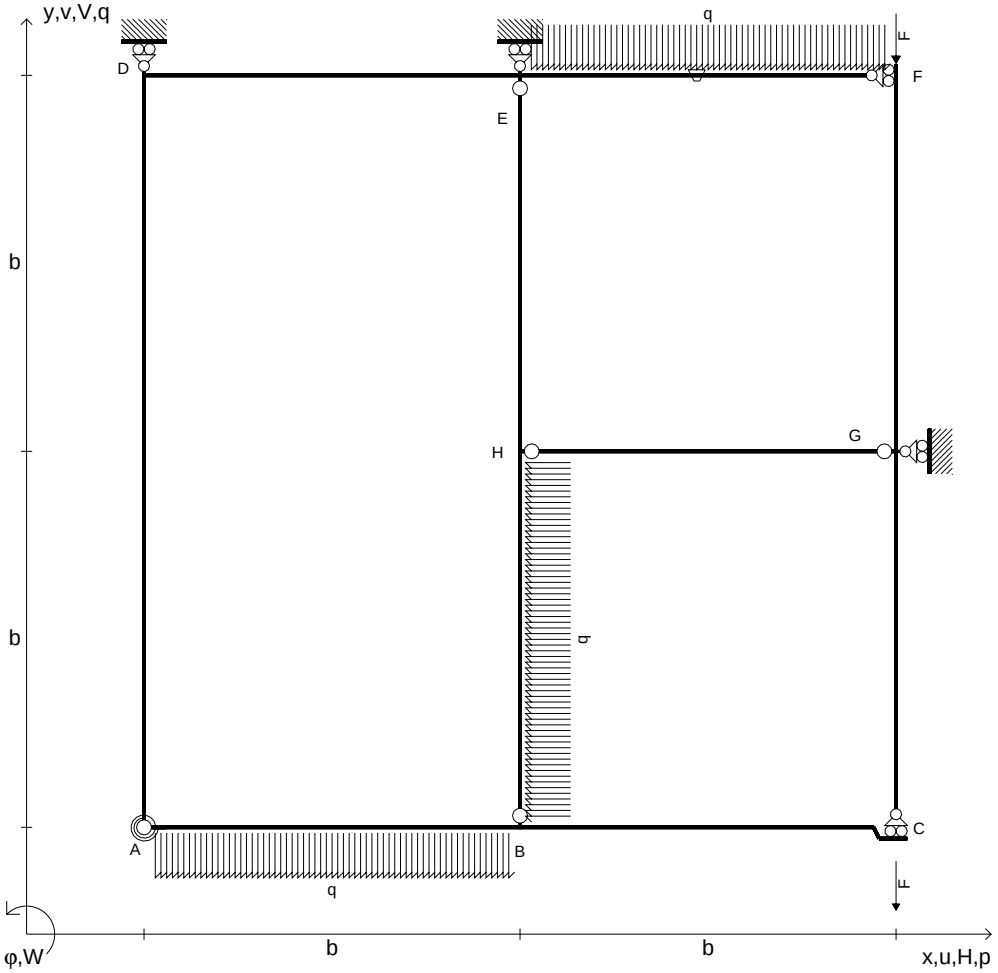

$V_{FG} = -F$
$V_{CB} = -F$
$q_{AB} = -q = -F/b$
$p_{HB} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$

$k_{AB} = 4EJ/b$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$

$EJ_{FG} = EJ$
$EJ_{GC} = EJ$
$EJ_{HG} = EJ$
$EJ_{HB} = EJ$
$EJ_{HE} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta EF positiva se convessa a destra con inizio E.

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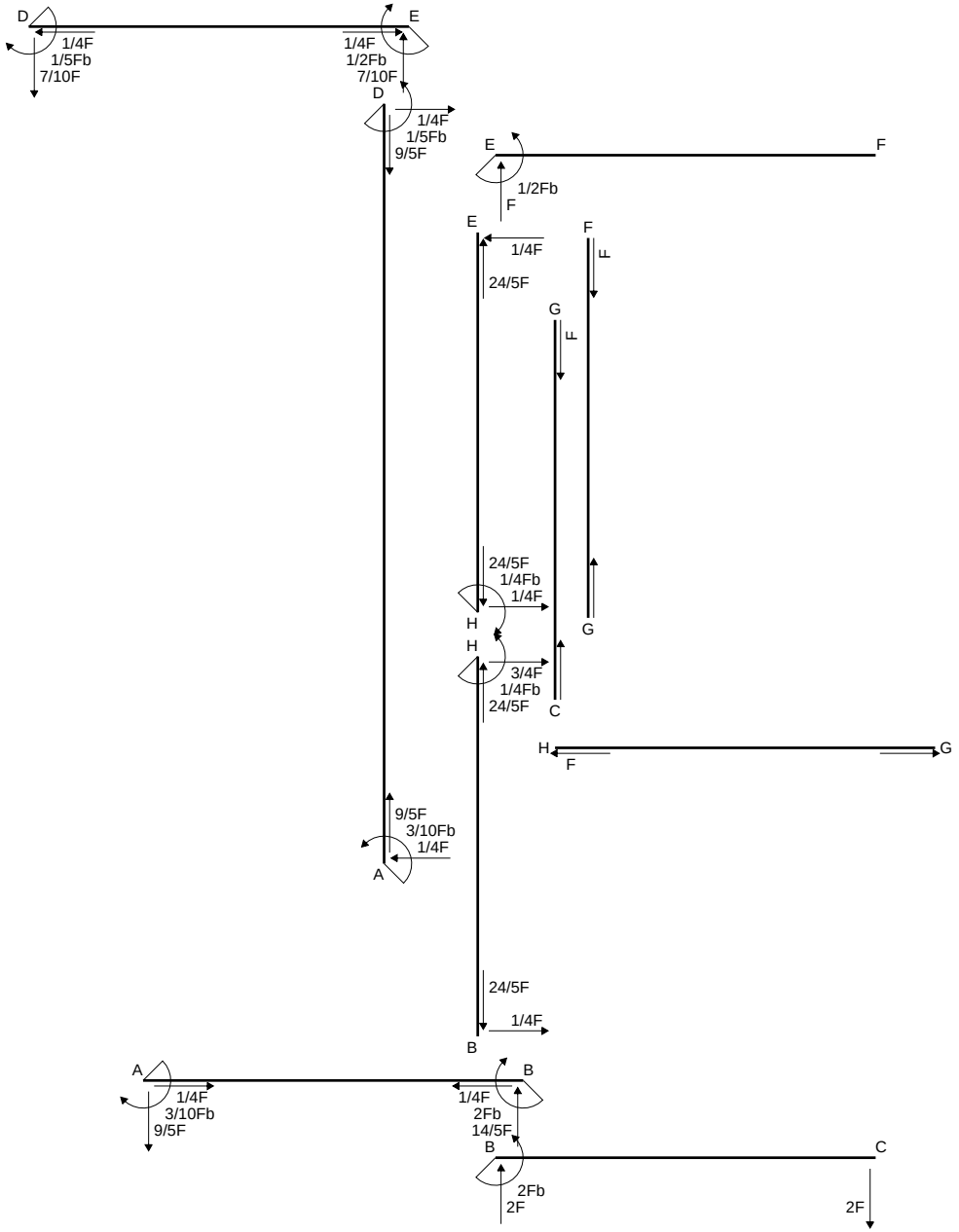

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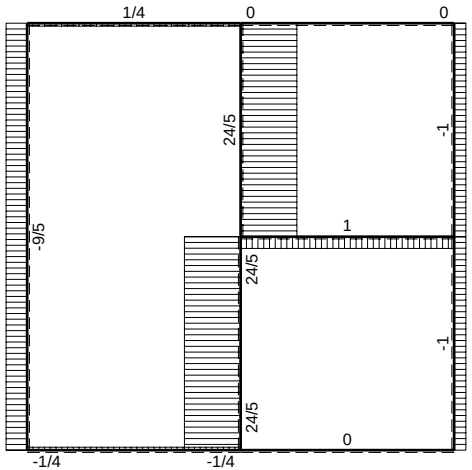
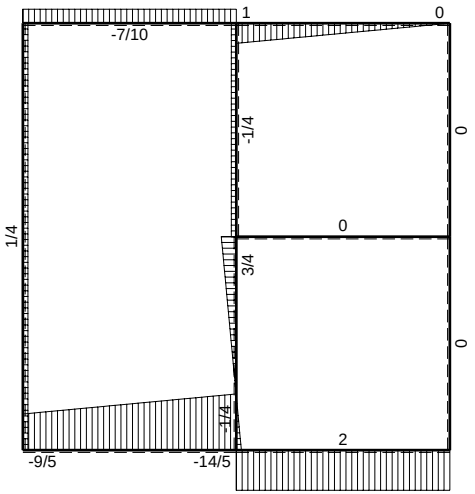
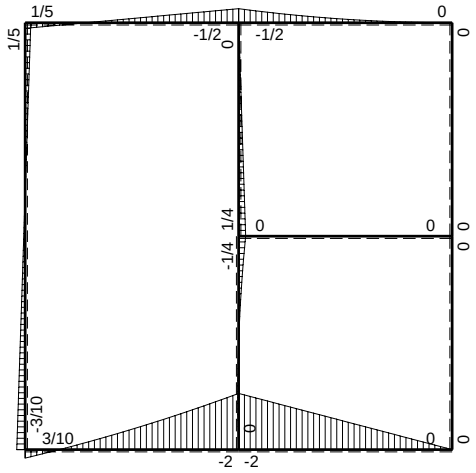


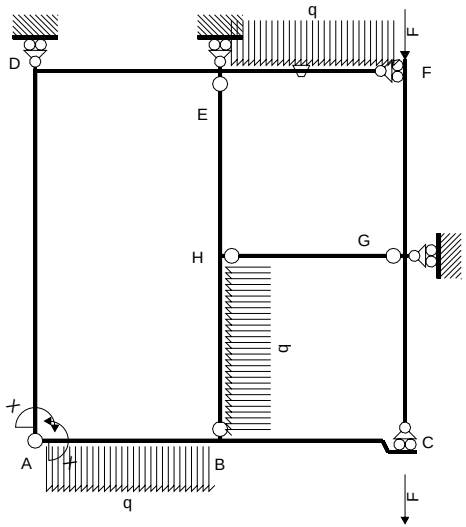


←□→ F

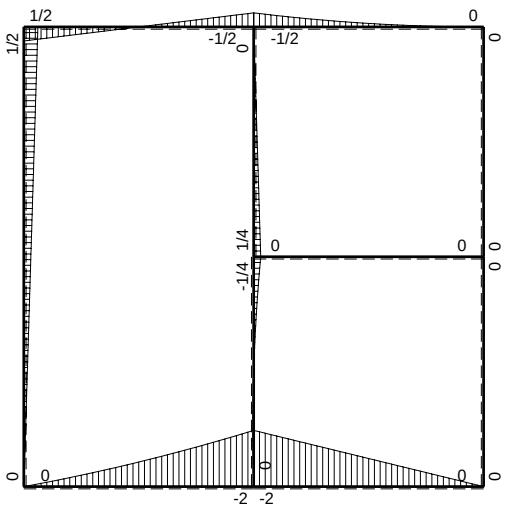
↑□↓ F



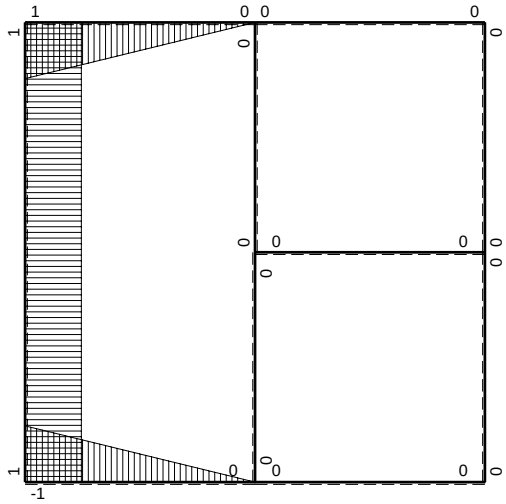
(□) Fb



Schema di calcolo iperstatico



(□) M_o flessione da carichi assegnati



(□) M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	$1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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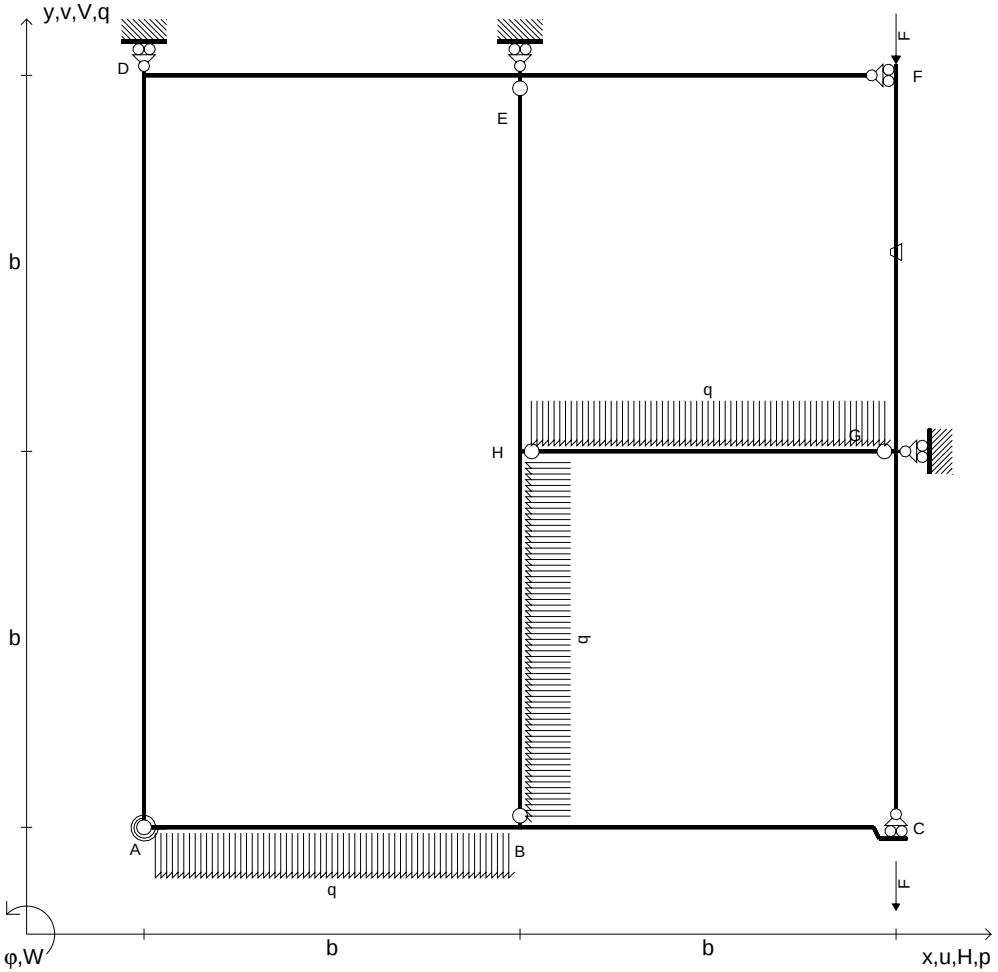

$V_{FG} = -F$ $k_{AB} = 4EJ/b$ $EJ_{FG} = EJ$

$V_{CB} = -F$ $EJ_{AB} = EJ$ $EJ_{GC} = EJ$

$q_{AB} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{HG} = EJ$

$p_{HB} = -q = -F/b$ $EJ_{AD} = EJ$ $EJ_{HB} = EJ$

$q_{HG} = -q = -F/b$ $EJ_{DE} = EJ$ $EJ_{HE} = EJ$

$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$ $EJ_{EF} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

Curvatura θ asta FG positiva se convessa a destra con inizio F.

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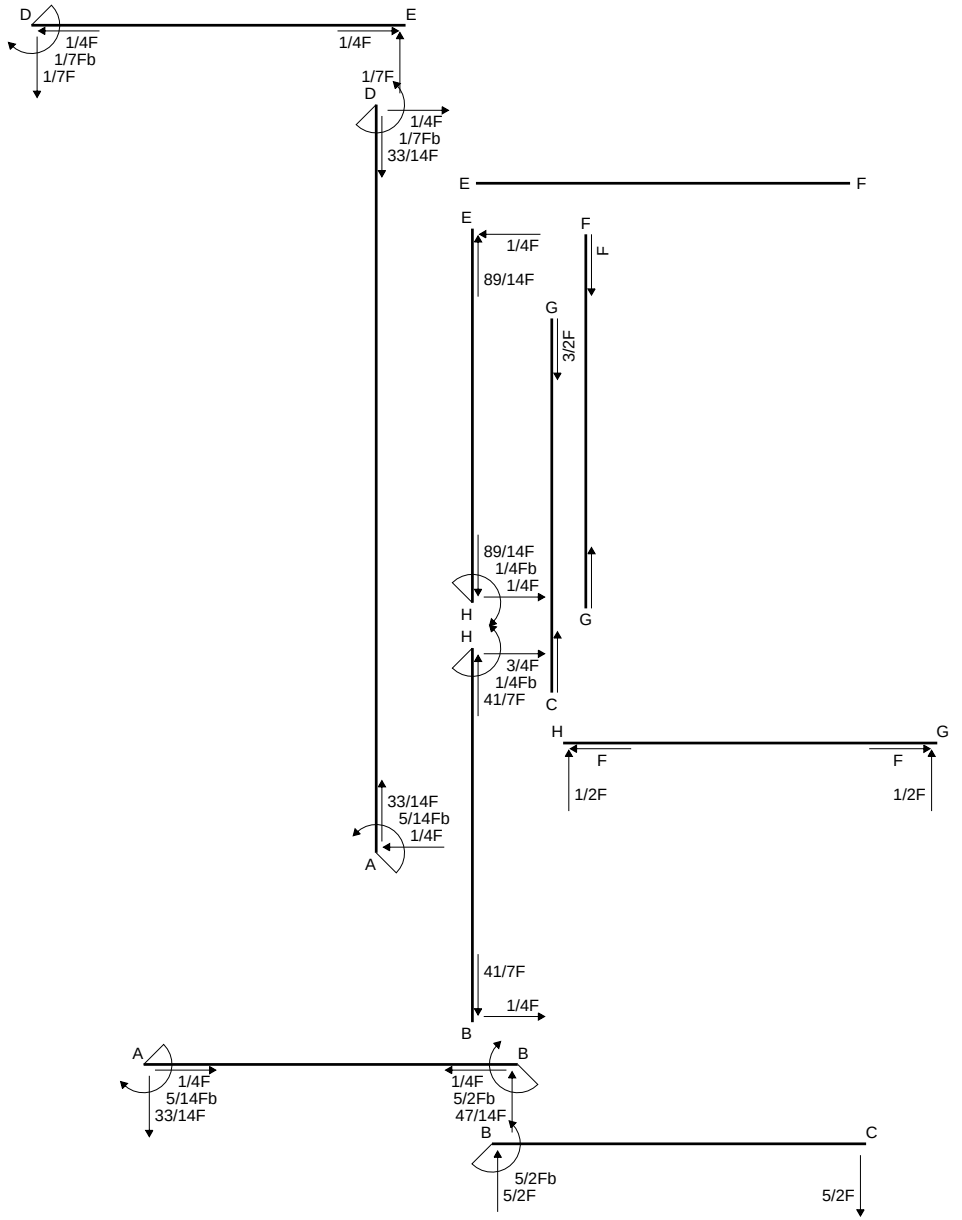

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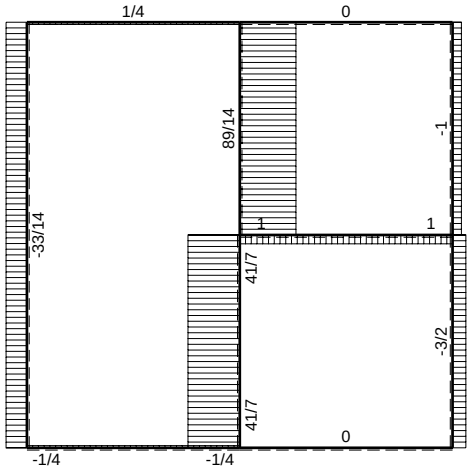
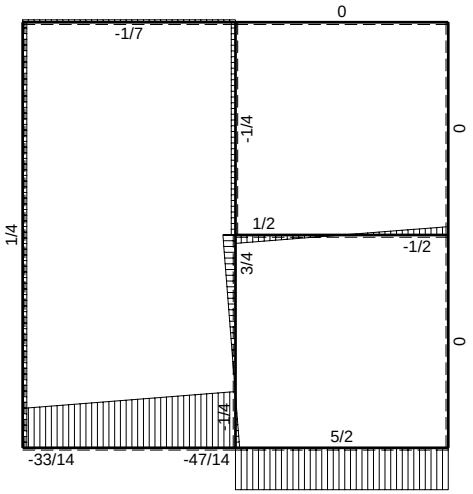
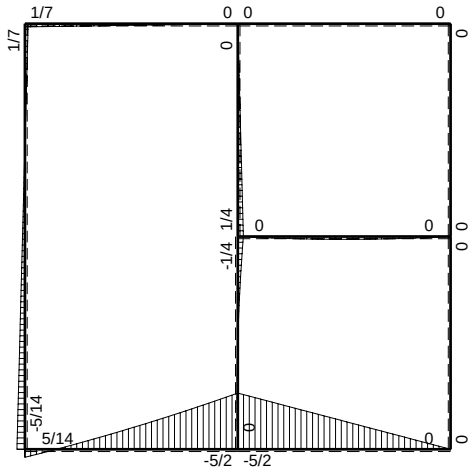


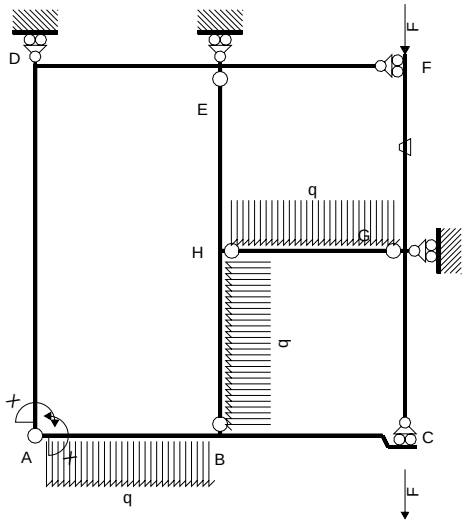


←□→ F

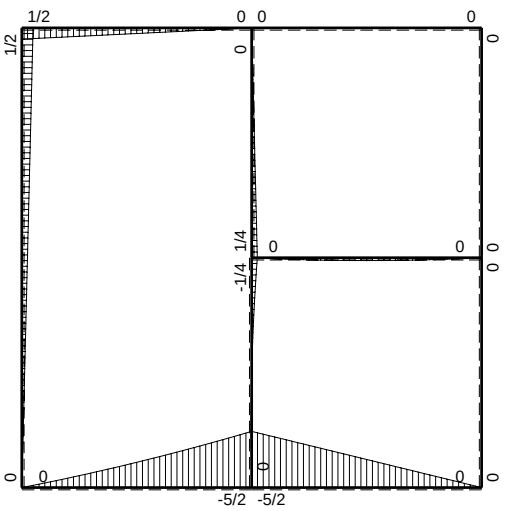
↑□↓ F



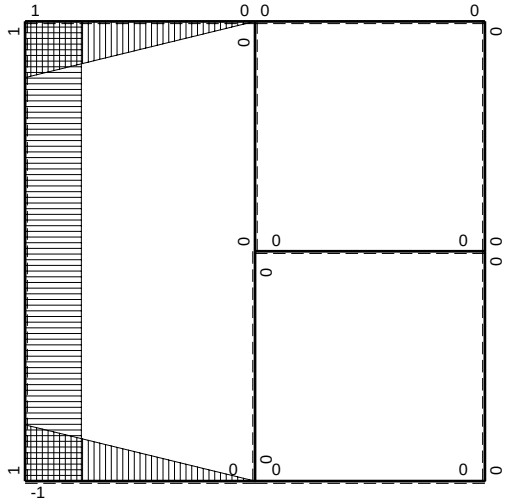
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-2Fx-1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$5/2Fb-3Fx+1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-5/2Fb+5/2Fx$	0	0	0	0	0+0	0
CB b	0	$5/2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HB b	0	$-1/4Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
EH b	0	$-1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-5/14Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (2\,x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (5/2\,x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (5/4\ b - b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 3/8\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\,x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238395 ESΣ03.xxxx.027

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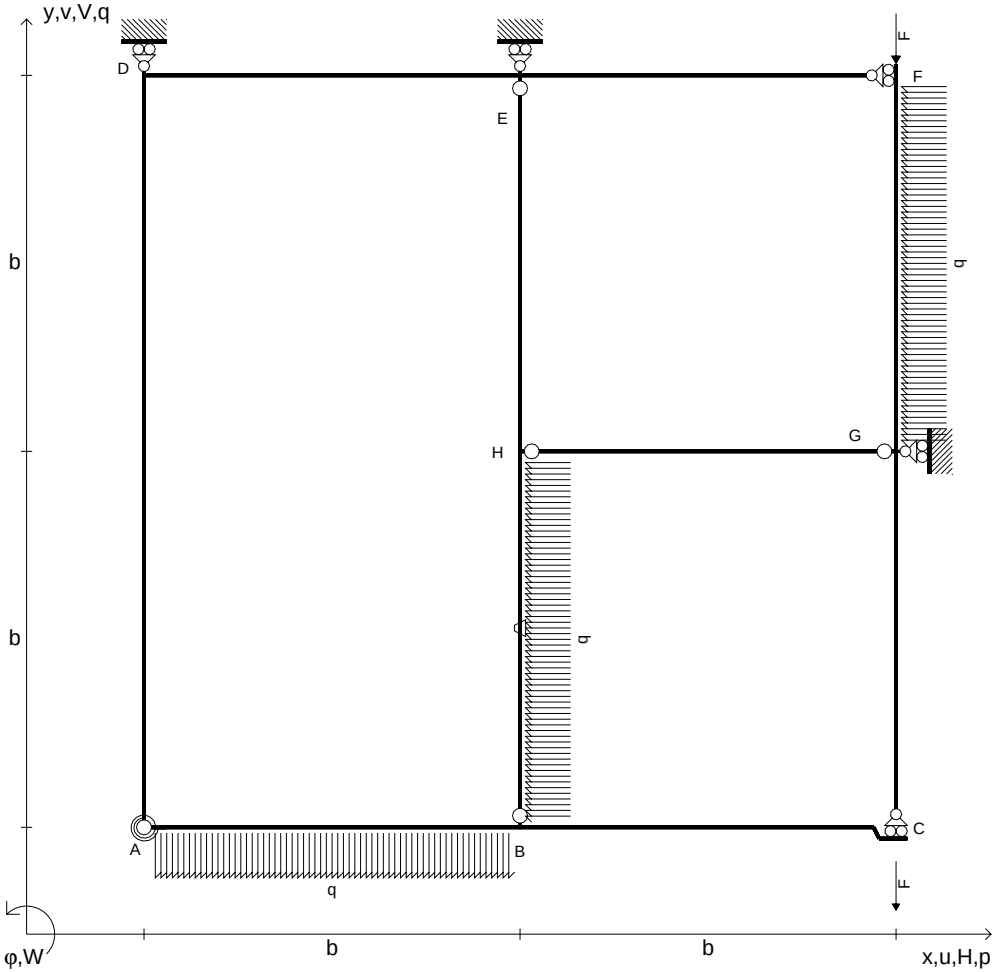

$V_{FG} = -F$
$V_{CB} = -F$
$q_{AB} = -q = -F/b$
$p_{HB} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{HB} = -\theta = -\alpha T/b = -bF/EJ$

$k_{AB} = 4EJ/b$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$

$EJ_{FG} = EJ$
$EJ_{GC} = EJ$
$EJ_{HG} = EJ$
$EJ_{HB} = EJ$
$EJ_{HE} = EJ$

Reazioni iperstatiche in soluzione: $X=V_{BA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta HB positiva se convessa a destra con inizio H.

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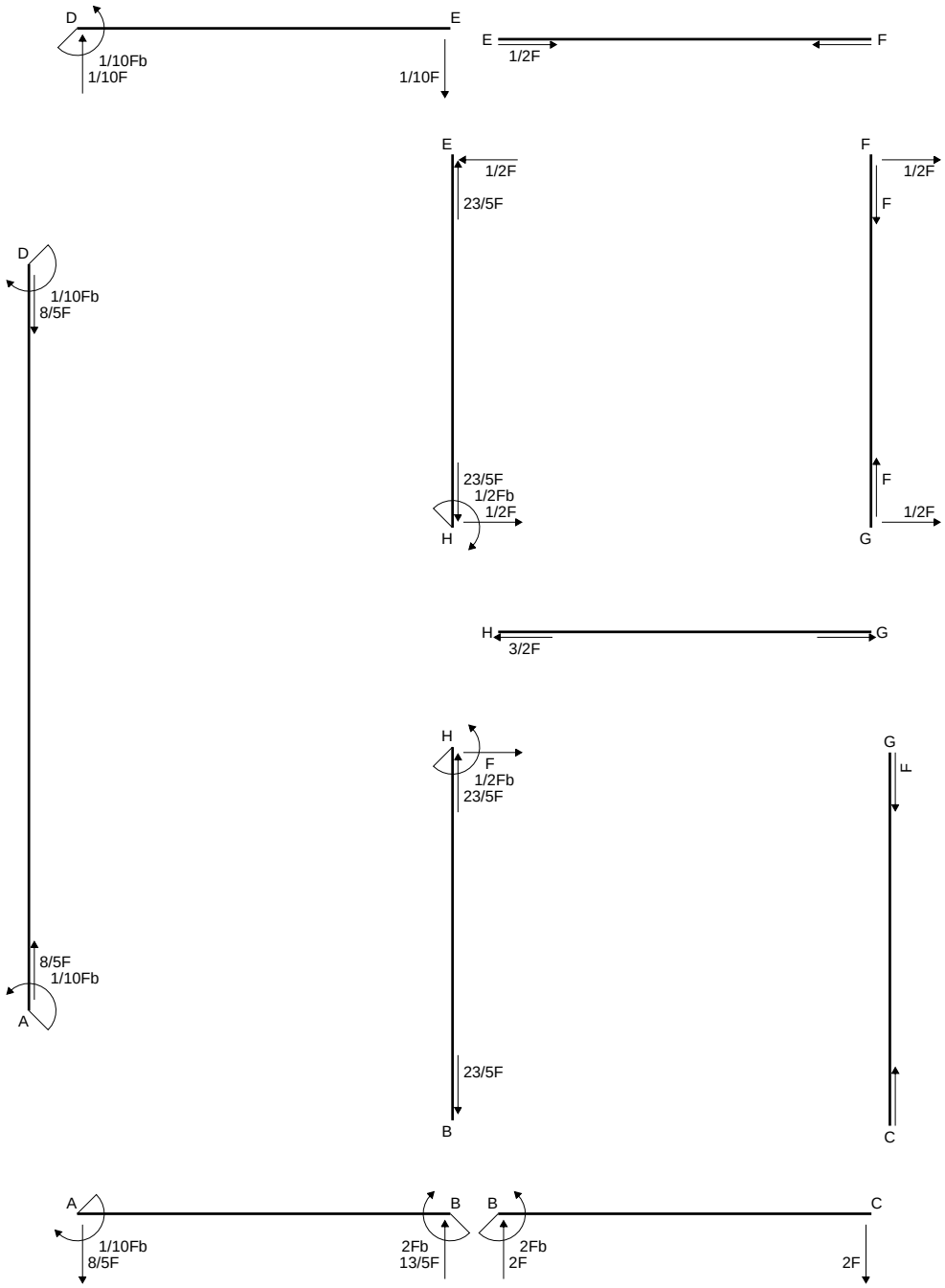

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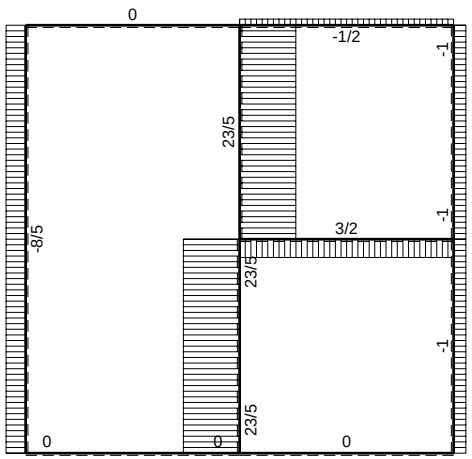
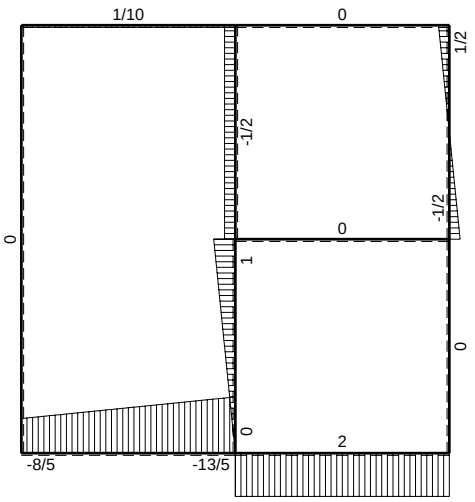
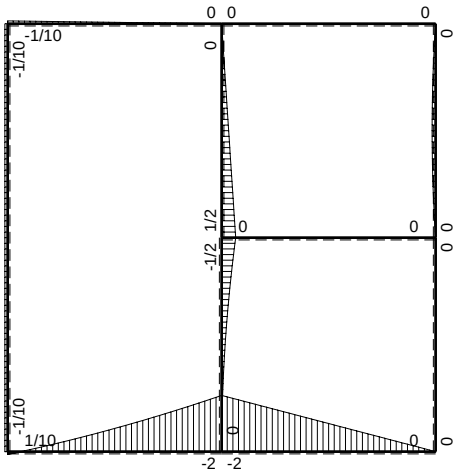


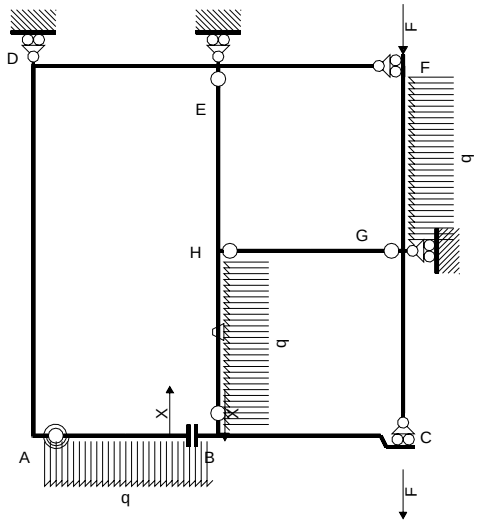
$\leftarrow \boxplus \rightarrow F$



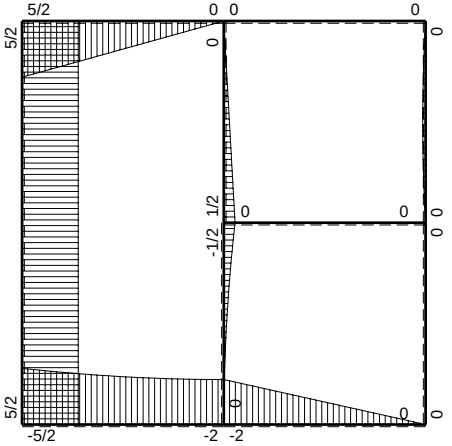
$\uparrow \boxplus \downarrow F$



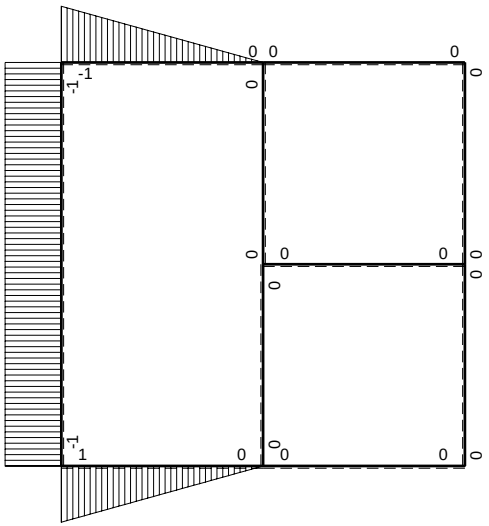
$\circlearrowleft \boxplus \circlearrowright$ Fb



Schema di calcolo iperstatico



$\circlearrowleft \boxplus \circlearrowright$ M_o flessione da carichi assegnati



$\circlearrowleft \boxplus \circlearrowright$ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_{BA}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	b-x	$-5/2Fb+Fx-1/2qx^2$	0	$-5/2Fb^2+7/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(-9/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	-x	$2Fb+1/2qx^2$	0	$-2Fbx-1/2qx^3$	0	x^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	-b	$5/2Fb$	0	$-5/2Fb^2$	0	b^2	$(-5+0)Fb^3/EJ$	$2Xb^3/EJ$
DA 2b	b	$-5/2Fb$	0	$-5/2Fb^2$	0	b^2		
DE b	-b+x	$5/2Fb-5/2Fx$	0	$-5/2Fb^2+5Fbx-5/2Fx^2$	0	$b^2-2bx+x^2$	$(-5/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
ED b	x	$-5/2Fx$	0	$-5/2Fx^2$	0	x^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-1/2Fb+Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BH b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HE b	0	$1/2Fb-1/2Fx$	0	0	0	0	0+0	0
EH b	0	$-1/2Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$						$-5/8Fb^3/EJ$	$1/4Xb^3/EJ$
	totali						$-91/12Fb^3/EJ$	$35/12Xb^3/EJ$
	iperstatica $X=V_{BA}$						13/5F	

PROCEDIMENTO E RISULTATI 309819 ESΣ03.xxxx.036

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx - 1\ \ (-1)\ \ 1/4\ \ b^3/EJ$

$= \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ\ \ - 1\ \ (-1)\ \ 1/4\ \ b^3/EJ$

$= \left(\ b - b +1/3\ b\ \right) b^2\ 1/EJ\ \ - 1\ \ (-1)\ \ 1/4\ \ b^3/EJ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx - 1\ \ (-1)\ \ 1/4\ \ b^3/EJ = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ\ \ - 1\ \ (-1)\ \ 1/4\ \ b^3/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ \ - 1\ \ (-1)\ \ 1/4\ \ b^3/EJ = 7/12\ \ b^3/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right) b^2\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b} b^2\ 1/EJ$

$= \left(2\ b\ \right) b^2\ 1/EJ\ \ = 2\ \ b^3/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right) b^2\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b} b^2\ 1/EJ$

$= \left(2\ b\ \right) b^2\ 1/EJ\ \ = 2\ \ b^3/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right) b^2\ 1/EJ\ \ = 1/3\ \ b^3/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ \ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-5/2 +7/2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx - 1\ \ 5/2\ \ 1/4\ \ Fb^3/EJ$

$= \left[-5/2\ x +7/4\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ\ \ - 1\ \ 5/2\ \ 1/4\ \ Fb^3/EJ$

$= \left(-5/2\ b +7/4\ b -1/2\ b +1/8\ b\ \right) Fb^2\ 1/EJ\ \ - 1\ \ 5/2\ \ 1/4\ \ Fb^3/EJ = -7/4\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-2\ x/b -1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx - 1\ \ 5/2\ \ 1/4\ \ Fb^3/EJ$

$= \left[- x^2/b -1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ\ \ - 1\ \ 5/2\ \ 1/4\ \ Fb^3/EJ$

$= \left(-\ b -1/8\ b\ \right) Fb^2\ 1/EJ\ \ - 1\ \ 5/2\ \ 1/4\ \ Fb^3/EJ = -7/4\ \ Fb^3/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/2\ \right) Fb^2\ 1/EJ\ dx = \left[-5/2\ x\ \right]_o^{2b} Fb^2\ 1/EJ$

$= \left(-5\ b\ \right) Fb^2\ 1/EJ\ \ = -5\ \ Fb^3/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-5/2\ \right) Fb^2\ 1/EJ\ dx = \left[-5/2\ x\ \right]_o^{2b} Fb^2\ 1/EJ$

$= \left(-5\ b\ \right) Fb^2\ 1/EJ\ \ = -5\ \ Fb^3/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-5/2 +5\ x/b -5/2\ x^2/b^2\ \right) Fb^2\ 1/EJ\ dx = \left[-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-5/2\ b +5/2\ b -5/6\ b\ \right) Fb^2\ 1/EJ\ \ = -5/6\ \ Fb^3/EJ$

$L^{Xo}_{ED} = \int_o^b \left(-5/2\ x^2/b^2\ \right) Fb^2\ 1/EJ\ dx = \left[-5/6\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-5/6\ b\ \right) Fb^2\ 1/EJ\ \ = -5/6\ \ Fb^3/EJ$

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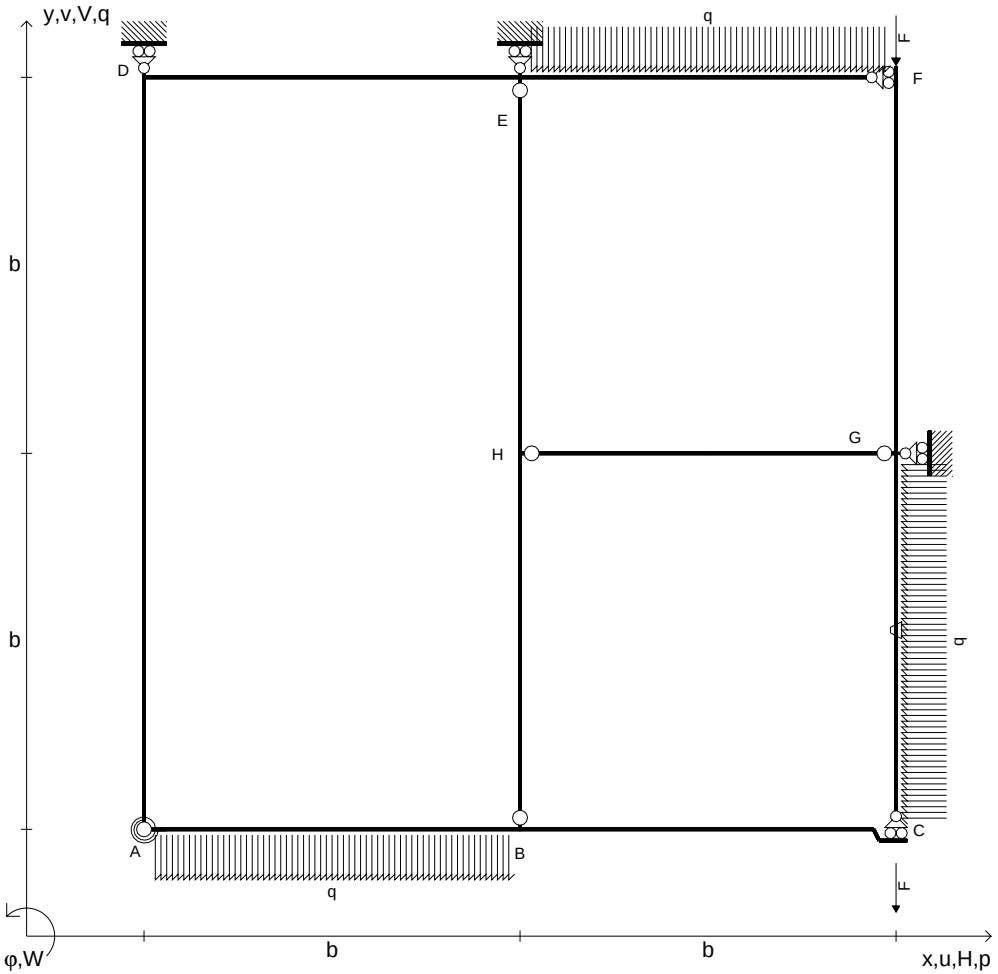

$V_{FG} = -F$ $k_{AB} = 4EJ/b$ $EJ_{FG} = EJ$

$V_{CB} = -F$ $EJ_{AB} = EJ$ $EJ_{GC} = EJ$

$q_{AB} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{HG} = EJ$

$p_{GC} = -q = -F/b$ $EJ_{AD} = EJ$ $EJ_{HB} = EJ$

$q_{EF} = -q = -F/b$ $EJ_{DE} = EJ$ $EJ_{HE} = EJ$

$\theta_{GC} = -\theta = -\alpha T/b = -bF/EJ$ $EJ_{EF} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

Curvatura θ asta GC positiva se convessa a destra con inizio G.

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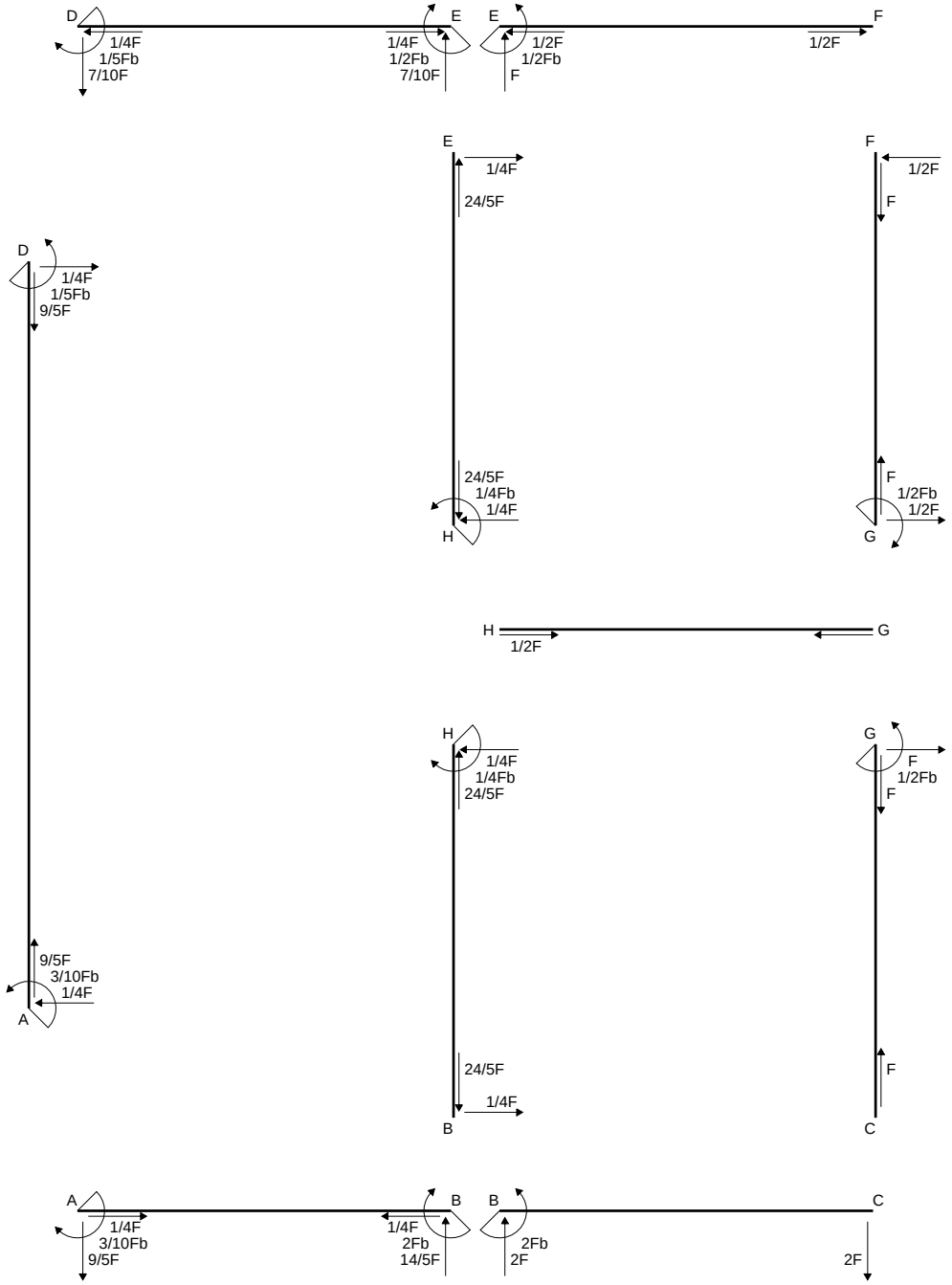

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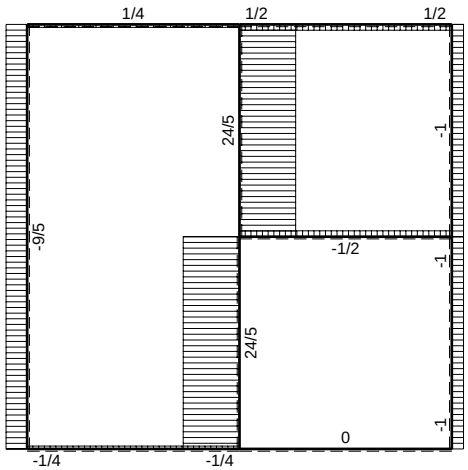
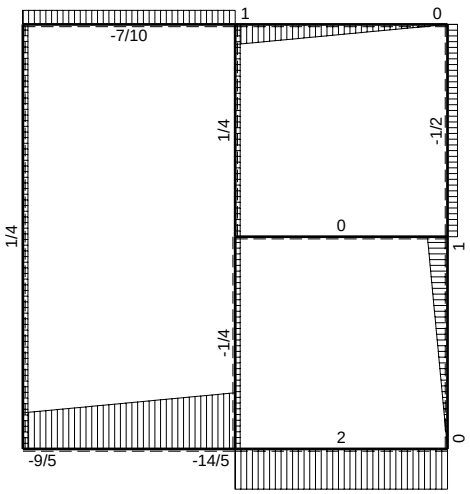
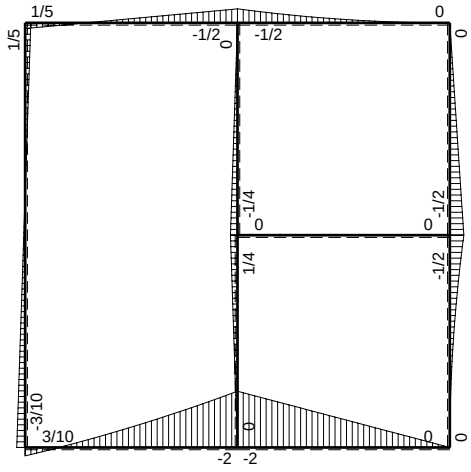


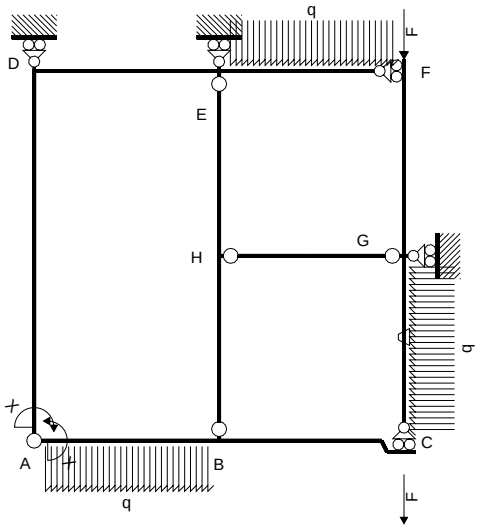


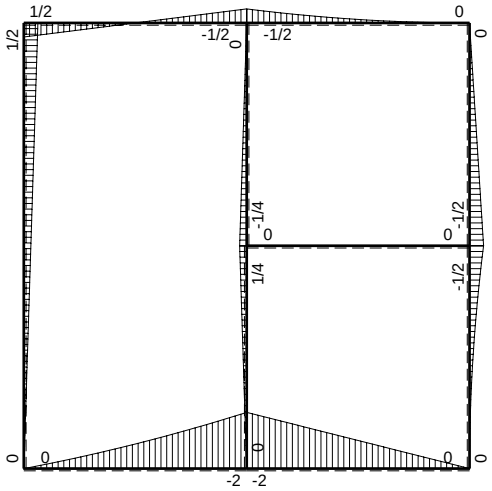
←[+]→ F

↑[+]↓ F

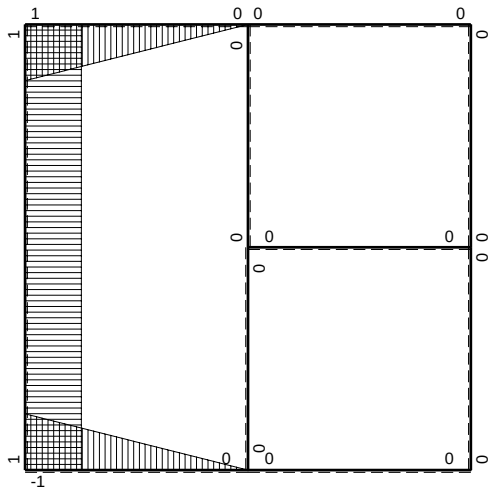


[+] Fb





[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236422 ESΣ03.xxxx.044

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$1/4Fx$	0	$1/4Fx$	0	1	$(1/2+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+1/4Fx$	0	$1/2Fb-1/4Fx$	0	1		
DE b	$1-x/b$	$1/2Fb-Fx$	0	$1/2Fb-3/2Fx+Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/12+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fb-Fx$	0	$-1/2Fx+Fx^2/b$	0	x^2/b^2		
EF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	$1/2qx^2$	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$1/4Fb-1/4Fx$	0	0	0	0	0+0	0
BH b	0	$-1/4Fx$	0	0	0	0		
HE b	0	$-1/4Fb+1/4Fx$	0	0	0	0	0+0	0
EH b	0	$1/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/8Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-3/10Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/8\ x^2/b\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 3/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-1/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 236422 ESΣ03.xxxx.044

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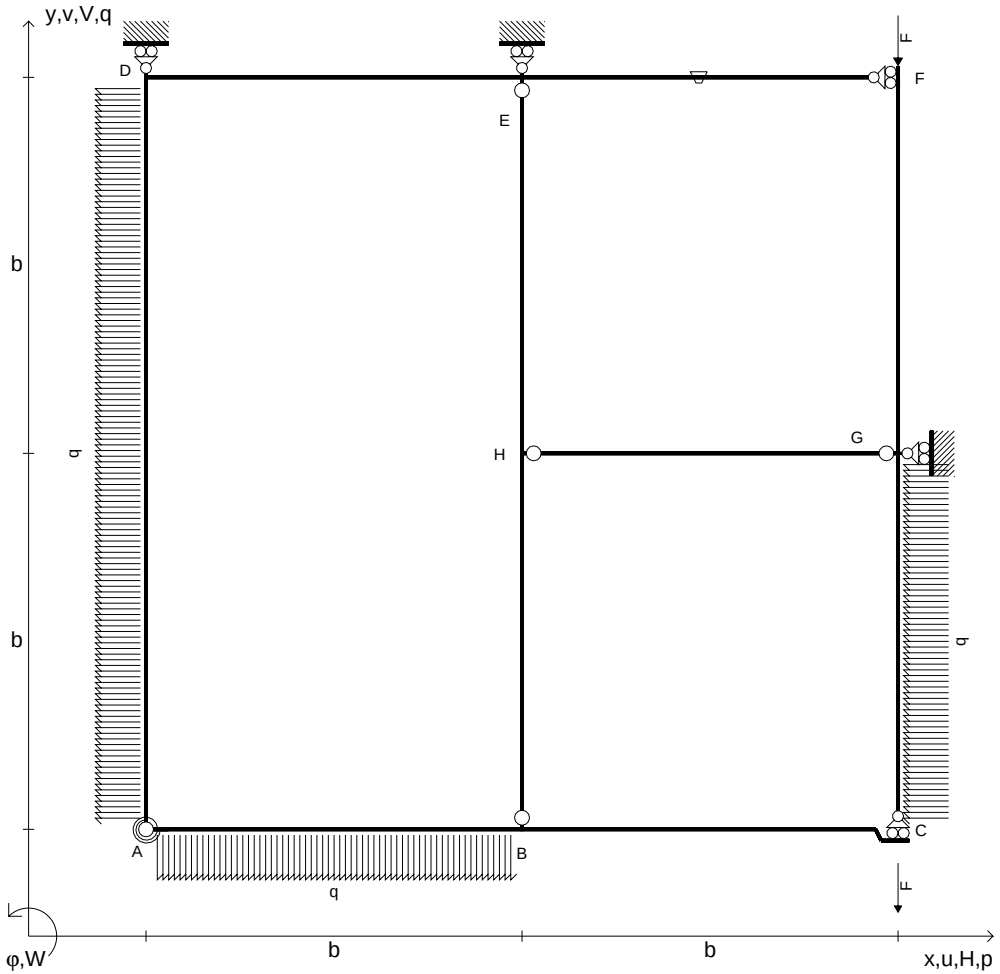

$V_{FG} = -F$ $k_{AB} = 4EJ/b$ $EJ_{FG} = EJ$

$V_{CB} = -F$ $EJ_{AB} = EJ$ $EJ_{GC} = EJ$

$q_{AB} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{HG} = EJ$

$p_{GC} = -q = -F/b$ $EJ_{AD} = EJ$ $EJ_{HB} = EJ$

$p_{AD} = -q = -F/b$ $EJ_{DE} = EJ$ $EJ_{HE} = EJ$

$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$ $EJ_{EF} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

Curvatura θ asta EF positiva se convessa a destra con inizio E.

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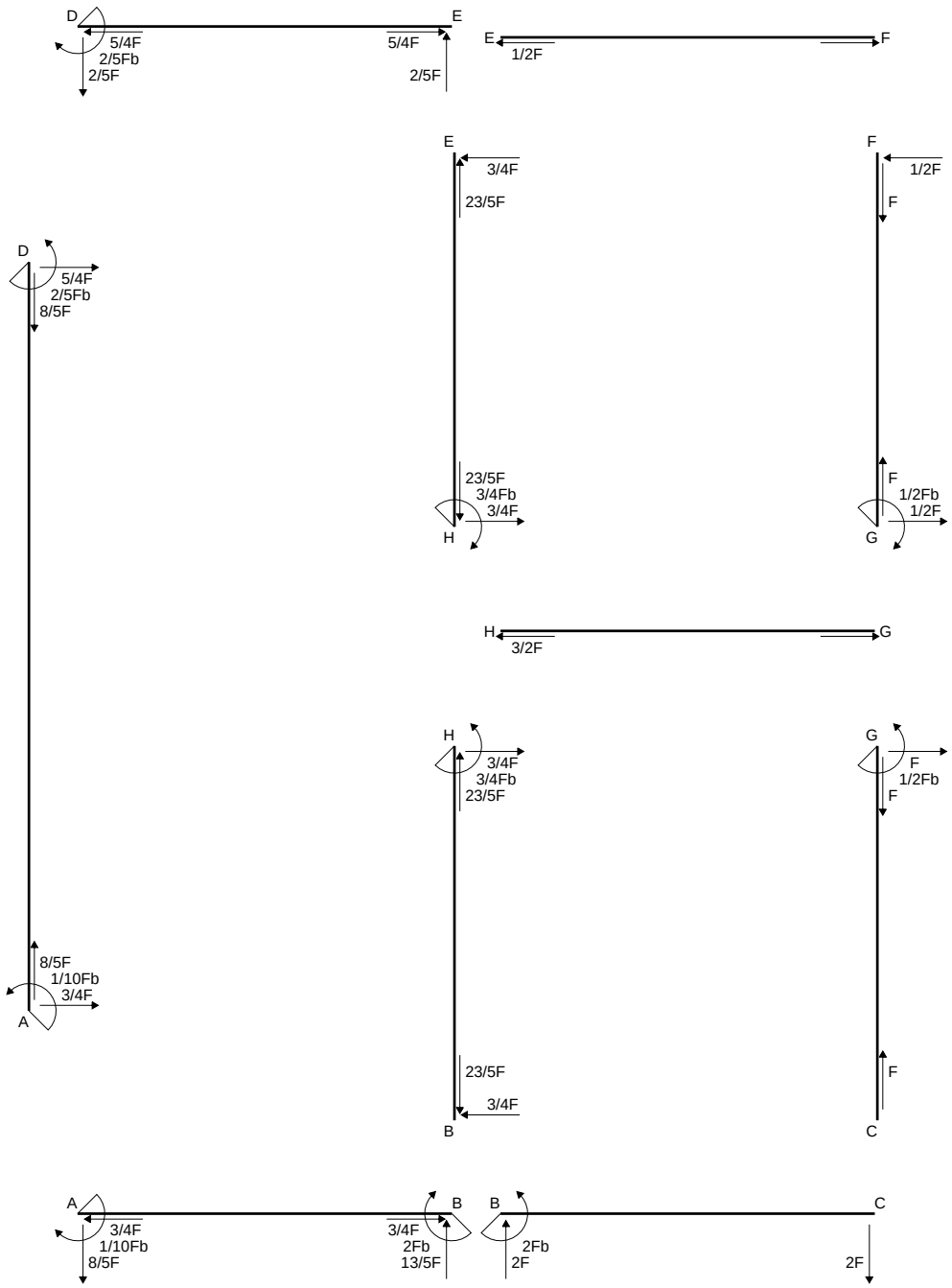

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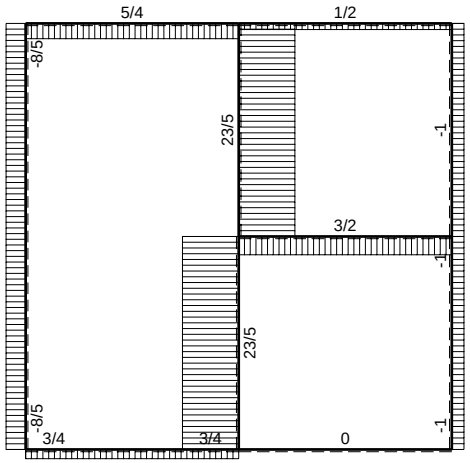
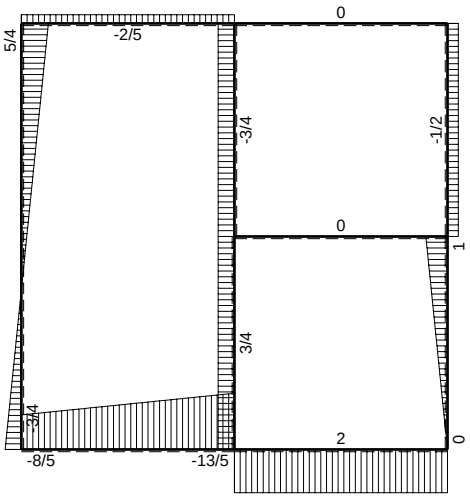
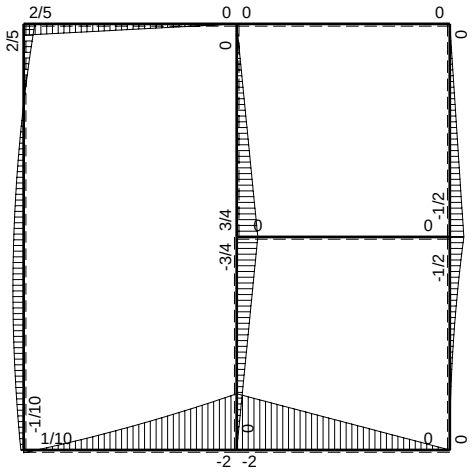


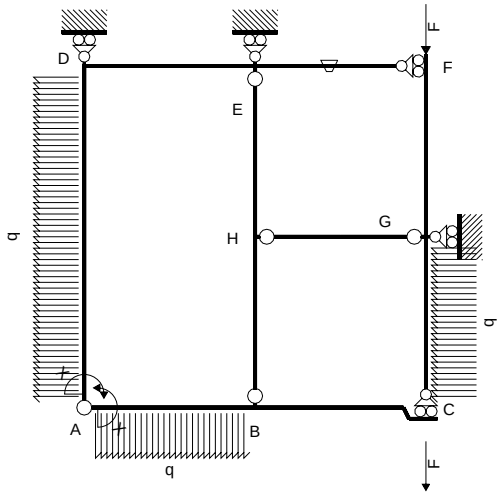


←□→ F

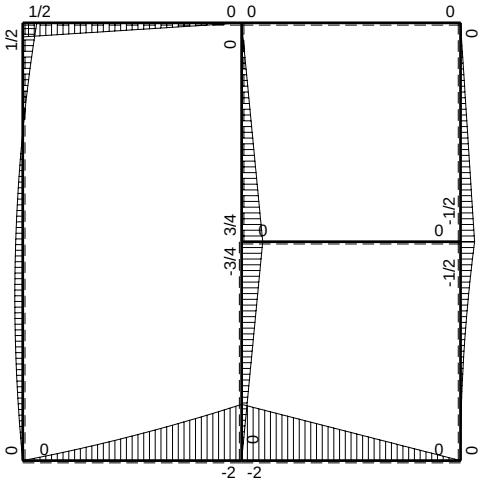
↑□↓ F



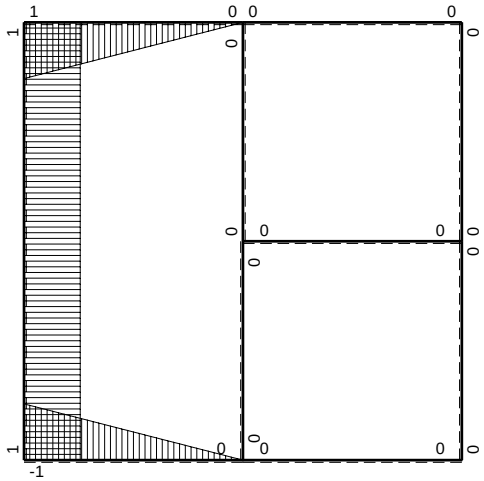
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-3/4Fx+1/2qx^2$	0	$-3/4Fx+1/2Fx^2/b$	0	1	$(-1/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$1/2Fb-5/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$1/2Fb-1/2Fx$	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$-1/2Fx$	0	$1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	0	Fb/EJ	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-3/4Fb+3/4Fx$	0	0	0	0	0+0	0
BH b	0	$3/4Fx$	0	0	0	0		
HE b	0	$3/4Fb-3/4Fx$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238485 ESΣ03.xxxx.053

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$L^{XX}_{AB} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= (1/3\ b)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b (3/2\ x/b - x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [3/4\ x^2/b - 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (3/4\ b - 1/3\ b - 1/8\ b)\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (2\ x/b - 5/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= [\ x^2/b - 5/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= (\ b - 5/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} (-3/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-3/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} (1/2 - 5/4\ x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 5/8\ x^2/b + 1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (\ b - 5/2\ b + 4/3\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238485 ESΣ03.xxxx.053

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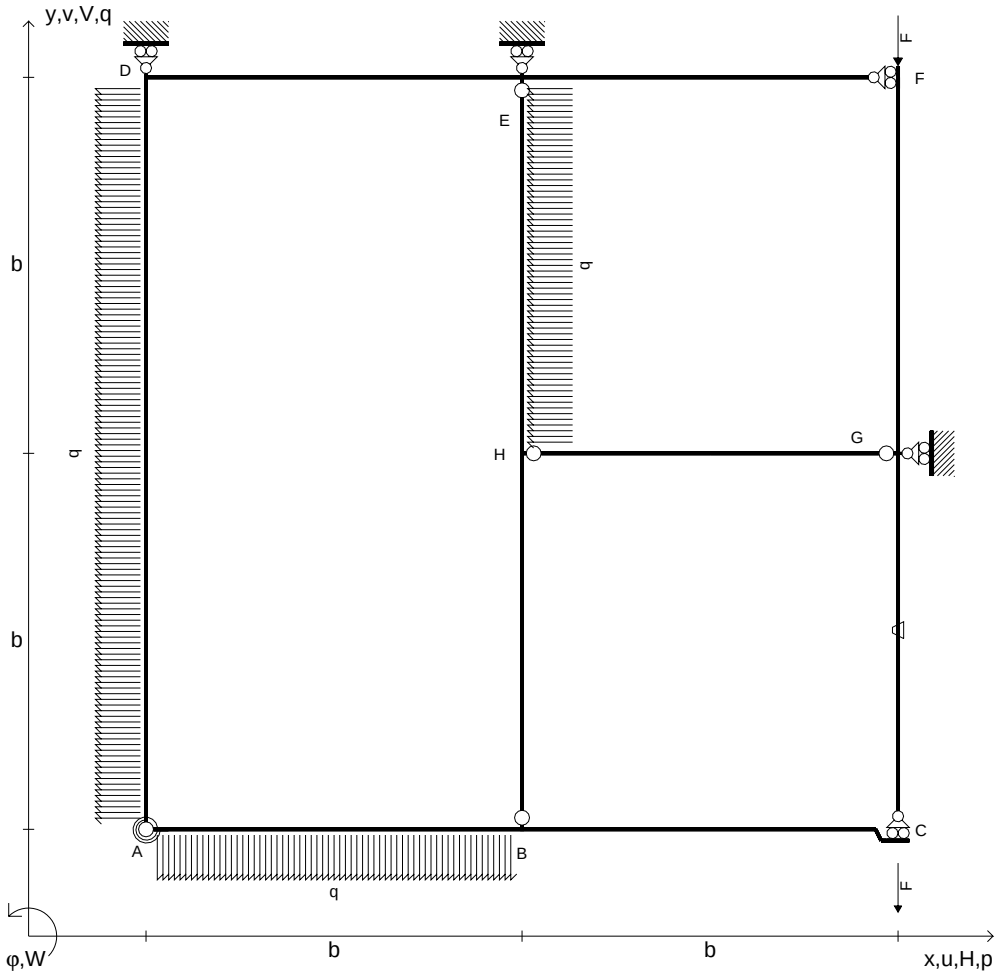

$V_{FG} = -F$ $k_{AB} = 4EJ/b$ $EJ_{FG} = EJ$

$V_{CB} = -F$ $EJ_{AB} = EJ$ $EJ_{GC} = EJ$

$q_{AB} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{HG} = EJ$

$p_{AD} = -q = -F/b$ $EJ_{AD} = EJ$ $EJ_{HB} = EJ$

$p_{HE} = -q = -F/b$ $EJ_{DE} = EJ$ $EJ_{HE} = EJ$

$\theta_{GC} = -\theta = -\alpha T/b = -bF/EJ$ $EJ_{EF} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

Curvatura θ asta GC positiva se convessa a destra con inizio G.

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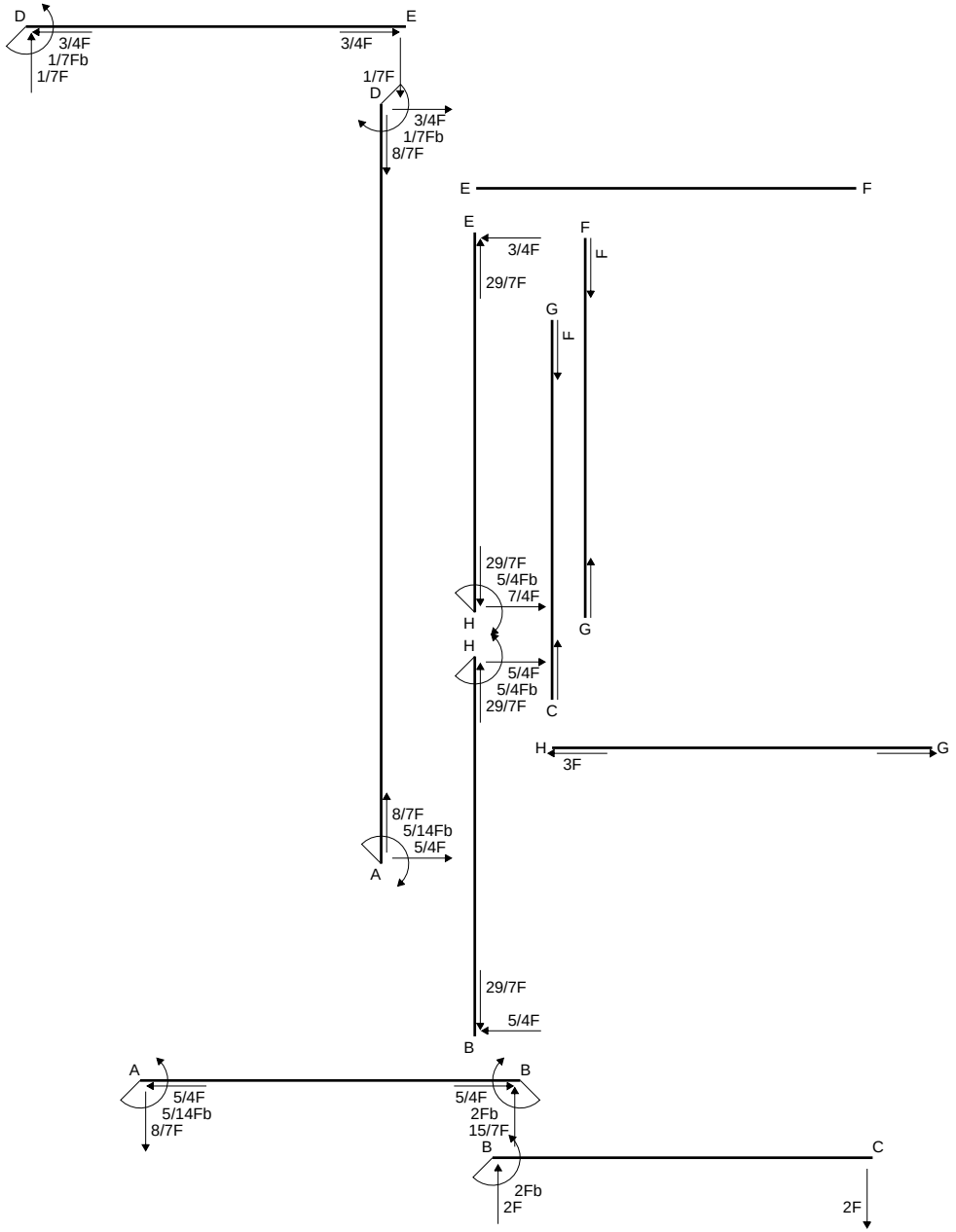

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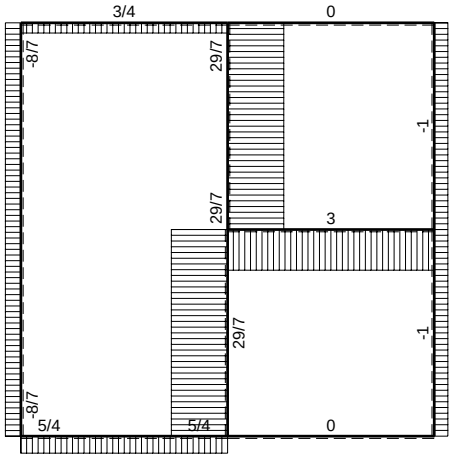

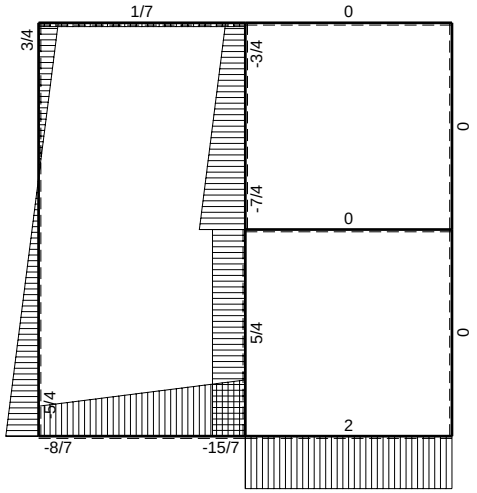

←⊞→ F

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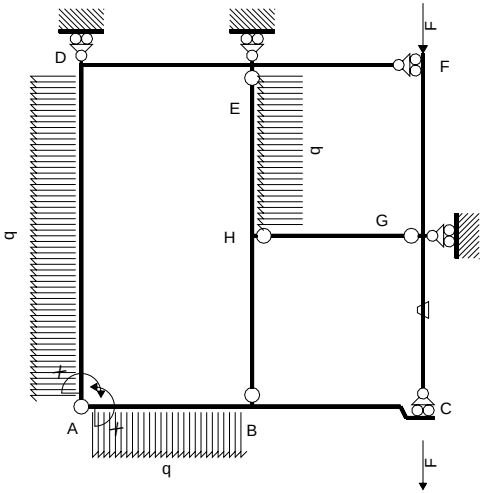
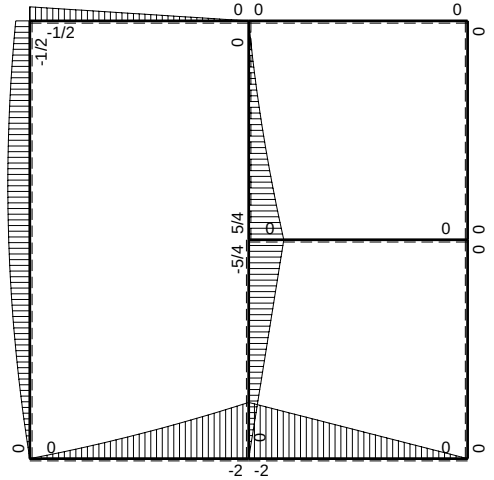
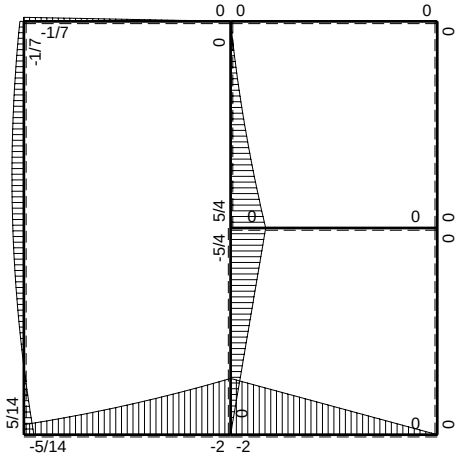


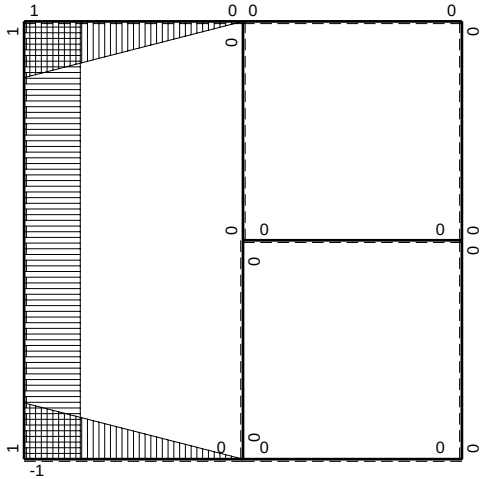
Schema di calcolo iperstatico



↺⊞↻ M_o flessione da carichi assegnati



↺⊞↻ Fb



↺⊞↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	$-5/4Fx+1/2qx^2$	0	$-5/4Fx+1/2Fx^2/b$	0	1	$(-7/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	-1	$1/2Fb+3/4Fx-1/2qx^2$	0	$-1/2Fb-3/4Fx+1/2Fx^2/b$	0	1		
DE b	$1-x/b$	$-1/2Fb+1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	$-Fb/EJ$	0	0	0	0+0	0
CG b	0	0	Fb/EJ	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+5/4Fx$	0	0	0	0	0+0	0
BH b	0	$5/4Fx$	0	0	0	0		
HE b	0	$5/4Fb-7/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EH b	0	$-3/4Fx-1/2qx^2$	0	0	0	0		
AB	molla asta $-W_{1AB}(W_{oAB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-25/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$5/14Fb$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 239191 ESΣ03.xxxx.120
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 12.12.25

$L^{XX}_{AB} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx + 1\ \ 1\ \ 1/4\ \ b/EJ = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ + 1\ \ 1\ \ 1/4\ \ b/EJ = 7/12\ \ b/EJ$

$L^{XX}_{AD} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DA} = \int_o^{2b} \left(1\ \right)\ 1/EJ\ dx = \left[\ x\ \right]_o^{2b}\ 1/EJ$

$= \left(2\ b\ \right)\ 1/EJ\ = 2\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b \left(1 -2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b +1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b \left(\ x^2/b^2\ \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2\ x/b - x^2/b^2 -1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[3/4\ x^2/b -1/3\ x^3/b^2 -1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(3/4\ b -1/3\ b -1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(2\ x/b -5/2\ x^2/b^2 +1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left[\ x^2/b -5/6\ x^3/b^2 +1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ$

$= \left(\ b -5/6\ b +1/8\ b\ \right) Fb\ 1/EJ\ + 1\ \ 0\ \ 1/4\ \ Fb^2/EJ = 7/24\ \ Fb^2/EJ$

$L^{Xo}_{AD} = \int_o^{2b} \left(-5/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-5/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-5/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DA} = \int_o^{2b} \left(-1/2 -3/4\ x/b +1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x -3/8\ x^2/b +1/6\ x^3/b^2\ \right]_o^{2b}\ Fb\ 1/EJ$

$= \left(-\ b -3/2\ b +4/3\ b\ \right) Fb\ 1/EJ\ = -7/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b \left(-1/2 + x/b -1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/2\ b +1/2\ b -1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b \left(-1/2\ x^2/b^2\ \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/6\ b\ \right) Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

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