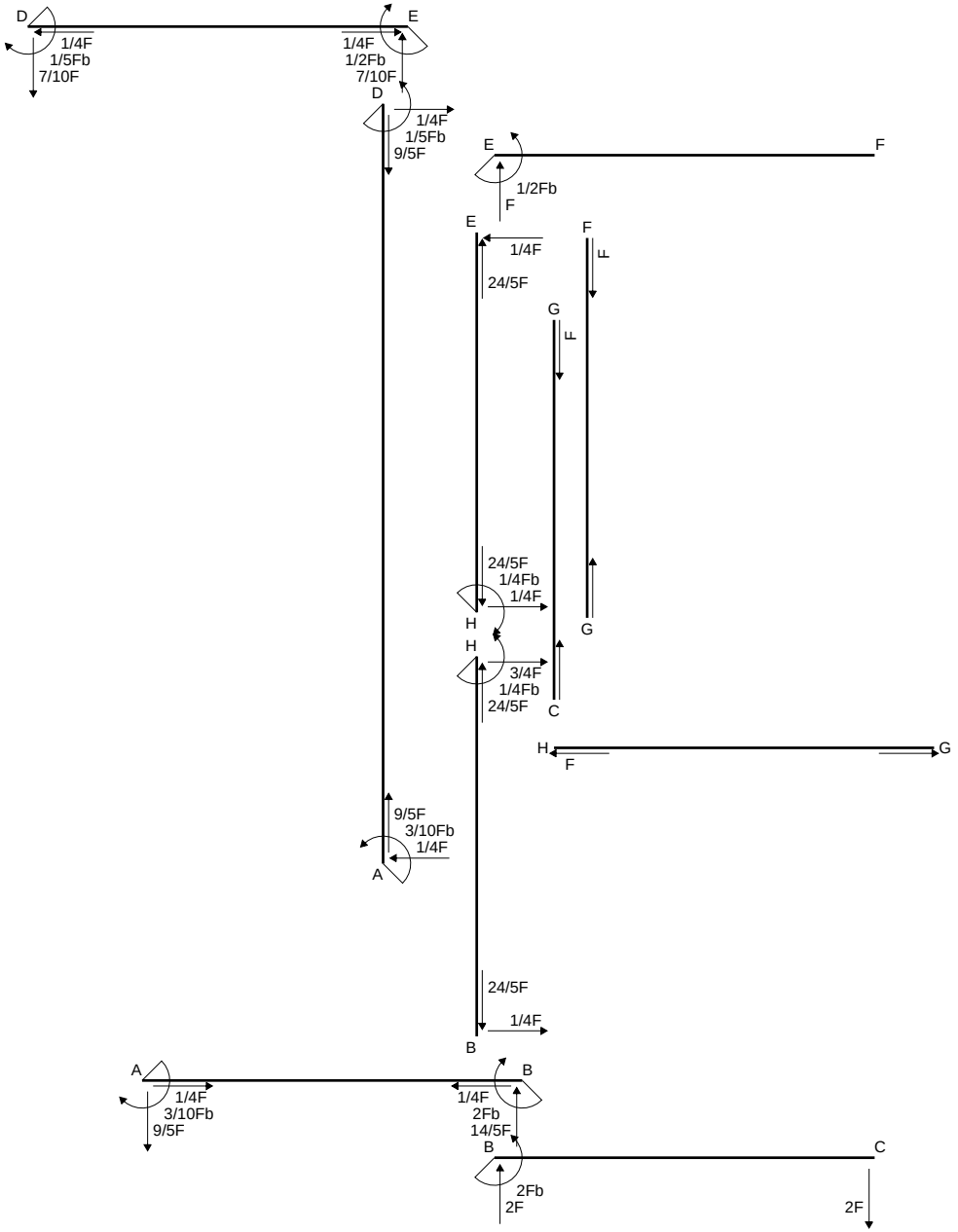
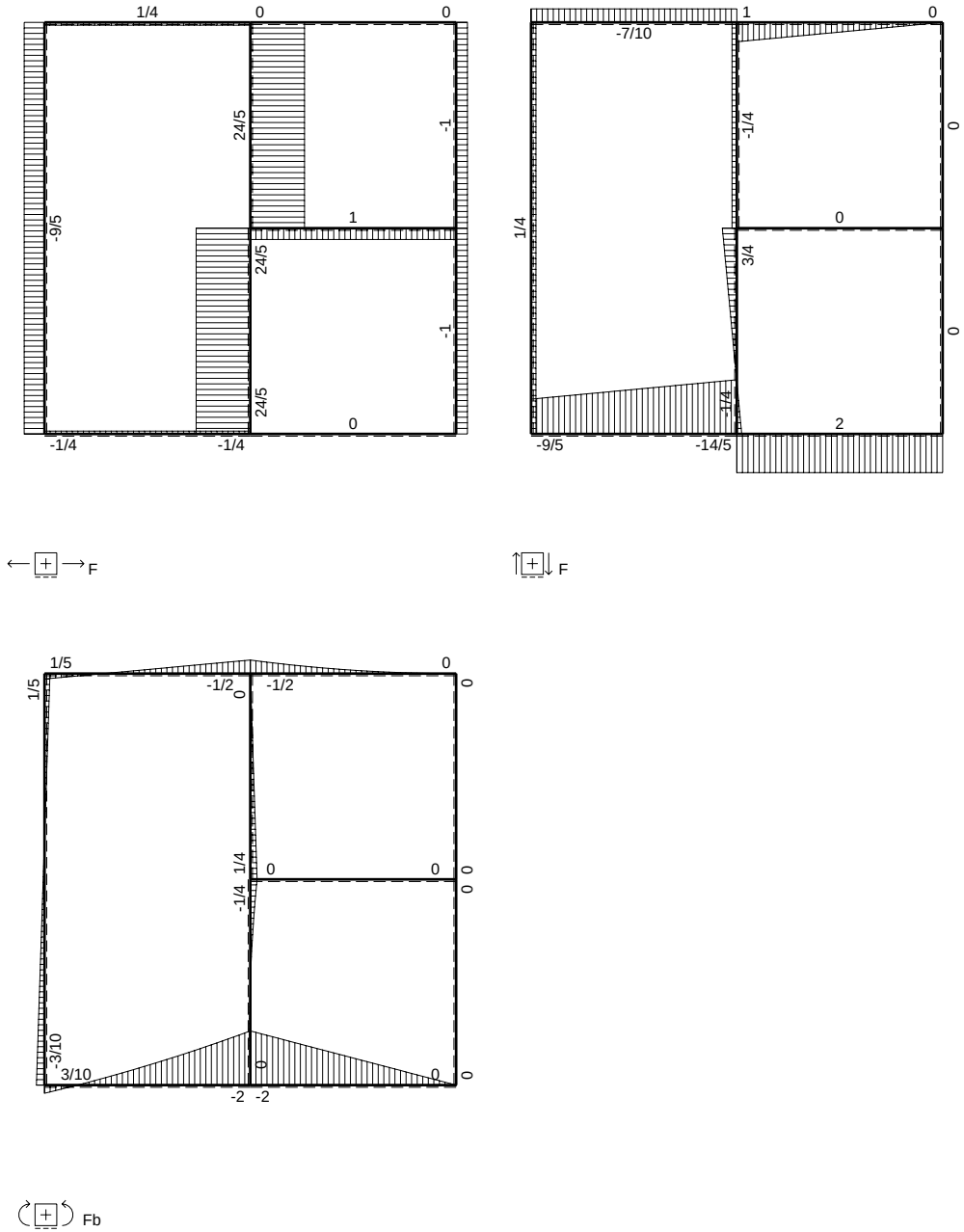
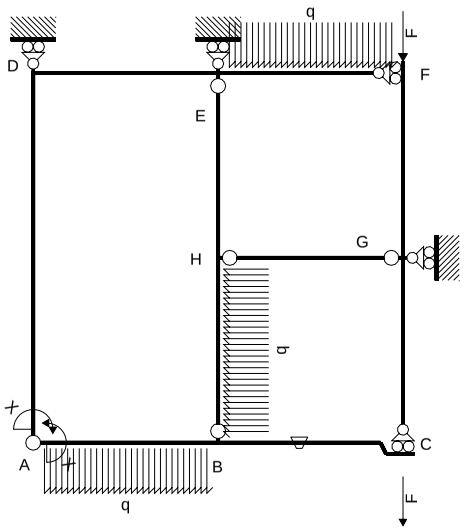
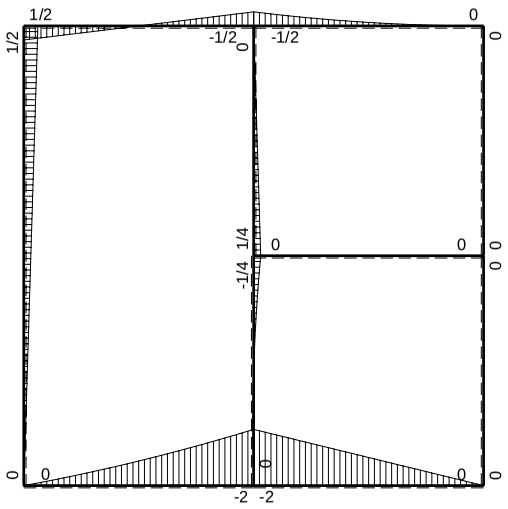

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7/10F
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1/5Fb
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2F
2F

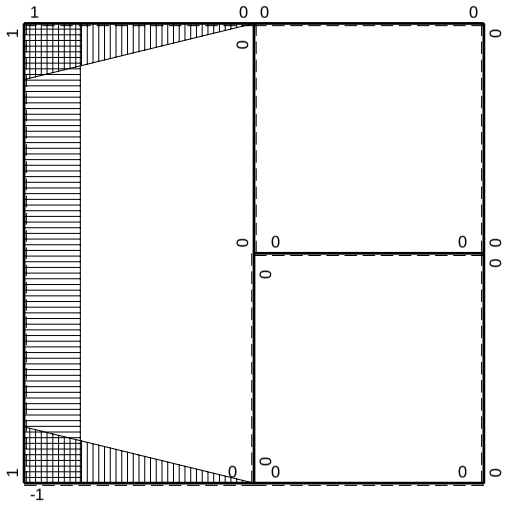

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24/5
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0
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F
1/5
0
-1/2
-1/2
0
0
1/4
0
0
-1/4
0
0
0
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

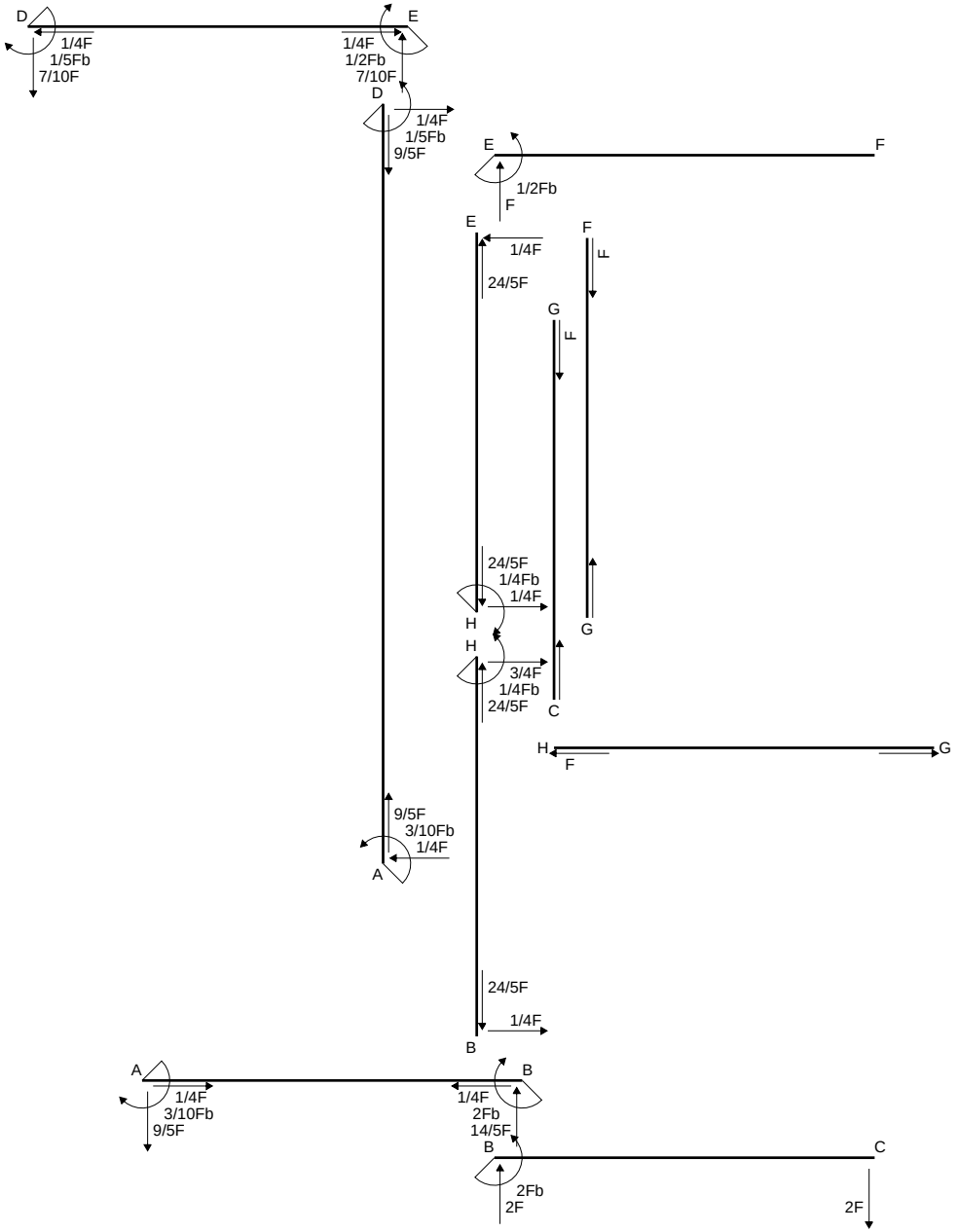
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

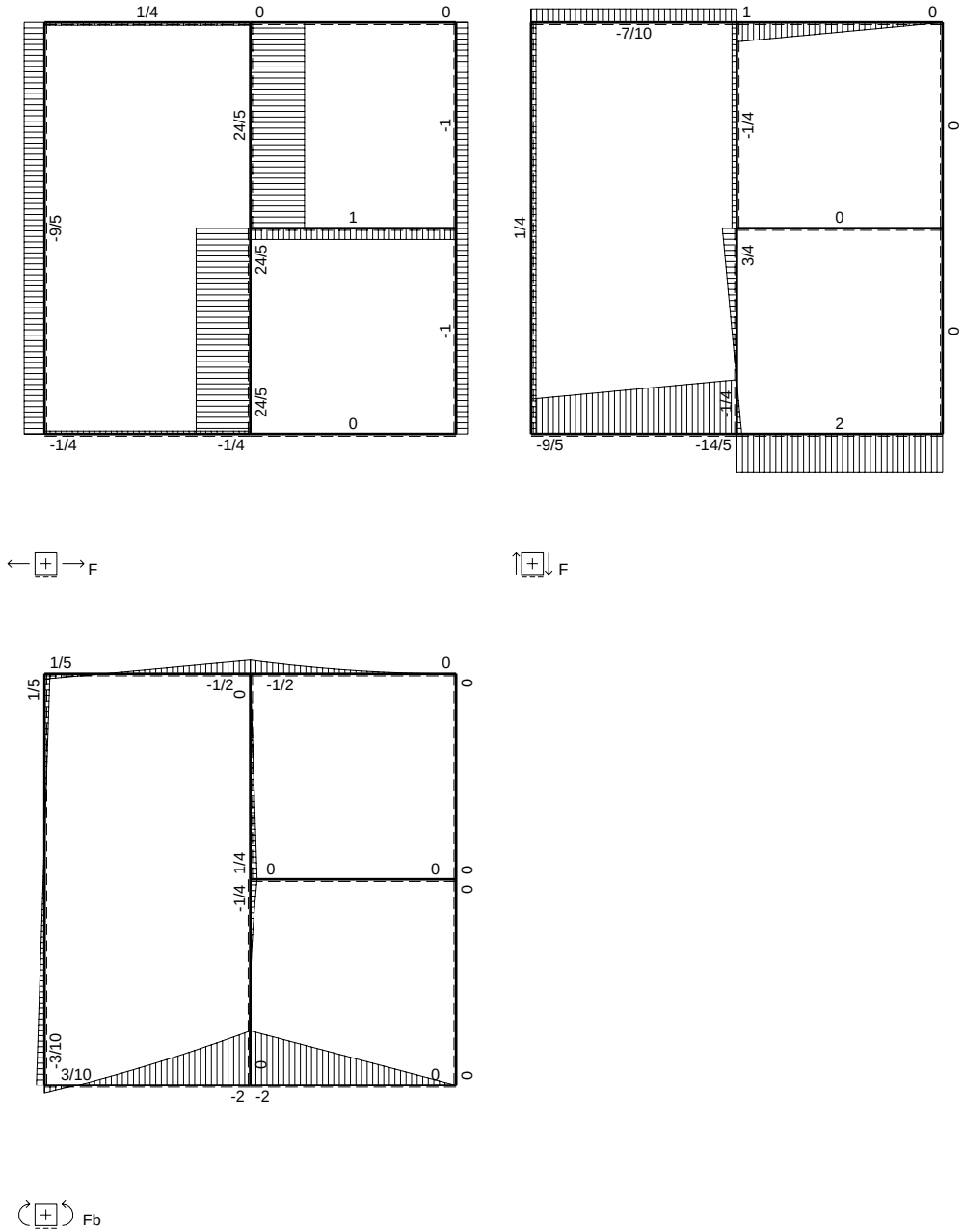
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

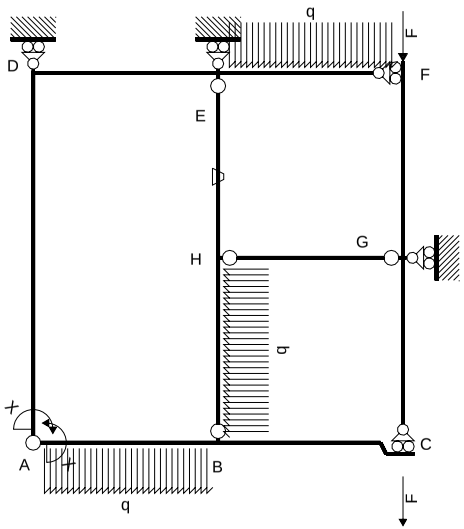
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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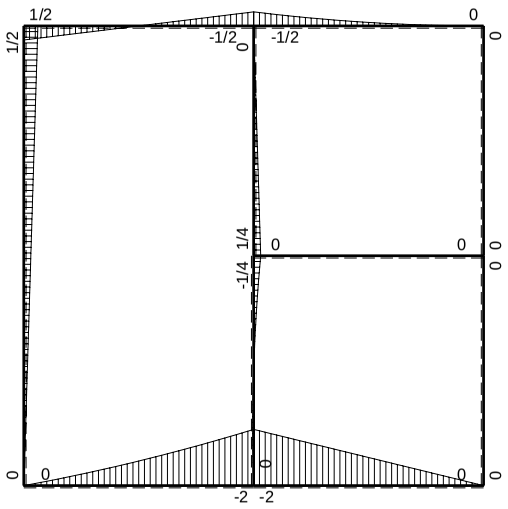
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E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
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1/2Fb
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C
2Fb
2F
2F

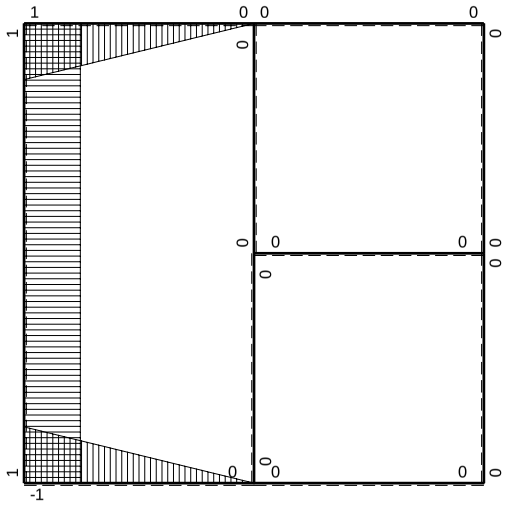

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24/5
-1
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F
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-7/10
0
1/4
-1/4
0
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0
-1/4
2
-9/5
-14/5
F
1/5
0
1/5
-1/2
0
-1/2
0
0
1/4
0
0
-1/4
0
0
-3/10
3/10
0
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238024 ESΣ03.xxxx.002

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	1/4Fb-1/4Fx	-Fb/EJ	0	0	0	0+0	0
EH b	0	-1/4Fx	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

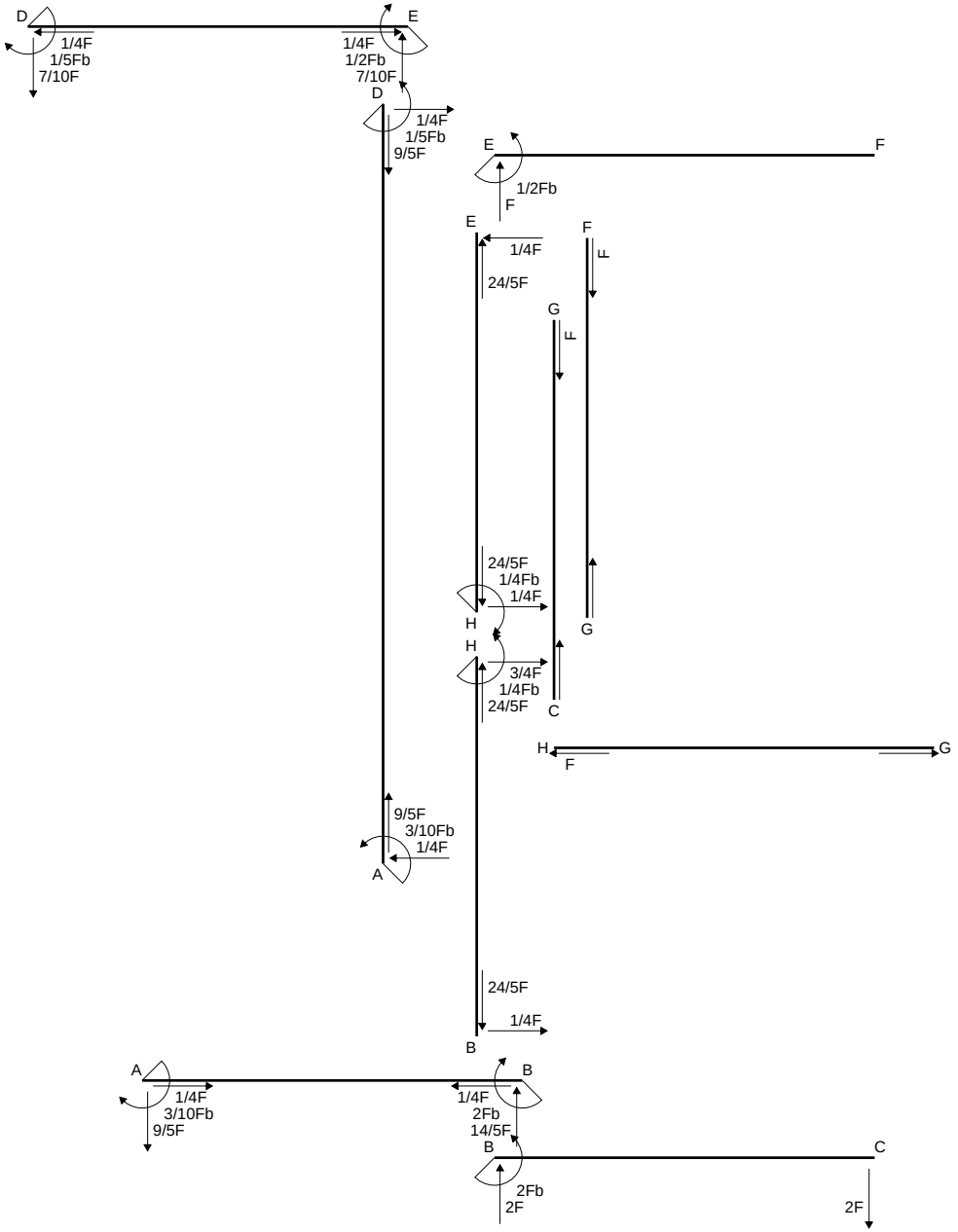
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

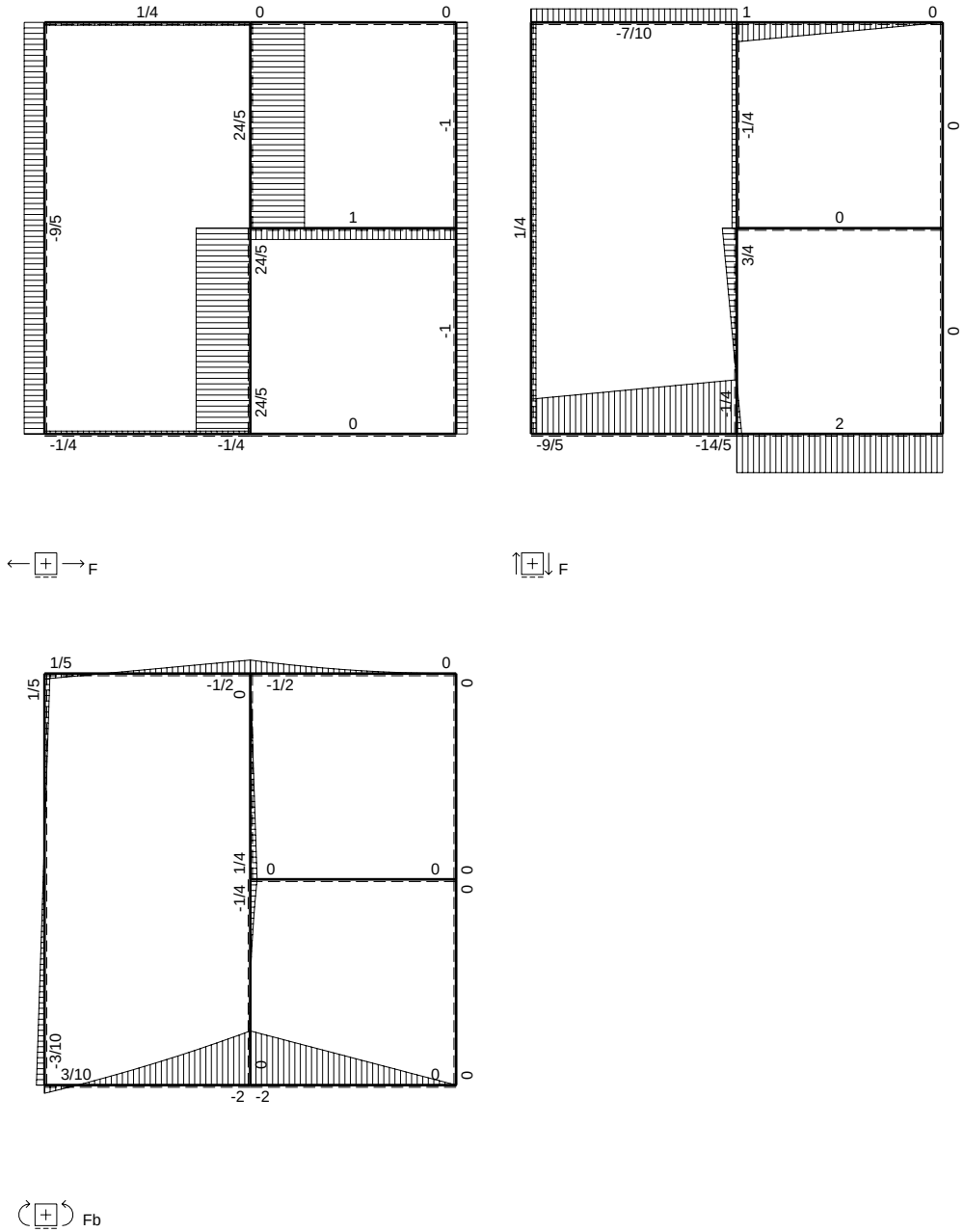
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

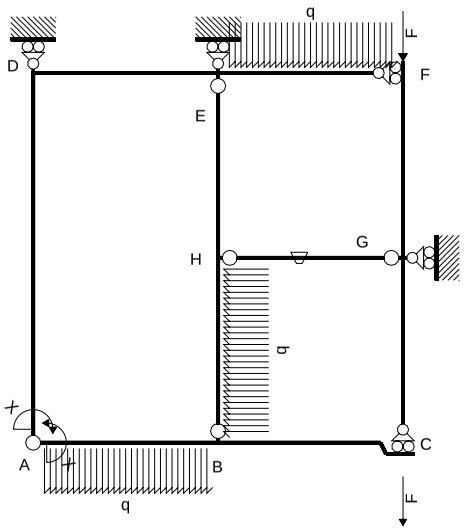
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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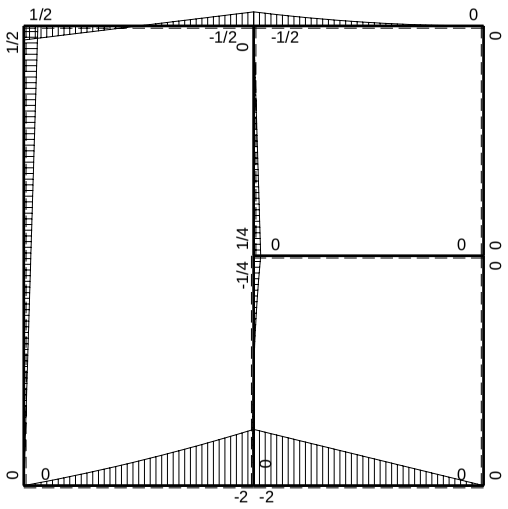
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1/5Fb
7/10F
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1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
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1/2Fb
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G
F
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1/4F
H
G
H
3/4F
1/4Fb
24/5F
C
H
F
G
9/5F
3/10Fb
1/4F
A
24/5F
1/4F
B
A
B
1/4F
3/10Fb
9/5F
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2Fb
14/5F
B
C
2Fb
2F
2F

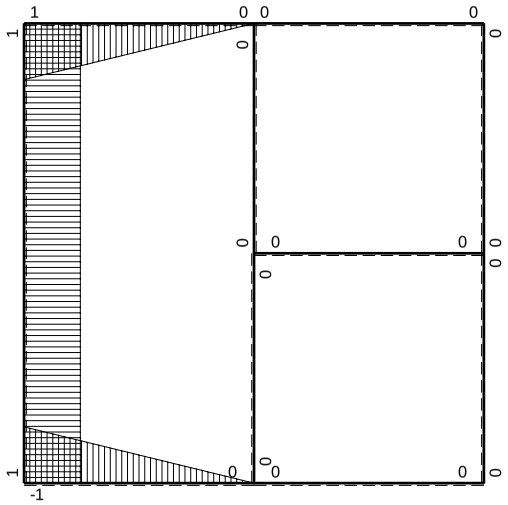

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24/5
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-9/5
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24/5
-1
24/5
0
-1/4
-1/4
F
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0
-7/10
-1/4
0
1/4
3/4
0
-1/4
2
-9/5
-14/5
F
1/5
0
-1/2
-1/2
0
0
1/4
0
0
-1/4
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239148 ESΣ03.xxxx.003

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
EH b	0	-1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

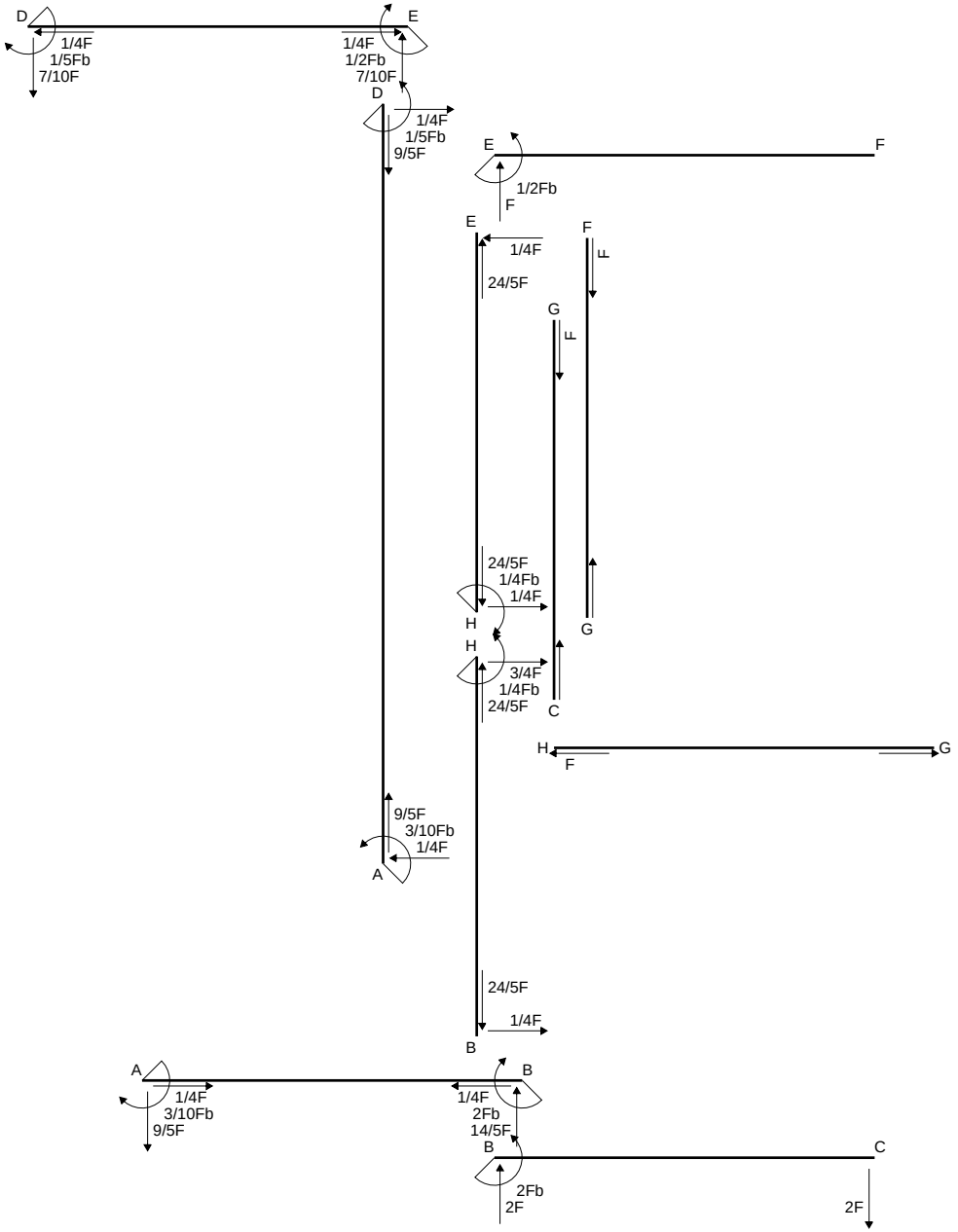
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

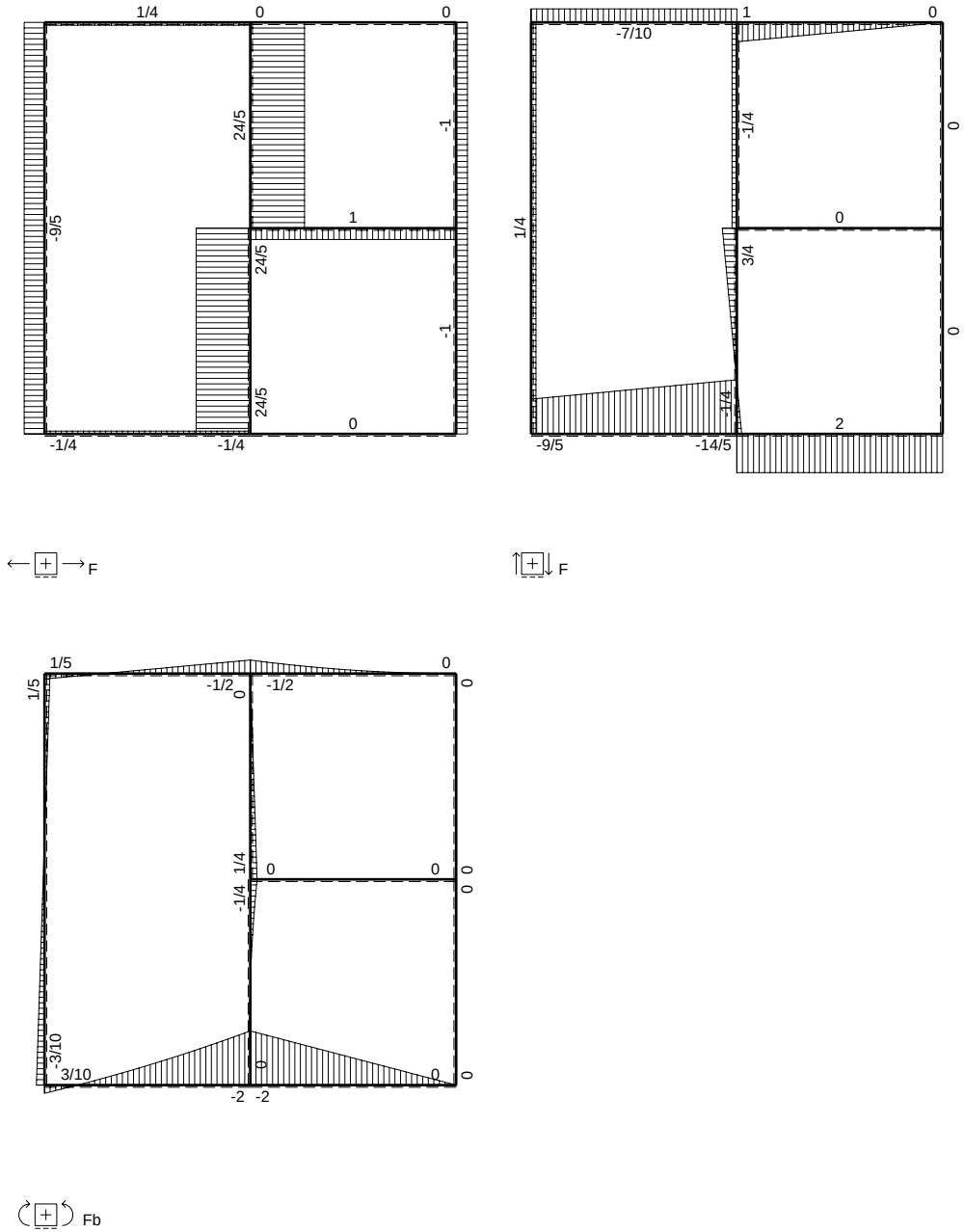
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

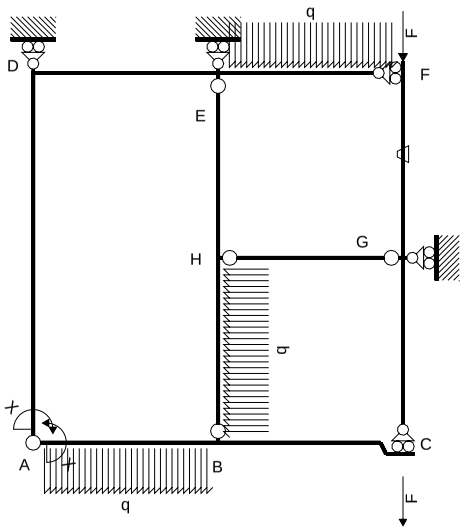
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 239148 ESΣ03.xxxx.003

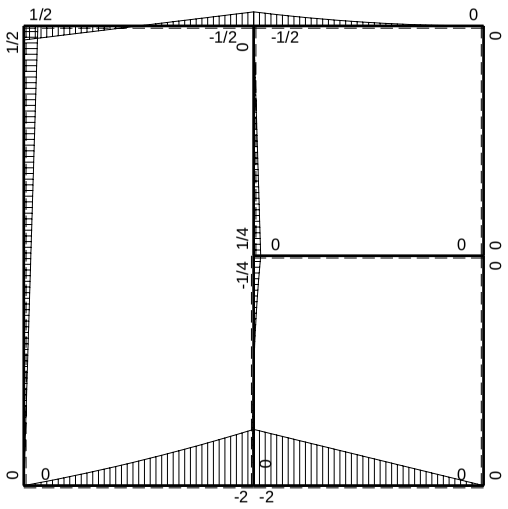
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D
E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
E
F
1/2Fb
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E
1/4F
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2F
2F

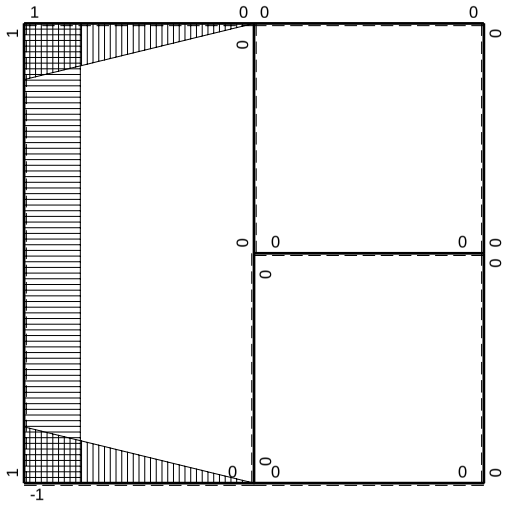

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0
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-1
-9/5
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24/5
-1
24/5
0
-1/4
-1/4
F
1
-7/10
0
1/4
-1/4
0
3/4
0
-1/4
2
-9/5
-14/5
F
1/5
0
1/5
-1/2
0
-1/2
0
1/4
0
0
-1/4
0
0
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238207 ESΣ03.xxxx.004

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238207 ESΣ03.xxxx.004
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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

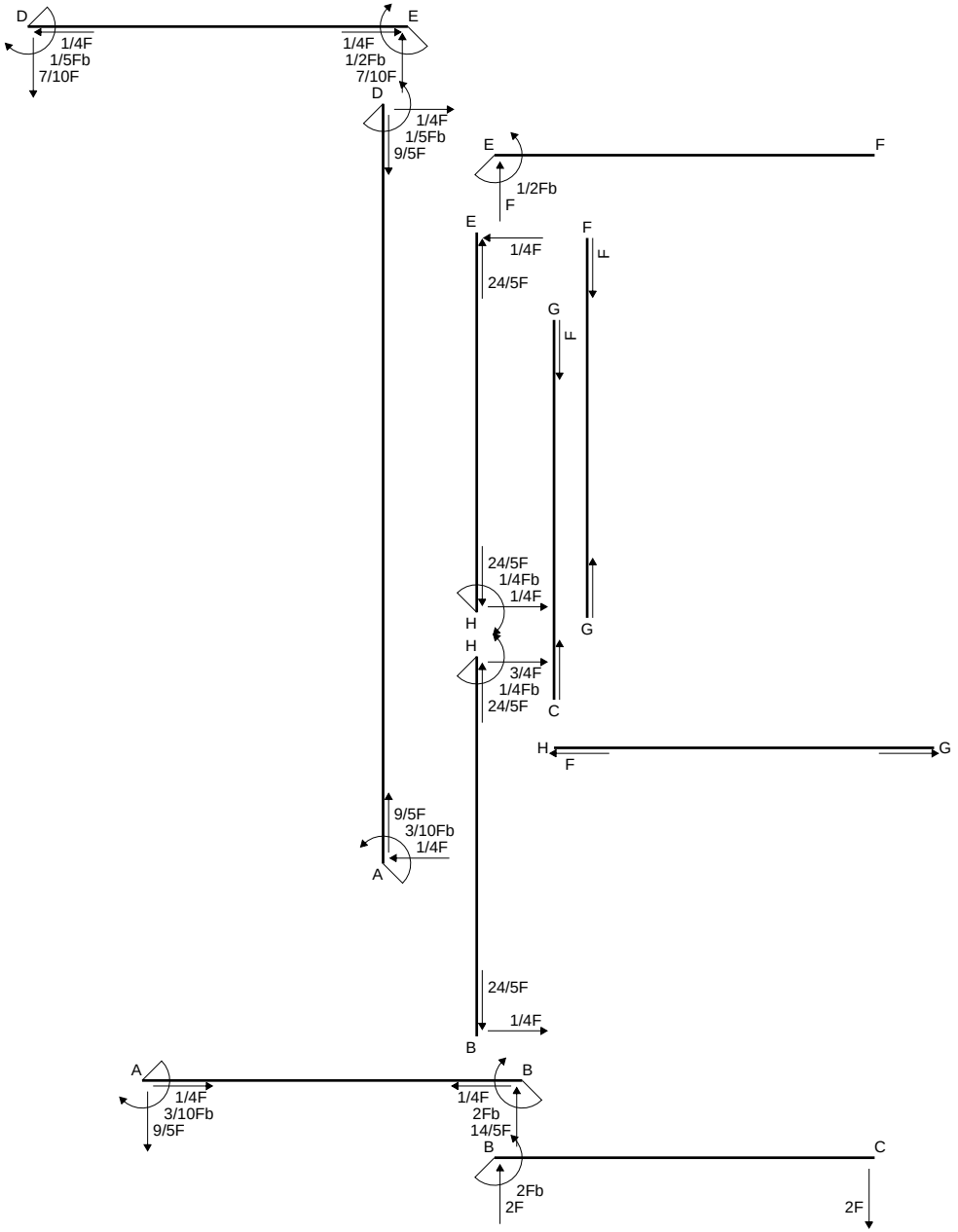
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

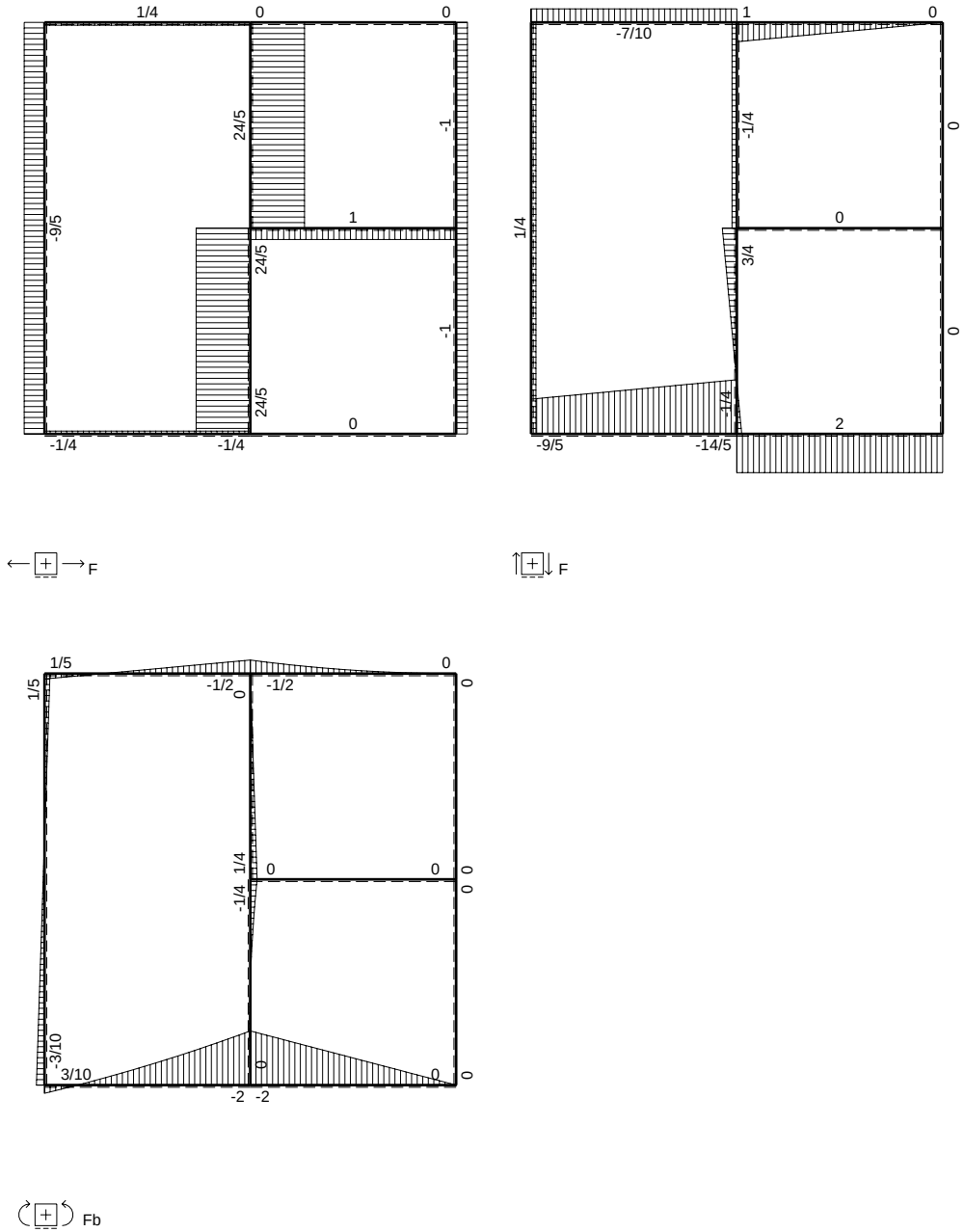
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

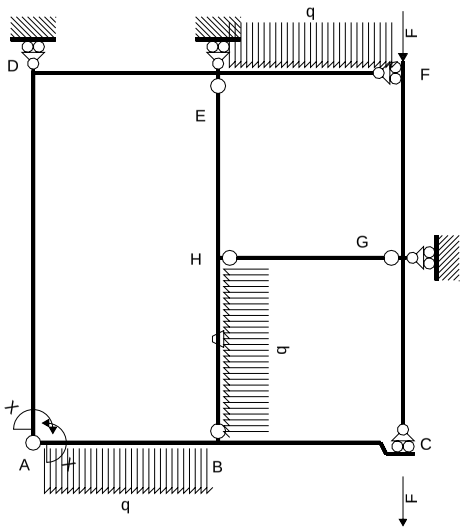
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238207 ESΣ03.xxxx.004

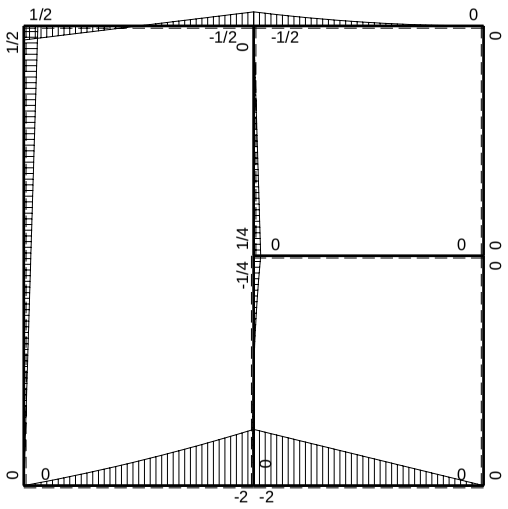
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D
E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
E
F
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F
E
1/4F
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G
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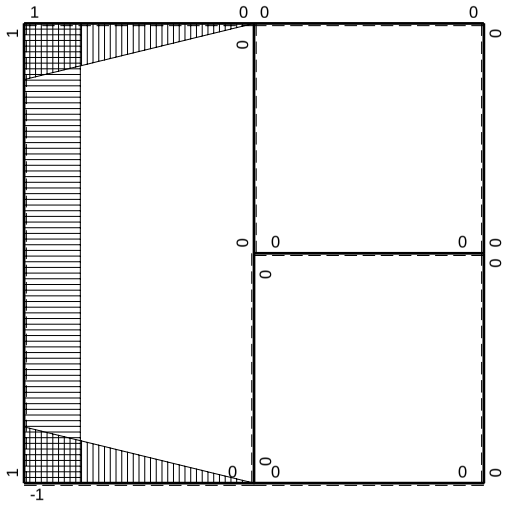

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24/5
-1
24/5
0
-1/4
-1/4
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-7/10
0
1/4
-1/4
0
3/4
0
-1/4
2
-9/5
-14/5
F
1/5
0
1/5
-1/2
0
-1/2
0
1/4
0
0
-1/4
0
0
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238304 ESΣ03.xxxx.005

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
EH b	0	-1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238304 ESΣ03.xxxx.005
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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

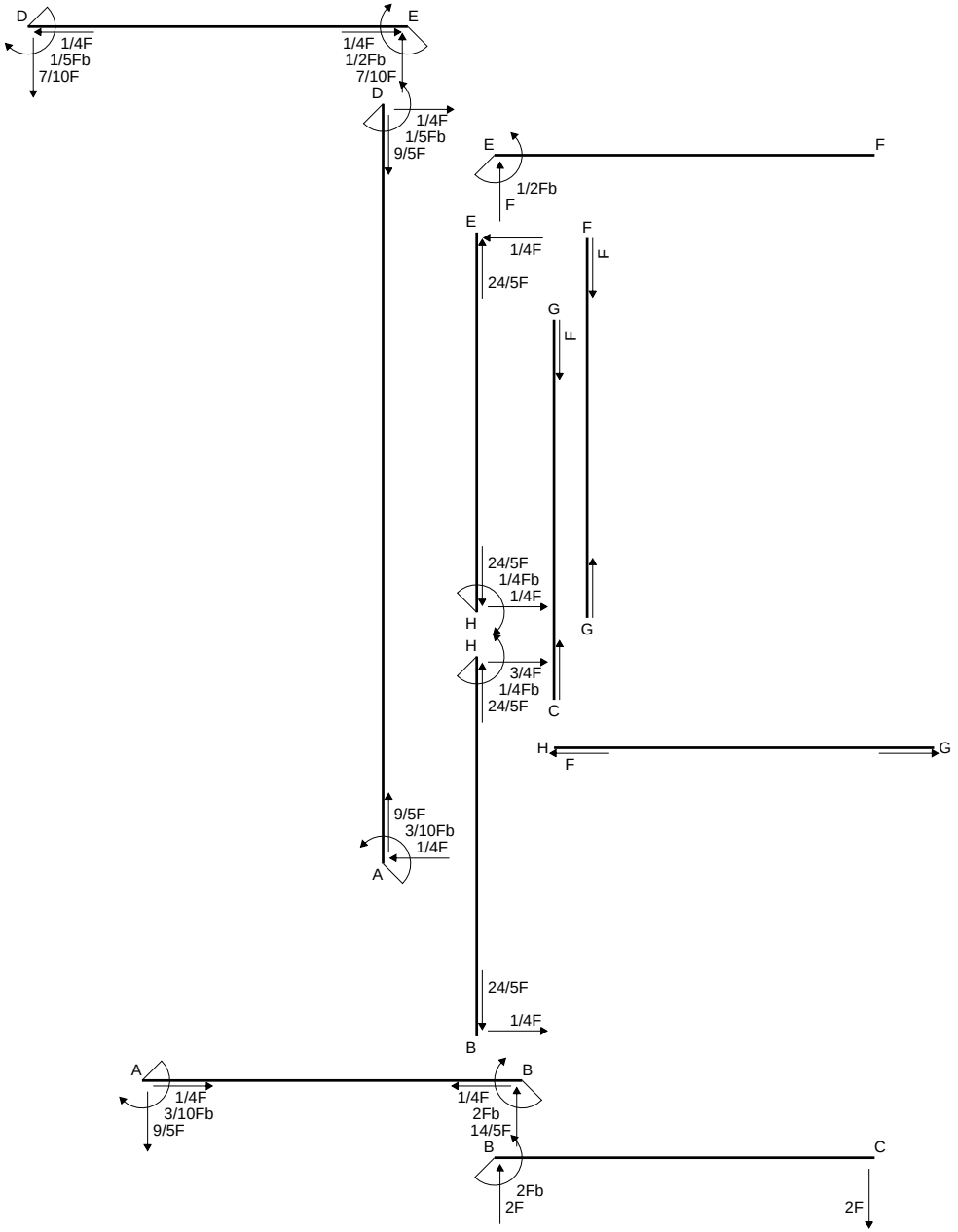
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

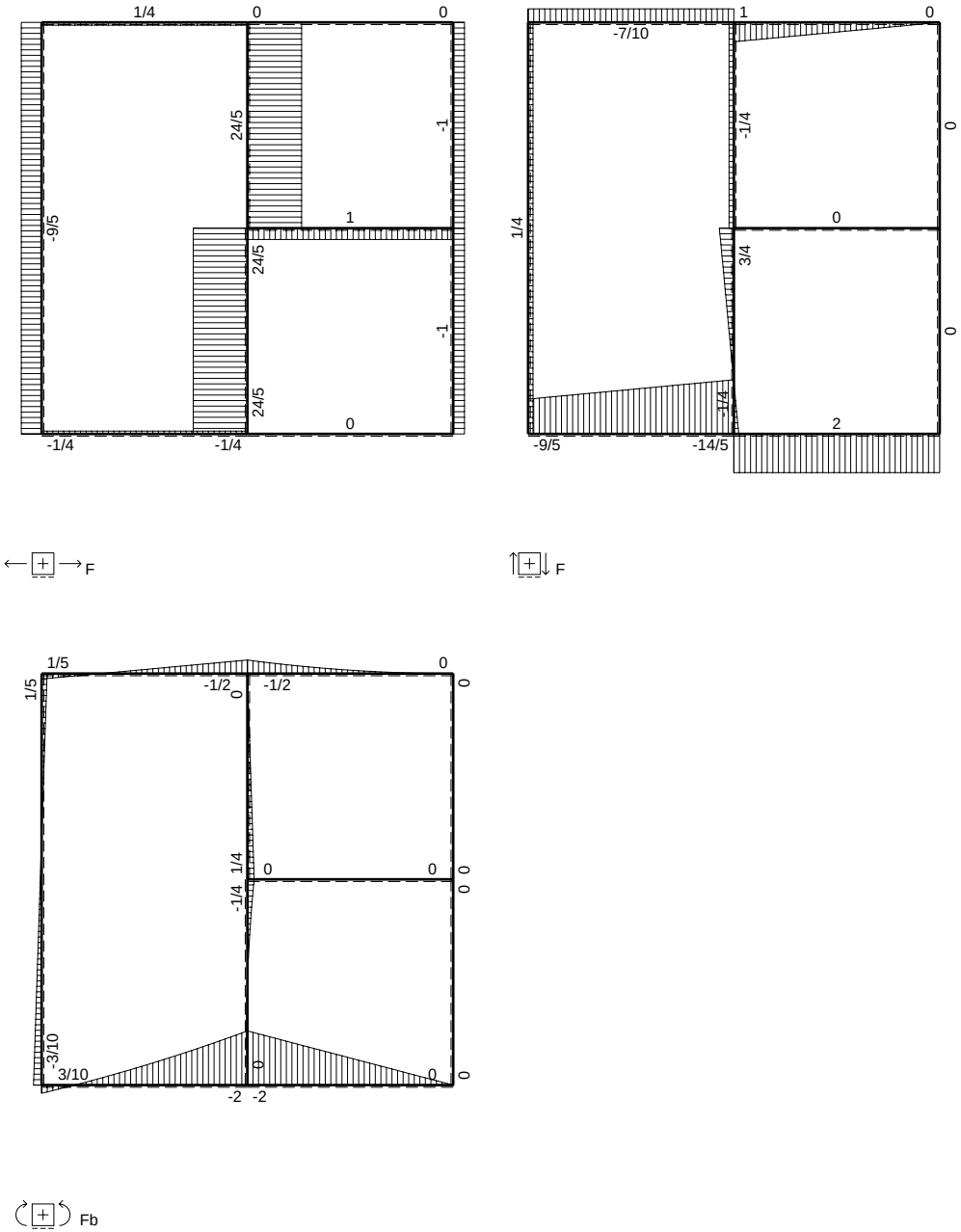
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

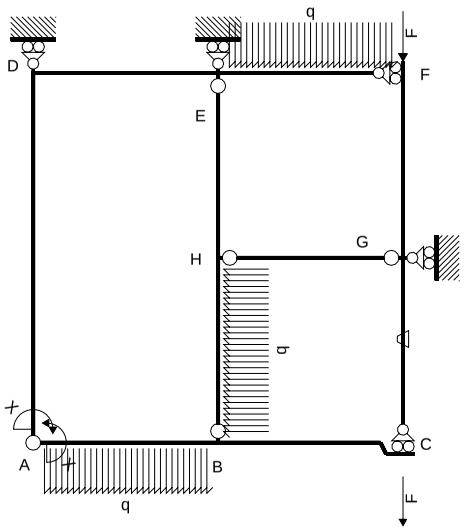
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238304 ESΣ03.xxxx.005

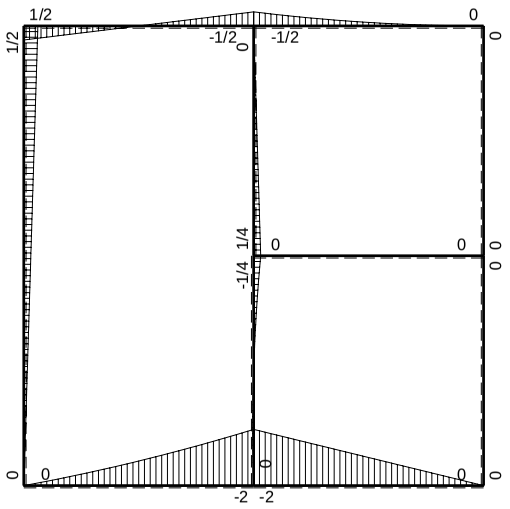
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D
E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
E
F
1/2Fb
F
E
1/4F
24/5F
F
F
G
F
24/5F
1/4Fb
1/4F
H
G
H
3/4F
1/4Fb
24/5F
C
H
F
G
9/5F
3/10Fb
1/4F
A
24/5F
1/4F
B
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B
C
2Fb
2F
2F

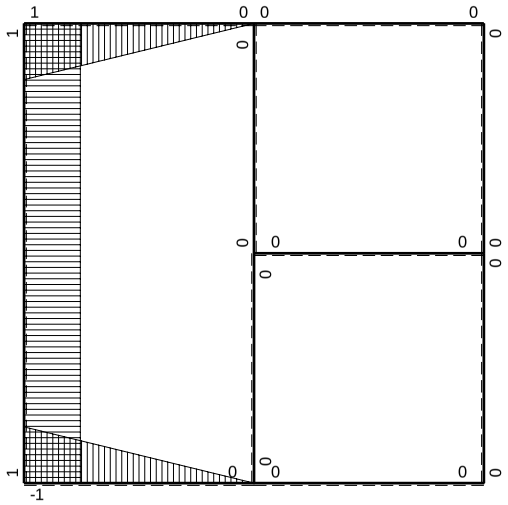

1/4
0
0
24/5
-1
-9/5
1
24/5
-1
24/5
0
-1/4
-1/4
F
1
0
-7/10
-1/4
0
1/4
3/4
0
-1/4
2
-9/5
-14/5
F
1/5
0
-1/2
-1/2
0
0
1/5
0
0
1/4
-1/4
0
0
0
0
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 310182 ESΣ03.xxxx.006

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PROCEDIMENTO E RISULTATI 310182 ESΣ03.xxxx.006

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
EH b	0	-1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

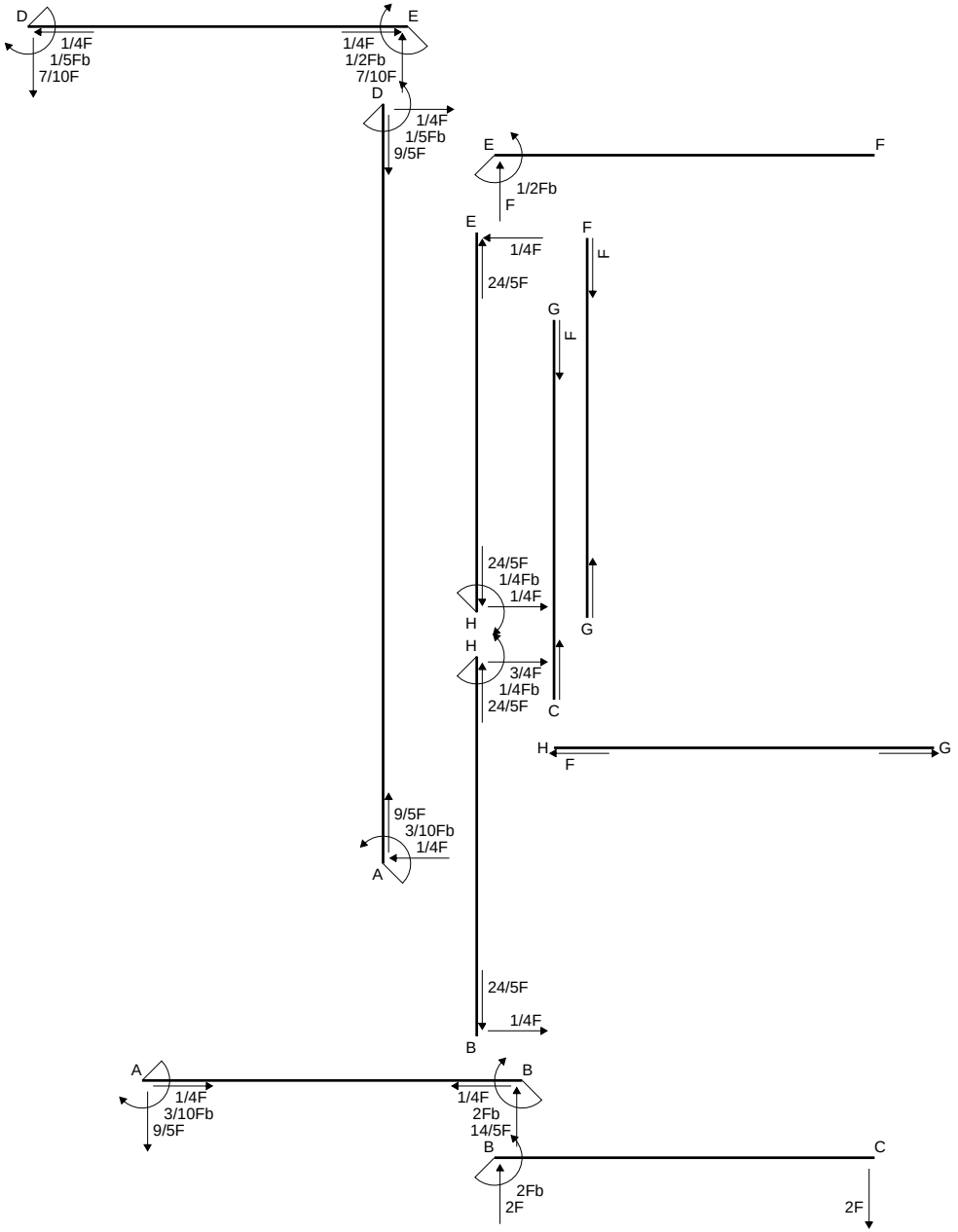
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

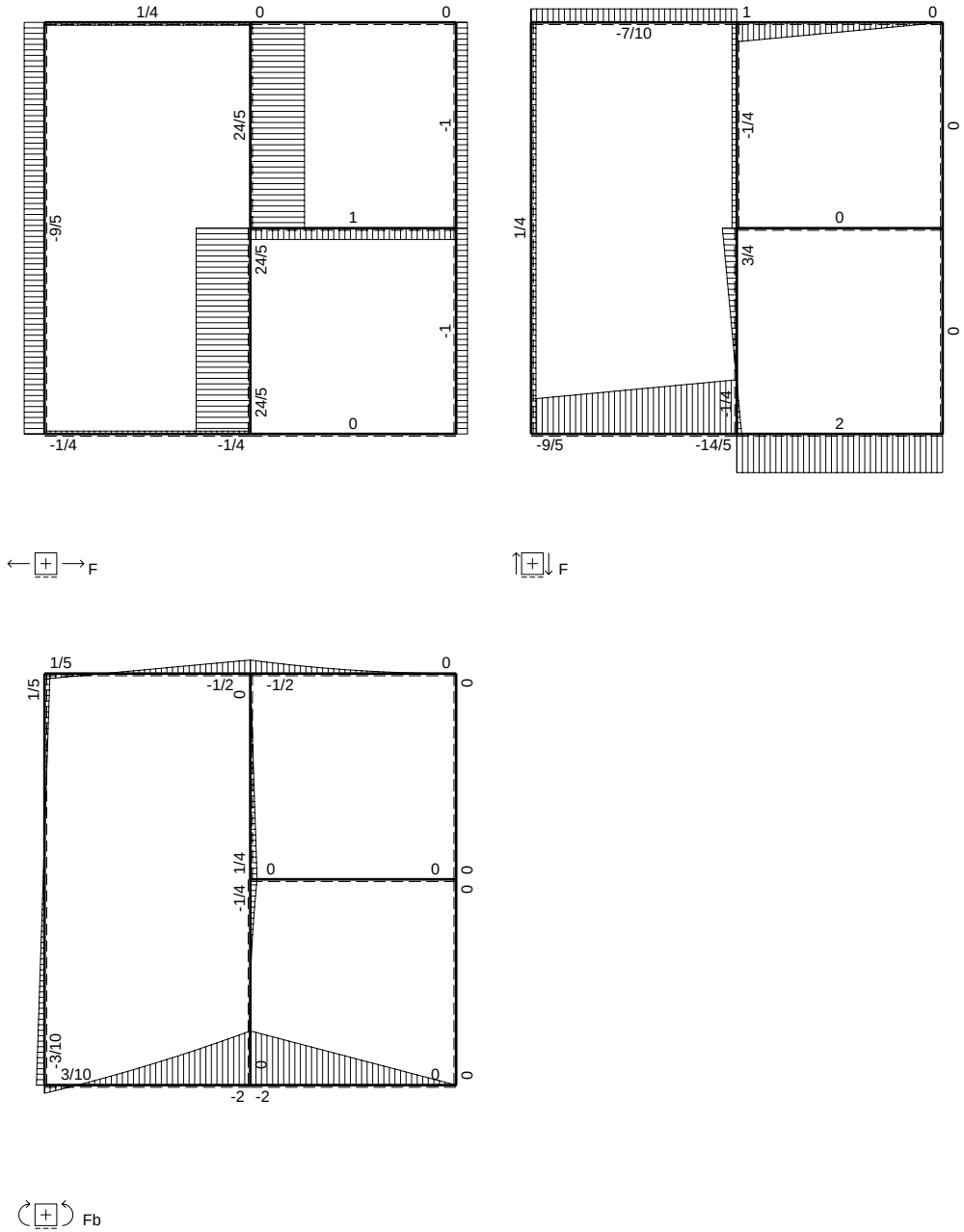
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

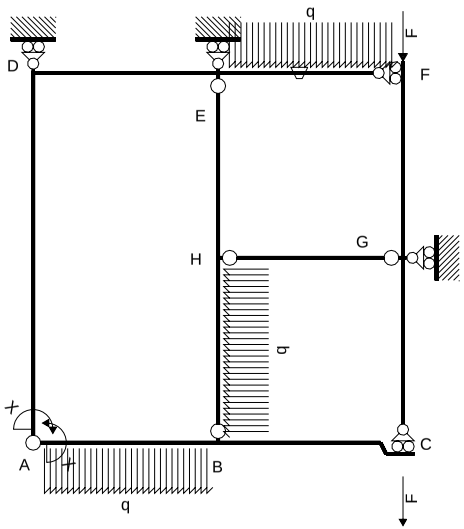
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 310182 ESΣ03.xxxx.006

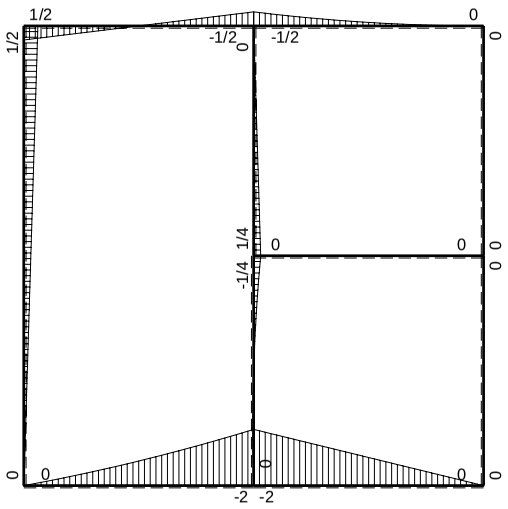
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D
E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
E
F
1/2Fb
F
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1/4F
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F
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G
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1/4Fb
24/5F
C
H
F
G
9/5F
3/10Fb
1/4F
A
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B
C
2Fb
2F
2F

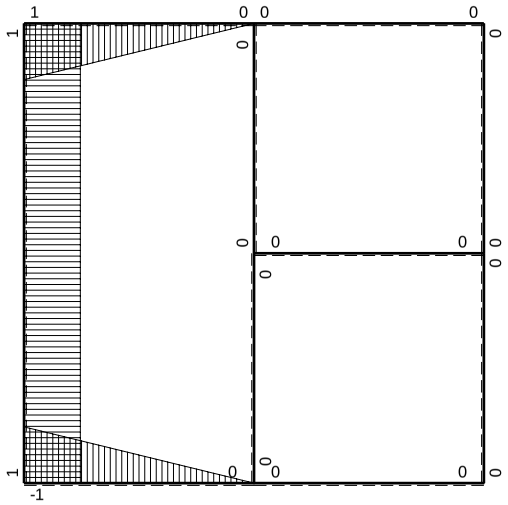

1/4
0
0
24/5
-1
-9/5
1
24/5
-1
24/5
0
-1/4
-1/4
F
1
-7/10
0
1/4
-1/4
0
3/4
0
-1/4
2
-9/5
-14/5
F
1/5
0
1/5
-1/2
0
-1/2
0
0
1/4
0
0
-1/4
0
0
-3/10
3/10
0
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236761 ESΣ03.xxxx.007

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PROCEDIMENTO E RISULTATI 236761 ESΣ03.xxxx.007

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
EH b	0	-1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

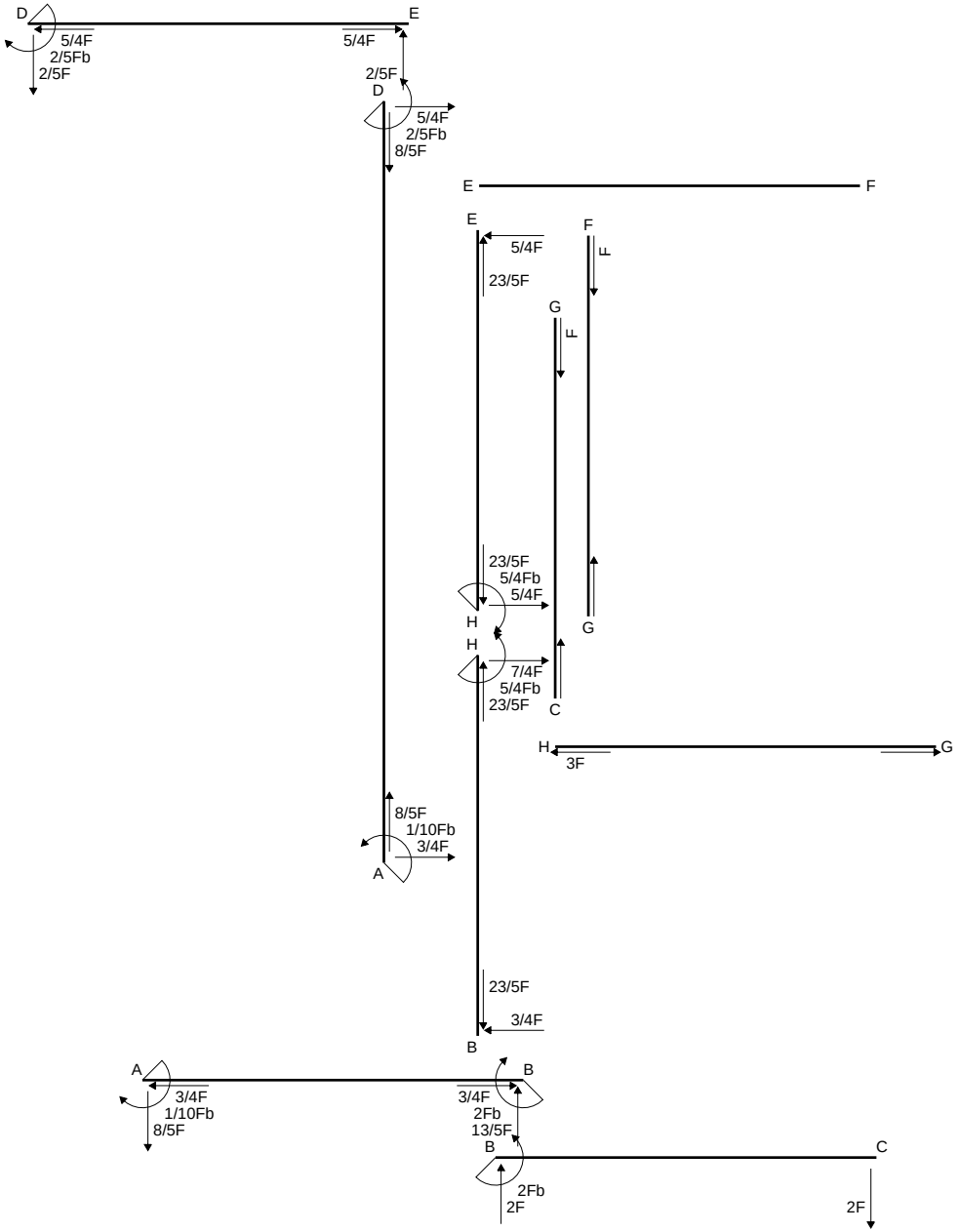
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

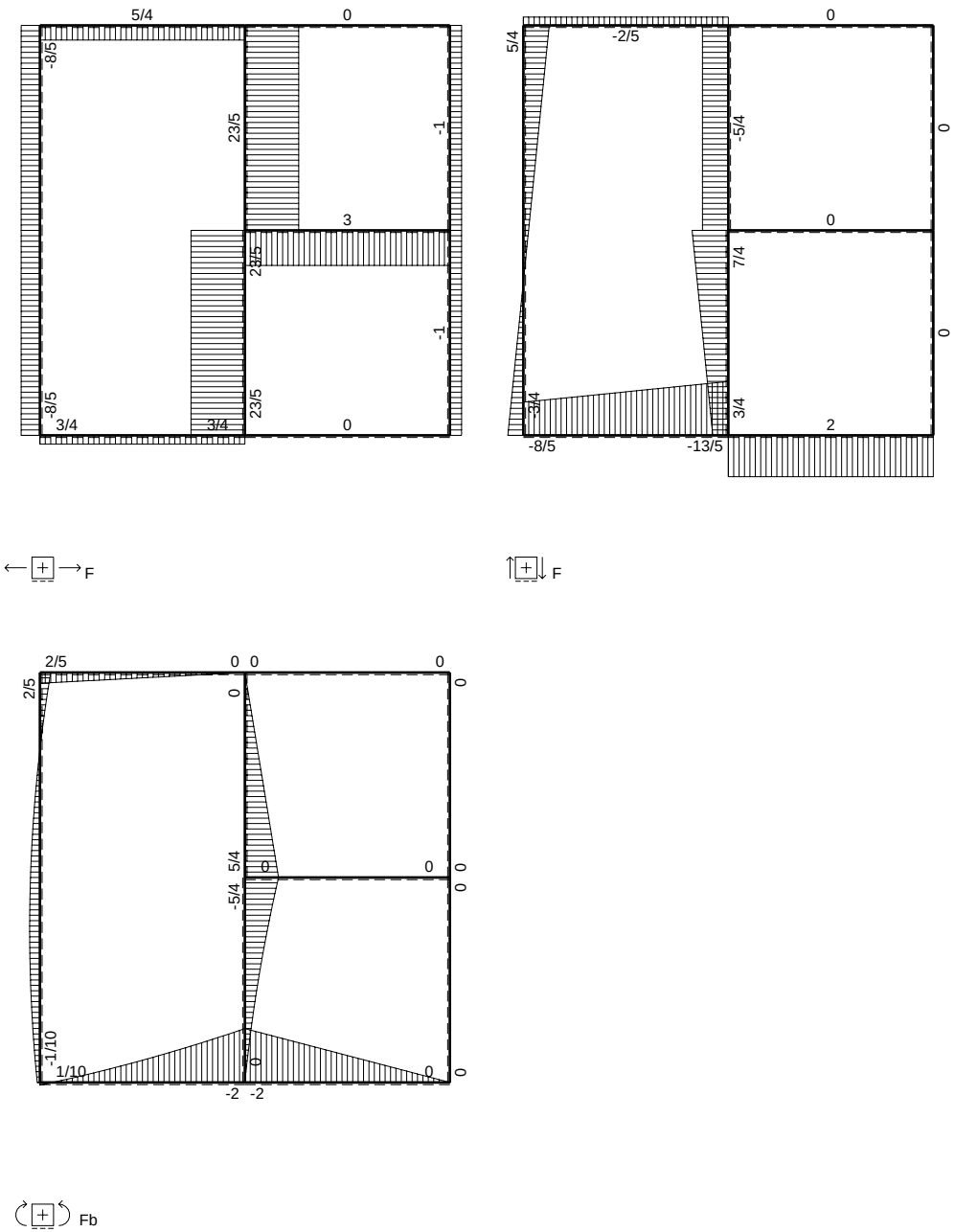
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

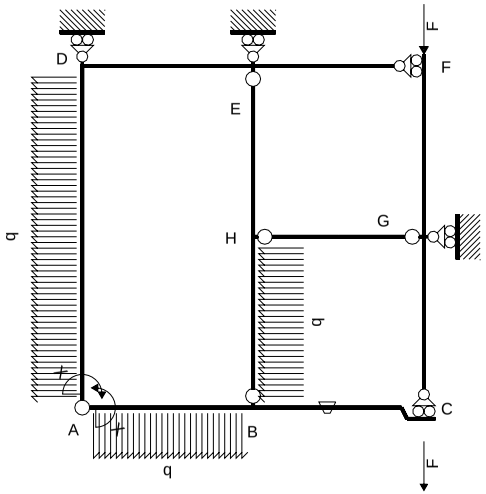
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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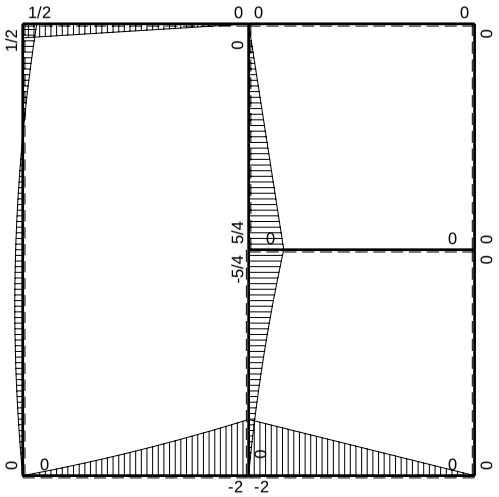
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5/4F
2/5F
D
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2/5Fb
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23/5F
F
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23/5F
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C
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2Fb
2F
2F

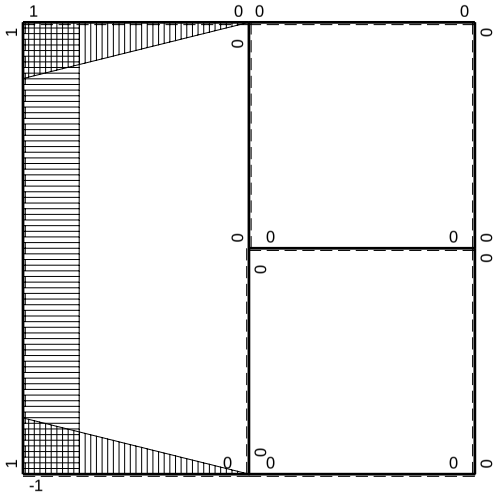

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0
0
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5/4
0
0
0
-5/4
0
0
-1/10
0
1/10
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$$L^{XX}_{AB} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right)\ 1/EJ\, dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$$

$$L^{XX}_{BA} = \int_o^b \left(\, x^2/b^2 \,\right)\ 1/EJ\, dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\, x^3/b^2 \,\right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$$

$$= \left(1/3\, b\right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$$

$$L^{XX}_{AD} = \int_o^{2b} \left(1\right)\ 1/EJ\, dx = \left[\, x \,\right]_o^{2b}\ 1/EJ$$

$$= \left(2\, b\right)\ 1/EJ\ = 2\ \ b/EJ$$

$$L^{XX}_{DA} = \int_o^{2b} \left(1\right)\ 1/EJ\, dx = \left[\, x \,\right]_o^{2b}\ 1/EJ$$

$$= \left(2\, b\right)\ 1/EJ\ = 2\ \ b/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right)\ 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b\ 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(\, x^2/b^2 \,\right)\ 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b\ 1/EJ$$

$$= \left(1/3\, b\right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L^{Xo}_{AB} = \int_o^b \left(3/2\, x/b - x^2/b^2 - 1/2\, x^3/b^3\right) Fb\ 1/EJ\, dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left[3/4\, x^2/b - 1/3\, x^3/b^2 - 1/8\, x^4/b^3 \,\right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left(3/4\, b - 1/3\, b - 1/8\, b\right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$$

$$L^{Xo}_{BA} = \int_o^b \left(2\, x/b - 5/2\, x^2/b^2 + 1/2\, x^3/b^3\right) Fb\ 1/EJ\, dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left[\, x^2/b - 5/6\, x^3/b^2 + 1/8\, x^4/b^3 \,\right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left(\, b - 5/6\, b + 1/8\, b\,\right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$$

$$L^{Xo}_{AD} = \int_o^{2b} \left(-3/4\, x/b + 1/2\, x^2/b^2\right) Fb\ 1/EJ\, dx = \left[-3/8\, x^2/b + 1/6\, x^3/b^2 \,\right]_o^{2b}\ Fb\ 1/EJ$$

$$= \left(-3/2\, b + 4/3\, b\right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L^{Xo}_{DA} = \int_o^{2b} \left(1/2 - 5/4\, x/b + 1/2\, x^2/b^2\right) Fb\ 1/EJ\, dx = \left[1/2\, x - 5/8\, x^2/b + 1/6\, x^3/b^2 \,\right]_o^{2b}\ Fb\ 1/EJ$$

$$= \left(\, b - 5/2\, b + 4/3\, b\,\right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L^{Xo}_{DE} = \int_o^b \left(1/2 - x/b + 1/2\, x^2/b^2\right) Fb\ 1/EJ\, dx = \left[1/2\, x - 1/2\, x^2/b + 1/6\, x^3/b^2 \,\right]_o^b\ Fb\ 1/EJ$$

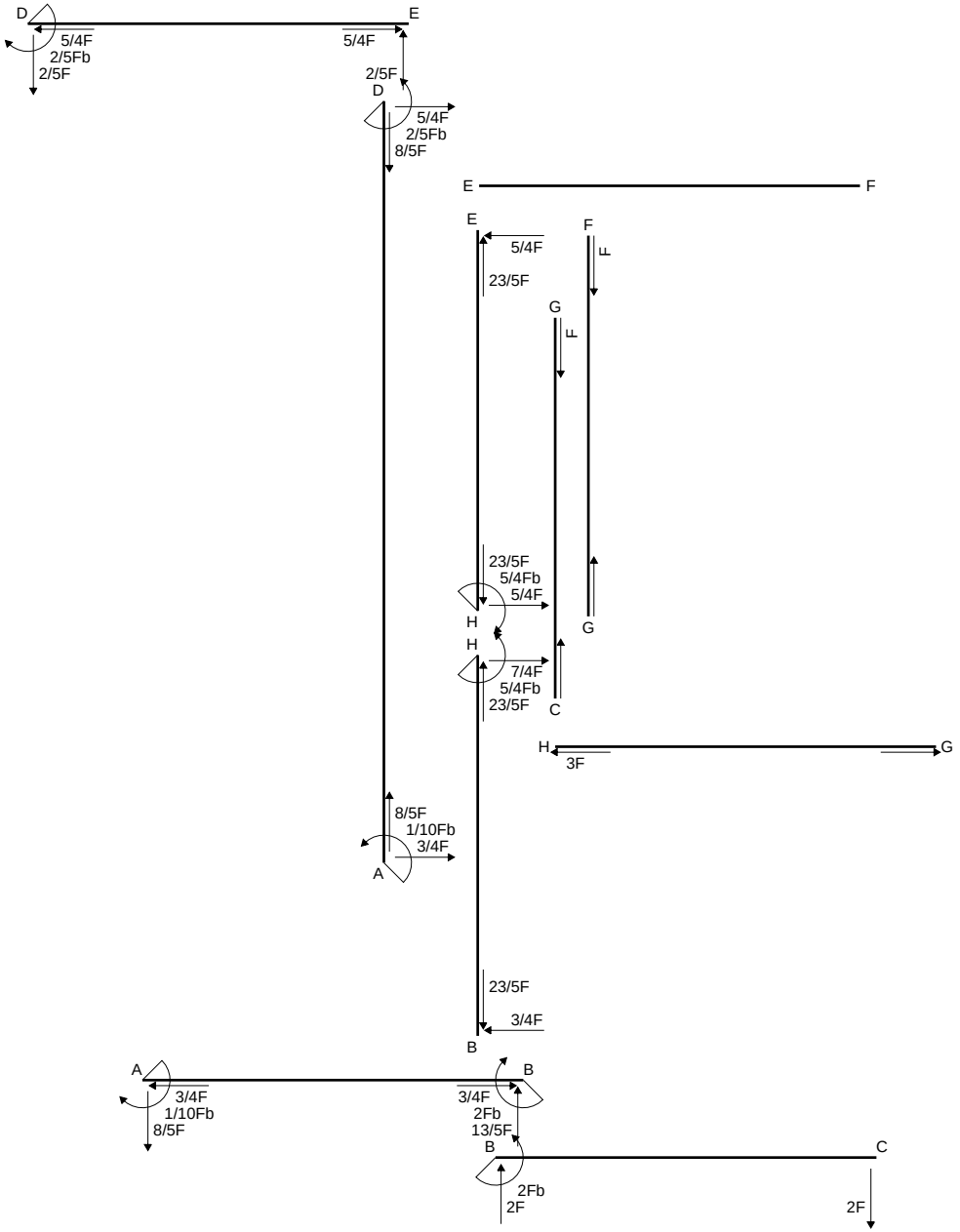
$$= \left(1/2\, b - 1/2\, b + 1/6\, b\right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$$

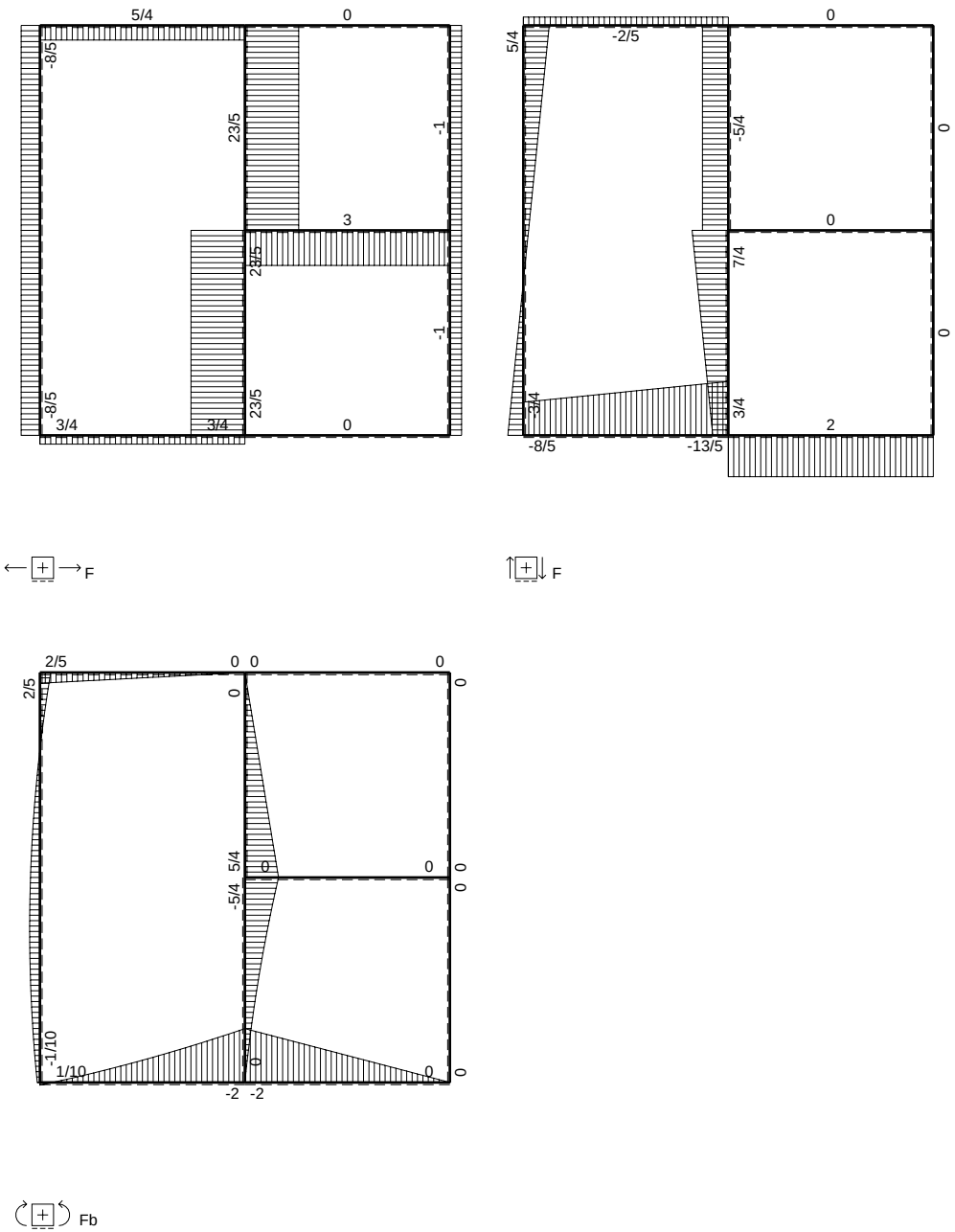
$$L^{Xo}_{ED} = \int_o^b \left(1/2\, x^2/b^2\right) Fb\ 1/EJ\, dx = \left[1/6\, x^3/b^2 \,\right]_o^b\ Fb\ 1/EJ$$

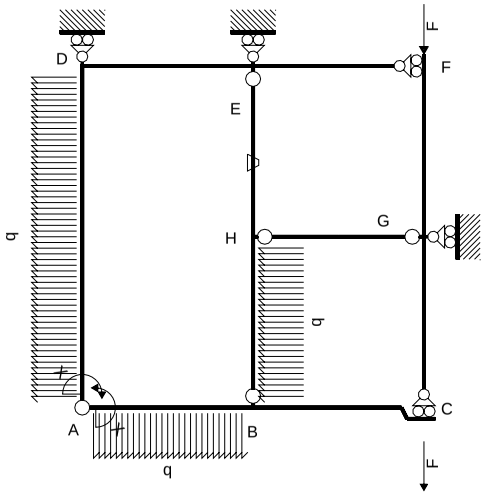
$$= \left(1/6\, b\right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$$

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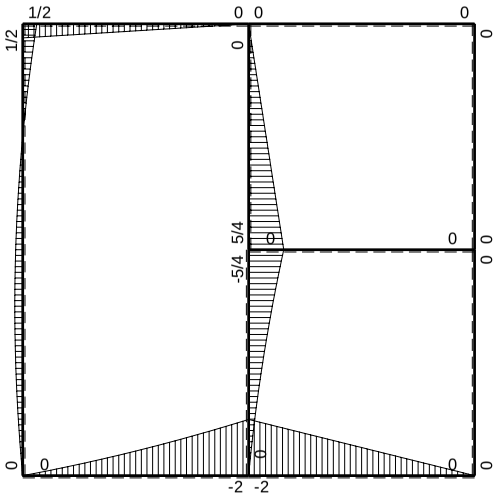
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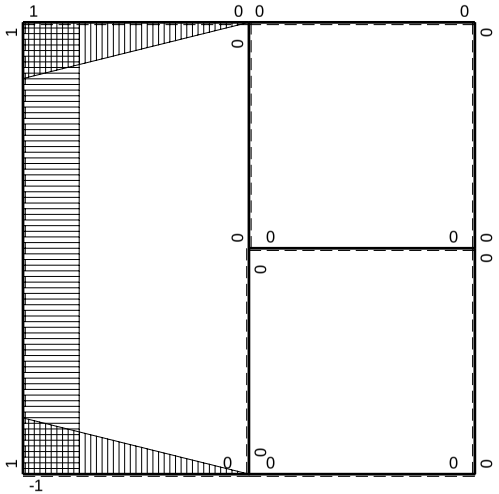

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0
0
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0
0 0
-5/4
0
-1/10
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1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-5/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(1/2 -5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x -5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(\ b -5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(1/2 - x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

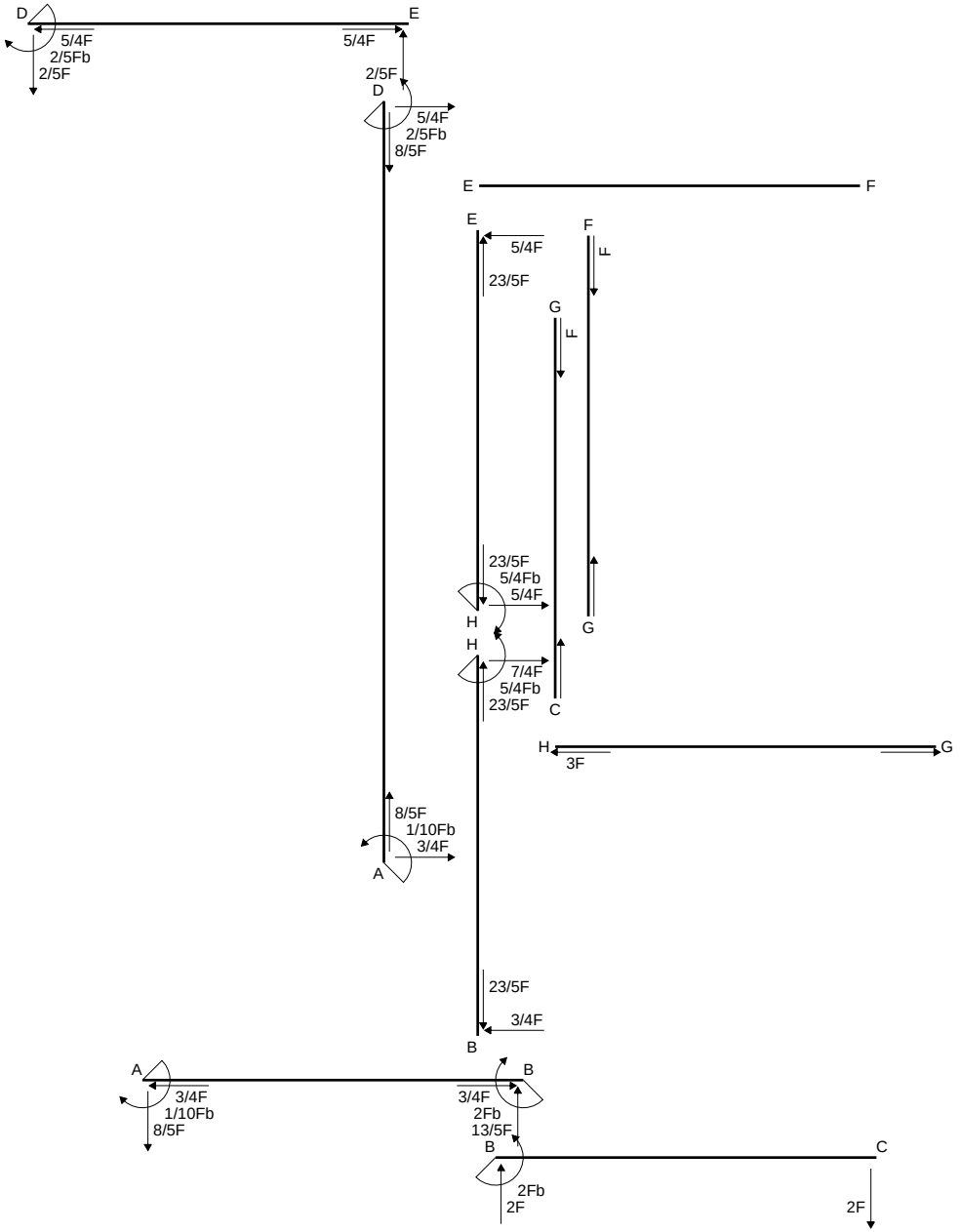
$= \left(1/2\ b -1/2\ b +1/6\ b\ \right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

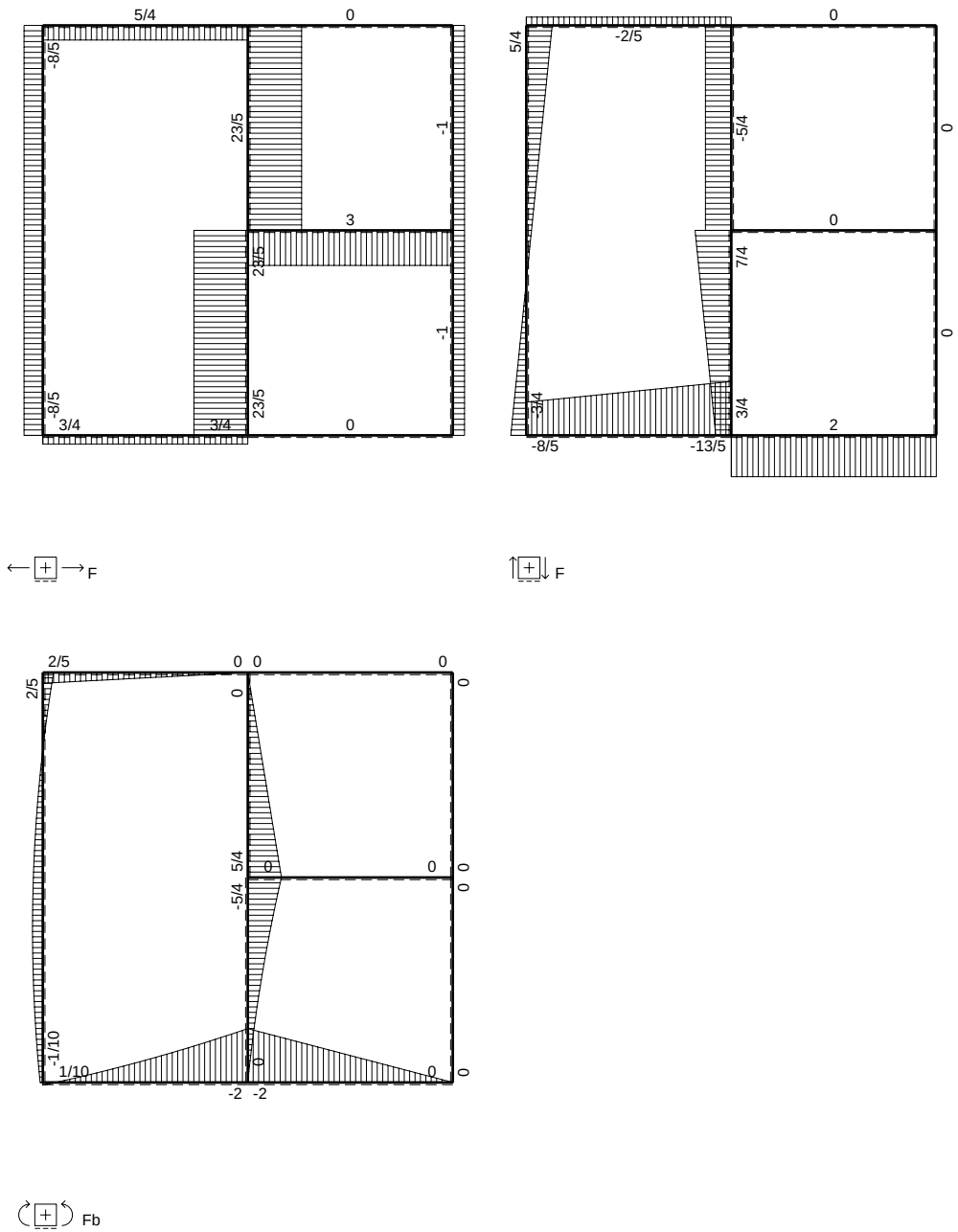
$L^{Xo}_{ED} = \int_o^b \left(1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

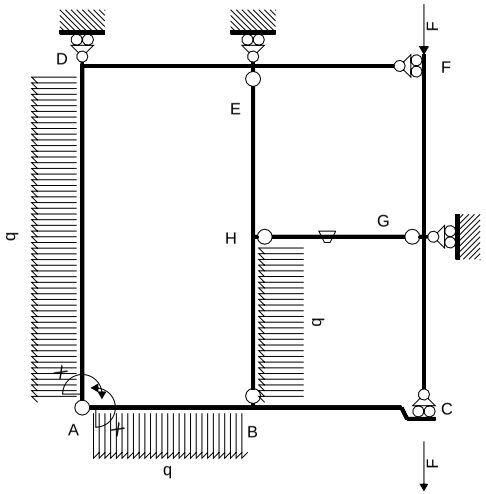
$= \left(1/6\ b\ \right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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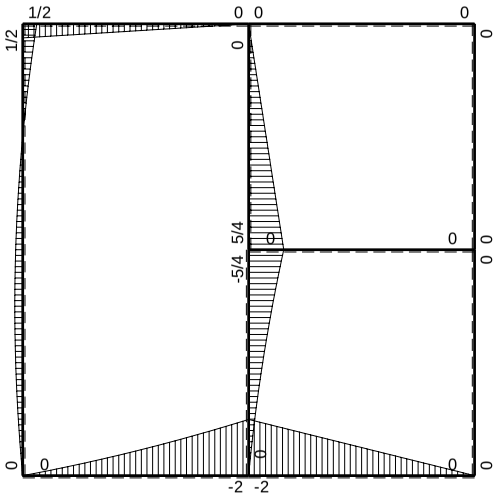
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D
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5/4F
2/5Fb
2/5F
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2/5F
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2/5Fb
8/5F
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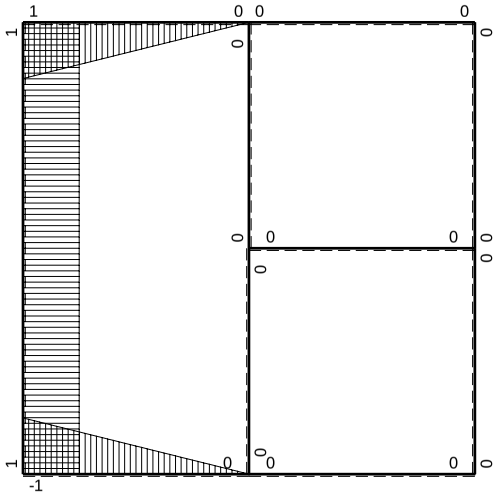

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0
0
5/4
0
0 0
-5/4
0
-1/10
0
1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

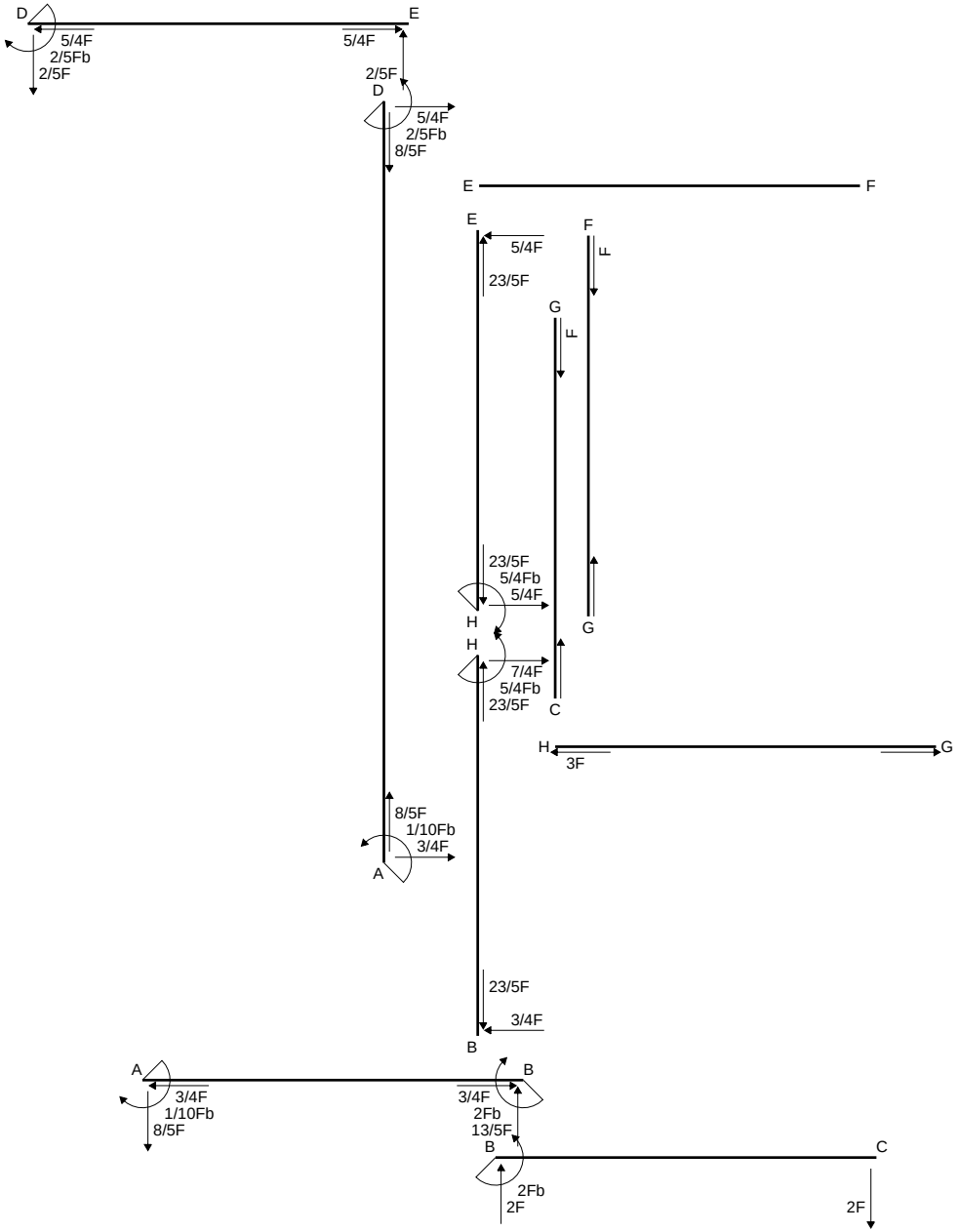
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

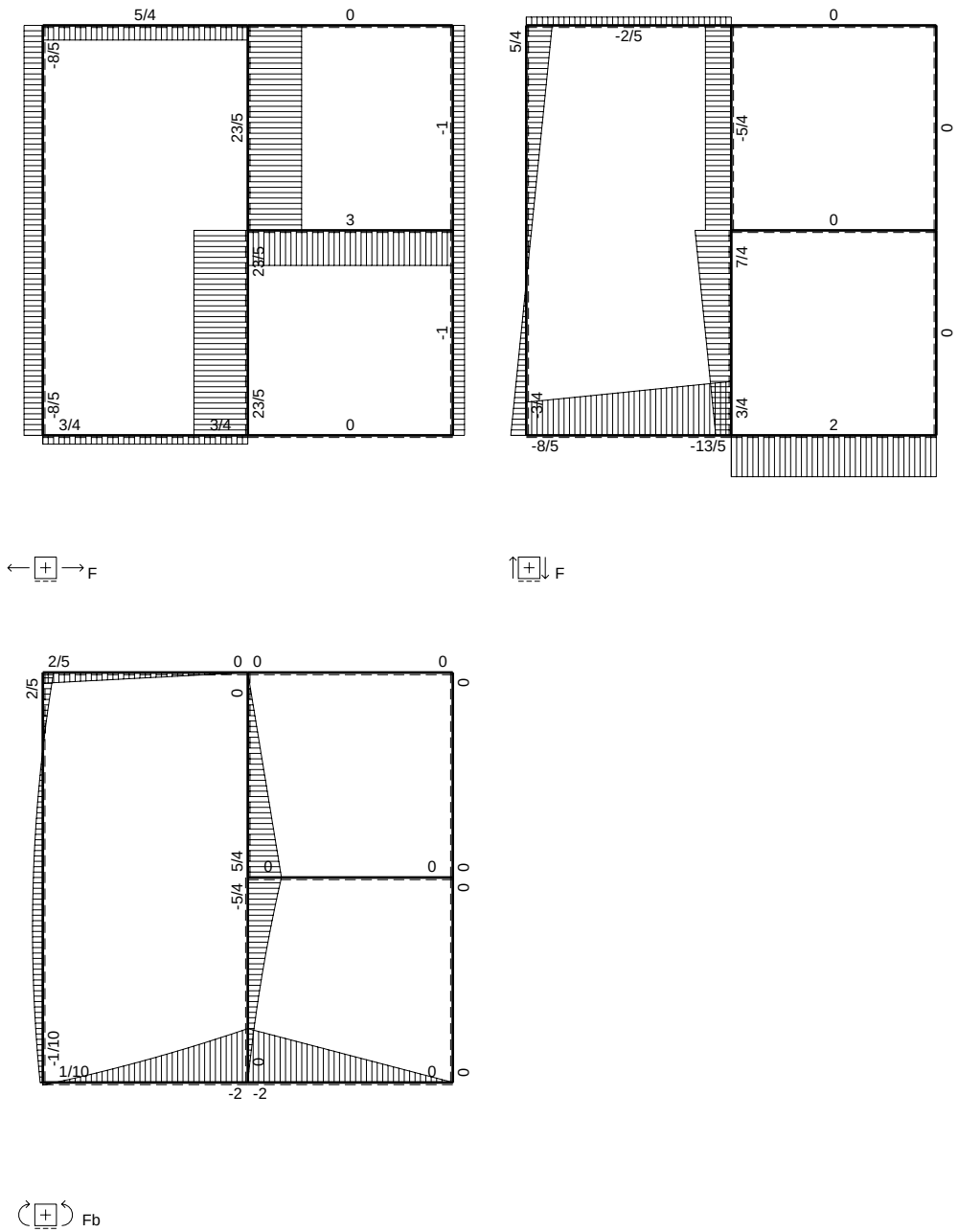
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

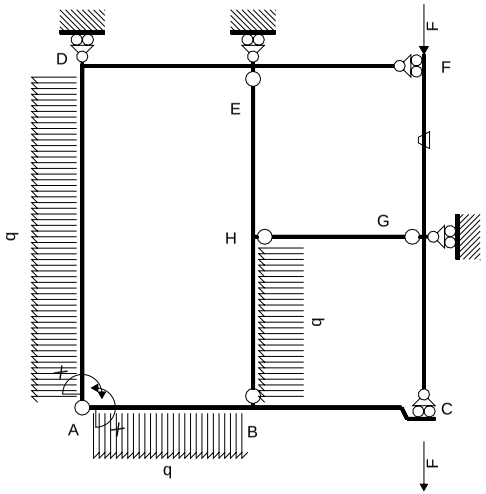
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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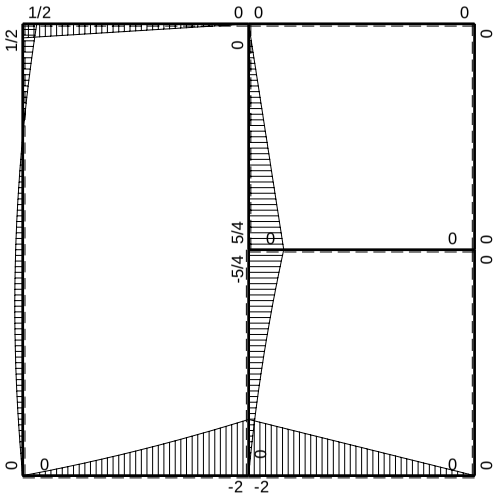
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2/5F
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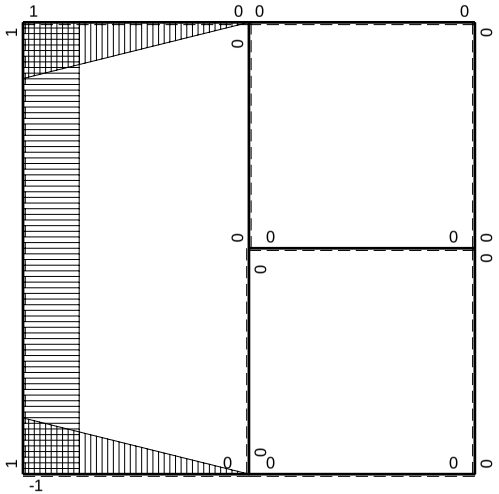

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-13/5
F
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0
2/5
0
0
0
5/4
0
0 0
-5/4
0
-1/10
0
1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$$L^{XX}_{AB} = \int_o^b \left(1 - 2\,x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$$

$$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$$

$$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$$

$$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$$

$$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$$

$$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 - 2\,x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left[3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left(3/4\ b - 1/3\ b - 1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$$

$$L^{Xo}_{BA} = \int_o^b \left(2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left[\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left(\ b - 5/6\ b + 1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$$

$$L^{Xo}_{AD} = \int_o^{2b} \left(-3/4\ x/b + 1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$$

$$= \left(-3/2\ b + 4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L^{Xo}_{DA} = \int_o^{2b} \left(1/2 - 5/4\ x/b + 1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$$

$$= \left(\ b - 5/2\ b + 4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L^{Xo}_{DE} = \int_o^b \left(1/2 - x/b + 1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

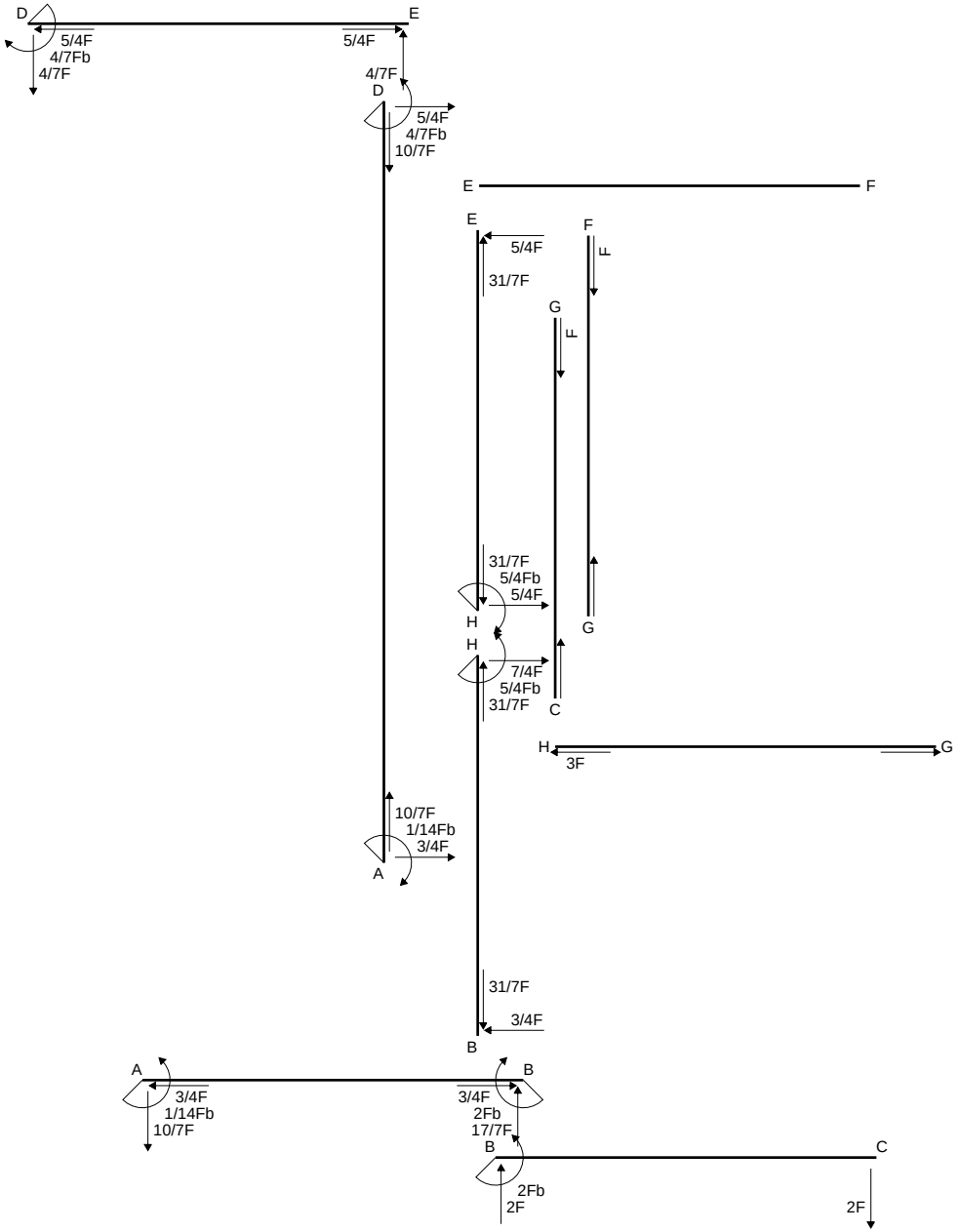
$$= \left(1/2\ b - 1/2\ b + 1/6\ b\ \right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$$

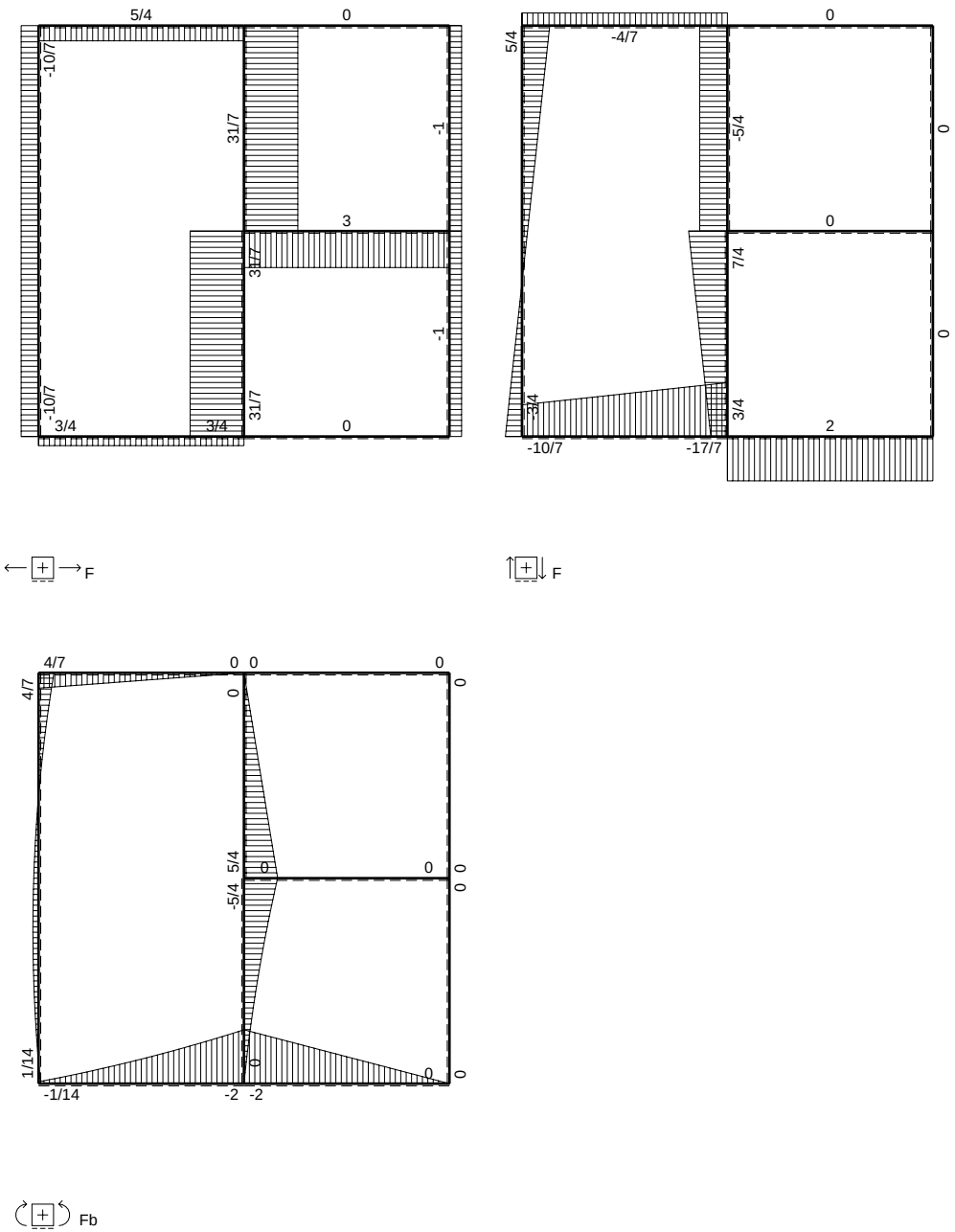
$$L^{Xo}_{ED} = \int_o^b \left(1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$$

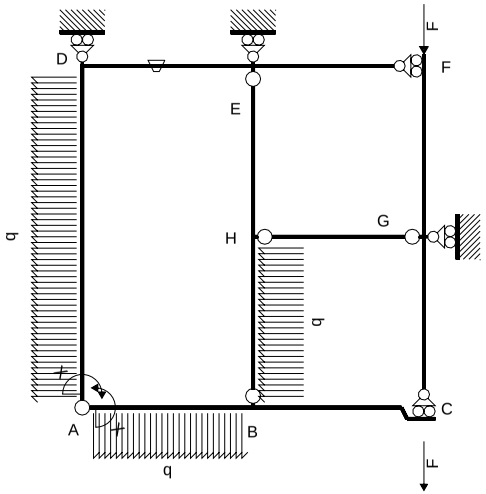
$$= \left(1/6\ b\ \right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$$

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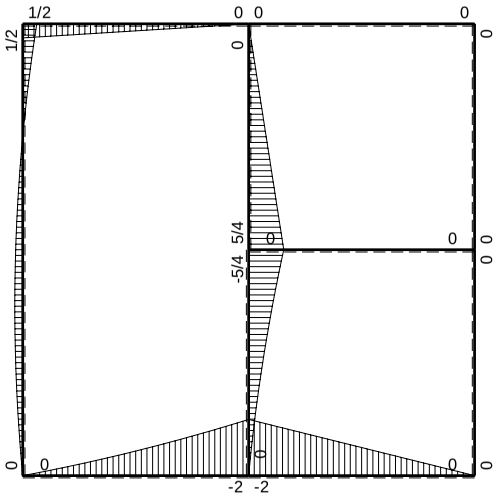
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4/7Fb
4/7F
5/4F
4/7F
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4/7Fb
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5/4Fb
31/7F
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3F
10/7F
1/14Fb
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B
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2F
2F

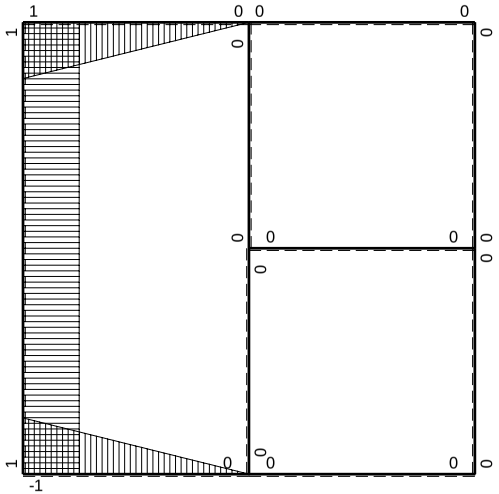

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0
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5/4
0
0
0
-5/4
0
0
1/14
0
0
-1/14
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	$-Fb/EJ$	$1/2Fb-Fx+1/2Fx^2/b$	$-Fb/EJ+Fx/EJ$	$1-2x/b+x^2/b^2$	$(1/6-1/2)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	Fb/EJ	$1/2Fx^2/b$	$-Fx/EJ$	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(1/2 -5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x -5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(\ b -5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(1/2 - x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

$= \left[1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x +1/2\ x^2/b\ \right]_o^b\ \theta$

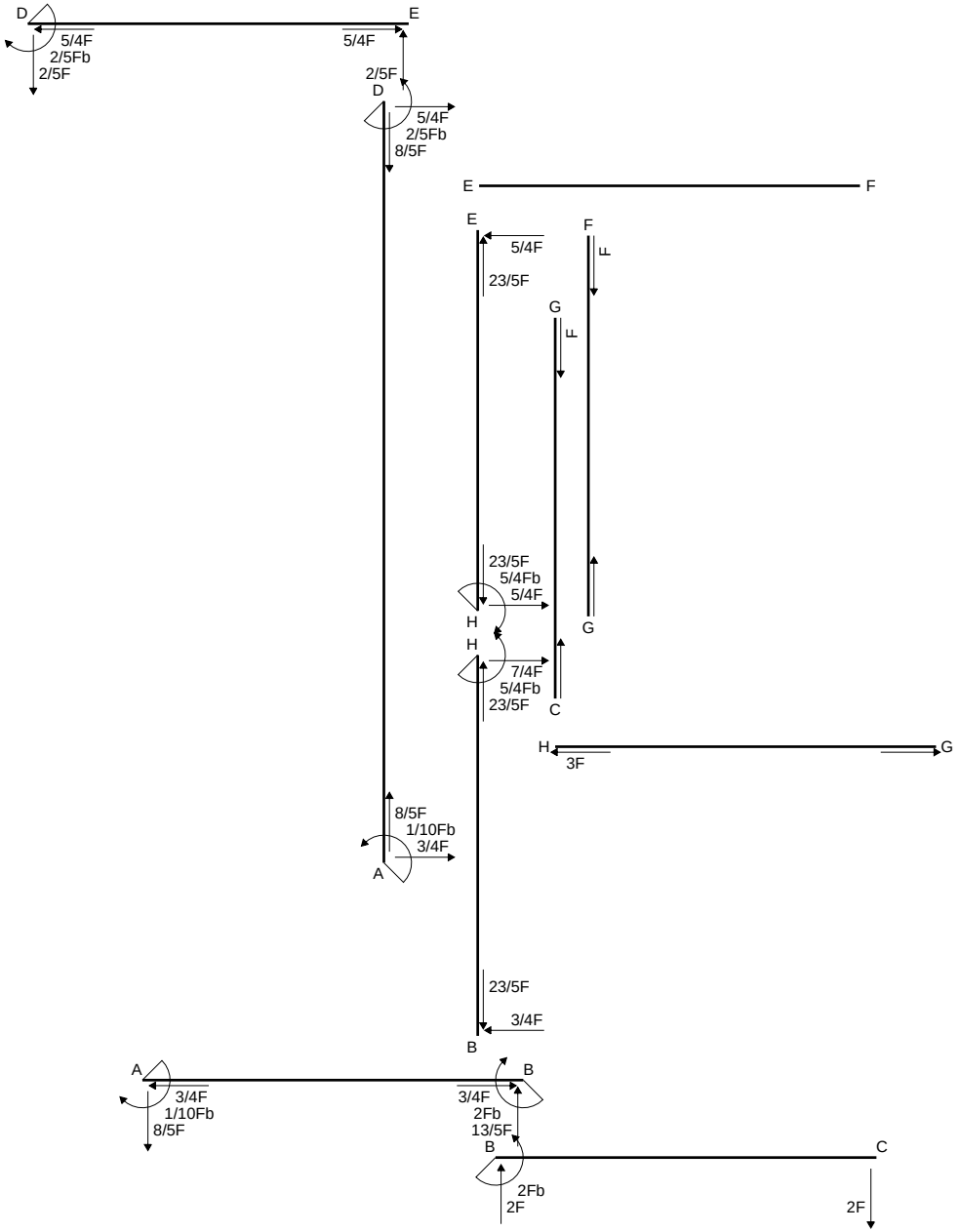
$= \left(1/2\ b -1/2\ b +1/6\ b\ \right) Fb\ 1/EJ\ + \left(-\ b +1/2\ b\ \right)\ \theta\ \ = -1/3\ \ Fb^2/EJ$

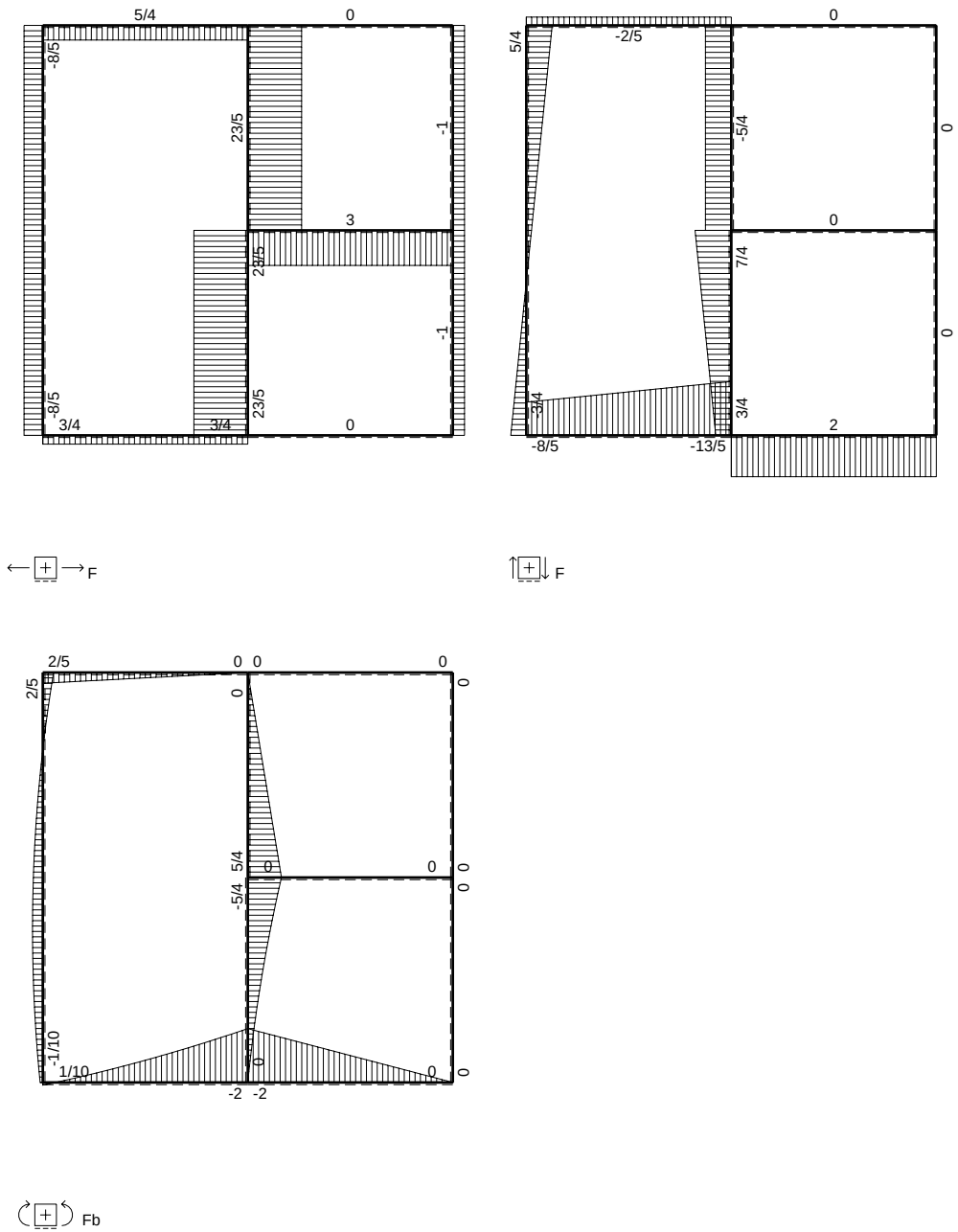
$L^{Xo}_{ED} = \int_o^b \left(1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx = \left[1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

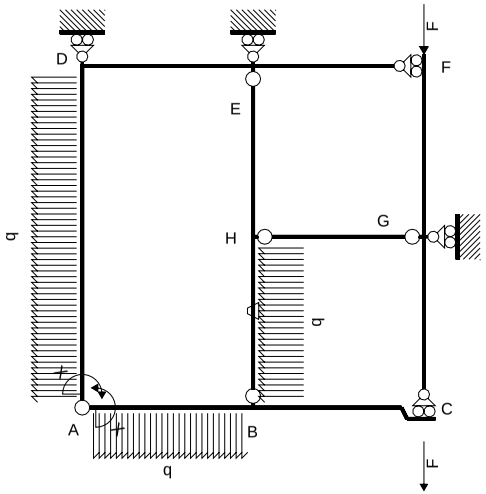
$= \left(1/6\ b\ \right) Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = -1/3\ \ Fb^2/EJ$

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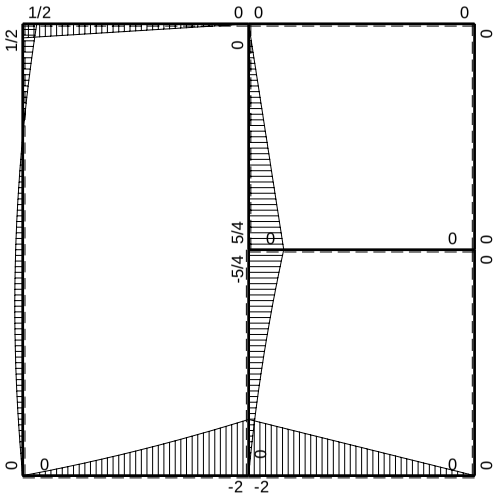
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2/5F
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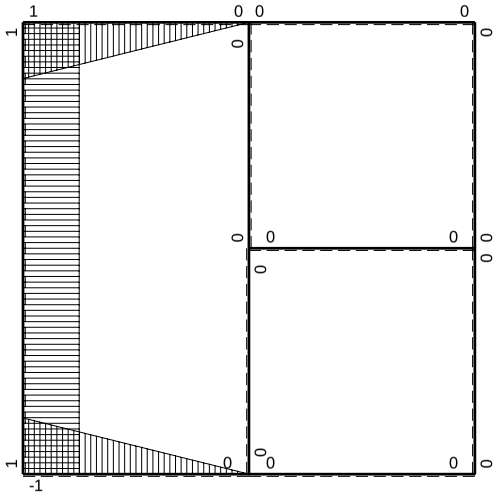

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0
0
5/4
0
0 0
-5/4
0
-1/10
0
1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

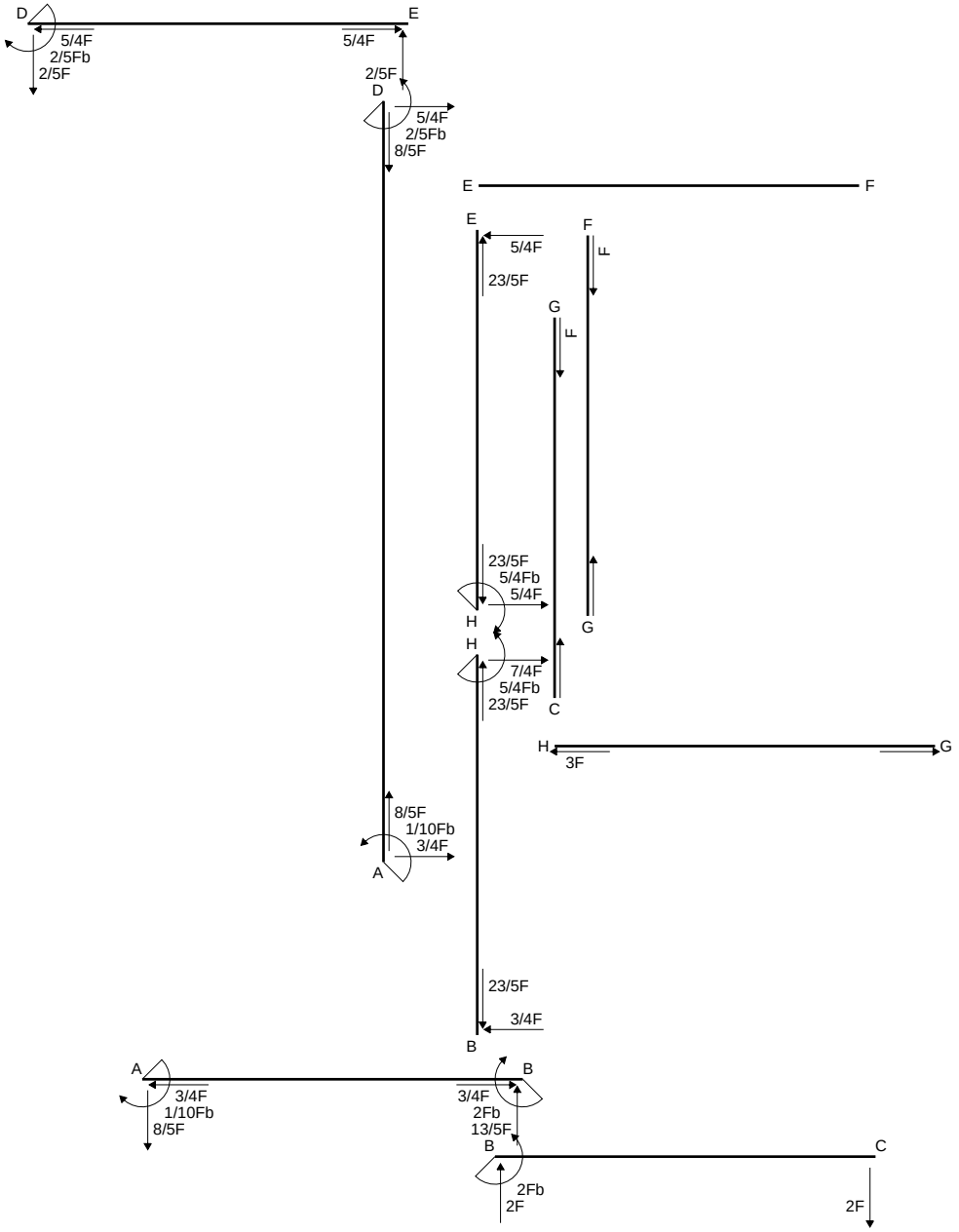
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

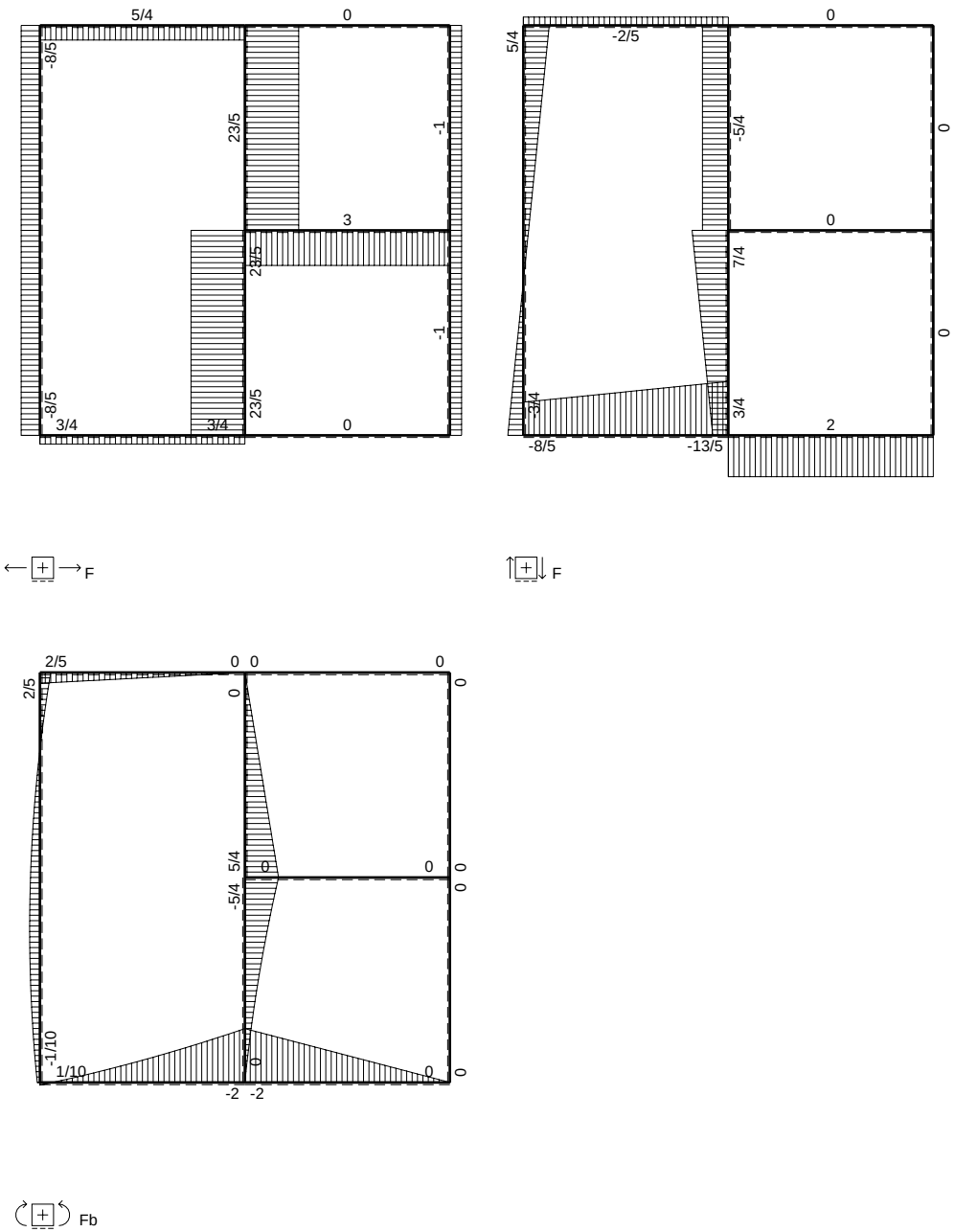
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

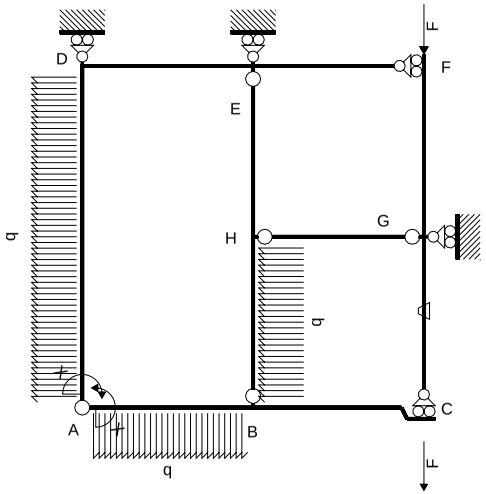
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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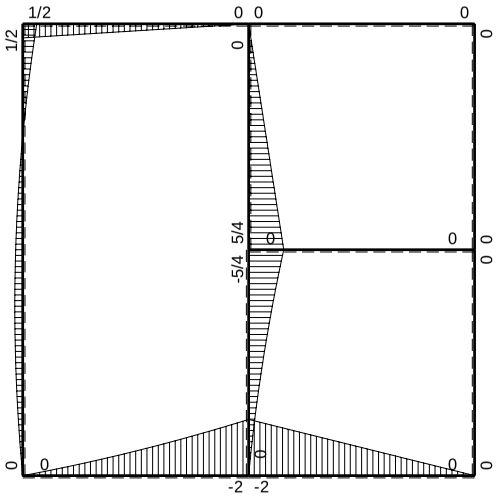
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2/5Fb
2/5F
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2/5F
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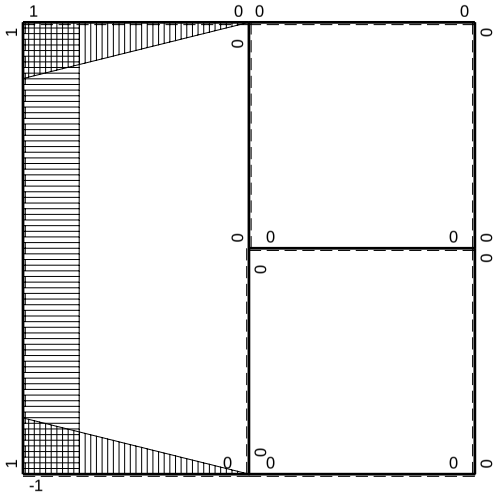

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0
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5/4
0
0
0
-5/4
0
0
-1/10
0
1/10
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

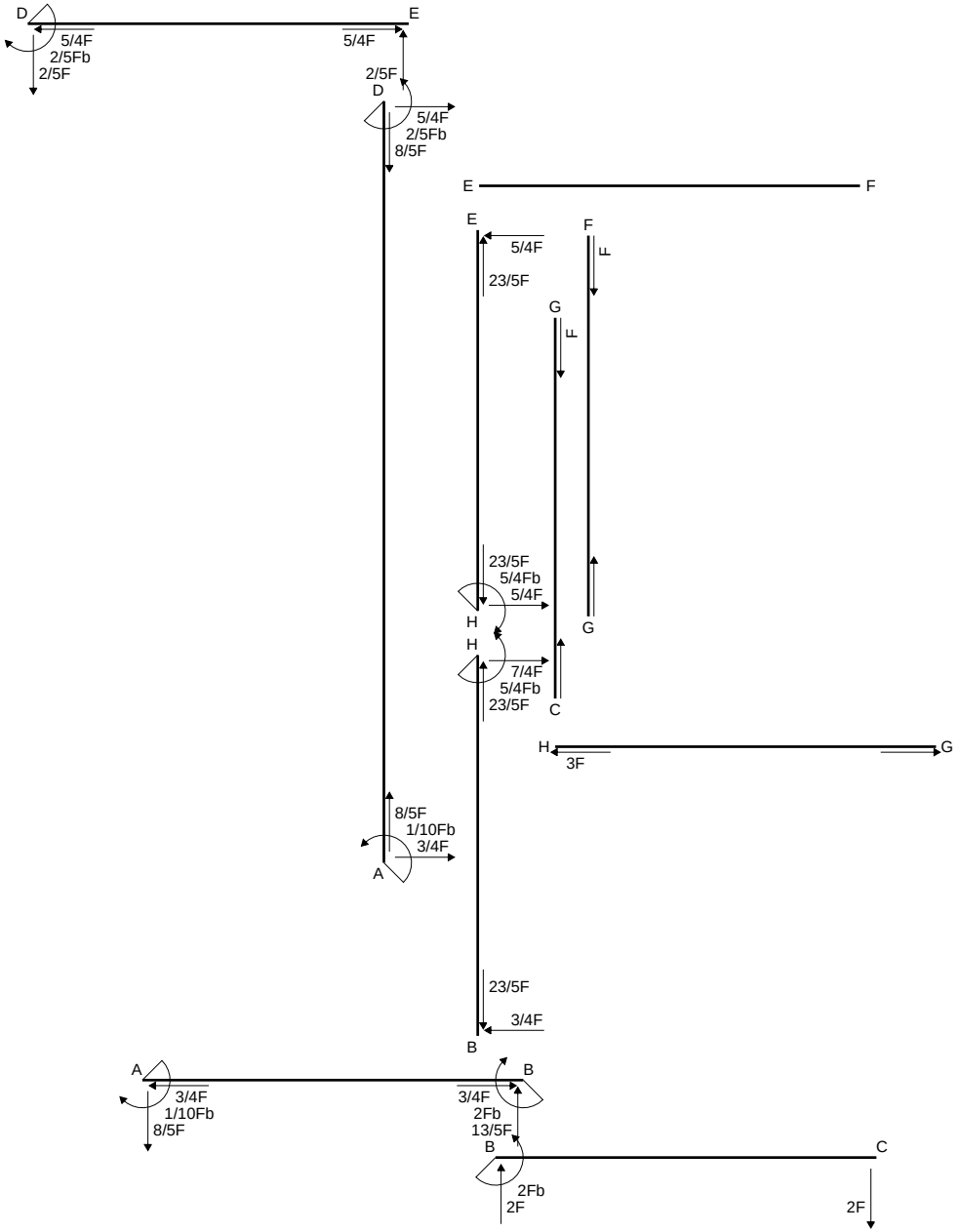
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

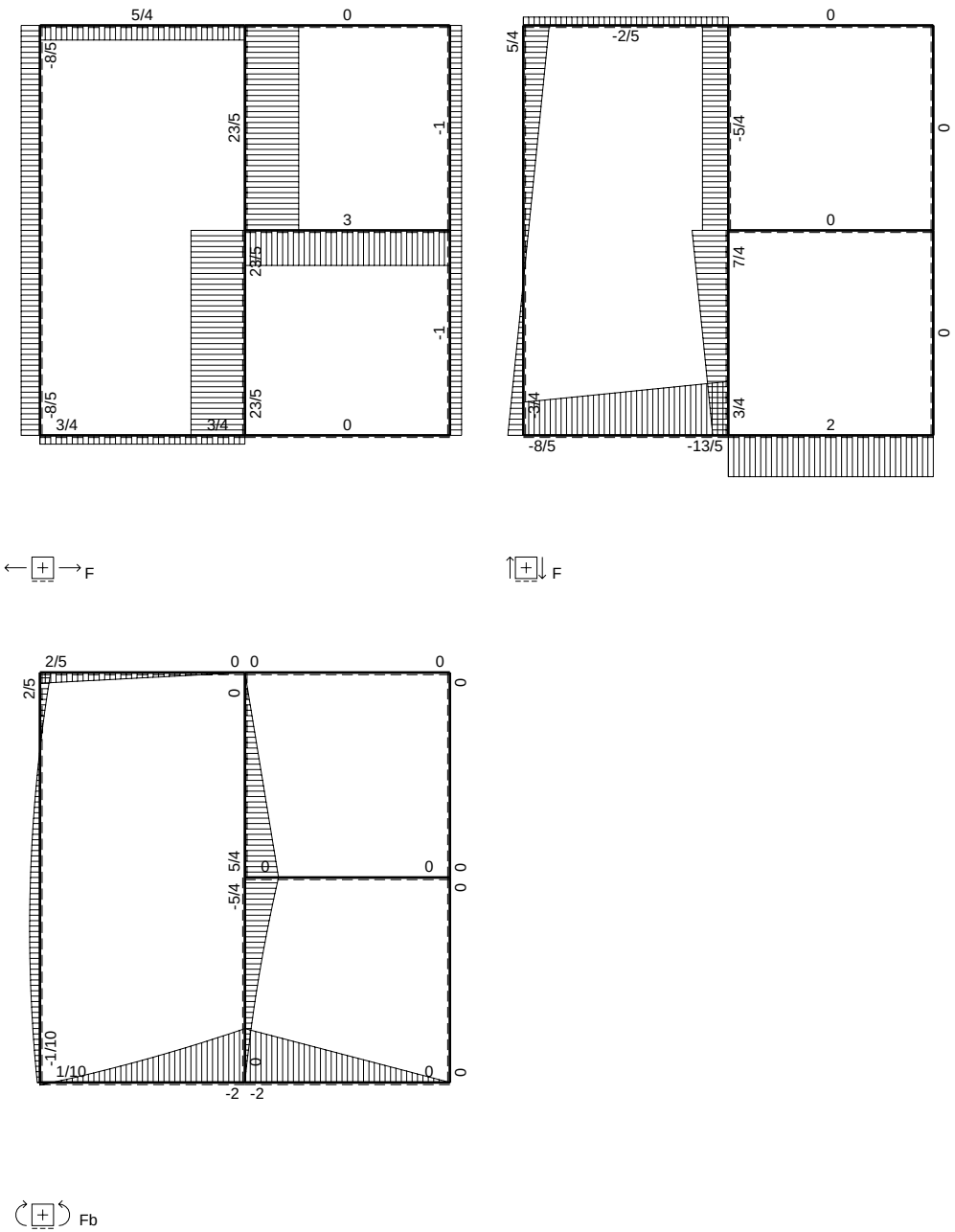
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

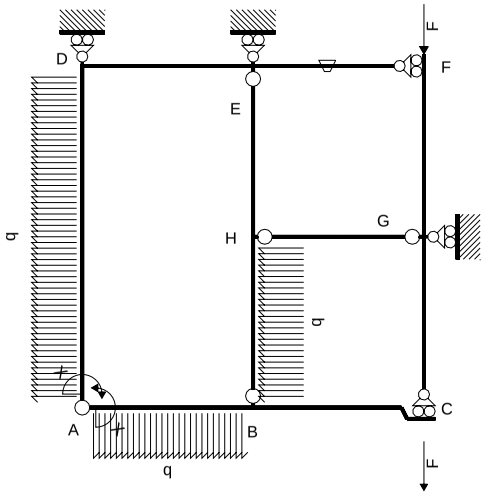
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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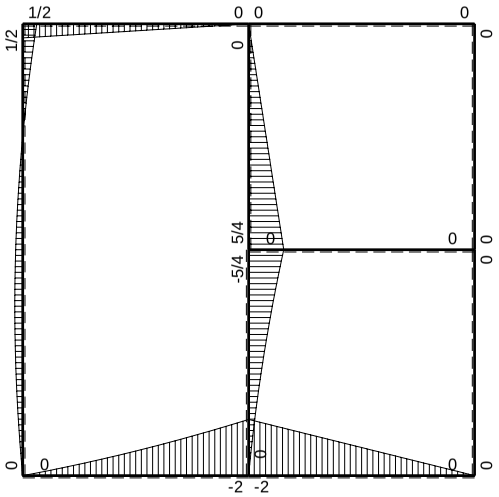
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2/5F
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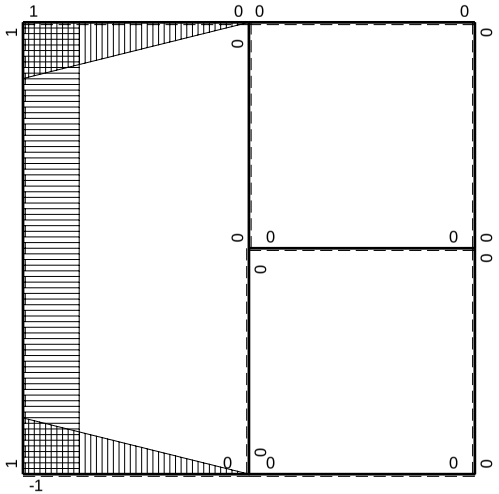

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F
2/5
0 0
0
2/5
0
0
0
5/4
0
0 0
-5/4
0
-1/10
0
1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	-1/2Fx	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	-Fb/EJ	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	5/4Fb-5/4Fx	0	0	0	0	0+0	0
EH b	0	-5/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

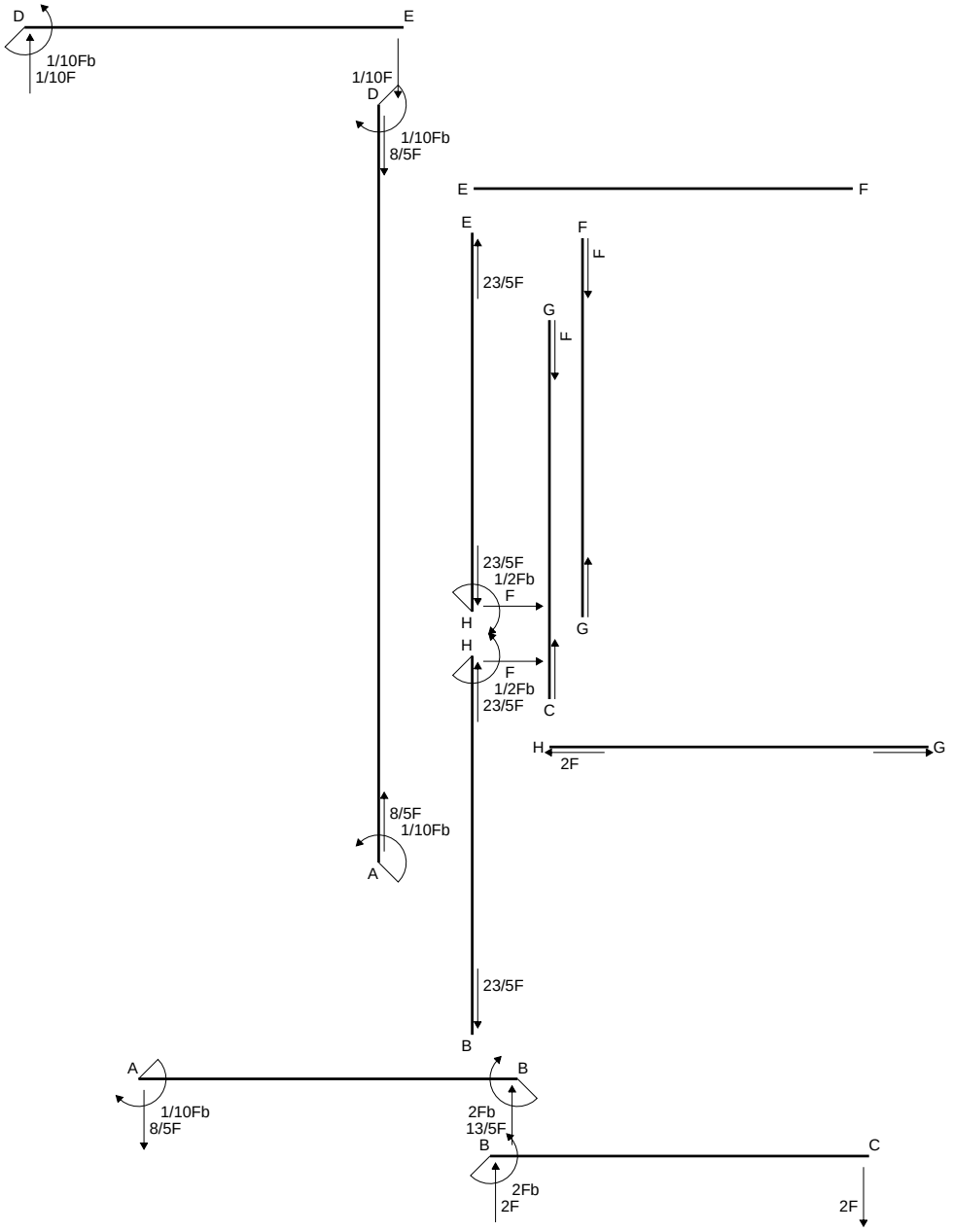
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

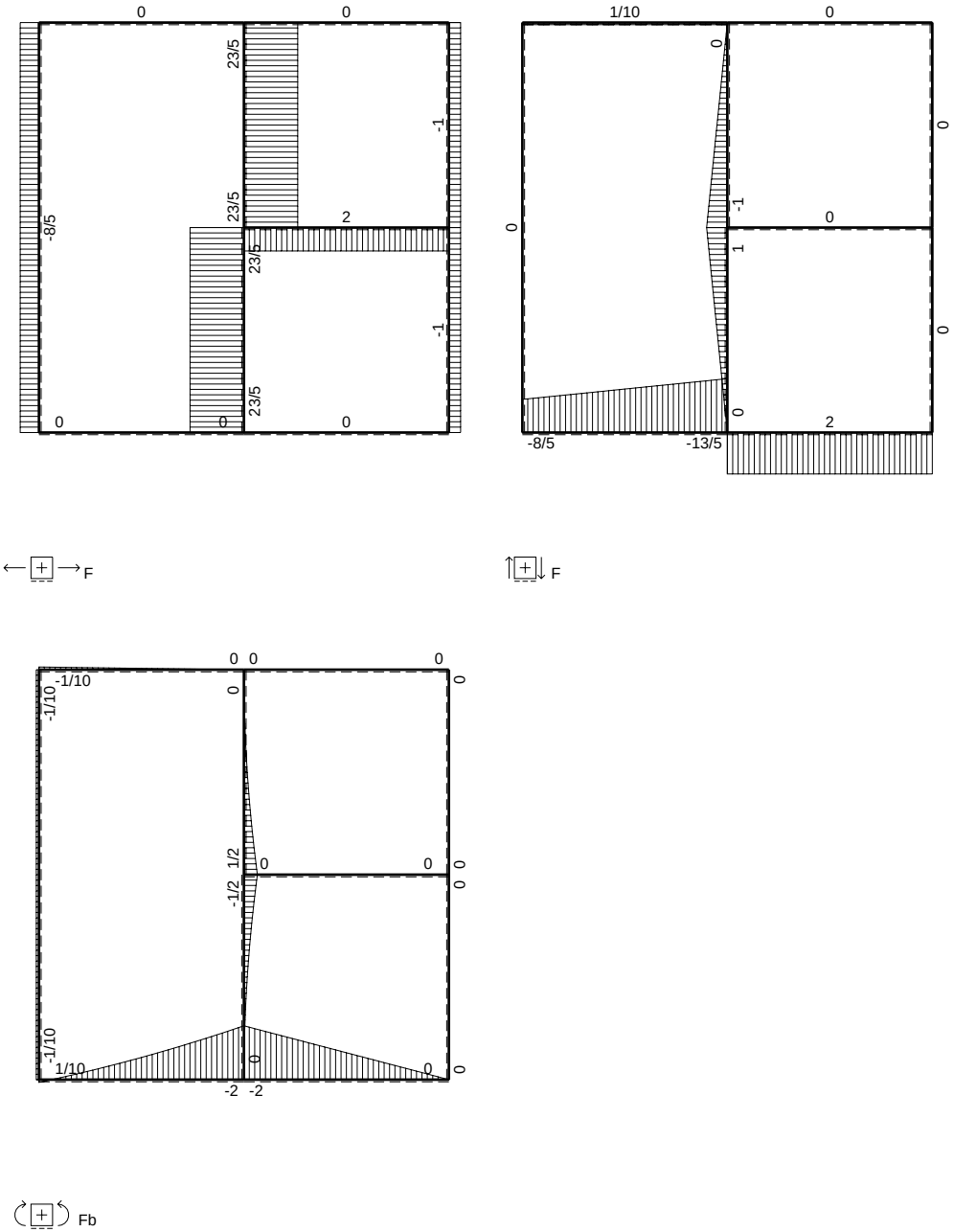
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

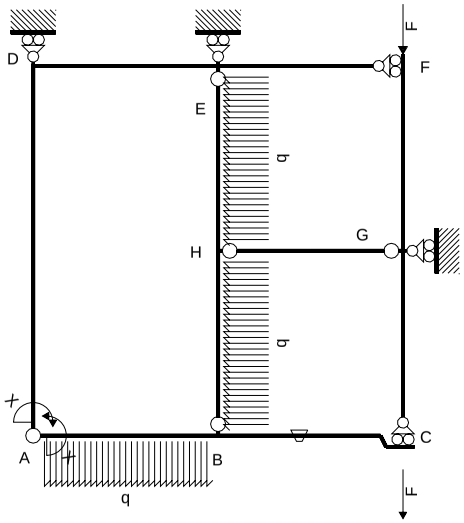
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 995596 ESΣ03.xxxx.015

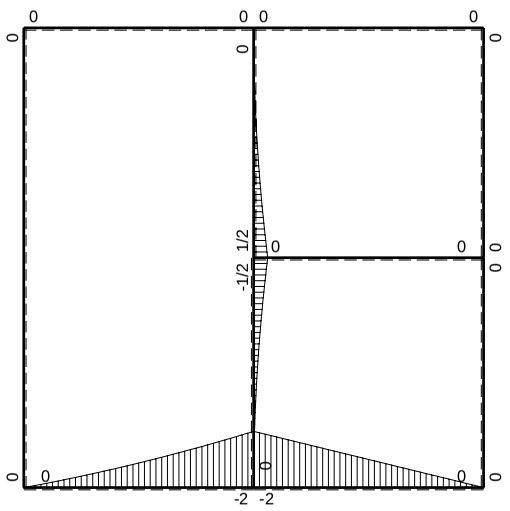
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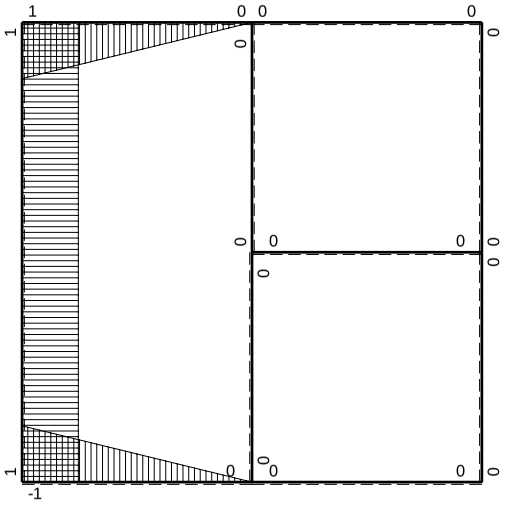

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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238641 ESΣ03.xxxx.016

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PROCEDIMENTO E RISULTATI 238641 ESΣ03.xxxx.016

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,3/4\,b - 1/3\,b - 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

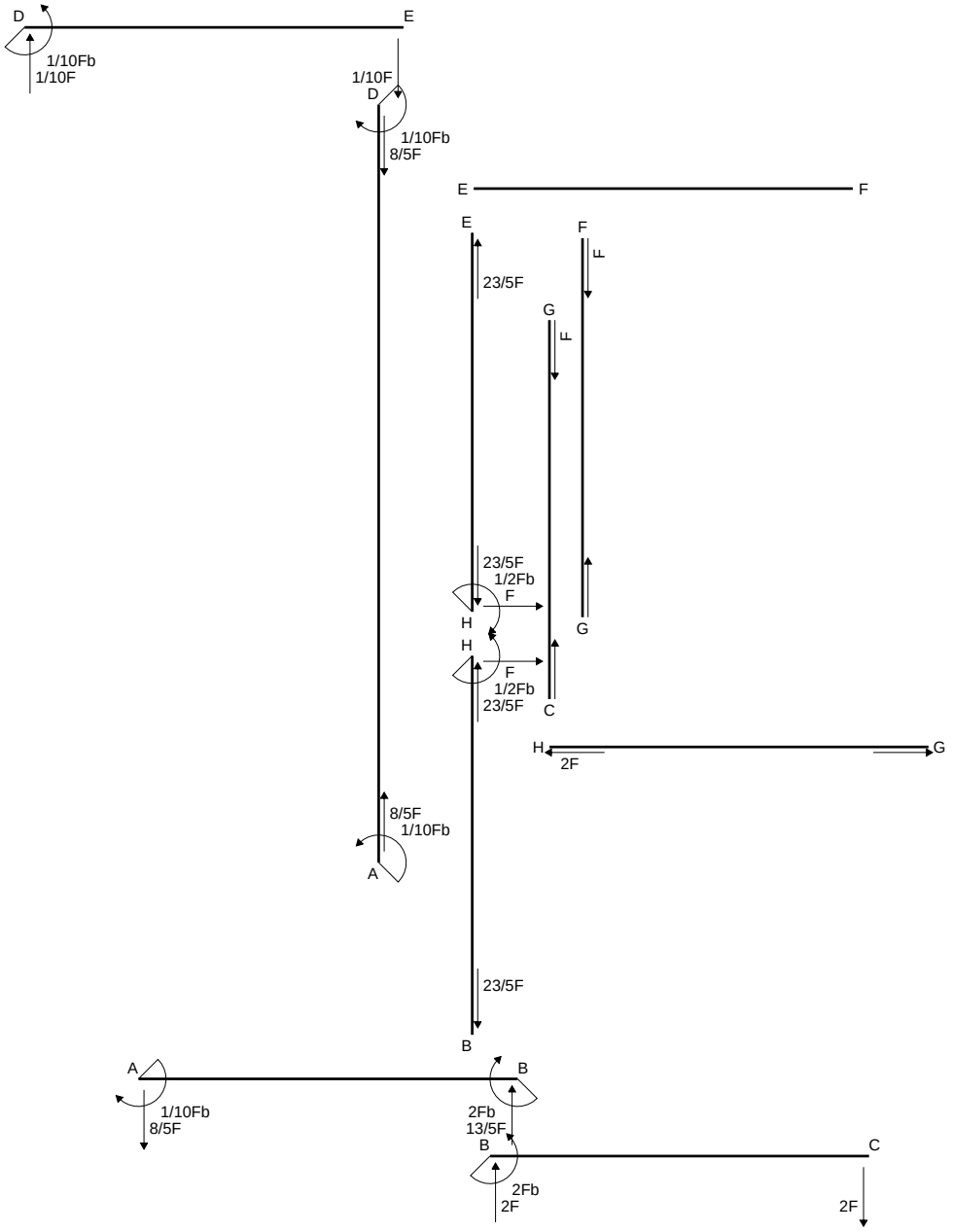
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

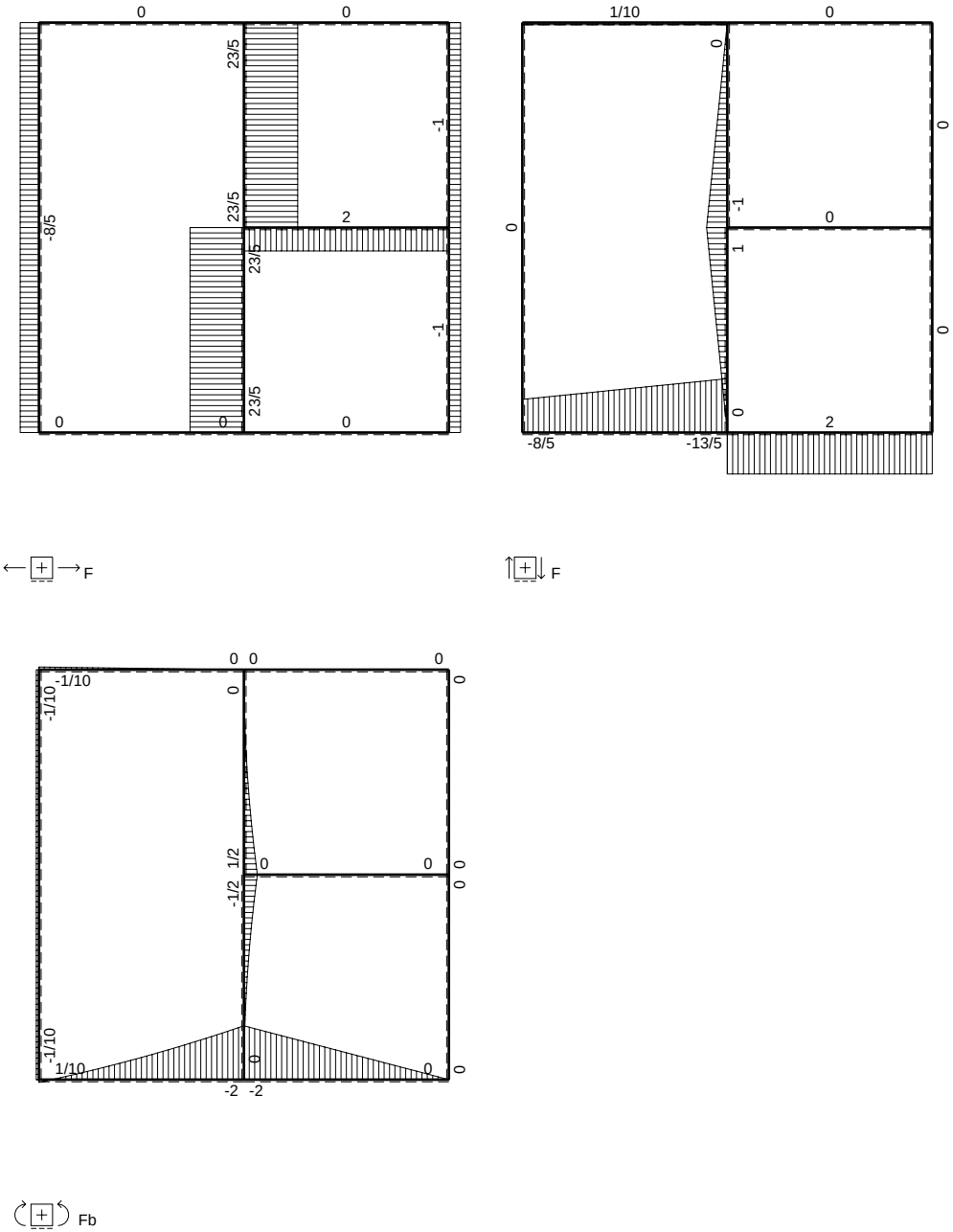
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

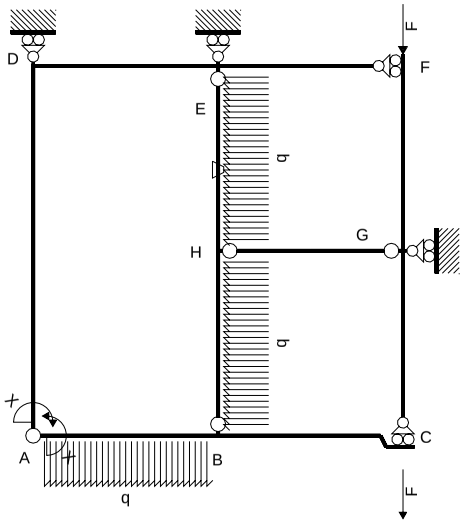
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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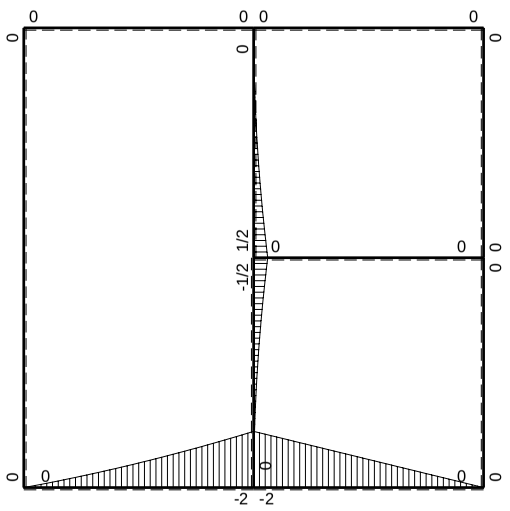
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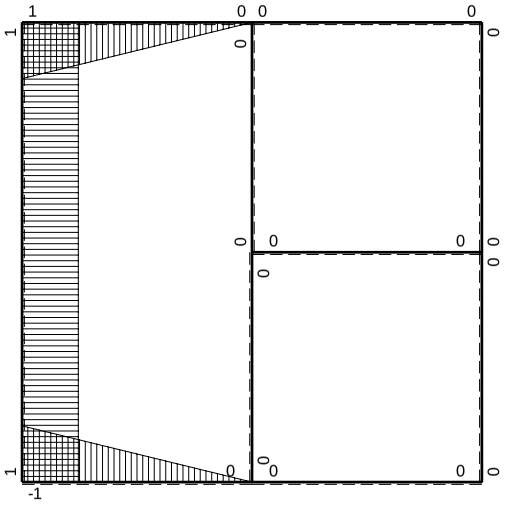

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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237997 ESΣ03.xxxx.017

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PROCEDIMENTO E RISULTATI 237997 ESΣ03.xxxx.017

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

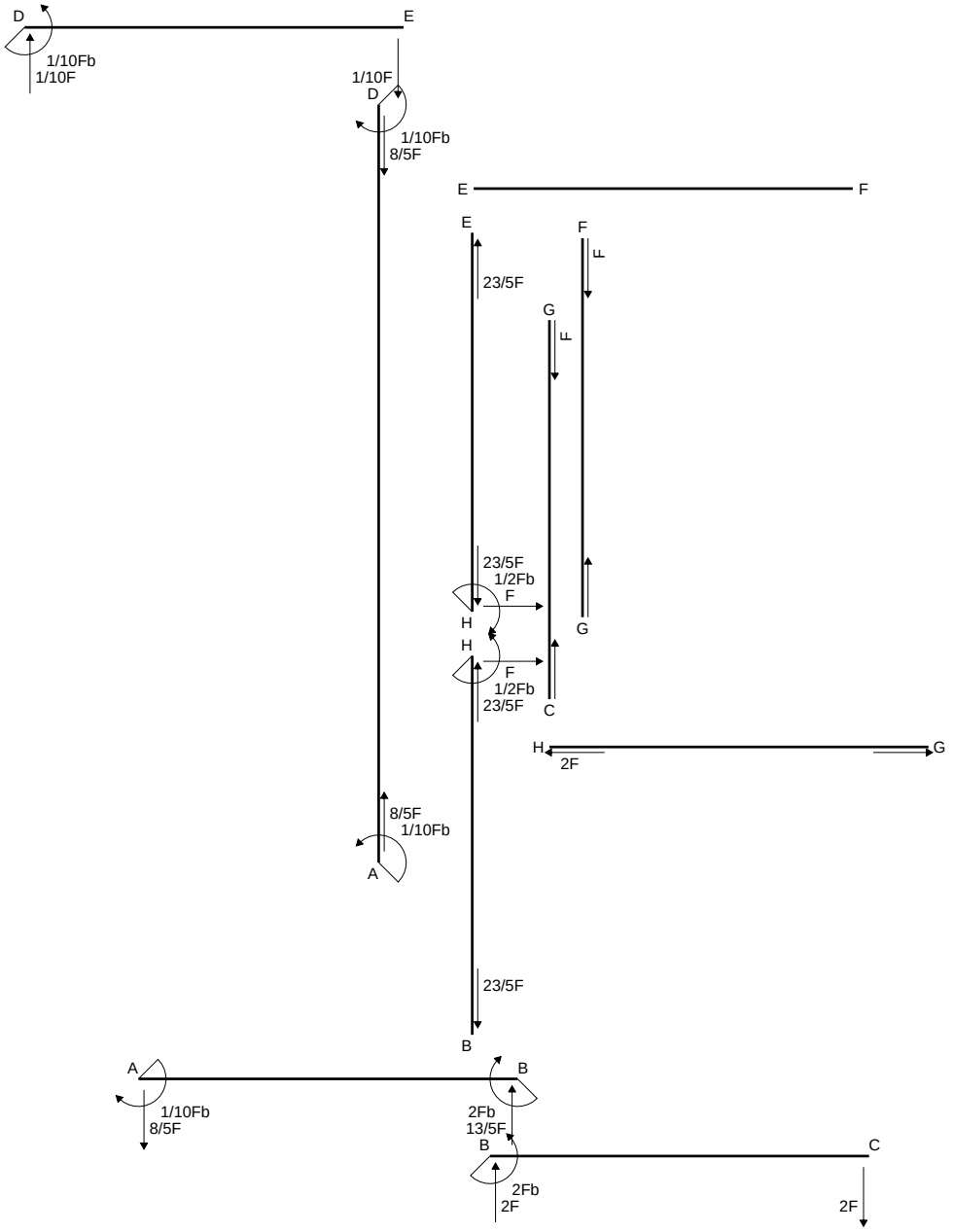
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

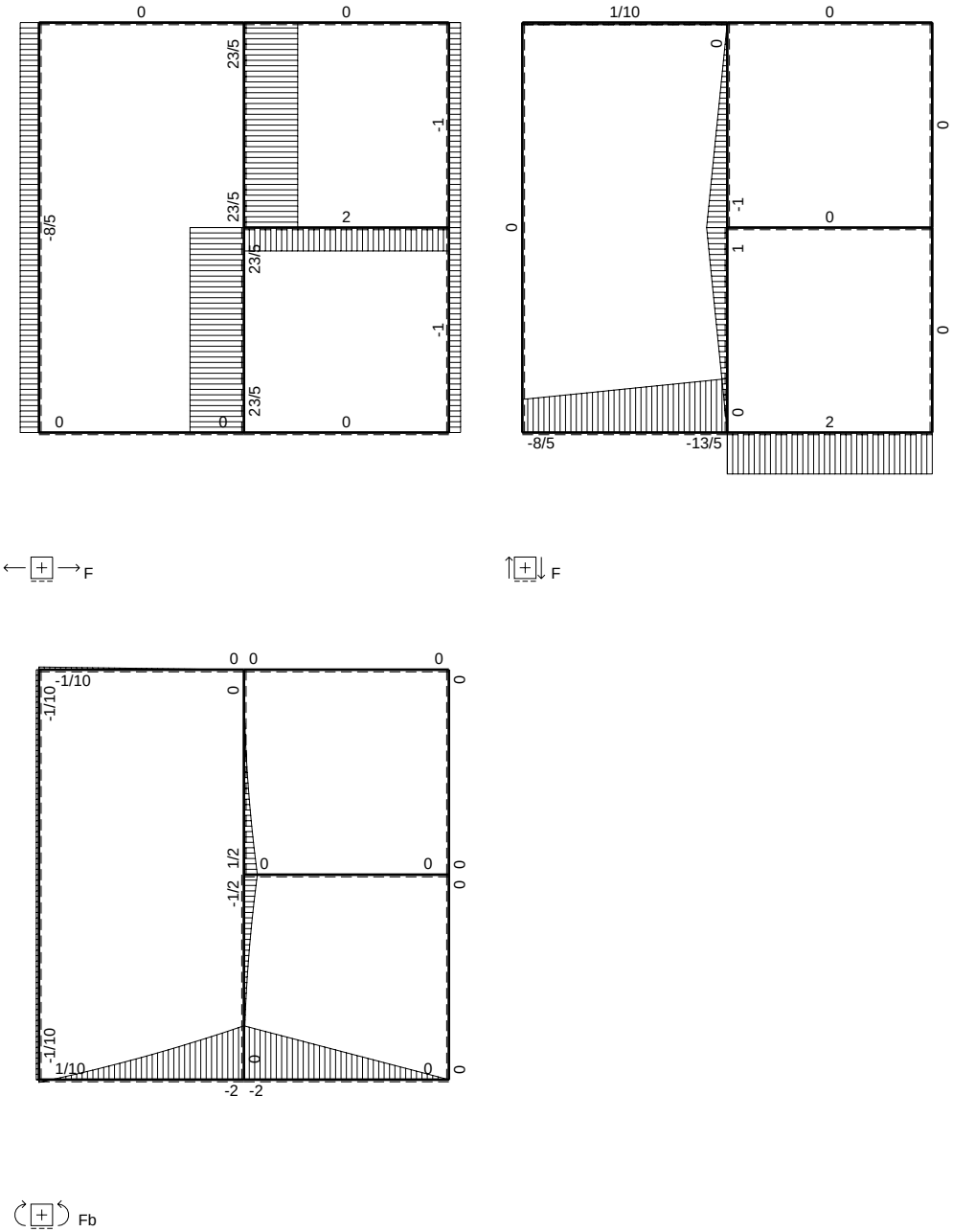
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

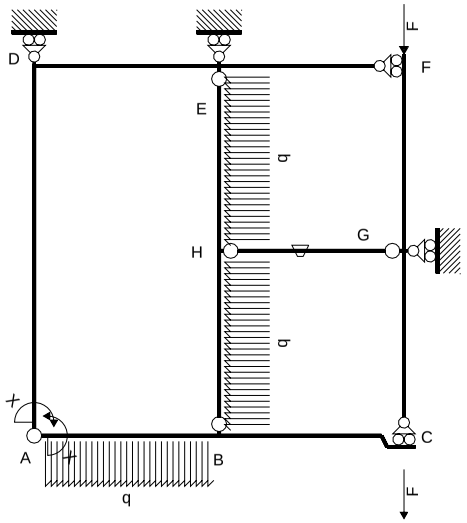
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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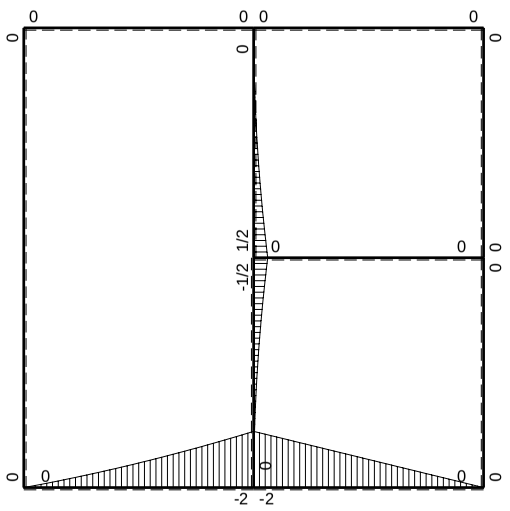
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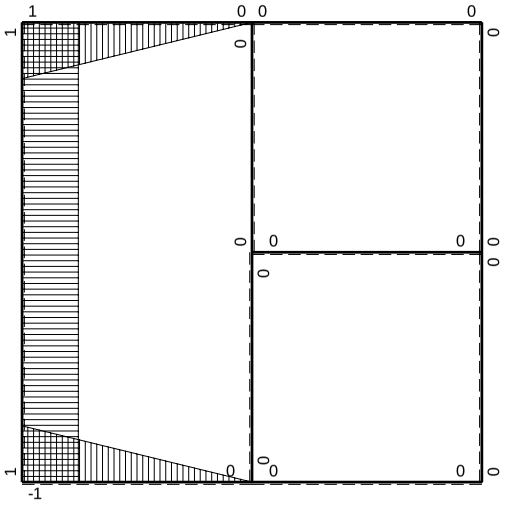

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0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237137 ESΣ03.xxxx.018

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PROCEDIMENTO E RISULTATI 237137 ESΣ03.xxxx.018

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

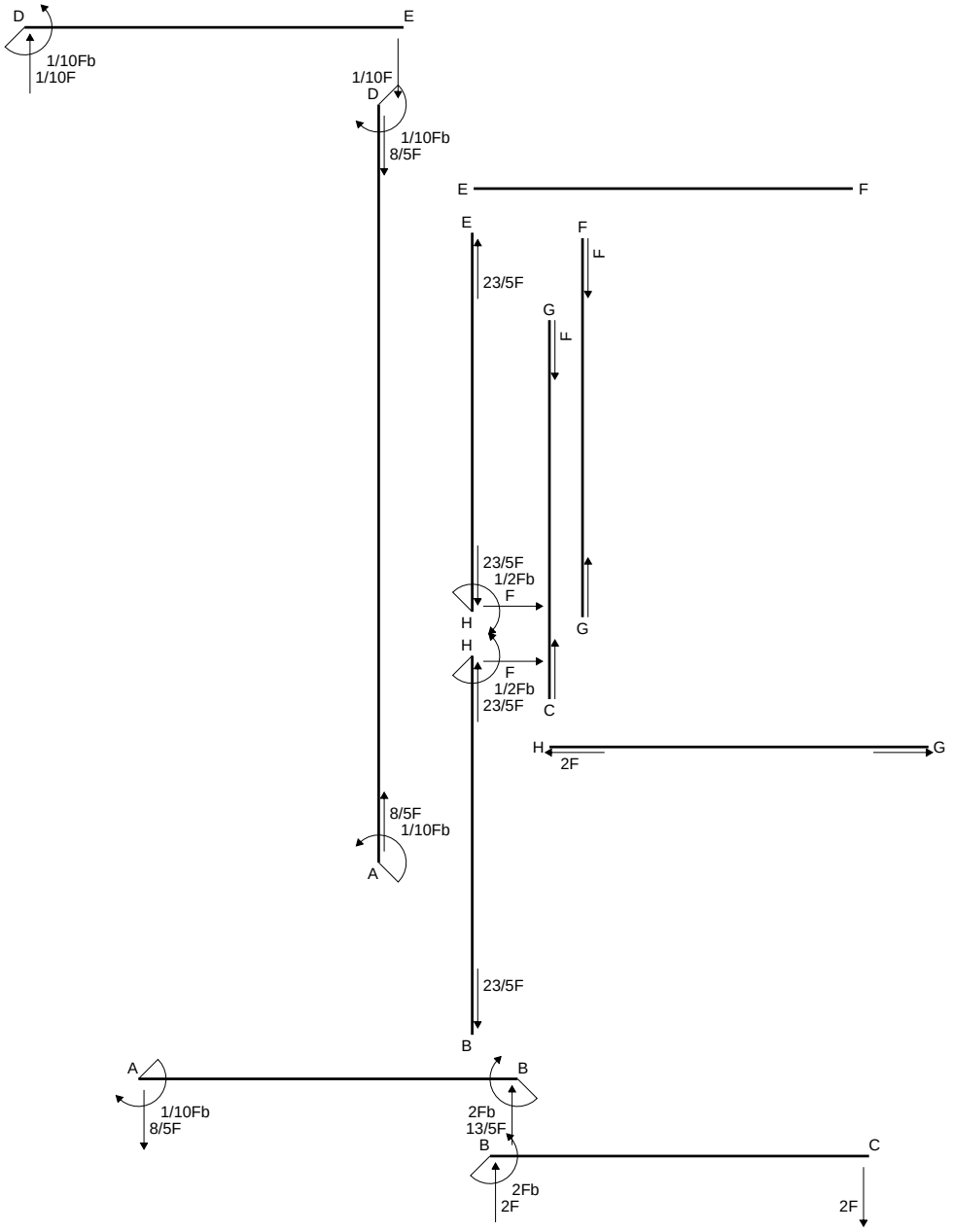
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

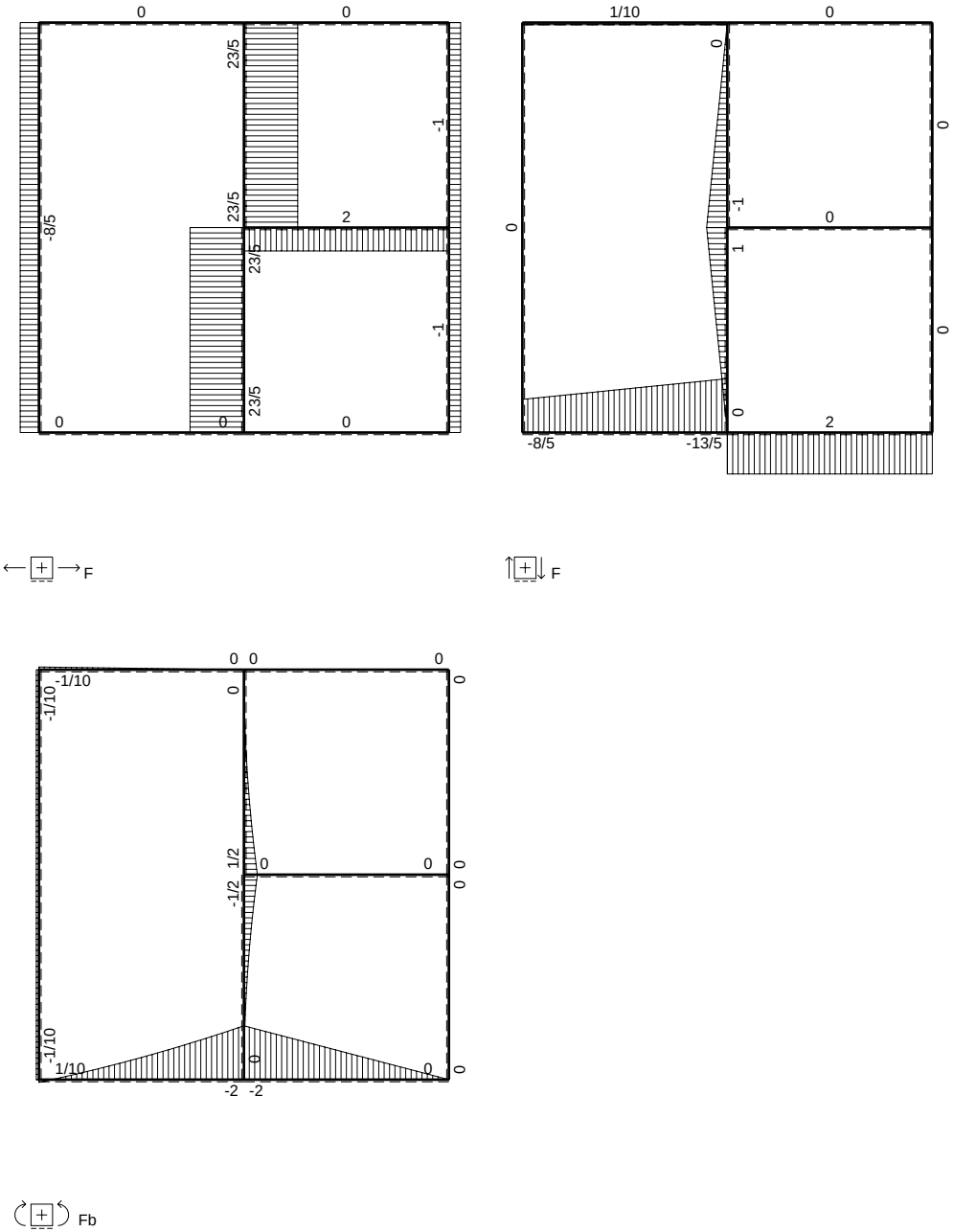
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

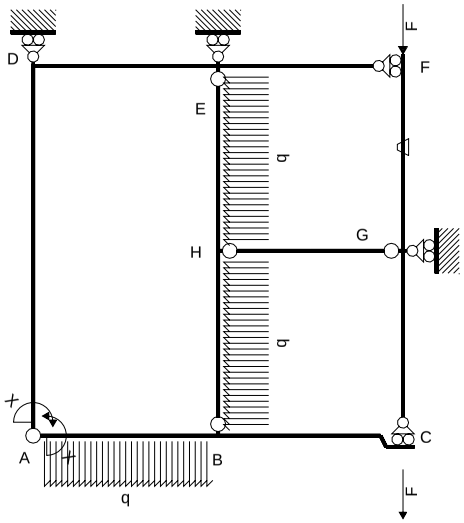
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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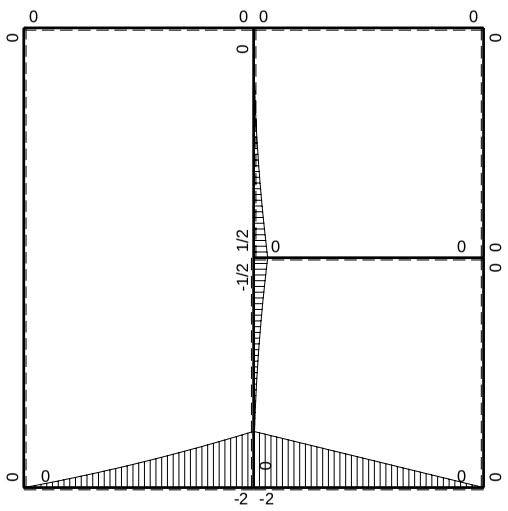
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D
E
1/10Fb
1/10F
1/10F
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1/10Fb
8/5F
E
F
E
F
F
23/5F
G
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23/5F
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2F

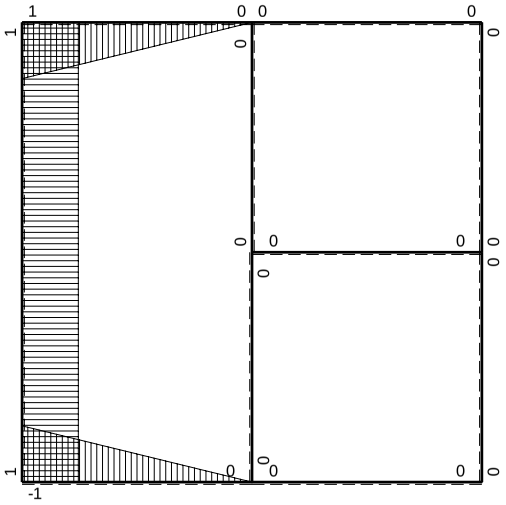

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0
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0
1
0
0
0
2
-8/5
-13/5
F
F
0 0
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-1/10
-1/10
0
0
0
1/2
0
0
-1/2
0 0
0
-1/10
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237596 ESΣ03.xxxx.019

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PROCEDIMENTO E RISULTATI 237596 ESΣ03.xxxx.019

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

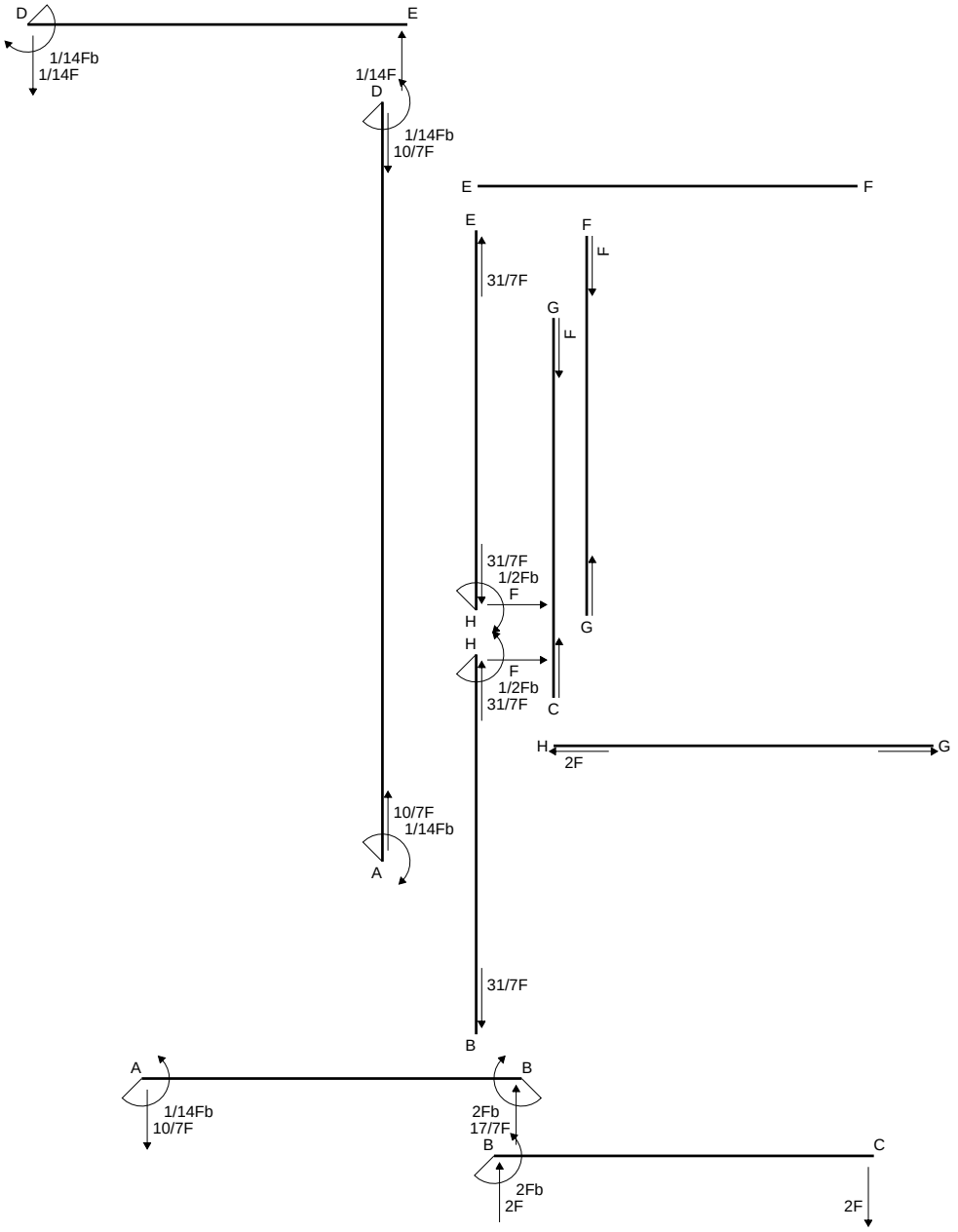
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

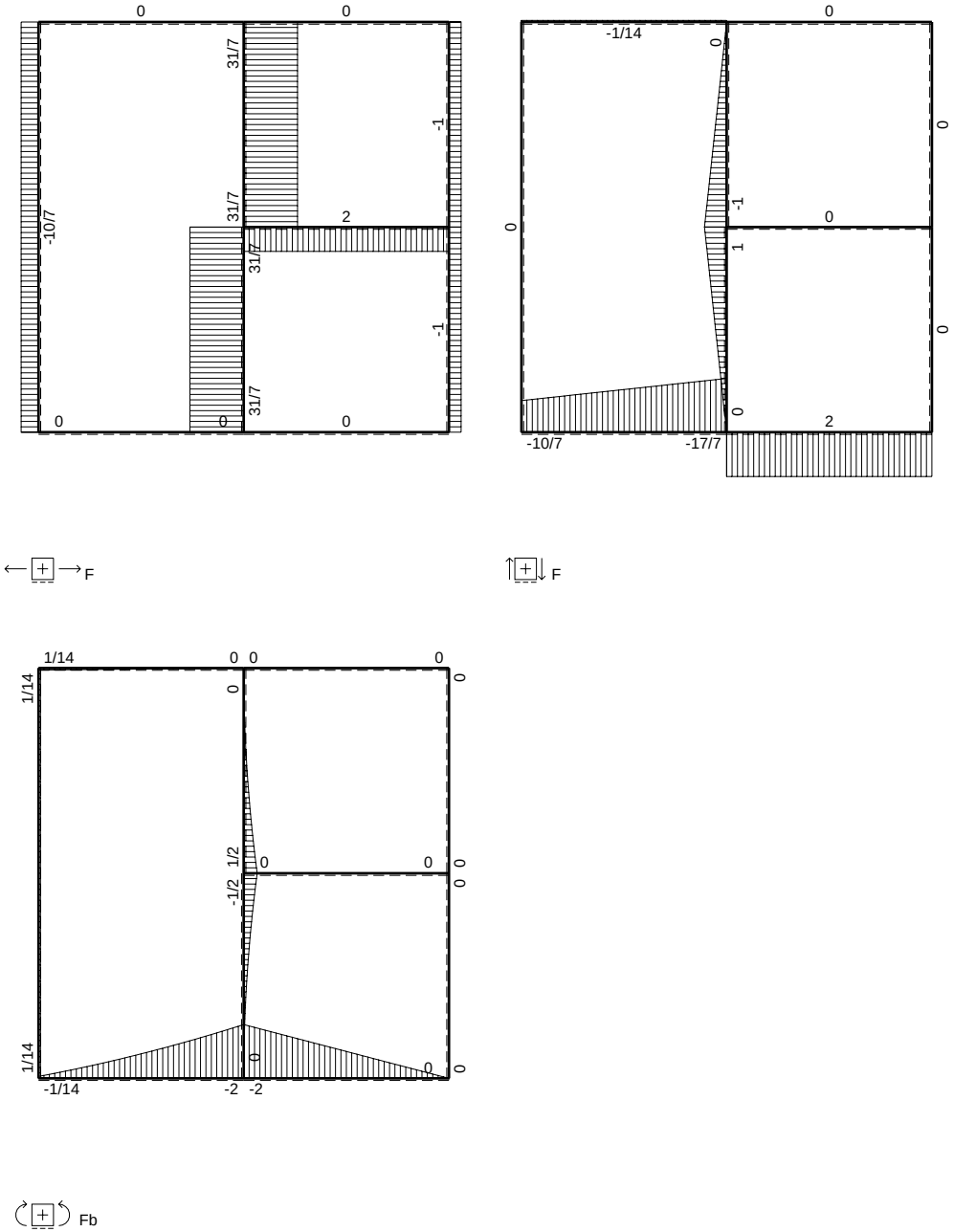
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

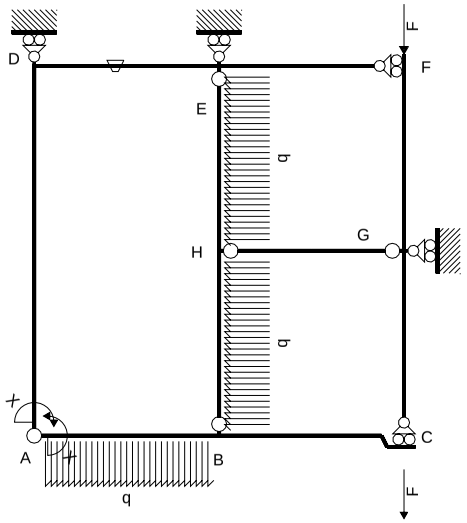
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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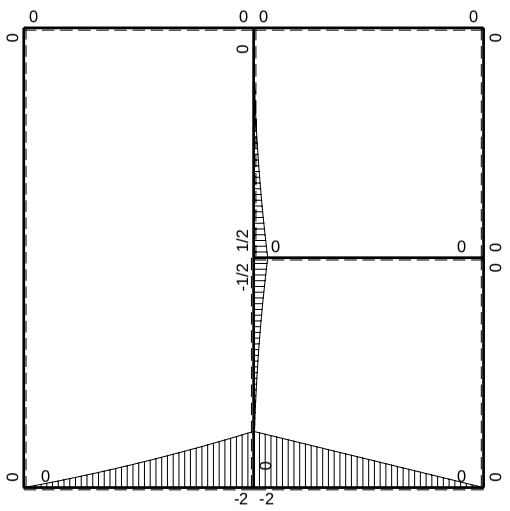
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D
E
1/14Fb
1/14F
1/14F
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1/14Fb
10/7F
E
F
E
F
31/7F
F
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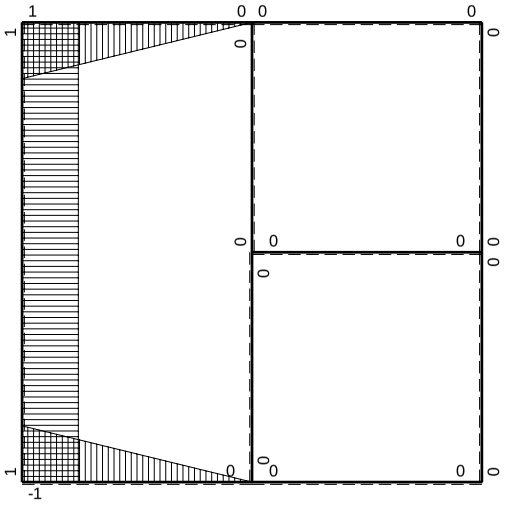

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0
0
-1
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1
0
0
0
2
-10/7
-17/7
F
1/14
0 0
0
0
1/14
0
0
1/2
0
0
-1/2
0
0
0
1/14
0
0
-1/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 984543 ESΣ03.xxxx.020

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	-Fb/EJ	0	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$	$(0-1/2)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	0	Fb/EJ	0	-Fx/EJ	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984543 ESΣ03.xxxx.020

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + x/b\ )\ \theta\ \ dx = [\ - x + 1/2\ x^2/b\ ]_o^b\ \theta$

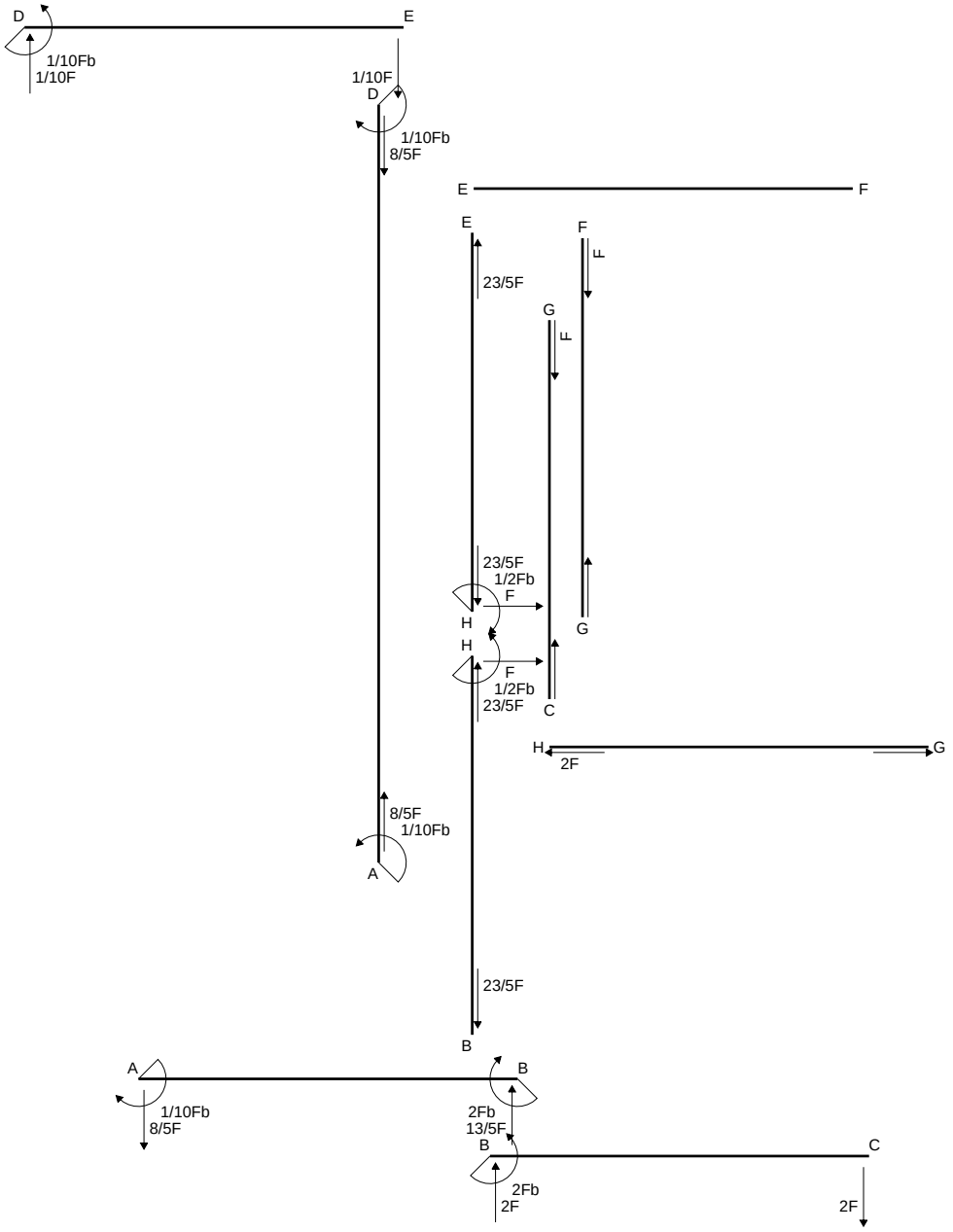
$= (\ - b + 1/2\ b\ )\ \theta\ \ = -1/2\ \ Fb^2/EJ$

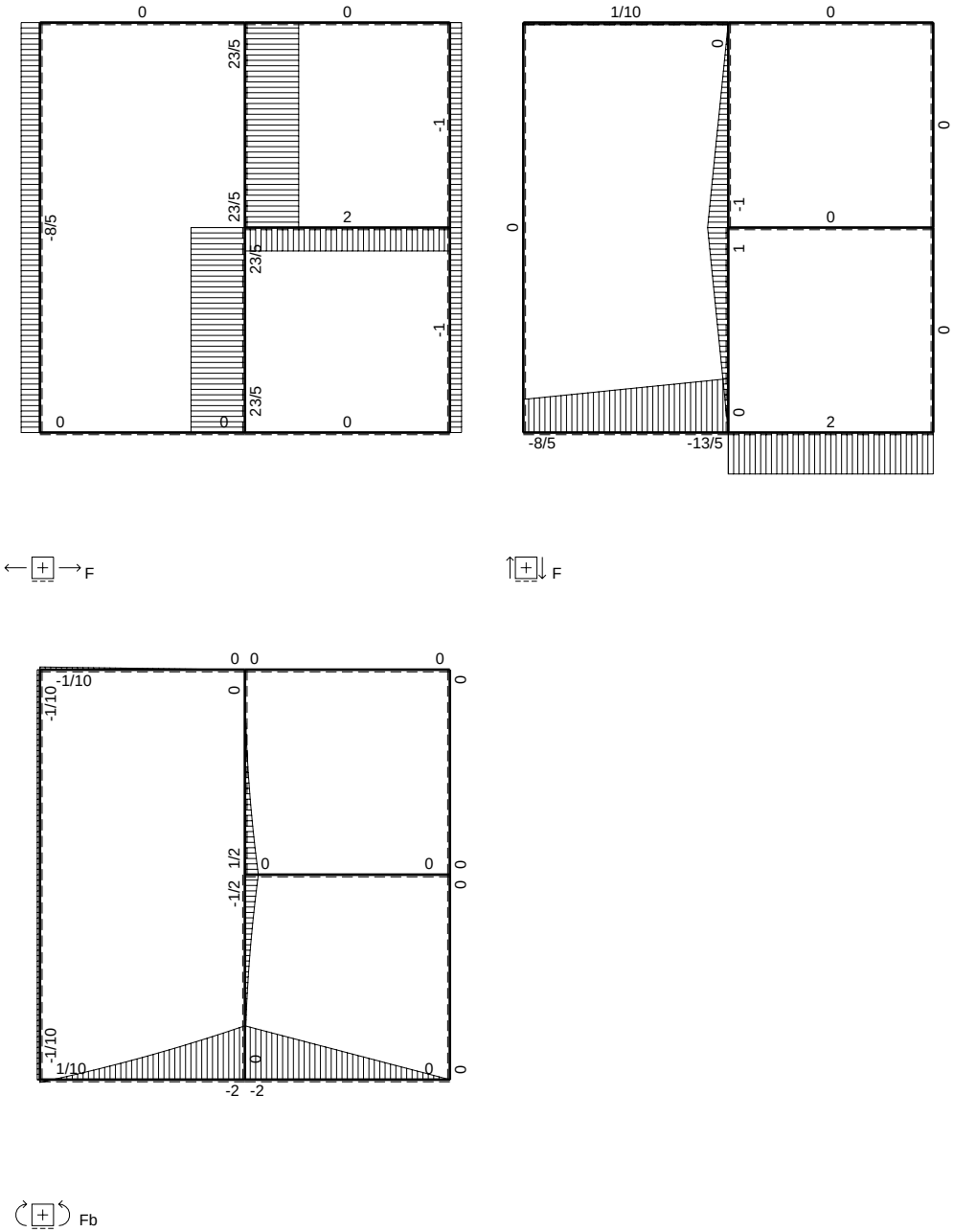
$L^{Xo}_{ED} = \int_o^b (\ x/b\ )\ \theta\ \ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

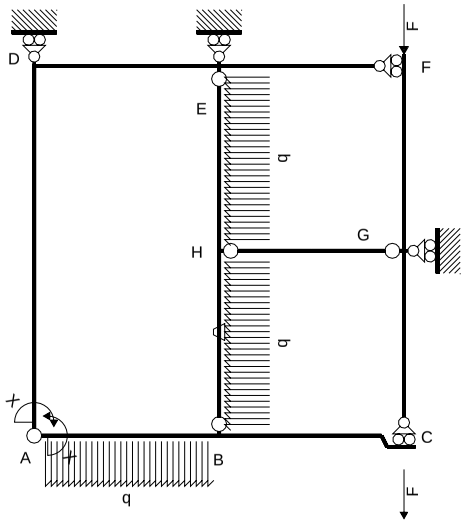
$= (1/2\ b\ )\ \theta\ \ = -1/2\ \ Fb^2/EJ$

ESΣ03.xxxx.020

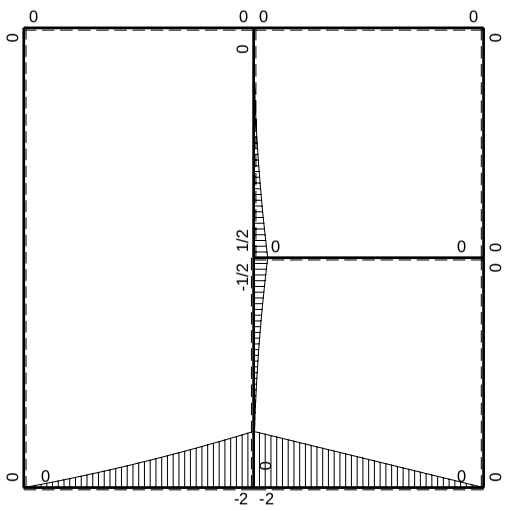
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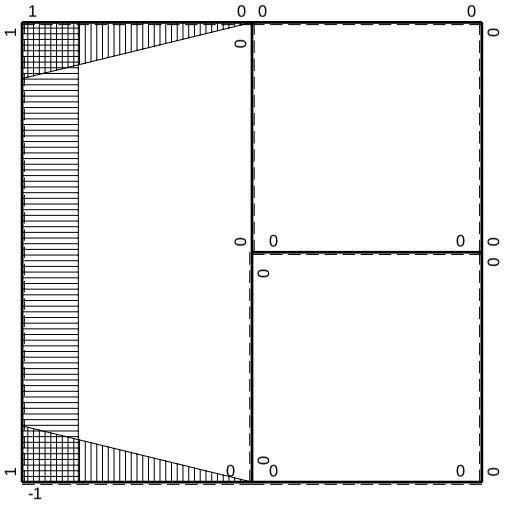

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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 250247 ESΣ03.xxxx.021

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PROCEDIMENTO E RISULTATI 250247 ESΣ03.xxxx.021

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

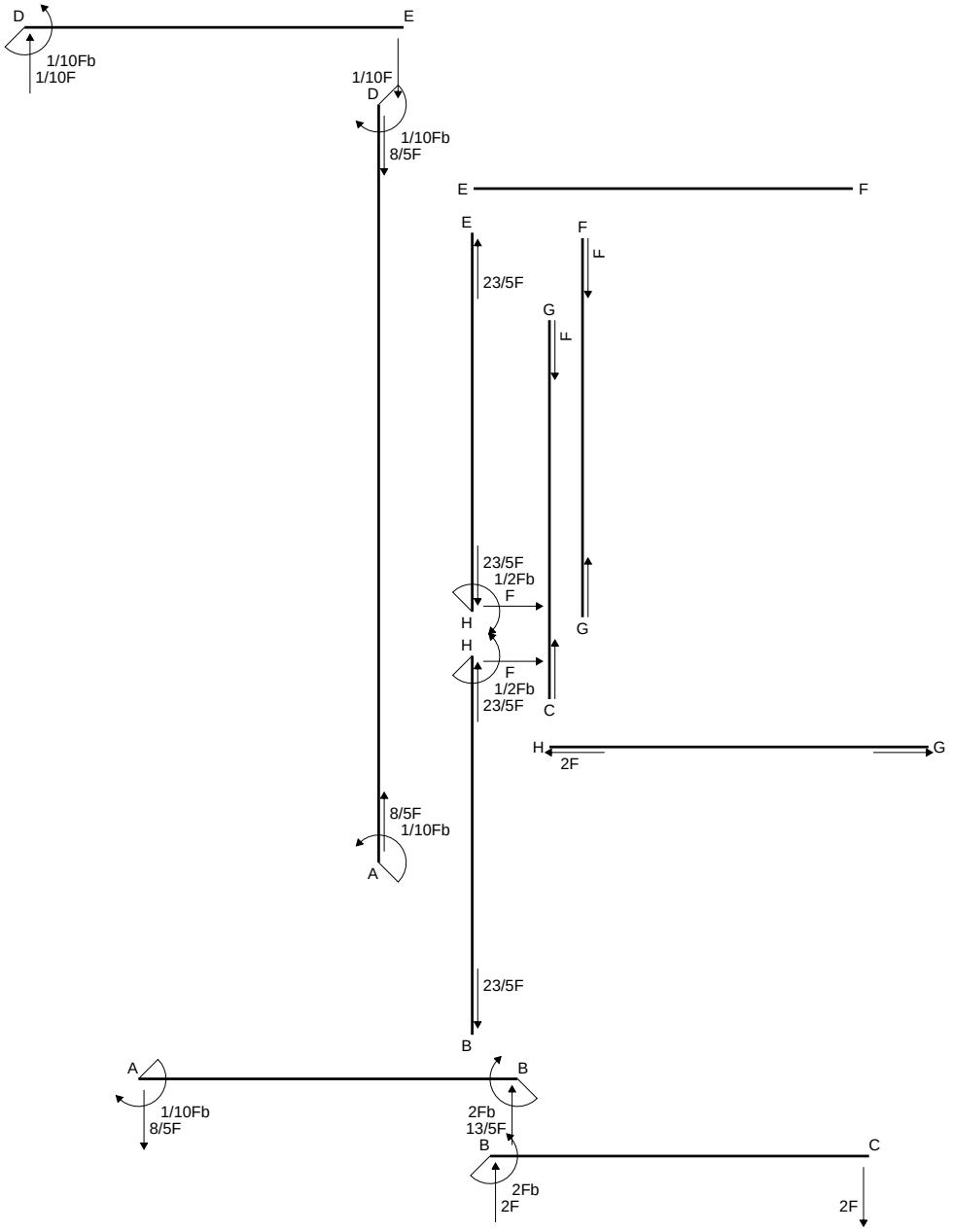
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

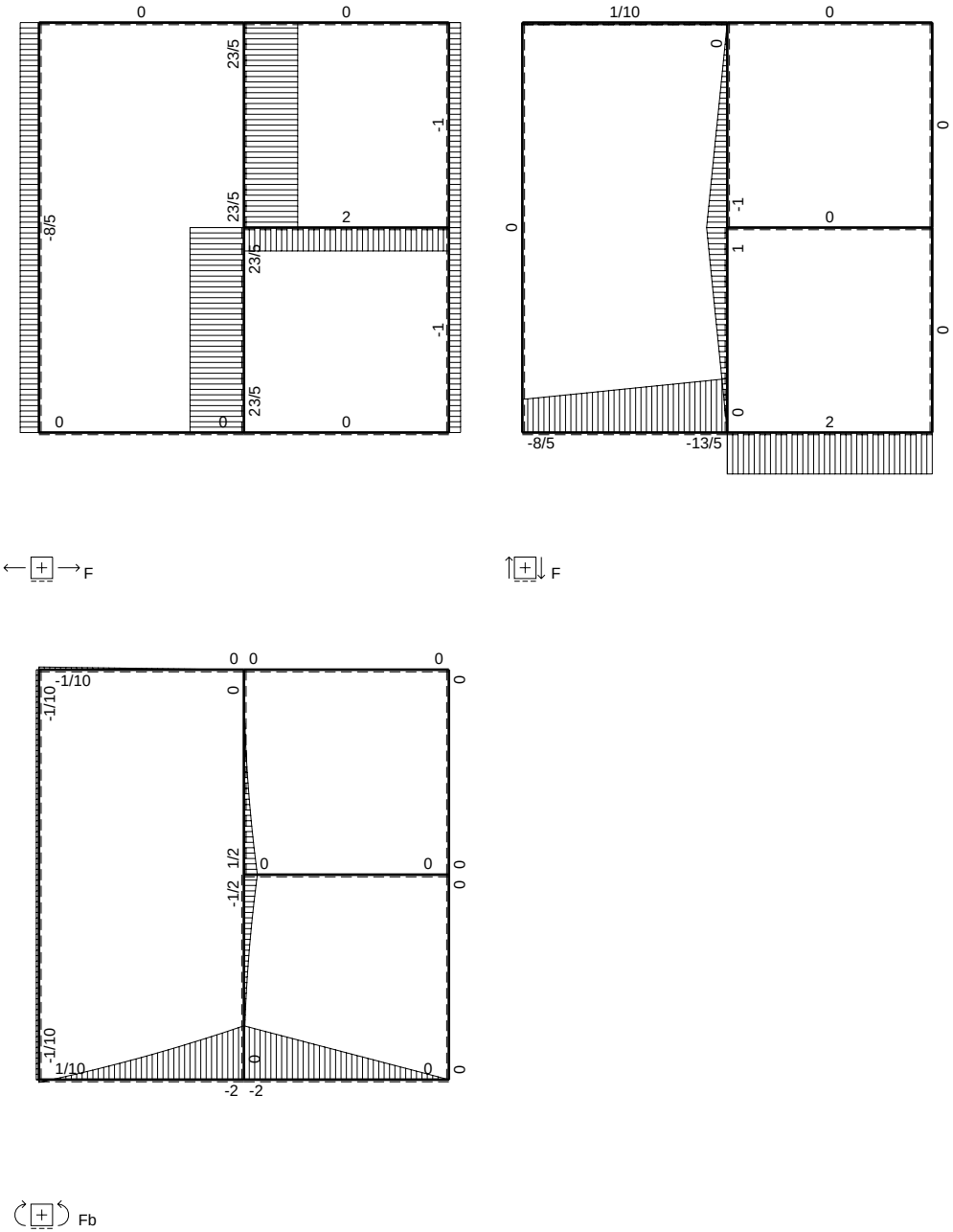
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

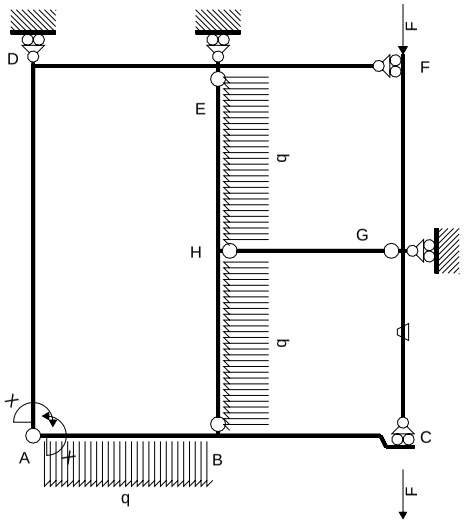
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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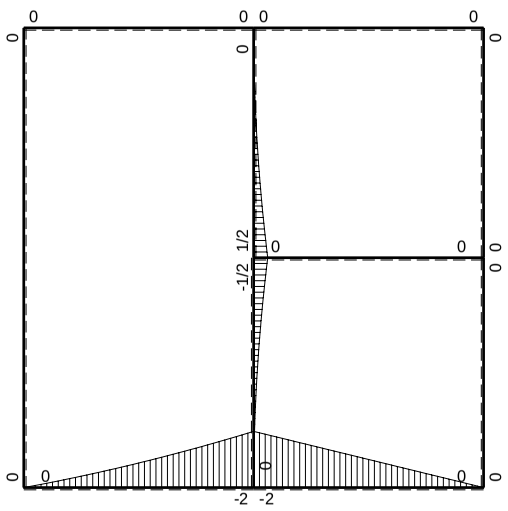
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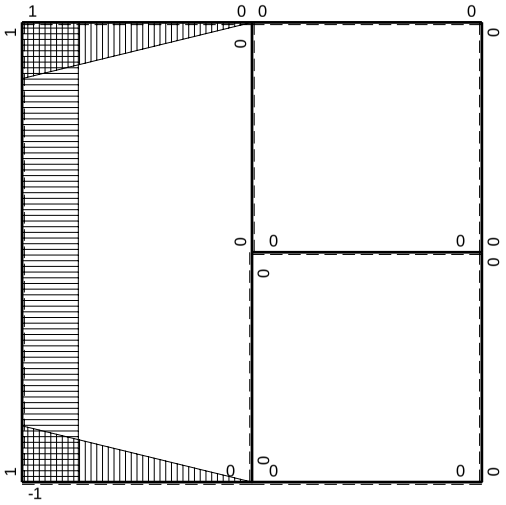

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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238694 ESΣ03.xxxx.022

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PROCEDIMENTO E RISULTATI 238694 ESΣ03.xxxx.022

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

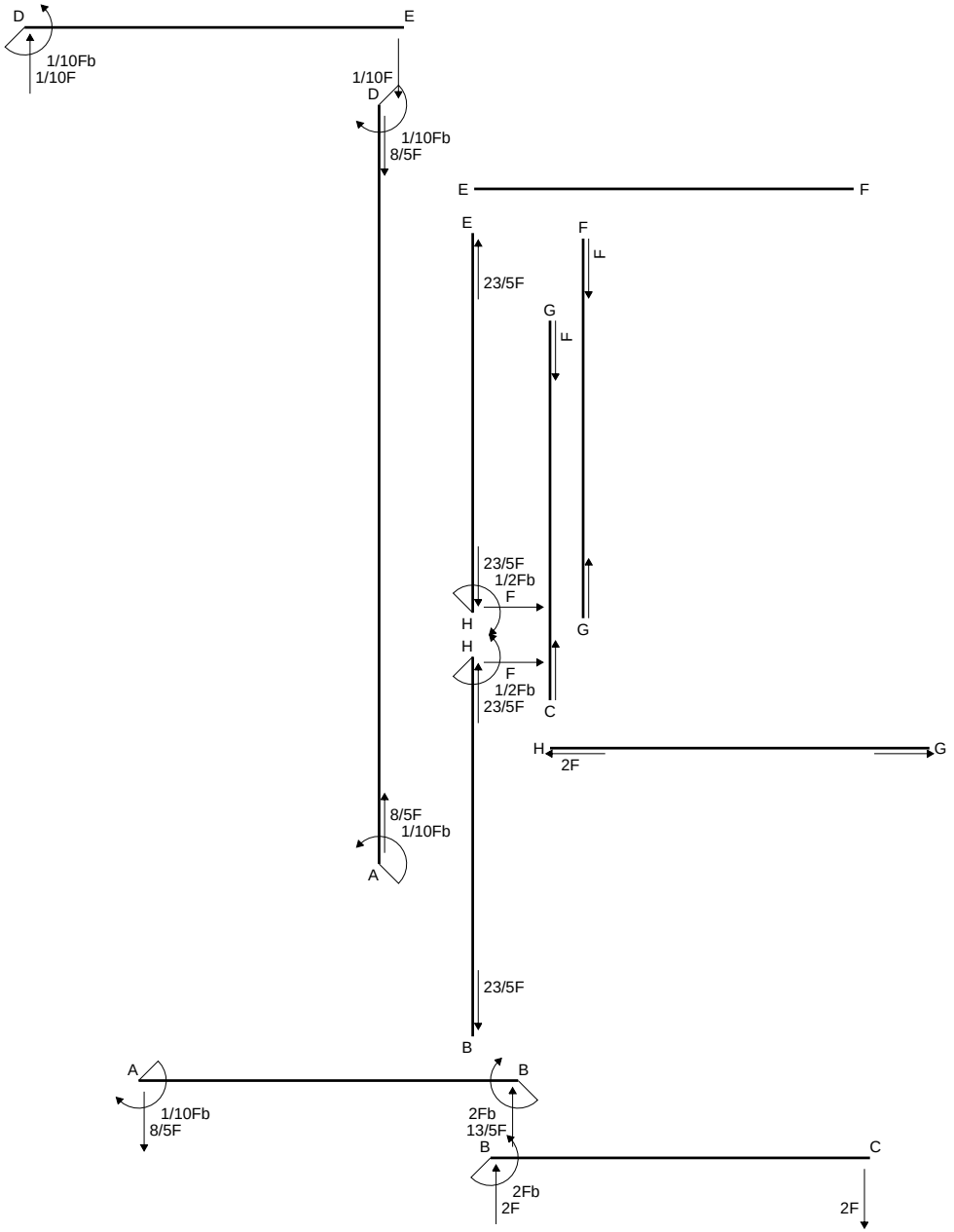
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

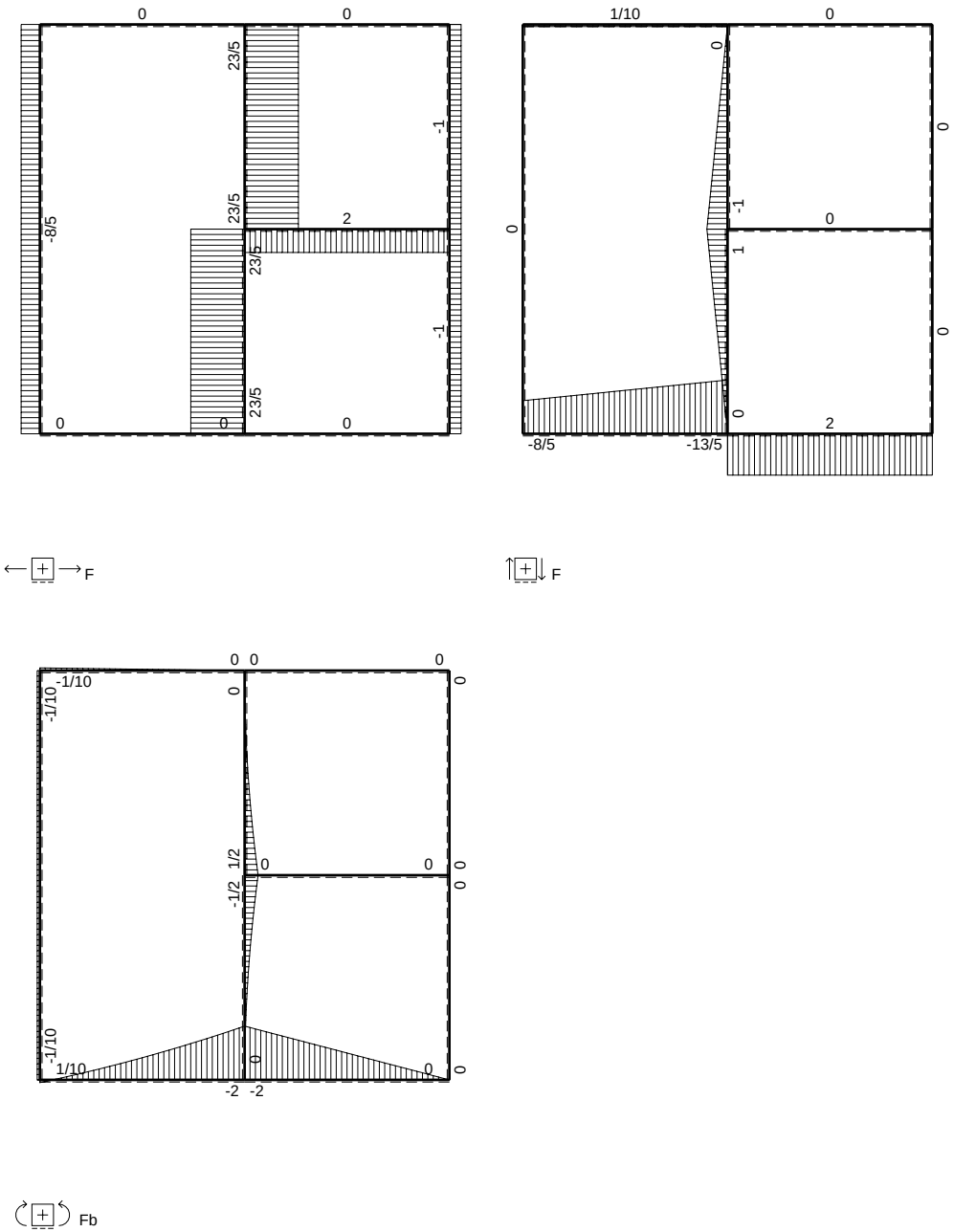
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

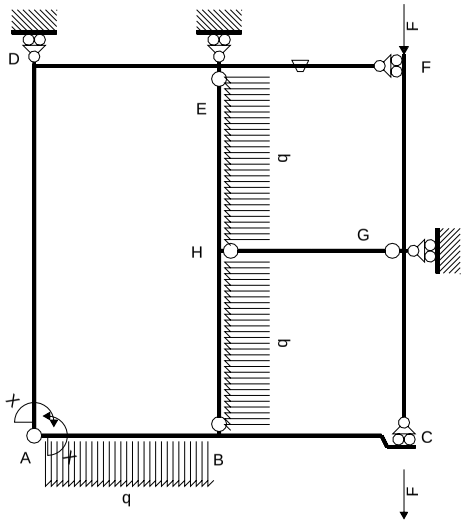
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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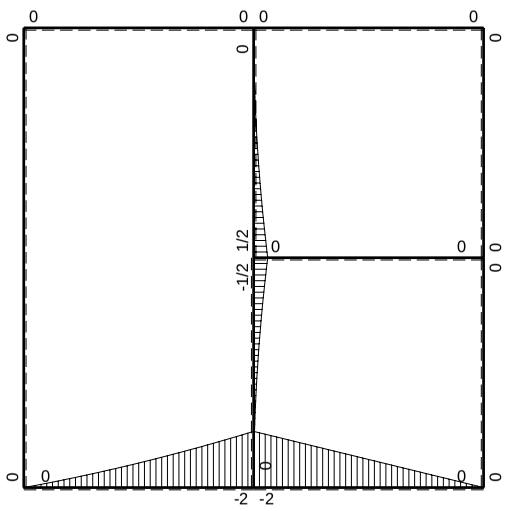
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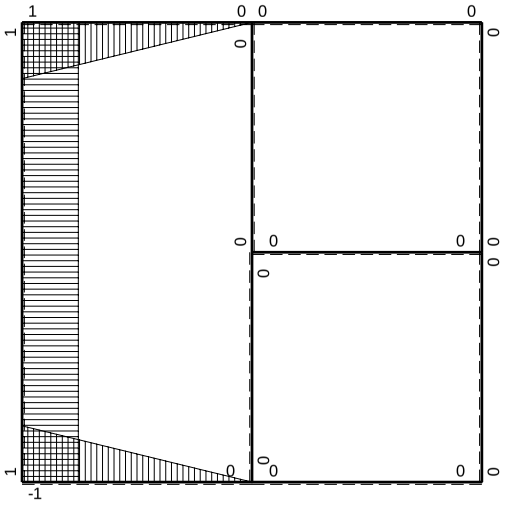

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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236433 ESΣ03.xxxx.023

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PROCEDIMENTO E RISULTATI 236433 ESΣ03.xxxx.023

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,3/4\,b - 1/3\,b - 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

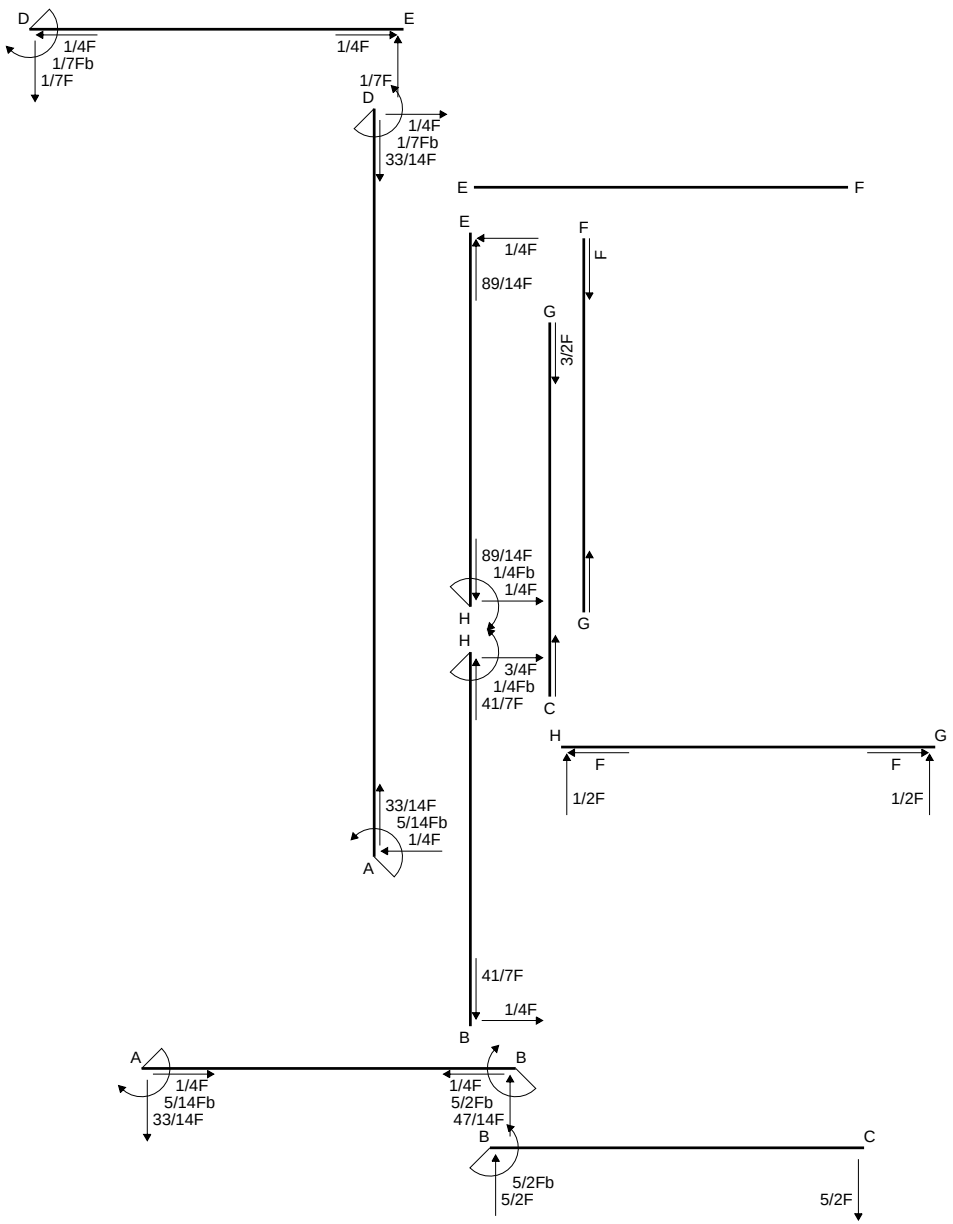
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

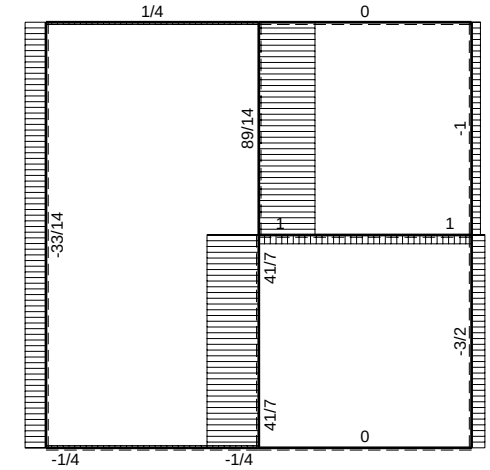
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

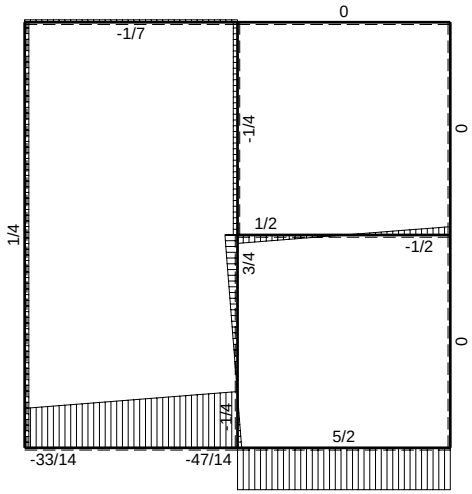
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

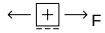
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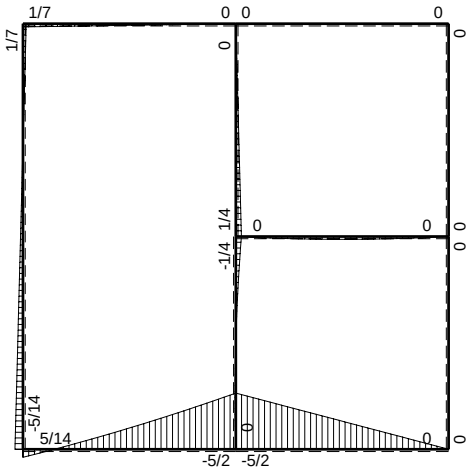






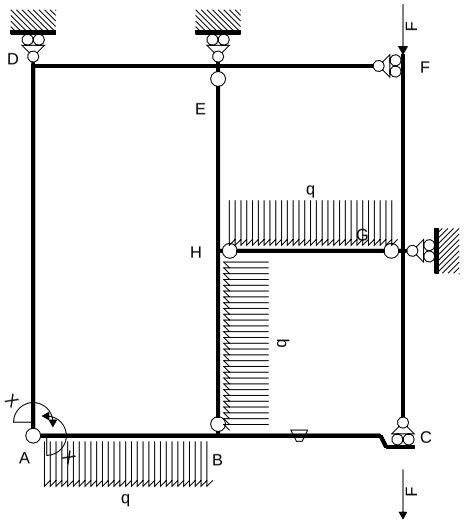




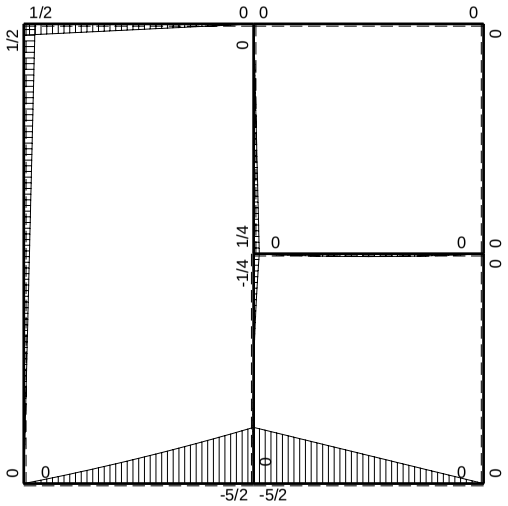




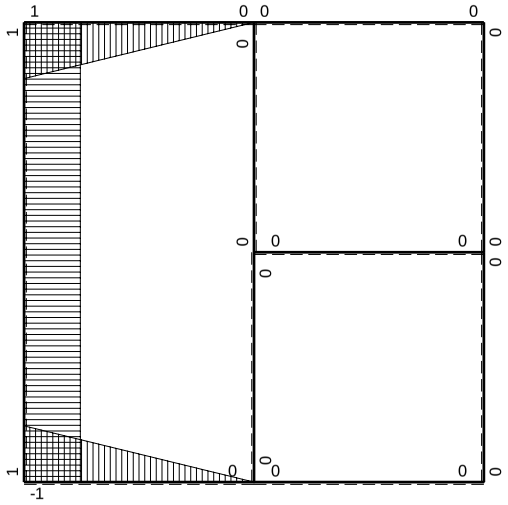
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Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$5/2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

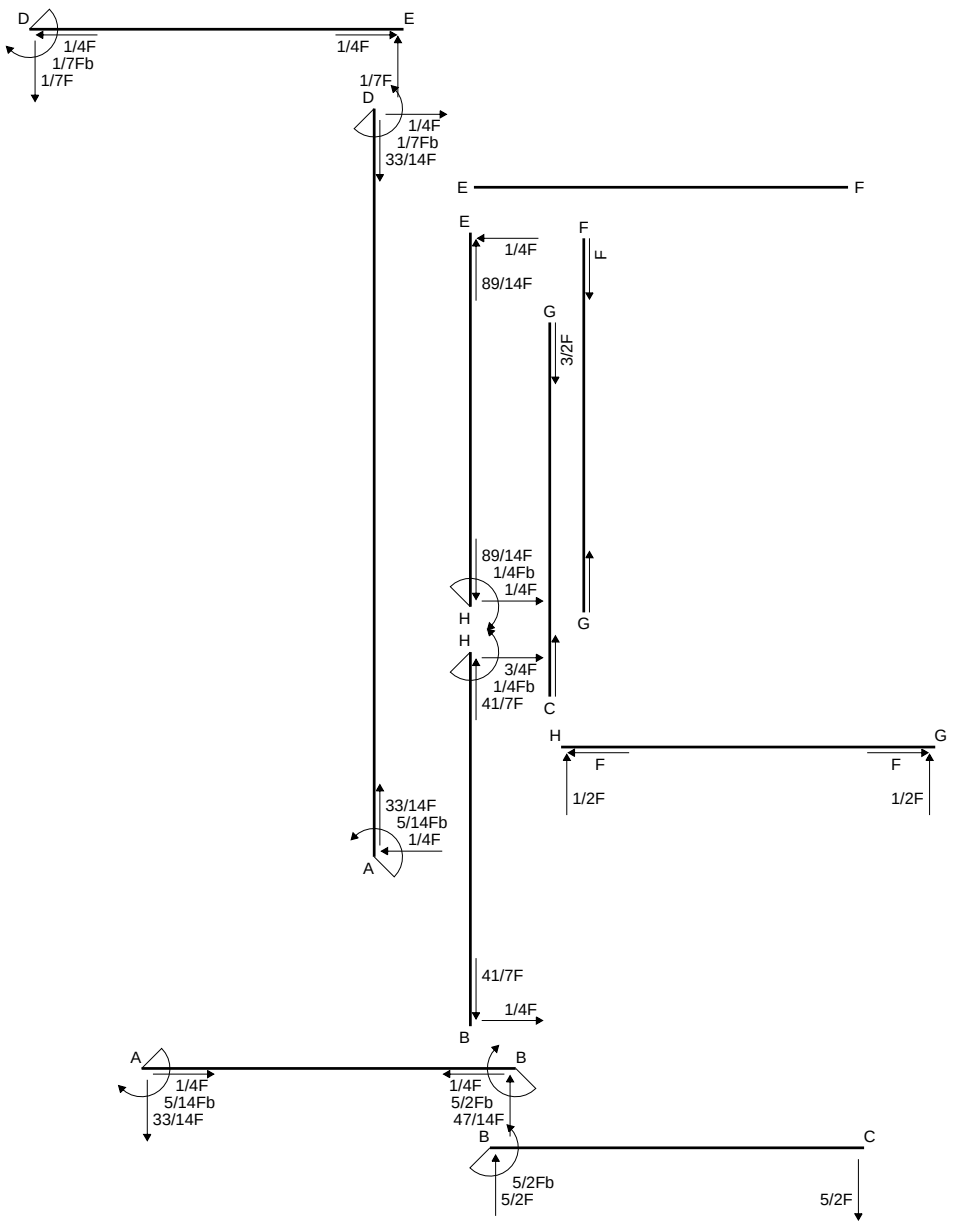
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

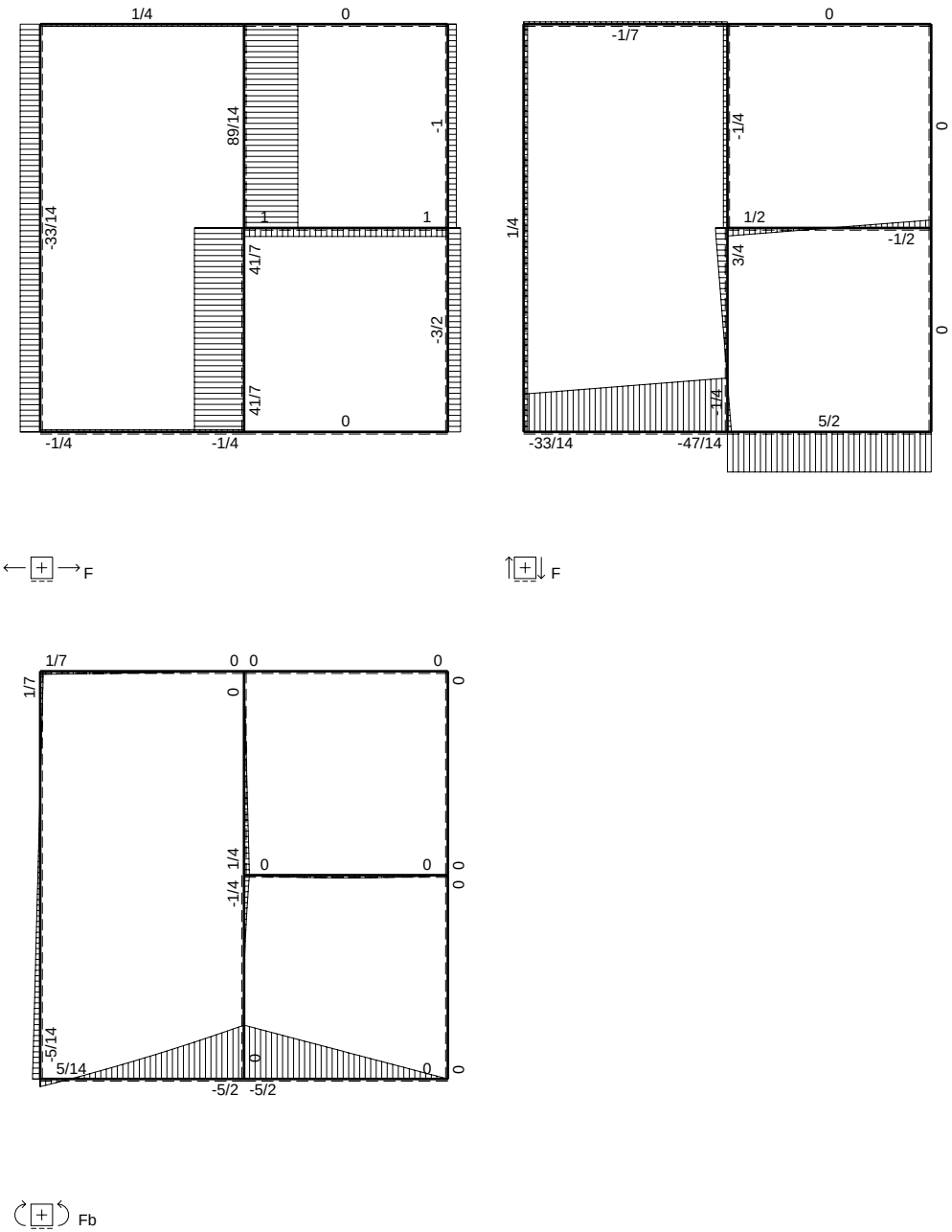
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

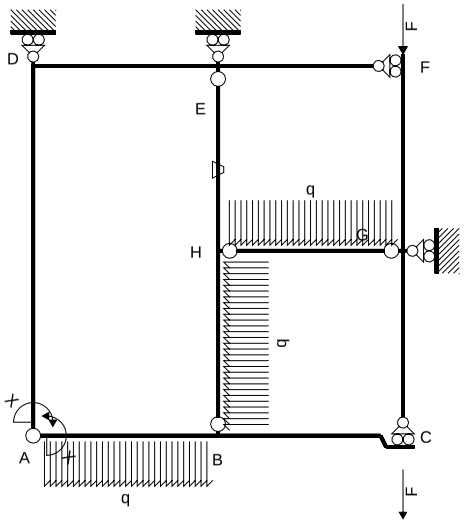
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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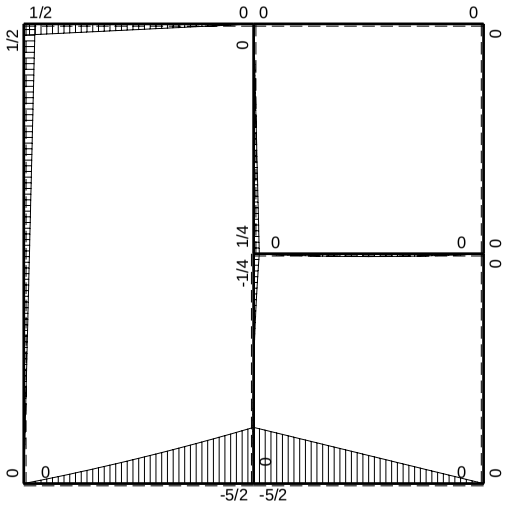
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
D
1/4F
1/7Fb
33/14F
E
F
E
1/4F
89/14F
F
F
G
3/2F
89/14F
1/4Fb
1/4F
H
G
H
3/4F
1/4Fb
41/7F
C
H
G
F
F
1/2F
1/2F
33/14F
5/14Fb
1/4F
A
41/7F
1/4F
B
A
B
1/4F
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1/4F
5/2Fb
47/14F
B
C
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5/2F

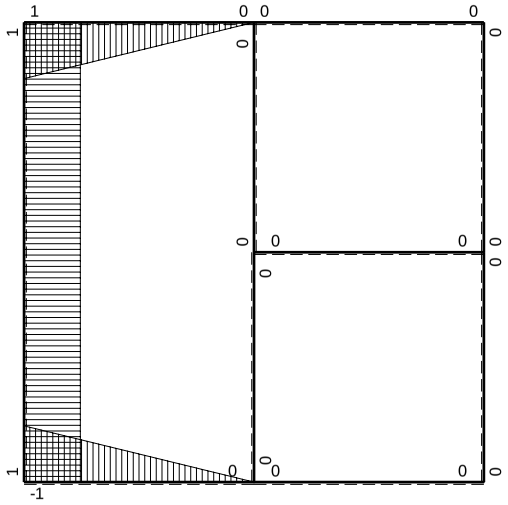

1/4
0
89/14
-1
-33/14
1
1
41/7
-3/2
41/7
0
-1/4
-1/4
F
-1/7
0
-1/4
0
1/2
3/4
-1/2
1/4
0
-1/4
5/2
-33/14
-47/14
F
1/7
0 0
0
1/7
0
0
1/4
0
0
-1/4
0 0
0
-5/14
5/14
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-1/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

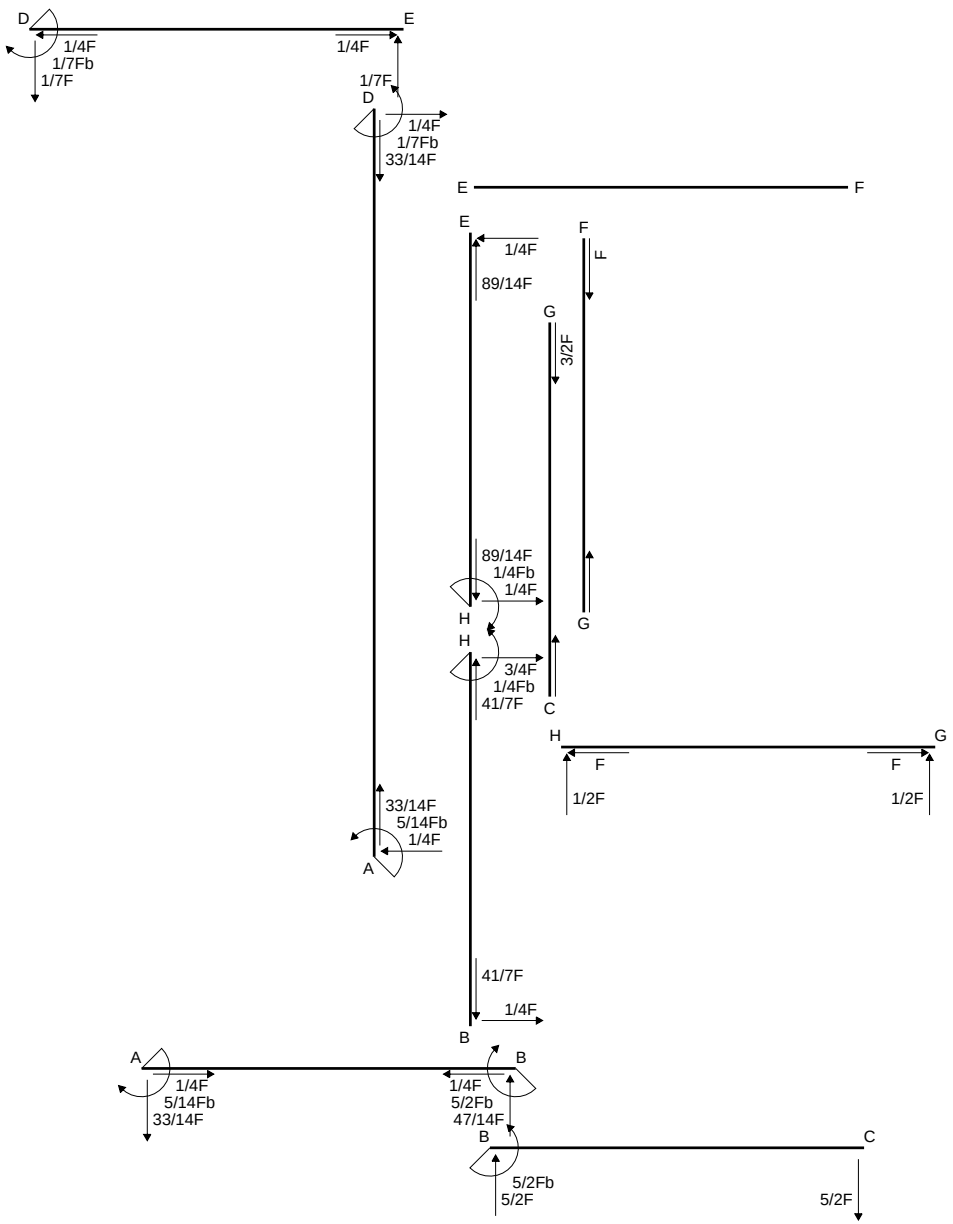
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

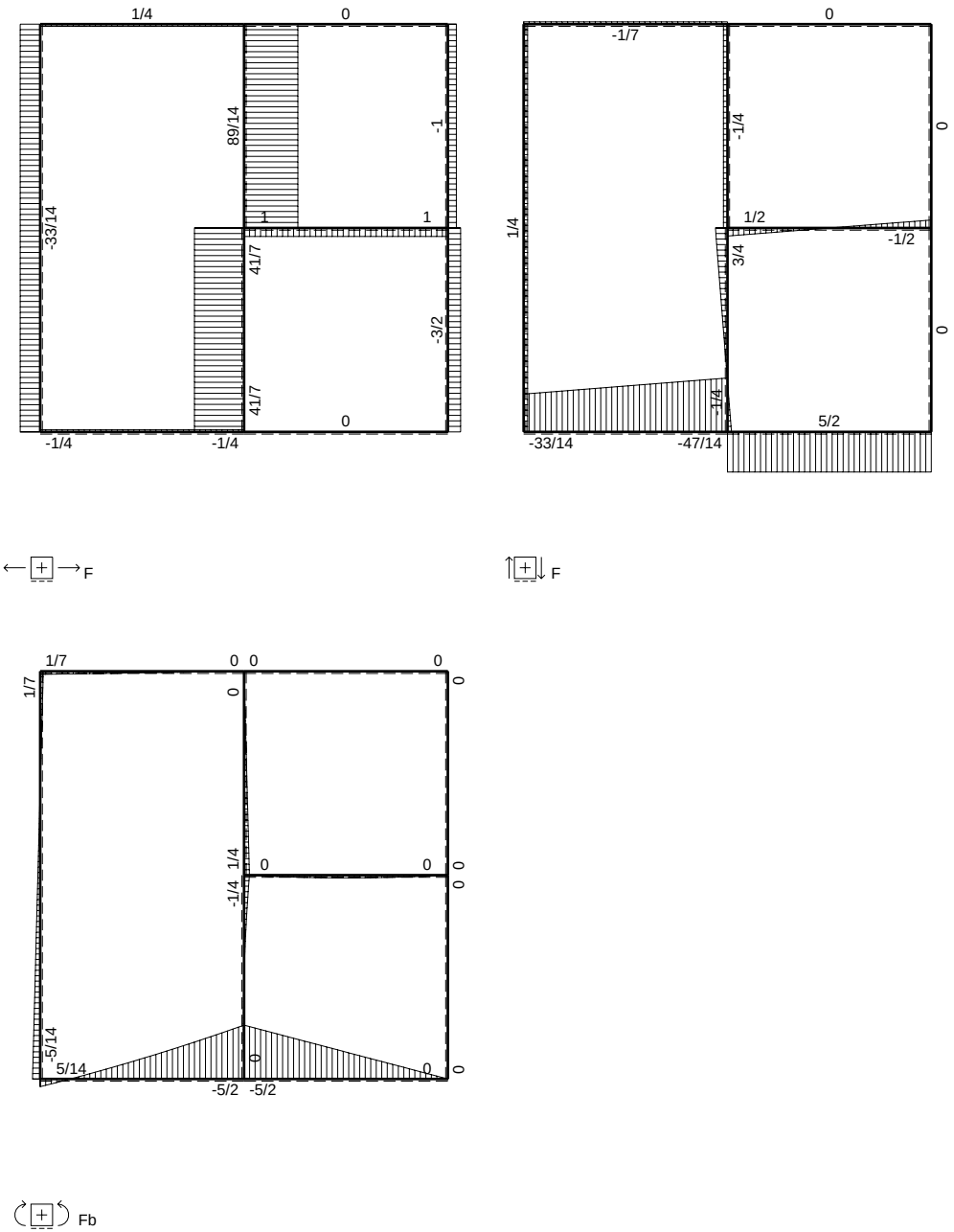
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

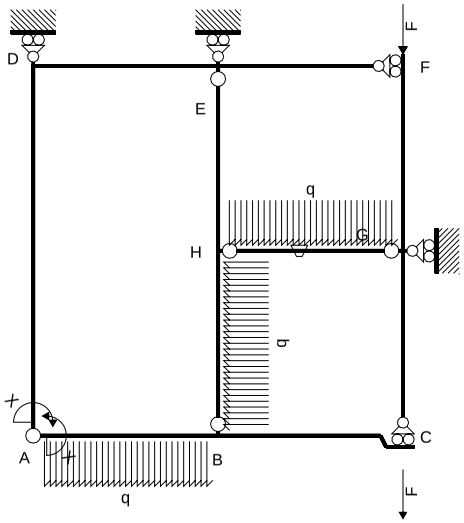
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 237330 ESΣ03.xxxx.025

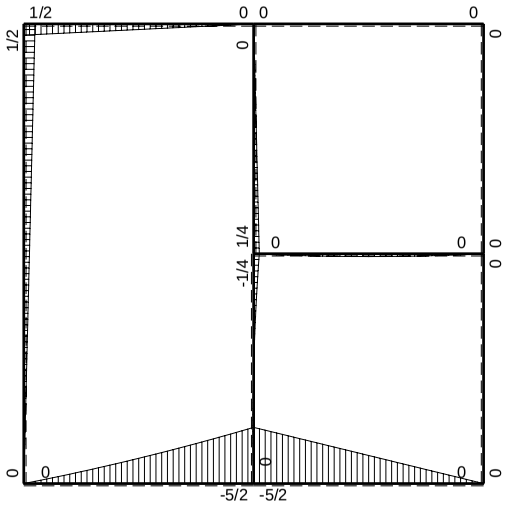
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
D
1/4F
1/7Fb
33/14F
E
F
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1/4F
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F
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3/4F
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41/7F
C
H
G
F
F
1/2F
1/2F
33/14F
5/14Fb
1/4F
A
41/7F
1/4F
B
A
B
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C
5/2Fb
5/2F
5/2F

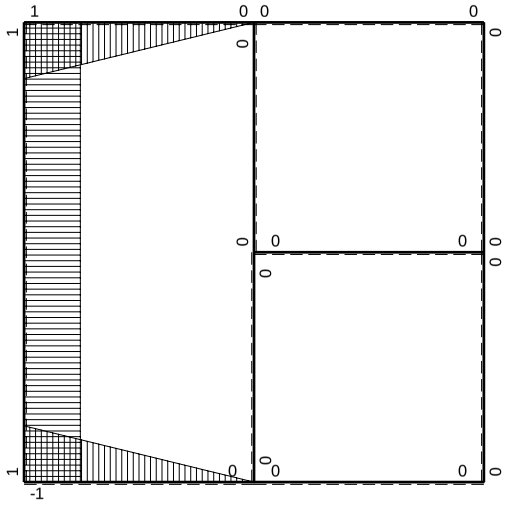

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1/7
0 0
0
1/7
0
0
1/4
0
0
-1/4
0 0
0
-5/14
5/14
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212625 ESΣ03.xxxx.026

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

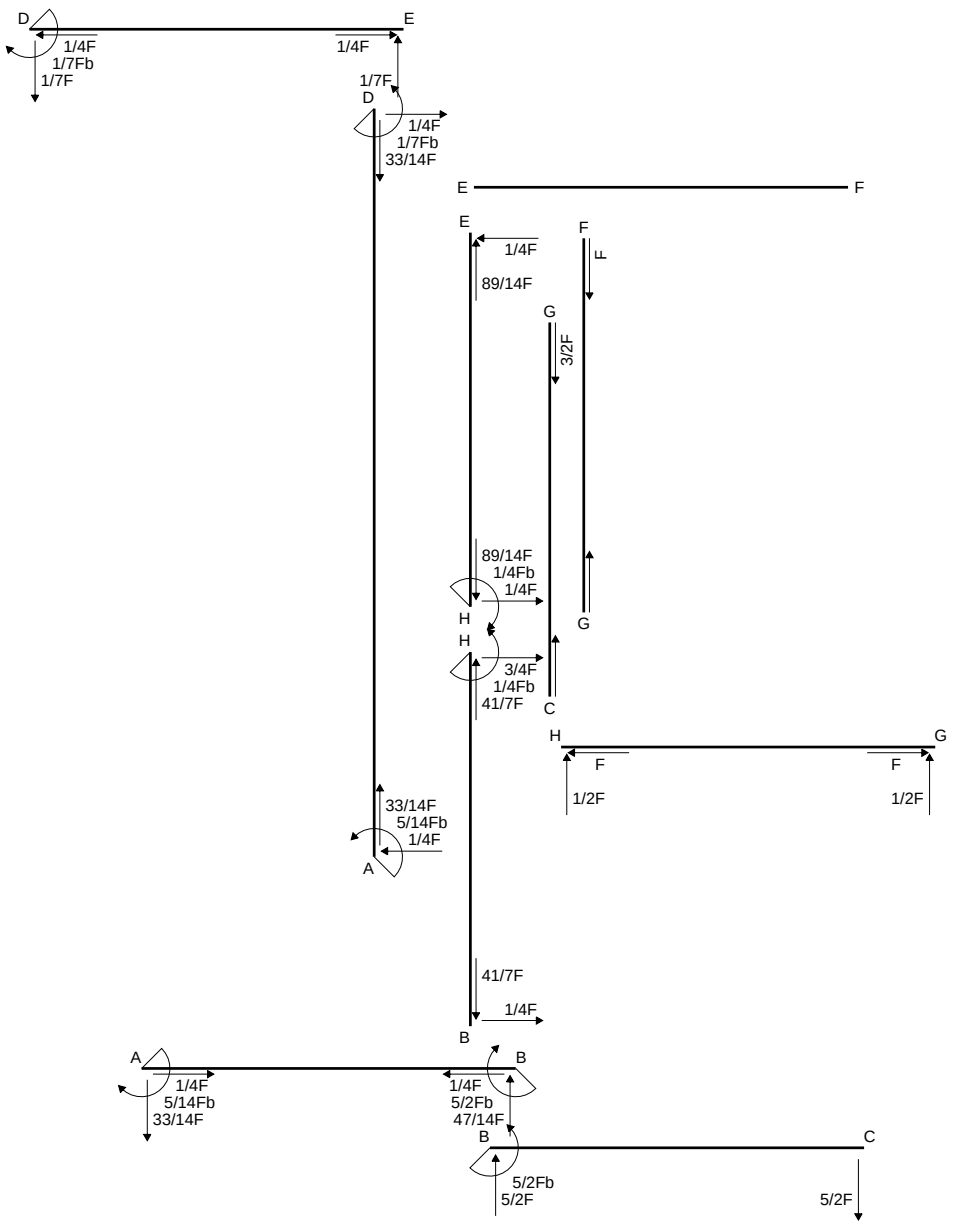
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

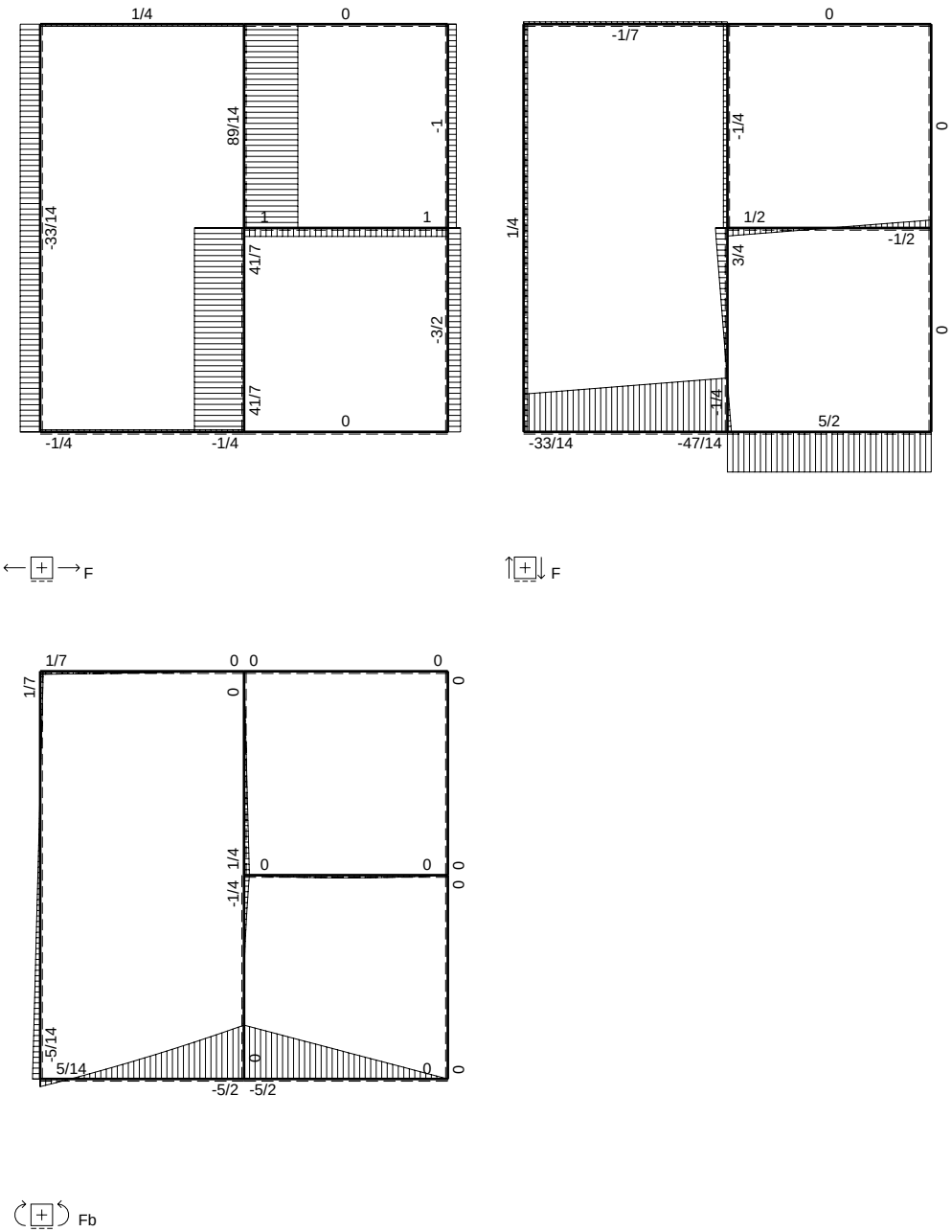
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

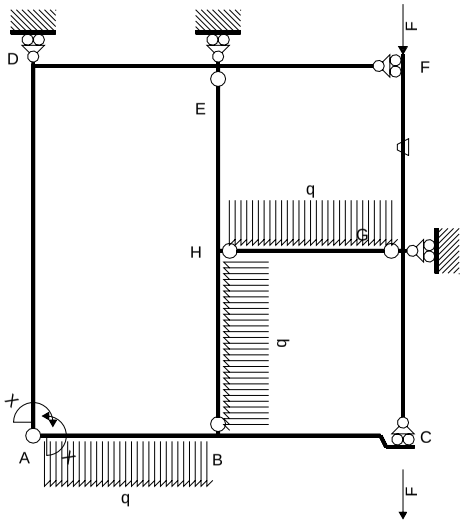
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 212625 ESΣ03.xxxx.026

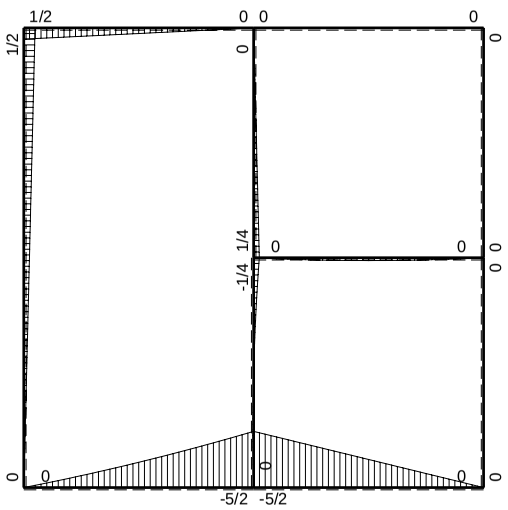
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D
E
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1/7Fb
1/7F
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1/7F
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1/7Fb
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G
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F
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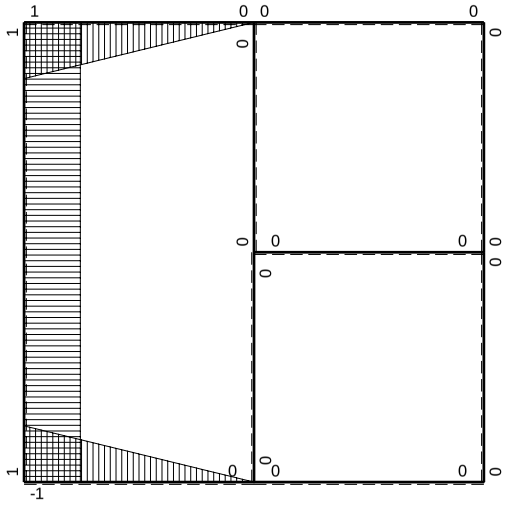

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41/7
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-1/4
-1/4
F
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1/2
3/4
-1/2
0
-1/4
5/2
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0
0
-1/4
0 0
0
-5/14
5/14
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238395 ESΣ03.xxxx.027

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PROCEDIMENTO E RISULTATI 238395 ESΣ03.xxxx.027

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

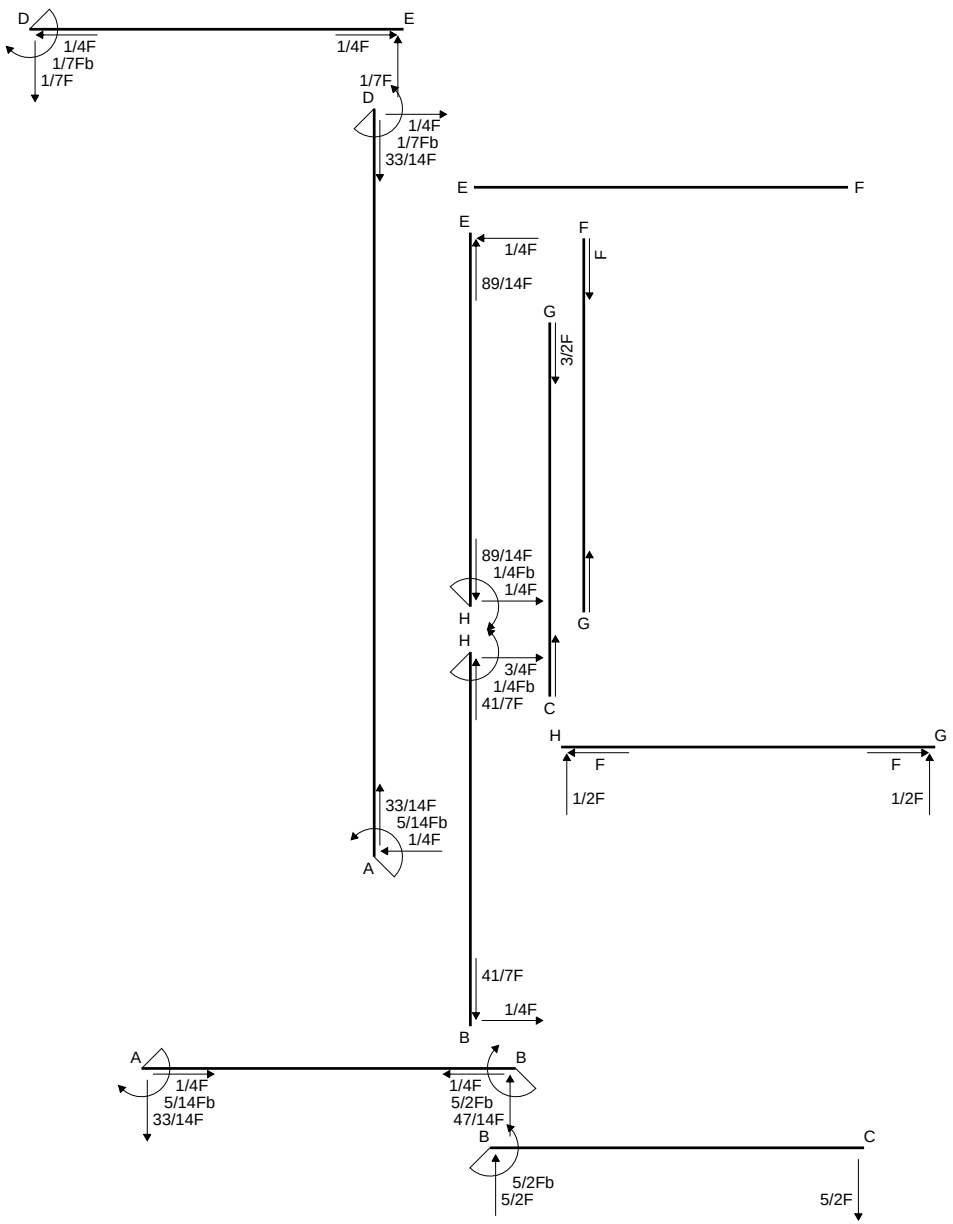
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

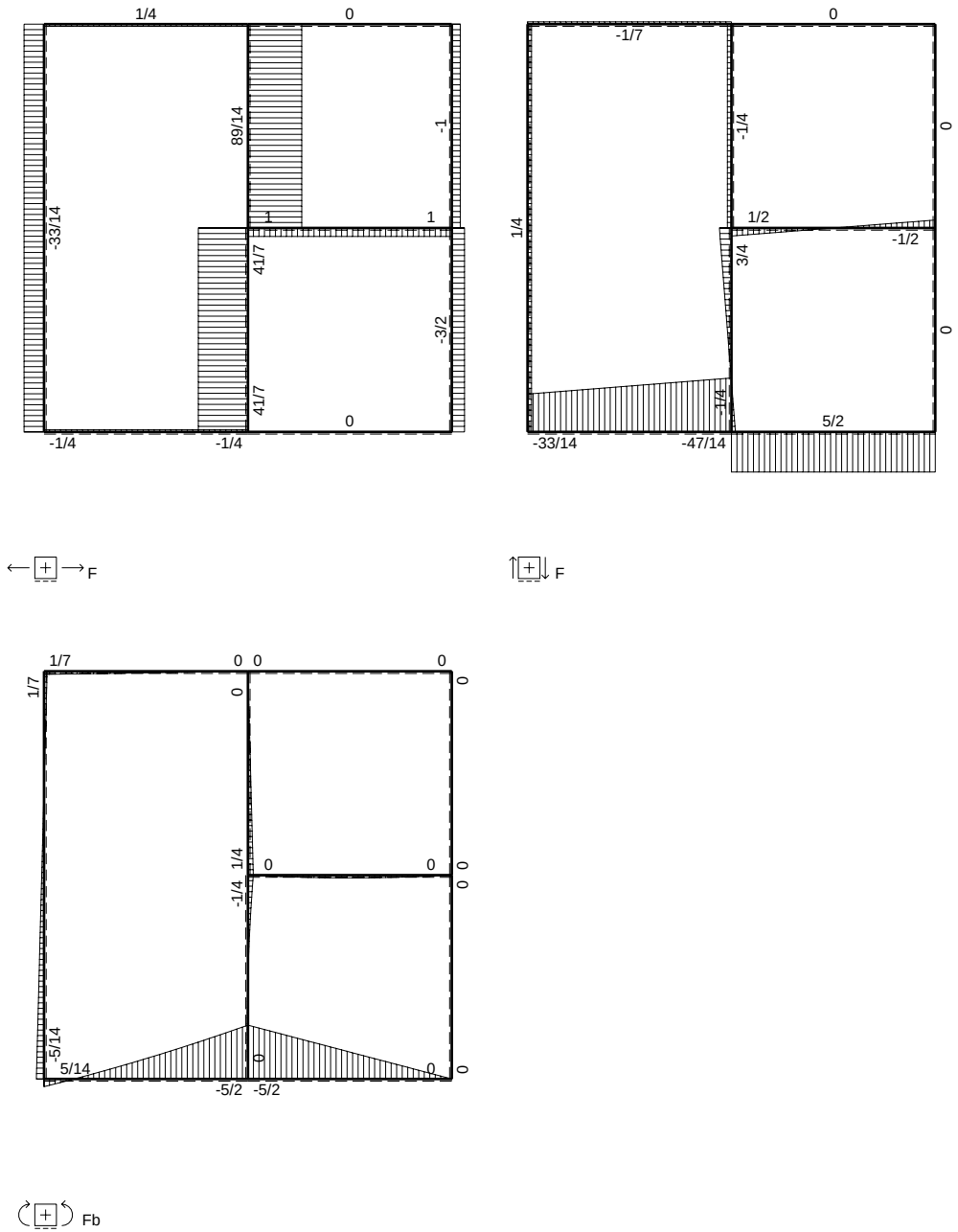
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

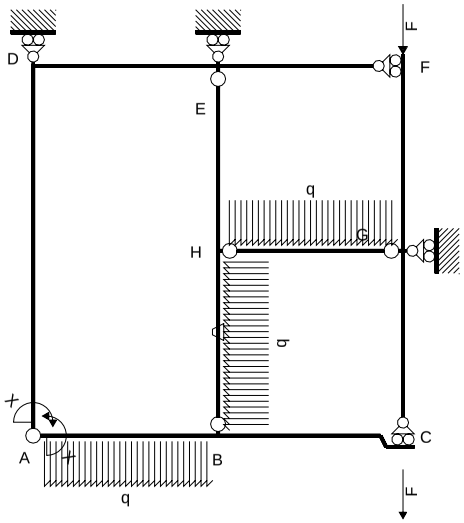
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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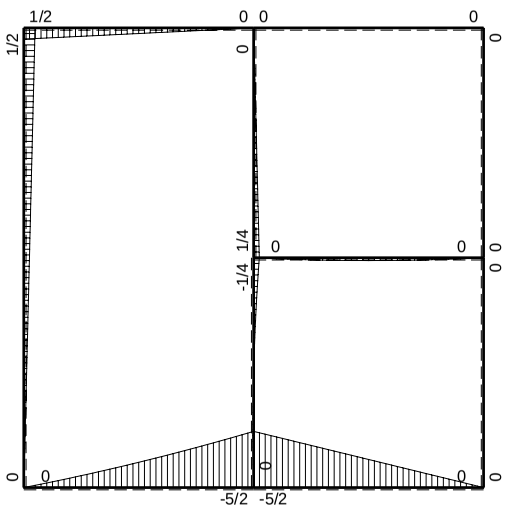
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
D
1/4F
1/7Fb
33/14F
E
F
E
1/4F
89/14F
F
F
G
3/2F
89/14F
1/4Fb
1/4F
H
G
H
3/4F
1/4Fb
41/7F
C
H
G
F
F
1/2F
1/2F
33/14F
5/14Fb
1/4F
A
41/7F
1/4F
B
A
B
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B
C
5/2Fb
5/2F
5/2F

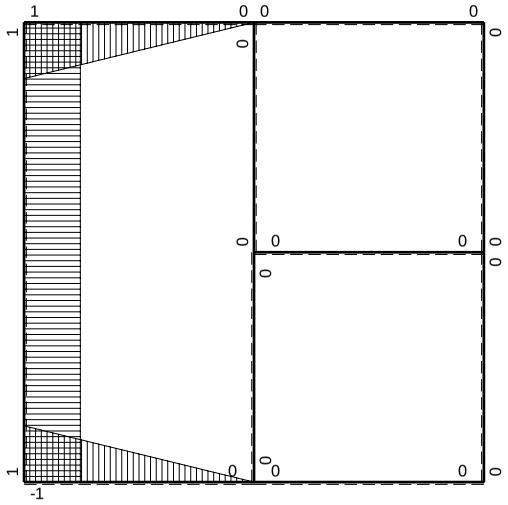

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-1/4
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-1/4
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1/2
3/4
-1/2
1/4
0
-1/4
5/2
-33/14
-47/14
F
1/7
0 0
0
1/7
0
0
1/4
0
0
-1/4
0 0
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

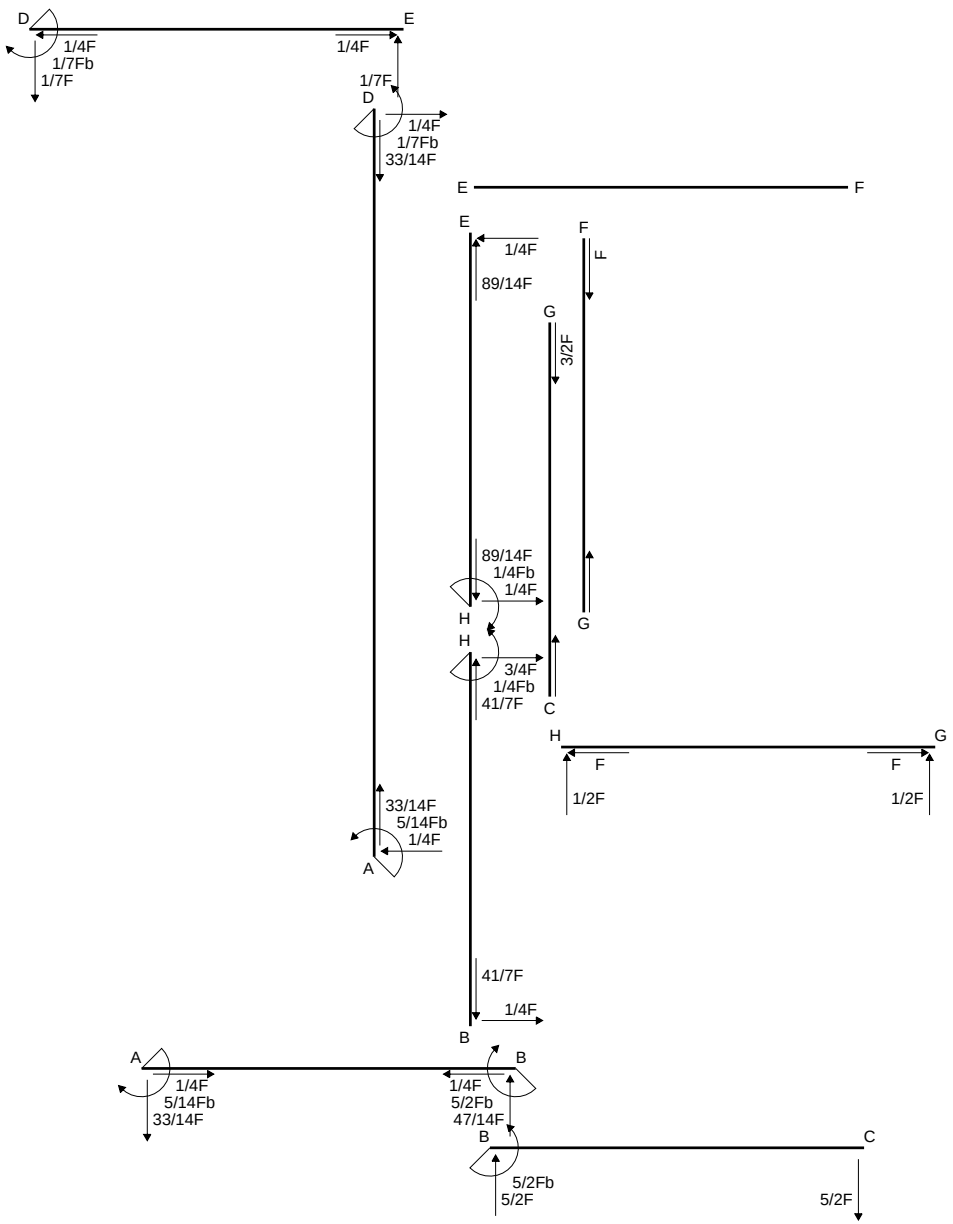
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

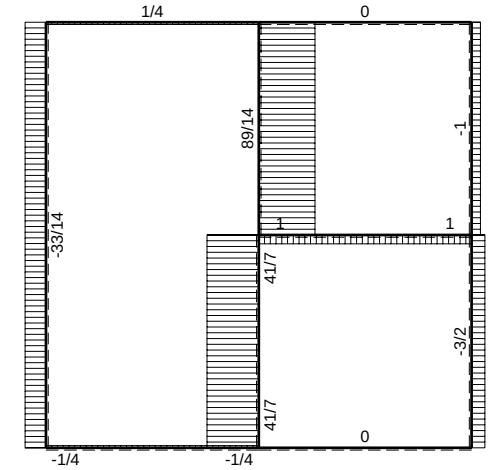
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

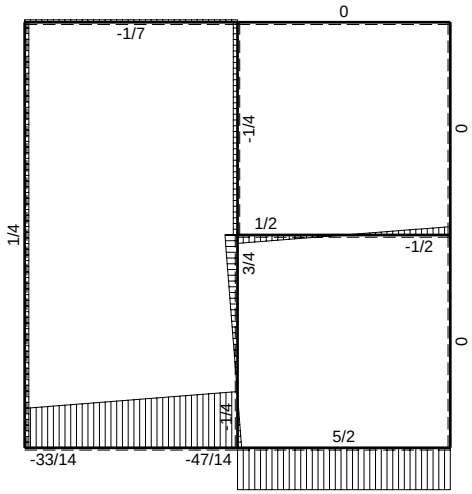
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

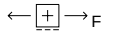
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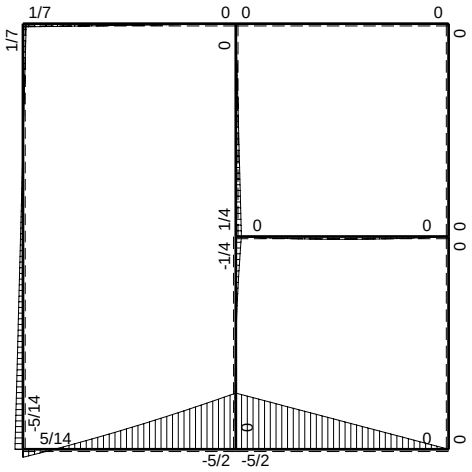

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1/7Fb
1/7F
1/4F
1/7F
D
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1/7Fb
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89/14F
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1/4F
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33/14F
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5/2Fb
47/14F
B
C
5/2Fb
5/2F
5/2F


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-1/4

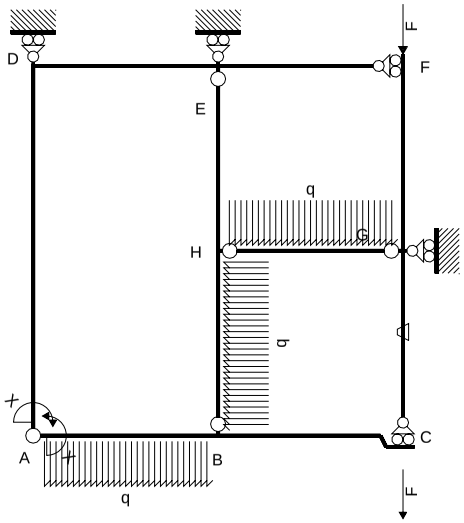

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5/2
-33/14
-47/14


F

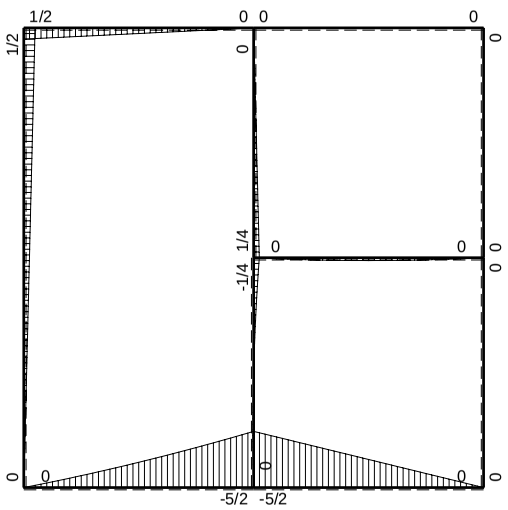

F


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1/7
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1/4
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5/14
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-5/2

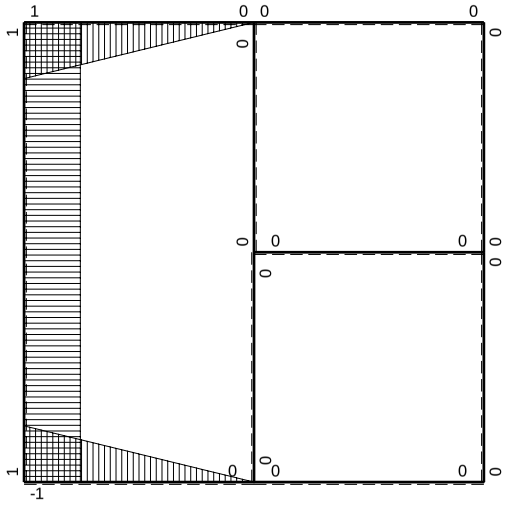

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236678 ESΣ03.xxxx.029

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

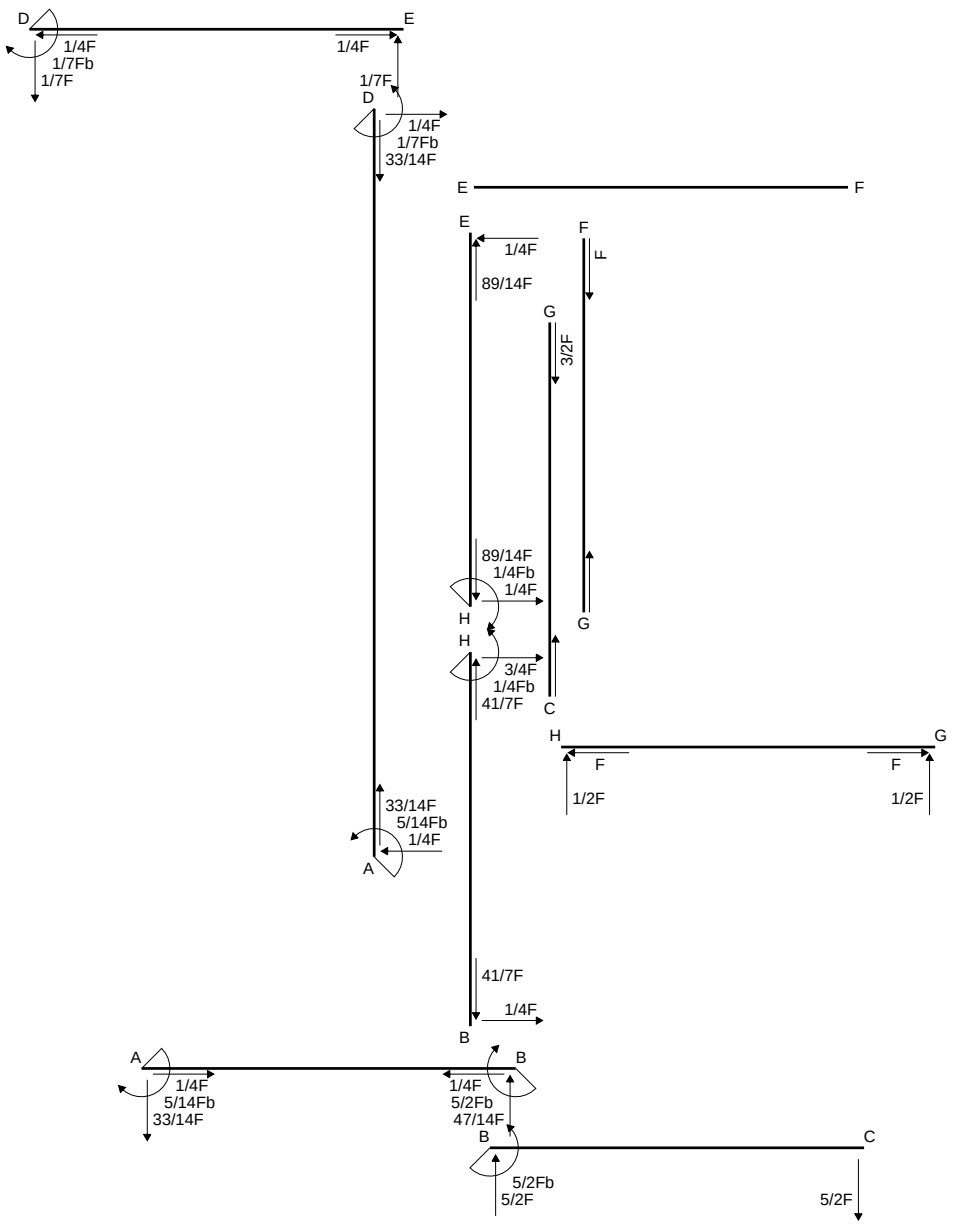
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

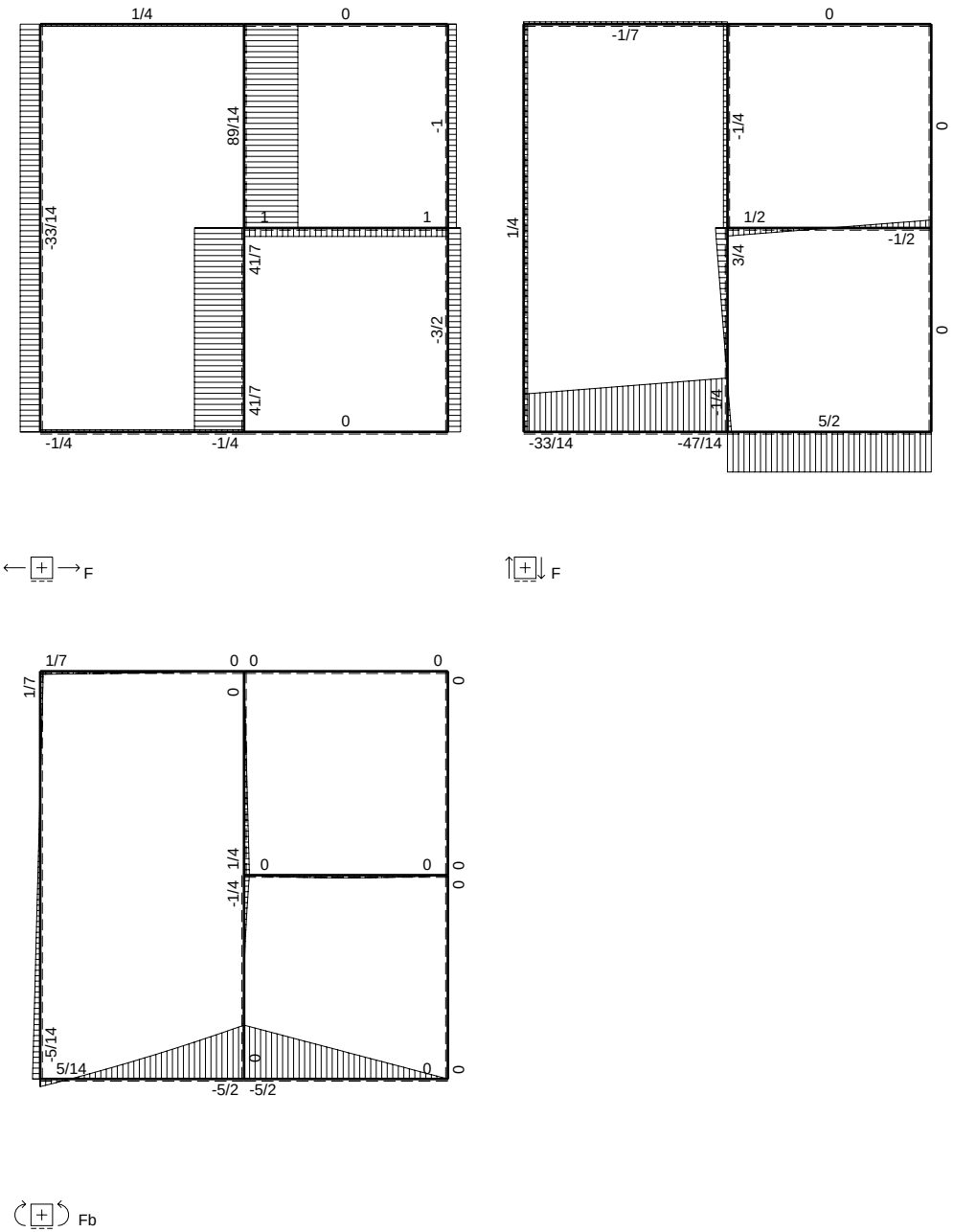
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

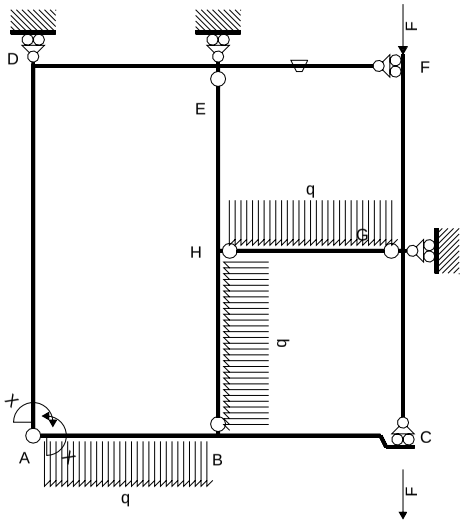
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 236678 ESΣ03.xxxx.029

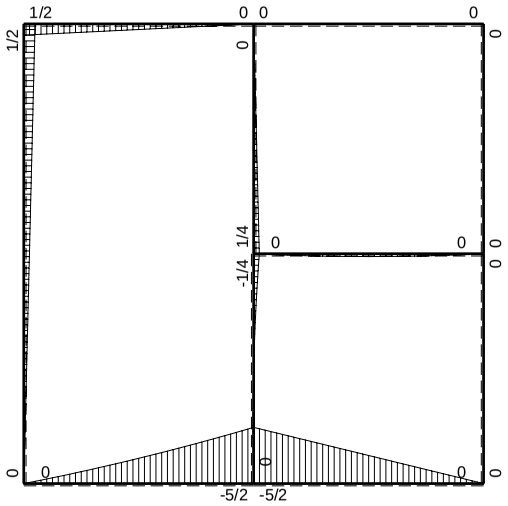
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1/4F
1/7F
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C
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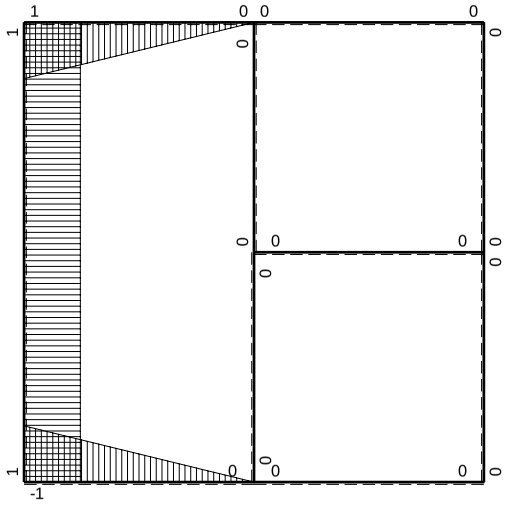

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0 0
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1/4
0
0
-1/4
0 0
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

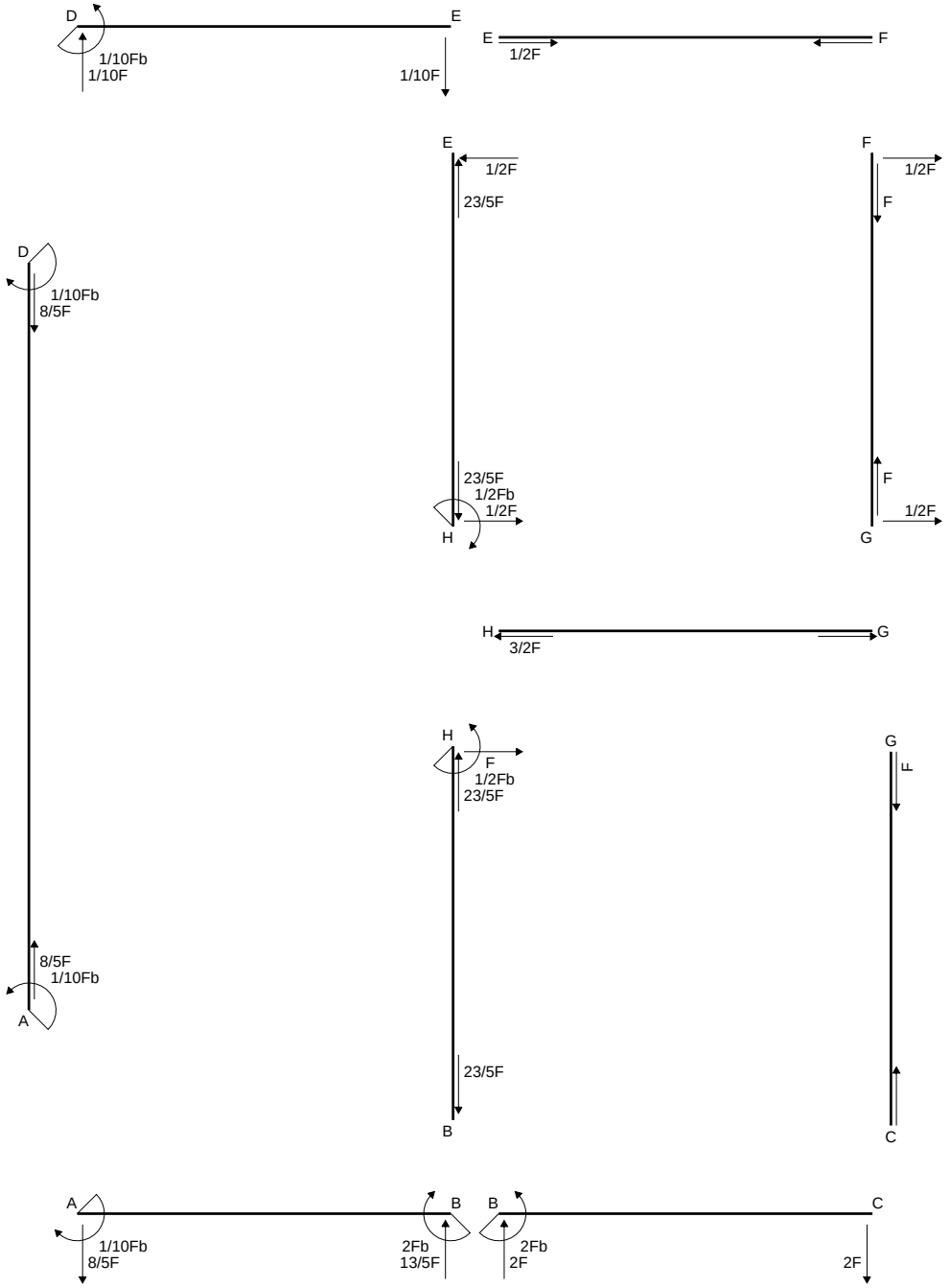
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

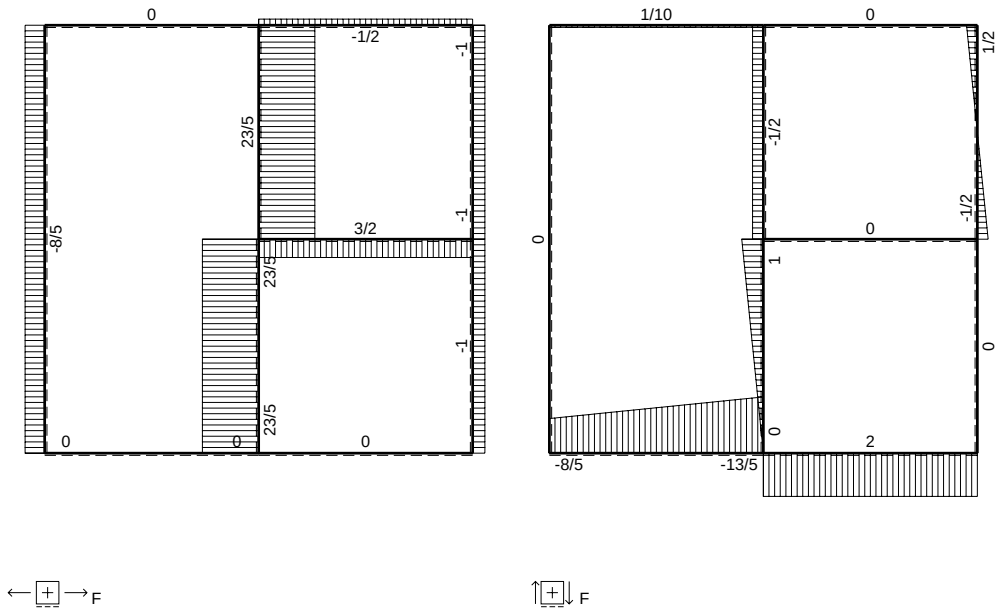
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

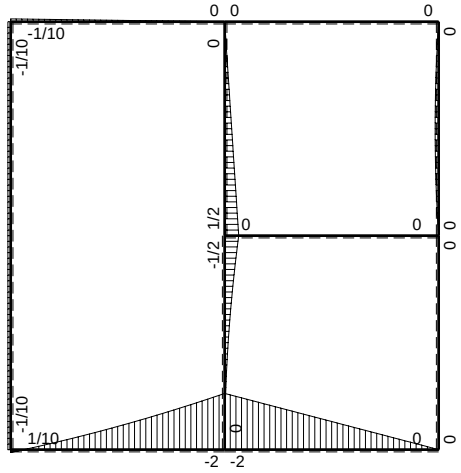
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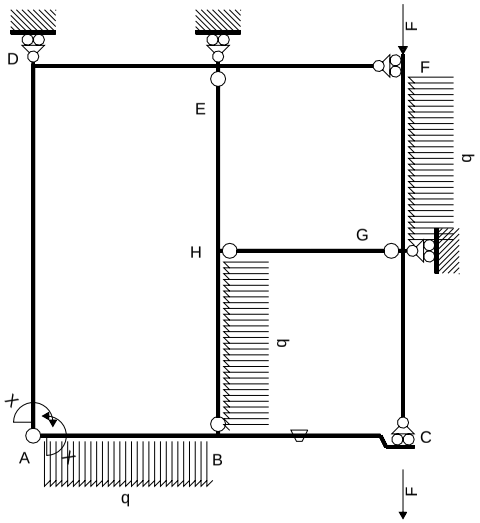

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C
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B
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C
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2F
2F


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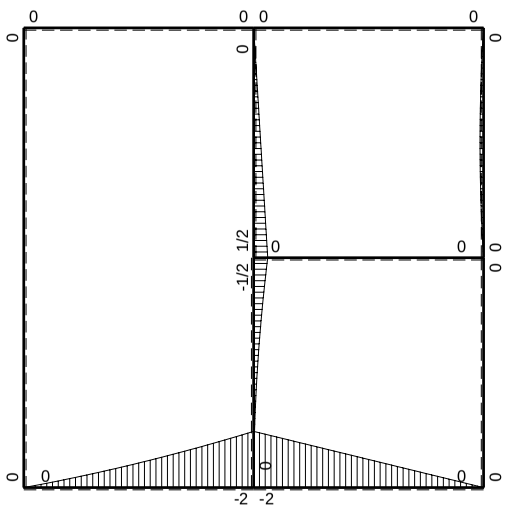
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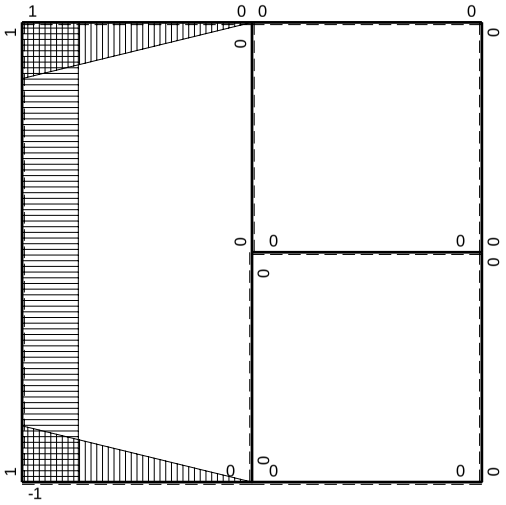
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 984894 ESΣ03.xxxx.031

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PROCEDIMENTO E RISULTATI 984894 ESΣ03.xxxx.031

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	-Fb/EJ	0	0	0	0+0	0
CB b	0	2Fx	Fb/EJ	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
EH b	0	-1/2Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

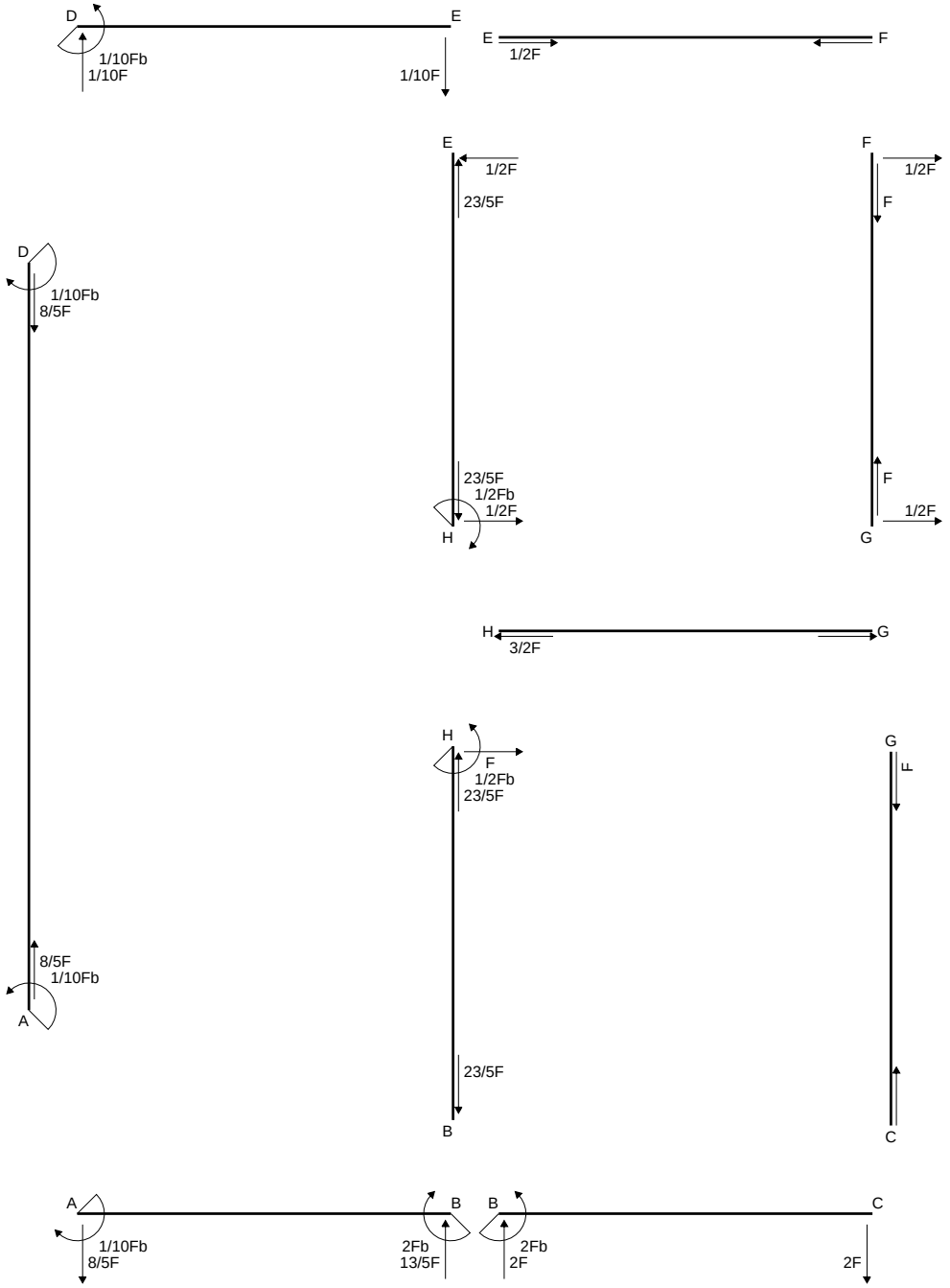
$L_{BA}^{Xo} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

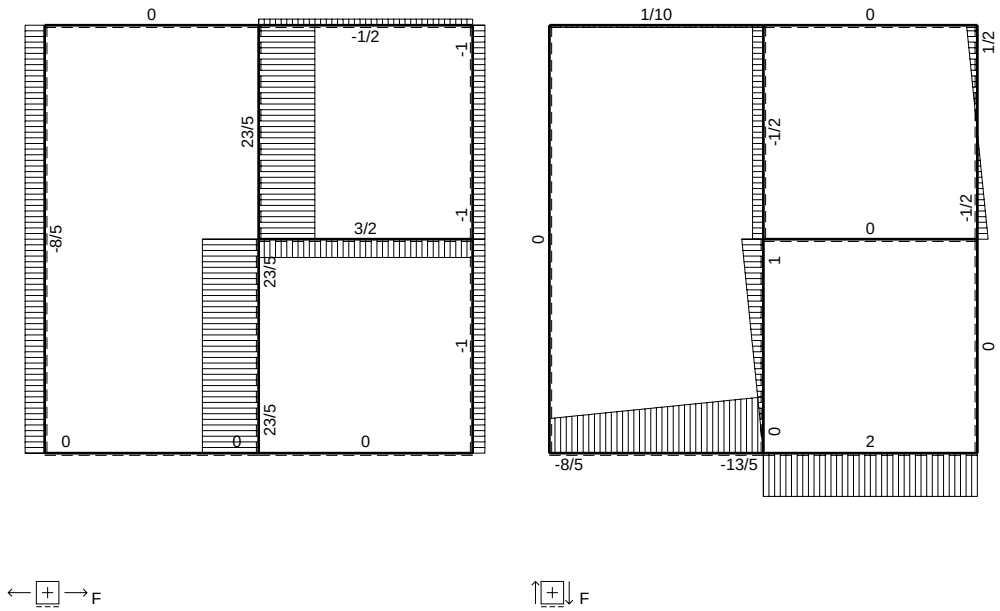
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

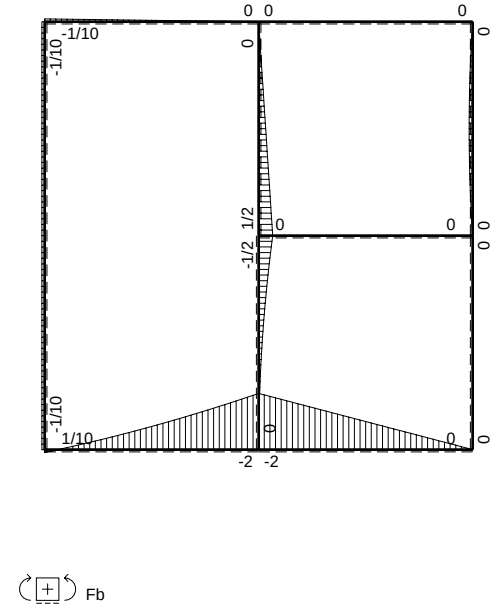
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

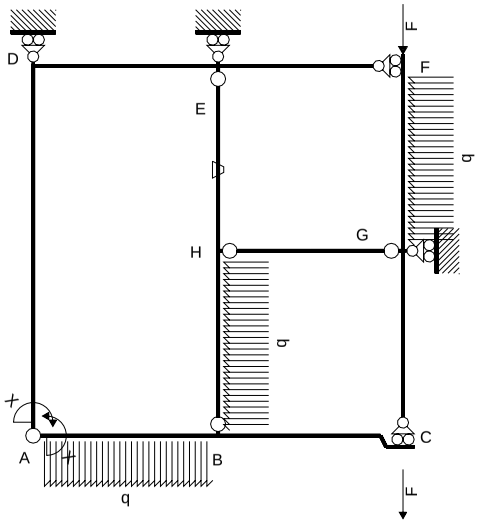
ESΣ03.xxxx.031

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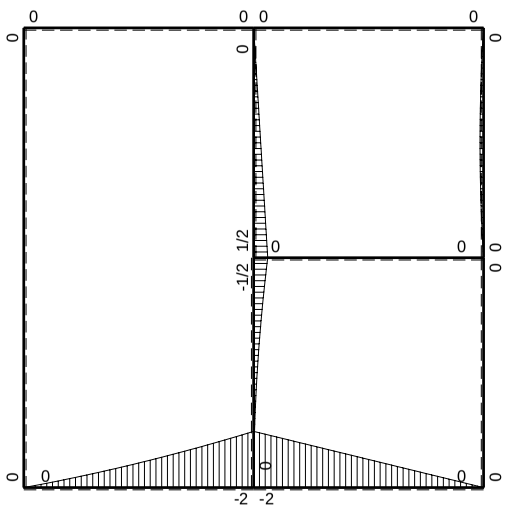

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1/10Fb
1/10F
1/10F
E
F
1/2F
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1/2F
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F
1/2F
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G
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B
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2F
2F


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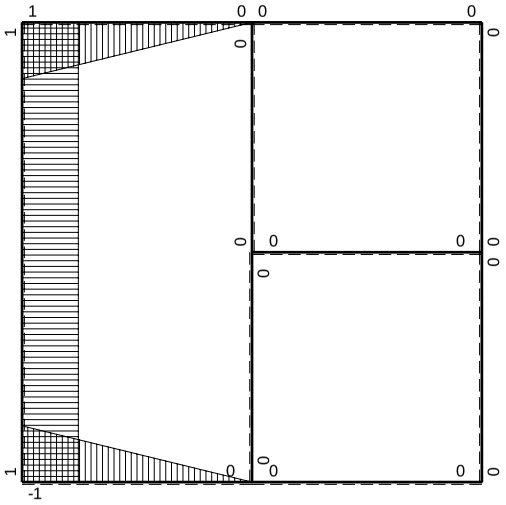

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1/10
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238135 ESΣ03.xxxx.032

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PROCEDIMENTO E RISULTATI 238135 ESΣ03.xxxx.032

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-1/2Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,3/4\,b - 1/3\,b - 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

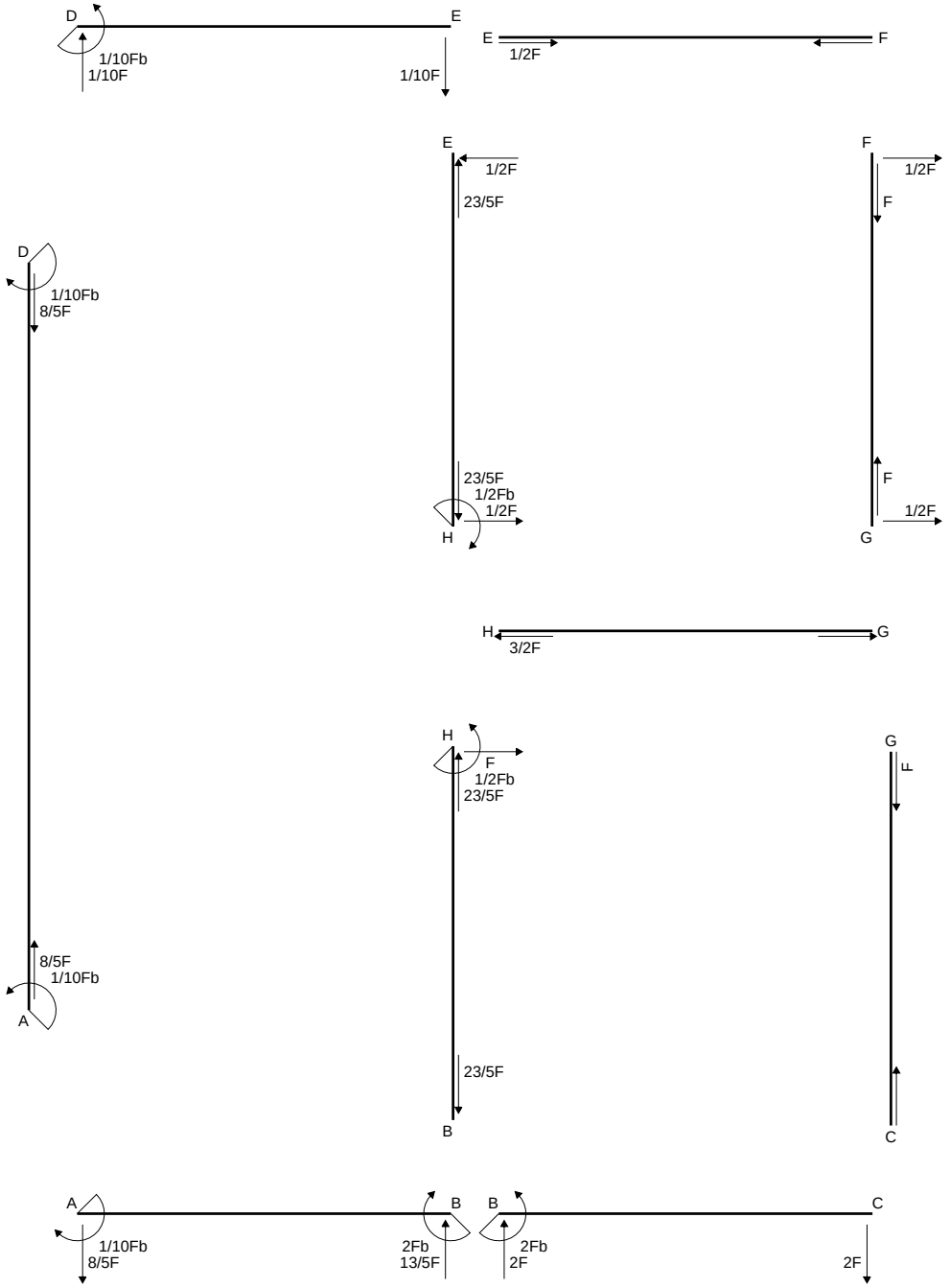
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

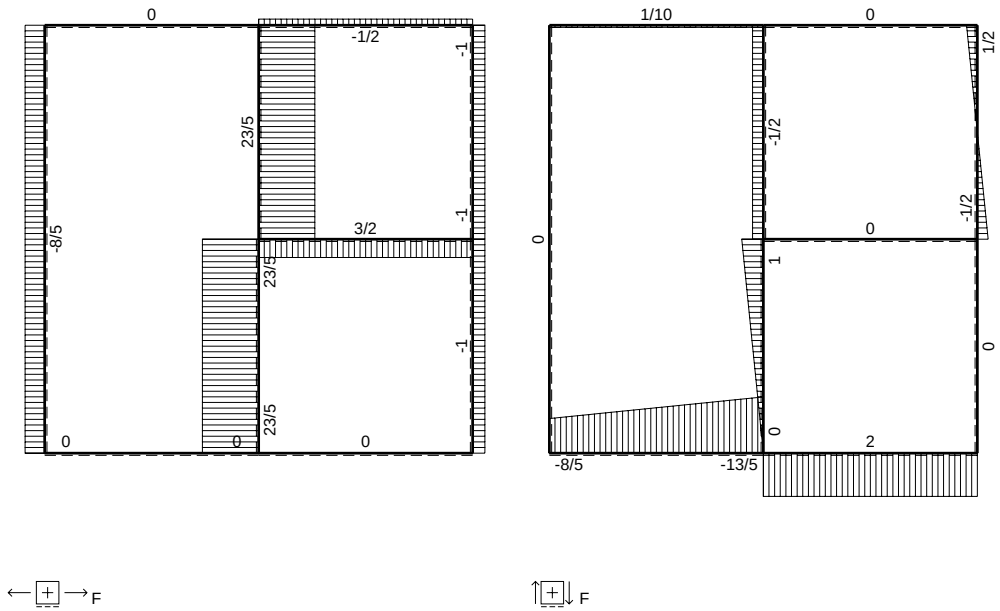
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

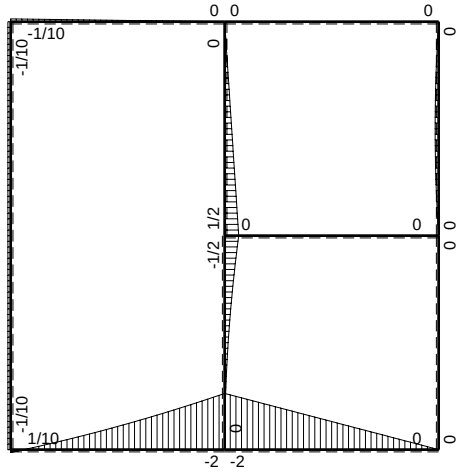
ESΣ03.xxxx.032

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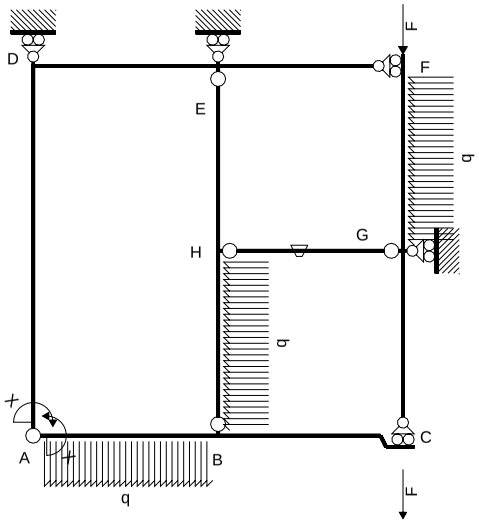

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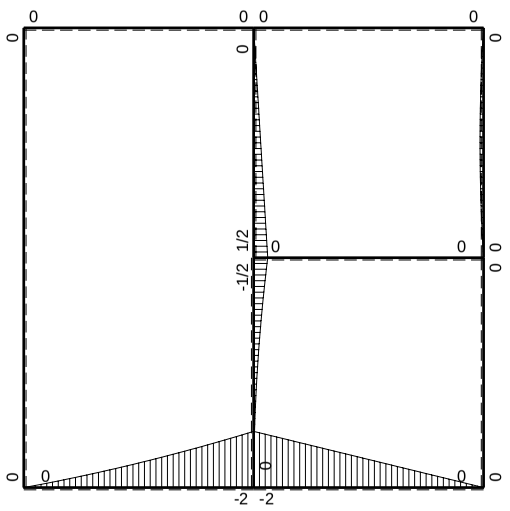
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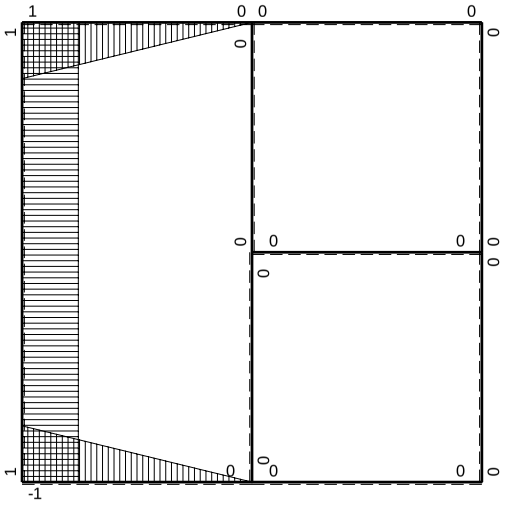
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238530 ESΣ03.xxxx.033

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PROCEDIMENTO E RISULTATI 238530 ESΣ03.xxxx.033

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
EH b	0	-1/2Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

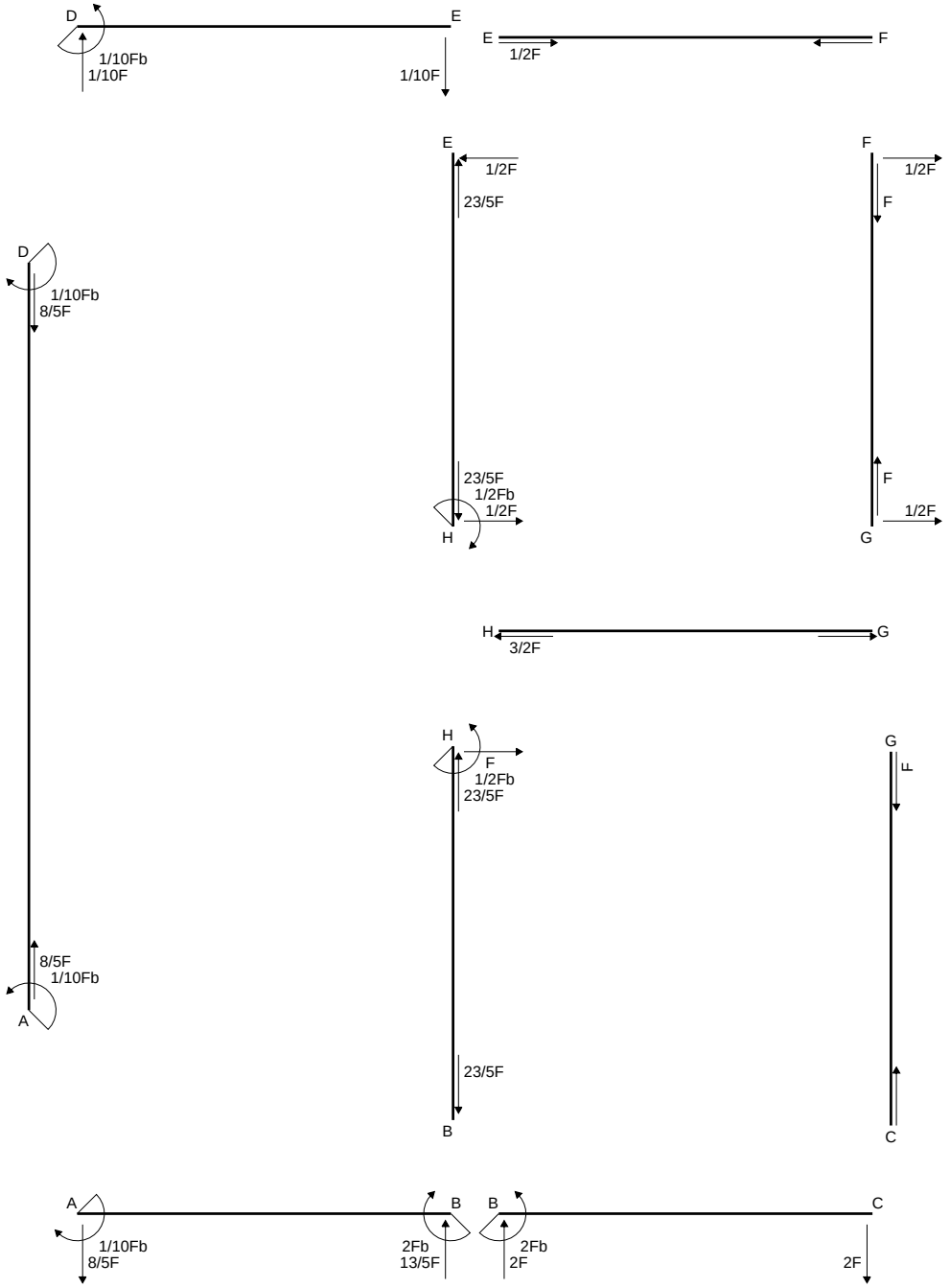
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

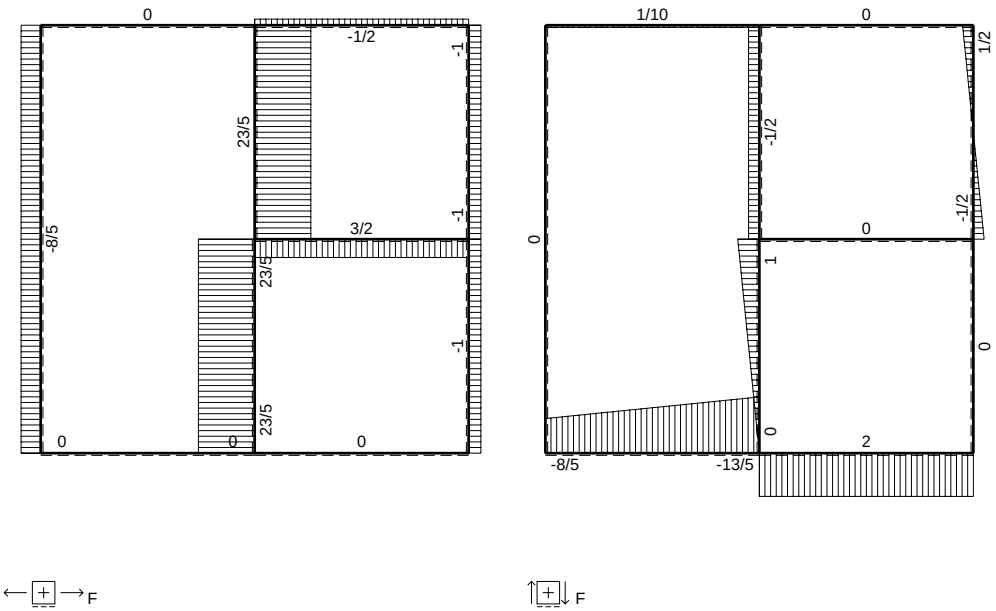
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

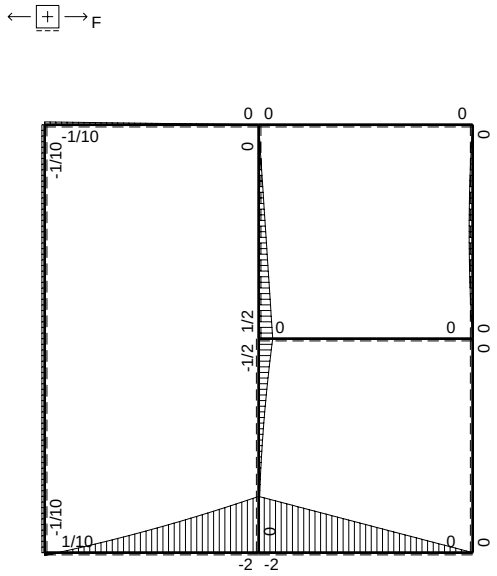
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

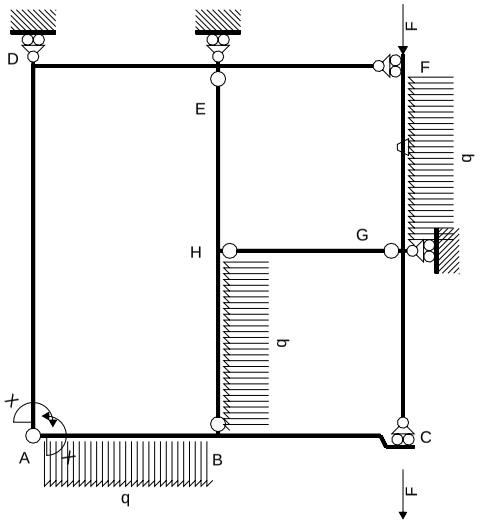
ESΣ03.xxxx.033

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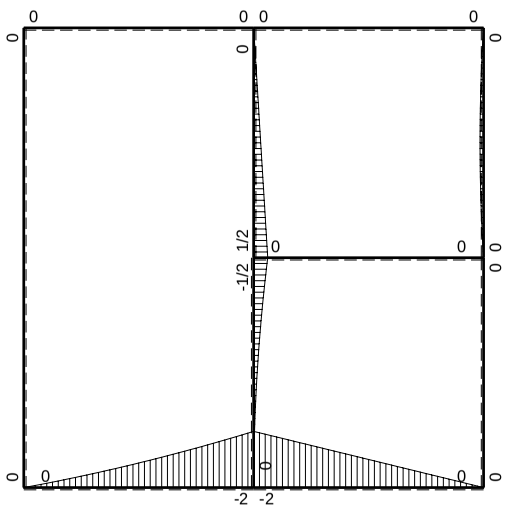

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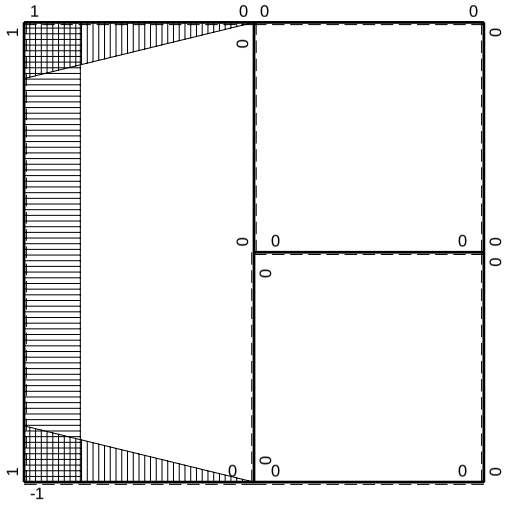

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-2 -2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236818 ESΣ03.xxxx.034

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PROCEDIMENTO E RISULTATI 236818 ESΣ03.xxxx.034

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-1/2Fx$	0	0	0	0	0+0	0
EH b	0	$-1/2Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (1/3\,b)\ \ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\,b - 1/3\,b - 1/8\,b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

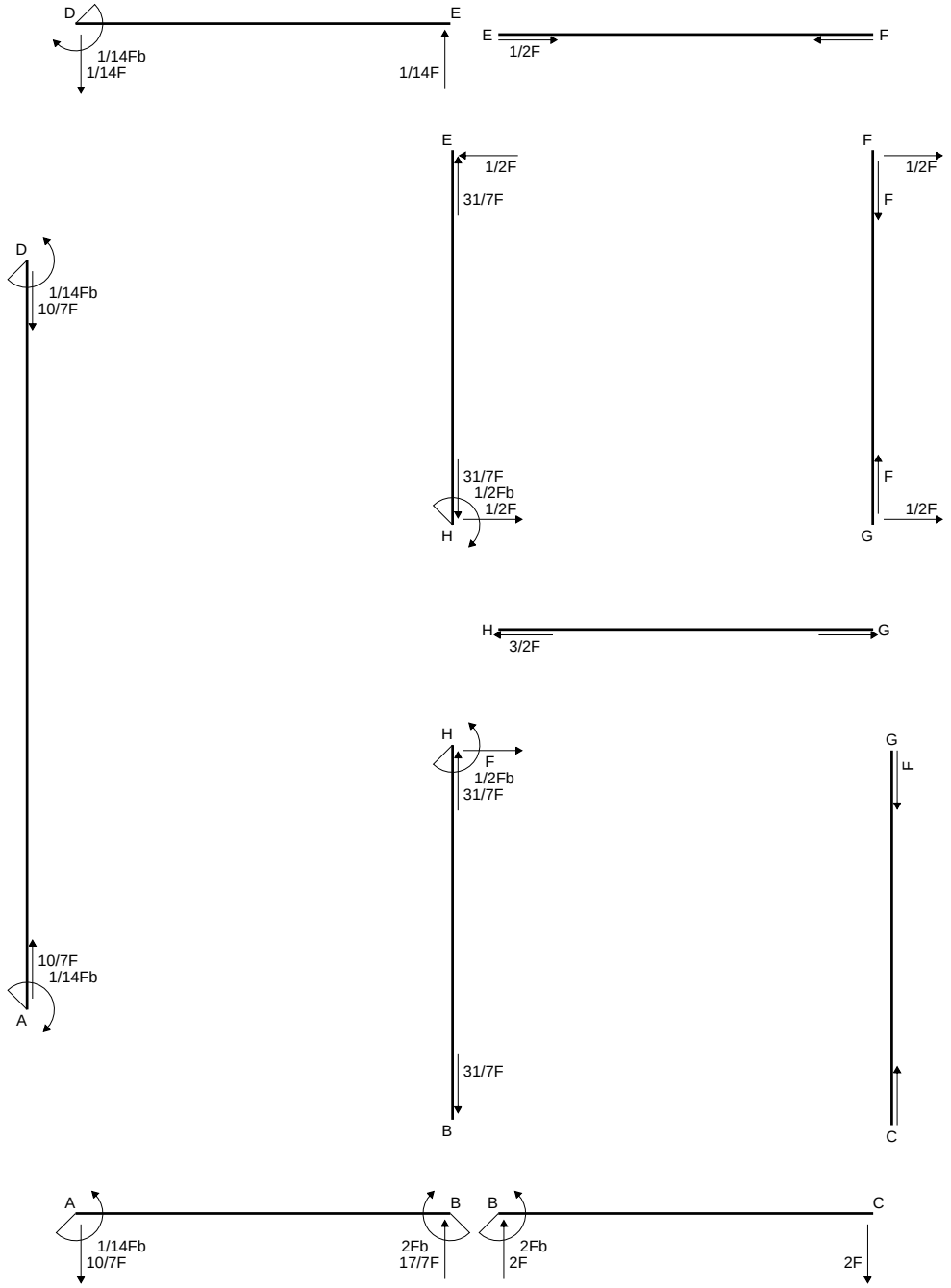
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

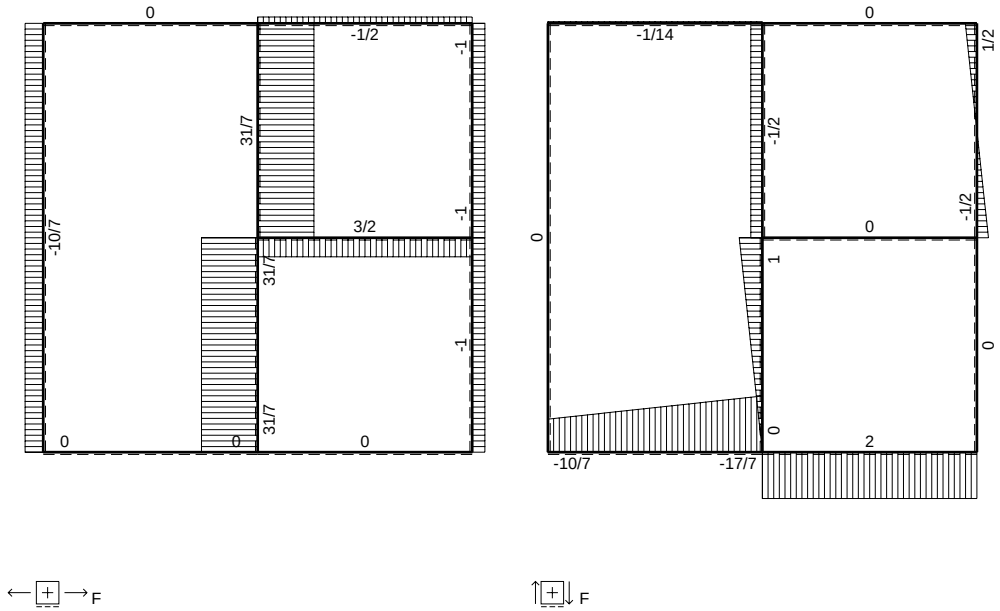
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

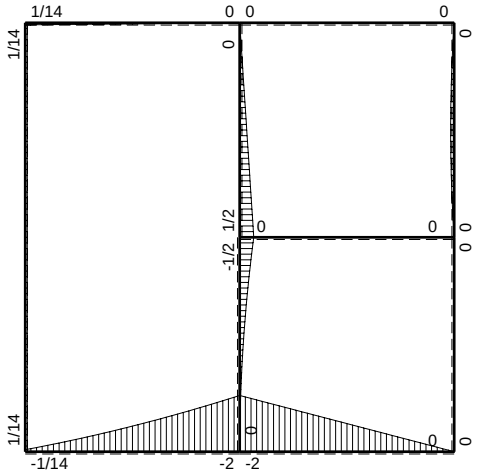
ESΣ03.xxxx.034

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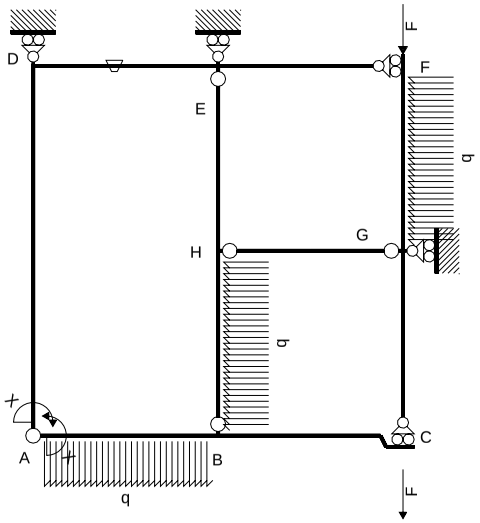

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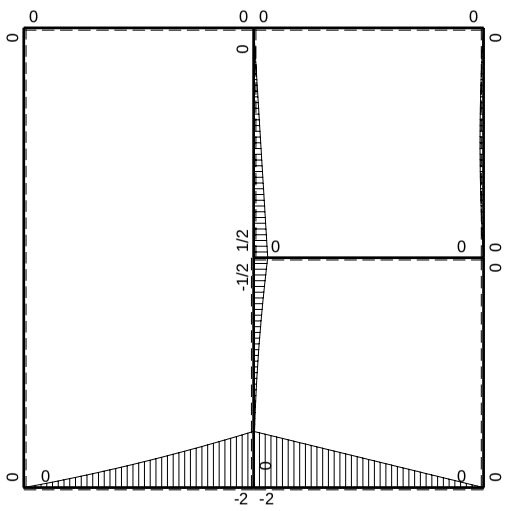
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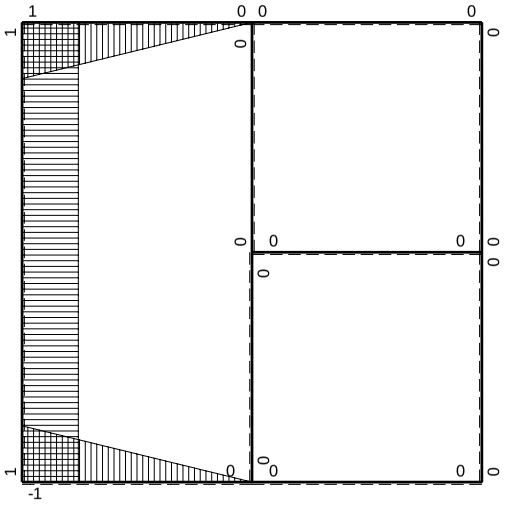
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210192 ESΣ03.xxxx.035

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	-Fb/EJ	0	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$	$(0-1/2)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	0	Fb/EJ	0	-Fx/EJ	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
EH b	0	-1/2Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210192 ESΣ03.xxxx.035
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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + x/b\ )\ \theta\ \ dx = [\ - x + 1/2\ x^2/b\ ]_o^b\ \theta$

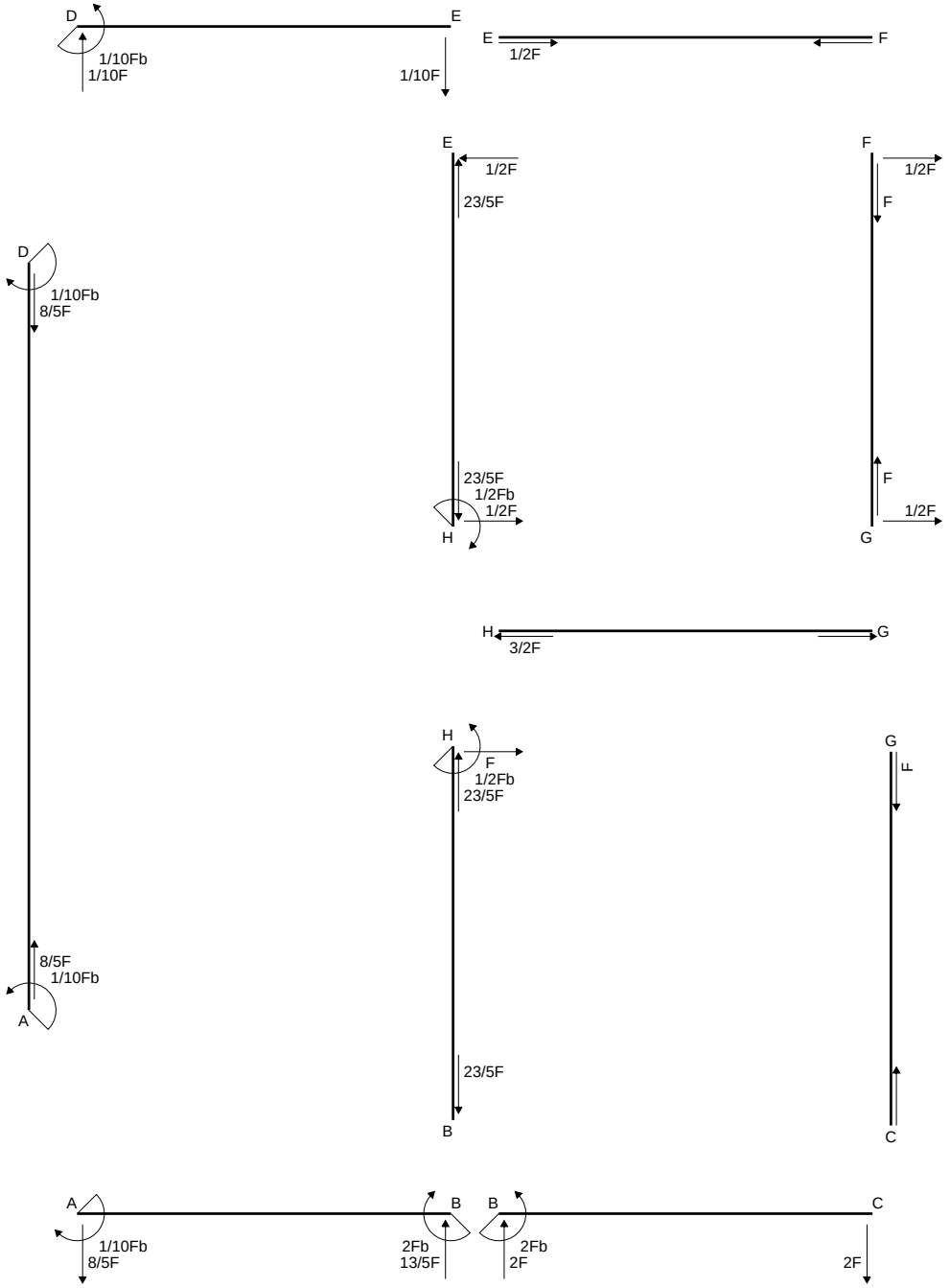
$= (\ - b + 1/2\ b\ )\ \theta\ \ = -1/2\ \ Fb^2/EJ$

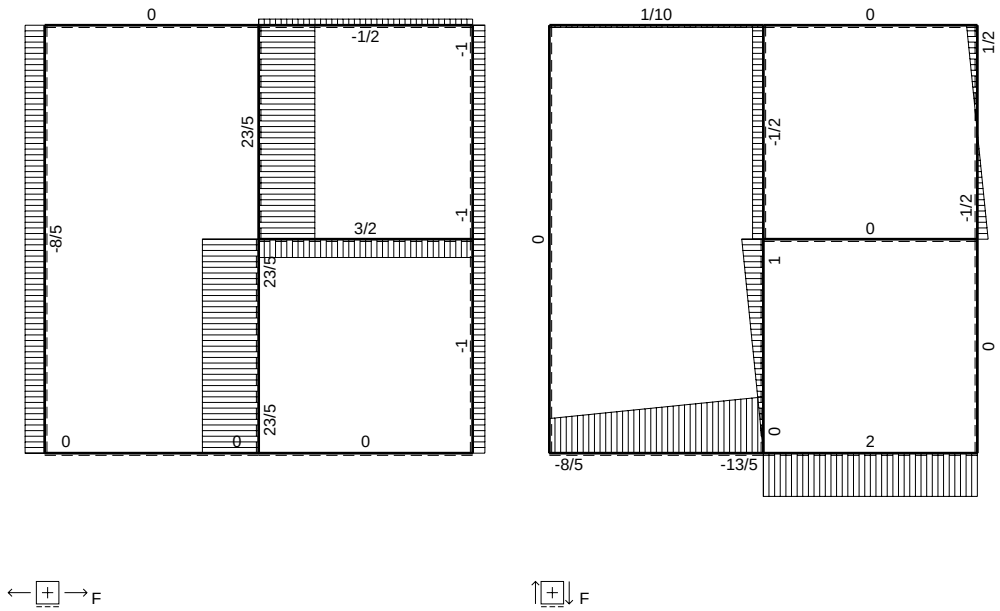
$L^{Xo}_{ED} = \int_o^b (\ x/b\ )\ \theta\ \ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

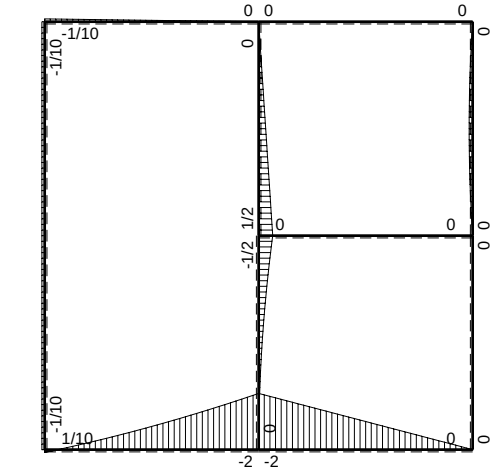
$= (1/2\ b)\ \theta\ \ = -1/2\ \ Fb^2/EJ$

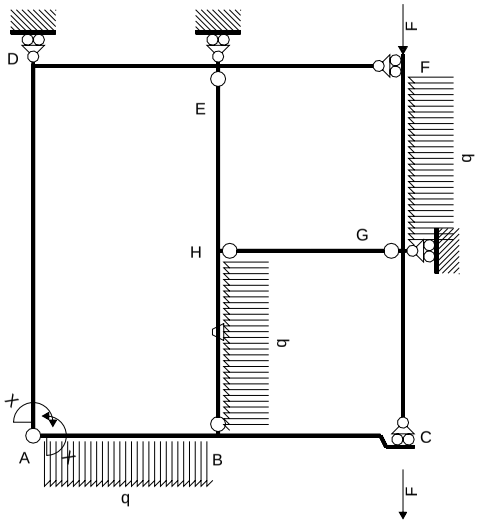
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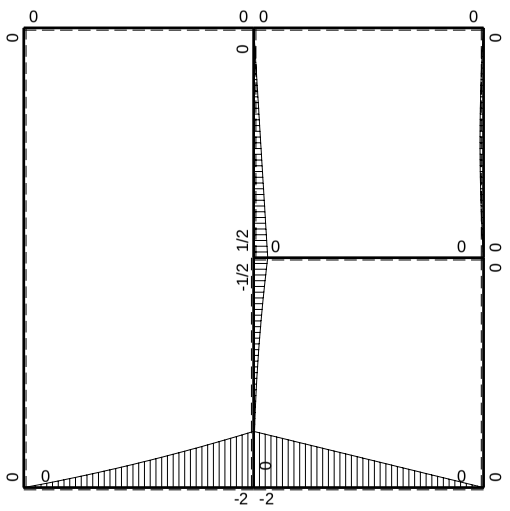

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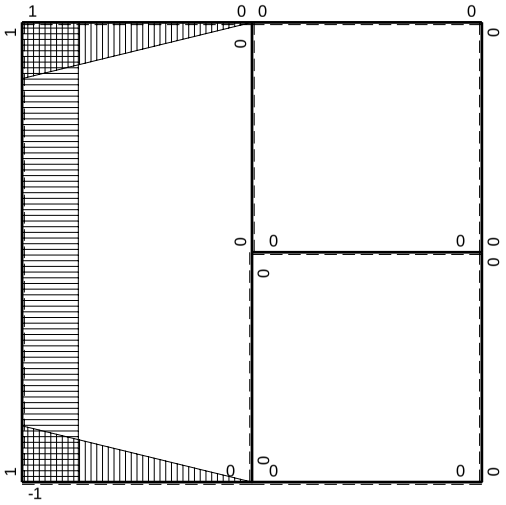

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0
-2 -2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 309819 ESΣ03.xxxx.036

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PROCEDIMENTO E RISULTATI 309819 ESΣ03.xxxx.036

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BH b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
EH b	0	-1/2Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

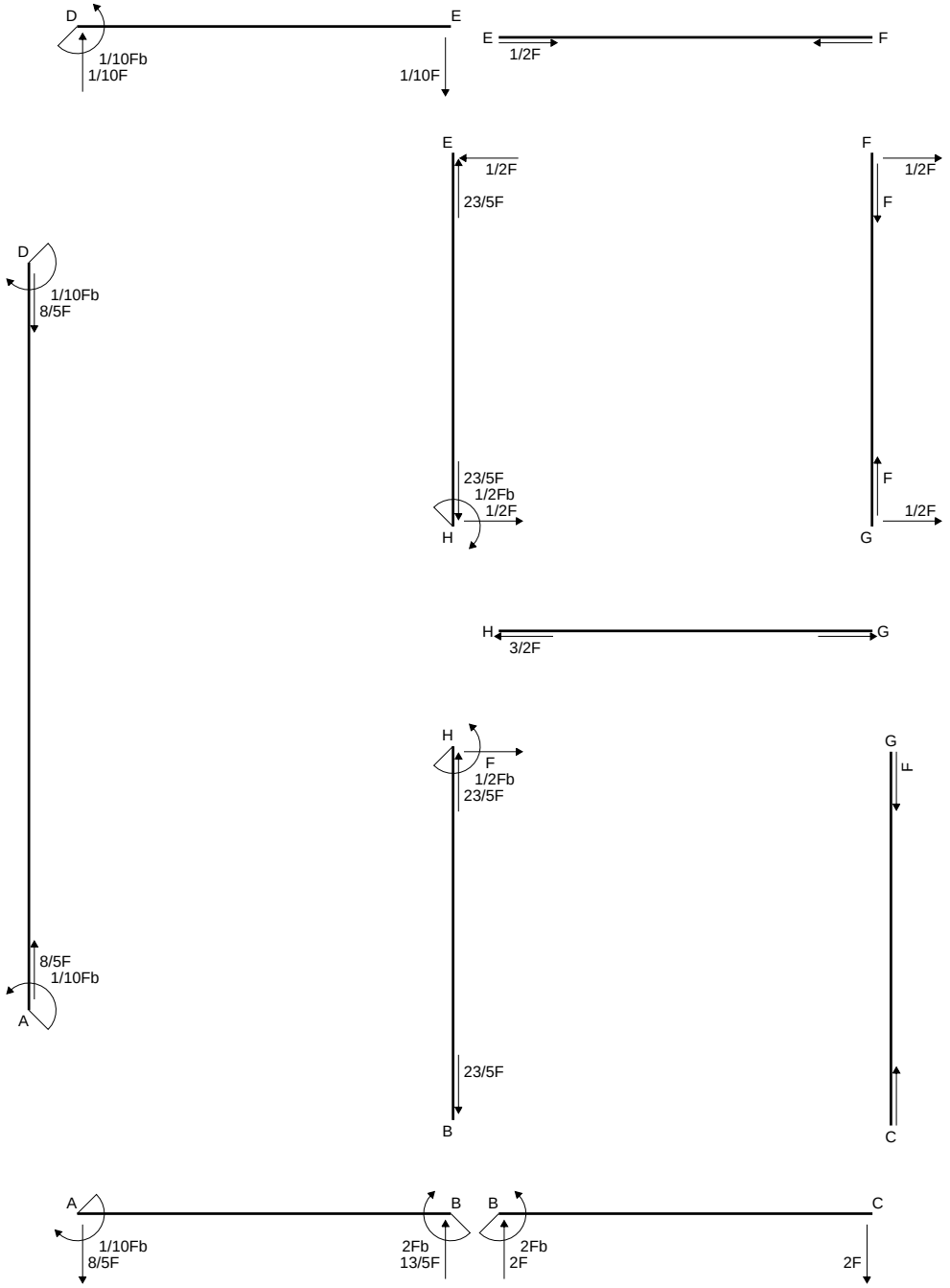
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

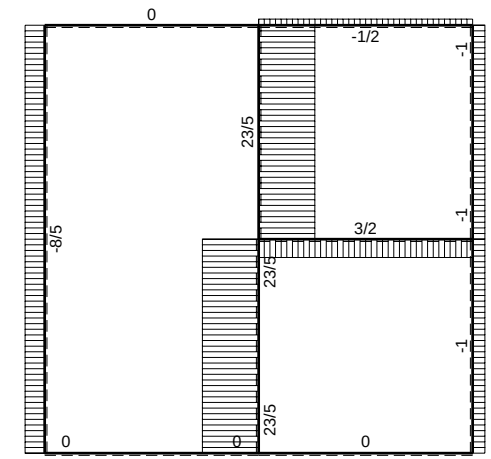
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

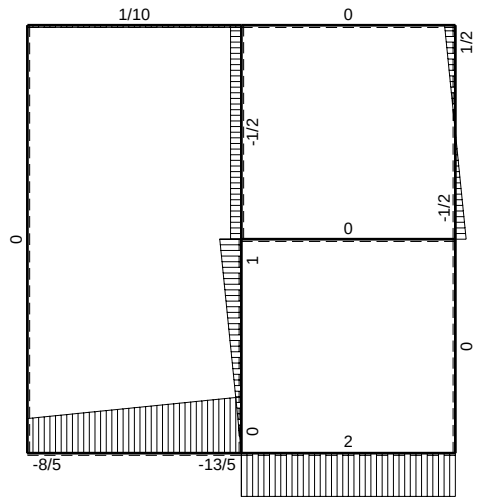
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

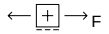
ESΣ03.xxxx.036

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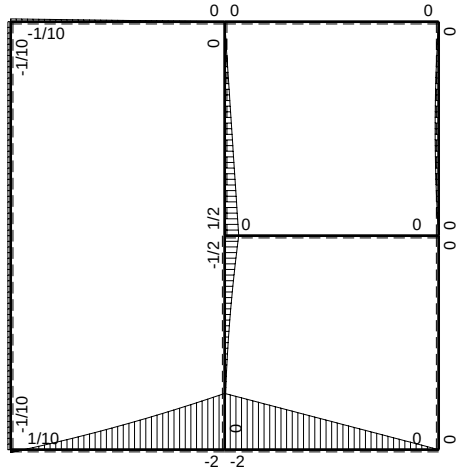

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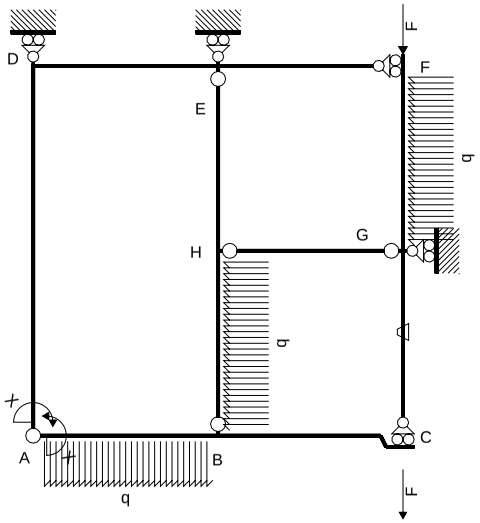

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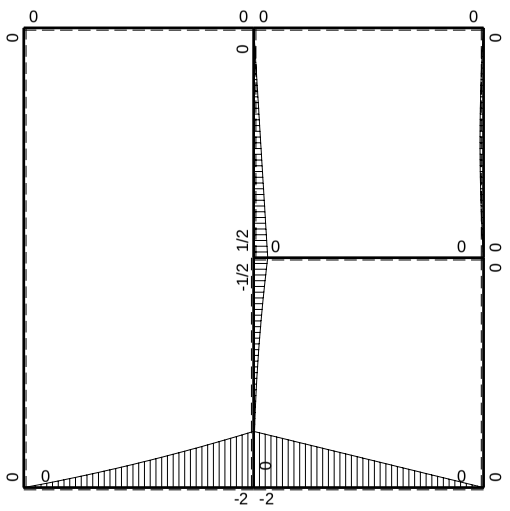

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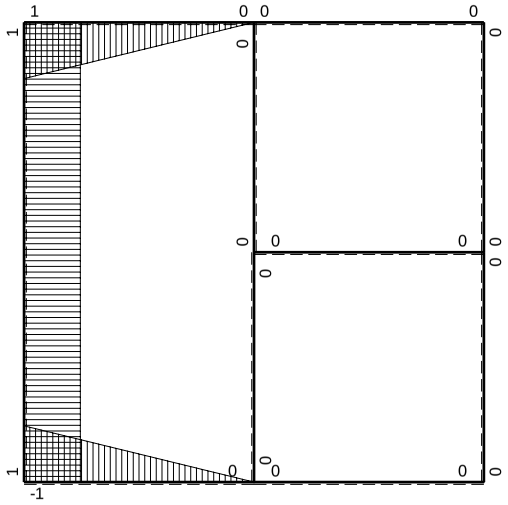

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239217 ESΣ03.xxxx.037

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-1/2Fx$	0	0	0	0	0+0	0
EH b	0	$-1/2Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,3/4\,b - 1/3\,b - 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

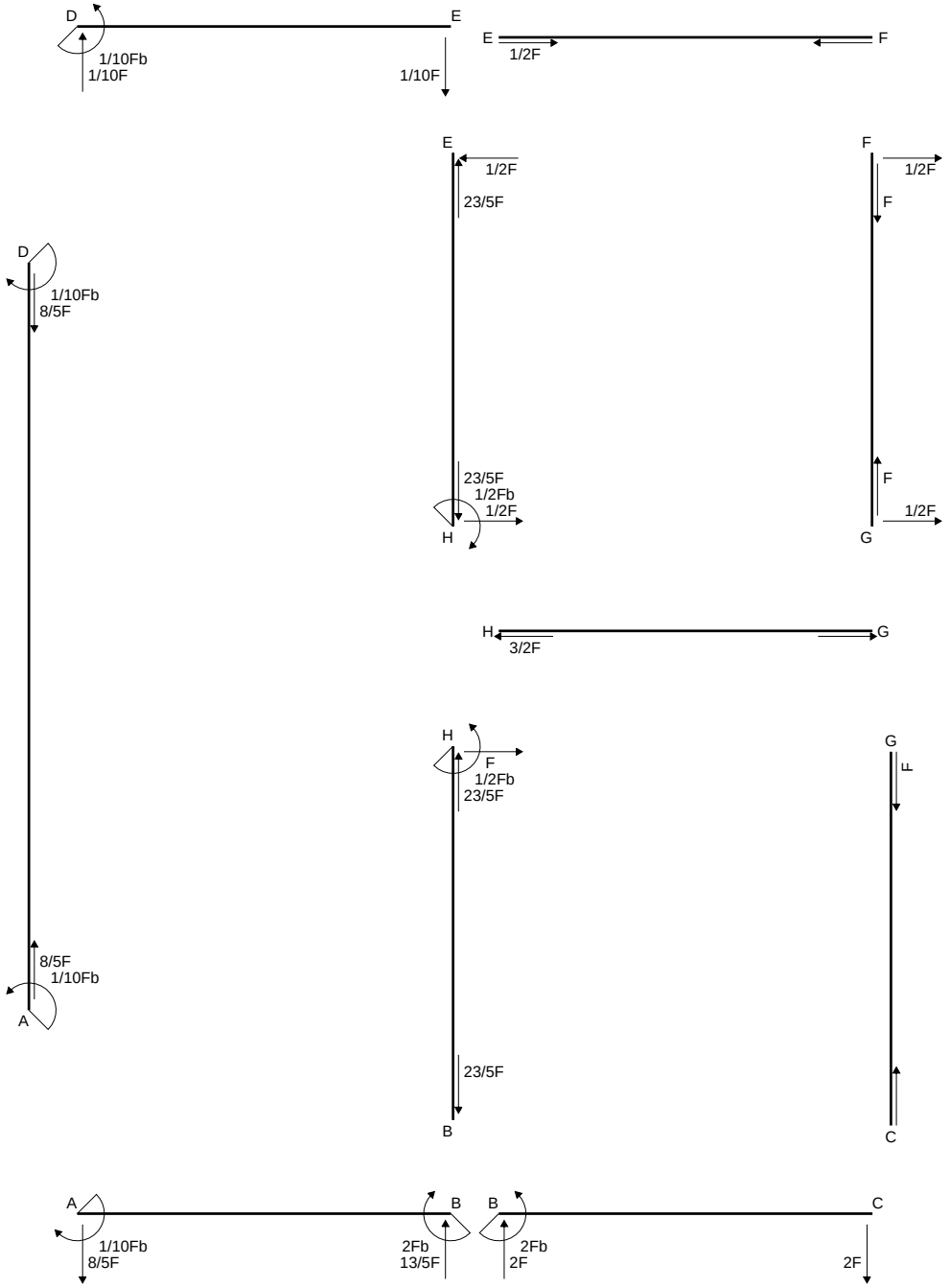
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

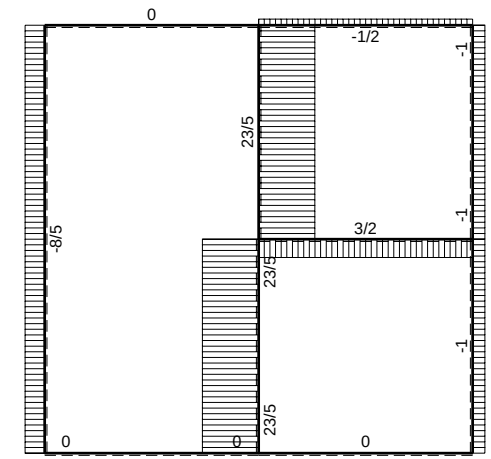
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

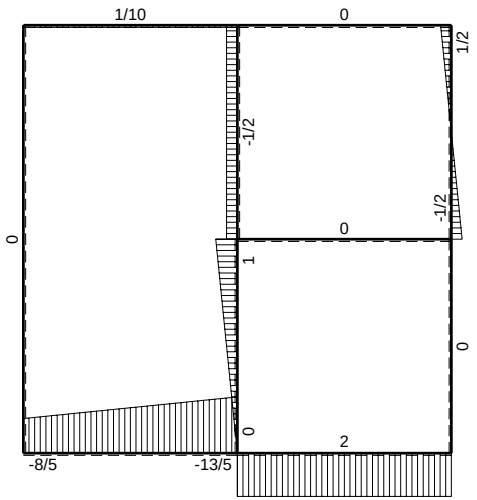
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

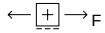
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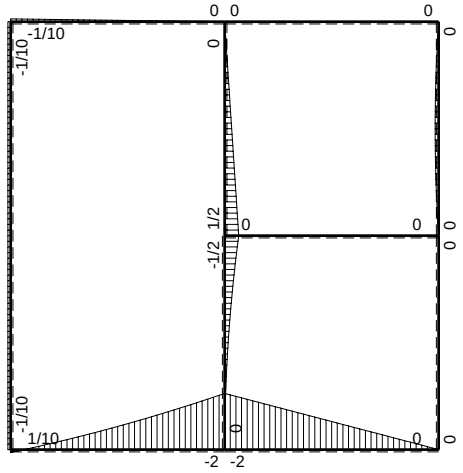

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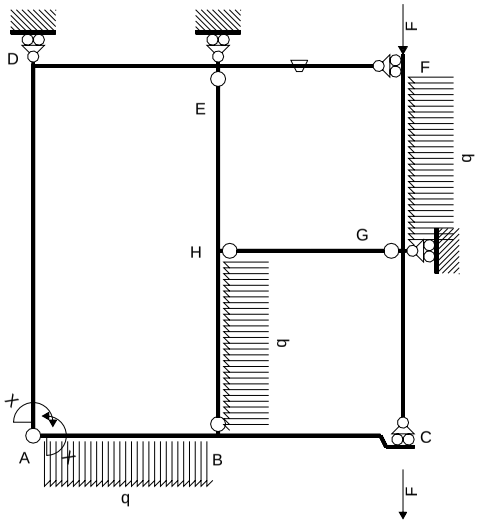

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F

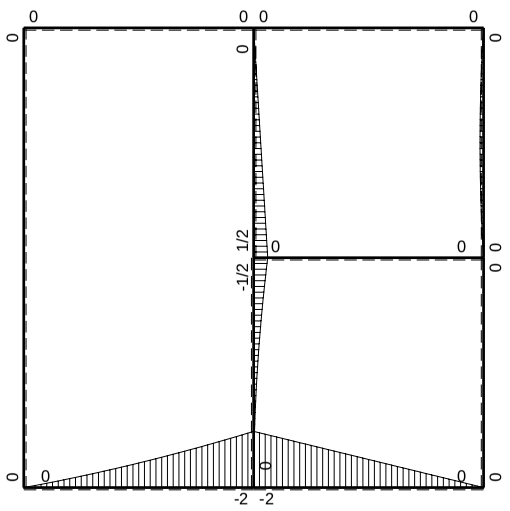

F


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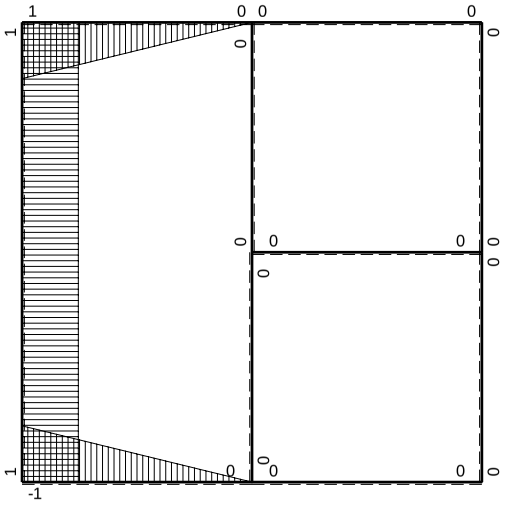

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238594 ESΣ03.xxxx.038

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PROCEDIMENTO E RISULTATI 238594 ESΣ03.xxxx.038

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-1/2Fx$	0	0	0	0	0+0	0
EH b	0	$-1/2Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

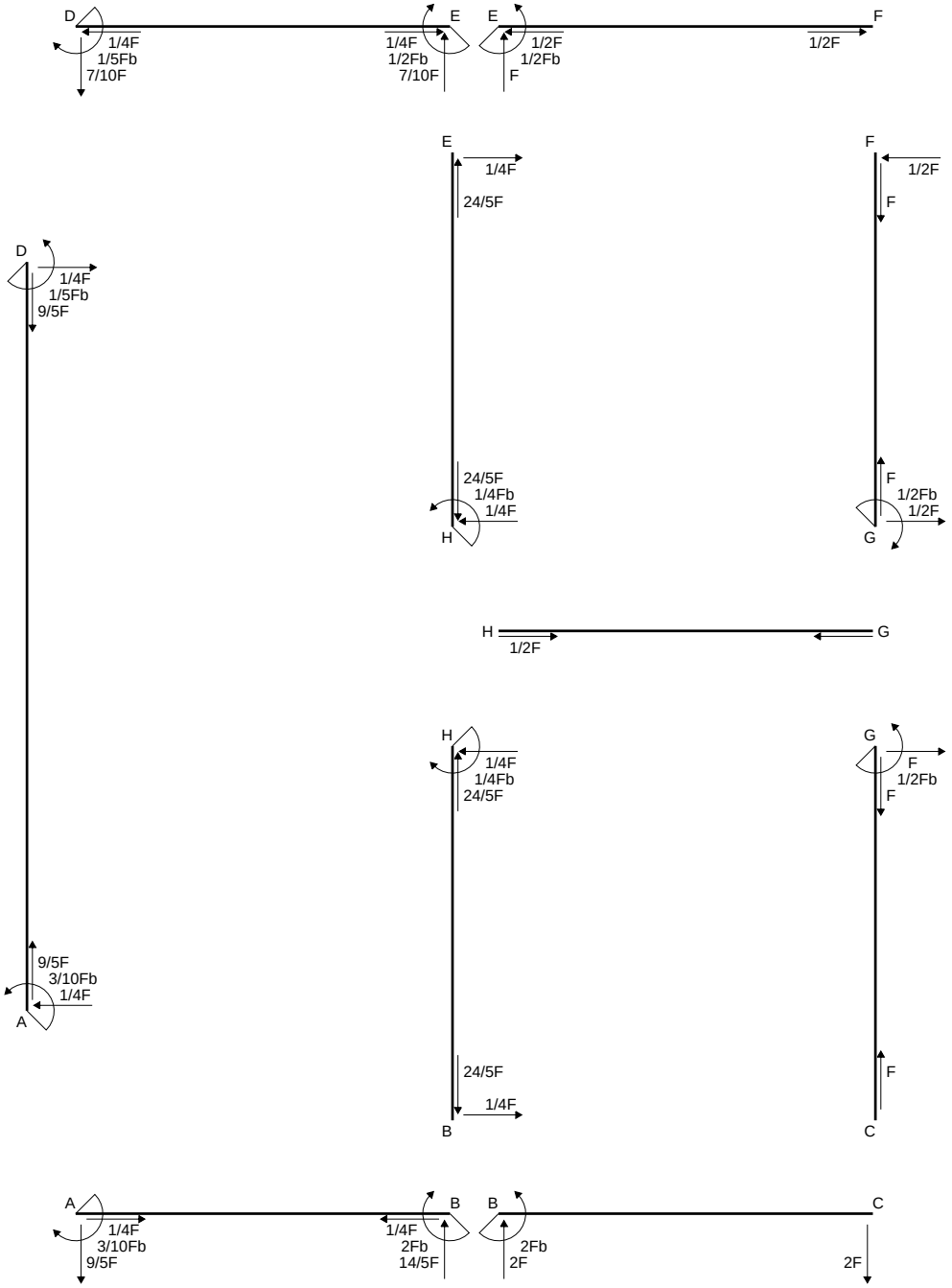
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

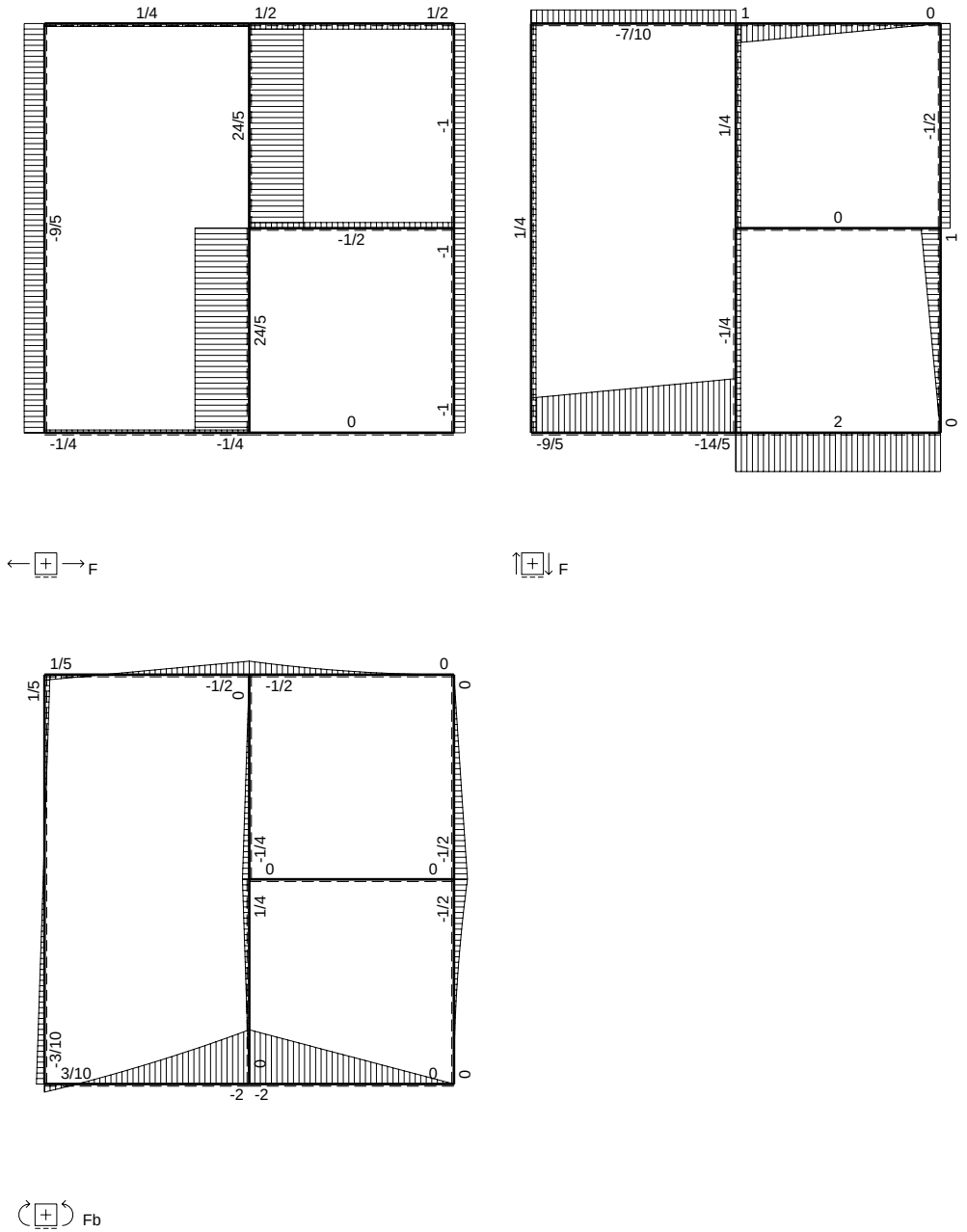
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

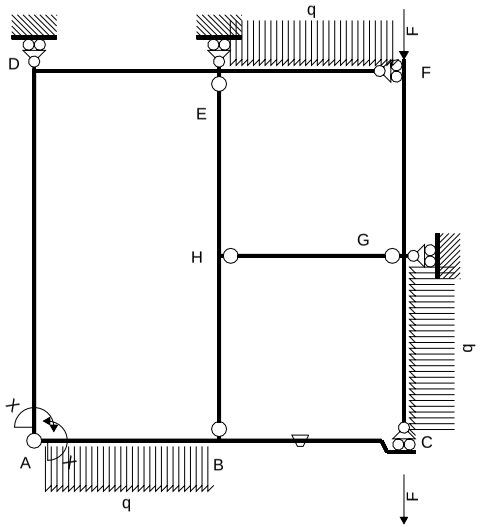
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

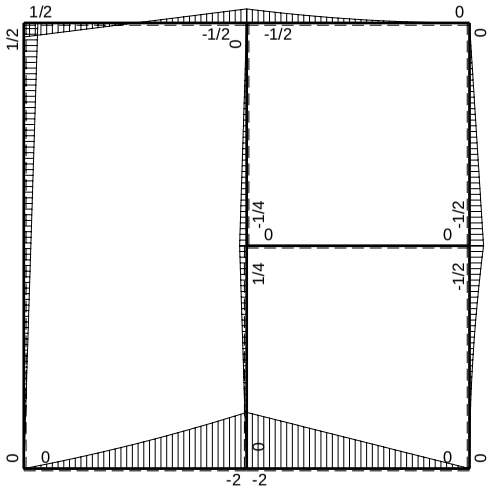
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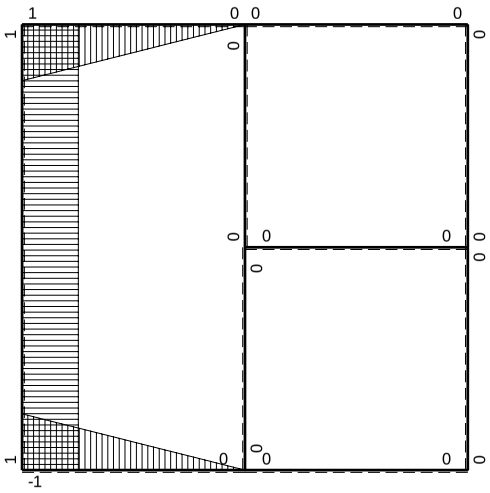

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-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240434 ESΣ03.xxxx.039

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	-Fb/EJ	0	0	0	0+0	0
CB b	0	2Fx	Fb/EJ	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	$1-x/b$	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
BH b	0	-1/4Fx	0	0	0	0		
HE b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
EH b	0	1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

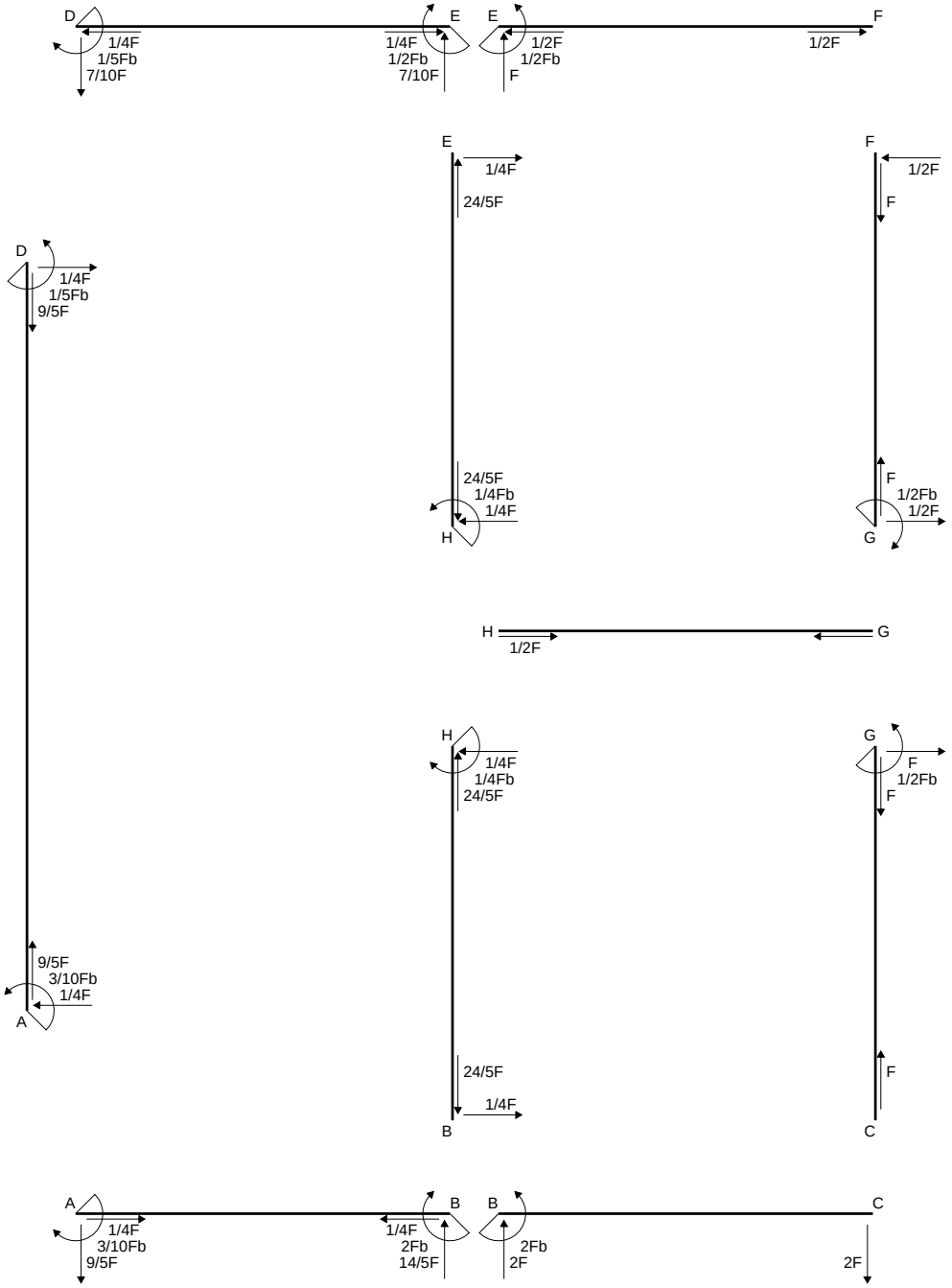
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

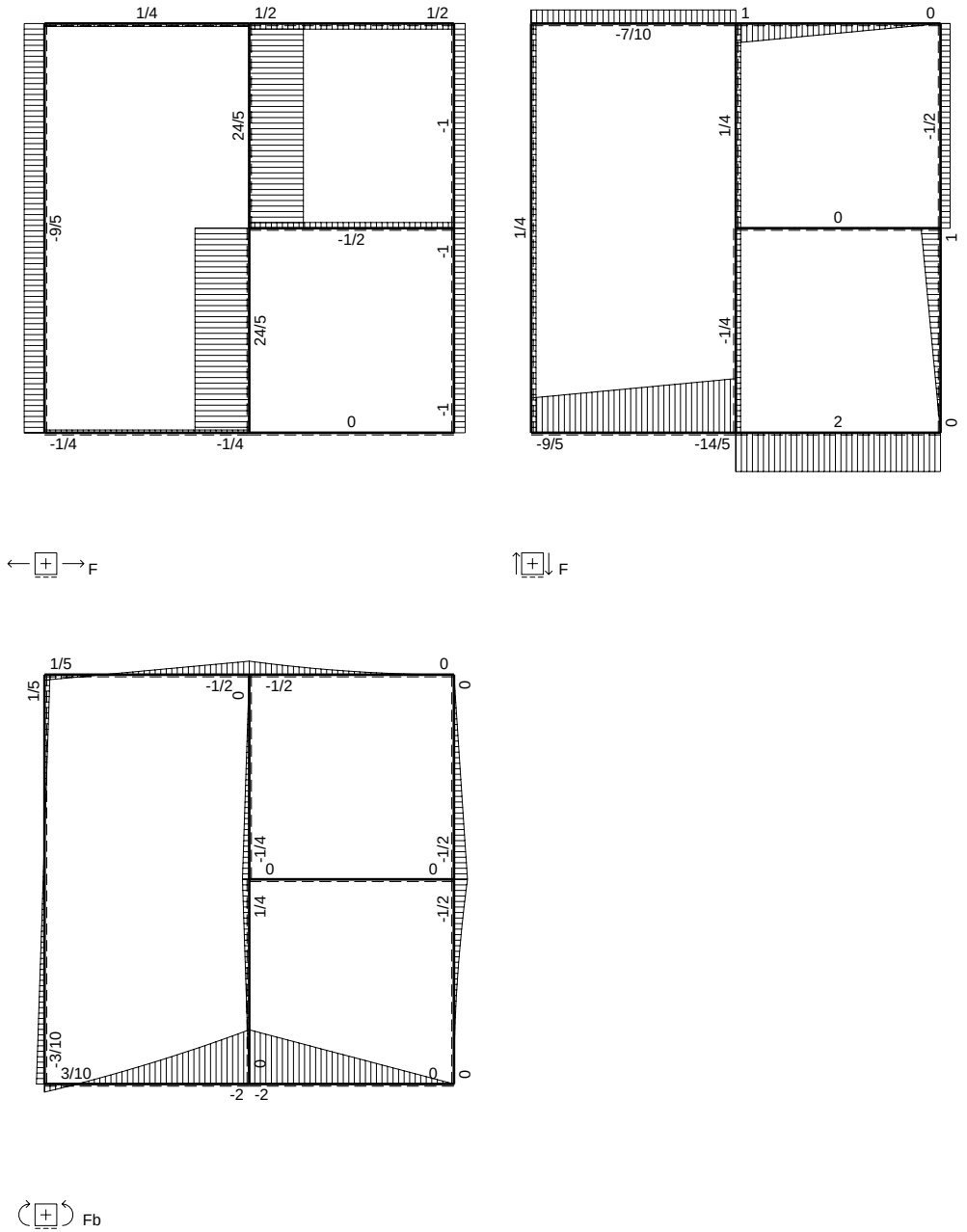
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

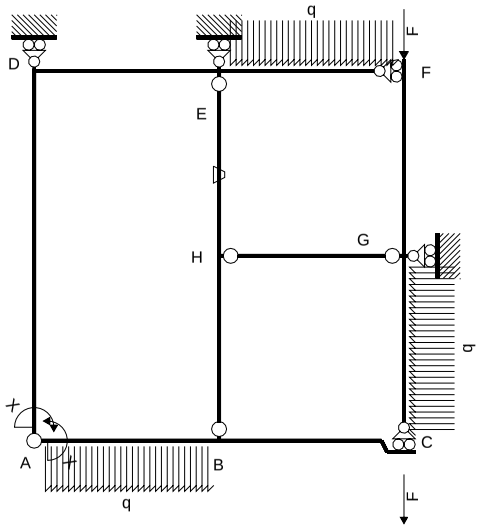
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

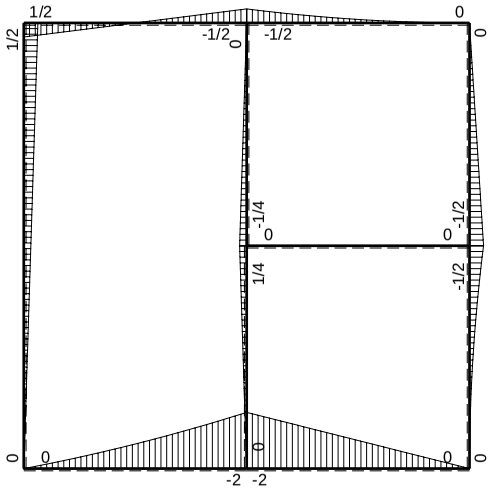
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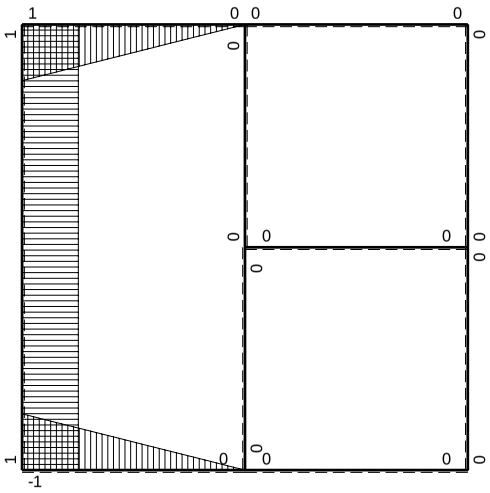

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1/2Fb
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C
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0
3/10
0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238922 ESΣ03.xxxx.040

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$1/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

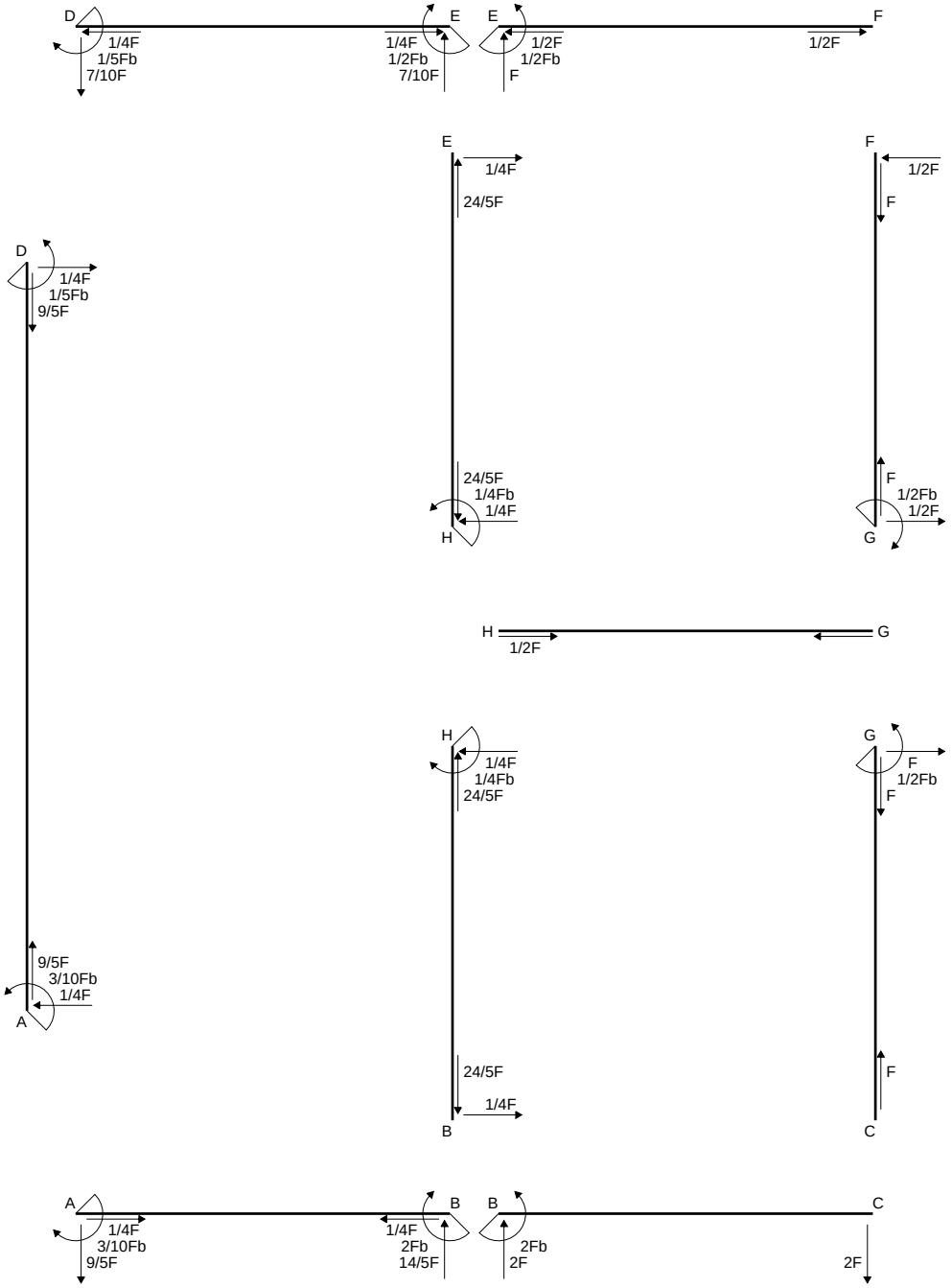
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

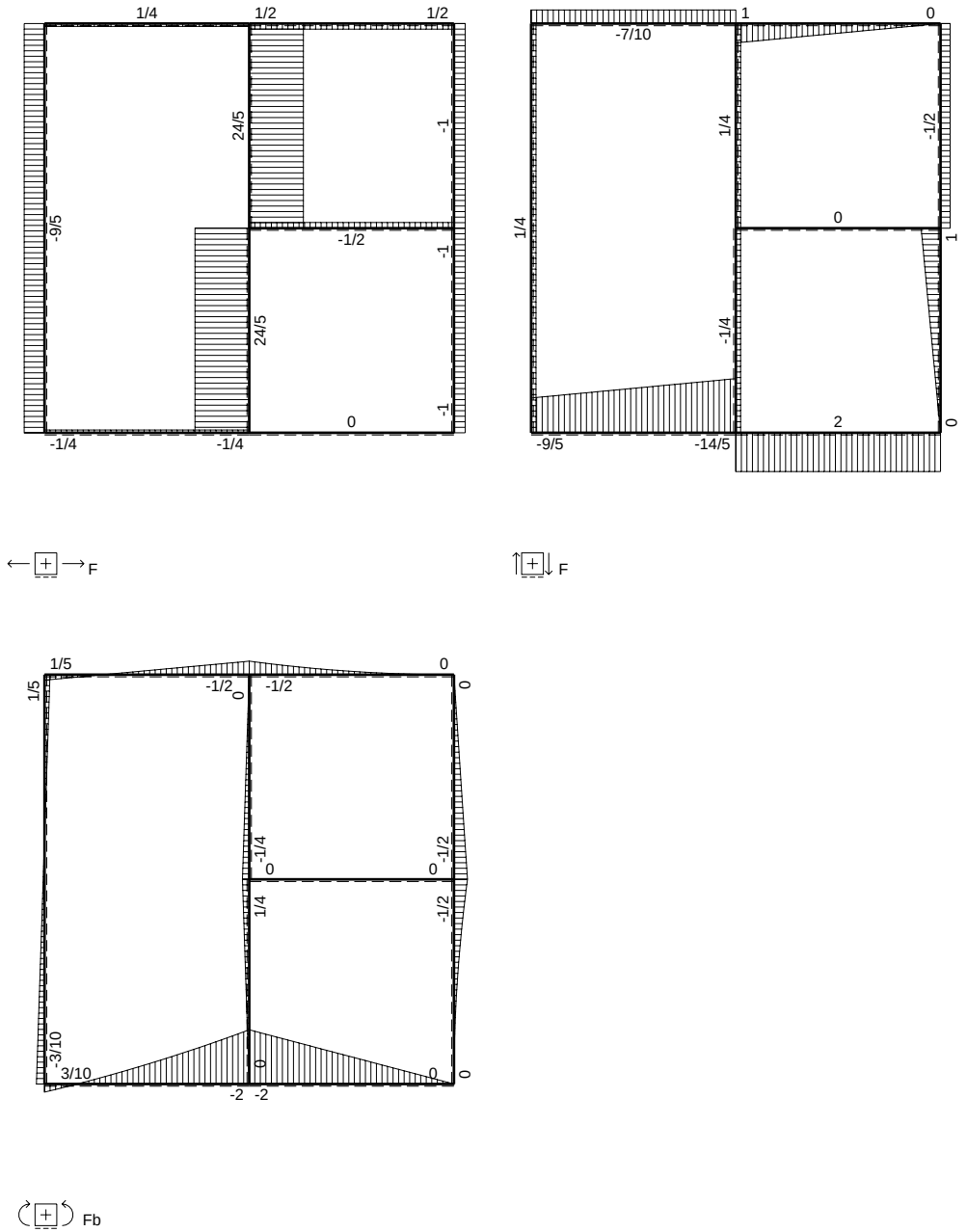
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

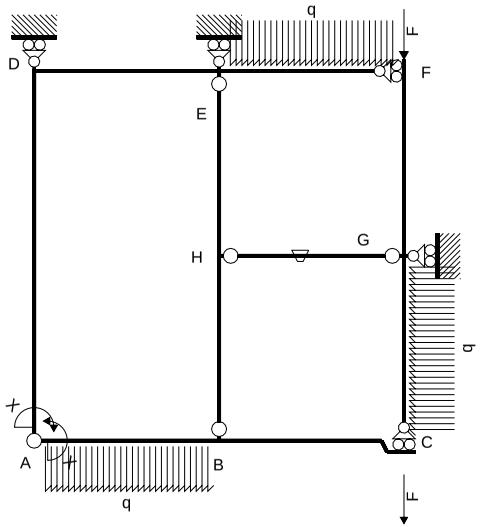
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

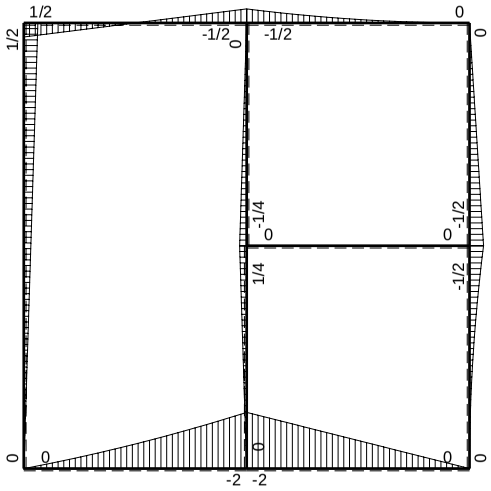
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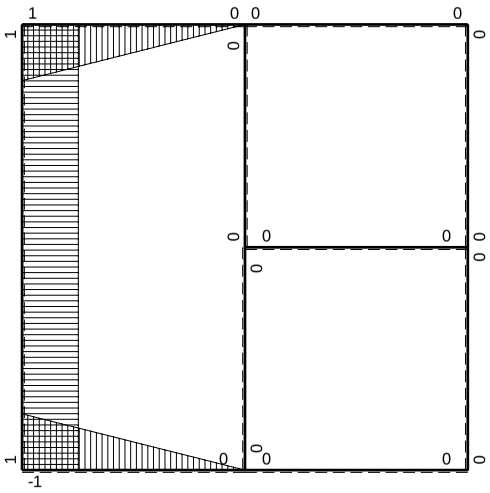

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 956982 ESΣ03.xxxx.041

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PROCEDIMENTO E RISULTATI 956982 ESΣ03.xxxx.041

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	$1-x/b$	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
BH b	0	-1/4Fx	0	0	0	0		
HE b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
EH b	0	1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

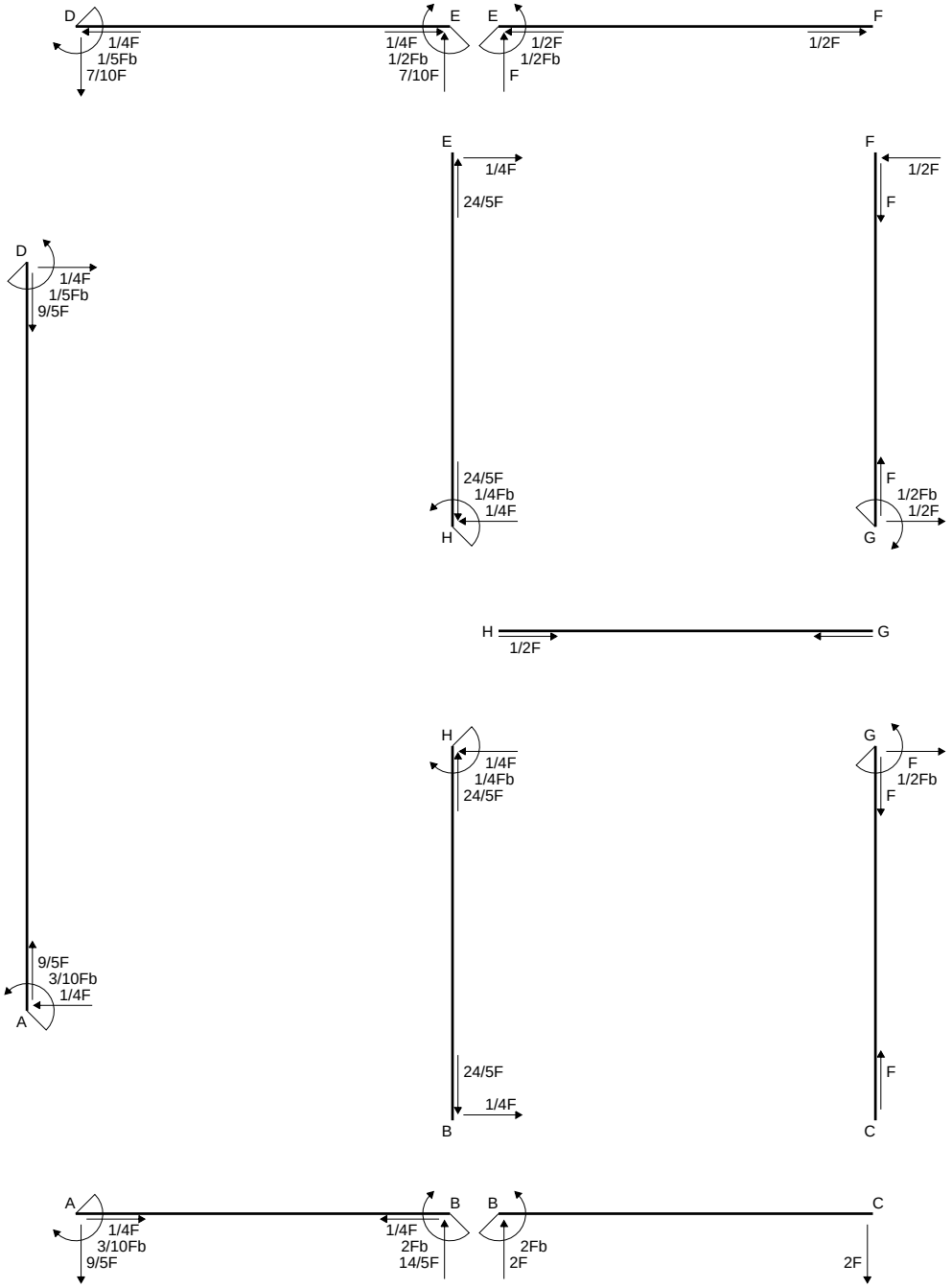
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

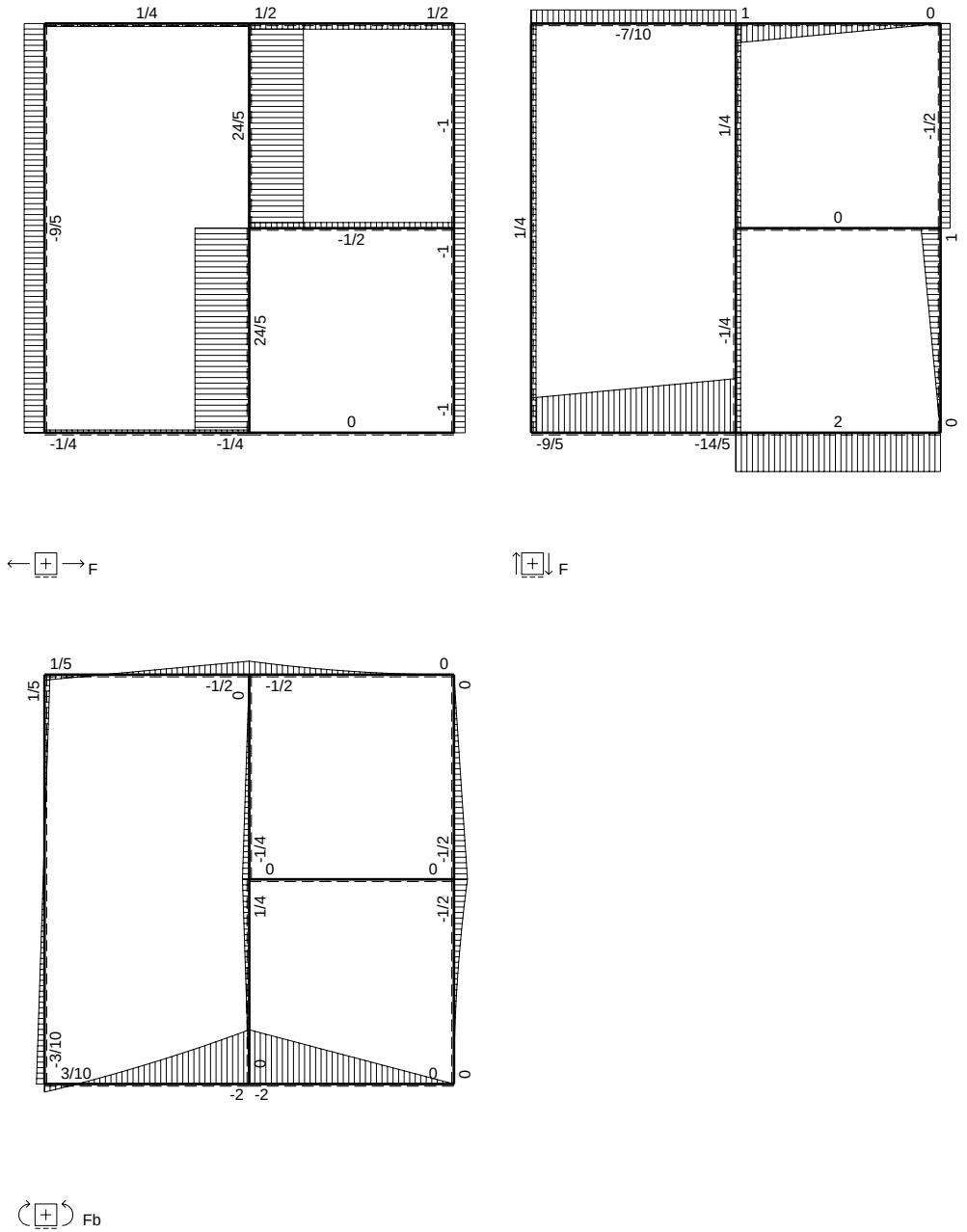
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

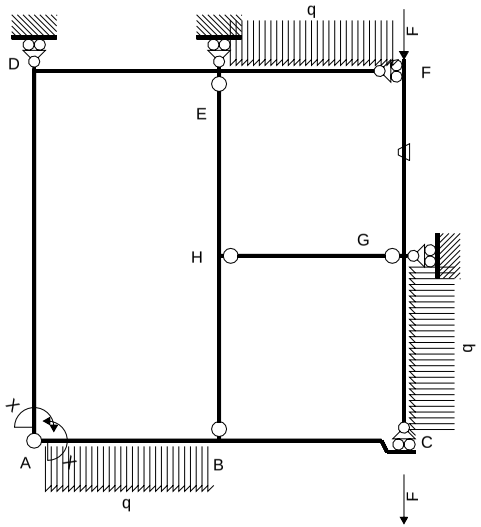
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

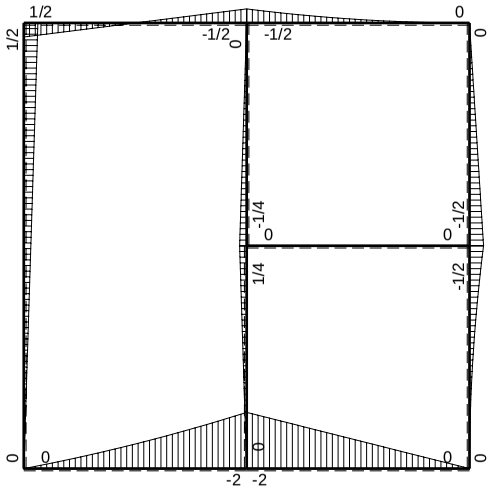
PROCEDIMENTO E RISULTATI 956982 ESΣ03.xxxx.041

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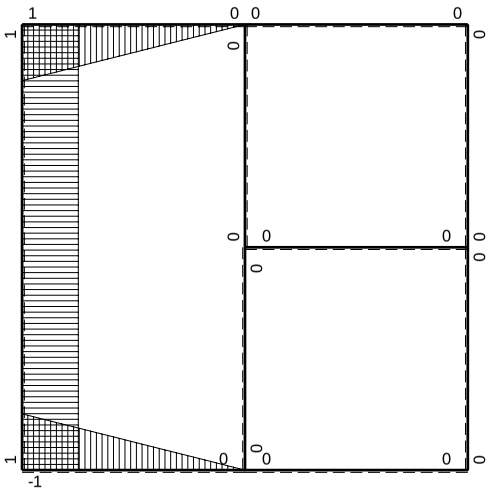

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7/10F
1/2F
1/2Fb
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F
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-1/2
0
0
1/4
-1/2
-3/10
0
0
3/10
0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211619 ESΣ03.xxxx.042

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	Fb/EJ	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

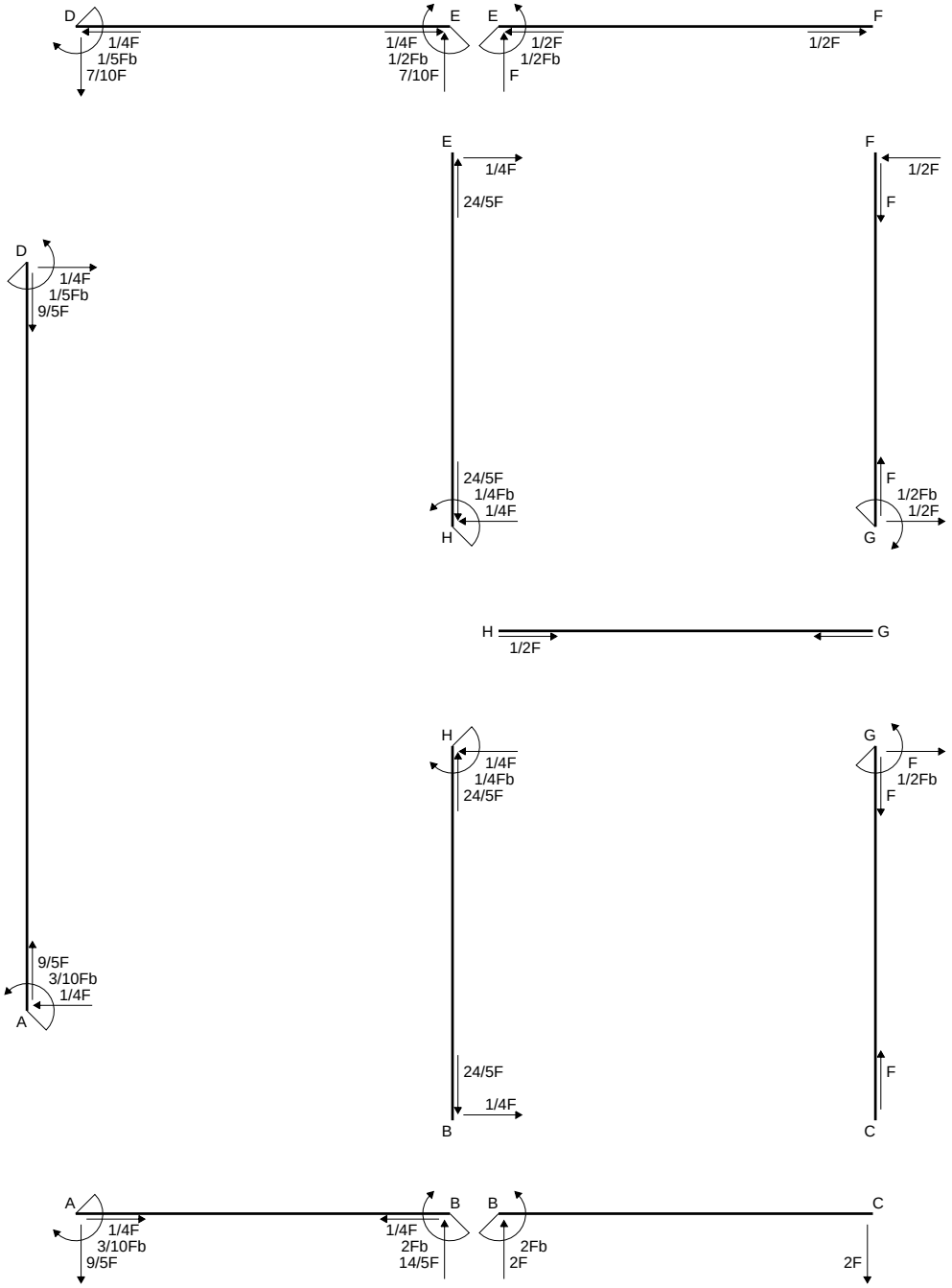
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

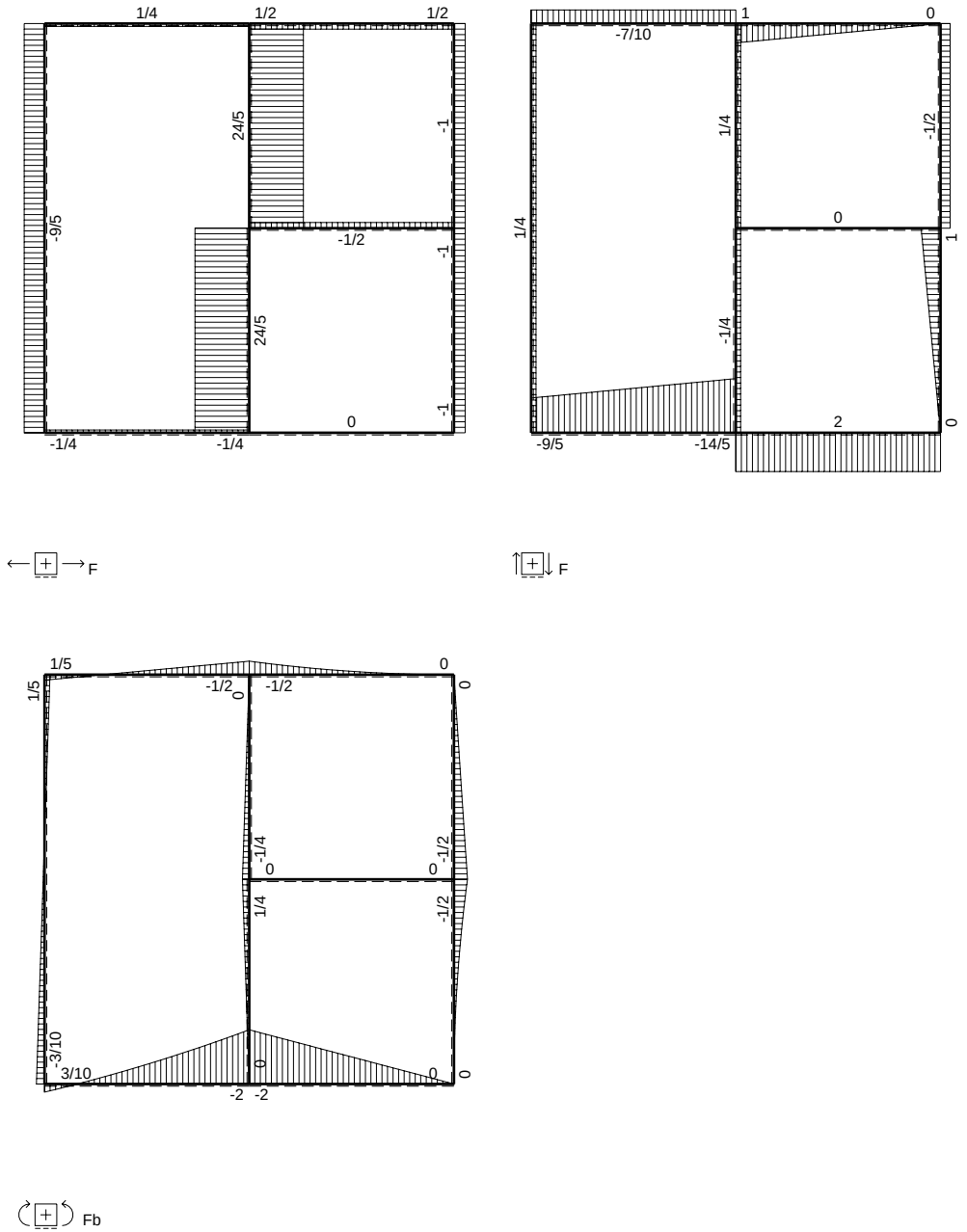
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

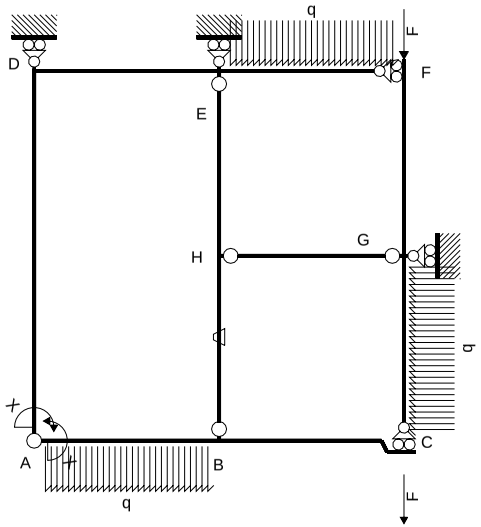
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

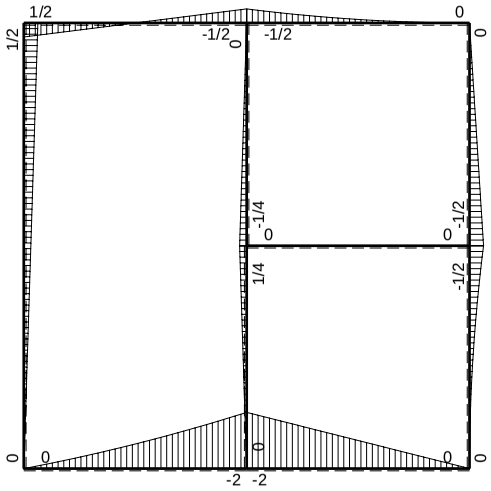
PROCEDIMENTO E RISULTATI 211619 ESΣ03.xxxx.042

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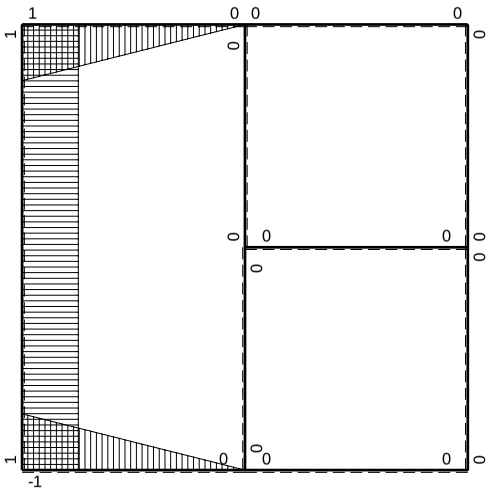

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1/4
-1/2
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0
3/10
0
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-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238581 ESΣ03.xxxx.043

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PROCEDIMENTO E RISULTATI 238581 ESΣ03.xxxx.043

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	$1-x/b$	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	1/4Fb-1/4Fx	-Fb/EJ	0	0	0	0+0	0
BH b	0	-1/4Fx	Fb/EJ	0	0	0		
HE b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
EH b	0	1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

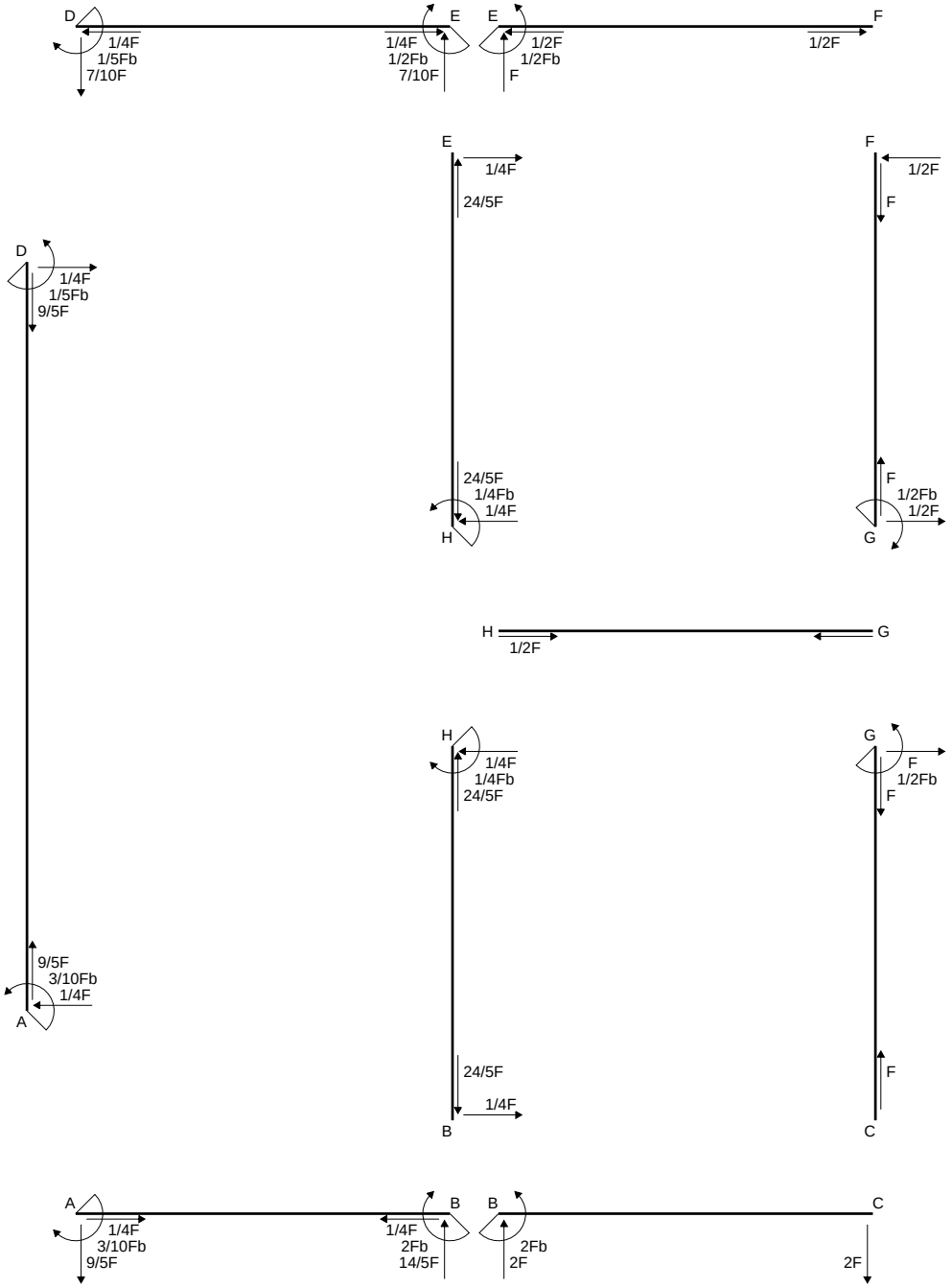
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

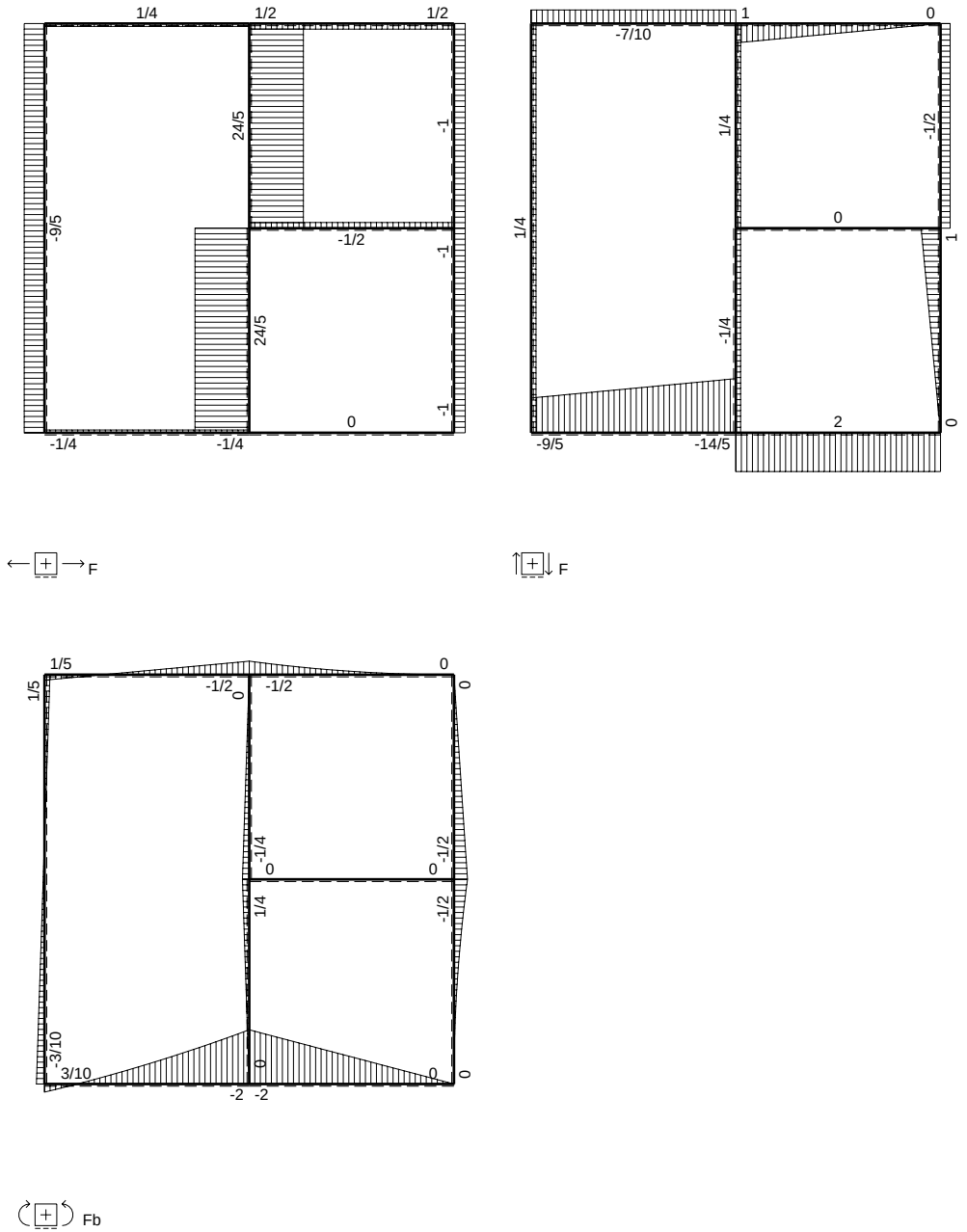
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

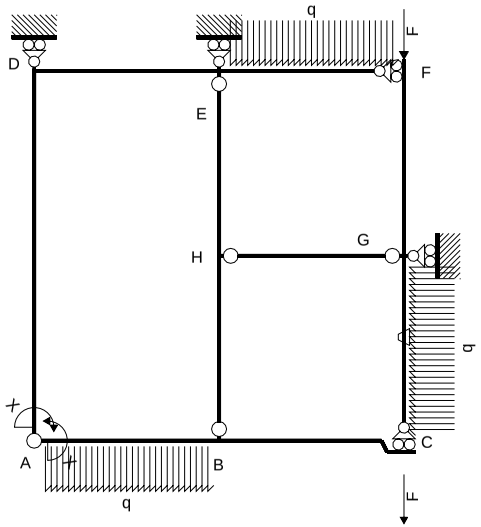
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

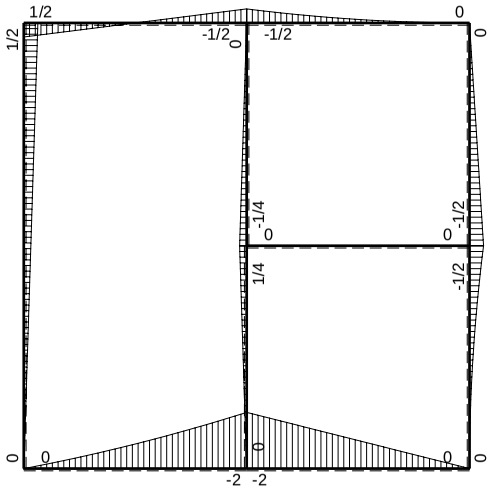
PROCEDIMENTO E RISULTATI 238581 ESΣ03.xxxx.043

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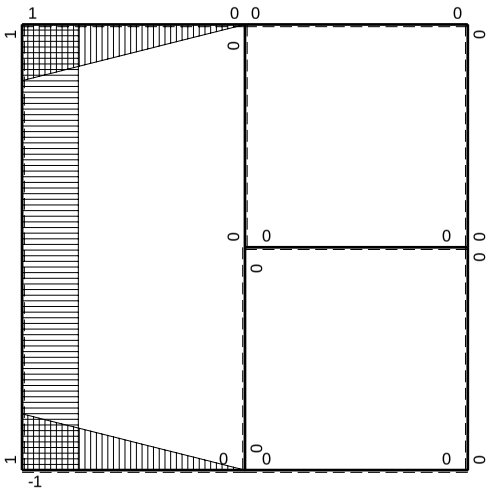

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1/5Fb
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1/2Fb
7/10F
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0
0
1/4
-1/2
-3/10
0
0
3/10
0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236422 ESΣ03.xxxx.044

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PROCEDIMENTO E RISULTATI 236422 ESΣ03.xxxx.044

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\ b + 1/8\ b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b)\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (\,b - 1/2\ b\,)\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

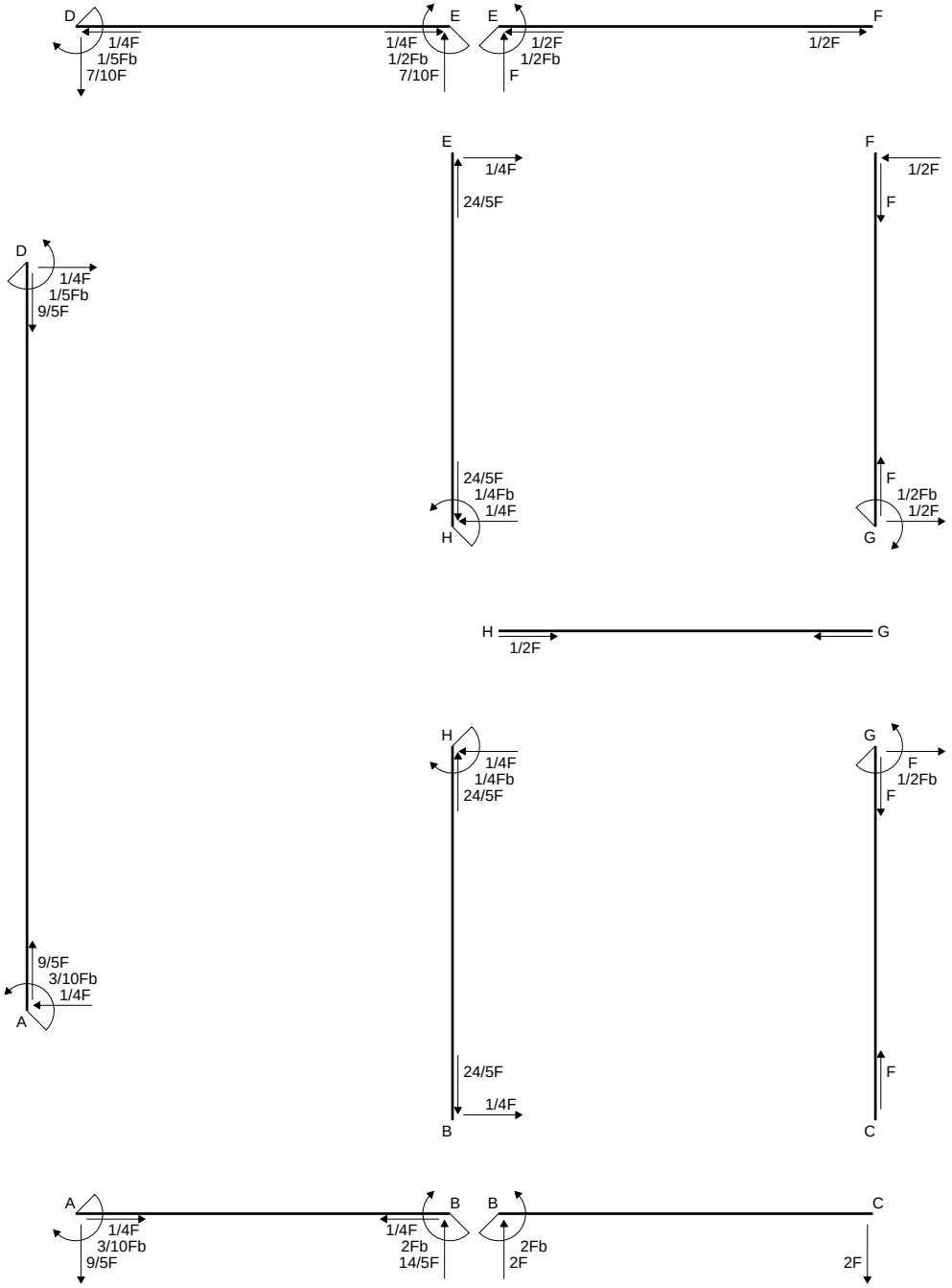
$= (1/2\ b - 3/4\ b + 1/3\ b)\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

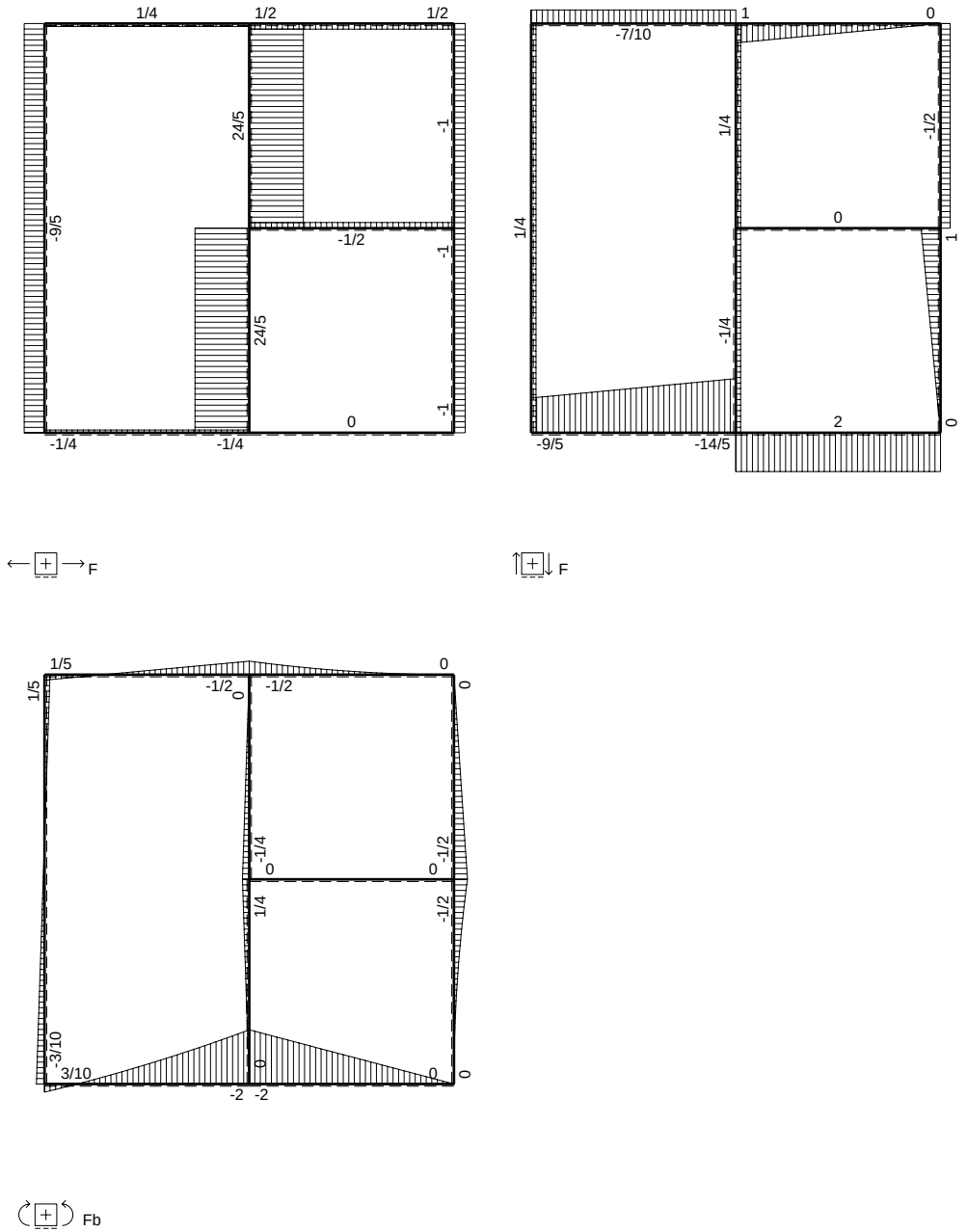
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

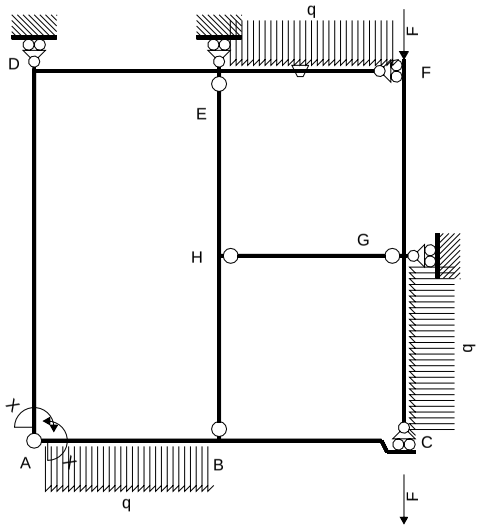
$= (-1/4\ b + 1/3\ b)\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

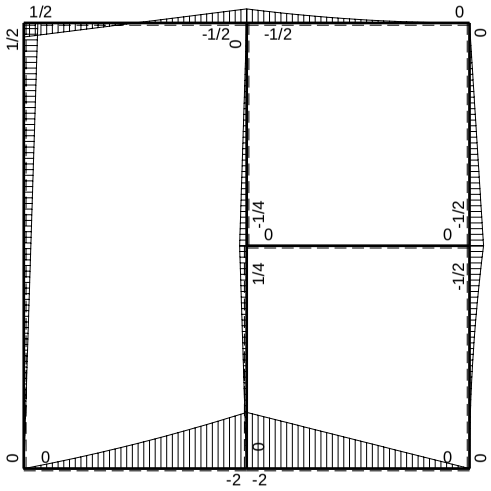
PROCEDIMENTO E RISULTATI 236422 ESΣ03.xxxx.044

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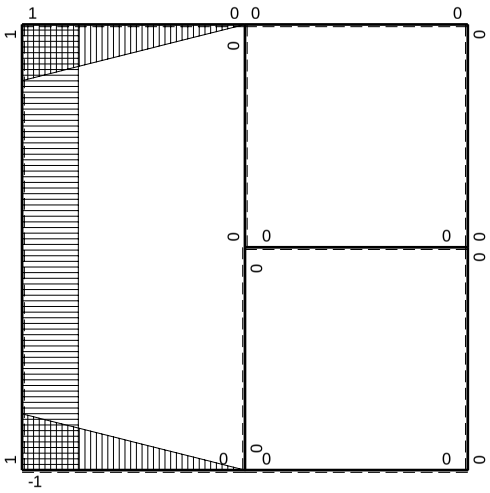

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0
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-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237273 ESΣ03.xxxx.045

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PROCEDIMENTO E RISULTATI 237273 ESΣ03.xxxx.045

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	$1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

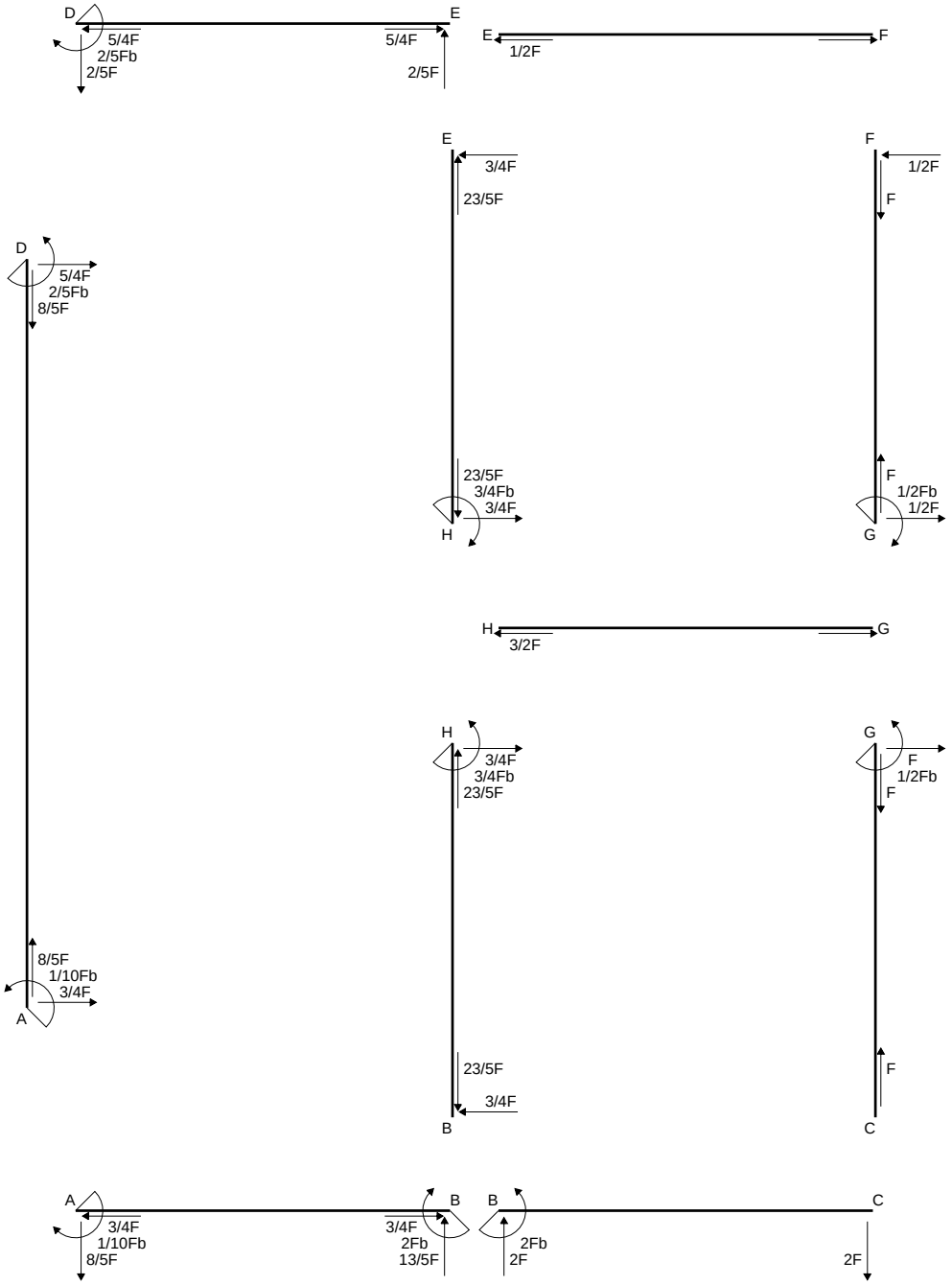
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

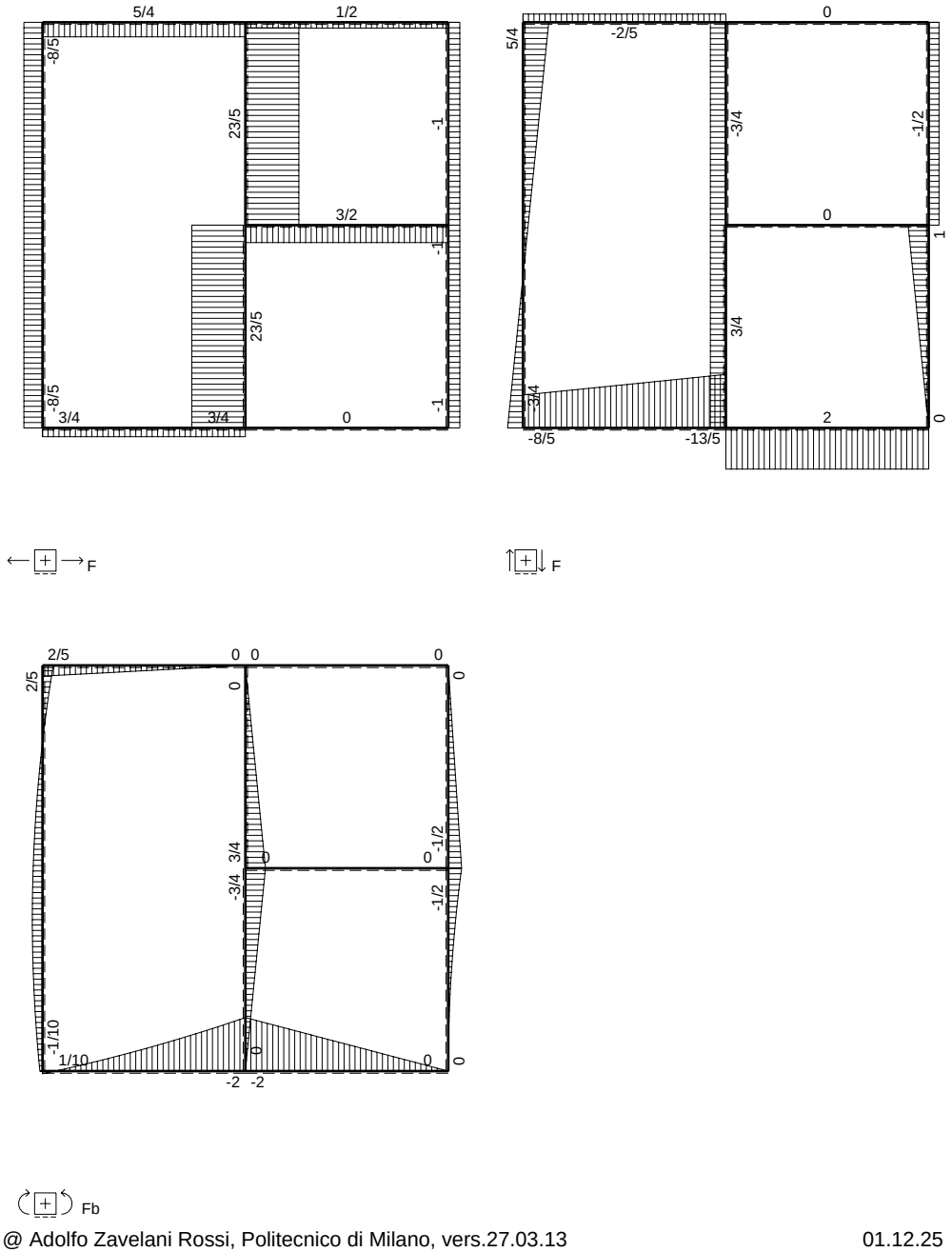
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

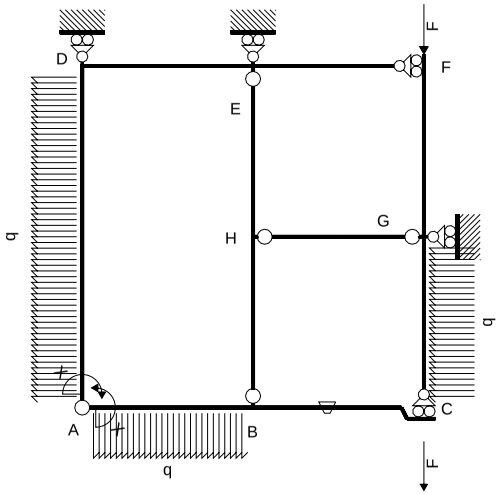
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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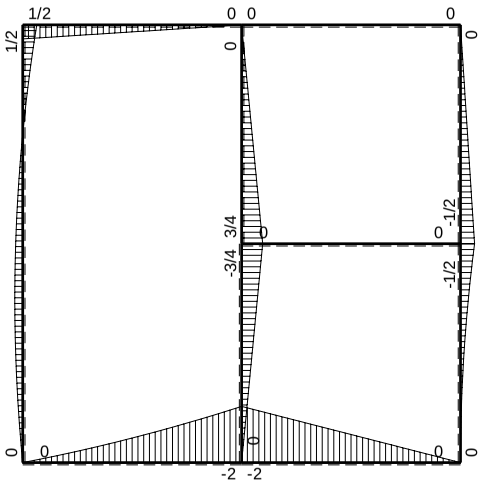
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
E
F
1/2F
E
3/4F
23/5F
F
1/2F
F
D
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2/5Fb
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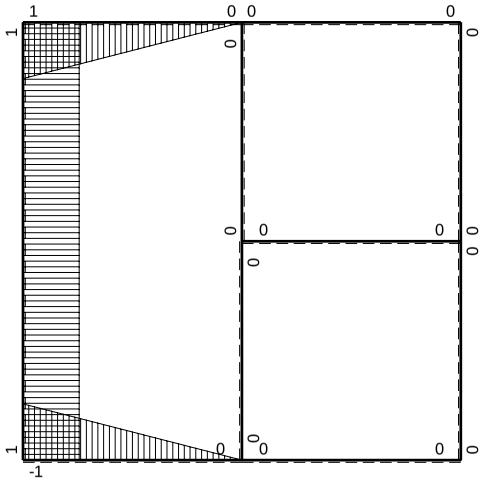

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0
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2/5
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0
0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	-Fb/EJ	0	0	0	0+0	0
CB b	0	2Fx	Fb/EJ	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-3/4Fb+3/4Fx	0	0	0	0	0+0	0
BH b	0	3/4Fx	0	0	0	0		
HE b	0	3/4Fb-3/4Fx	0	0	0	0	0+0	0
EH b	0	-3/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$$L^{XX}_{AB} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\, x - x^2/b +1/3\, x^3/b^2 \,\right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$$

$$= \left(\, b - b +1/3\, b \,\right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$$

$$L^{XX}_{BA} = \int_o^b \left(\, x^2/b^2 \,\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\, x^3/b^2 \,\right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$$

$$= \left(1/3\, b\right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$$

$$L^{XX}_{AD} = \int_o^{2b} \left(1\right)\ 1/EJ\ dx = \left[\, x \,\right]_o^{2b}\ 1/EJ$$

$$= \left(2\, b\right)\ 1/EJ\ = 2\ \ b/EJ$$

$$L^{XX}_{DA} = \int_o^{2b} \left(1\right)\ 1/EJ\ dx = \left[\, x \,\right]_o^{2b}\ 1/EJ$$

$$= \left(2\, b\right)\ 1/EJ\ = 2\ \ b/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\, x - x^2/b +1/3\, x^3/b^2 \,\right]_o^b\ 1/EJ$$

$$= \left(\, b - b +1/3\, b \,\right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(\, x^2/b^2 \,\right)\ 1/EJ\ dx = \left[1/3\, x^3/b^2 \,\right]_o^b\ 1/EJ$$

$$= \left(1/3\, b\right)\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L^{Xo}_{AB} = \int_o^b \left(3/2\, x/b - x^2/b^2 -1/2\, x^3/b^3\right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left[3/4\, x^2/b -1/3\, x^3/b^2 -1/8\, x^4/b^3 \,\right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left(3/4\, b -1/3\, b -1/8\, b\right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$$

$$L^{Xo}_{BA} = \int_o^b \left(2\, x/b -5/2\, x^2/b^2 +1/2\, x^3/b^3\right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left[\, x^2/b -5/6\, x^3/b^2 +1/8\, x^4/b^3 \,\right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$$

$$= \left(\, b -5/6\, b +1/8\, b\,\right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$$

$$L^{Xo}_{AD} = \int_o^{2b} \left(-3/4\, x/b +1/2\, x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-3/8\, x^2/b +1/6\, x^3/b^2 \,\right]_o^{2b}\ Fb\ 1/EJ$$

$$= \left(-3/2\, b +4/3\, b\,\right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L^{Xo}_{DA} = \int_o^{2b} \left(1/2 -5/4\, x/b +1/2\, x^2/b^2\right) Fb\ 1/EJ\ dx = \left[1/2\, x -5/8\, x^2/b +1/6\, x^3/b^2 \,\right]_o^{2b}\ Fb\ 1/EJ$$

$$= \left(\, b -5/2\, b +4/3\, b\,\right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$$

$$L^{Xo}_{DE} = \int_o^b \left(1/2 - x/b +1/2\, x^2/b^2\right) Fb\ 1/EJ\ dx = \left[1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2 \,\right]_o^b\ Fb\ 1/EJ$$

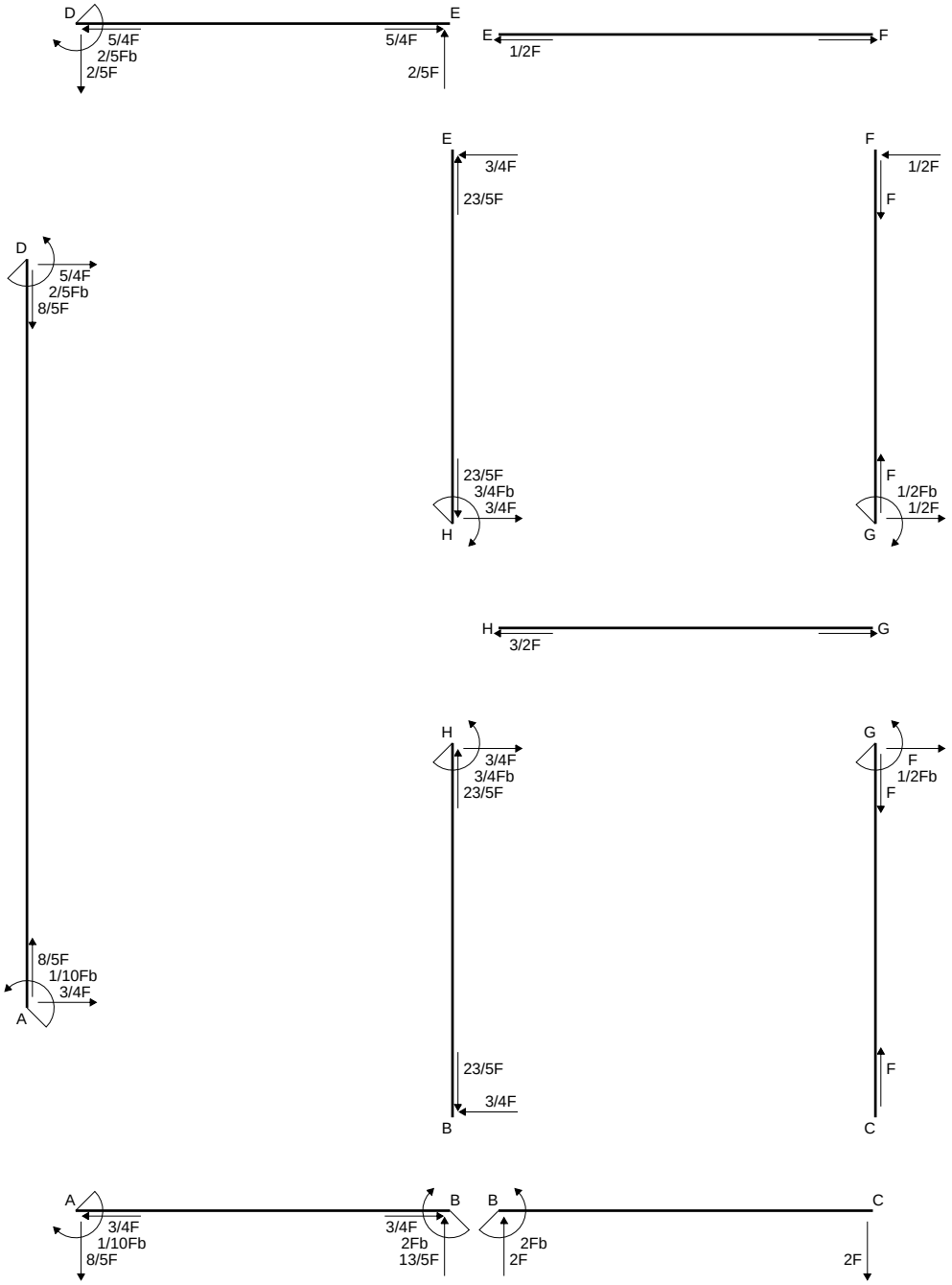
$$= \left(1/2\, b -1/2\, b +1/6\, b\,\right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$$

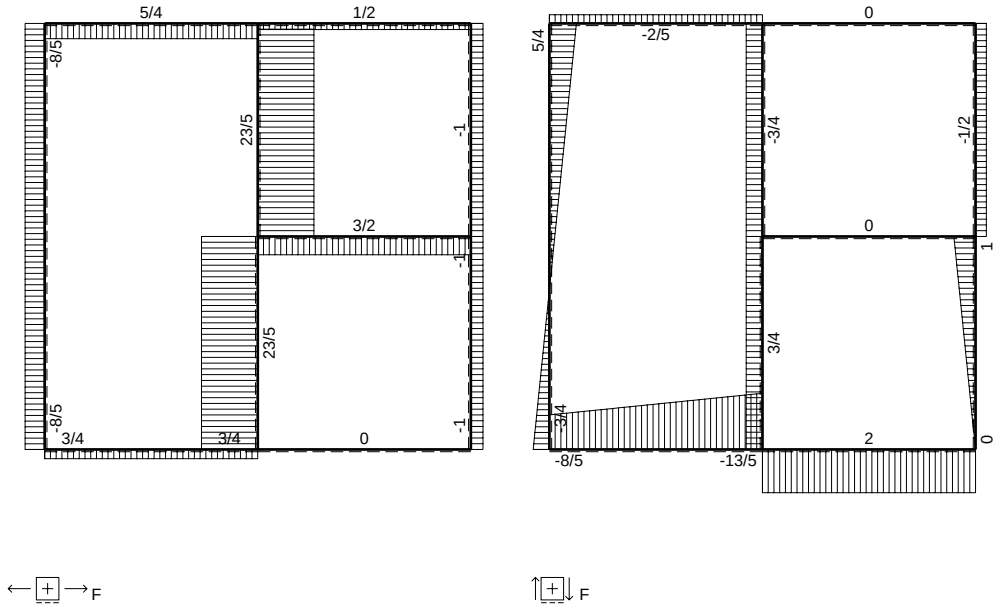
$$L^{Xo}_{ED} = \int_o^b \left(1/2\, x^2/b^2\right) Fb\ 1/EJ\ dx = \left[1/6\, x^3/b^2 \,\right]_o^b\ Fb\ 1/EJ$$

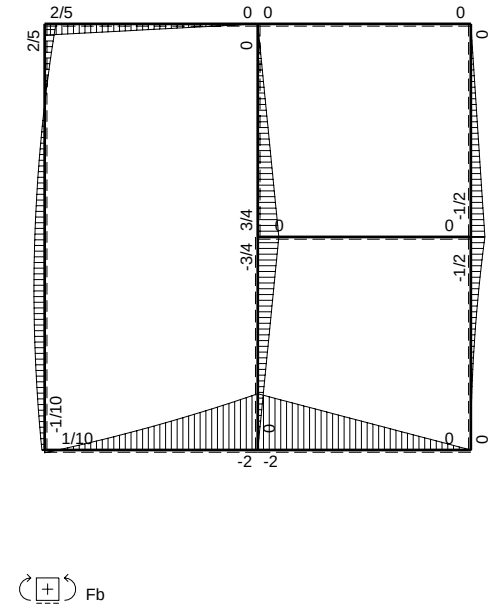
$$= \left(1/6\, b\right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$$

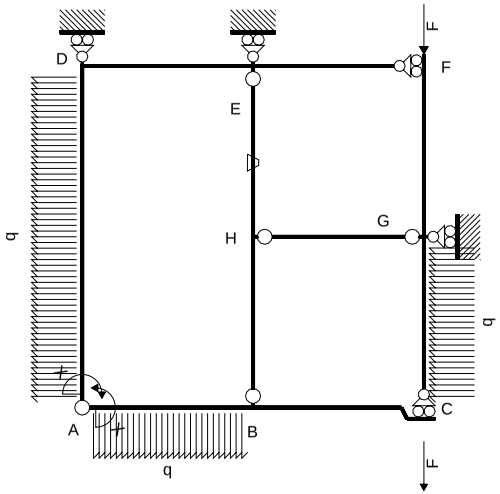
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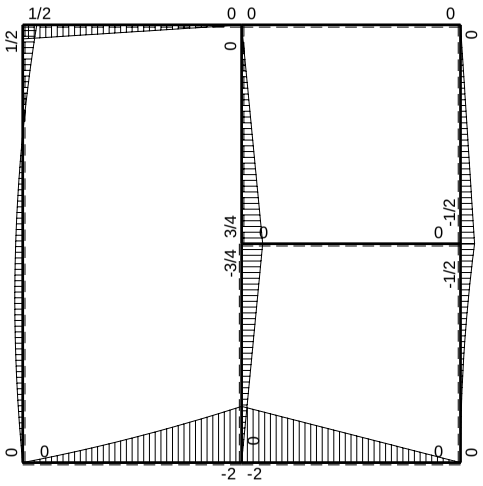

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D
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2/5Fb
8/5F
23/5F
3/4Fb
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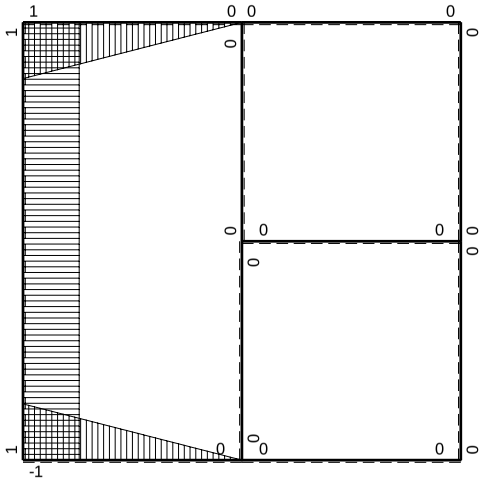

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-1/2
-3/4
-1/2
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0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	0	0	0	0	0+0	0
BH b	0	$3/4Fx$	0	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-3/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

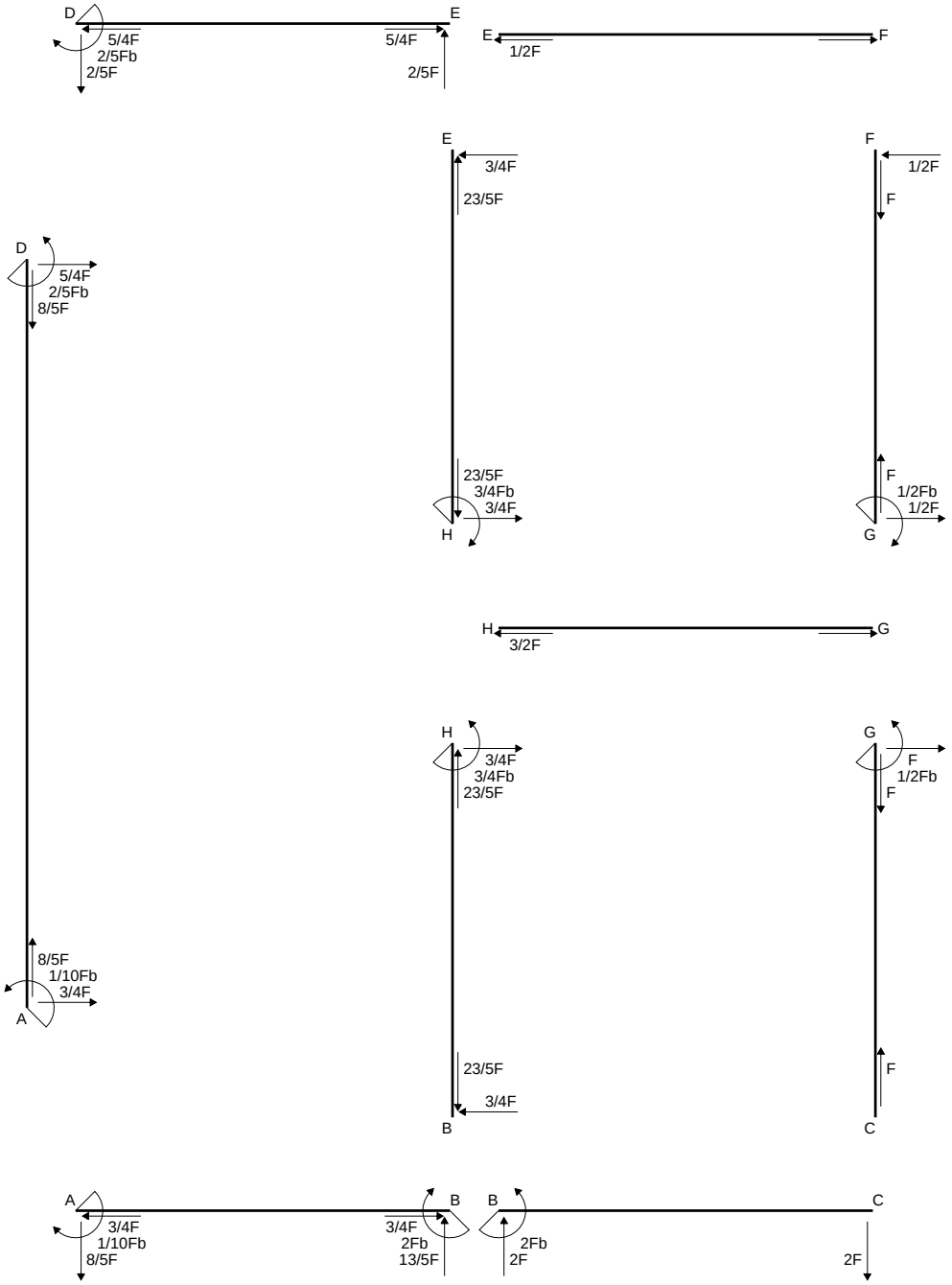
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

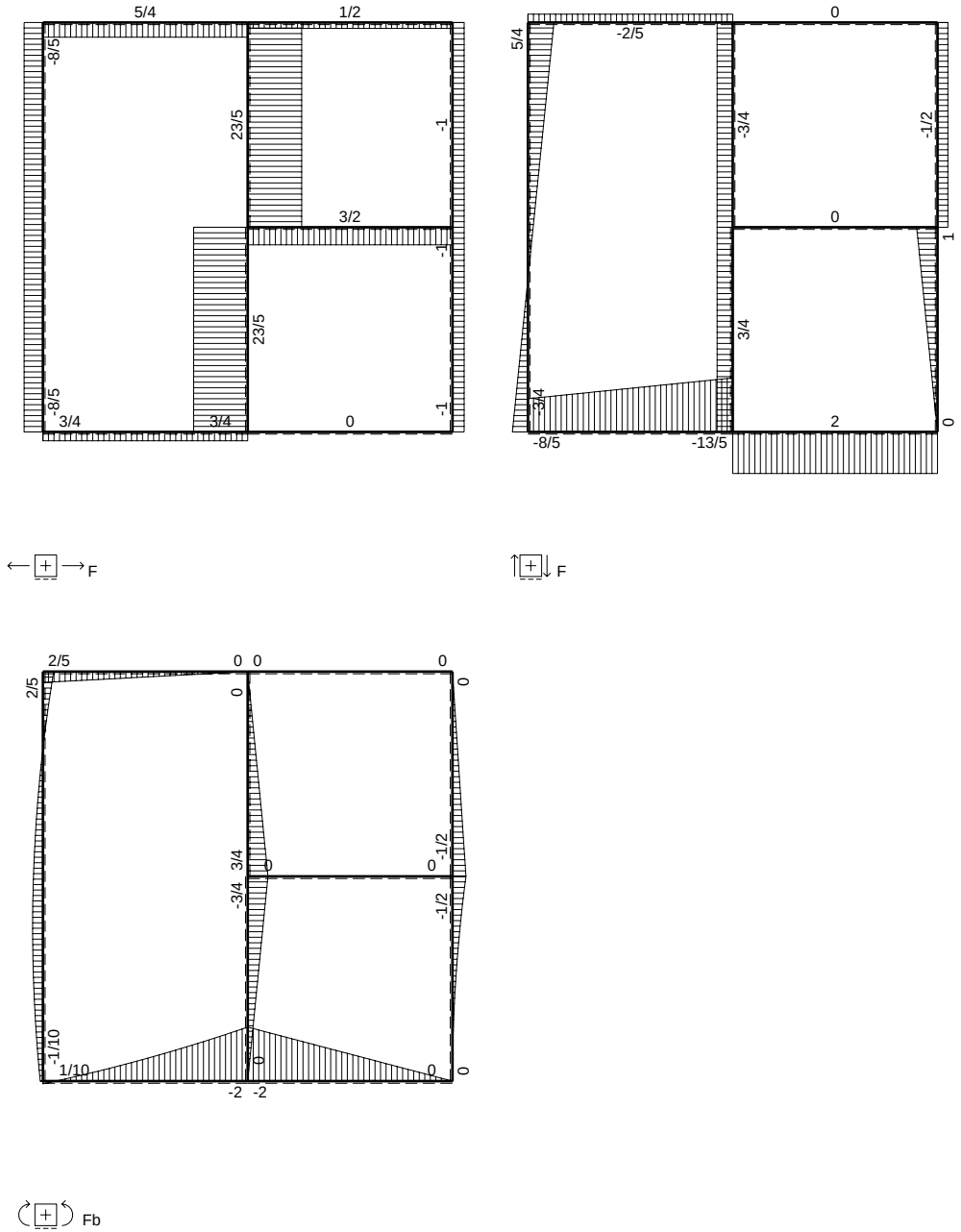
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

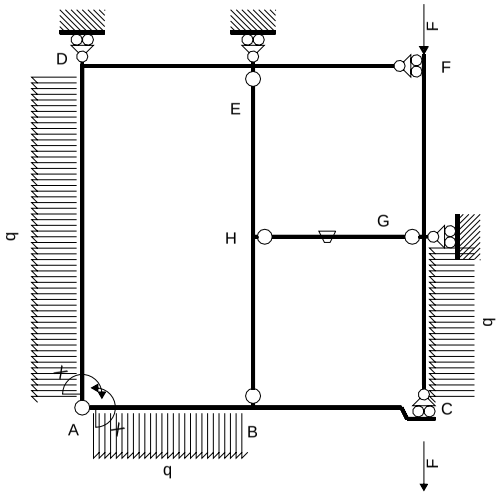
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238341 ESΣ03.xxxx.047

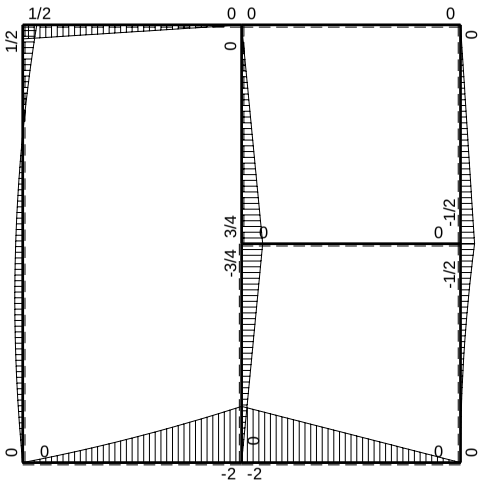
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D
E
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2/5Fb
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5/4F
2/5F
E
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E
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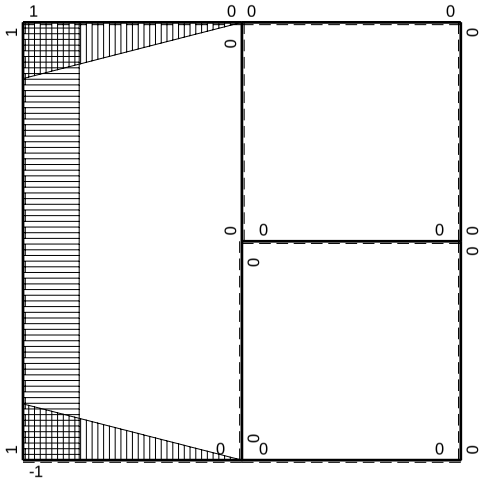

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0
-1/2
-3/4
0
-1/2
-1/10
1/10
0
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	0	0	0	0	0+0	0
BH b	0	$3/4Fx$	0	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

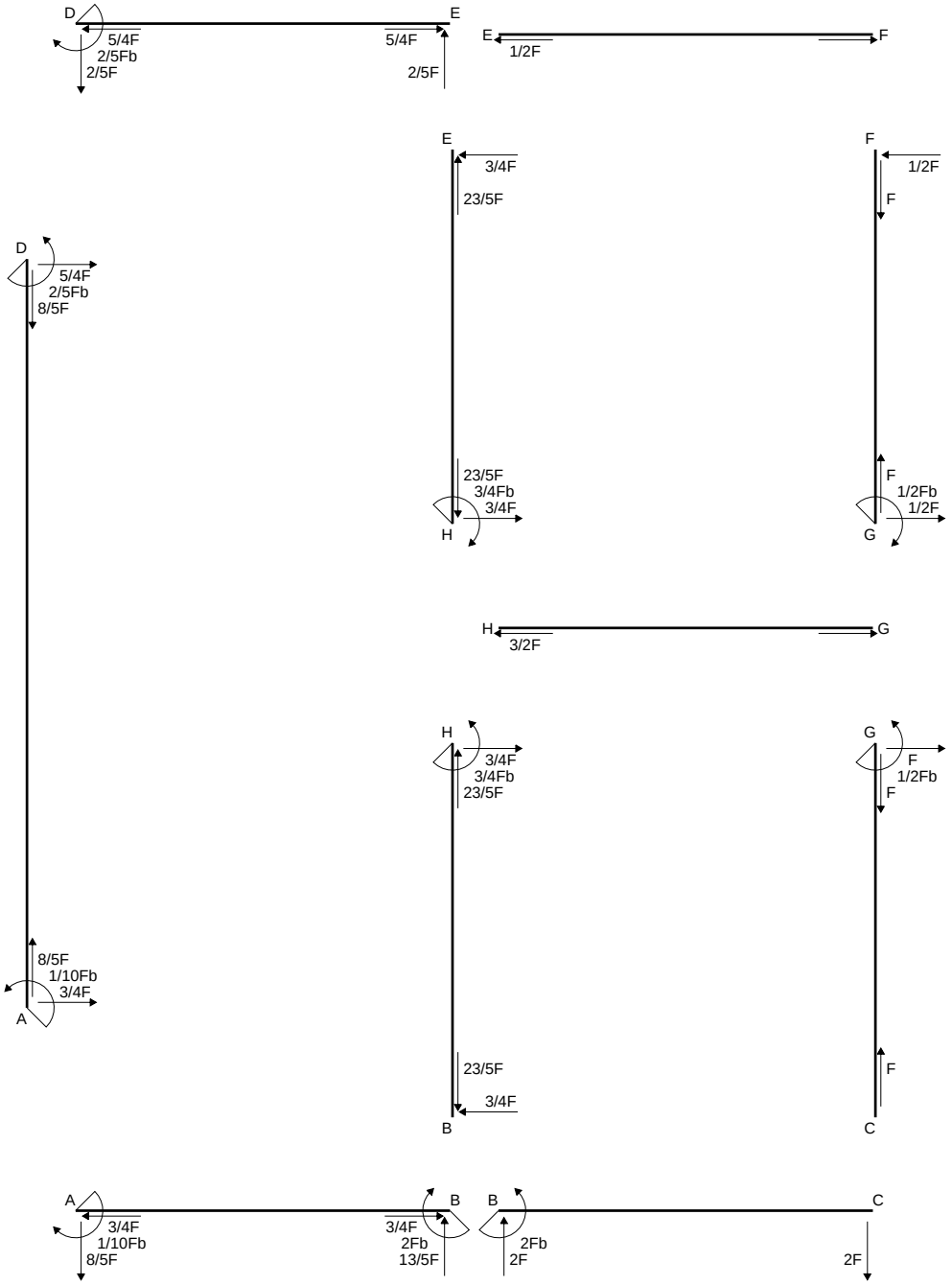
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

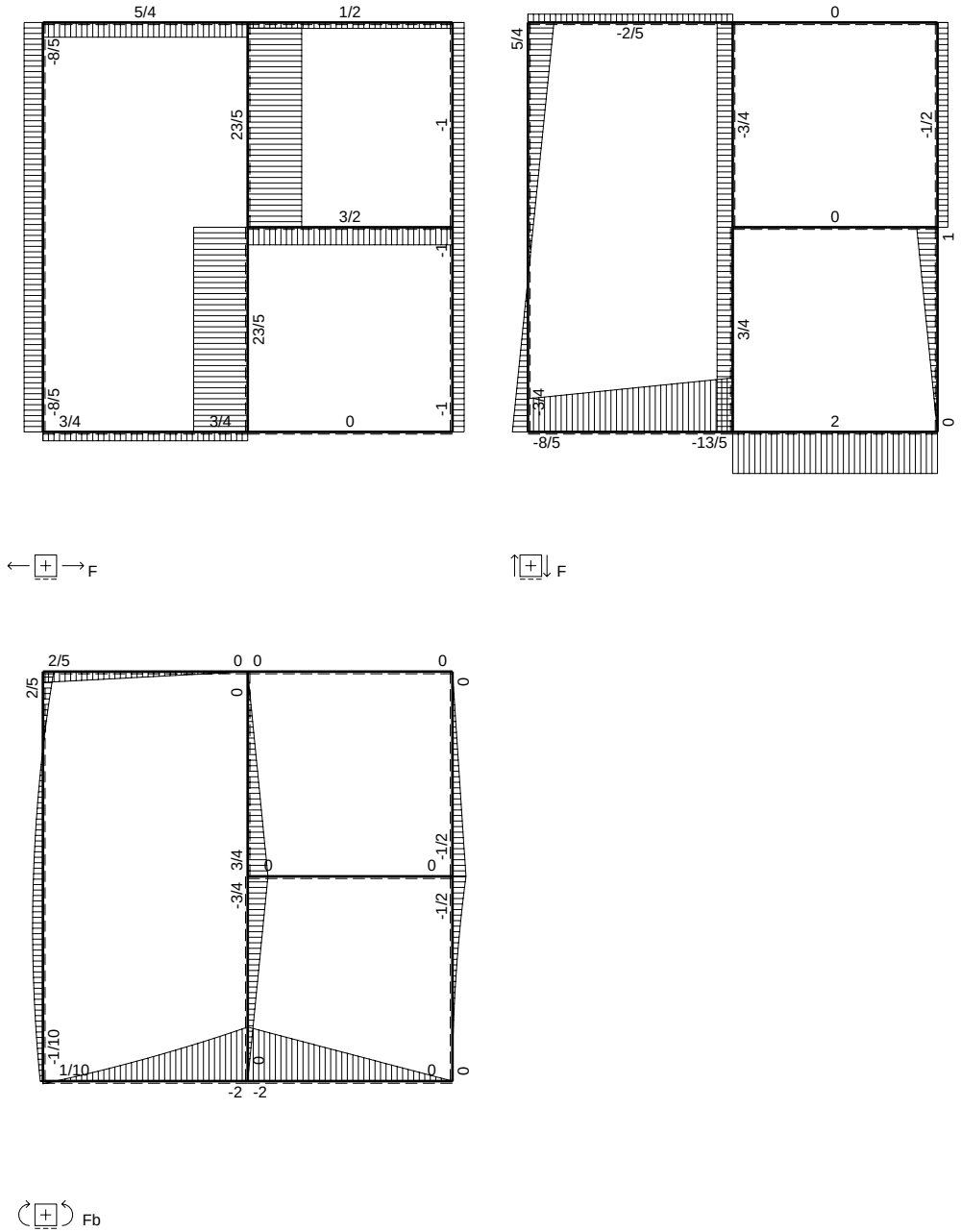
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

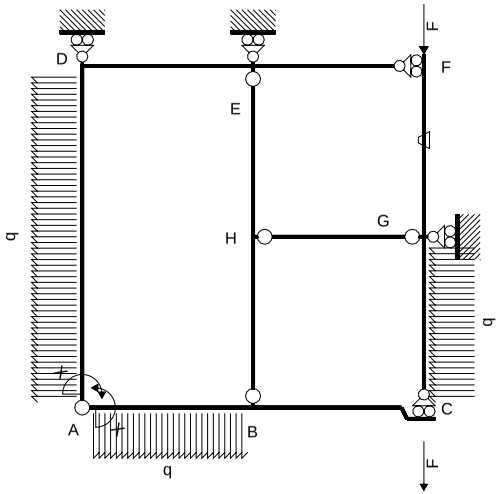
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 934507 ESΣ03.xxxx.048

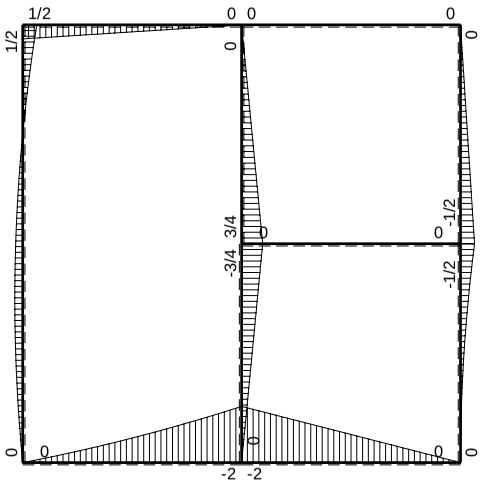
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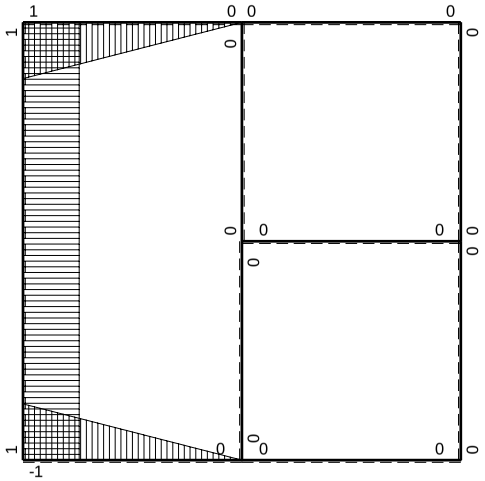

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0
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-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 280338 ESΣ03.xxxx.049

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PROCEDIMENTO E RISULTATI 280338 ESΣ03.xxxx.049

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	-Fb/EJ	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	Fb/EJ	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-3/4Fb+3/4Fx	0	0	0	0	0+0	0
BH b	0	3/4Fx	0	0	0	0		
HE b	0	3/4Fb-3/4Fx	0	0	0	0	0+0	0
EH b	0	-3/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

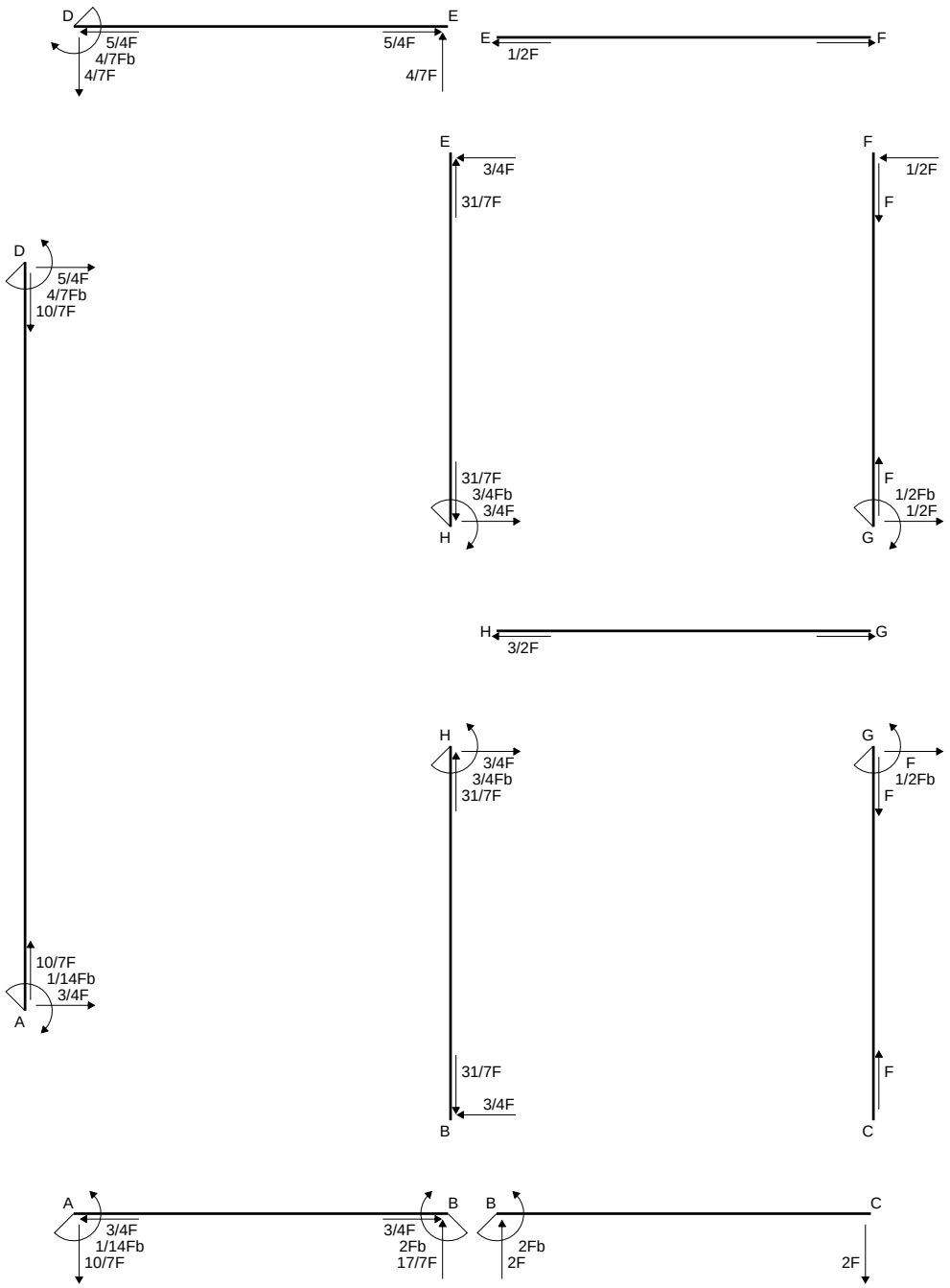
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

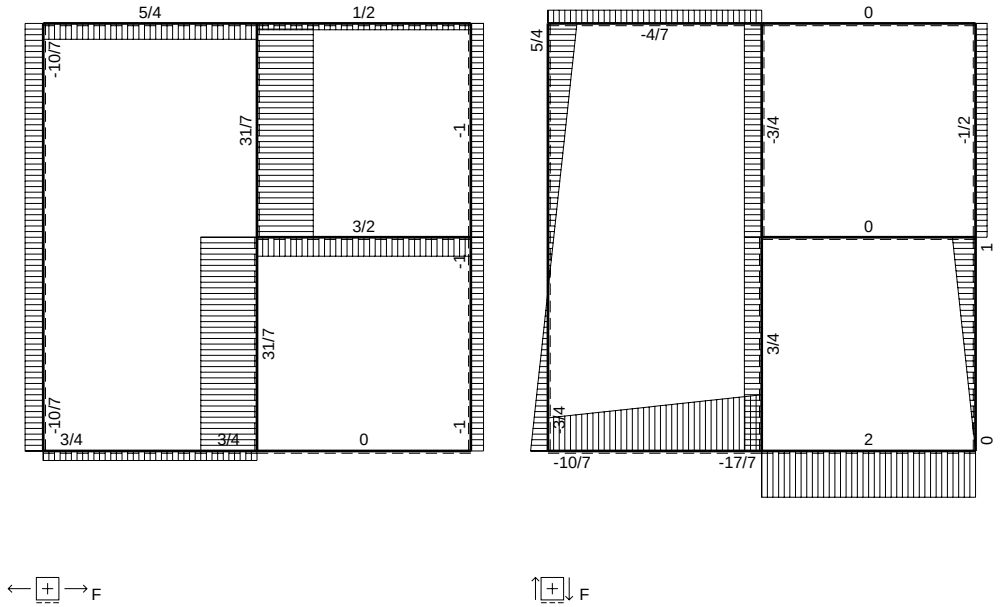
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

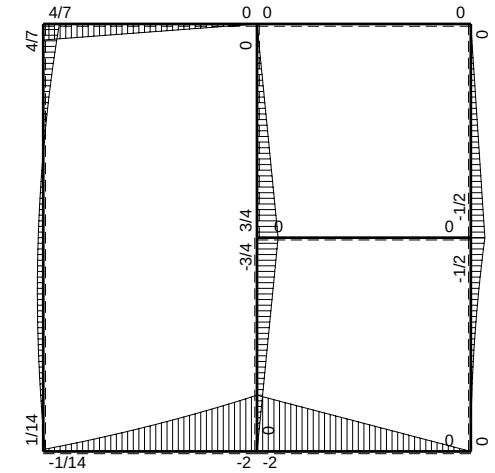
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

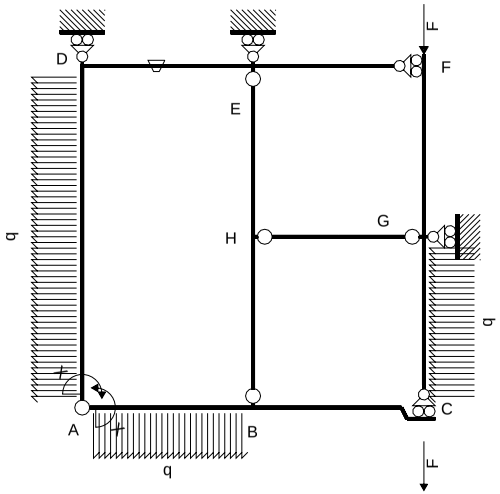
PROCEDIMENTO E RISULTATI 280338 ESΣ03.xxxx.049

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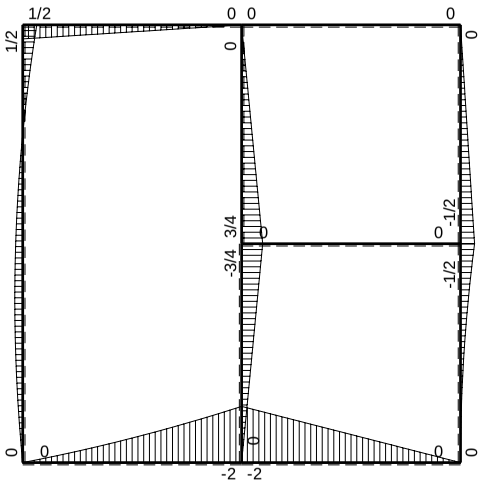

D
E
5/4F
4/7Fb
4/7F
5/4F
4/7F
E
F
1/2F
E
3/4F
31/7F
F
1/2F
F
D
5/4F
4/7Fb
10/7F
31/7F
3/4Fb
3/4F
H
F
1/2Fb
1/2F
G
H
G
3/2F
H
3/4F
3/4Fb
31/7F
G
F
1/2Fb
F
10/7F
1/14Fb
3/4F
A
31/7F
3/4F
B
F
C
A
3/4F
1/14Fb
10/7F
B
3/4F
2Fb
17/7F
B
2Fb
2F
C
2F


5/4
1/2
-10/7
31/7
-1
3/2
-1
31/7
-10/7
3/4
3/4
0
-1
F
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-3/4
-1/2
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3/4
2
0
-10/7
-17/7
F

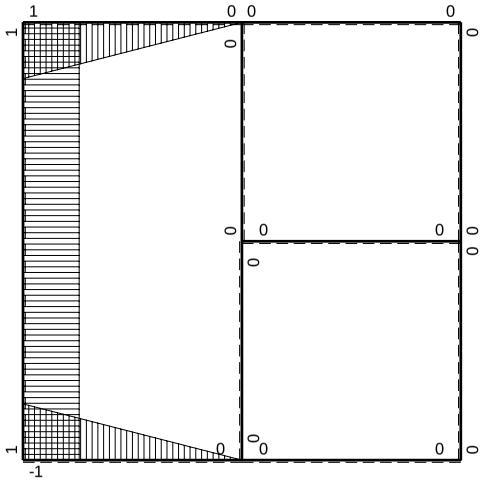

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0
4/7
0
0
0
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3/4
0
0
-3/4
-1/2
1/14
0
0
-1/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 242960 ESΣ03.xxxx.050

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PROCEDIMENTO E RISULTATI 242960 ESΣ03.xxxx.050

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	$-Fb/EJ$	$1/2Fb-Fx+1/2Fx^2/b$	$-Fb/EJ+Fx/EJ$	$1-2x/b+x^2/b^2$	$(1/6-1/2)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	Fb/EJ	$1/2Fx^2/b$	$-Fx/EJ$	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	0	0	0	0	0+0	0
BH b	0	$3/4Fx$	0	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-5/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(1/2 -5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x -5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(\ b -5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(1/2 - x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

$= \left[1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[- x +1/2\ x^2/b\ \right]_o^b\ \theta$

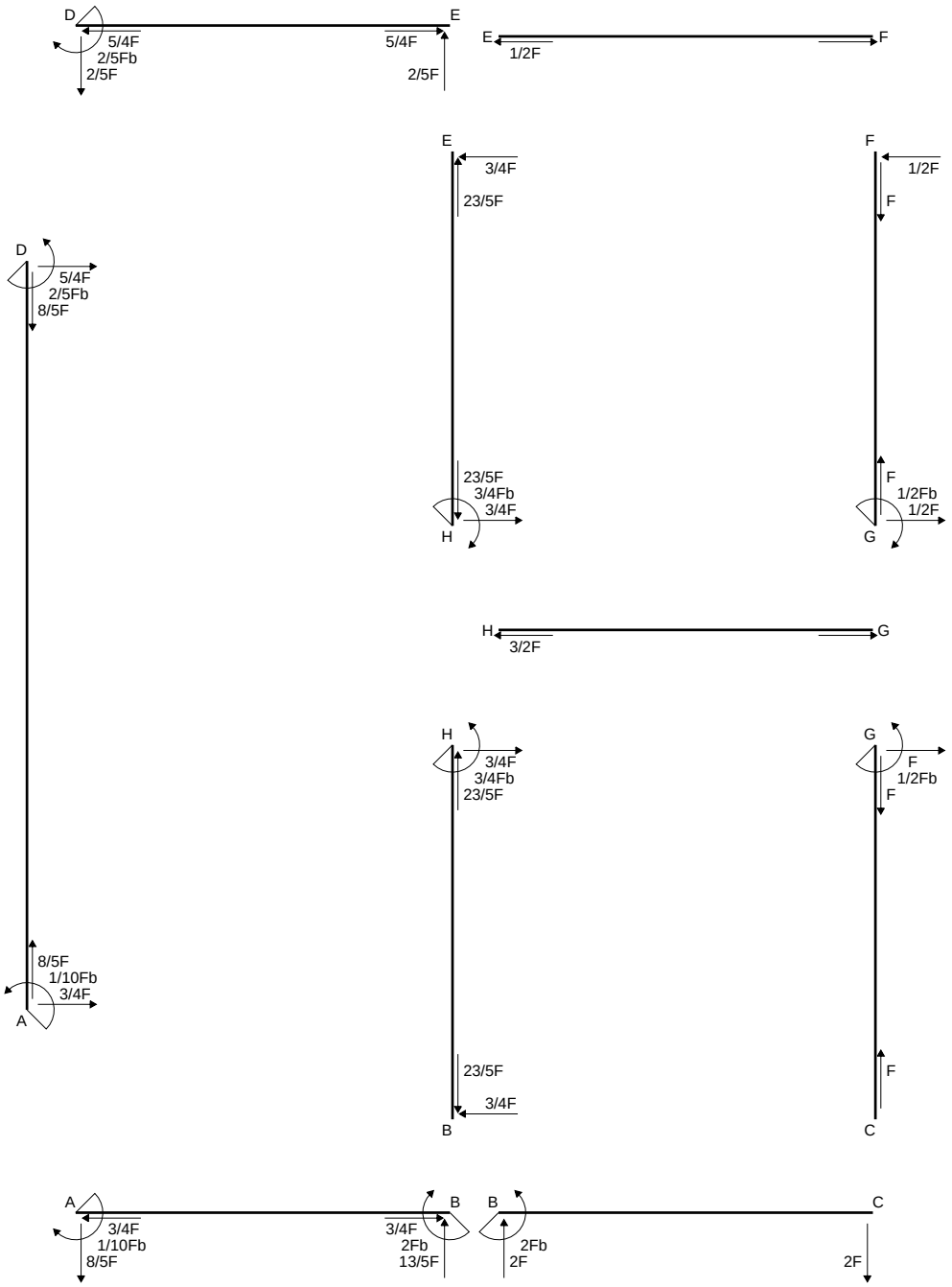
$= \left(1/2\ b -1/2\ b +1/6\ b\ \right) Fb\ 1/EJ\ + \left(- b +1/2\ b\ \right)\ \theta\ \ = -1/3\ \ Fb^2/EJ$

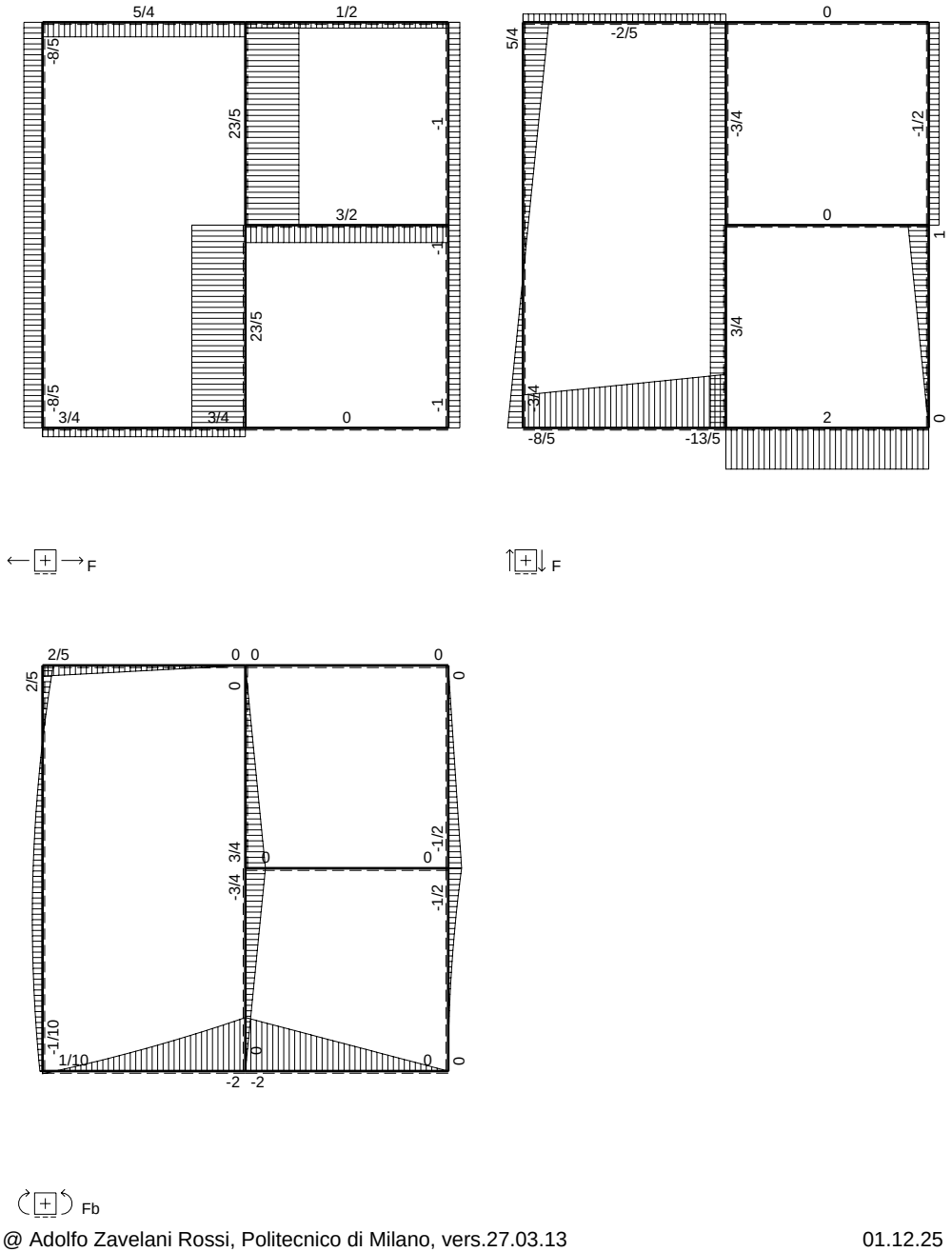
$L^{Xo}_{ED} = \int_o^b \left(1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx = \left[1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

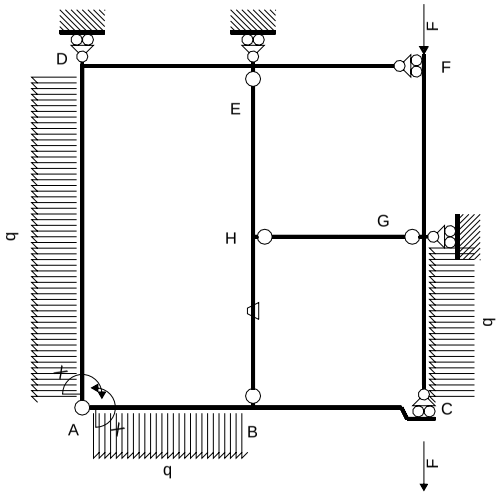
$= \left(1/6\ b\ \right) Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = -1/3\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 242960 ESΣ03.xxxx.050

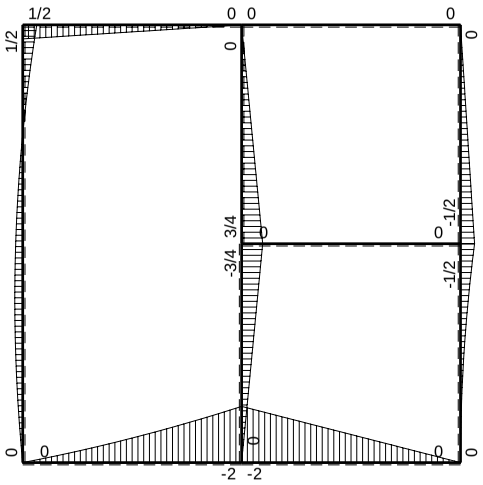
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D
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5/4F
2/5Fb
2/5F
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2/5F
E
F
1/2F
E
3/4F
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1/2F
F
D
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13/5F
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2F
2F

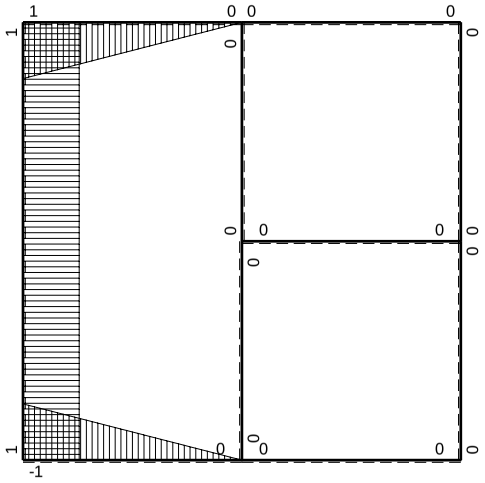

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0
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0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237721 ESΣ03.xxxx.051

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PROCEDIMENTO E RISULTATI 237721 ESΣ03.xxxx.051

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$3/4Fx$	Fb/EJ	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

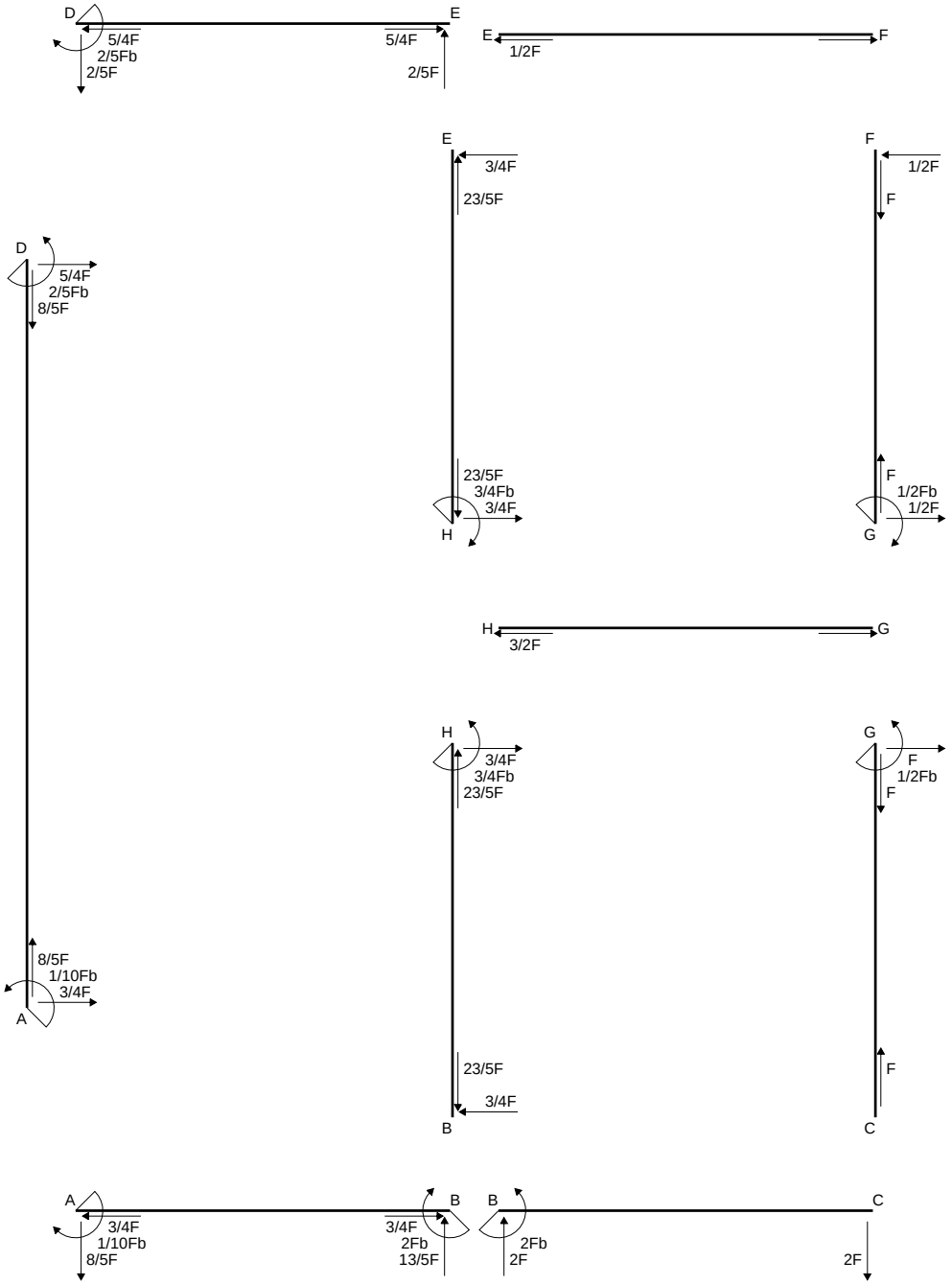
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

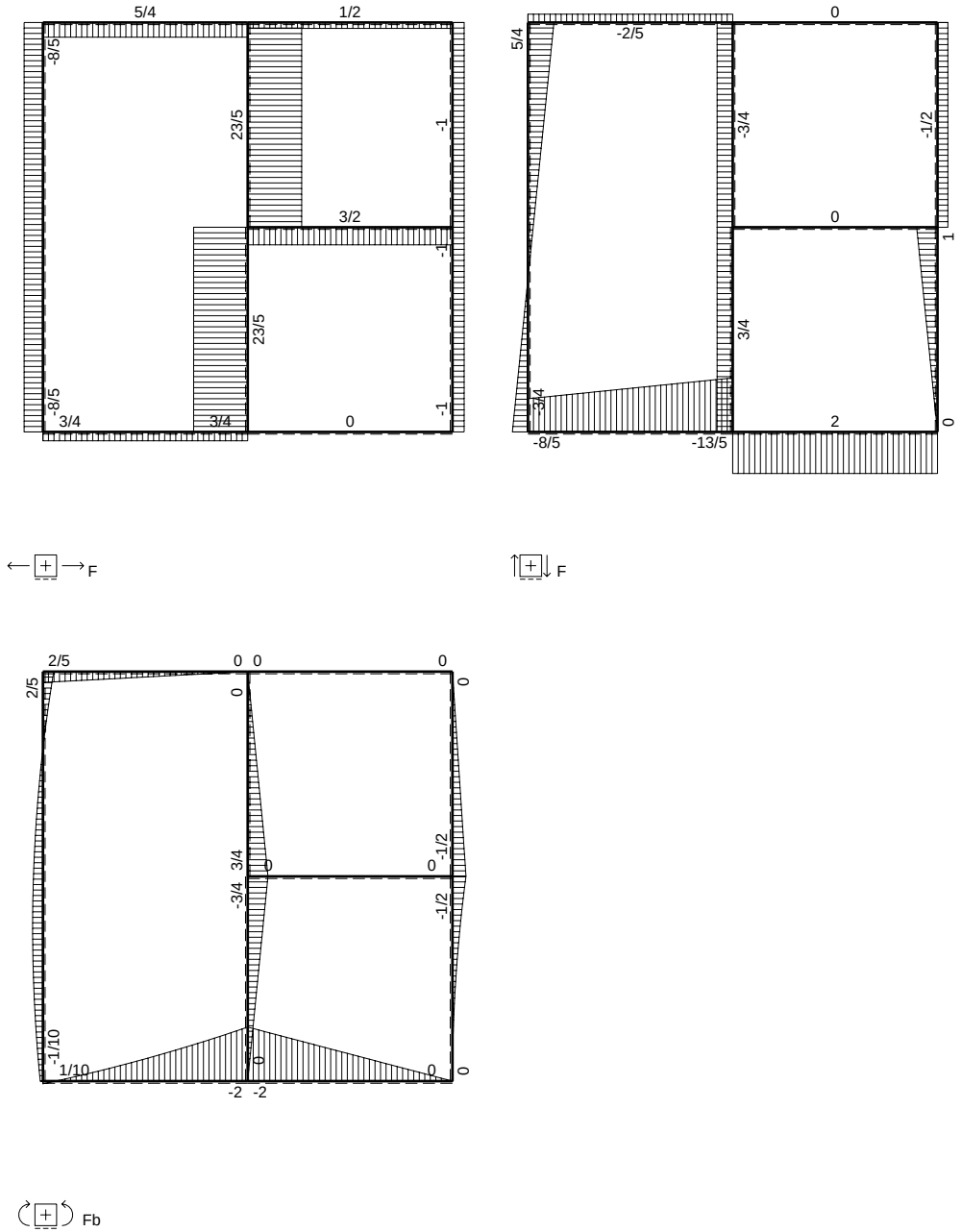
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

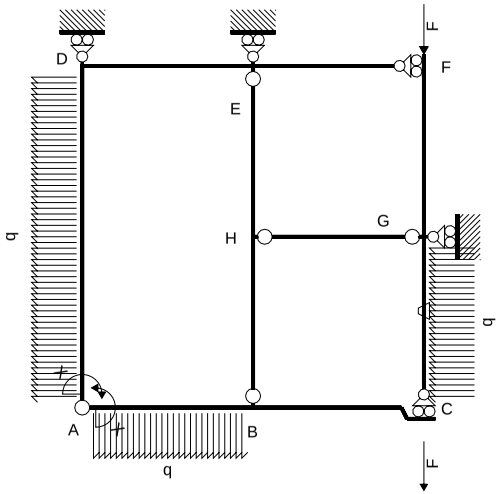
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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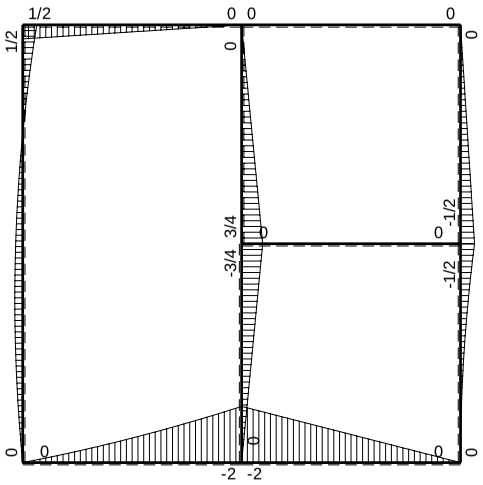
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
E
F
1/2F
E
3/4F
23/5F
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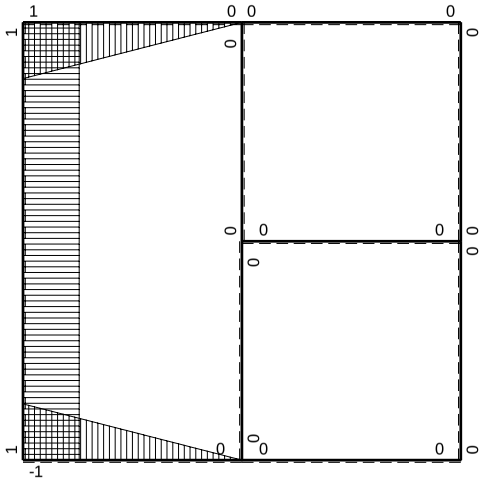

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2/5
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0
0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238387 ESΣ03.xxxx.052

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	0	0	0	0	0+0	0
BH b	0	$3/4Fx$	0	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

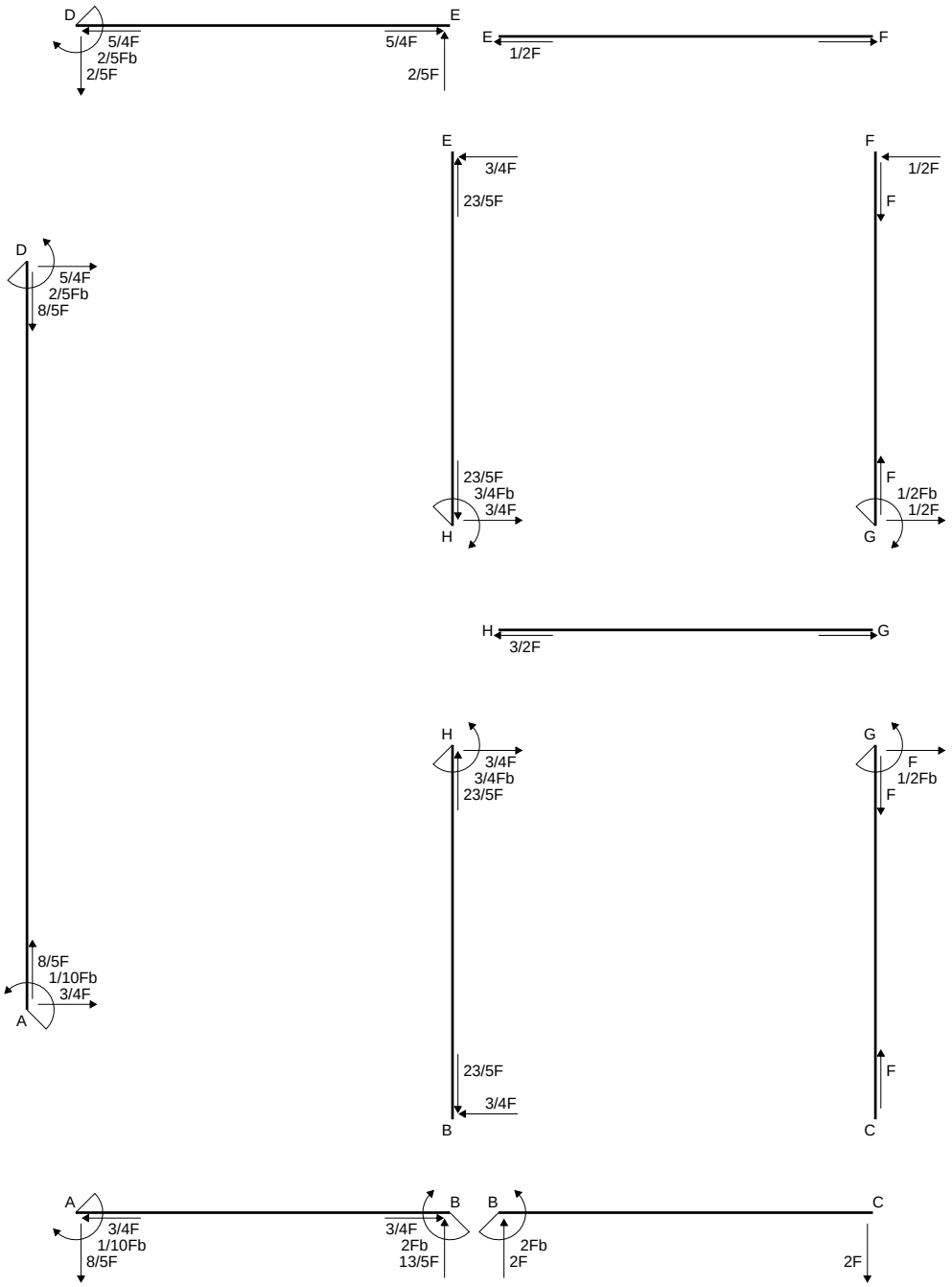
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

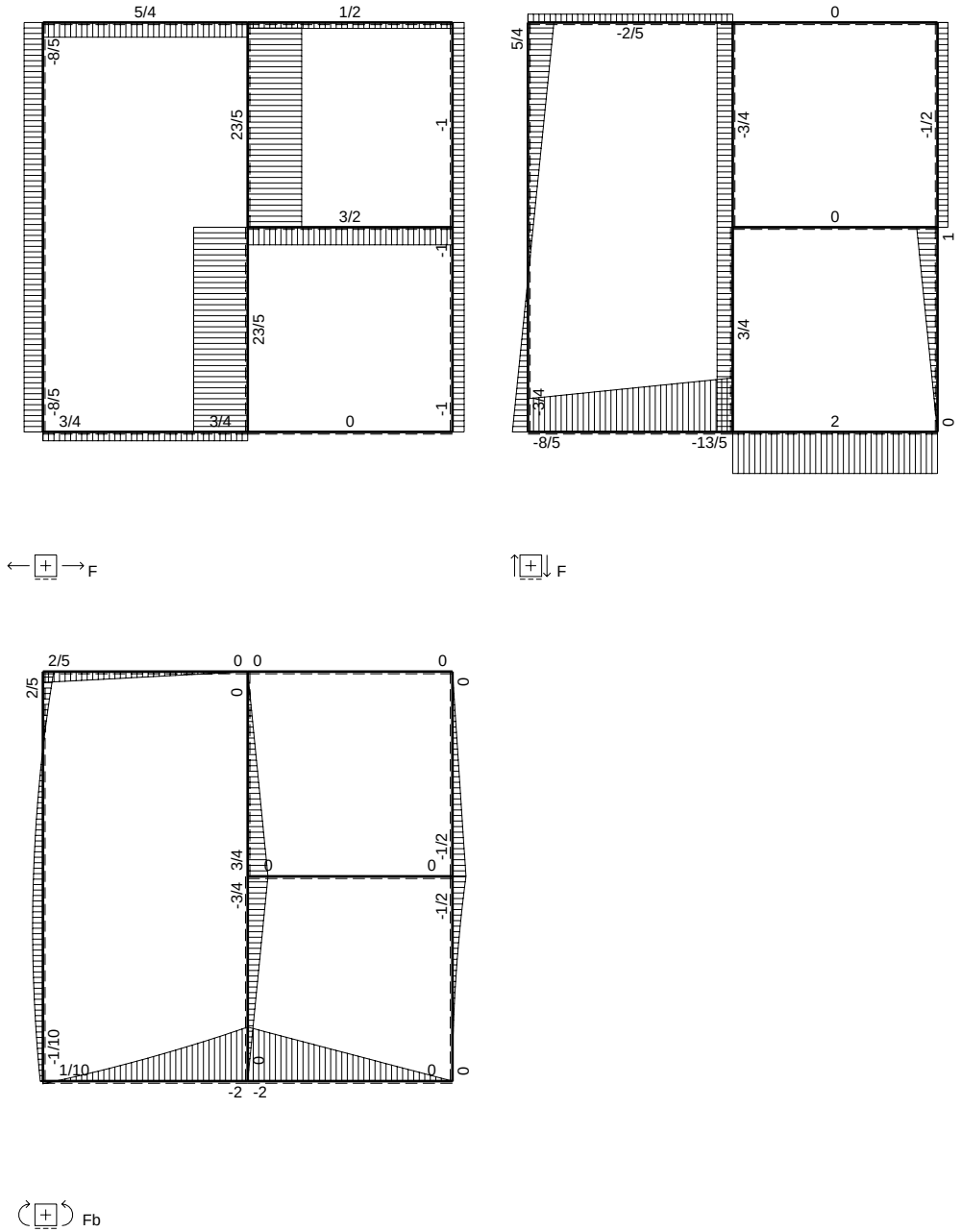
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

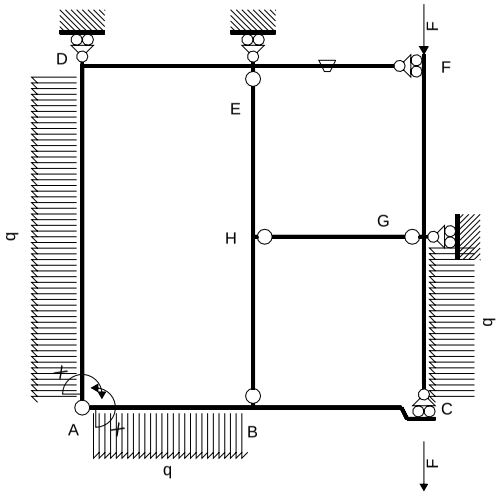
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238387 ESΣ03.xxxx.052

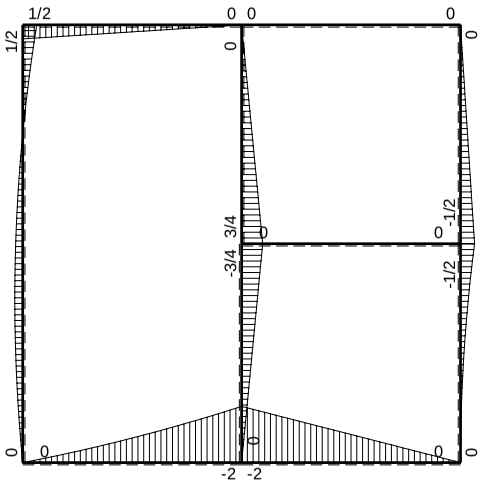
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2/5Fb
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2/5F
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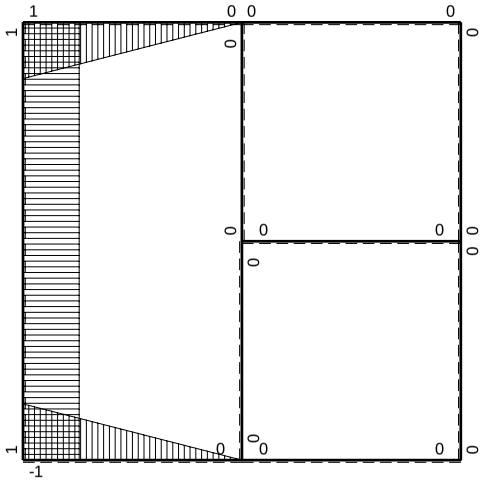

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0
0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238485 ESΣ03.xxxx.053

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	-Fb/EJ	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-3/4Fb+3/4Fx	0	0	0	0	0+0	0
BH b	0	3/4Fx	0	0	0	0		
HE b	0	3/4Fb-3/4Fx	0	0	0	0	0+0	0
EH b	0	-3/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

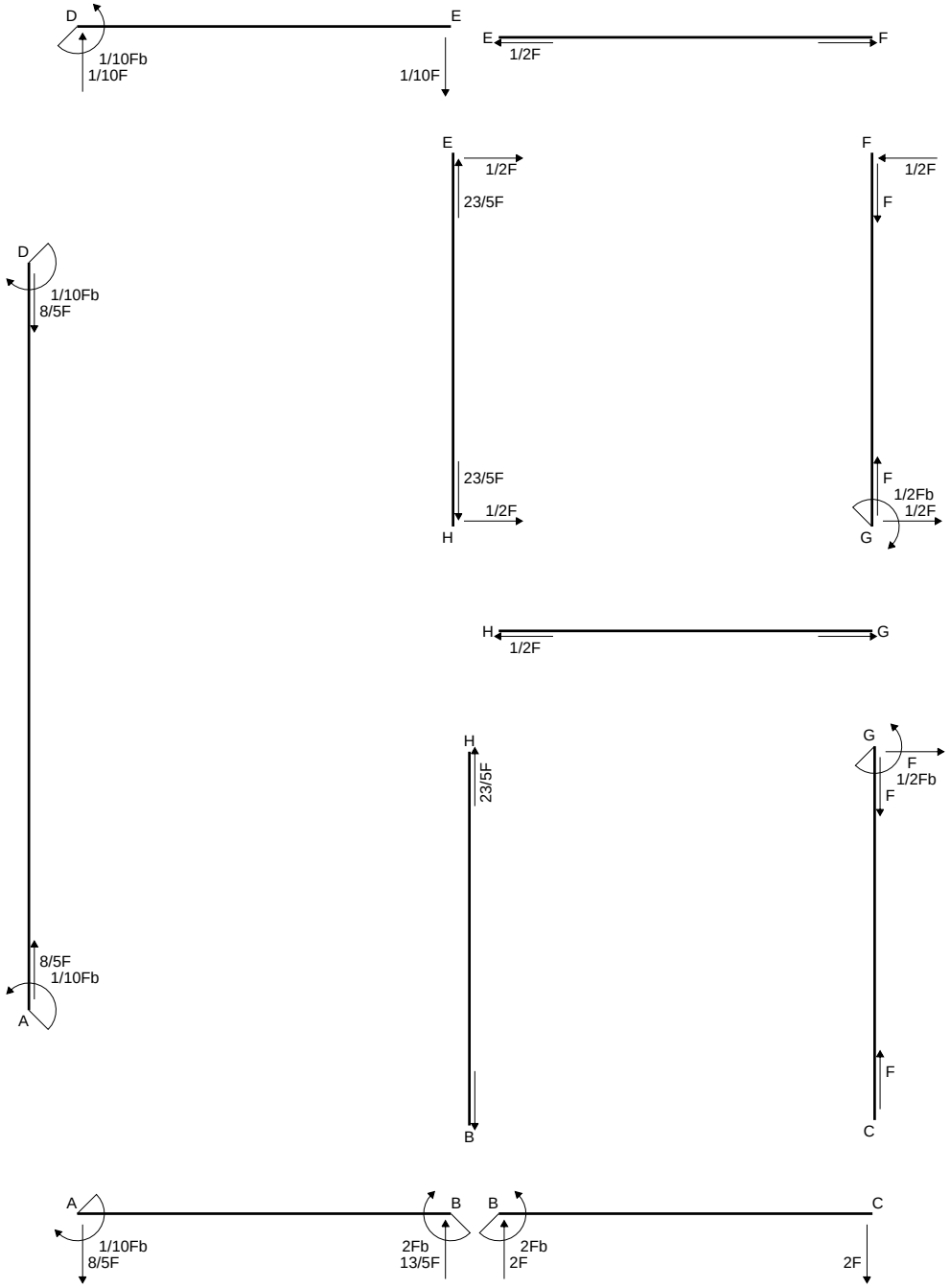
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

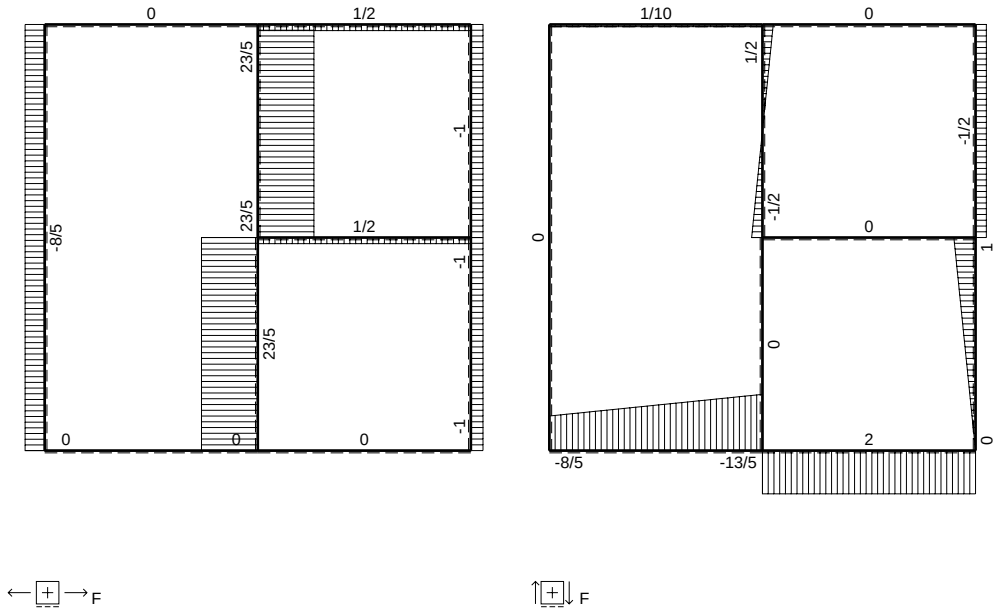
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

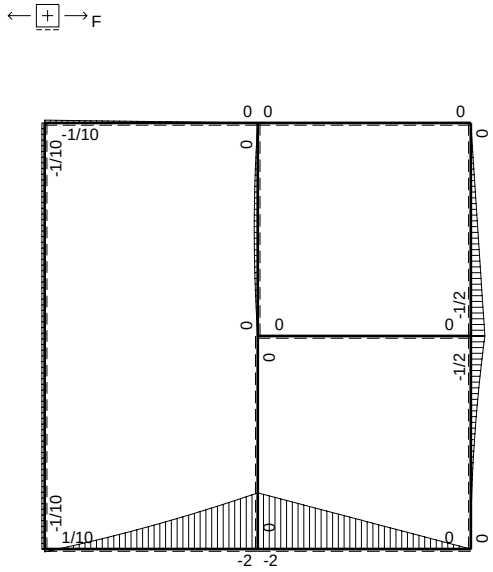
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

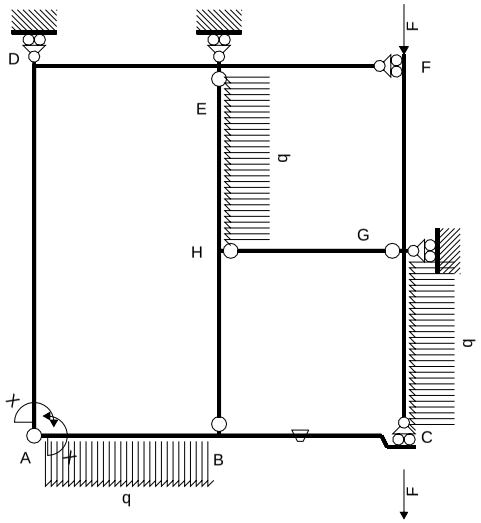
PROCEDIMENTO E RISULTATI 238485 ESΣ03.xxxx.053

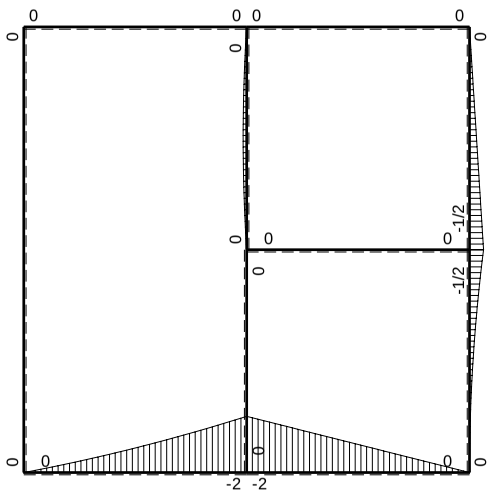
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D
E
1/10Fb
1/10F
1/10F
E
F
1/2F
E
1/2F
23/5F
23/5F
1/2F
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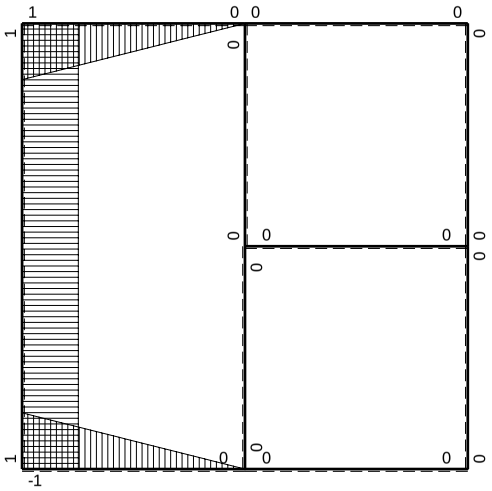

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2
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-8/5
-13/5


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-1/2
0
0
-1/2
-1/10
-1/10
0
0
0
-2 -2





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 214395 ESΣ03.xxxx.054

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	-Fb/EJ	0	0	0	0+0	0
CB b	0	2Fx	Fb/EJ	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

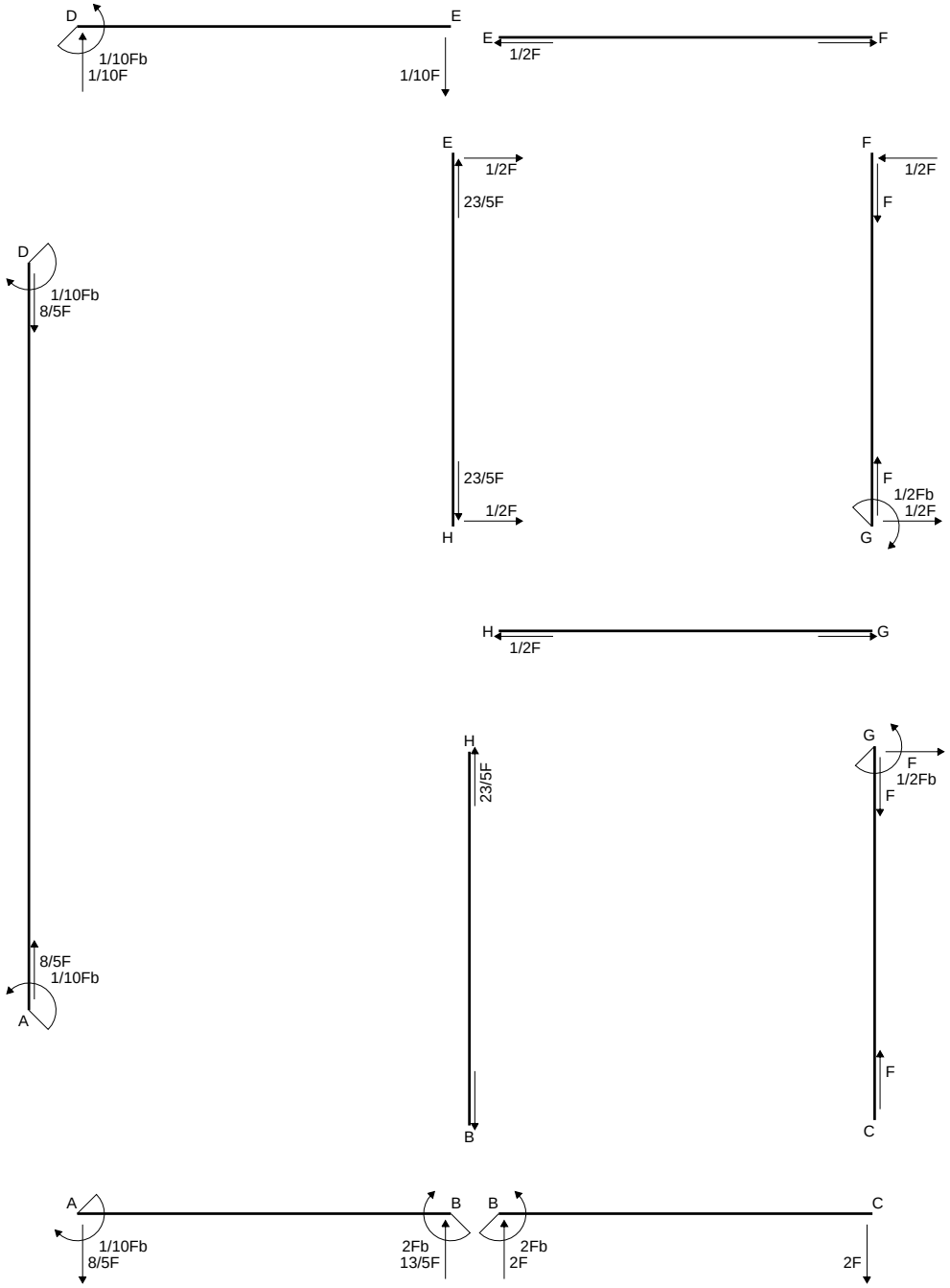
$L_{BA}^{Xo} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

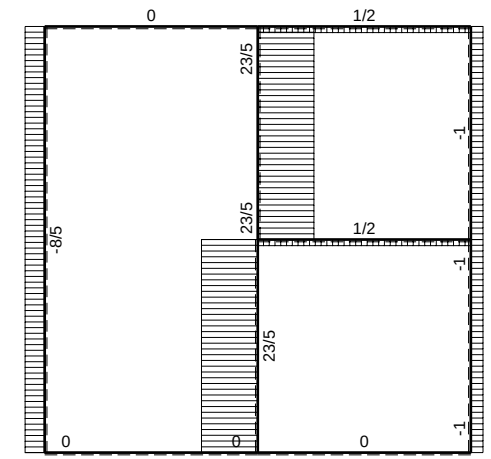
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

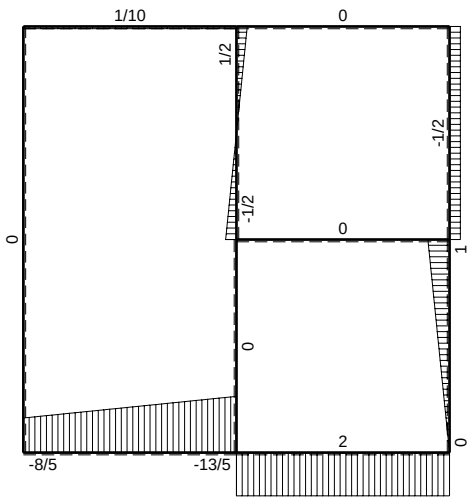
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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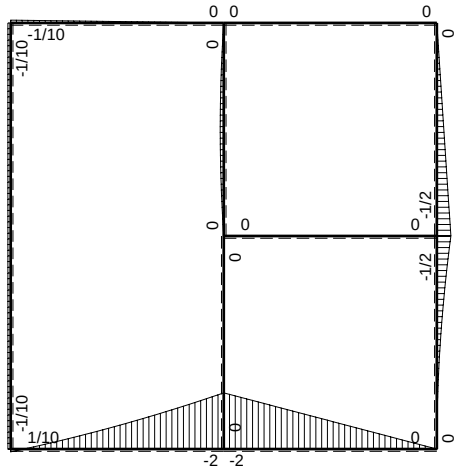

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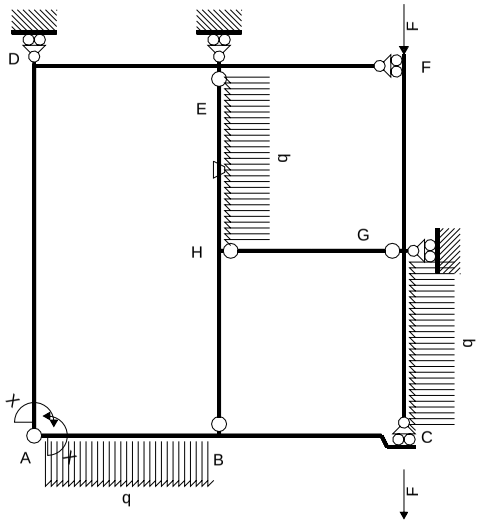

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-13/5

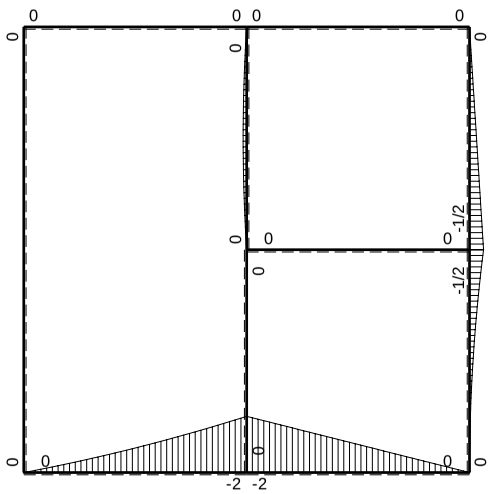
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F

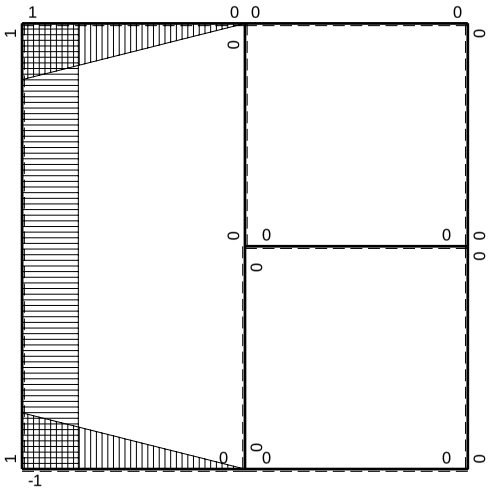

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-1/10
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0
0
-2 -2

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (1/3\,b)\ \ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,3/4\,b - 1/3\,b - 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

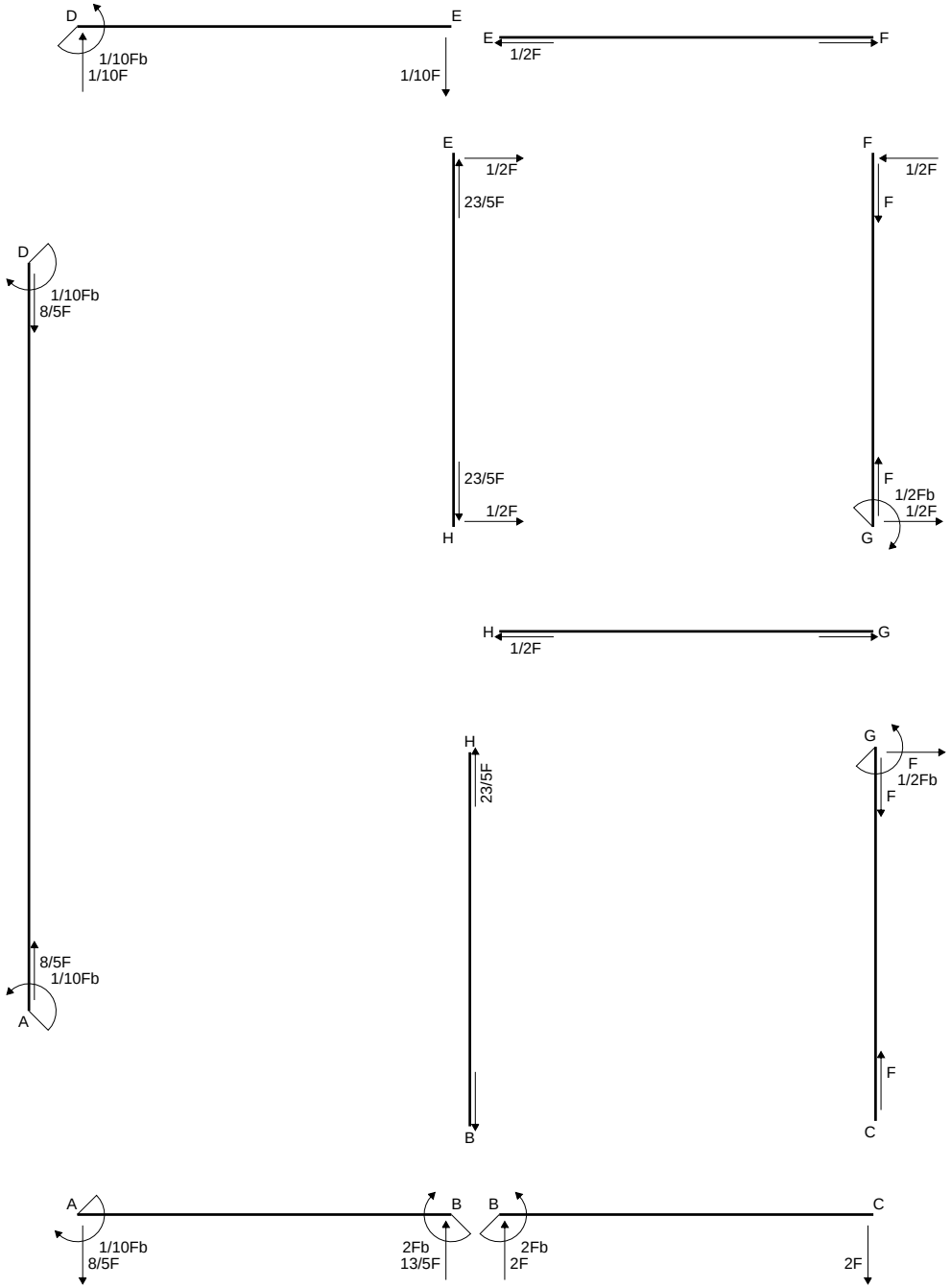
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

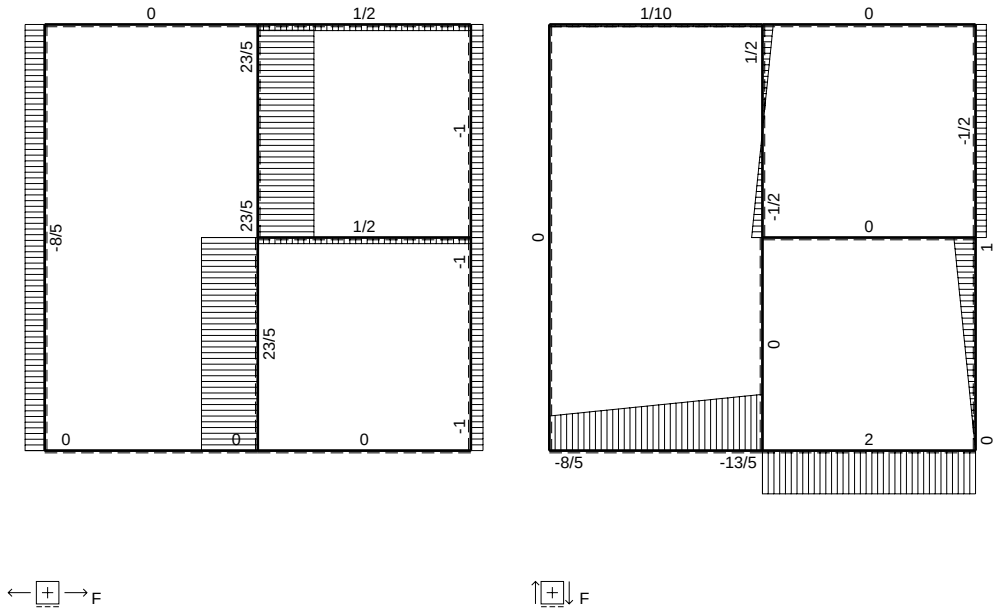
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

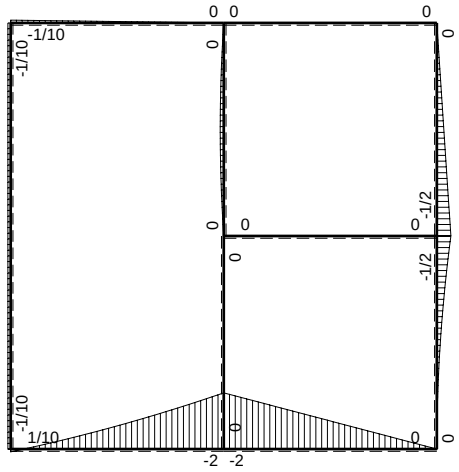
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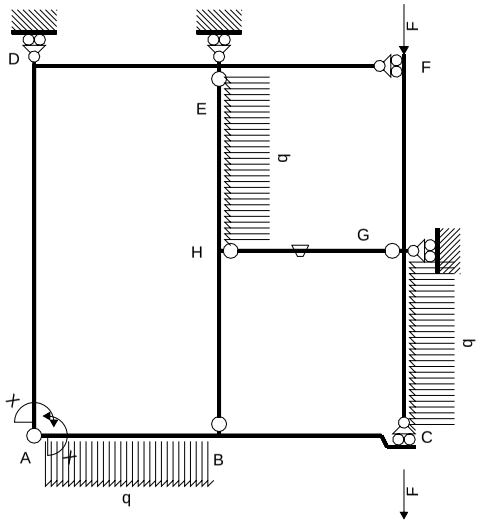

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G
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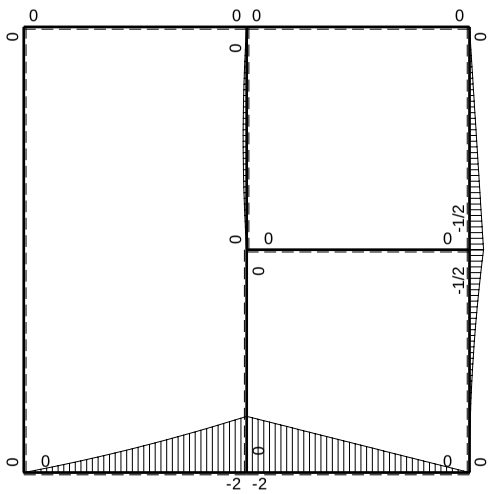

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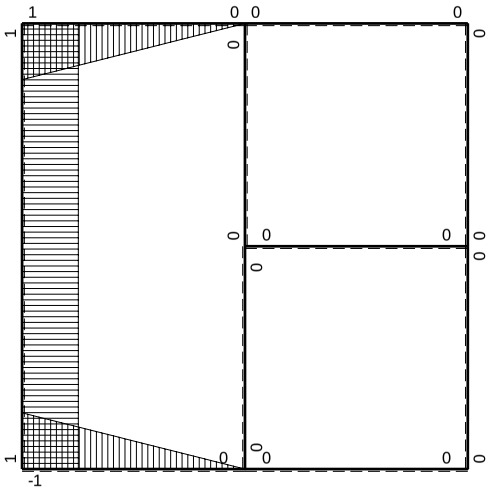

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238123 ESΣ03.xxxx.056

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

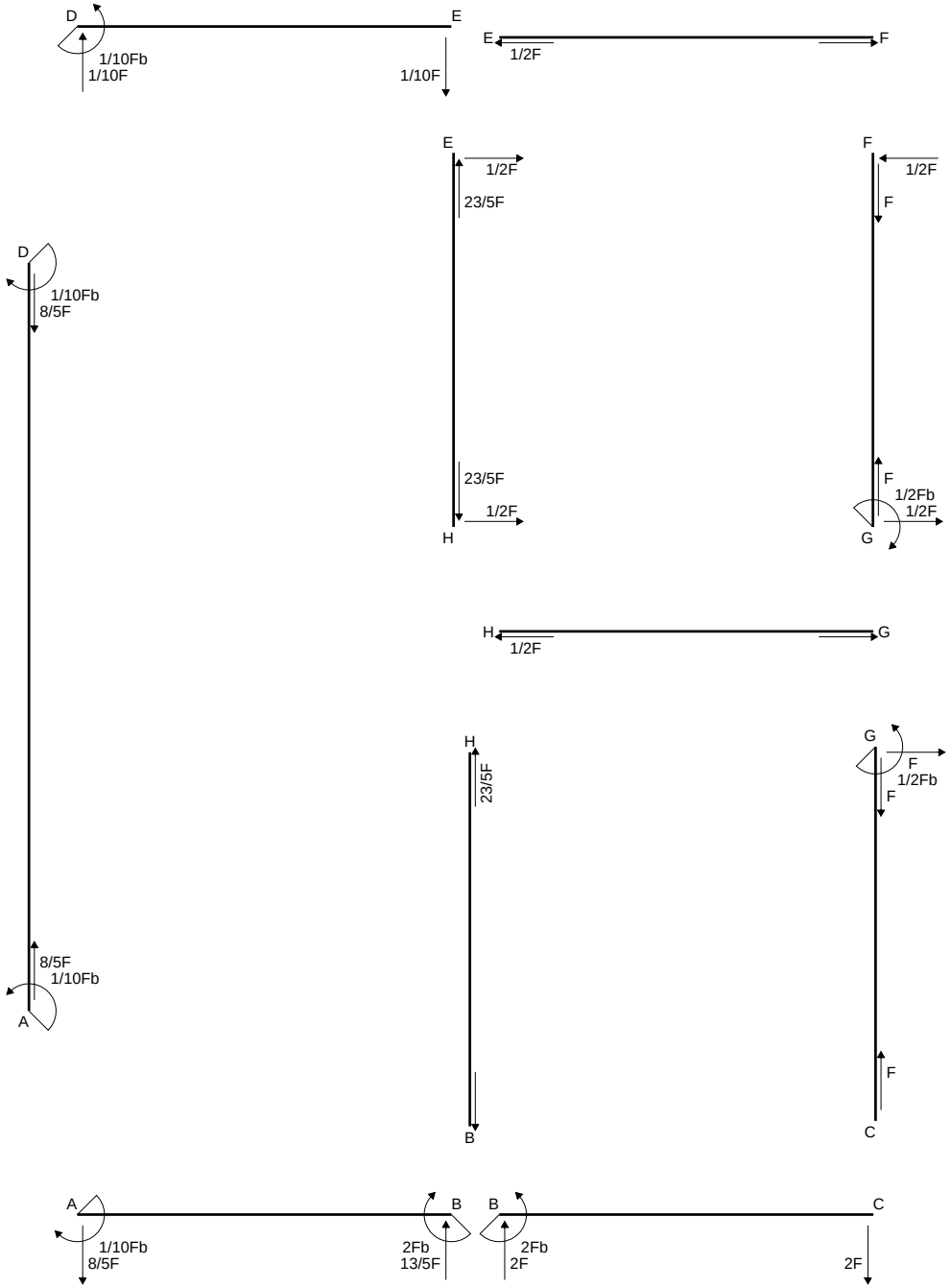
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

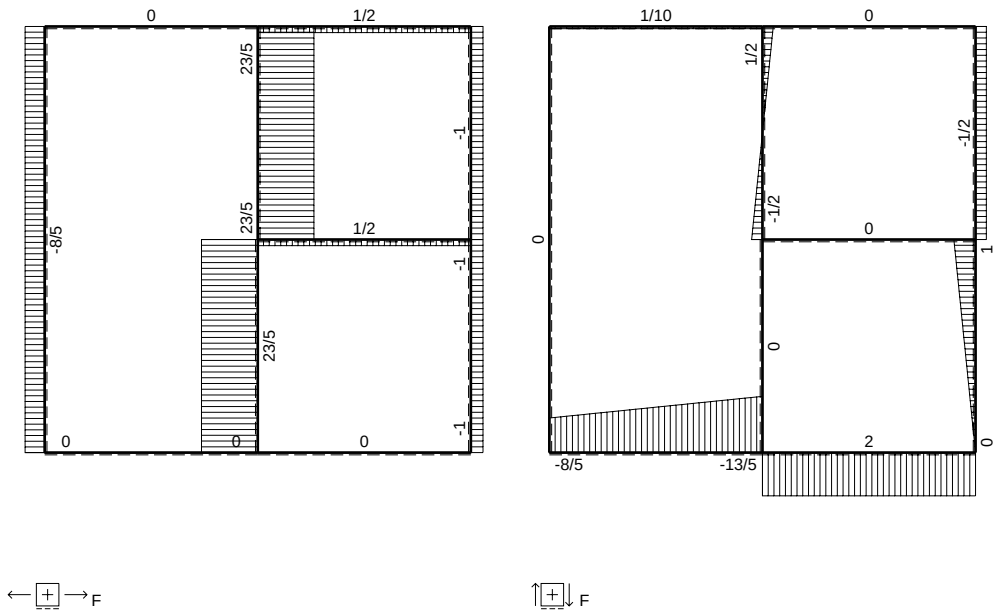
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

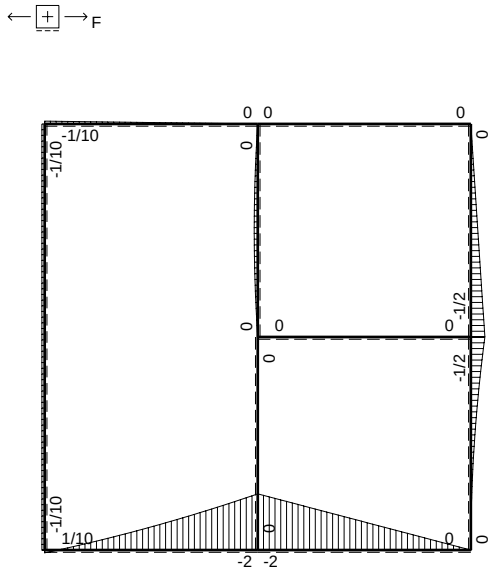
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

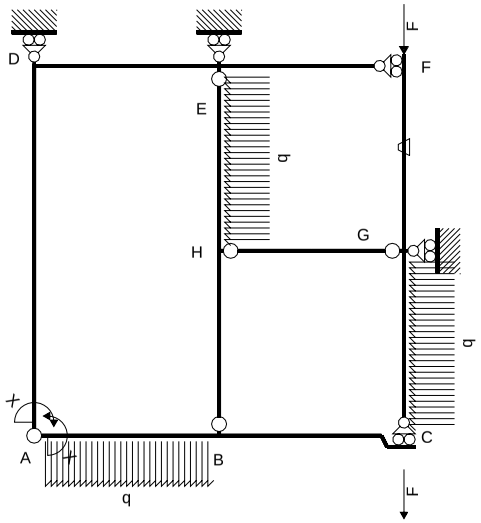
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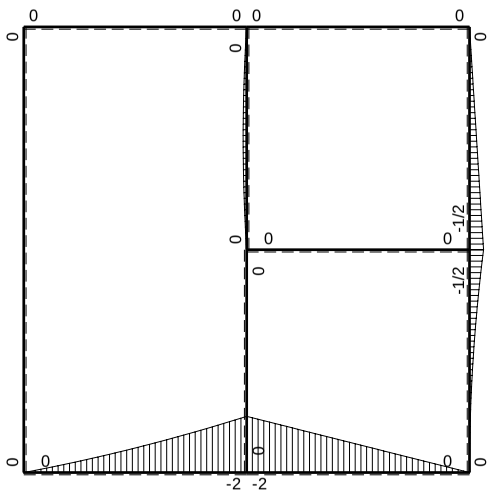
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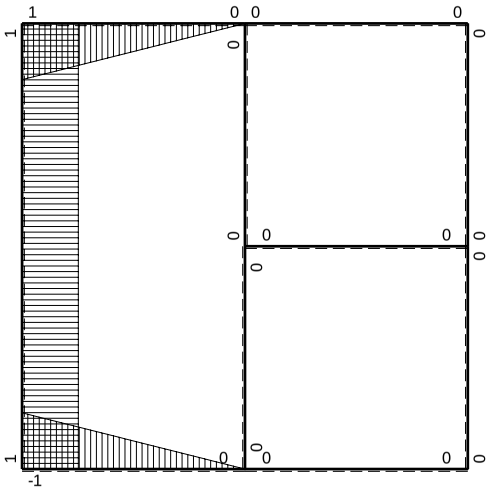

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-2 -2





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 955375 ESΣ03.xxxx.057

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PROCEDIMENTO E RISULTATI 955375 ESΣ03.xxxx.057

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	Fb/EJ	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\,b - 1/3\,b - 1/8\,b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

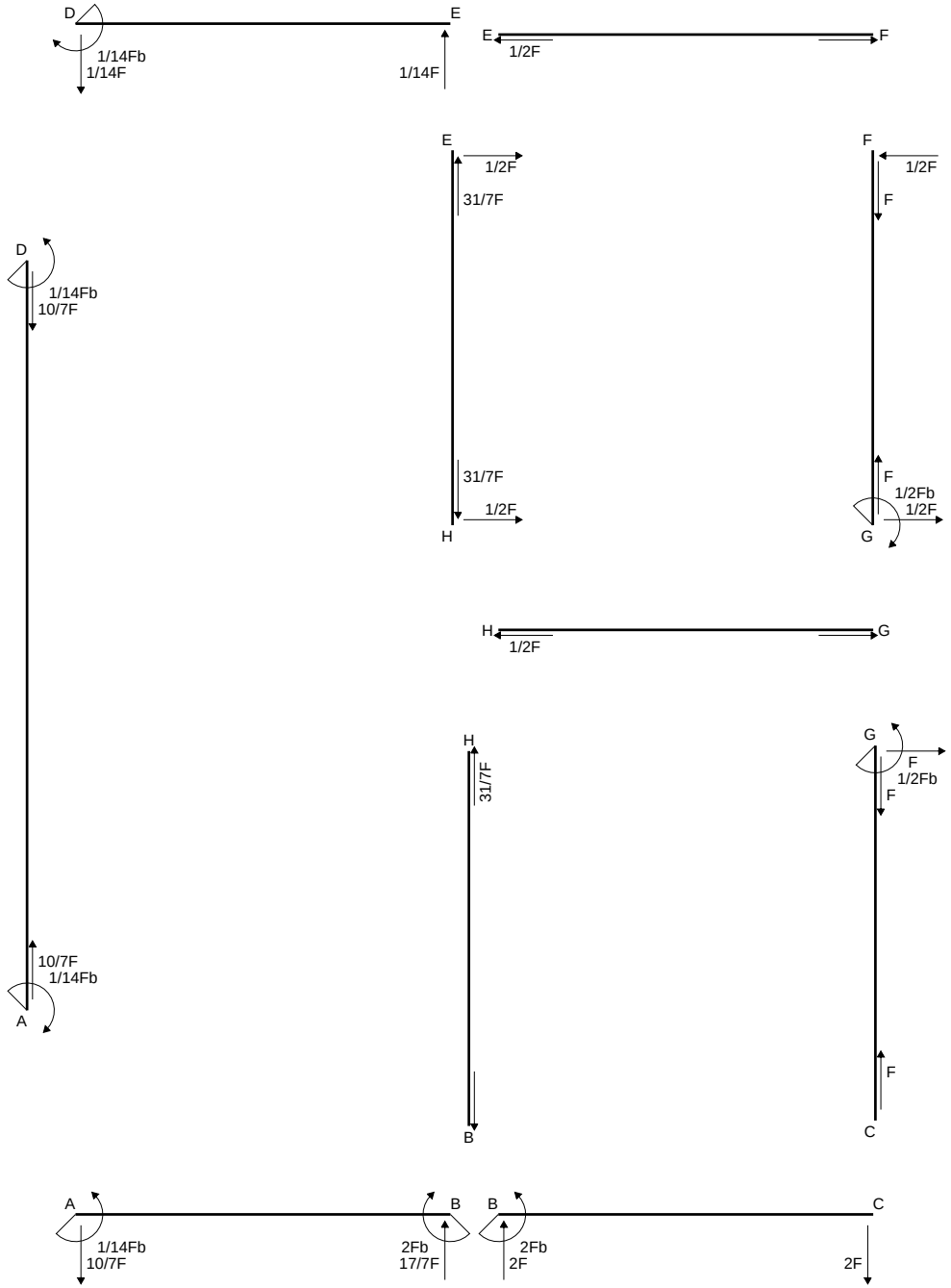
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

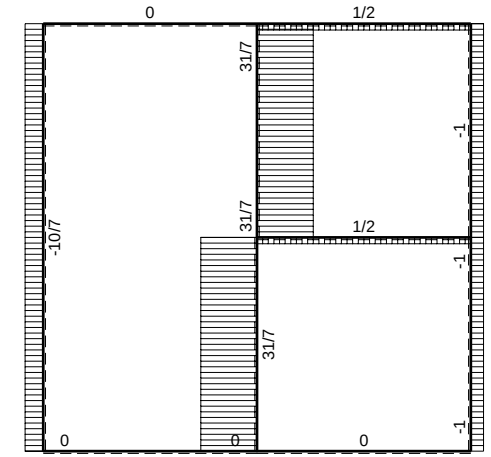
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

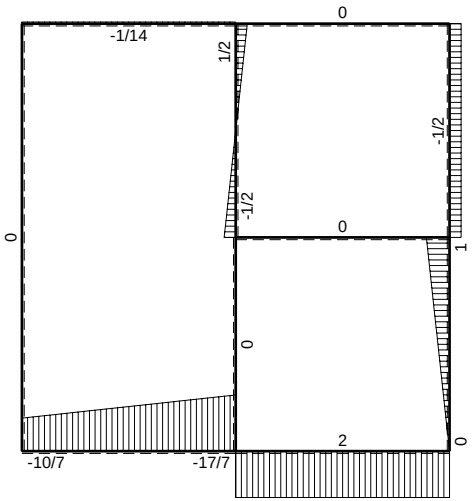
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

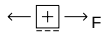
ESΣ03.xxxx.057

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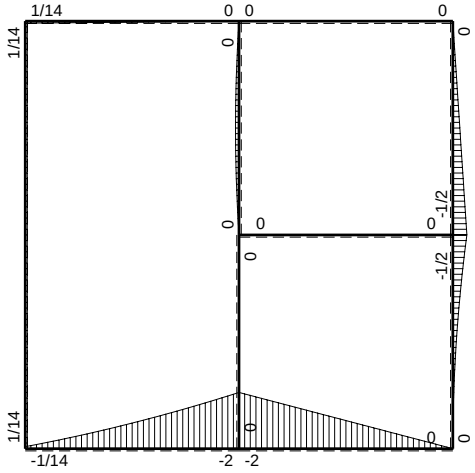

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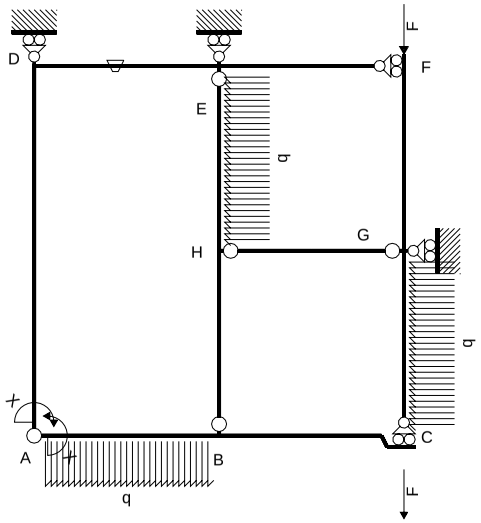

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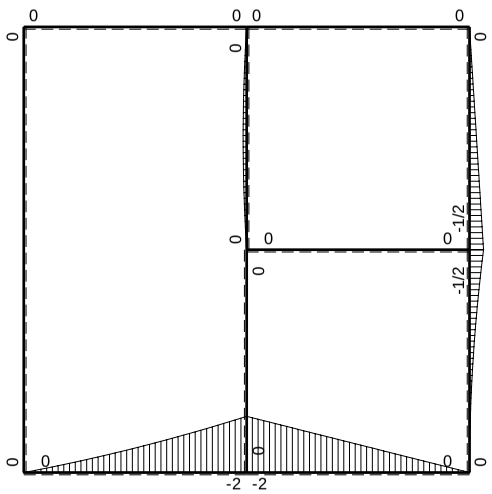

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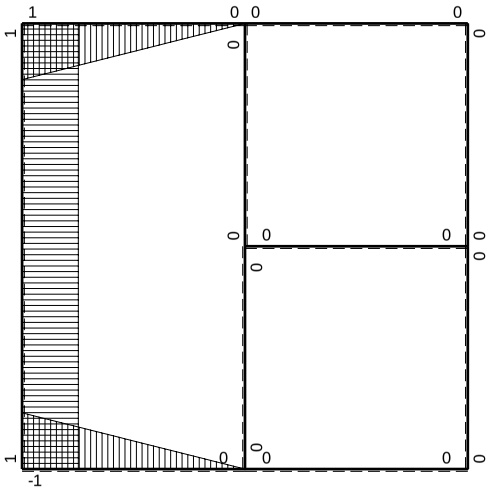

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-2


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240378 ESΣ03.xxxx.058

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	-Fb/EJ	0	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$	$(0-1/2)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	0	Fb/EJ	0	-Fx/EJ	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 240378 ESΣ03.xxxx.058
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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + x/b)\ \theta\ dx = [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

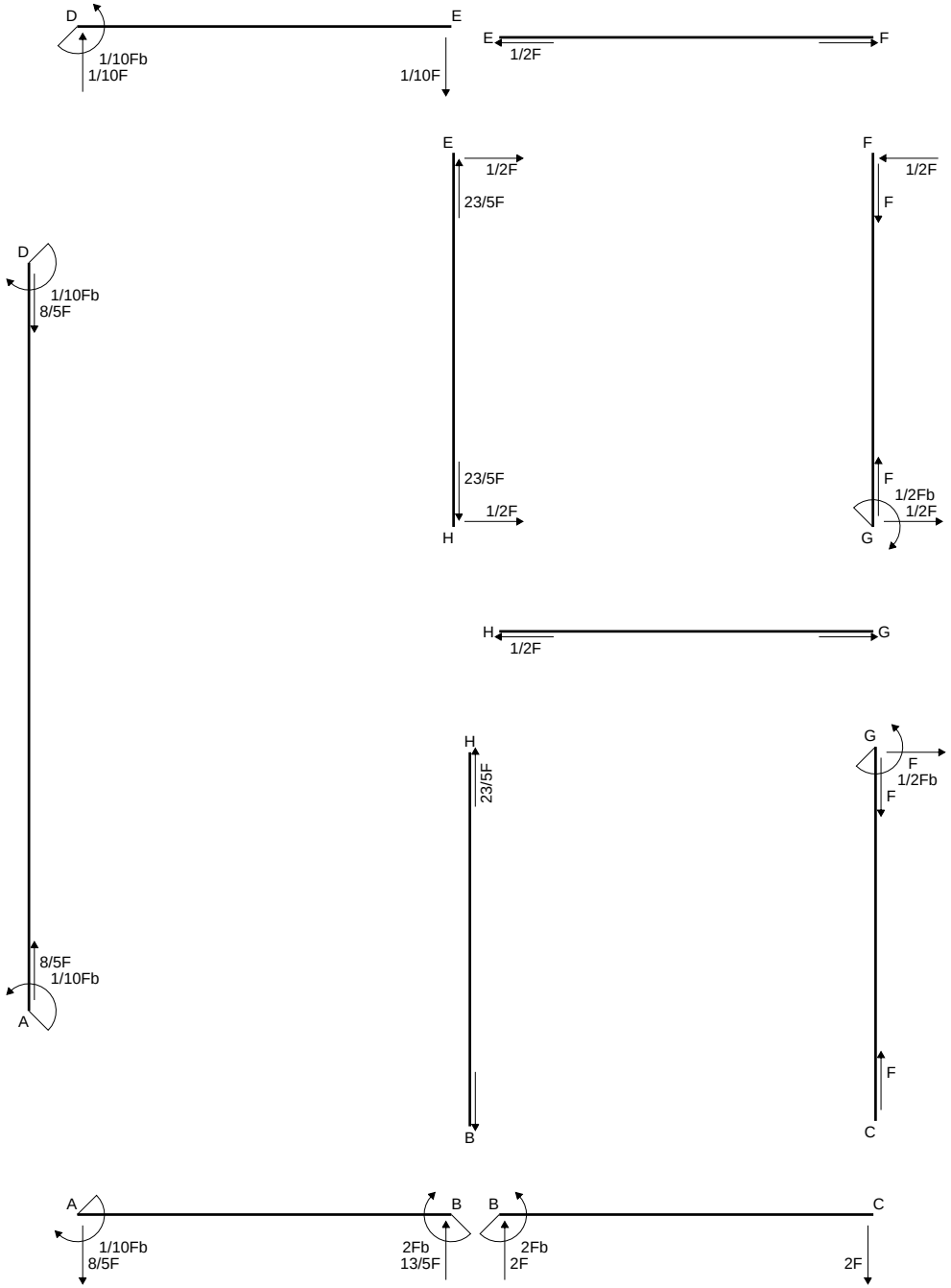
$= (-\ b + 1/2\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

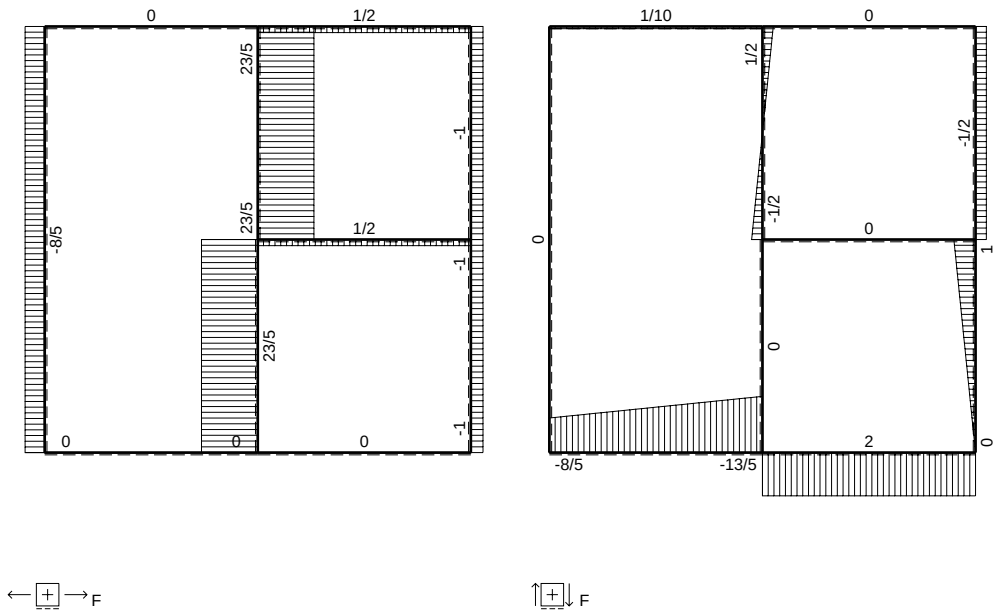
$L^{Xo}_{ED} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

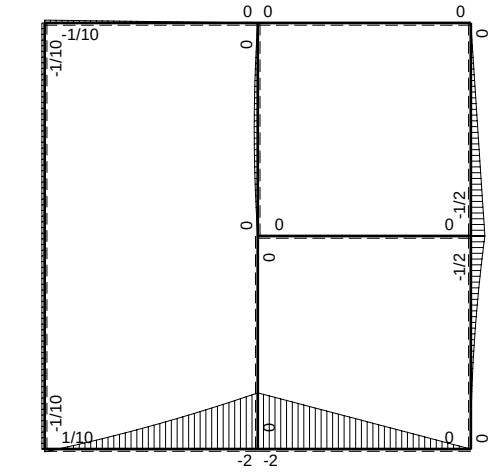
$= (1/2\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

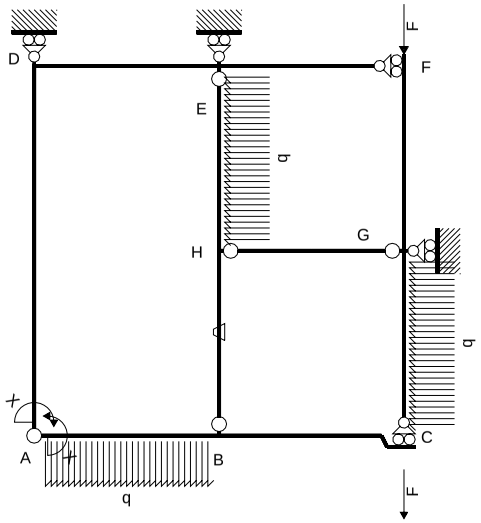
ESΣ03.xxxx.058

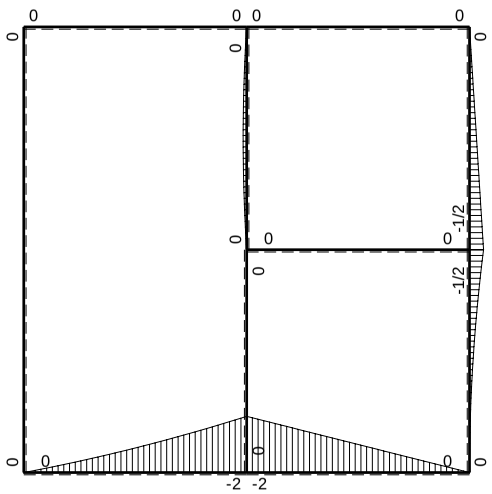
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D
E
1/10Fb
1/10F
1/10F
E
F
1/2F
E
1/2F
23/5F
23/5F
1/2F
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2F

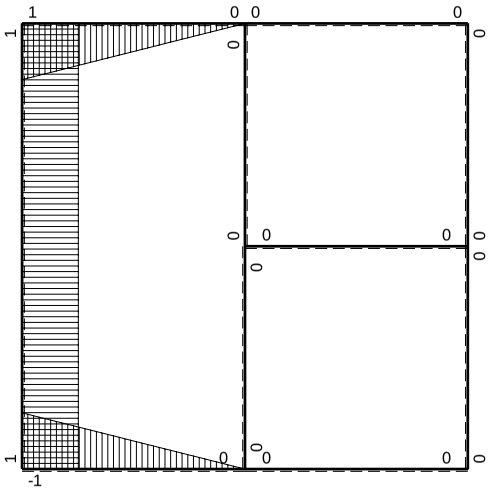

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1
0
0
2
0
-8/5
-13/5
F
F


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0
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-1/10
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0
-1/2
0
0
-1/2
-1/10
-1/10
0
0
0
-2 -2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240138 ESΣ03.xxxx.059

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PROCEDIMENTO E RISULTATI 240138 ESΣ03.xxxx.059

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	0	Fb/EJ	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,3/4\,b - 1/3\,b - 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

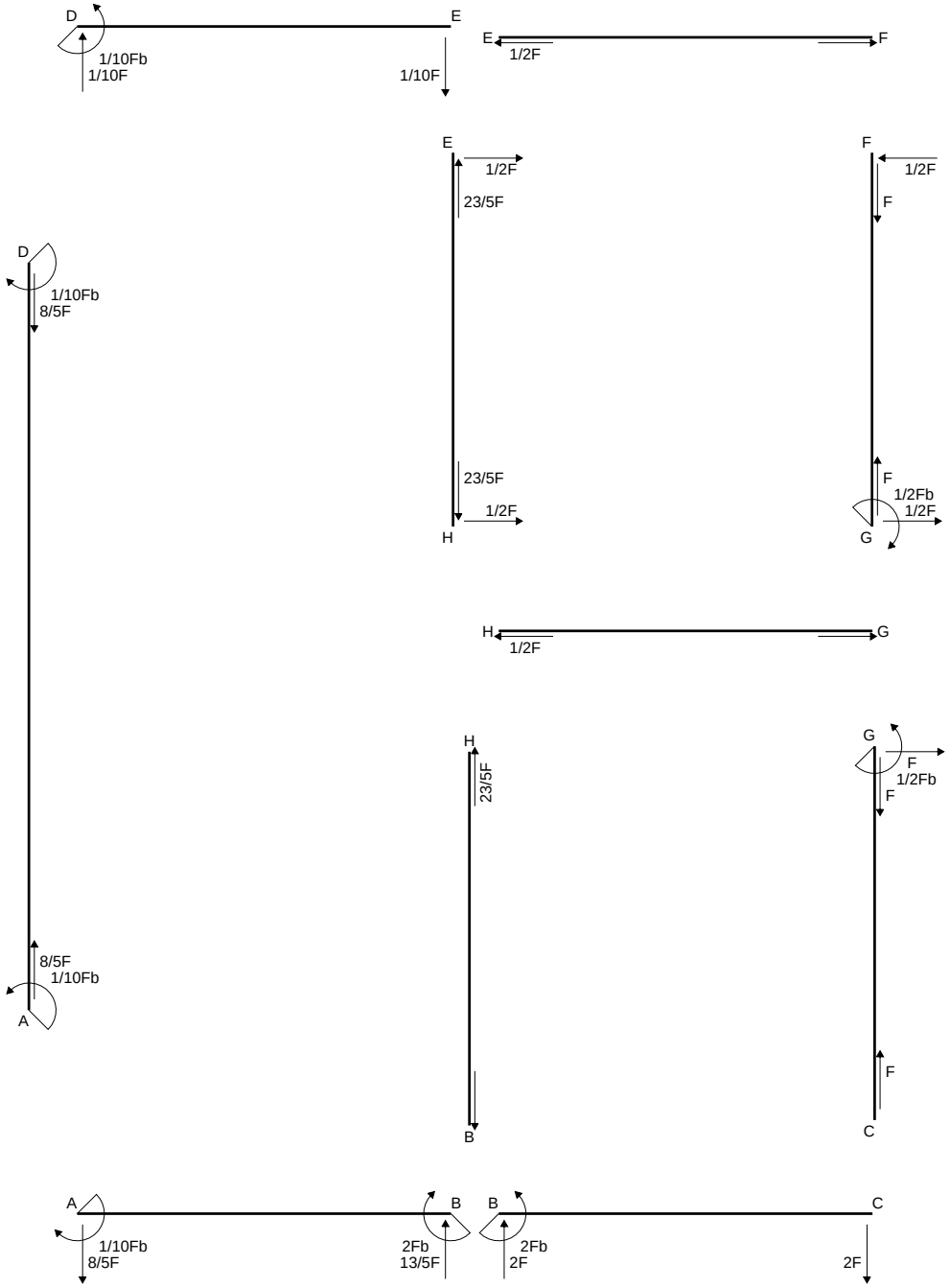
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

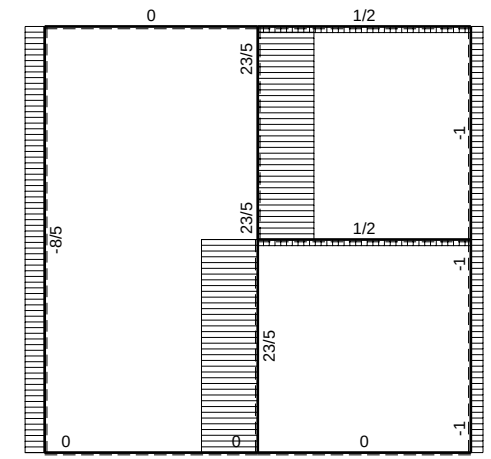
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

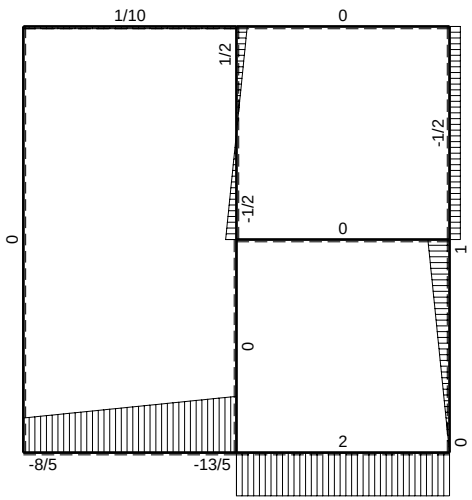
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

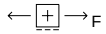
ESΣ03.xxxx.059

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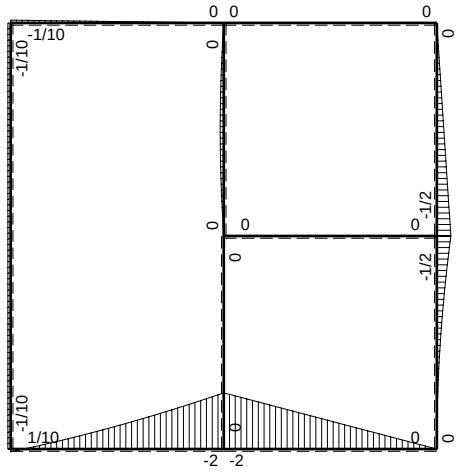

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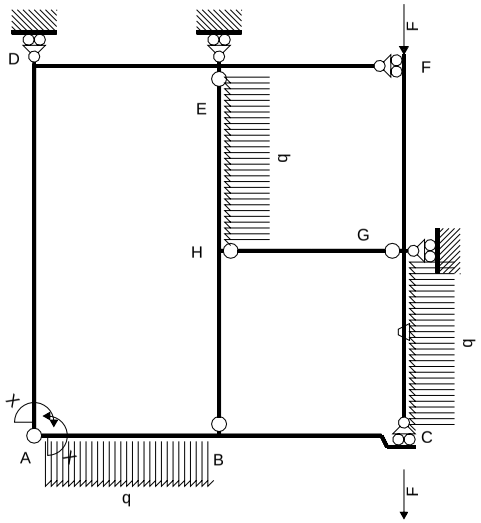

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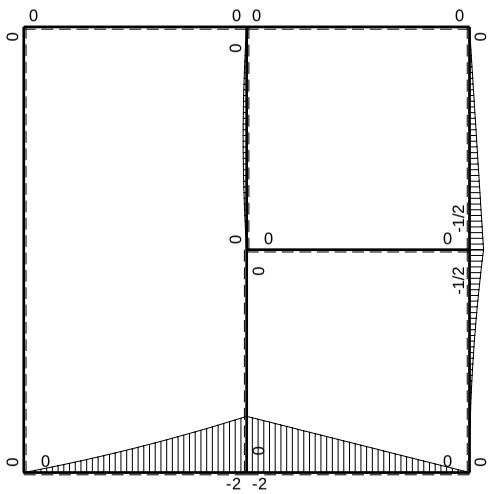

F


F

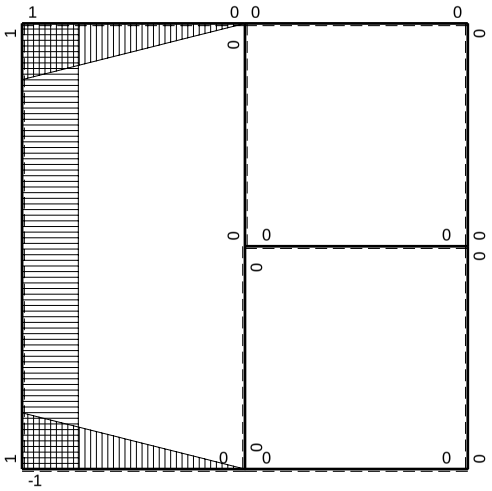

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0
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-1/10
-1/10
0
0
0
-2 -2


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237067 ESΣ03.xxxx.060

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PROCEDIMENTO E RISULTATI 237067 ESΣ03.xxxx.060

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

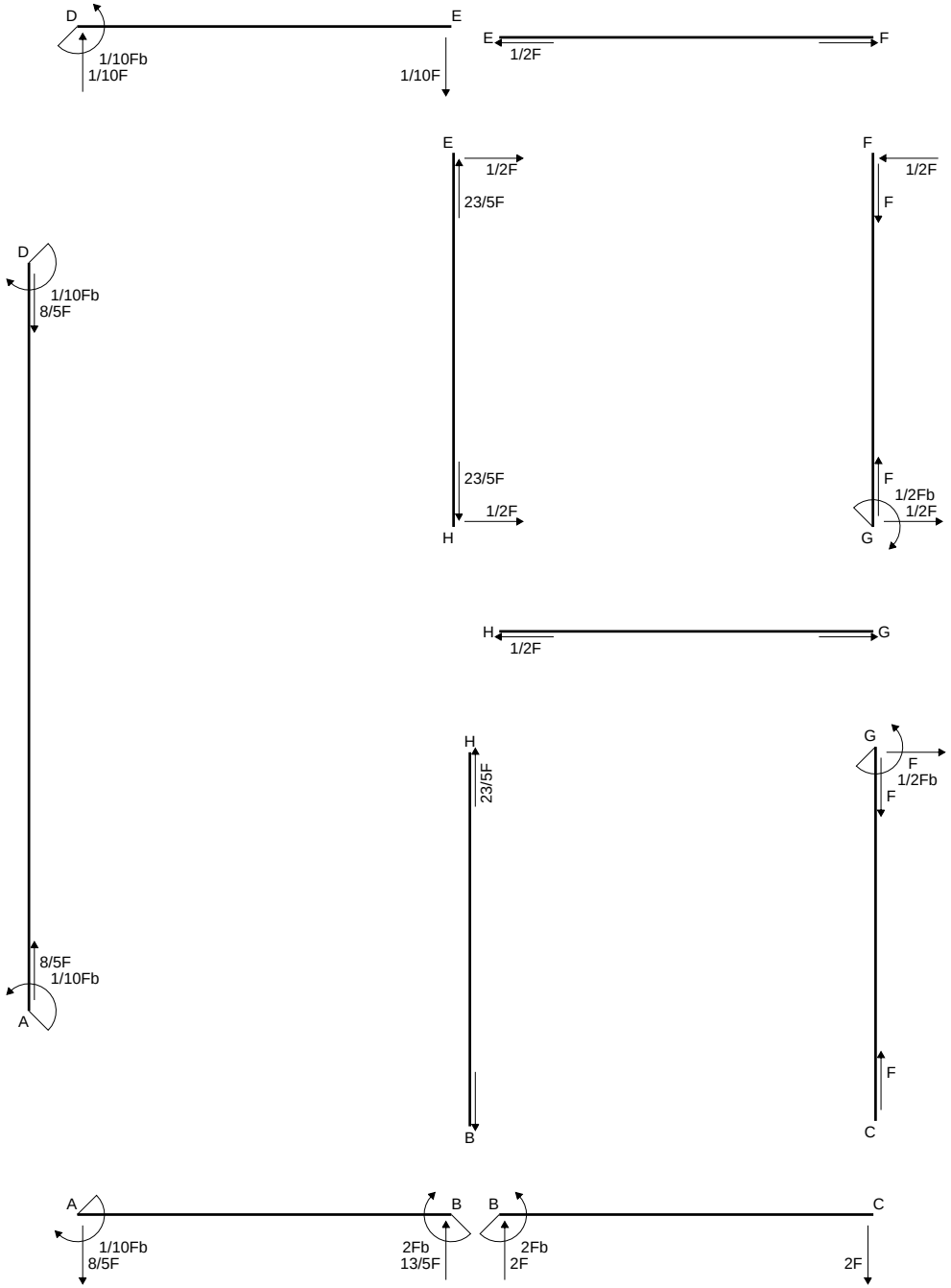
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

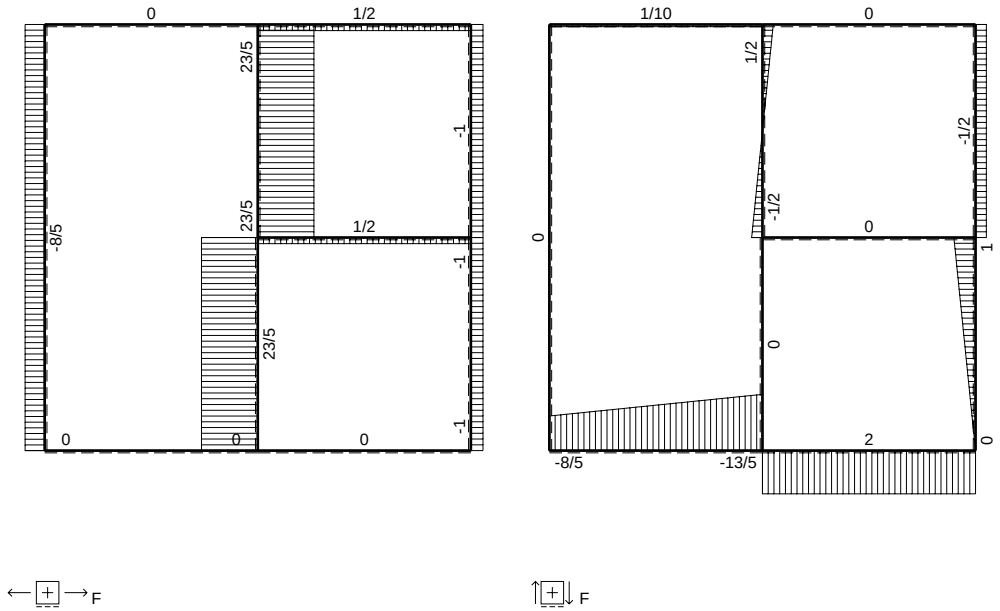
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

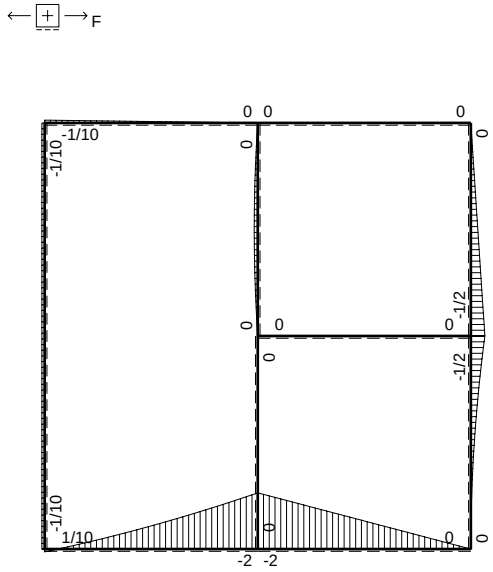
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

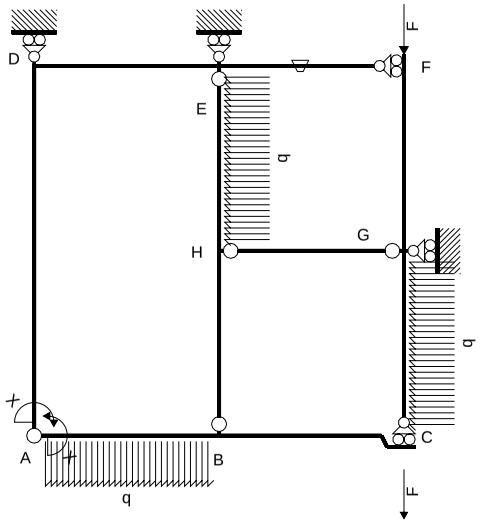
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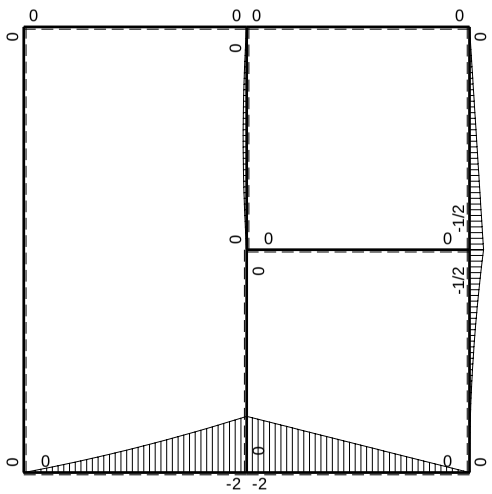
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D
E
1/10Fb
1/10F
1/10F
E
F
1/2F
E
1/2F
23/5F
23/5F
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H
F
1/2F
F
1/2Fb
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G
D
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C
2F

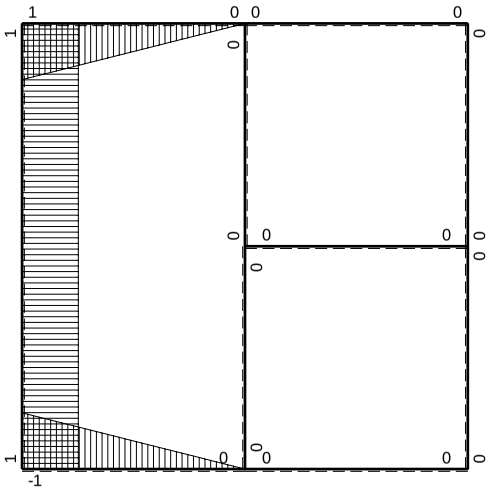

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-1/2
-1/2
0
0
1
0
0
2
0
-8/5
-13/5


0 0
0
-1/10
-1/10
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0
0
0
-1/2
0
0
-1/2
-1/10
-1/10
0
0
0
-2 -2





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

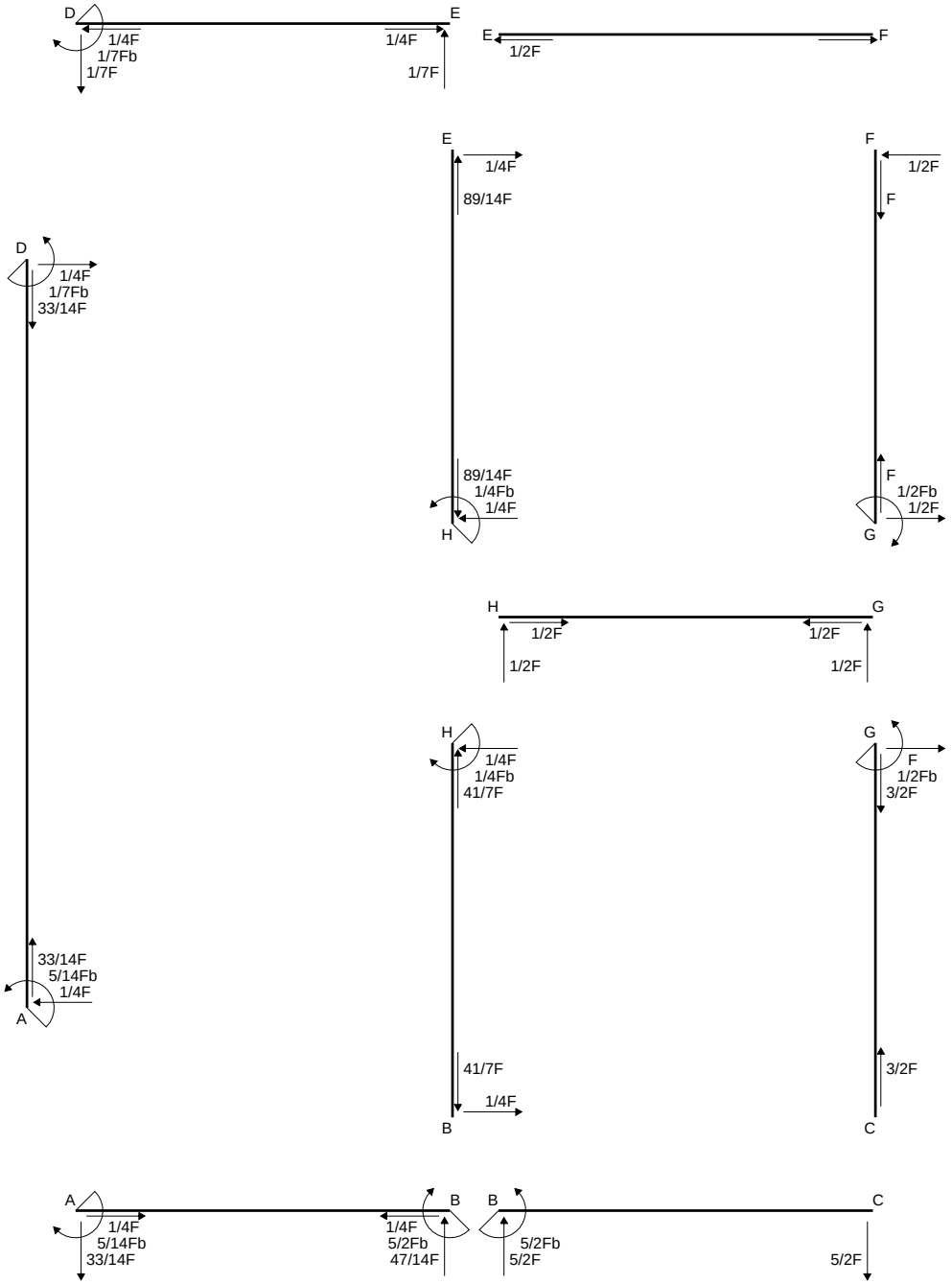
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

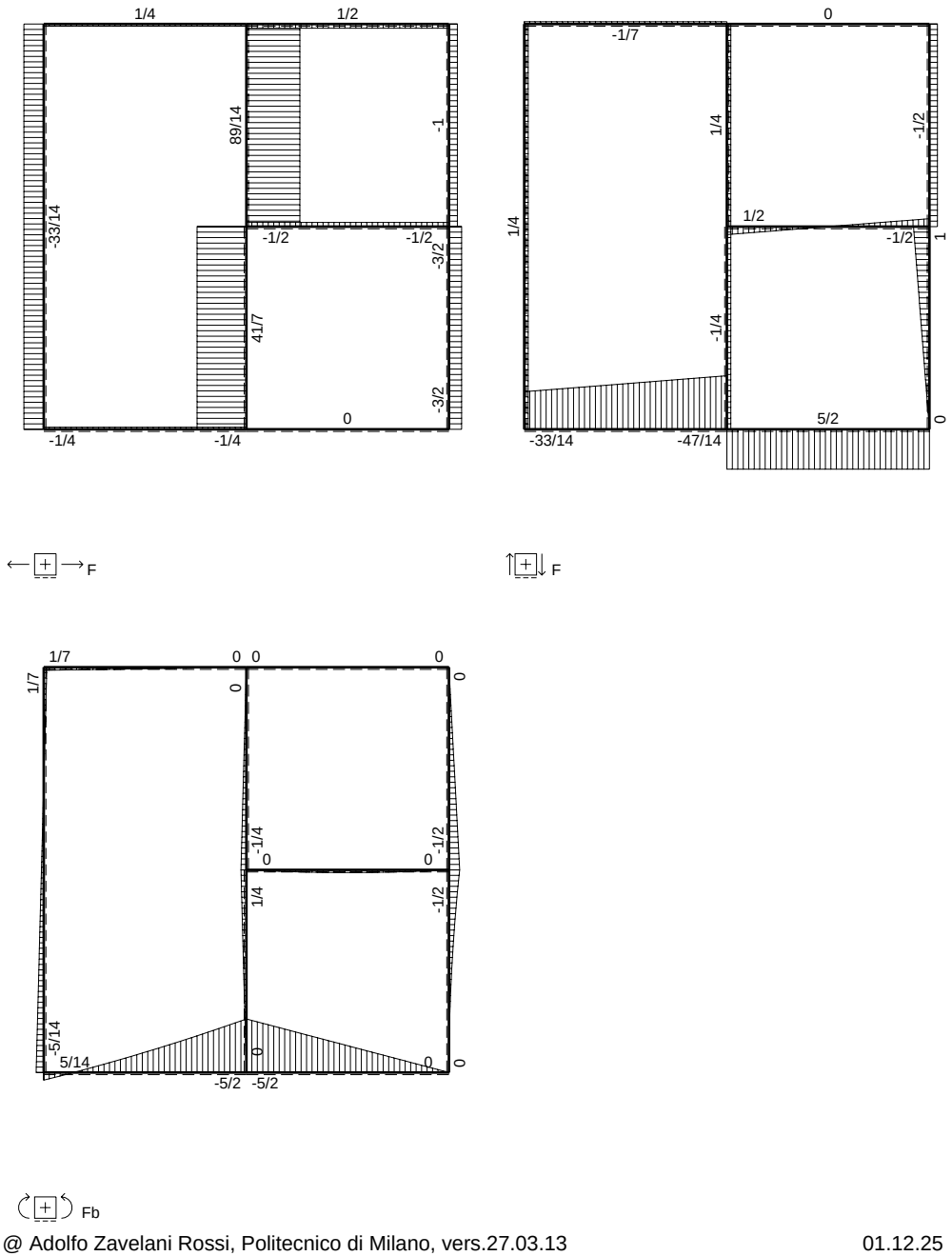
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

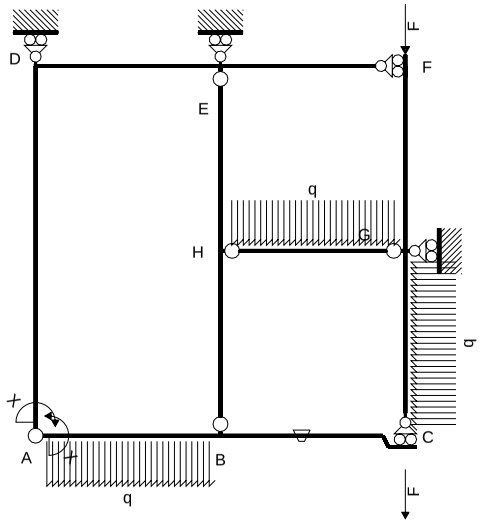
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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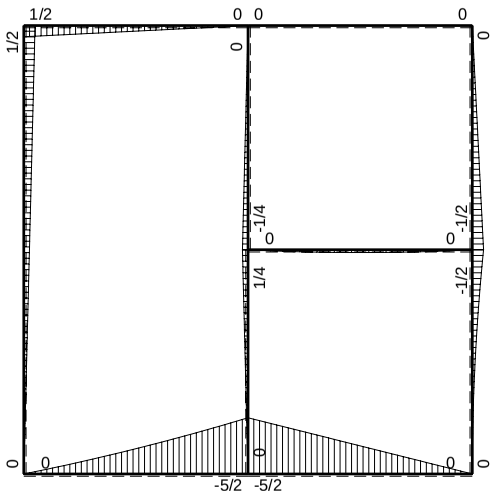
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
E
F
1/2F
E
1/4F
89/14F
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G
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5/14Fb
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A
A
1/4F
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B
B
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5/2Fb
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5/2F

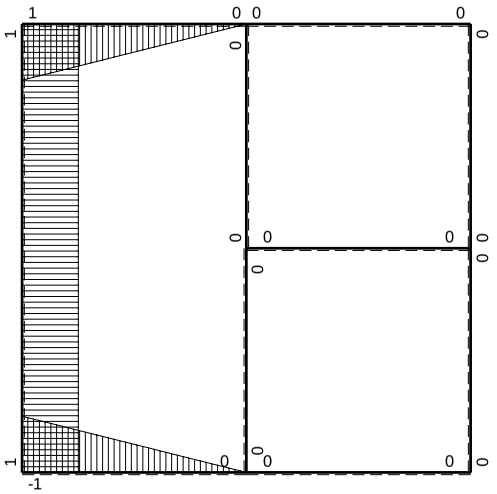

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-3/2
41/7
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-3/2
-1/4
-1/4
F
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1/2
-1/2
1/4
1
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5/2
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-47/14
F
1/7
0 0
0
1/7
0
0
-1/4
0
0
-1/2
1/4
-1/2
-5/14
5/14
0
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	-Fb/EJ	0	0	0	0+0	0
CB b	0	5/2Fx	Fb/EJ	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	$1-x/b$	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
EH b	0	1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

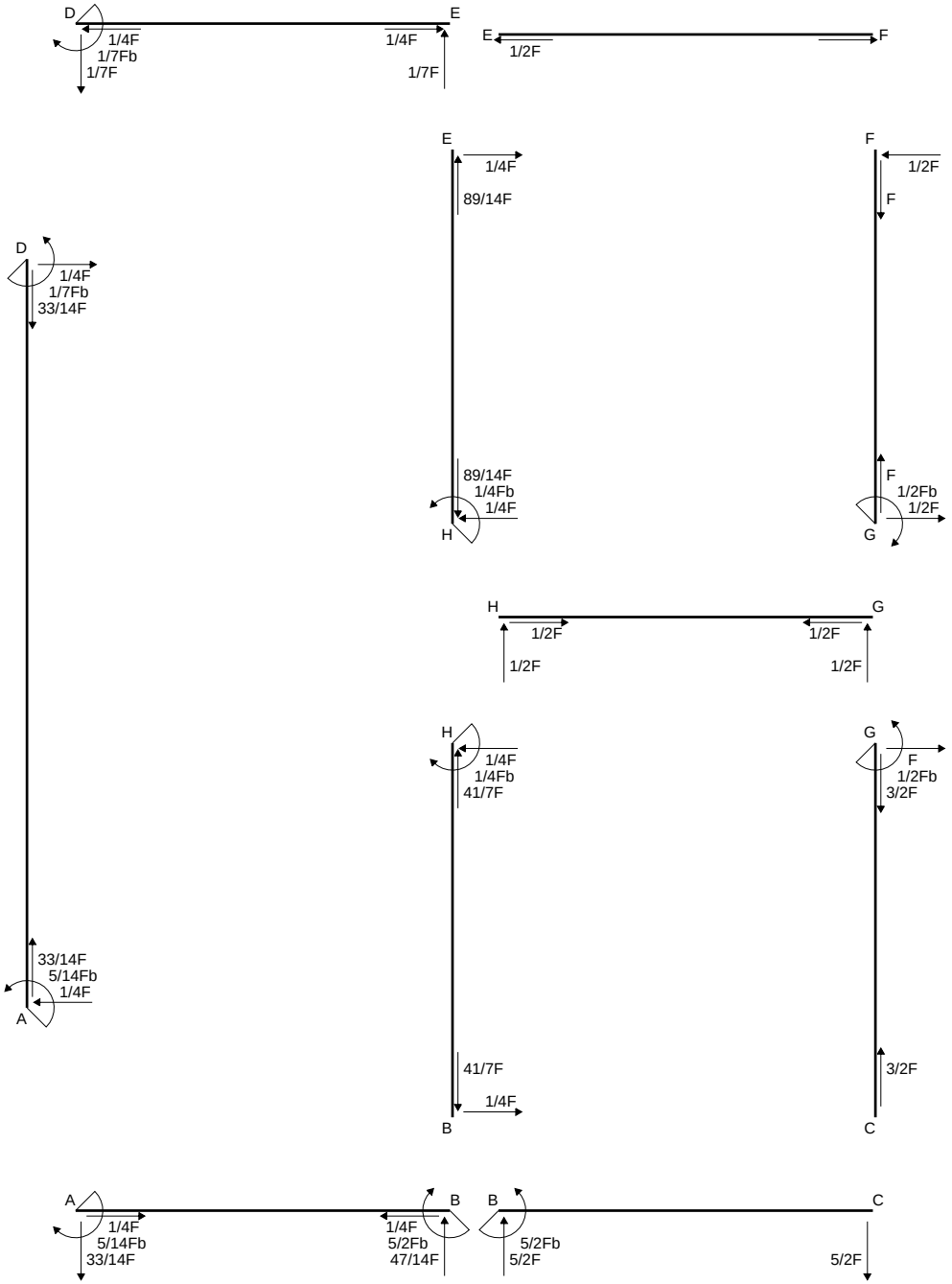
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

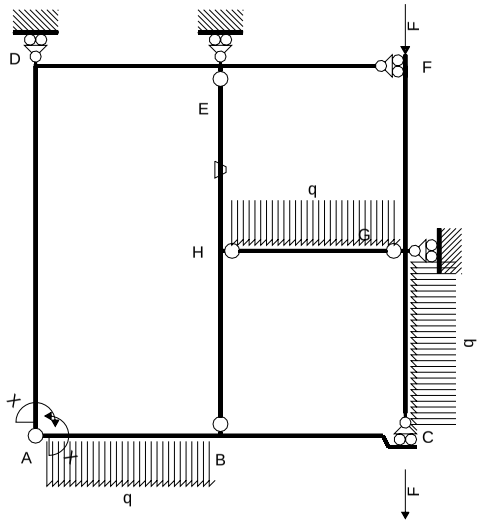
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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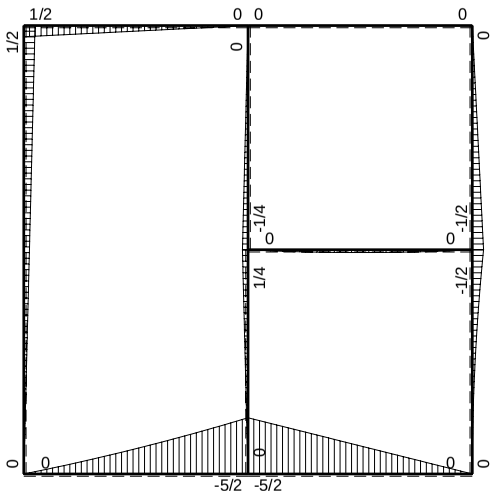
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D
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1/7Fb
1/7F
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1/7F
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1/4Fb
41/7F
41/7F
1/4F
B
G
F
1/2Fb
3/2F
3/2F
C
33/14F
5/14Fb
1/4F
A
A
1/4F
5/14Fb
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B
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1/4F
5/2Fb
47/14F
5/2Fb
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C
5/2F

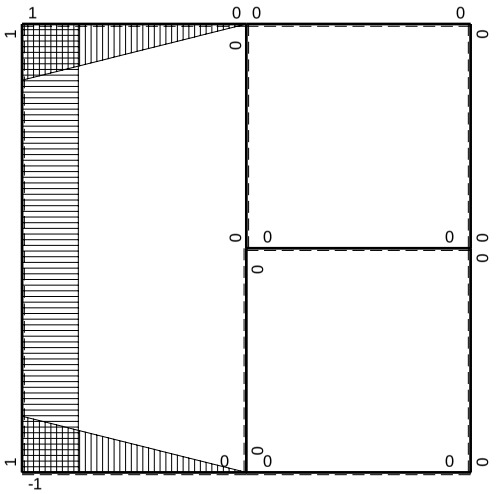

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-5/14
5/14
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0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$1/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

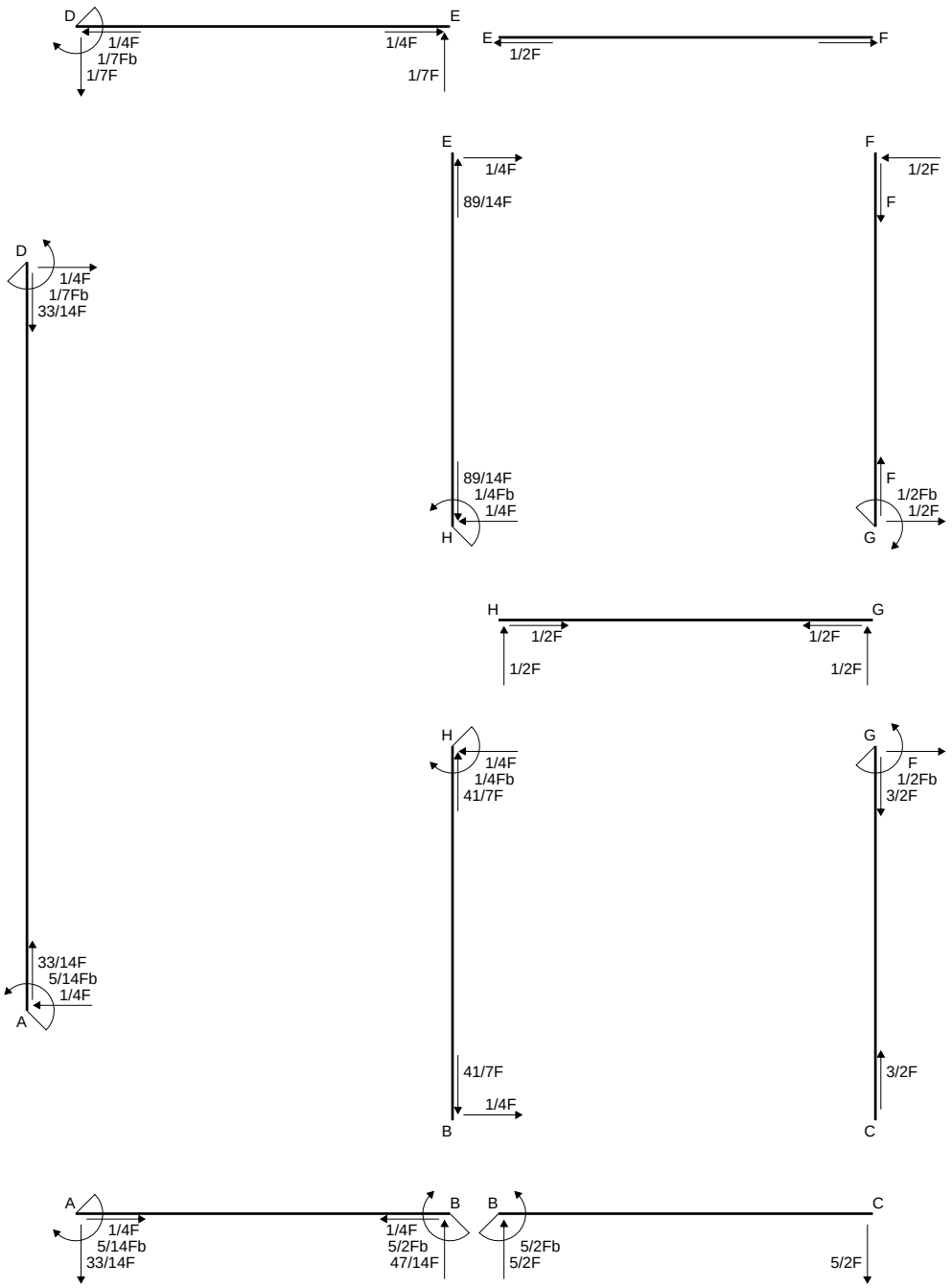
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

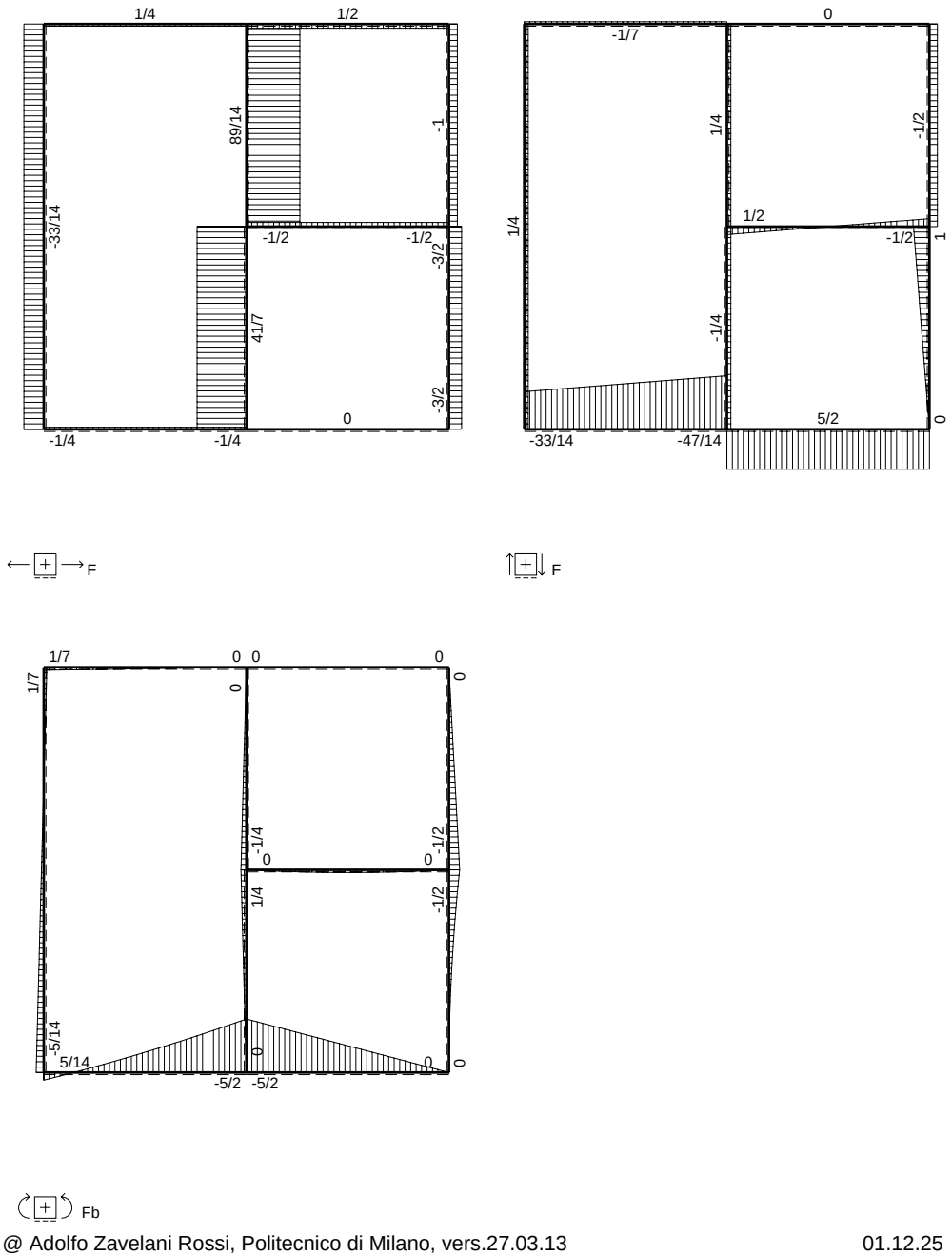
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

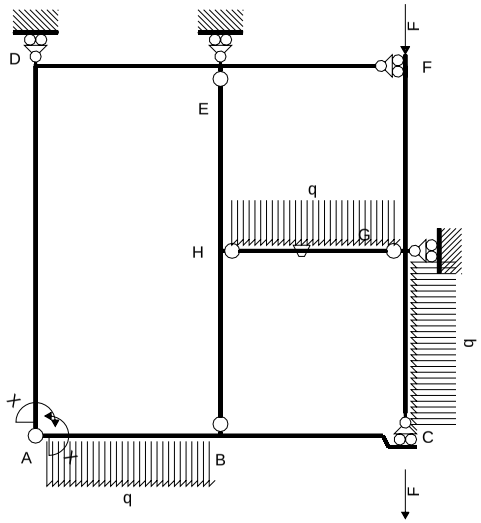
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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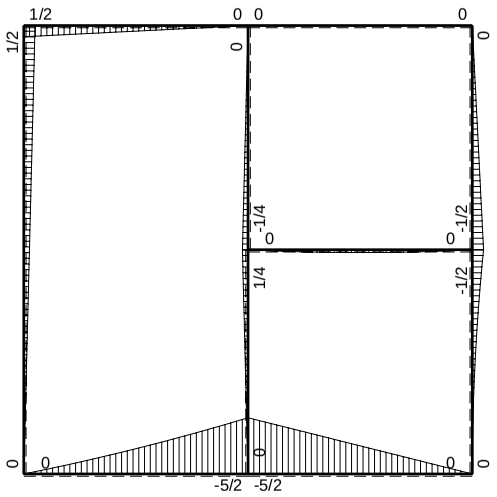
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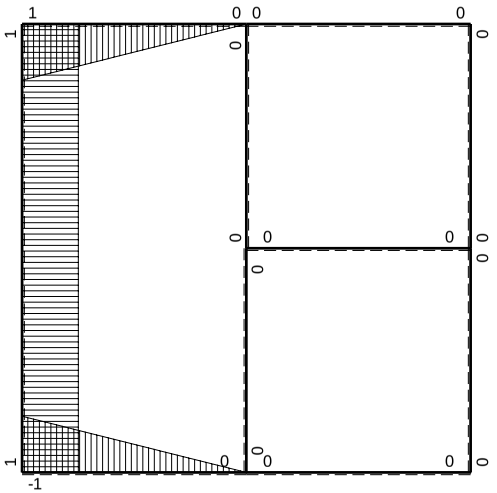

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-1/2
-5/14
5/14
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0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

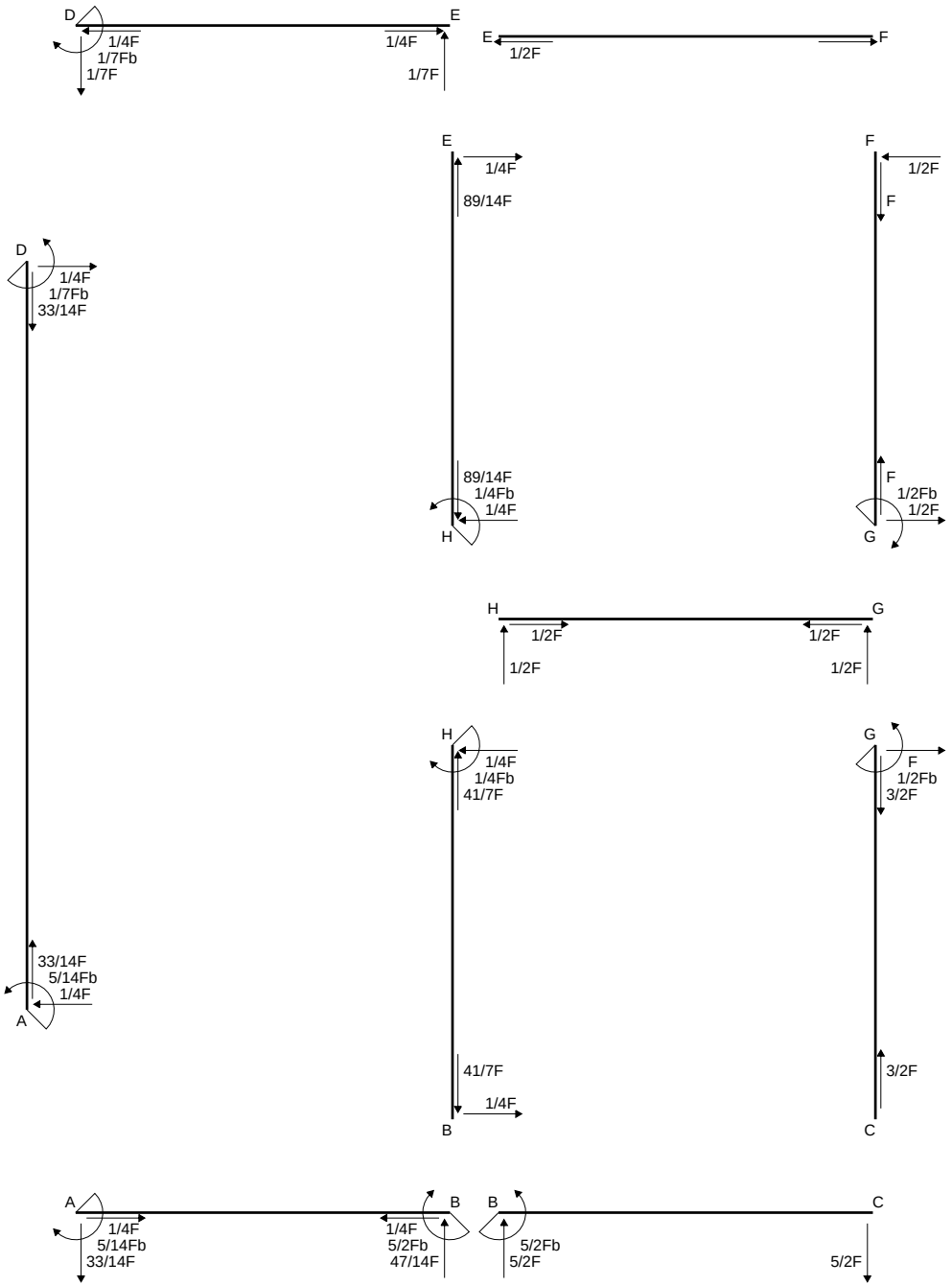
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

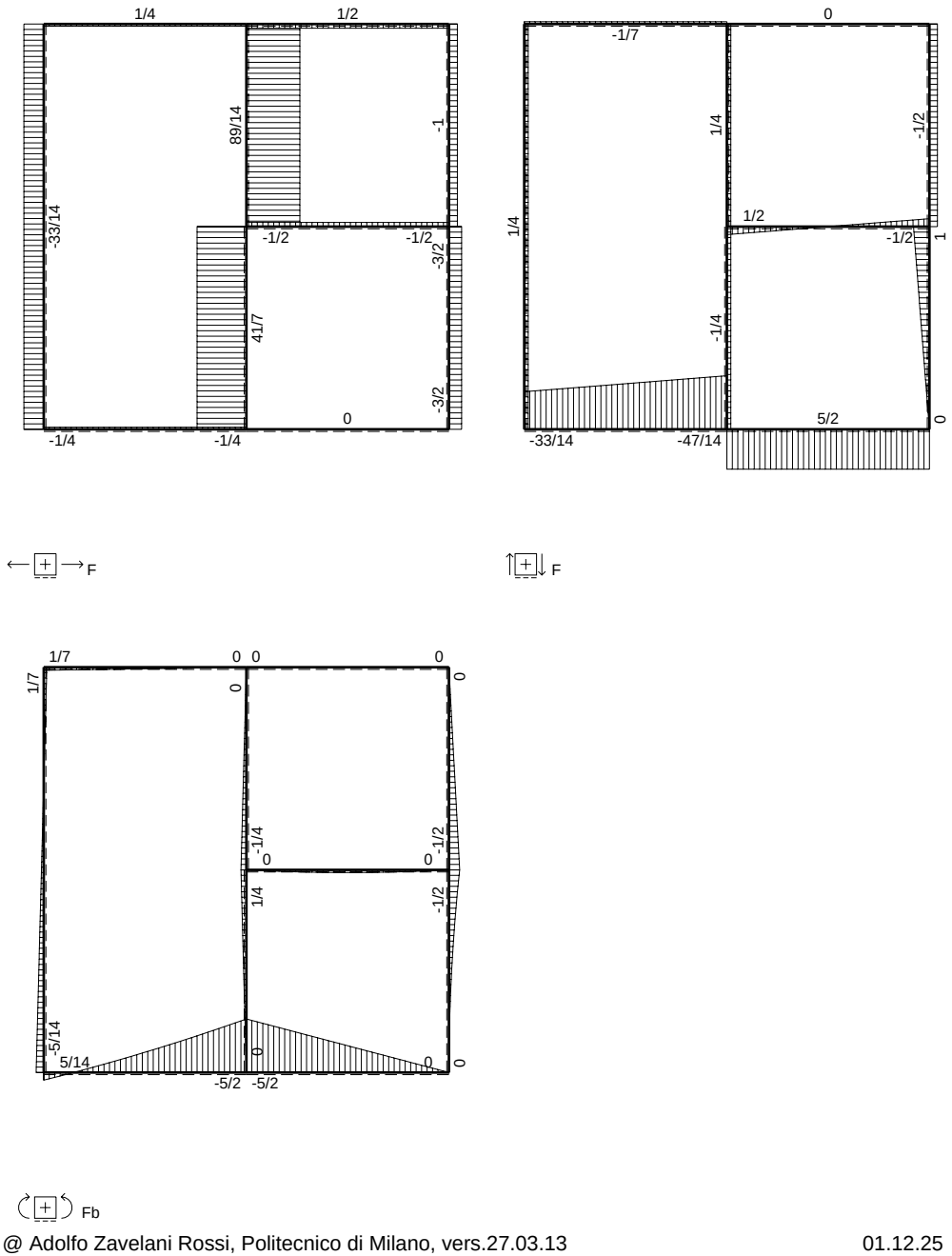
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

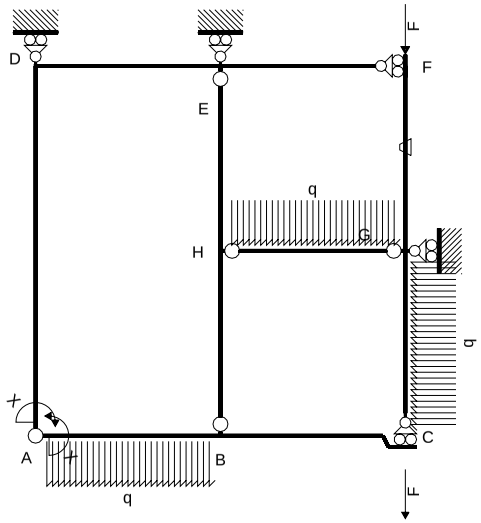
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 982517 ESΣ03.xxxx.064

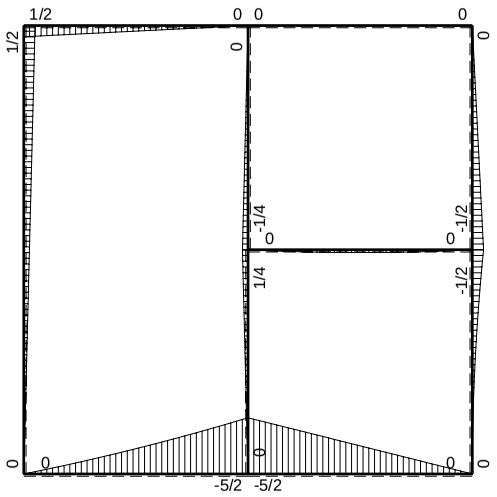
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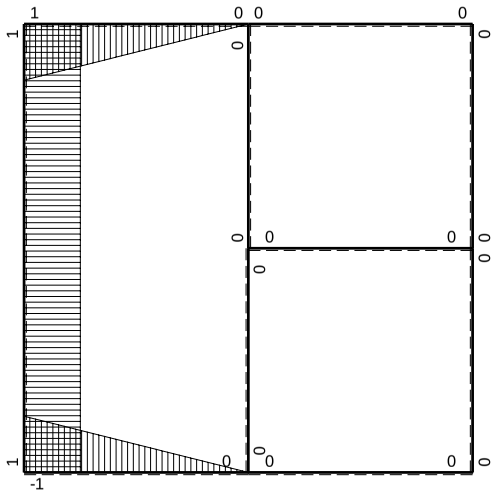

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0 0
0
1/7
0
0
-1/4
0
0
-1/2
1/4
-1/2
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238072 ESΣ03.xxxx.065

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	Fb/EJ	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

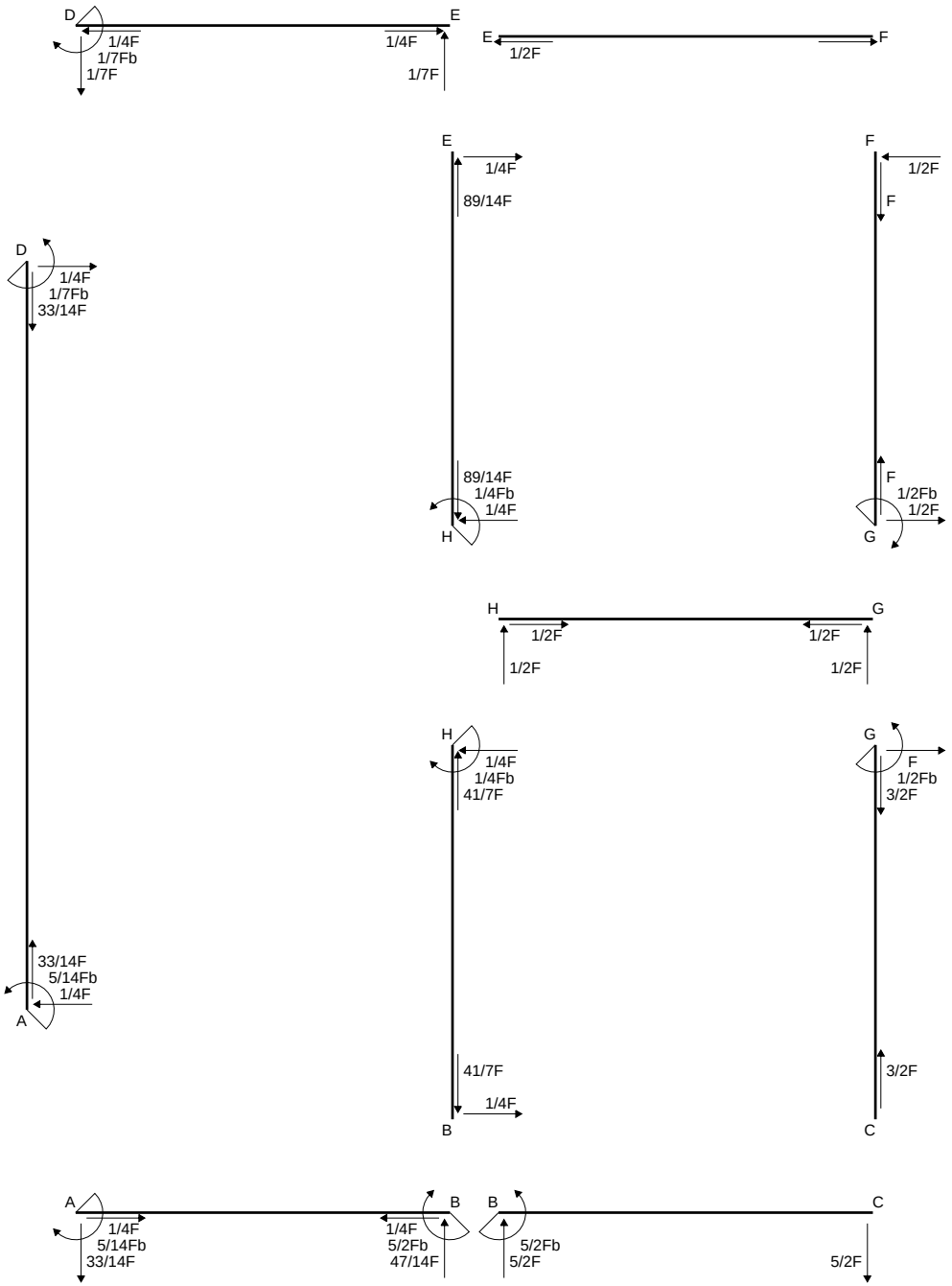
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

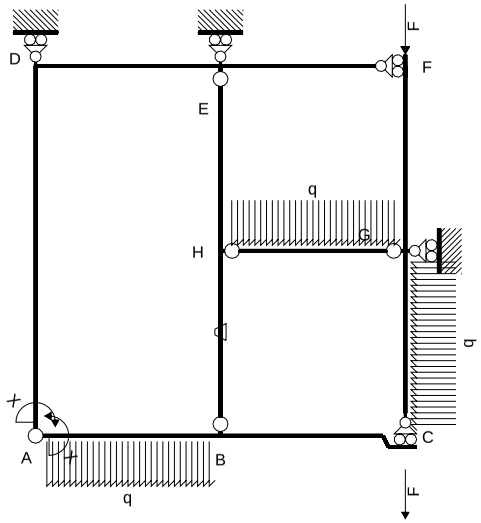
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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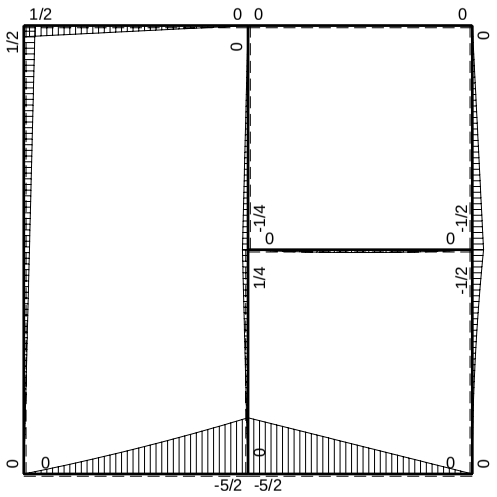
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E
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1/7Fb
1/7F
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1/7F
E
F
1/2F
E
1/4F
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B
G
F
1/2Fb
3/2F
3/2F
C
33/14F
5/14Fb
1/4F
A
A
1/4F
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B
B
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5/2Fb
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5/2Fb
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C
5/2F

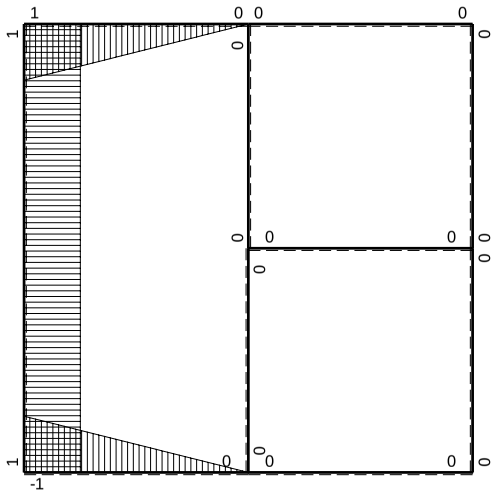

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-1
-33/14
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-1/2
-3/2
41/7
-3/2
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-1/4
-1/4
F
0
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1/4
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1/2
-1/2
1
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-1/4
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1/7
0
0
-1/4
0
0
-1/2
1/4
-1/2
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236438 ESΣ03.xxxx.066

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PROCEDIMENTO E RISULTATI 236438 ESΣ03.xxxx.066

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$-1/4Fx$	Fb/EJ	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

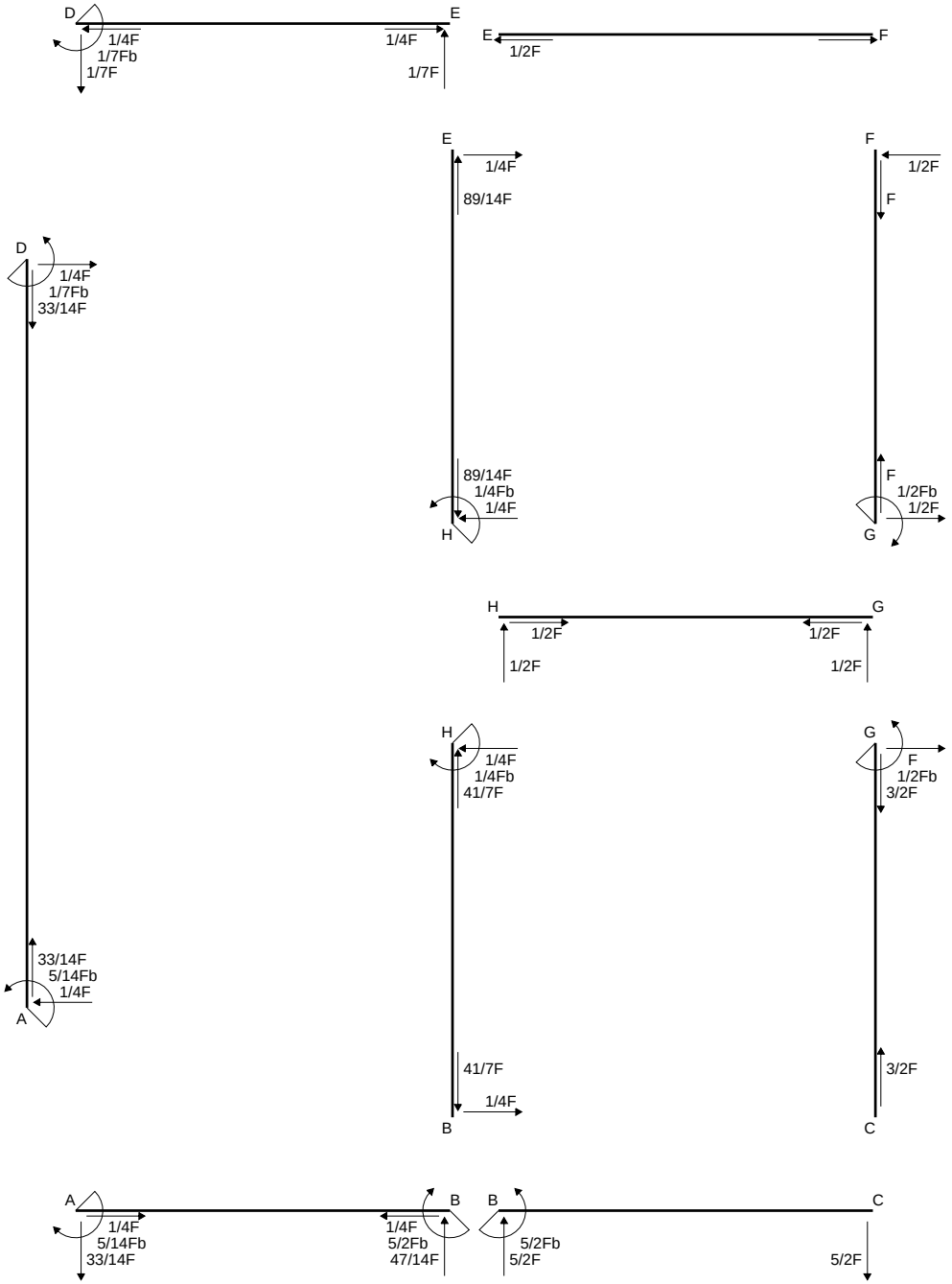
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

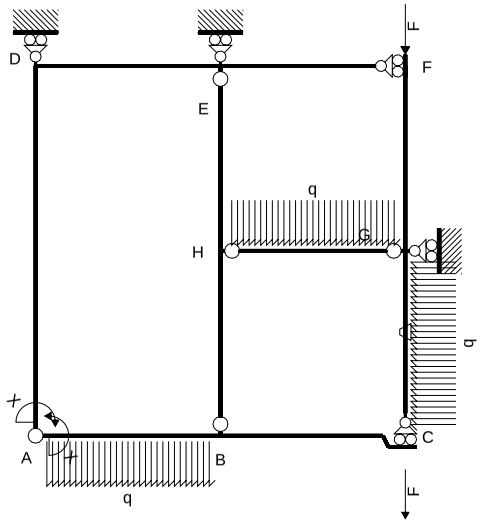
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 236438 ESΣ03.xxxx.066

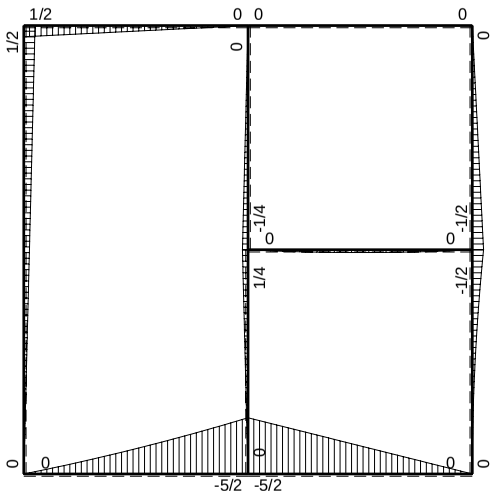
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E
1/4F
1/7Fb
1/7F
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1/7F
E
F
1/2F
E
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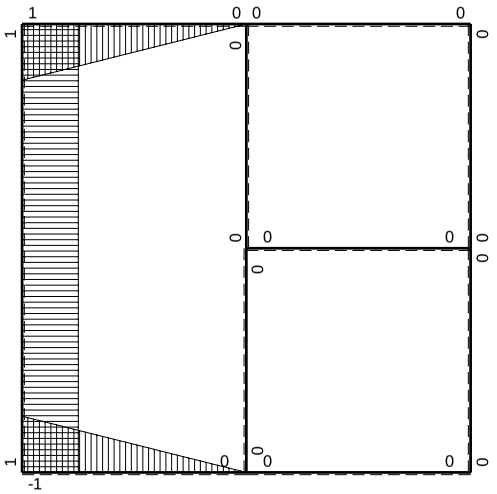

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0
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1/4
-1/2
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 280947 ESΣ03.xxxx.067

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PROCEDIMENTO E RISULTATI 280947 ESΣ03.xxxx.067

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

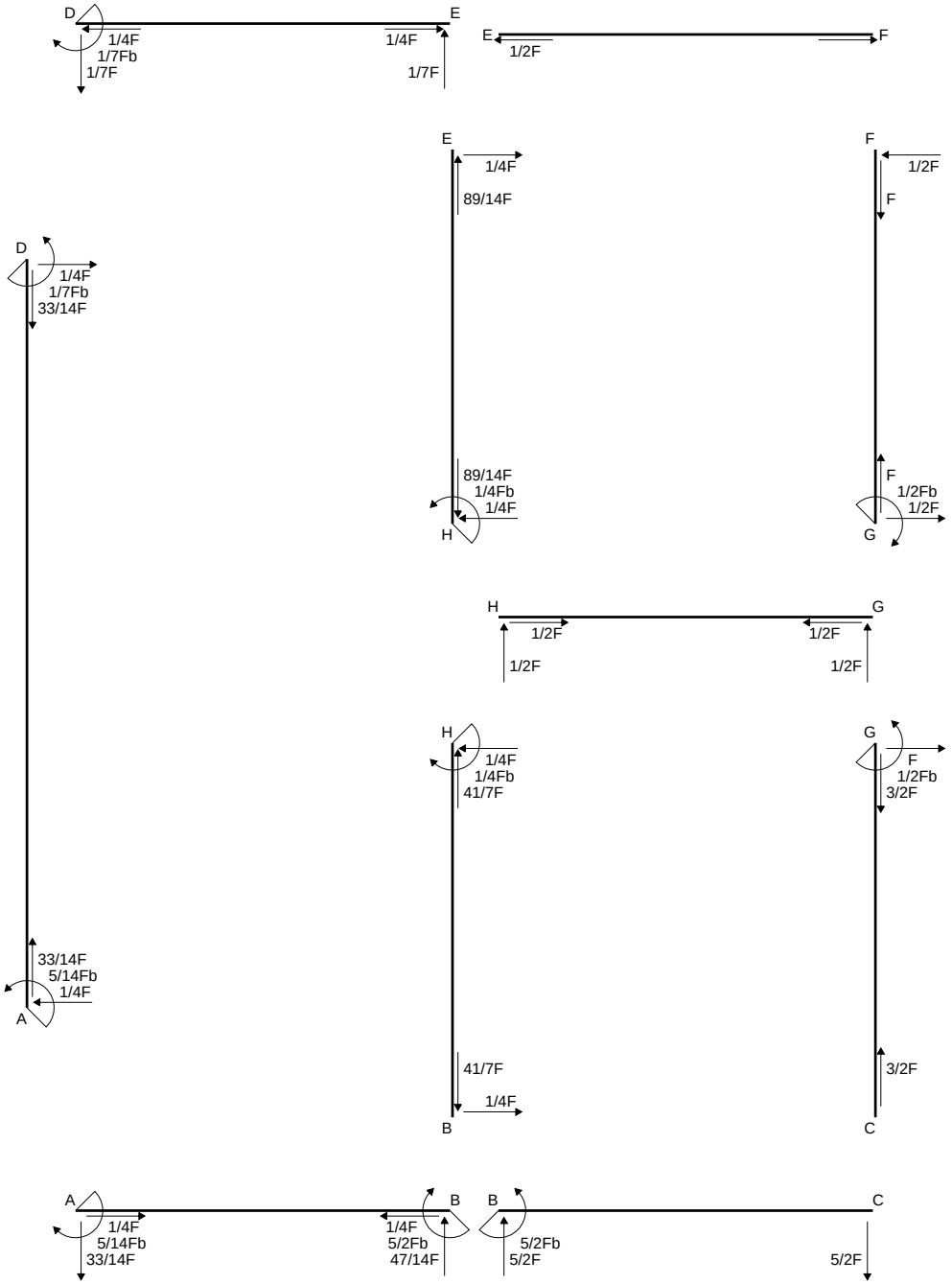
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

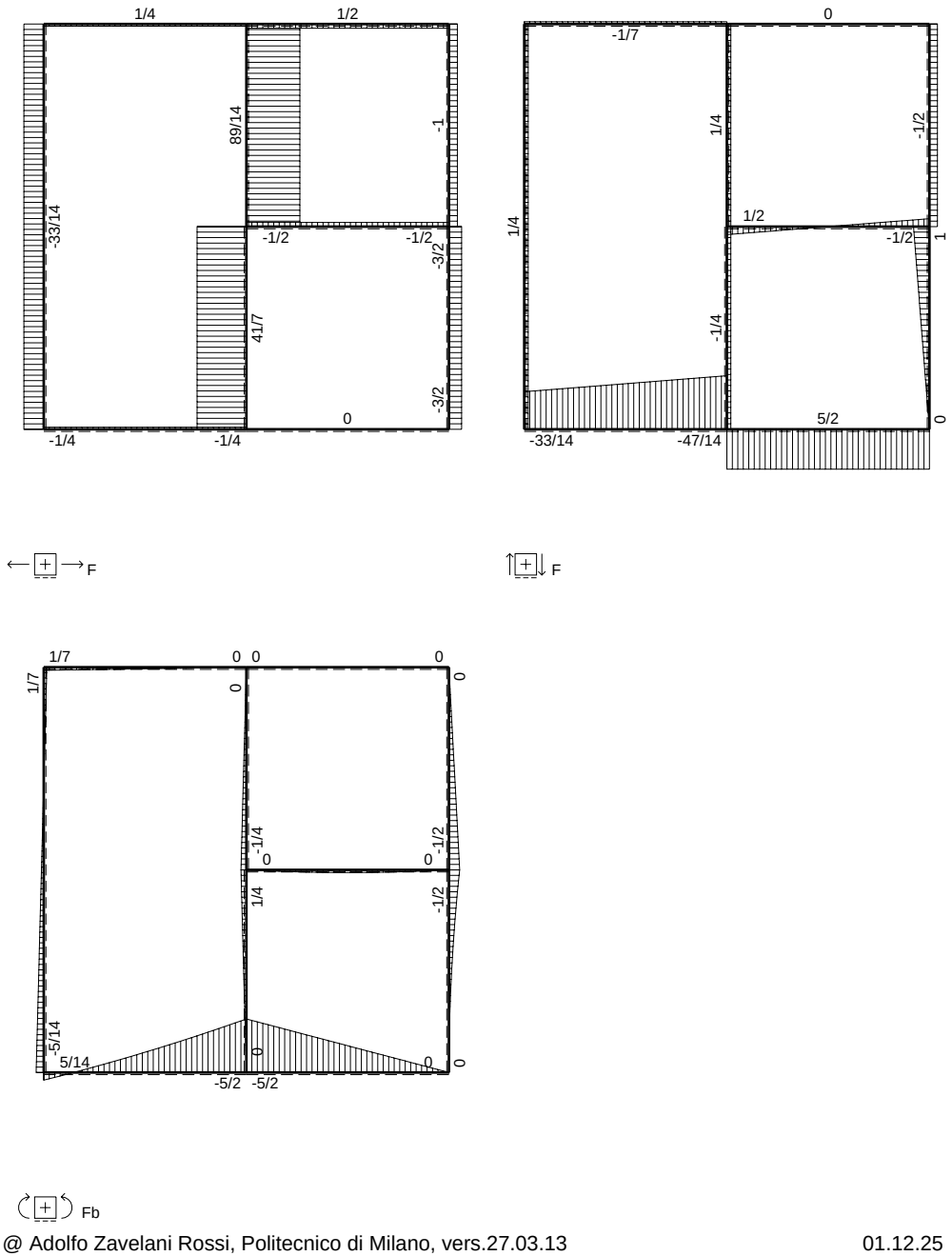
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

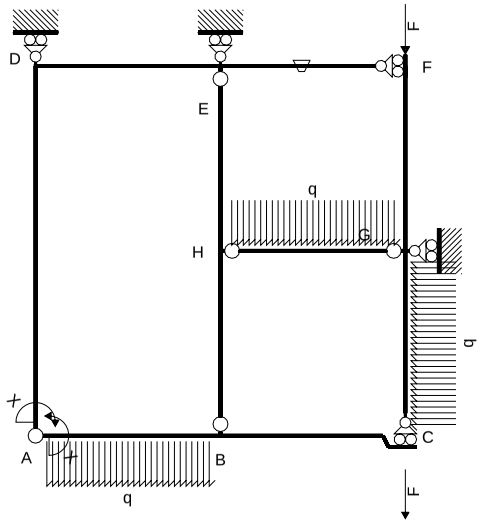
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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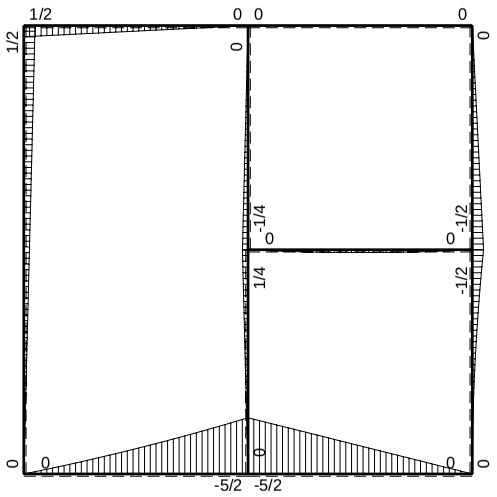
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
E
F
1/2F
E
1/4F
89/14F
89/14F
1/4Fb
1/4F
H
F
1/2F
F
1/2Fb
1/2F
G
D
1/4F
1/7Fb
33/14F
H
G
1/2F
1/2F
1/2F
1/2F
H
1/4F
1/4Fb
41/7F
41/7F
1/4F
B
G
F
1/2Fb
3/2F
3/2F
C
33/14F
5/14Fb
1/4F
A
A
1/4F
5/14Fb
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B
B
1/4F
5/2Fb
47/14F
5/2Fb
5/2F
C
5/2F

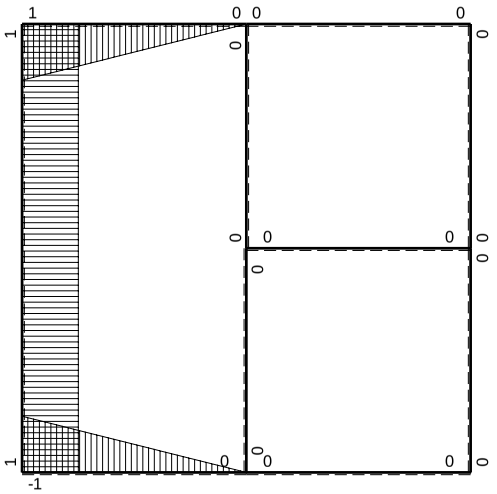

1/4
1/2
89/14
-1
-33/14
-1/2
-1/2
-3/2
41/7
-3/2
0
-1/4
-1/4
F
0
-1/7
1/4
-1/2
1/2
-1/2
1/4
1
-1/4
5/2
0
-33/14
-47/14
F
1/7
0 0
0
1/7
0
0
-1/4
0
0
-1/2
1/4
-1/2
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

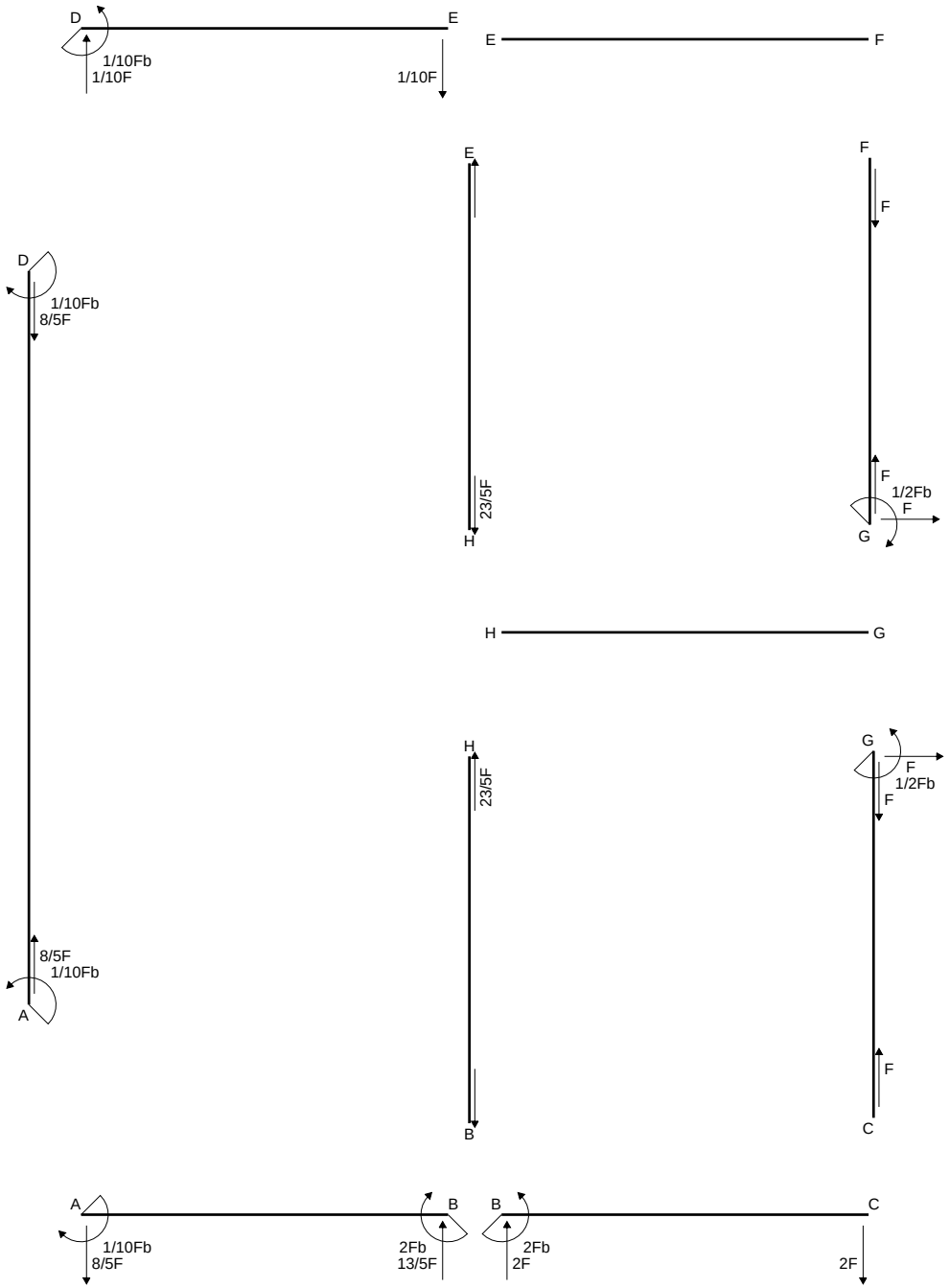
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

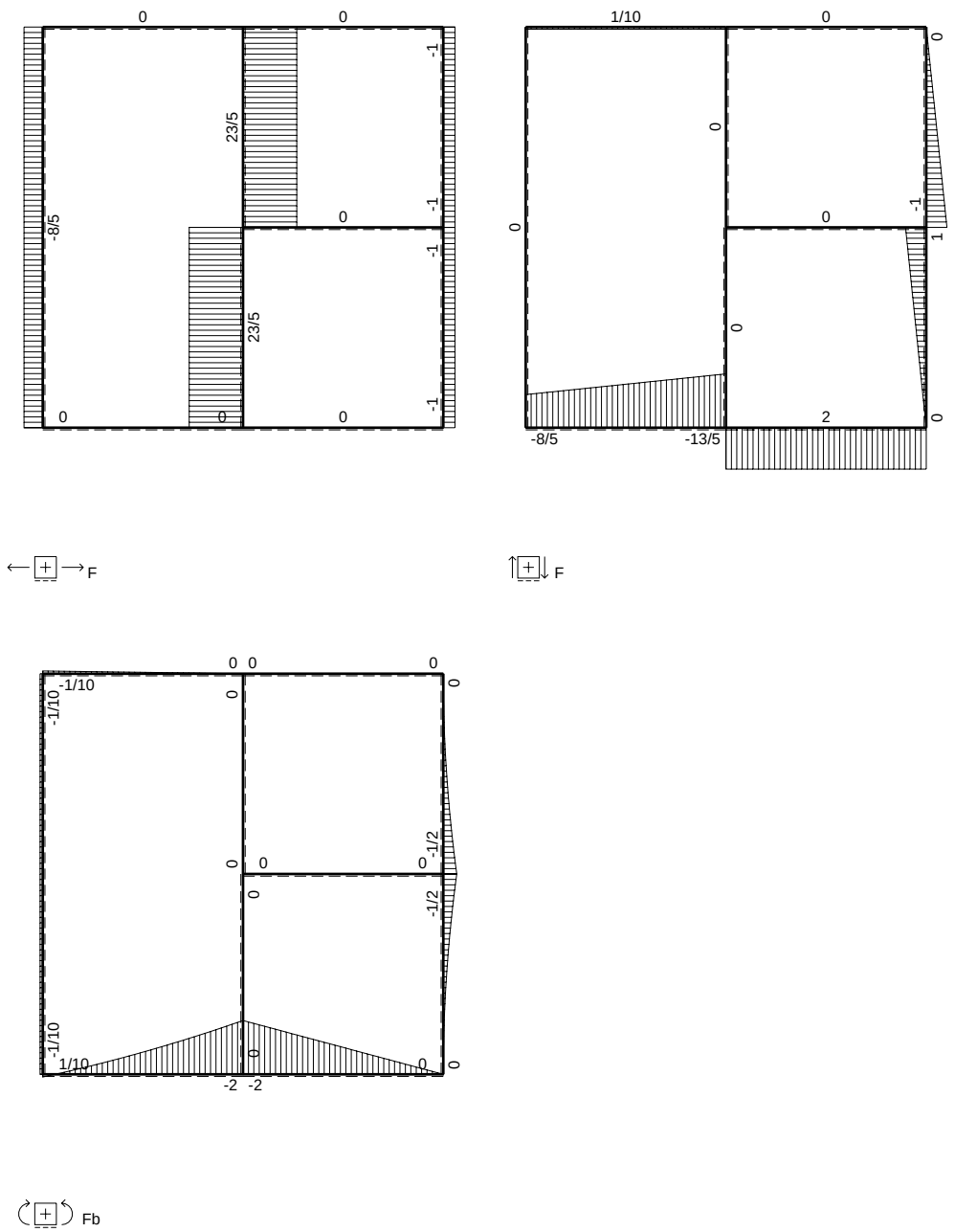
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

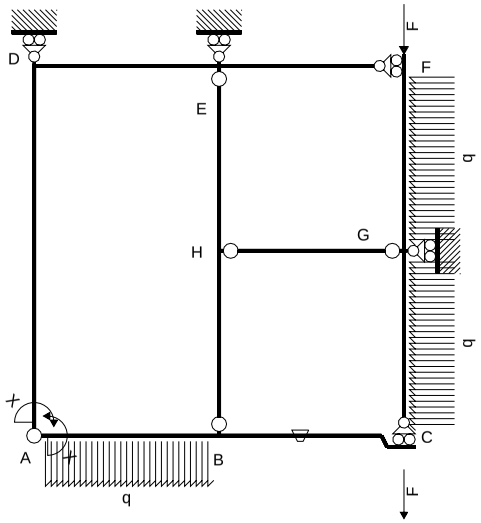
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

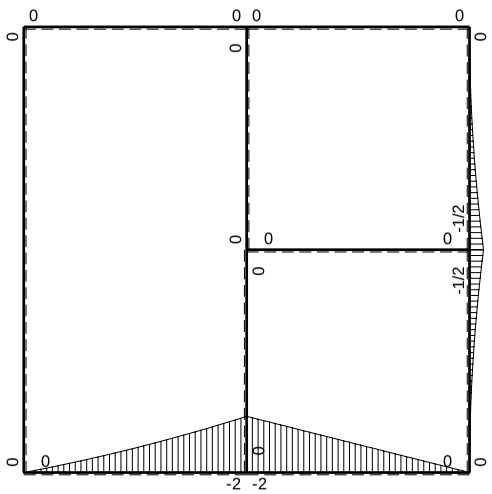
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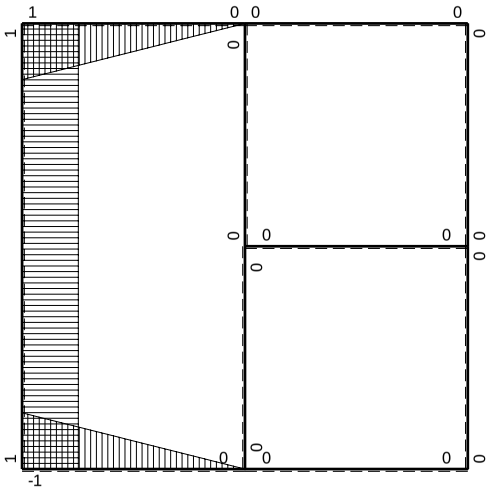

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E
1/10Fb
1/10F
1/10F
E
F
E
F
F
D
1/10Fb
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F
1/2Fb
F
G
H
G
H
G
F
1/2Fb
F
8/5F
1/10Fb
A
F
B
C
A
B
B
C
1/10Fb
8/5F
2Fb
13/5F
2Fb
2F
2F


0
0
-1
23/5
-8/5
0
-1
0
-1
23/5
0
0
0
-1
F
1/10
0
0
0
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-8/5
-13/5
F
0 0
0
-1/10
-1/10
0
0
0
0
-1/2
0
-1/2
-1/10
1/10
0
0
-2 -2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

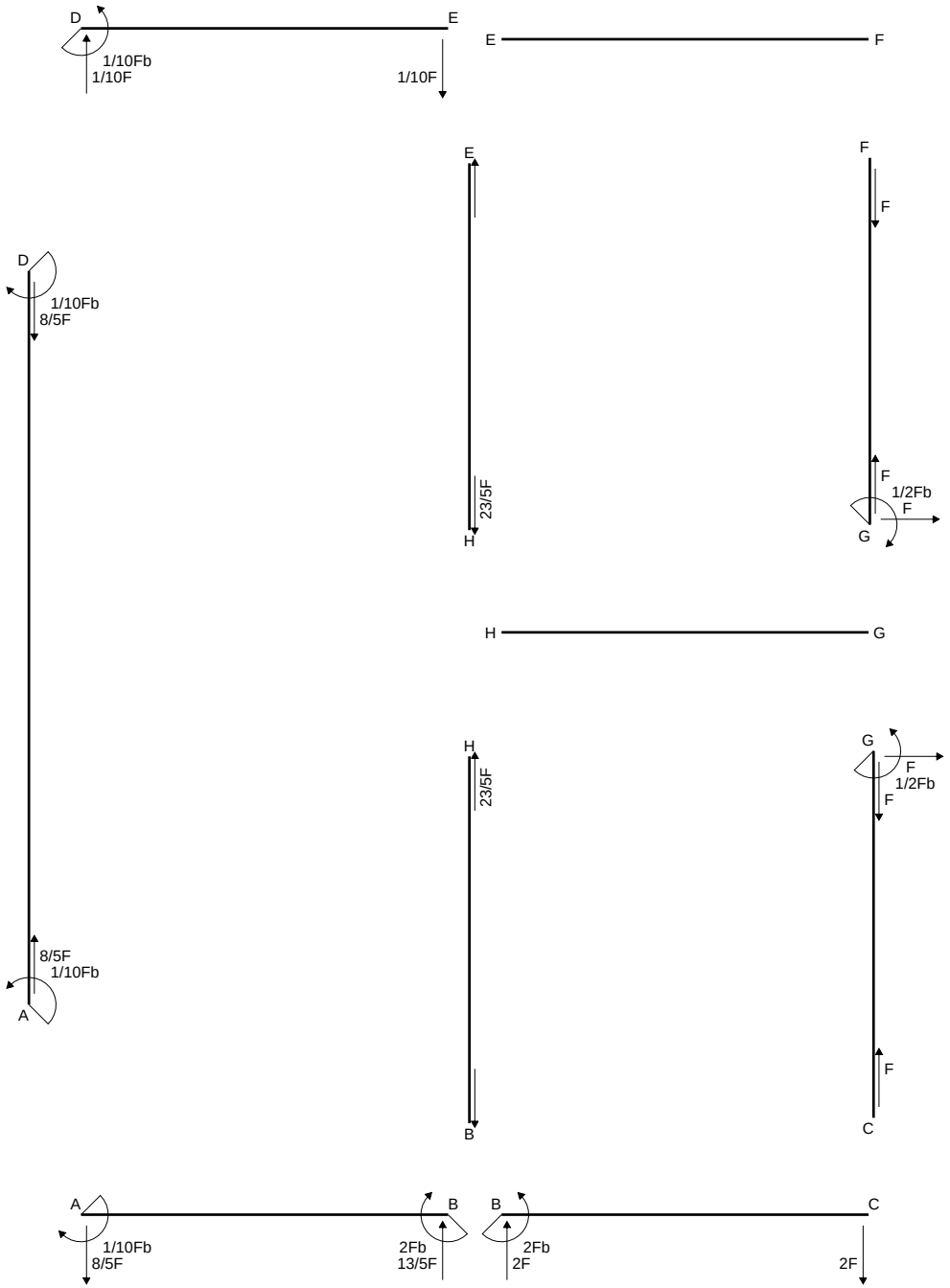
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

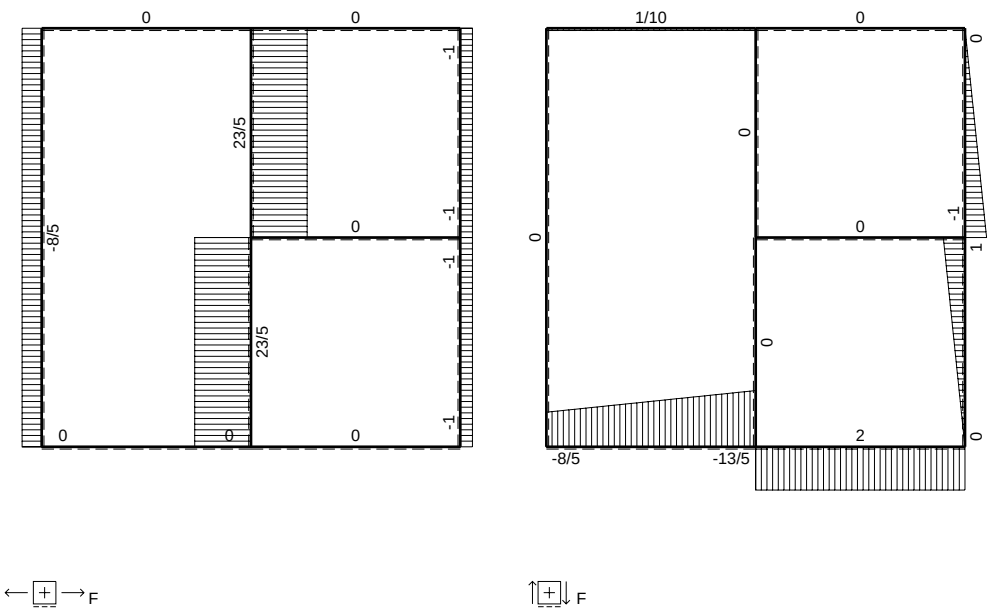
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

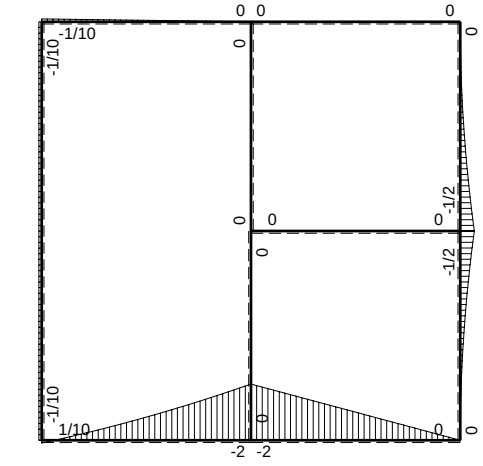
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

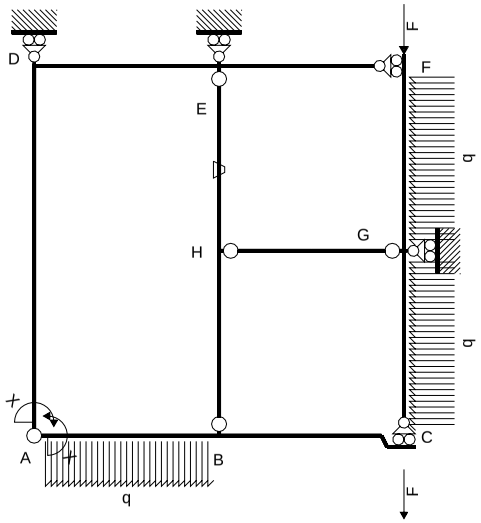
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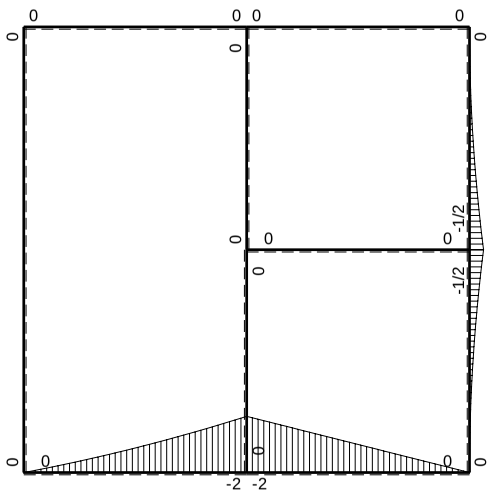
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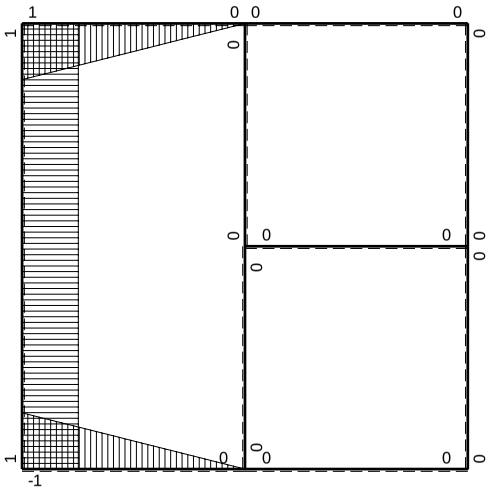

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-1/10
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-2 -2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982506 ESΣ03.xxxx.070

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PROCEDIMENTO E RISULTATI 982506 ESΣ03.xxxx.070

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	-Fb/EJ	0	0	0	0+0	0
EH b	0	0	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (1/3\,b)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\,b - 1/3\,b - 1/8\,b)\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

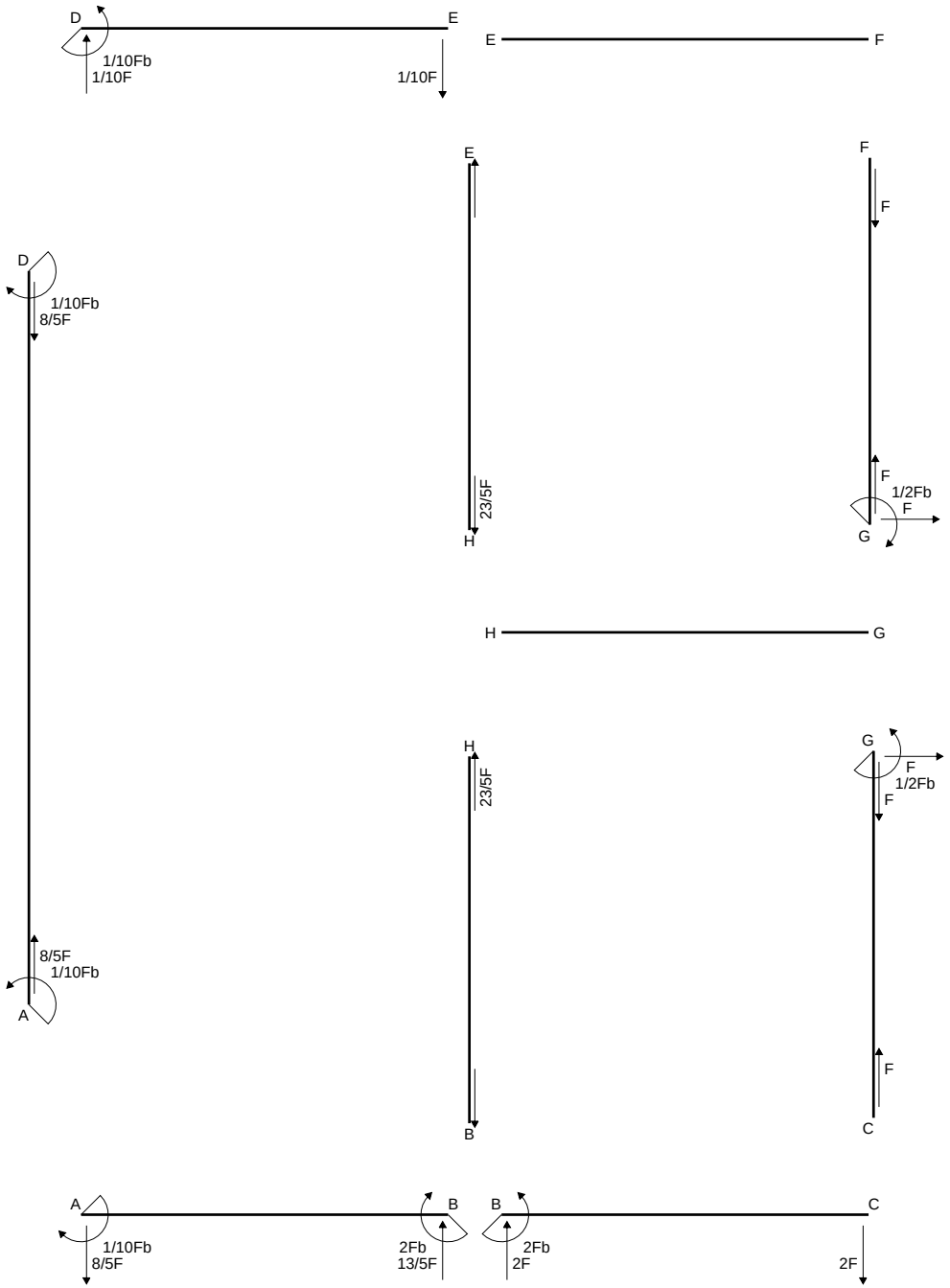
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

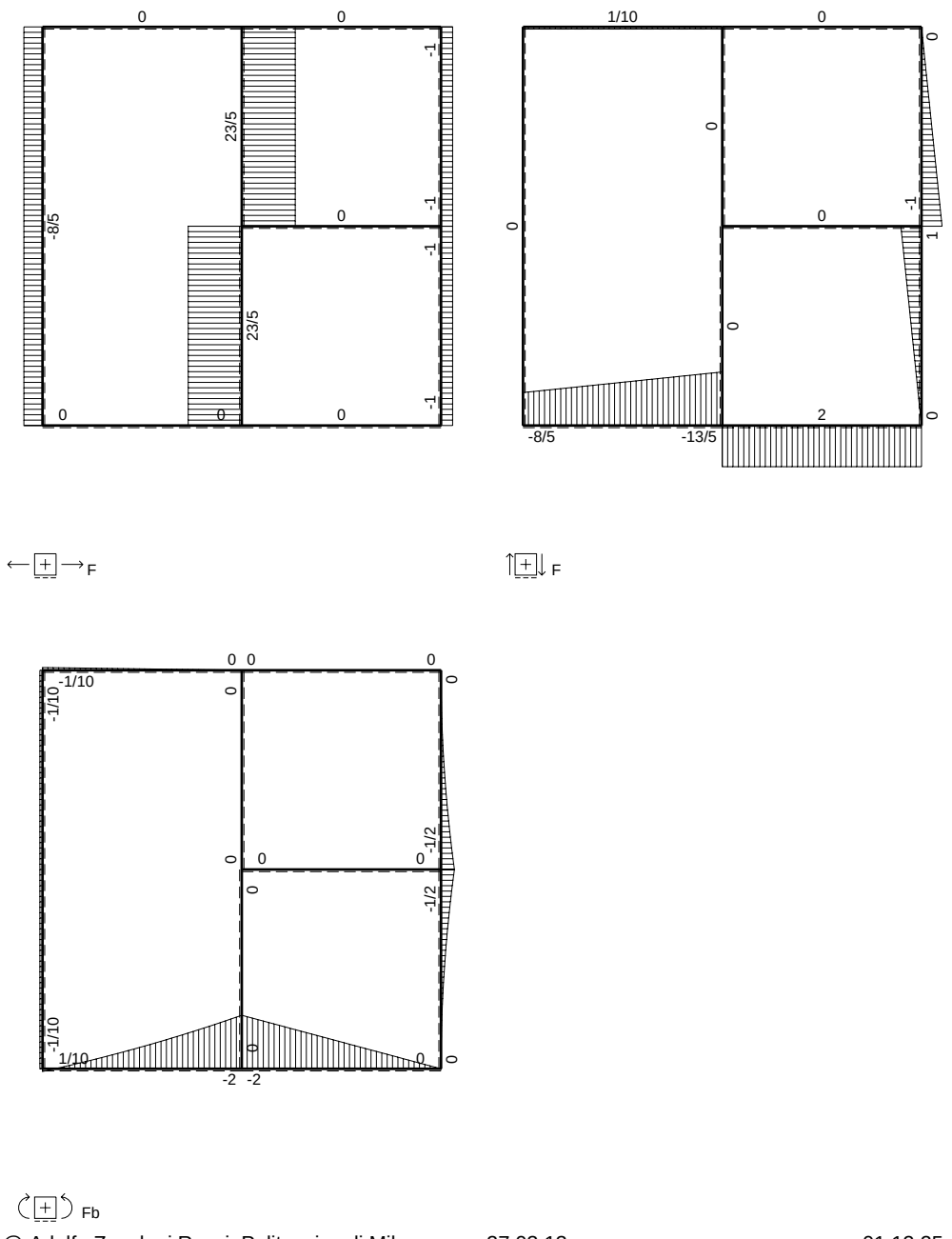
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

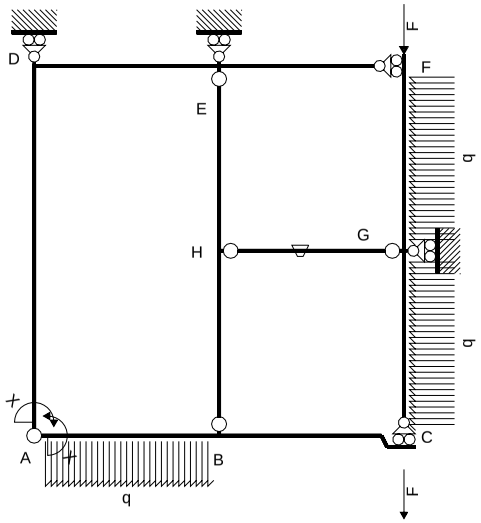
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

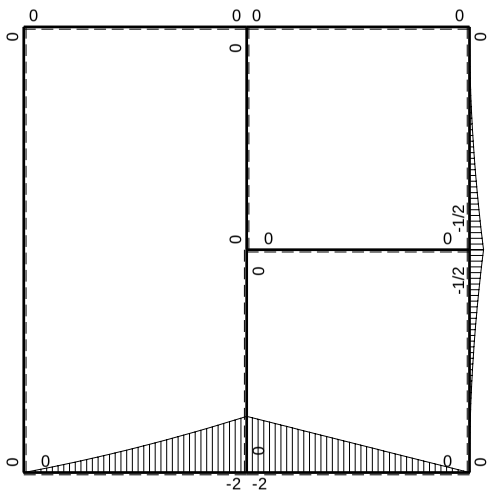
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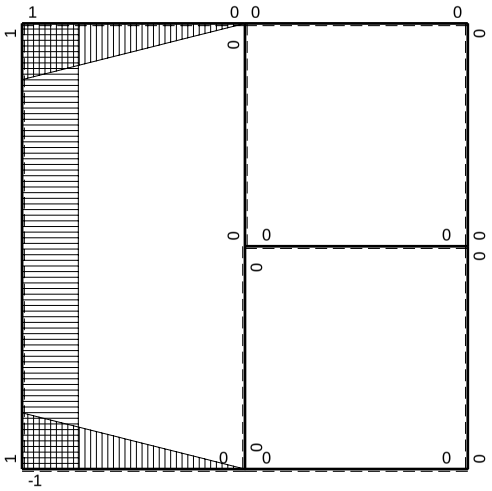

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 961051 ESΣ03.xxxx.071

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PROCEDIMENTO E RISULTATI 961051 ESΣ03.xxxx.071

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,3/4\,b - 1/3\,b - 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

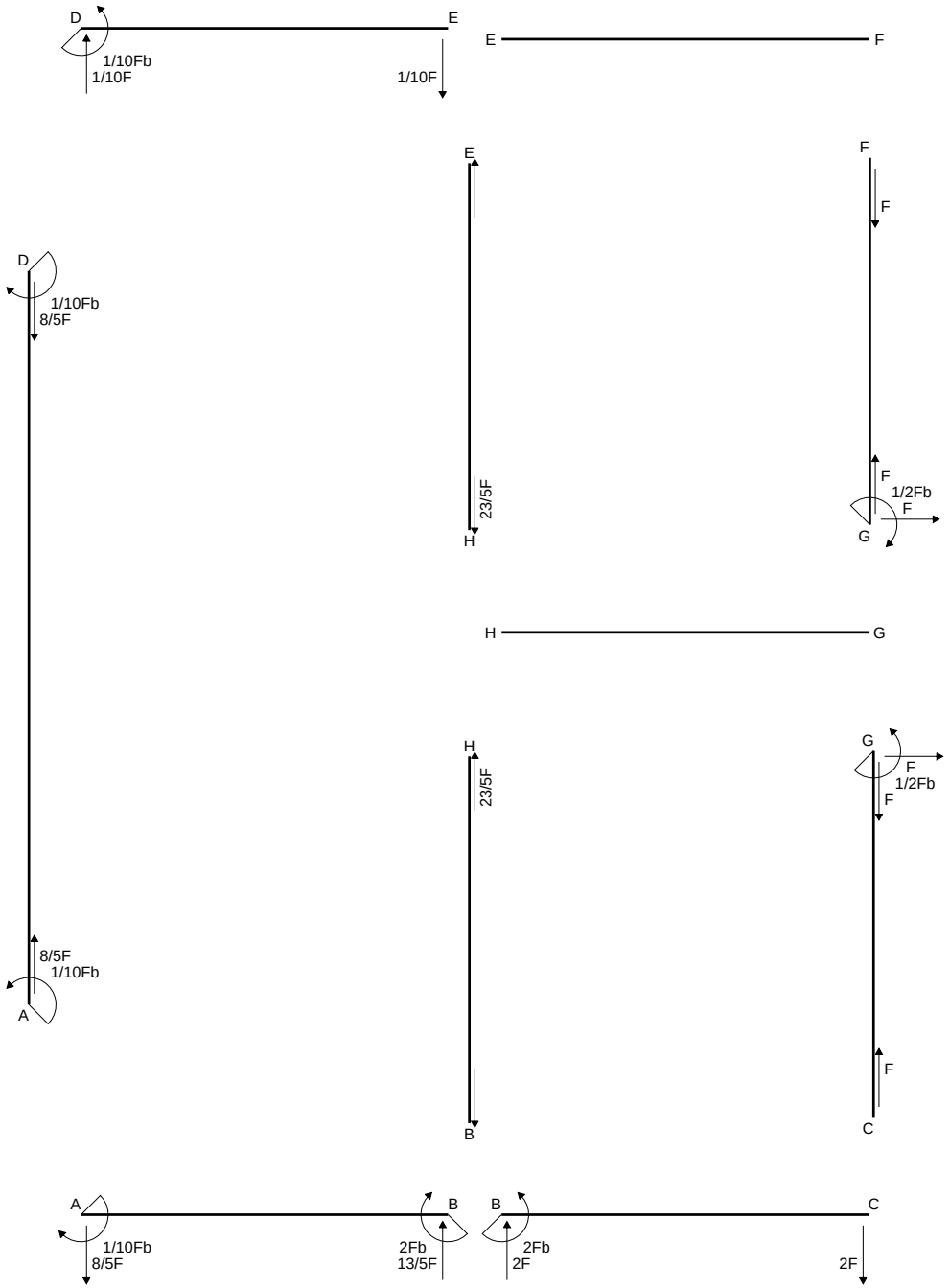
$L_{BA}^{Xo} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

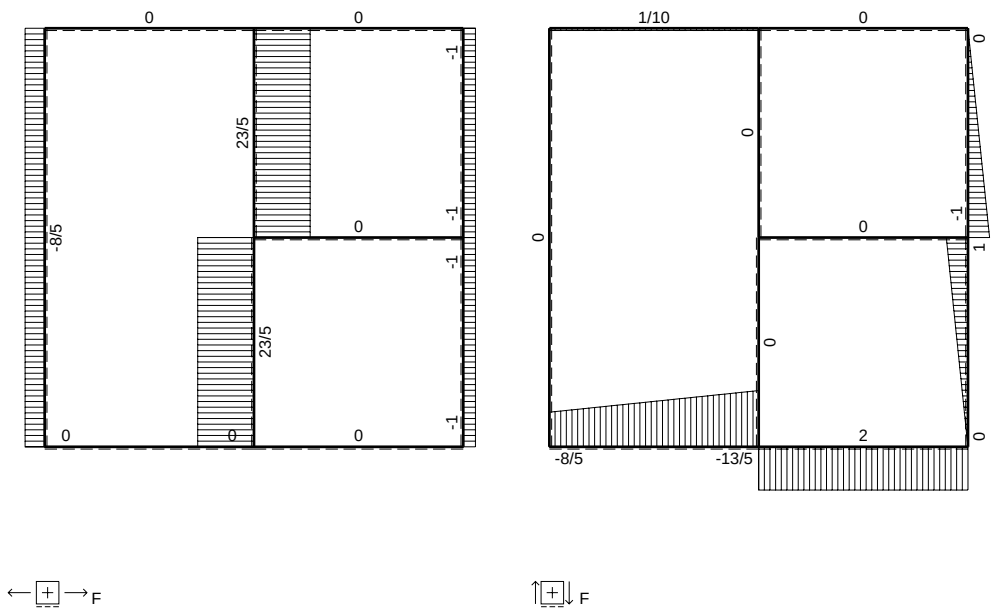
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

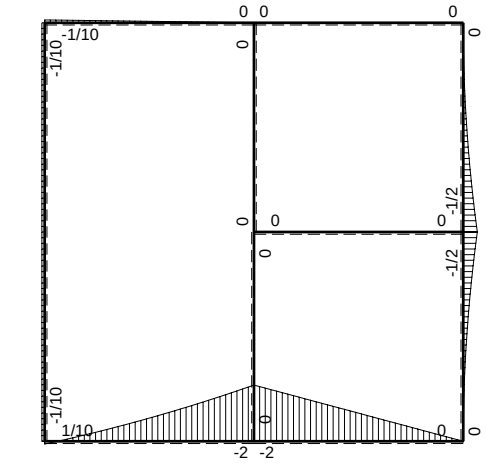
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

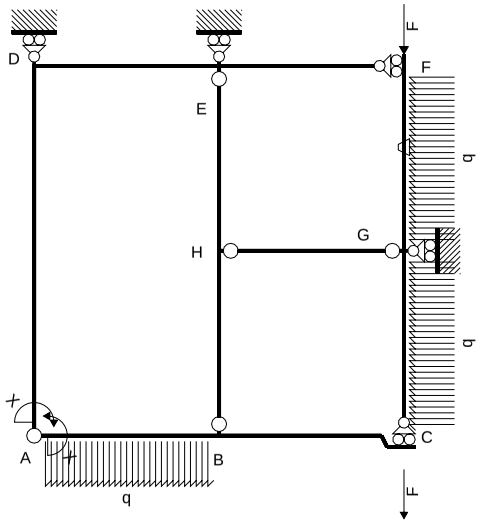
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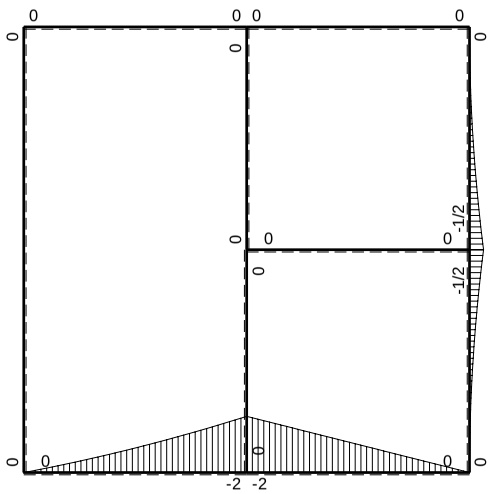
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2Fb
13/5F
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2F
2F

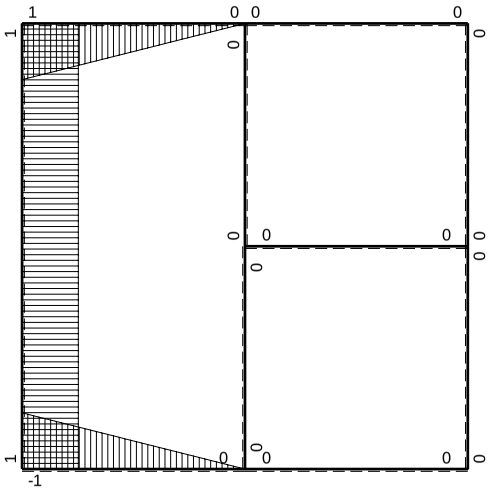

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 957197 ESΣ03.xxxx.072

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PROCEDIMENTO E RISULTATI 957197 ESΣ03.xxxx.072

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
GF b	0	$1/2Fb-Fx+1/2qx^2$	Fb/EJ	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

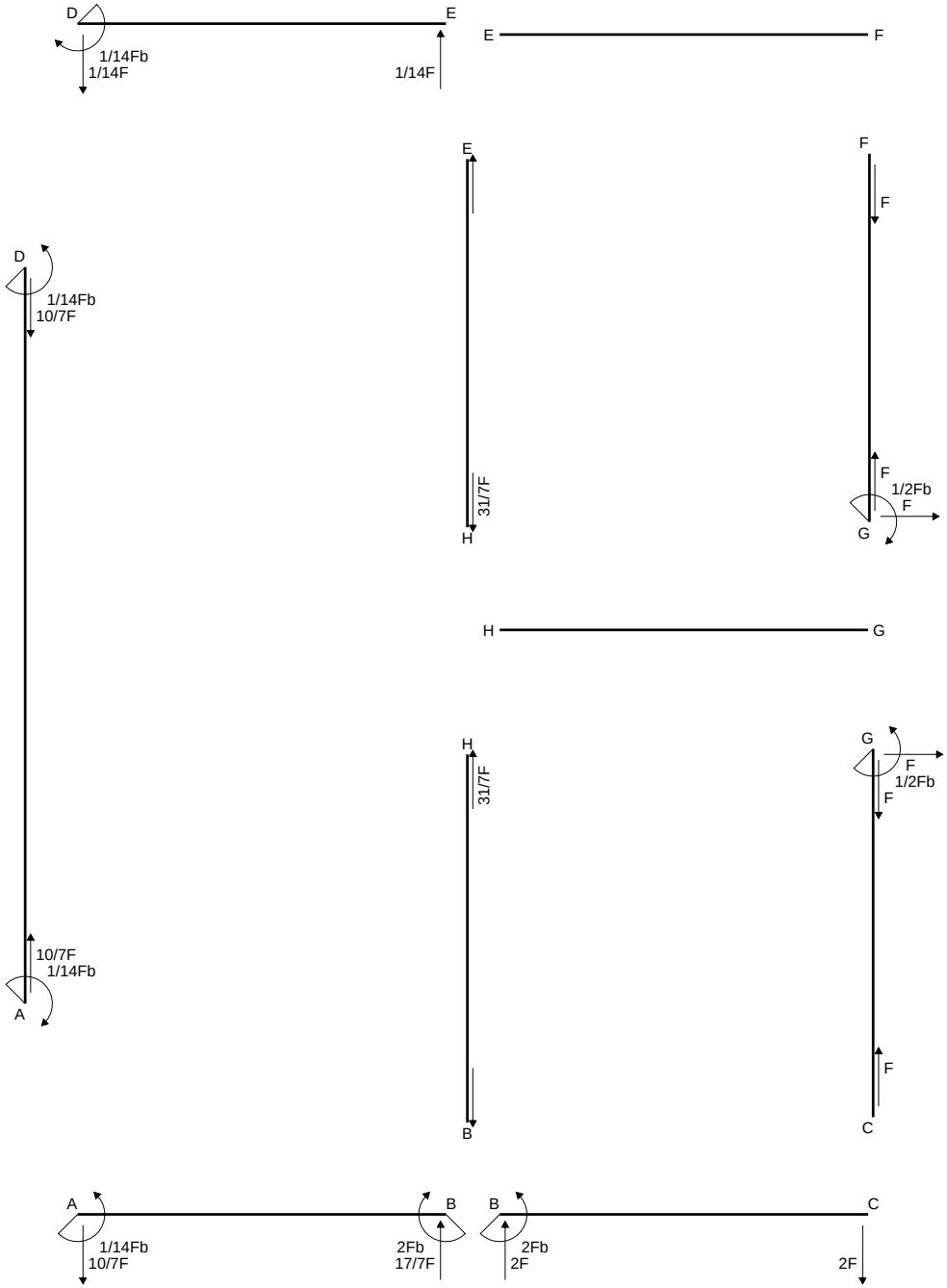
$L_{BA}^{Xo} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

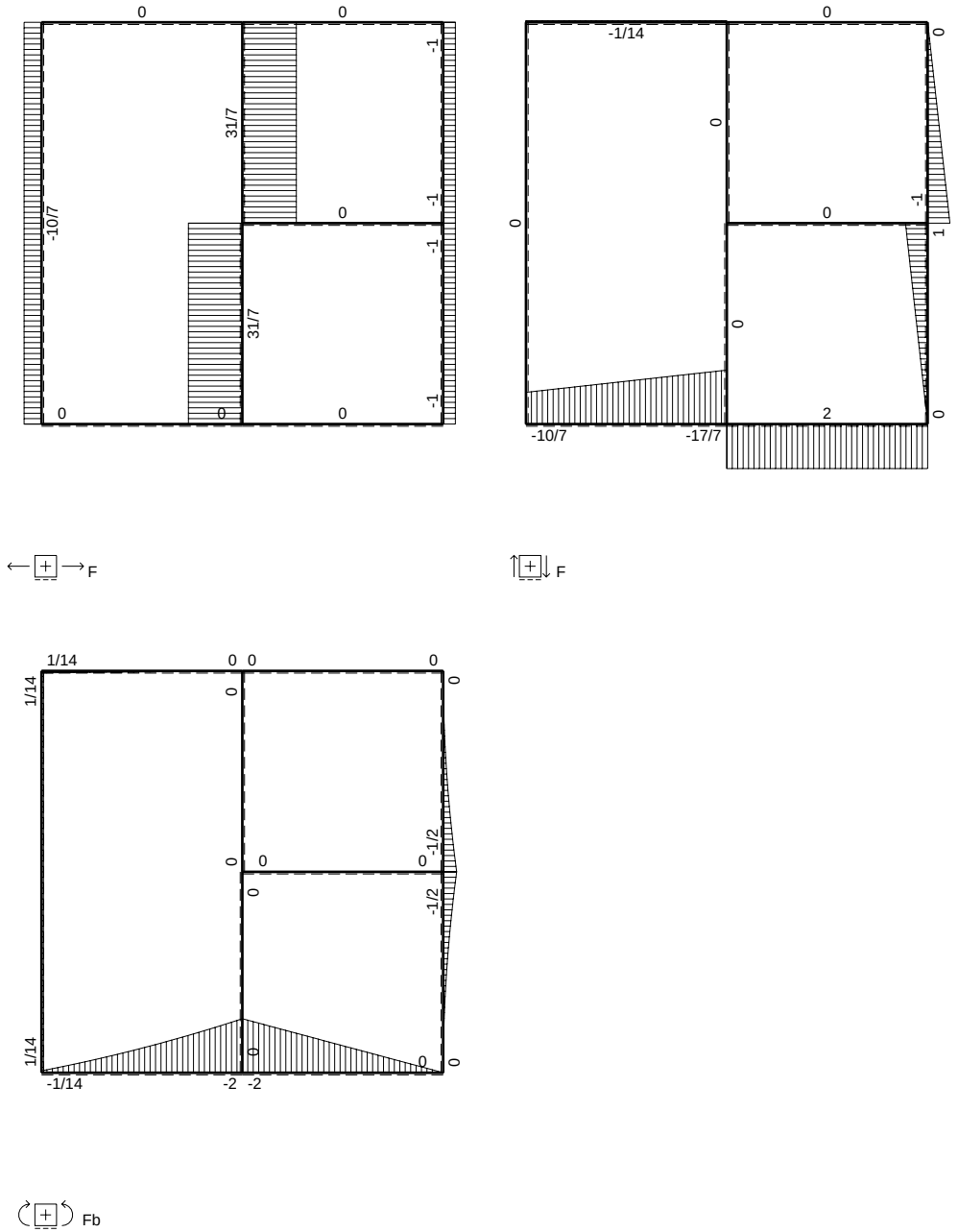
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

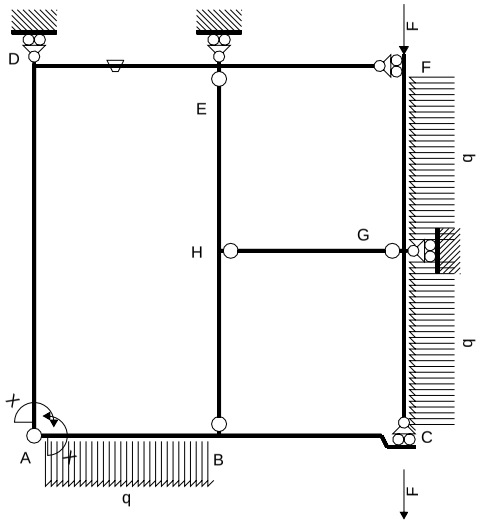
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

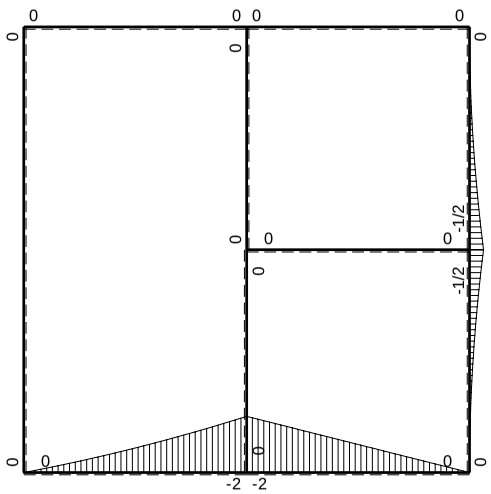
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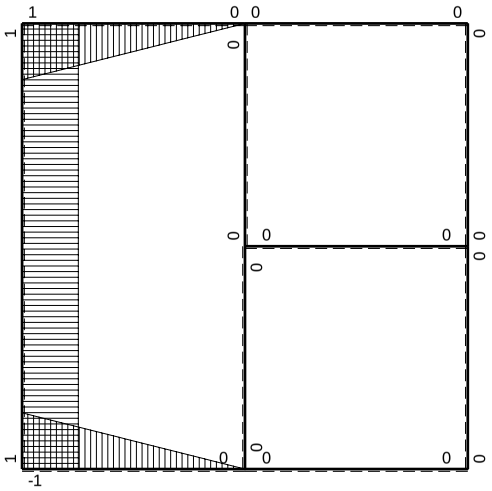

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-1/14
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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 984792 ESΣ03.xxxx.073

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	-Fb/EJ	0	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$	$(0-1/2)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	0	Fb/EJ	0	-Fx/EJ	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984792 ESΣ03.xxxx.073
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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + x/b\ )\ \theta\ \ dx = [\ - x + 1/2\ x^2/b\ ]_o^b\ \theta$

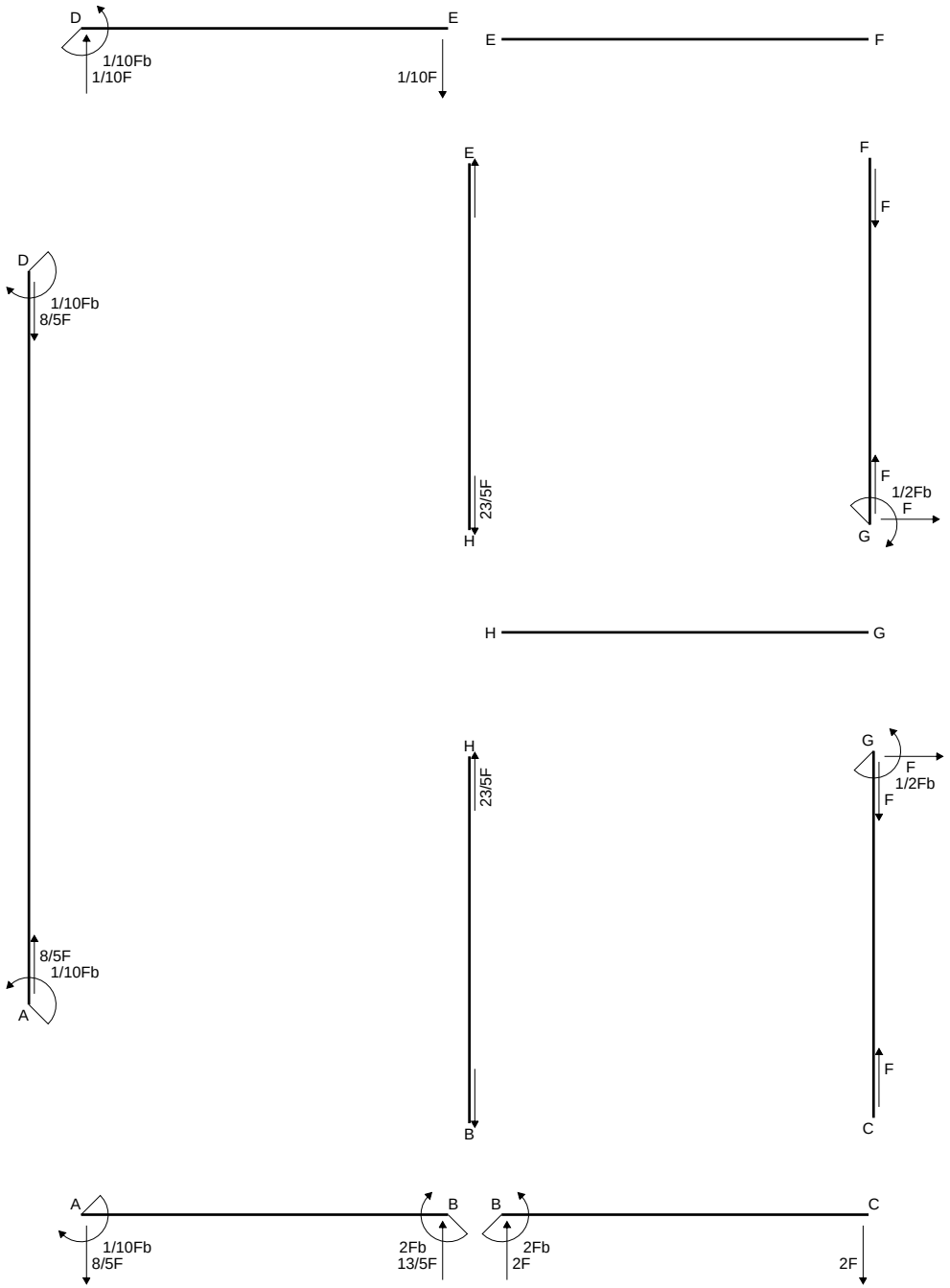
$= (\ - b + 1/2\ b\ )\ \theta\ \ = -1/2\ \ Fb^2/EJ$

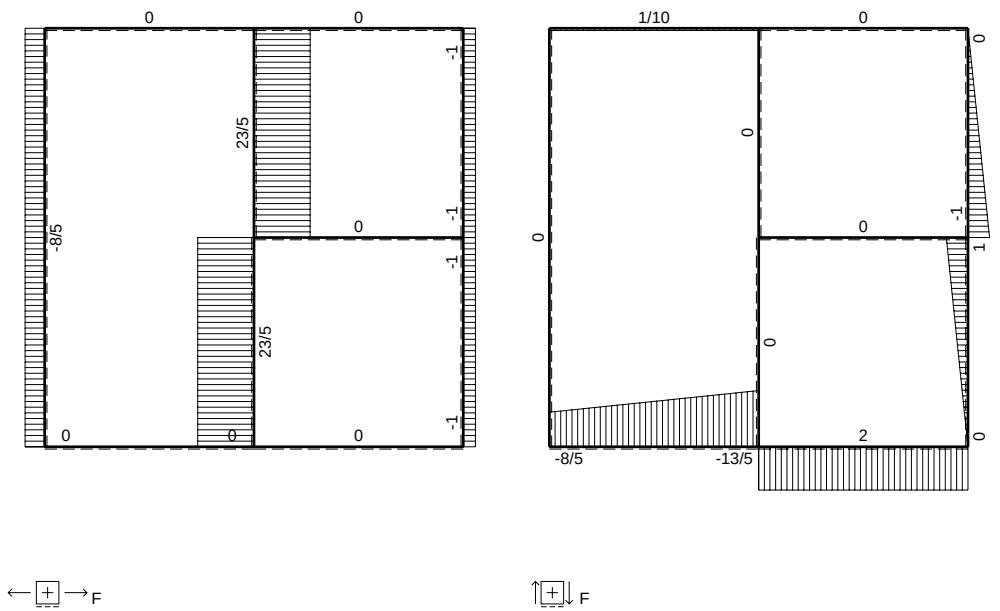
$L^{Xo}_{ED} = \int_o^b (\ x/b\ )\ \theta\ \ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

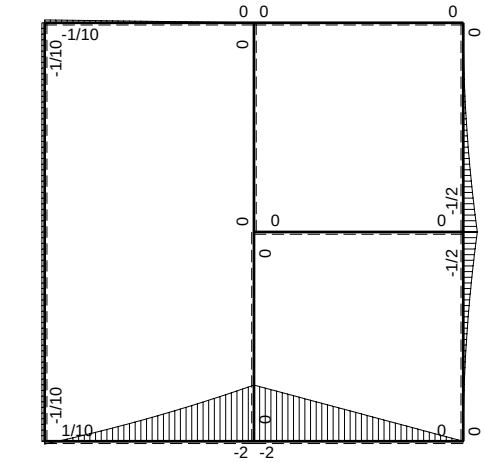
$= (1/2\ b\ )\ \theta\ \ = -1/2\ \ Fb^2/EJ$

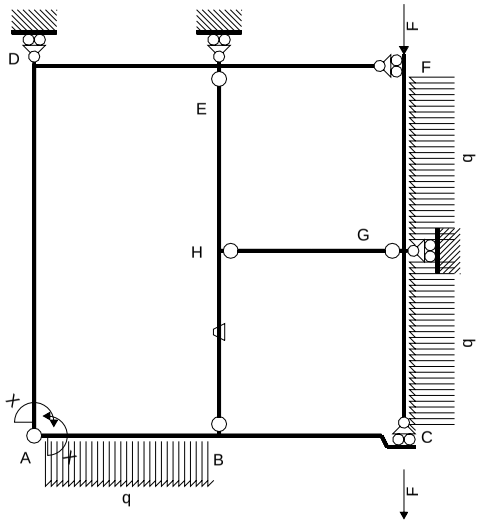
ESΣ03.xxxx.073

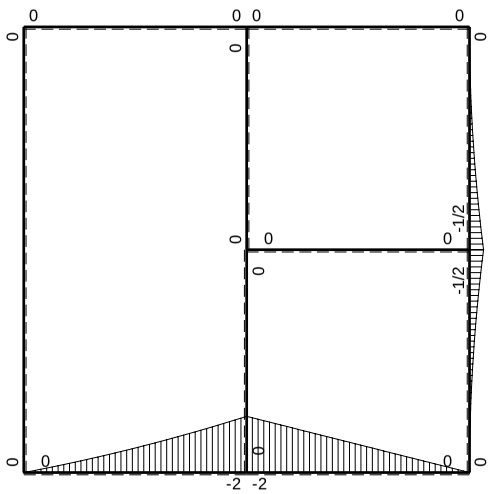
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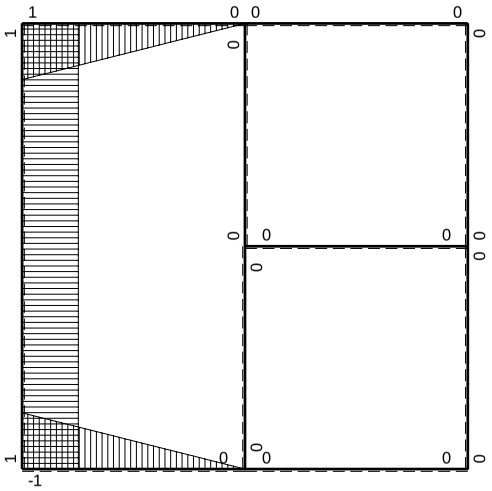

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-1/10
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-2 -2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237122 ESΣ03.xxxx.074

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PROCEDIMENTO E RISULTATI 237122 ESΣ03.xxxx.074

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	0	Fb/EJ	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

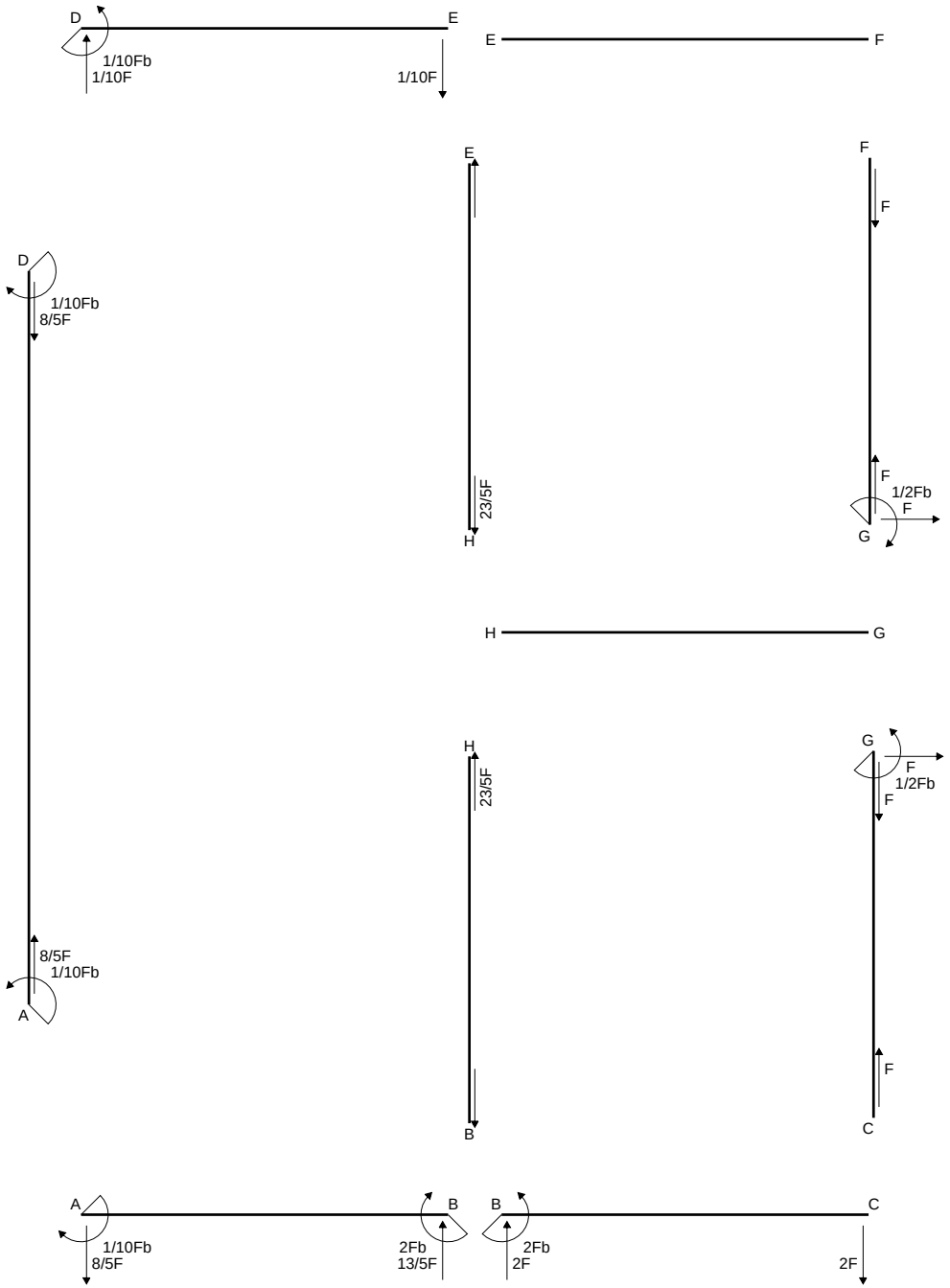
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

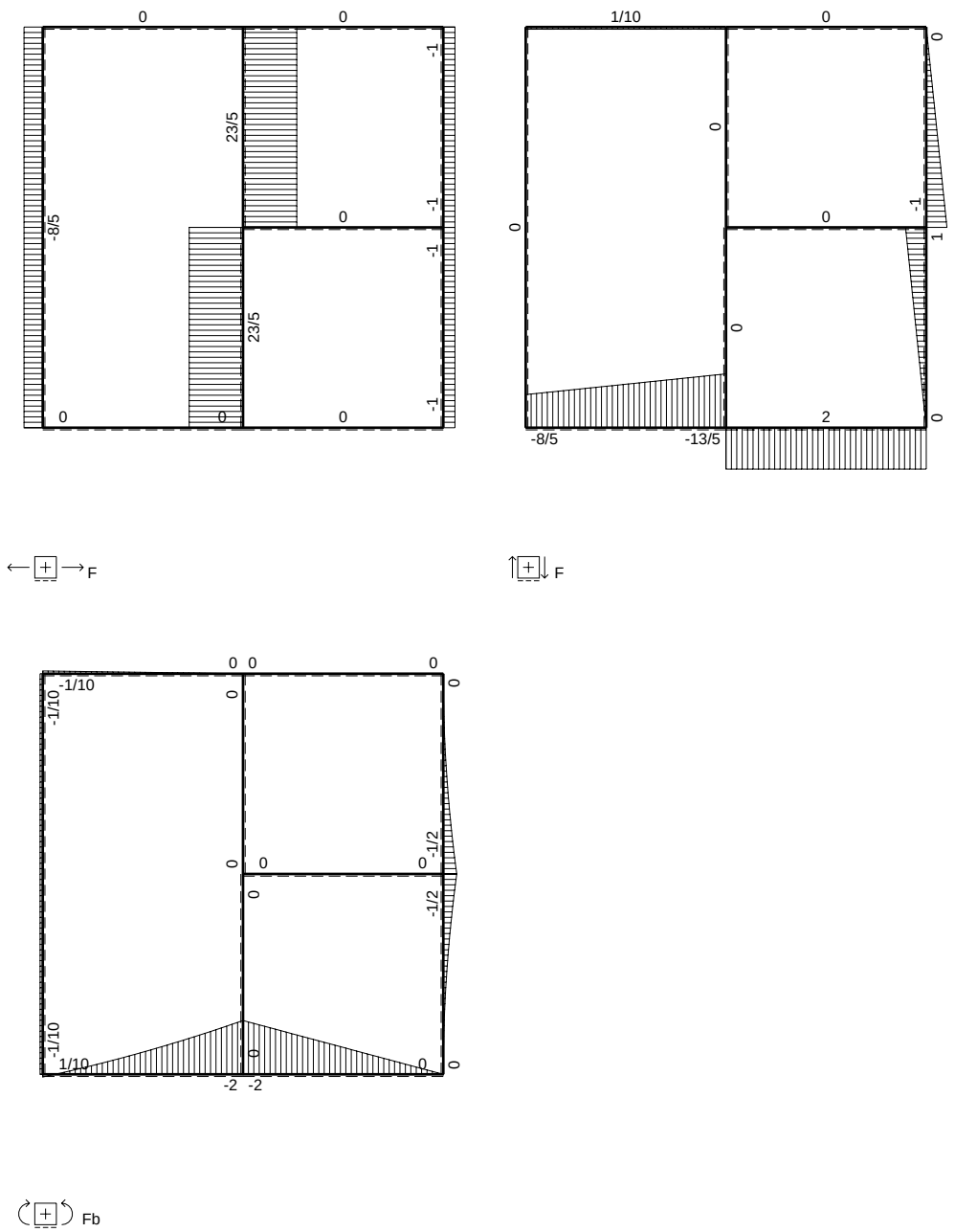
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

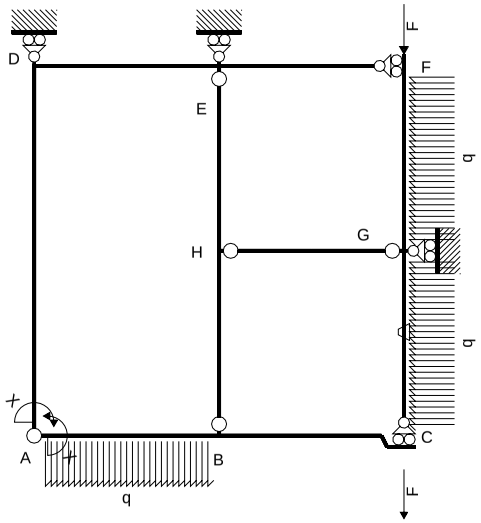
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

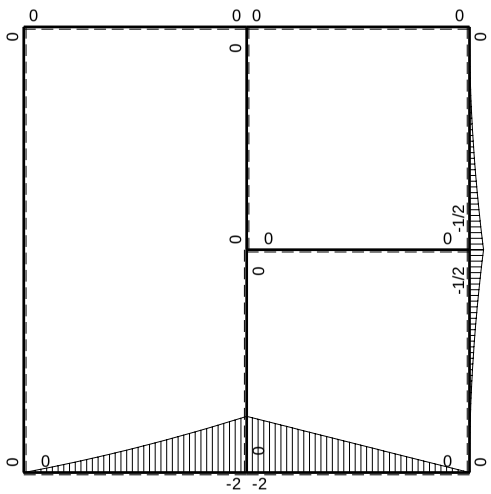
ESΣ03.xxxx.074

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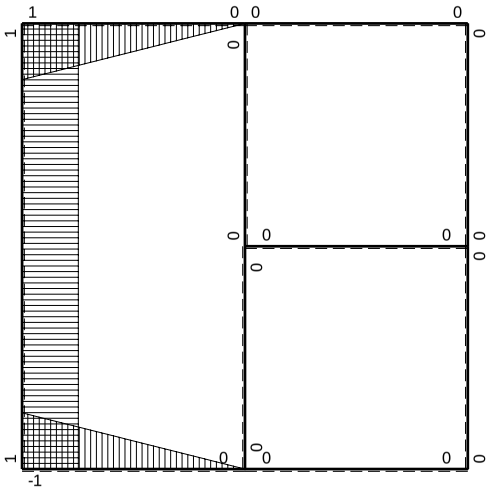

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Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238160 ESΣ03.xxxx.075

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PROCEDIMENTO E RISULTATI 238160 ESΣ03.xxxx.075

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (1/3\,b)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\,b - 1/3\,b - 1/8\,b)\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

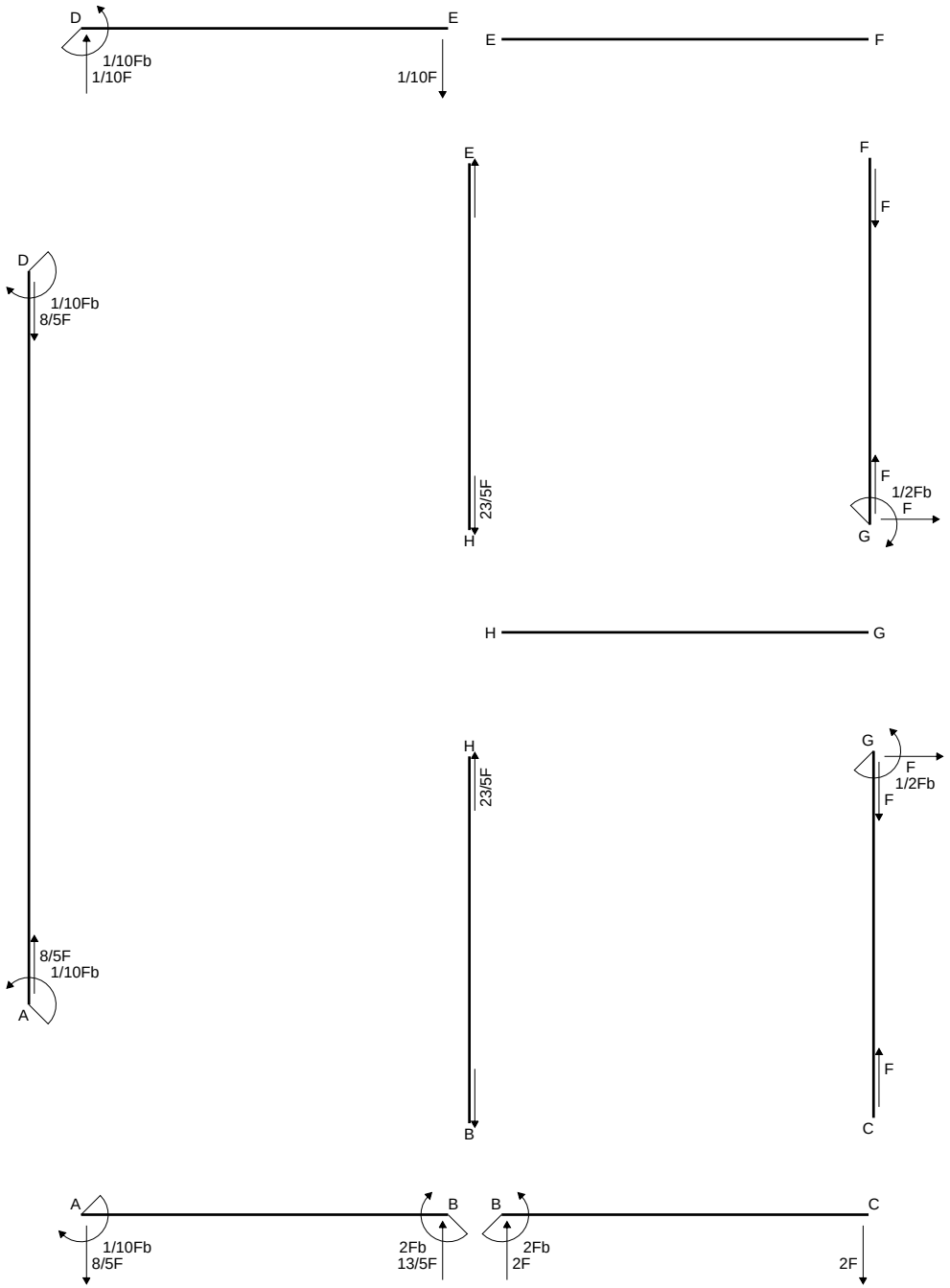
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

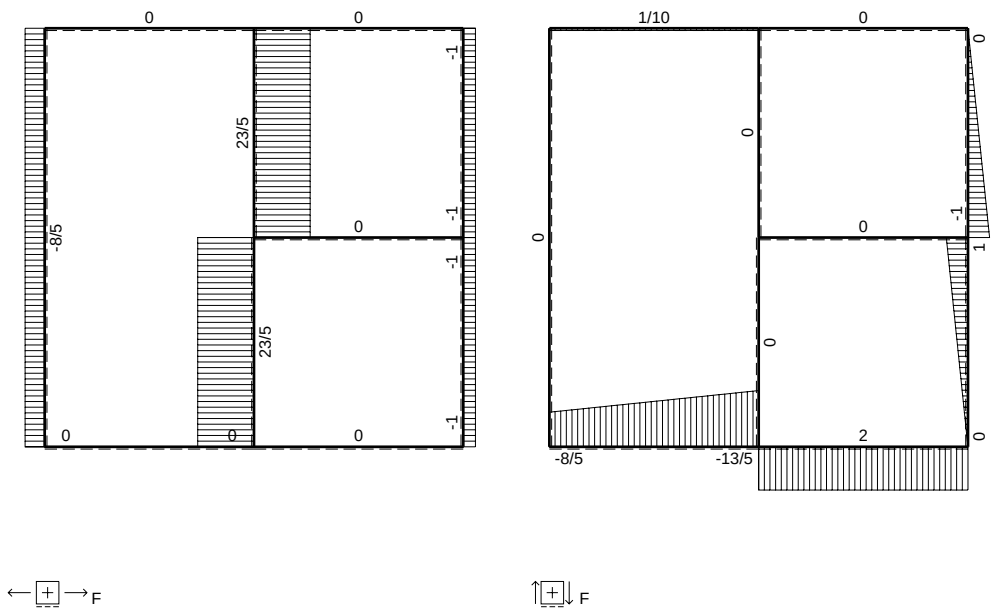
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

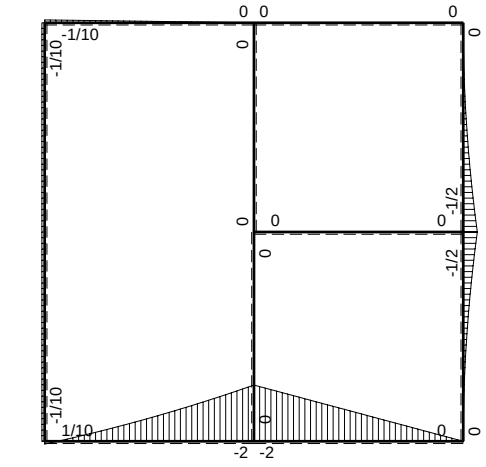
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

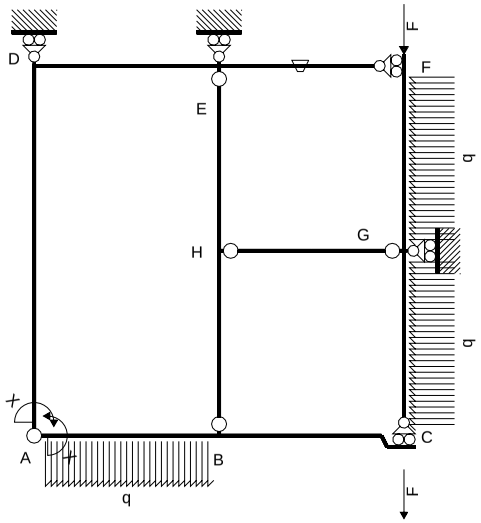
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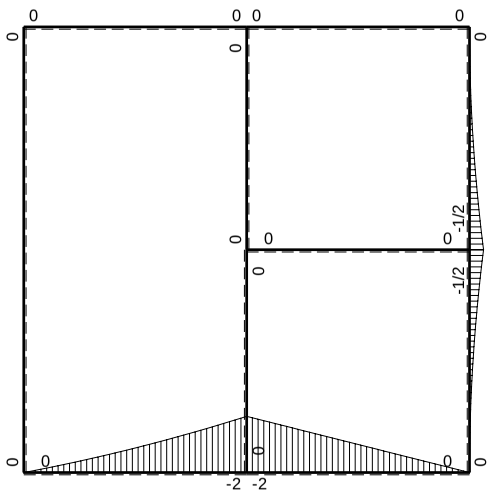
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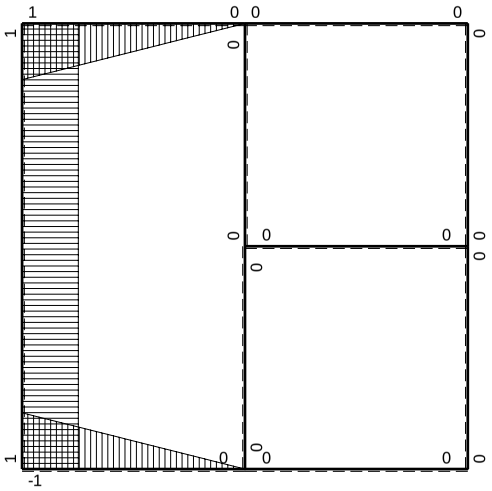

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0
-1/10
1/10
0
0
-2 -2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212411 ESΣ03.xxxx.076

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PROCEDIMENTO E RISULTATI 212411 ESΣ03.xxxx.076

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	-Fb/EJ	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

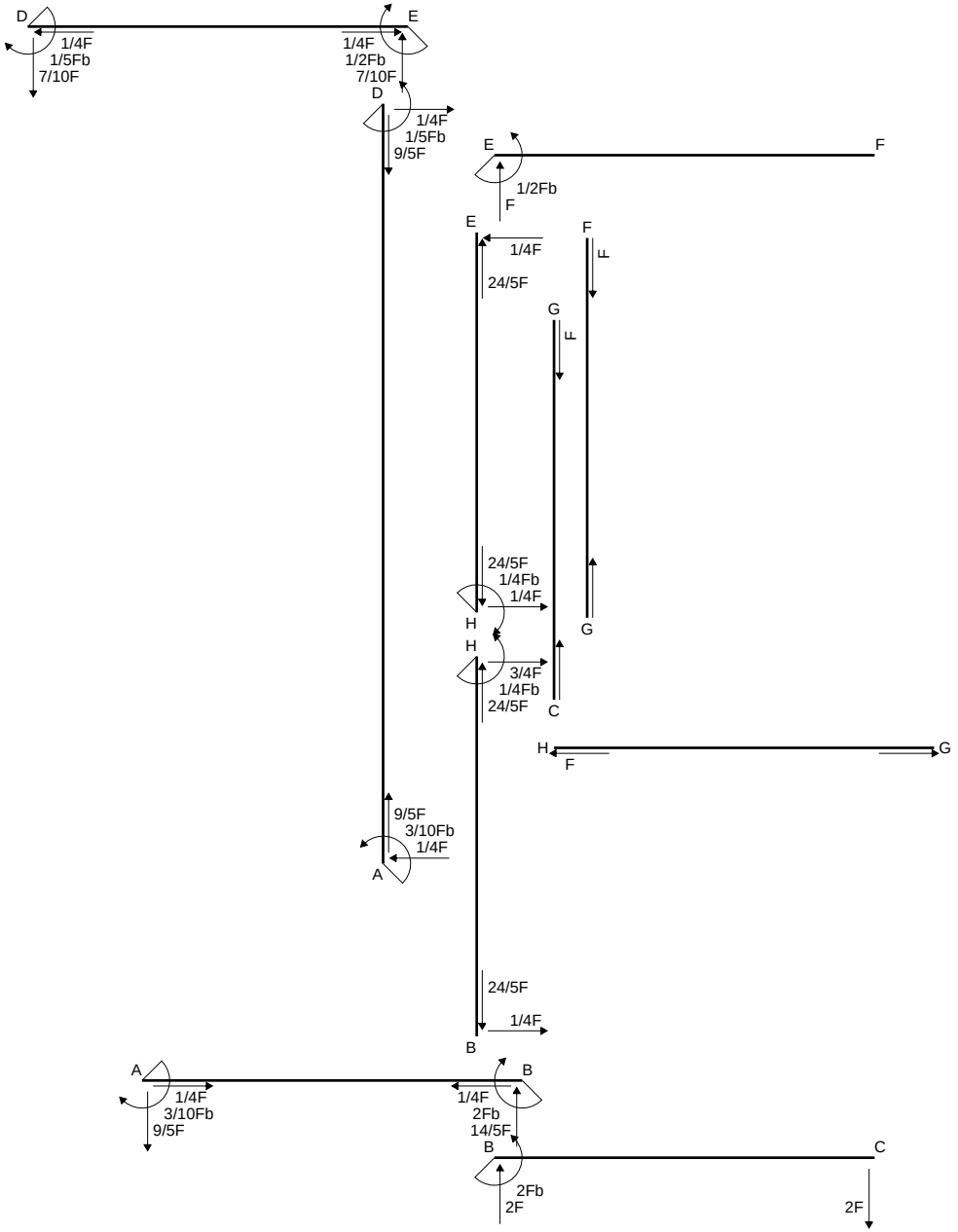
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

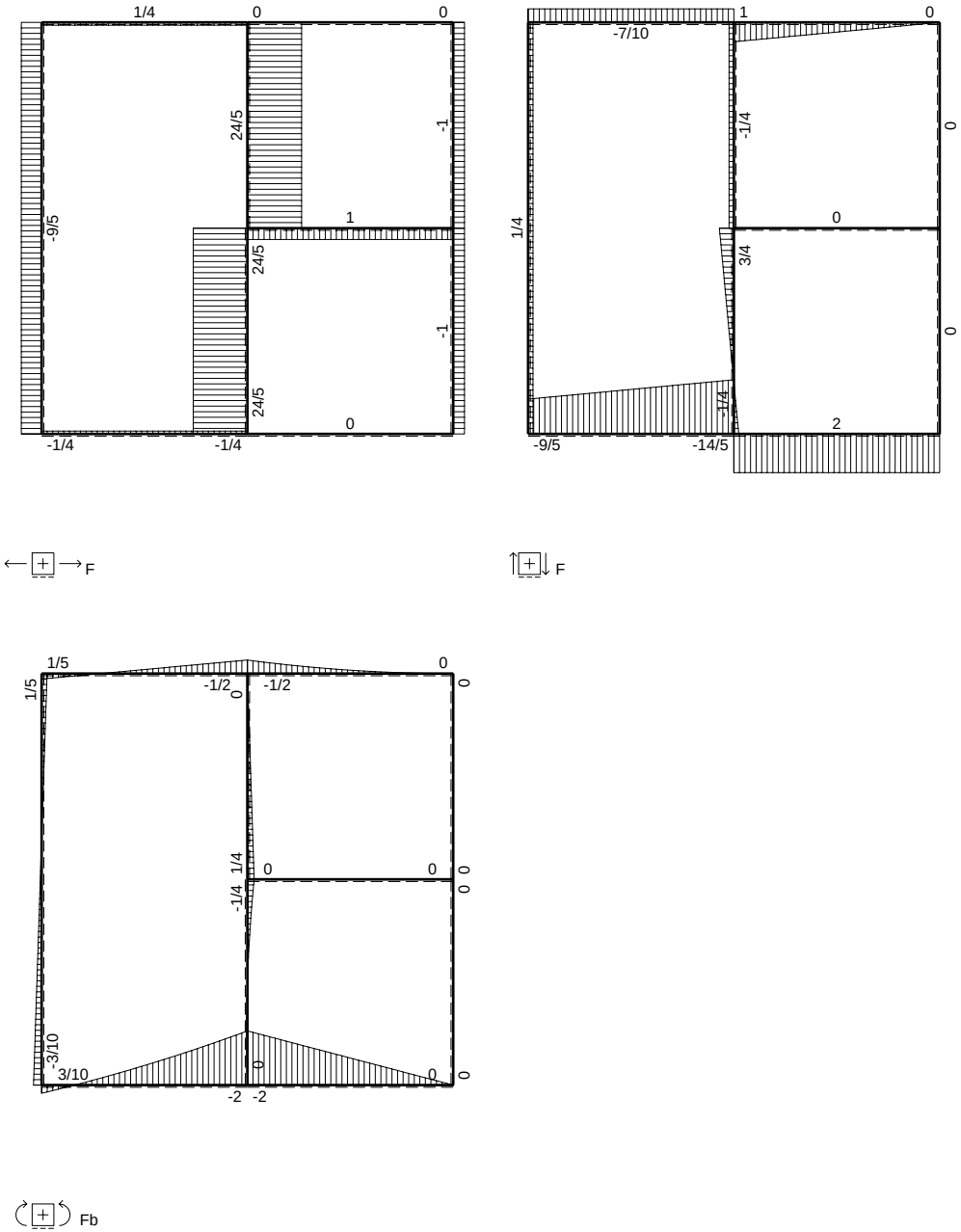
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

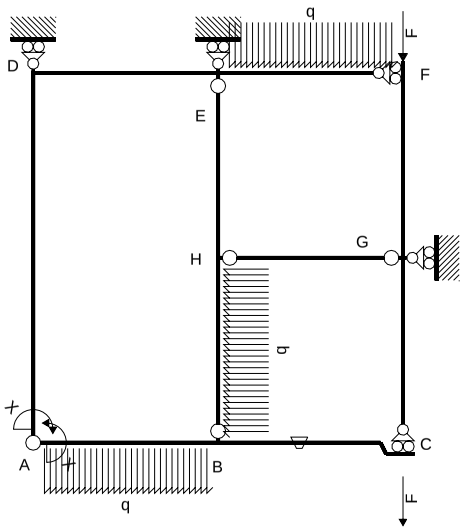
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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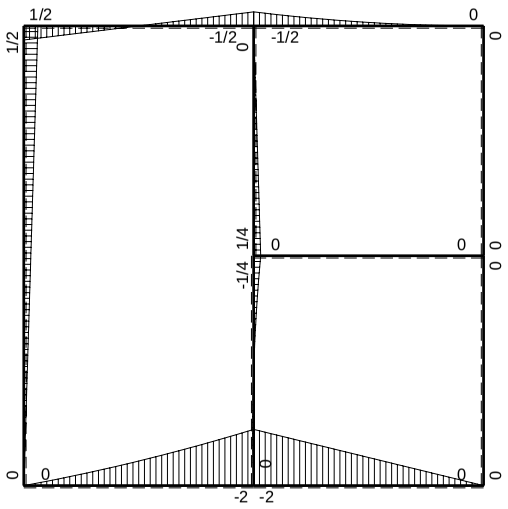
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1/2Fb
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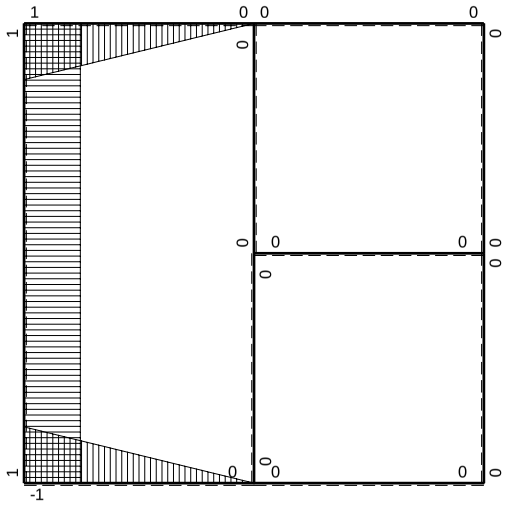

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0
-1/4
0
0
0
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238342 ESΣ03.xxxx.077

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PROCEDIMENTO E RISULTATI 238342 ESΣ03.xxxx.077

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

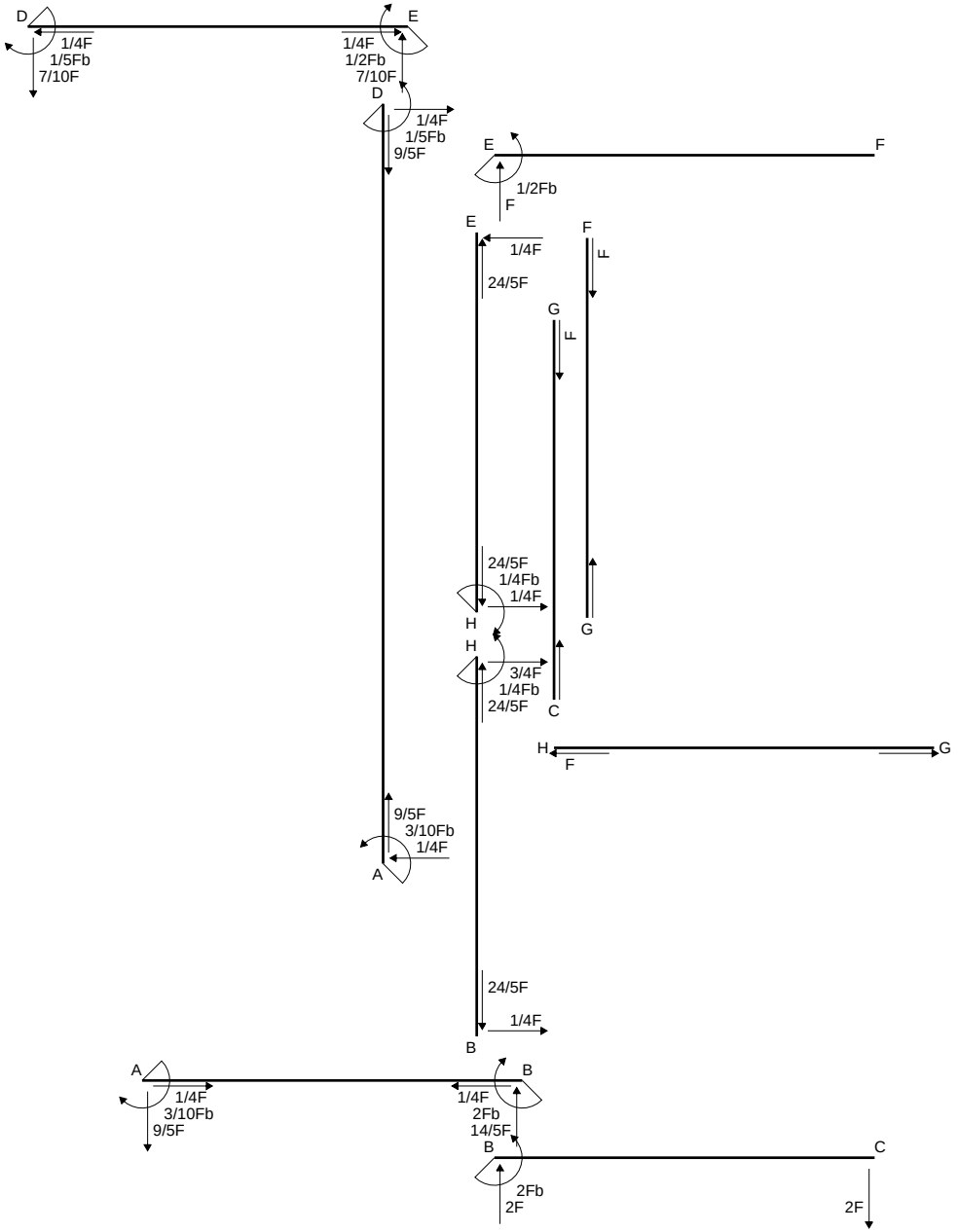
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

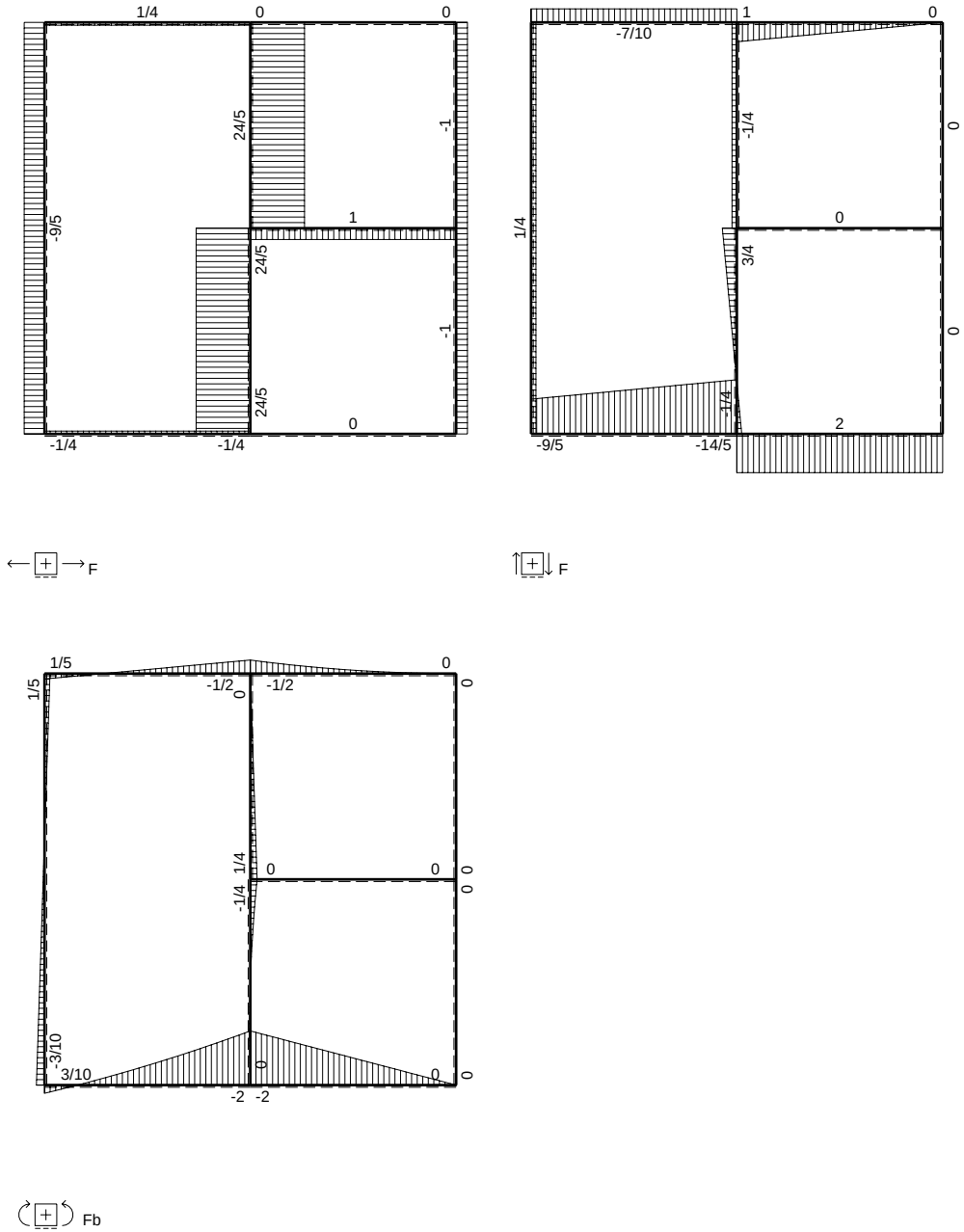
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

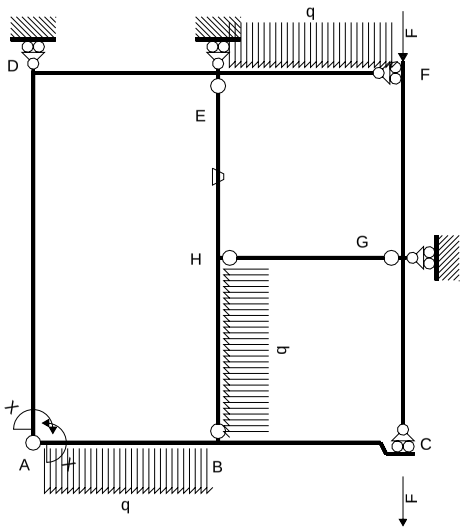
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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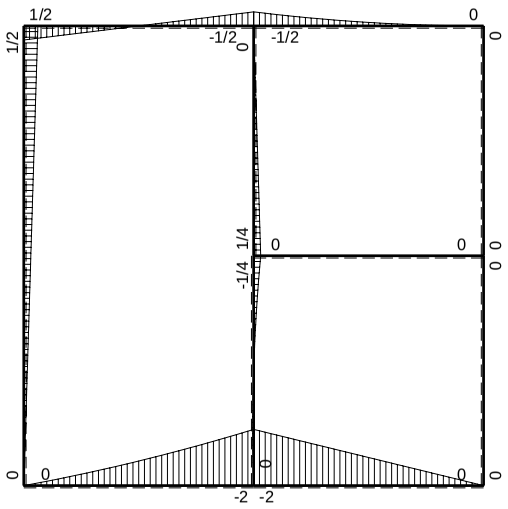
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D
E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
E
F
1/2Fb
F
E
1/4F
24/5F
F
F
G
F
24/5F
1/4Fb
1/4F
H
G
H
3/4F
1/4Fb
24/5F
C
H
F
G
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3/10Fb
1/4F
A
24/5F
1/4F
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B
C
2Fb
2F
2F

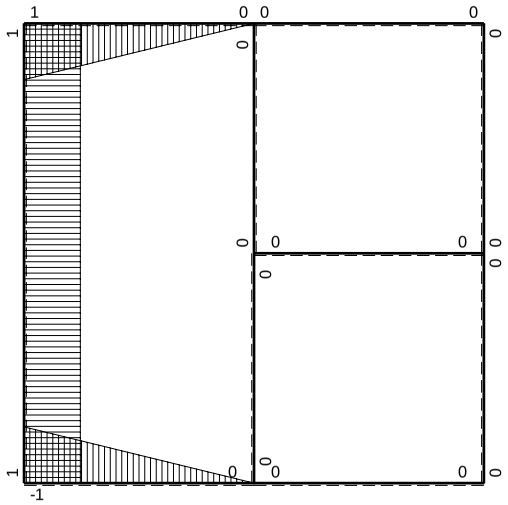

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0
24/5
-1
-9/5
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24/5
-1
24/5
0
-1/4
-1/4
F
1
0
-7/10
-1/4
0
1/4
3/4
0
-1/4
2
-9/5
-14/5
F
1/5
0
-1/2
-1/2
0
0
1/4
0
0
-1/4
0
0
0
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-1/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

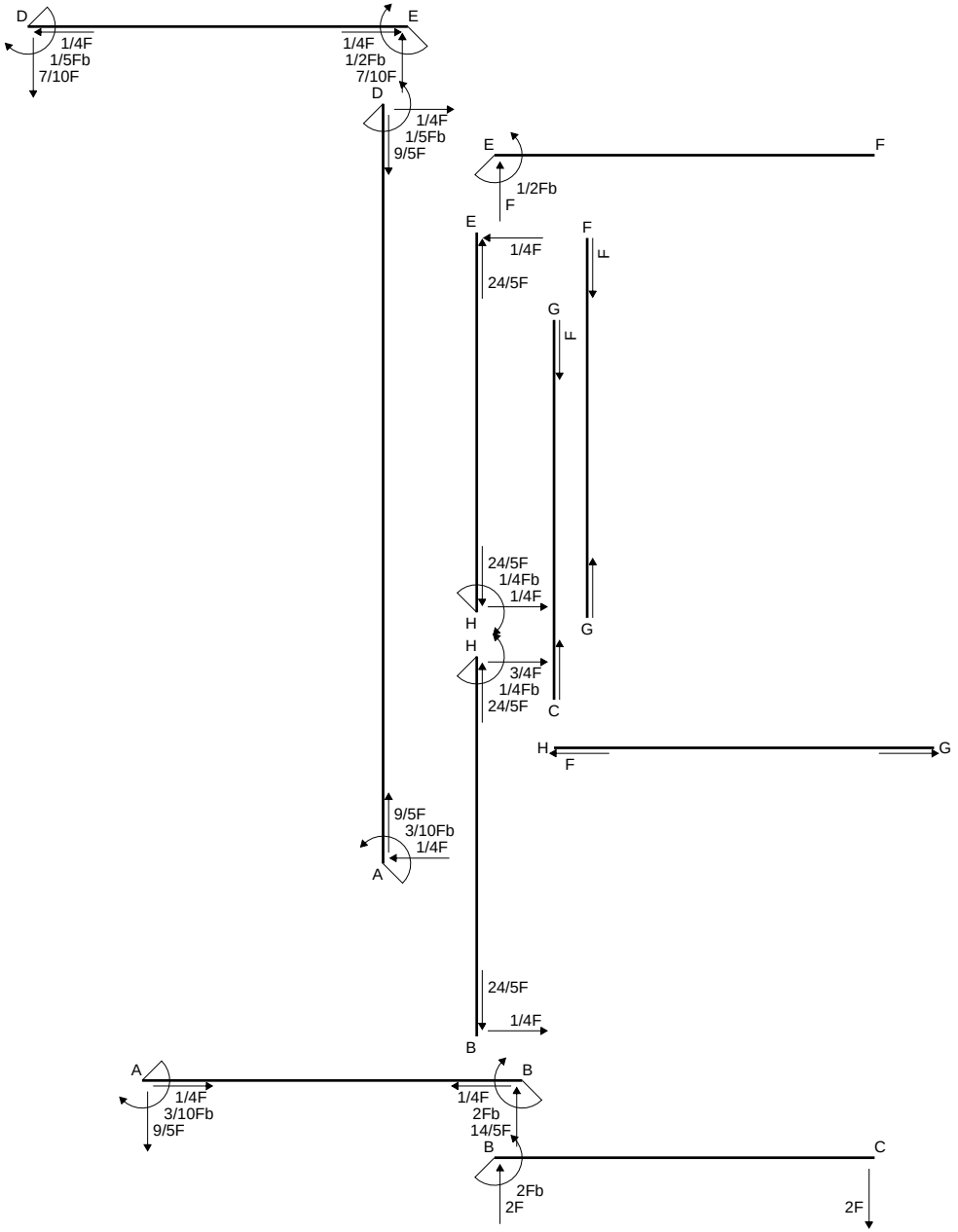
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

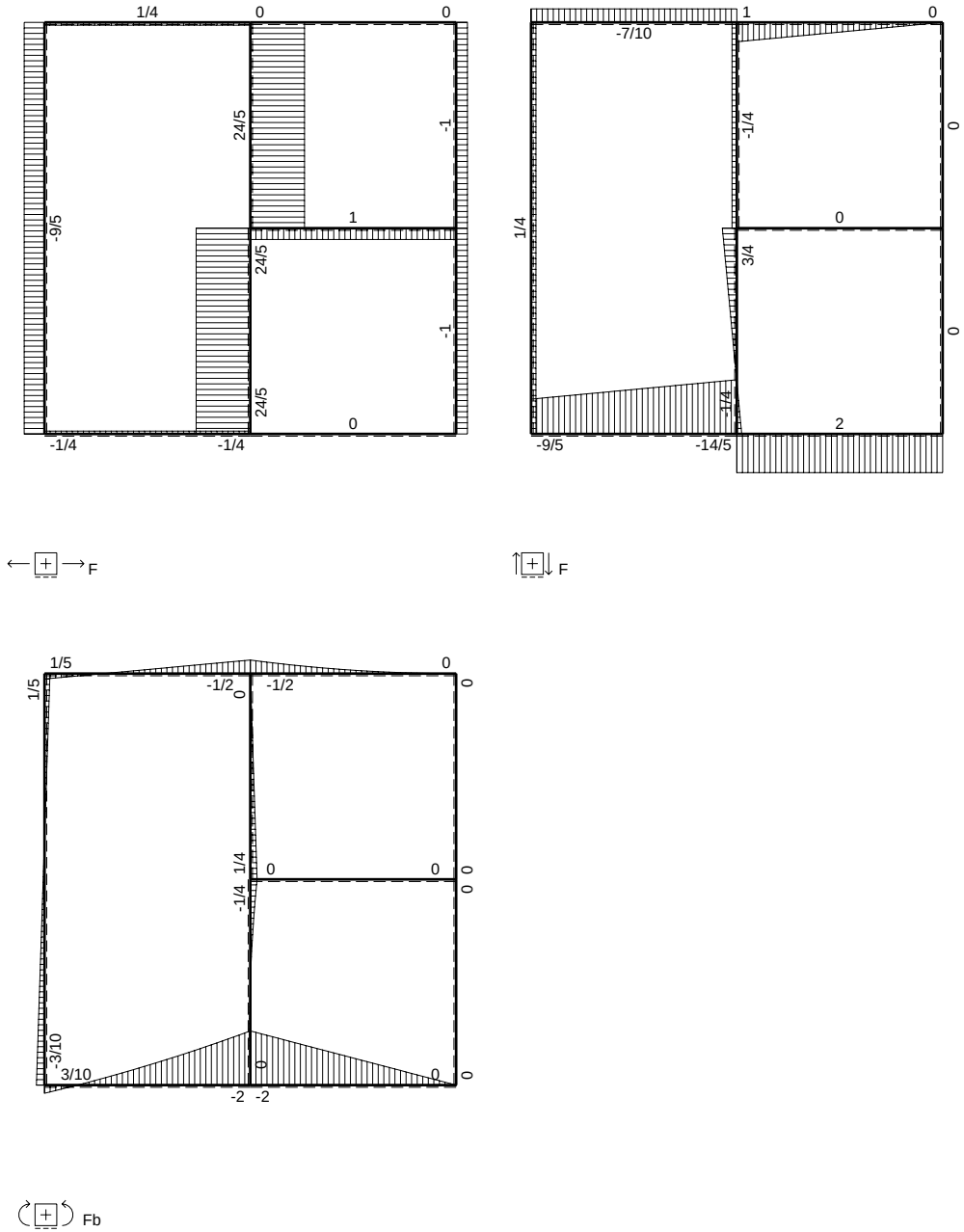
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

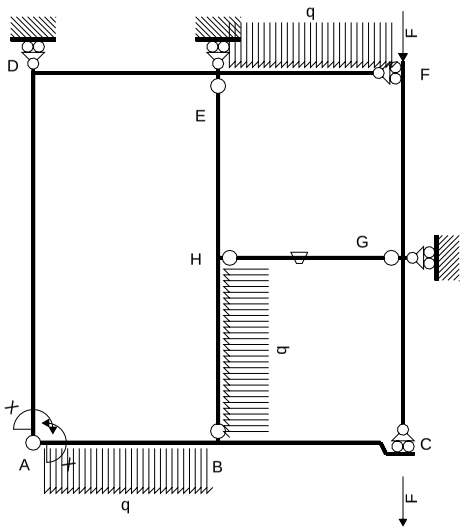
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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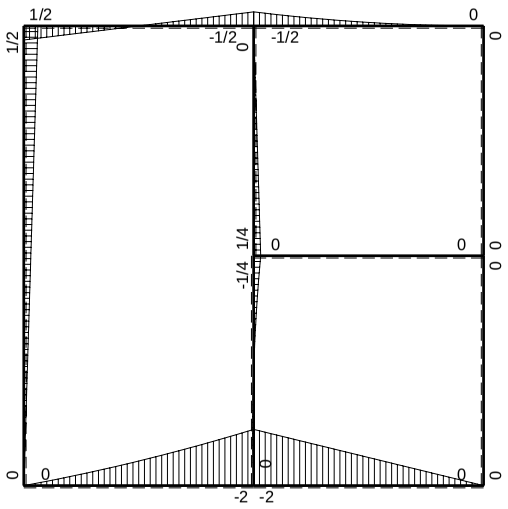
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D
E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
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1/4Fb
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C
H
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9/5F
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C
2Fb
2F
2F

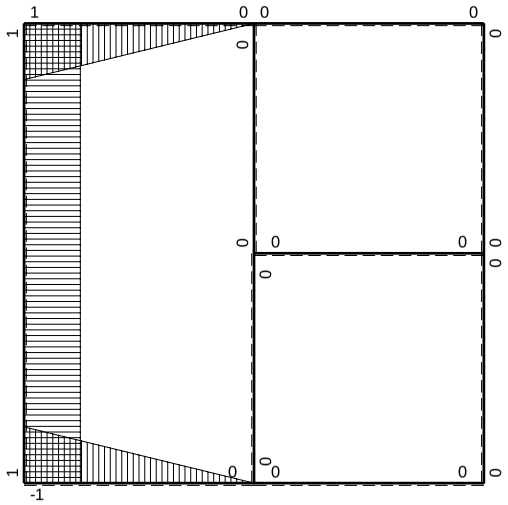

1/4
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-9/5
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24/5
-1
24/5
0
-1/4
-1/4
F
1
0
-7/10
-1/4
0
1/4
3/4
0
-1/4
2
-9/5
-14/5
F
1/5
0
-1/2
-1/2
0
0
1/4
0
0
-1/4
0
0
0
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211257 ESΣ03.xxxx.079

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
EH b	0	-1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

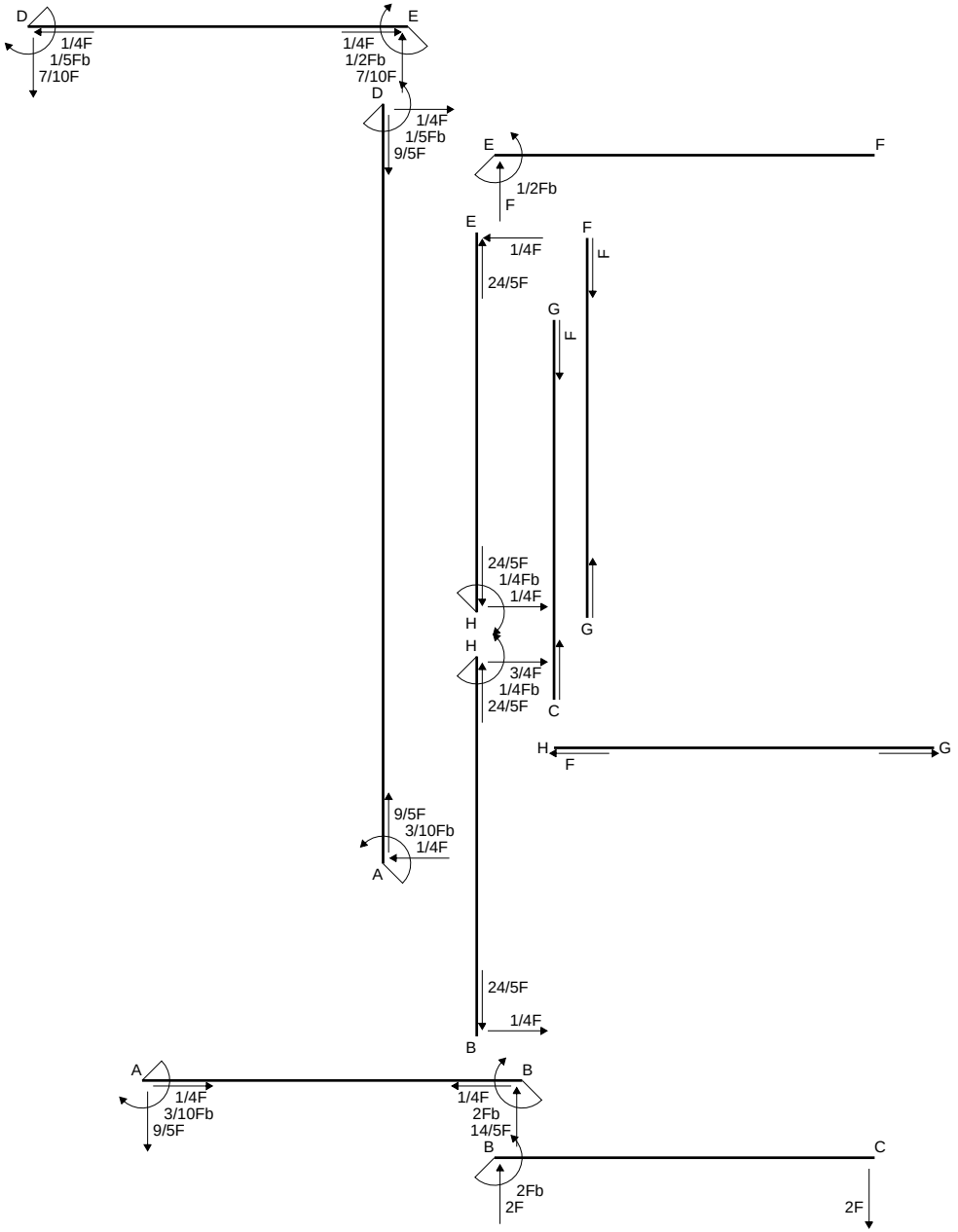
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

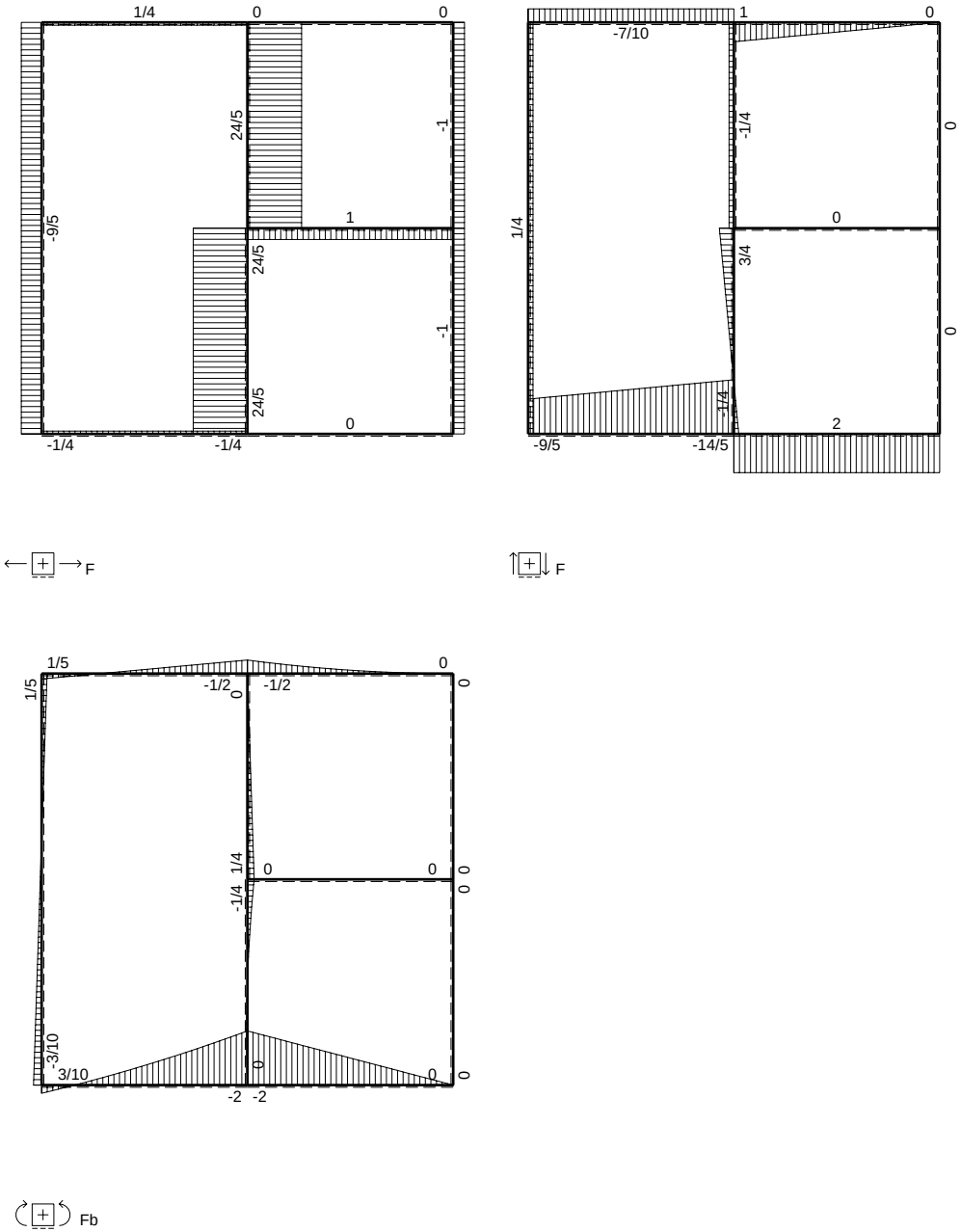
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

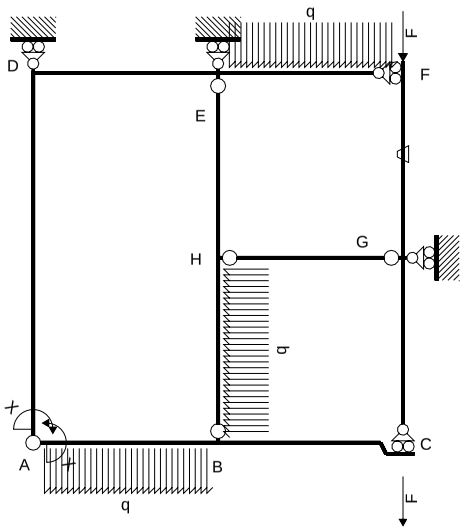
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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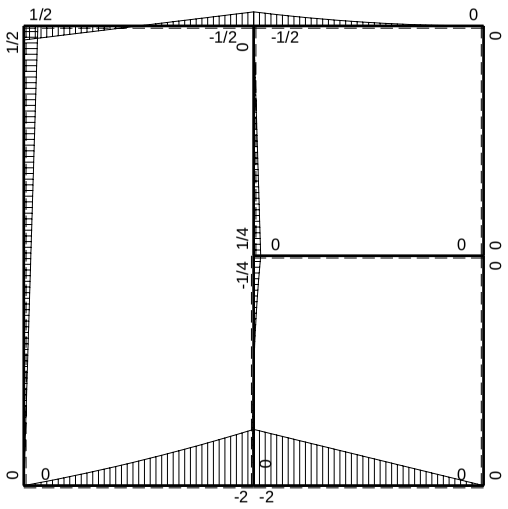
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D
E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
E
F
1/2Fb
F
E
1/4F
24/5F
F
F
G
F
24/5F
1/4Fb
1/4F
H
G
H
3/4F
1/4Fb
24/5F
C
H
F
G
9/5F
3/10Fb
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A
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C
2Fb
2F
2F

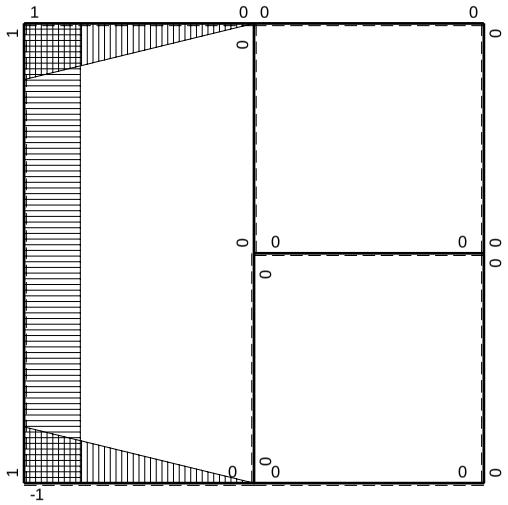

1/4
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24/5
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24/5
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24/5
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-1/4
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2
-9/5
-14/5
F
1/5
0
-1/2
-1/2
0
0
1/4
0
0
-1/4
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238991 ESΣ03.xxxx.080

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

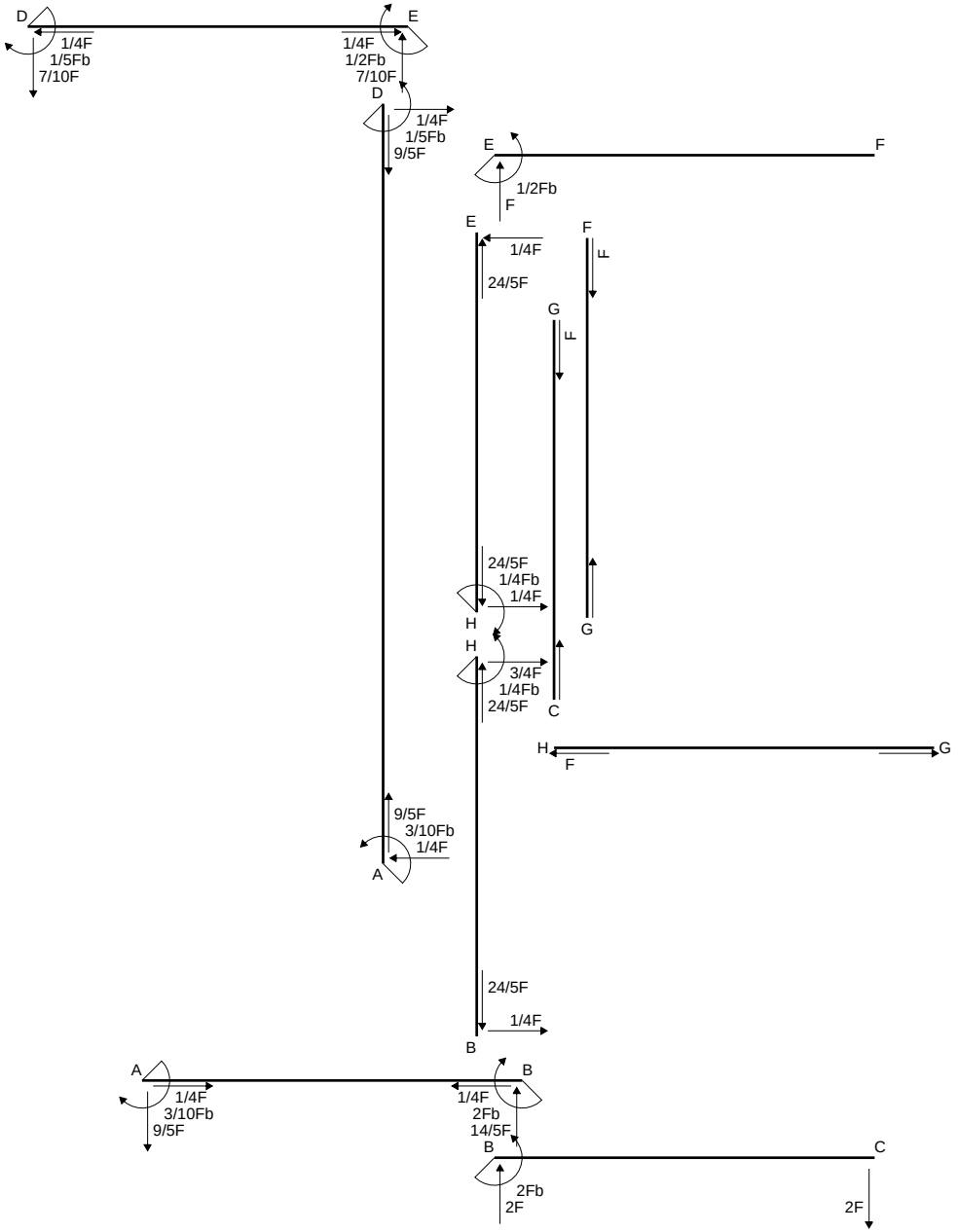
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

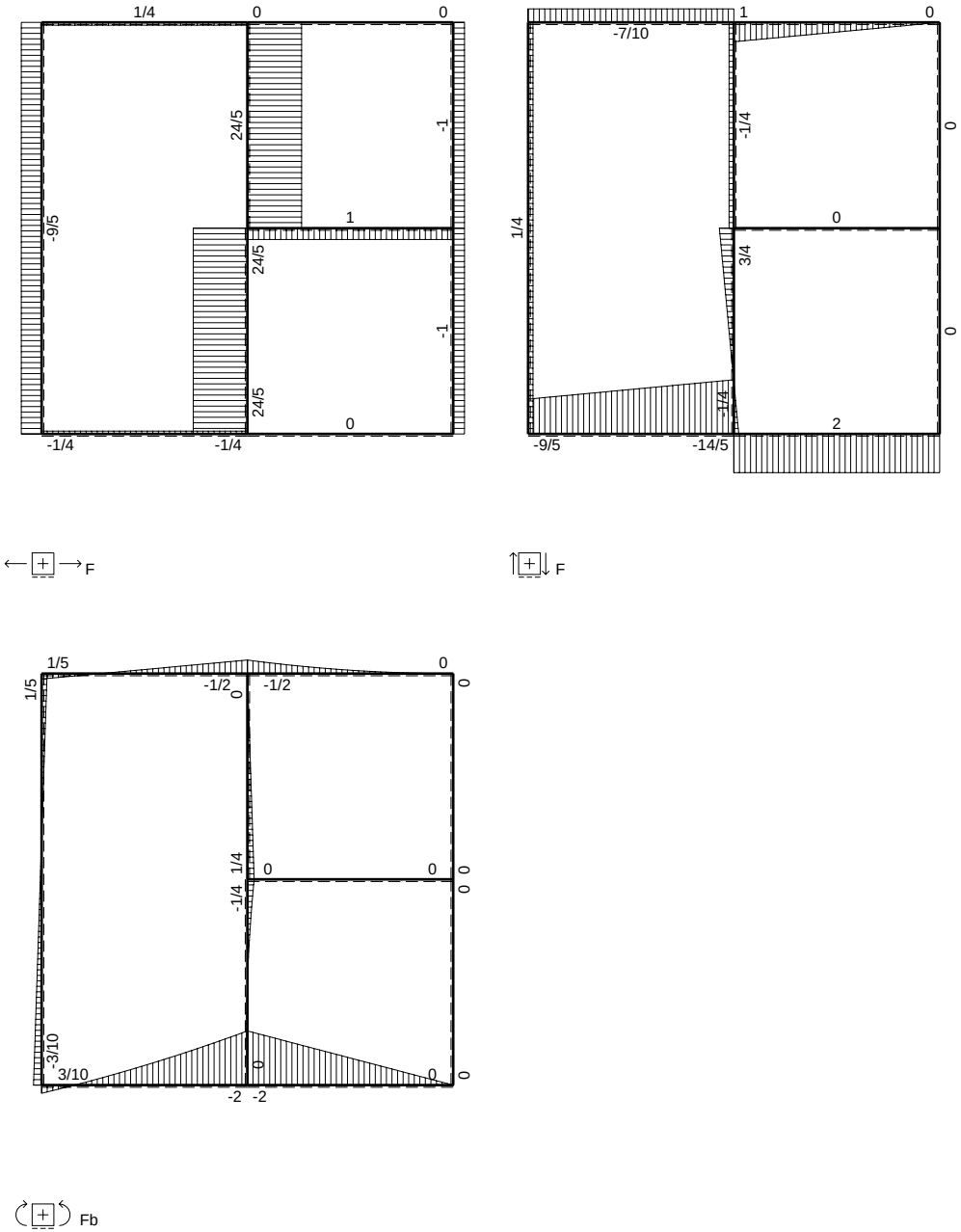
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

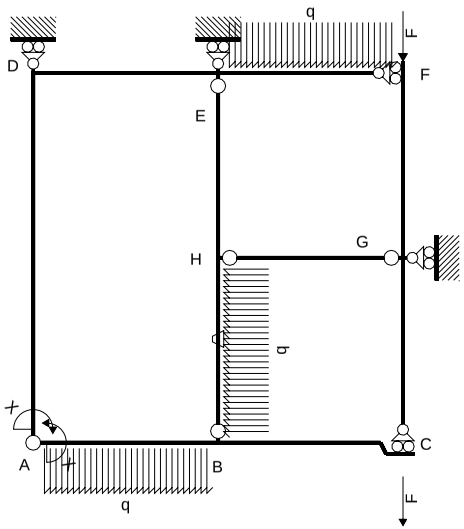
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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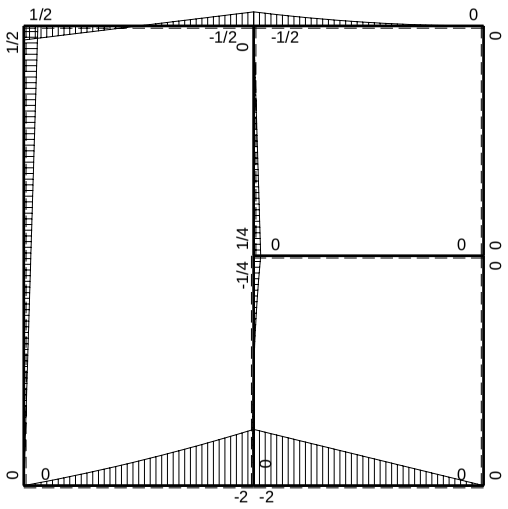
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D
E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
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2Fb
2F
2F

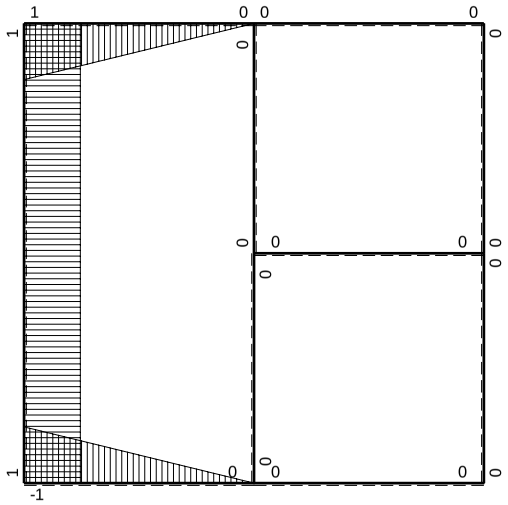

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-1
-9/5
1
24/5
-1
24/5
0
-1/4
-1/4
F
1
-7/10
0
1/4
-1/4
0
3/4
0
-1/4
2
-9/5
-14/5
F
1/5
0
1/5
-1/2
0
-1/2
0
0
1/4
0
0
-1/4
0
0
-3/10
3/10
0
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237525 ESΣ03.xxxx.081

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
EH b	0	-1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 237525 ESΣ03.xxxx.081
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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

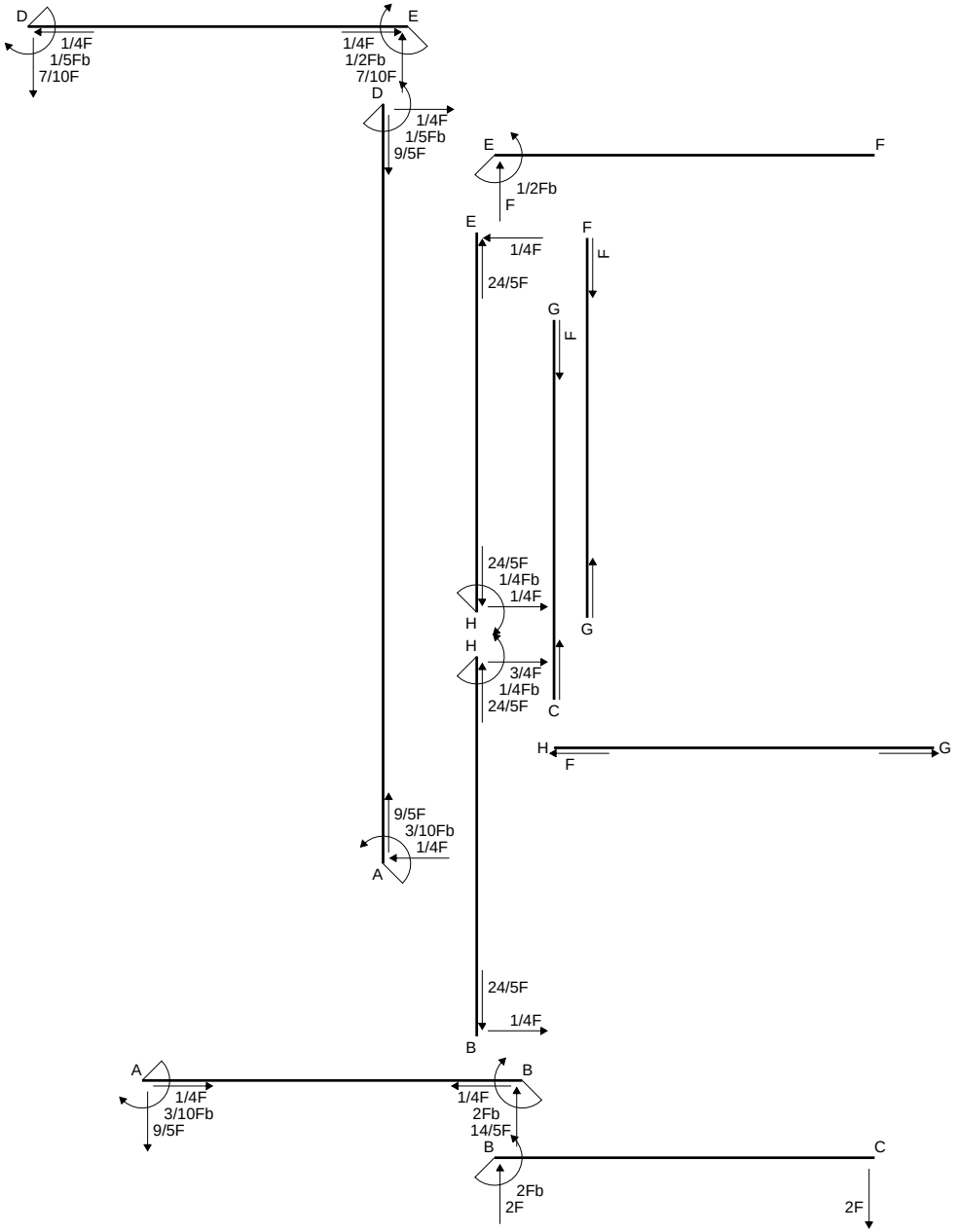
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

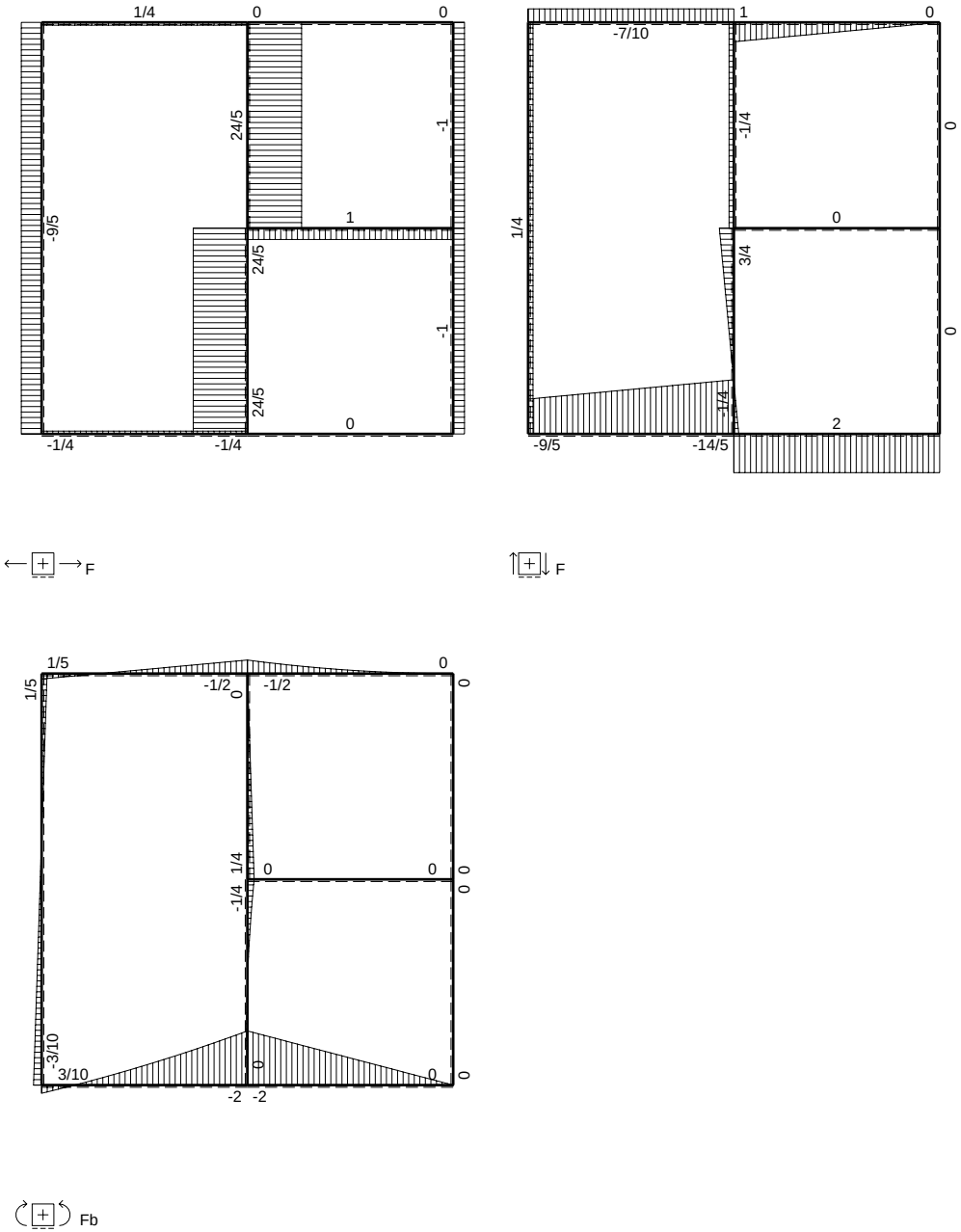
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

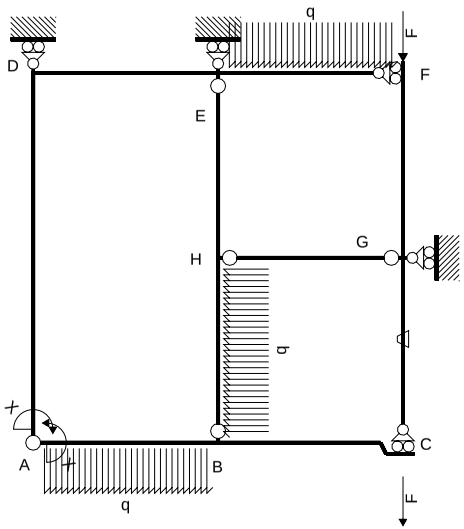
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 237525 ESΣ03.xxxx.081

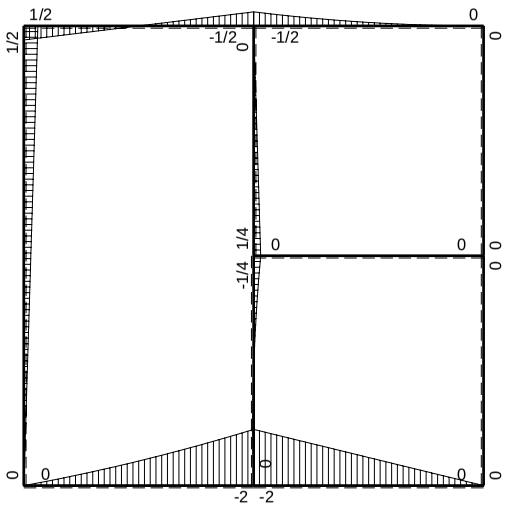
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D
E
1/4F
1/5Fb
7/10F
1/4F
1/2Fb
7/10F
D
1/4F
1/5Fb
9/5F
E
F
1/2Fb
F
E
1/4F
24/5F
F
F
G
F
24/5F
1/4Fb
1/4F
H
G
H
3/4F
1/4Fb
24/5F
C
H
F
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C
2Fb
2F
2F

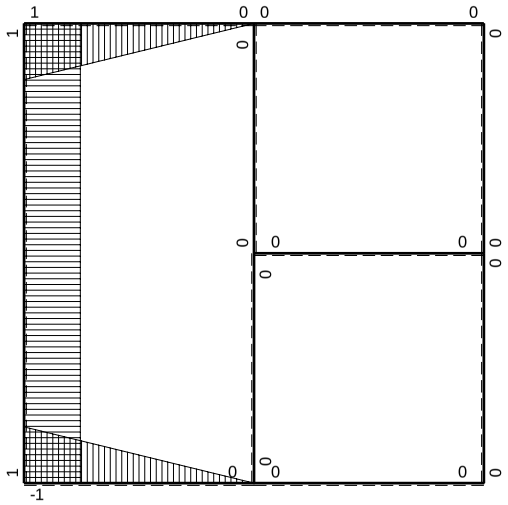

1/4
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-1
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24/5
-1
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-14/5
F
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0
-1/2
-1/2
0
0
1/4
0
0
-1/4
0
0
0
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 252216 ESΣ03.xxxx.082

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 252216 ESΣ03.xxxx.082

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

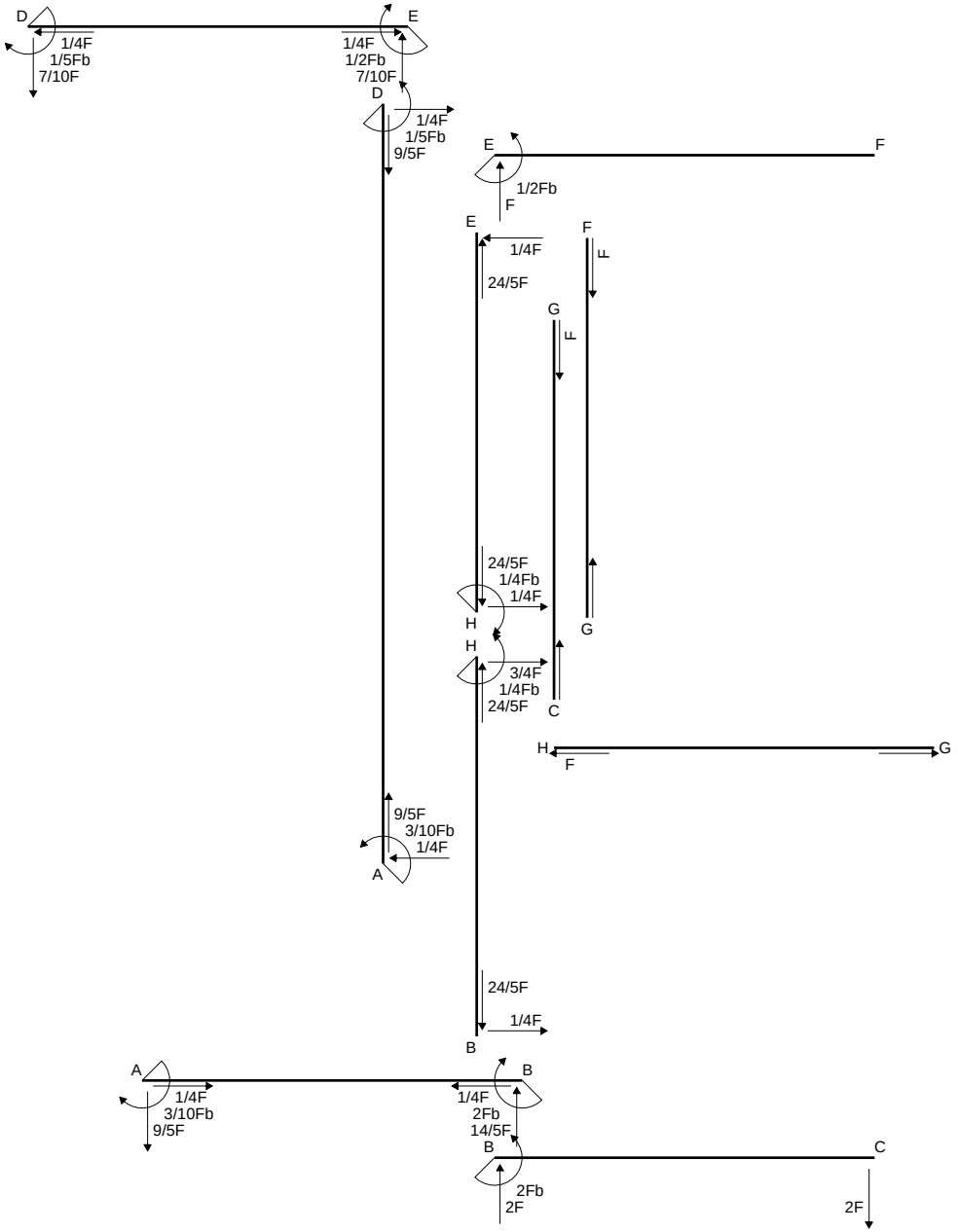
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

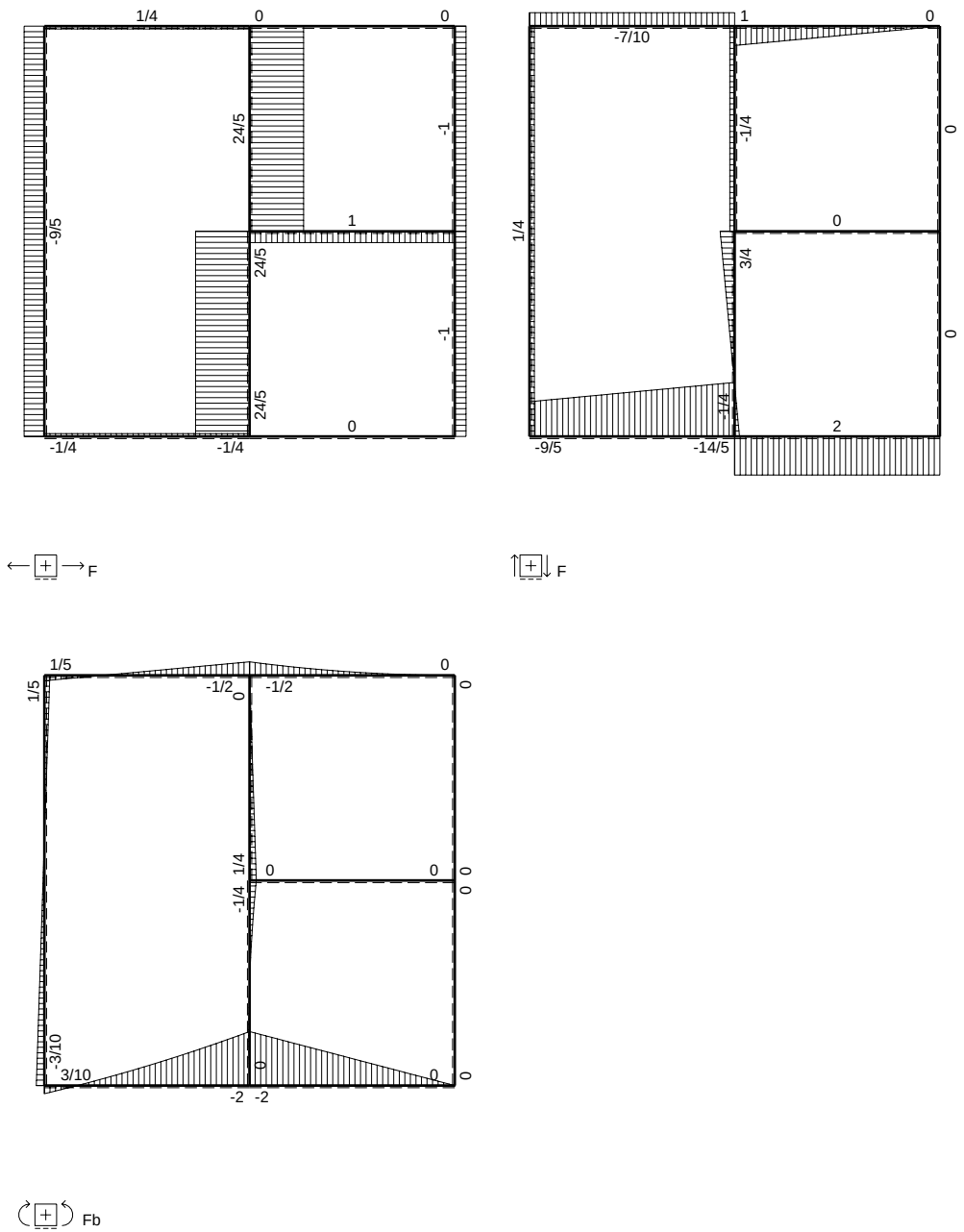
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

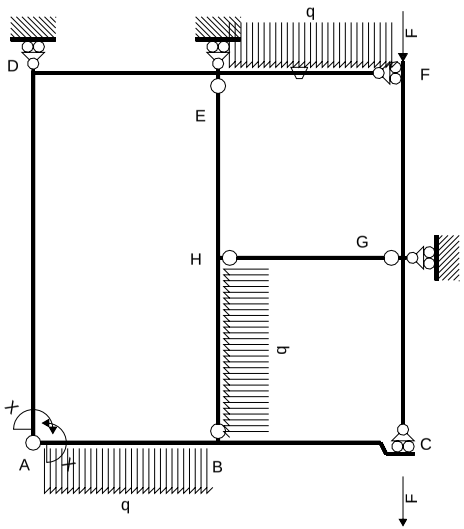
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 252216 ESΣ03.xxxx.082

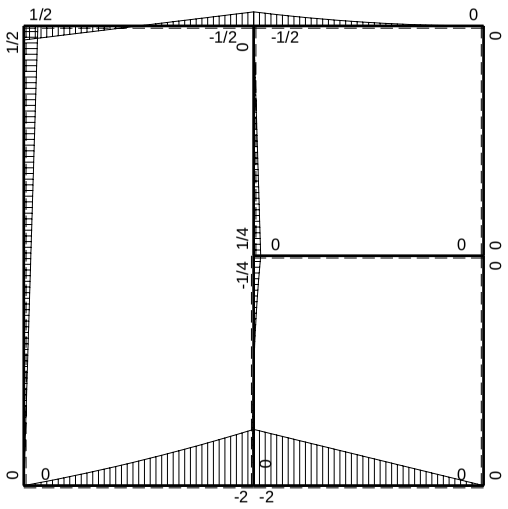
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D
E
1/4F
1/5Fb
7/10F
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1/2Fb
7/10F
D
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2F
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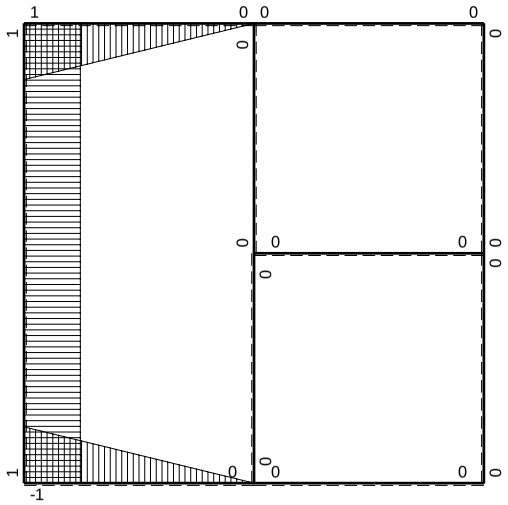

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24/5
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3/4
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2
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-14/5
F
1/5
0
-1/2
-1/2
0
0
1/4
0
0
-1/4
0
0
0
-3/10
3/10
0
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236511 ESΣ03.xxxx.083

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PROCEDIMENTO E RISULTATI 236511 ESΣ03.xxxx.083

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
FE b	0	$1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
EH b	0	-1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

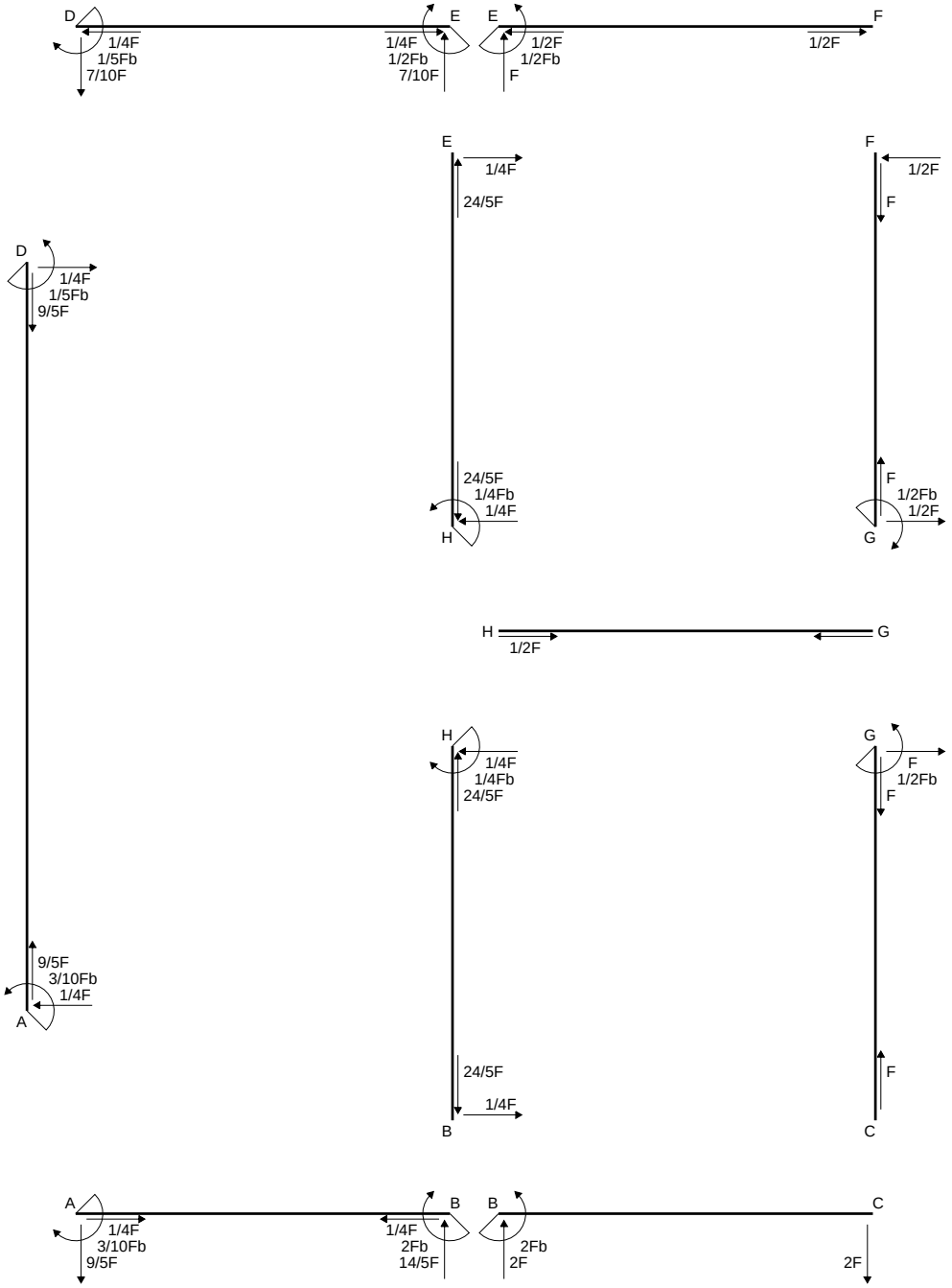
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

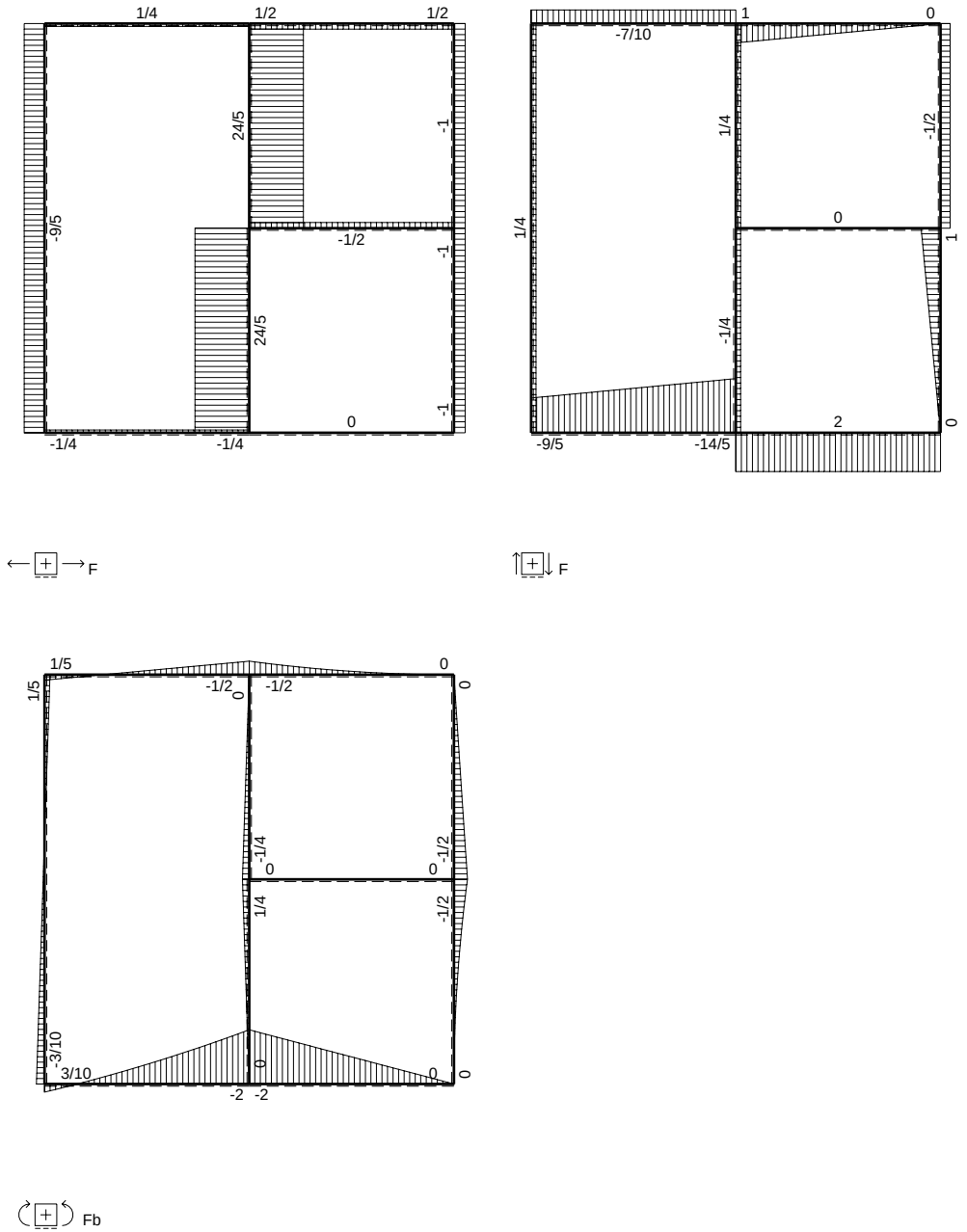
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

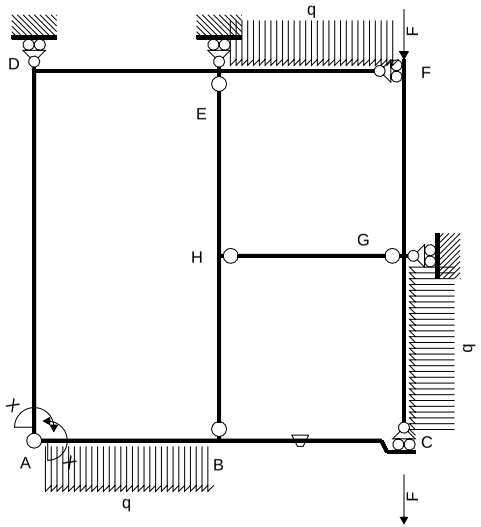
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

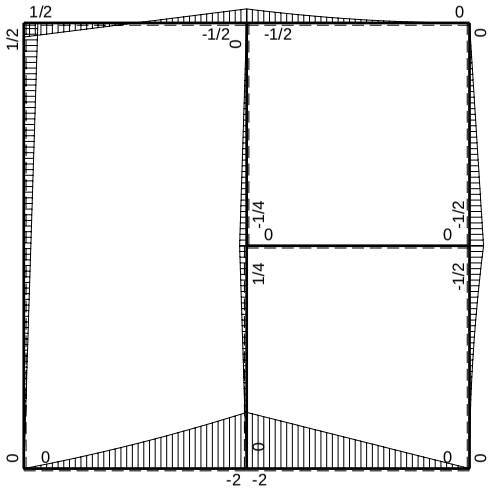
PROCEDIMENTO E RISULTATI 236511 ESΣ03.xxxx.083

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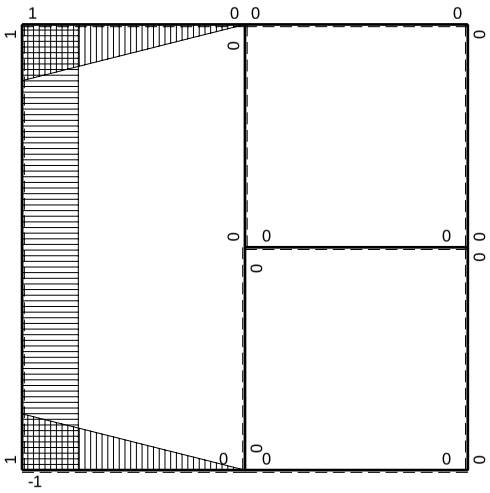

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-1/2
0
0
1/4
-1/2
-3/10
0
0
3/10
0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236427 ESΣ03.xxxx.084

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PROCEDIMENTO E RISULTATI 236427 ESΣ03.xxxx.084

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\ b + 1/8\ b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b)\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (\,b - 1/2\ b\,)\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

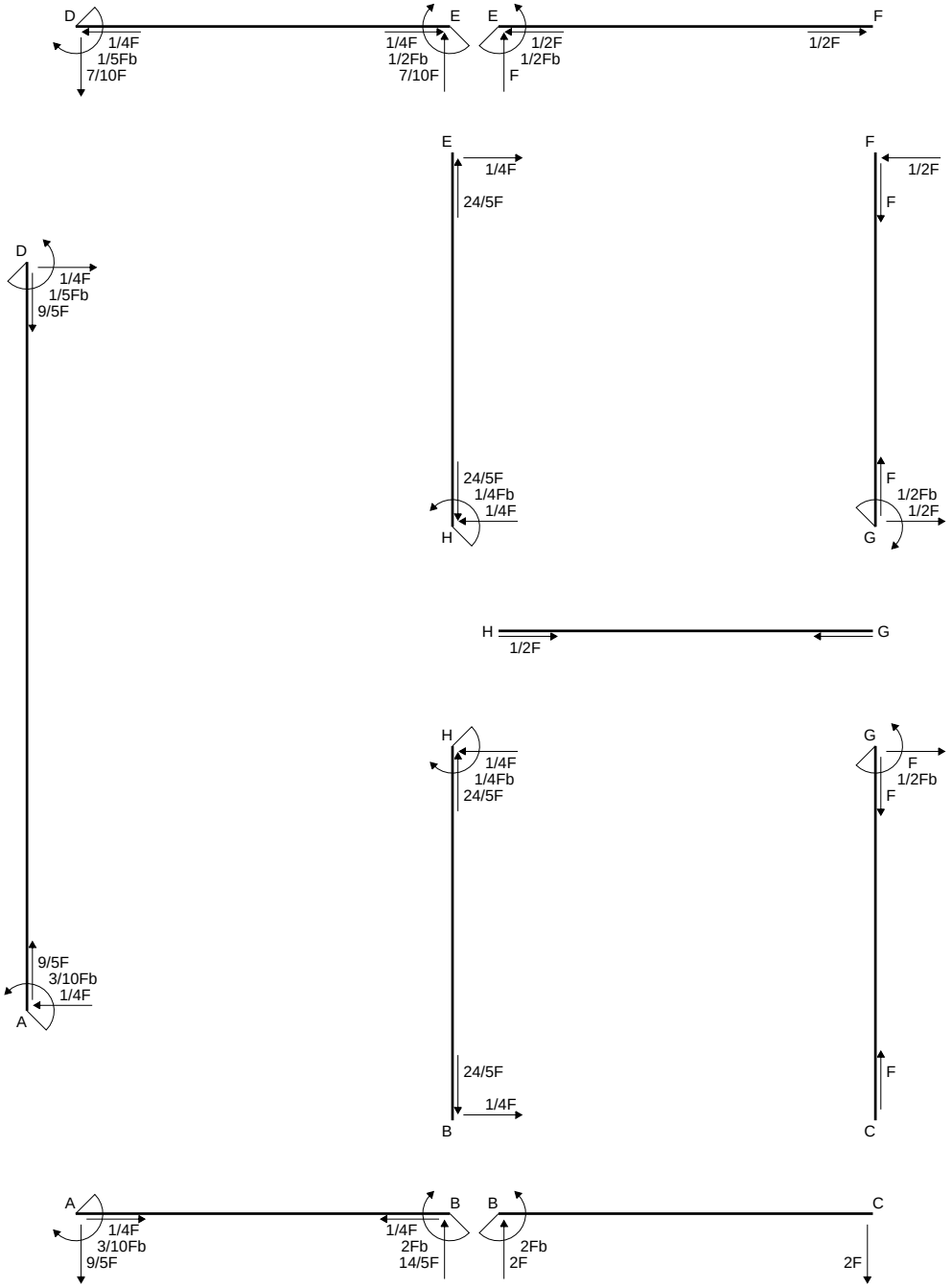
$= (1/2\ b - 3/4\ b + 1/3\ b)\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

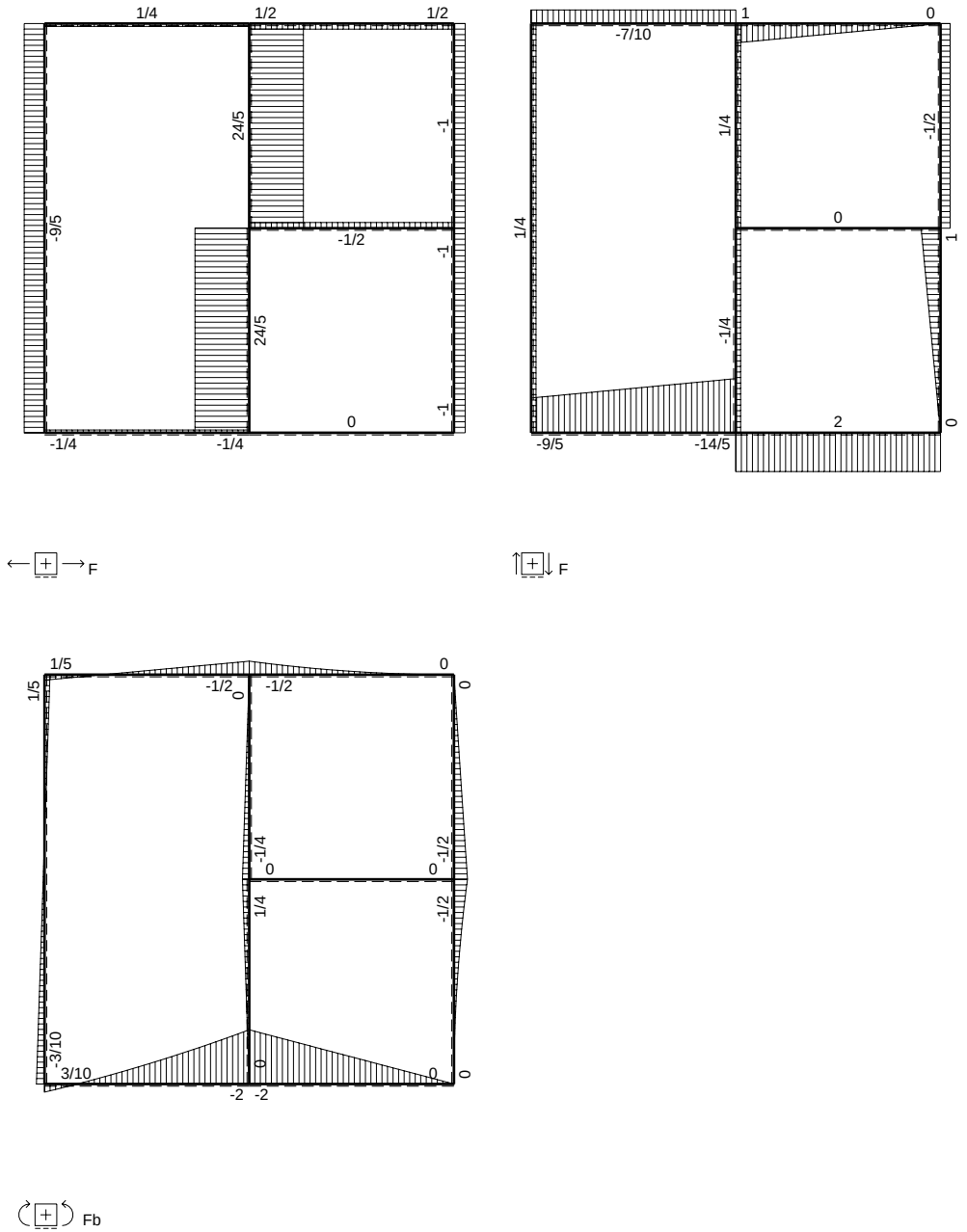
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

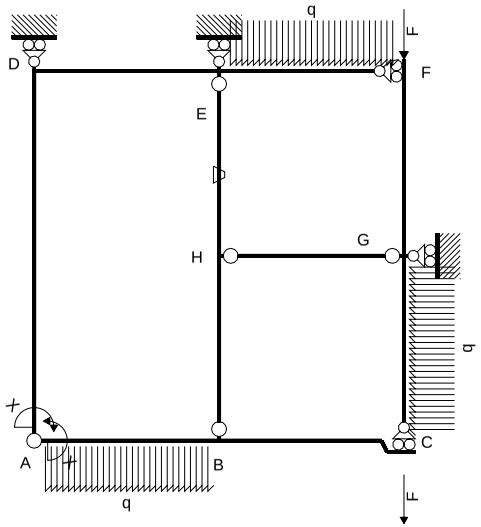
$= (-1/4\ b + 1/3\ b)\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

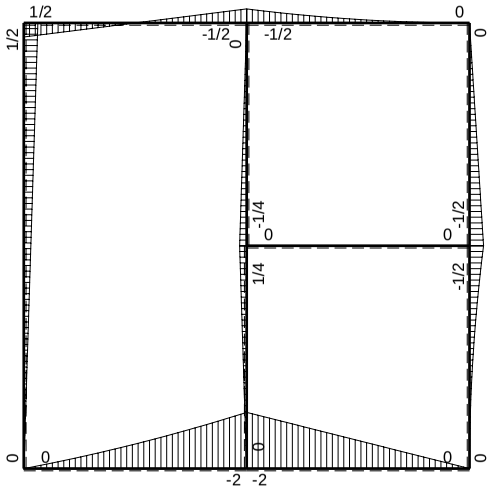
PROCEDIMENTO E RISULTATI 236427 ESΣ03.xxxx.084

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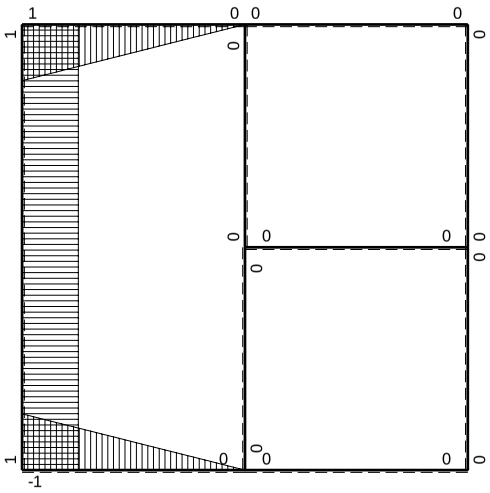

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2F
C
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0
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0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237813 ESΣ03.xxxx.085

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PROCEDIMENTO E RISULTATI 237813 ESΣ03.xxxx.085

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$1/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

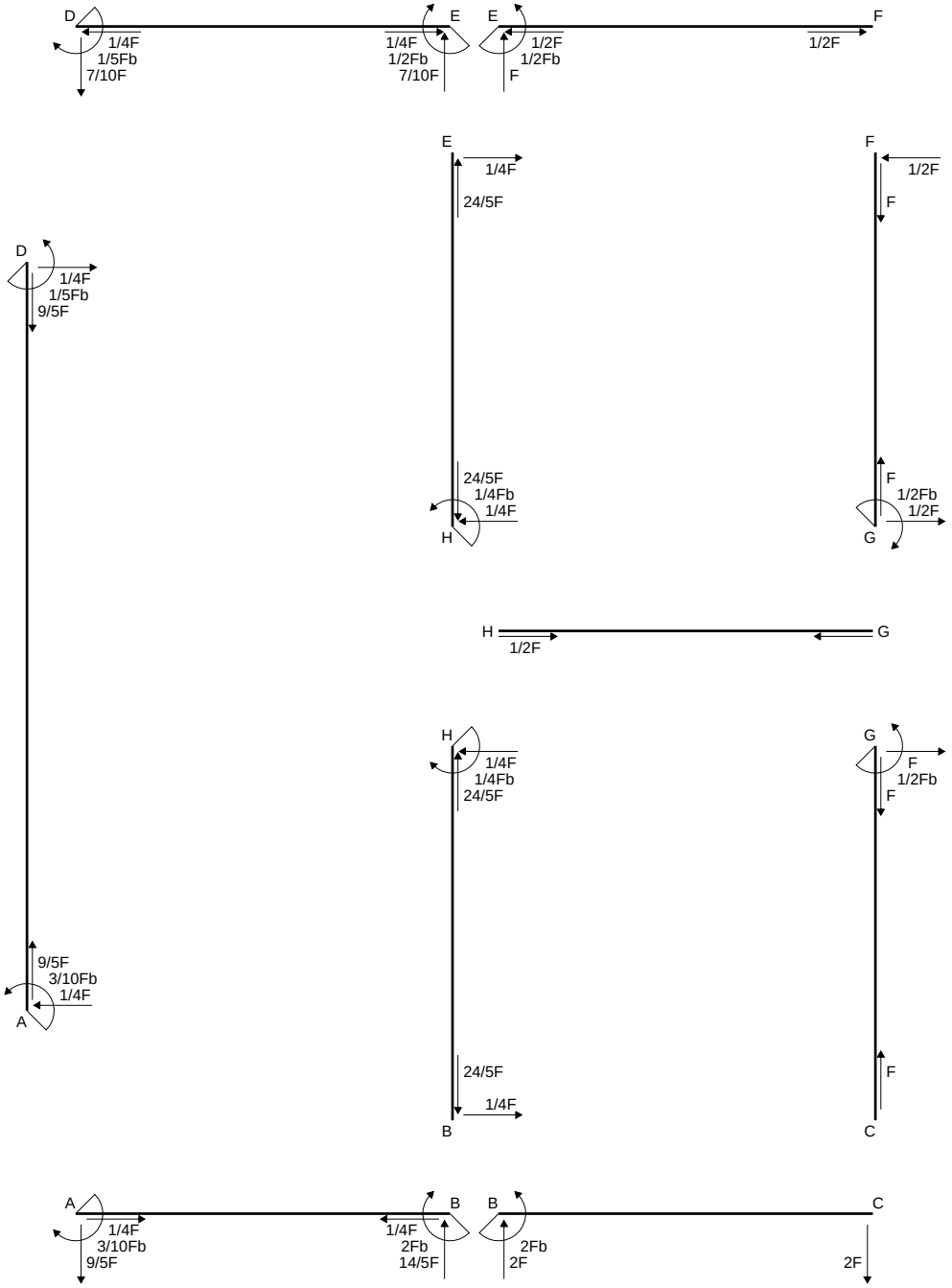
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

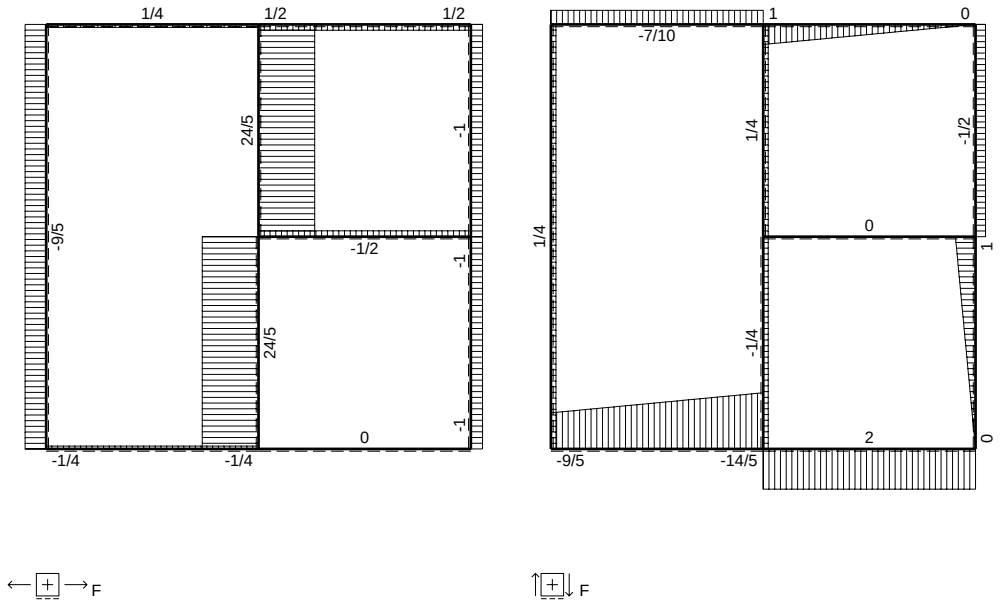
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

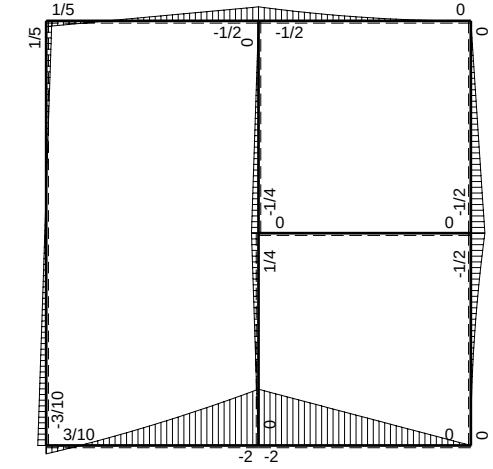
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

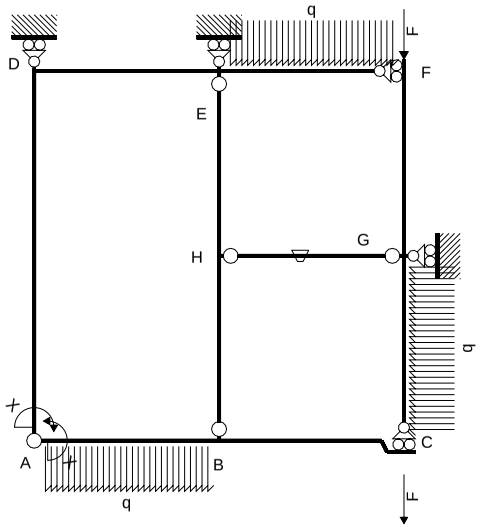
PROCEDIMENTO E RISULTATI 237813 ESΣ03.xxxx.085

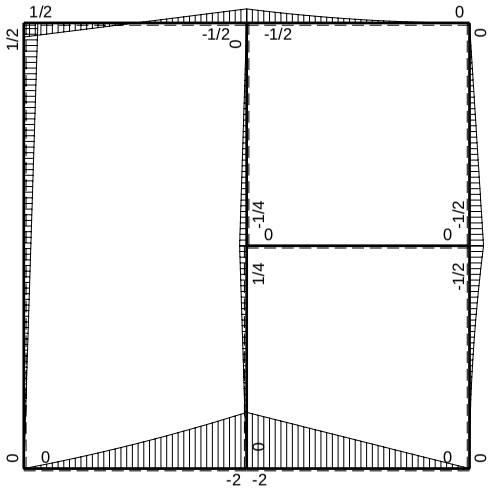
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D
1/4F
1/5Fb
7/10F
E
1/4F
1/2Fb
7/10F
E
1/2F
1/2Fb
F
F
1/2F
E
1/4F
24/5F
F
D
1/4F
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F
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C
2F

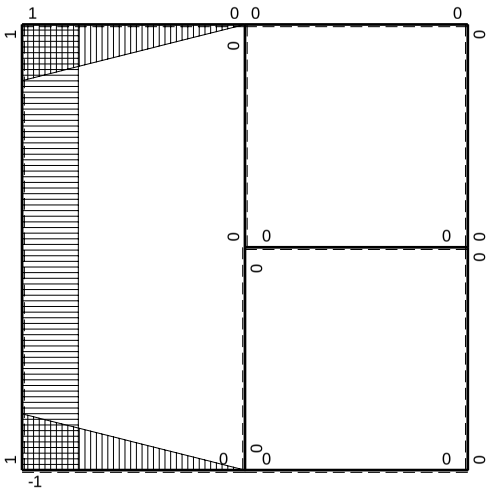

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24/5
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1
0
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1/4
-1/2
0
1/4
1
-1/4
0
2
-9/5
-14/5
F
F


1/5
0
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-1/2
1/5
0
0
-1/4
-1/2
0
0
1/4
-1/2
-3/10
0
0
3/10
0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237504 ESΣ03.xxxx.086

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	$1-x/b$	1/2Fb-Fx	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	1/2Fb-Fx	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
BH b	0	-1/4Fx	0	0	0	0		
HE b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
EH b	0	1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/8Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-3/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

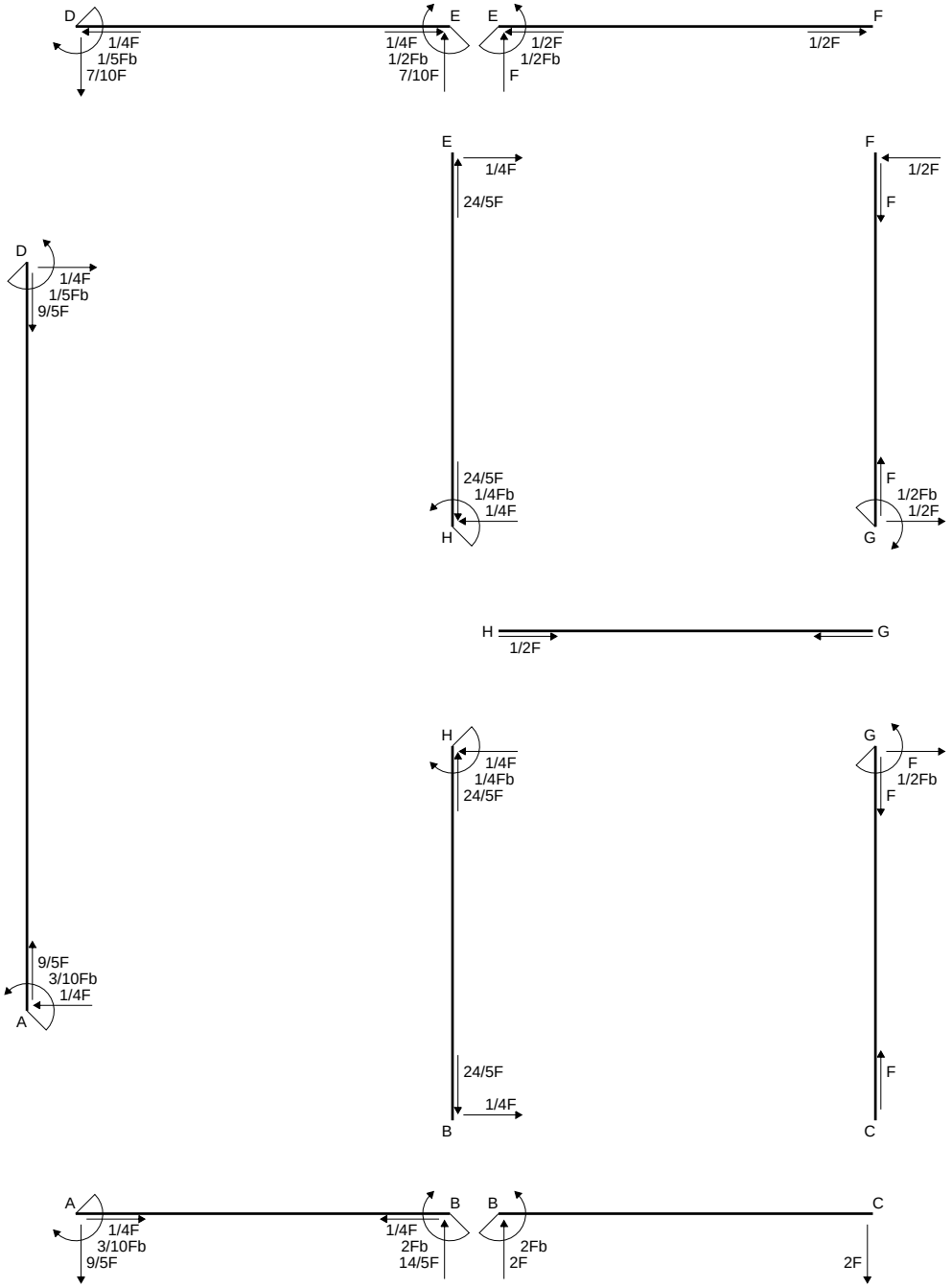
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

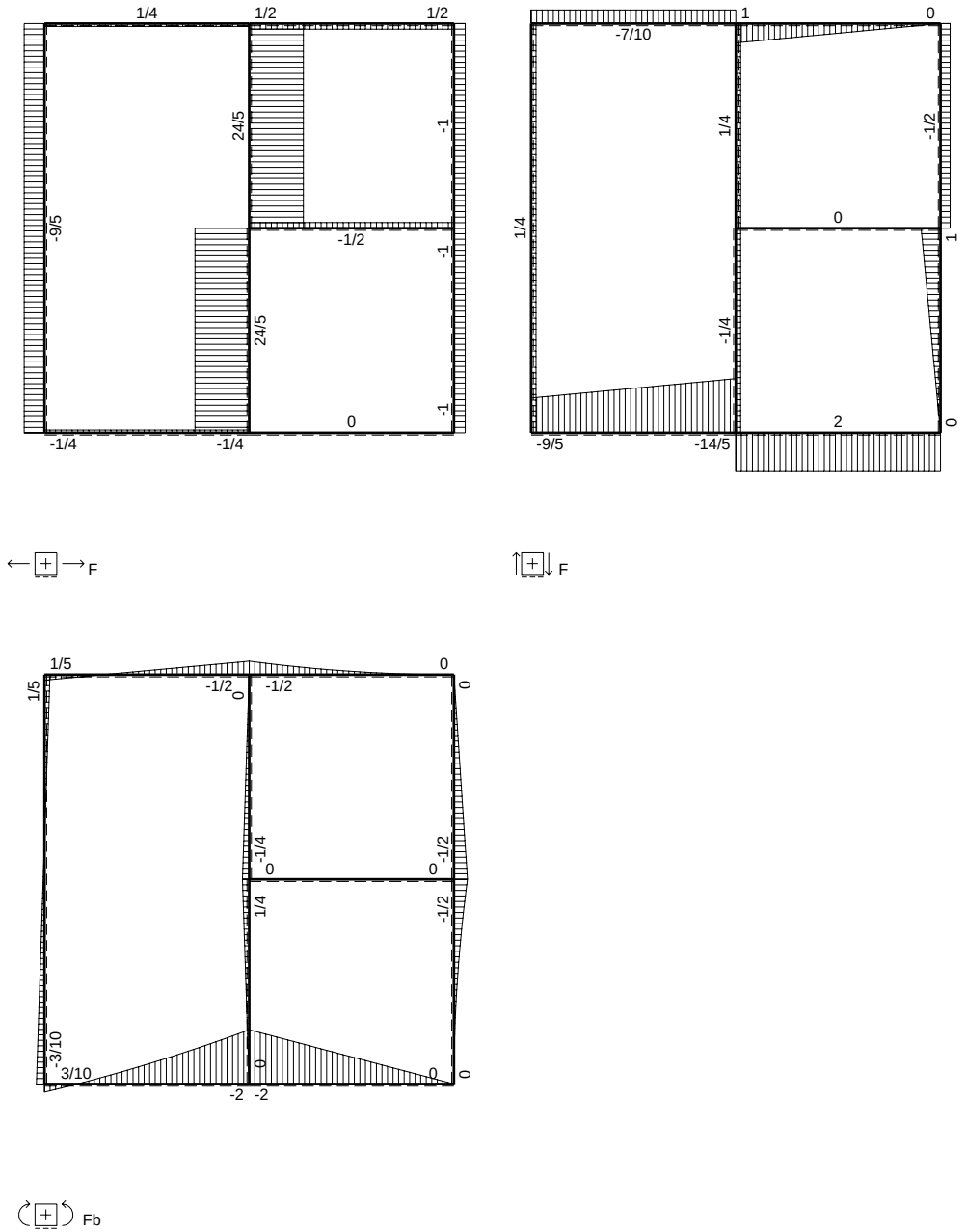
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

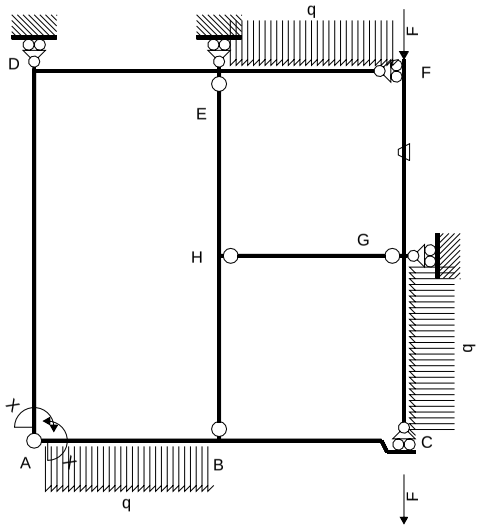
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

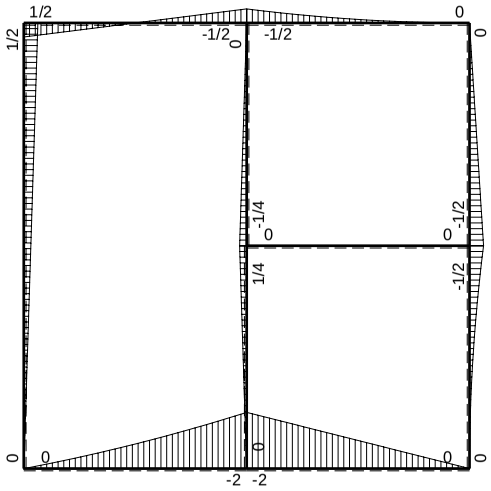
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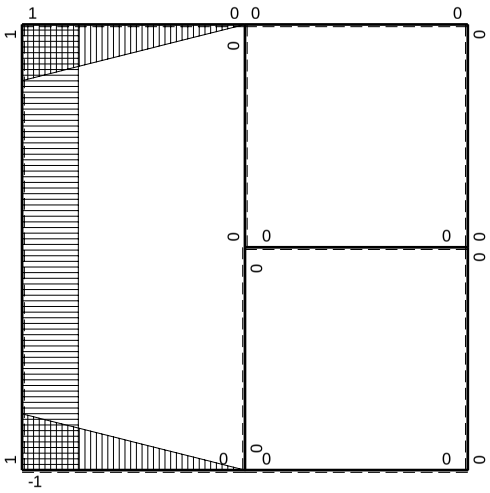

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1/5Fb
7/10F
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-1/2
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3/10
0
0
0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237785 ESΣ03.xxxx.087

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PROCEDIMENTO E RISULTATI 237785 ESΣ03.xxxx.087

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	Fb/EJ	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

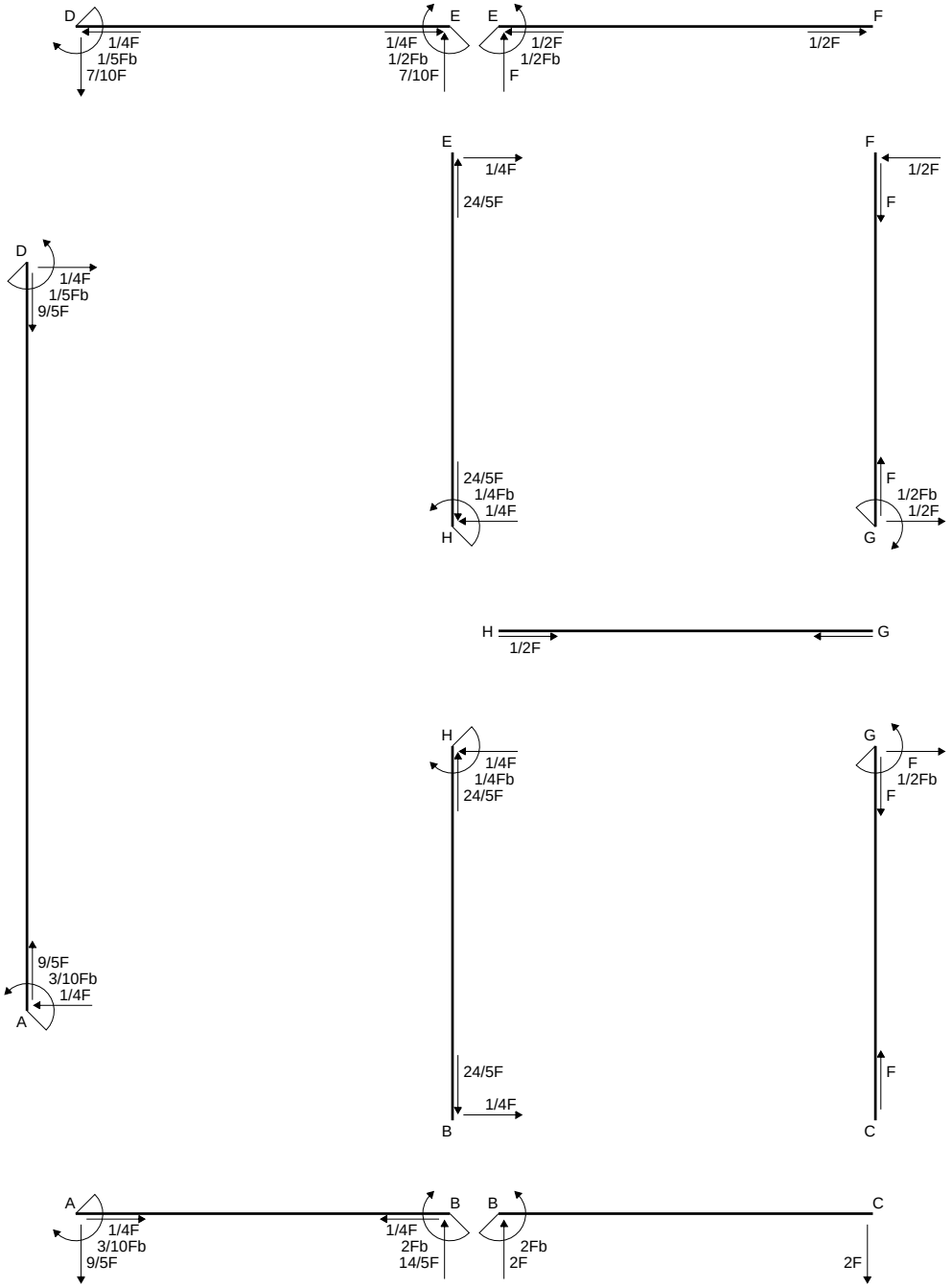
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

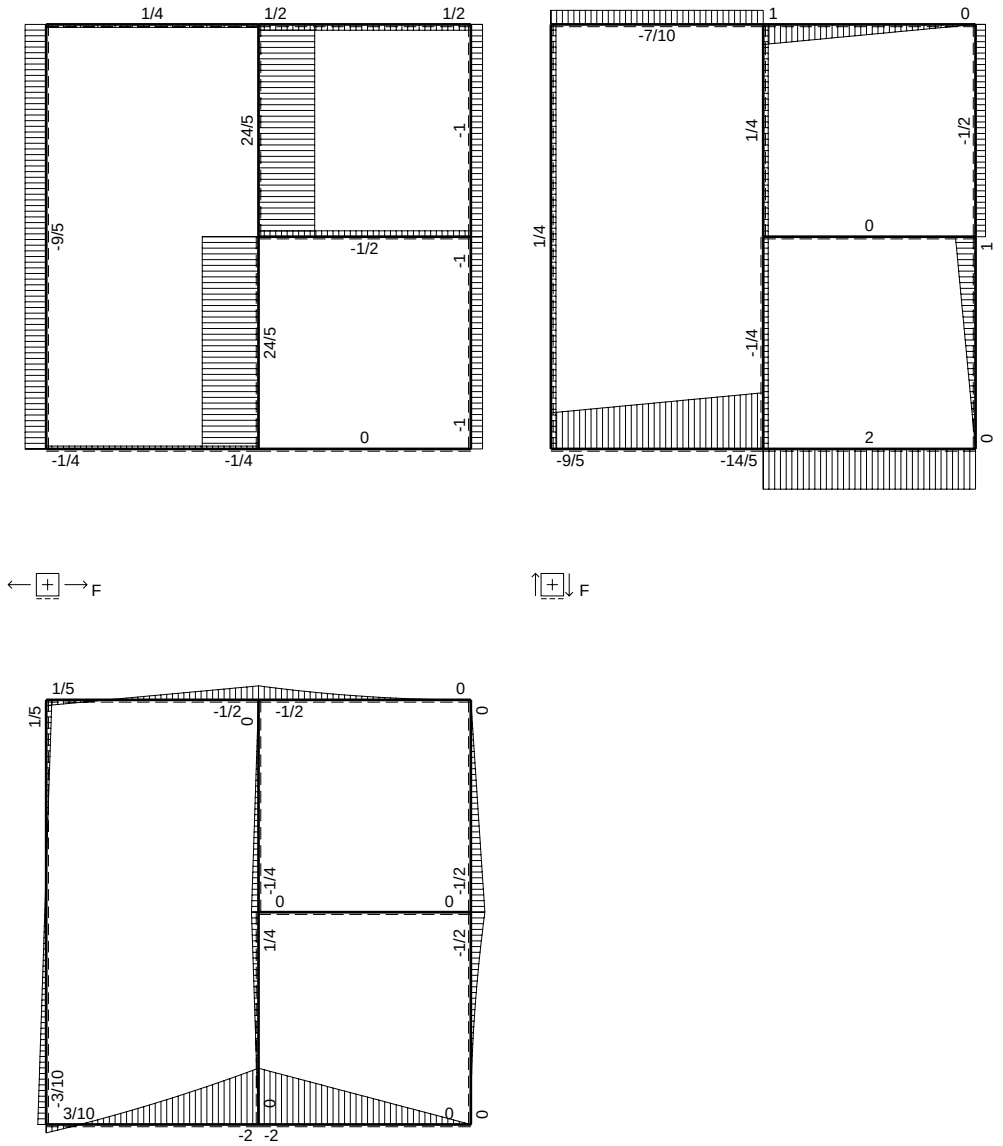
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

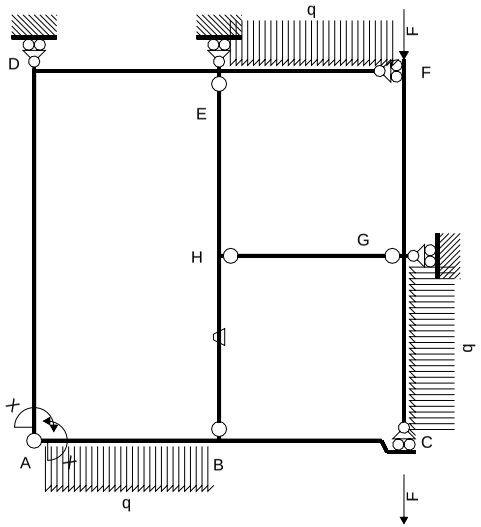
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

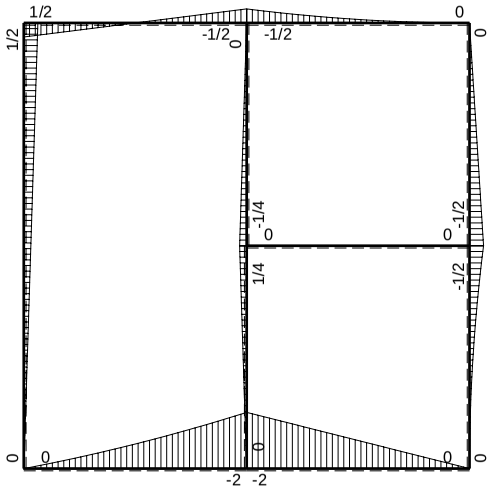
PROCEDIMENTO E RISULTATI 237785 ESΣ03.xxxx.087

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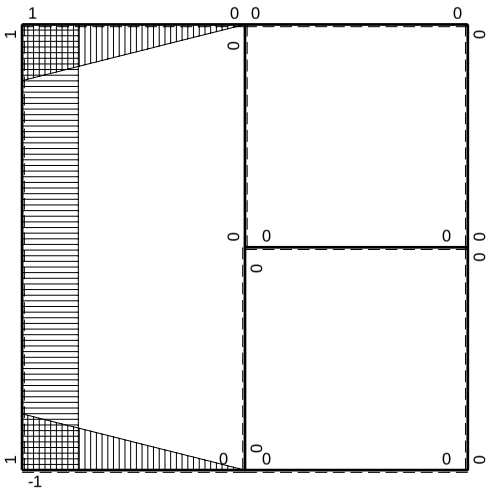

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1/5Fb
7/10F
E
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1/2Fb
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E
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2Fb
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C
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0
0
1/4
-1/2
-3/10
3/10
0
0
0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236384 ESΣ03.xxxx.088

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$-1/4Fx$	Fb/EJ	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

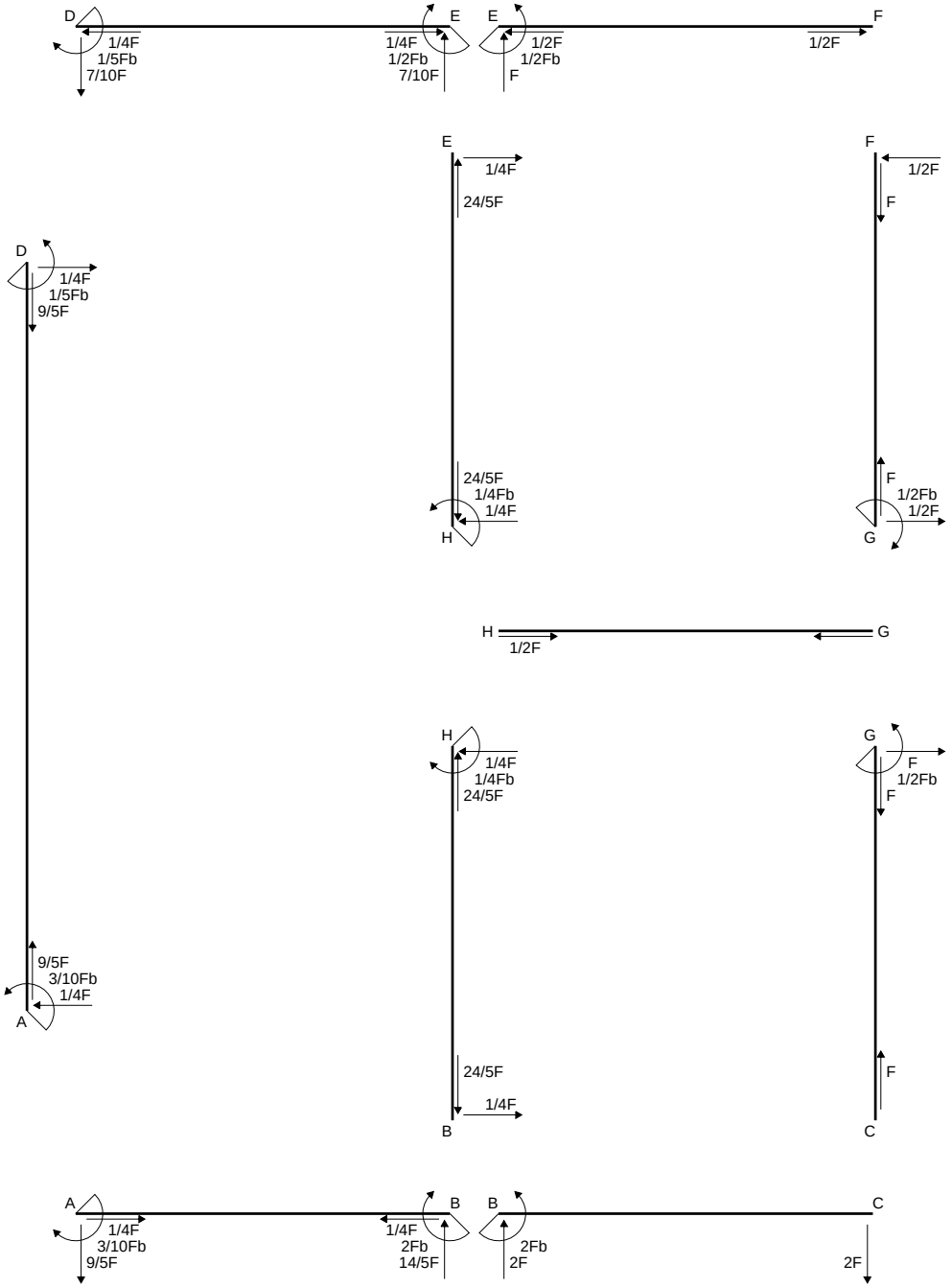
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

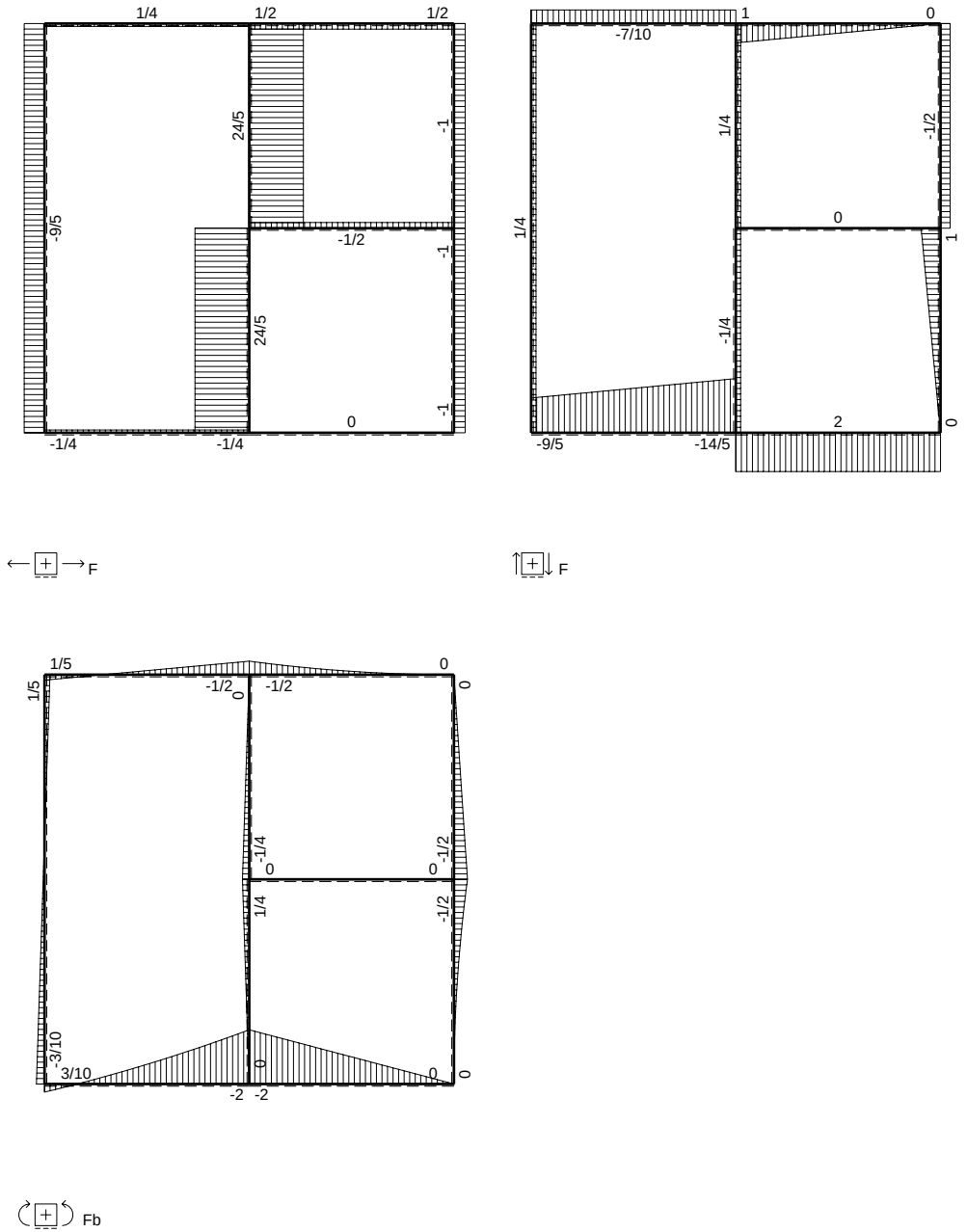
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

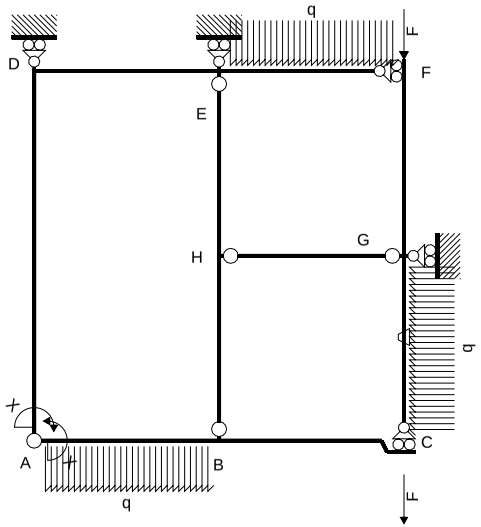
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

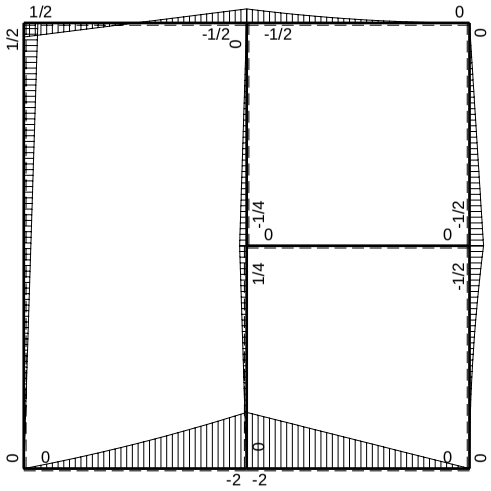
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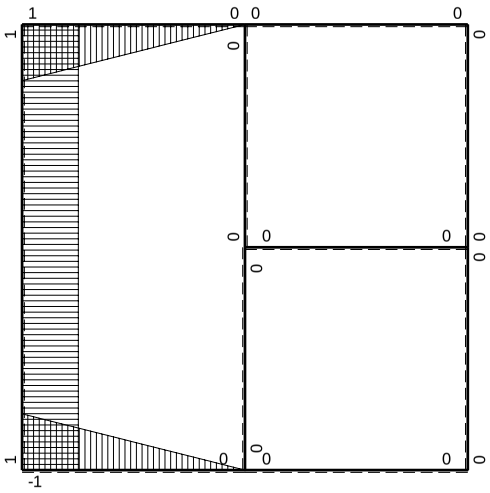

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-3/10
0
0
3/10
0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239159 ESΣ03.xxxx.089

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

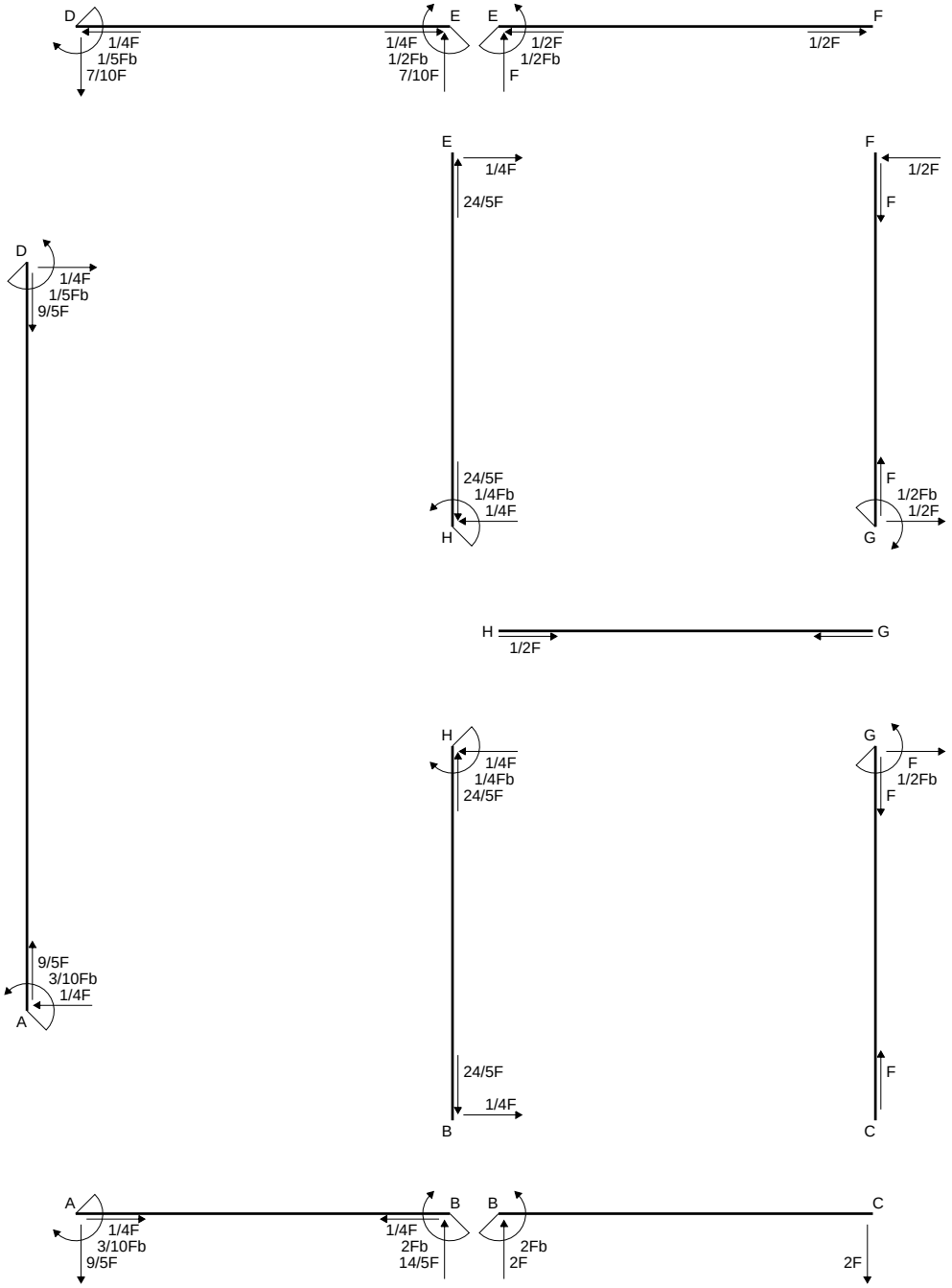
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

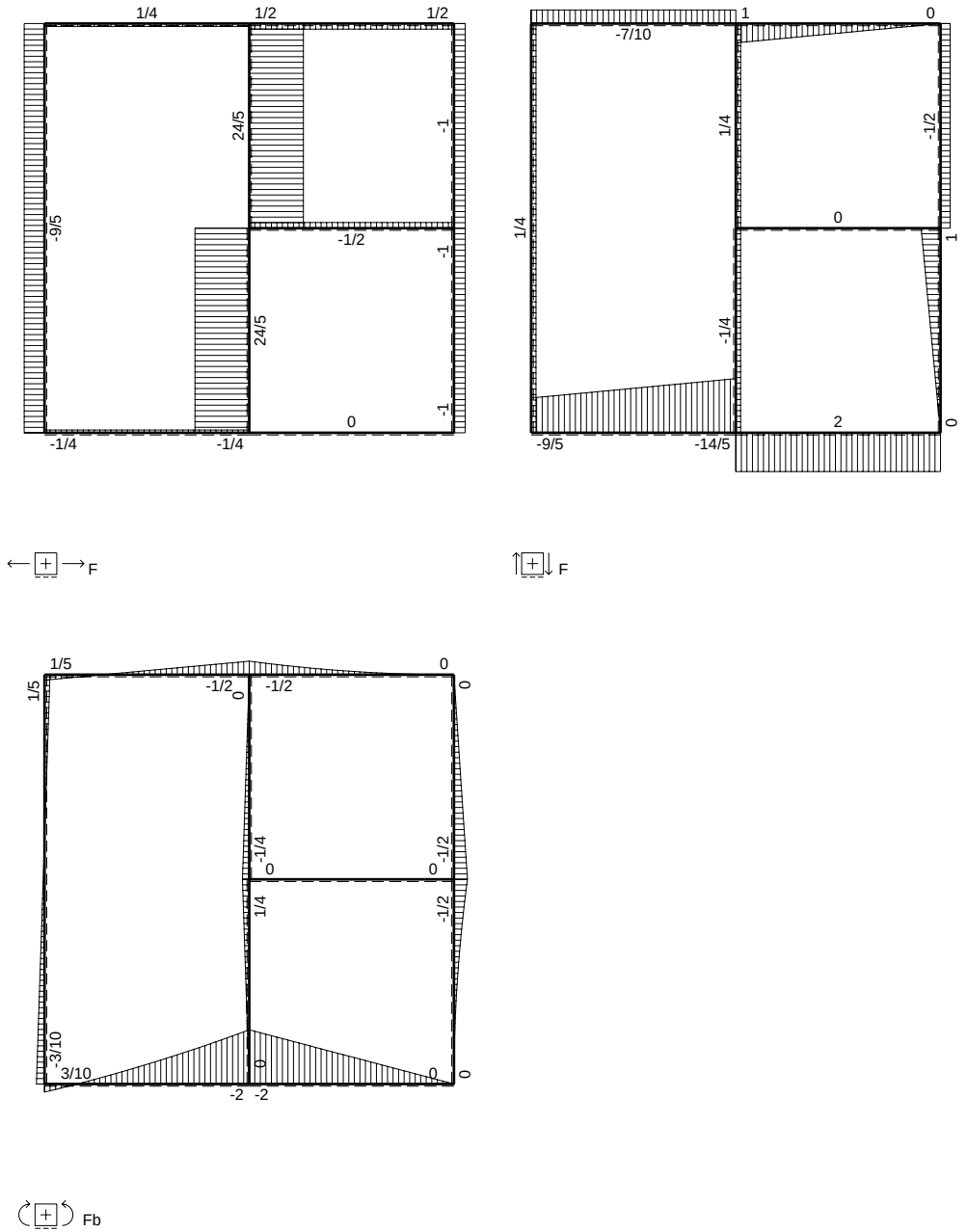
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

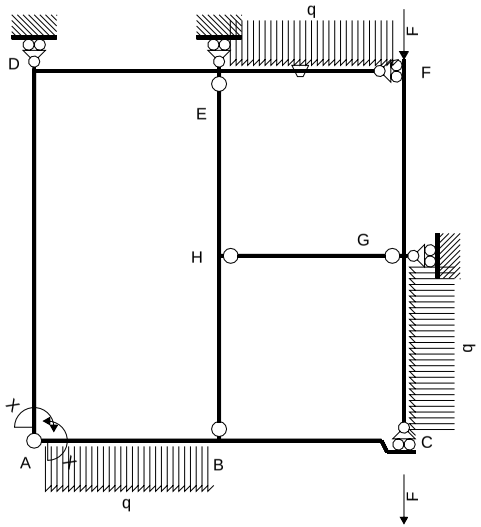
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

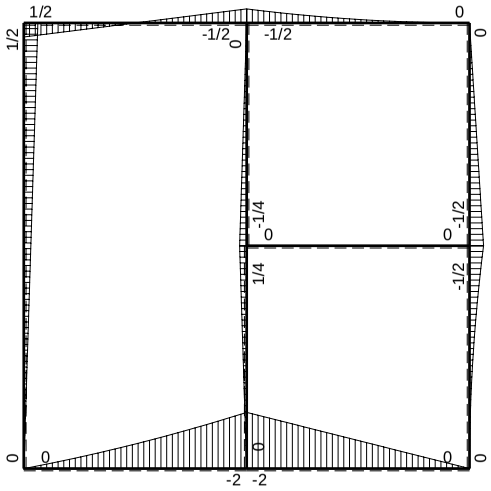
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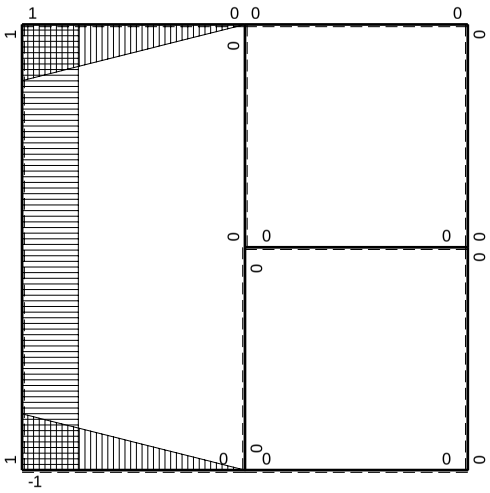

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1/5Fb
7/10F
E
1/4F
1/2Fb
7/10F
E
1/2F
1/2Fb
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F
1/2F
E
1/4F
24/5F
F
1/2F
F
D
1/4F
1/5Fb
9/5F
24/5F
1/4Fb
1/4F
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F
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F
C
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9/5F
B
1/4F
2Fb
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B
2Fb
2F
C
2F


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1/2
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0
-1/4
-1/2
0
0
1/4
-1/2
-3/10
3/10
0
0
0
-2
-2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	$1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

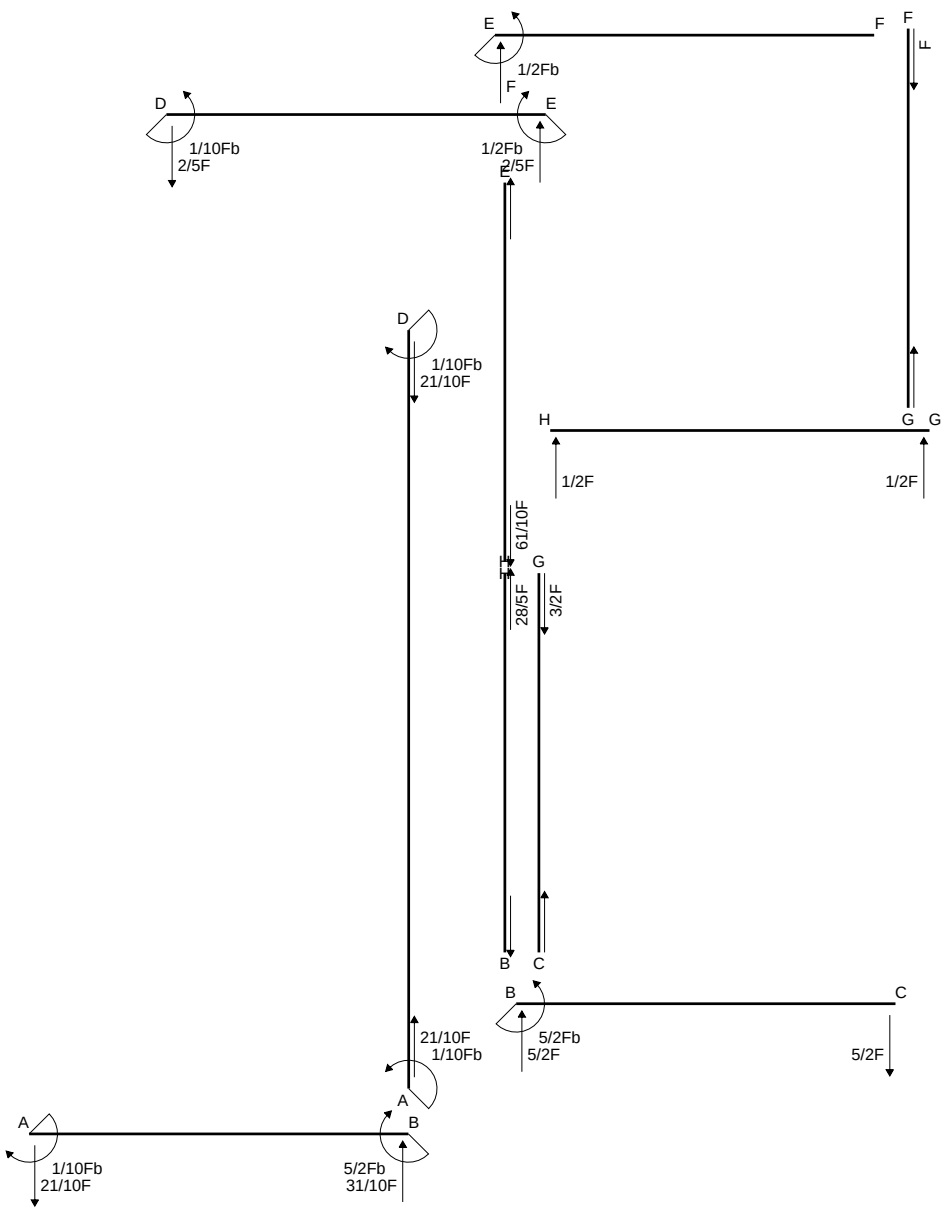
$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

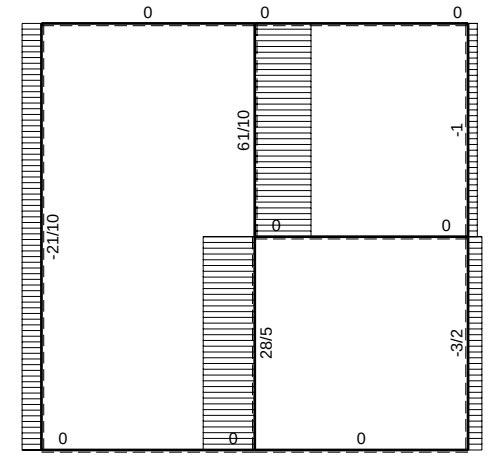
$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

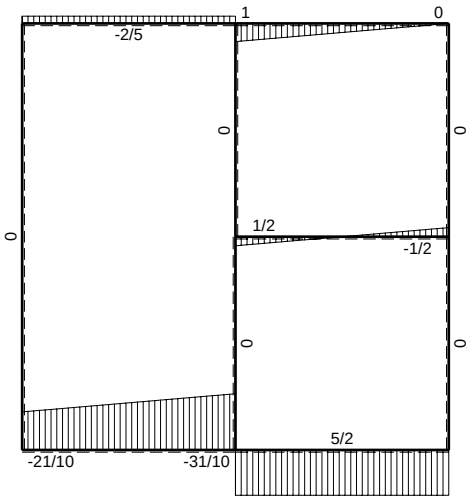
$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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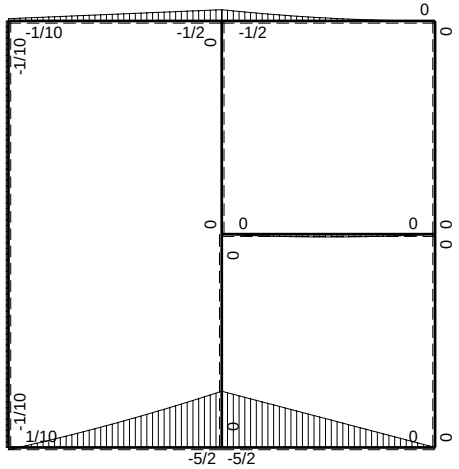

E
F
F
F
1/2Fb
F
E
D
1/10Fb
2/5F
1/2Fb
2/5F
E
D
1/10Fb
21/10F
H
G
G
1/2F
1/2F
61/10F
F
G
28/5F
3/2F
B
C
B
C
21/10F
1/10Fb
5/2Fb
5/2F
5/2F
A
B
A
1/10Fb
21/10F
5/2Fb
31/10F


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0
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61/10
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-21/10
0
0
28/5
-3/2
0
0
0

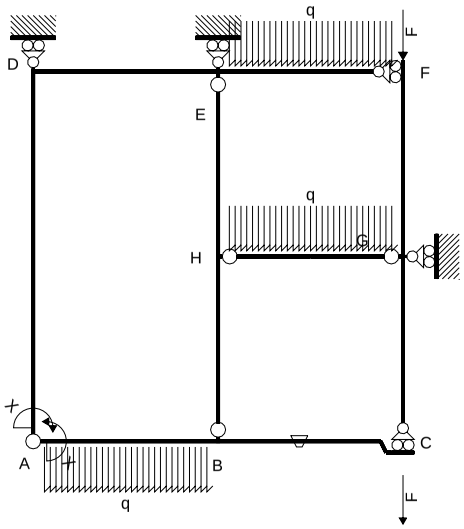

1
0
-2/5
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0
1/2
-1/2
0
0
0
5/2
-21/10
-31/10

←□→ F

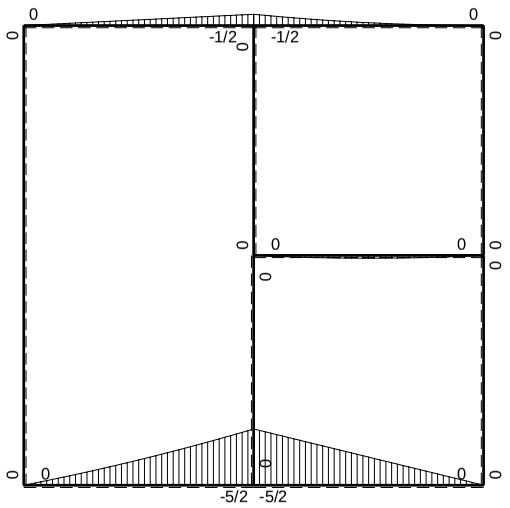
↑□↓ F


0
-1/10
-1/2
-1/2
-1/10
0
0
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1/10
0
-5/2
-5/2

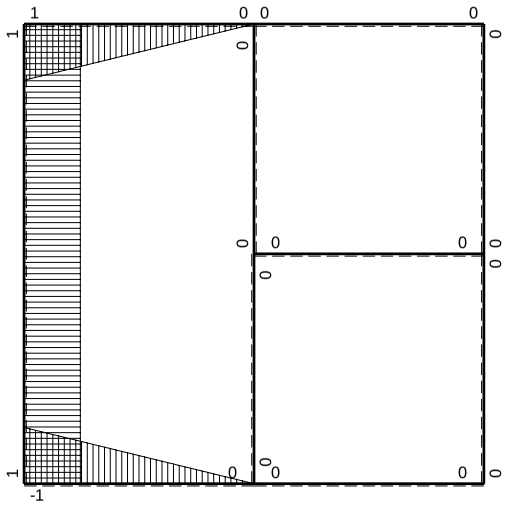
↻□↺ Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210287 ESΣ03.xxxx.091

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$5/2Fx$	Fb/EJ	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

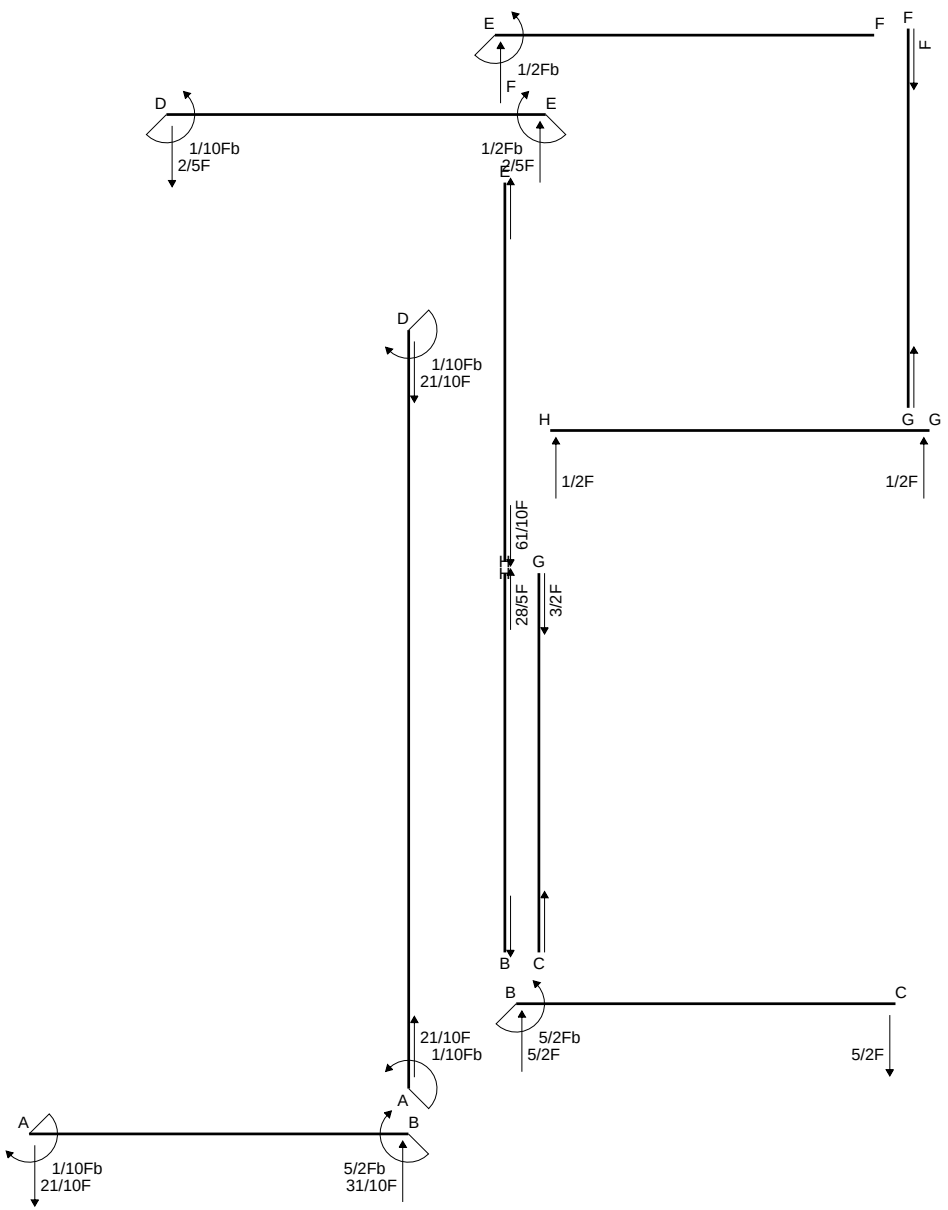
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

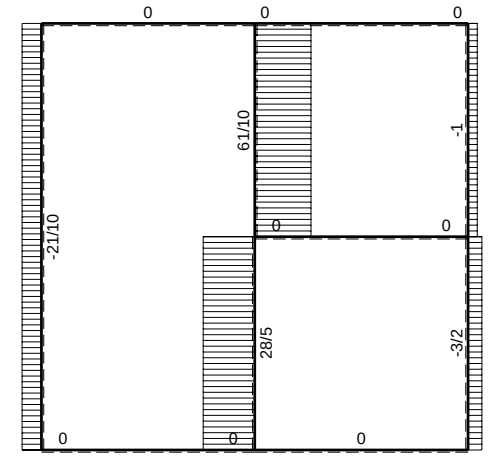
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

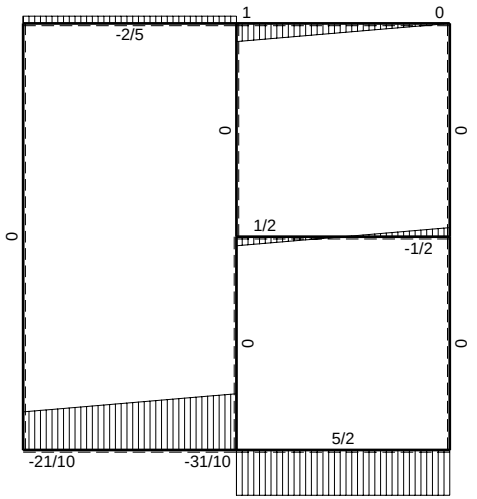
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

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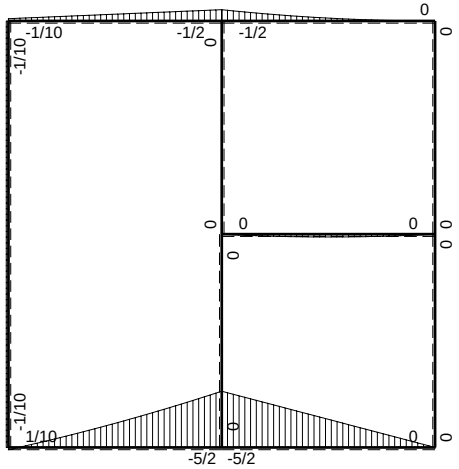

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H
G
G
1/2F
1/2F
61/10F
F
G
28/5F
3/2F
B
C
B
C
21/10F
1/10Fb
5/2Fb
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A
B
1/10Fb
21/10F
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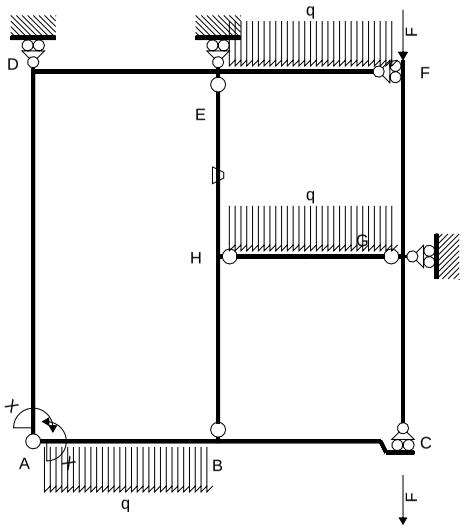

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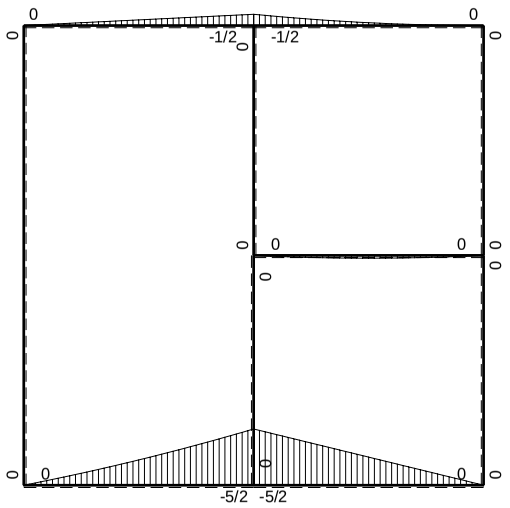
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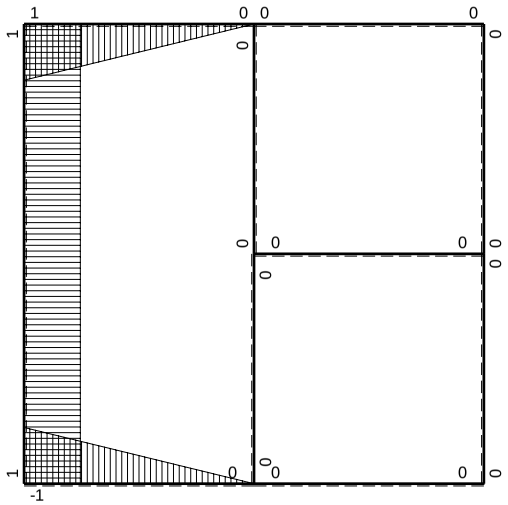
↻□↺ Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	0	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

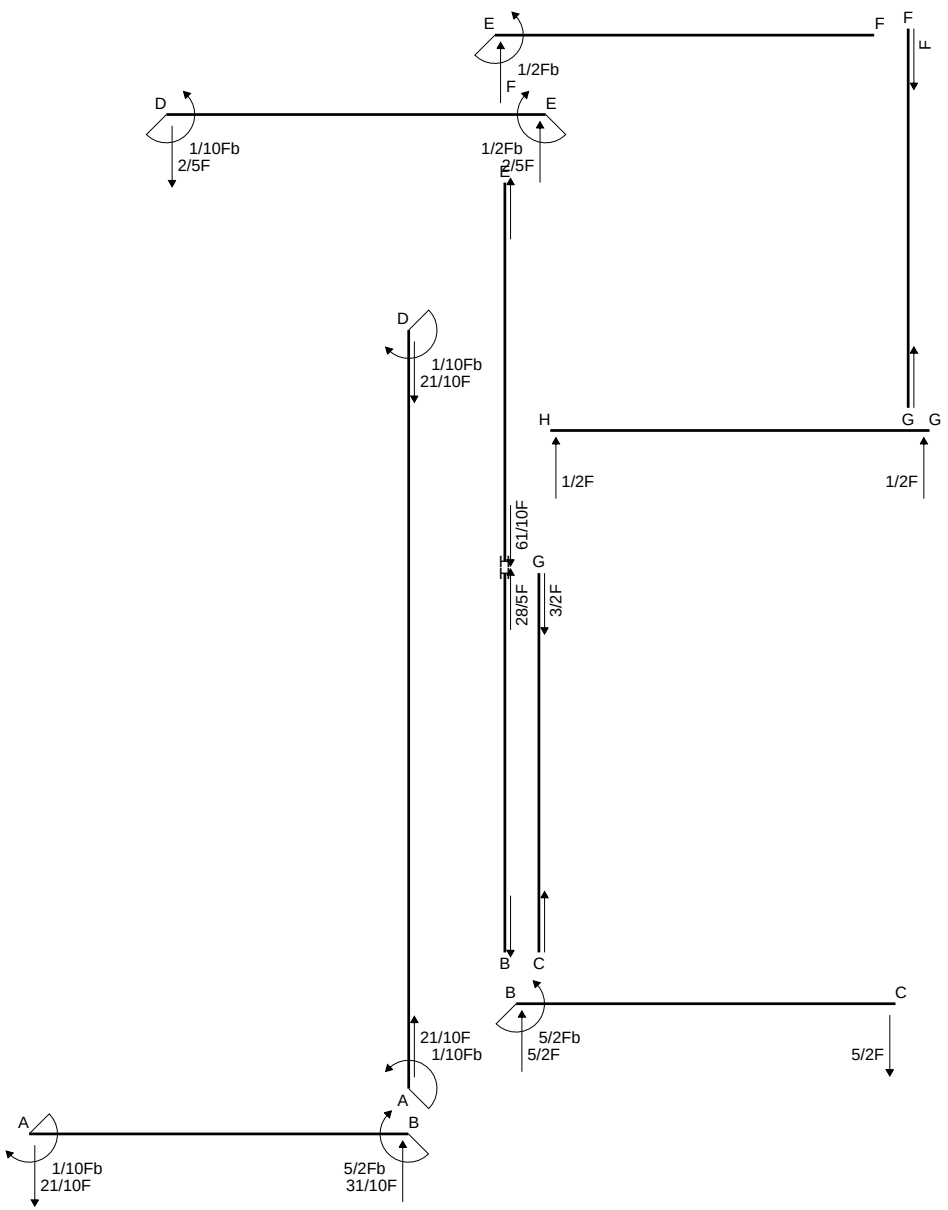
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

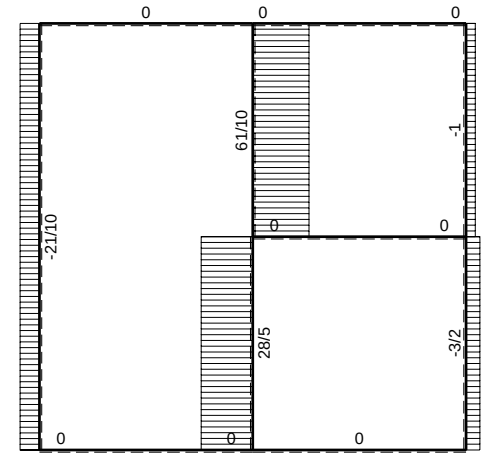
$L_{ED}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

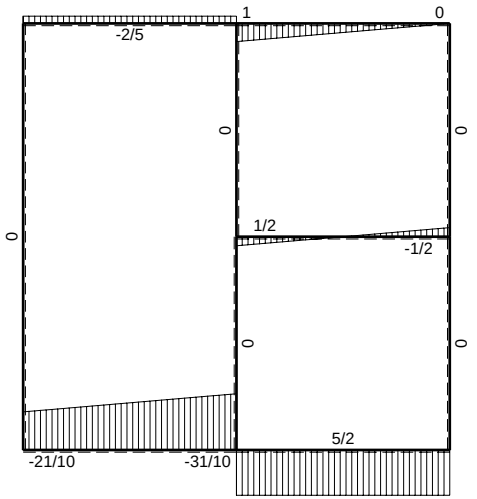
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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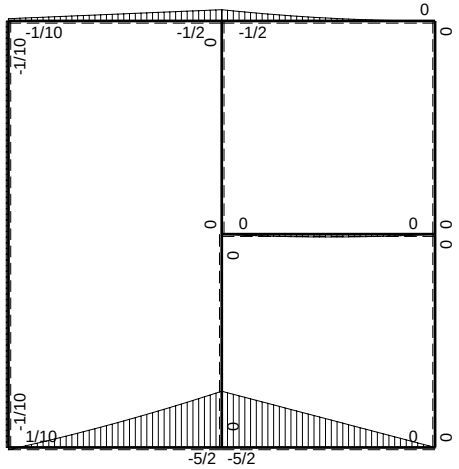

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1/2F
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3/2F
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5/2Fb
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5/2Fb
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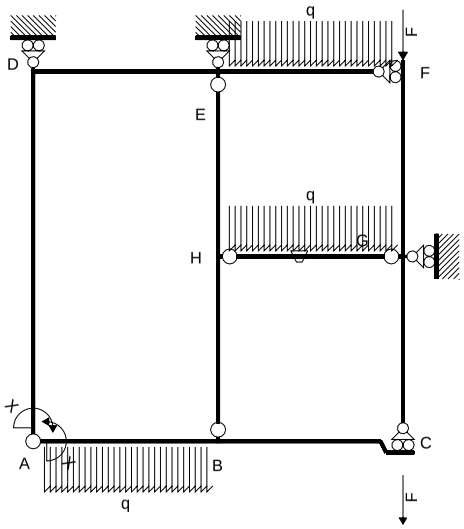

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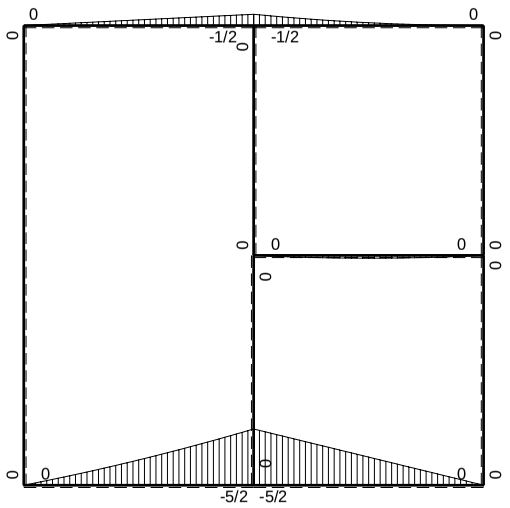
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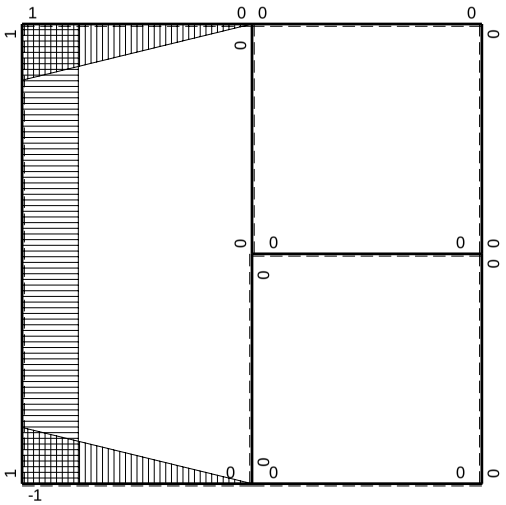
↻□↺ Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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PROCEDIMENTO E RISULTATI 210483 ESΣ03.xxxx.093

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

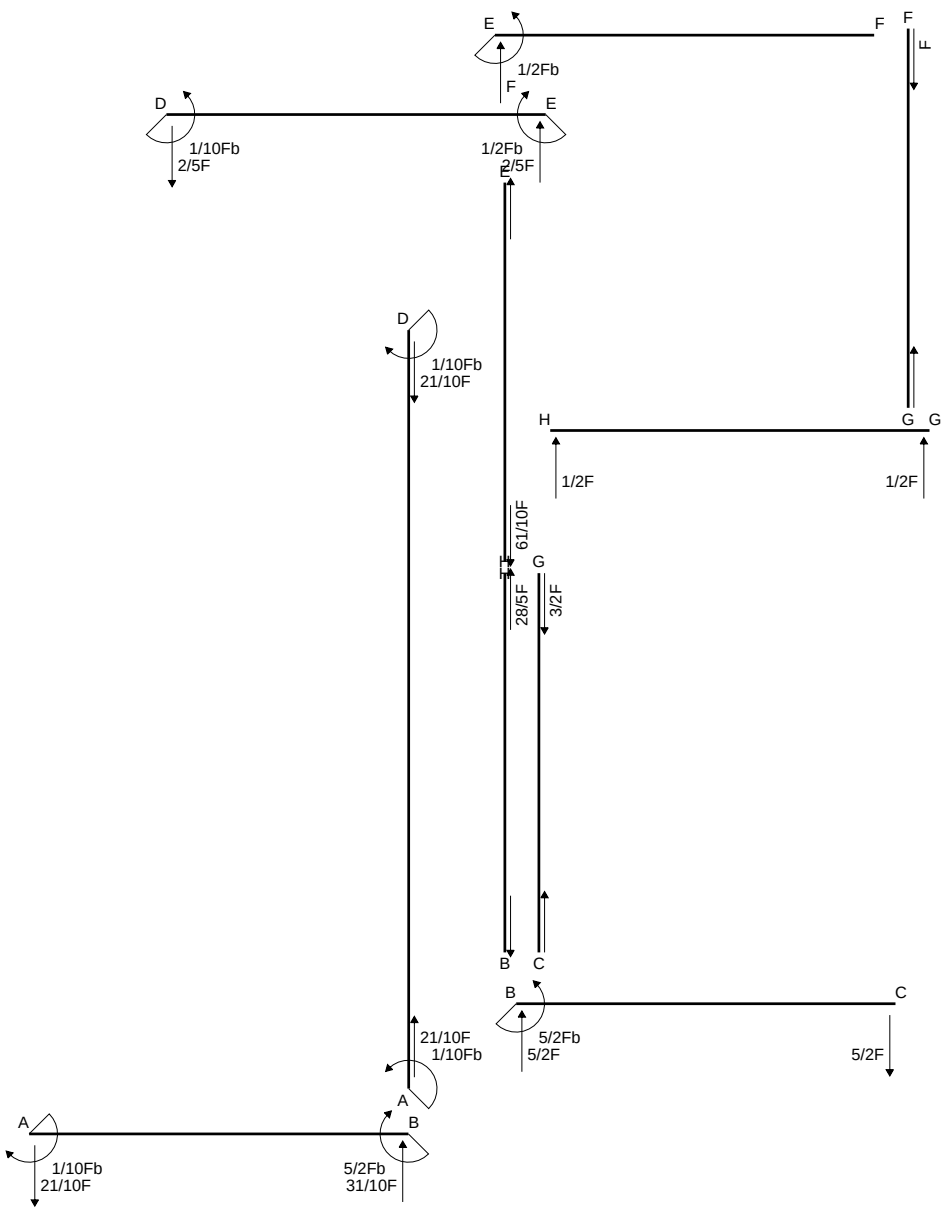
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

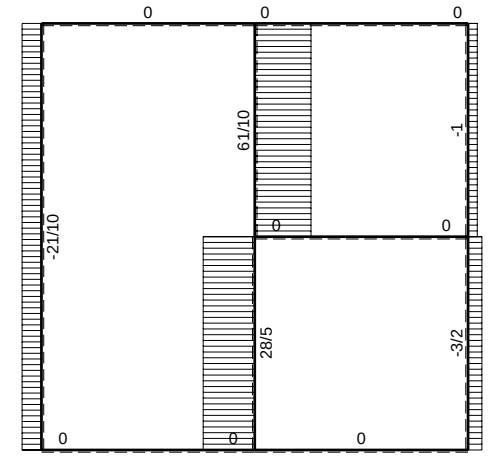
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

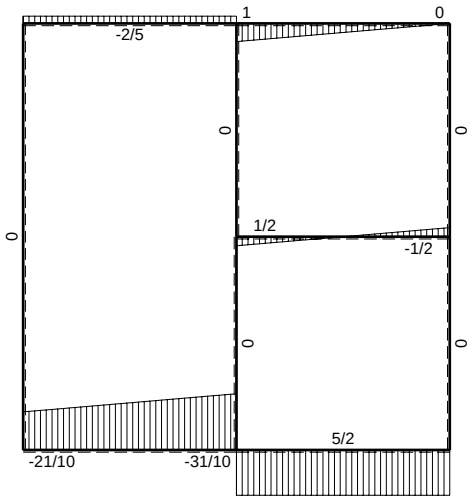
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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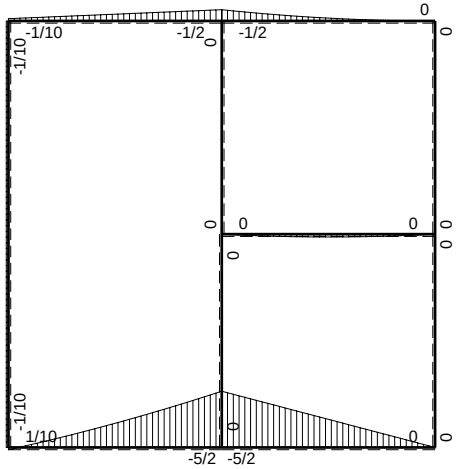

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B
1/10Fb
21/10F
5/2Fb
31/10F


0
0
0
61/10
-1
-21/10
0
0
28/5
-3/2
0
0
0

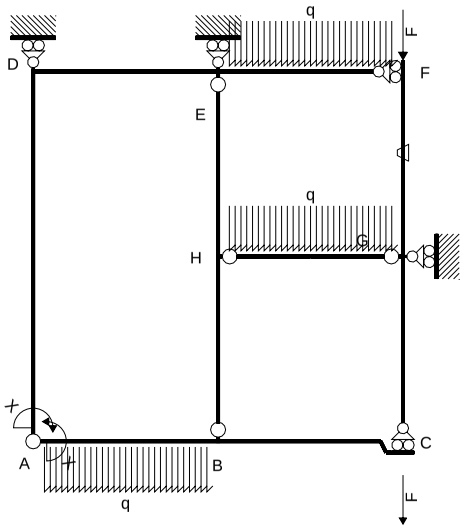

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-2/5
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0
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←□→ F

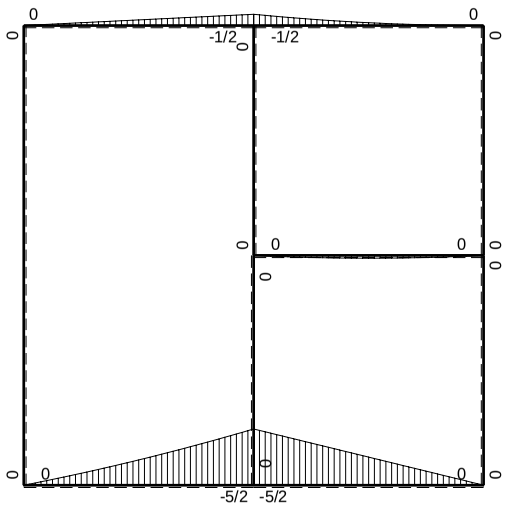
↑□↓ F


0
-1/10
-1/2
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-5/2
-5/2

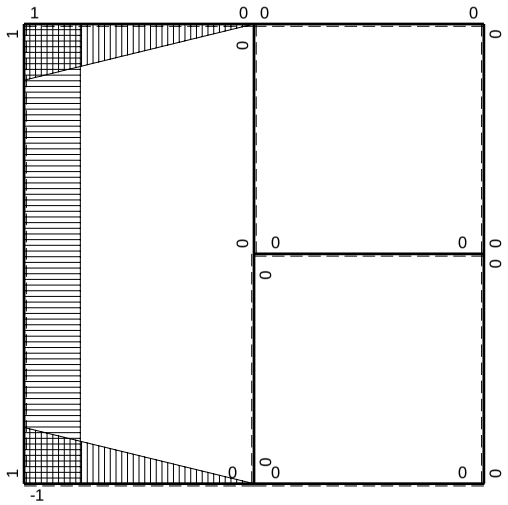
↻□↺ Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

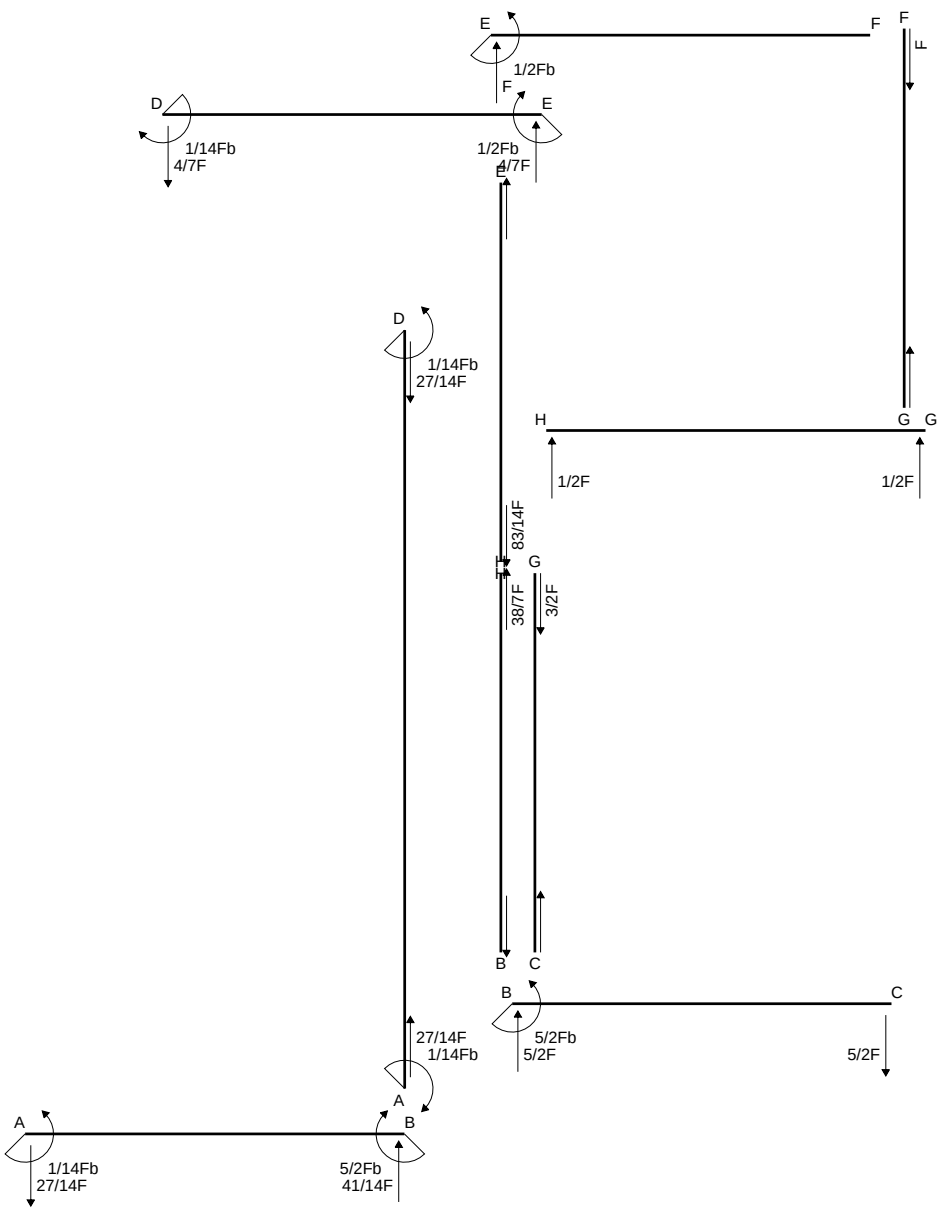
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

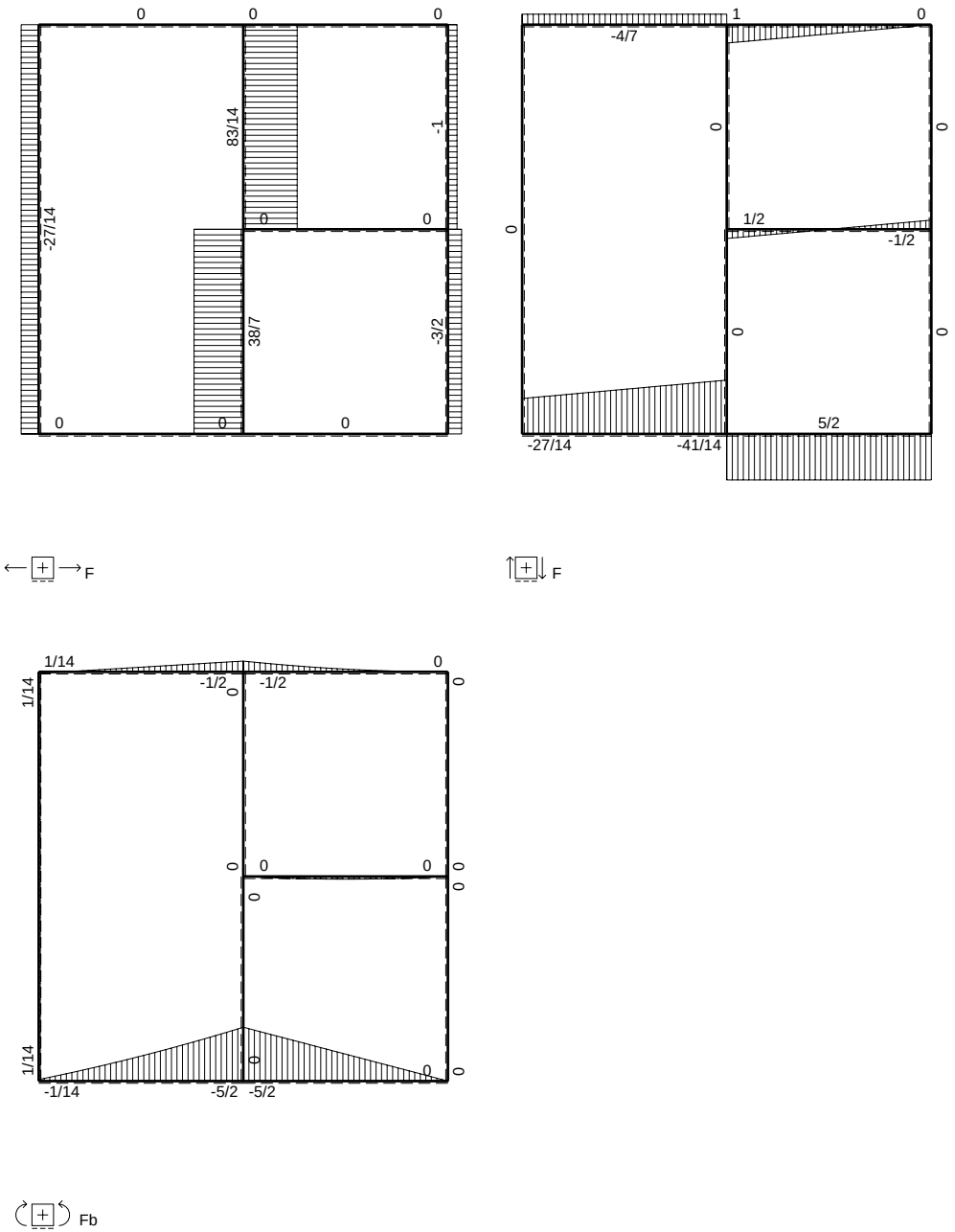
$L_{ED}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

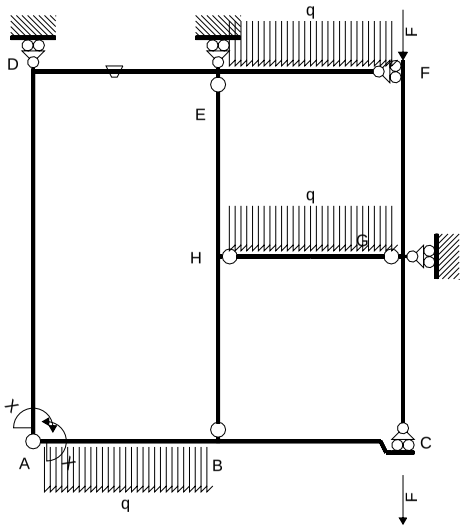
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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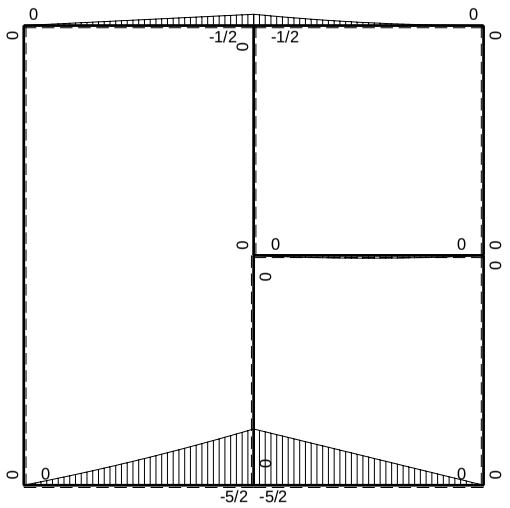
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E
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F
1/2Fb
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D
E
1/14Fb
4/7F
1/2Fb
4/7F
E
D
1/14Fb
27/14F
H
G
G
1/2F
1/2F
83/14F
F
G
38/7F
3/2F
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C
27/14F
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5/2Fb
41/14F

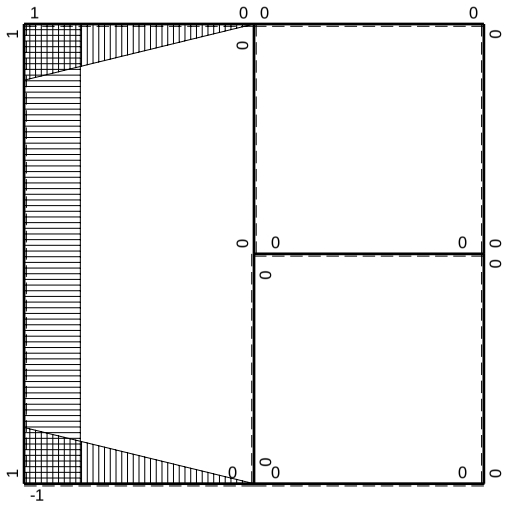

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1/2
-1/2
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5/2
-27/14
-41/14
F
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1/14
-1/2
-1/2
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0
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0
0
0
0
0
0
0
0
1/14
-1/14
-5/2
-5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	$-Fb/EJ$	$-1/2Fx+1/2Fx^2/b$	$-Fb/EJ+Fx/EJ$	$1-2x/b+x^2/b^2$	$(-1/12-1/2)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	Fb/EJ	$-1/2Fx+1/2Fx^2/b$	$-Fx/EJ$	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-5/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ \ = -7/12\ \ Fb^2/EJ$

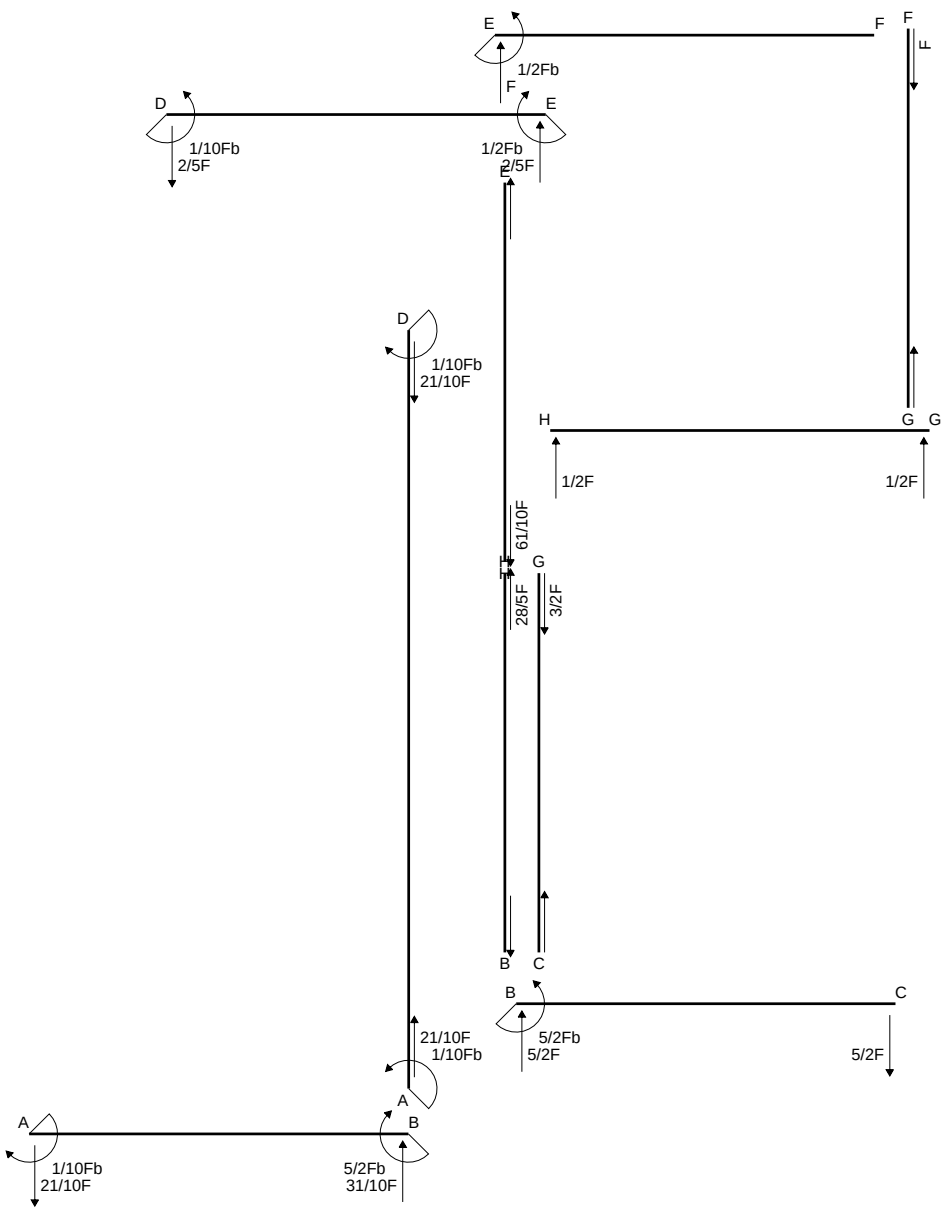
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

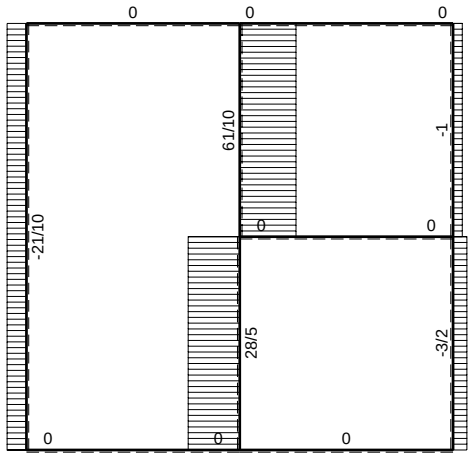
$= [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

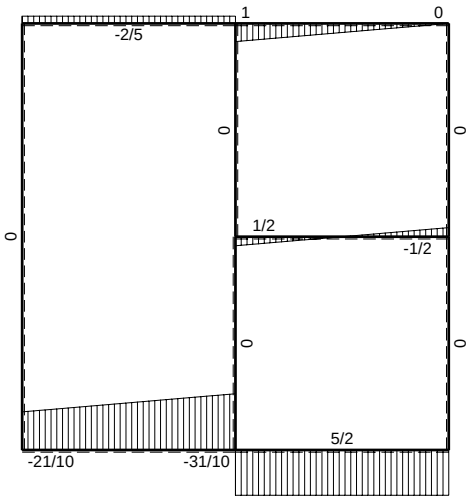
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ \ = -7/12\ \ Fb^2/EJ$

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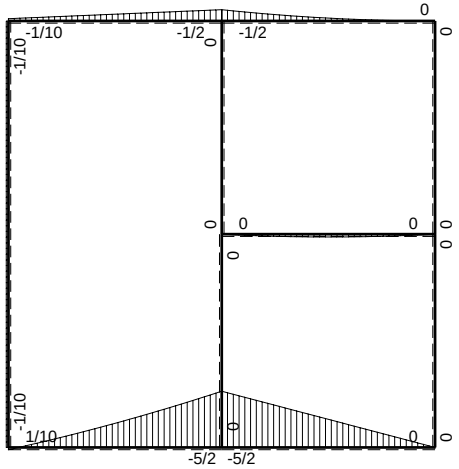

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21/10F
H
G
G
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1/2F
61/10F
F
G
28/5F
3/2F
B
C
B
C
21/10F
1/10Fb
5/2Fb
5/2F
5/2F
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A
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21/10F
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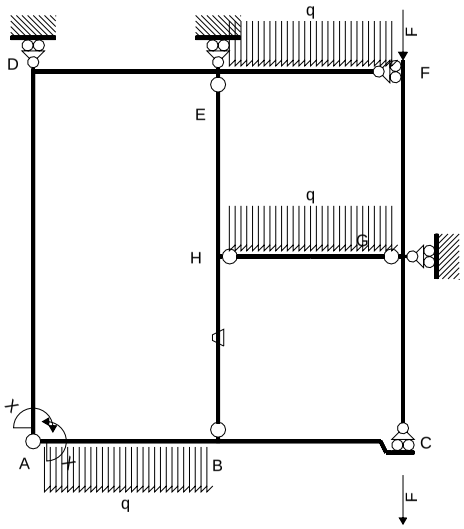

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5/2
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←□→ F

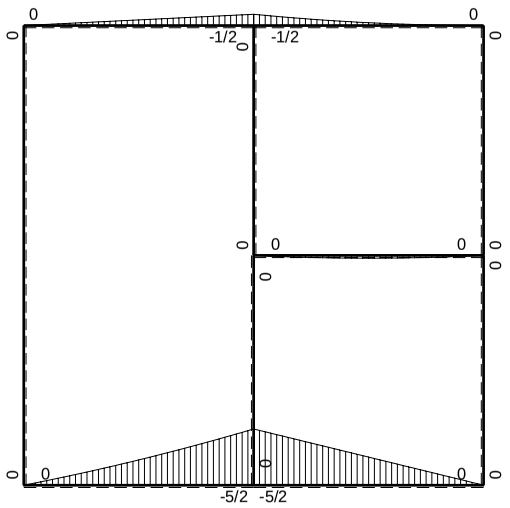
↑□↓ F


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-1/10
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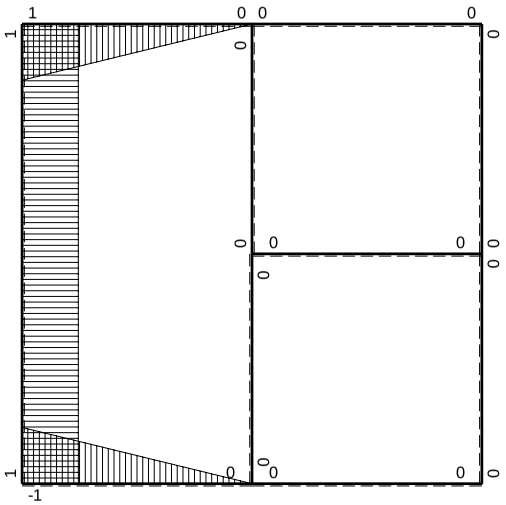
↻□↺ Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 955615 ESΣ03.xxxx.096

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PROCEDIMENTO E RISULTATI 955615 ESΣ03.xxxx.096

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	0	Fb/EJ	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

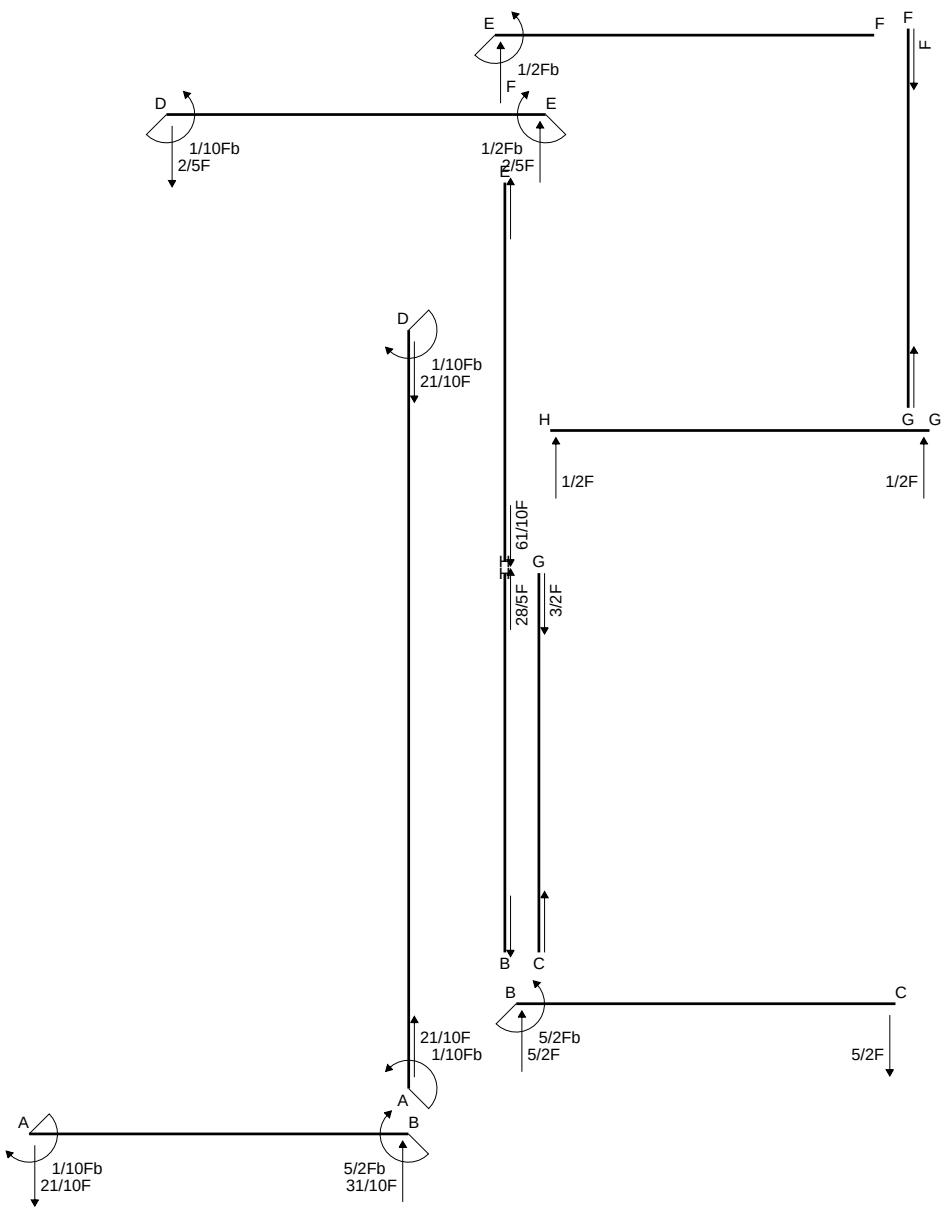
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

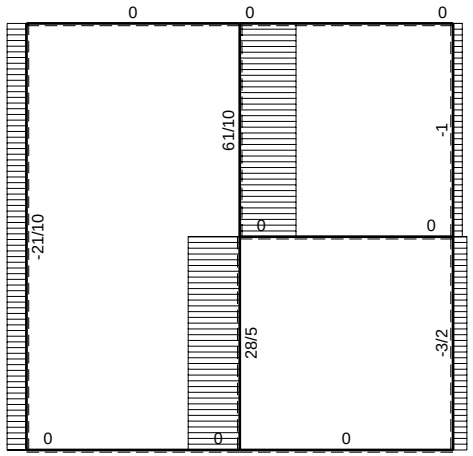
$L_{ED}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

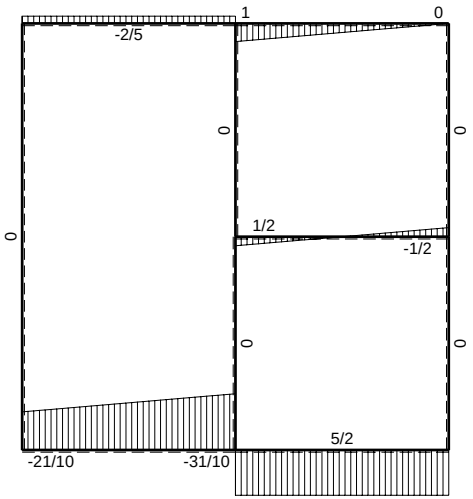
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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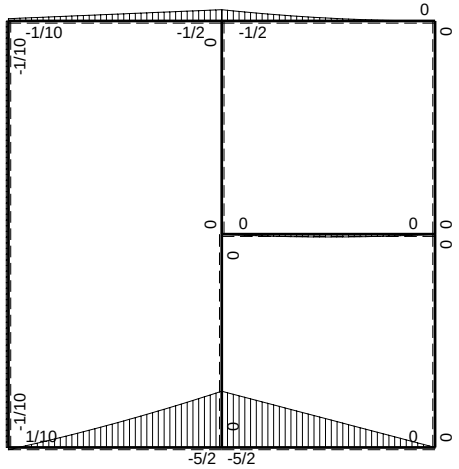

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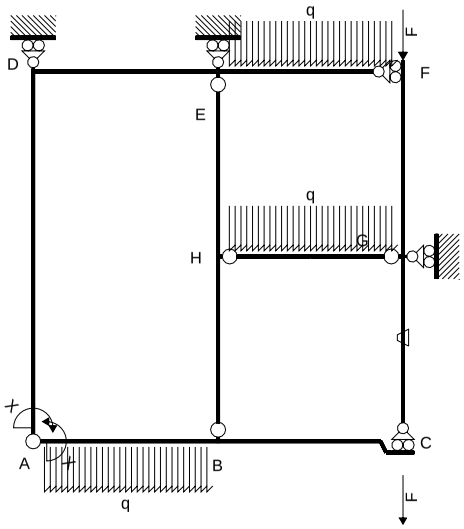

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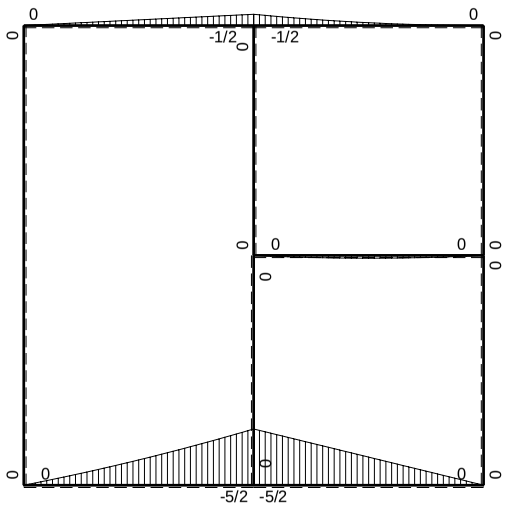
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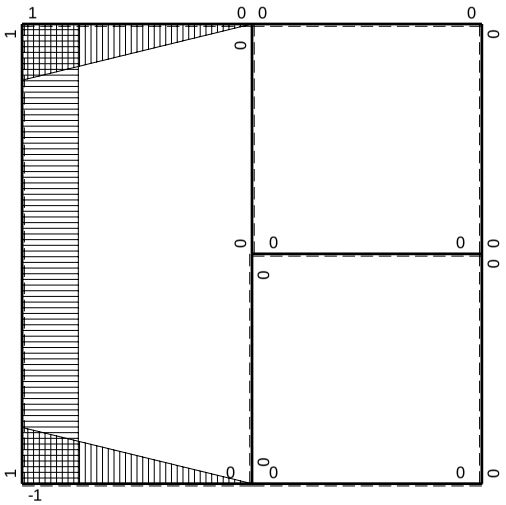
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Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

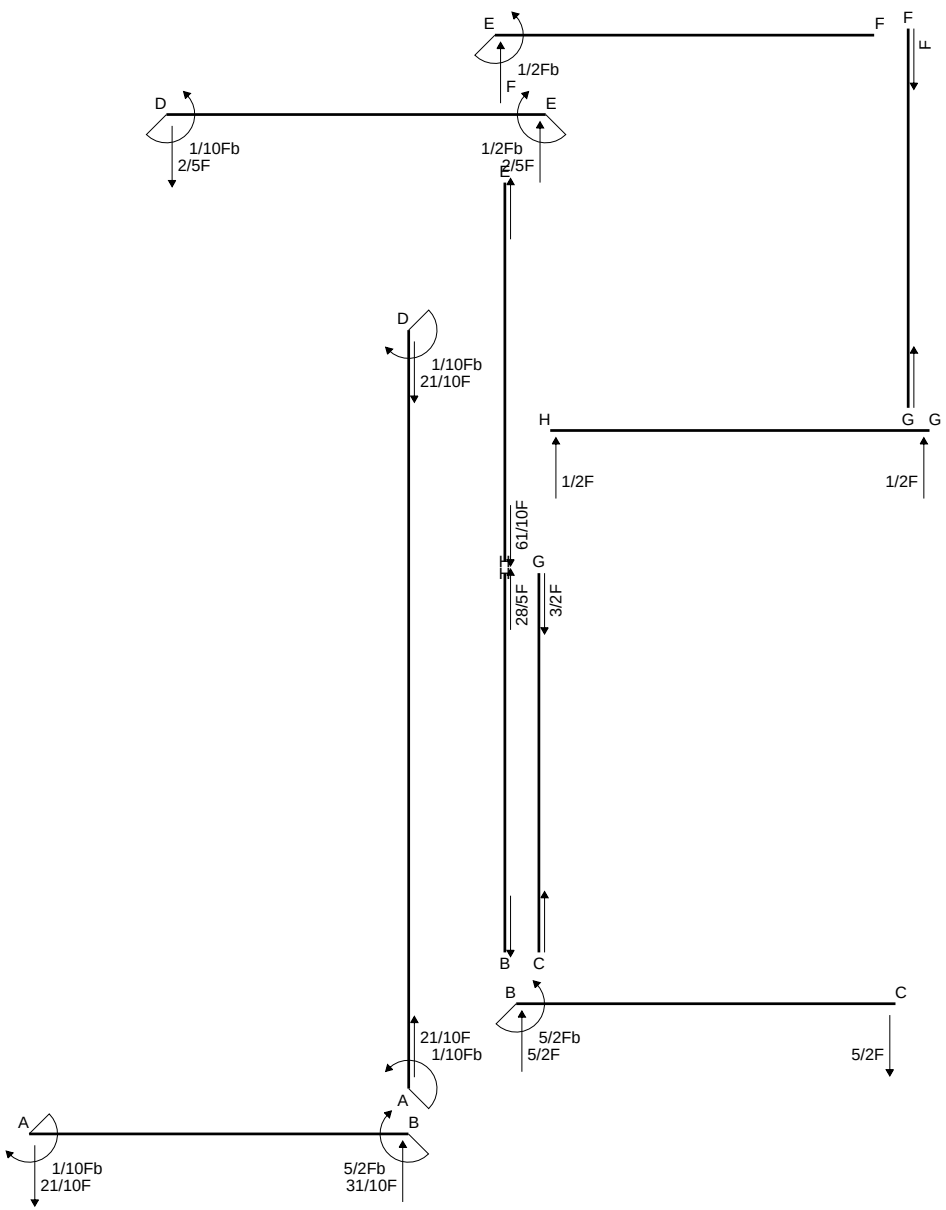
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

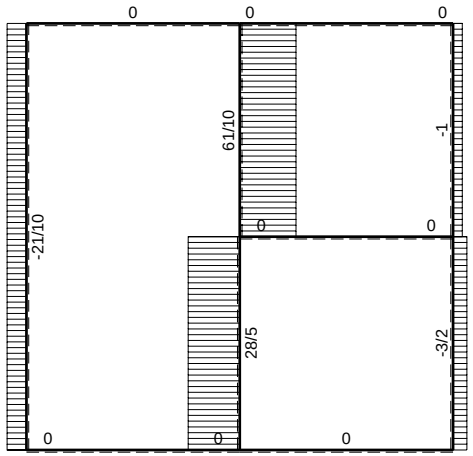
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

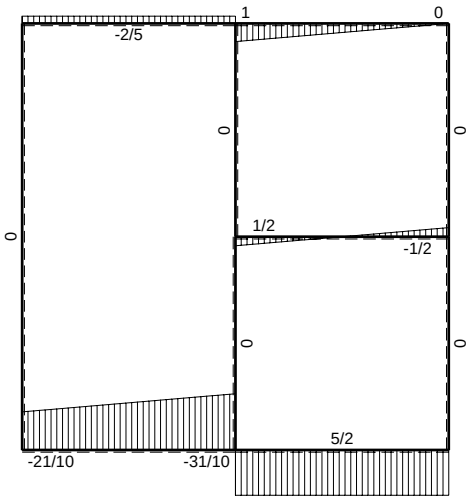
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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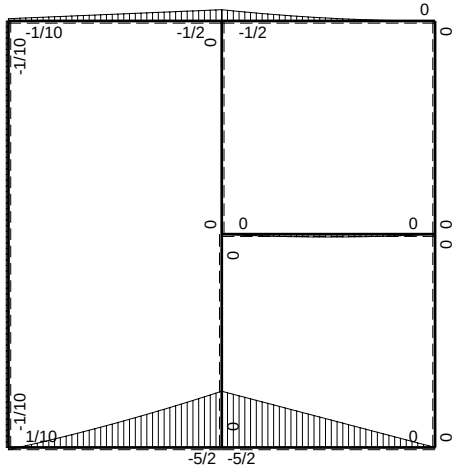

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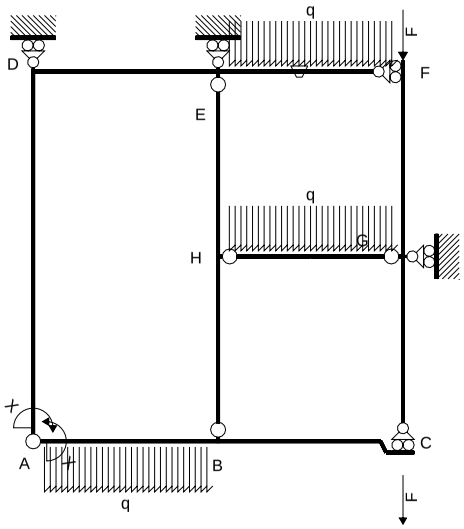

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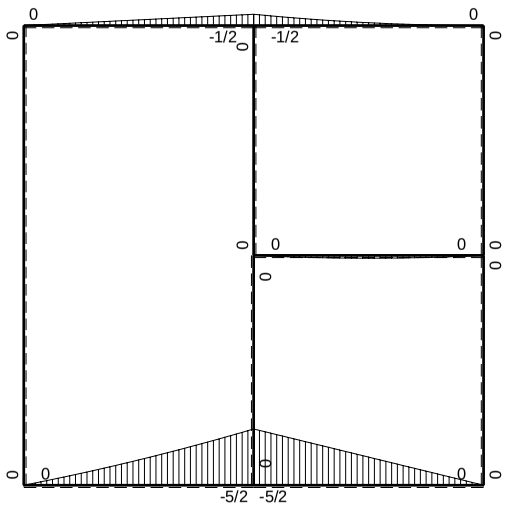
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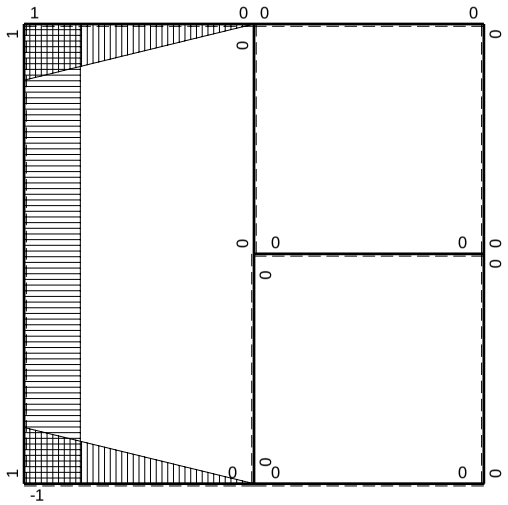
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Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	$1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

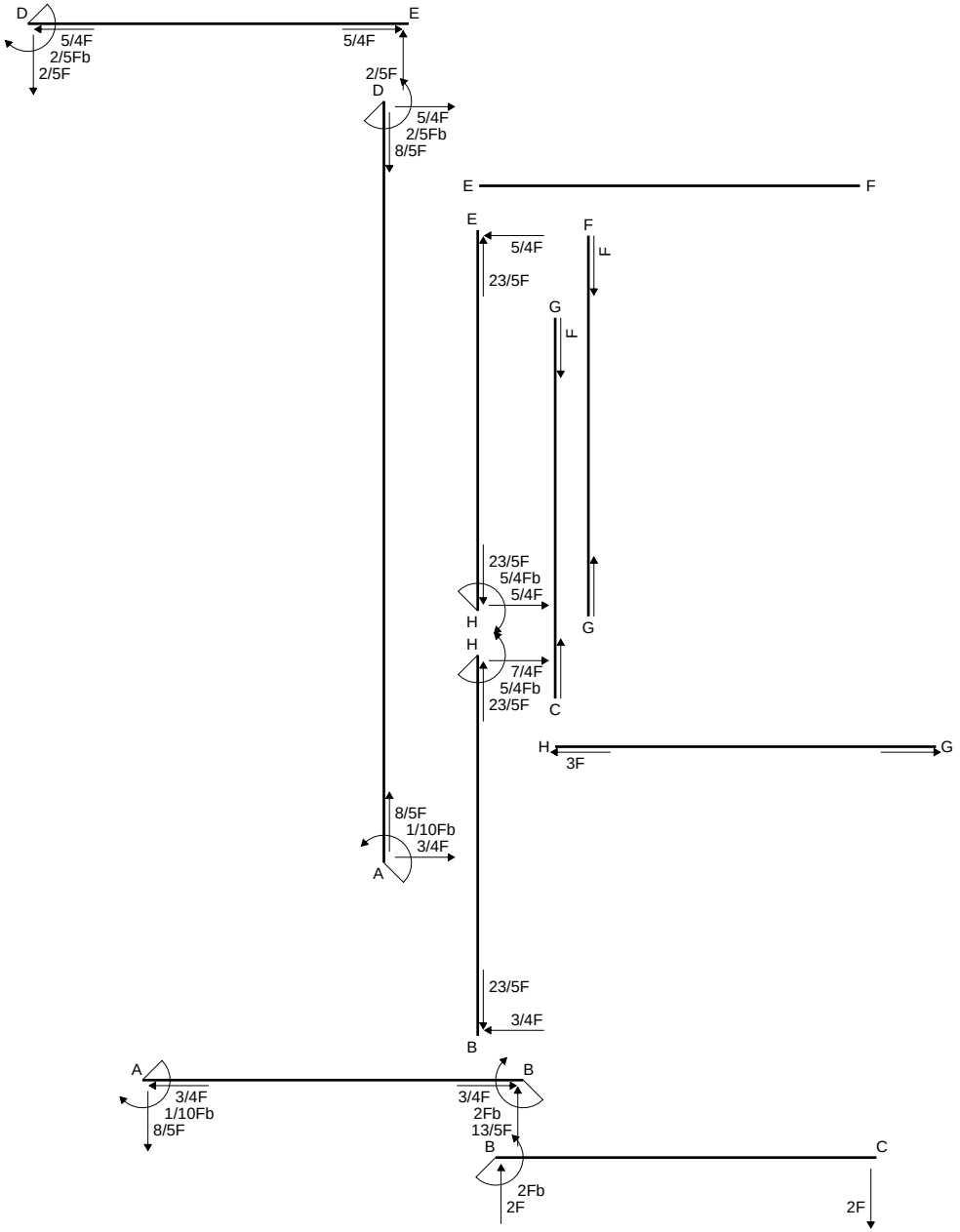
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

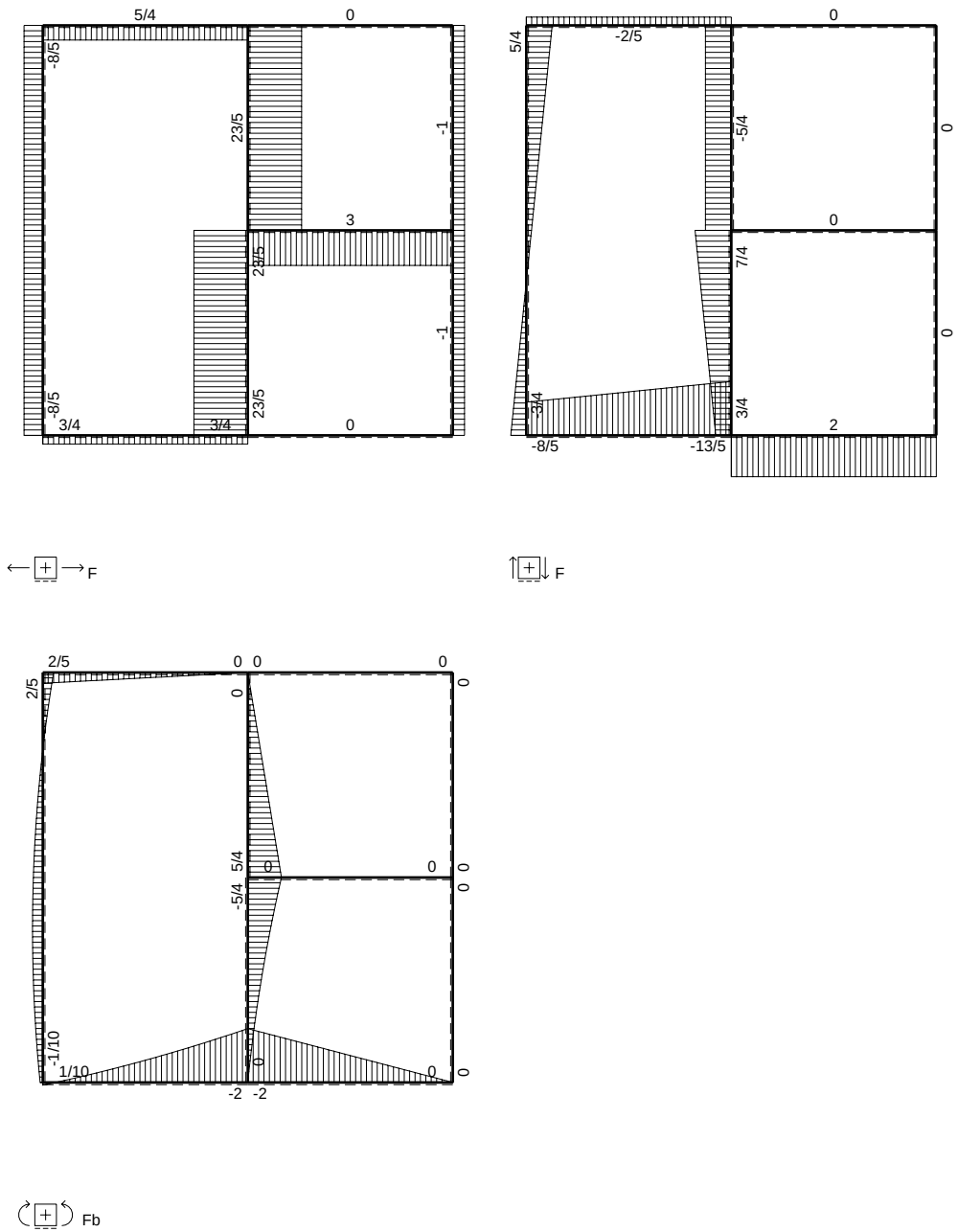
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

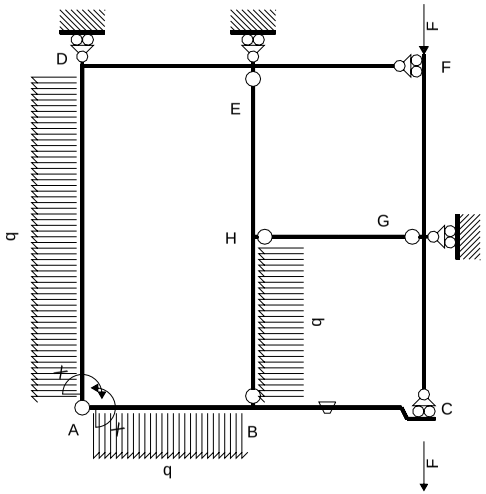
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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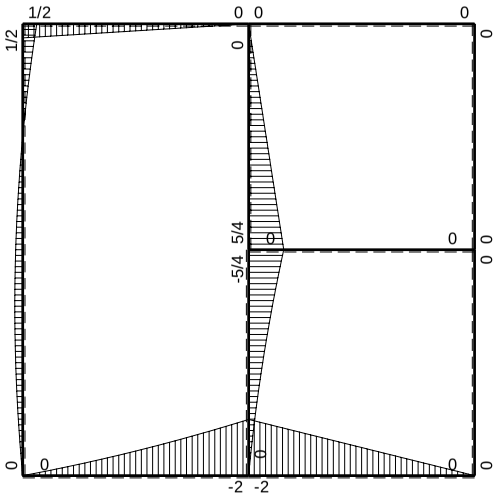
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2/5Fb
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2/5F
D
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2/5Fb
8/5F
E
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F
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2F
2F

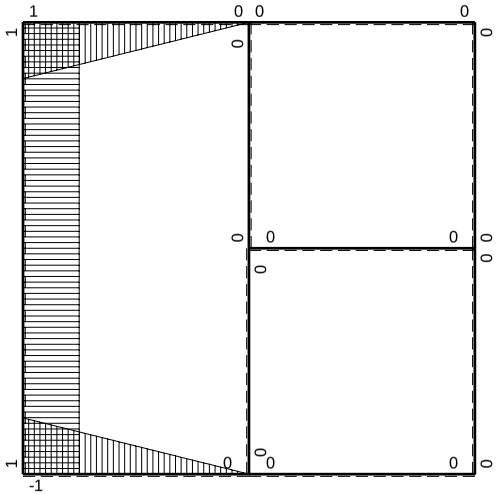

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F
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2/5
0
0
0
5/4
0
0 0
-5/4
0
-1/10
0
1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\ b + 1/8\ b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\,]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\,]_o^{2b}\ Fb\ 1/EJ$

$= (\,b - 5/2\ b + 4/3\ b\,)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

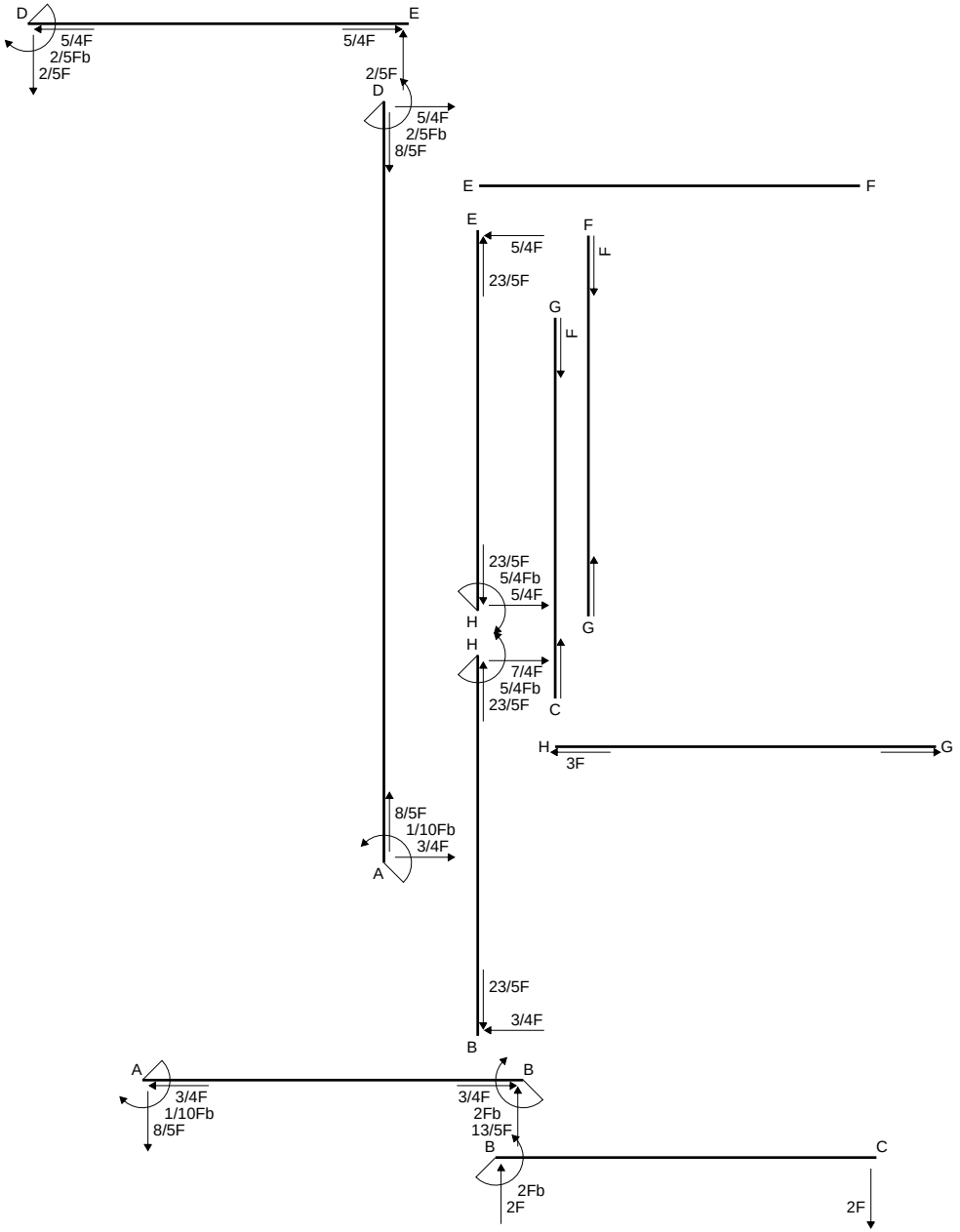
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

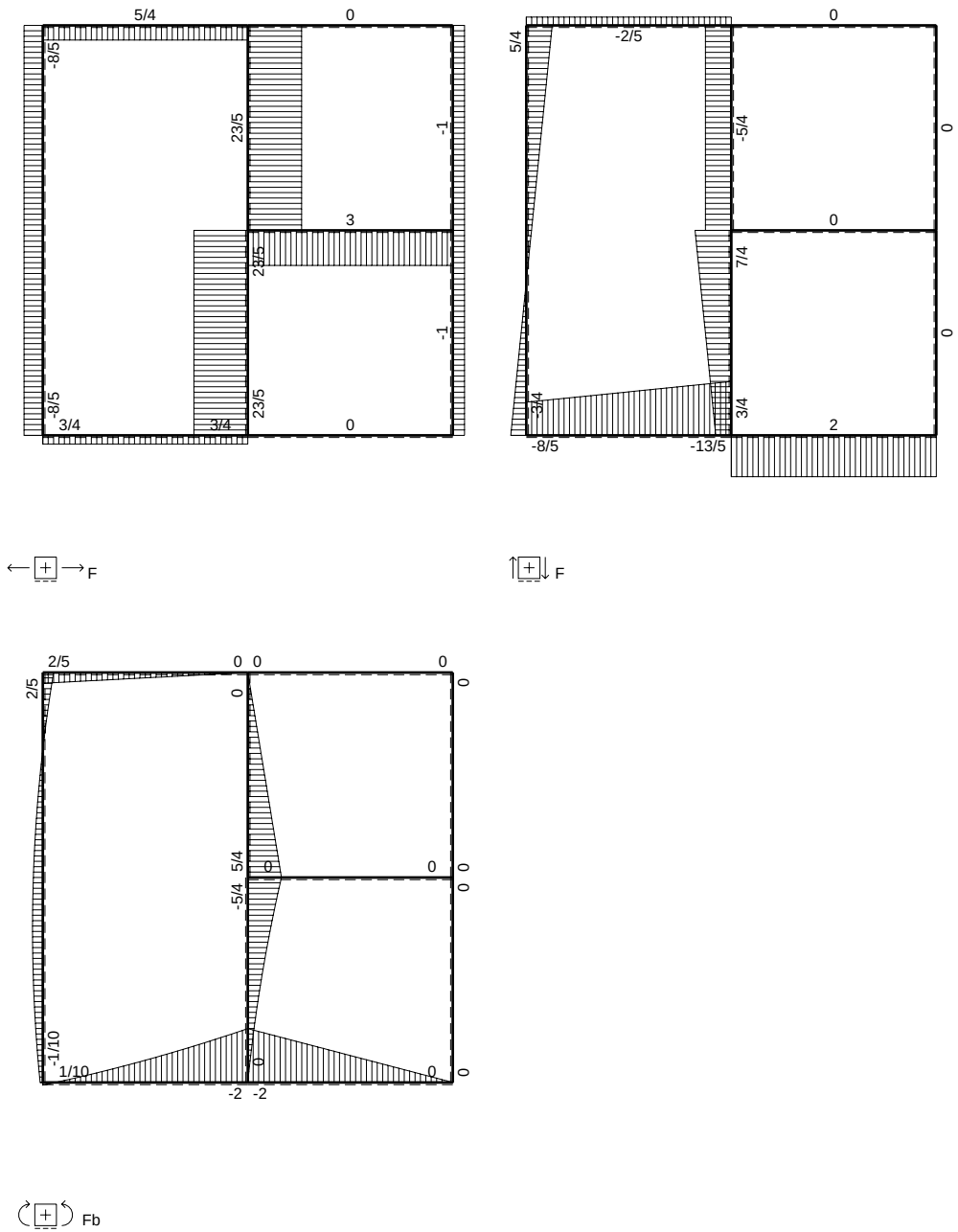
$L_{ED}^{Xo} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

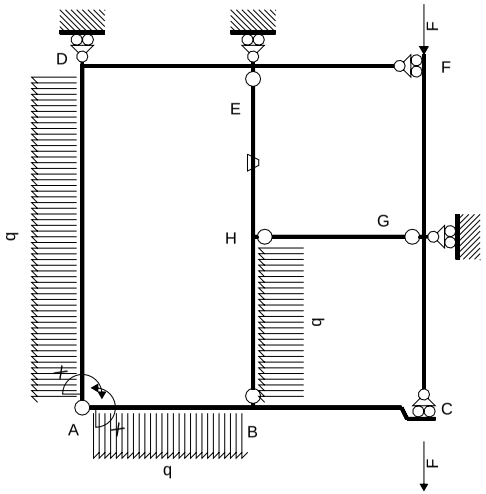
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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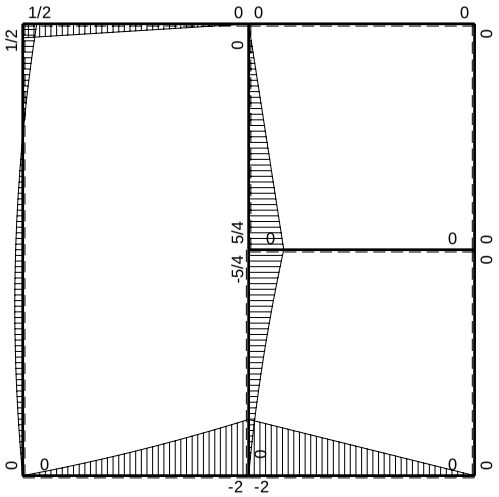
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2/5F
D
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8/5F
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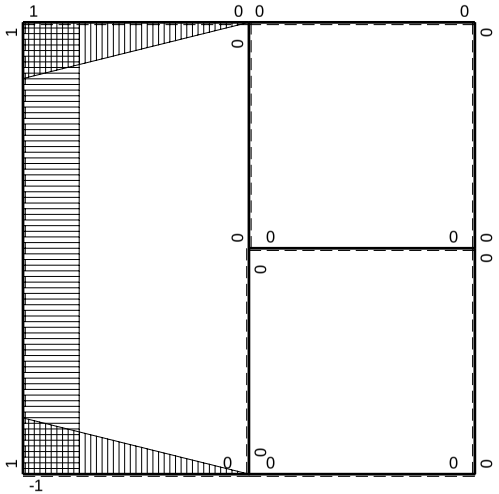

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-13/5
F
2/5
0 0
0
2/5
0
0
0
5/4
0
0 0
-5/4
0
-1/10
0
1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	5/4Fb-5/4Fx	-Fb/EJ	0	0	0	0+0	0
EH b	0	-5/4Fx	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

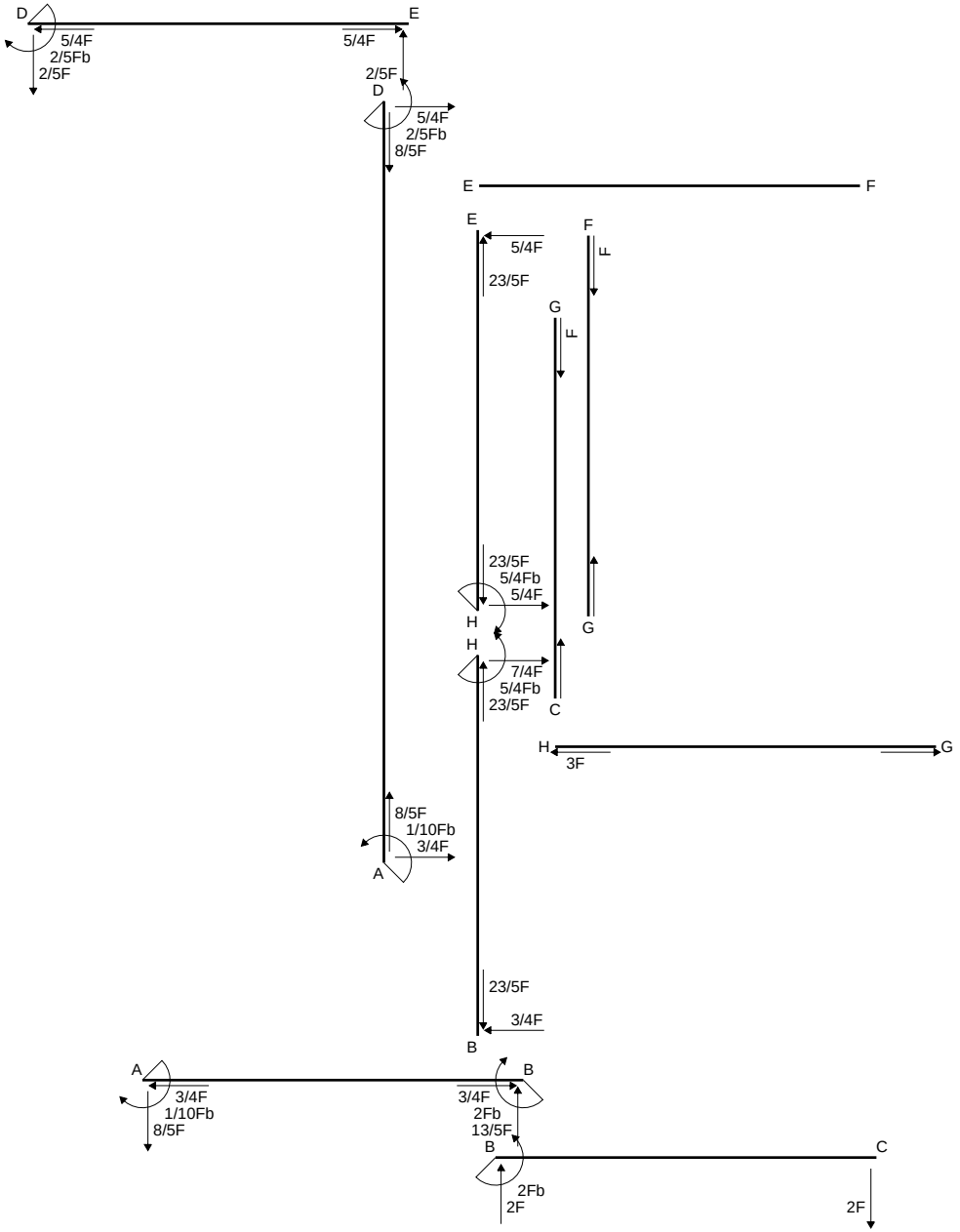
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

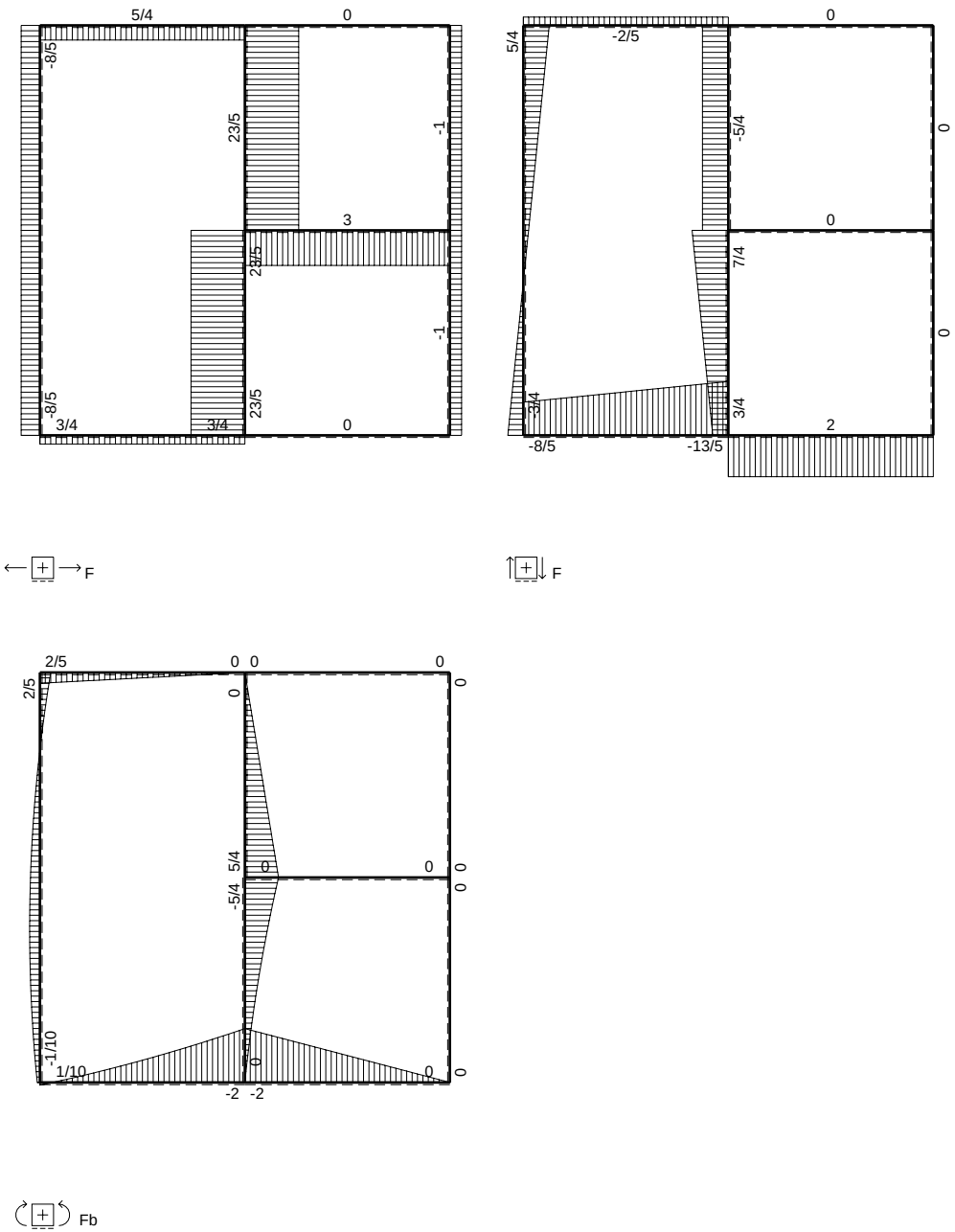
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

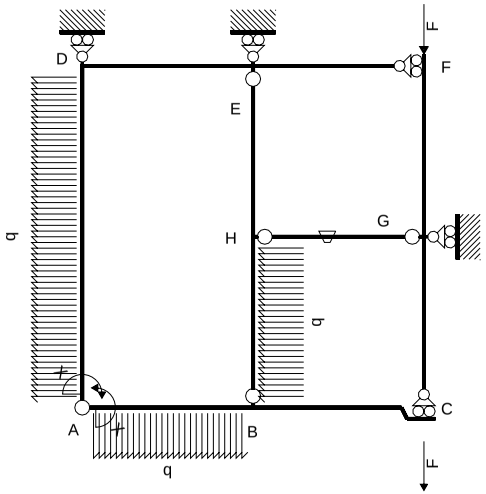
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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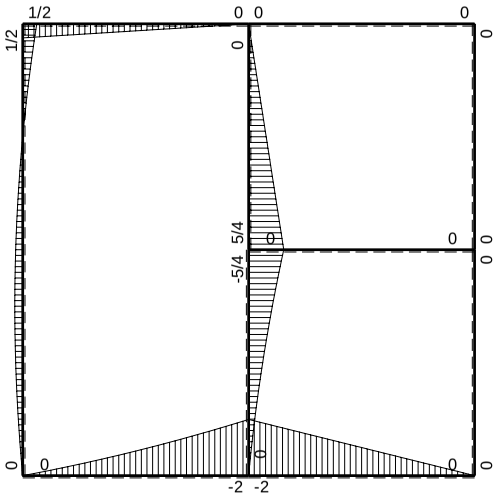
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5/4F
2/5Fb
2/5F
5/4F
2/5F
D
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2/5Fb
8/5F
E
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G
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H
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C
2Fb
2F
2F

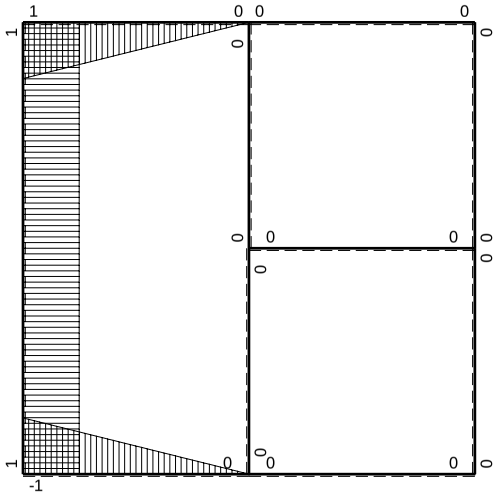

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23/5
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-8/5
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3/4
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3/4
3/4
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-8/5
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F
2/5
0
0
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2/5
0
0
0
5/4
0
0
0
-5/4
0
0
-1/10
0
1/10
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

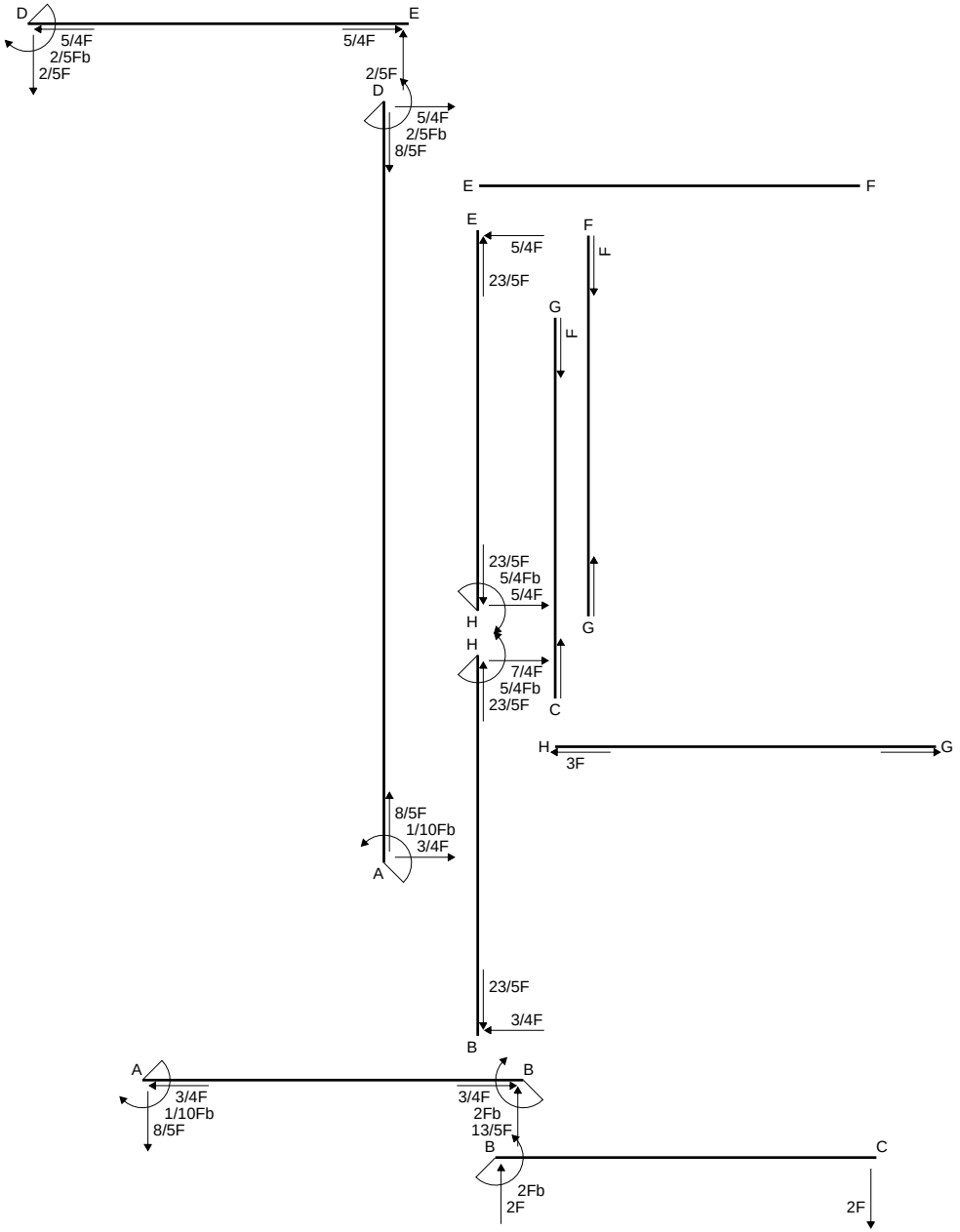
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

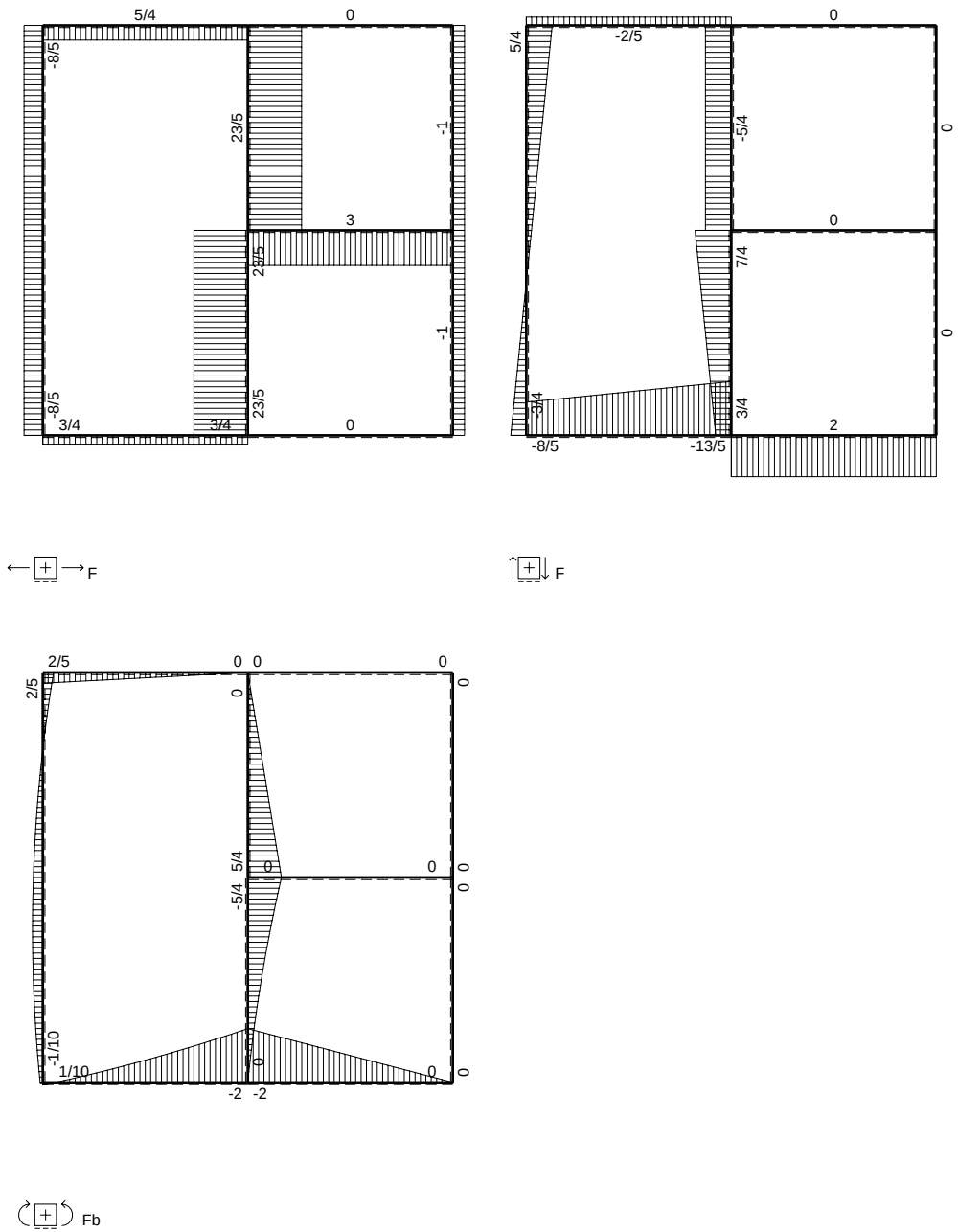
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

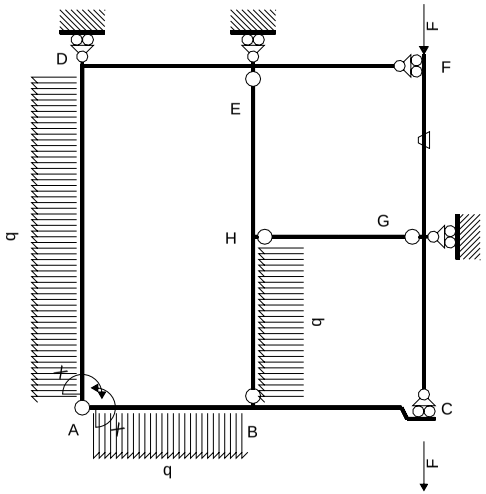
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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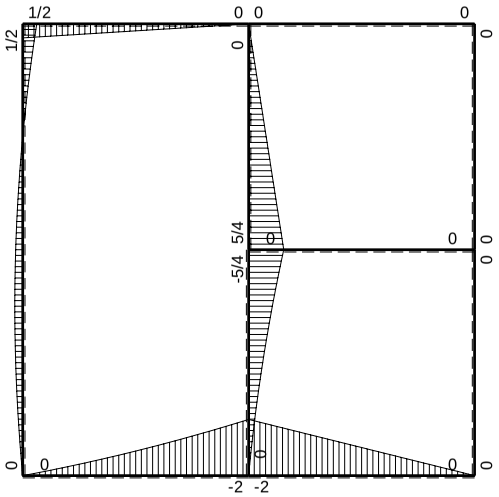
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2/5F
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2/5Fb
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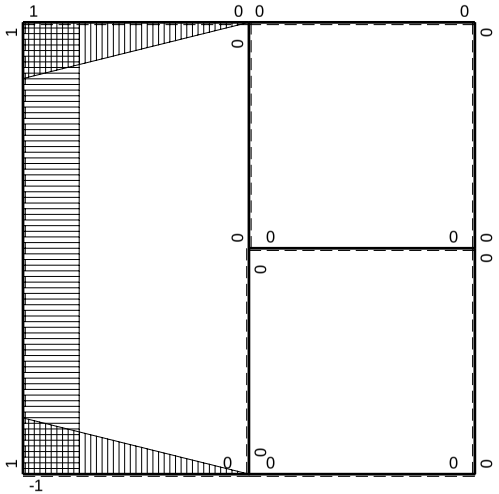

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3/4
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-8/5
-13/5
F
2/5
0 0
0
2/5
0
0
0
5/4
0
0 0
-5/4
0
-1/10
0
1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

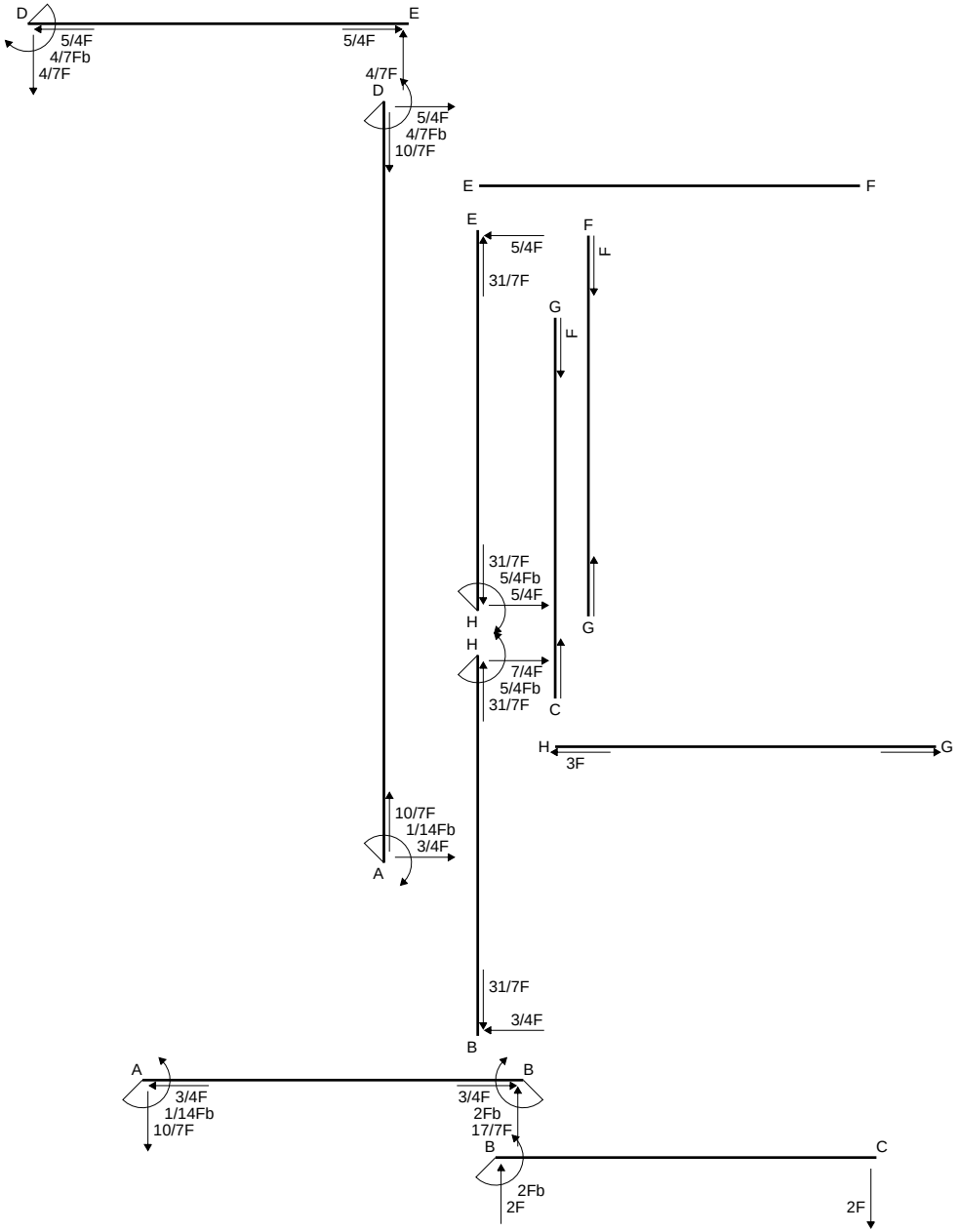
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

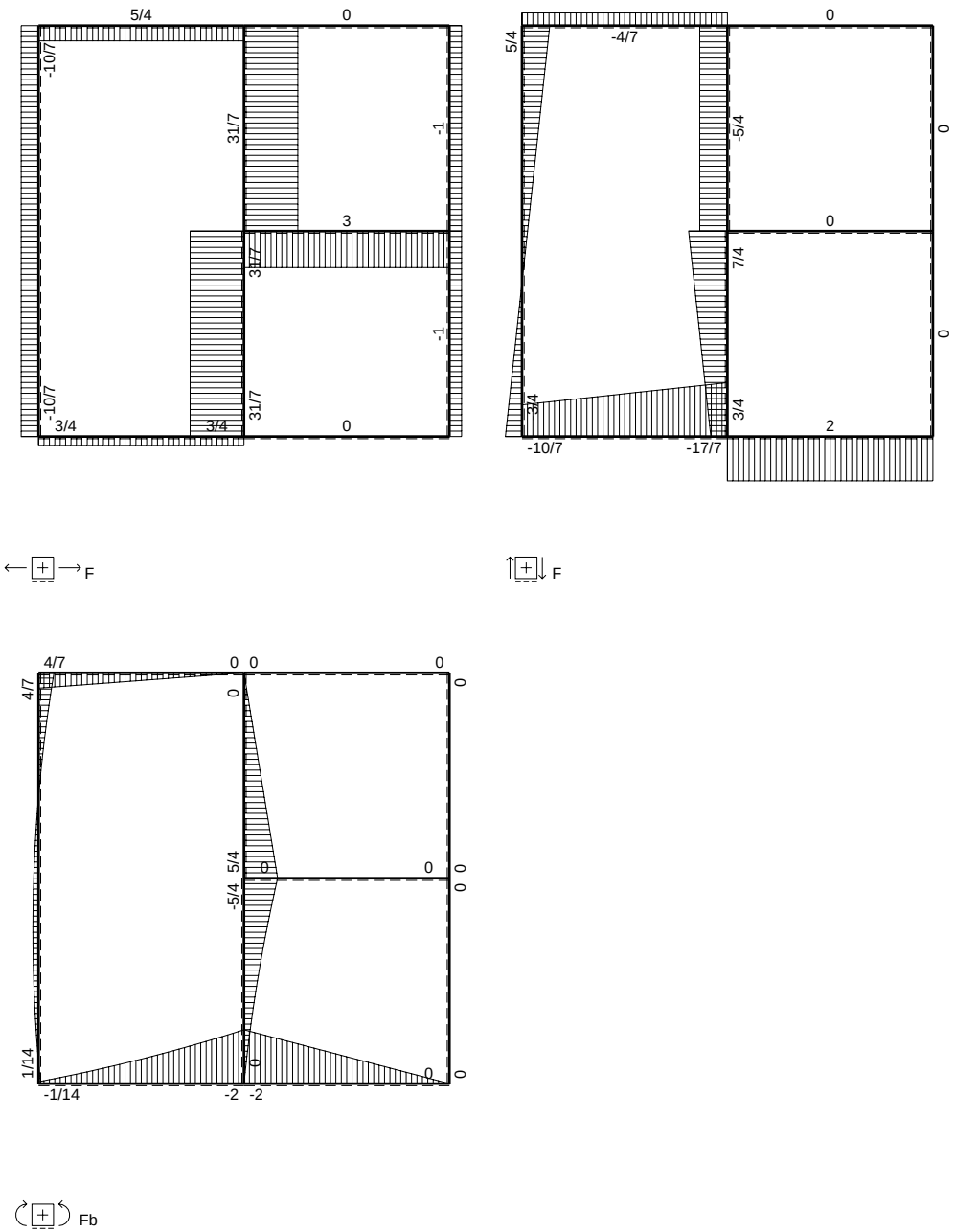
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

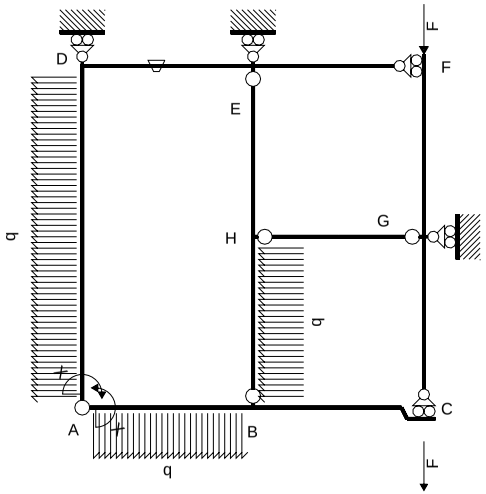
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 236508 ESΣ03.xxxx.102

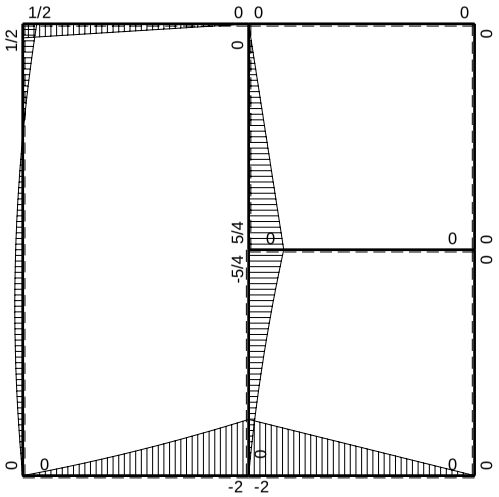
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D
E
5/4F
4/7Fb
4/7F
5/4F
4/7F
D
5/4F
4/7Fb
10/7F
E
F
E
F
5/4F
31/7F
G
F
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31/7F
5/4Fb
5/4F
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G
H
7/4F
5/4Fb
31/7F
C
H
G
3F
10/7F
1/14Fb
3/4F
A
31/7F
3/4F
B
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B
C
2Fb
2F
2F

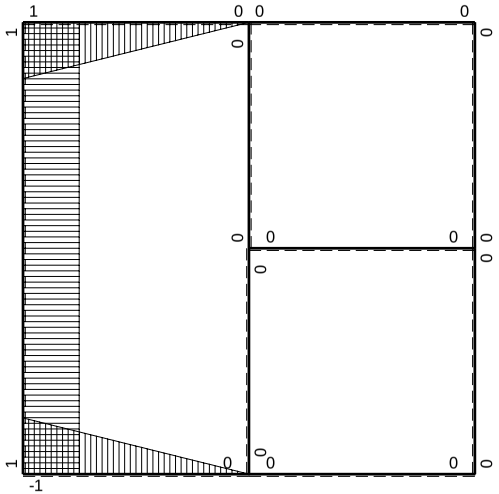

5/4
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31/7
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3/4
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0
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31/7
F
5/4
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2
-10/7
-17/7
F
4/7
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0
0
4/7
0
0
5/4
0
0
-5/4
0
0
1/14
0
-1/14
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236538 ESΣ03.xxxx.103

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	$-Fb/EJ$	$1/2Fb-Fx+1/2Fx^2/b$	$-Fb/EJ+Fx/EJ$	$1-2x/b+x^2/b^2$	$(1/6-1/2)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	Fb/EJ	$1/2Fx^2/b$	$-Fx/EJ$	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(1/2 -5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x -5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(\ b -5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(1/2 - x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

$= \left[1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[- x +1/2\ x^2/b\ \right]_o^b\ \theta$

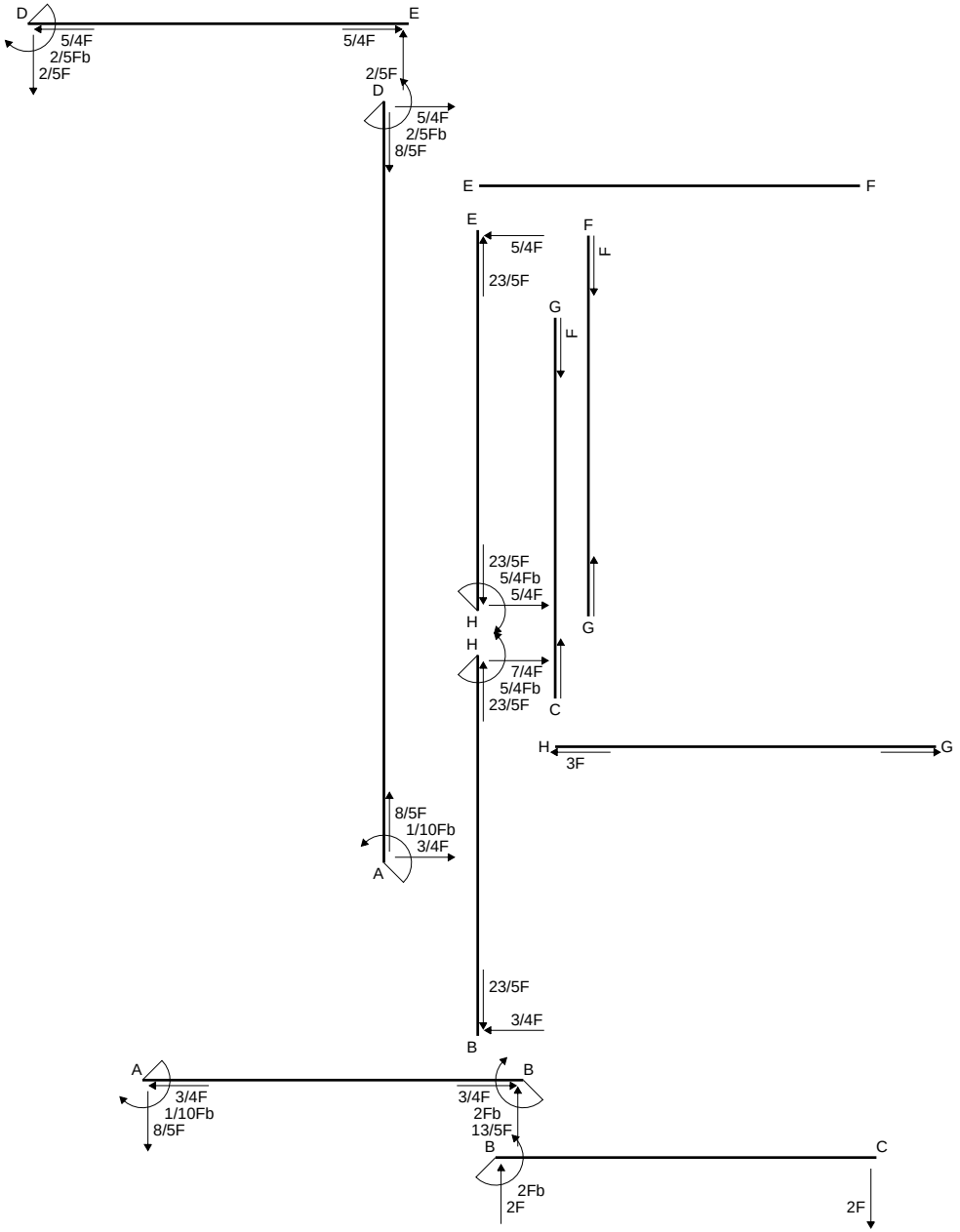
$= \left(1/2\ b -1/2\ b +1/6\ b\ \right) Fb\ 1/EJ\ + \left(- b +1/2\ b\ \right)\ \theta\ \ = -1/3\ \ Fb^2/EJ$

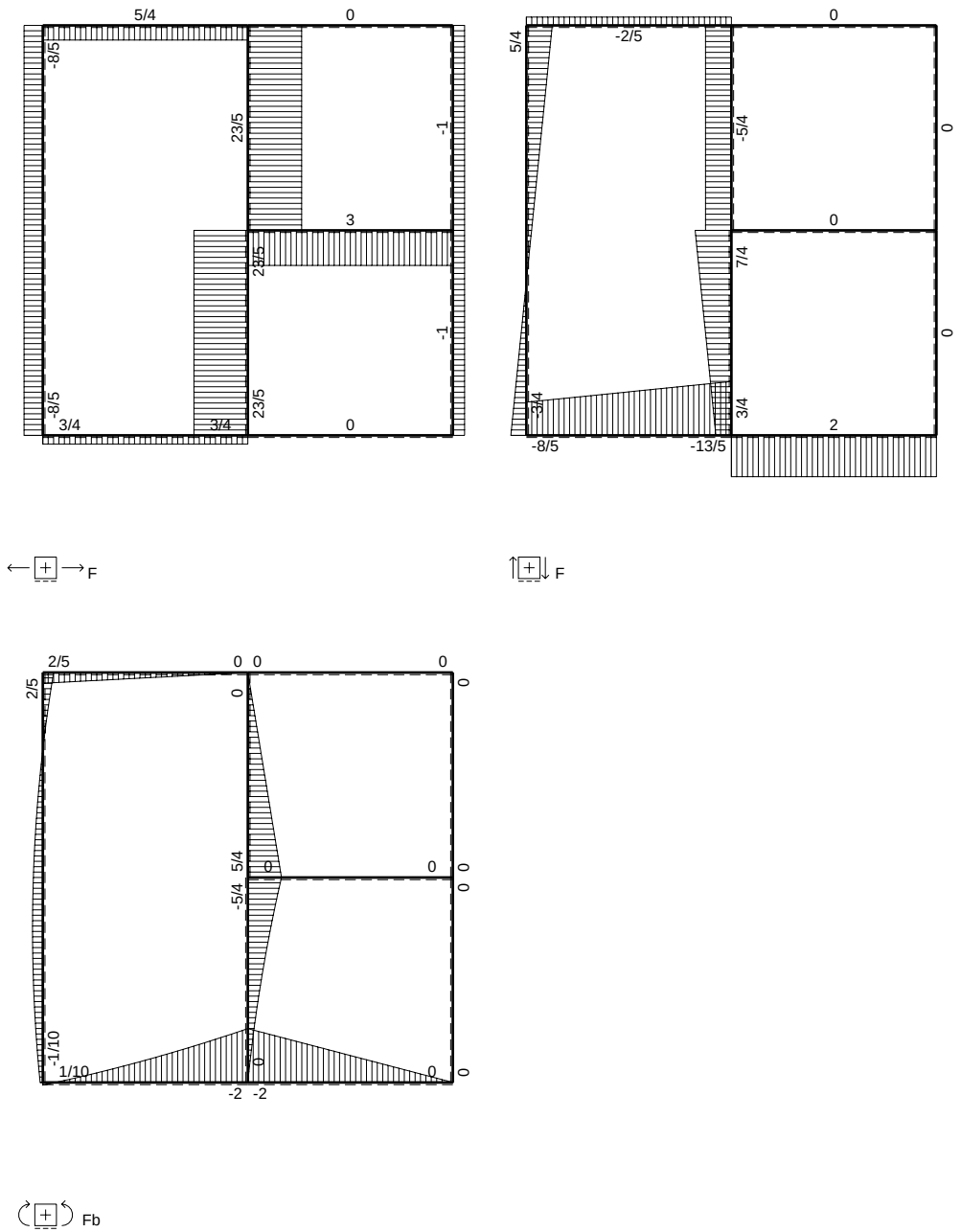
$L^{Xo}_{ED} = \int_o^b \left(1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx = \left[1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

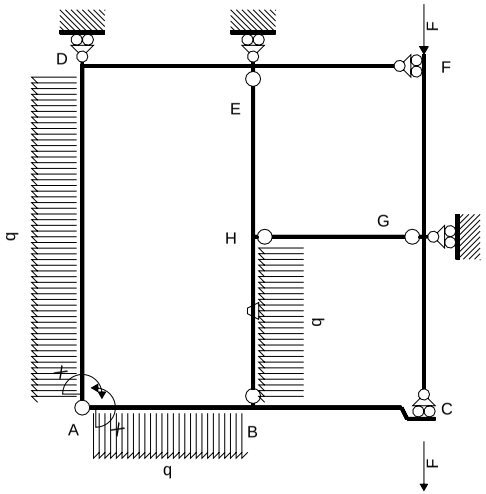
$= \left(1/6\ b\ \right) Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = -1/3\ \ Fb^2/EJ$

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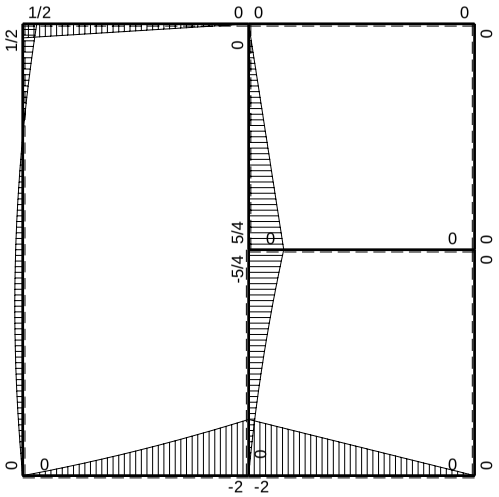
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
D
5/4F
2/5Fb
8/5F
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F
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23/5F
C
H
G
3F
8/5F
1/10Fb
3/4F
A
23/5F
3/4F
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B
C
2Fb
2F
2F

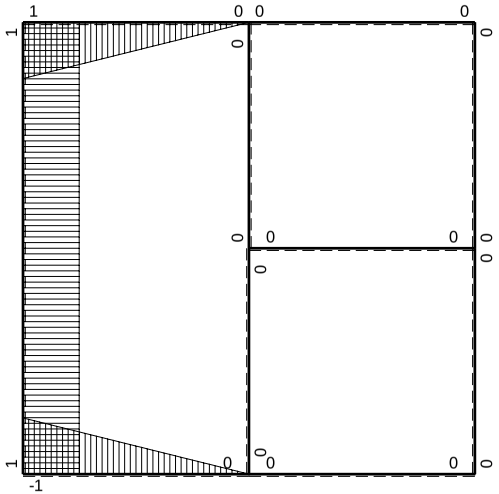

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23/5
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-8/5
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0
0
7/4
0
3/4
3/4
2
-8/5
-13/5
F
2/5
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0
2/5
0
0
0
5/4
0
0 0
-5/4
0
-1/10
0
1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236835 ESΣ03.xxxx.104

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PROCEDIMENTO E RISULTATI 236835 ESΣ03.xxxx.104

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

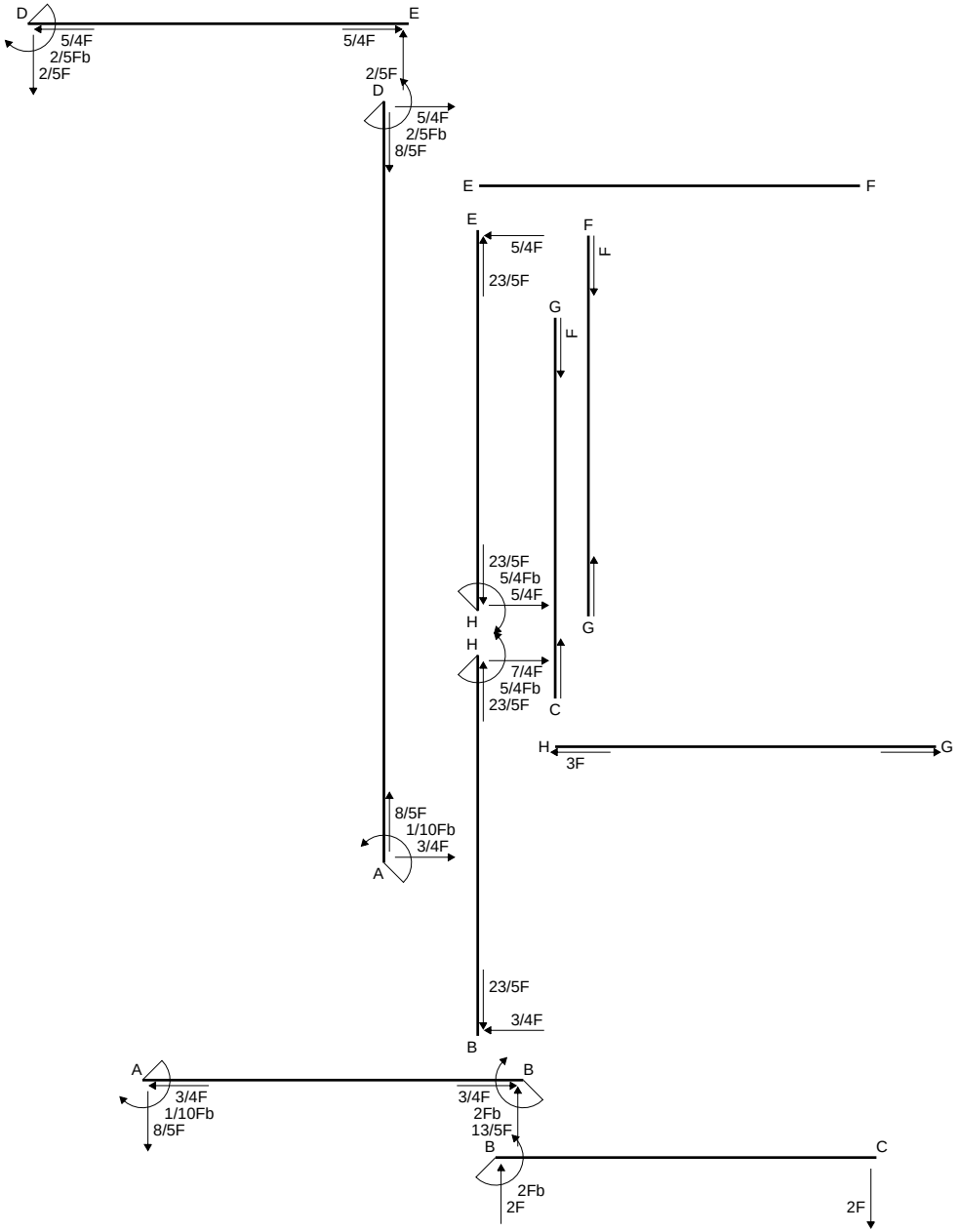
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

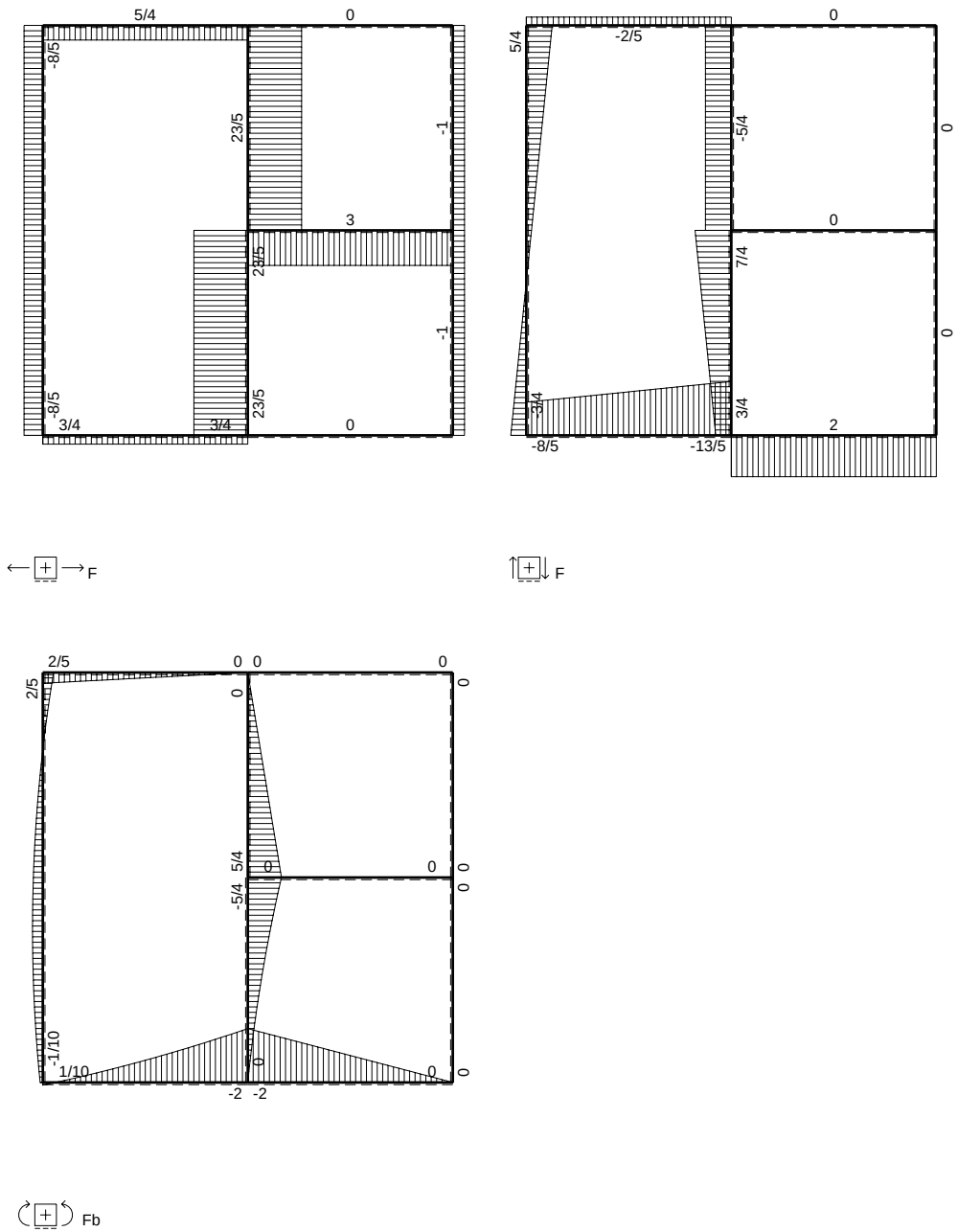
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

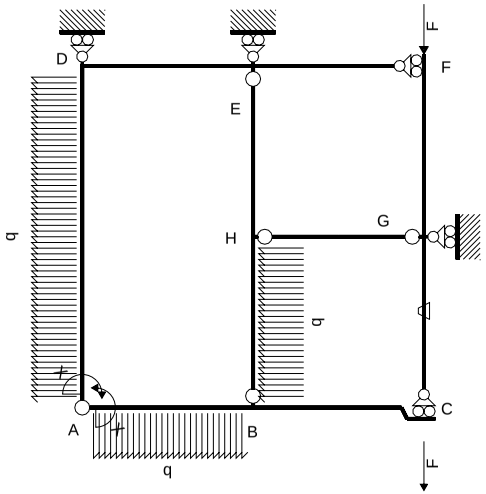
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 236835 ESΣ03.xxxx.104

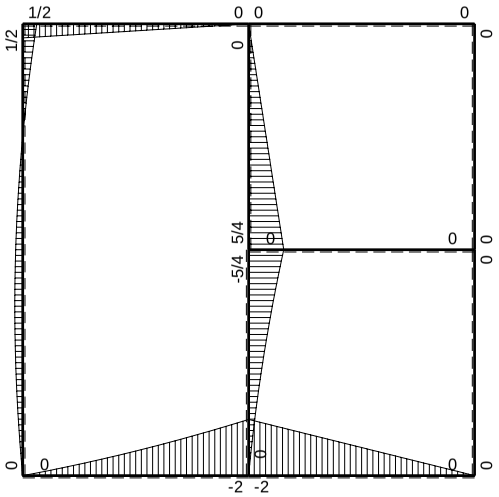
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
D
5/4F
2/5Fb
8/5F
E
F
E
F
5/4F
23/5F
F
G
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23/5F
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5/4F
H
G
H
7/4F
5/4Fb
23/5F
C
H
G
3F
8/5F
1/10Fb
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A
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3/4F
B
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C
2Fb
2F
2F

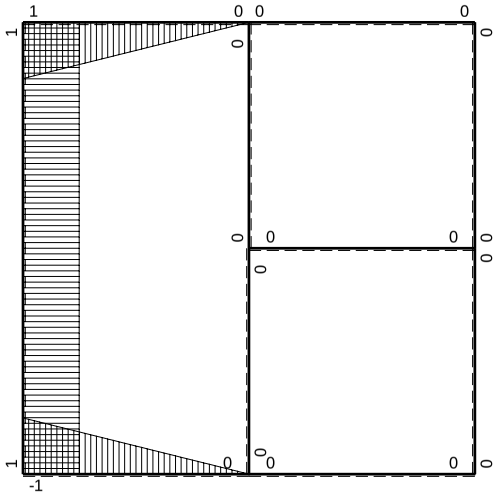

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23/5
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23/5
-1
-8/5
23/5
3/4
3/4
0
F
5/4
0
-2/5
-5/4
0
0
7/4
0
3/4
3/4
2
-8/5
-13/5
F
2/5
0
0
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2/5
0
0
0
5/4
0
0
0
-5/4
0
0
-1/10
0
1/10
0
-2
-2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

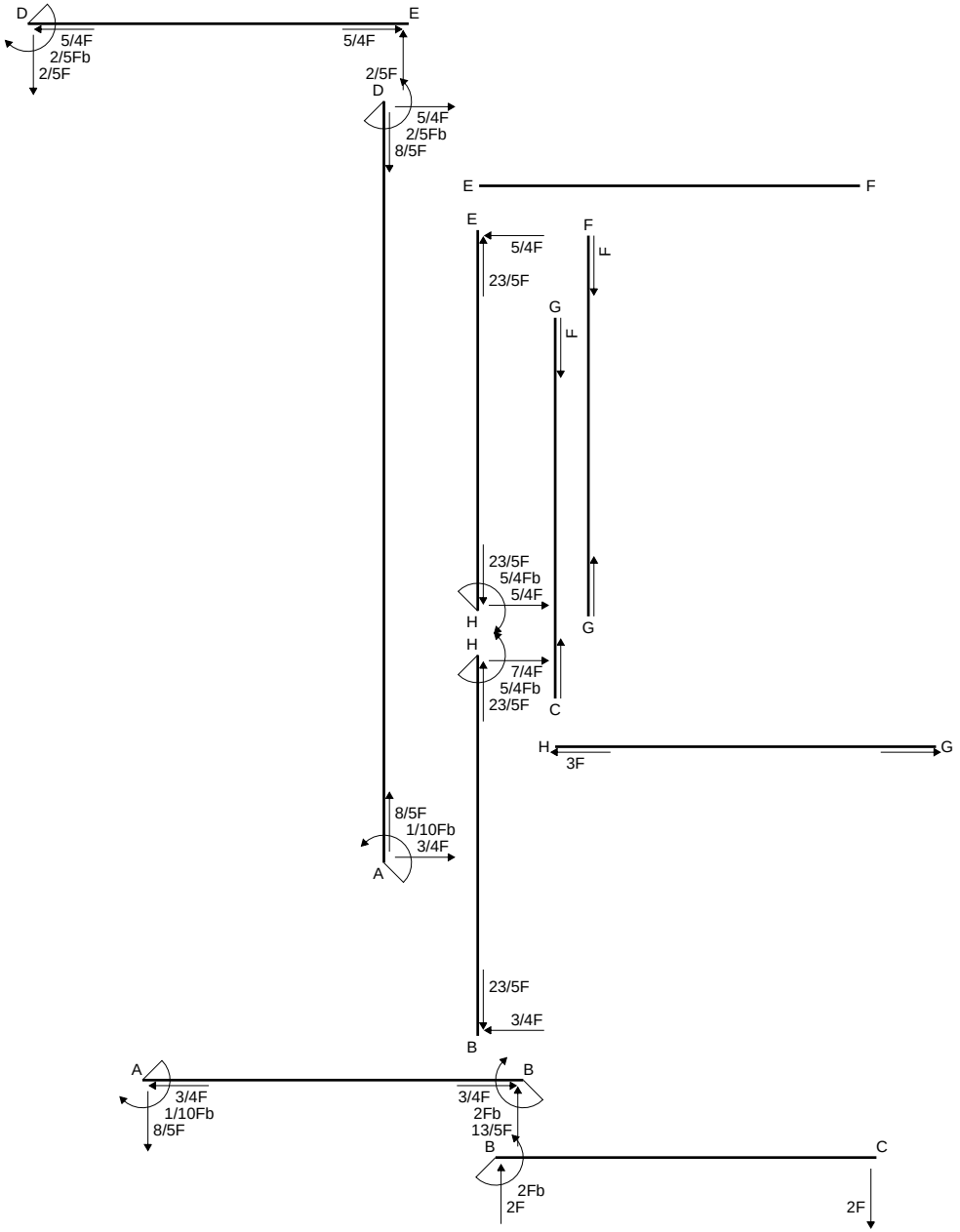
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

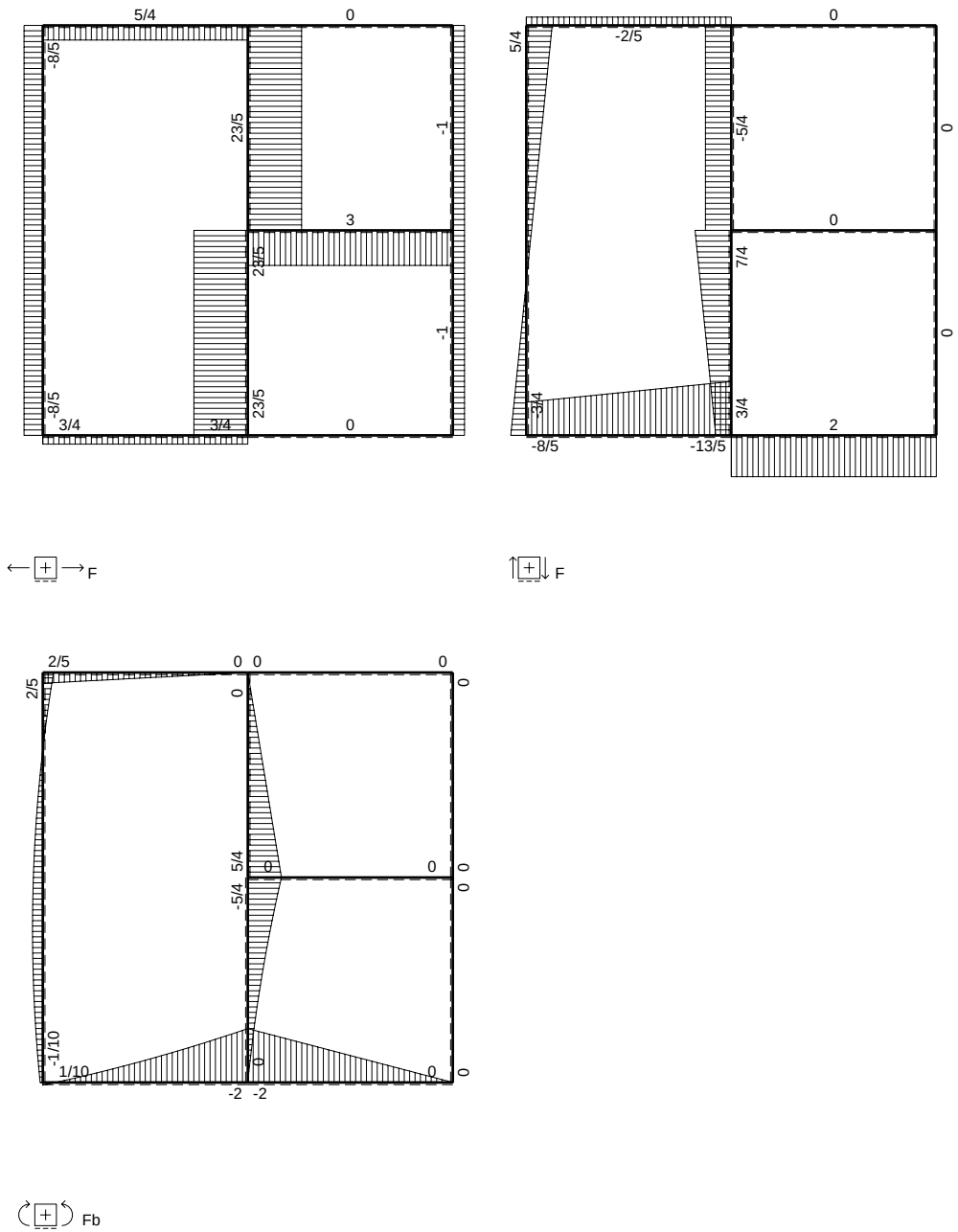
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

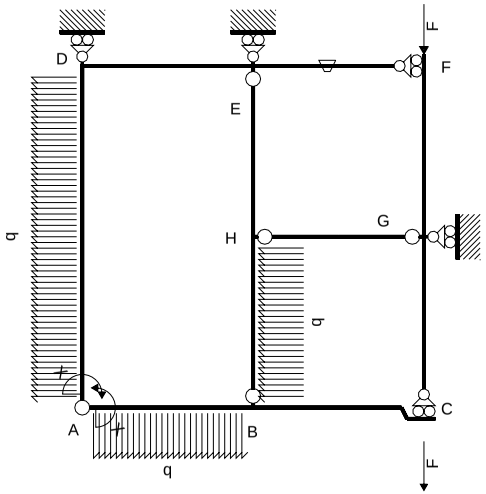
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 237131 ESΣ03.xxxx.105

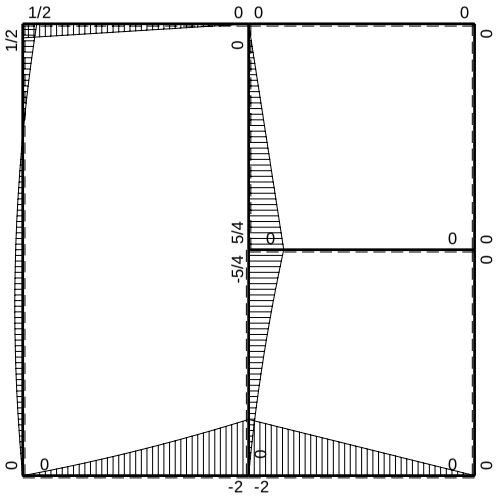
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
D
5/4F
2/5Fb
8/5F
E
F
E
F
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23/5F
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23/5F
C
H
G
3F
8/5F
1/10Fb
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A
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3/4F
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B
C
2Fb
2F
2F

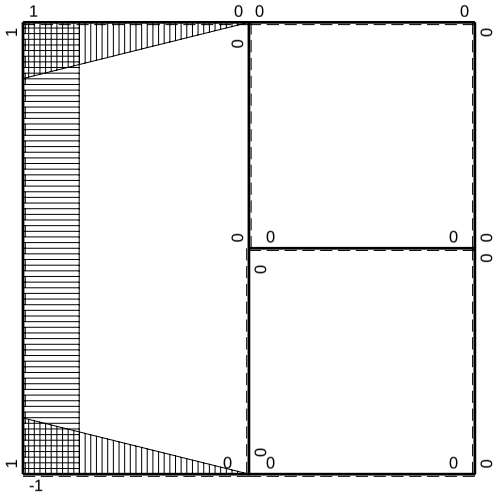

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F
5/4
0
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-5/4
0
0
7/4
0
3/4
3/4
2
-8/5
-13/5
F
2/5
0 0
0
2/5
0
0
0
5/4
0
0 0
-5/4
0
-1/10
0
1/10
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239106 ESΣ03.xxxx.106

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PROCEDIMENTO E RISULTATI 239106 ESΣ03.xxxx.106

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

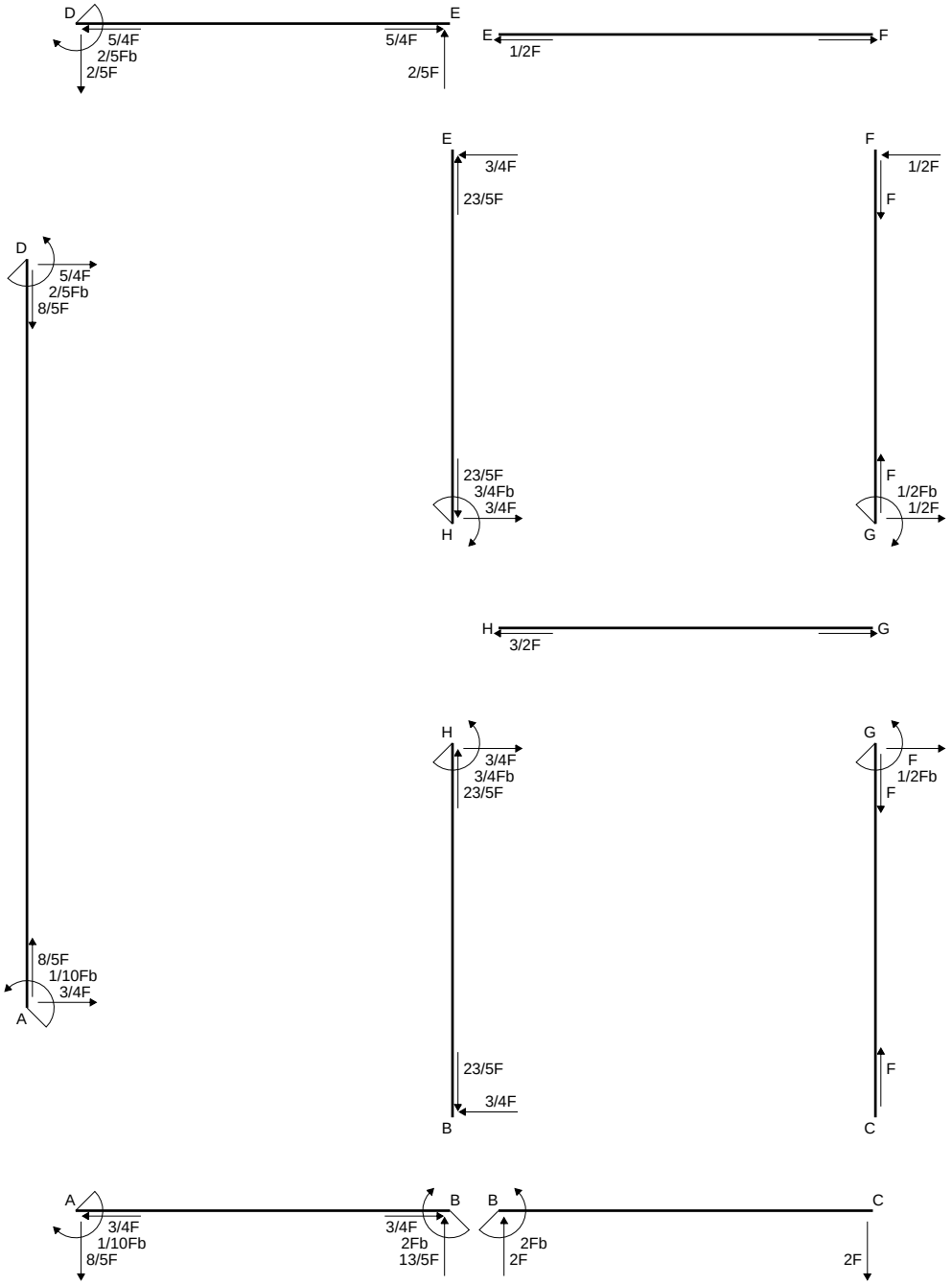
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

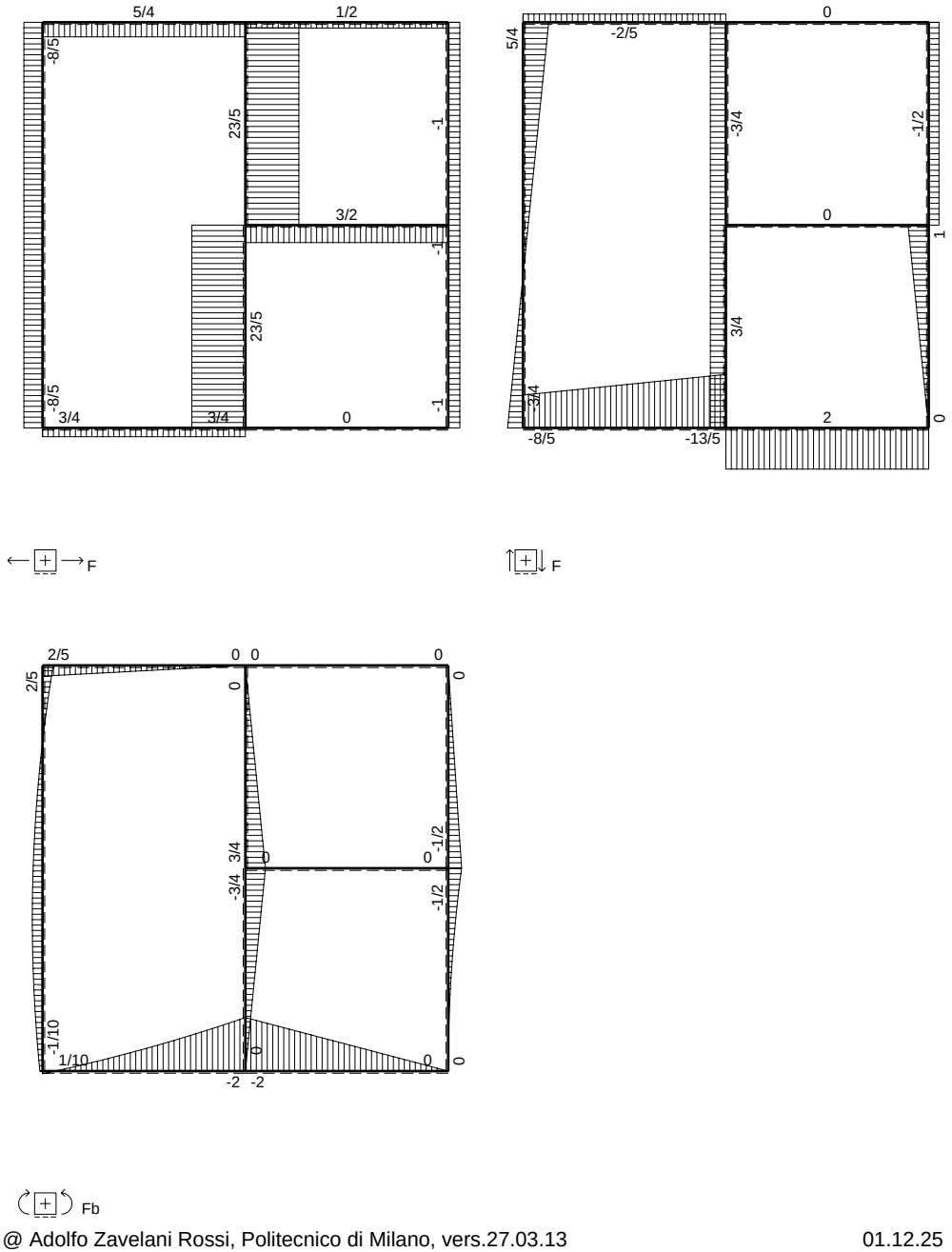
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

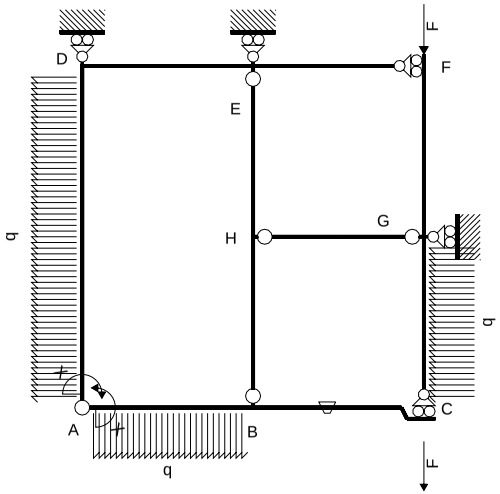
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 239106 ESΣ03.xxxx.106

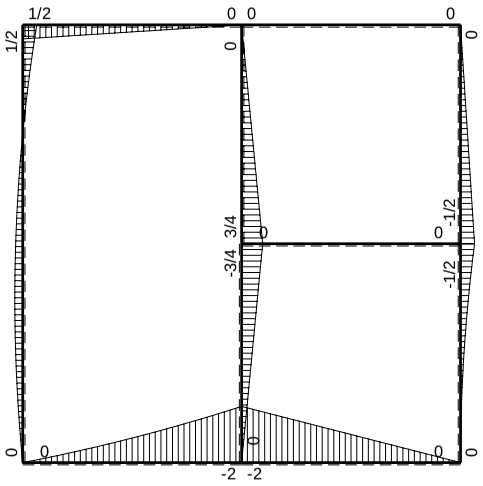
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
E
F
1/2F
E
3/4F
23/5F
F
1/2F
F
D
5/4F
2/5Fb
8/5F
23/5F
3/4Fb
3/4F
H
F
1/2Fb
1/2F
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3/2F
H
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3/4Fb
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G
F
1/2Fb
F
8/5F
1/10Fb
3/4F
A
23/5F
3/4F
B
F
C
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13/5F
2Fb
2F
2F

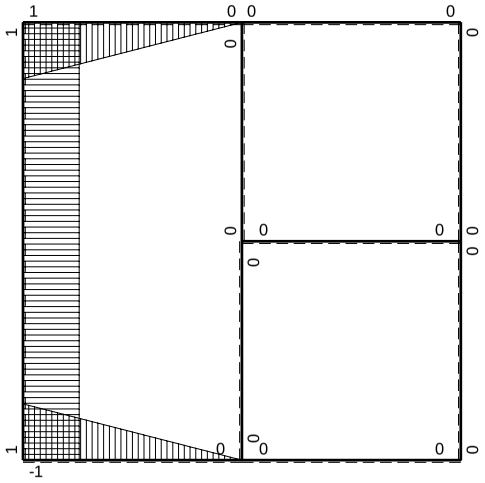

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23/5
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3/4
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F
5/4
0
-2/5
-3/4
-1/2
0
1
3/4
-3/4
2
0
-8/5
-13/5
F
2/5
0 0
0
2/5
0
0
0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 983430 ESΣ03.xxxx.107

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	-Fb/EJ	0	0	0	0+0	0
CB b	0	2Fx	Fb/EJ	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	-1/2Fx	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-3/4Fb+3/4Fx	0	0	0	0	0+0	0
BH b	0	3/4Fx	0	0	0	0		
HE b	0	3/4Fb-3/4Fx	0	0	0	0	0+0	0
EH b	0	-3/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

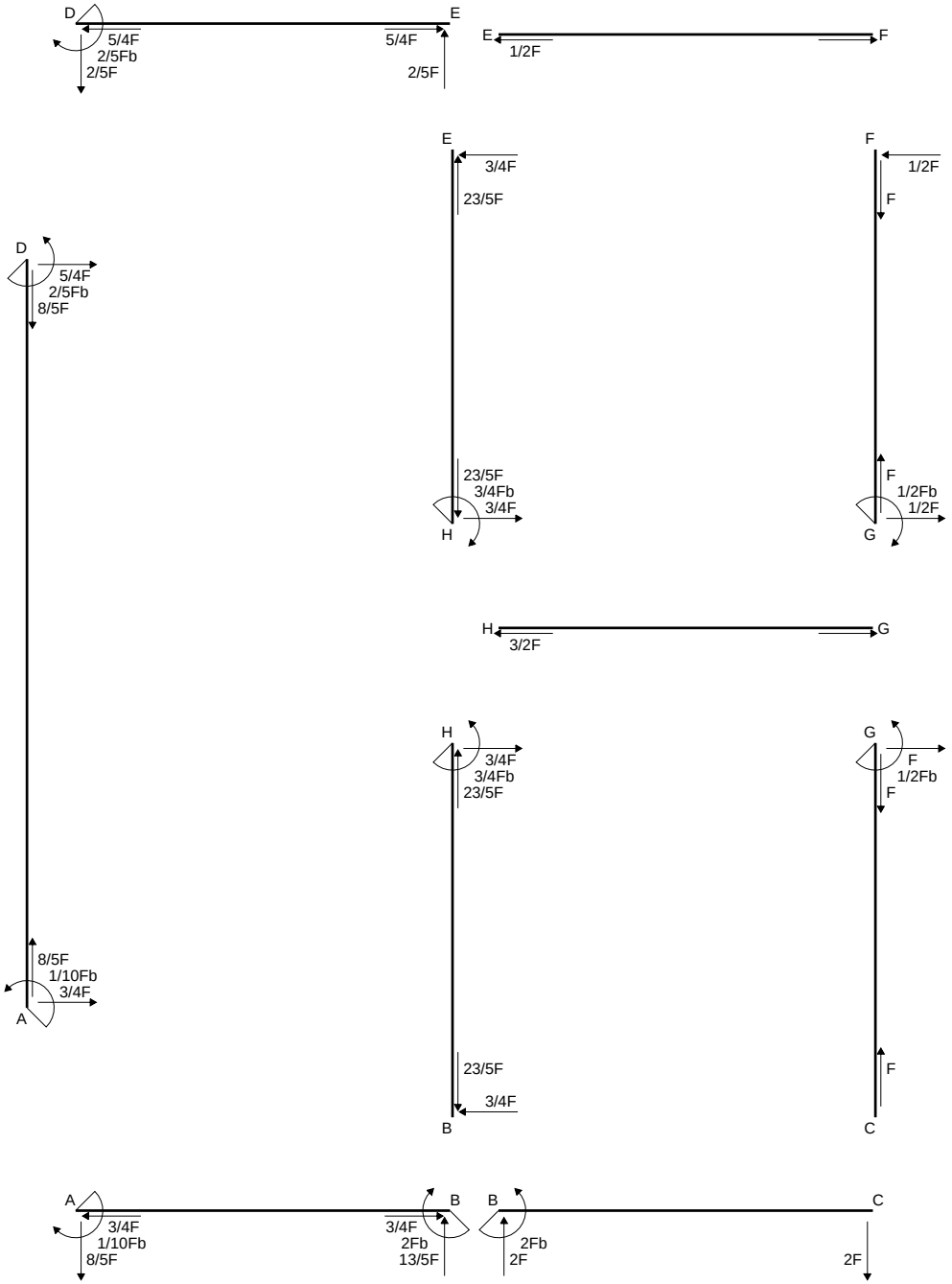
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

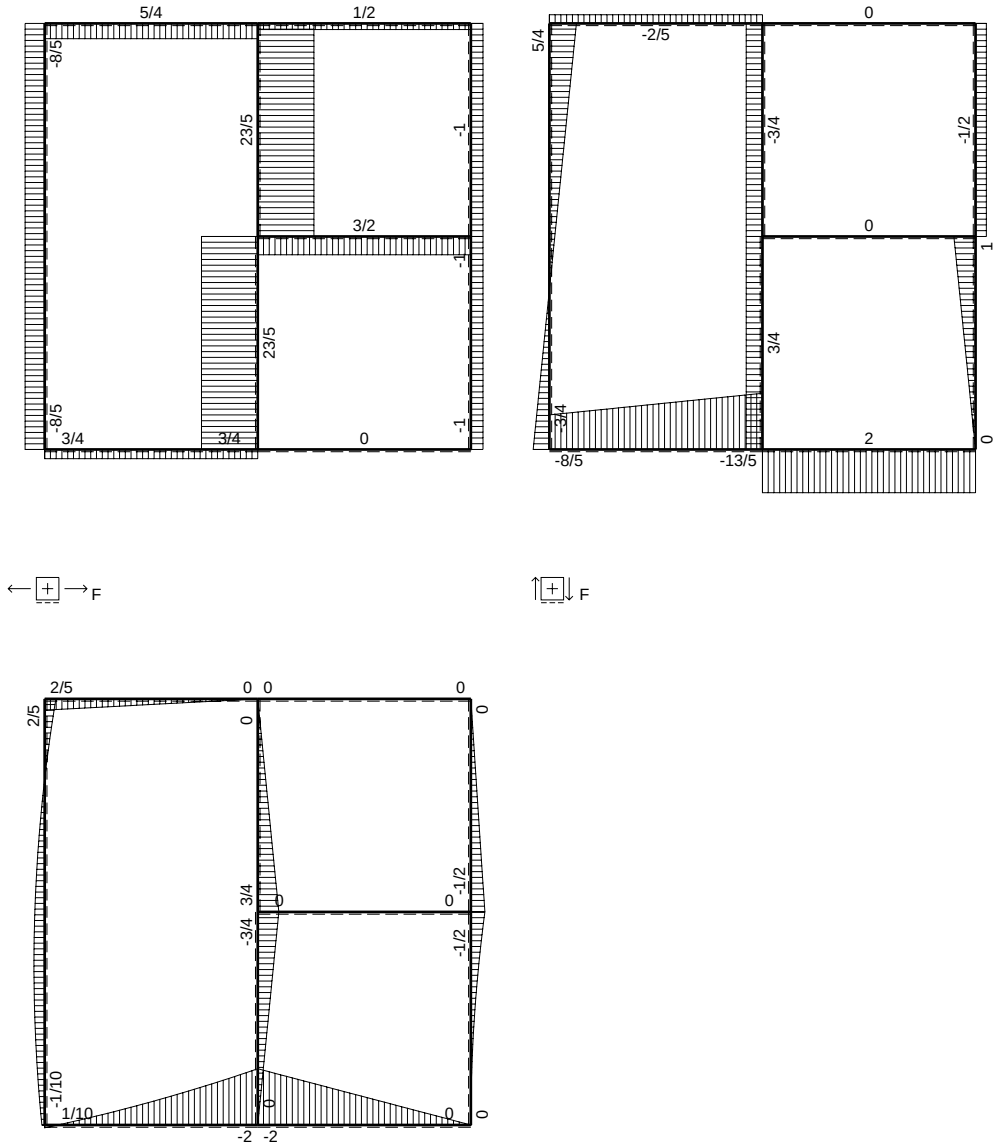
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

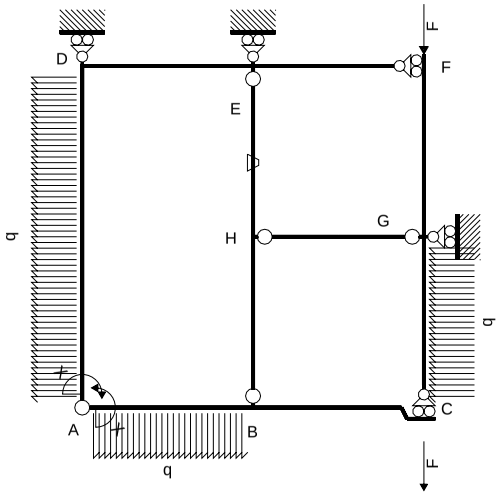
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 983430 ESΣ03.xxxx.107

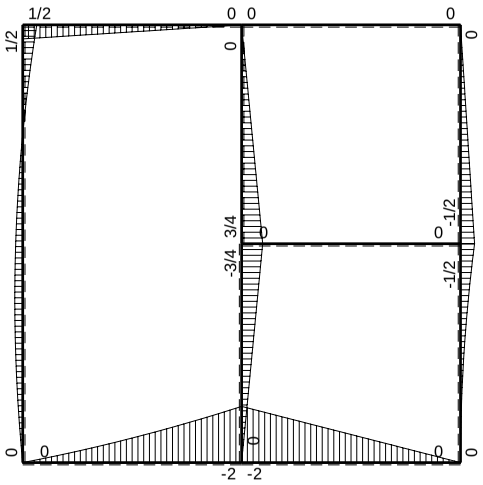
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
E
F
1/2F
E
3/4F
23/5F
F
1/2F
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2/5Fb
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1/2Fb
F
8/5F
1/10Fb
3/4F
A
23/5F
3/4F
B
F
C
A
B
B
C
3/4F
1/10Fb
8/5F
3/4F
2Fb
13/5F
2Fb
2F
2F

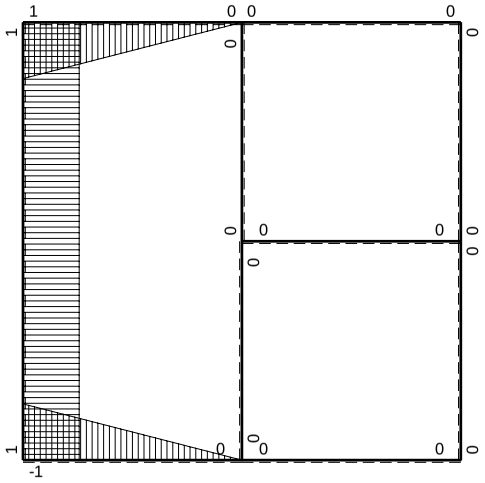

5/4
1/2
-8/5
23/5
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3/2
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23/5
-8/5
3/4
3/4
0
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F
5/4
0
-2/5
-3/4
-1/2
0
1
3/4
-3/4
2
0
-8/5
-13/5
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2/5
0
0
0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237985 ESΣ03.xxxx.108

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PROCEDIMENTO E RISULTATI 237985 ESΣ03.xxxx.108

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	0	0	0	0	0+0	0
BH b	0	$3/4Fx$	0	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-3/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

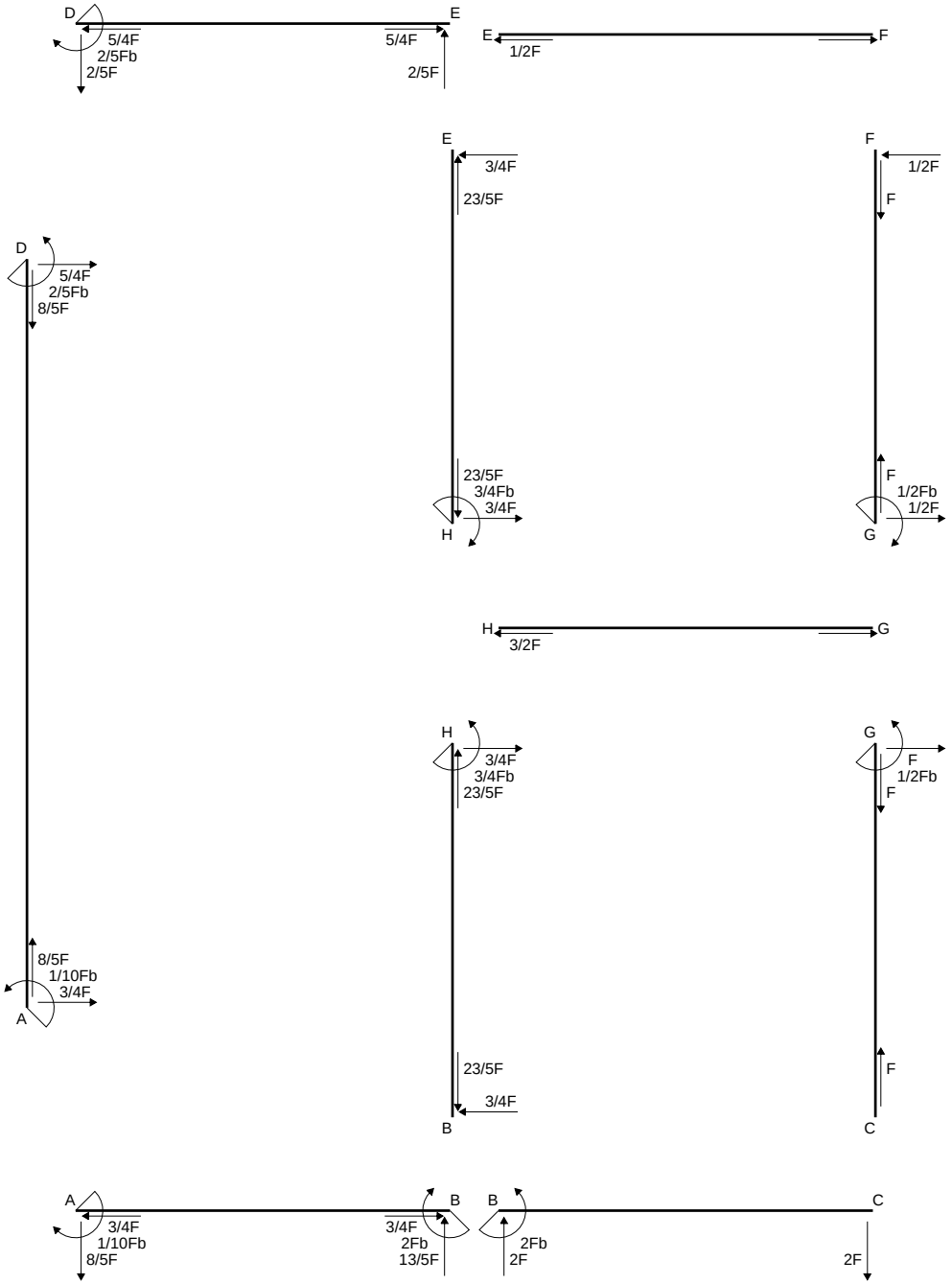
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

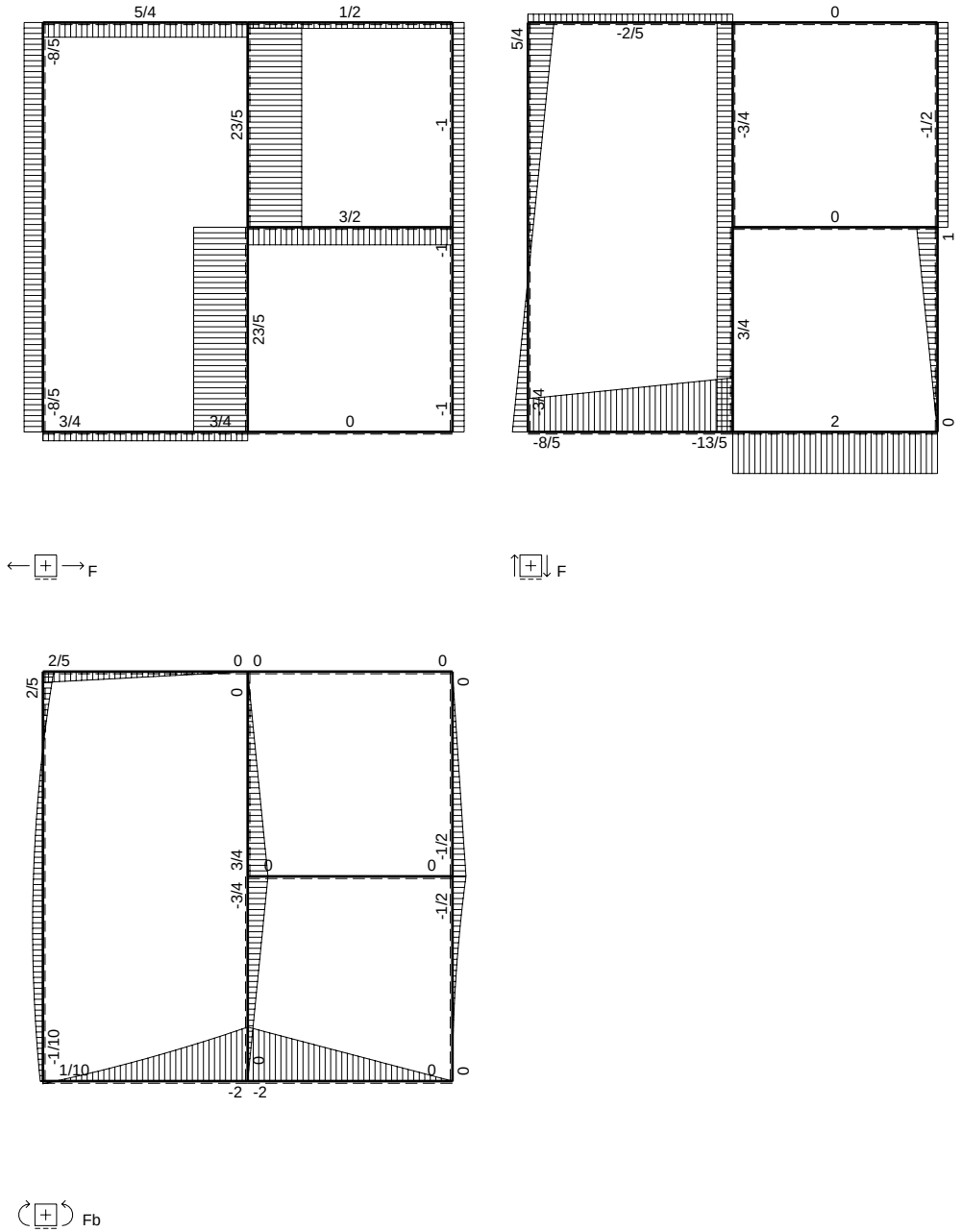
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

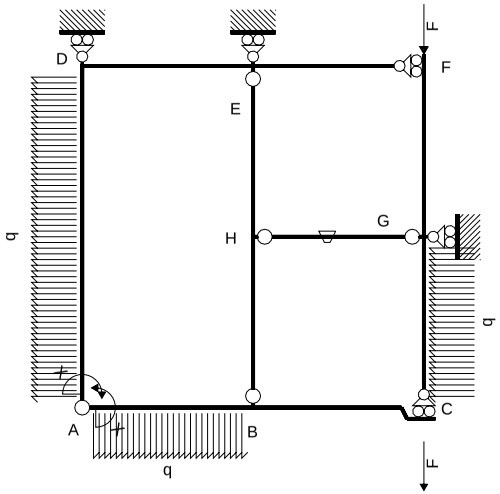
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 237985 ESΣ03.xxxx.108

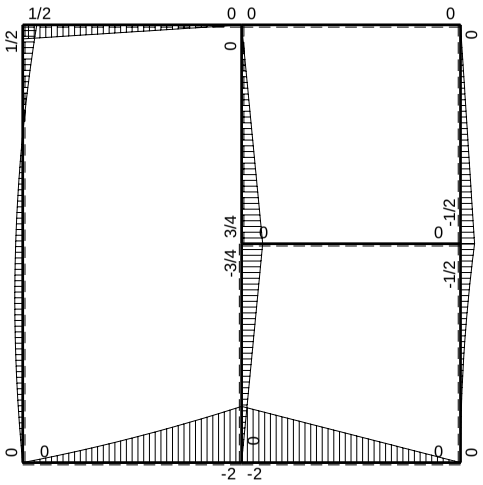
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
E
F
1/2F
E
3/4F
23/5F
F
1/2F
F
D
5/4F
2/5Fb
8/5F
23/5F
3/4Fb
3/4F
H
F
1/2Fb
1/2F
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3/2F
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3/4F
3/4Fb
23/5F
G
F
1/2Fb
F
8/5F
1/10Fb
3/4F
A
23/5F
3/4F
B
F
C
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13/5F
2Fb
2F
2F

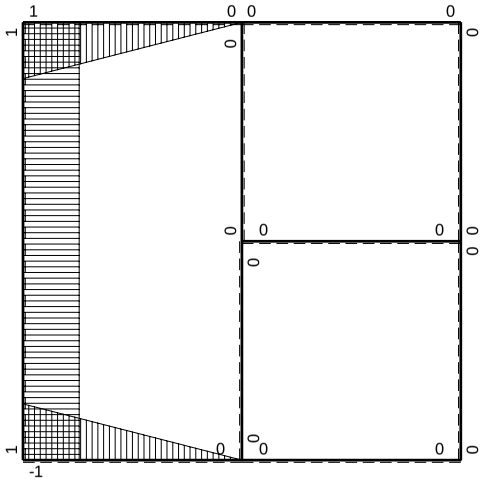

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23/5
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23/5
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3/4
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-1
F
5/4
0
-2/5
-3/4
-1/2
0
1
3/4
-3/4
2
0
-8/5
-13/5
F
2/5
0 0
0
2/5
0
0
0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 309494 ESΣ03.xxxx.109

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	-3/4Fb+3/4Fx	0	0	0	0	0+0	0
BH b	0	3/4Fx	0	0	0	0		
HE b	0	3/4Fb-3/4Fx	0	0	0	0	0+0	0
EH b	0	-3/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

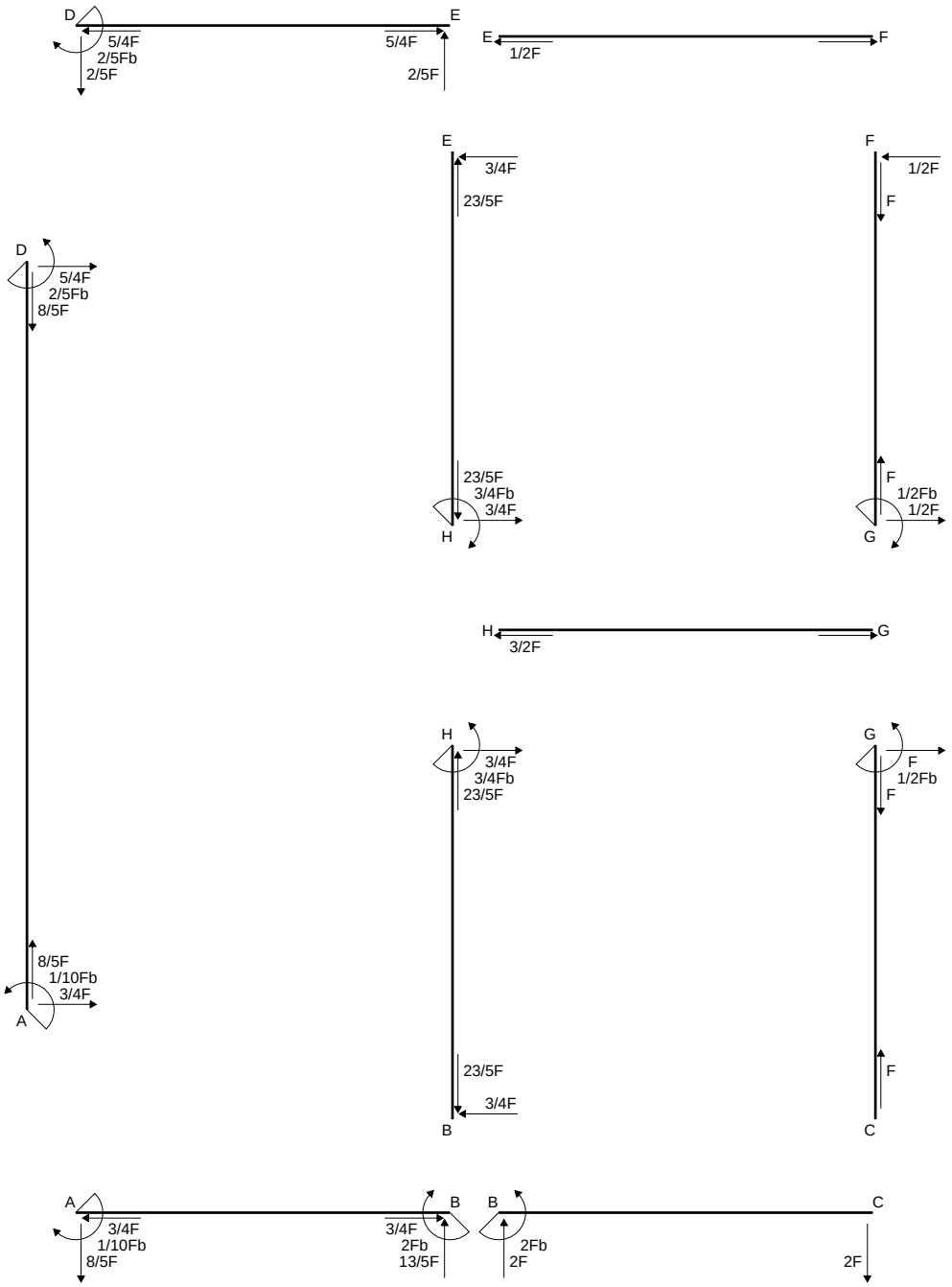
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

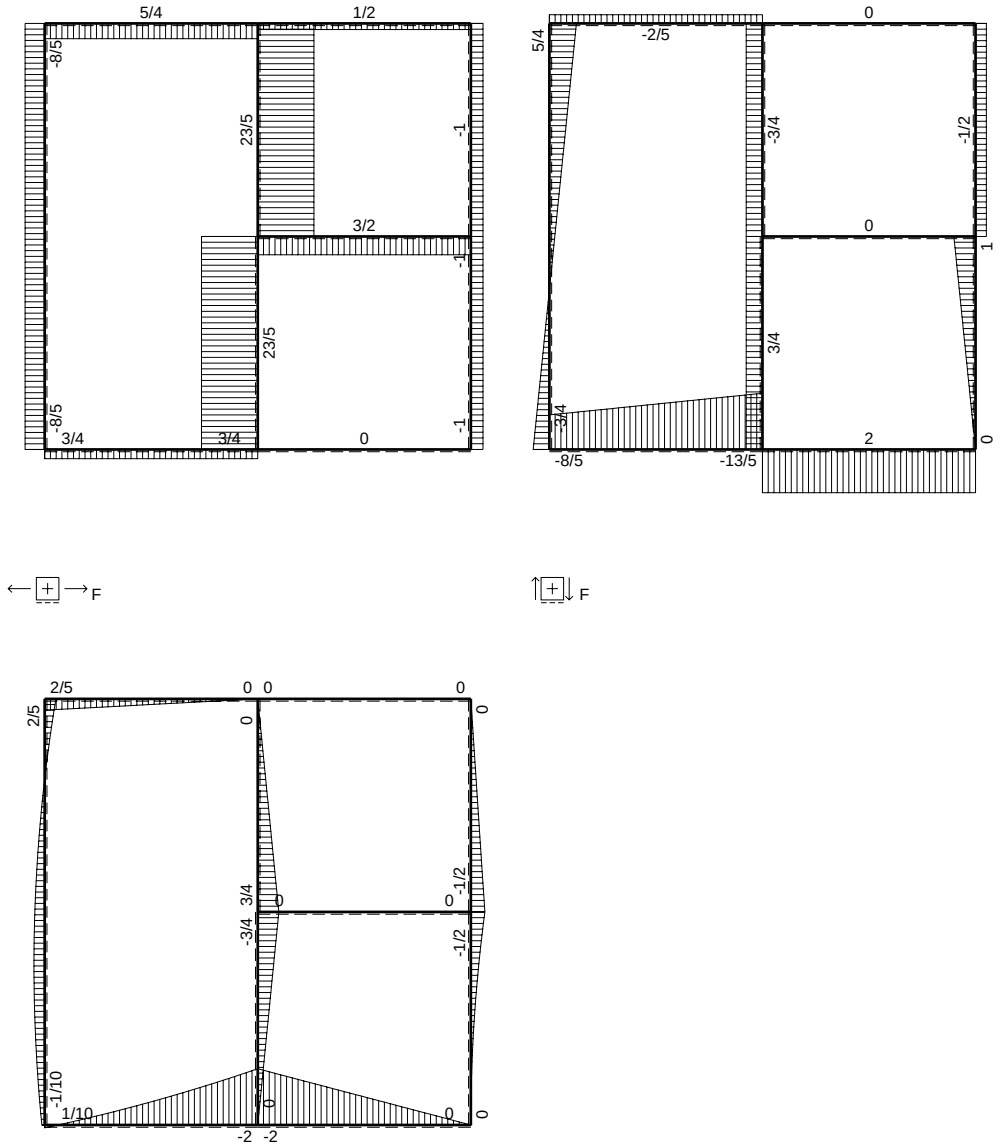
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

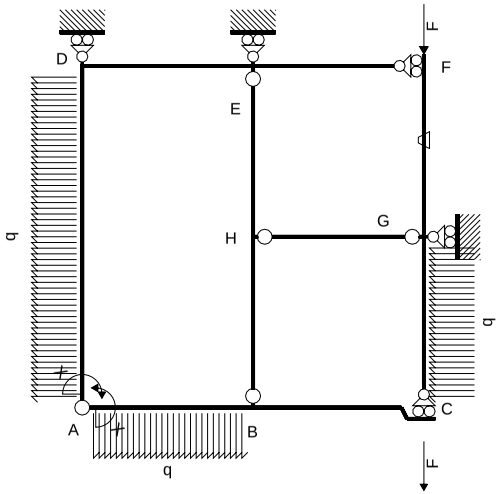
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 309494 ESΣ03.xxxx.109

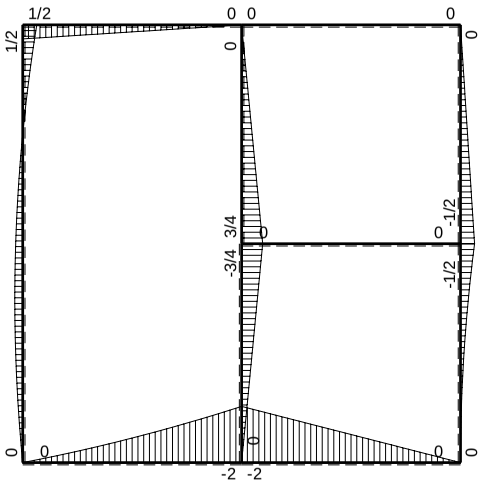
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
E
F
1/2F
E
3/4F
23/5F
F
1/2F
F
D
5/4F
2/5Fb
8/5F
23/5F
3/4Fb
3/4F
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F
1/2Fb
F
8/5F
1/10Fb
3/4F
A
23/5F
3/4F
B
F
C
A
B
B
C
3/4F
1/10Fb
8/5F
3/4F
2Fb
13/5F
2Fb
2F
2F

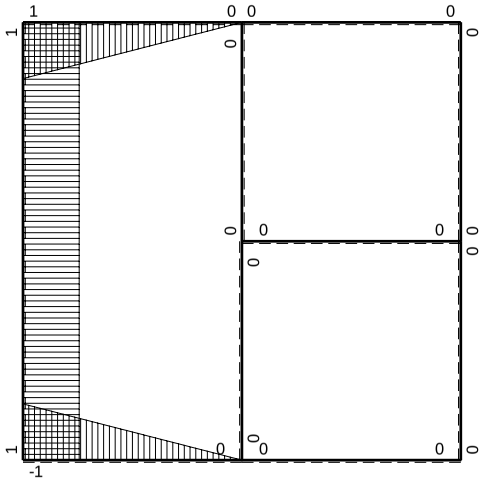

5/4
1/2
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23/5
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3/2
-1
23/5
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3/4
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0
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F
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0
-2/5
-3/4
-1/2
0
1
3/4
-3/4
0
2
0
-8/5
-13/5
F
2/5
0 0
0
2/5
0
0
0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 245588 ESΣ03.xxxx.110

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PROCEDIMENTO E RISULTATI 245588 ESΣ03.xxxx.110

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	-Fb/EJ	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	Fb/EJ	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-3/4Fb+3/4Fx	0	0	0	0	0+0	0
BH b	0	3/4Fx	0	0	0	0		
HE b	0	3/4Fb-3/4Fx	0	0	0	0	0+0	0
EH b	0	-3/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

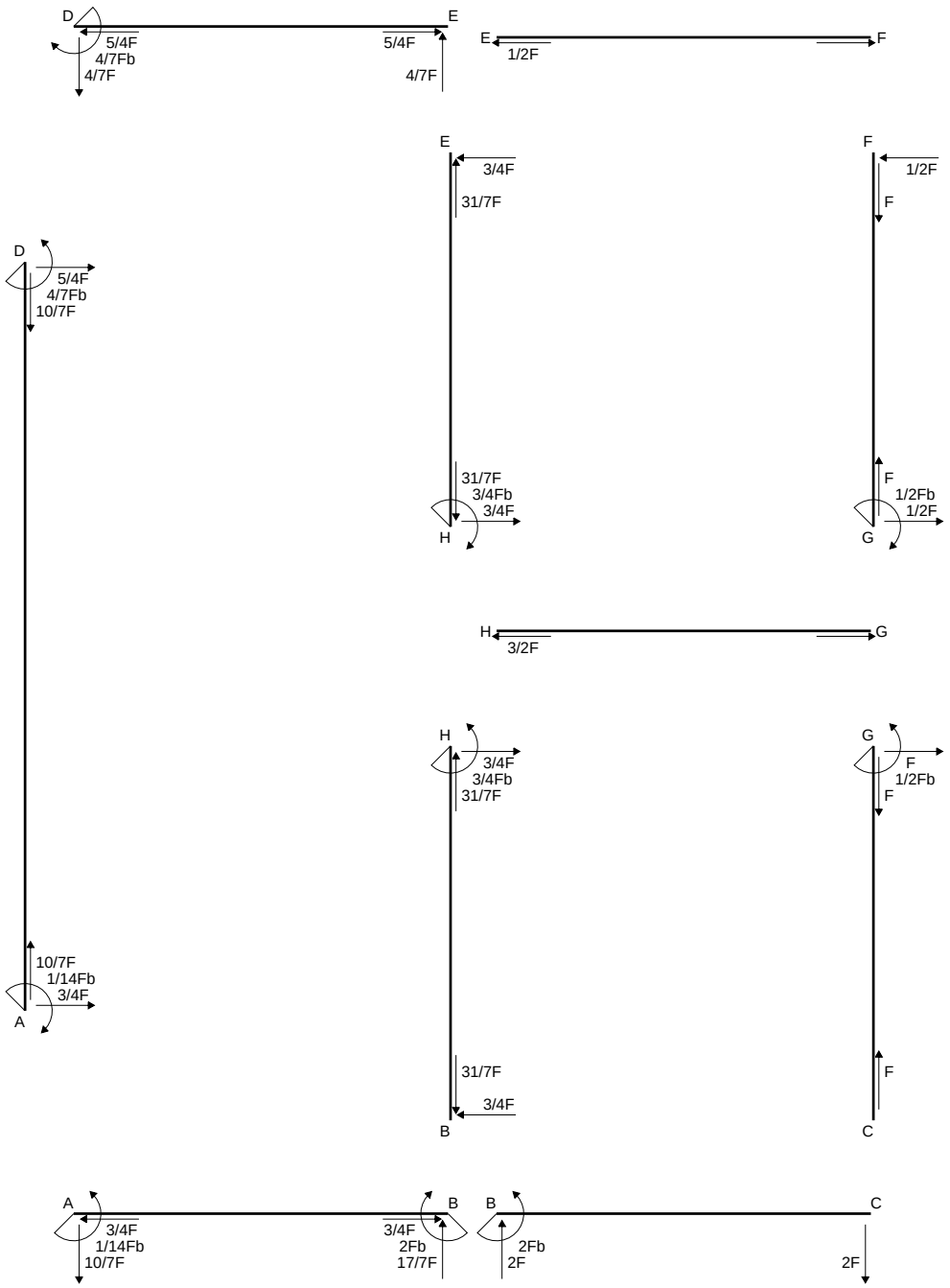
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

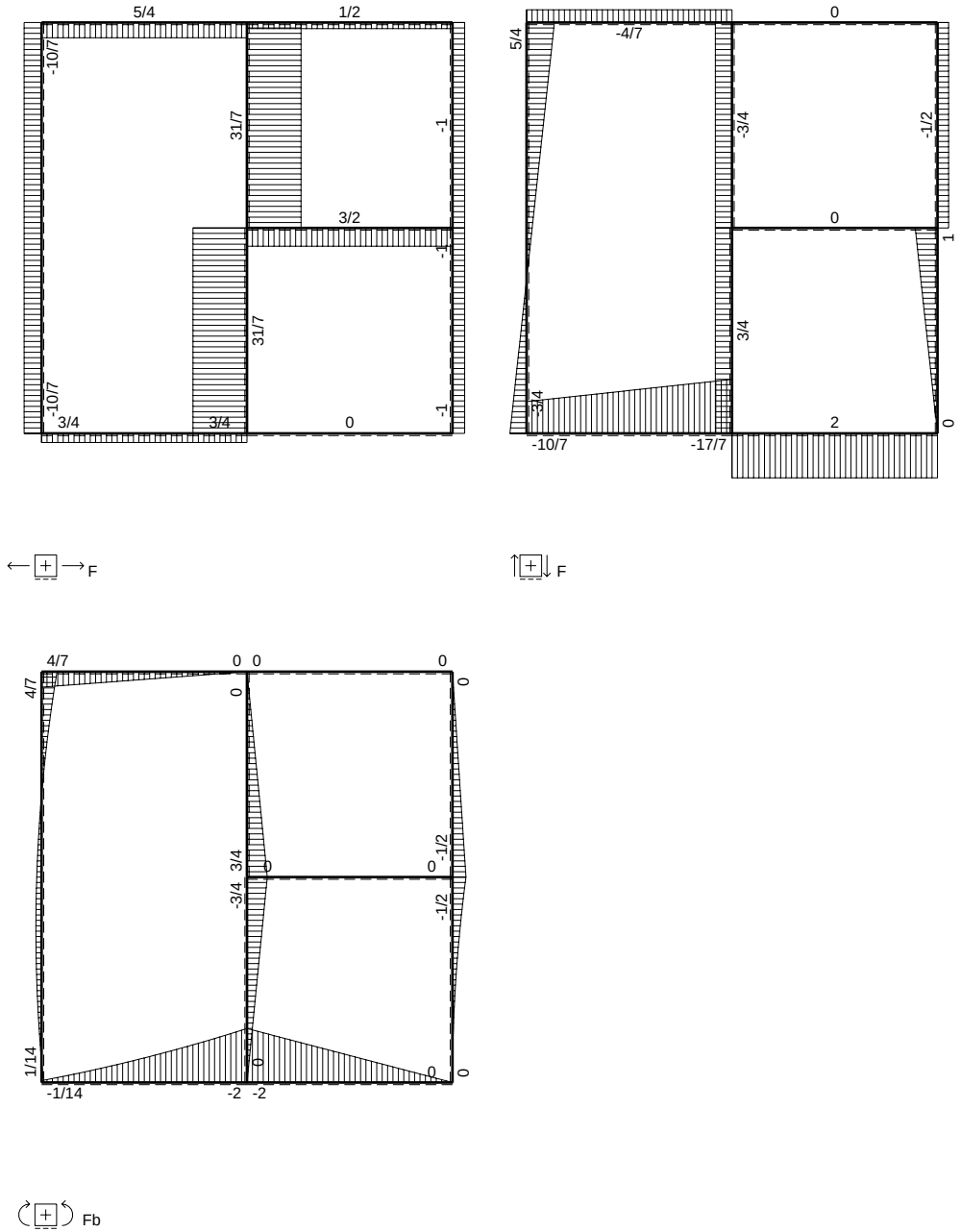
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

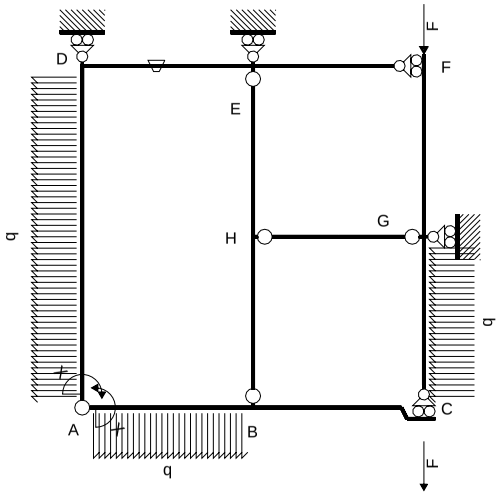
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 245588 ESΣ03.xxxx.110

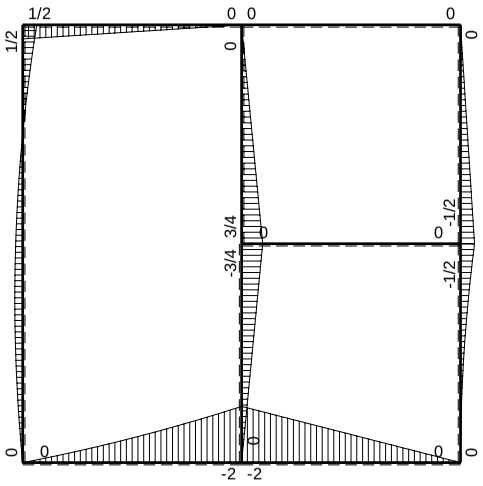
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D
E
5/4F
4/7Fb
4/7F
5/4F
4/7F
E
F
1/2F
E
3/4F
31/7F
F
1/2F
F
D
5/4F
4/7Fb
10/7F
31/7F
3/4Fb
3/4F
H
F
1/2Fb
1/2F
G
H
G
3/2F
H
3/4F
3/4Fb
31/7F
G
F
1/2Fb
F
10/7F
1/14Fb
3/4F
A
31/7F
3/4F
B
F
C
A
B
B
C
3/4F
1/14Fb
10/7F
3/4F
2Fb
17/7F
2Fb
2F
2F

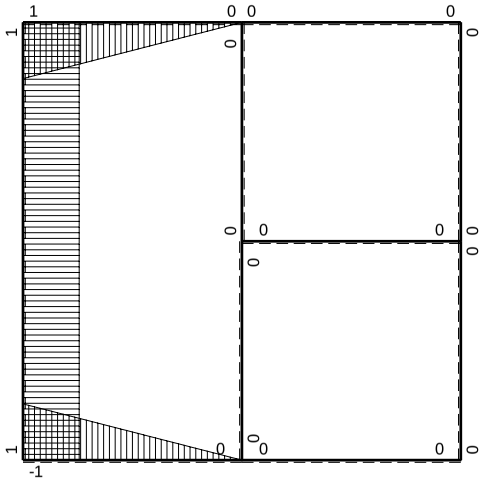

5/4
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-10/7
31/7
-1
3/2
-1
31/7
-10/7
3/4
3/4
0
-1
F
5/4
-4/7
0
-3/4
-1/2
0
1
3/4
3/4
2
0
-10/7
-17/7
F
4/7
0 0
0
4/7
0
0
0
3/4
0
-1/2
-3/4
0
-1/2
1/14
0
0
-1/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237870 ESΣ03.xxxx.111

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	$-Fb/EJ$	$1/2Fb-Fx+1/2Fx^2/b$	$-Fb/EJ+Fx/EJ$	$1-2x/b+x^2/b^2$	$(1/6-1/2)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	Fb/EJ	$1/2Fx^2/b$	$-Fx/EJ$	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	0	0	0	0	0+0	0
BH b	0	$3/4Fx$	0	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-5/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(1/2 -5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x -5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(\ b -5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(1/2 - x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right)\ \theta\ \ dx$

$= \left[1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[-\ x +1/2\ x^2/b\ \right]_o^b\ \theta$

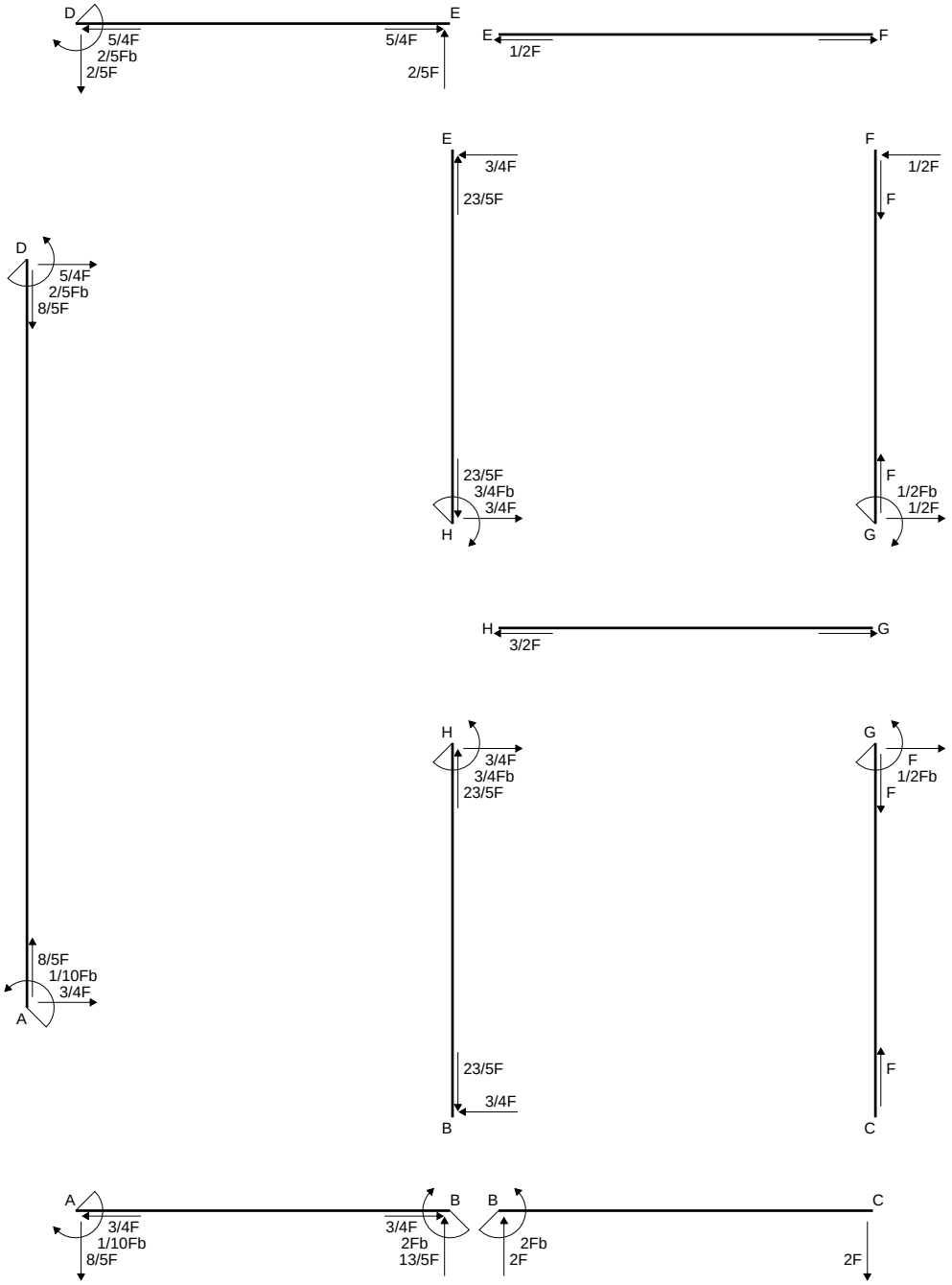
$= \left(1/2\ b -1/2\ b +1/6\ b\ \right) Fb\ 1/EJ\ + \left(-\ b +1/2\ b\ \right)\ \theta\ \ = -1/3\ \ Fb^2/EJ$

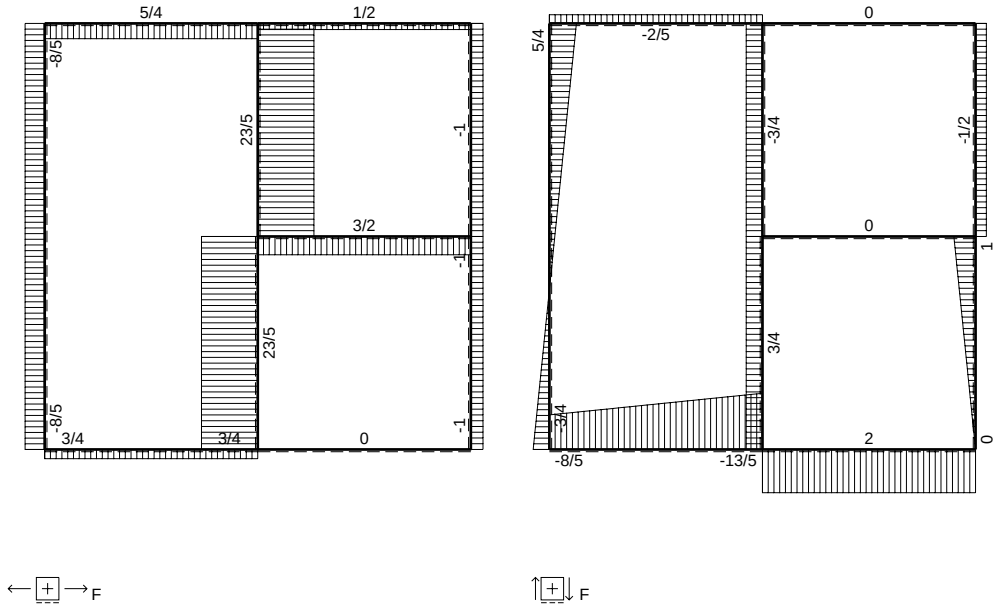
$L^{Xo}_{ED} = \int_o^b \left(1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\ \theta\ \ dx = \left[1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b\ \theta$

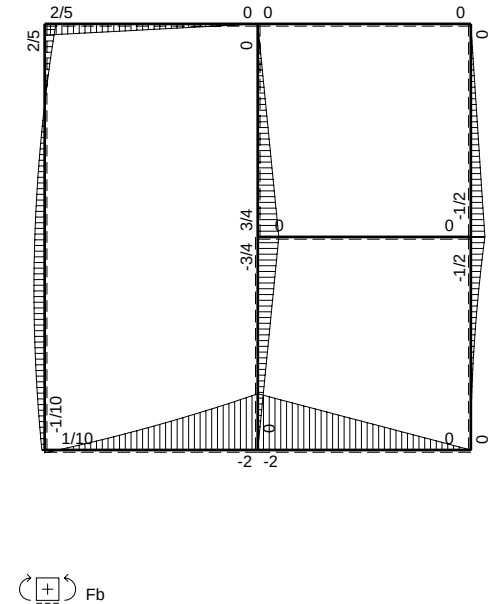
$= \left(1/6\ b\ \right) Fb\ 1/EJ\ + \left(1/2\ b\ \right)\ \theta\ \ = -1/3\ \ Fb^2/EJ$

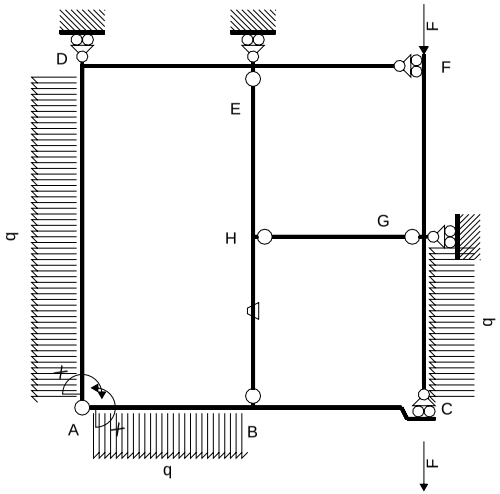
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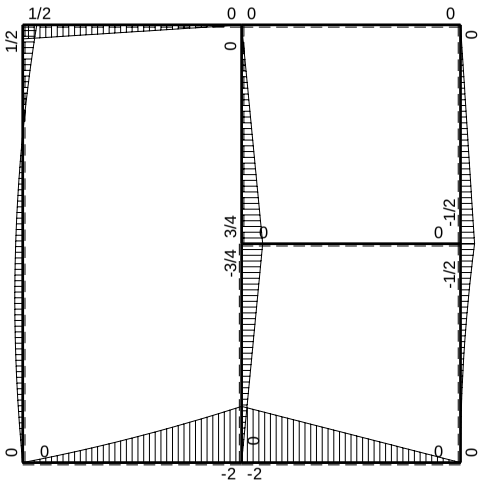

D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
E
F
1/2F
E
3/4F
23/5F
F
1/2F
F
D
5/4F
2/5Fb
8/5F
23/5F
3/4Fb
3/4F
H
F
1/2Fb
1/2F
G
H
G
3/2F
H
3/4F
3/4Fb
23/5F
G
F
1/2Fb
F
8/5F
1/10Fb
3/4F
A
23/5F
3/4F
B
F
C
A
B
B
C
3/4F
1/10Fb
8/5F
3/4F
2Fb
13/5F
2Fb
2F
2F


5/4
1/2
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-13/5
F

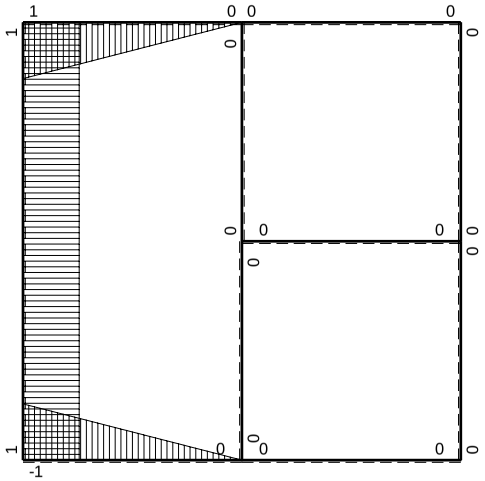

2/5
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2/5
0
0
0
3/4
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0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211786 ESΣ03.xxxx.112

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$3/4Fx$	Fb/EJ	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

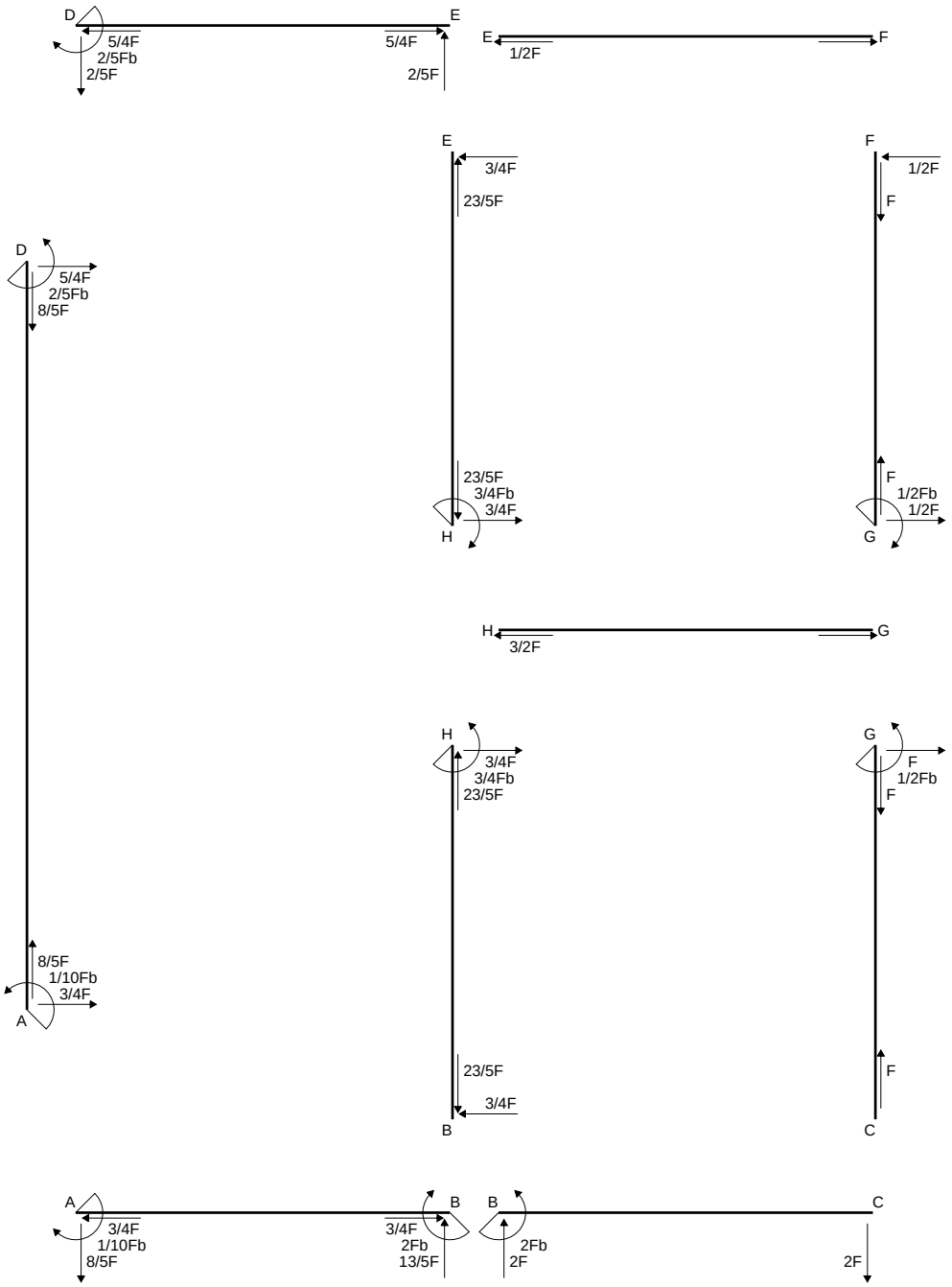
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

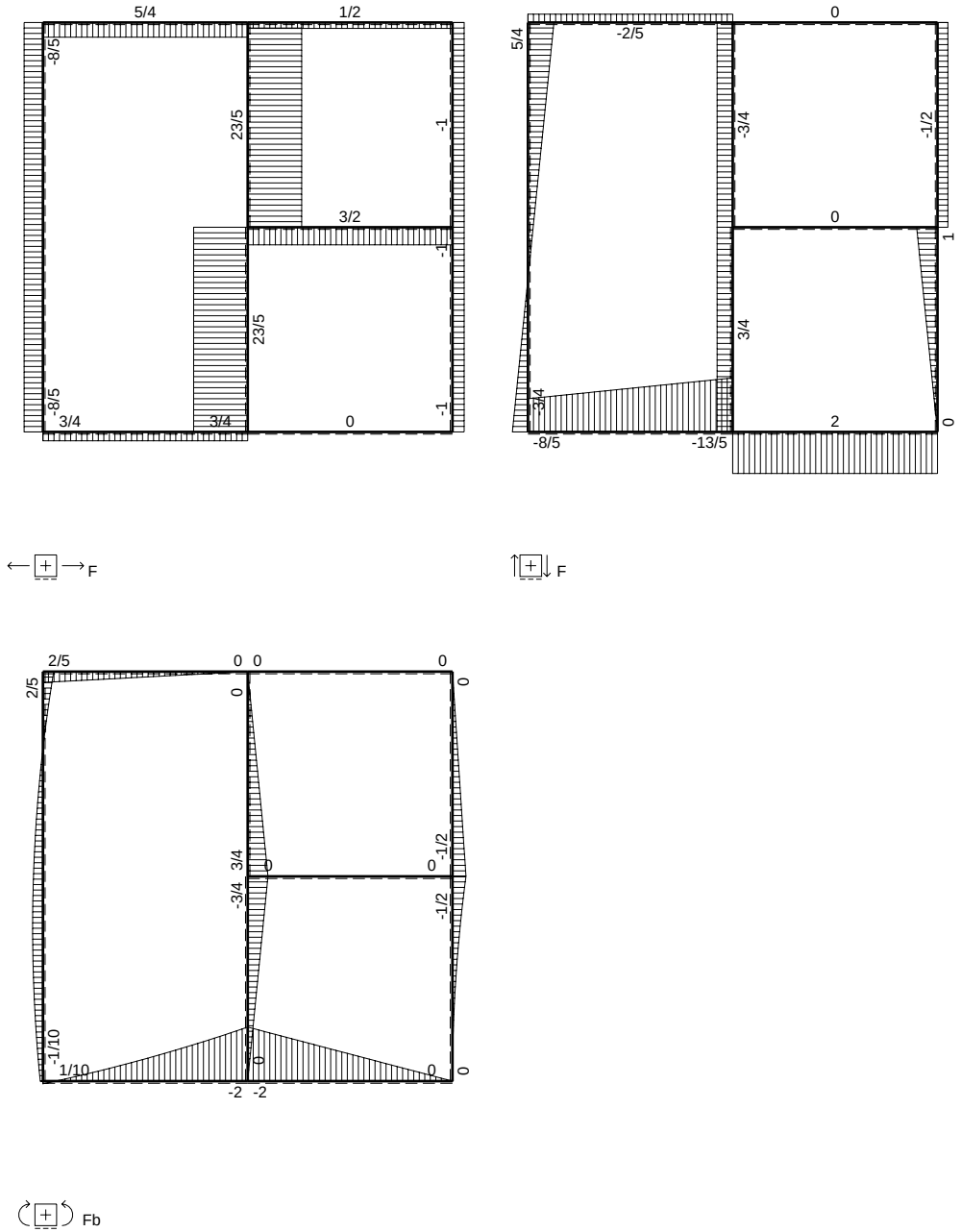
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

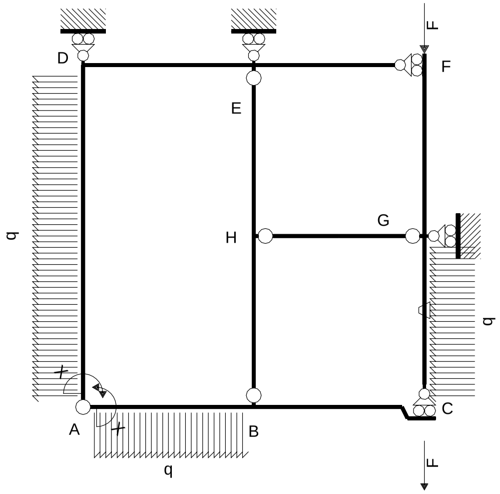
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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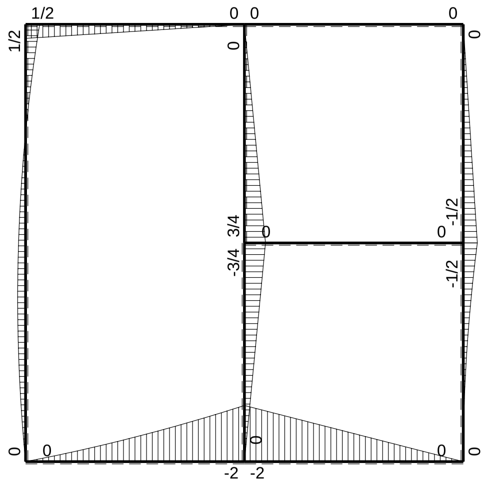
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D
E
5/4F
2/5Fb
2/5F
5/4F
2/5F
E
F
1/2F
E
3/4F
23/5F
F
1/2F
F
D
5/4F
2/5Fb
8/5F
23/5F
3/4Fb
3/4F
H
F
1/2Fb
1/2F
G
H
G
3/2F
H
3/4F
3/4Fb
23/5F
G
F
1/2Fb
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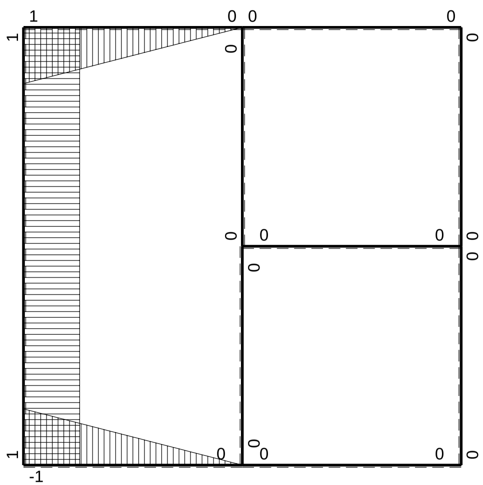

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23/5
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0
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-3/4
-1/2
0
1
3/4
-3/4
2
0
-8/5
-13/5
F
2/5
0 0
0
2/5
0
0
0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	0	0	0	0	0+0	0
BH b	0	$3/4Fx$	0	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

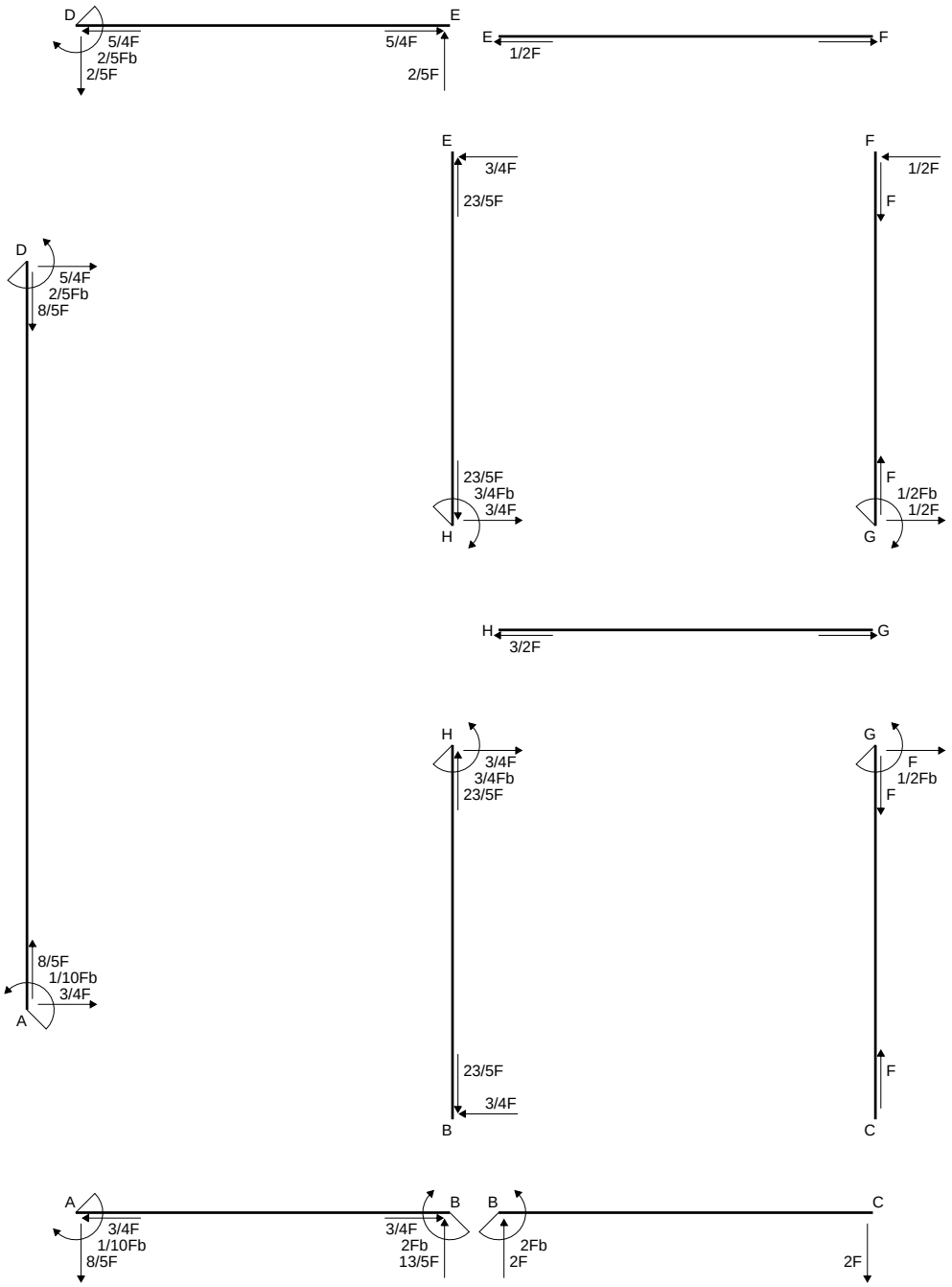
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

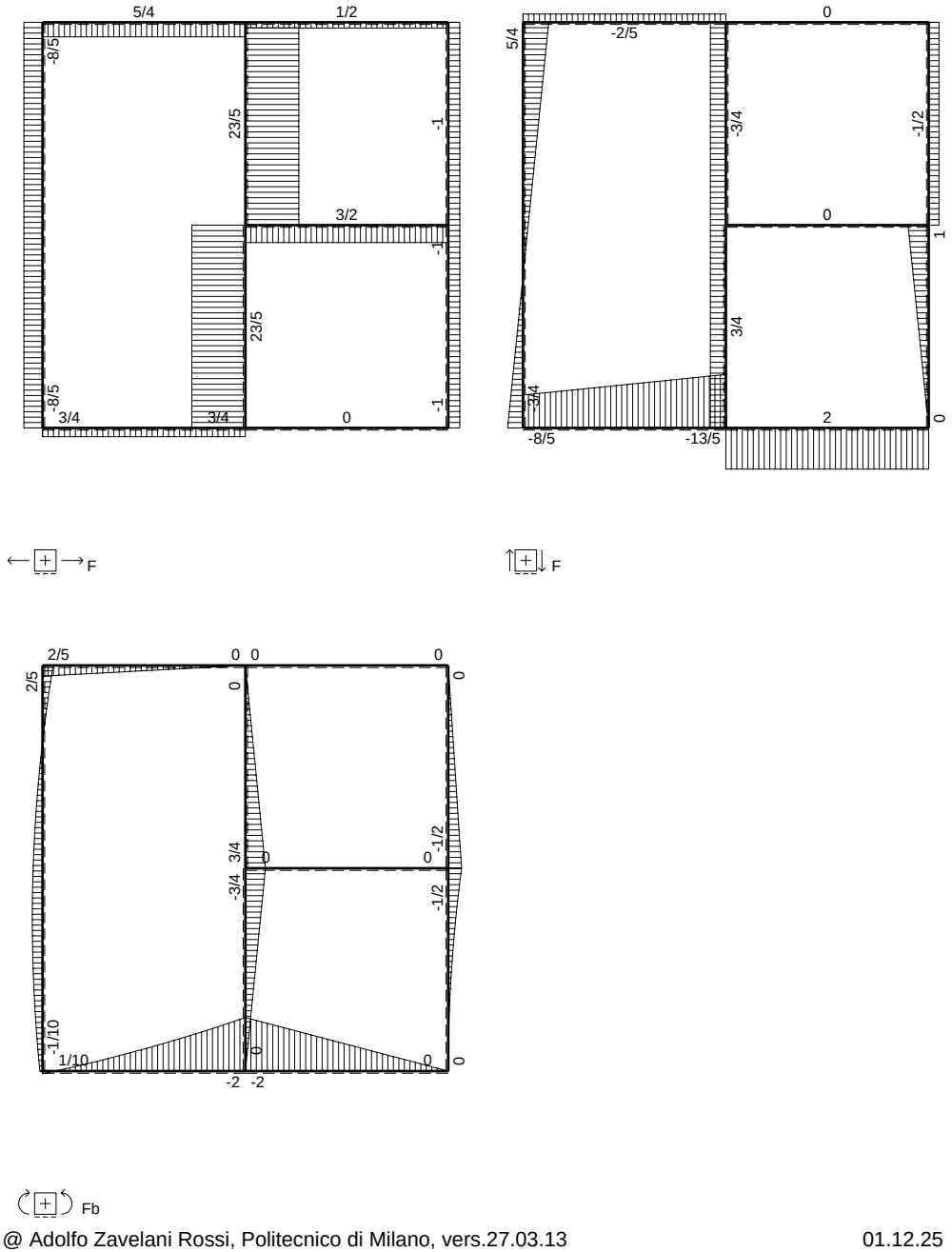
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

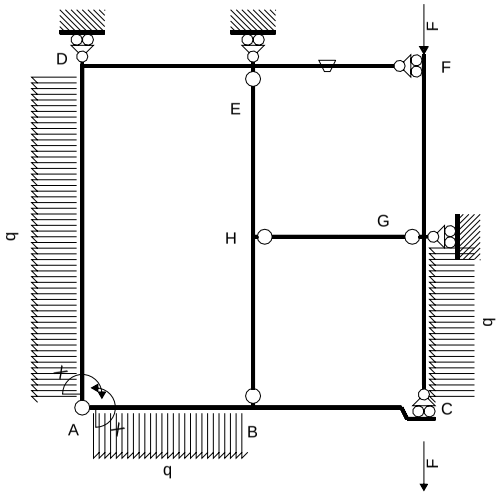
$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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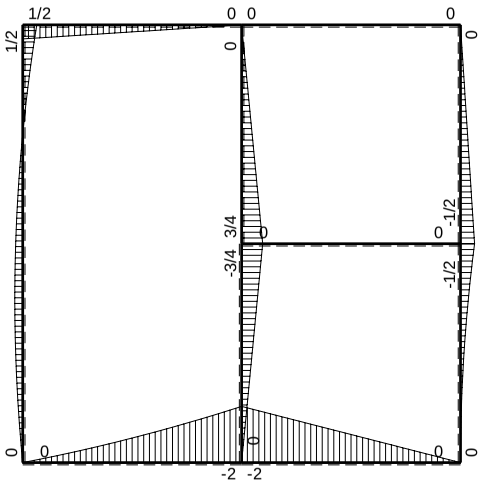
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D
E
5/4F
2/5Fb
2/5F
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2/5F
E
F
1/2F
E
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3/4F
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23/5F
3/4F
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F
C
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13/5F
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2F
2F

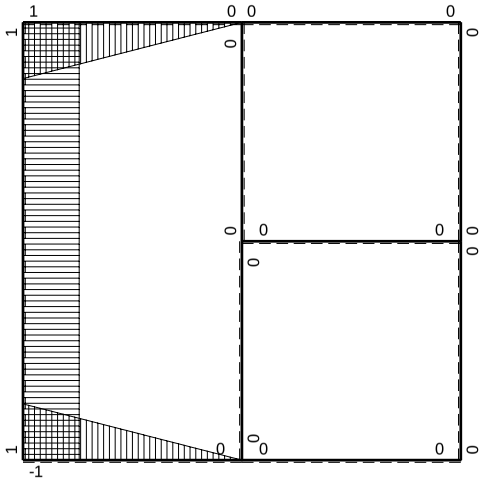

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0
-2/5
-3/4
-1/2
0
1
3/4
-3/4
2
0
-8/5
-13/5
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2/5
0
0
0
3/4
0
0
-1/2
-3/4
-1/2
-1/10
0
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	0	0	0	0	0+0	0
BH b	0	$3/4Fx$	0	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

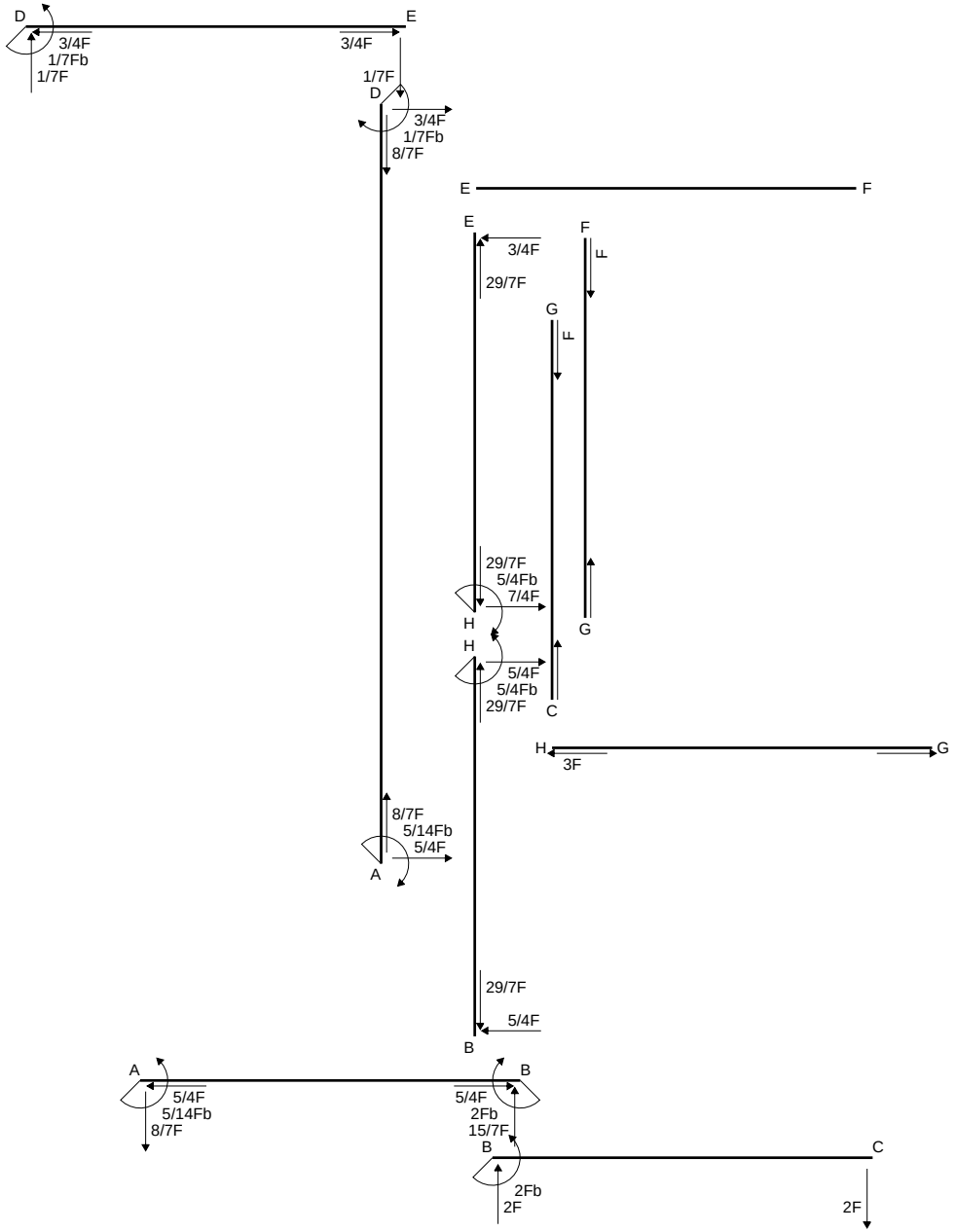
$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

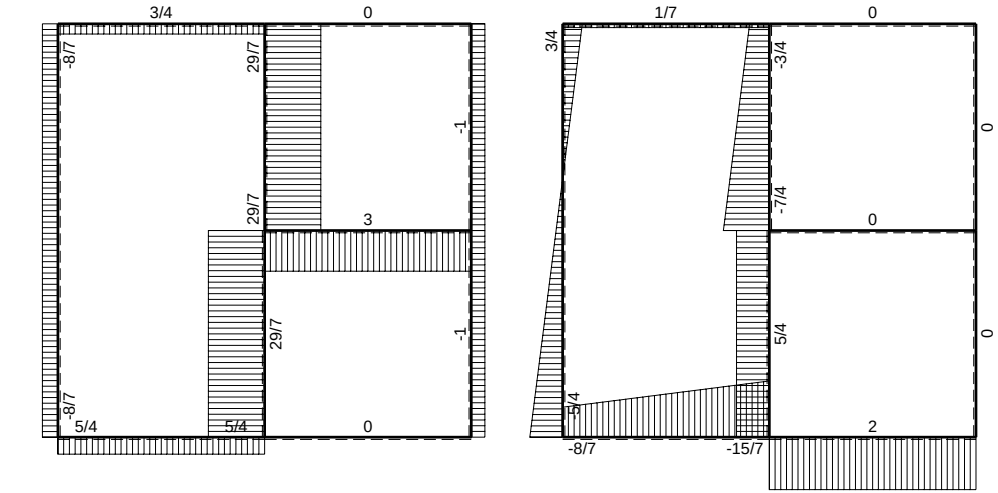
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238196 ESΣ03.xxxx.114

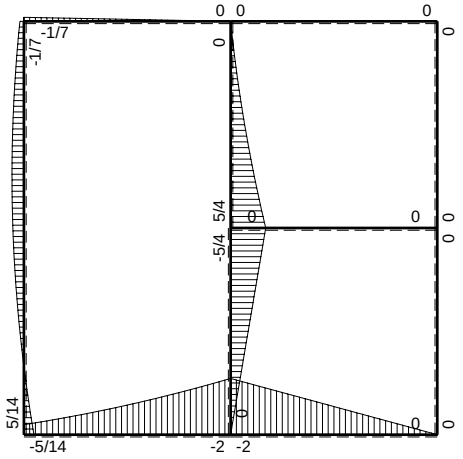
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
8/7F
E
F
E
F
3/4F
29/7F
G
F
F
29/7F
5/4Fb
7/4F
H
G
H
5/4F
5/4Fb
29/7F
C
H
G
3F
8/7F
5/14Fb
5/4F
A
29/7F
5/4F
B
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C
2Fb
2F
2F

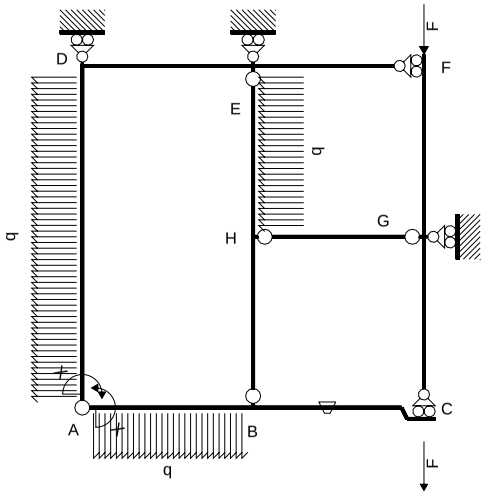

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29/7
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29/7
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29/7
-1
-8/7
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5/4
0
1/7
0
3/4
-3/4
0
-7/4
0
5/4
0
5/4
2
-8/7
-15/7


F

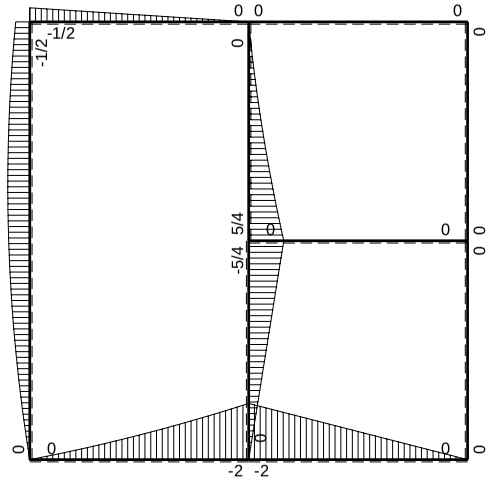

F


0 0
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-1/7
-1/7
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0
-5/14
-2 -2

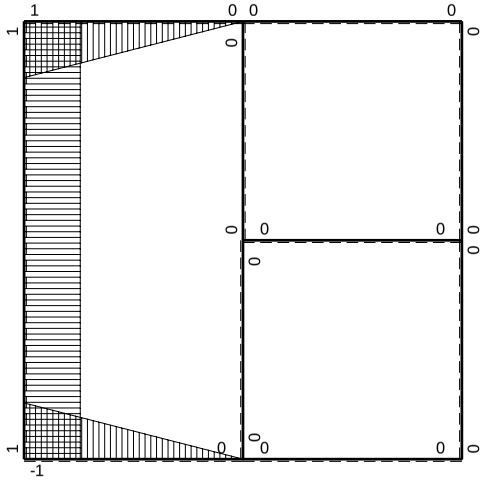

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238776 ESΣ03.xxxx.115

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-5/2\ b +4/3\ b\ \right)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 -3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x -3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b -3/2\ b +4/3\ b\ \right)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

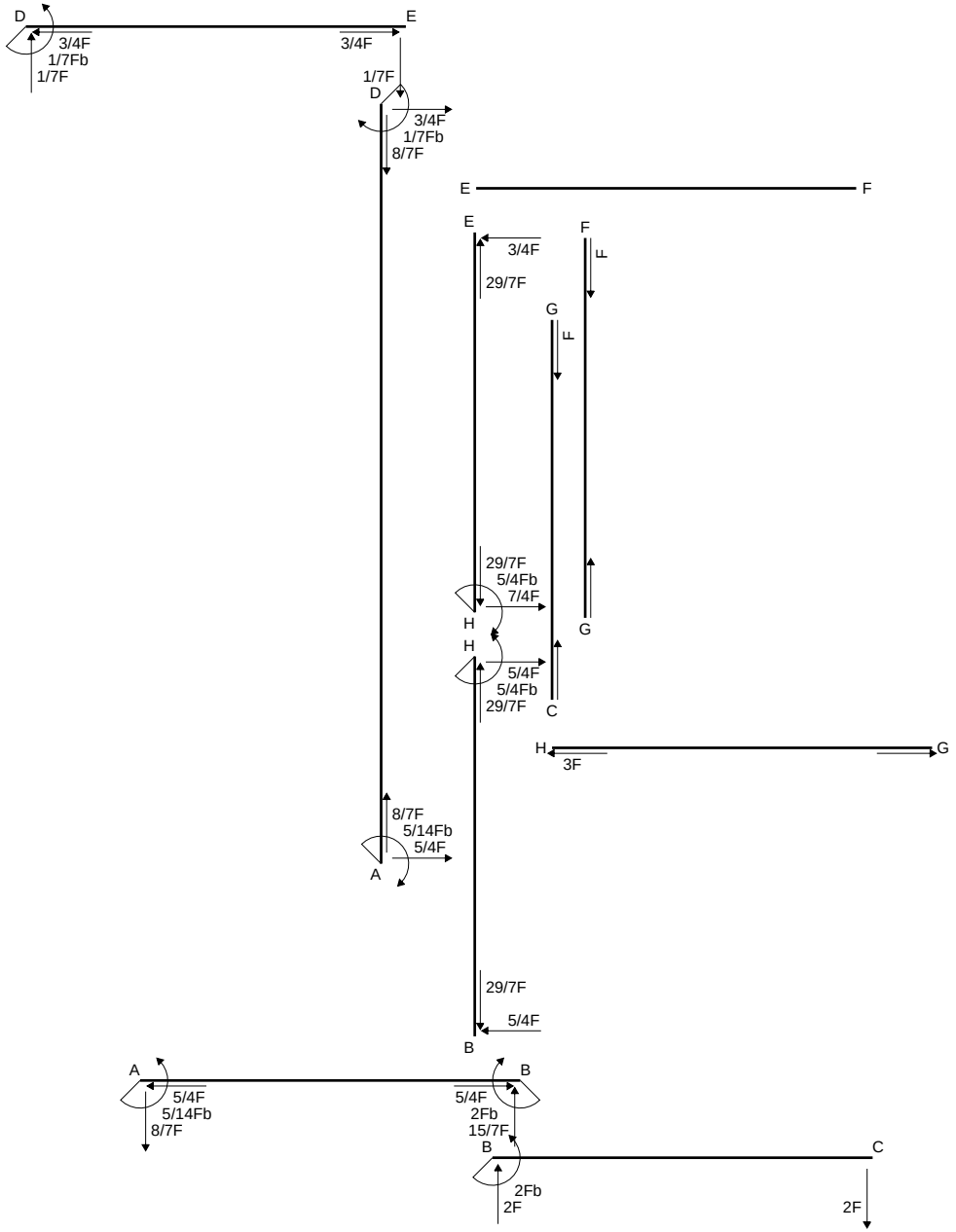
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

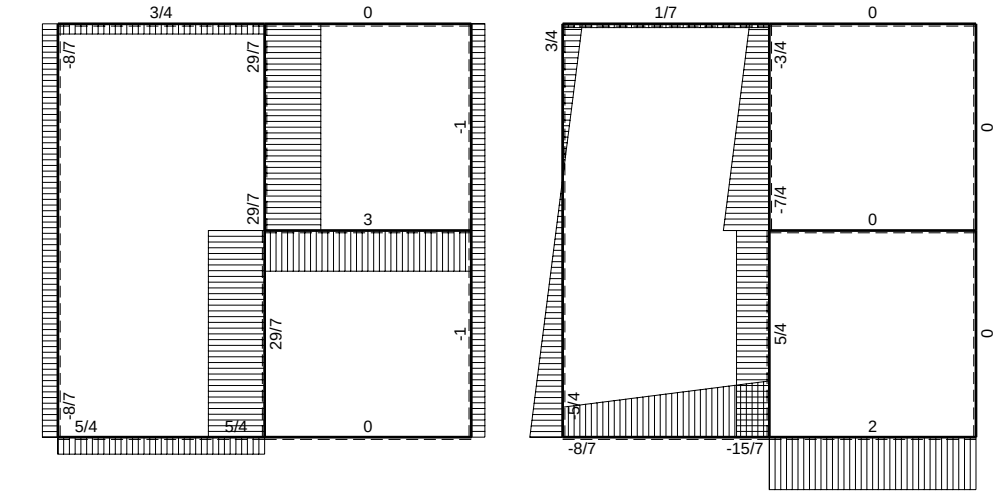
$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

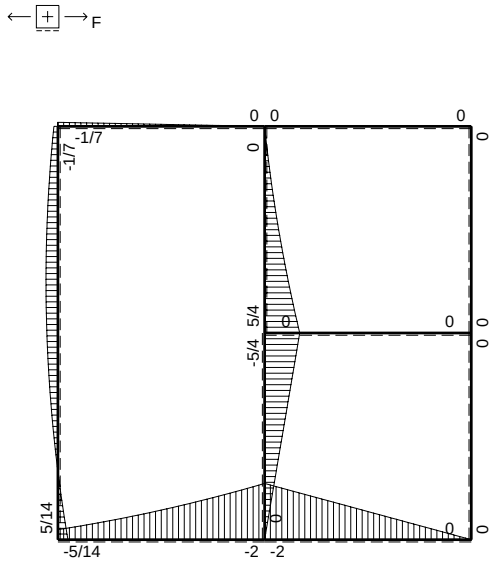
$= \left(-1/6\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

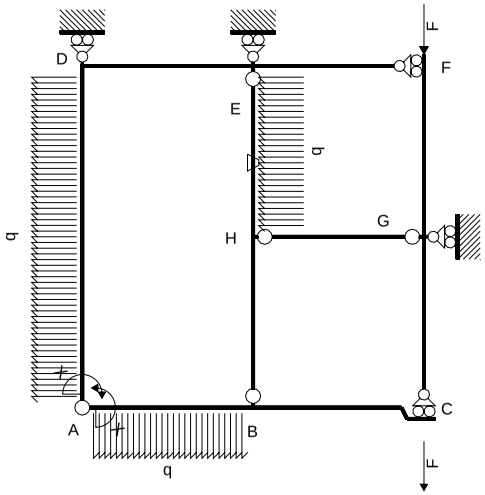
PROCEDIMENTO E RISULTATI 238776 ESΣ03.xxxx.115

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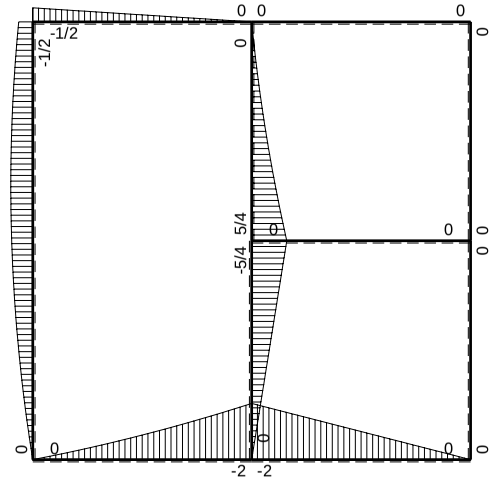

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15/7F
B
C
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2F
2F


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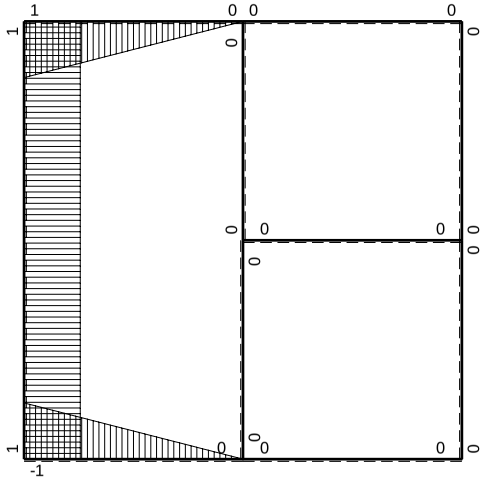

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0
-5/14
-2 -2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 209691 ESΣ03.xxxx.116

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PROCEDIMENTO E RISULTATI 209691 ESΣ03.xxxx.116

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-5/4Fb+5/4Fx	0	0	0	0	0+0	0
BH b	0	5/4Fx	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 -3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x -3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b -3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

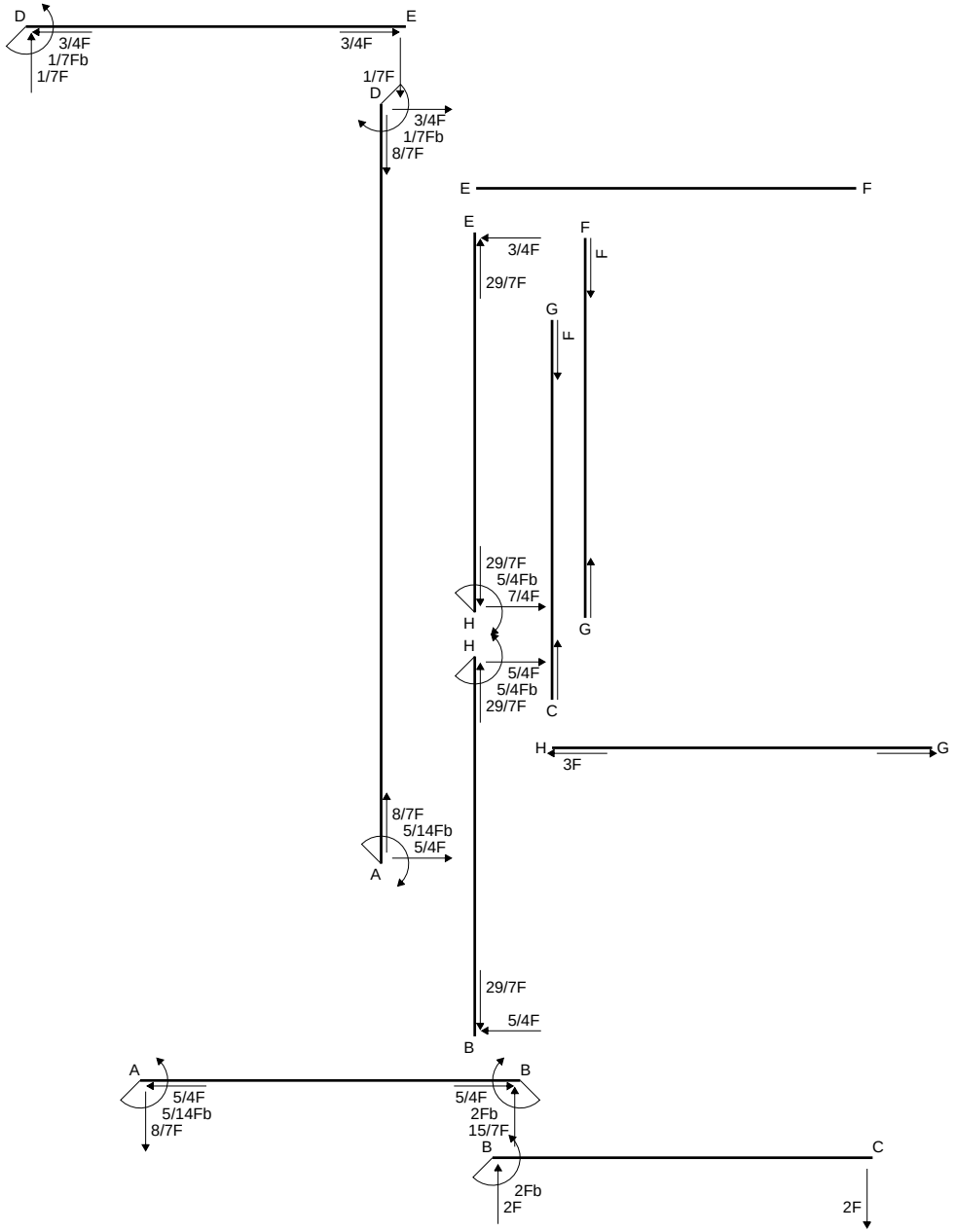
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

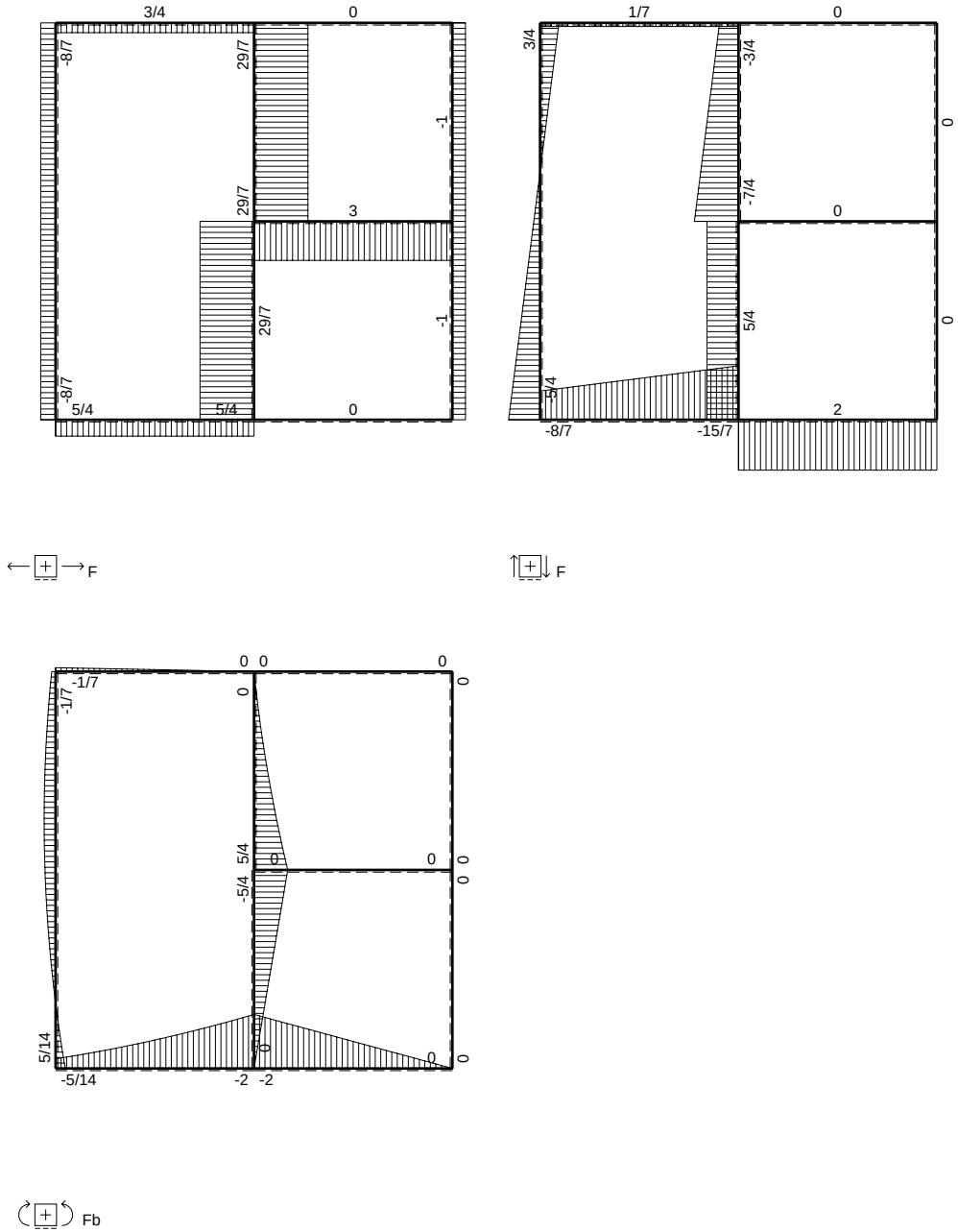
$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

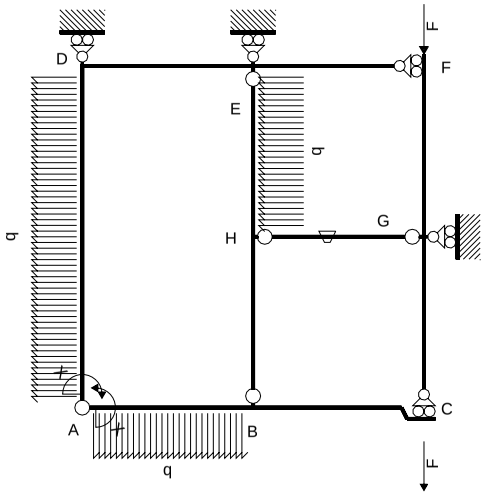
$= \left(-1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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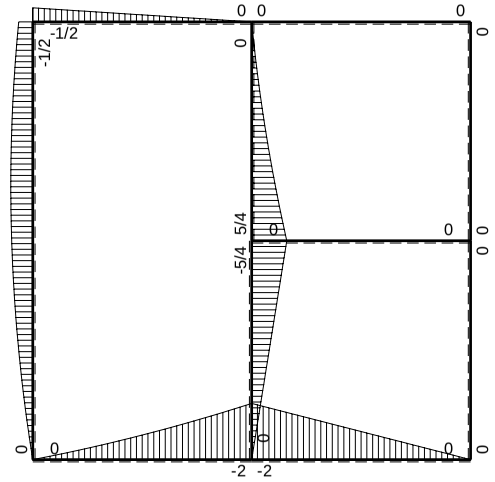
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
8/7F
E
F
E
F
3/4F
29/7F
G
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5/4F
5/4Fb
29/7F
C
H
G
3F
8/7F
5/14Fb
5/4F
A
29/7F
5/4F
B
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B
C
2Fb
2F
2F

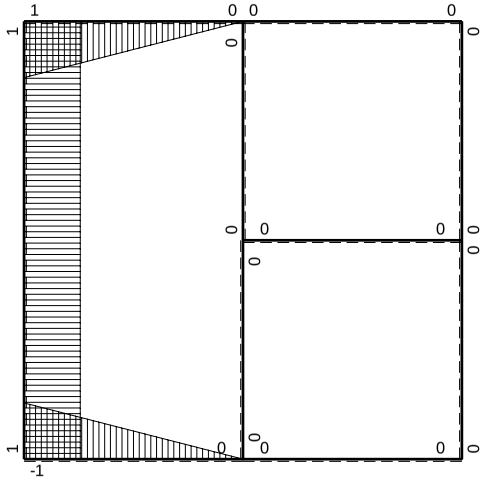

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29/7
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-1
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F
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3/4
-3/4
0
-7/4
0
5/4
0
5/4
-8/7
-15/7
2
F
0 0
0
-1/7
-1/7
0
0
0
5/4
0
0
0
-5/4
0
0
5/14
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	-5/4Fb+5/4Fx	0	0	0	0	0+0	0
BH b	0	5/4Fx	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

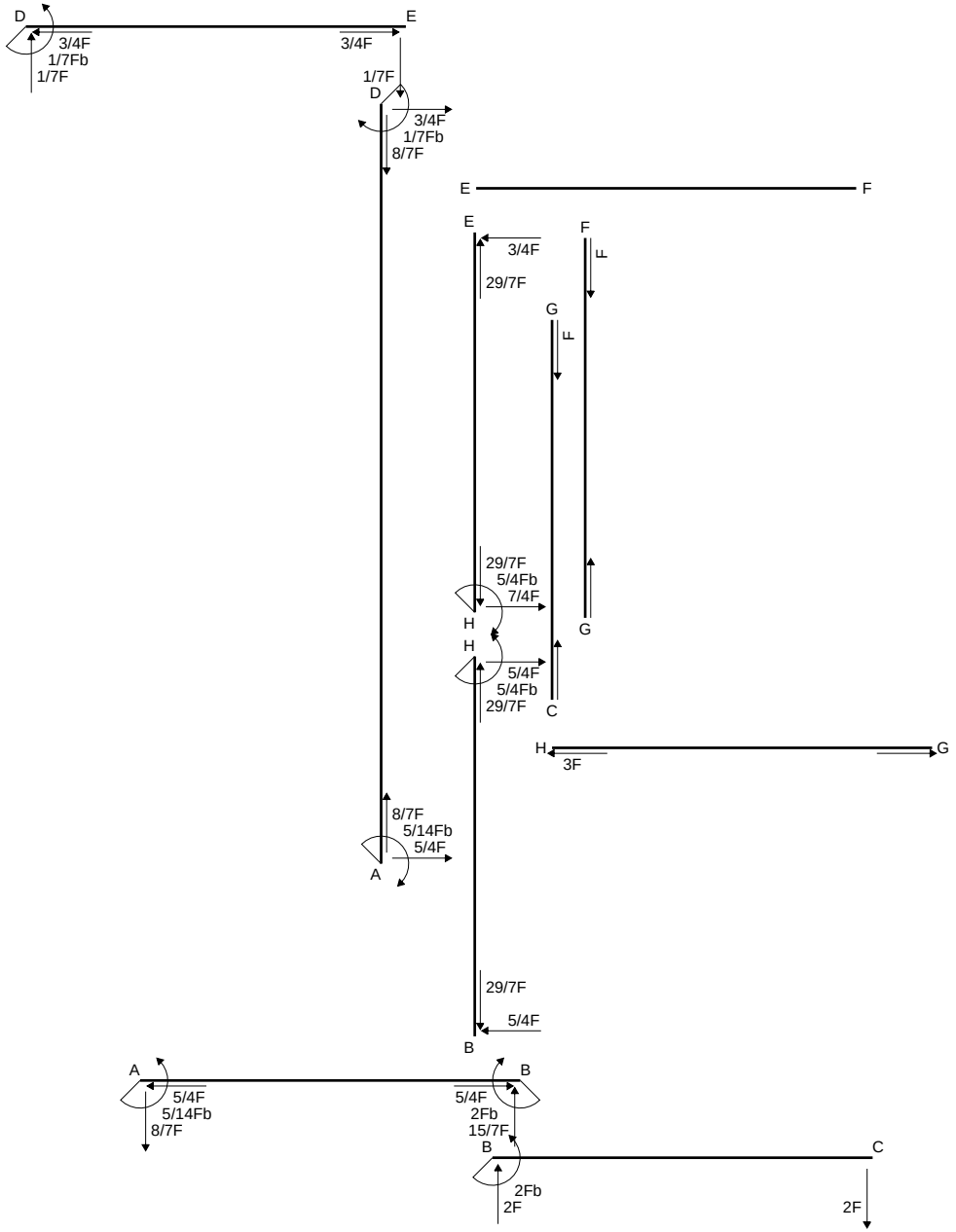
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

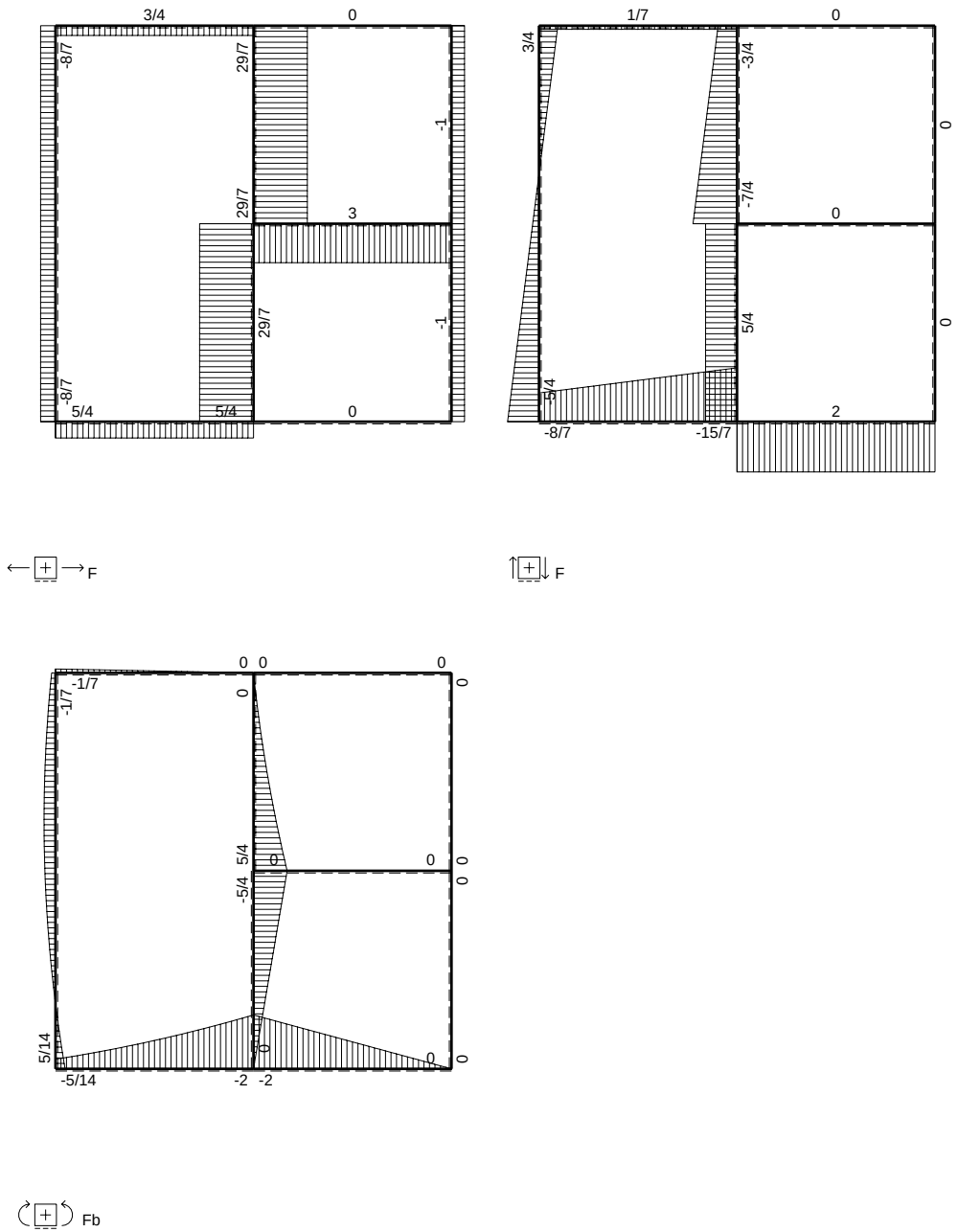
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

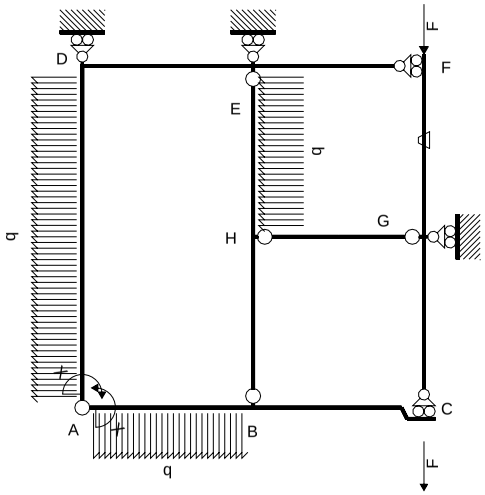
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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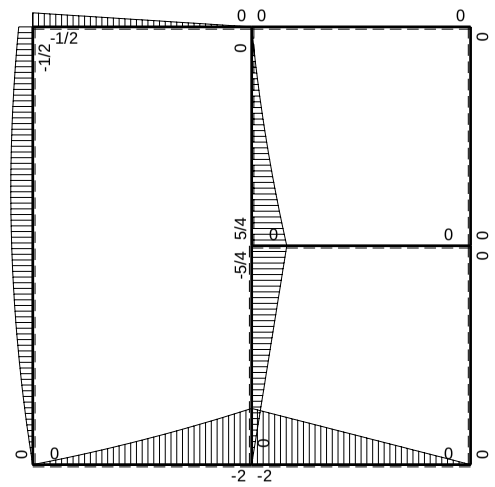
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1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
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5/4Fb
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G
3F
8/7F
5/14Fb
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B
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2F
2F

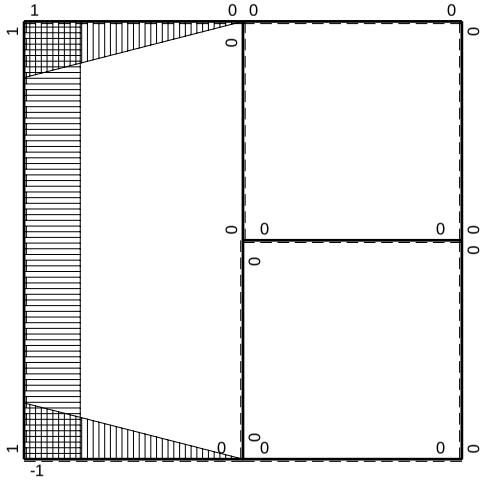

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-15/7
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0
-1/7
-1/7
0
0
0
5/4
0
0
0
-5/4
0
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

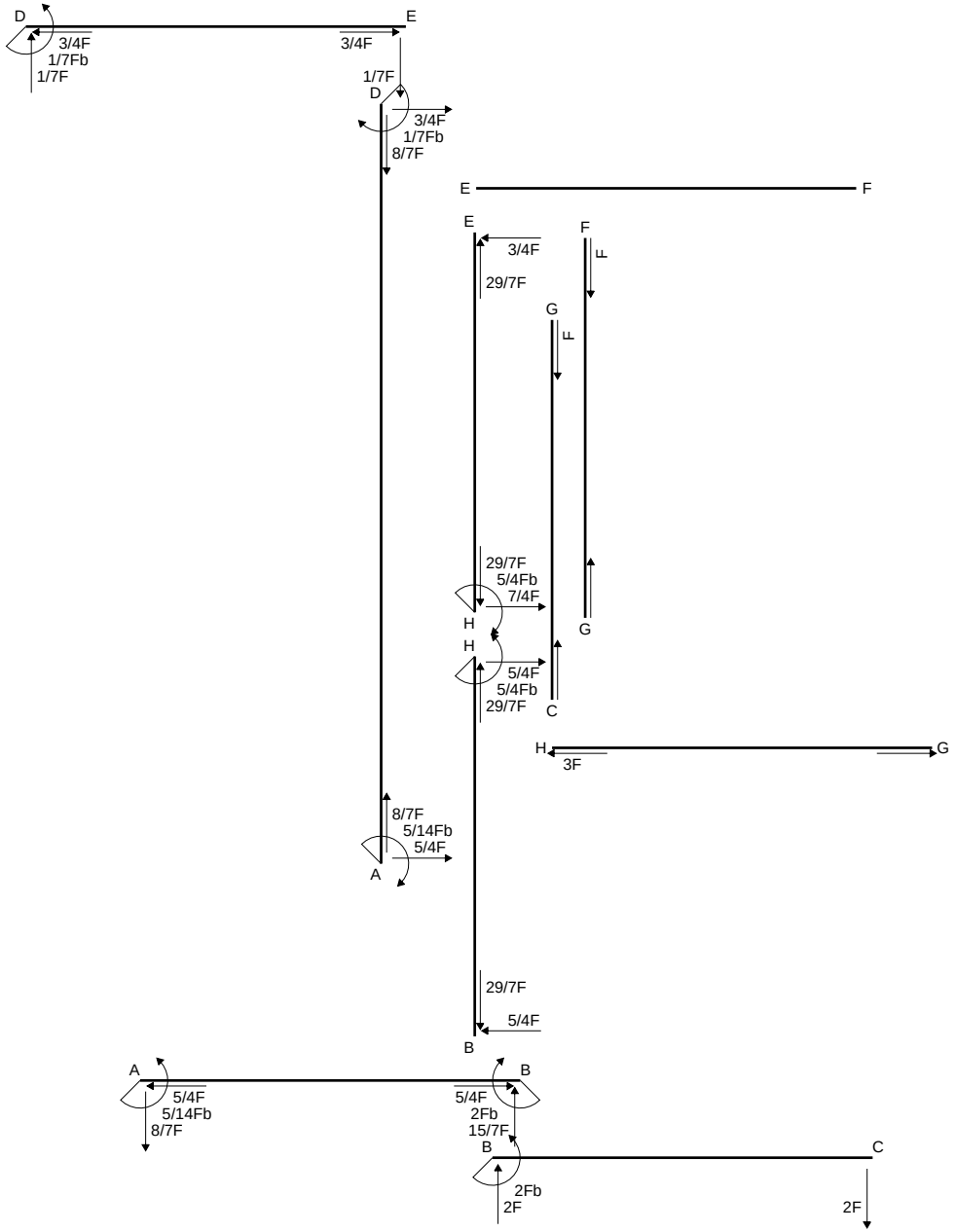
$= (-1/2\ b + 1/2\ b - 1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

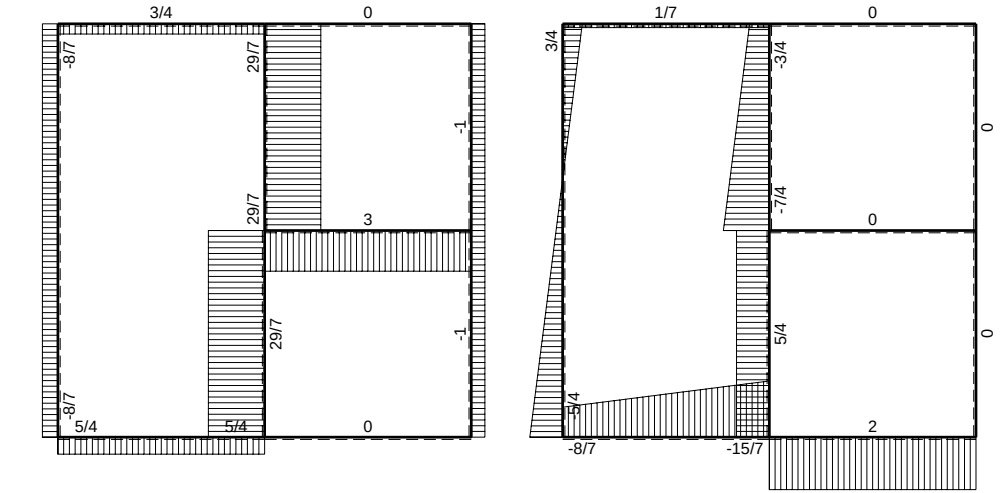
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

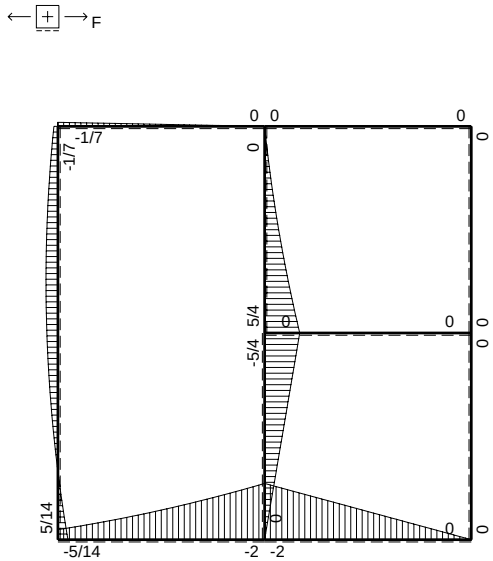
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

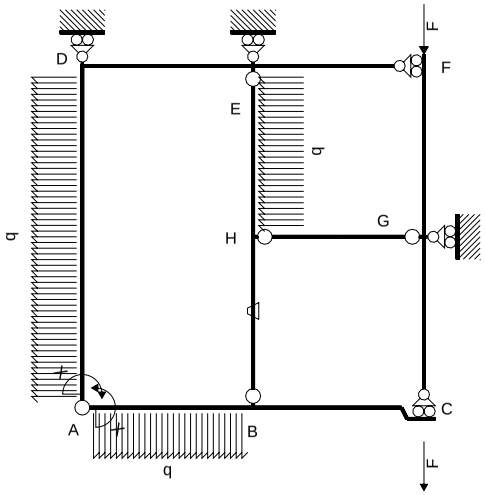
PROCEDIMENTO E RISULTATI 236903 ESΣ03.xxxx.118

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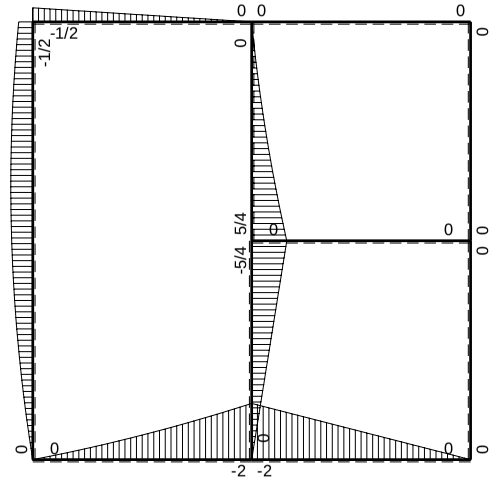

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3F
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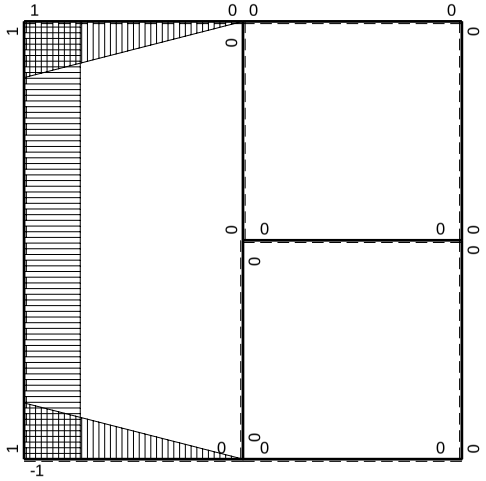

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0
-5/14
-2 -2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238107 ESΣ03.xxxx.119

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$5/4Fx$	Fb/EJ	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

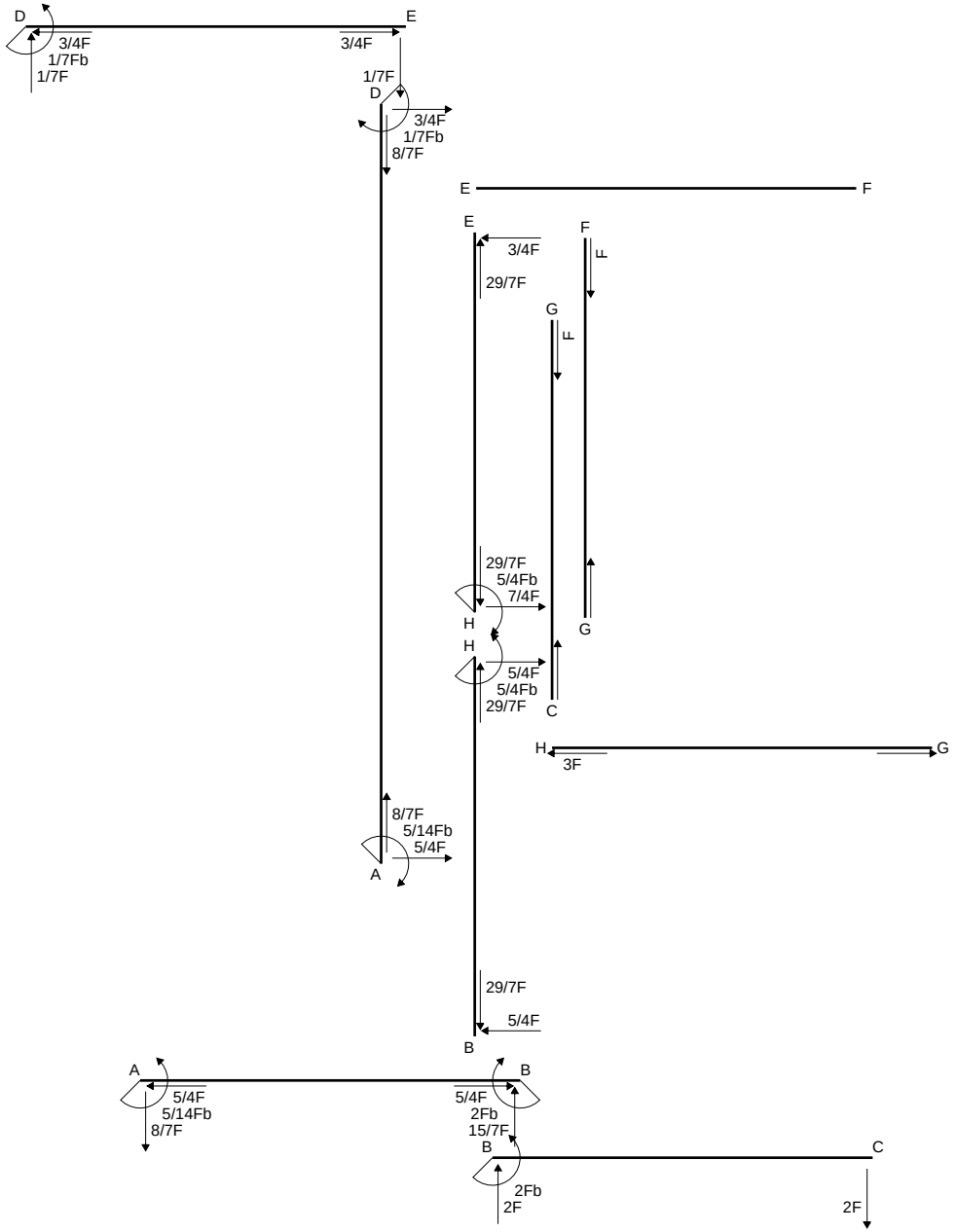
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

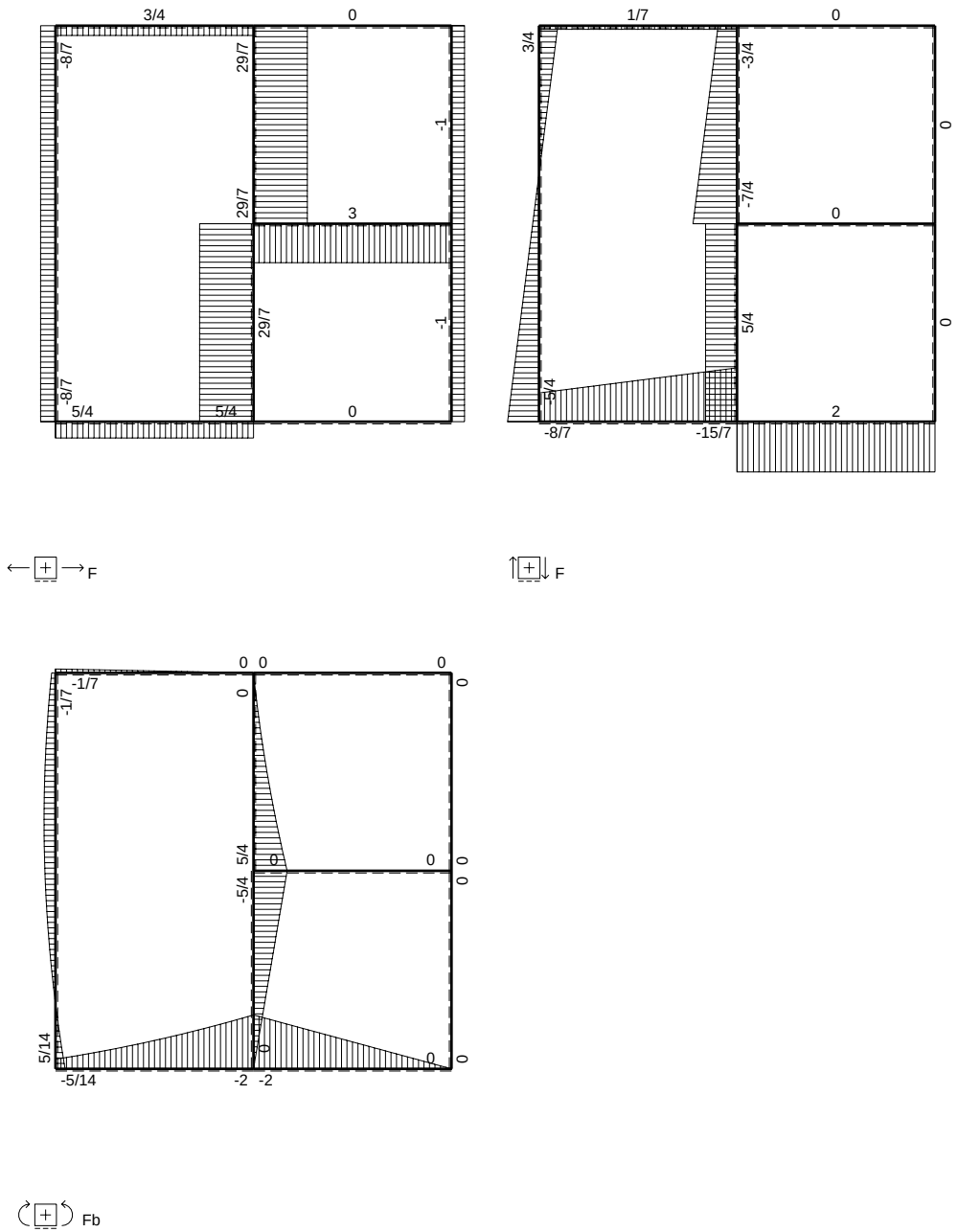
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

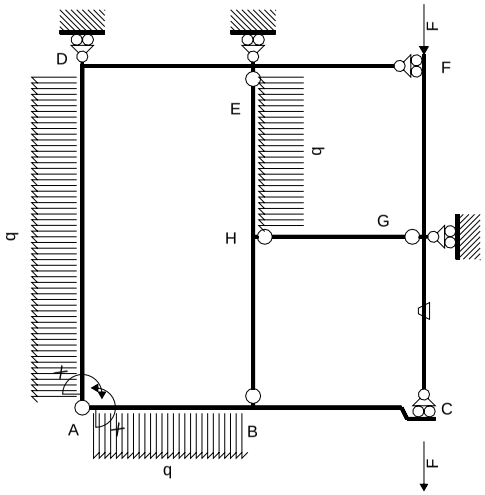
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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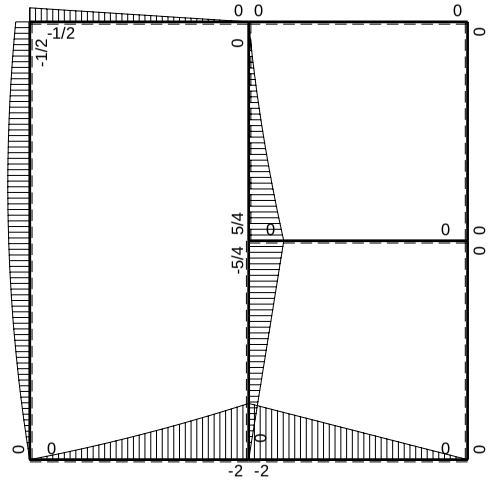
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1/7F
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5/4Fb
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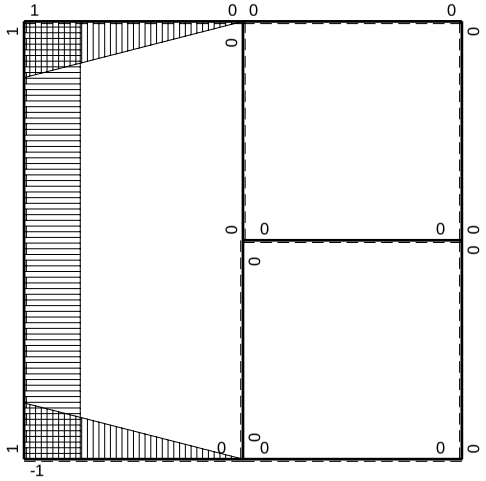

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-1/7
-1/7
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0
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0 0
-5/4
0
5/14
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-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

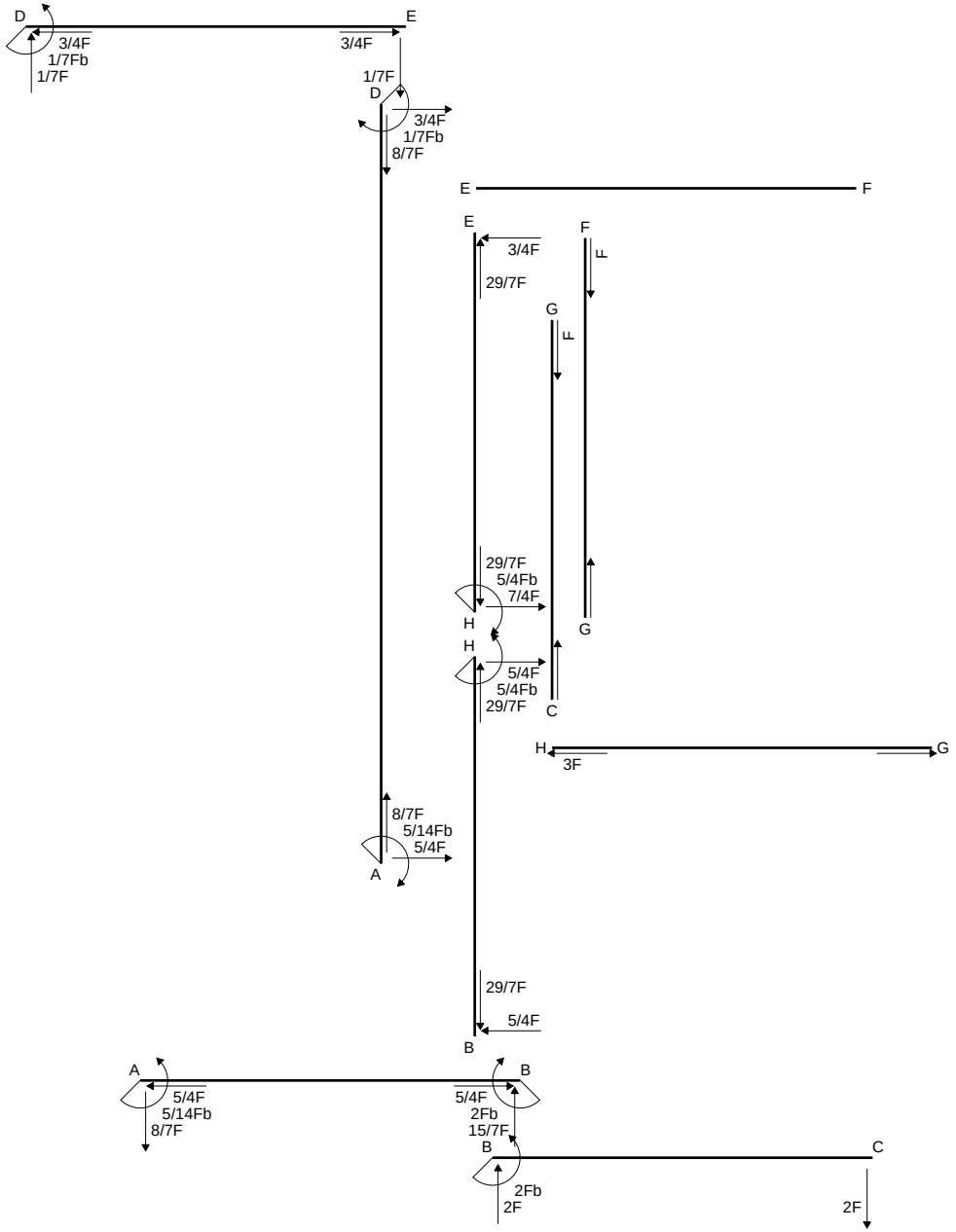
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

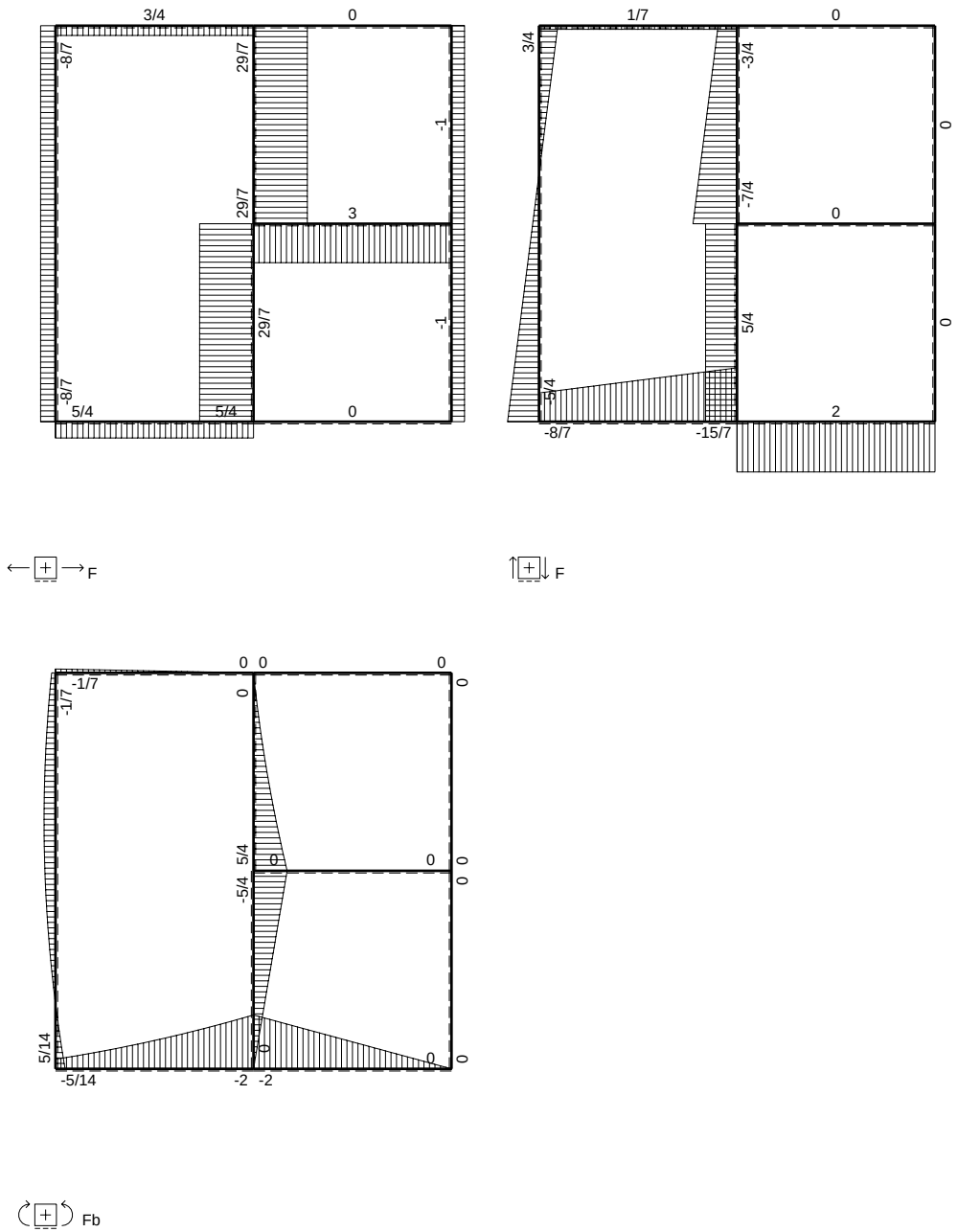
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

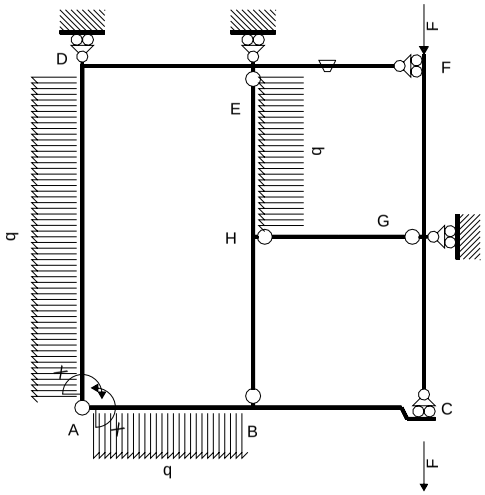
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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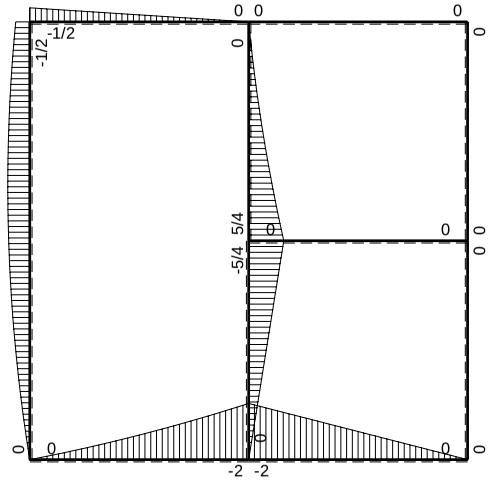
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1/7F
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1/7Fb
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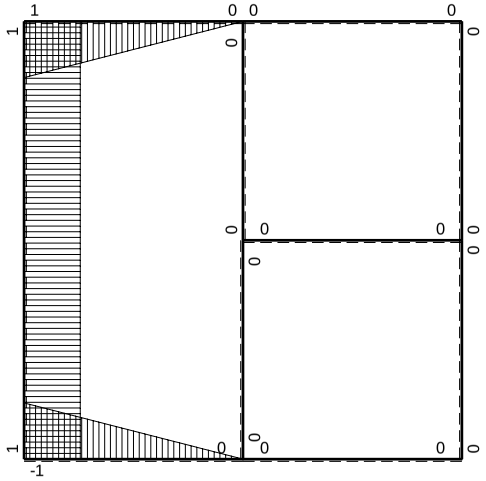

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-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 -3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x -3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b -3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

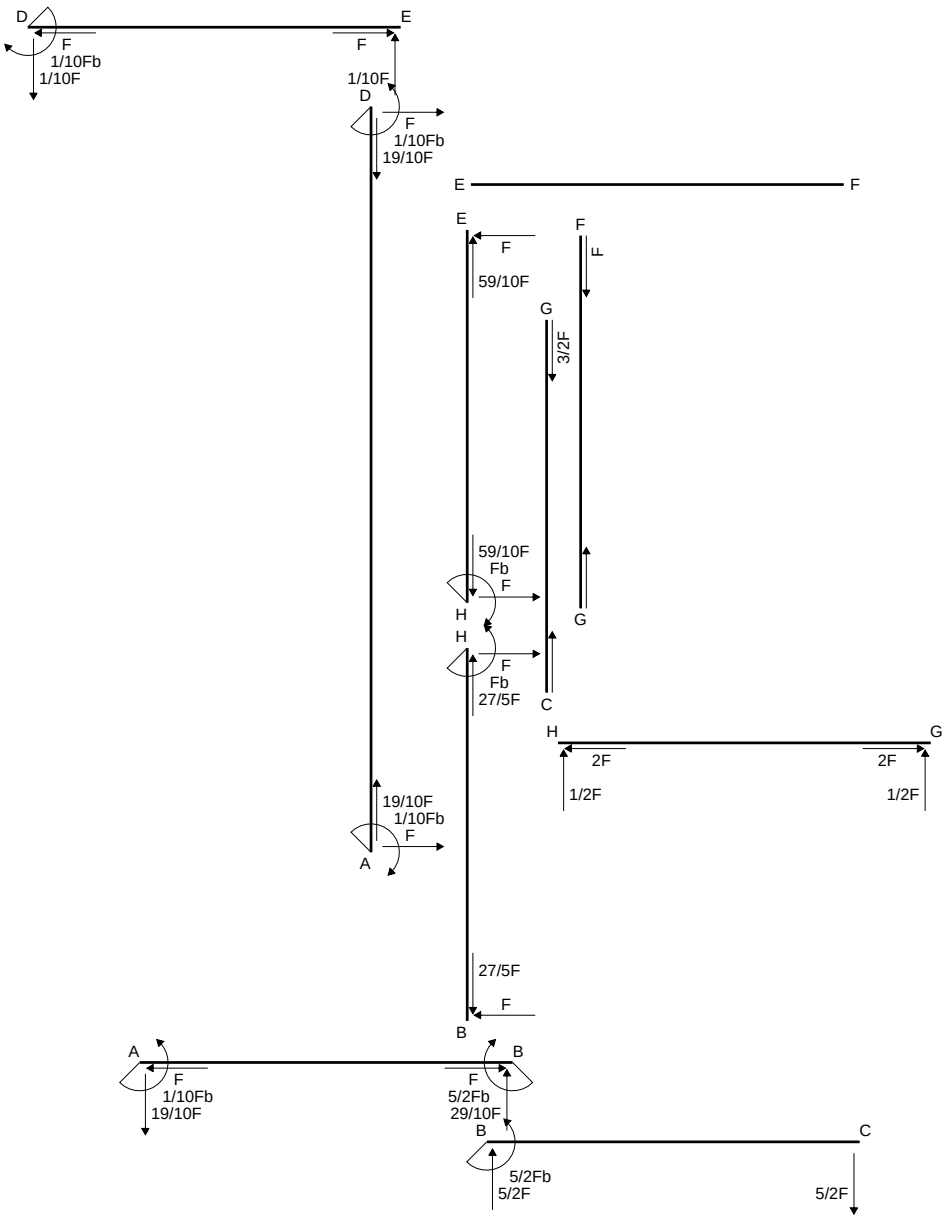
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

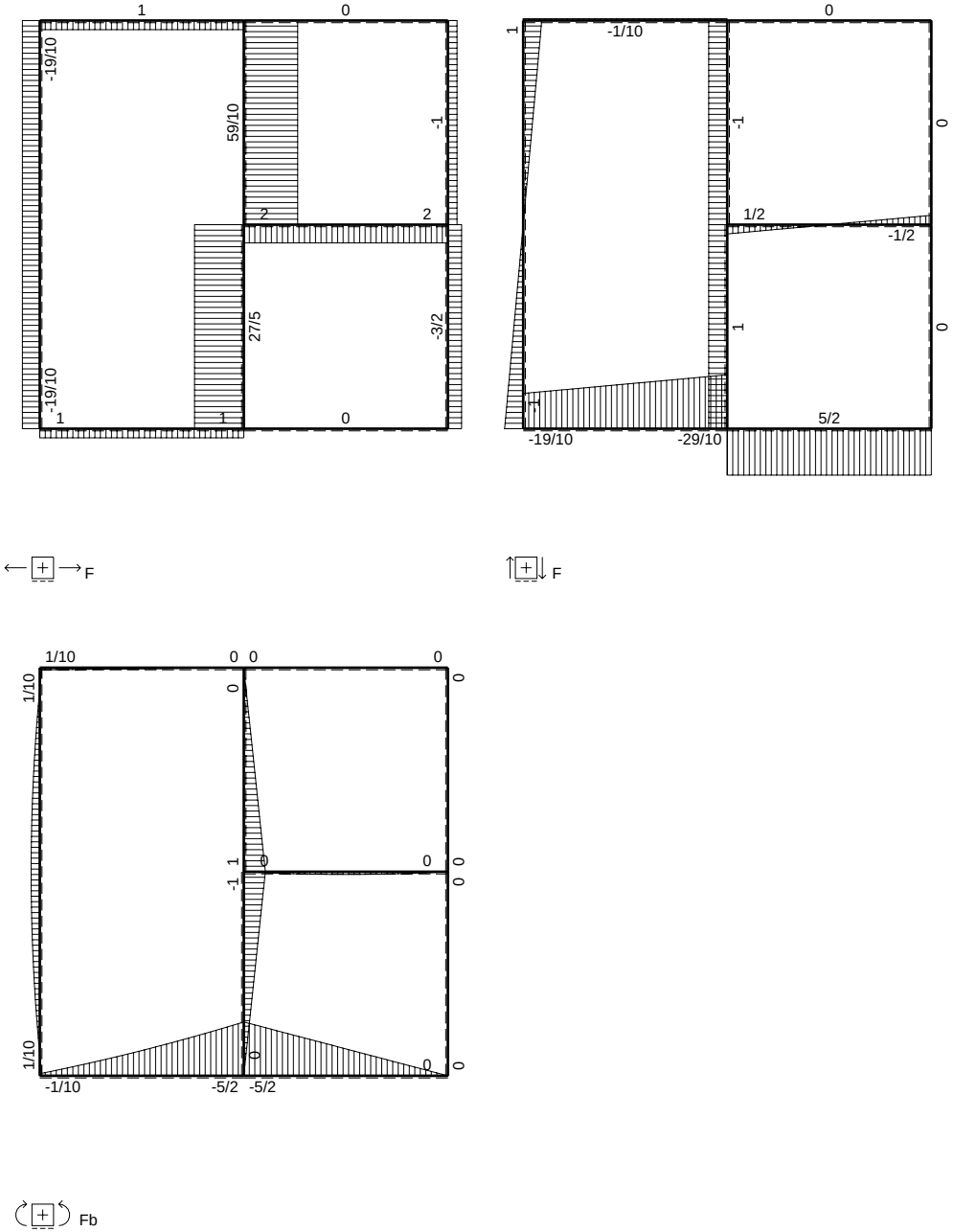
$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

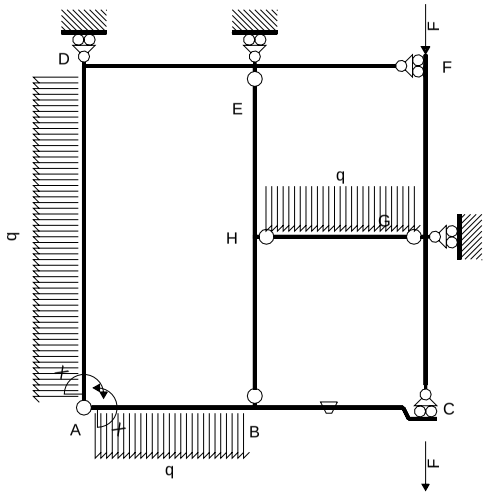
$= \left(-1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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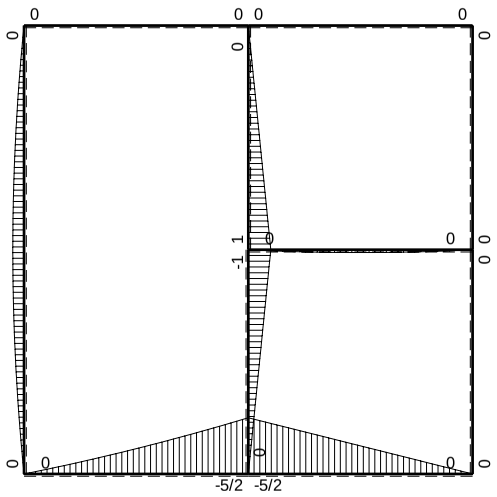
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1/10Fb
1/10F
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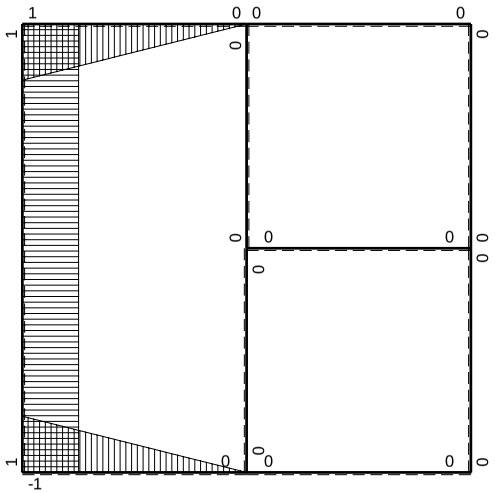

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1/10
0
0
0
-1 1
0
0
0 0
0
1/10
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	-Fb/EJ	0	0	0	0+0	0
CB b	0	5/2Fx	Fb/EJ	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-Fb+Fx	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	Fb-Fx	0	0	0	0	0+0	0
EH b	0	-Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

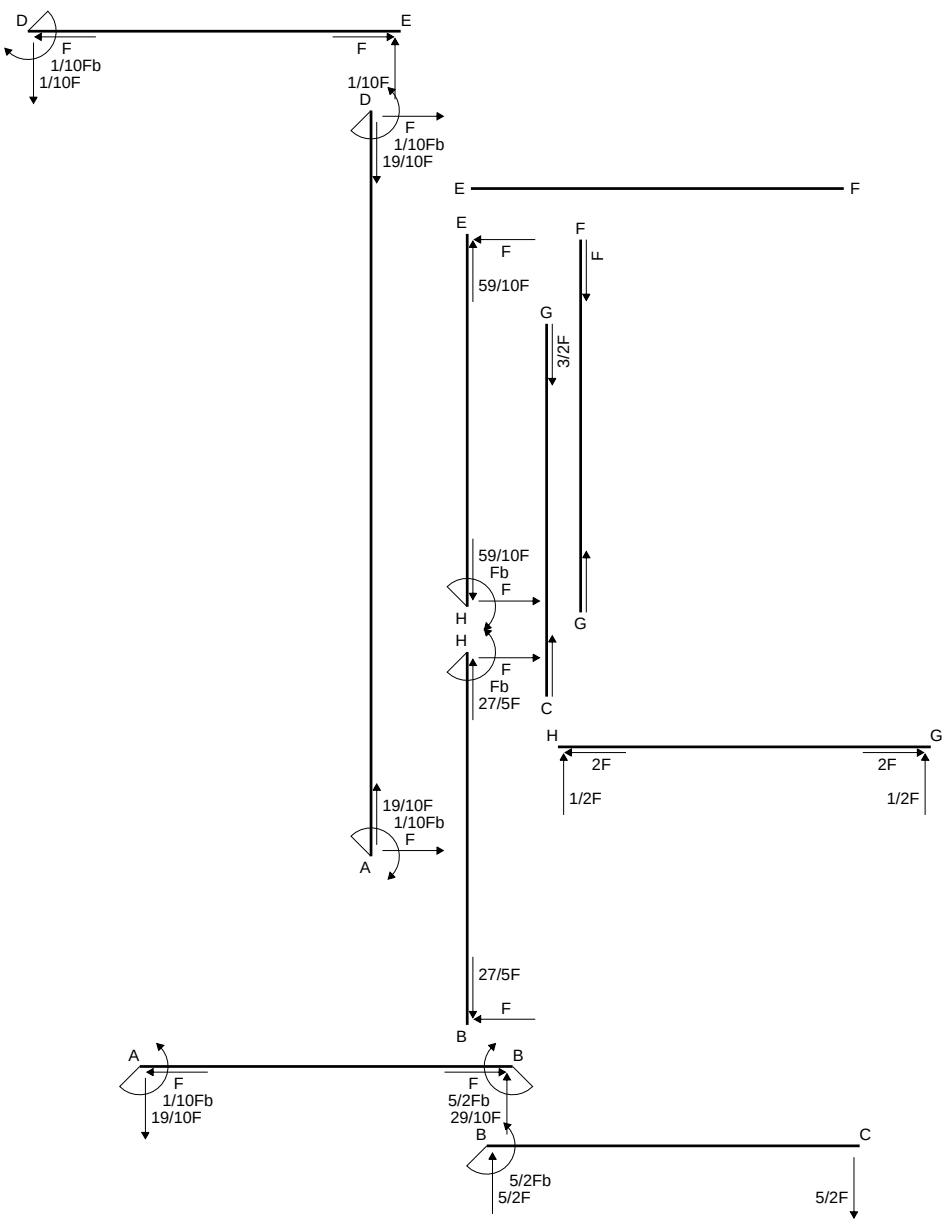
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

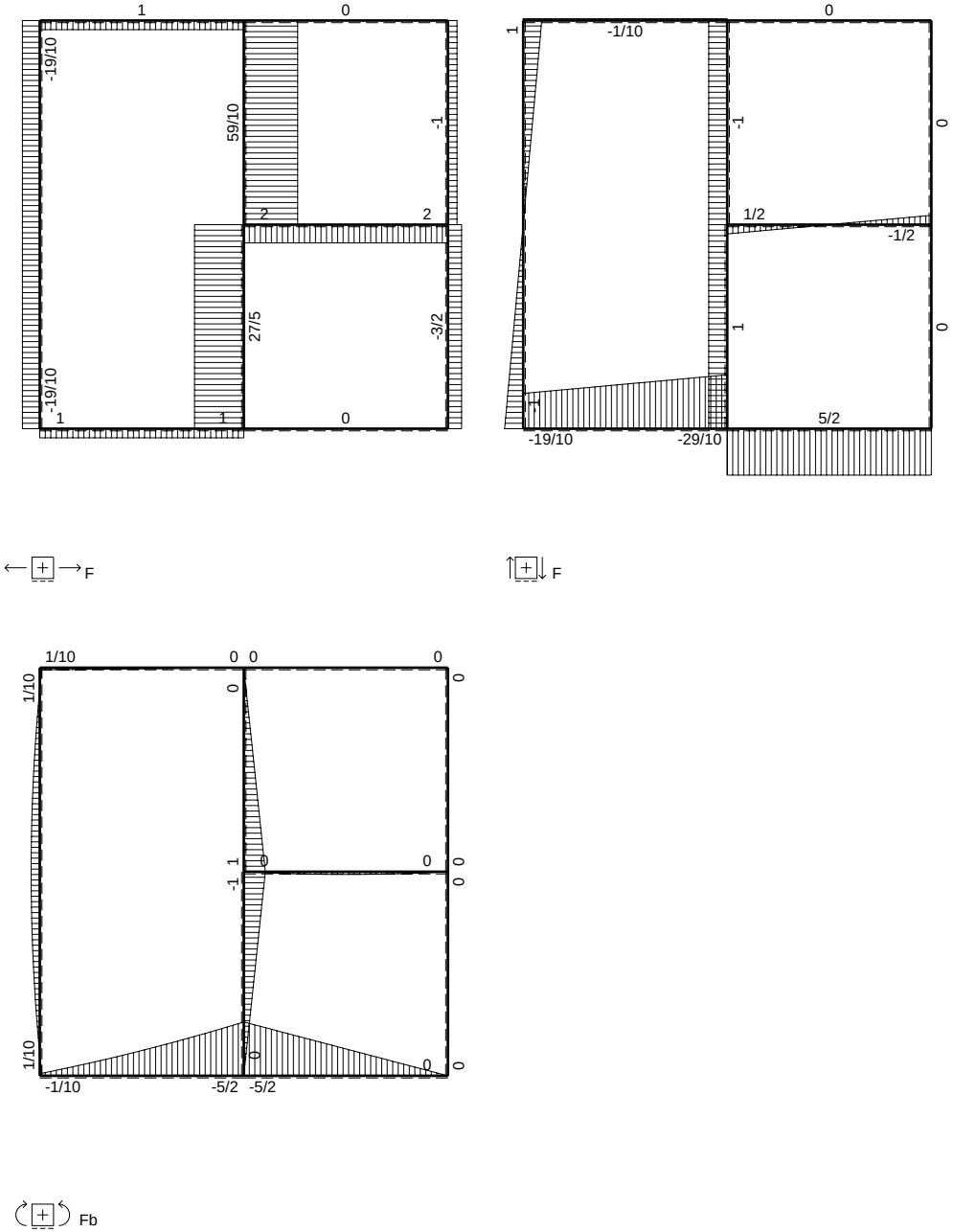
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

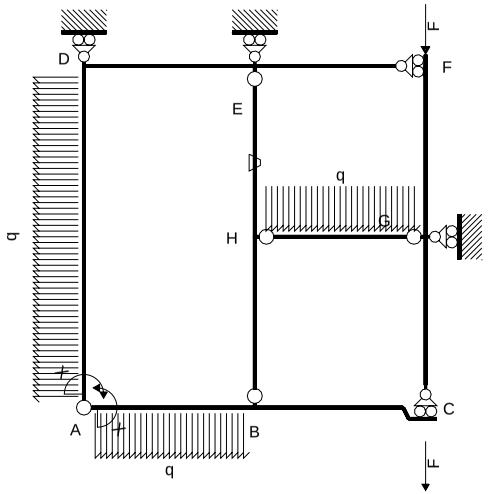
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

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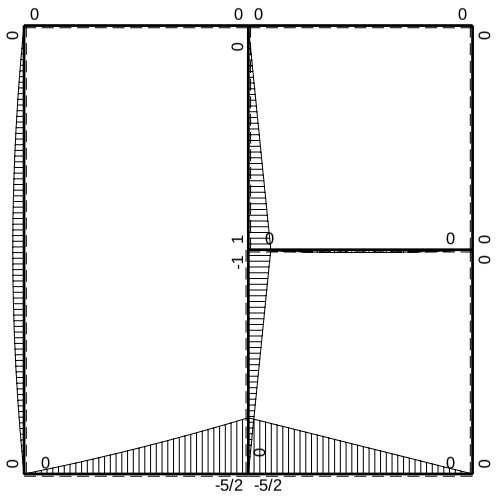
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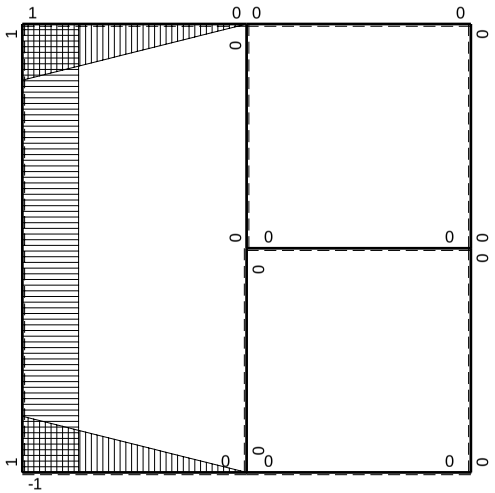

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-1 1
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0 0
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1/10
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-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-Fb+Fx$	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	$Fb-Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

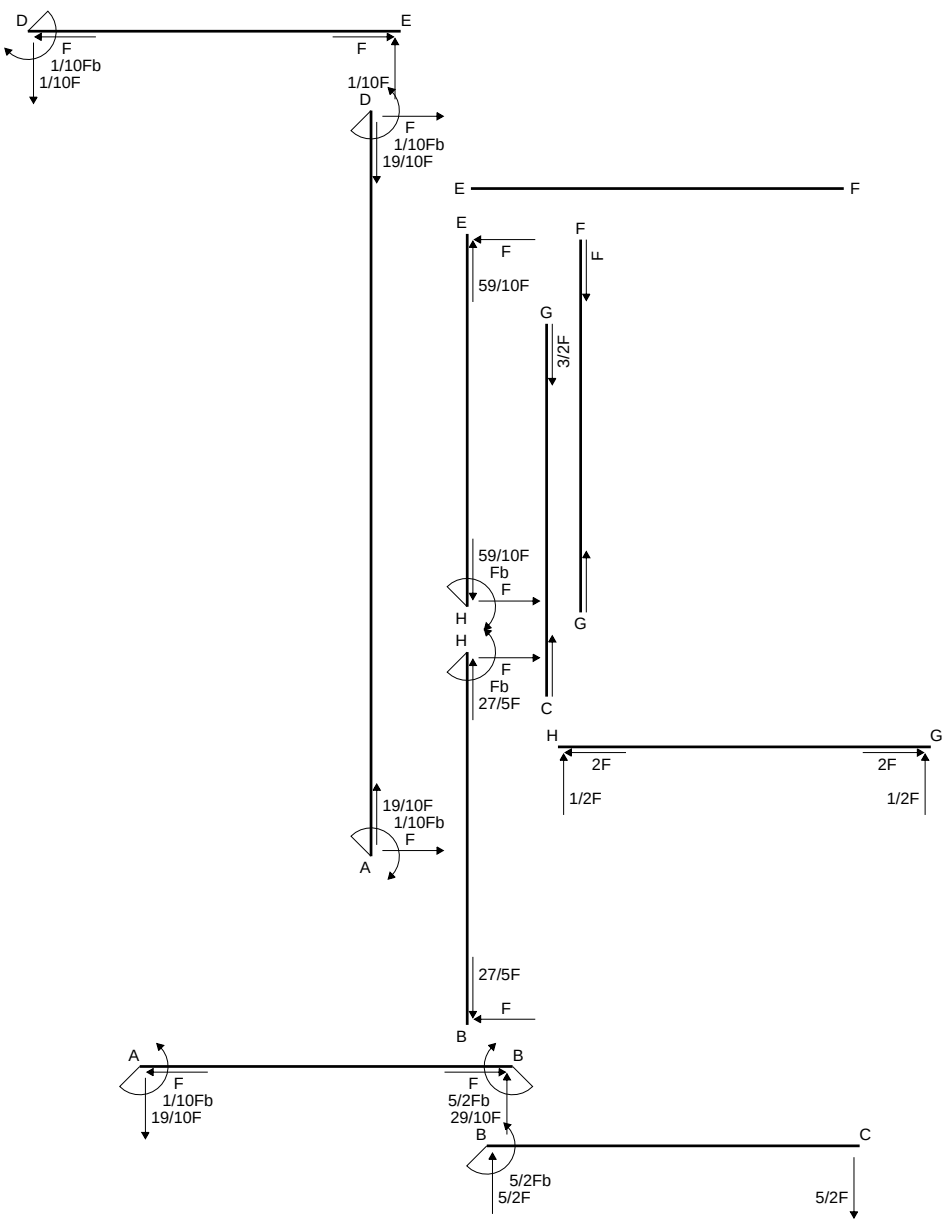
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

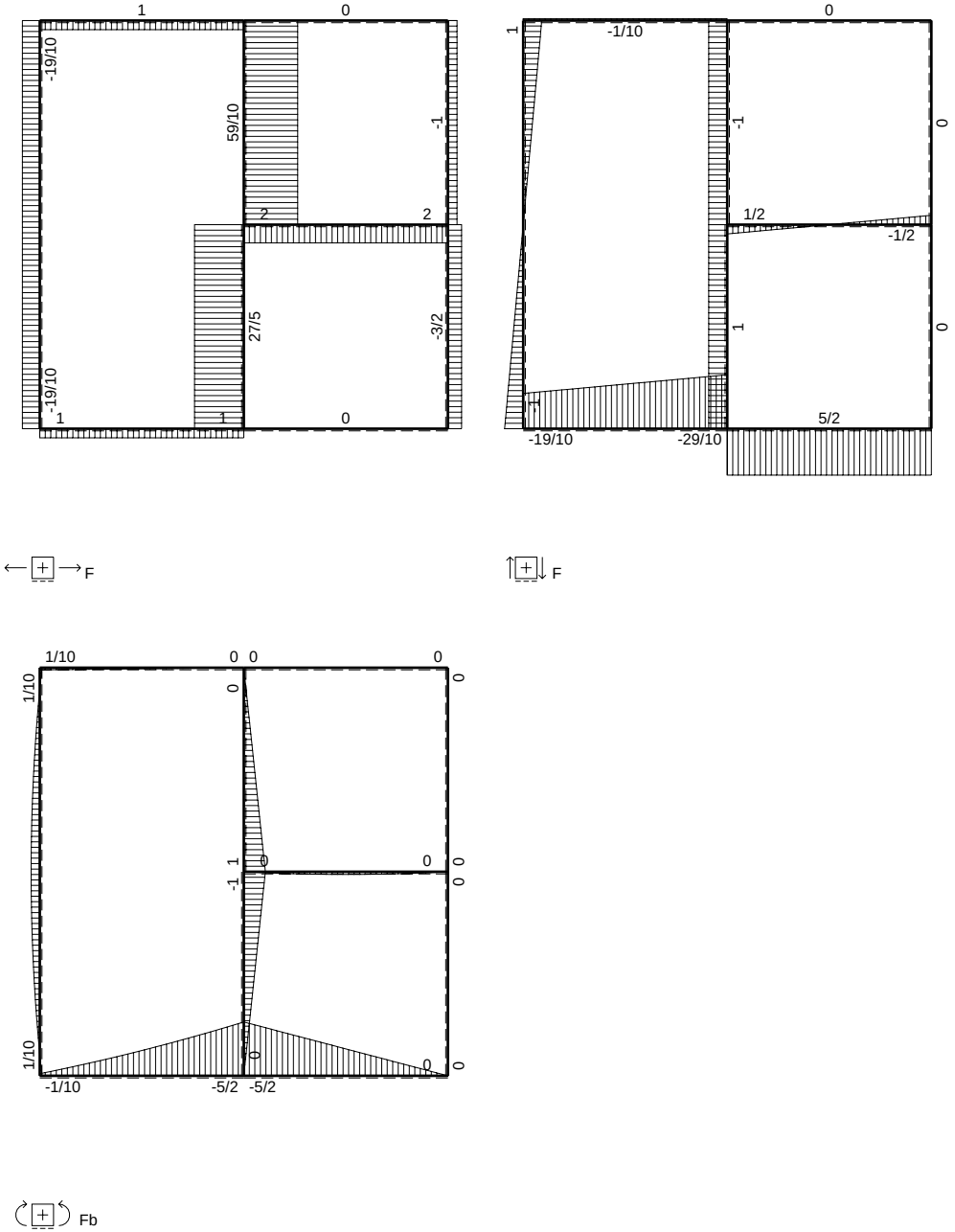
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

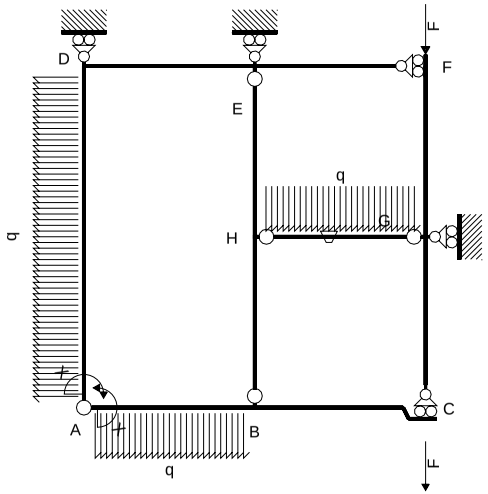
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

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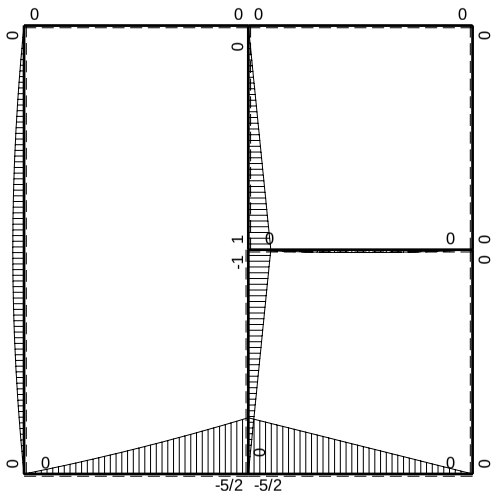
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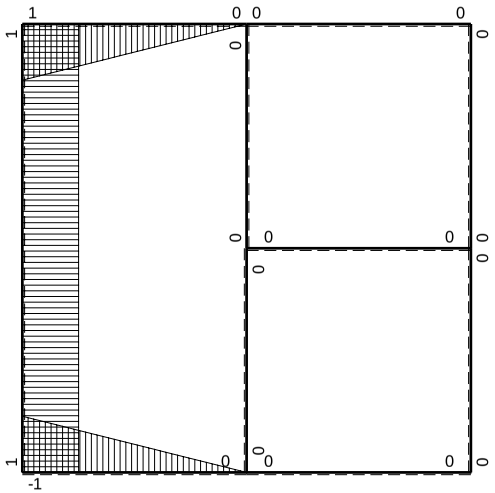

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0
-1 1
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0
0
0
0
1/10
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240392 ESΣ03.xxxx.124

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	$-Fb+Fx$	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	$Fb-Fx$	0	0	0	0	0+0	0
EH b	0	$-Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

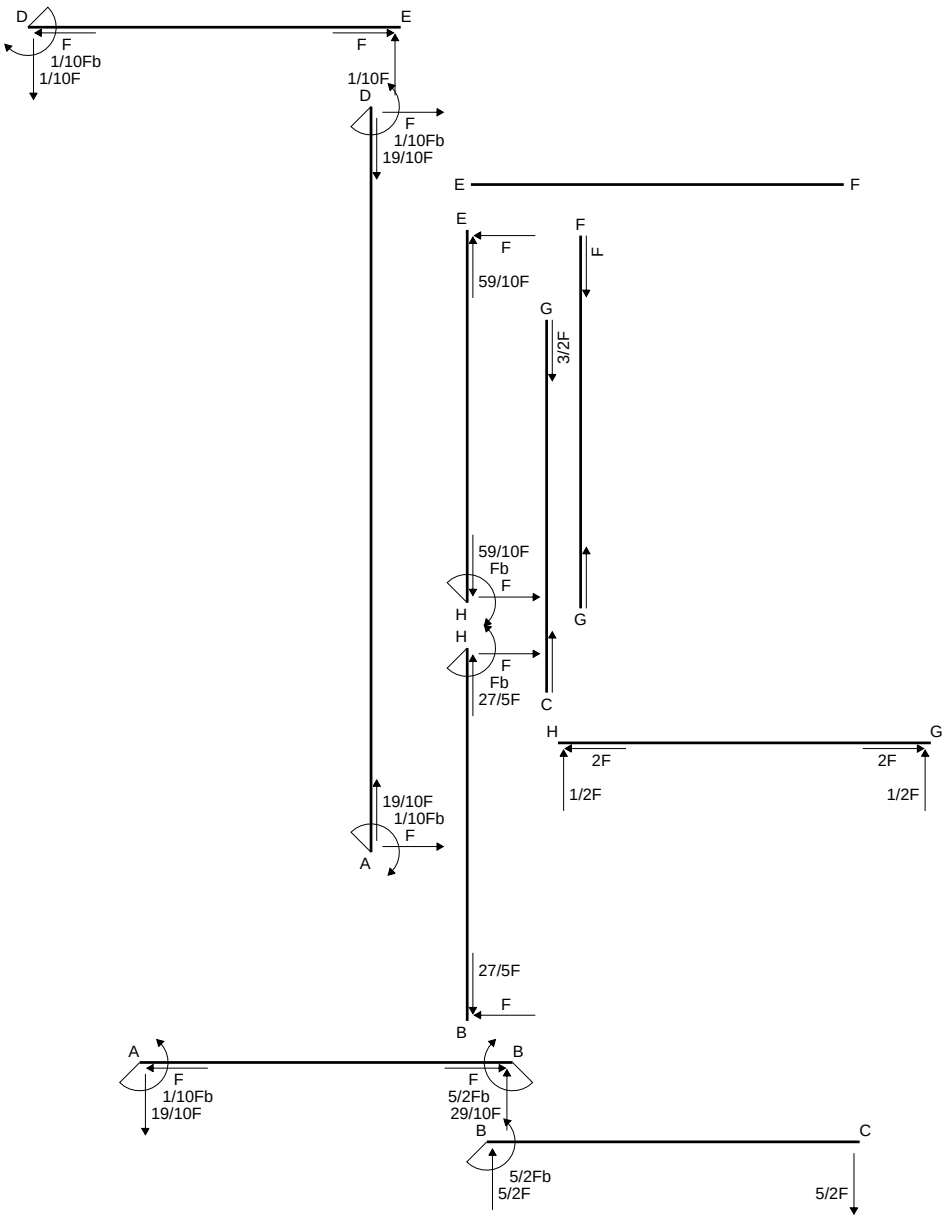
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

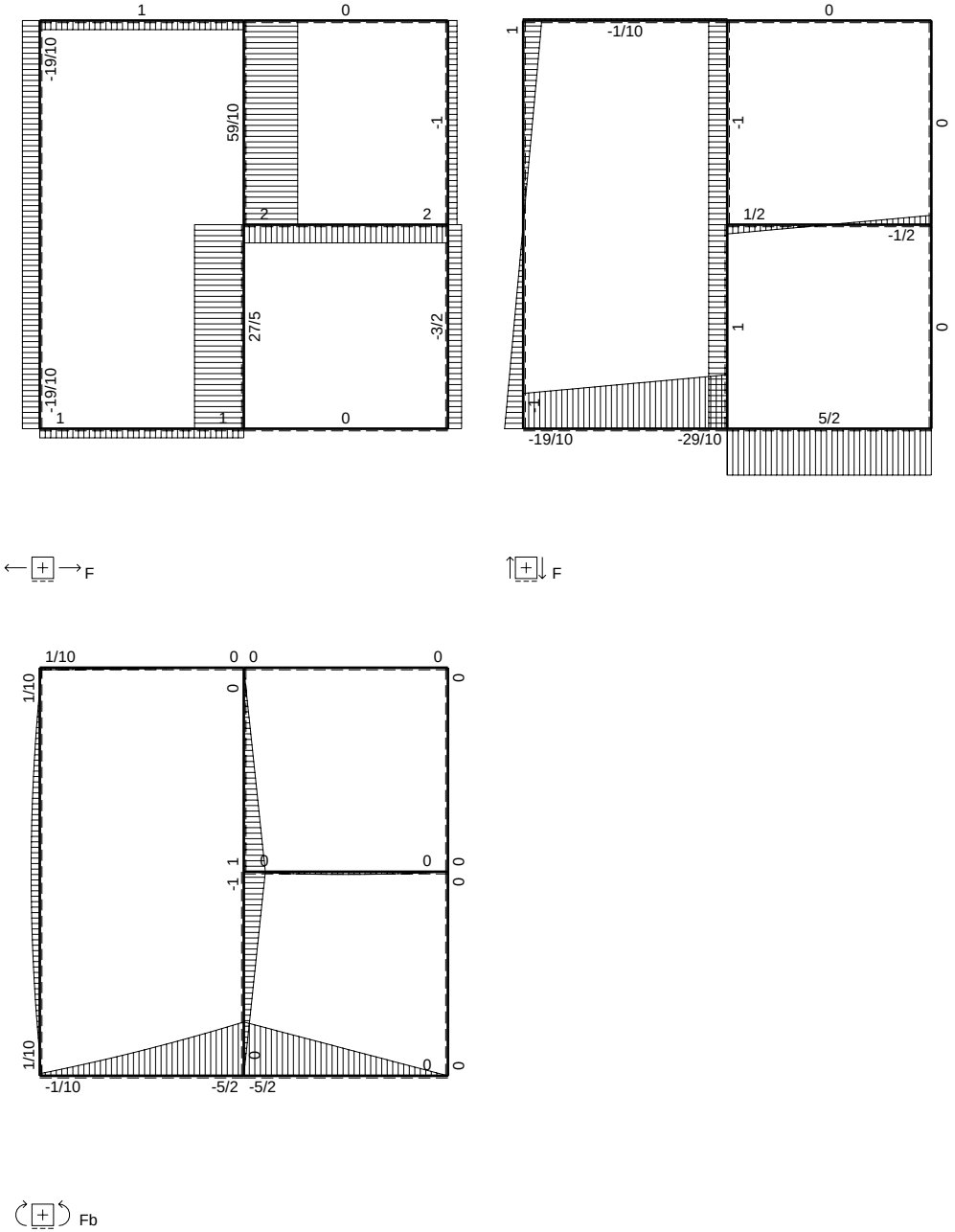
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

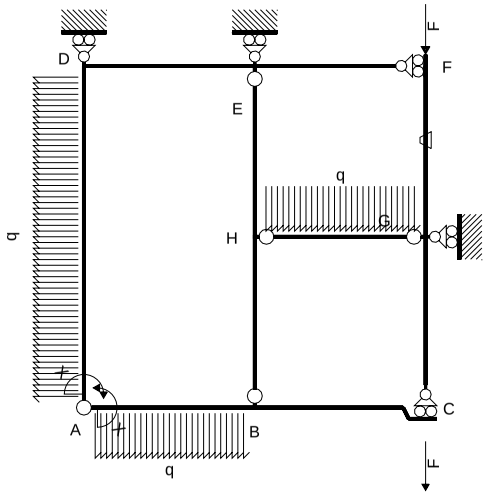
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

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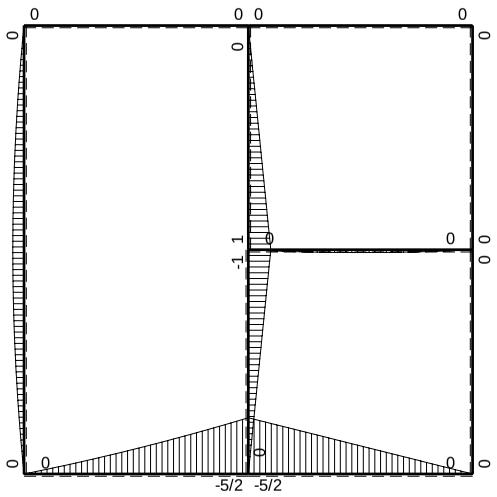
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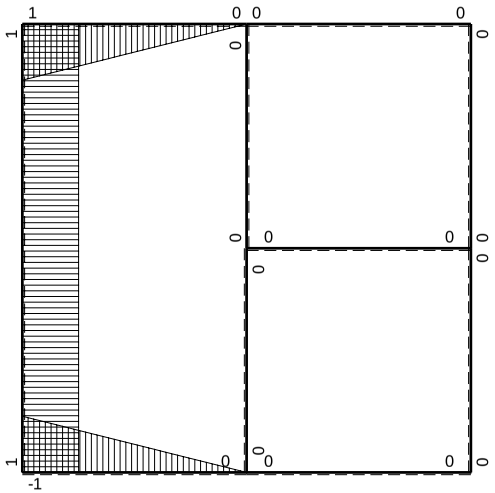

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0 0
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-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211461 ESΣ03.xxxx.125

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PROCEDIMENTO E RISULTATI 211461 ESΣ03.xxxx.125

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-Fb+Fx	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	Fb-Fx	0	0	0	0	0+0	0
EH b	0	-Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

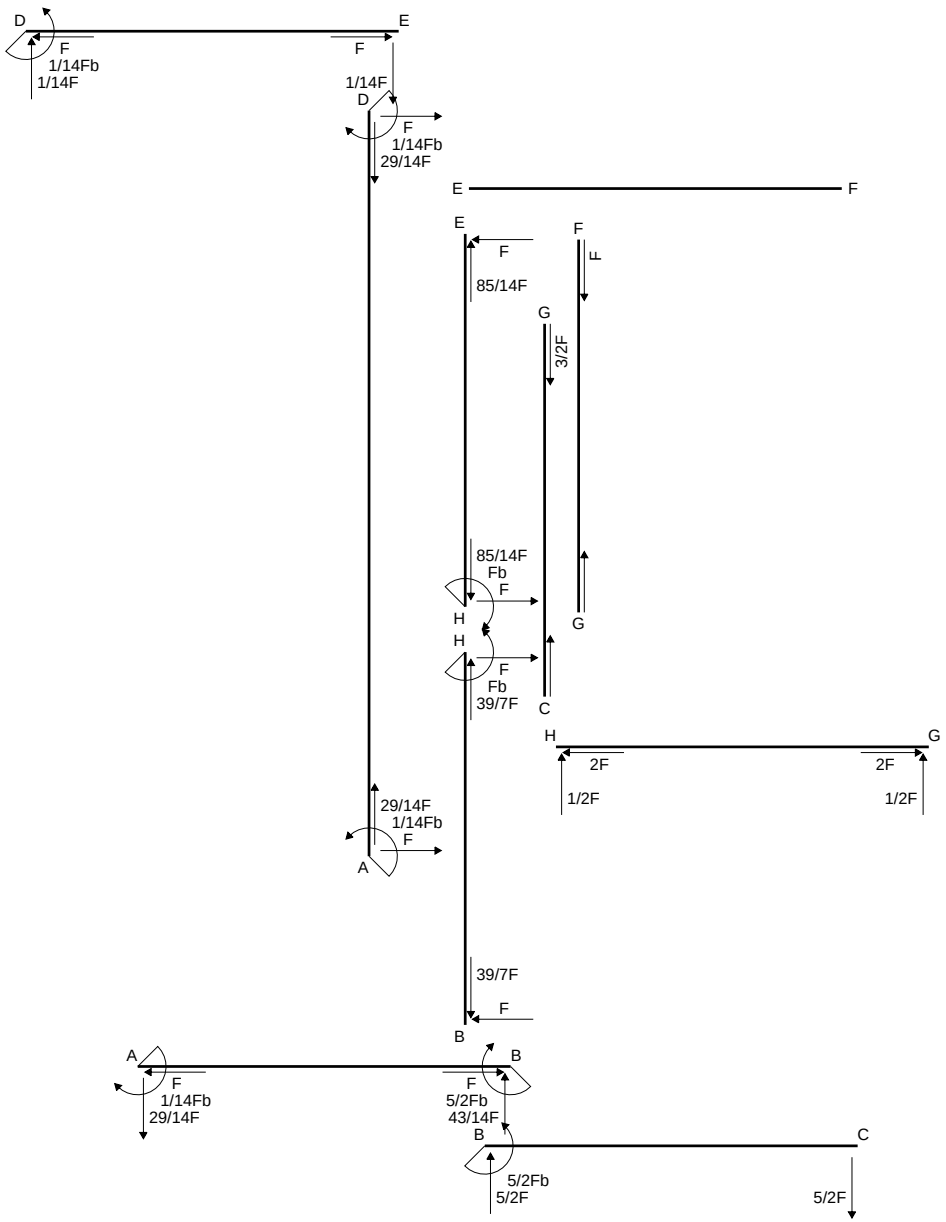
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

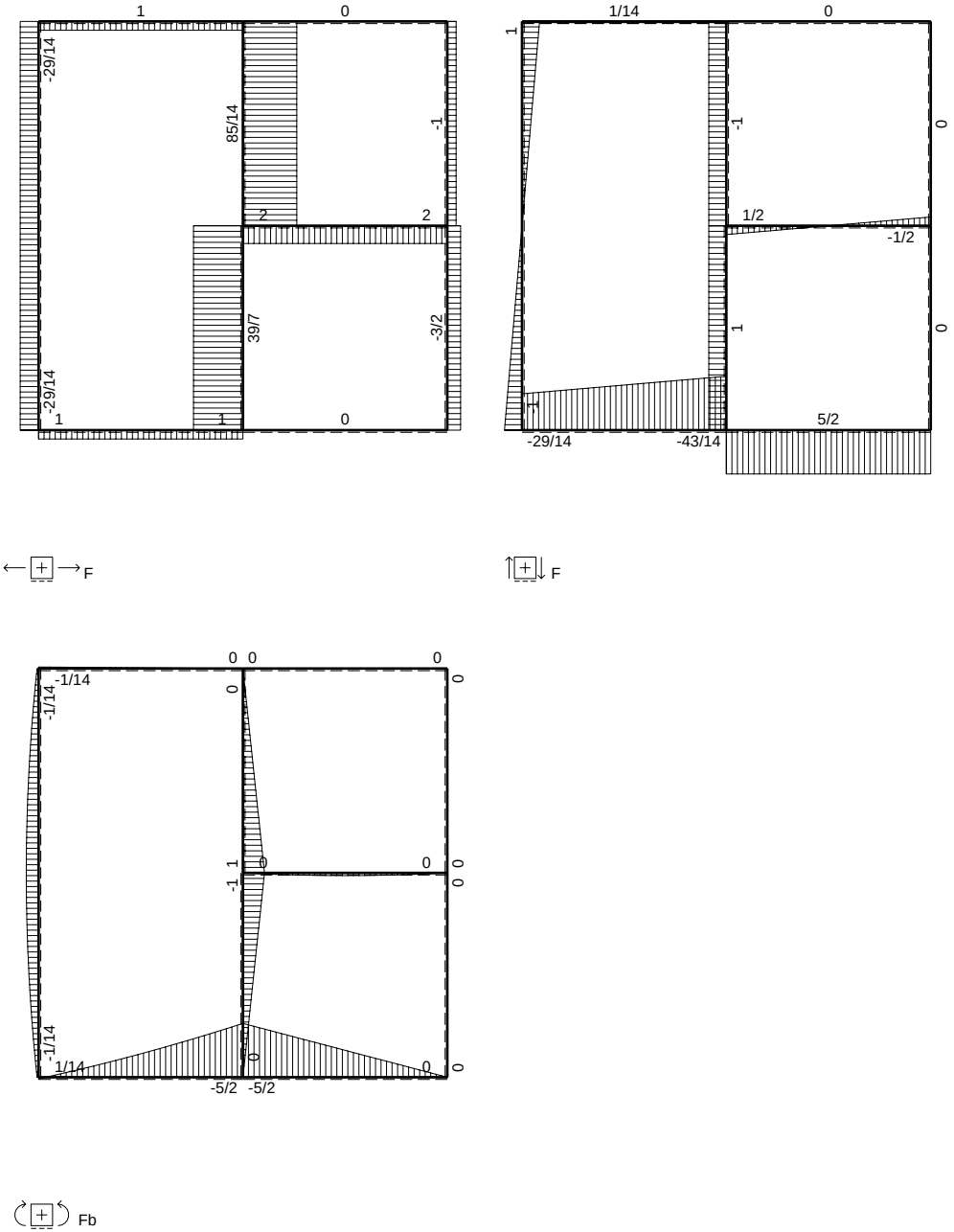
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

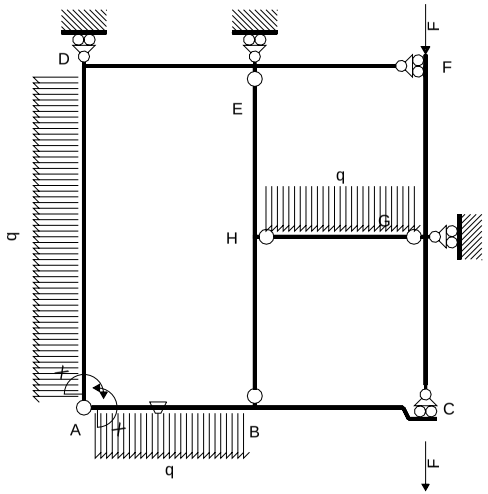
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

ESΣ03.xxxx.125

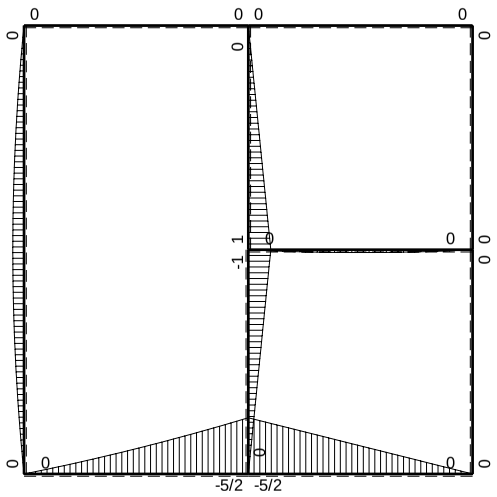
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D
E
F
1/14Fb
1/14F
F
1/14F
D
F
1/14Fb
29/14F
E
F
E
F
F
85/14F
G
3/2F
85/14F
Fb
F
H
G
H
F
Fb
39/7F
C
H
G
2F
2F
1/2F
1/2F
29/14F
1/14Fb
F
A
39/7F
F
B
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5/2F
5/2F

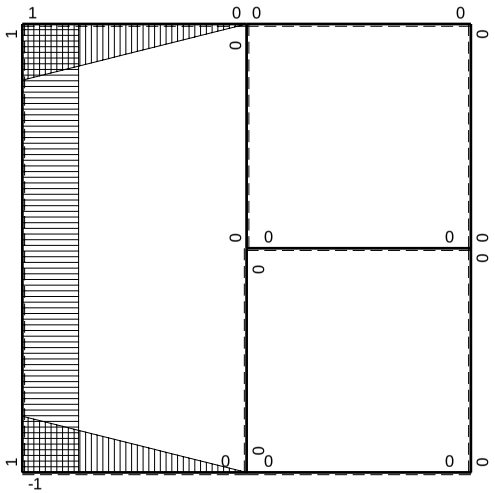

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85/14
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39/7
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-29/14
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1/2
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5/2
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-43/14
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-1/14
-1/14
0
0
0
-1 1
0
0
0 0
-1/14
1/14
0
-5/2 -5/2
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240146 ESΣ03.xxxx.126

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	$-Fb/EJ$	$2Fx-3/2Fx^2/b-1/2qx^3/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$	$(3/8+1/2)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	$5/2Fx-3Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-Fb+Fx$	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	$Fb-Fx$	0	0	0	0	0+0	0
EH b	0	$-Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$5/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/14Fb$	

Sviluppi di calcolo iperstatica

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ESΣ03.xxxx.126

$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [\ x - 1/2\ x^2/b\ ]_o^b\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + (\ b - 1/2\ b\ )\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\,x/b\ )\ \theta\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/8\ \ Fb^2/EJ$

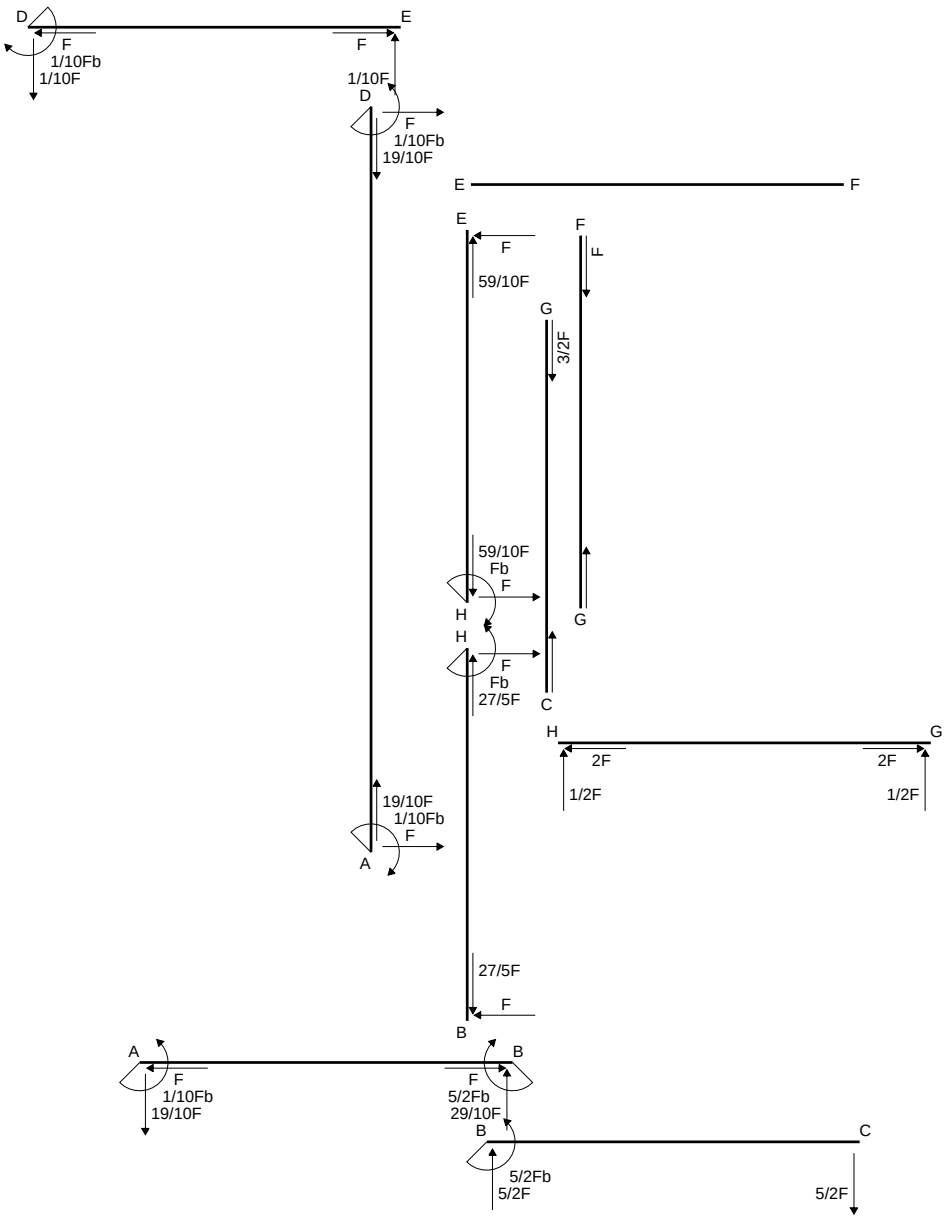
$L^{Xo}_{AD} = \int_o^{2b} (-\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

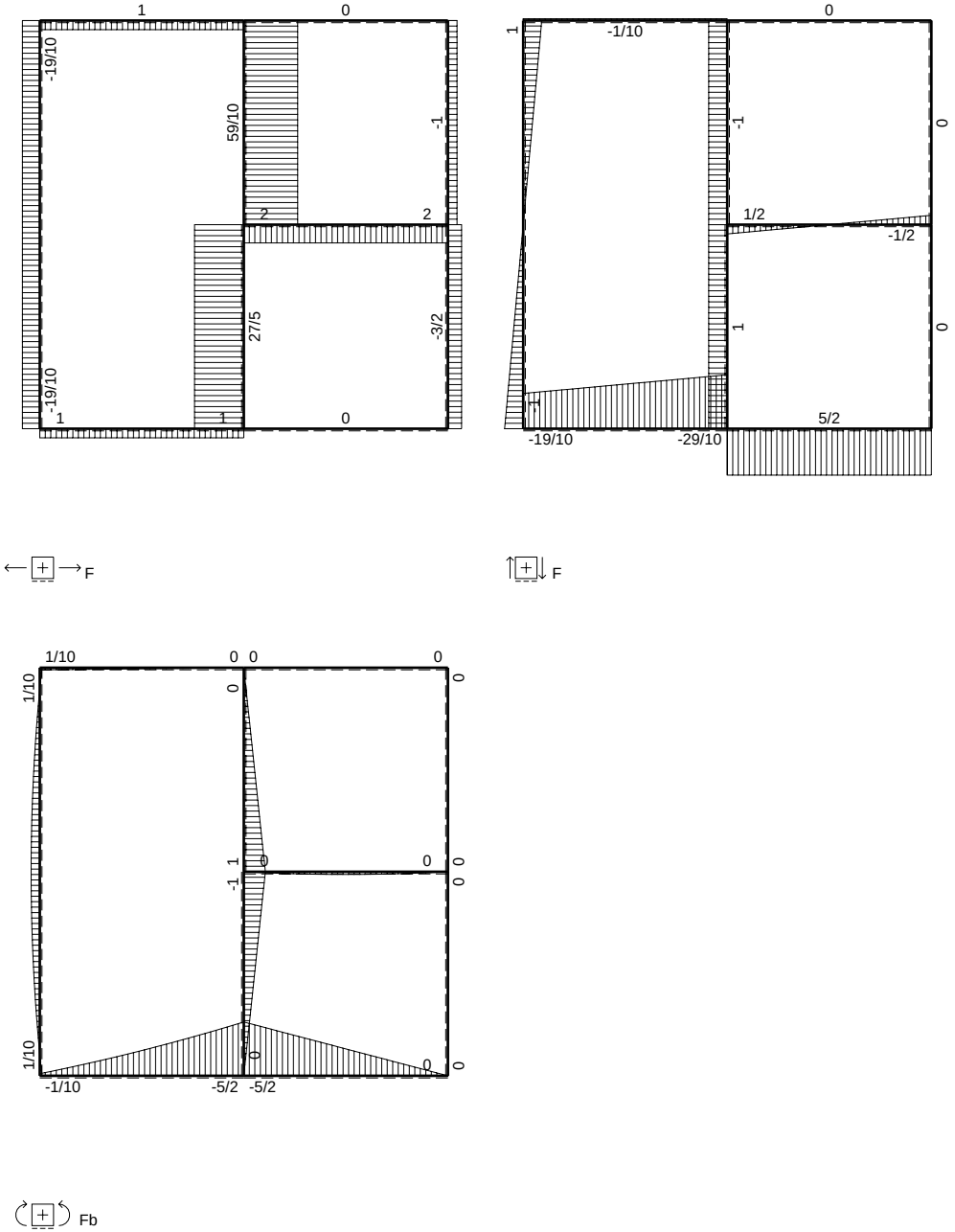
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -2/3\ \ Fb^2/EJ$

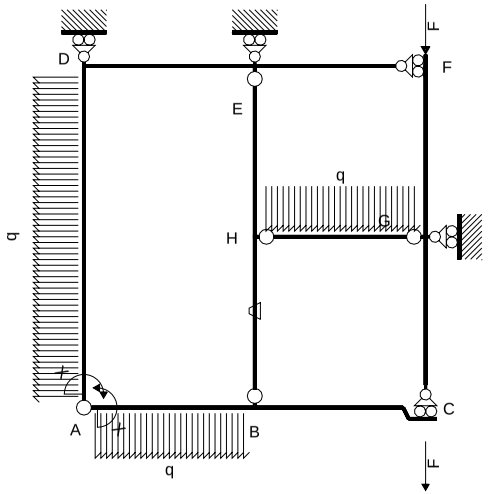
$L^{Xo}_{DA} = \int_o^{2b} (-\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -2/3\ \ Fb^2/EJ$

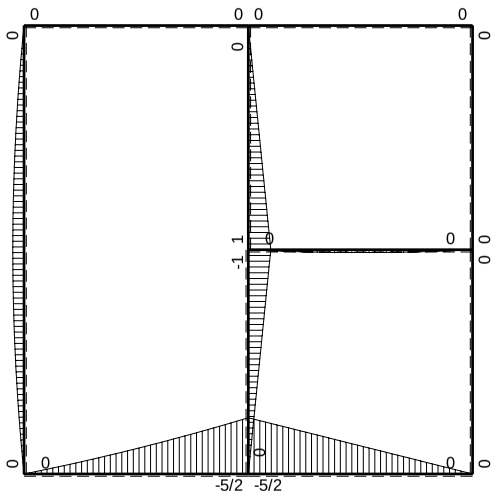
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D
E
F
1/10Fb
1/10F
1/10F
D
F
1/10Fb
19/10F
E
F
E
F
F
59/10F
G
3/2F
59/10F
Fb
F
H
G
H
F
Fb
27/5F
C
H
G
2F
2F
1/2F
1/2F
19/10F
1/10Fb
F
A
27/5F
F
B
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5/2Fb
5/2F
5/2F

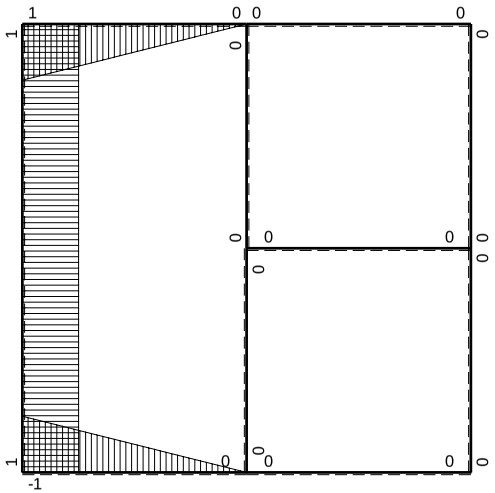

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59/10
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2
27/5
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0
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-1
1/2
-1/2
1
5/2
-19/10
-29/10
F
1/10
0 0
0
1/10
0
0
0
-1 1
0
0
0 0
0
1/10
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239766 ESΣ03.xxxx.127

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PROCEDIMENTO E RISULTATI 239766 ESΣ03.xxxx.127

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-Fb+Fx$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	Fx	Fb/EJ	0	0	0		
HE b	0	$Fb-Fx$	0	0	0	0	0+0	0
EH b	0	$-Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

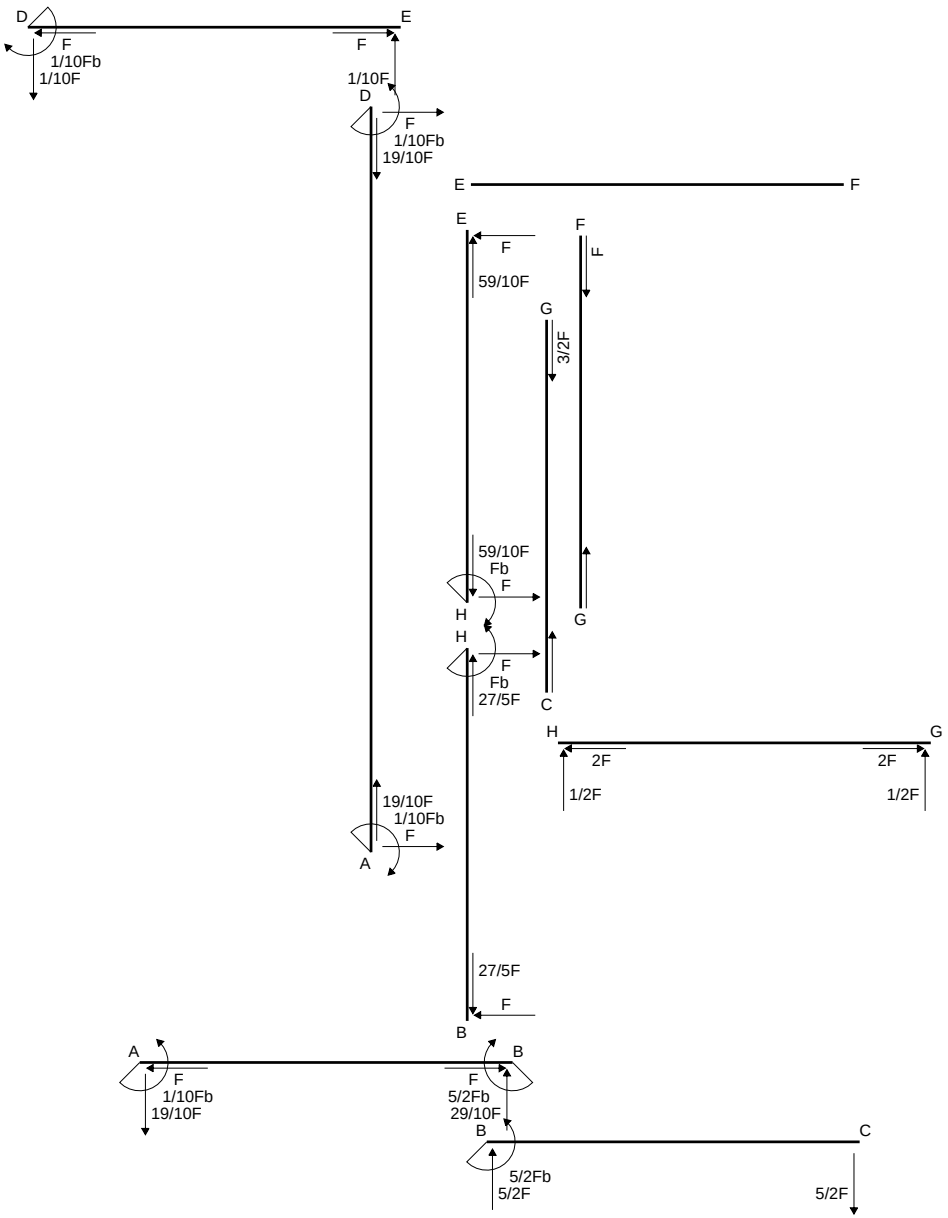
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

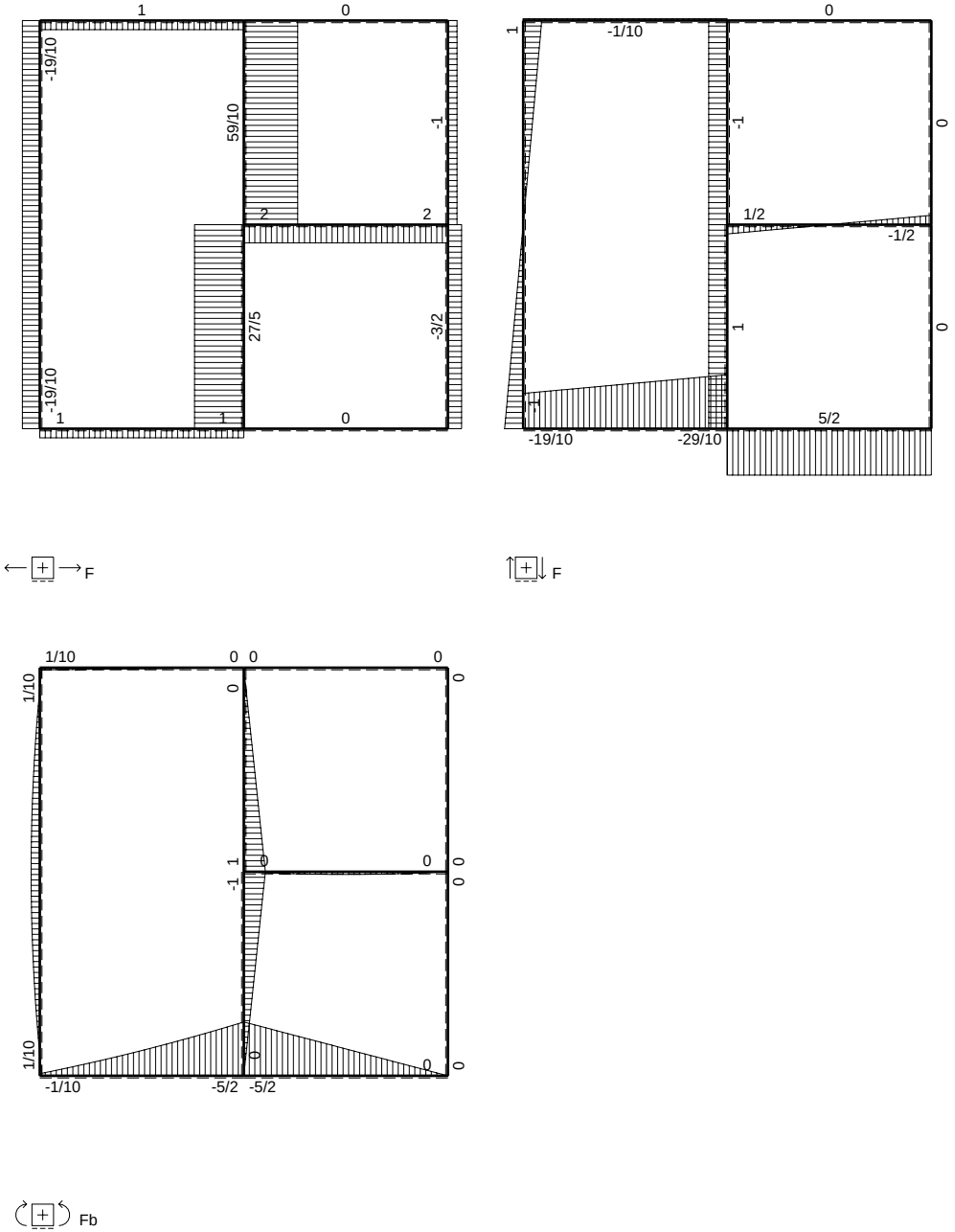
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

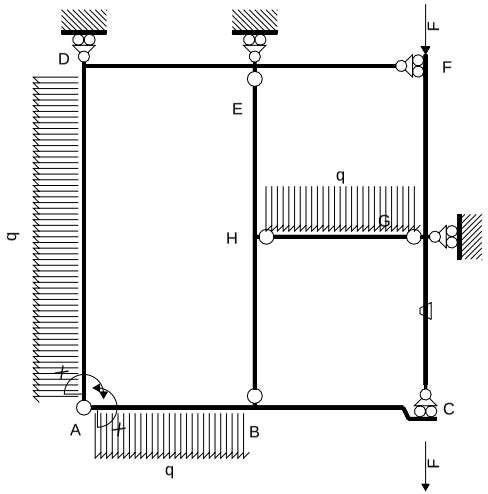
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

ESΣ03.xxxx.127

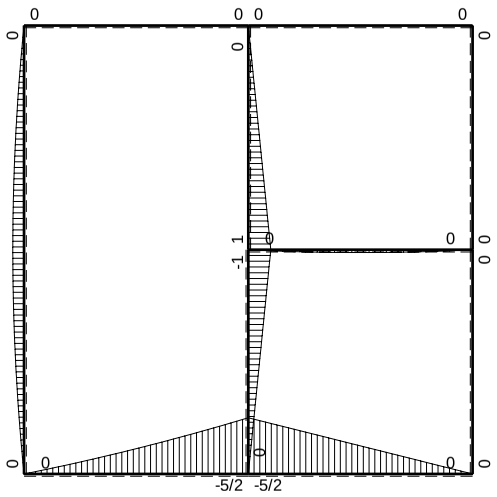
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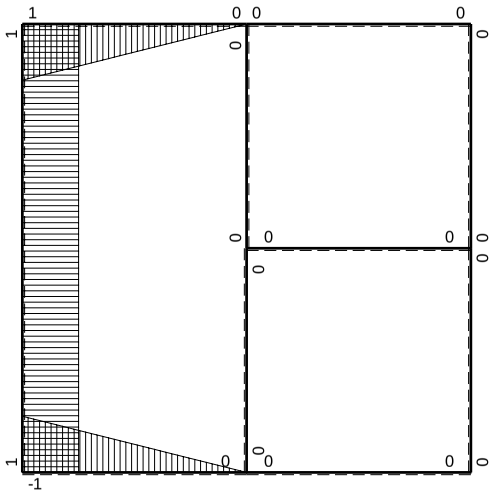

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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211653 ESΣ03.xxxx.128

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-Fb+Fx$	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	$Fb-Fx$	0	0	0	0	0+0	0
EH b	0	$-Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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ESΣ03.xxxx.128

$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

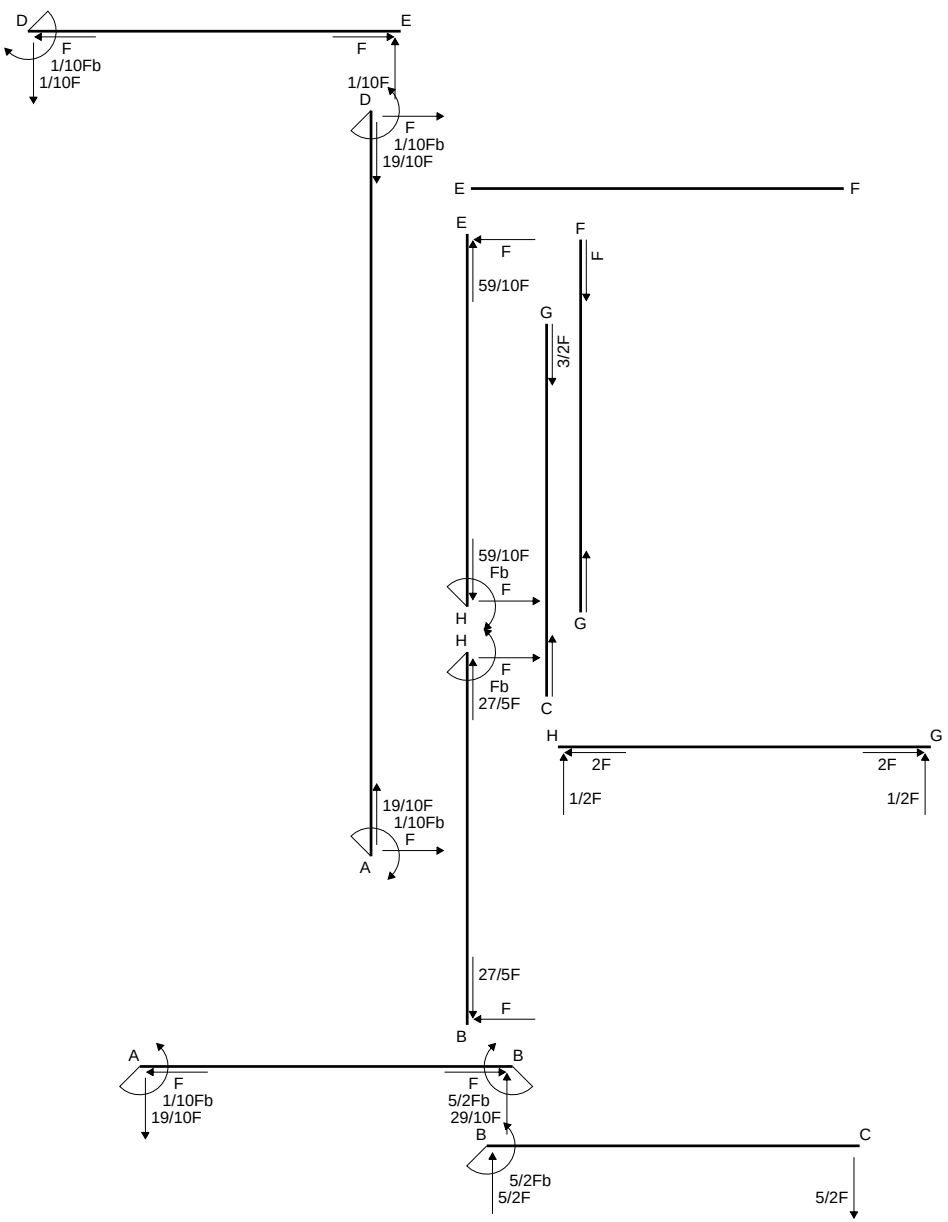
$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

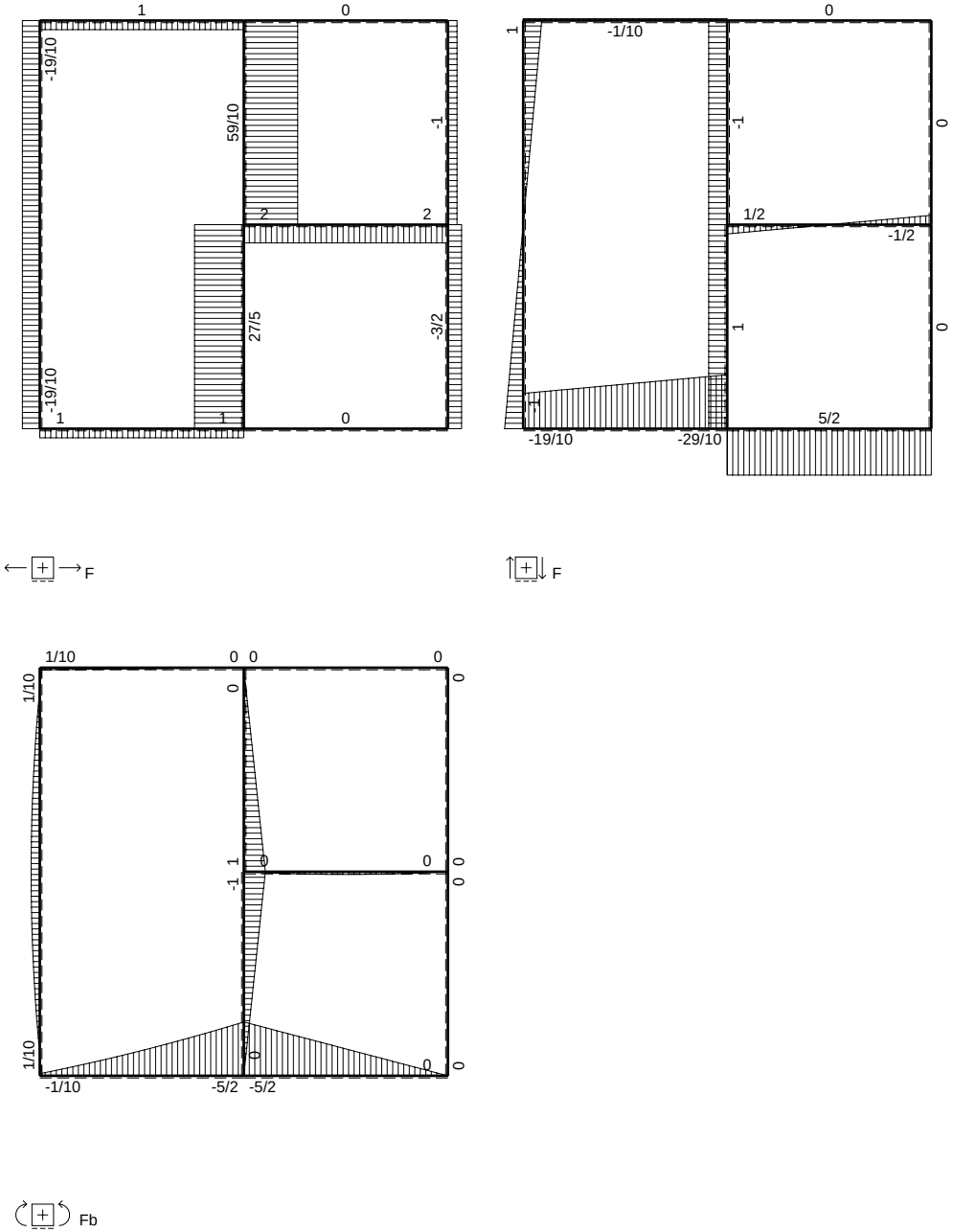
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

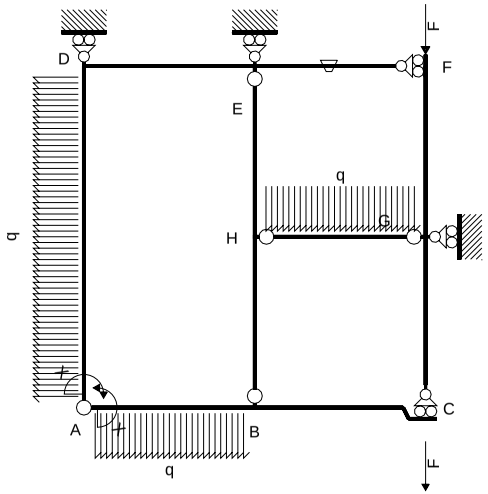
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

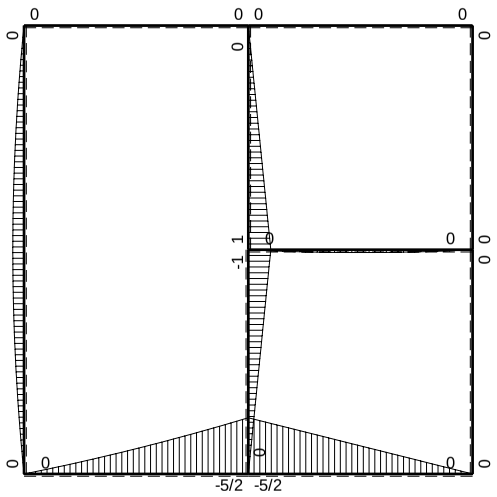
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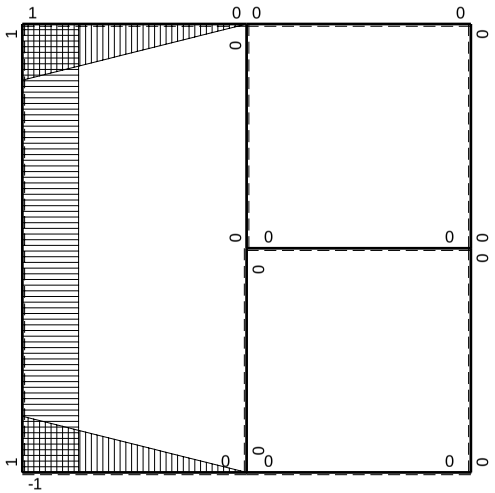

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0
0
0 0
0
1/10
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238810 ESΣ03.xxxx.129

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PROCEDIMENTO E RISULTATI 238810 ESΣ03.xxxx.129

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-Fb+Fx$	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	$Fb-Fx$	0	0	0	0	0+0	0
EH b	0	$-Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

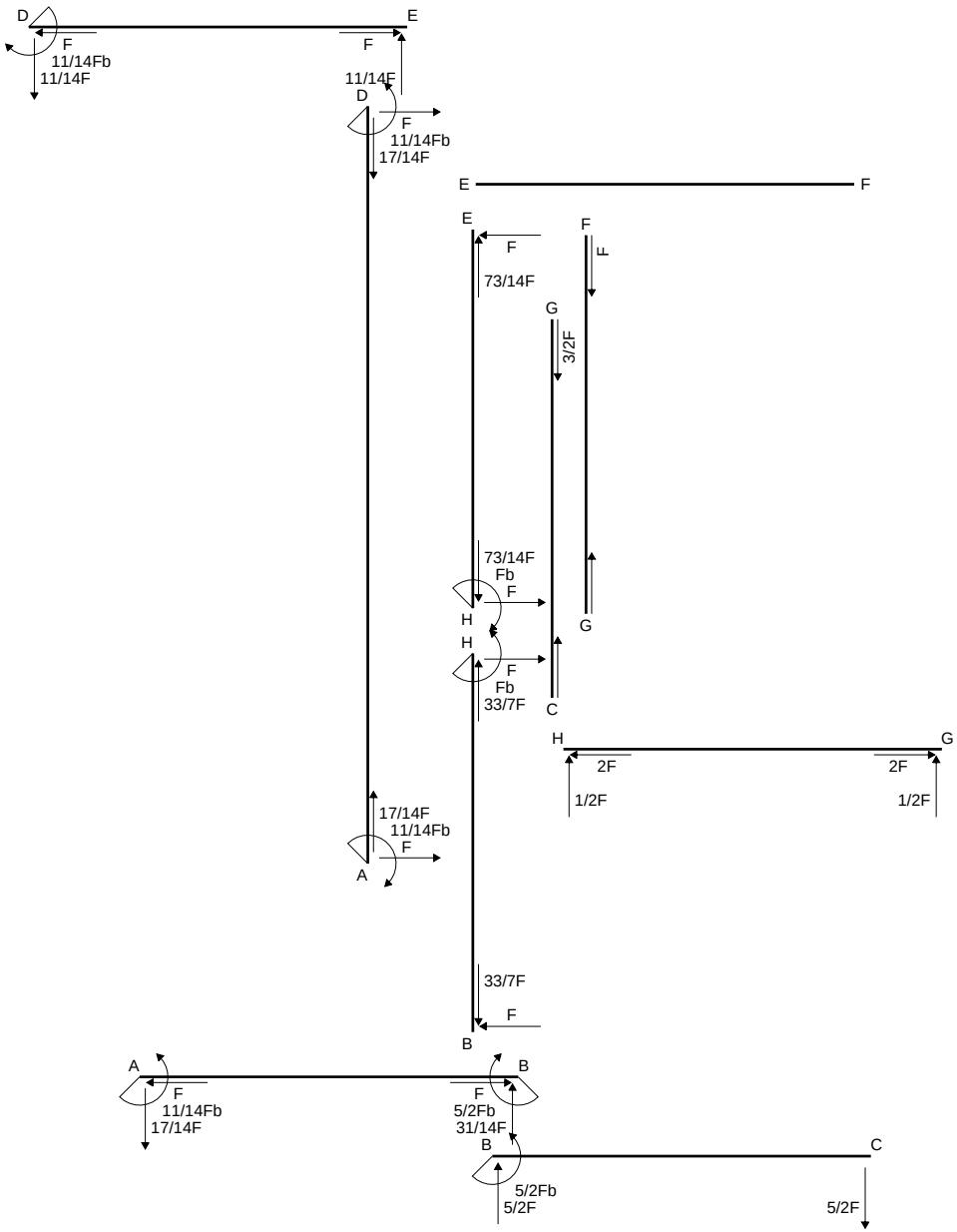
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

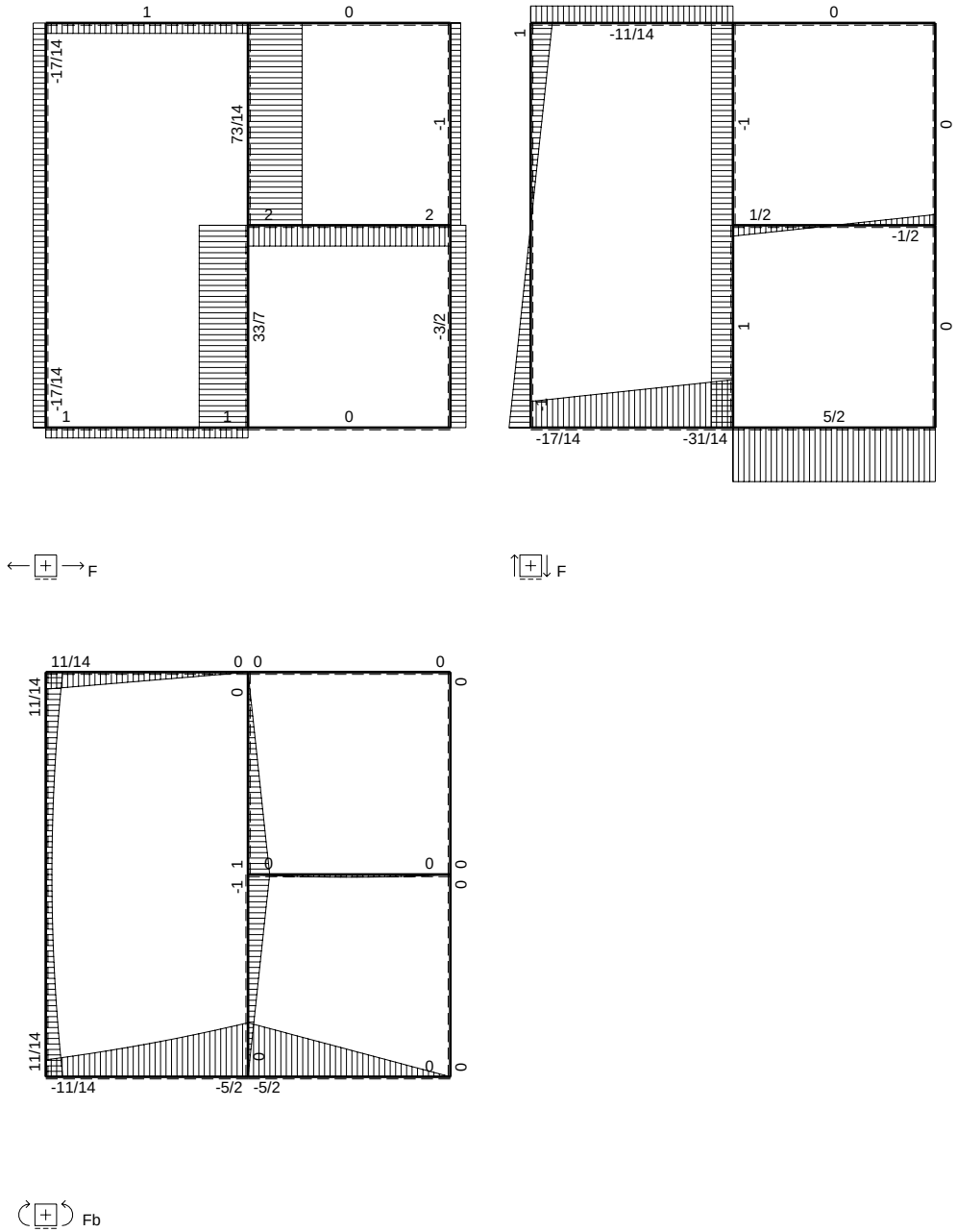
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

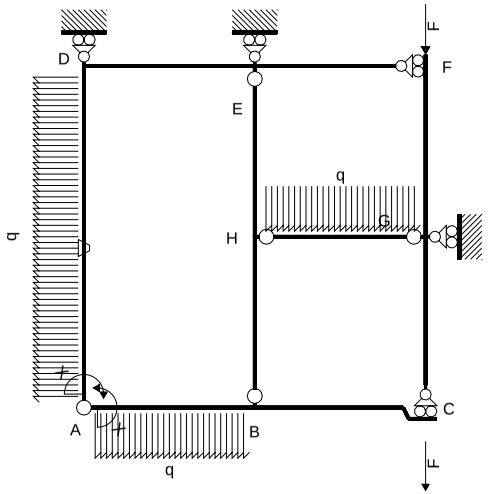
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

ESΣ03.xxxx.129

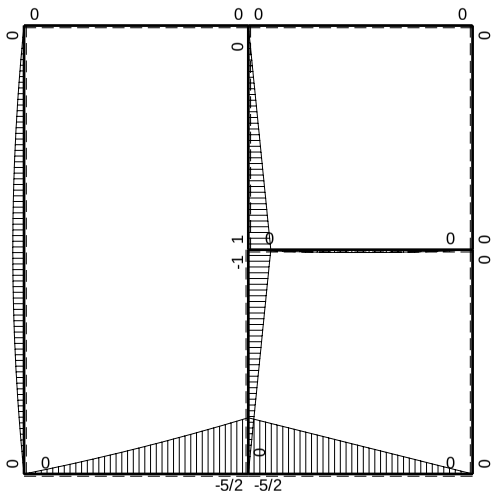
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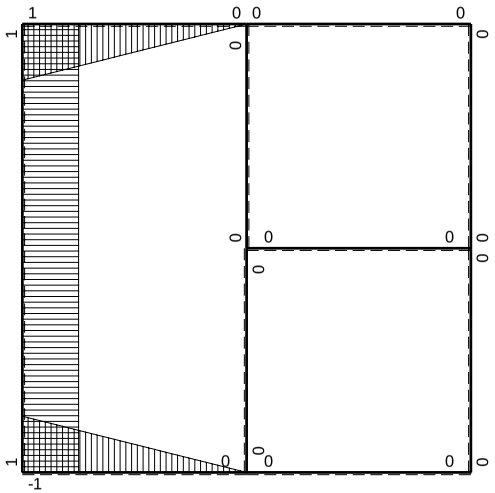

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-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239279 ESΣ03.xxxx.130

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	$-Fb/EJ$	$-Fx+1/2Fx^2/b$	$-Fb/EJ$	1	$(-2/3-2)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	Fb/EJ	$-Fx+1/2Fx^2/b$	$-Fb/EJ$	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-Fb+Fx$	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	$Fb-Fx$	0	0	0	0	0+0	0
EH b	0	$-Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-55/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						11/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^{2b} (-1\ )\ \theta\ \ dx$

$= [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ\ \ + [-\ x\ ]_o^{2b}\ \theta$

$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ + (-2\ b\ )\ \theta\ \ = -8/3\ \ Fb^2/EJ$

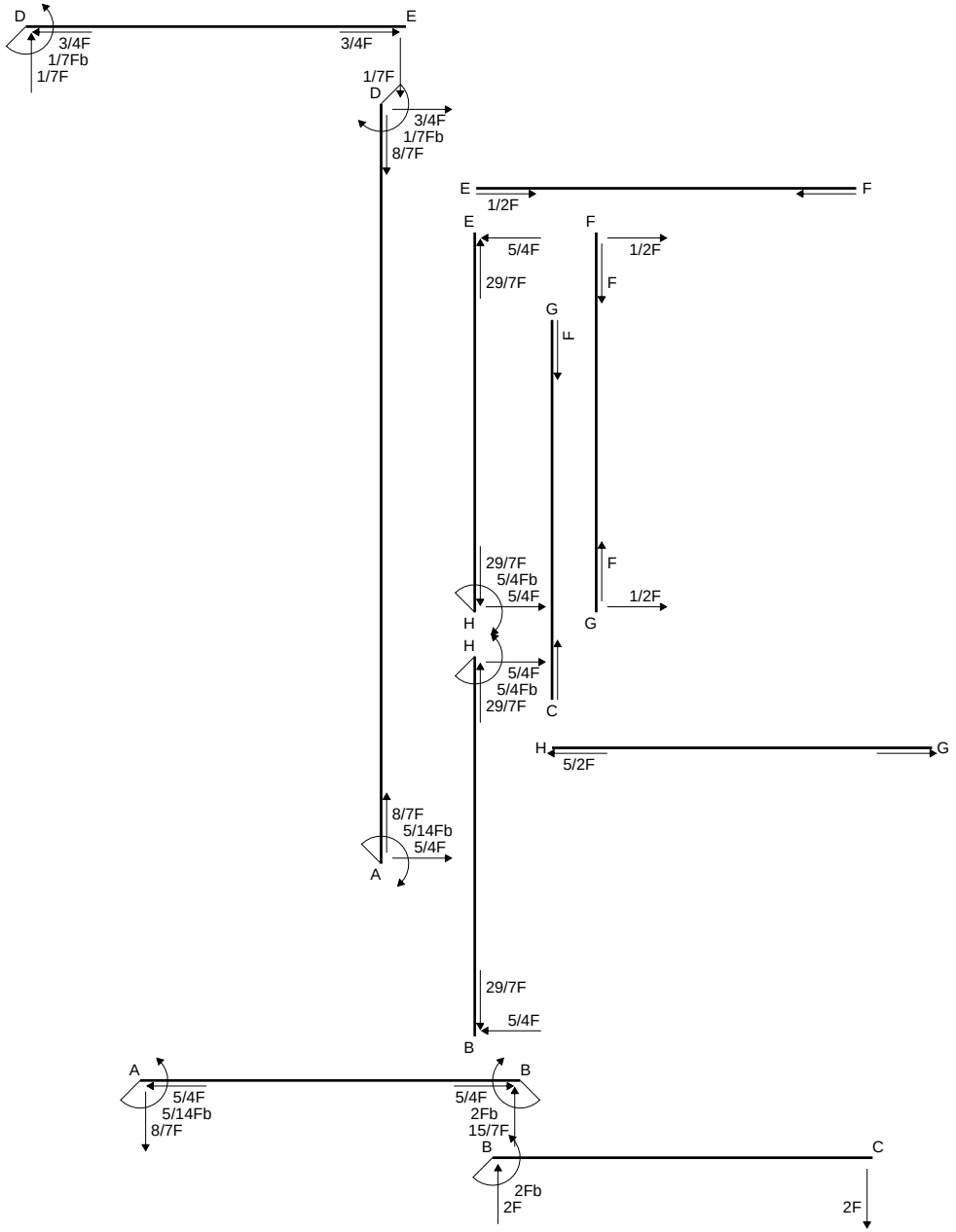
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^{2b} (1\ )\ \theta\ \ dx$

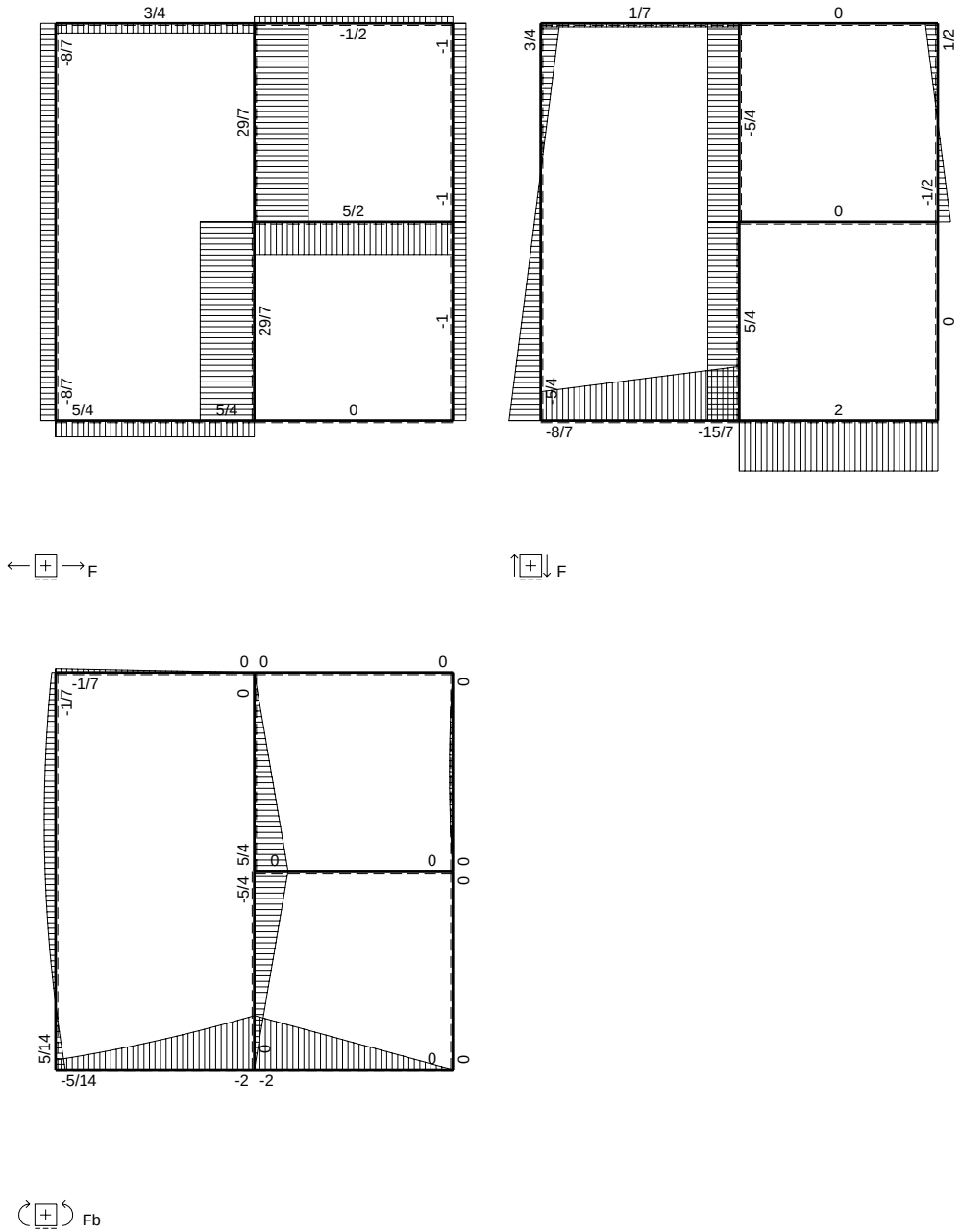
$= [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ\ \ + [\ x\ ]_o^{2b}\ \theta$

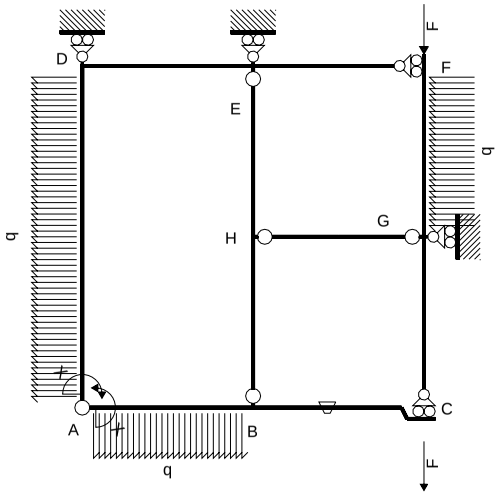
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ + (2\ b\ )\ \theta\ \ = -8/3\ \ Fb^2/EJ$

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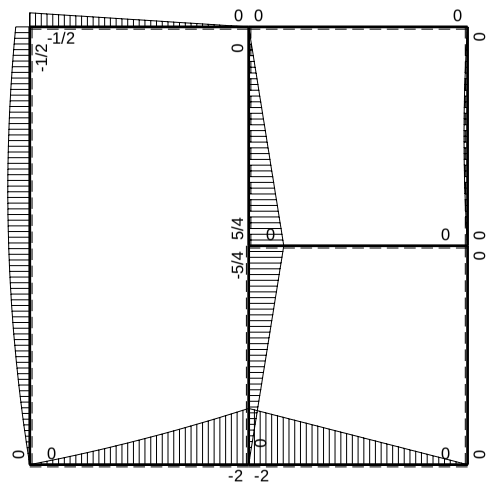
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E
3/4F
1/7Fb
1/7F
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1/7F
D
3/4F
1/7Fb
8/7F
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2F

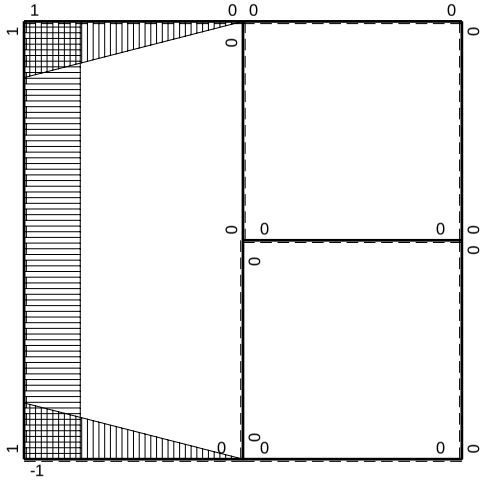

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-1/7
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0
0
0
5/4
0
0
0 0
-5/4
0
5/14
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 -3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x -3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b -3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

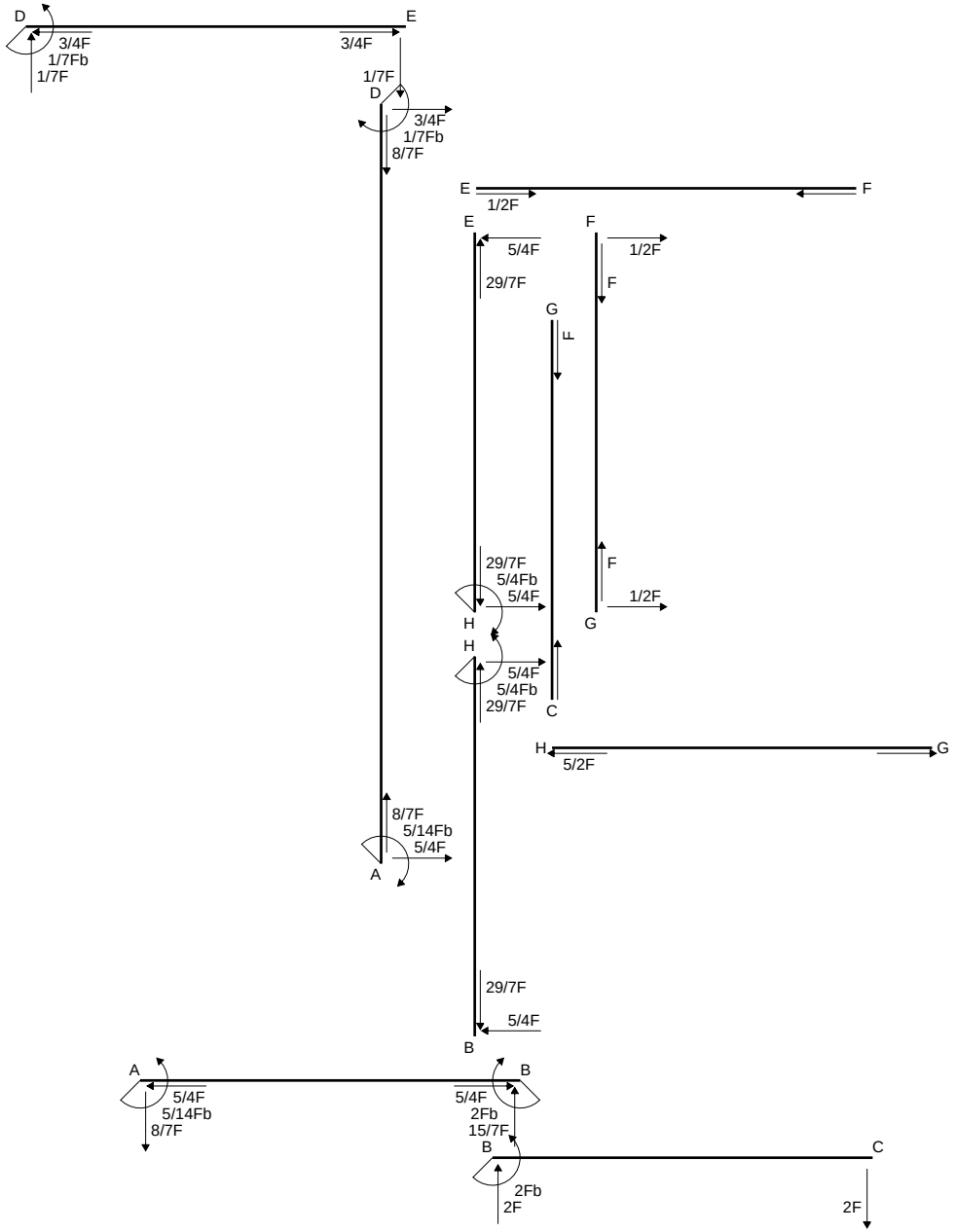
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

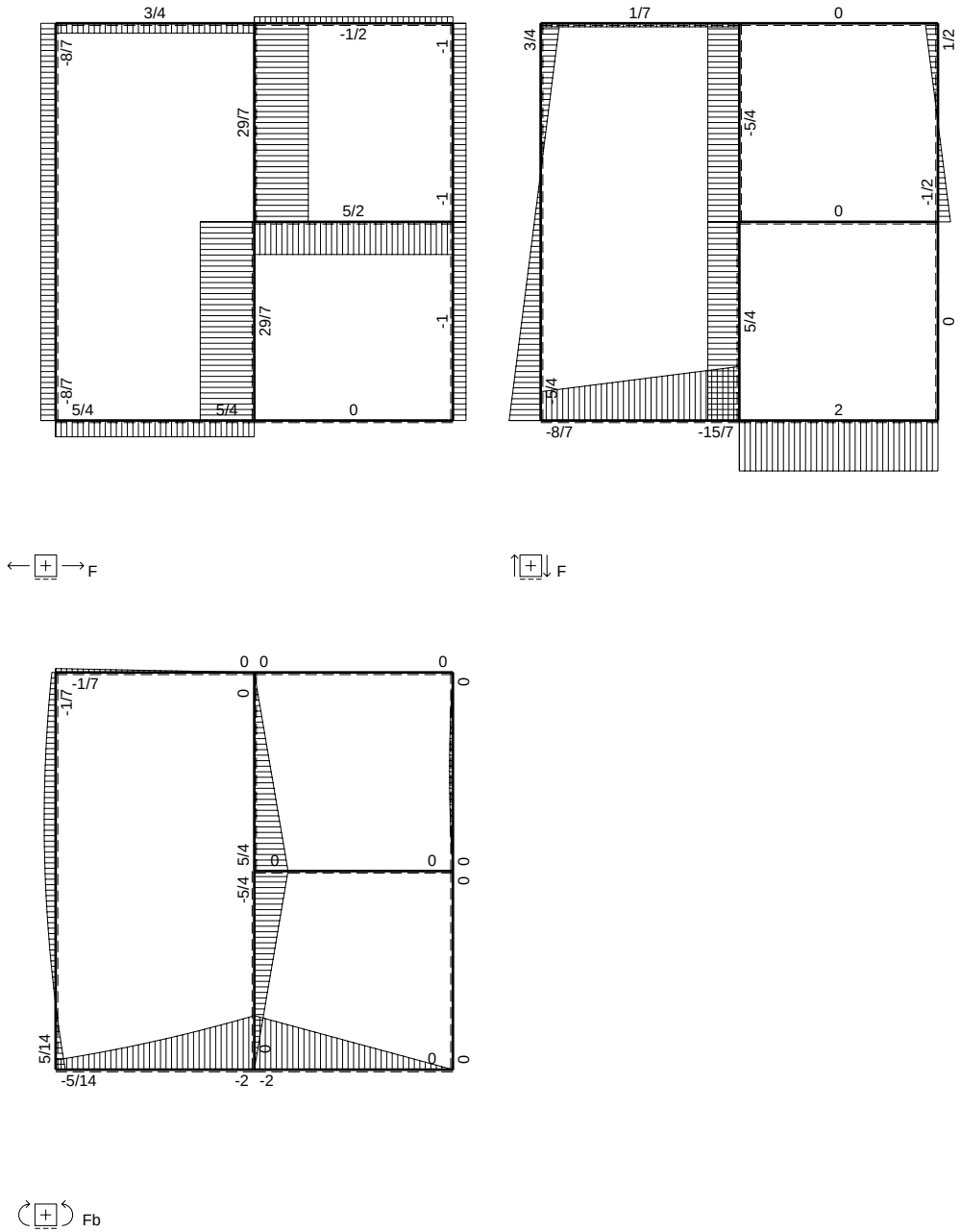
$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

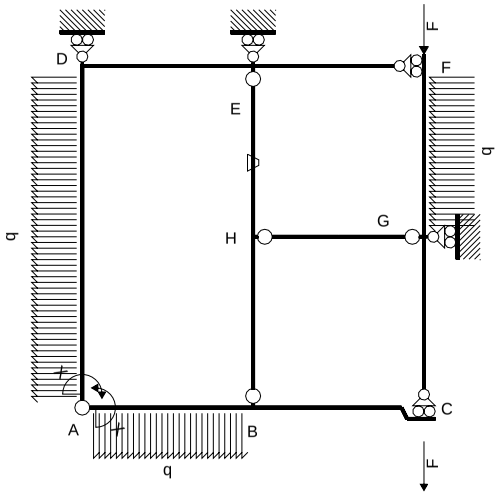
$= \left(-1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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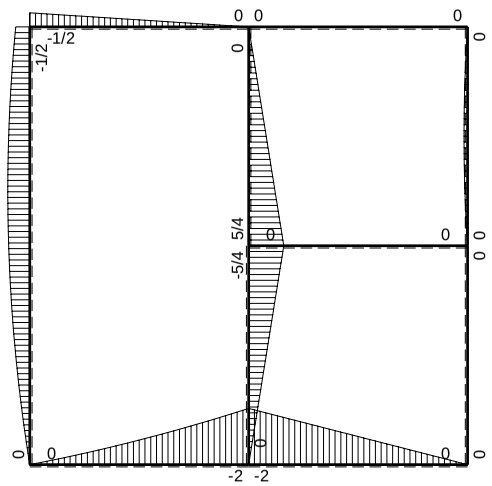
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
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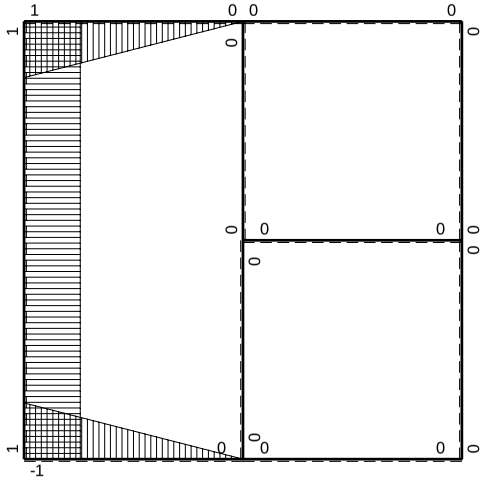

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0
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0
-5/4
0 0
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-5/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

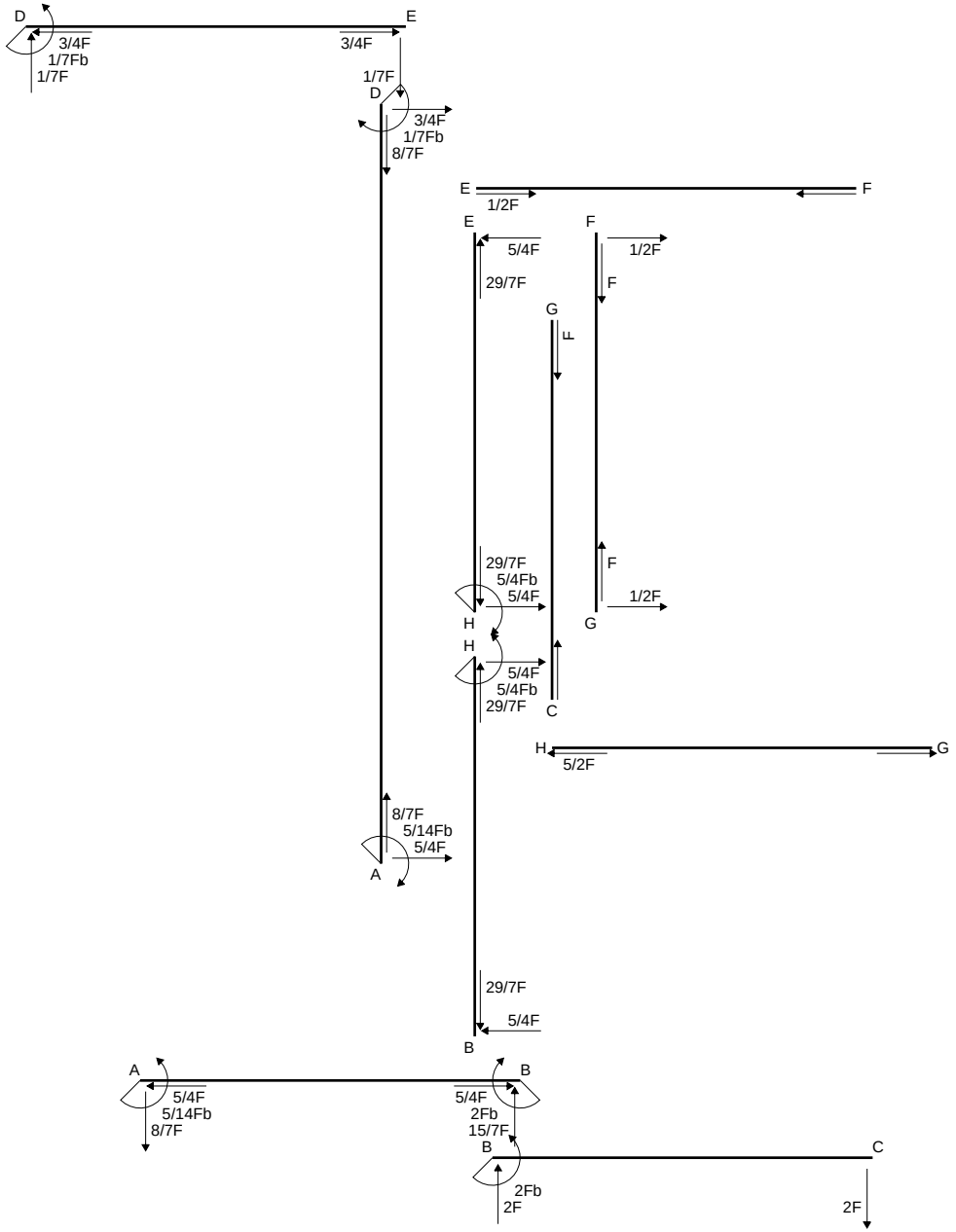
$= (-1/2\ b + 1/2\ b - 1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

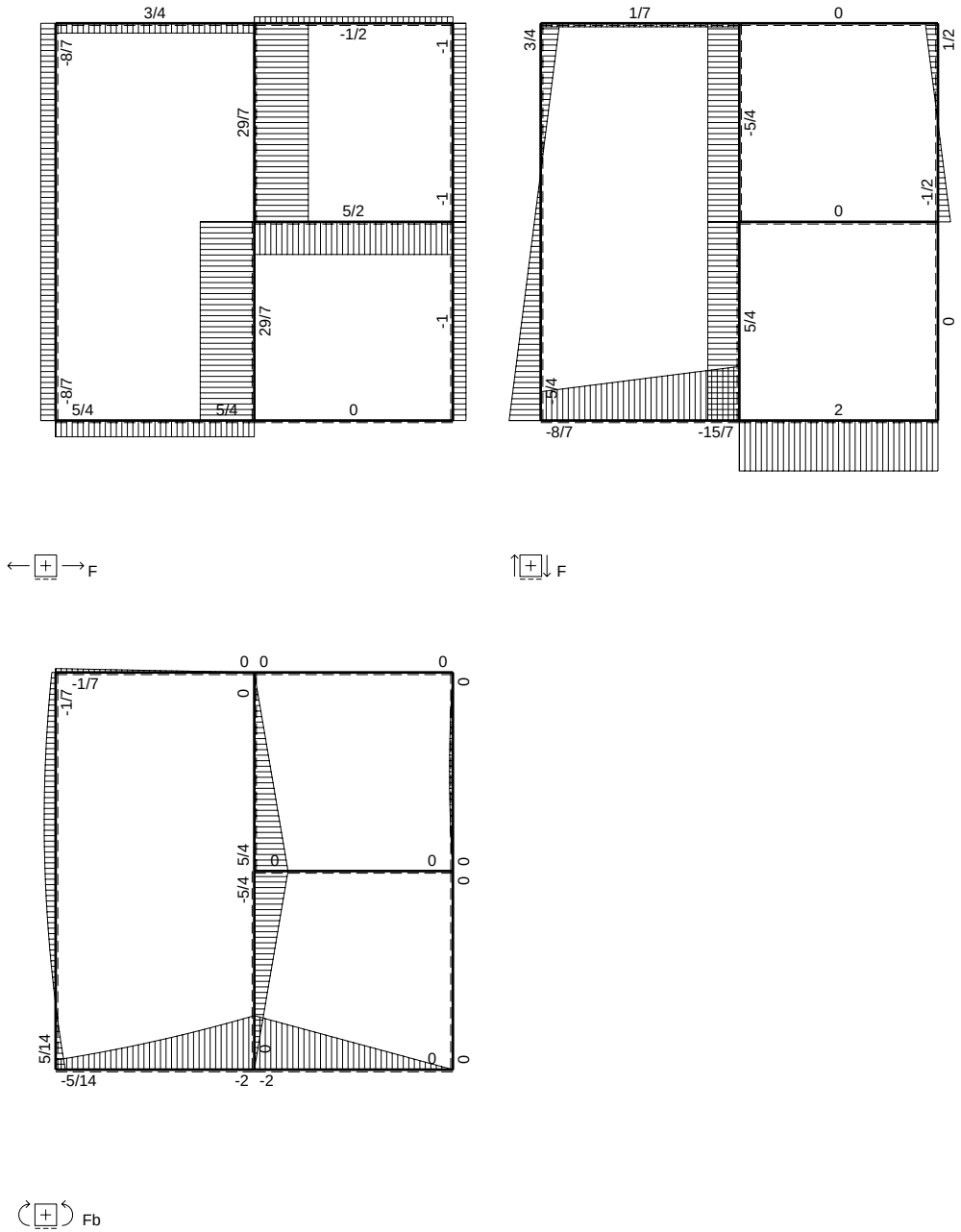
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

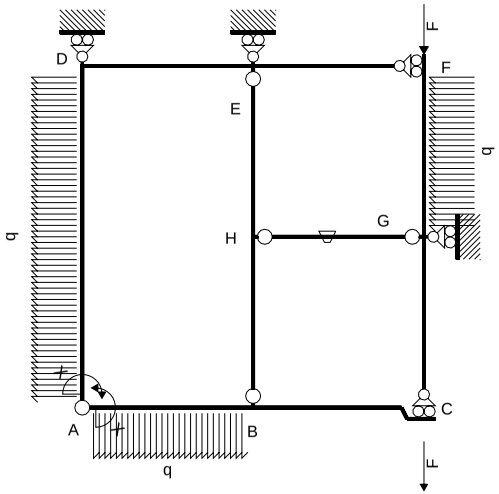
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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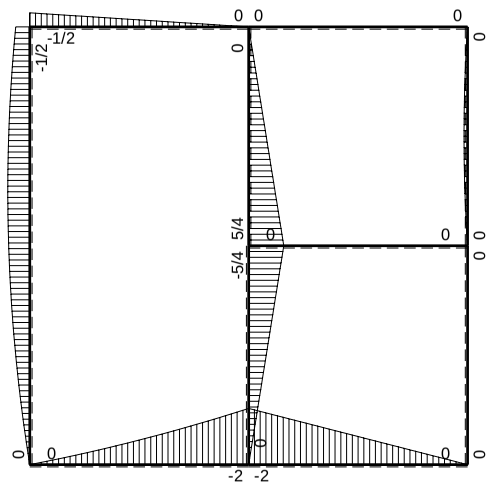
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
8/7F
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2F
2F

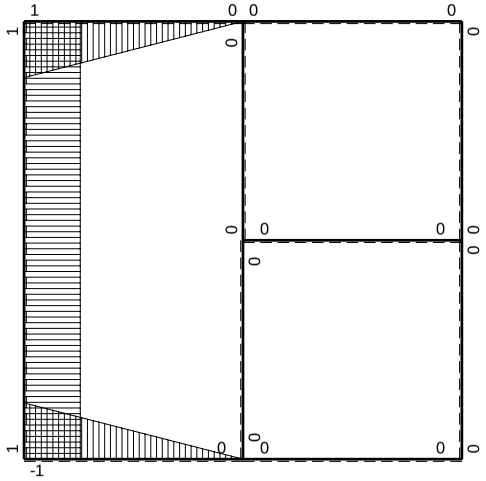

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0
0
5/4
0
0
0 0
-5/4
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-5/2\ b +4/3\ b\ \right)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 -3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x -3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b -3/2\ b +4/3\ b\ \right)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

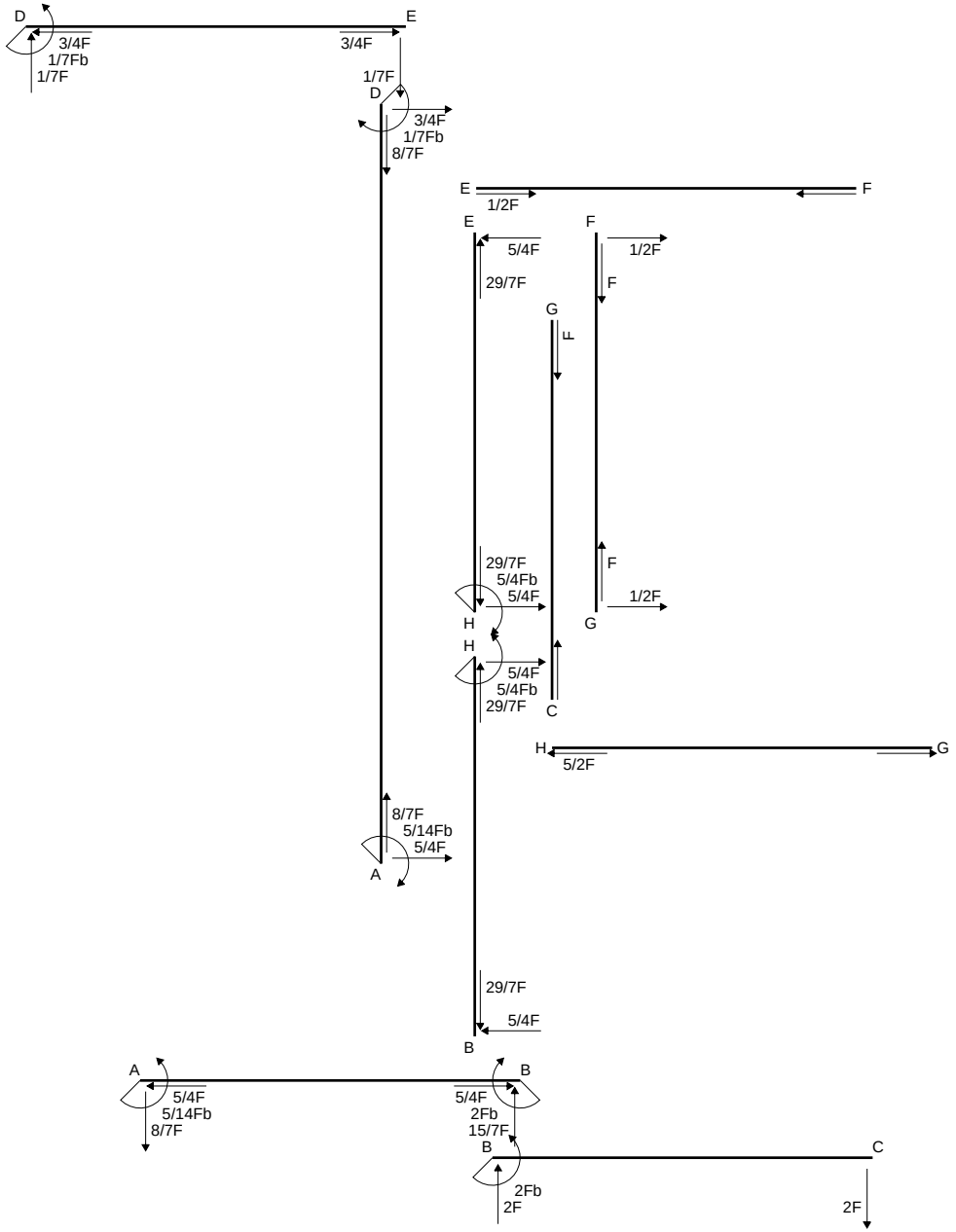
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

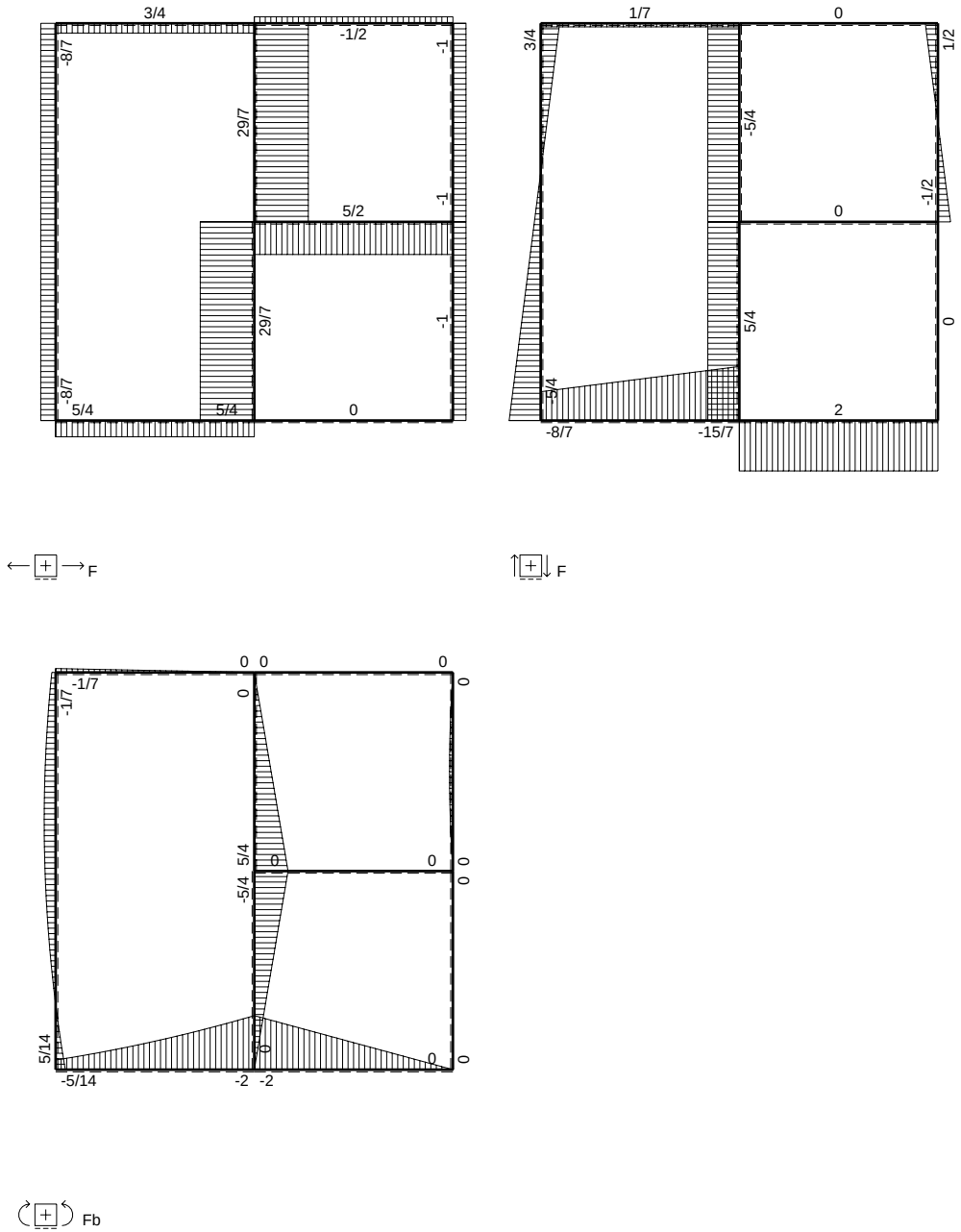
$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

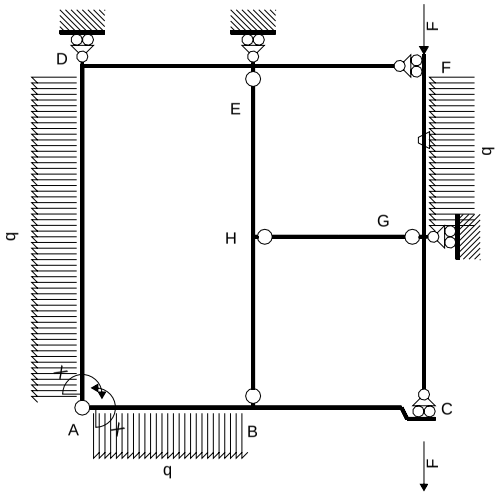
$= \left(-1/6\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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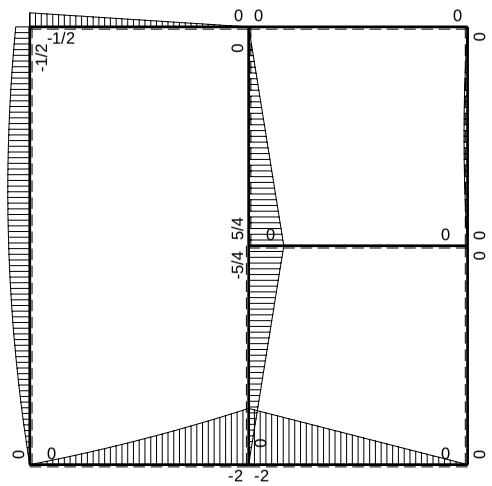
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D
E
3/4F
1/7Fb
1/7F
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1/7F
D
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1/7Fb
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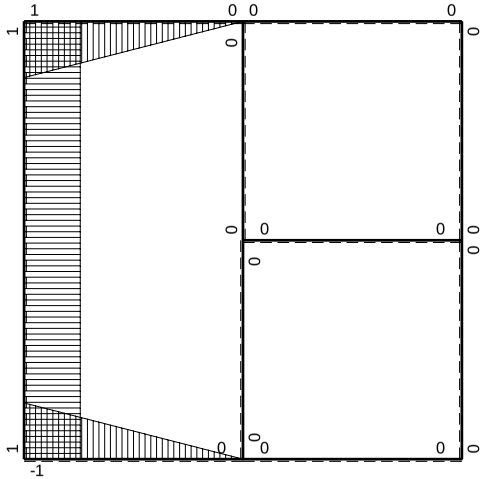

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1/2
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5/4
0
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-1/7
0
0
0
0
5/4
0
0
0
-5/4
0 0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	0	0	0	0	0+0	0
EH b	0	$-5/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\,b - 1/3\,b - 1/8\,b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\,x/b + 1/2\,x^2/b^2)\ Fb\ 1/EJ\,dx = [-5/8\,x^2/b + 1/6\,x^3/b^2\,]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\,b + 4/3\,b)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\,x/b + 1/2\,x^2/b^2)\ Fb\ 1/EJ\,dx = [-1/2\,x - 3/8\,x^2/b + 1/6\,x^3/b^2\,]_o^{2b}\ Fb\ 1/EJ$

$= (-\,b - 3/2\,b + 4/3\,b)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\,x^2/b^2)\ Fb\ 1/EJ\,dx = [-1/2\,x + 1/2\,x^2/b - 1/6\,x^3/b^2\,]_o^b\ Fb\ 1/EJ$

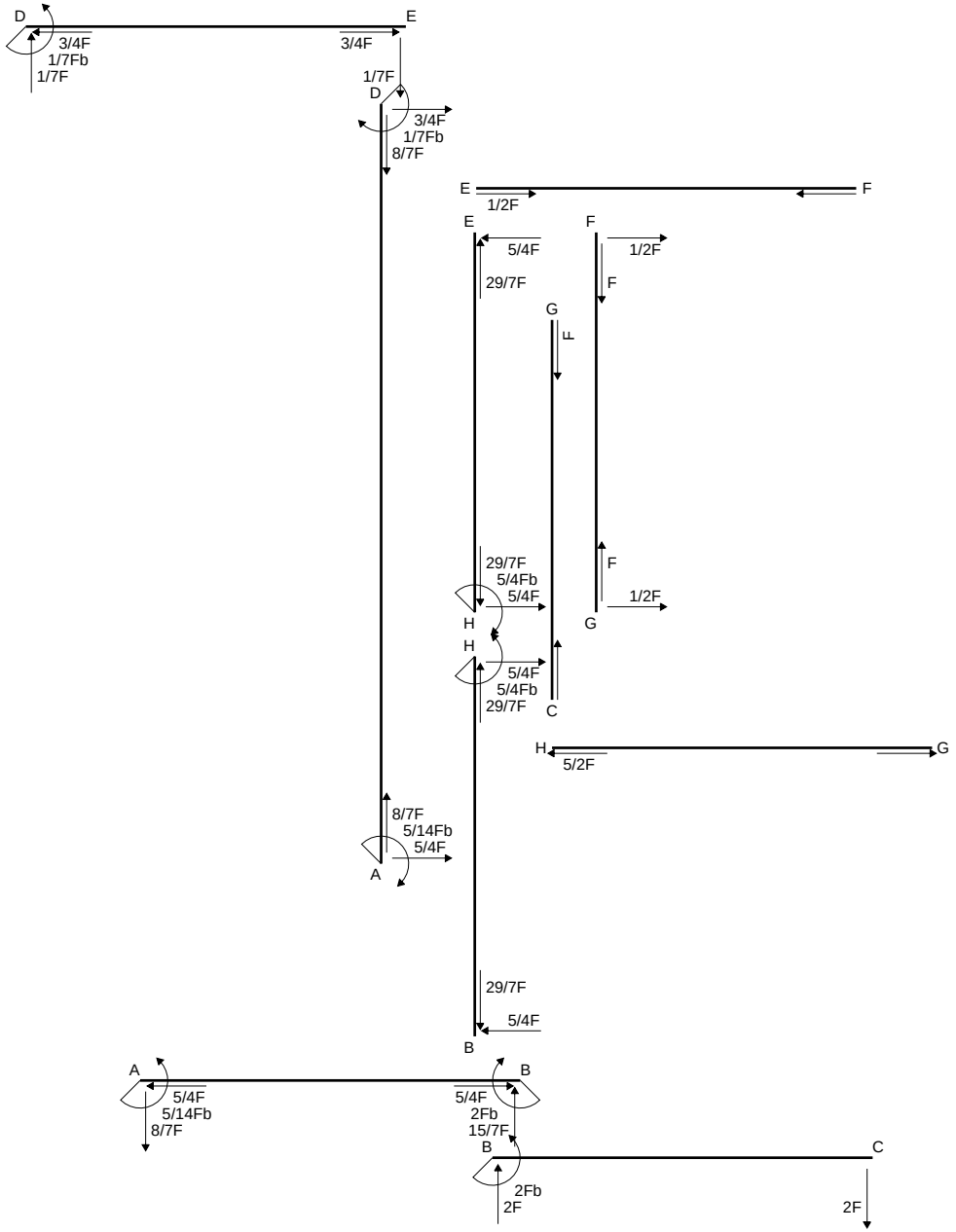
$= (-1/2\,b + 1/2\,b - 1/6\,b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

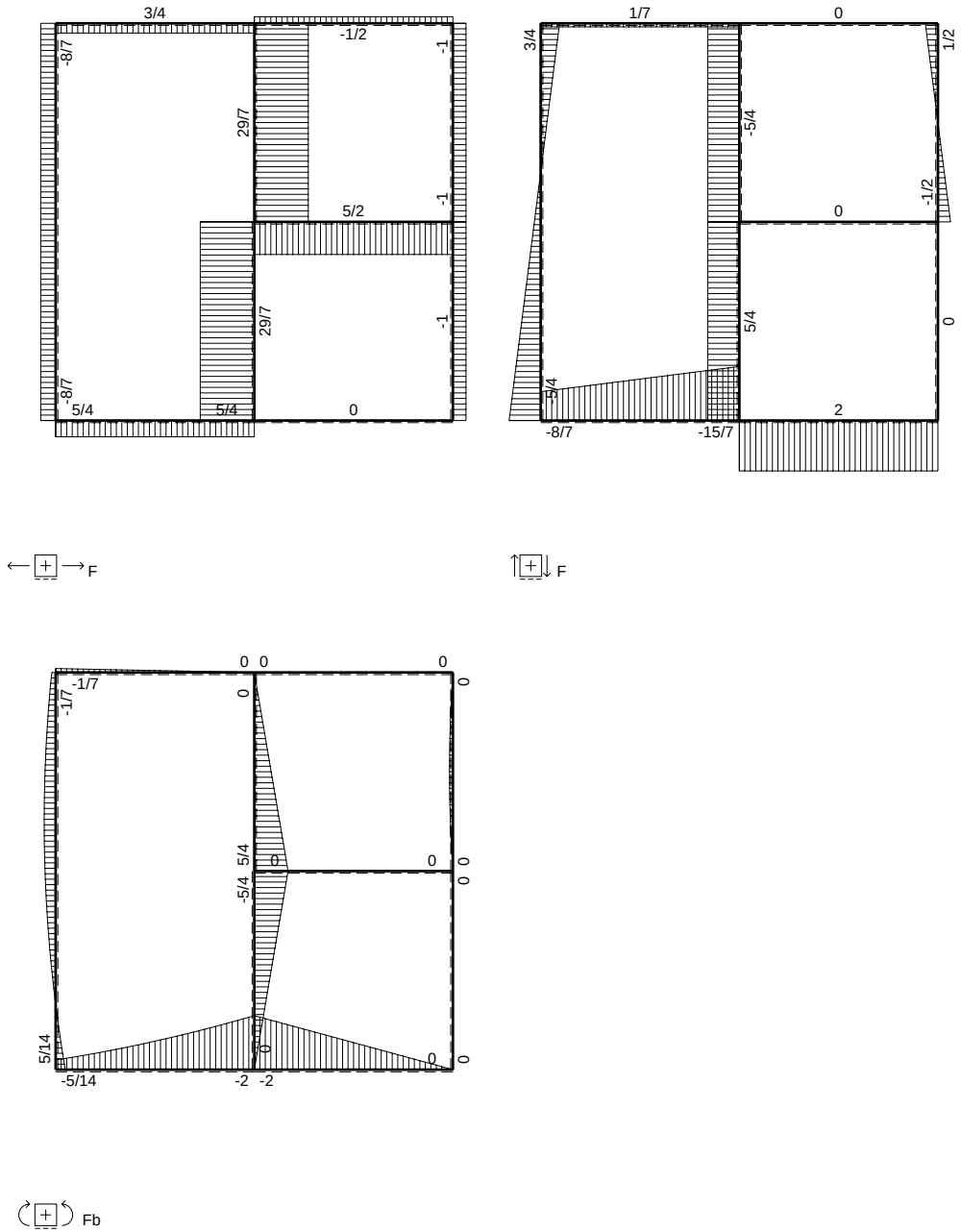
$L^{Xo}_{ED} = \int_o^b (-1/2\,x^2/b^2)\ Fb\ 1/EJ\,dx = [-1/6\,x^3/b^2\,]_o^b\ Fb\ 1/EJ$

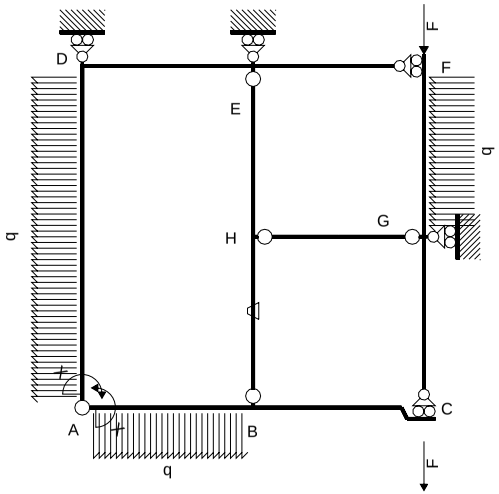
$= (-1/6\,b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 981992 ESΣ03.xxxx.134

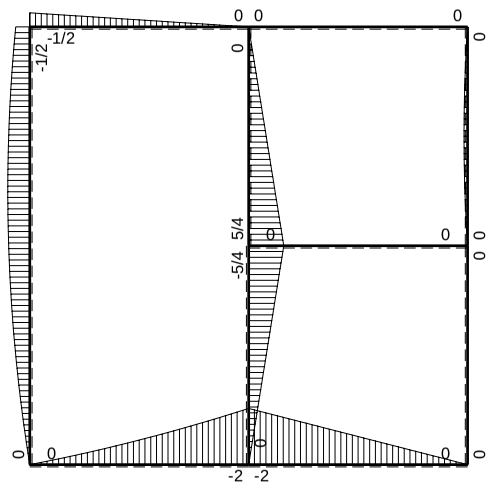
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
8/7F
E
F
1/2F
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F
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C
2Fb
2F
2F

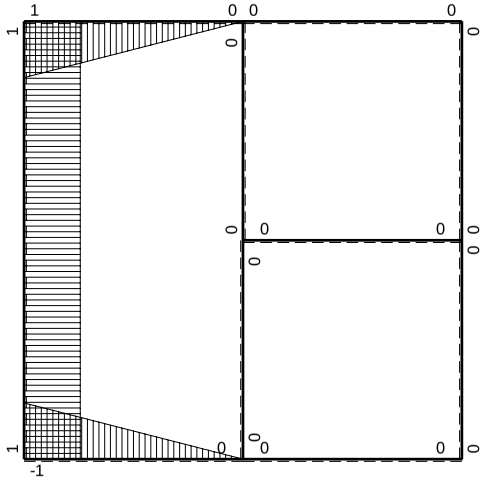

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-8/7
-15/7
2
F
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-1/7
0
0
0
0
5/4
0
0
0
-5/4
0
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-5/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
BH b	0	5/4Fx	Fb/EJ	0	0	0		
HE b	0	5/4Fb-5/4Fx	0	0	0	0	0+0	0
EH b	0	-5/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\ b + 1/8\ b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\,]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\,]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

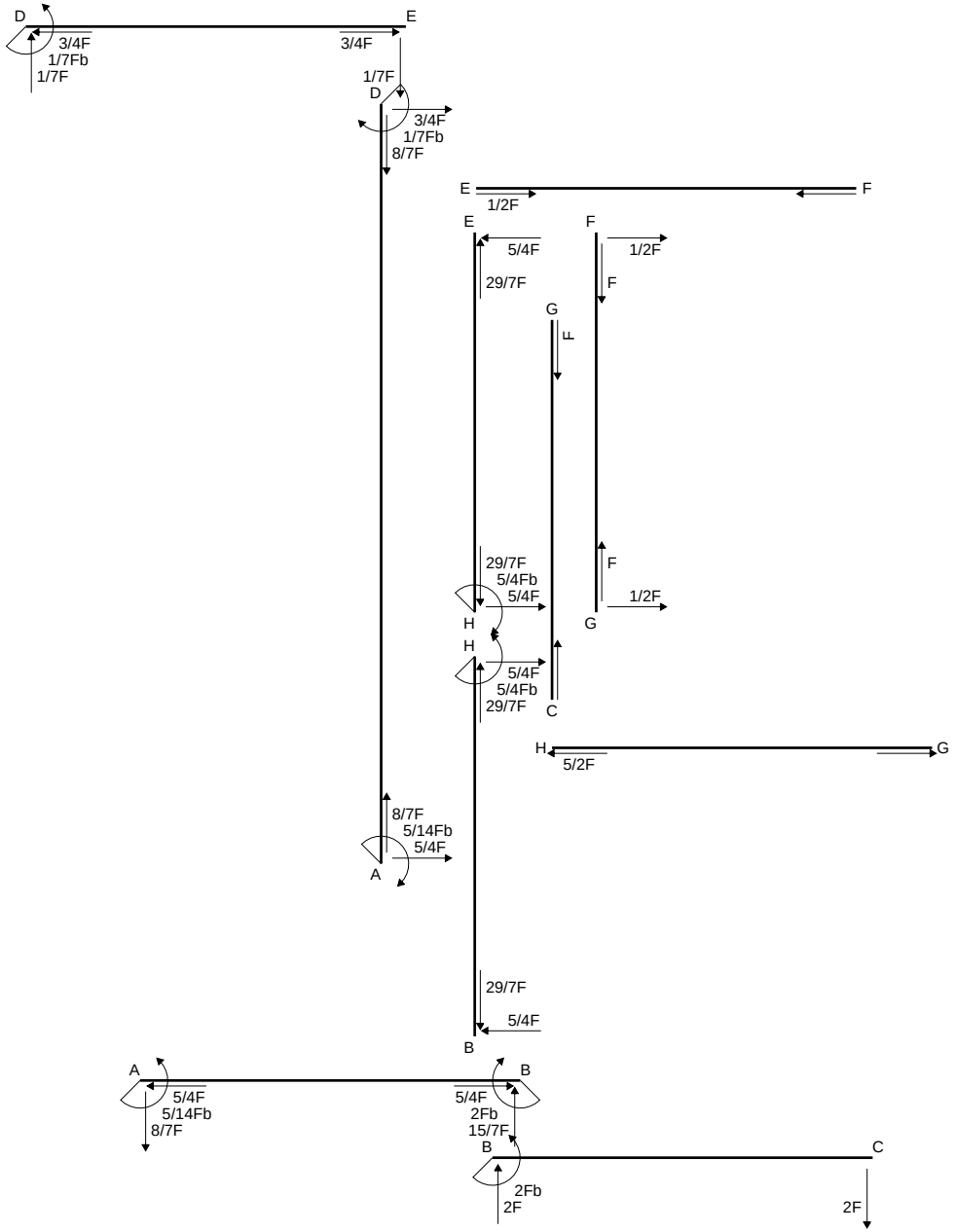
$= (-1/2\ b + 1/2\ b - 1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

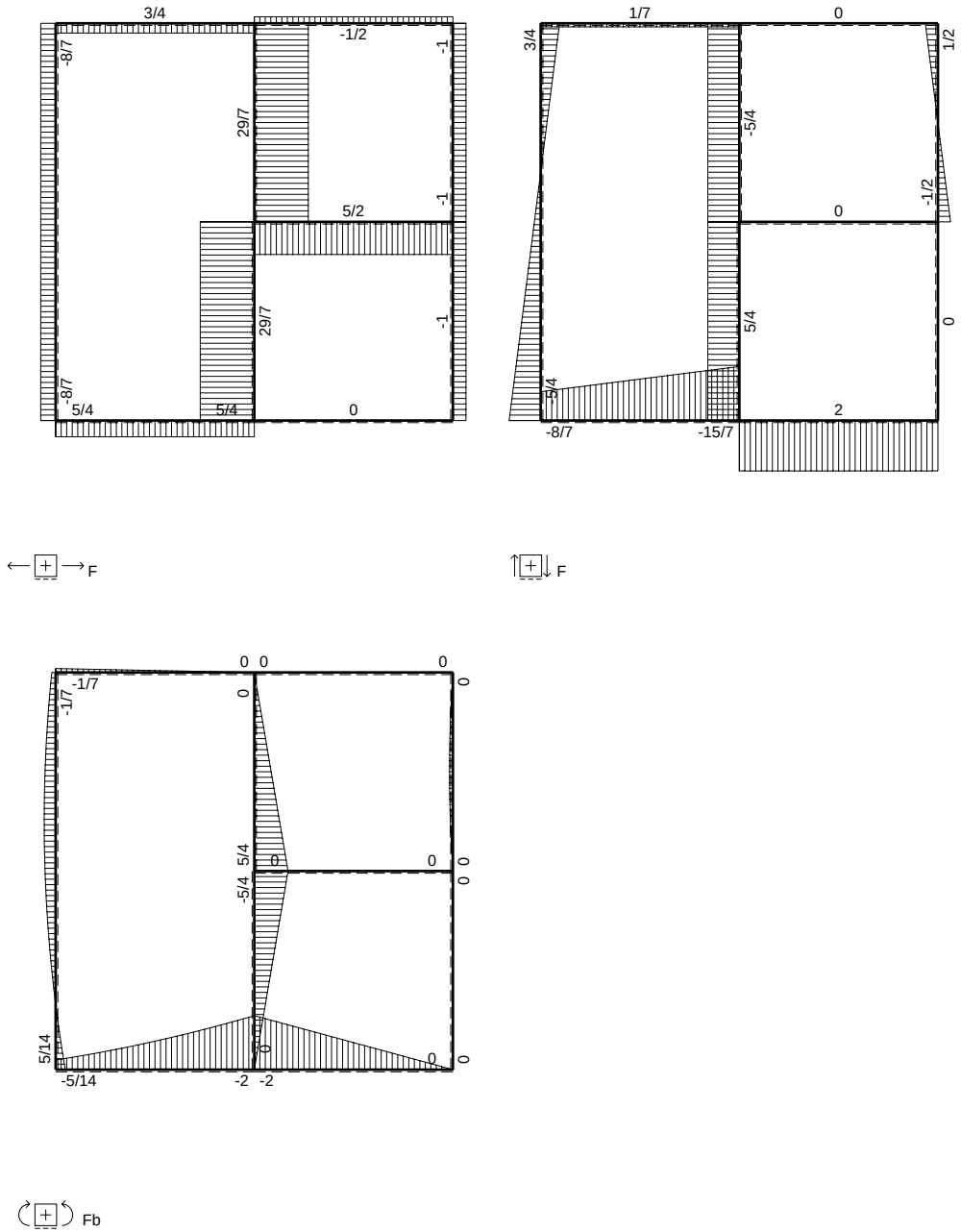
$L_{ED}^{Xo} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

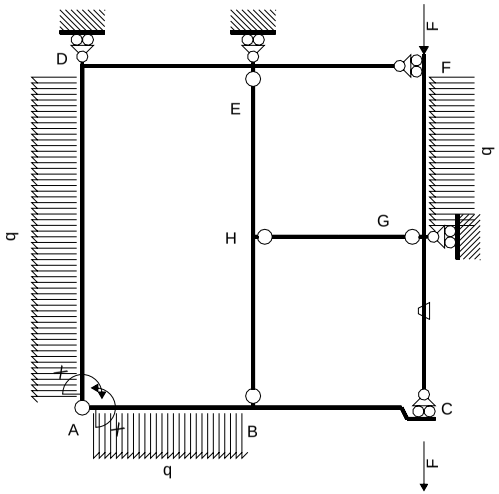
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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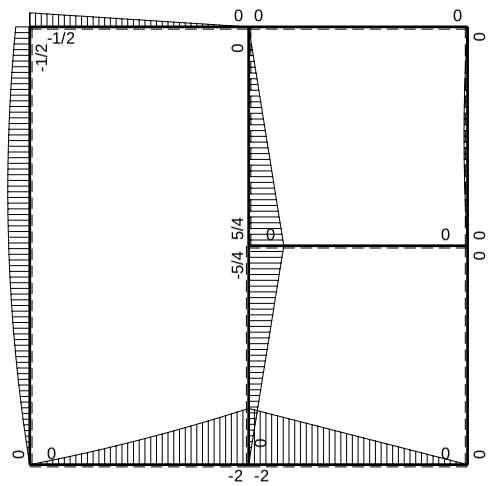
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1/7Fb
1/7F
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1/7F
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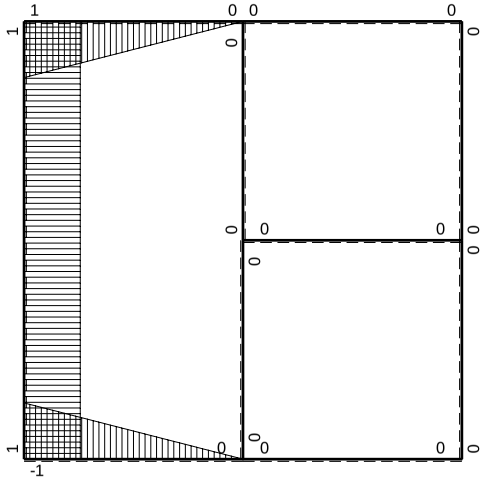

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0
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-15/7
F
F
0 0
0
-1/7
-1/7
0
0
0
5/4
0
0
0 0
-5/4
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 991200 ESΣ03.xxxx.136

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PROCEDIMENTO E RISULTATI 991200 ESΣ03.xxxx.136

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-5/4Fb+5/4Fx	0	0	0	0	0+0	0
BH b	0	5/4Fx	0	0	0	0		
HE b	0	5/4Fb-5/4Fx	0	0	0	0	0+0	0
EH b	0	-5/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

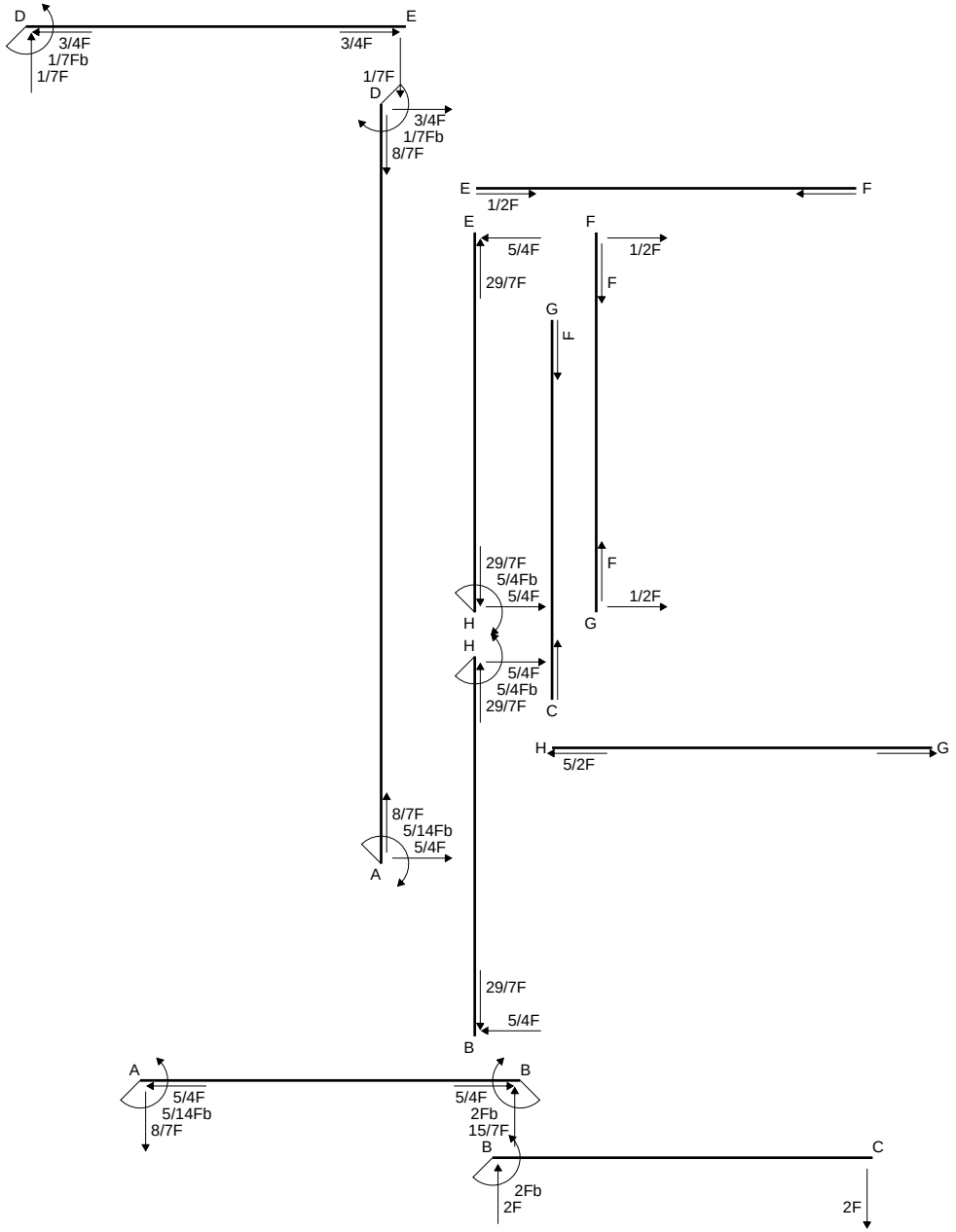
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

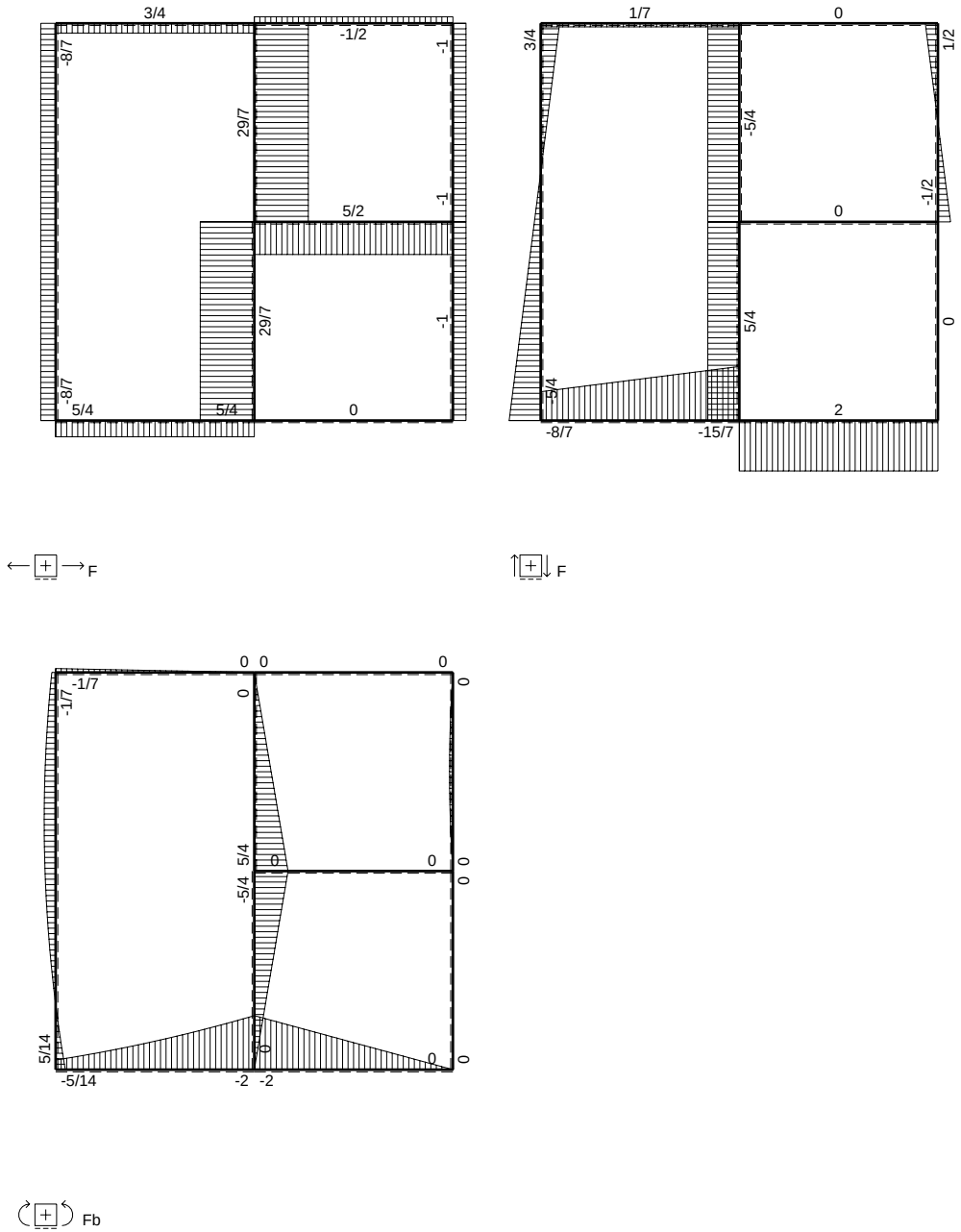
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

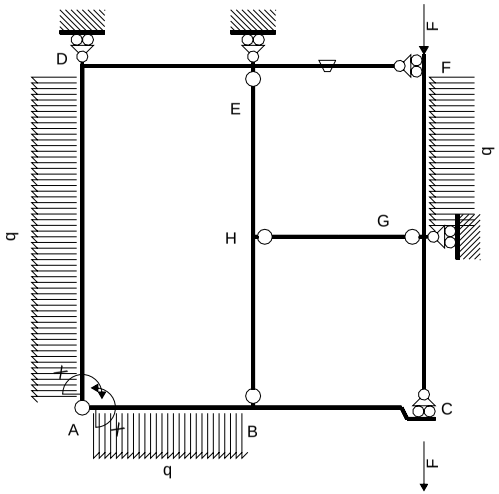
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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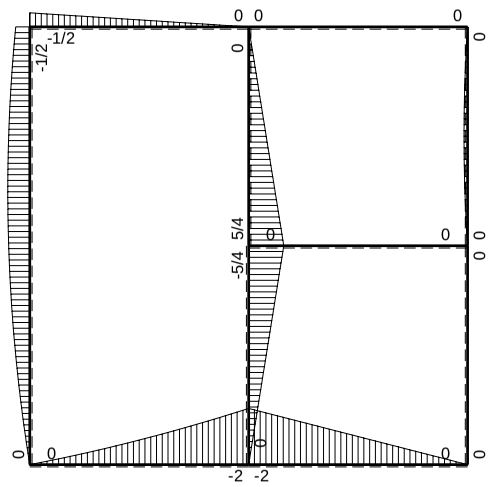
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
8/7F
E
F
1/2F
E
F
5/4F
1/2F
29/7F
F
G
F
29/7F
5/4Fb
5/4F
F
1/2F
H
G
H
5/4F
5/4Fb
29/7F
C
H
G
5/2F
8/7F
5/14Fb
5/4F
A
29/7F
5/4F
B
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B
C
2Fb
2F
2F

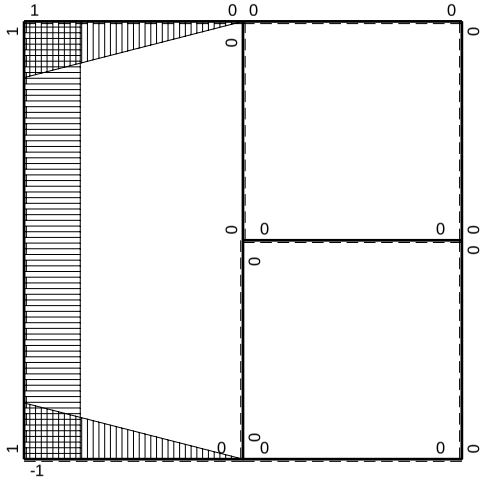

3/4
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-1/2
-1
29/7
5/2
-1
29/7
-1
-8/7
5/4
5/4
0
F
1/7
0
3/4
1/2
-5/4
0
-1/2
5/4
0
5/4
-8/7
-15/7
2
F
0 0
0
-1/7
-1/7
0
0
0
0
0
5/4
0
0
-5/4
0 0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	-Fb/EJ	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-5/4Fb+5/4Fx	0	0	0	0	0+0	0
BH b	0	5/4Fx	0	0	0	0		
HE b	0	5/4Fb-5/4Fx	0	0	0	0	0+0	0
EH b	0	-5/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

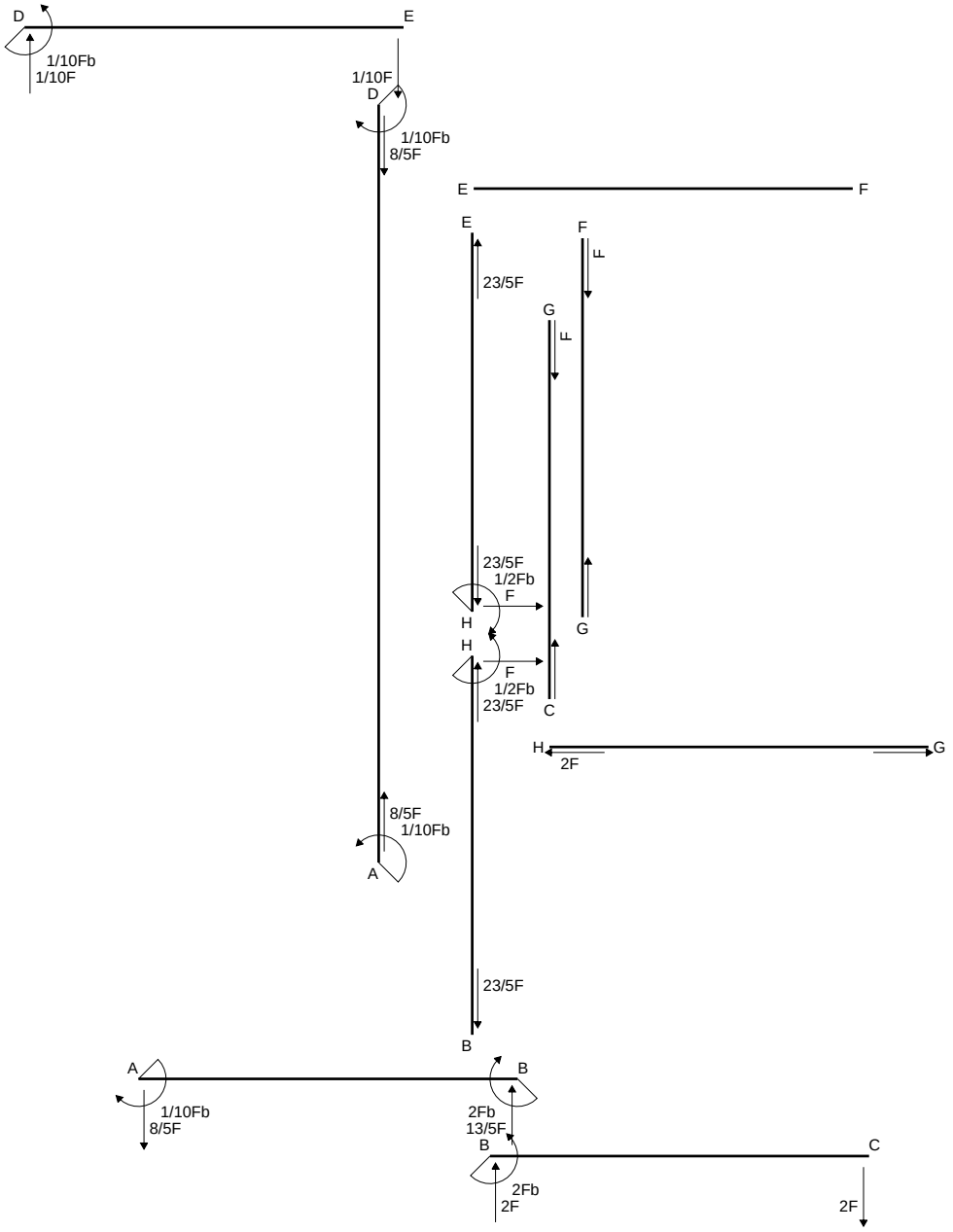
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

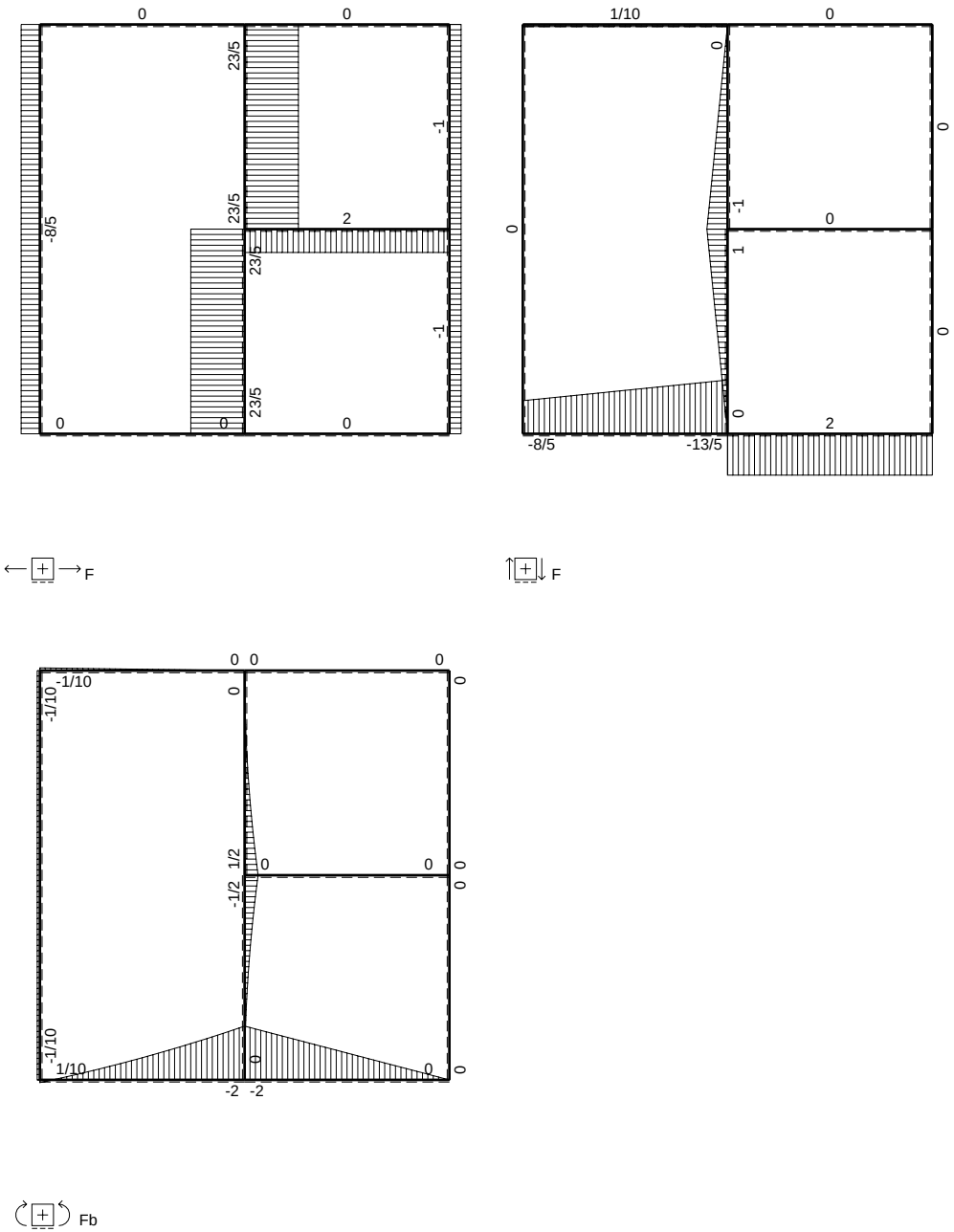
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

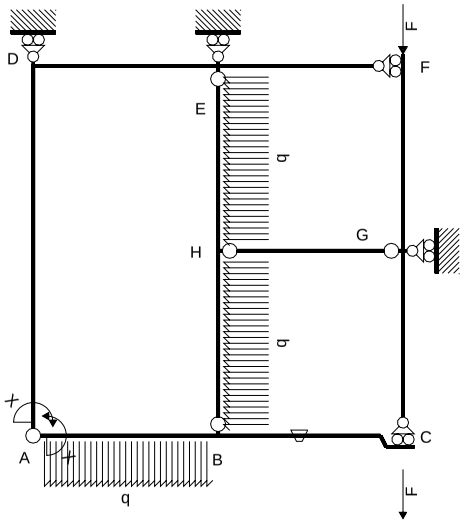
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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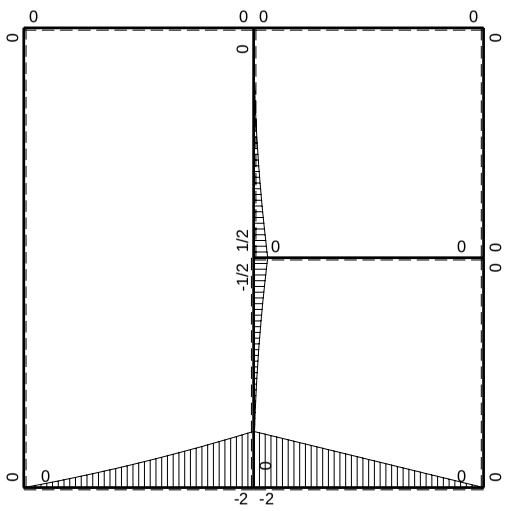
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D
E
1/10Fb
1/10F
1/10F
D
1/10Fb
8/5F
E
F
E
F
F
23/5F
G
F
23/5F
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F
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F
1/2Fb
23/5F
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B
C
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2F
2F

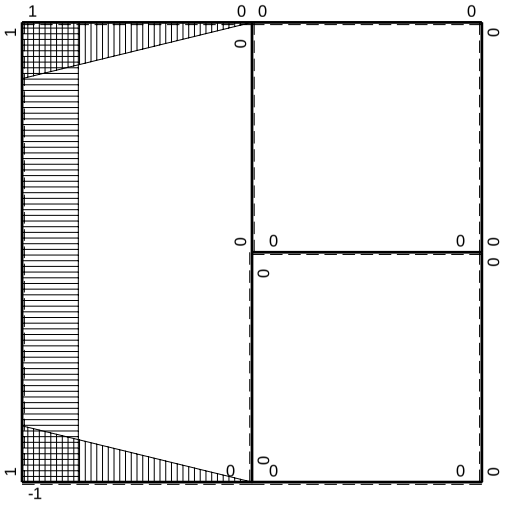

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-1
0
1
0
0
0
2
-8/5
-13/5
F
F
0 0
0
-1/10
-1/10
0
0
0
1/2
0
0
-1/2
0 0
0
-1/10
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240010 ESΣ03.xxxx.138

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PROCEDIMENTO E RISULTATI 240010 ESΣ03.xxxx.138

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

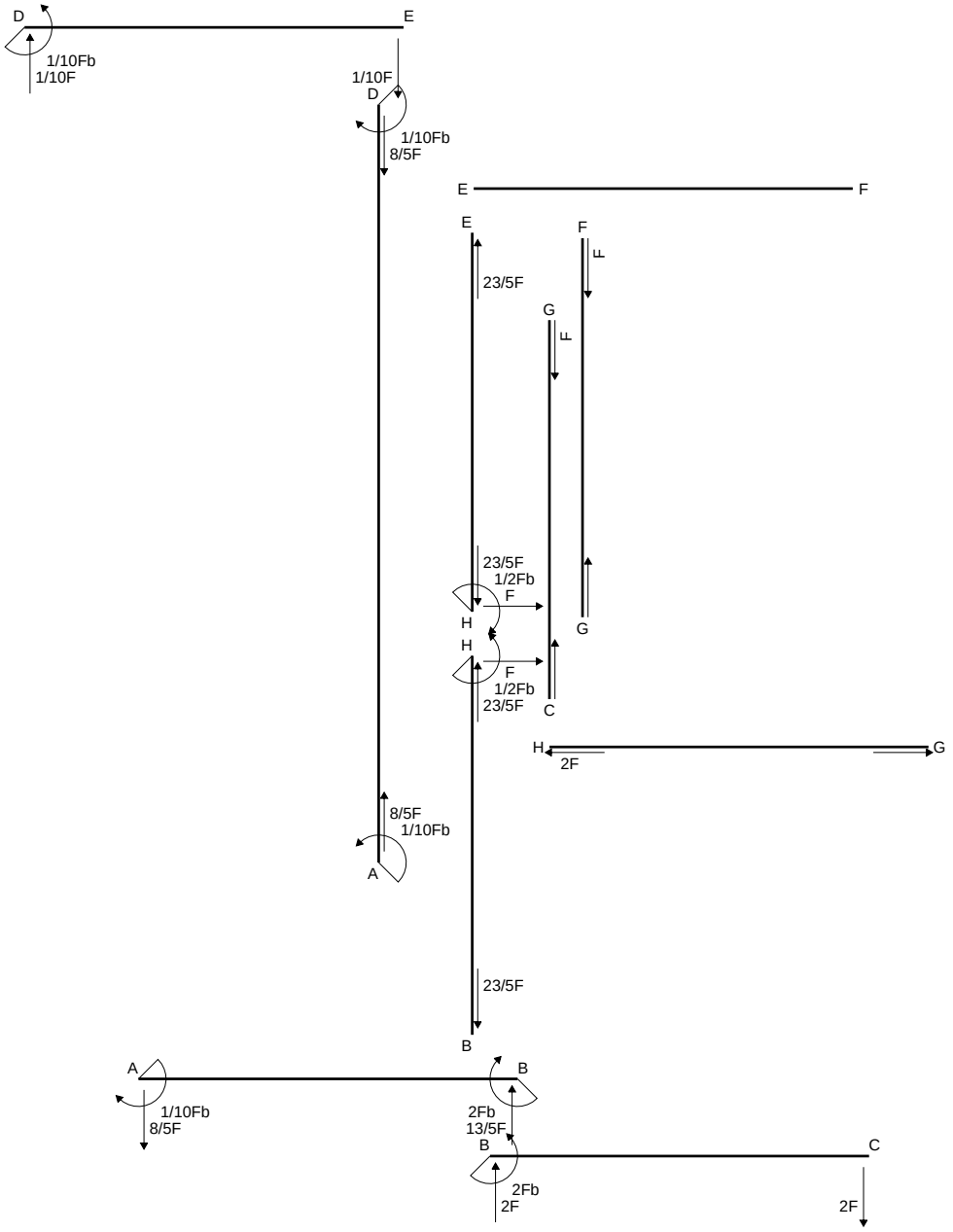
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

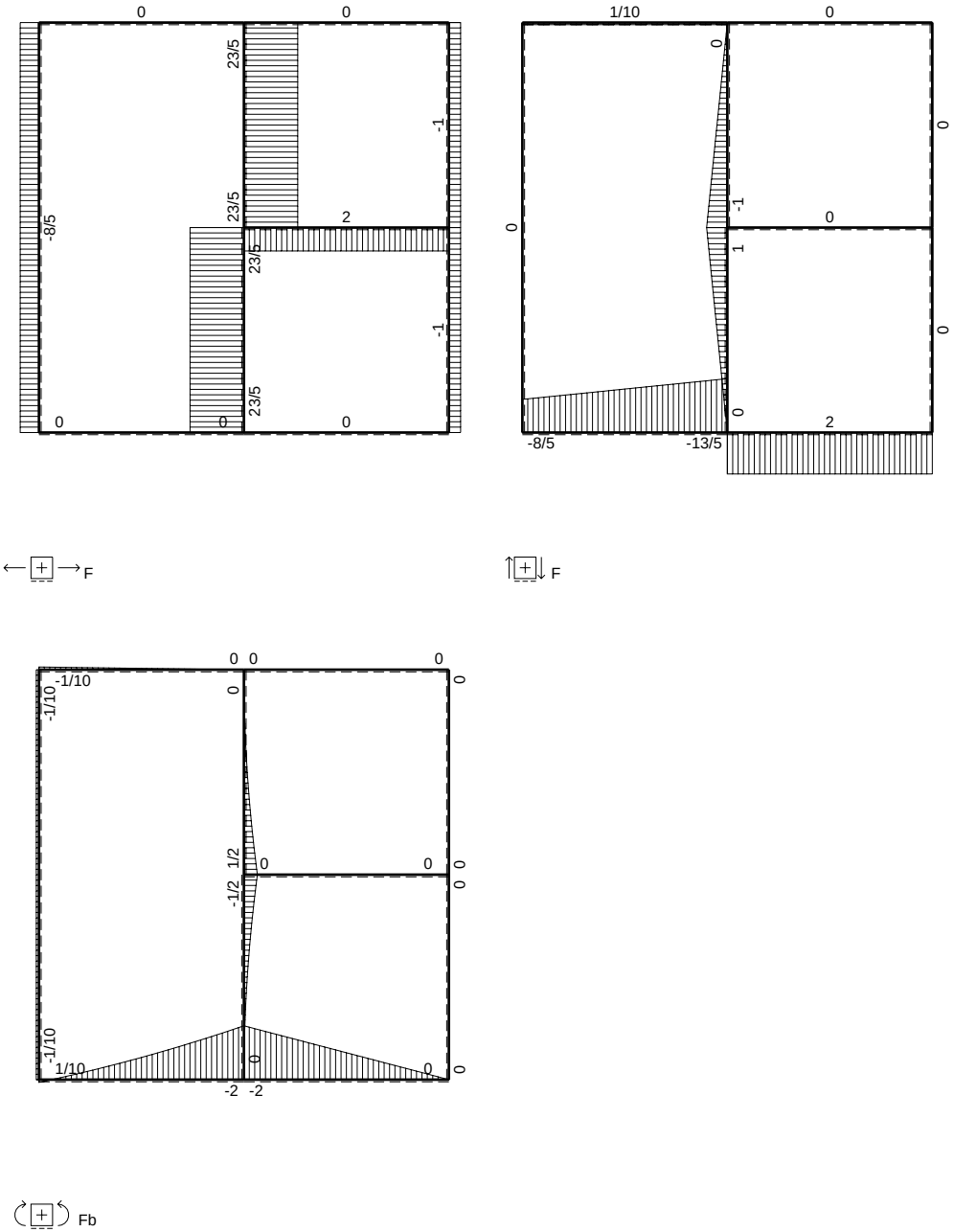
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

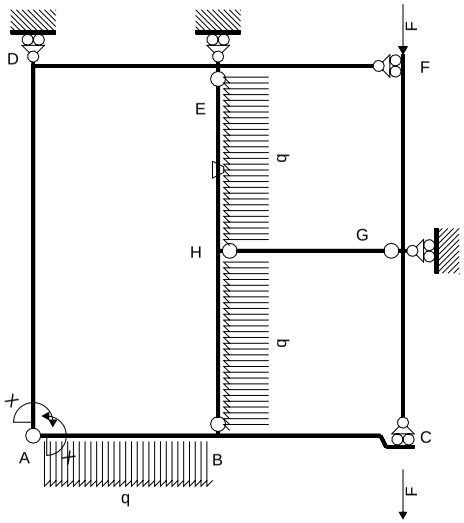
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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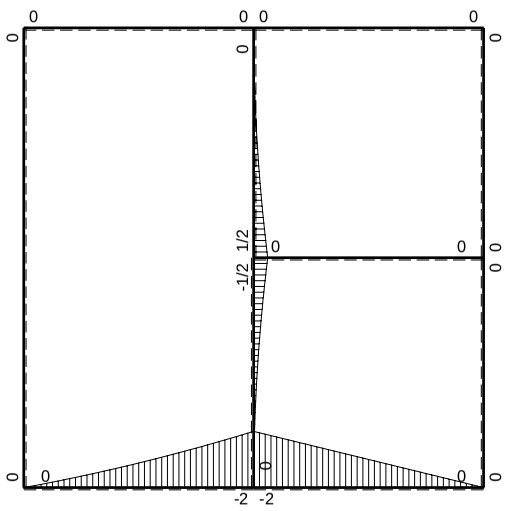
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D
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1/10Fb
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1/10F
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1/10Fb
8/5F
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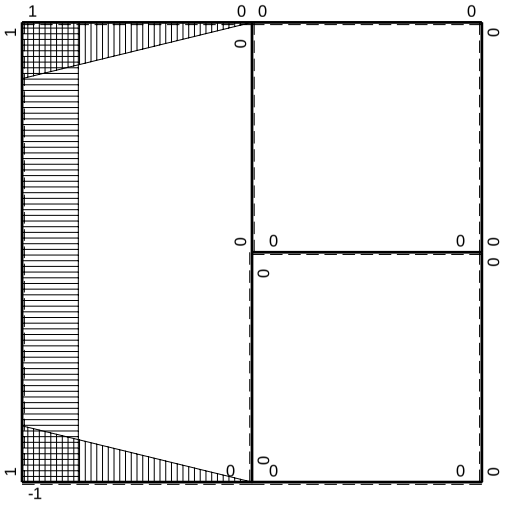

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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 241207 ESΣ03.xxxx.139

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PROCEDIMENTO E RISULTATI 241207 ESΣ03.xxxx.139

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

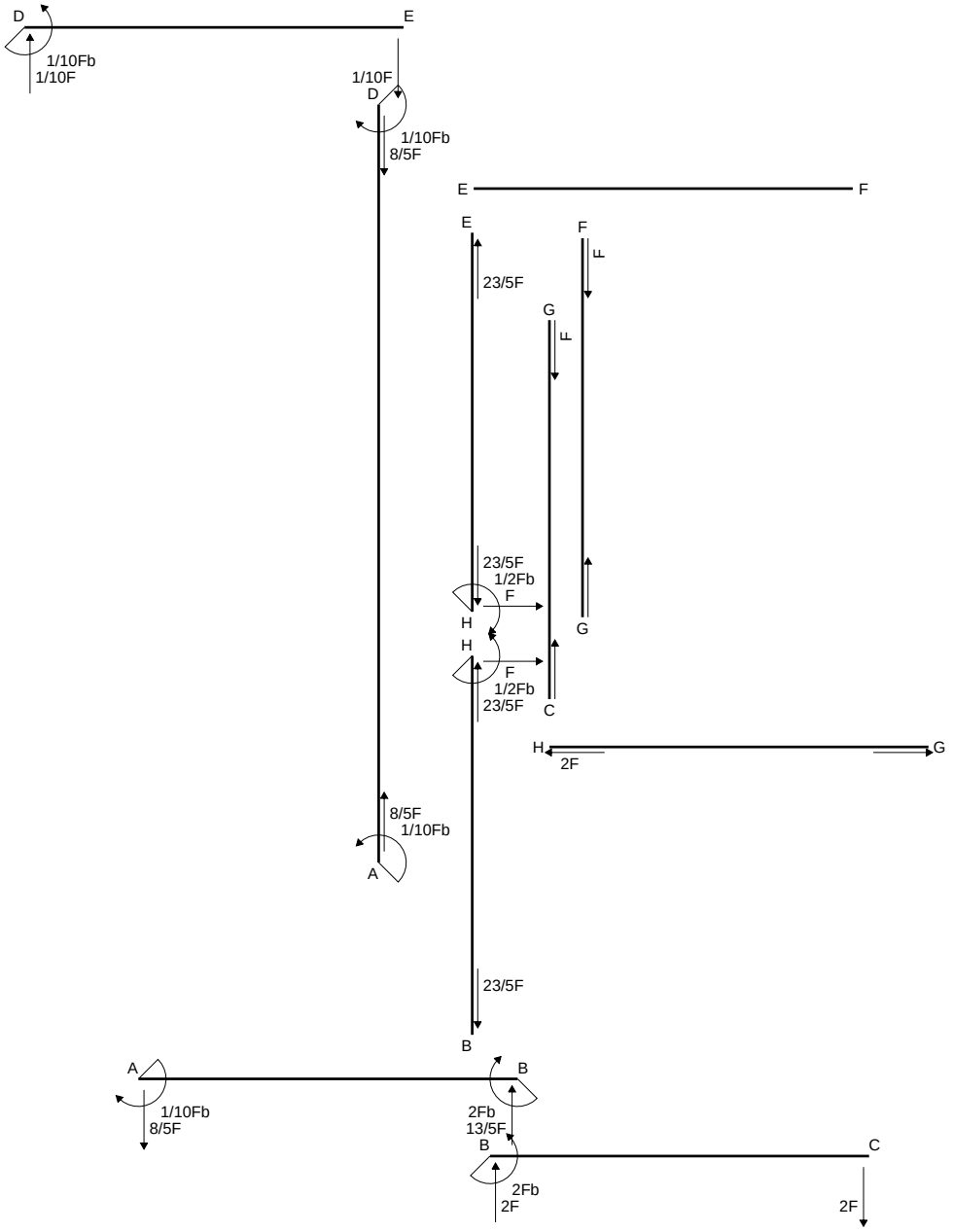
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

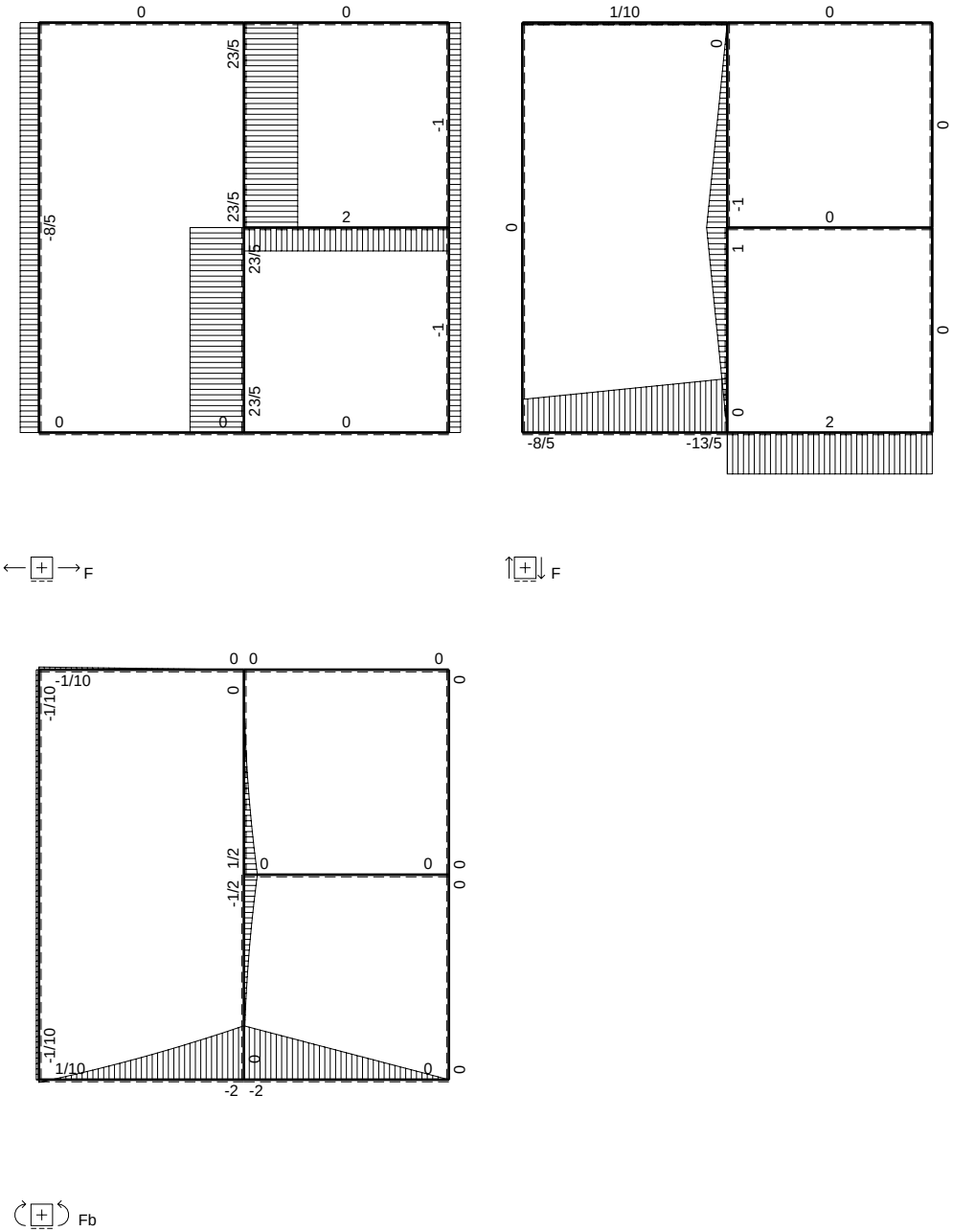
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

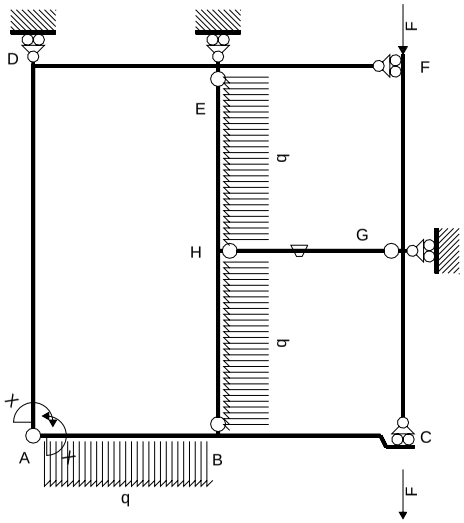
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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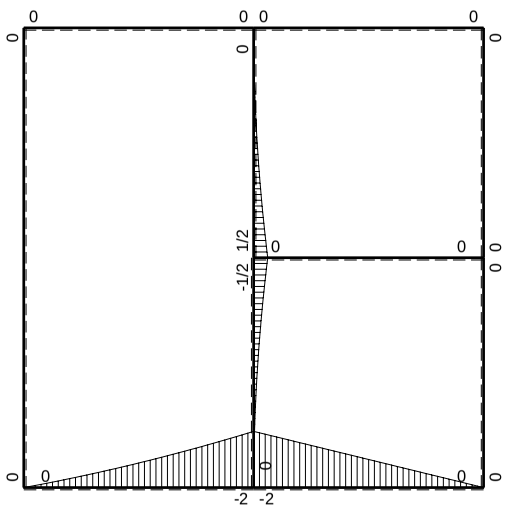
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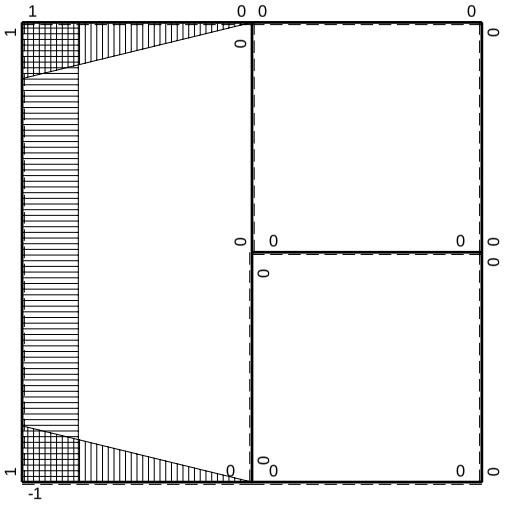

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0
-1/2
0 0
0 0
-1/10
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982534 ESΣ03.xxxx.140

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PROCEDIMENTO E RISULTATI 982534 ESΣ03.xxxx.140

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

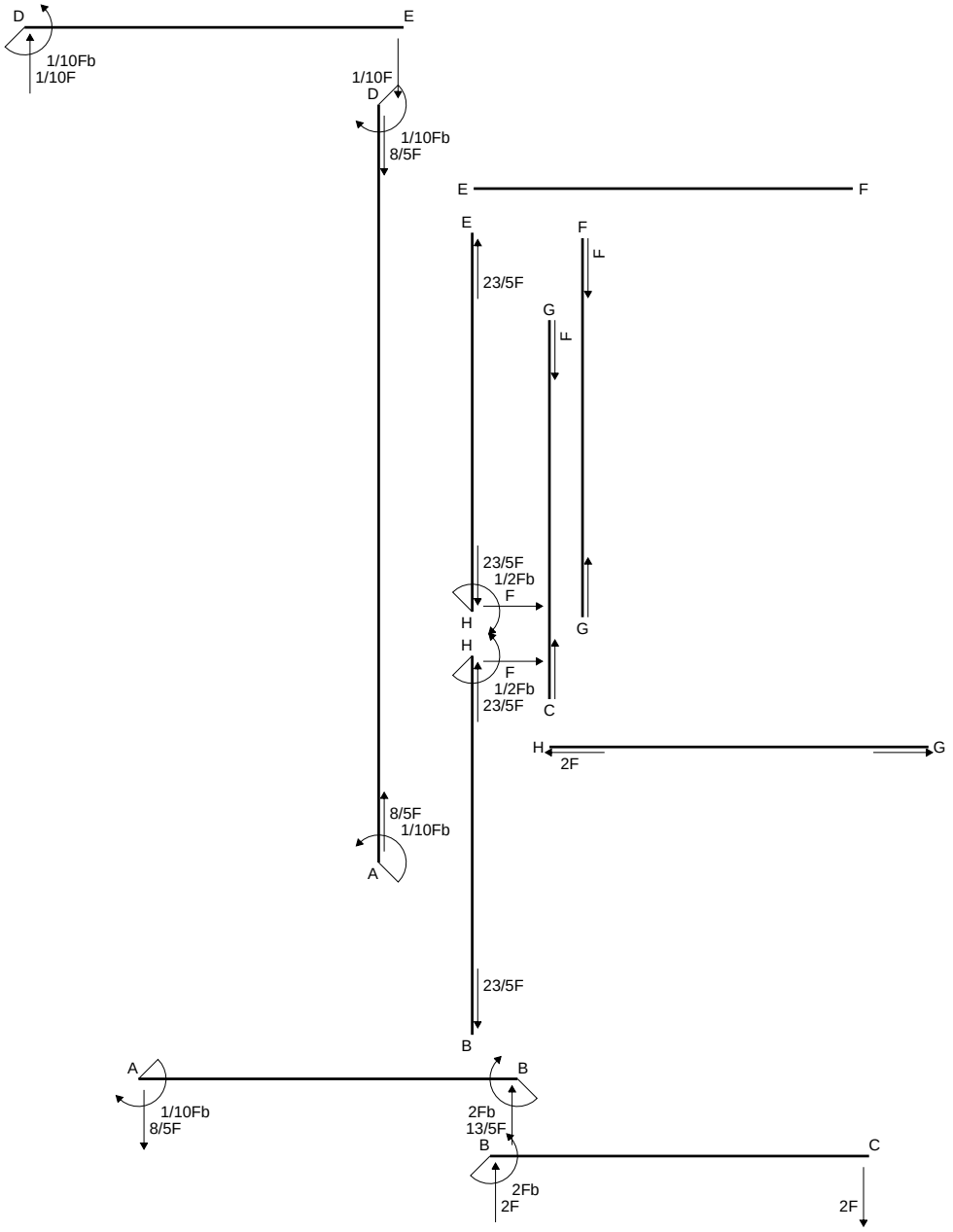
$L_{BA}^{Xo} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

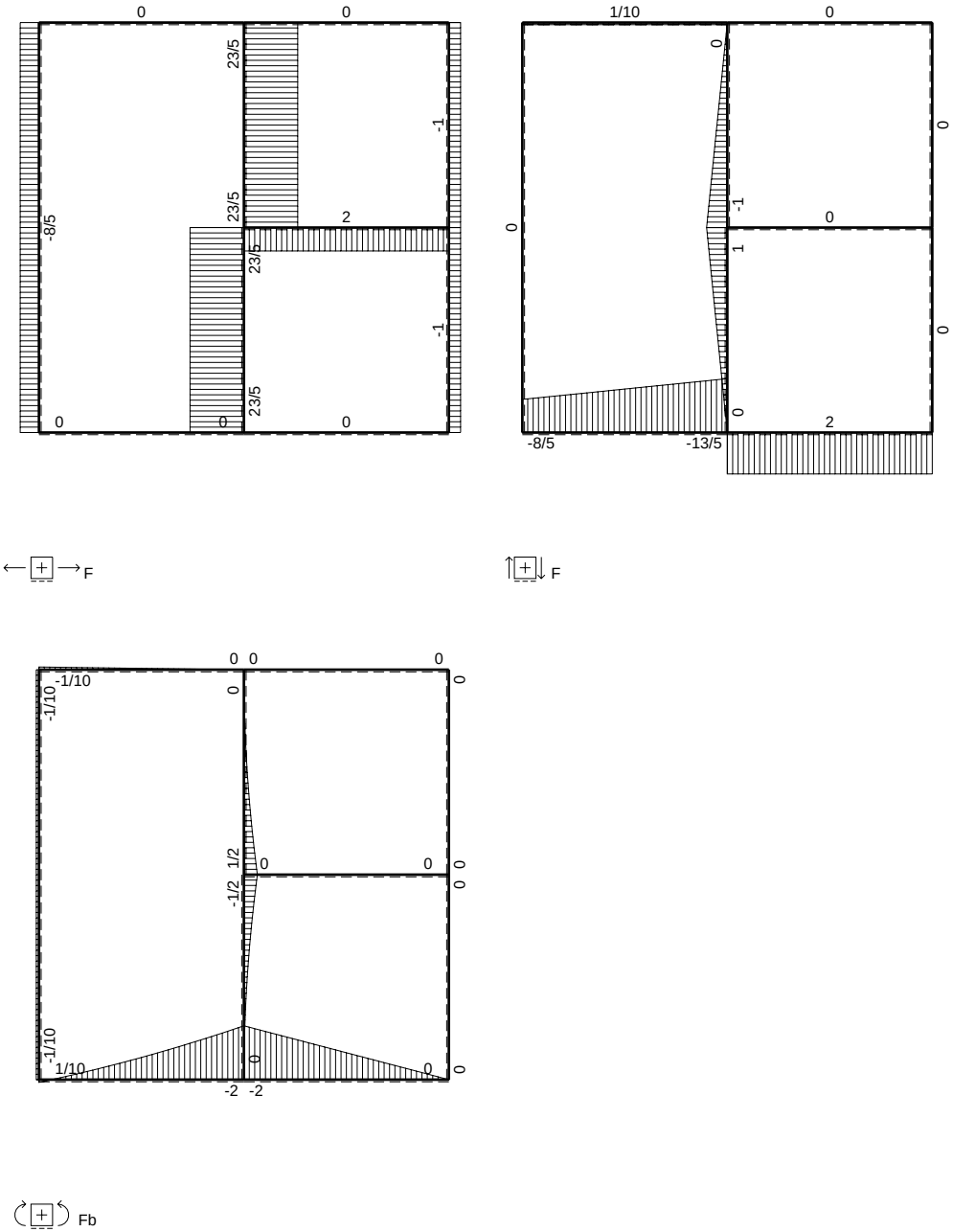
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

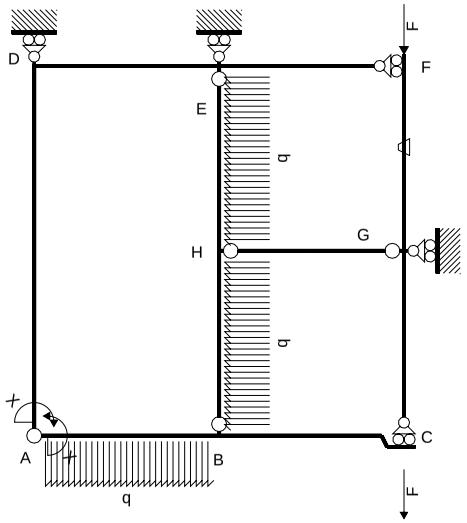
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

ESΣ03.xxxx.140

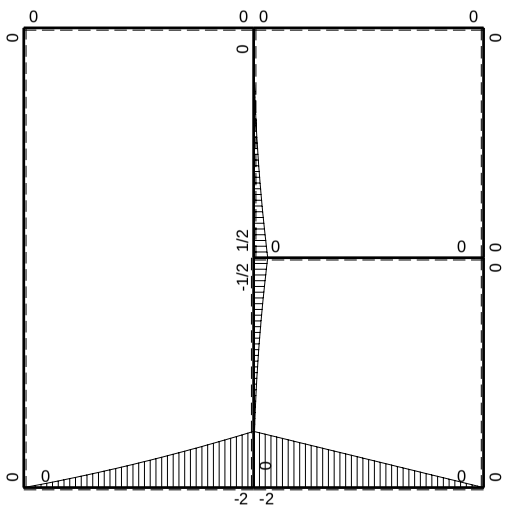
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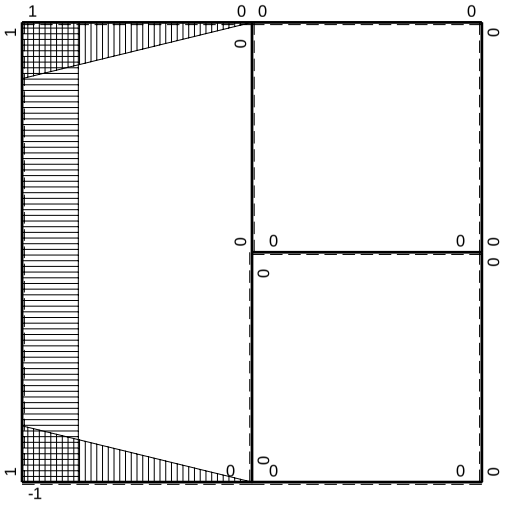

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0
-1/2
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-1/10
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0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236561 ESΣ03.xxxx.141

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PROCEDIMENTO E RISULTATI 236561 ESΣ03.xxxx.141

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (1/3\,b)\ \ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\,b - 1/3\,b - 1/8\,b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

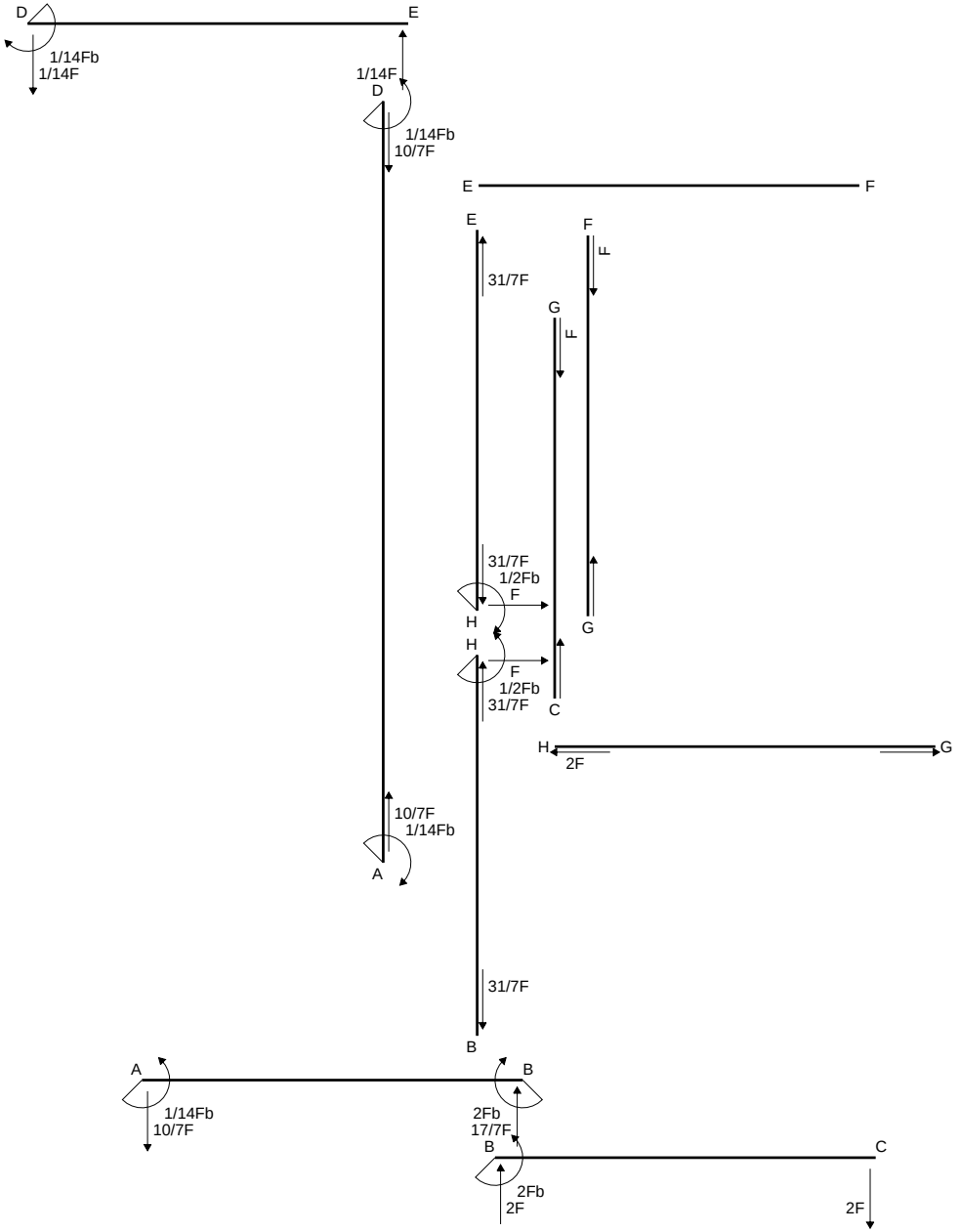
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

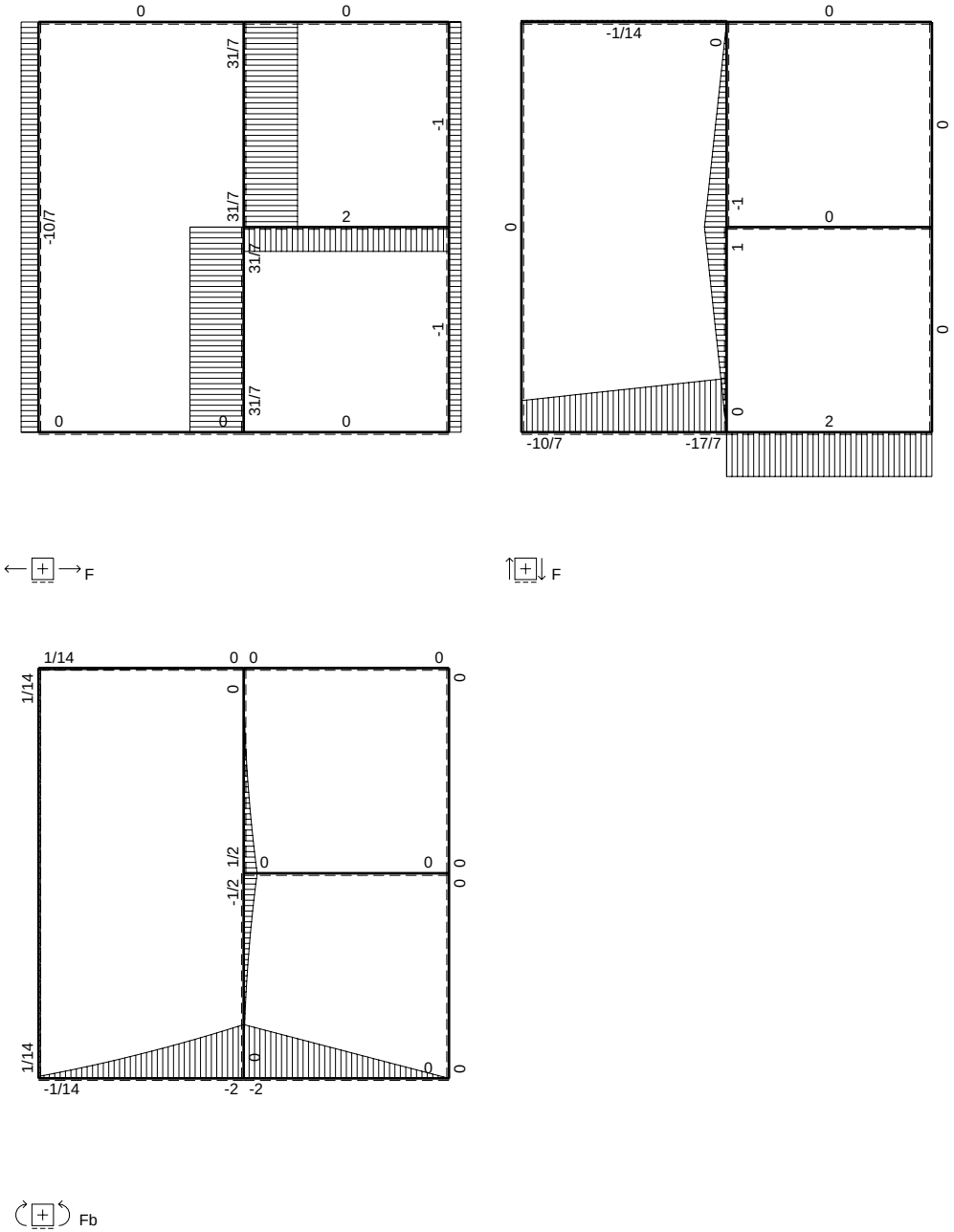
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

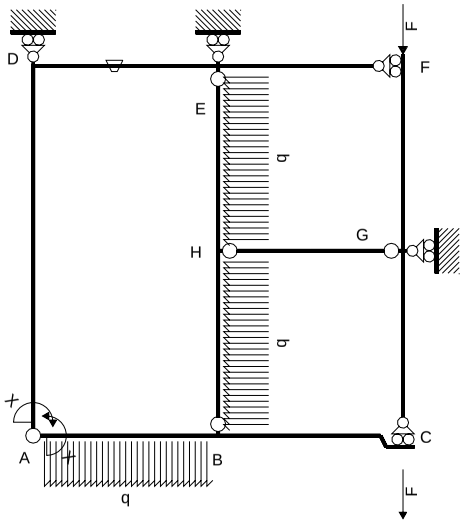
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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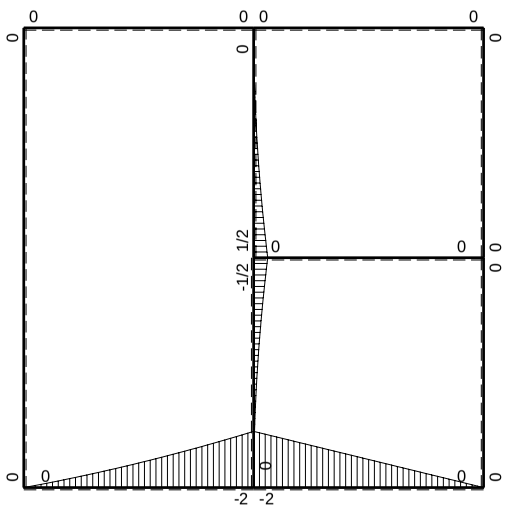
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D
E
1/14Fb
1/14F
1/14F
D
1/14Fb
10/7F
E
F
E
F
31/7F
F
G
F
31/7F
1/2Fb
F
H
H
F
1/2Fb
31/7F
C
H
G
2F
10/7F
1/14Fb
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B
C
2Fb
2F
2F

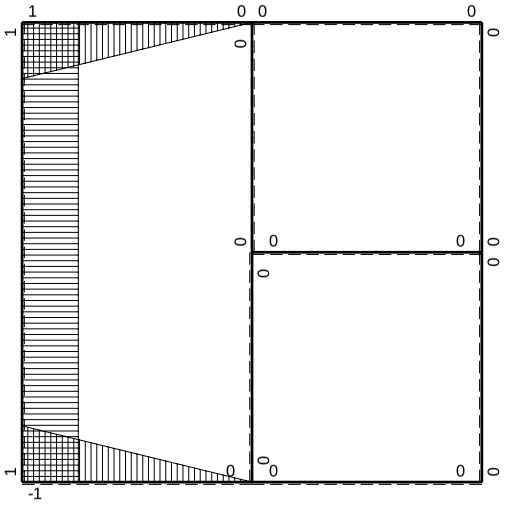

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31/7
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-10/7
31/7
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31/7
-1
31/7
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0
0
-1/14
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1
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0
0
2
-10/7
-17/7
F
F
1/14
0 0
0
0
0
1/14
1/2
0
0
-1/2
0 0
0
1/14
0
0
-1/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238186 ESΣ03.xxxx.142

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	-Fb/EJ	0	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$	$(0-1/2)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	0	Fb/EJ	0	-Fx/EJ	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238186 ESΣ03.xxxx.142
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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + x/b\ )\ \theta\ \ dx = [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

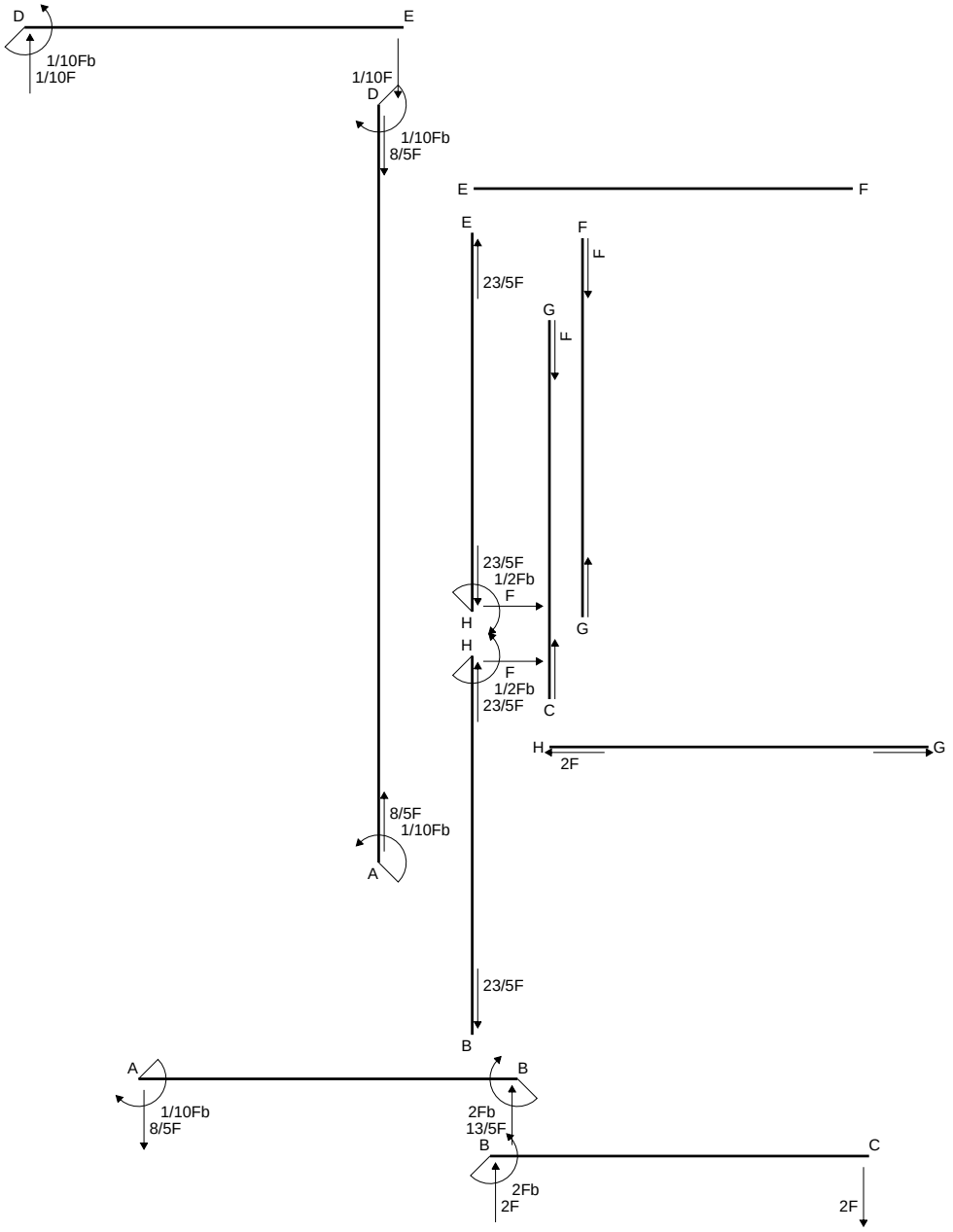
$= (-\ b + 1/2\ b\ )\ \theta\ \ = -1/2\ \ Fb^2/EJ$

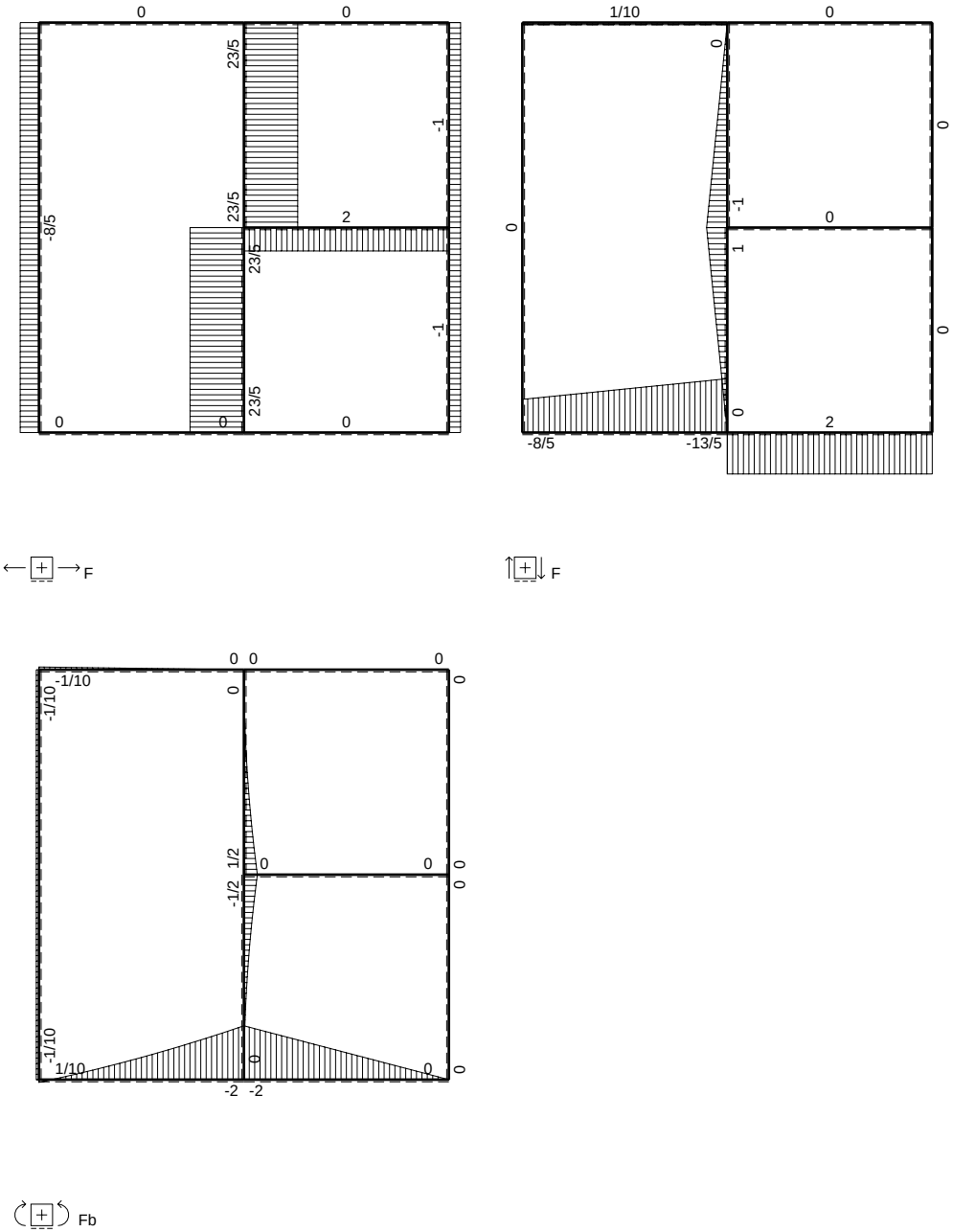
$L^{Xo}_{ED} = \int_o^b (\ x/b\ )\ \theta\ \ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

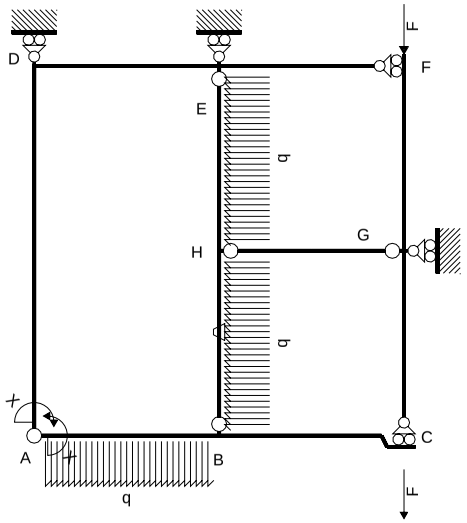
$= (1/2\ b\ )\ \theta\ \ = -1/2\ \ Fb^2/EJ$

ESΣ03.xxxx.142

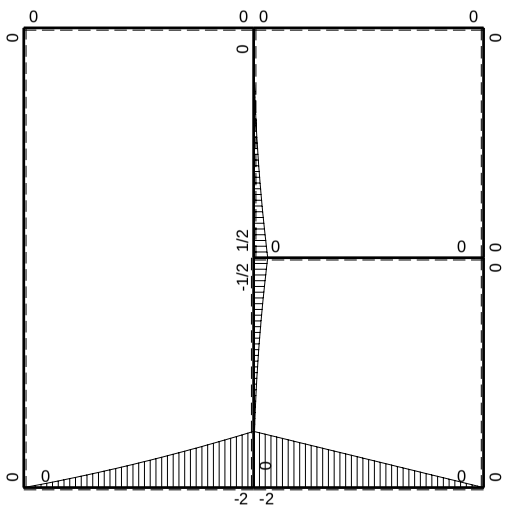
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D
E
1/10Fb
1/10F
1/10F
D
1/10Fb
8/5F
E
F
E
F
F
23/5F
G
F
23/5F
1/2Fb
F
H
G
H
F
1/2Fb
23/5F
C
H
G
2F
8/5F
1/10Fb
A
23/5F
B
A
B
1/10Fb
8/5F
2Fb
13/5F
B
C
2Fb
2F
2F

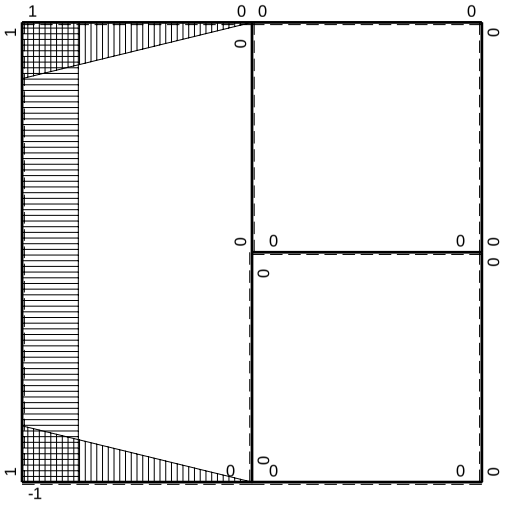

0
0
23/5
-1
-8/5
23/5
2
23/5
-1
23/5
0
0
0
1/10
0
0
0
-1
0
1
0
0
0
2
-8/5
-13/5
F
F
0 0
0
-1/10
-1/10
0
0
1/2
0
0
0 0
-1/2
0
0
-1/10
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237118 ESΣ03.xxxx.143

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PROCEDIMENTO E RISULTATI 237118 ESΣ03.xxxx.143

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

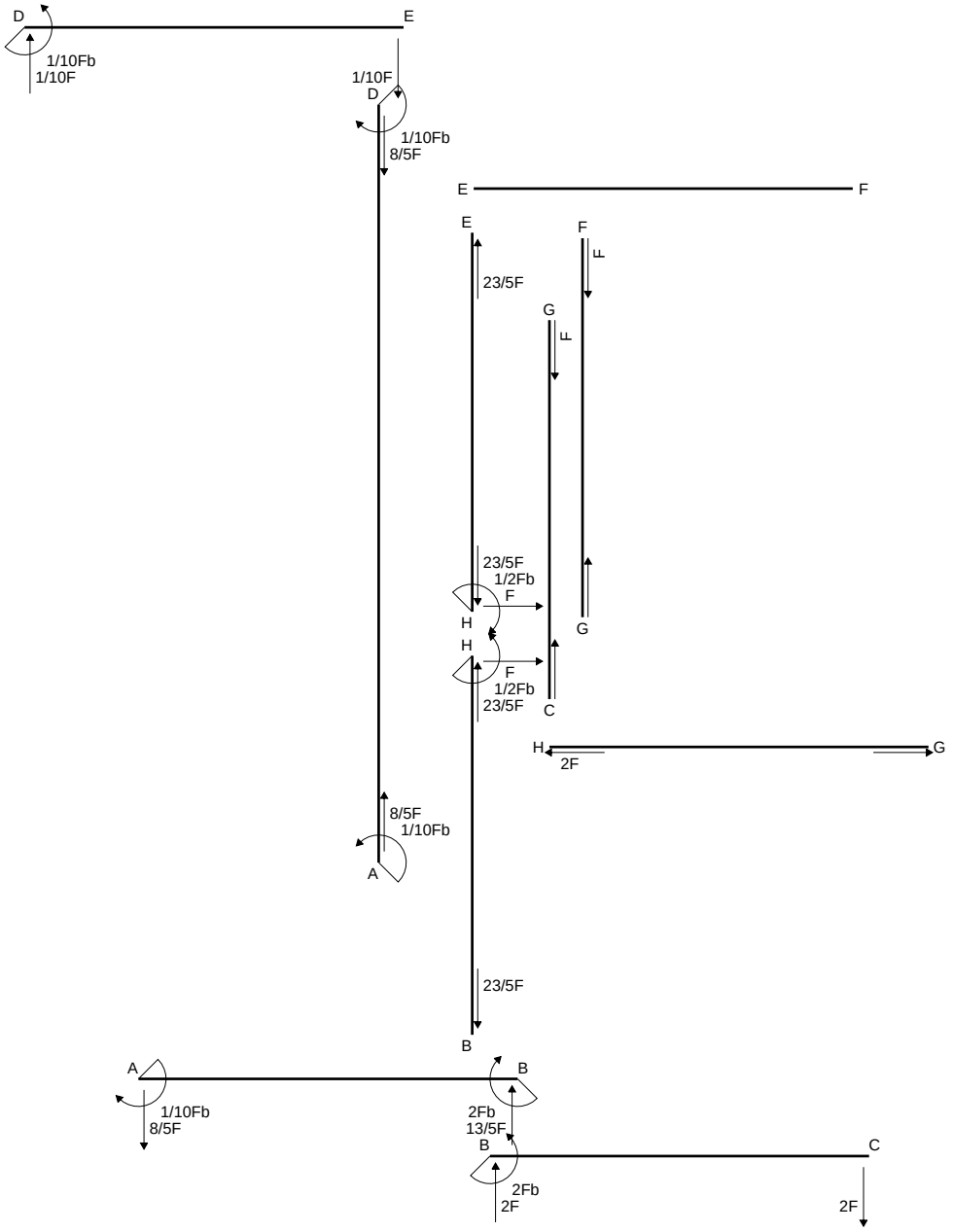
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

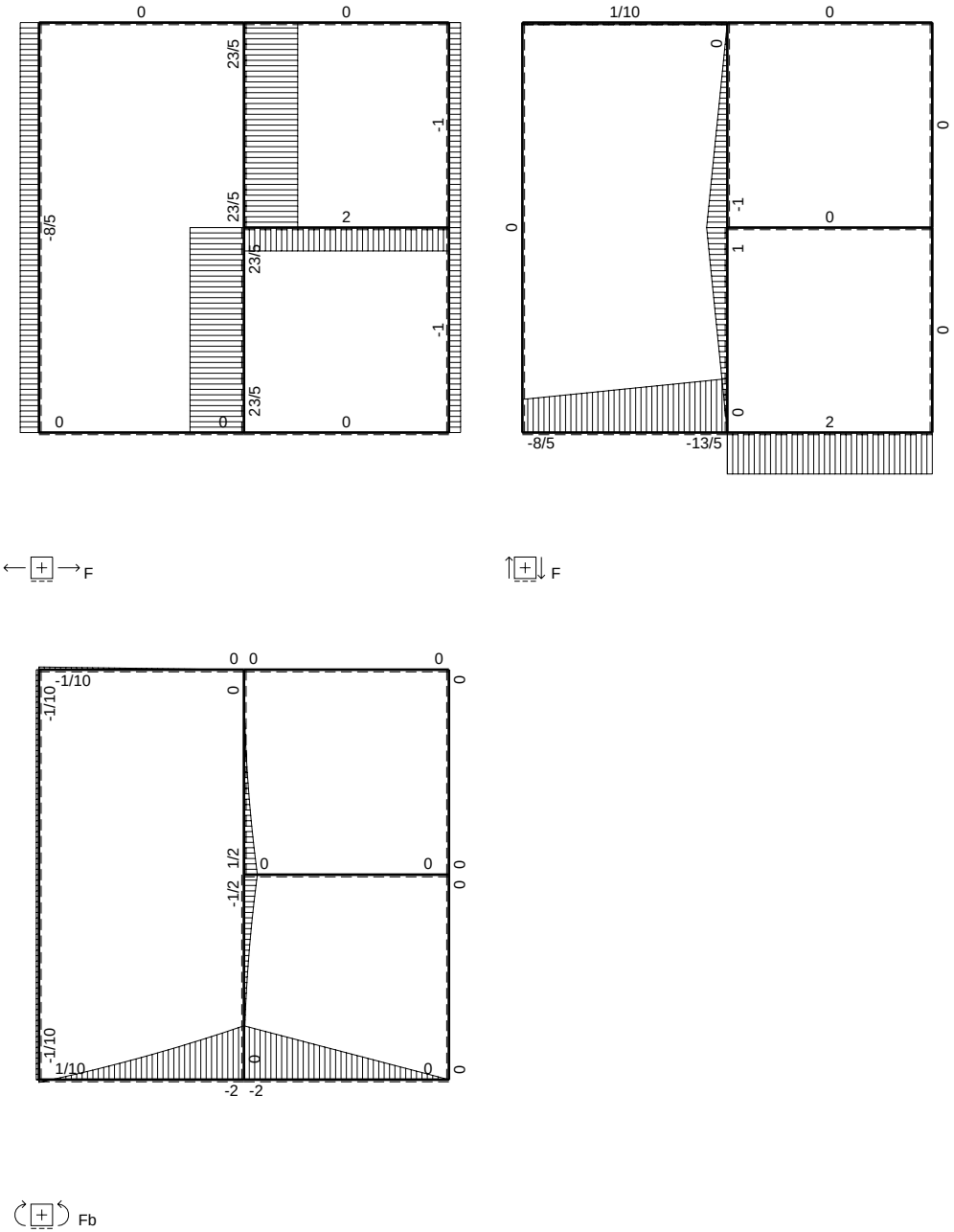
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

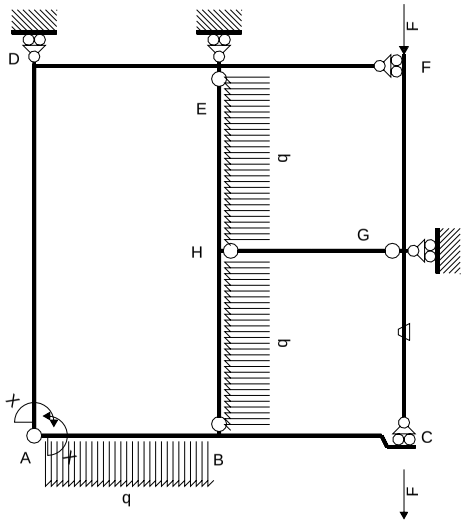
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

ESΣ03.xxxx.143

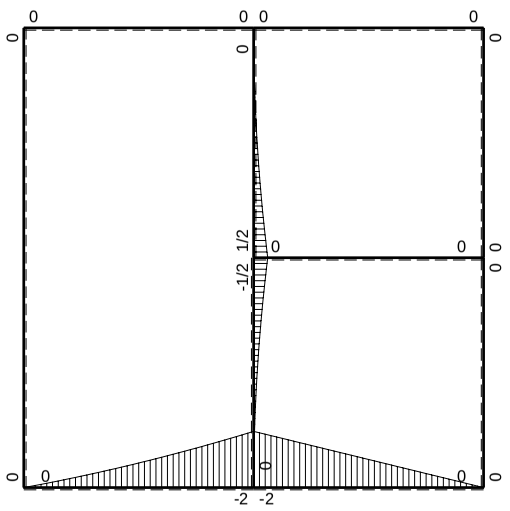
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 01.12.25


D
E
1/10Fb
1/10F
1/10F
D
1/10Fb
8/5F
E
F
E
F
F
23/5F
G
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23/5F
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2F

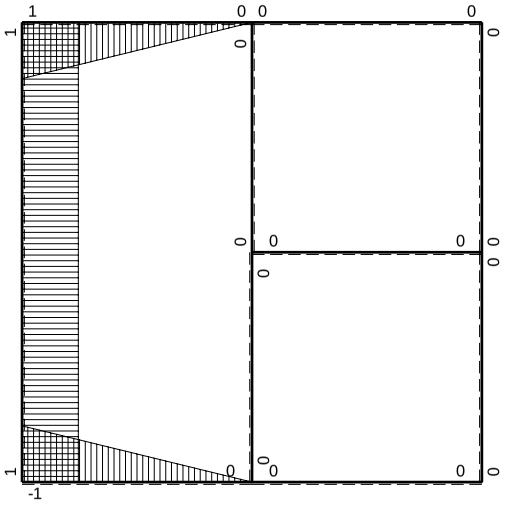

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-1/2
0 0
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0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 241258 ESΣ03.xxxx.144

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PROCEDIMENTO E RISULTATI 241258 ESΣ03.xxxx.144

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

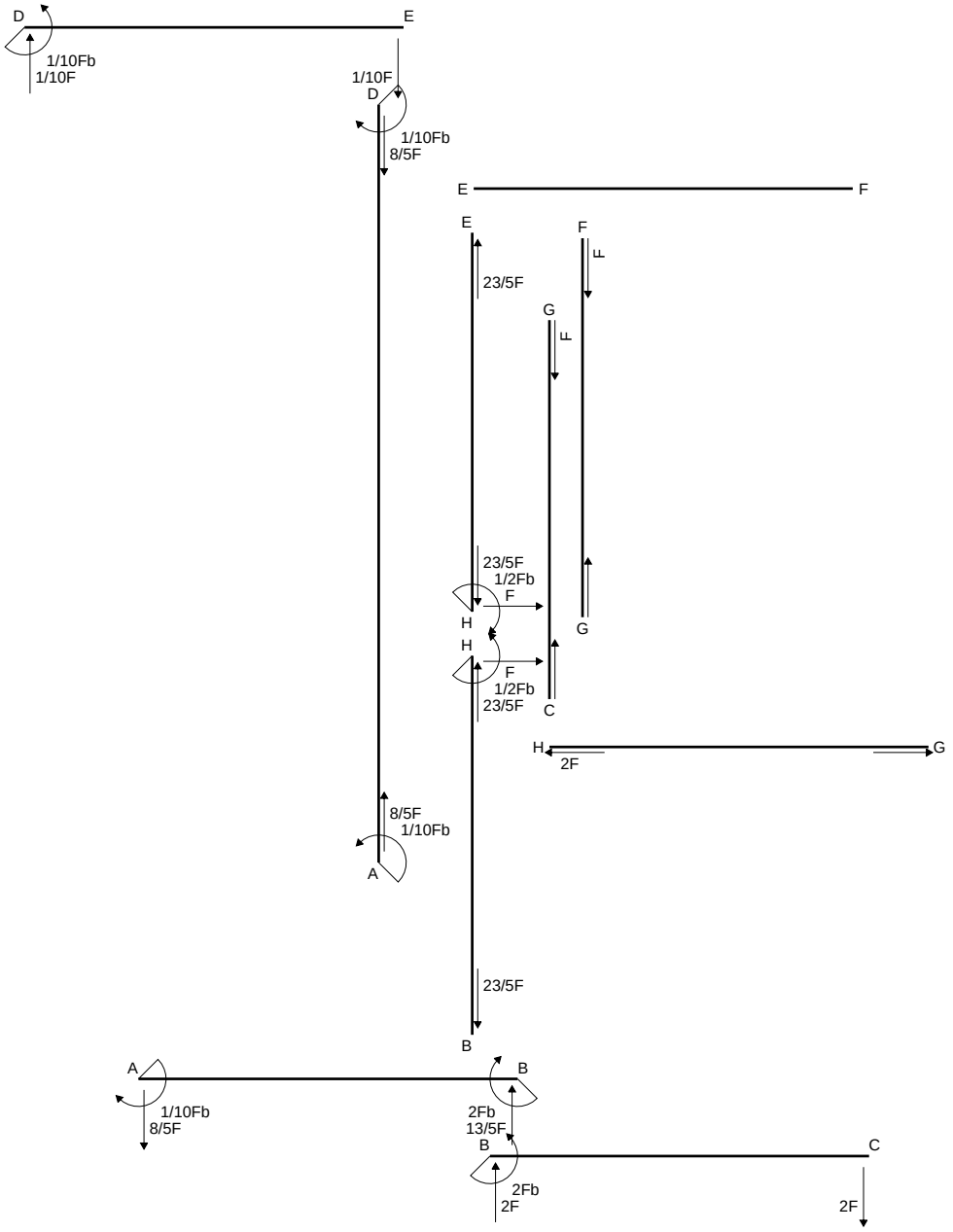
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

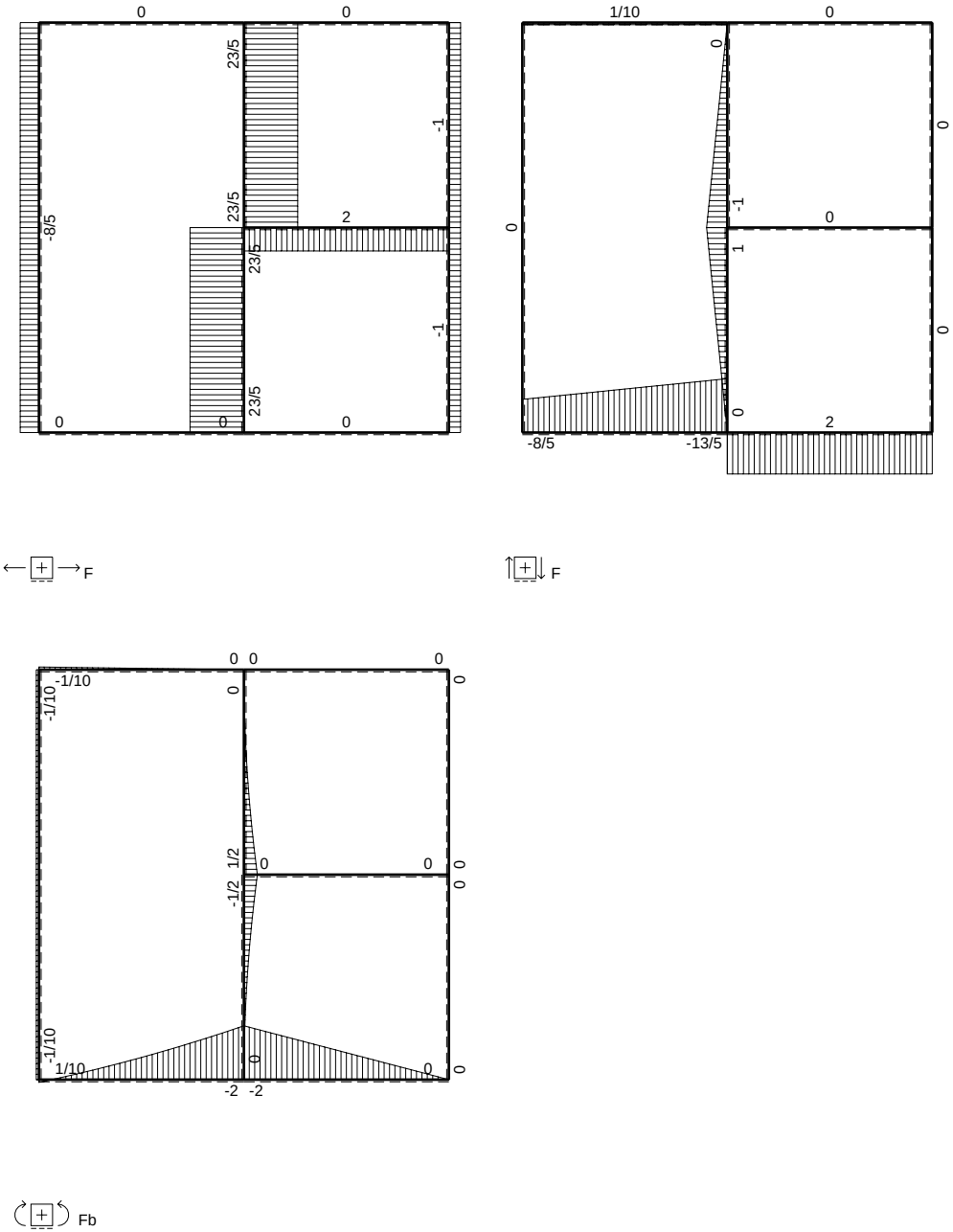
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

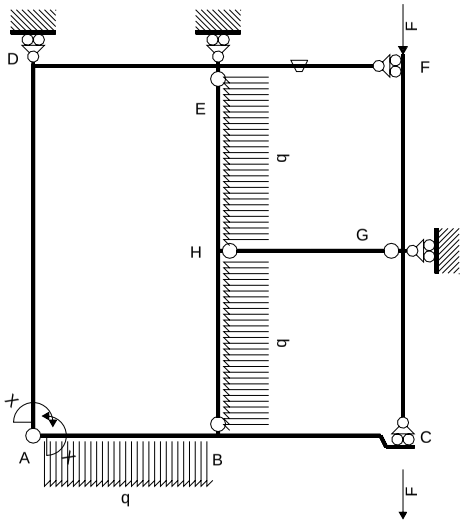
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

ESΣ03.xxxx.144

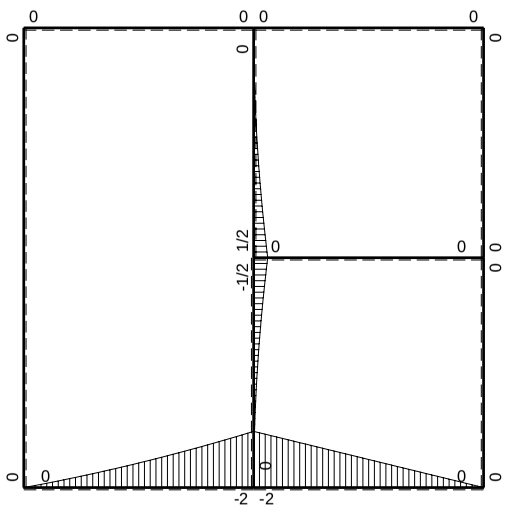
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1/10Fb
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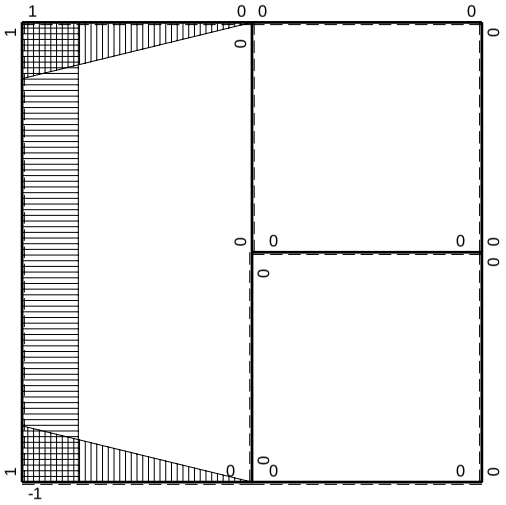

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0
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1/2
0
0
-1/2
0 0
0
-1/10
1/10
0
0
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236825 ESΣ03.xxxx.145

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PROCEDIMENTO E RISULTATI 236825 ESΣ03.xxxx.145

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$1/2qx^2$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

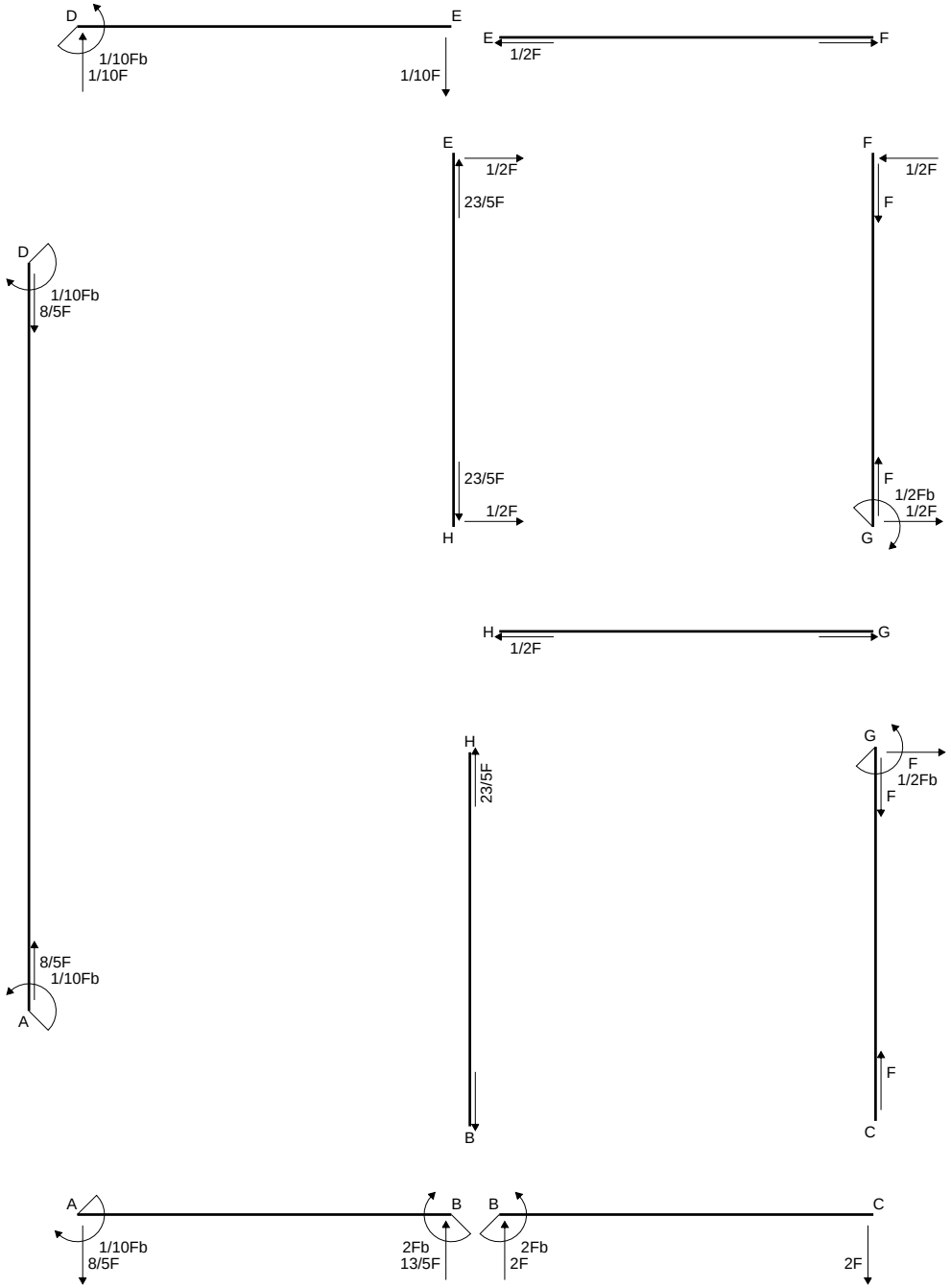
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

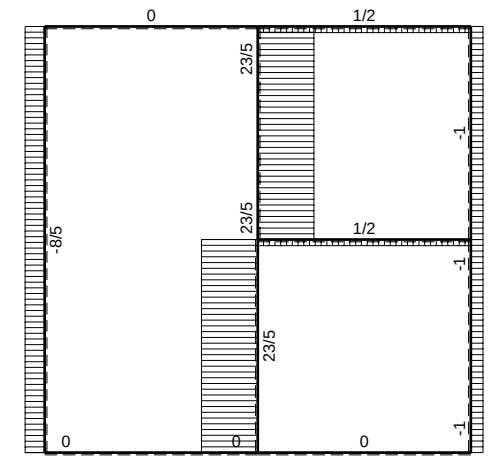
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

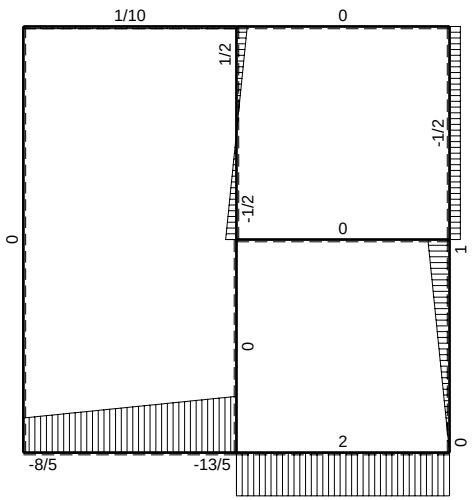
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

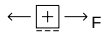
ESΣ03.xxxx.145

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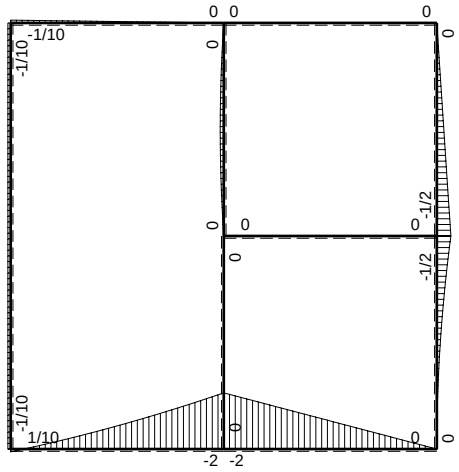

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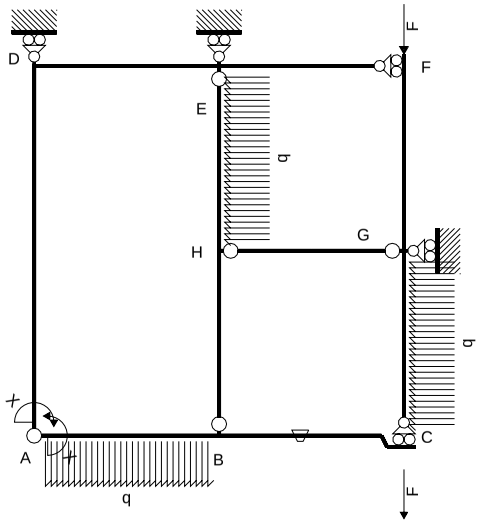

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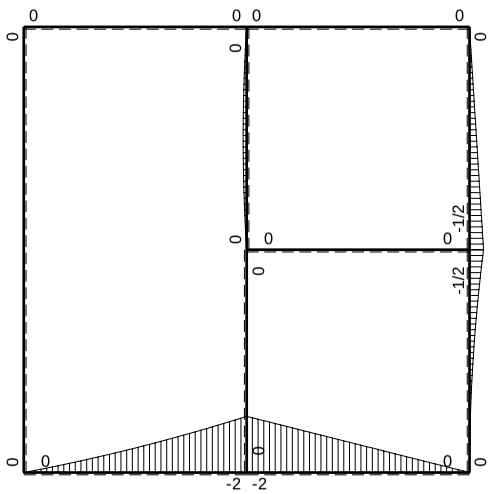

F


F

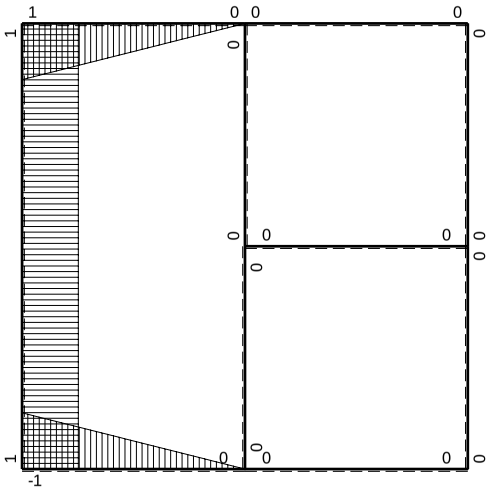

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-1/10
-1/10
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-2 -2


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238665 ESΣ03.xxxx.146

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PROCEDIMENTO E RISULTATI 238665 ESΣ03.xxxx.146

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

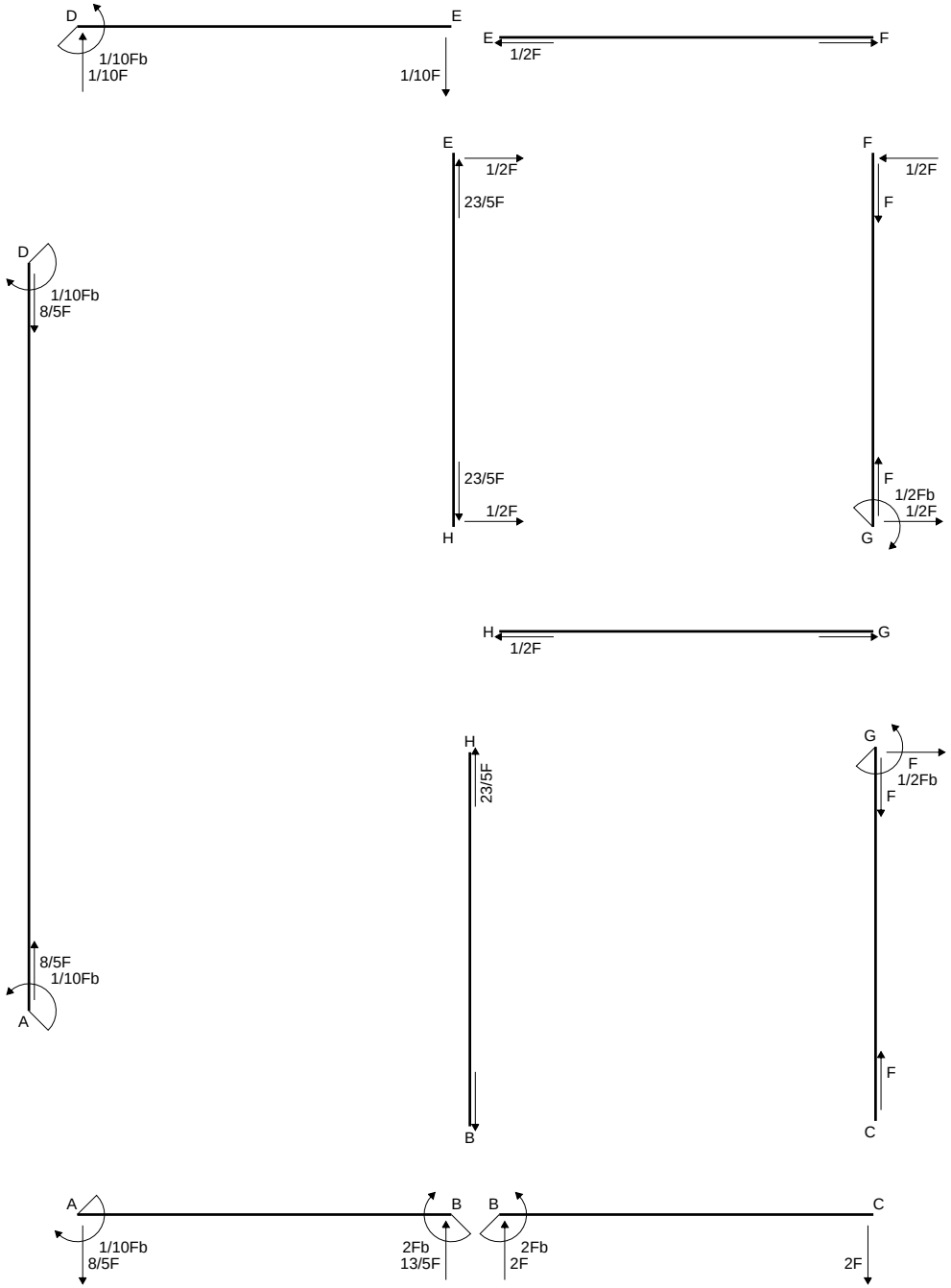
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

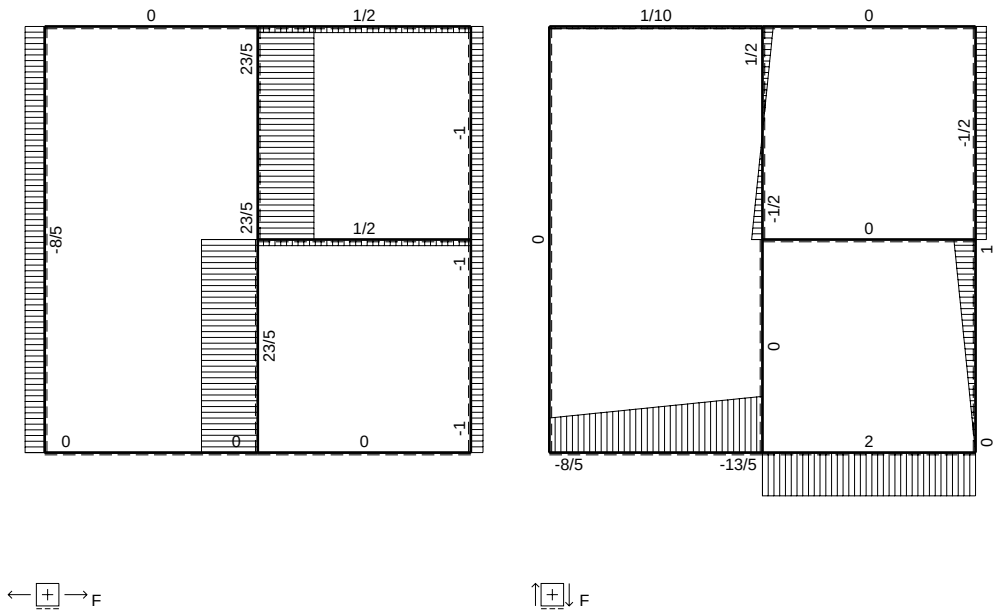
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

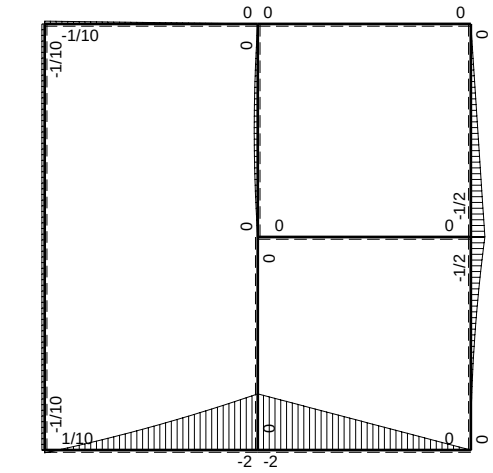
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

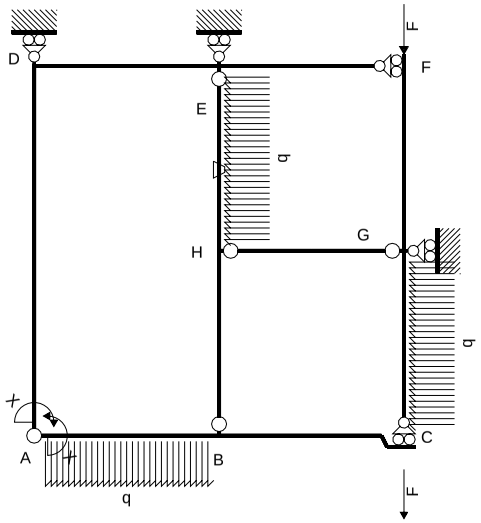
ESΣ03.xxxx.146

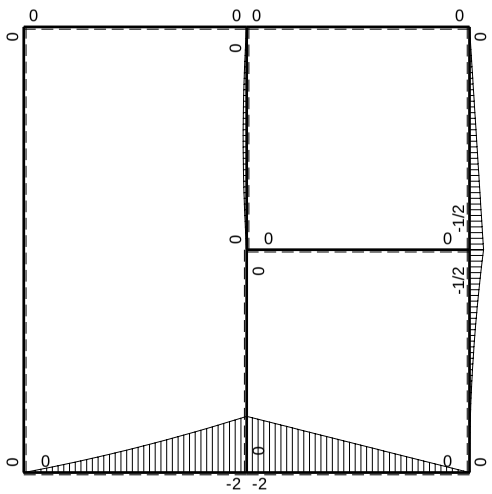
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F
1/2F
E
1/2F
23/5F
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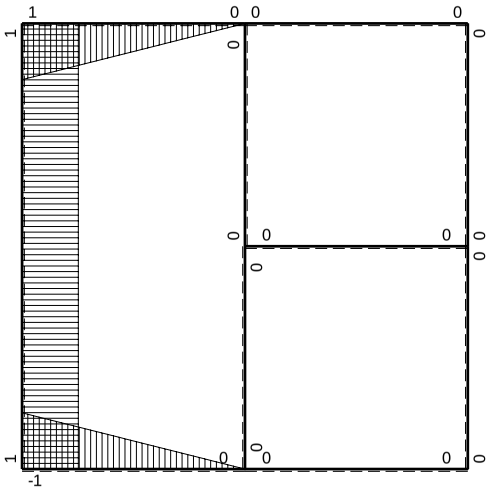

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-13/5
F
F


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0
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-1/2
0
0
-1/2
-1/10
-1/10
0
0
0
0
-2 -2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212317 ESΣ03.xxxx.147

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ \ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (1/3\,b)\ \ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\,b - 1/3\,b - 1/8\,b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

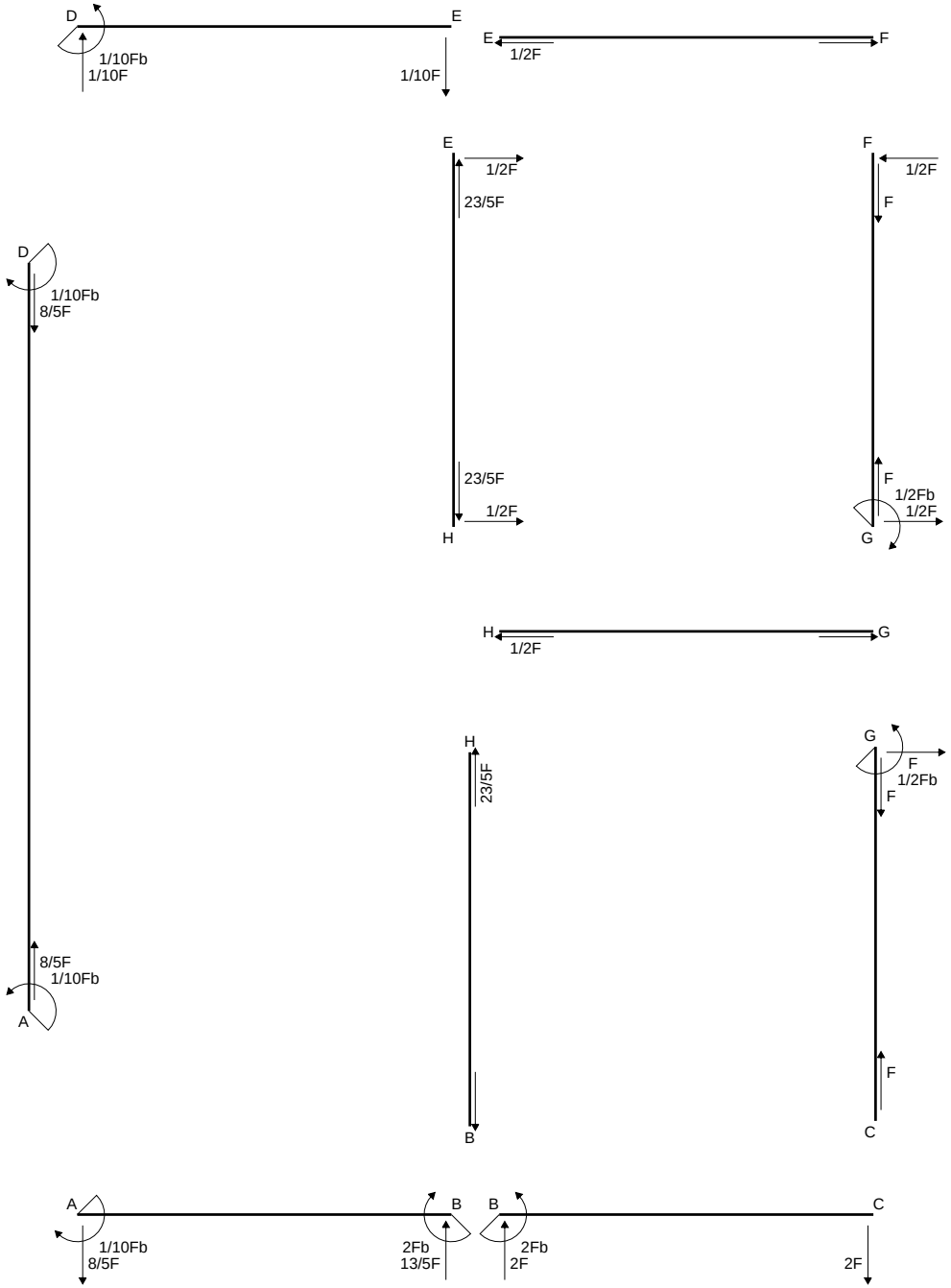
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

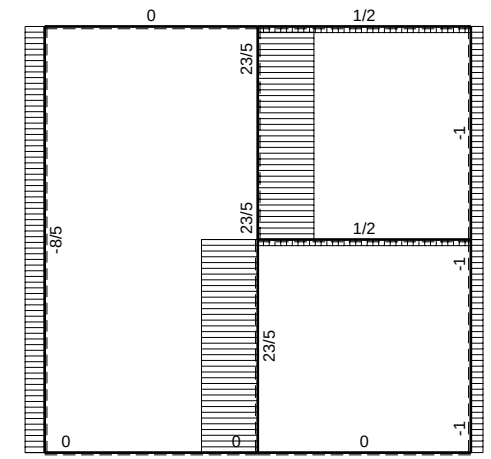
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

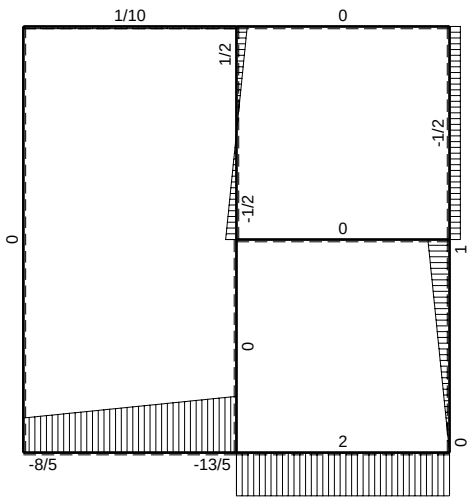
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

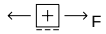
ESΣ03.xxxx.147

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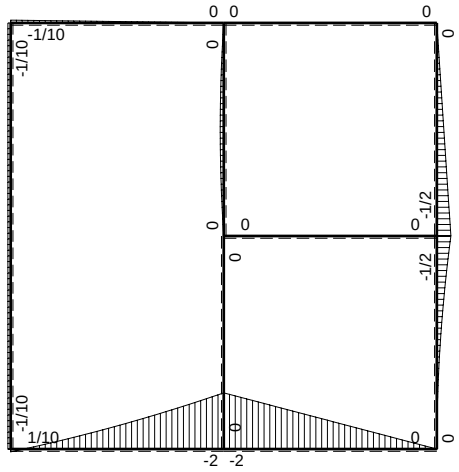

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C
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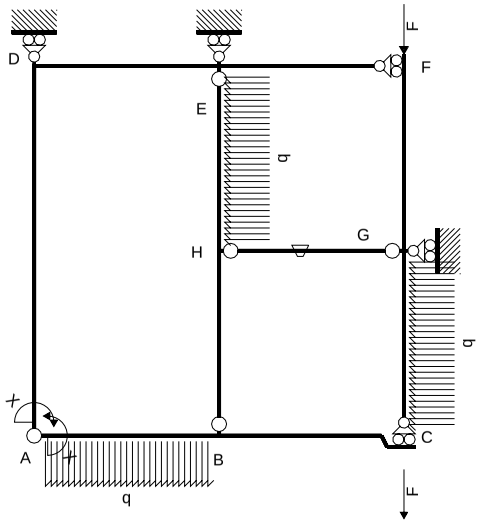

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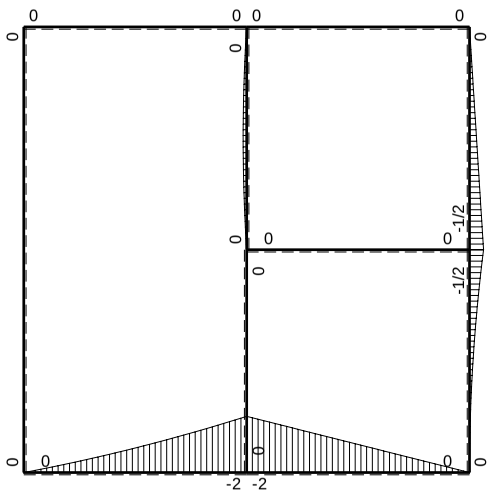

F


F

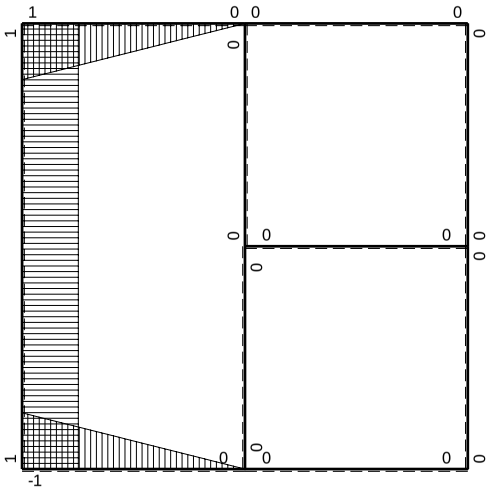

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-1/10
-1/10
0
0
0
-2 -2


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236719 ESΣ03.xxxx.148

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PROCEDIMENTO E RISULTATI 236719 ESΣ03.xxxx.148

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

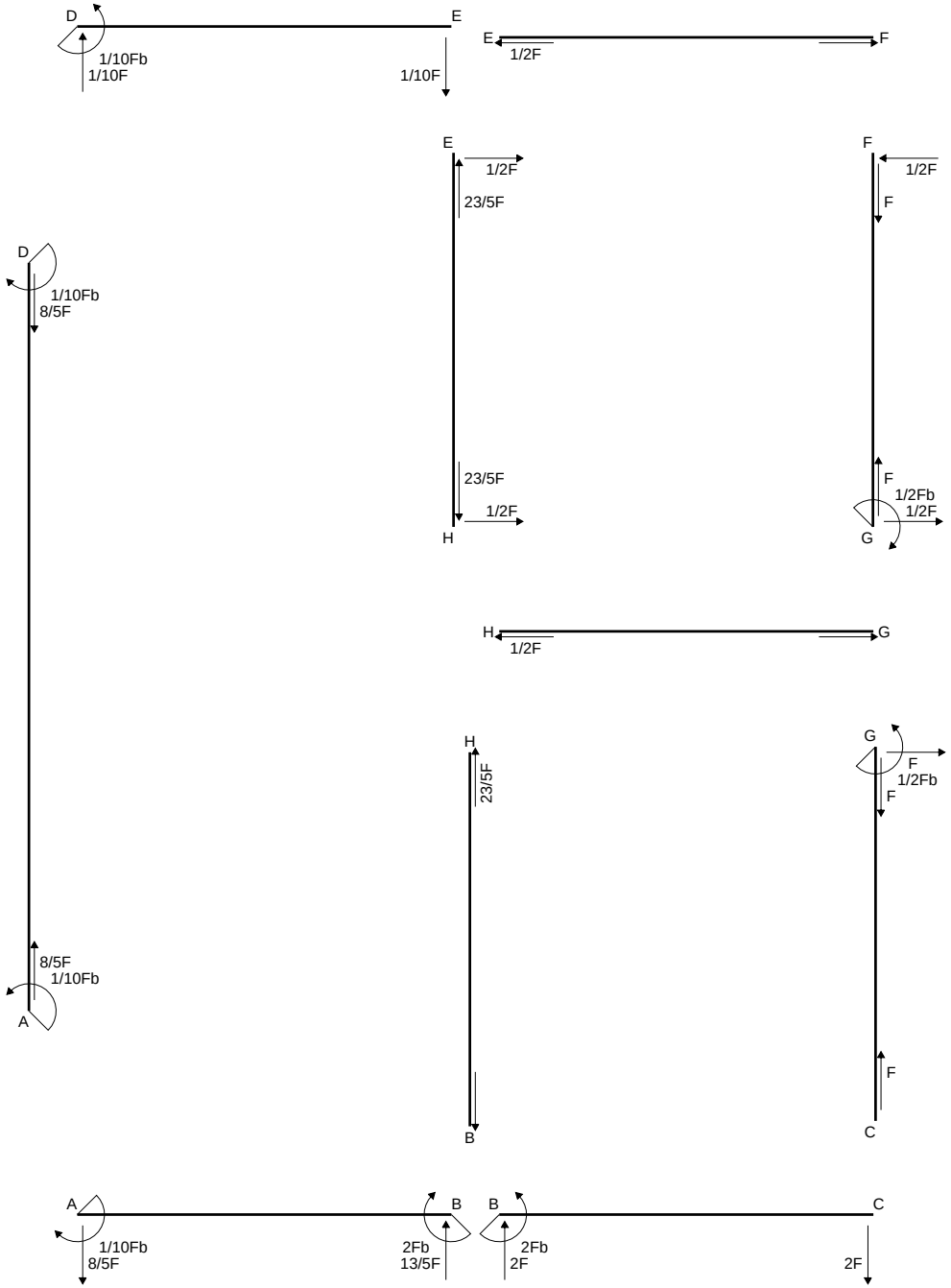
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

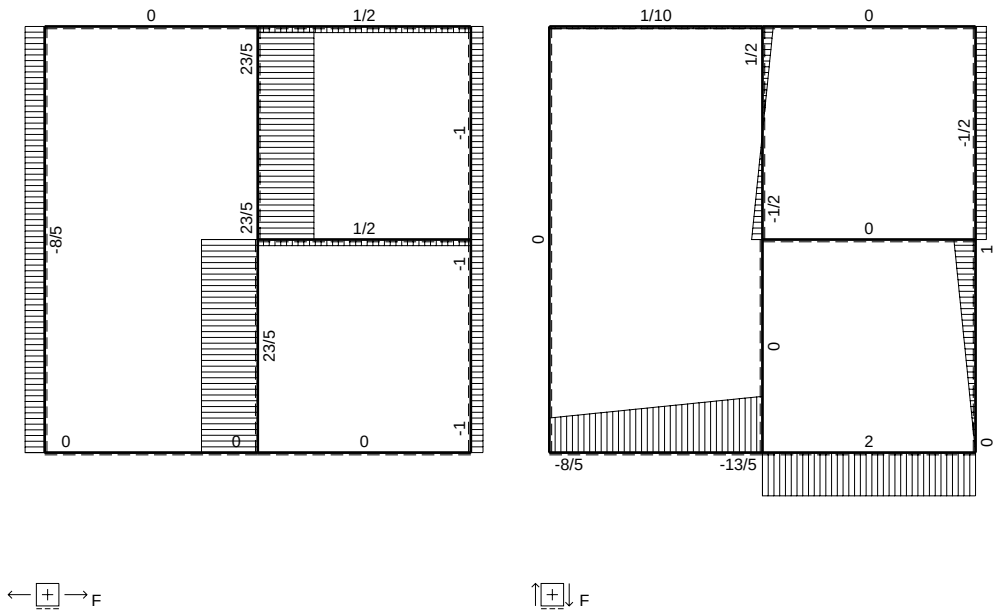
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

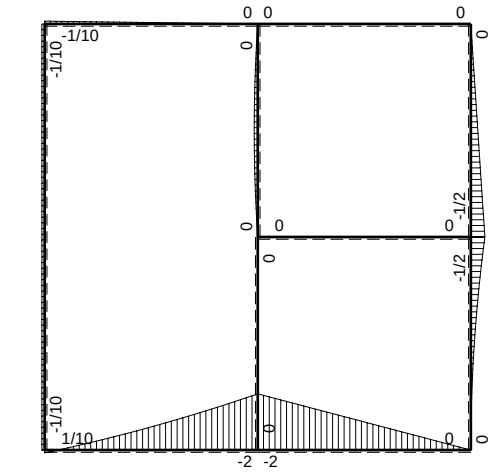
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

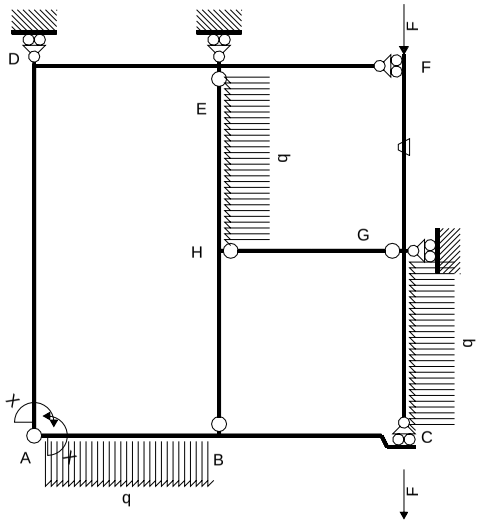
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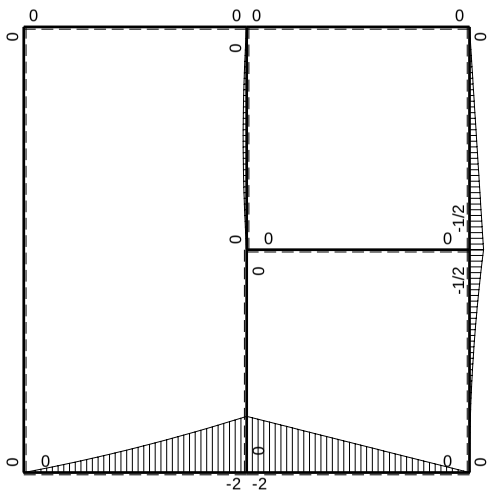
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D
E
1/10Fb
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E
F
1/2F
E
1/2F
23/5F
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C
2F

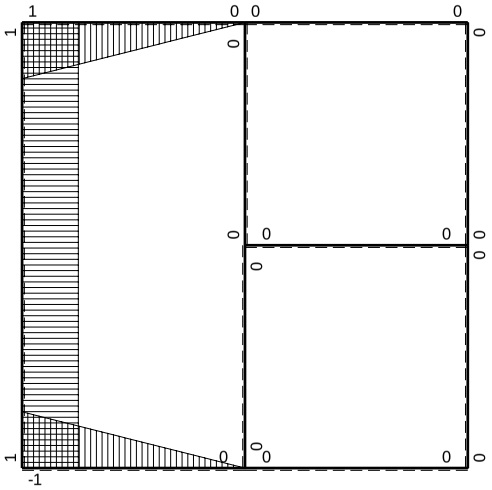

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1/2
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0
1
0
0
2
0
-8/5
-13/5
F
F


0 0
0
-1/10
-1/10
0
0
-1/2
0
0
0
0
-1/2
-1/10
-1/10
0
0
0
0
-2 -2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237735 ESΣ03.xxxx.149

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PROCEDIMENTO E RISULTATI 237735 ESΣ03.xxxx.149

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	Fb/EJ	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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ESΣ03.xxxx.149

$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

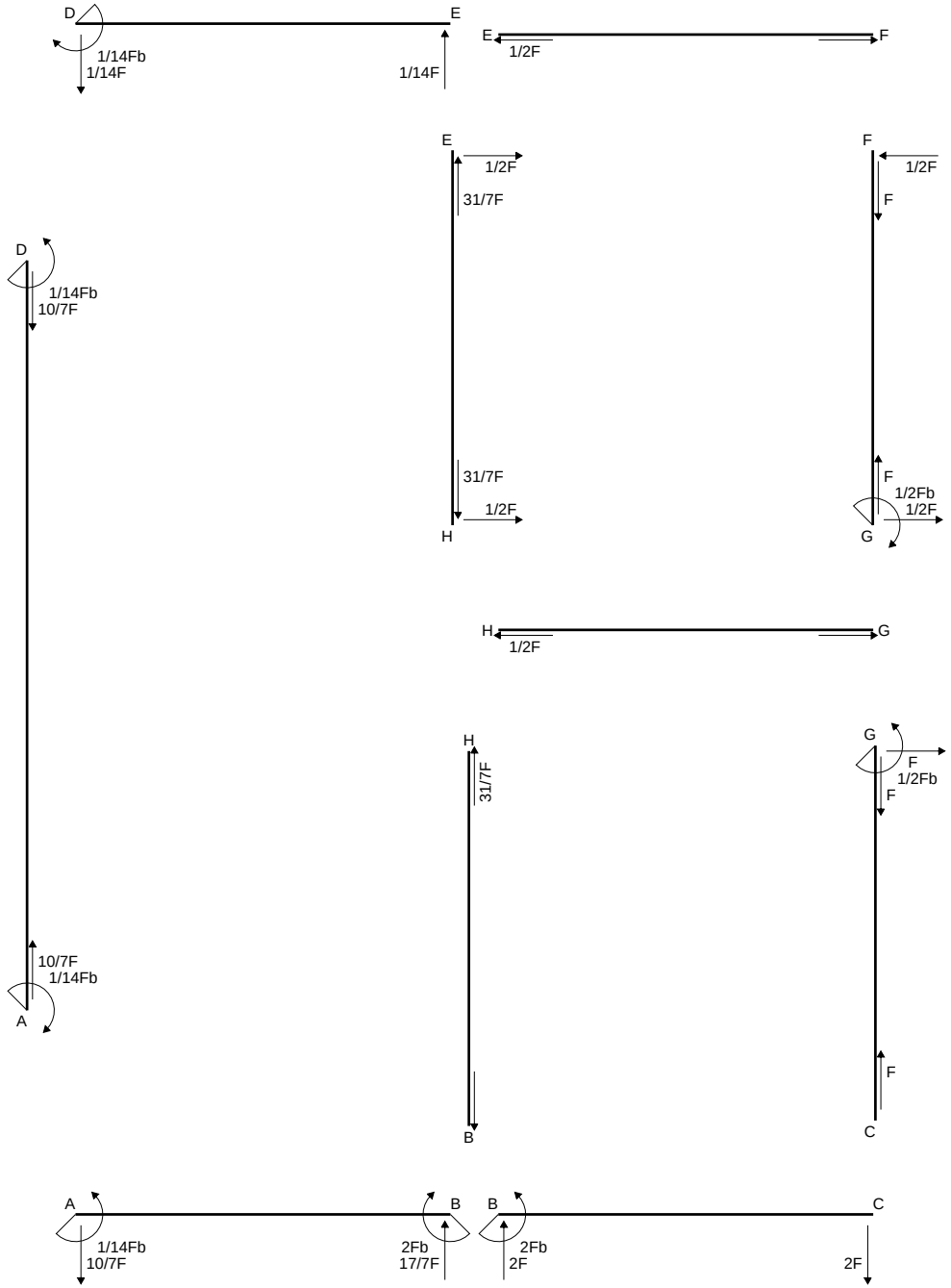
$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

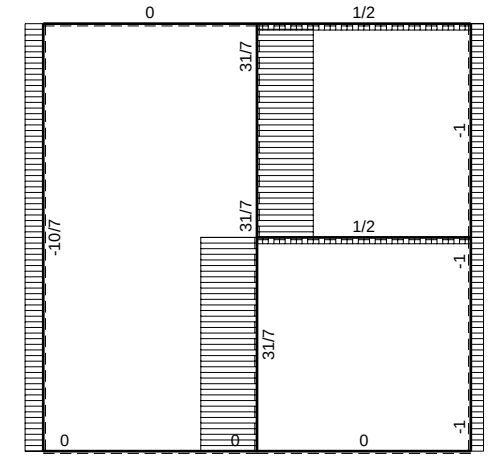
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

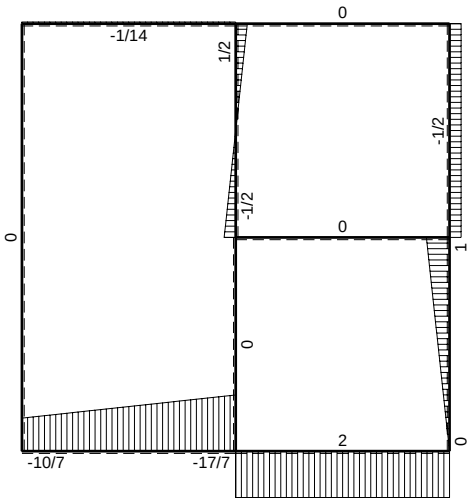
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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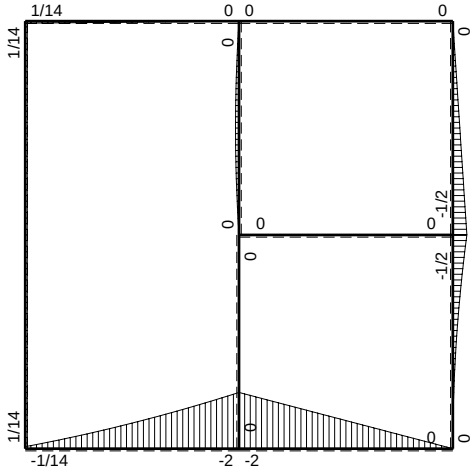

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2F
C
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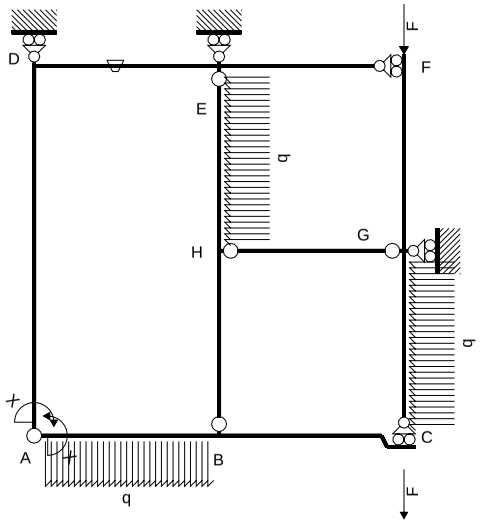

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-17/7

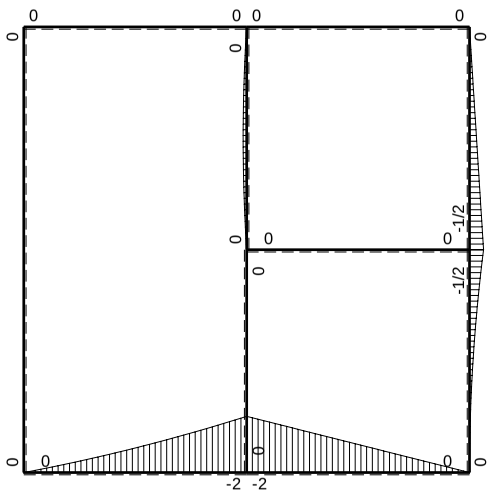
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F

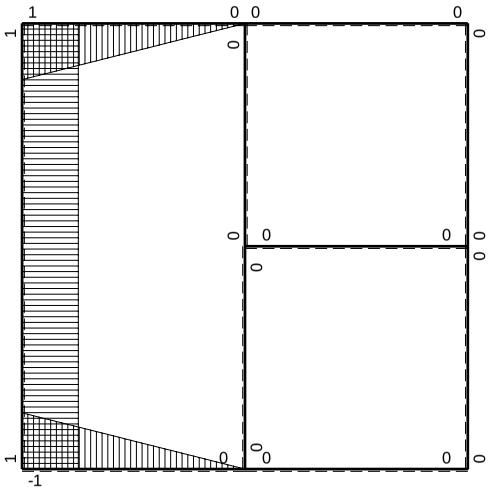

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0
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-1/2
0
0
0
1/14
-1/14
-2
-2
0
0

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212534 ESΣ03.xxxx.150

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	0	-Fb/EJ	0	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$	$(0-1/2)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	0	Fb/EJ	0	-Fx/EJ	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	-1/2Fx	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212534 ESΣ03.xxxx.150
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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1 + x/b)\ \theta\ dx = [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

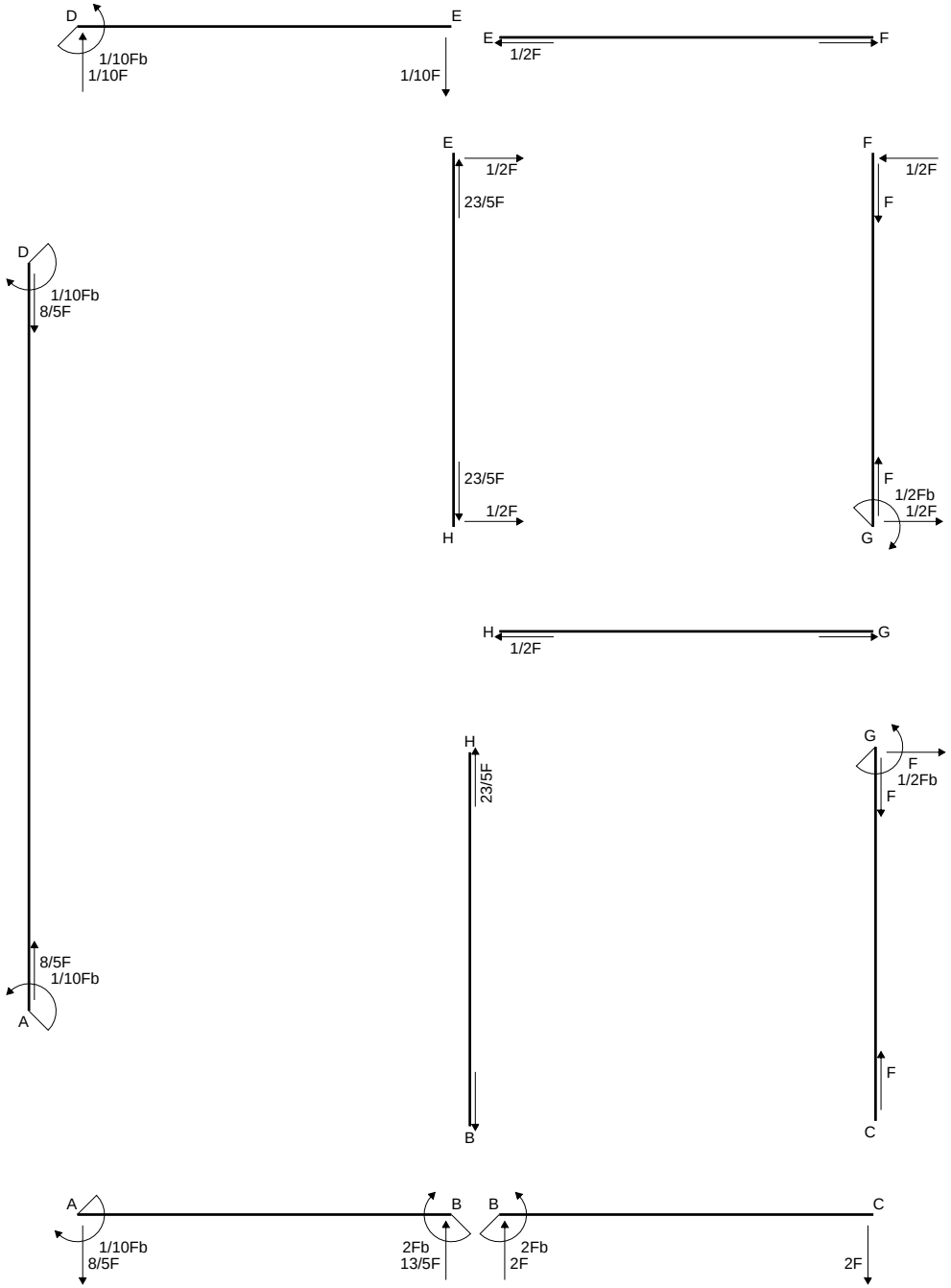
$= (-\ b + 1/2\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

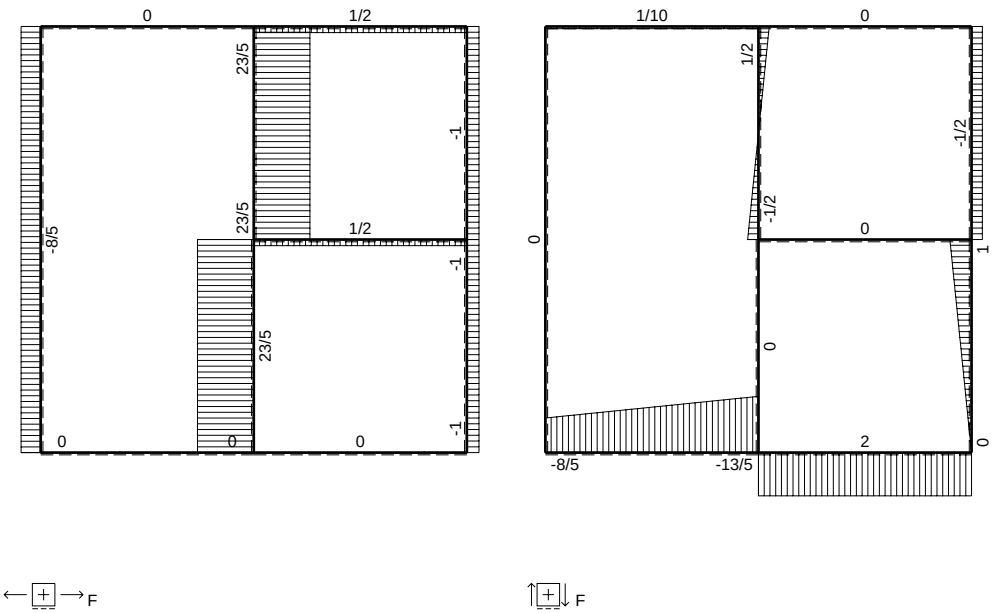
$L_{ED}^{Xo} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

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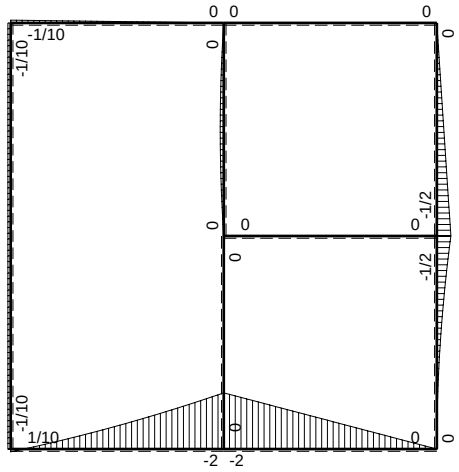
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1/10Fb
1/10F
1/10F
E
F
1/2F
E
1/2F
23/5F
23/5F
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F
1/2Fb
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G
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1/2Fb
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8/5F
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B
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A
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B
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8/5F
2Fb
13/5F
2Fb
2F
2F

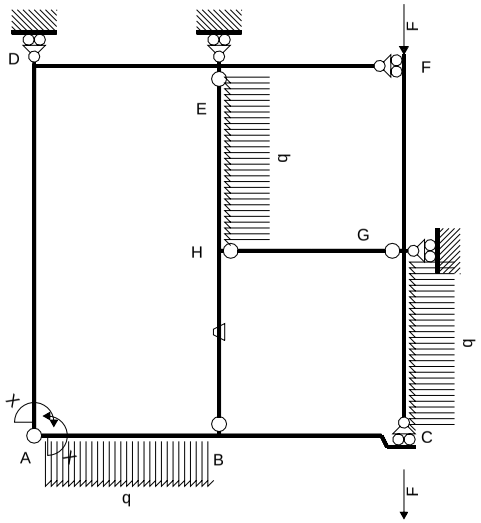

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-13/5

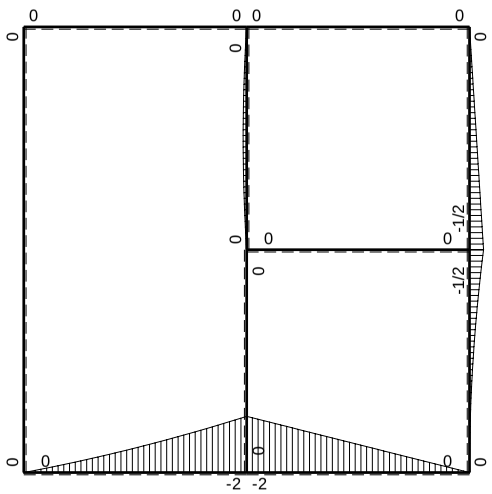
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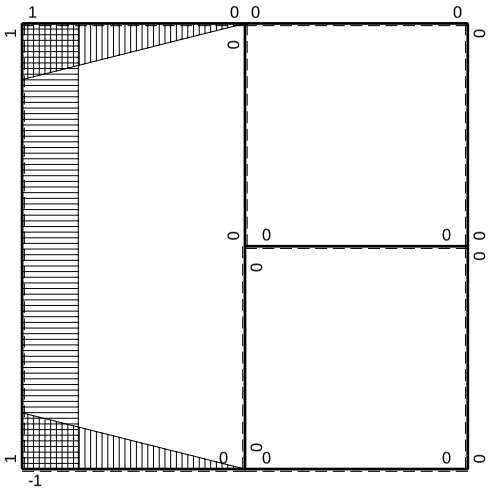

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0
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0
0
-1/2
-1/10
-1/10
0
0
0
-2 -2


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237895 ESΣ03.xxxx.151

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PROCEDIMENTO E RISULTATI 237895 ESΣ03.xxxx.151

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	0	Fb/EJ	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

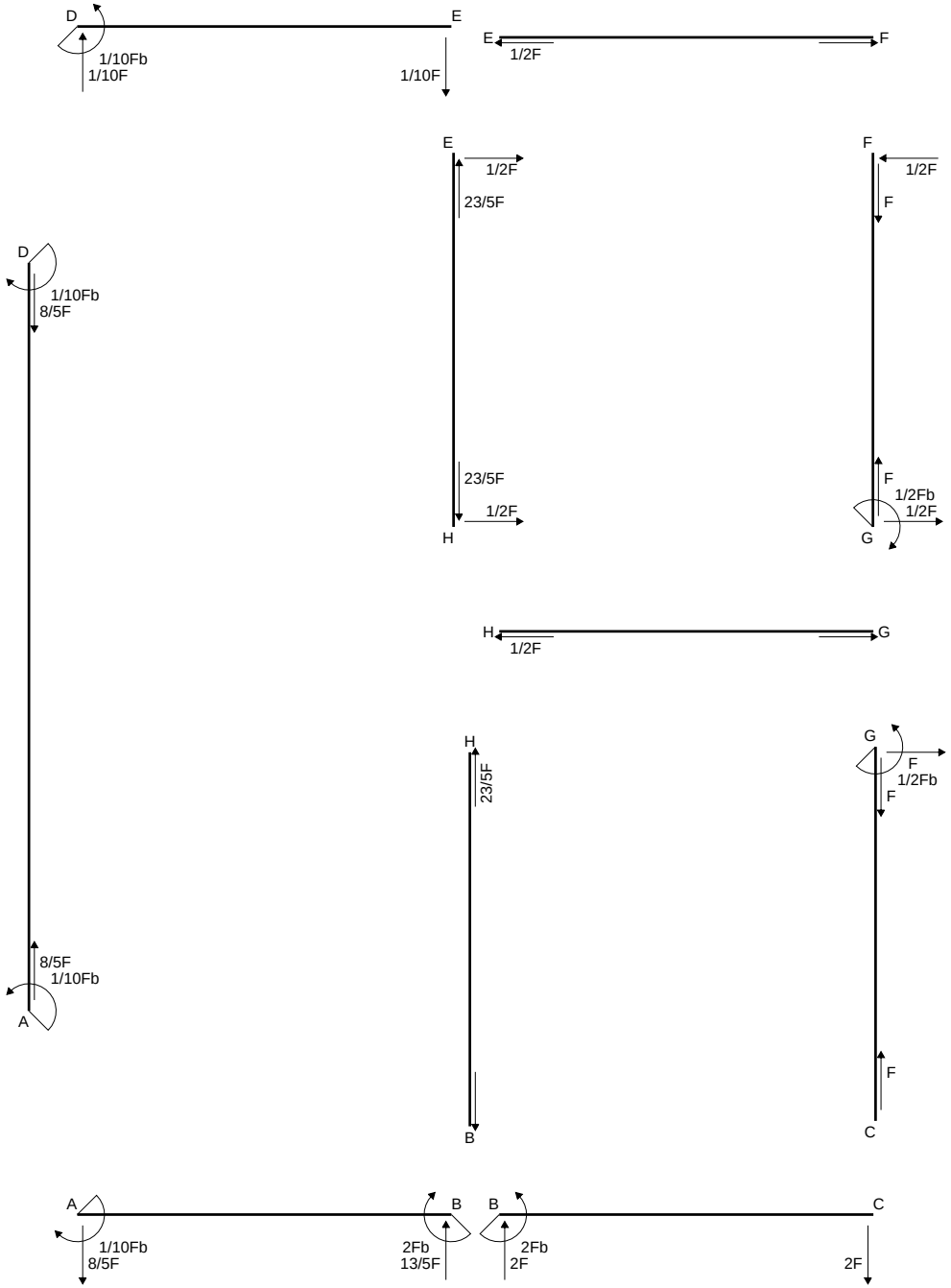
$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

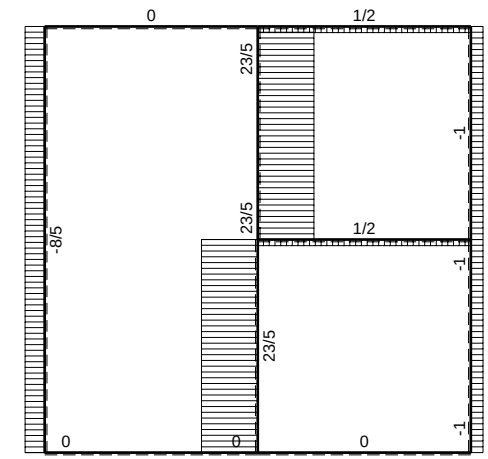
$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

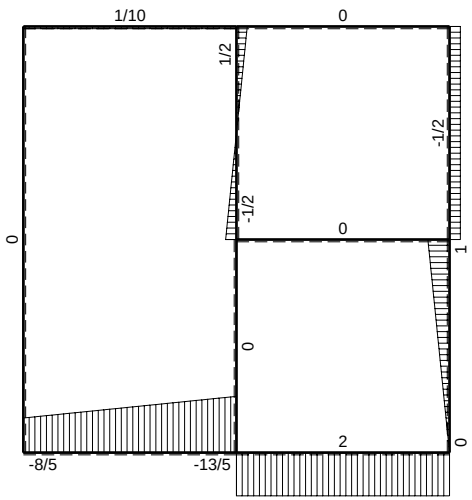
$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

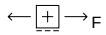
ESΣ03.xxxx.151

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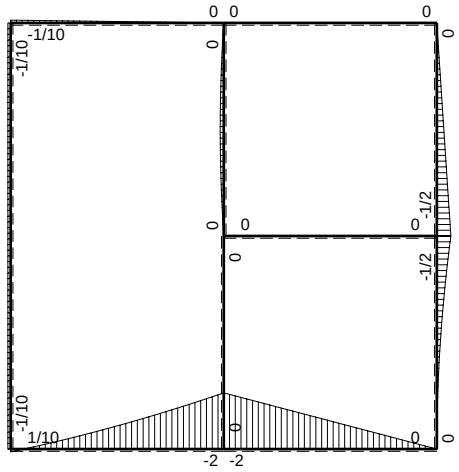

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F
B
C
A
B
B
C
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8/5F
2Fb
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2Fb
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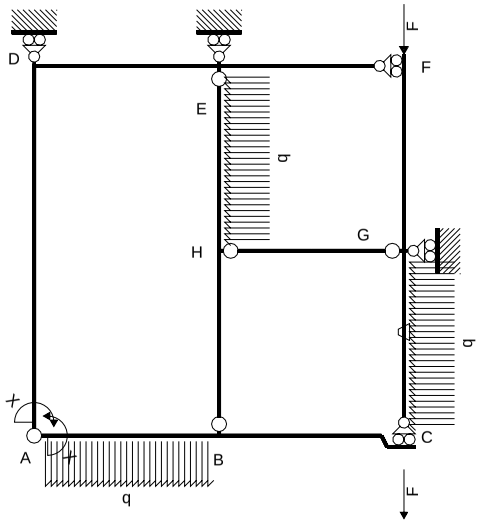

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-8/5
-13/5

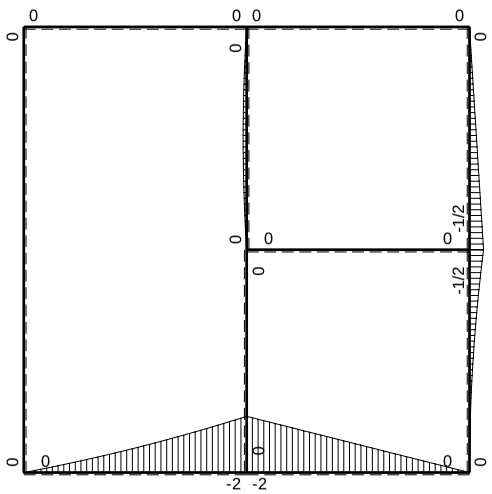

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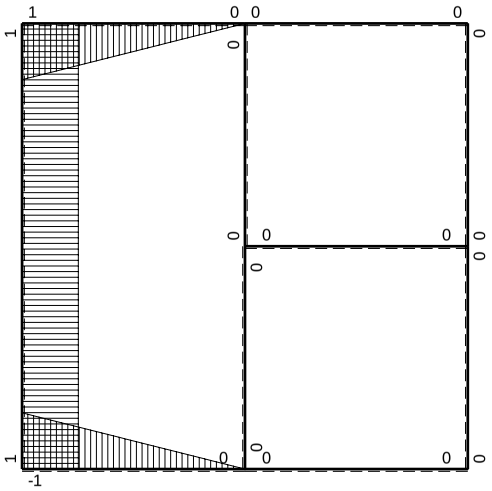

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-1/10
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0
0
-1/2
0
-1/2
-1/10
-1/10
0
0
0
-2 -2


Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237862 ESΣ03.xxxx.152

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PROCEDIMENTO E RISULTATI 237862 ESΣ03.xxxx.152

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\,x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\,dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\,b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\,dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,3/4\,b - 1/3\,b - 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

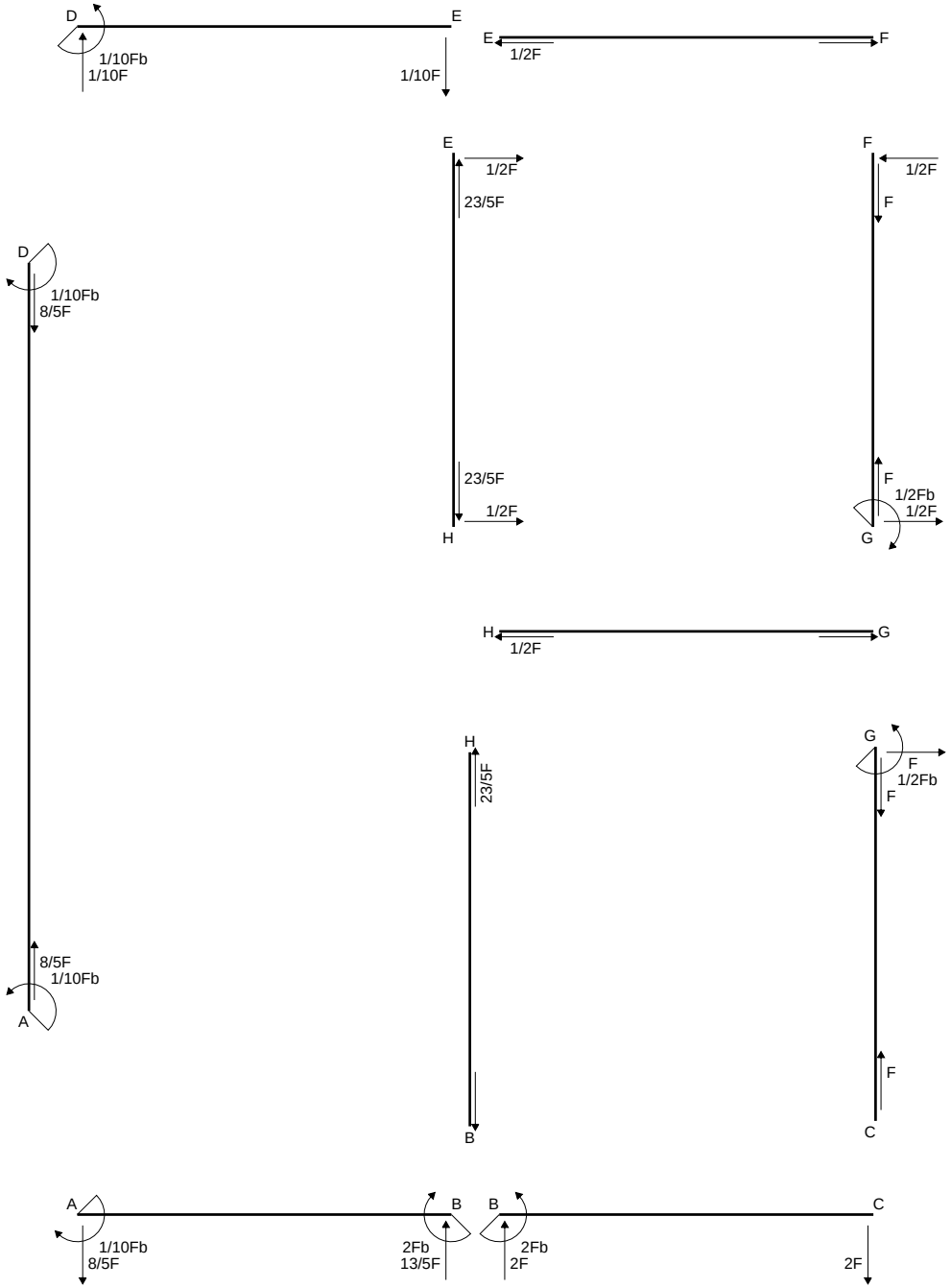
$L_{BA}^{Xo} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

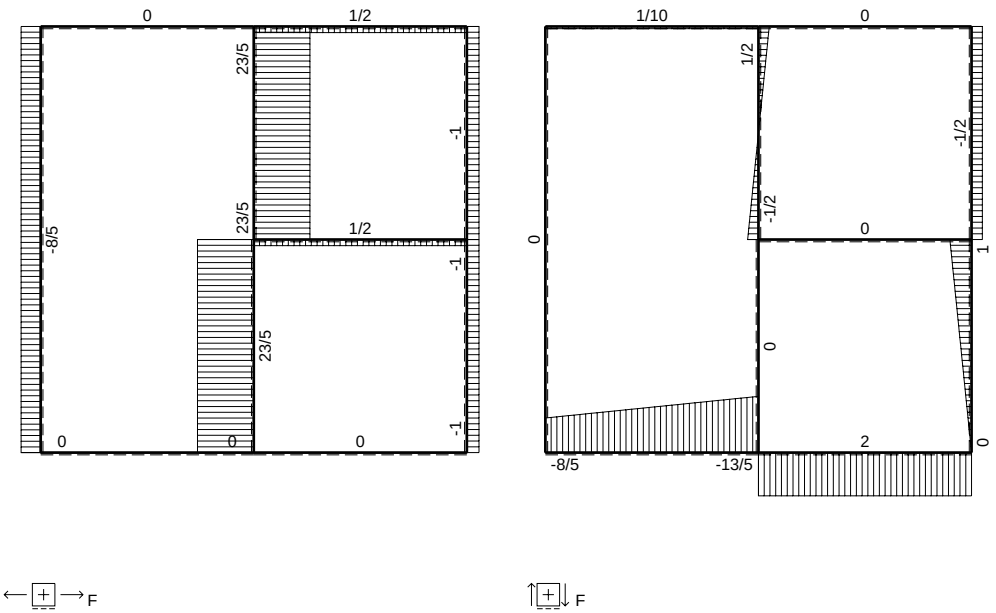
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

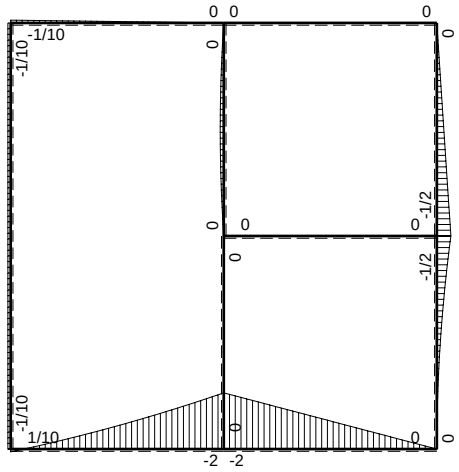
ESΣ03.xxxx.152

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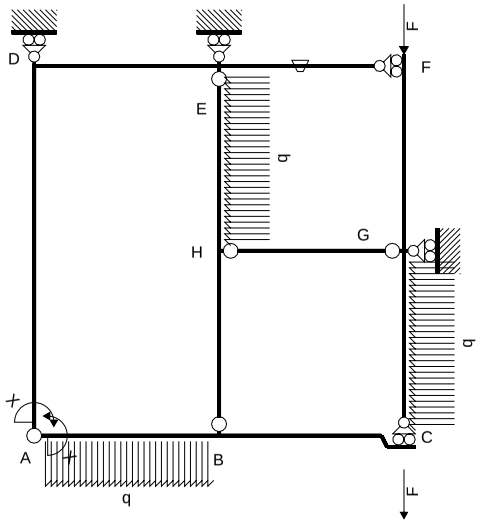

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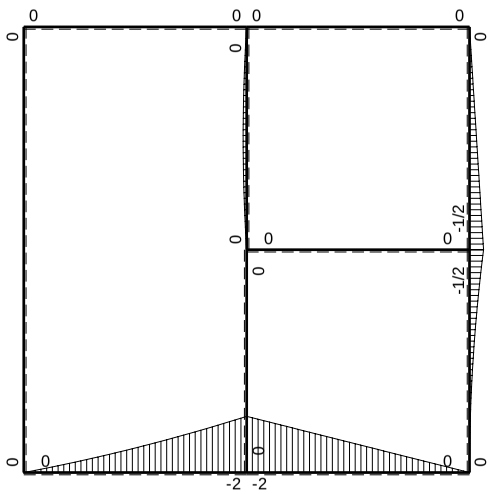

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-8/5
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F F

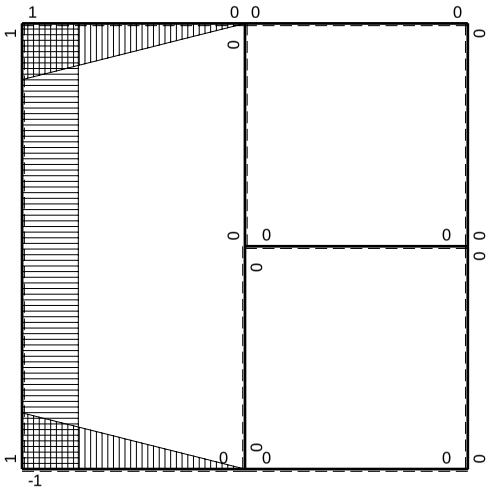

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-1/10
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0
0
0
-2 -2

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238254 ESΣ03.xxxx.153

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PROCEDIMENTO E RISULTATI 238254 ESΣ03.xxxx.153

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\,b)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\,dx = [\,x\,]_o^{2b}\ \ 1/EJ$

$= (2\,b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\,dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\,x^2/b^2\,)\ \ 1/EJ\,dx = [\,1/3\,x^3/b^2\,]_o^b\ \ 1/EJ$

$= (1/3\,b)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,3/4\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\,b - 1/3\,b - 1/8\,b)\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

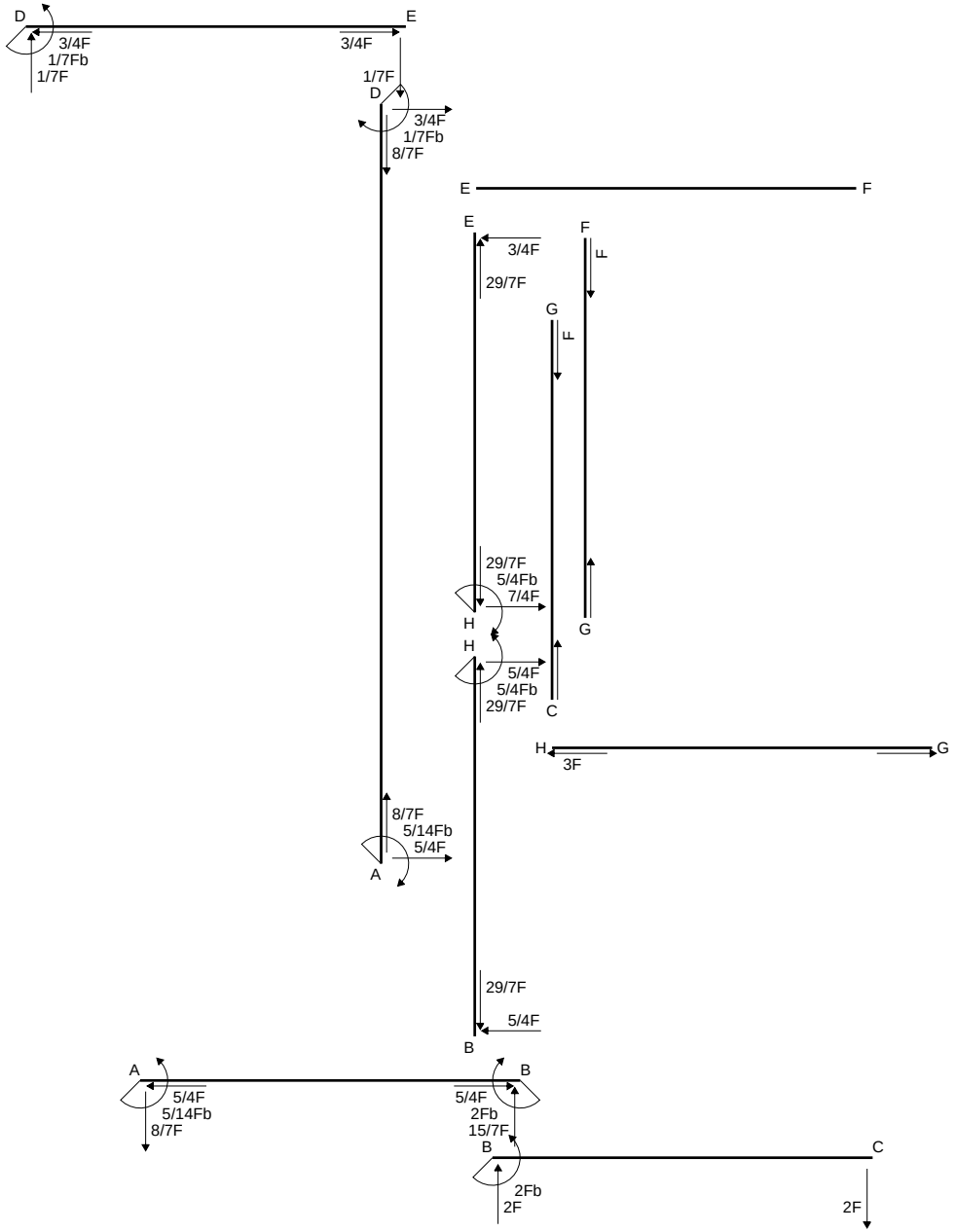
$L^{Xo}_{BA} = \int_o^b (2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\ Fb\ 1/EJ\,dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

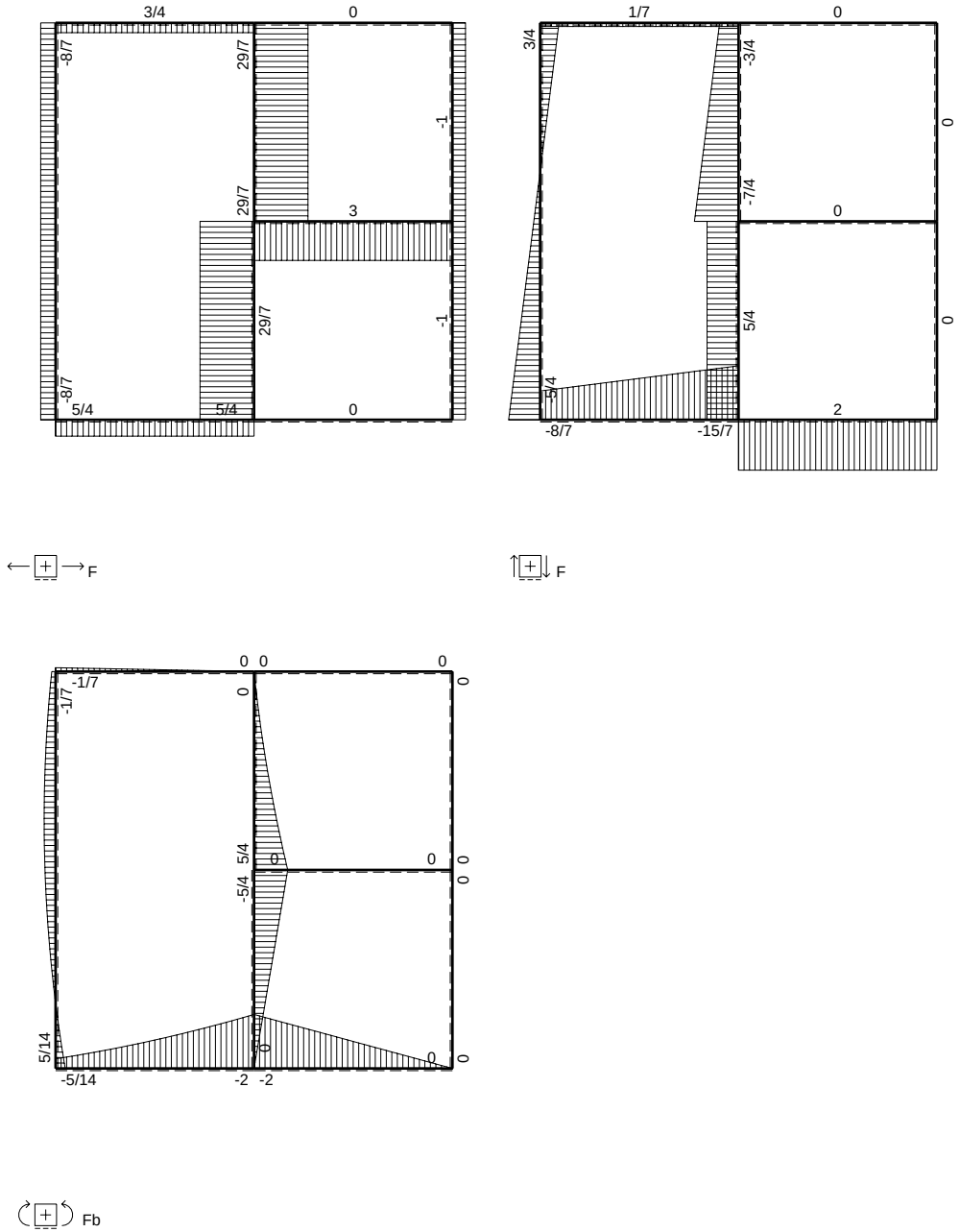
$= [\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3\,]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

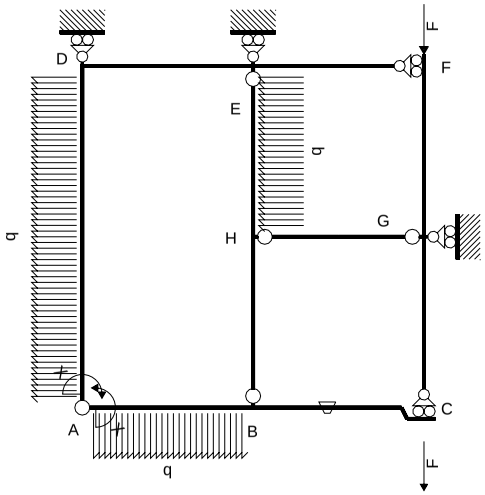
$= (\,b - 5/6\,b + 1/8\,b\,)\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

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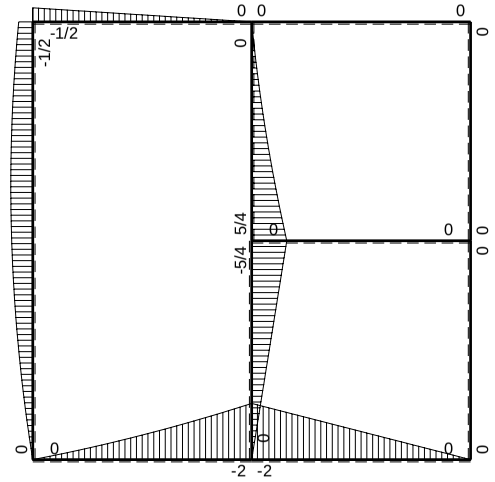
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
8/7F
E
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C
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B
C
2Fb
2F
2F

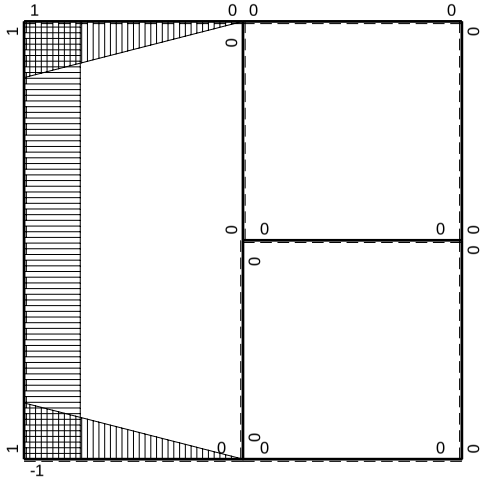

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5/4
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-8/7
-15/7
F
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-1/7
-1/7
0
0
5/4
0
0
0 0
-5/4
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-5/2\ b +4/3\ b\ \right)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 -3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x -3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b -3/2\ b +4/3\ b\ \right)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

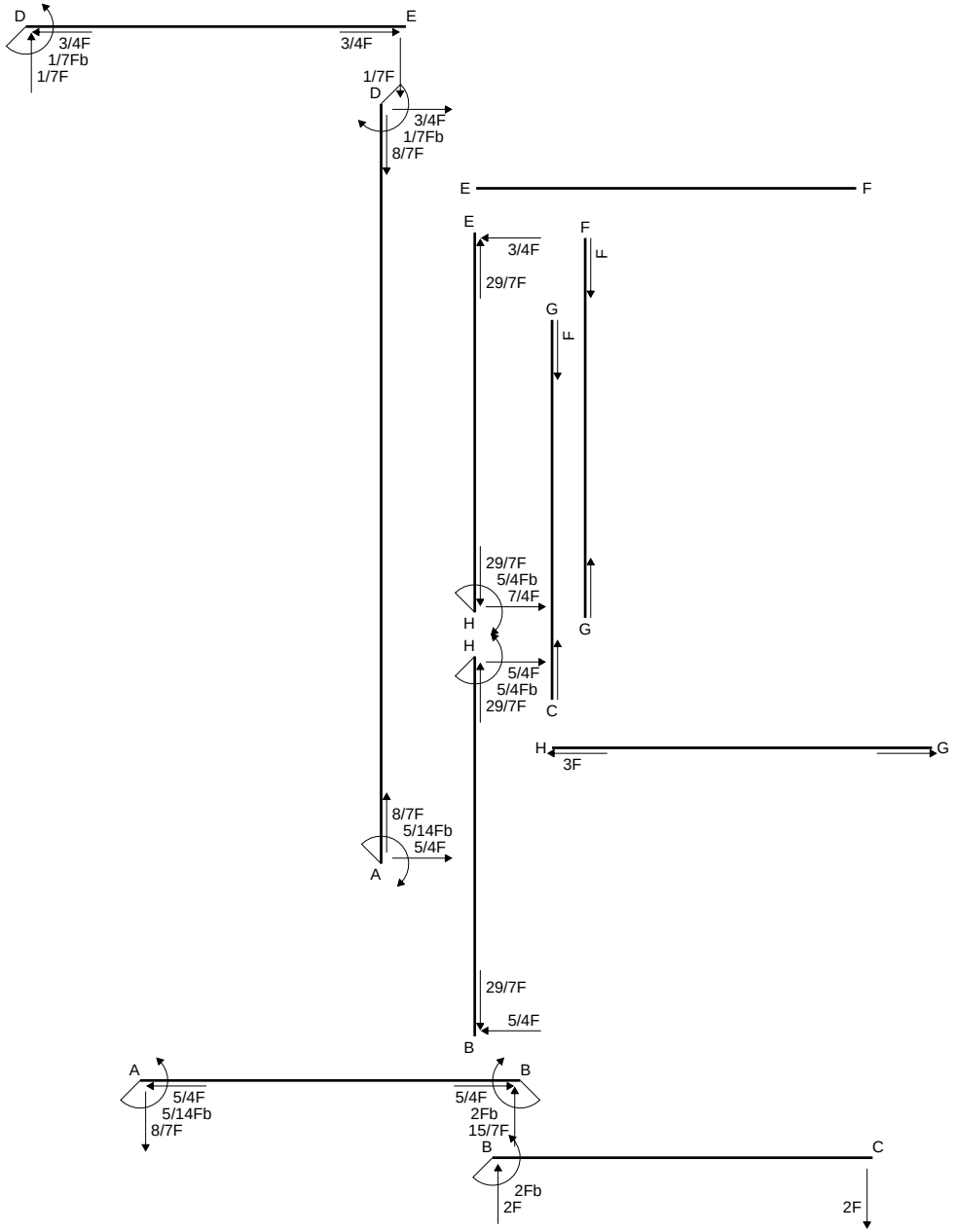
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

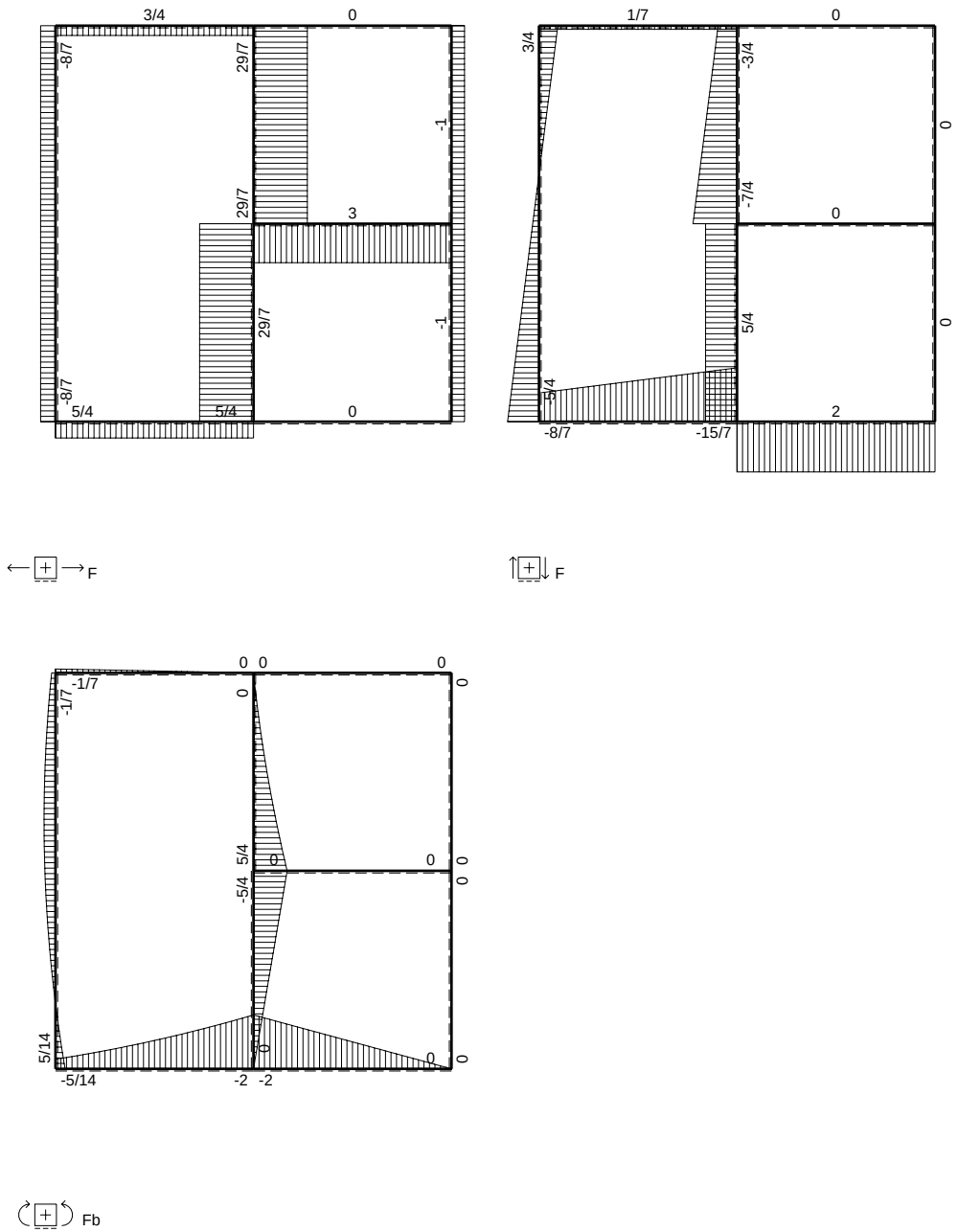
$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

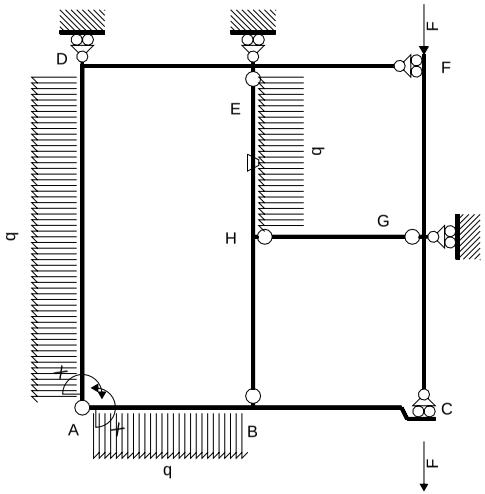
$= \left(-1/6\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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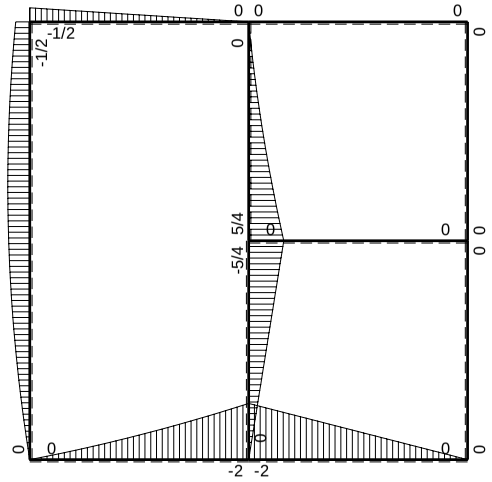
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1/7F
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2F
2F

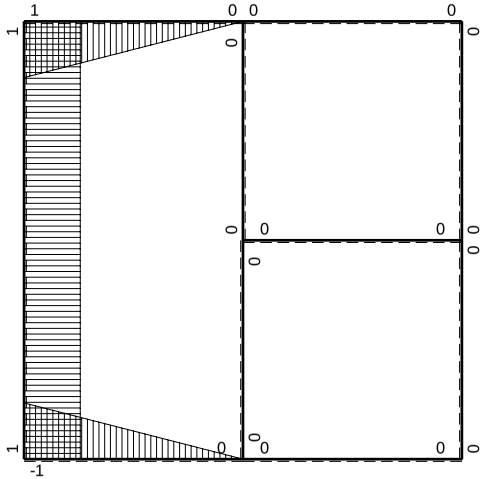

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F
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0
-1/7
-1/7
0
0
5/4
0
0
0 0
-5/4
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-5/4Fb+5/4Fx	0	0	0	0	0+0	0
BH b	0	5/4Fx	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 -3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x -3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b -3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

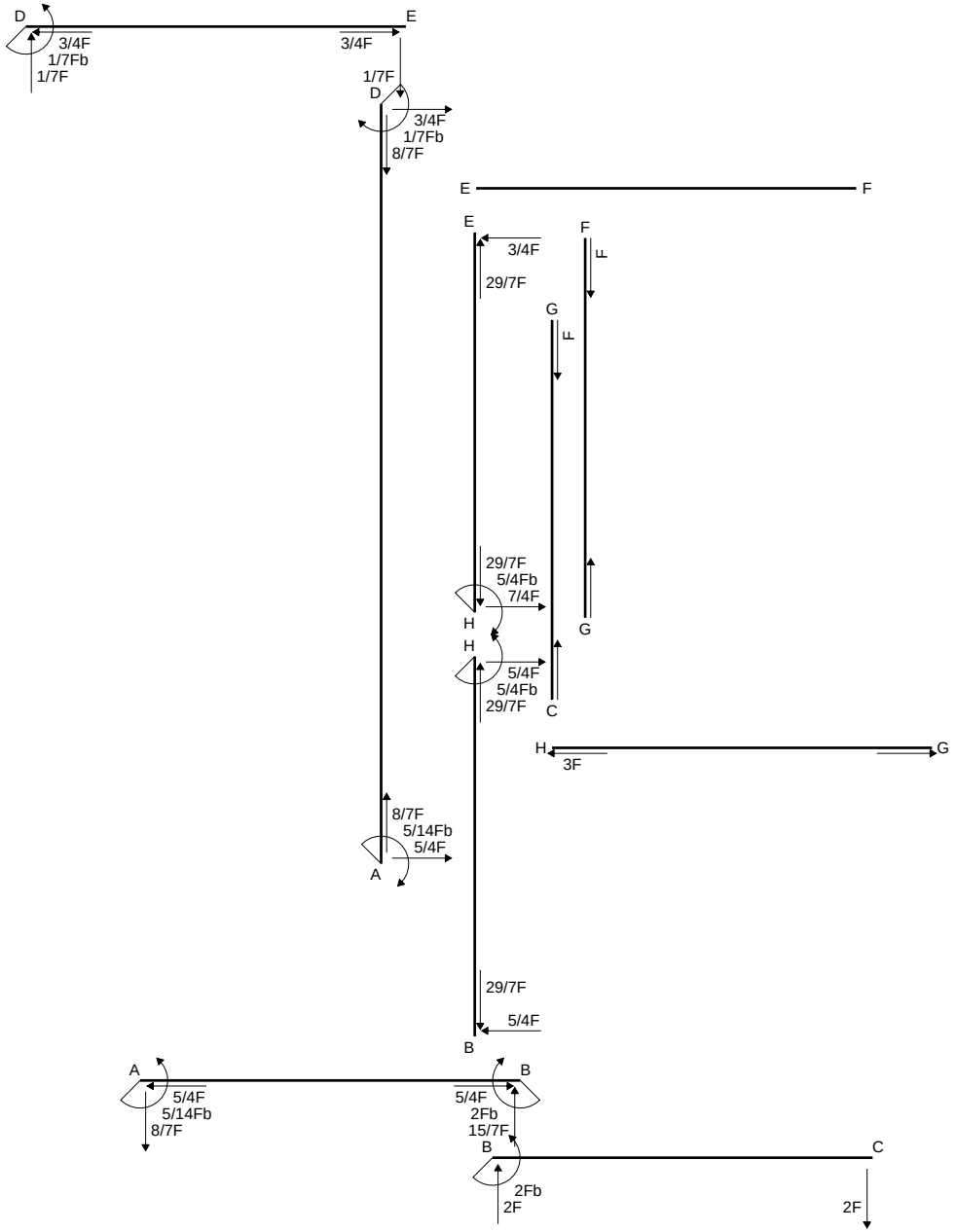
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

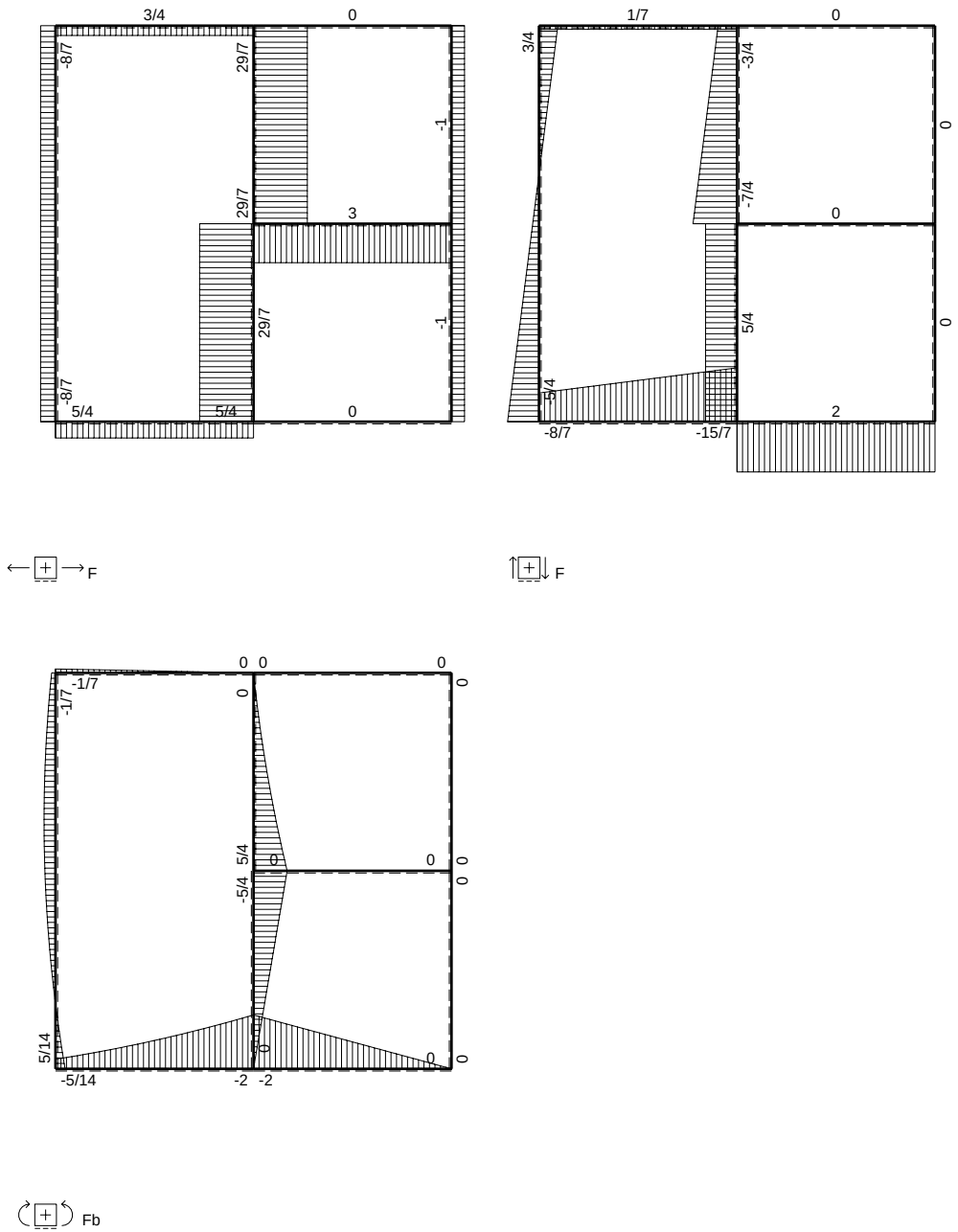
$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

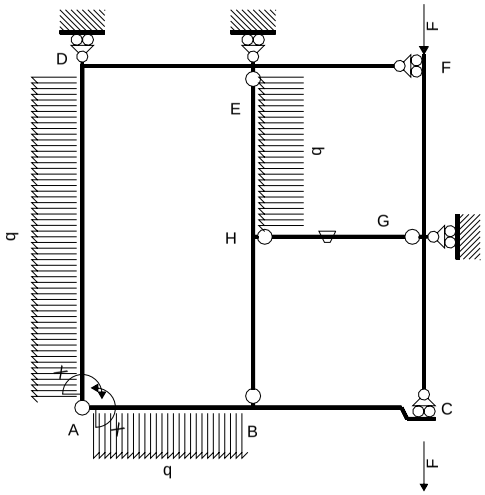
$= \left(-1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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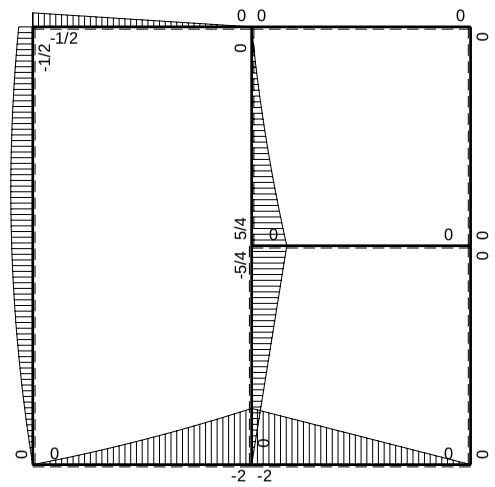
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
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E
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F
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5/4F
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C
H
G
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B
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B
C
2Fb
2F
2F

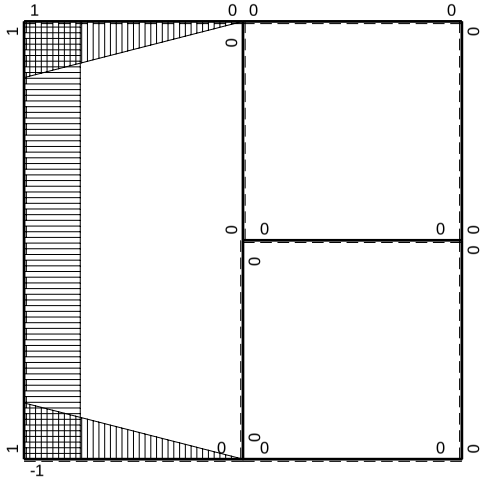

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-8/7
29/7
-1
29/7
3
29/7
-1
-8/7
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F
1/7
0
3/4
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-7/4
0
5/4
0
5/4
2
-8/7
-15/7
F
0 0
0
-1/7
-1/7
0
0
5/4
0
0
0 0
-5/4
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

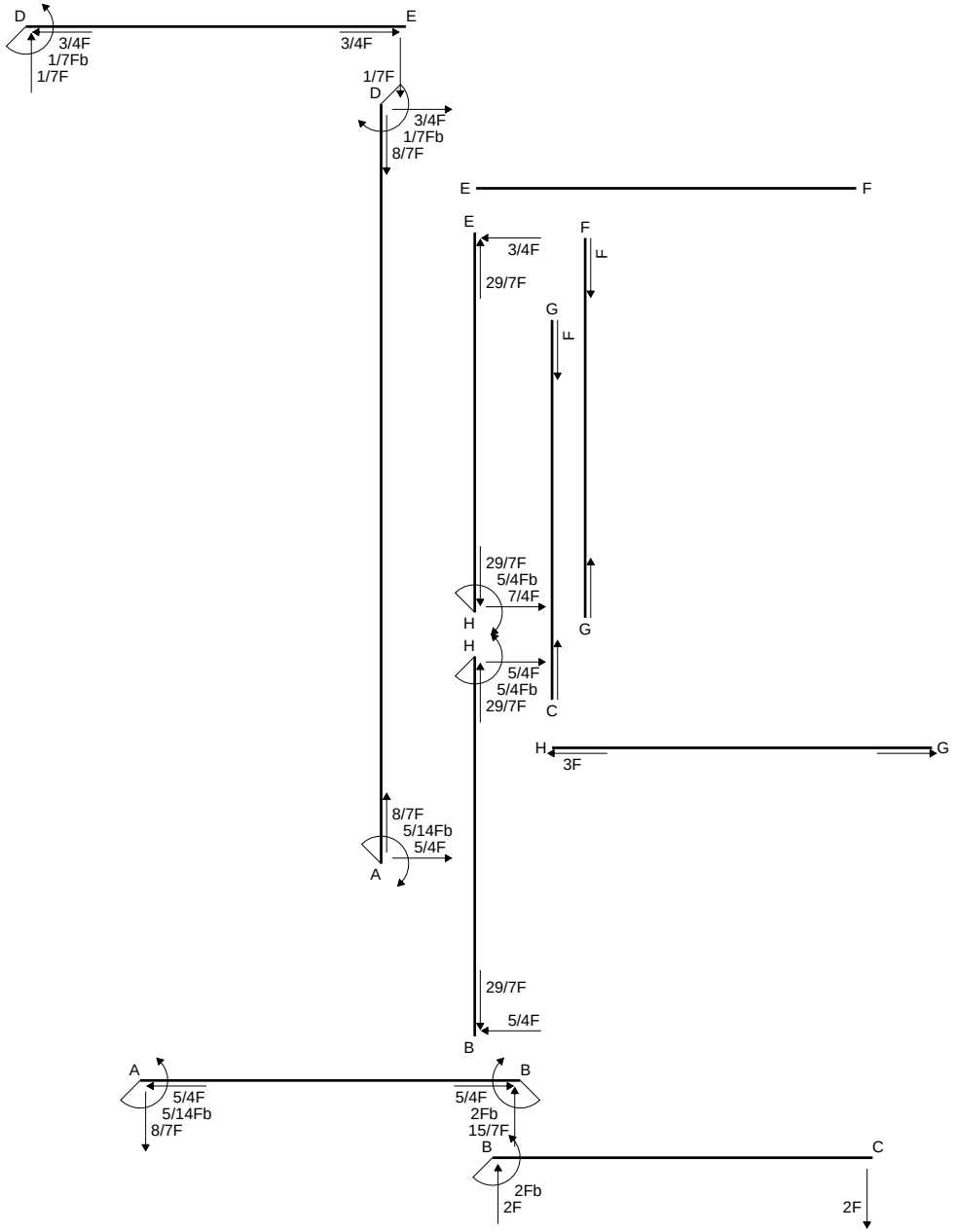
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

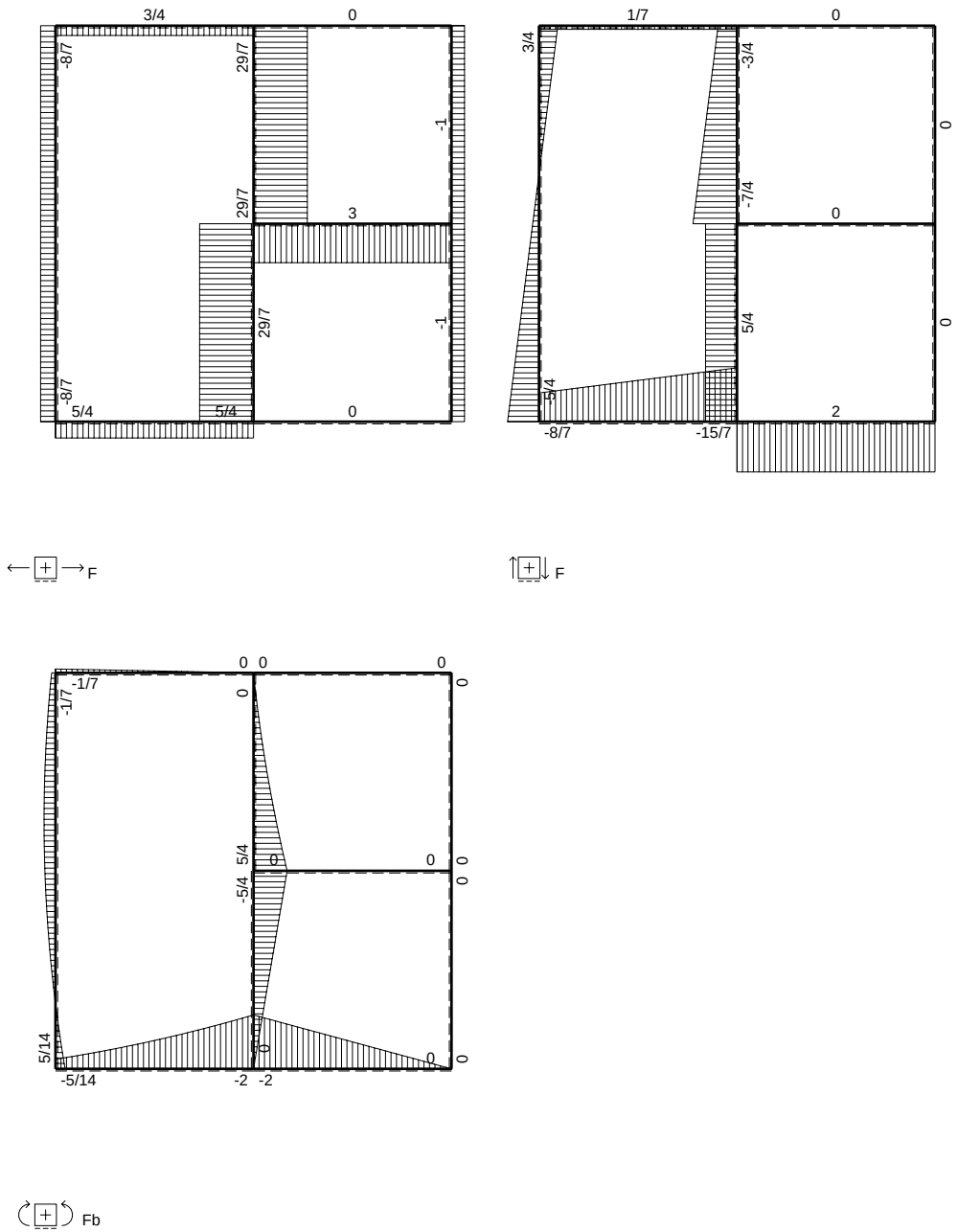
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

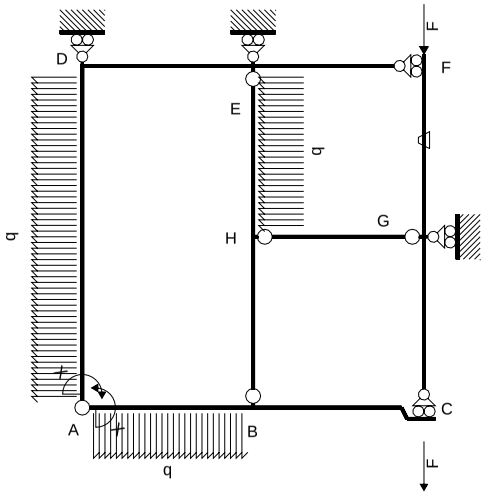
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 211694 ESΣ03.xxxx.156

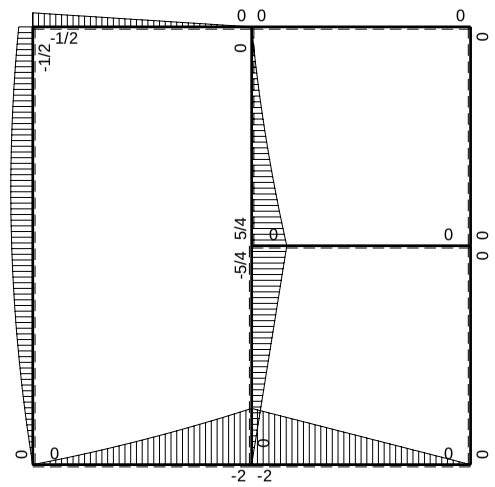
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
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G
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29/7F
C
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G
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A
29/7F
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B
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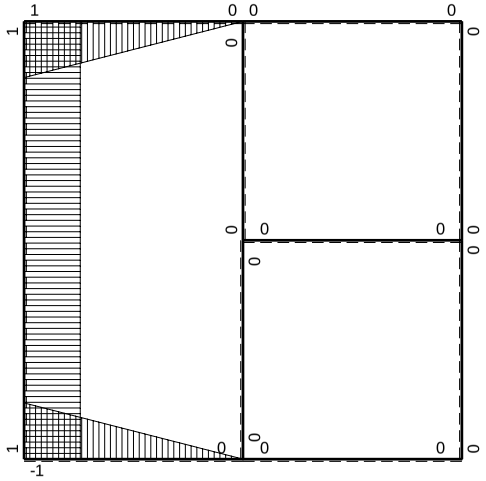

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-3/4
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5/4
0
5/4
2
-8/7
-15/7
F
0 0
0
-1/7
-1/7
0
0
0
5/4
0
0
0
-5/4
0
0
5/14
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238079 ESΣ03.xxxx.157

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

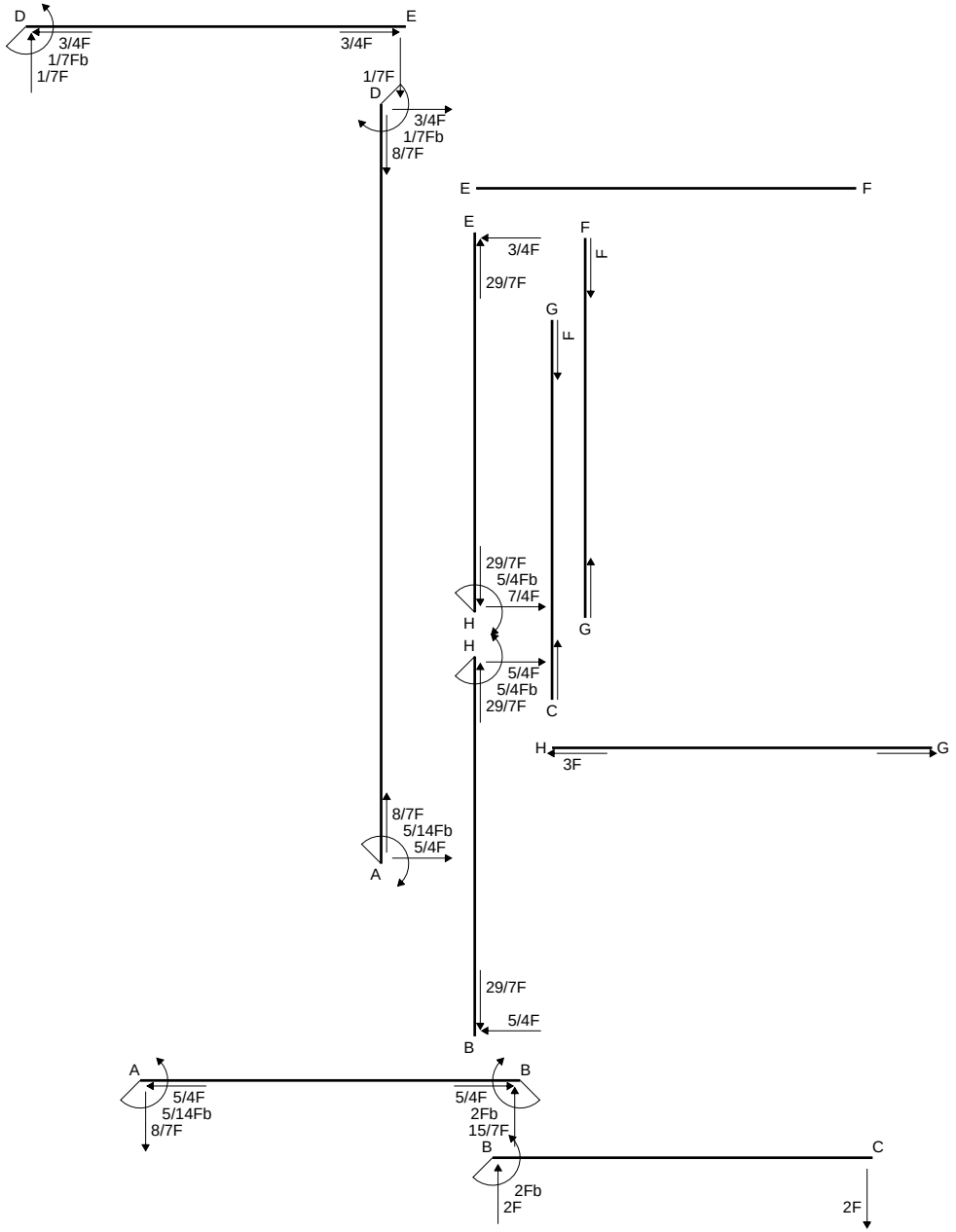
$= (-1/2\ b + 1/2\ b - 1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

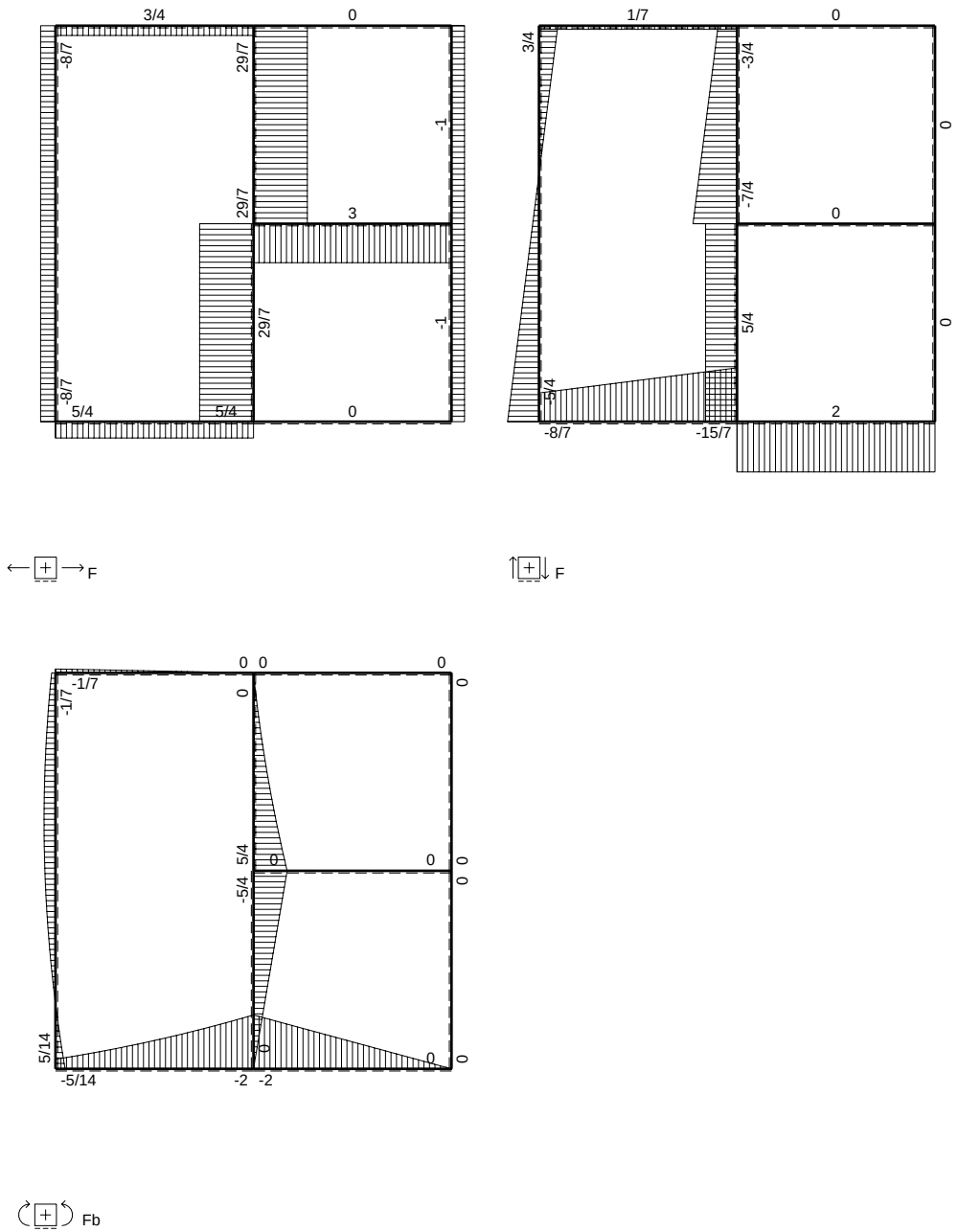
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

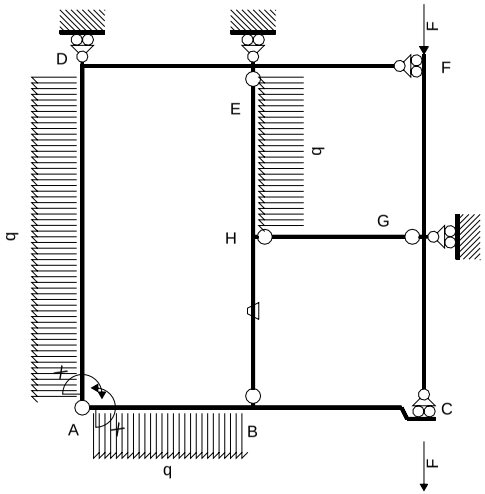
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238079 ESΣ03.xxxx.157

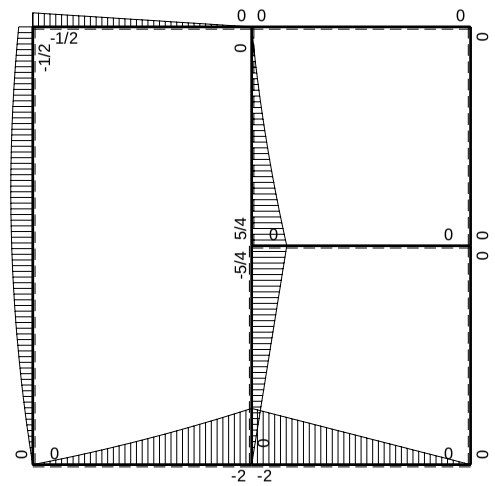
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
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F
E
F
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29/7F
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H
G
H
5/4F
5/4Fb
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C
H
G
3F
8/7F
5/14Fb
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A
29/7F
5/4F
B
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C
2Fb
2F
2F

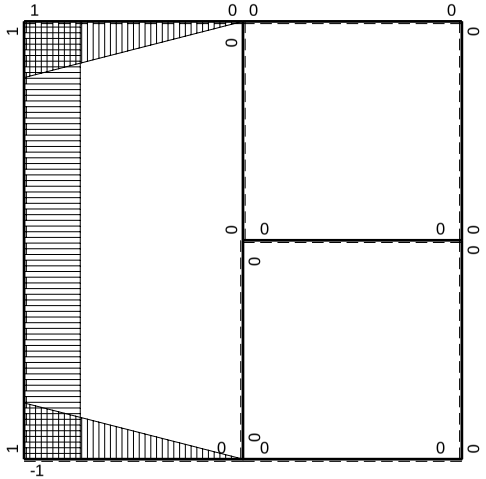

3/4
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-8/7
29/7
-1
29/7
3
29/7
-1
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5/4
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0
F
1/7
0
3/4
-3/4
0
-7/4
0
5/4
0
5/4
2
-8/7
-15/7
F
0 0
0
-1/7
-1/7
0
0
5/4
0
0
0 0
-5/4
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-5/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
BH b	0	5/4Fx	Fb/EJ	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

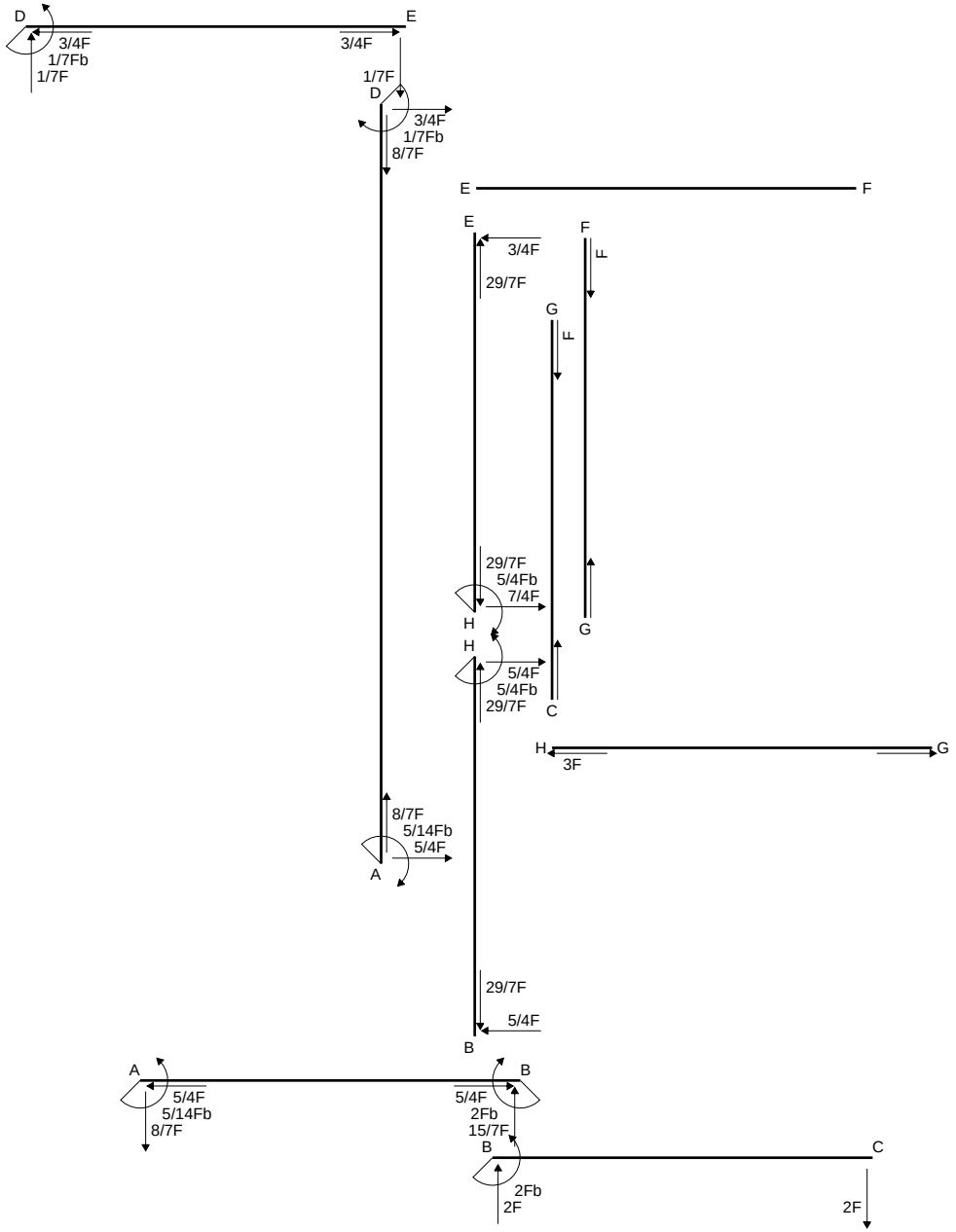
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

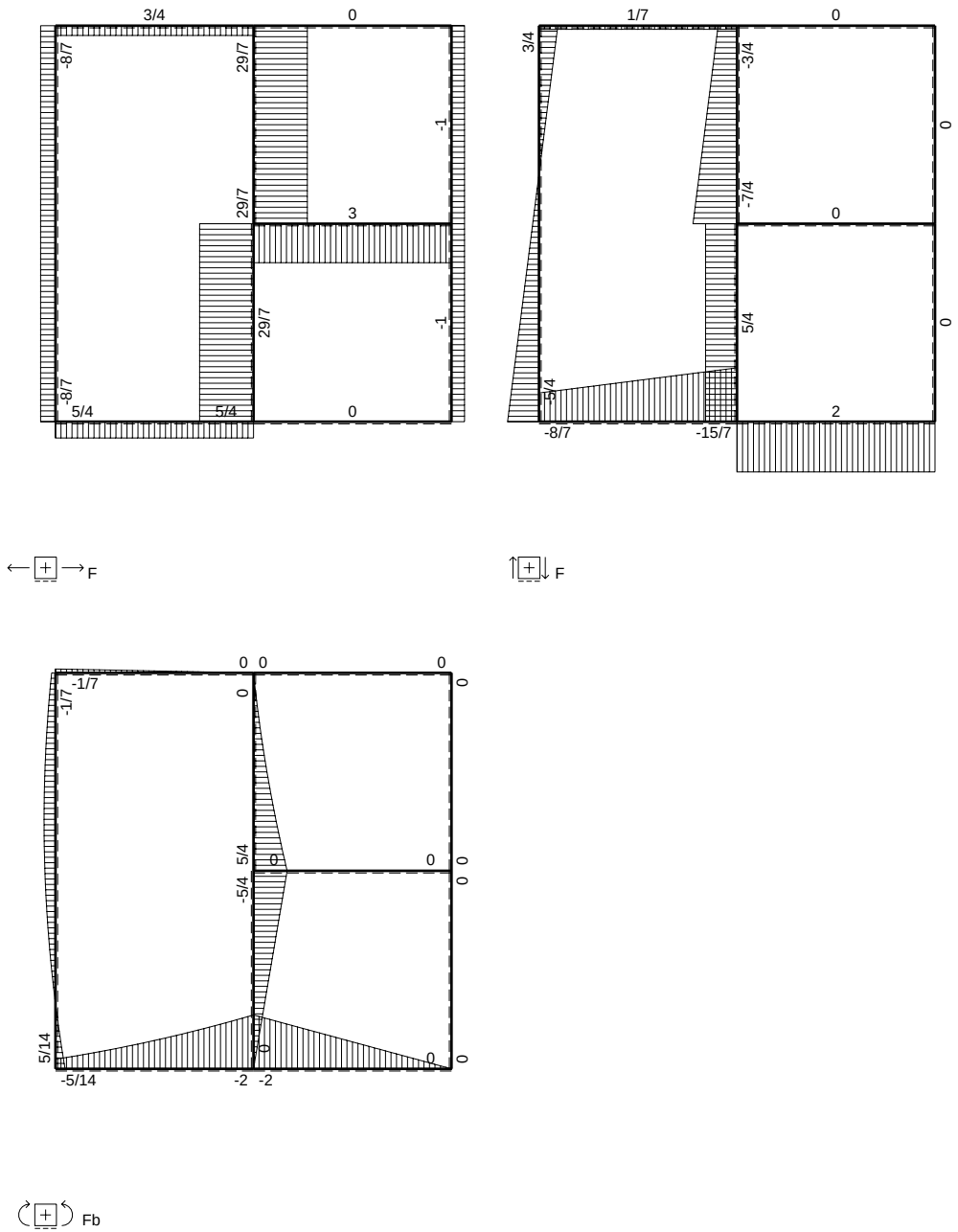
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

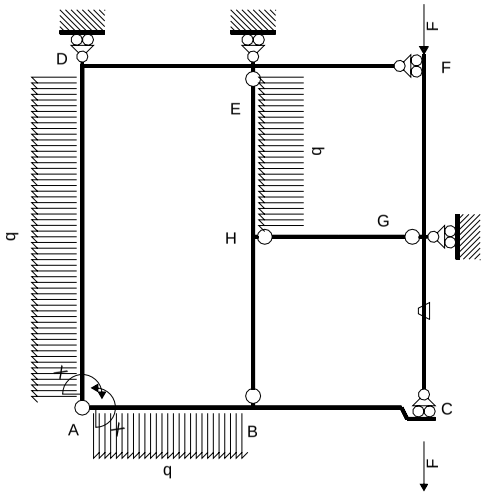
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 237078 ESΣ03.xxxx.158

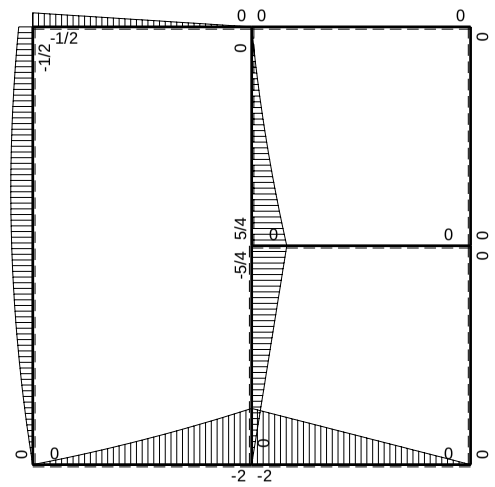
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
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F
G
29/7F
5/4Fb
7/4F
H
H
5/4F
5/4Fb
29/7F
C
H
G
3F
8/7F
5/14Fb
5/4F
A
29/7F
5/4F
B
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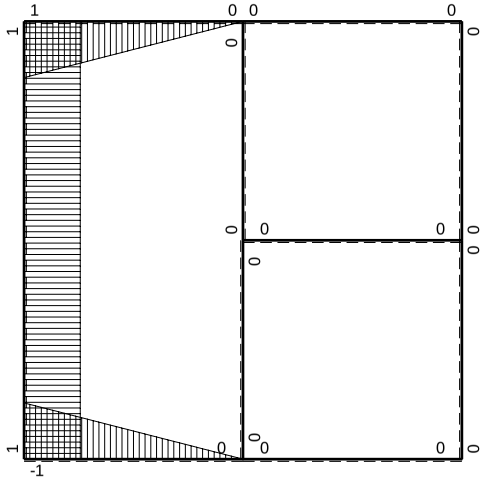

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-3/4
0
-7/4
0
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0
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-8/7
-15/7
2
F
0 0
0
-1/7
-1/7
0
0
0
5/4
0
0
-5/4
0
0
0
5/14
0
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982585 ESΣ03.xxxx.159

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-5/4Fb+5/4Fx	0	0	0	0	0+0	0
BH b	0	5/4Fx	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 - 3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x - 3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b - 3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

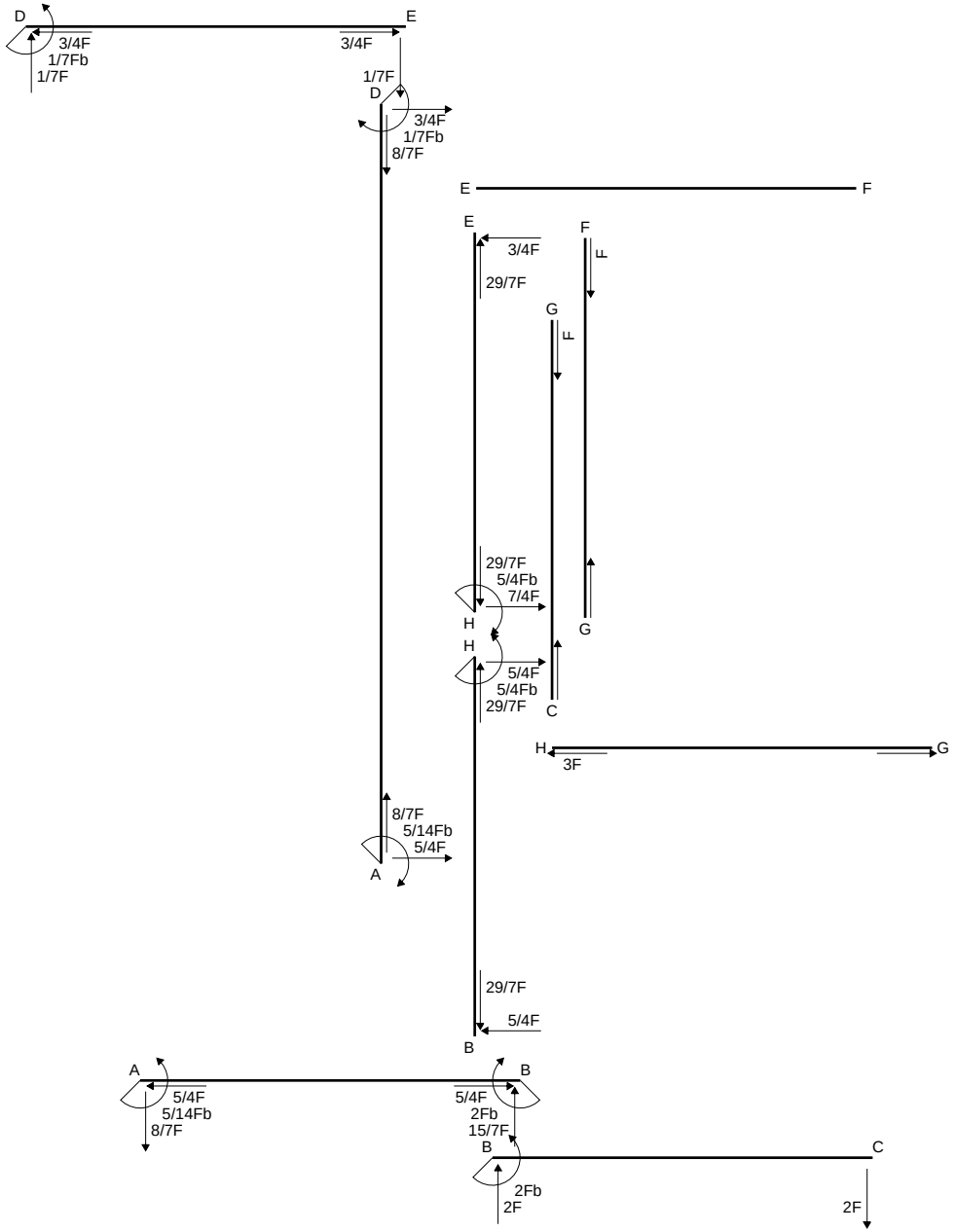
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

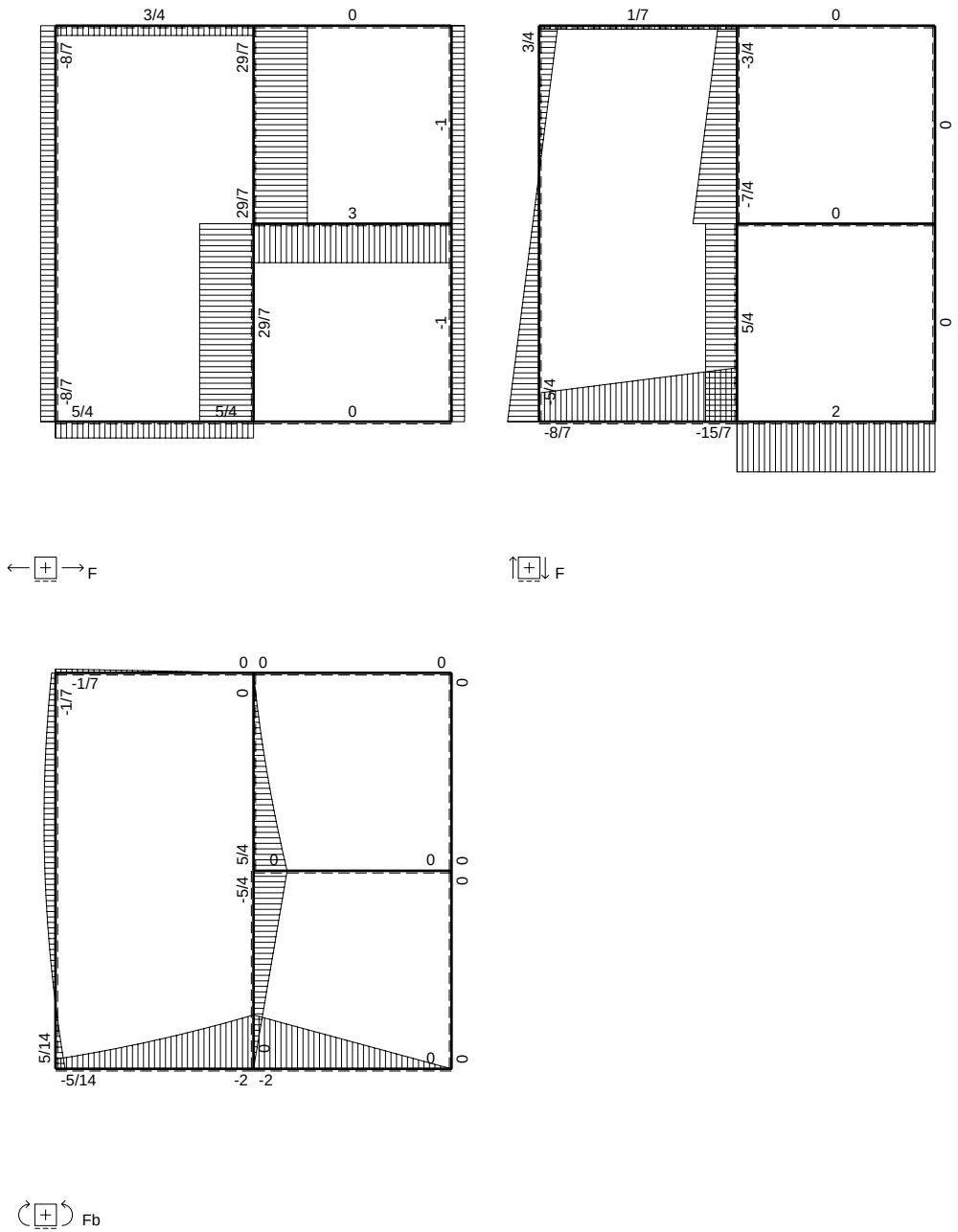
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

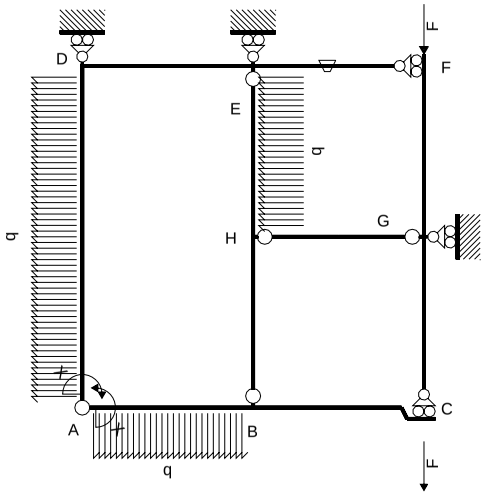
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 982585 ESΣ03.xxxx.159

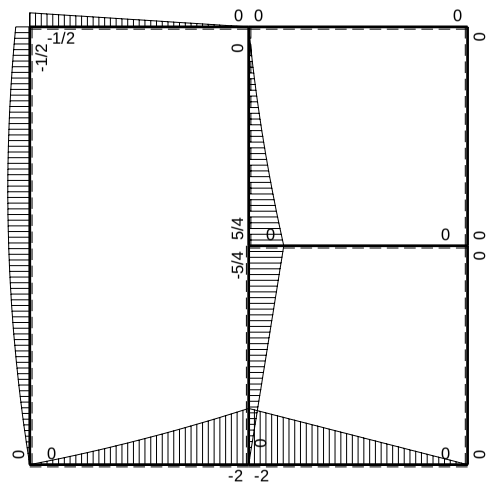
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D
E
3/4F
1/7Fb
1/7F
3/4F
1/7F
D
3/4F
1/7Fb
8/7F
E
F
E
F
3/4F
29/7F
G
F
F
G
29/7F
5/4Fb
7/4F
H
H
5/4F
5/4Fb
29/7F
C
H
G
3F
8/7F
5/14Fb
5/4F
A
29/7F
5/4F
B
A
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B
C
2Fb
2F
2F

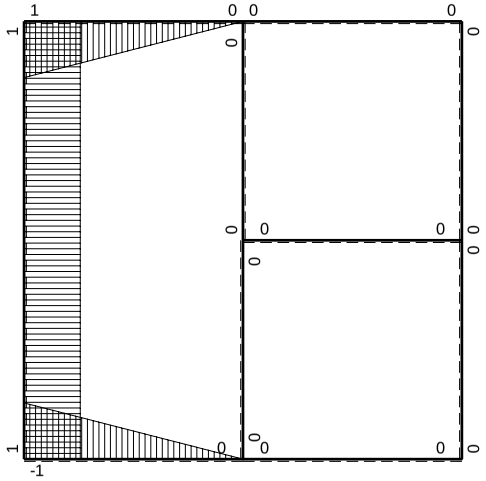

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29/7
-1
29/7
3
29/7
-1
-8/7
5/4
5/4
0
F
1/7
0
3/4
-3/4
0
-7/4
0
5/4
0
5/4
2
-8/7
-15/7
F
0 0
0
-1/7
-1/7
0
0
5/4
0
0
0 0
-5/4
0
5/14
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210489 ESΣ03.xxxx.160

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/4\ x/b +1/2\ x^2/b^2\ \right)\ Fb\ 1/EJ\ dx = \left[-5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-5/2\ b +4/3\ b\ \right)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 -3/4\ x/b +1/2\ x^2/b^2\ \right)\ Fb\ 1/EJ\ dx = \left[-1/2\ x -3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b -3/2\ b +4/3\ b\ \right)\ Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right)\ Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

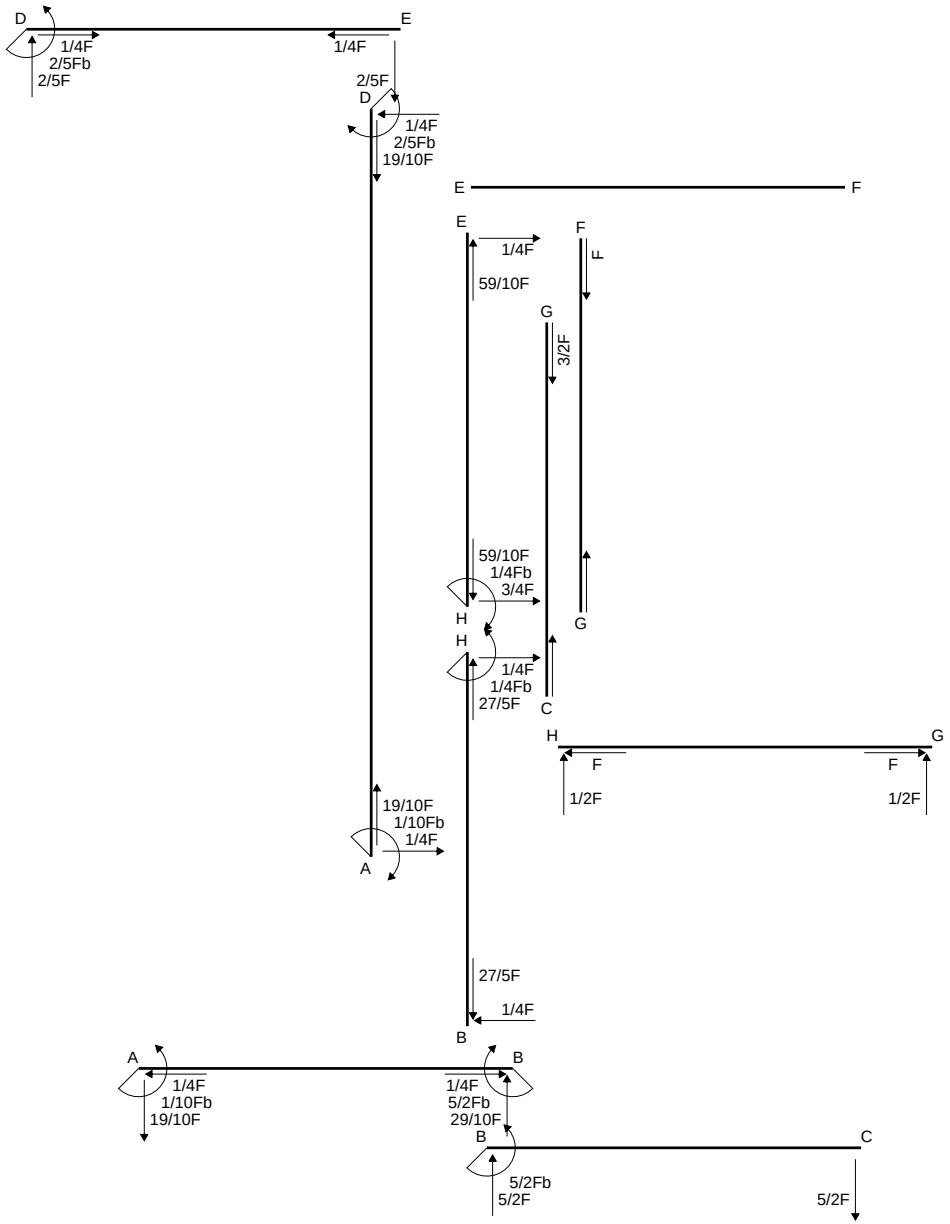
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

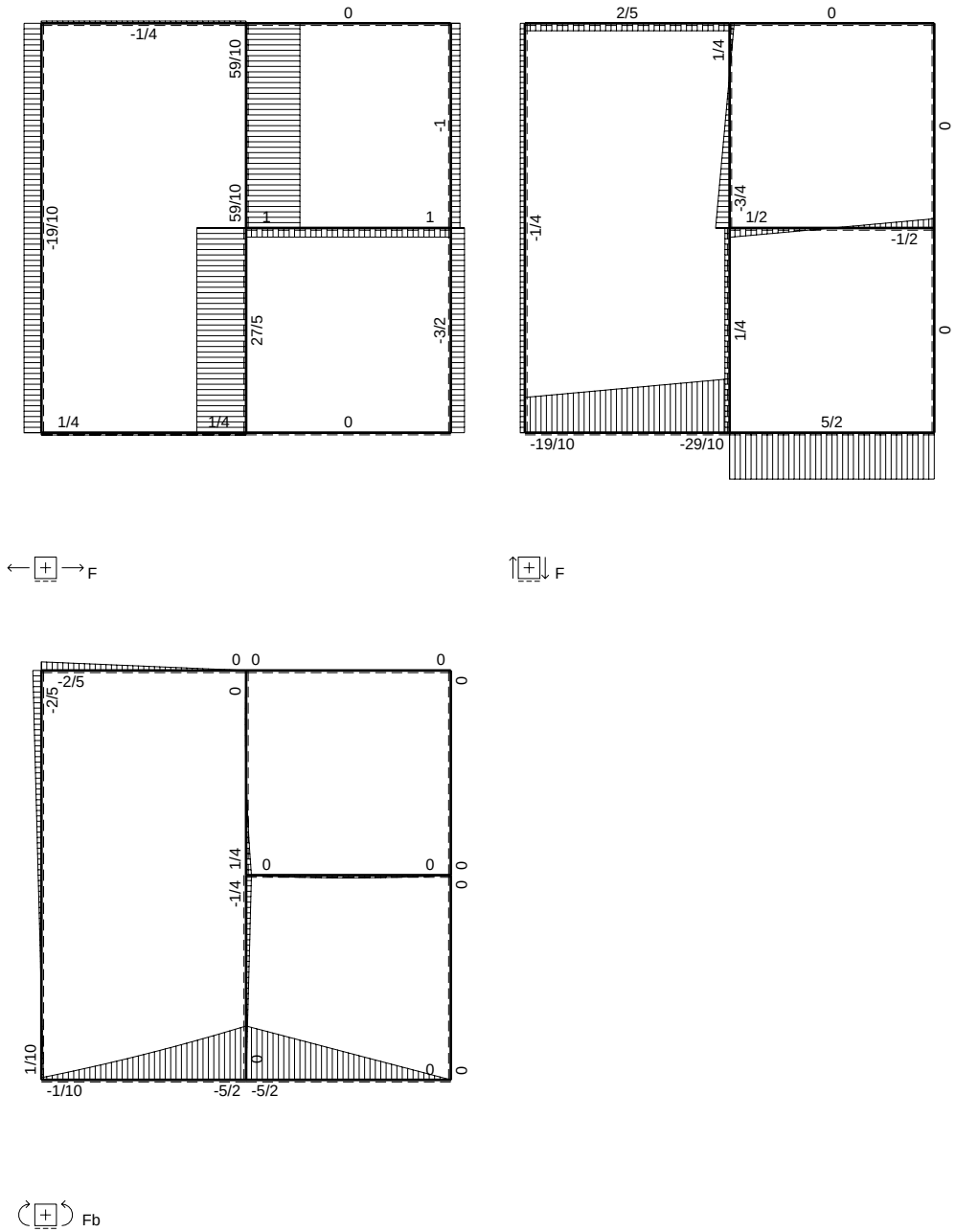
$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right)\ Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

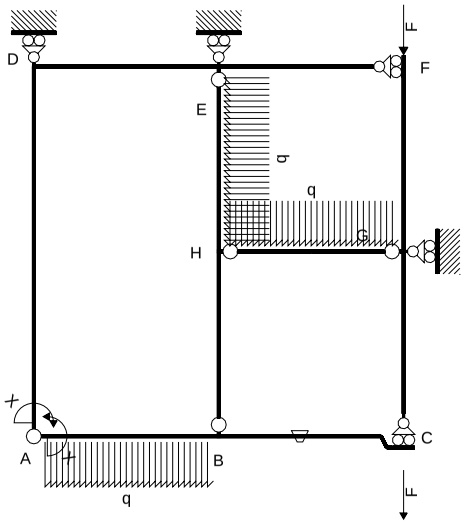
$= \left(-1/6\ b\ \right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 210489 ESΣ03.xxxx.160

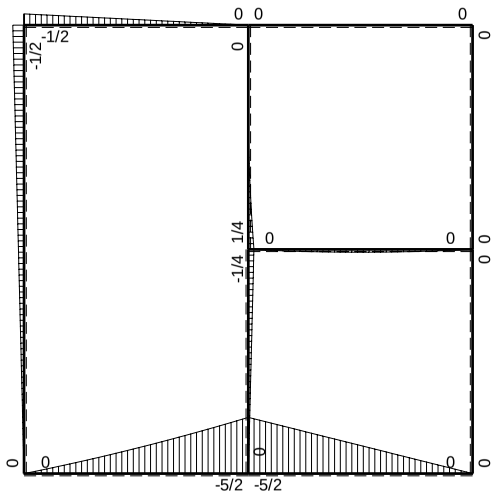
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D
E
1/4F
2/5Fb
2/5F
1/4F
2/5F
D
1/4F
2/5Fb
19/10F
E
F
E
1/4F
59/10F
F
F
G
3/2F
59/10F
1/4Fb
3/4F
H
G
H
1/4F
1/4Fb
27/5F
C
H
G
F
F
1/2F
1/2F
19/10F
1/10Fb
1/4F
A
27/5F
1/4F
B
A
B
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5/2Fb
5/2F
5/2F

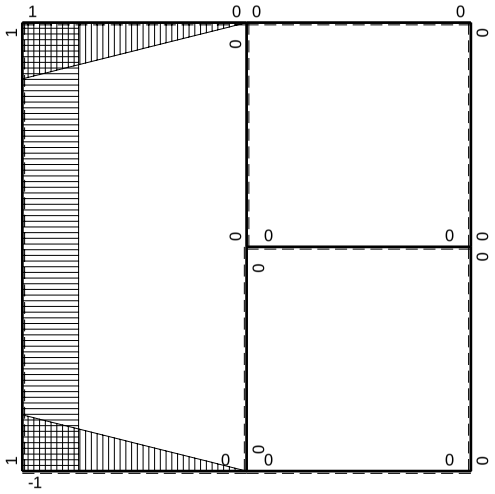

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59/10
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27/5
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F
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0
1/4
-3/4
1/2
-1/4
-1/2
1/4
0
0
5/2
-19/10
-29/10
F
0 0 0
-2/5
-2/5
0
0
0
1/4
0
0
-1/4
0 0
0
0
1/10
0
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237549 ESΣ03.xxxx.161

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$5/2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$-1/4Fx$	0	$-1/4Fx$	0	1	$(-1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb-1/4Fx$	0	$-1/2Fb+1/4Fx$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
BH b	0	$1/4Fx$	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

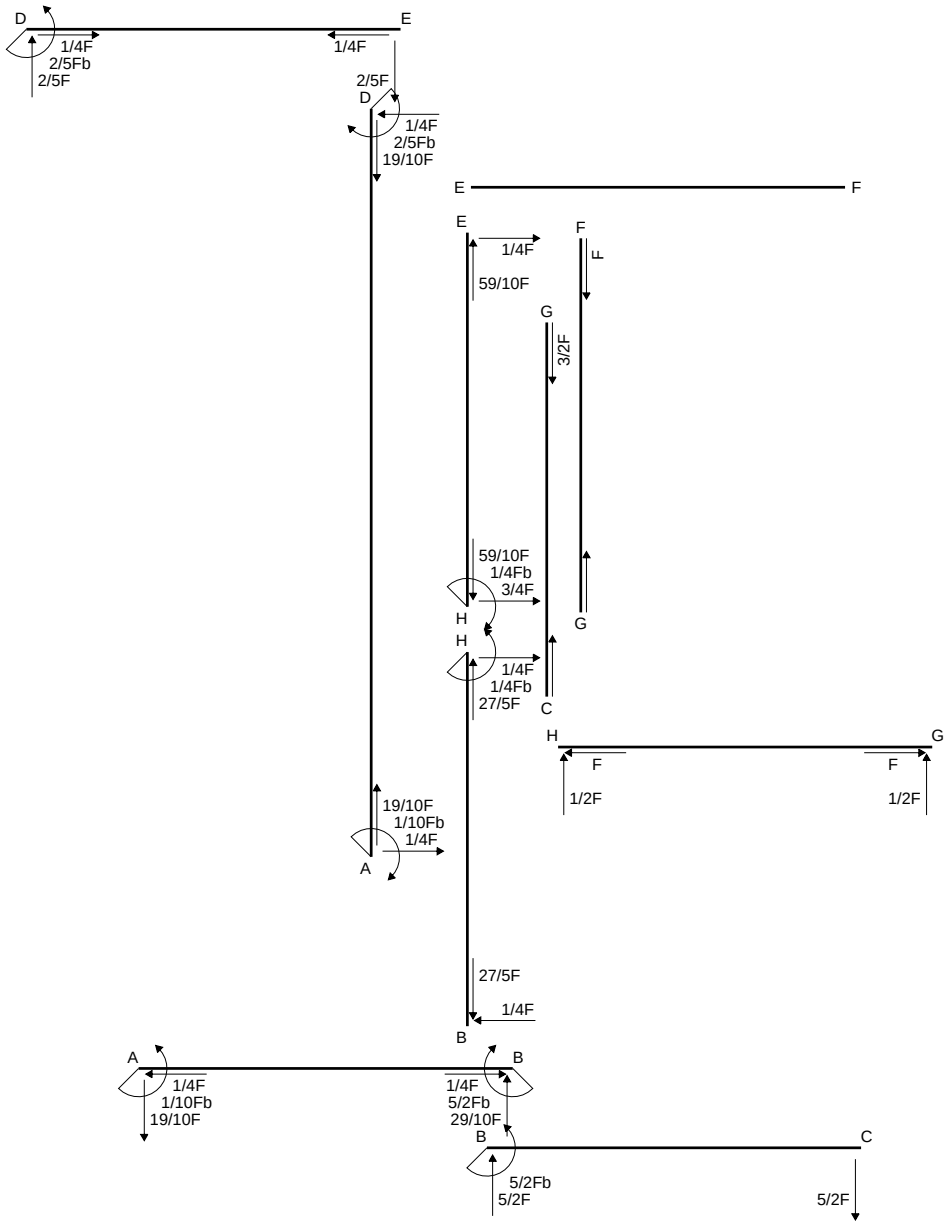
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

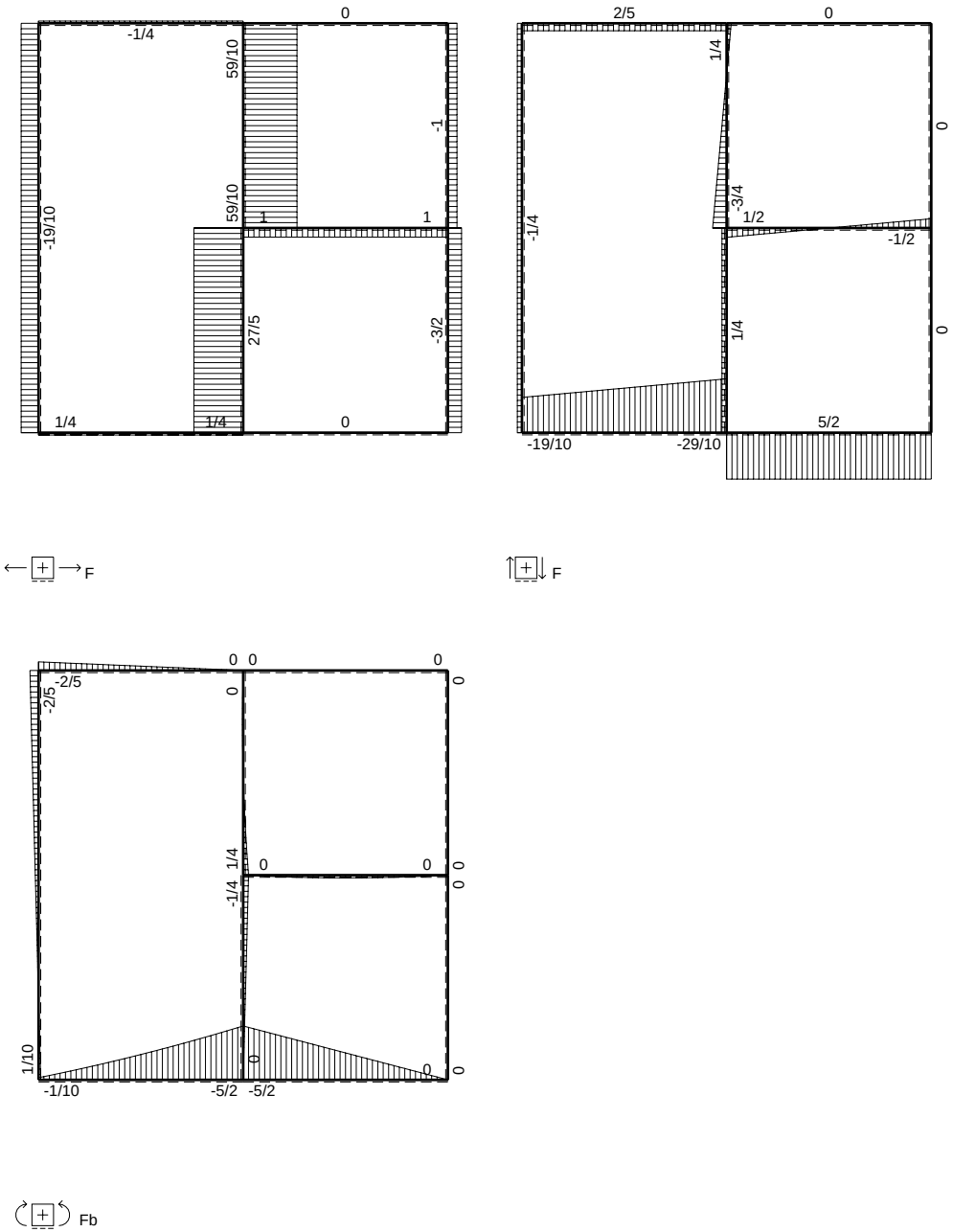
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

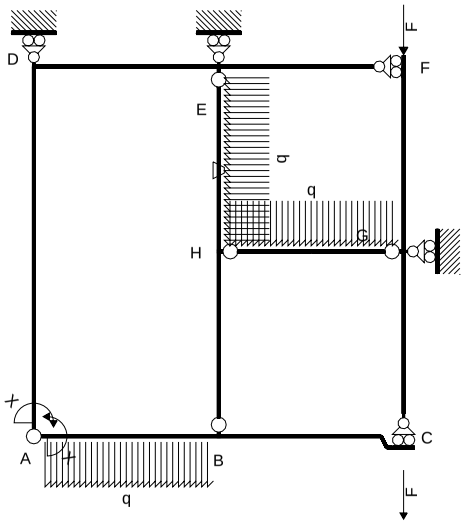
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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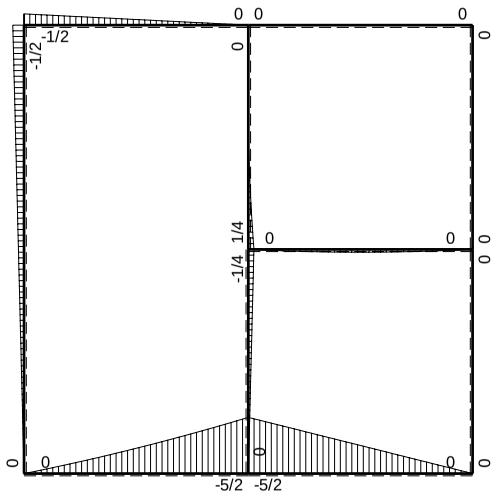
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D
E
1/4F
2/5Fb
2/5F
1/4F
2/5F
D
1/4F
2/5Fb
19/10F
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F
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1/4F
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F
F
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5/2F

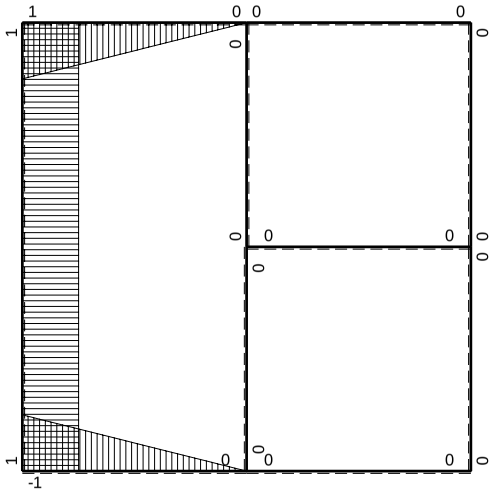

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-19/10
-29/10
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-2/5
0
0
0
1/4
0
0
-1/4
0 0
0
0
1/10
0
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
BH b	0	1/4Fx	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

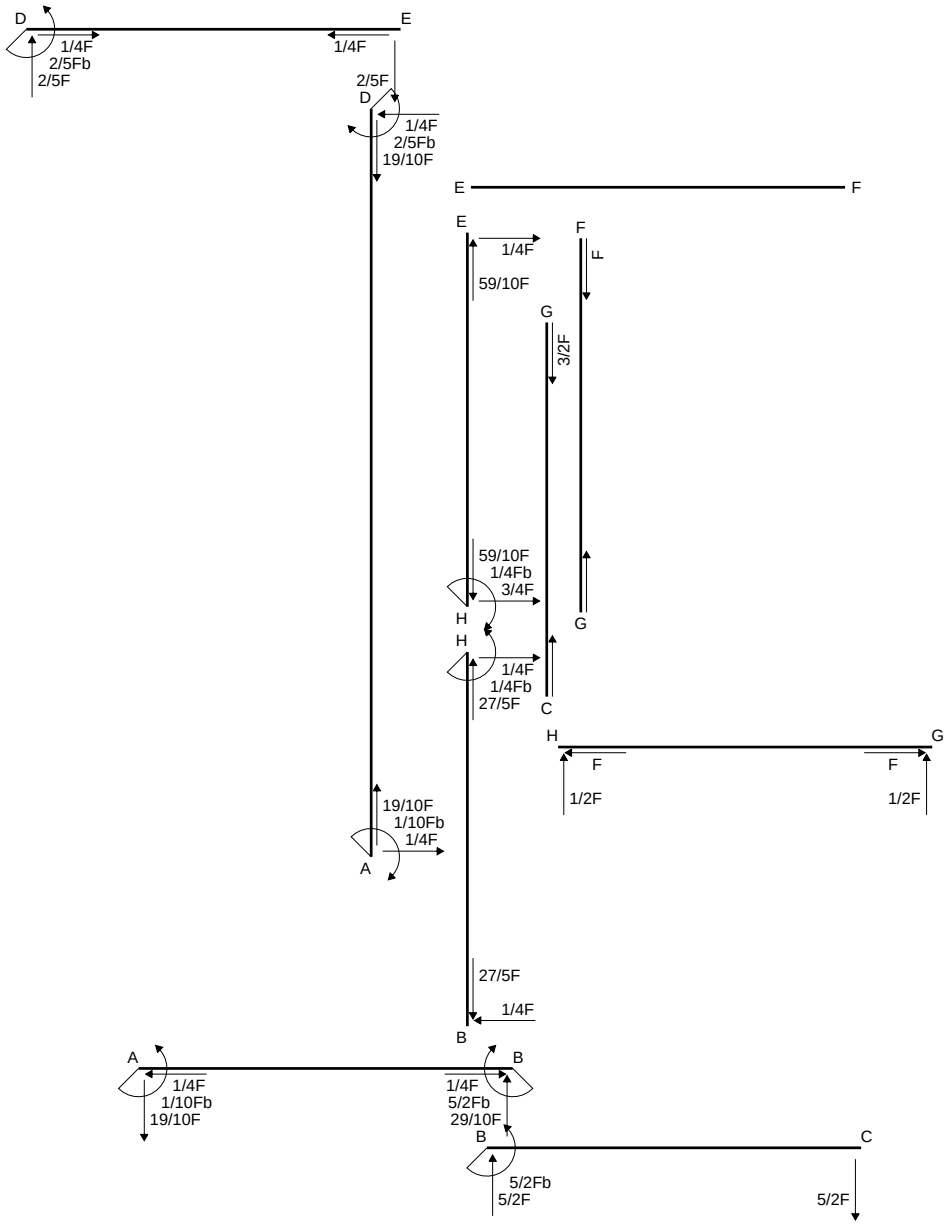
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

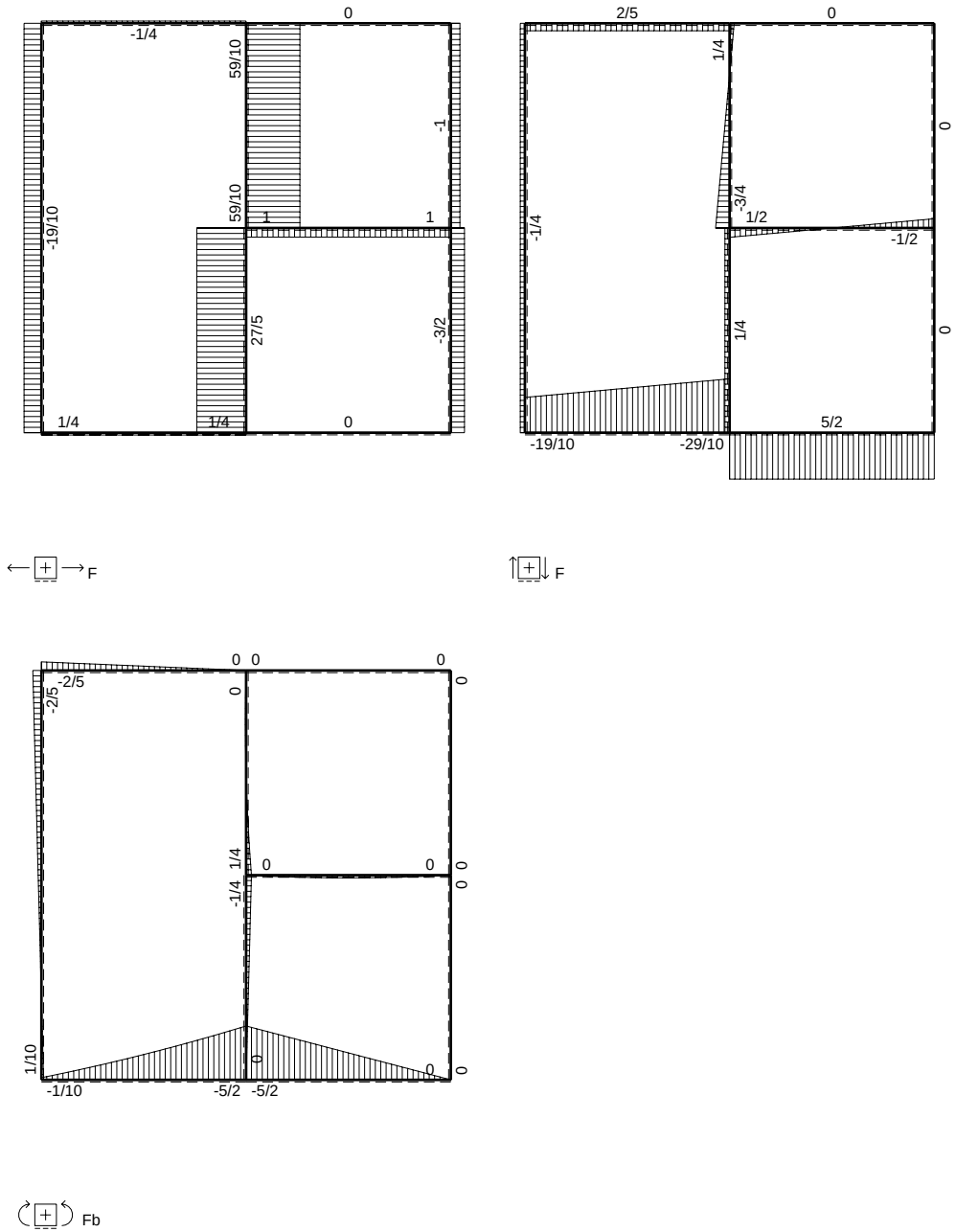
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

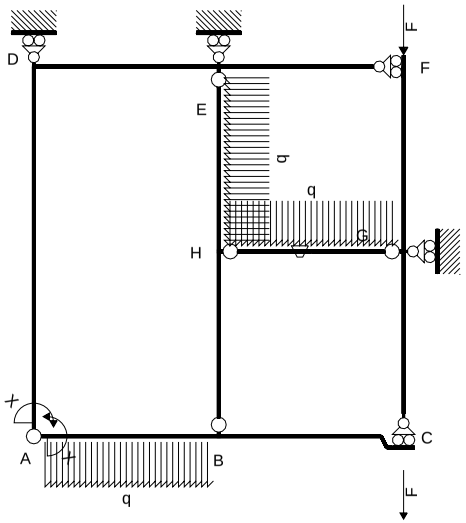
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 966407 ESΣ03.xxxx.162

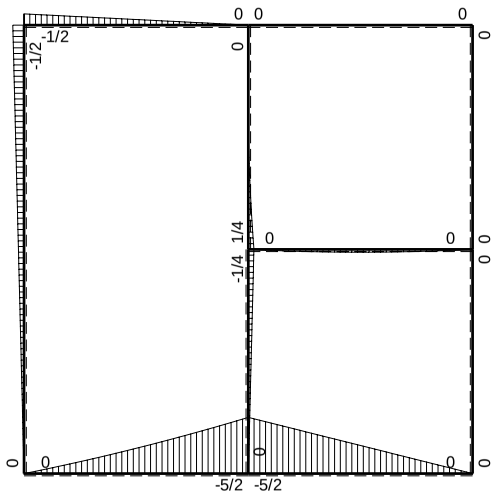
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D
E
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2/5Fb
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2/5F
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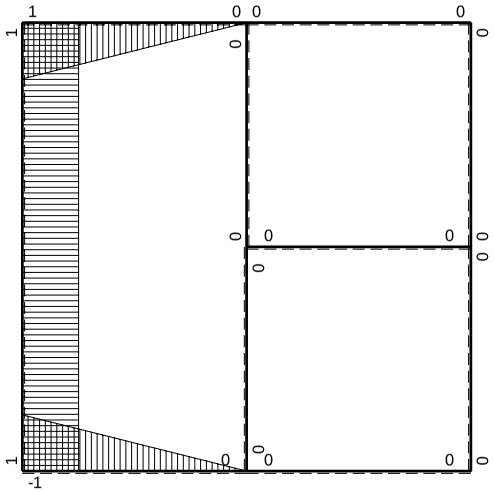

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59/10
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1/4
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1/4
-3/4
1/2
-1/4
-1/2
1/4
0
0
5/2
-19/10
-29/10
F
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-2/5
-2/5
0
0
0
1/4
0
0
-1/4
0 0
0
0
1/10
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 213077 ESΣ03.xxxx.163

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PROCEDIMENTO E RISULTATI 213077 ESΣ03.xxxx.163

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
BH b	0	1/4Fx	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

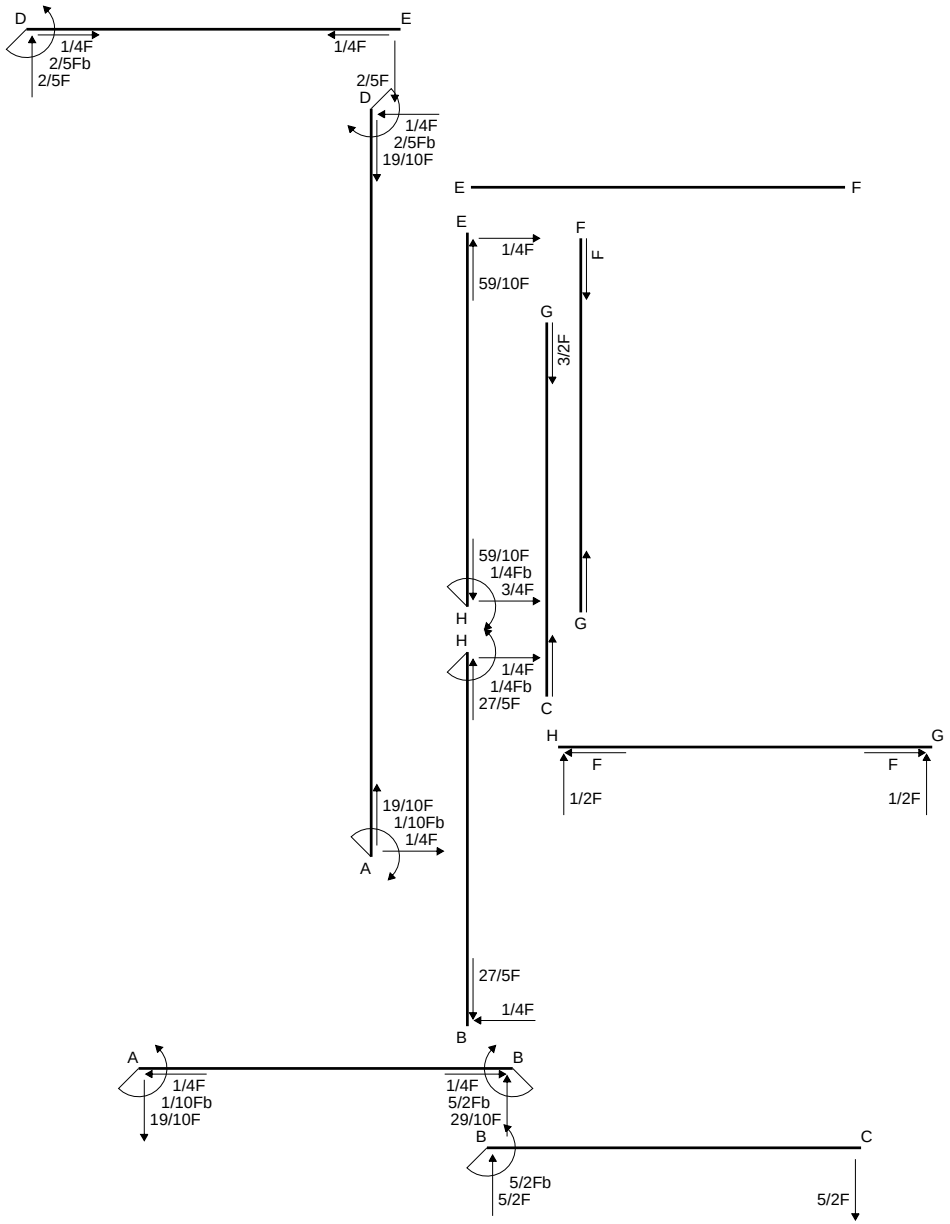
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

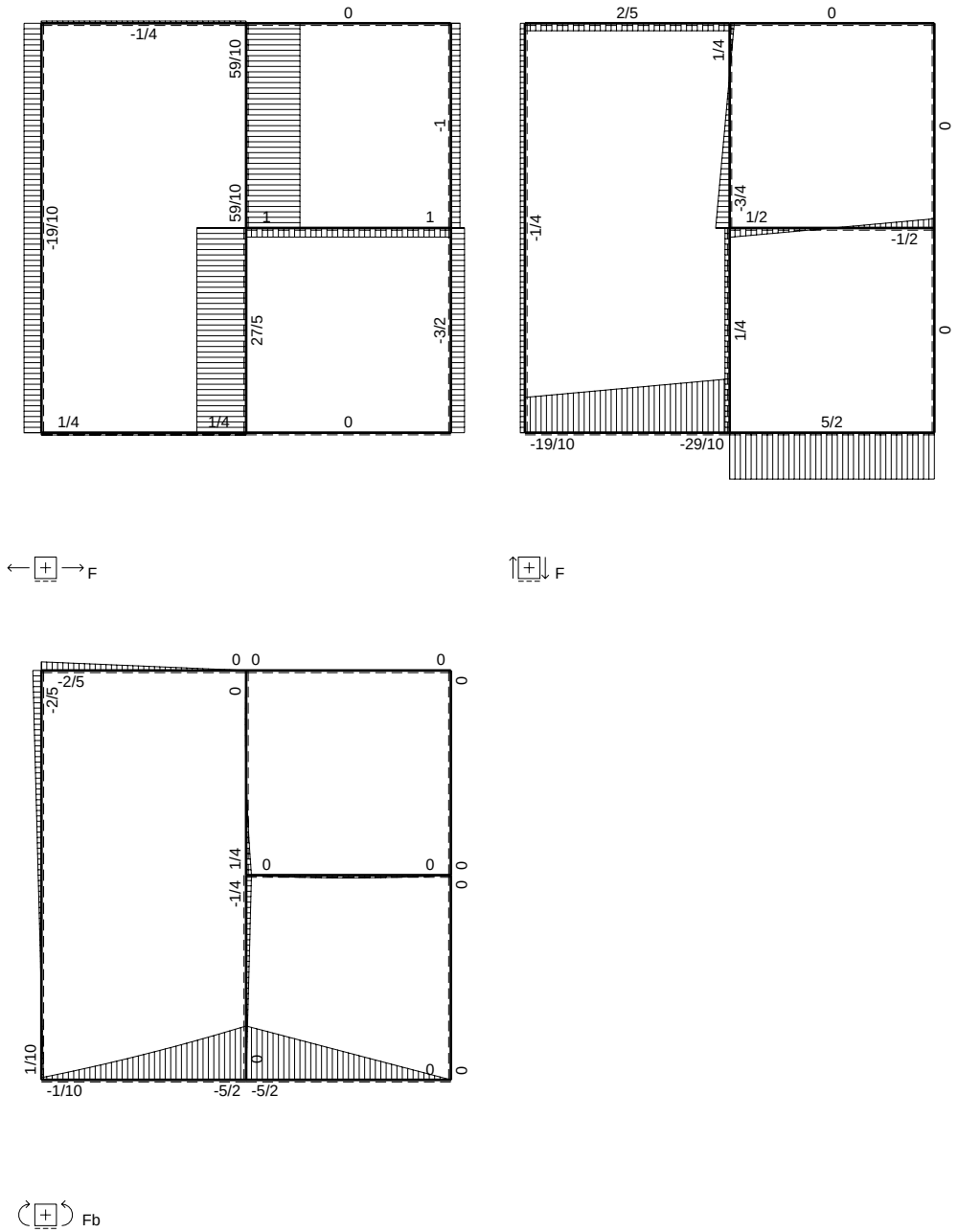
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

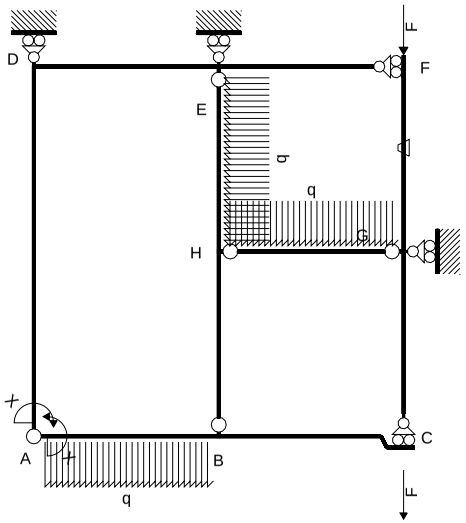
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 213077 ESΣ03.xxxx.163

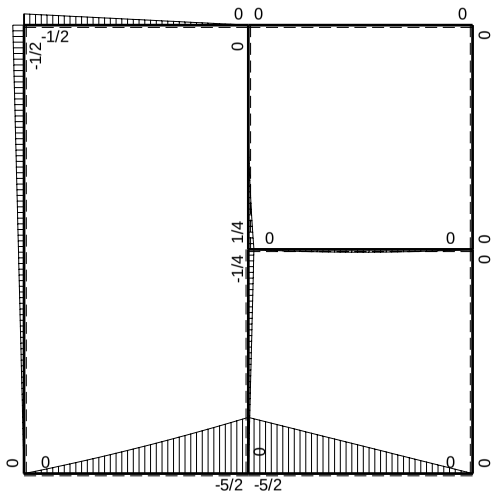
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2/5Fb
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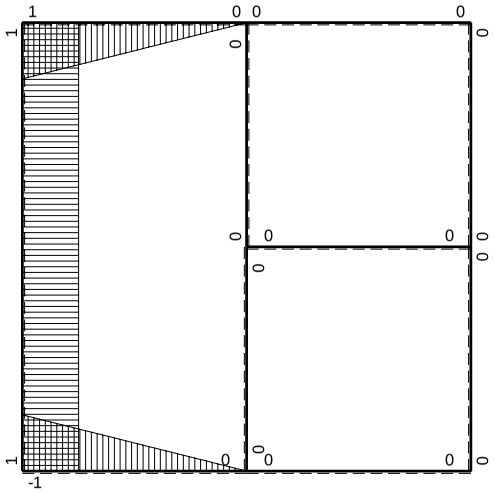

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-29/10
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-2/5
-2/5
0
0
0
1/4
0
0
-1/4
0 0
0
0
1/10
0
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-1/4Fx$	0	$-1/4Fx$	0	1	$(-1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb-1/4Fx$	0	$-1/2Fb+1/4Fx$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
BH b	0	$1/4Fx$	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

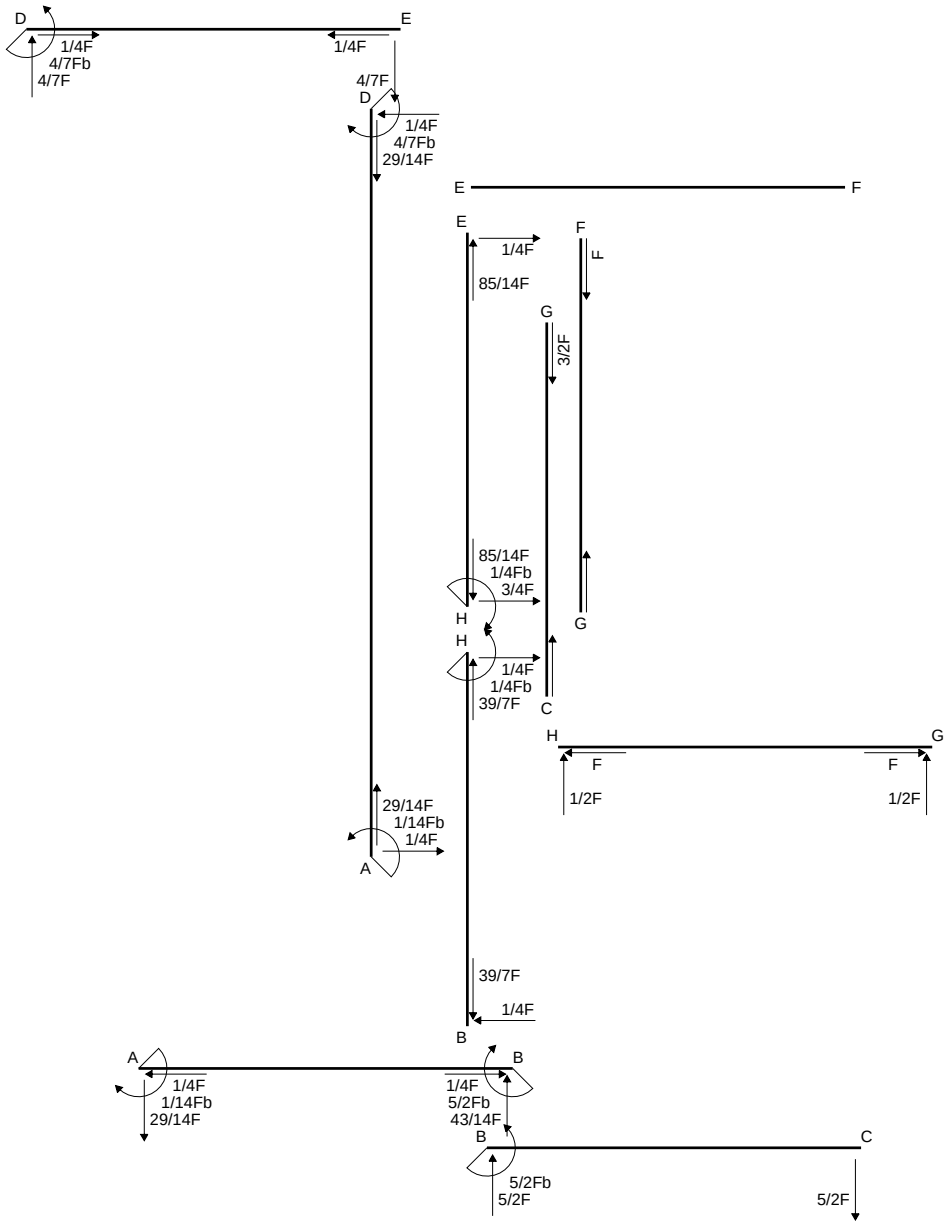
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

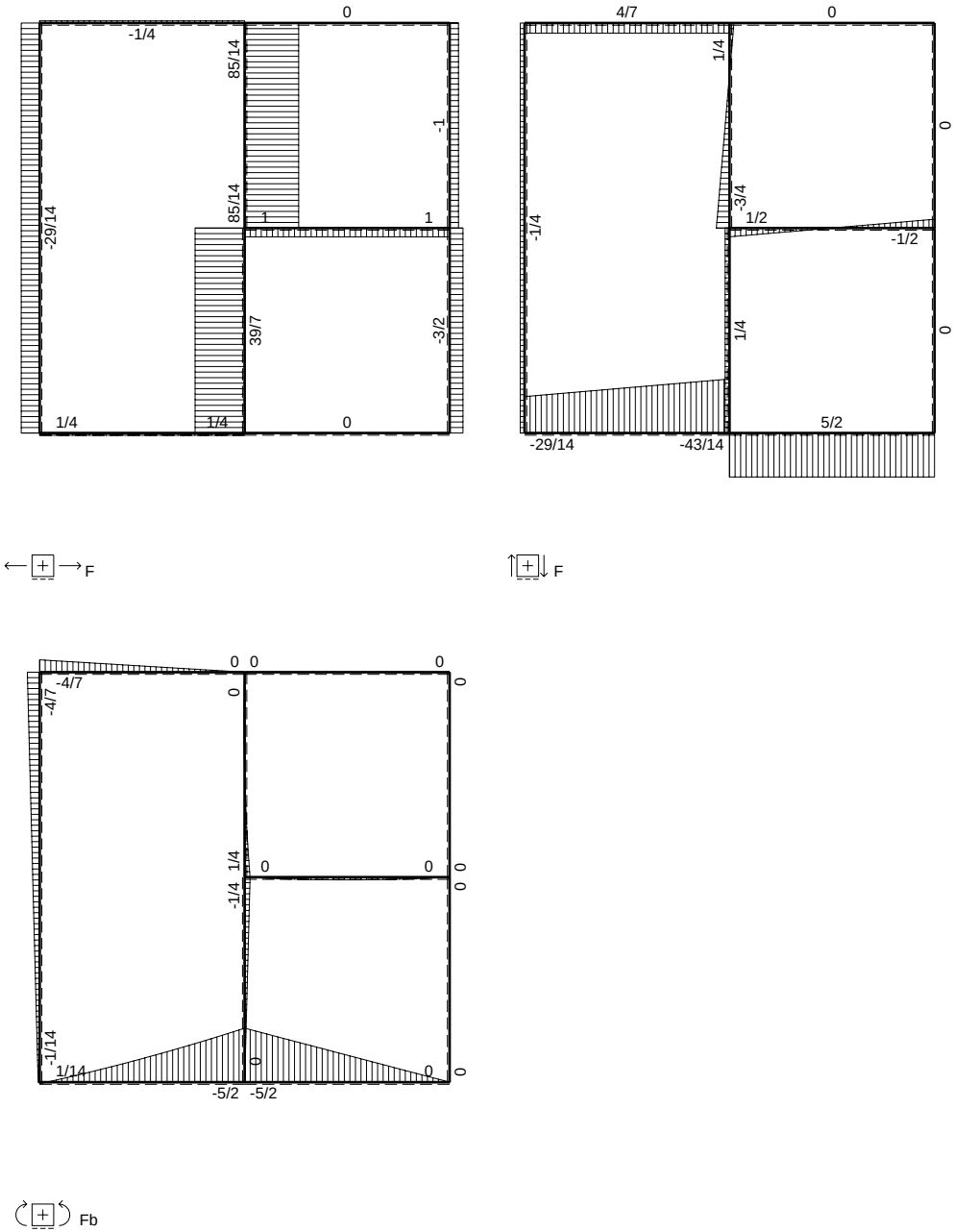
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

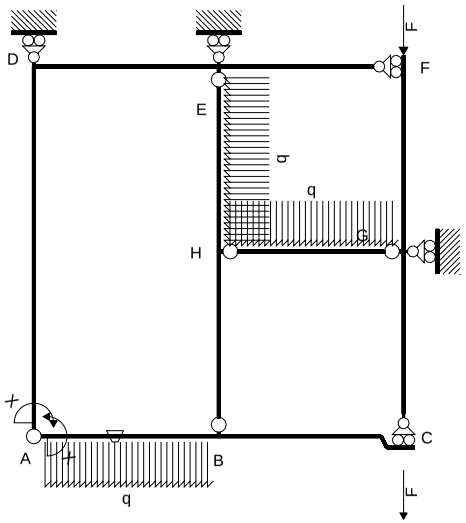
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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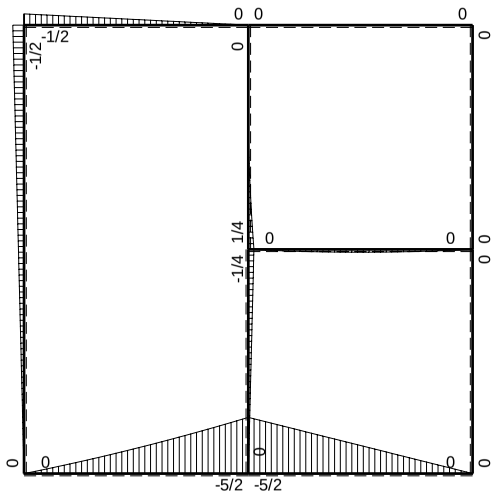
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D
E
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4/7Fb
4/7F
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4/7F
D
1/4F
4/7Fb
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5/2F

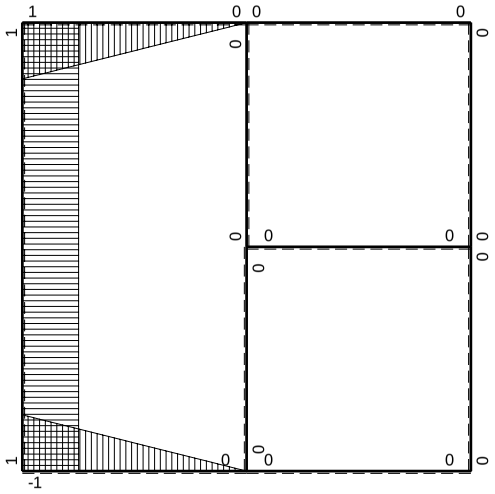

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0
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0
0
0
-1/14
0
1/14
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$	$(3/8+1/2)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	$5/2Fx-3Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
BH b	0	1/4Fx	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b)\ \theta\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [\ x - 1/2\ x^2/b\ ]_o^b\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + (\ b - 1/2\ b\ )\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\,x/b)\ \theta\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b)\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\,b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

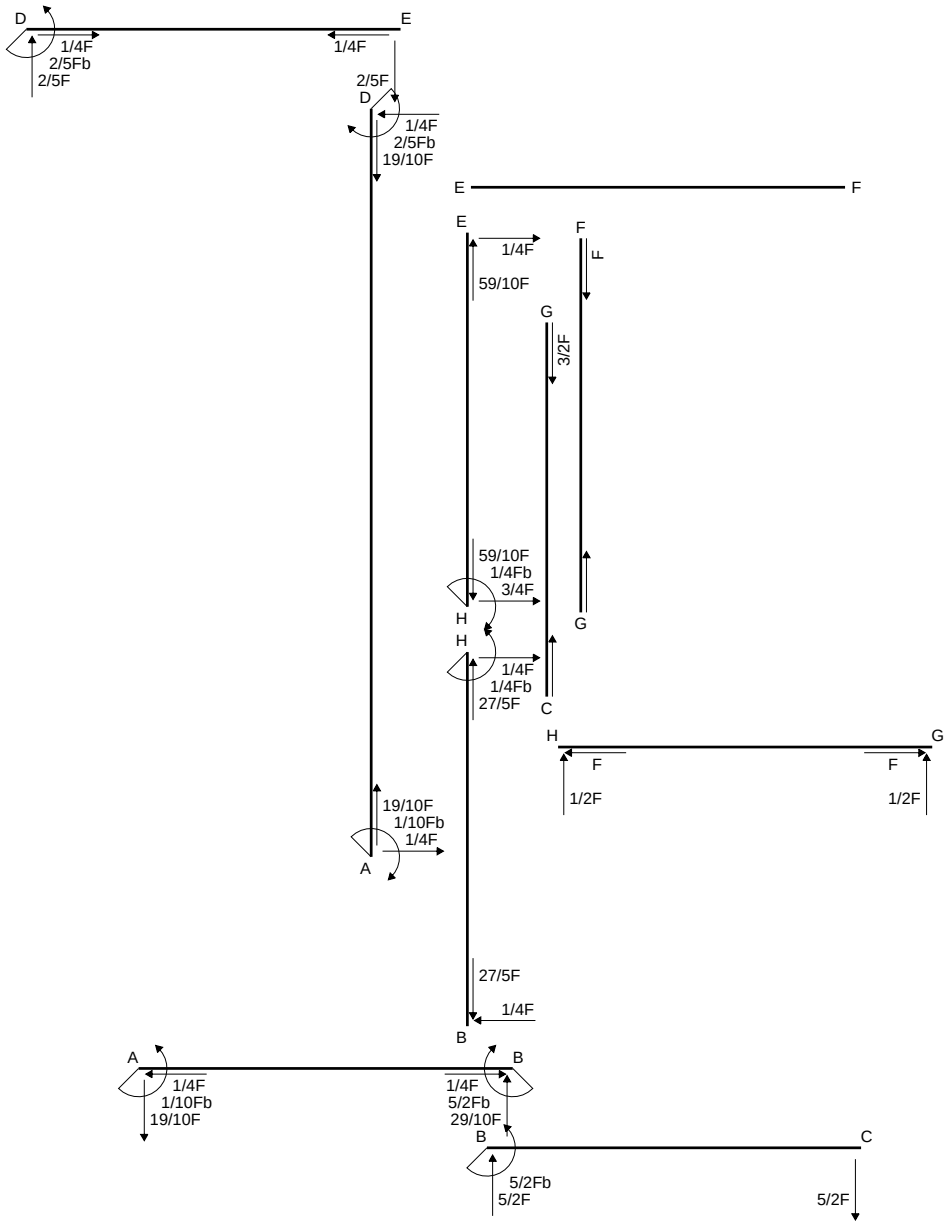
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

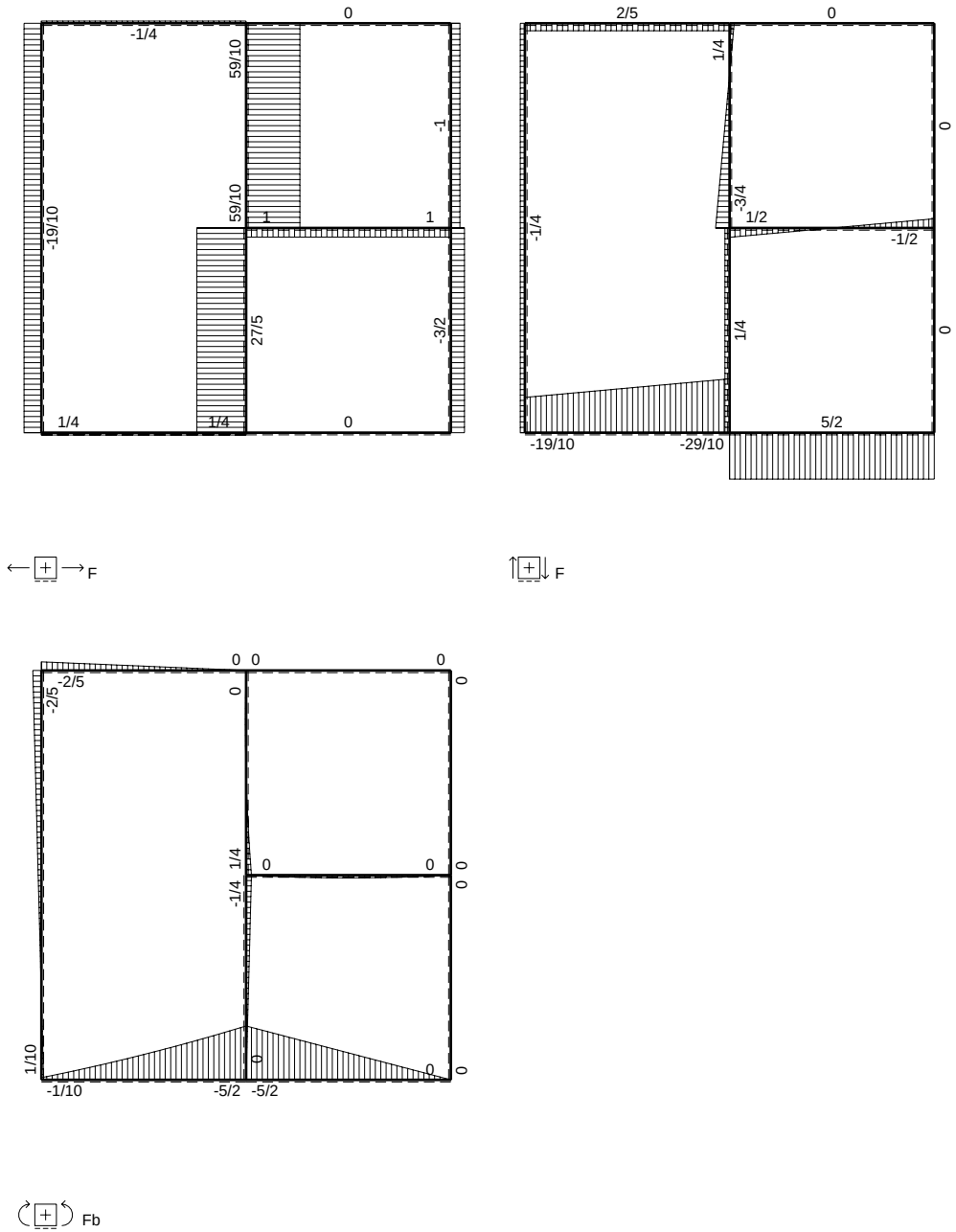
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

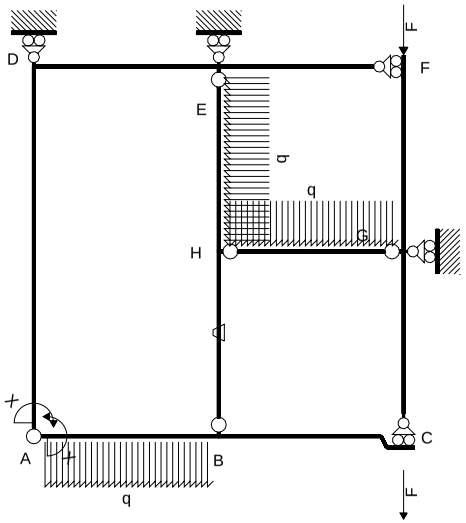
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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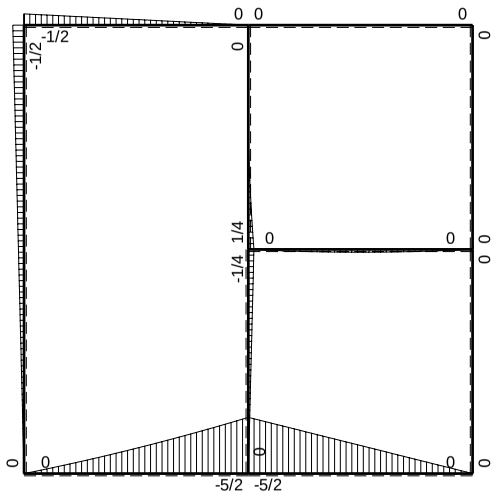
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D
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2/5Fb
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G
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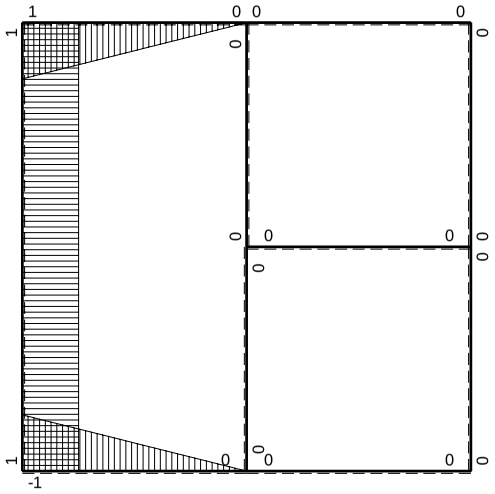

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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-1/4Fb+1/4Fx	-Fb/EJ	0	0	0	0+0	0
BH b	0	1/4Fx	Fb/EJ	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

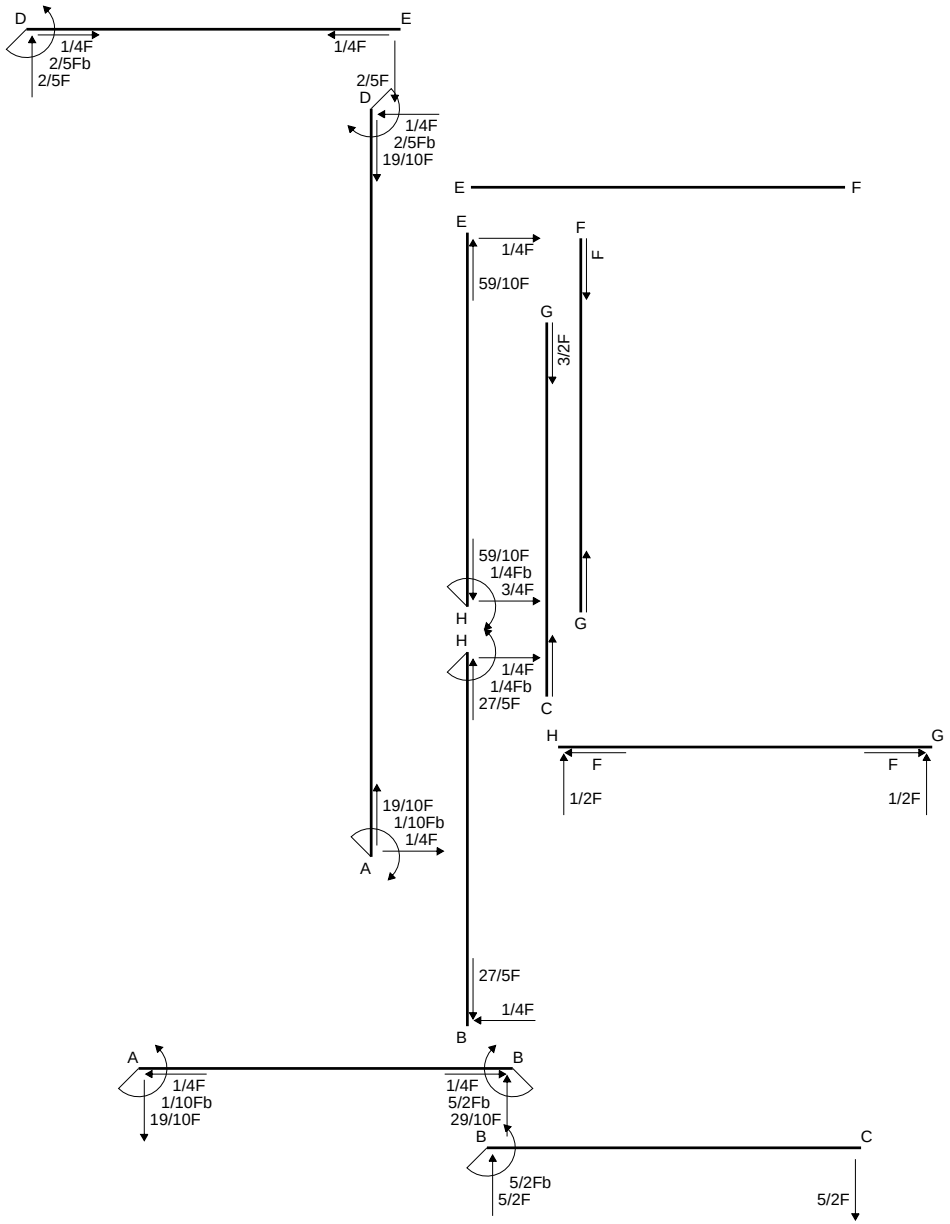
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

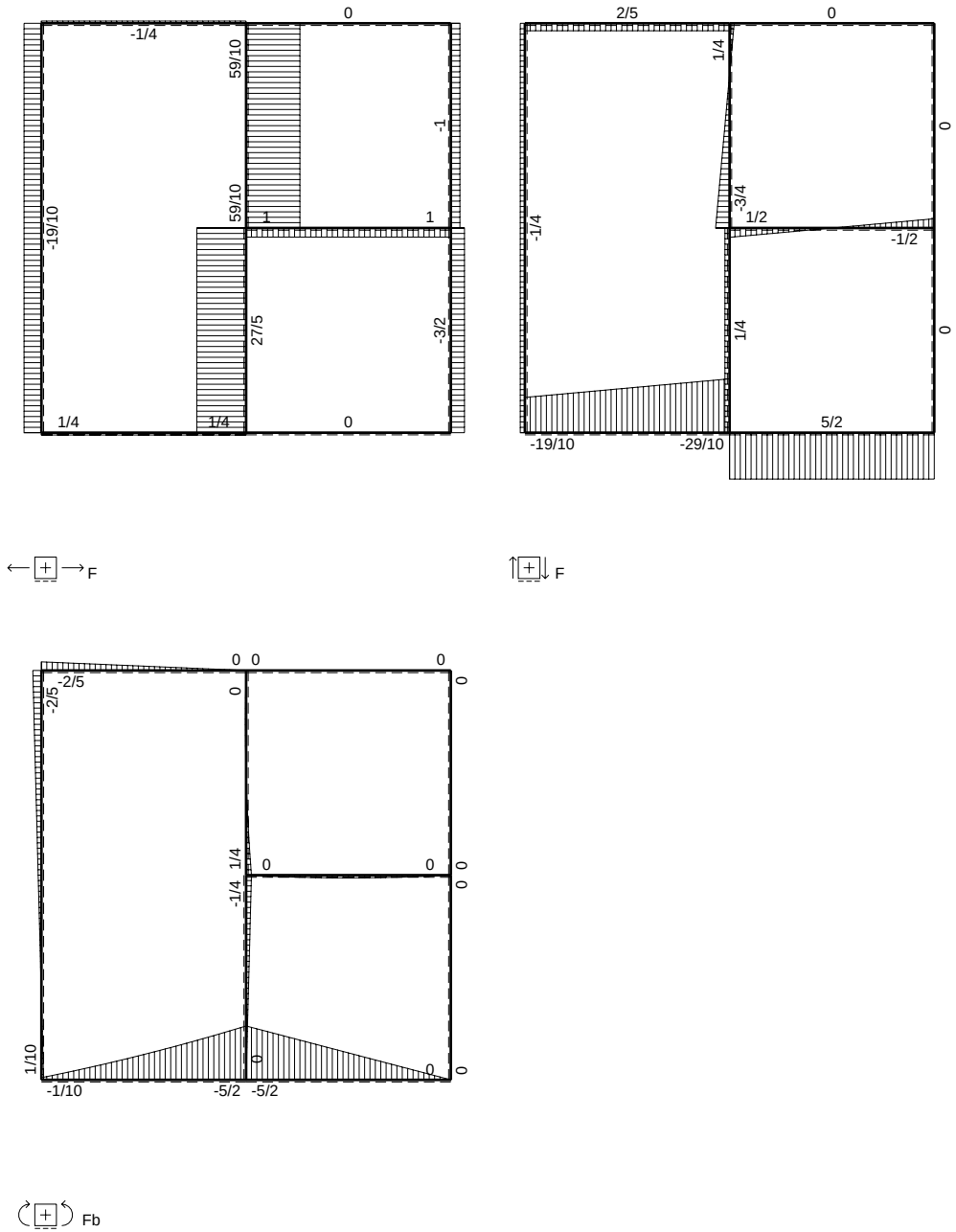
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

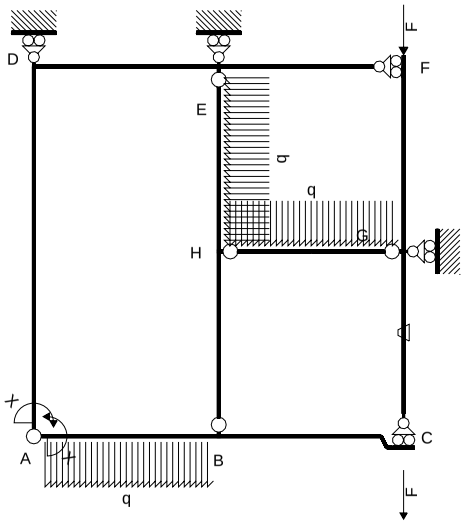
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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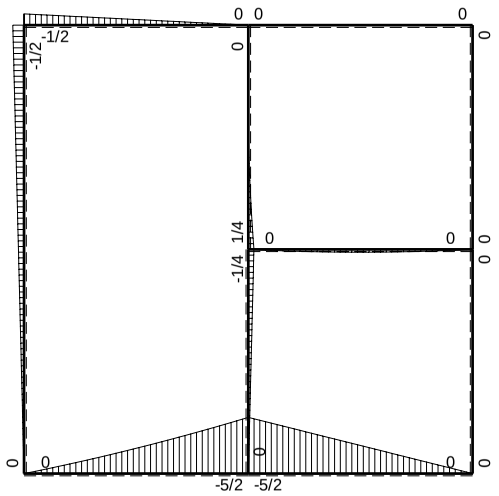
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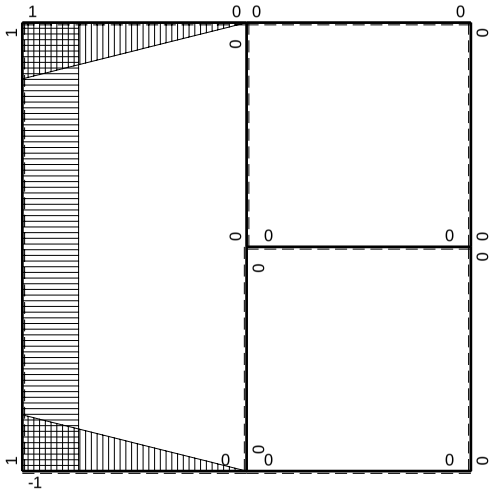

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-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-1/4Fx$	0	$-1/4Fx$	0	1	$(-1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb-1/4Fx$	0	$-1/2Fb+1/4Fx$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
BH b	0	$1/4Fx$	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

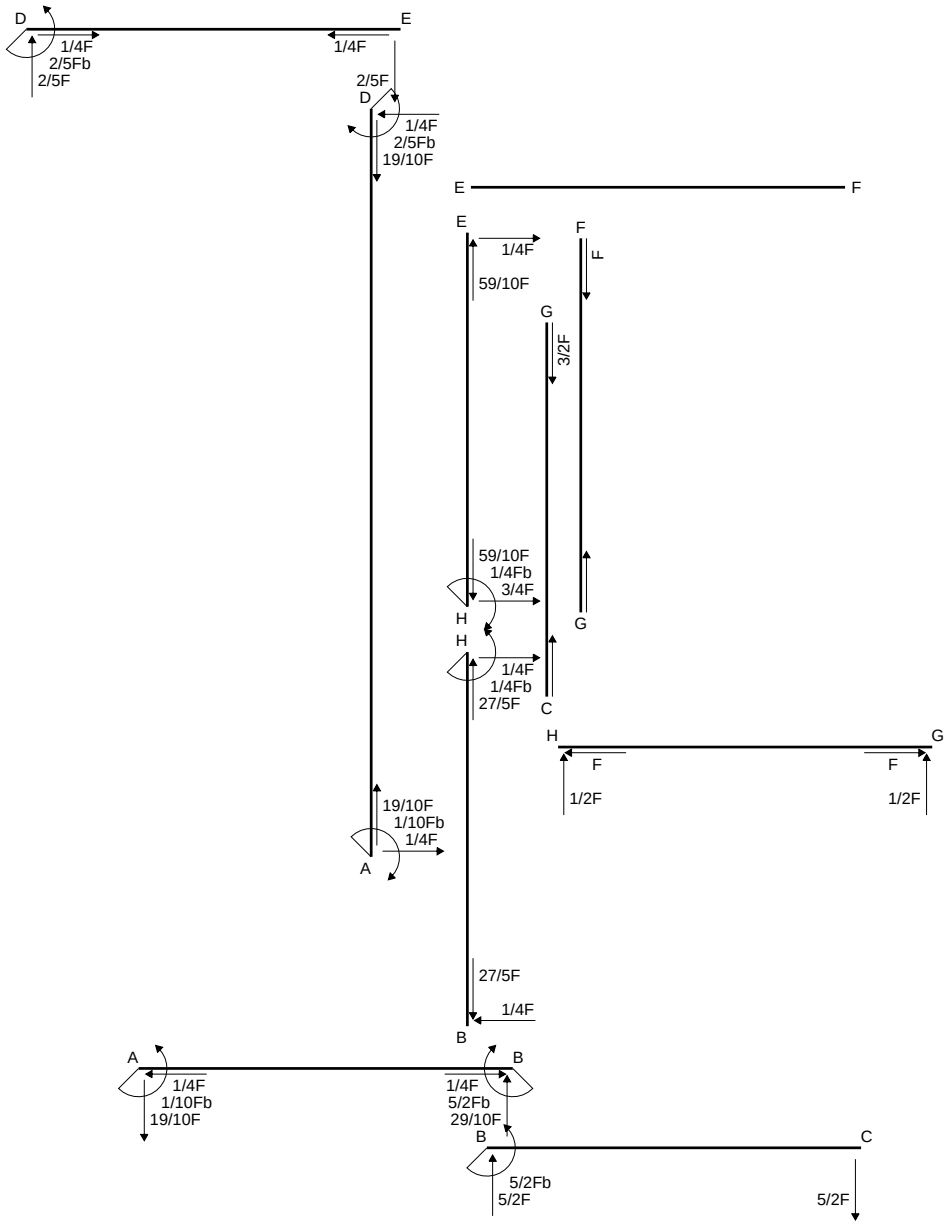
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

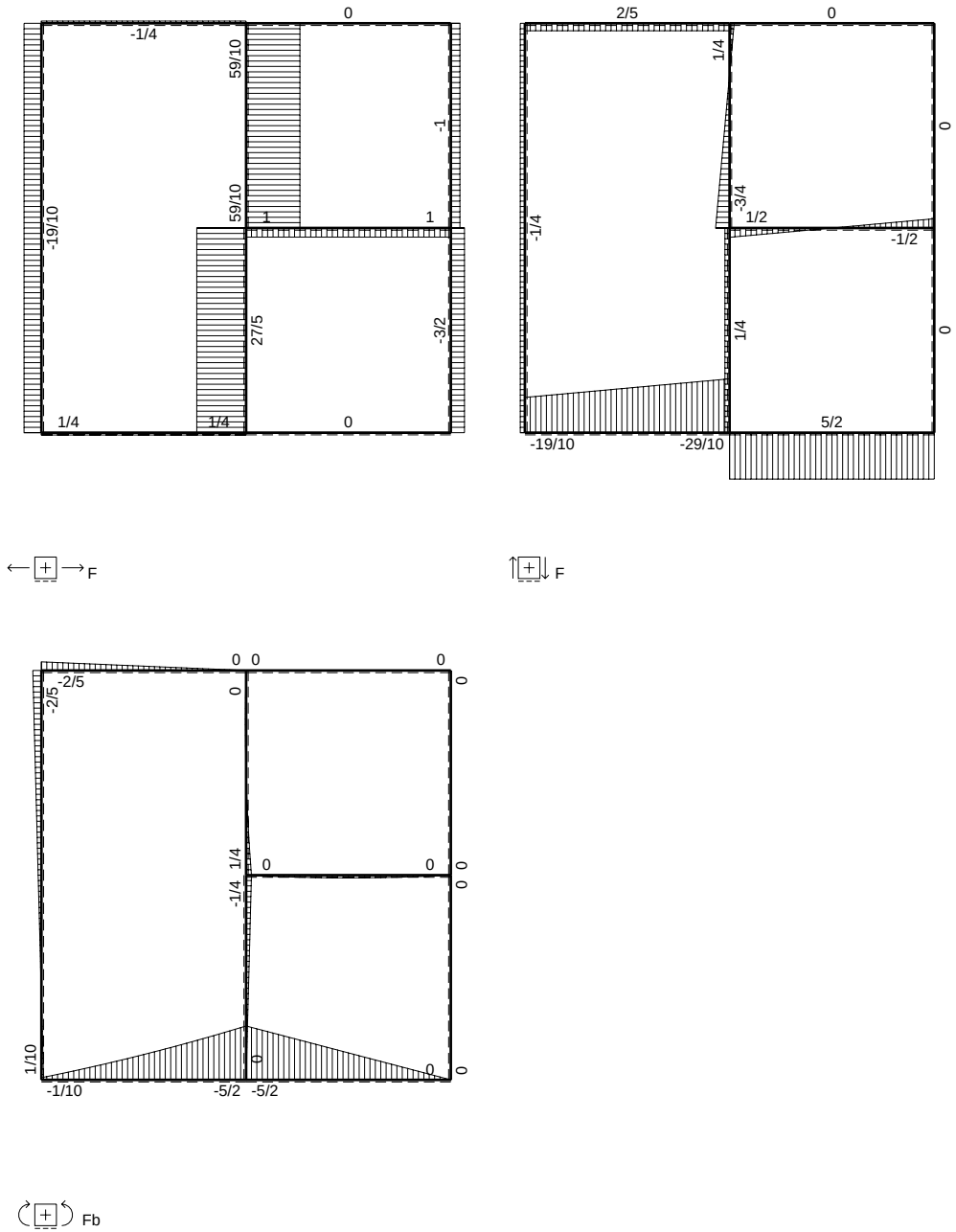
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

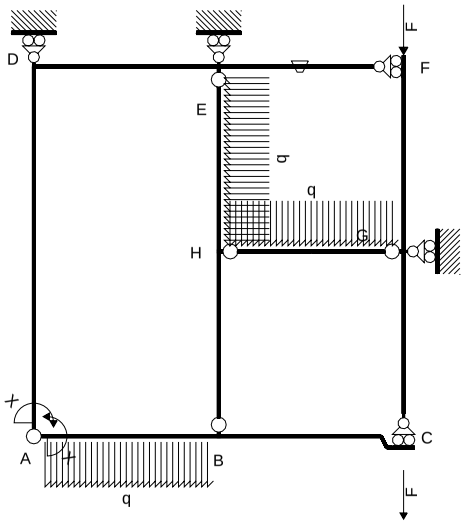
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238444 ESΣ03.xxxx.167

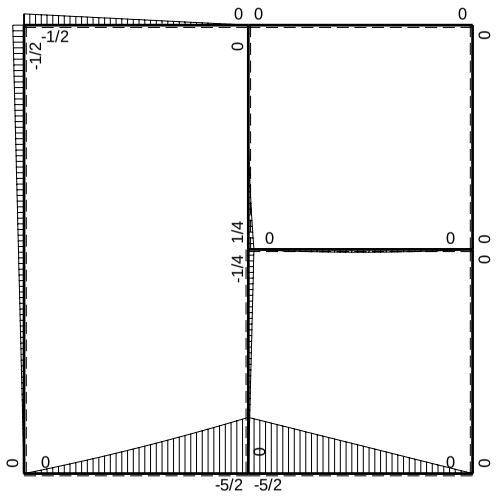
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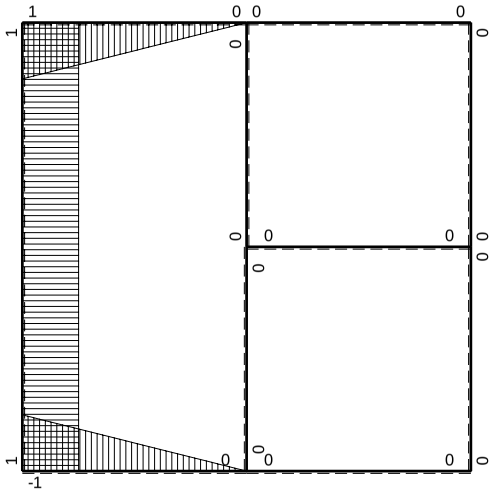

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1/10
0
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237042 ESΣ03.xxxx.168

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-1/4Fx$	0	$-1/4Fx$	0	1	$(-1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb-1/4Fx$	0	$-1/2Fb+1/4Fx$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
BH b	0	$1/4Fx$	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

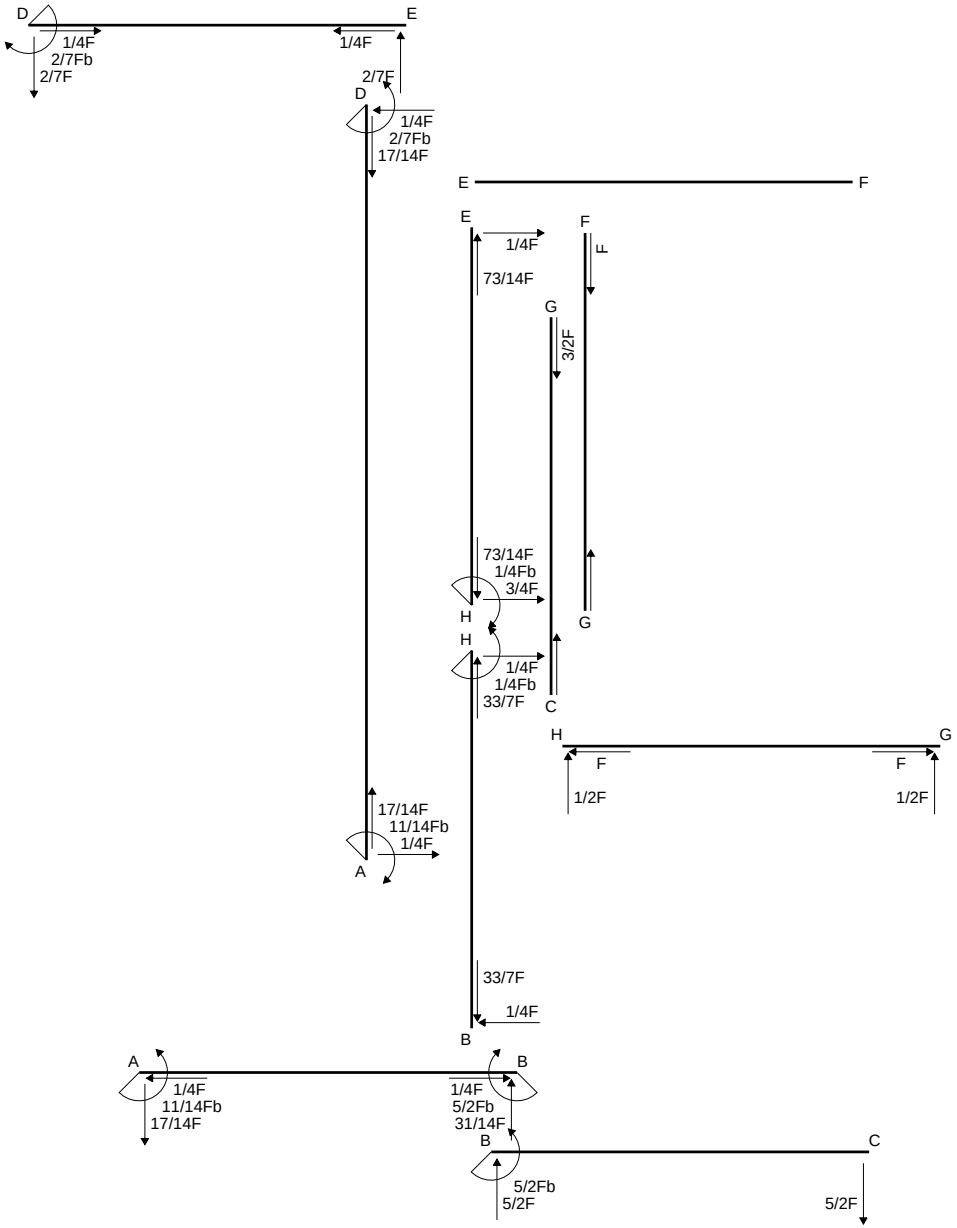
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

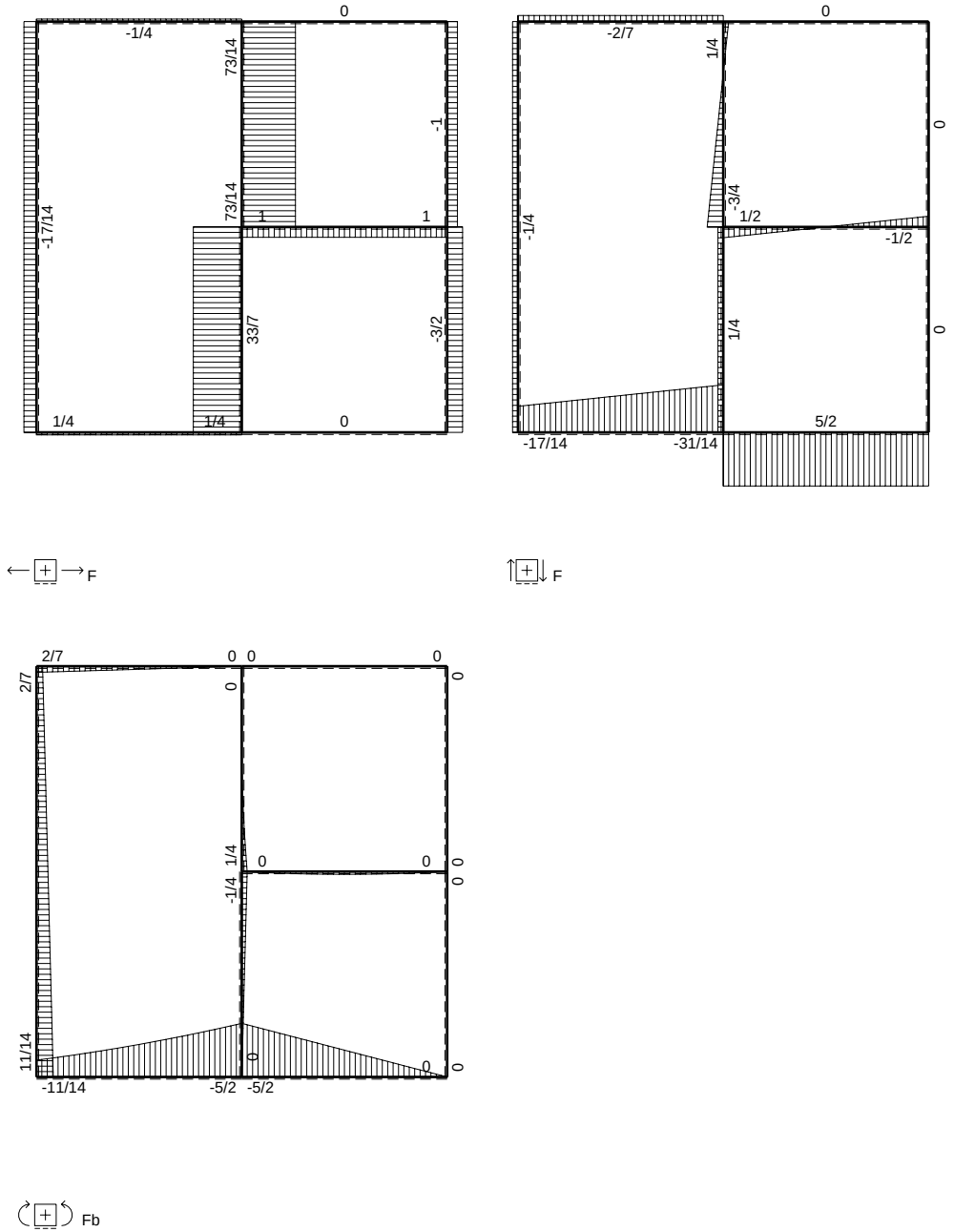
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

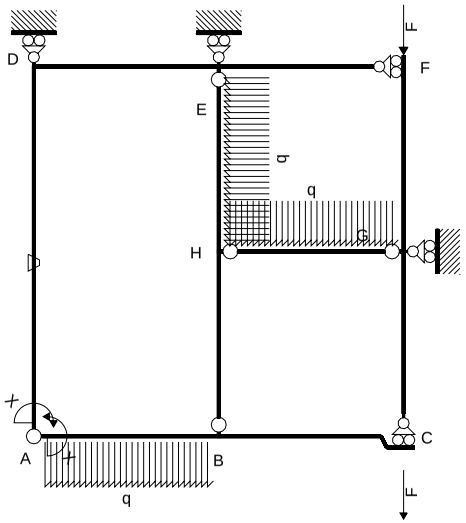
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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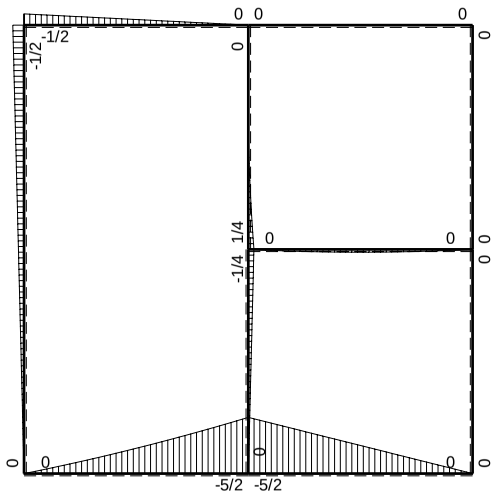
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D
E
1/4F
2/7Fb
2/7F
1/4F
2/7F
D
1/4F
2/7Fb
17/14F
E
F
E
1/4F
F
73/14F
G
3/2F
73/14F
1/4Fb
3/4F
H
G
H
1/4F
1/4Fb
33/7F
C
H
G
F
F
1/2F
1/2F
17/14F
11/14Fb
1/4F
A
33/7F
1/4F
B
A
1/4F
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5/2Fb
5/2F
5/2F

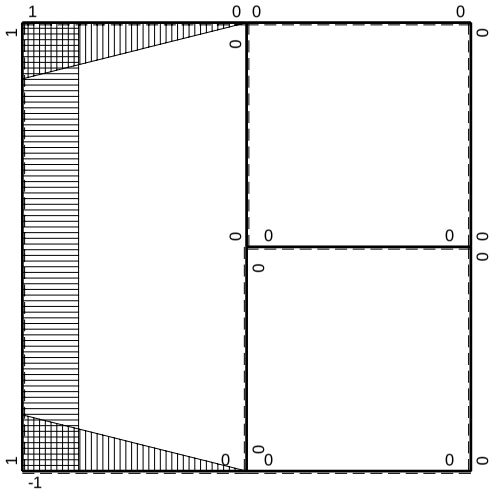

0
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73/14
-1
73/14
-17/14
1
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33/7
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1/4
1/4
0
0
-2/7
1/4
-3/4
1/2
-1/4
-1/2
1/4
0
0
5/2
-17/14
-31/14
F
F
2/7
0 0
0
2/7
0
0
1/4
0
0
-1/4
0 0
0
11/14
0
0
-11/14
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-1/4Fx$	$-Fb/EJ$	$-1/4Fx$	$-Fb/EJ$	1	$(-1/2-2)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb-1/4Fx$	Fb/EJ	$-1/2Fb+1/4Fx$	$-Fb/EJ$	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
BH b	0	$1/4Fx$	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-55/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$11/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx + \int_o^{2b} (-1\ )\ \theta\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ\ + [-\ x\ ]_o^{2b}\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-2\ b\ )\ \theta\ = -5/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx + \int_o^{2b} (1\ )\ \theta\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ\ + [\ x\ ]_o^{2b}\ \theta$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ + (2\ b\ )\ \theta\ = -5/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

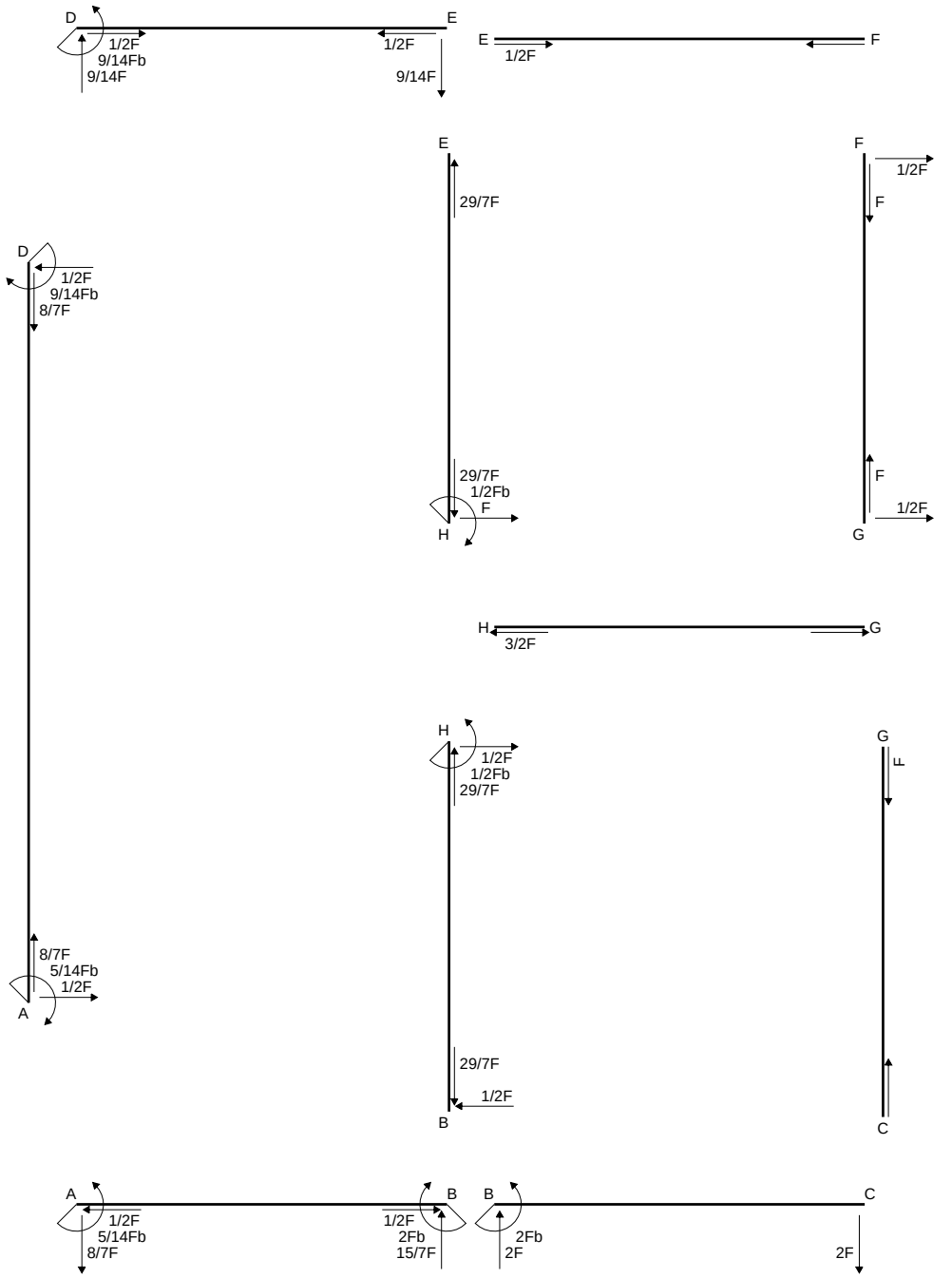
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

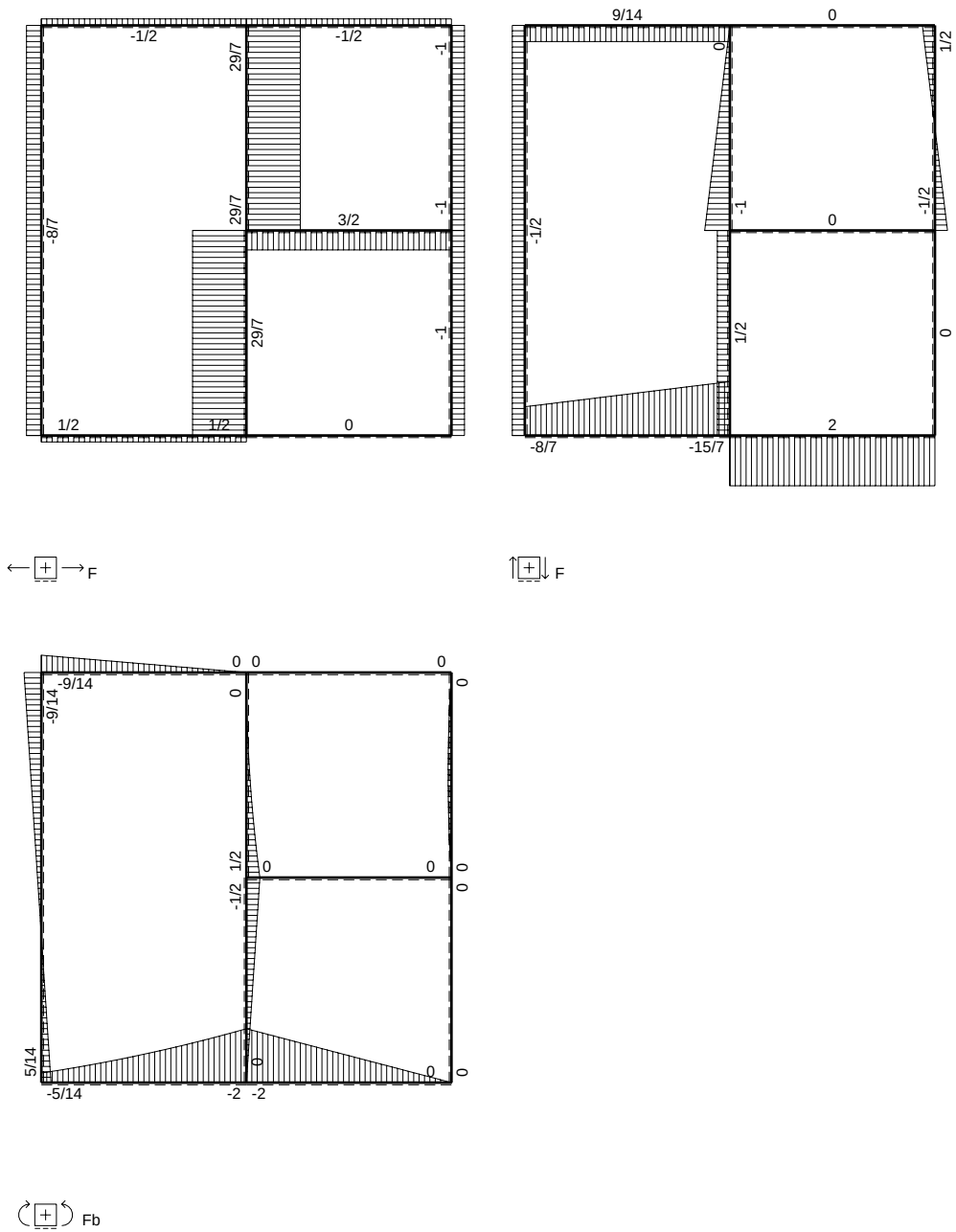
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

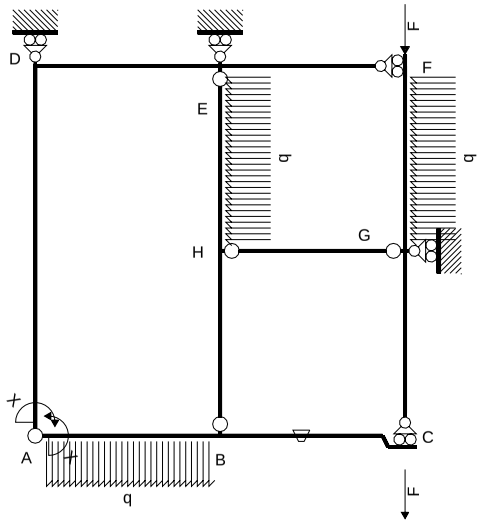
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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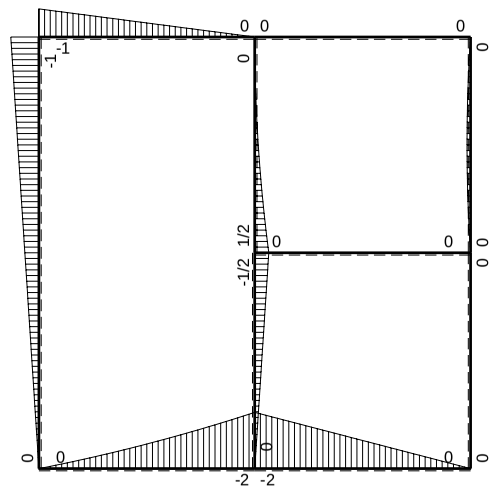
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D
E
1/2F
9/14Fb
9/14F
1/2F
9/14F
E
F
1/2F
E
29/7F
F
1/2F
F
D
1/2F
9/14Fb
8/7F
29/7F
1/2Fb
F
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3/2F
H
1/2F
1/2Fb
29/7F
G
F
8/7F
5/14Fb
1/2F
A
29/7F
1/2F
B
C
A
1/2F
5/14Fb
8/7F
B
1/2F
2Fb
15/7F
B
2Fb
2F
C
2F

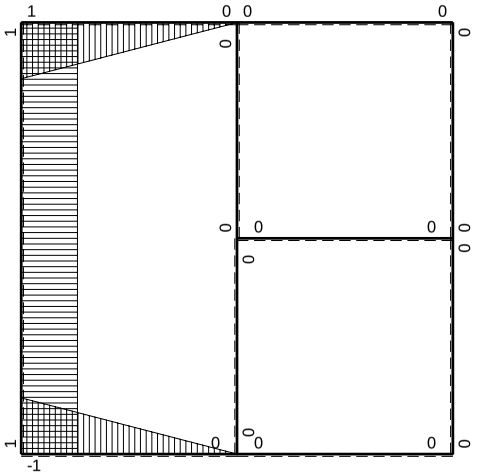

-1/2
29/7
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29/7
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29/7
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0 0
0
-9/14
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0
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1/2
0
0
-1/2
0
0
0
5/14
0
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	-Fb/EJ	0	0	0	0+0	0
CB b	0	2Fx	Fb/EJ	0	0	0		
AD 2b	1	-1/2Fx	0	-1/2Fx	0	1	$(-1+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	Fb-1/2Fx	0	-Fb+1/2Fx	0	1		
DE b	1-x/b	-Fb+Fx	0	$-Fb+2Fx-Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/3+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	Fx	0	$-Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BH b	0	1/2Fx	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1 + 1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-\ x + 1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-2\ b + b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + 2\,x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x + x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

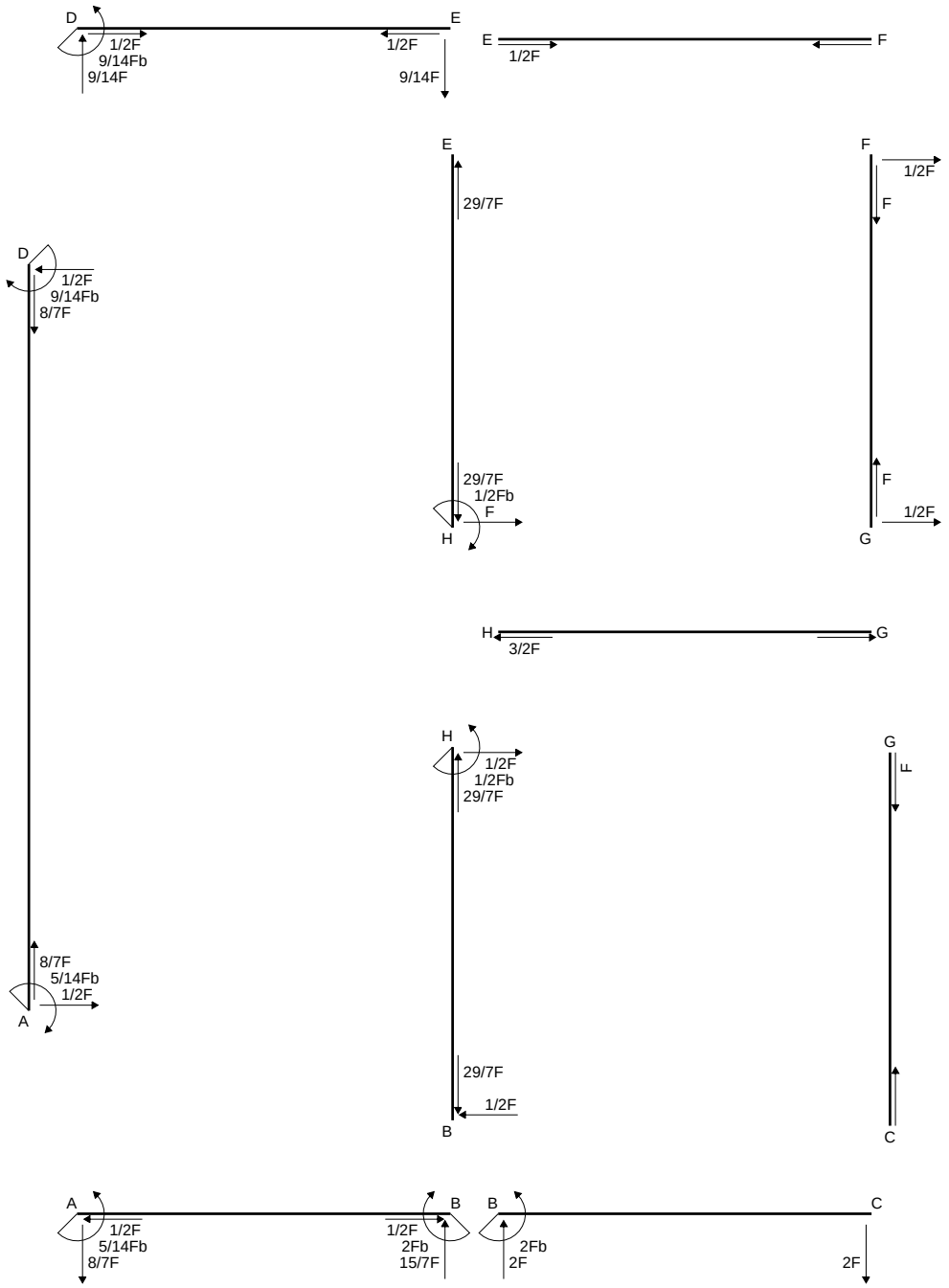
$= (-\ b + b - 1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

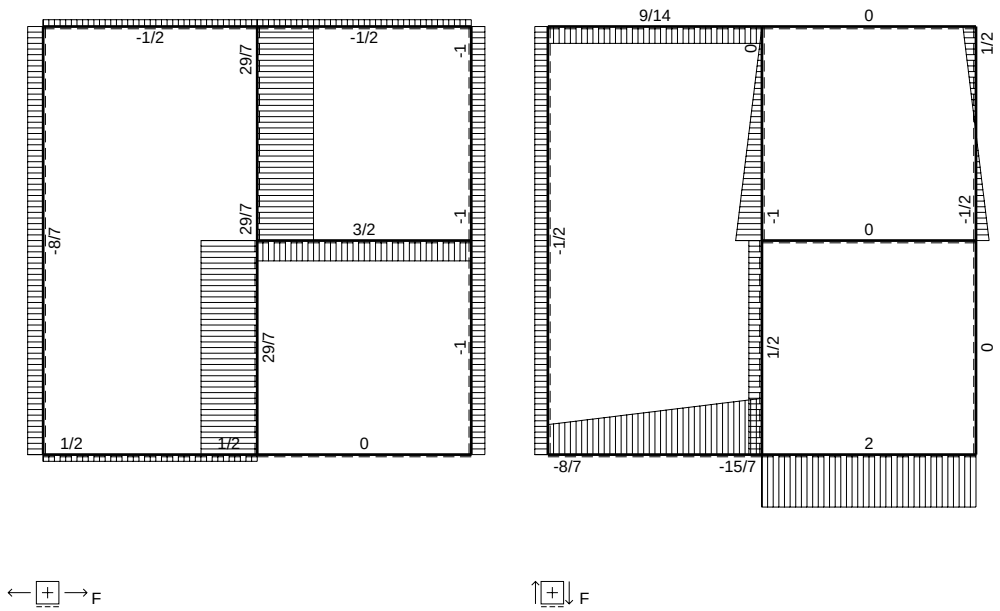
$L^{Xo}_{ED} = \int_o^b (-\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

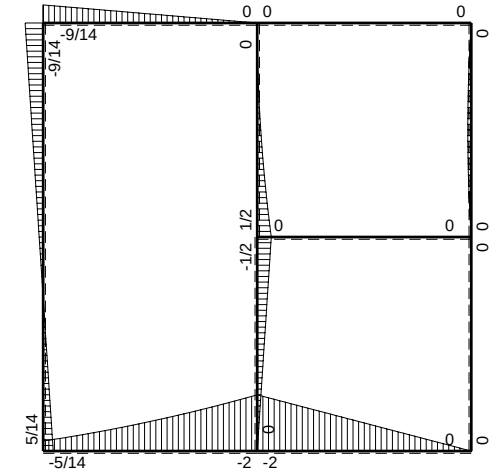
$= (-1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

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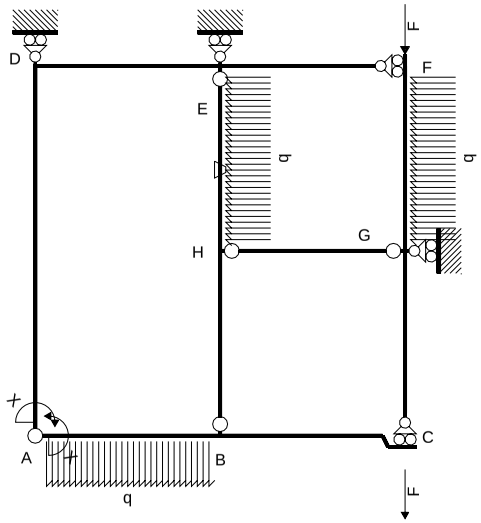
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9/14F
E
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E
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2F

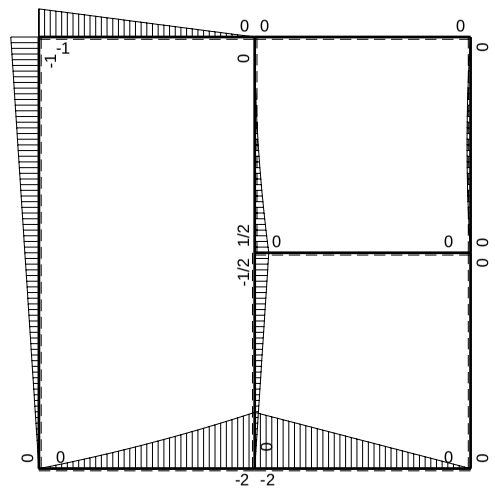

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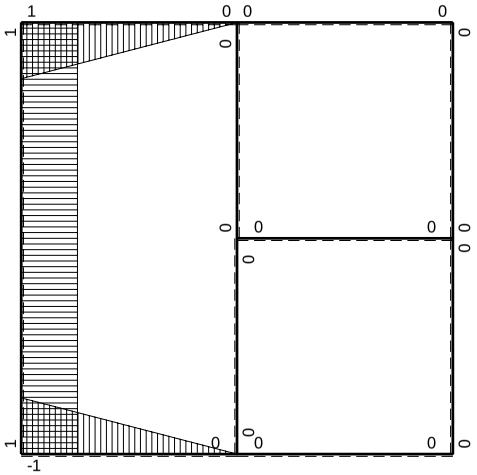
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-1/2Fx$	0	$-1/2Fx$	0	1	$(-1+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fb-1/2Fx$	0	$-Fb+1/2Fx$	0	1		
DE b	$1-x/b$	$-Fb+Fx$	0	$-Fb+2Fx-Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/3+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	Fx	0	$-Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BH b	0	$1/2Fx$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\ b + 1/8\ b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-1/2\ x/b)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (-1 + 1/2\ x/b)\ Fb\ 1/EJ\ dx = [-\ x + 1/4\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (-2\ b + b)\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1 + 2\,x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x + x^2/b - 1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

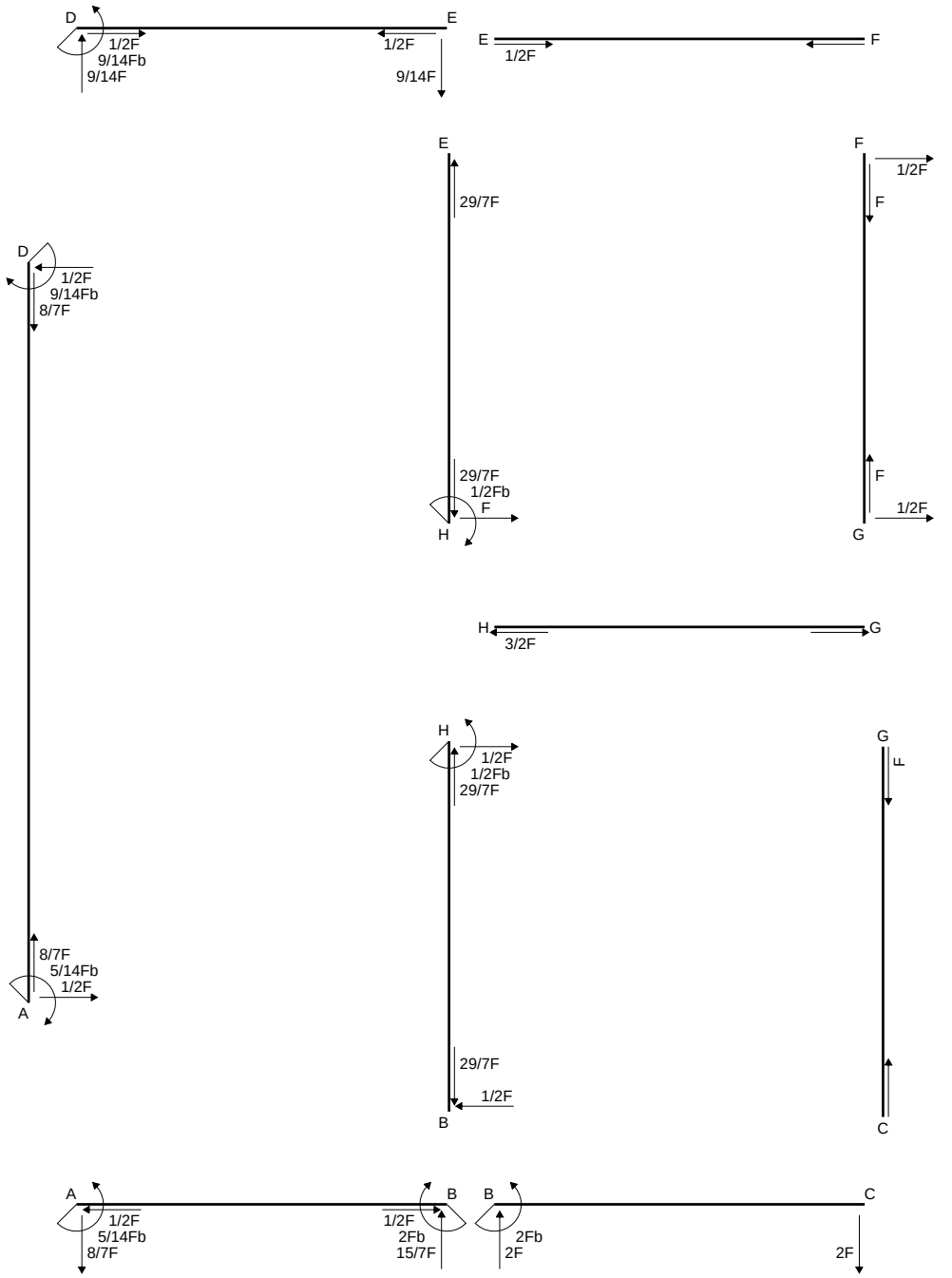
$= (-\ b + b - 1/3\ b)\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

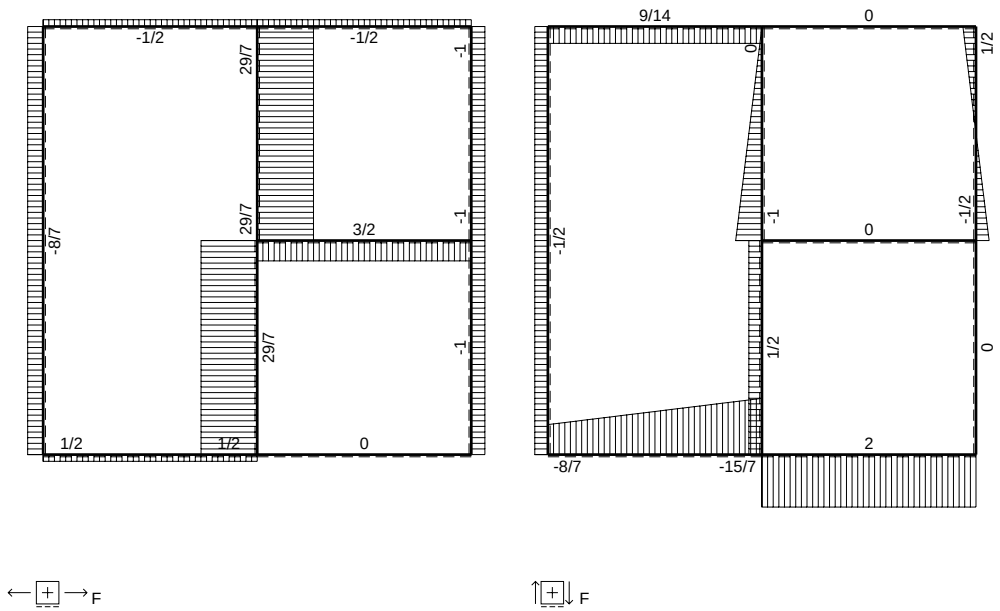
$L_{ED}^{Xo} = \int_o^b (-\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

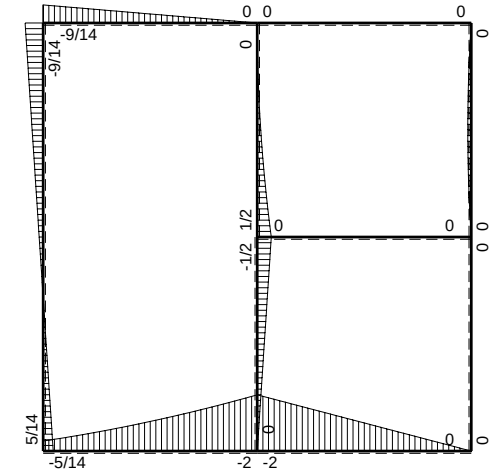
$= (-1/3\ b)\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

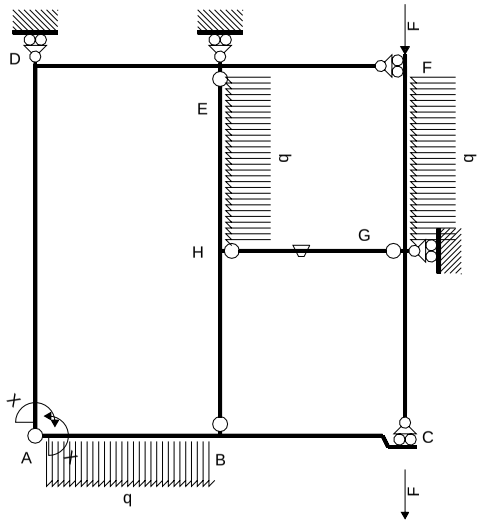
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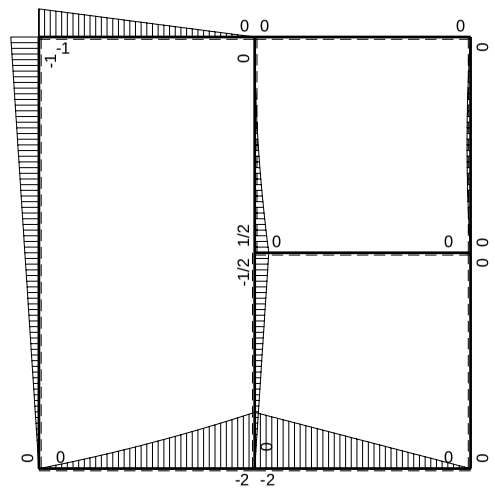

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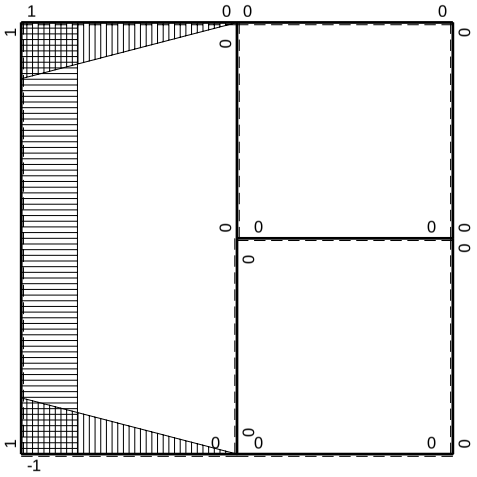

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0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	-1/2Fx	0	-1/2Fx	0	1	$(-1+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	Fb-1/2Fx	0	-Fb+1/2Fx	0	1		
DE b	$1-x/b$	-Fb+Fx	0	$-Fb+2Fx-Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/3+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	Fx	0	$-Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	-Fb/EJ	0	0	0	0+0	0
GH b	0	0	Fb/EJ	0	0	0		
HB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BH b	0	1/2Fx	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 5/6\ b + 1/8\ b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-1/2\ x/b)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (-1 + 1/2\ x/b)\ Fb\ 1/EJ\ dx = [-\ x + 1/4\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (-2\ b + b)\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1 + 2\,x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x + x^2/b - 1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

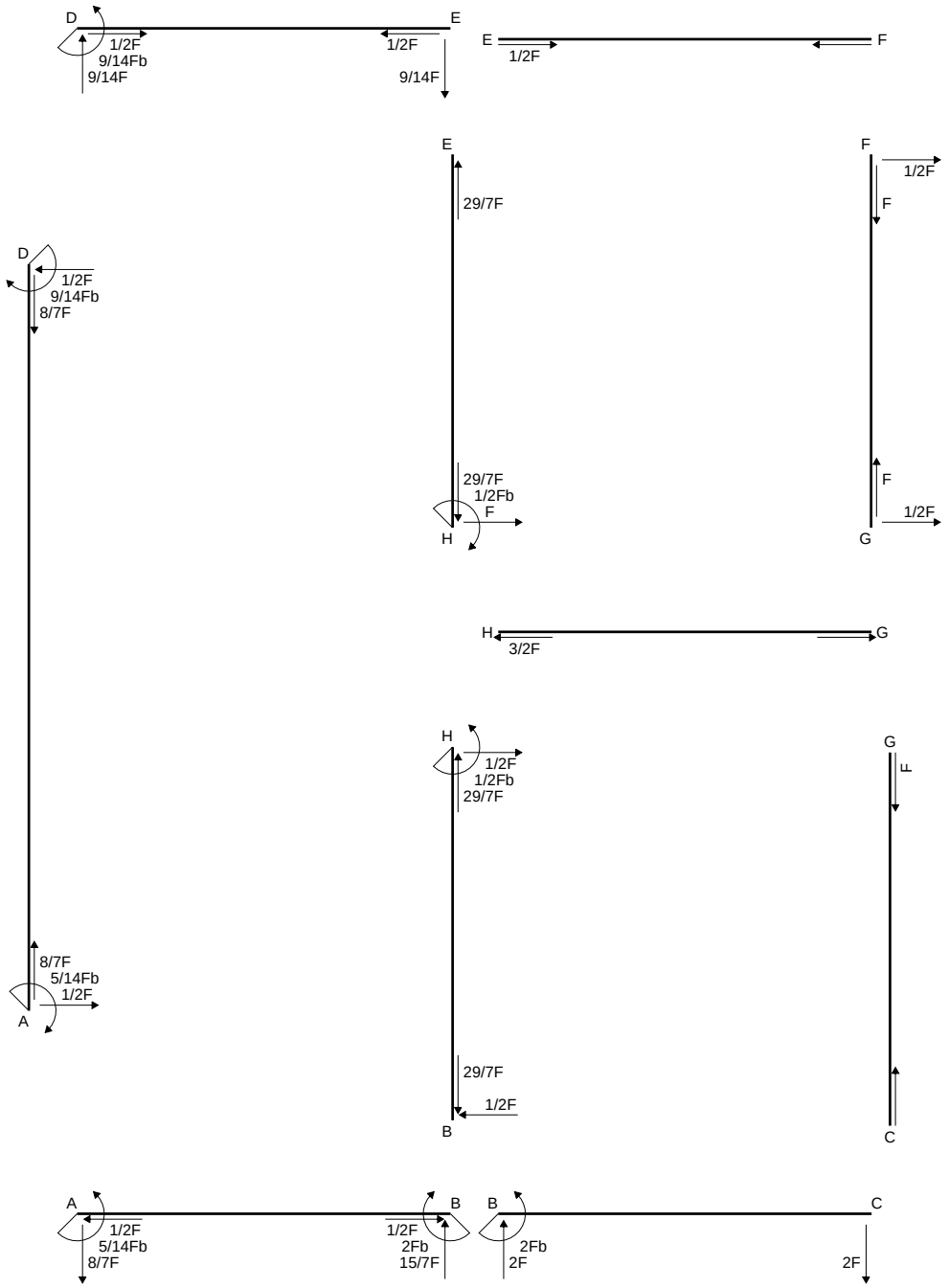
$= (-\ b + b - 1/3\ b)\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

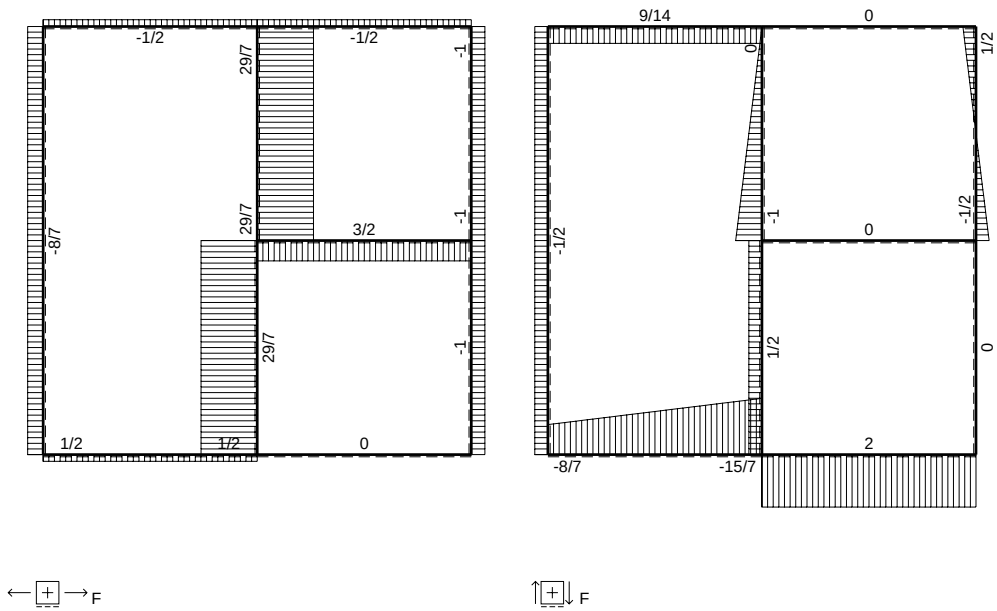
$L_{ED}^{Xo} = \int_o^b (-\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

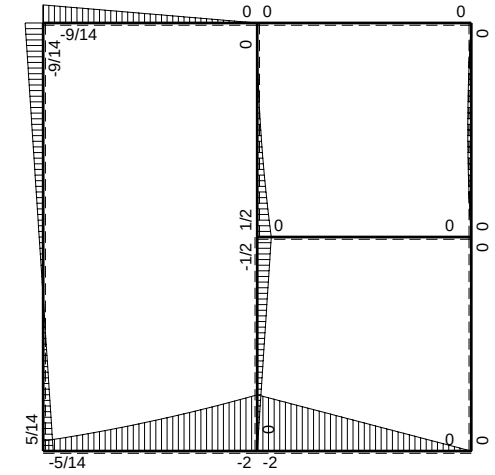
$= (-1/3\ b)\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

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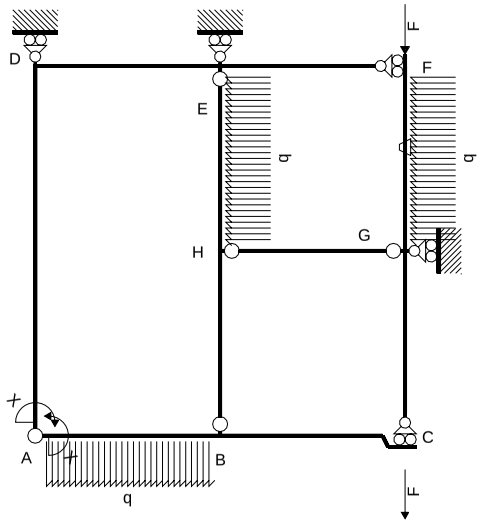
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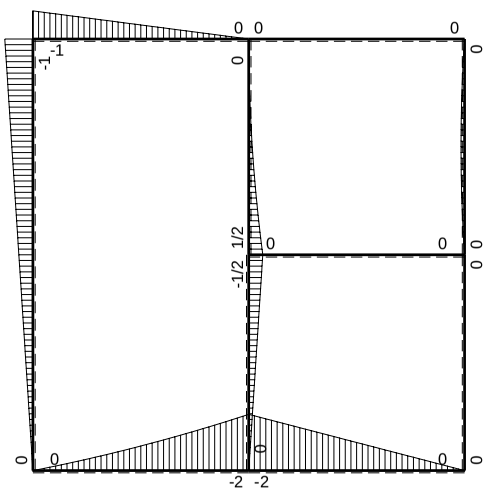

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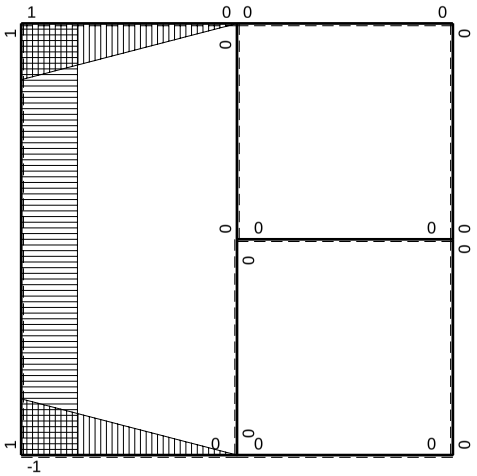
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238219 ESΣ03.xxxx.173

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-2Fb+2Fx	0	0	0	0	0+0	0
CB b	0	2Fx	0	0	0	0		
AD 2b	1	-1/2Fx	0	-1/2Fx	0	1	$(-1+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	Fb-1/2Fx	0	-Fb+1/2Fx	0	1		
DE b	1-x/b	-Fb+Fx	0	$-Fb+2Fx-Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/3+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	Fx	0	$-Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BH b	0	1/2Fx	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						5/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1 + 1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-\ x + 1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-2\ b + b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + 2\,x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x + x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

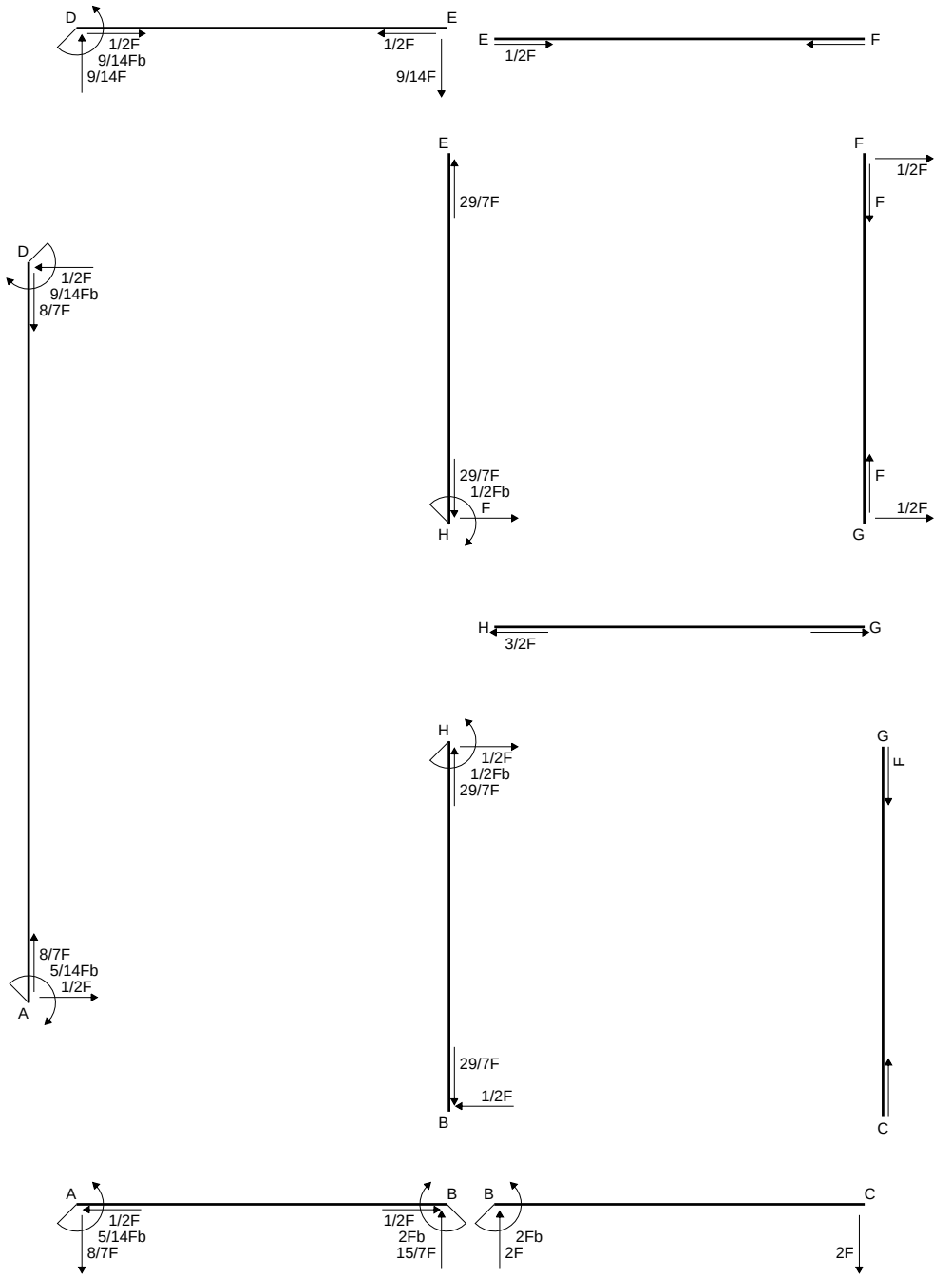
$= (-\ b + b - 1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

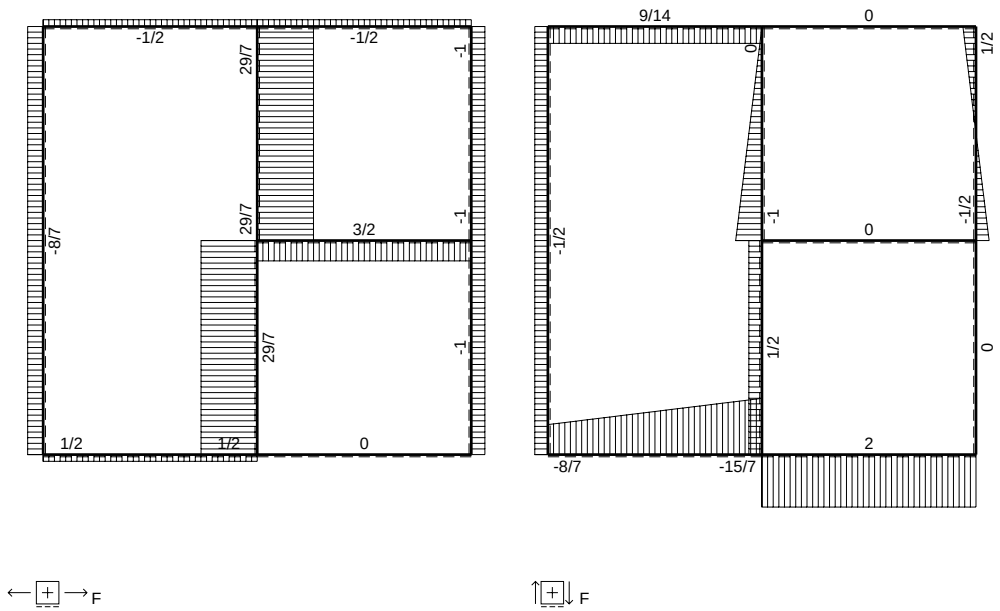
$L^{Xo}_{ED} = \int_o^b (-\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

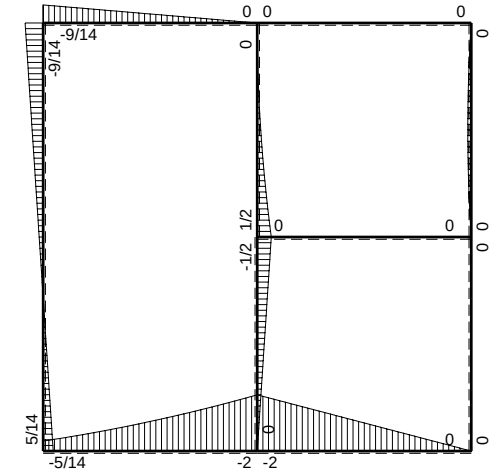
$= (-1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

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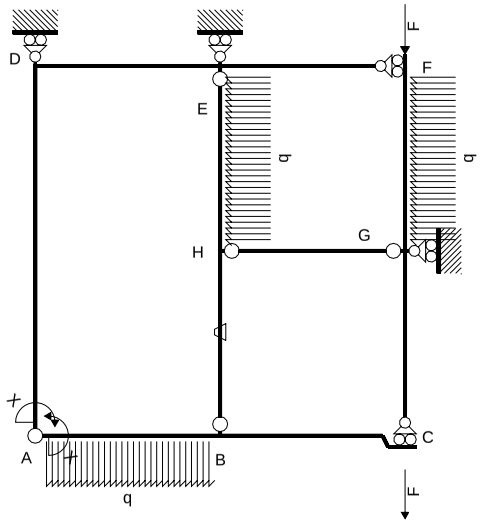
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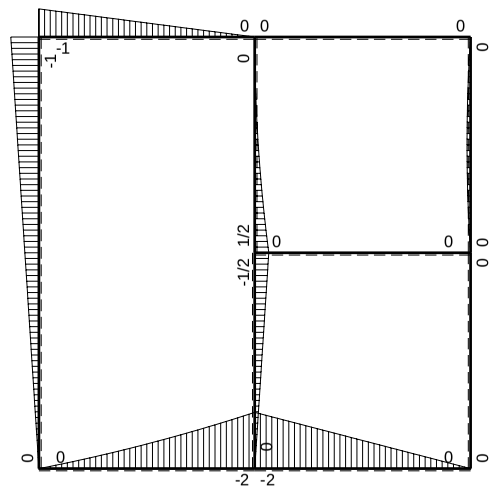

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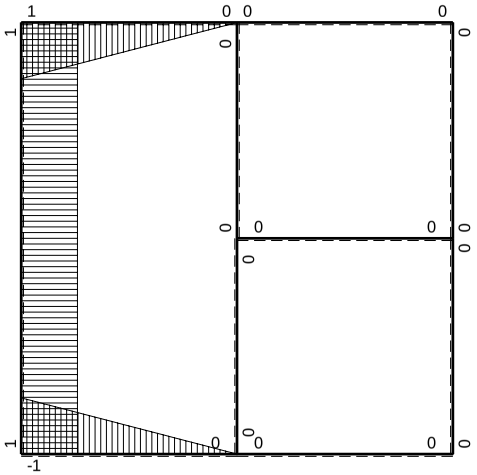
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-1/2Fx$	0	$-1/2Fx$	0	1	$(-1+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fb-1/2Fx$	0	$-Fb+1/2Fx$	0	1		
DE b	$1-x/b$	$-Fb+Fx$	0	$-Fb+2Fx-Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/3+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	Fx	0	$-Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$1/2Fx$	Fb/EJ	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1 + 1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-\ x + 1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-2\ b + b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + 2\,x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x + x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

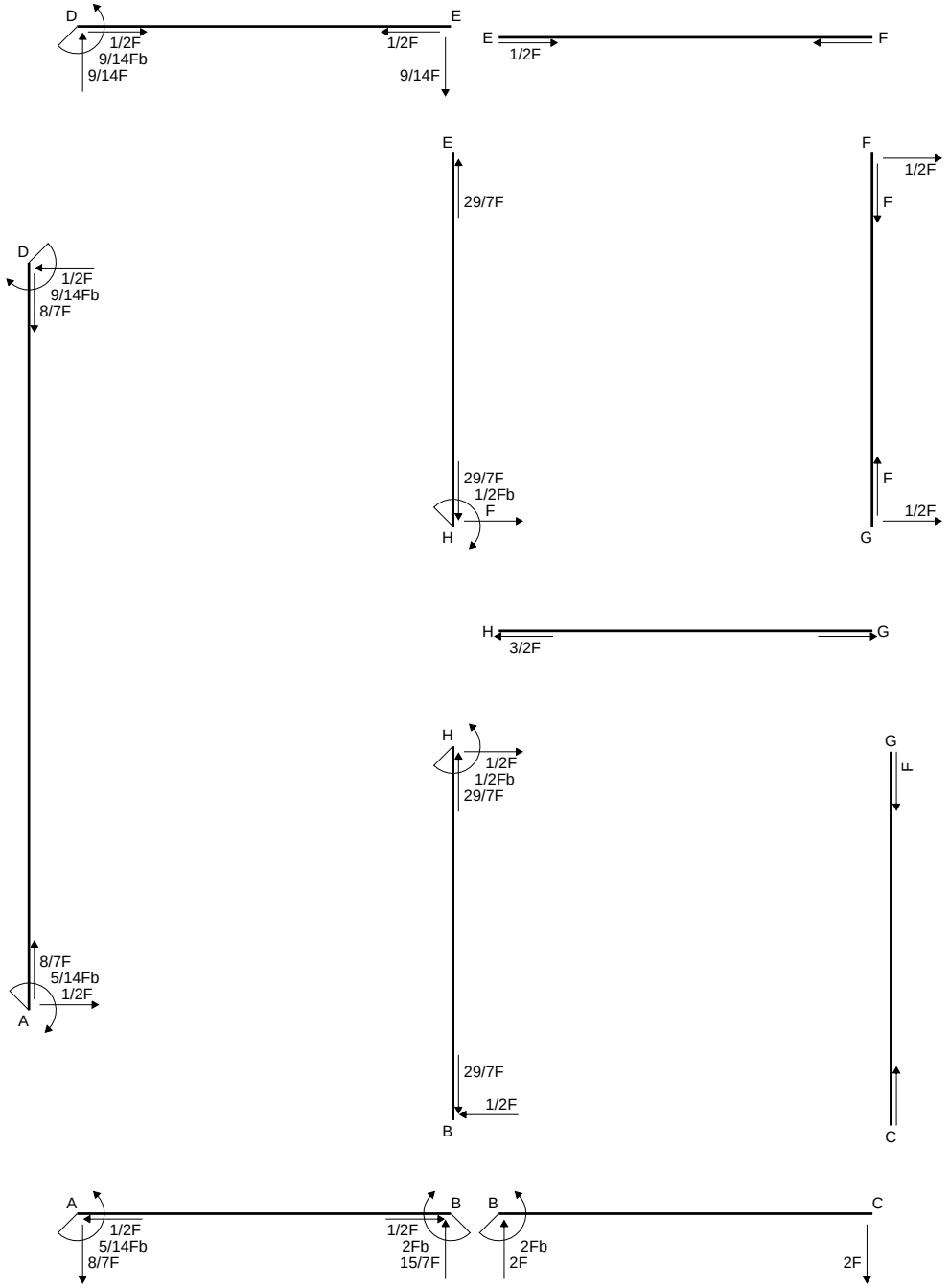
$= (-\ b + b - 1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

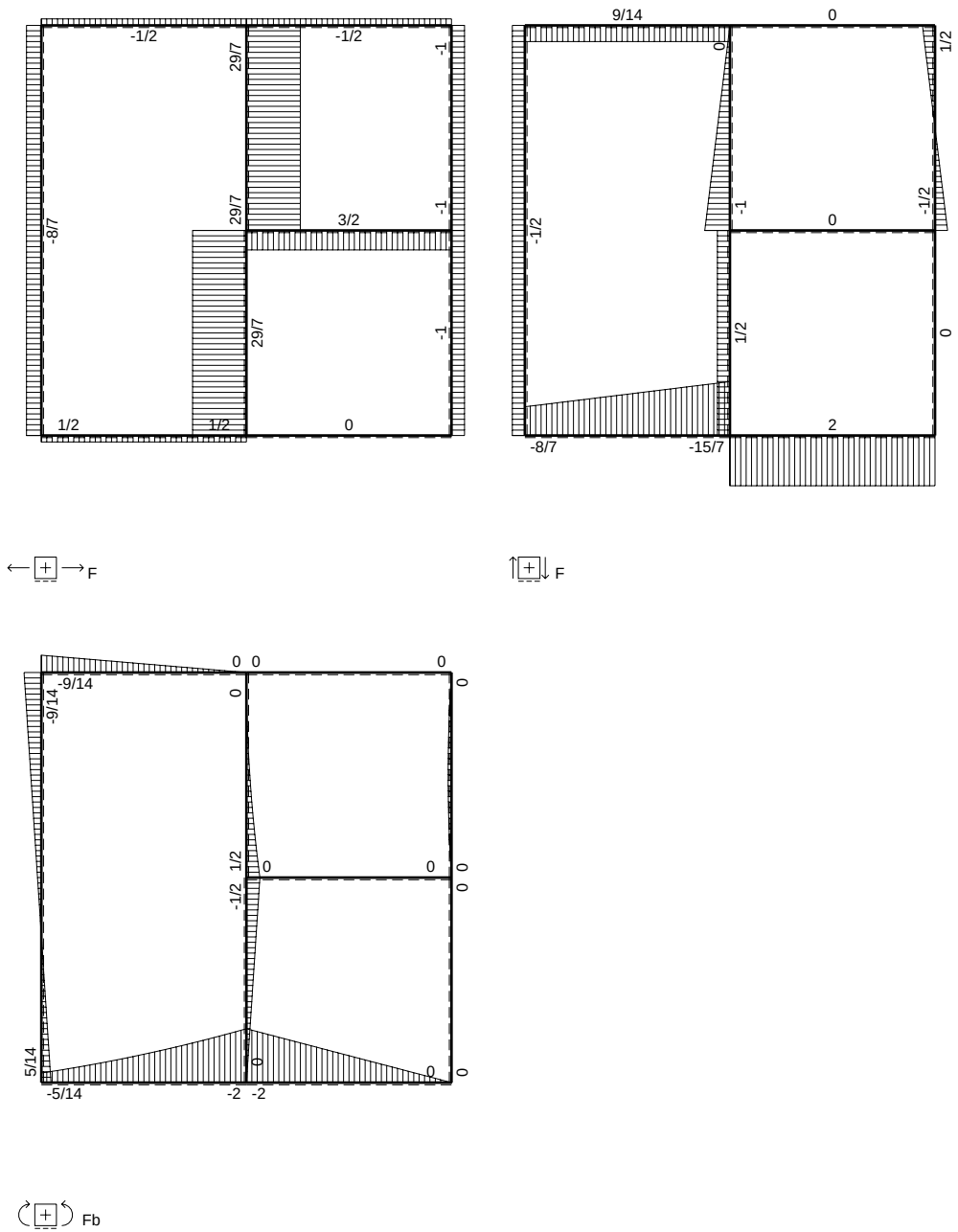
$L^{Xo}_{ED} = \int_o^b (-\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

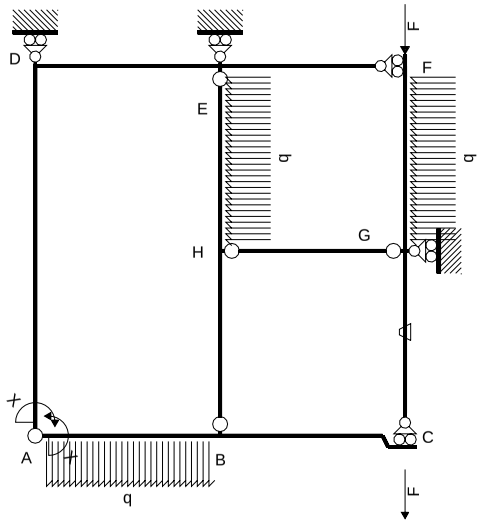
$= (-1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

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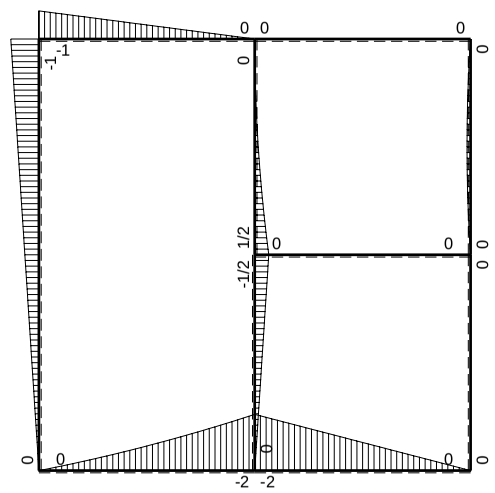
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D
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9/14F
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9/14F
E
F
1/2F
E
29/7F
F
1/2F
F
D
1/2F
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F
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29/7F
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B
C
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C
2F

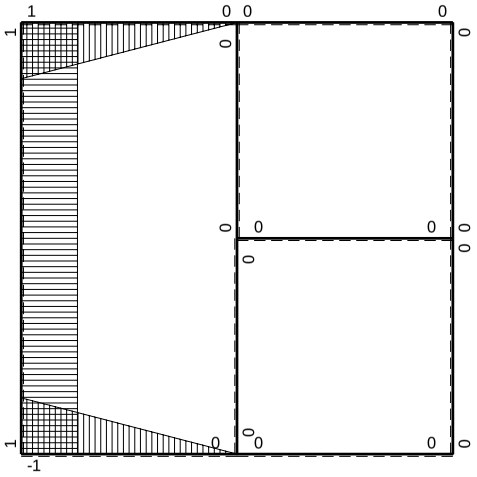

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0
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-9/14
0
0
0
1/2
0
0
-1/2
0
0
0
5/14
0
0
-5/14
-2 -2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-1/2Fx$	0	$-1/2Fx$	0	1	$(-1+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fb-1/2Fx$	0	$-Fb+1/2Fx$	0	1		
DE b	$1-x/b$	$-Fb+Fx$	0	$-Fb+2Fx-Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/3+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	Fx	0	$-Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BH b	0	$1/2Fx$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1 + 1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-\ x + 1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-2\ b + b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + 2\,x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x + x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

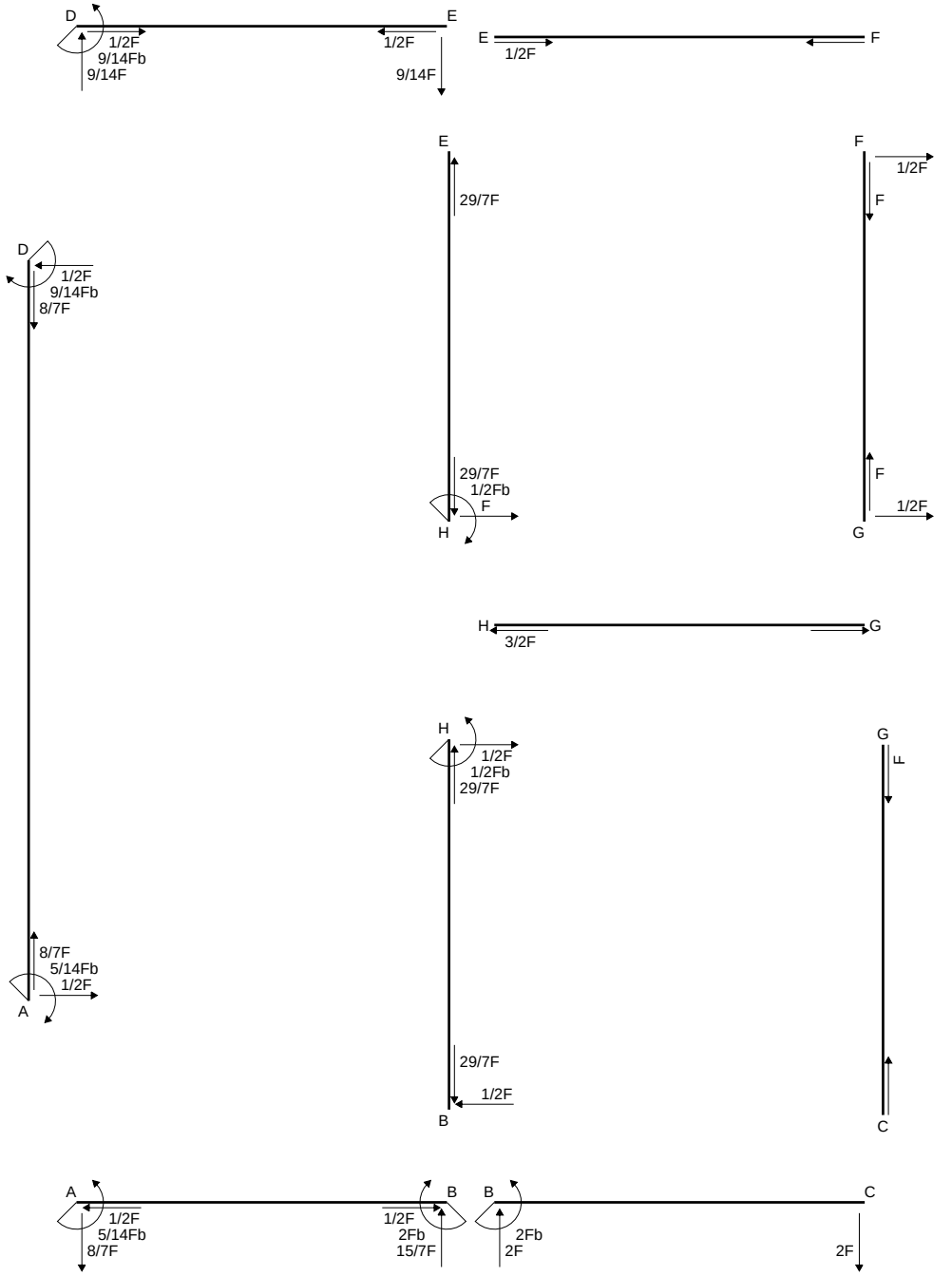
$= (-\ b + b - 1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

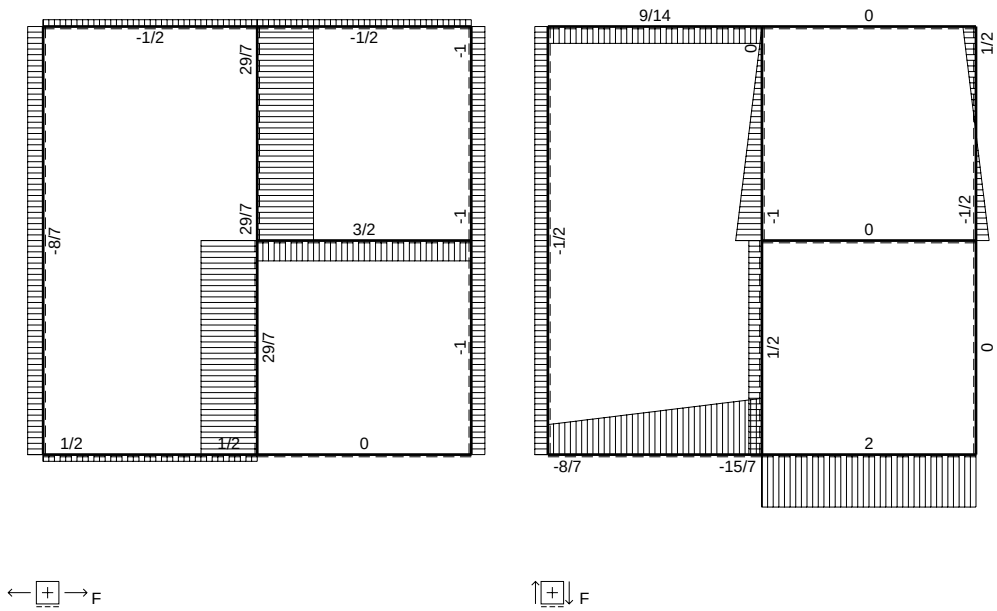
$L^{Xo}_{ED} = \int_o^b (-\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

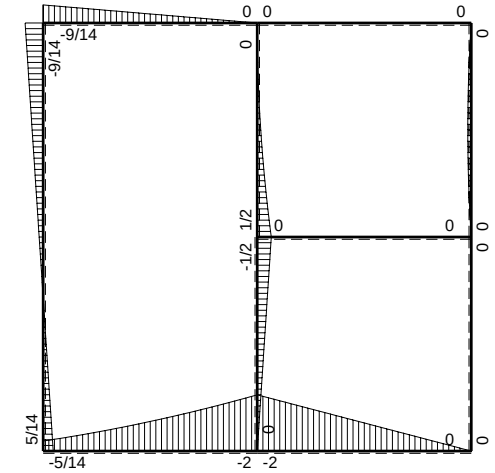
$= (-1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

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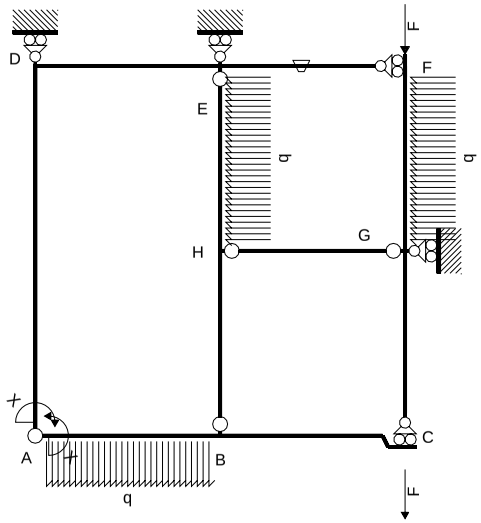
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9/14F
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C
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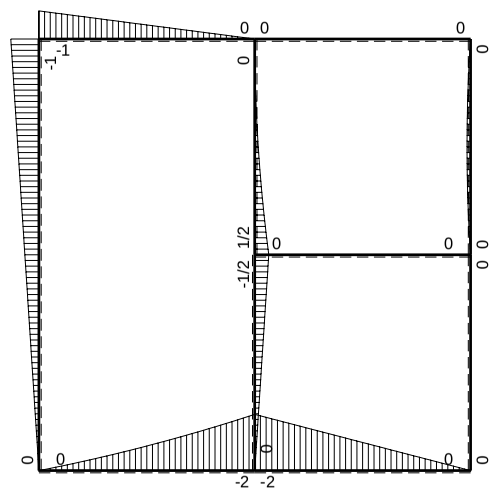

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-15/7
2
F
F


0 0
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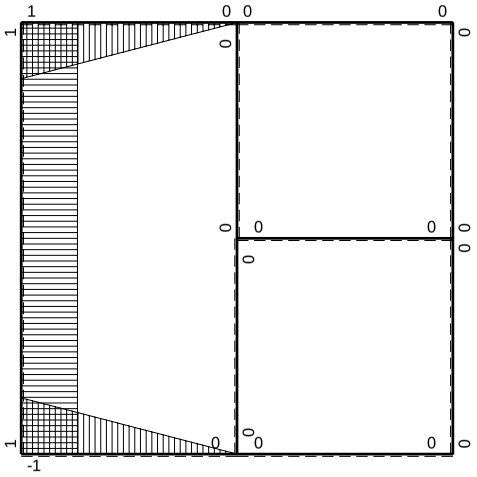

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.176

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-1/2Fx$	0	$-1/2Fx$	0	1	$(-1+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fb-1/2Fx$	0	$-Fb+1/2Fx$	0	1		
DE b	$1-x/b$	$-Fb+Fx$	0	$-Fb+2Fx-Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/3+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	Fx	0	$-Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BH b	0	$1/2Fx$	0	0	0	0		
HE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1 + 1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-\ x + 1/4\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-2\ b + b\ )\ Fb\ 1/EJ\ = -\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1 + 2\,x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x + x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

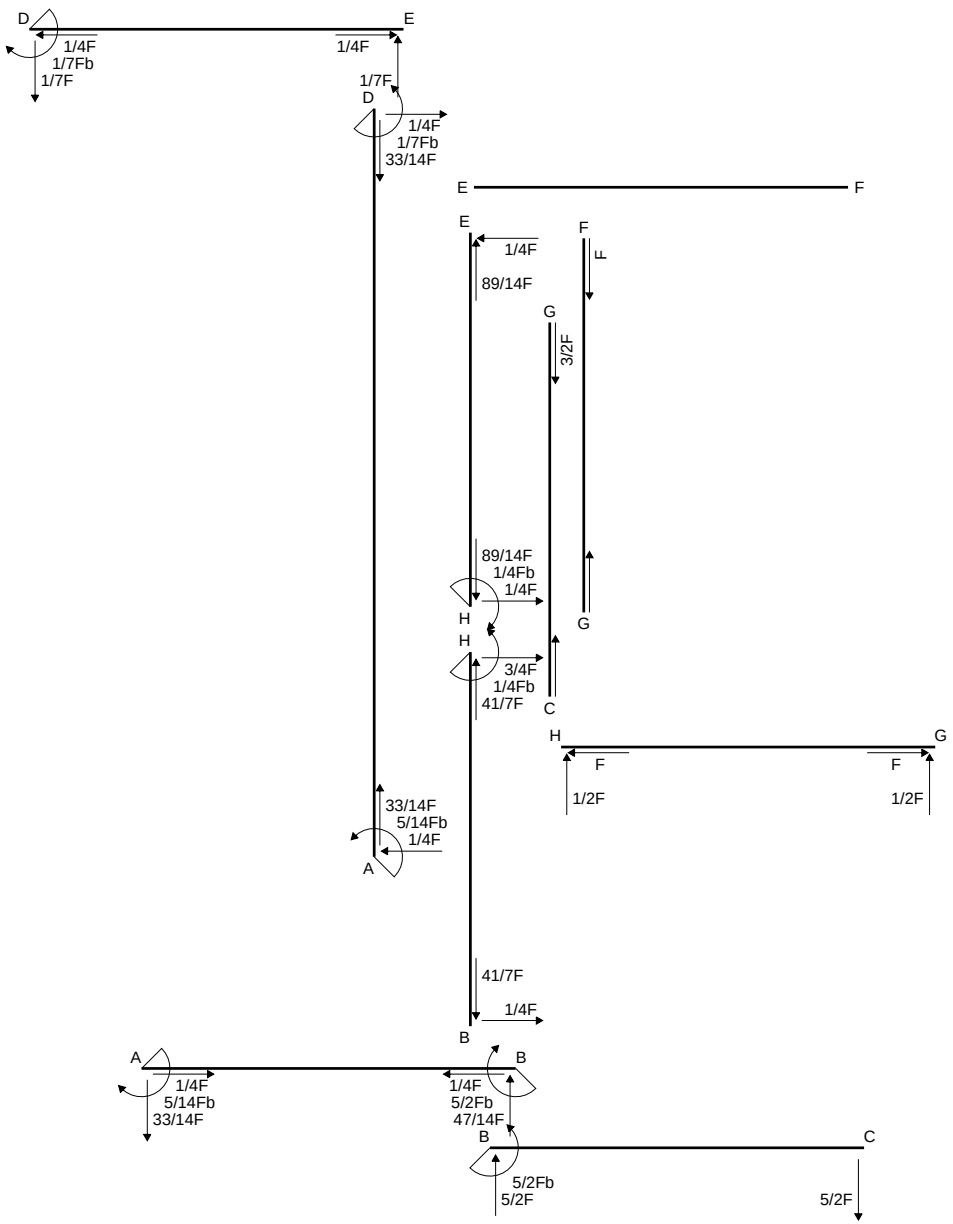
$= (-\ b + b - 1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

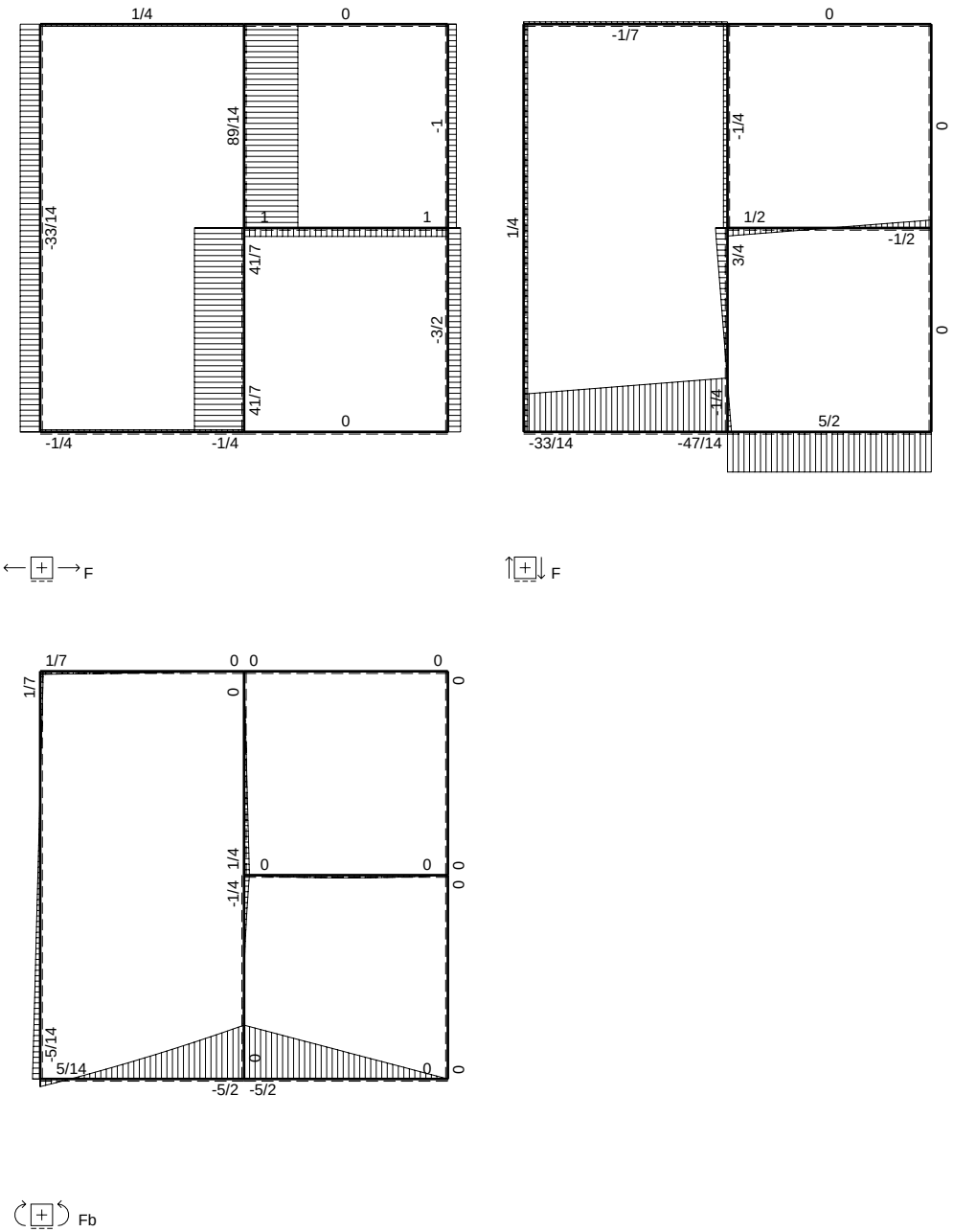
$L^{Xo}_{ED} = \int_o^b (-\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

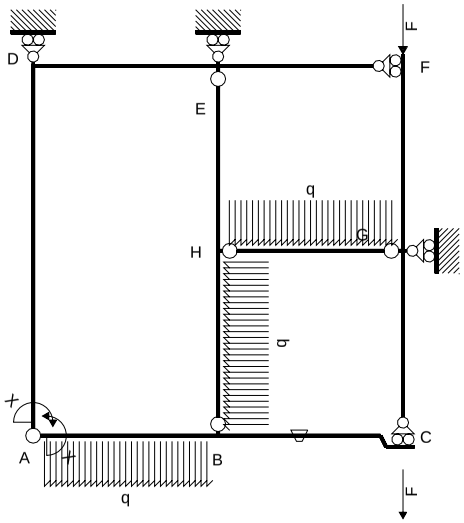
$= (-1/3\ b\ )\ Fb\ 1/EJ\ = -1/3\ \ Fb^2/EJ$

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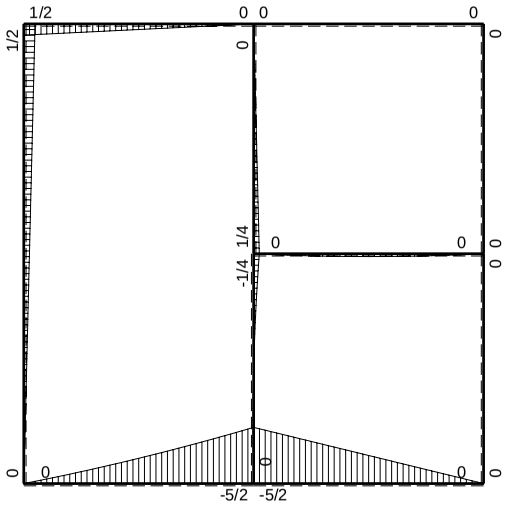
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
D
1/4F
1/7Fb
33/14F
E
F
E
1/4F
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F
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3/4F
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C
H
G
F
F
1/2F
1/2F
33/14F
5/14Fb
1/4F
A
41/7F
1/4F
B
A
B
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B
C
5/2Fb
5/2F
5/2F

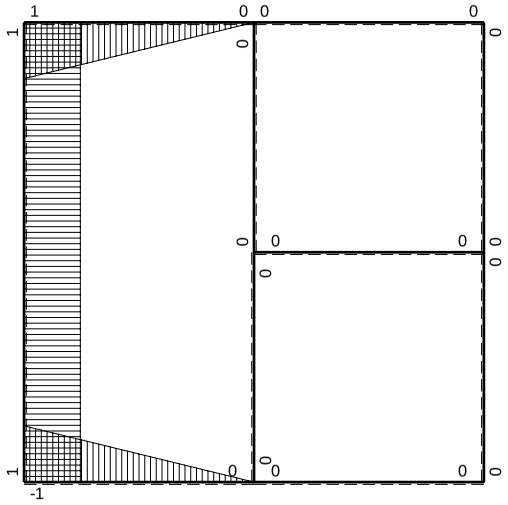

1/4
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89/14
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-33/14
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41/7
-3/2
41/7
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-1/4
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-1/2
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5/2
-33/14
-47/14
F
1/7
0 0
0
1/7
0
0
1/4
0
0
-1/4
0 0
0
-5/14
5/14
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.177

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$5/2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.177

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

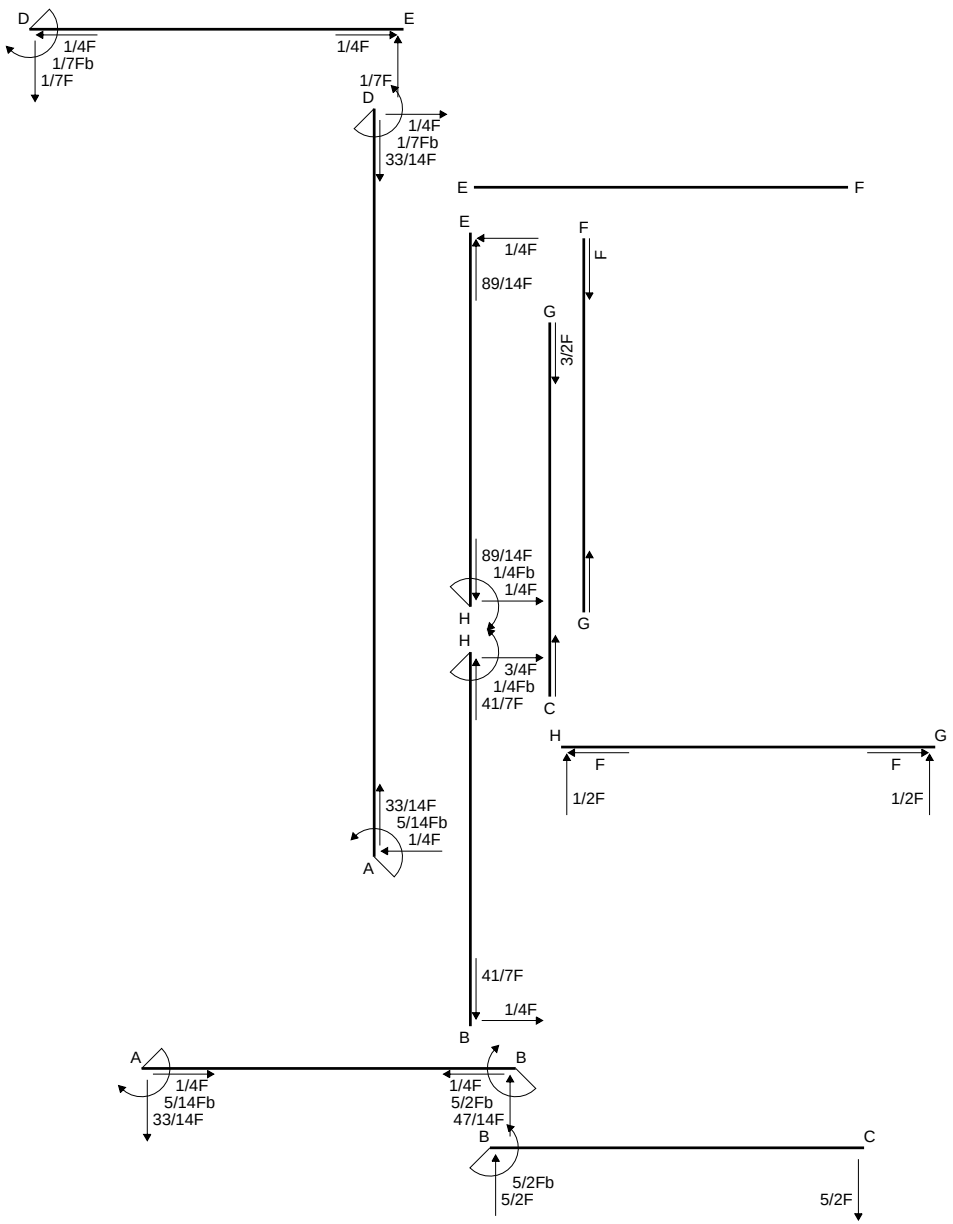
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

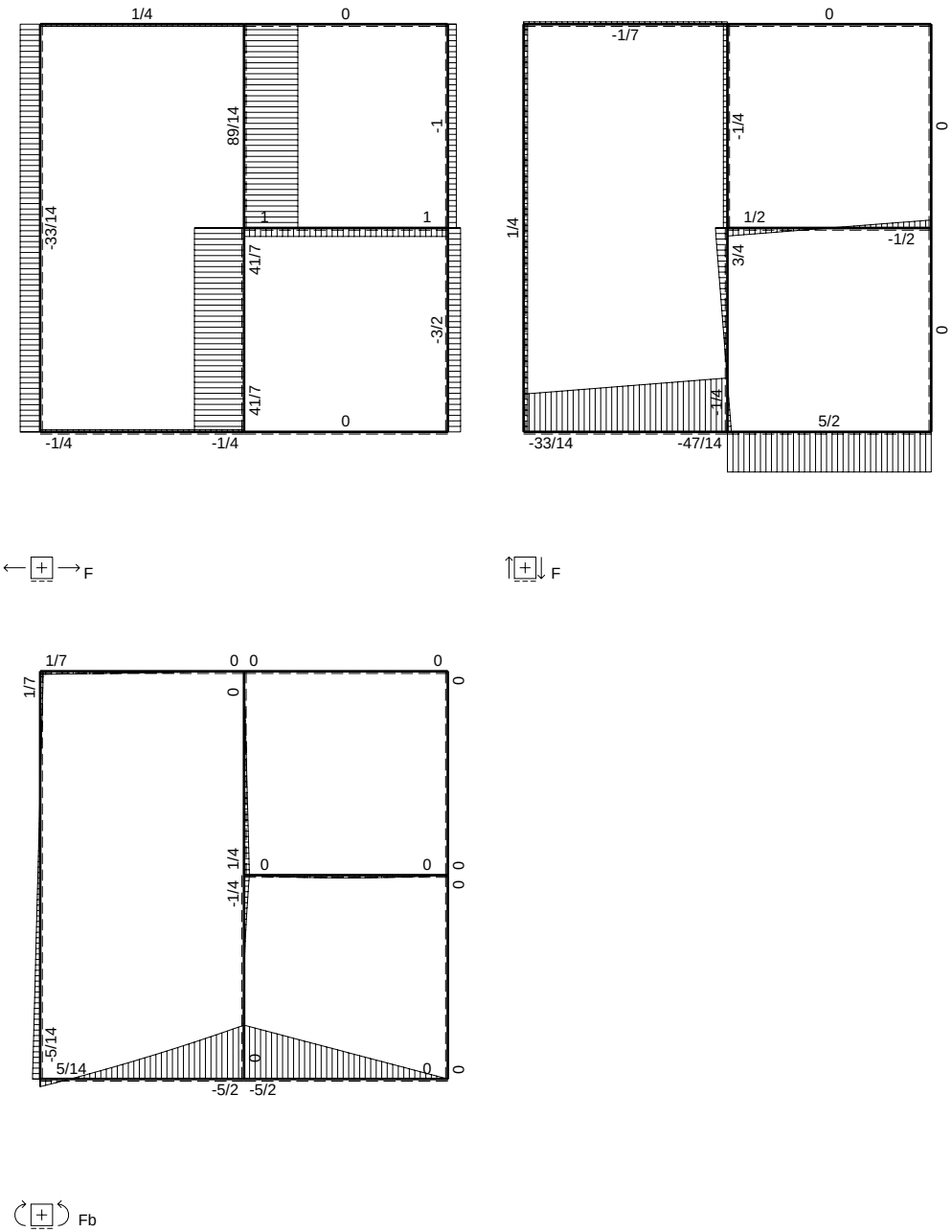
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

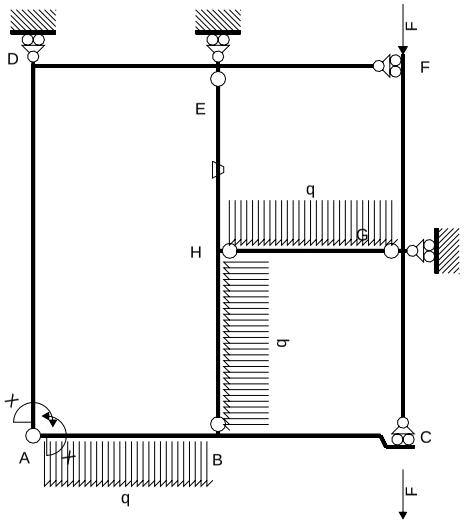
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.177

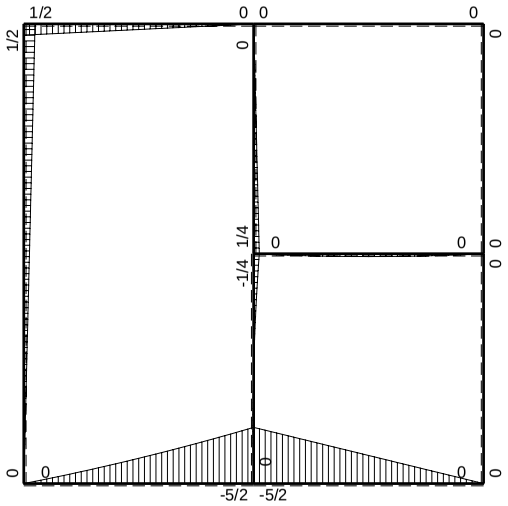
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D
E
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1/7Fb
1/7F
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1/7F
D
1/4F
1/7Fb
33/14F
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1/4F
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5/2F

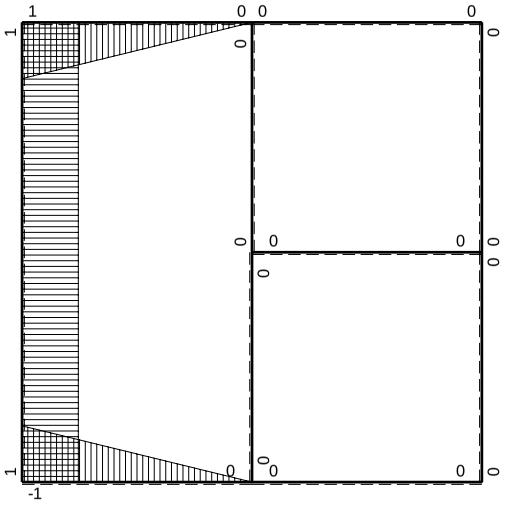

1/4
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1
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41/7
-3/2
41/7
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-1/4
-1/4
F
-1/7
0
-1/4
0
1/2
3/4
-1/2
1/4
0
-1/4
5/2
-33/14
-47/14
F
1/7
0 0
0
1/7
0
0
1/4
0
0
-1/4
0 0
0
-5/14
5/14
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.178

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	1/4Fb-1/4Fx	-Fb/EJ	0	0	0	0+0	0
EH b	0	-1/4Fx	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-5/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.178

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

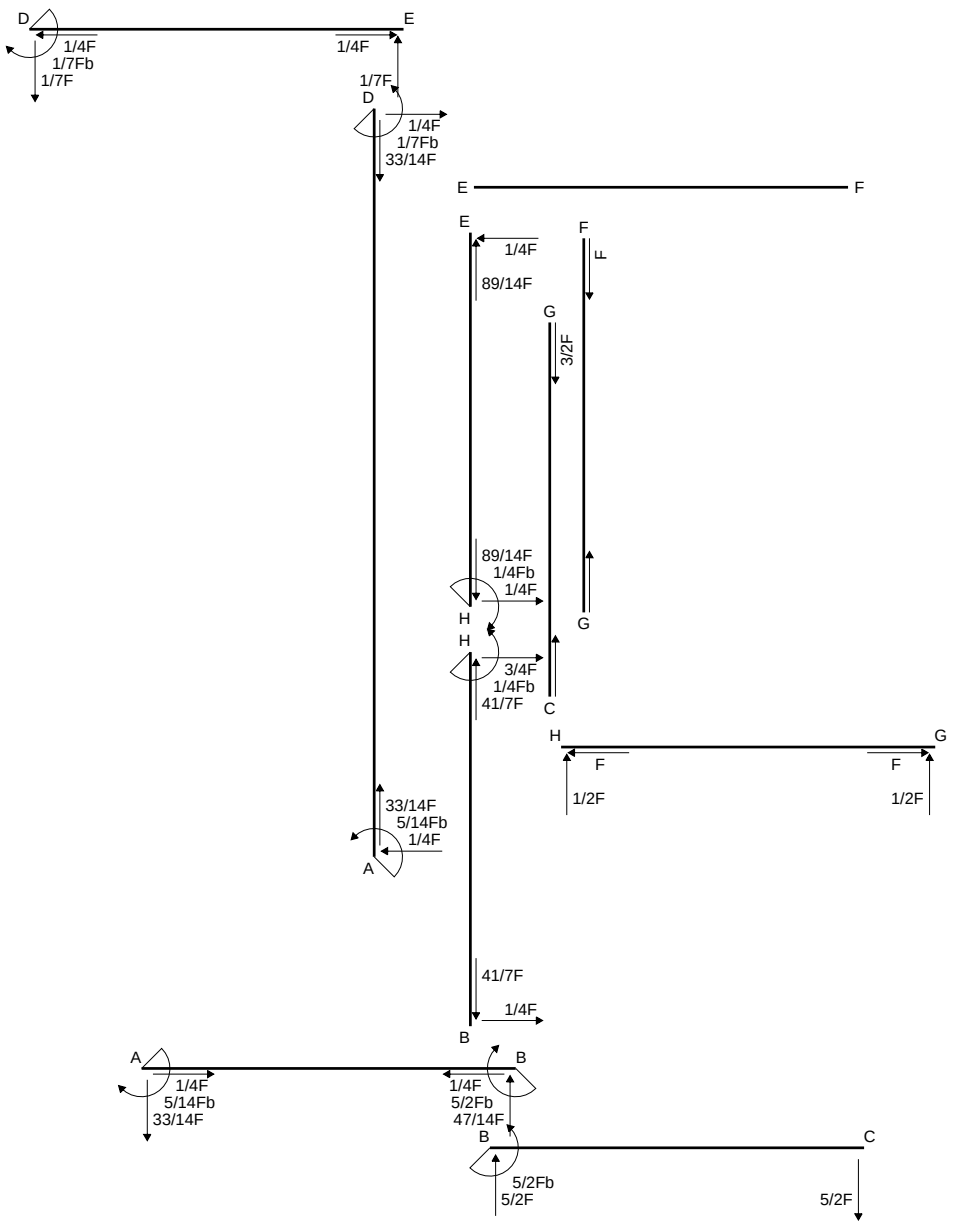
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

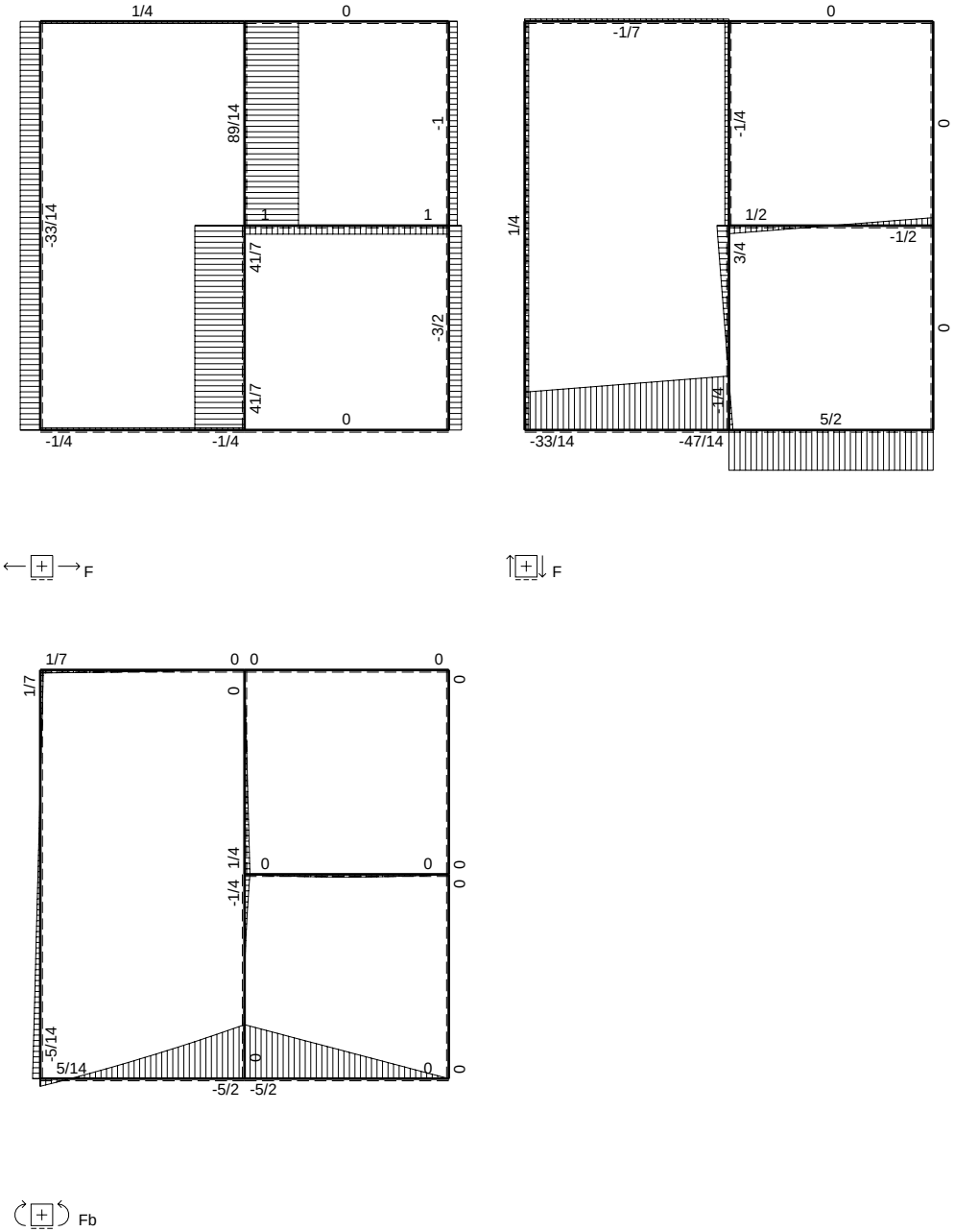
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

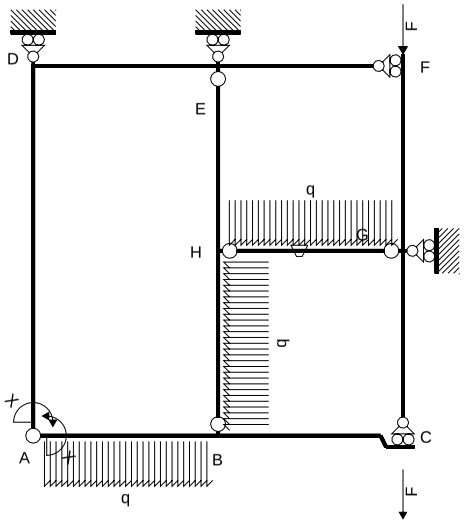
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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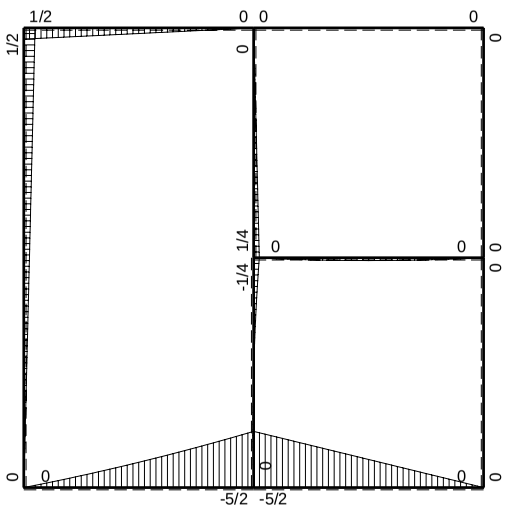
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
D
1/4F
1/7Fb
33/14F
E
F
E
1/4F
89/14F
F
F
G
3/2F
89/14F
1/4Fb
1/4F
G
H
H
3/4F
1/4Fb
41/7F
C
H
G
F
F
1/2F
1/2F
33/14F
5/14Fb
1/4F
A
41/7F
1/4F
B
A
B
1/4F
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33/14F
1/4F
5/2Fb
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B
C
5/2Fb
5/2F
5/2F

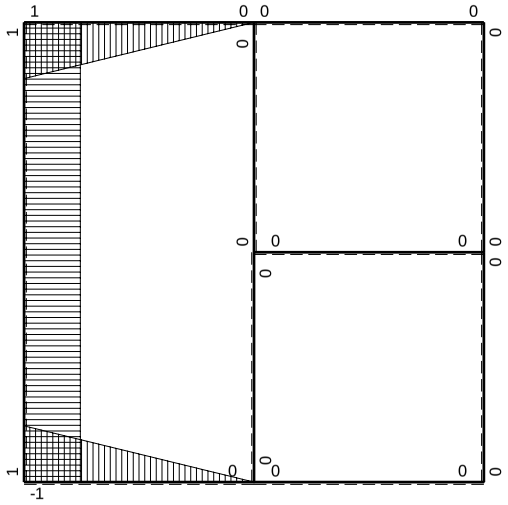

1/4
0
89/14
-1
-33/14
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1
41/7
-3/2
41/7
0
-1/4
-1/4
F
0
-1/7
-1/4
0
1/4
1/2
3/4
-1/2
0
-1/4
5/2
-33/14
-47/14
F
1/7
0 0
0
1/7
0
0
1/4
0
0
-1/4
0 0
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

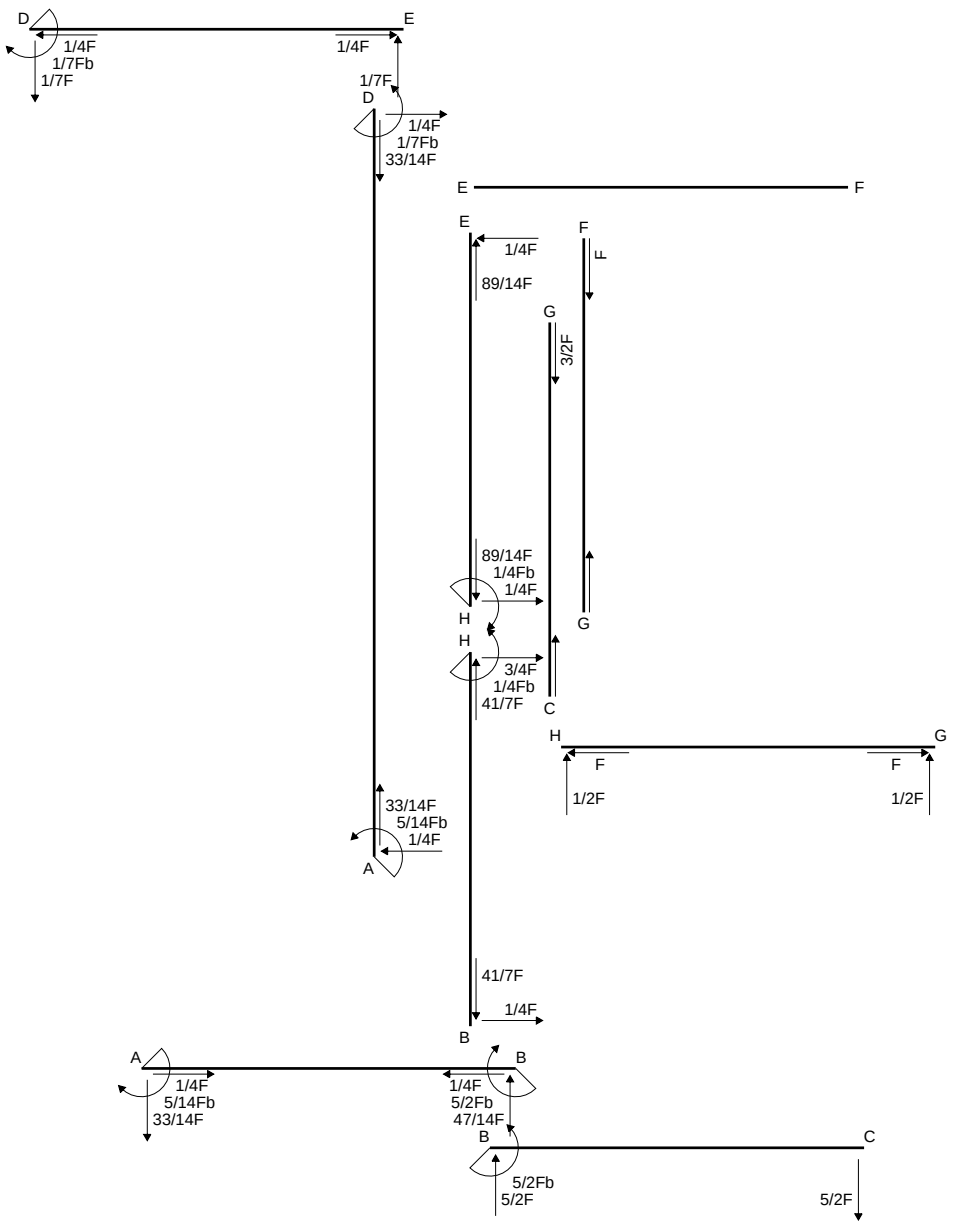
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

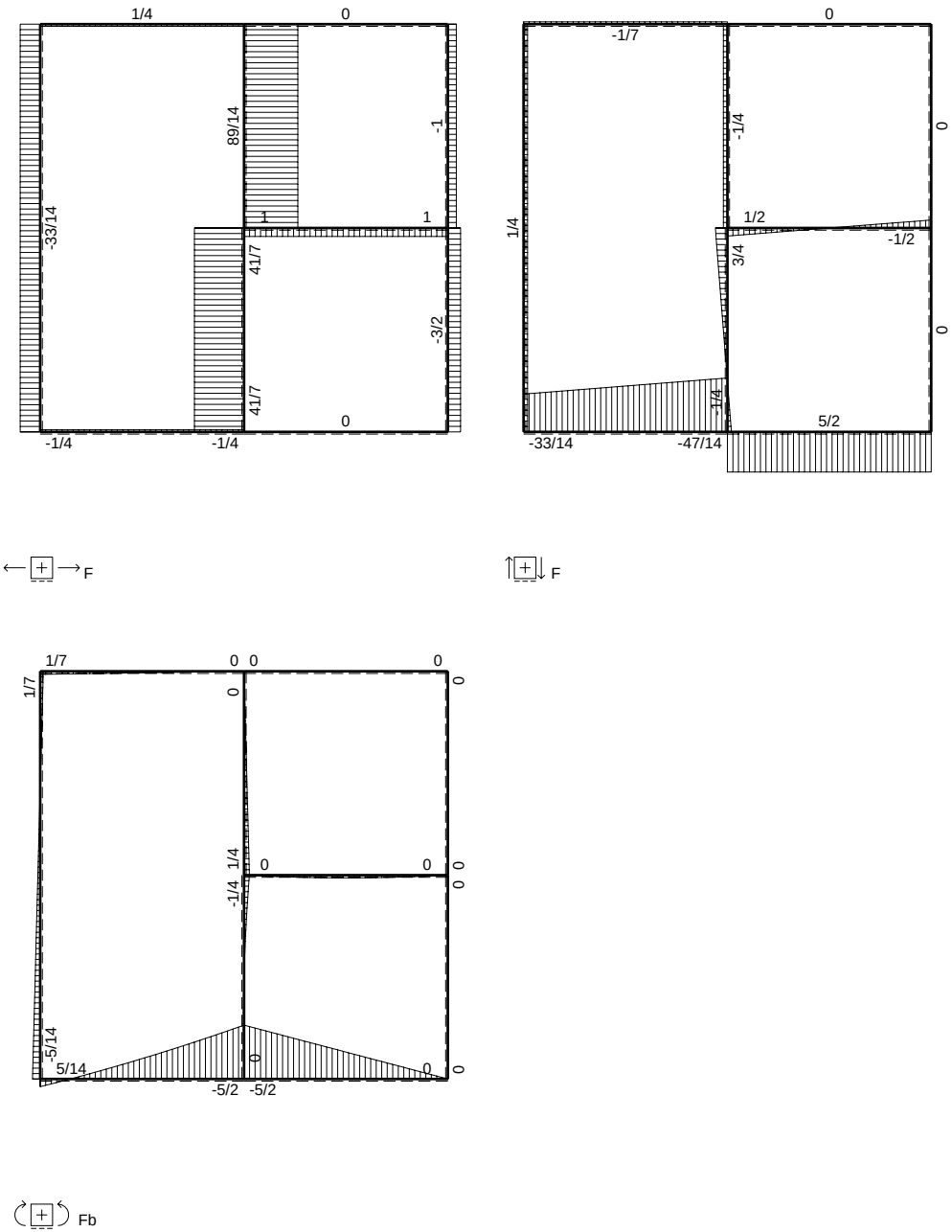
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

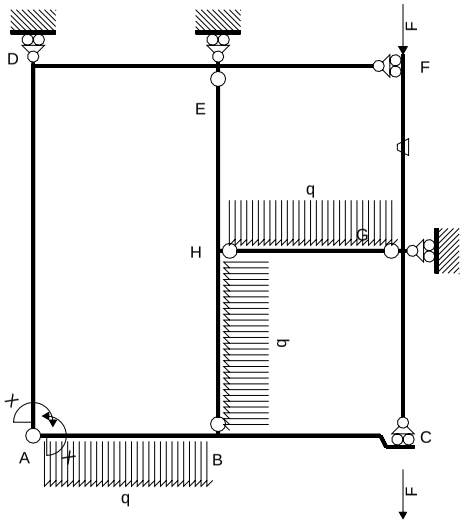
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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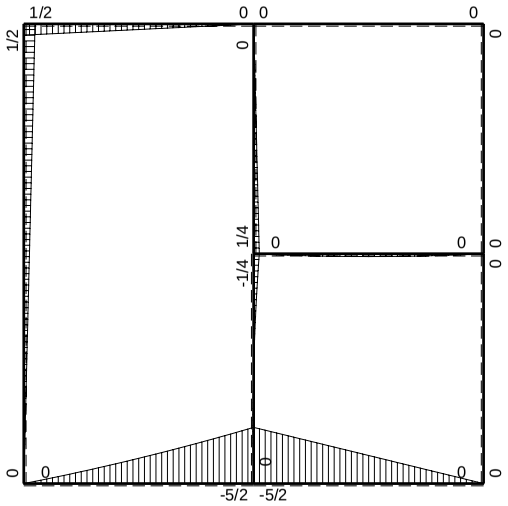
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
D
1/4F
1/7Fb
33/14F
E
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5/2F
5/2F

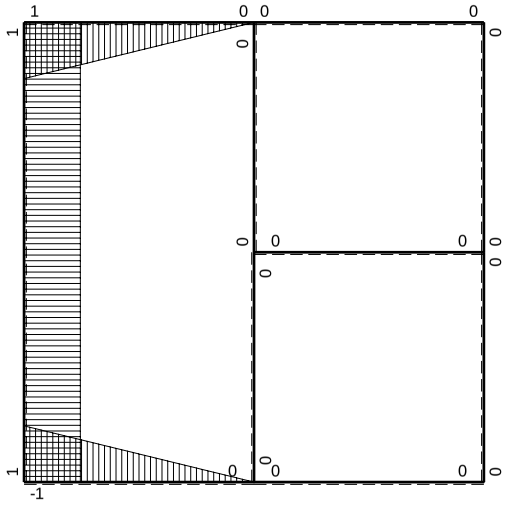

1/4
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41/7
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-1/4
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-1/7
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-1/2
0
-1/4
5/2
-33/14
-47/14
F
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0
1/7
0
0
1/4
0
0
-1/4
0 0
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.180

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

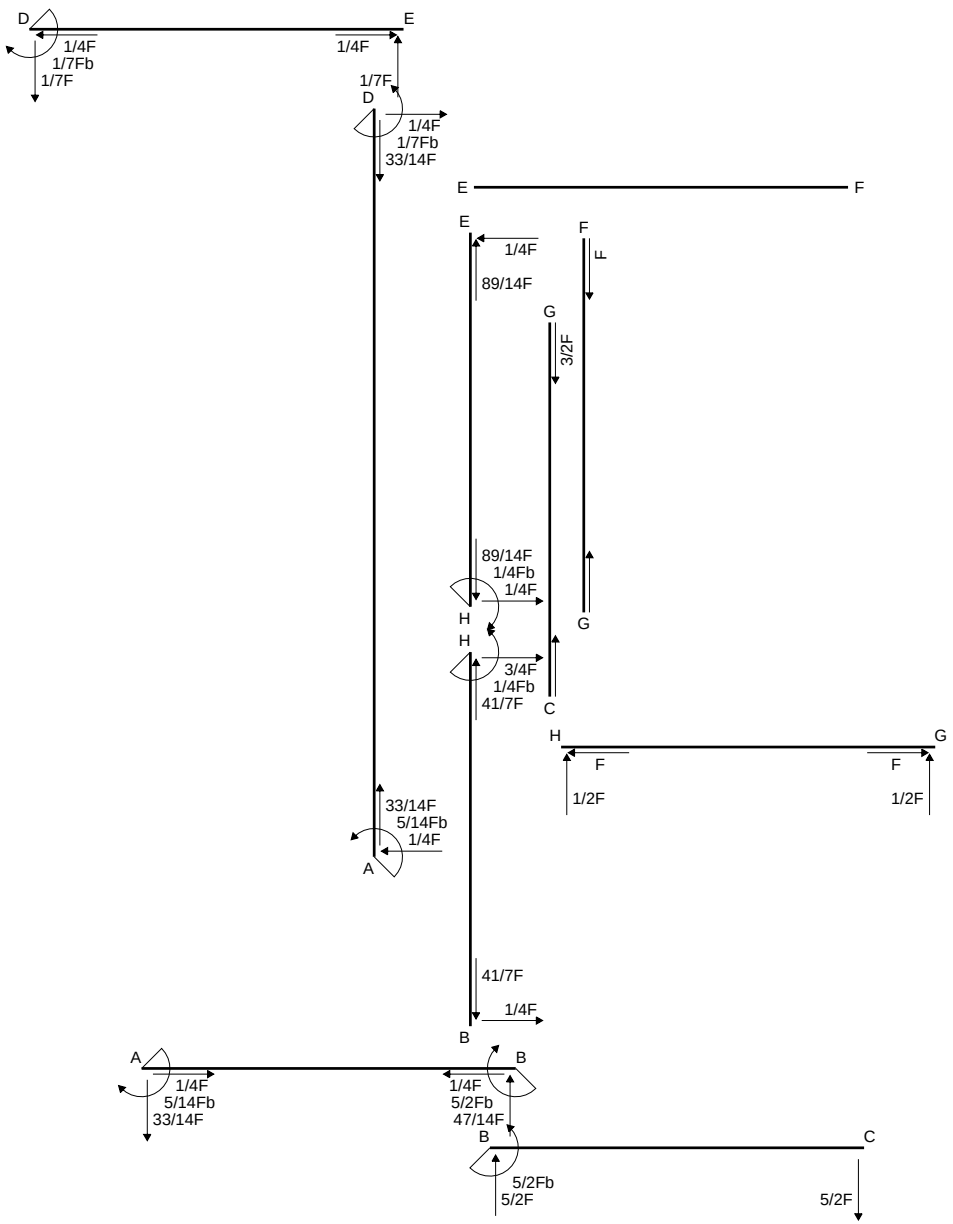
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

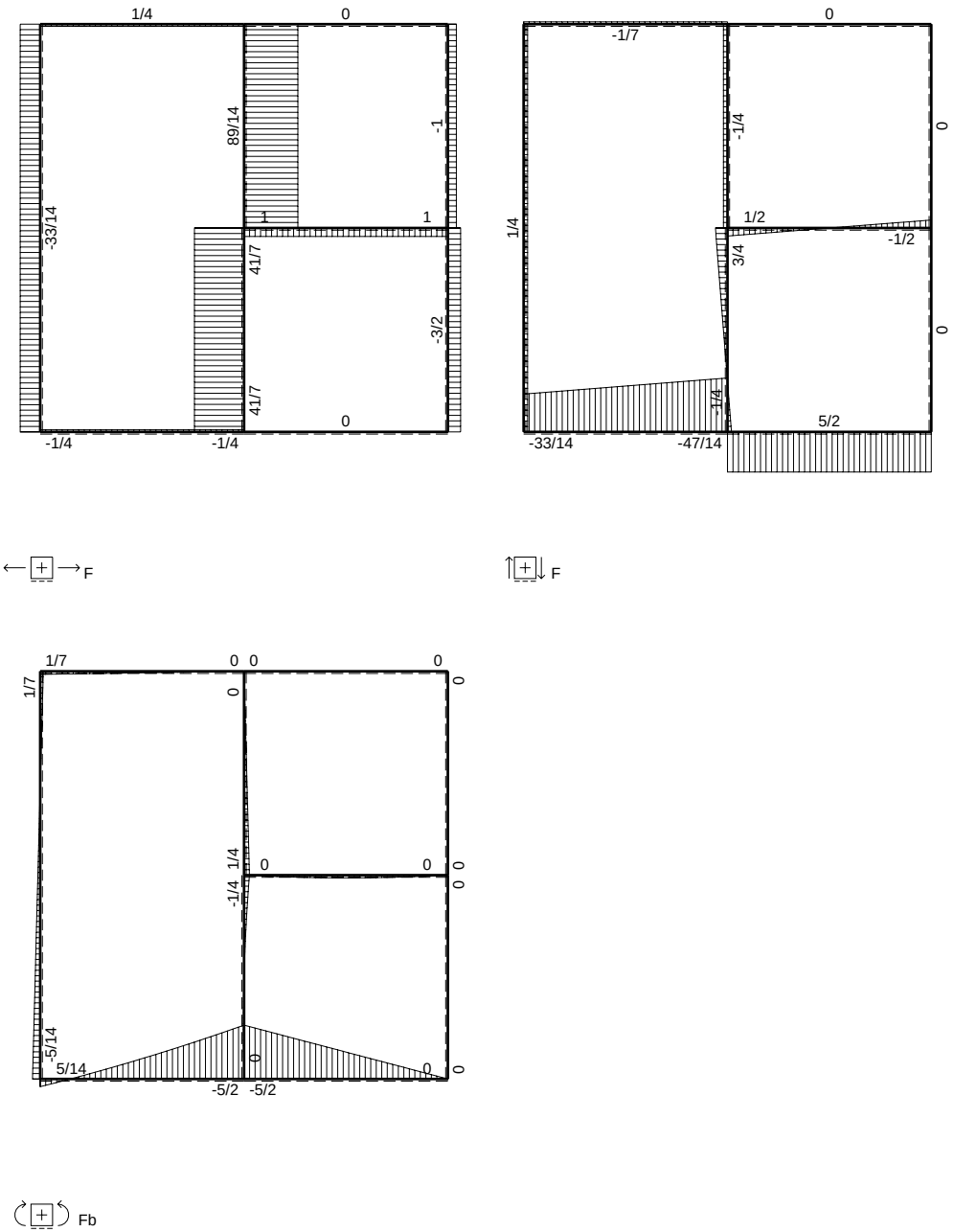
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

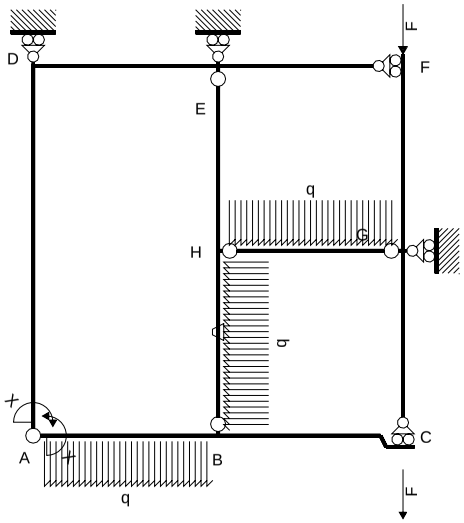
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.180

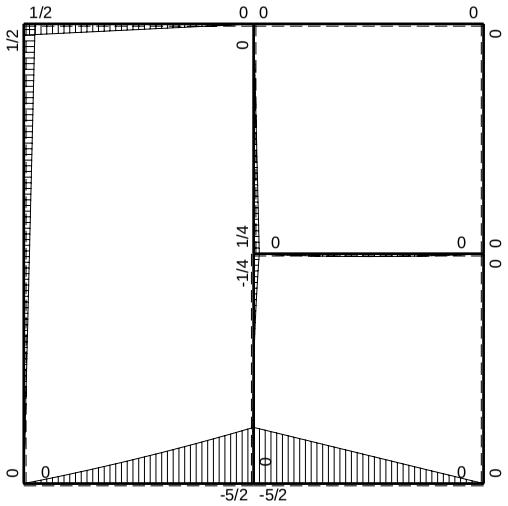
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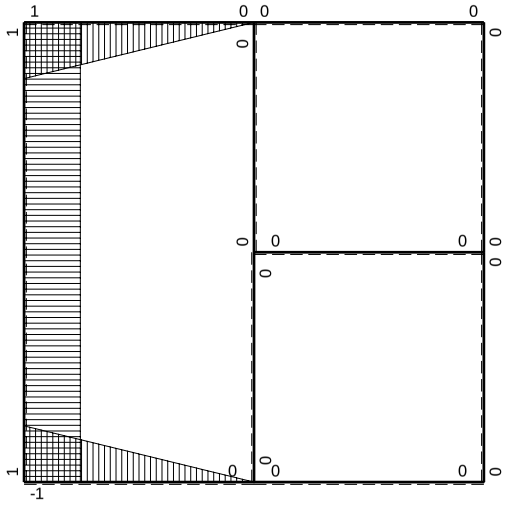

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-5/14
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0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.181

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
EH b	0	-1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-5/14Fb	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

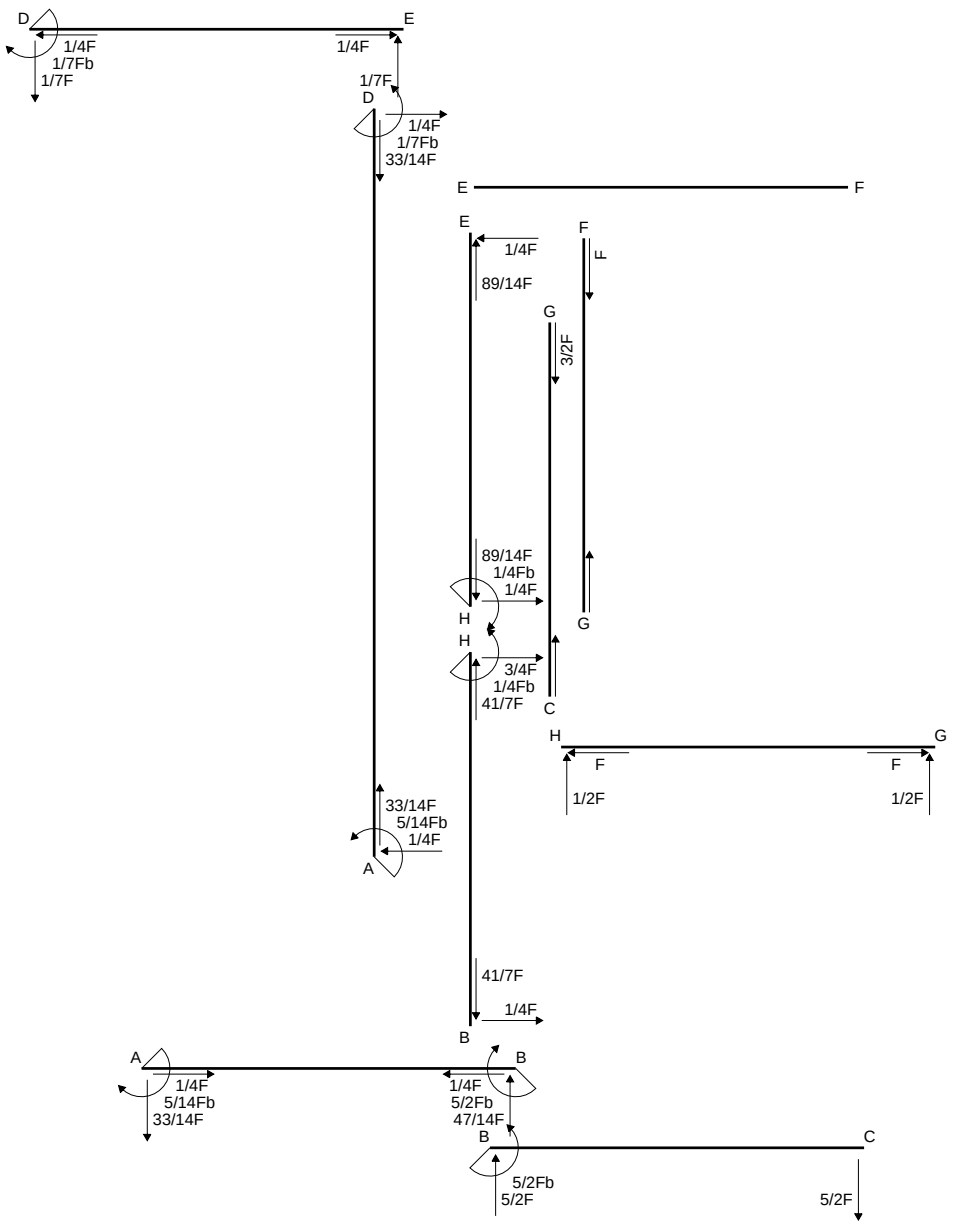
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

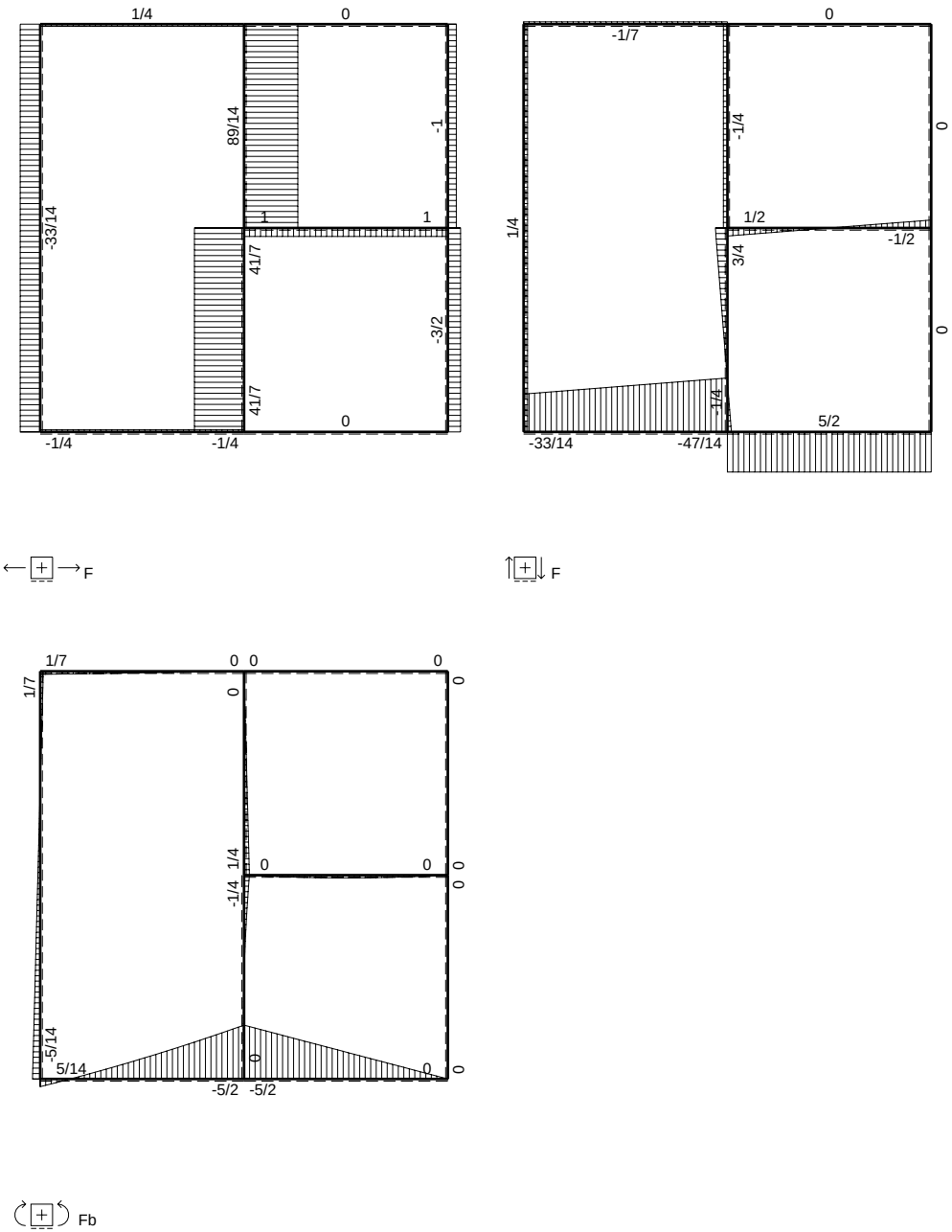
$L_{ED}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

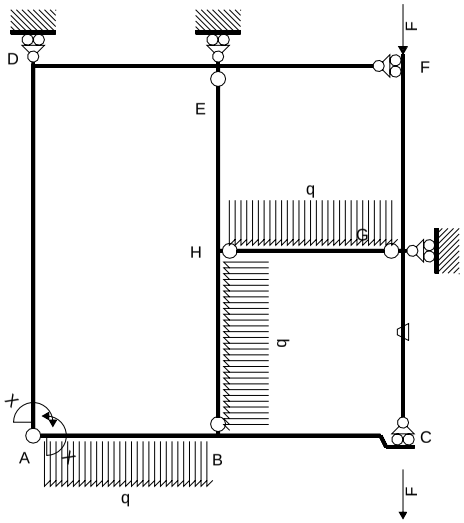
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.181

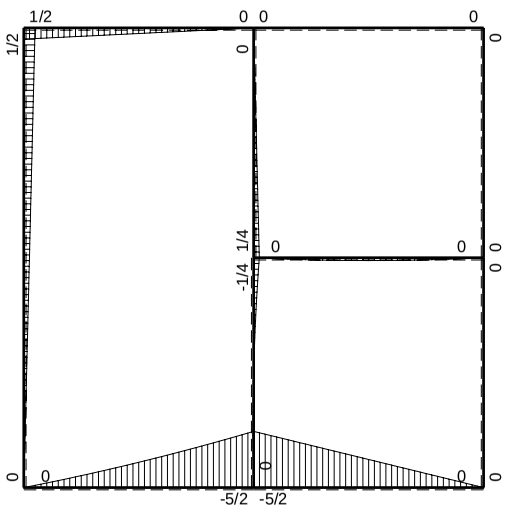
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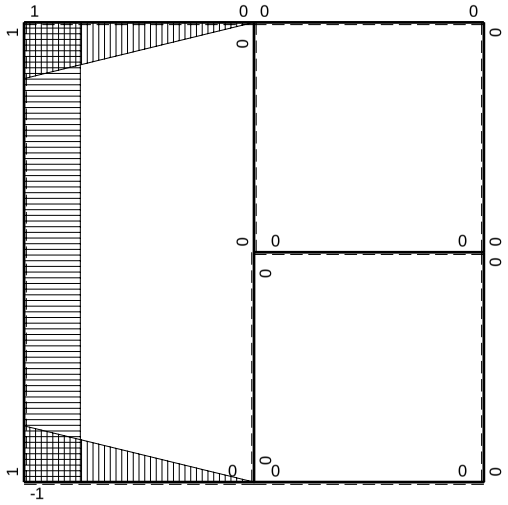

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-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.182

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PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.182

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

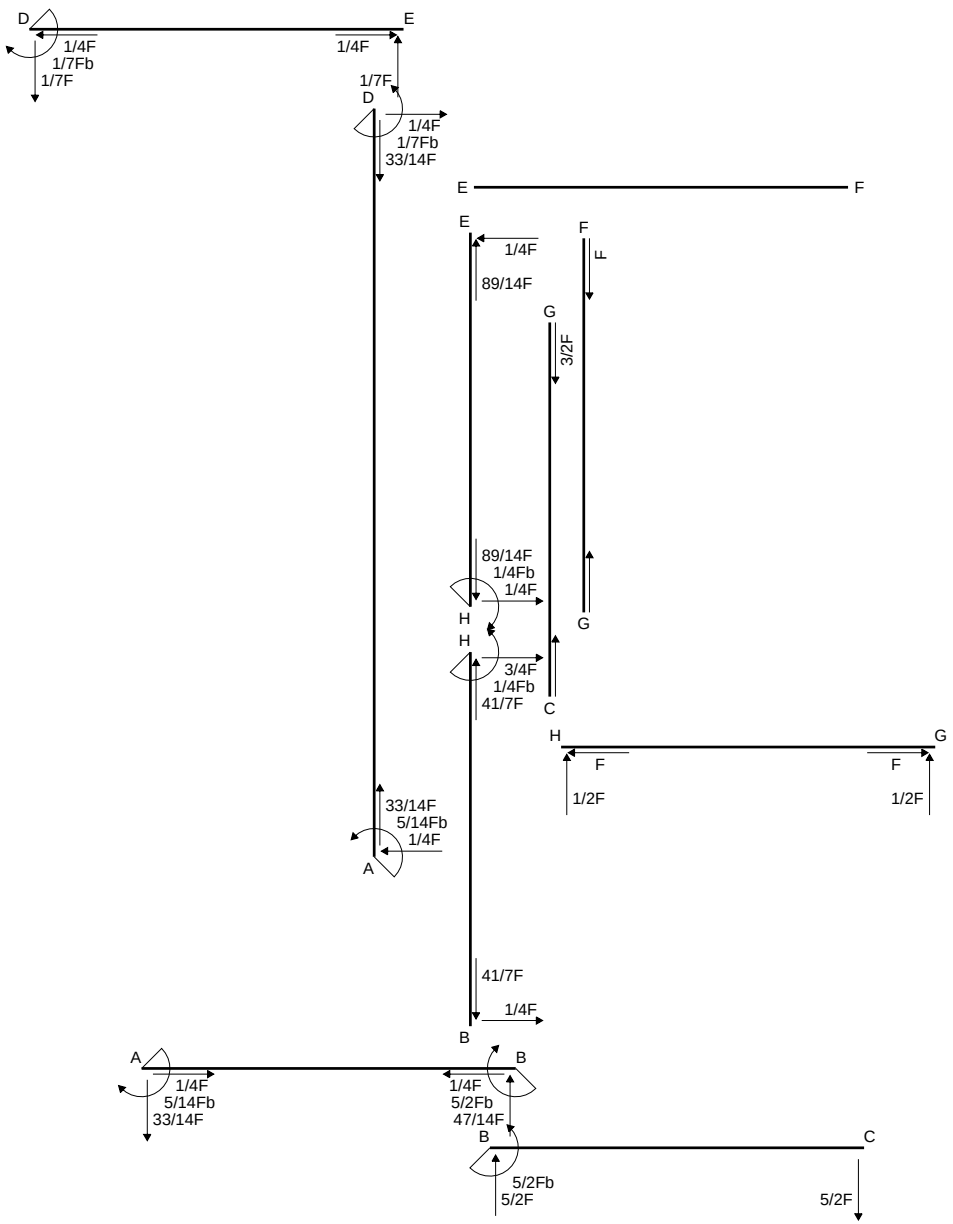
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

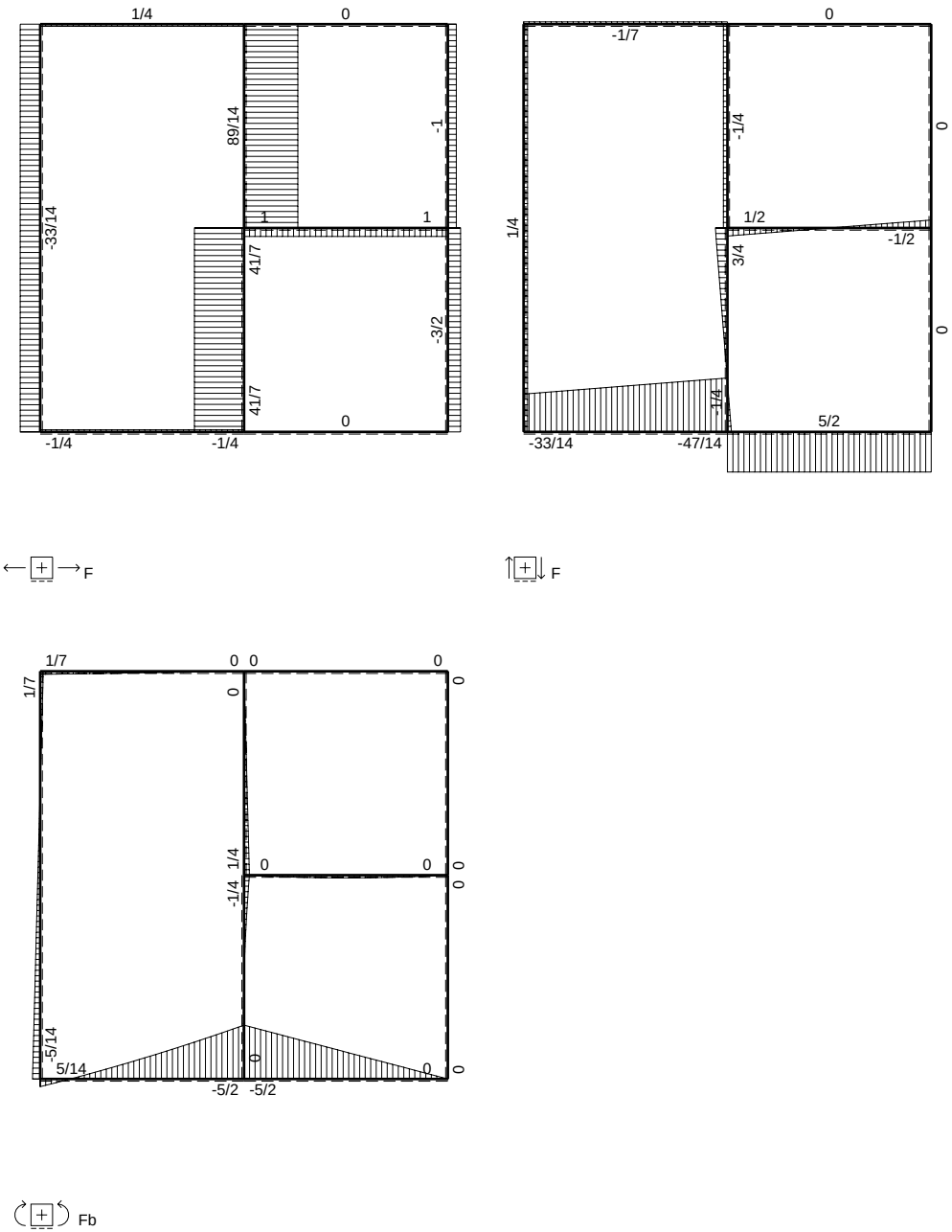
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

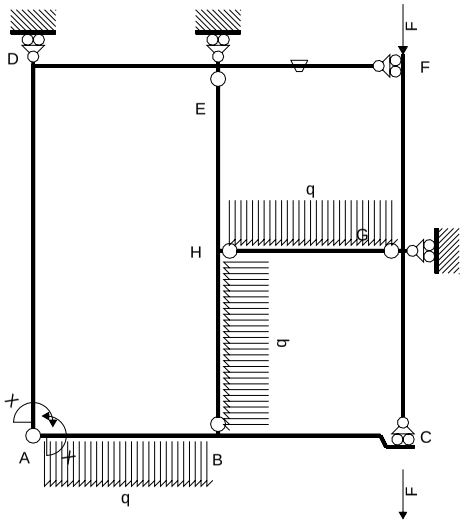
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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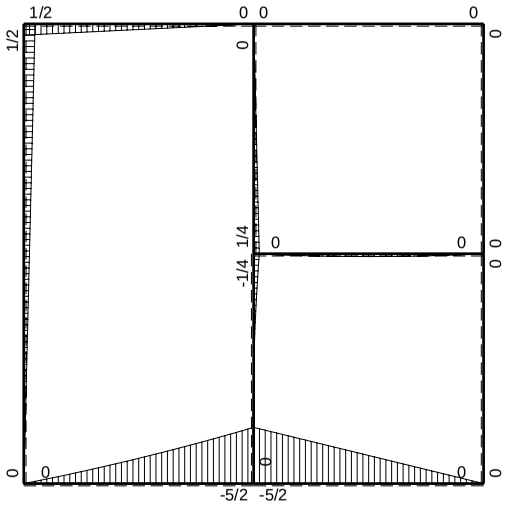
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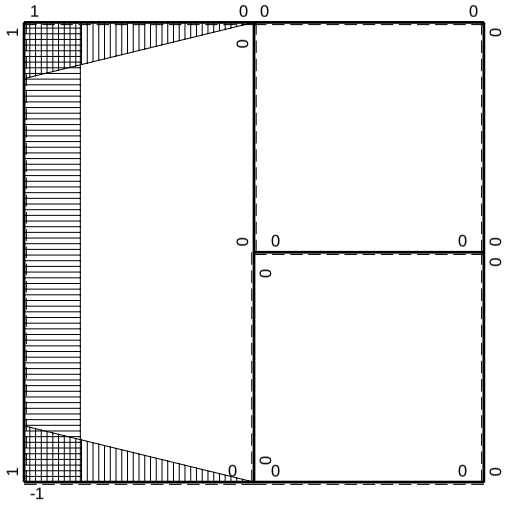

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-1
-33/14
1
1
41/7
-3/2
41/7
0
-1/4
-1/4
F
0
-1/7
-1/4
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1/4
1/2
3/4
-1/2
0
-1/4
5/2
-33/14
-47/14
F
1/7
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0
1/4
0
0
-1/4
0 0
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.183

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.183

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

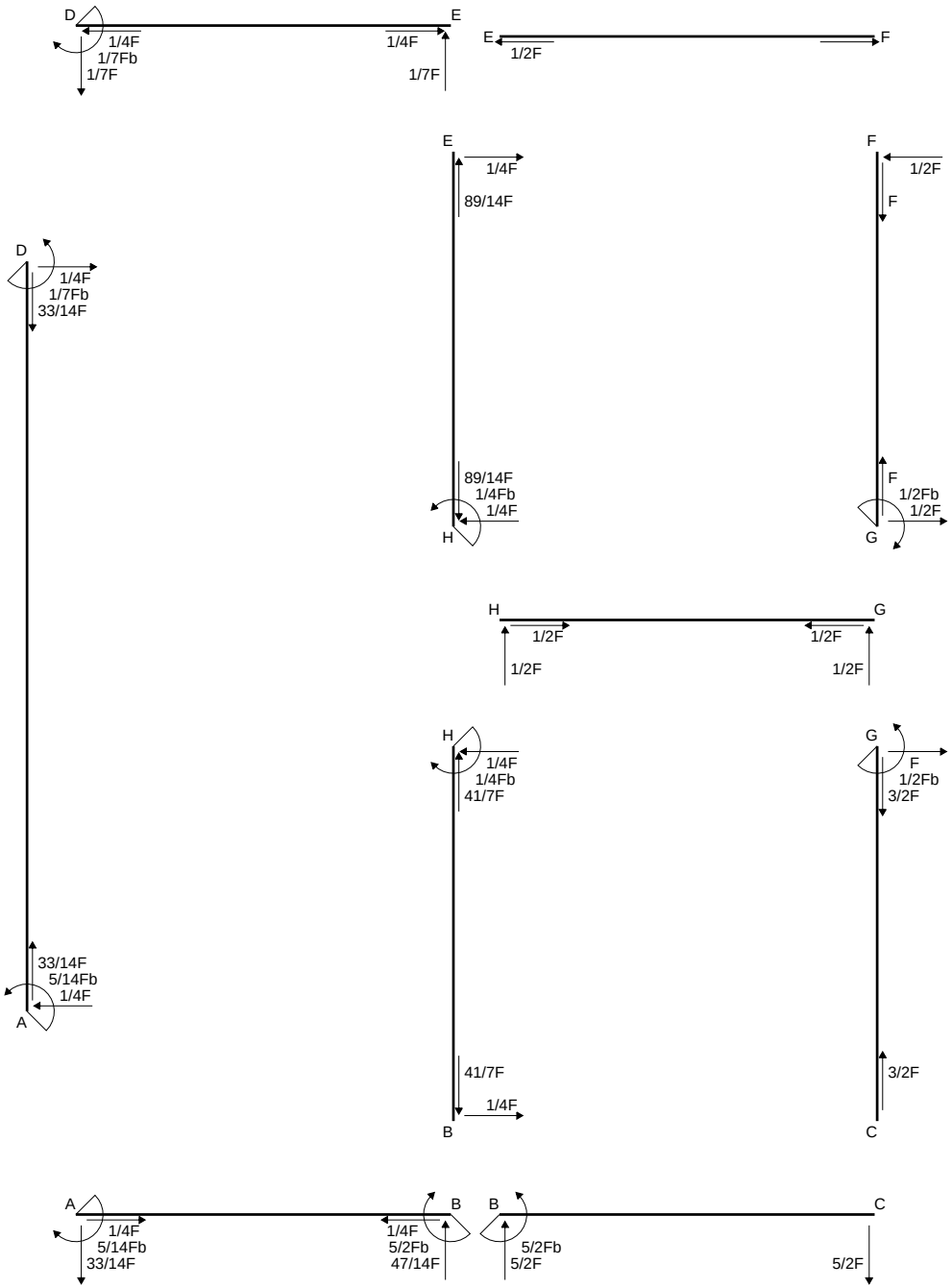
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

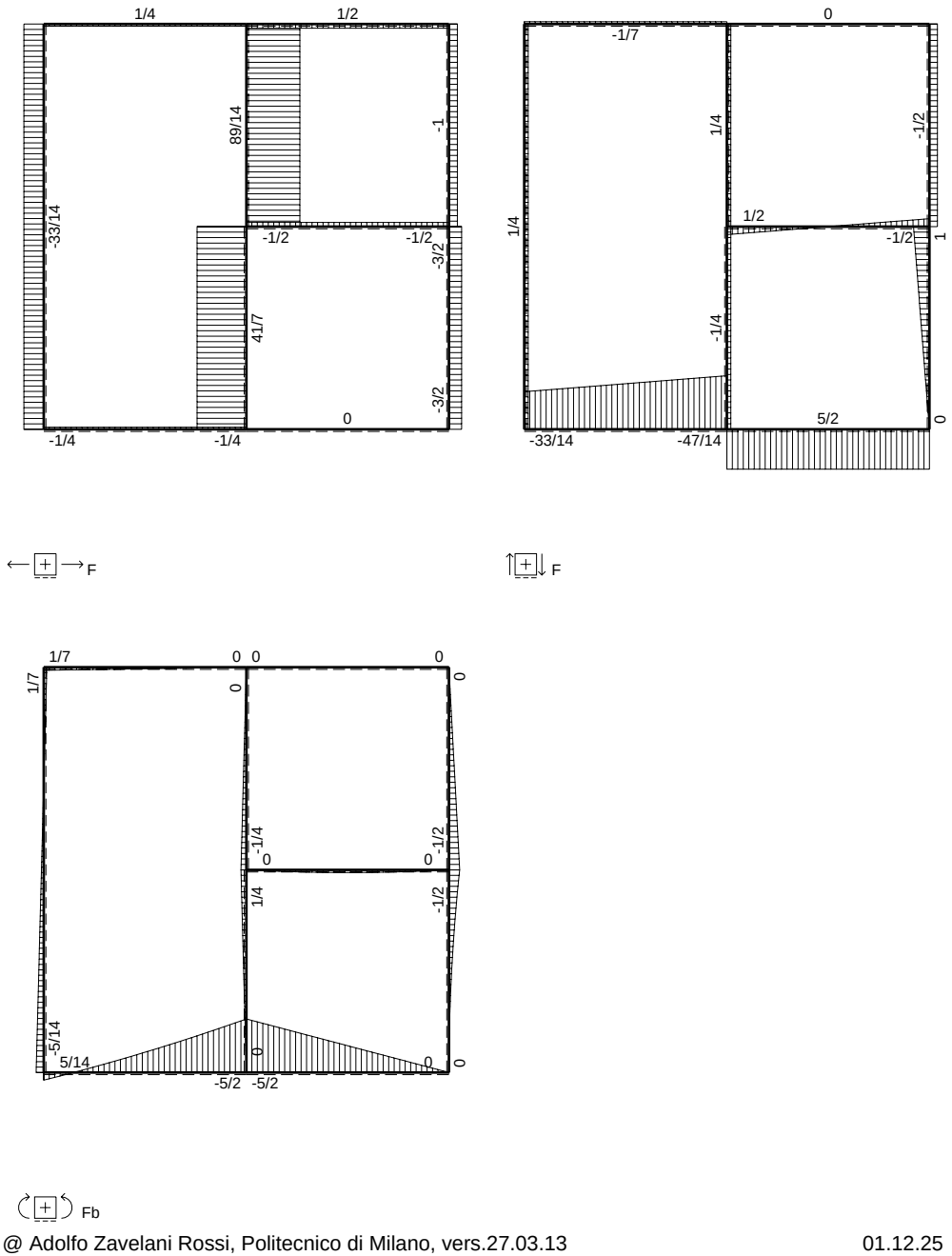
$L_{ED}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

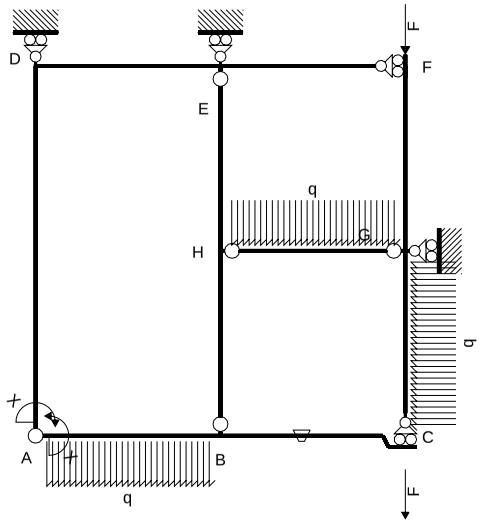
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.183

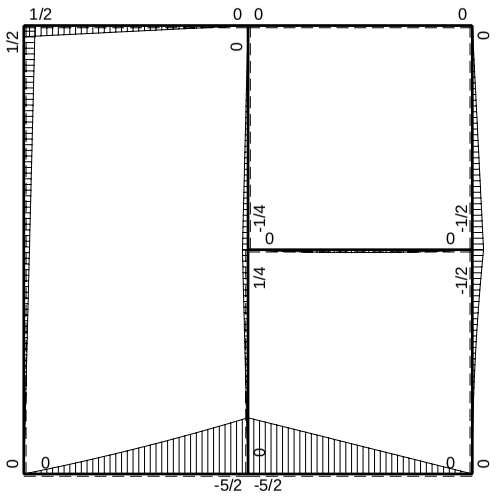
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1/2F
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89/14F
1/4Fb
1/4F
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F
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D
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5/14Fb
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A
1/4F
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1/4F
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5/2Fb
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C
5/2F

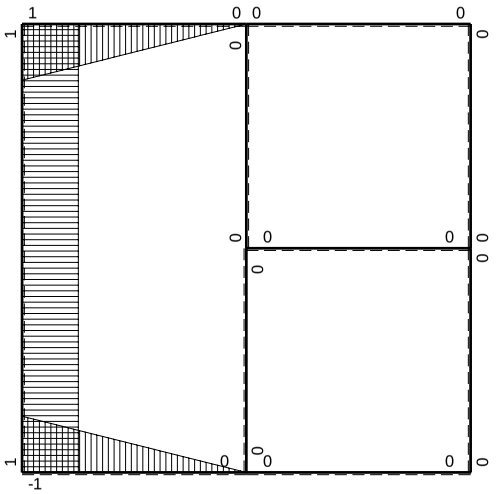

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1/4
-1/2
-5/14
5/14
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0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.184

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$5/2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.184

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

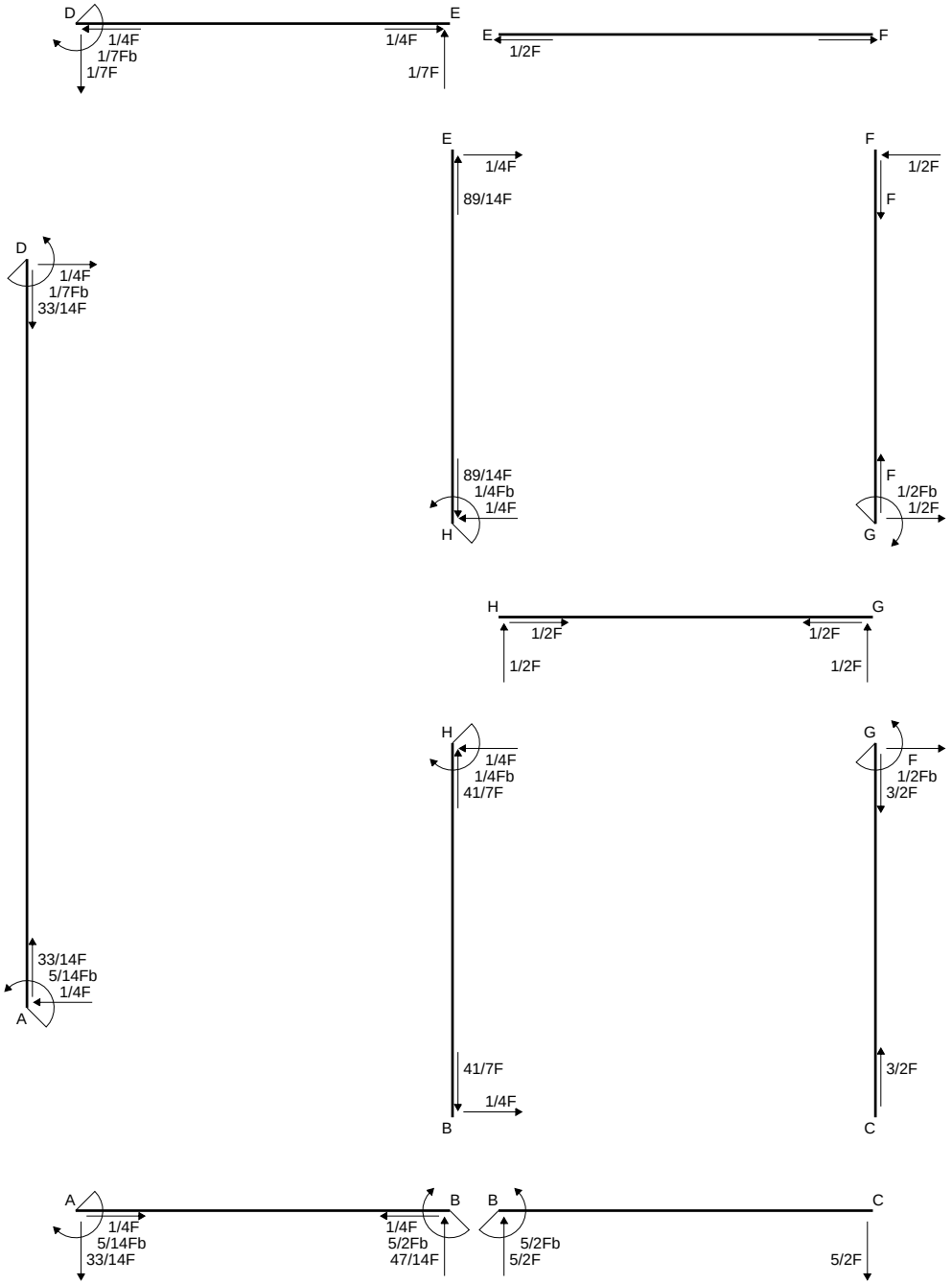
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

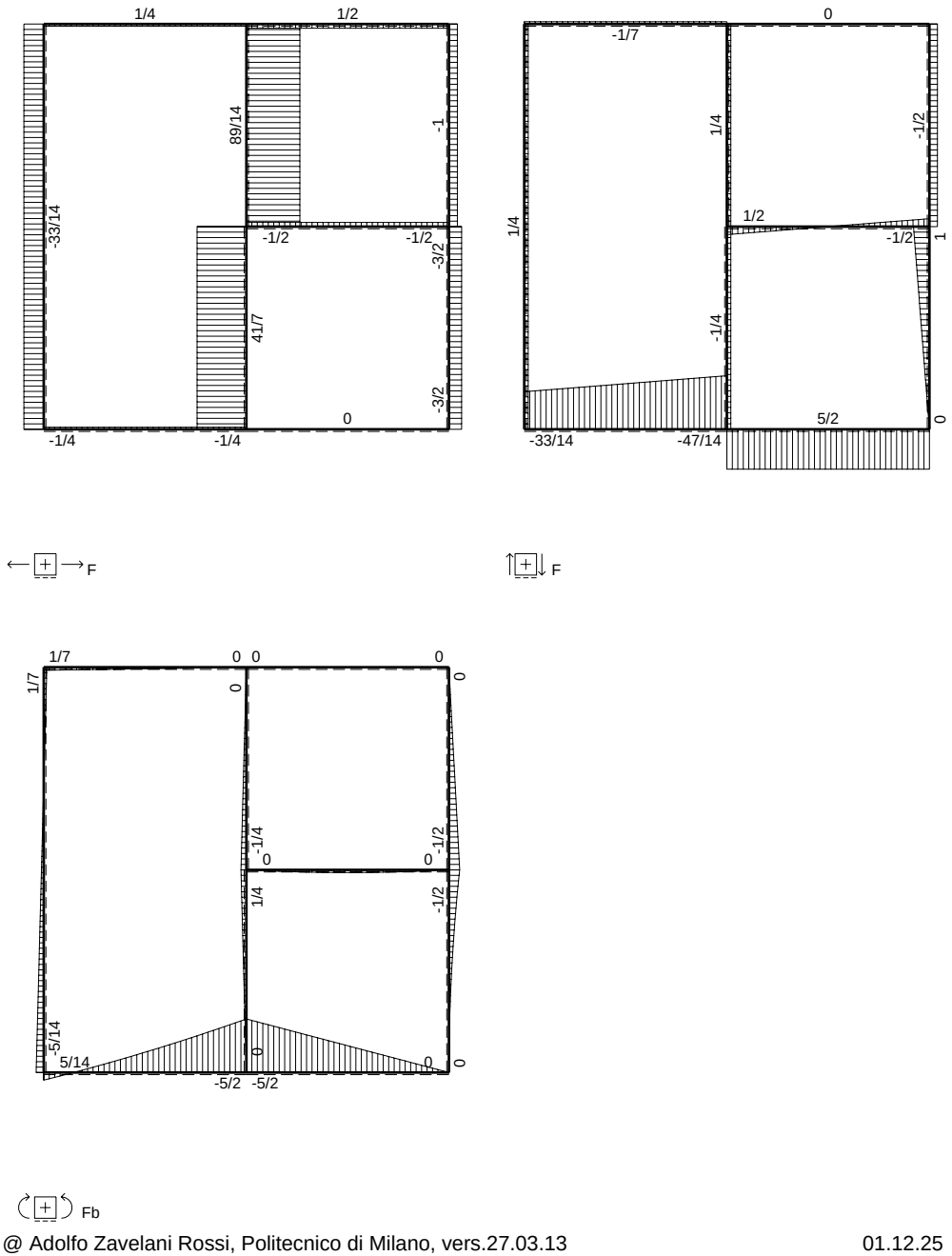
$L_{ED}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

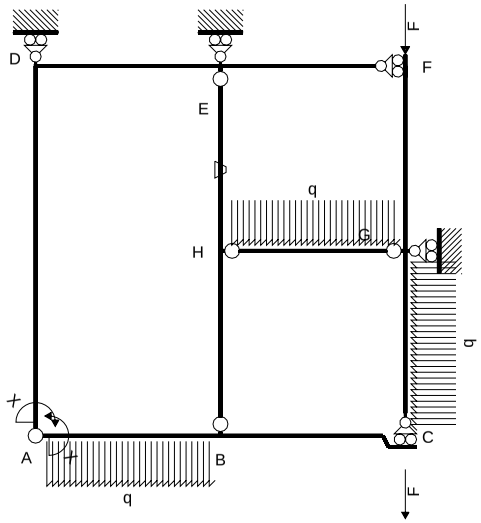
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.184

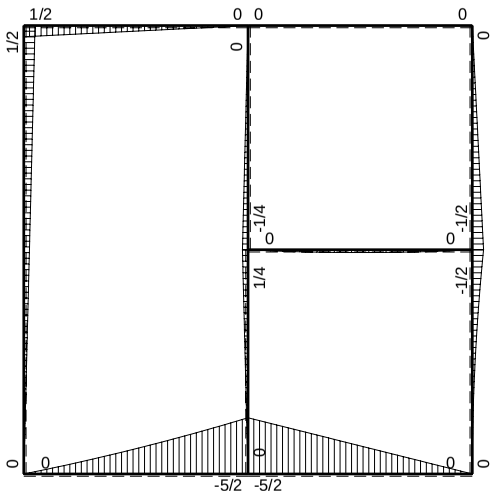
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D
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1/4F
1/7Fb
1/7F
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1/7F
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1/2F
E
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89/14F
89/14F
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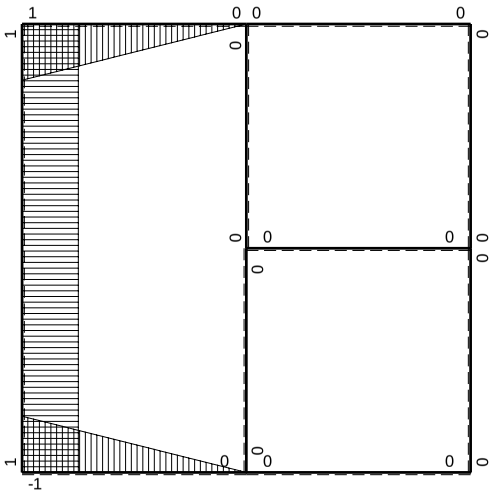

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-5/14
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0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.185

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$1/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.185

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

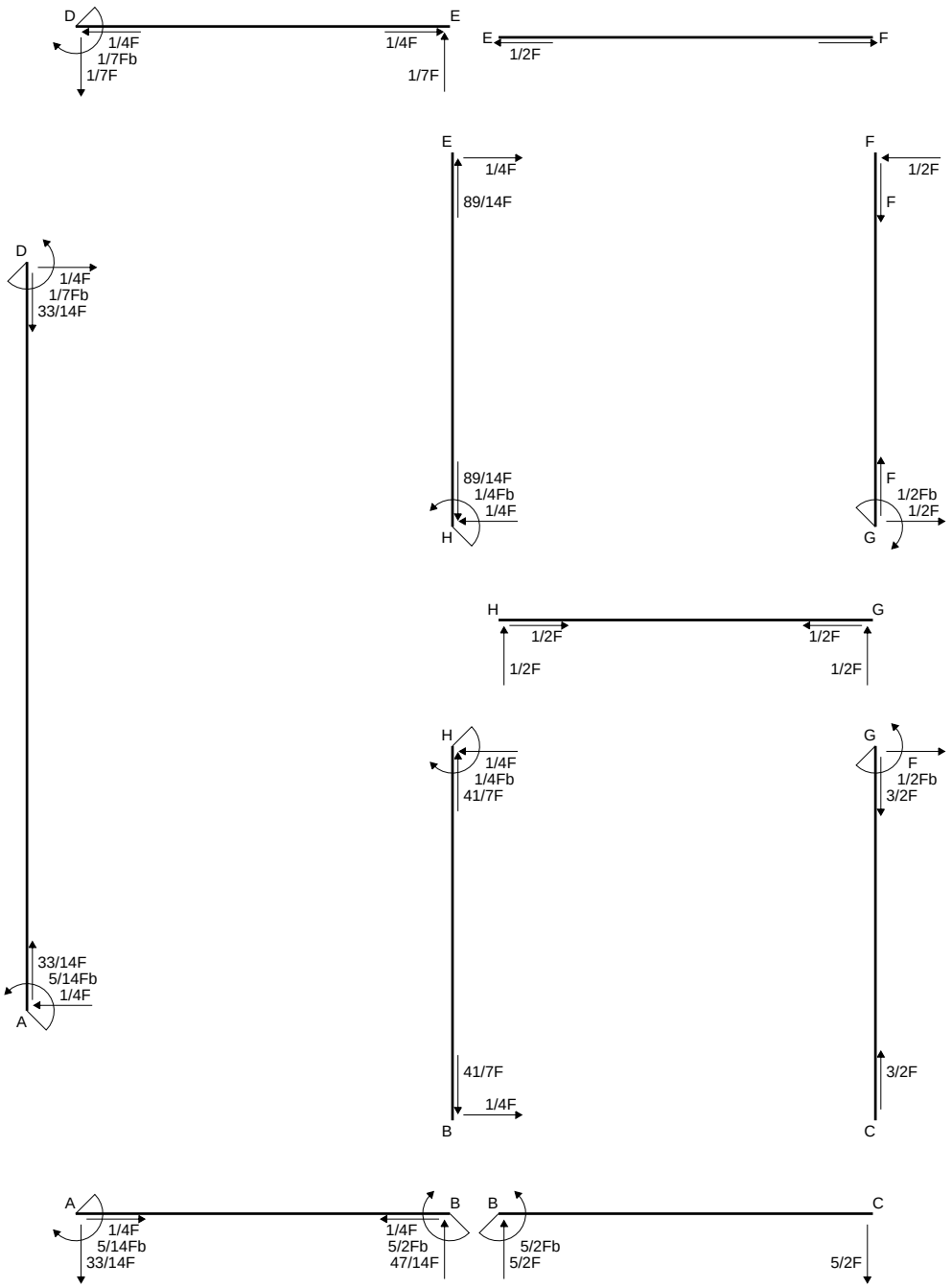
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

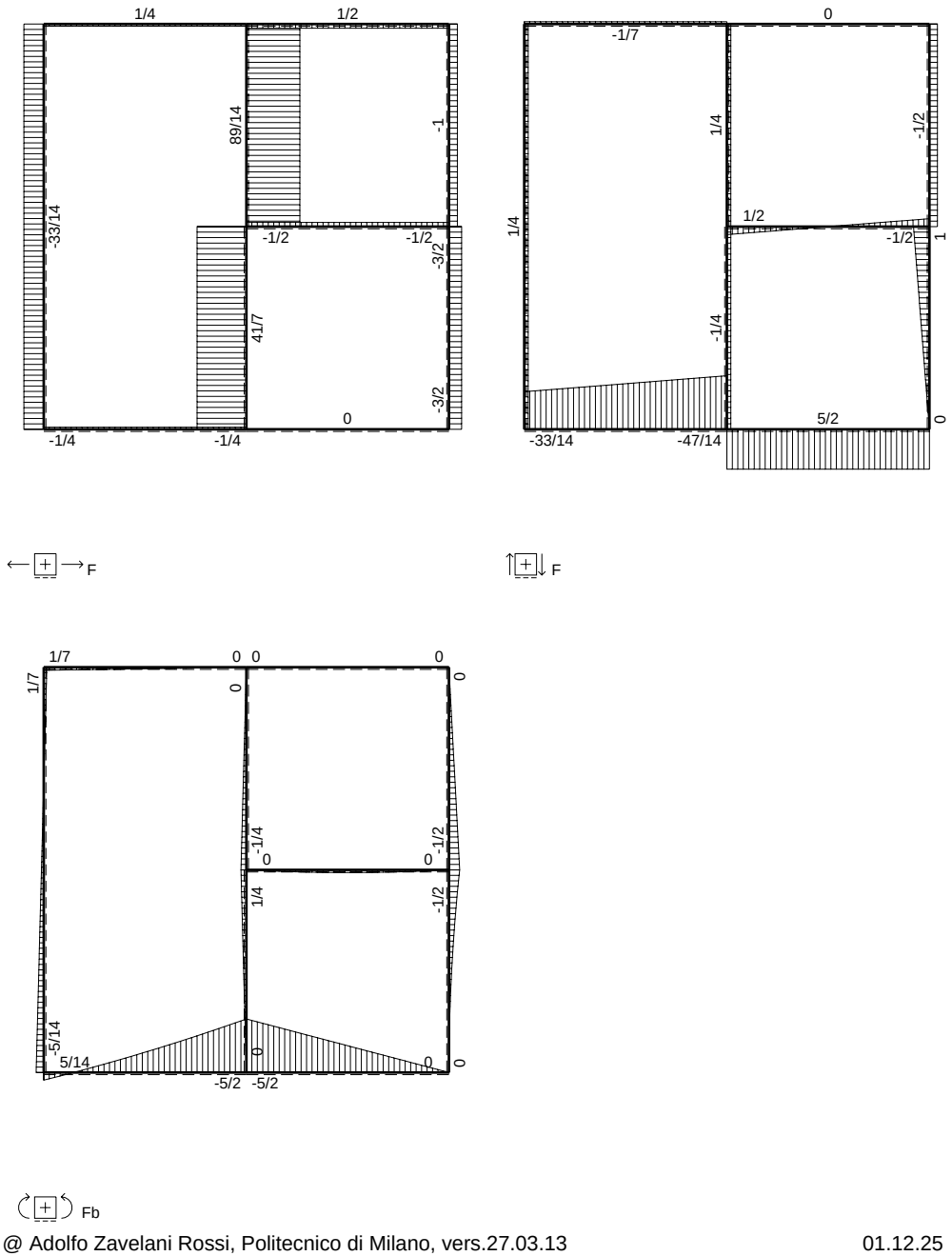
$L_{ED}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

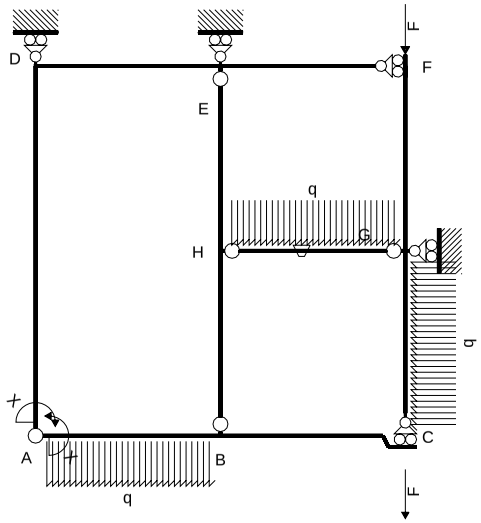
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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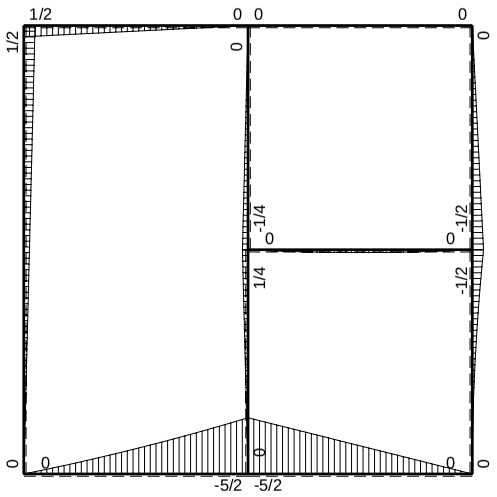
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
E
F
1/2F
E
F
1/4F
89/14F
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F
D
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41/7F
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B
3/2F
C
A
B
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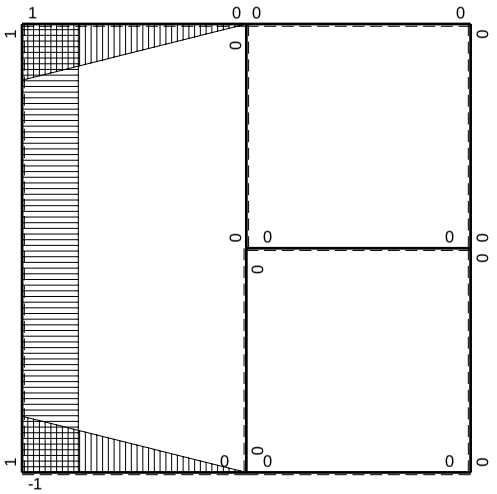

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0
0
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1/4
-1/2
-5/14
5/14
0
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.186

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.186

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(2\ x/b -3/2\ x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -1/2\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -1/2\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(5/2\ x/b -3\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[5/4\ x^2/b - x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(5/4\ b - b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(1/4\ x/b\ \right) Fb\ 1/EJ\ dx = \left[1/8\ x^2/b\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(1/2\ b\ \right) Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(1/2\ -1/4\ x/b\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x\ -1/8\ x^2/b\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(\ b\ -1/2\ b\ \right) Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(1/2\ -\ x/b\ +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x\ -1/2\ x^2/b\ +1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

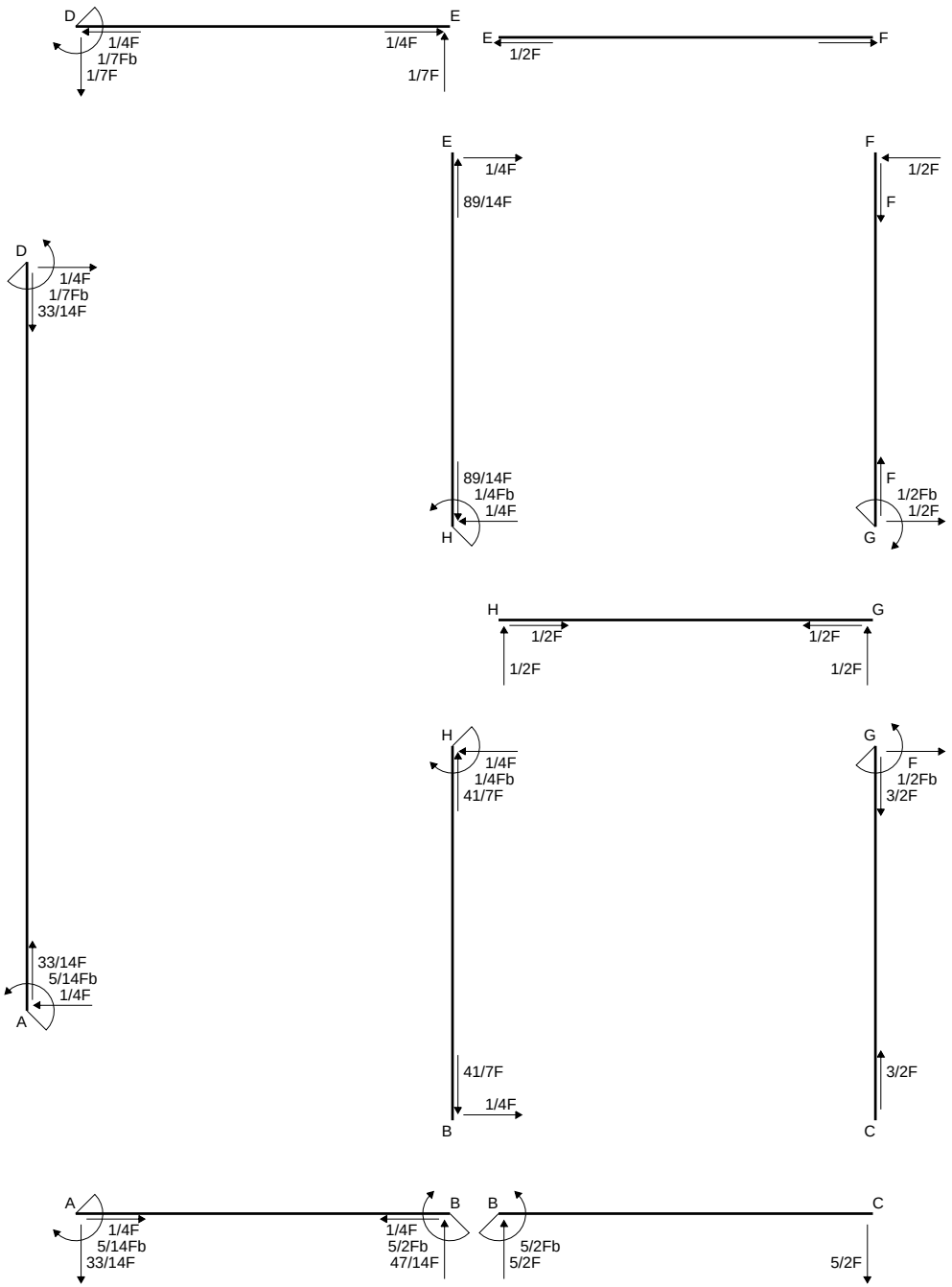
$= \left(1/2\ b\ -1/2\ b\ +1/6\ b\ \right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

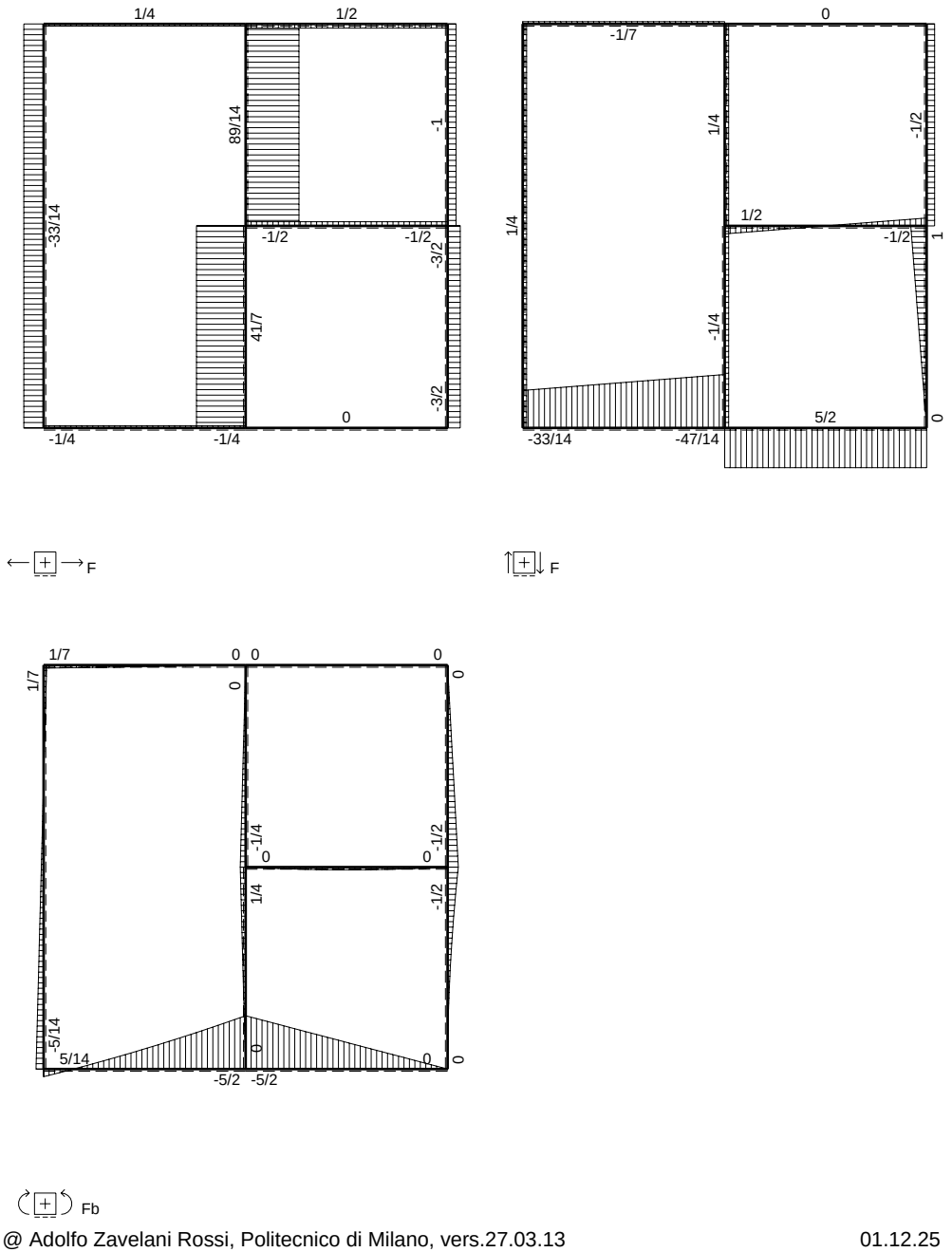
$L^{Xo}_{ED} = \int_o^b \left(1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

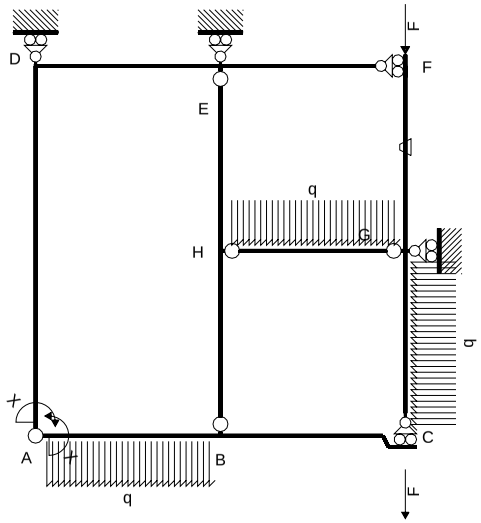
$= \left(1/6\ b\ \right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.186

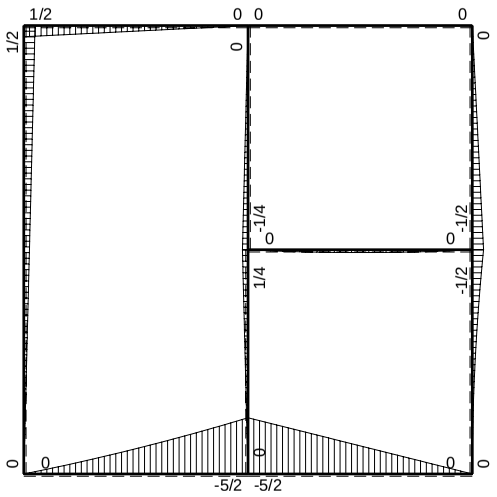
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E
1/4F
1/7Fb
1/7F
1/4F
1/7F
E
F
1/2F
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3/2F
3/2F
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5/14Fb
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A
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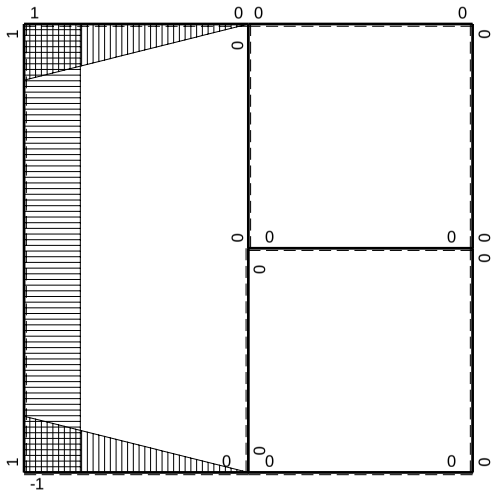

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0
0
-1/2
1/4
-1/2
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.187

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	Fb/EJ	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.187

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

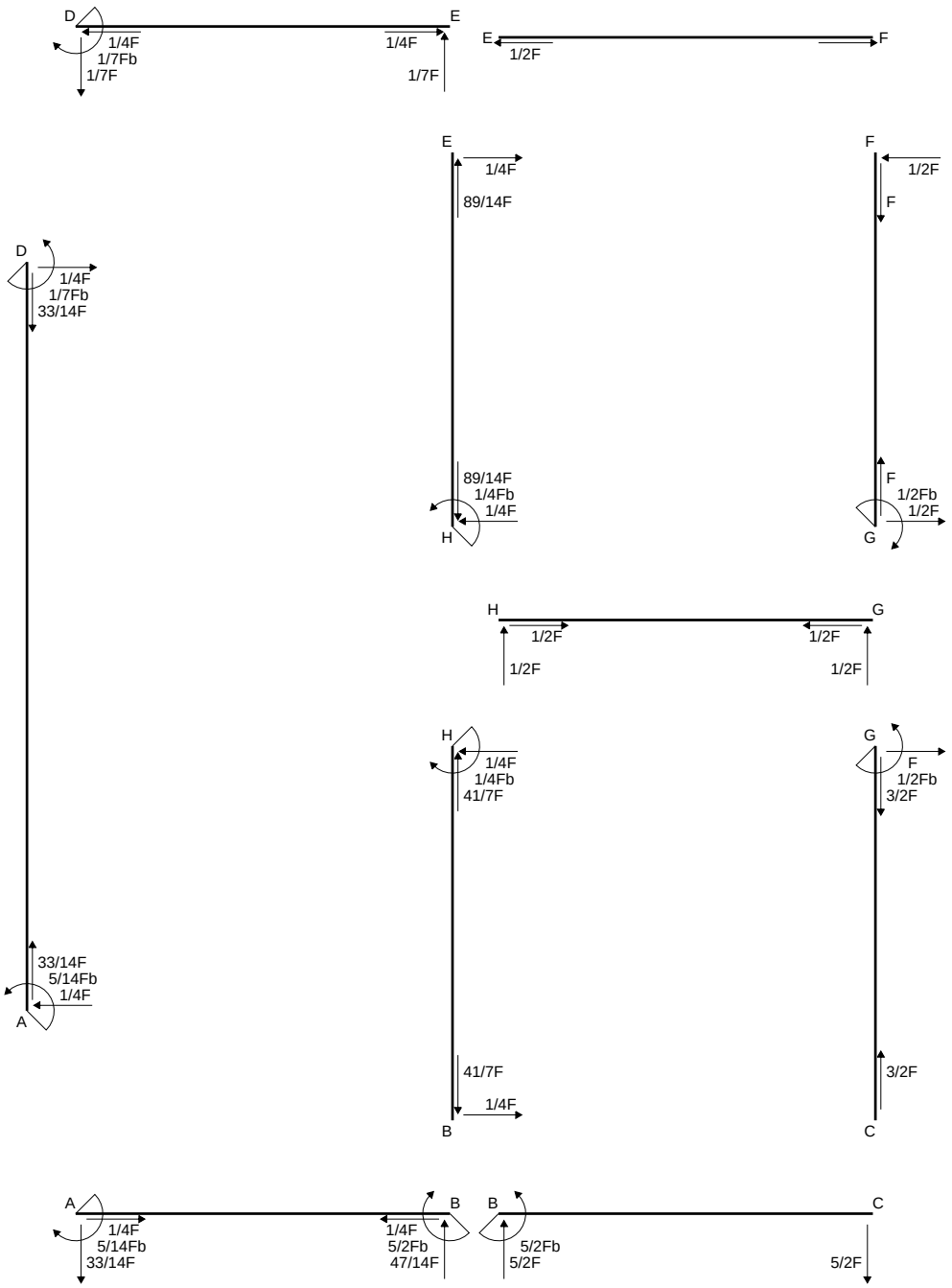
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

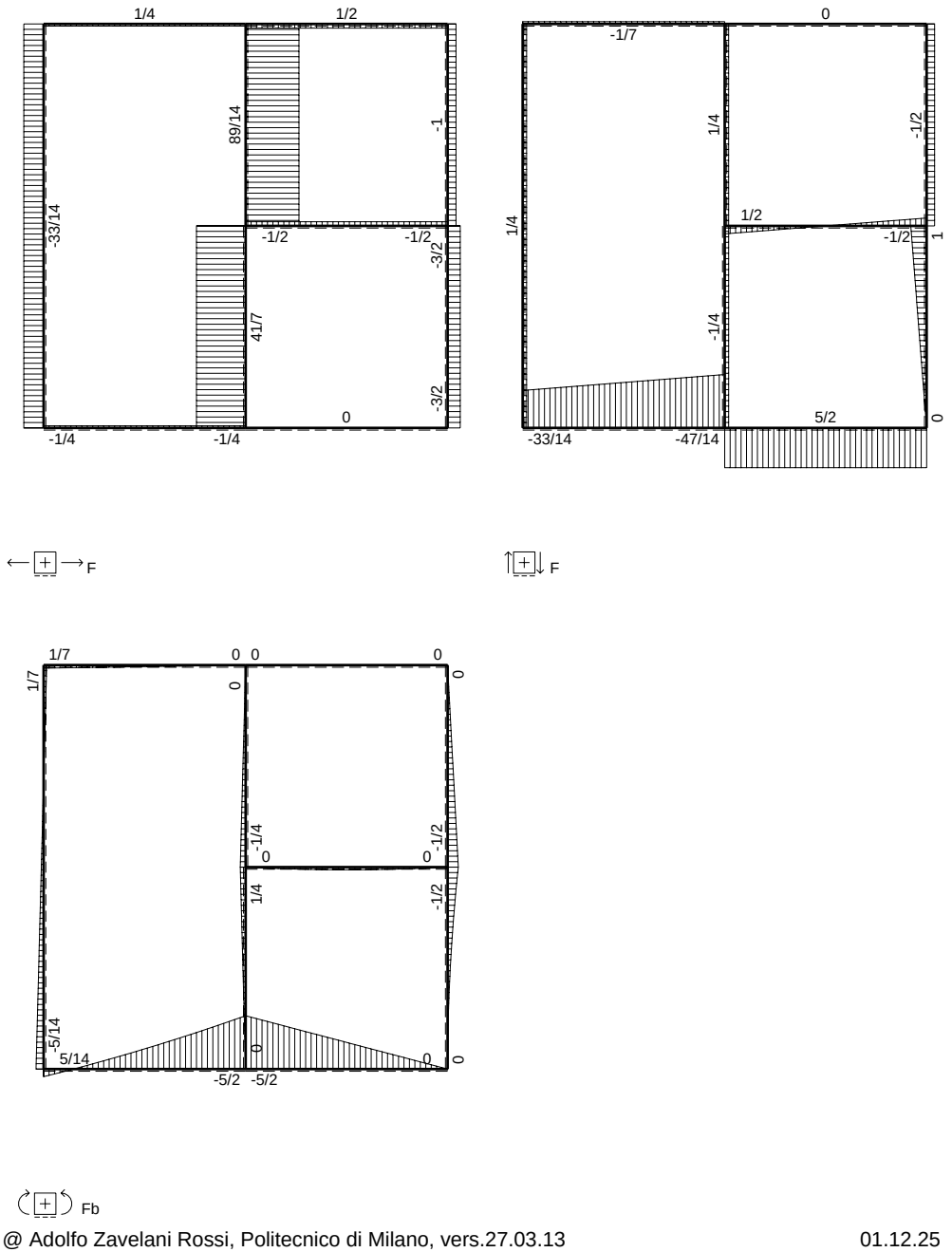
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

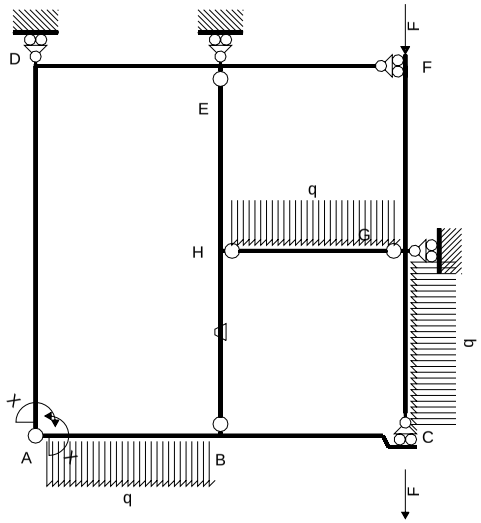
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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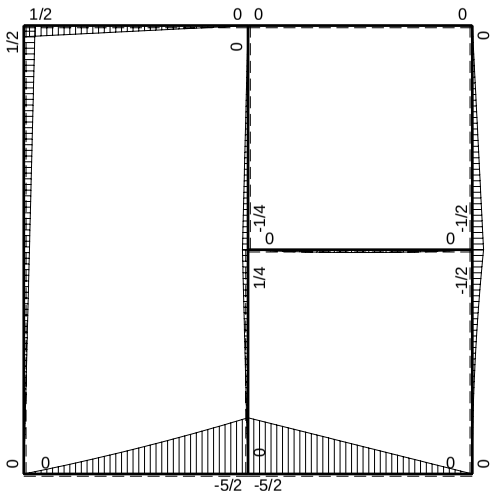
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
E
F
1/2F
E
1/4F
89/14F
89/14F
1/4Fb
1/4F
H
F
1/2F
F
1/2Fb
1/2F
G
D
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B
G
F
1/2Fb
3/2F
3/2F
C
33/14F
5/14Fb
1/4F
A
A
1/4F
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B
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C
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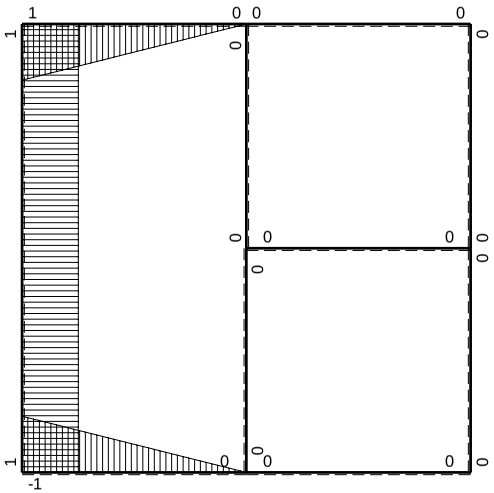

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F
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0
0
-1/2
1/4
-1/2
-5/14
5/14
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.188

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	$-1/4Fx$	Fb/EJ	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.188

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

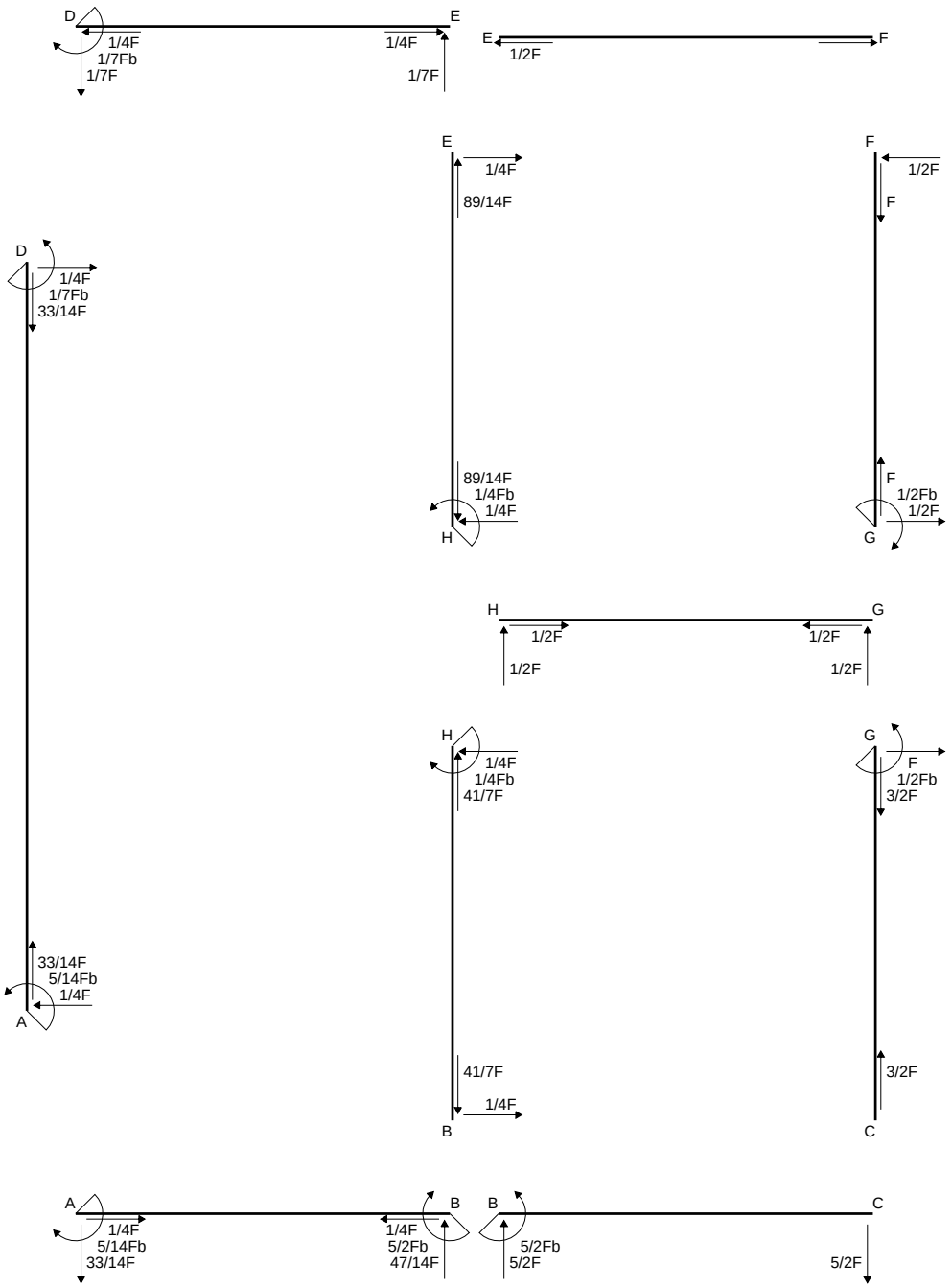
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

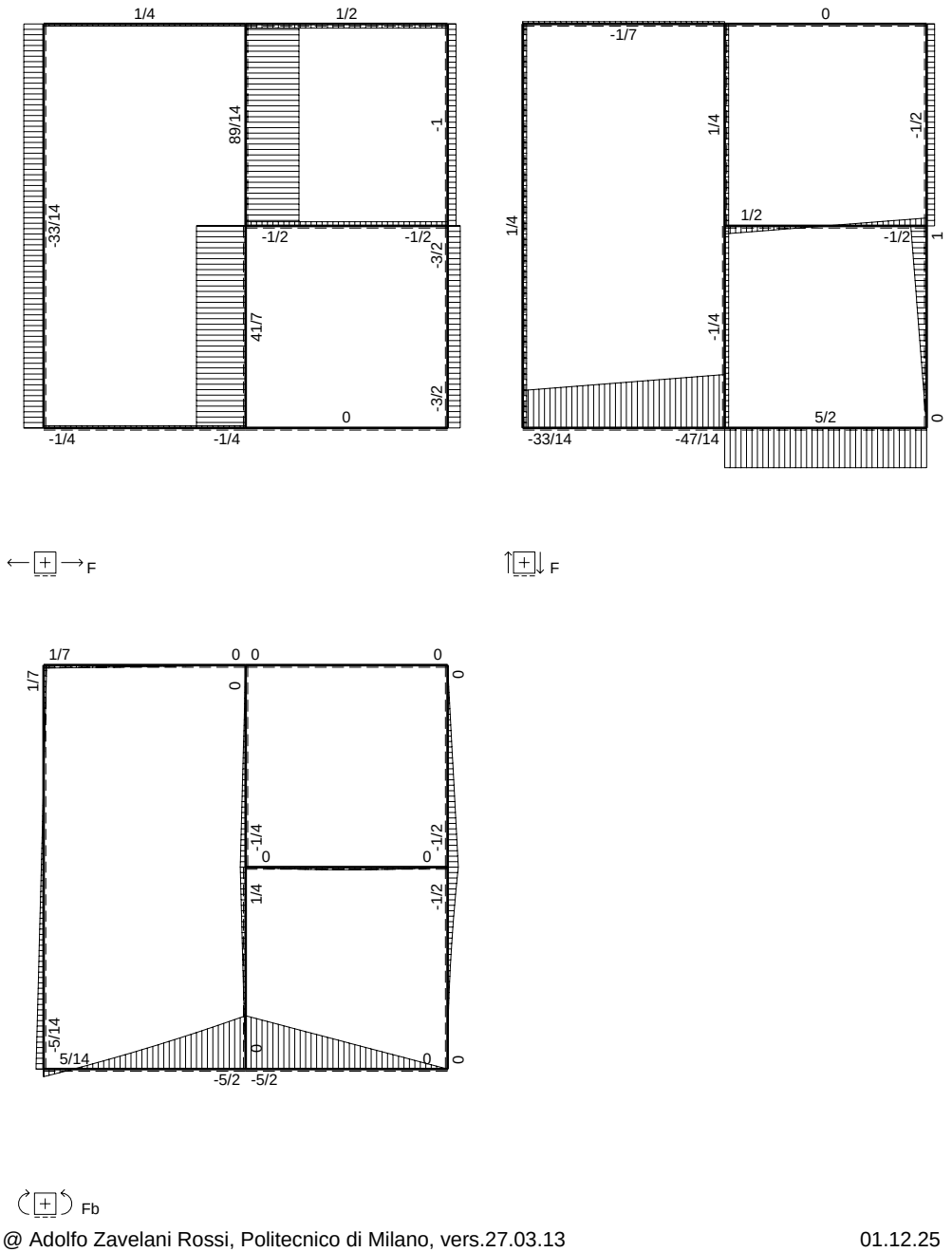
$L_{ED}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

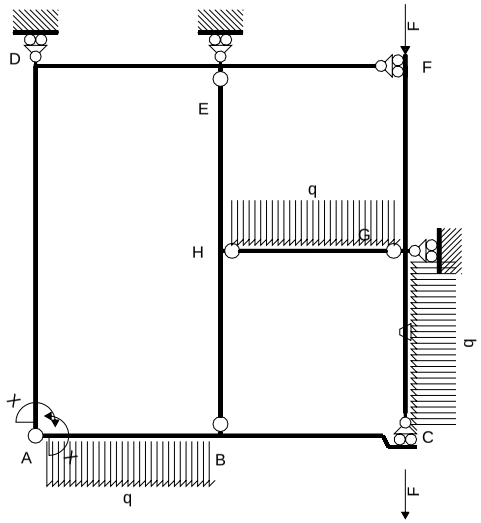
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.188

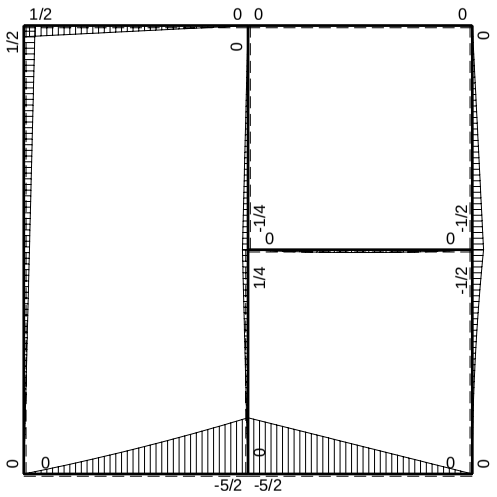
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
E
F
1/2F
E
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89/14F
89/14F
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5/14Fb
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1/4F
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C
5/2F

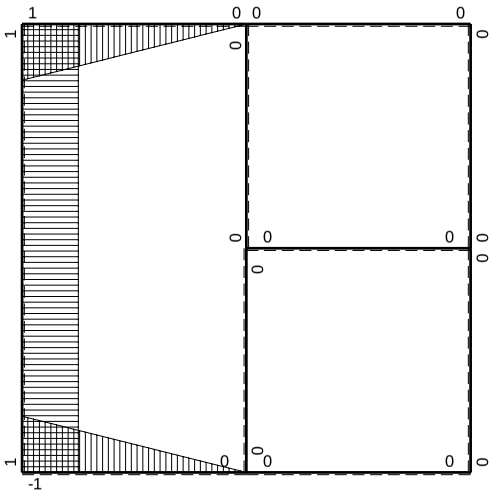

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1
-1/4
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F
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0
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0
0
-1/2
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-1/2
-5/14
5/14
0
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.189

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	1/4Fx	0	1/4Fx	0	1	$(1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	-1/2Fb+1/4Fx	0	1/2Fb-1/4Fx	0	1		
DE b	$1-x/b$	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	1/2Fb-1/2Fx	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0
BH b	0	-1/4Fx	0	0	0	0		
HE b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
EH b	0	1/4Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$25/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-5/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.189

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

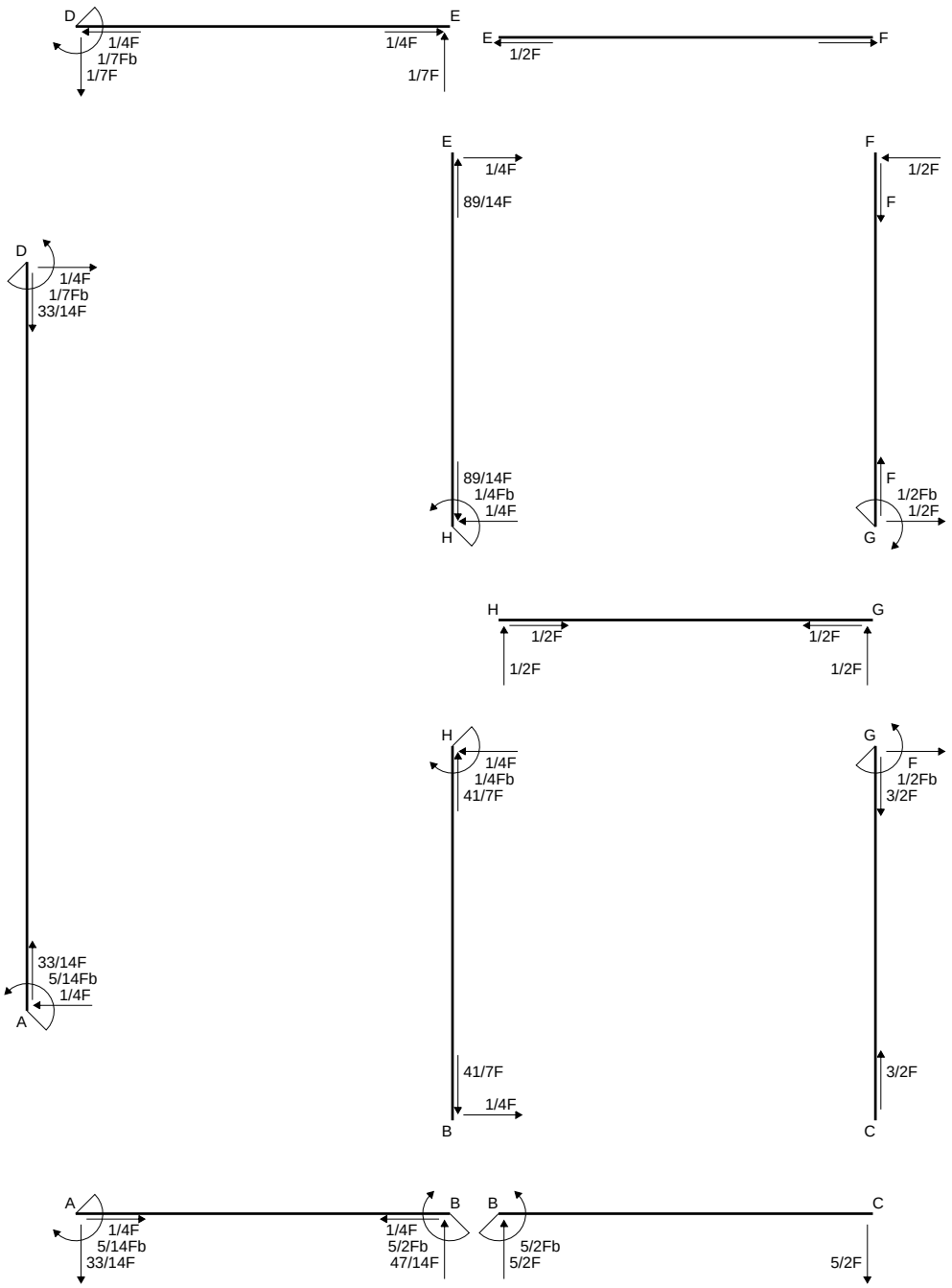
$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

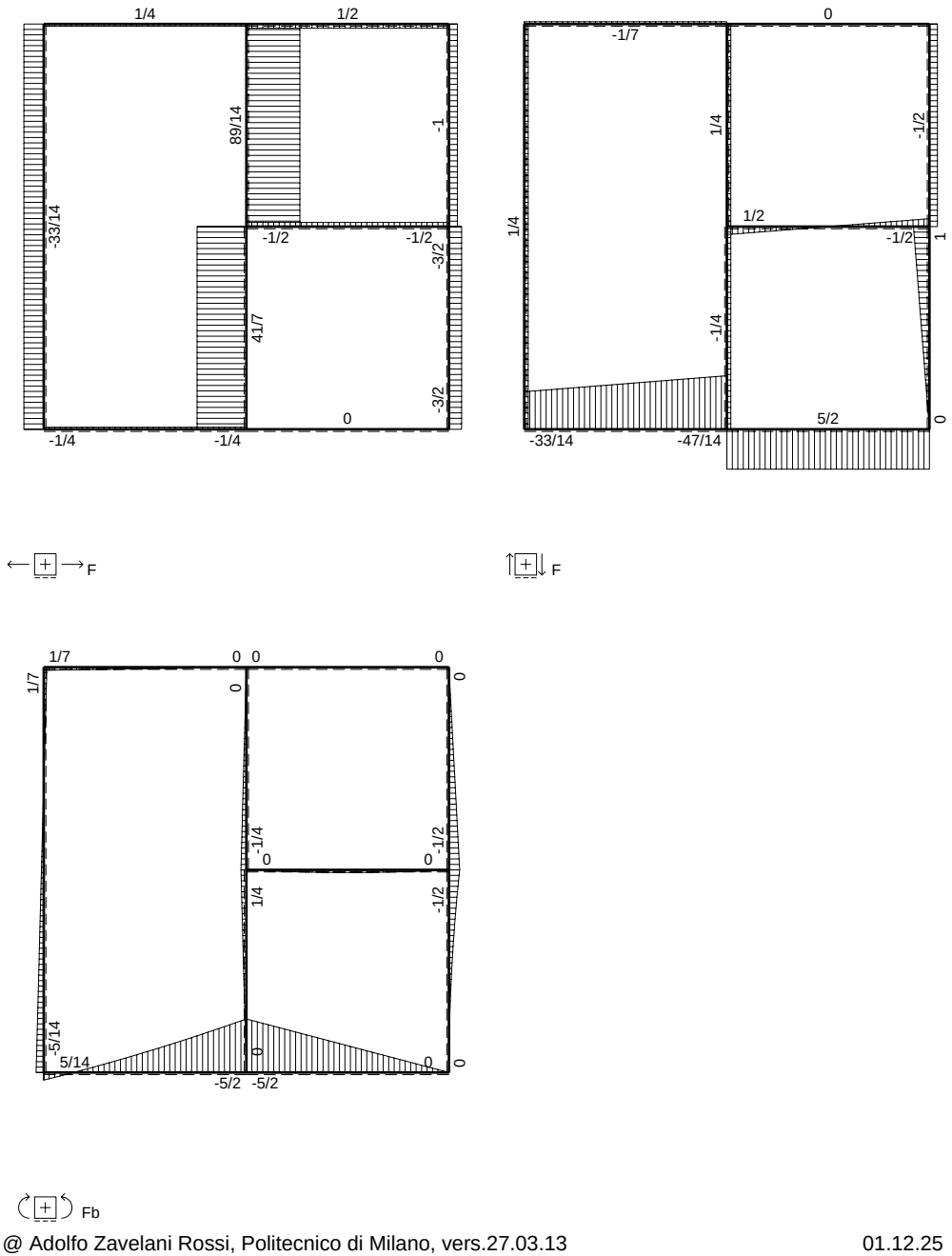
$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

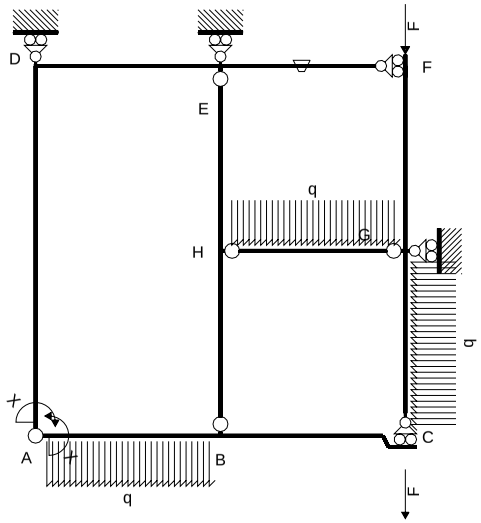
$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.189

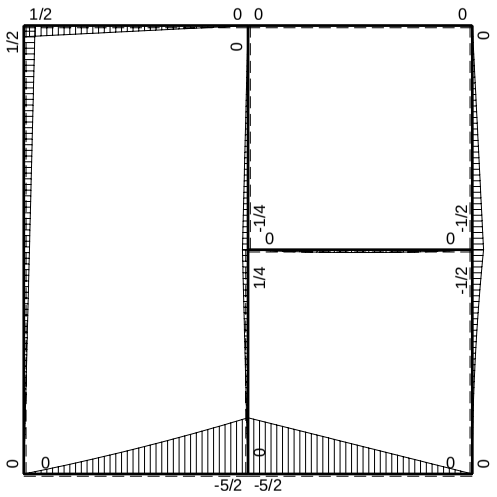
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D
E
1/4F
1/7Fb
1/7F
1/4F
1/7F
E
F
1/2F
E
1/4F
89/14F
89/14F
1/4Fb
1/4F
H
F
1/2F
F
1/2Fb
1/2F
G
D
1/4F
1/7Fb
33/14F
H
G
1/2F
1/2F
1/2F
1/2F
H
1/4F
1/4Fb
41/7F
41/7F
1/4F
B
G
F
1/2Fb
3/2F
3/2F
C
33/14F
5/14Fb
1/4F
A
A
1/4F
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B
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5/2Fb
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5/2Fb
5/2F
C
5/2F

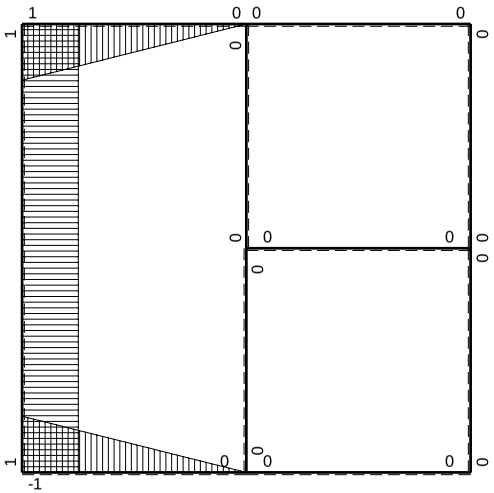

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F
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F
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1/7
0
0
-1/4
0
0
-1/2
1/4
-1/2
-5/14
5/14
0
0
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.190

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.190

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(2\ x/b -3/2\ x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -1/2\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -1/2\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(5/2\ x/b -3\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[5/4\ x^2/b - x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(5/4\ b - b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(1/4\ x/b\ \right) Fb\ 1/EJ\ dx = \left[1/8\ x^2/b\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(1/2\ b\ \right) Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(1/2\ -1/4\ x/b\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x\ -1/8\ x^2/b\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(\ b\ -1/2\ b\ \right) Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(1/2\ -\ x/b\ +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/2\ x\ -1/2\ x^2/b\ +1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

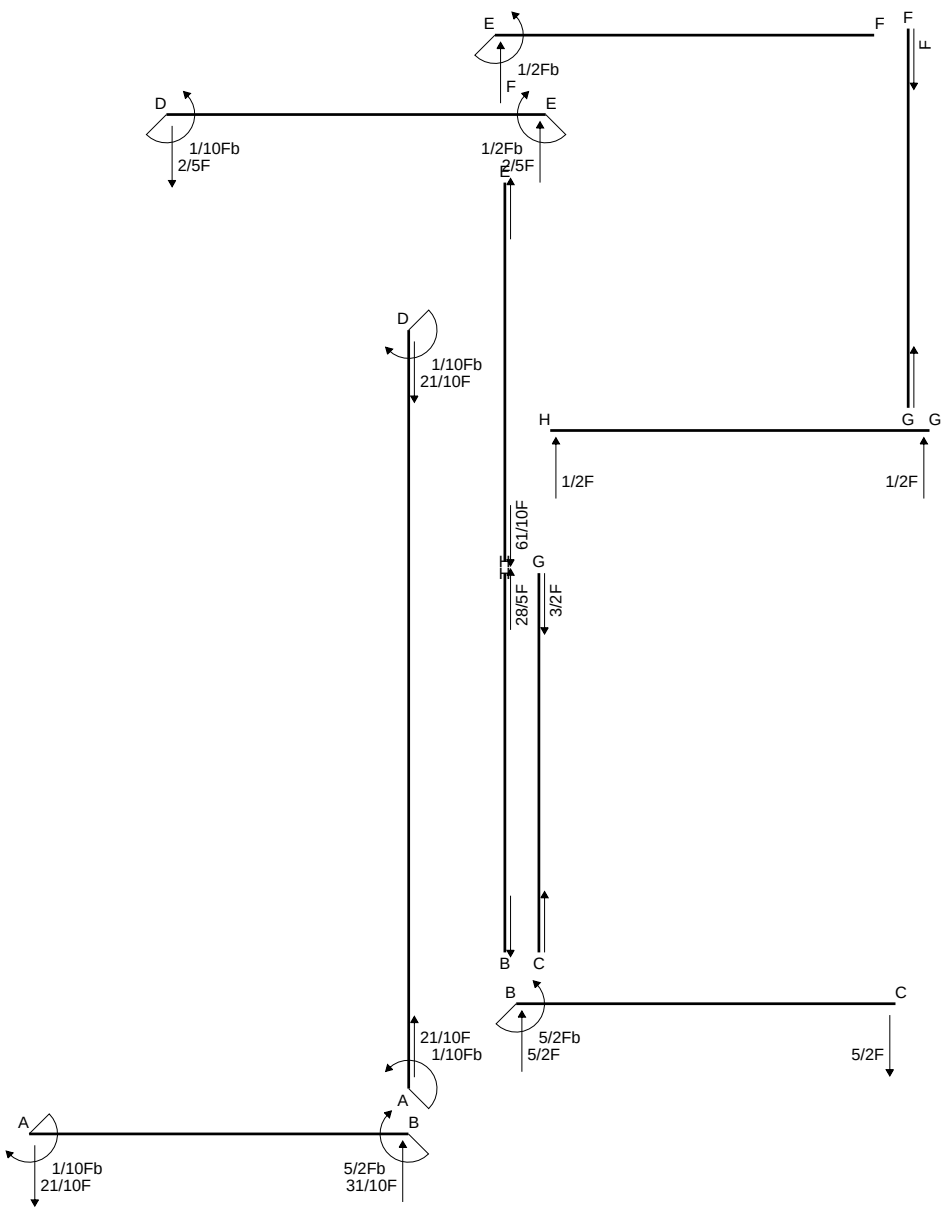
$= \left(1/2\ b\ -1/2\ b\ +1/6\ b\ \right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

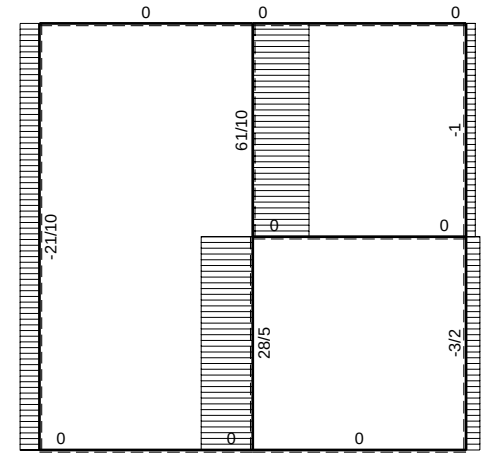
$L^{Xo}_{ED} = \int_o^b \left(1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

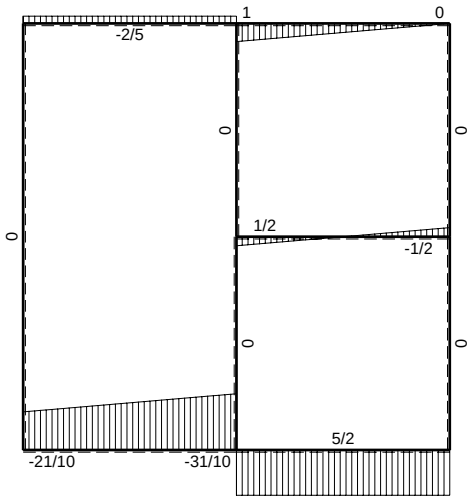
$= \left(1/6\ b\ \right) Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.190

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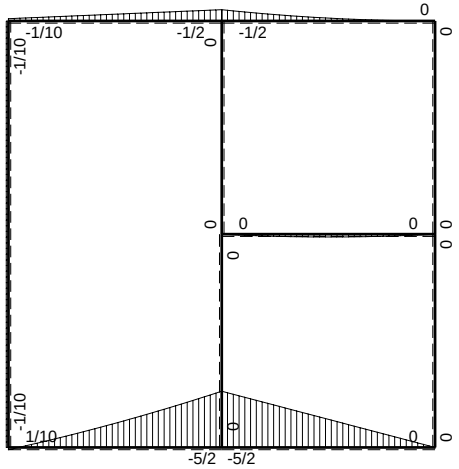

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F
E
D
1/10Fb
2/5F
1/2Fb
2/5F
E
D
1/10Fb
21/10F
H
G
G
1/2F
1/2F
61/10F
F
G
28/5F
3/2F
B
C
B
C
21/10F
1/10Fb
5/2Fb
5/2F
5/2F
A
B
A
1/10Fb
21/10F
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61/10
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28/5
-3/2
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0
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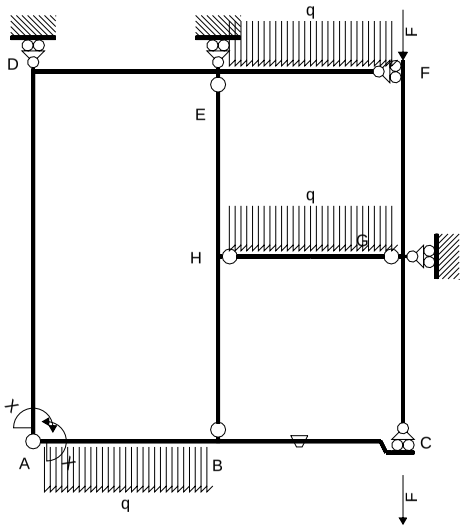

1
0
-2/5
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0
1/2
-1/2
0
0
0
5/2
-21/10
-31/10

←□→ F

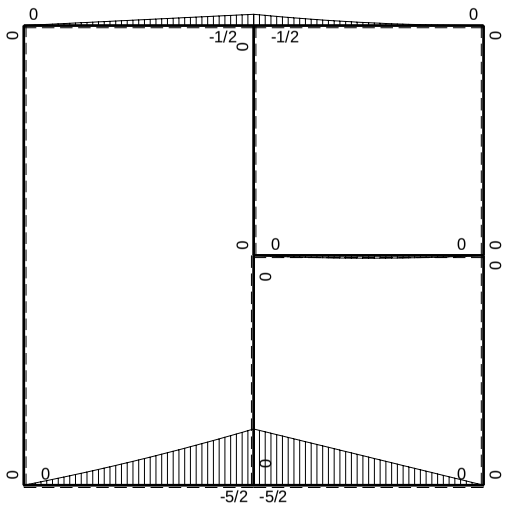
↑□↓ F


0
-1/10
-1/2
-1/2
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0
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1/10
0
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-5/2
-5/2

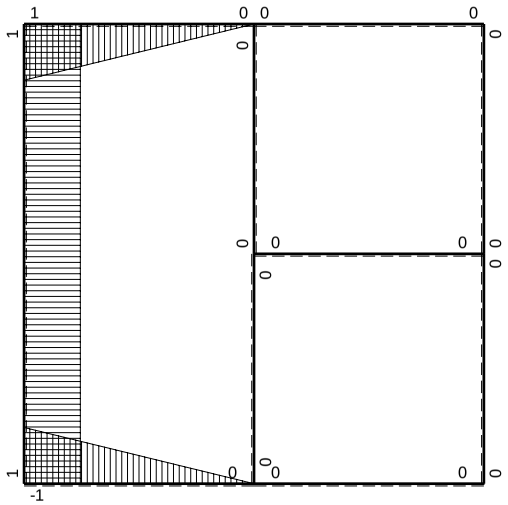
↻□↺ Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.191

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$5/2Fx$	Fb/EJ	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.191

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

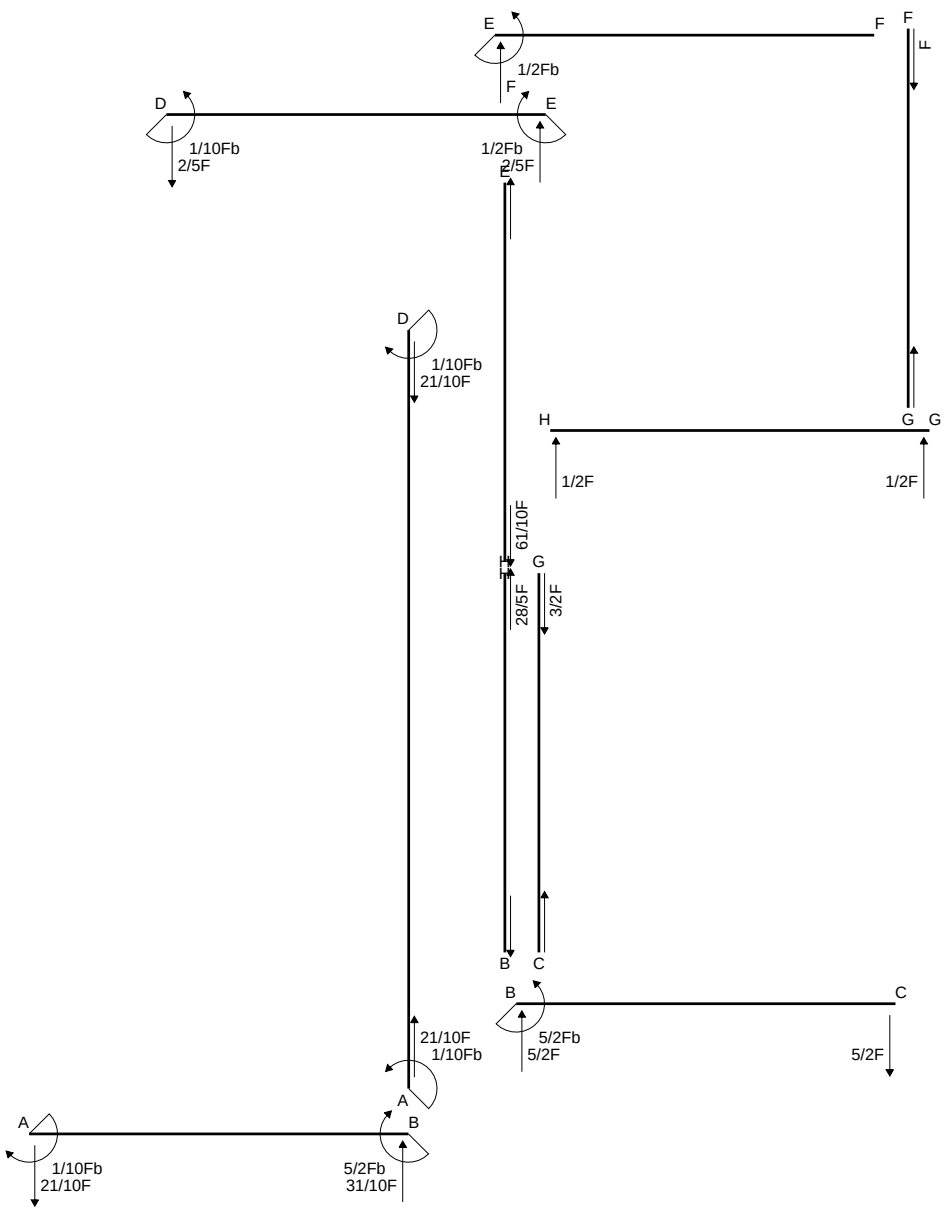
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

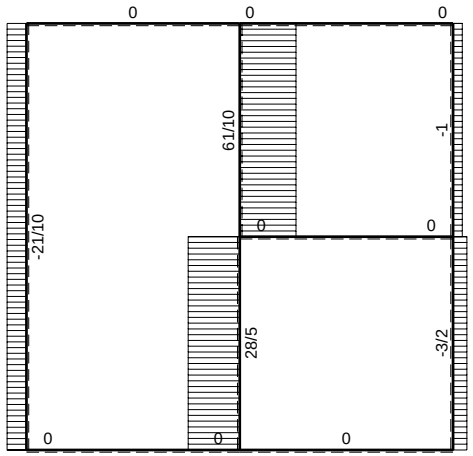
$L_{ED}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

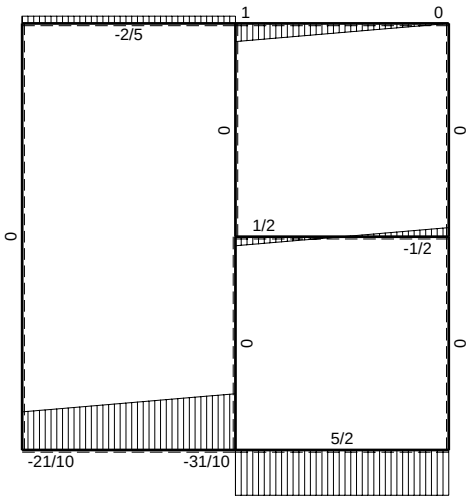
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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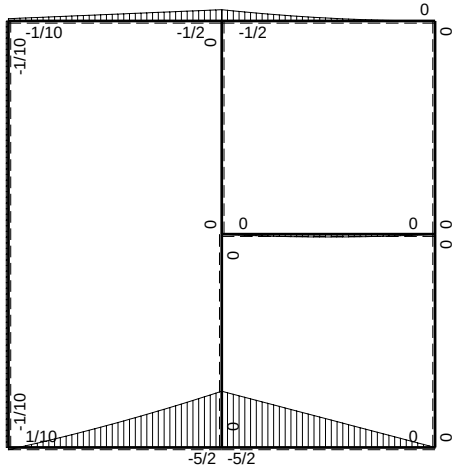

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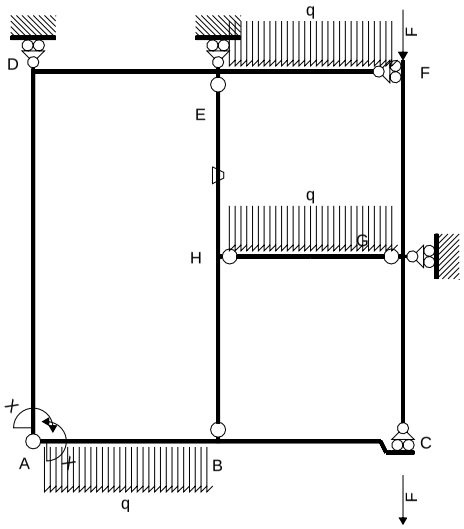

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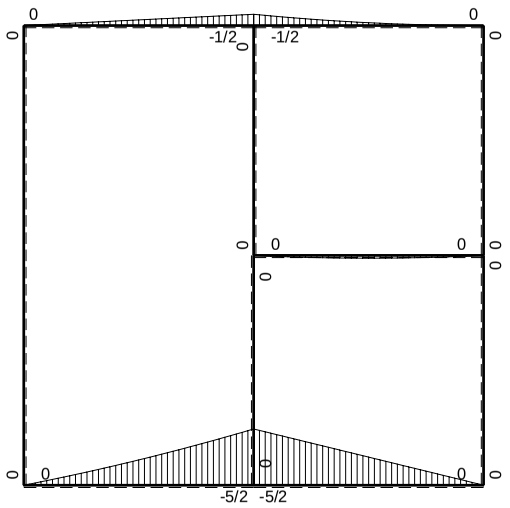
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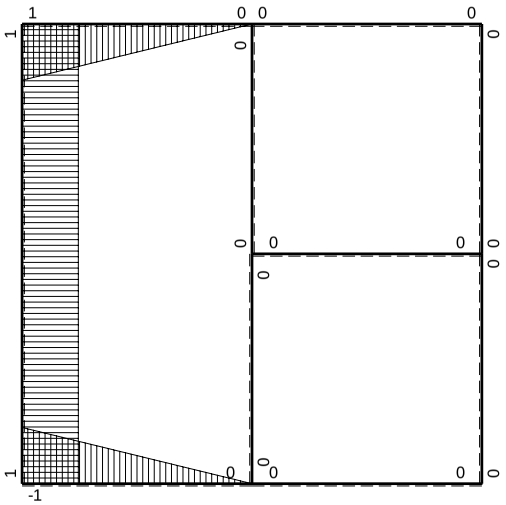
↻+↺ Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	-1/2Fx	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-1/2Fx	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	-Fb/EJ	0	0	0	0+0	0
EH b	0	0	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.192

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

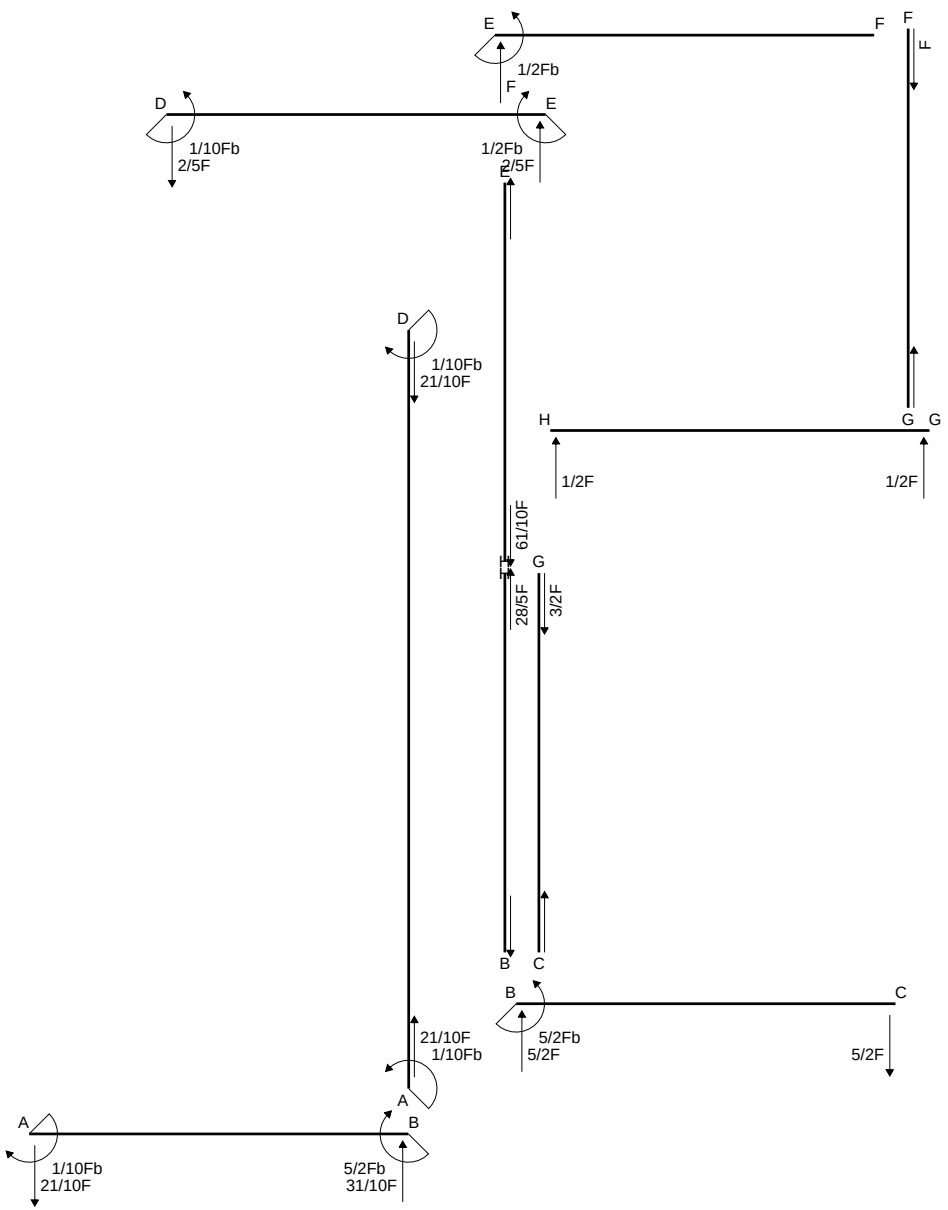
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

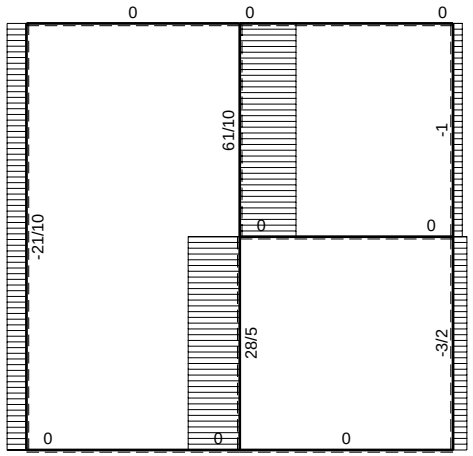
$L_{ED}^{Xo} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

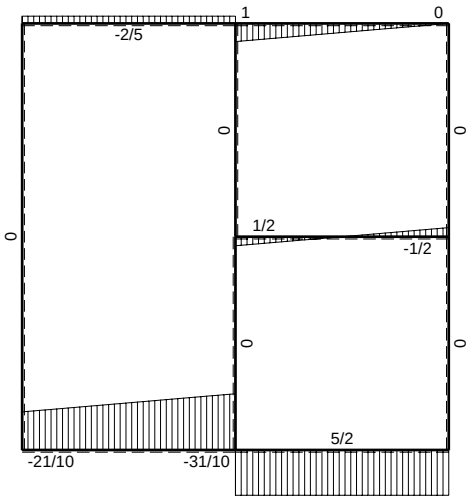
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

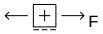
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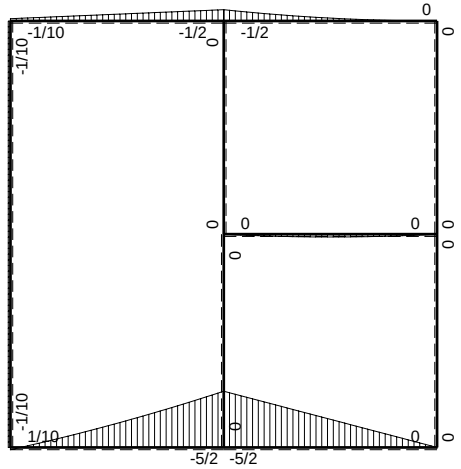

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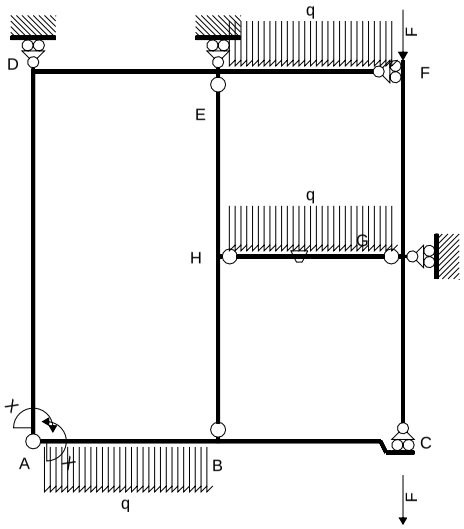

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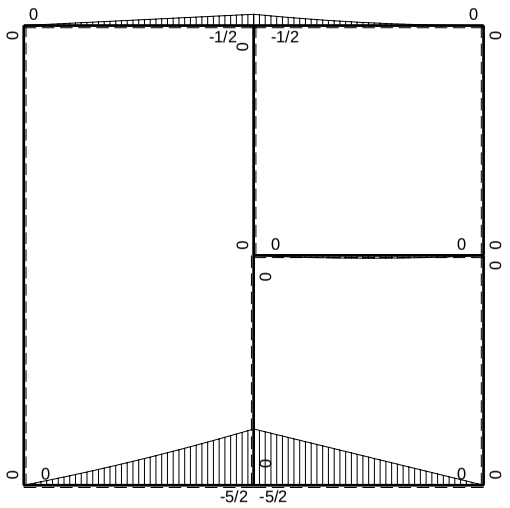

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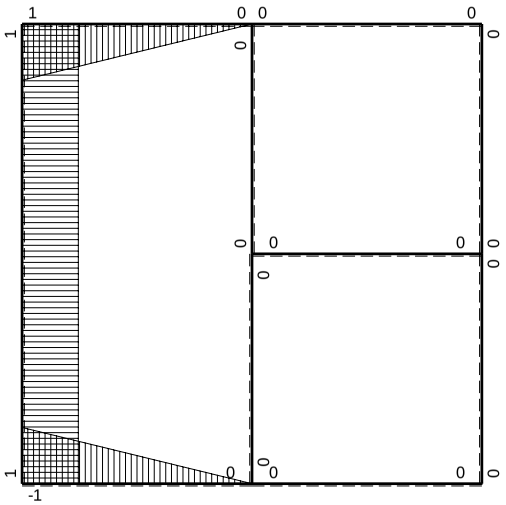

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.193

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PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.193

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

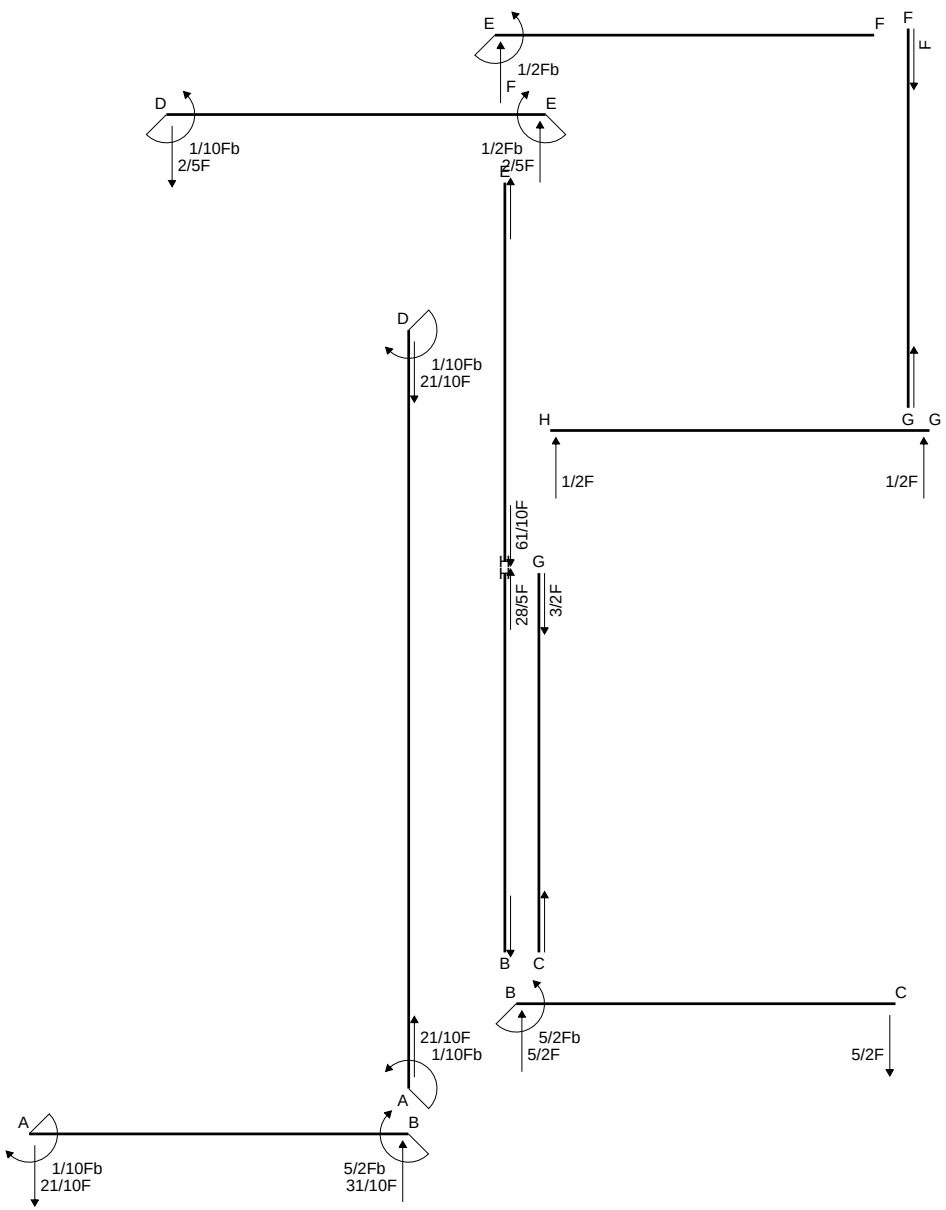
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

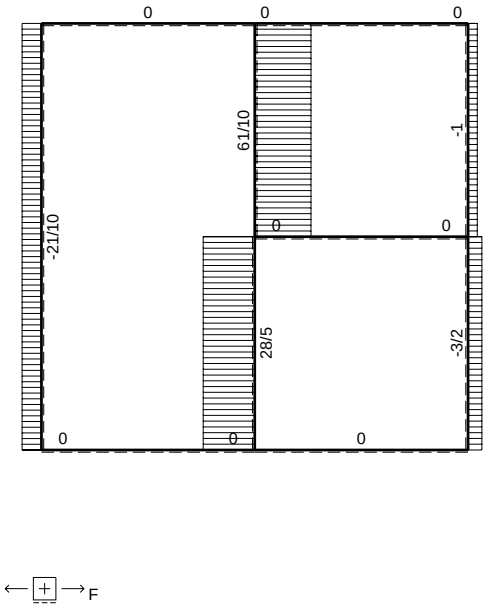
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

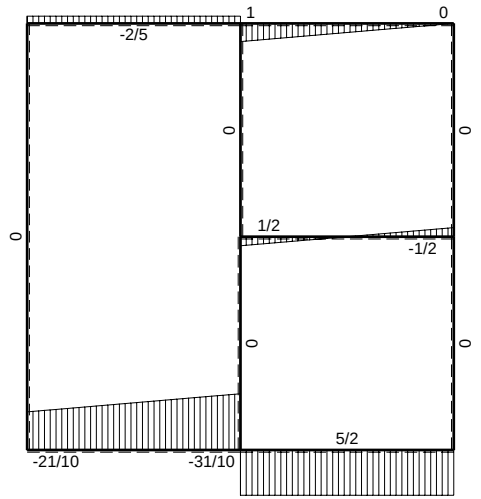
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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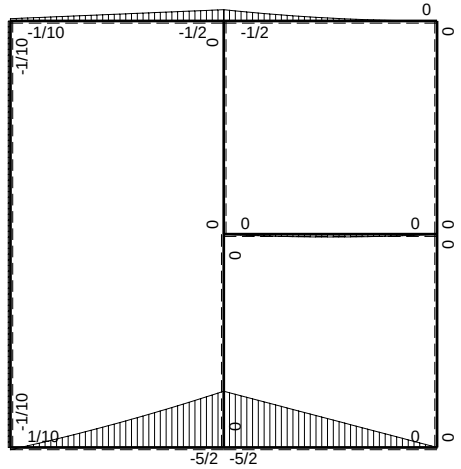

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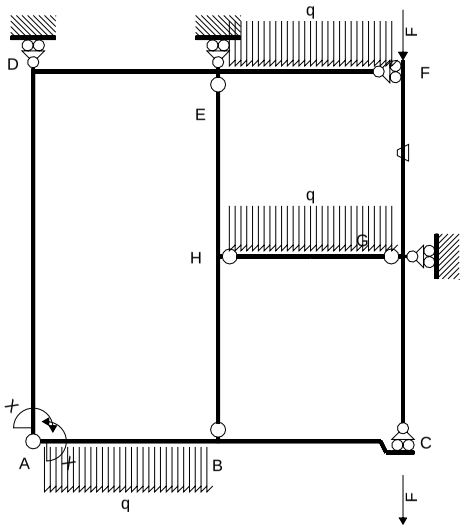

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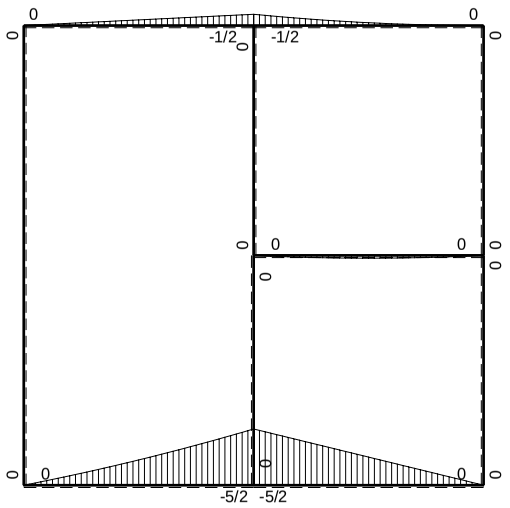

F


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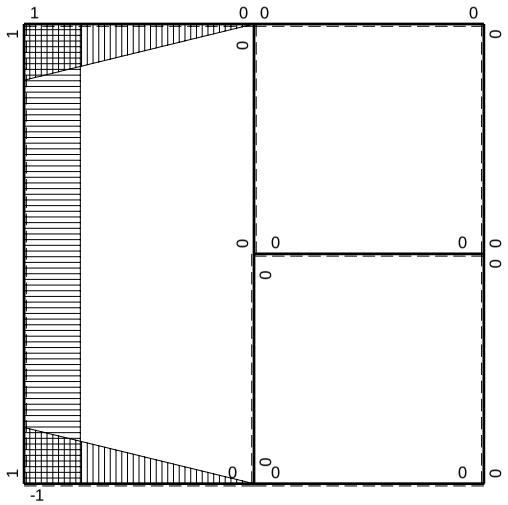

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.194

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.194

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

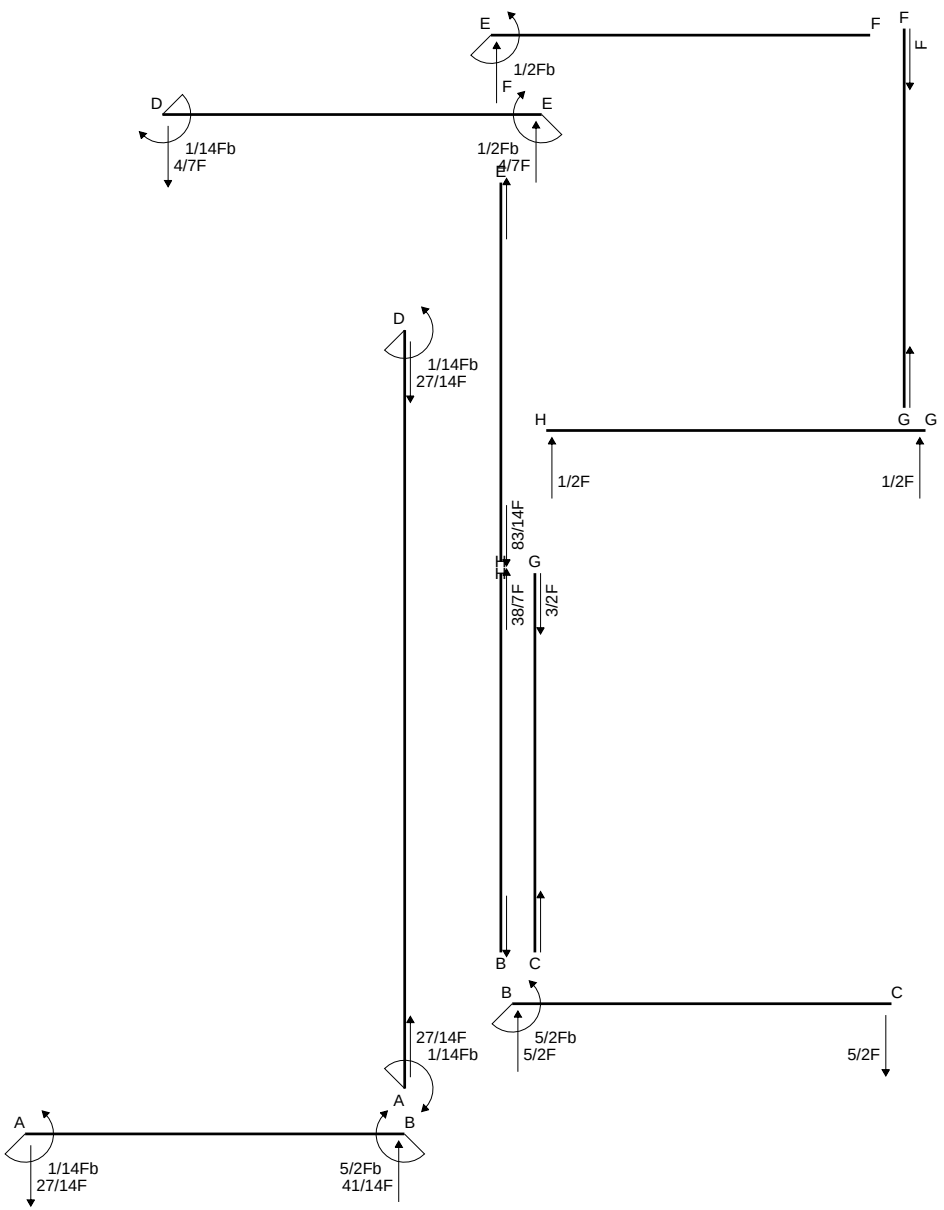
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

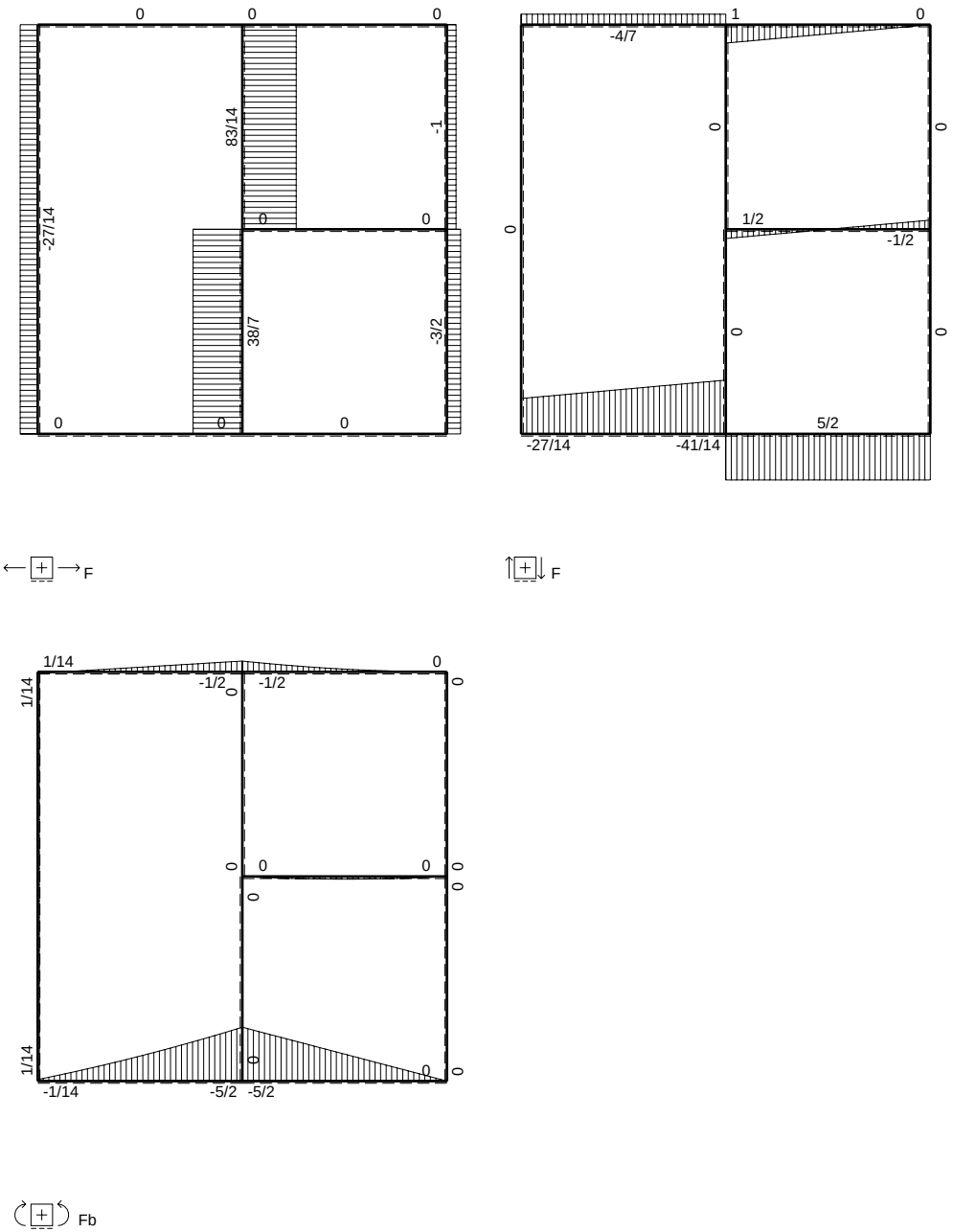
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

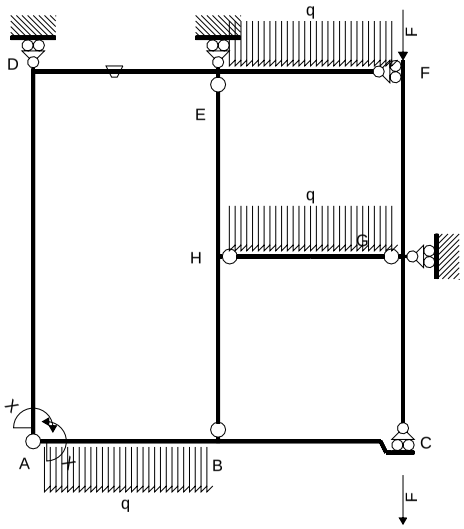
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

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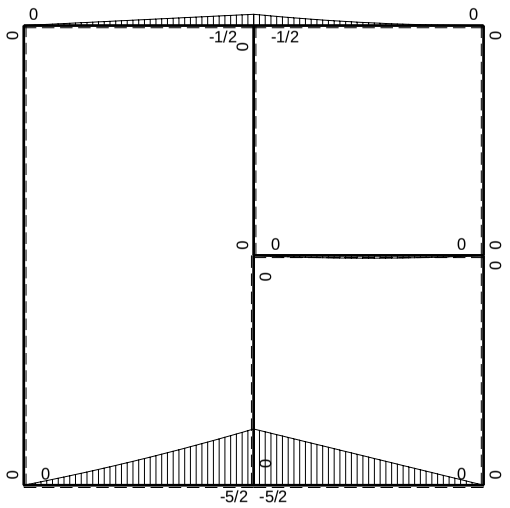
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4/7F
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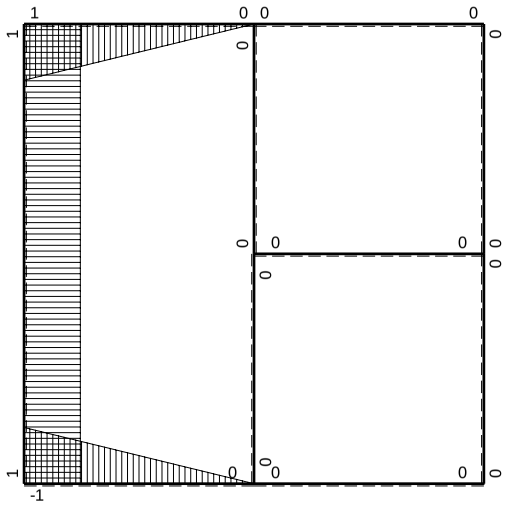

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0
0
1/14
-1/14
-5/2
-5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.195

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	$-Fb/EJ$	$-1/2Fx+1/2Fx^2/b$	$-Fb/EJ+Fx/EJ$	$1-2x/b+x^2/b^2$	$(-1/12-1/2)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	Fb/EJ	$-1/2Fx+1/2Fx^2/b$	$-Fx/EJ$	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-5/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.195

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ = -7/12\ \ Fb^2/EJ$

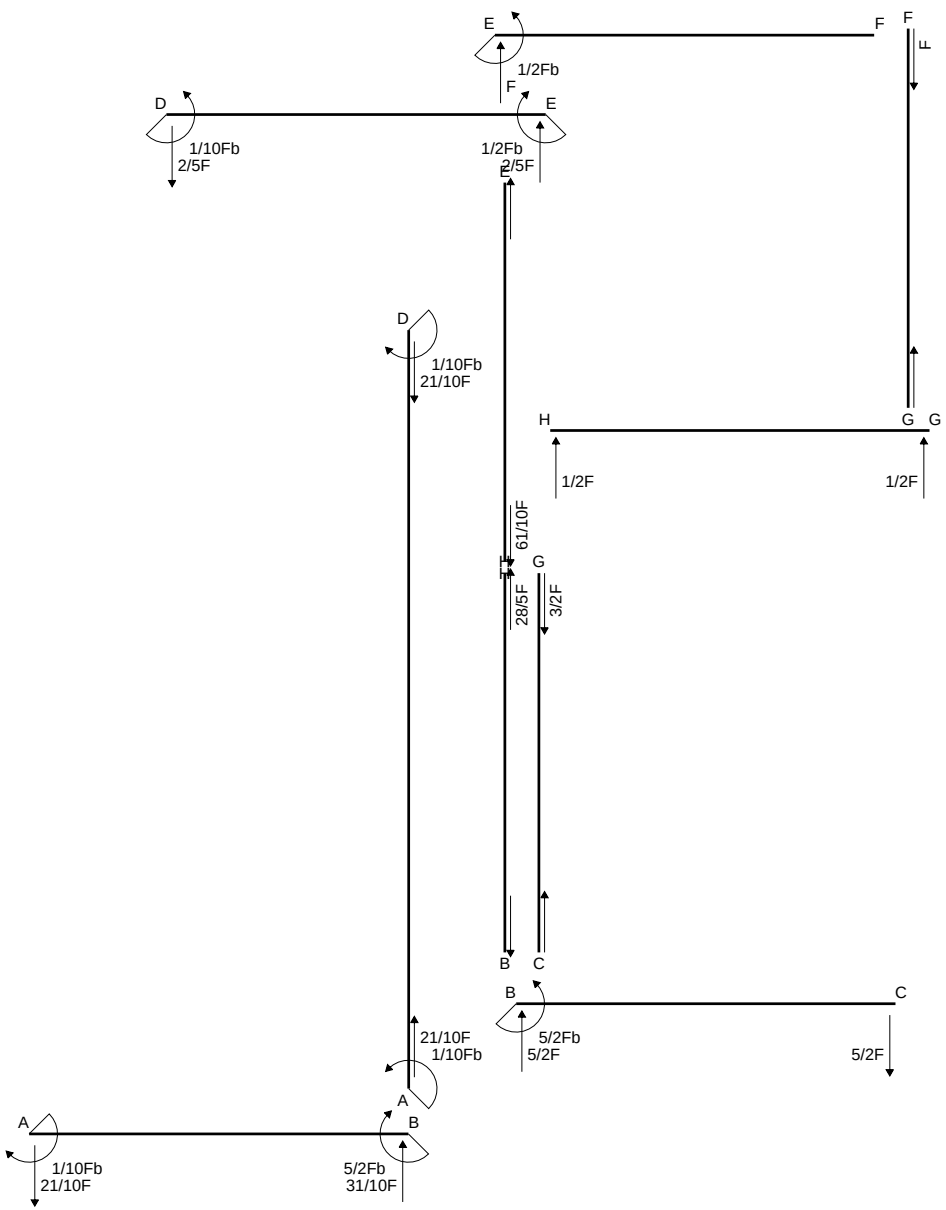
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

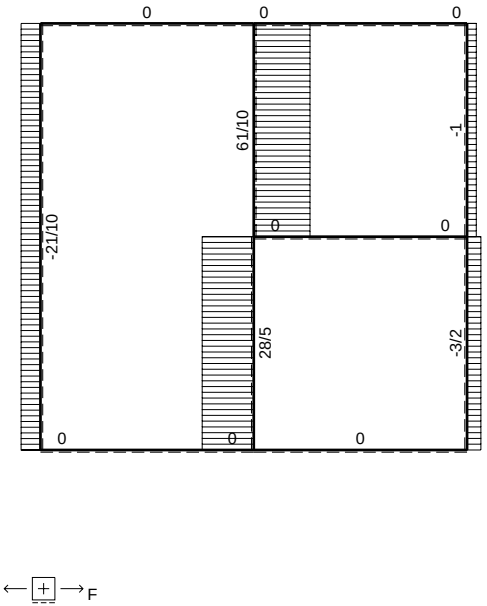
$= [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

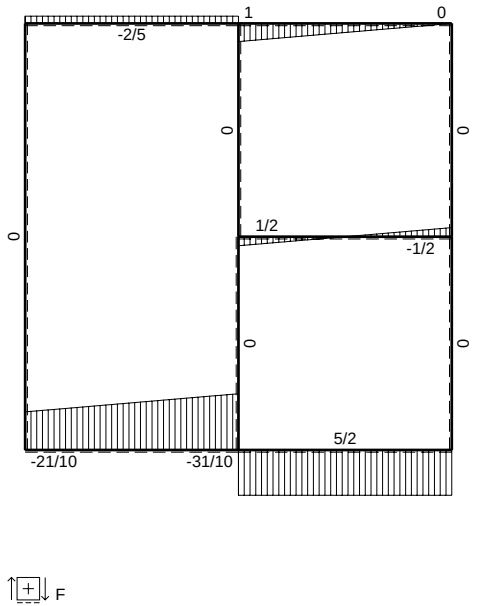
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = -7/12\ \ Fb^2/EJ$

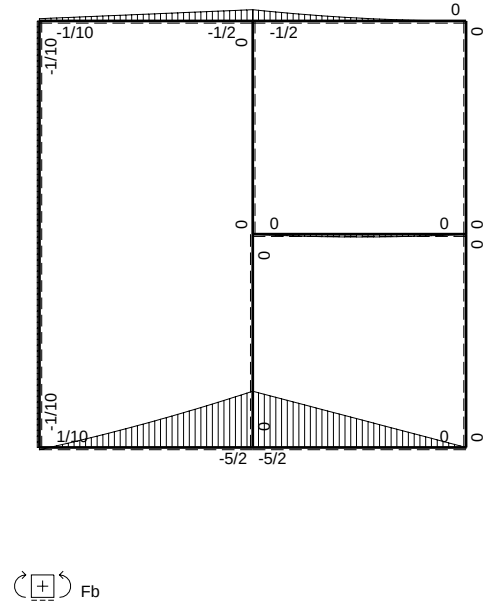
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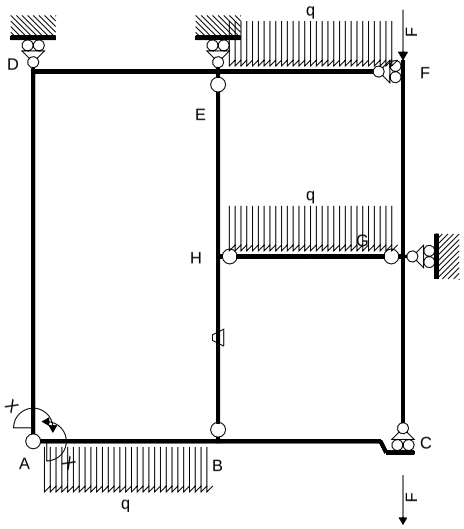
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D
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2/5F
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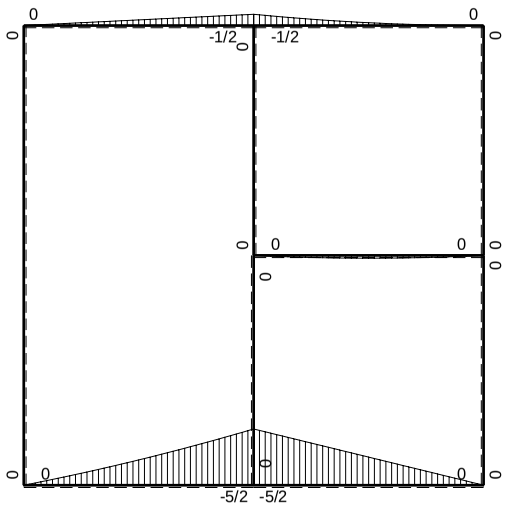

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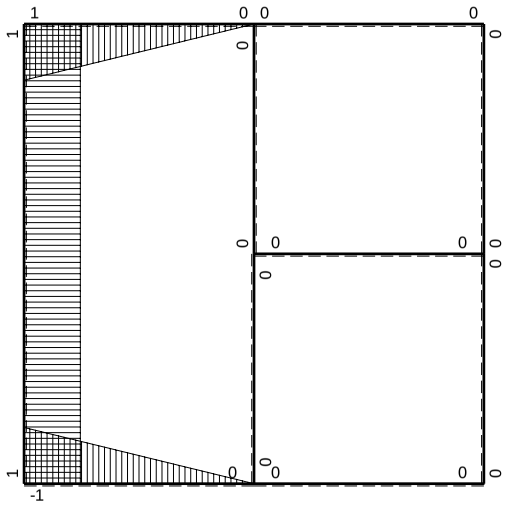

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1/10
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0
-5/2
-5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.196

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	2Xb/EJ
DA 2b	-1	0	0	0	0	1		
DE b	1-x/b	-1/2Fx	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	1/2Fb-1/2Fx	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	-Fb/EJ	0	0	0	0+0	0
BH b	0	0	Fb/EJ	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.196

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

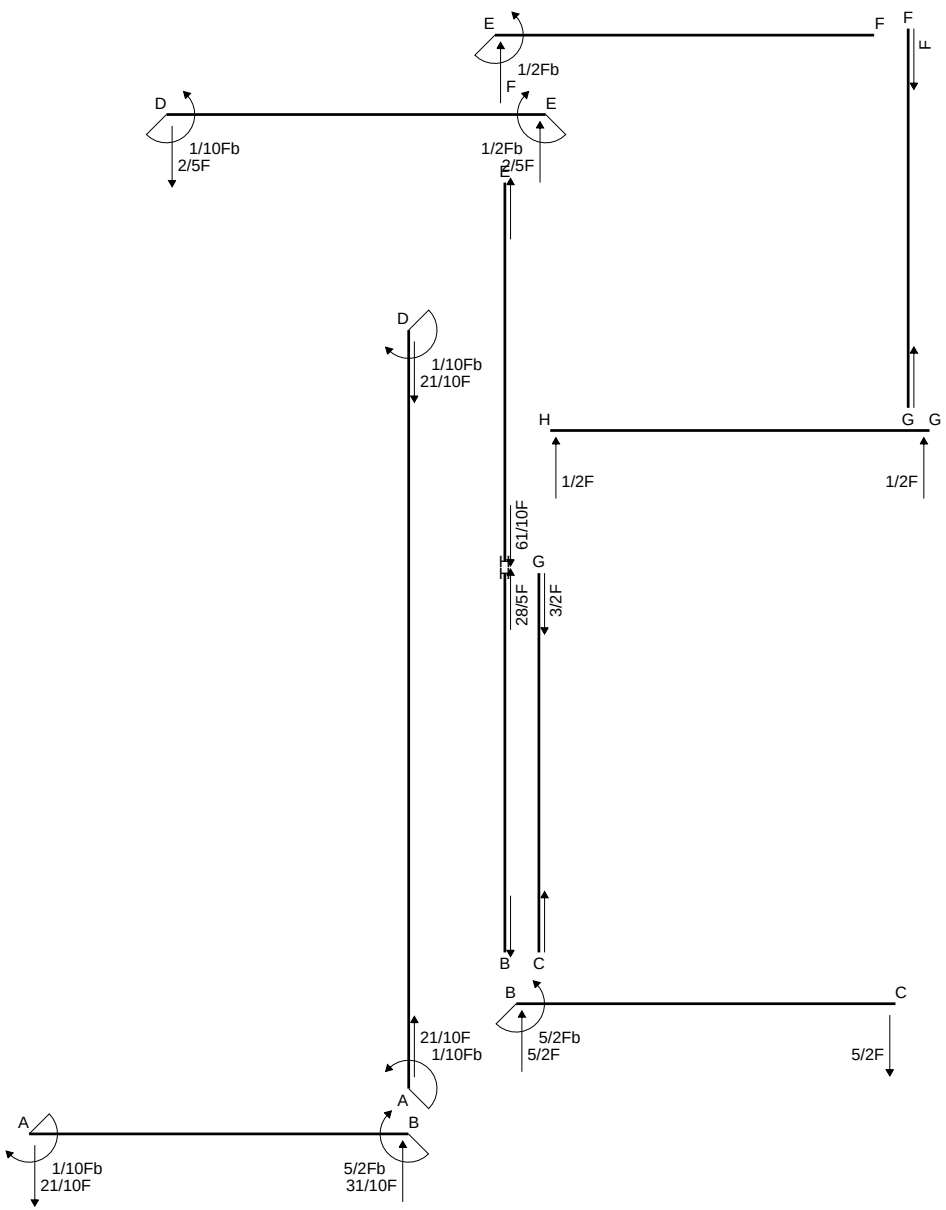
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

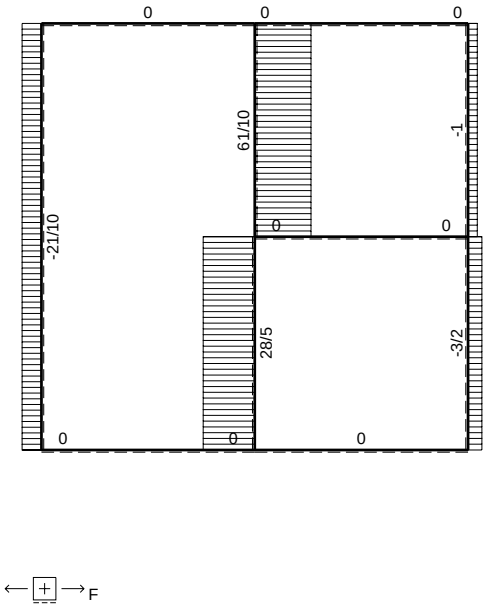
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

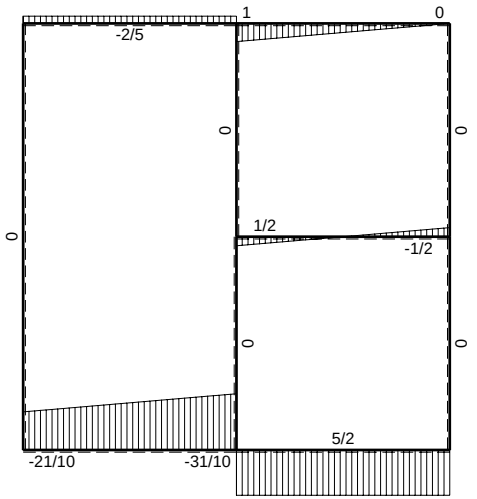
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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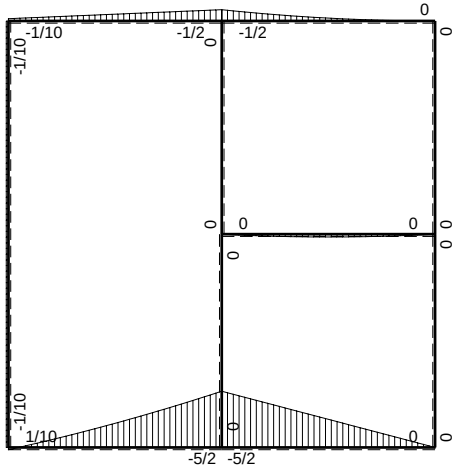

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A
B
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21/10F
5/2Fb
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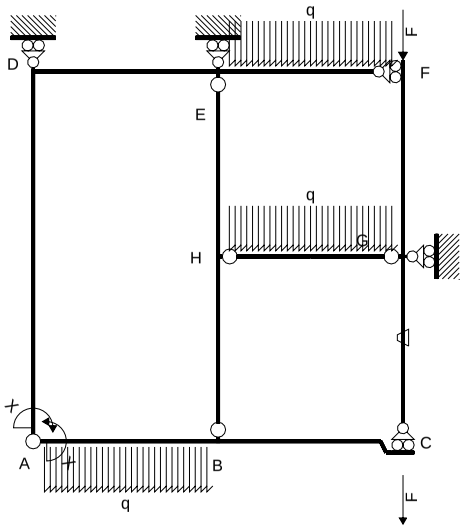

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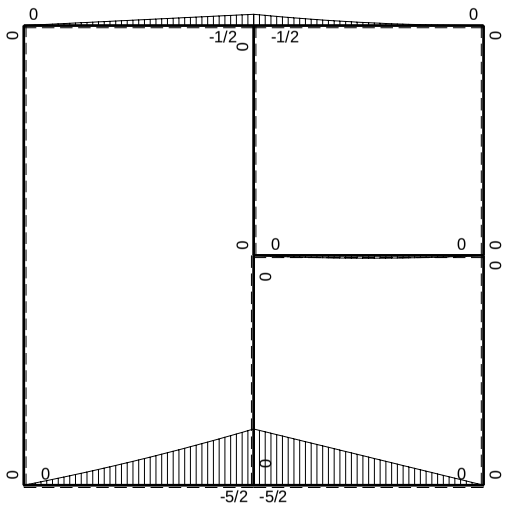

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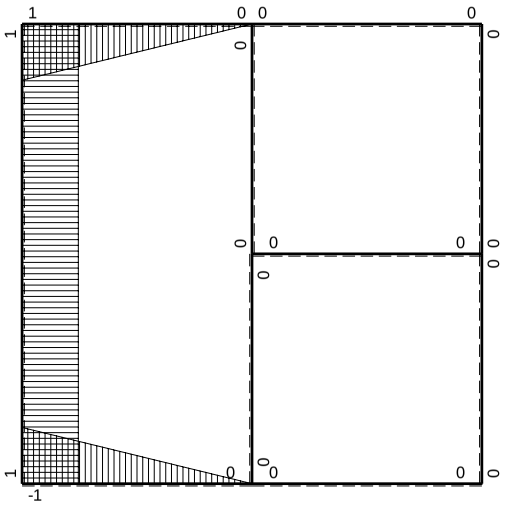

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.197

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.197

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

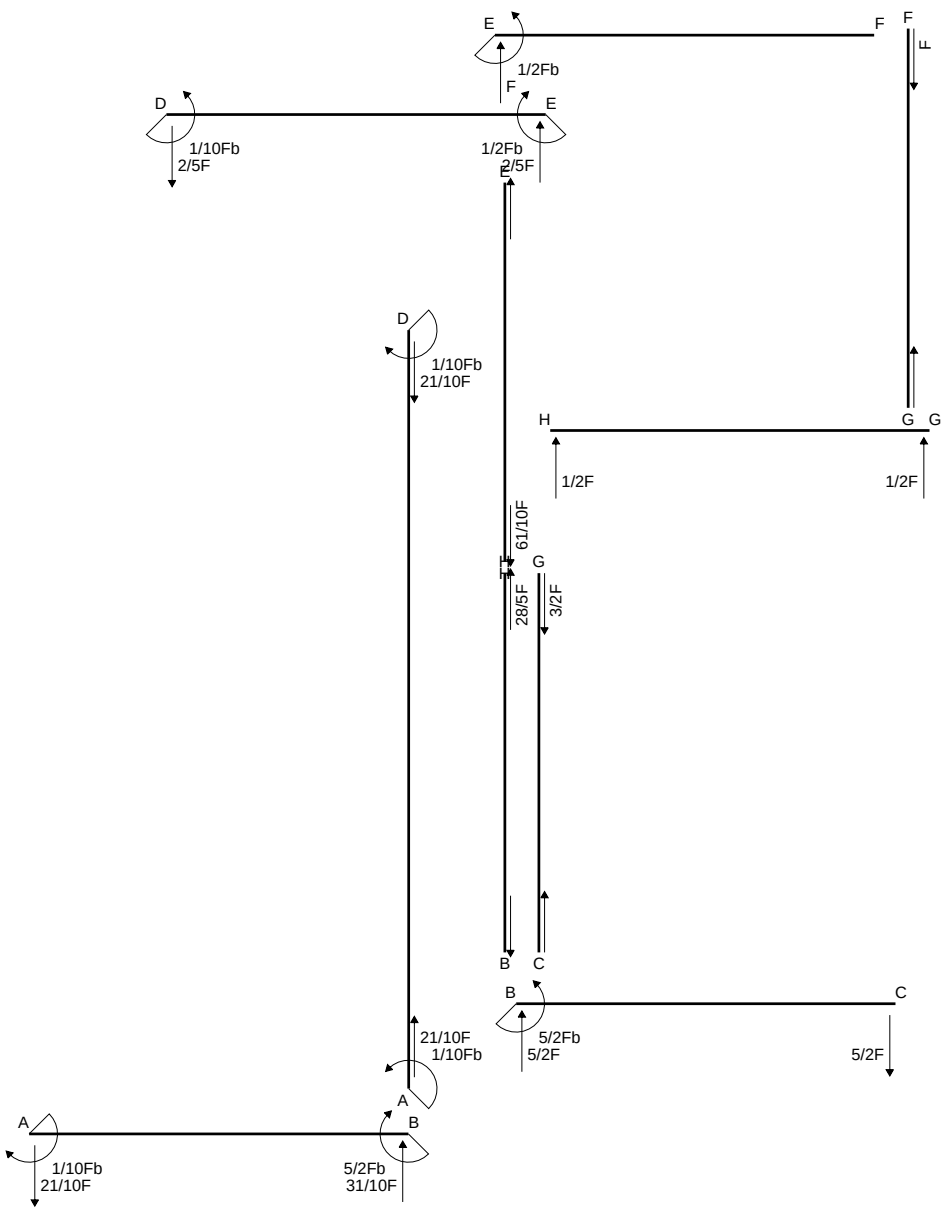
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

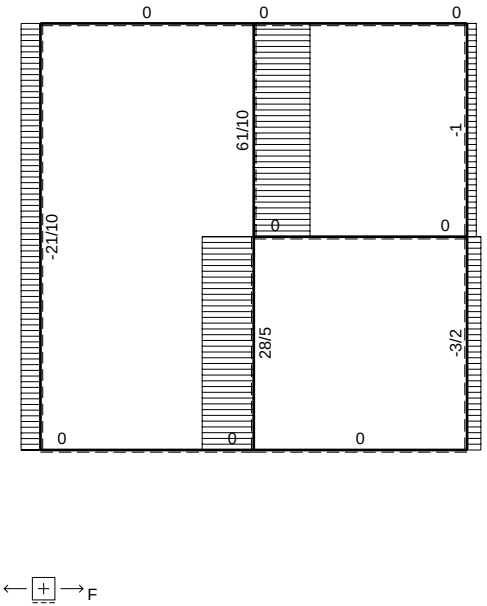
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

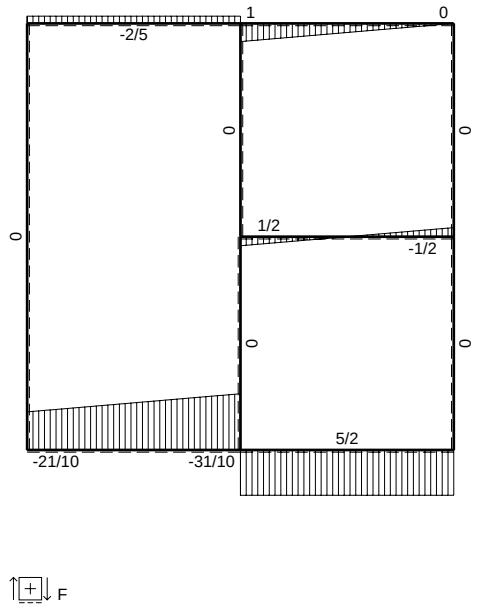
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

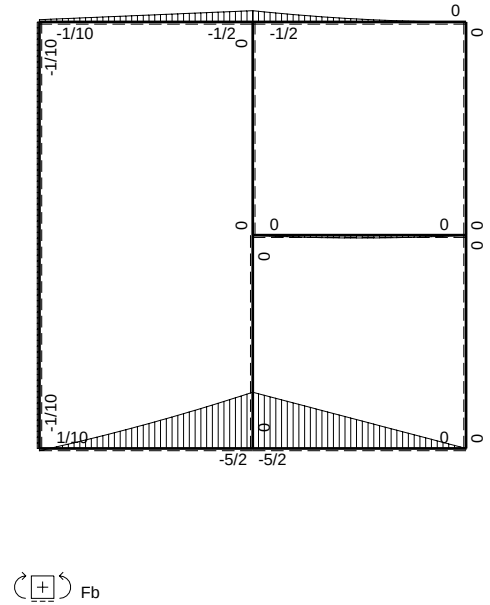
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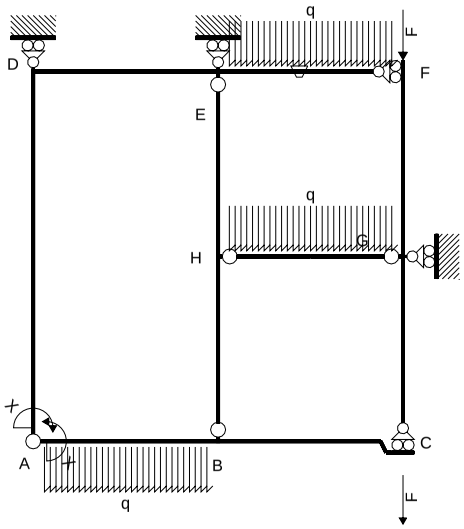
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1/10Fb
2/5F
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2/5F
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1/2F
61/10F
F
G
28/5F
3/2F
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C
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31/10F

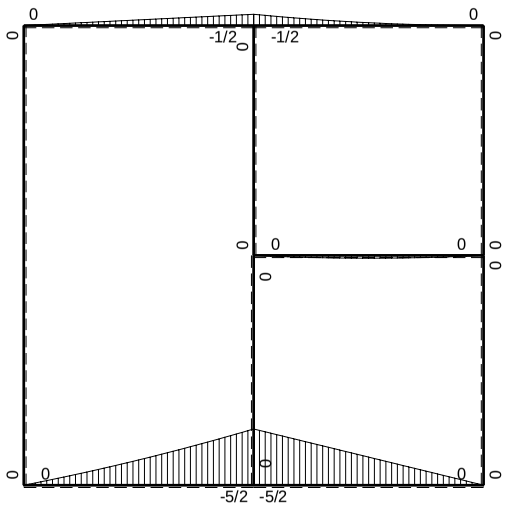

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-31/10
F

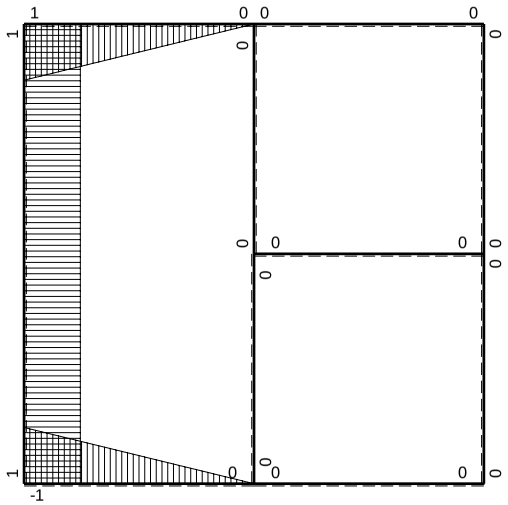

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Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.198

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	$-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	$1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	0	0	0	0	0	0+0	0
EH b	0	0	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.198

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

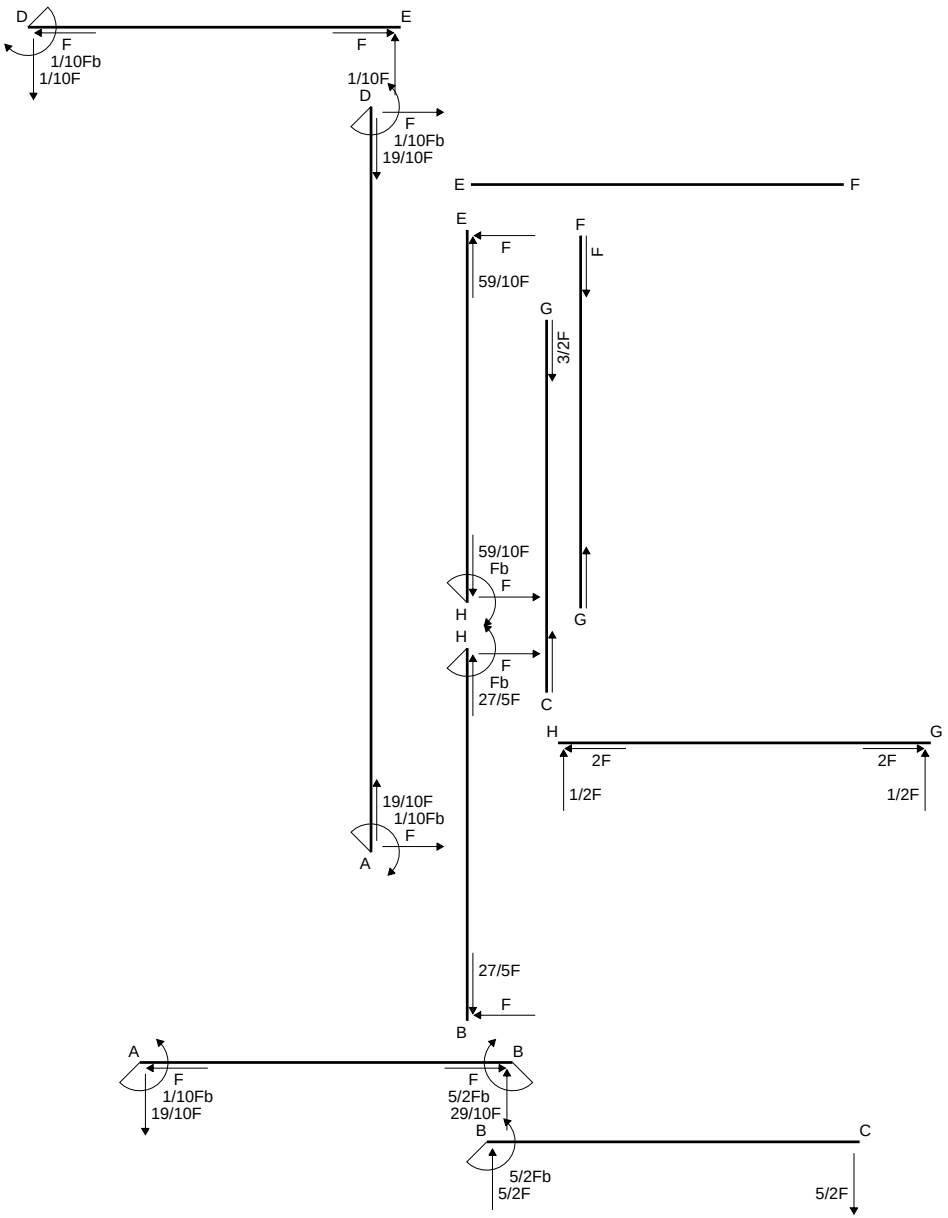
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

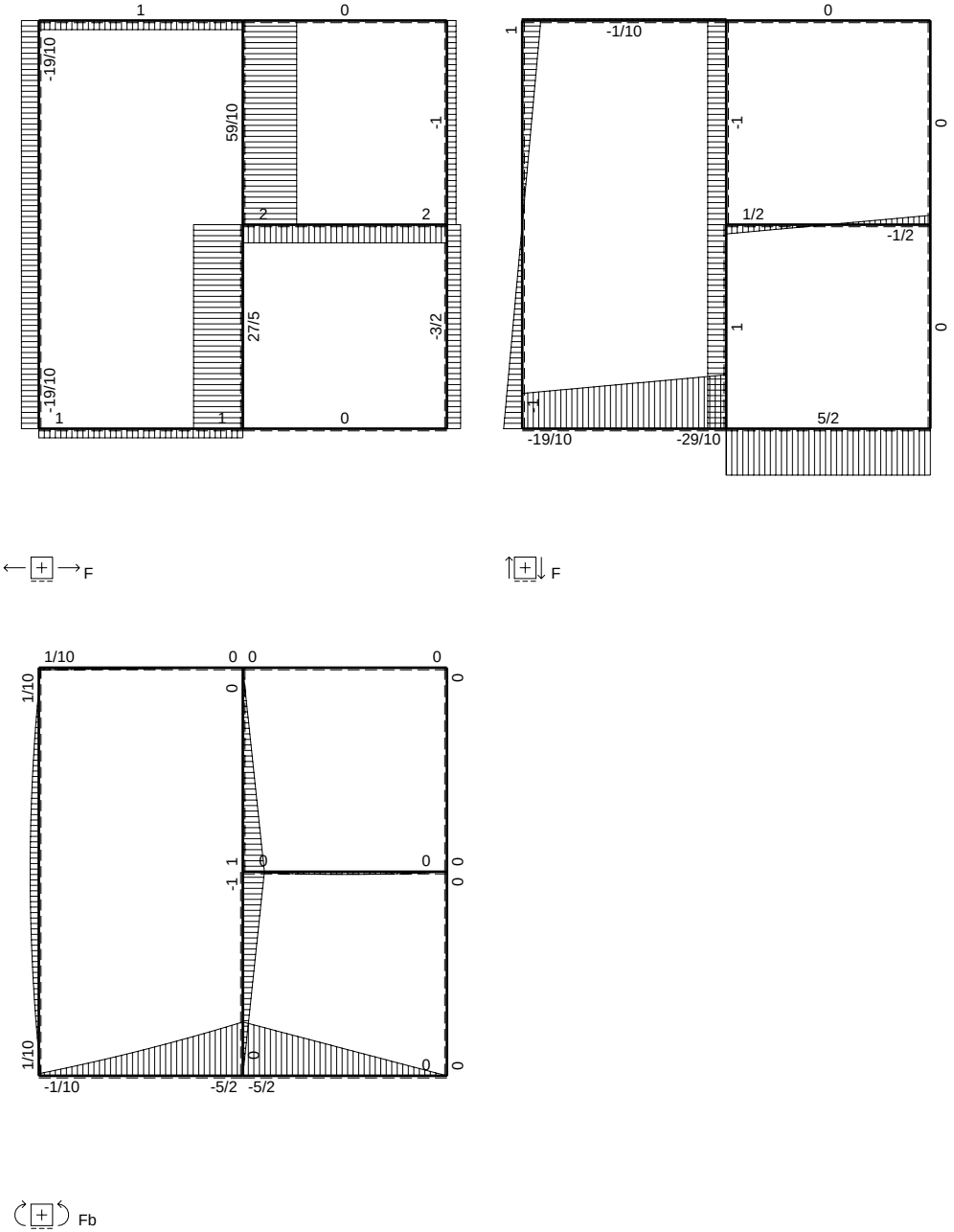
$L^{Xo}_{ED} = \int_o^b (-1/2\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

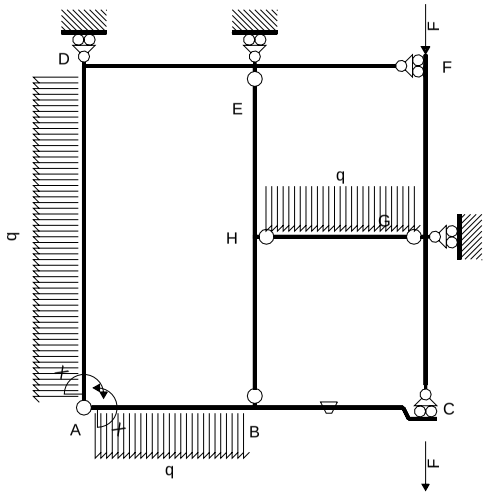
$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ \ = -1/12\ \ Fb^2/EJ$

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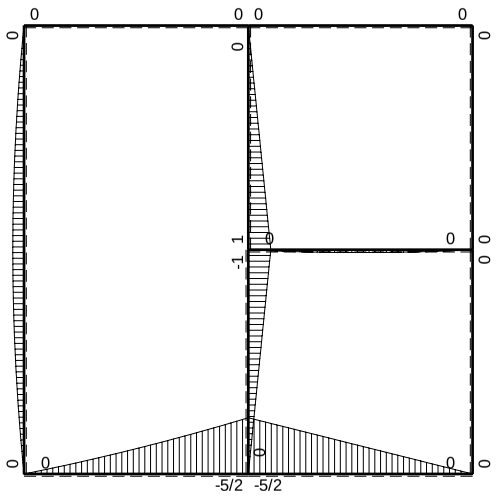
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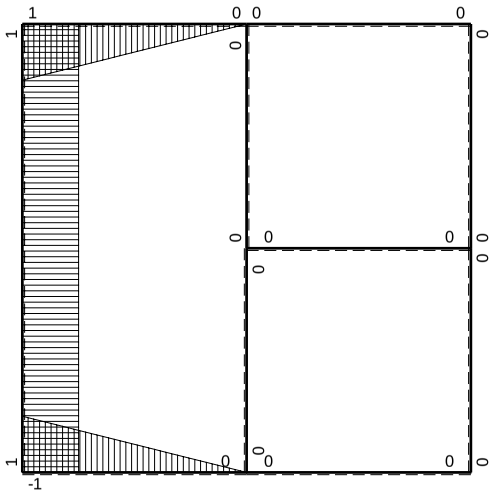

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0
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0
-1 1
0
0
0 0
0
1/10
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.199

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	-Fb/EJ	0	0	0	0+0	0
CB b	0	5/2Fx	Fb/EJ	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-Fb+Fx	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	Fb-Fx	0	0	0	0	0+0	0
EH b	0	-Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.199

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

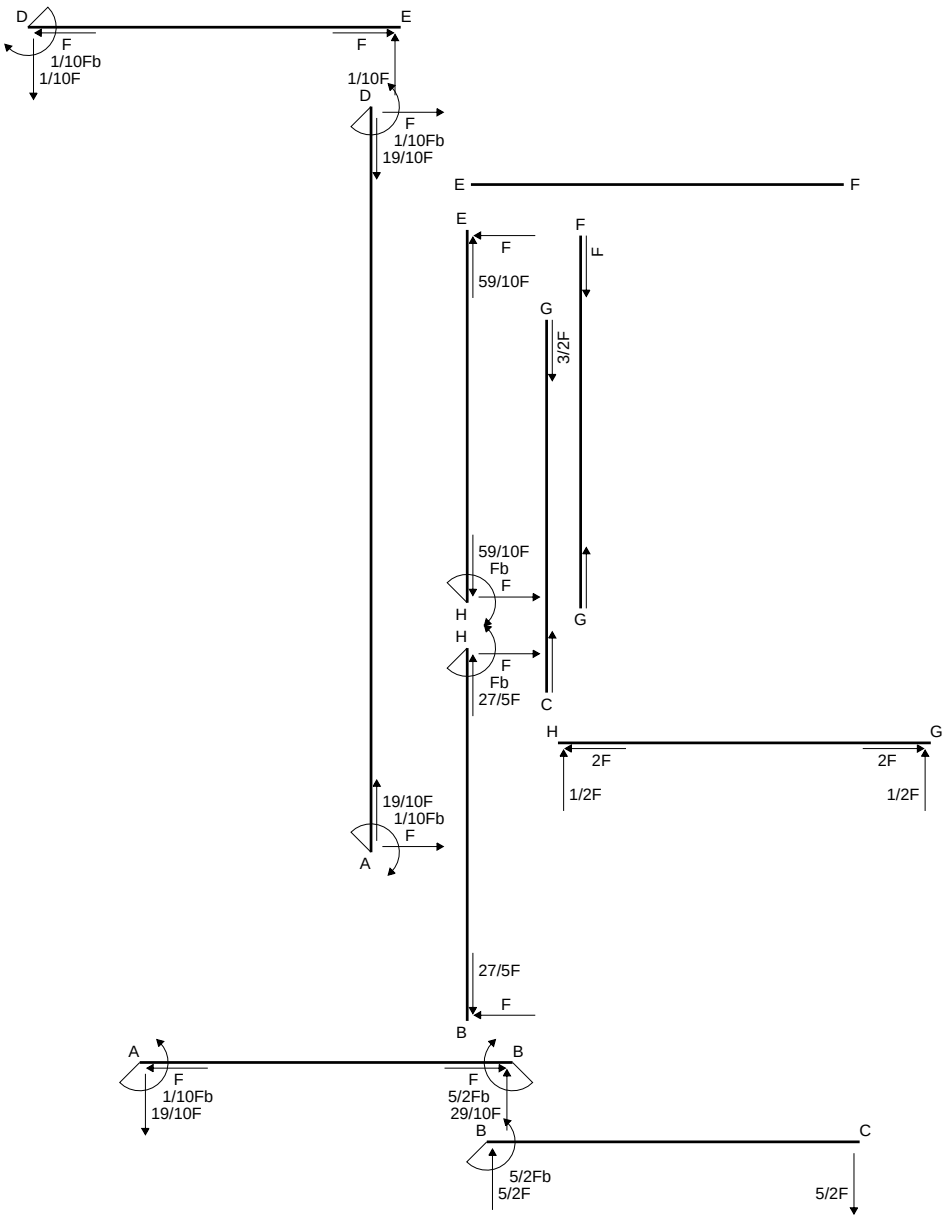
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

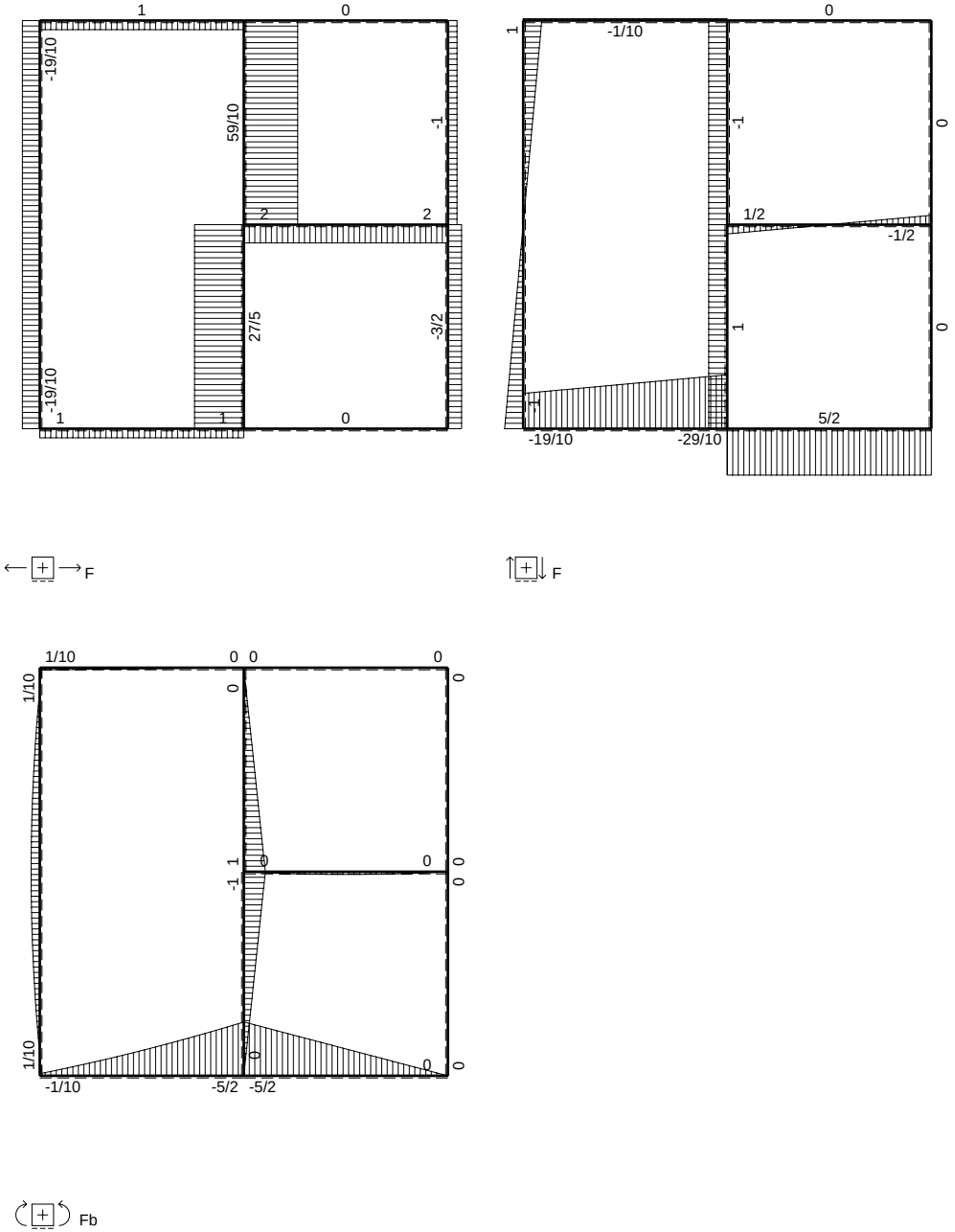
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

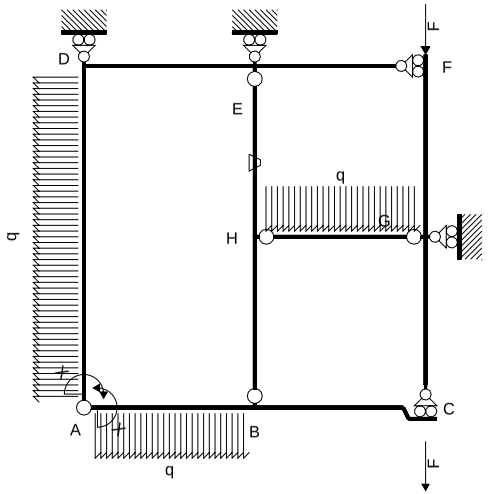
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

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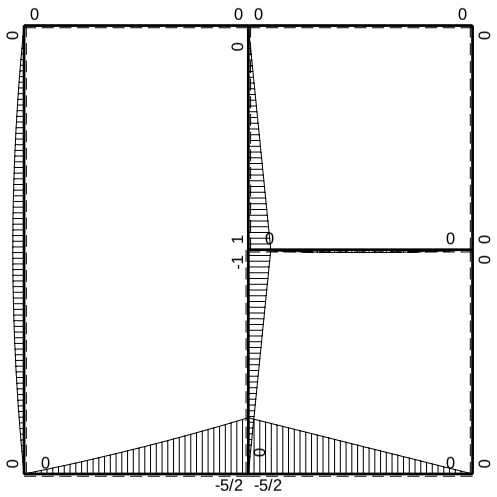
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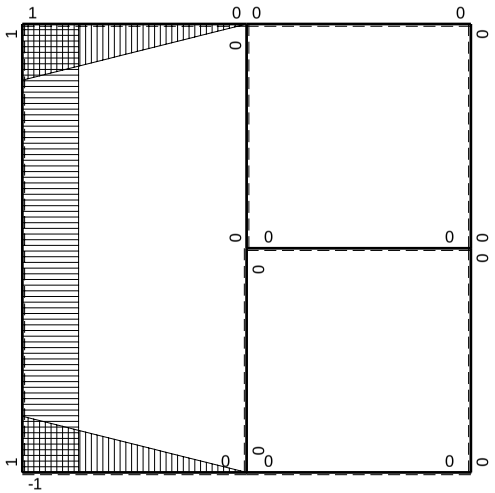

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-29/10
F
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-1 1
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0 0
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1/10
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.200

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-Fb+Fx	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
EH b	0	-Fx	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.200

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

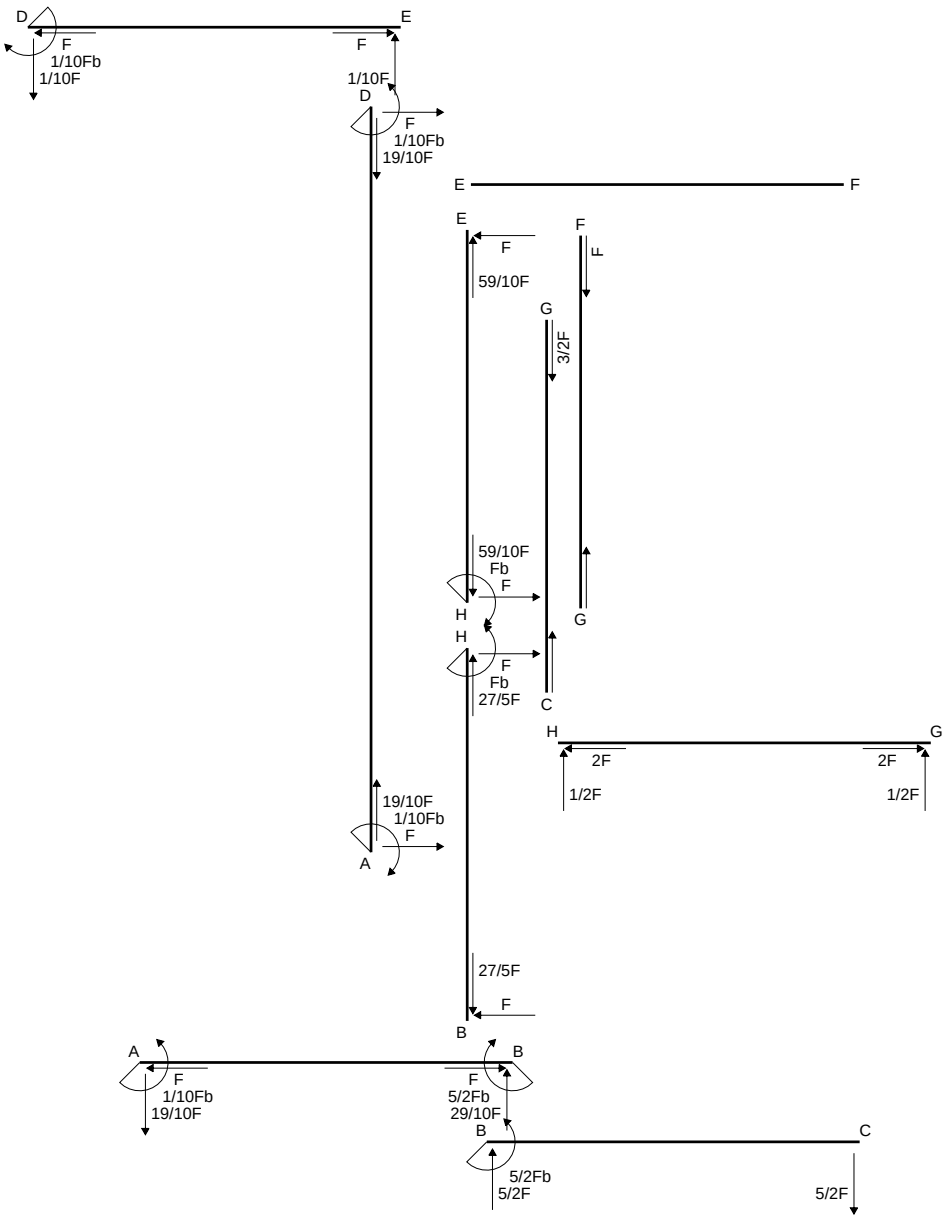
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

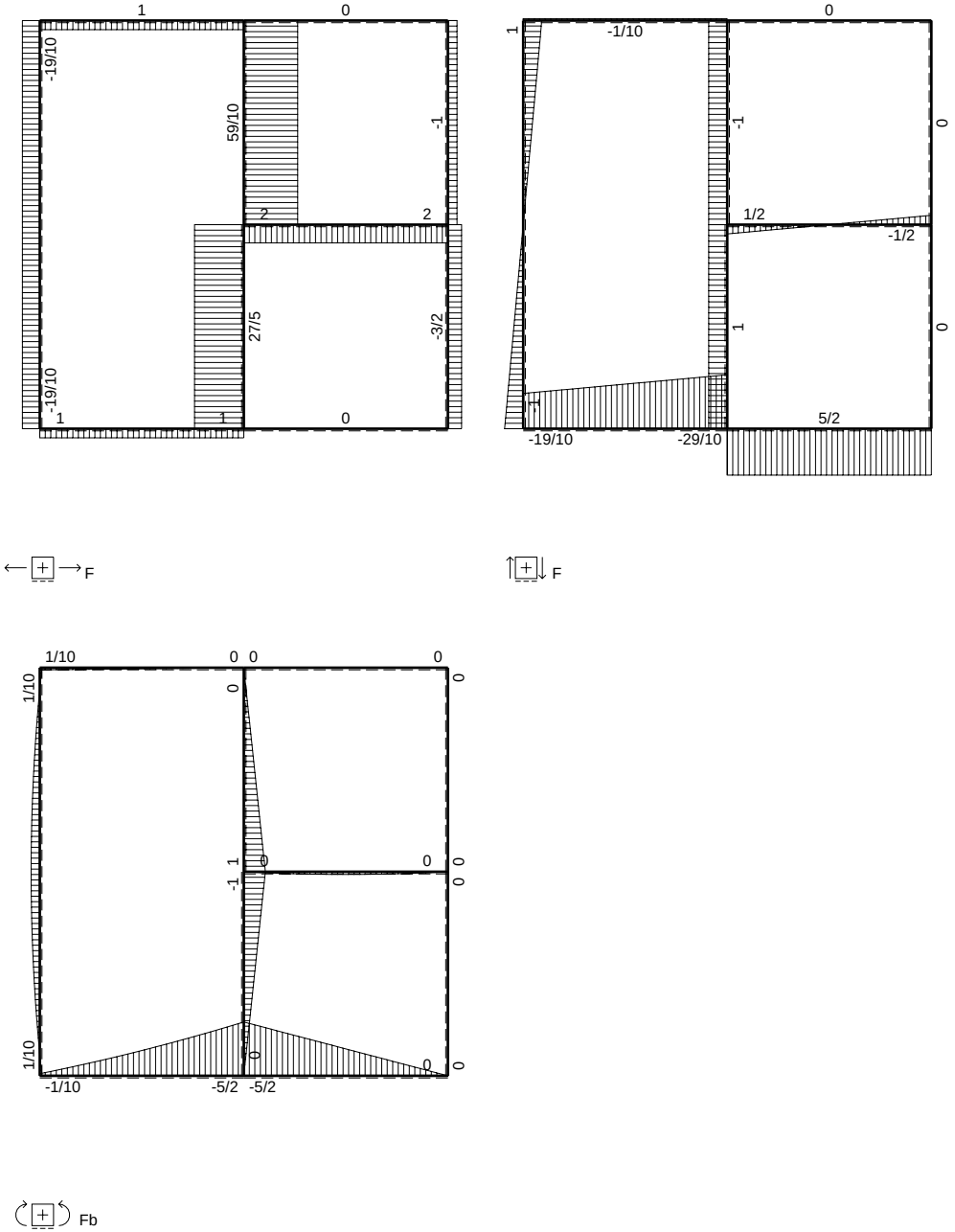
$L_{DA}^{Xo} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

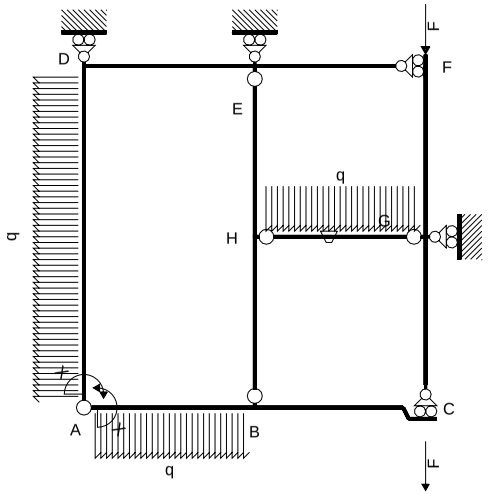
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

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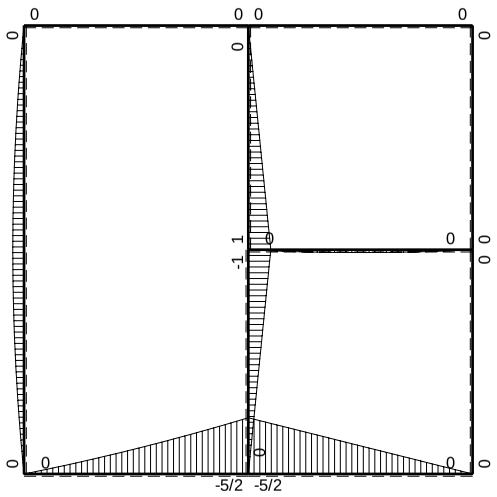
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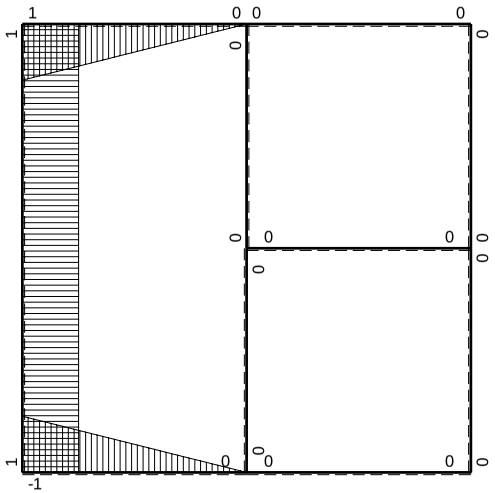

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-1 1
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0 0
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-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	$-Fb+Fx$	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	$Fb-Fx$	0	0	0	0	0+0	0
EH b	0	$-Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.201

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

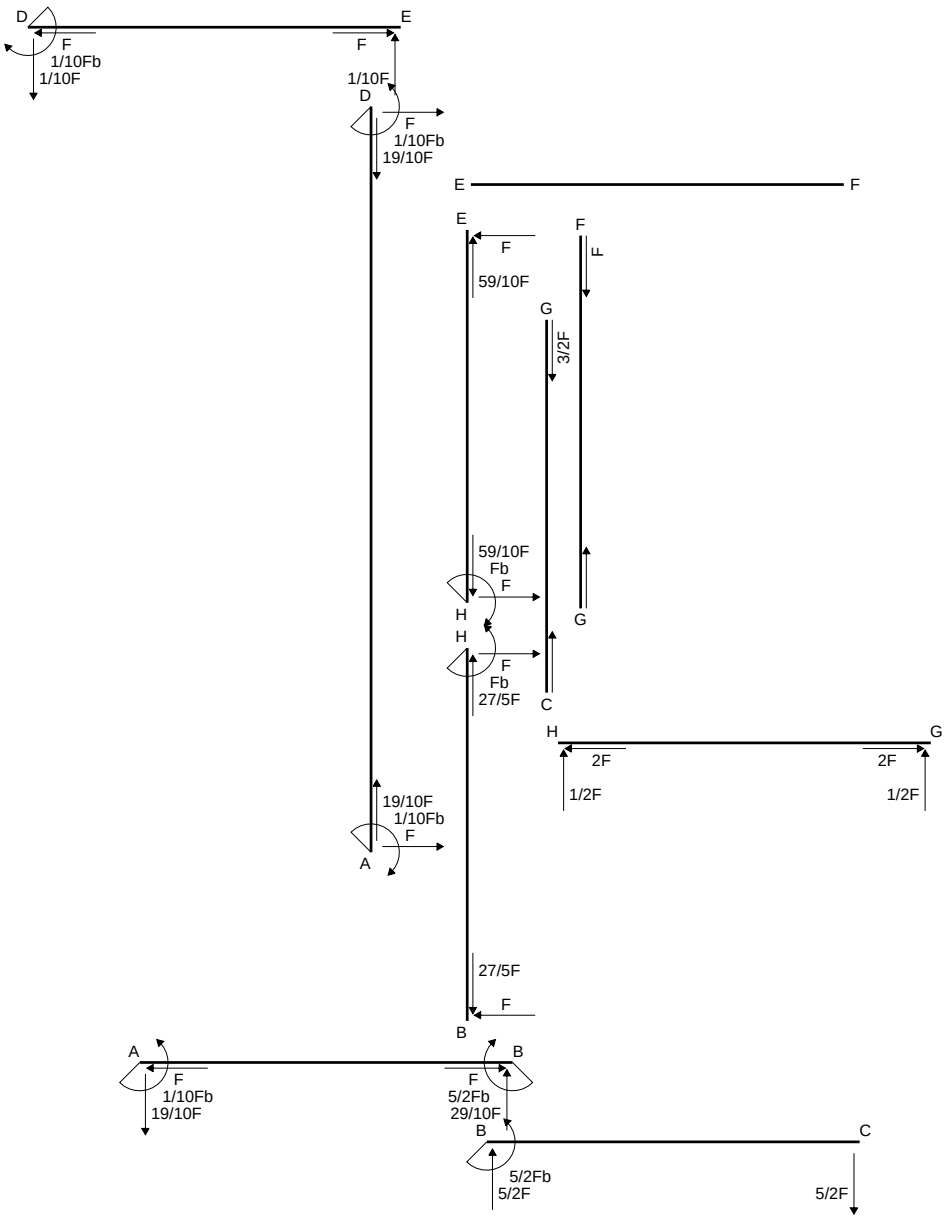
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

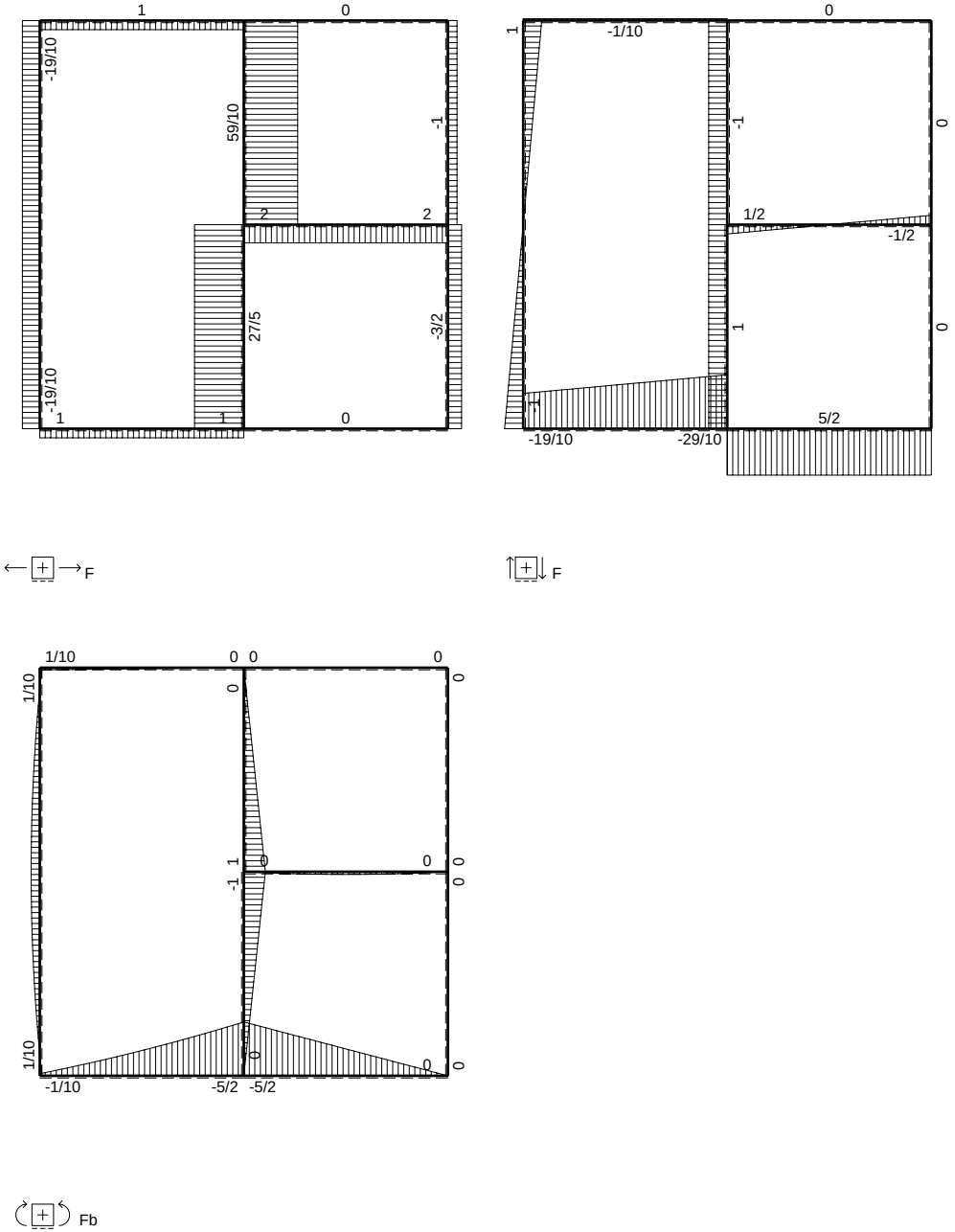
$L_{DA}^{Xo} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

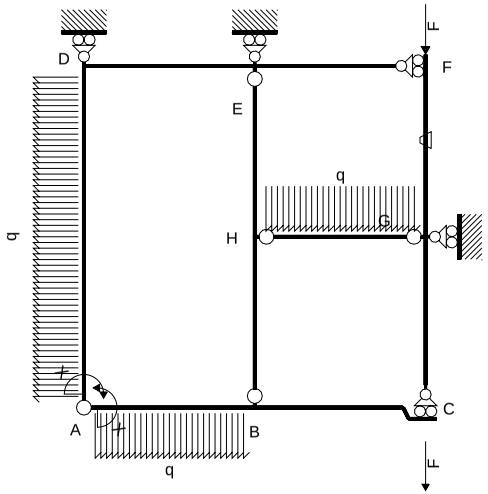
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

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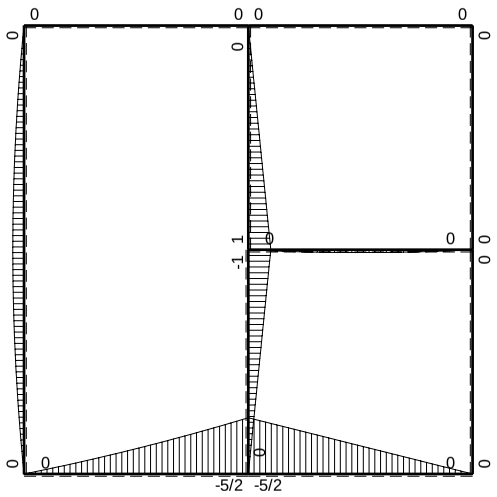
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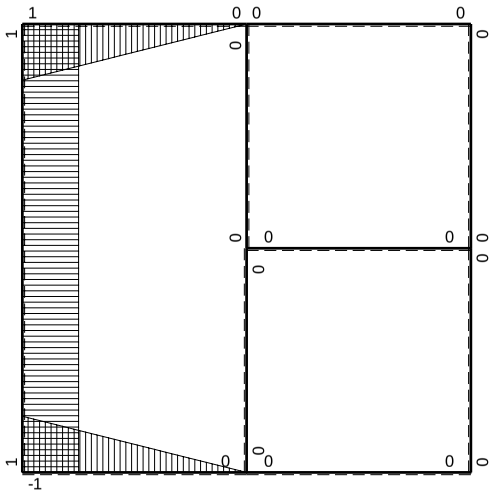

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-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.202

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-Fb+Fx	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	Fb-Fx	0	0	0	0	0+0	0
EH b	0	-Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.202

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

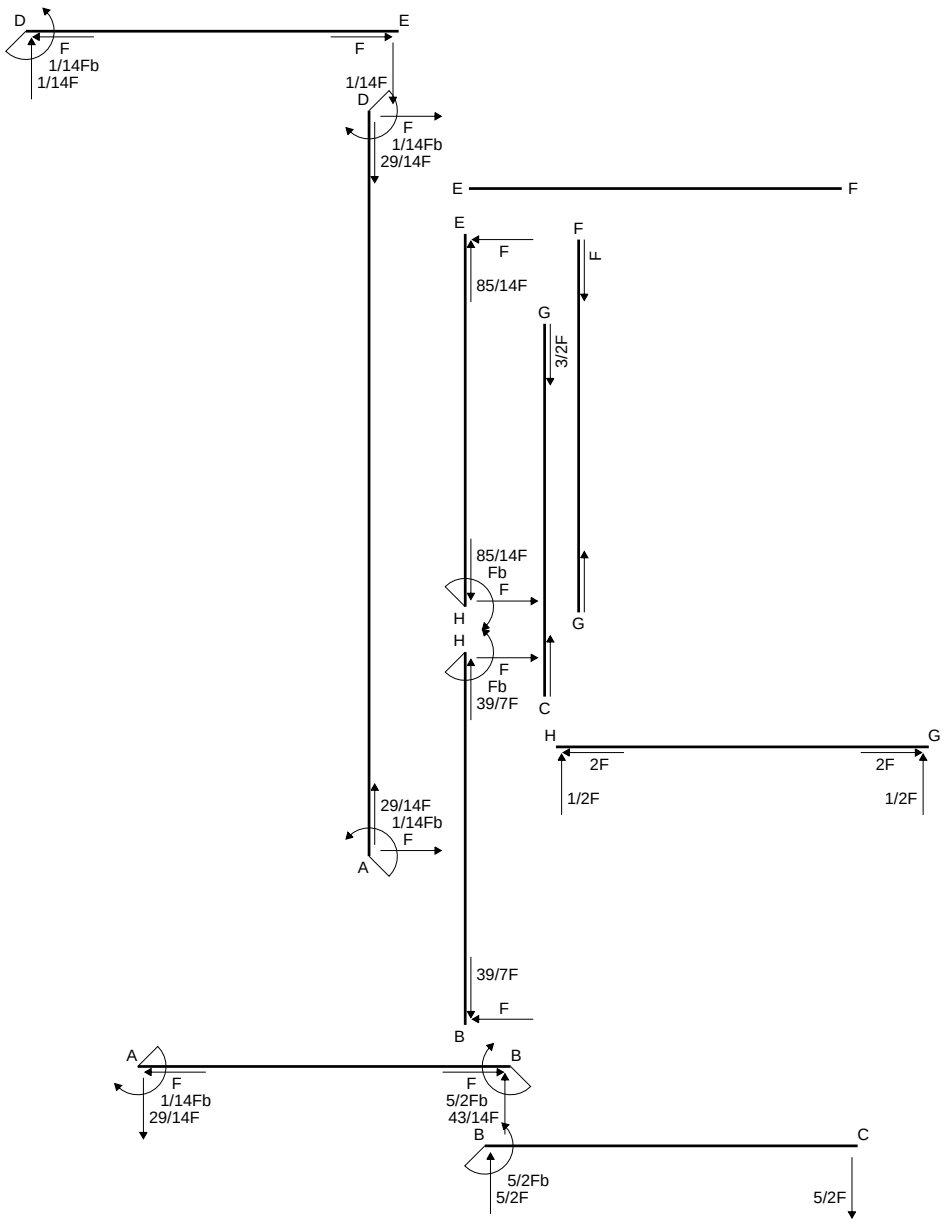
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

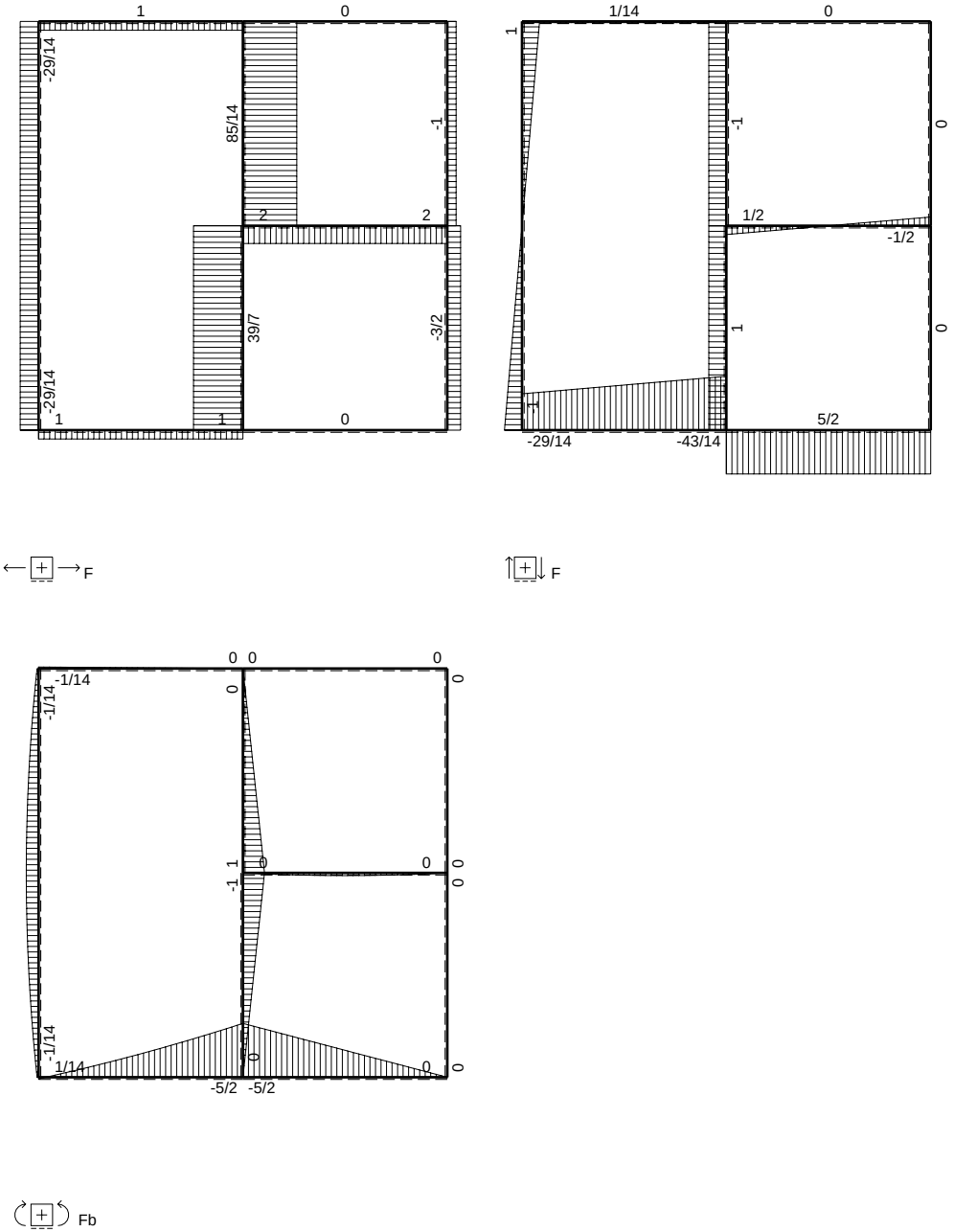
$L_{DA}^{Xo} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

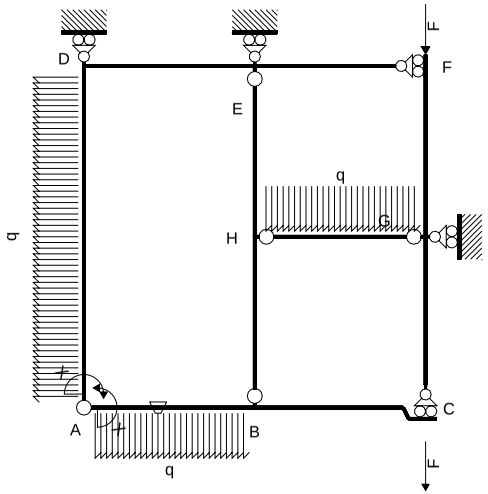
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

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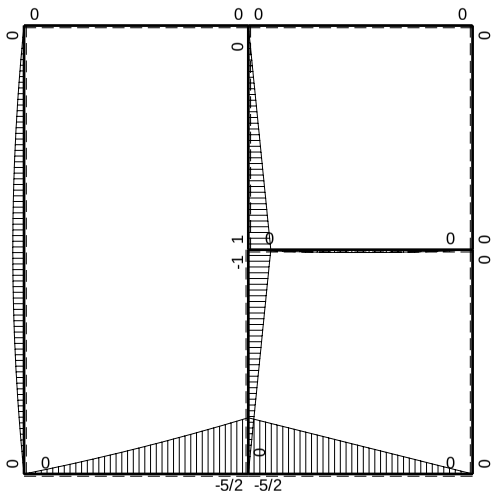
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D
E
F
1/14Fb
1/14F
F
1/14F
D
F
1/14Fb
29/14F
E
F
E
F
F
85/14F
G
3/2F
85/14F
Fb
F
H
G
H
F
Fb
39/7F
C
H
G
2F
2F
1/2F
1/2F
29/14F
1/14Fb
F
A
39/7F
F
B
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5/2Fb
5/2F
5/2F

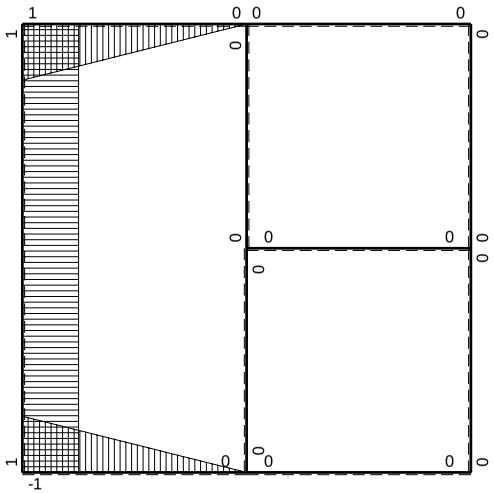

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85/14
-1
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2
39/7
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-29/14
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1
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1/2
-1/2
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0
5/2
-29/14
-43/14
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-1/14
-1/14
0
0
0
-1 1
0
0
0 0
-1/14
1/14
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.203

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$	$(3/8+1/2)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	$5/2Fx-3Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-Fb+Fx	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	Fb-Fx	0	0	0	0	0+0	0
EH b	0	-Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.203

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [\ x - 1/2\ x^2/b\ ]_o^b\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + (\ b - 1/2\ b\ )\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\,x/b\ )\ \theta\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

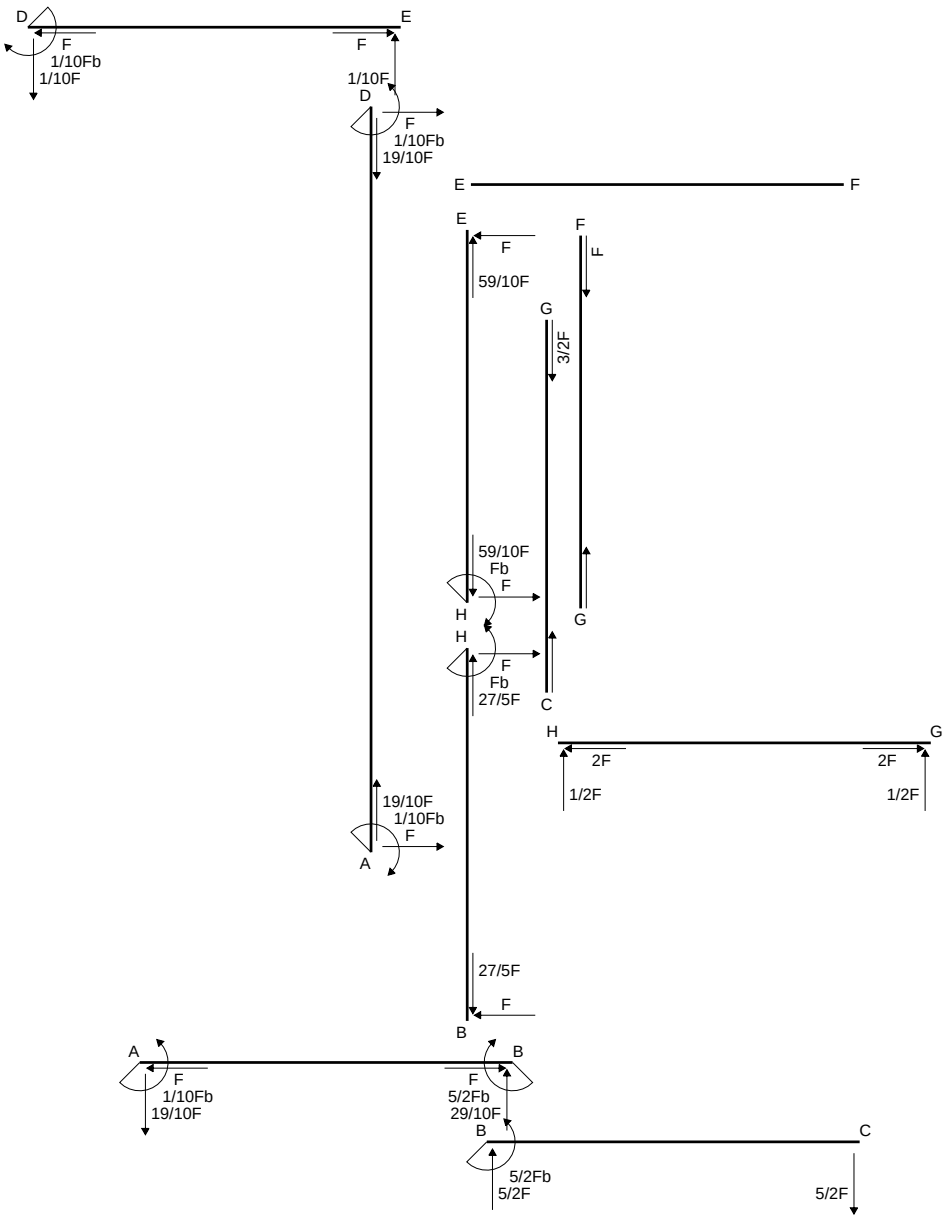
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -2/3\ \ Fb^2/EJ$

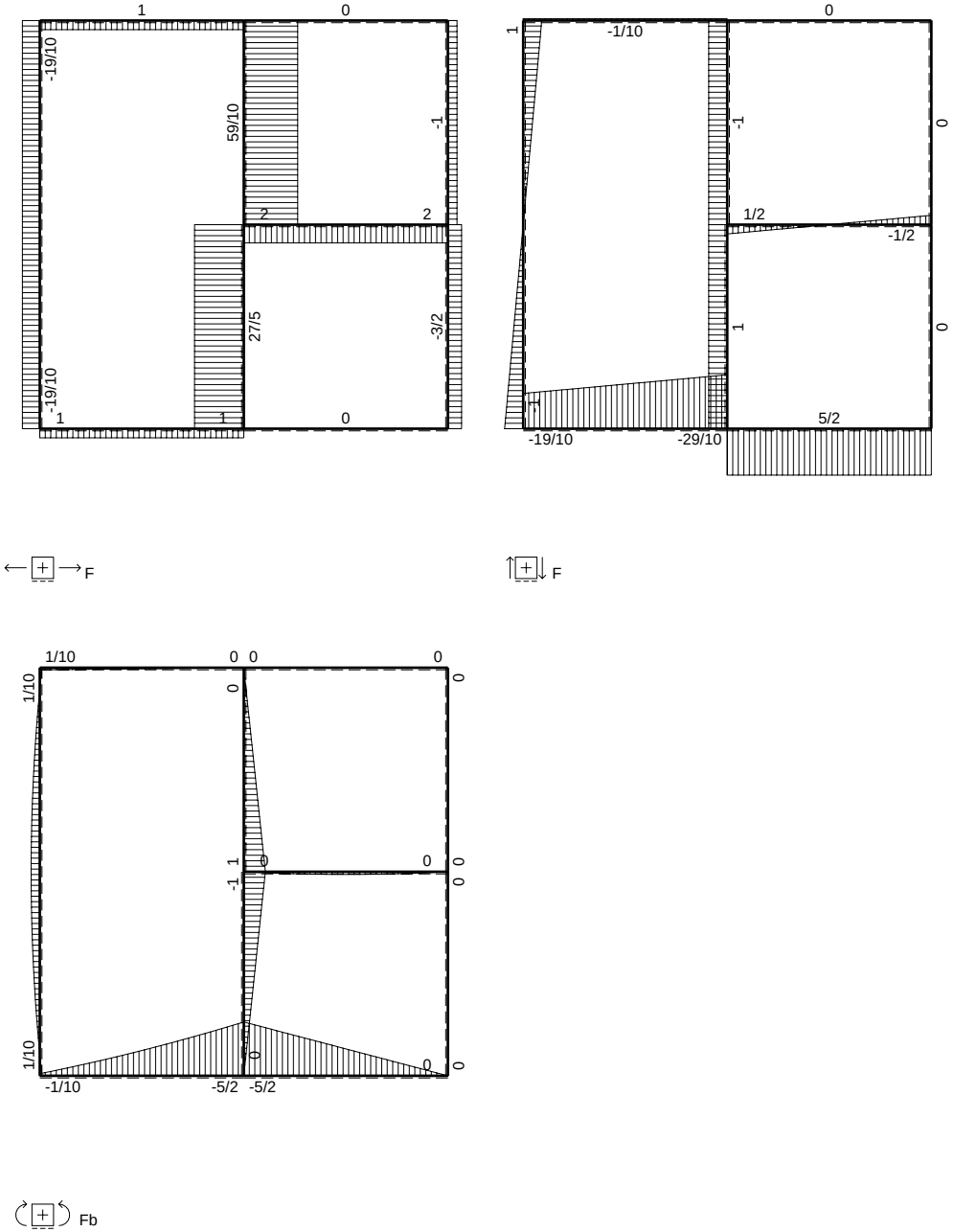
$L^{Xo}_{DA} = \int_o^{2b} (-\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

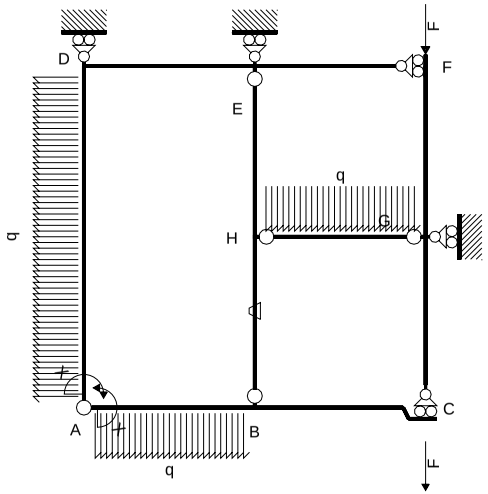
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -2/3\ \ Fb^2/EJ$

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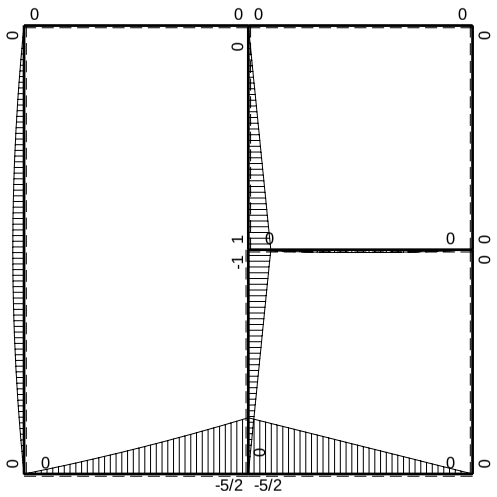
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D
E
F
1/10Fb
1/10F
F
1/10F
D
F
1/10Fb
19/10F
E
F
E
F
F
59/10F
G
3/2F
59/10F
Fb
F
H
G
H
F
Fb
27/5F
C
H
G
2F
2F
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1/2F
19/10F
1/10Fb
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5/2Fb
5/2F
5/2F

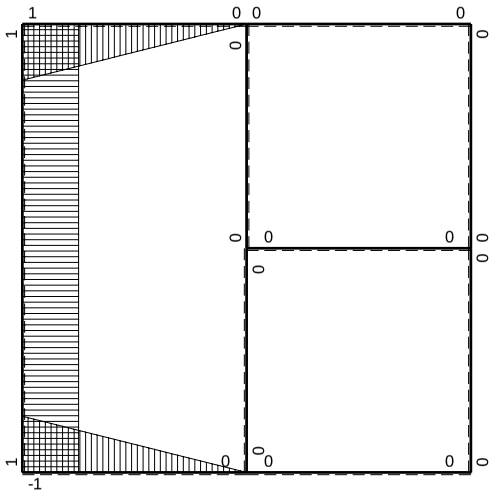

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59/10
-1
2
2
27/5
-3/2
-19/10
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1
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-1
1/2
-1/2
1
5/2
-19/10
-29/10
F
F
1/10
0 0
0
1/10
0
0
0
-1 1
0
0
0 0
0
0
1/10
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.204

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-Fb+Fx$	$-Fb/EJ$	0	0	0	0+0	0
BH b	0	Fx	Fb/EJ	0	0	0		
HE b	0	$Fb-Fx$	0	0	0	0	0+0	0
EH b	0	$-Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.204

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b\ )\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

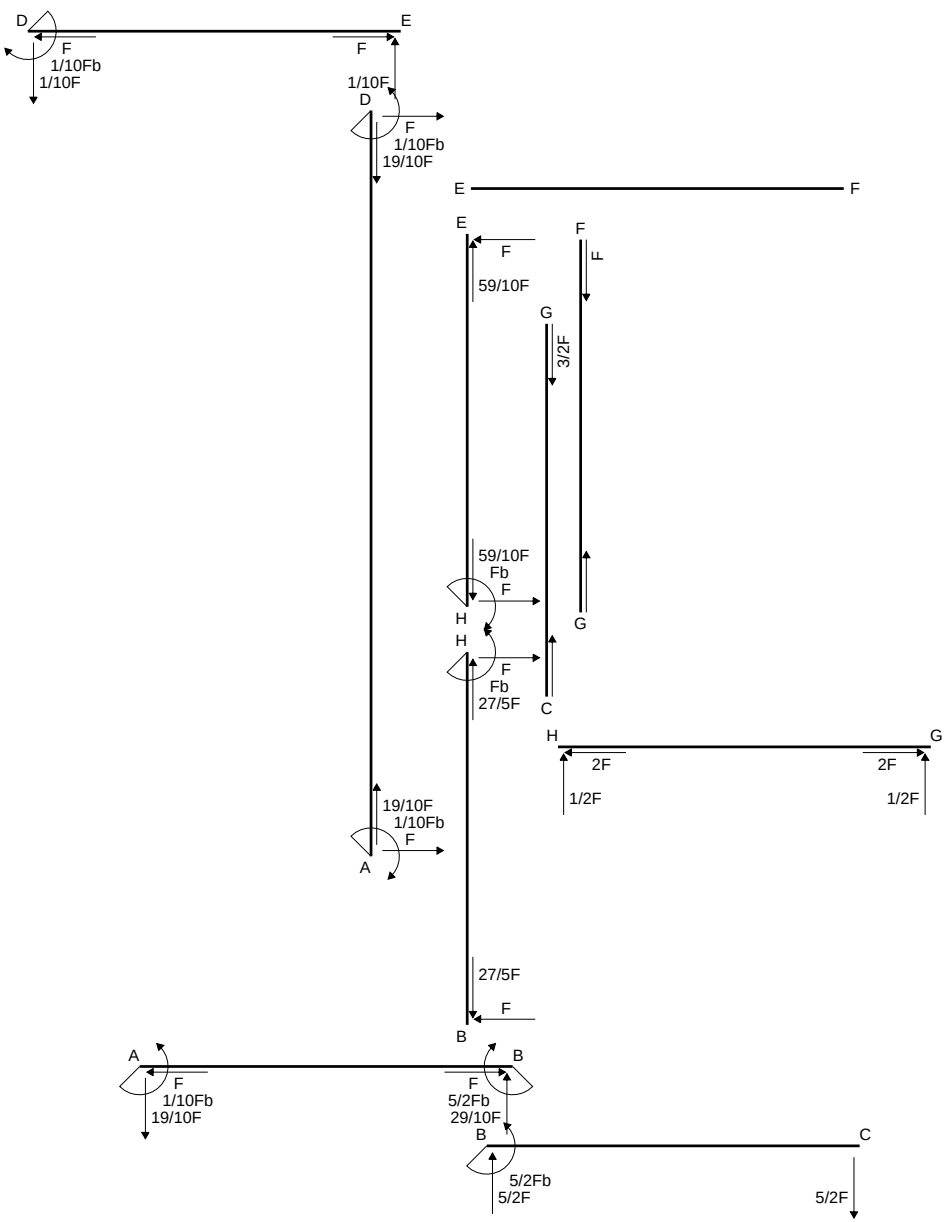
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

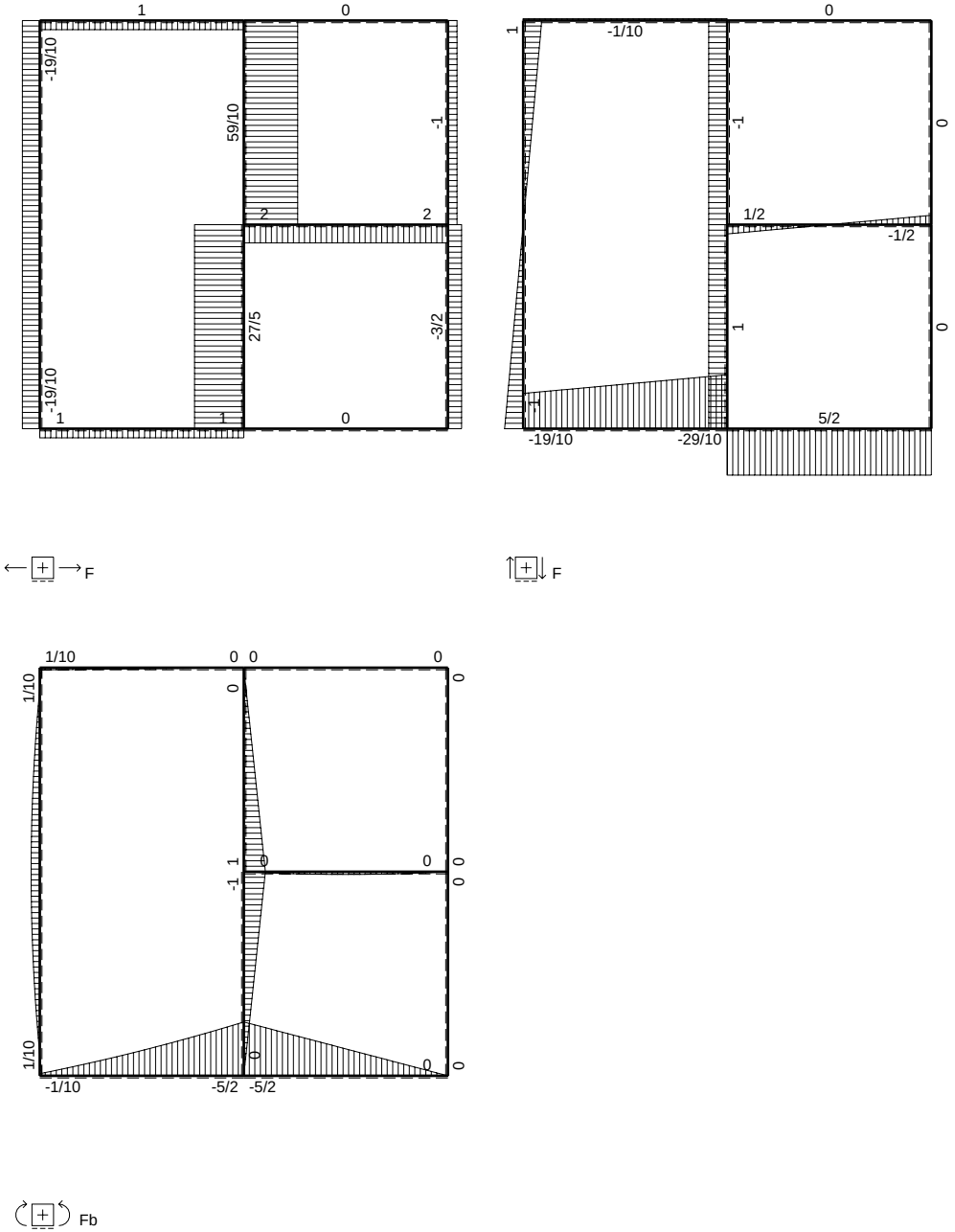
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

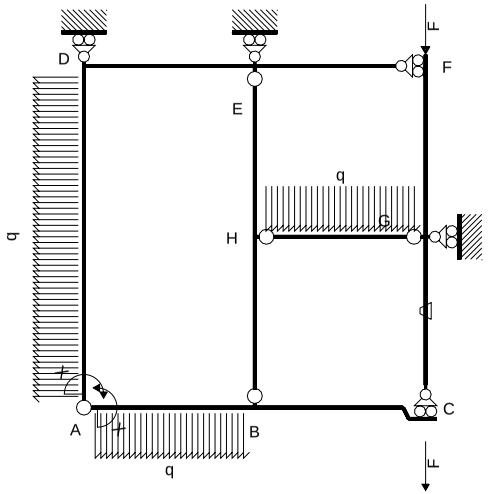
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

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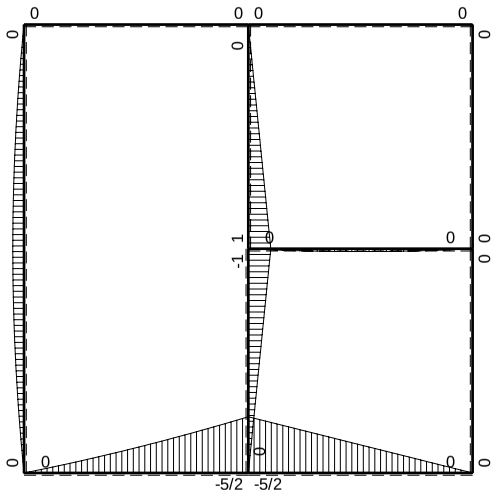
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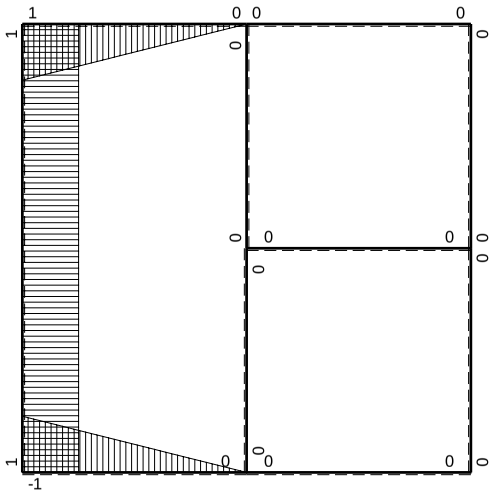

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-1 1
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0
1/10
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.205

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-Fb+Fx$	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	$Fb-Fx$	0	0	0	0	0+0	0
EH b	0	$-Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.205

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ \ 1/EJ\ \ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ \ 1/EJ\ dx = [\ x\ ]_o^{2b}\ \ 1/EJ$

$= (2\ b)\ \ 1/EJ\ \ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ \ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ \ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ \ 1/EJ$

$= (1/3\ b)\ \ 1/EJ\ \ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

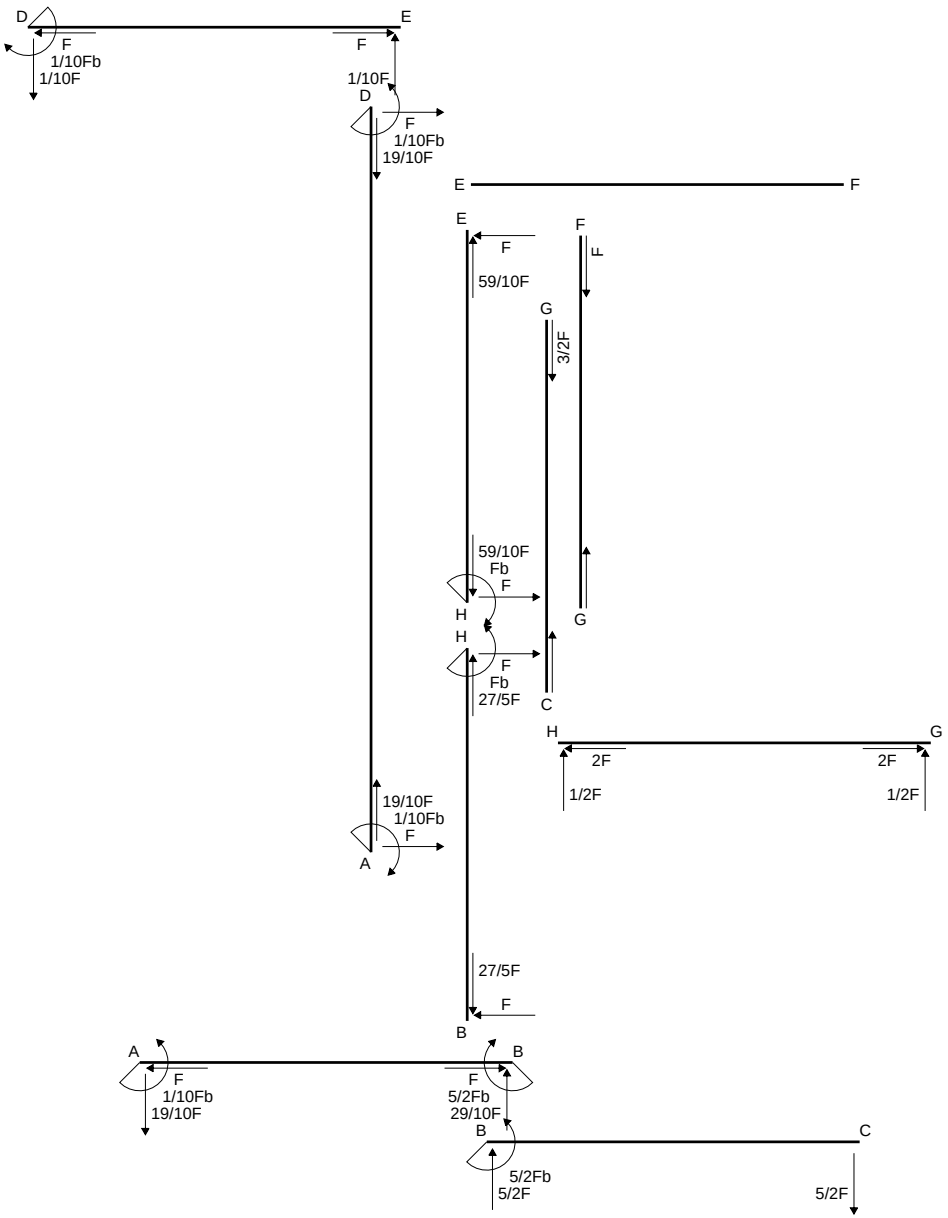
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

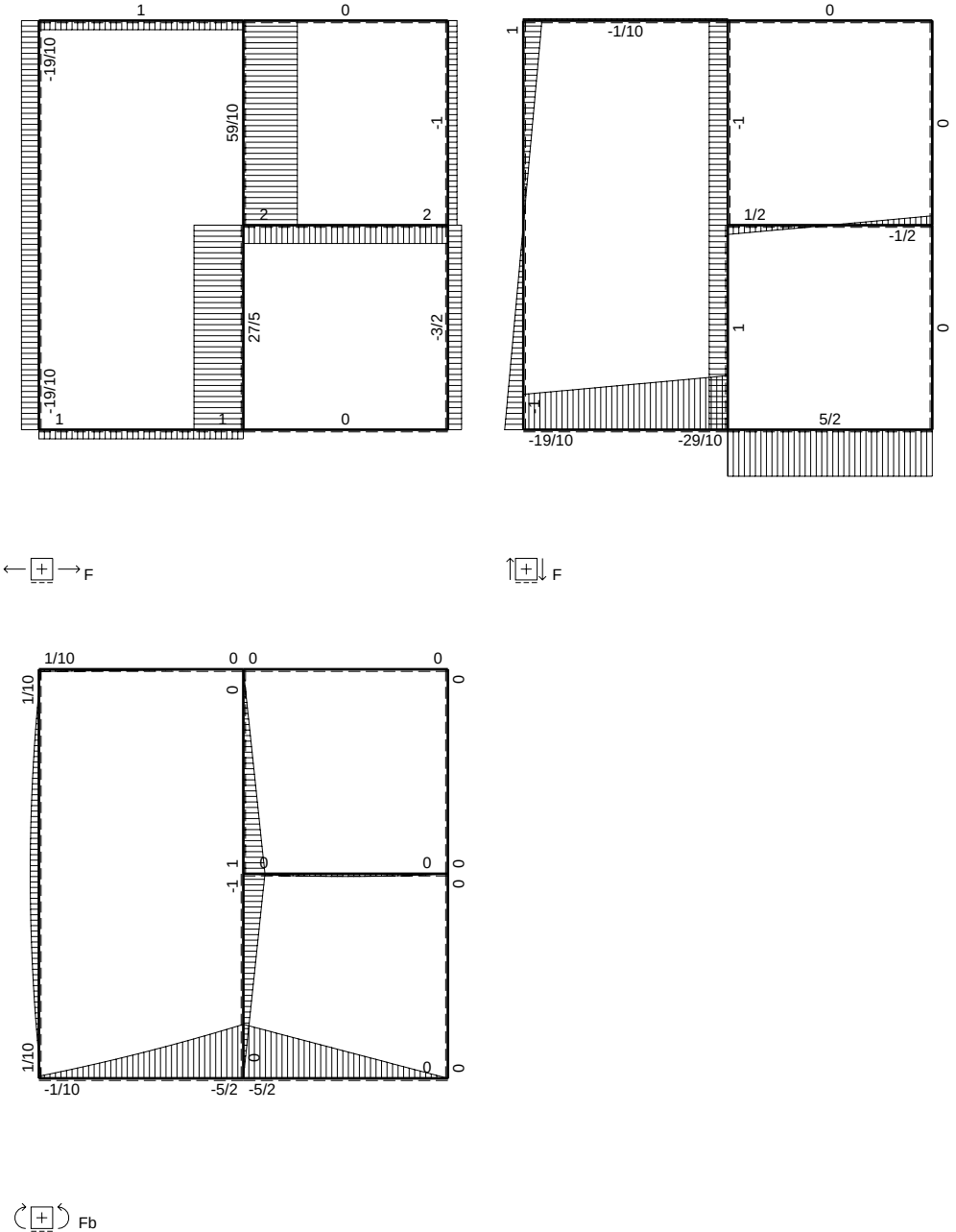
$L^{Xo}_{DA} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

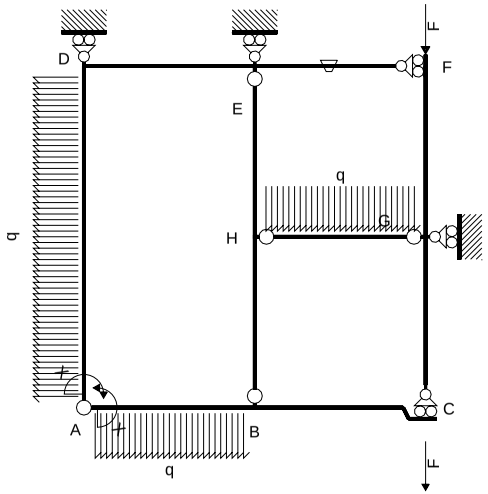
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ \ = -2/3\ \ Fb^2/EJ$

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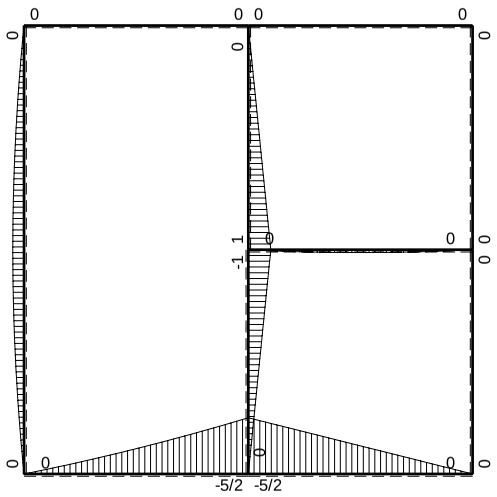
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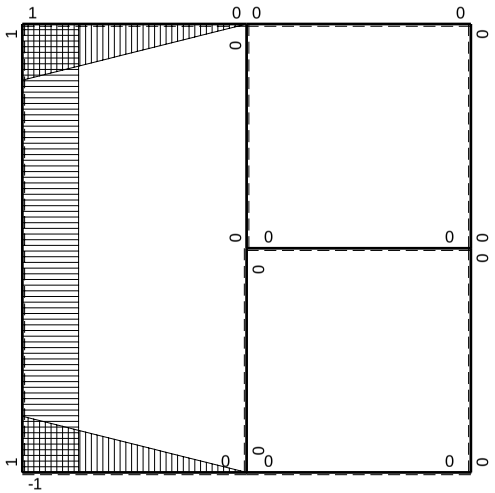

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-1 1
0
0
0 0
0
1/10
0
-1/10
-5/2 -5/2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.206

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	$(-2/3+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$Fx-1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	-Fb/EJ	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-Fb+Fx	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	Fb-Fx	0	0	0	0	0+0	0
EH b	0	-Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.206

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

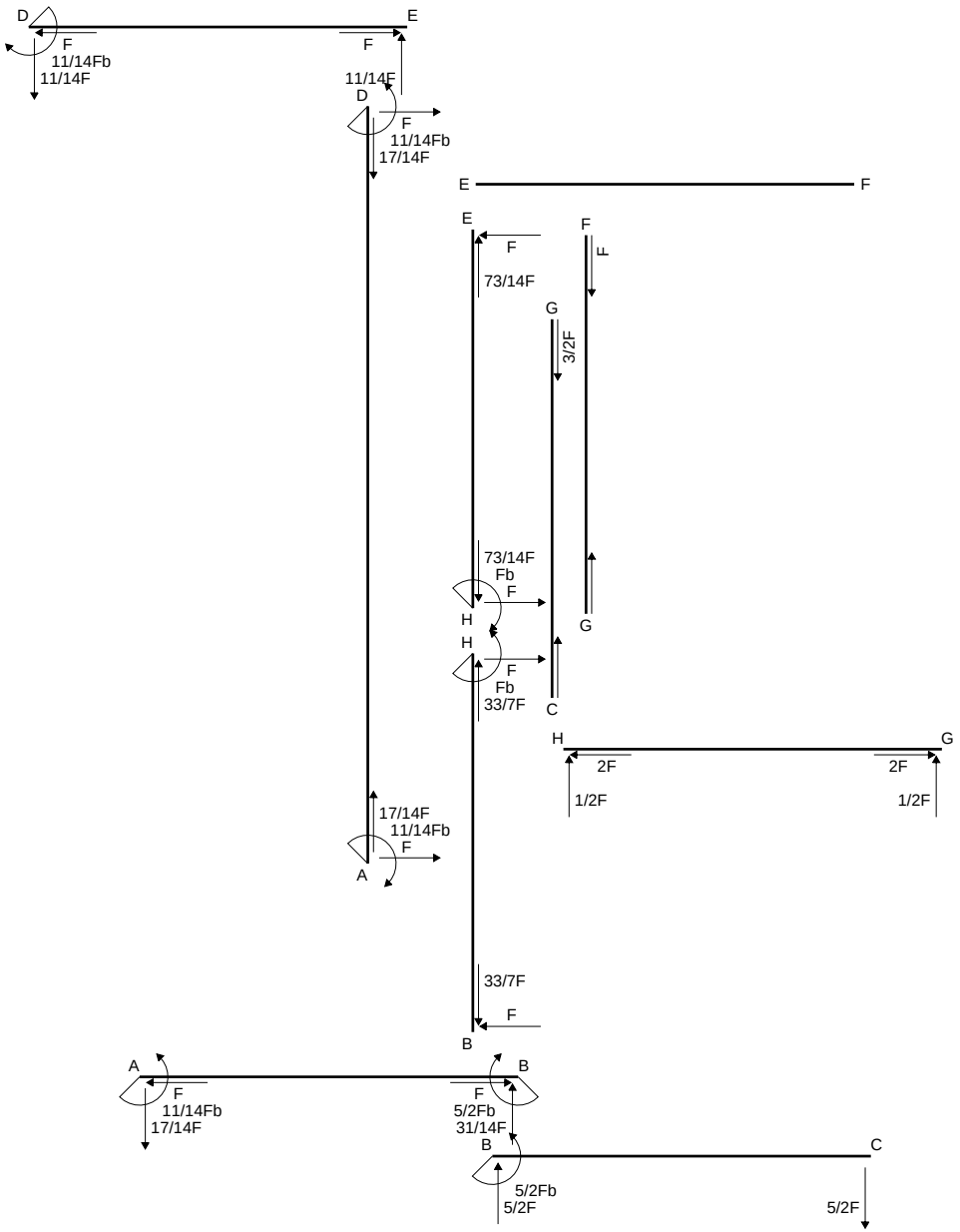
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -2/3\ \ Fb^2/EJ$

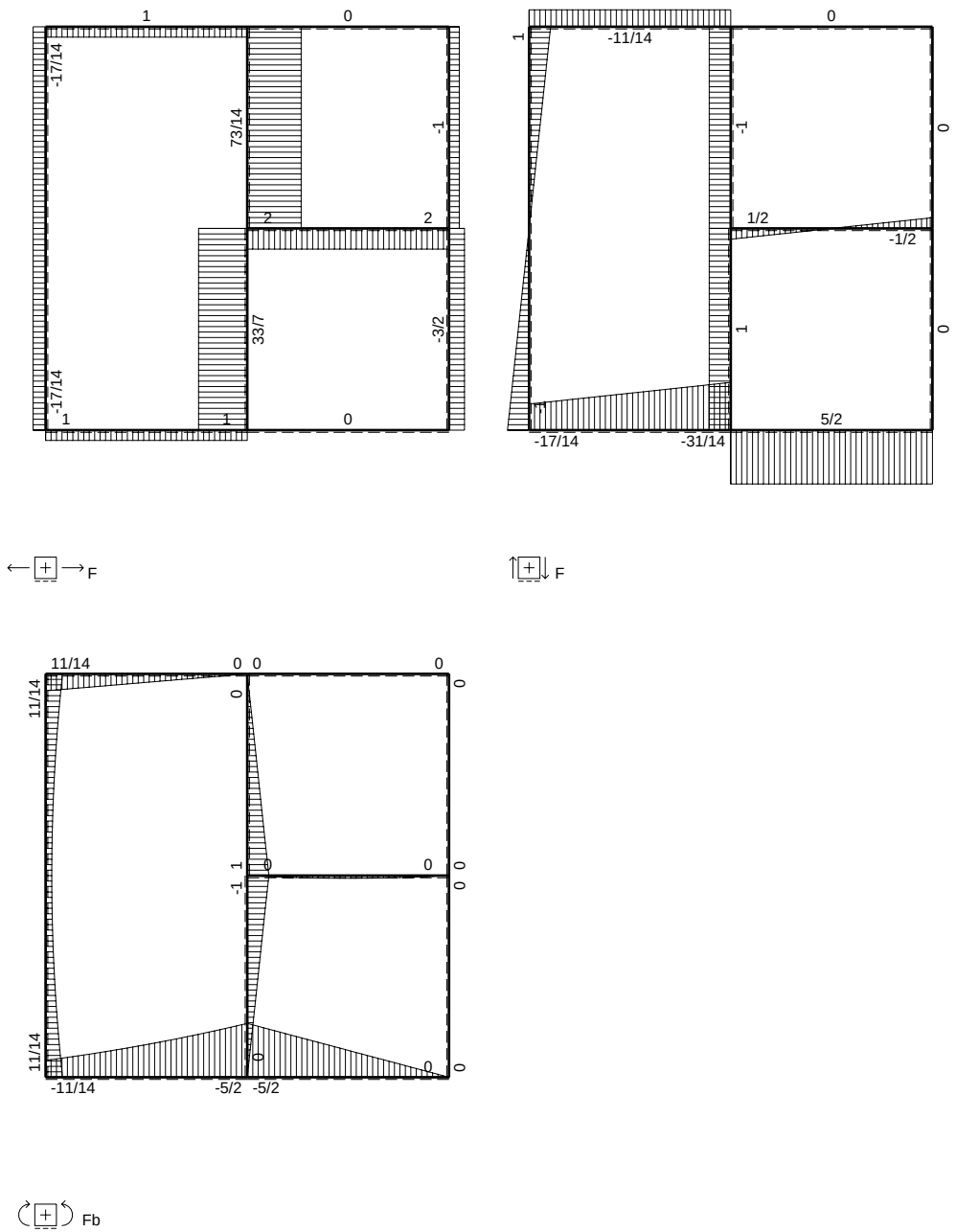
$L_{DA}^{Xo} = \int_o^{2b} (-\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

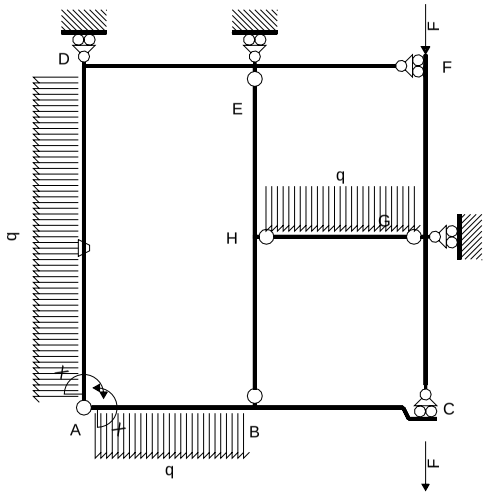
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -2/3\ \ Fb^2/EJ$

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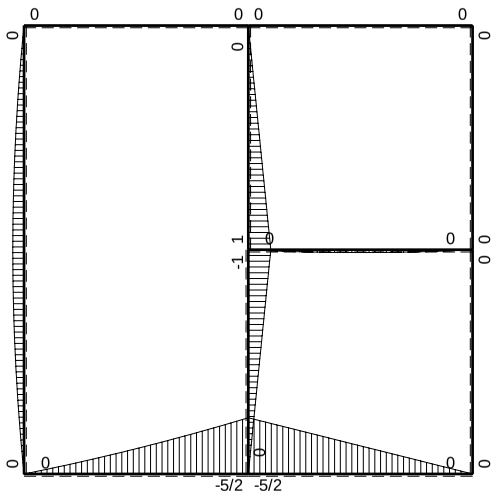
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11/14Fb
73/14F
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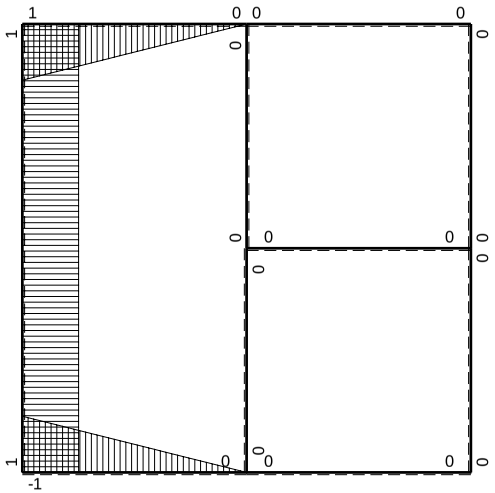

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1/2
-1/2
5/2
-31/14
11/14
-5/2
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.207

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PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.207

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	$-Fx+1/2qx^2$	-Fb/EJ	$-Fx+1/2Fx^2/b$	-Fb/EJ	1	$(-2/3-2)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	$Fx-1/2qx^2$	Fb/EJ	$-Fx+1/2Fx^2/b$	-Fb/EJ	1		
DE b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
ED b	-x/b	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-Fb+Fx	0	0	0	0	0+0	0
BH b	0	Fx	0	0	0	0		
HE b	0	Fb-Fx	0	0	0	0	0+0	0
EH b	0	-Fx	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-55/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						11/14Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^{2b} (-1)\ \theta\ dx$

$= [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ\ + [-\,x\ ]_o^{2b}\ \theta$

$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ + (-2\ b\ )\ \theta\ = -8/3\ \ Fb^2/EJ$

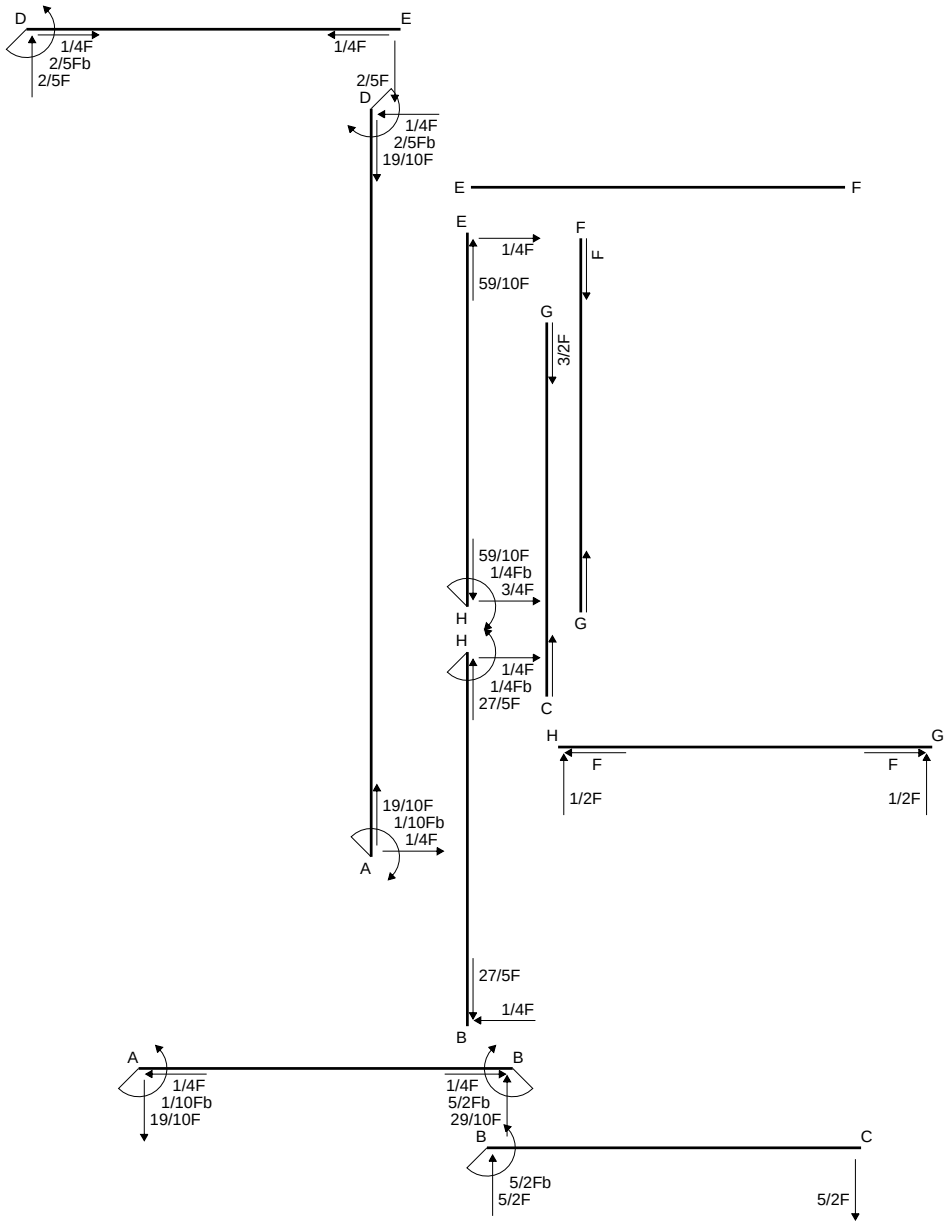
$L^{Xo}_{DA} = \int_o^{2b} (-\,x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^{2b} (1)\ \theta\ dx$

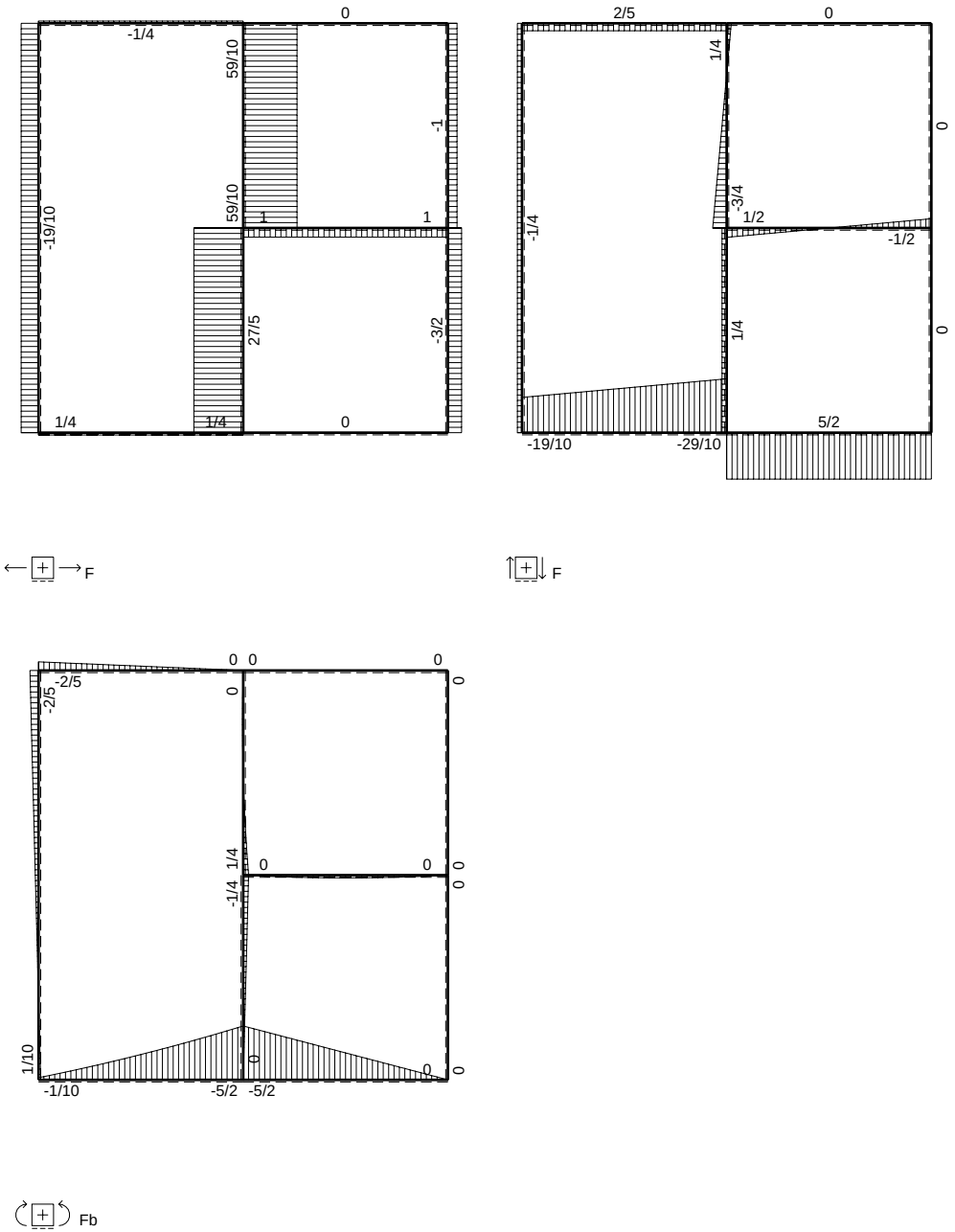
$= [-1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ\ + [\ x\ ]_o^{2b}\ \theta$

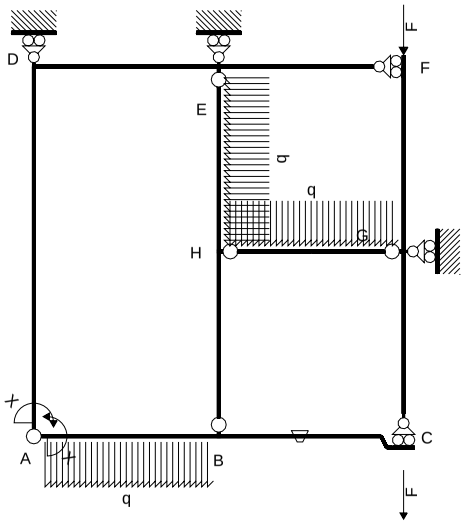
$= (-2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ + (2\ b\ )\ \theta\ = -8/3\ \ Fb^2/EJ$

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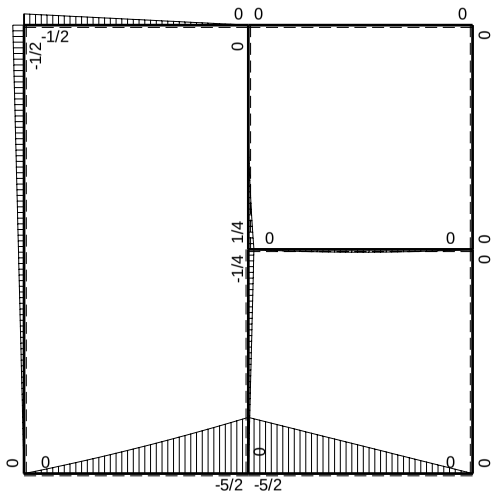
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2/5Fb
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2/5F
D
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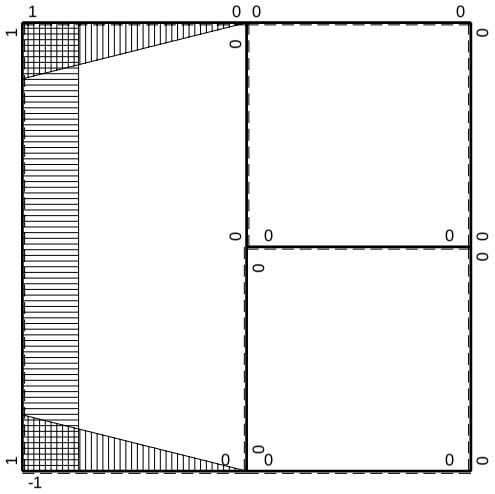

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0
0
-1/4
0 0
0
0
1/10
0
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.208

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$5/2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$-1/4Fx$	0	$-1/4Fx$	0	1	$(-1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb-1/4Fx$	0	$-1/2Fb+1/4Fx$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
BH b	0	$1/4Fx$	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

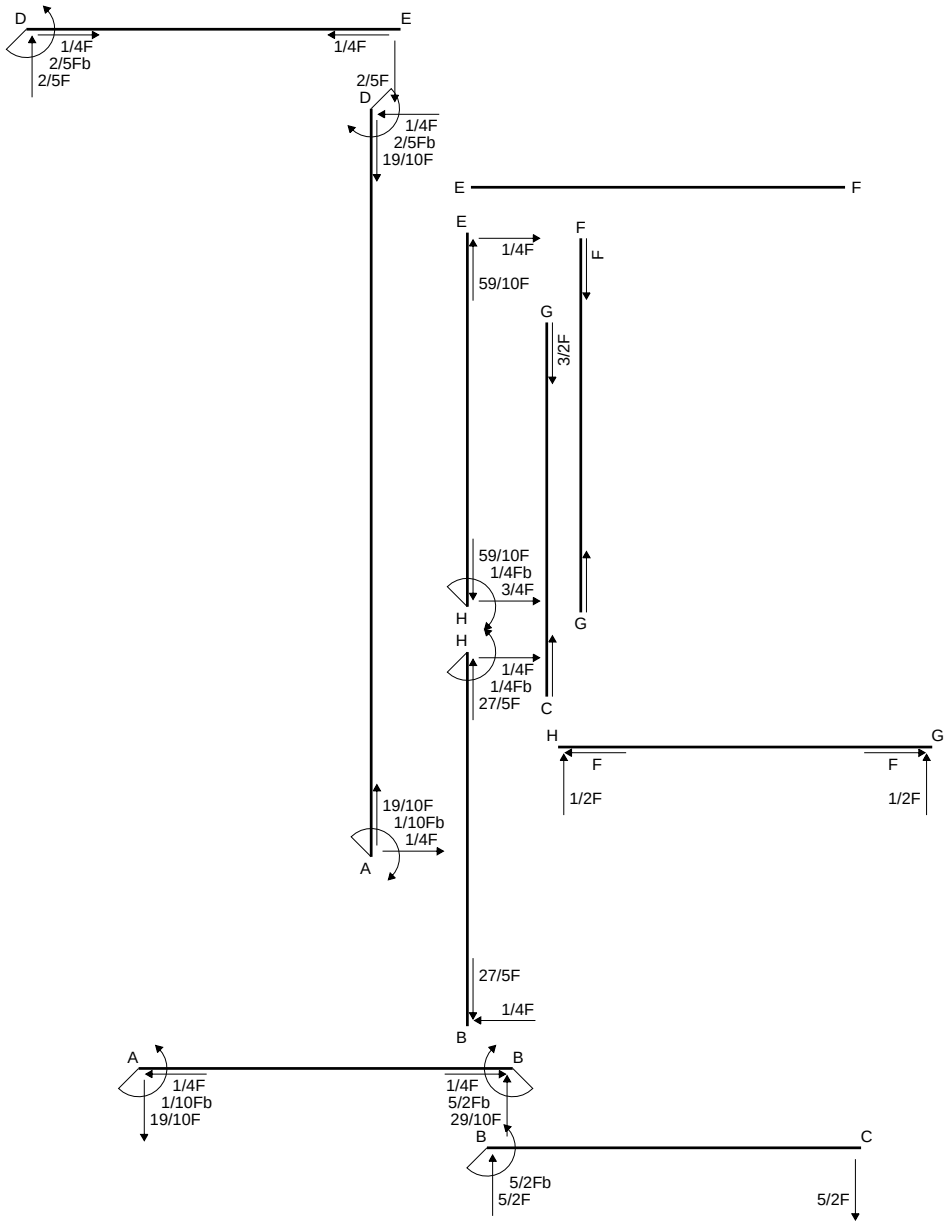
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

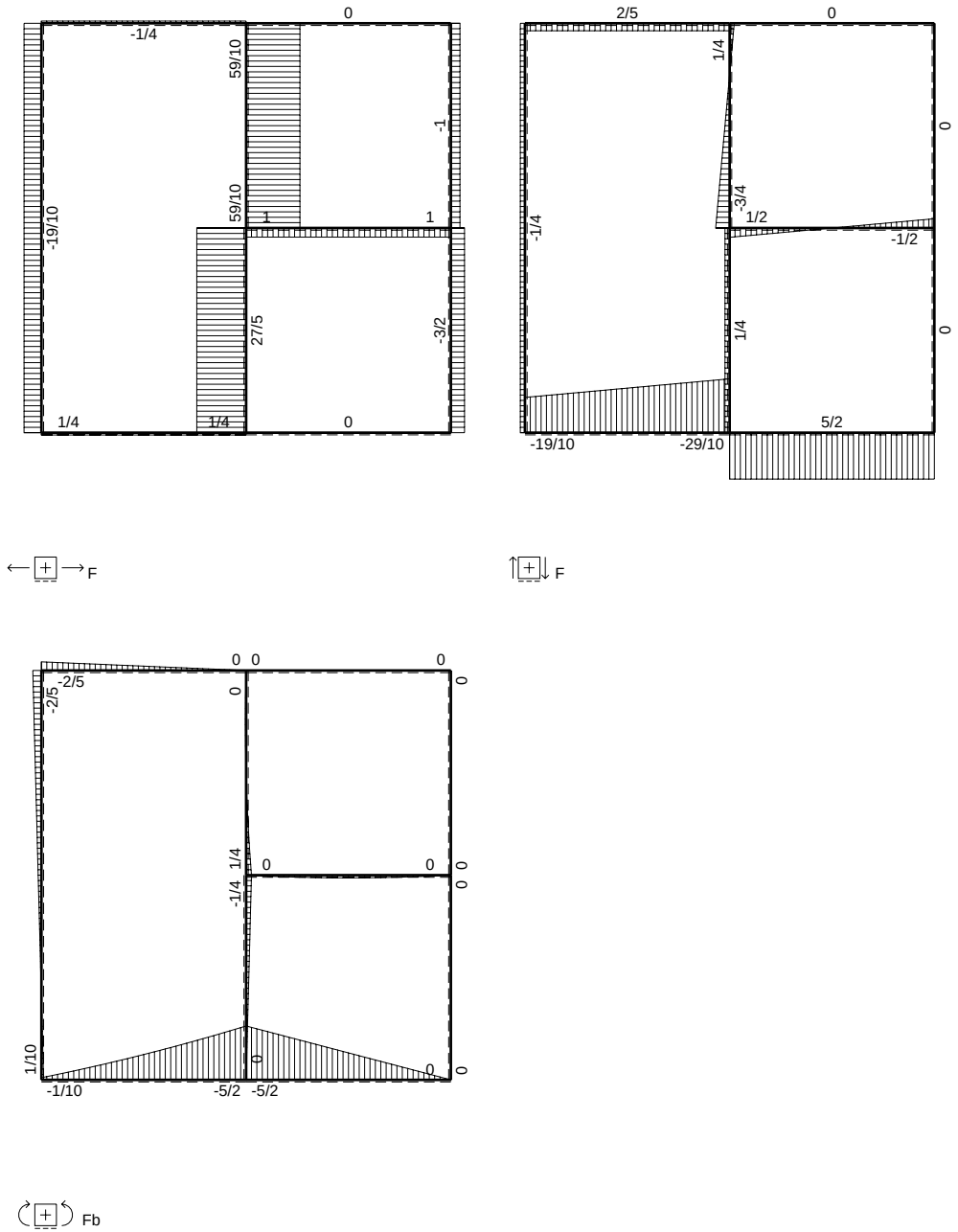
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

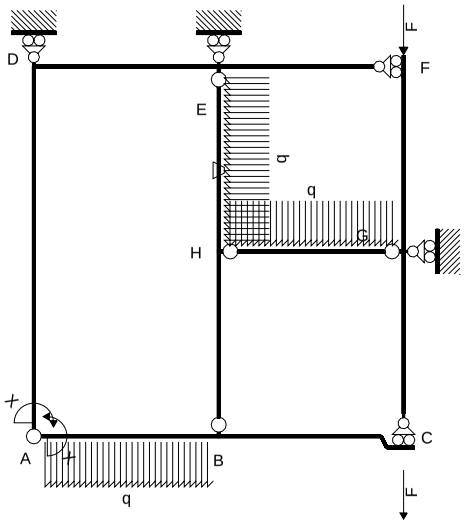
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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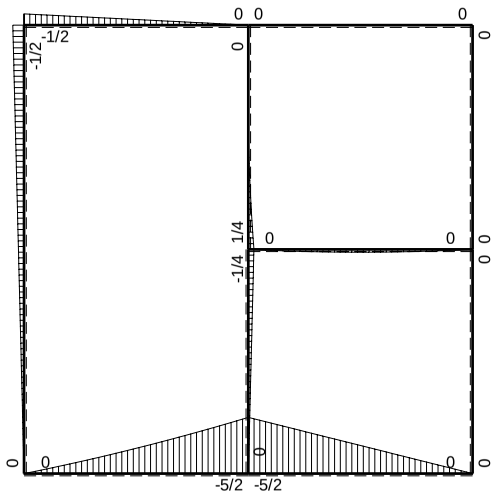
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2/5F
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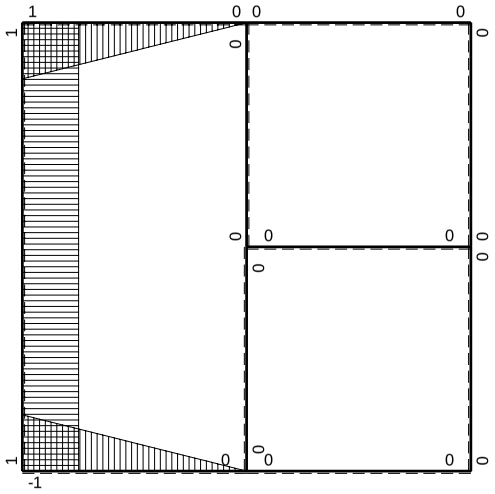

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-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
BH b	0	1/4Fx	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.209

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(2\ x/b -3/2\ x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -1/2\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -1/2\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(5/2\ x/b -3\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[5/4\ x^2/b - x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(5/4\ b - b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-1/4\ x/b\ \right) Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-1/2\ b\ \right) Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 +1/4\ x/b\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/8\ x^2/b\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b +1/2\ b\ \right) Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

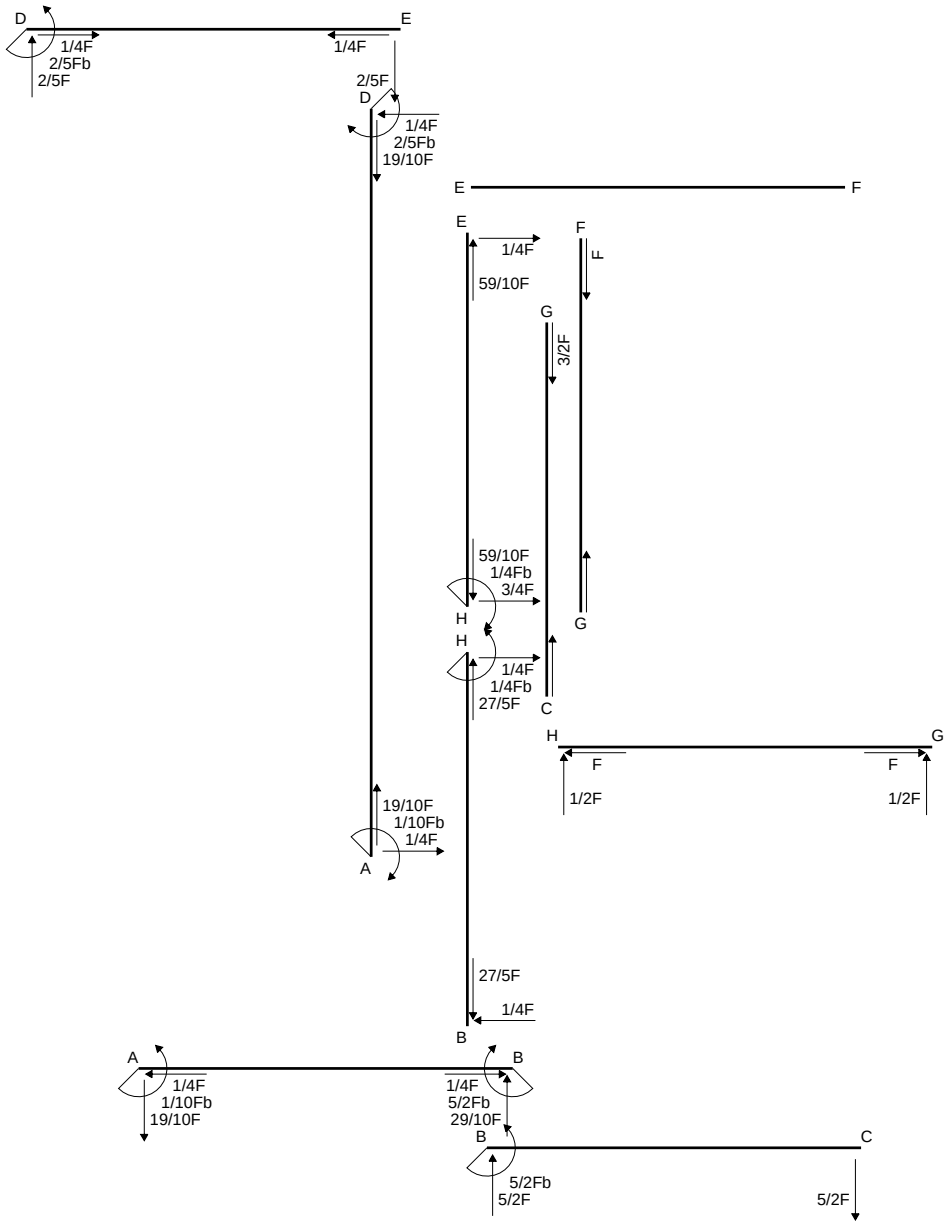
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

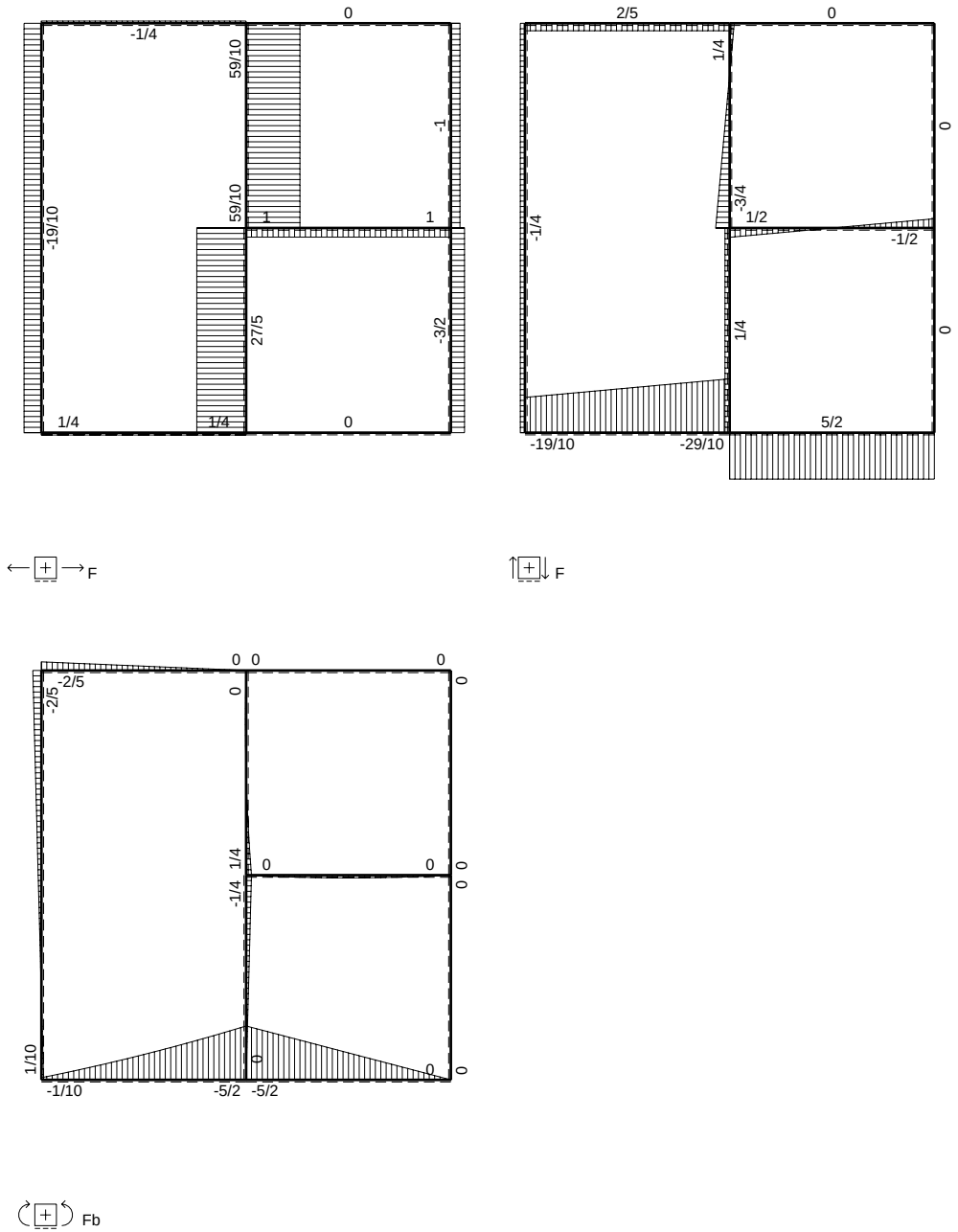
$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

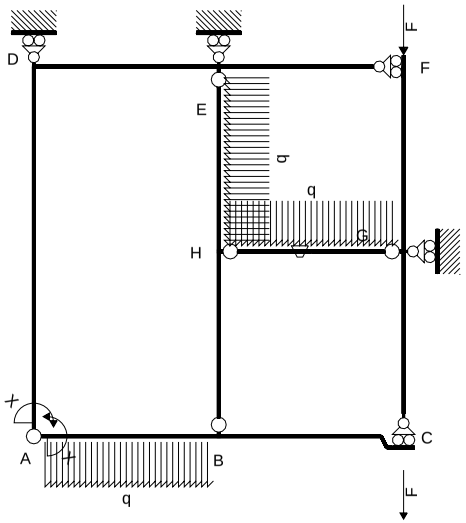
$= \left(-1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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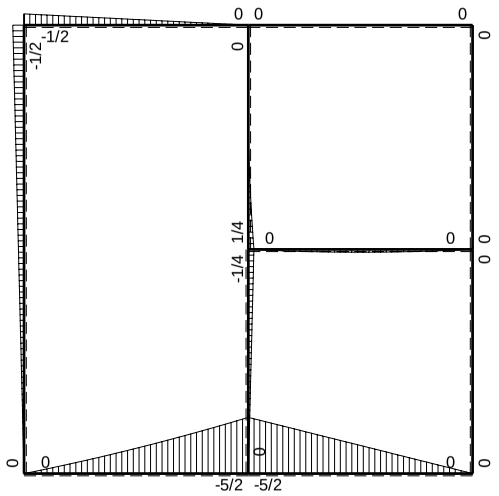
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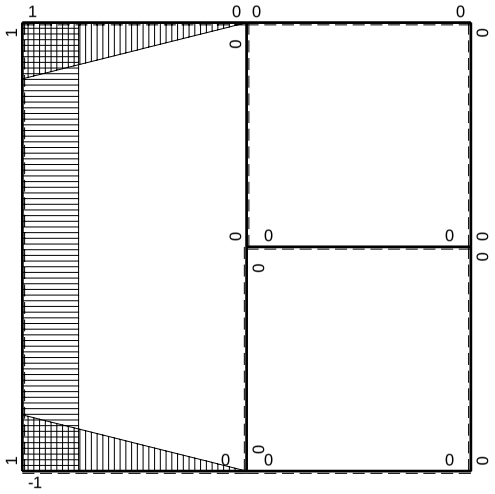

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-2/5
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0
0
1/4
0
0
-1/4
0 0
0 0
1/10
0
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.210

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	Fb/EJ	0	0	0		
HB b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
BH b	0	1/4Fx	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

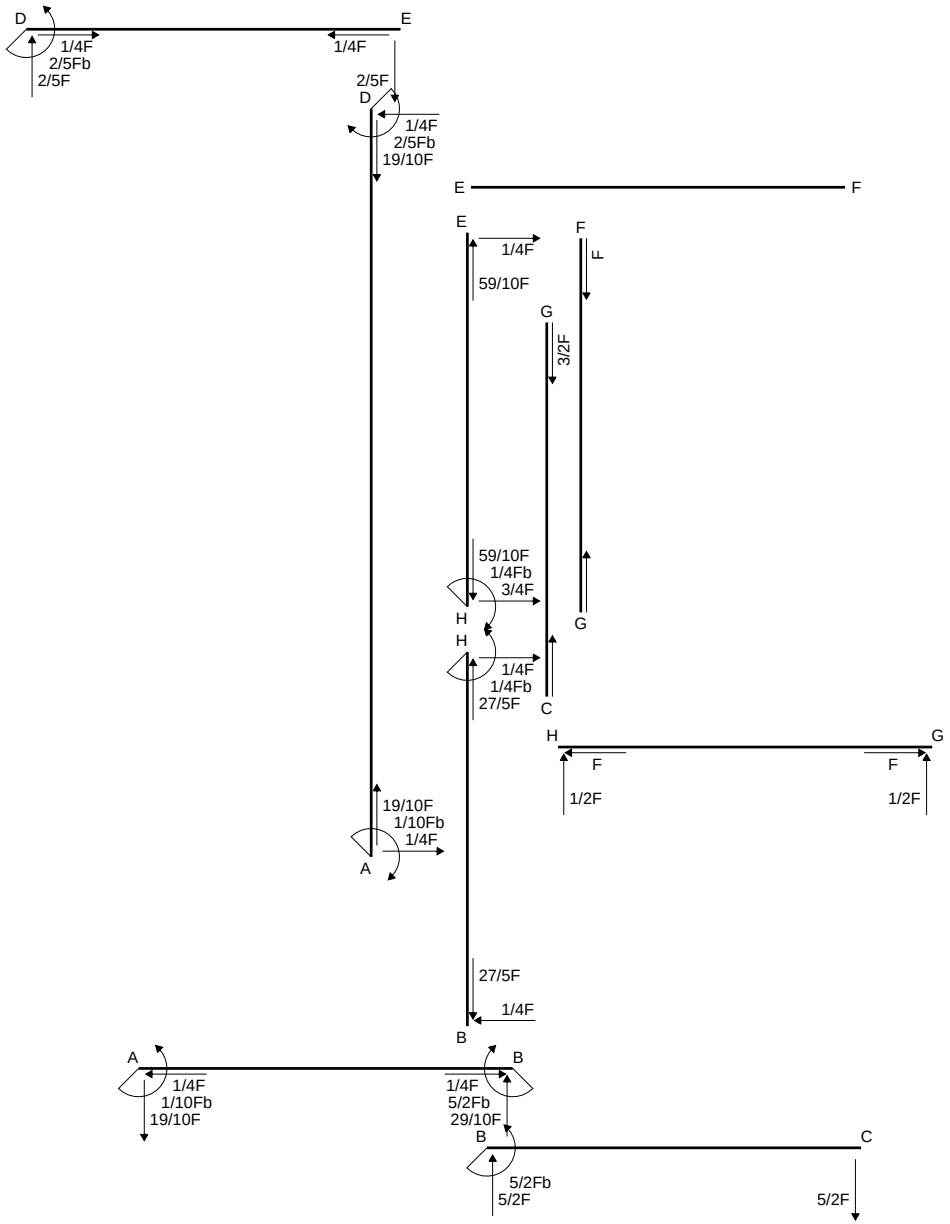
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

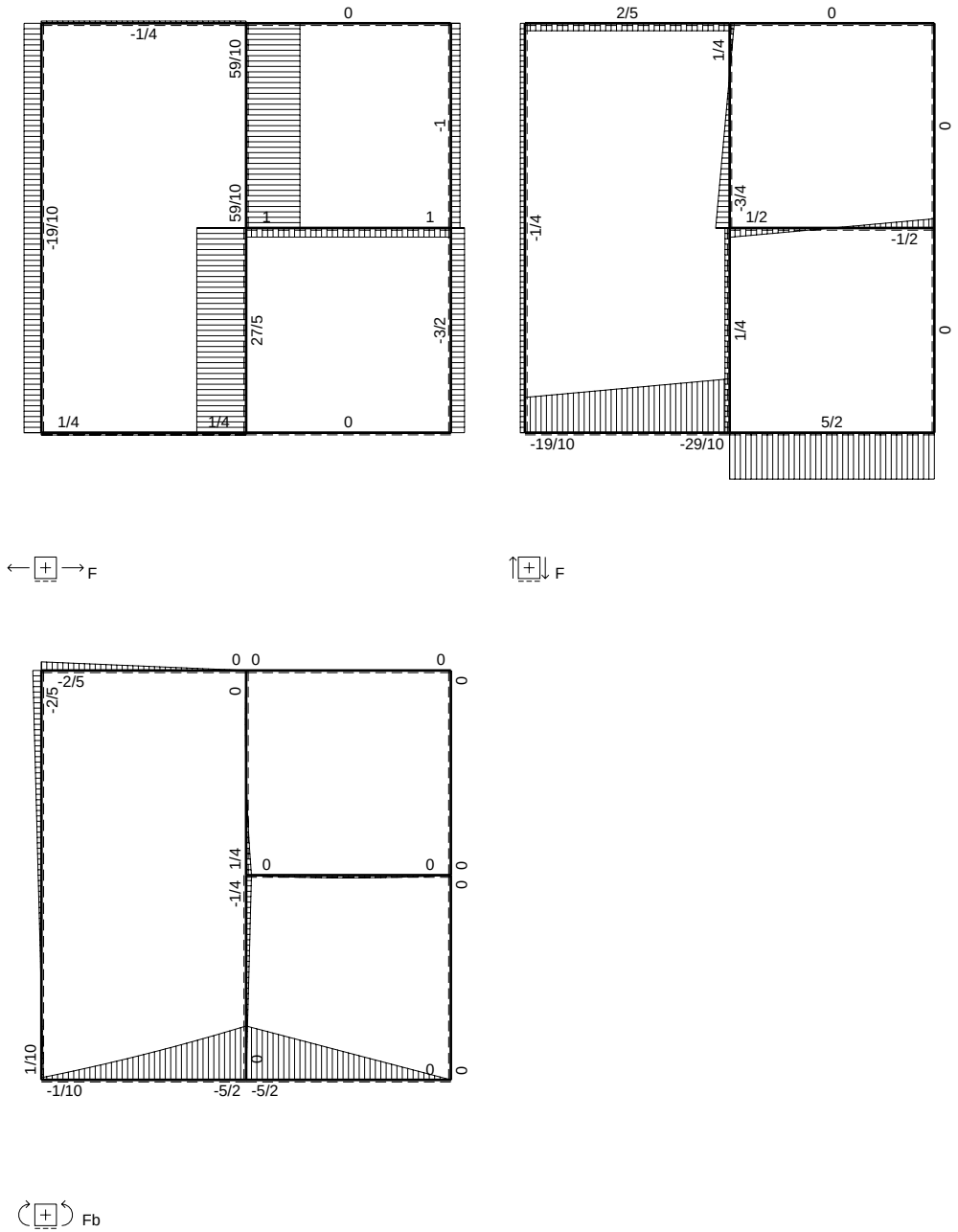
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

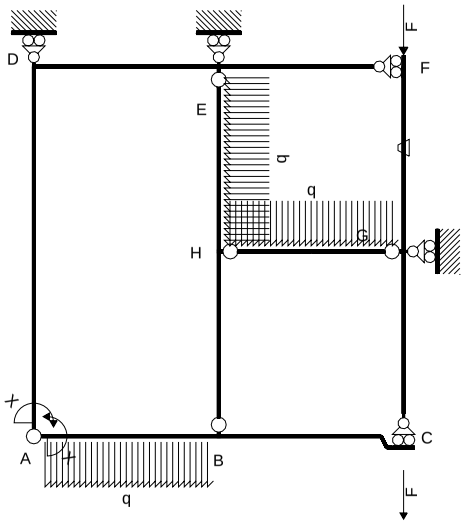
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.210

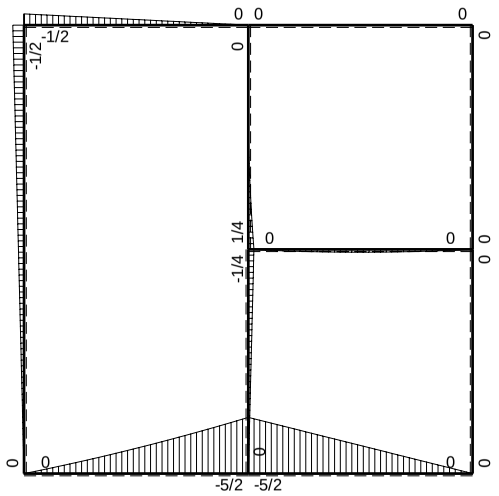
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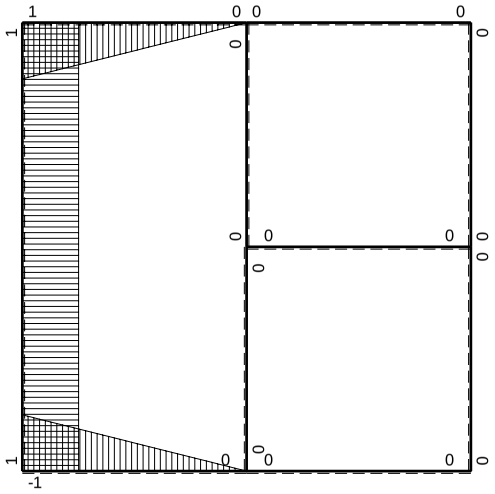

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1/10
0
0
-1/10
-5/2
-5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.211

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
BH b	0	1/4Fx	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.211

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

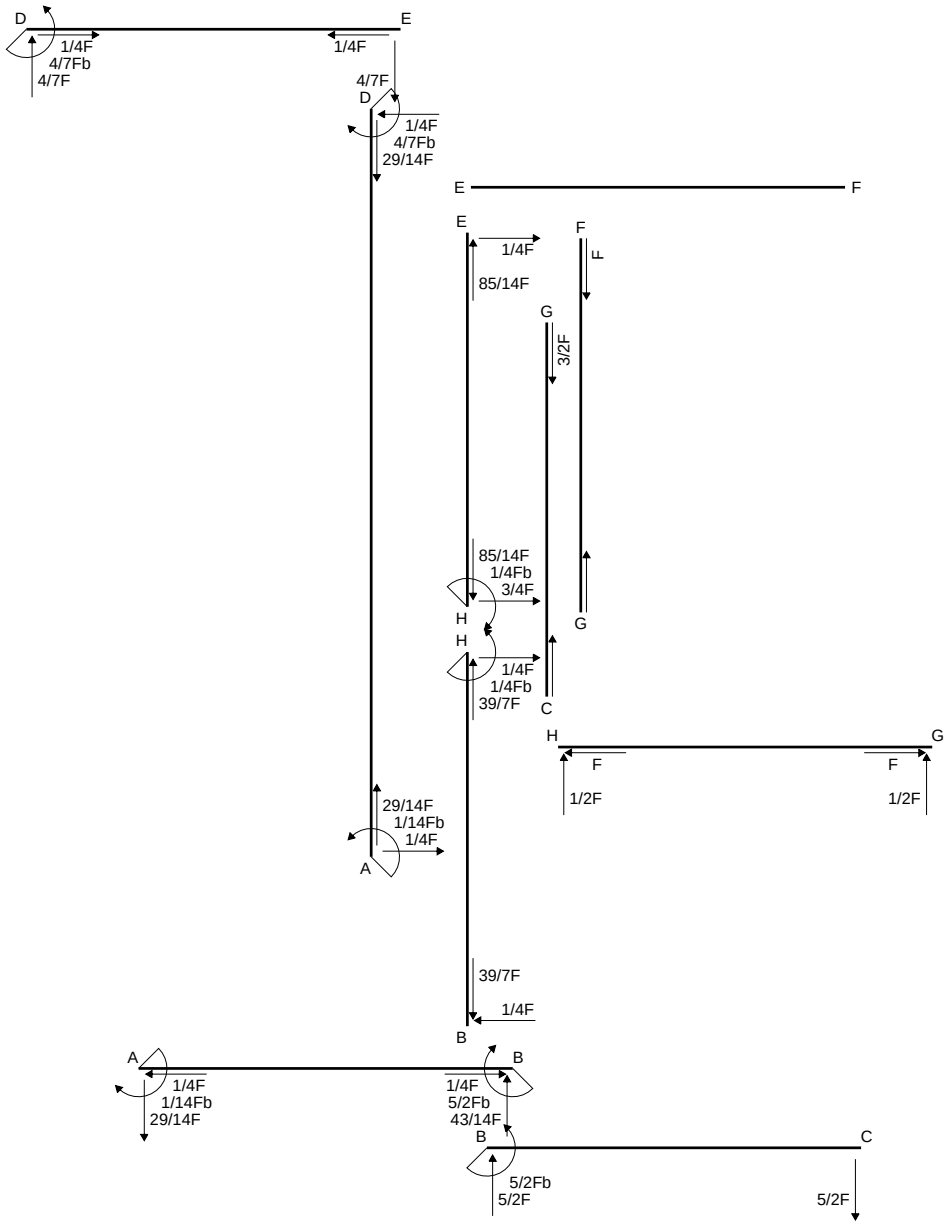
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

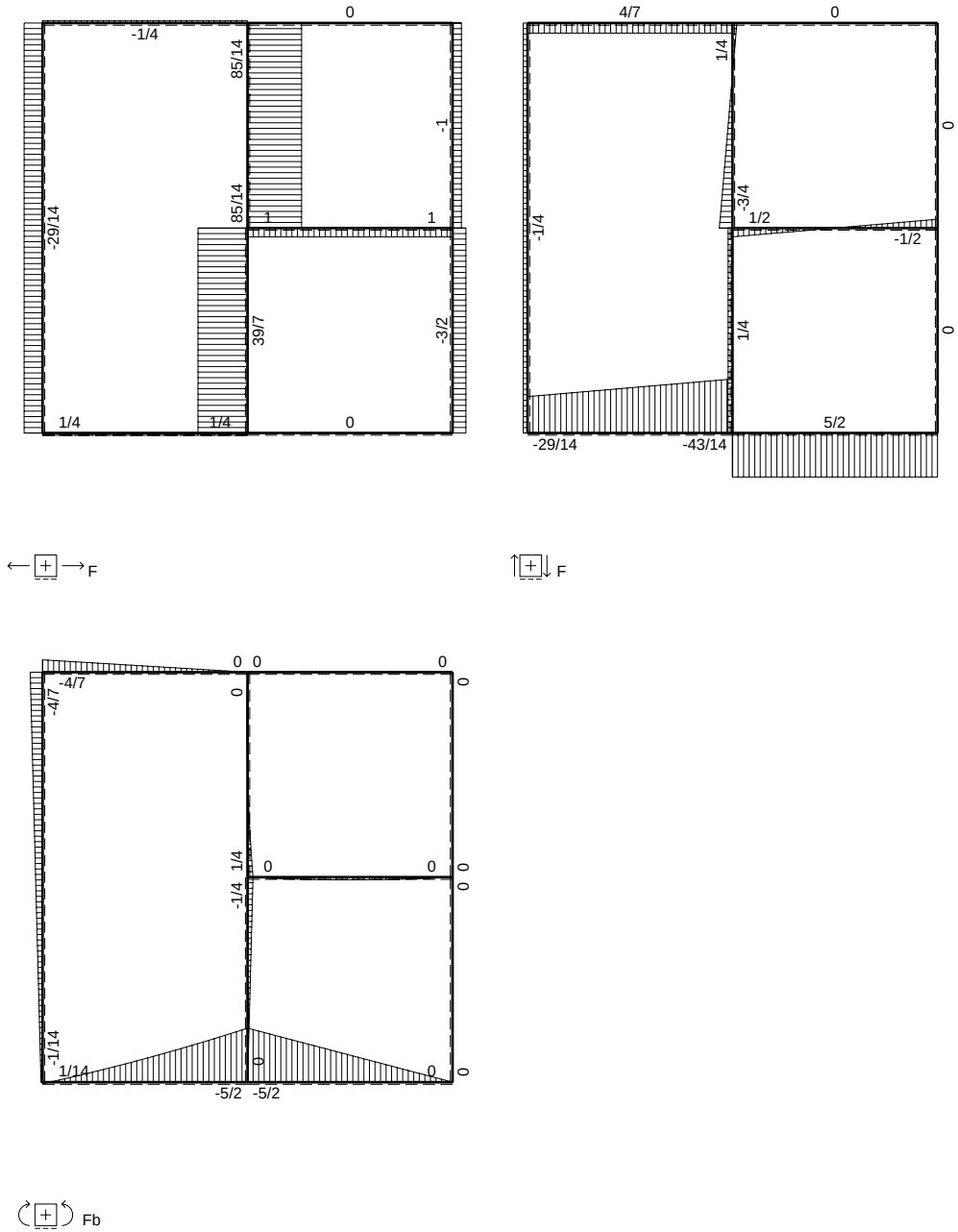
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

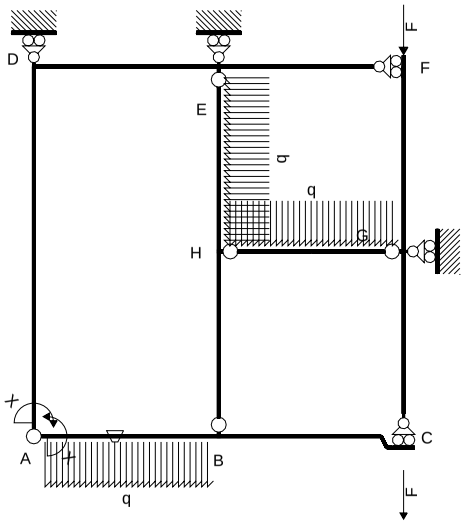
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.211

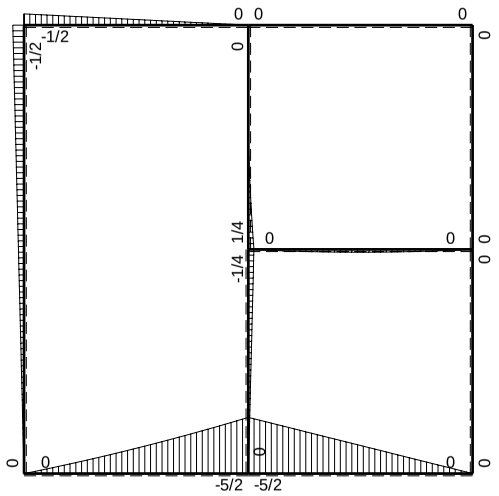
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D
E
1/4F
4/7Fb
4/7F
1/4F
4/7F
D
1/4F
4/7Fb
29/14F
E
F
E
F
1/4F
85/14F
F
G
3/2F
85/14F
1/4Fb
3/4F
H
G
H
1/4F
1/4Fb
39/7F
C
H
G
F
F
1/2F
1/2F
29/14F
1/14Fb
1/4F
A
39/7F
1/4F
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B
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5/2Fb
5/2F
5/2F

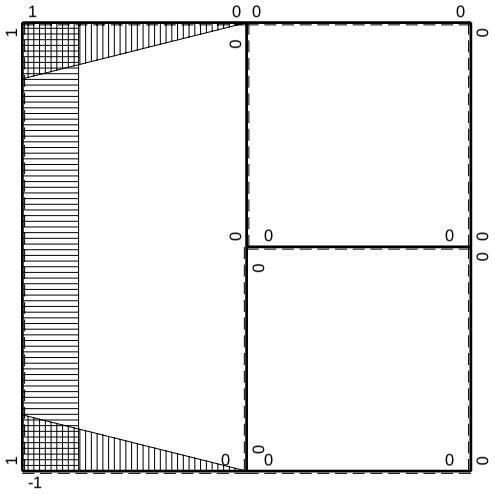

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-43/14
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-4/7
-4/7
0
0
0
0
1/4
-1/4
0
0
0
0
-1/14
1/14
0
0
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.212

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$	$(3/8+1/2)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	$5/2Fx-3Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
BH b	0	1/4Fx	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$5/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						-1/14Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.212

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b)\ \theta\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [\ x - 1/2\ x^2/b\ ]_o^b\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + (\ b - 1/2\ b\ )\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\,x/b)\ \theta\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b)\ \theta\ \ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

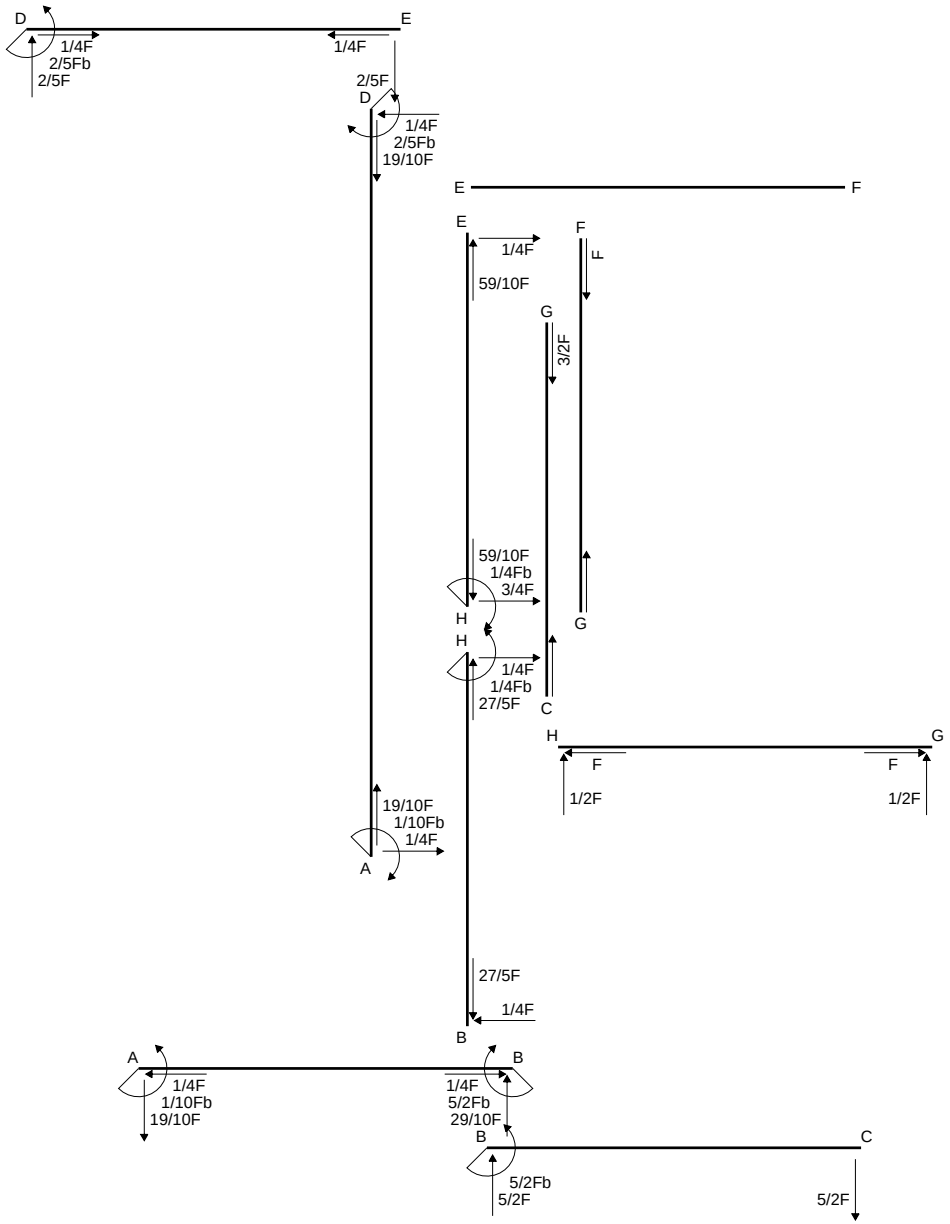
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

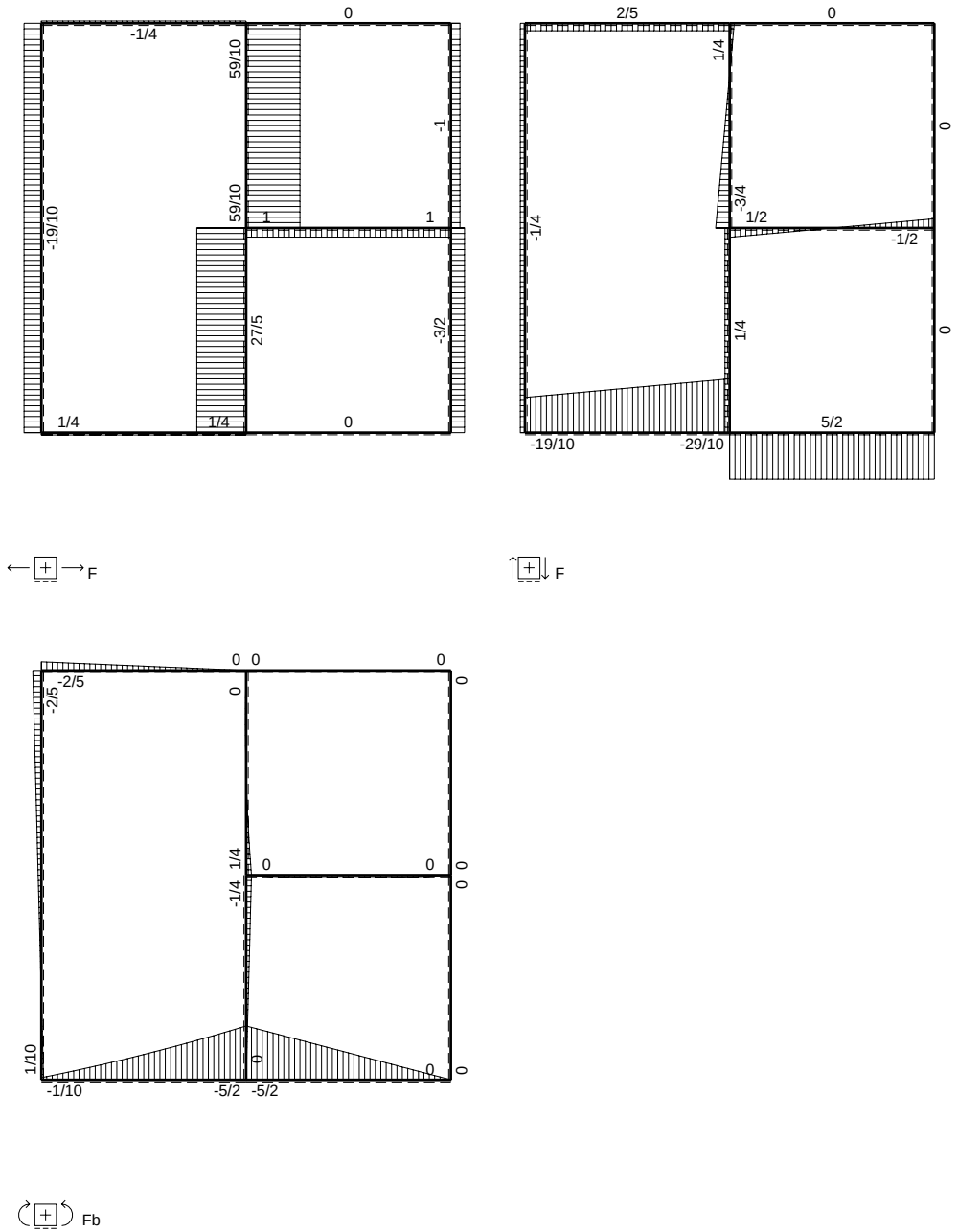
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

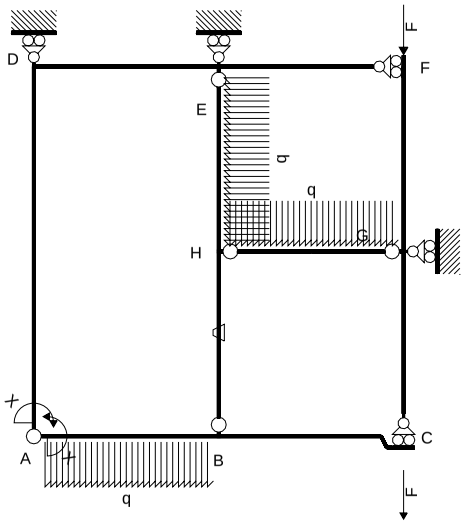
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.212

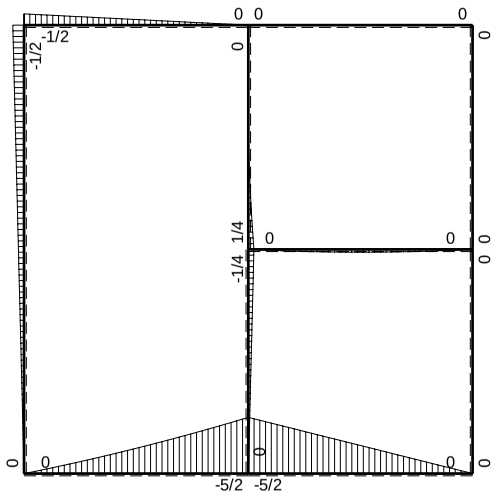
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D
E
1/4F
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2/5F
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2/5F
D
1/4F
2/5Fb
19/10F
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59/10F
F
F
G
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5/2F

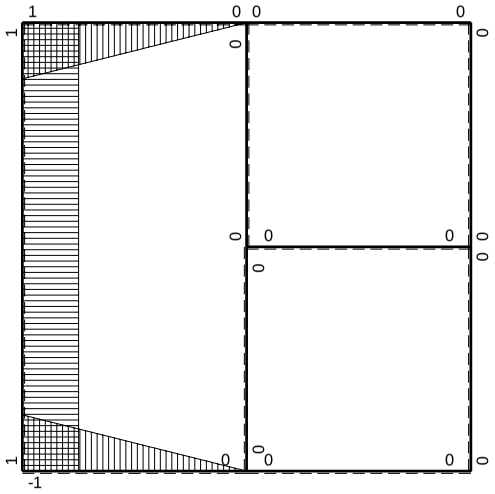

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0
5/2
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-29/10
F
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-2/5
0
0
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1/4
0
0
-1/4
0 0
0
0
1/10
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.213

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-1/4Fb+1/4Fx	-Fb/EJ	0	0	0	0+0	0
BH b	0	1/4Fx	Fb/EJ	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.213

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

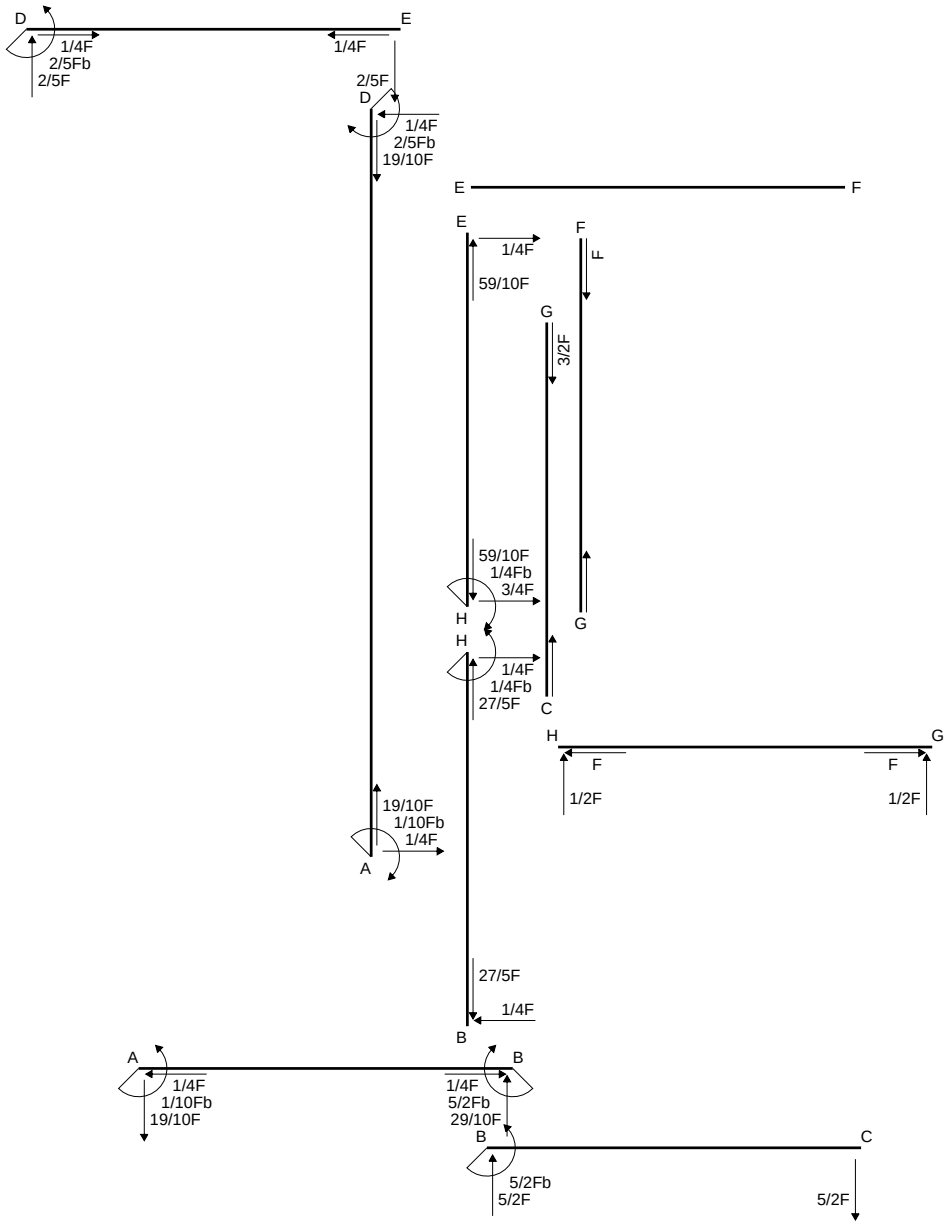
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

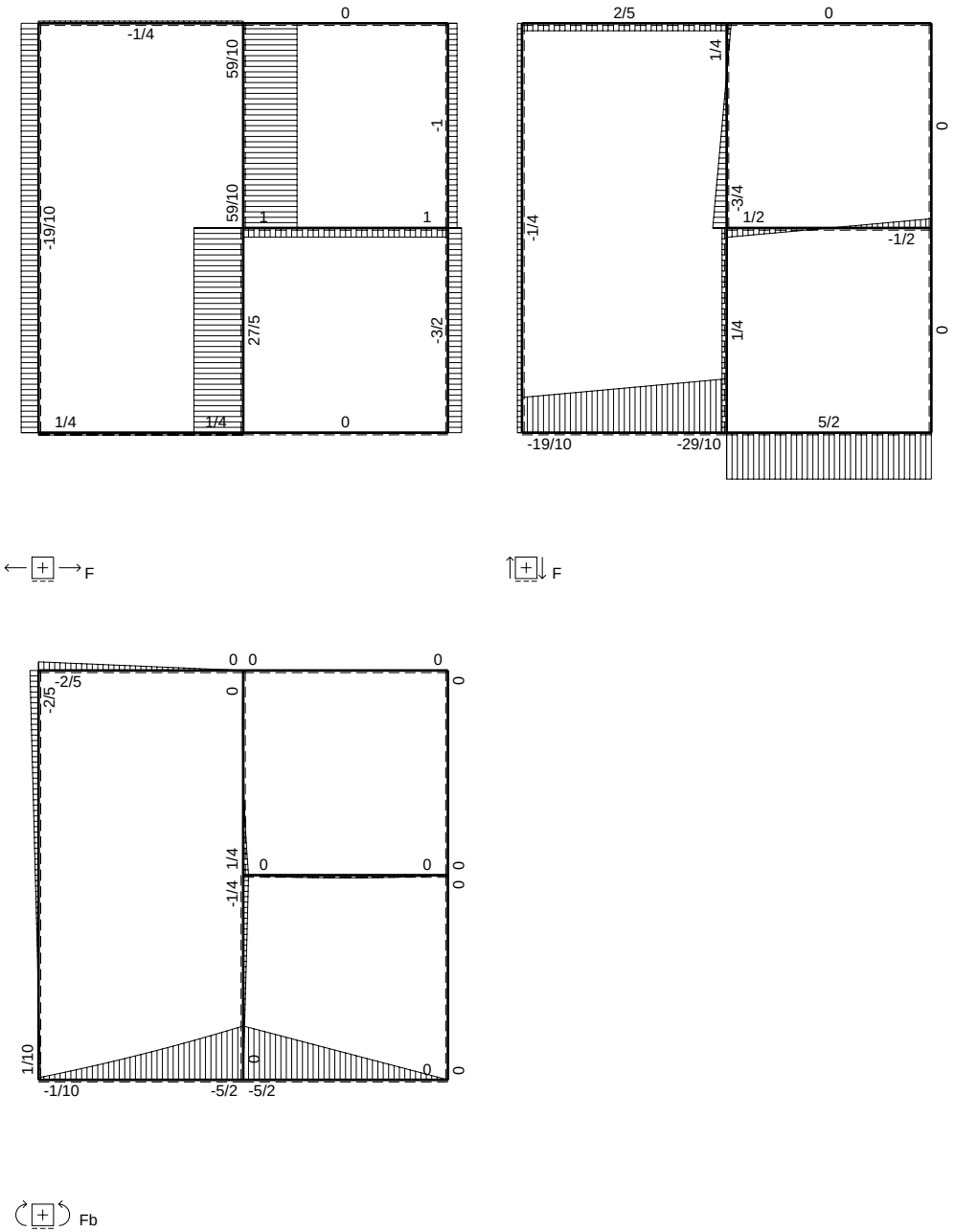
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

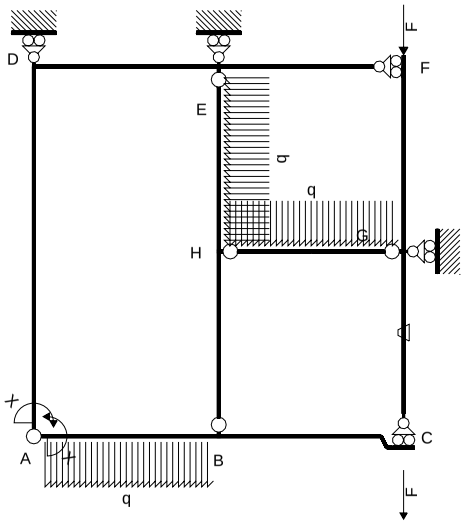
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.213

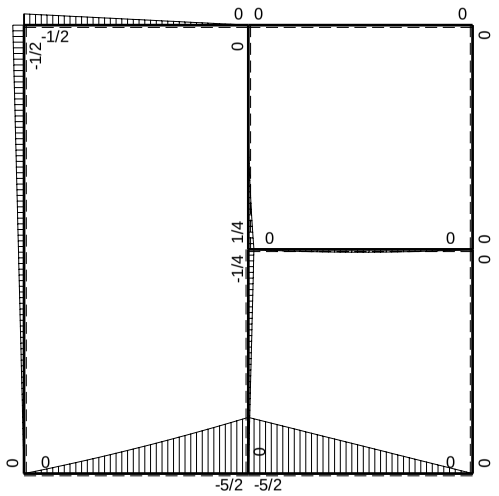
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2/5Fb
2/5F
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2/5F
D
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2/5Fb
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E
F
E
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G
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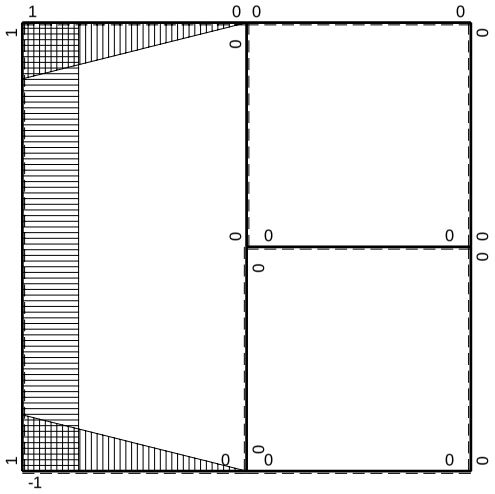

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F
F
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-2/5
0
0
0
1/4
0
0
-1/4
0
0
0
0
1/10
0
0
-1/10
-5/2
-5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.214

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+x/b	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	-5/2Fb+5/2Fx	0	0	0	0	0+0	0
CB b	0	5/2Fx	0	0	0	0		
AD 2b	1	-1/4Fx	0	-1/4Fx	0	1	$(-1/2+0)Fb^2/EJ$	2Xb/EJ
DA 2b	-1	1/2Fb-1/4Fx	0	-1/2Fb+1/4Fx	0	1		
DE b	1-x/b	-1/2Fb+1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	-Fb/EJ	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	-1/4Fb+1/4Fx	0	0	0	0	0+0	0
BH b	0	1/4Fx	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							1/4Xb/EJ
	totali						$-7/24Fb^2/EJ$	35/12Xb/EJ
	iperstatica $X=W_{AB}$						1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.214

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\,]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\,x\,]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\,x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\,b - 1/2\ b - 1/8\ b\,)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\,]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-1/4\,x/b)\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b)\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (-1/2 + 1/4\,x/b)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\,]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b)\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

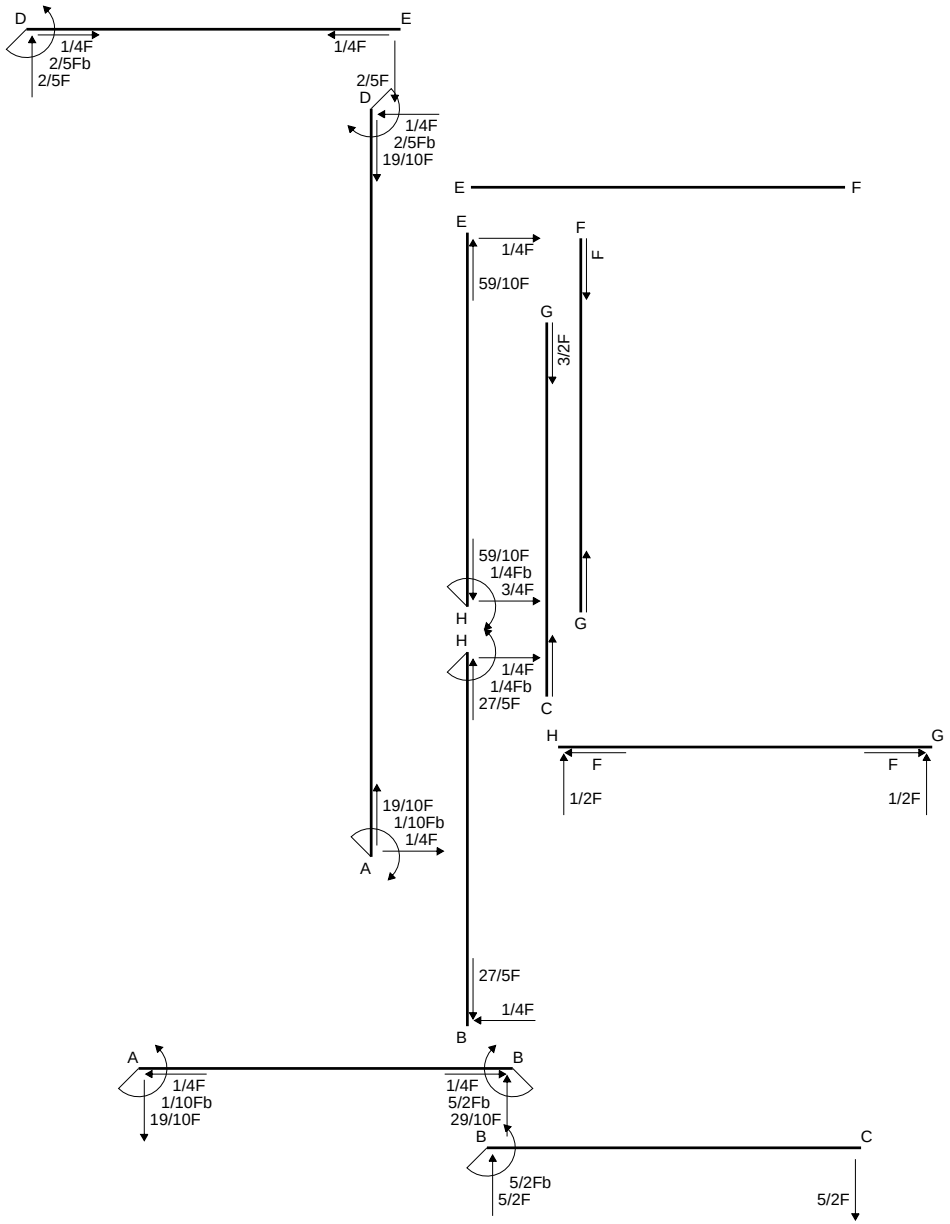
$= (-1/2\ b + 1/2\ b - 1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

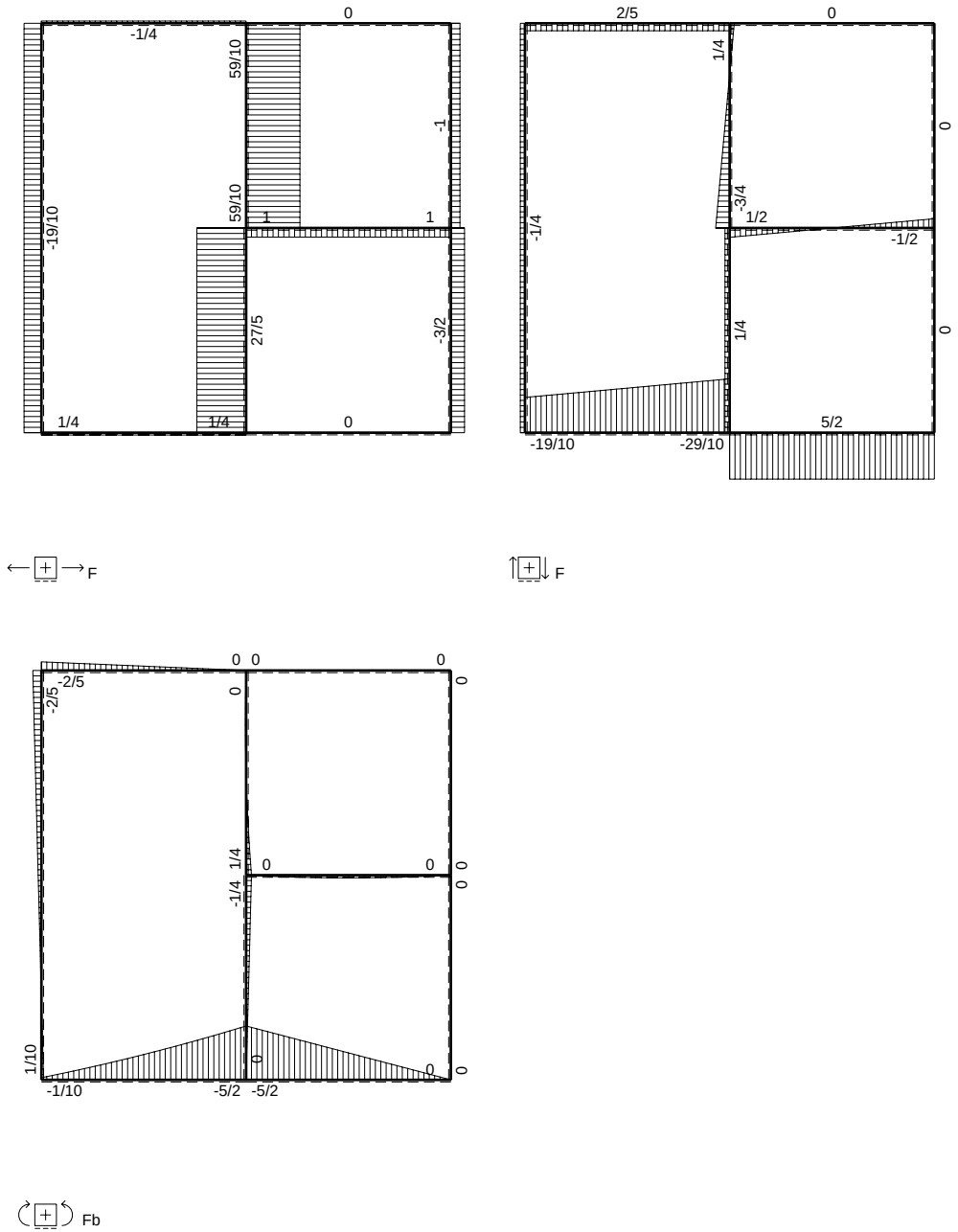
$L_{ED}^{Xo} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

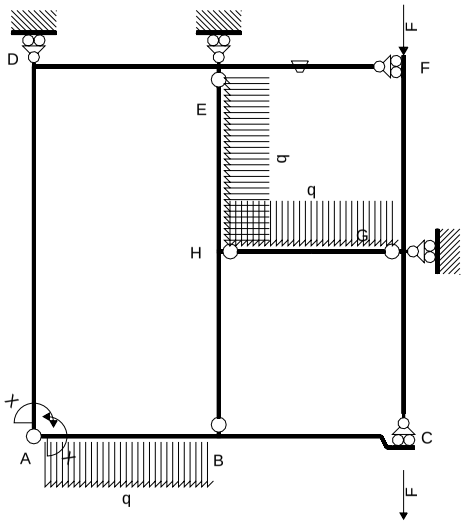
$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.214

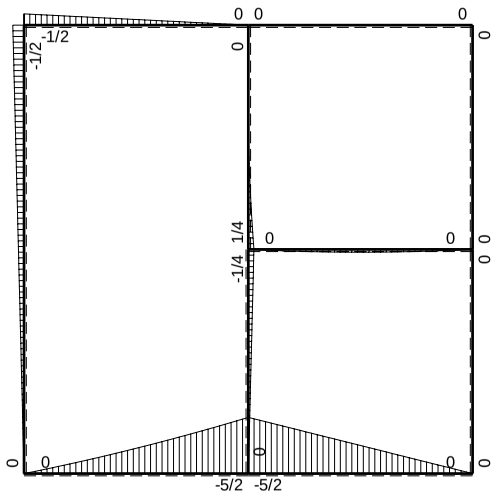
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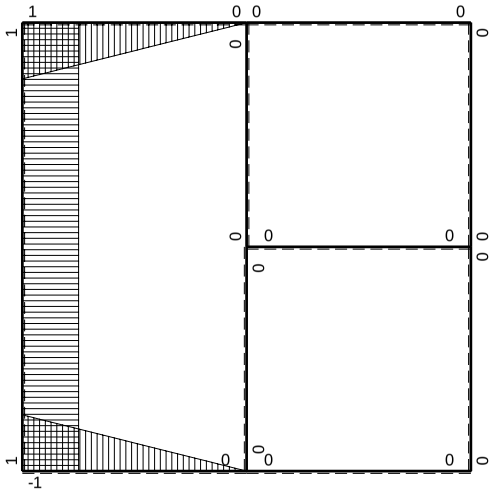

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-2/5
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0
0
-1/4
0 0
0
0
1/10
0
0
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.215

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-1/4Fx$	0	$-1/4Fx$	0	1	$(-1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb-1/4Fx$	0	$-1/2Fb+1/4Fx$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
BH b	0	$1/4Fx$	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.215

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$L_{AB}^{XX} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(2\ x/b -3/2\ x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -1/2\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -1/2\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(5/2\ x/b -3\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[5/4\ x^2/b - x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(5/4\ b - b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} \left(-1/4\ x/b\ \right) Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-1/2\ b\ \right) Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} \left(-1/2 +1/4\ x/b\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/8\ x^2/b\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b +1/2\ b\ \right) Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

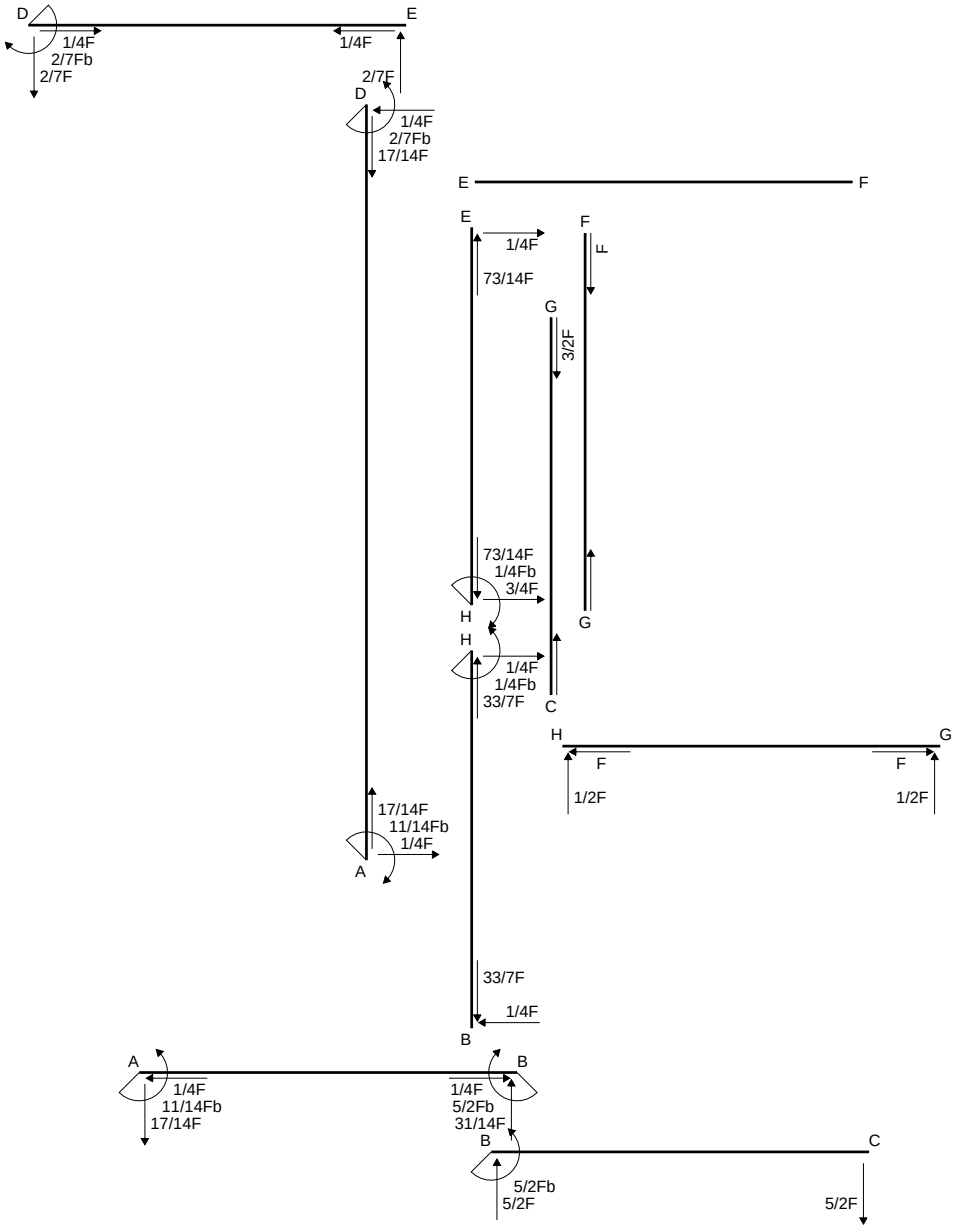
$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

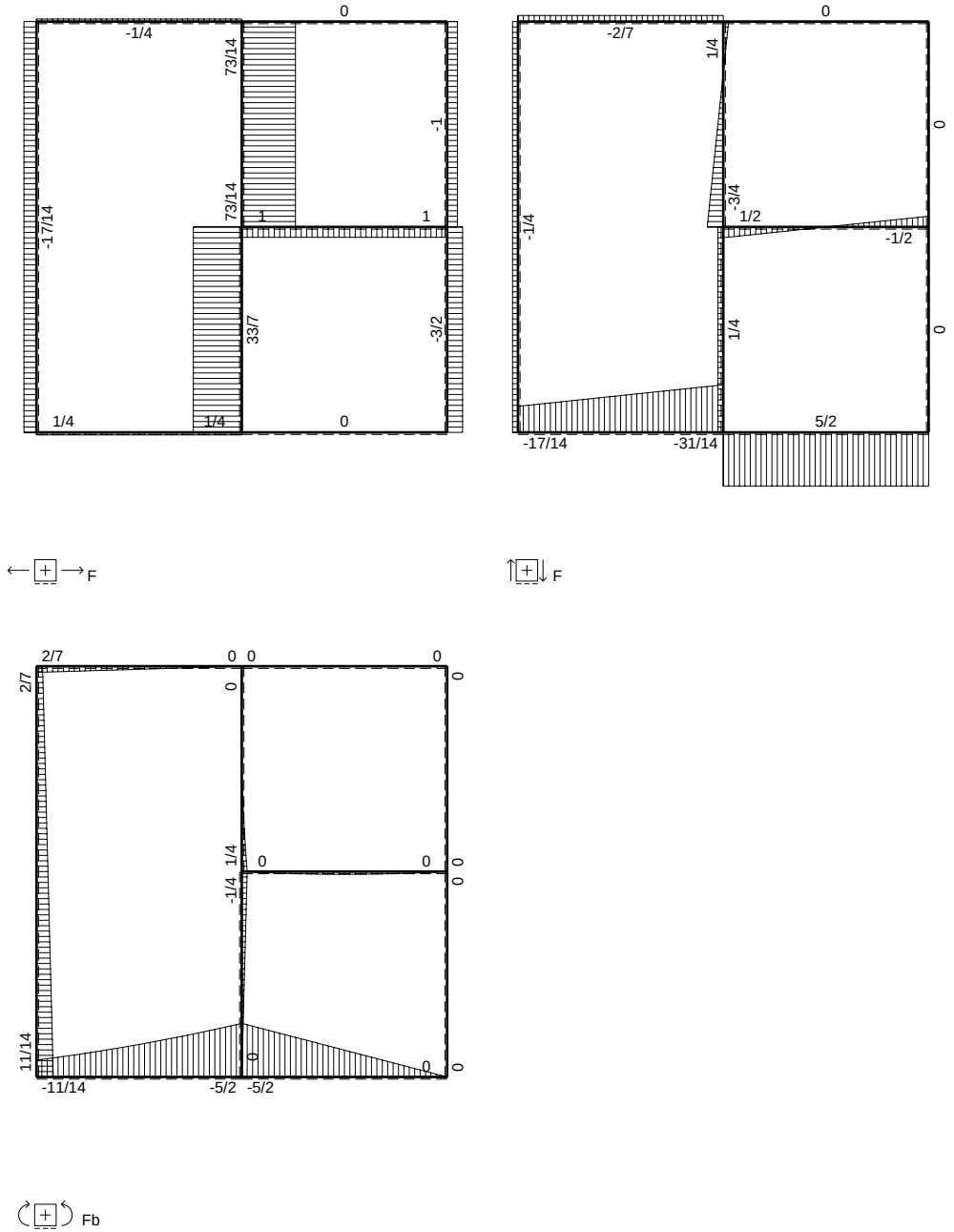
$L_{ED}^{Xo} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

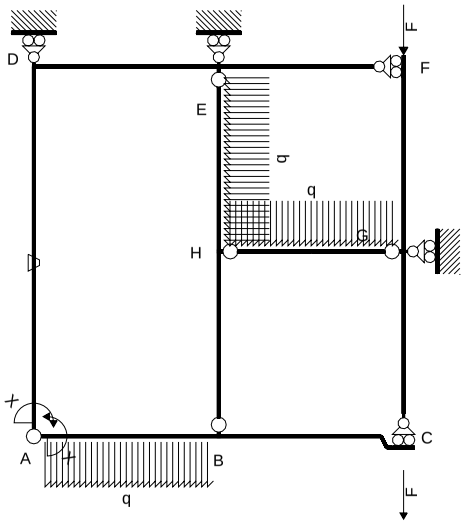
$= \left(-1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.215

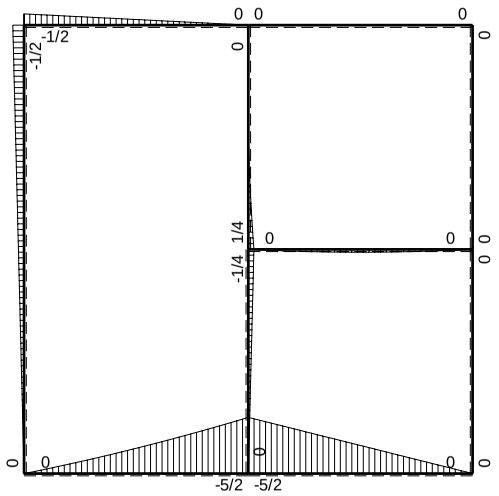
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D
E
1/4F
2/7Fb
2/7F
1/4F
2/7F
D
1/4F
2/7Fb
17/14F
E
F
E
F
1/4F
73/14F
F
G
3/2F
73/14F
1/4Fb
3/4F
H
G
H
1/4F
1/4Fb
33/7F
C
H
G
F
F
1/2F
1/2F
17/14F
11/14Fb
1/4F
A
33/7F
1/4F
B
A
B
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C
5/2Fb
5/2F
5/2F

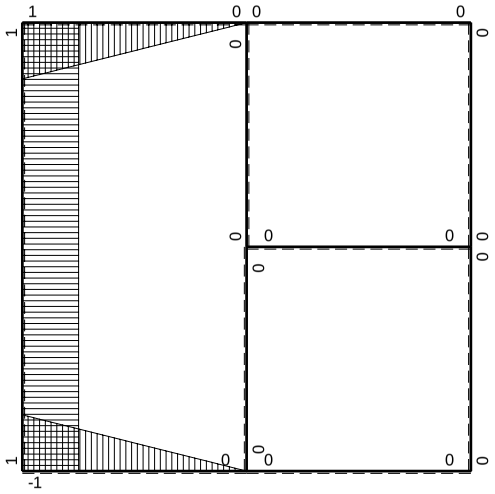

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73/14
-1
73/14
-17/14
1
1
33/7
-3/2
1/4
1/4
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F
0
-2/7
1/4
-3/4
1/2
-1/2
-1/4
1/4
0
0
5/2
-17/14
-31/14
F
2/7
0 0
0
2/7
0
0
0
1/4
0
0
-1/4
0 0
0
11/14
0
0
-11/14
-5/2
-5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.216

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$-1/4Fx$	$-Fb/EJ$	$-1/4Fx$	$-Fb/EJ$	1	$(-1/2-2)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb-1/4Fx$	Fb/EJ	$-1/2Fb+1/4Fx$	$-Fb/EJ$	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
BH b	0	$1/4Fx$	0	0	0	0		
HE b	0	$1/4Fb-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$1/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-55/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$11/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.216

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-1/4\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^{2b} (-1\ )\ \theta\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ\ + [-\ x\ ]_o^{2b}\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-2\ b\ )\ \theta\ = -5/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (-1/2 + 1/4\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^{2b} (1\ )\ \theta\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ\ + [\ x\ ]_o^{2b}\ \theta$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ + (2\ b\ )\ \theta\ = -5/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

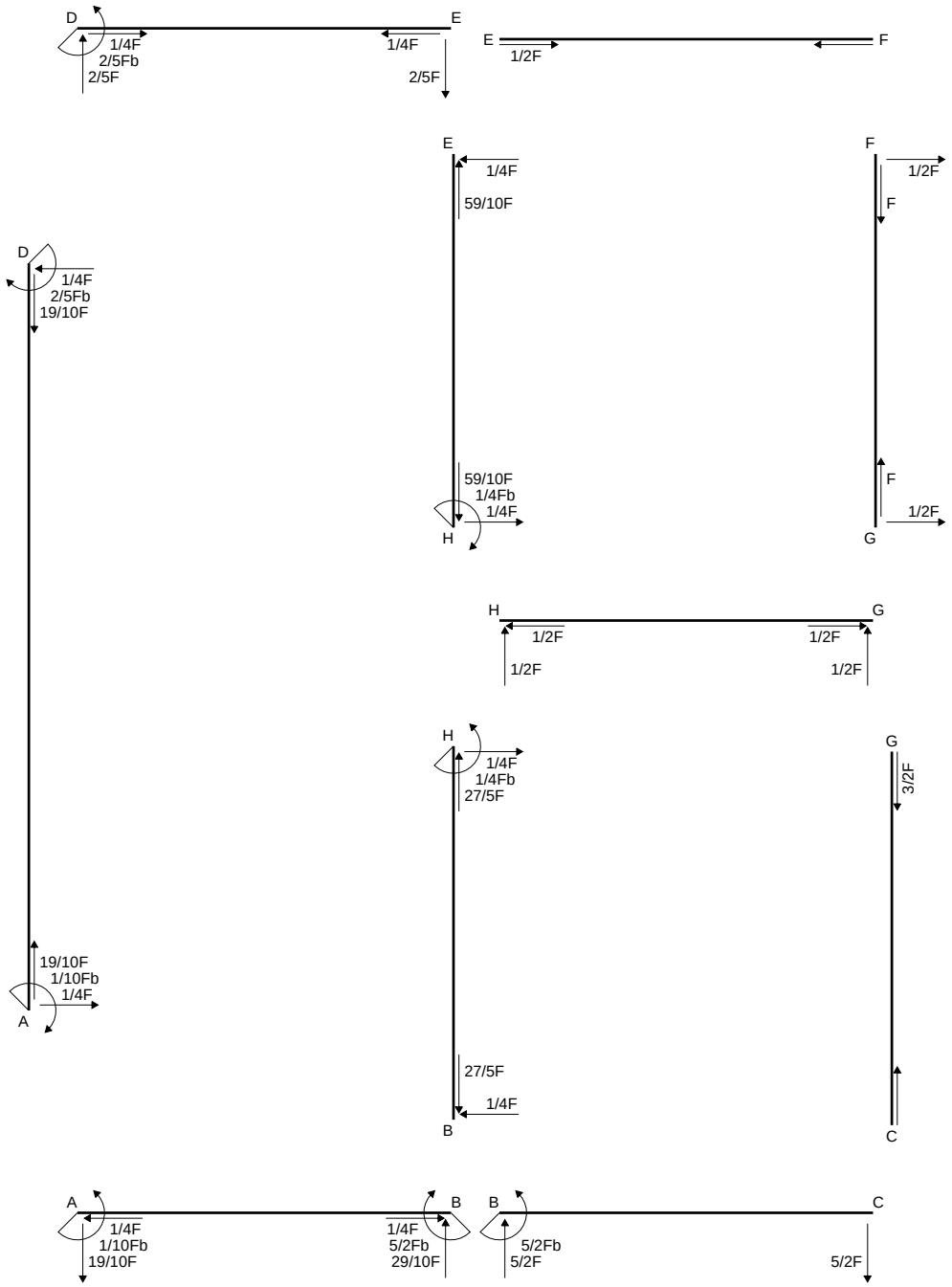
$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

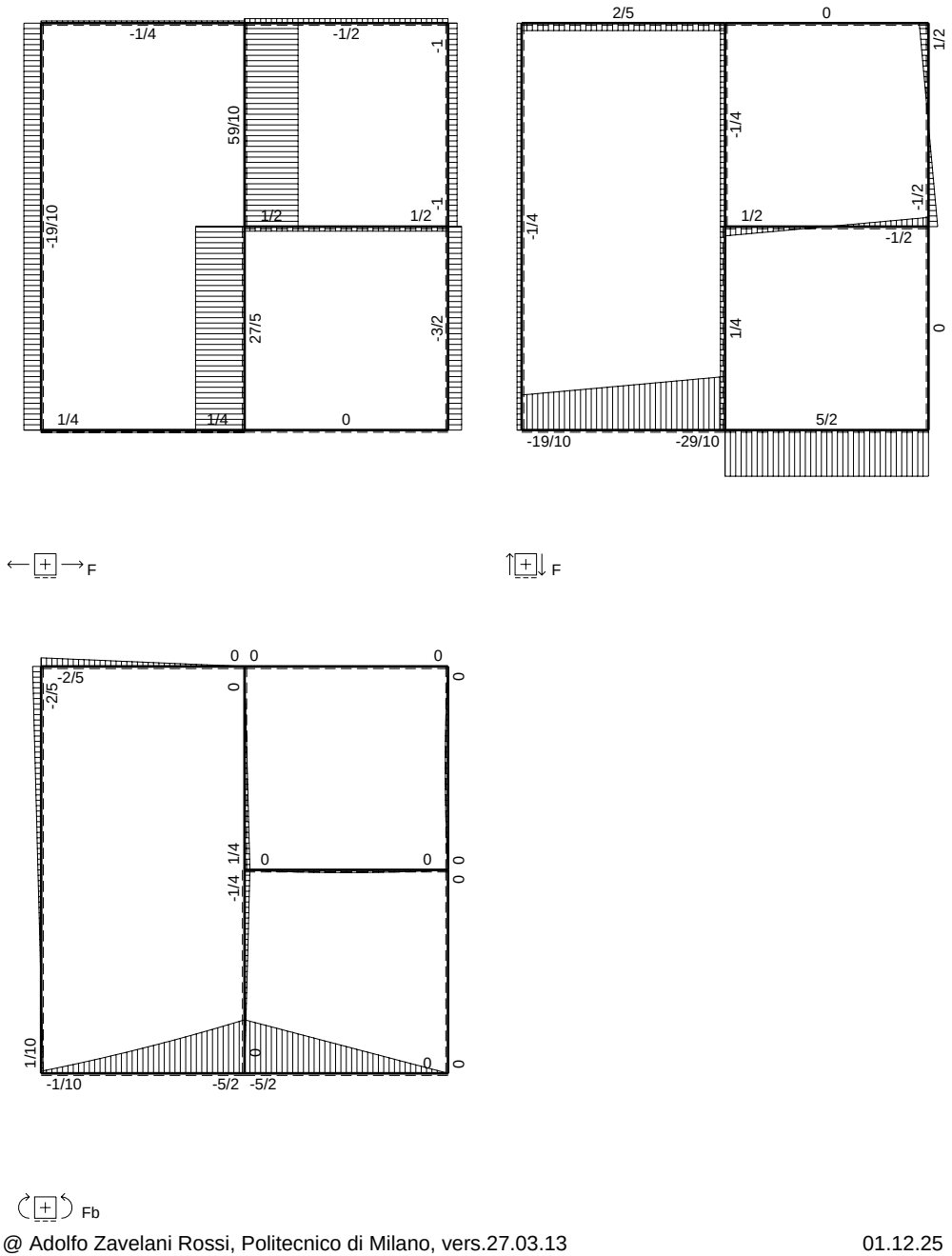
$L^{Xo}_{ED} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

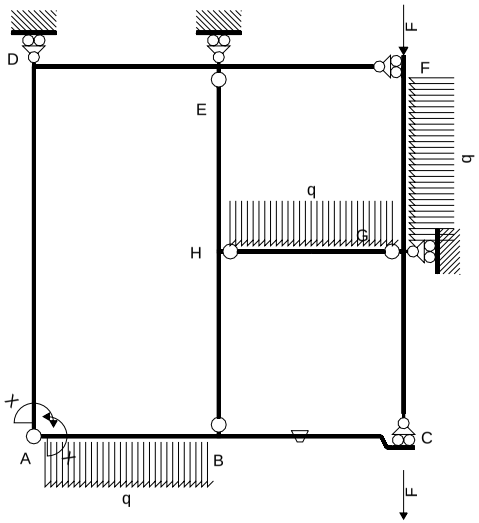
$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.216

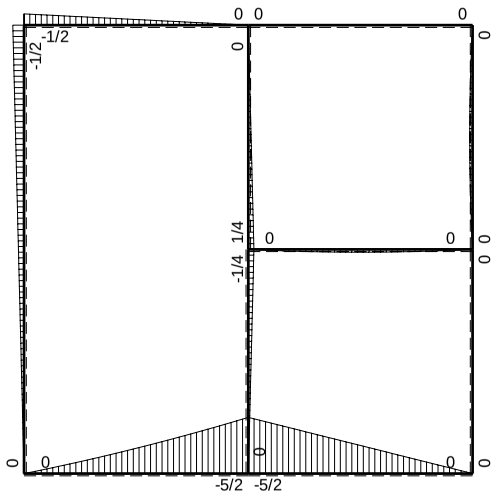
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D
E
1/4F
2/5Fb
2/5F
1/4F
2/5F
E
F
1/2F
E
1/4F
59/10F
F
1/2F
F
D
1/4F
2/5Fb
19/10F
59/10F
1/4Fb
1/4F
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1/2F
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1/2F
1/2F
H
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1/4Fb
27/5F
3/2F
19/10F
1/10Fb
1/4F
A
27/5F
1/4F
B
C
A
B
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5/2Fb
5/2F
5/2F

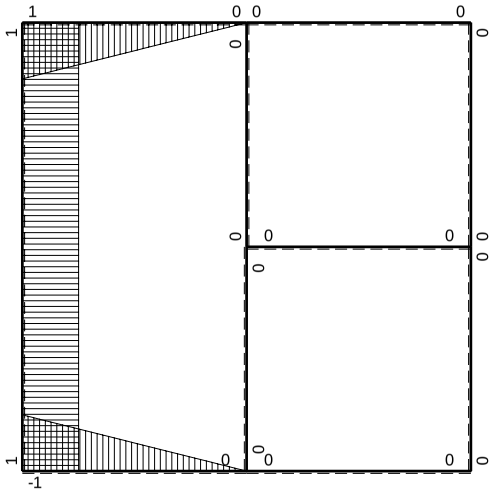

-1/4
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59/10
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F
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-1/4
-1/2
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-29/10
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-2/5
0
0
0
1/4
-1/4
0
0
0
0
0
0
1/10
-1/10
-5/2 -5/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.217

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
CB b	0	$5/2Fx$	Fb/EJ	0	0	0		
AD 2b	1	$-1/4Fx$	0	$-1/4Fx$	0	1	$(-1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb-1/4Fx$	0	$-1/2Fb+1/4Fx$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
BH b	0	$1/4Fx$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.217

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$L_{AB}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L_{AD}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DA}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^{2b} (-1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{DA}^{Xo} = \int_o^{2b} (-1/2 + 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-\ b + 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ03.xxxx.217

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