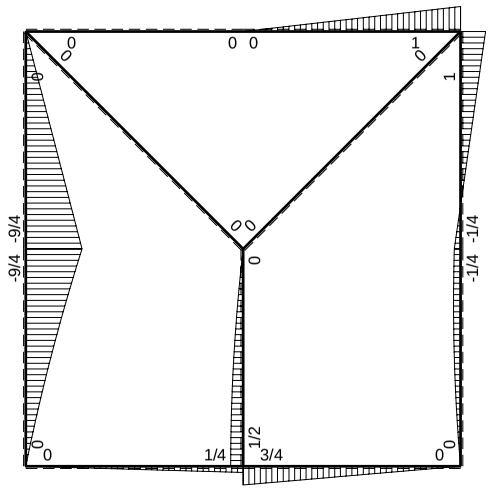
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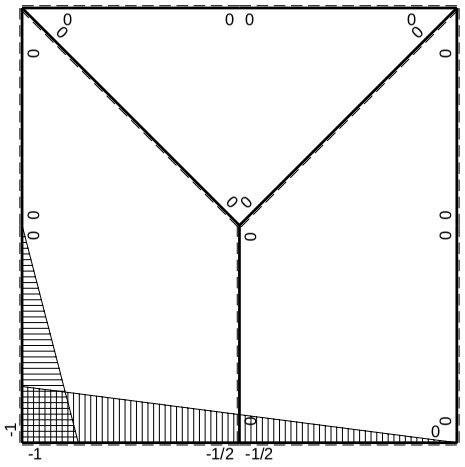

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↻□↺ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.213

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PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.213

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/4Fb+1/4Fx$	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	Fb/EJ	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

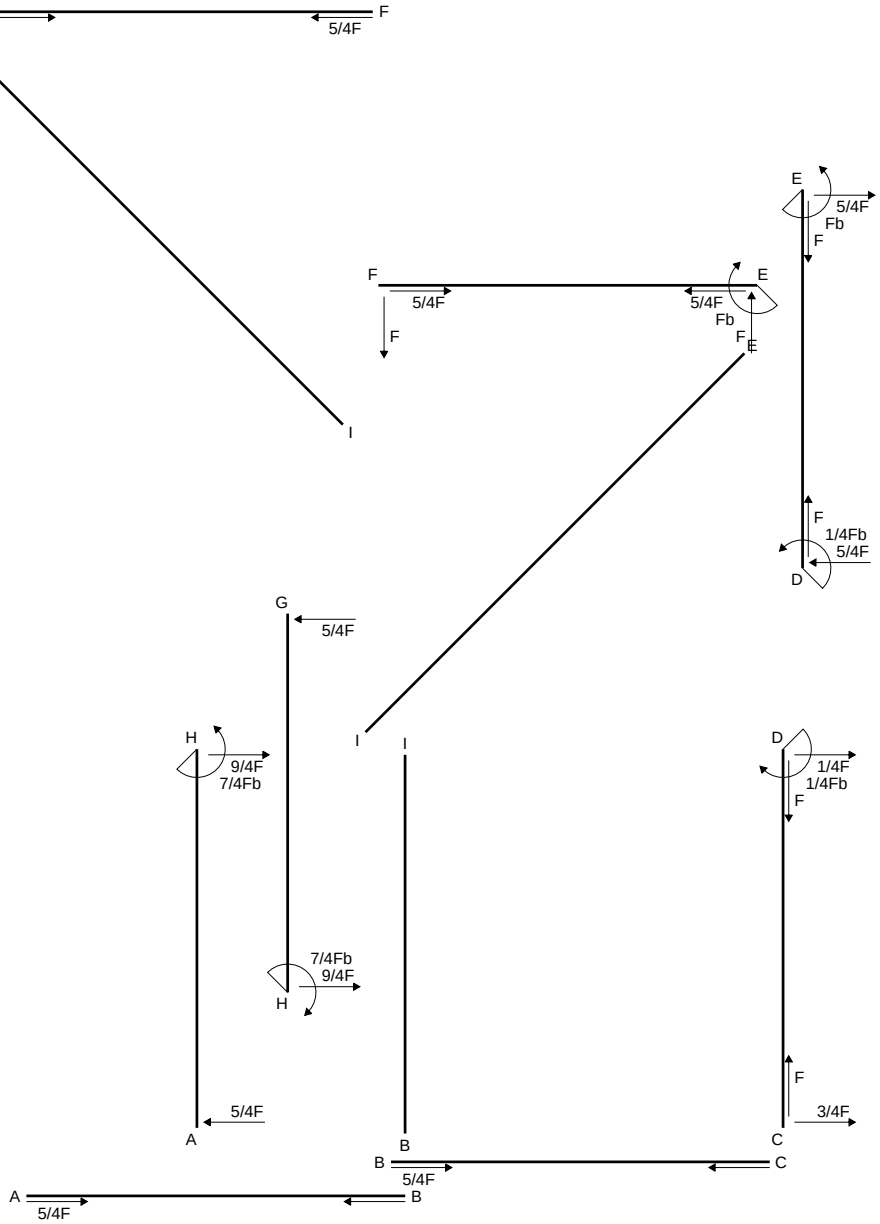
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

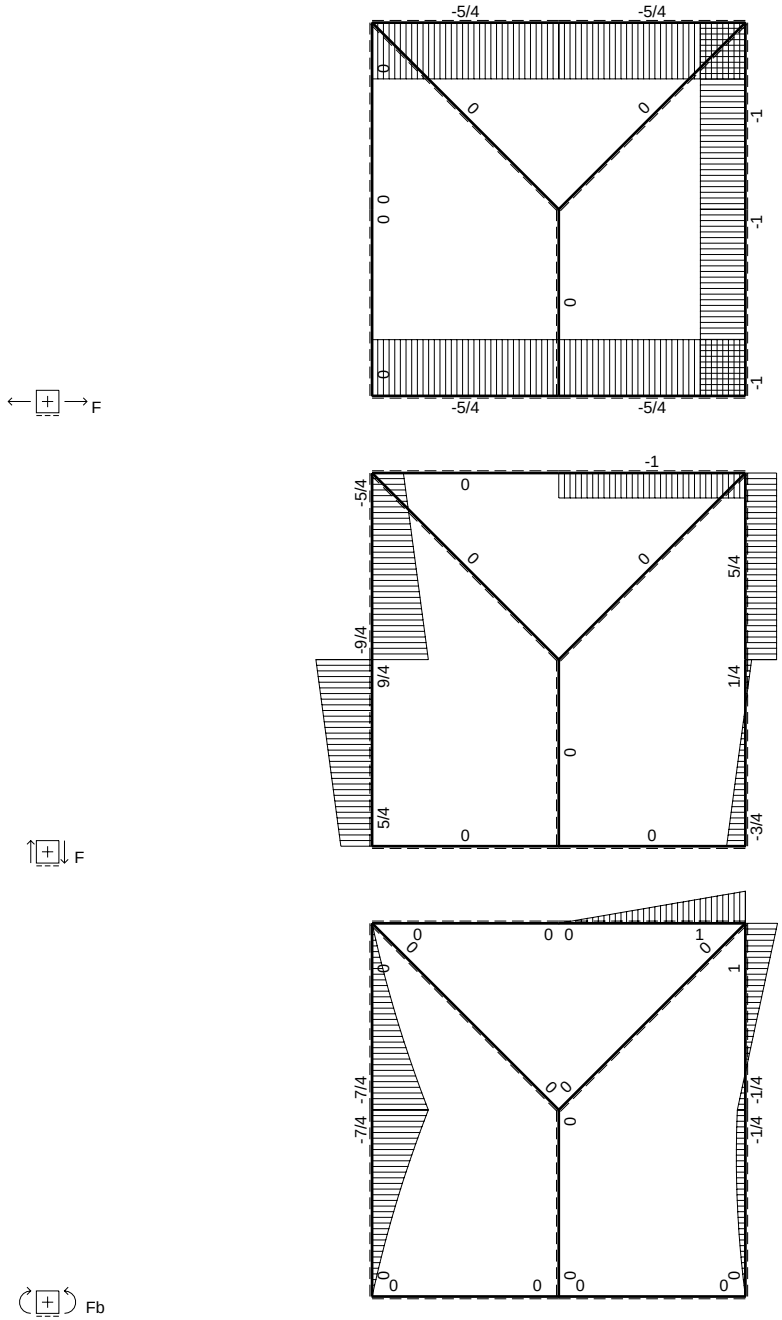
$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

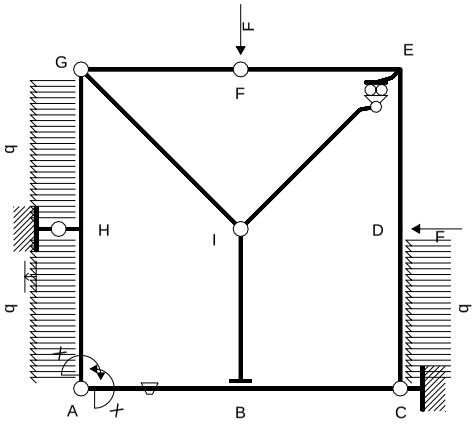
$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

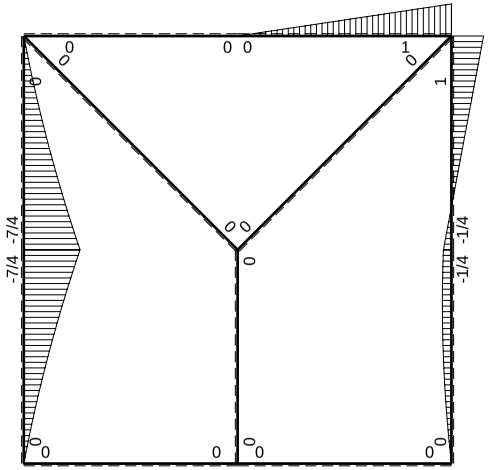
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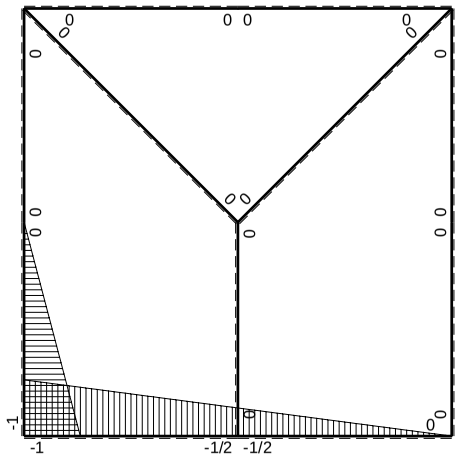

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-9/4
9/4
1/4
-3/4
5/4
Fb
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-1/4
-1/4
1
1





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.214

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	-Fb/EJ	0	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(0+3/4)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	0	Fb/EJ	0	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						0	Xb/EJ
	iperstatica $X=W_{AB}$						0	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(1 - 1/2\ x/b\ \right)\theta\ \ dx = \left[\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/2\ b - 1/4\ b\ \right)\theta\ \ = 3/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

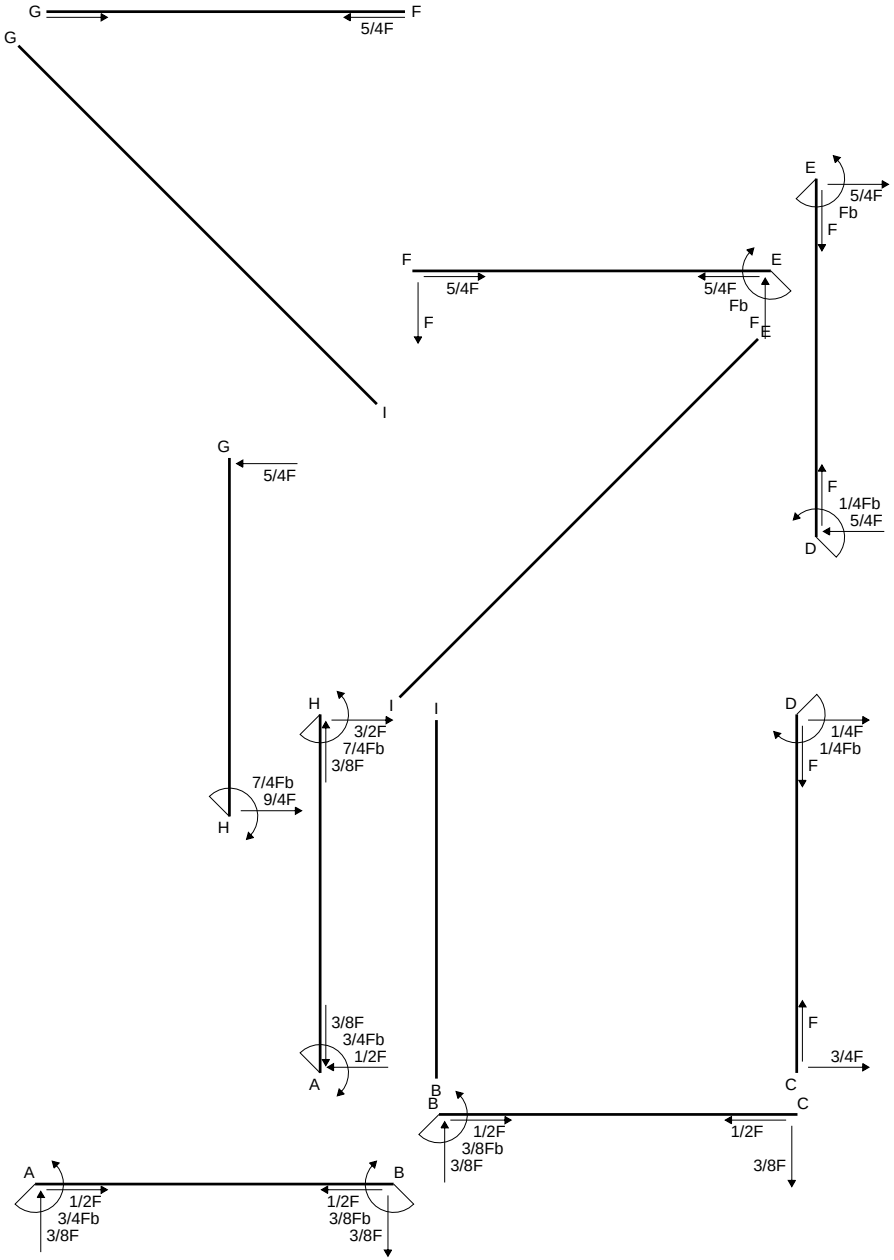
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

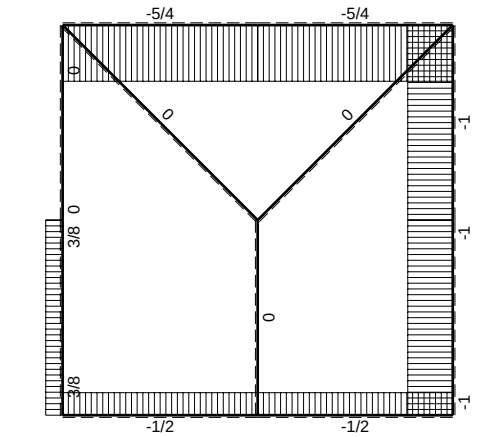
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

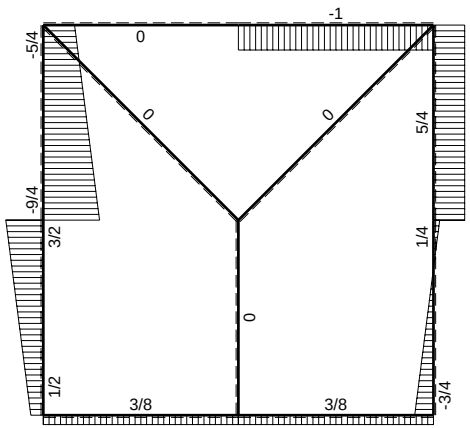
$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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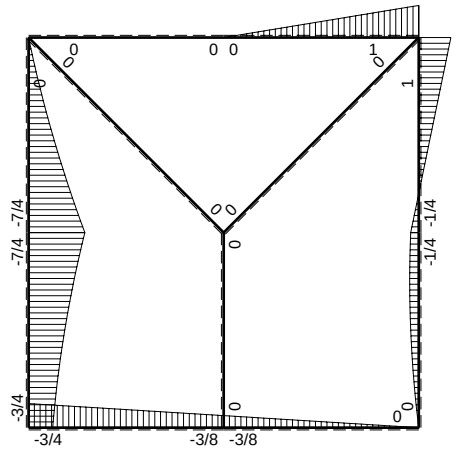

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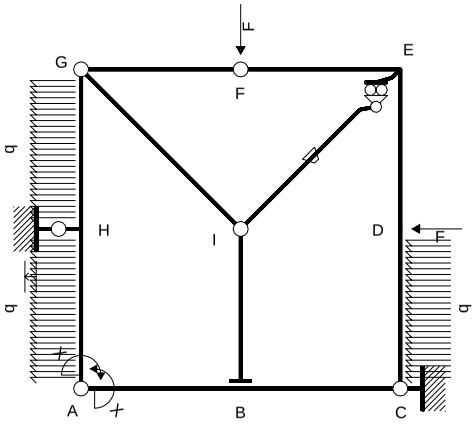

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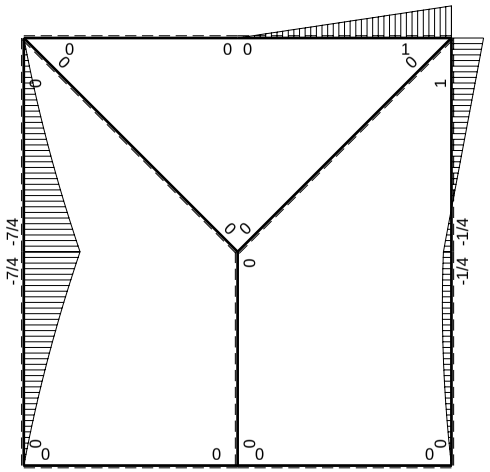
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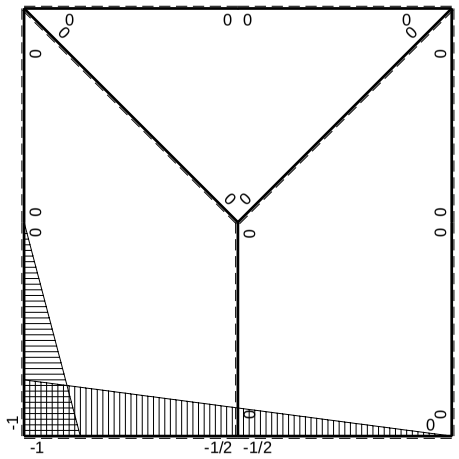

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-3/4
-3/8
-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ04.xxxx.215

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

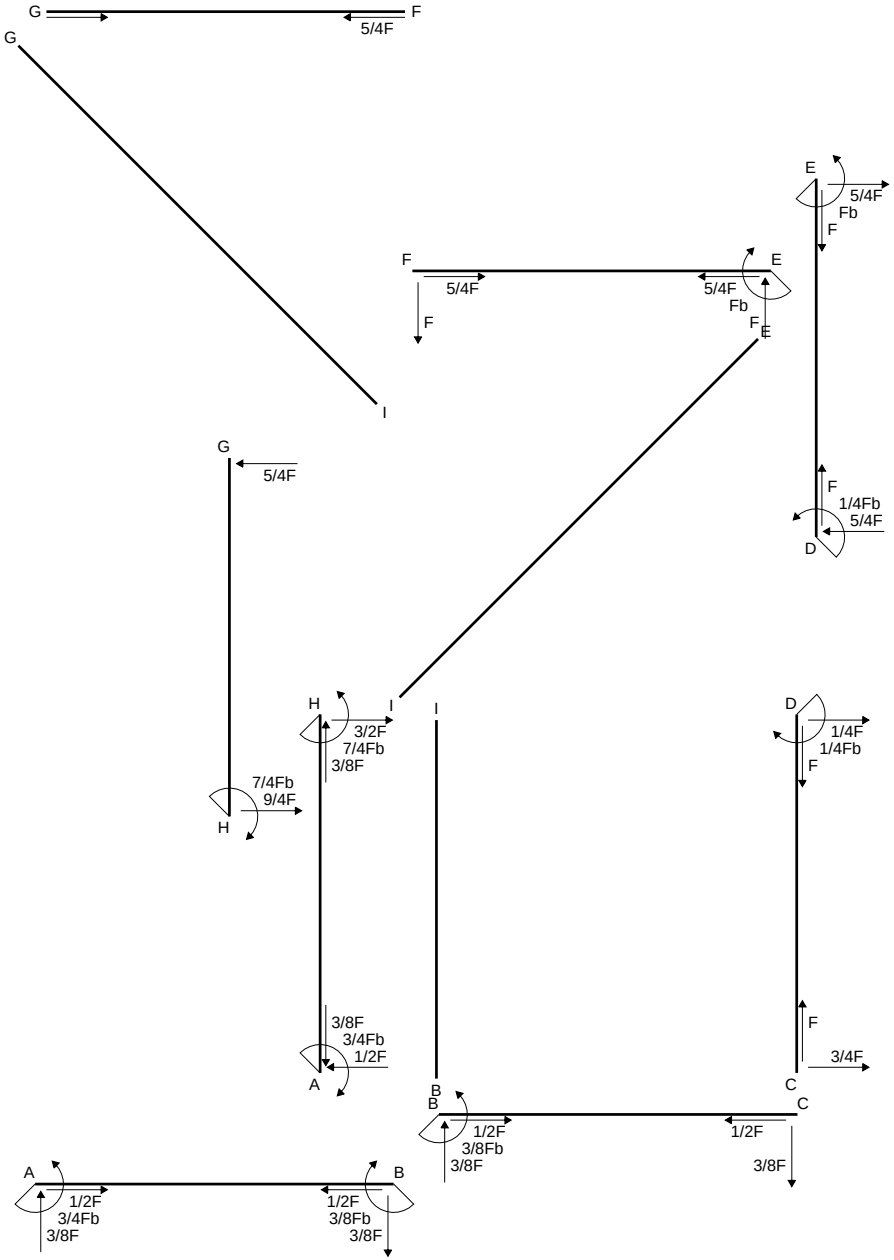
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

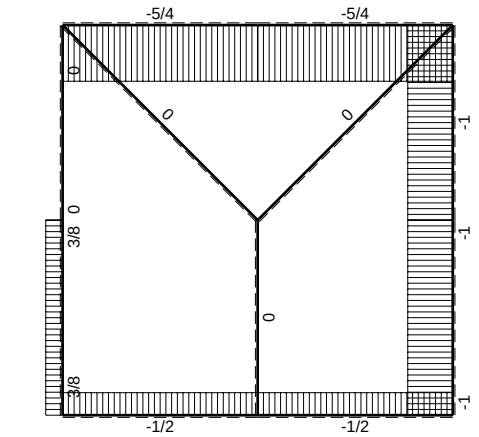
$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

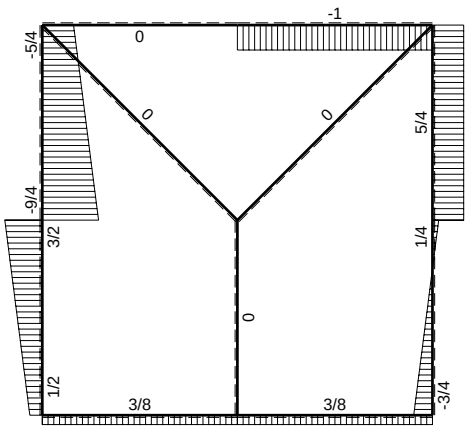
$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

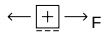
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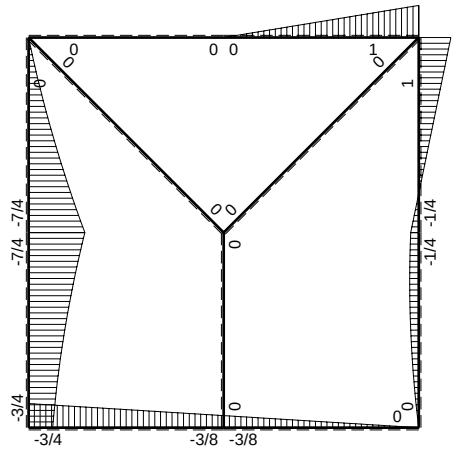

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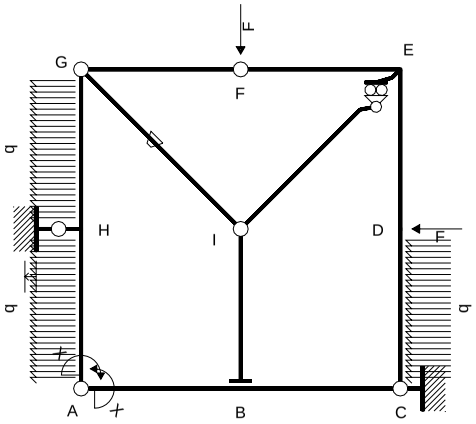

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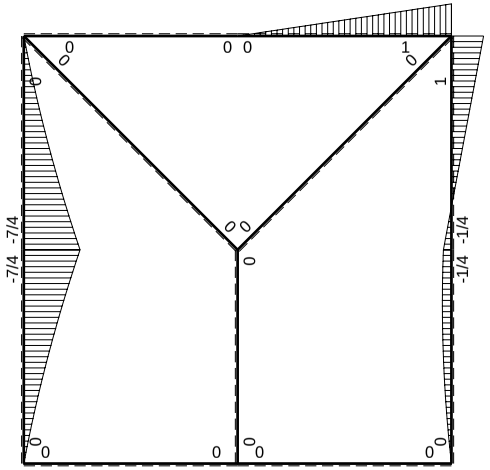

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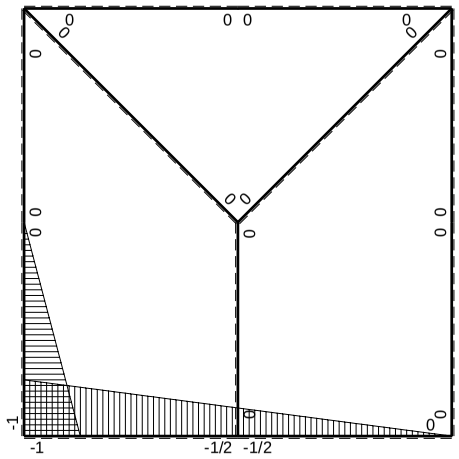

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\ \right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{HA} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

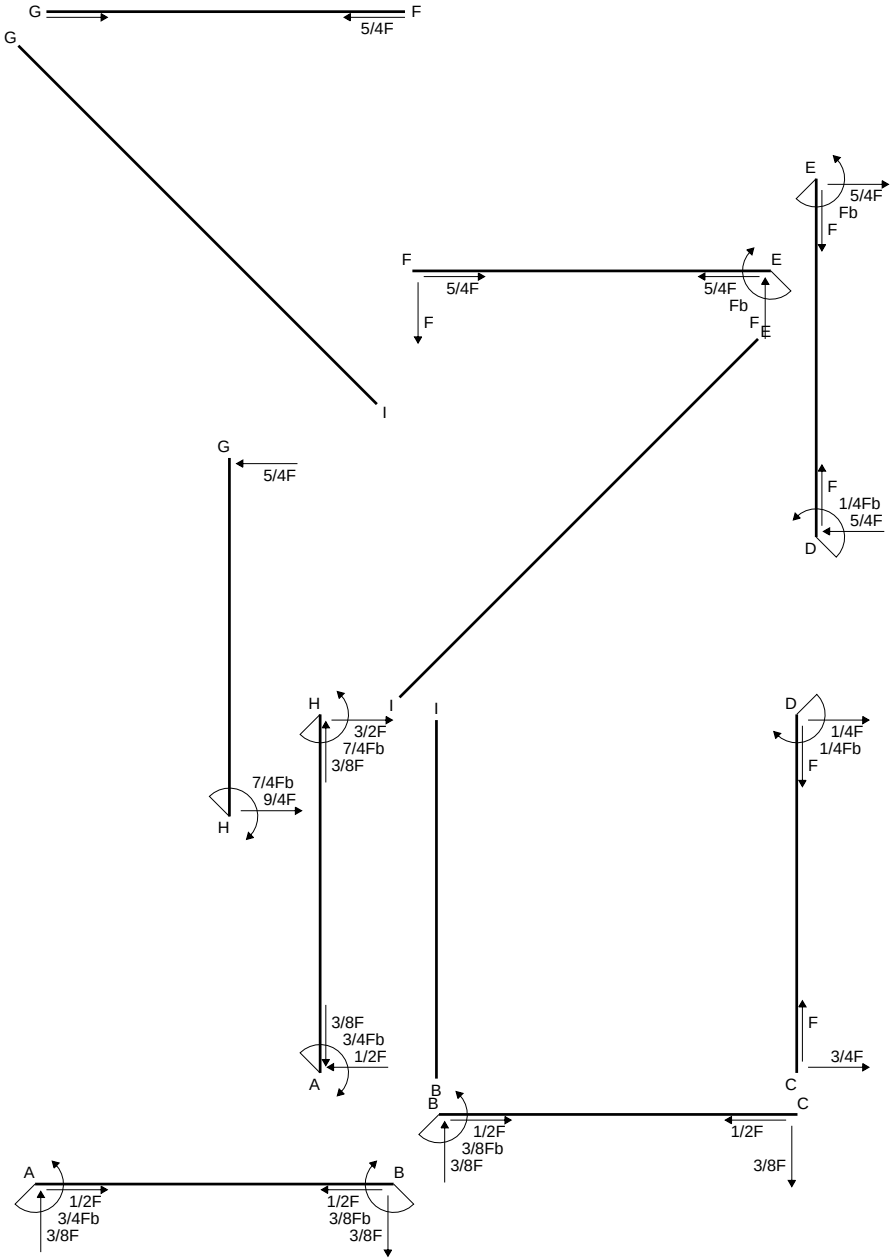
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

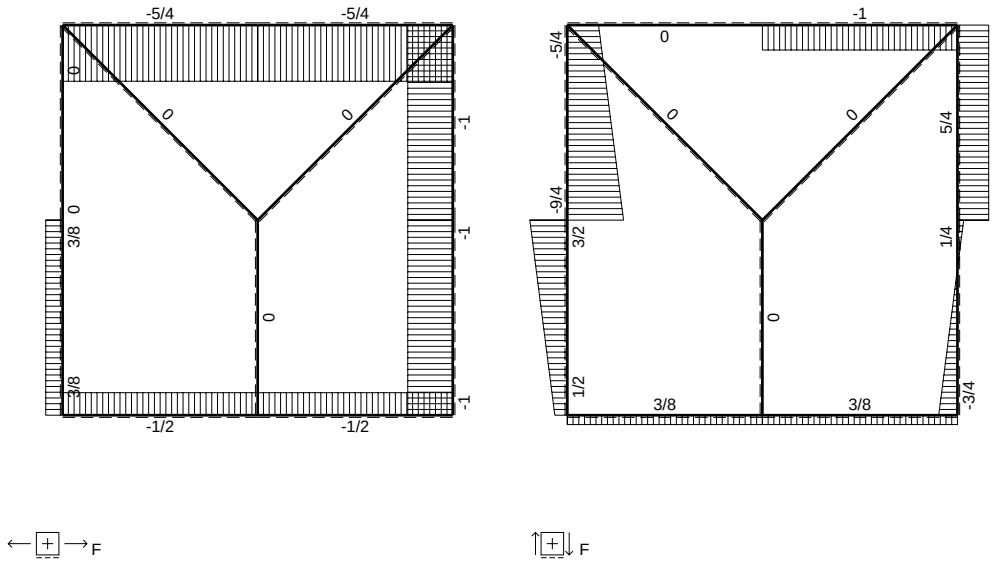
$L^{Xo}_{AH} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ$

$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

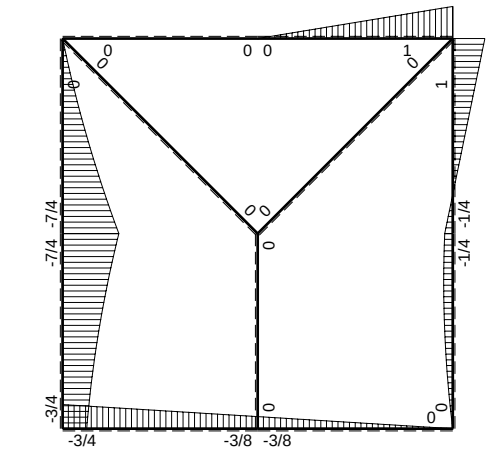
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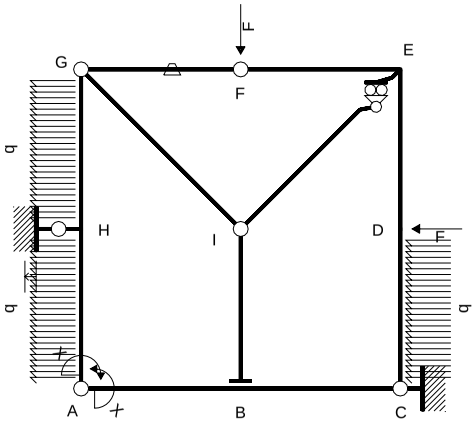

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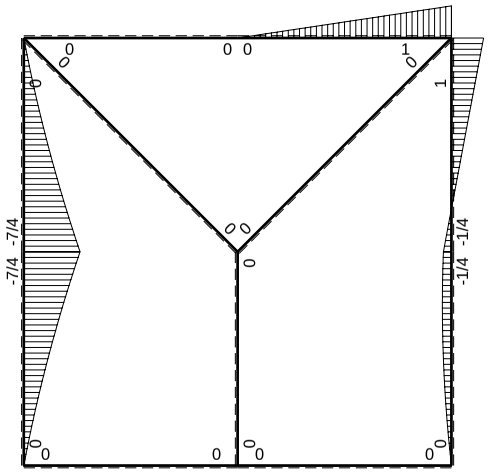

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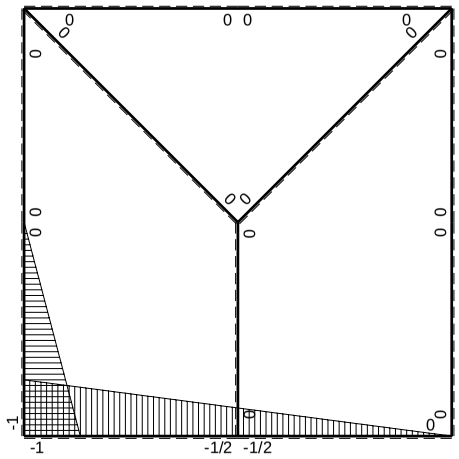

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-3/4
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-3/8

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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