

$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$\rho_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$\rho_{IB} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$\rho_{GH} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{HA} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

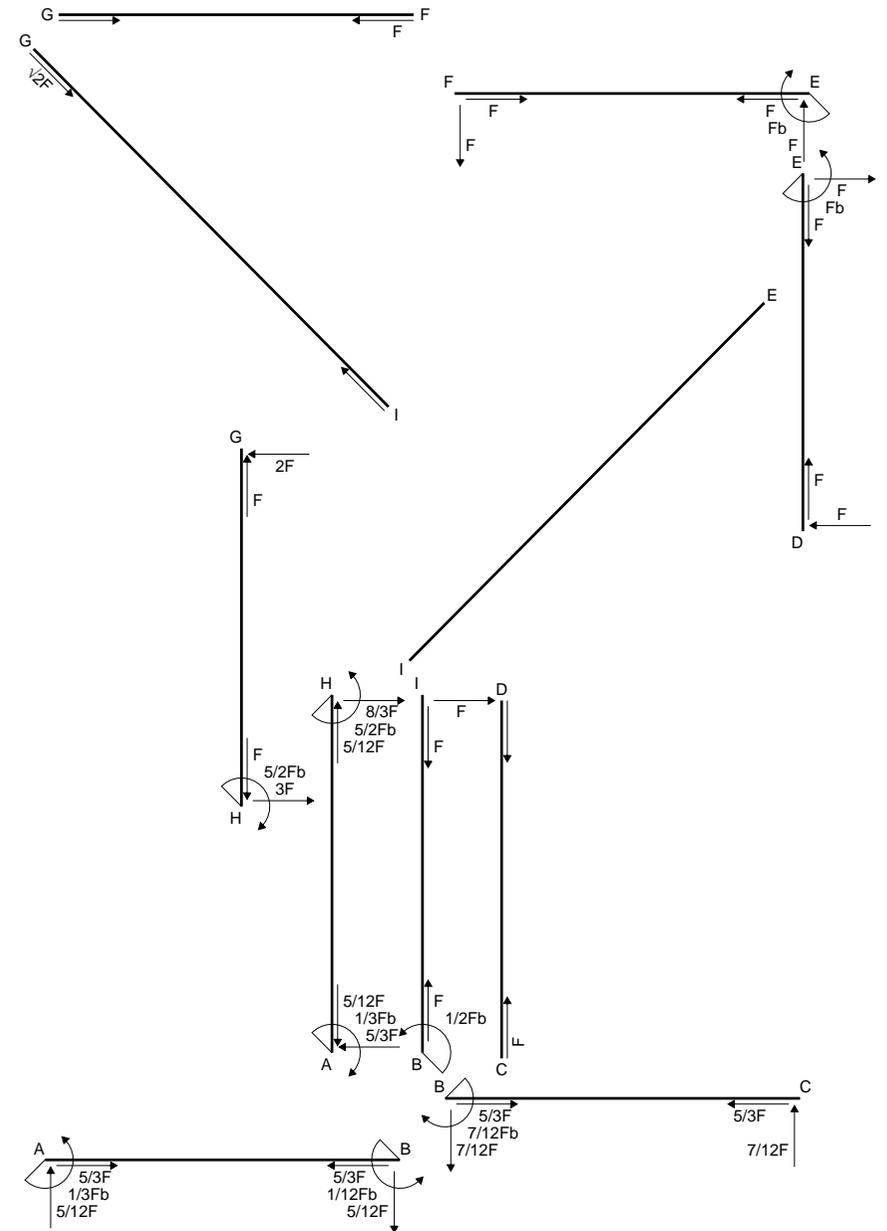
Carichi di aste curve misurati in proiezione sugli assi x,y.

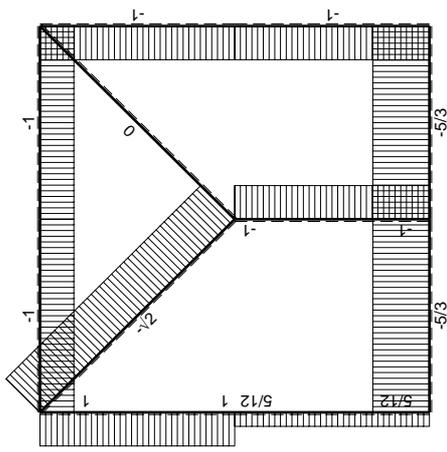
$J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta HA positiva se convessa a destra con inizio H.

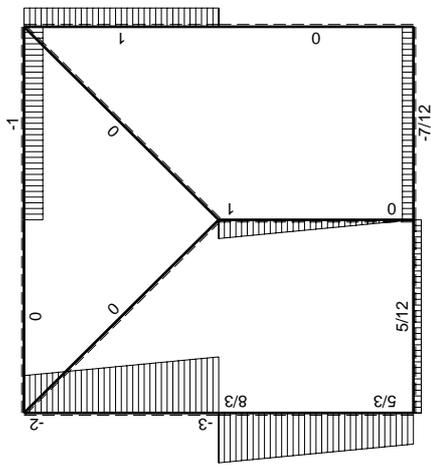
Spostamento orizzontale assoluto u imposto al nodo H.

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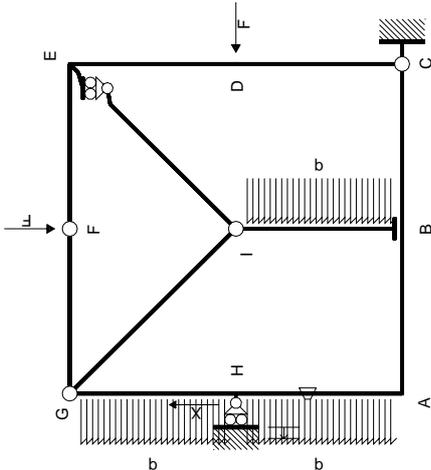




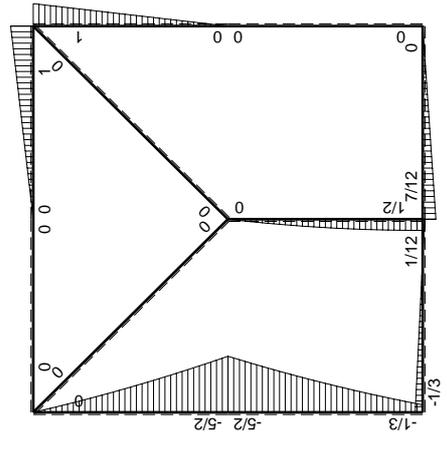
$\left[\begin{array}{c} + \\ - \end{array} \right]$ \rightarrow F



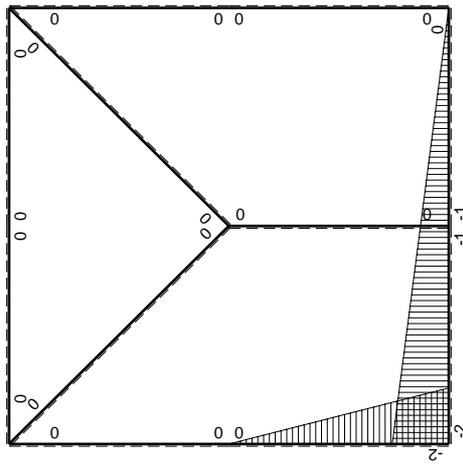
\uparrow $\left[\begin{array}{c} + \\ - \end{array} \right]$ \rightarrow F



$\left[\begin{array}{c} + \\ - \end{array} \right]$ \curvearrowright M_0 flessione da carichi assegnati



$\left[\begin{array}{c} + \\ - \end{array} \right]$ \curvearrowright F_b



$\left[\begin{array}{c} + \\ - \end{array} \right]$ \curvearrowright M_x flessione da iperstatica $X=1$

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	$-2b+x$	$-3/2Fb+Fx$	0	$3Fb^2-7/2Fbx+Fx^2$	0	$4b^2-4bx+x^2$	$(19/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$b+x$	$1/2Fb+Fx$	0	$1/2Fb^2+3/2Fbx+Fx^2$	0	$b^2+2bx+x^2$			
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$	
CB b	x	0	0	0	0	x^2			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	$-Fb+Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-2x$	$-5/2Fb+3/2Fx-1/2qx^2$	$-Fb/EJ$	$5Fbx-3Fx^2+qx^3$	$2Fxb/EJ$	$4x^2$	$(7/4+1)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$3/2Fb+1/2Fx+1/2qx^2$	Fb/EJ	$3Fb^2-2Fbx-qx^3$	$2Fb^2/EJ-2Fxb/EJ$	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$7/3Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							$-7/12F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (3 - 7/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [3x - 7/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3b - 7/4 b + 1/3 b) Fb^2 1/EJ = 19/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 + 3/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [1/2 x + 3/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b + 3/4 b + 1/3 b) Fb^2 1/EJ = 19/12 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (5x/b - 3x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx + \int_0^b (2x/b) \theta dx$$

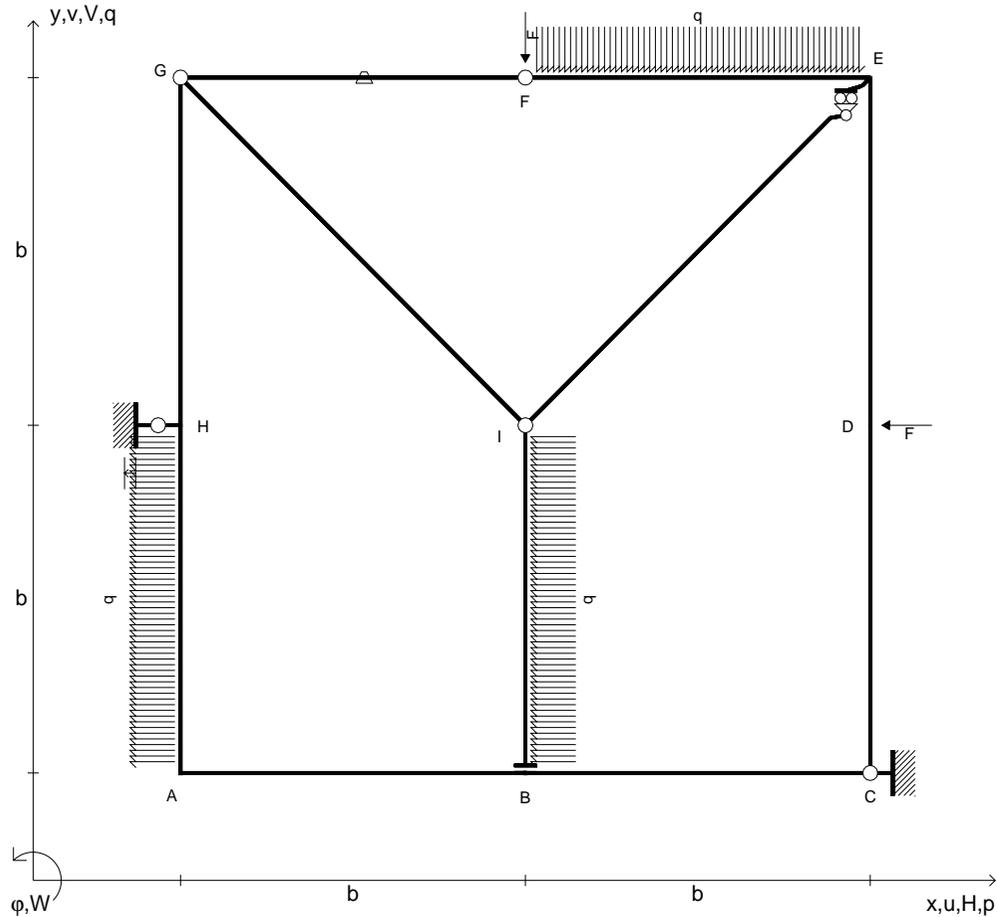
$$= [5/2 x^2/b - x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ + [x^2/b]_0^b \theta$$

$$= (5/2 b - b + 1/4 b) Fb^2 1/EJ + (b) \theta = 11/4 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (3 - 2x/b - x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-2 + 2x/b) \theta dx$$

$$= [3x - x^2/b - 1/4 x^4/b^3]_0^b Fb^2 1/EJ + [-2x + x^2/b]_0^b \theta$$

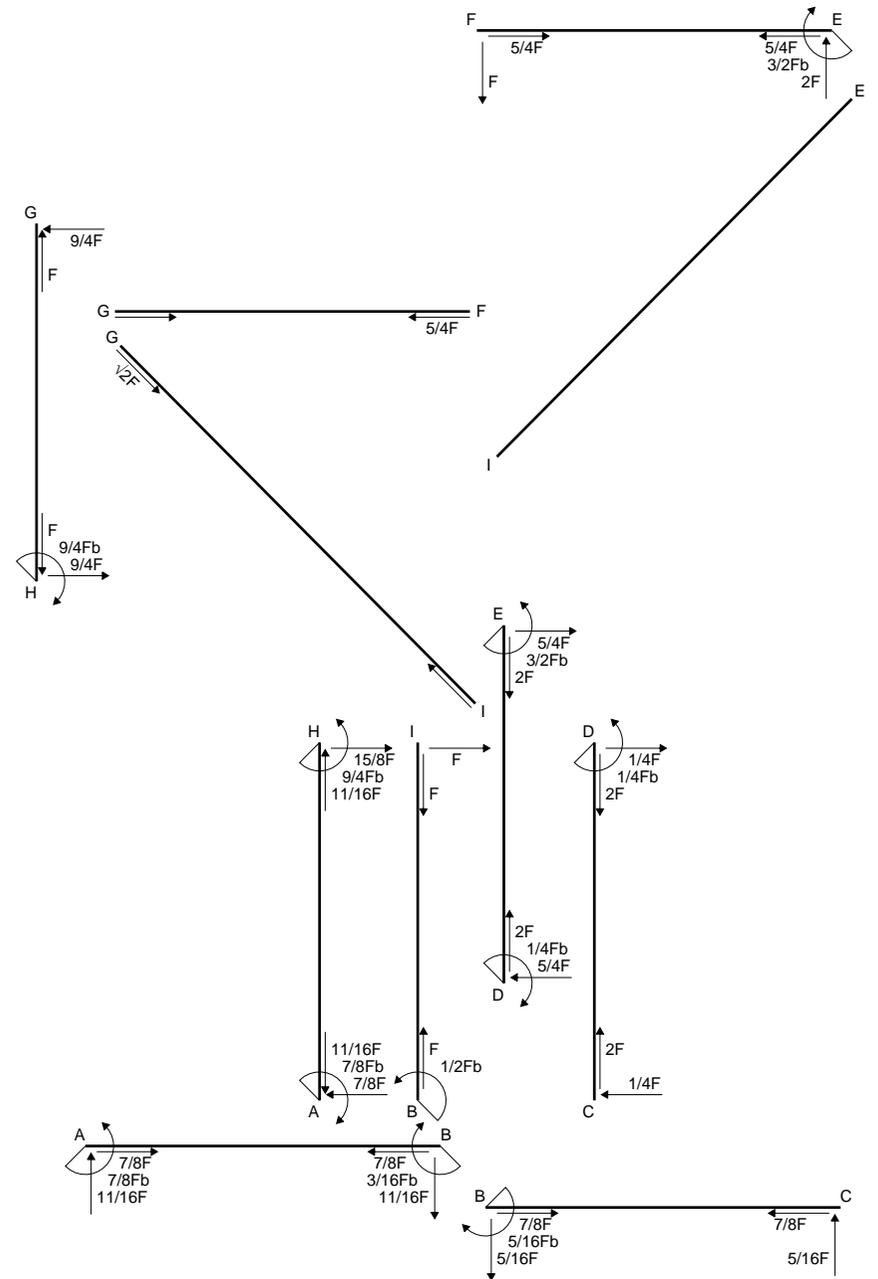
$$= (3b - b - 1/4 b) Fb^2 1/EJ + (-2b + b) \theta = 11/4 Fb^3/EJ$$



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$\rho_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
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$q_{EF} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

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 Curvatura θ asta FG positiva se convessa a destra con inizio F.
 Spostamento orizzontale assoluto u imposto al nodo H.



Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	$-3/2Fb+Fx$	0	$3Fb^2-7/2Fbx+Fx^2$	0	$4b^2-4bx+x^2$	$(19/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$b+x$	$1/2Fb+Fx$	0	$1/2Fb^2+3/2Fbx+Fx^2$	0	$b^2+2bx+x^2$			
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$	
CB b	x	0	0	0	0	x^2			
CD b	0	$1/4Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0			
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-Fx-1/2qx^2$	0	0	0	0			
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0	
GF b	0	0	Fb/EJ	0	0	0			
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0	
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-2x$	$-9/4Fb+5/4Fx-1/2qx^2$	0	$9/2Fbx-5/2Fx^2+qx^3$	0	$4x^2$	$(5/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$3/2Fb+1/4Fx+1/2qx^2$	0	$3Fb^2-5/2Fbx+1/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$5/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							$-5/16F$	

Sviluppi di calcolo iperstatica

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$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

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$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

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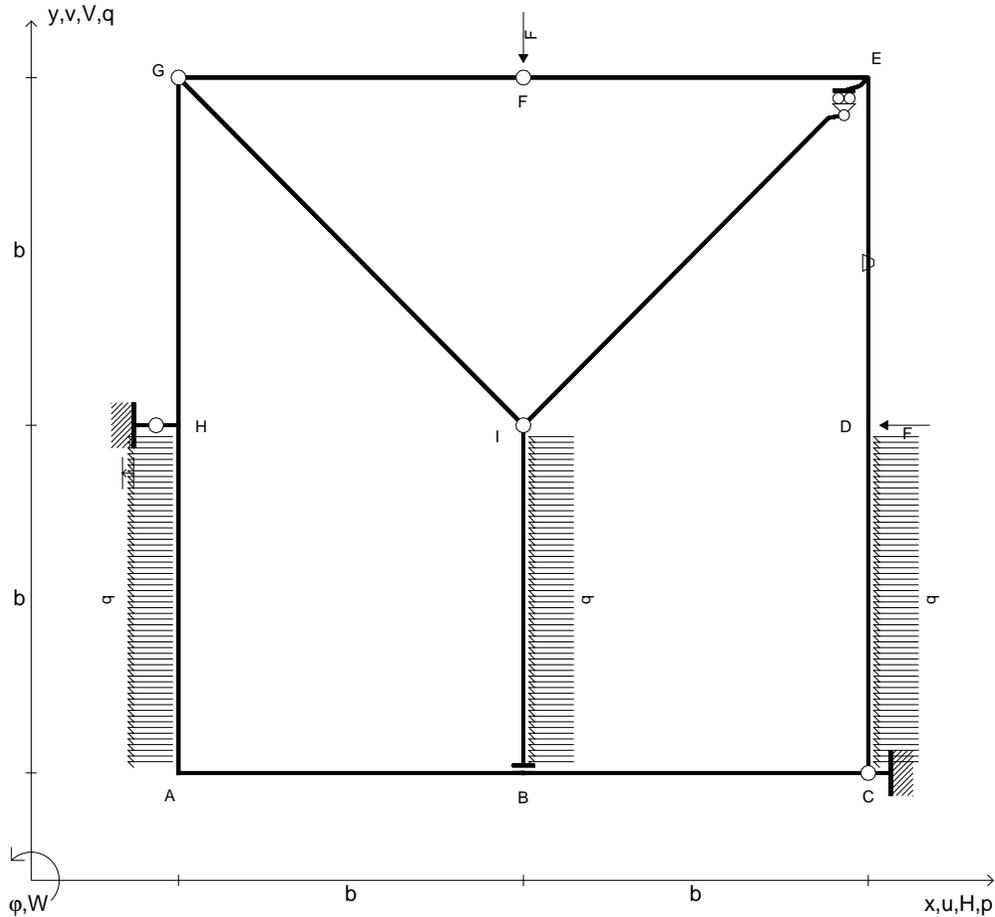
$$= (1/2 b + 3/4 b + 1/3 b) Fb^2 1/EJ = 19/12 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (9/2 x/b - 5/2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [9/4 x^2/b - 5/6 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (9/4 b - 5/6 b + 1/4 b) Fb^2 1/EJ = 5/3 Fb^3/EJ$$

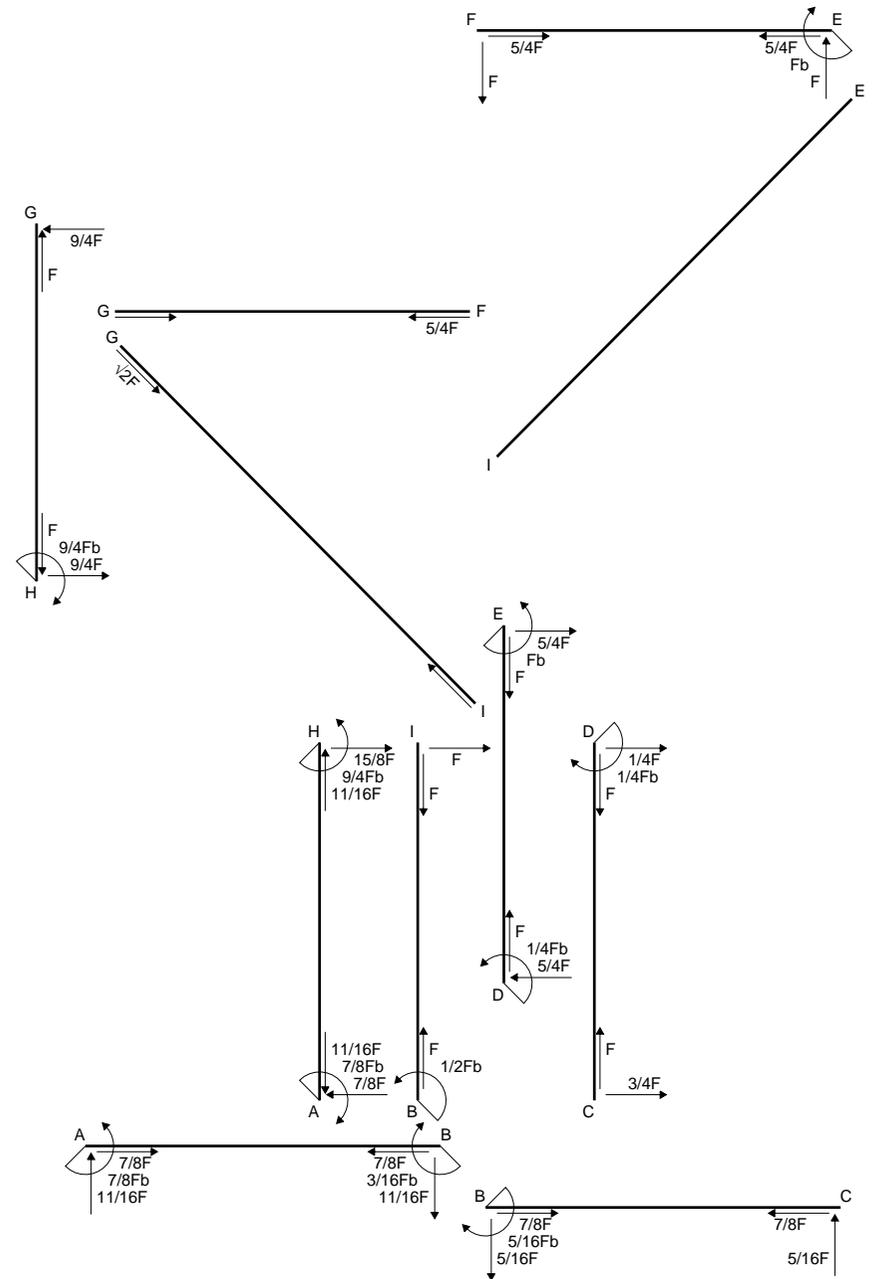
$$L_{AH}^{xo} = \int_0^b (3 - 5/2 x/b + 1/2 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [3x - 5/4 x^2/b + 1/6 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3b - 5/4 b + 1/6 b - 1/4 b) Fb^2 1/EJ = 5/3 Fb^3/EJ$$



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$p_{CD} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	$-3/2Fb+Fx$	0	$3Fb^2-7/2Fbx+Fx^2$	0	$4b^2-4bx+x^2$	$(19/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$b+x$	$1/2Fb+Fx$	0	$1/2Fb^2+3/2Fbx+Fx^2$	0	$b^2+2bx+x^2$			
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$	
CB b	x	0	0	0	0	x^2			
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0	
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0			
DE b	0	$-1/4Fb+5/4Fx$	$-Fb/EJ$	0	0	0	0+0	0	
ED b	0	$-Fb+5/4Fx$	Fb/EJ	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0	
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0			
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BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
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AH b	$2b-2x$	$3/2Fb+1/4Fx+1/2qx^2$	0	$3Fb^2-5/2Fbx+1/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
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$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (3 - 7/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [3x - 7/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3b - 7/4 b + 1/3 b) Fb^2 1/EJ = 19/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 + 3/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [1/2 x + 3/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b + 3/4 b + 1/3 b) Fb^2 1/EJ = 19/12 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (9/2 x/b - 5/2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [9/4 x^2/b - 5/6 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (9/4 b - 5/6 b + 1/4 b) Fb^2 1/EJ = 5/3 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (3 - 5/2 x/b + 1/2 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [3x - 5/4 x^2/b + 1/6 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3b - 5/4 b + 1/6 b - 1/4 b) Fb^2 1/EJ = 5/3 Fb^3/EJ$$

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	$-5/2Fb+2Fx$	0	$5Fb^2-13/2Fbx+2Fx^2$	0	$4b^2-4bx+x^2$	$(29/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$b+x$	$1/2Fb+2Fx$	0	$1/2Fb^2+5/2Fbx+2Fx^2$	0	$b^2+2bx+x^2$			
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$	
CB b	x	0	0	0	0	x^2			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	$-Fb+Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-3Fx$	0	0	0	0	0+0	0	
HG b	0	$3Fb-3Fx$	0	0	0	0			
GI $\sqrt{2}b$	0	0	$-Fb/EJ$	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0	
HA b	$-2x$	$-3Fb+Fx-1/2qx^2$	0	$6Fbx-2Fx^2+qx^3$	0	$4x^2$	$(31/12+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$5/2Fb+1/2qx^2$	0	$5Fb^2-5Fbx+Fx^2-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$3Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							$-3/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (5 - 13/2 x/b + 2x^2/b^2) Fb^2 1/EJ dx = [5x - 13/4 x^2/b + 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (5b - 13/4 b + 2/3 b) Fb^2 1/EJ = 29/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 + 5/2 x/b + 2x^2/b^2) Fb^2 1/EJ dx = [1/2 x + 5/4 x^2/b + 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

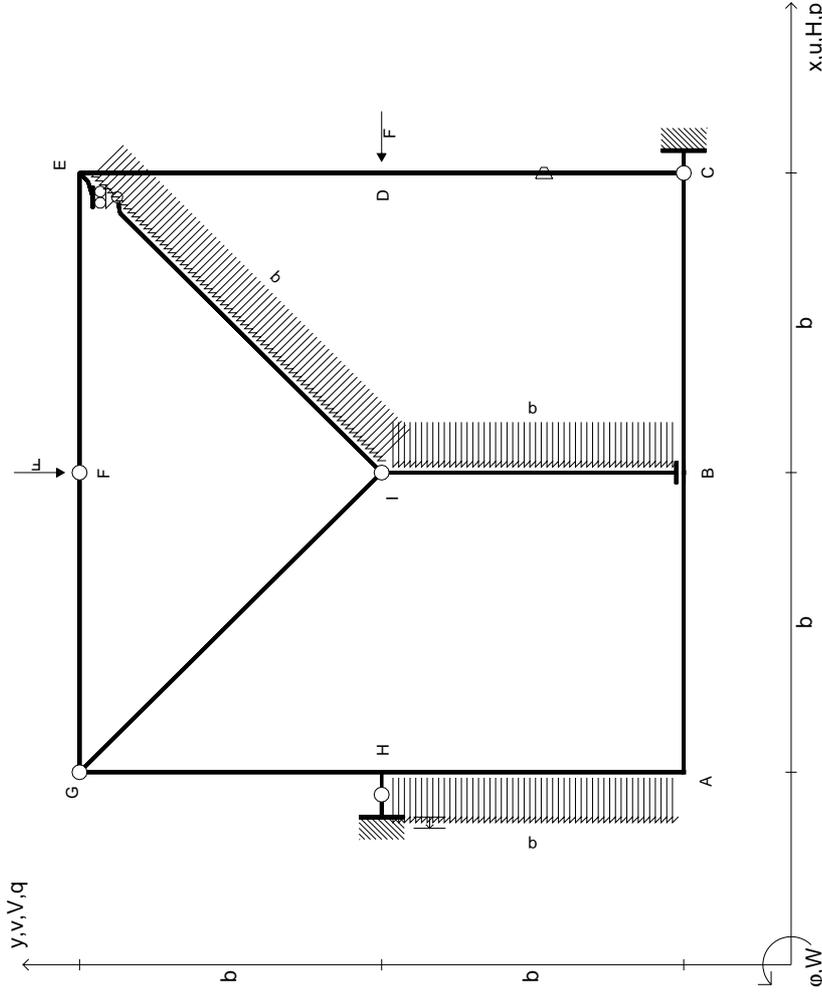
$$= (1/2 b + 5/4 b + 2/3 b) Fb^2 1/EJ = 29/12 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (6x/b - 2x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [3x^2/b - 2/3 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3b - 2/3 b + 1/4 b) Fb^2 1/EJ = 31/12 Fb^3/EJ$$

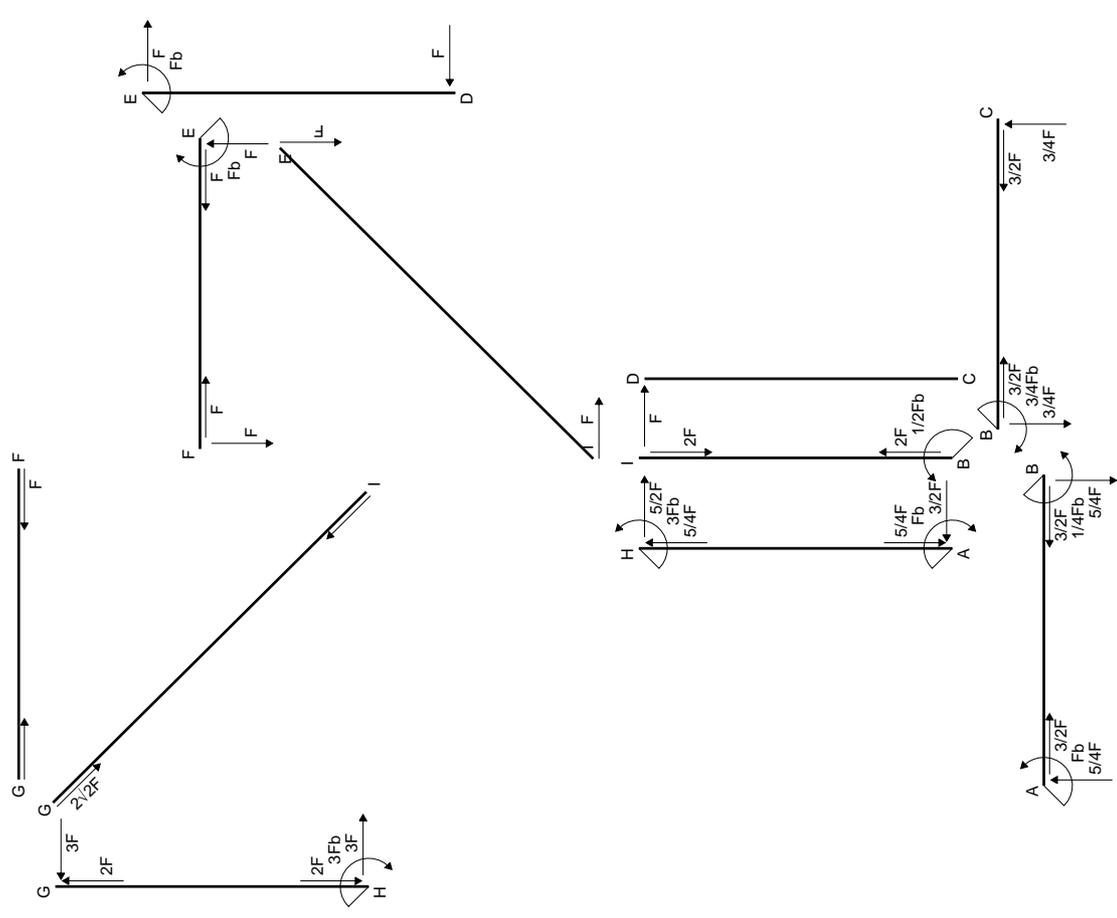
$$L_{AH}^{xo} = \int_0^b (5 - 5x/b + x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [5x - 5/2 x^2/b + 1/3 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (5b - 5/2 b + 1/3 b - 1/4 b) Fb^2 1/EJ = 31/12 Fb^3/EJ$$



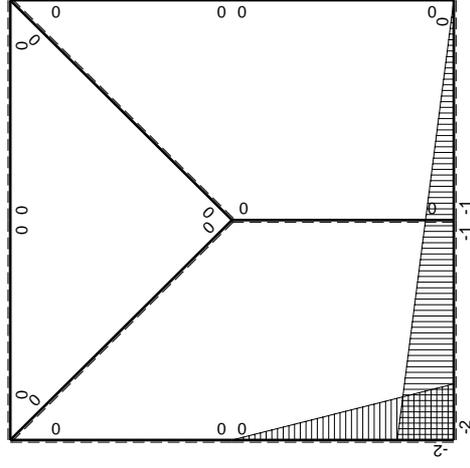
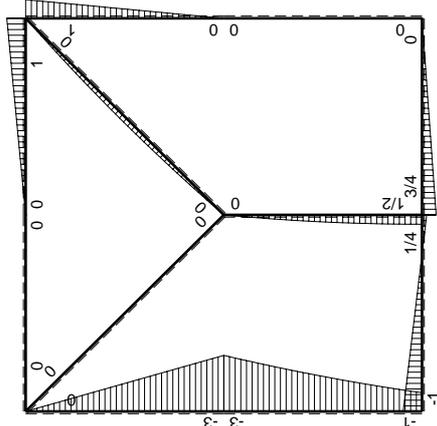
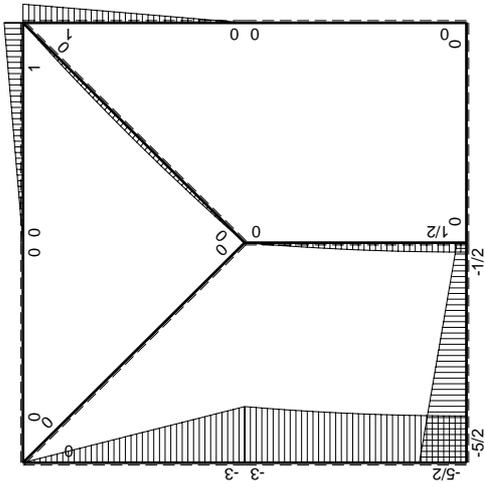
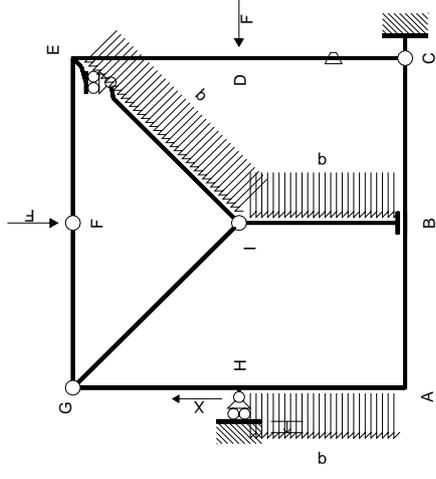
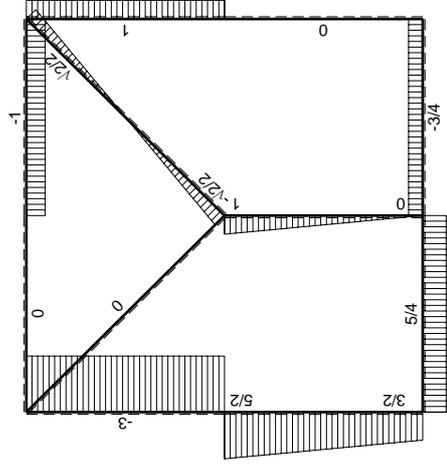
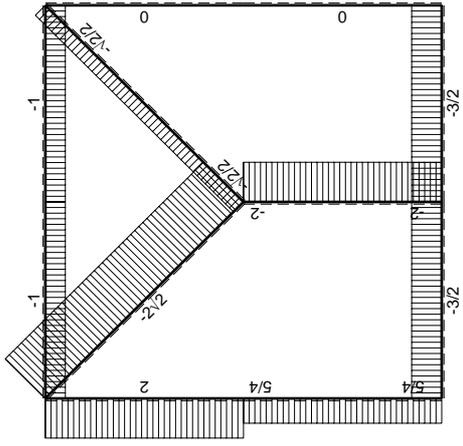
$$\begin{aligned}
 V_F &= -F \\
 H_b &= -F \\
 P_{HA} &= -q = -F/b \\
 P_{IB} &= -q = -F/b \\
 P_{IE} &= -q = -F/b \\
 q_{IE} &= q = F/b \\
 \theta_{CD} &= -\theta = -\alpha T/b = -bF/EJ \\
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=V_H$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 Diagrammi di carico con valori riferiti ad asse della trave.
 Componenti di carico distribuito riferiti ad assi ortogonali.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta CD positiva se convessa a destra con inizio C.
 Spostamento orizzontale assoluto u imposto al nodo H.



$$\begin{aligned}
 EJ_{GH} &= EJ \\
 EJ_{GI} &= EJ \\
 EJ_{IB} &= EJ \\
 EJ_{IE} &= EJ \\
 EJ_{HA} &= EJ
 \end{aligned}$$

$$\begin{aligned}
 u_H &= -\delta = -b^3 F/EJ \\
 EJ_{AB} &= EJ \\
 EJ_{BC} &= EJ \\
 EJ_{CD} &= EJ \\
 EJ_{DE} &= EJ \\
 EJ_{EF} &= EJ \\
 EJ_{FG} &= EJ
 \end{aligned}$$



Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	$-5/2Fb+2Fx$	0	$5Fb^2-13/2Fbx+2Fx^2$	0	$4b^2-4bx+x^2$	$(29/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$b+x$	$1/2Fb+2Fx$	0	$1/2Fb^2+5/2Fbx+2Fx^2$	0	$b^2+2bx+x^2$			
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$	
CB b	x	0	0	0	0	x^2			
CD b	0	0	$-Fb/EJ$	0	0	0	0+0	0	
DC b	0	0	Fb/EJ	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	$-Fb+Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-3Fx$	0	0	0	0	0+0	0	
HG b	0	$3Fb-3Fx$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0	
HA b	$-2x$	$-3Fb+Fx-1/2qx^2$	0	$6Fbx-2Fx^2+qx^3$	0	$4x^2$	$(31/12+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$5/2Fb+1/2qx^2$	0	$5Fb^2-5Fbx+Fx^2-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$3Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							$-3/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (5 - 13/2 x/b + 2x^2/b^2) Fb^2 1/EJ dx = [5x - 13/4 x^2/b + 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (5b - 13/4 b + 2/3 b) Fb^2 1/EJ = 29/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 + 5/2 x/b + 2x^2/b^2) Fb^2 1/EJ dx = [1/2 x + 5/4 x^2/b + 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

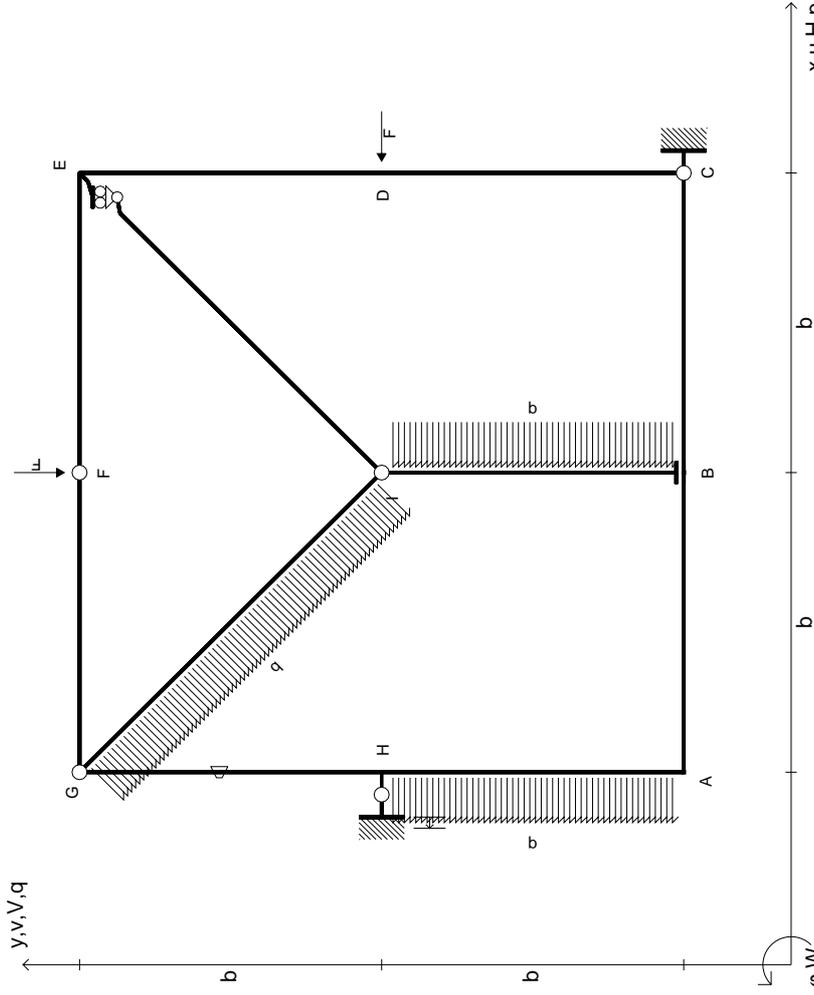
$$= (1/2 b + 5/4 b + 2/3 b) Fb^2 1/EJ = 29/12 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (6x/b - 2x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [3x^2/b - 2/3 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3b - 2/3 b + 1/4 b) Fb^2 1/EJ = 31/12 Fb^3/EJ$$

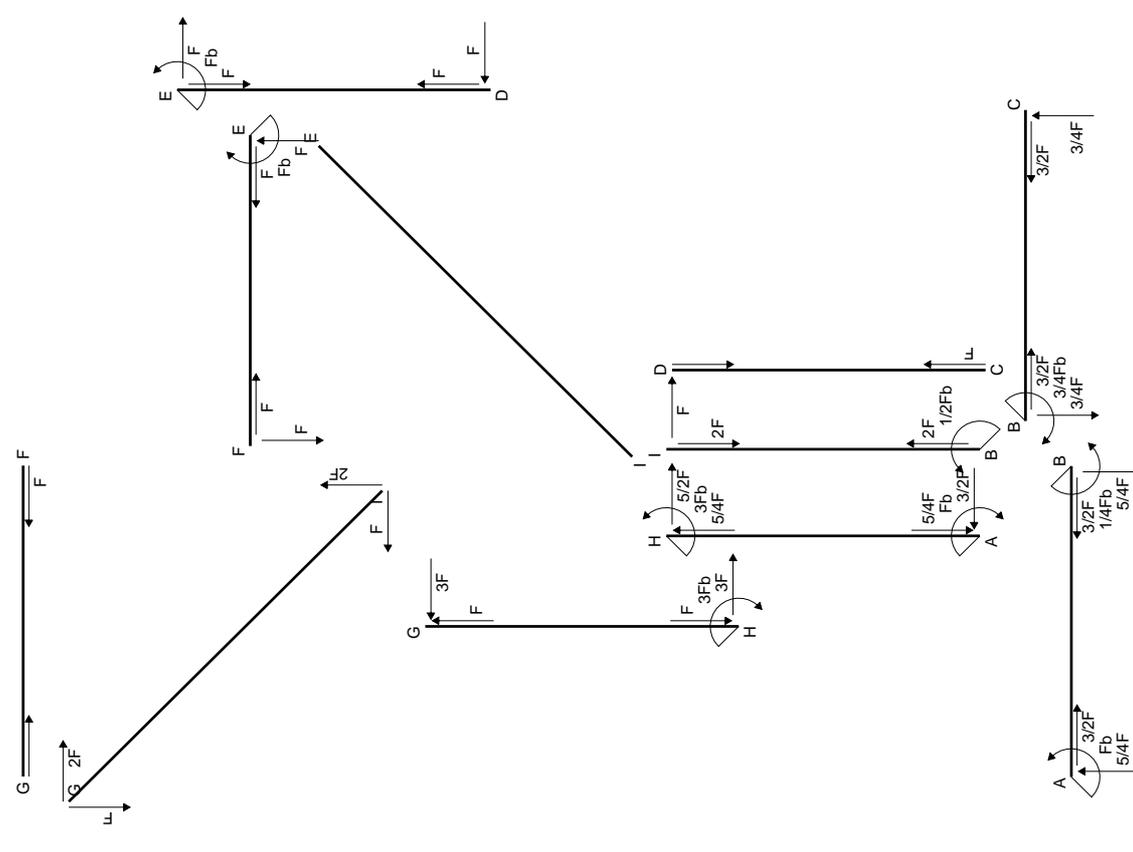
$$L_{AH}^{xo} = \int_0^b (5 - 5x/b + x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [5x - 5/2 x^2/b + 1/3 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

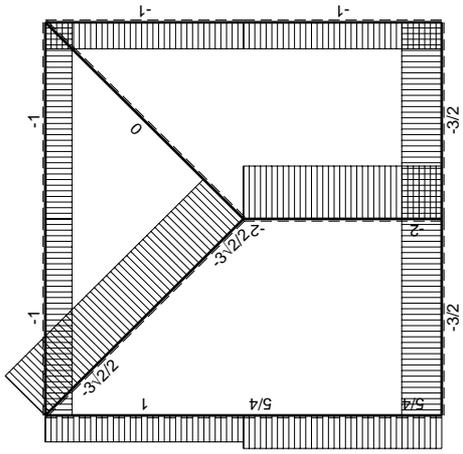
$$= (5b - 5/2 b + 1/3 b - 1/4 b) Fb^2 1/EJ = 31/12 Fb^3/EJ$$



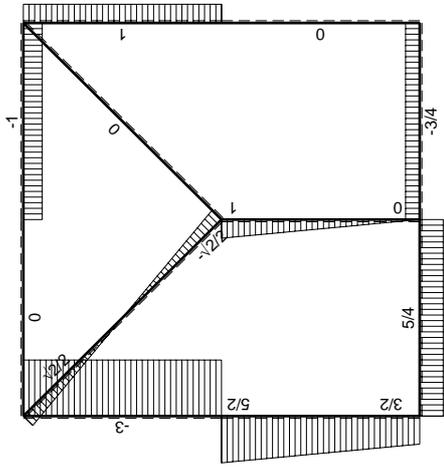
$V_F = -F$
 $H_b = -F$
 $P_{HA} = -q = -F/b$
 $P_{LB} = -q = -F/b$
 $P_{GI} = -q = -F/b$
 $q_{GI} = -q = -F/b$
 $\theta_{GH} = -\theta = -\alpha T/b = -bF/EJ$
 $EJ_{GH} = EJ$
 $EJ_{GI} = EJ$
 $EJ_{JB} = EJ$
 $EJ_{IE} = EJ$
 $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$
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 Diagrammi di carico con valori riferiti ad asse della trave.
 Componenti di carico distribuito riferiti ad assi ortogonali.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta GH positiva se convessa a destra con inizio G.
 Spostamento orizzontale assoluto u imposto al nodo H.
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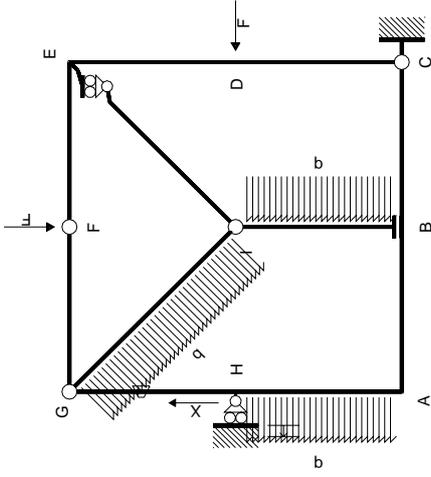




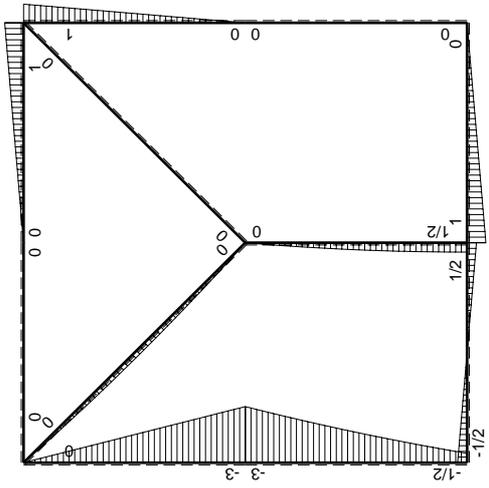
$\left[\begin{array}{c} \oplus \\ \ominus \end{array} \right] \rightarrow F$



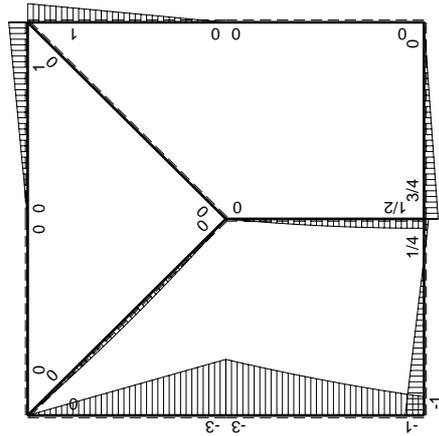
$\uparrow \left[\begin{array}{c} \oplus \\ \ominus \end{array} \right] F$



$\left(\left[\begin{array}{c} \oplus \\ \ominus \end{array} \right] \right) M_0$ flessione da carichi assegnati

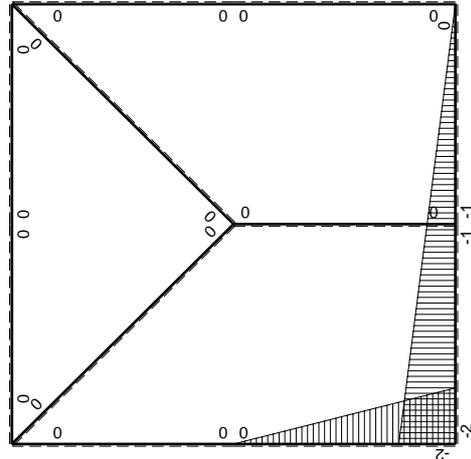


$\left(\left[\begin{array}{c} \oplus \\ \ominus \end{array} \right] \right) F_b$



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$\left(\left[\begin{array}{c} \oplus \\ \ominus \end{array} \right] \right) M_x$ flessione da iperstatica X=1



Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	$-1/2Fb+Fx$	0	$Fb^2-5/2Fbx+Fx^2$	0	$4b^2-4bx+x^2$	$(1/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$b+x$	$-1/2Fb+Fx$	0	$-1/2Fb^2+1/2Fbx+Fx^2$	0	$b^2+2bx+x^2$			
BC b	$-b+x$	$Fb-Fx$	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
CB b	x	$-Fx$	0	$-Fx^2$	0	x^2			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	$-Fb+Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-3Fx$	$-Fb/EJ$	0	0	0	0+0	0	
HG b	0	$3Fb-3Fx$	Fb/EJ	0	0	0			
GI $\sqrt{2}b$	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-2x$	$-3Fb+3Fx-1/2qx^2$	0	$6Fbx-6Fx^2+qx^3$	0	$4x^2$	$(5/4+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$1/2Fb+2Fx+1/2qx^2$	0	$Fb^2+3Fbx-3Fx^2-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$-Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							$1/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (1 - 5/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [x - 5/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (b - 5/4 b + 1/3 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 + 1/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 1/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 1/4 b + 1/3 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1 + 2x/b - x^2/b^2) Fb^2 1/EJ dx = [-x + x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-b + b - 1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-x^2/b^2) Fb^2 1/EJ dx = [-1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

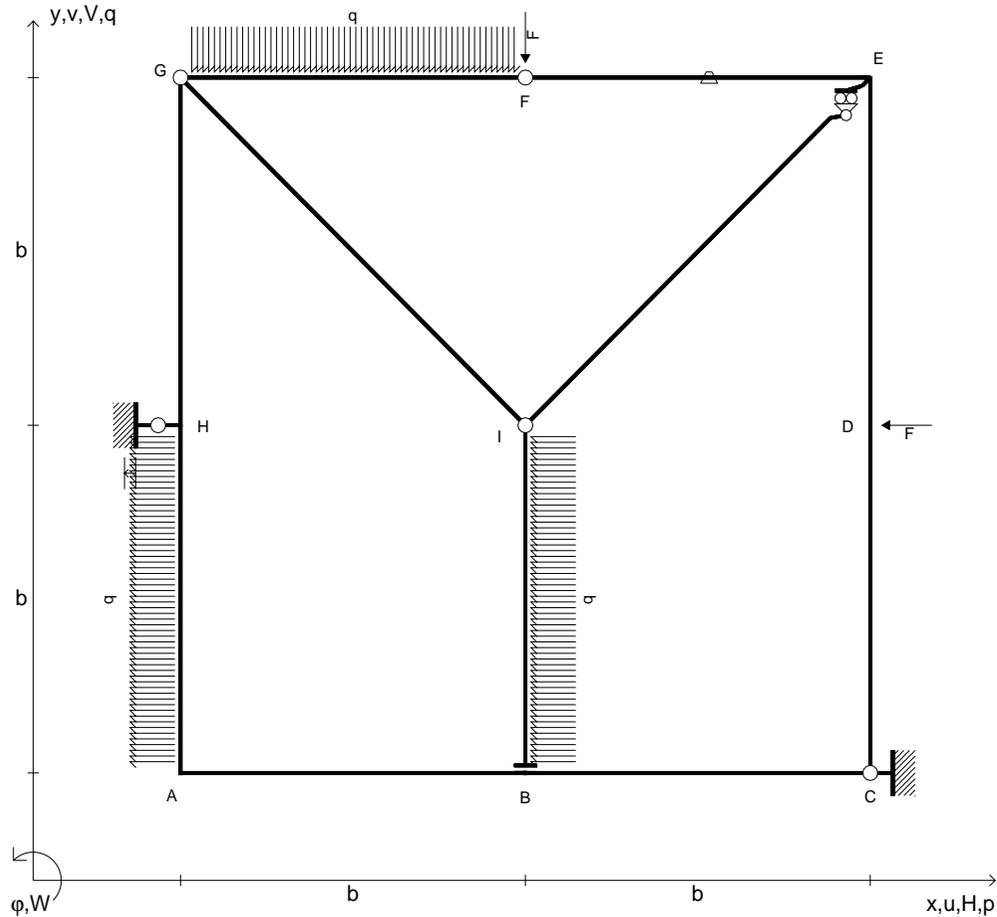
$$= (-1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (6x/b - 6x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [3x^2/b - 2x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3b - 2b + 1/4 b) Fb^2 1/EJ = 5/4 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (1 + 3x/b - 3x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [x + 3/2 x^2/b - x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (b + 3/2 b - b - 1/4 b) Fb^2 1/EJ = 5/4 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$\rho_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$\rho_{IB} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{FG} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

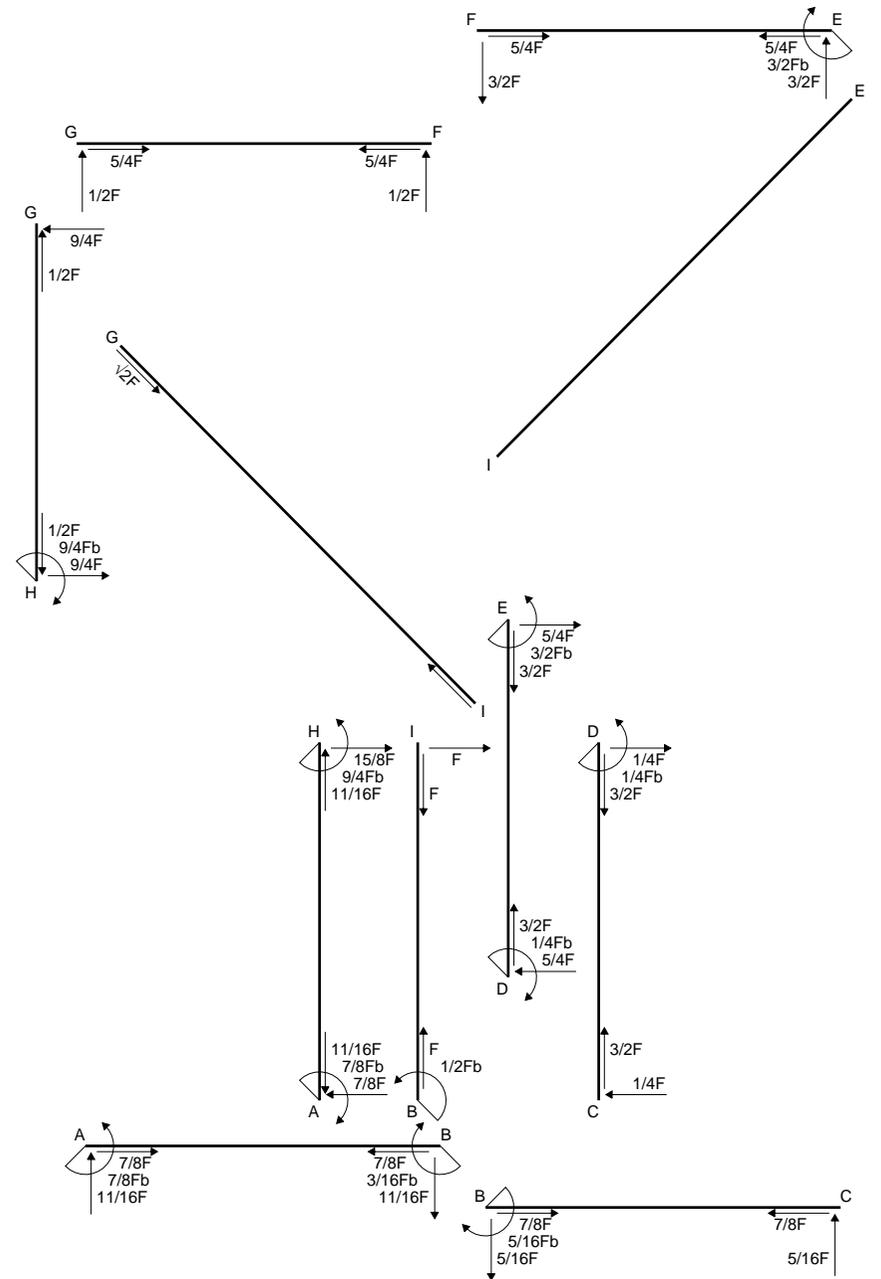
Carichi di aste curve misurati in proiezione sugli assi x,y.

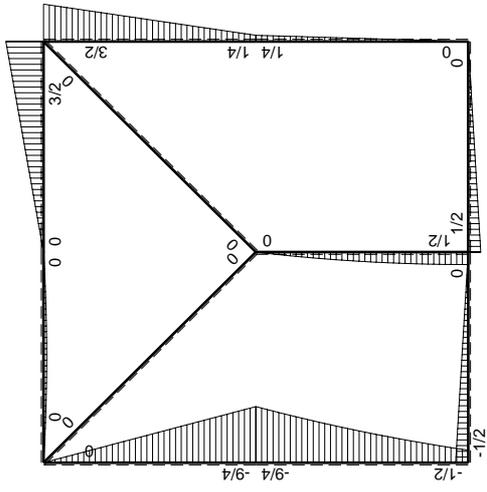
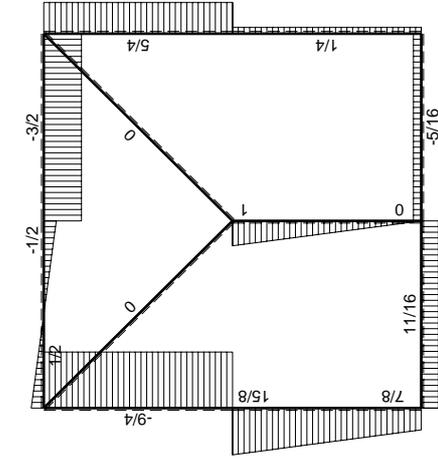
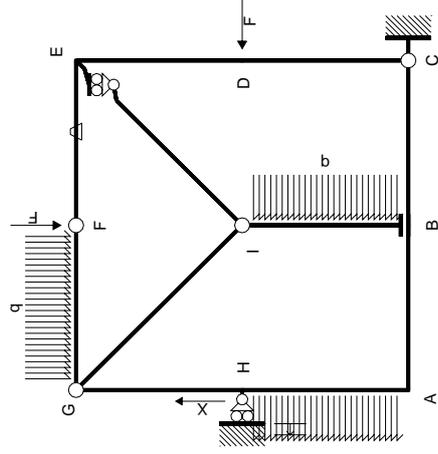
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta EF positiva se convessa a destra con inizio E.

Spostamento orizzontale assoluto u imposto al nodo H.

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

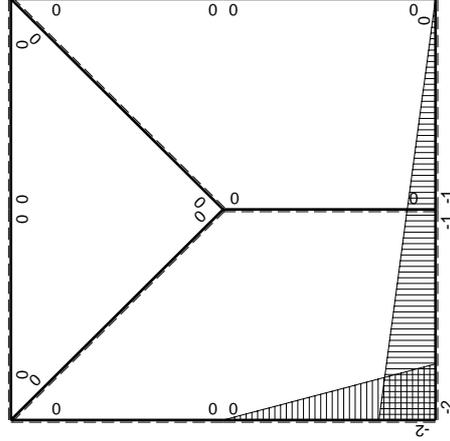
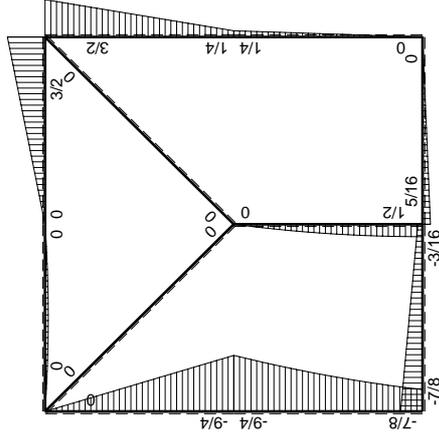
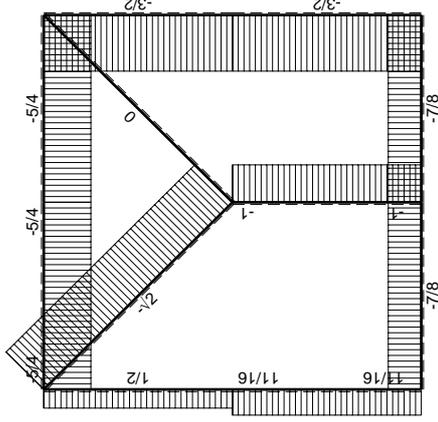




← ⊕ → F

↑ ⊕ ↓ F

⊕ M₀ flessione da carichi assegnati



⊕ F_b

⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	$-2b+x$	$-1/2Fb+1/2Fx$	0	$Fb^2-3/2Fbx+1/2Fx^2$	0	$4b^2-4bx+x^2$	$(5/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	$b+x$	$1/2Fx$	0	$1/2Fbx+1/2Fx^2$	0	$b^2+2bx+x^2$		
BC b	$-b+x$	$1/2Fb-1/2Fx$	0	$-1/2Fb^2+Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-1/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	x	$-1/2Fx$	0	$-1/2Fx^2$	0	x^2		
CD b	0	$1/4Fx$	0	0	0	0	0+0	0
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0		
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0		
EF b	0	$3/2Fb-3/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	$-3/2Fx$	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0		
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0
HA b	$-2x$	$-9/4Fb+9/4Fx-1/2qx^2$	0	$9/2Fbx-9/2Fx^2+qx^3$	0	$4x^2$	$(1+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	$2b-2x$	$1/2Fb+5/4Fx+1/2qx^2$	0	$Fb^2+3/2Fbx-3/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$-3/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						$3/16F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (1 - 3/2 x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [x - 3/4 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (b - 3/4 b + 1/6 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/4 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b + 1/6 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 1/2 b - 1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

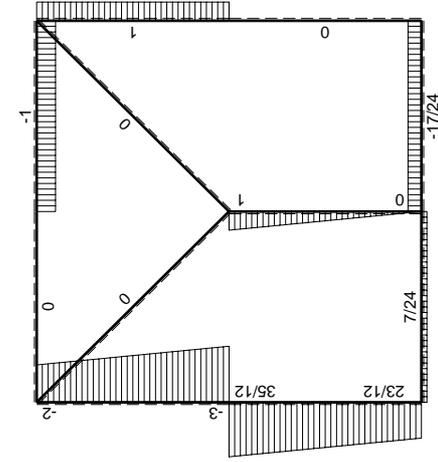
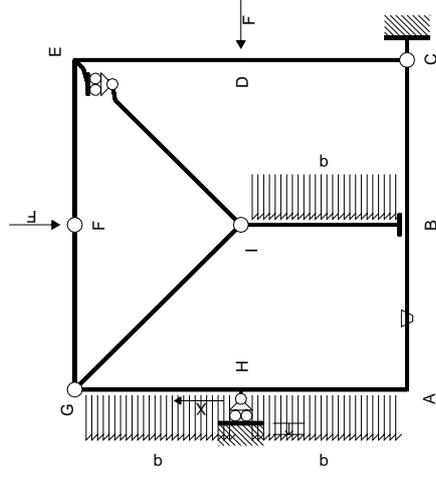
$$= (-1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (9/2 x/b - 9/2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [9/4 x^2/b - 3/2 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

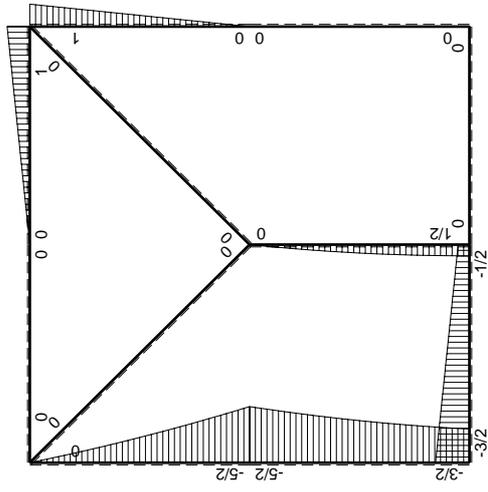
$$= (9/4 b - 3/2 b + 1/4 b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (1 + 3/2 x/b - 3/2 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [x + 3/4 x^2/b - 1/2 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

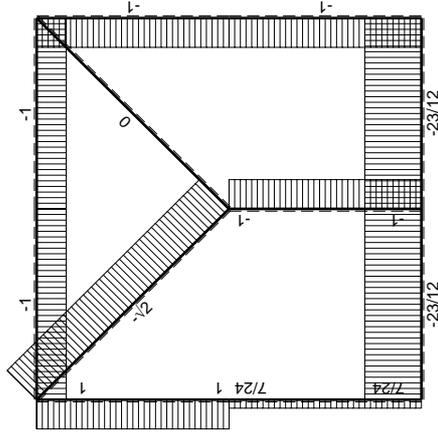
$$= (b + 3/4 b - 1/2 b - 1/4 b) Fb^2 1/EJ = Fb^3/EJ$$



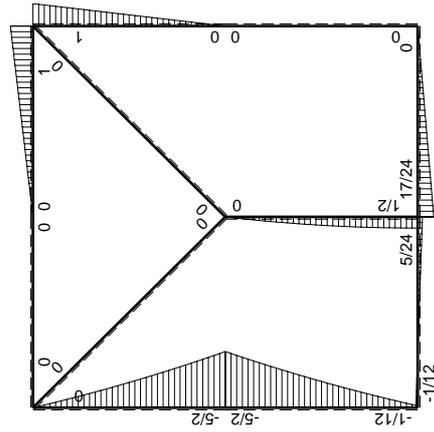
$\left[\begin{matrix} + \\ - \end{matrix} \right] F$



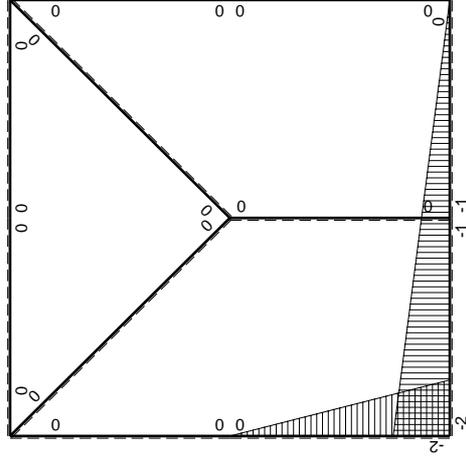
$\left[\begin{matrix} + \\ - \end{matrix} \right] M_y$ flessione da carichi assegnati



$\left[\begin{matrix} + \\ - \end{matrix} \right] F_b$



$\left[\begin{matrix} + \\ - \end{matrix} \right] F_x$



$\left[\begin{matrix} + \\ - \end{matrix} \right] M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	$-3/2Fb+Fx$	$-Fb/EJ$	$3Fb^2-7/2Fbx+Fx^2$	$2Fb^2/EJ-Fxb/EJ$	$4b^2-4bx+x^2$	$(19/12+3/2)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$b+x$	$1/2Fb+Fx$	Fb/EJ	$1/2Fb^2+3/2Fbx+Fx^2$	$Fb^2/EJ+Fxb/EJ$	$b^2+2bx+x^2$			
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	1/3Xb ³ /EJ	
CB b	x	0	0	0	0	x^2			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	$-Fb+Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-2x$	$-5/2Fb+3/2Fx-1/2qx^2$	0	$5Fbx-3Fx^2+qx^3$	0	$4x^2$	$(7/4+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$3/2Fb+1/2Fx+1/2qx^2$	0	$3Fb^2-2Fbx-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$		
	totali						$17/6Fb^3/EJ$	$4Xb^3/EJ$	
	iperstatica $X=V_H$						$-17/24F$		

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (3 - 7/2 x/b + x^2/b^2) Fb^2 1/EJ dx + \int_0^b (2 - x/b) \theta dx$$

$$= [3x - 7/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ + [2x - 1/2 x^2/b]_0^b \theta$$

$$= (3b - 7/4 b + 1/3 b) Fb^2 1/EJ + (2b - 1/2 b) \theta = 37/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 + 3/2 x/b + x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1 - x/b) \theta dx$$

$$= [1/2 x + 3/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ + [-x - 1/2 x^2/b]_0^b \theta$$

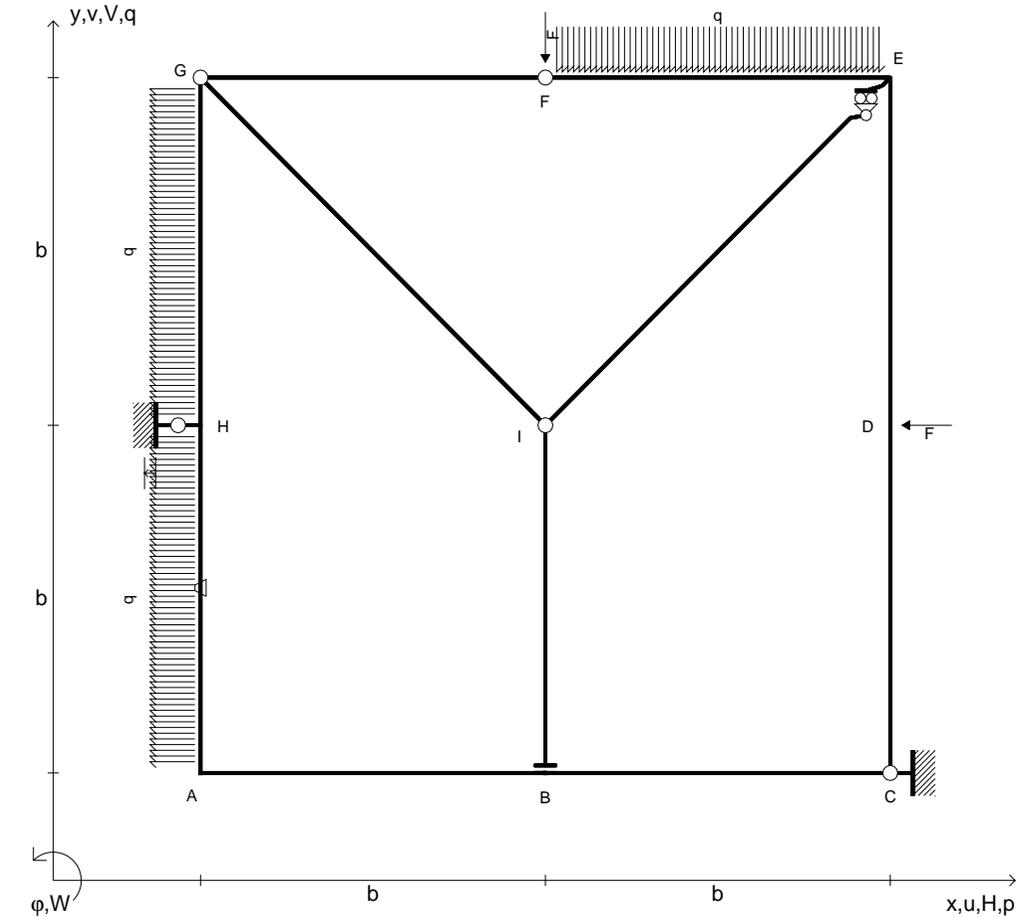
$$= (1/2 b + 3/4 b + 1/3 b) Fb^2 1/EJ + (-b - 1/2 b) \theta = 37/12 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (5x/b - 3x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [5/2 x^2/b - x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (5/2 b - b + 1/4 b) Fb^2 1/EJ = 7/4 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (3 - 2x/b - x^3/b^3) Fb^2 1/EJ dx = [3x - x^2/b - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

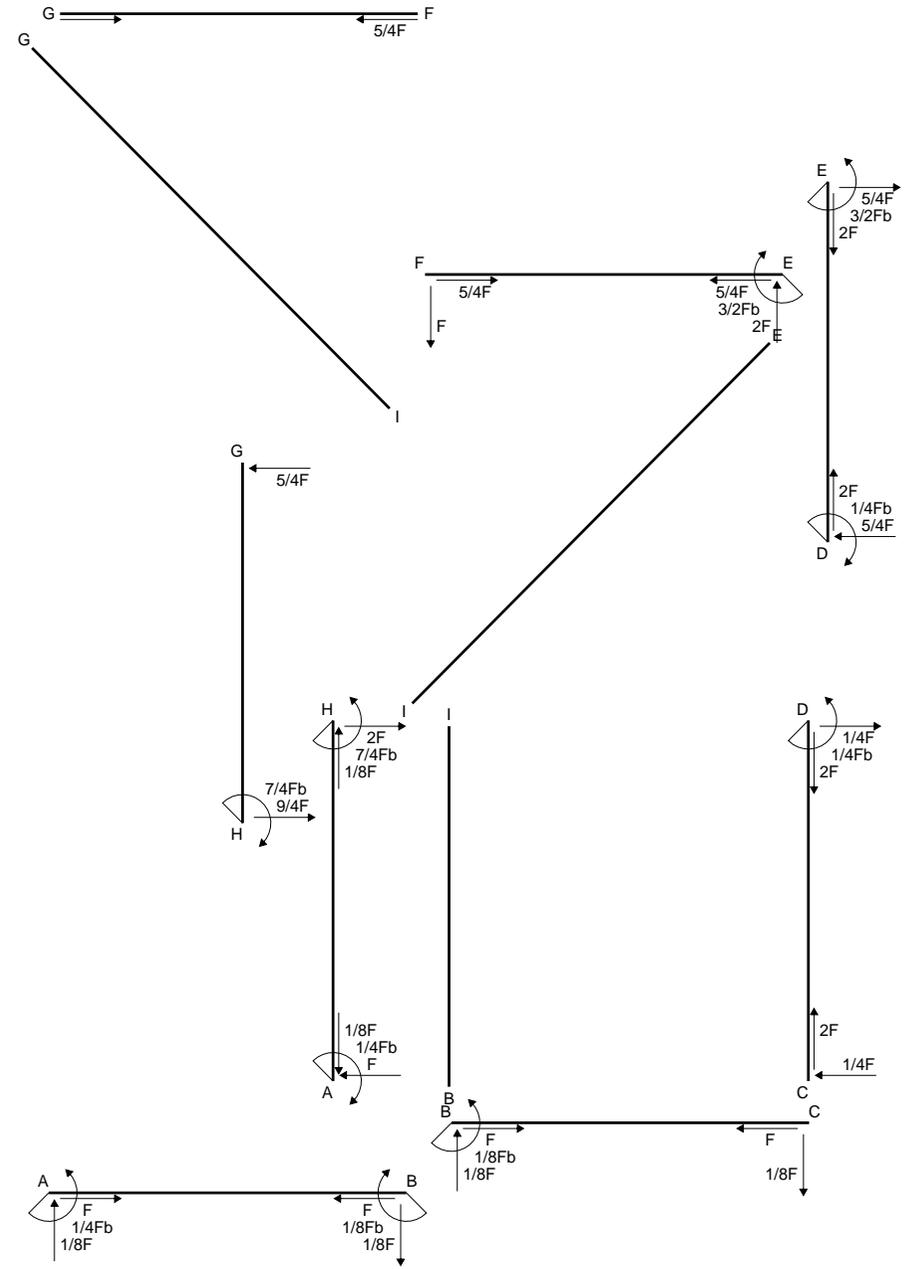
$$= (3b - b - 1/4 b) Fb^2 1/EJ = 7/4 Fb^3/EJ$$

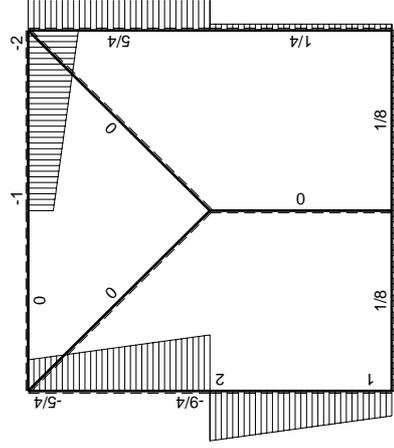
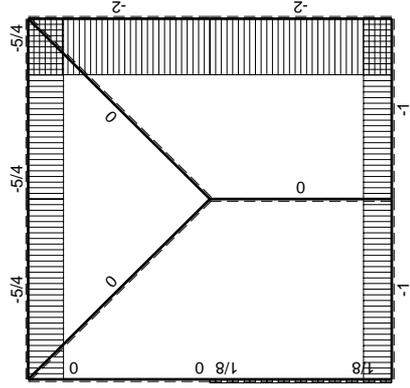
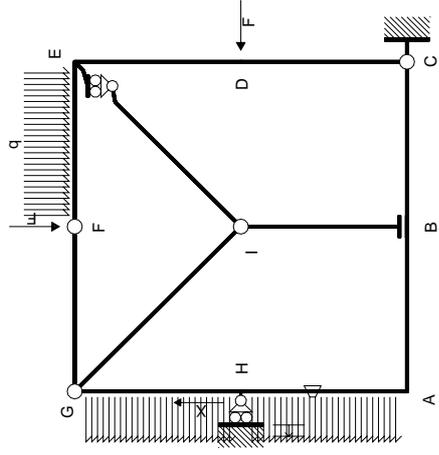


$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$\rho_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$\rho_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{HA} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

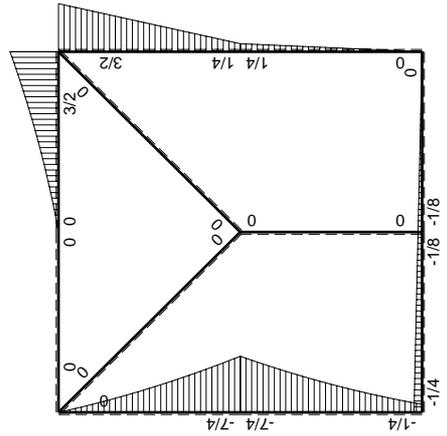
Carichi e deformazioni date hanno verso efficace in disegno.
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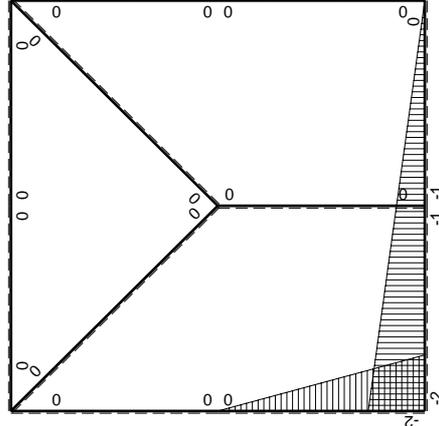


$\left[\begin{array}{c} + \\ - \end{array} \right]$ F

$\left[\begin{array}{c} + \\ - \end{array} \right]$ F



$\left[\begin{array}{c} + \\ - \end{array} \right]$ M_0 flessione da carichi assegnati



$\left[\begin{array}{c} + \\ - \end{array} \right]$ Fb

$\left[\begin{array}{c} + \\ - \end{array} \right]$ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	-2b+x	0	0	0	0	$4b^2-4bx+x^2$	0+0	$7/3Xb^3/EJ$	
BA b	b+x	0	0	0	0	$b^2+2bx+x^2$			
BC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$	
CB b	x	0	0	0	0	x^2			
CD b	0	1/4Fx	0	0	0	0	0+0	0	
DC b	0	-1/4Fb+1/4Fx	0	0	0	0			
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0	
ED b	0	-3/2Fb+5/4Fx	0	0	0	0			
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-Fx-1/2qx^2$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	-2x	$-7/4Fb+9/4Fx-1/2qx^2$	-Fb/EJ	$7/2Fbx-9/2Fx^2+qx^3$	2Fxb/EJ	$4x^2$	$(1/2+1)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	2b-2x	$5/4Fx+1/2qx^2$	Fb/EJ	$5/2Fbx-3/2Fx^2-qx^3$	$2Fb^2/EJ-2Fxb/EJ$	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$-1/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							1/8F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (7/2 x/b - 9/2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx + \int_0^b (2x/b) \theta dx$$

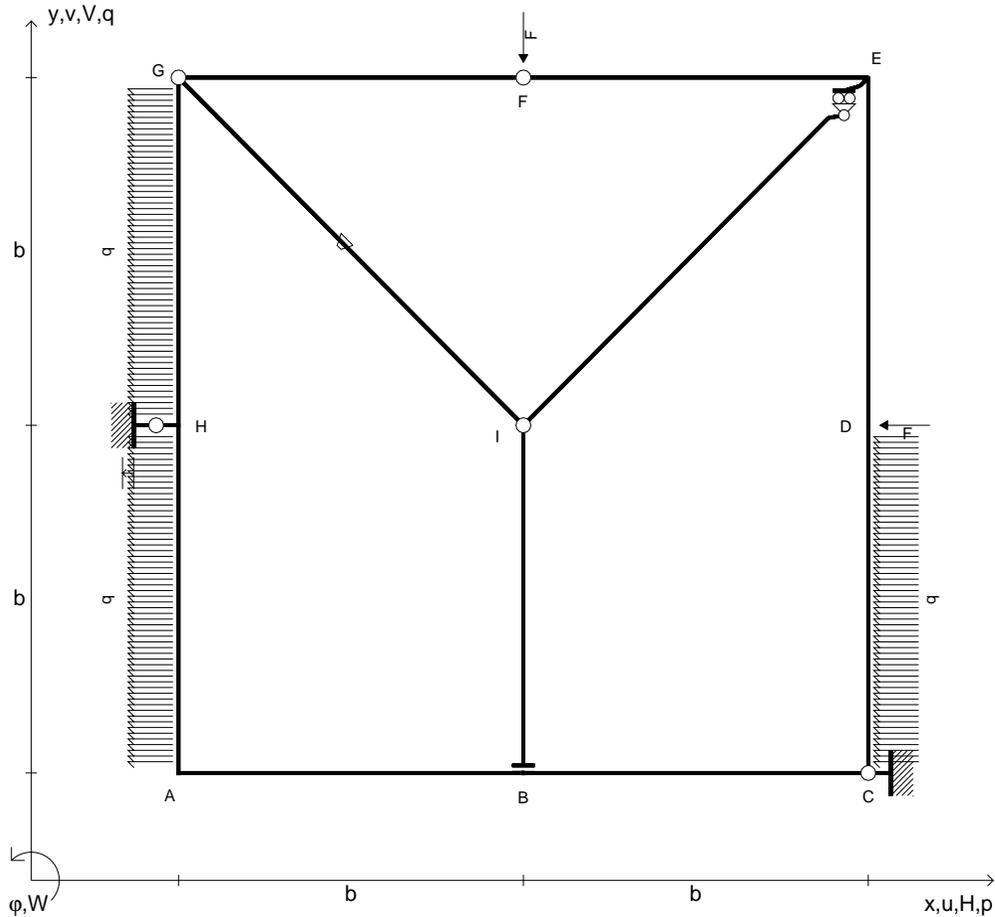
$$= [7/4 x^2/b - 3/2 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ + [x^2/b]_0^b \theta$$

$$= (7/4 b - 3/2 b + 1/4 b) Fb^2 1/EJ + (b) \theta = 3/2 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (5/2 x/b - 3/2 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-2 + 2x/b) \theta dx$$

$$= [5/4 x^2/b - 1/2 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ + [-2x + x^2/b]_0^b \theta$$

$$= (5/4 b - 1/2 b - 1/4 b) Fb^2 1/EJ + (-2b + b) \theta = 3/2 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$p_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$p_{CD} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{GI} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

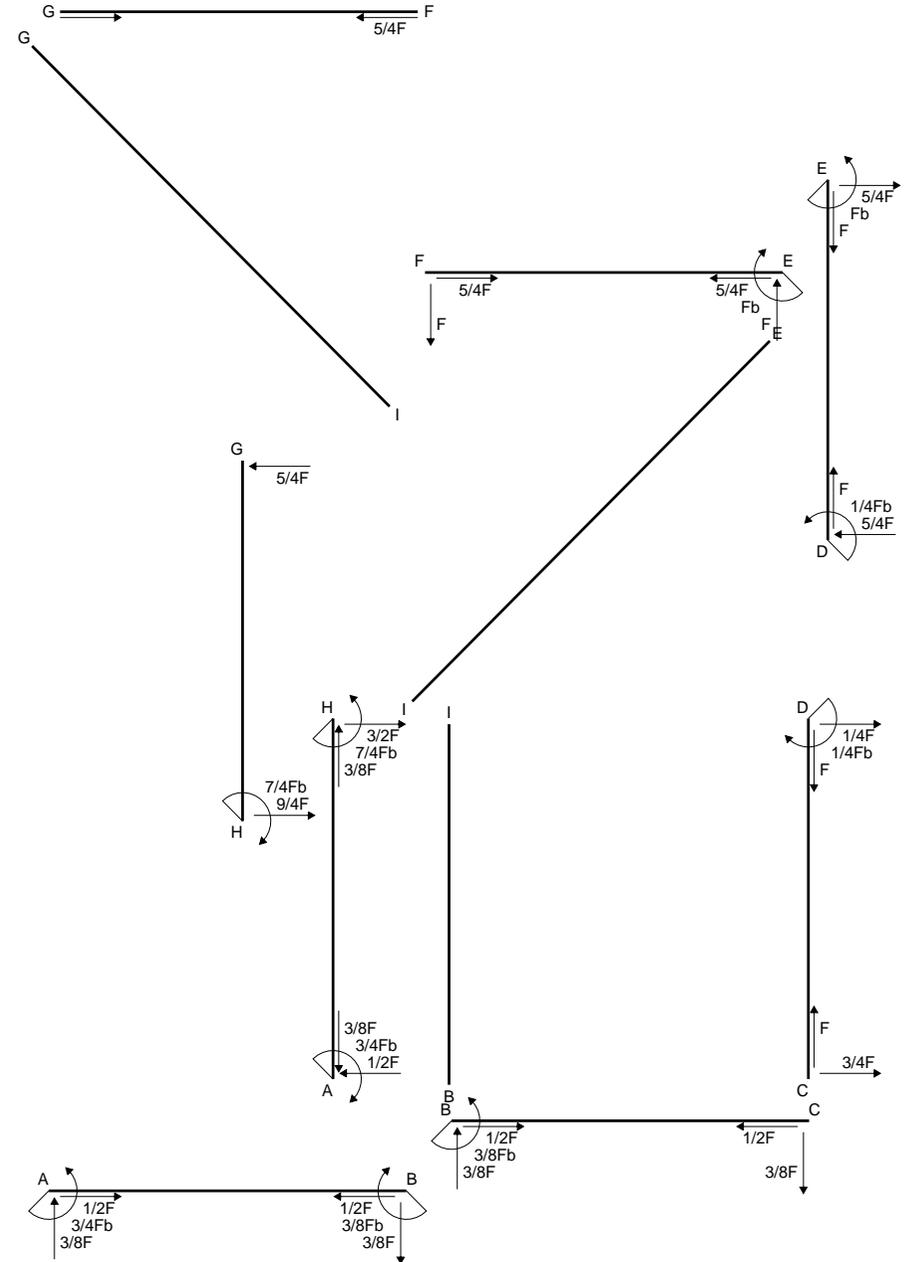
Carichi di aste curve misurati in proiezione sugli assi x,y.

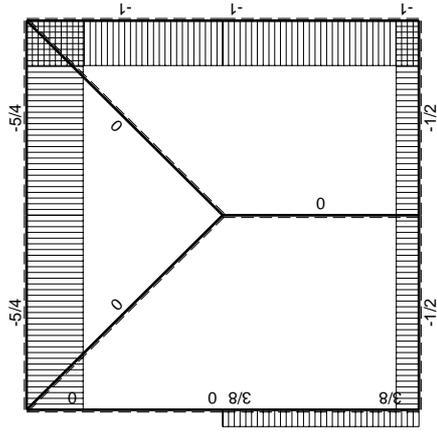
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta GI positiva se convessa a destra con inizio G.

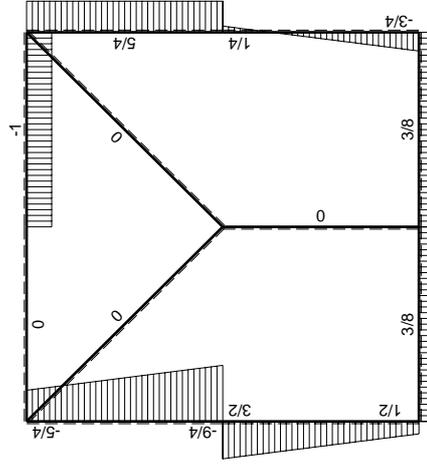
Spostamento orizzontale assoluto u imposto al nodo H.

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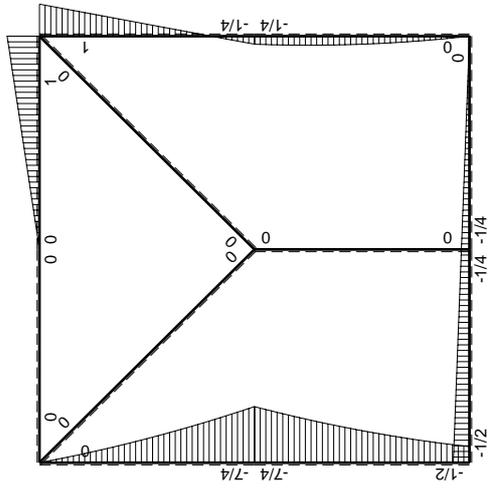
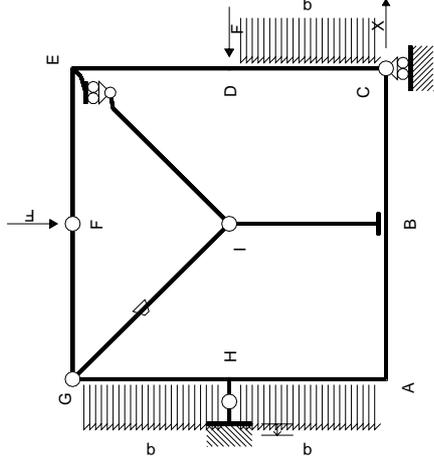


$\left[\begin{array}{c} + \\ - \end{array} \right] \rightarrow F$

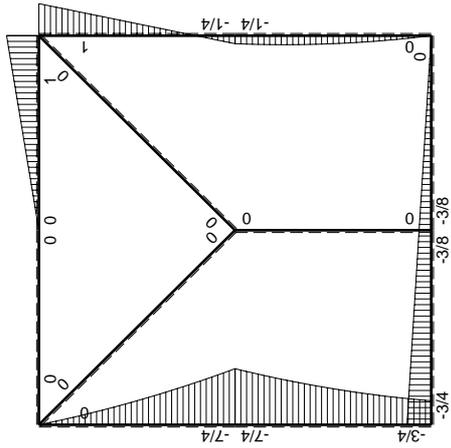


$\left[\begin{array}{c} + \\ - \end{array} \right] \rightarrow F$

$\left[\begin{array}{c} + \\ - \end{array} \right] F_b$



$\left[\begin{array}{c} + \\ - \end{array} \right] M_0$ flessione da carichi assegnati



$\left[\begin{array}{c} + \\ - \end{array} \right] M_x$ flessione da iperstatica $X=1$

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	$-b+1/2x$	$-1/2Fb+1/4Fx$	0	$1/2Fb^2-1/2Fbx+1/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
BA b	$1/2b+1/2x$	$1/4Fb+1/4Fx$	0	$1/8Fb^2+1/4Fbx+1/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
BC b	$-1/2b+1/2x$	$-1/4Fb+1/4Fx$	0	$1/8Fb^2-1/4Fbx+1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
CB b	$1/2x$	$1/4Fx$	0	$1/8Fx^2$	0	$1/4x^2$			
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0	
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0			
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-Fb+5/4Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	$-Fb/EJ$	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x$	$-7/4Fb+7/4Fx-1/2qx^2$	0	$7/4Fbx-7/4Fx^2+1/2qx^3$	0	x^2	$(5/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
AH b	$b-x$	$1/2Fb+3/4Fx+1/2qx^2$	0	$1/2Fb^2+1/4Fbx-1/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^3/EJ$	
	totali							$-1/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$							$1/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 - 1/2 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/4 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/4 b + 1/24 b) Fb^2 1/EJ = 7/24 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/8 + 1/4 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x + 1/8 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b + 1/8 b + 1/24 b) Fb^2 1/EJ = 7/24 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/8 - 1/4 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x - 1/8 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b - 1/8 b + 1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/8 x^2/b^2) Fb^2 1/EJ dx = [1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

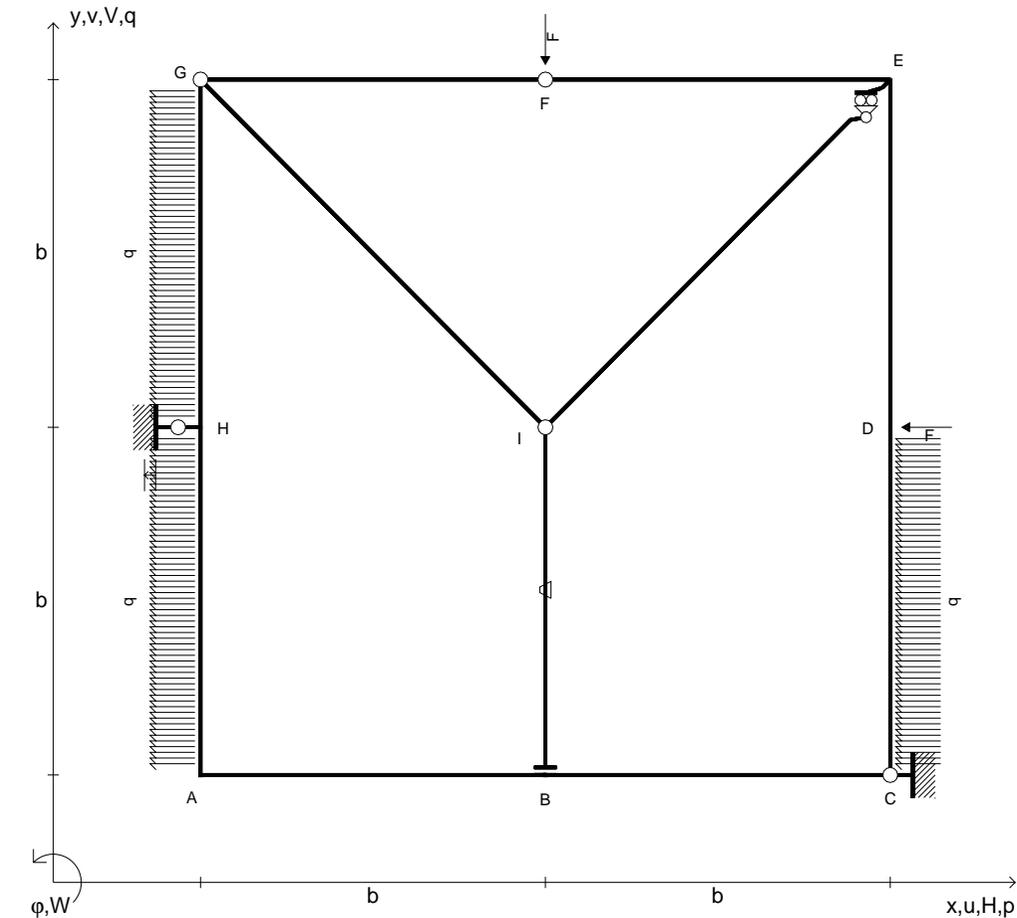
$$L_{HA}^{xo} = \int_0^b (7/4 x/b - 7/4 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [7/8 x^2/b - 7/12 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (7/8 b - 7/12 b + 1/8 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (1/2 + 1/4 x/b - 1/4 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [1/2 x + 1/8 x^2/b - 1/12 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (1/2 b + 1/8 b - 1/12 b - 1/8 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$p_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$p_{CD} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{IB} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

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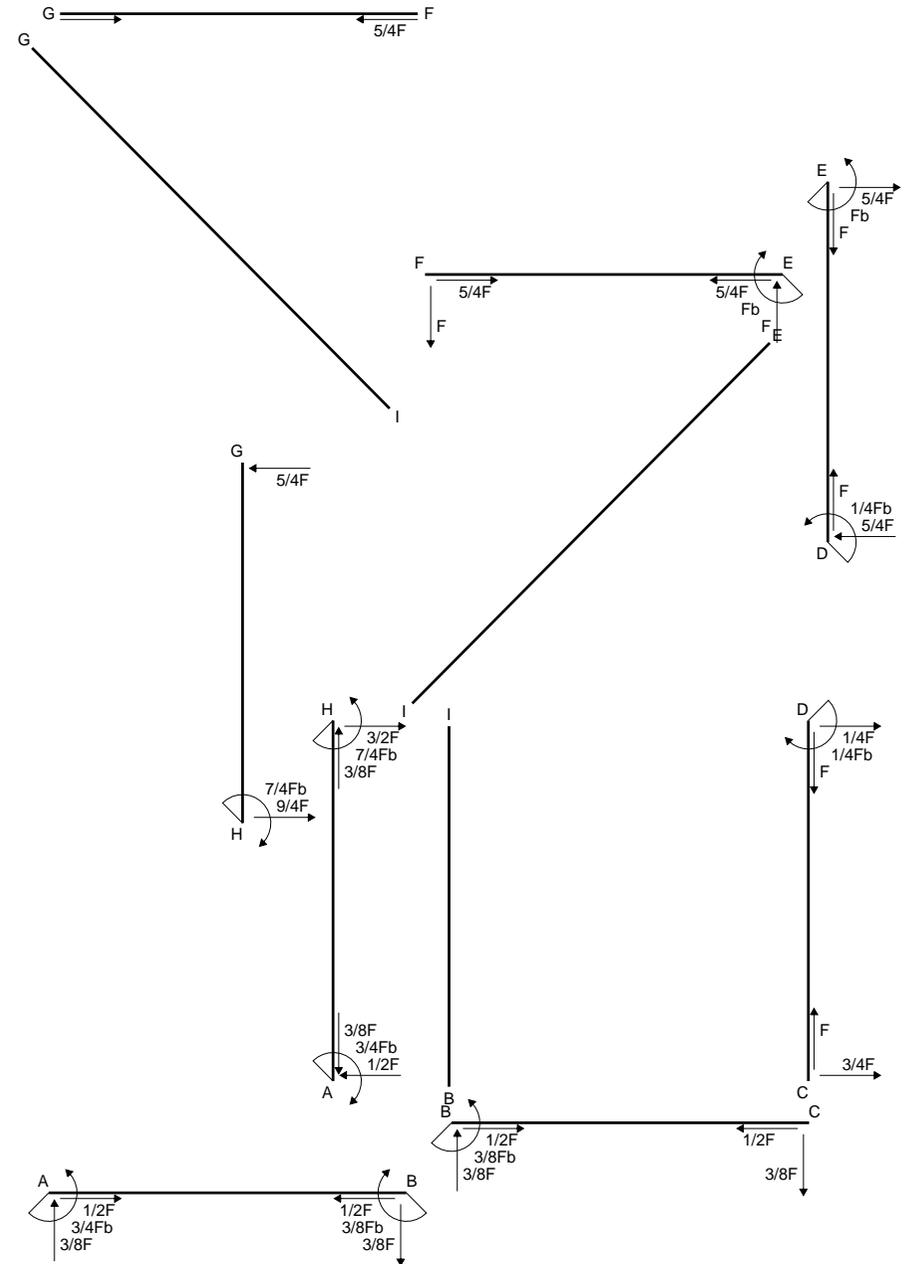
Carichi di aste curve misurati in proiezione sugli assi x,y.

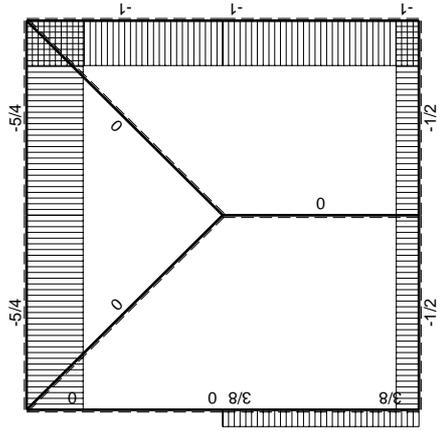
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta IB positiva se convessa a destra con inizio I.

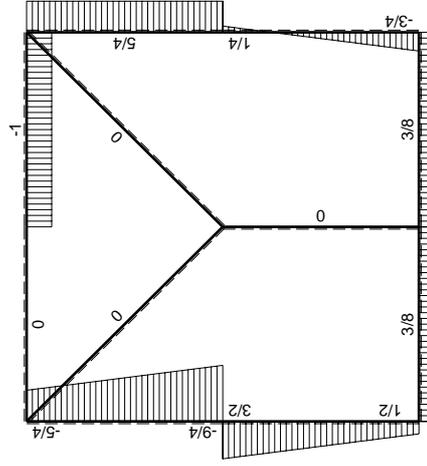
Spostamento orizzontale assoluto u imposto al nodo H.

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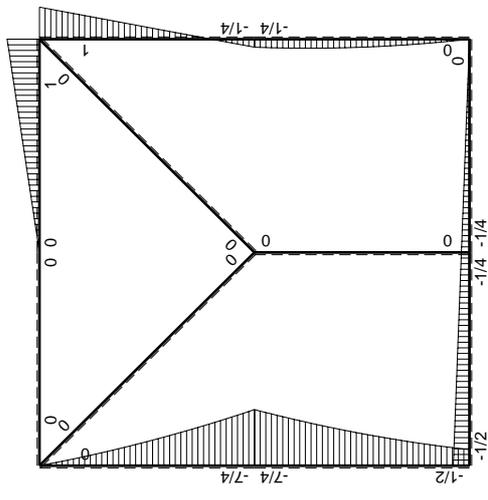
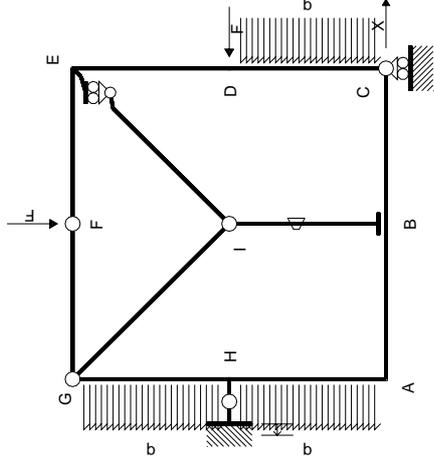


$\left[\begin{array}{c} + \\ - \end{array} \right] \rightarrow F$

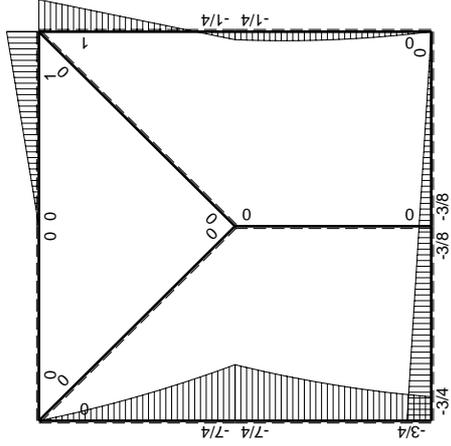


$\left[\begin{array}{c} + \\ - \end{array} \right] \rightarrow F$

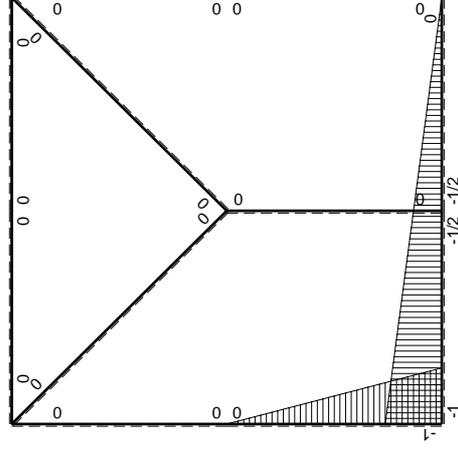
$\left[\begin{array}{c} + \\ - \end{array} \right] F_b$



$\left[\begin{array}{c} + \\ - \end{array} \right] M_0$ flessione da carichi assegnati



$\left[\begin{array}{c} + \\ - \end{array} \right] M_x$ flessione da iperstatica $X=1$



Quadro contributi PLV per iperstatica X=H_C

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫xM _x M _x /EJdx	
AB b	-b+1/2x	-1/2Fb+1/4Fx	0	1/2Fb ² -1/2Fbx+1/8Fx ²	0	b ² -bx+1/4x ²	(7/24+0)Fb ³ /EJ	7/12Xb ³ /EJ	
BA b	1/2b+1/2x	1/4Fb+1/4Fx	0	1/8Fb ² +1/4Fbx+1/8Fx ²	0	1/4b ² +1/2bx+1/4x ²			
BC b	-1/2b+1/2x	-1/4Fb+1/4Fx	0	1/8Fb ² -1/4Fbx+1/8Fx ²	0	1/4b ² -1/2bx+1/4x ²	(1/24+0)Fb ³ /EJ	1/12Xb ³ /EJ	
CB b	1/2x	1/4Fx	0	1/8Fx ²	0	1/4x ²			
CD b	0	-3/4Fx+1/2qx ²	0	0	0	0	0+0	0	
DC b	0	1/4Fb+1/4Fx-1/2qx ²	0	0	0	0			
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0	
ED b	0	-Fb+5/4Fx	0	0	0	0			
EF b	0	Fb-Fx	0	0	0	0	0+0	0	
FE b	0	-Fx	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	-5/4Fx-1/2qx ²	0	0	0	0	0+0	0	
HG b	0	7/4Fb-9/4Fx+1/2qx ²	0	0	0	0			
GI √2b	0	0	0	0	0	0	0	0	
IB b	0	0	-Fb/EJ	0	0	0	0+0	0	
BI b	0	0	Fb/EJ	0	0	0			
IE √2b	0	0	0	0	0	0	0	0	
HA b	-x	-7/4Fb+7/4Fx-1/2qx ²	0	7/4Fbx-7/4Fx ² +1/2qx ³	0	x ²	(5/12+0)Fb ³ /EJ	1/3Xb ³ /EJ	
AH b	b-x	1/2Fb+3/4Fx+1/2qx ²	0	1/2Fb ² +1/4Fbx-1/4Fx ² -1/2qx ³	0	b ² -2bx+x ²			
H	cedimento nodo -H _{1H} u _H							-Fb ³ /EJ	
	totali							-1/4Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _C							1/4F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 - 1/2 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/4 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/4 b + 1/24 b) Fb^2 1/EJ = 7/24 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/8 + 1/4 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x + 1/8 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b + 1/8 b + 1/24 b) Fb^2 1/EJ = 7/24 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/8 - 1/4 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x - 1/8 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b - 1/8 b + 1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/8 x^2/b^2) Fb^2 1/EJ dx = [1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

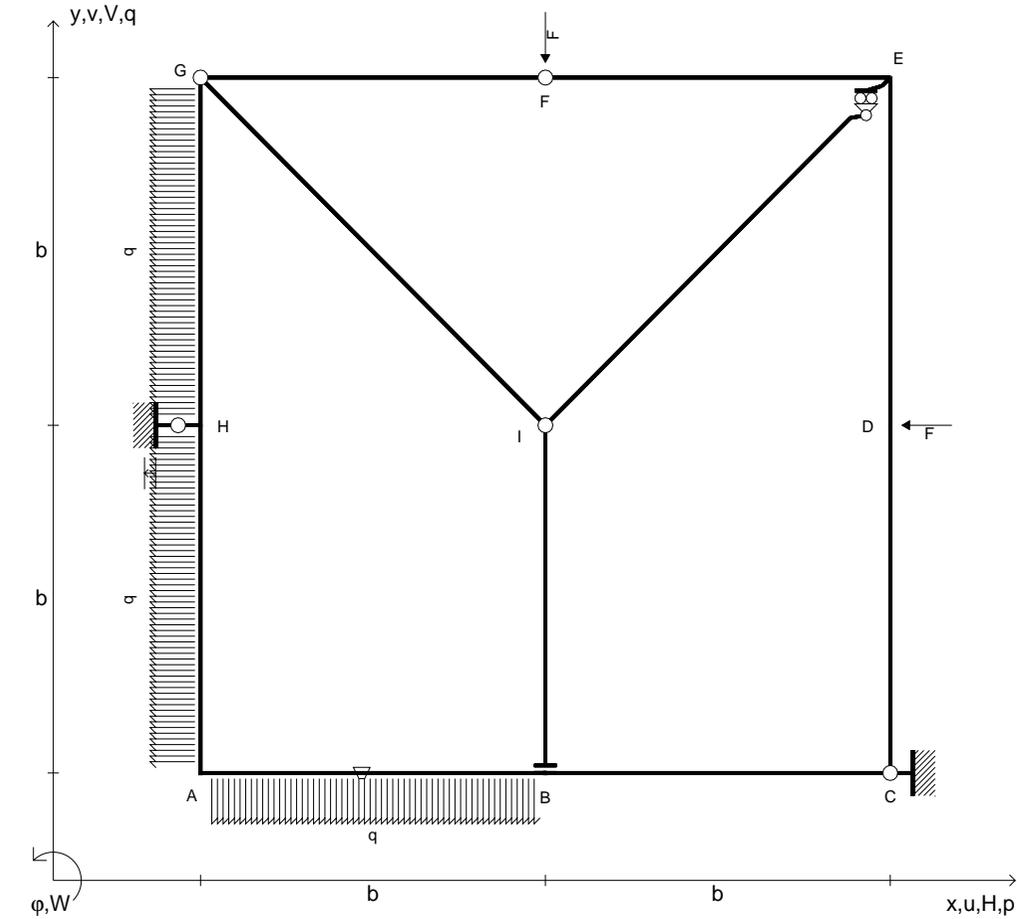
$$L_{HA}^{xo} = \int_0^b (7/4 x/b - 7/4 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [7/8 x^2/b - 7/12 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (7/8 b - 7/12 b + 1/8 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (1/2 + 1/4 x/b - 1/4 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [1/2 x + 1/8 x^2/b - 1/12 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (1/2 b + 1/8 b - 1/12 b - 1/8 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$\rho_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$\rho_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{AB} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

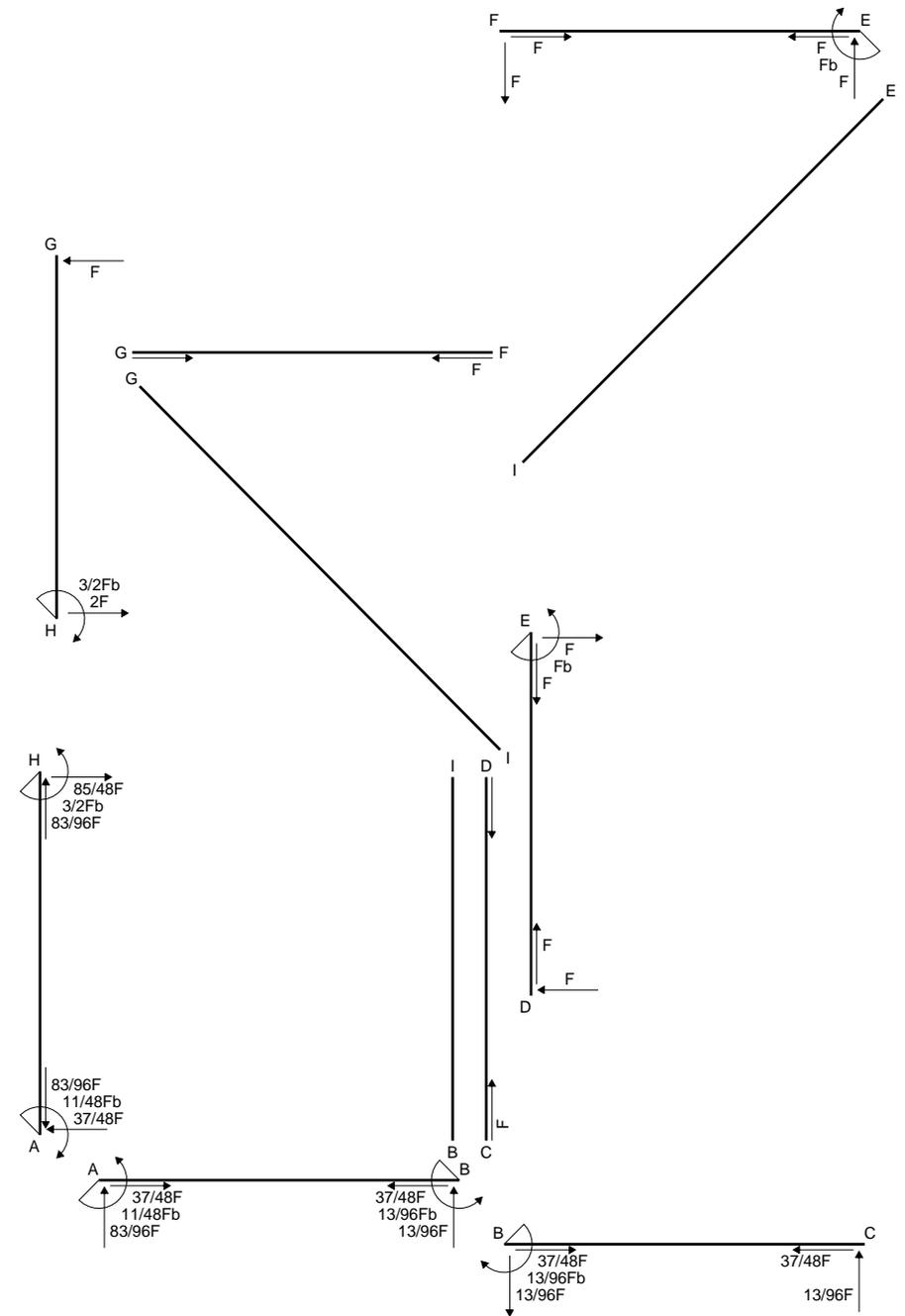
Carichi di aste curve misurati in proiezione sugli assi x,y.

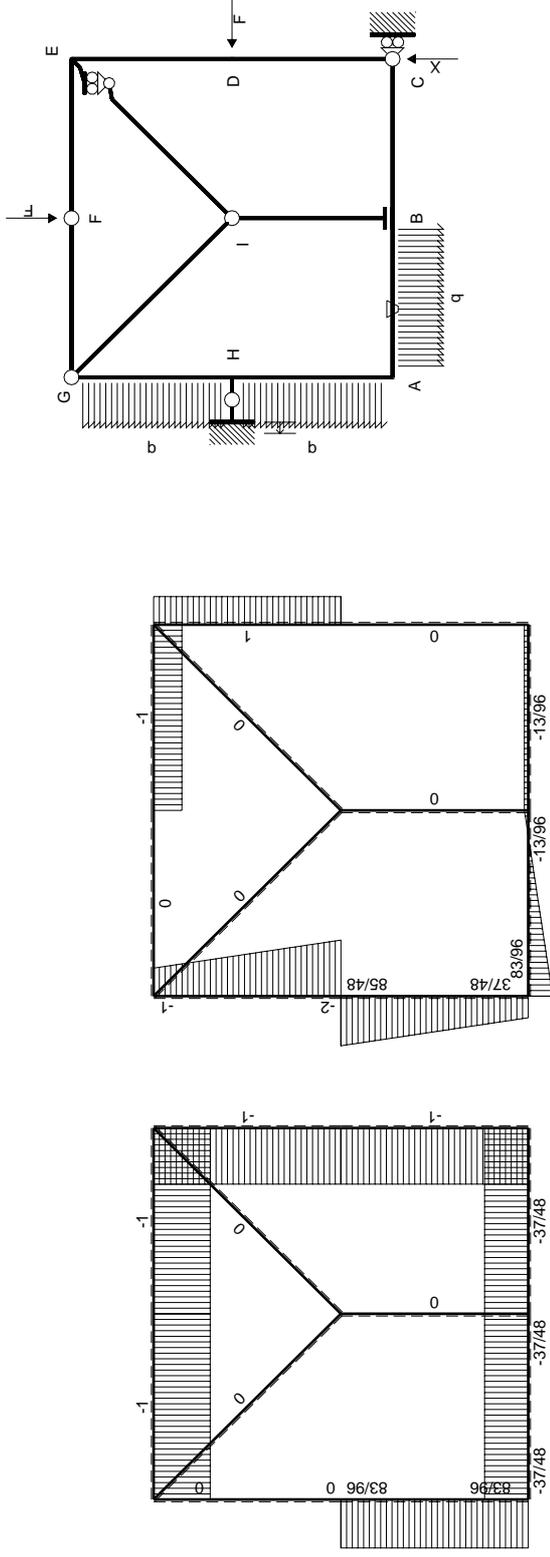
$J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta AB positiva se convessa a destra con inizio A.

Spostamento orizzontale assoluto u imposto al nodo H.

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



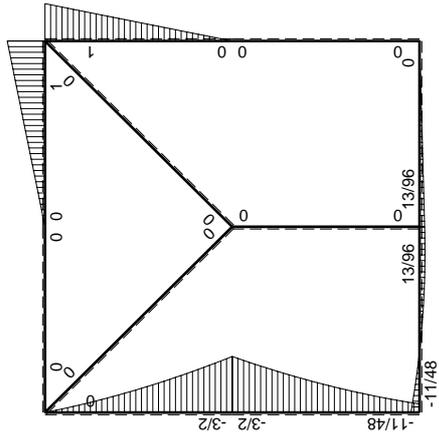


$\left[\begin{array}{c} \square \\ + \\ \square \end{array} \right] \rightarrow F$

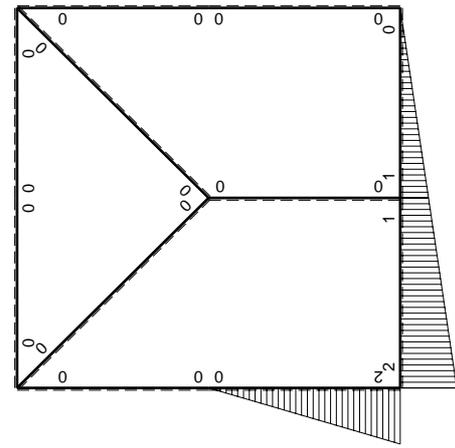
$\uparrow \left[\begin{array}{c} \square \\ + \\ \square \end{array} \right] F$

Schema di calcolo iperstatico

$\left(\left[\begin{array}{c} \square \\ + \\ \square \end{array} \right] \right) M_0$ flessione da carichi assegnati



$\left(\left[\begin{array}{c} \square \\ + \\ \square \end{array} \right] \right) F_b$



$\left(\left[\begin{array}{c} \square \\ + \\ \square \end{array} \right] \right) M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$	
AB b	$2b-x$	$-5/2Fb+2Fx-1/2qx^2$	$-Fb/EJ$	$-5Fb^2+13/2Fbx-3Fx^2+1/2qx^3$	$-2Fb^2/EJ+Fx/EJ$	$4b^2-4bx+x^2$	$(-21/8-3/2)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$-b-x$	$Fb+Fx+1/2qx^2$	Fb/EJ	$-Fb^2-2Fbx-3/2Fx^2-1/2qx^3$	$-Fb^2/EJ-Fx/EJ$	$b^2+2bx+x^2$			
BC b	$b-x$	$-Fb+Fx$	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
CB b	$-x$	Fx	0	$-Fx^2$	0	x^2			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	$-Fb+Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$2x$	$-3/2Fb-1/2Fx-1/2qx^2$	0	$-3Fbx-Fx^2-qx^3$	0	$4x^2$	$(-25/12+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$-2b+2x$	$5/2Fb-3/2Fx+1/2qx^2$	0	$-5Fb^2+8Fbx-4Fx^2+qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$2Fb^3/EJ$	
	totali							$-109/24Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_C$							$109/96F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-5 + 13/2 x/b - 3x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-2 + x/b) \theta dx$$

$$= [-5x + 13/4 x^2/b - x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [-2x + 1/2 x^2/b]_0^b \theta$$

$$= (-5b + 13/4 b - b + 1/8 b) Fb^2 1/EJ + (-2b + 1/2 b) \theta = -33/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 - 2x/b - 3/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (1 + x/b) \theta dx$$

$$= [-x - x^2/b - 1/2 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [x + 1/2 x^2/b]_0^b \theta$$

$$= (-b - b - 1/2 b - 1/8 b) Fb^2 1/EJ + (b + 1/2 b) \theta = -33/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1 + 2x/b - x^2/b^2) Fb^2 1/EJ dx = [-x + x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-b + b - 1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-x^2/b^2) Fb^2 1/EJ dx = [-1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

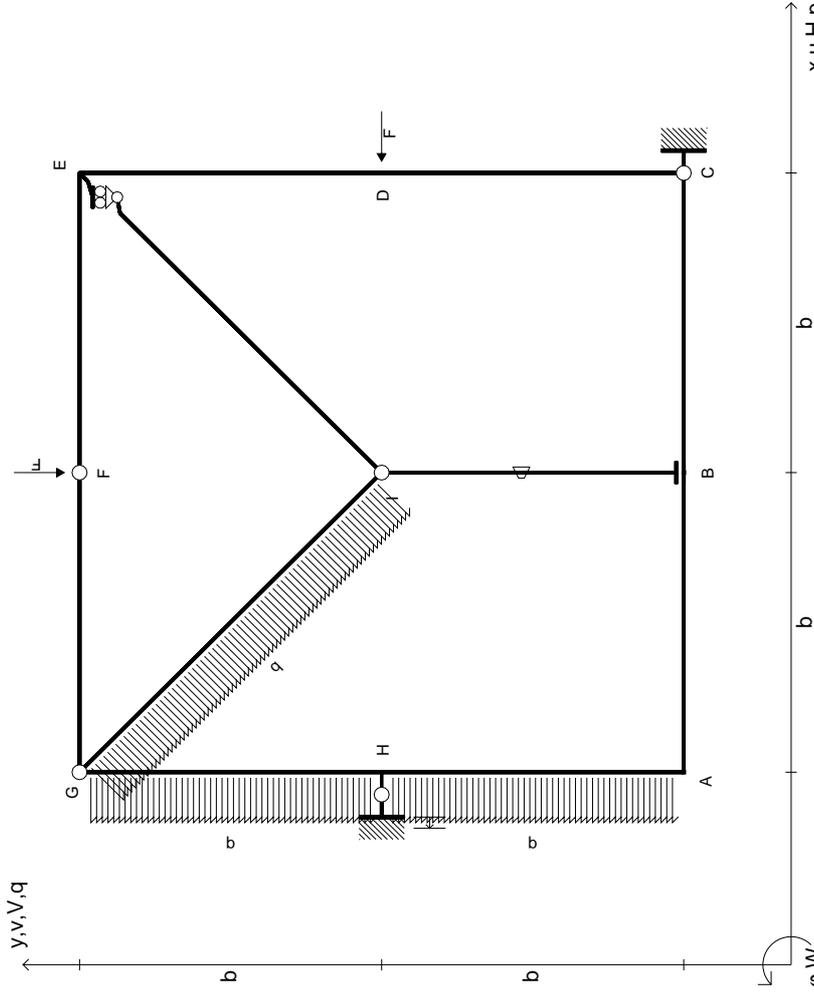
$$= (-1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (-3x/b - x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [-3/2 x^2/b - 1/3 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-3/2 b - 1/3 b - 1/4 b) Fb^2 1/EJ = -25/12 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (-5 + 8x/b - 4x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [-5x + 4x^2/b - 4/3 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-5b + 4b - 4/3 b + 1/4 b) Fb^2 1/EJ = -25/12 Fb^3/EJ$$

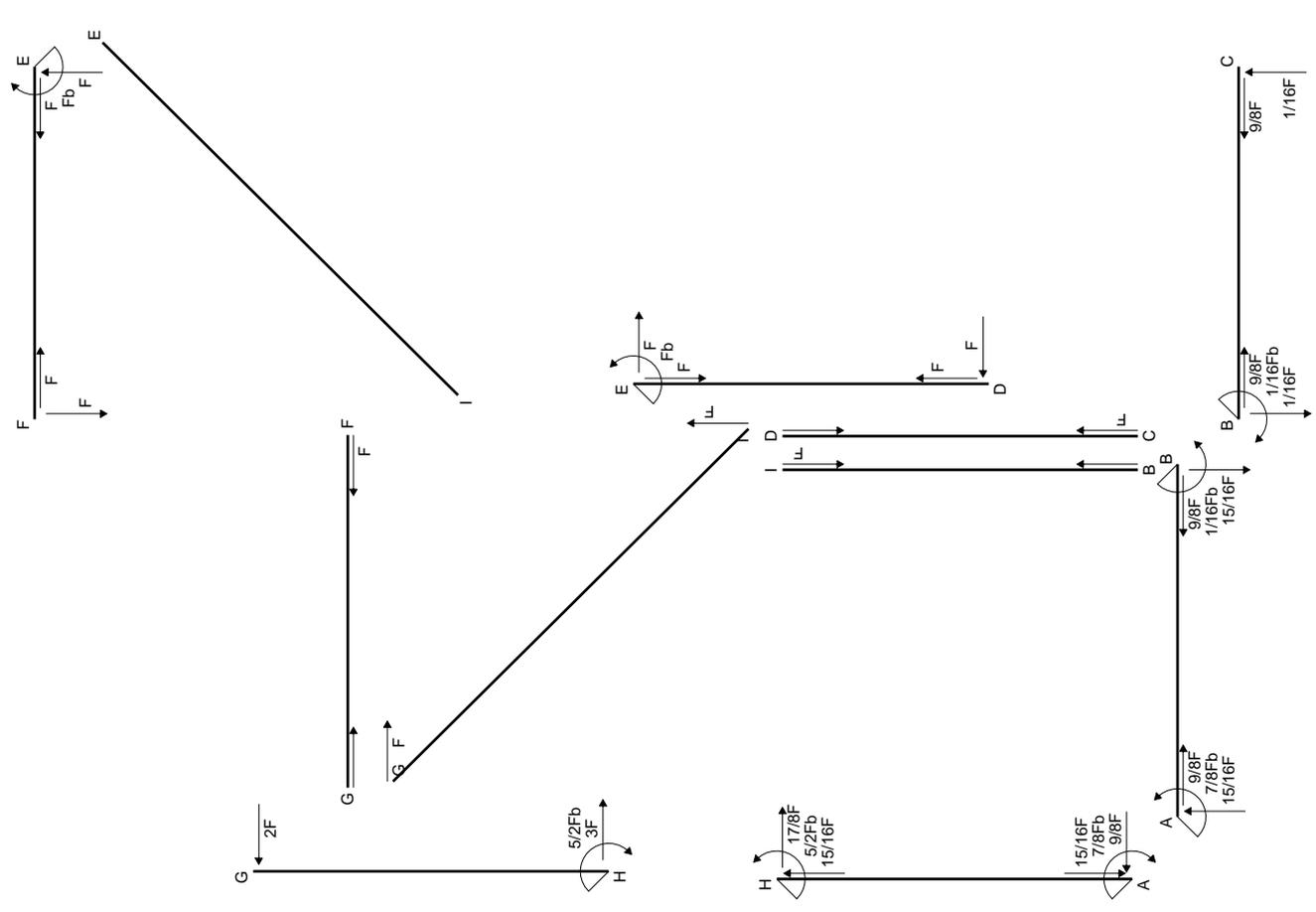


$$\begin{aligned}
 V_F &= -F \\
 H_D &= -F \\
 P_{HA} &= -q = -F/b \\
 P_{GH} &= -q = -F/b \\
 P_{GI} &= -q = -F/b \\
 q_{GI} &= -q = -F/b \\
 \theta_B &= -\theta = -\alpha T/b = -bF/EJ \\
 \end{aligned}$$

$$\begin{aligned}
 U_H &= -\delta = -b^3 F/EJ \\
 E V_{AB} &= EJ \\
 E V_{BC} &= EJ \\
 E V_{CD} &= EJ \\
 E V_{DE} &= EJ \\
 E V_{EF} &= EJ \\
 E V_{FG} &= EJ \\
 \end{aligned}$$

$$\begin{aligned}
 E J_{GH} &= EJ \\
 E J_{GI} &= EJ \\
 E J_{JB} &= EJ \\
 E J_{JE} &= EJ \\
 E J_{HA} &= EJ \\
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_C$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste misurati in proiezione sugli assi x,y.
 Diagrammi di carico con valori riferiti ad asse della trave.
 Componenti di carico distribuito riferiti ad assi ortogonali.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta IB positiva se convessa a destra con inizio I.
 Spostamento orizzontale assoluto u imposto al nodo H.



Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-b+1/2x$	$-2Fb+3/2Fx$	0	$2Fb^2-5/2Fbx+3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
BA b	$1/2b+1/2x$	$1/2Fb+3/2Fx$	0	$1/4Fb^2+Fbx+3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
BC b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
CB b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	$-Fb+Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0	
IB b	0	0	$-Fb/EJ$	0	0	0	0+0	0	
BI b	0	0	Fb/EJ	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x$	$-5/2Fb+Fx-1/2qx^2$	0	$5/2Fbx-Fx^2+1/2qx^3$	0	x^2	$(25/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
AH b	$b-x$	$2Fb+1/2qx^2$	0	$2Fb^2-2Fbx+1/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^3/EJ$	
	totali							$9/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$							$-9/8F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (2 - 5/2 x/b + 3/4 x^2/b^2) Fb^2 1/EJ dx = [2x - 5/4 x^2/b + 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (2b - 5/4 b + 1/4 b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/4 + x/b + 3/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x + 1/2 x^2/b + 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b + 1/2 b + 1/4 b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

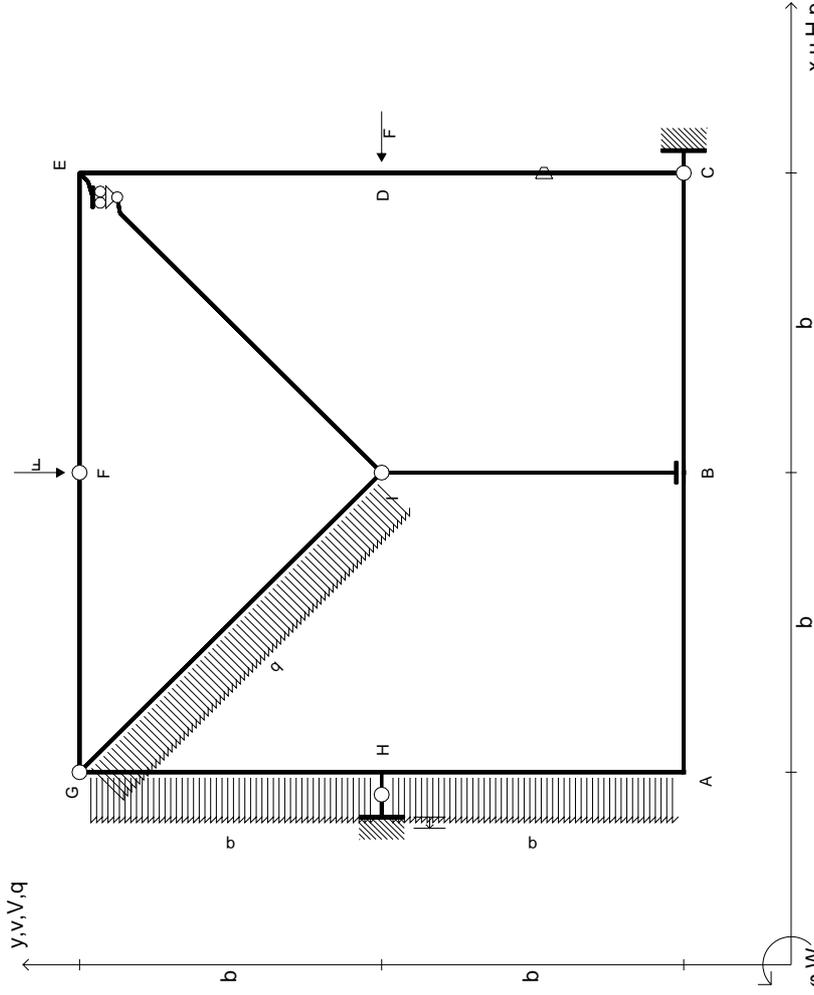
$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (5/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [5/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (5/4 b - 1/3 b + 1/8 b) Fb^2 1/EJ = 25/24 Fb^3/EJ$$

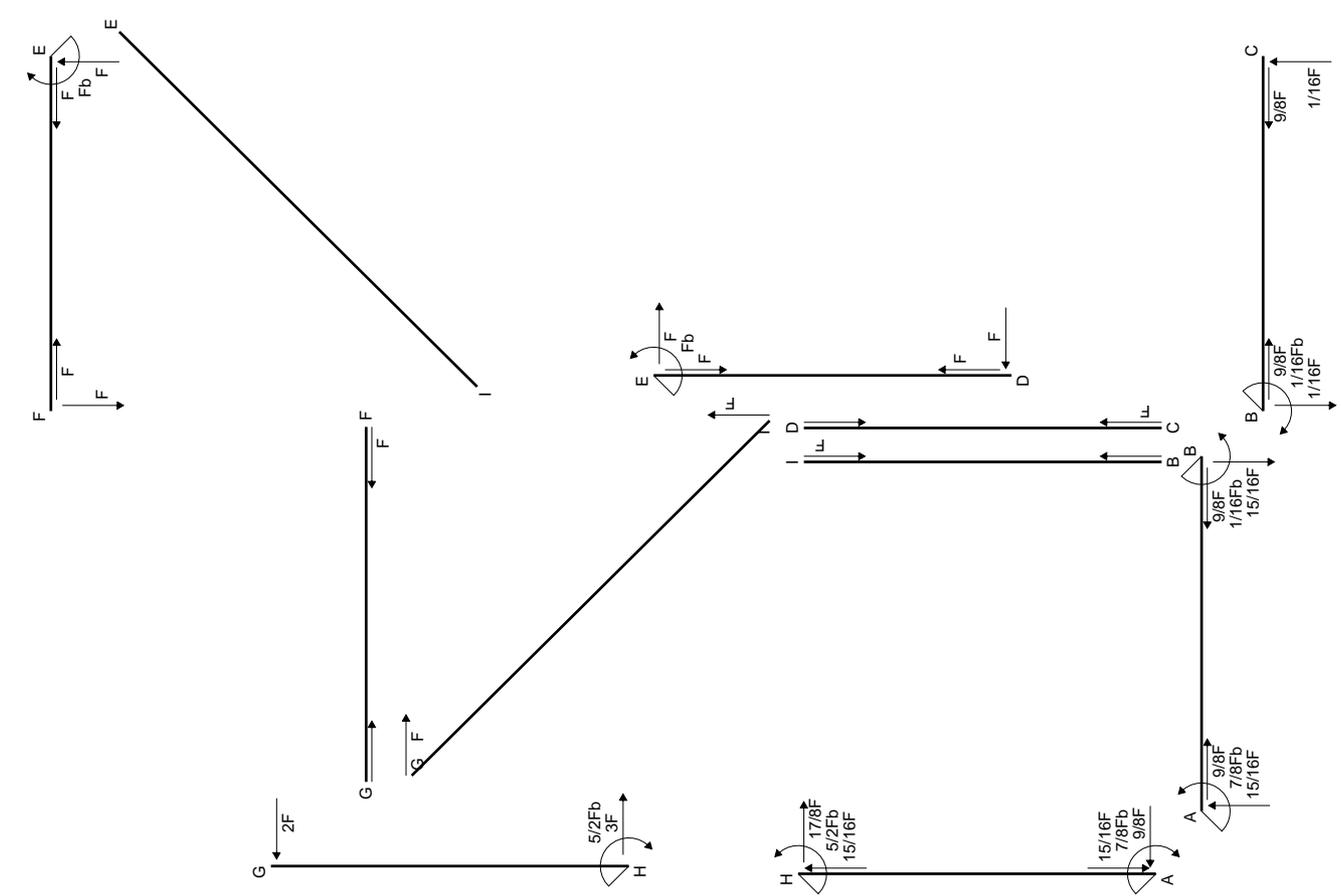
$$L_{AH}^{xo} = \int_0^b (2 - 2x/b + 1/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [2x - x^2/b + 1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

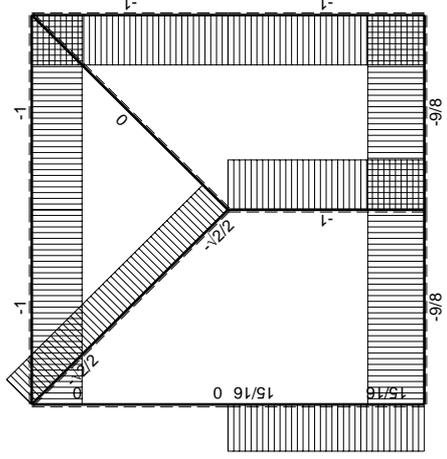
$$= (2b - b + 1/6 b - 1/8 b) Fb^2 1/EJ = 25/24 Fb^3/EJ$$



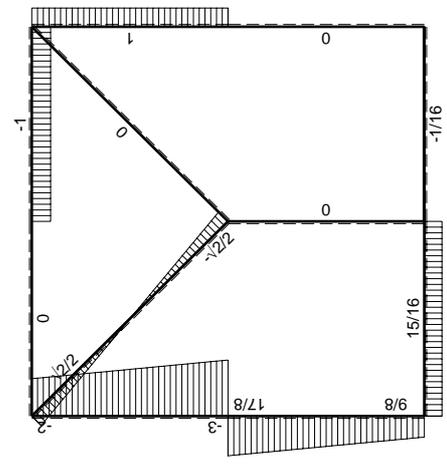
$$\begin{aligned}
 V_F &= -F \\
 H_D &= -F \\
 P_{HA} &= -q = -F/b \\
 P_{EH} &= -q = -F/b \\
 P_{GI} &= -q = -F/b \\
 q_{GI} &= -q = -F/b \\
 \theta_{CD} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_H &= -\delta = -b^3 F/EJ \\
 EJ_{AB} &= EJ \\
 EJ_{BC} &= EJ \\
 EJ_{CD} &= EJ \\
 EJ_{DE} &= EJ \\
 EJ_{EF} &= EJ \\
 EJ_{FG} &= EJ \\
 EJ_{GH} &= EJ \\
 EJ_{GI} &= EJ \\
 EJ_{IB} &= EJ \\
 EJ_{IE} &= EJ \\
 EJ_{HA} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_C$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste misurati in proiezione sugli assi x,y.
 Diagrammi di carico con valori riferiti ad asse della trave.
 Componenti di carico distribuito riferiti ad assi ortogonali.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta CD positiva se convessa a destra con inizio C.
 Spostamento orizzontale assoluto u imposto al nodo H.

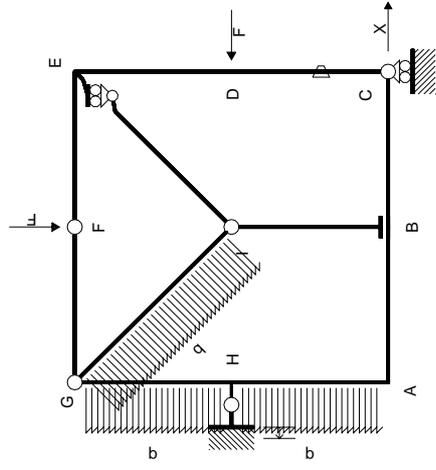




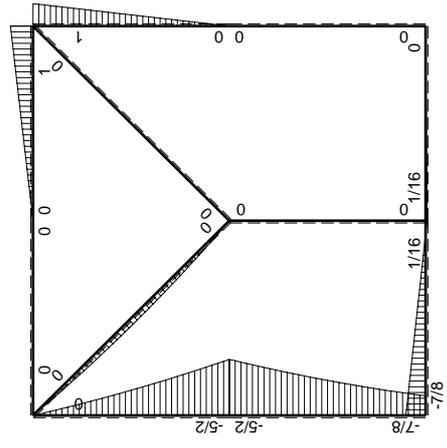
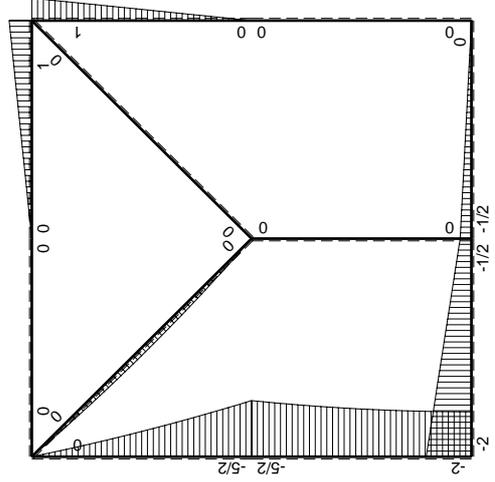
$\left[\begin{array}{c} + \\ - \end{array} \right]$ \rightarrow F



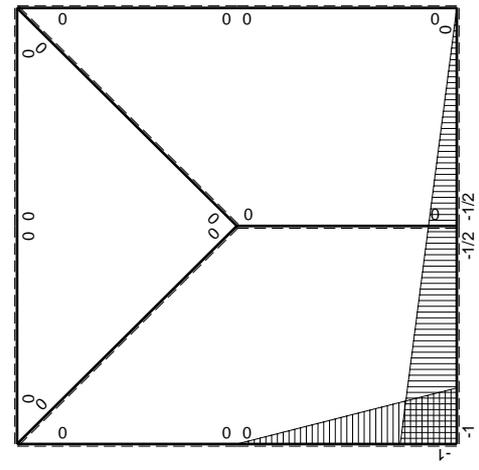
$\left[\begin{array}{c} + \\ - \end{array} \right]$ \rightarrow F



$\left[\begin{array}{c} + \\ - \end{array} \right]$ \curvearrowright M_0 flessione da carichi assegnati



$\left[\begin{array}{c} + \\ - \end{array} \right]$ Fb



$\left[\begin{array}{c} + \\ - \end{array} \right]$ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_C

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	-b+1/2x	-2Fb+3/2Fx	0	2Fb ² -5/2Fbx+3/4Fx ²	0	b ² -bx+1/4x ²	(1+0)Fb ³ /EJ	7/12Xb ³ /EJ	
BA b	1/2b+1/2x	1/2Fb+3/2Fx	0	1/4Fb ² +Fbx+3/4Fx ²	0	1/4b ² +1/2bx+1/4x ²			
BC b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb ² -1/2Fbx+1/4Fx ²	0	1/4b ² -1/2bx+1/4x ²	(1/12+0)Fb ³ /EJ	1/12Xb ³ /EJ	
CB b	1/2x	1/2Fx	0	1/4Fx ²	0	1/4x ²			
CD b	0	0	-Fb/EJ	0	0	0	0+0	0	
DC b	0	0	Fb/EJ	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	-Fb+Fx	0	0	0	0			
EF b	0	Fb-Fx	0	0	0	0	0+0	0	
FE b	0	-Fx	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	-2Fx-1/2qx ²	0	0	0	0	0+0	0	
HG b	0	5/2Fb-3Fx+1/2qx ²	0	0	0	0			
GI √2b	0	√2/2Fx-1/2qx ²	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE √2b	0	0	0	0	0	0	0	0	
HA b	-x	-5/2Fb+Fx-1/2qx ²	0	5/2Fbx-Fx ² +1/2qx ³	0	x ²	(25/24+0)Fb ³ /EJ	1/3Xb ³ /EJ	
AH b	b-x	2Fb+1/2qx ²	0	2Fb ² -2Fbx+1/2Fx ² -1/2qx ³	0	b ² -2bx+x ²			
H	cedimento nodo -H _{1H} u _H							-Fb ³ /EJ	
	totali							9/8Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _C							-9/8F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (2 - 5/2 x/b + 3/4 x^2/b^2) Fb^2 1/EJ dx = [2x - 5/4 x^2/b + 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (2b - 5/4 b + 1/4 b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/4 + x/b + 3/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x + 1/2 x^2/b + 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b + 1/2 b + 1/4 b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

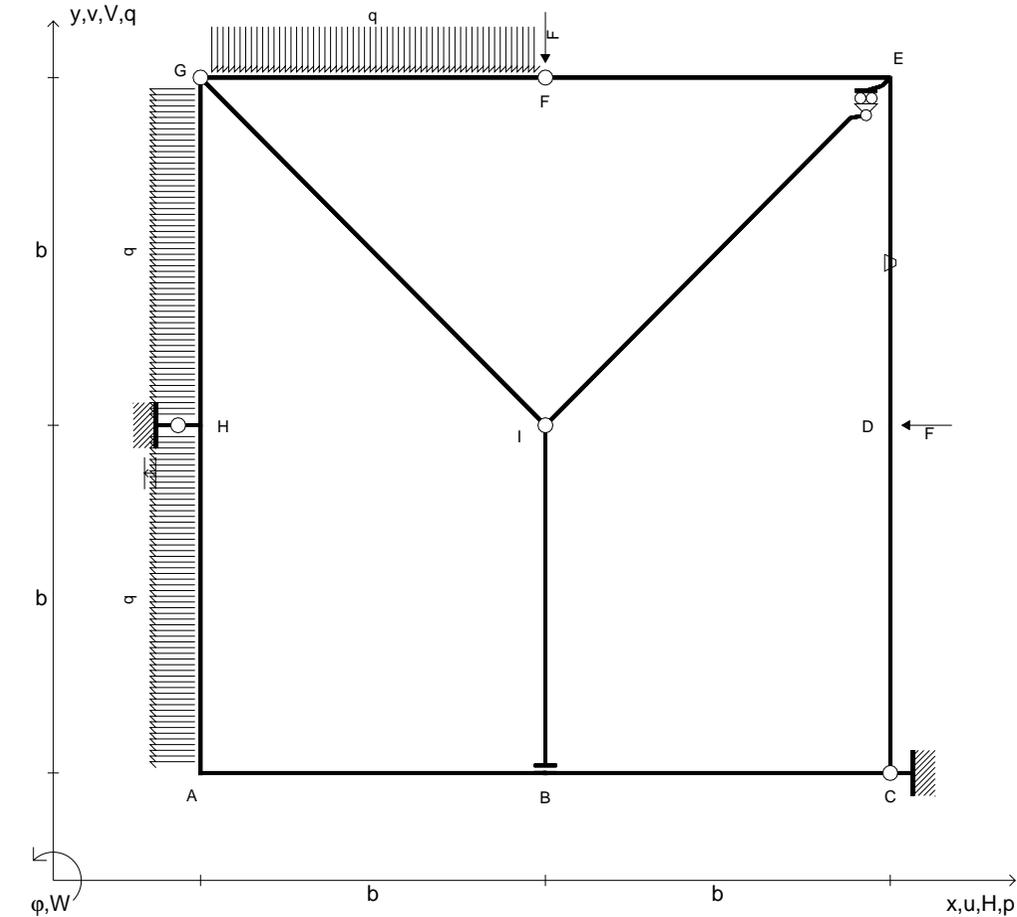
$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (5/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [5/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (5/4 b - 1/3 b + 1/8 b) Fb^2 1/EJ = 25/24 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (2 - 2x/b + 1/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [2x - x^2/b + 1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (2b - b + 1/6 b - 1/8 b) Fb^2 1/EJ = 25/24 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$p_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{FG} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{DE} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

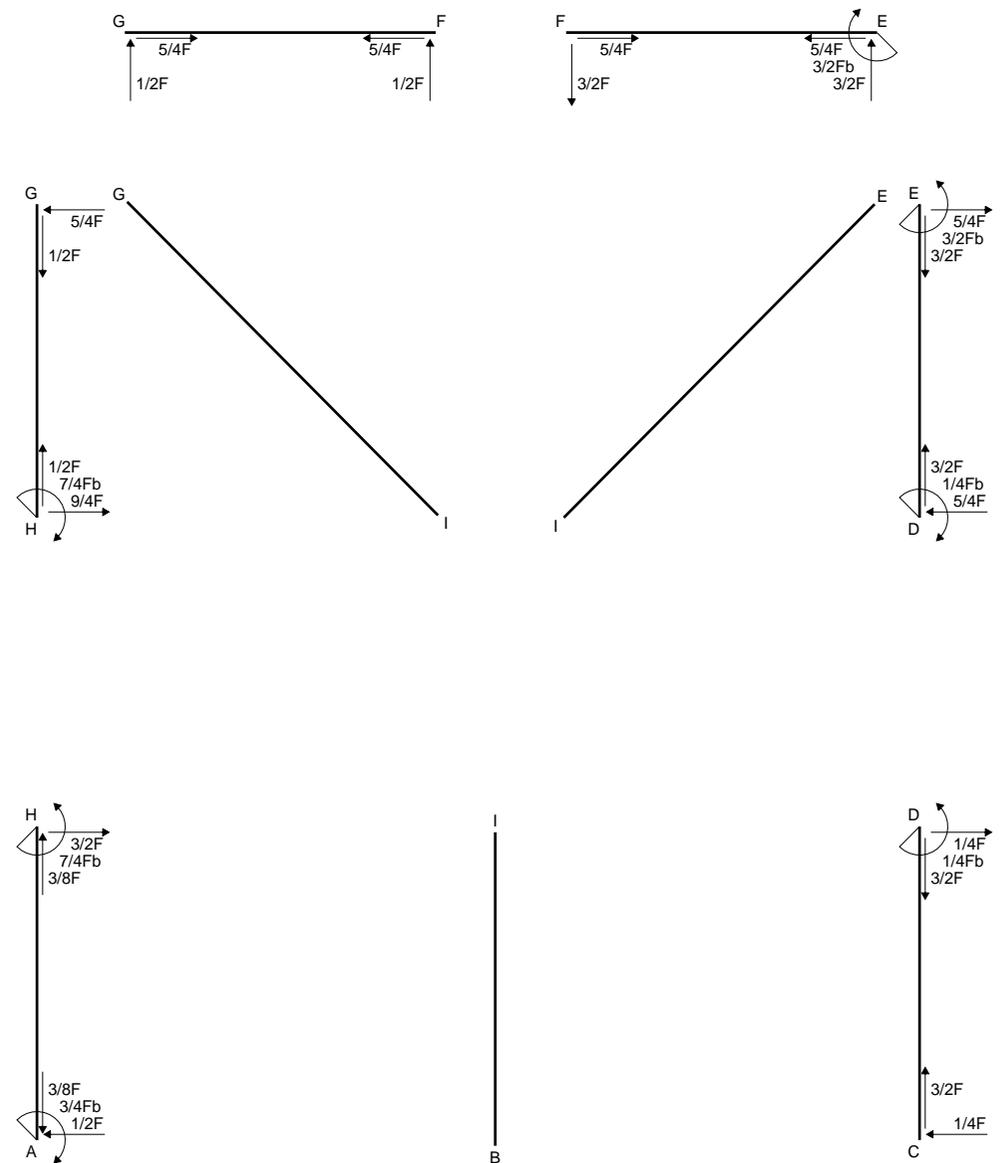
Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta DE positiva se convessa a destra con inizio D.

Spostamento orizzontale assoluto u imposto al nodo H.

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	$Fb-1/2Fx$	0	$-2Fb^2+2Fbx-1/2Fx^2$	0	$4b^2-4bx+x^2$	$(-7/6+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$b+x$	$-1/2Fb-1/2Fx$	0	$-1/2Fb^2-Fbx-1/2Fx^2$	0	$b^2+2bx+x^2$			
BC b	$-b+x$	$1/2Fb-1/2Fx$	0	$-1/2Fb^2+Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-1/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
CB b	x	$-1/2Fx$	0	$-1/2Fx^2$	0	x^2			
CD b	0	$1/4Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	$-Fb/EJ$	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	Fb/EJ	0	0	0			
EF b	0	$3/2Fb-3/2Fx$	0	0	0	0	0+0	0	
FE b	0	$-3/2Fx$	0	0	0	0			
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-2x$	$-7/4Fb+13/4Fx-1/2qx^2$	0	$7/2Fbx-13/2Fx^2+qx^3$	0	$4x^2$	$(-1/6+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$-Fb+9/4Fx+1/2qx^2$	0	$-2Fb^2+13/2Fbx-7/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$-7/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							$7/8F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-2 + 2x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-2x + x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2b + b - 1/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 - x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x - 1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b - 1/2 b - 1/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 1/2 b - 1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

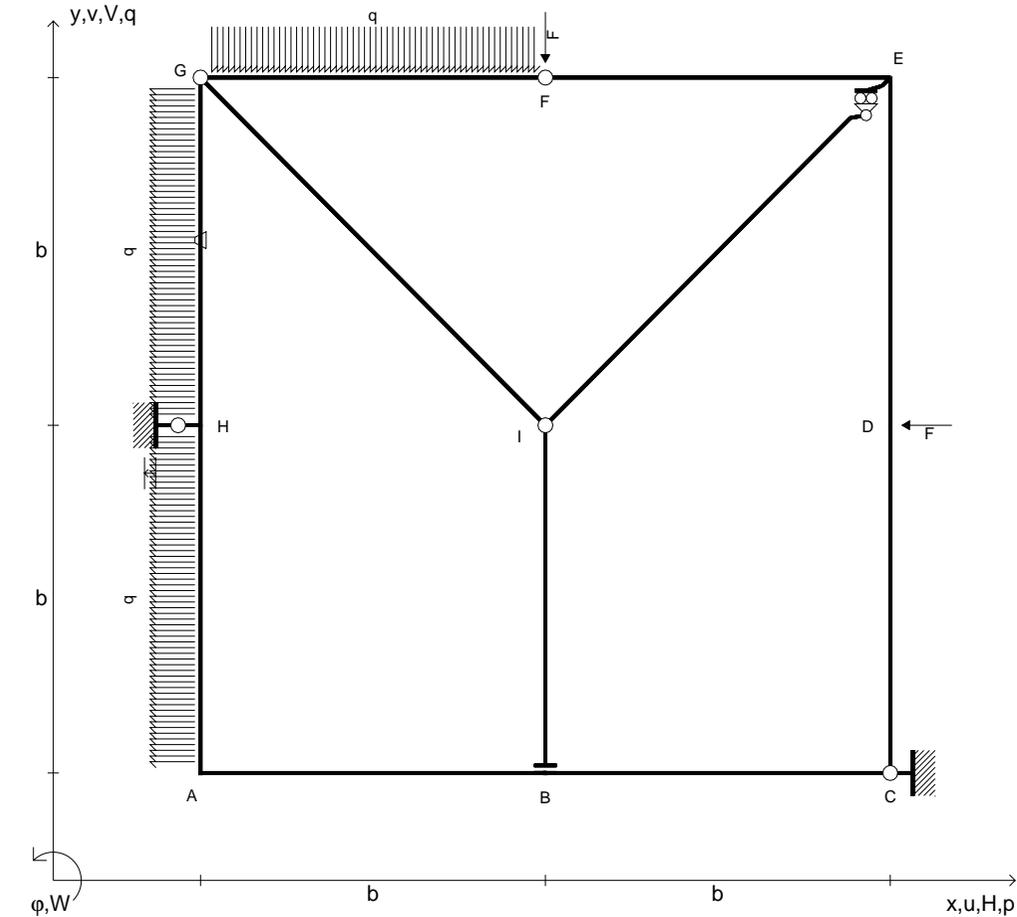
$$= (-1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (7/2 x/b - 13/2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [7/4 x^2/b - 13/6 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (7/4 b - 13/6 b + 1/4 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (-2 + 13/2 x/b - 7/2 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [-2x + 13/4 x^2/b - 7/6 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-2b + 13/4 b - 7/6 b - 1/4 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{FG} = EJ$
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$q_{FG} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{GH} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

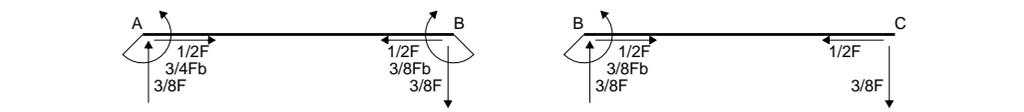
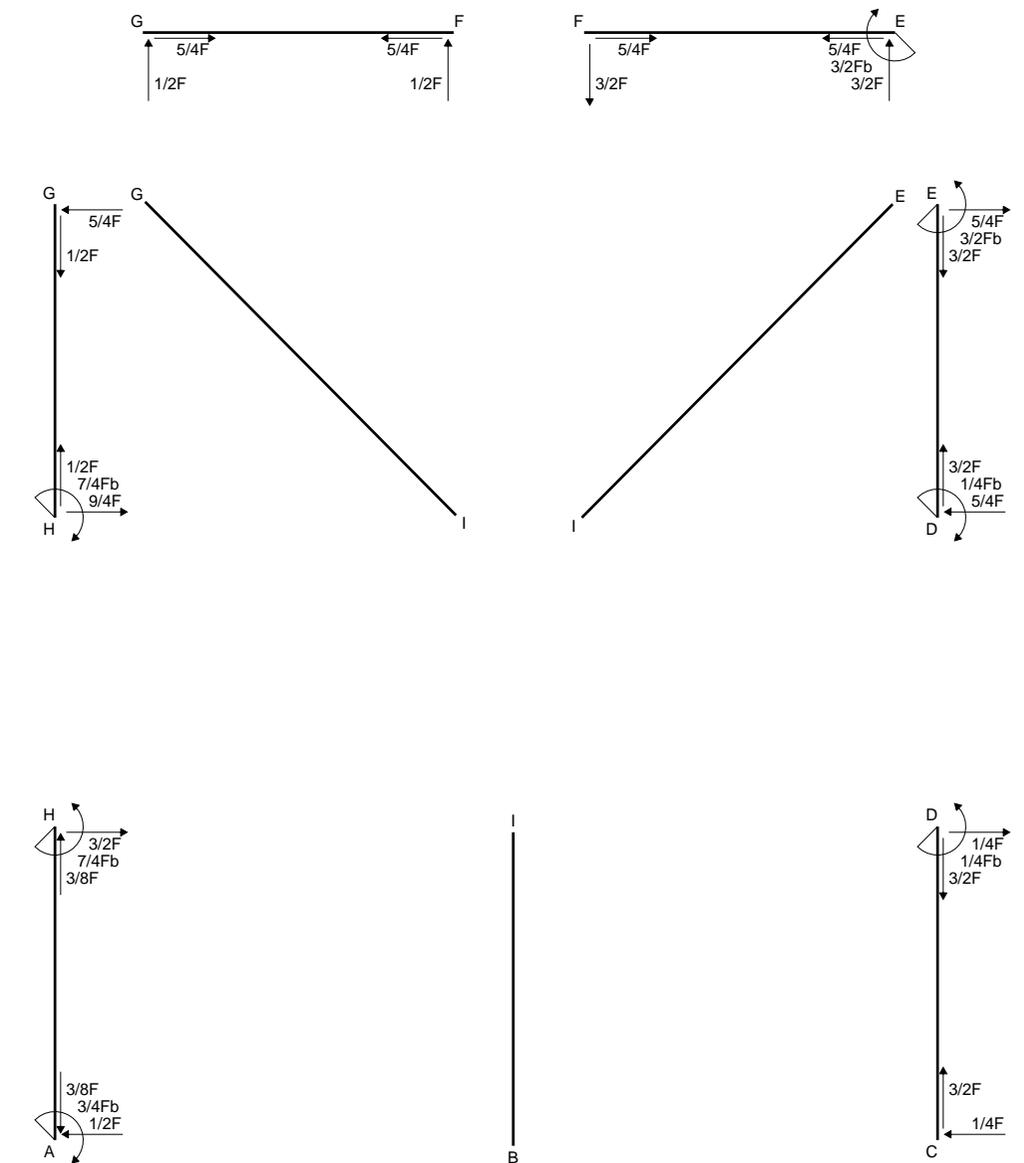
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta GH positiva se convessa a destra con inizio G.

Spostamento orizzontale assoluto u imposto al nodo H.

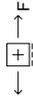
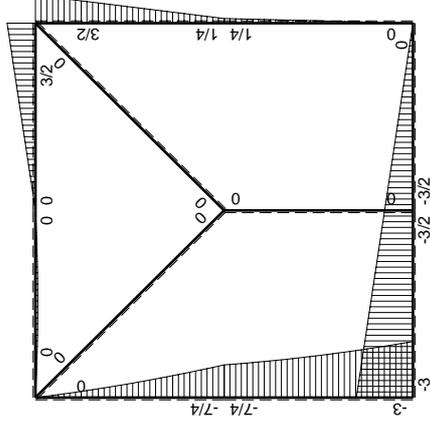
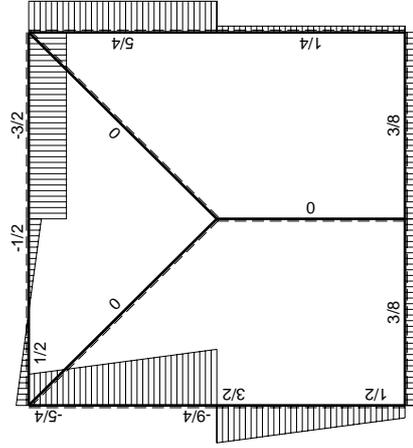
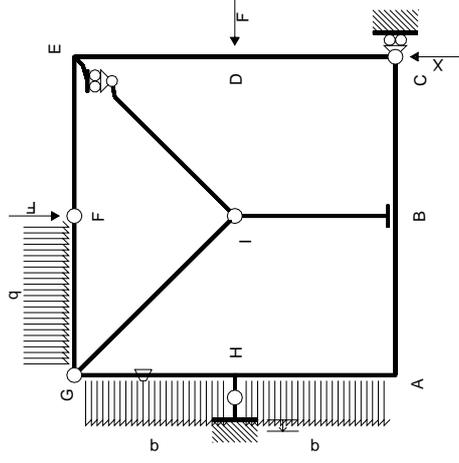
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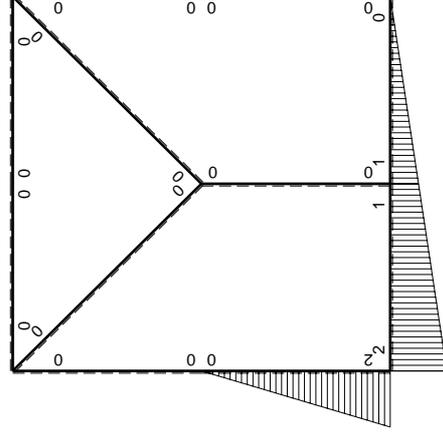
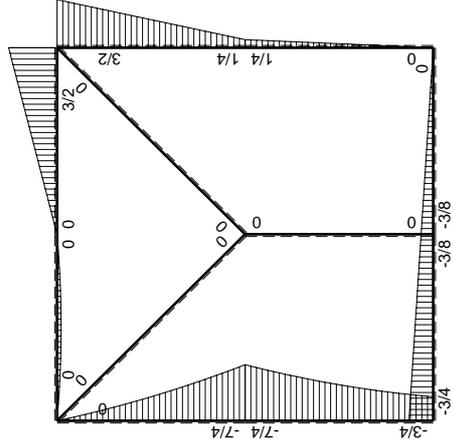


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Schema di calcolo iperstatico



Quadro contributi PLV per iperstatica $X=V_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$2b-x$	$-3Fb+3/2Fx$	0	$-6Fb^2+6Fbx-3/2Fx^2$	0	$4b^2-4bx+x^2$	$(-7/2+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$-b-x$	$3/2Fb+3/2Fx$	0	$-3/2Fb^2-3Fbx-3/2Fx^2$	0	$b^2+2bx+x^2$			
BC b	$b-x$	$-3/2Fb+3/2Fx$	0	$-3/2Fb^2+3Fbx-3/2Fx^2$	0	$b^2-2bx+x^2$	$(-1/2+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
CB b	$-x$	$3/2Fx$	0	$-3/2Fx^2$	0	x^2			
CD b	0	$1/4Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0			
EF b	0	$3/2Fb-3/2Fx$	0	0	0	0	0+0	0	
FE b	0	$-3/2Fx$	0	0	0	0			
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	Fb/EJ	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$2x$	$-7/4Fb-3/4Fx-1/2qx^2$	0	$-7/2Fbx-3/2Fx^2-qx^3$	0	$4x^2$	$(-5/2+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$-2b+2x$	$3Fb-7/4Fx+1/2qx^2$	0	$-6Fb^2+19/2Fbx-9/2Fx^2+qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$2Fb^3/EJ$	
	totali							$-9/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_C$							$9/8F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-6 + 6x/b - 3/2 x^2/b^2) Fb^2 1/EJ dx = [-6x + 3x^2/b - 1/2 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-6b + 3b - 1/2b) Fb^2 1/EJ = -7/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-3/2 - 3x/b - 3/2 x^2/b^2) Fb^2 1/EJ dx = [-3/2 x - 3/2 x^2/b - 1/2 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-3/2b - 3/2b - 1/2b) Fb^2 1/EJ = -7/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-3/2 + 3x/b - 3/2 x^2/b^2) Fb^2 1/EJ dx = [-3/2 x + 3/2 x^2/b - 1/2 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-3/2b + 3/2b - 1/2b) Fb^2 1/EJ = -1/2 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-3/2 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2b) Fb^2 1/EJ = -1/2 Fb^3/EJ$$

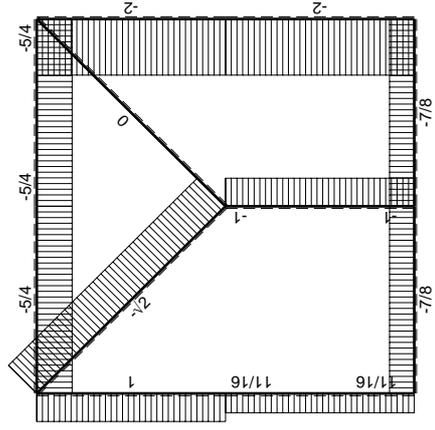
$$L_{HA}^{xo} = \int_0^b (-7/2 x/b - 3/2 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [-7/4 x^2/b - 1/2 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-7/4b - 1/2b - 1/4b) Fb^2 1/EJ = -5/2 Fb^3/EJ$$

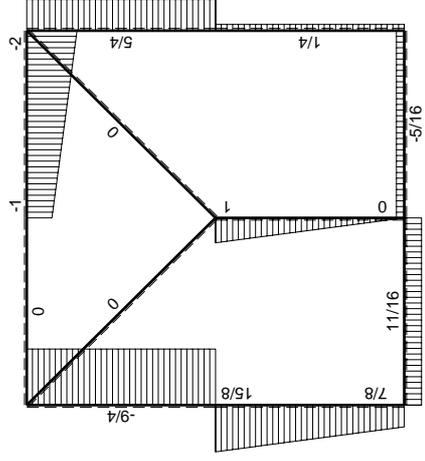
$$L_{AH}^{xo} = \int_0^b (-6 + 19/2 x/b - 9/2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx$$

$$= [-6x + 19/4 x^2/b - 3/2 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

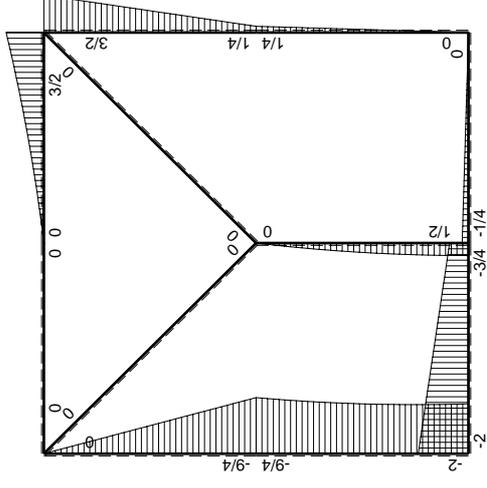
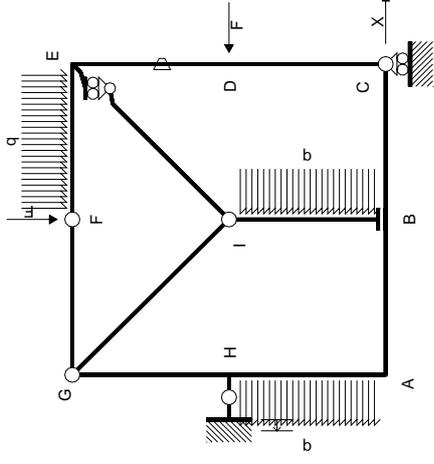
$$= (-6b + 19/4b - 3/2b + 1/4b) Fb^2 1/EJ = -5/2 Fb^3/EJ$$



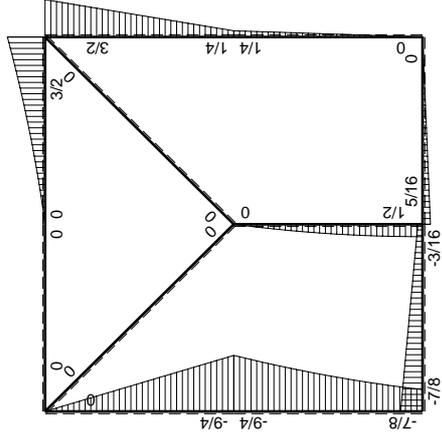
← → F



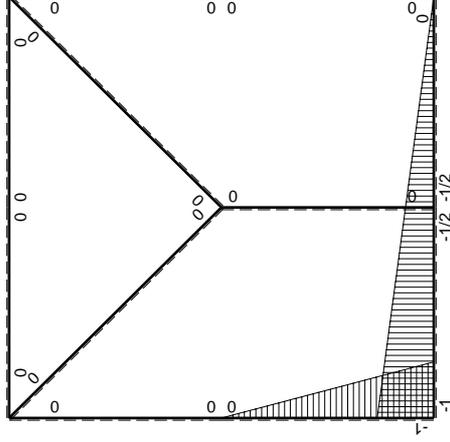
↑ ↓ F



↺ ↻ M₀ flessione da carichi assegnati



← → F_b



↺ ↻ M_y flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-b+1/2x$	$-2Fb+5/4Fx$	0	$2Fb^2-9/4Fbx+5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(13/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
BA b	$1/2b+1/2x$	$3/4Fb+5/4Fx$	0	$3/8Fb^2+Fbx+5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
BC b	$-1/2b+1/2x$	$-1/4Fb+1/4Fx$	0	$1/8Fb^2-1/4Fbx+1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
CB b	$1/2x$	$1/4Fx$	0	$1/8Fx^2$	0	$1/4x^2$			
CD b	0	$1/4Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	$-Fb/EJ$	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	Fb/EJ	0	0	0			
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-Fx-1/2qx^2$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0	
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x$	$-9/4Fb+3/4Fx-1/2qx^2$	0	$9/4Fbx-3/4Fx^2+1/2qx^3$	0	x^2	$(1+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
AH b	$b-x$	$2Fb-1/4Fx+1/2qx^2$	0	$2Fb^2-9/4Fbx+3/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^3/EJ$	
	totali							$9/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$							$-9/8F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (2 - 9/4 x/b + 5/8 x^2/b^2) Fb^2 1/EJ dx = [2x - 9/8 x^2/b + 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (2b - 9/8 b + 5/24 b) Fb^2 1/EJ = 13/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (3/8 + x/b + 5/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x + 1/2 x^2/b + 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b + 1/2 b + 5/24 b) Fb^2 1/EJ = 13/12 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/8 - 1/4 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x - 1/8 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b - 1/8 b + 1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/8 x^2/b^2) Fb^2 1/EJ dx = [1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

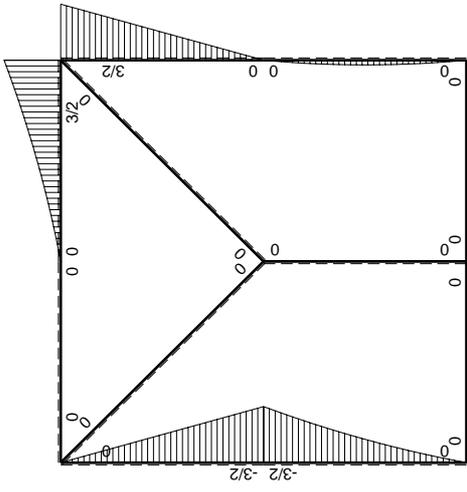
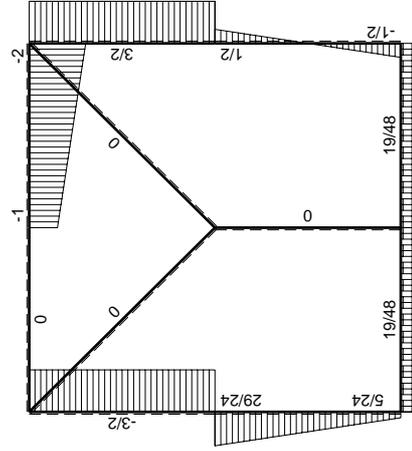
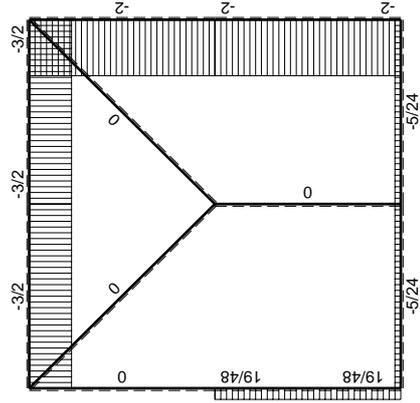
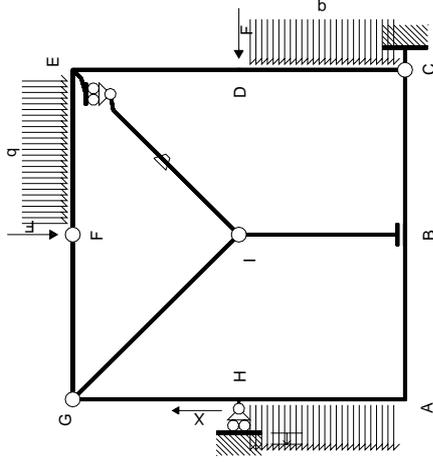
$$= (1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (9/4 x/b - 3/4 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [9/8 x^2/b - 1/4 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (9/8 b - 1/4 b + 1/8 b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (2 - 9/4 x/b + 3/4 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [2x - 9/8 x^2/b + 1/4 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

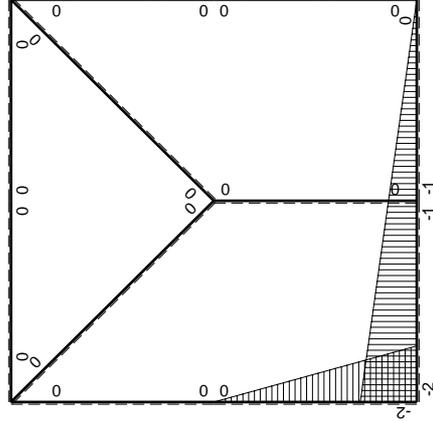
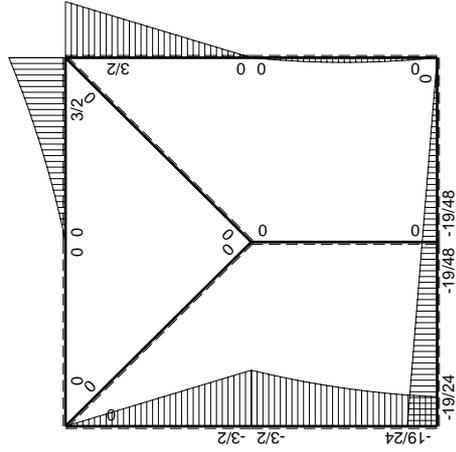
$$= (2b - 9/8 b + 1/4 b - 1/8 b) Fb^2 1/EJ = Fb^3/EJ$$



← → F

↑ ↓ F

↺ ↻ M₀ flessione da carichi assegnati



↺ ↻ F_b

↺ ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	0	0	0	0	$4b^2-4bx+x^2$	0+0	$7/3Xb^3/EJ$	
BA b	$b+x$	0	0	0	0	$b^2+2bx+x^2$			
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$	
CB b	x	0	0	0	0	x^2			
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0			
DE b	0	$3/2Fx$	0	0	0	0	0+0	0	
ED b	0	$-3/2Fb+3/2Fx$	0	0	0	0			
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-Fx-1/2qx^2$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-3/2Fx$	0	0	0	0	0+0	0	
HG b	0	$3/2Fb-3/2Fx$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	$-Fb/EJ$	0	0	0	0	0	
HA b	$-2x$	$-3/2Fb+2Fx-1/2qx^2$	0	$3Fbx-4Fx^2+qx^3$	0	$4x^2$	$(5/12+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$Fx+1/2qx^2$	0	$2Fbx-Fx^2-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$-19/12Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							$19/48F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = \left[4x - 2x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = \left[x + x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = \left[1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = \left[4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = \left[4x - 4x^2/b + 4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (3x/b - 4x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = \left[3/2 x^2/b - 4/3 x^3/b^2 + 1/4 x^4/b^3 \right]_0^b Fb^2 1/EJ$$

$$= (3/2 b - 4/3 b + 1/4 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (2x/b - x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = \left[x^2/b - 1/3 x^3/b^2 - 1/4 x^4/b^3 \right]_0^b Fb^2 1/EJ$$

$$= (b - 1/3 b - 1/4 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$