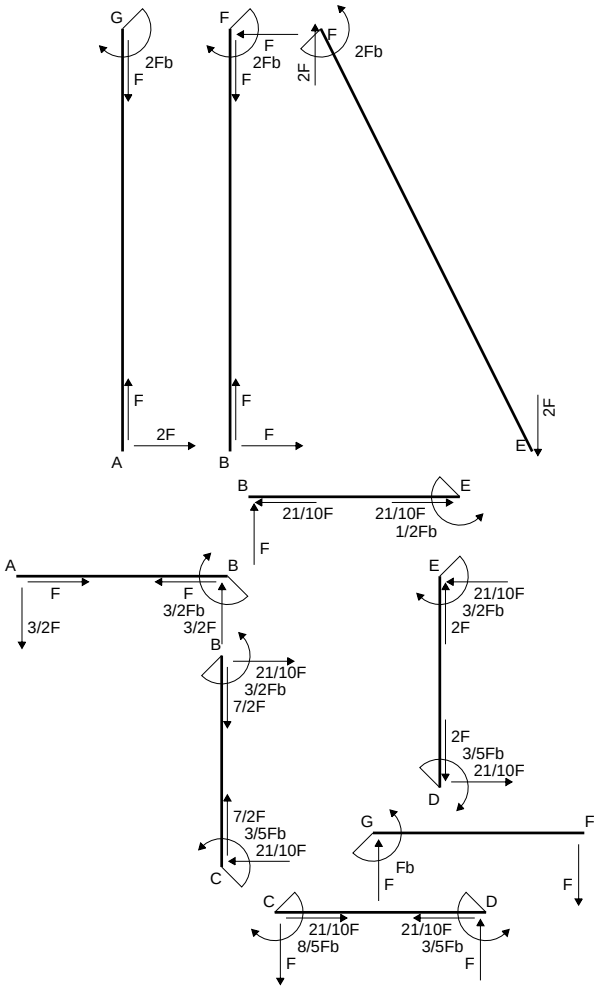
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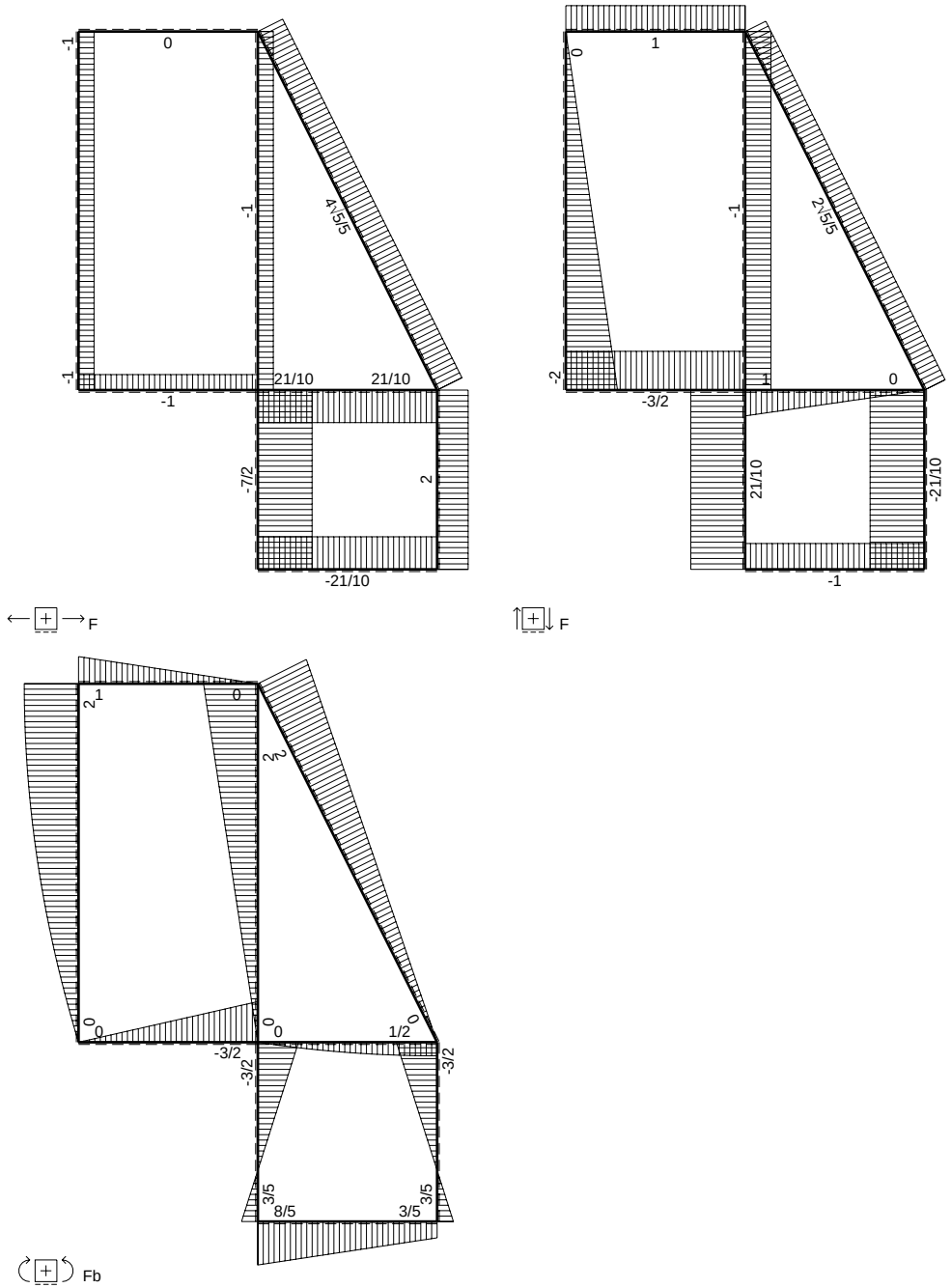
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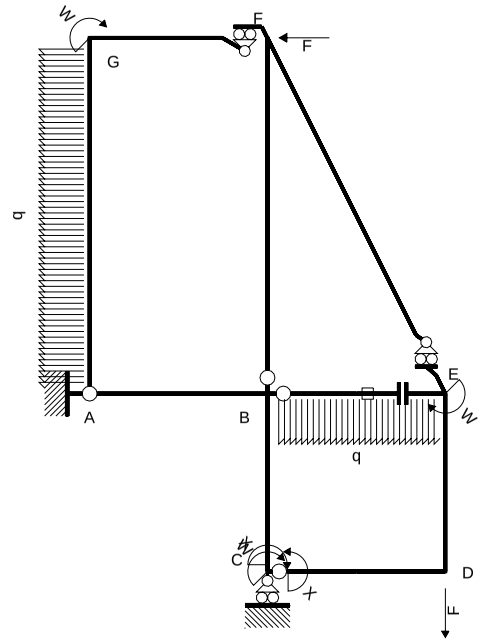
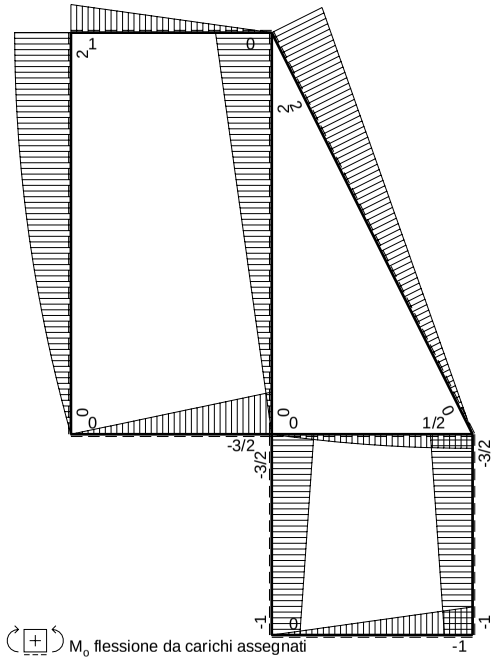
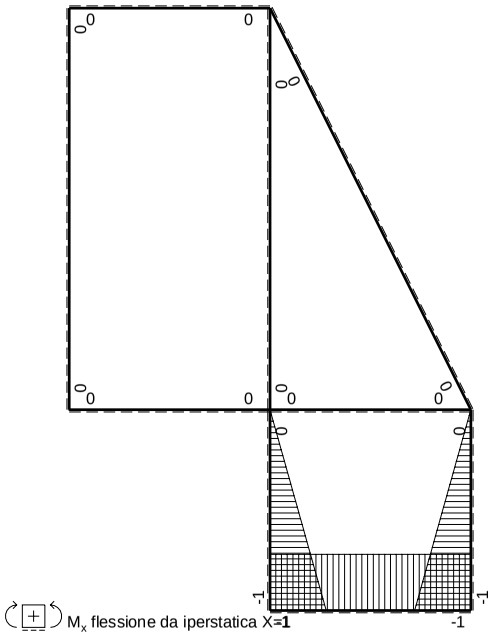


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fb-1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	2Fb-1/2qx^2	0	0	0	0
AG 2b	0	-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				8/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-8/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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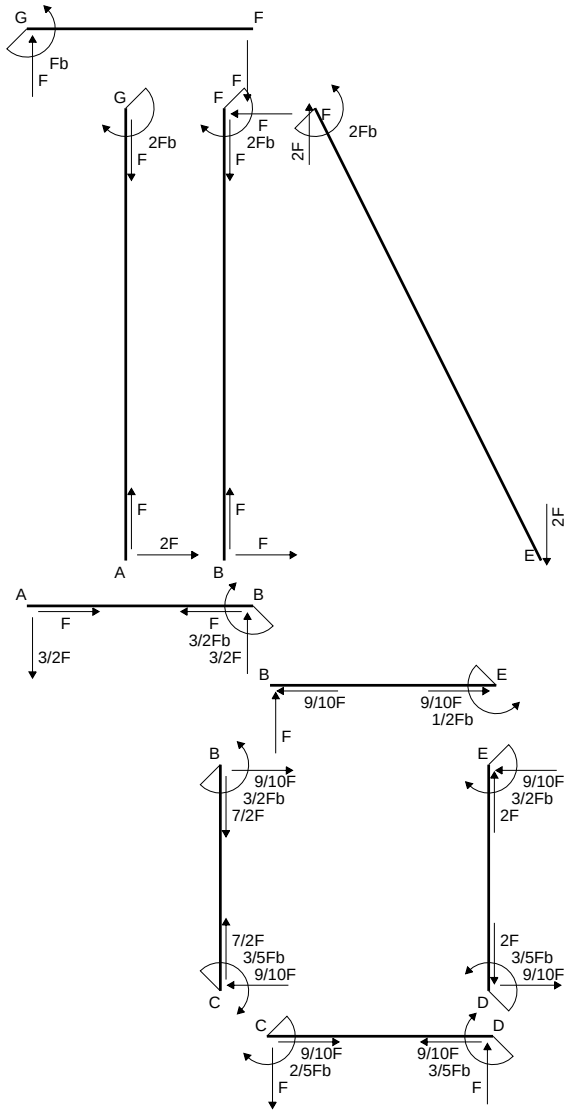
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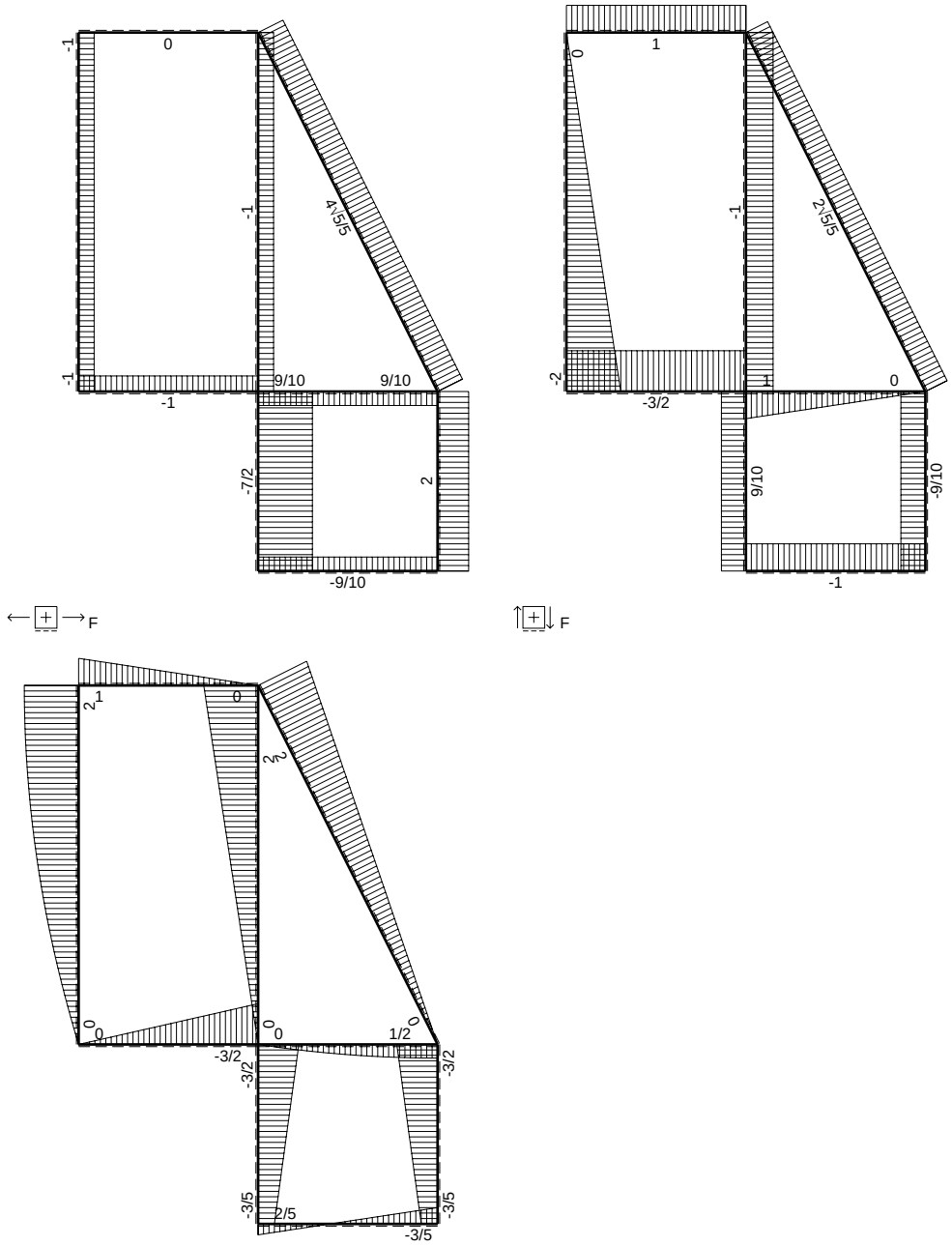
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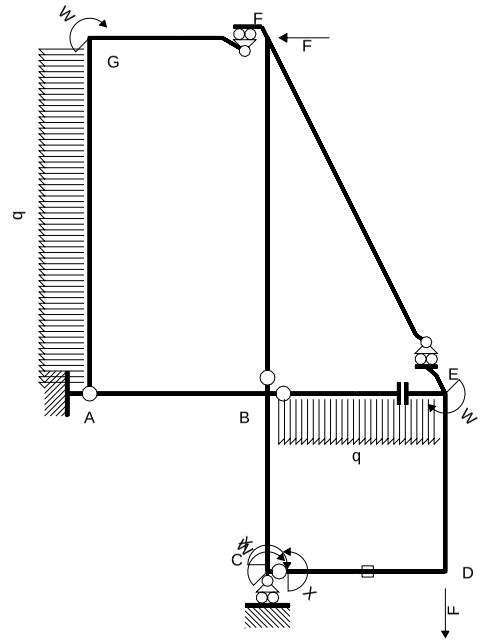
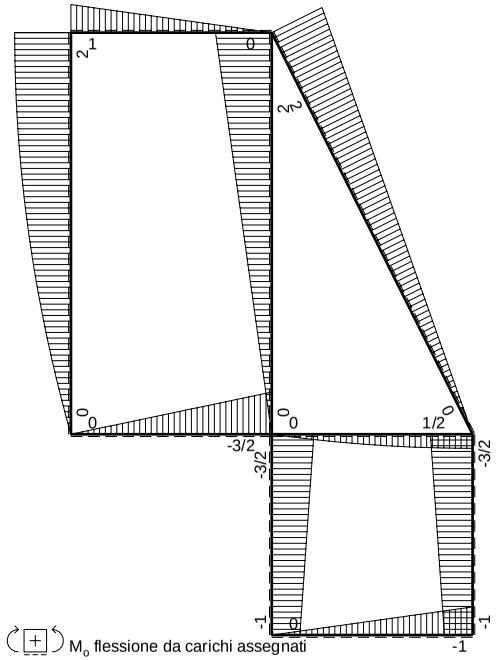
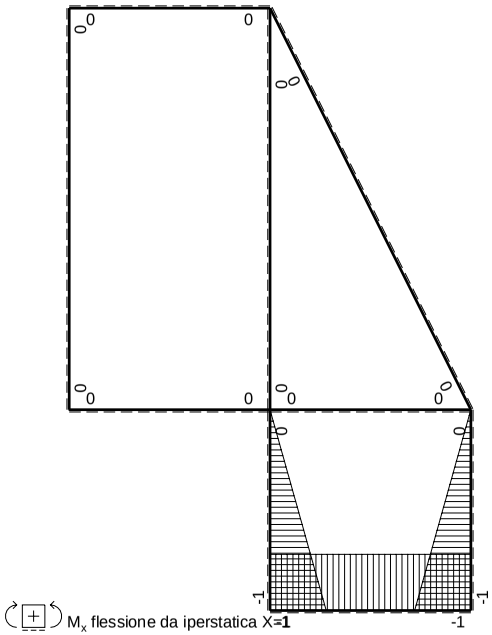


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	3/2Fb-1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$2/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-2/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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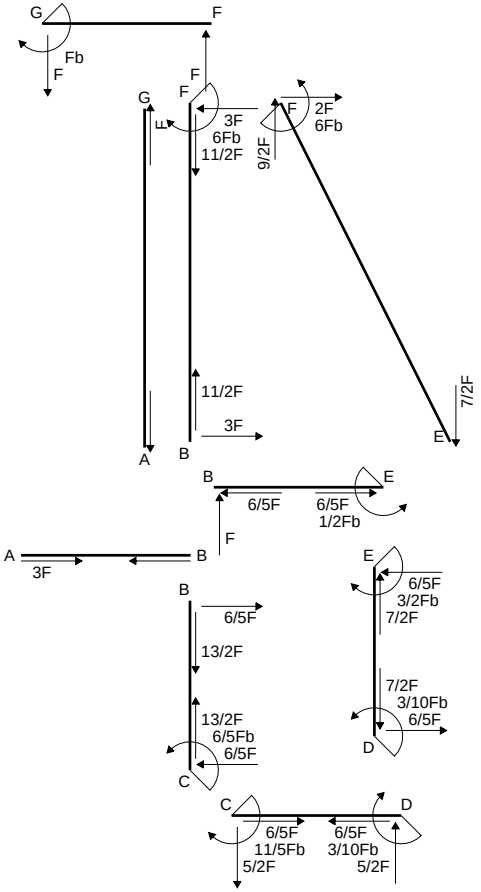
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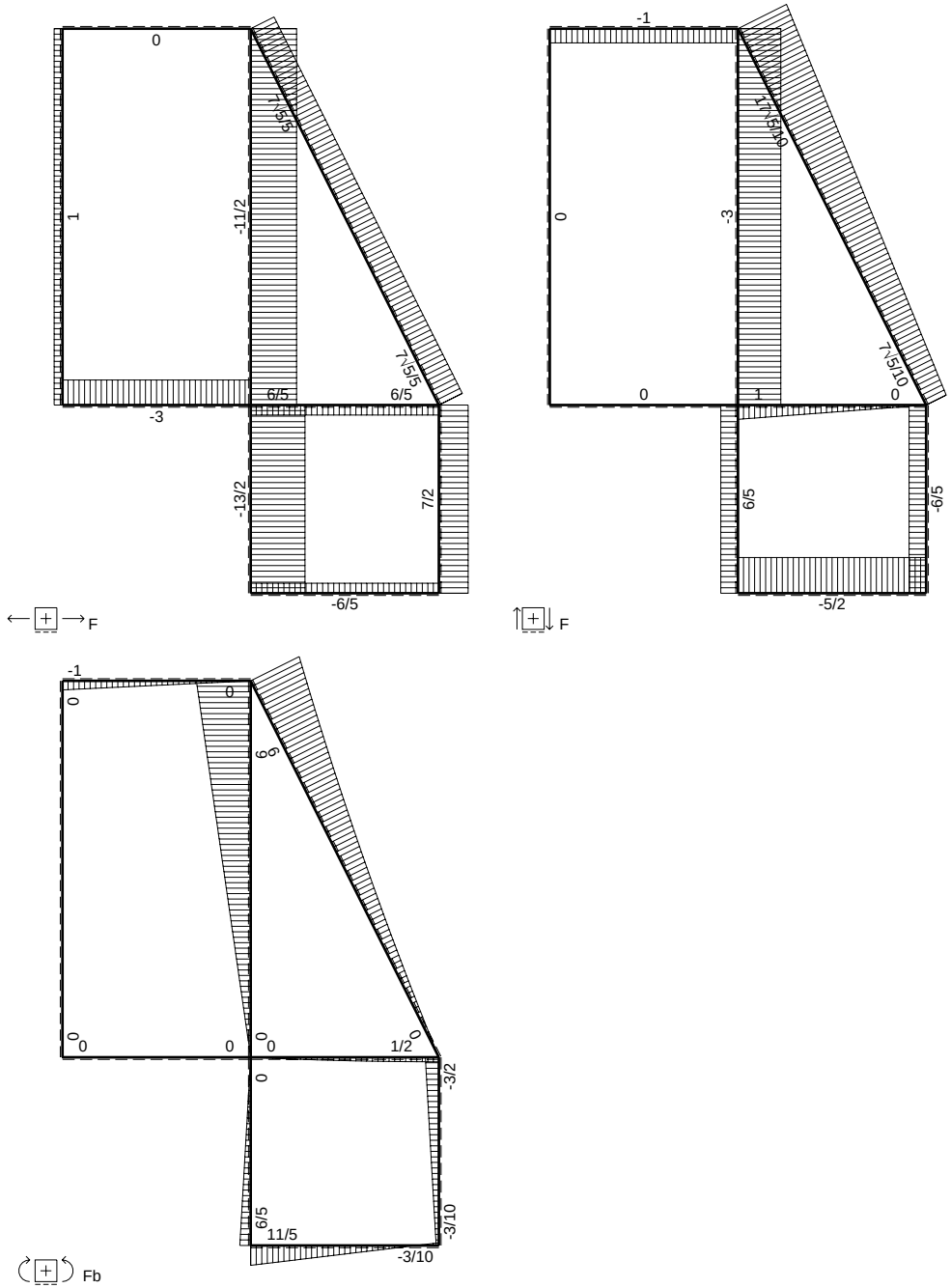
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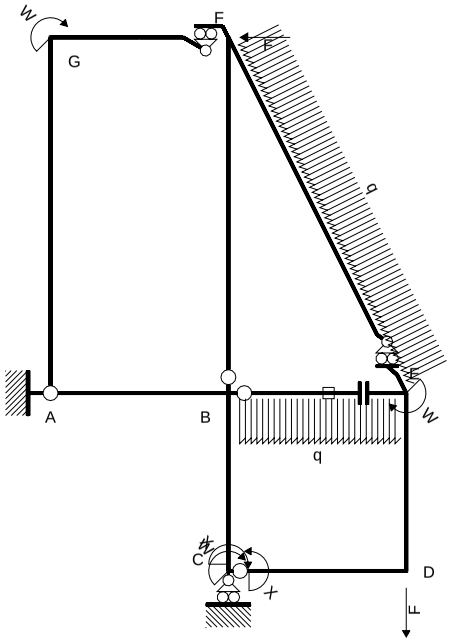
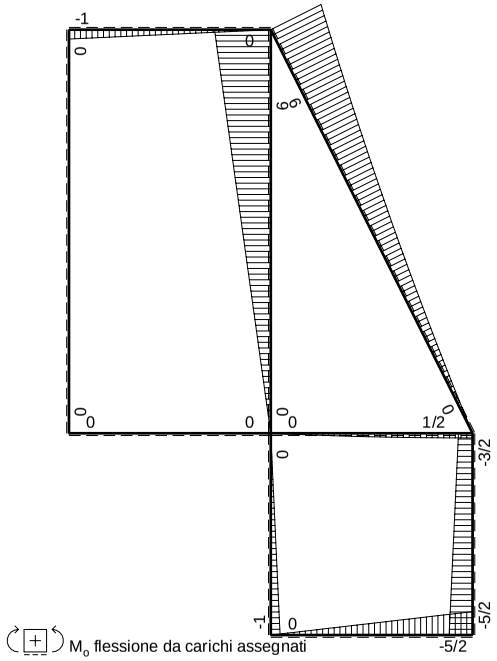
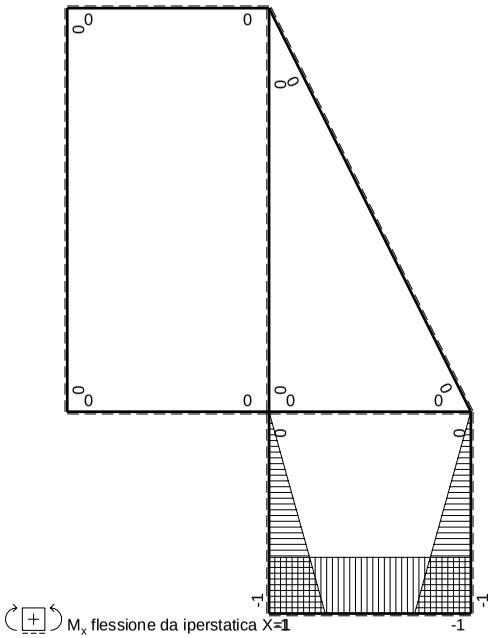
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+Fx	$5/2Fb-7/2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$13/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	3/2Fb+Fx	$3/2Fx+Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$11/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-11/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -7/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -7/4\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -7/4\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 13/12\ Fb^2/EJ$

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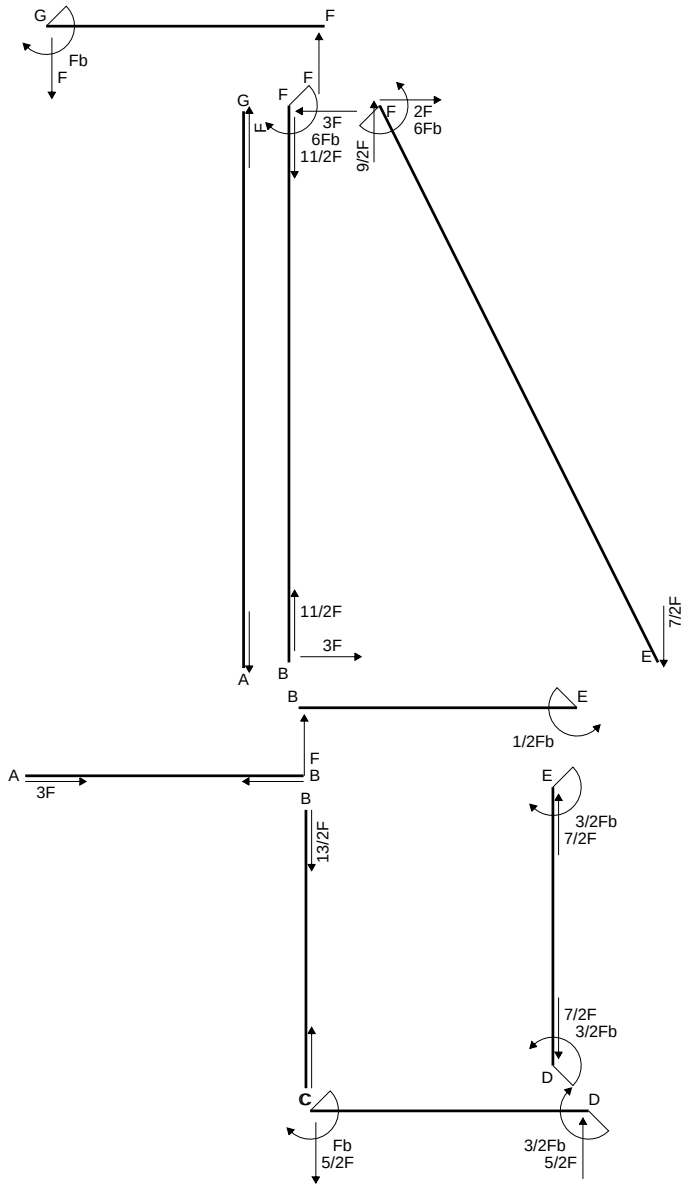
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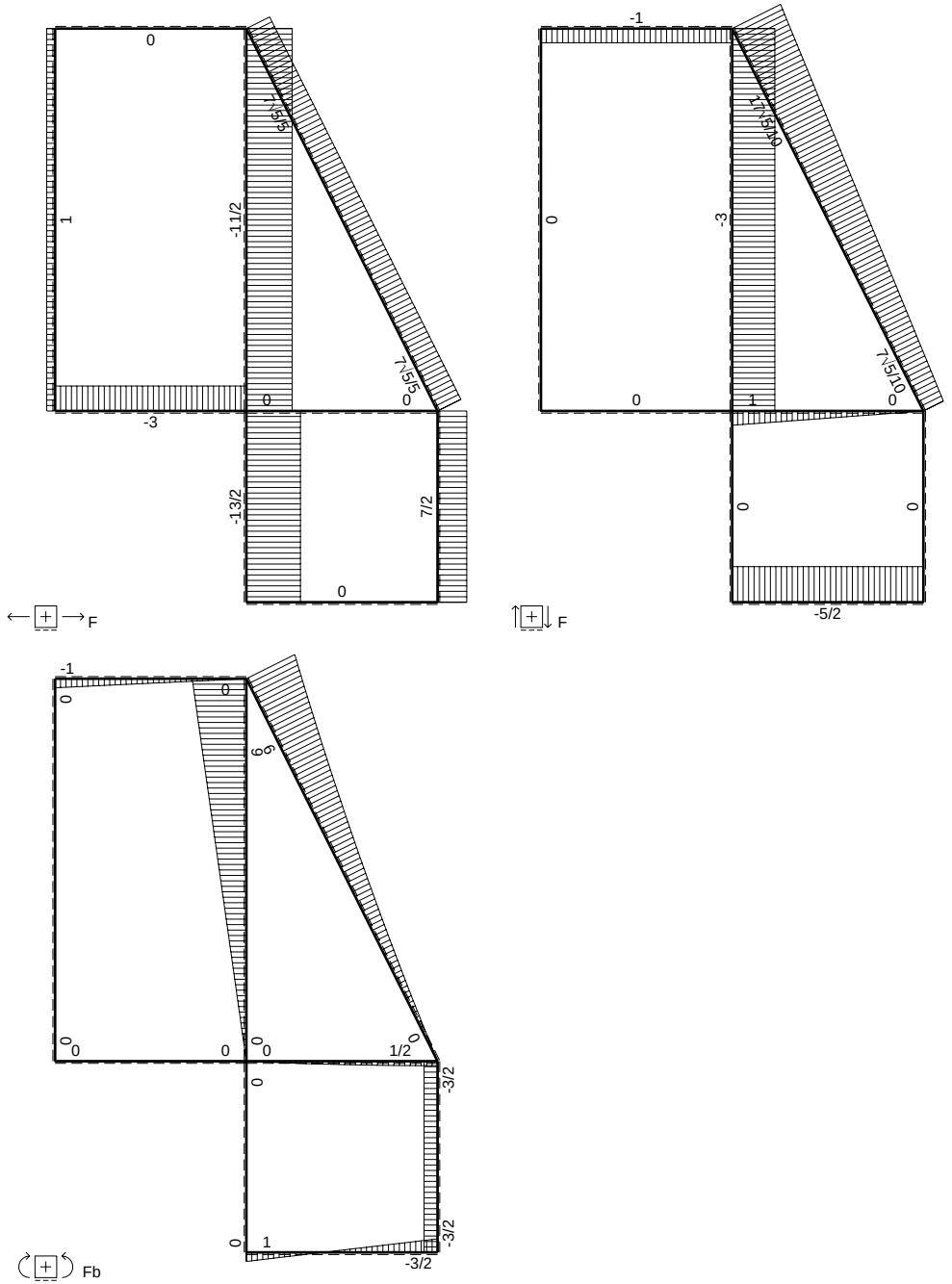
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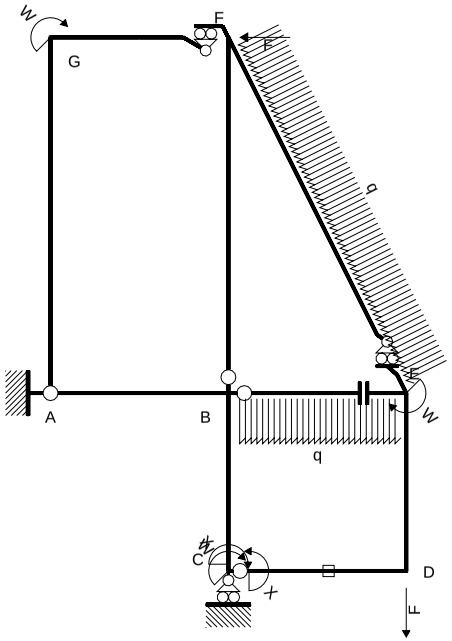
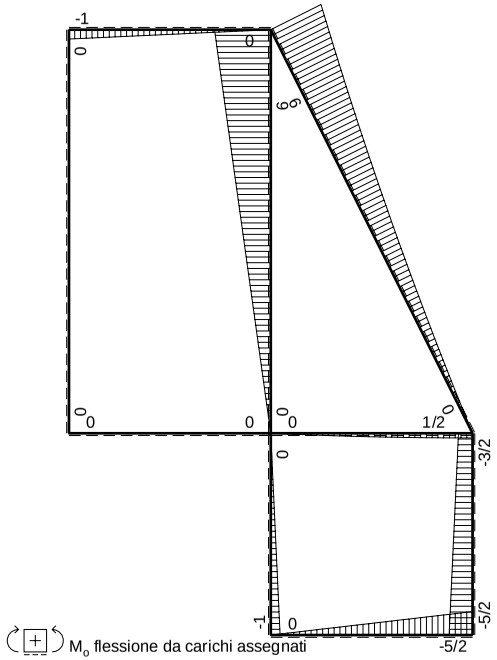
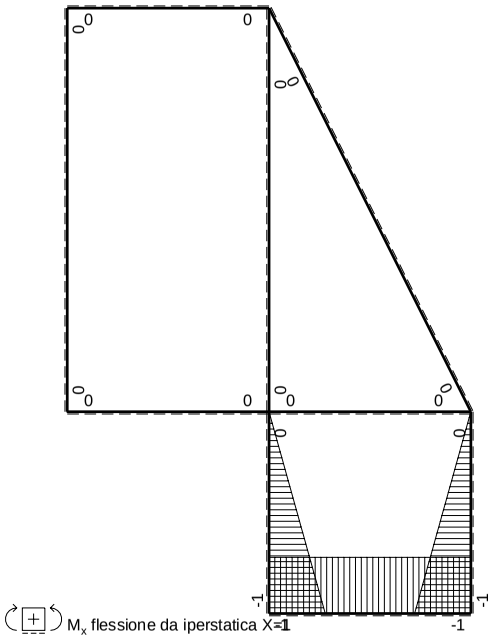


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+Fx	$5/2Fb-7/2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$13/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	3/2Fb+Fx	$3/2Fx+Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -7/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -7/4\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -7/4\ b +1/3\ b)\ Fb\ 1/EJ\ = 13/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/3\ b)\ Fb\ 1/EJ\ = 13/12\ \ Fb^2/EJ$

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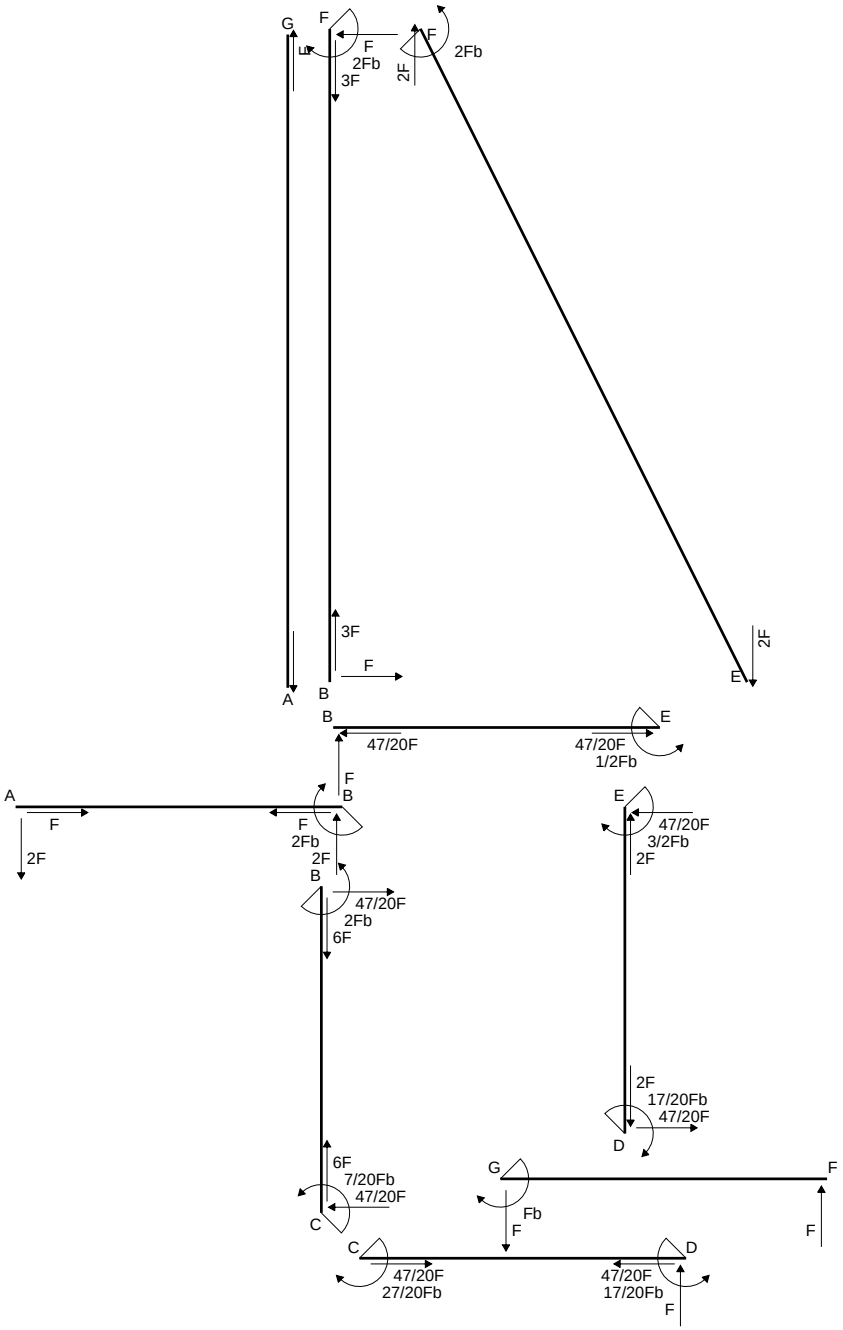
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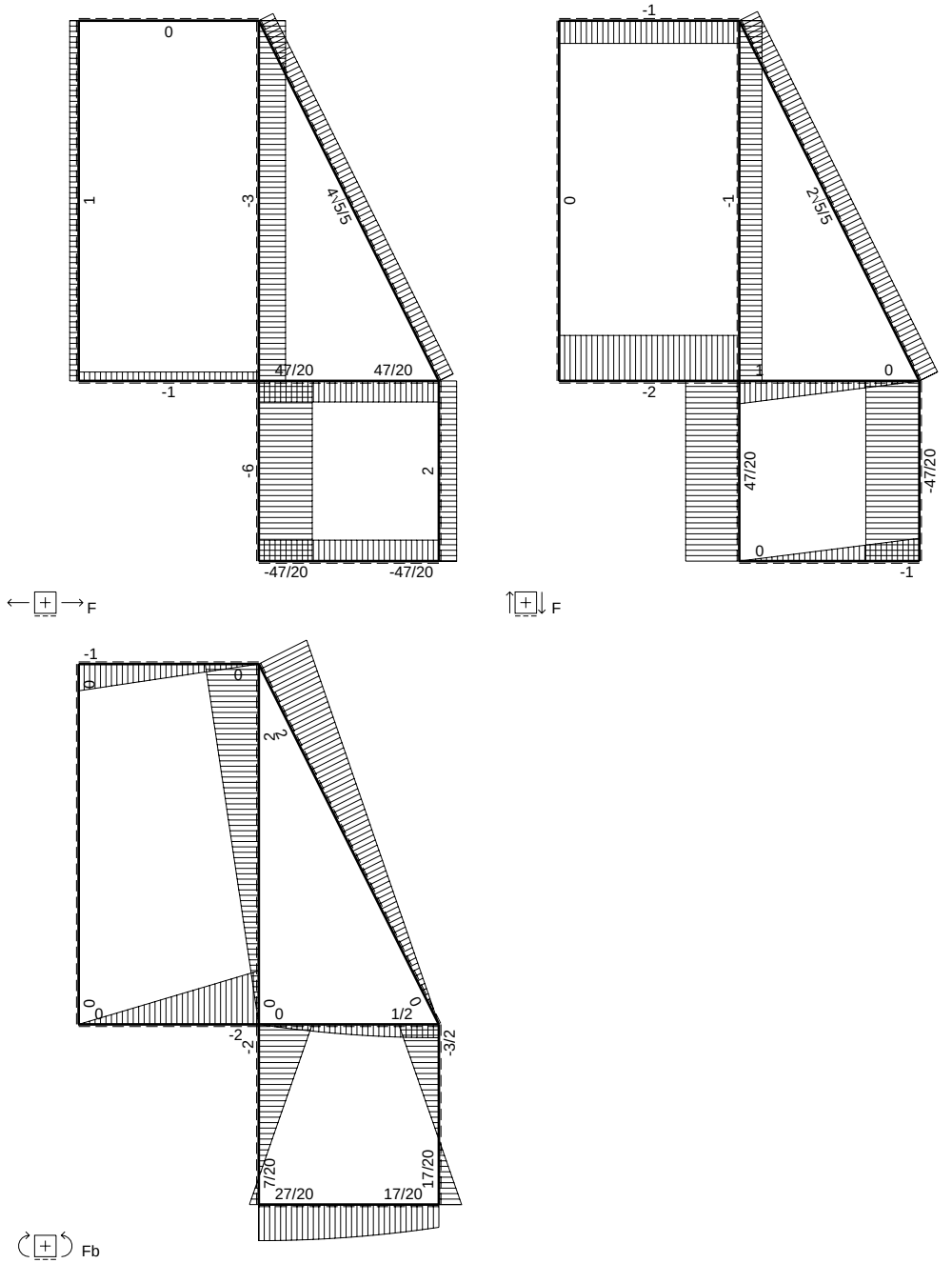
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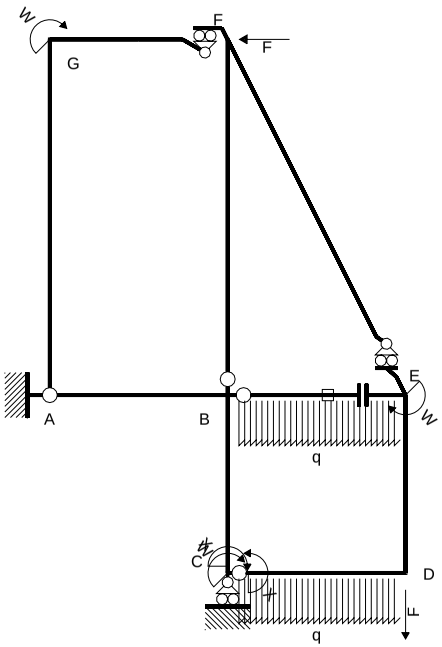
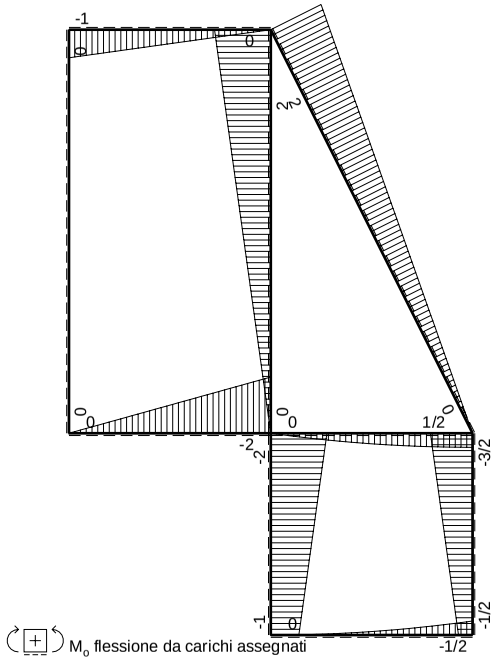
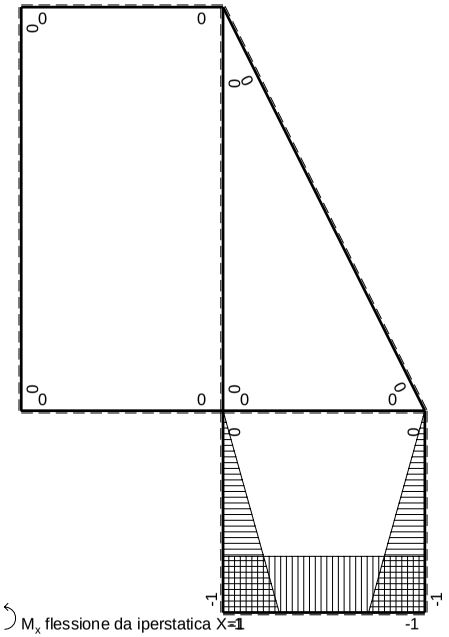


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	$2Fx-Fx^2/b$	x^2/b^2	$2/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+Fx	$Fb-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb-Fx	$1/2Fb+1/2Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	3/2Fb-Fx	$3/2Fx-Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	-1/2Fb+1/2qx²	0	0	0	0
EB b	0	Fx-1/2qx²	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$9/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-27/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (2\ x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 +1/2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x +1/4\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/4\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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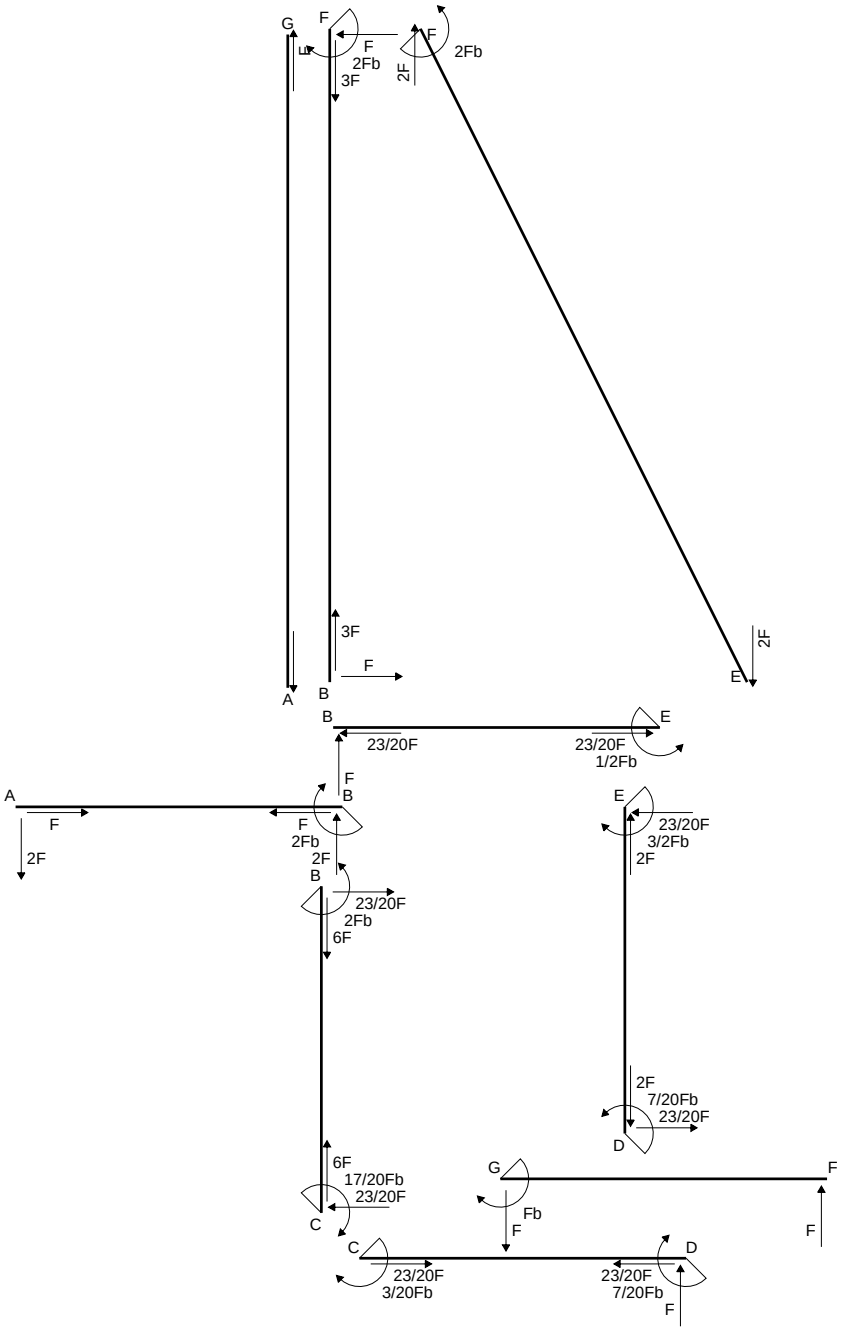
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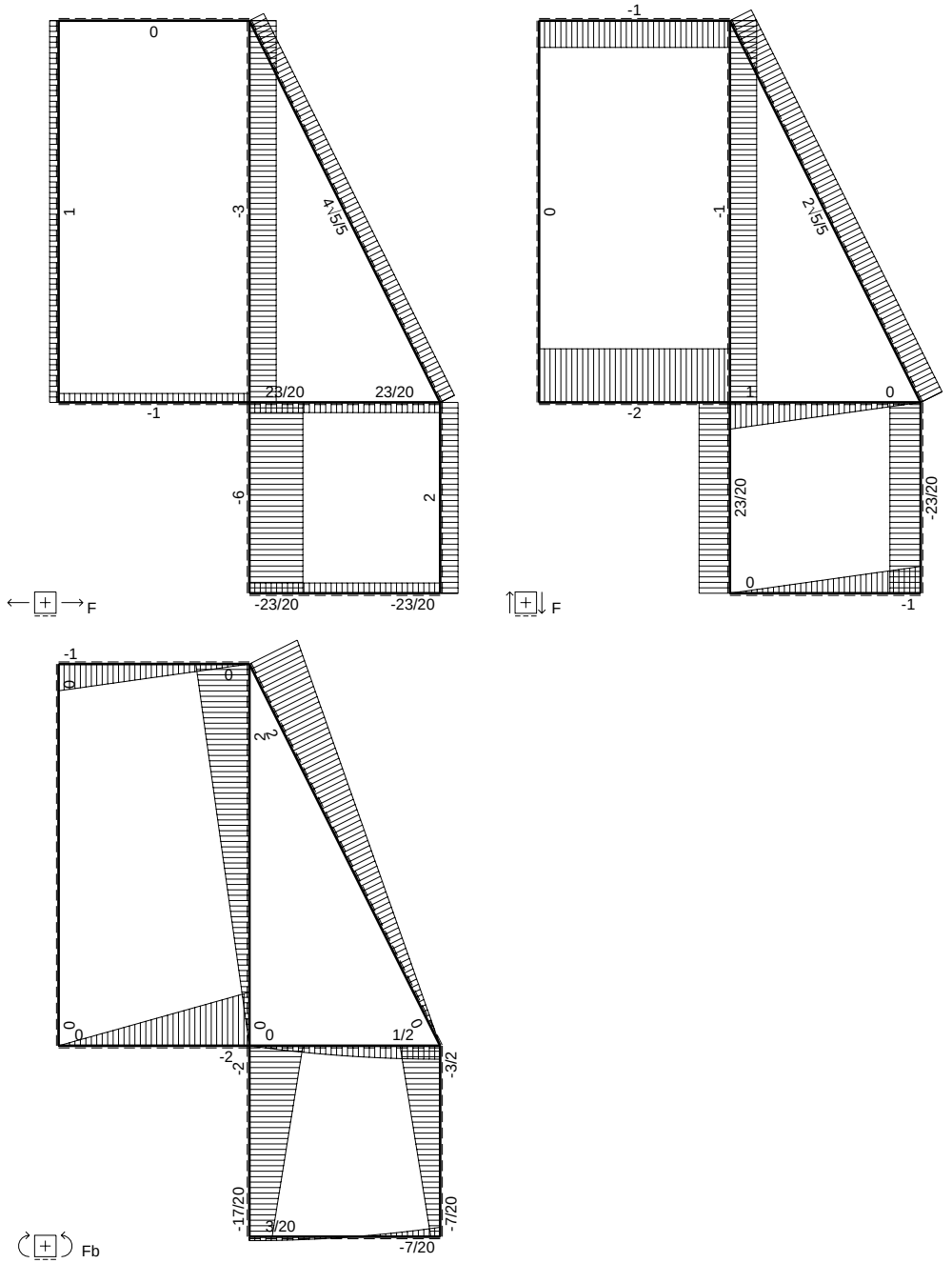
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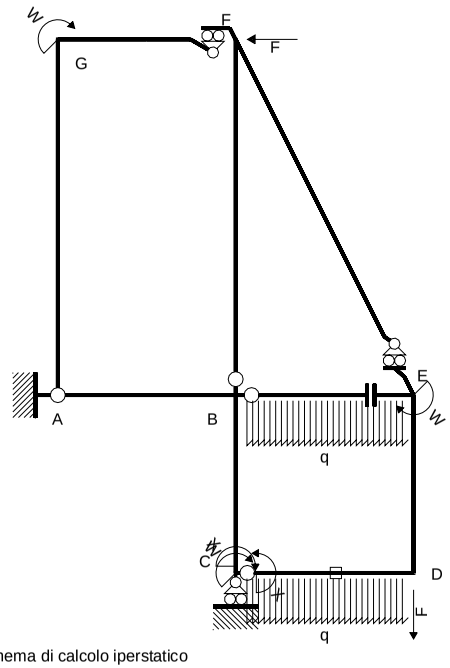
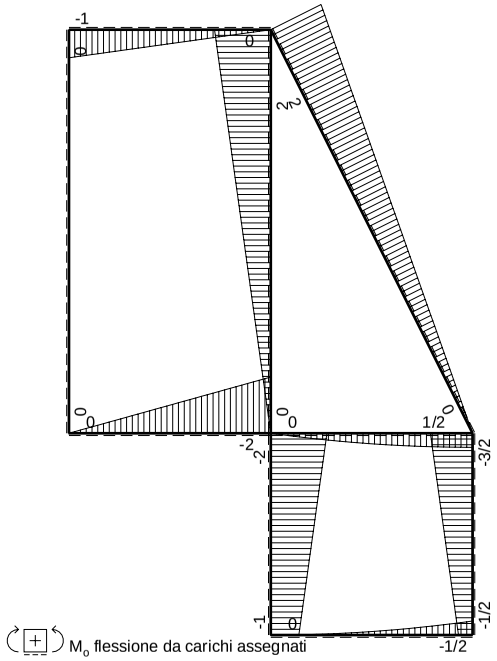
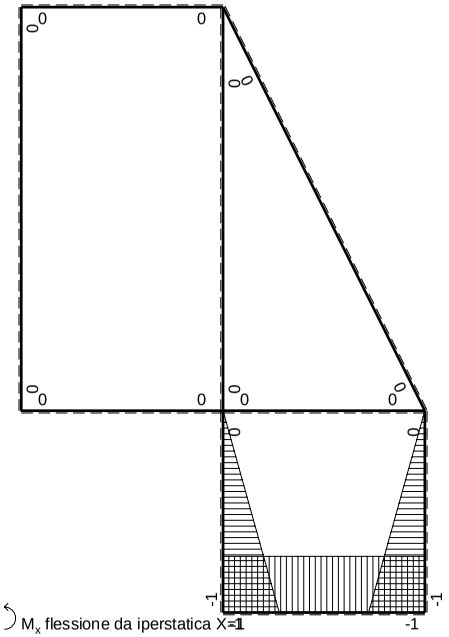


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	$2Fx-Fx^2/b$	x^2/b^2	$2/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+Fx	$Fb-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb-Fx	$1/2Fb+1/2Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	3/2Fb-Fx	$3/2Fx-Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 +1/2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x +1/4\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/4\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 5/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 5/12\ \ Fb^2/EJ$

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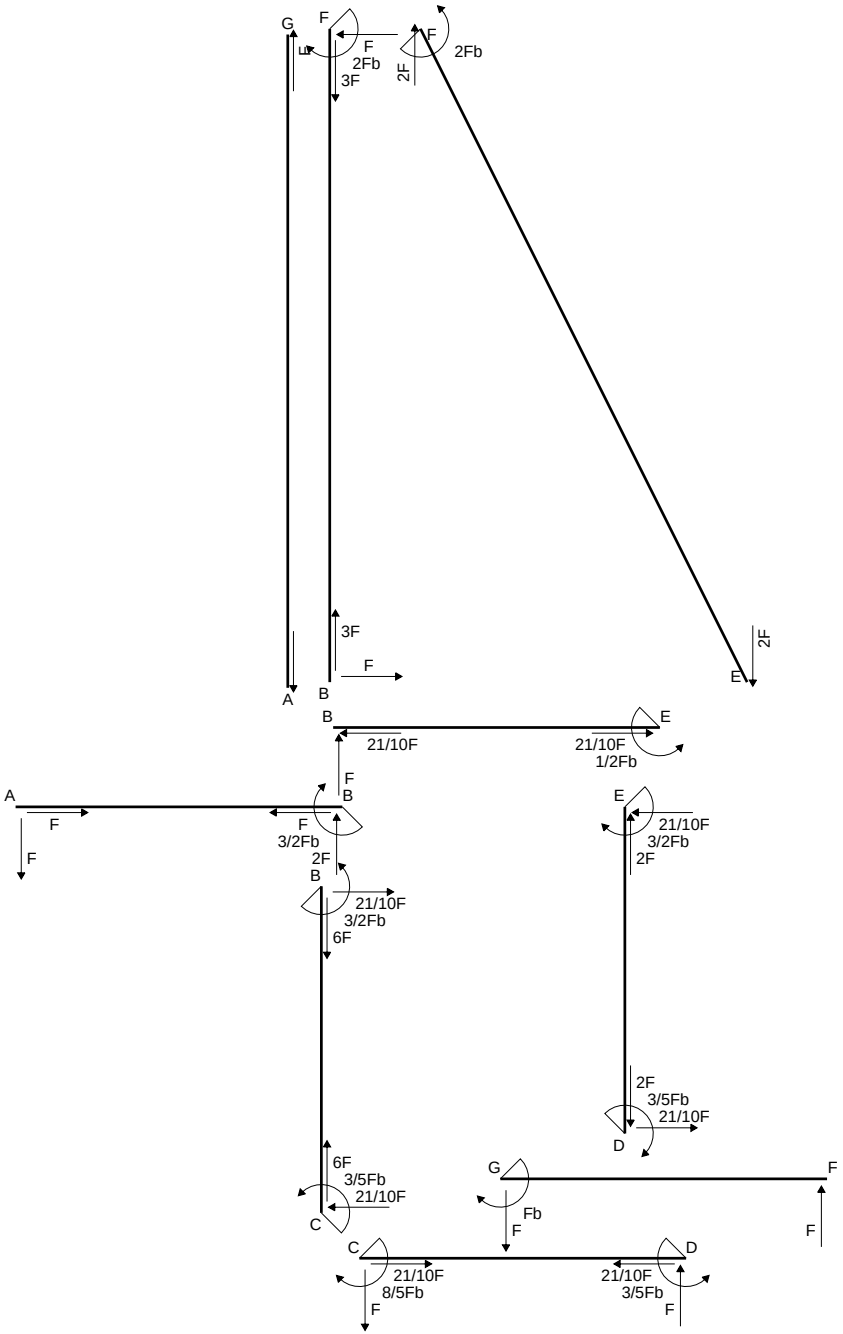
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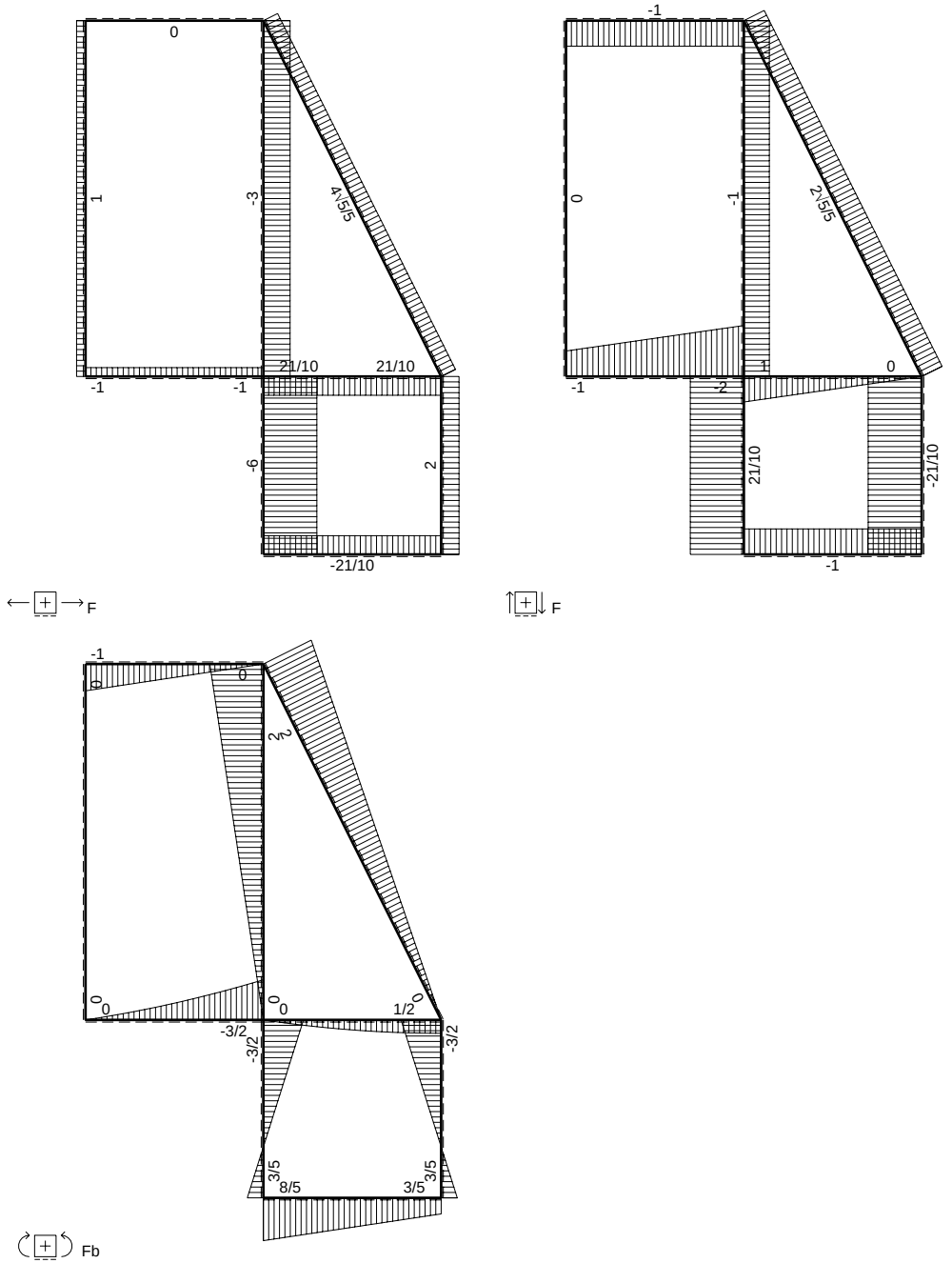
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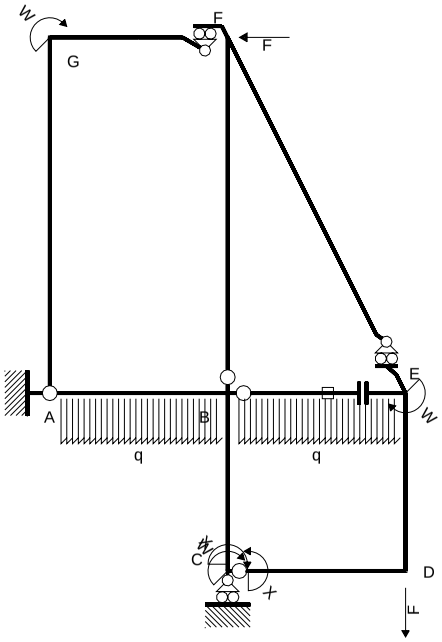
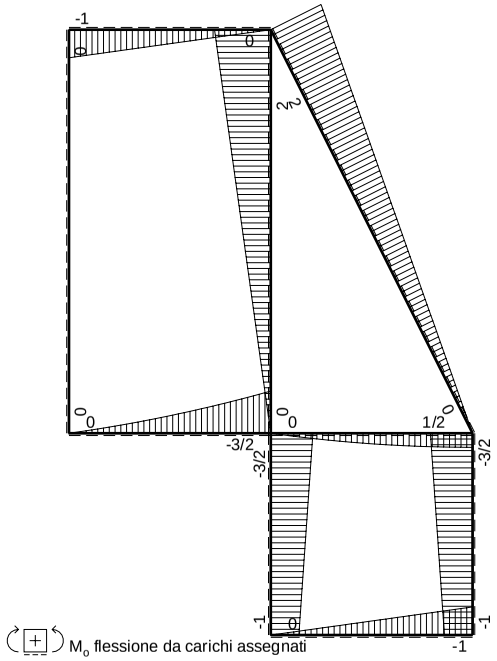
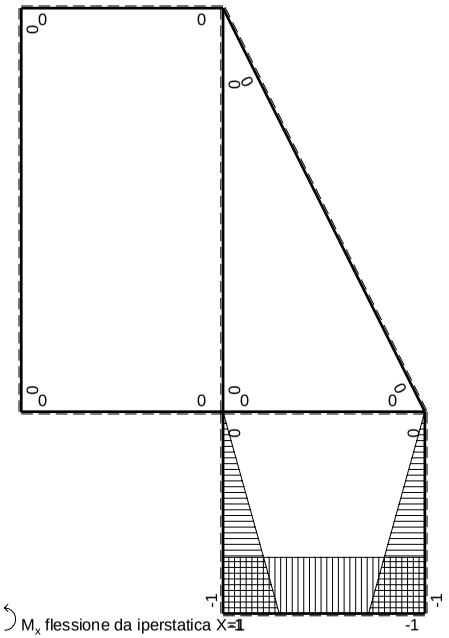


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$3/2Fb-1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$8/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-8/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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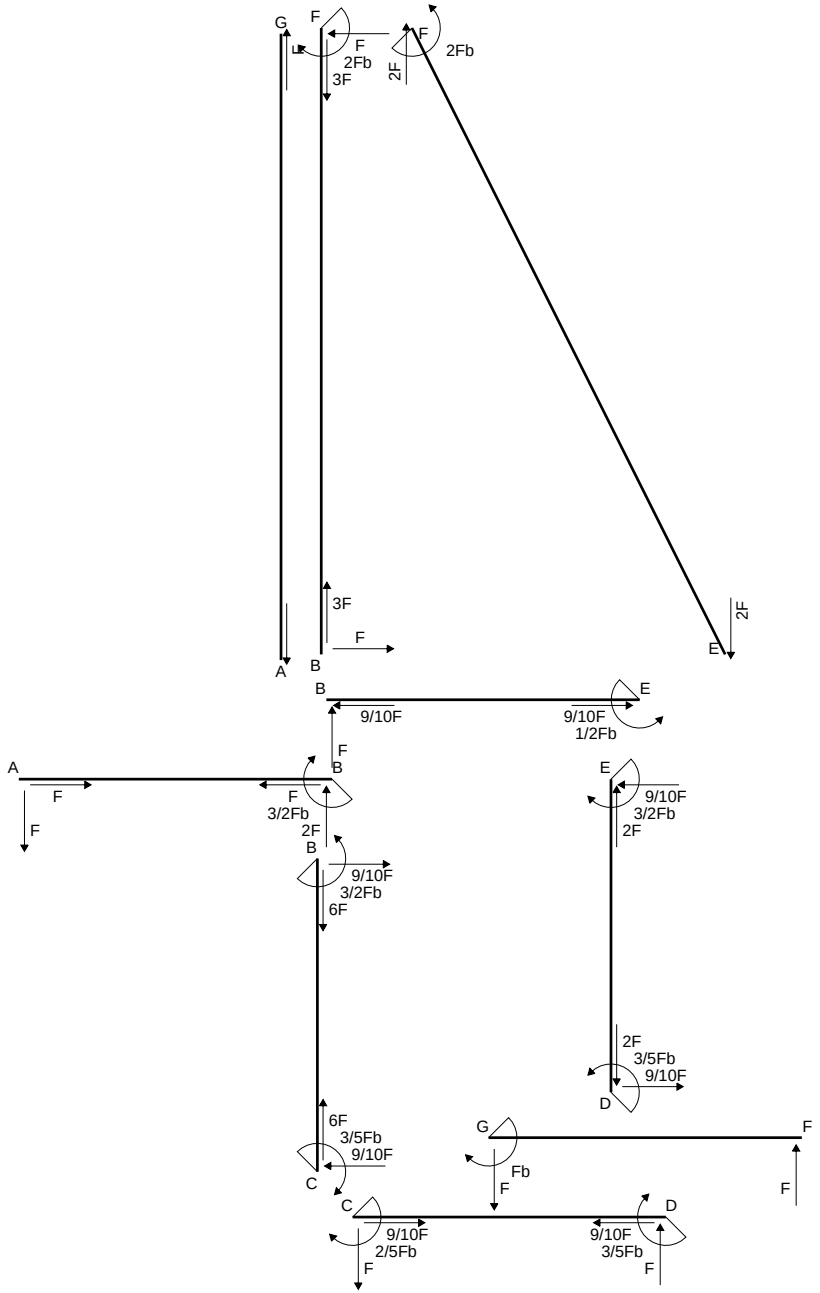
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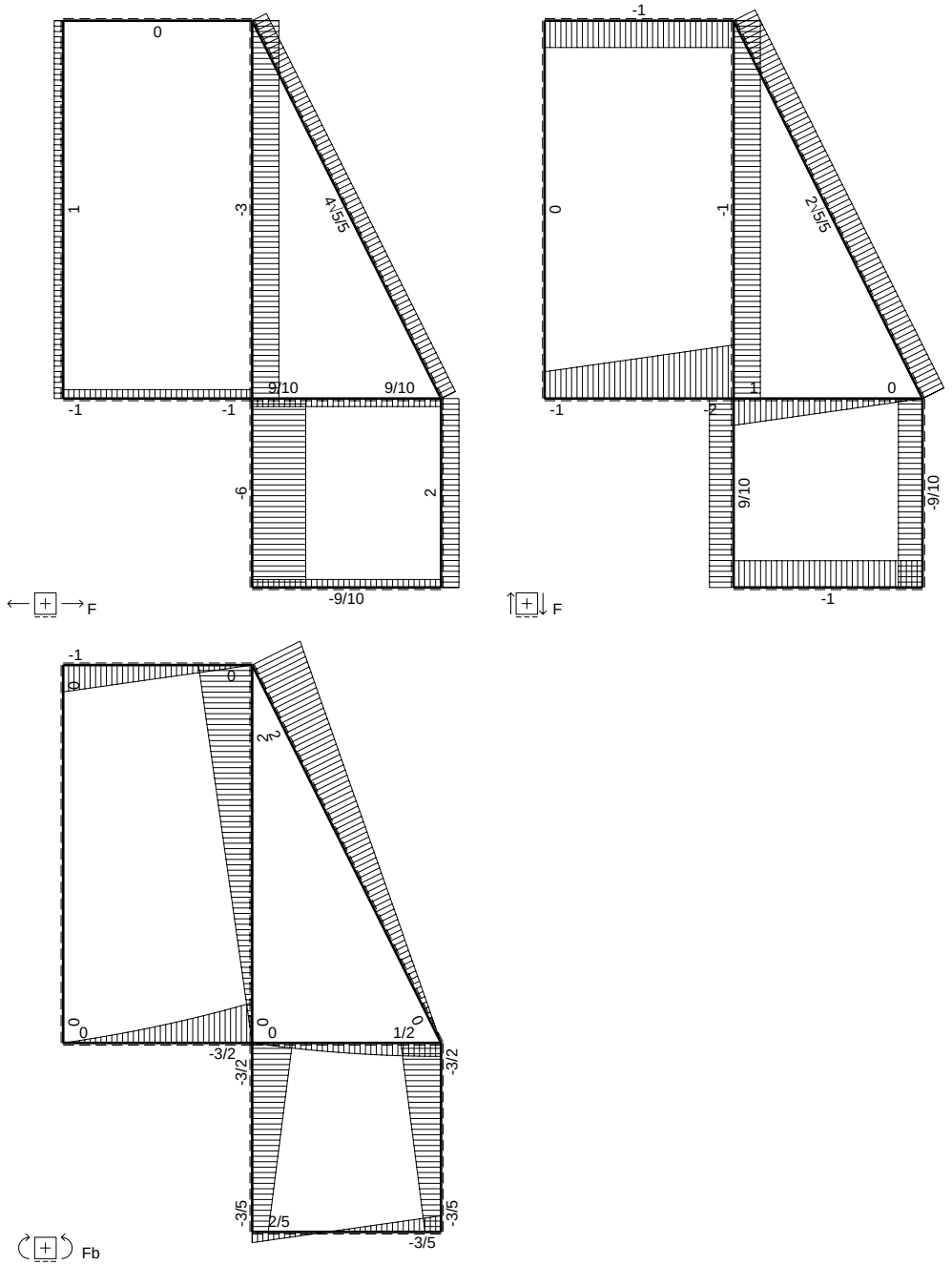
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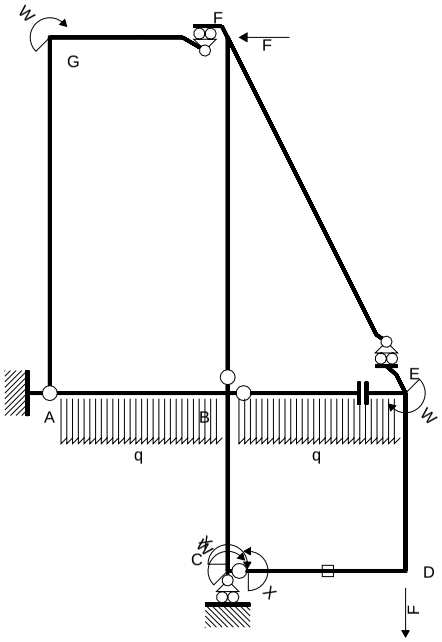
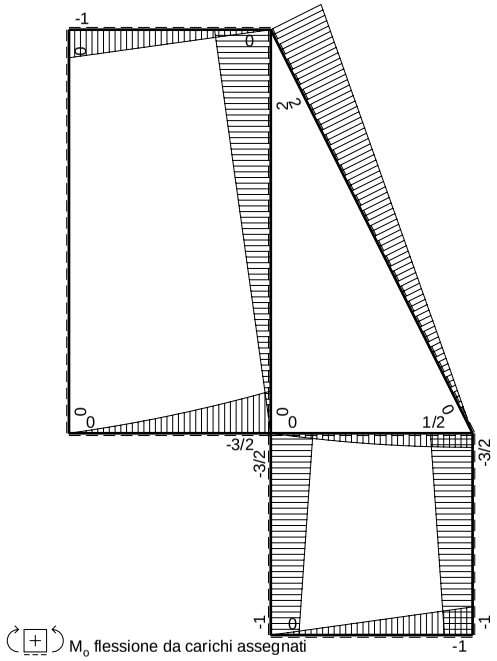
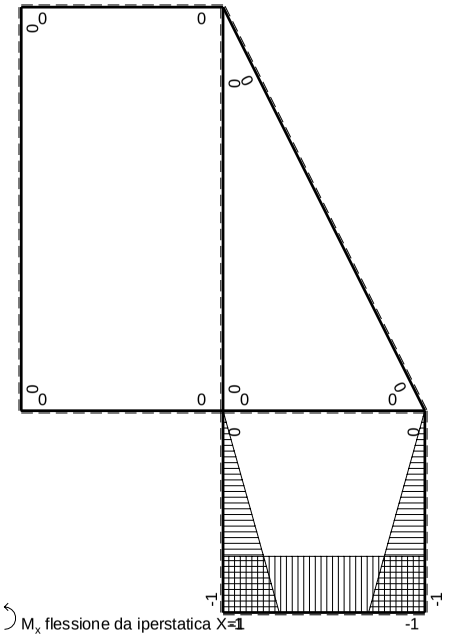
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$3/2Fb-1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$2/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-2/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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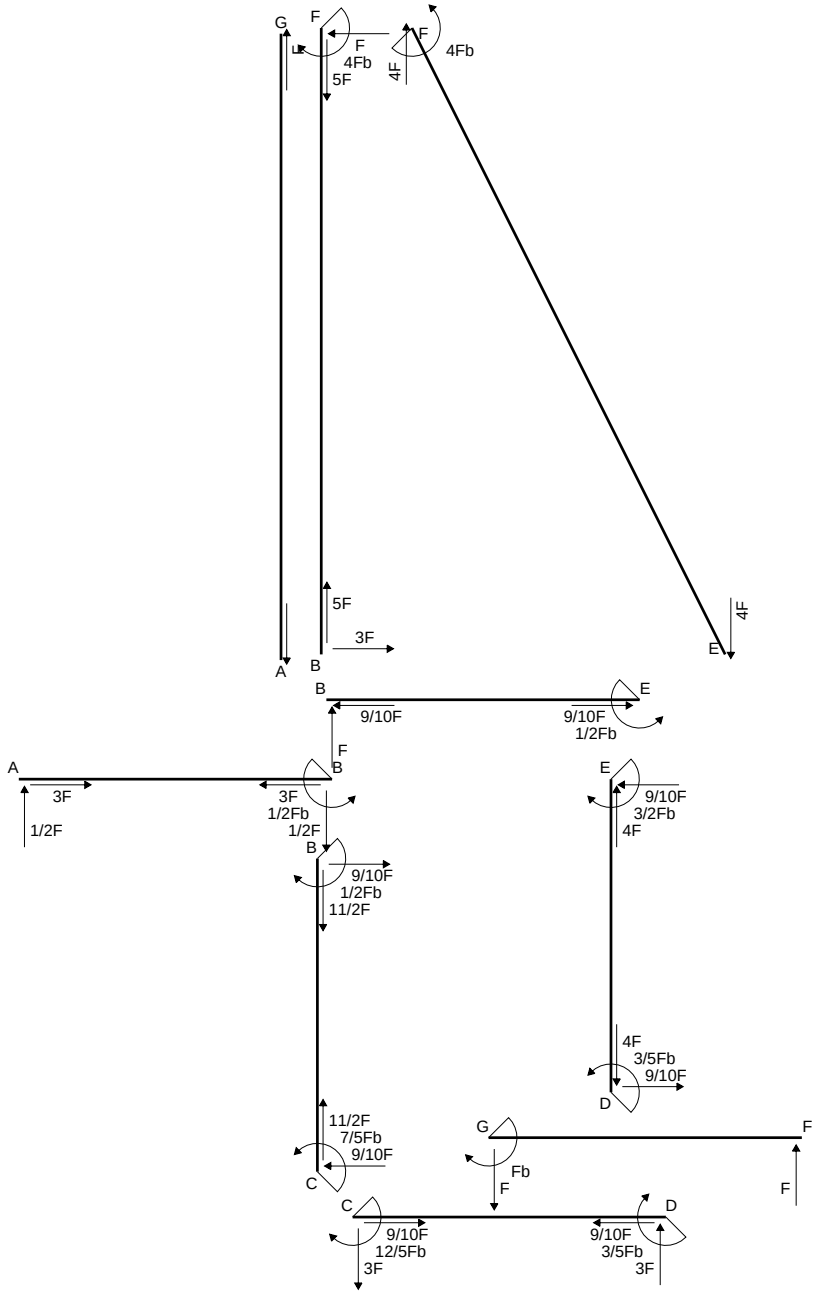
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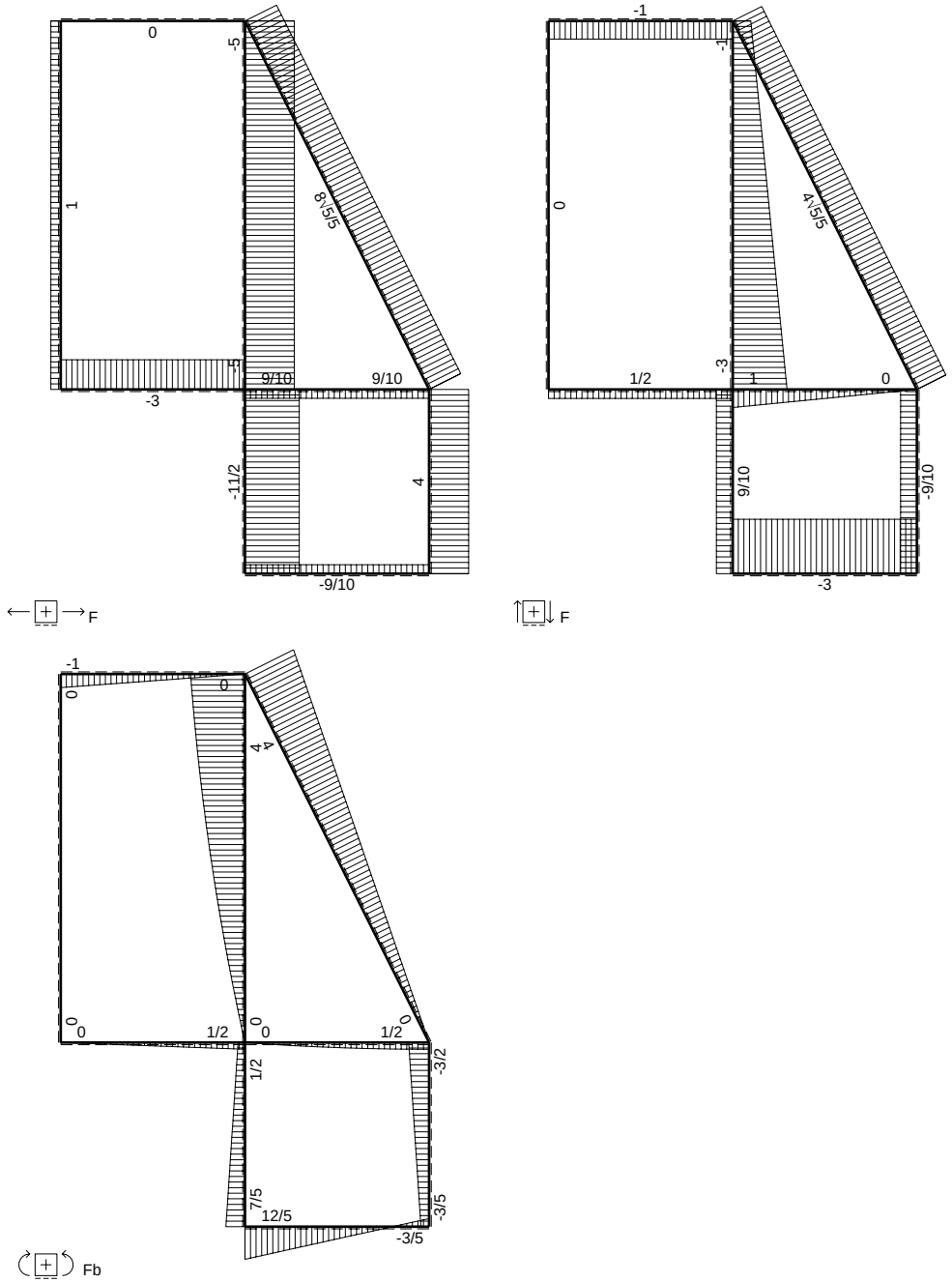
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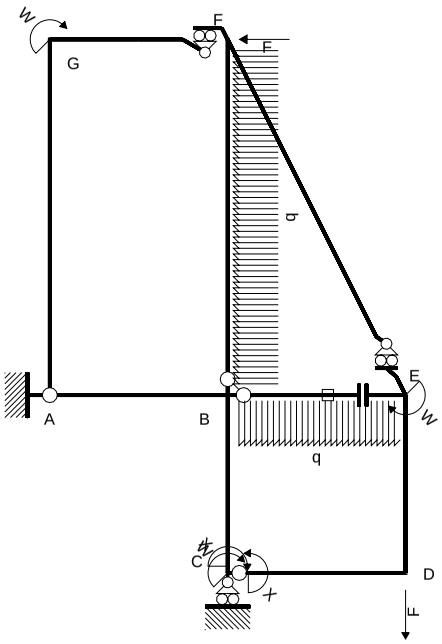
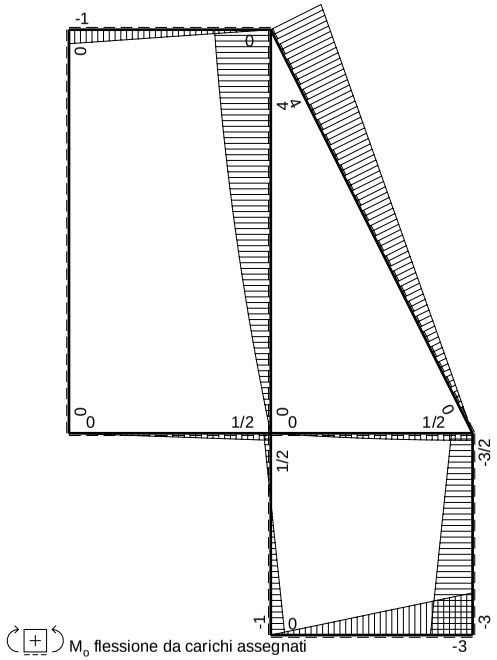
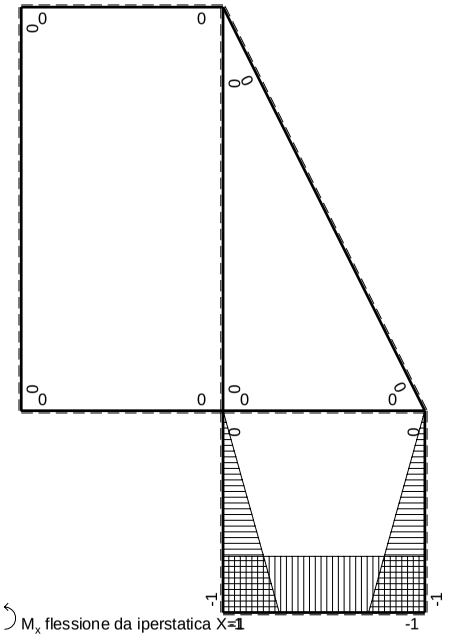


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3/2Fx	$3Fb-9/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/4Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	3/2Fb+3/2Fx	$3/2Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-12/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3\ x/b)\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b)\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b)\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b)\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -9/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -9/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -9/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

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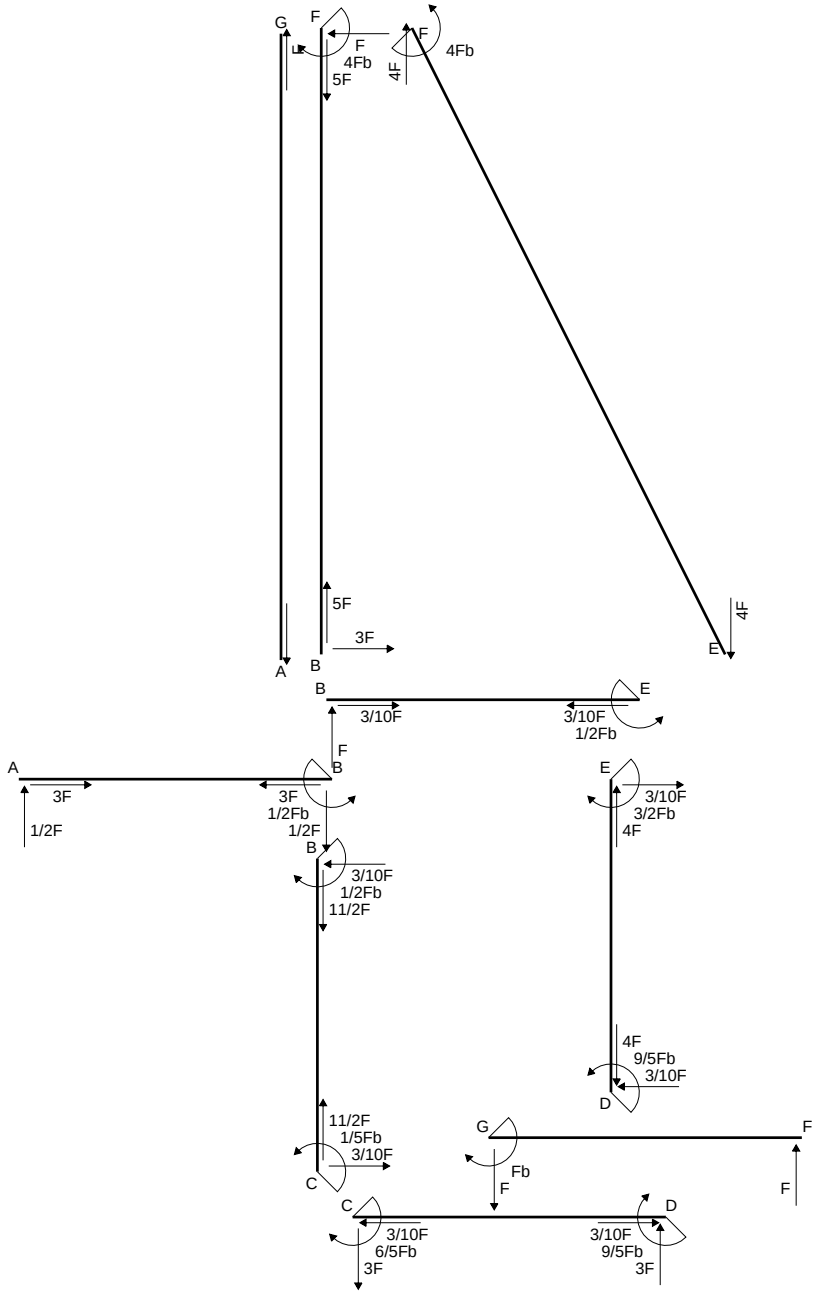
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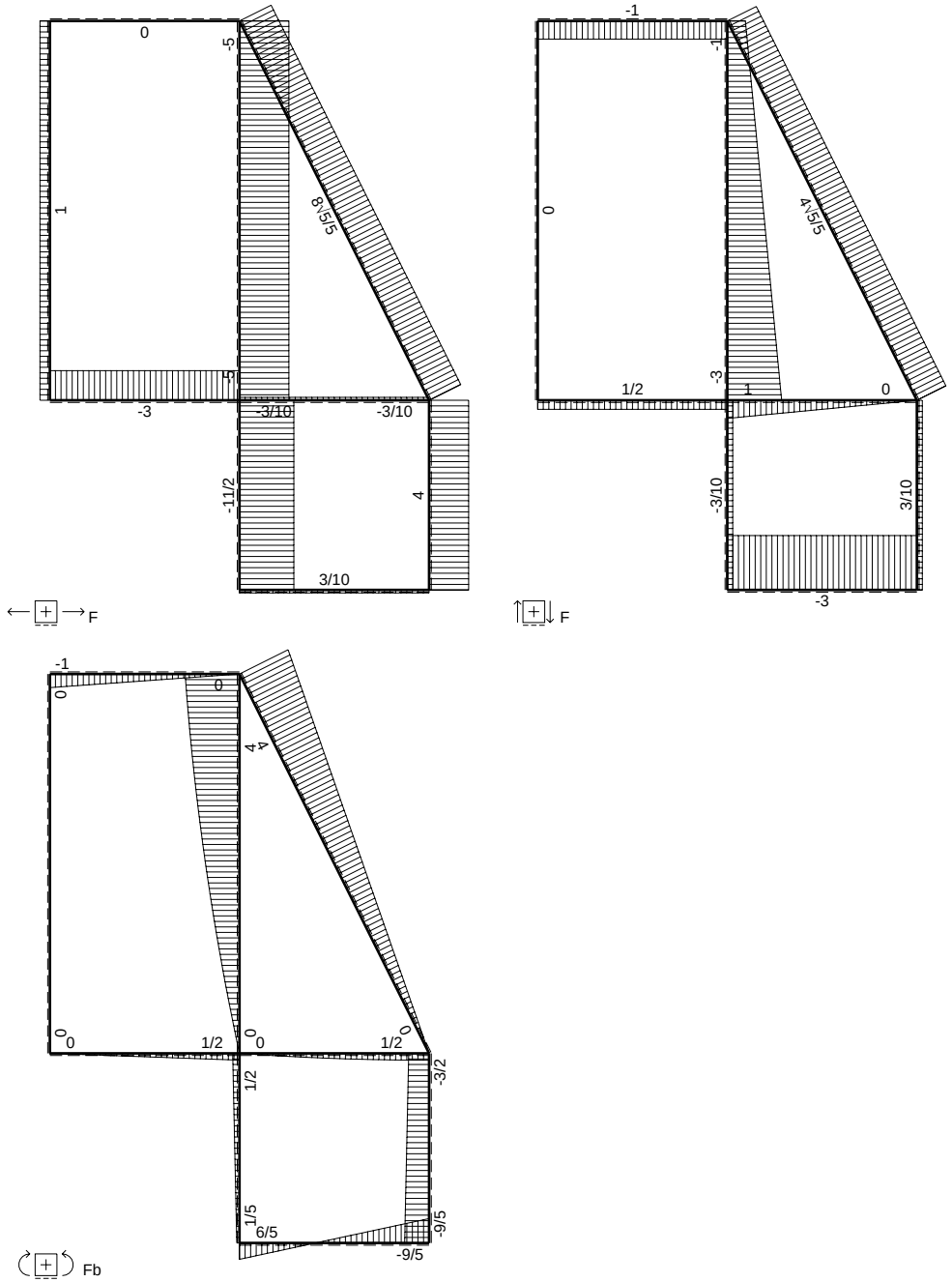
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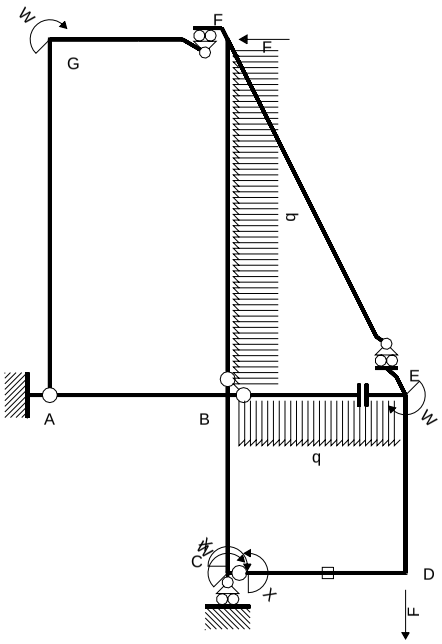
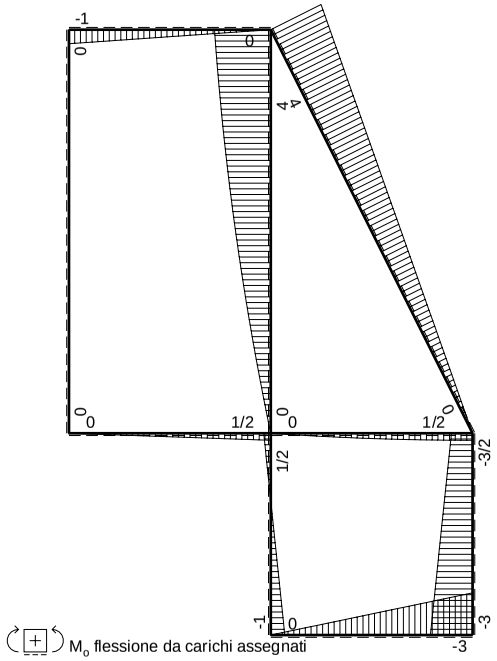
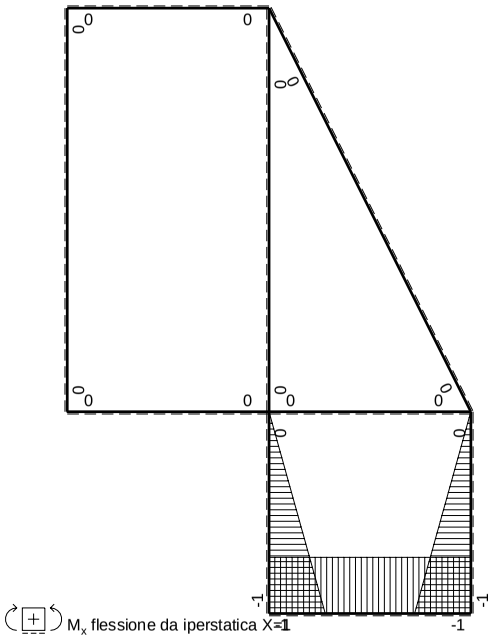


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	$-3Fb+3/2Fx$	$3Fb-9/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/4Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$3/2Fb+3/2Fx$	$3/2Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-6/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -9/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -9/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -9/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

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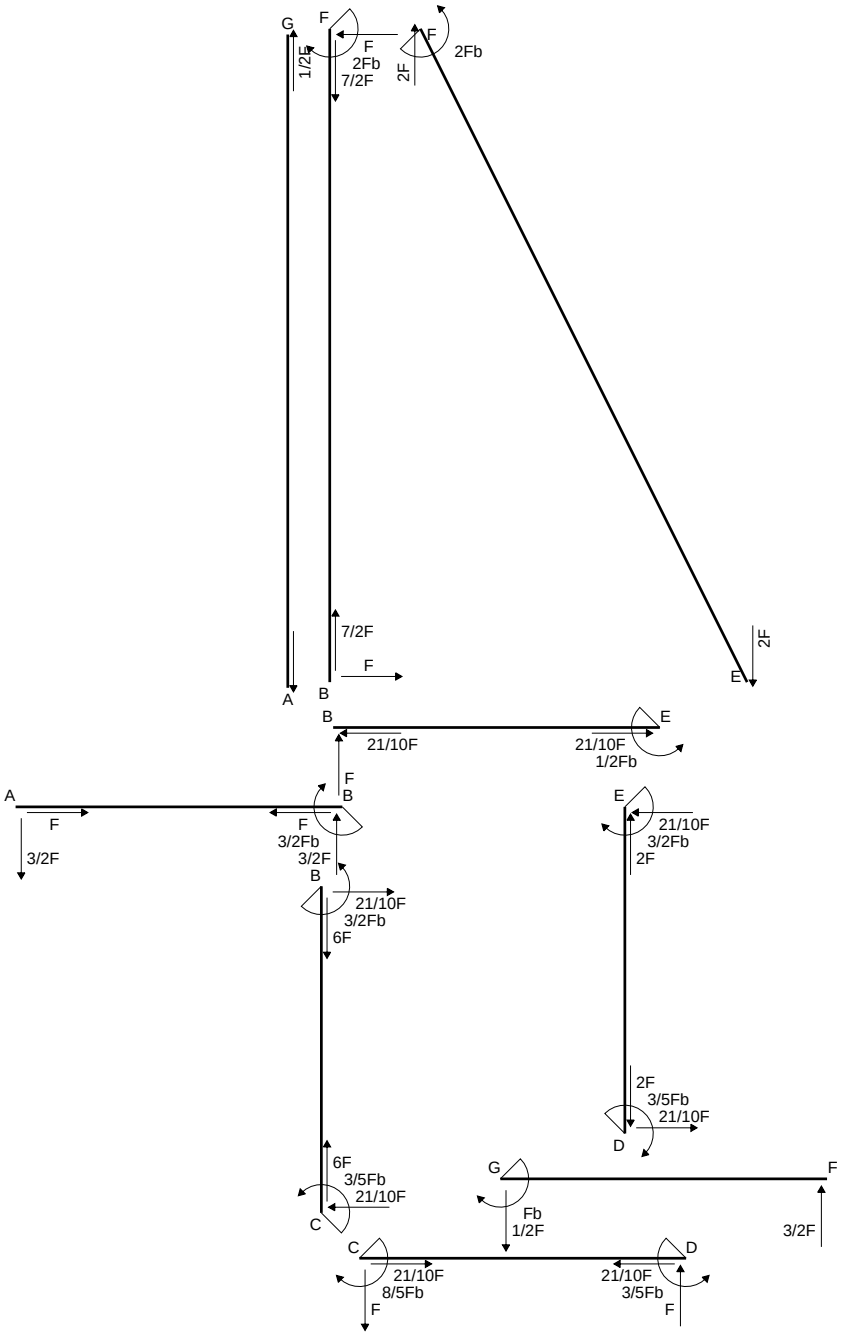
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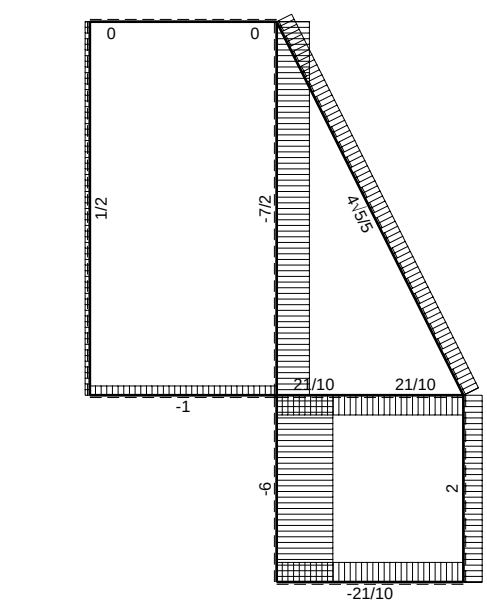
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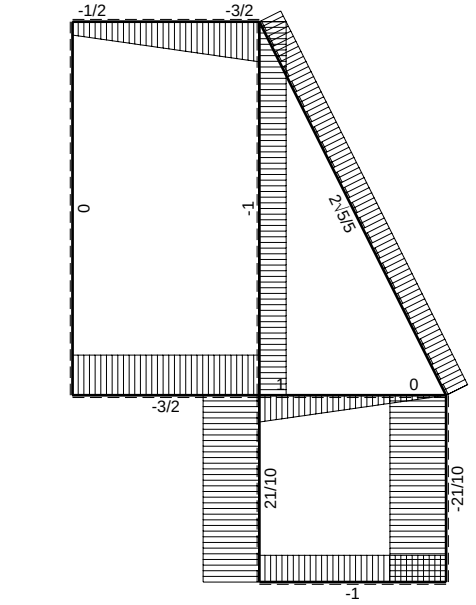
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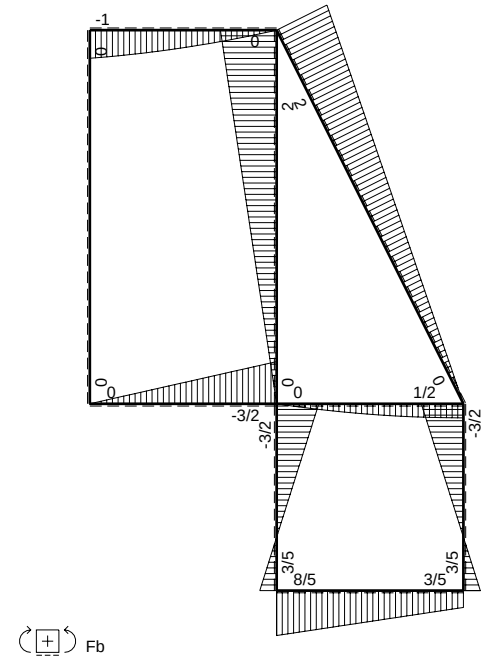
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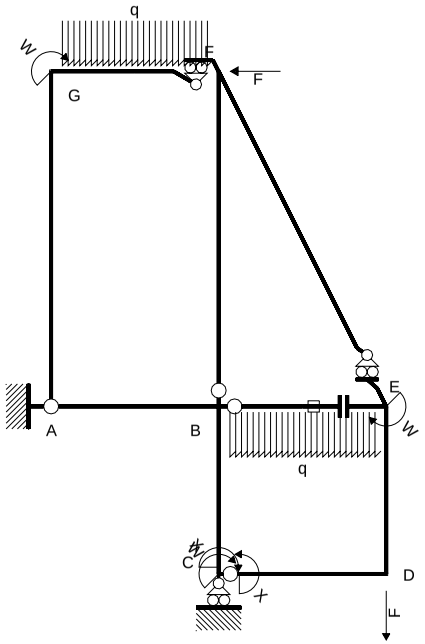
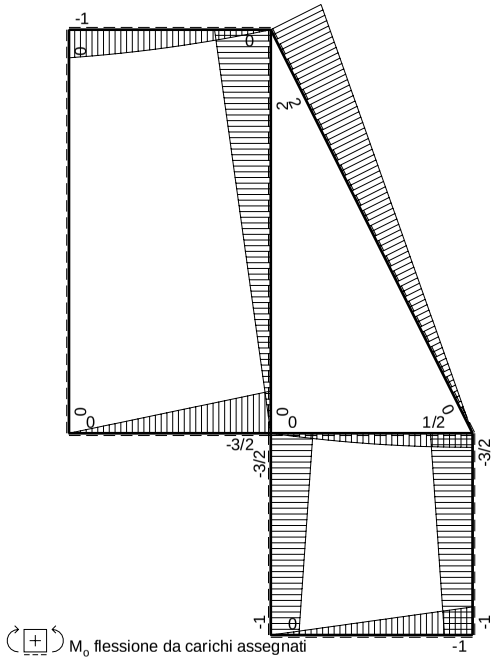
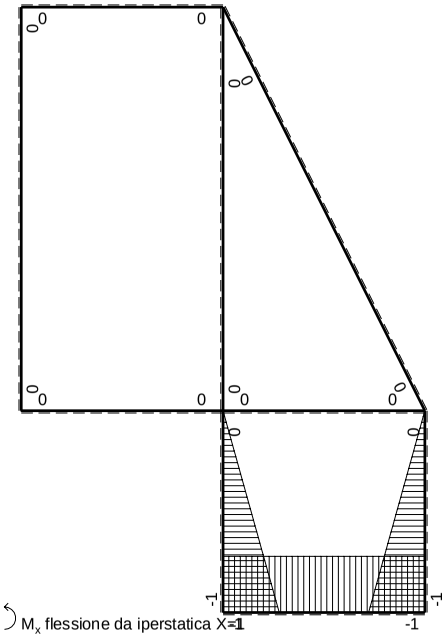


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fb-1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-3/2Fx+1/2qx^2	0	0	0	0
GF b	0	Fb-1/2Fx-1/2qx^2	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				8/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-8/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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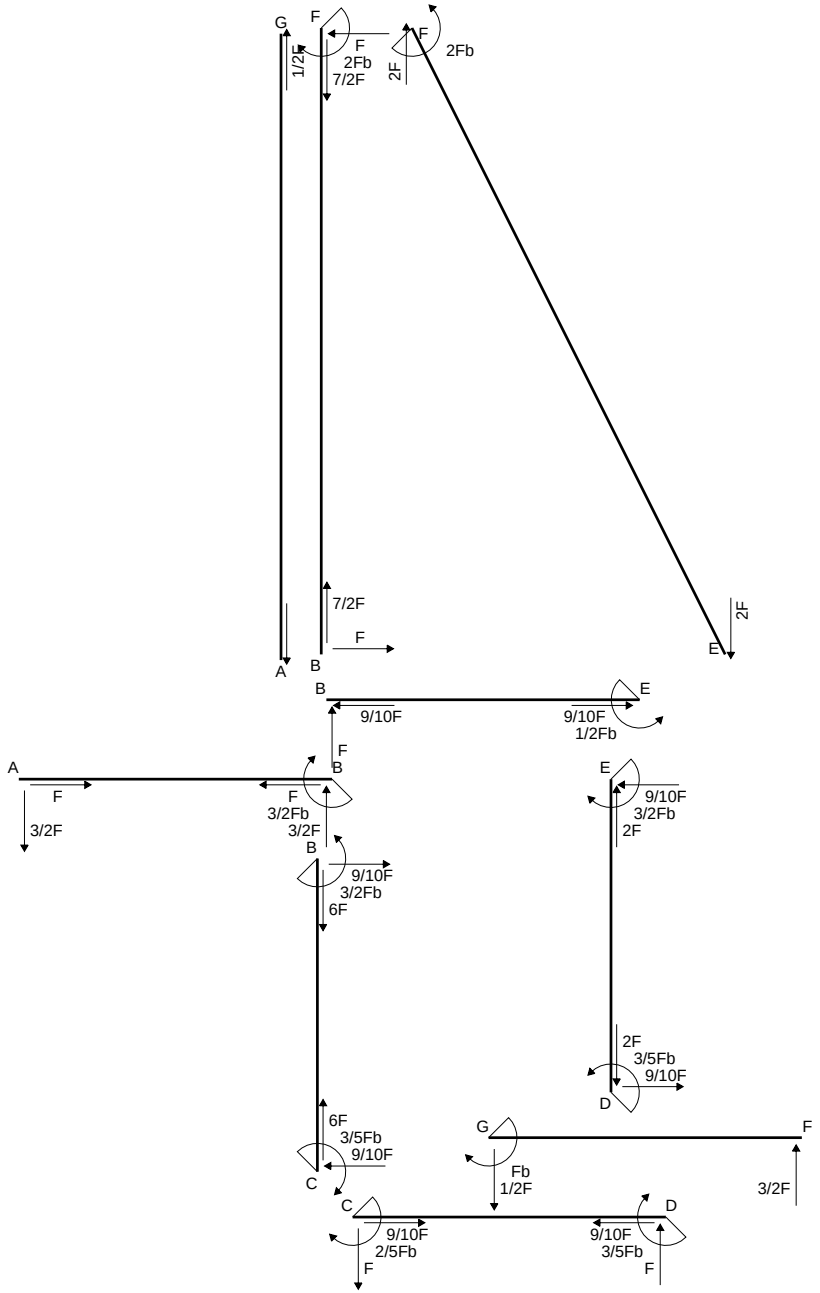
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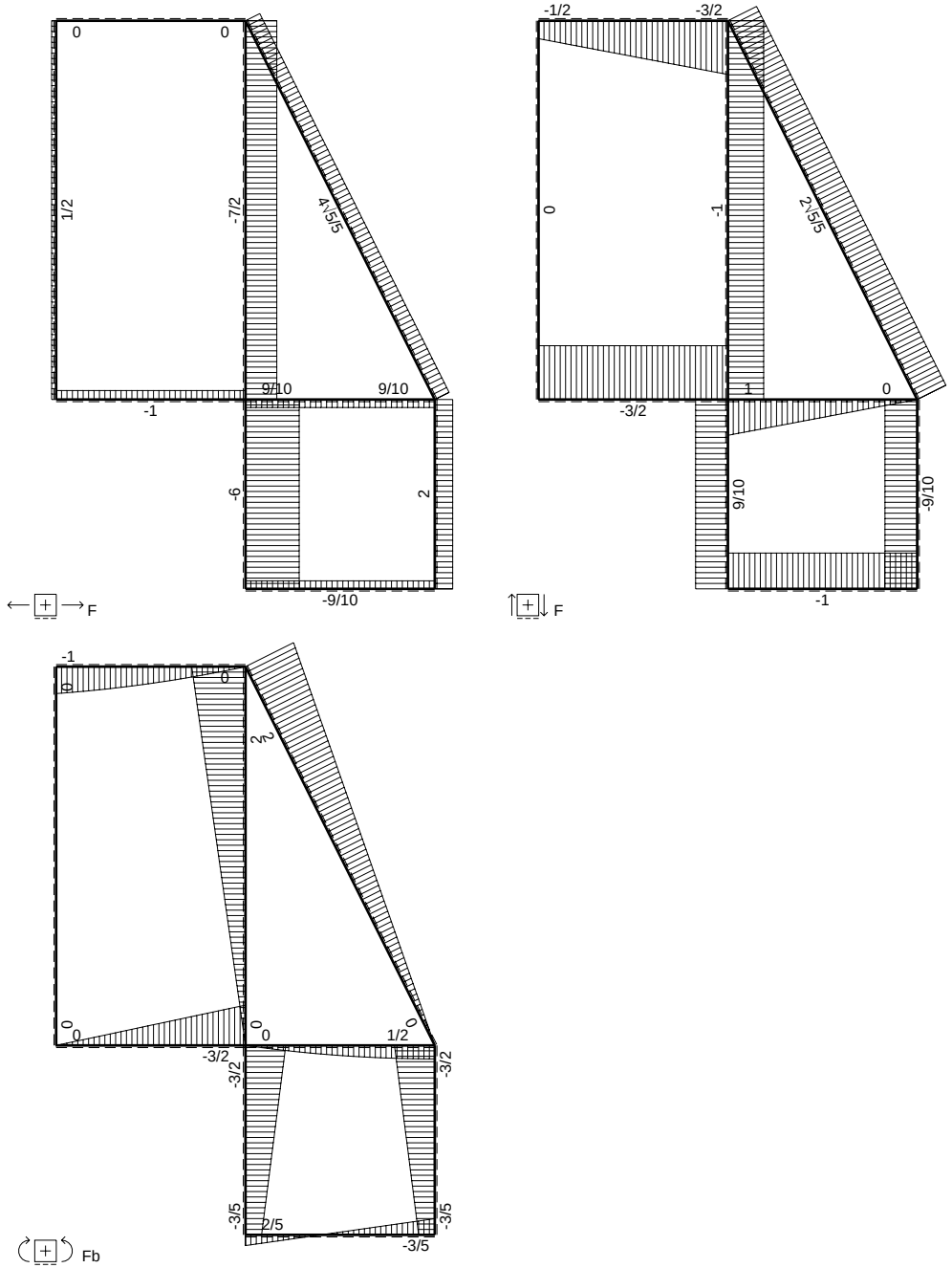
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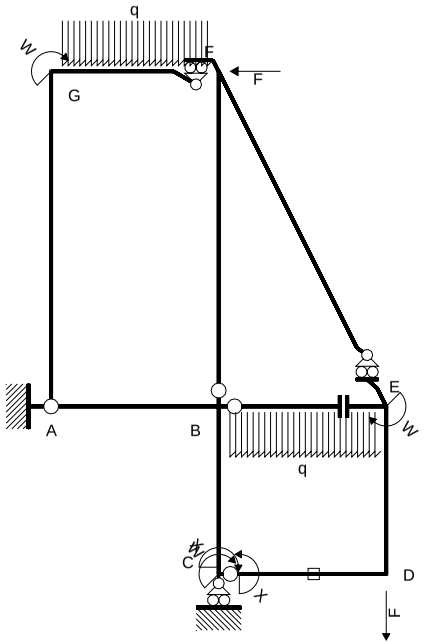
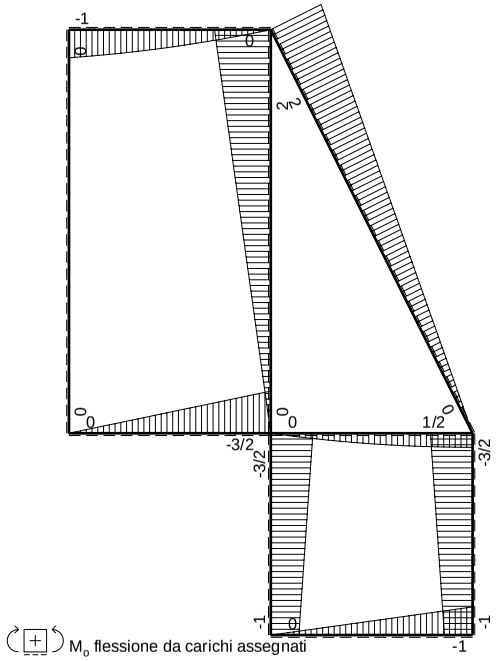
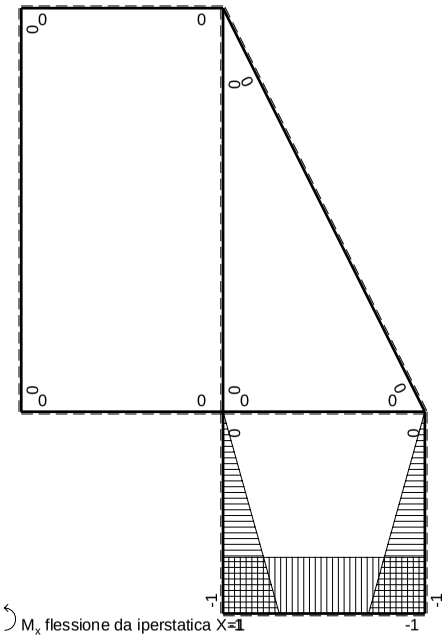


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fb-1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-3/2Fx+1/2qx^2	0	0	0	0
GF b	0	Fb-1/2Fx-1/2qx^2	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				2/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-2/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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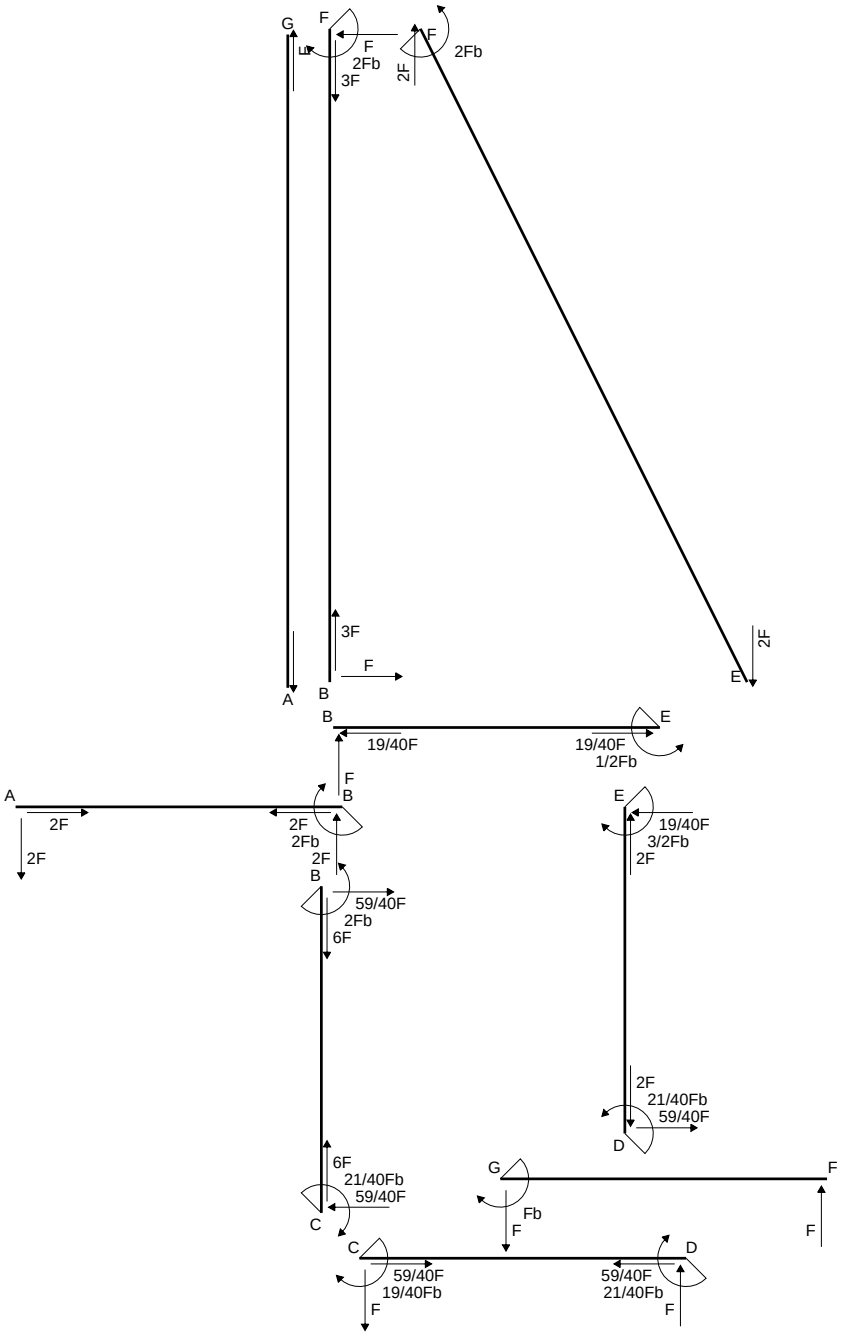
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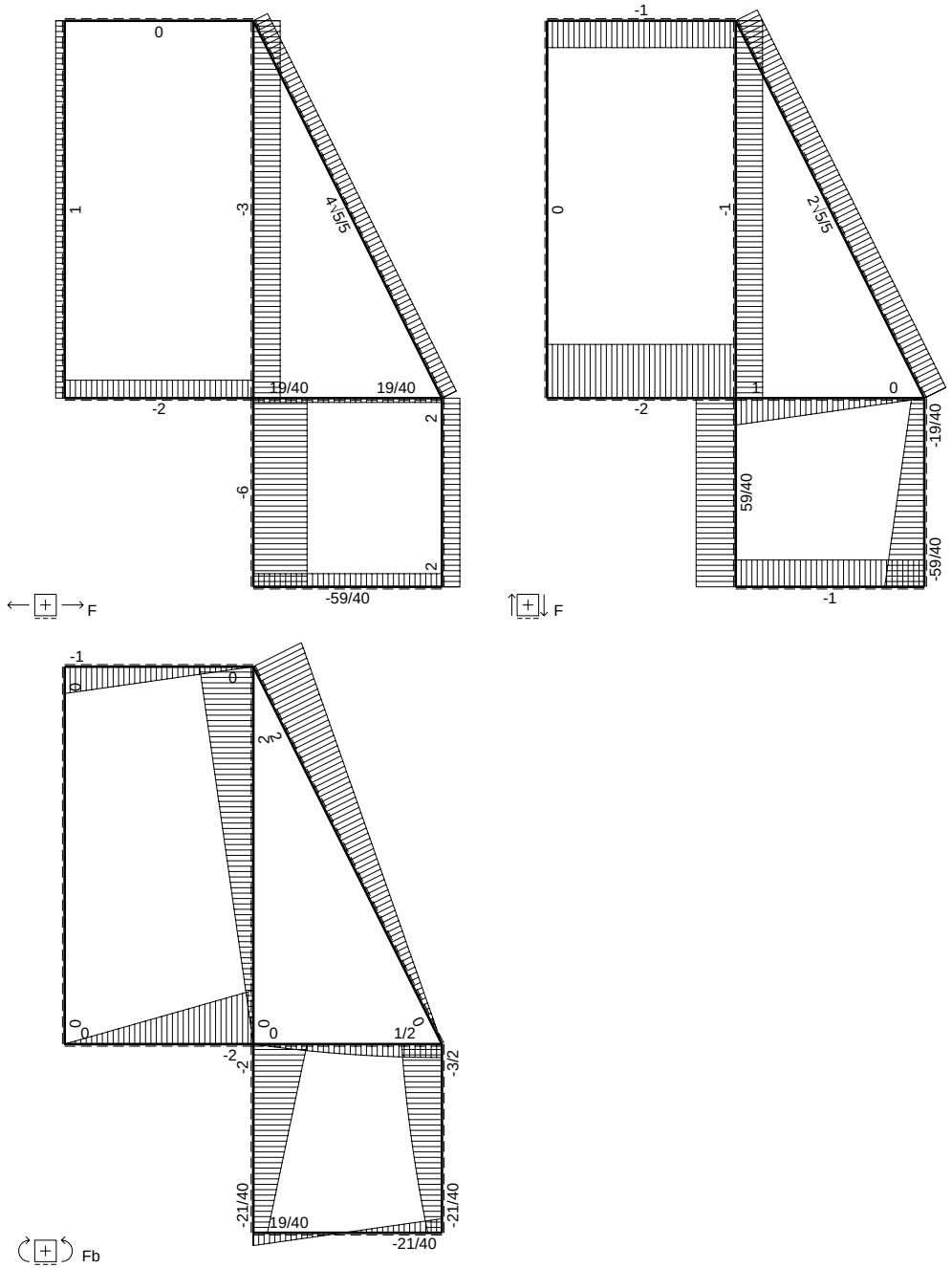
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-1
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-2
1
0
59/40
-19/40
-59/40
-1
F
-1
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2
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0
0
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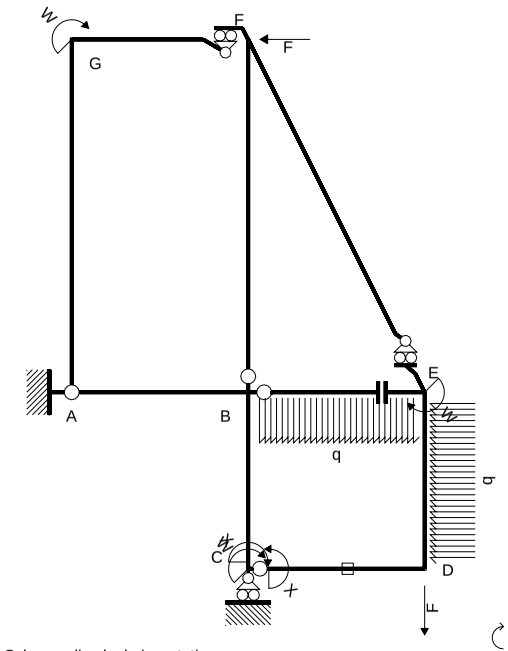
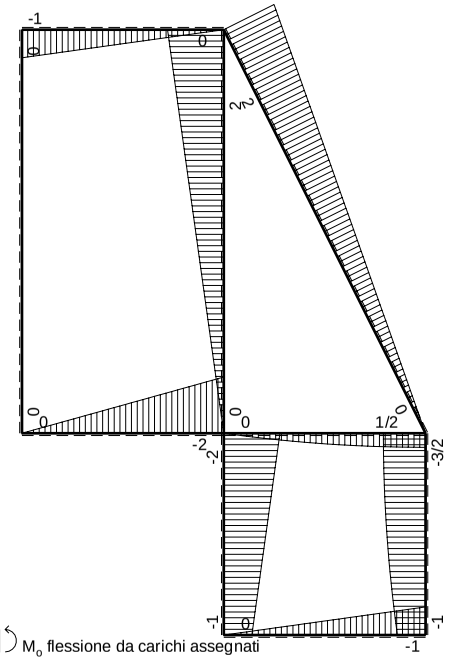
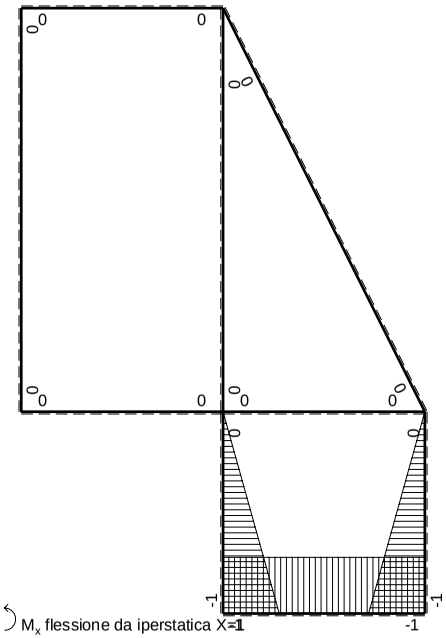
-21/40
-21/40
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	$2Fx-Fx^2/b$	x^2/b^2	$2/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+Fx	$Fb-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-Fx+1/2qx^2$	$Fb-3/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$5/8Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$3/2Fb-1/2qx^2$	$3/2Fx-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$19/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-19/40Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ \ Fb^2/EJ$

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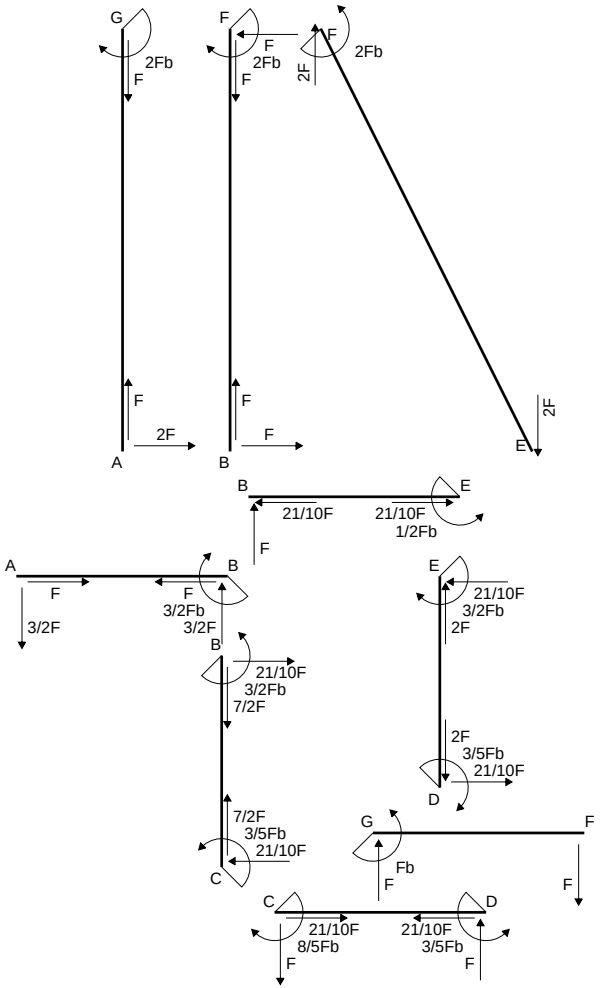
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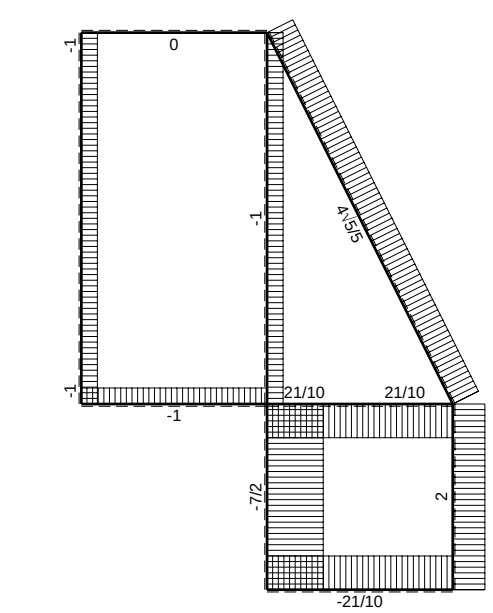
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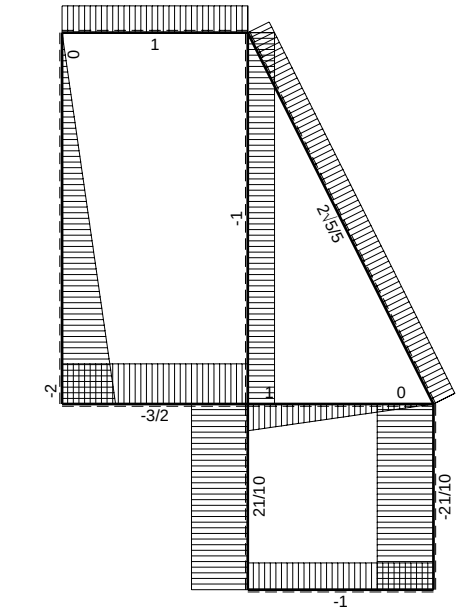
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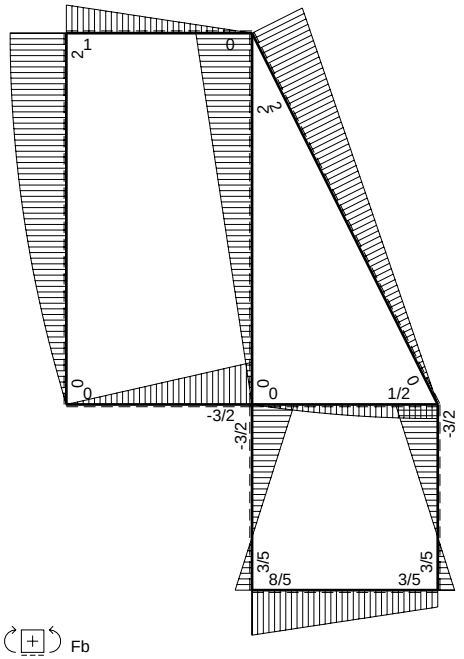
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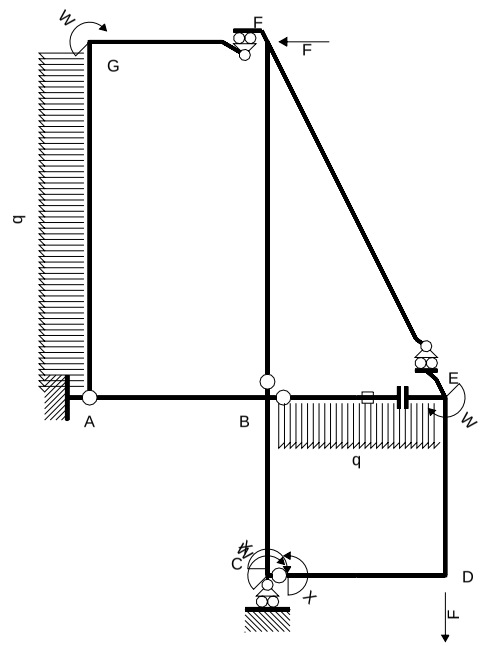
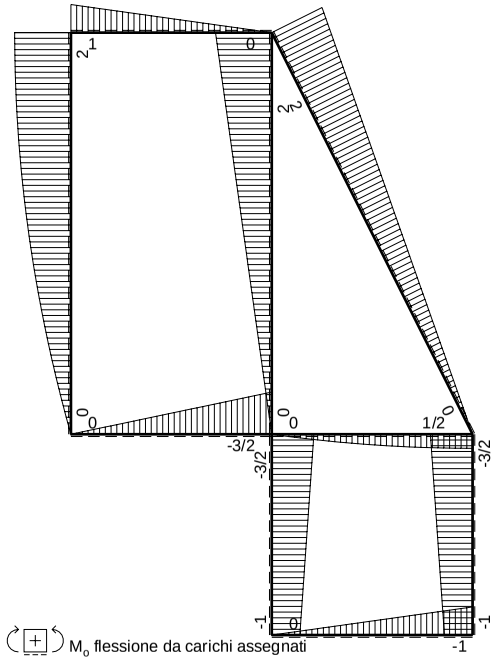
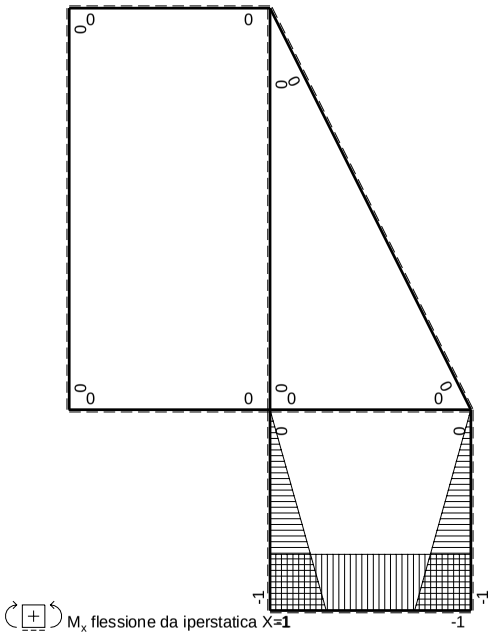


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	3/2Fb-1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$8/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-8/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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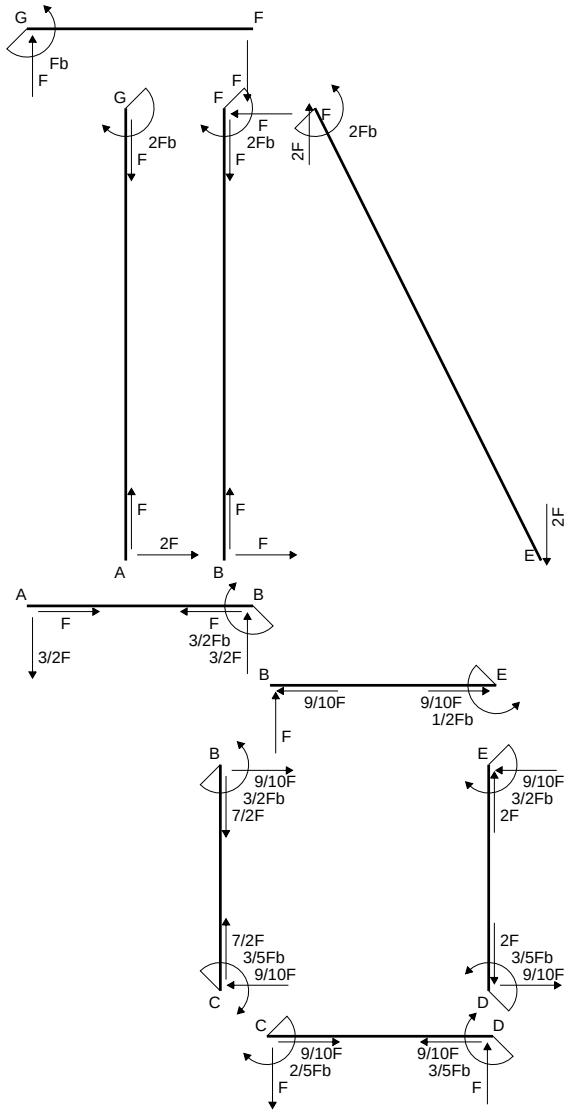
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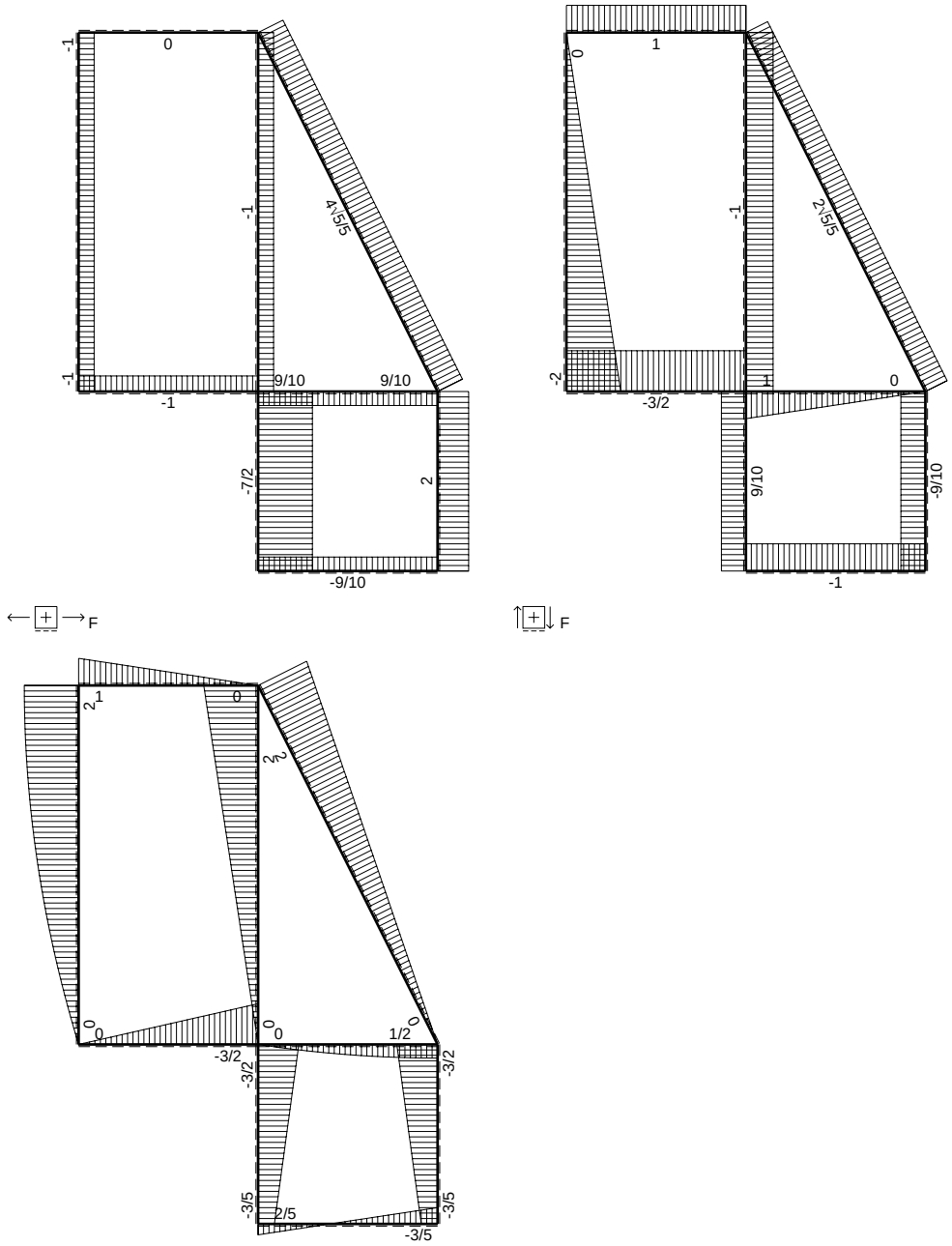
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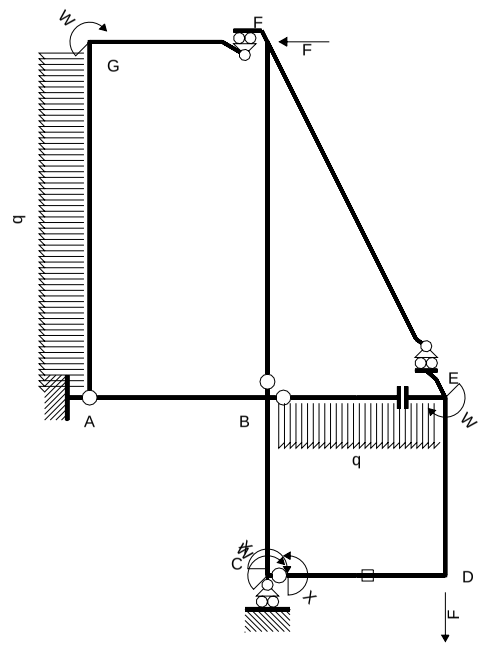
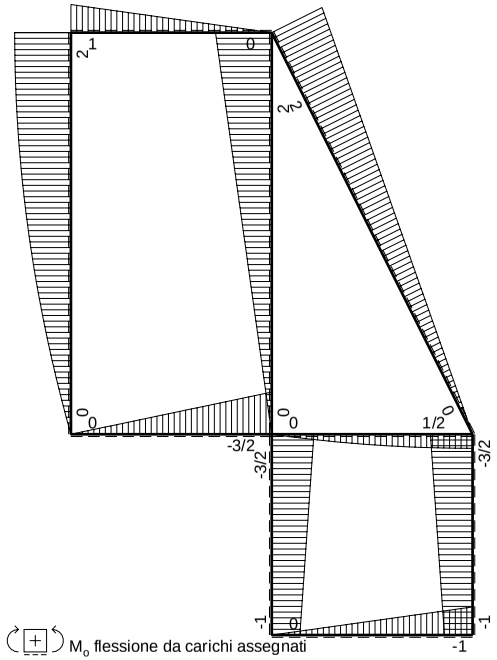
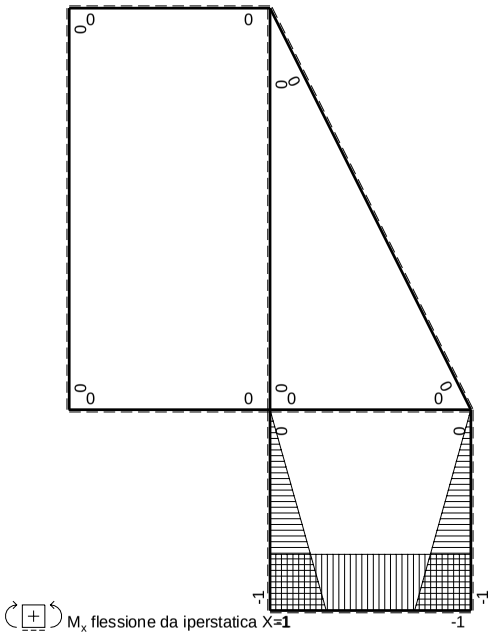


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fb-1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	2Fb-1/2qx^2	0	0	0	0
AG 2b	0	-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				2/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-2/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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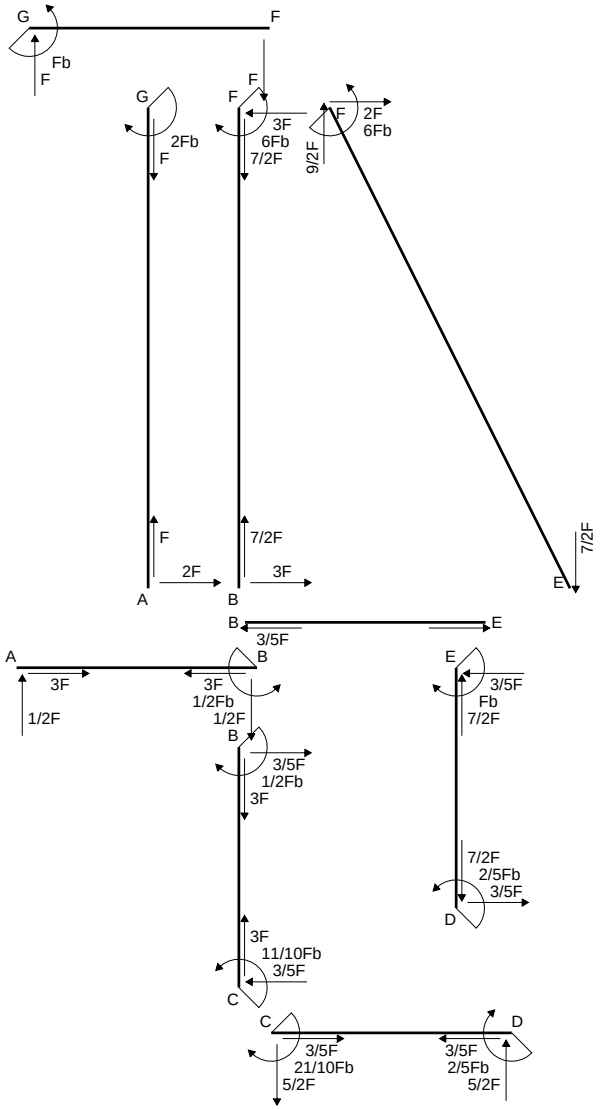
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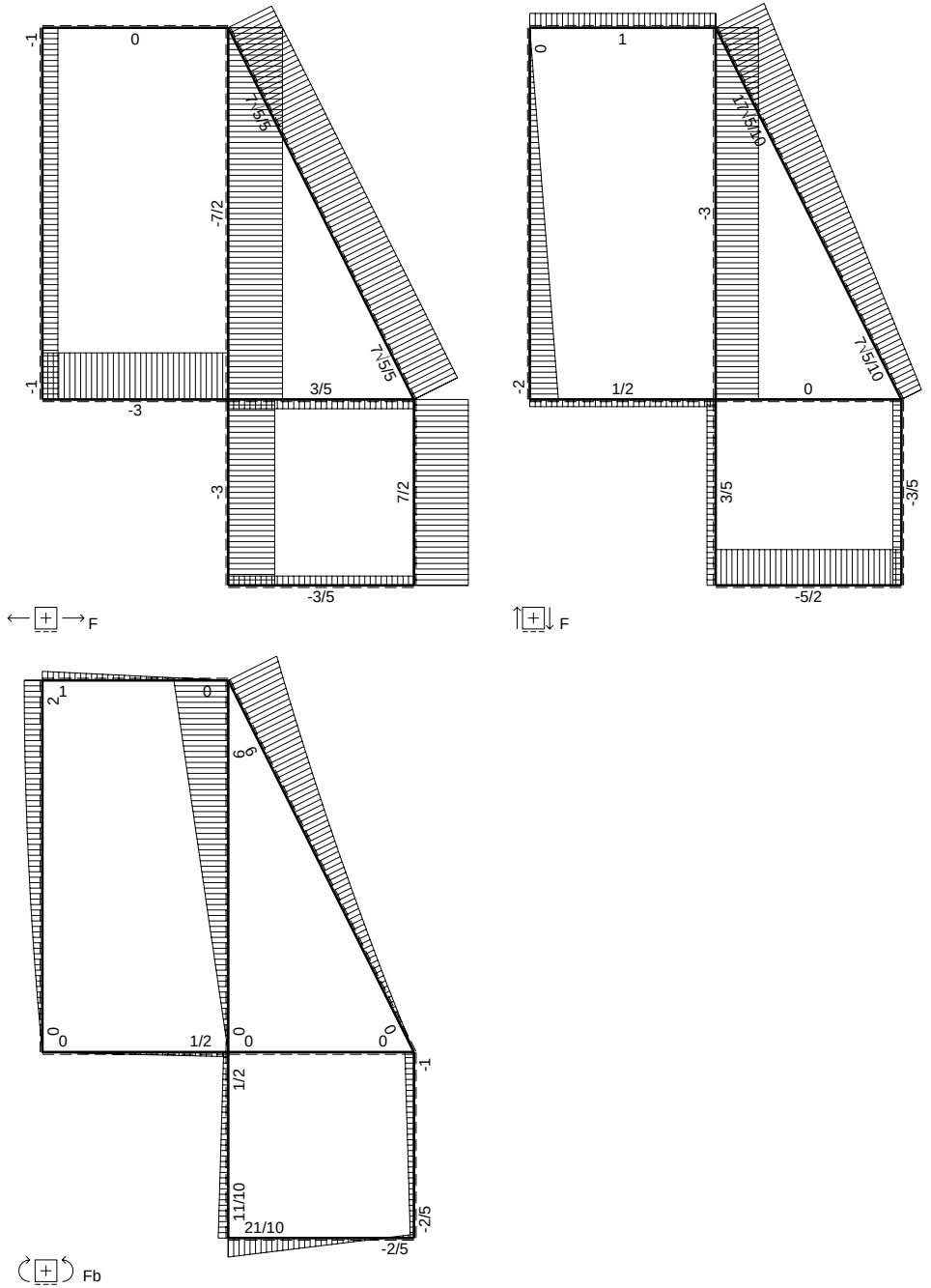
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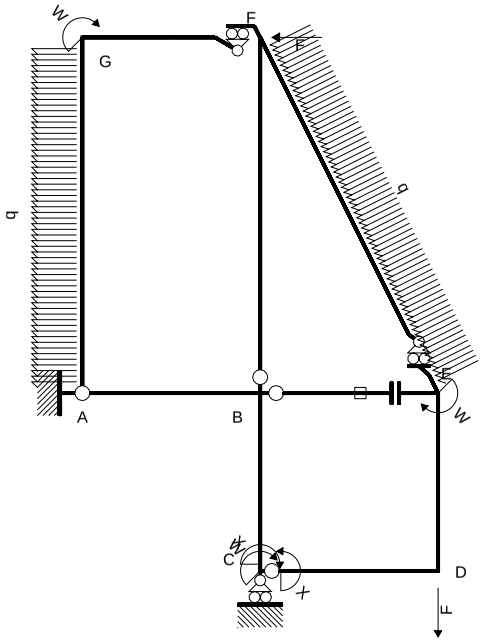
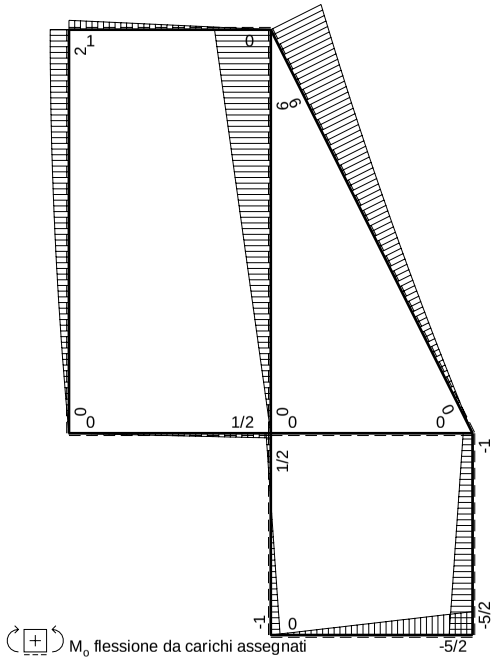
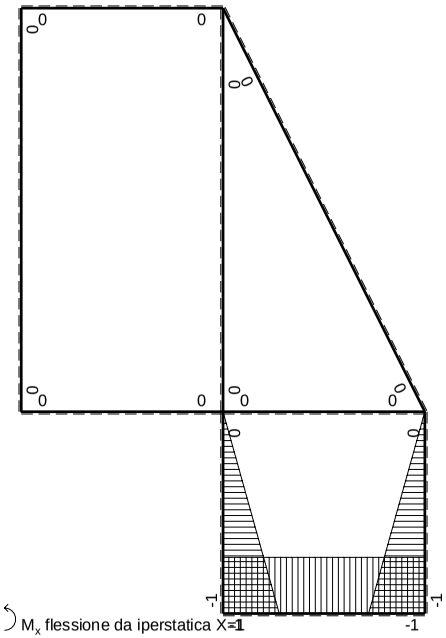


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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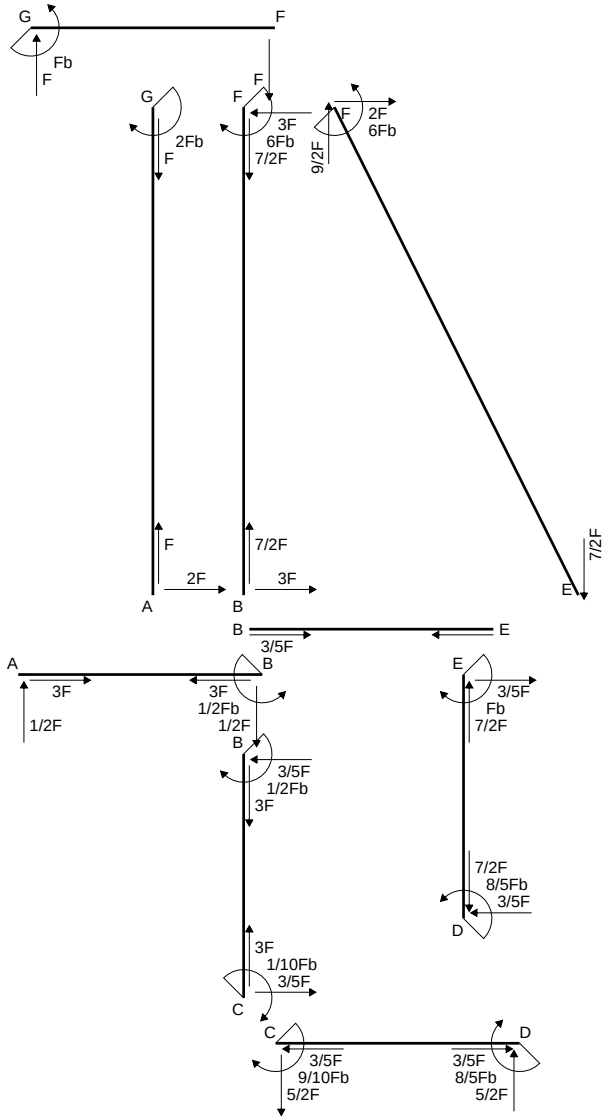
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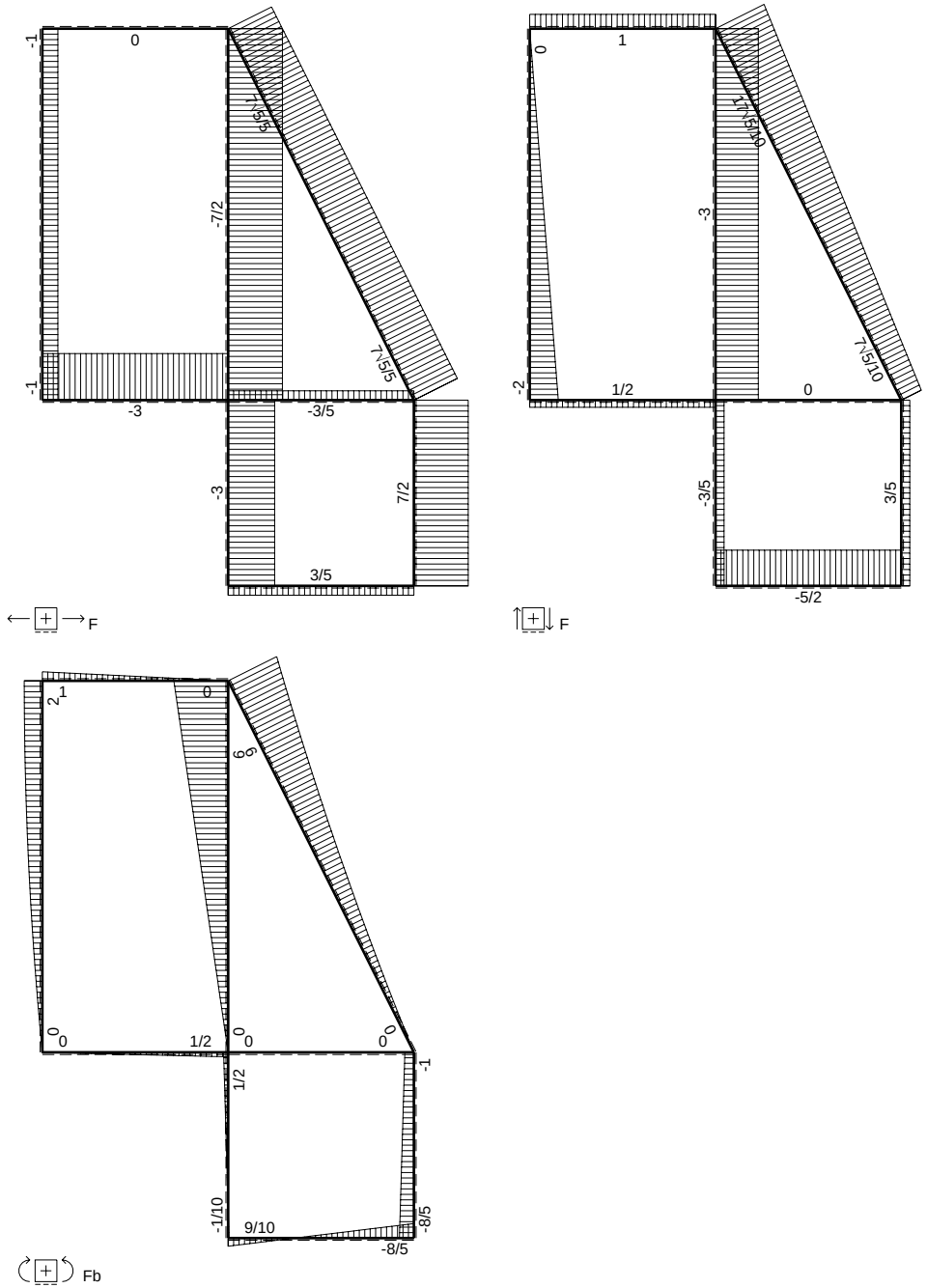
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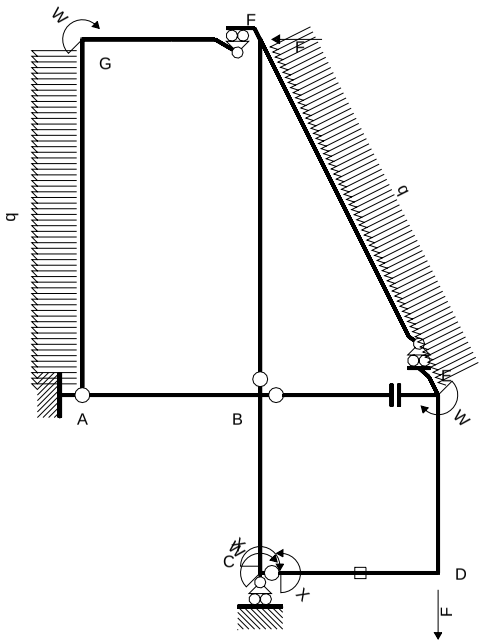
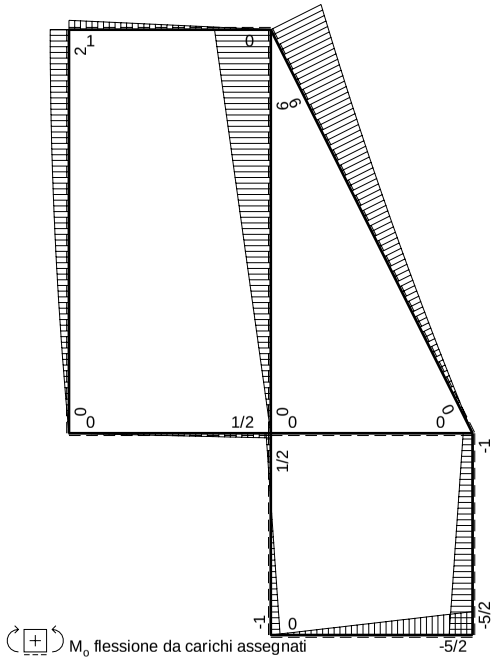
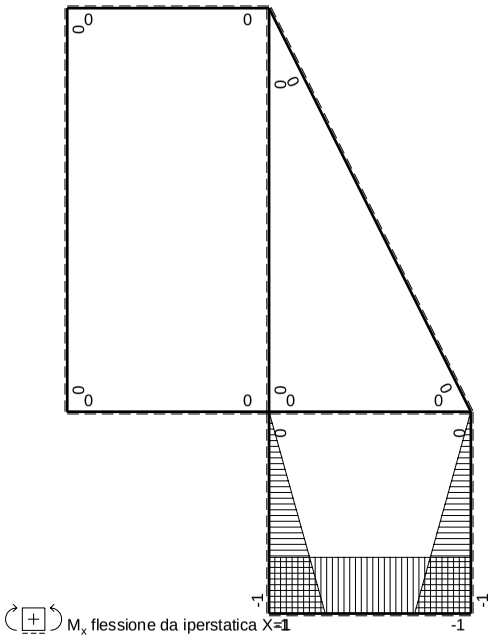


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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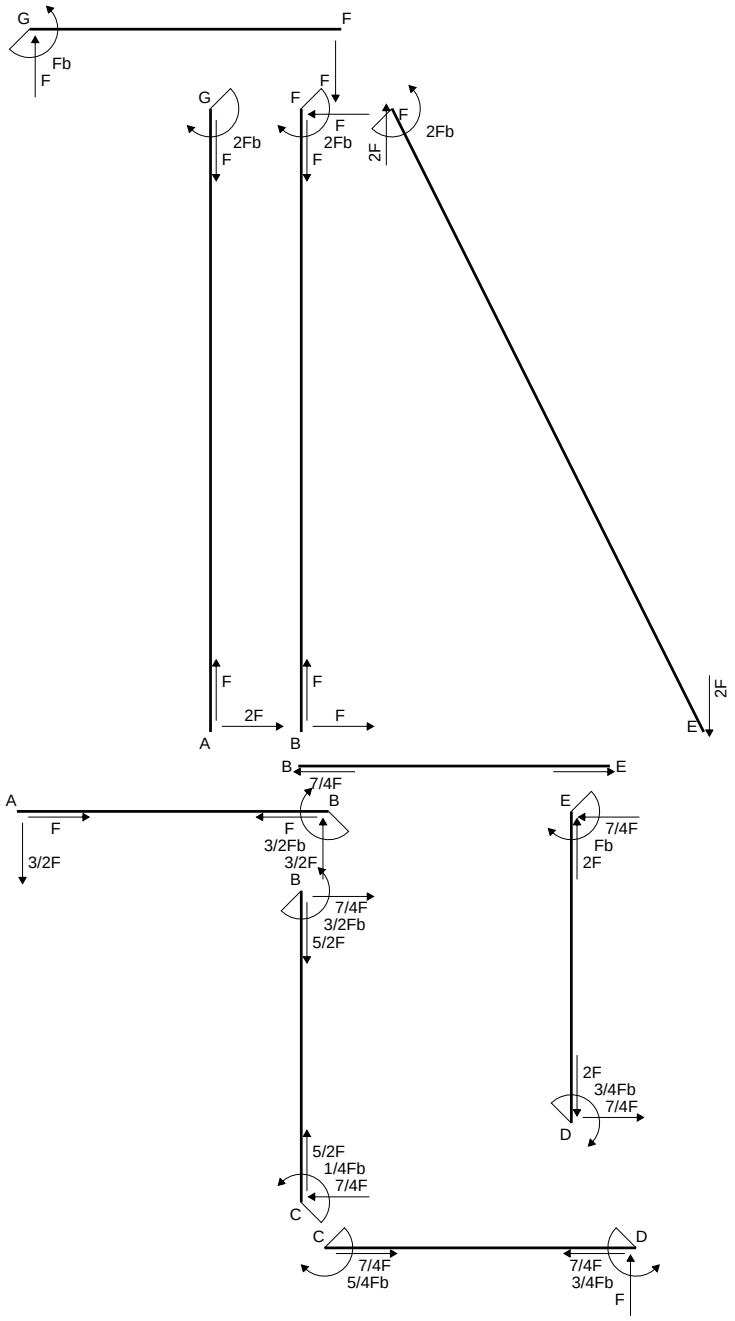
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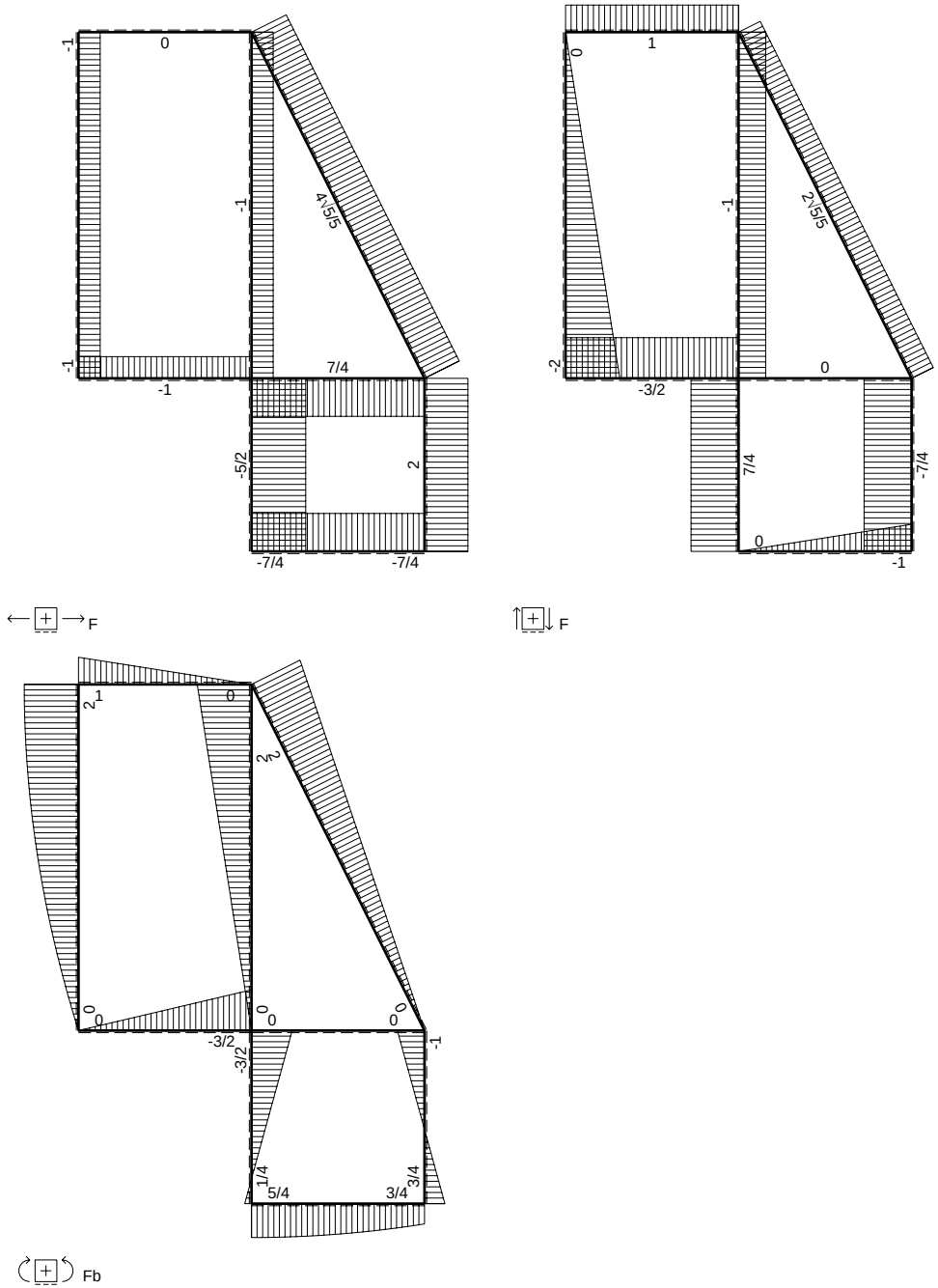
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2*5/5
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-3/2
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7/4
-7/4
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-1
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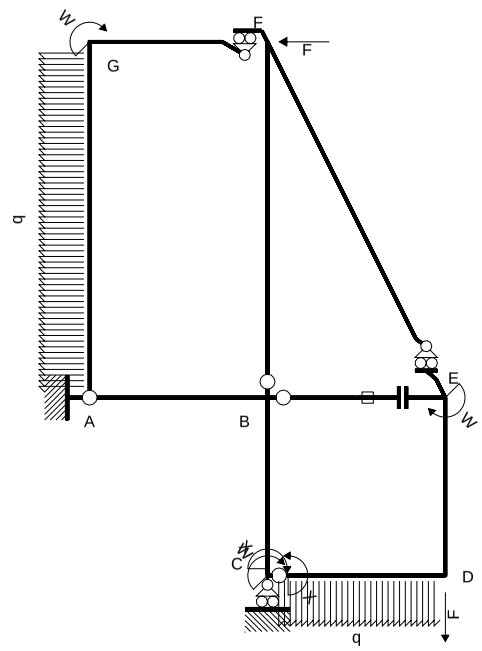
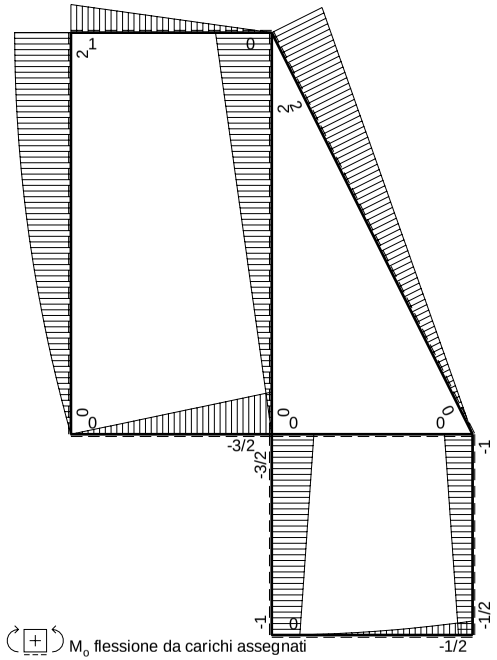
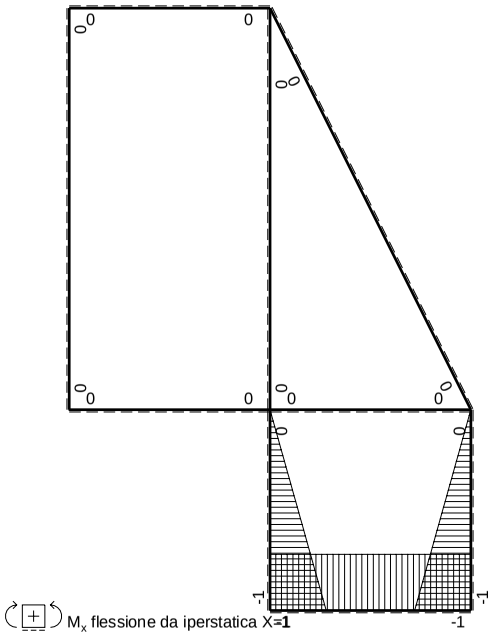


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb-1/2Fx	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb-1/2Fx	$Fx-1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$25/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-5/4Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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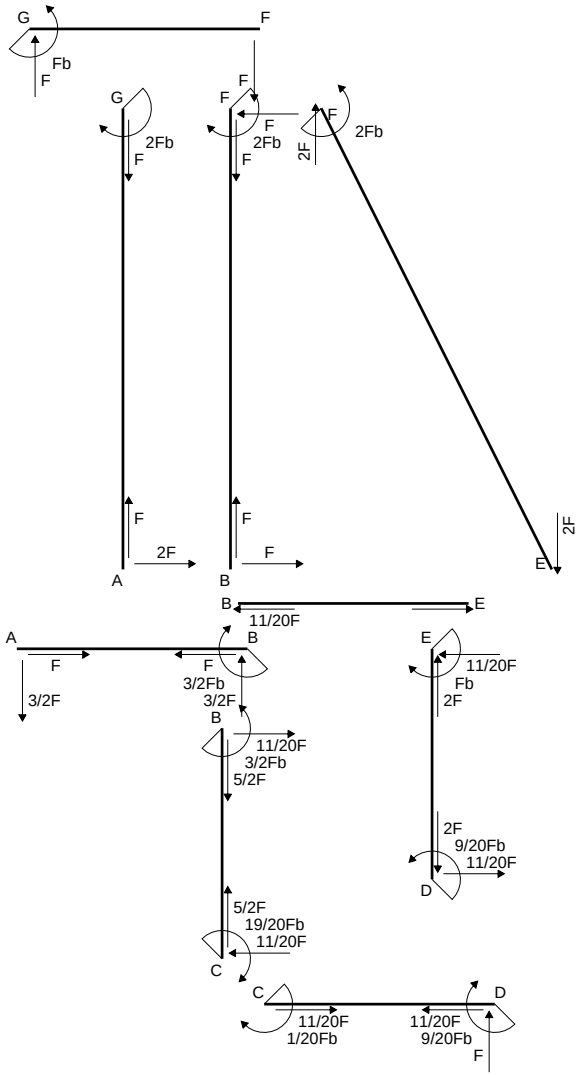
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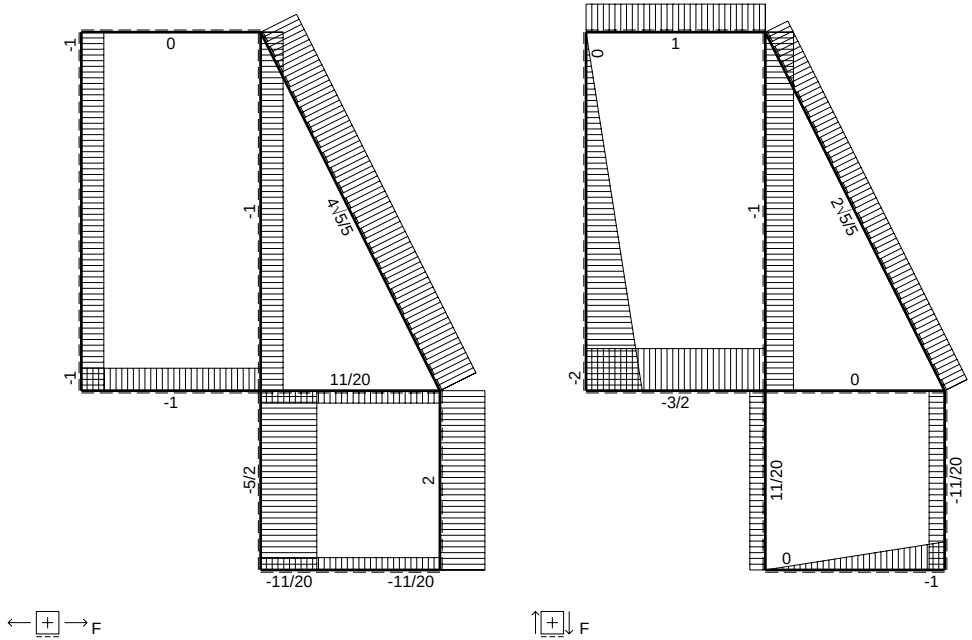
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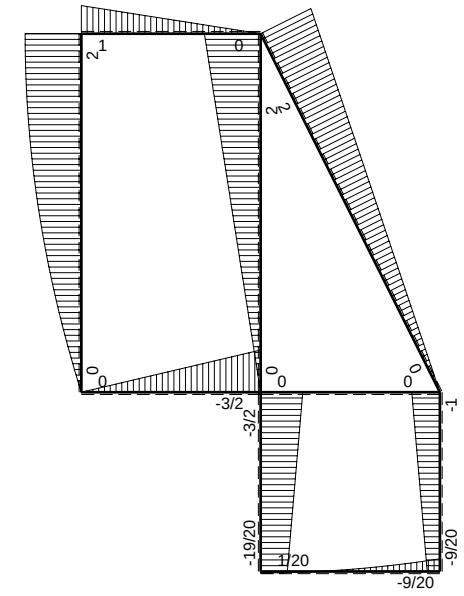
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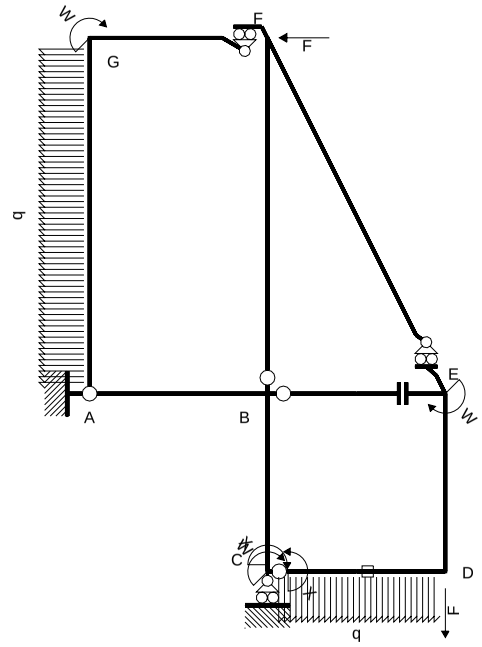
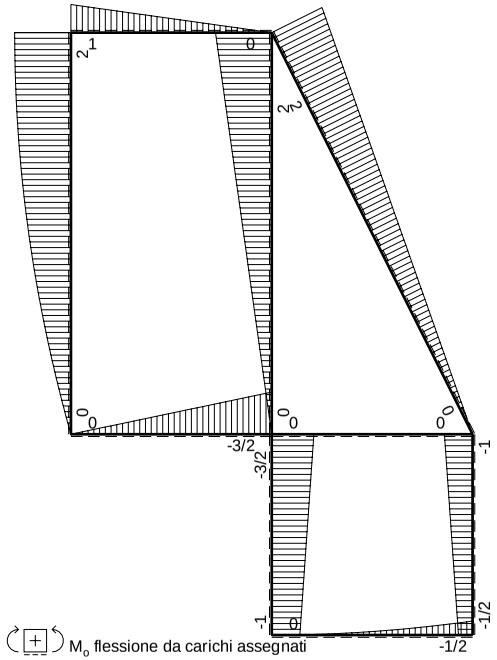
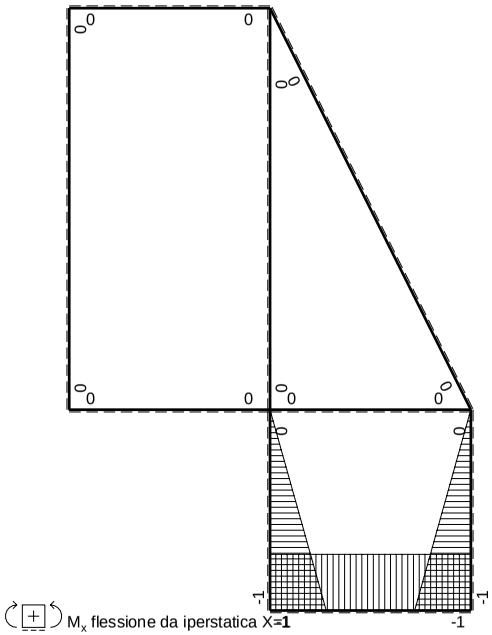


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb-1/2Fx	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb-1/2Fx	$Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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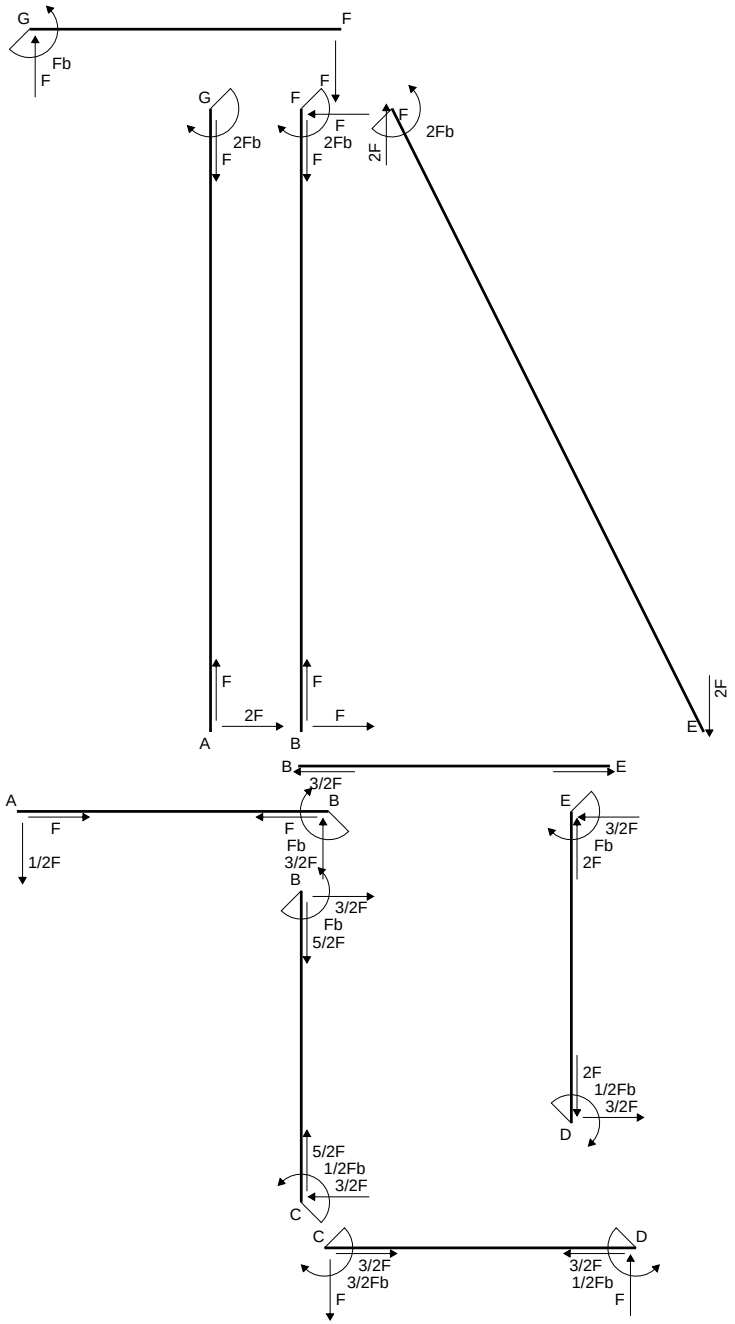
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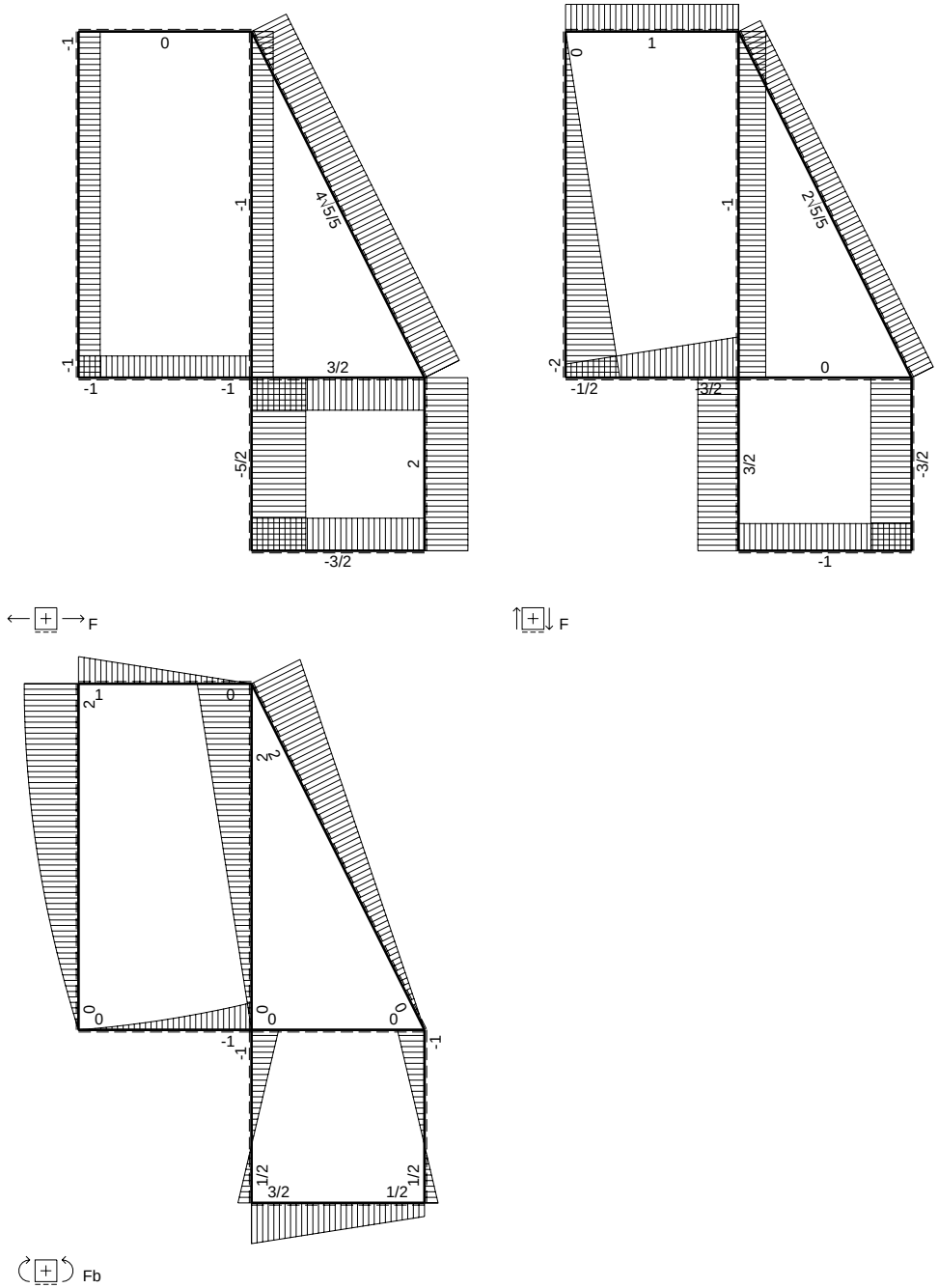
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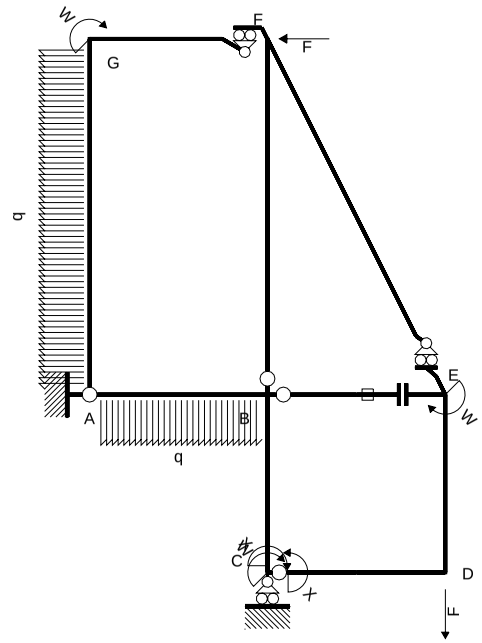
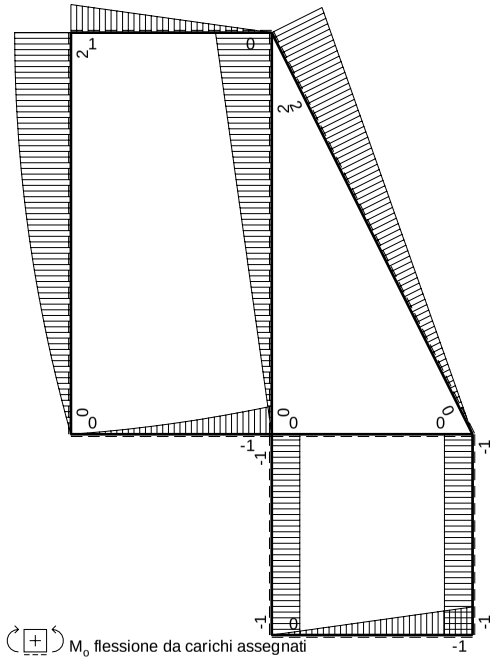
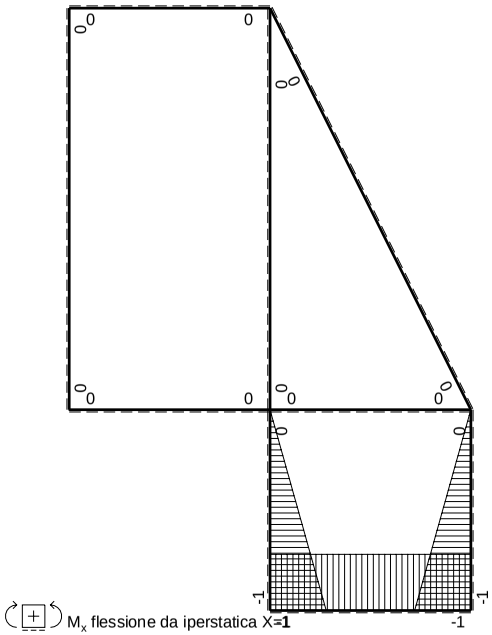


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=**1**

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$5/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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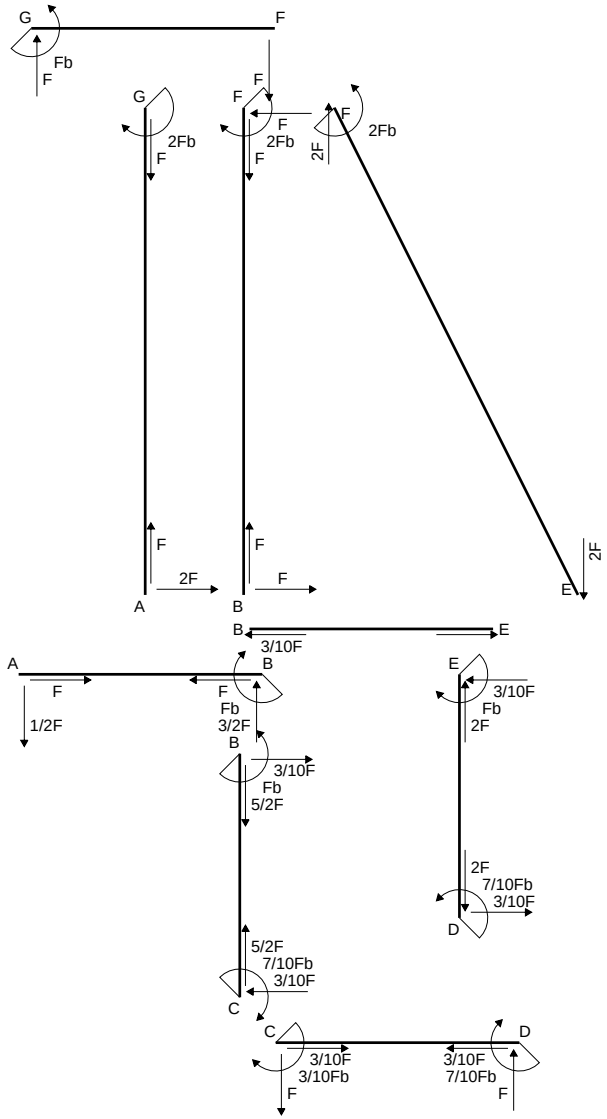
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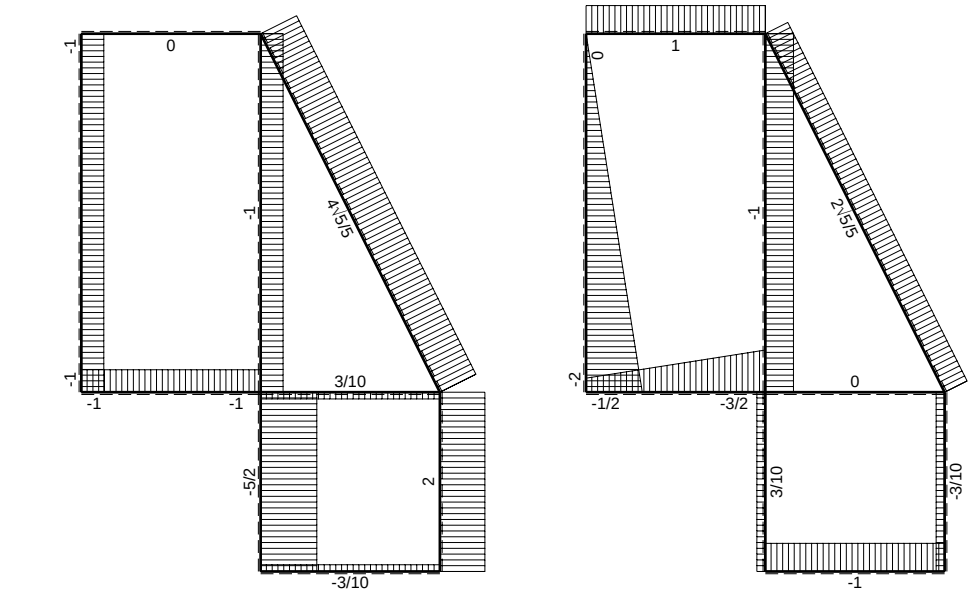
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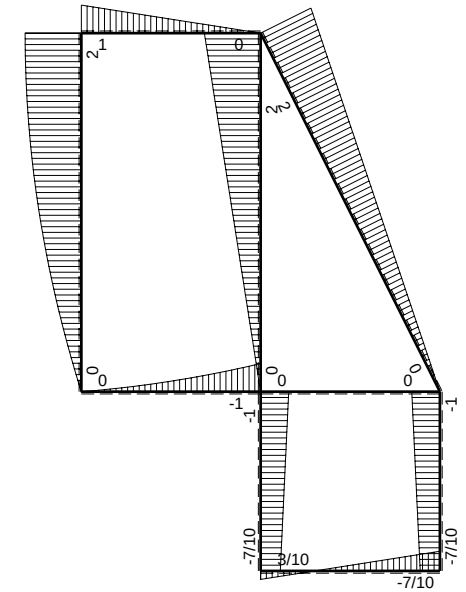
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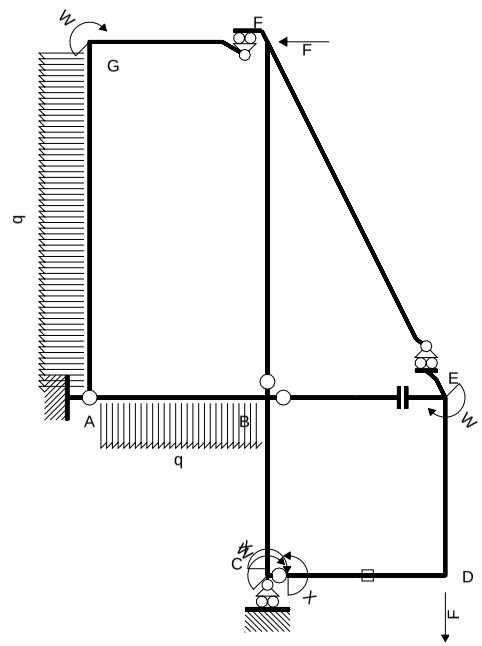
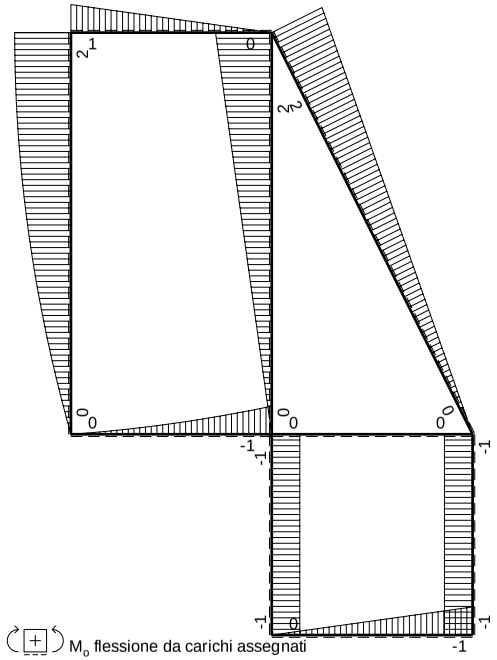
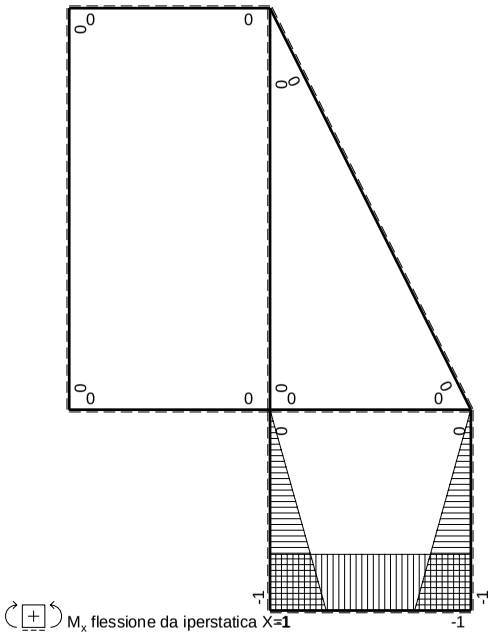


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-3/10Fb$	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

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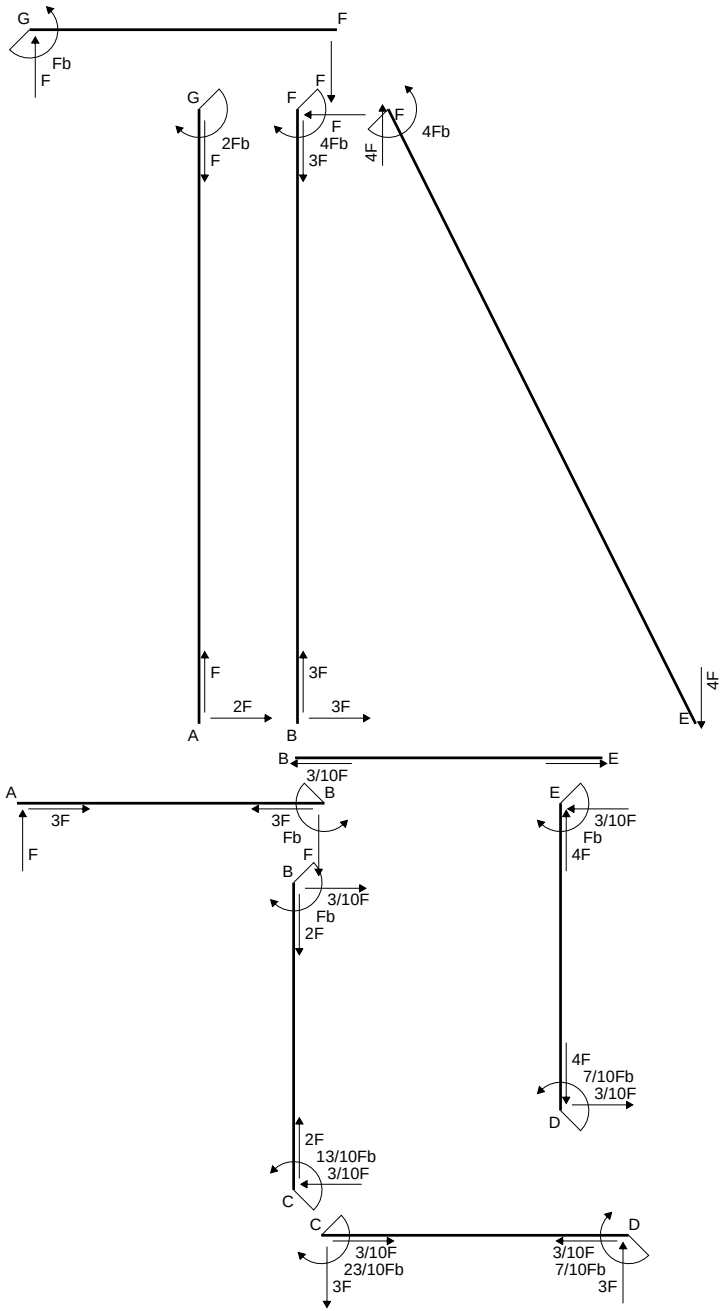
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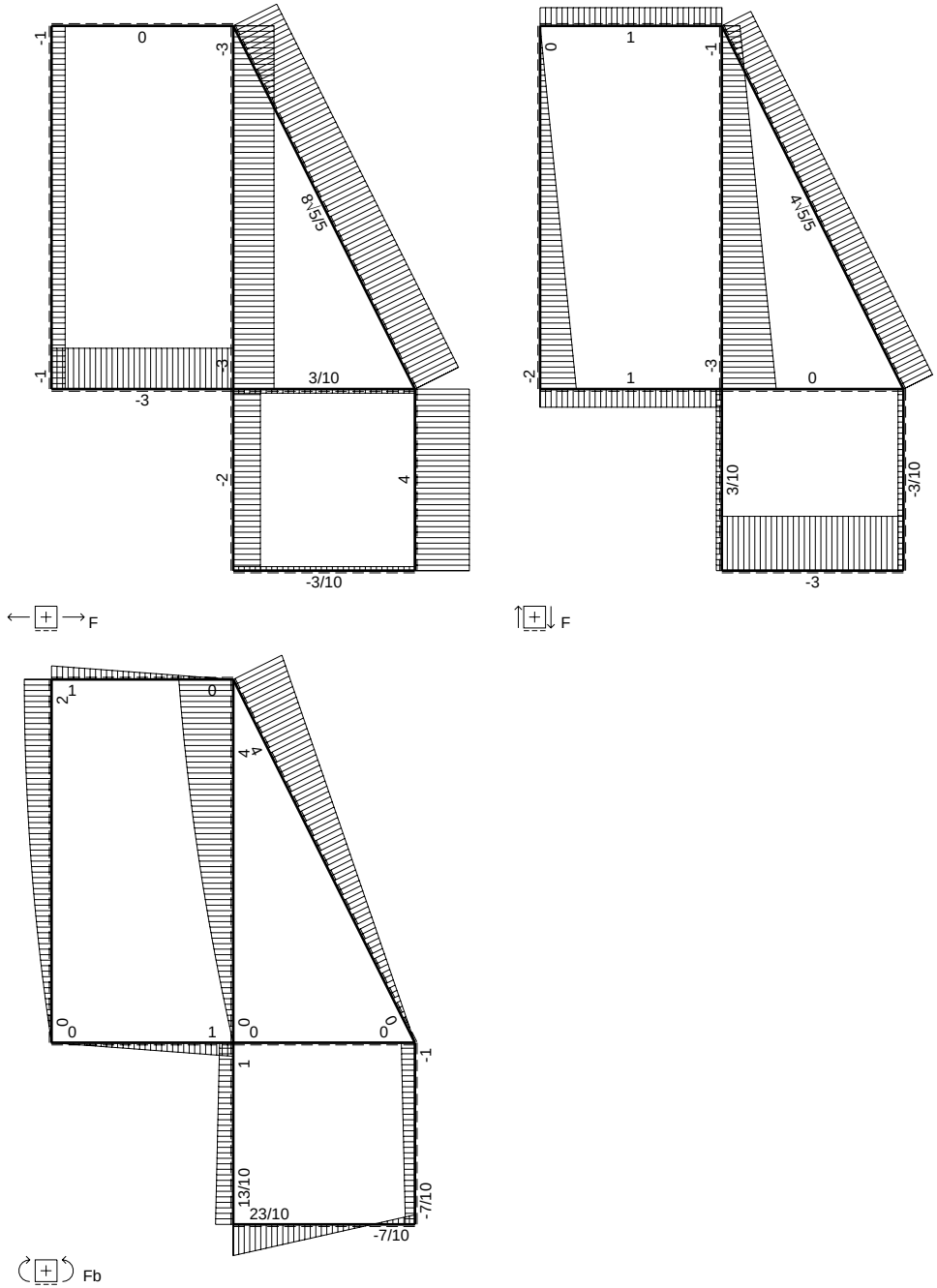
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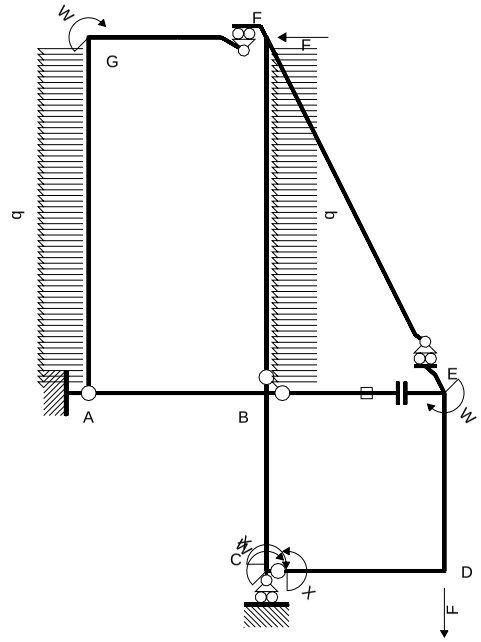
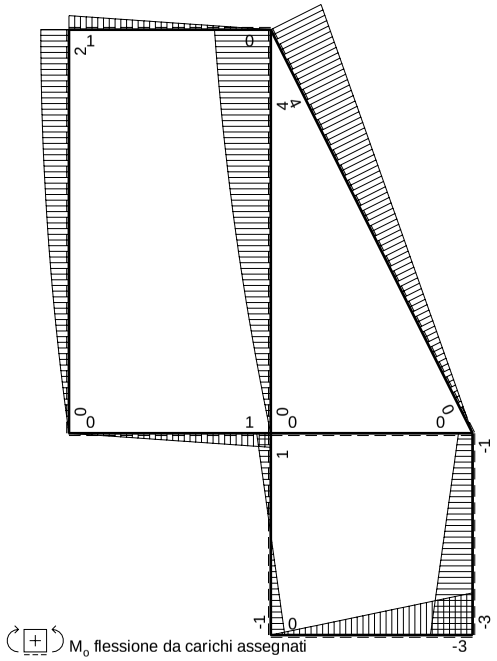
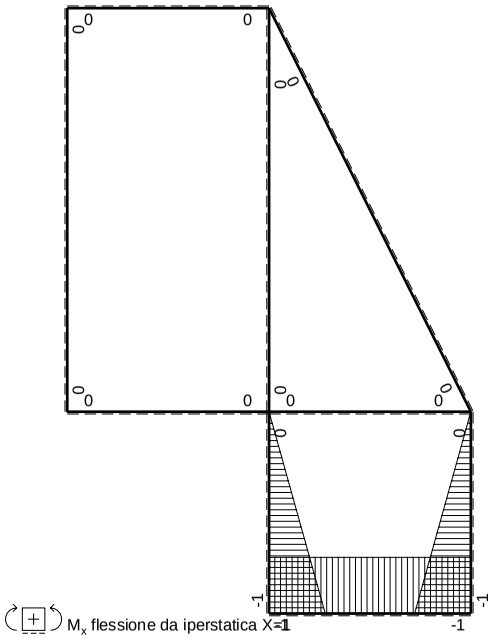


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$23/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-23/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b)\ Fb\ 1/EJ\ dx = [3/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (3/2\ b)\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b)\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b)\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

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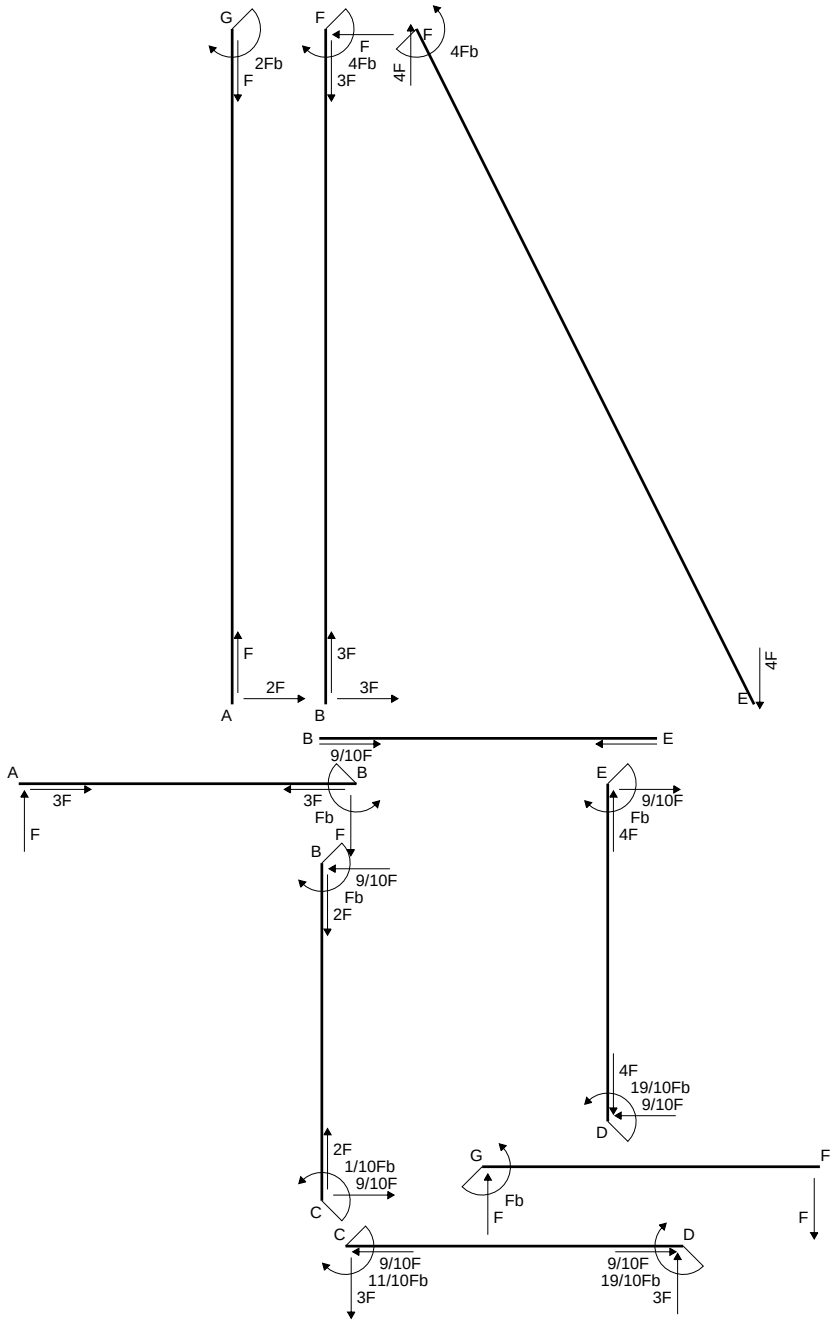
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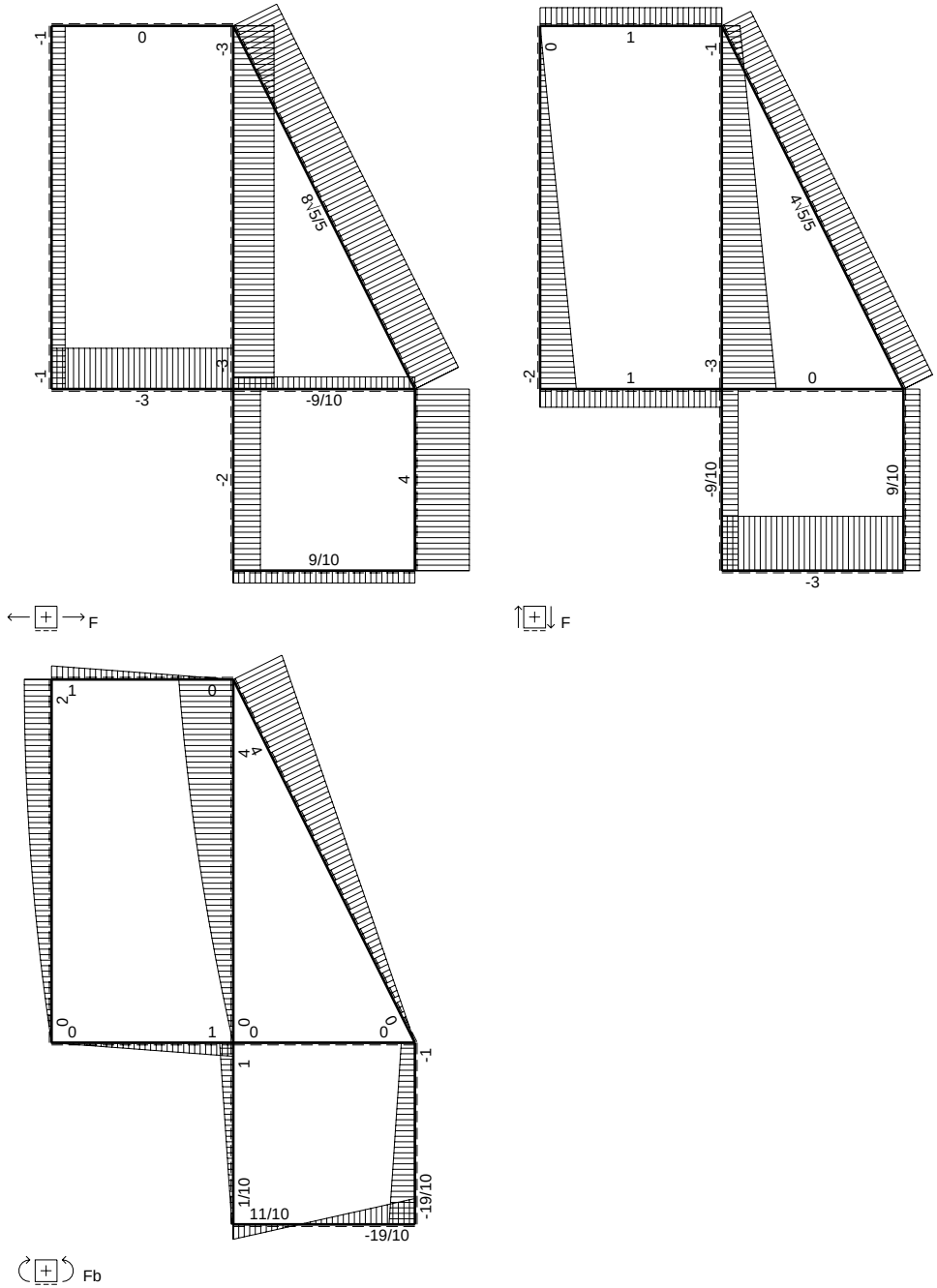
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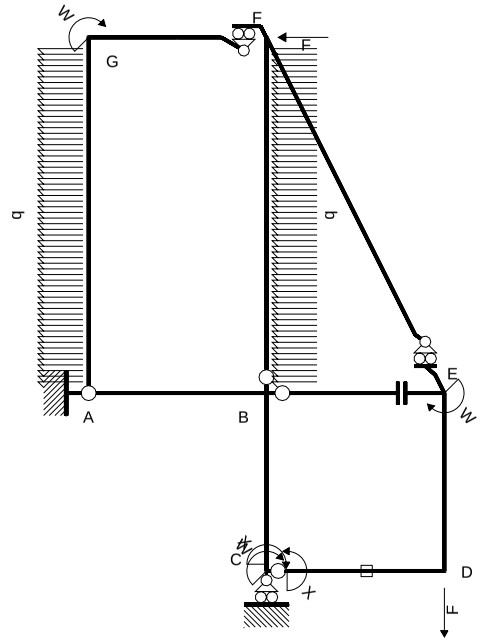
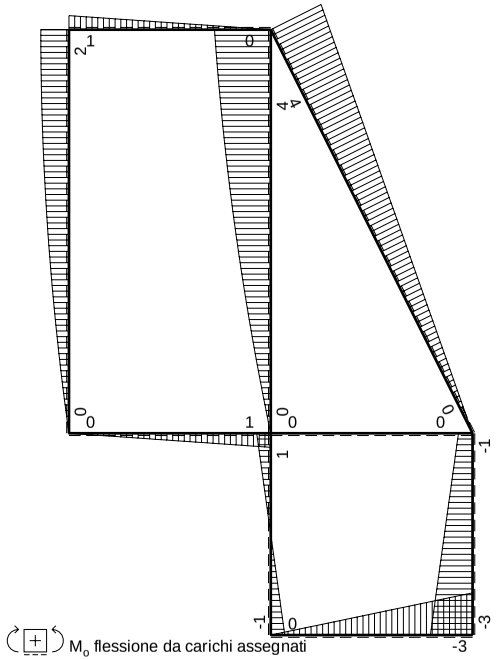
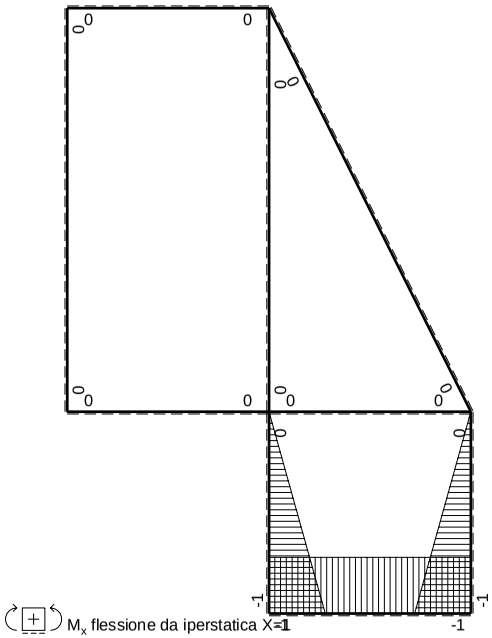


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$11/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-11/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

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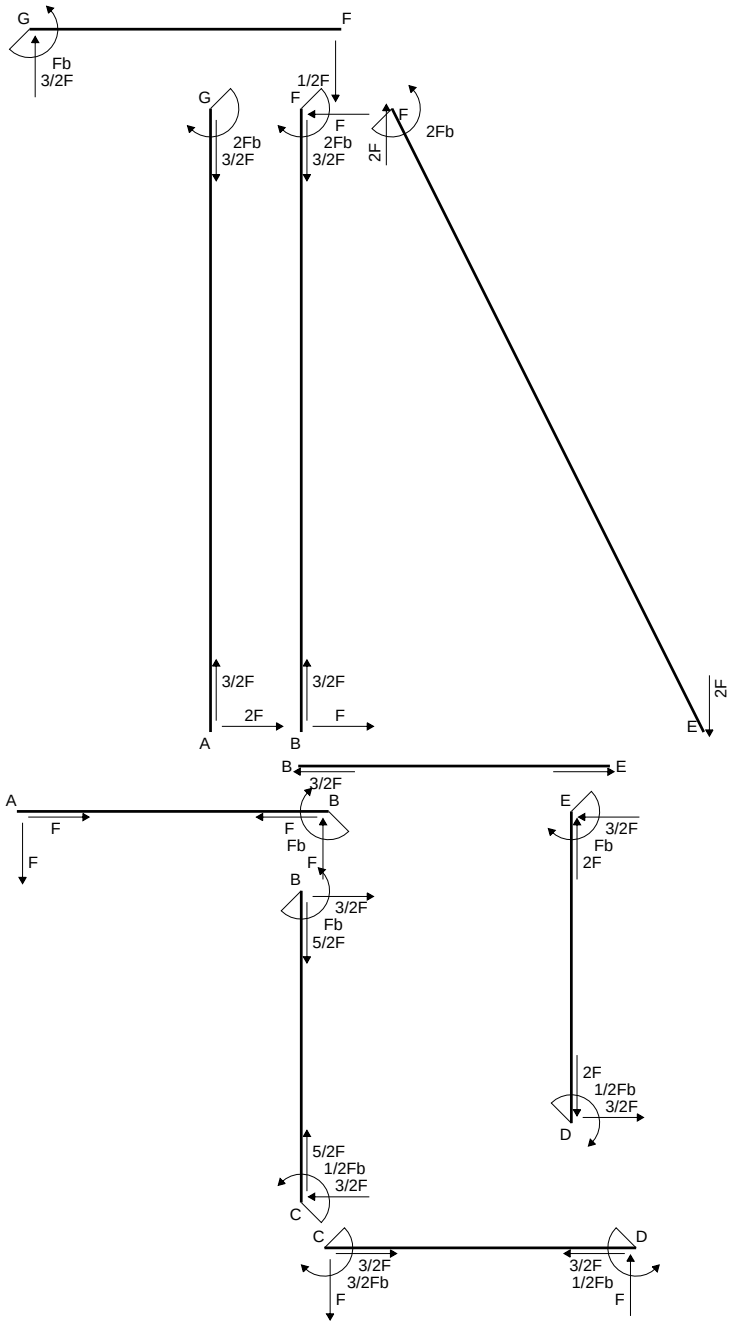
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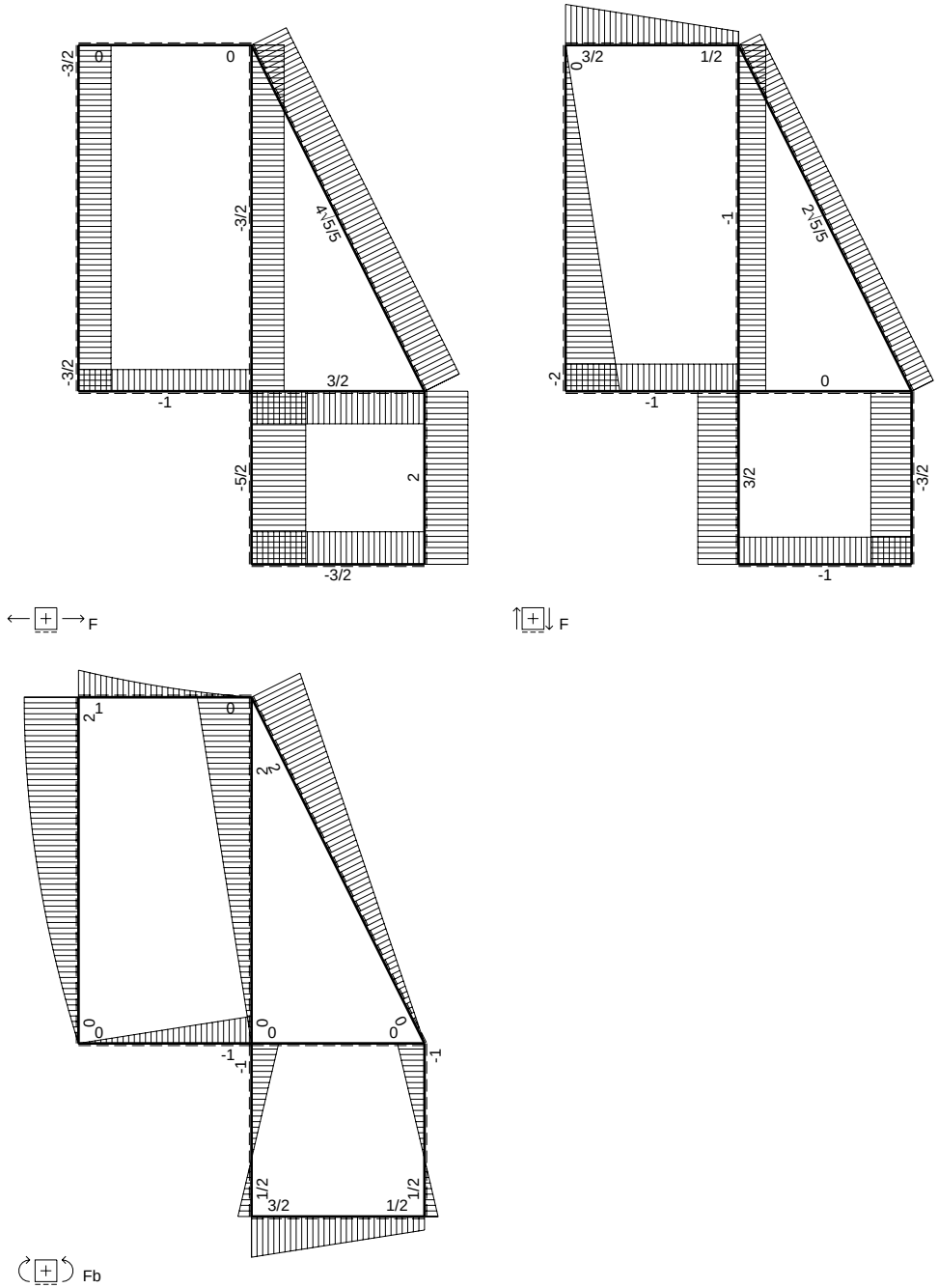
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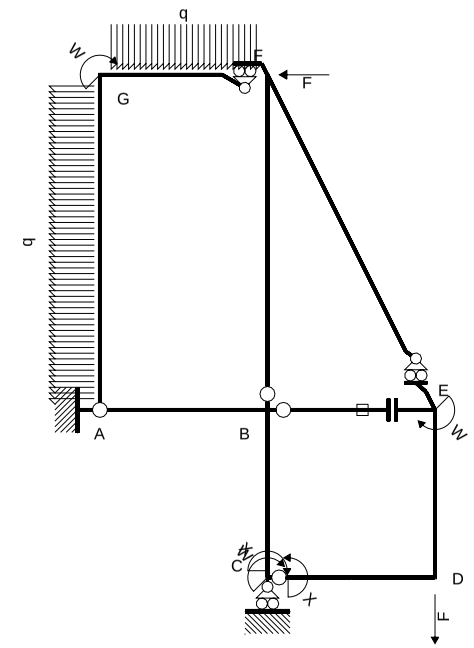
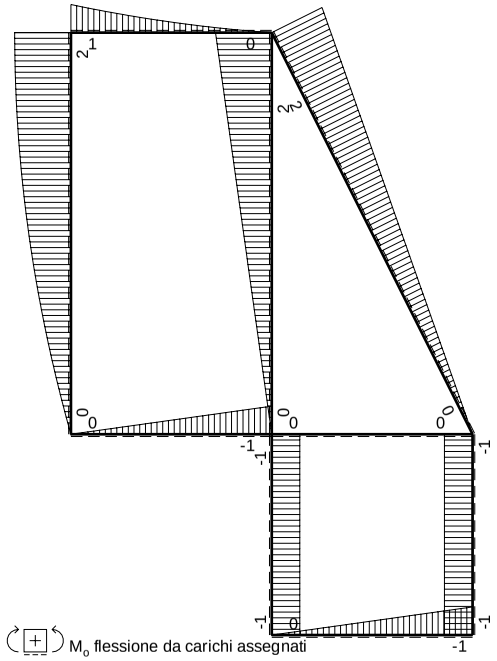
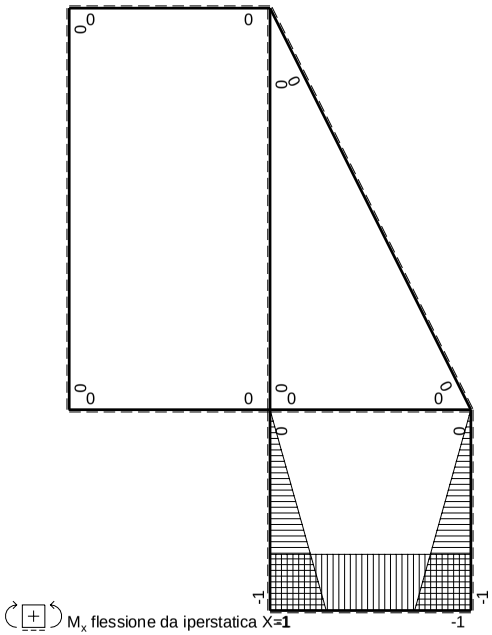


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=**1**

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$-Fb+3/2Fx-1/2qx^2$	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$5/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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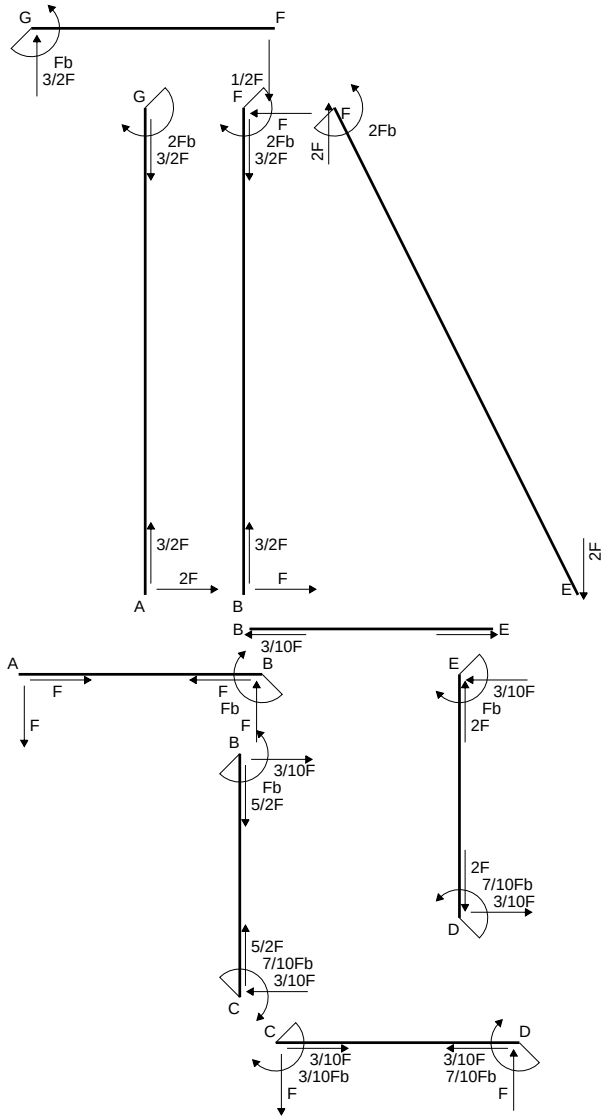
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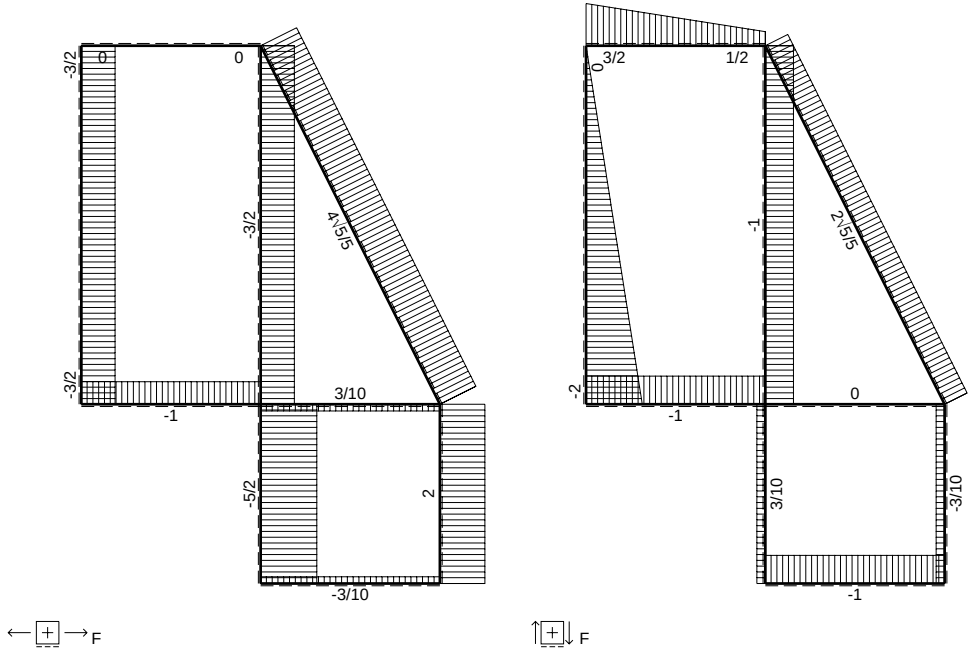
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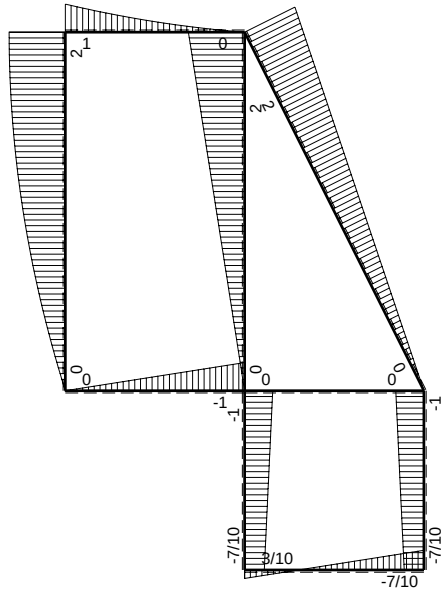
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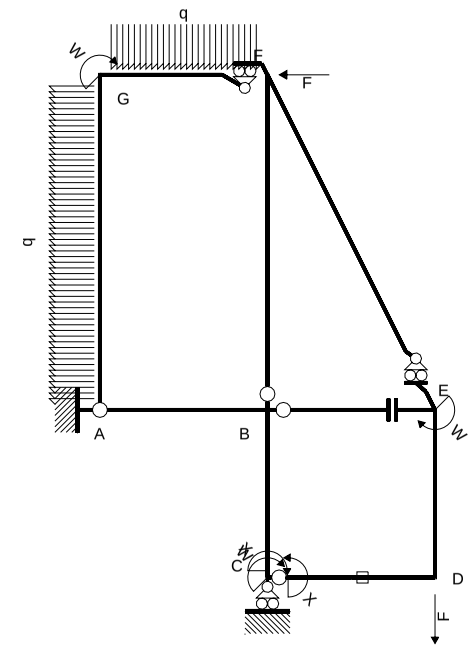
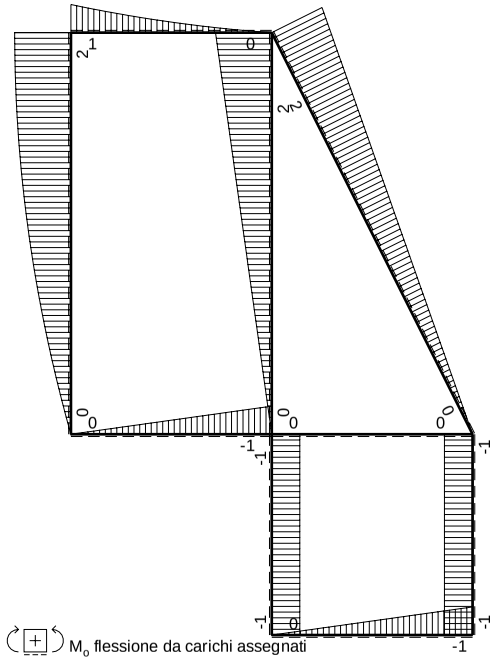
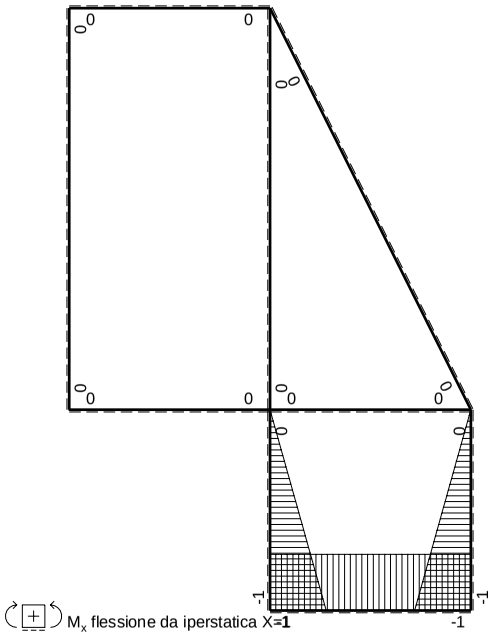


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$-Fb+3/2Fx-1/2qx^2$	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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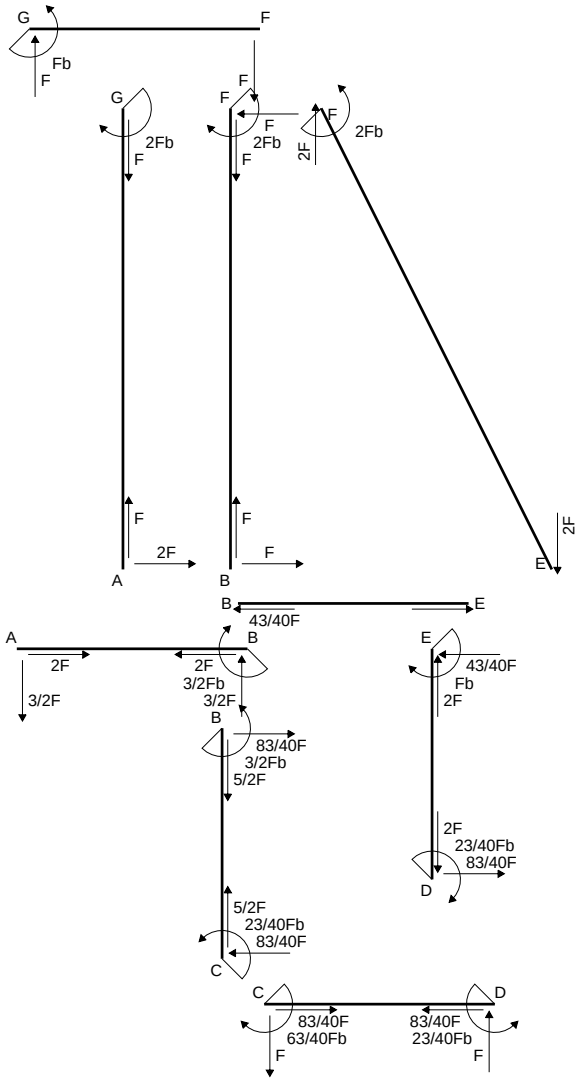
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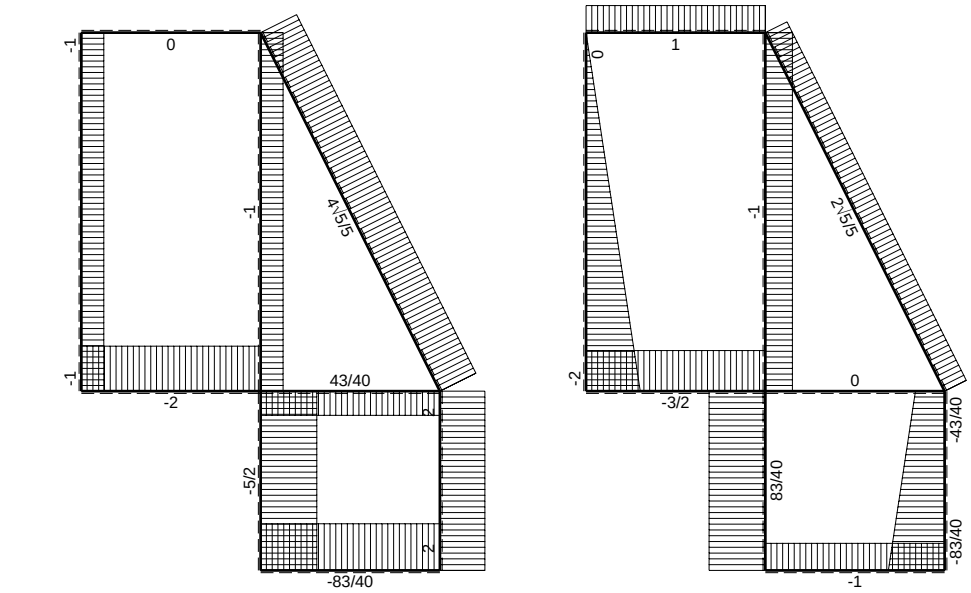
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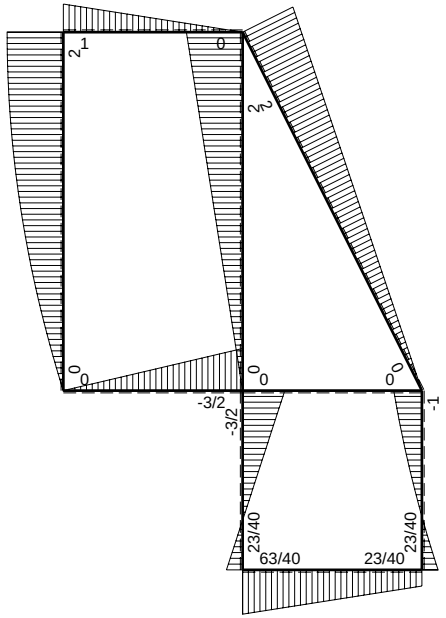
-83/40
-1

F F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23/40
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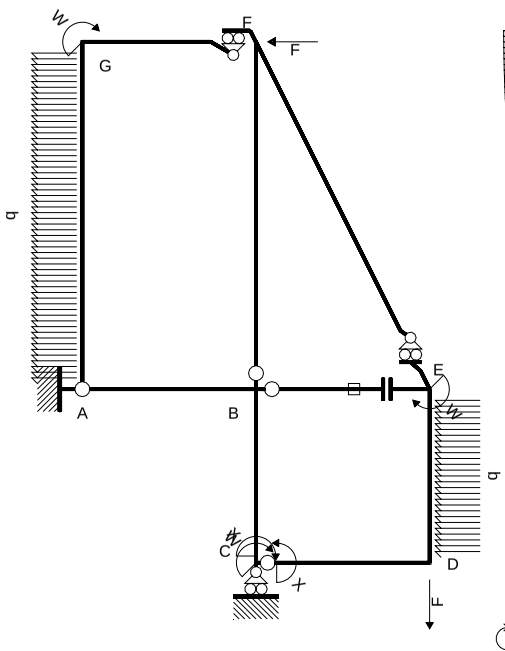
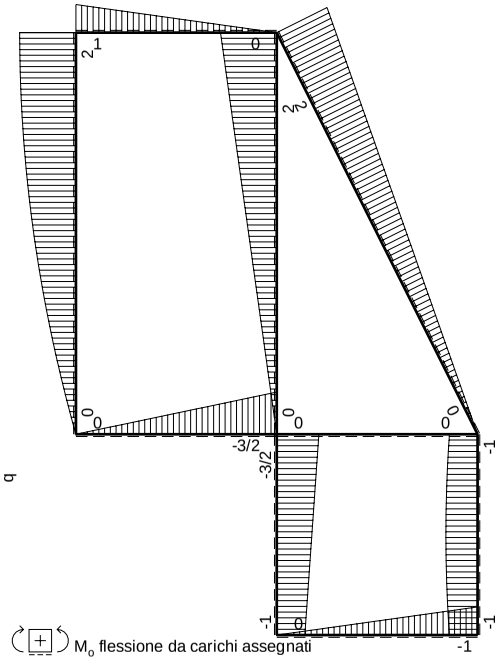
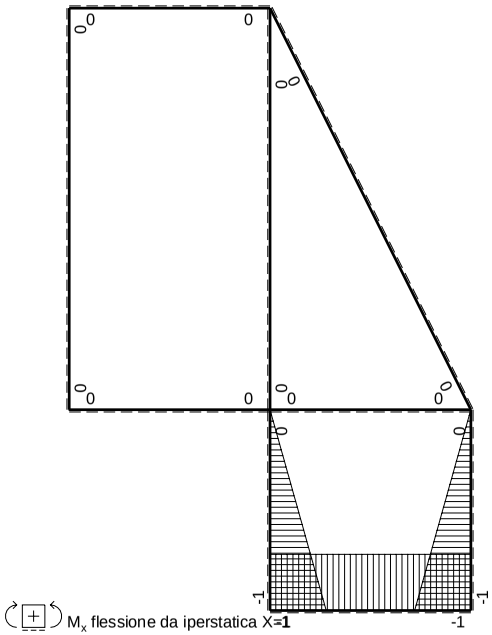
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=**1**

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$21/8Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-63/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

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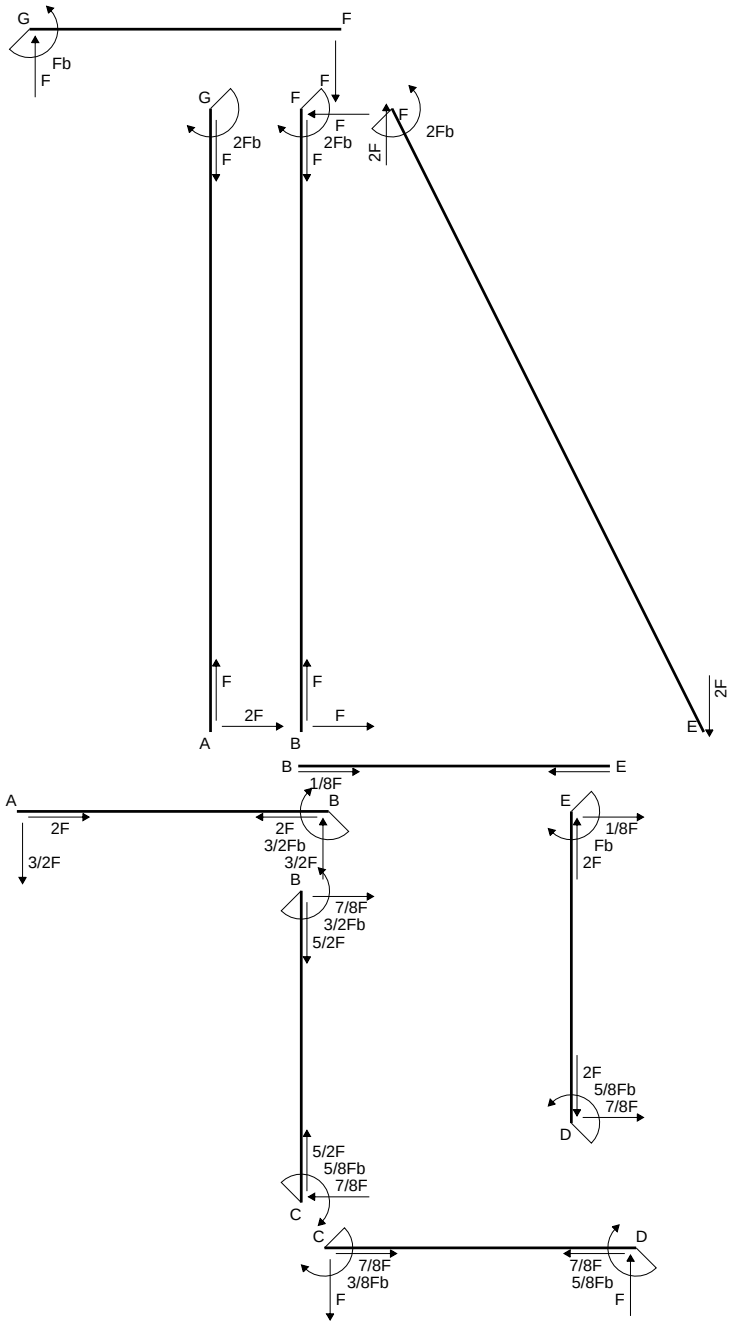
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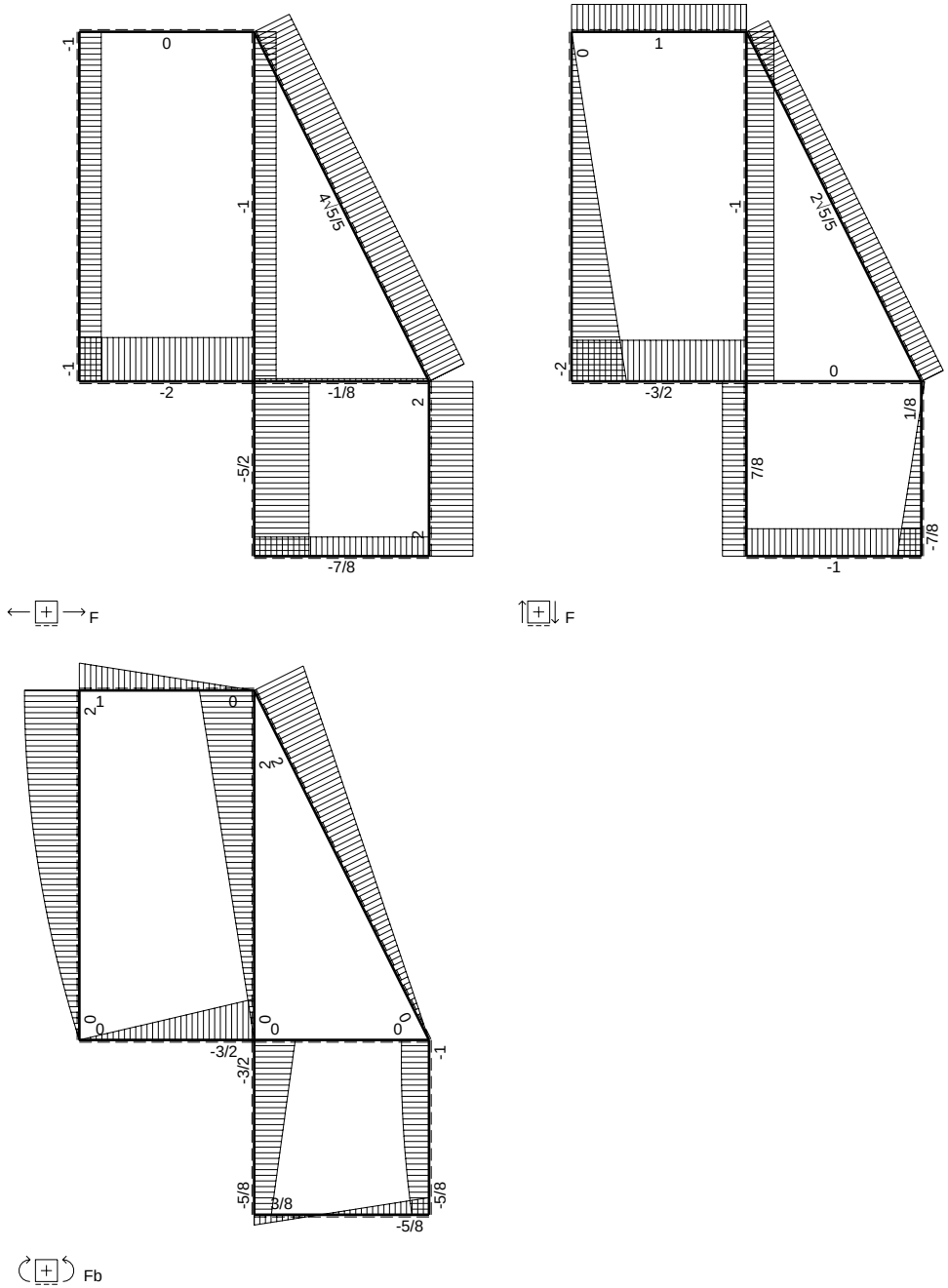
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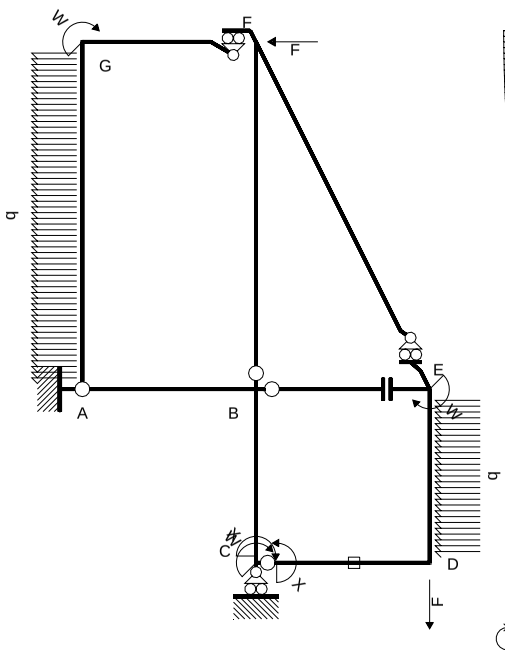
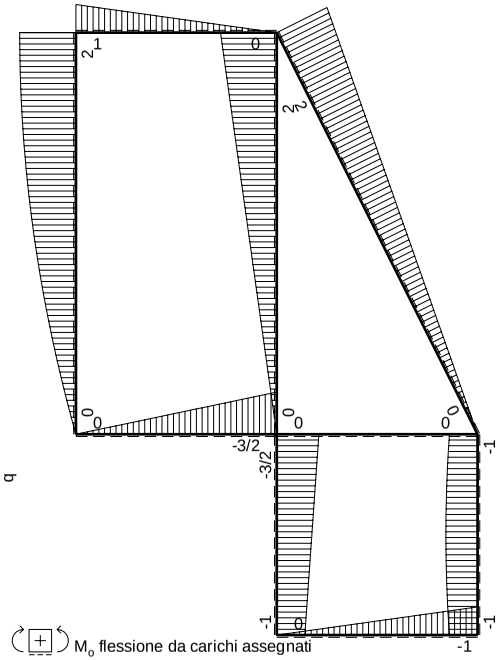
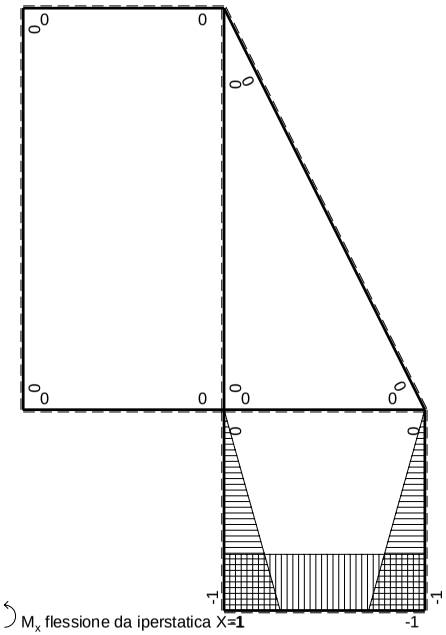
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=**1**

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx+1/2qx^2	Fb-1/2Fx-Fx^2/b+1/2qx^3/b	1-2x/b+x^2/b^2	13/24Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+1/2Fx-1/2qx^2	Fx+1/2Fx^2/b-1/2qx^3/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	2Fb-1/2qx^2	0	0	0	0
AG 2b	0	-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				5/8Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

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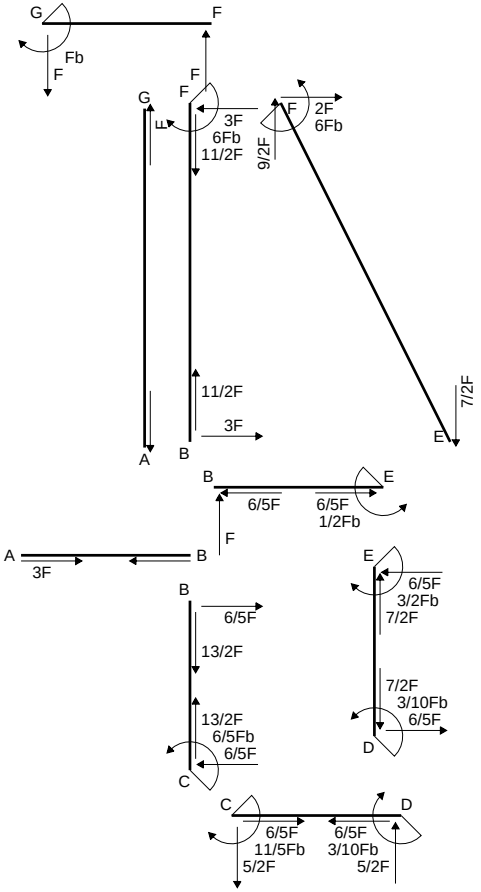
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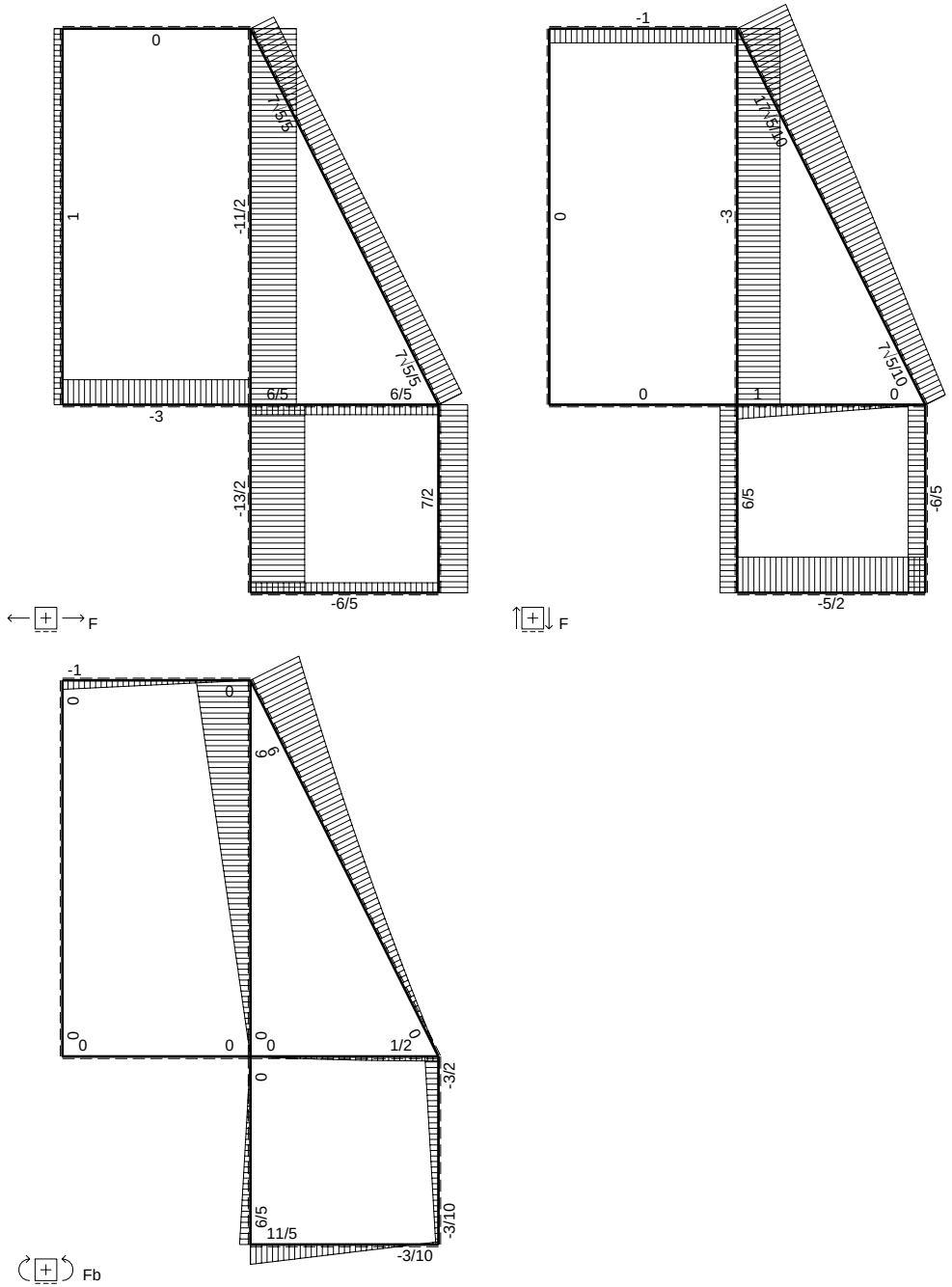
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-6/5
F
-1
0
-3
7√5/10
0
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0
6/5
-6/5
-5/2
F
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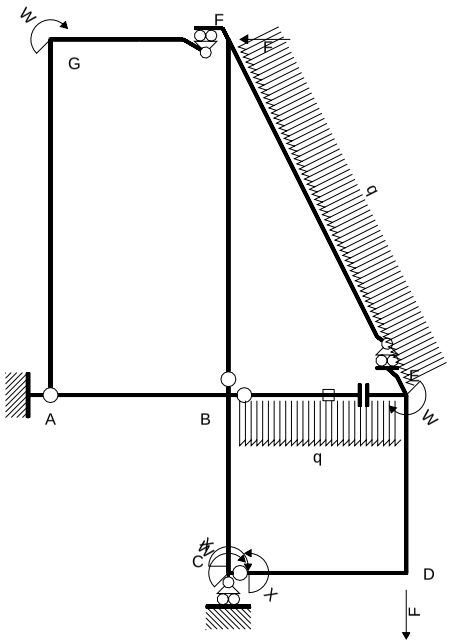
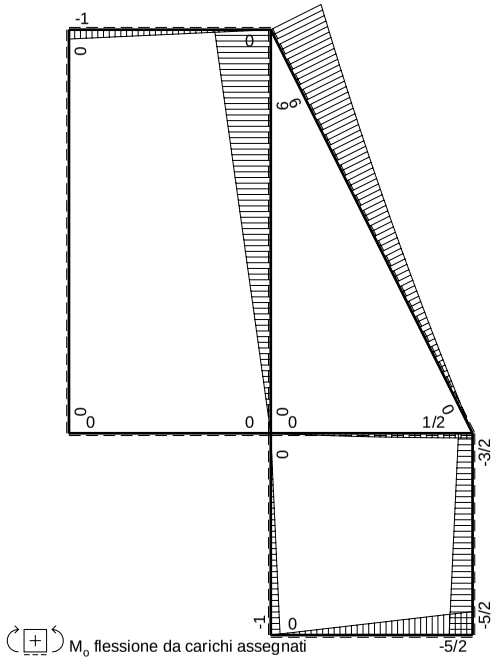
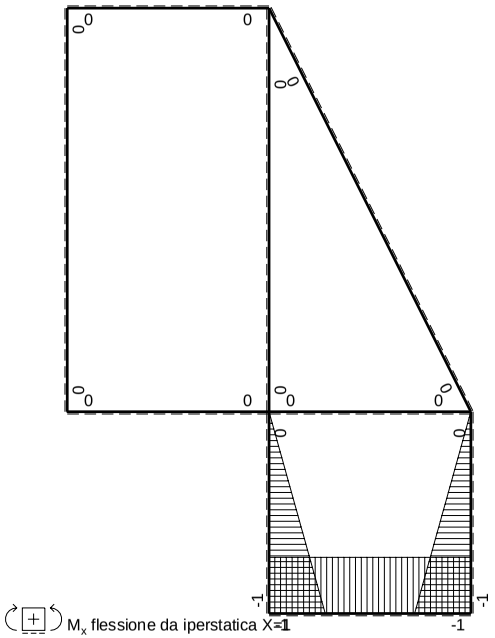
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+Fx	$5/2Fb-7/2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$13/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	3/2Fb+Fx	$3/2Fx+Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$11/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-11/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -7/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -7/4\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -7/4\ b +1/3\ b)\ Fb\ 1/EJ\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/3\ b)\ Fb\ 1/EJ\ = 13/12\ Fb^2/EJ$

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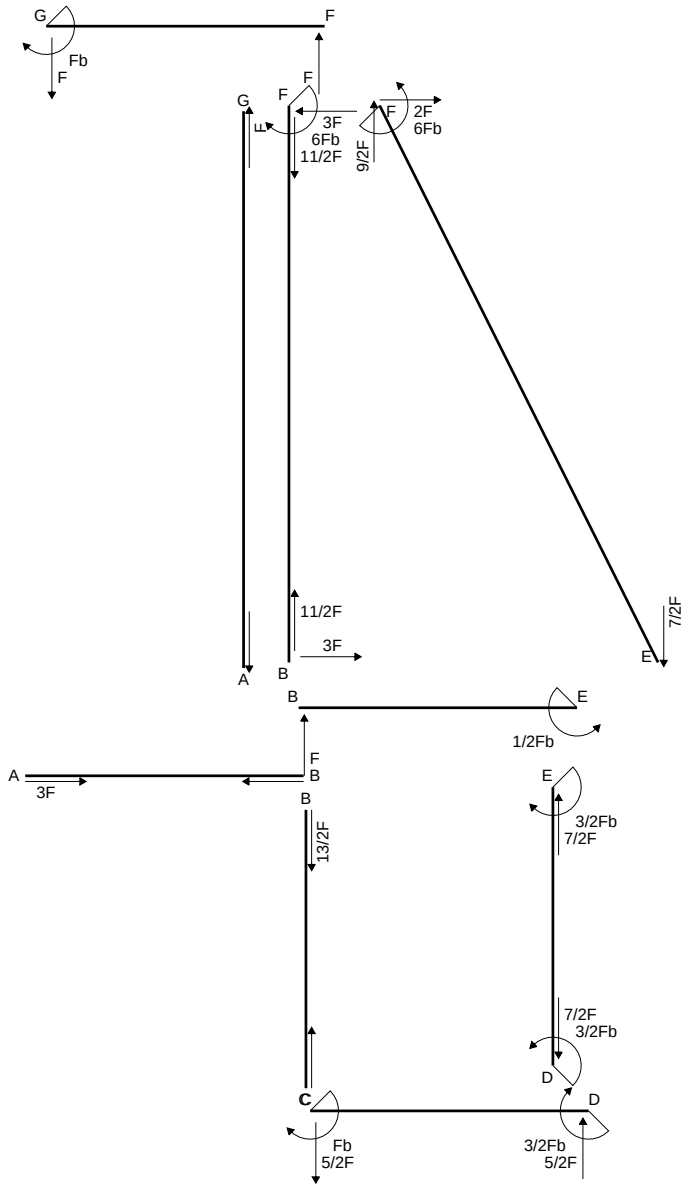
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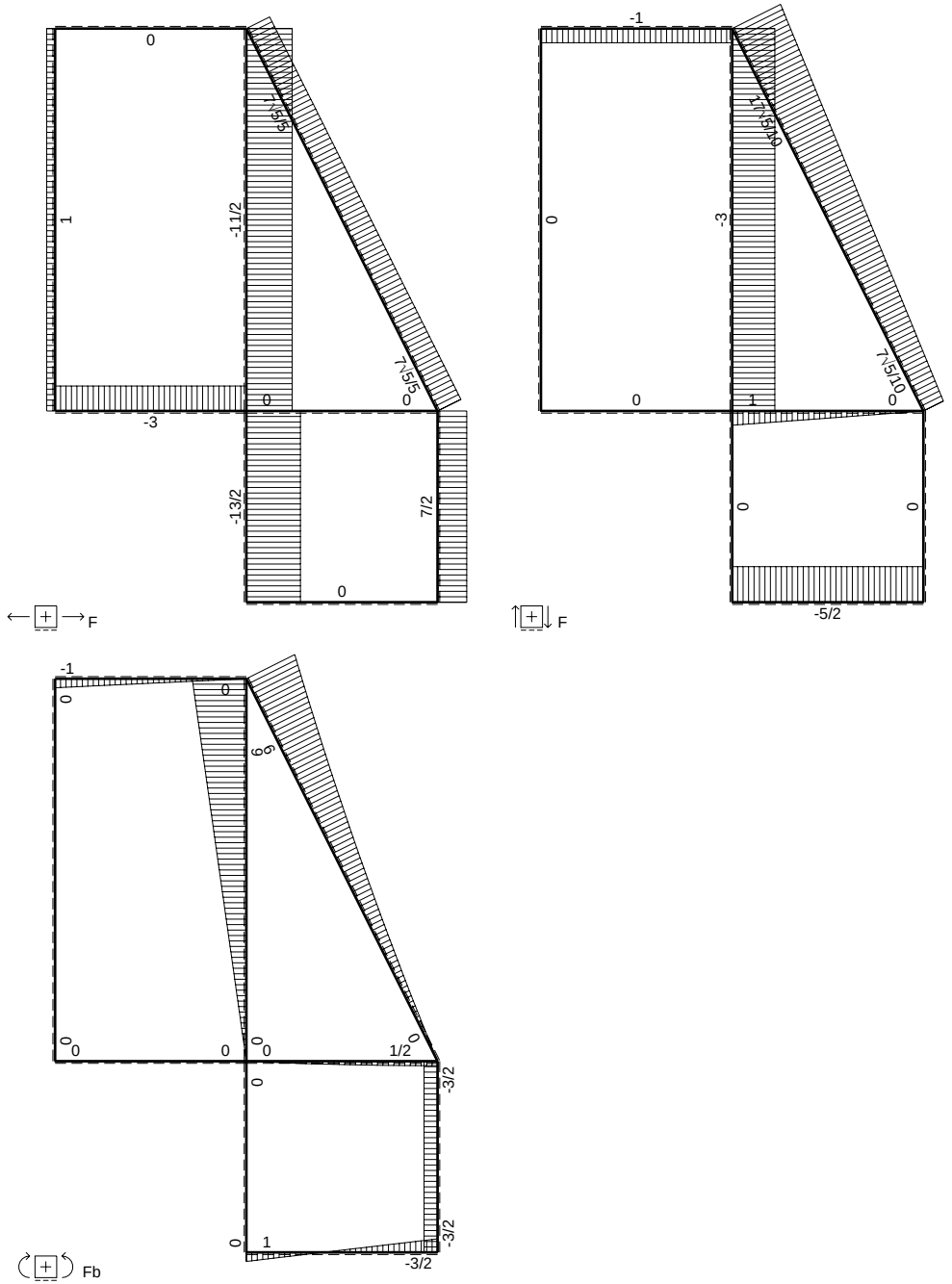
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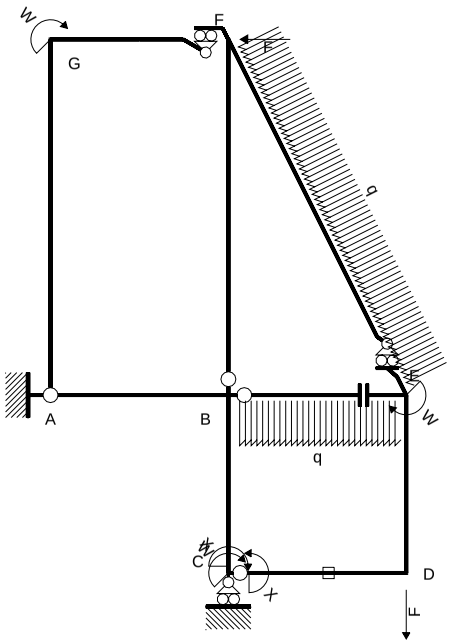
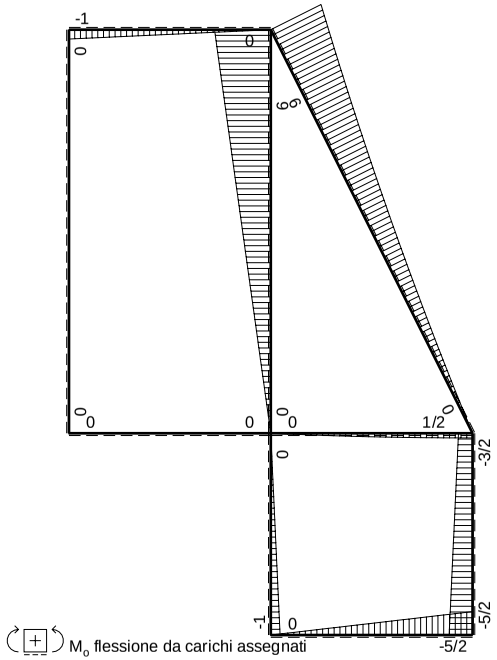
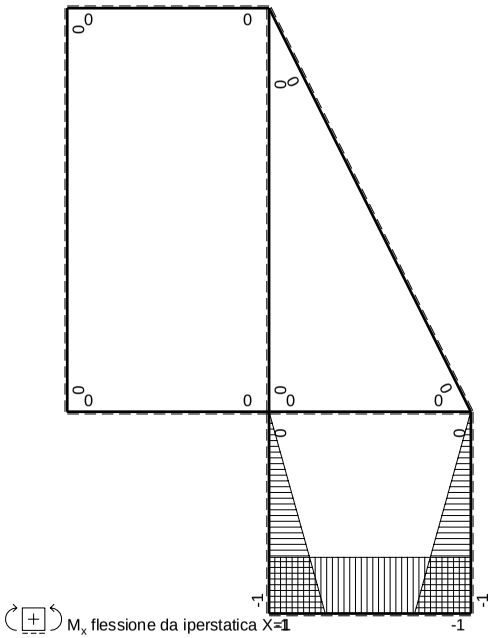


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+Fx	$5/2Fb-7/2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$13/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	3/2Fb+Fx	$3/2Fx+Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -7/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -7/4\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -7/4\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 13/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 13/12\ \ Fb^2/EJ$

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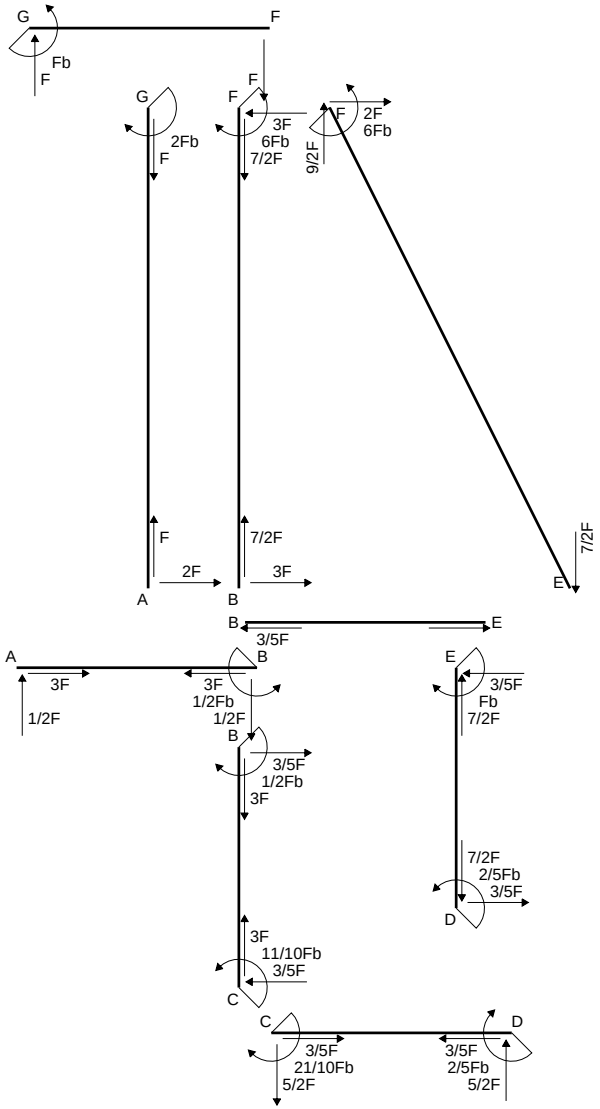
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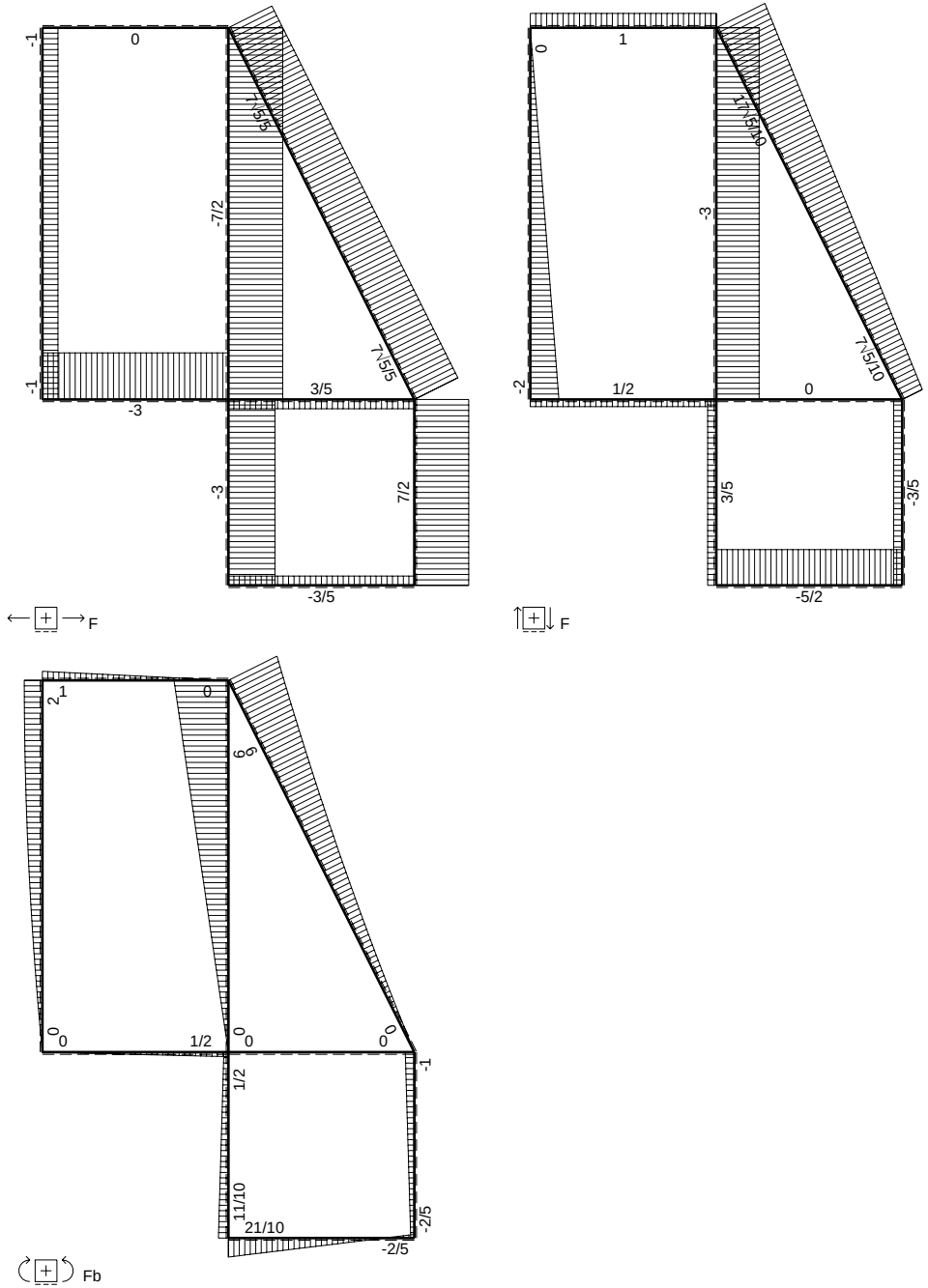
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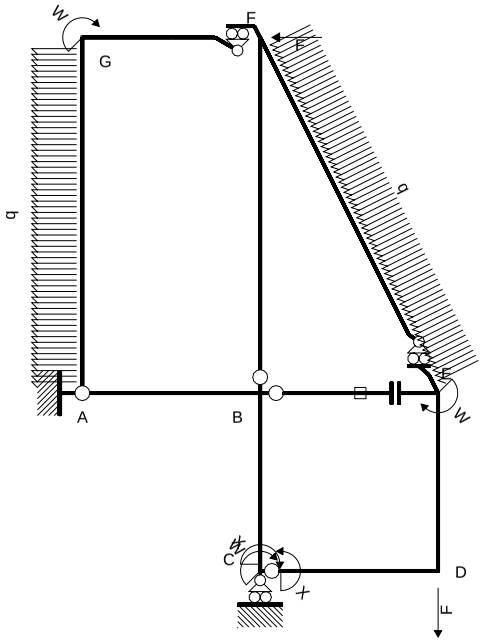
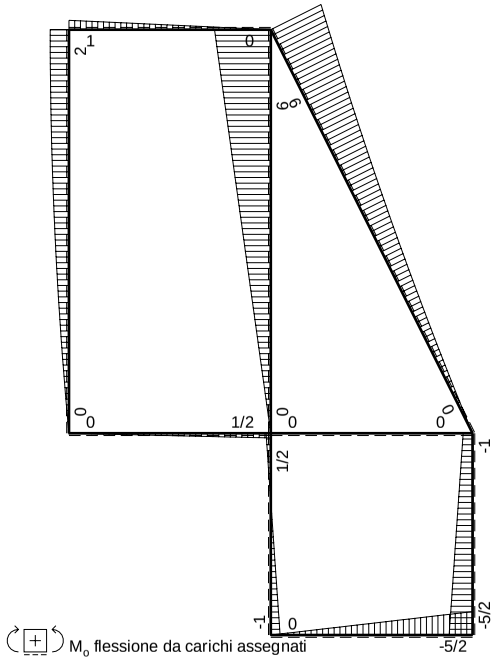
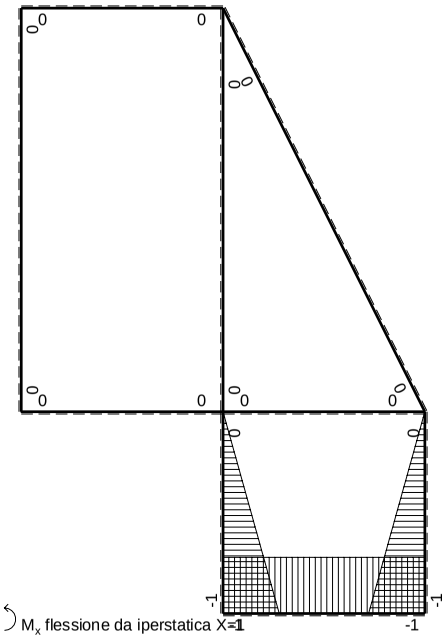


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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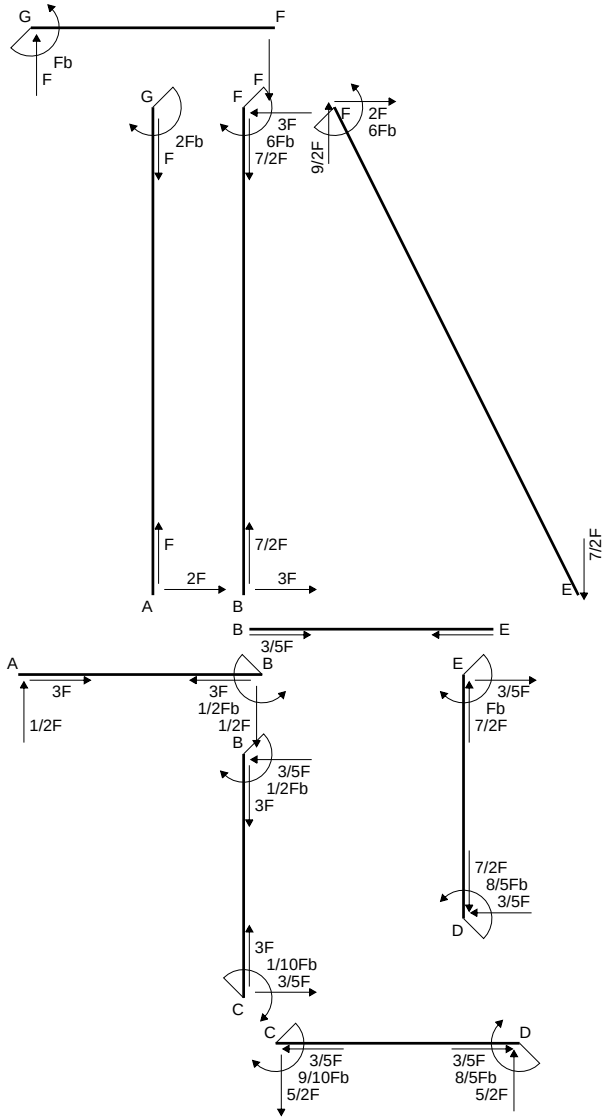
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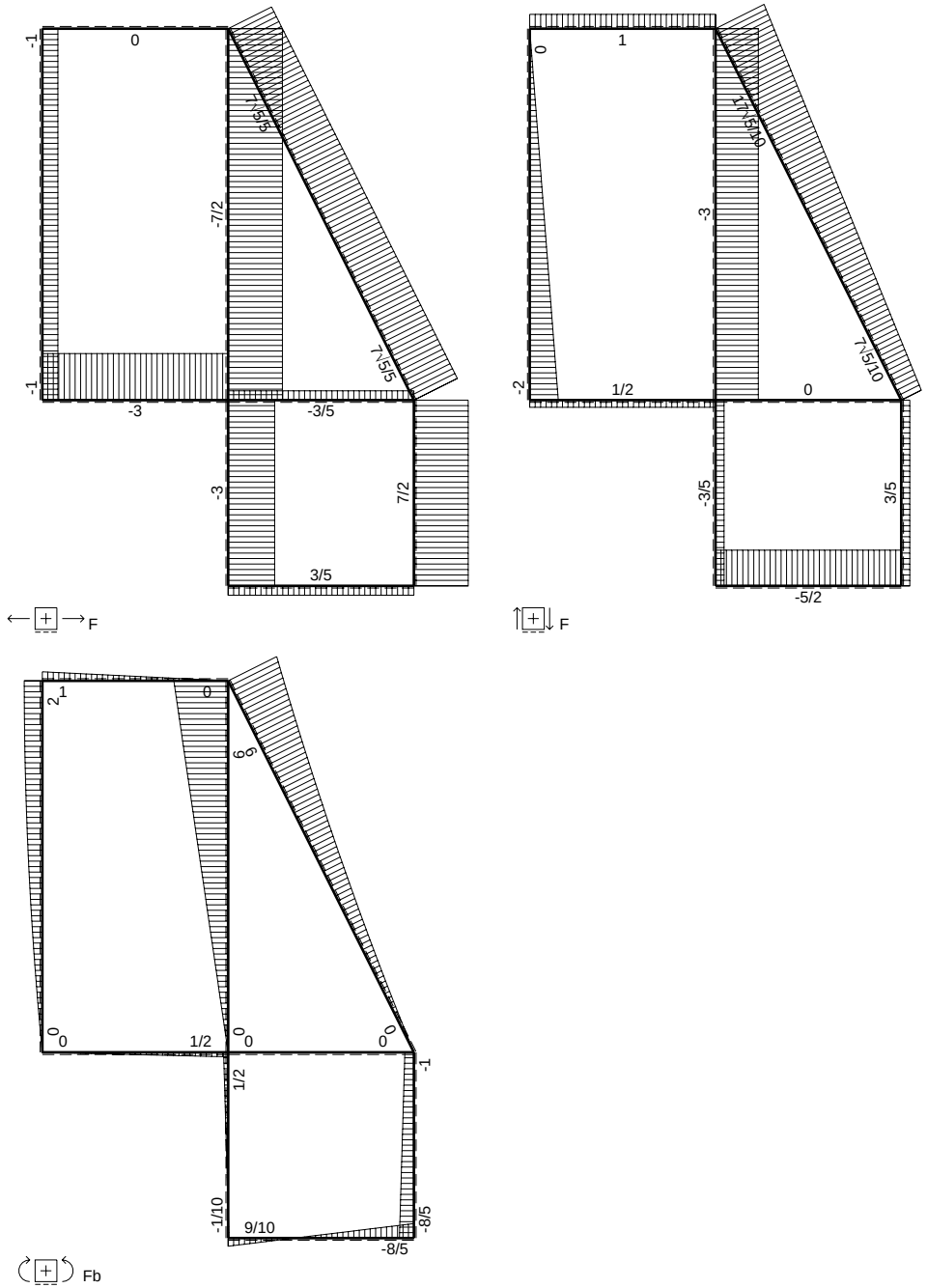
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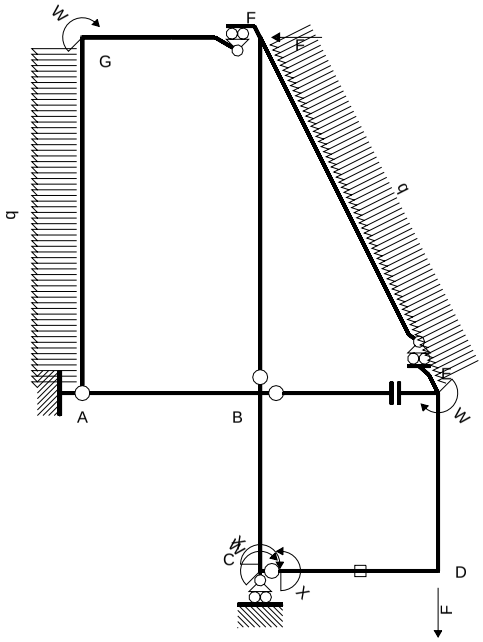
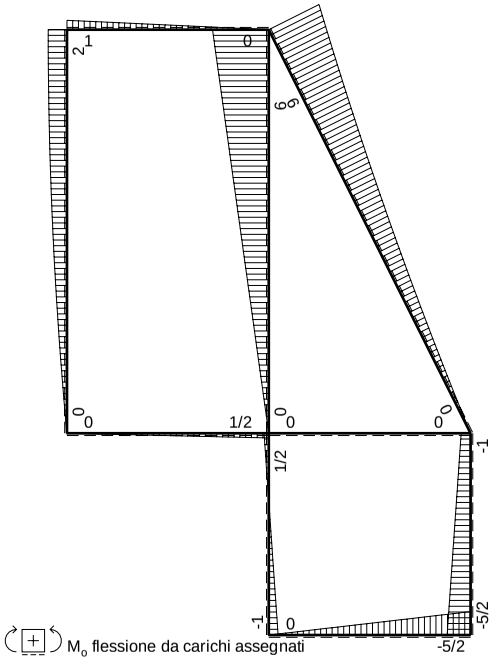
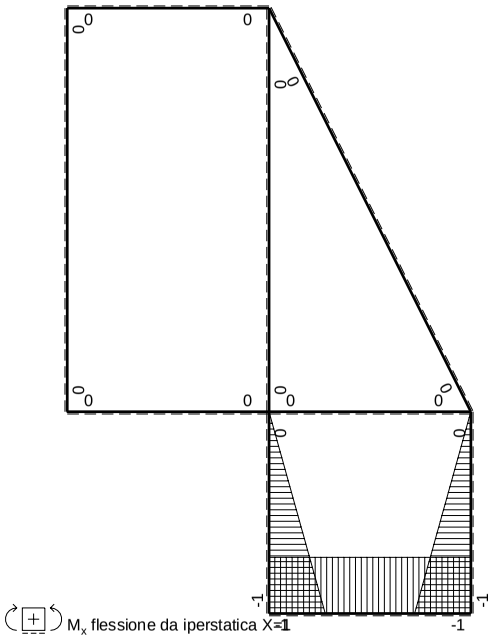


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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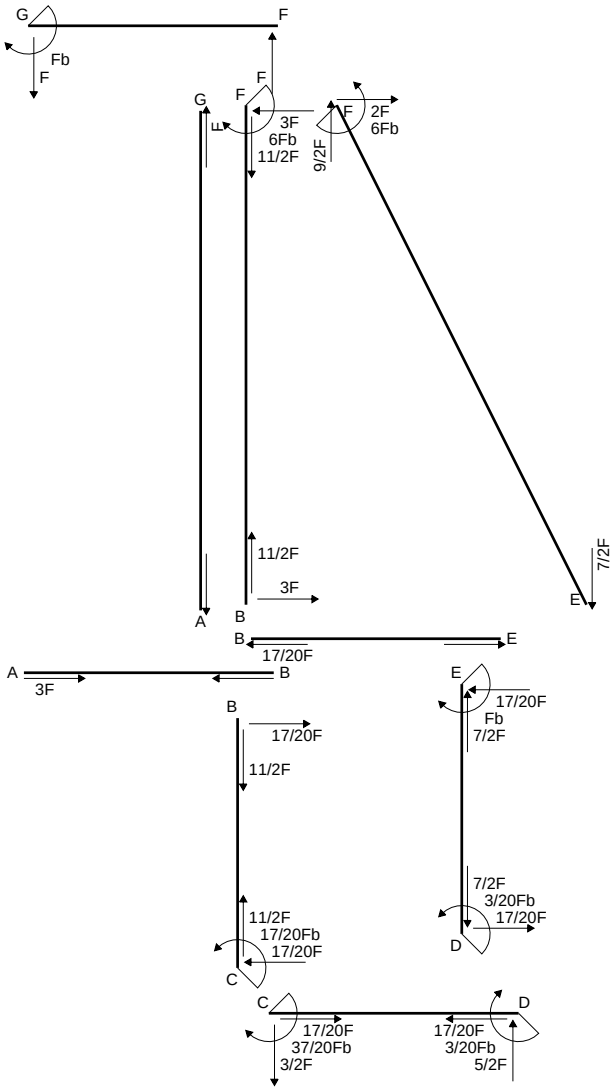
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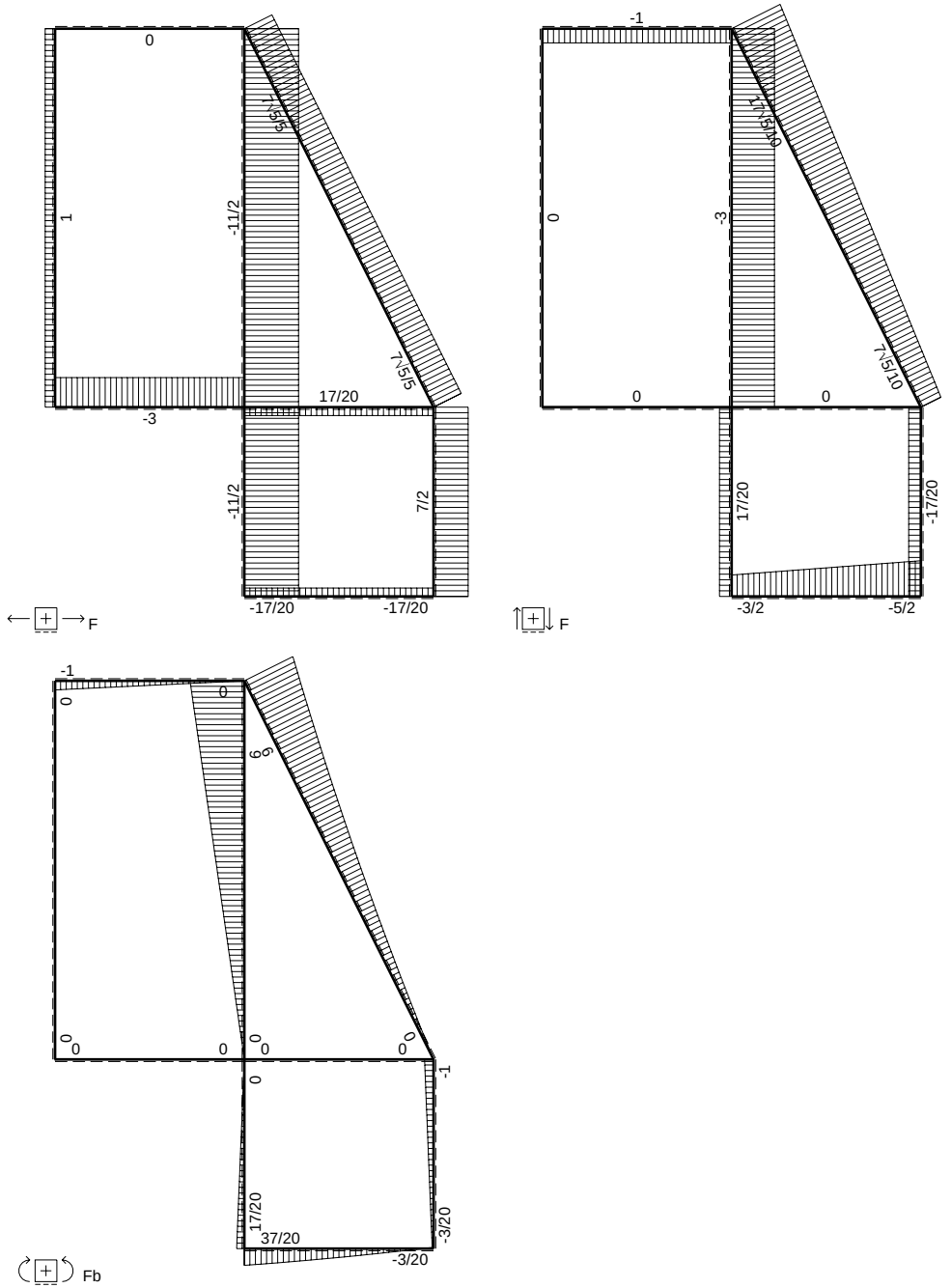
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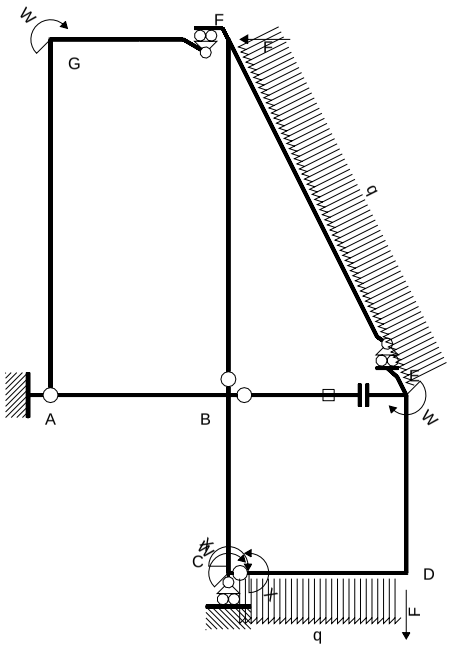
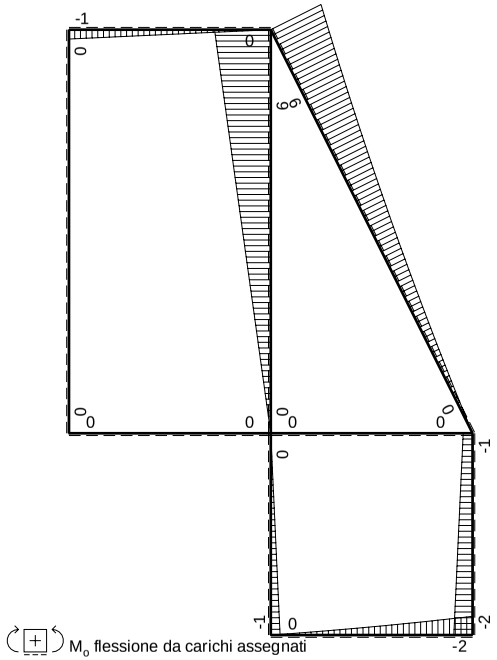
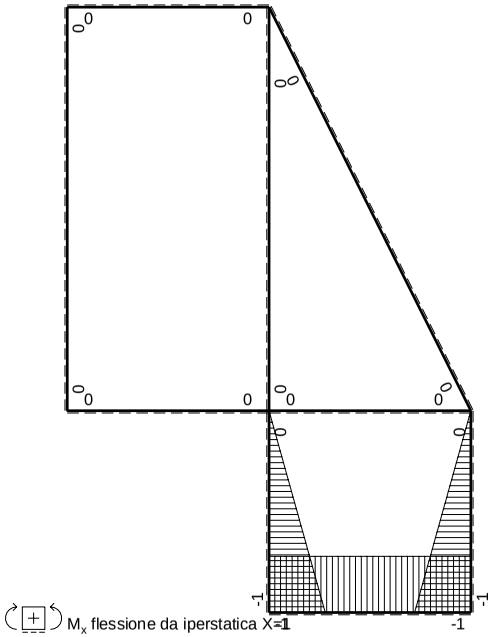


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-3/2Fx-1/2qx^2$	$3/2Fx+1/2Fx^2/b$	1	$11/12Fb^2/EJ$	Xb/EJ
DC b	1	$2Fb-5/2Fx+1/2qx^2$	$2Fb-5/2Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-2Fb+Fx	$2Fb-3Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+Fx	$Fx+Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$37/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-37/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (2 -5/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [2\ x -5/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -5/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (2 -3\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [2\ x -3/2\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -3/2\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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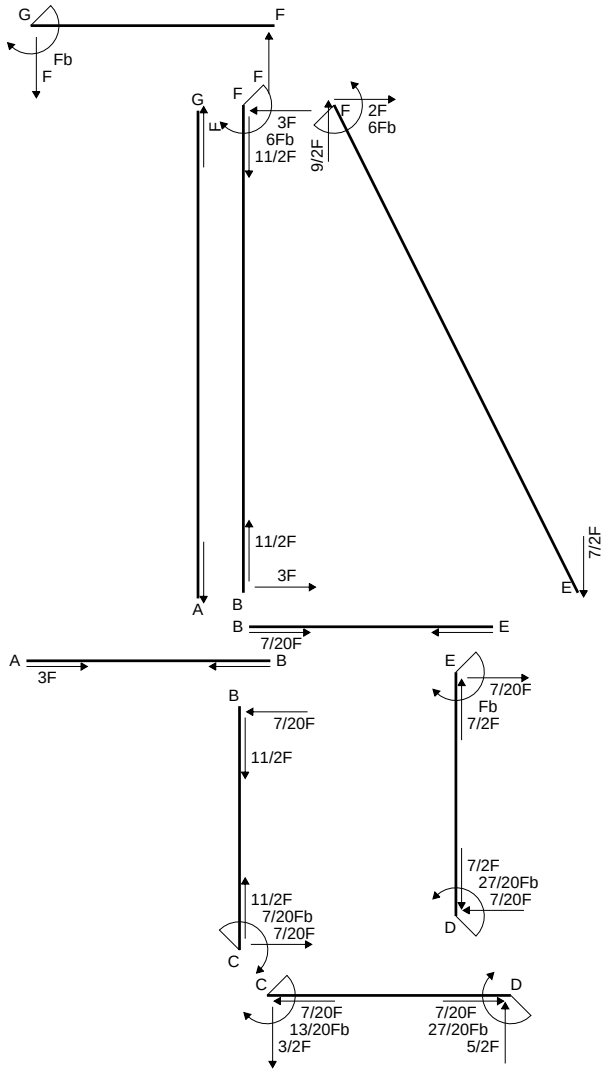
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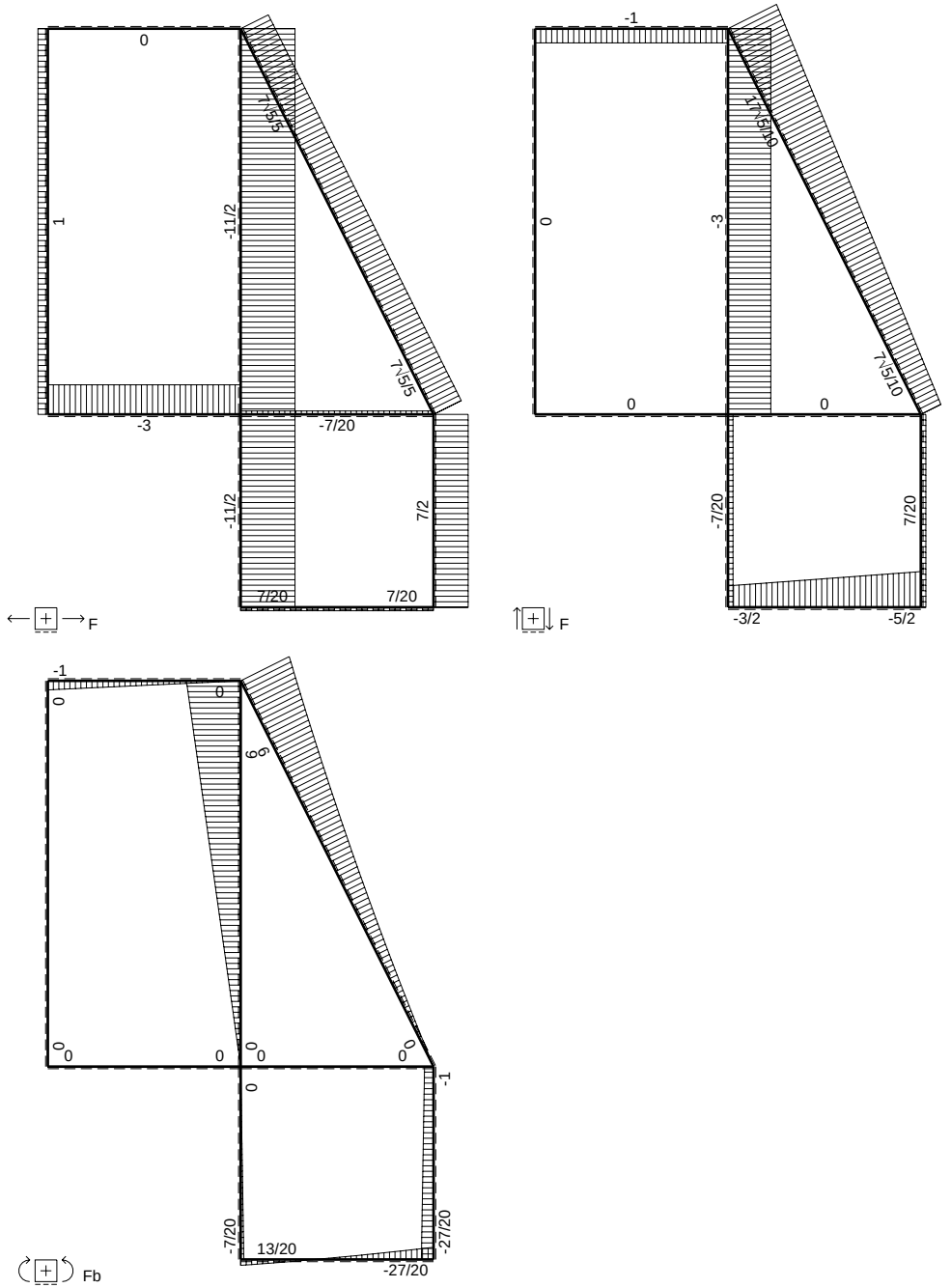
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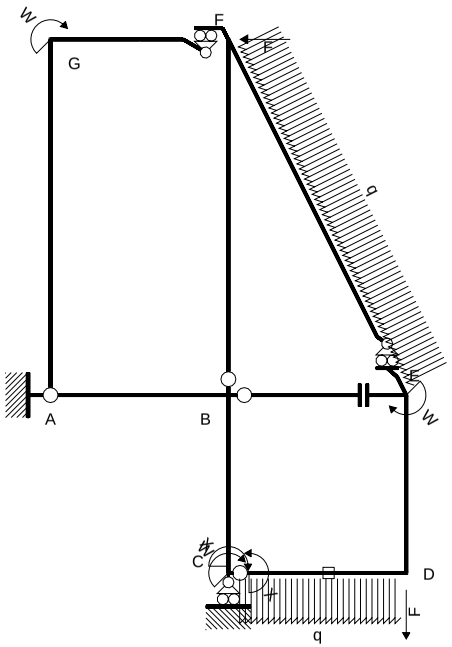
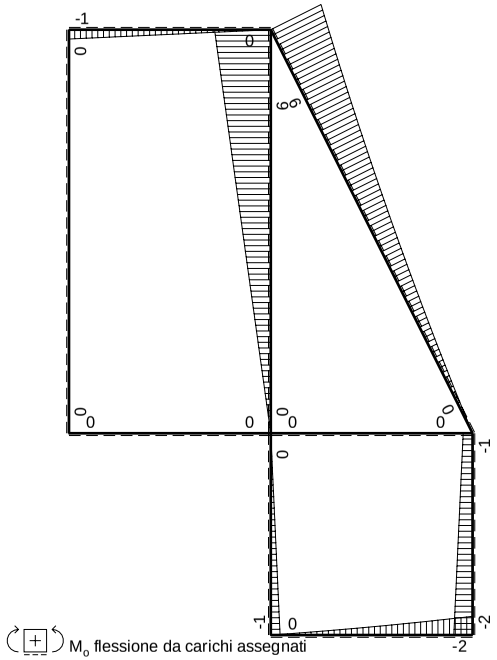
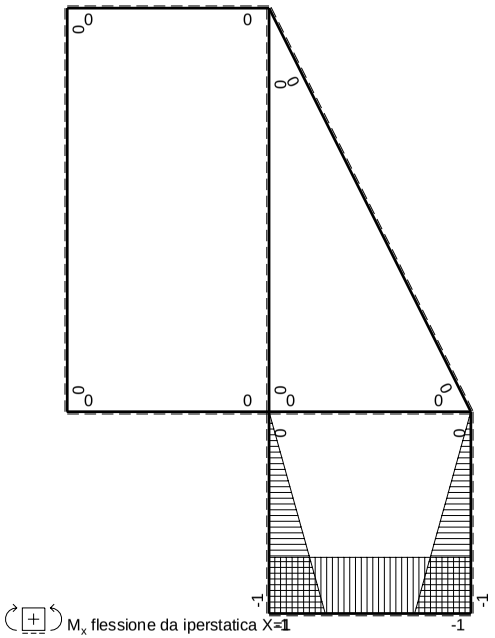


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-3/2Fx-1/2qx^2$	$3/2Fx+1/2Fx^2/b$	1	$11/12Fb^2/EJ$	Xb/EJ
DC b	1	$2Fb-5/2Fx+1/2qx^2$	$2Fb-5/2Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-2Fb+Fx	$2Fb-3Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+Fx	$Fx+Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$13/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-13/20Fb$	

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/12\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (2 -5/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [2\ x -5/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (2\ b -5/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/12\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (2 -3\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [2\ x -3/2\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -3/2\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238530 ESΣ02.xxxx.033

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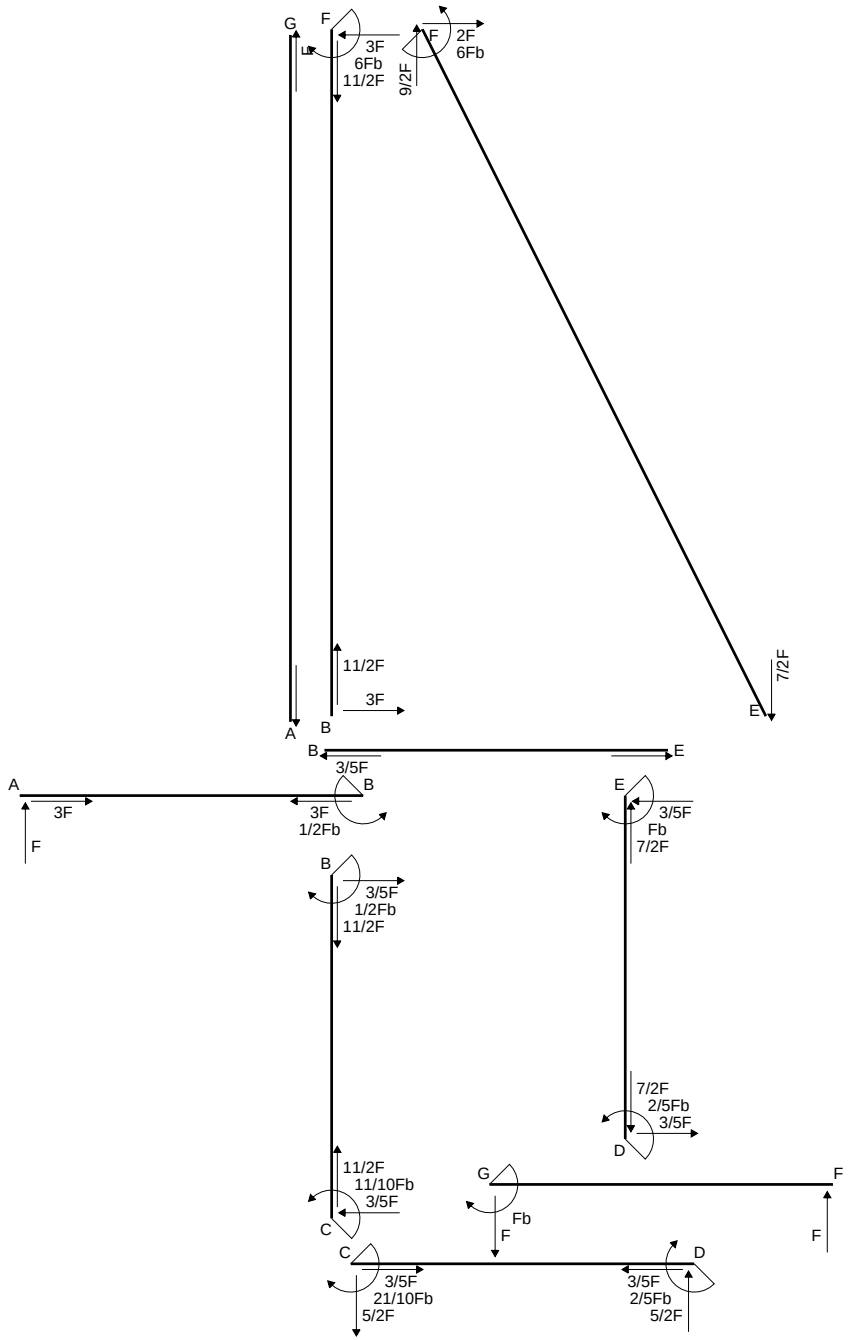
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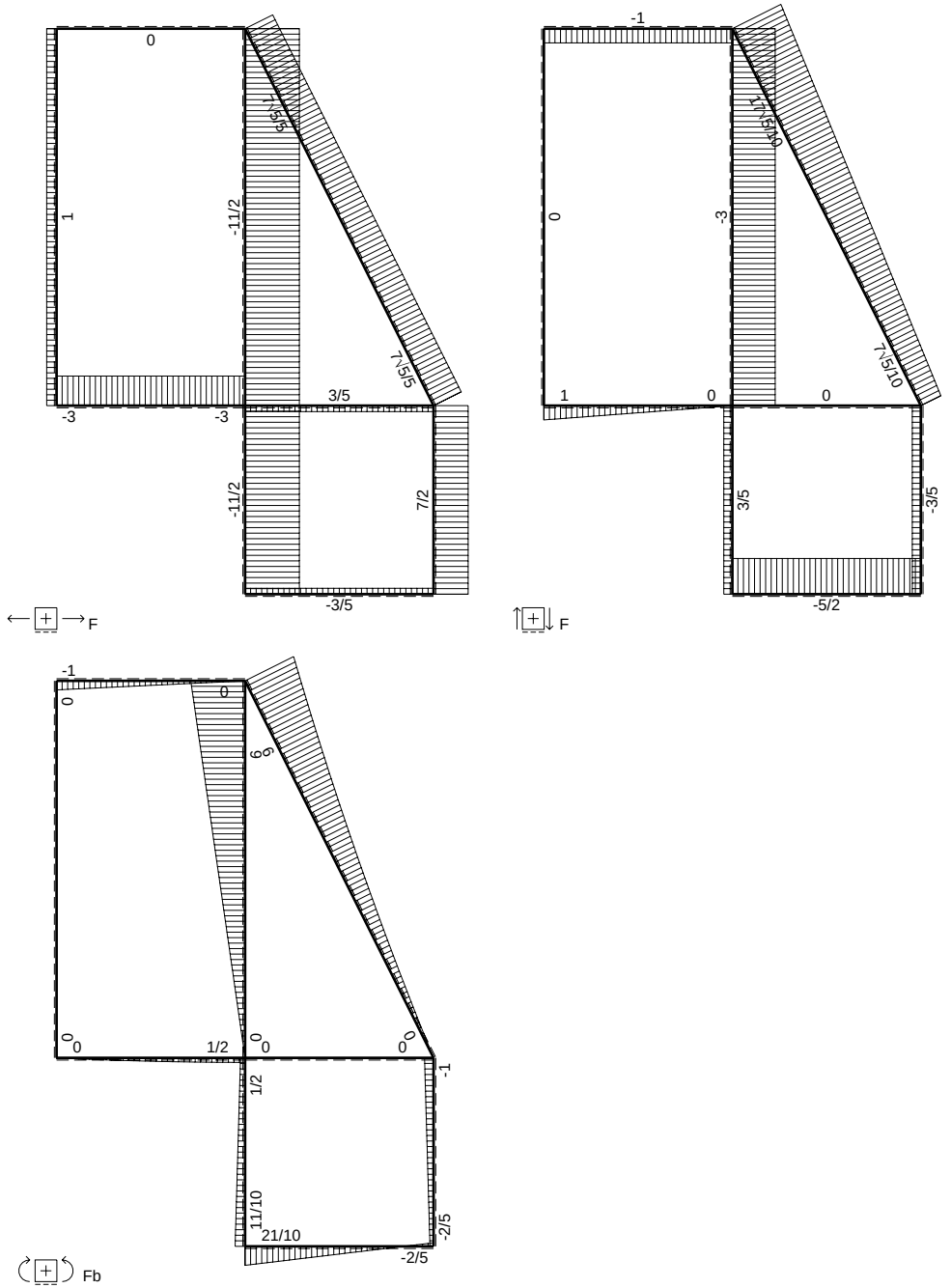
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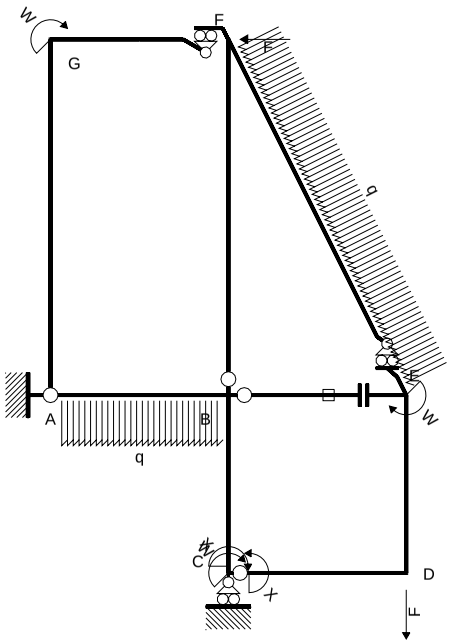
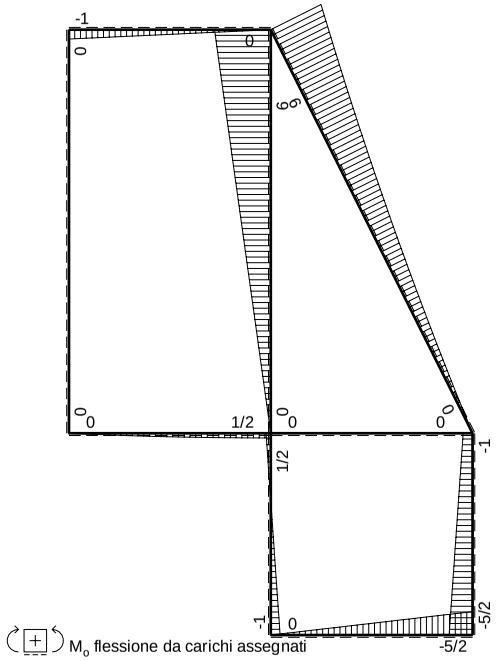
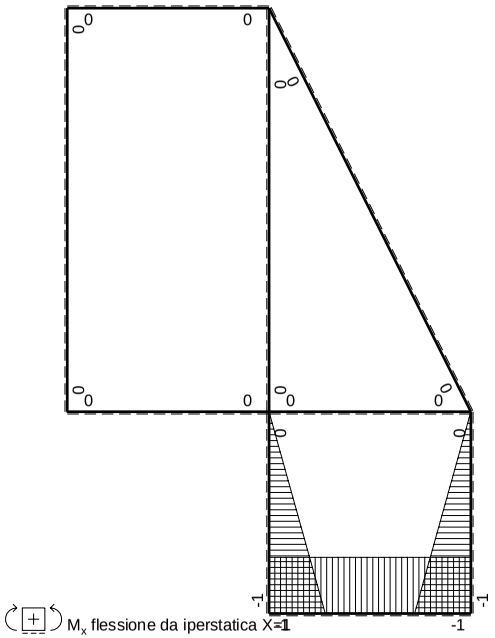


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fb+1/2qx^2$	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

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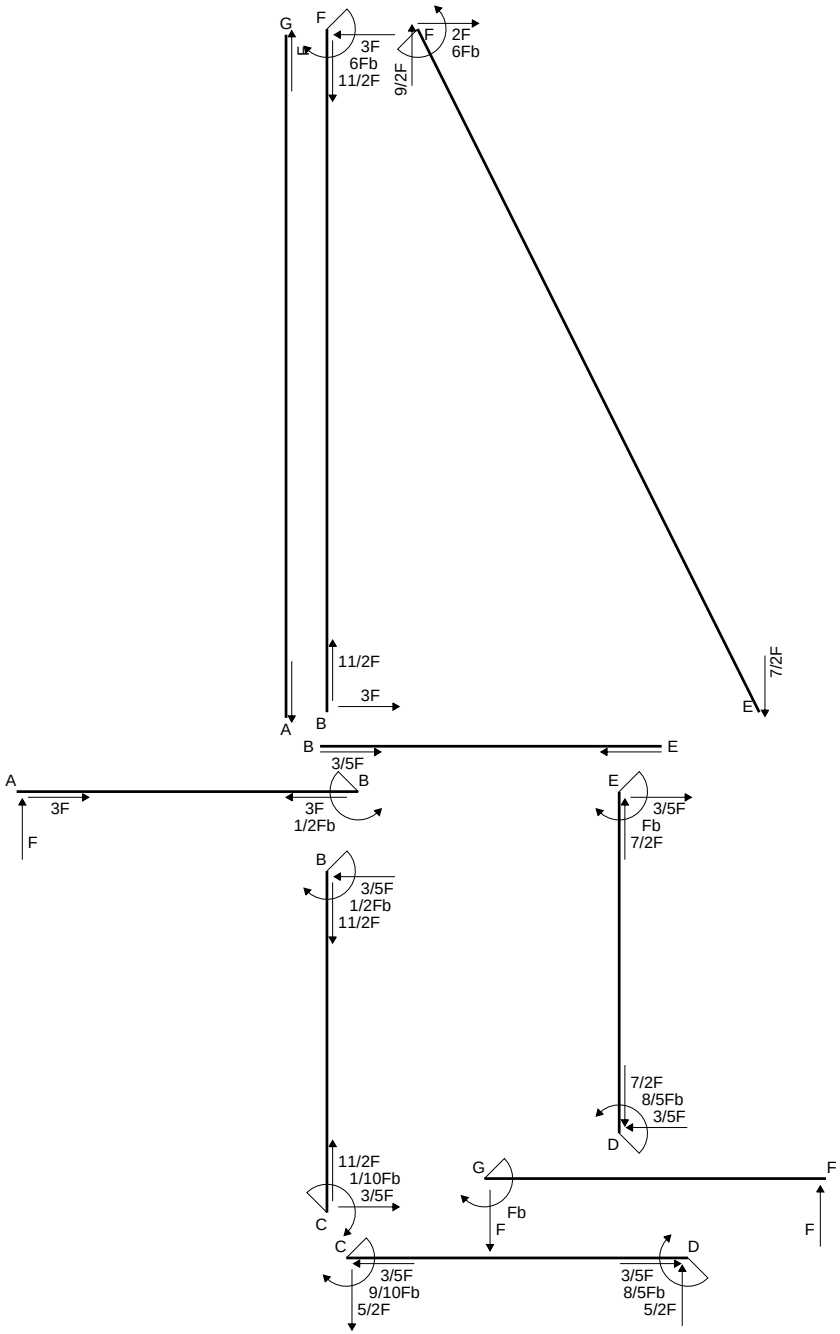
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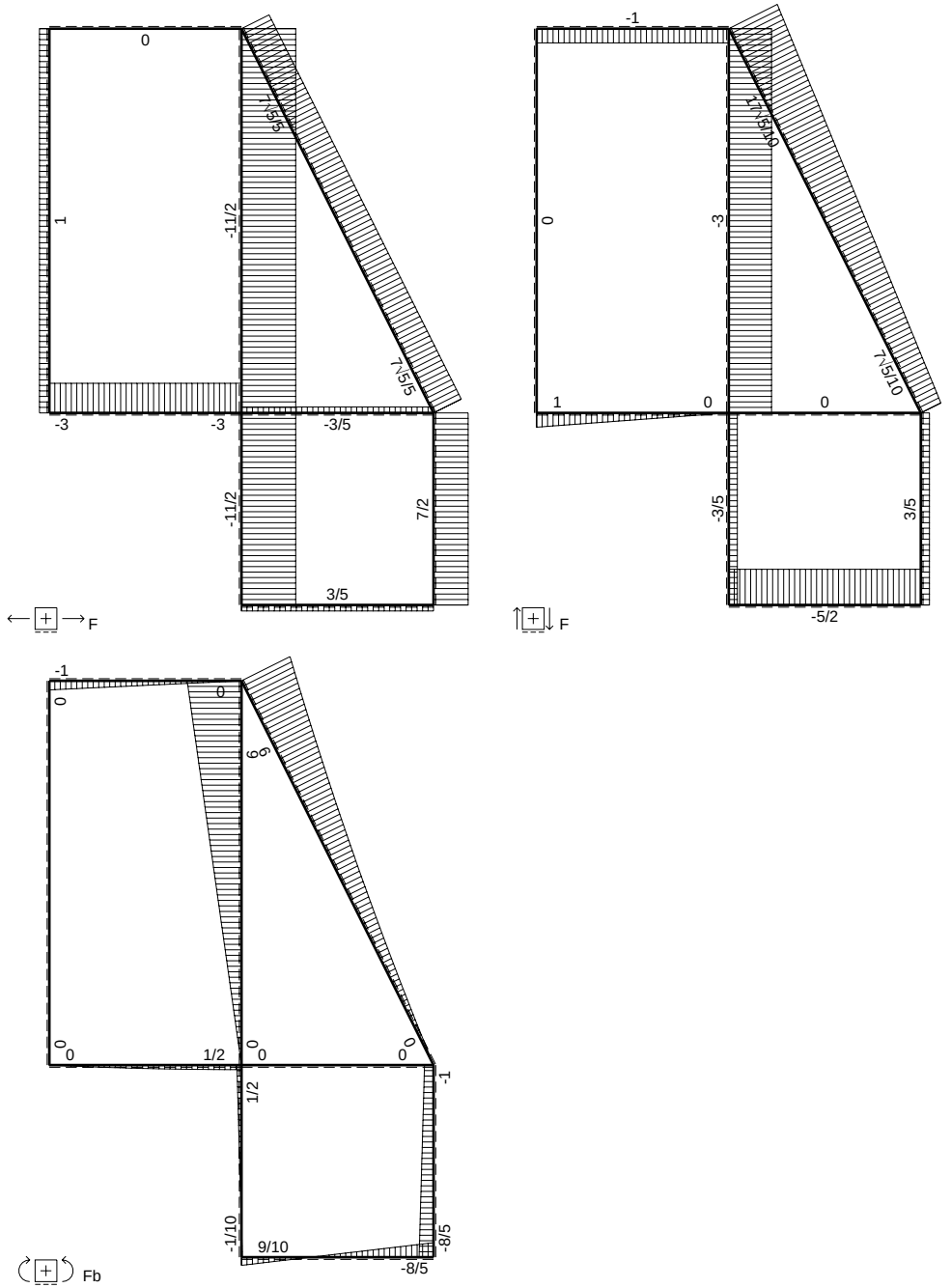
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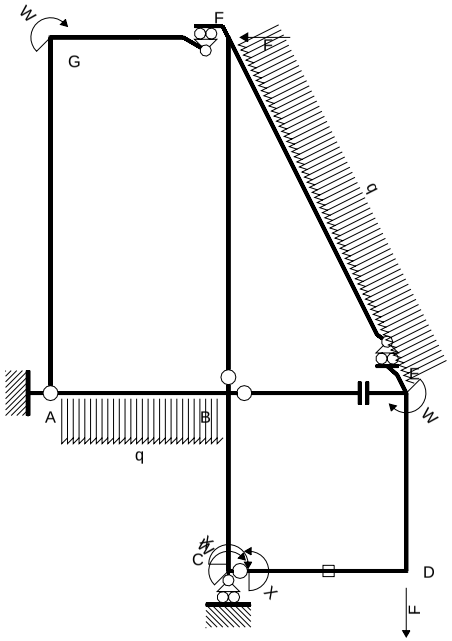
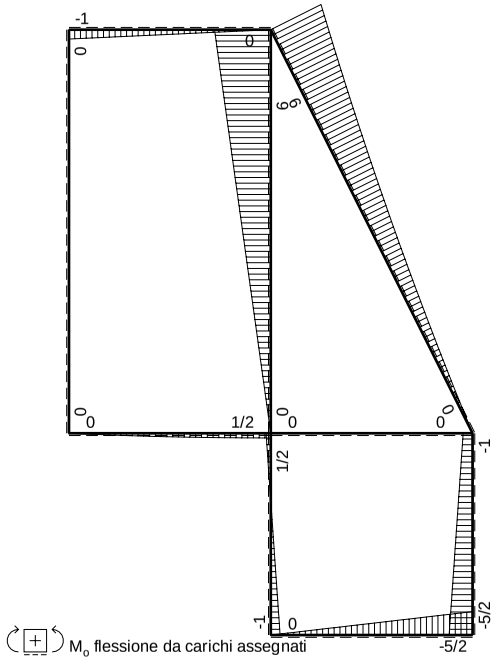
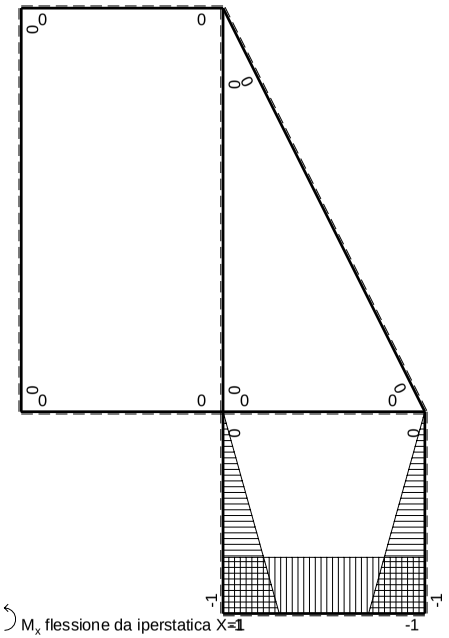


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fb+1/2qx^2$	0	0		
BC b	-x/b	$1/2Fb-3/2Fx$	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	$Fb-3/2Fx$	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-5/2Fx$	$5/2Fx$	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-5/2Fx$	$5/2Fb-5/2Fx$	1		
DE b	-1+x/b	$-5/2Fb+3/2Fx$	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	$Fb+3/2Fx$	$Fx+3/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-9/10Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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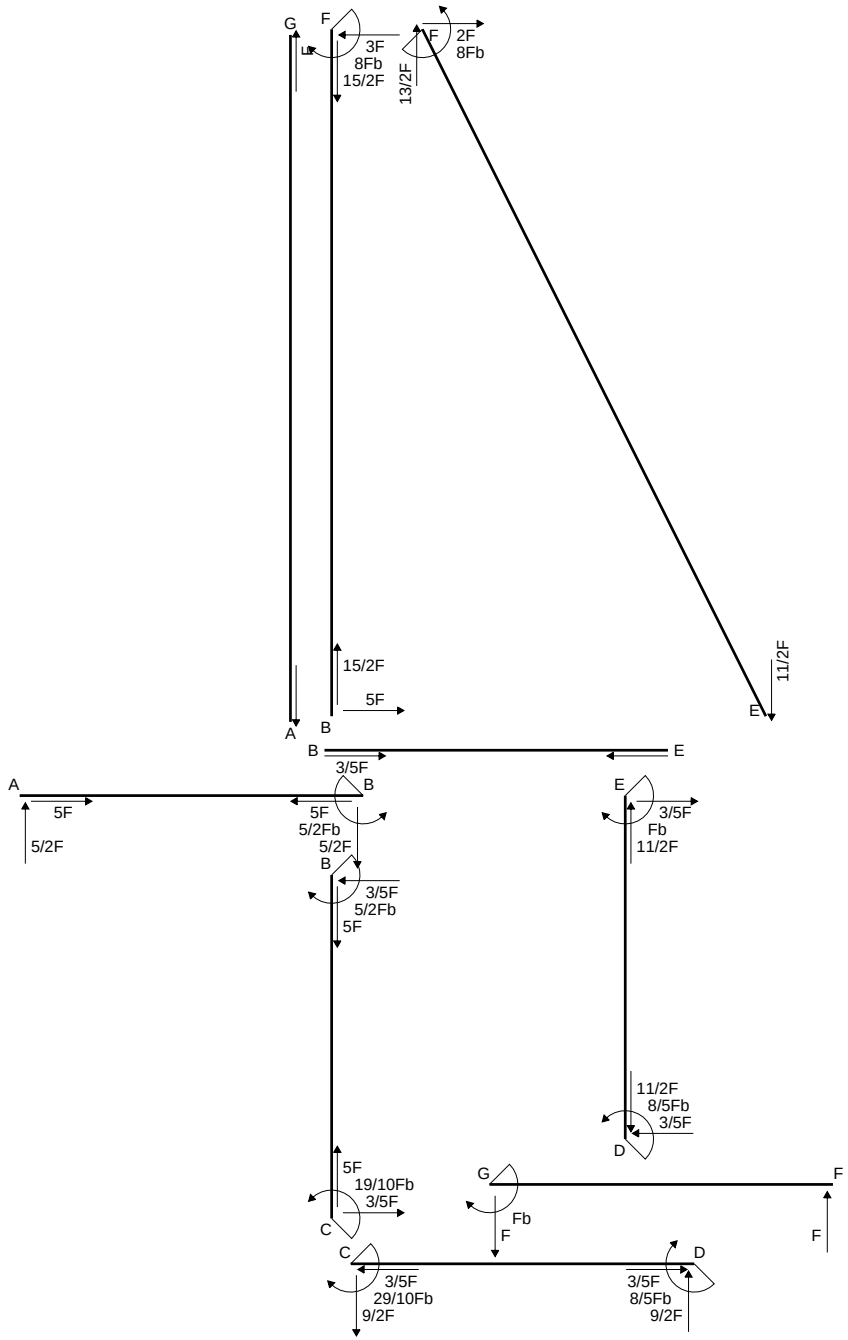
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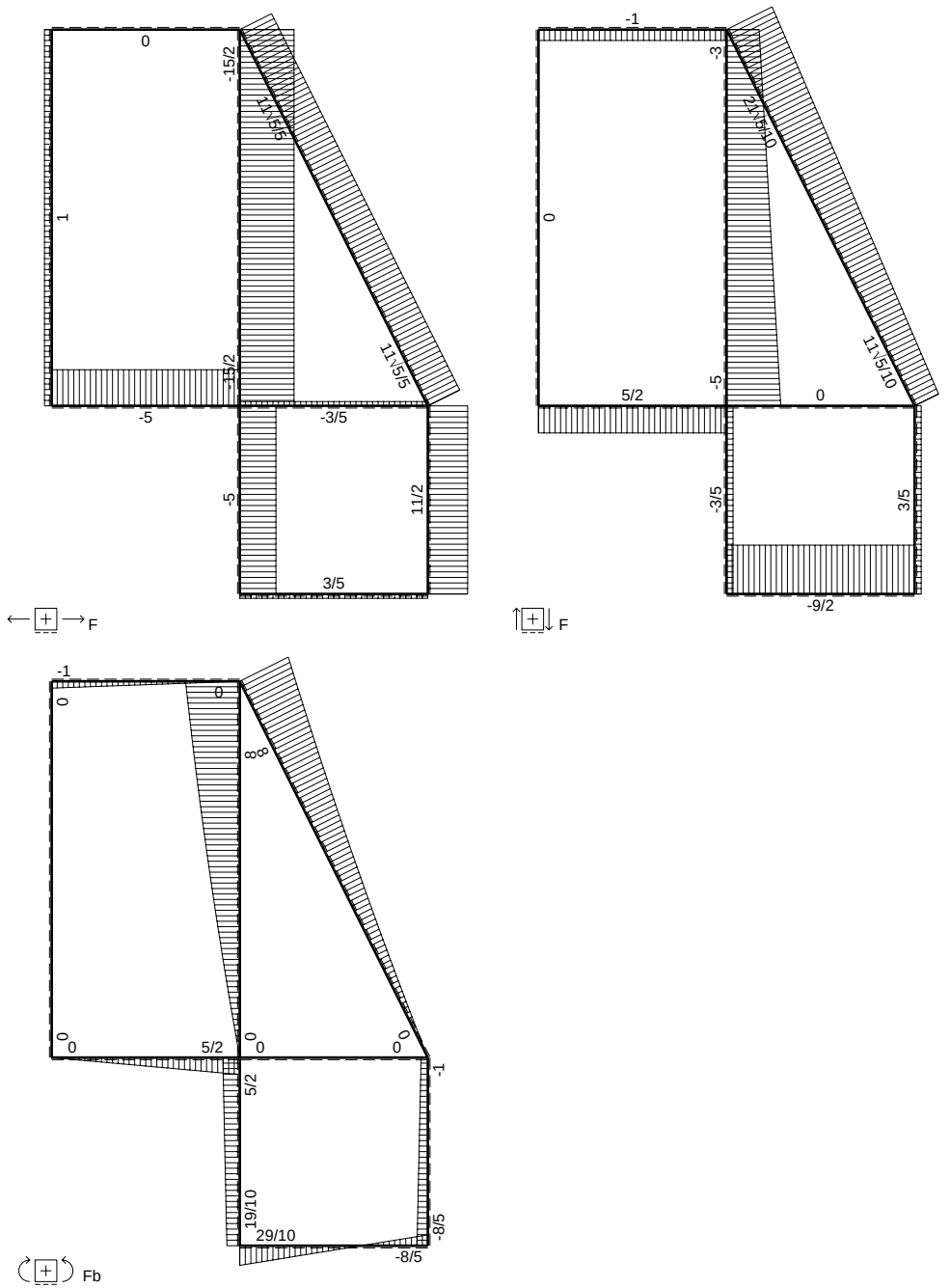
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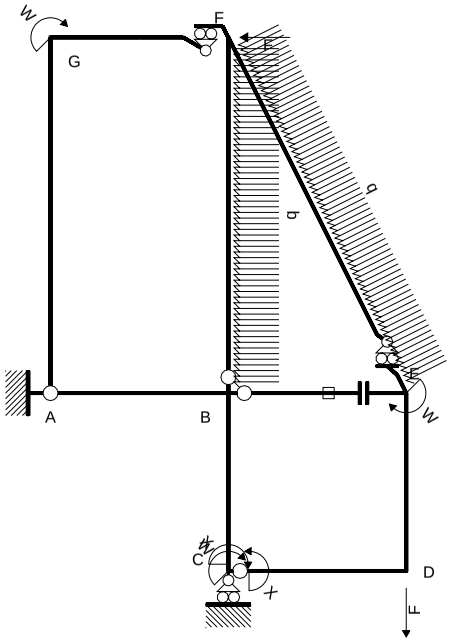
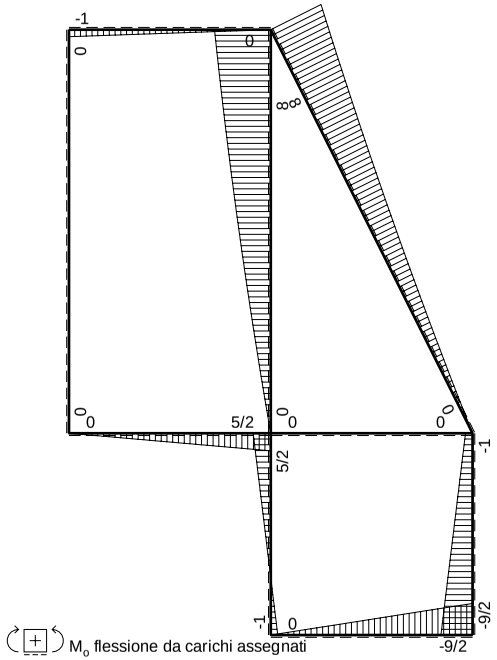
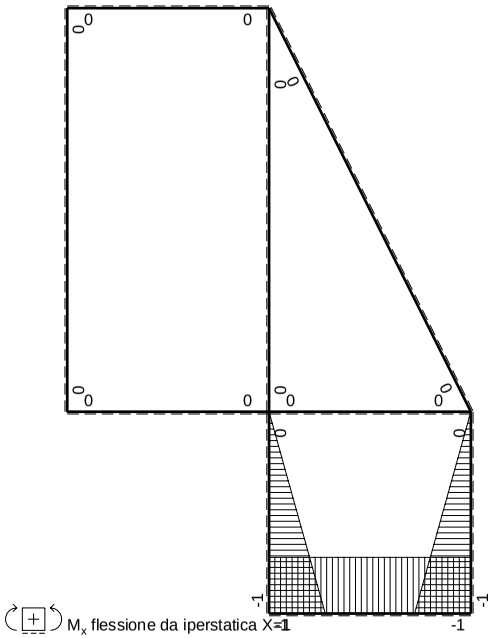
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	5/2Fx	0	0	0	0
BA b	0	-5/2Fb+5/2Fx	0	0		
BC b	-x/b	5/2Fb-7/2Fx	-5/2Fx+7/2Fx^2/b	x^2/b^2	-1/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-7/2Fx	Fb-9/2Fx+7/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-9/2Fx	9/2Fx	1	9/4Fb^2/EJ	Xb/EJ
DC b	1	9/2Fb-9/2Fx	9/2Fb-9/2Fx	1		
DE b	-1+x/b	-9/2Fb+7/2Fx	9/2Fb-8Fx+7/2Fx^2/b	1-2x/b+x^2/b^2	5/3Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+7/2Fx	Fx+7/2Fx^2/b	x^2/b^2		
EF √5b	0	11√5/10Fx+1/2qx^2	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	8Fb-3Fx-1/2qx^2	0	0	0	0
BF 2b	0	-5Fx+1/2qx^2	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				29/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-29/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-5/2\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -9/2\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -9/4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -9/4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (9/2\ x/b\ )\ Fb\ 1/EJ\ dx = [9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (9/4\ b\ )\ Fb\ 1/EJ\ = 9/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (9/2 -9/2\ x/b\ )\ Fb\ 1/EJ\ dx = [9/2\ x -9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -9/4\ b\ )\ Fb\ 1/EJ\ = 9/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (9/2 -8\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [9/2\ x -4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = 5/3\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +7/6\ b\ )\ Fb\ 1/EJ\ = 5/3\ Fb^2/EJ$

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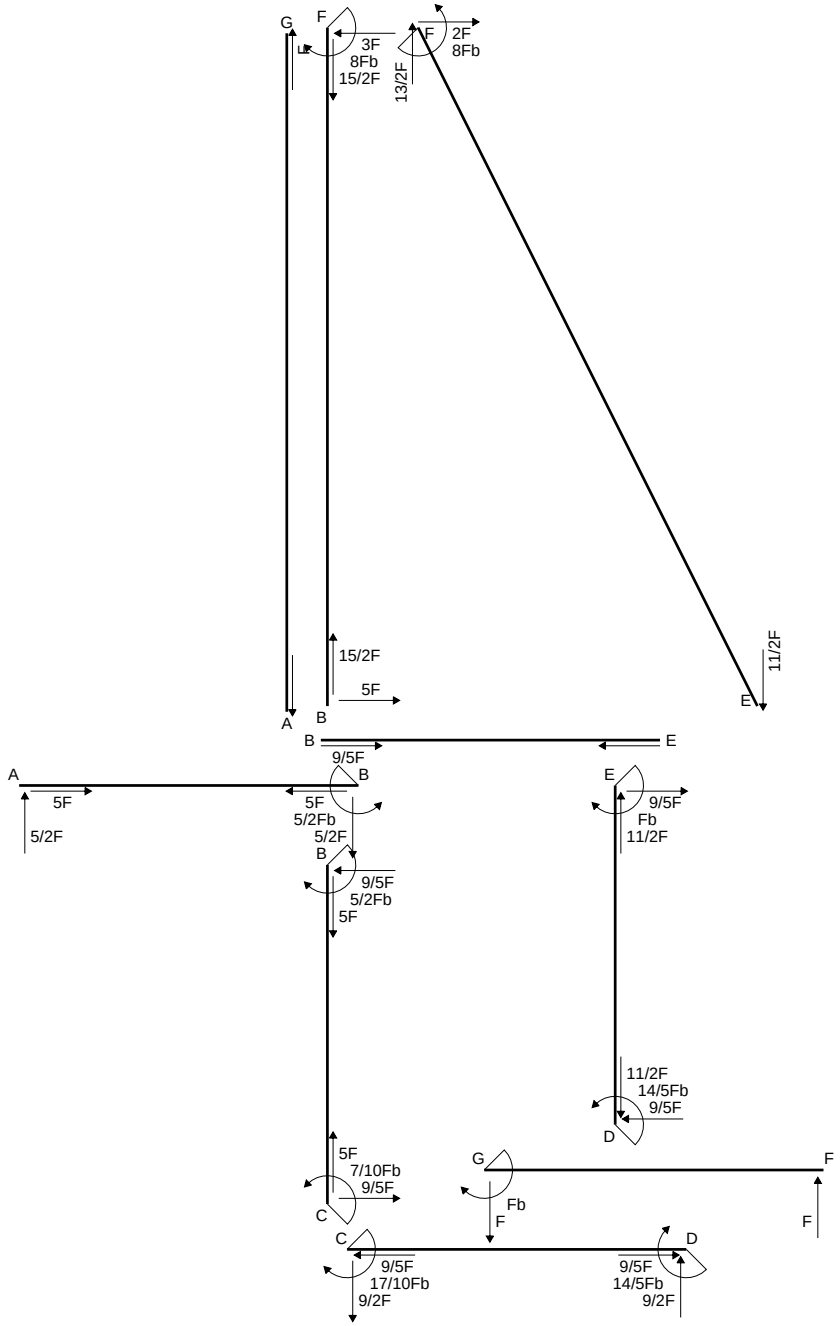
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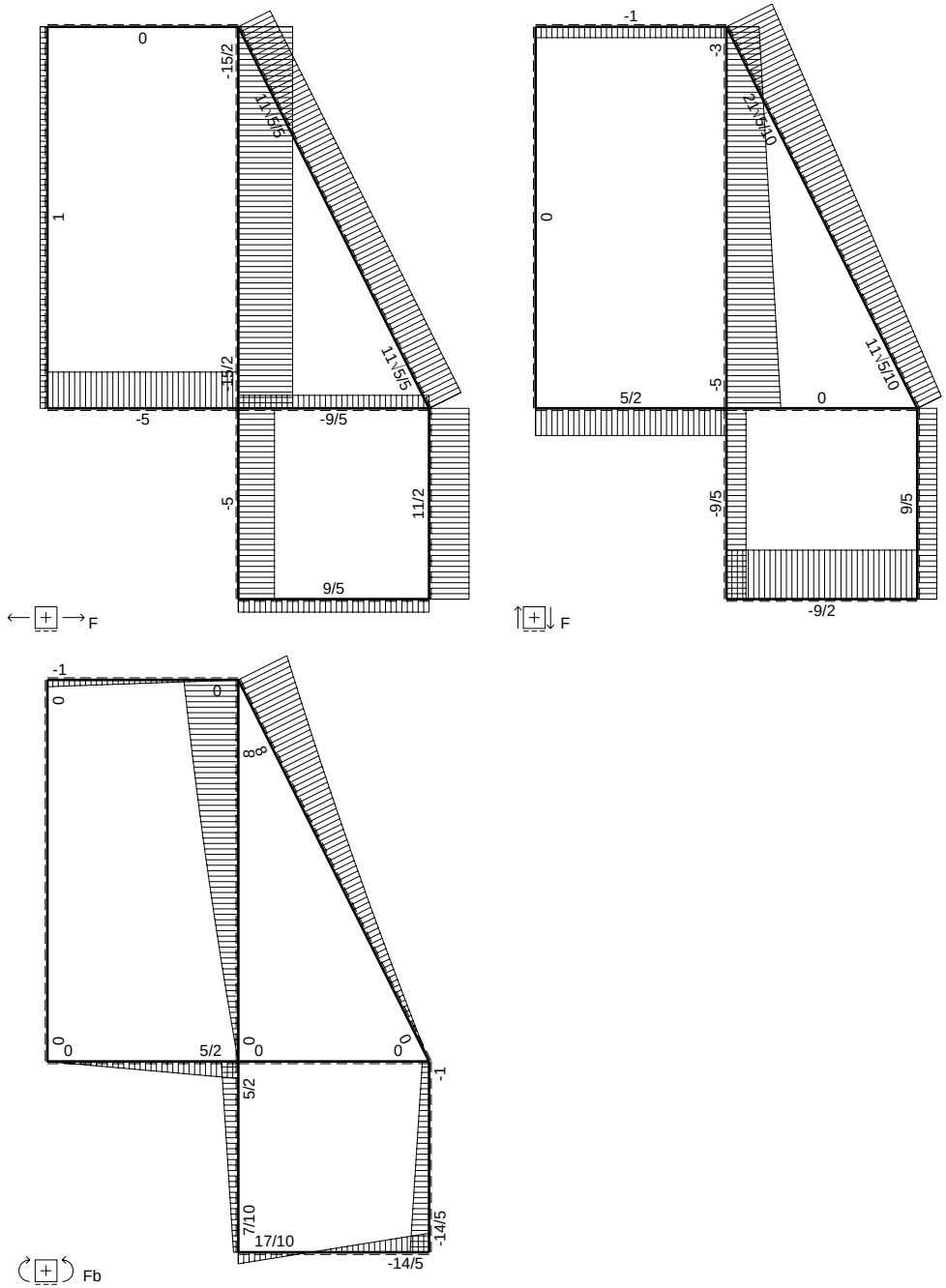
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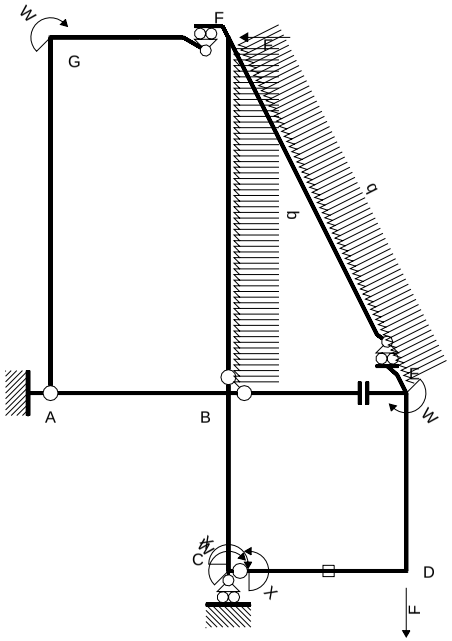
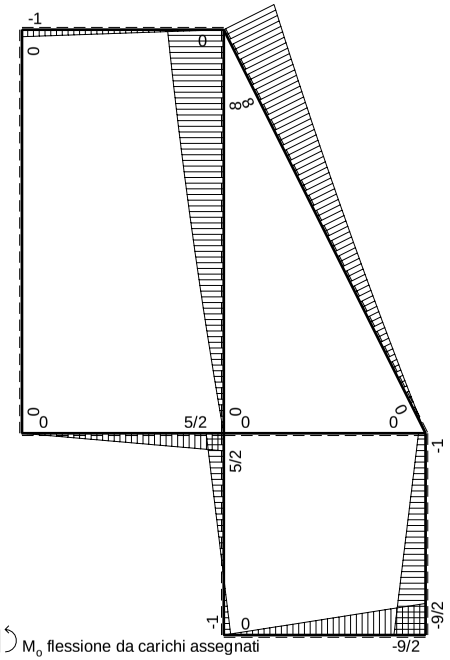


Schema di calcolo iperstatico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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-5/2\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -9/2\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -9/4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -9/4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (9/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (9/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 5/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (9/2 -9/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [9/2\ x -9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (9/2\ b -9/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 5/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (9/2 -8\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [9/2\ x -4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = 5/3\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +7/6\ b\ )\ Fb\ 1/EJ\ = 5/3\ \ Fb^2/EJ$

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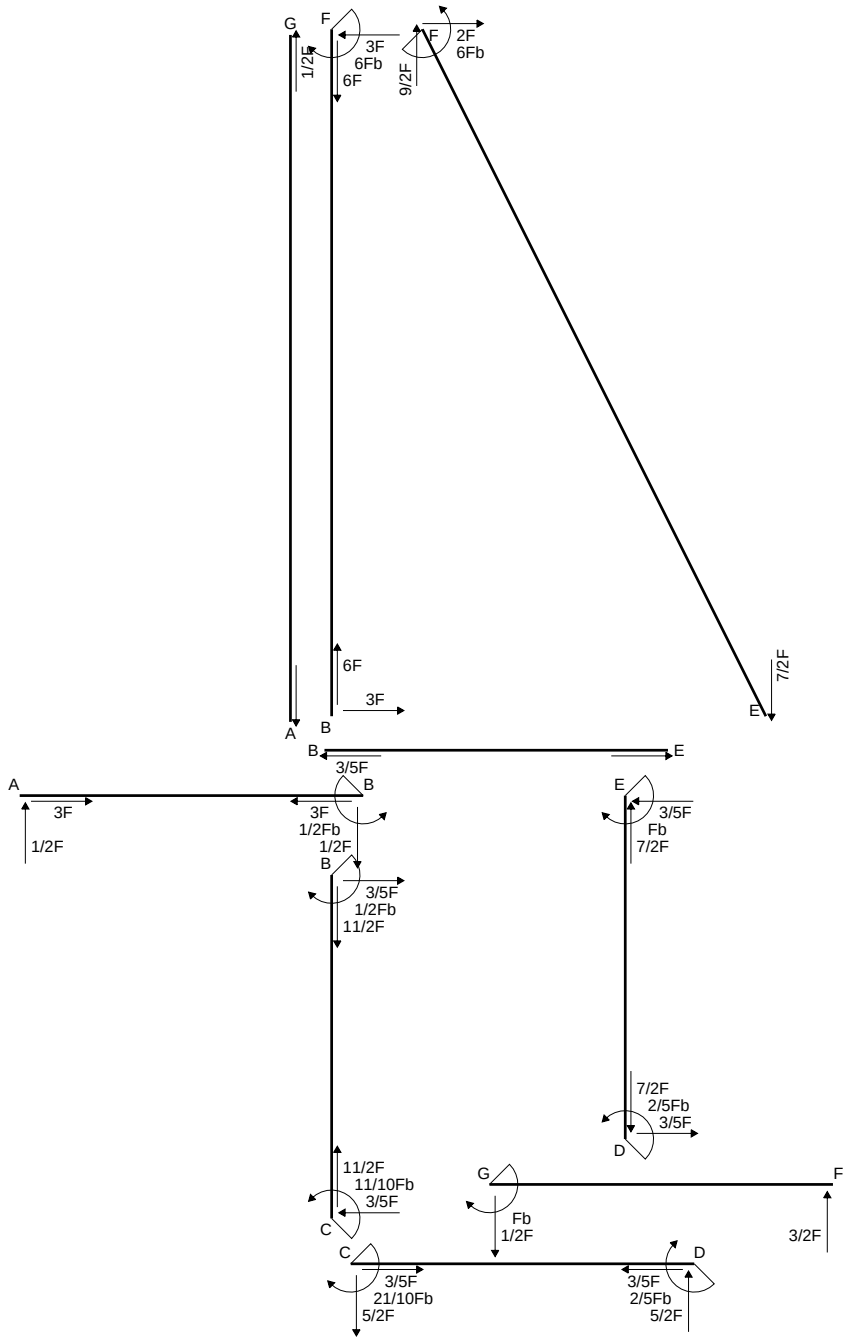
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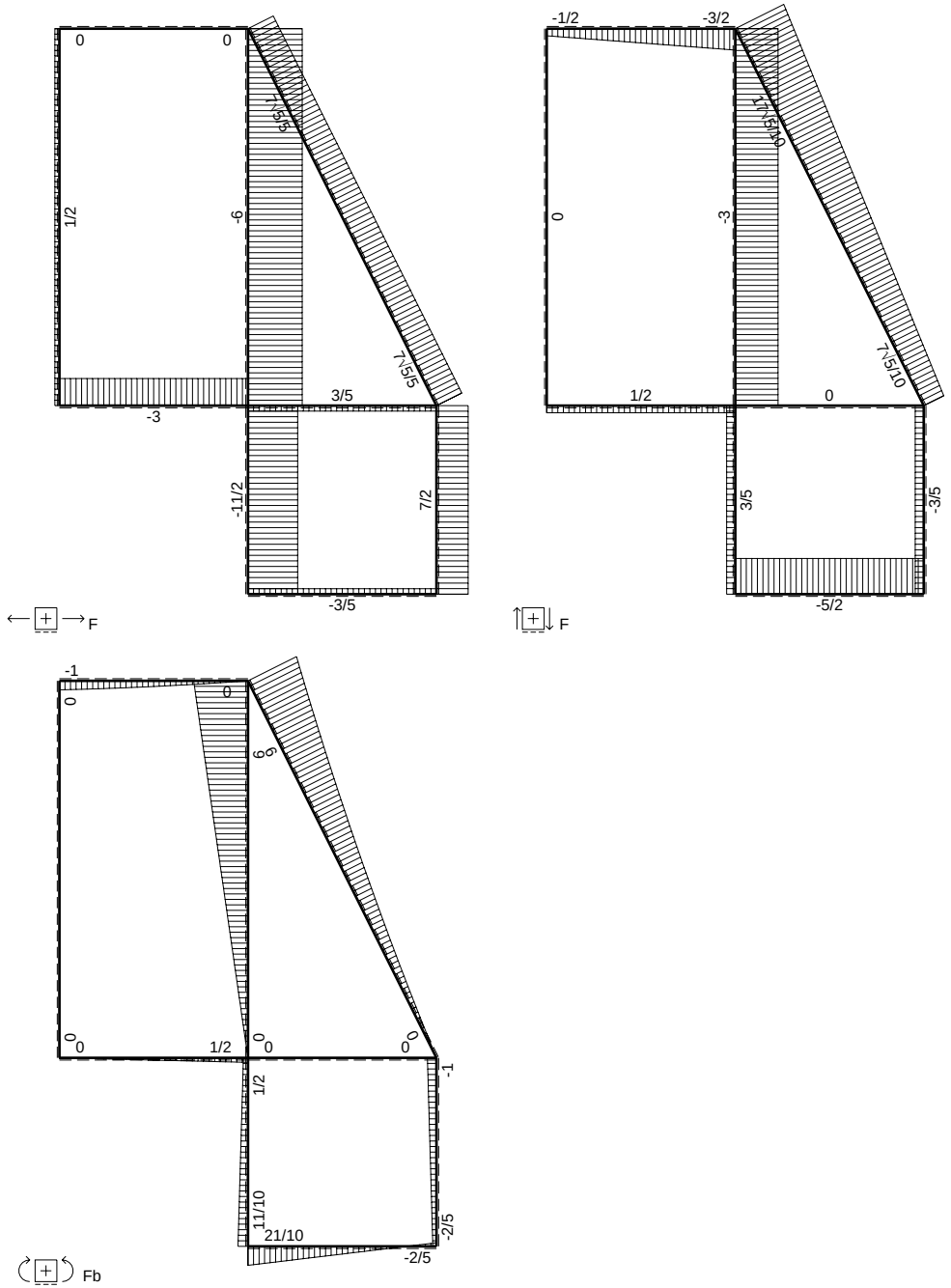
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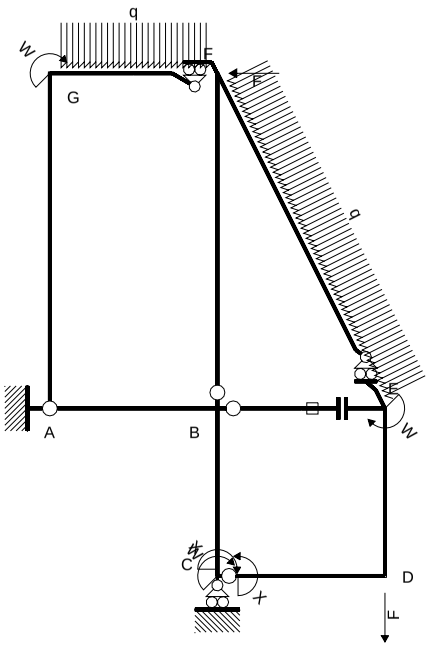
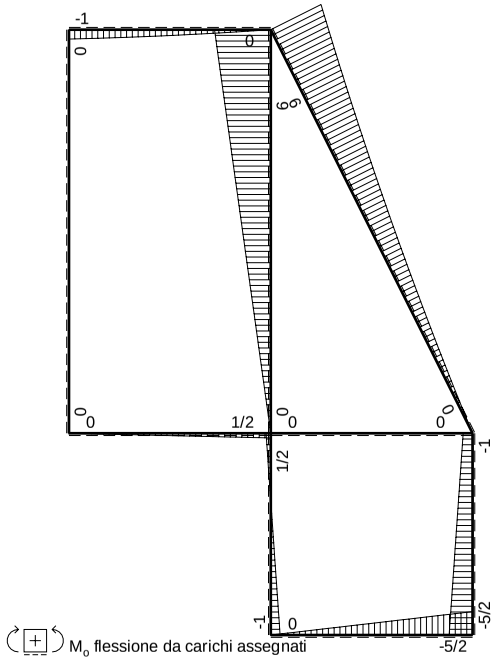
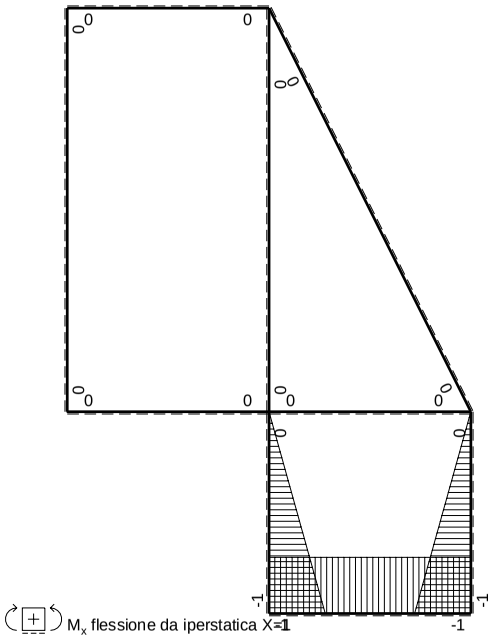


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

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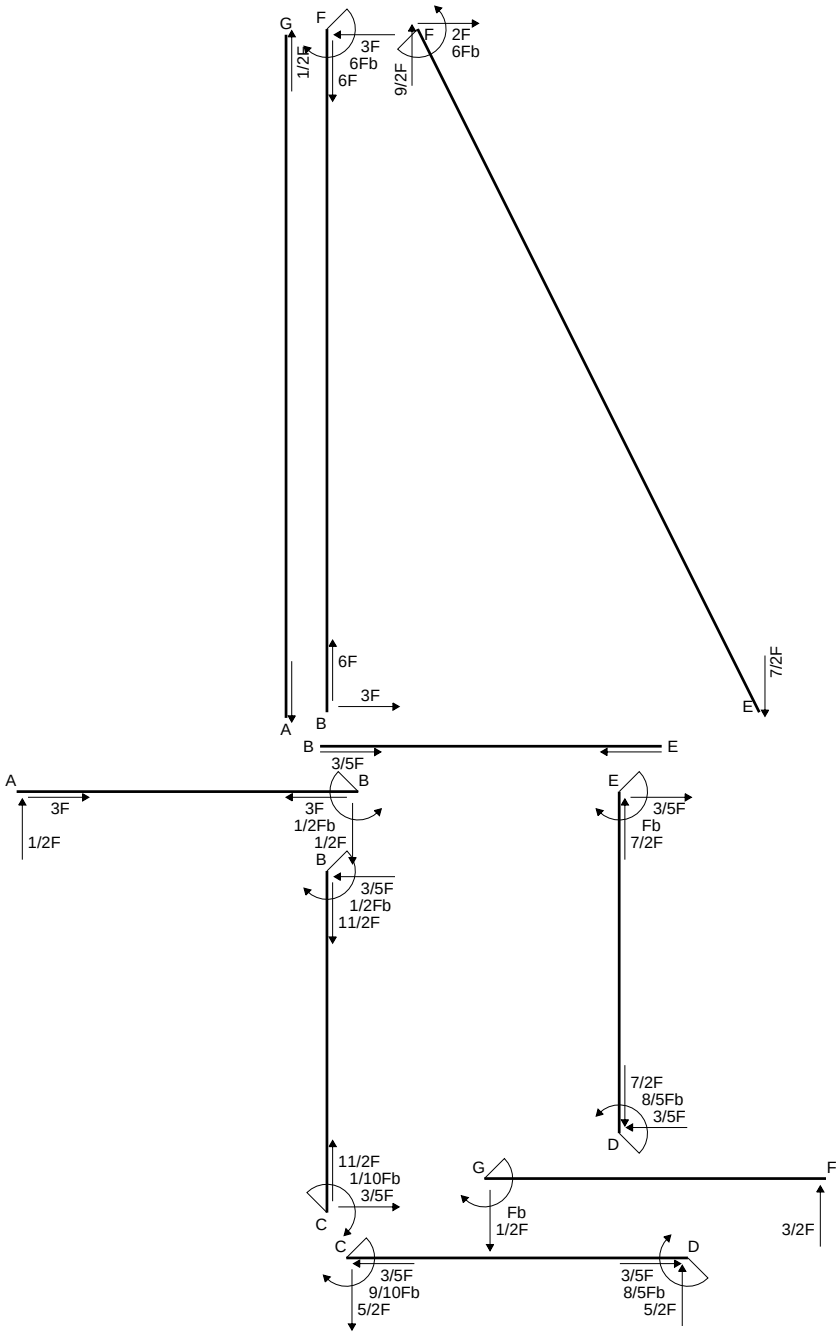
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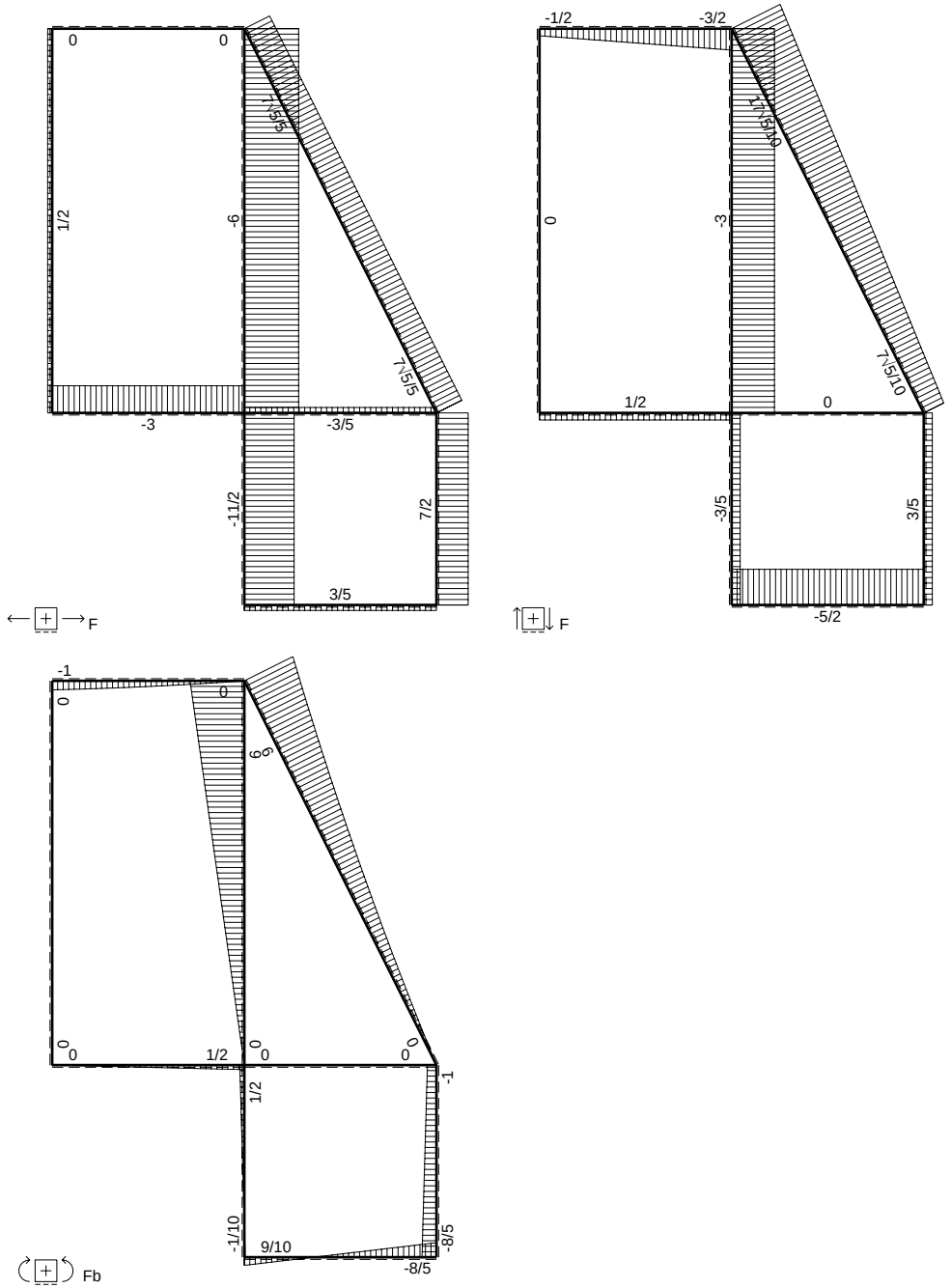
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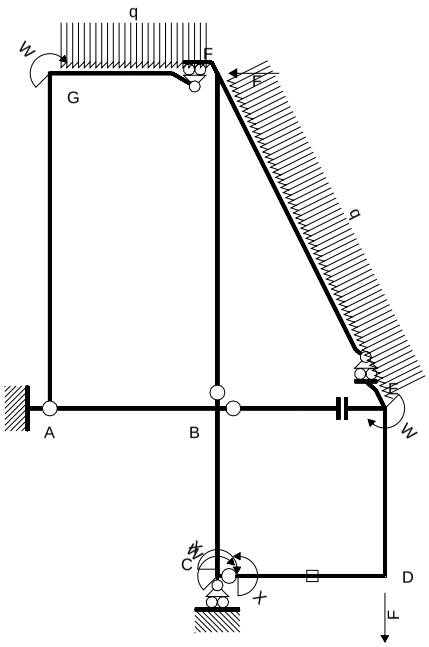
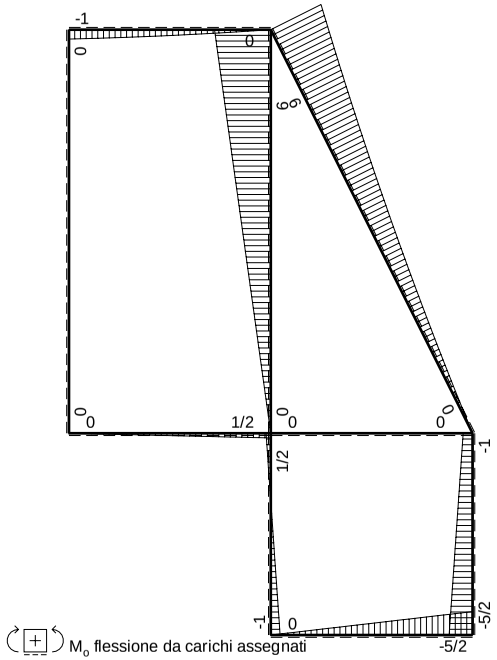
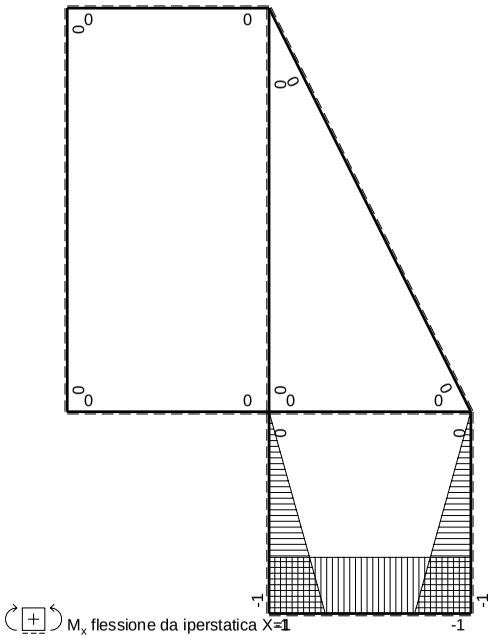
-8/5
-8/5
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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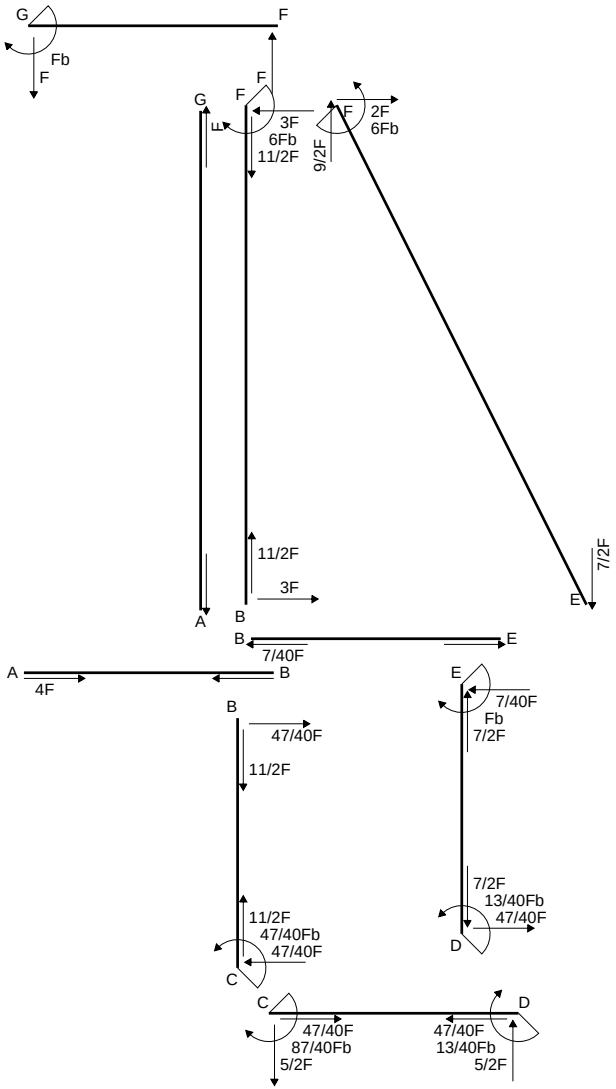
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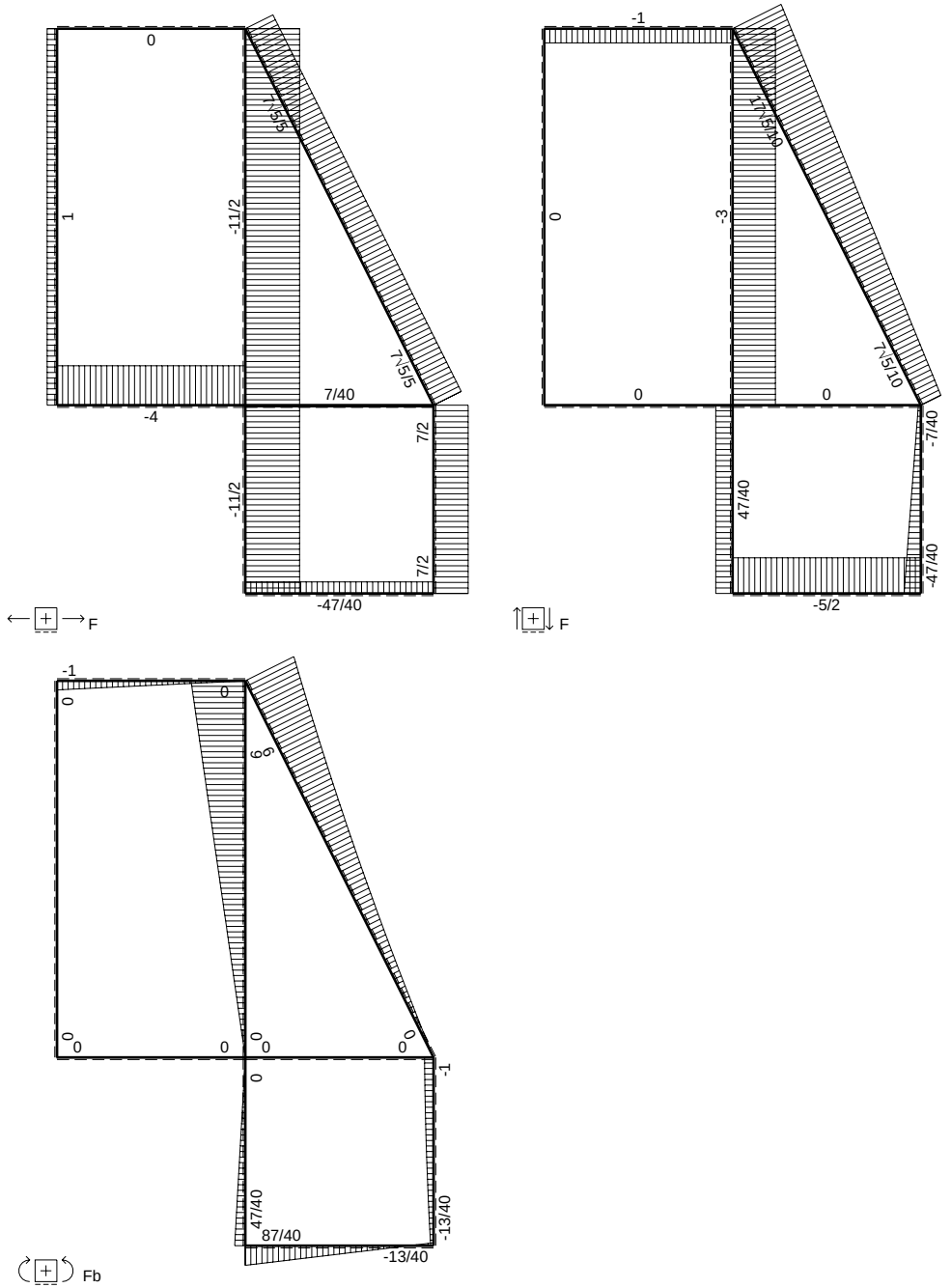
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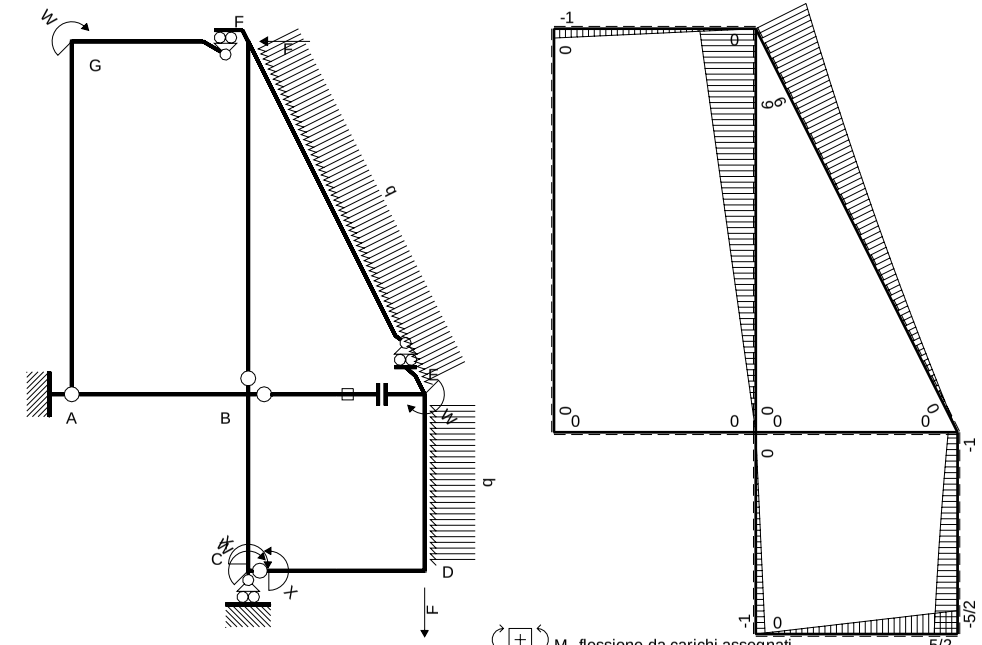
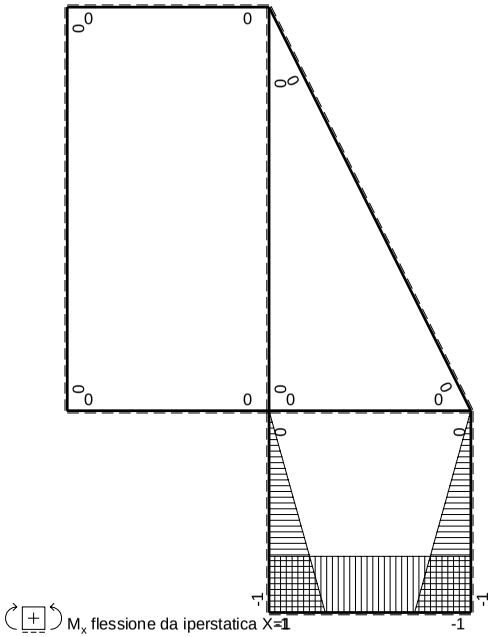
-13/40
-13/40
Fb



Schema di calcolo iperstatico

M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238922 ESΣ02.xxxx.040

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	$-5/2Fb+Fx+1/2qx^2$	$5/2Fb-7/2Fx+1/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$25/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb+2Fx-1/2qx^2$	$Fx+2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$29/8Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-87/40Fb	

PROCEDIMENTO E RISULTATI 238922 ESΣ02.xxxx.040

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -7/2\ x/b +1/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [5/2\ x -7/4\ x^2/b +1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -7/4\ b +1/6\ b +1/8\ b)\ Fb\ 1/EJ\ = 25/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b -1/8\ b)\ Fb\ 1/EJ\ = 25/24\ Fb^2/EJ$

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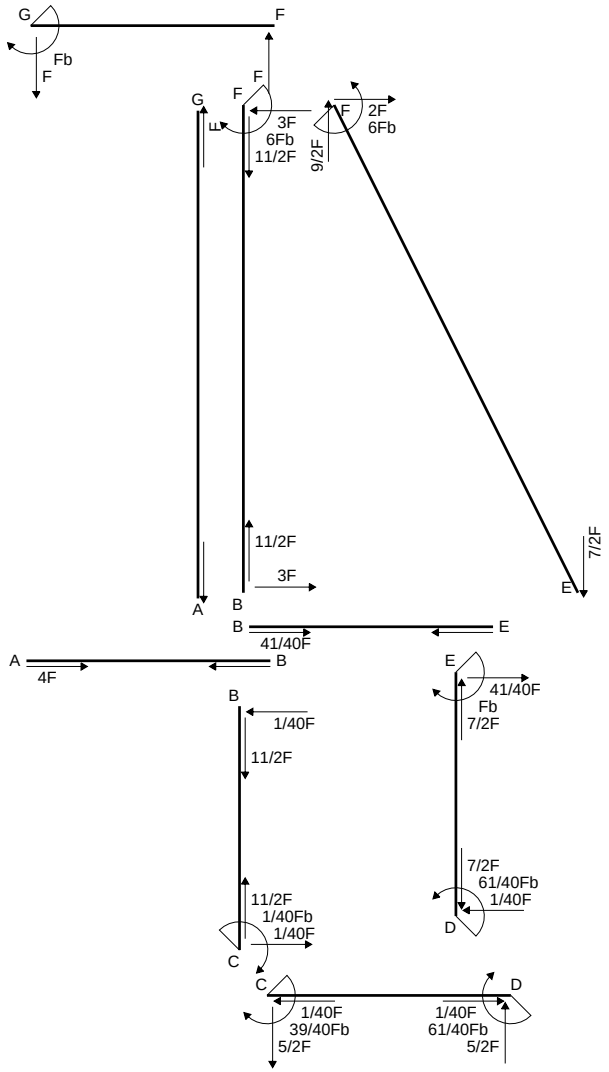
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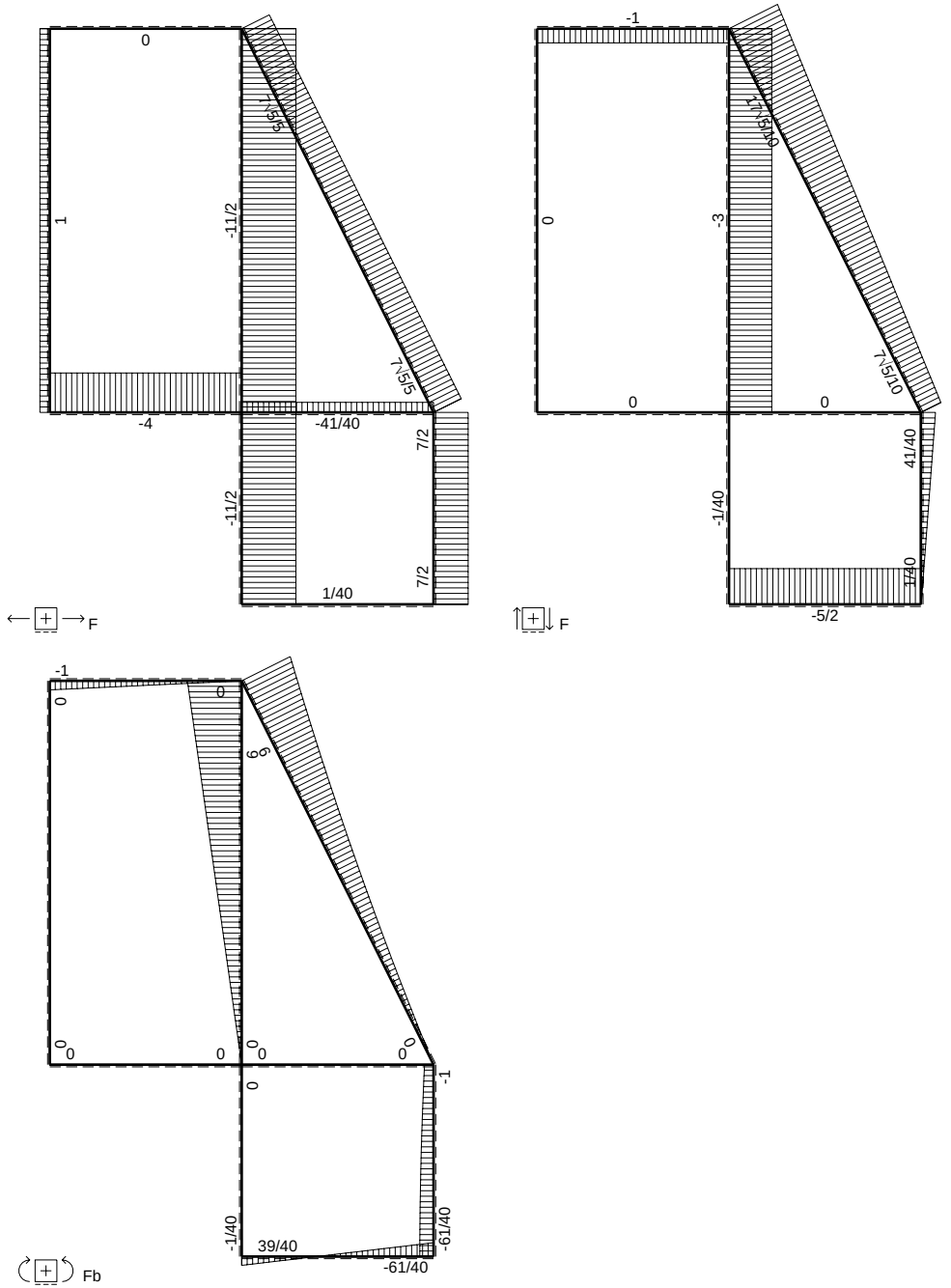
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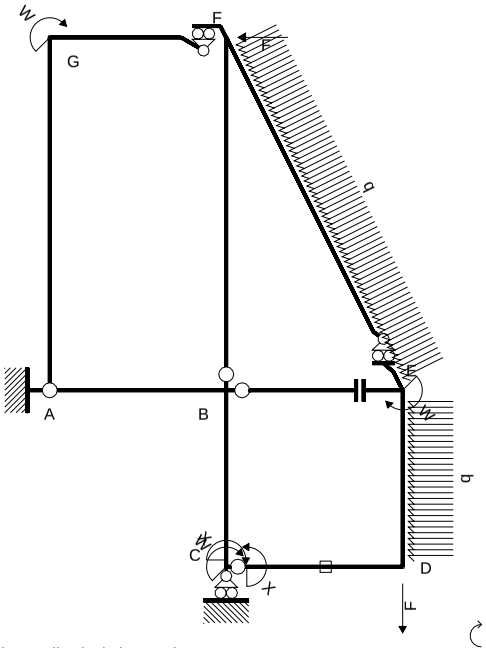
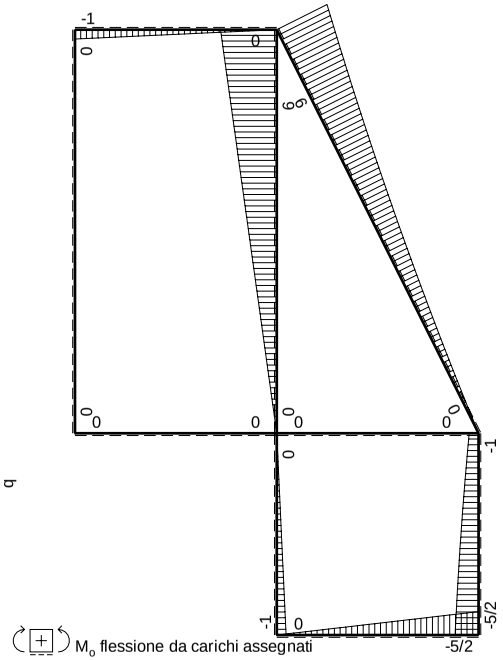
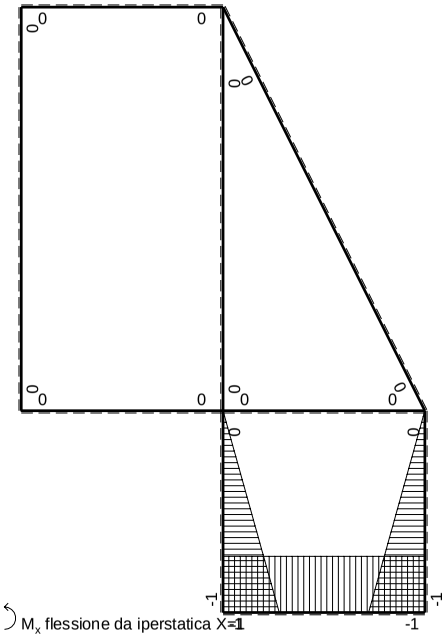


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 956982 ESΣ02.xxxx.041

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	$-5/2Fb+Fx+1/2qx^2$	$5/2Fb-7/2Fx+1/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$25/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb+2Fx-1/2qx^2$	$Fx+2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$13/8Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-39/40Fb	

PROCEDIMENTO E RISULTATI 956982 ESΣ02.xxxx.041

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -7/2\ x/b +1/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [5/2\ x -7/4\ x^2/b +1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -7/4\ b +1/6\ b +1/8\ b)\ Fb\ 1/EJ\ = 25/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b -1/8\ b)\ Fb\ 1/EJ\ = 25/24\ \ Fb^2/EJ$

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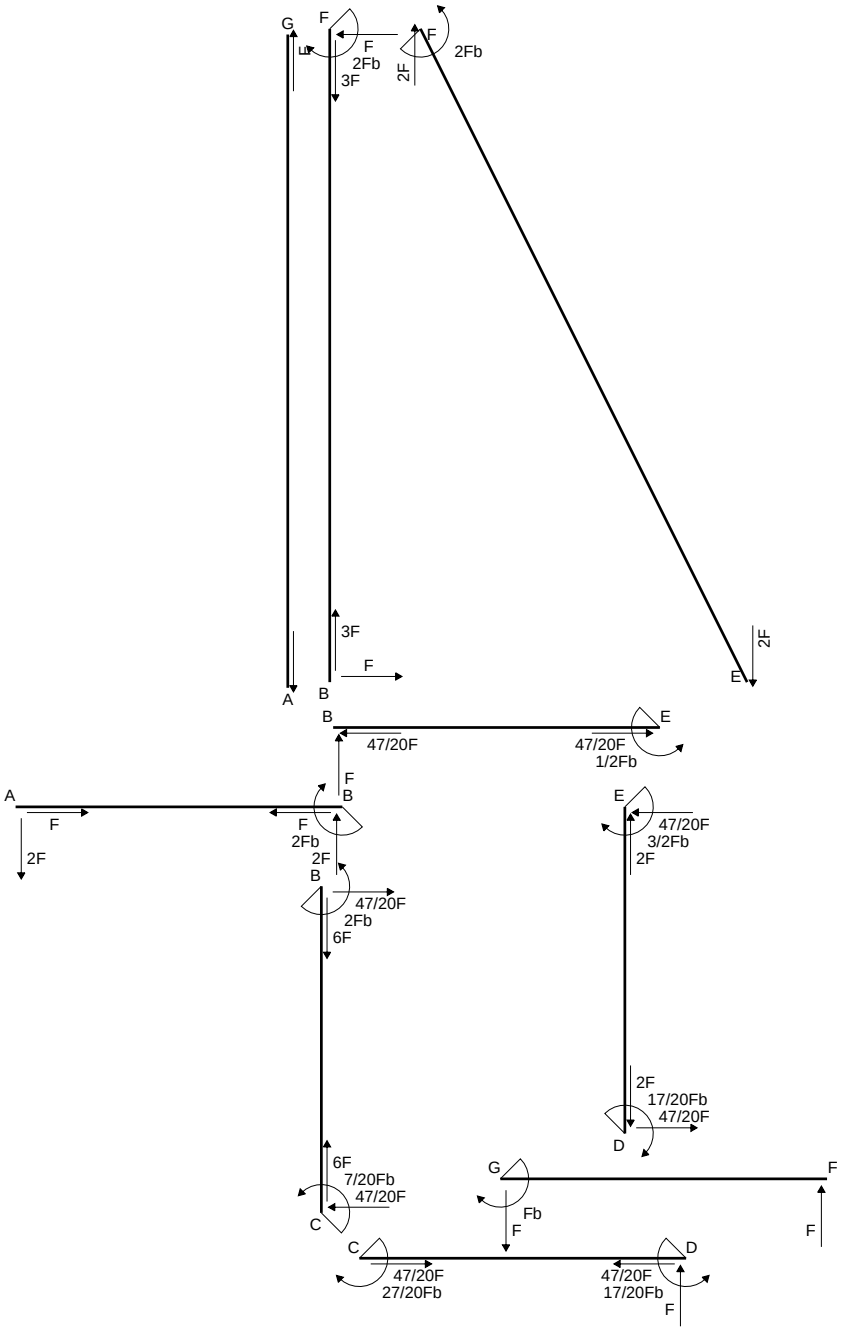
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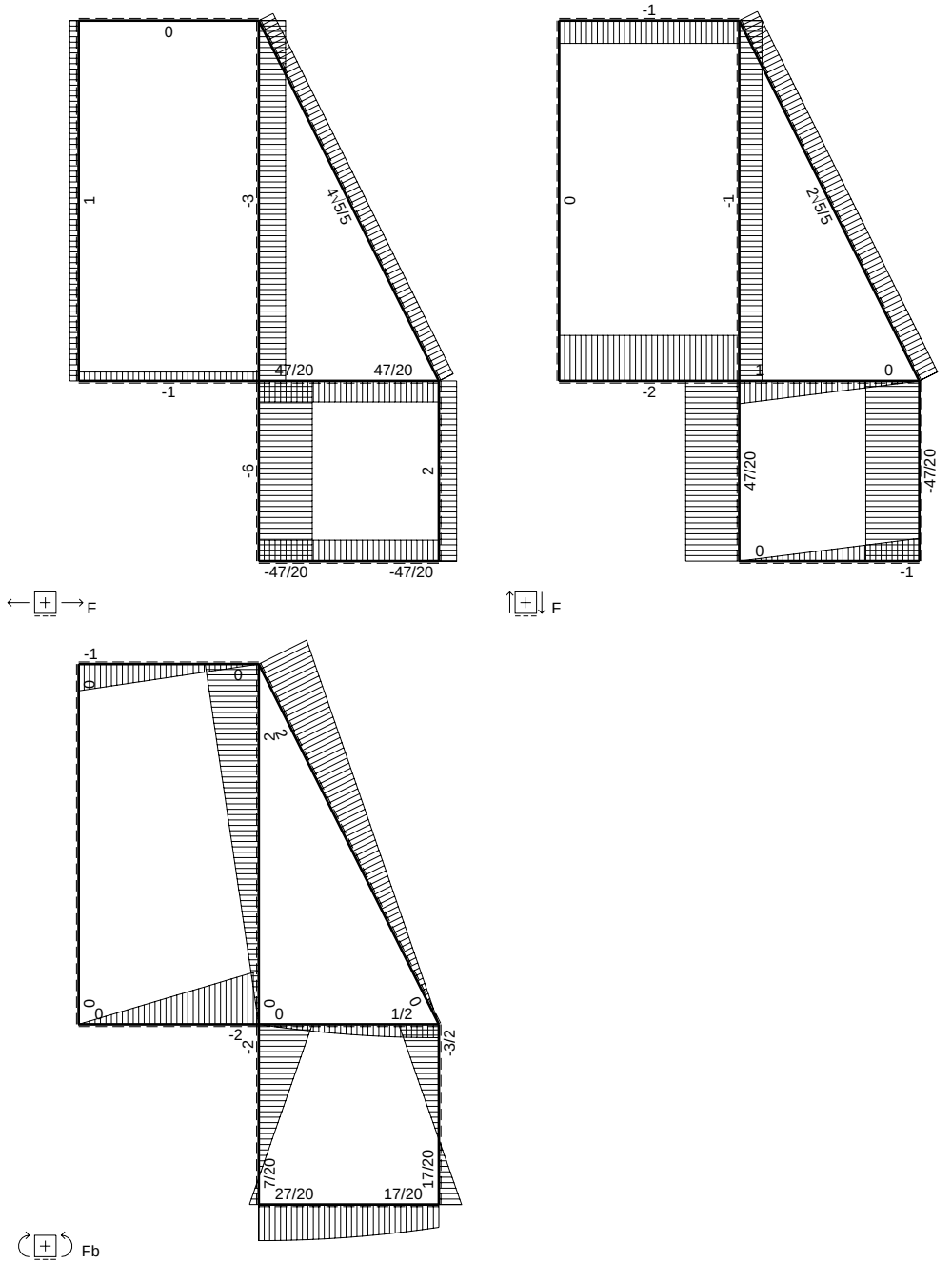
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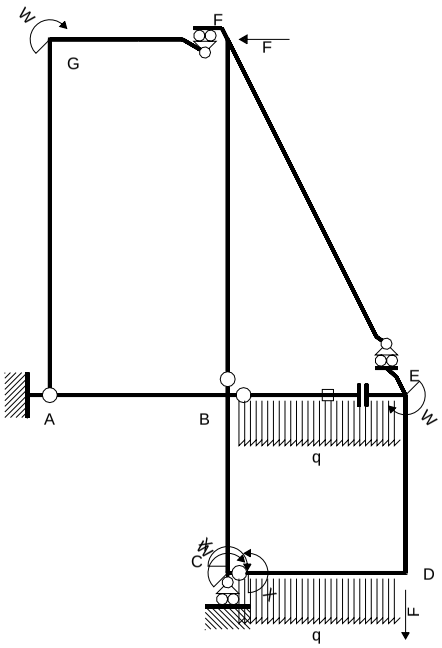
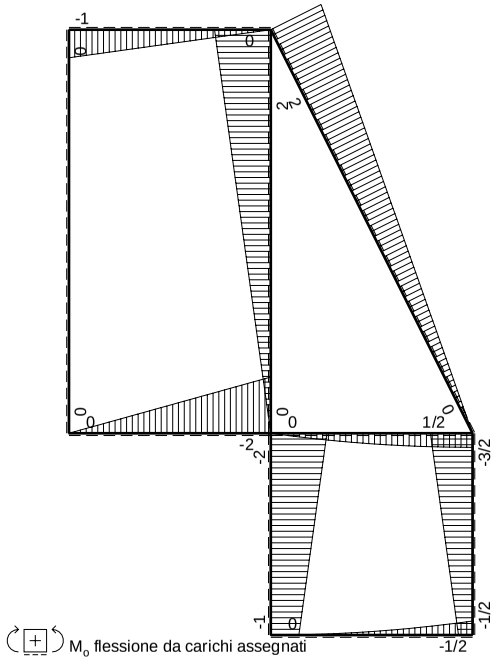
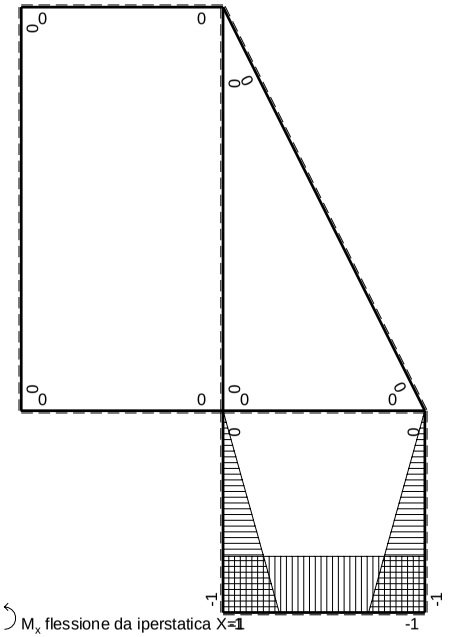


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	2Fx-Fx^2/b	x^2/b^2	2/3Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+Fx	Fb-Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-1/2qx^2	1/2Fx^2/b	1	1/6Fb^2/EJ	Xb/EJ
DC b	1	1/2Fb-Fx+1/2qx^2	1/2Fb-Fx+1/2Fx^2/b	1		
DE b	-1+x/b	-1/2Fb-Fx	1/2Fb+1/2Fx-Fx^2/b	1-2x/b+x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fb-Fx	3/2Fx-Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	-1/2Fb+1/2qx^2	0	0	0	0
EB b	0	Fx-1/2qx^2	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				9/4Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-27/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (2\ x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 +1/2\ x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x +1/4\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/4\ b -1/3\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/3\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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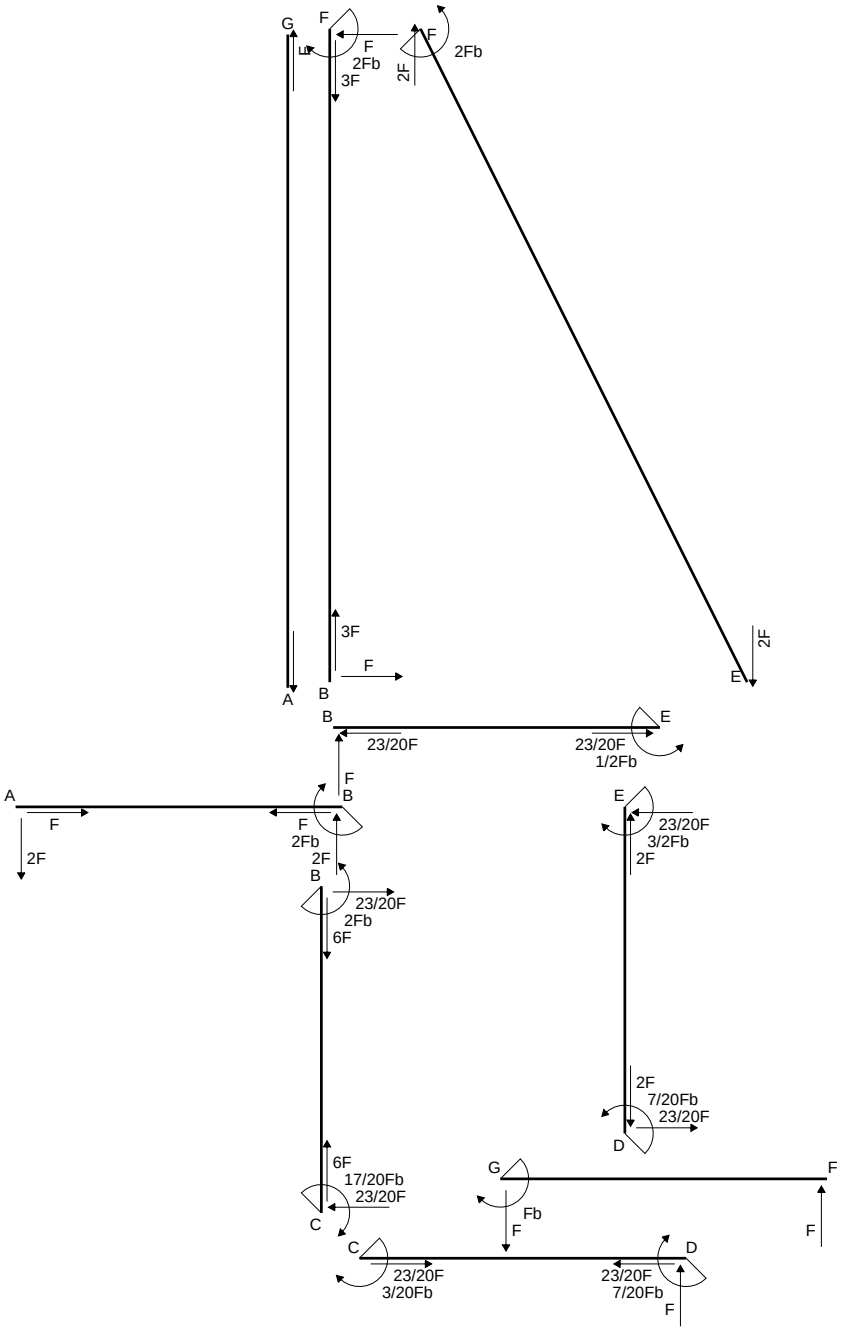
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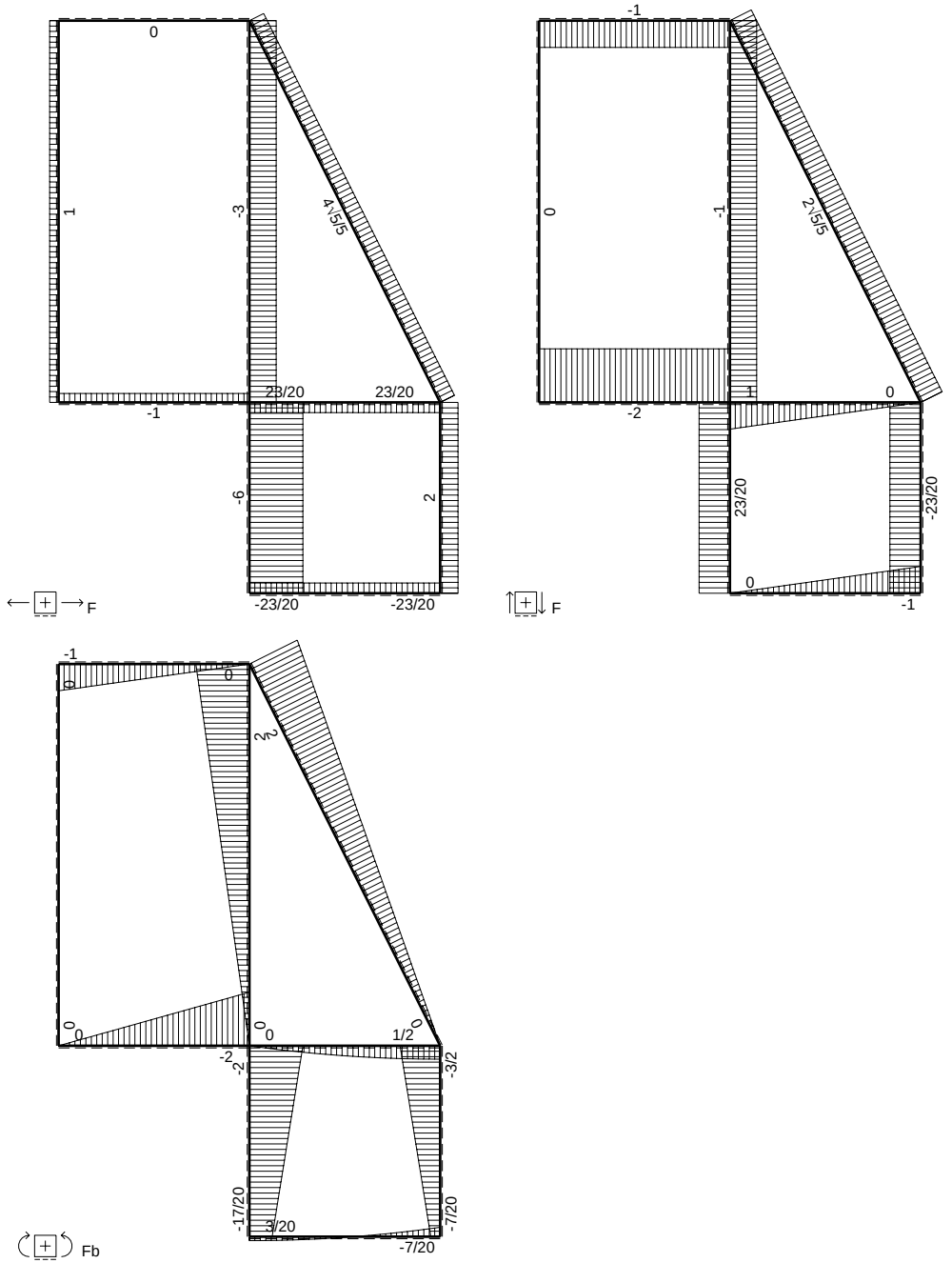
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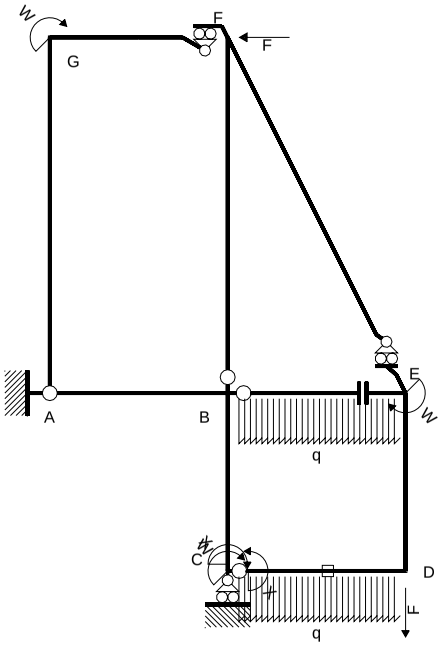
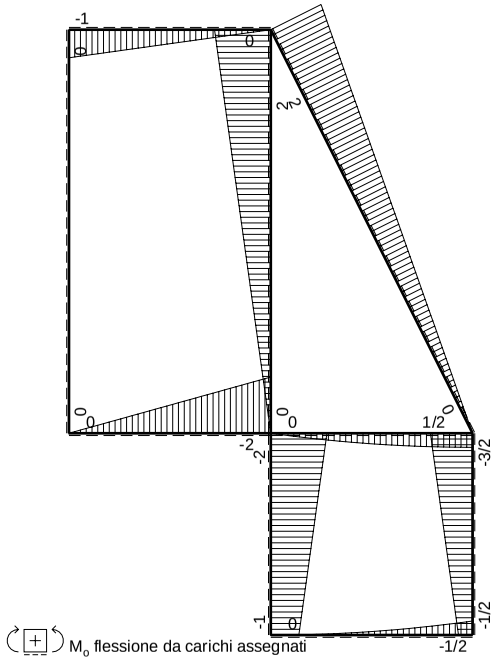
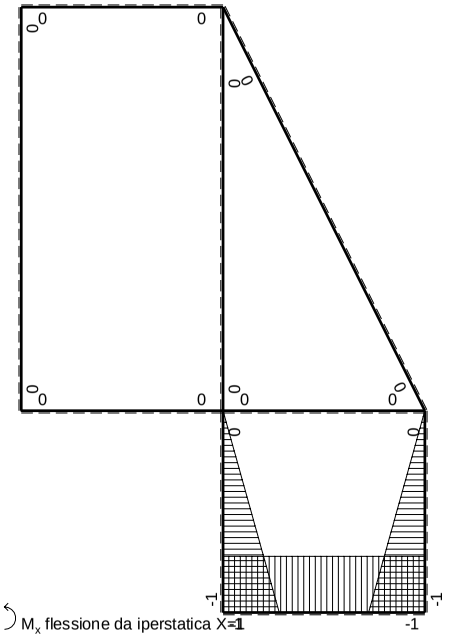


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	$2Fx-Fx^2/b$	x^2/b^2	$2/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+Fx	$Fb-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb-Fx	$1/2Fb+1/2Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	3/2Fb-Fx	$3/2Fx-Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 +1/2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x +1/4\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/4\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 5/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 5/12\ \ Fb^2/EJ$

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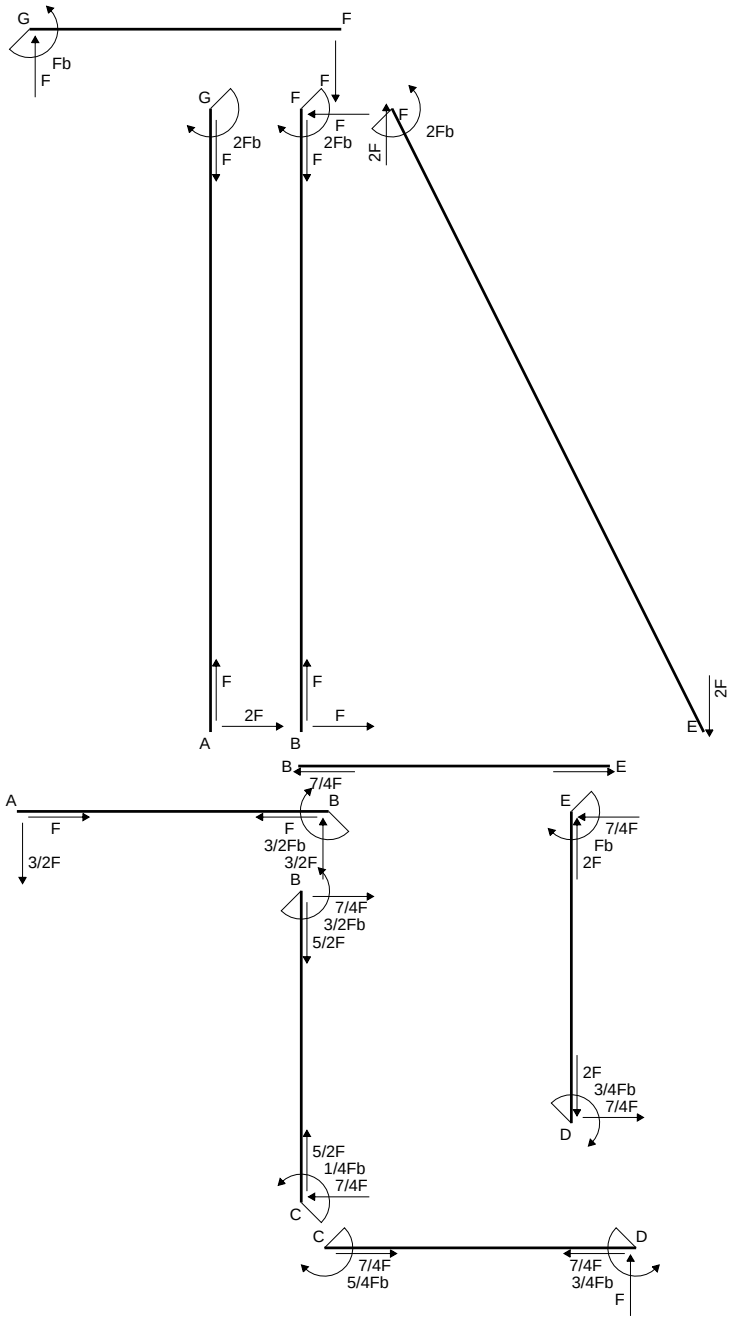
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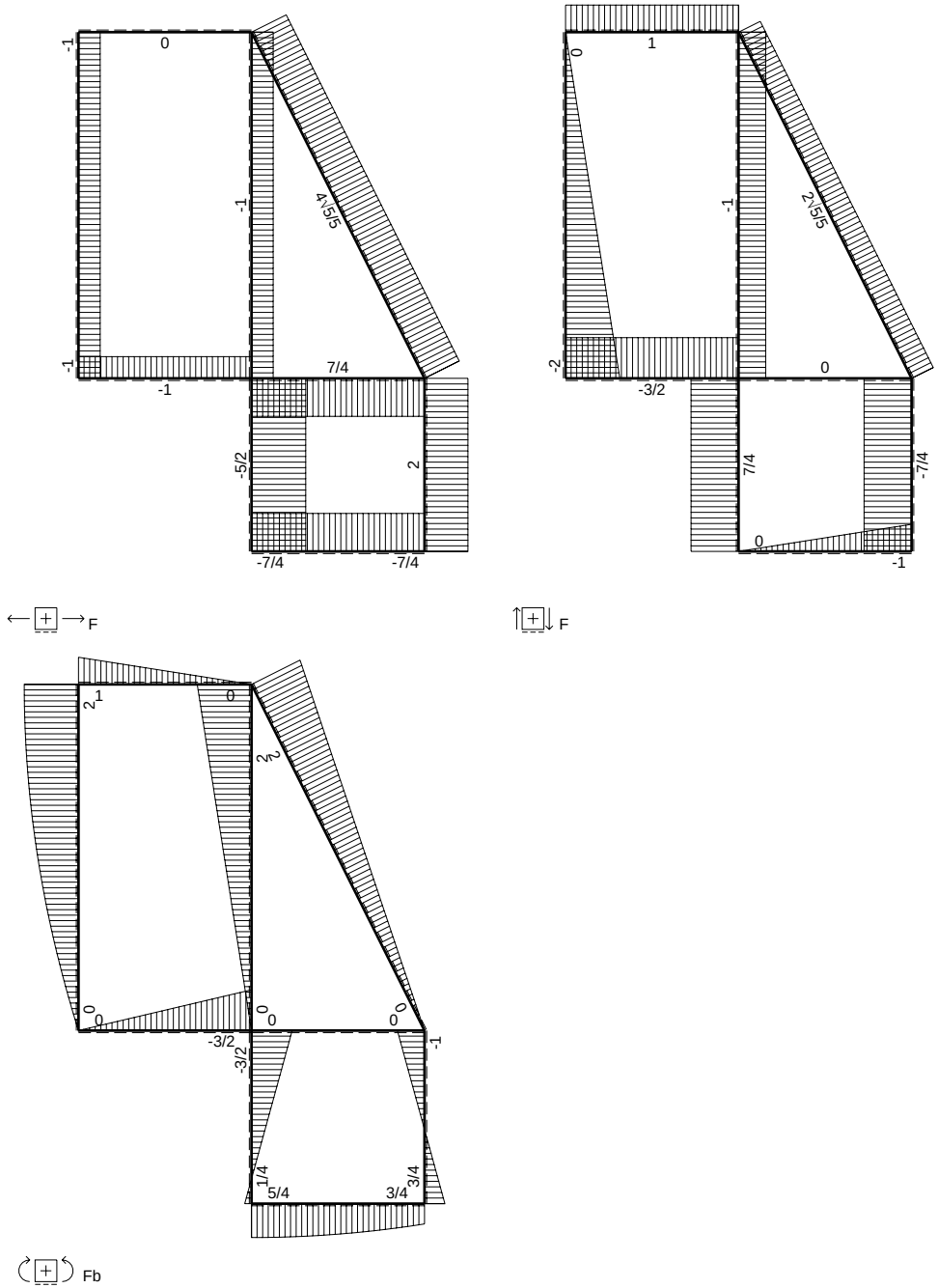
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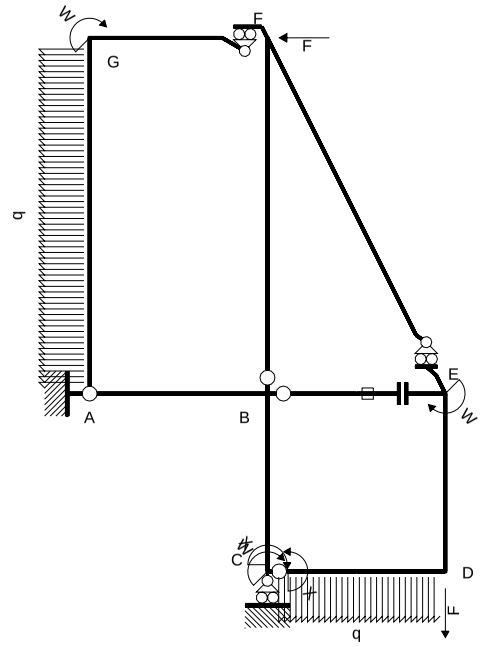
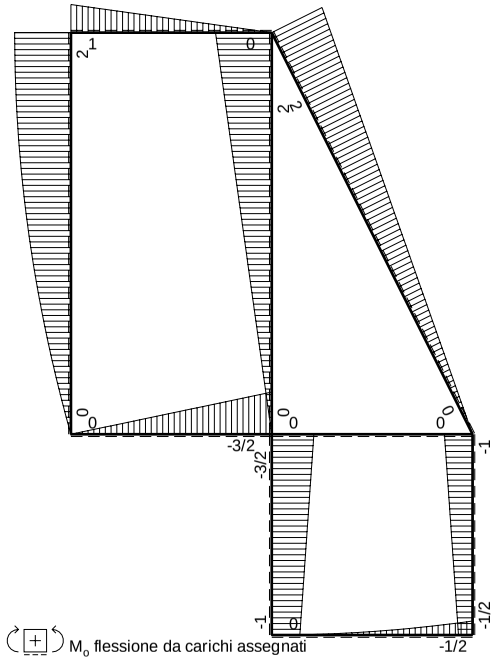
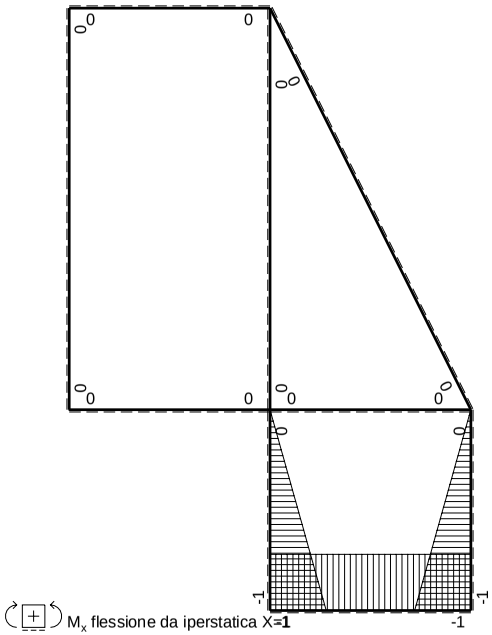


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-1/2qx^2	1/2Fx^2/b	1	1/6Fb^2/EJ	Xb/EJ
DC b	1	1/2Fb-Fx+1/2qx^2	1/2Fb-Fx+1/2Fx^2/b	1		
DE b	-1+x/b	-1/2Fb-1/2Fx	1/2Fb-1/2Fx^2/b	1-2x/b+x^2/b^2	1/3Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb-1/2Fx	Fx-1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	2Fb-1/2qx^2	0	0	0	0
AG 2b	0	-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				25/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-5/4Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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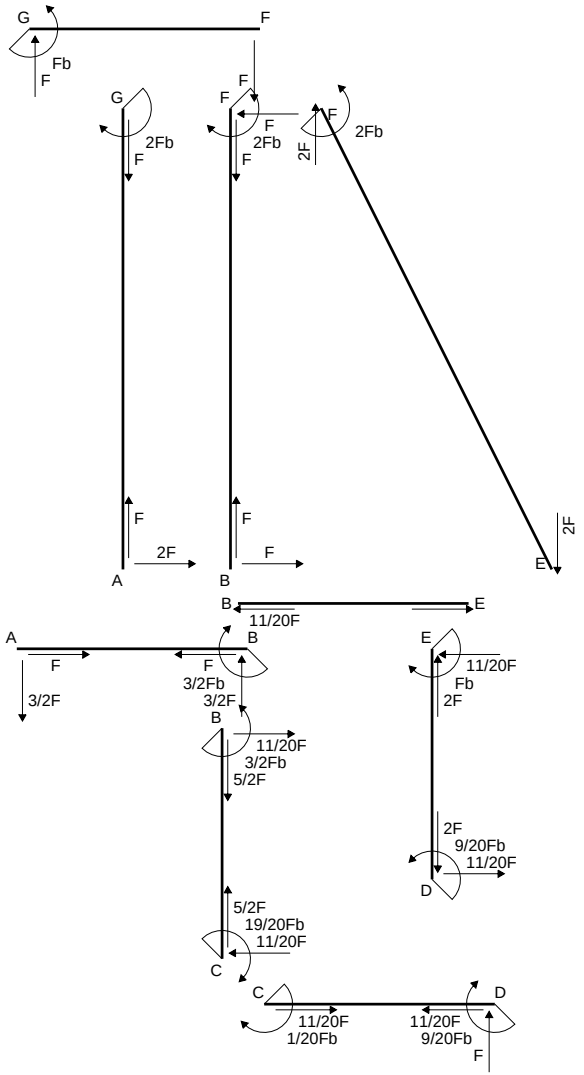
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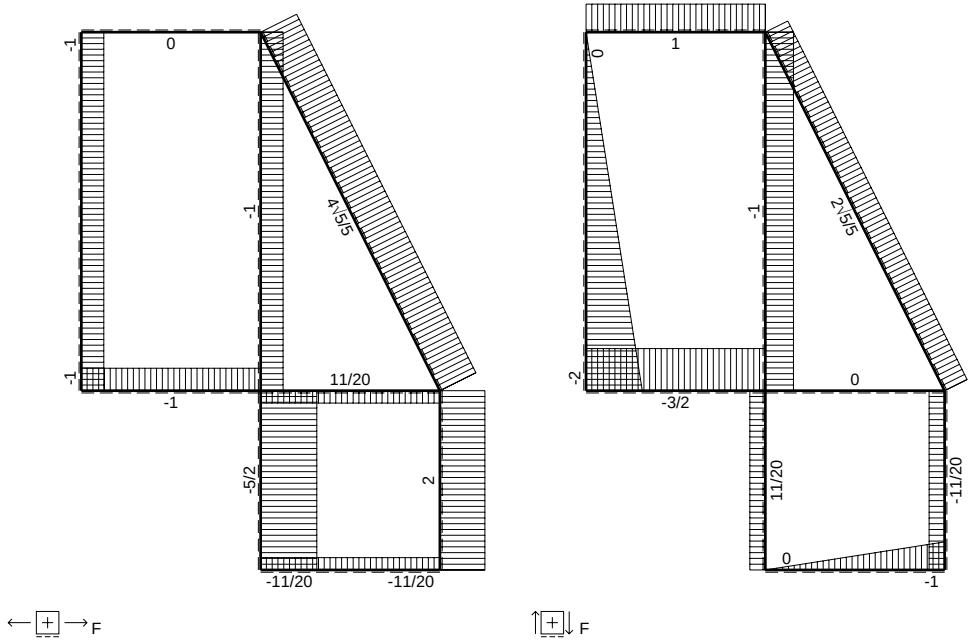
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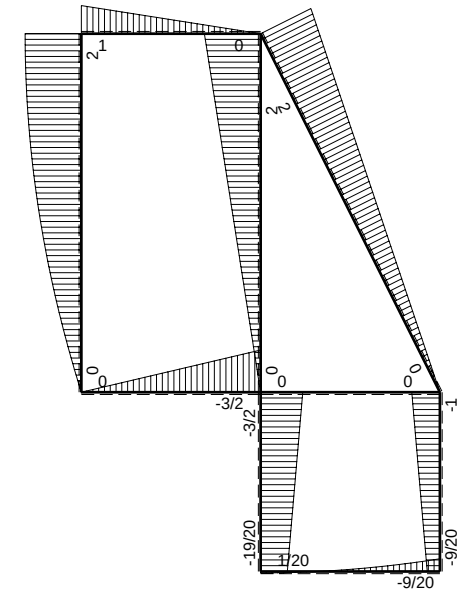
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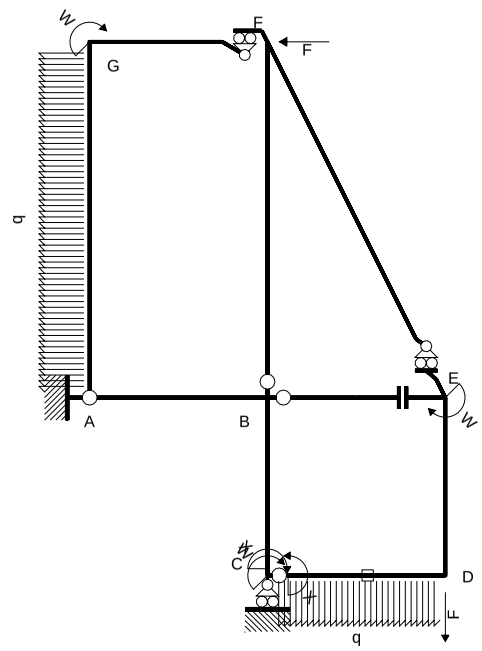
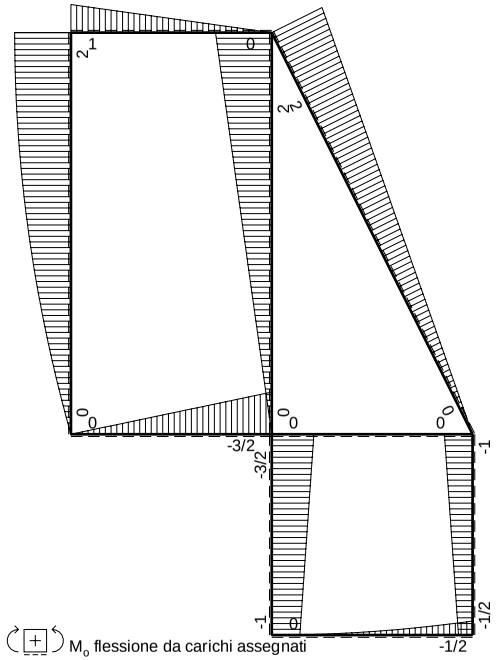
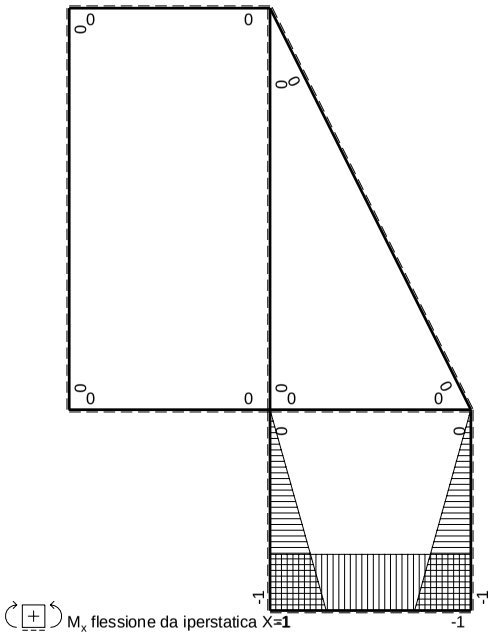


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb-1/2Fx	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb-1/2Fx	$Fx-1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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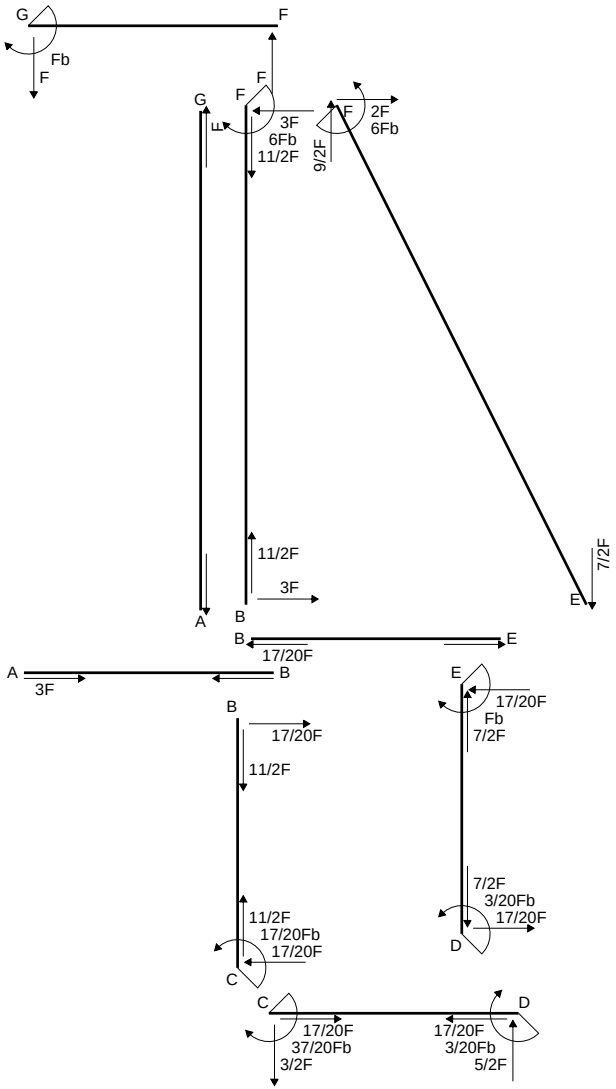
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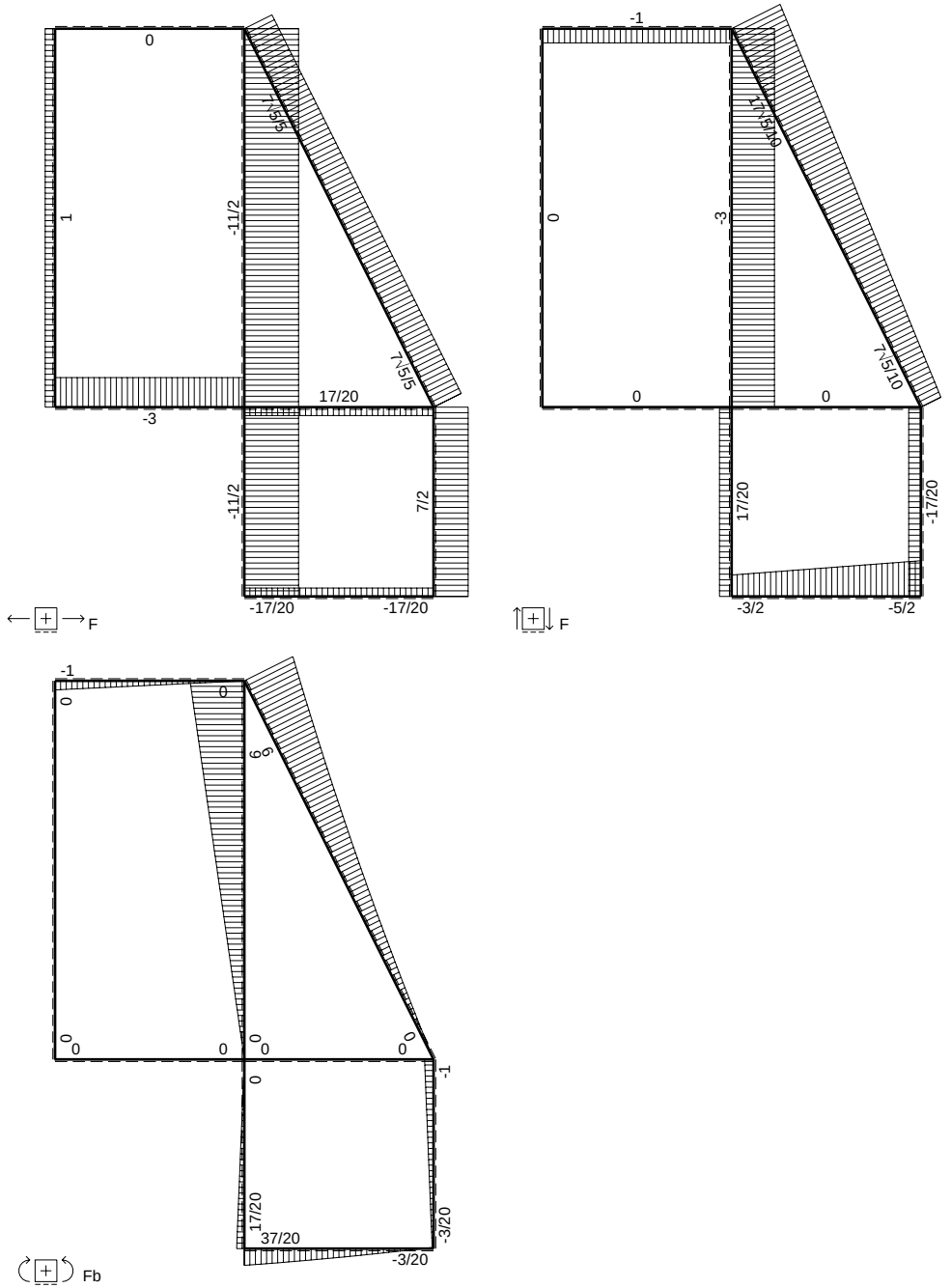
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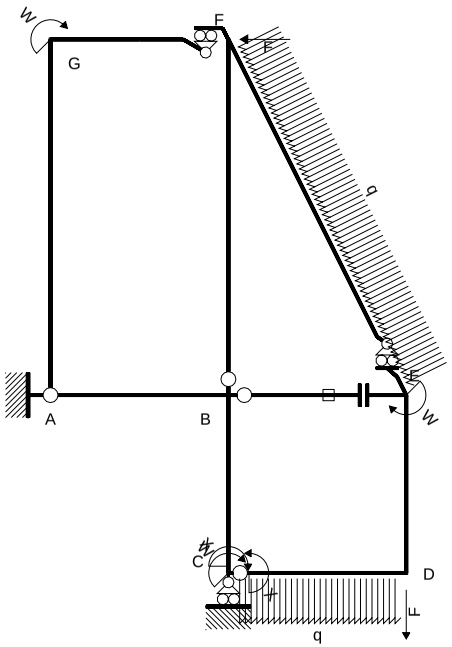
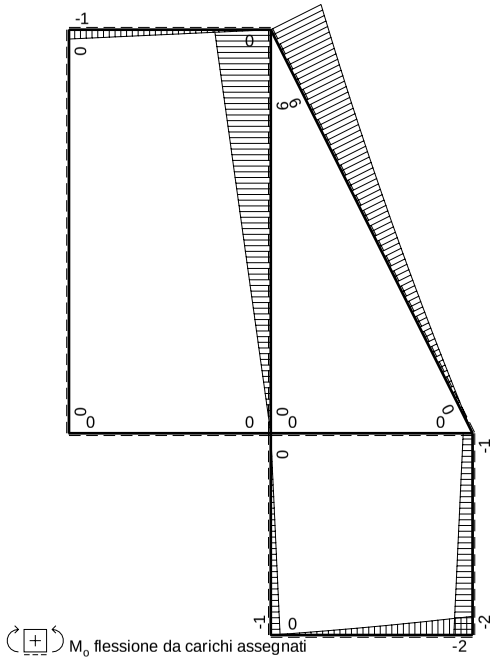
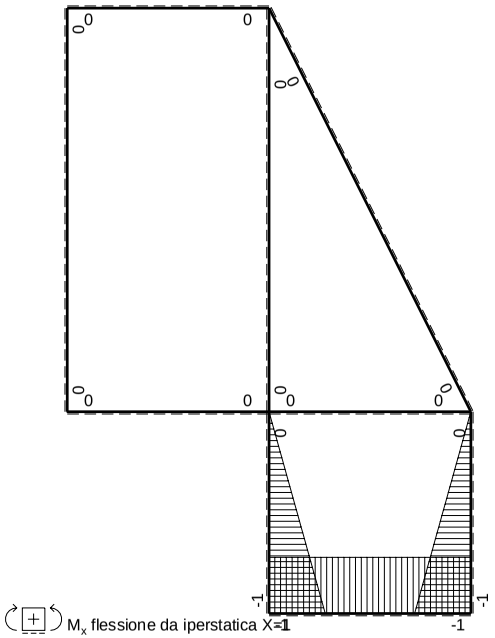
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-3/2Fx-1/2qx^2$	$3/2Fx+1/2Fx^2/b$	1	$11/12Fb^2/EJ$	Xb/EJ
DC b	1	$2Fb-5/2Fx+1/2qx^2$	$2Fb-5/2Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-2Fb+Fx	$2Fb-3Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+Fx	$Fx+Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$37/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-37/20Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (2 -5/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [2\ x -5/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -5/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (2 -3\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [2\ x -3/2\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -3/2\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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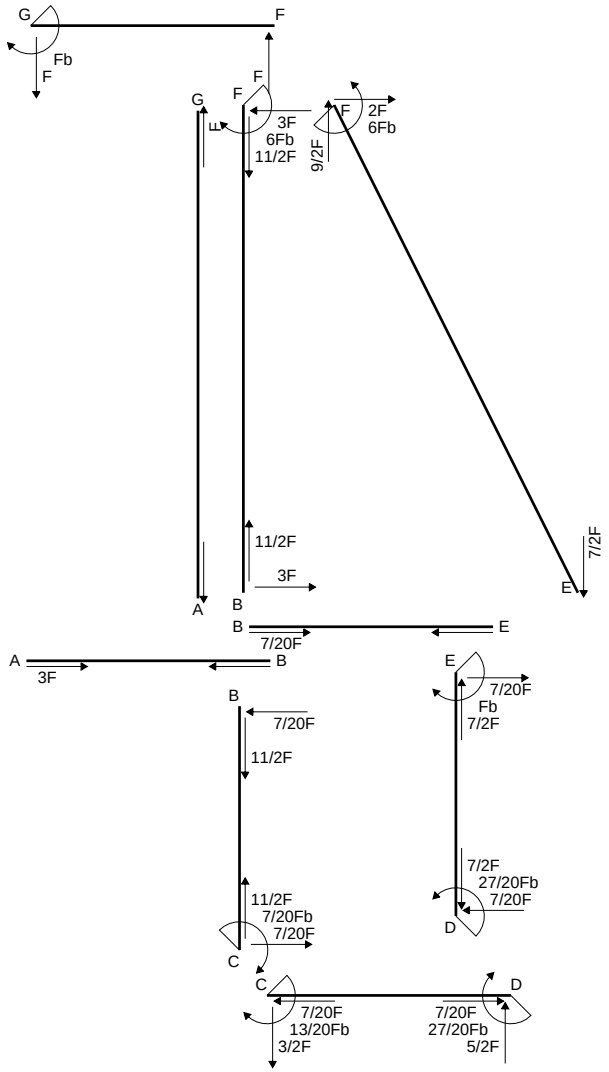
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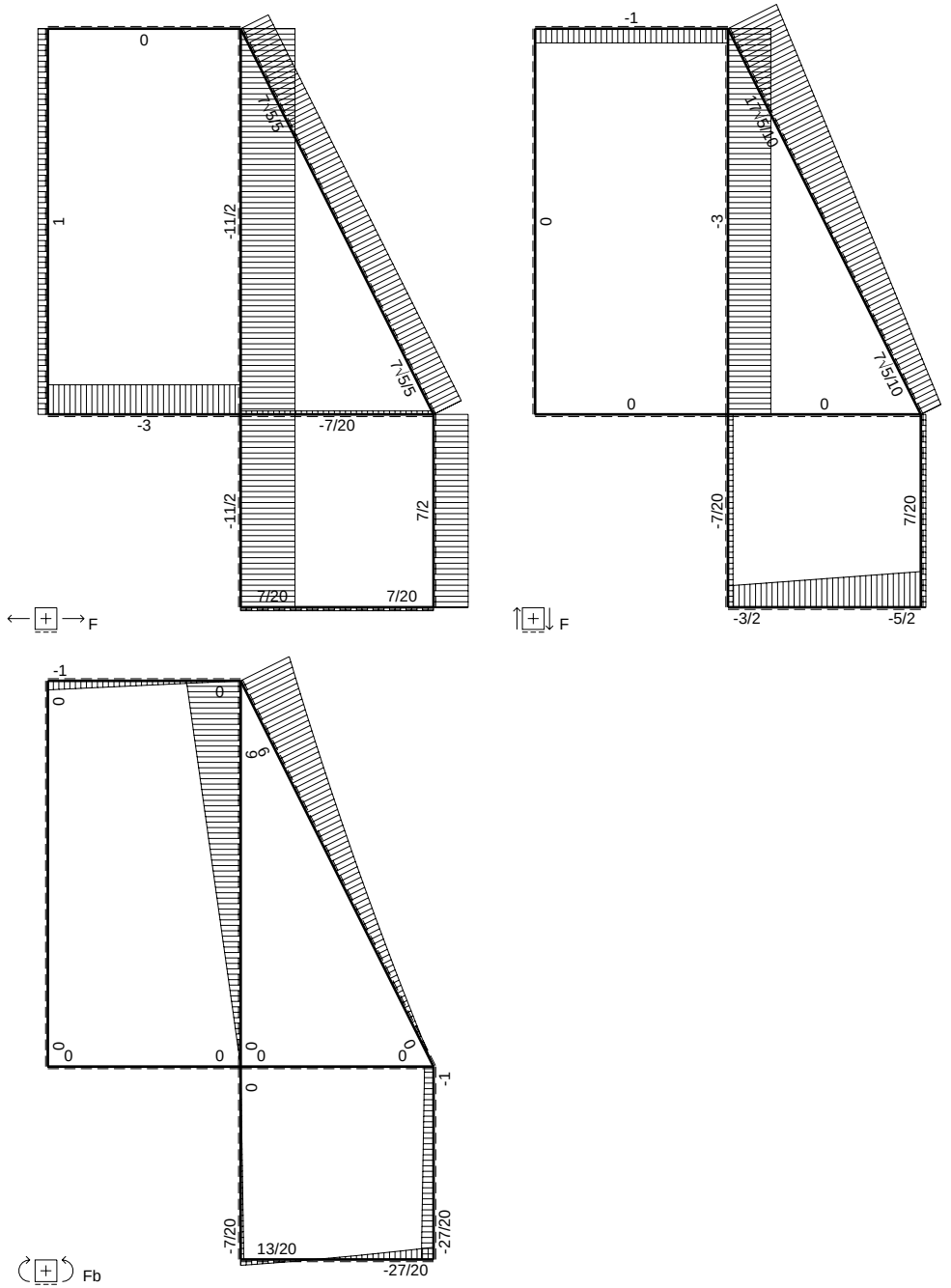
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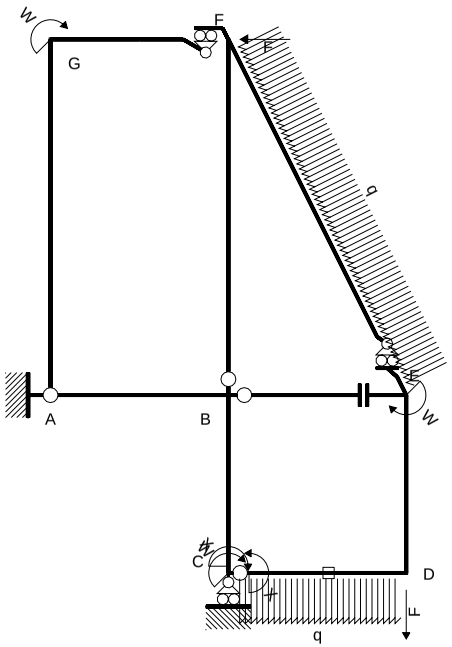
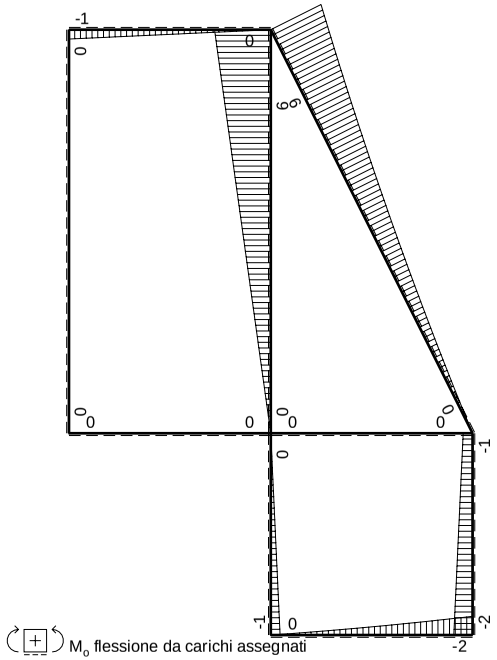
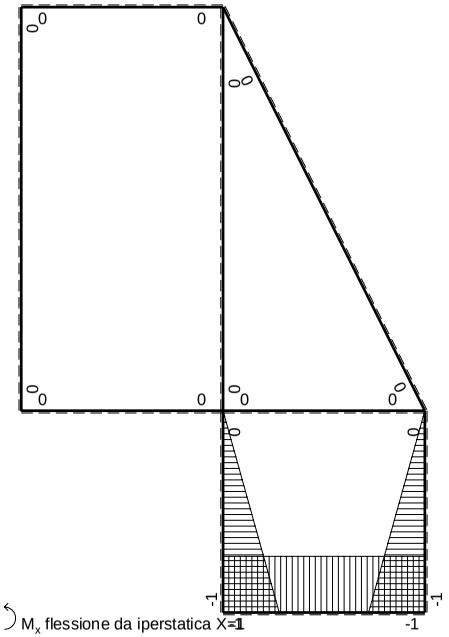


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-3/2Fx-1/2qx^2$	$3/2Fx+1/2Fx^2/b$	1	$11/12Fb^2/EJ$	Xb/EJ
DC b	1	$2Fb-5/2Fx+1/2qx^2$	$2Fb-5/2Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-2Fb+Fx	$2Fb-3Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+Fx	$Fx+Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$13/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-13/20Fb$	

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/12\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (2 -5/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [2\ x -5/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (2\ b -5/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/12\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (2 -3\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [2\ x -3/2\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -3/2\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/3\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

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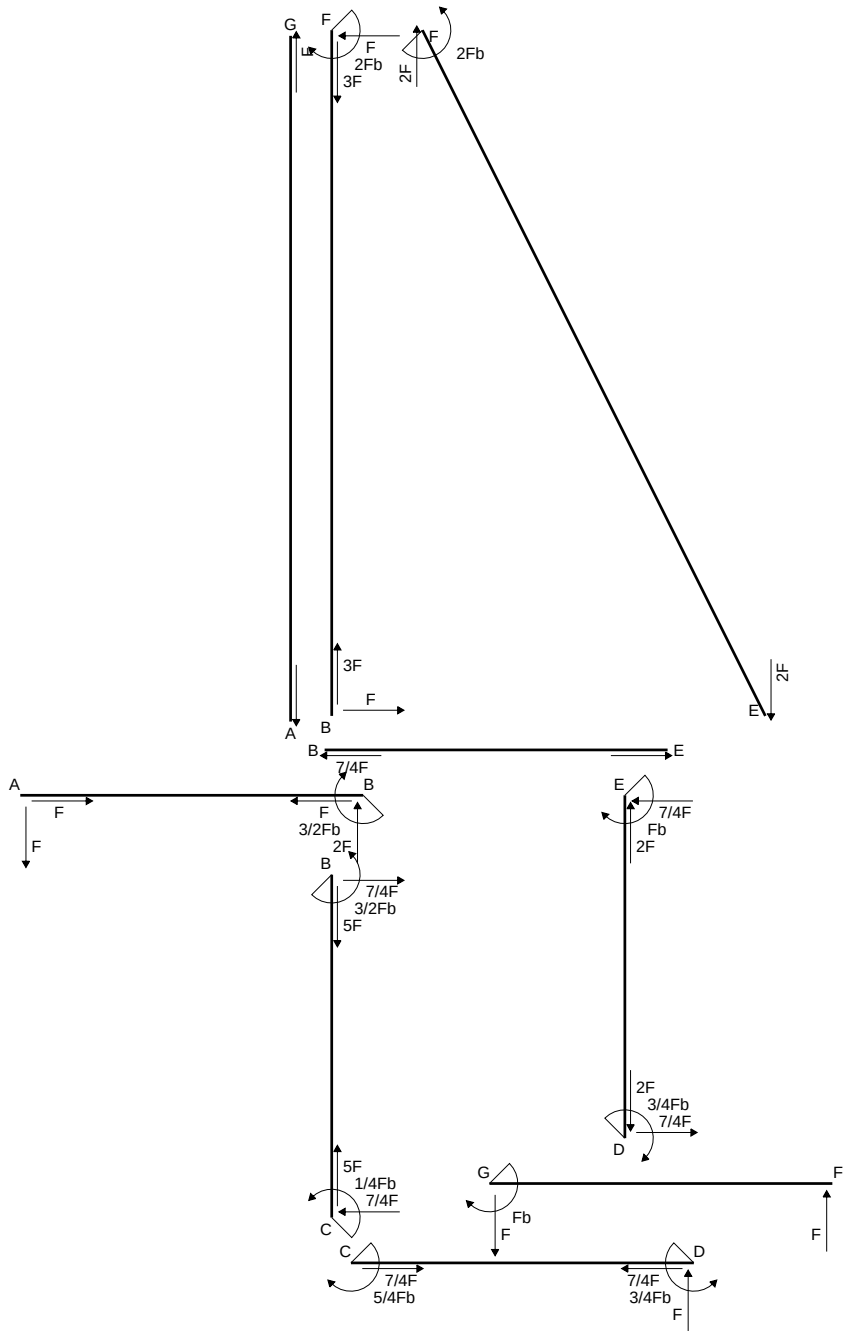
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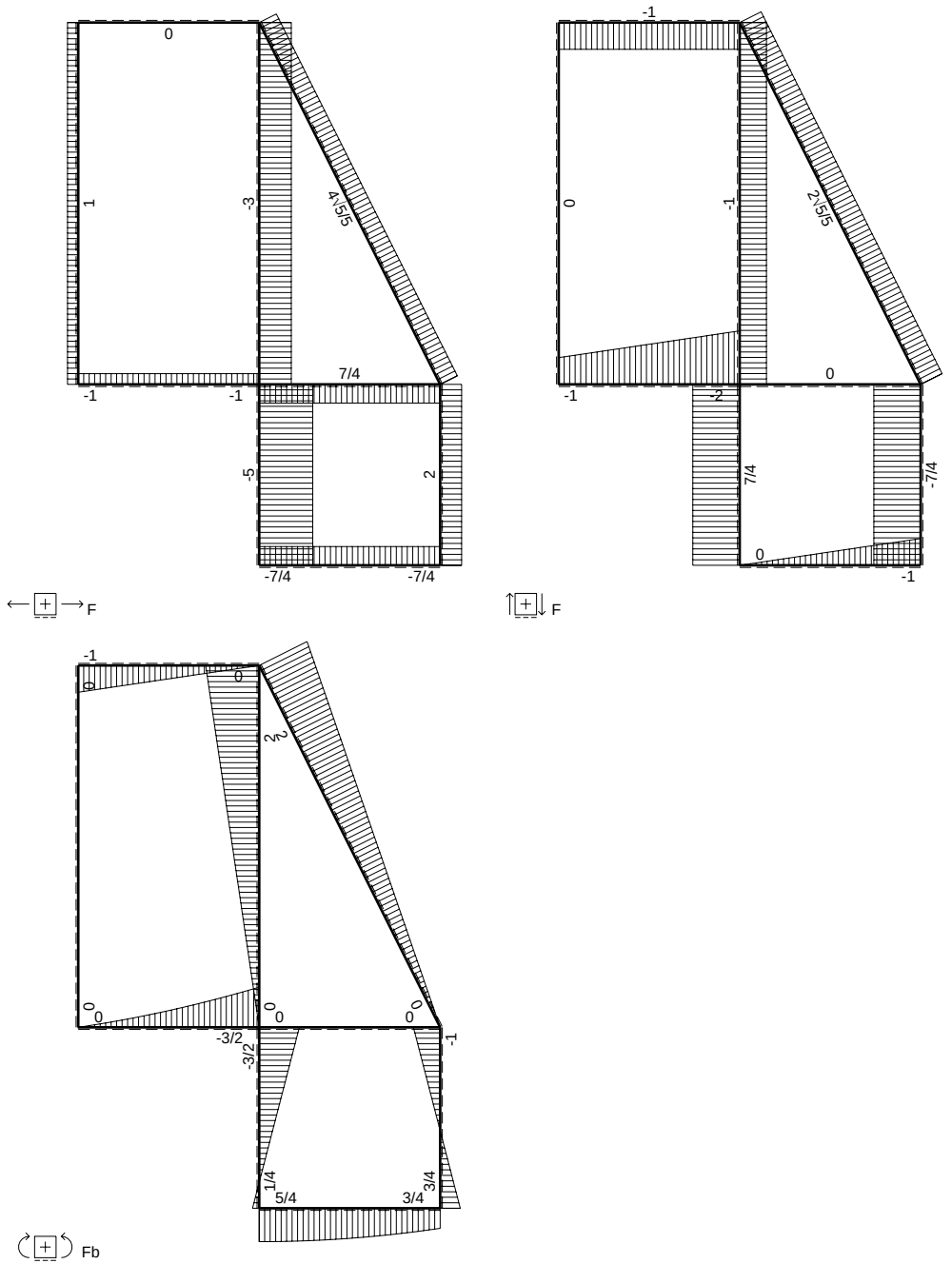
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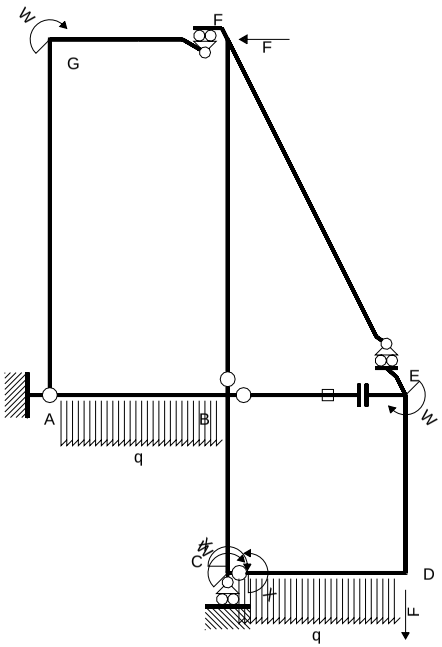
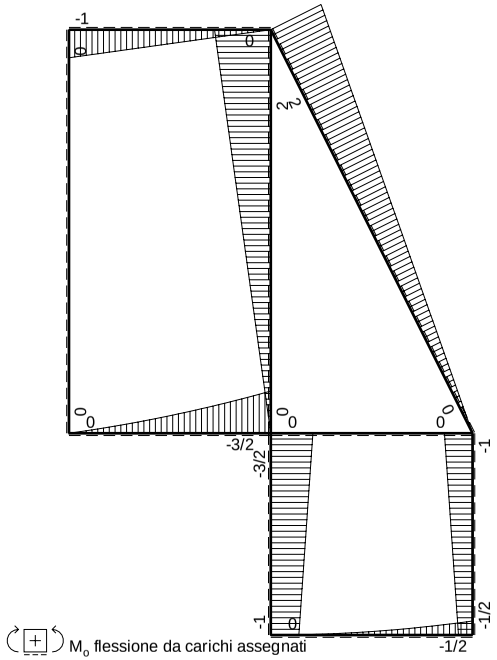
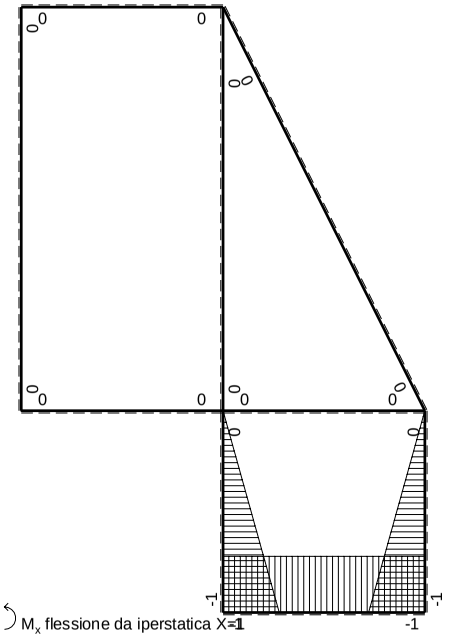


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb-1/2Fx$	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2Fx$	$Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$25/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-5/4Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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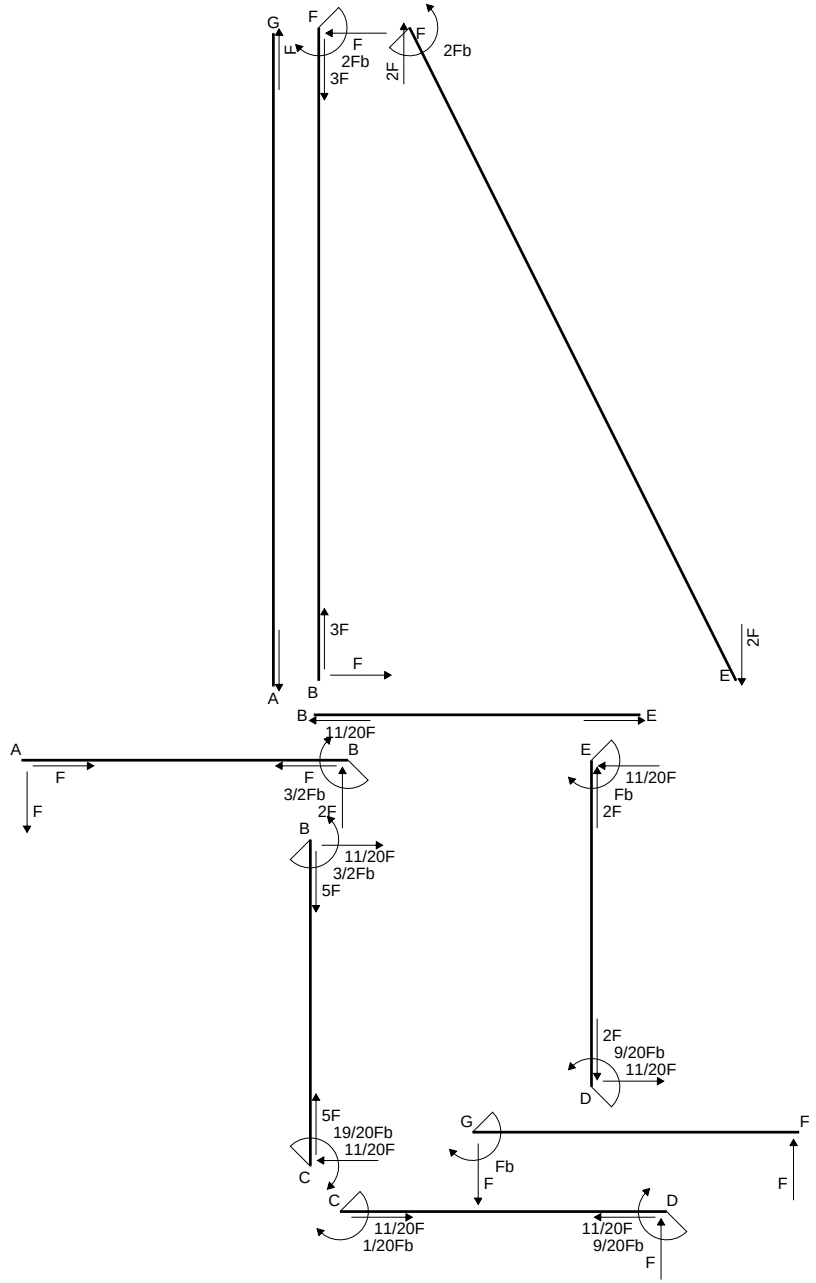
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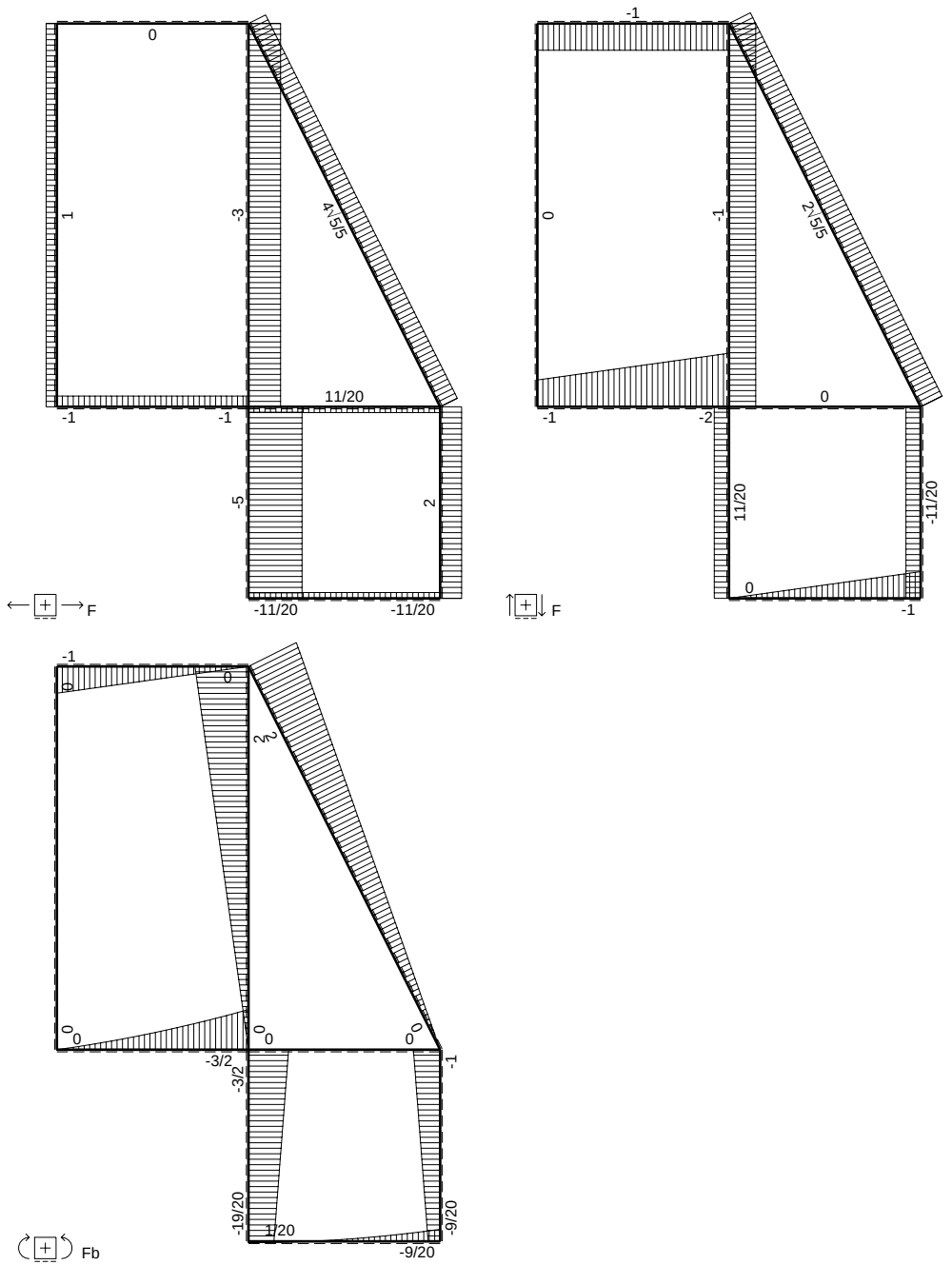
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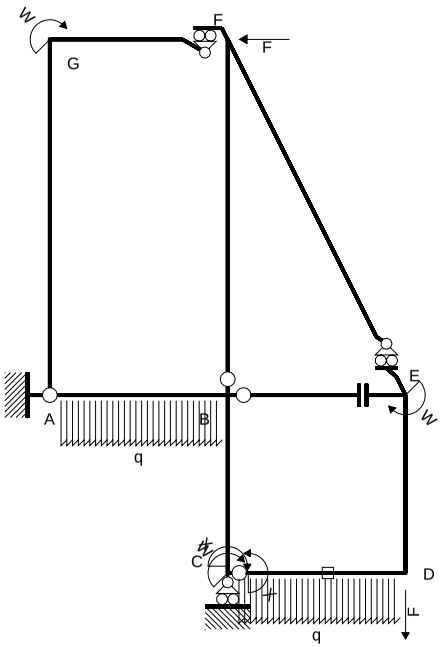
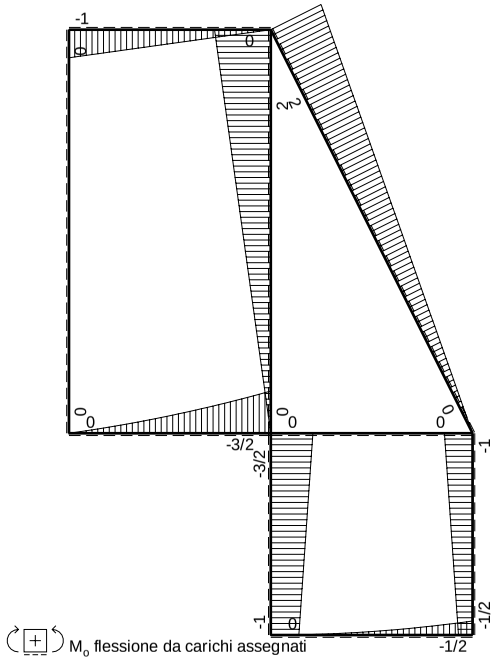
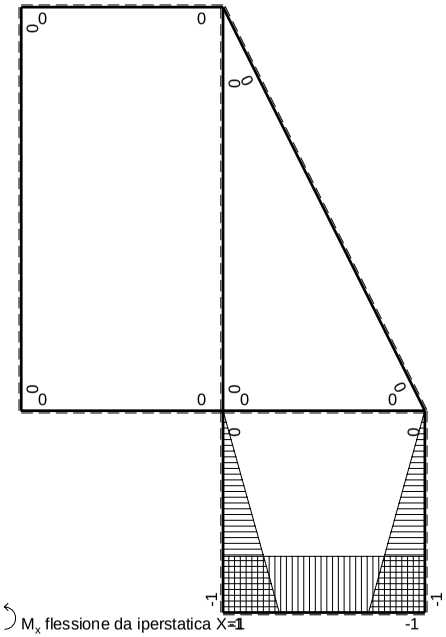


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb-1/2Fx$	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2Fx$	$Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-Fx$	0	0	0	0
GF b	0	$Fb-Fx$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-1/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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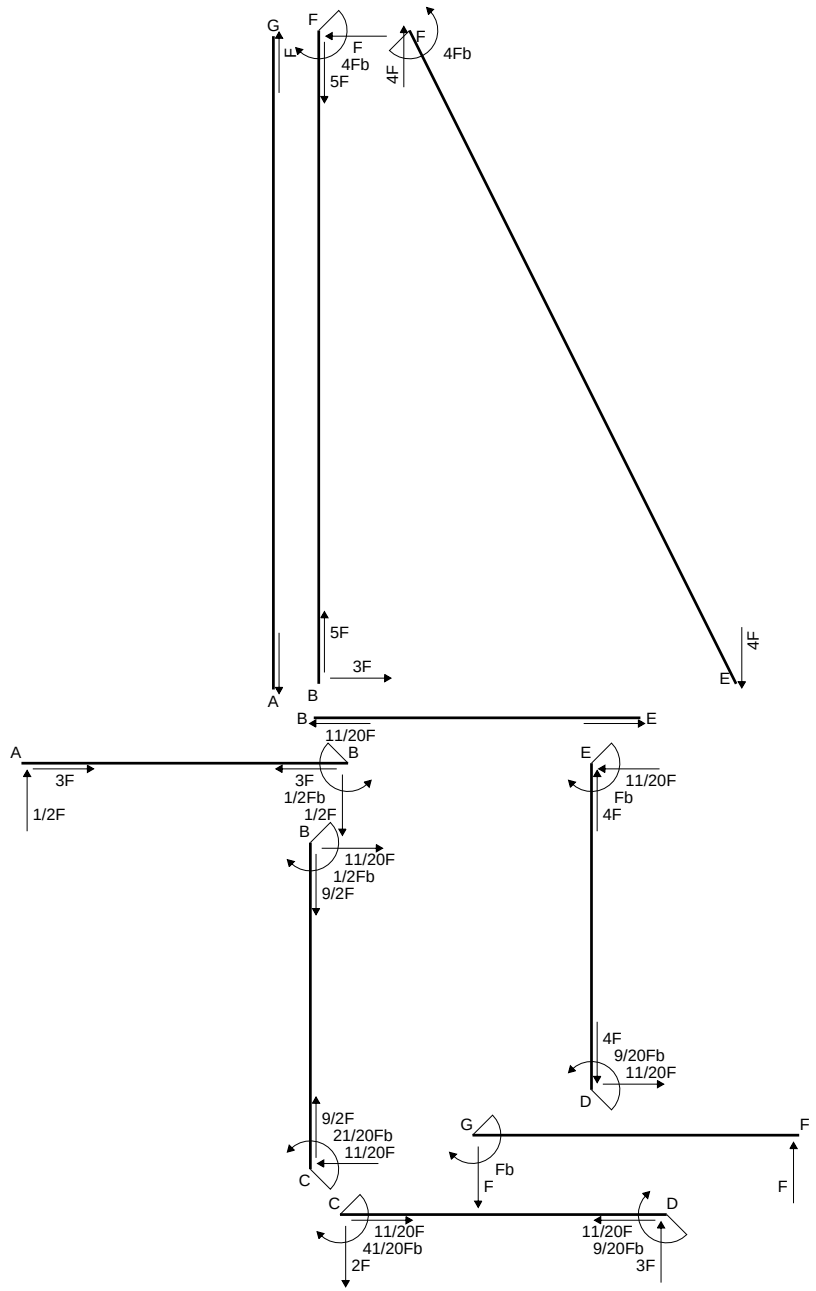
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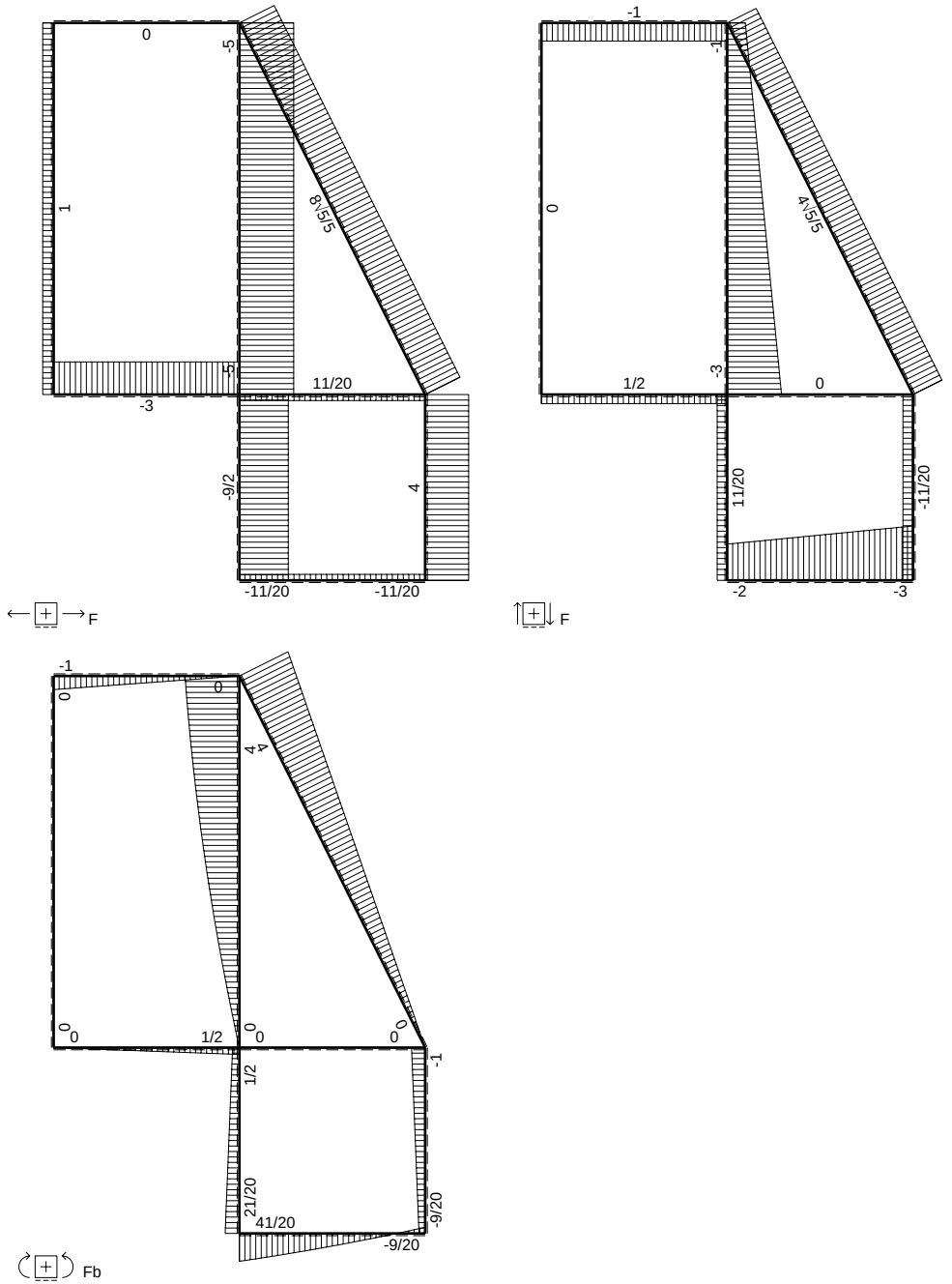
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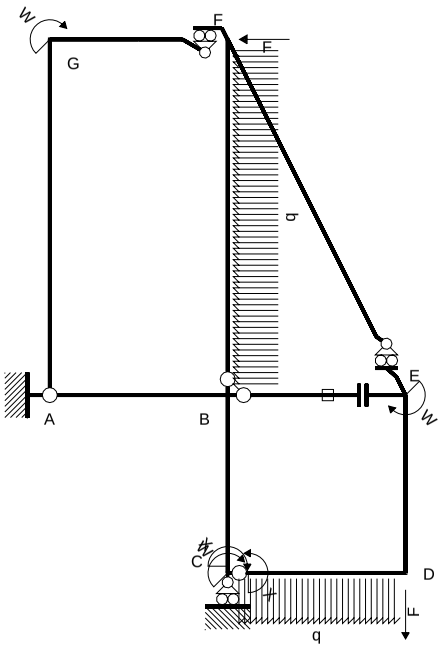
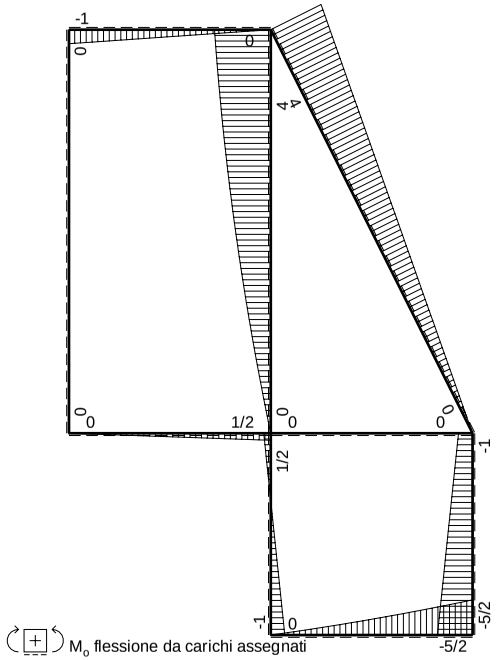
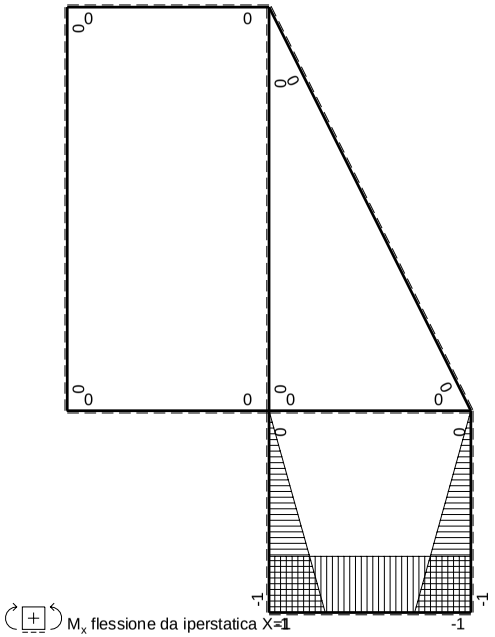


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-2Fx-1/2qx^2$	$2Fx+1/2Fx^2/b$	1	$7/6Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-3Fx+1/2qx^2$	$5/2Fb-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$41/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-41/20Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -3\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -3/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -3/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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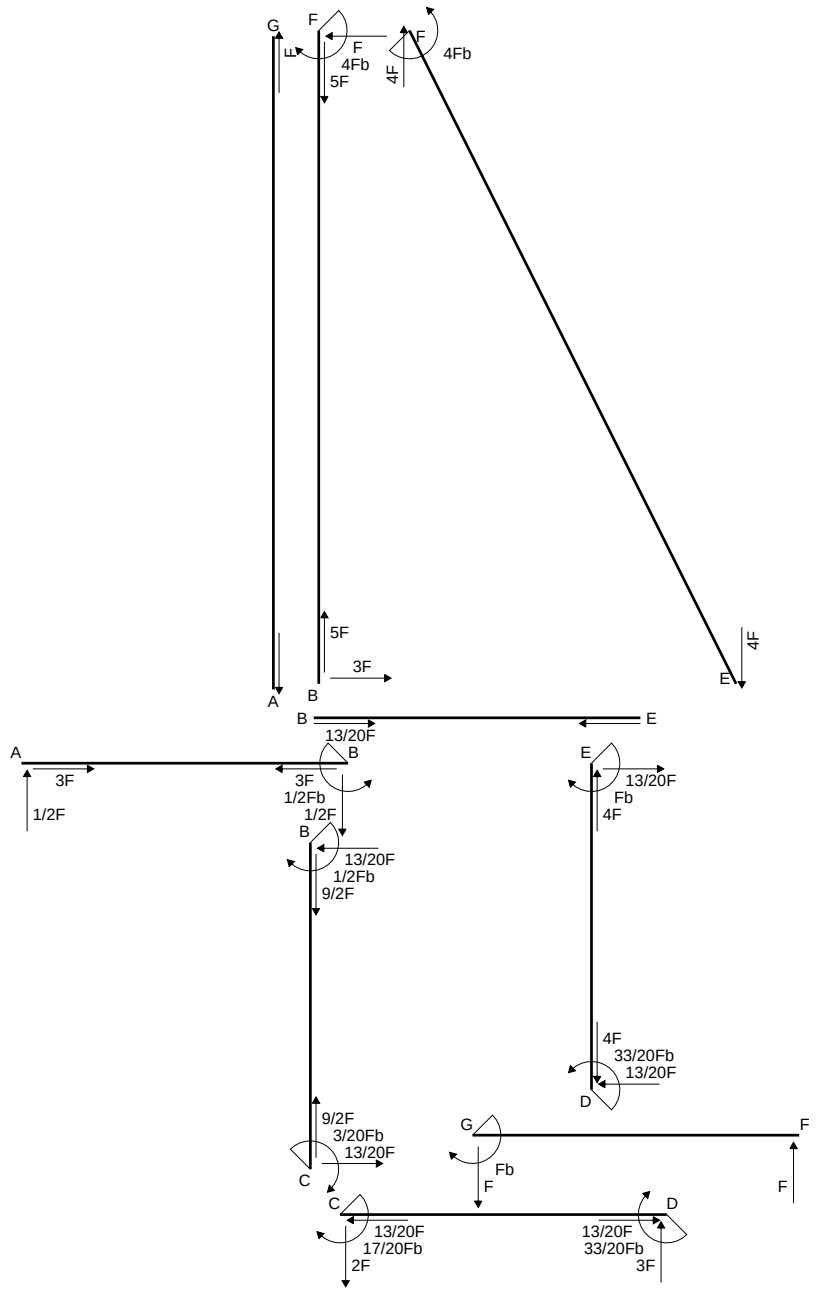
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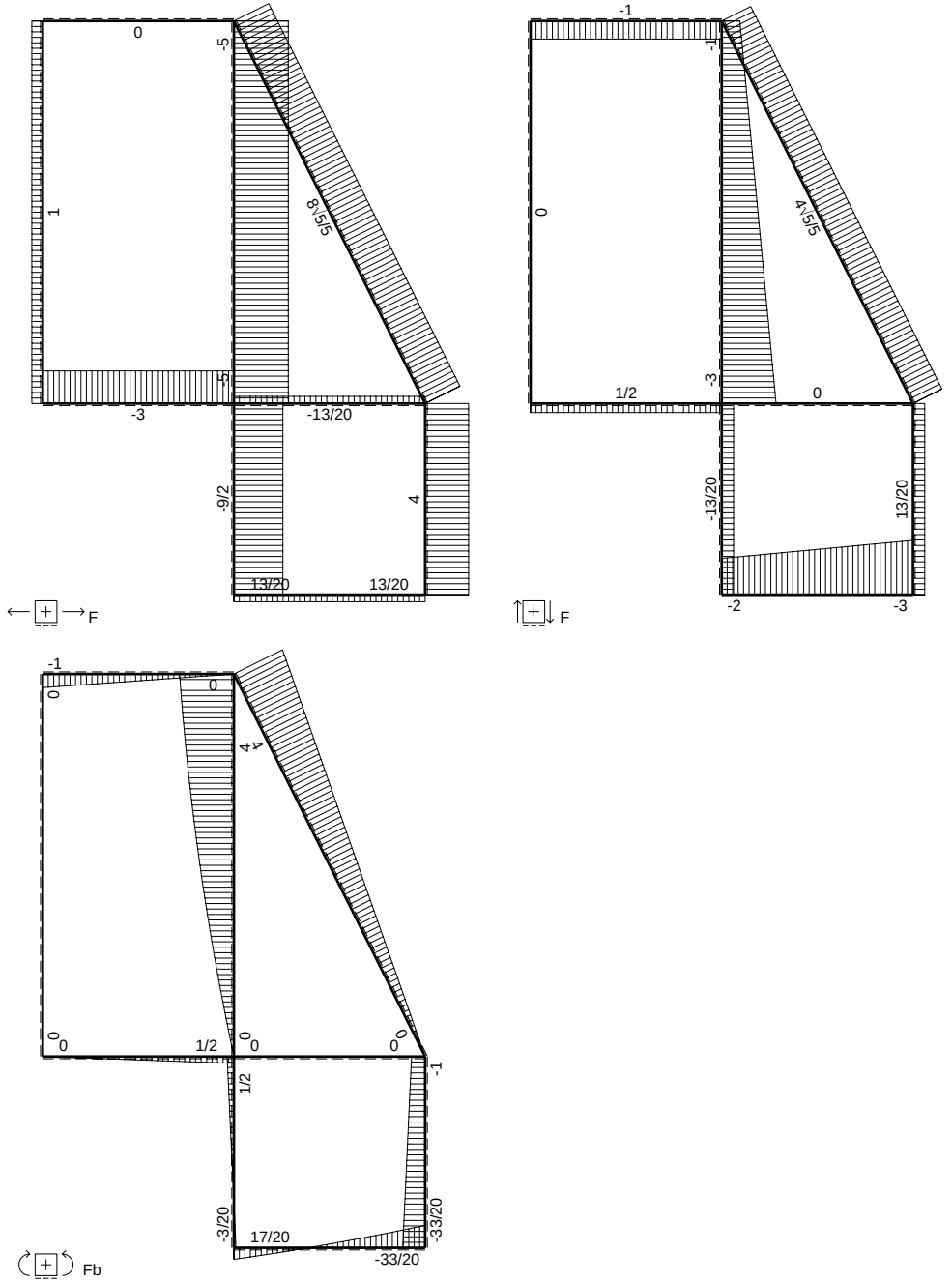
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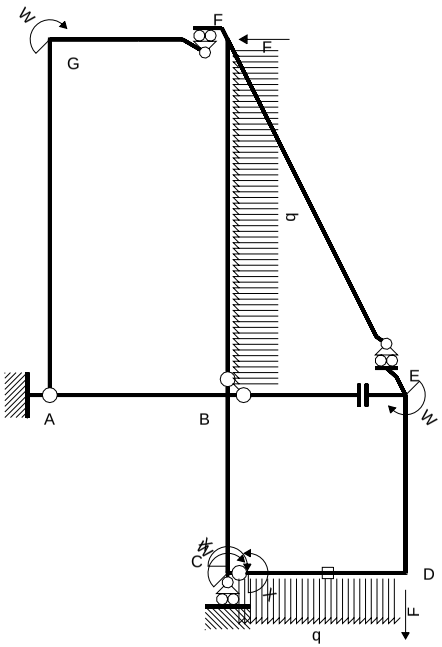
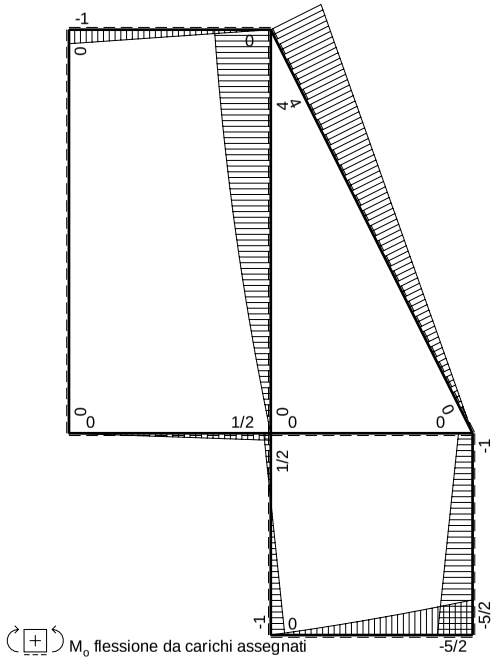
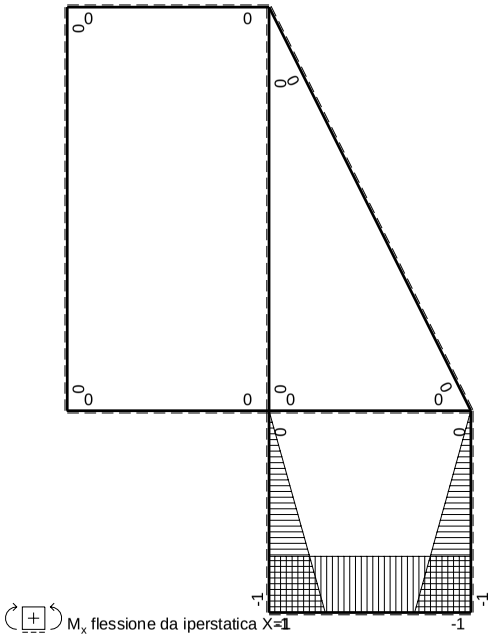


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-2Fx-1/2qx^2$	$2Fx+1/2Fx^2/b$	1	$7/6Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-3Fx+1/2qx^2$	$5/2Fb-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$17/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-17/20Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -3\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [5/2\ x -3/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/2\ b -3/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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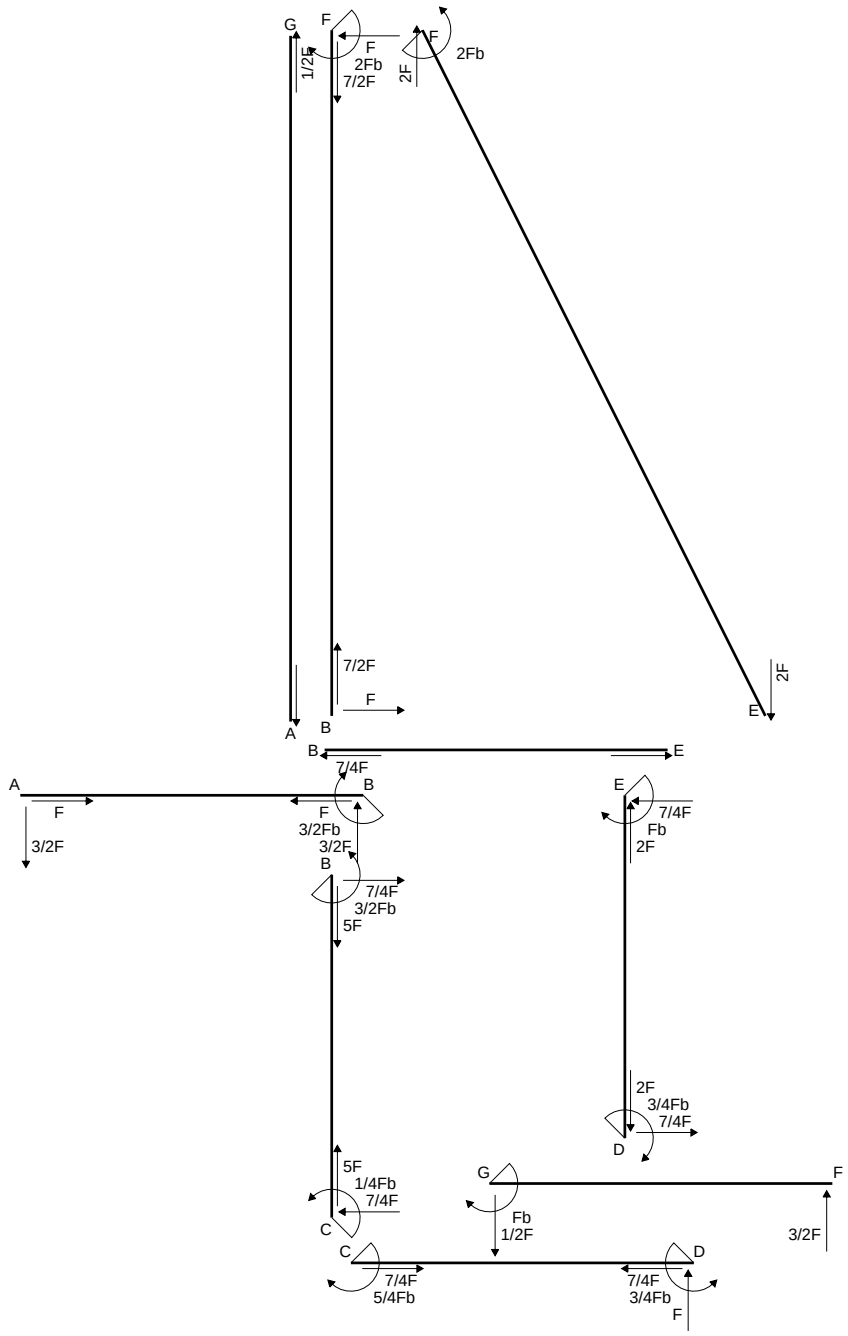
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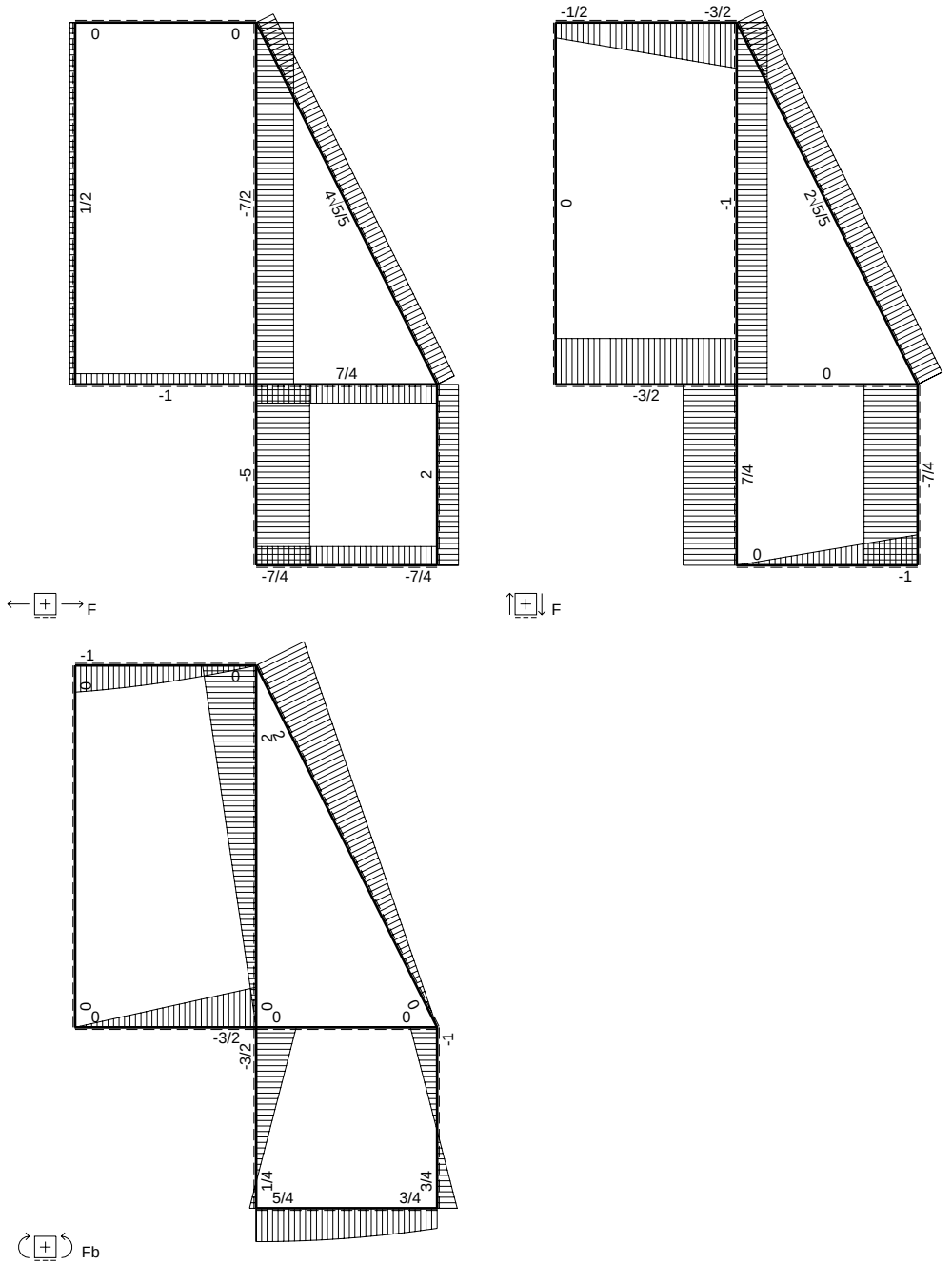
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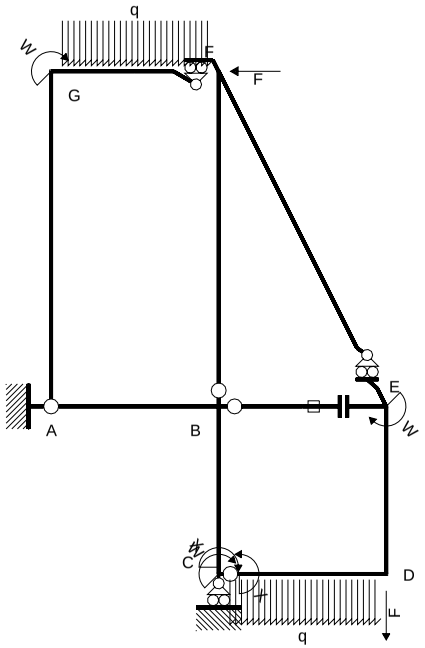
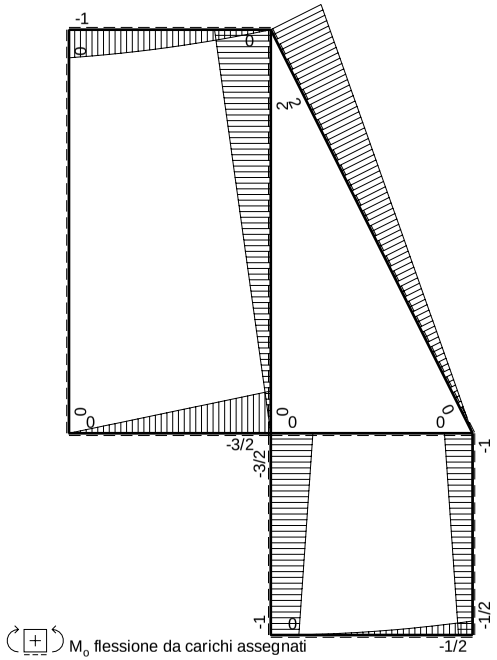
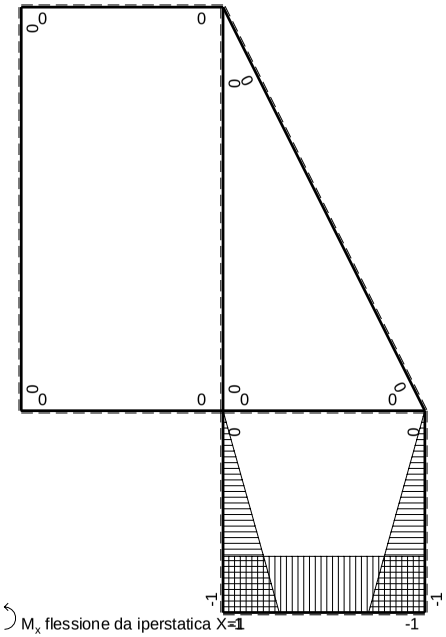


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb-1/2Fx	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb-1/2Fx	$Fx-1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$25/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-5/4Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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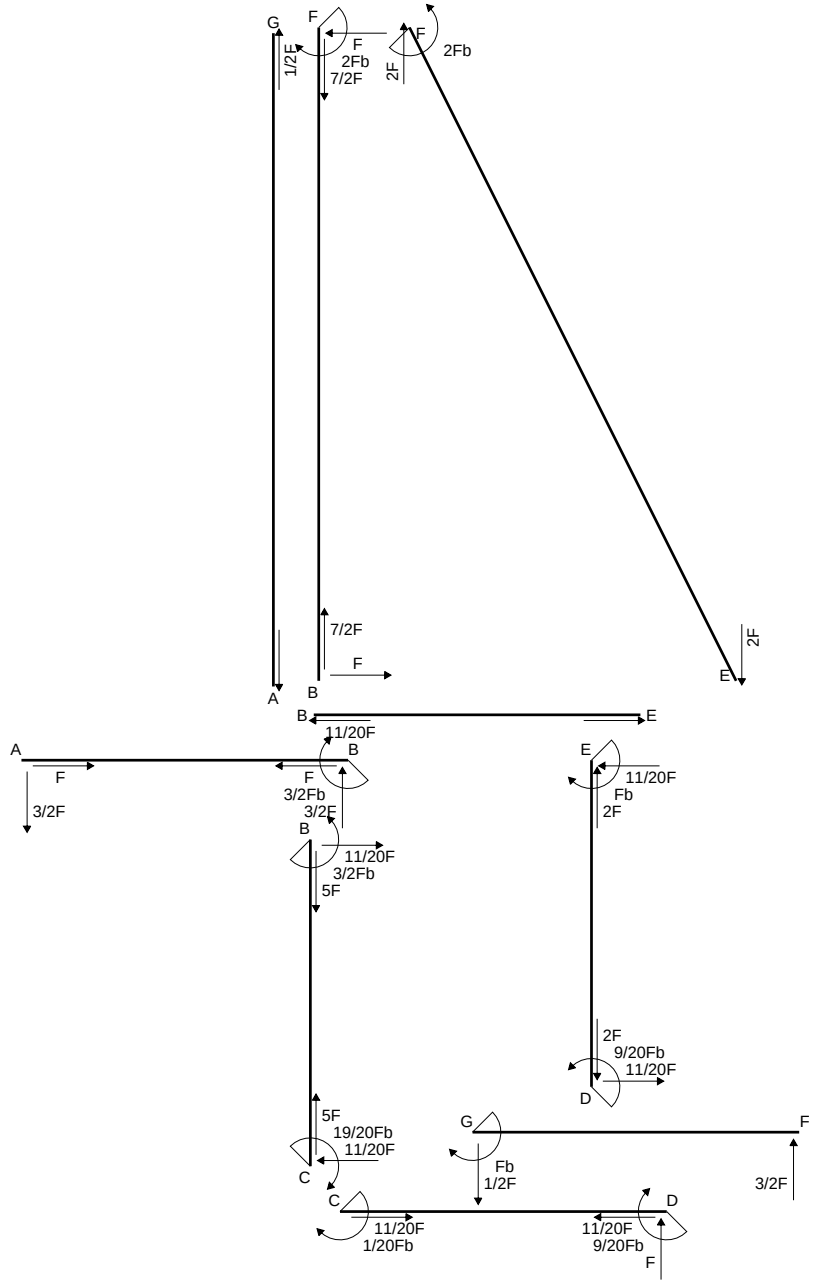
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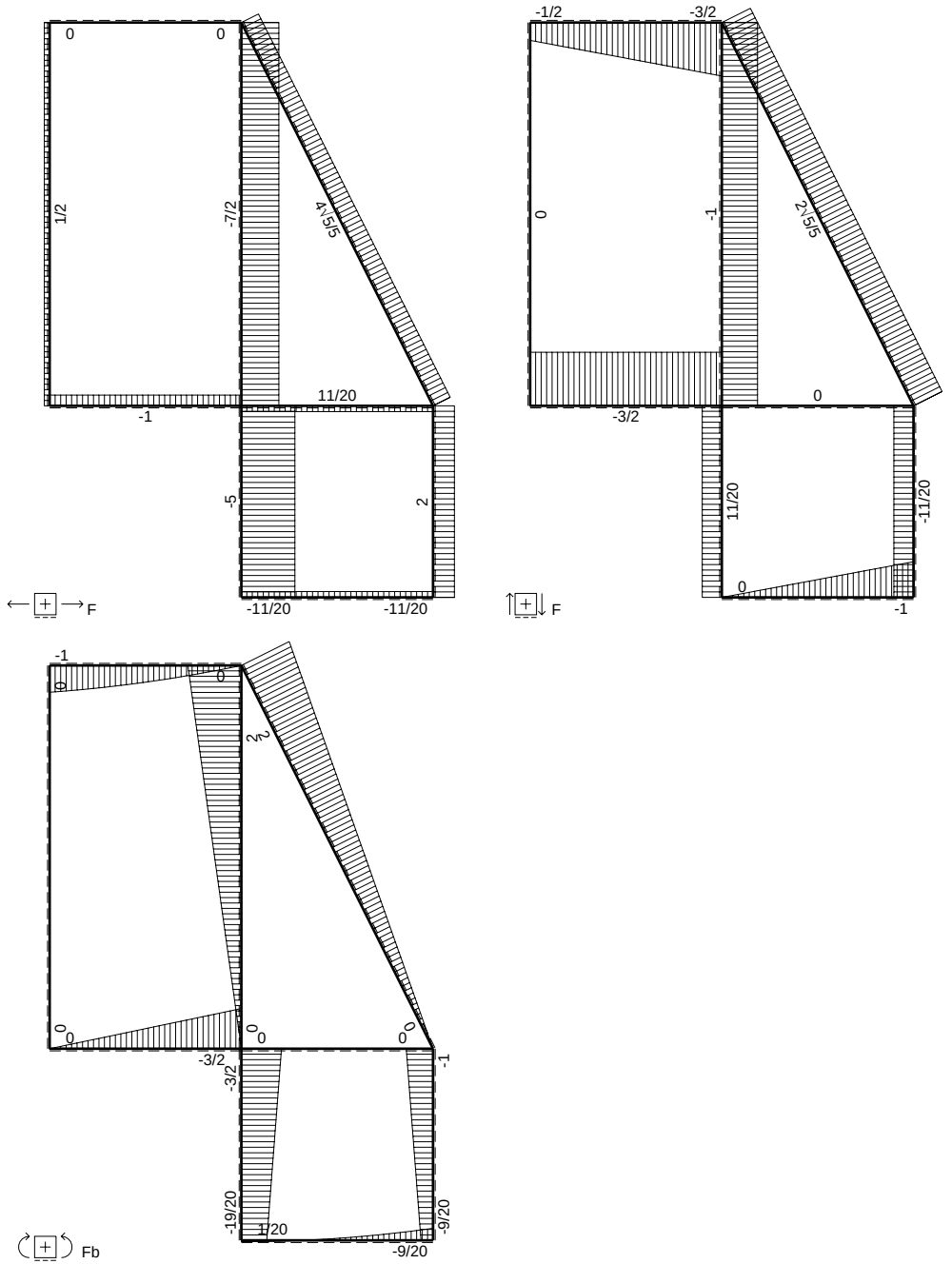
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0
-3/2
11/20
-11/20
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-1
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-9/20
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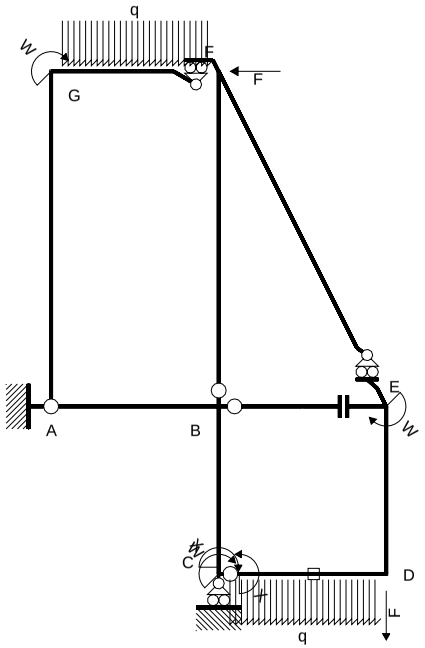
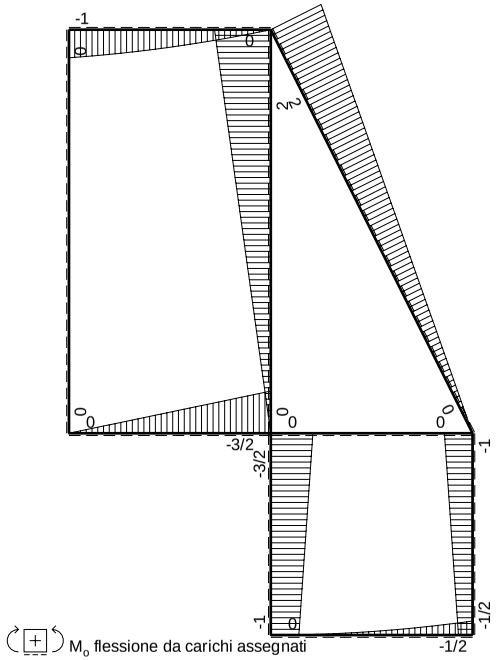
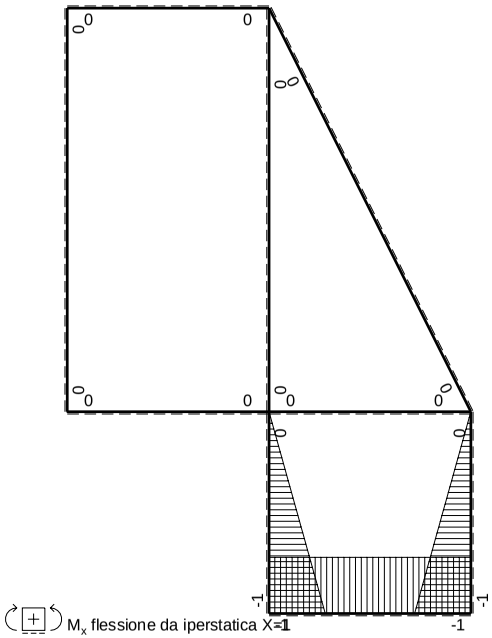
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb-1/2Fx	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb-1/2Fx	$Fx-1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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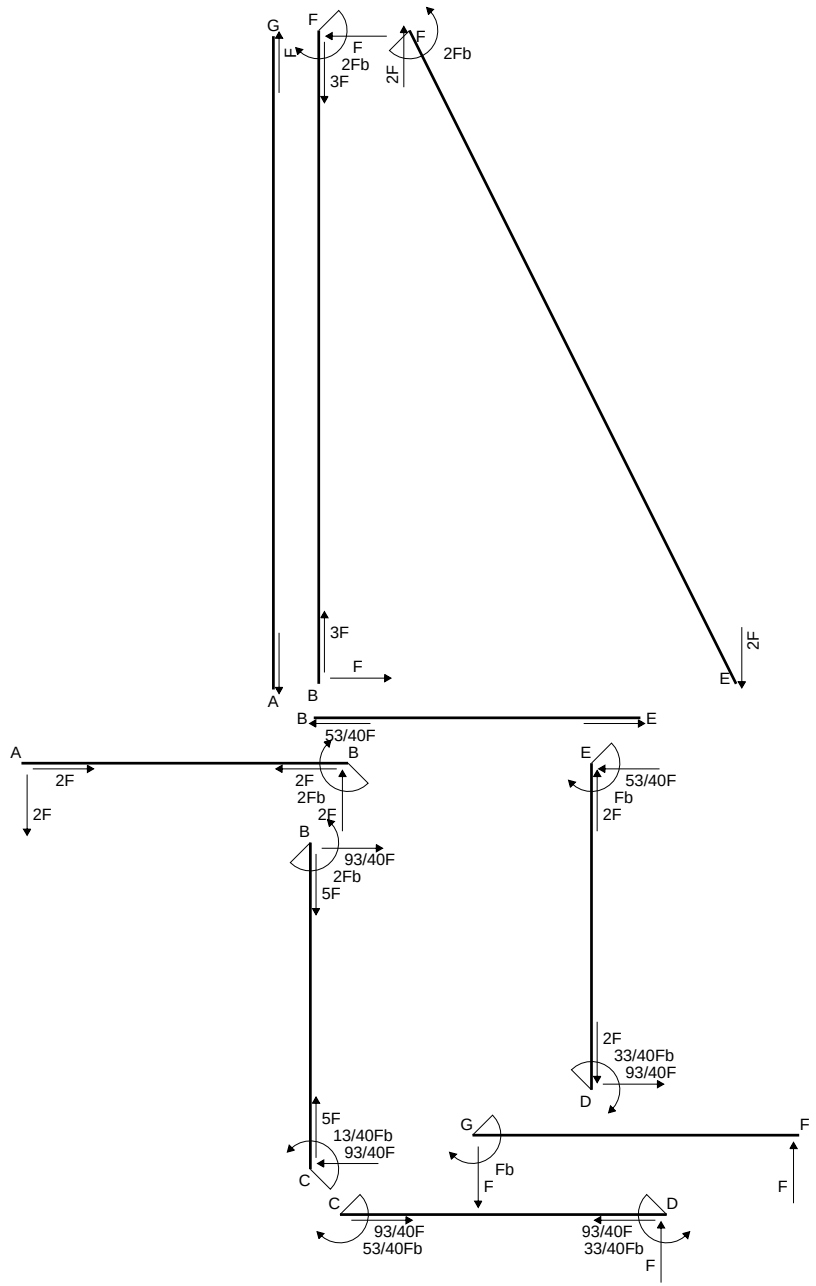
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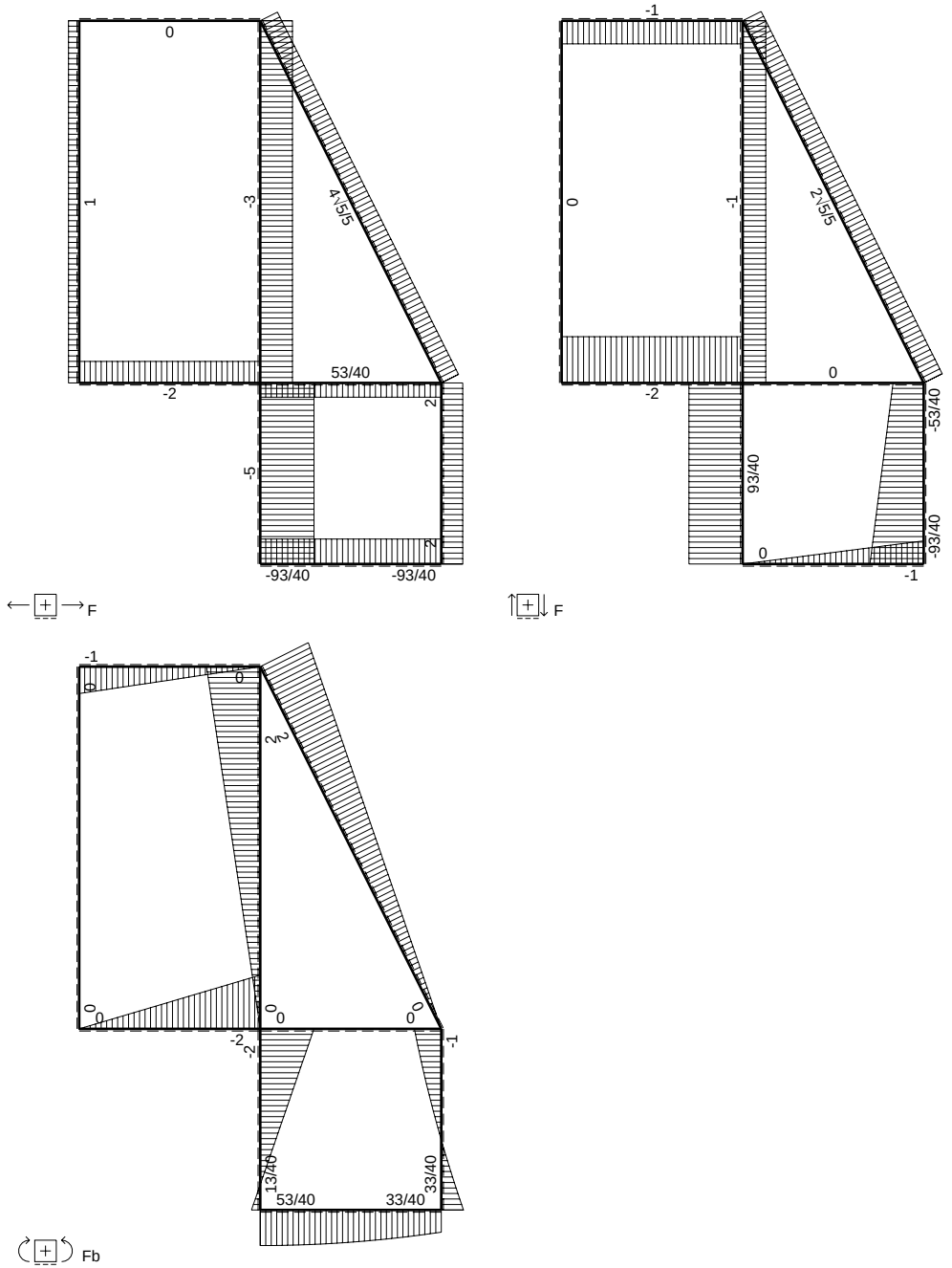
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-1
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0
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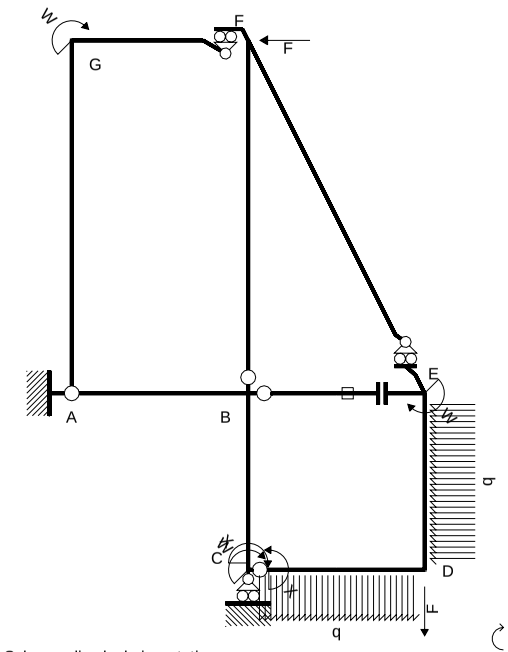
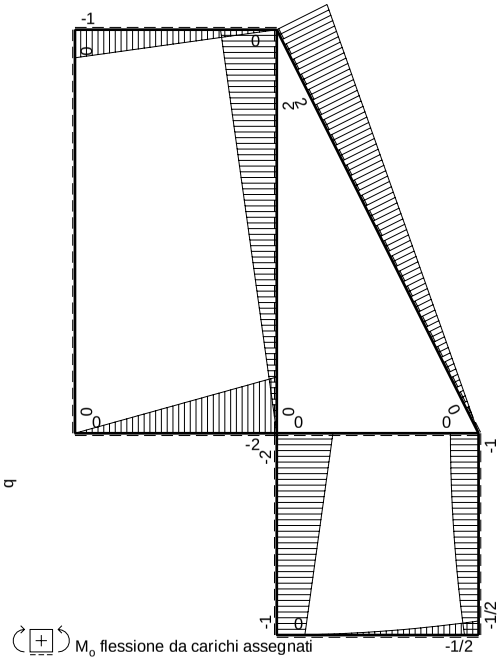
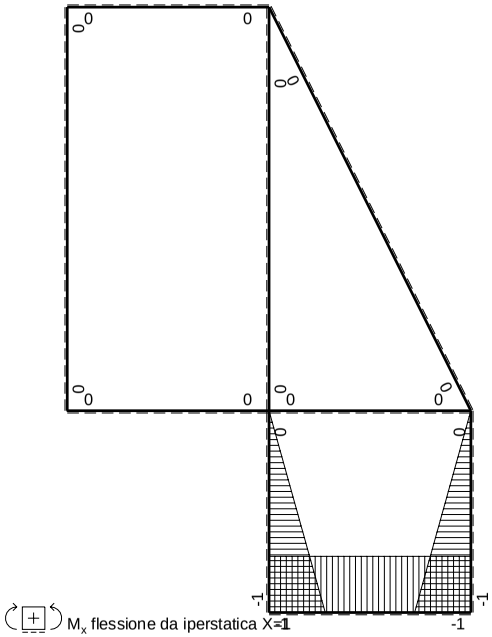
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 214395 ESΣ02.xxxx.054

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	$2Fx-Fx^2/b$	x^2/b^2	$2/3Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+Fx	$Fb-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb-Fx+1/2qx^2$	$1/2Fb+1/2Fx-3/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$3/8Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2qx^2$	$Fx-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$53/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-53/40Fb	

PROCEDIMENTO E RISULTATI 214395 ESΣ02.xxxx.054

Sviluppi di calcolo iperstatica

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$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (2\ x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b)\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b)\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 +1/2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [1/2\ x +1/4\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/4\ b -1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/8\ b)\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

ESΣ02.xxxx.054

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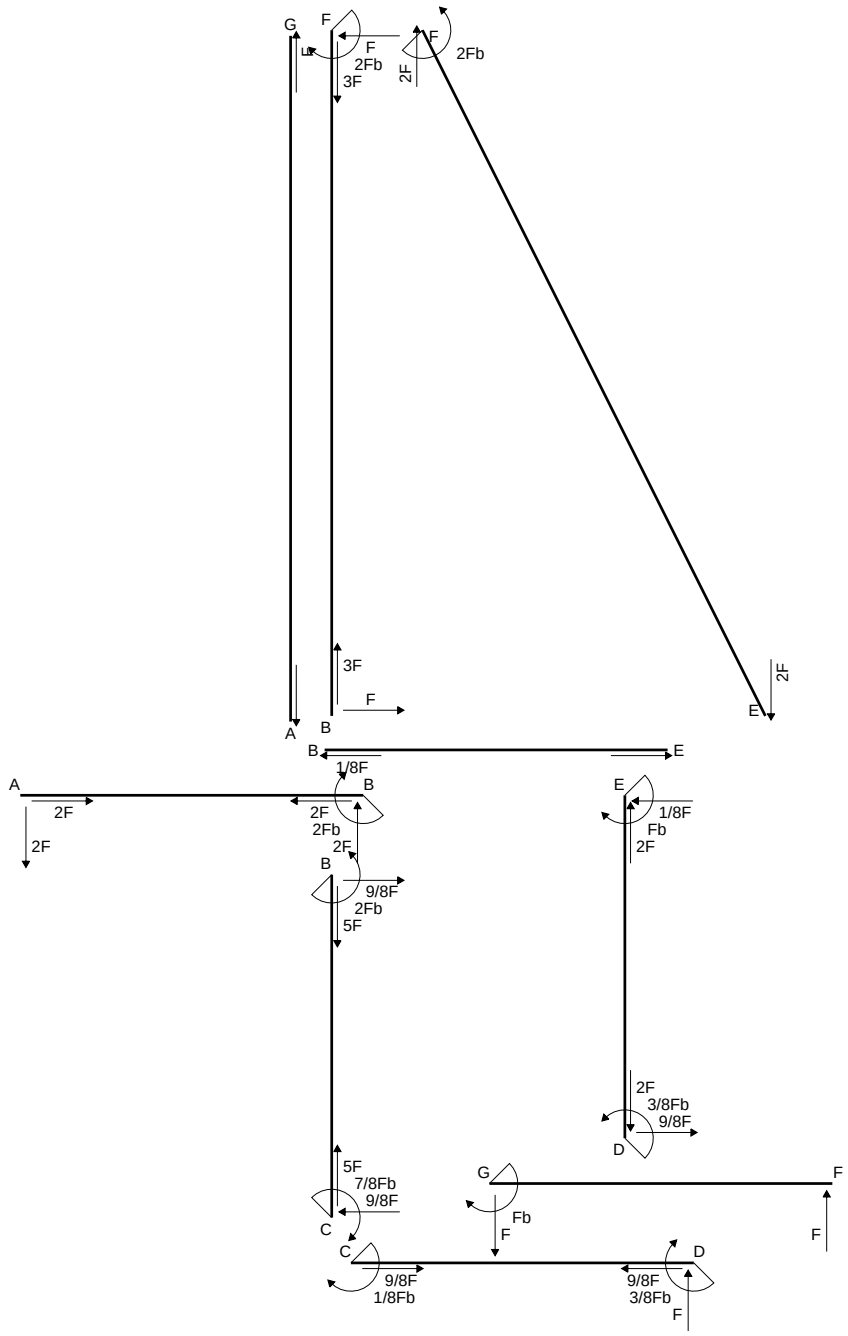
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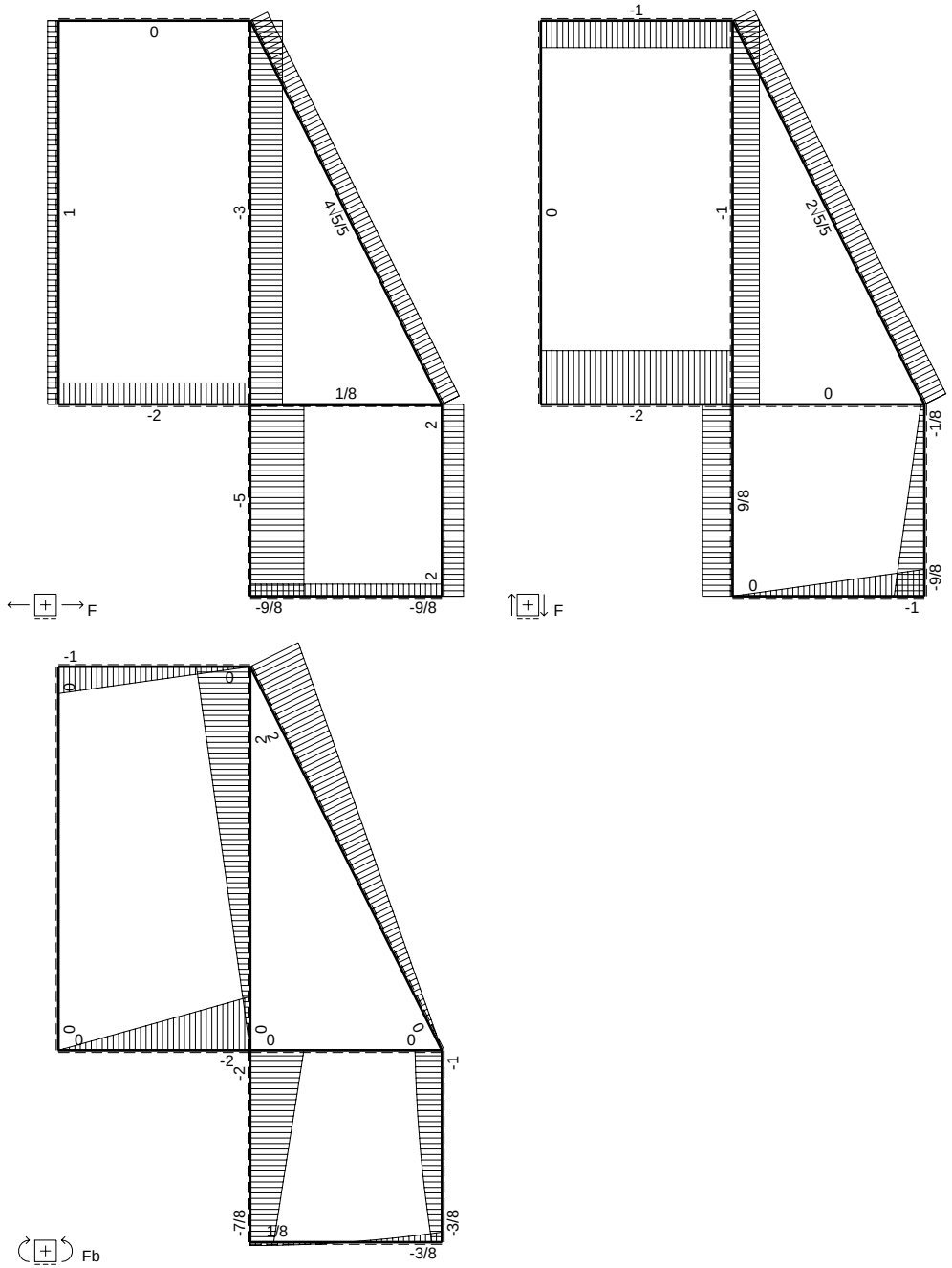
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2√5/5
0
-2
0
-1/8
9/8
0
-9/8
-1
F
-1
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0
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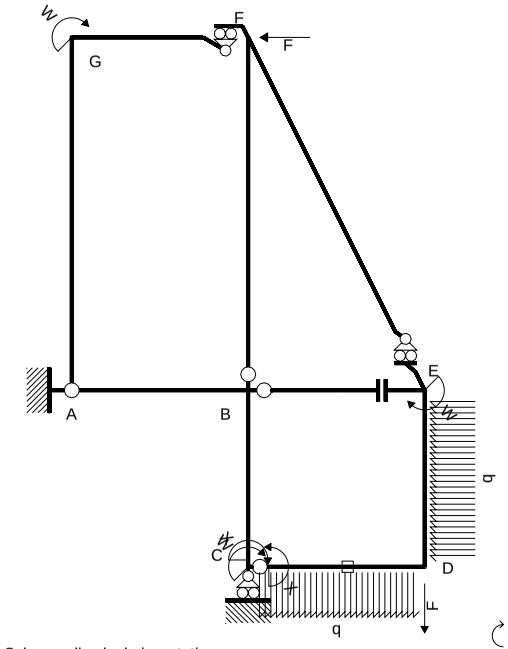
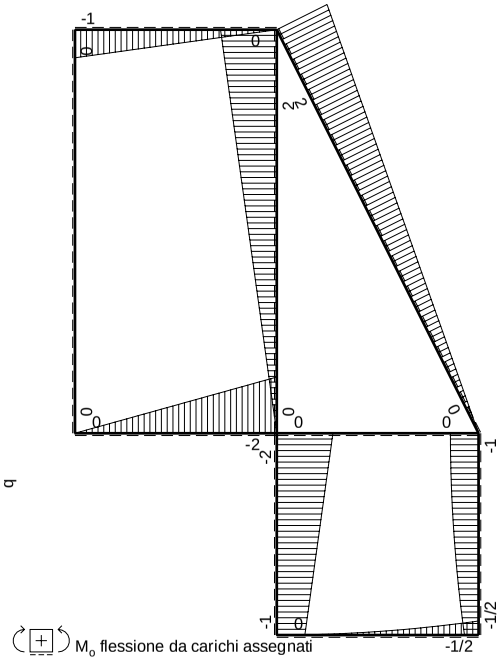
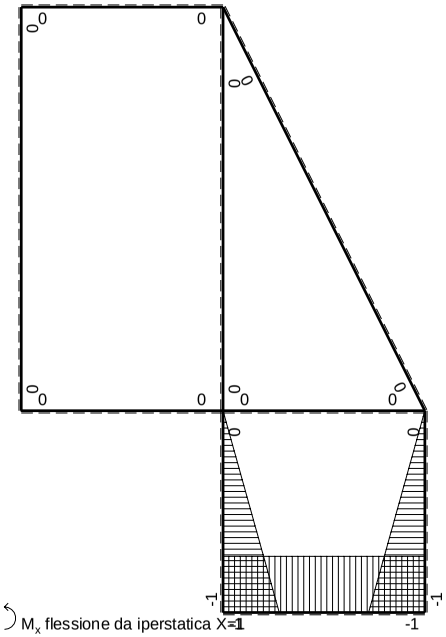
1/8
-3/8
-3/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	$2Fx-Fx^2/b$	x^2/b^2	$2/3Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+Fx	$Fb-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb-Fx+1/2qx^2$	$1/2Fb+1/2Fx-3/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$3/8Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2qx^2$	$Fx-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-1/8Fb	

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Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 +1/2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [1/2\ x +1/4\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/4\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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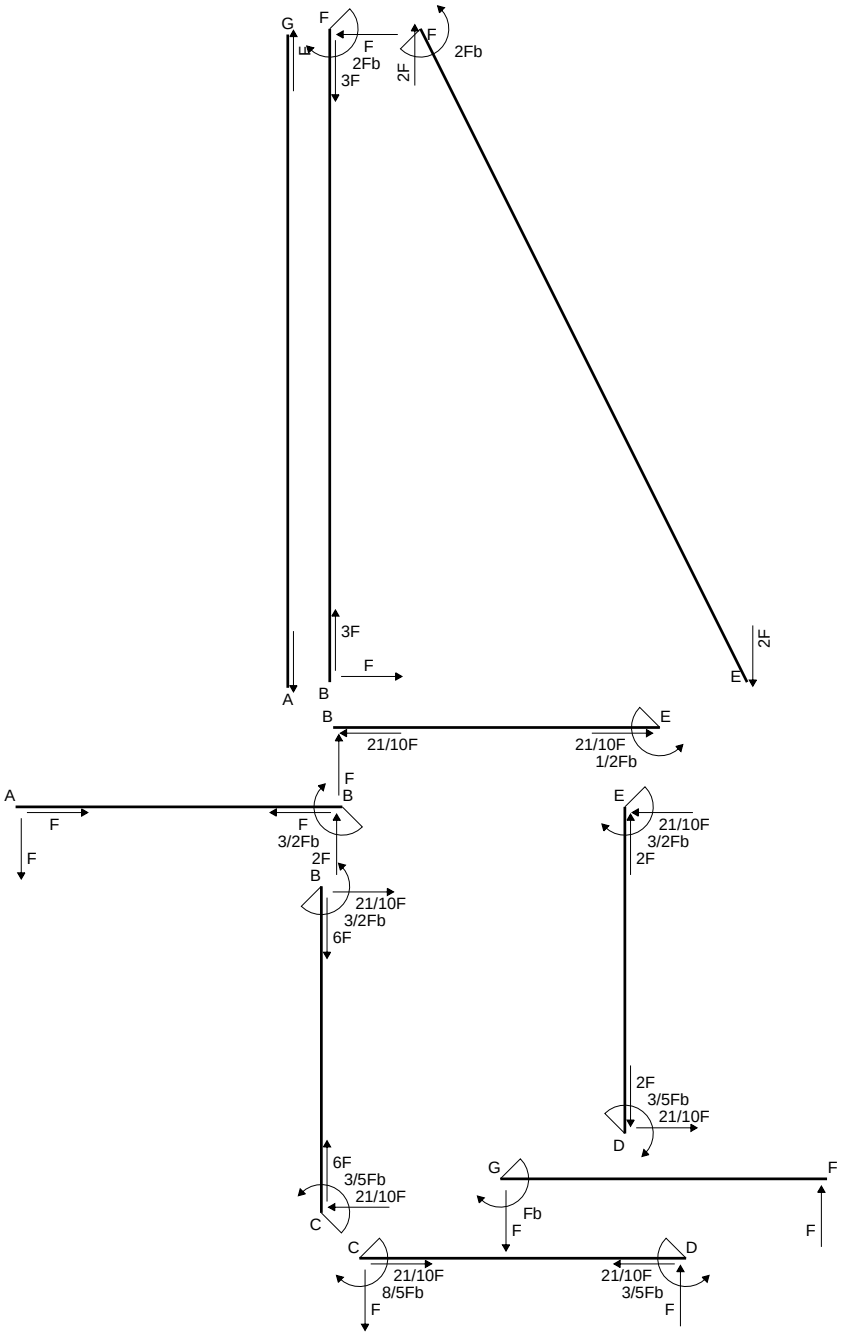
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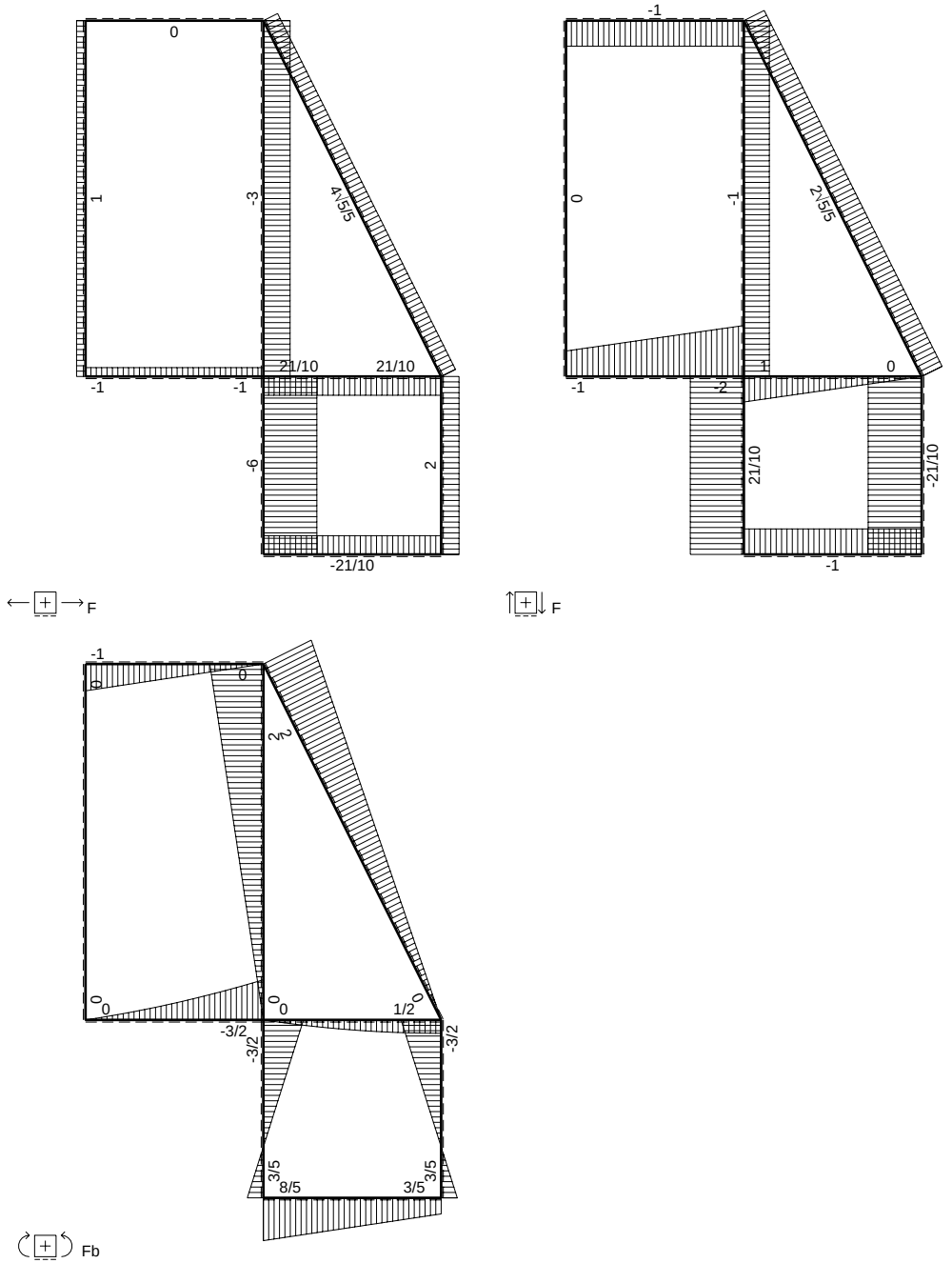
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-1
F
-1
0
2
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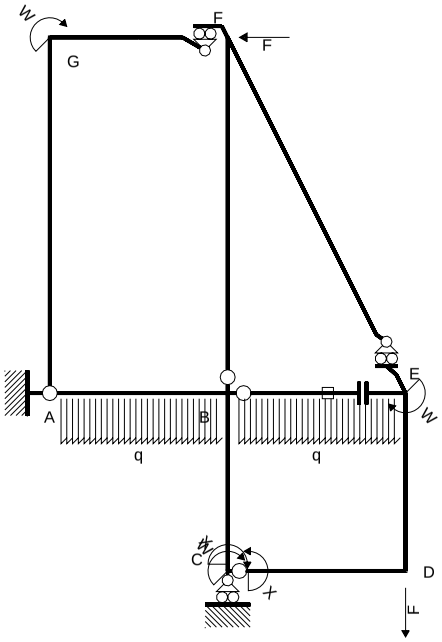
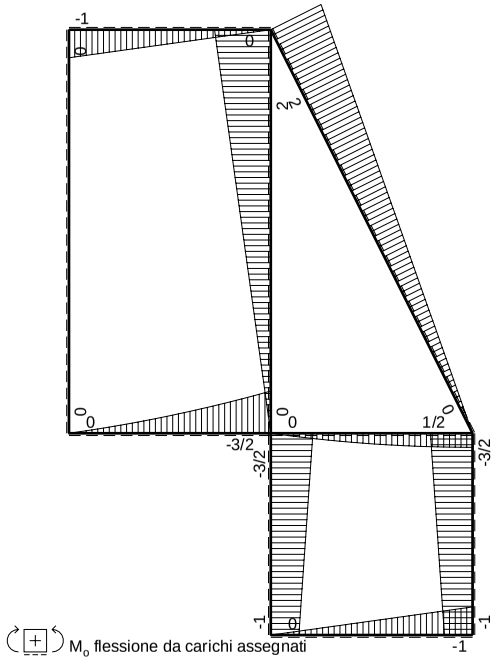
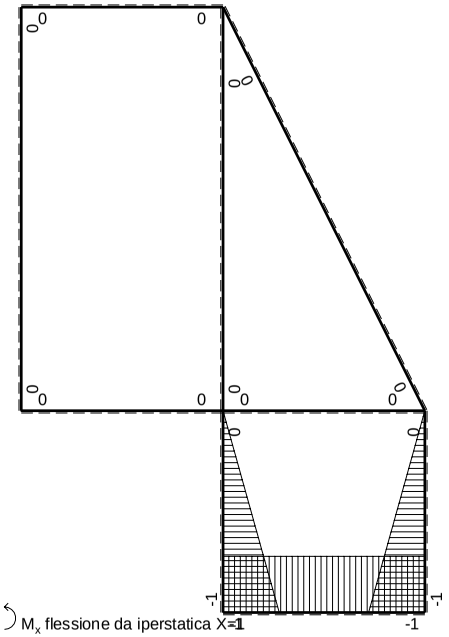
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$3/2Fb-1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$8/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-8/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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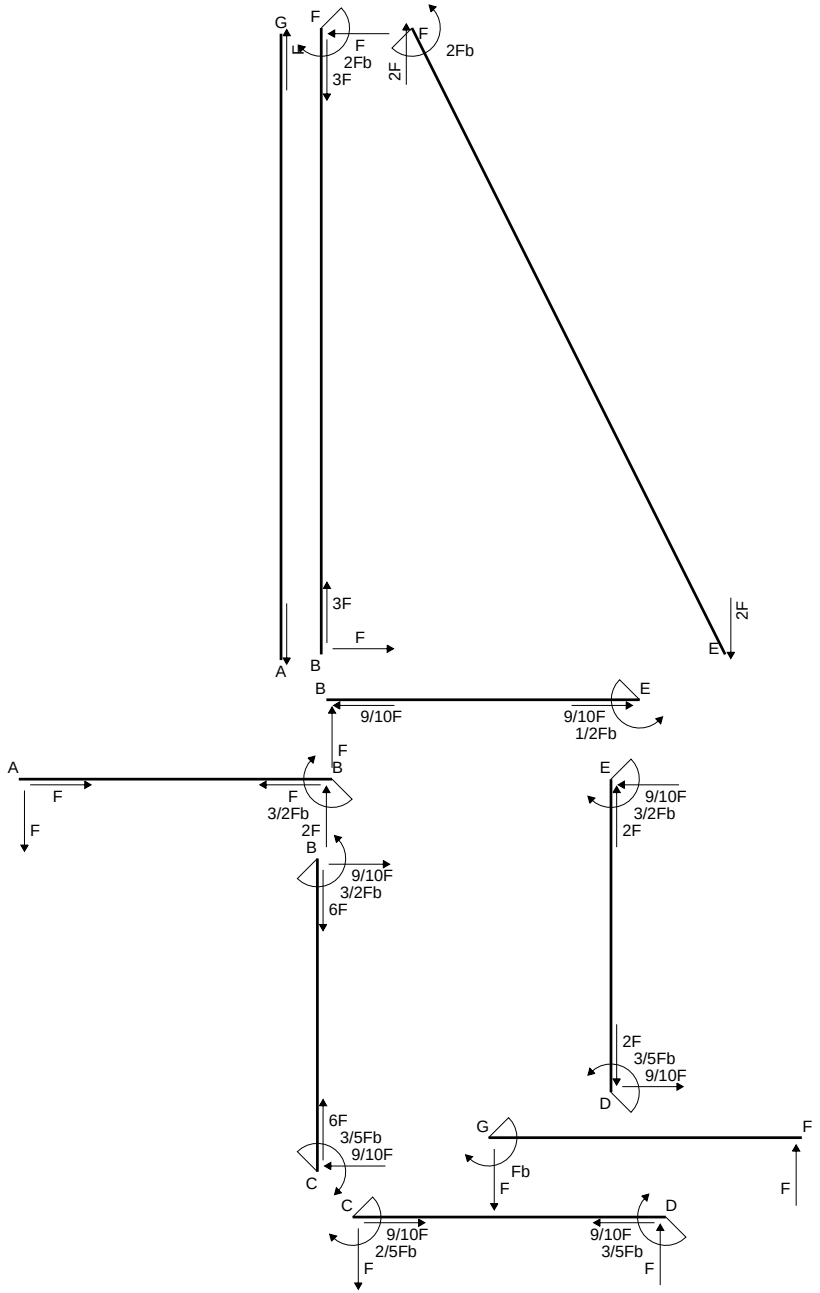
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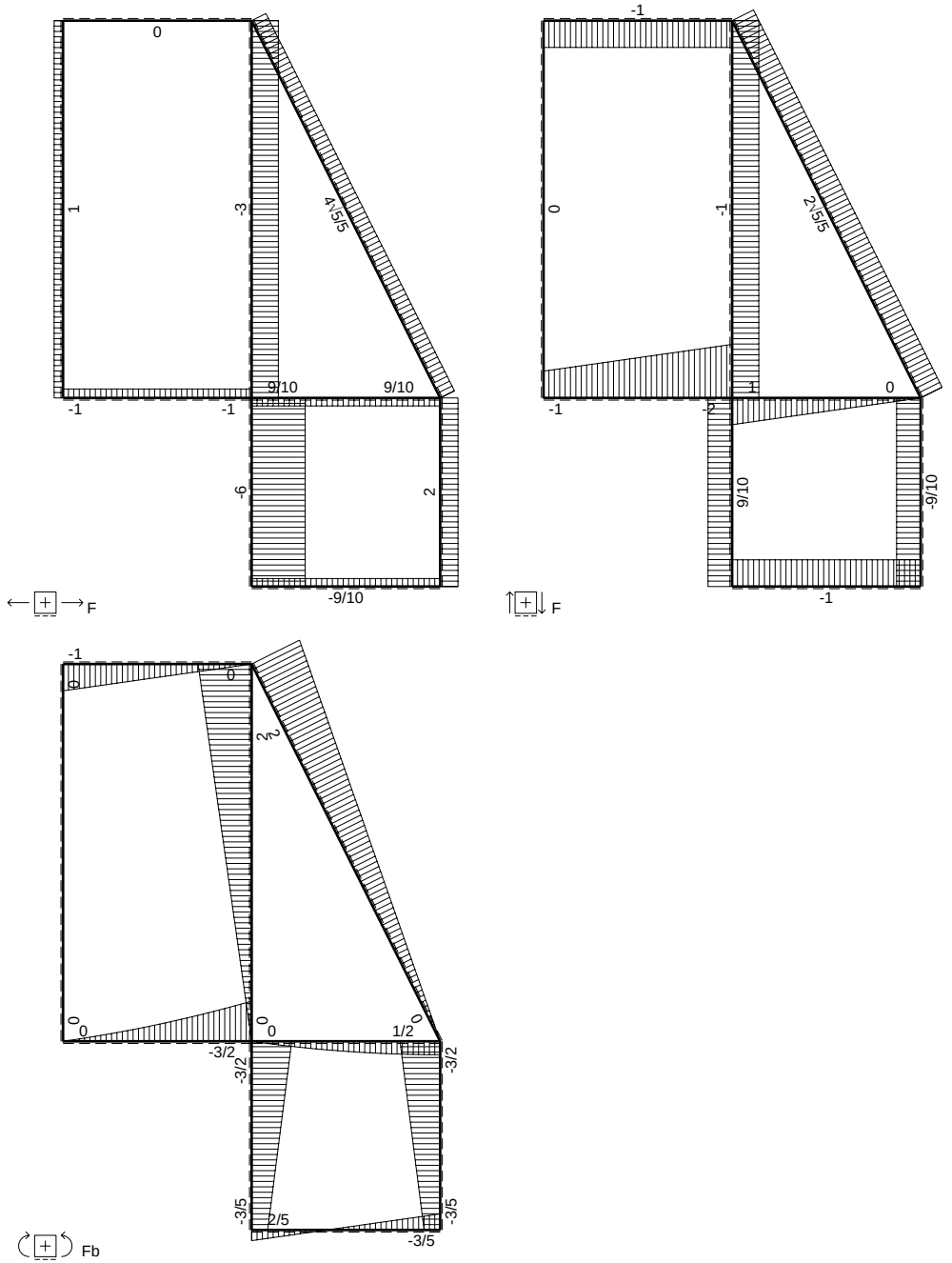
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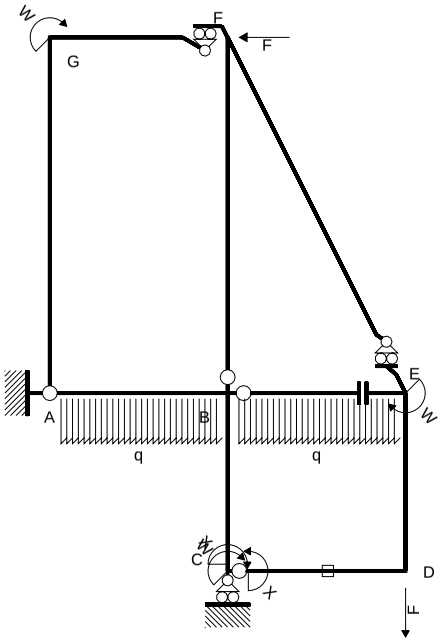
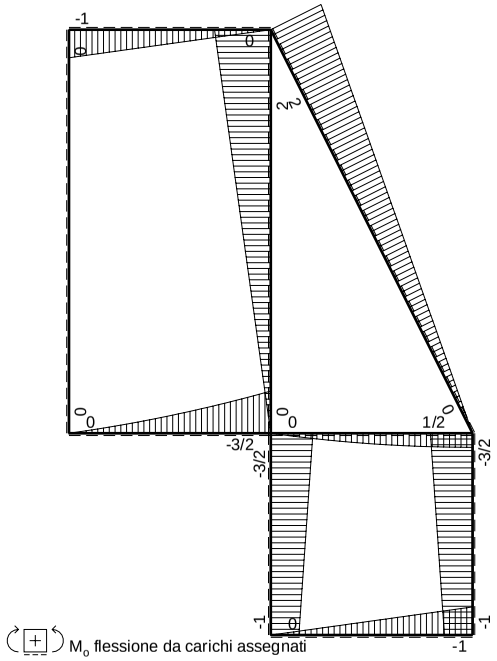
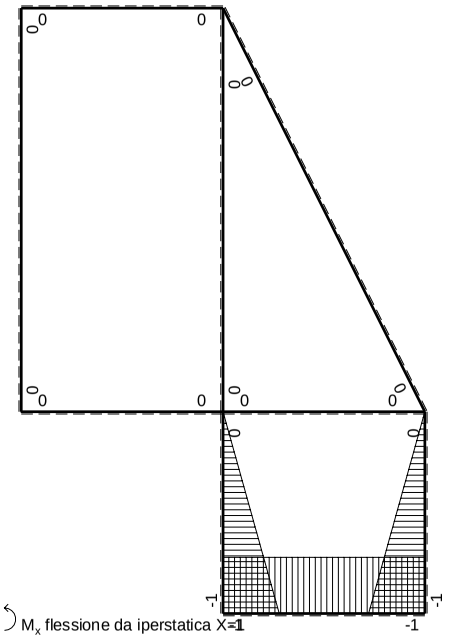


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$3/2Fb-1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$2/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-2/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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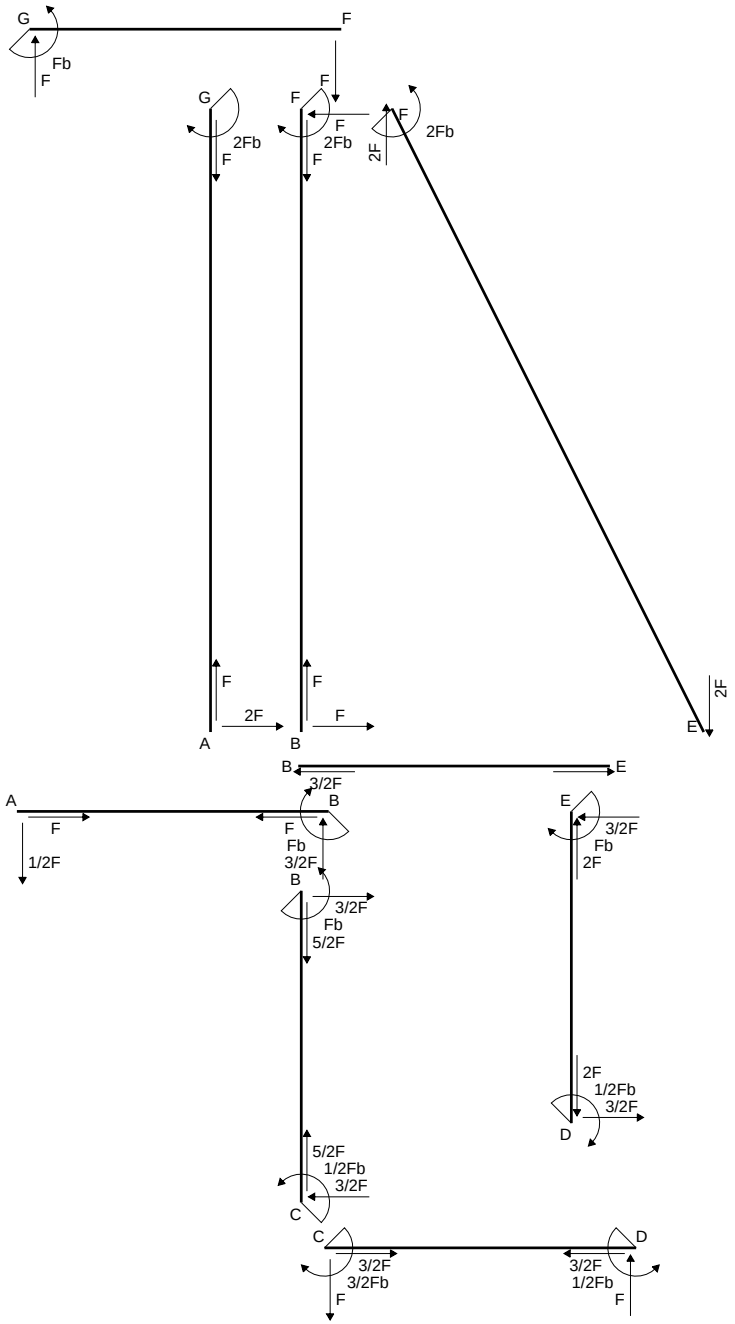
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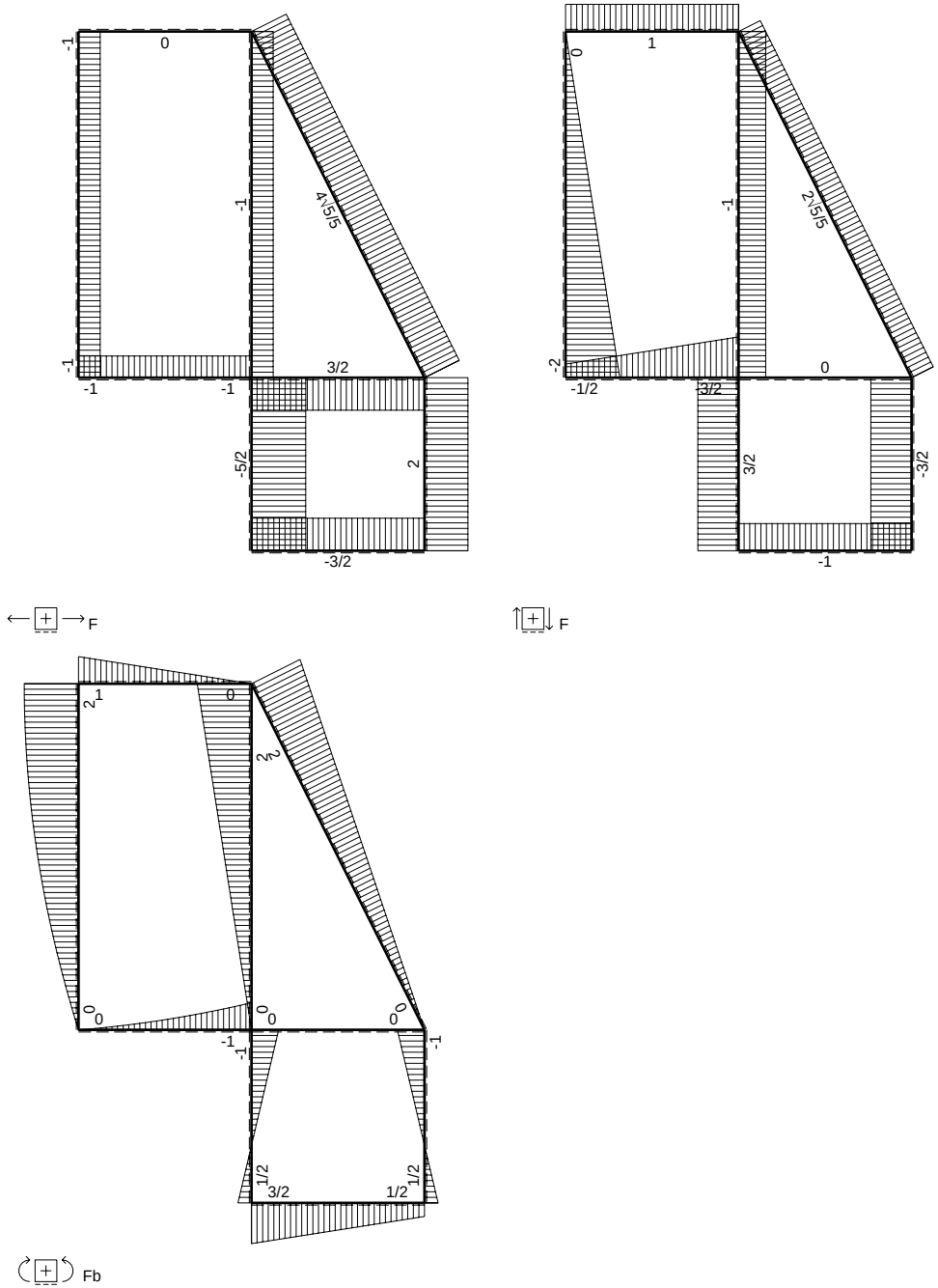
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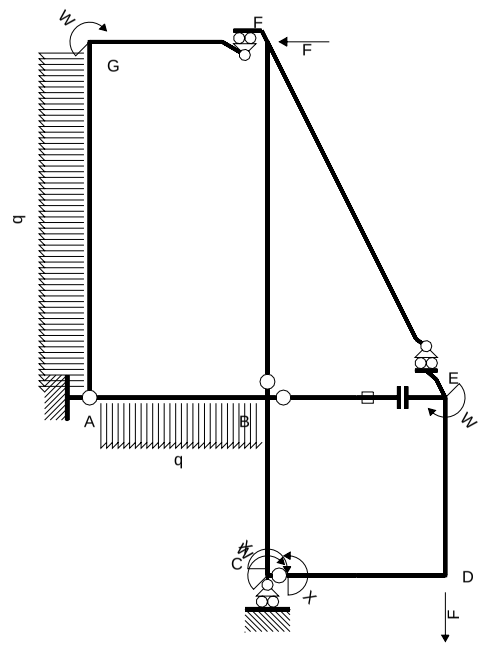
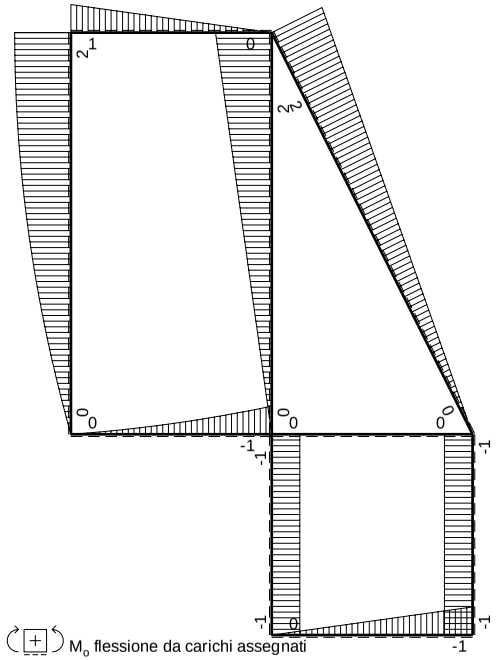
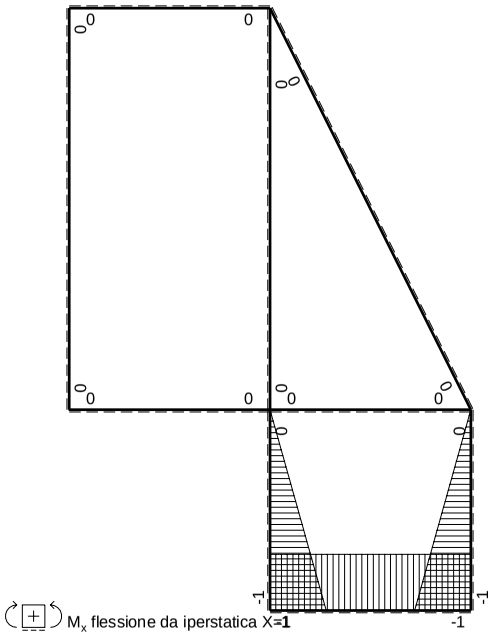


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$5/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/2Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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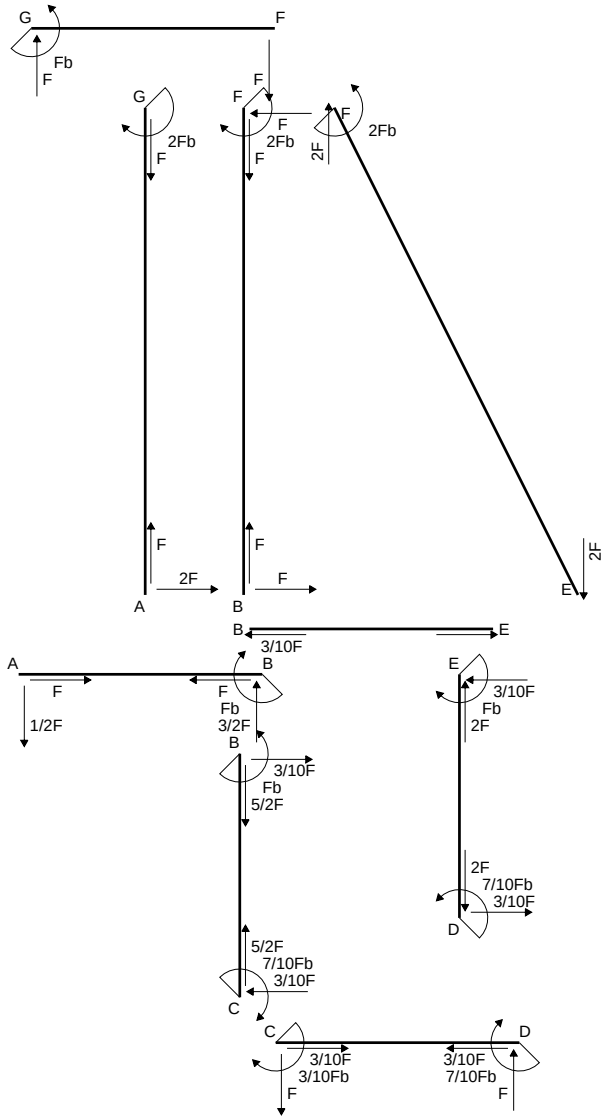

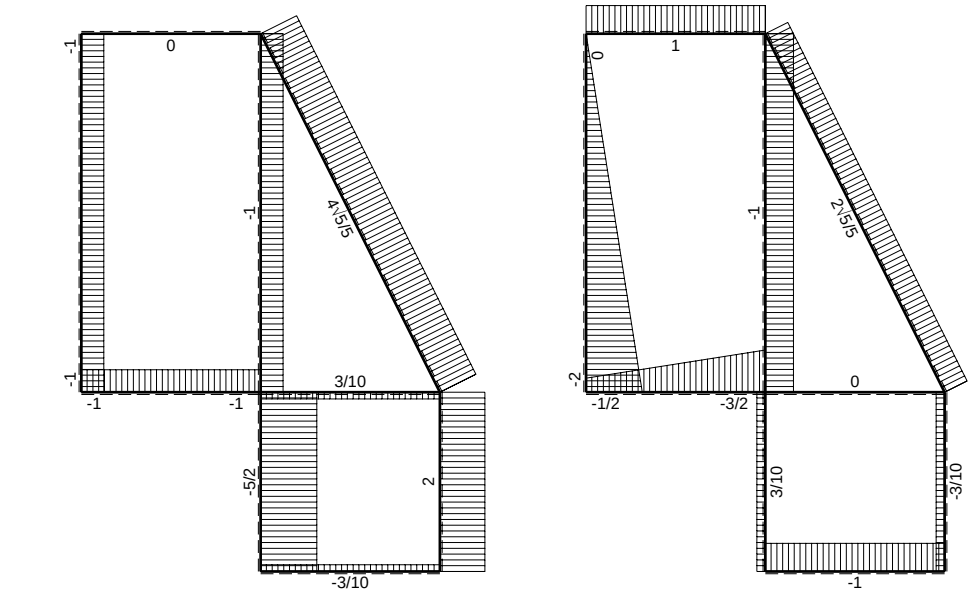

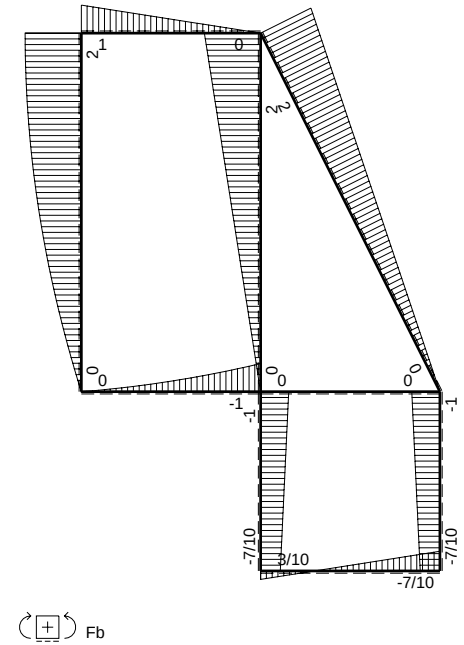

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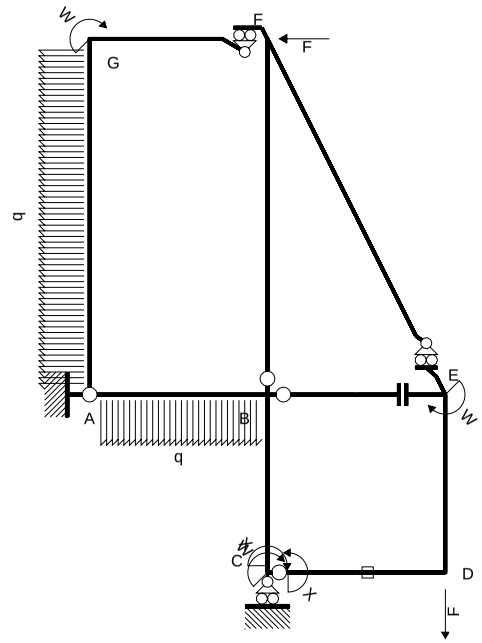
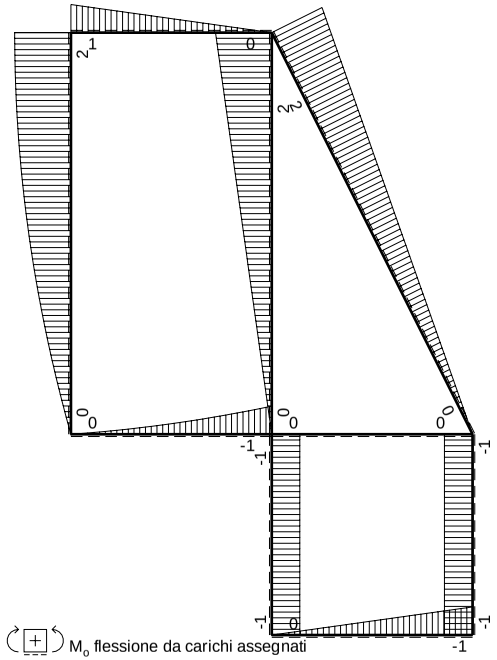
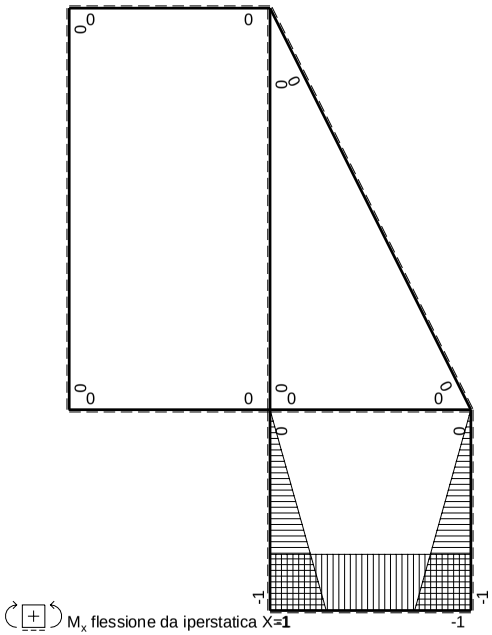


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-3/10Fb$	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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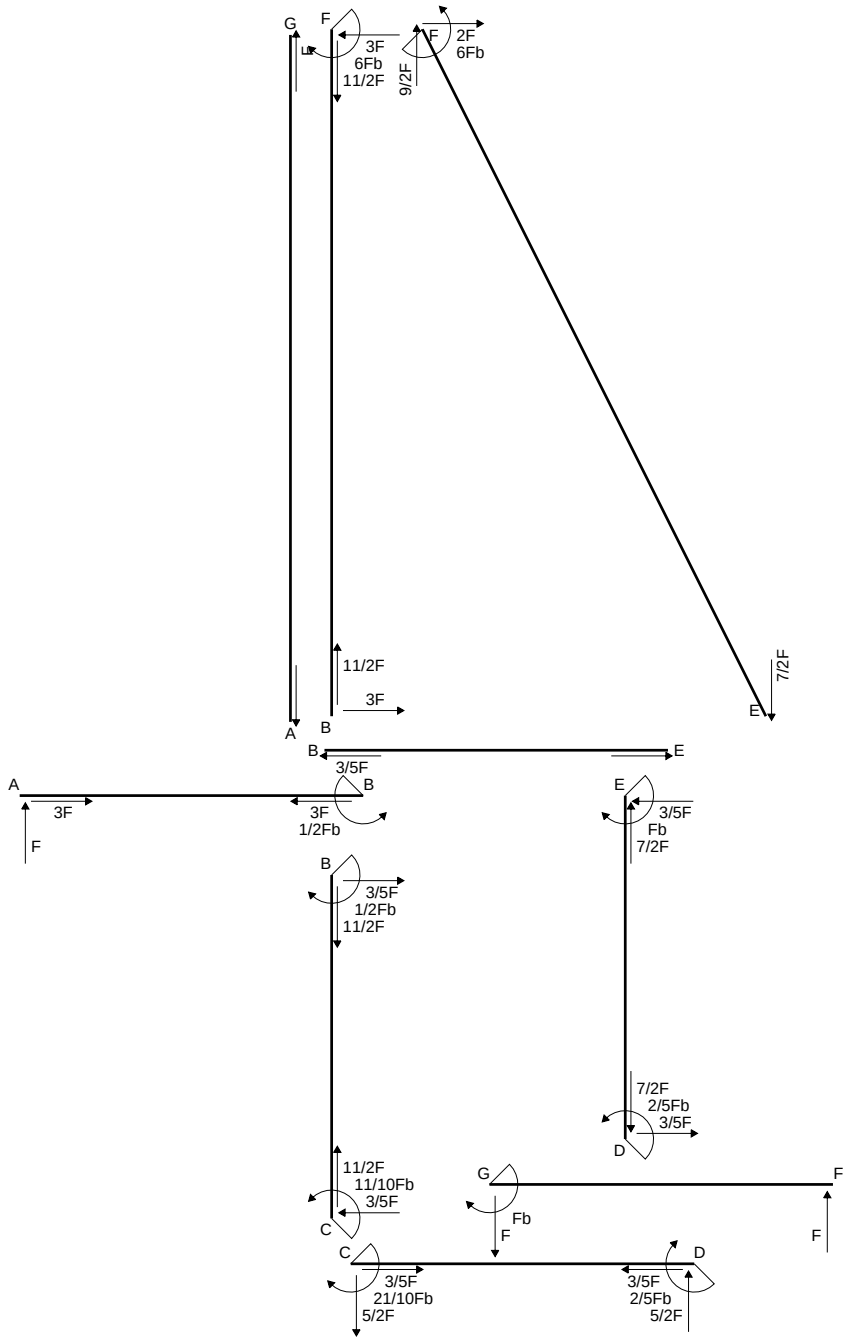
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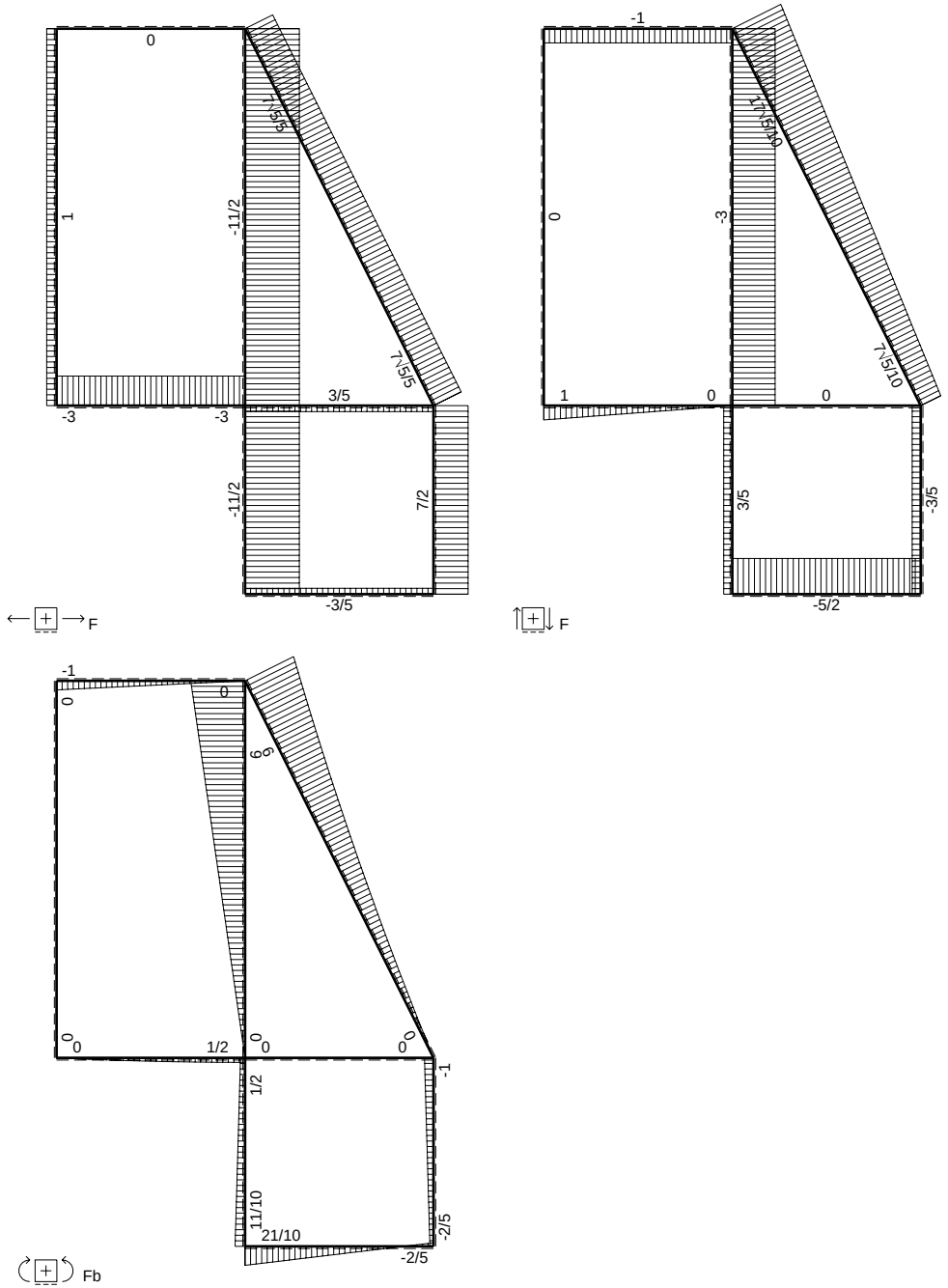
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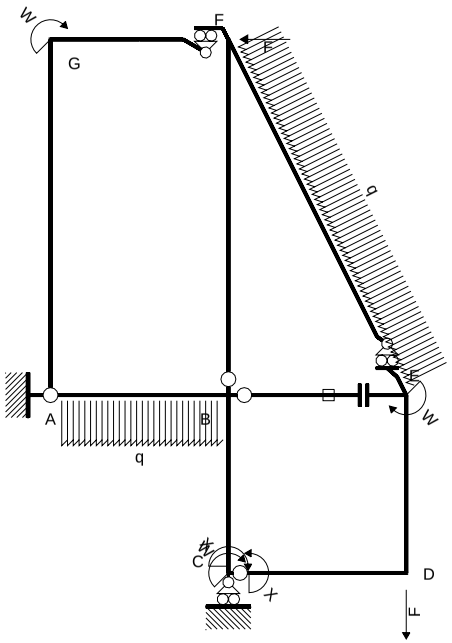
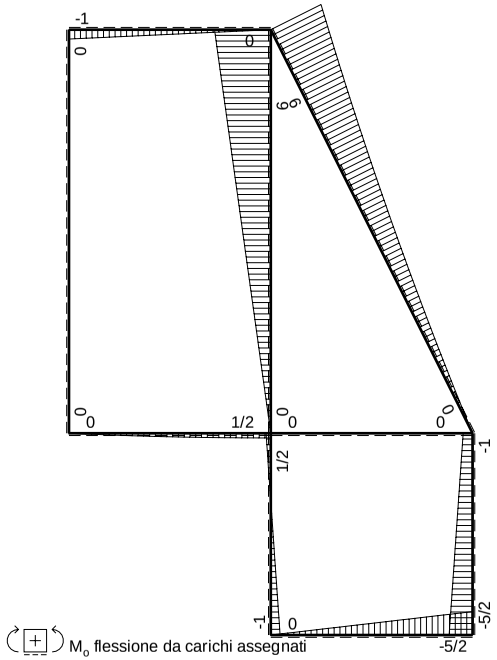
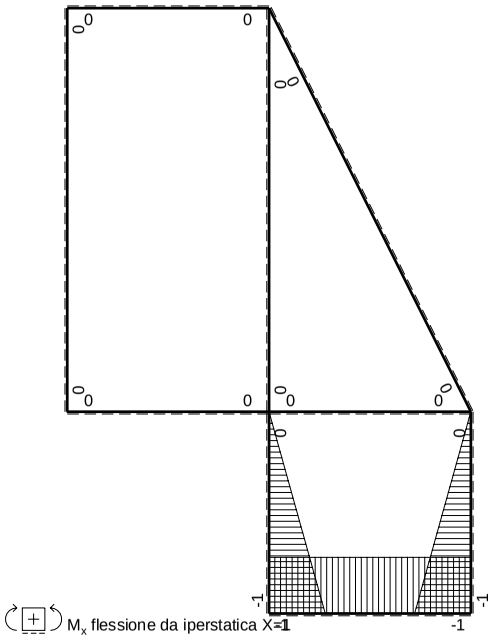


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fb+1/2qx^2$	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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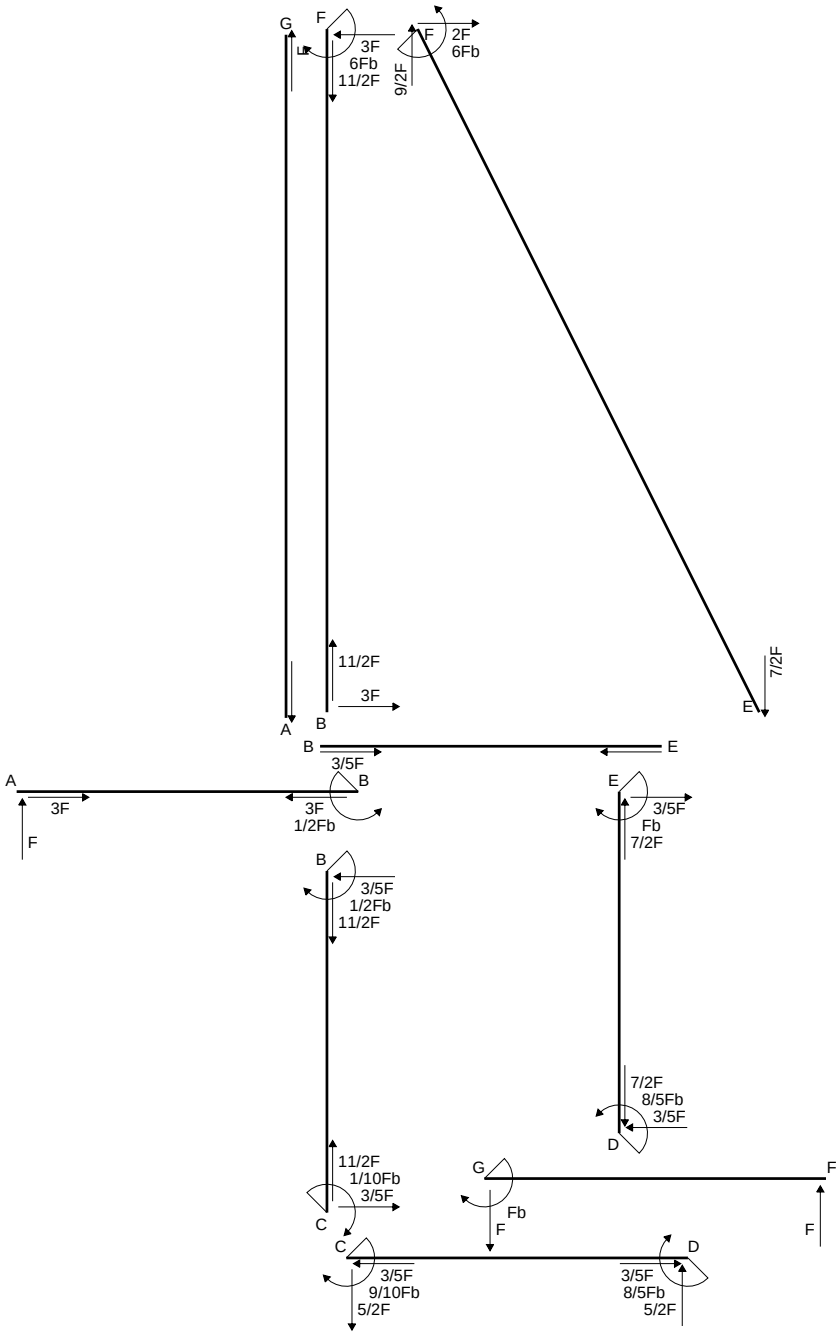
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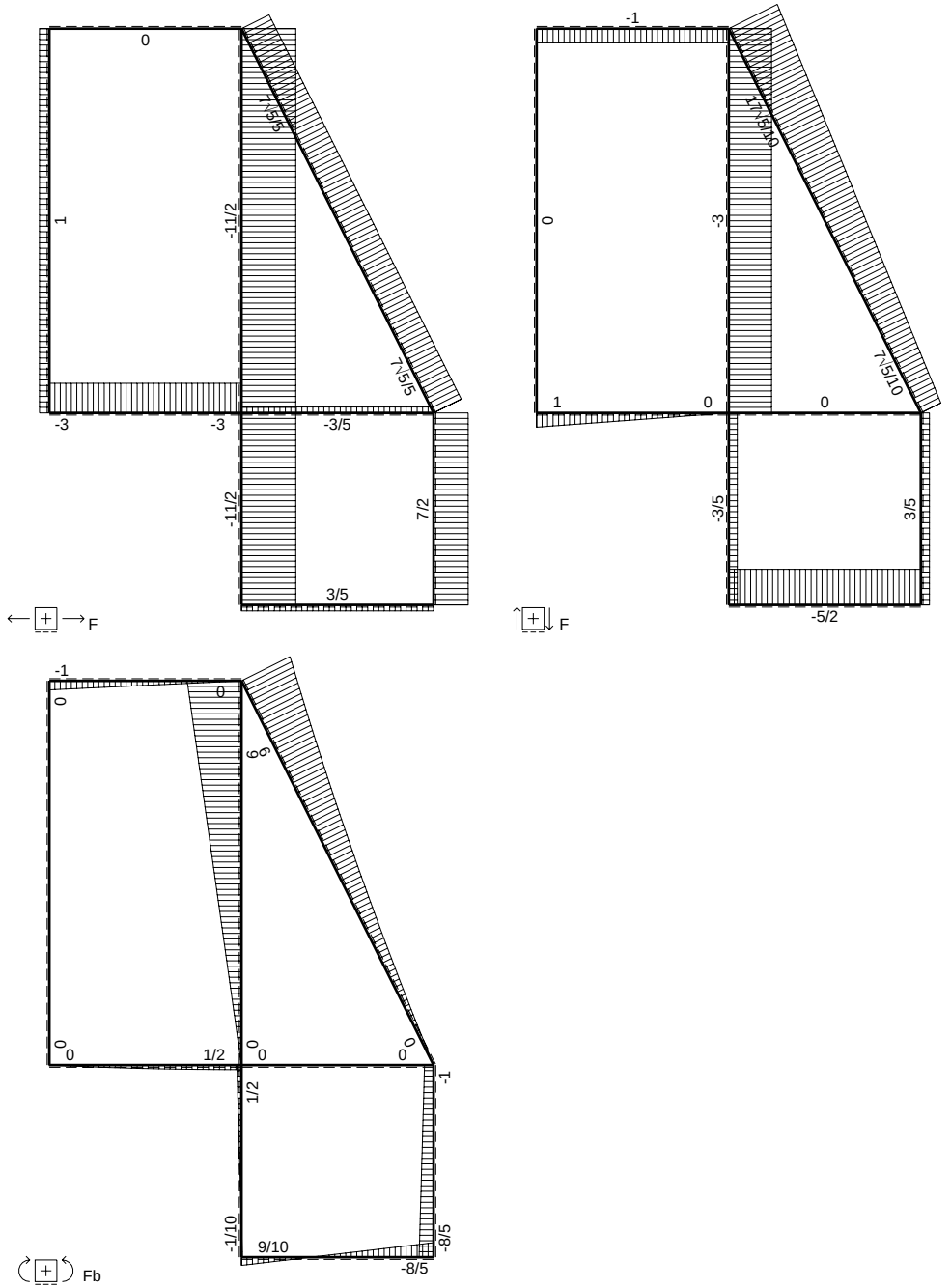
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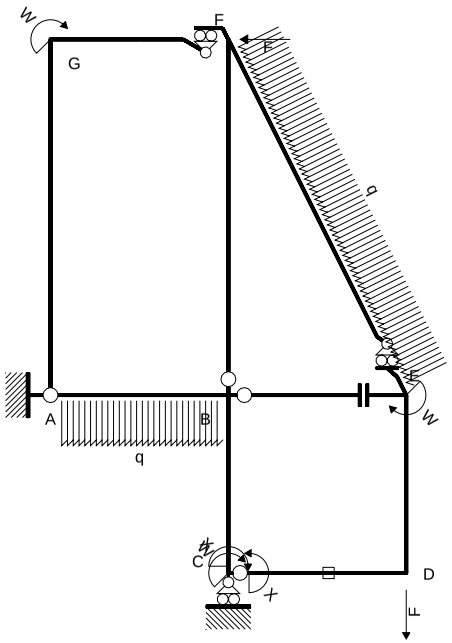
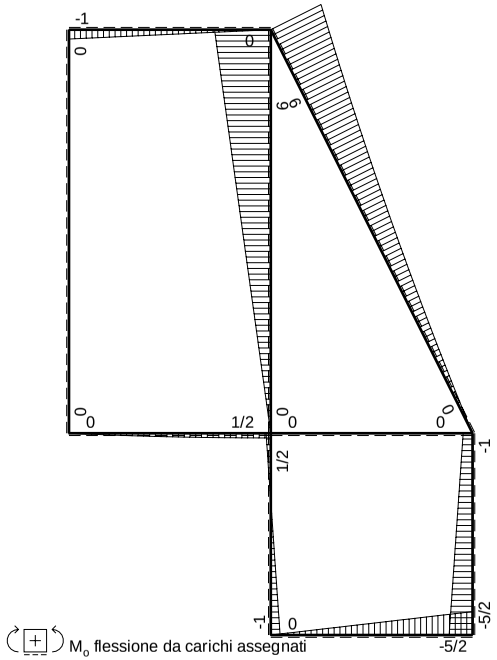
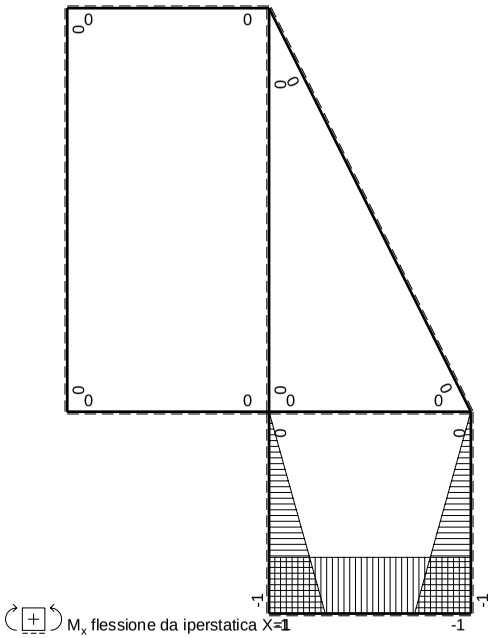


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fb+1/2qx^2$	0	0		
BC b	-x/b	$1/2Fb-3/2Fx$	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	$Fb-3/2Fx$	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-5/2Fx$	$5/2Fx$	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-5/2Fx$	$5/2Fb-5/2Fx$	1		
DE b	-1+x/b	$-5/2Fb+3/2Fx$	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	$Fb+3/2Fx$	$Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-9/10Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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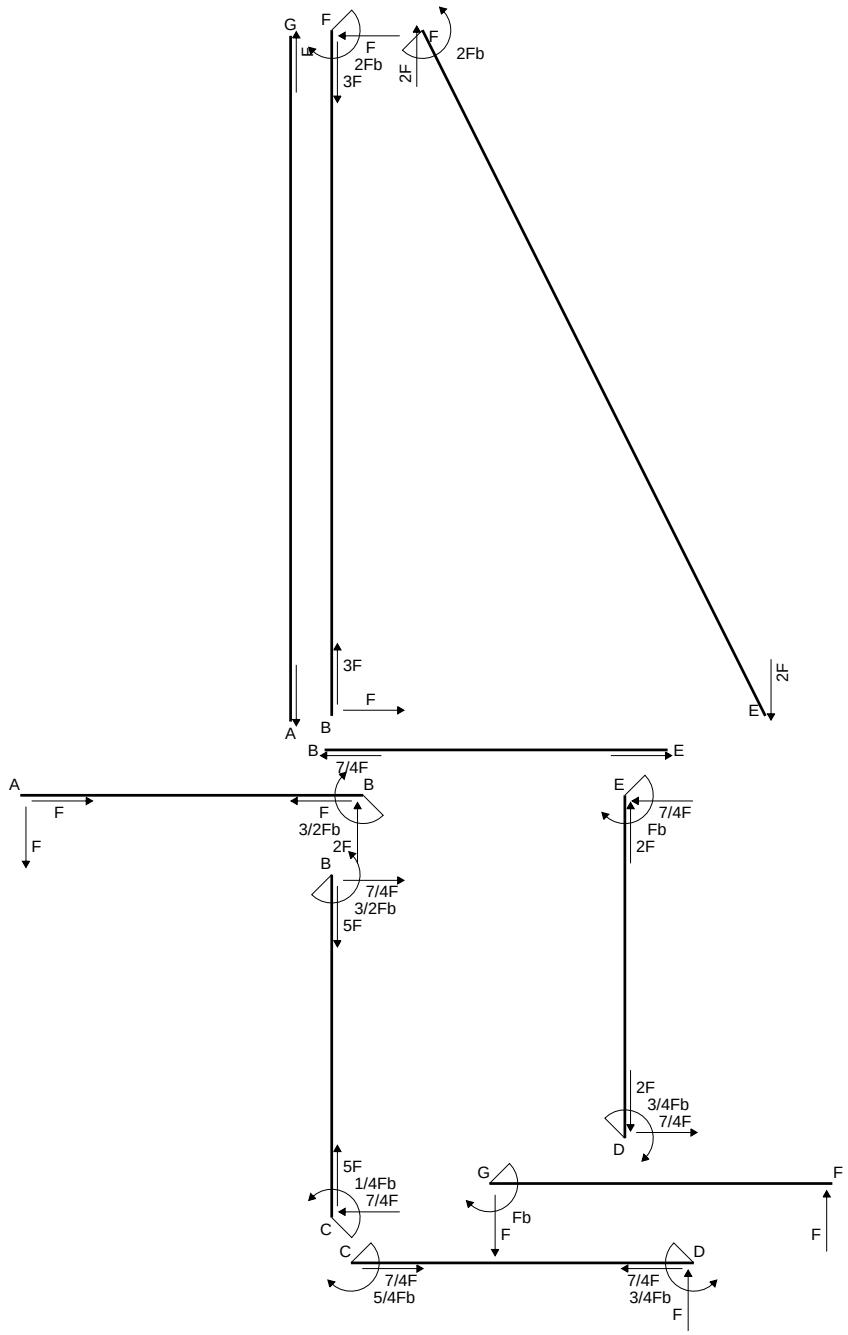
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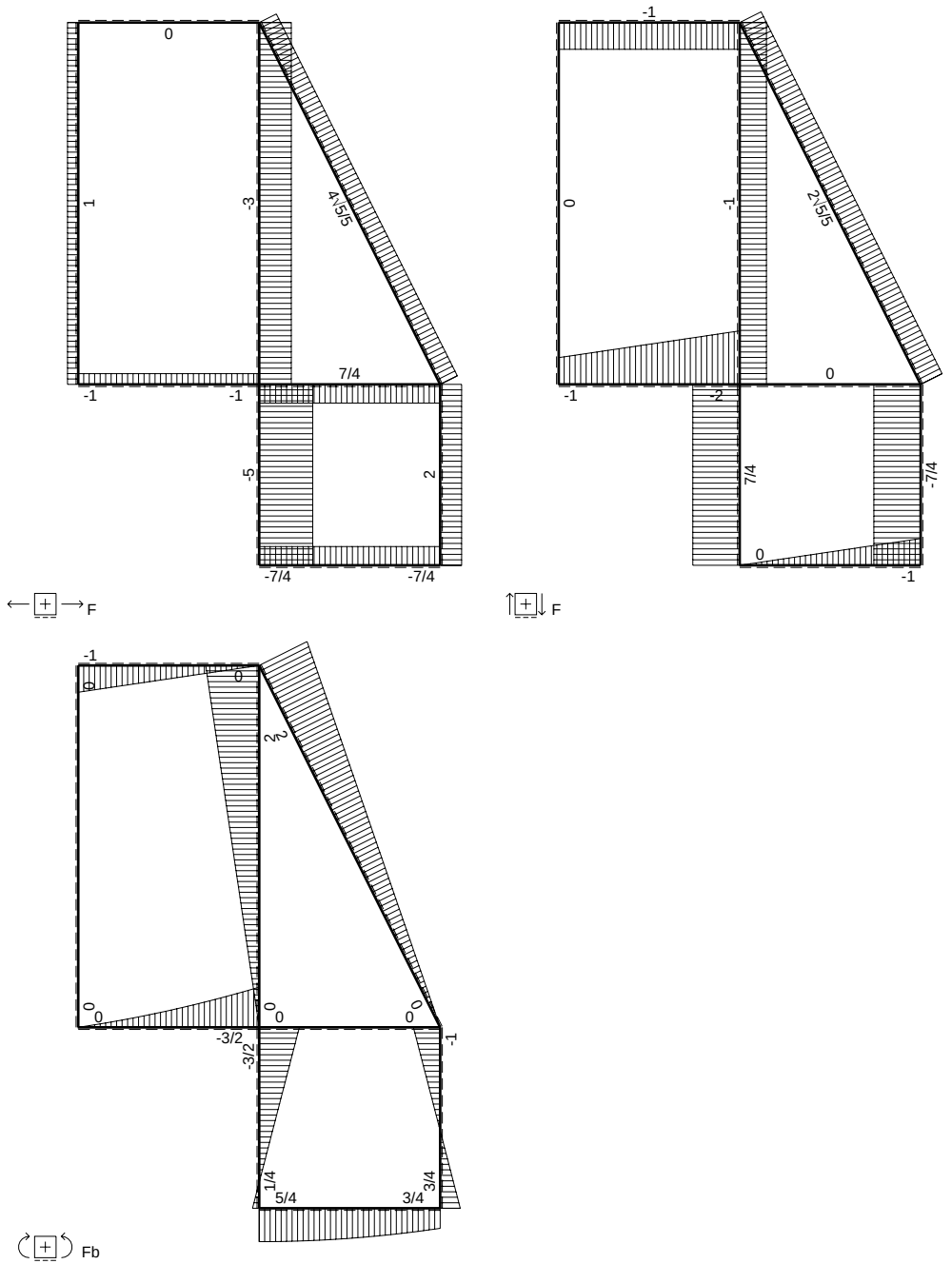
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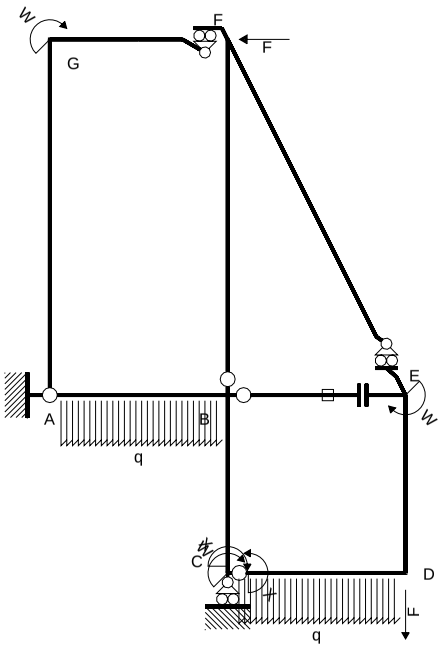
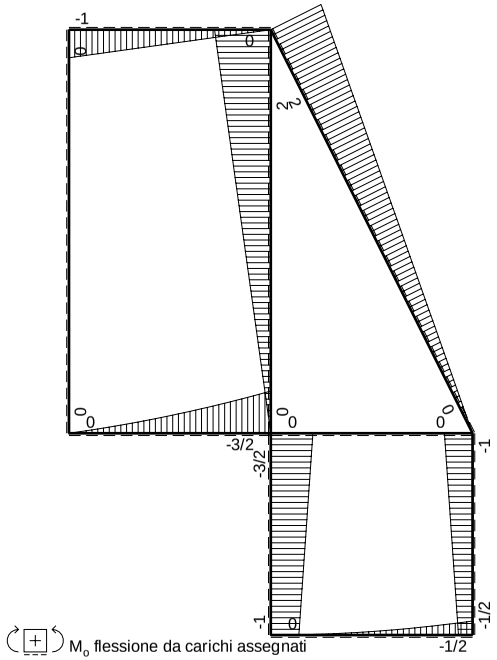
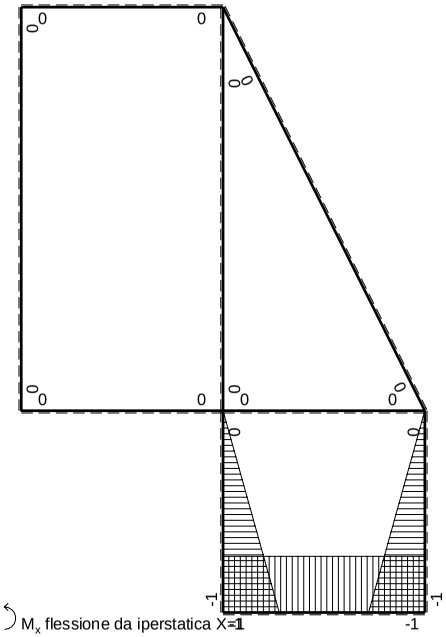


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb-1/2Fx$	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2Fx$	$Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$25/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-5/4Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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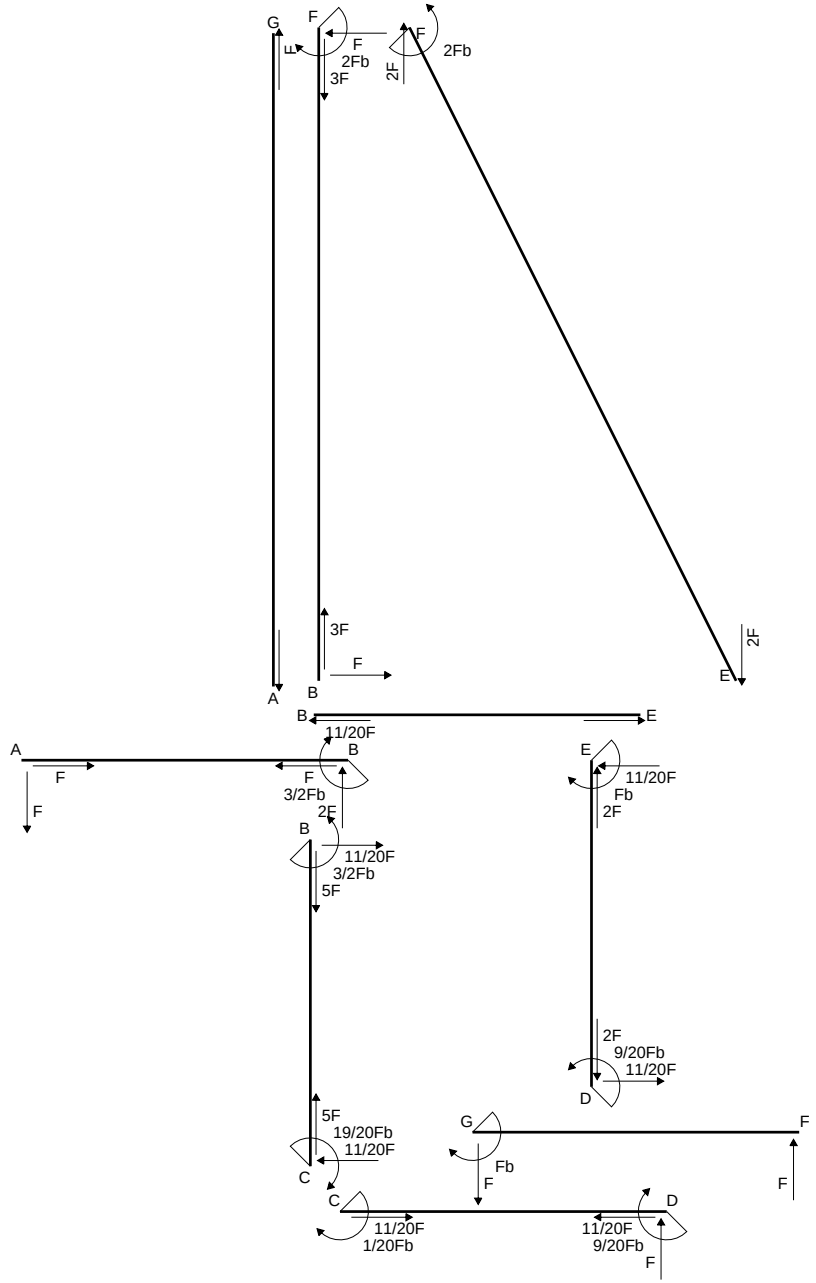
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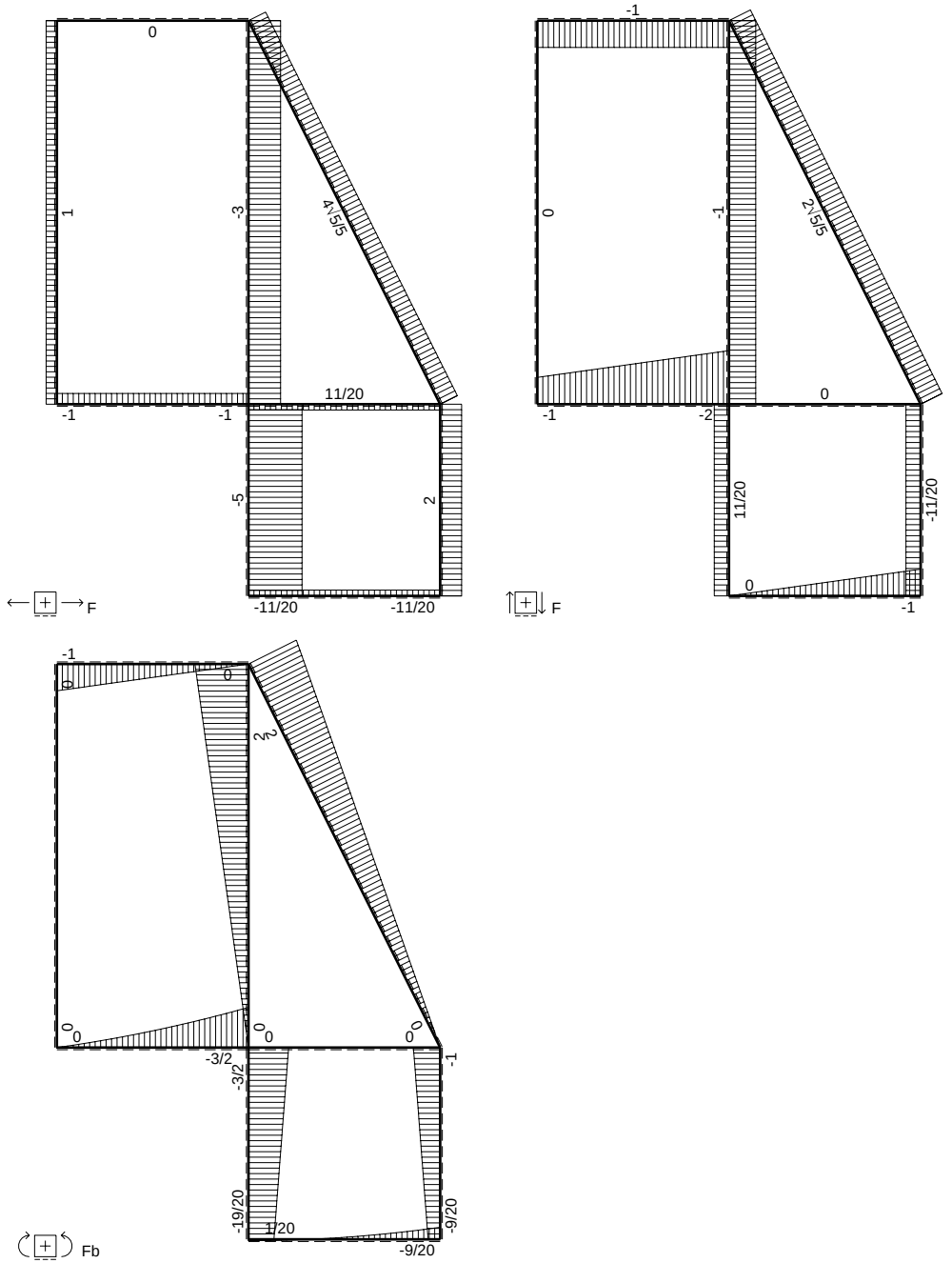
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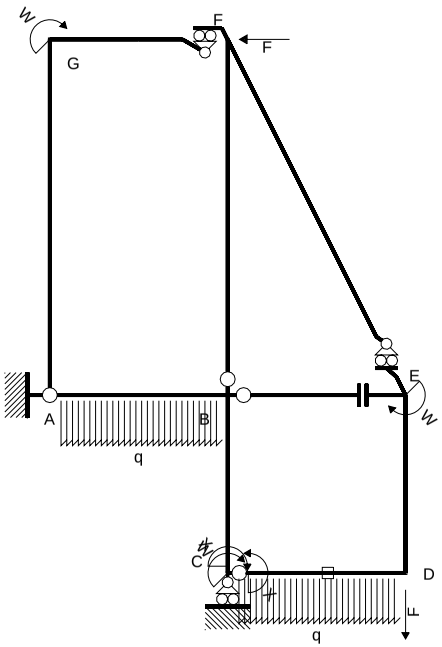
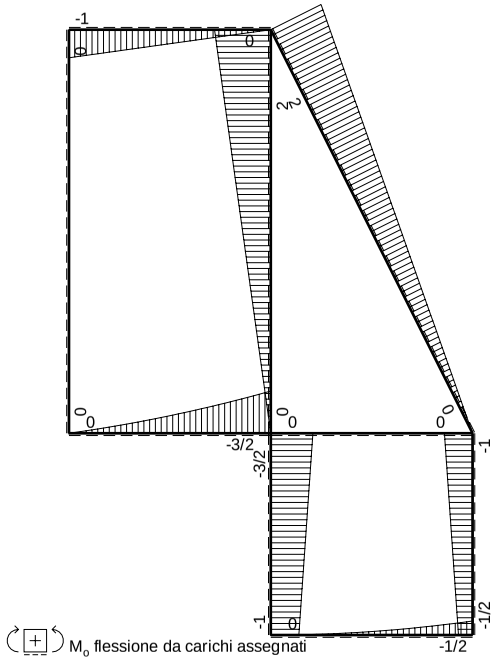
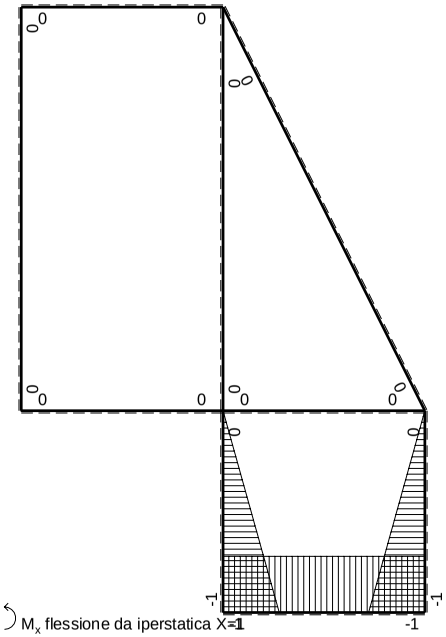


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb-1/2Fx$	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2Fx$	$Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-Fx$	0	0	0	0
GF b	0	$Fb-Fx$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$2Fb-Fx$	0	0	0	0
BF 2b	0	$-Fx$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-1/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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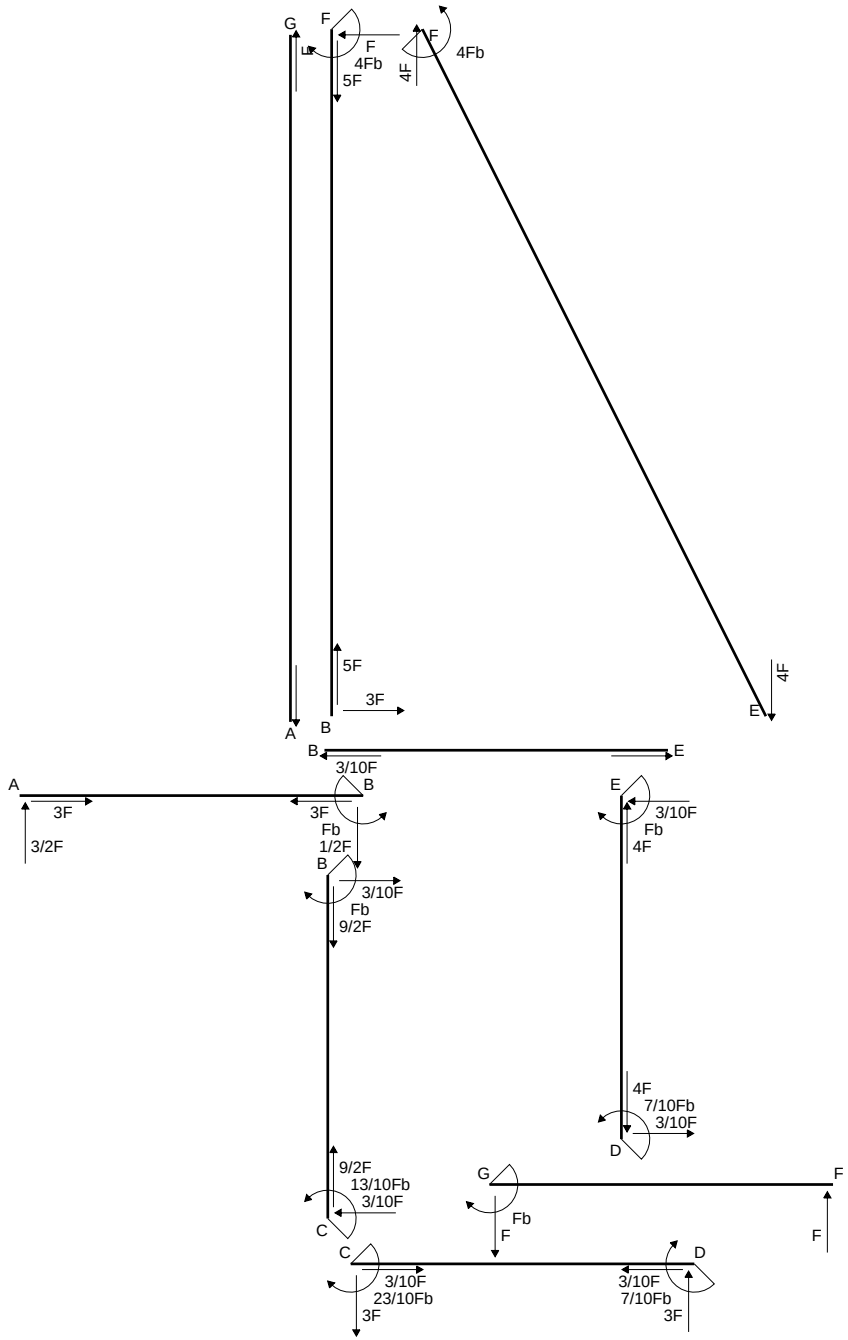
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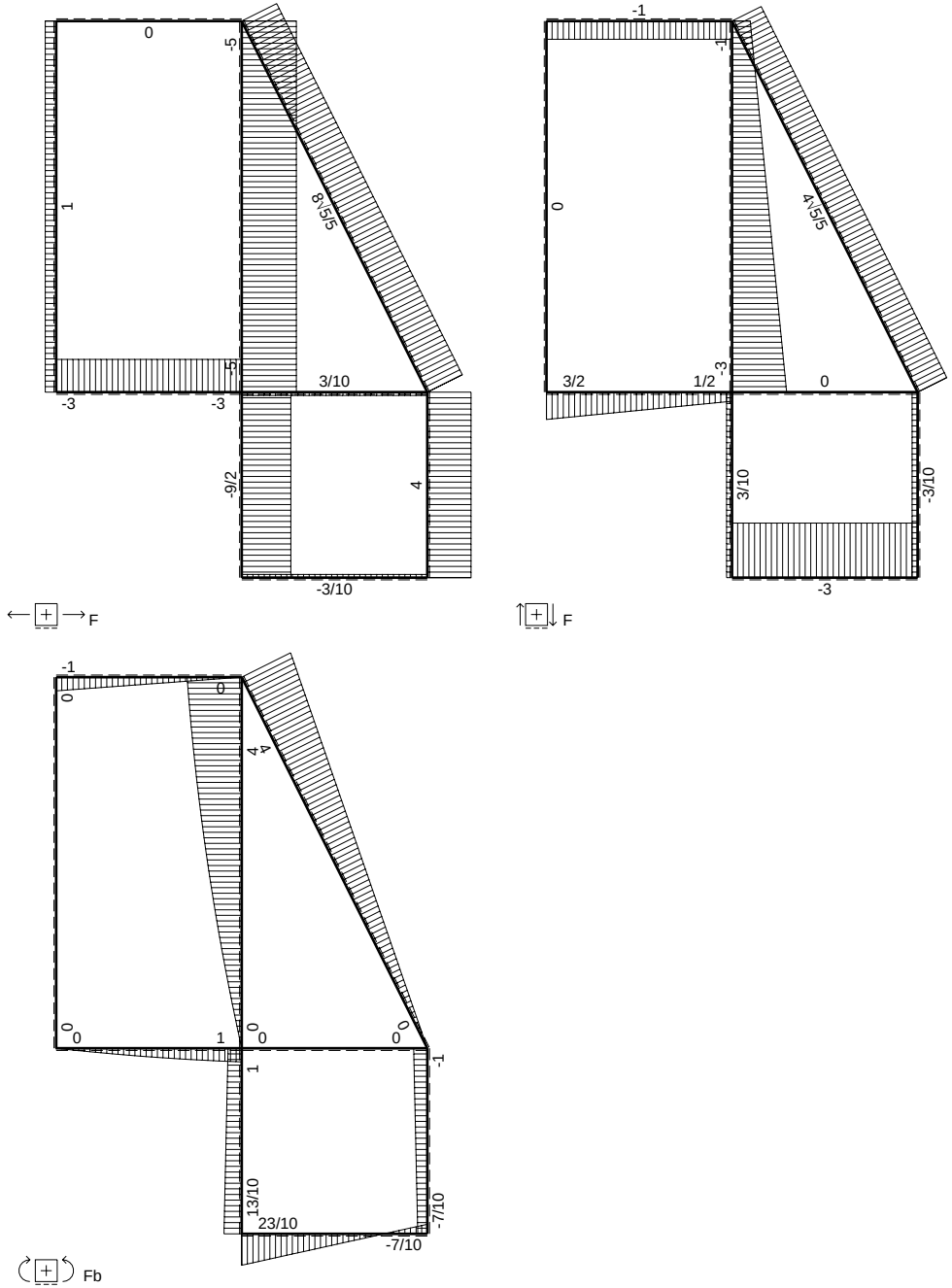
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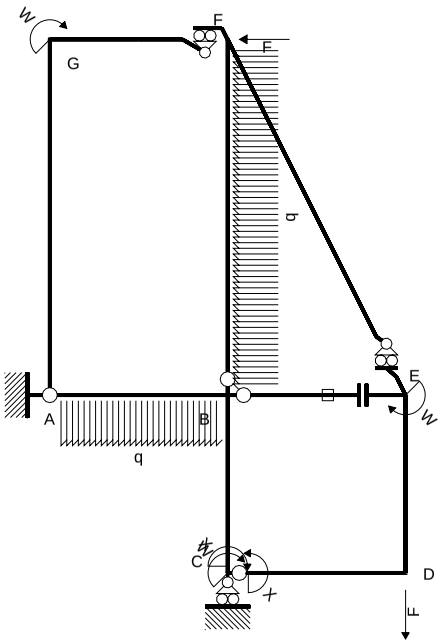
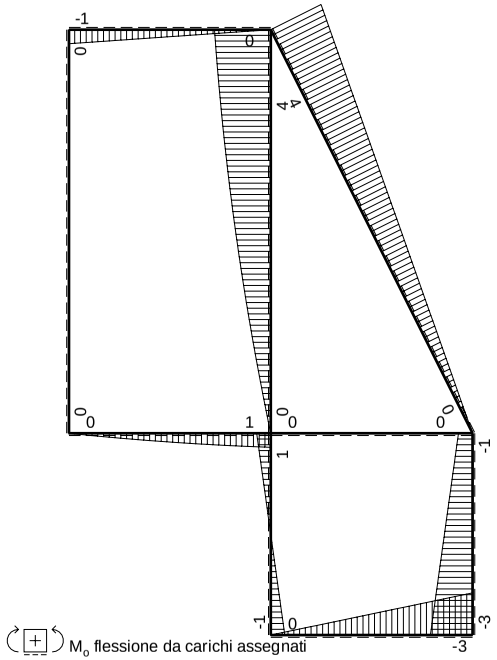
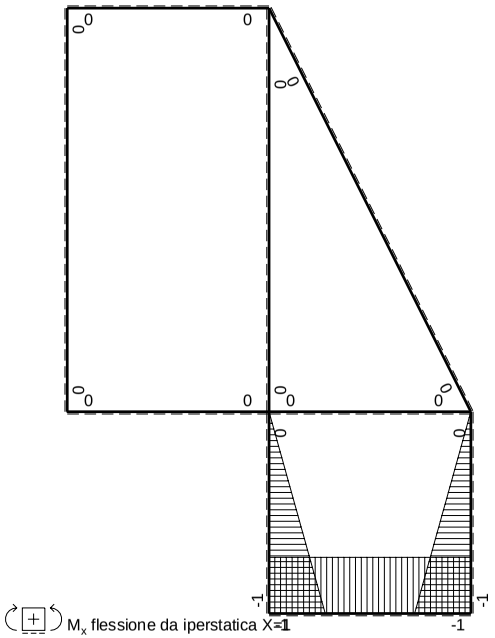
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$3/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-Fb+1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$23/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-23/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (- x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -5\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

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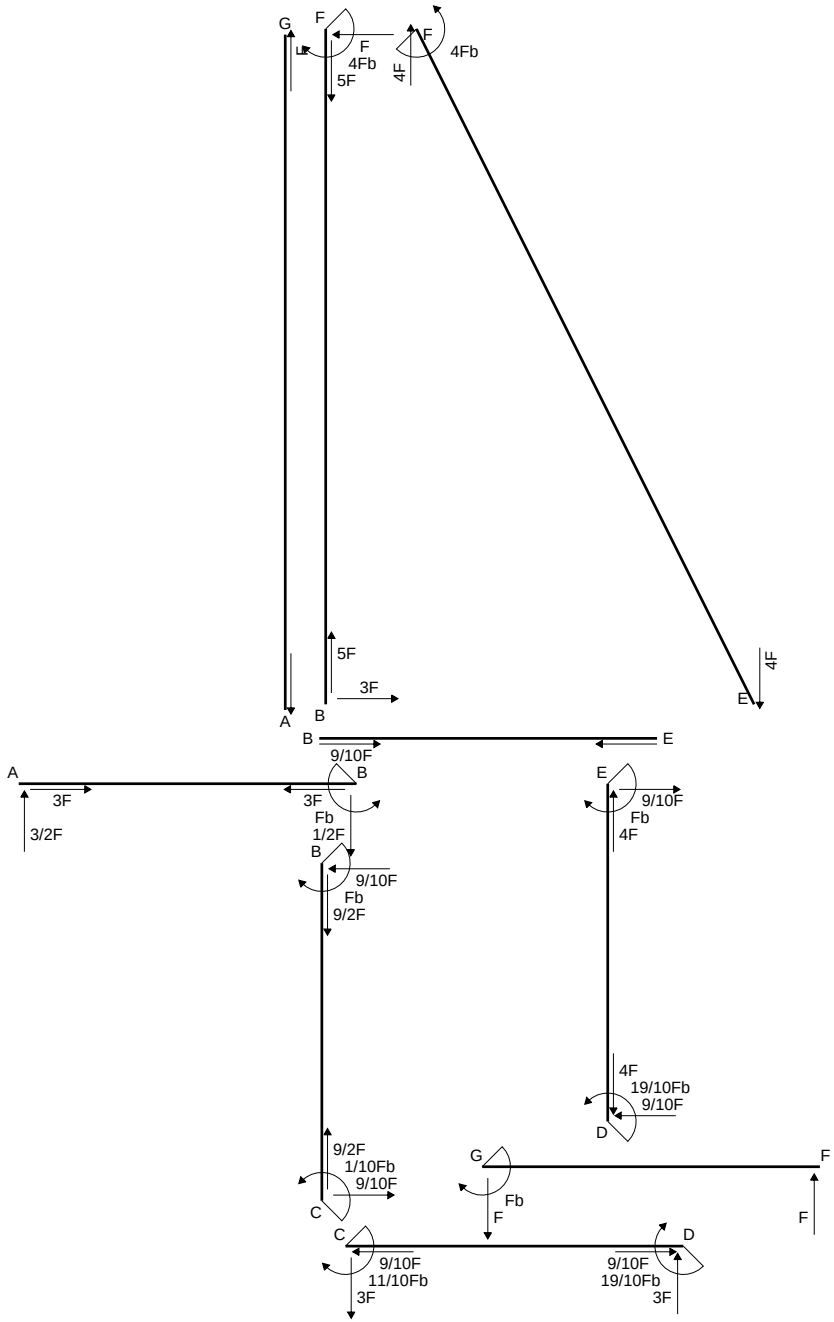
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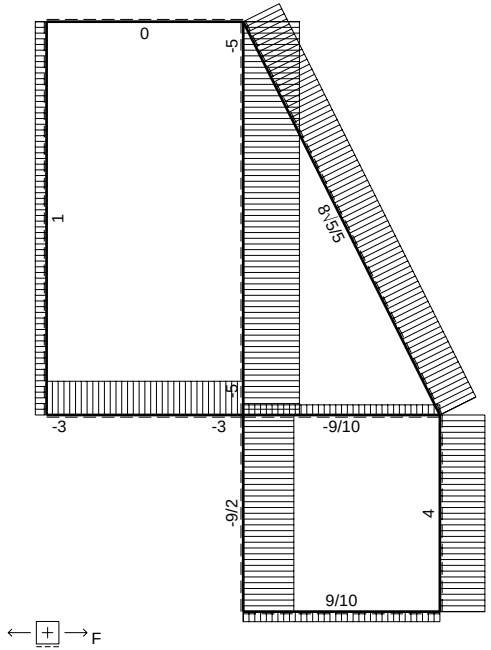
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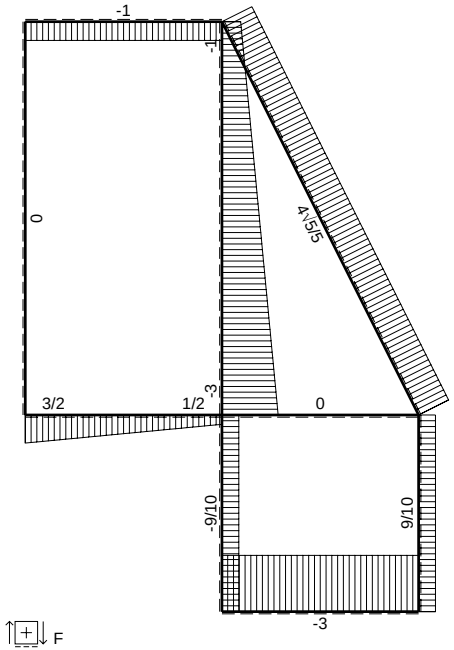
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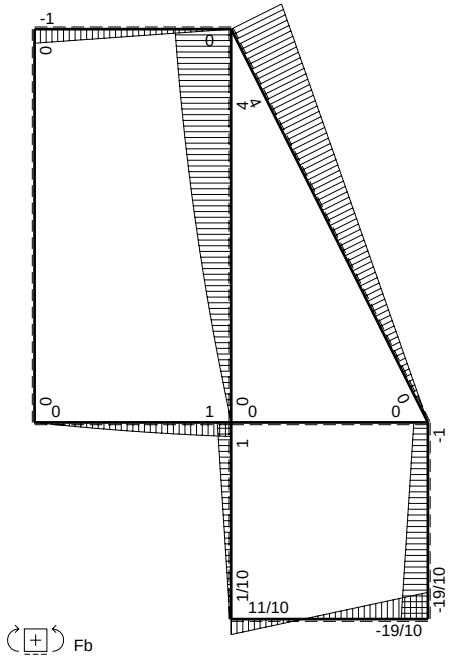
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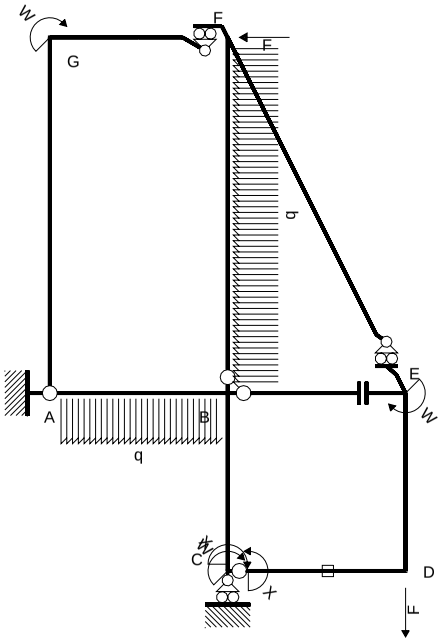
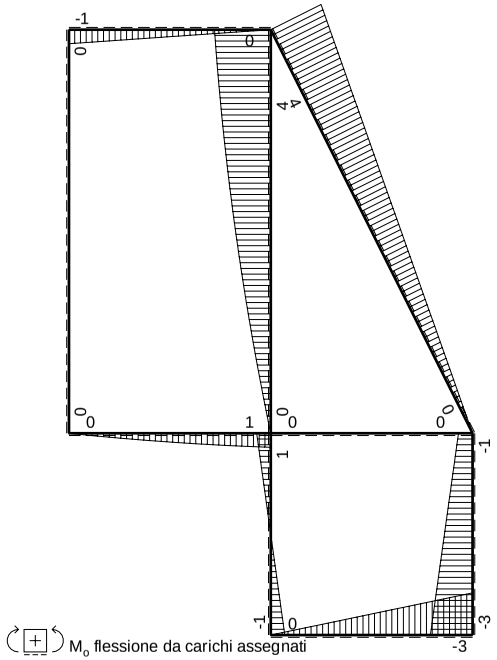
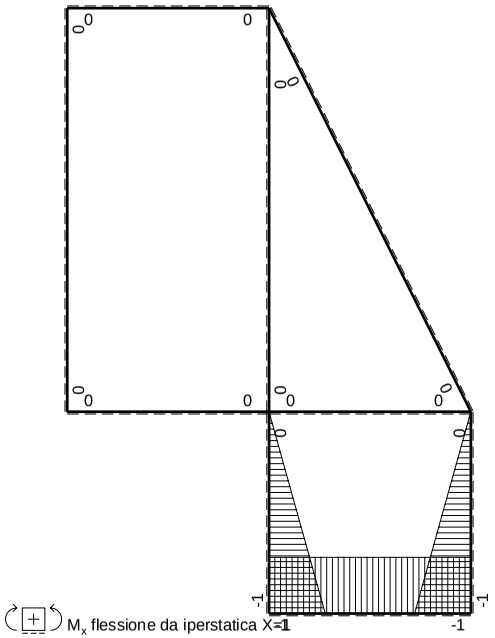


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$3/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-Fb+1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$11/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-11/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

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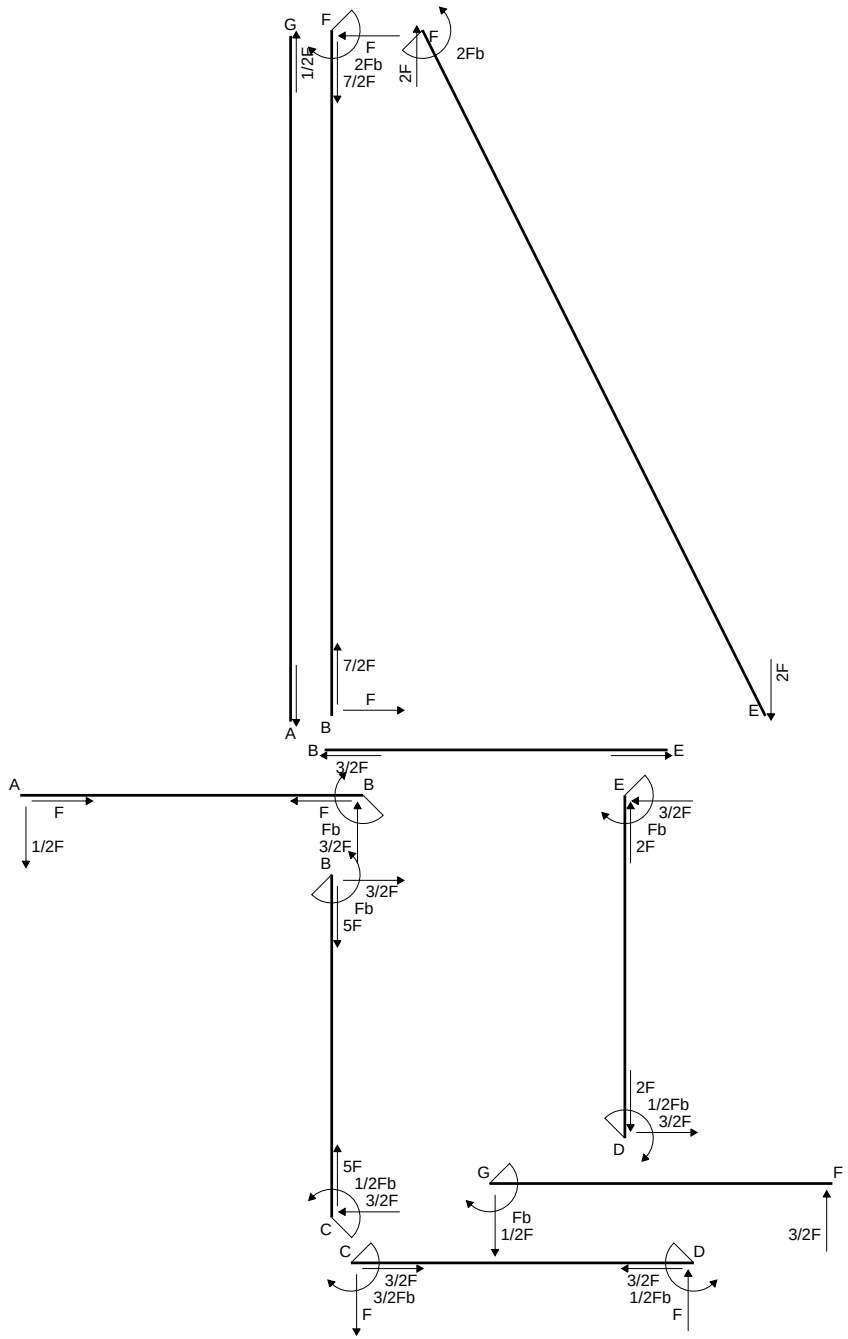
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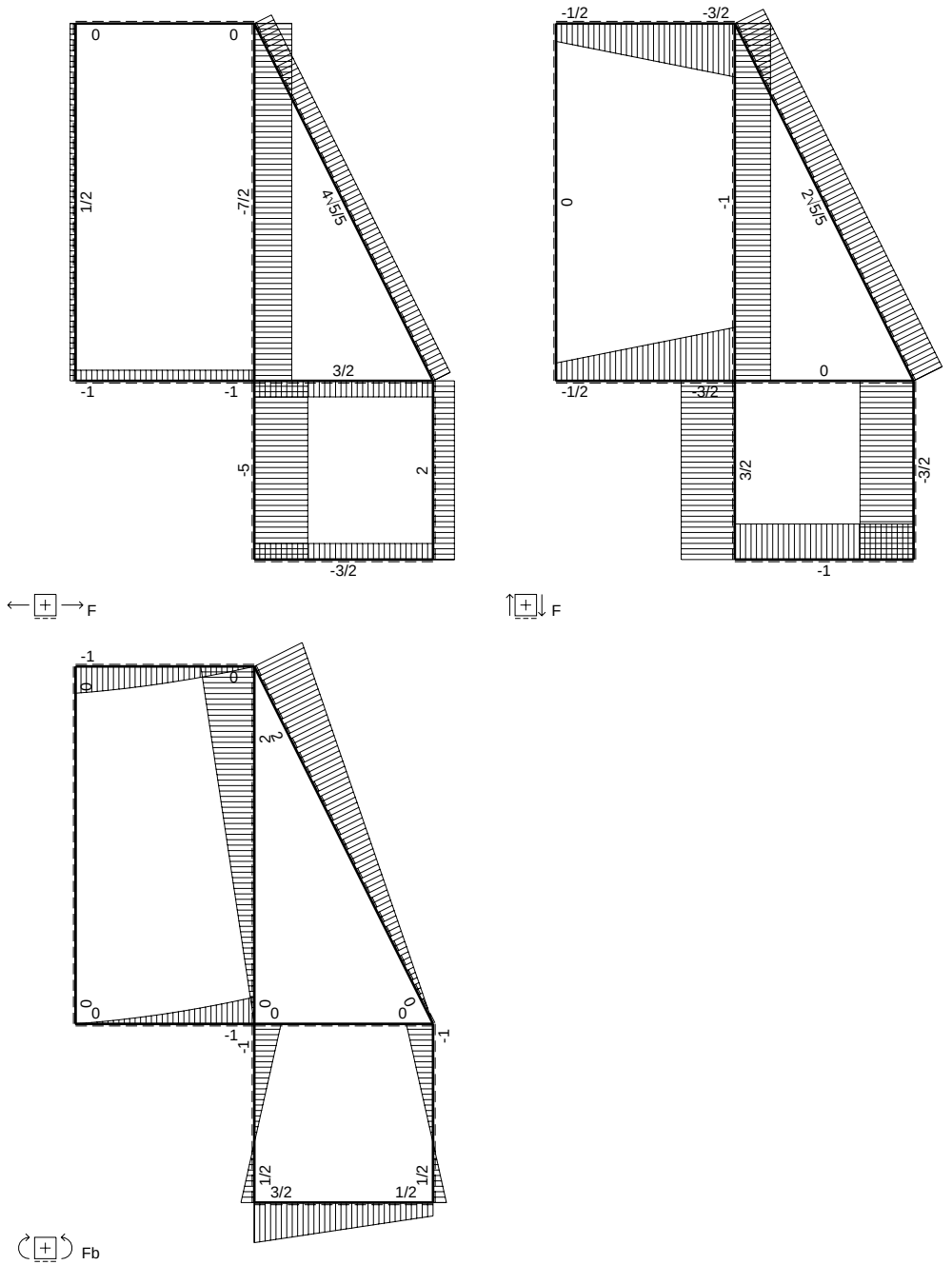
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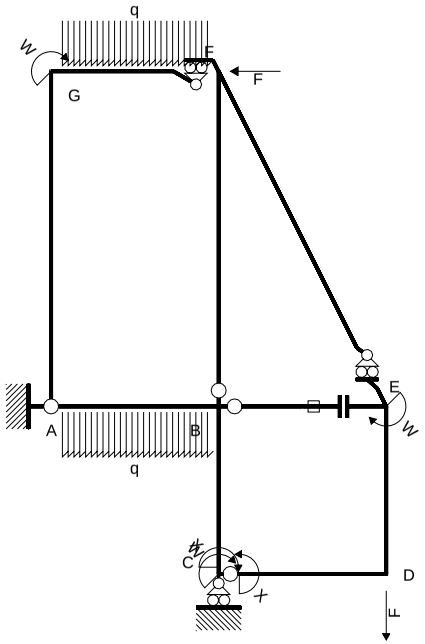
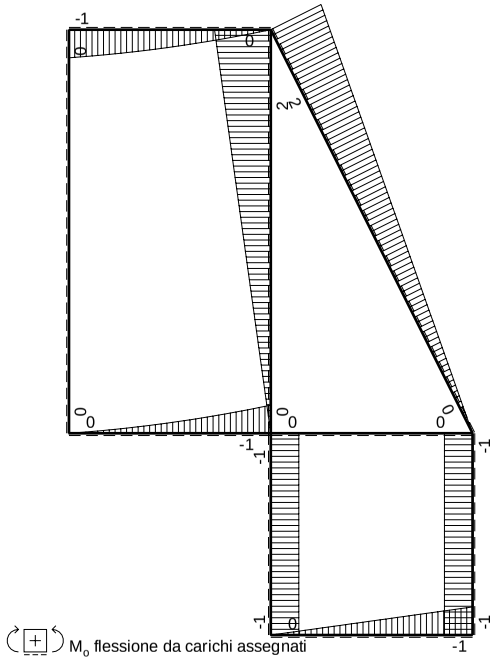
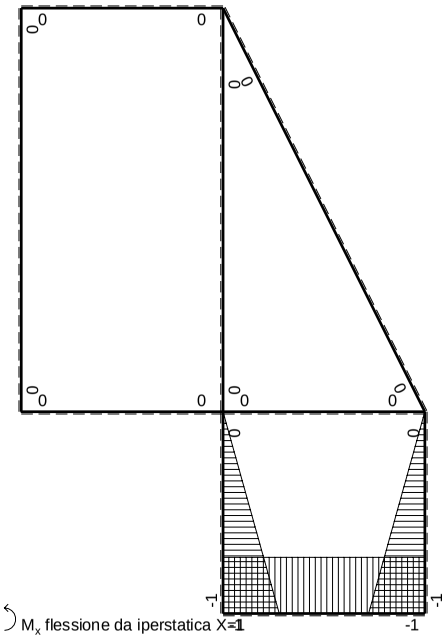


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$5/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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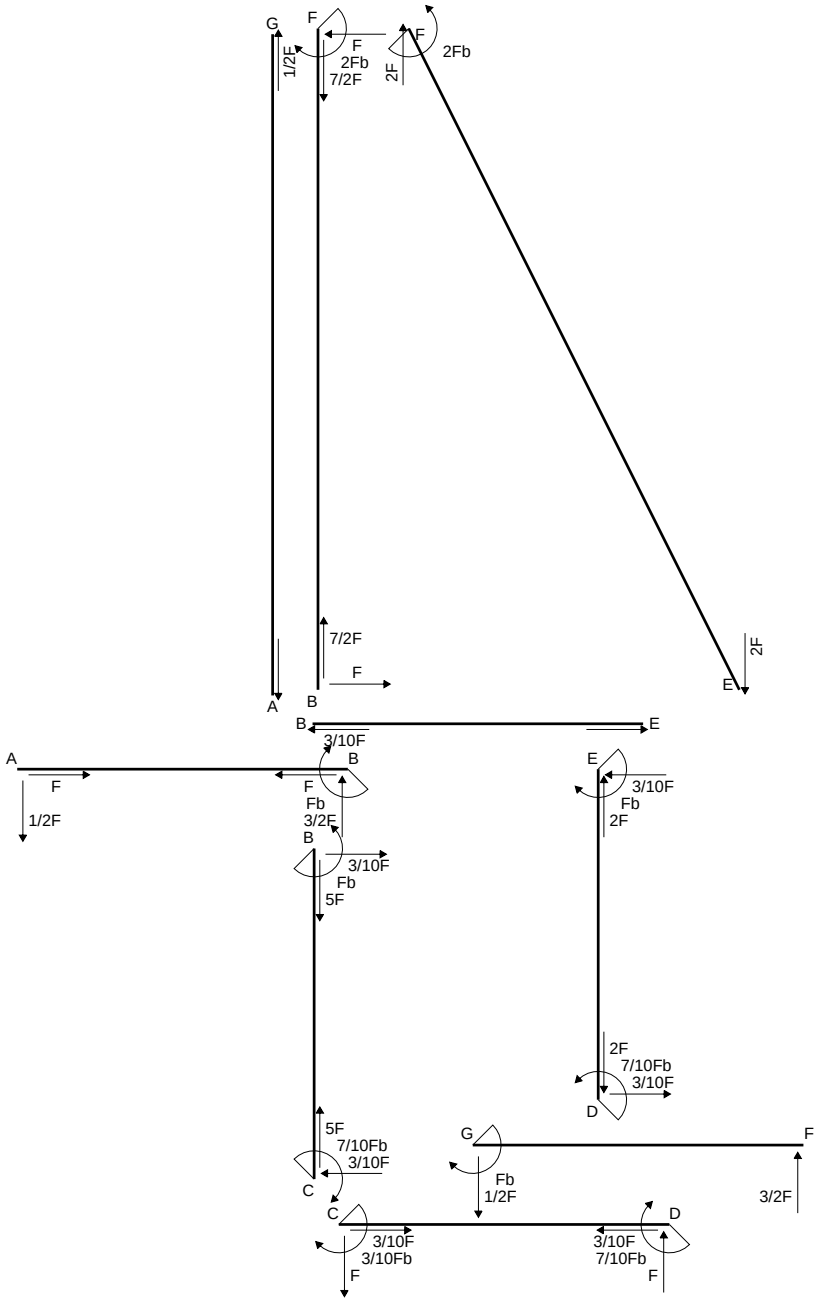
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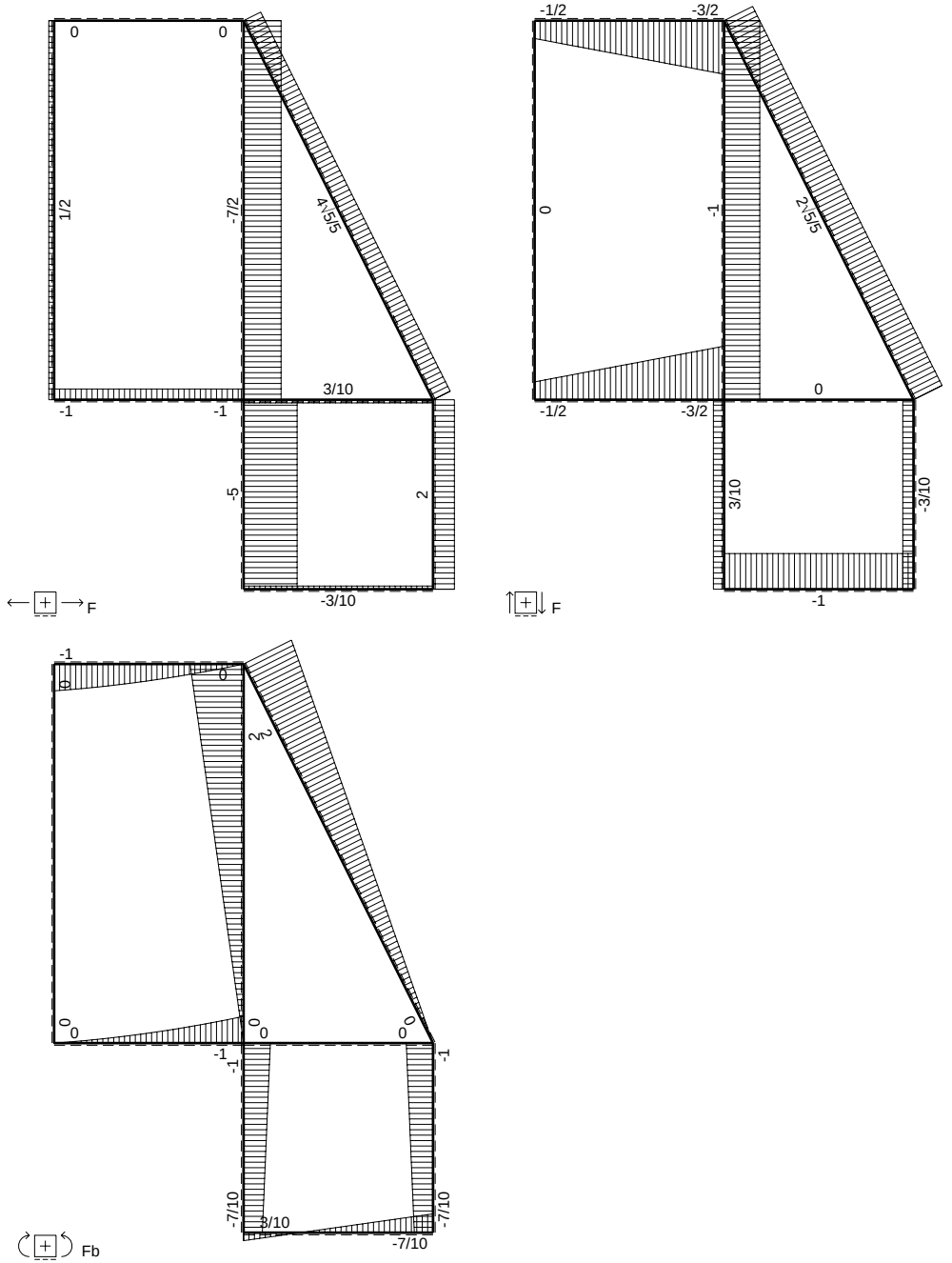
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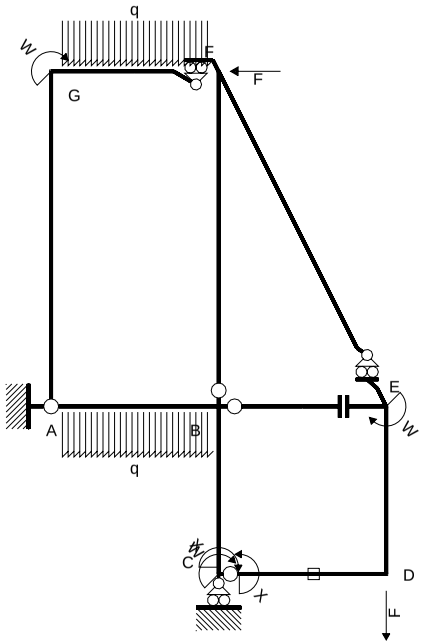
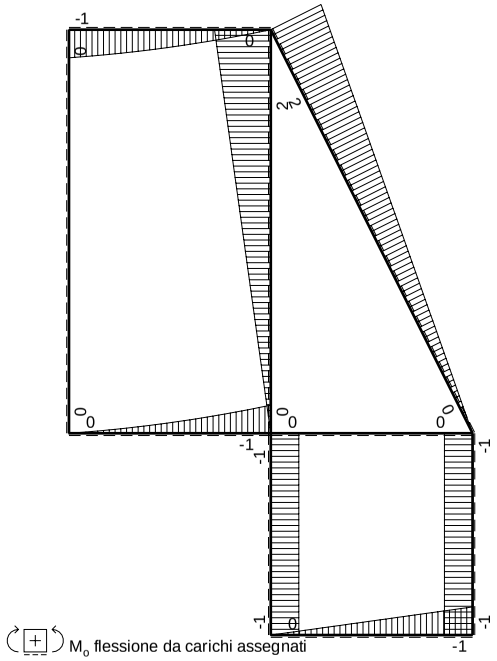
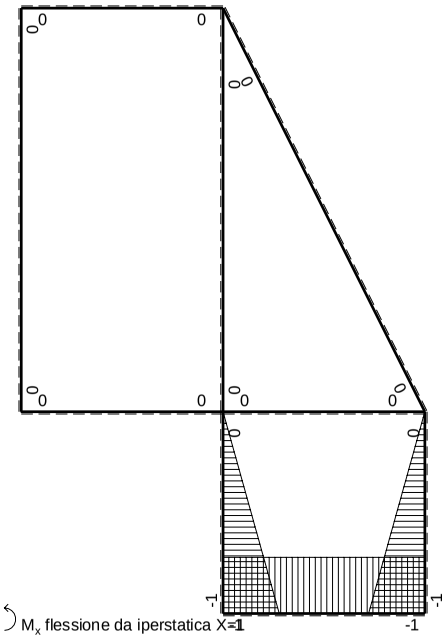


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-3/10Fb$	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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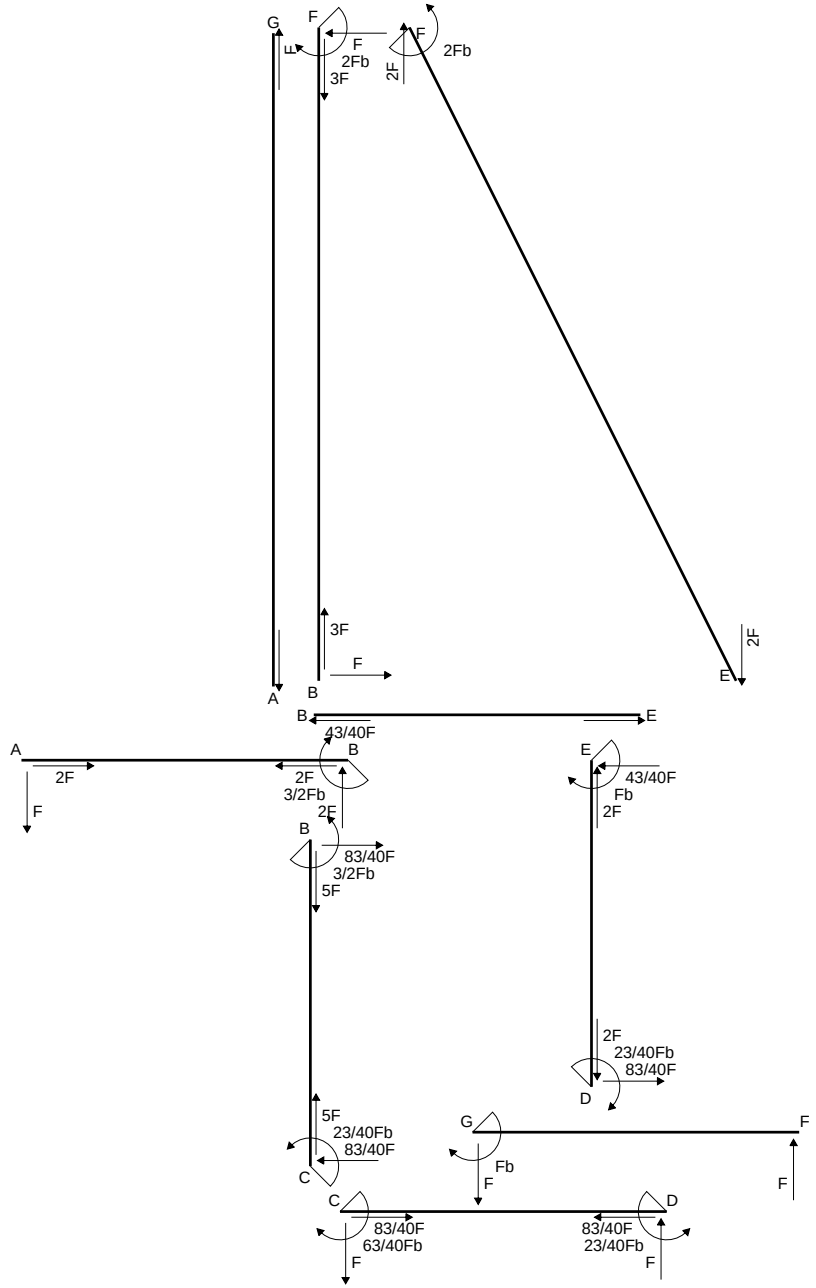
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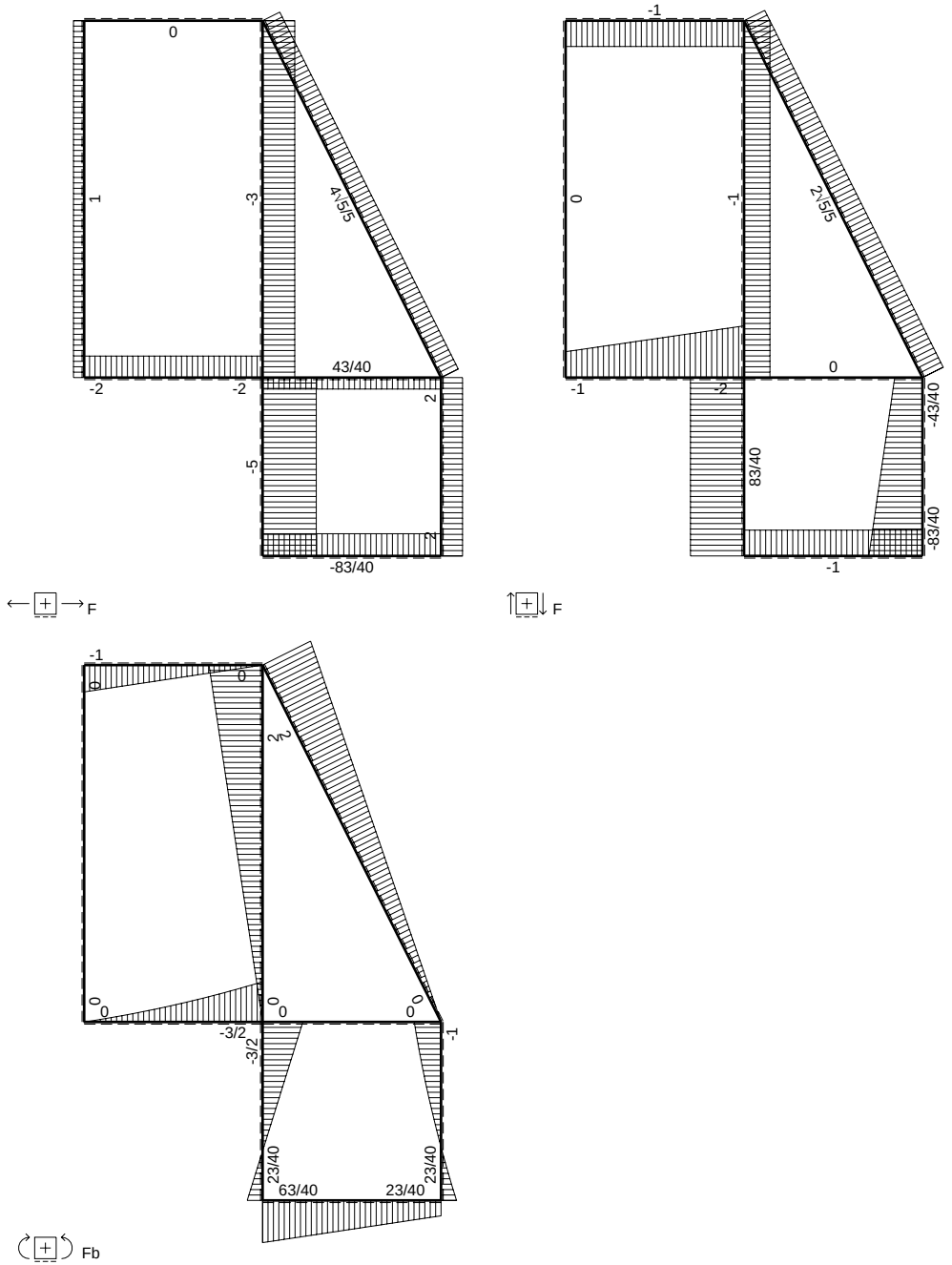
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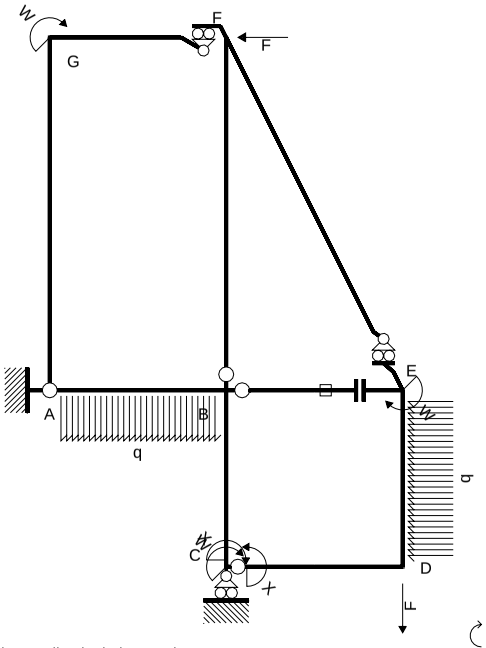
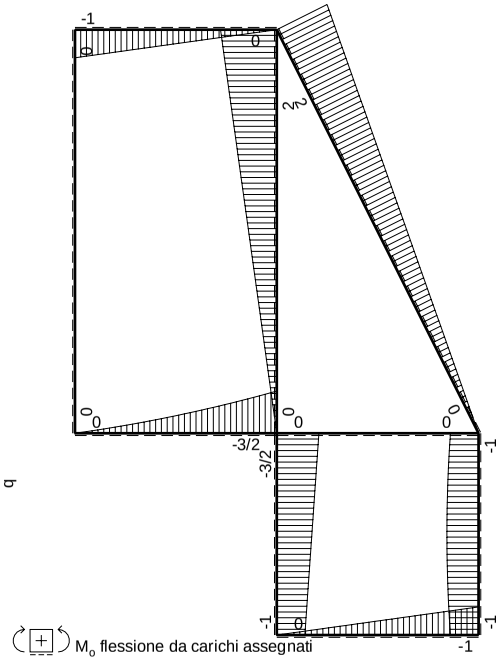
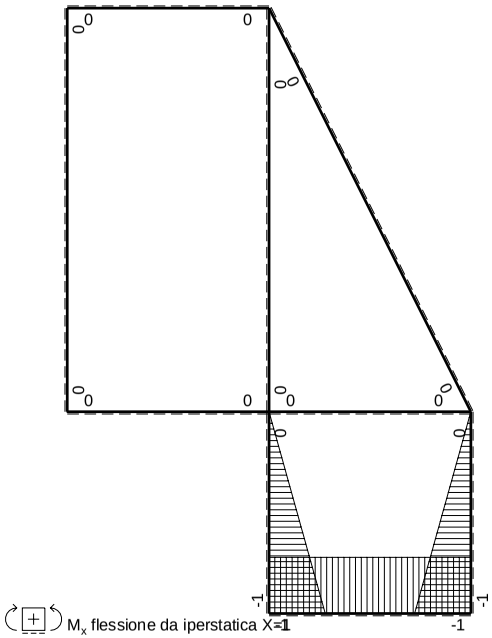


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$21/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-63/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

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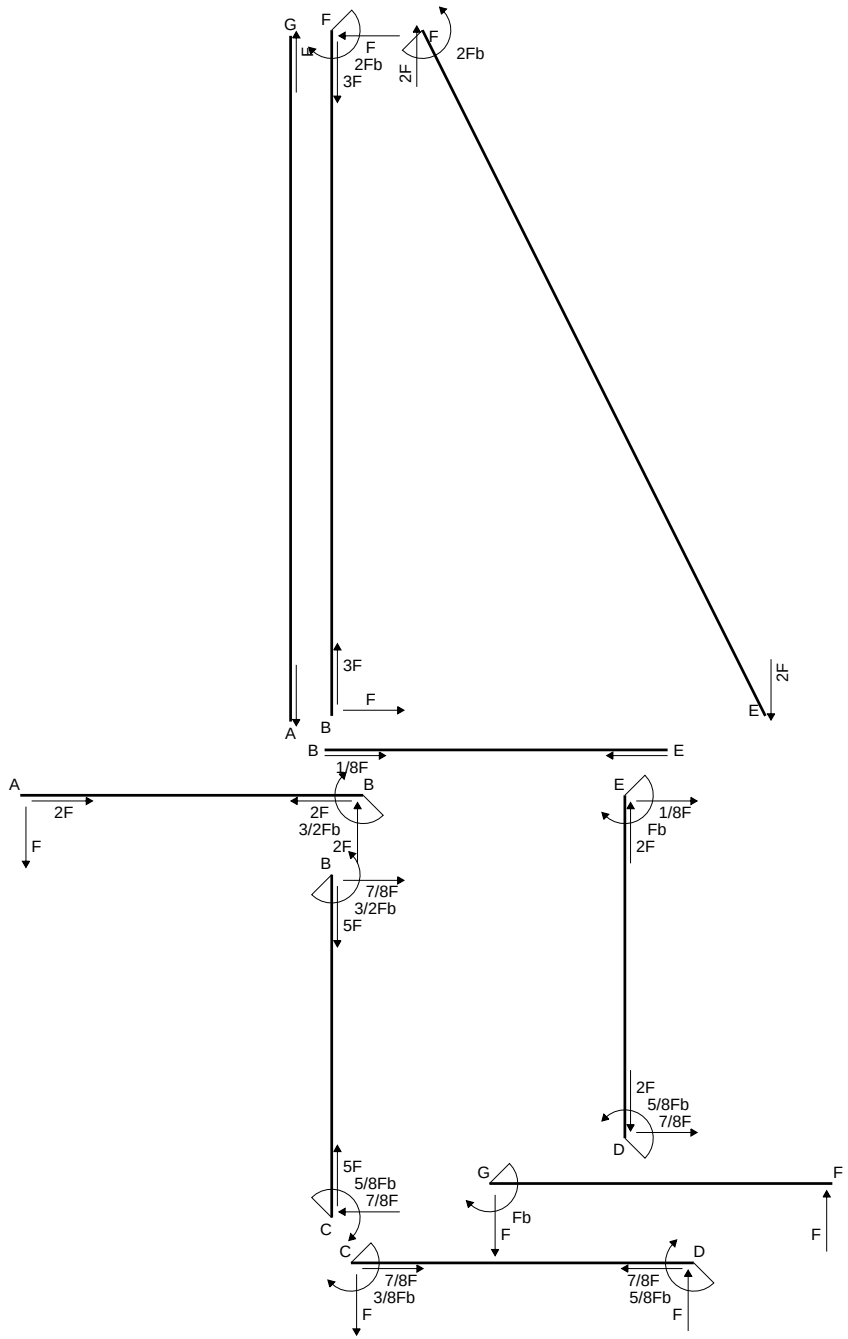
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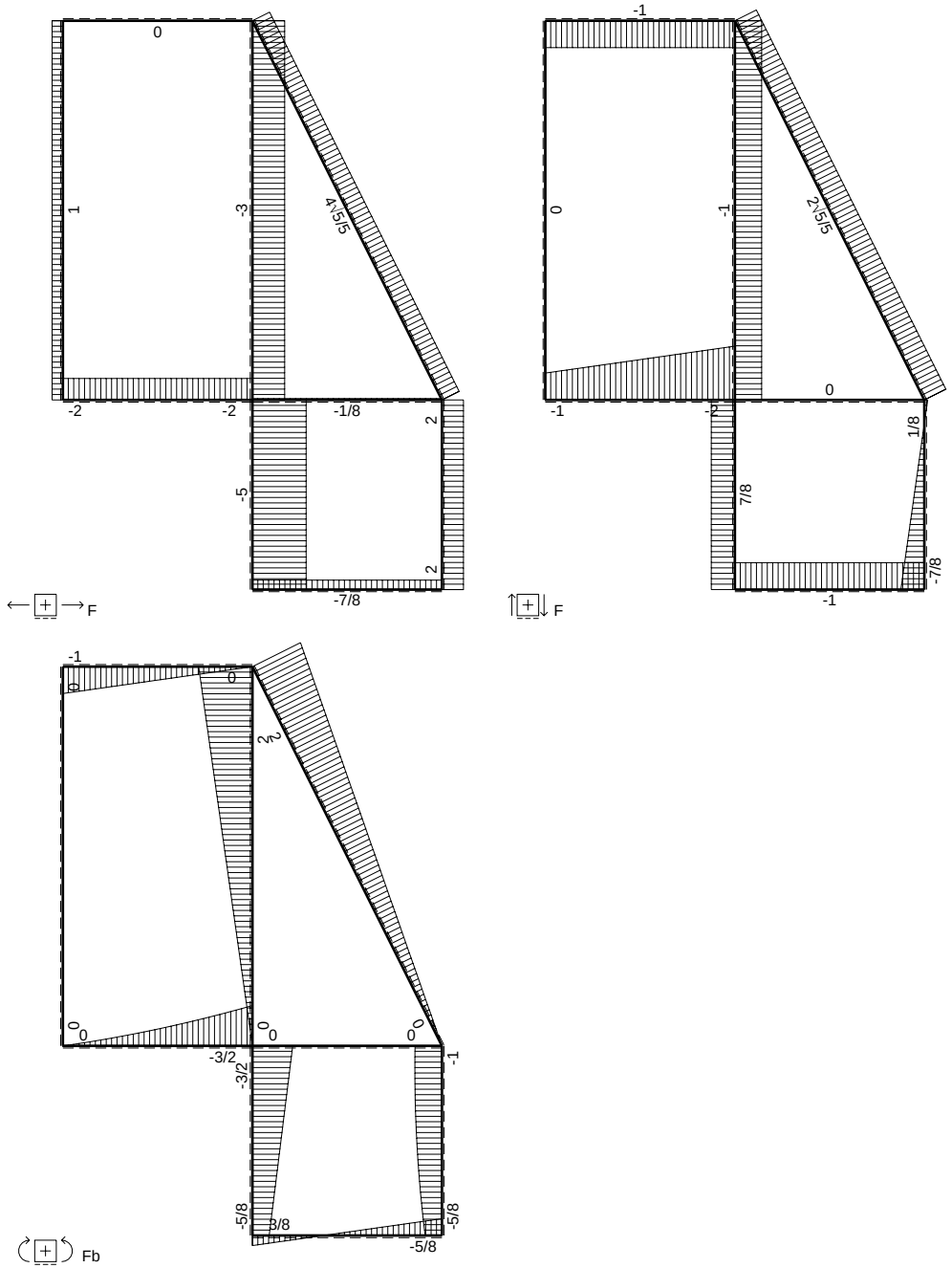
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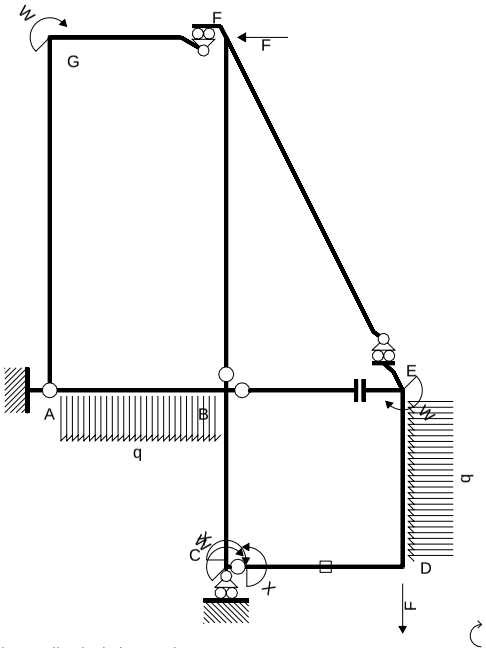
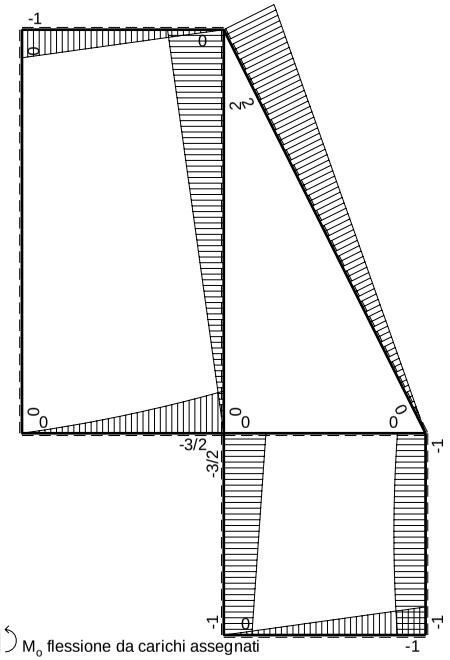
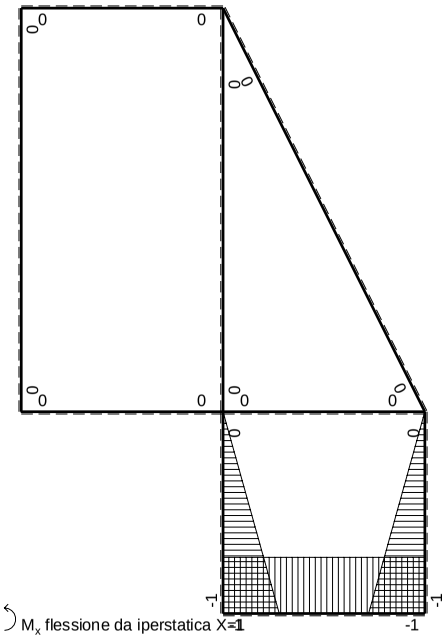


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

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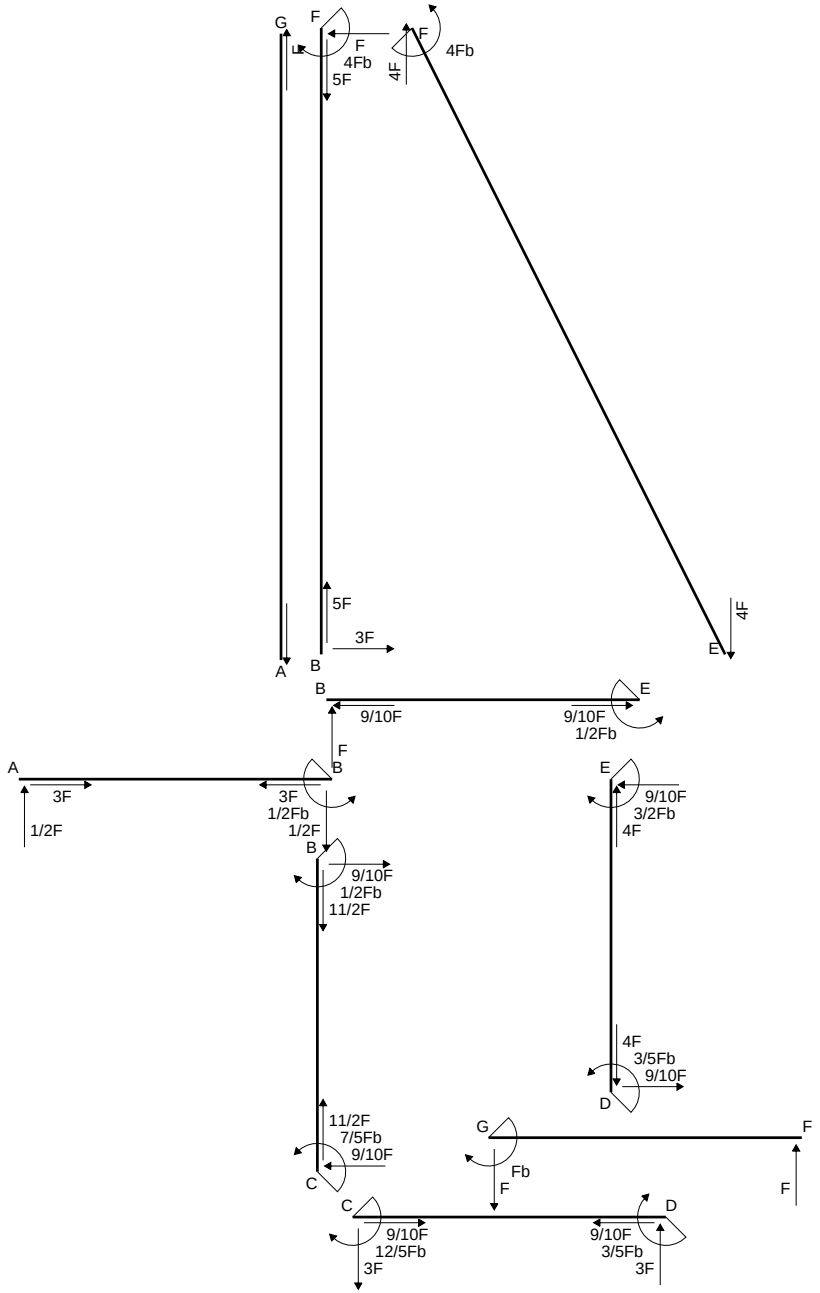
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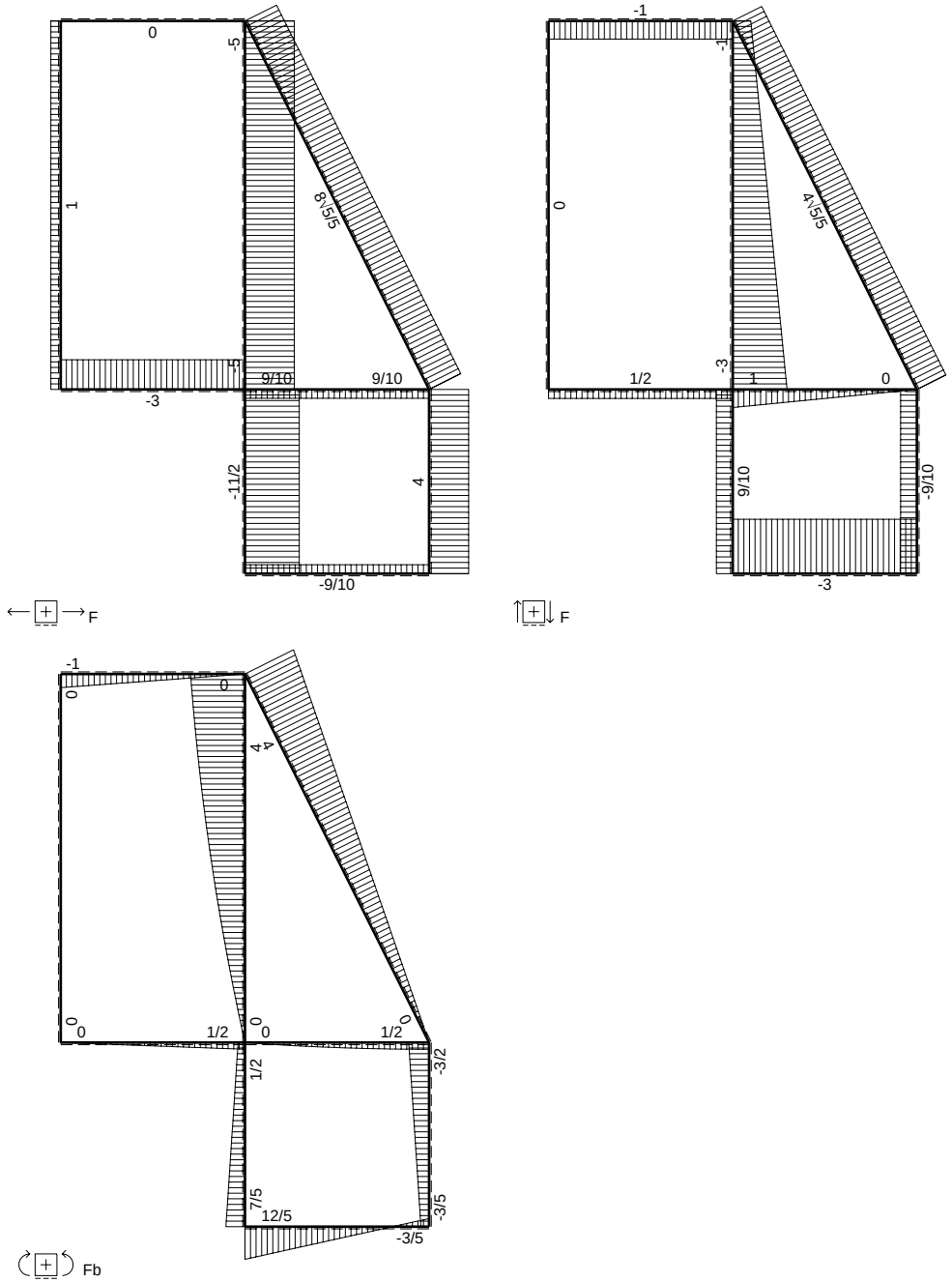
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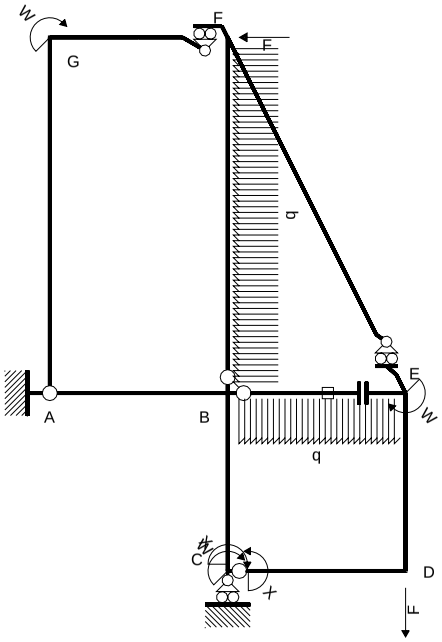
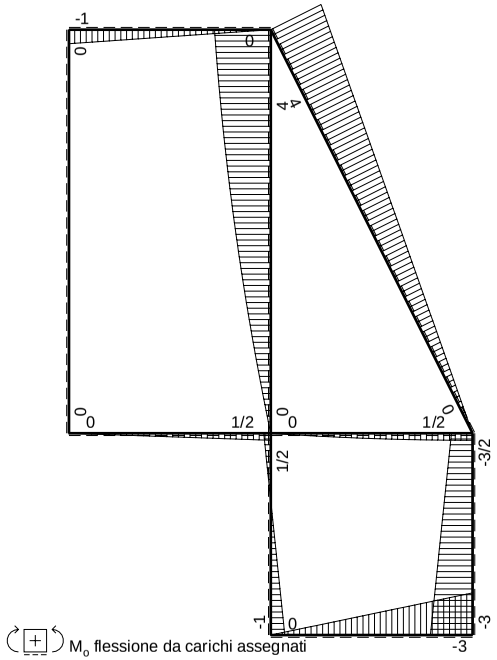
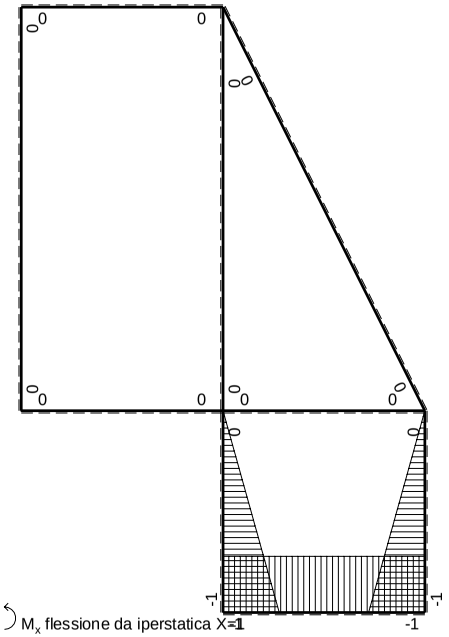


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3/2Fx	$3Fb-9/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/4Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	3/2Fb+3/2Fx	$3/2Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-12/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3\ x/b)\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b)\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b)\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b)\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -9/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -9/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -9/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

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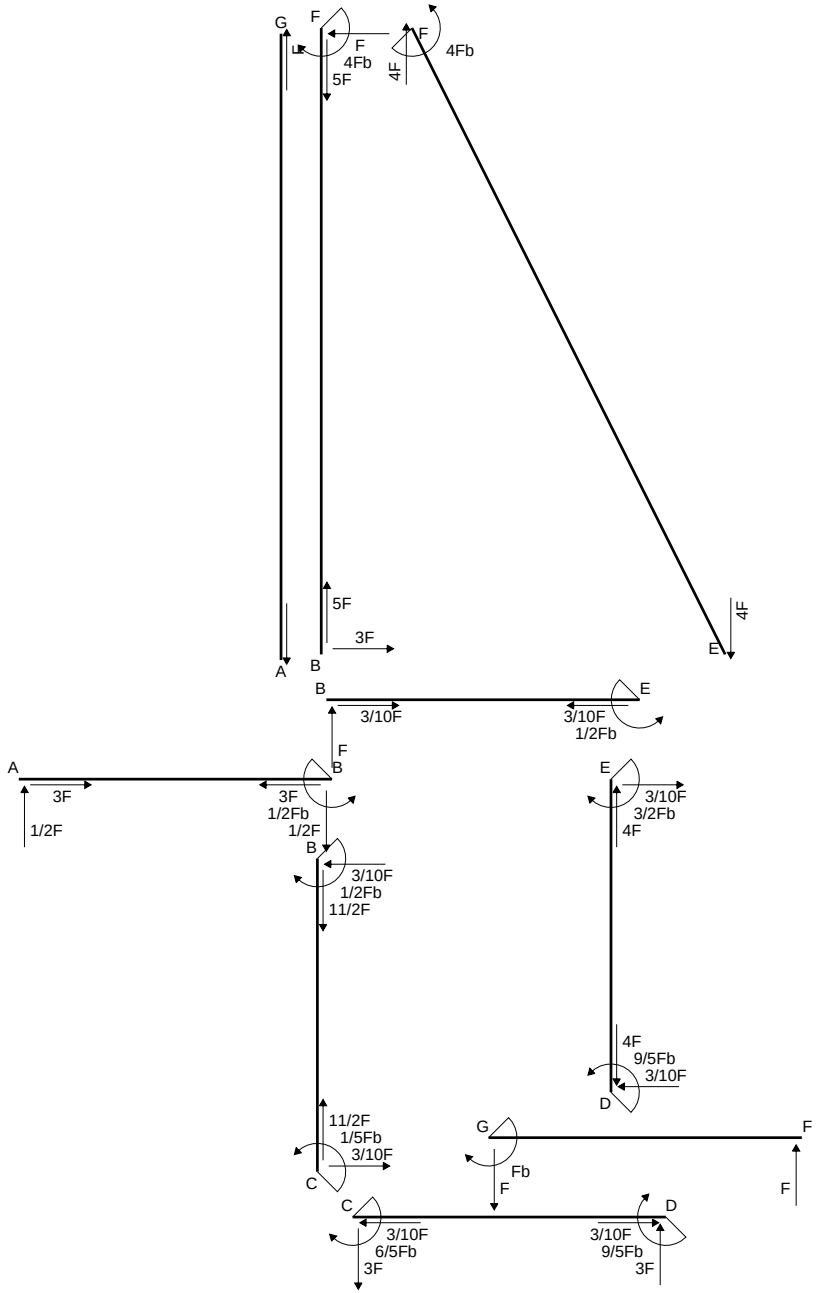
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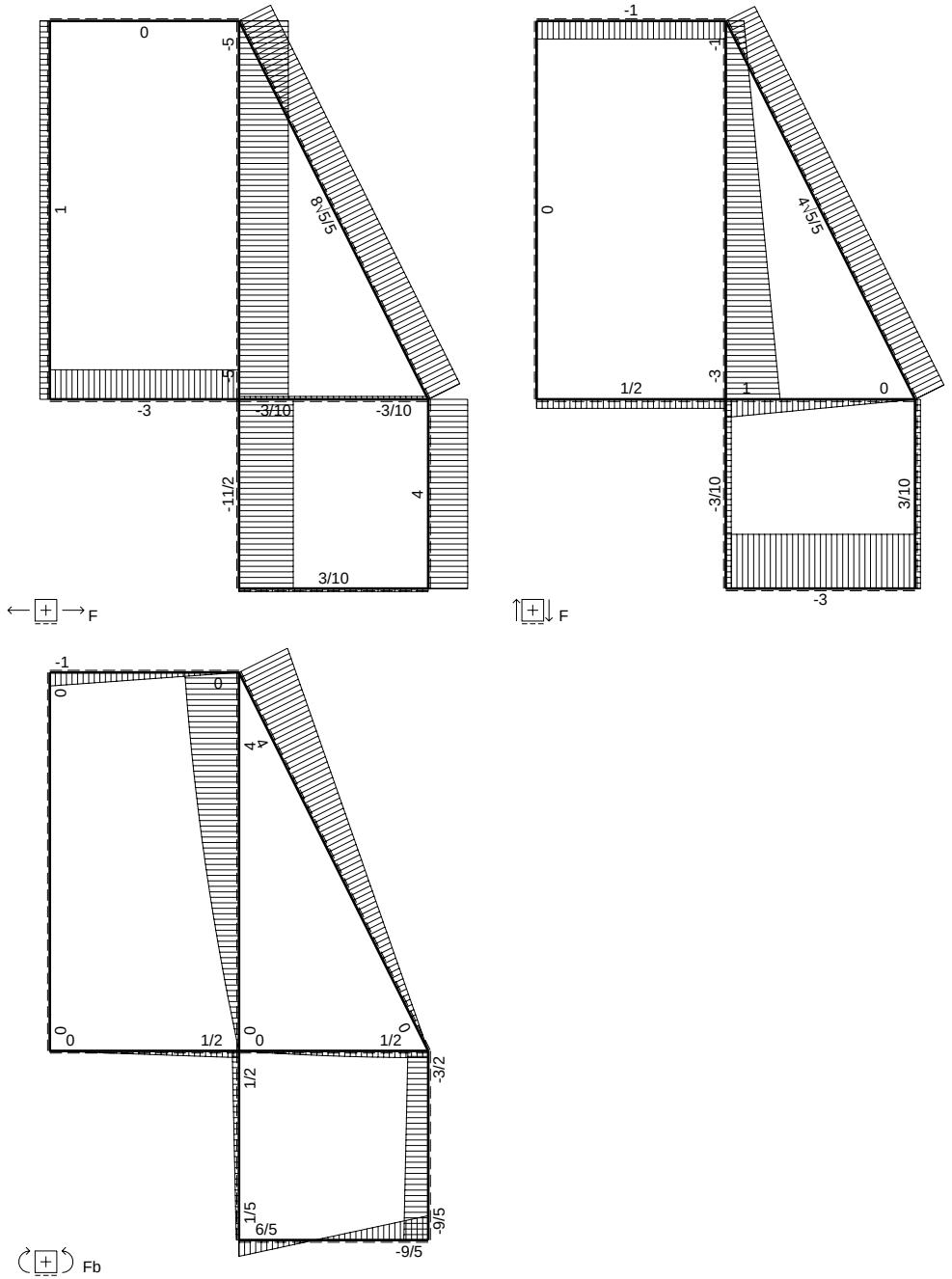
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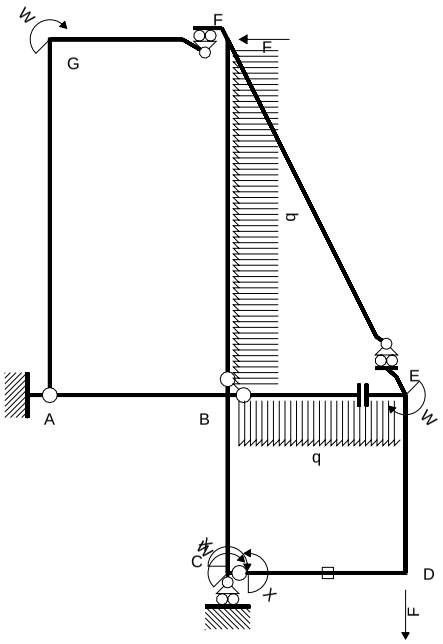
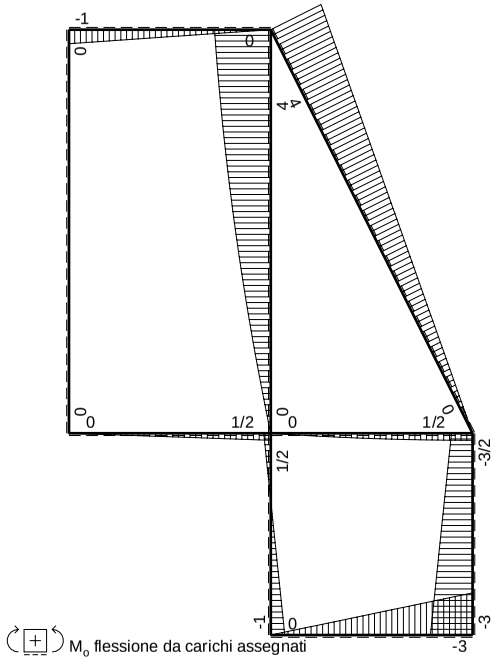
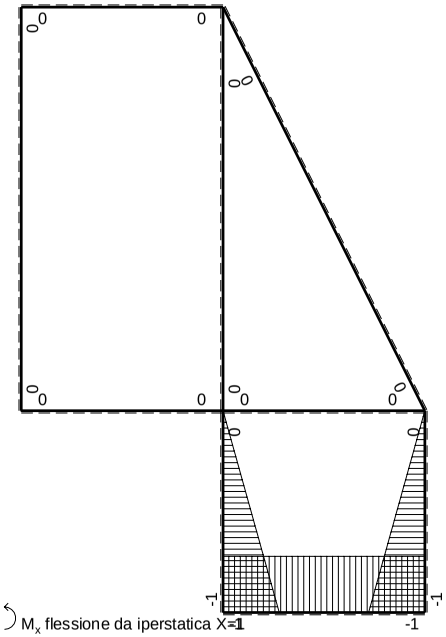


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	$-3Fb+3/2Fx$	$3Fb-9/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/4Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$3/2Fb+3/2Fx$	$3/2Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-6/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -9/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -9/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -9/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

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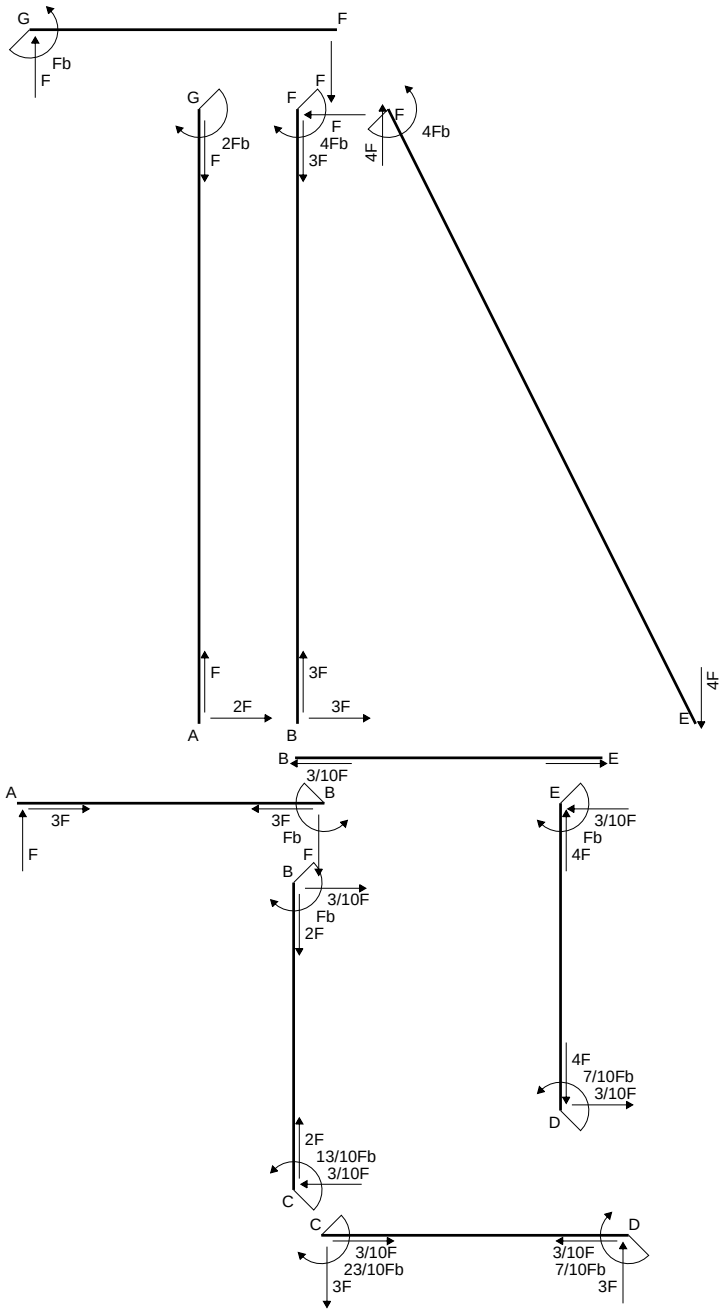
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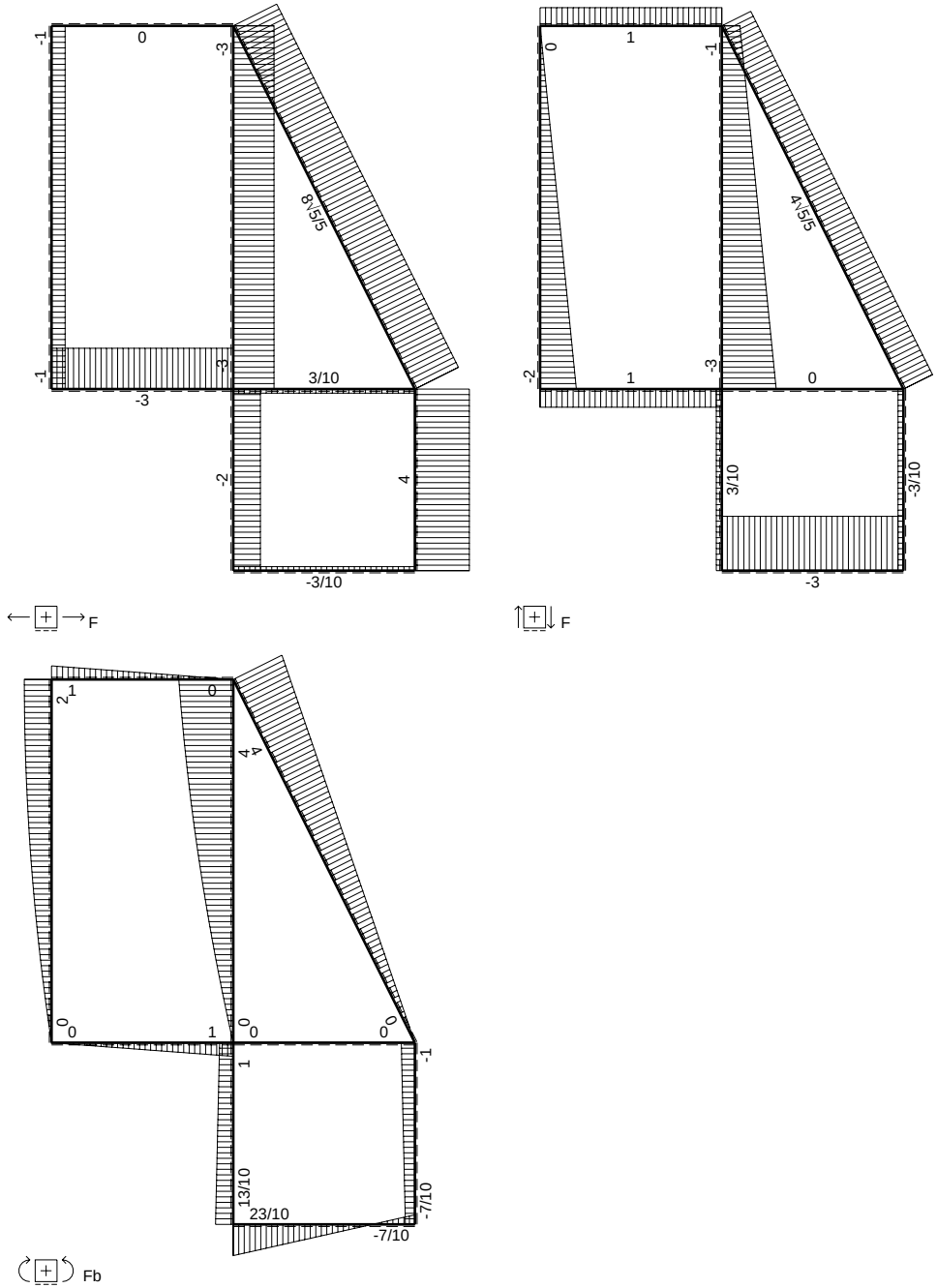
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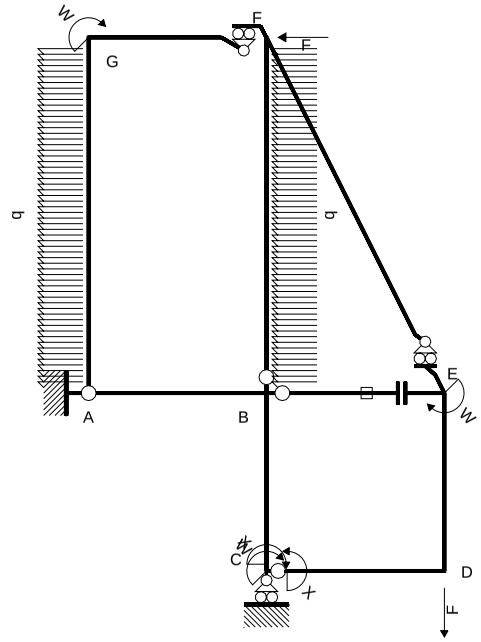
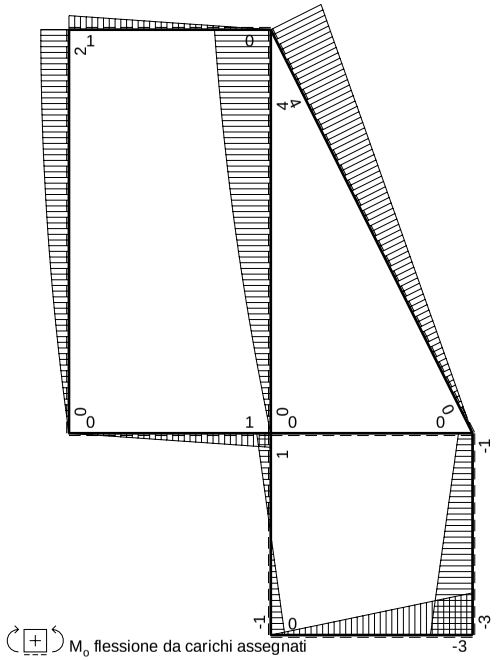
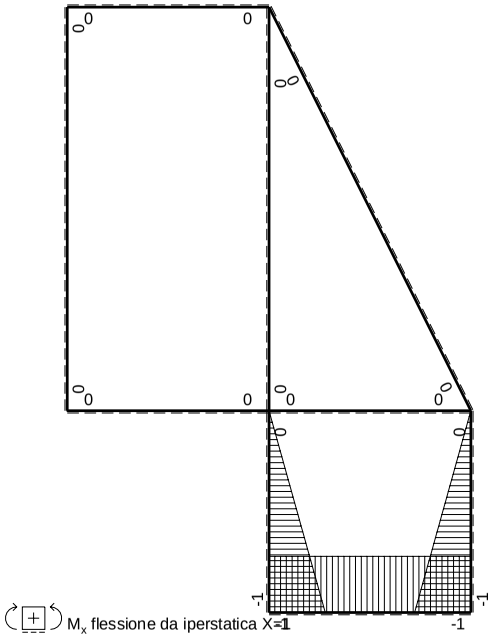
Fb



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$23/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-23/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (- x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -5\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

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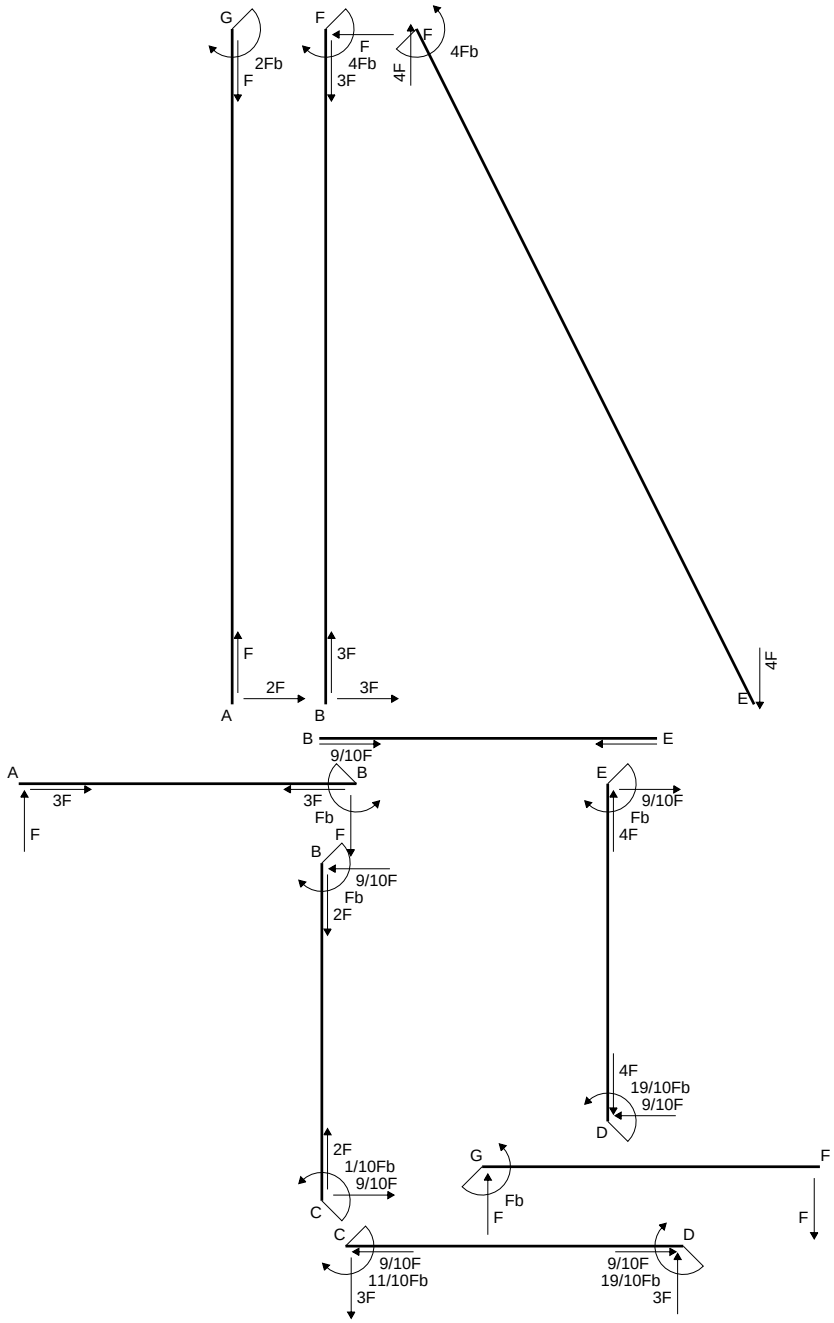
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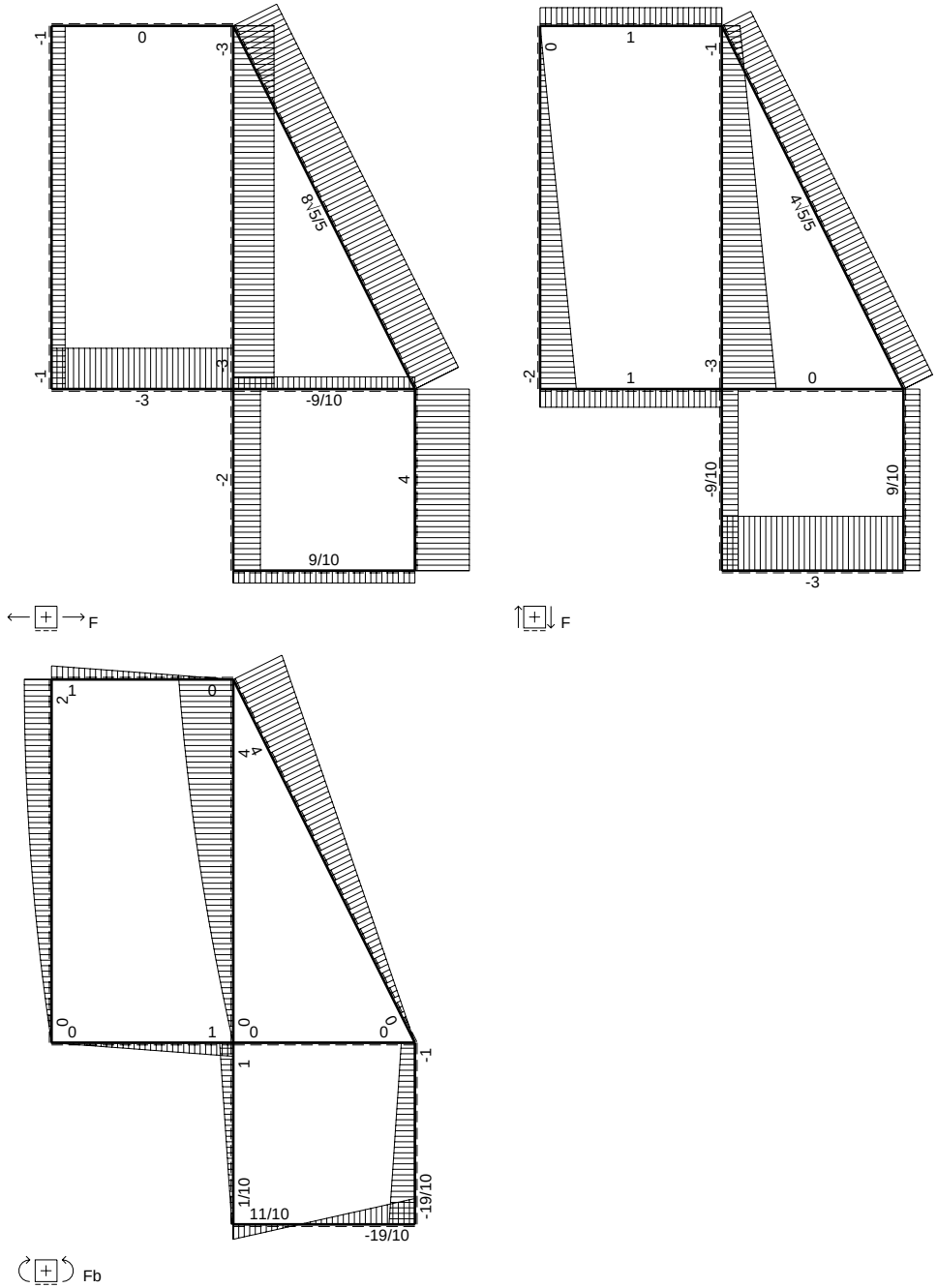
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F
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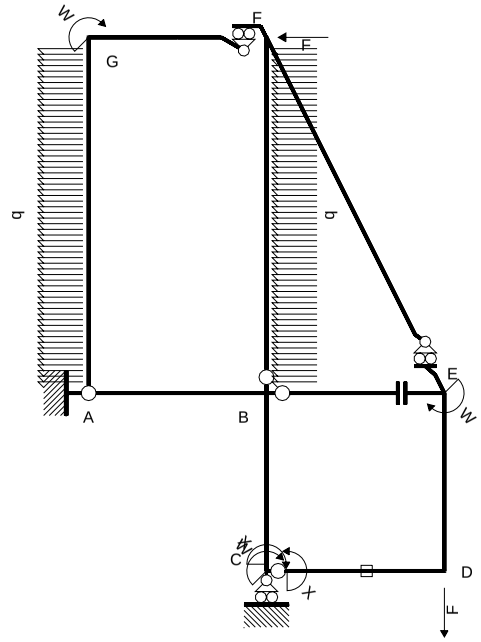
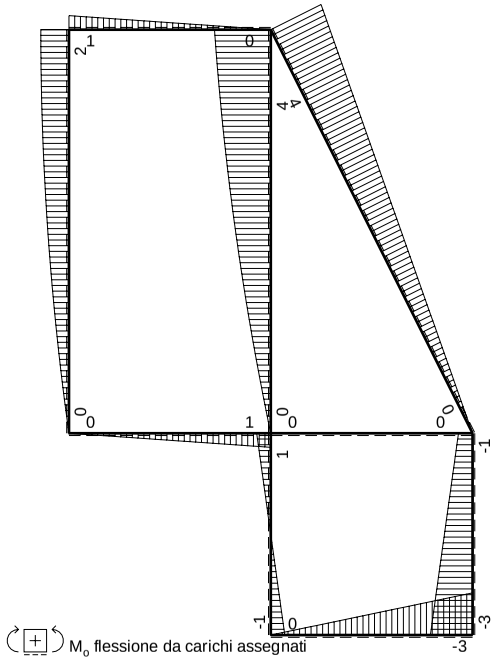
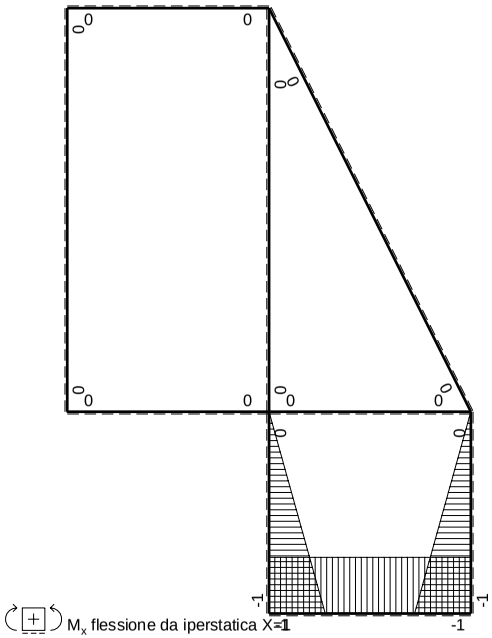
11/10
-19/10
-19/10
Fb



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$11/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-11/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

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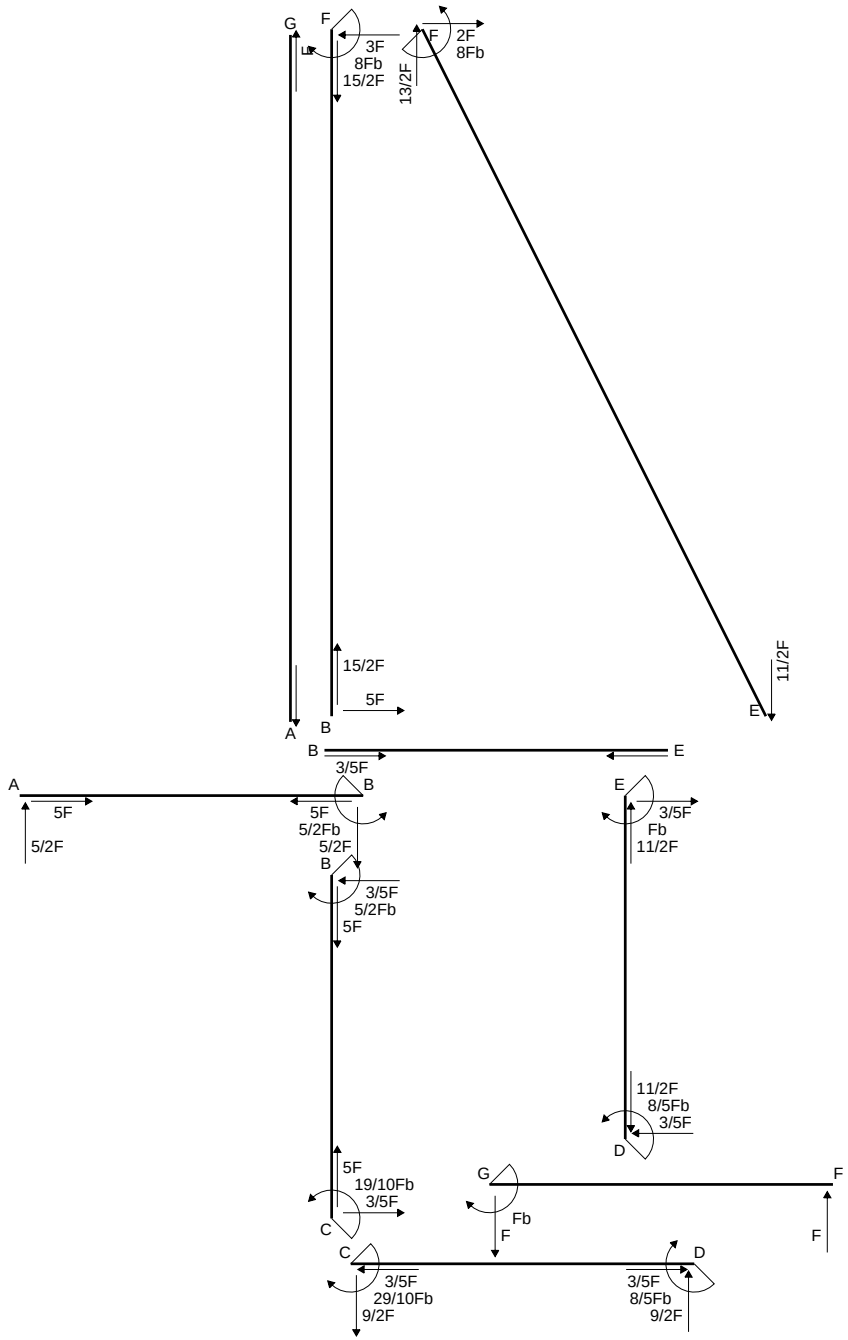
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5F
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5F
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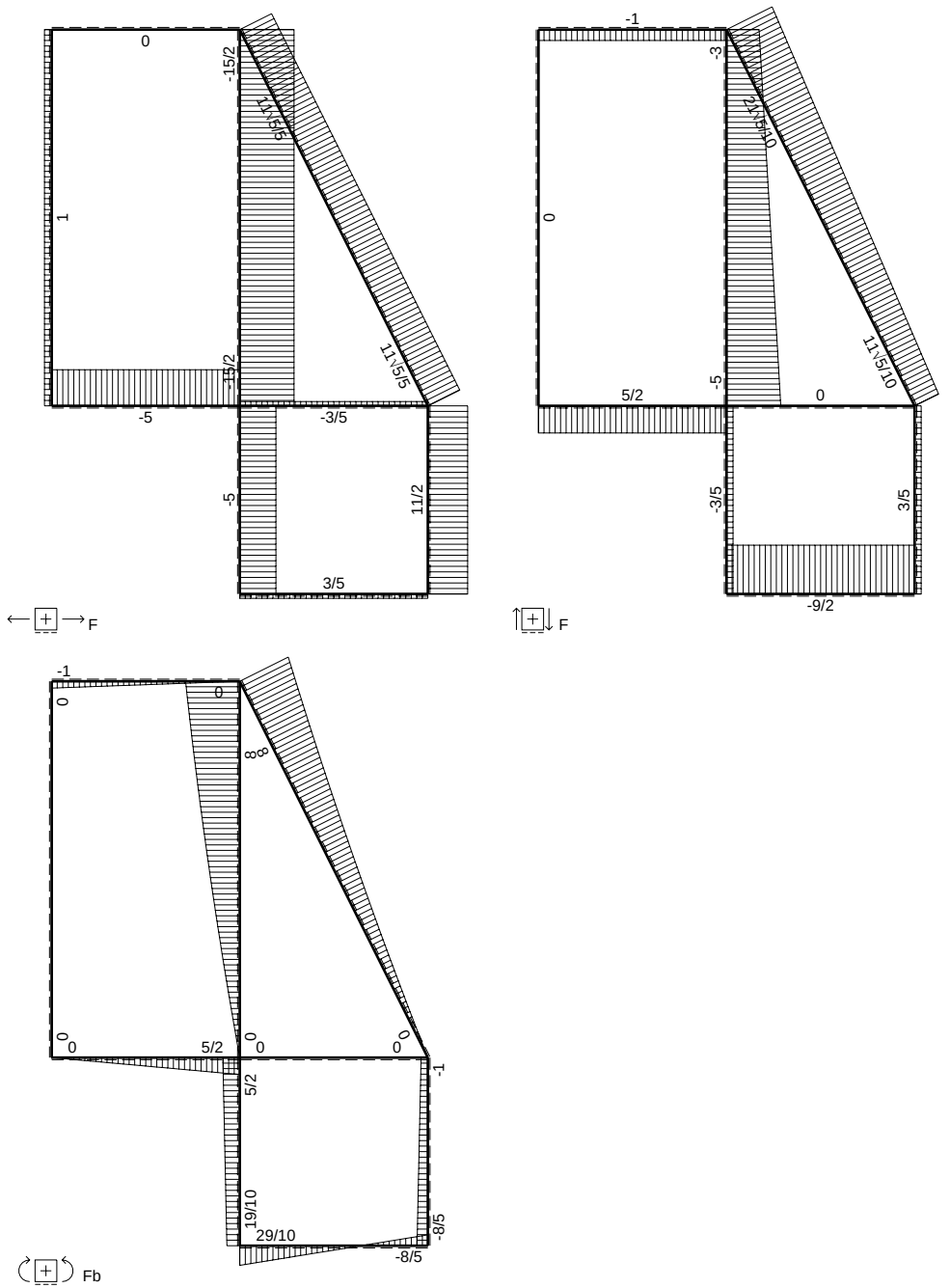
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-5
11/2
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0
11\5/10
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0
-3/5
3/5
-9/2
F
-1
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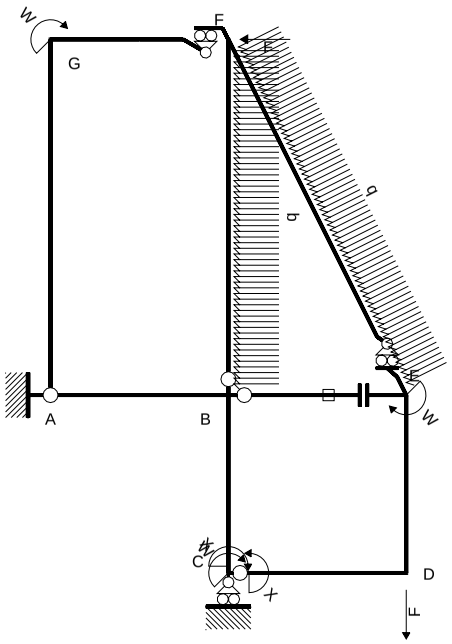
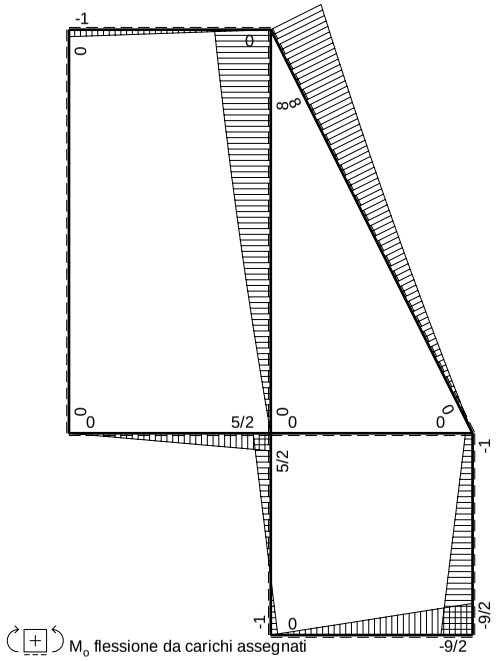
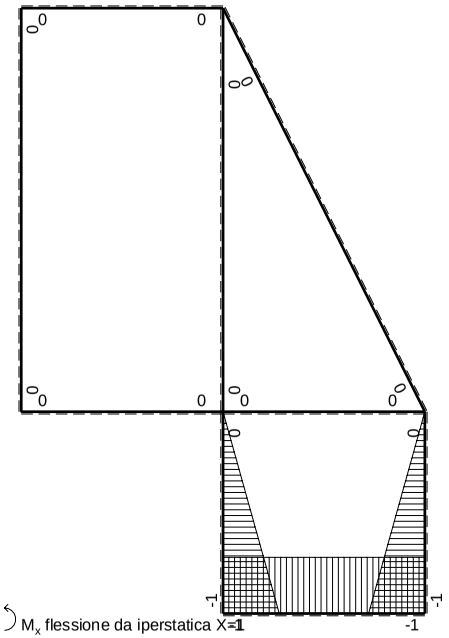
-8/5
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	5/2Fx	0	0	0	0
BA b	0	-5/2Fb+5/2Fx	0	0		
BC b	-x/b	5/2Fb-7/2Fx	-5/2Fx+7/2Fx^2/b	x^2/b^2	-1/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-7/2Fx	Fb-9/2Fx+7/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-9/2Fx	9/2Fx	1	9/4Fb^2/EJ	Xb/EJ
DC b	1	9/2Fb-9/2Fx	9/2Fb-9/2Fx	1		
DE b	-1+x/b	-9/2Fb+7/2Fx	9/2Fb-8Fx+7/2Fx^2/b	1-2x/b+x^2/b^2	5/3Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+7/2Fx	Fx+7/2Fx^2/b	x^2/b^2		
EF √5b	0	11√5/10Fx+1/2qx^2	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	8Fb-3Fx-1/2qx^2	0	0	0	0
BF 2b	0	-5Fx+1/2qx^2	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				29/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-29/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-5/2\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -9/2\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -9/4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -9/4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (9/2\ x/b\ )\ Fb\ 1/EJ\ dx = [9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (9/4\ b\ )\ Fb\ 1/EJ\ = 9/4\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (9/2 -9/2\ x/b\ )\ Fb\ 1/EJ\ dx = [9/2\ x -9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -9/4\ b\ )\ Fb\ 1/EJ\ = 9/4\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (9/2 -8\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [9/2\ x -4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = 5/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +7/6\ b\ )\ Fb\ 1/EJ\ = 5/3\ Fb^2/EJ$

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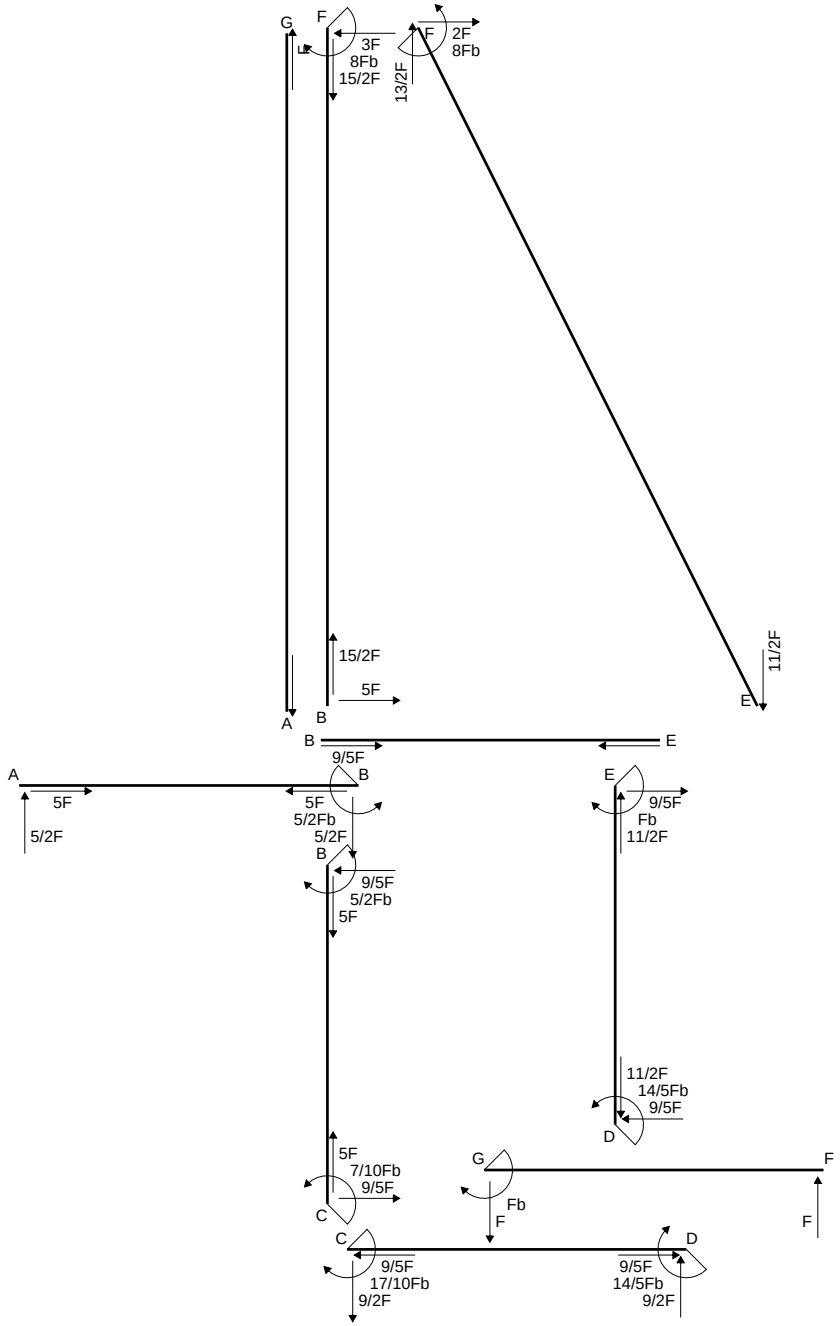
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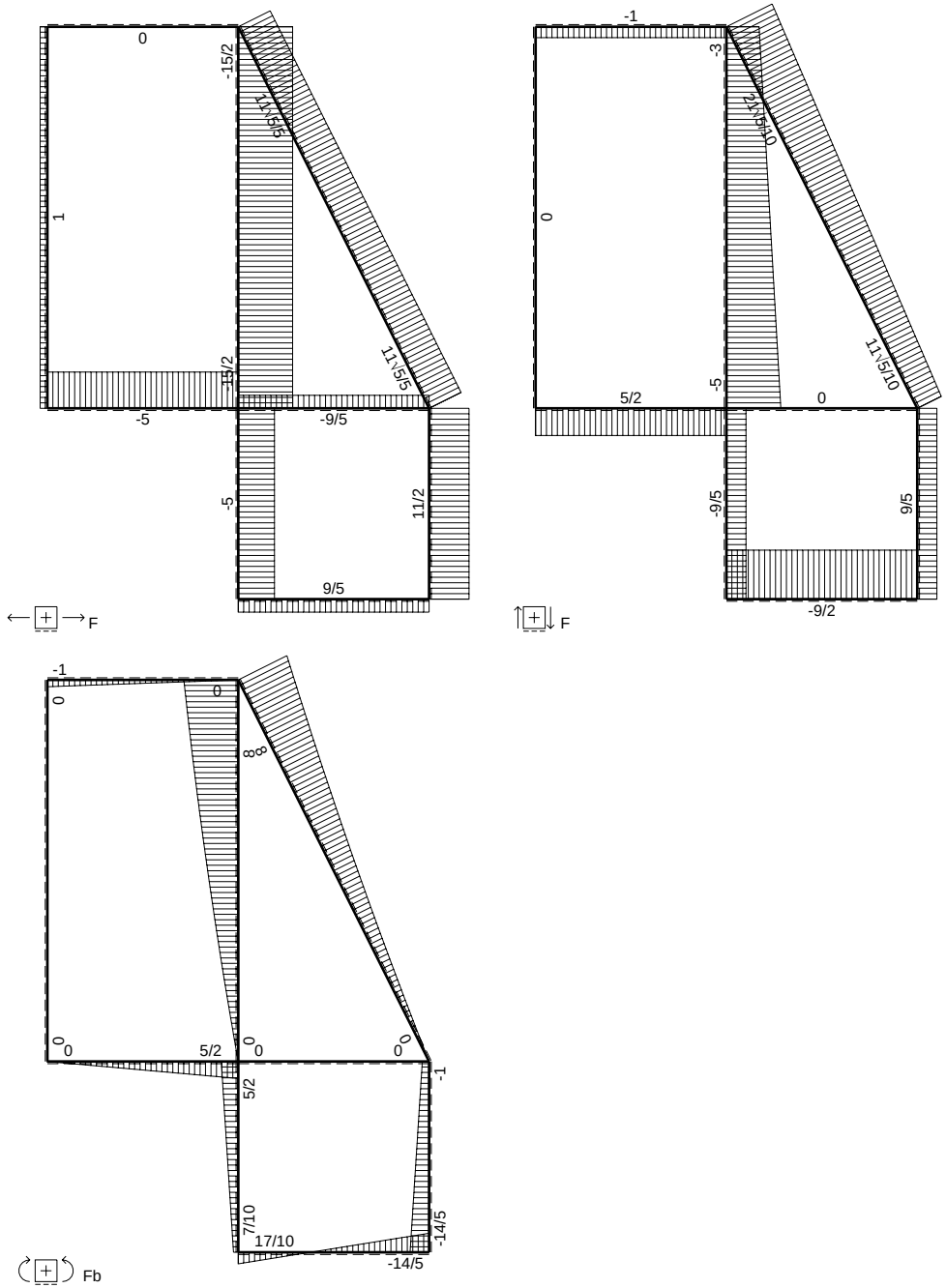
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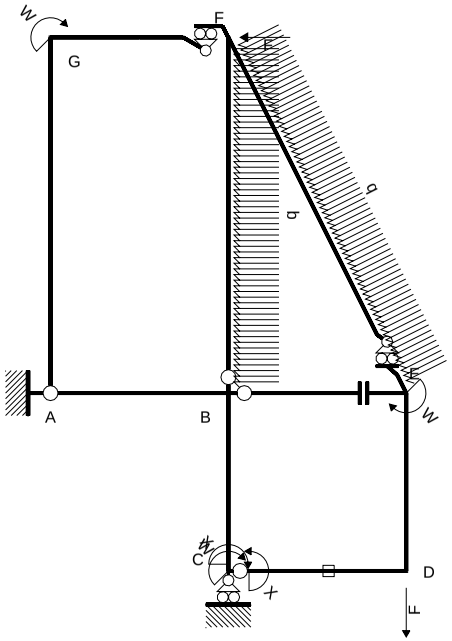
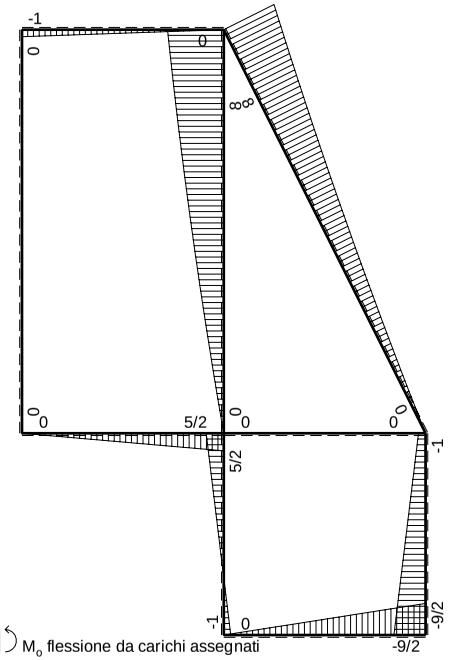
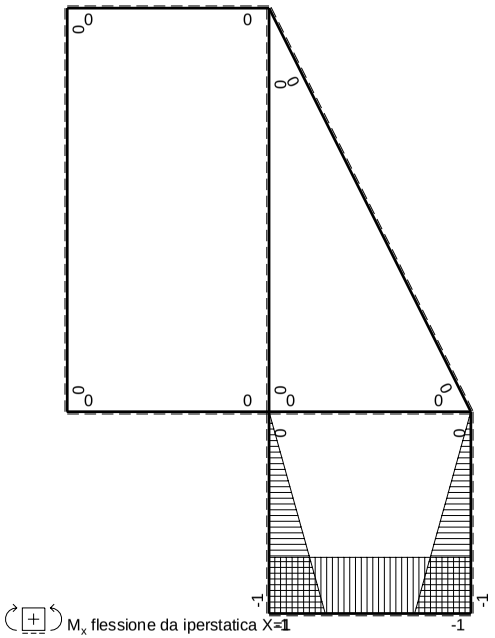


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	5/2Fx	0	0	0	0
BA b	0	-5/2Fb+5/2Fx	0	0		
BC b	-x/b	5/2Fb-7/2Fx	-5/2Fx+7/2Fx^2/b	x^2/b^2	-1/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-7/2Fx	Fb-9/2Fx+7/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-9/2Fx	9/2Fx	1	9/4Fb^2/EJ	Xb/EJ
DC b	1	9/2Fb-9/2Fx	9/2Fb-9/2Fx	1		
DE b	-1+x/b	-9/2Fb+7/2Fx	9/2Fb-8Fx+7/2Fx^2/b	1-2x/b+x^2/b^2	5/3Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+7/2Fx	Fx+7/2Fx^2/b	x^2/b^2		
EF √5b	0	11√5/10Fx+1/2qx^2	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	8Fb-3Fx-1/2qx^2	0	0	0	0
BF 2b	0	-5Fx+1/2qx^2	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				17/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-17/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-5/2\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -9/2\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -9/4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -9/4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (9/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (9/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 5/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (9/2 -9/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [9/2\ x -9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (9/2\ b -9/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 5/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (9/2 -8\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [9/2\ x -4\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -4\ b +7/6\ b\ )\ Fb\ 1/EJ\ = 5/3\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +7/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +7/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +7/6\ b\ )\ Fb\ 1/EJ\ = 5/3\ \ Fb^2/EJ$

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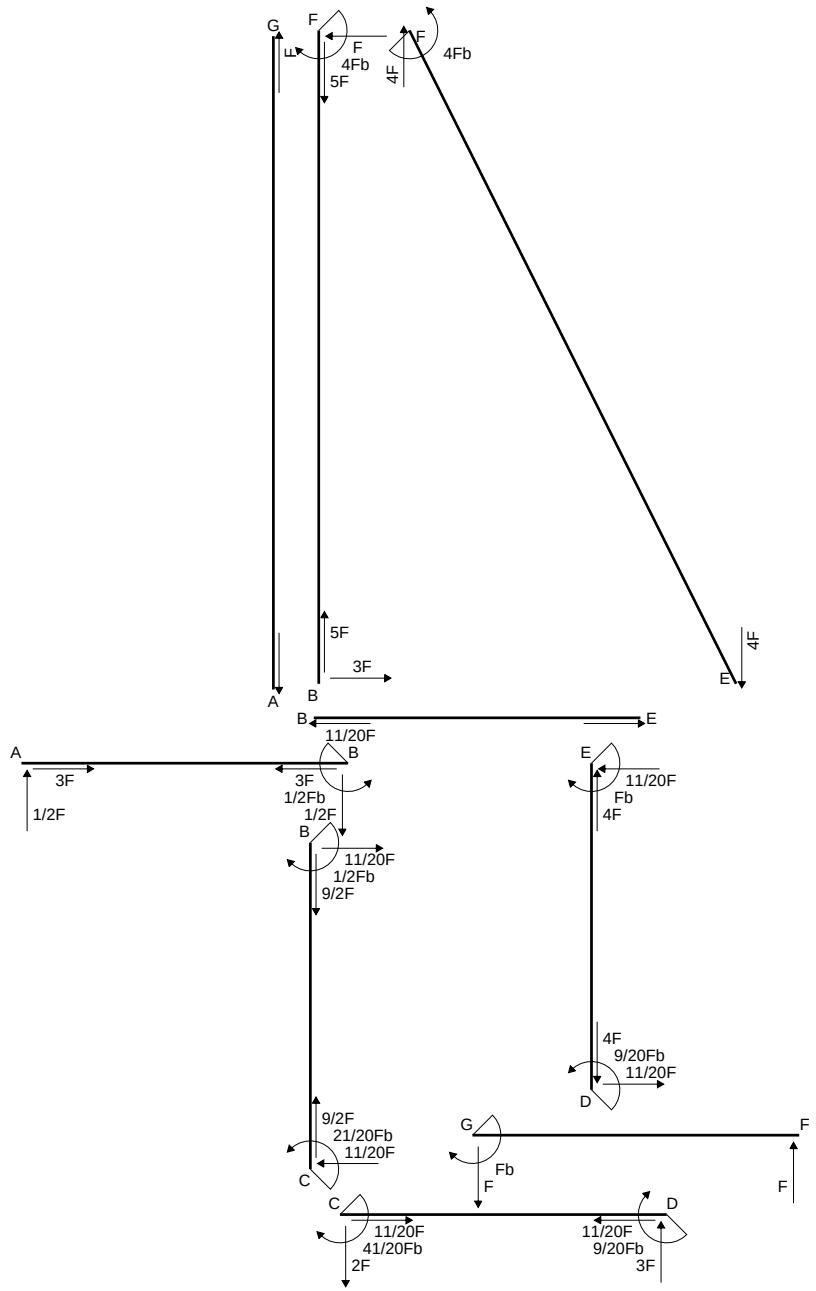
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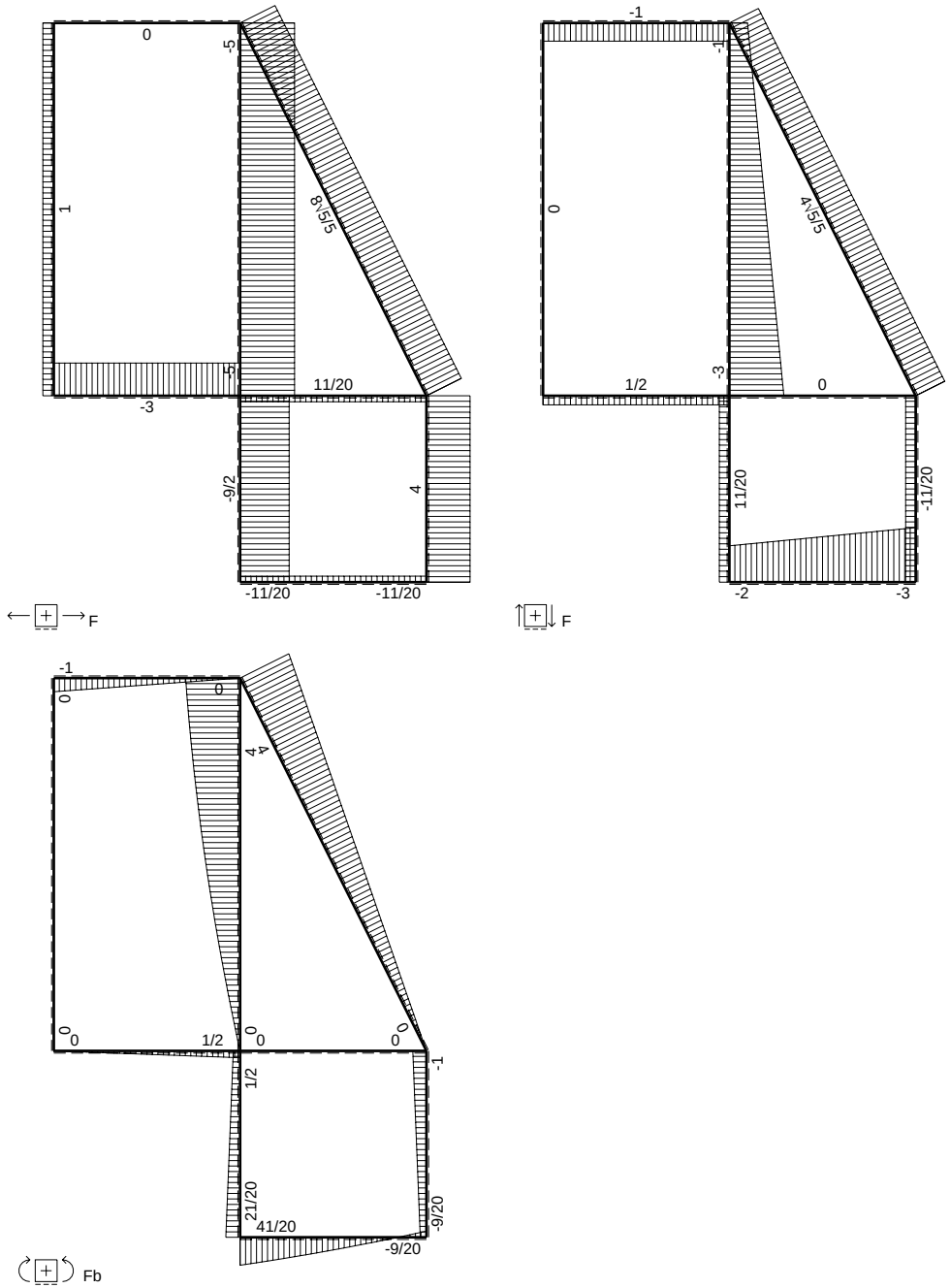
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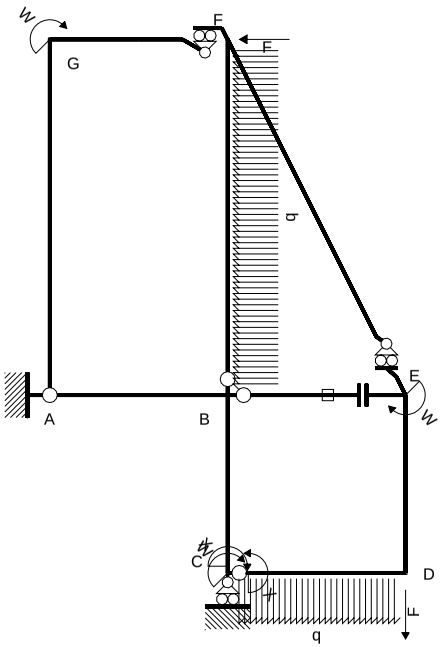
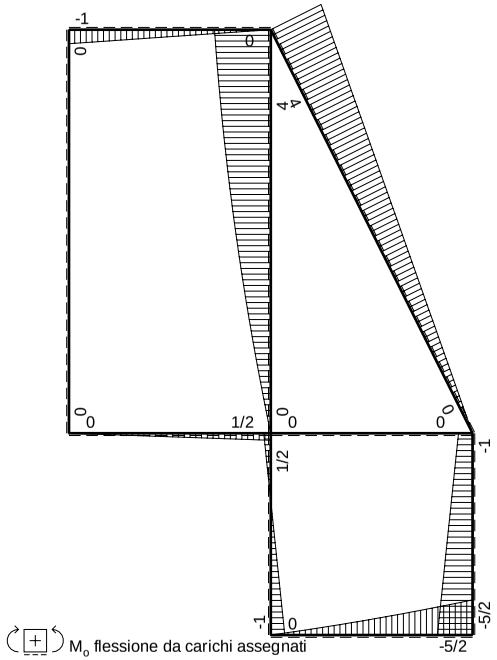
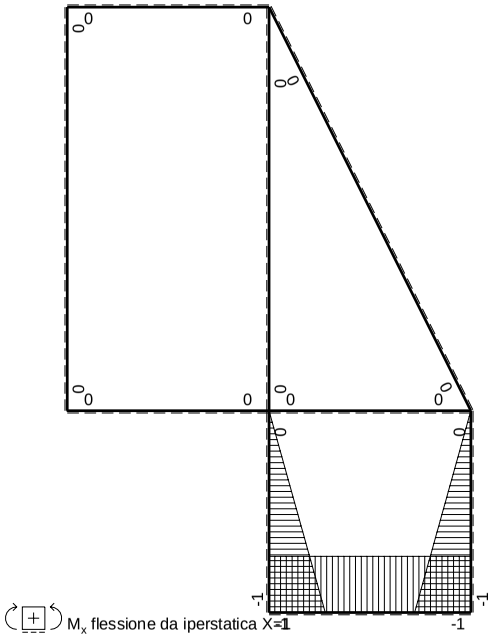


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-2Fx-1/2qx^2$	$2Fx+1/2Fx^2/b$	1	$7/6Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-3Fx+1/2qx^2$	$5/2Fb-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$41/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-41/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -3\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -3/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -3/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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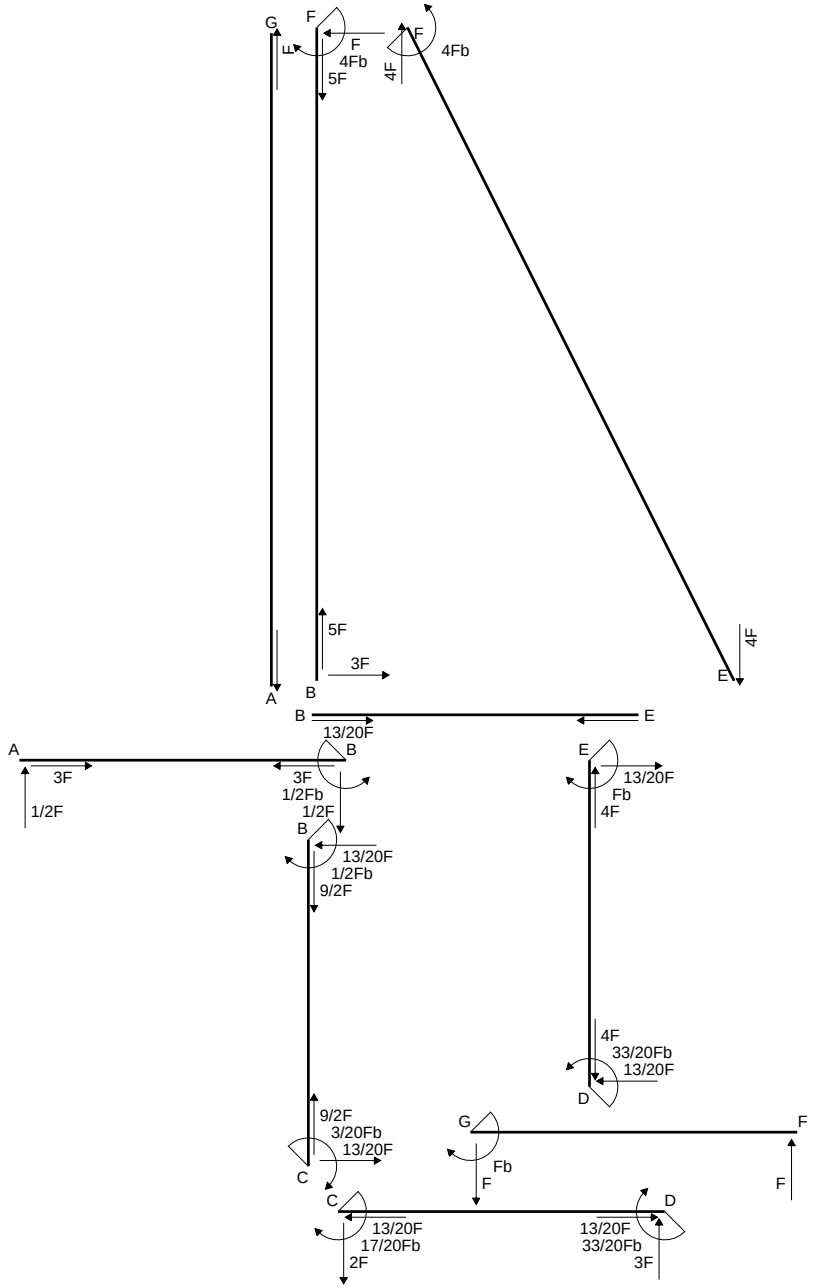
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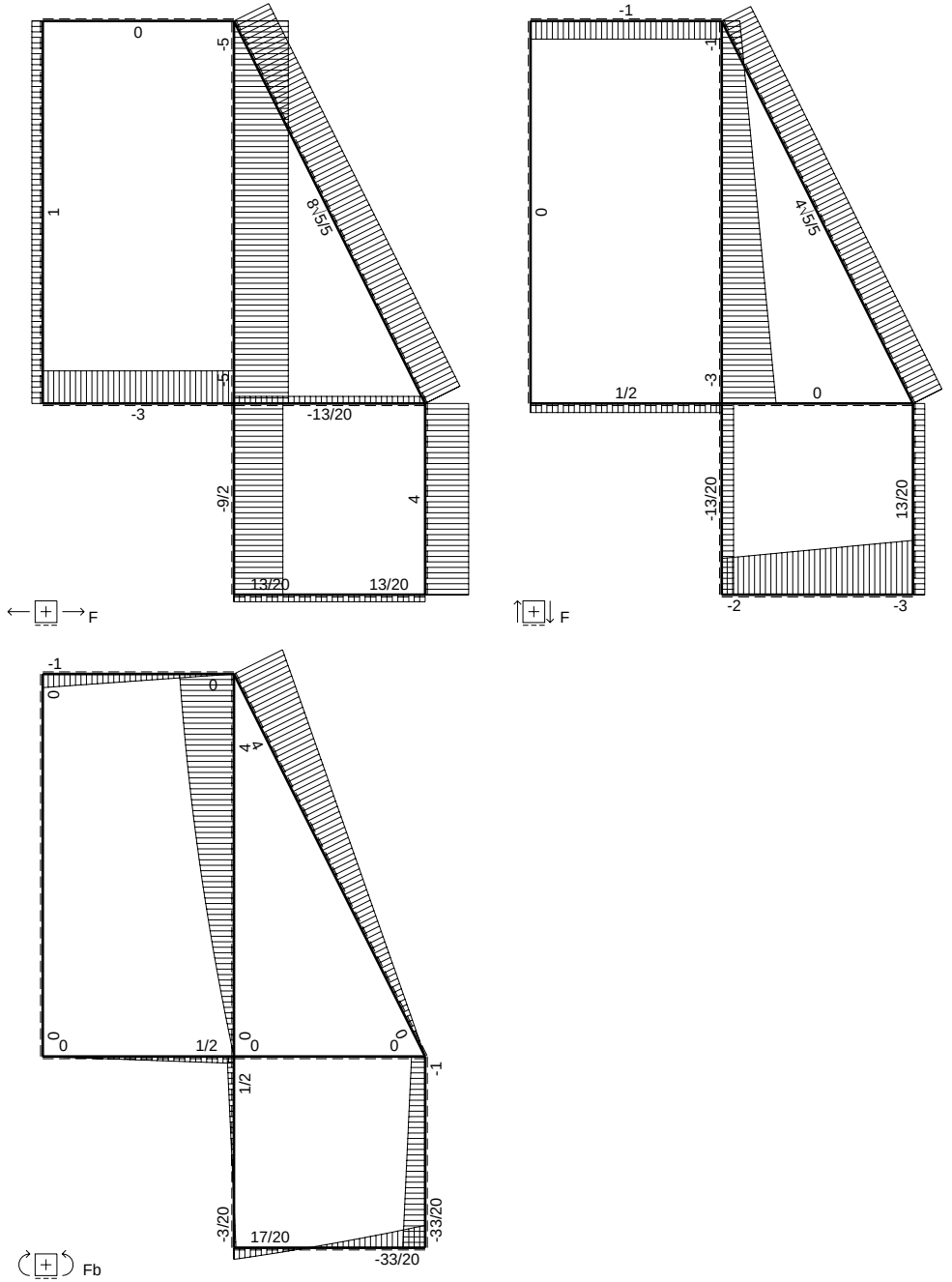
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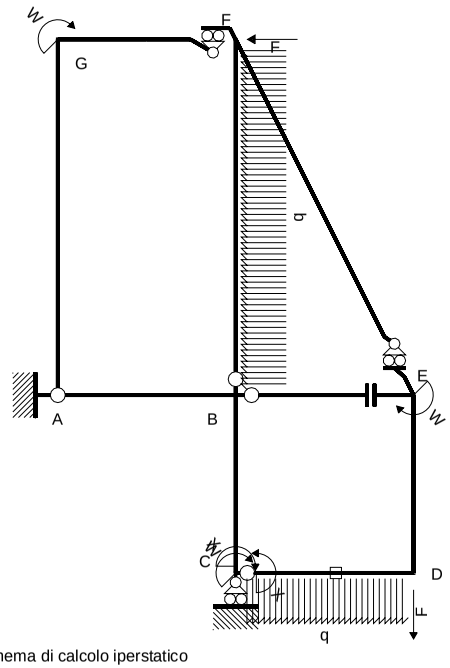
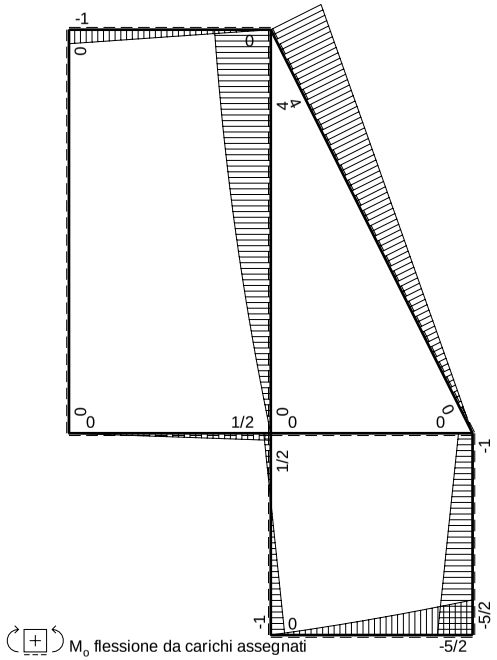
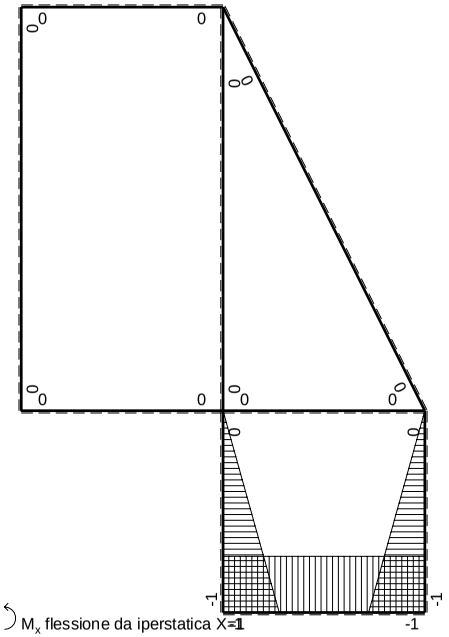


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-2Fx-1/2qx^2$	$2Fx+1/2Fx^2/b$	1	$7/6Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-3Fx+1/2qx^2$	$5/2Fb-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$17/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-17/20Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b +1/6\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -3\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [5/2\ x -3/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/2\ b -3/2\ b +1/6\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b)\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b)\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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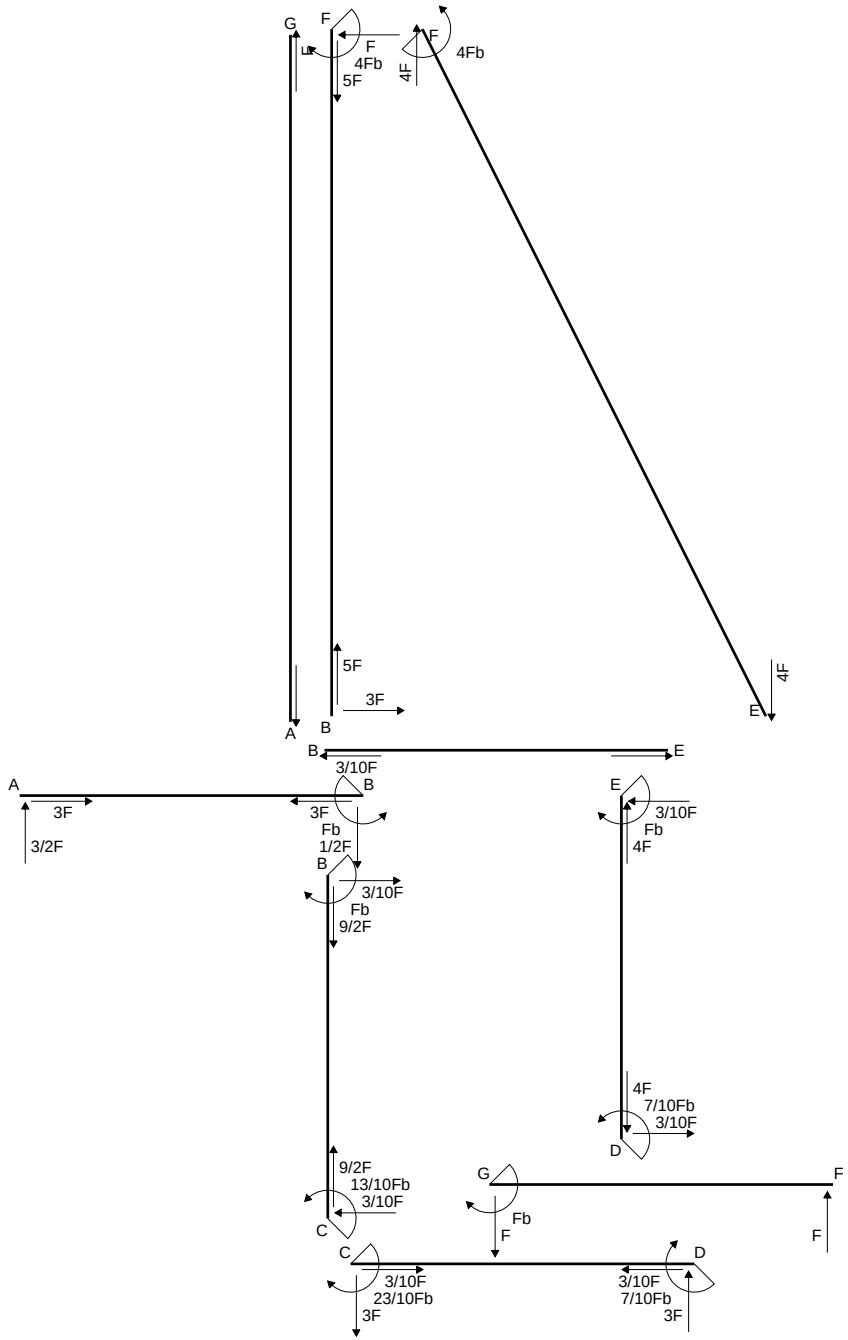
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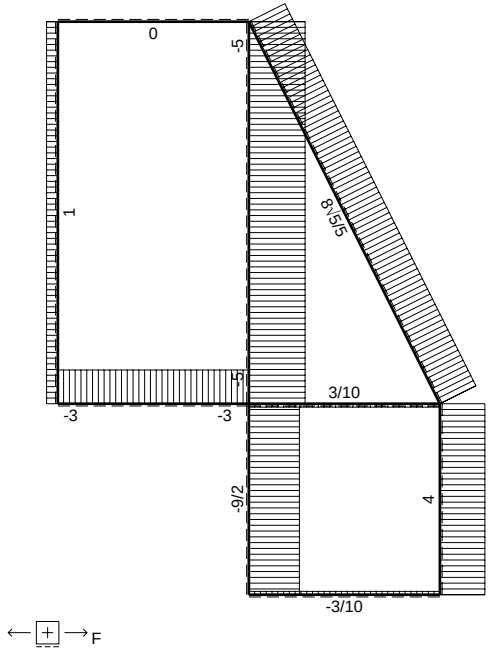
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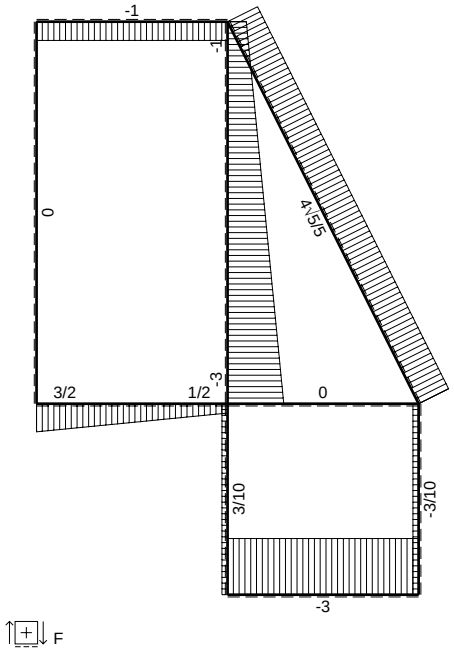
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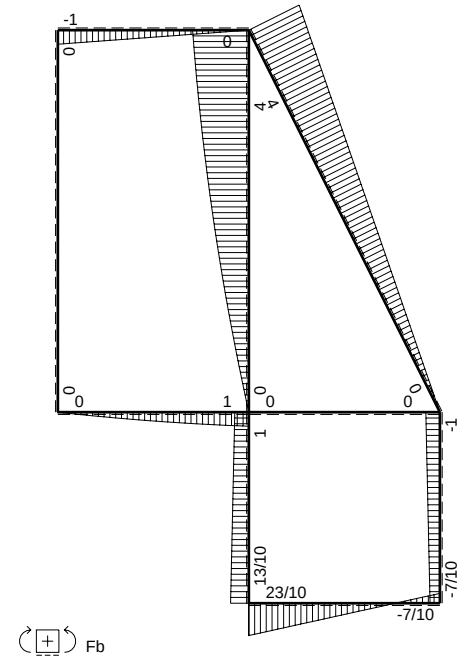
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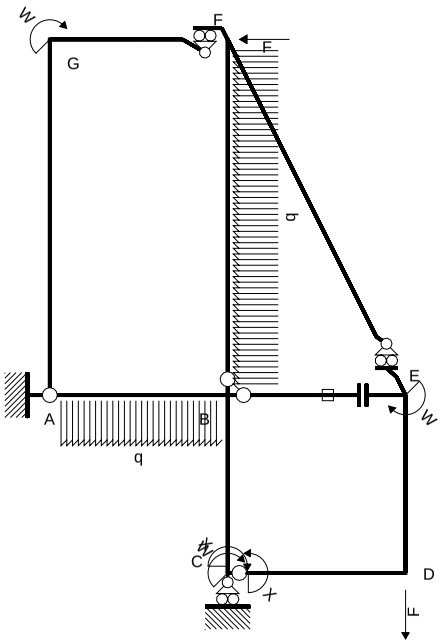
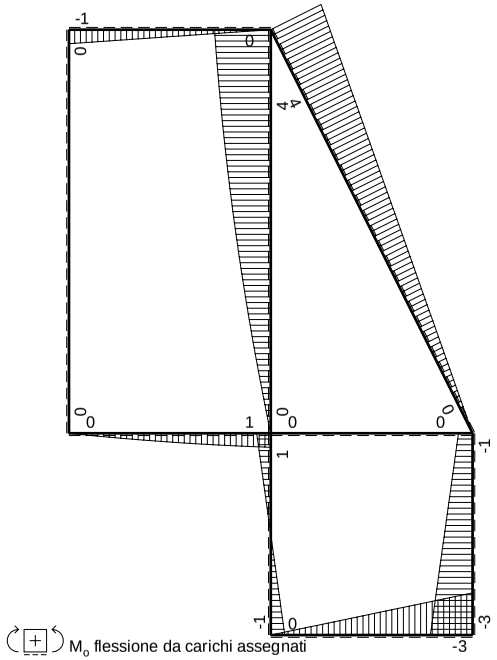
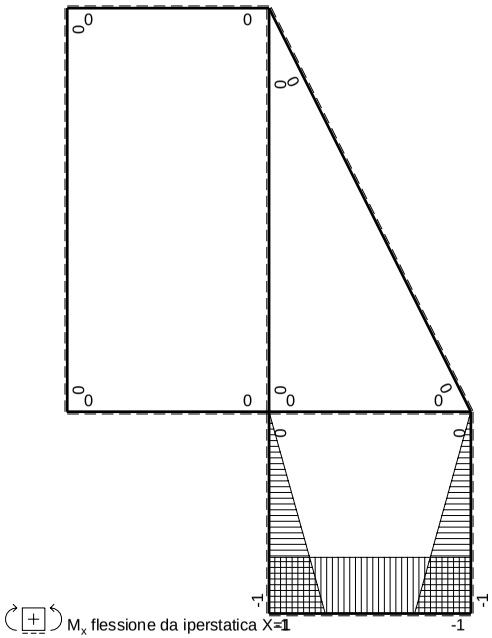


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$3/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-Fb+1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$23/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-23/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -5\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

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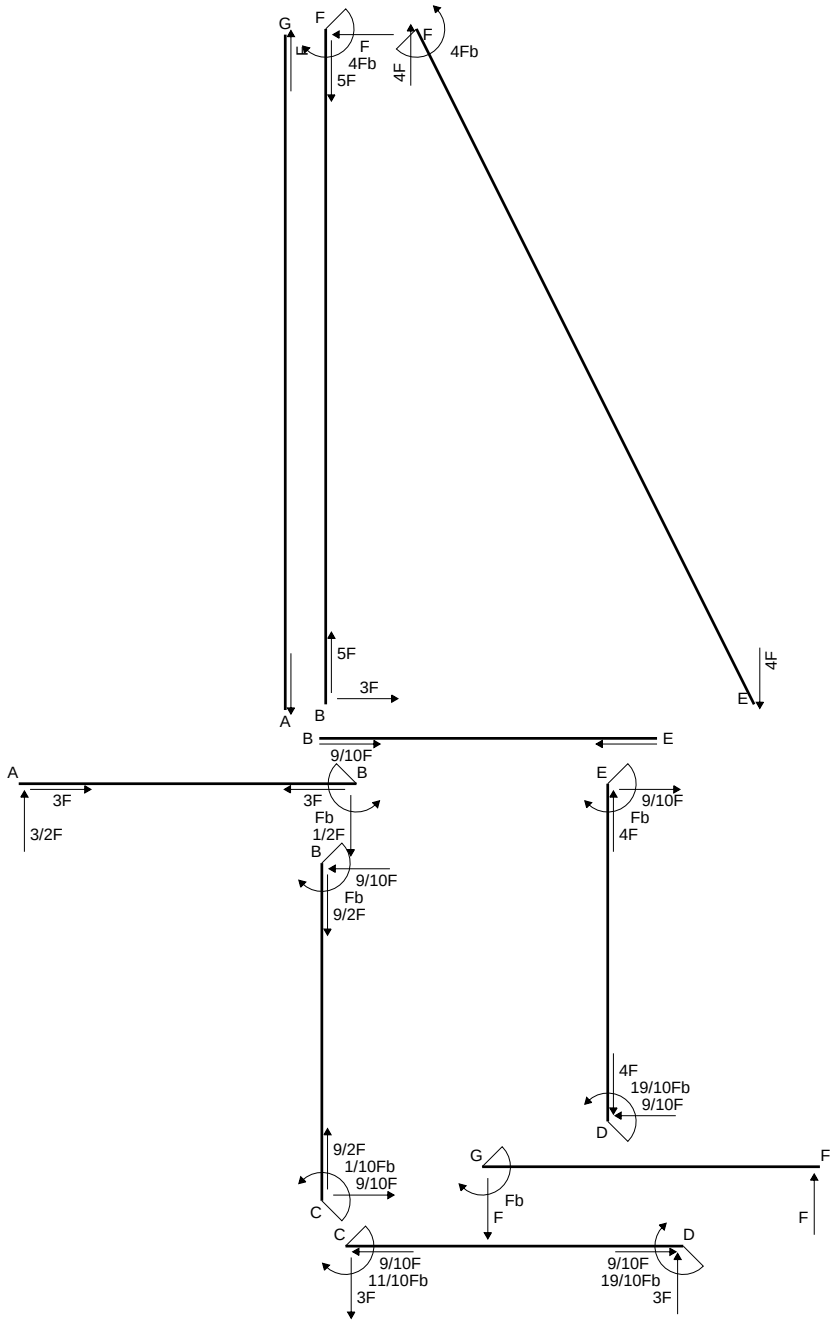
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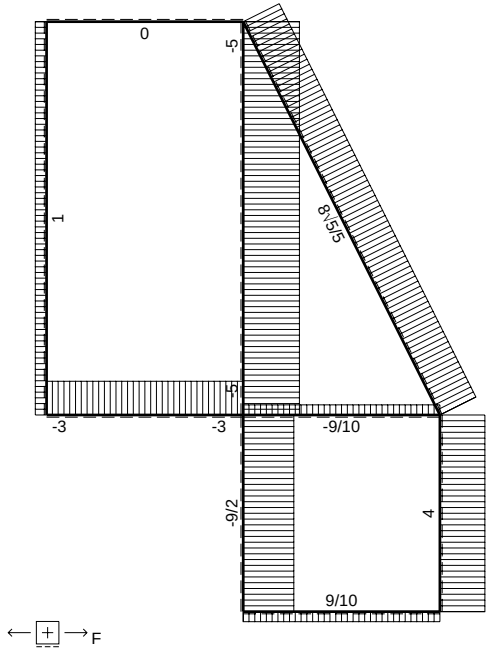
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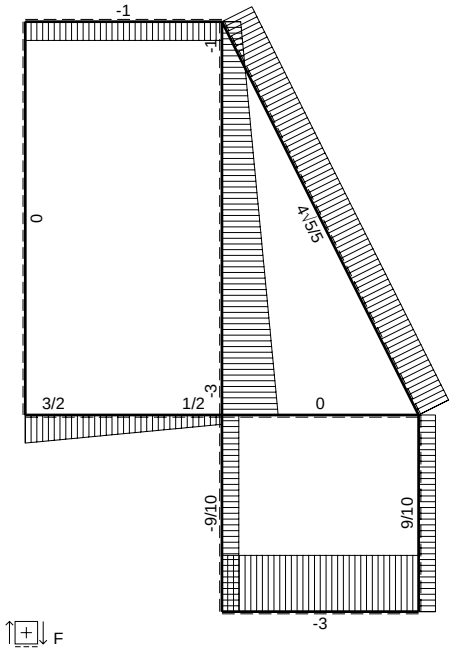
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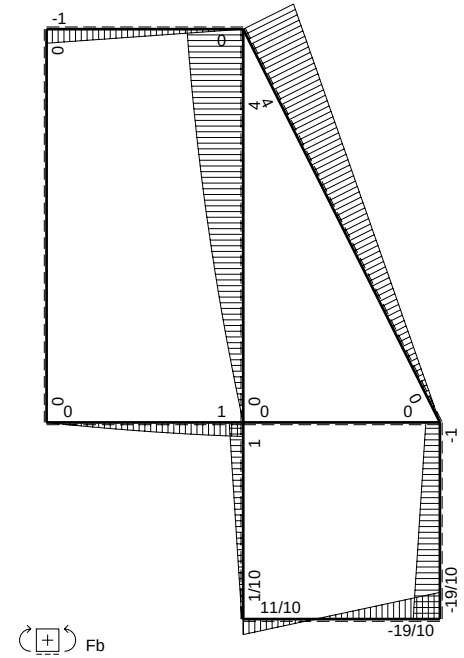
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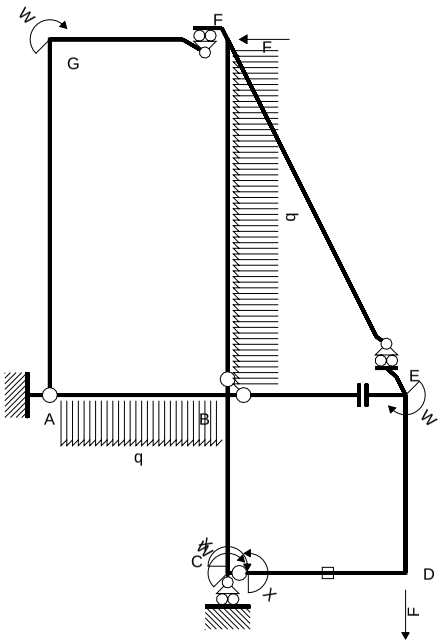
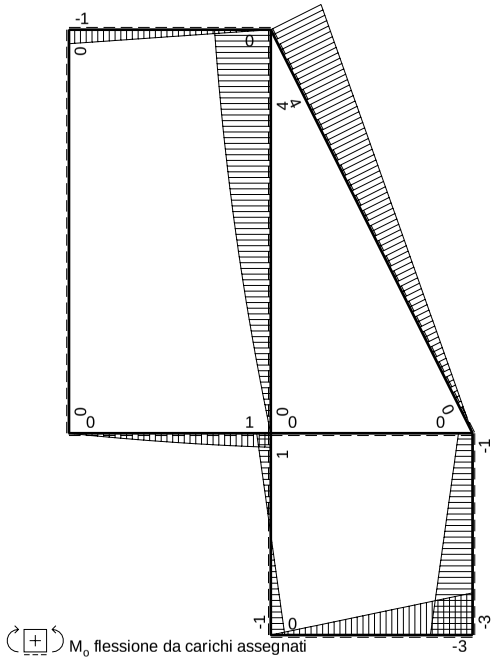
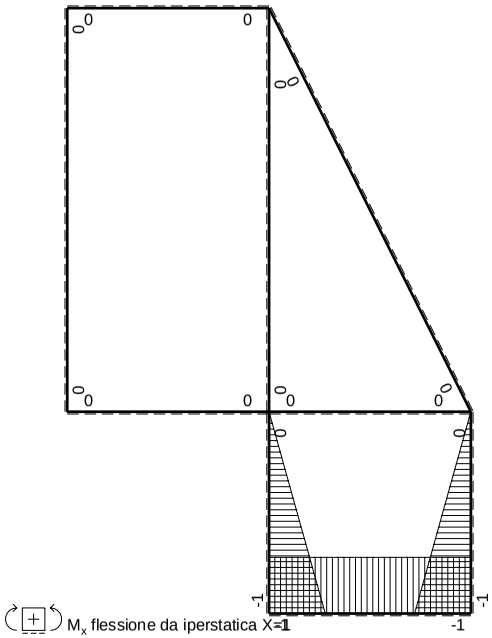
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$3/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-Fb+1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF √5b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$11/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-11/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

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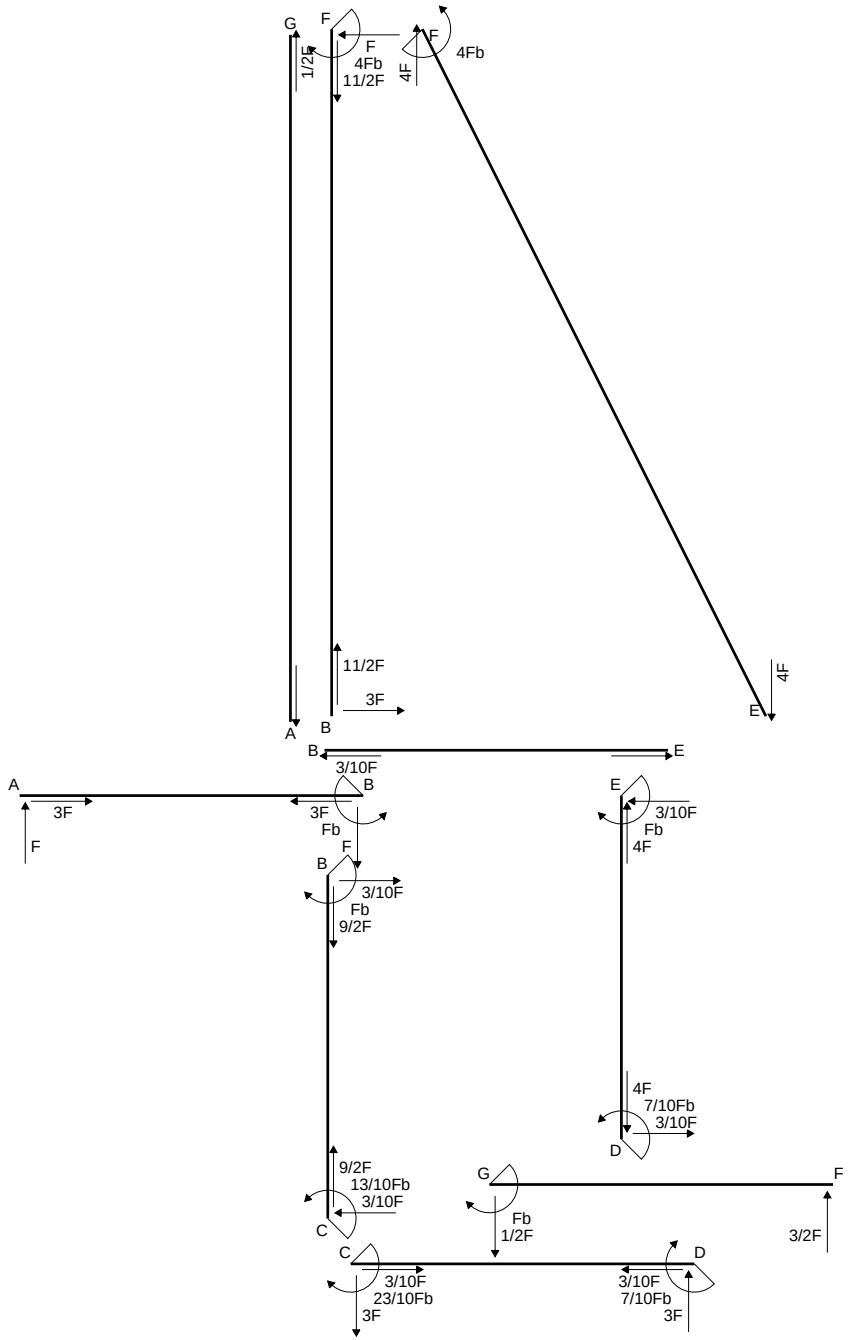
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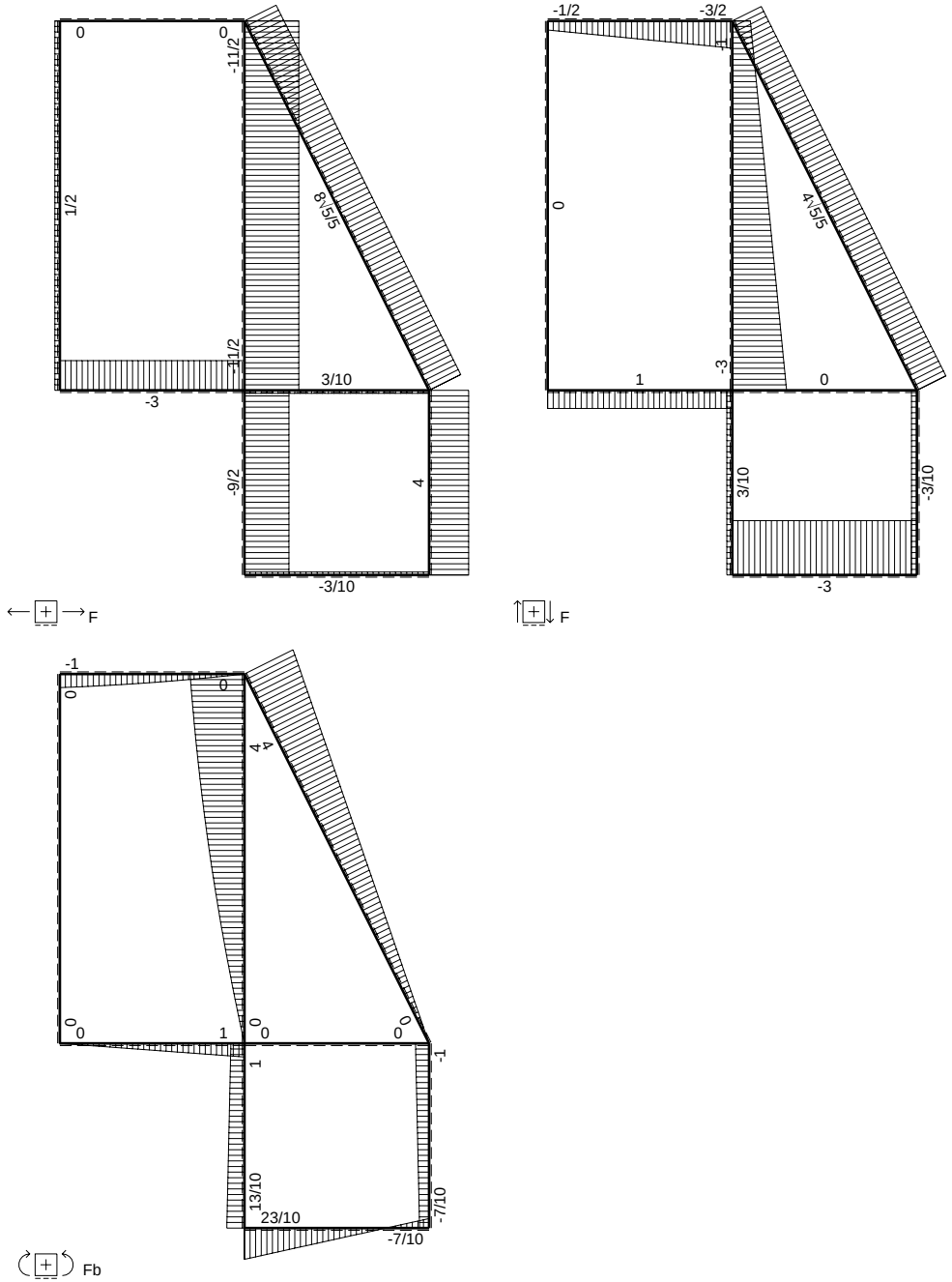
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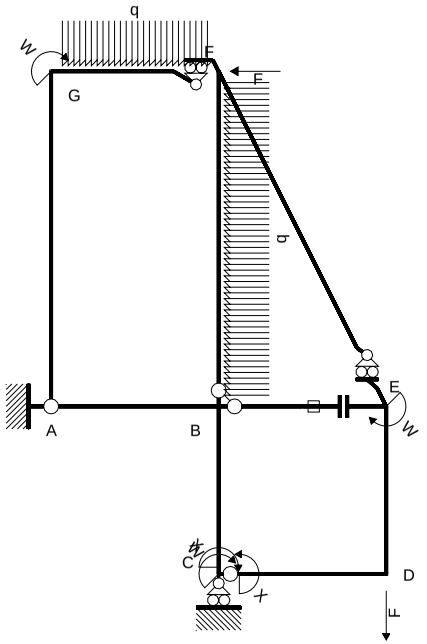
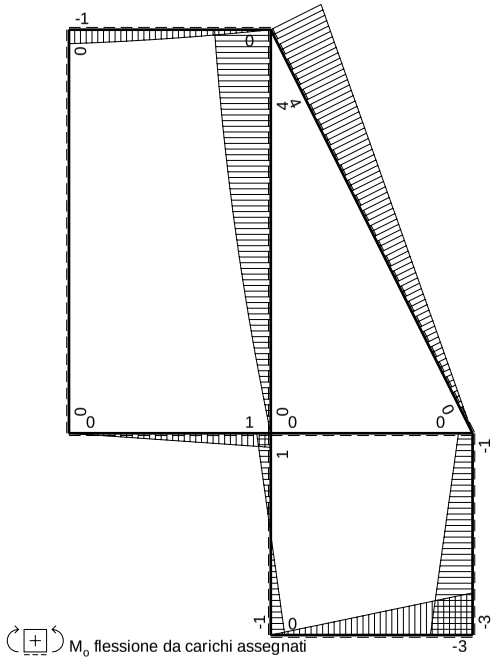
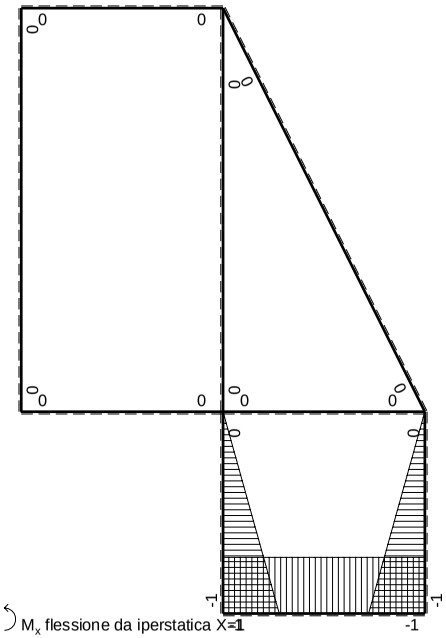


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$23/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-23/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b)\ Fb\ 1/EJ\ dx = [3/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (3/2\ b)\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b)\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b)\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

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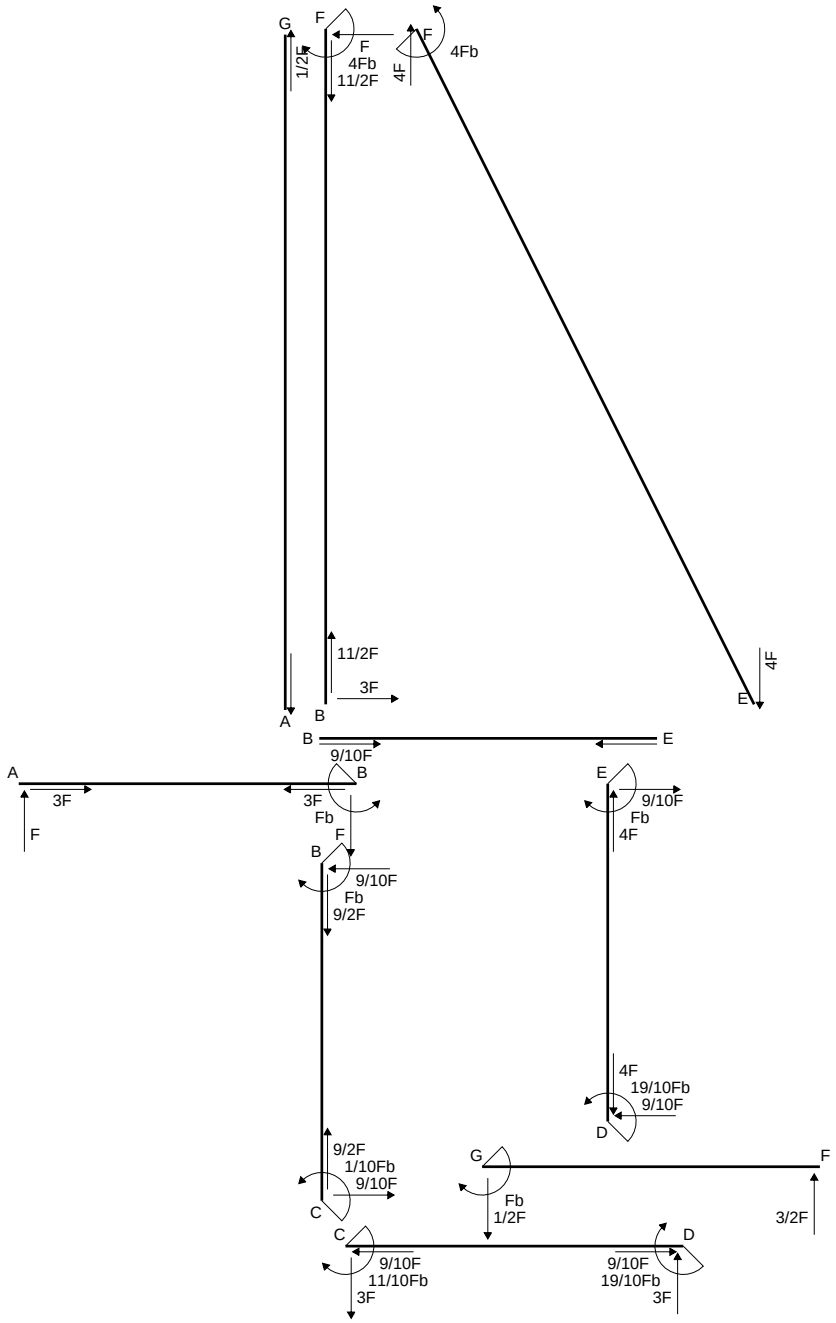
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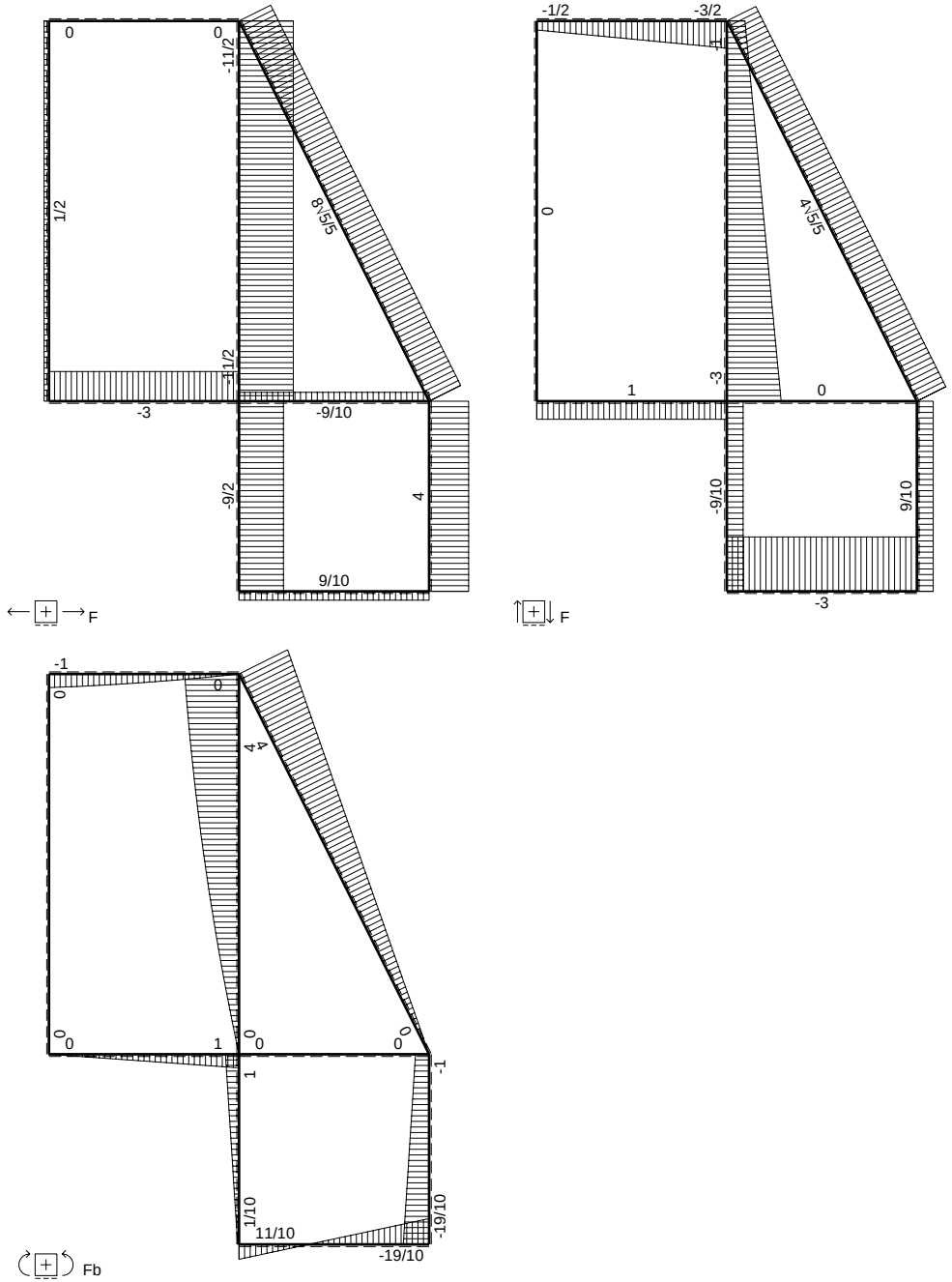
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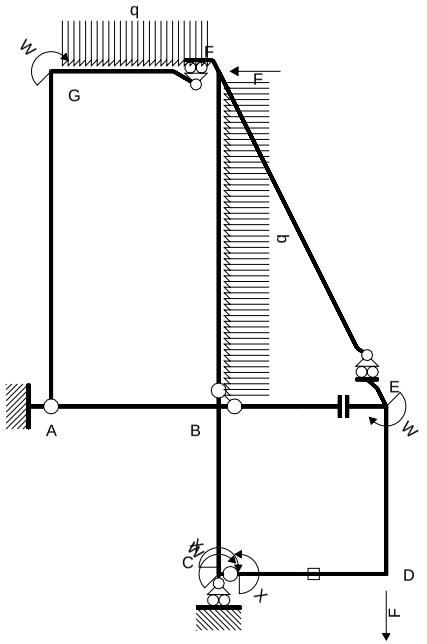
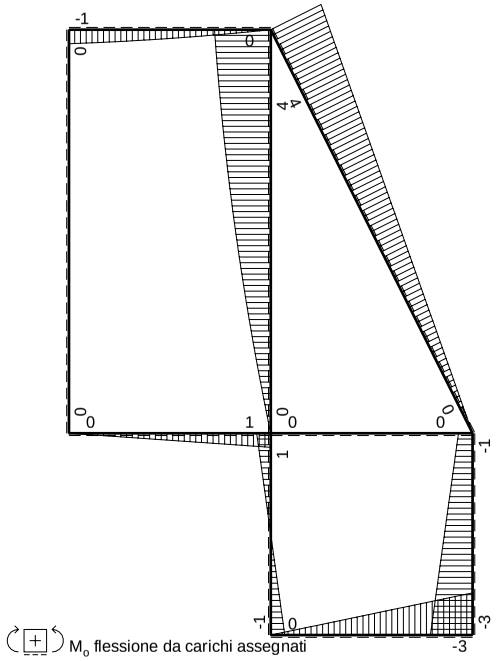
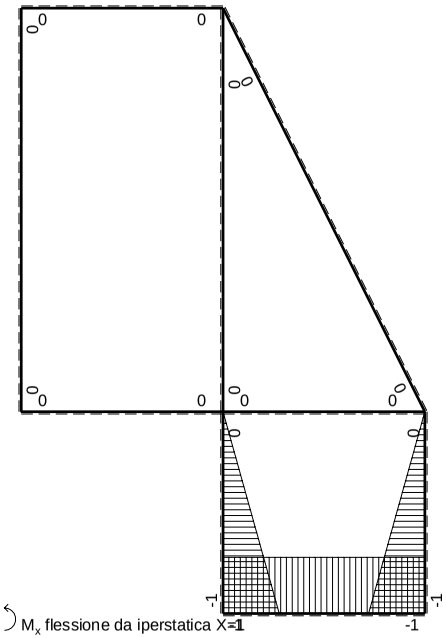


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$11/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-11/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

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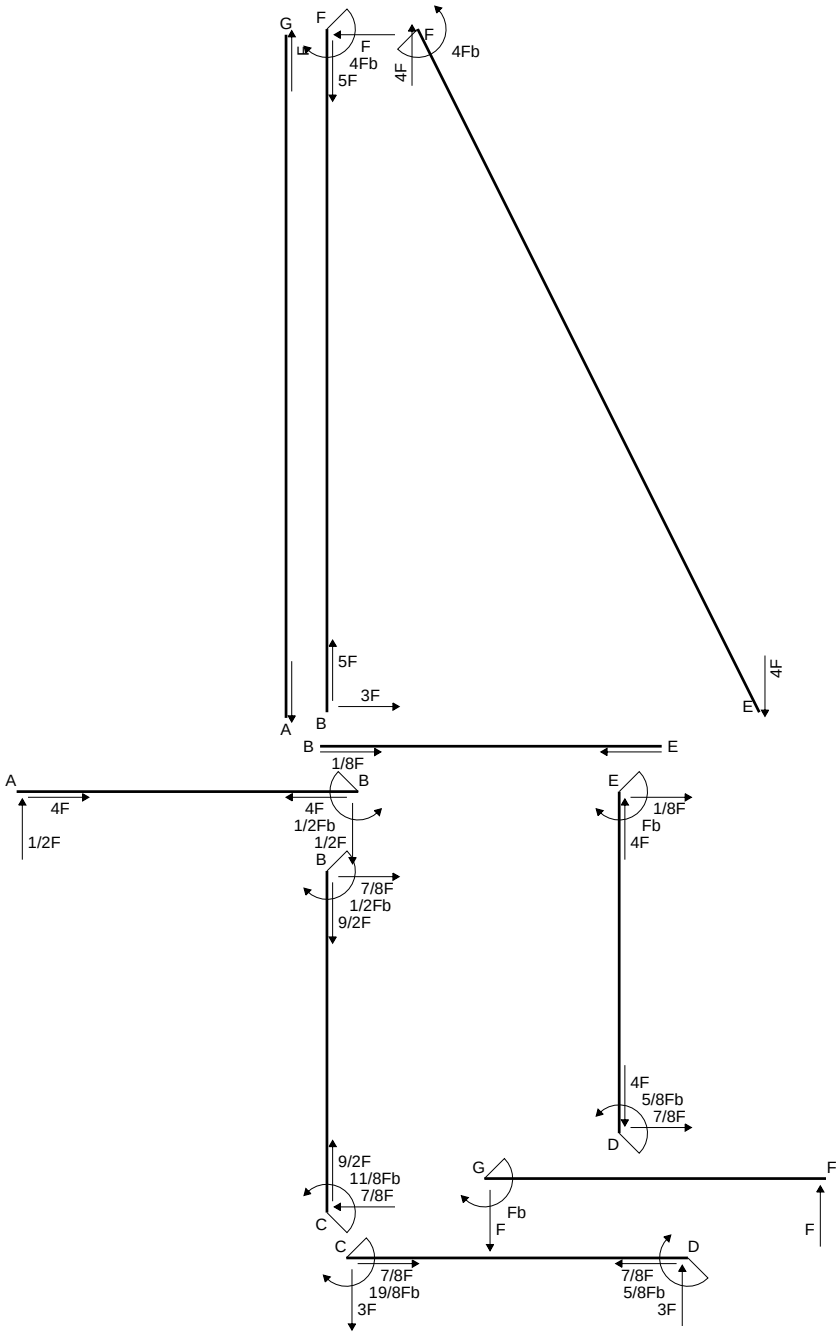
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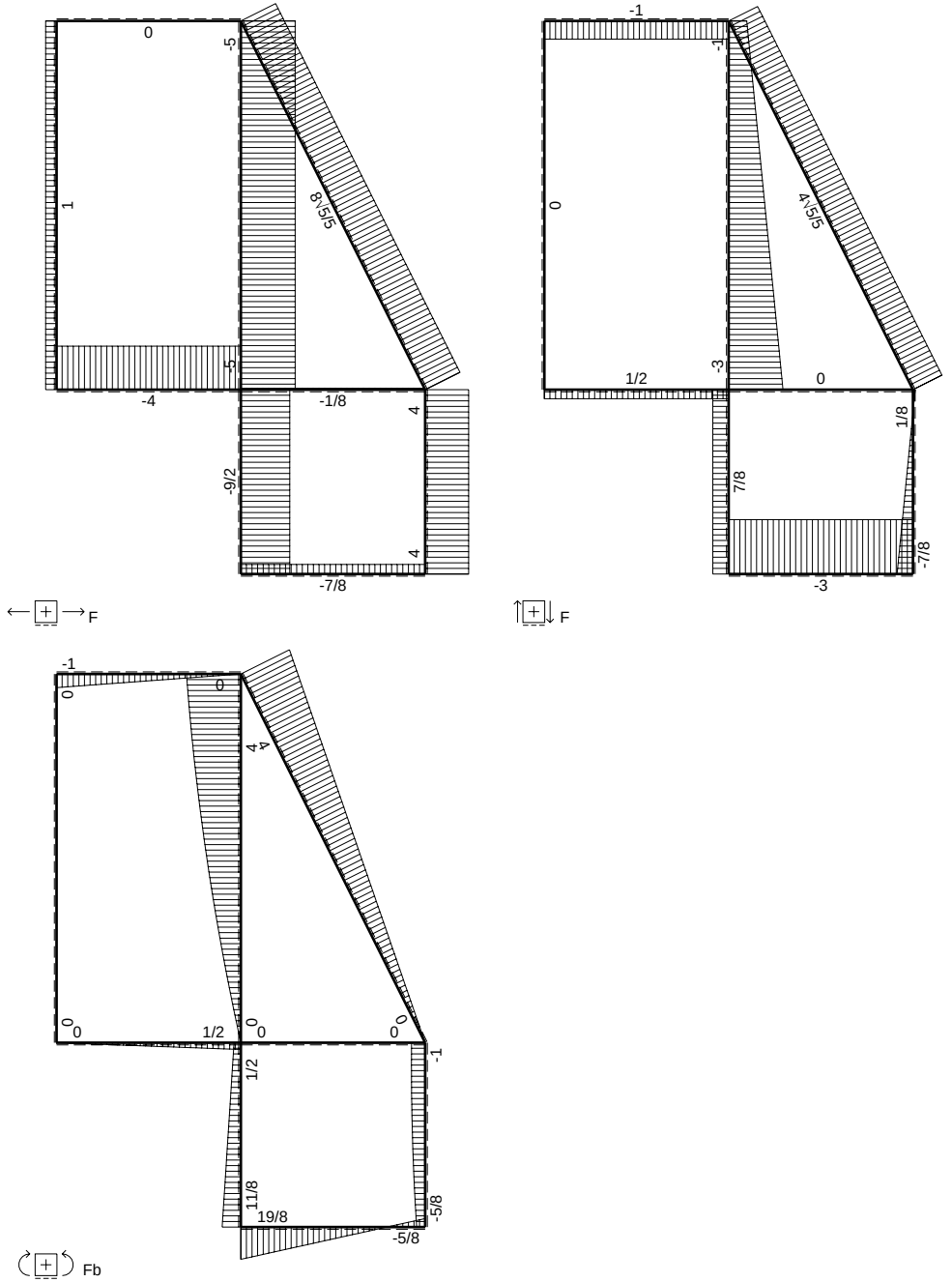
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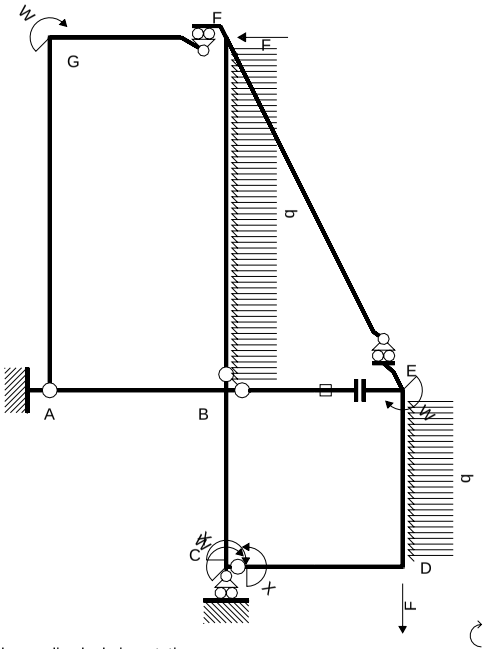
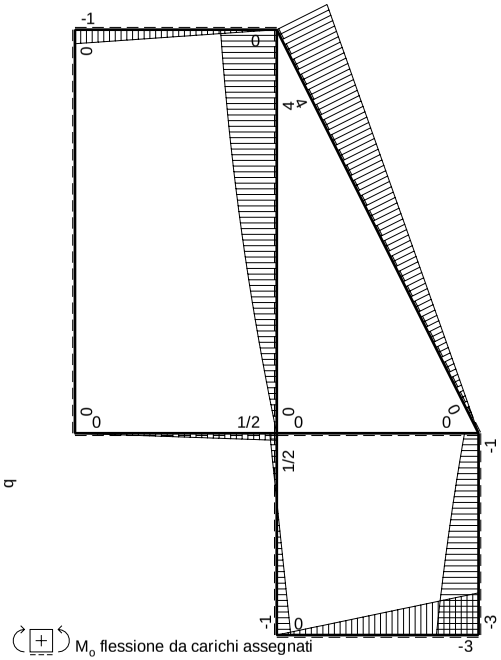
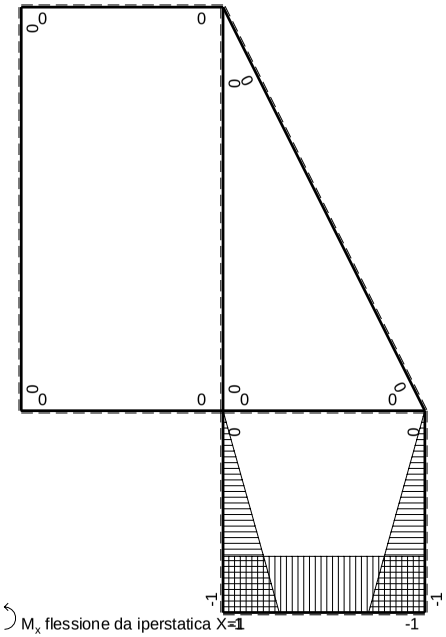


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 252216 ESΣ02.xxxx.082

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	$-3Fb+3/2Fx+1/2qx^2$	$3Fb-9/2Fx+Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$29/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+5/2Fx-1/2qx^2$	$Fx+5/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$95/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-19/8Fb	

PROCEDIMENTO E RISULTATI 252216 ESΣ02.xxxx.082

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 01.12.25

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -9/2\ x/b + x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [3\ x -9/4\ x^2/b +1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -9/4\ b +1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 29/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +5/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +5/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +5/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 29/24\ Fb^2/EJ$

ESΣ02.xxxx.082

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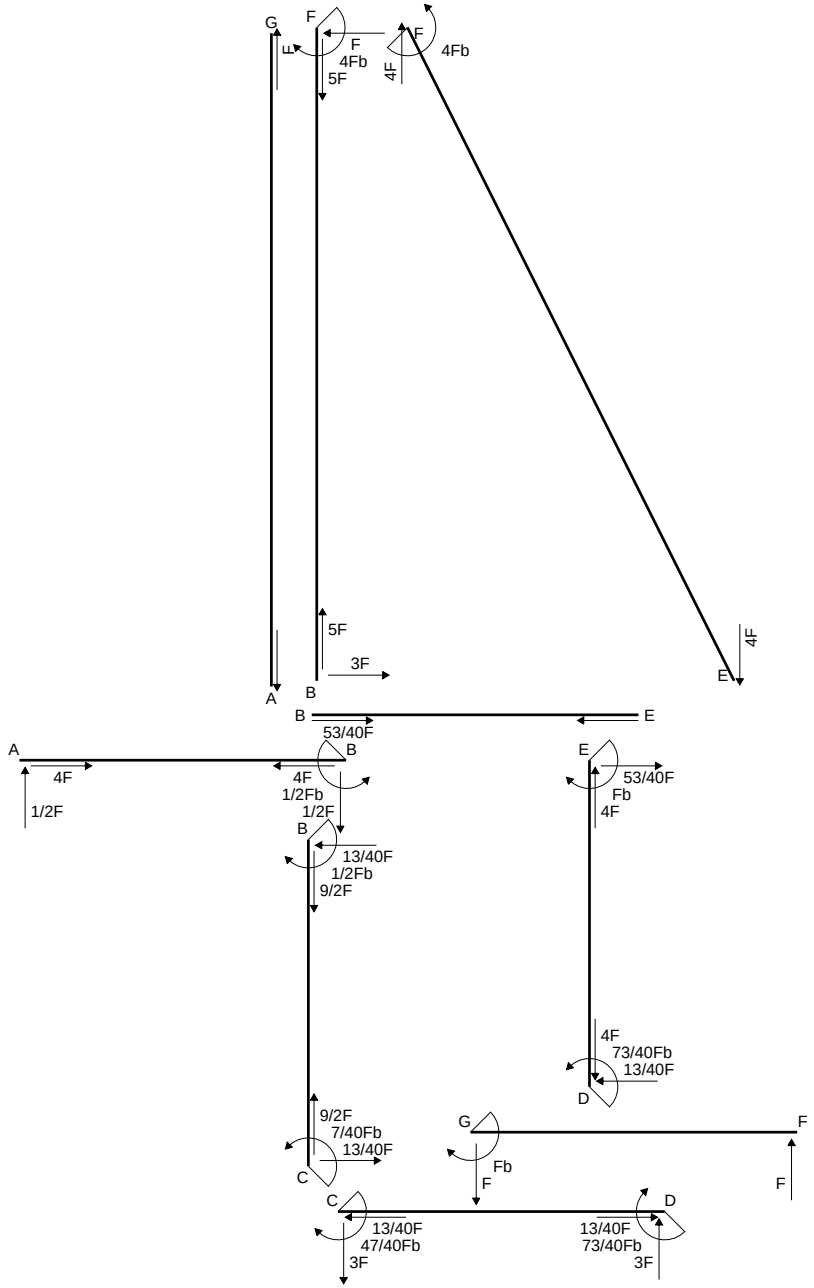
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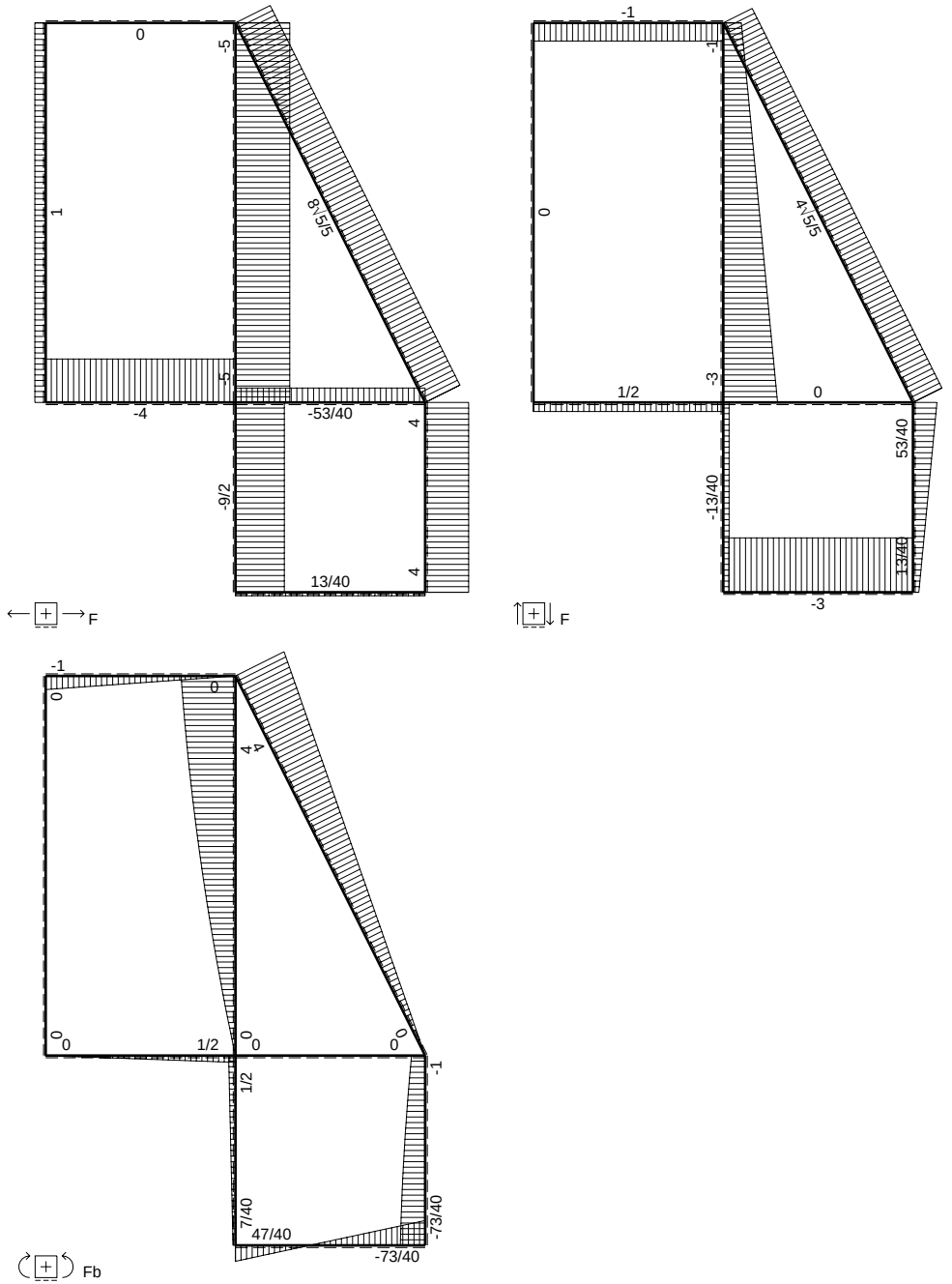
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F
-1
0
0
4√5
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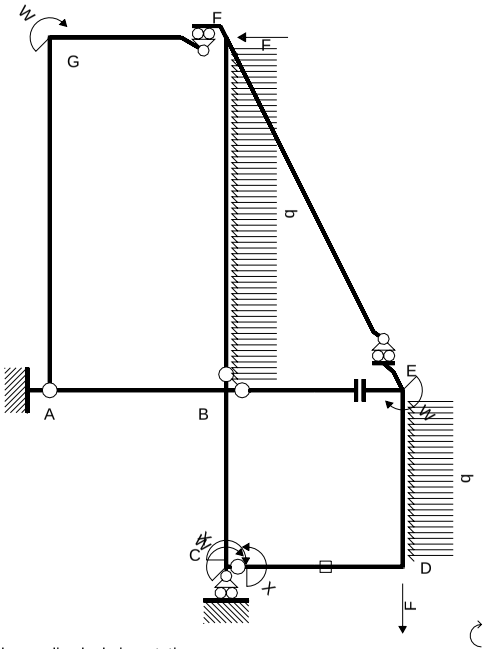
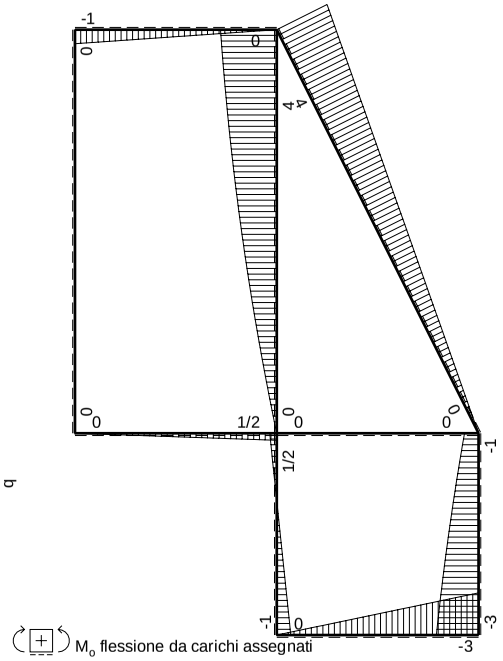
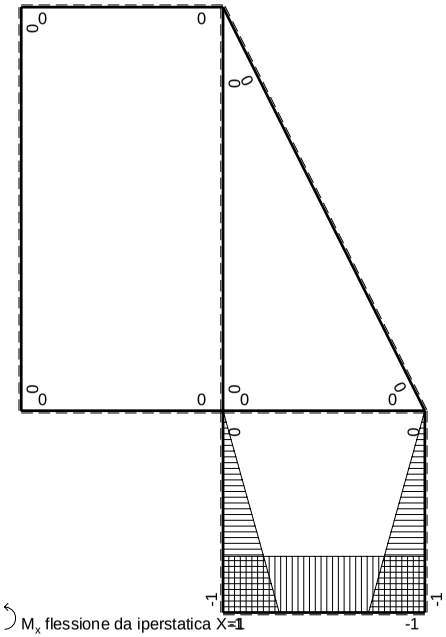
-73/40
-73/40
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236511 ESΣ02.xxxx.083

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	$-3Fb+3/2Fx+1/2qx^2$	$3Fb-9/2Fx+Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$29/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+5/2Fx-1/2qx^2$	$Fx+5/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$47/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-47/40Fb	

PROCEDIMENTO E RISULTATI 236511 ESΣ02.xxxx.083

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 01.12.25

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -9/2\ x/b + x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [3\ x -9/4\ x^2/b +1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -9/4\ b +1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 29/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +5/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +5/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +5/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 29/24\ \ Fb^2/EJ$

ESΣ02.xxxx.083

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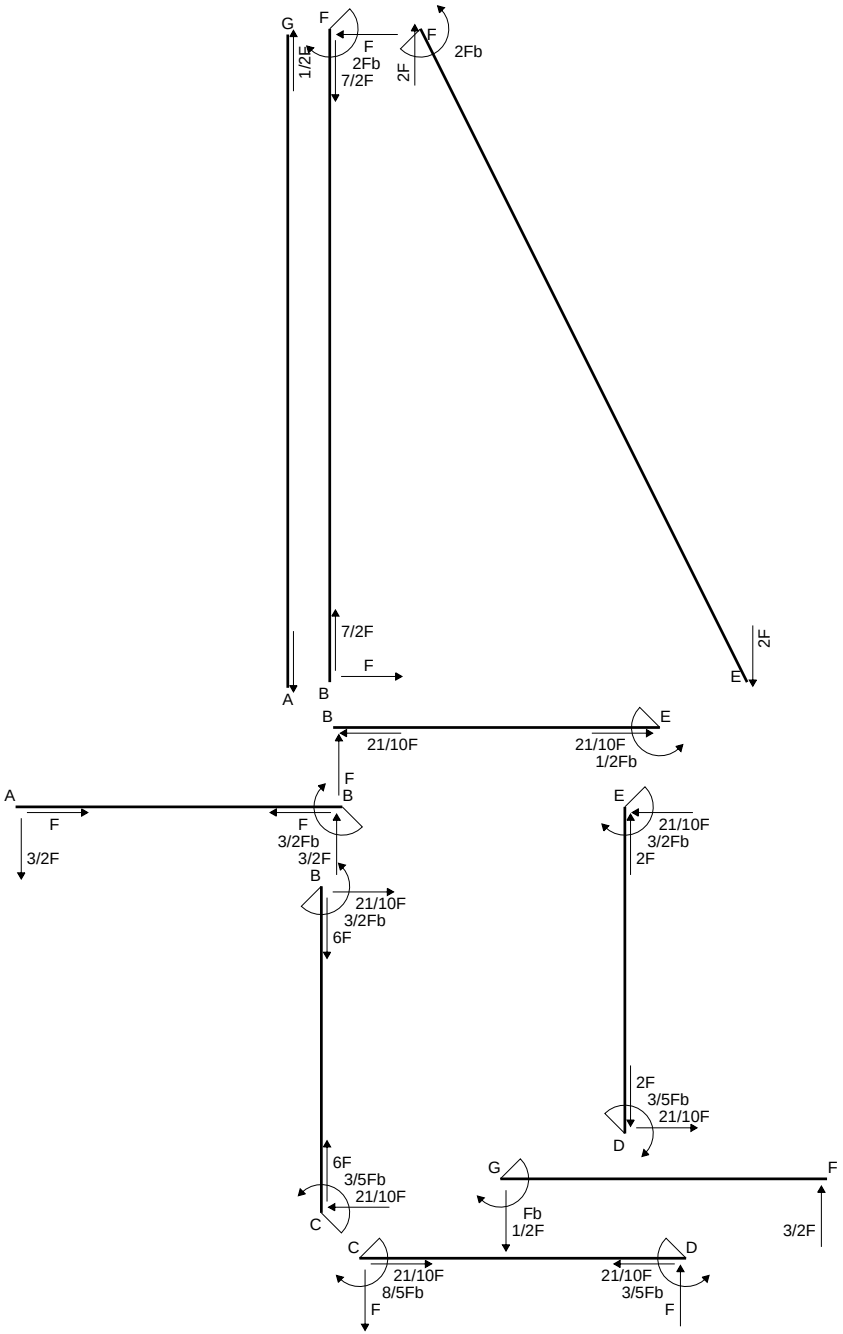
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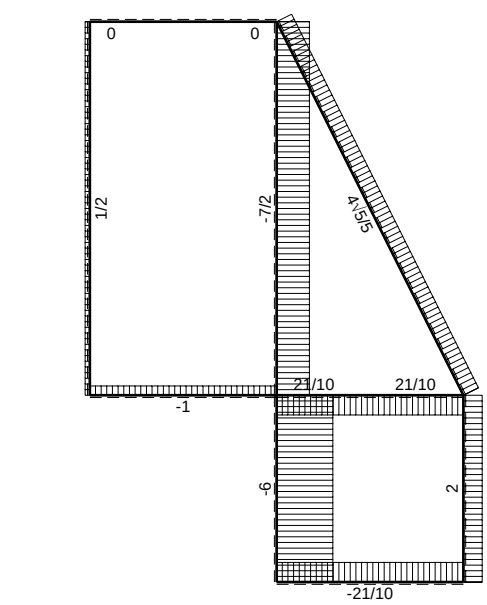
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2
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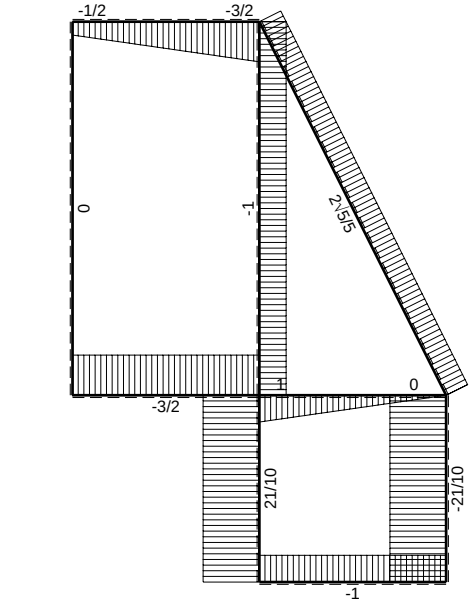
F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1
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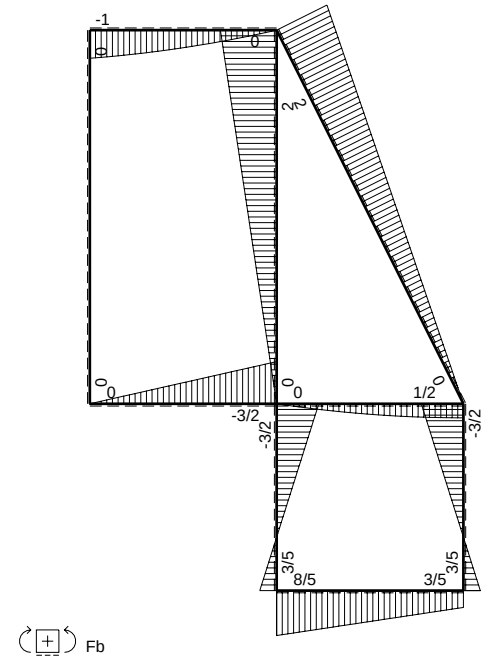
-21/10
-1
F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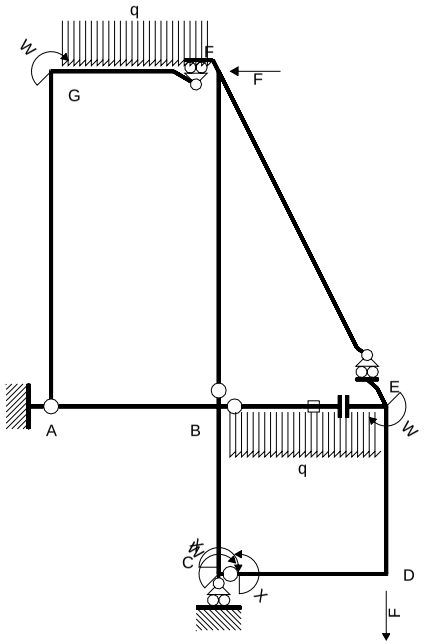
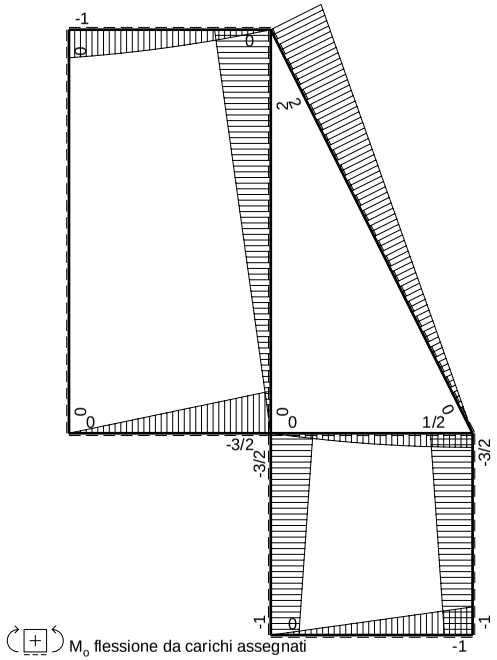
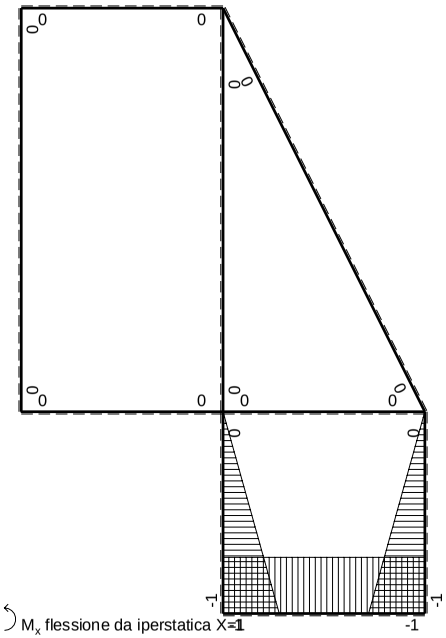
8/5
3/5
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fb-1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-3/2Fx+1/2qx^2	0	0	0	0
GF b	0	Fb-1/2Fx-1/2qx^2	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				8/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-8/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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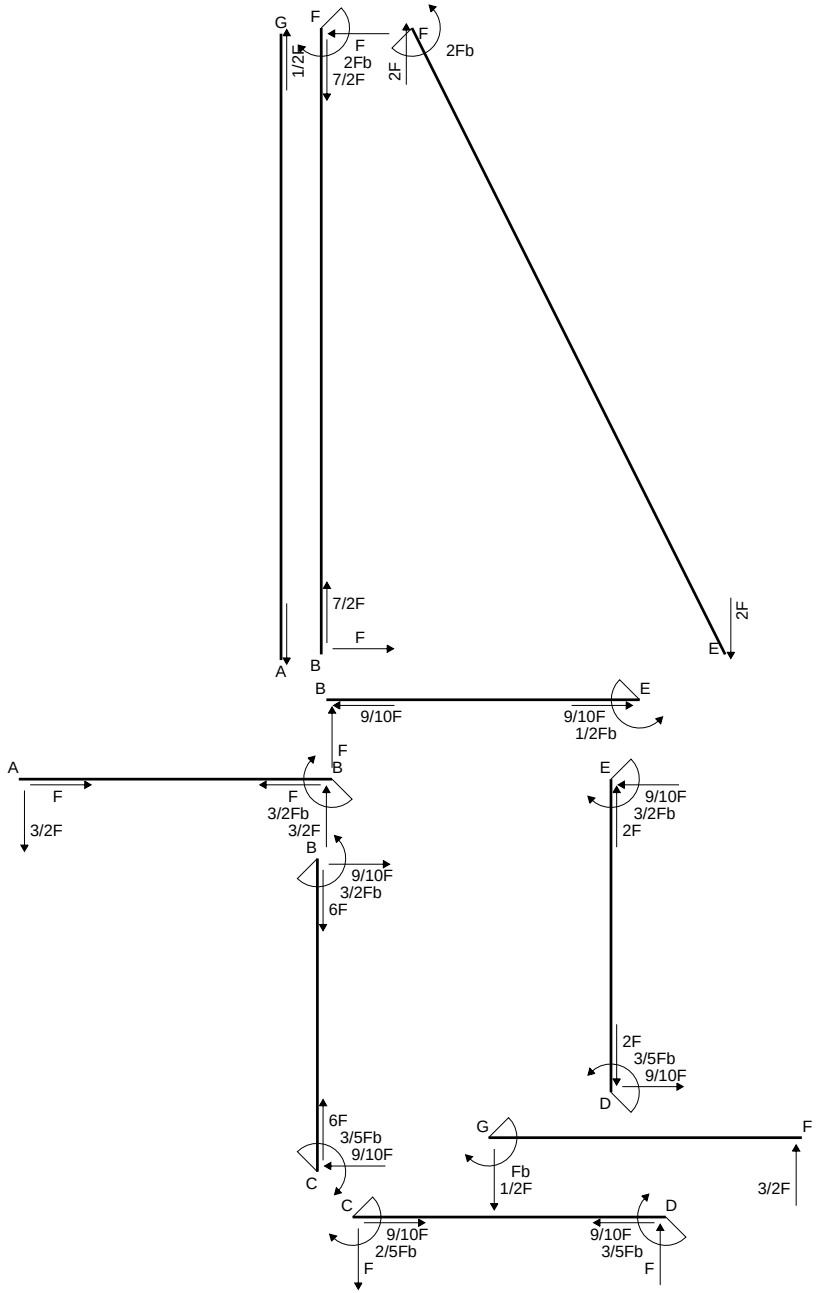
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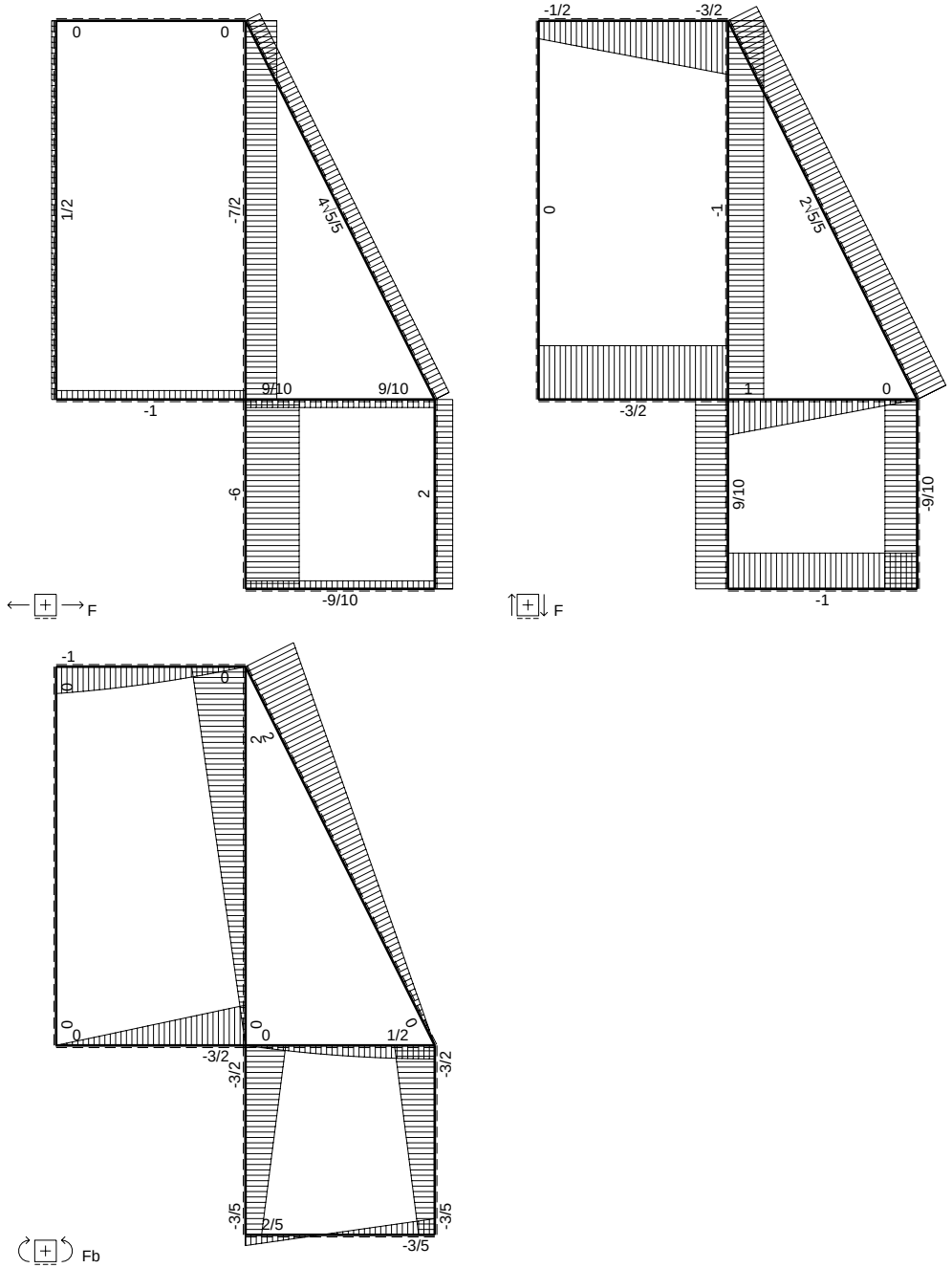
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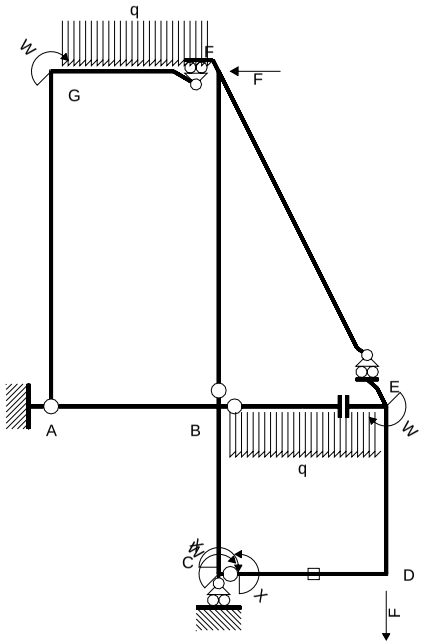
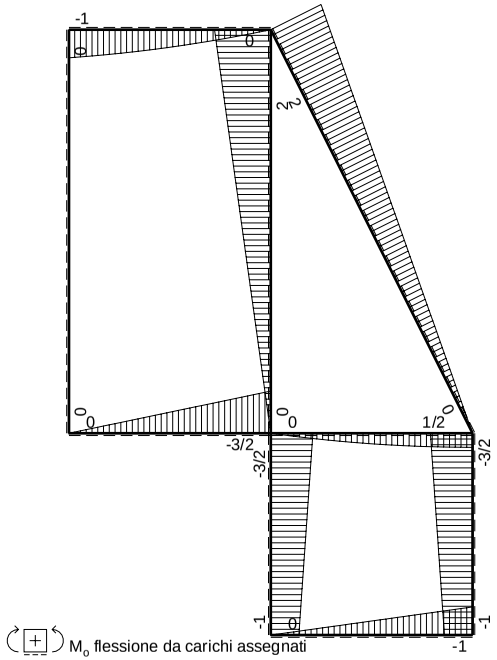
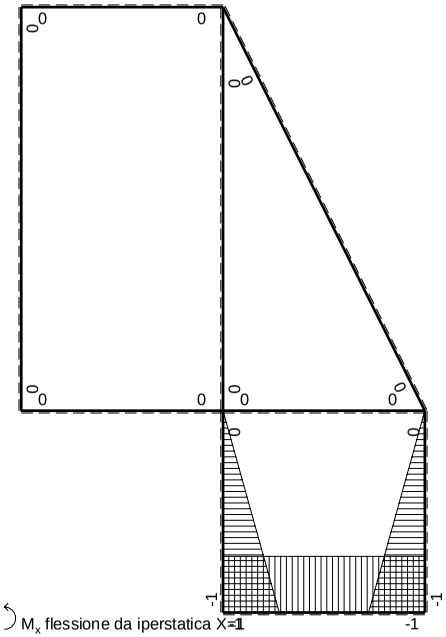


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fb-1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-3/2Fx+1/2qx^2	0	0	0	0
GF b	0	Fb-1/2Fx-1/2qx^2	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				2/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-2/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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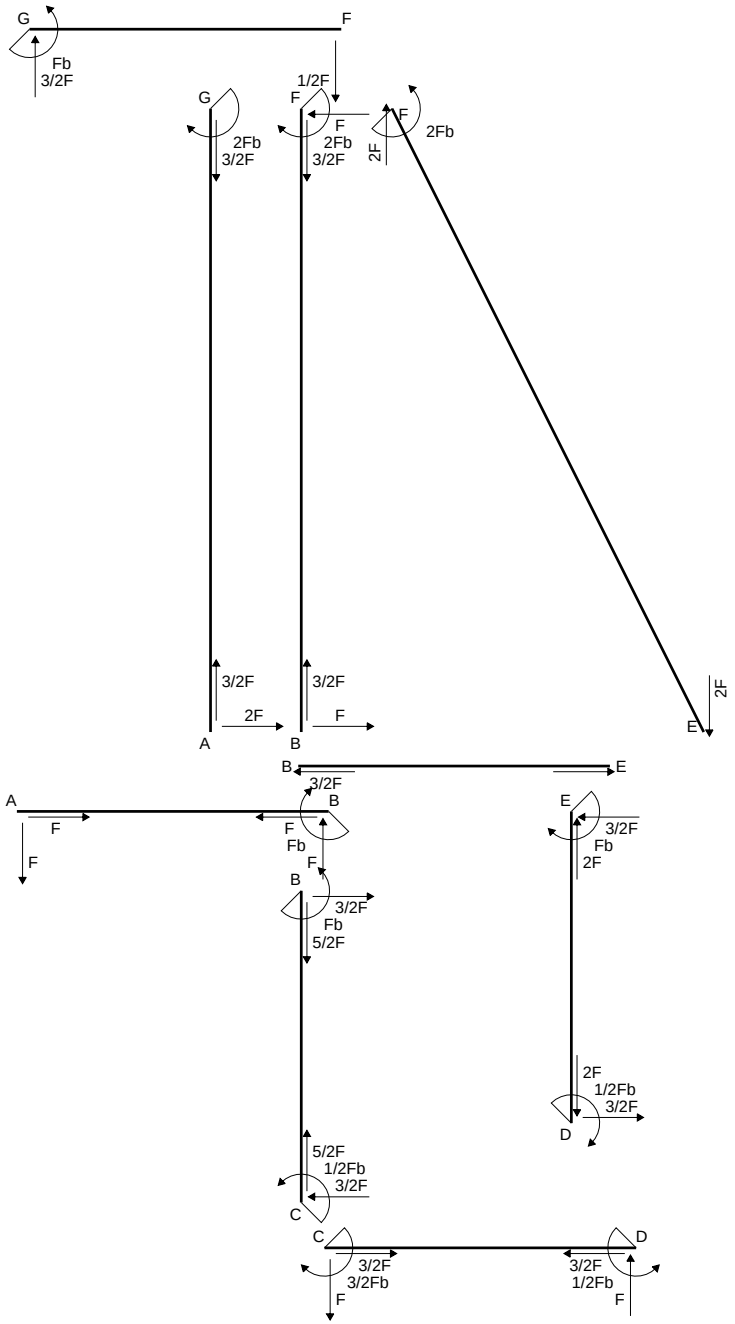
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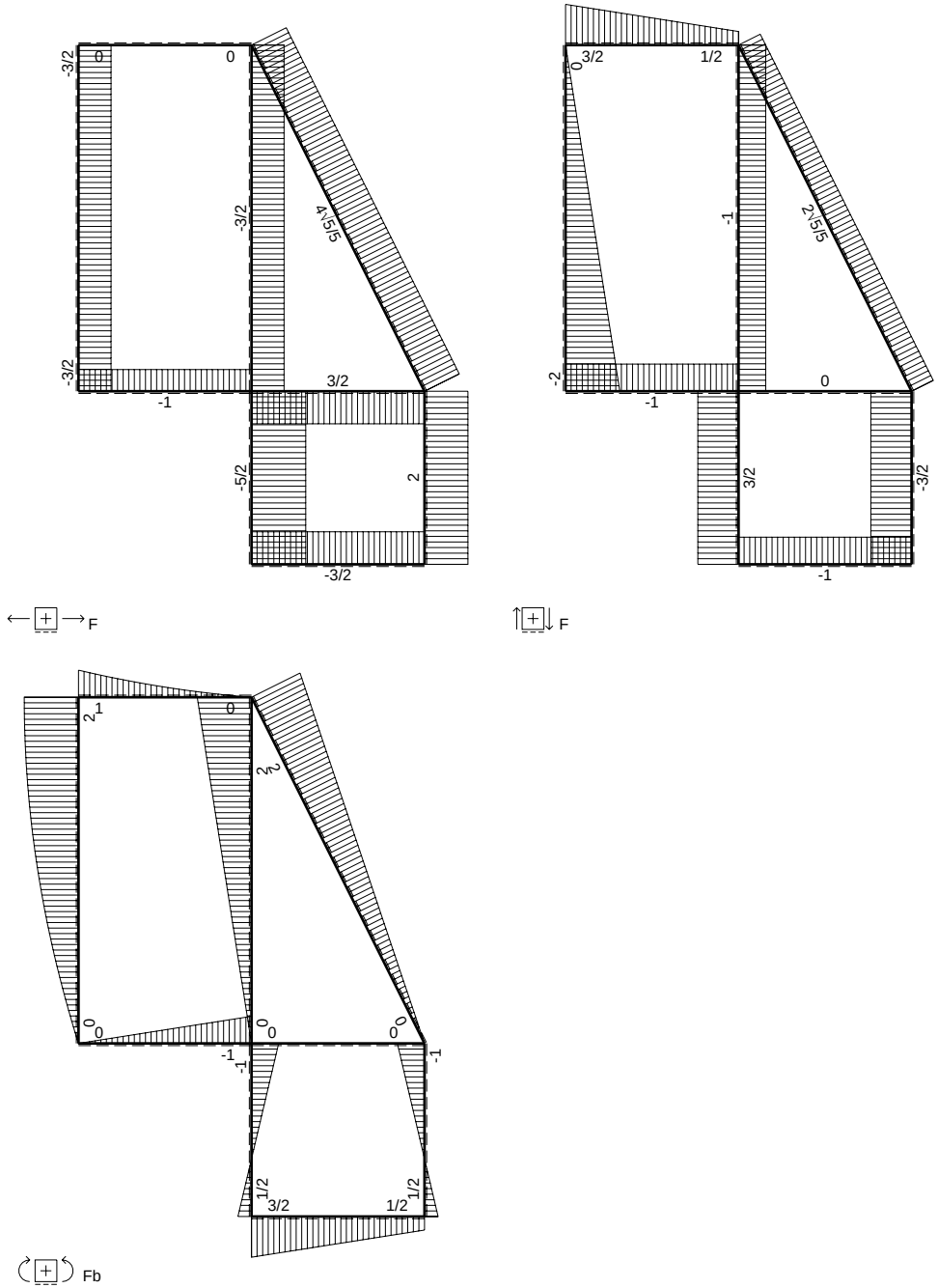
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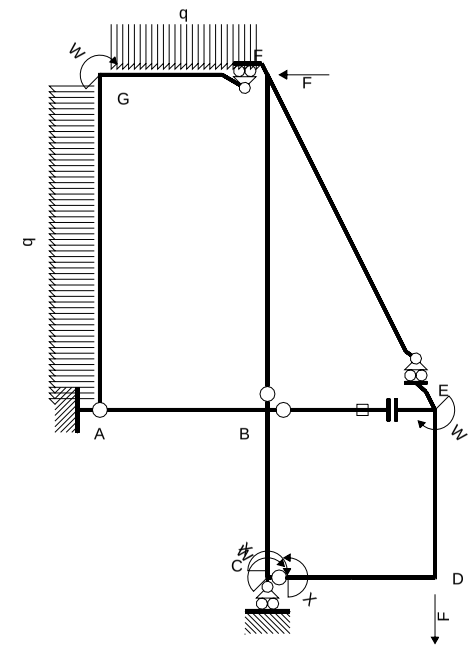
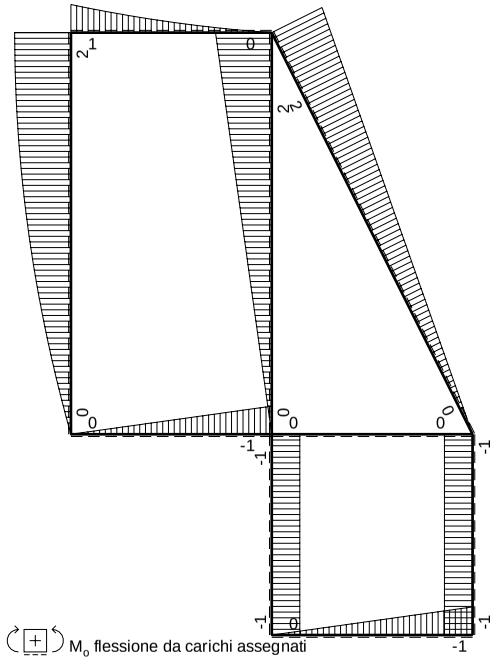
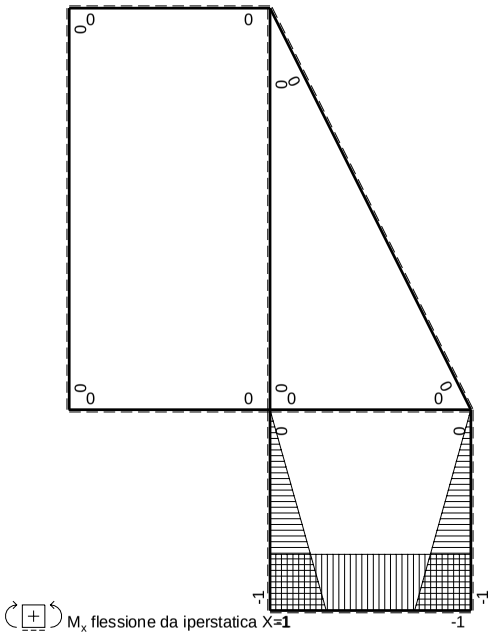


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=**1**

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$-Fb+3/2Fx-1/2qx^2$	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$5/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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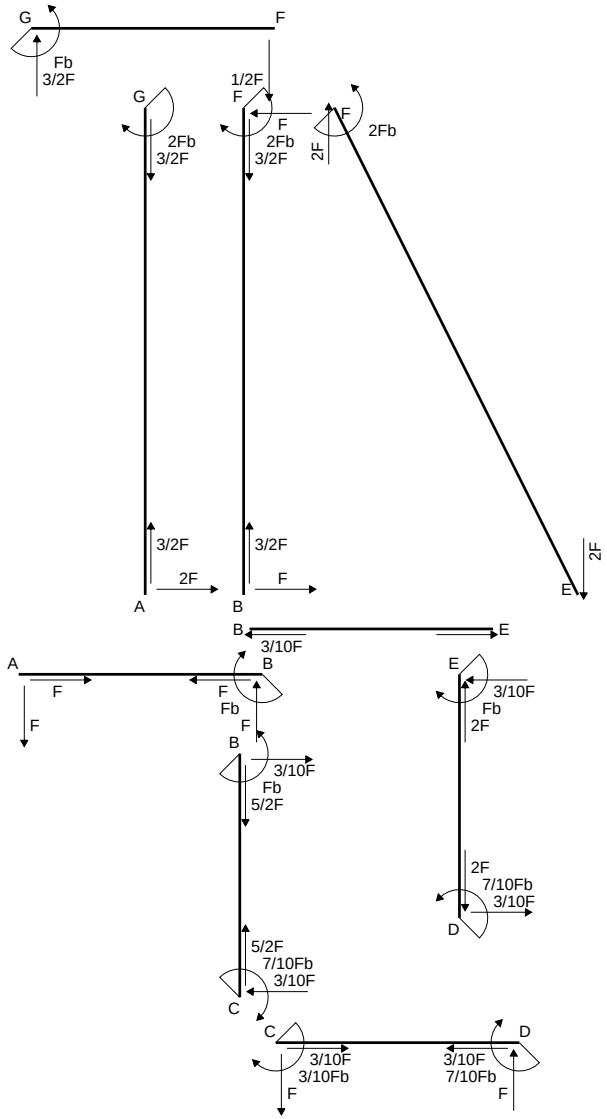
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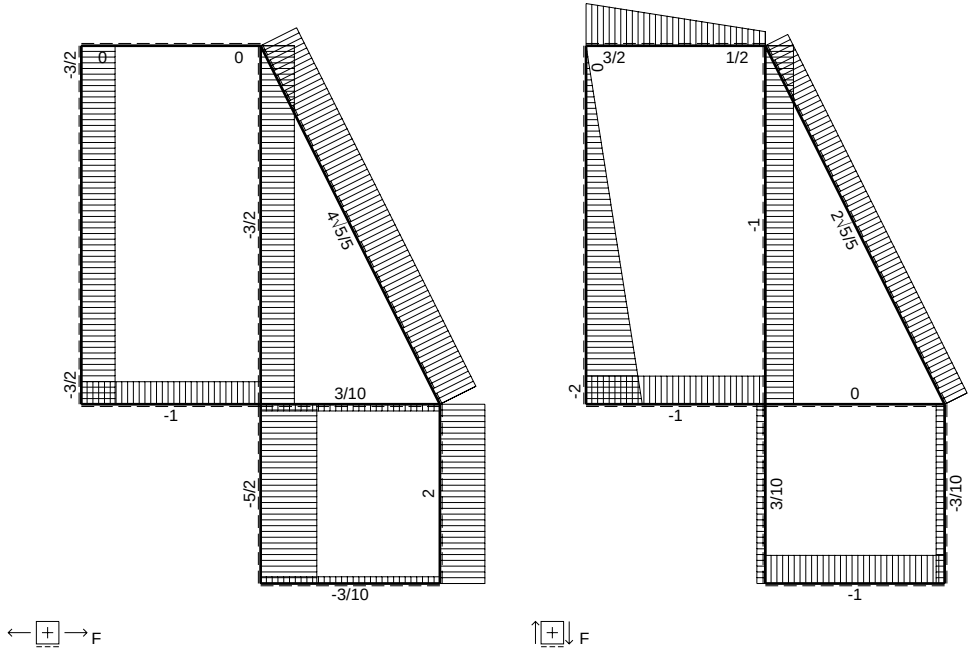
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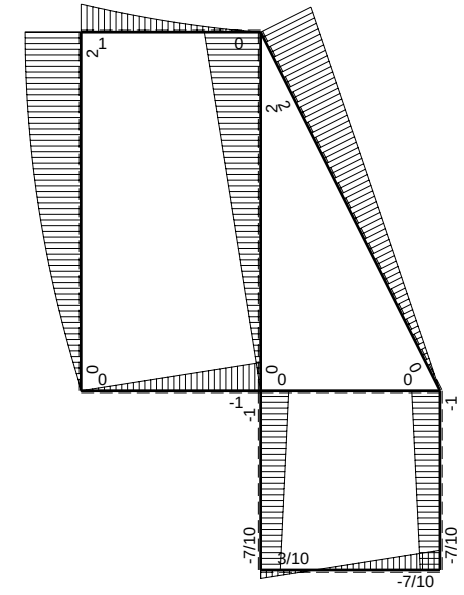
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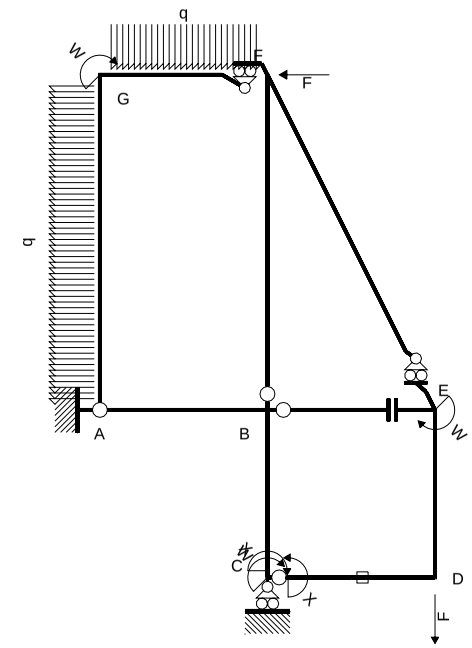
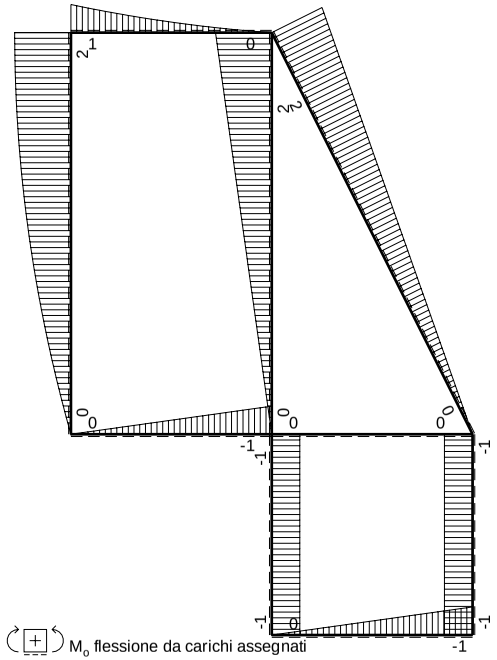
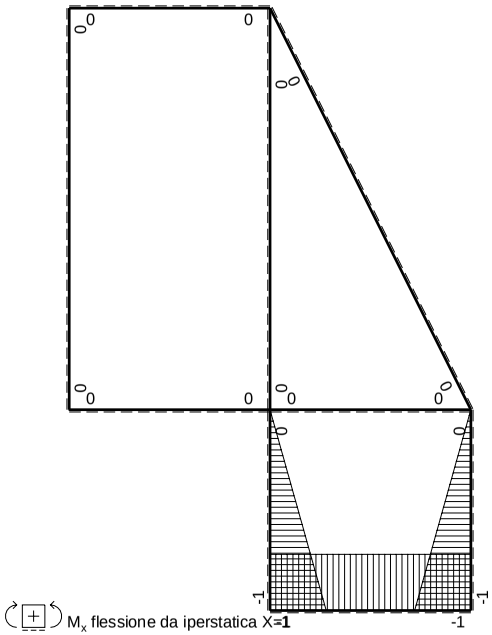
Fb



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$-Fb+3/2Fx-1/2qx^2$	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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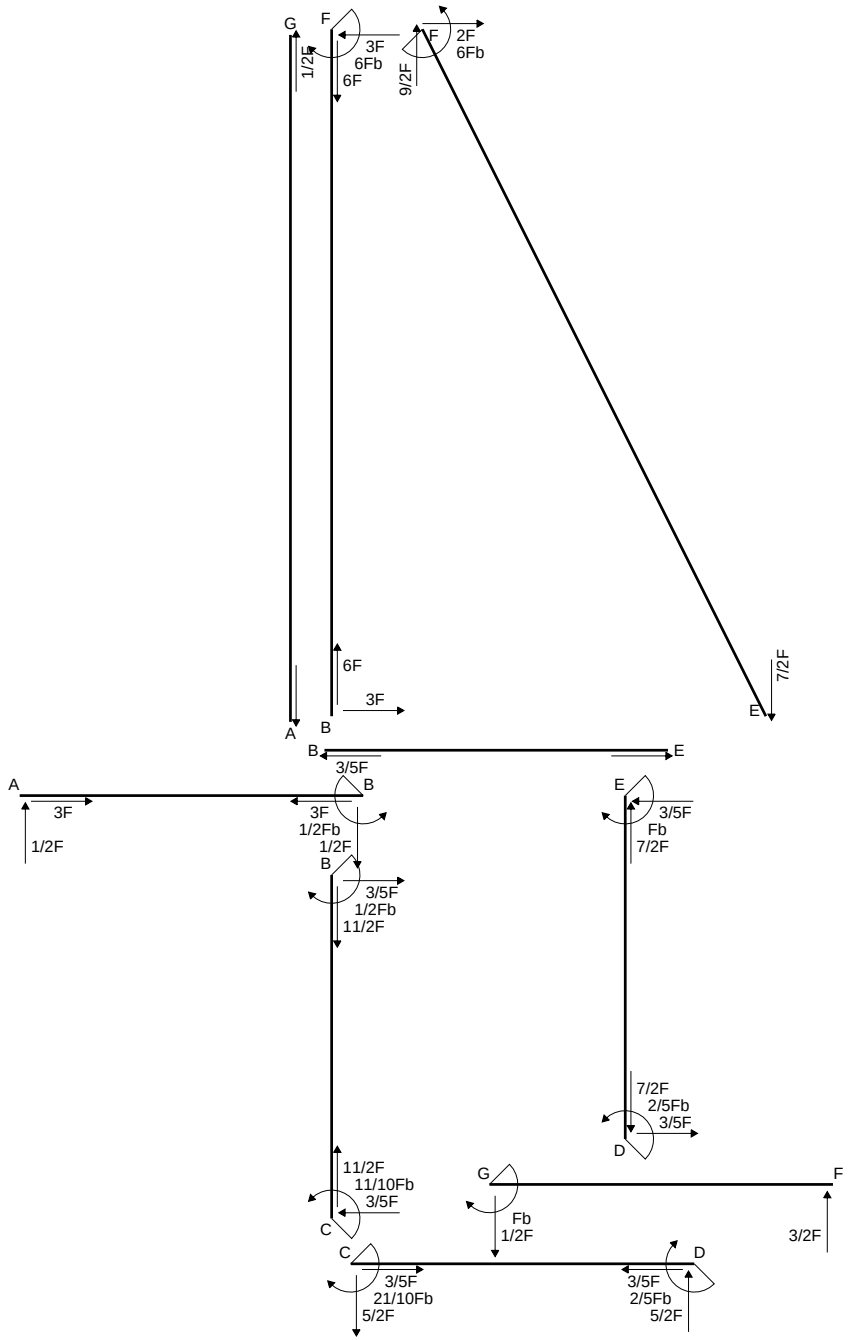
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2/5Fb
3/5F
D
11/2F
11/10Fb
3/5F
C
G
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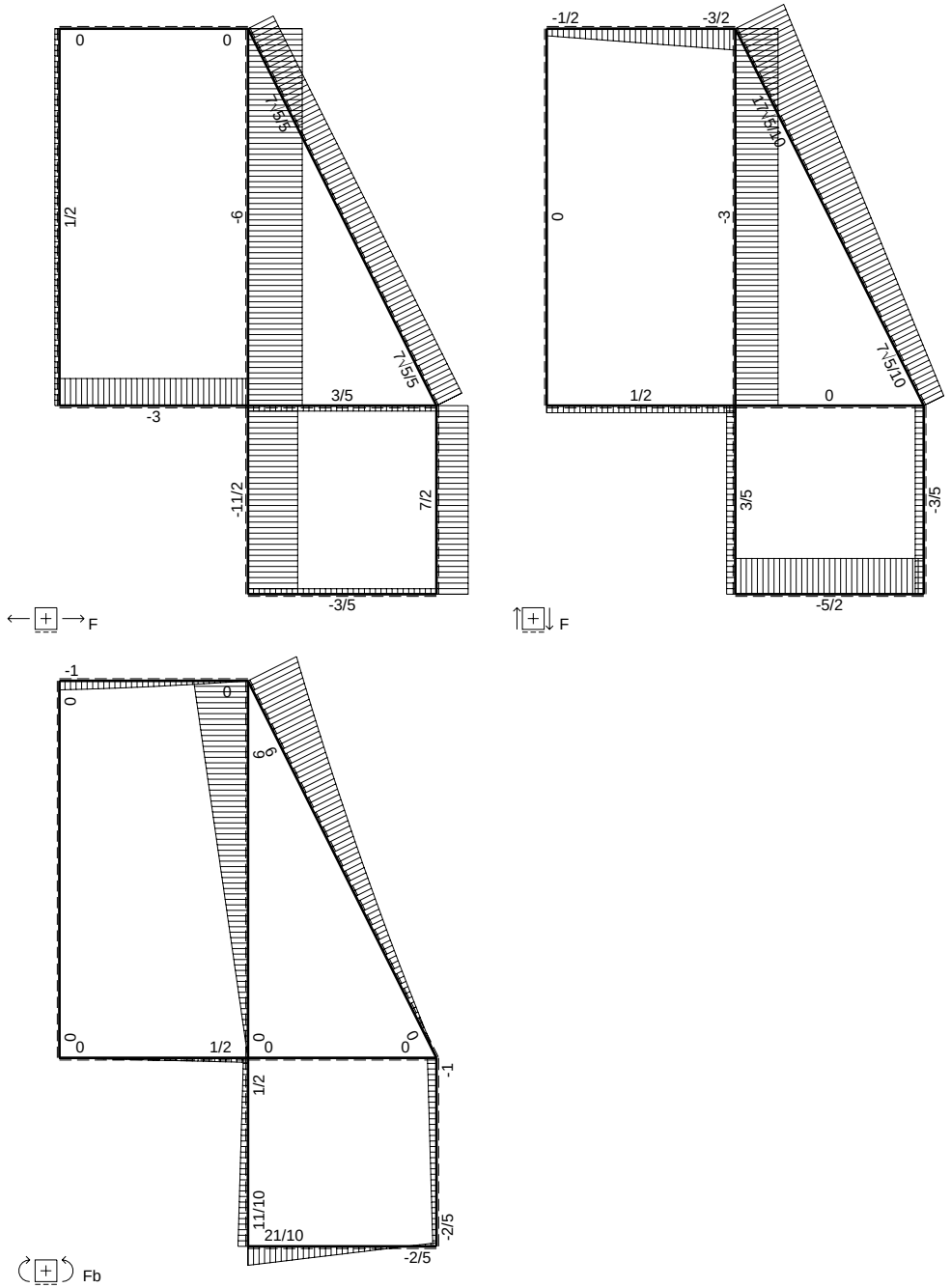
5/2F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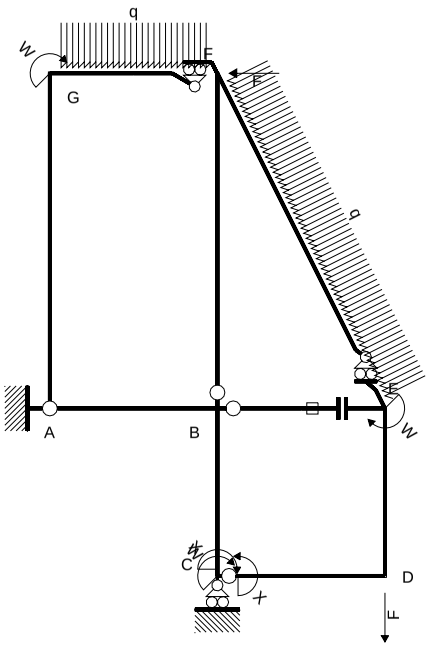
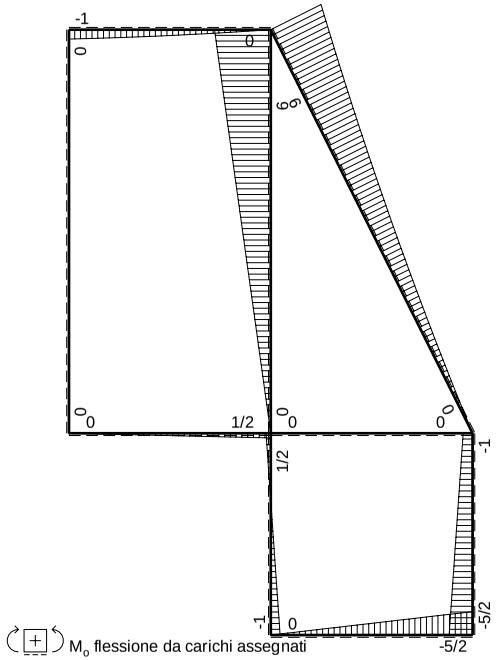
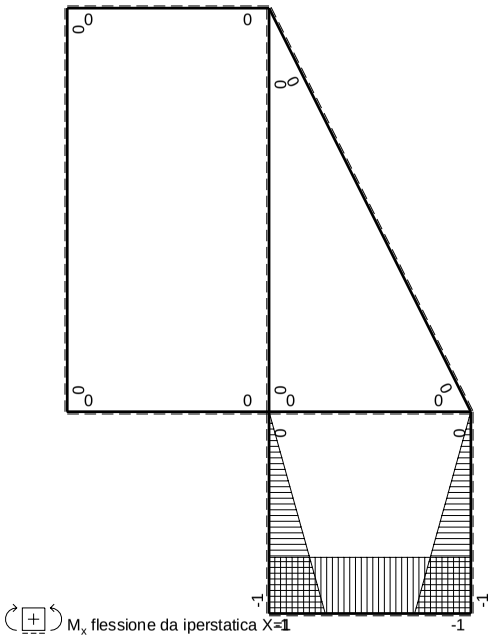
-2/5
-2/5
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

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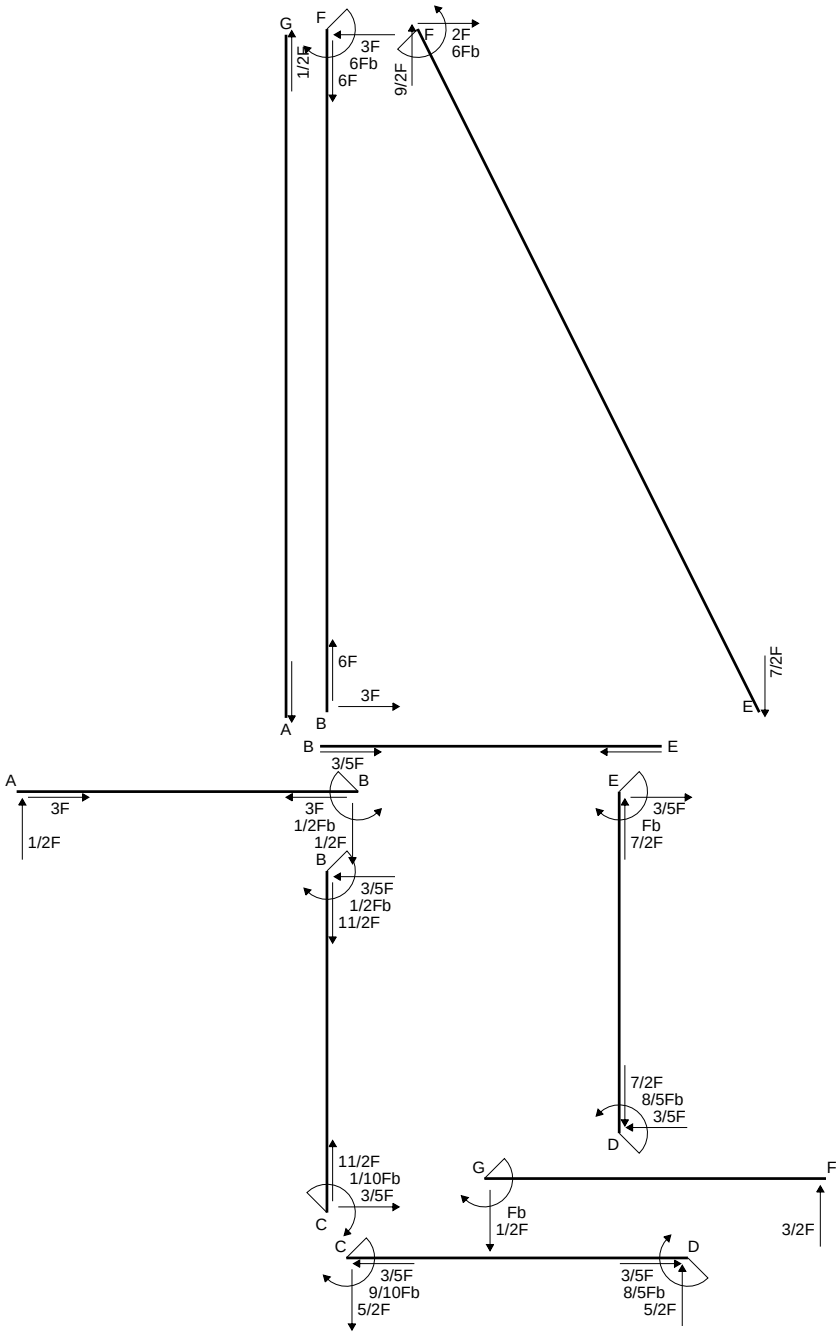
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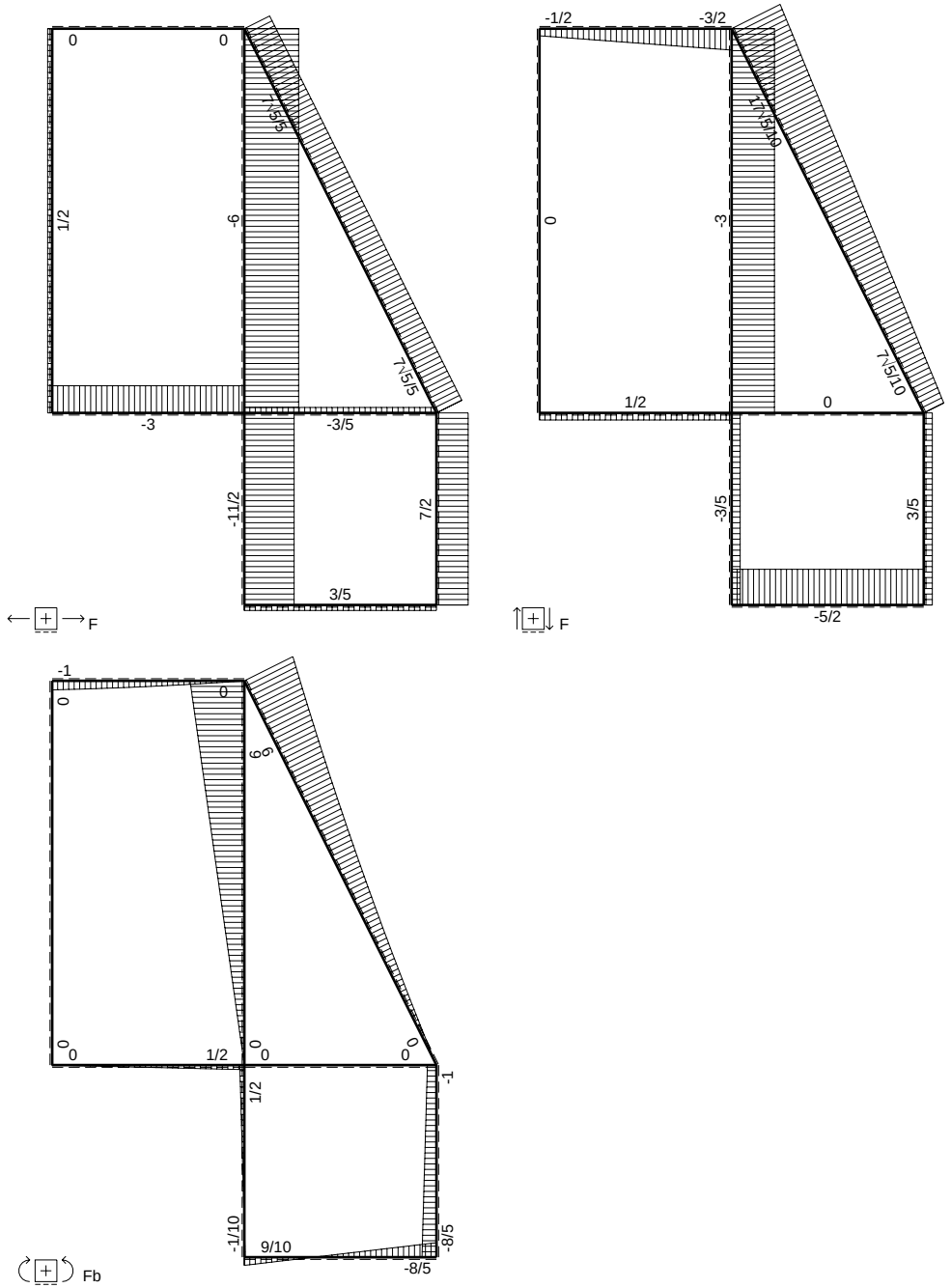
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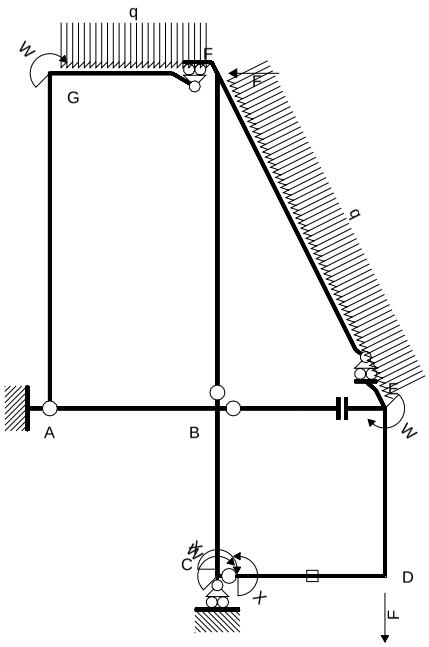
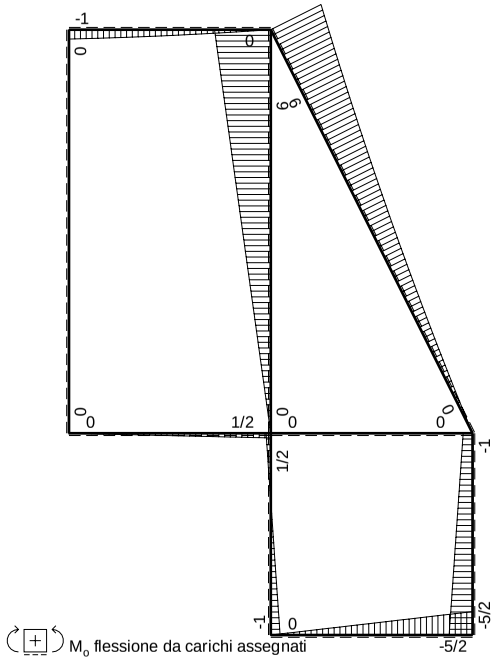
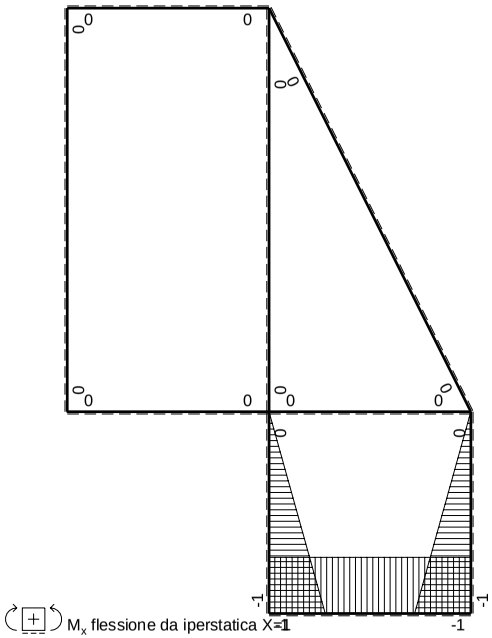
-8/5
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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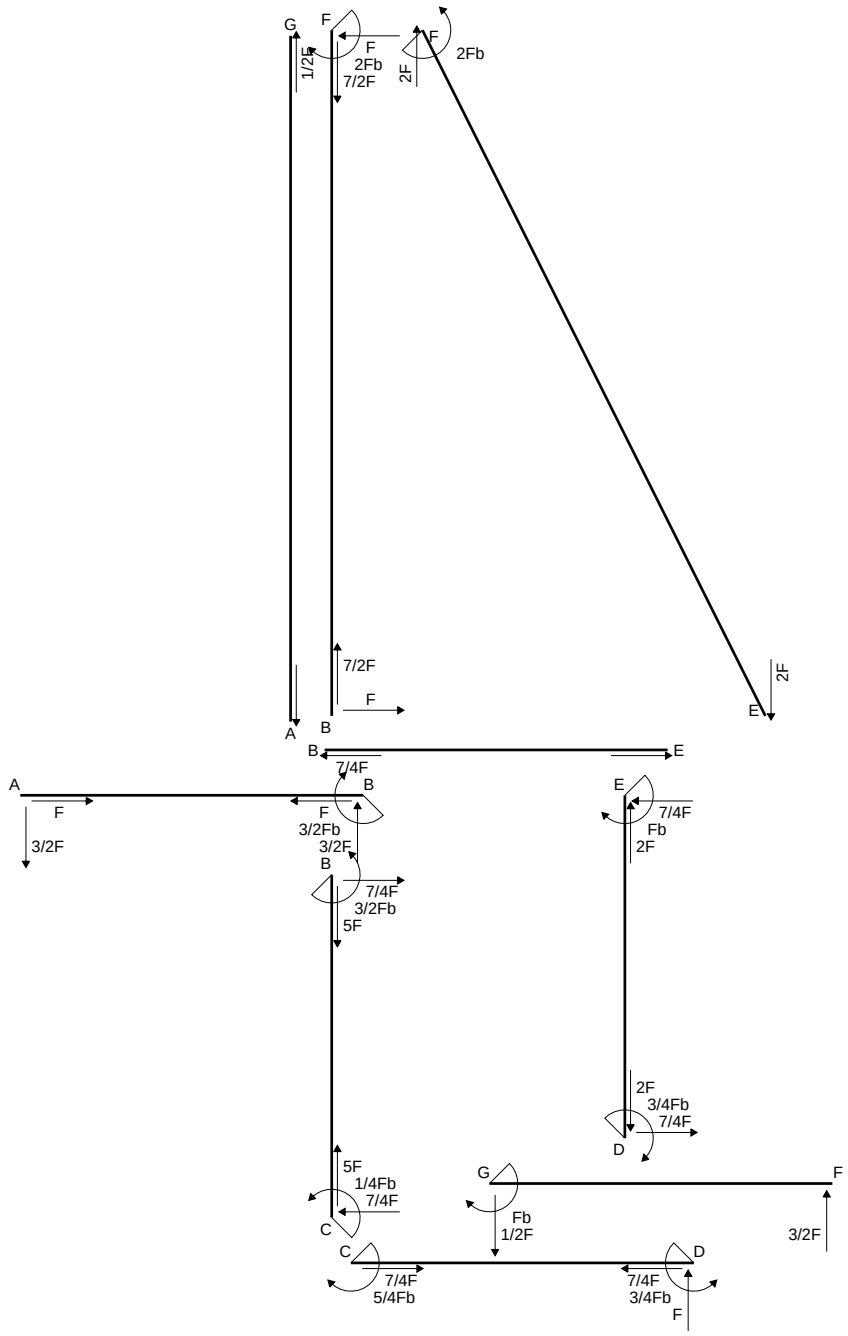
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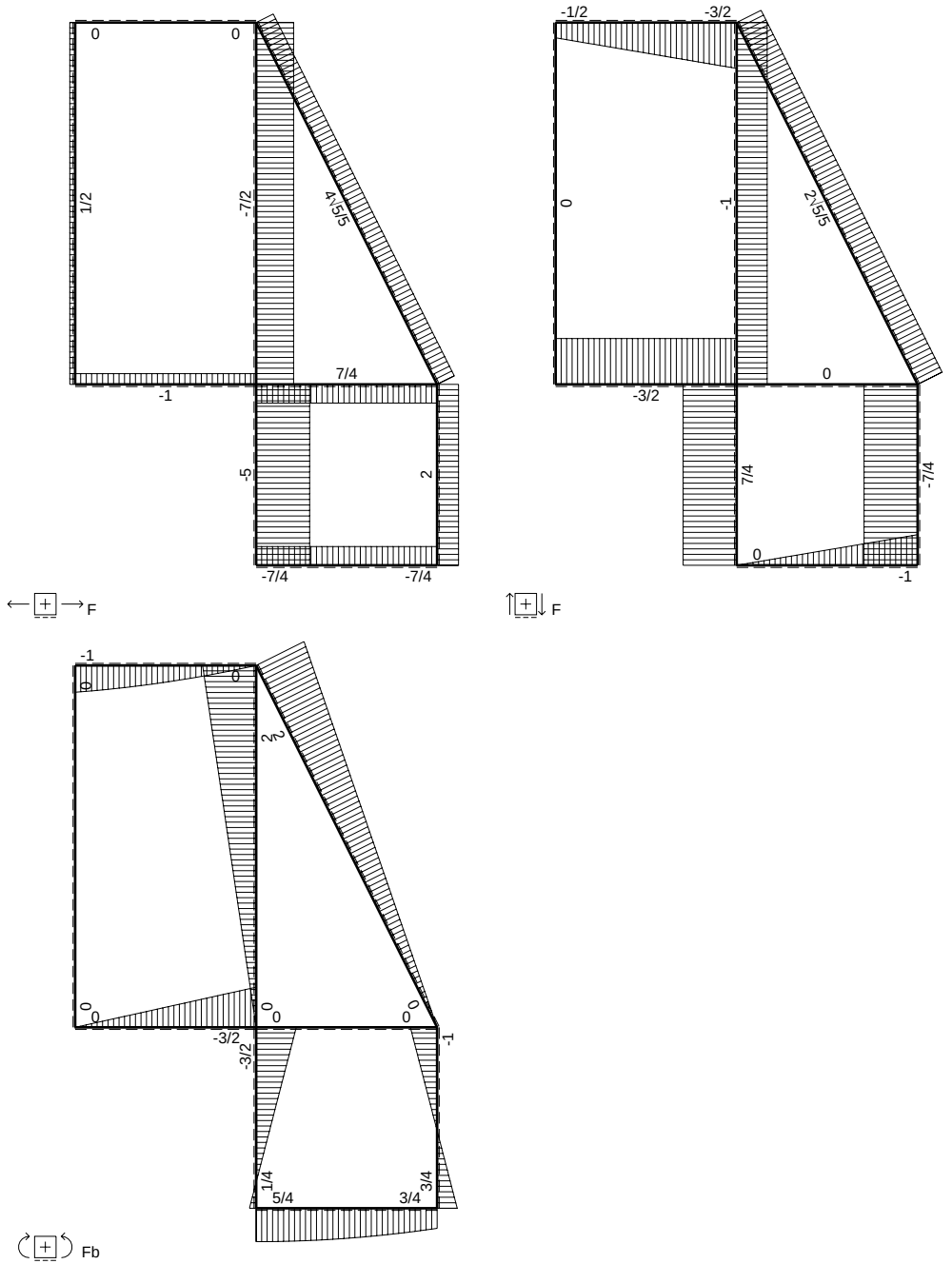
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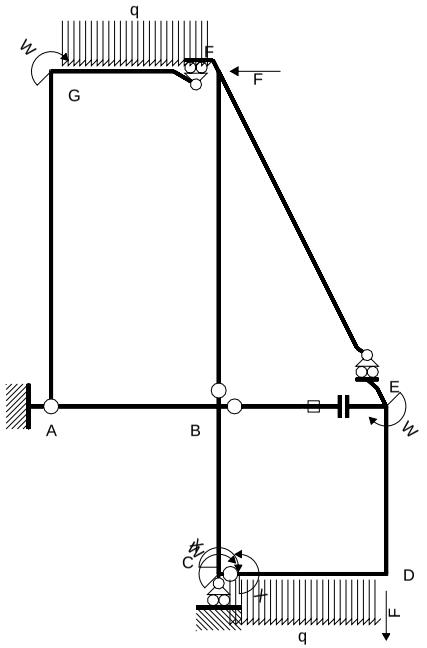
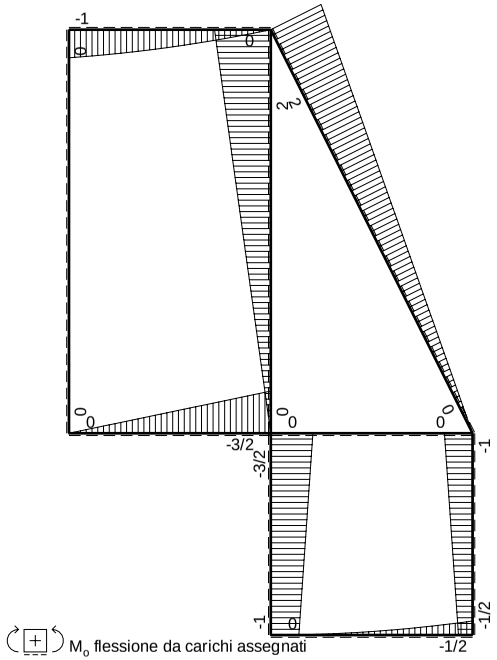
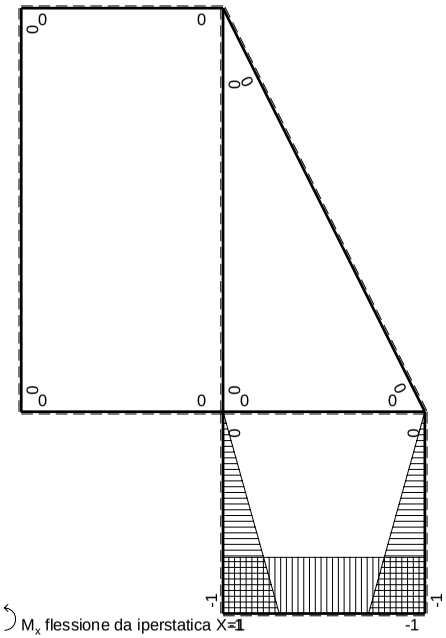


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-1/2qx^2	1/2Fx^2/b	1	1/6Fb^2/EJ	Xb/EJ
DC b	1	1/2Fb-Fx+1/2qx^2	1/2Fb-Fx+1/2Fx^2/b	1		
DE b	-1+x/b	-1/2Fb-1/2Fx	1/2Fb-1/2Fx^2/b	1-2x/b+x^2/b^2	1/3Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb-1/2Fx	Fx-1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-3/2Fx+1/2qx^2	0	0	0	0
GF b	0	Fb-1/2Fx-1/2qx^2	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				25/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-5/4Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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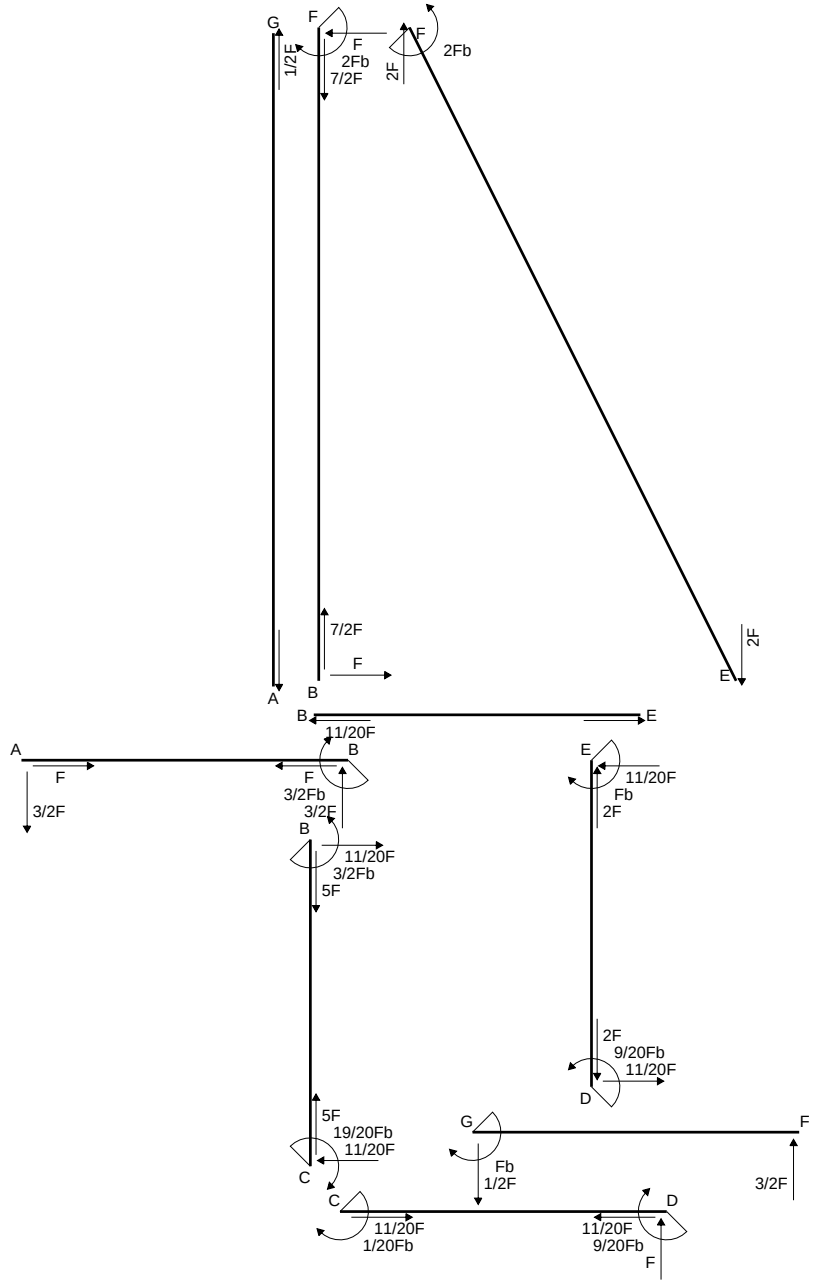
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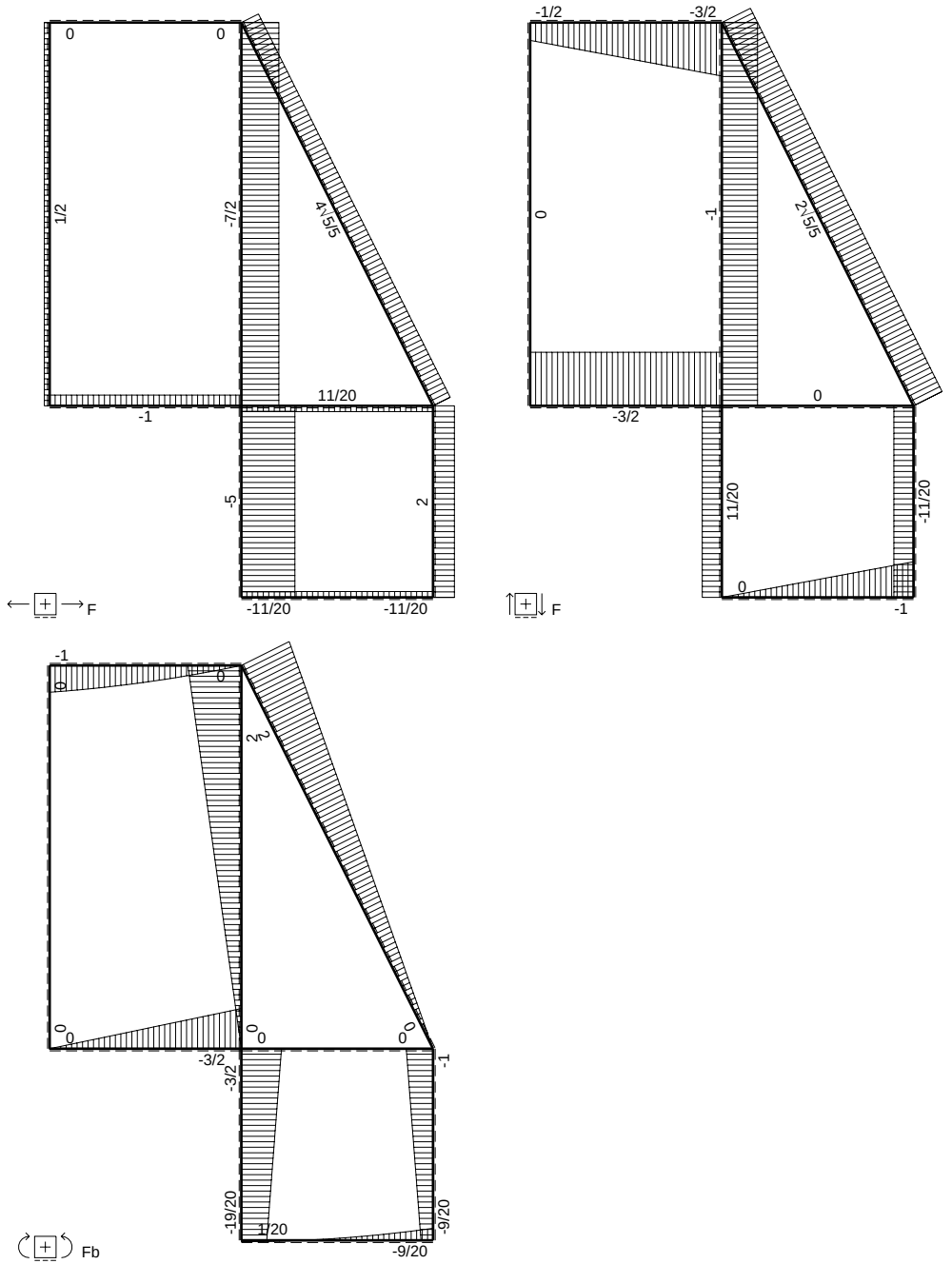
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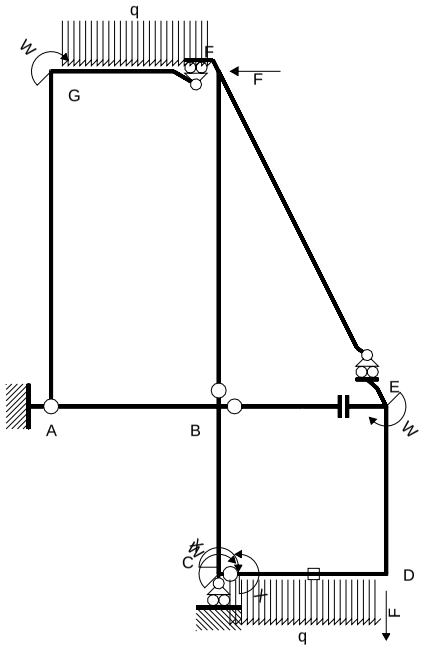
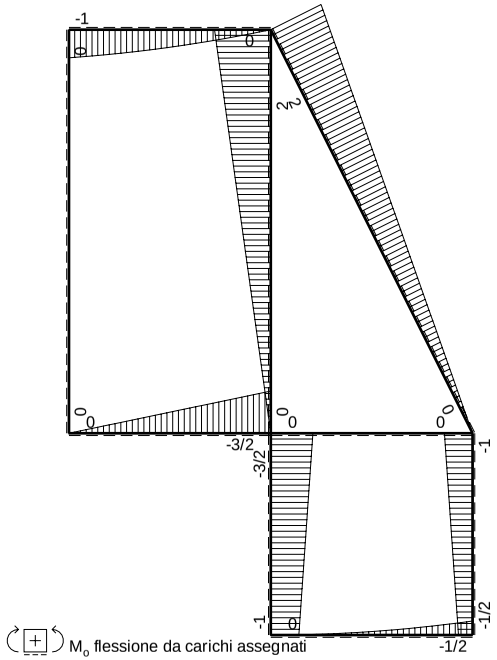
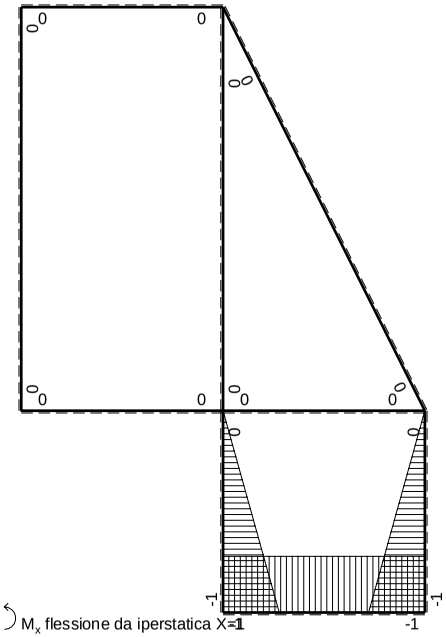


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb-1/2Fx	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb-1/2Fx	$Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-1/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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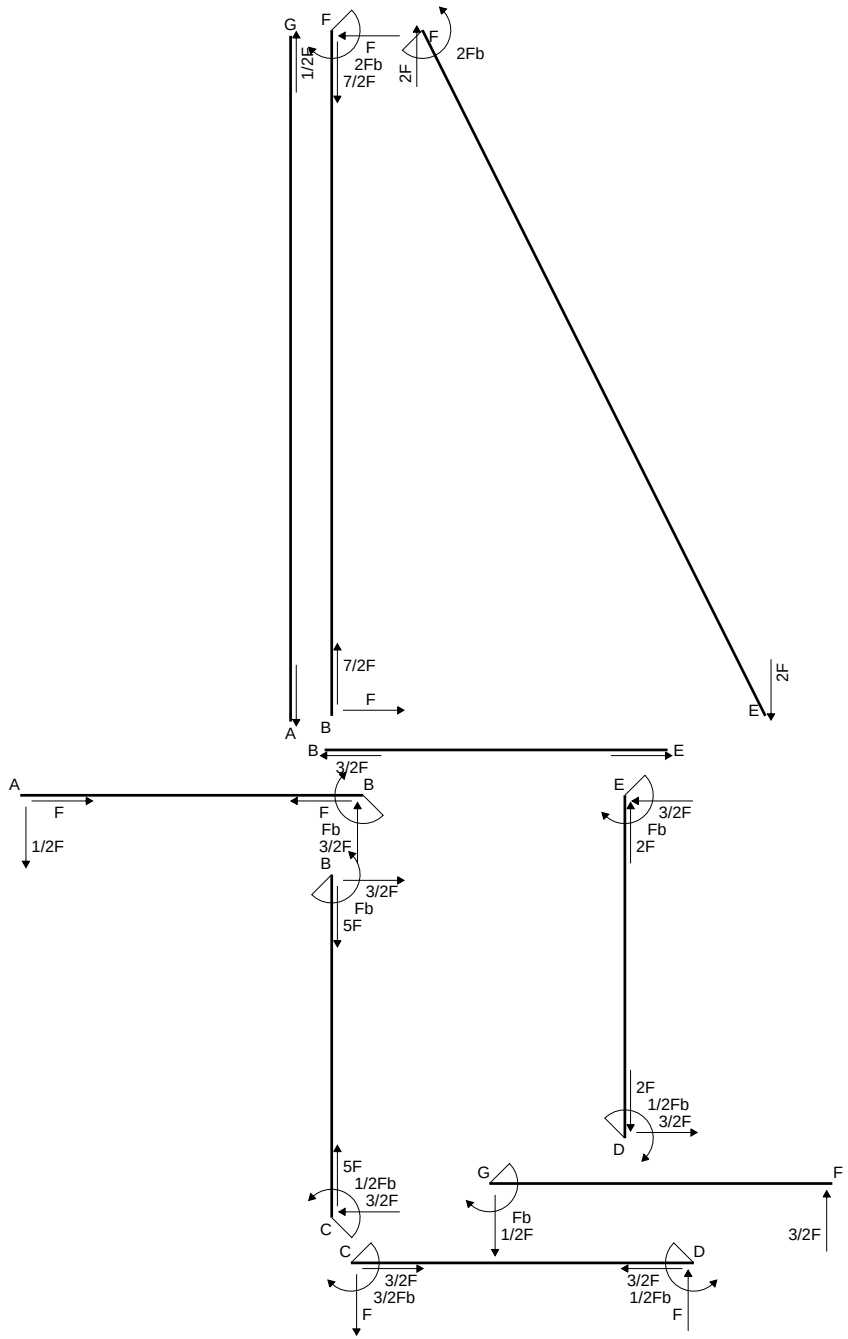
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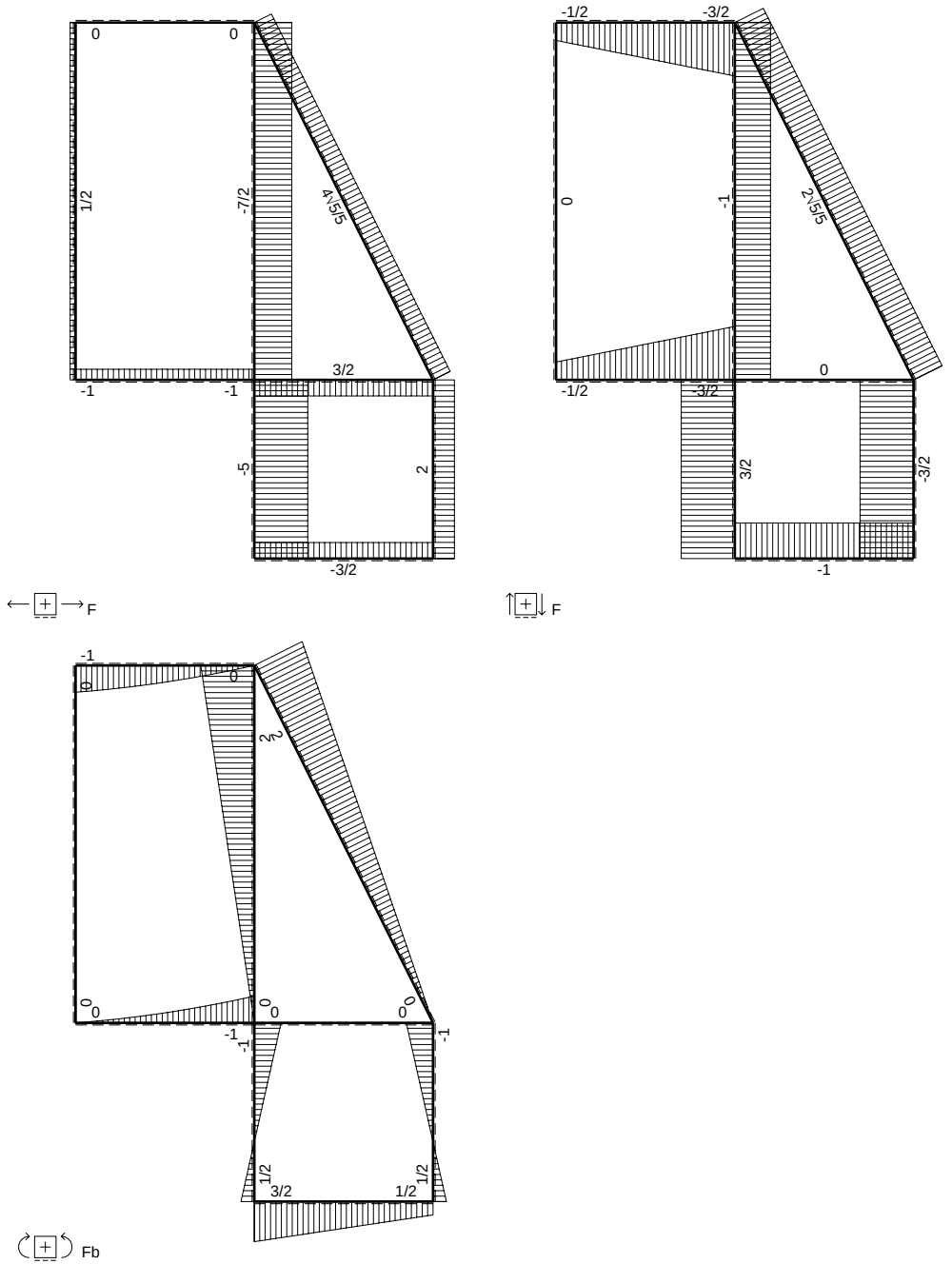
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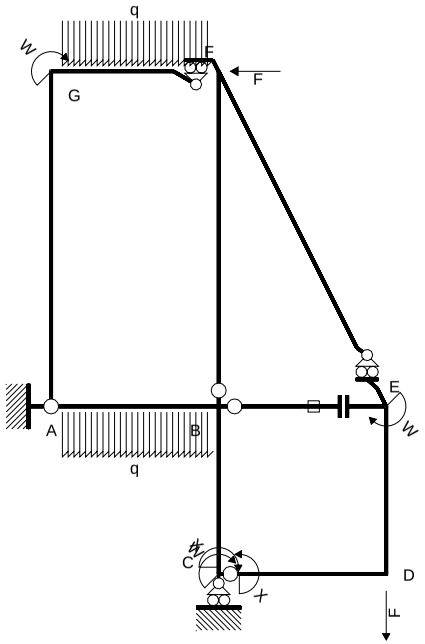
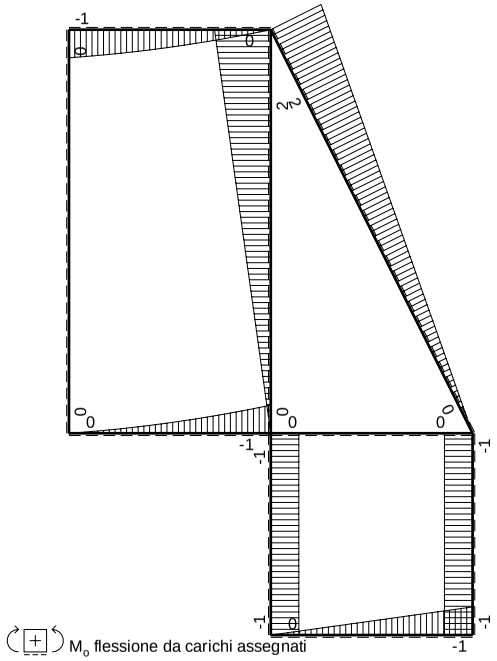
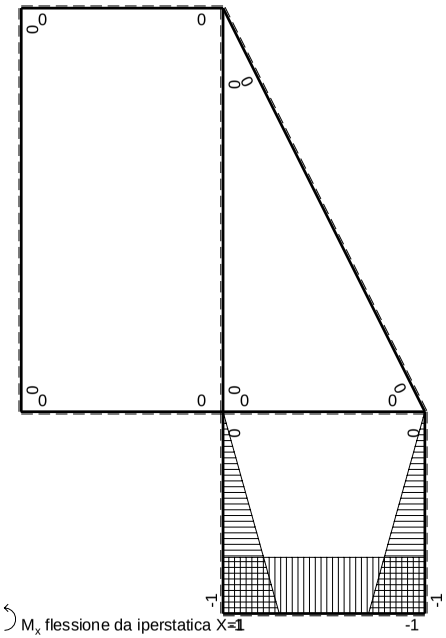
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$5/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-3/2Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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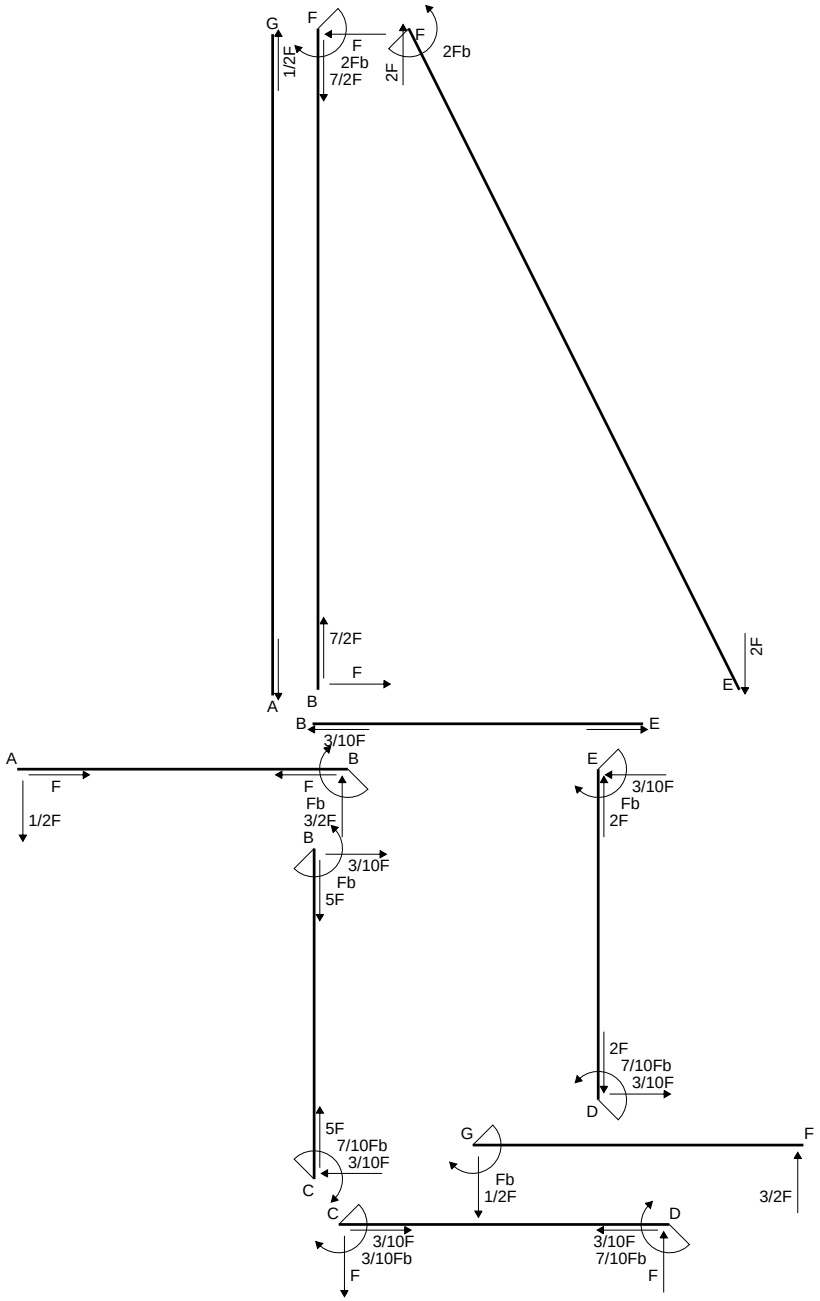
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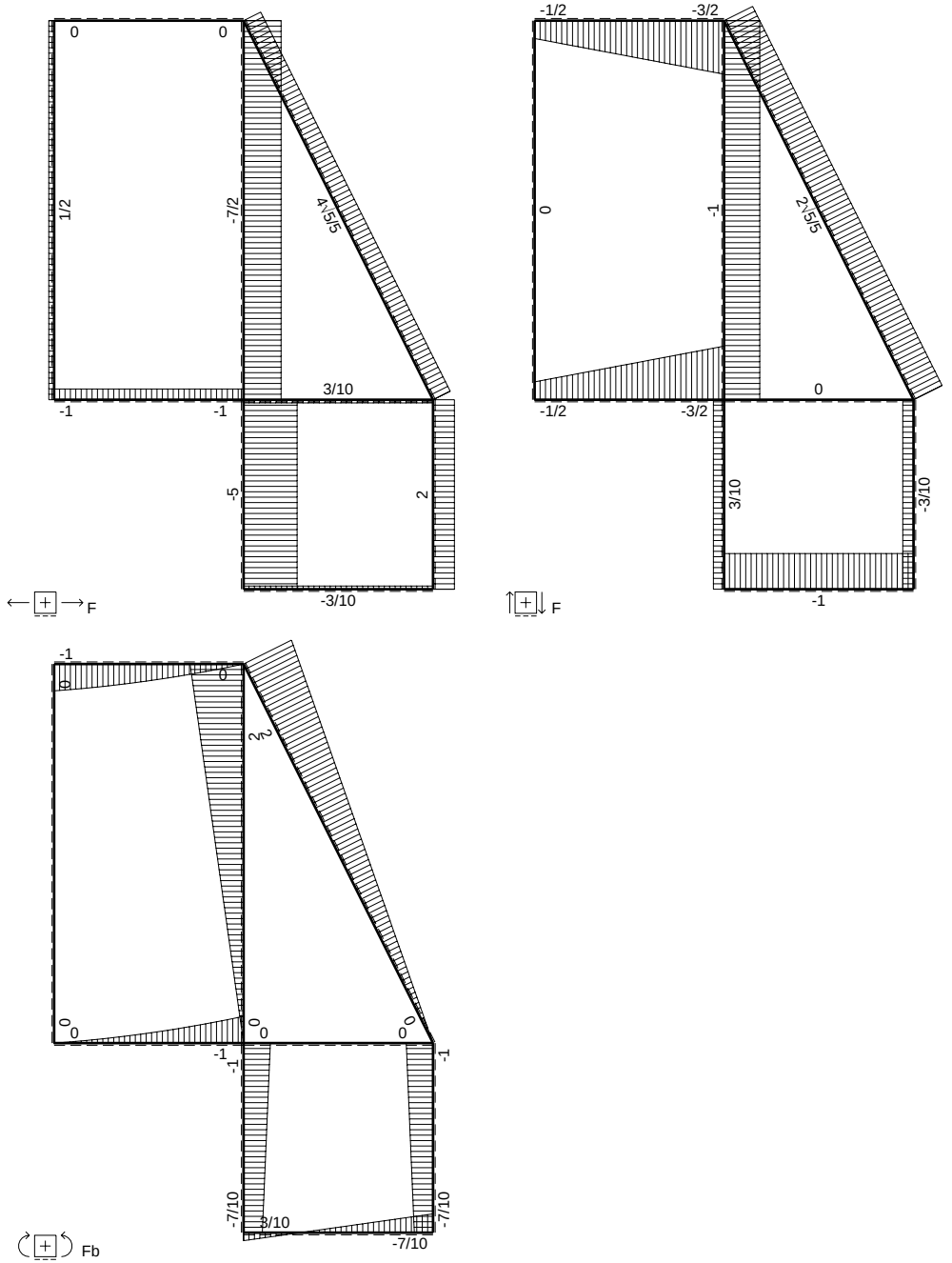
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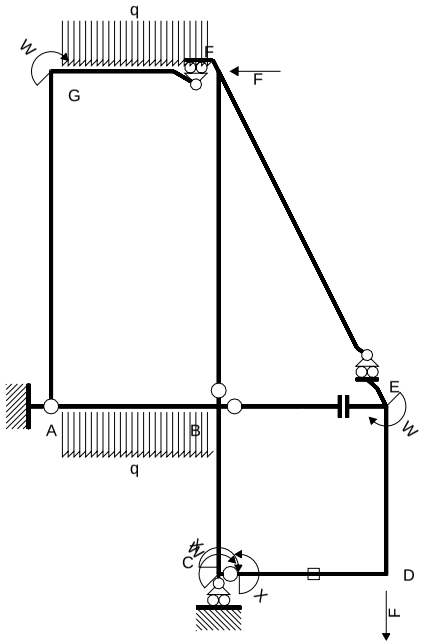
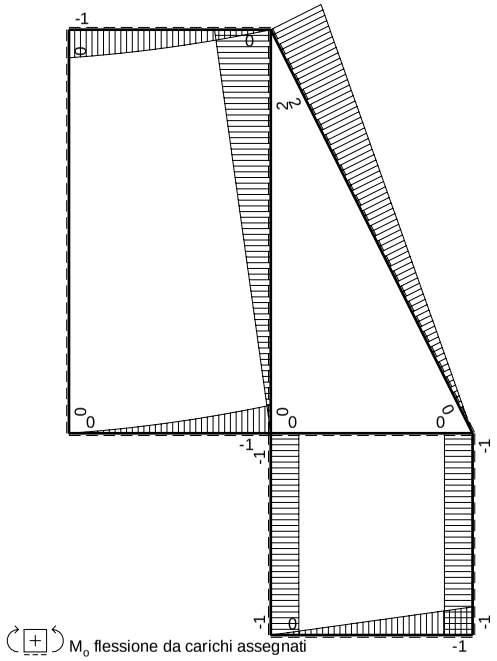
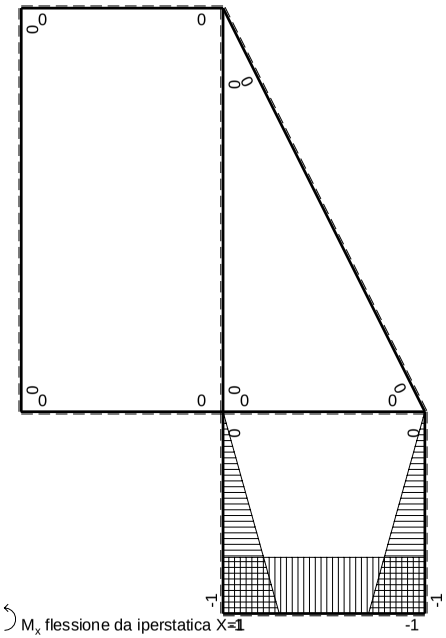


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-3/10Fb$	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

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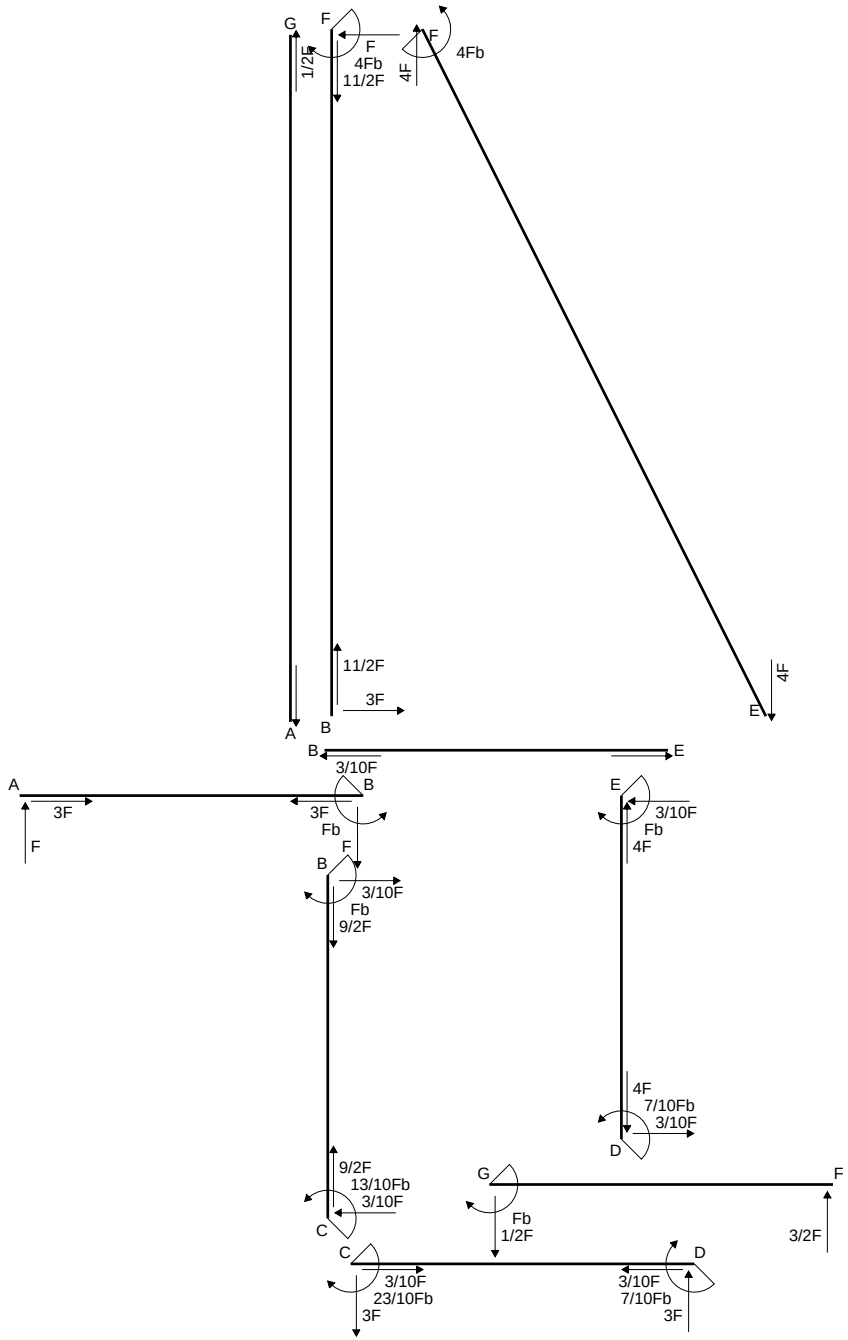
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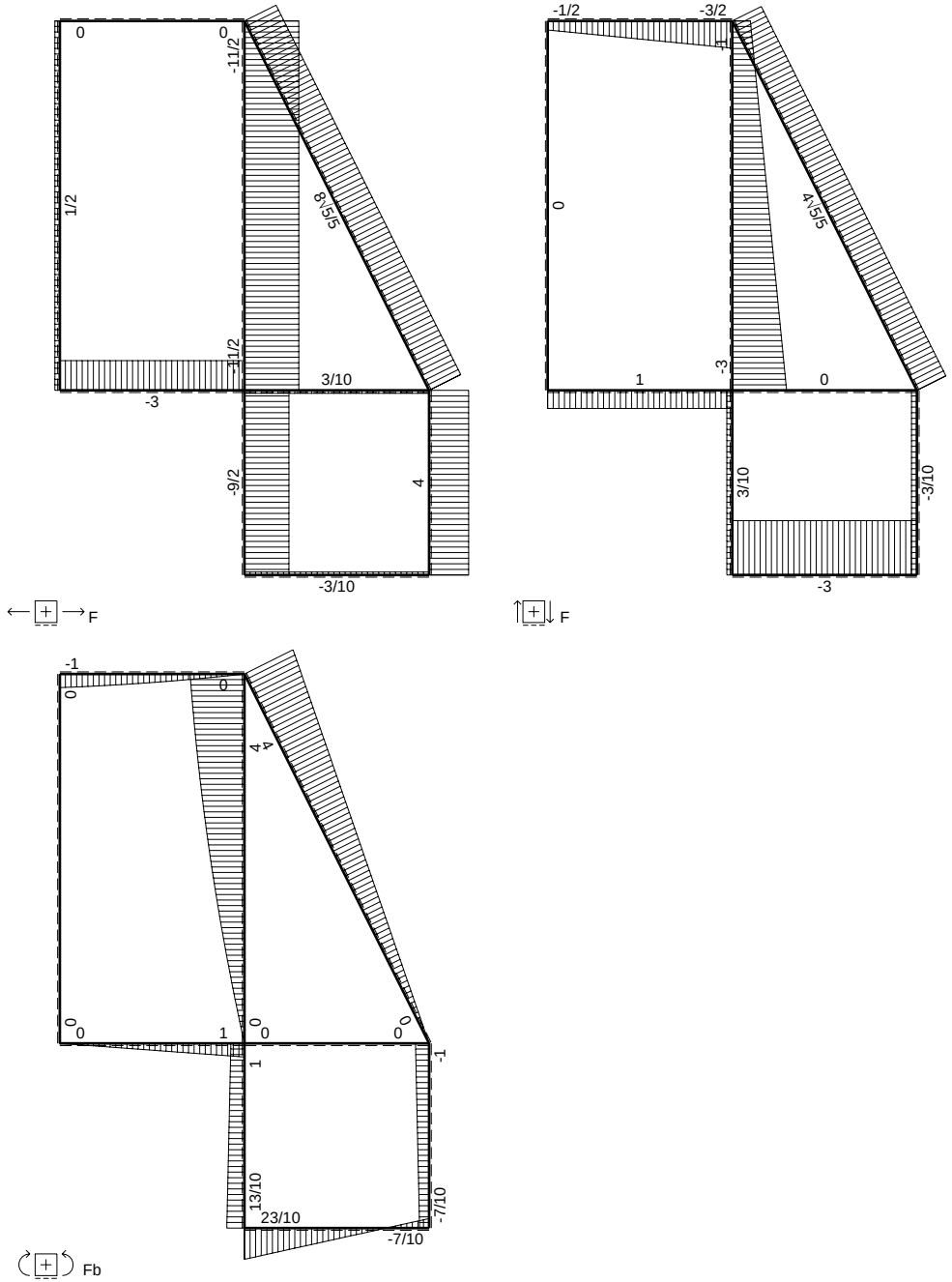
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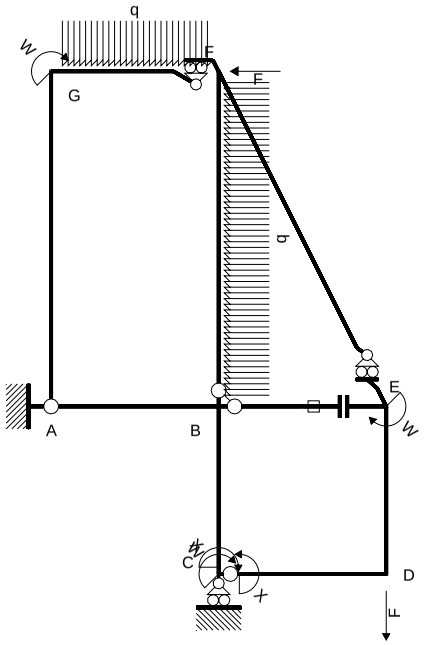
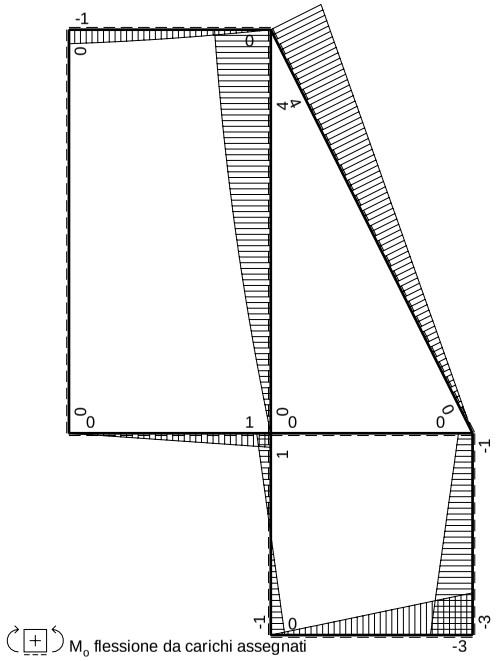
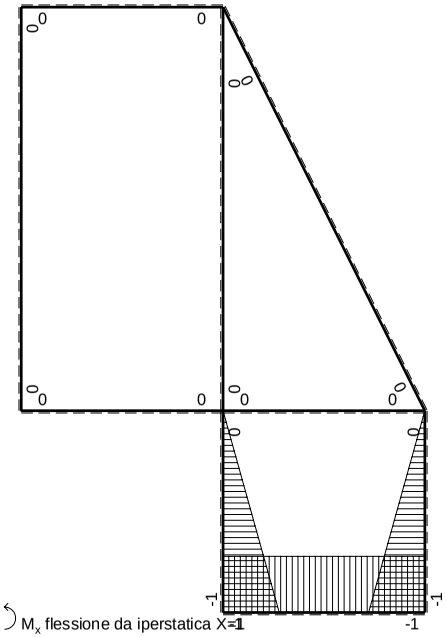


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$23/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-23/10Fb$	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

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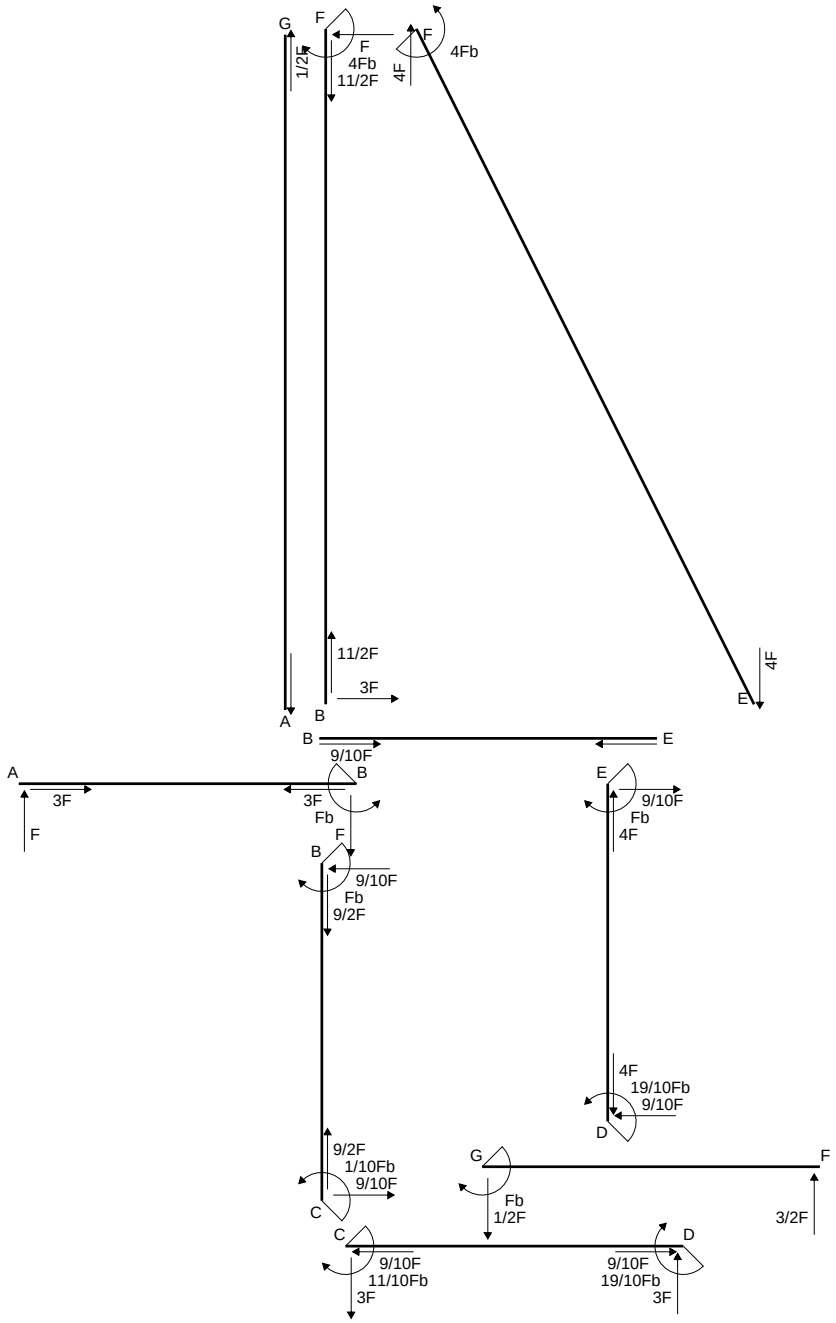
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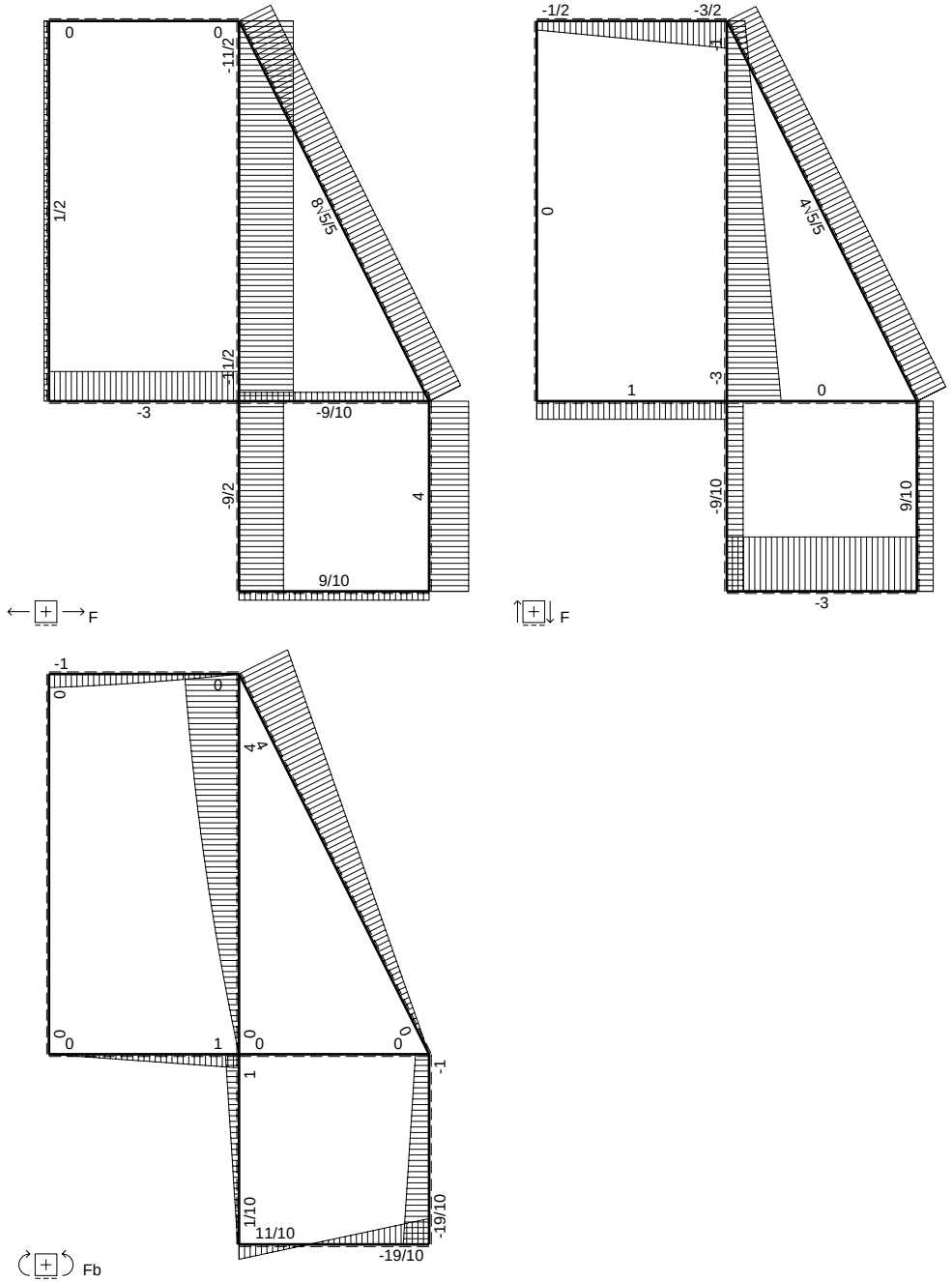
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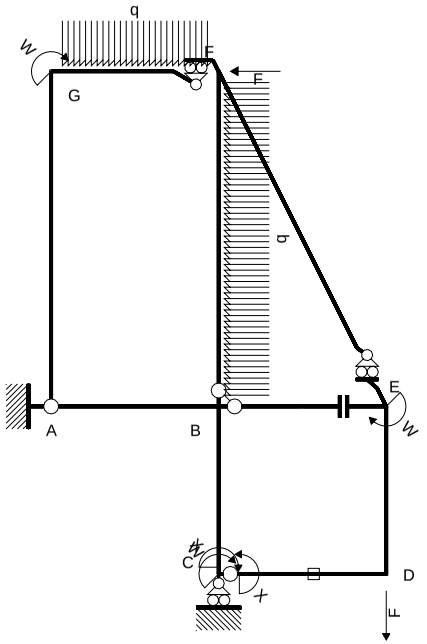
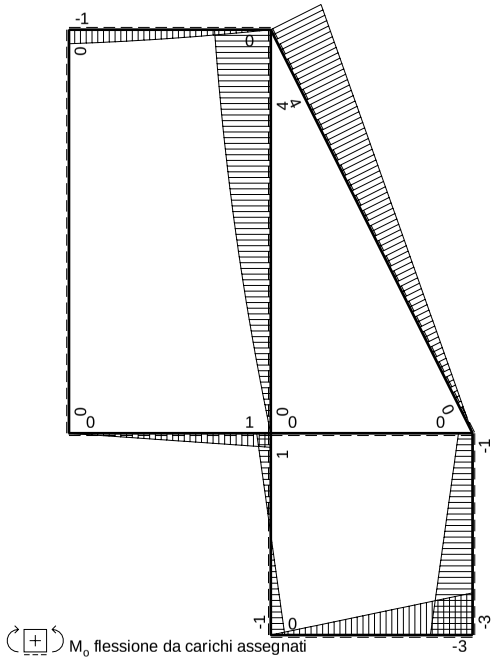
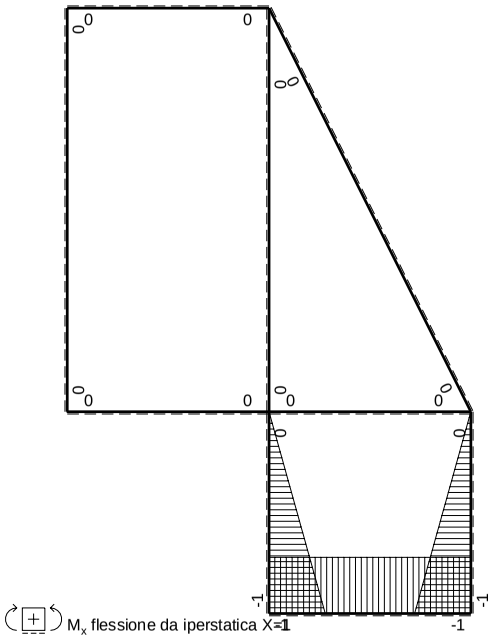


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	$3Fb-5Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb+2Fx	$Fx+2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$11/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-11/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

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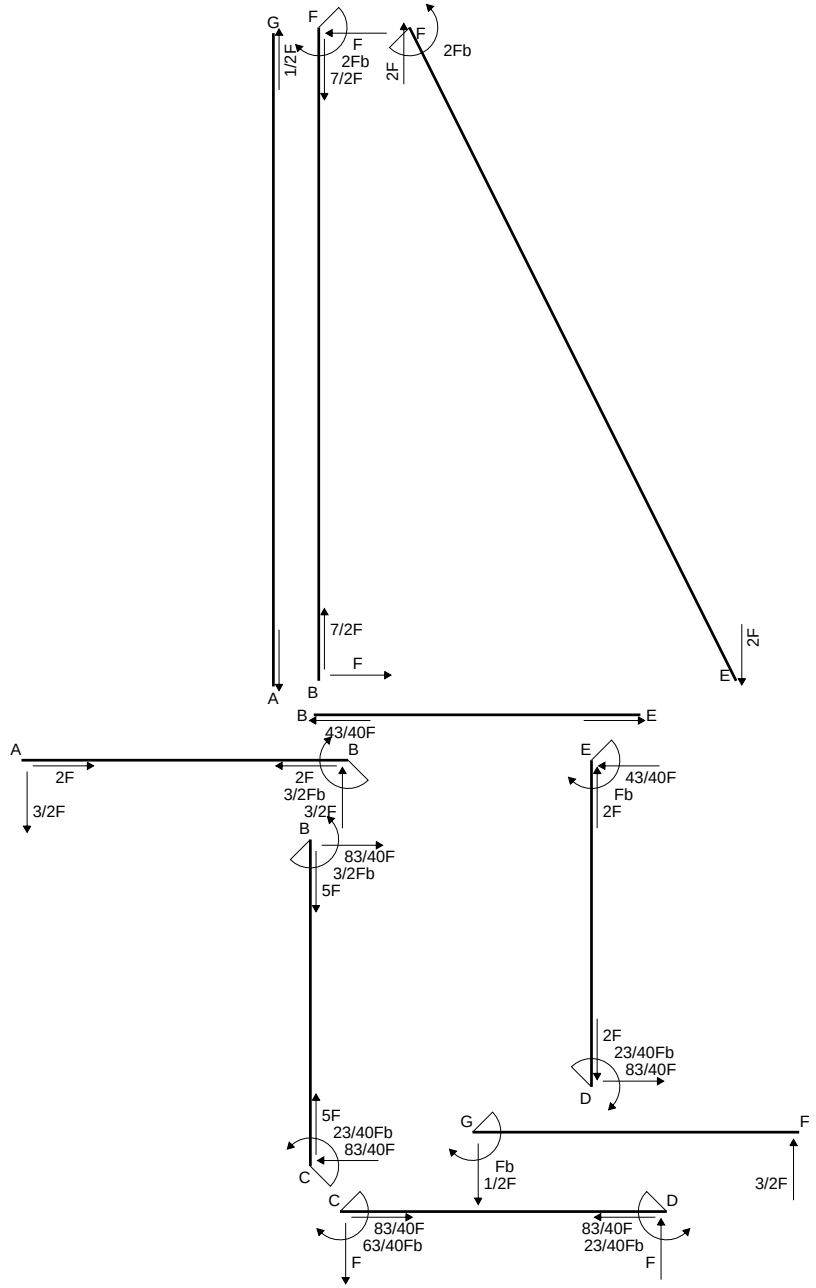
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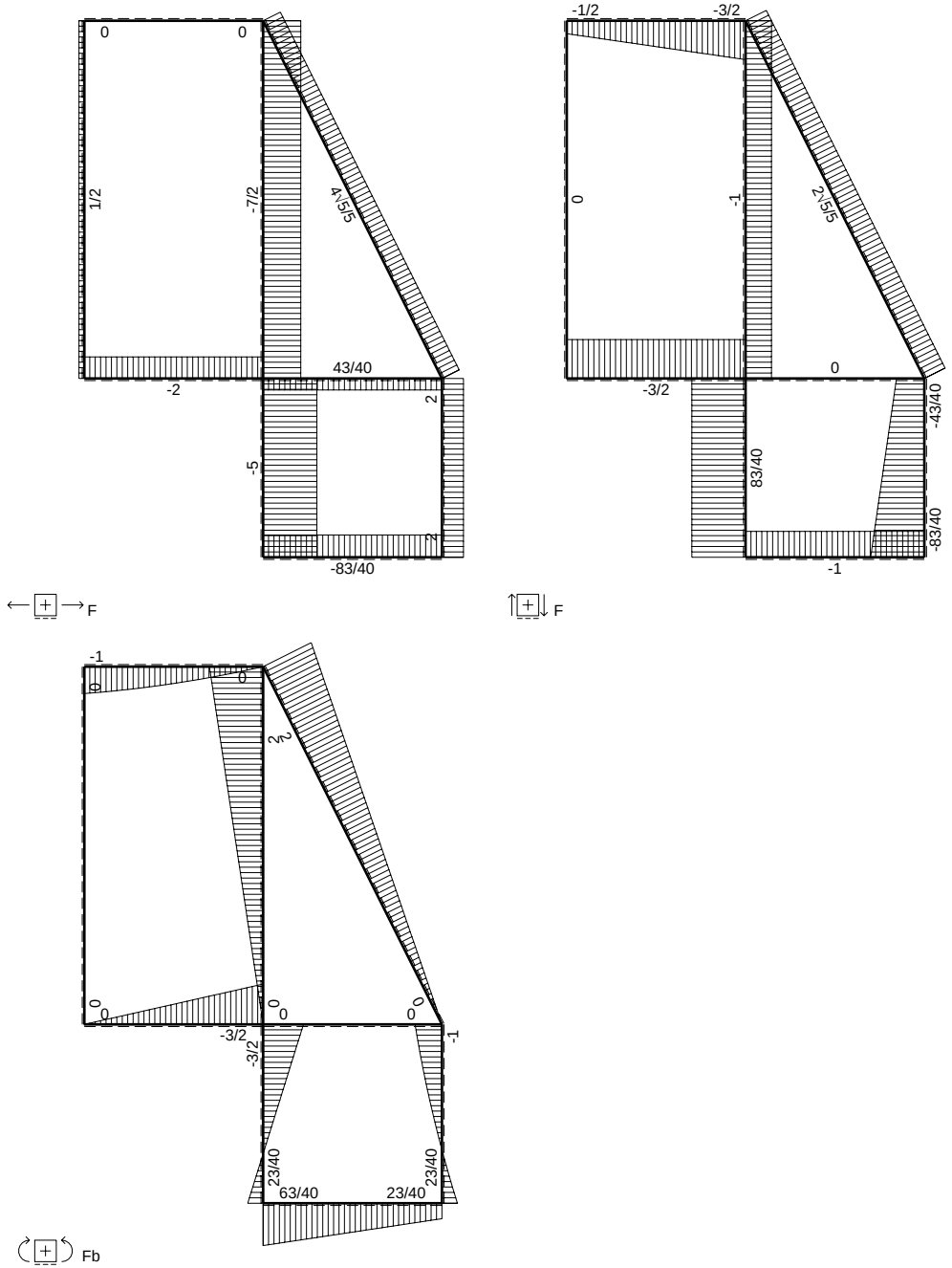
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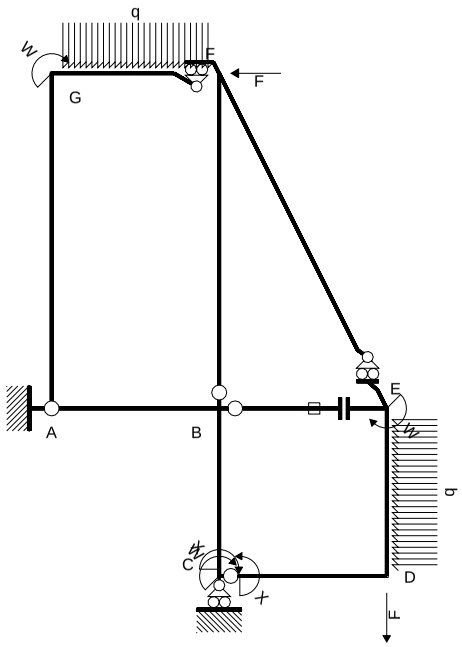
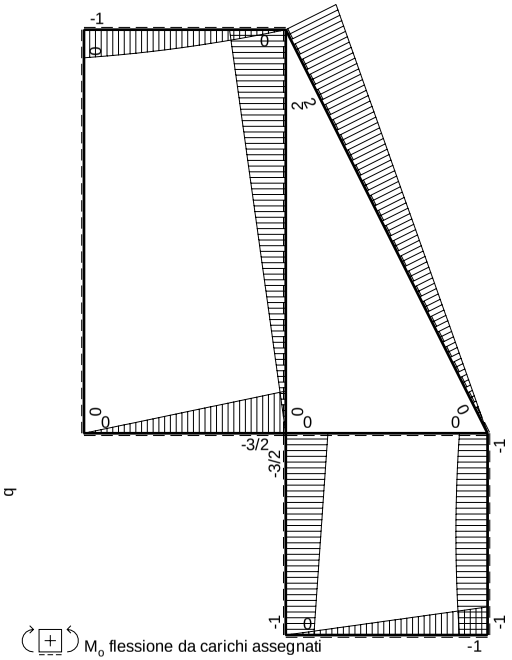
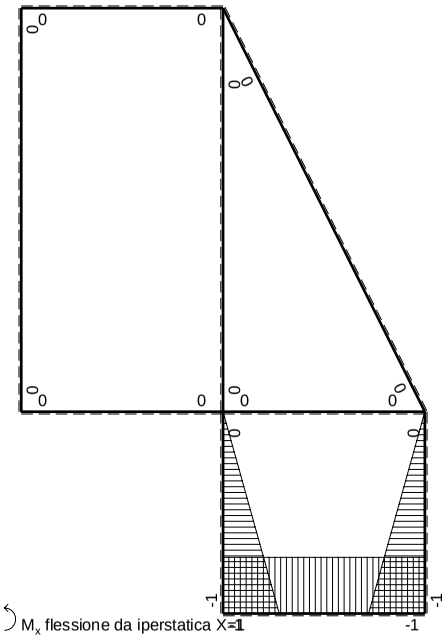


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$21/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-63/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

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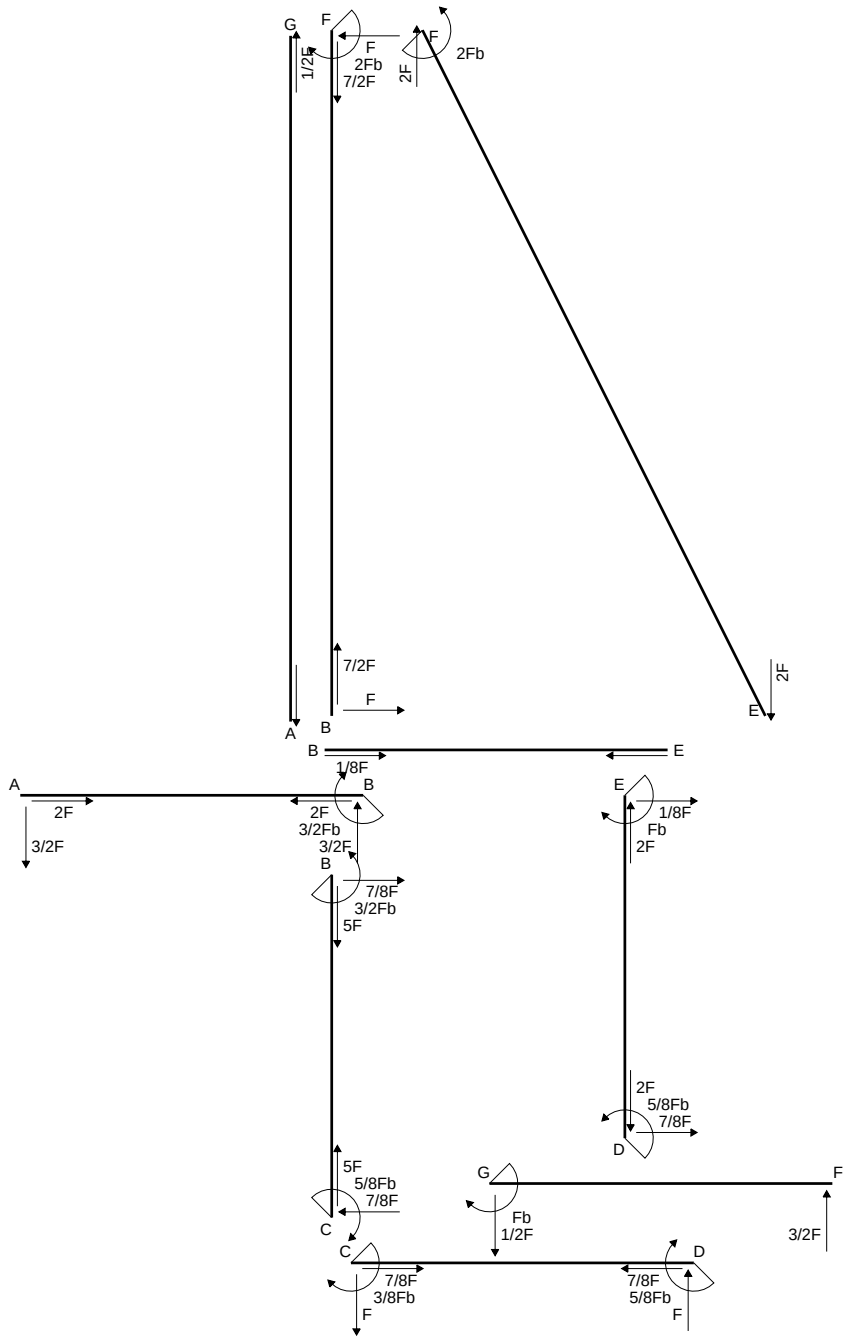
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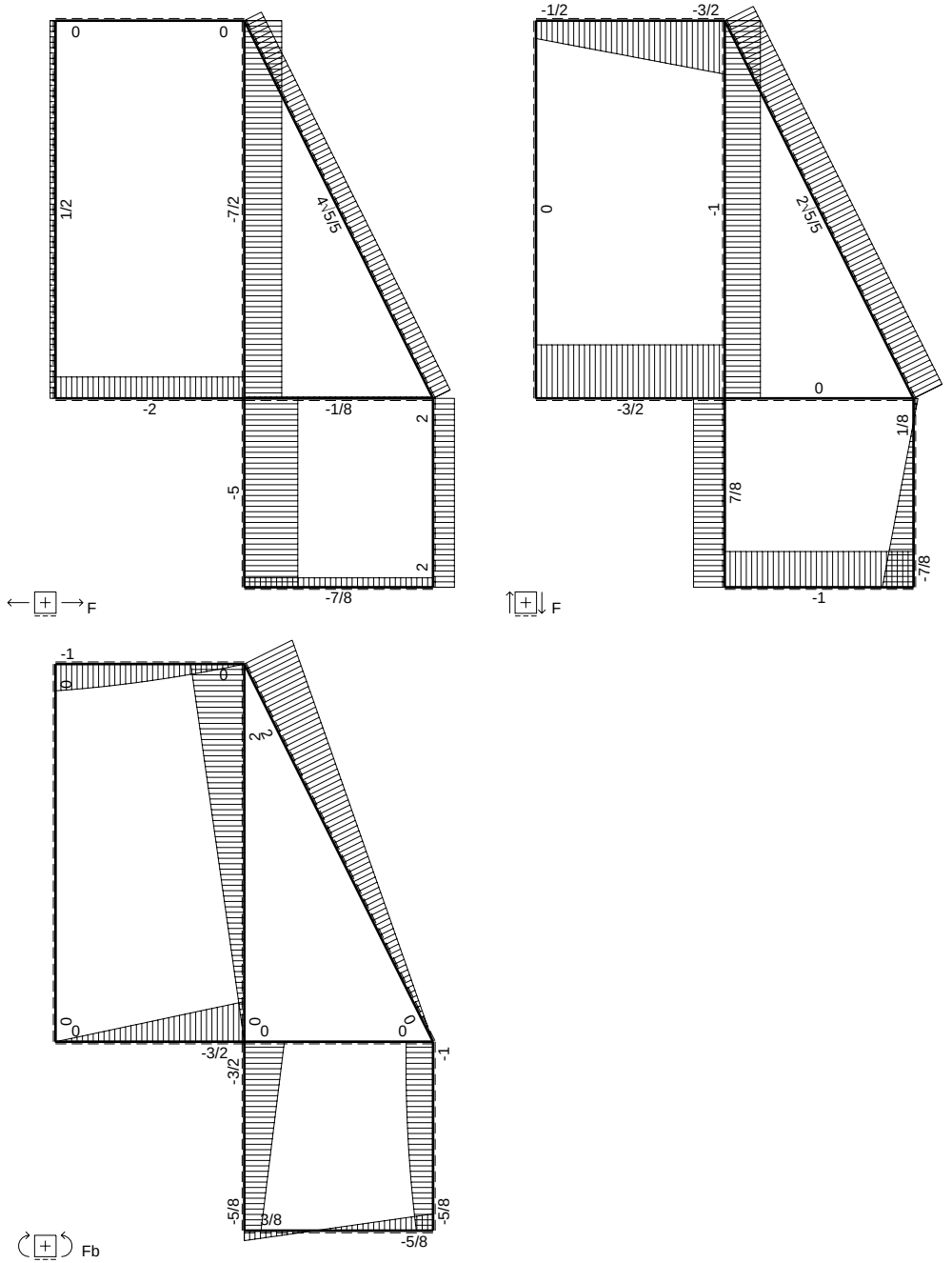
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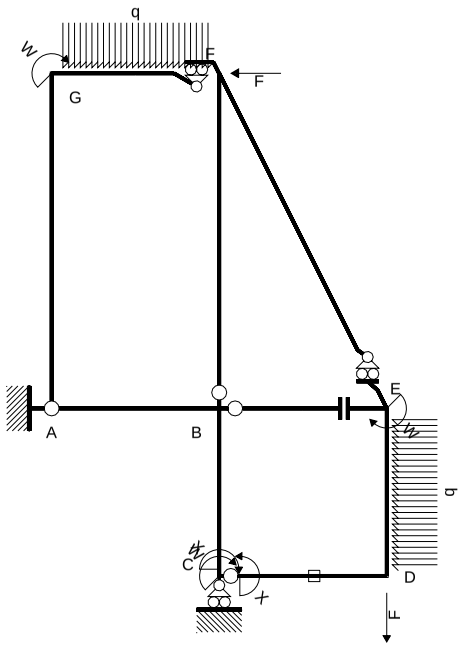
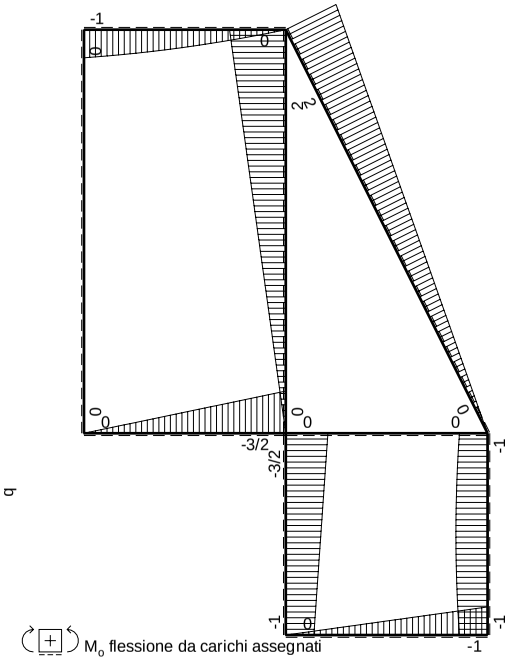
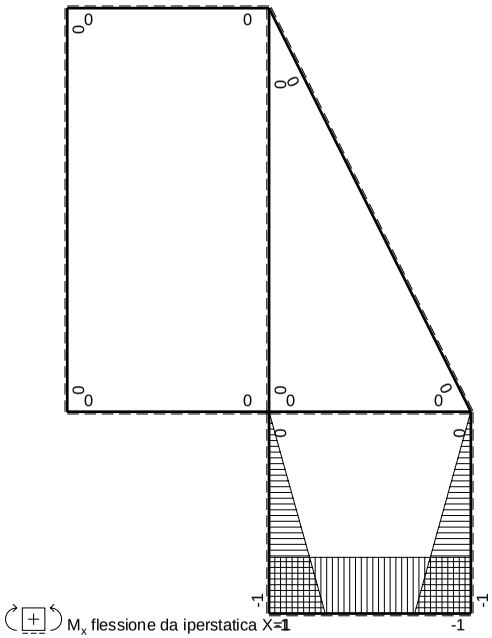


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

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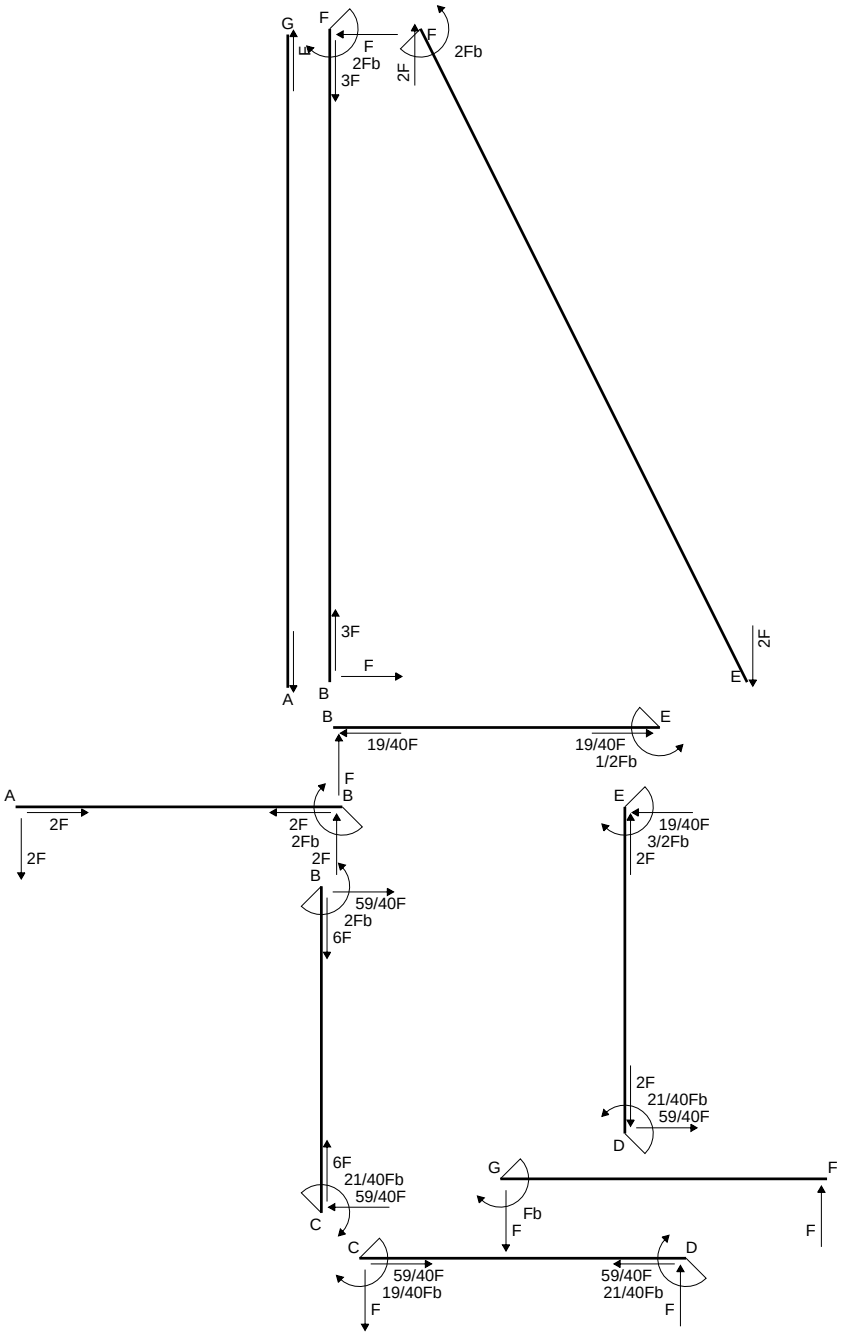
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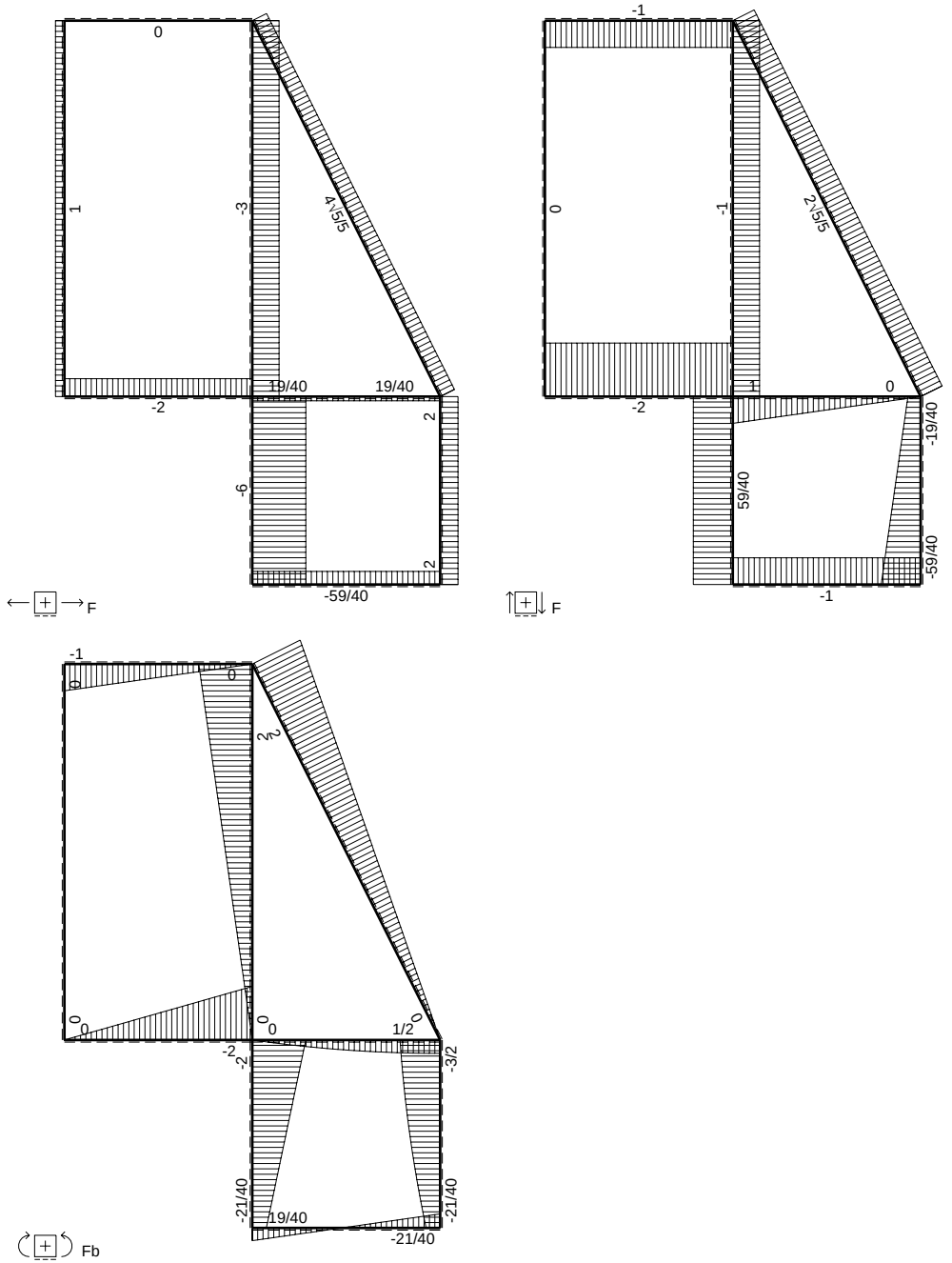
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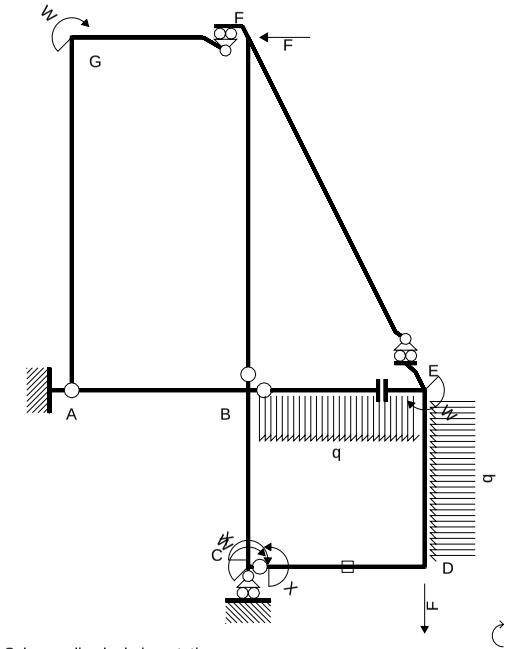
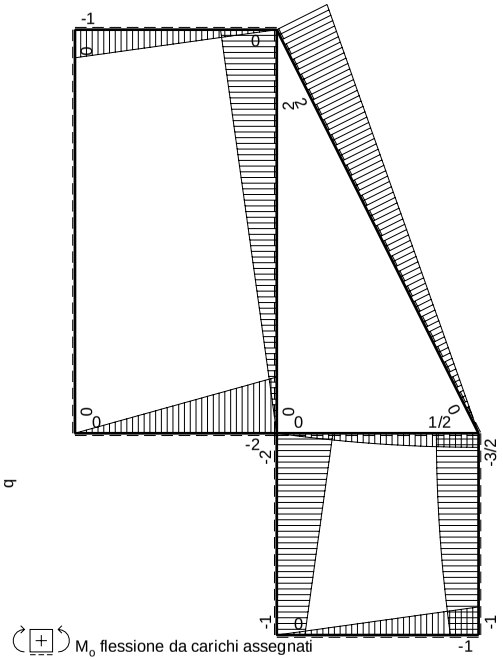
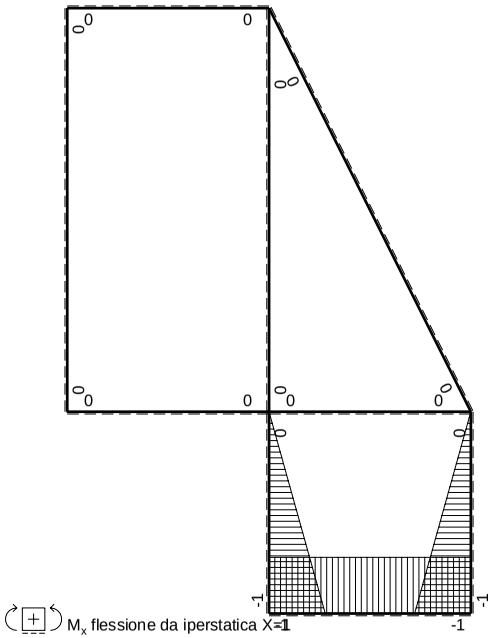


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	$2Fx-Fx^2/b$	x^2/b^2	$2/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+Fx	$Fb-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-Fx+1/2qx^2$	$Fb-3/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$5/8Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$3/2Fb-1/2qx^2$	$3/2Fx-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$19/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-19/40Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ \ Fb^2/EJ$

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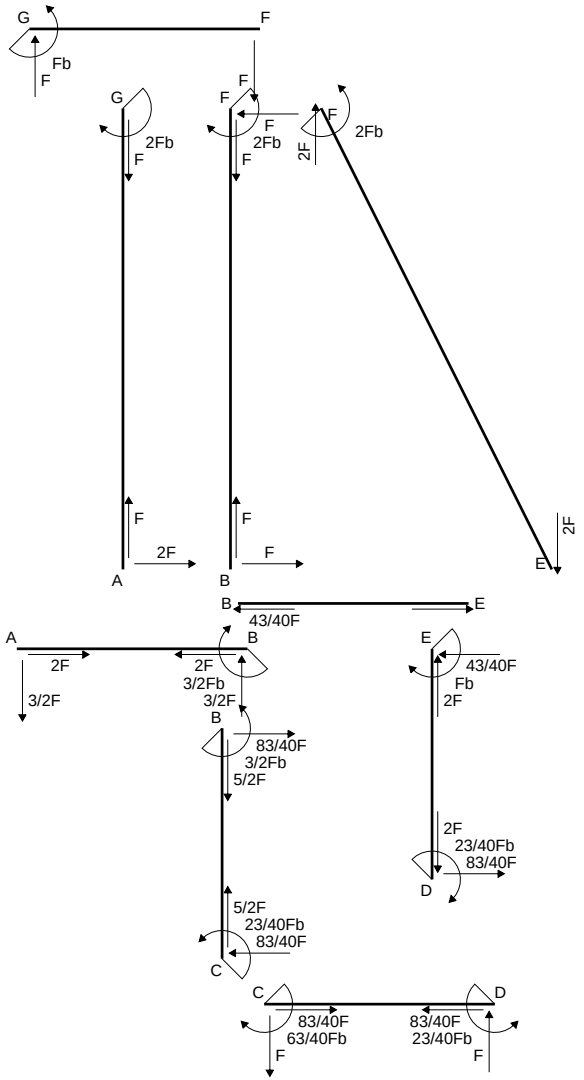
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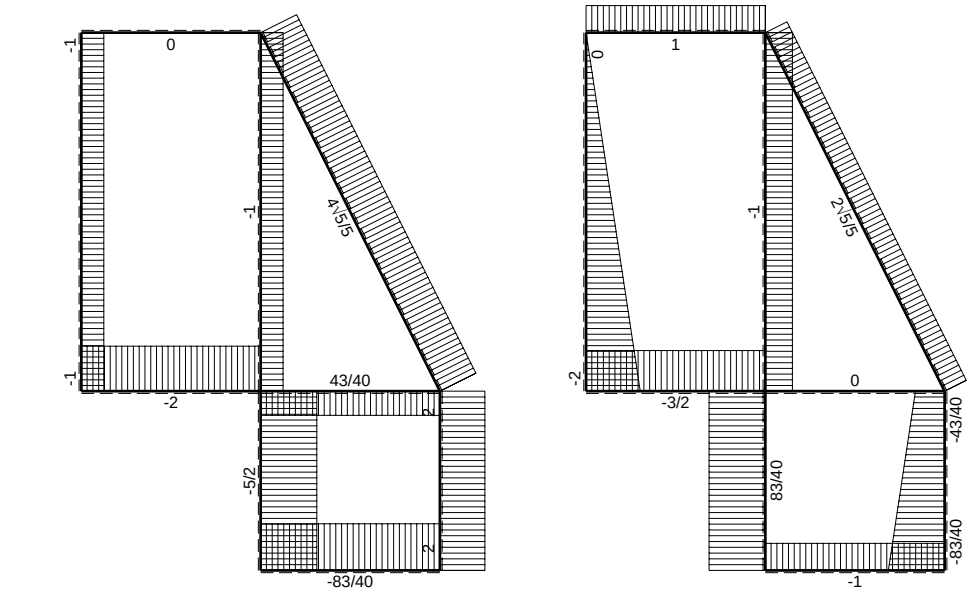
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-83/40
-1

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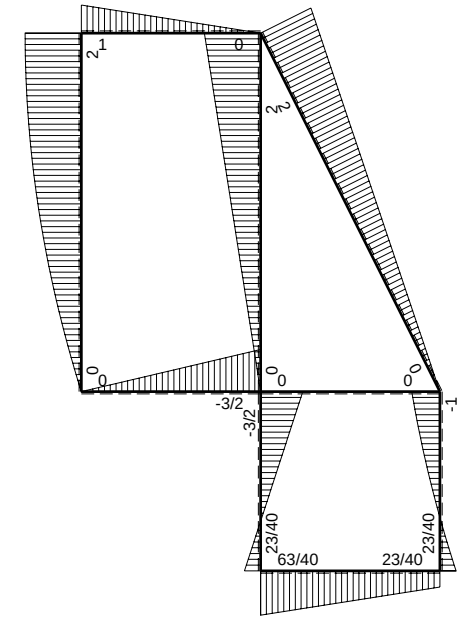
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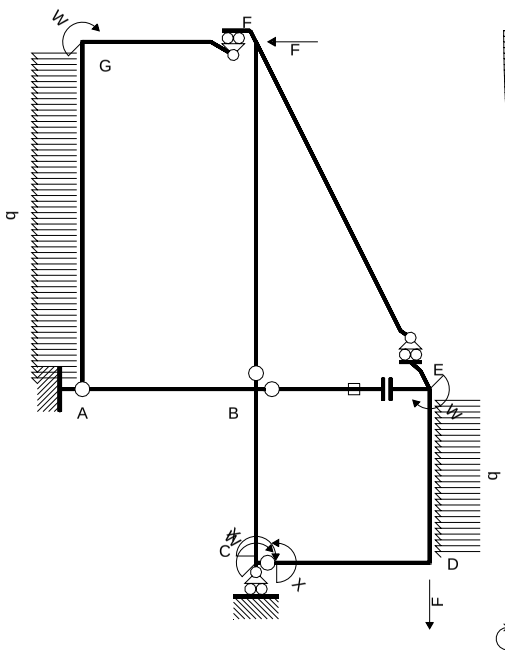
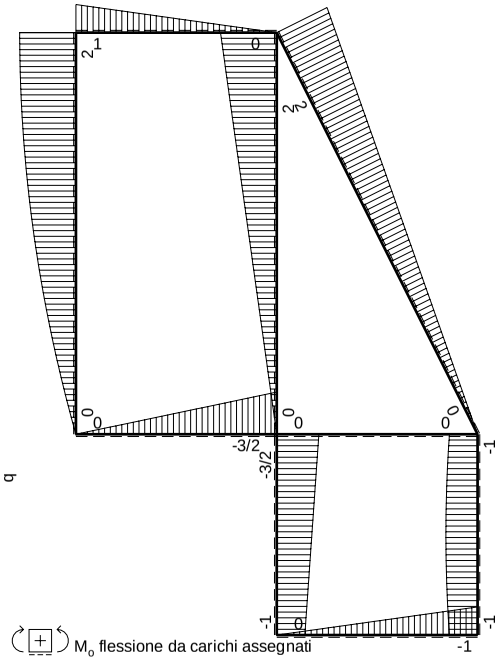
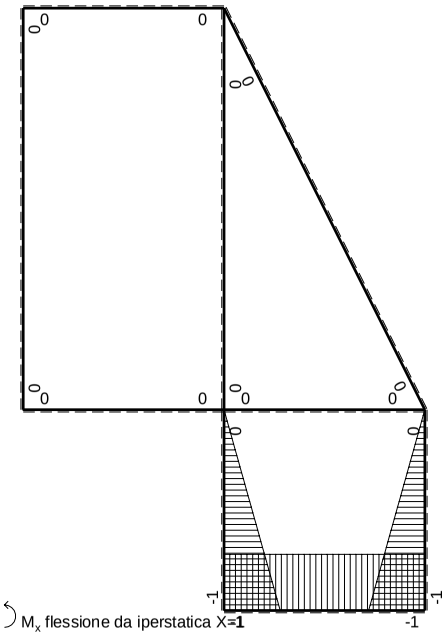
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=**1**

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx+1/2qx^2	Fb-1/2Fx-Fx^2/b+1/2qx^3/b	1-2x/b+x^2/b^2	13/24Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+1/2Fx-1/2qx^2	Fx+1/2Fx^2/b-1/2qx^3/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	2Fb-1/2qx^2	0	0	0	0
AG 2b	0	-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				21/8Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-63/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

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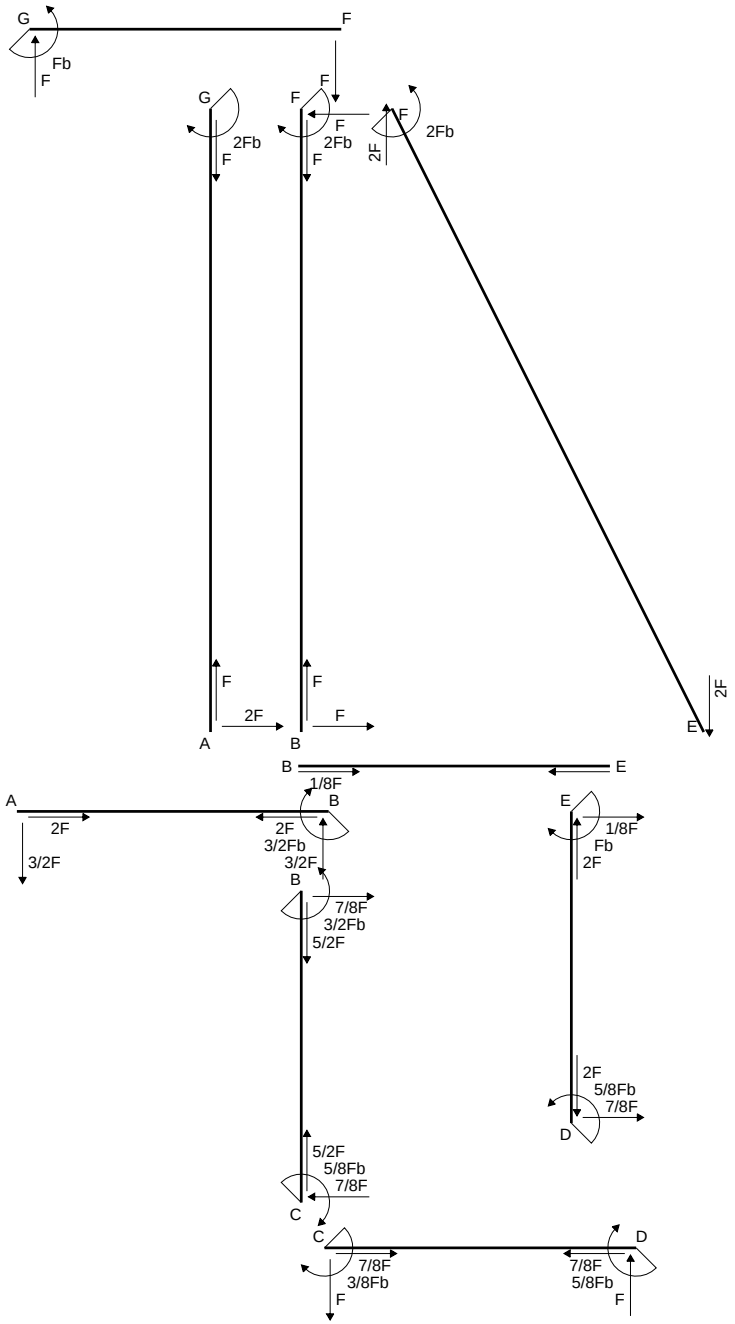
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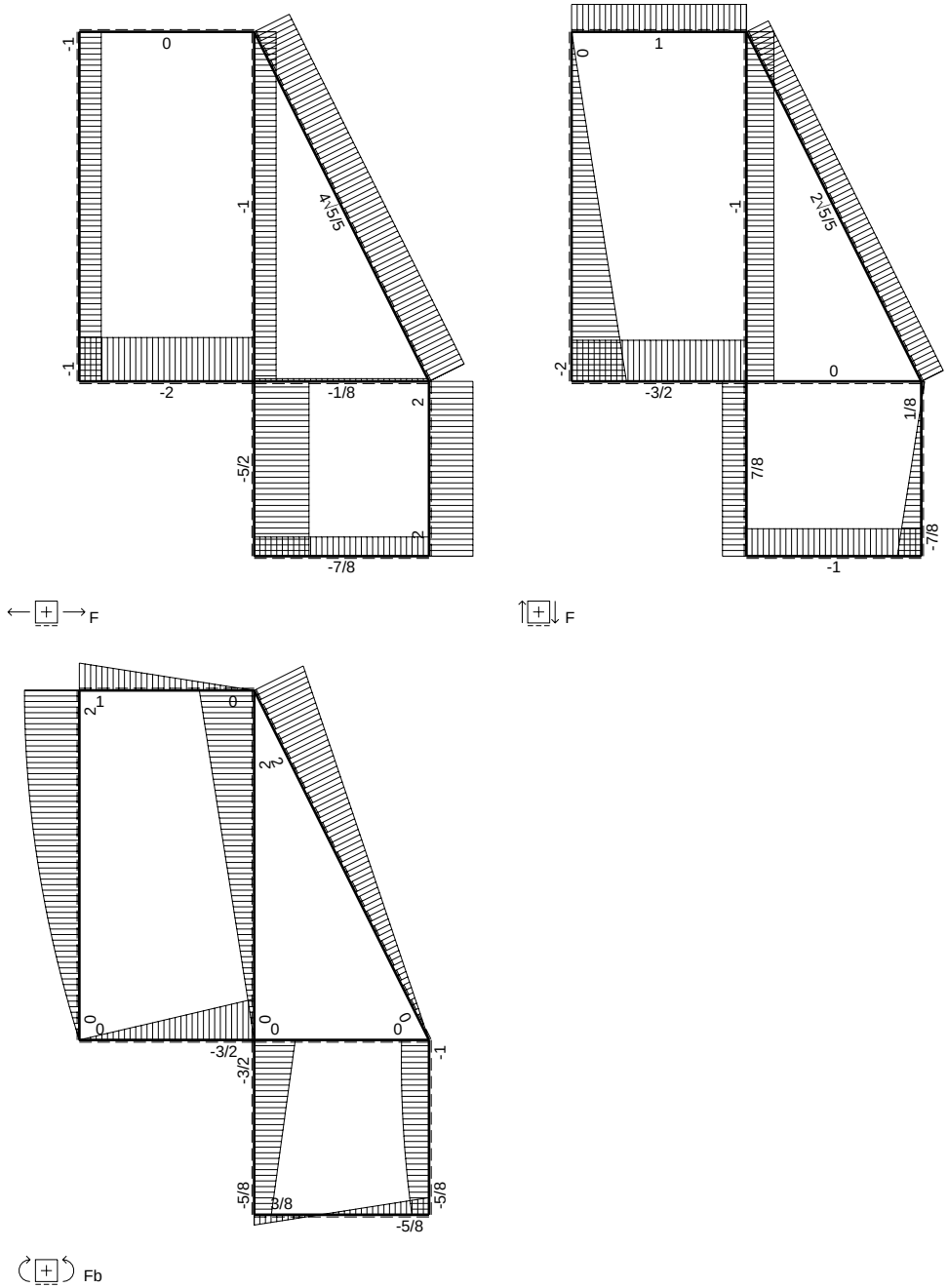
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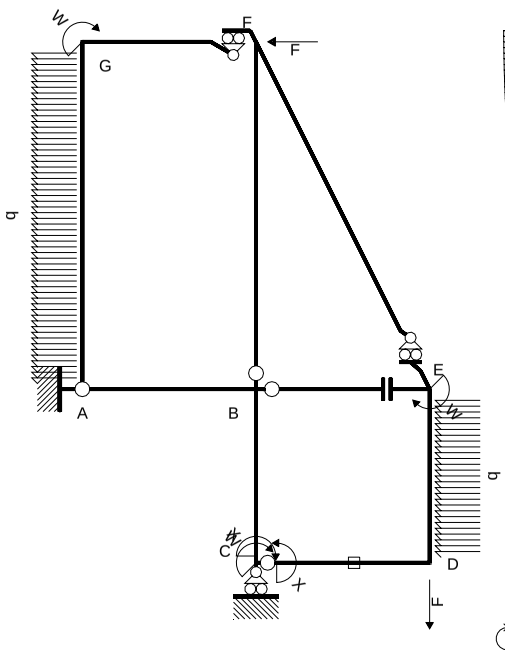
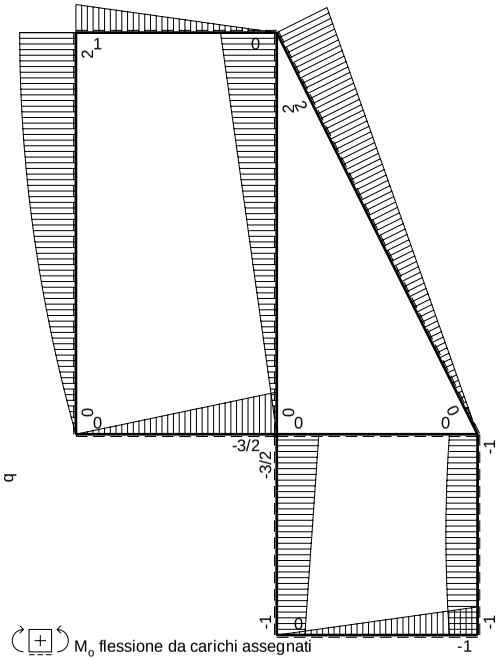
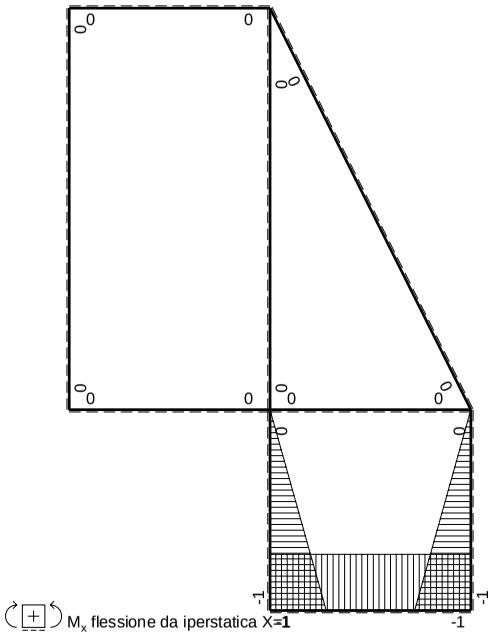


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=**1**

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx+1/2qx^2	Fb-1/2Fx-Fx^2/b+1/2qx^3/b	1-2x/b+x^2/b^2	13/24Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+1/2Fx-1/2qx^2	Fx+1/2Fx^2/b-1/2qx^3/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	2Fb-1/2qx^2	0	0	0	0
AG 2b	0	-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				5/8Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

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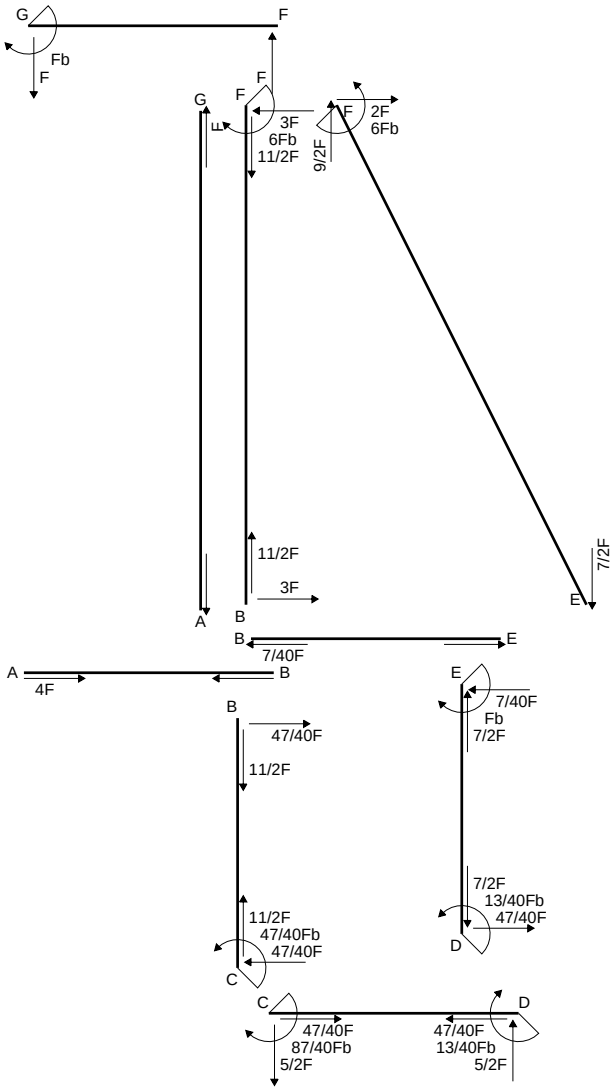
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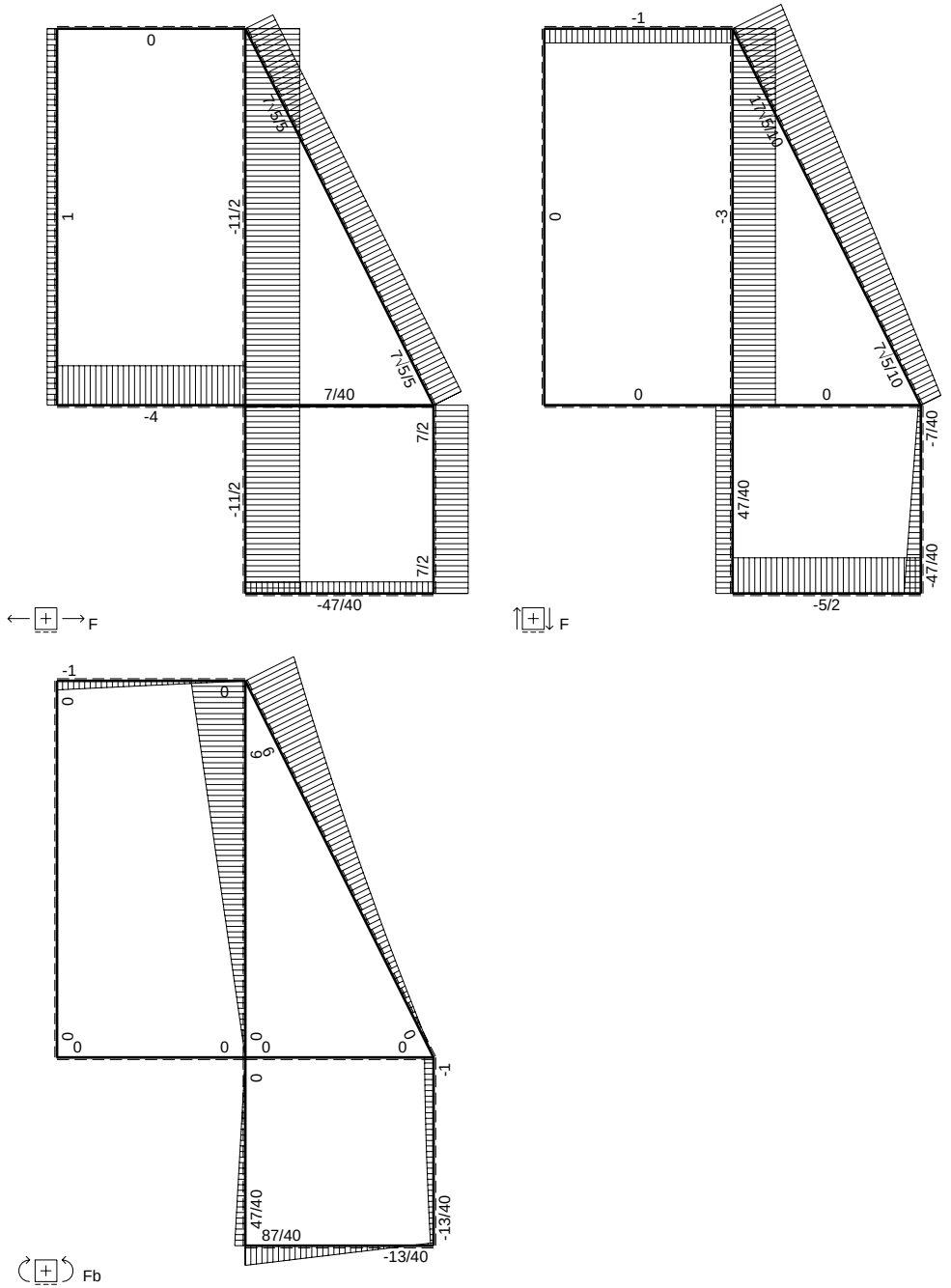
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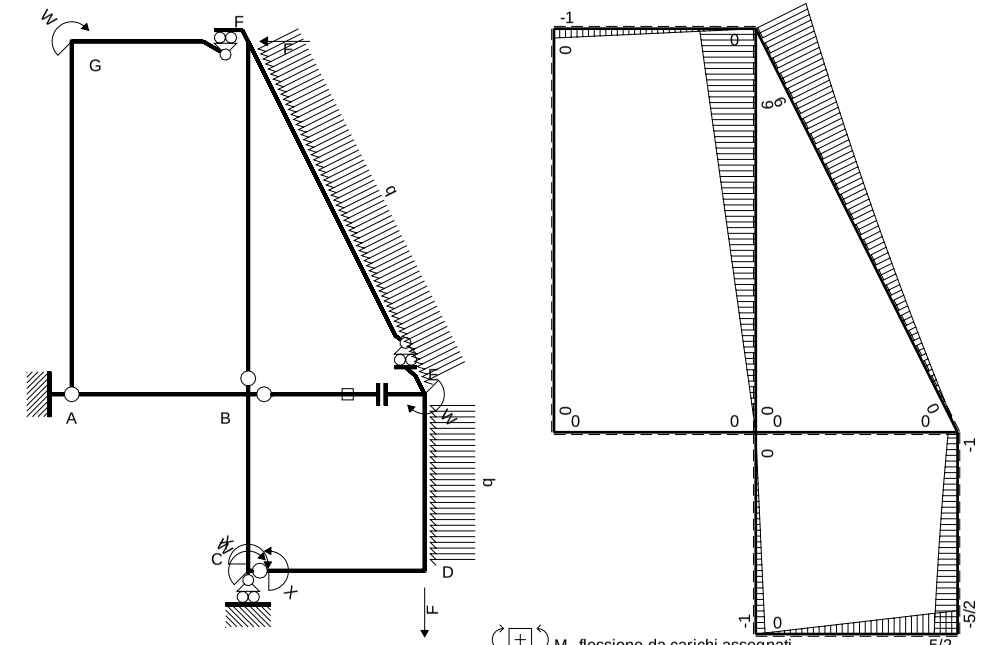
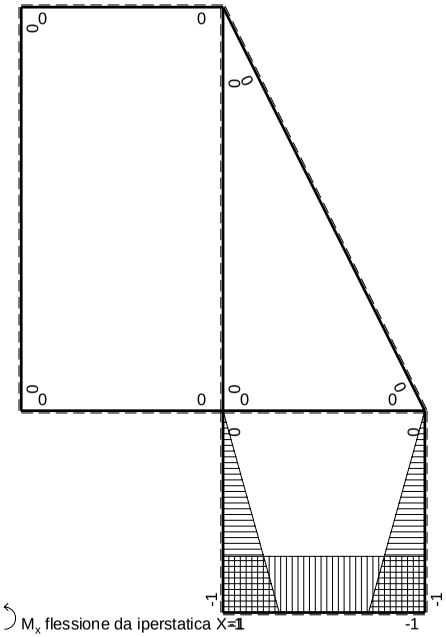


Schema di calcolo iperstatico

M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	$-5/2Fb+Fx+1/2qx^2$	$5/2Fb-7/2Fx+1/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$25/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb+2Fx-1/2qx^2$	$Fx+2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$29/8Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-87/40Fb	

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Sviluppi di calcolo iperstatica

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$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -7/2\ x/b +1/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [5/2\ x -7/4\ x^2/b +1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -7/4\ b +1/6\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b +2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 25/24\ Fb^2/EJ$

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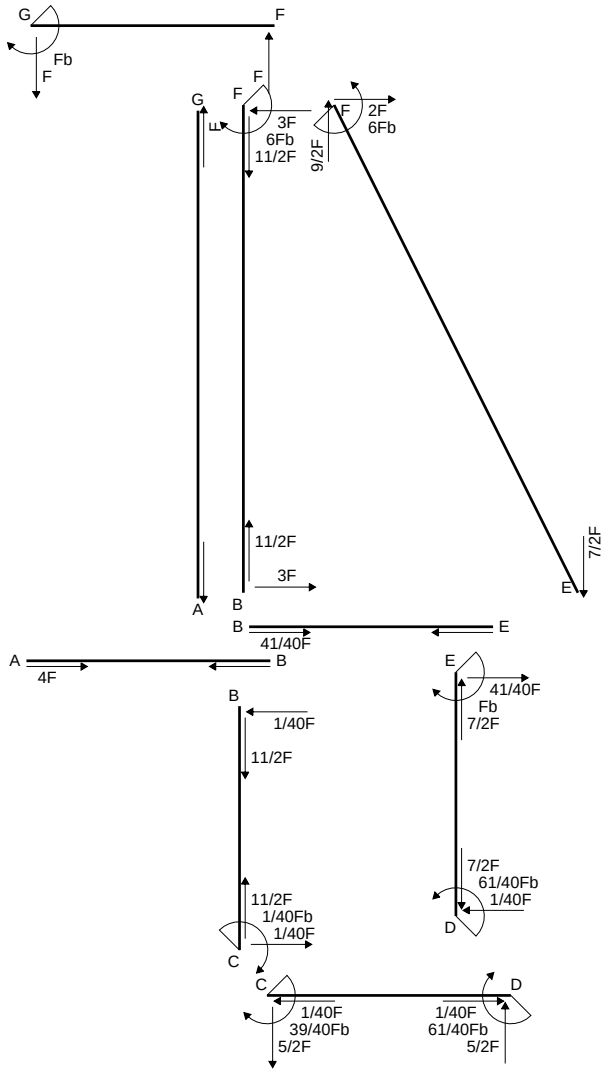
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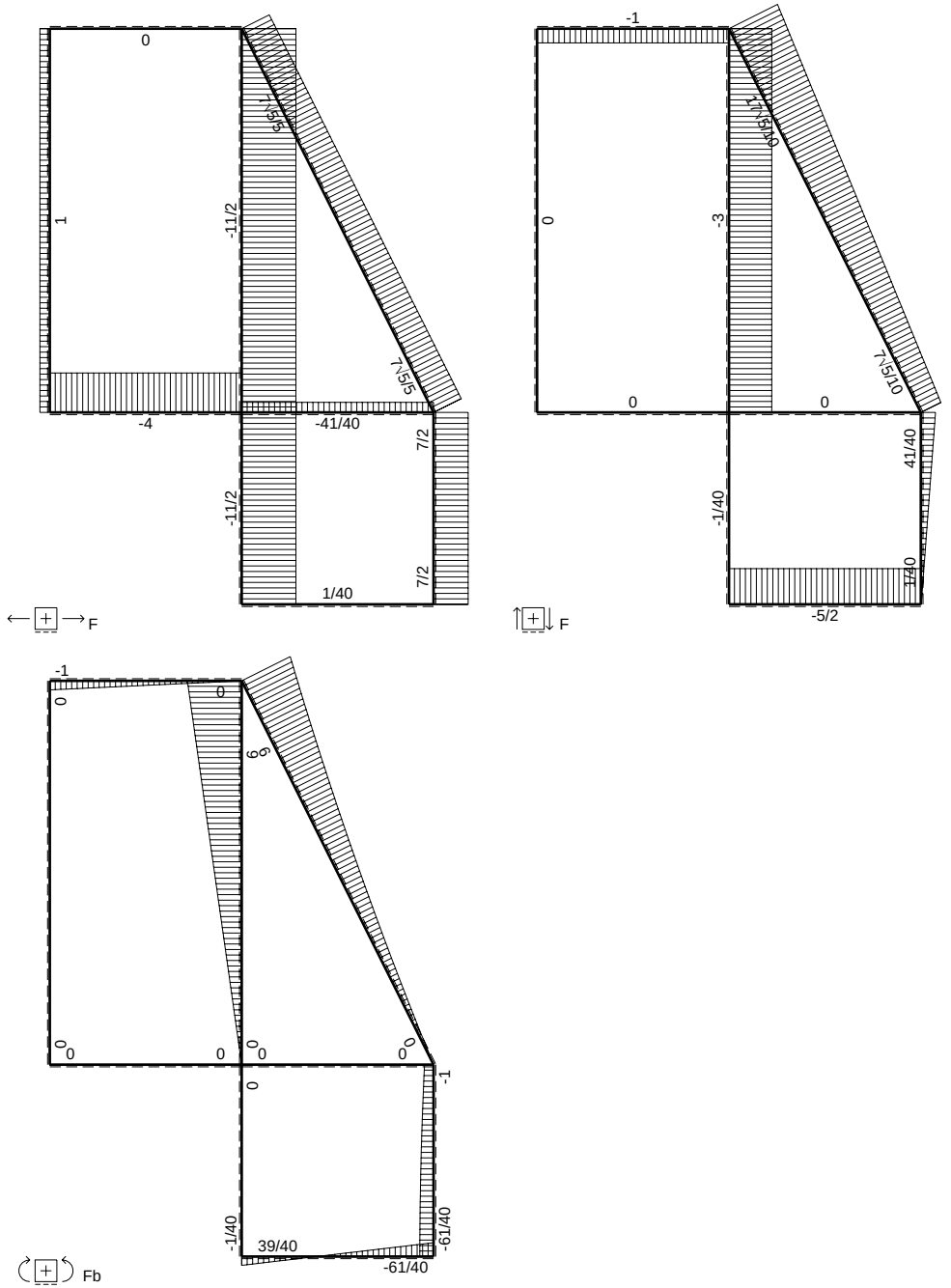
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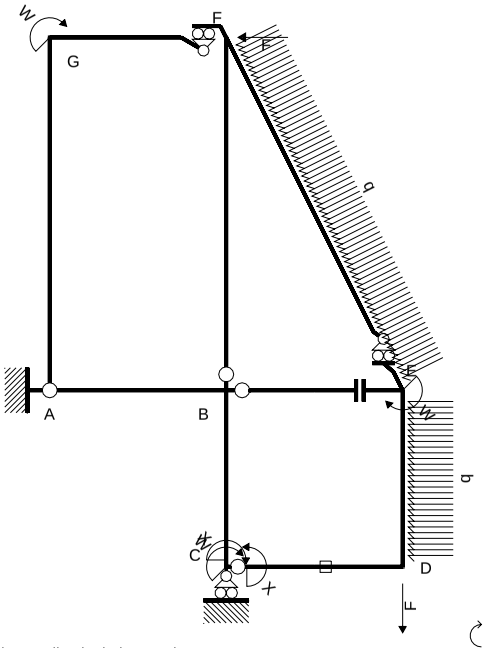
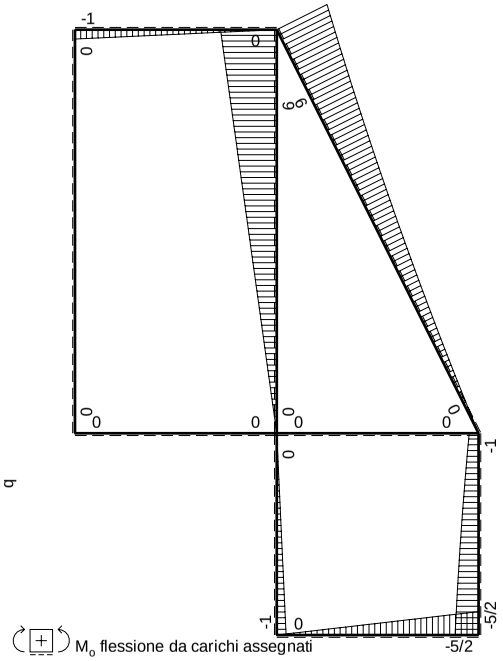
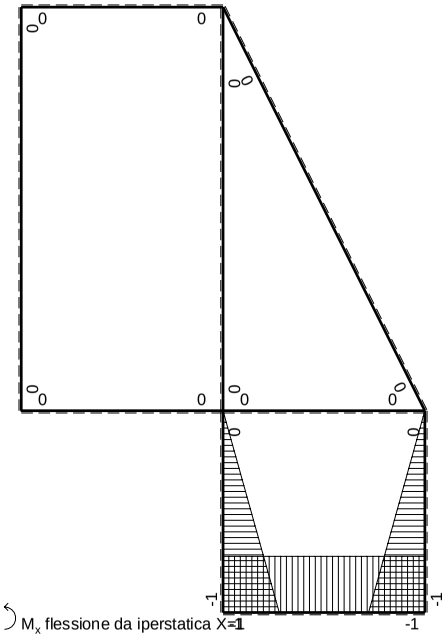


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	$-5/2Fb+Fx+1/2qx^2$	$5/2Fb-7/2Fx+1/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$25/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb+2Fx-1/2qx^2$	$Fx+2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$13/8Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-39/40Fb	

PROCEDIMENTO E RISULTATI 236508 ESΣ02.xxxx.102

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -7/2\ x/b +1/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [5/2\ x -7/4\ x^2/b +1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -7/4\ b +1/6\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 25/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 25/24\ \ Fb^2/EJ$

ESΣ02.xxxx.102

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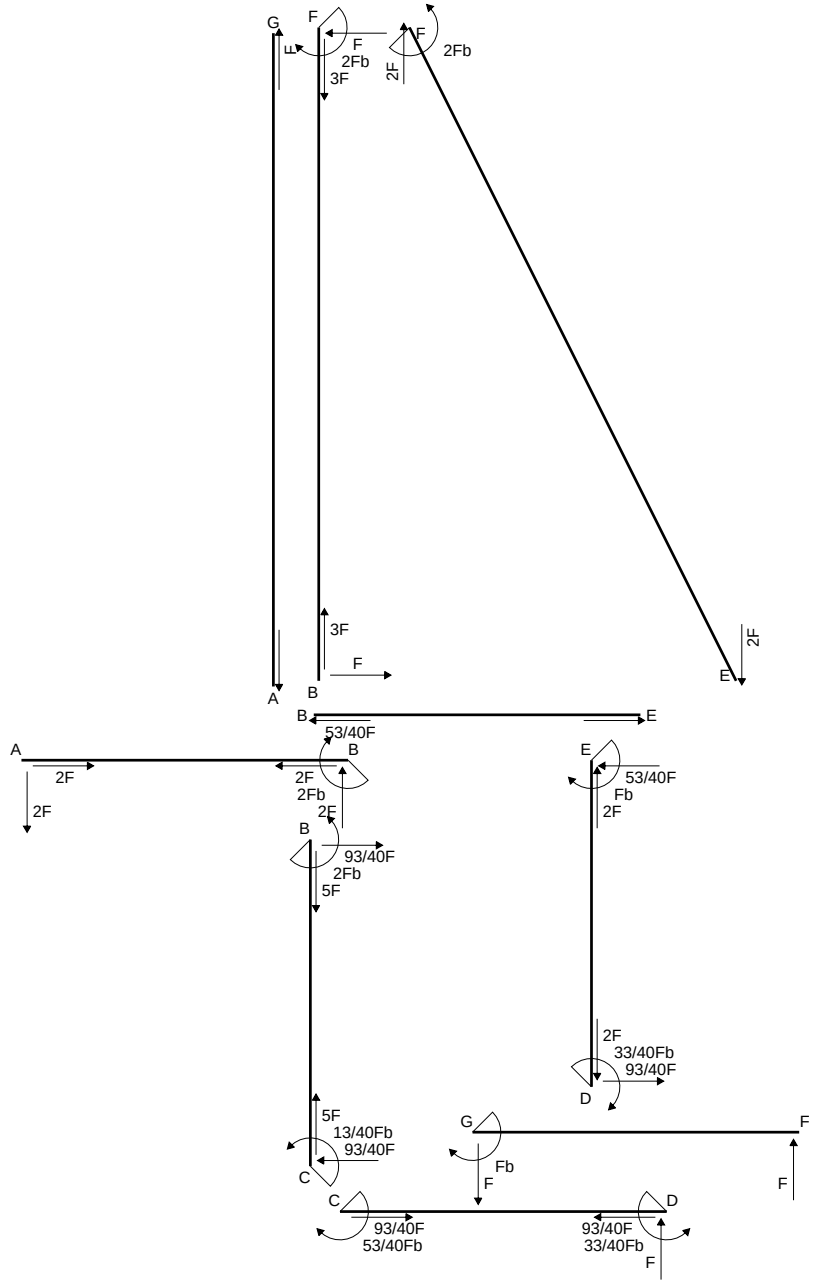
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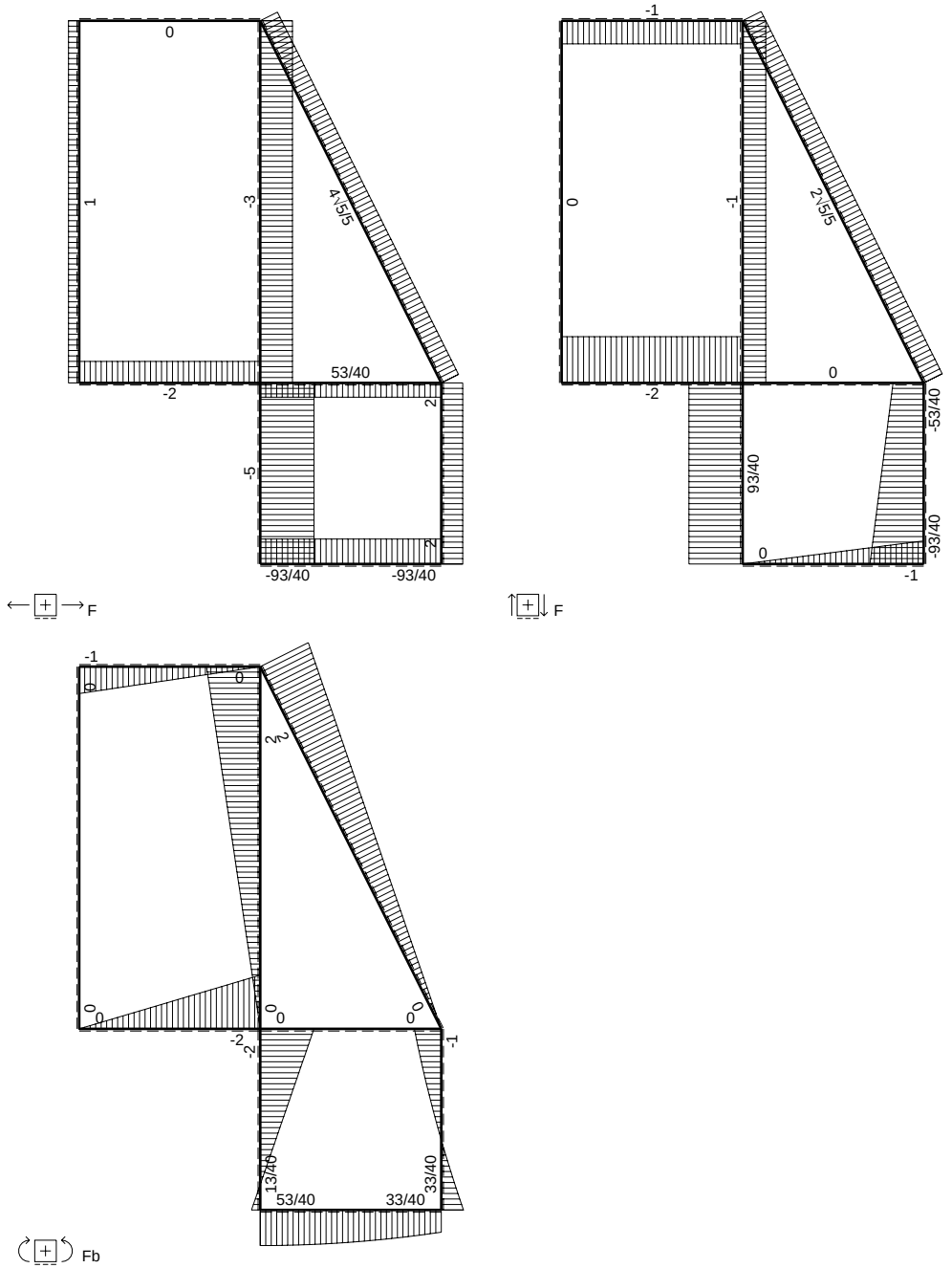
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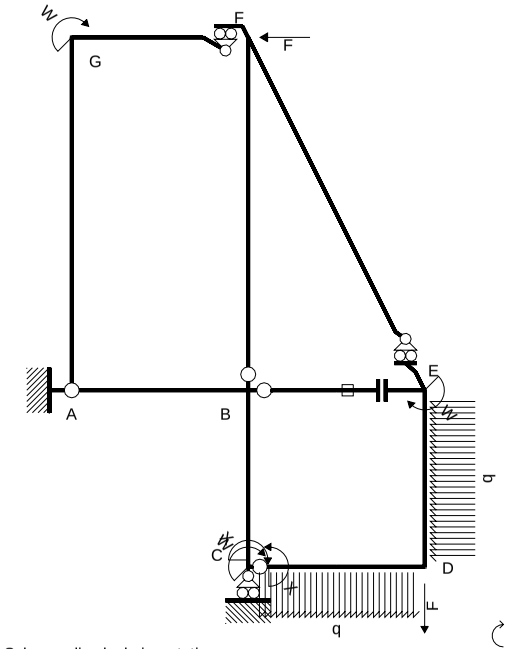
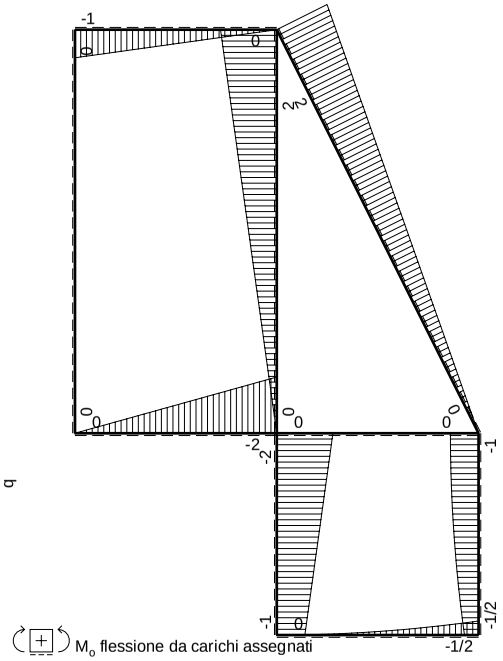
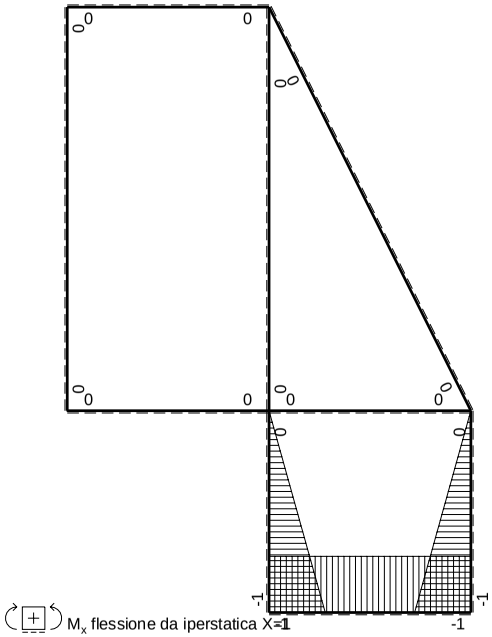


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236538 ESΣ02.xxxx.103

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	$2Fx-Fx^2/b$	x^2/b^2	$2/3Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+Fx	$Fb-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb-Fx+1/2qx^2$	$1/2Fb+1/2Fx-3/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$3/8Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2qx^2$	$Fx-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$53/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-53/40Fb	

PROCEDIMENTO E RISULTATI 236538 ESΣ02.xxxx.103

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 01.12.25

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (2\ x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b)\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b)\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 +1/2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [1/2\ x +1/4\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/4\ b -1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/8\ b)\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

ESΣ02.xxxx.103

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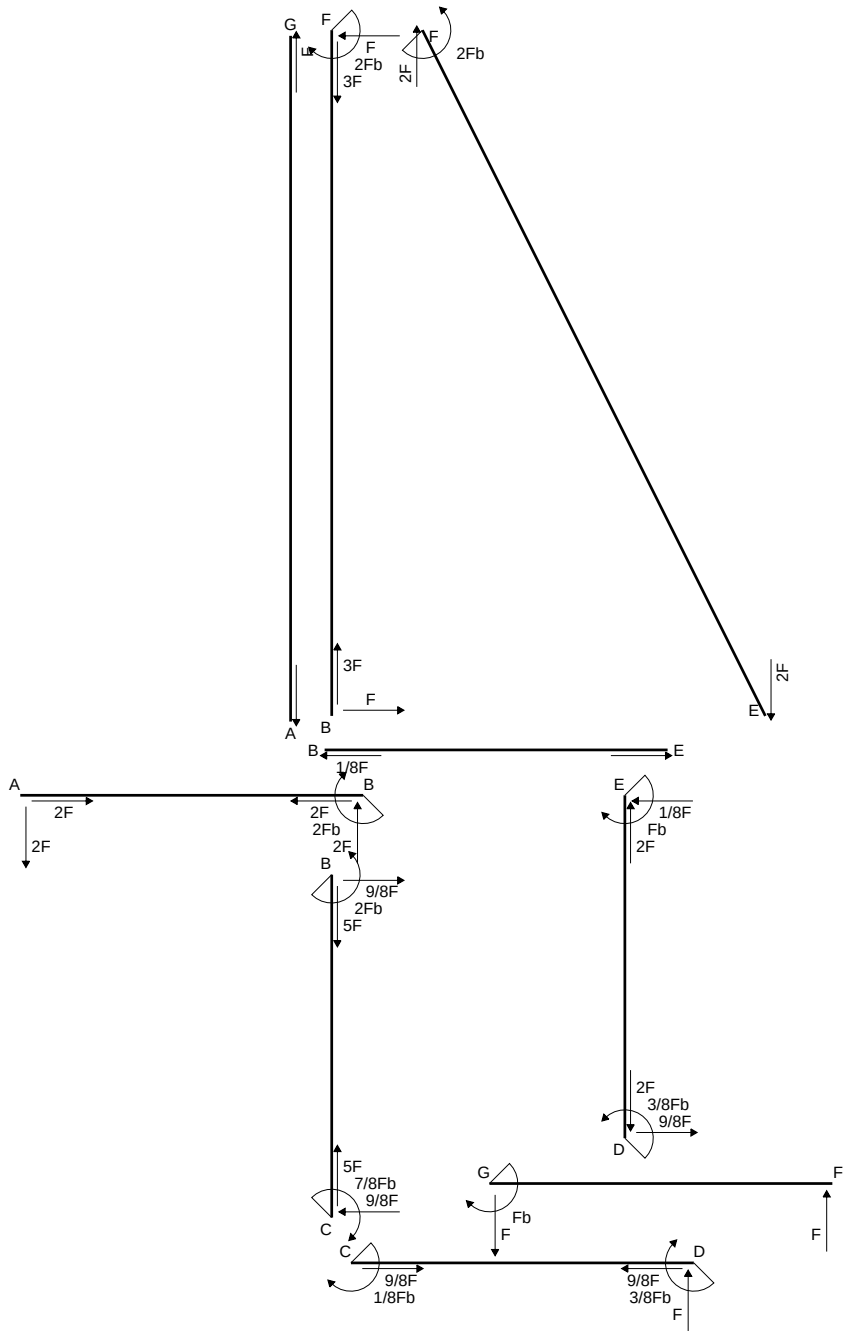
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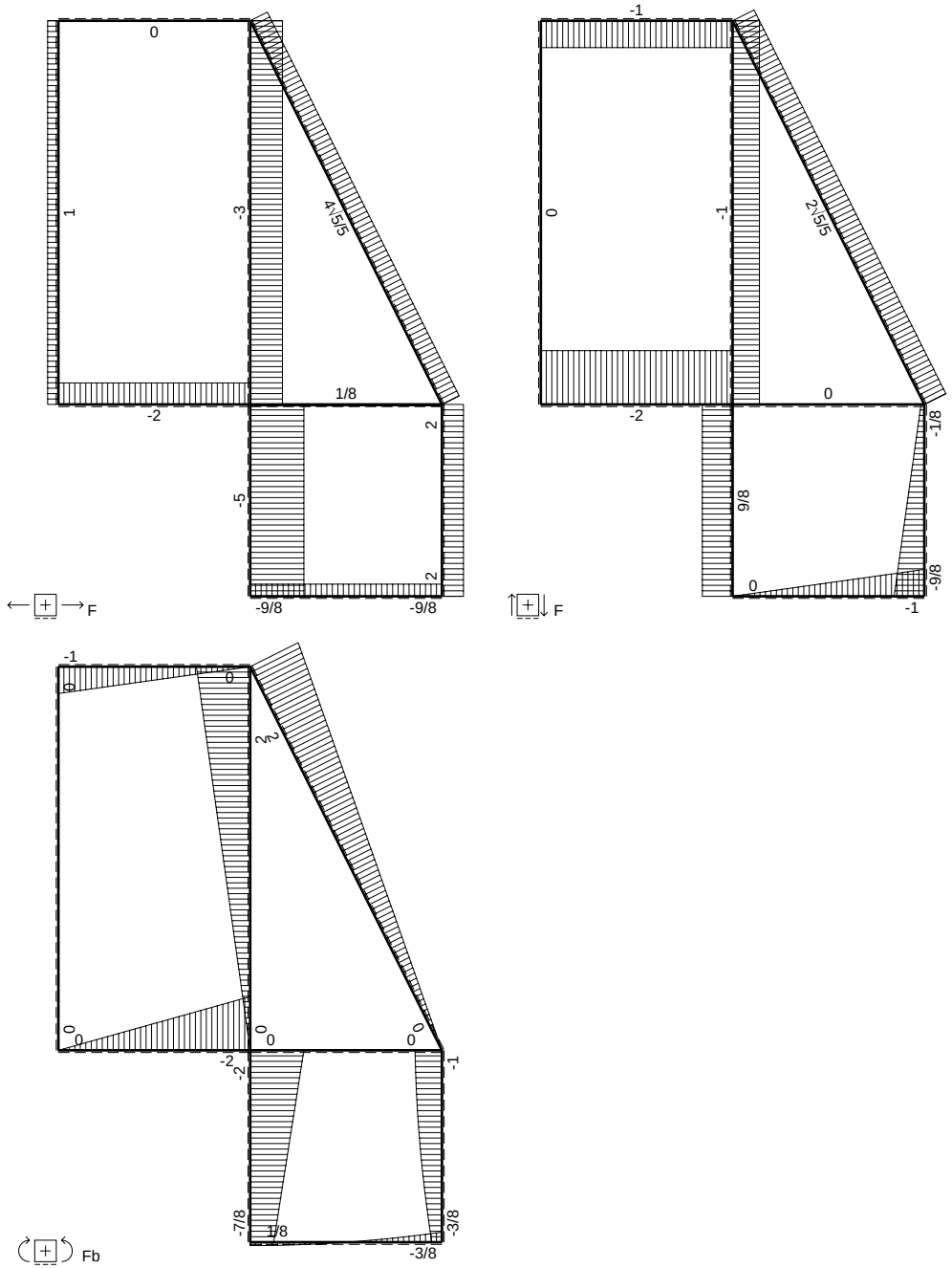
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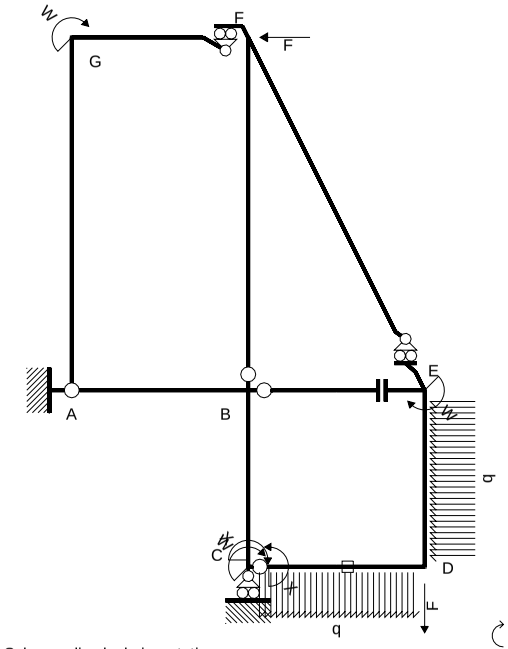
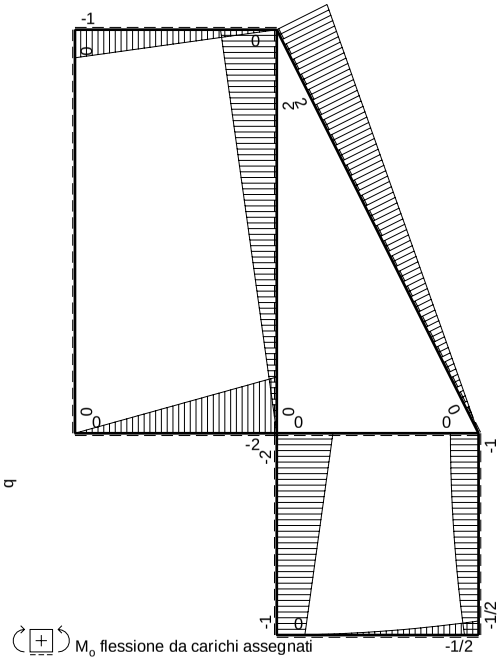
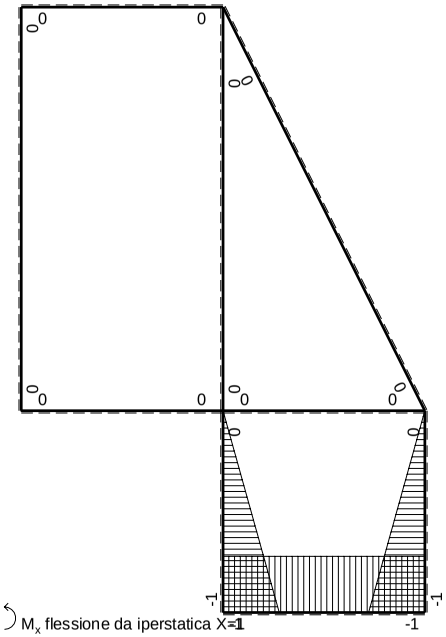


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 236835 ESΣ02.xxxx.104

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	-2Fb+Fx	$2Fx-Fx^2/b$	x^2/b^2	$2/3Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+Fx	$Fb-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb-Fx+1/2qx^2$	$1/2Fb+1/2Fx-3/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$3/8Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2qx^2$	$Fx-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-1/8Fb	

PROCEDIMENTO E RISULTATI 236835 ESΣ02.xxxx.104

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (2\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 +1/2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [1/2\ x +1/4\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/4\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 236835 ESΣ02.xxxx.104

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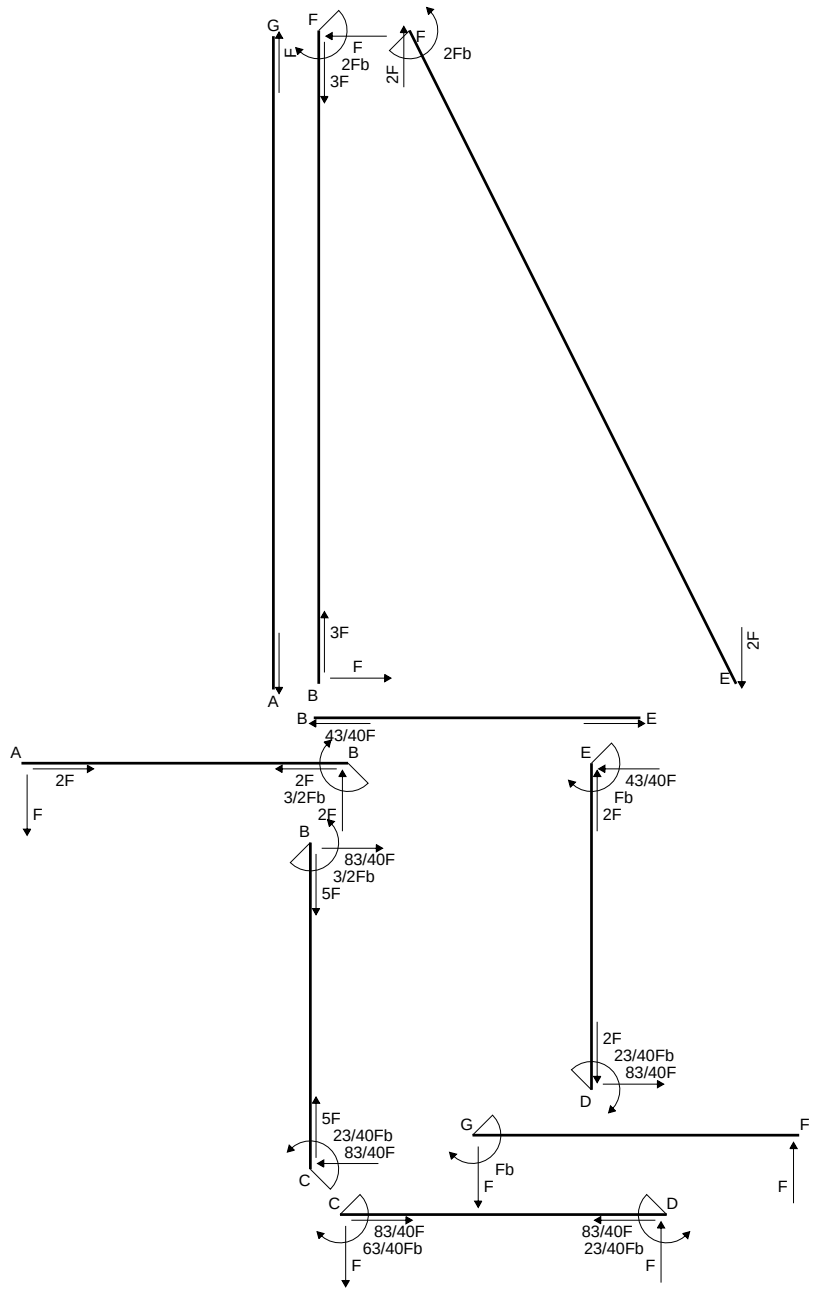
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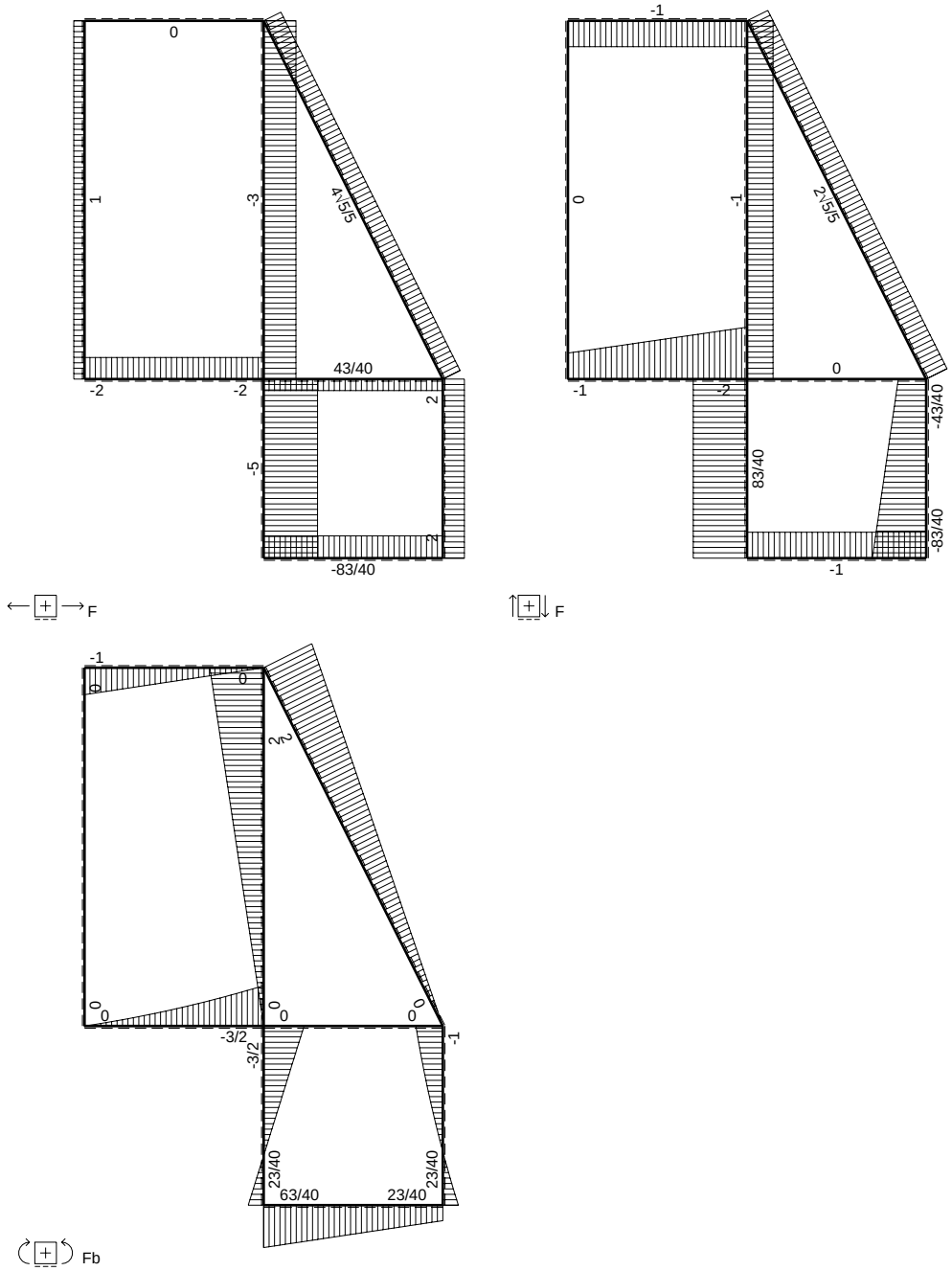
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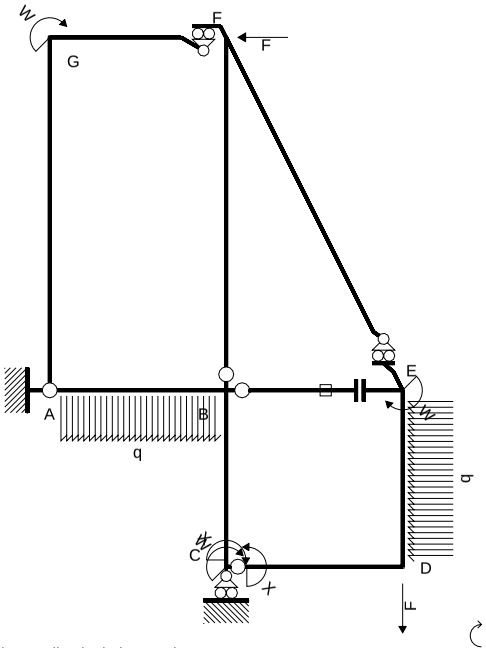
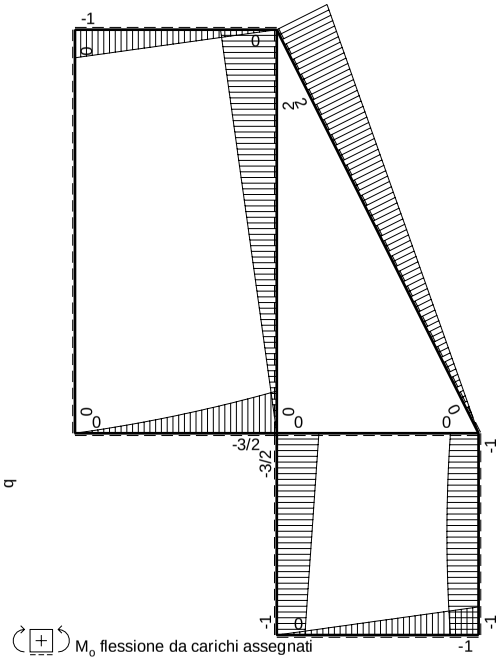
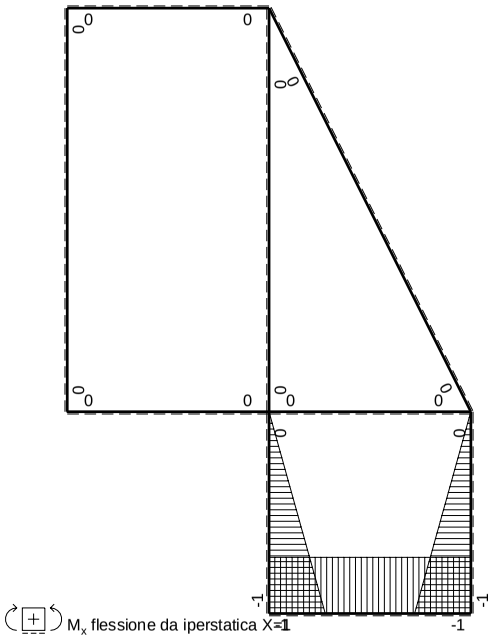


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-Fx$	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	$Fb-Fx$	$Fb-Fx$	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-Fx$	0	0	0	0
GF b	0	$Fb-Fx$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$2Fb-Fx$	0	0	0	0
BF 2b	0	$-Fx$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$21/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-63/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

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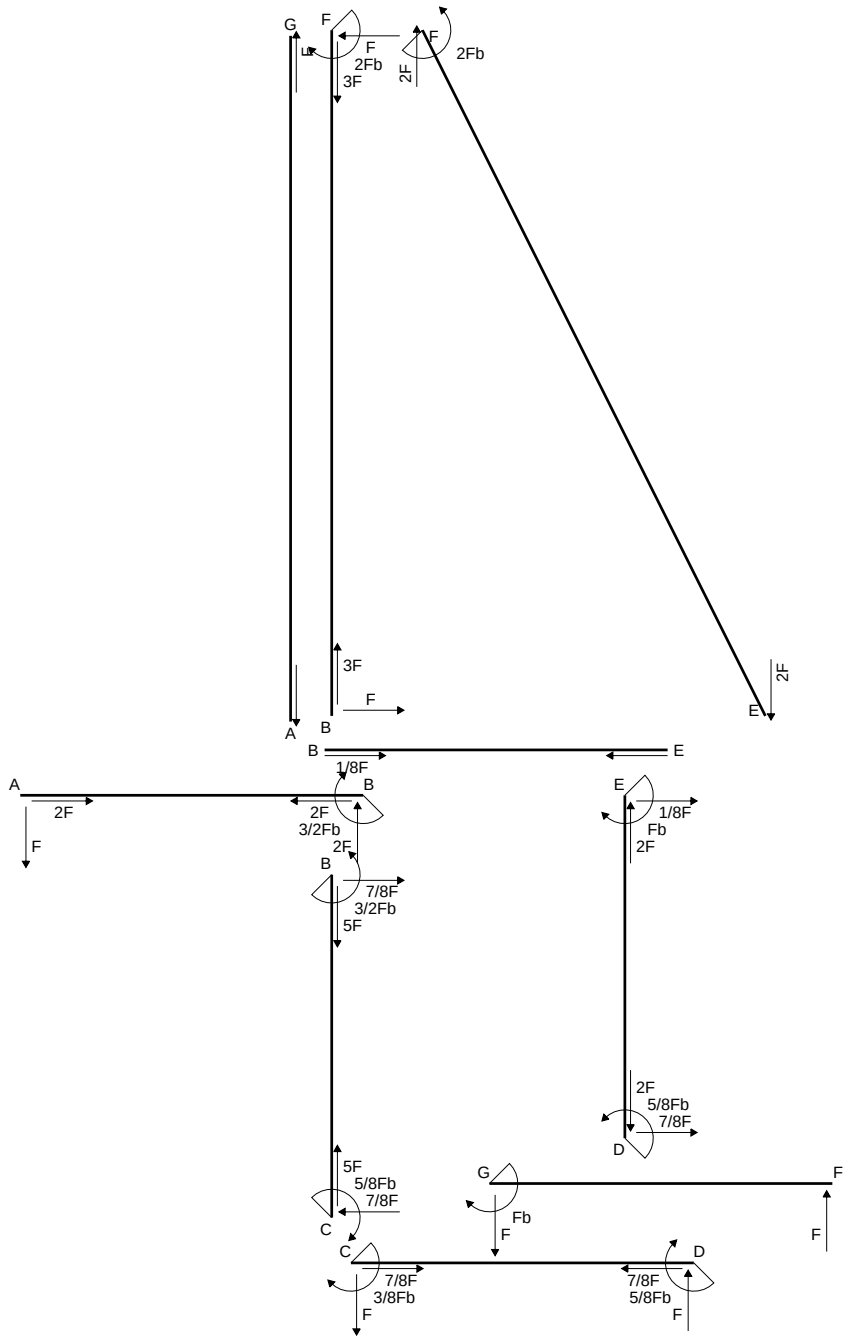
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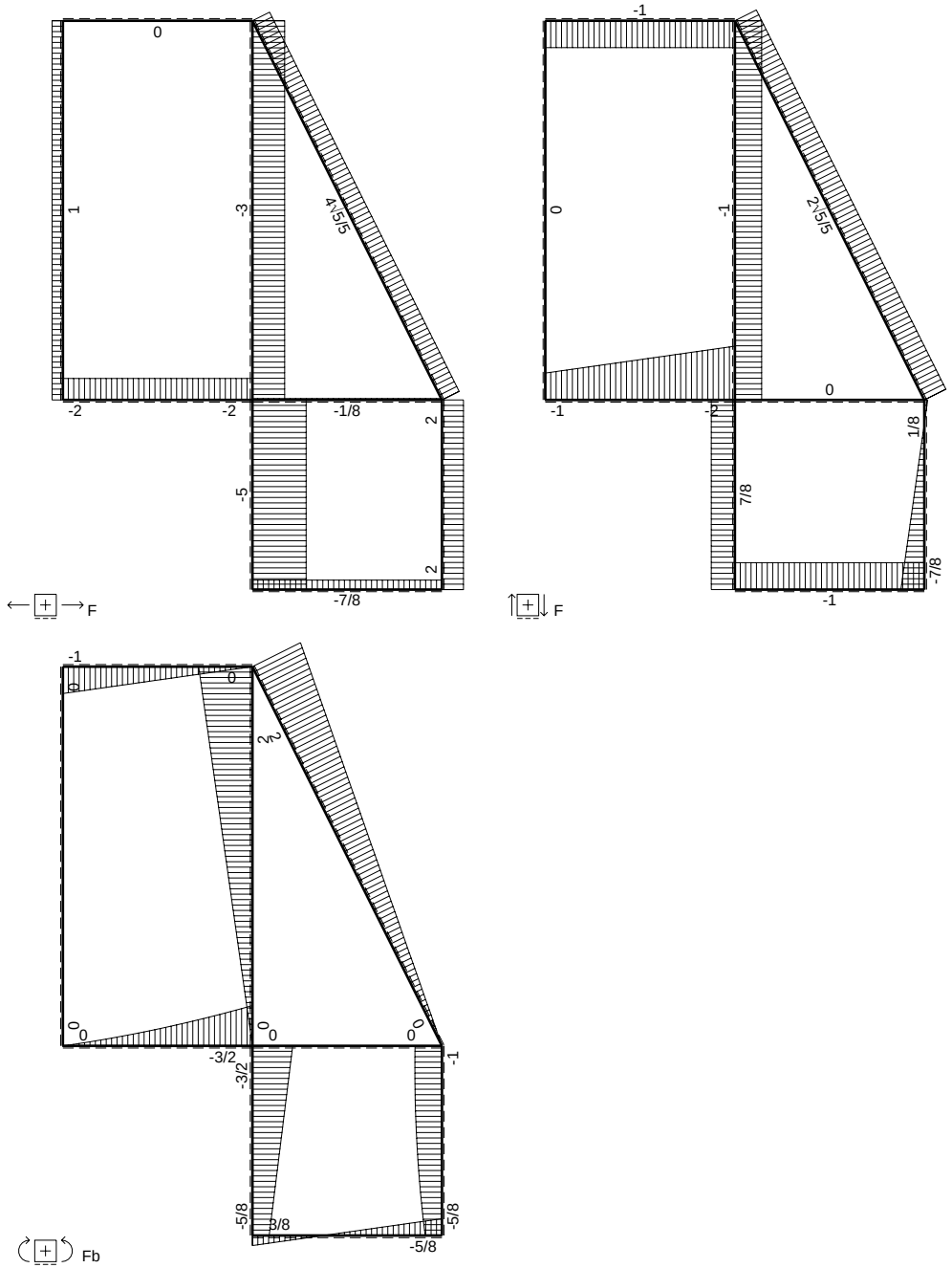
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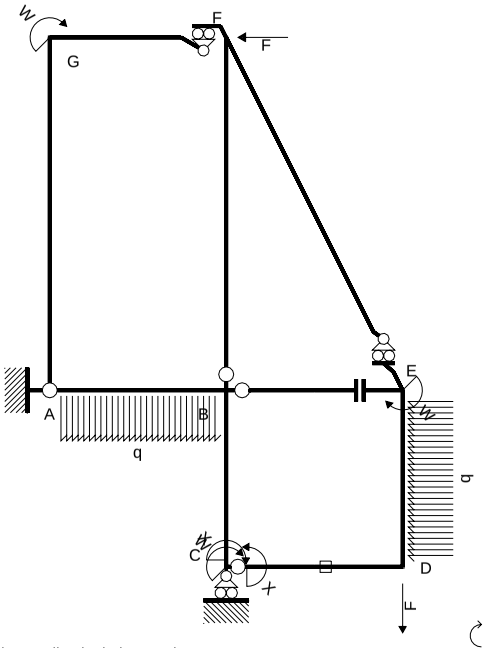
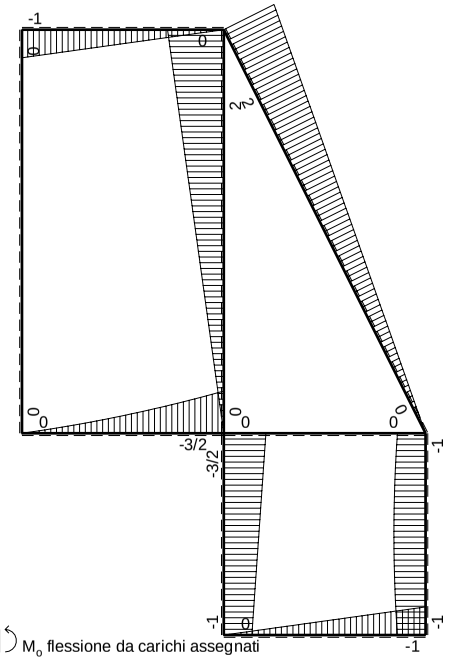
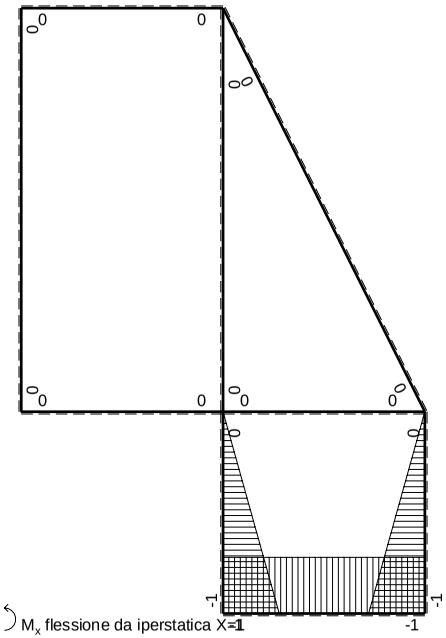


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	-x/b	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

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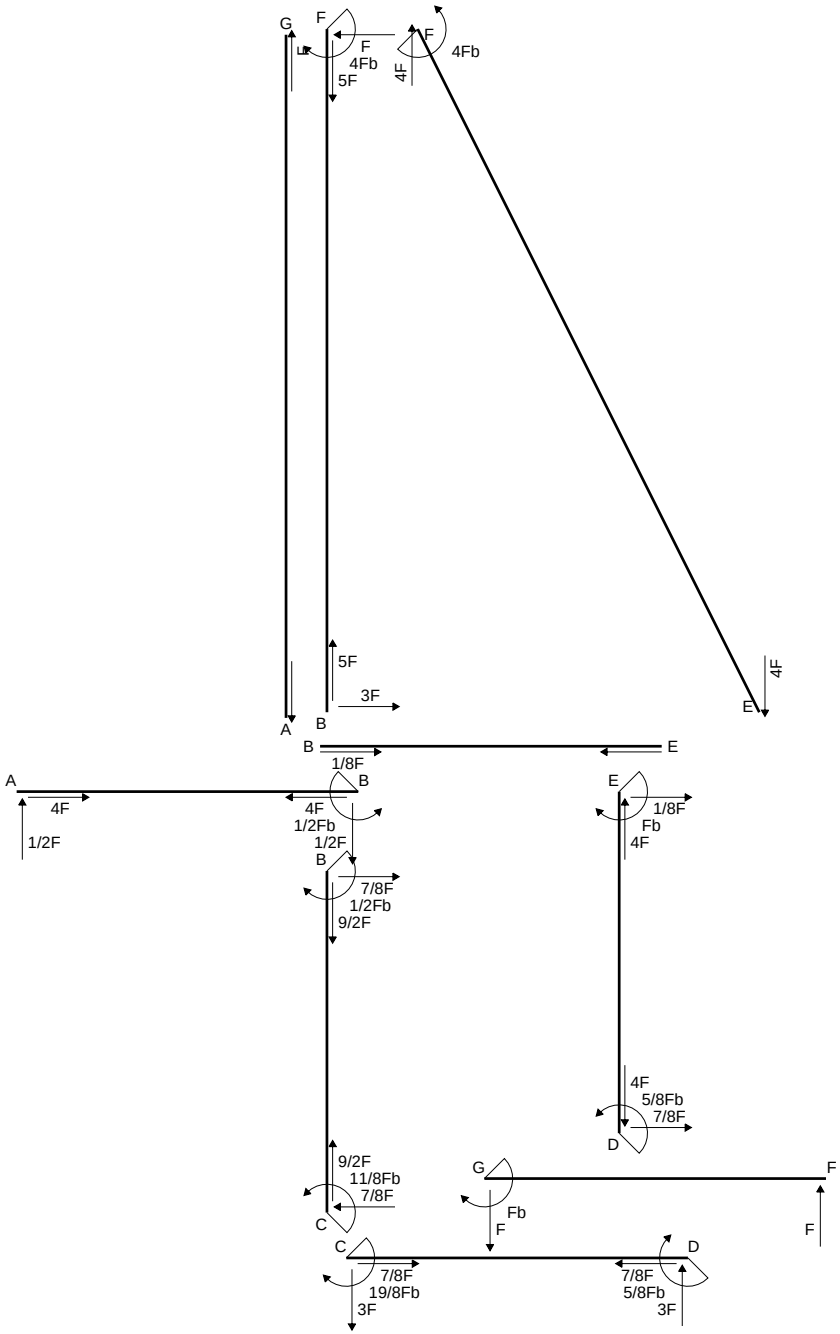
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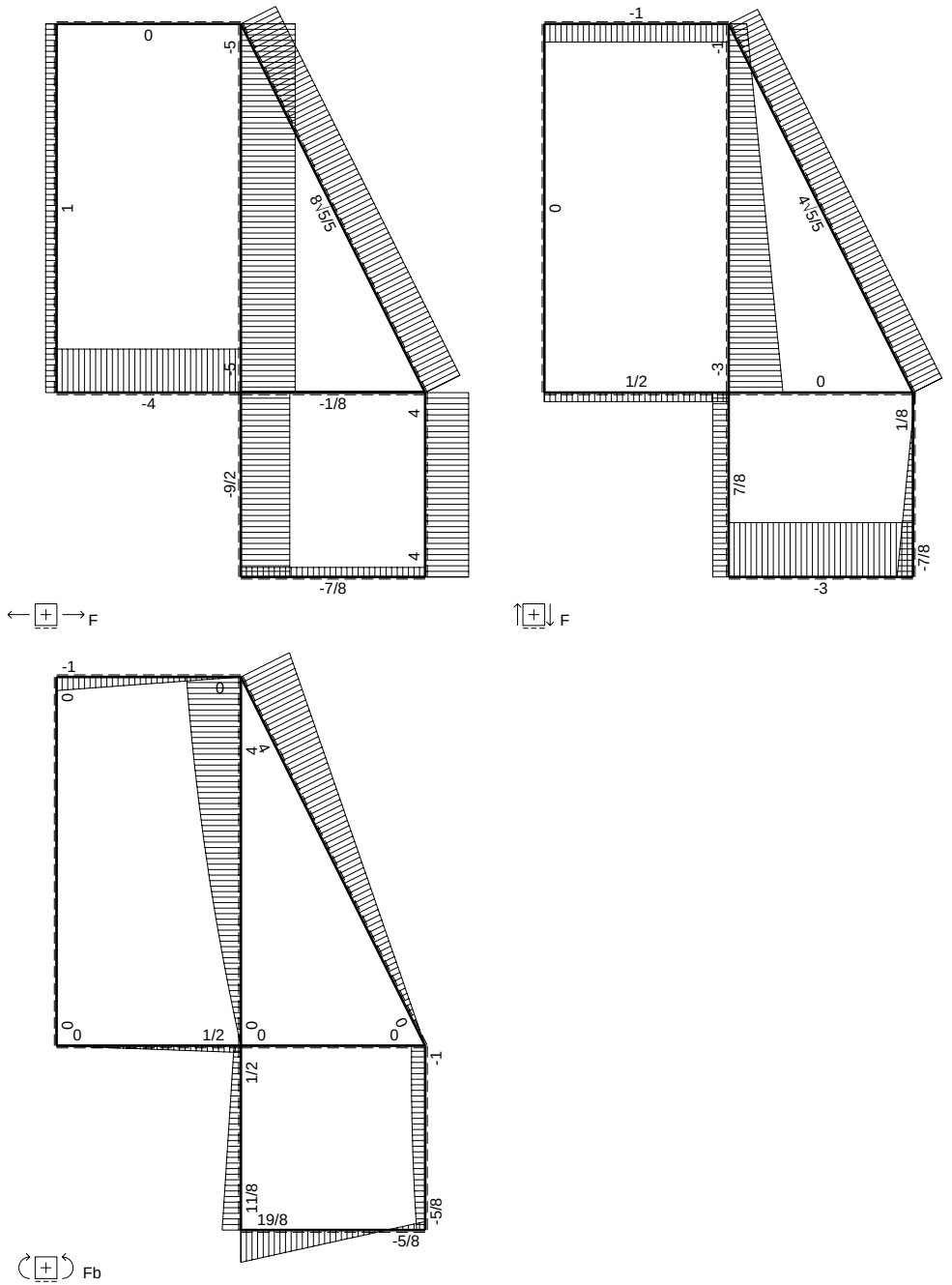
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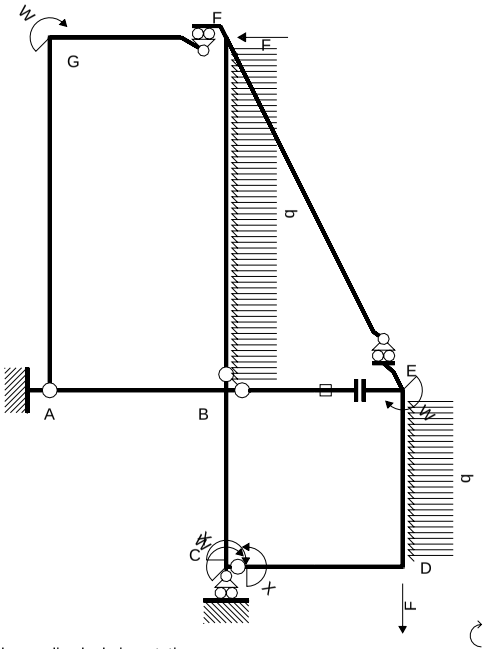
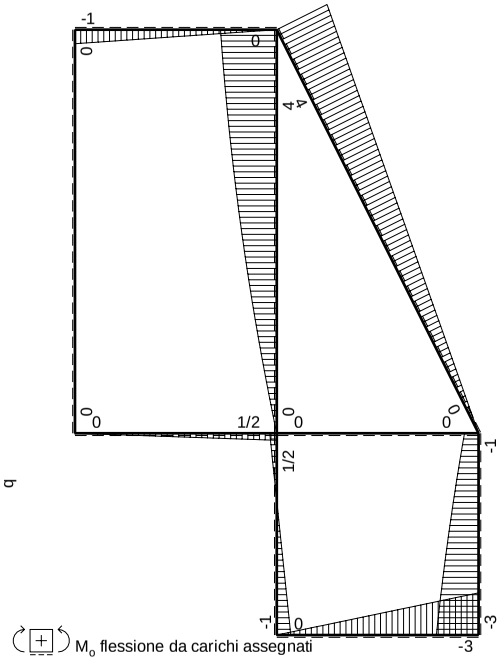
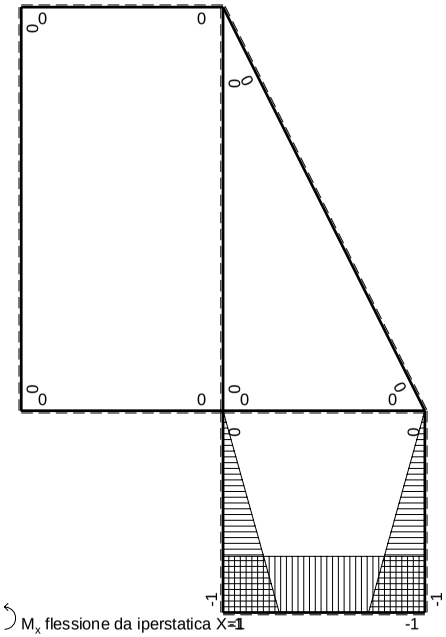


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 983430 ESΣ02.xxxx.107

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	$-3Fb+3/2Fx+1/2qx^2$	$3Fb-9/2Fx+Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$29/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb+5/2Fx-1/2qx^2$	$Fx+5/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$95/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-19/8Fb	

PROCEDIMENTO E RISULTATI 983430 ESΣ02.xxxx.107

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -9/2\ x/b + x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [3\ x -9/4\ x^2/b +1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -9/4\ b +1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 29/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +5/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +5/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +5/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 29/24\ Fb^2/EJ$

ESΣ02.xxxx.107

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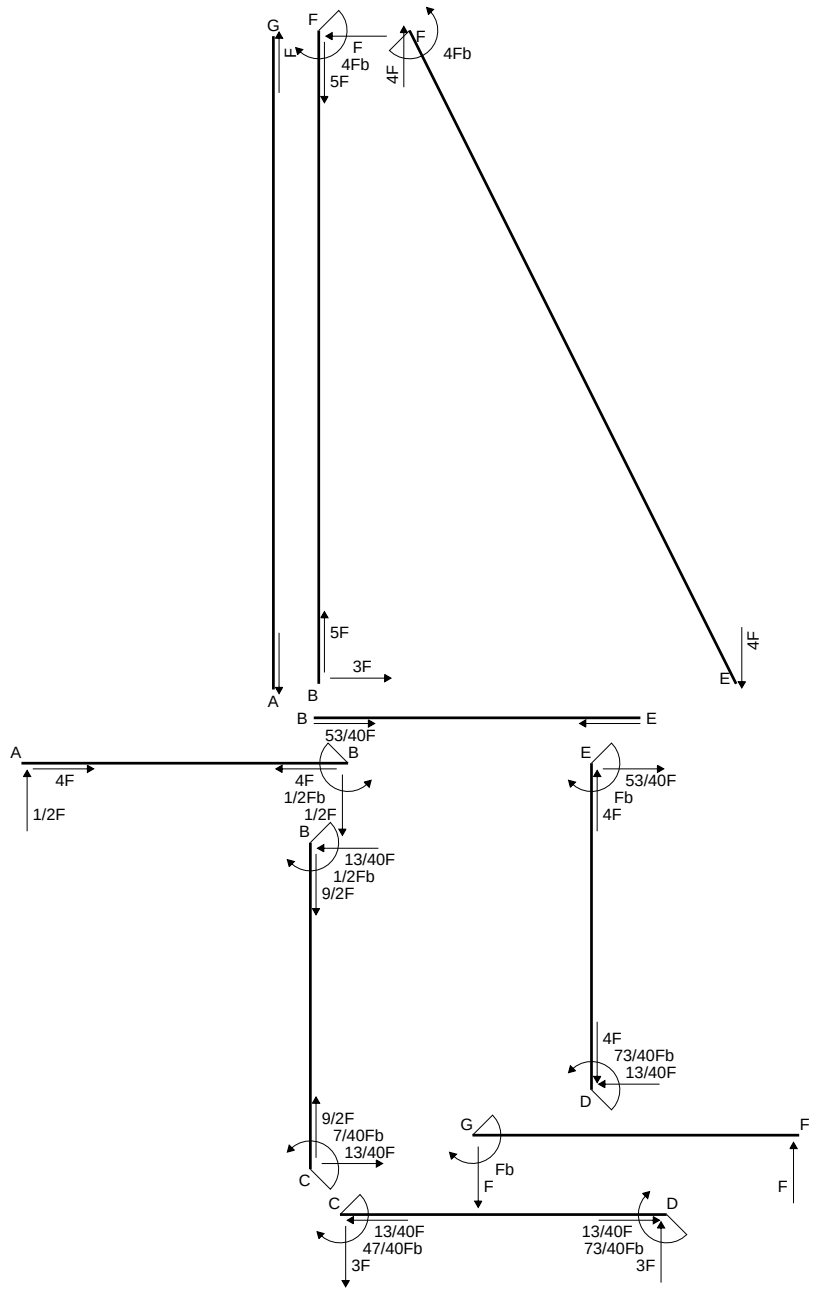
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9/2F
4F
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D
9/2F
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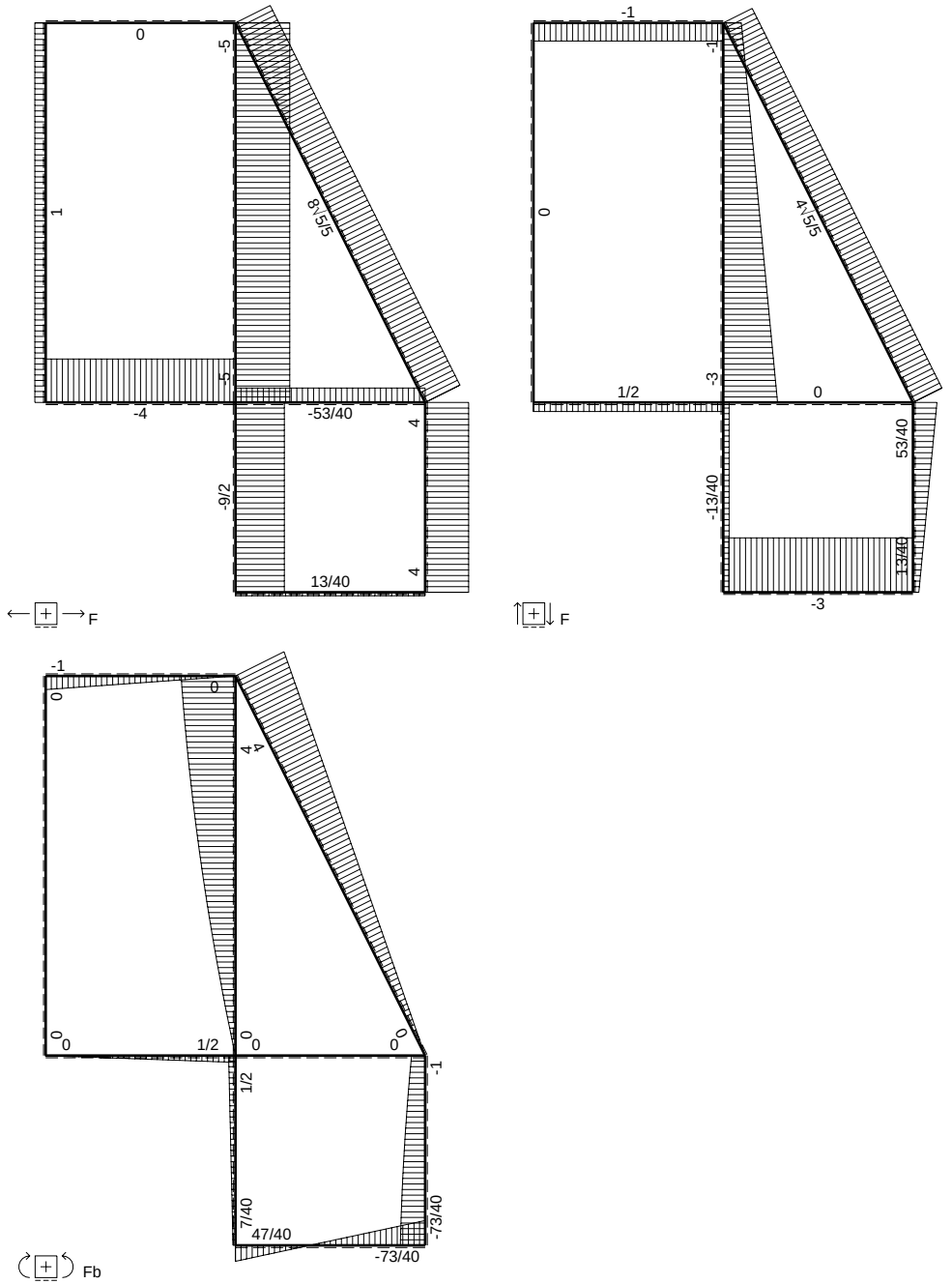
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-1
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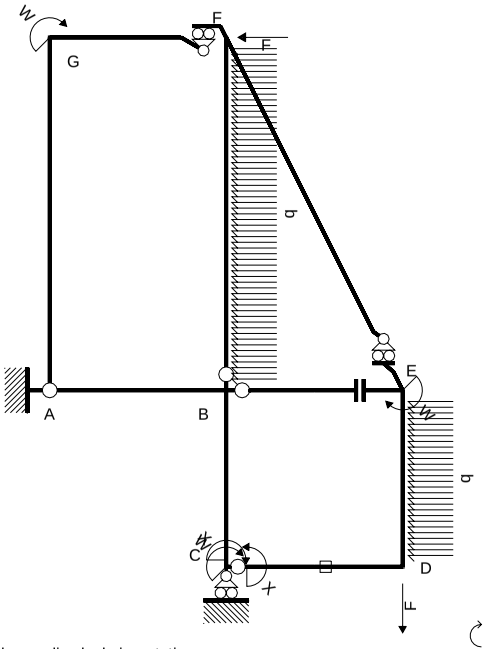
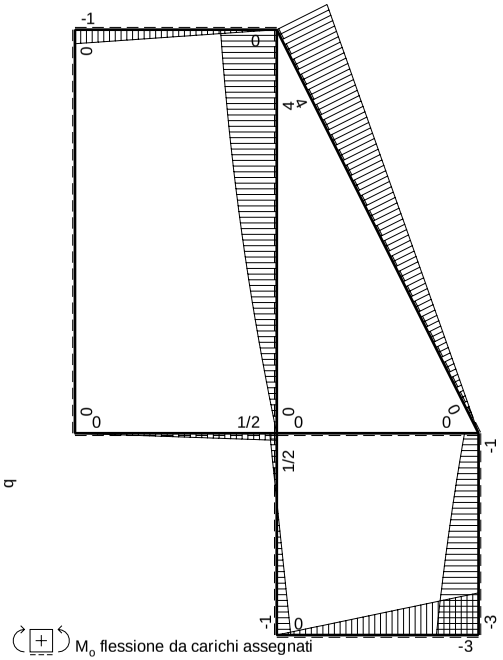
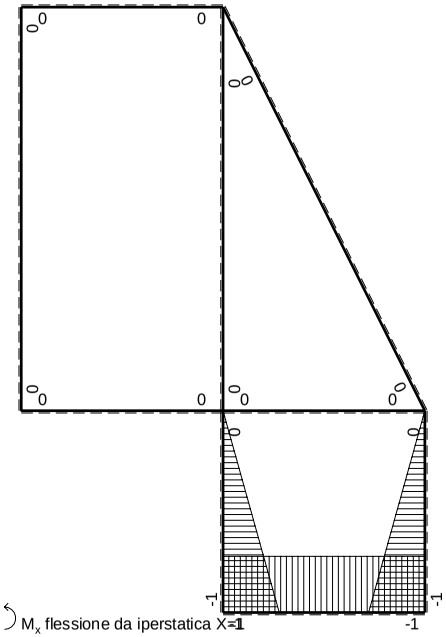
-73/40
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237985 ESΣ02.xxxx.108

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-3/2Fx	$-1/2Fx+3/2Fx^2/b$	x^2/b^2	$1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-3/2Fx	$Fb-5/2Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	$-3Fb+3/2Fx+1/2qx^2$	$3Fb-9/2Fx+Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$29/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb+5/2Fx-1/2qx^2$	$Fx+5/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$47/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-47/40Fb	

PROCEDIMENTO E RISULTATI 237985 ESΣ02.xxxx.108

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -9/2\ x/b + x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [3\ x -9/4\ x^2/b +1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -9/4\ b +1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 29/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +5/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +5/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +5/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 29/24\ \ Fb^2/EJ$

ESΣ02.xxxx.108

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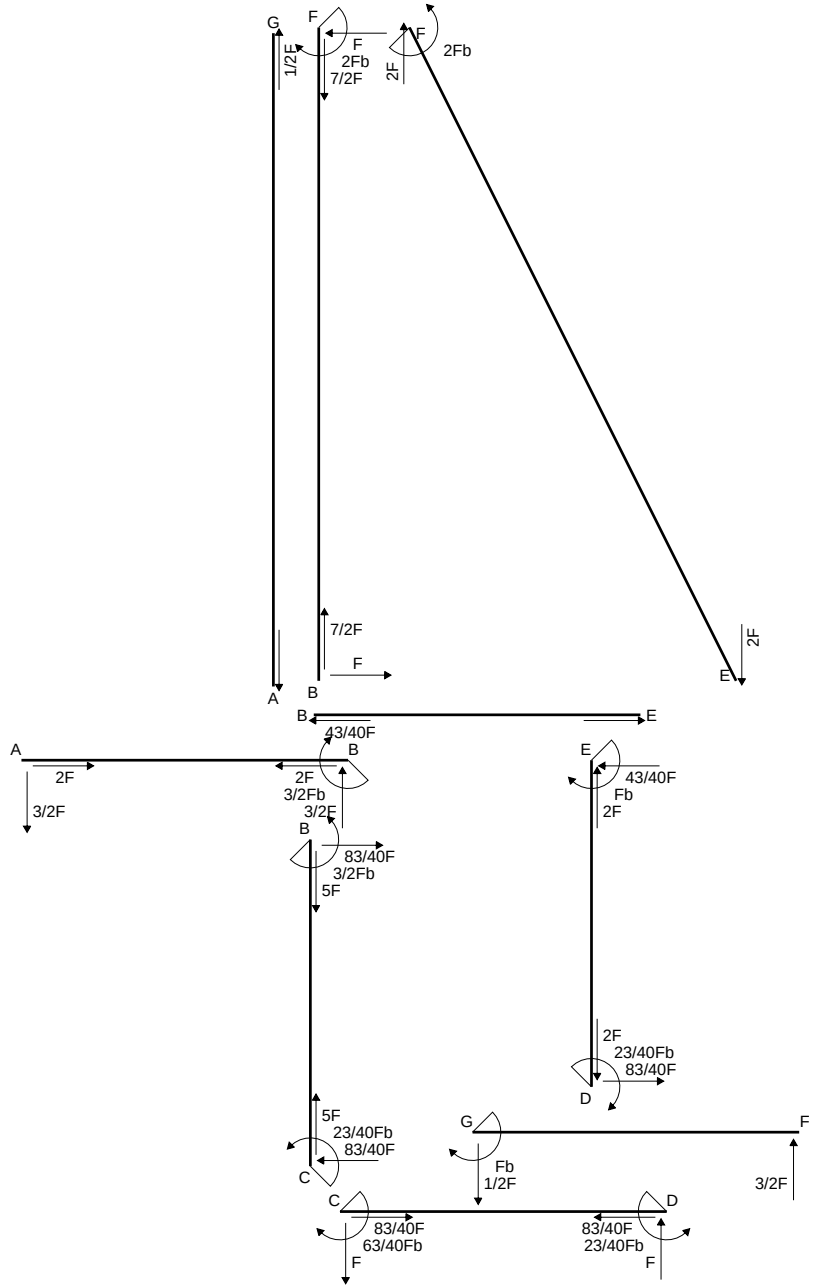
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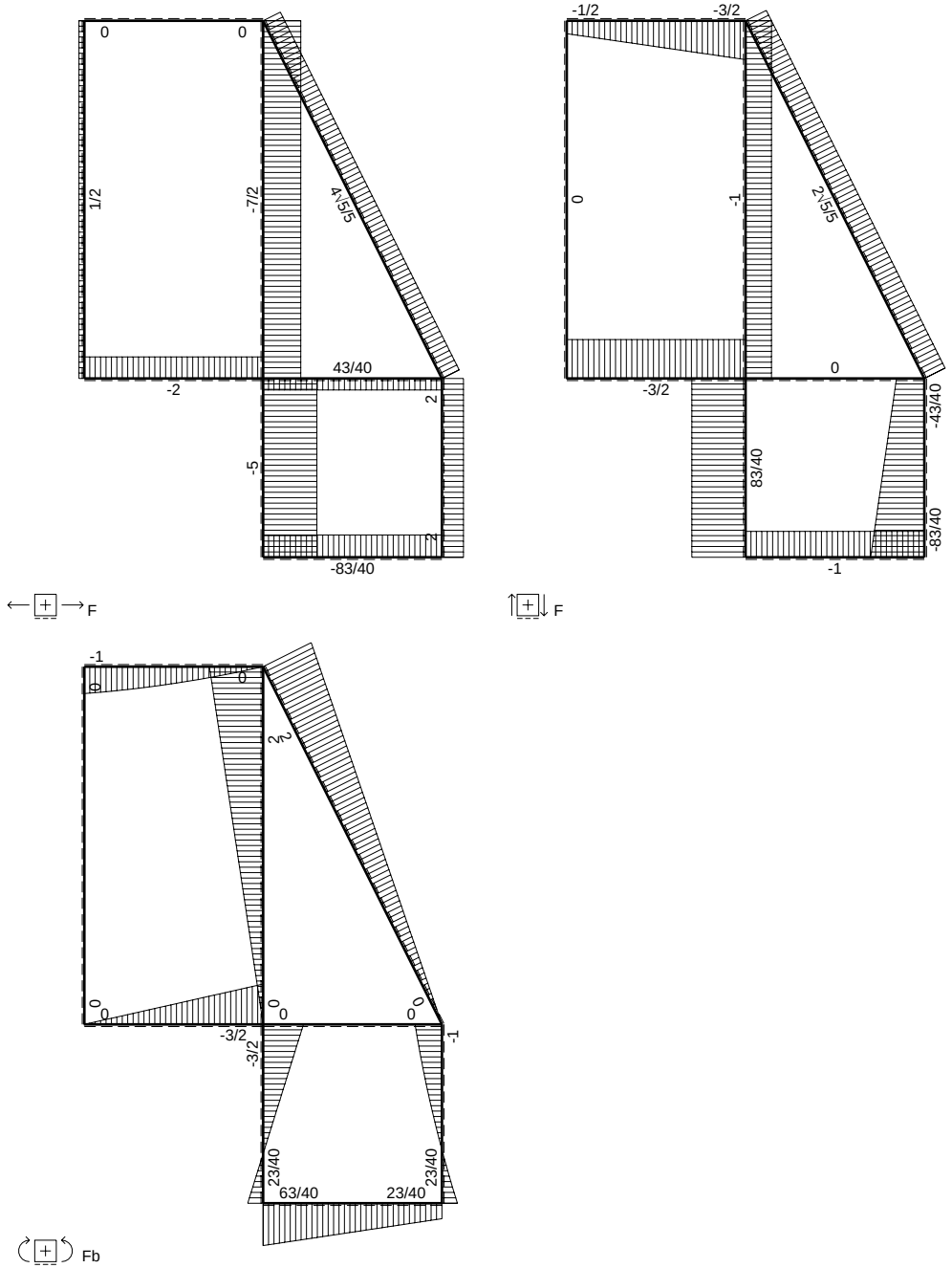
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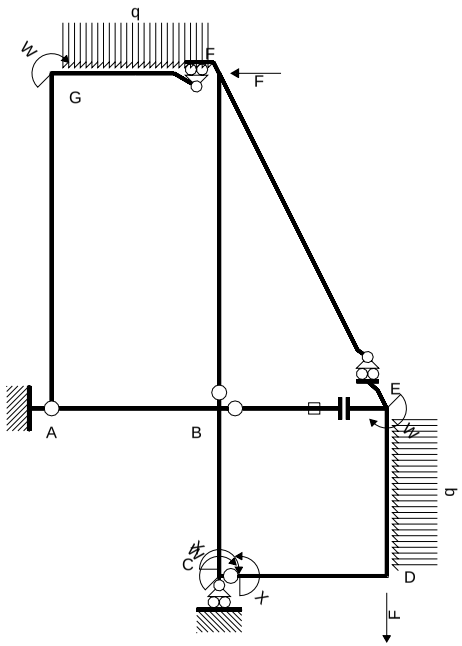
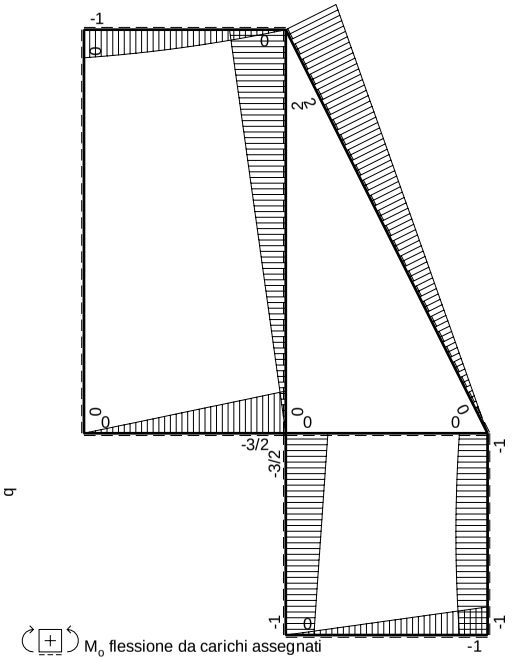
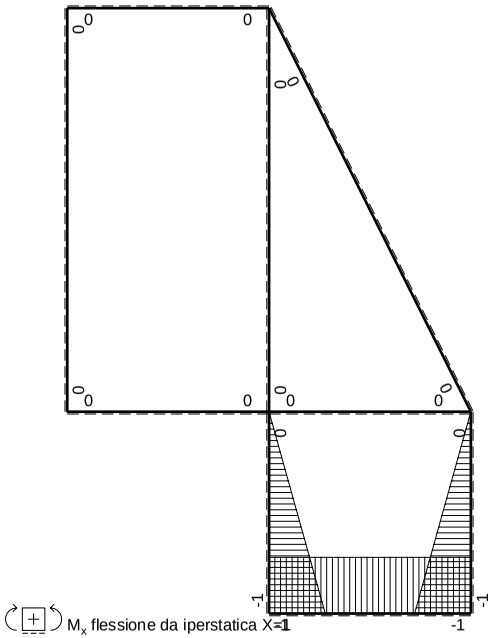
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$21/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-63/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

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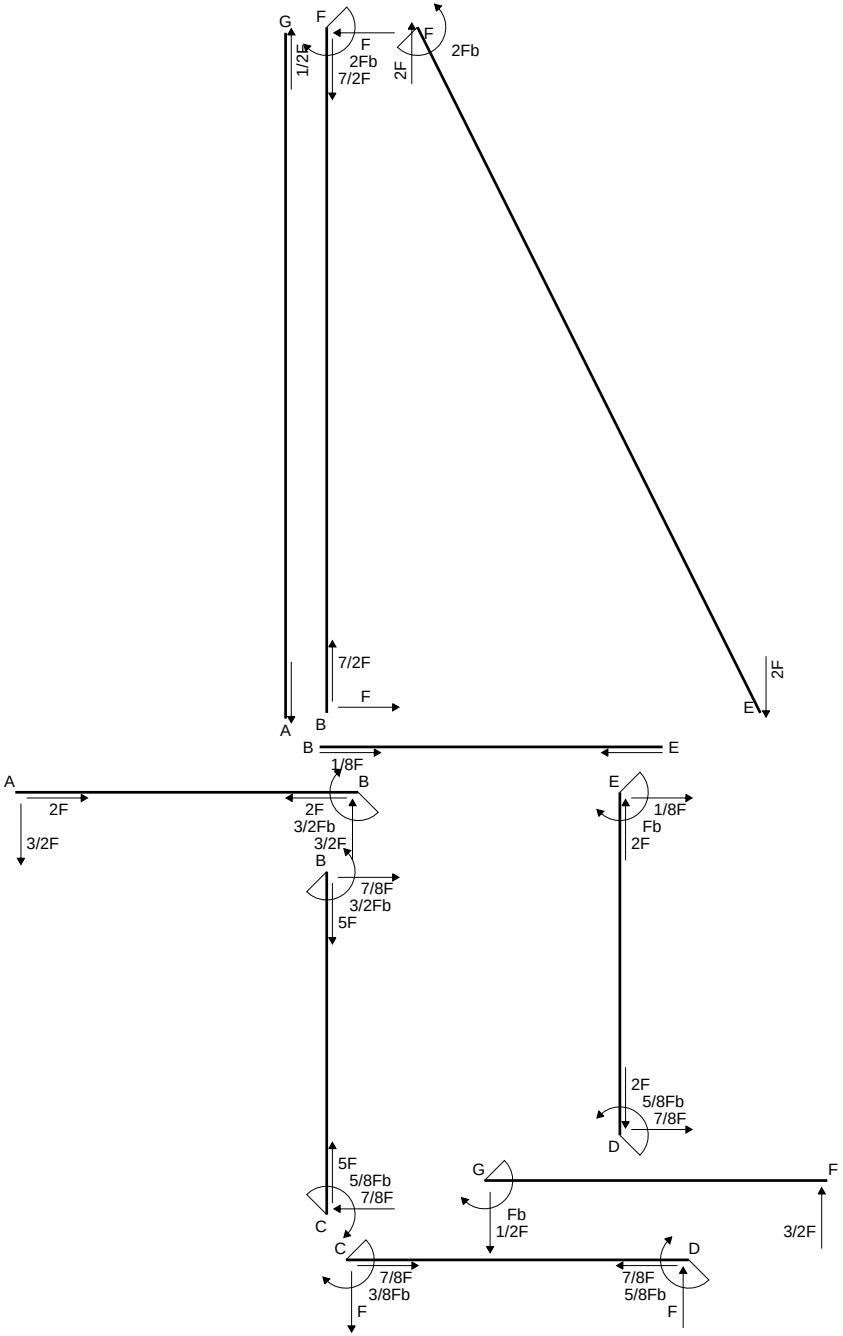
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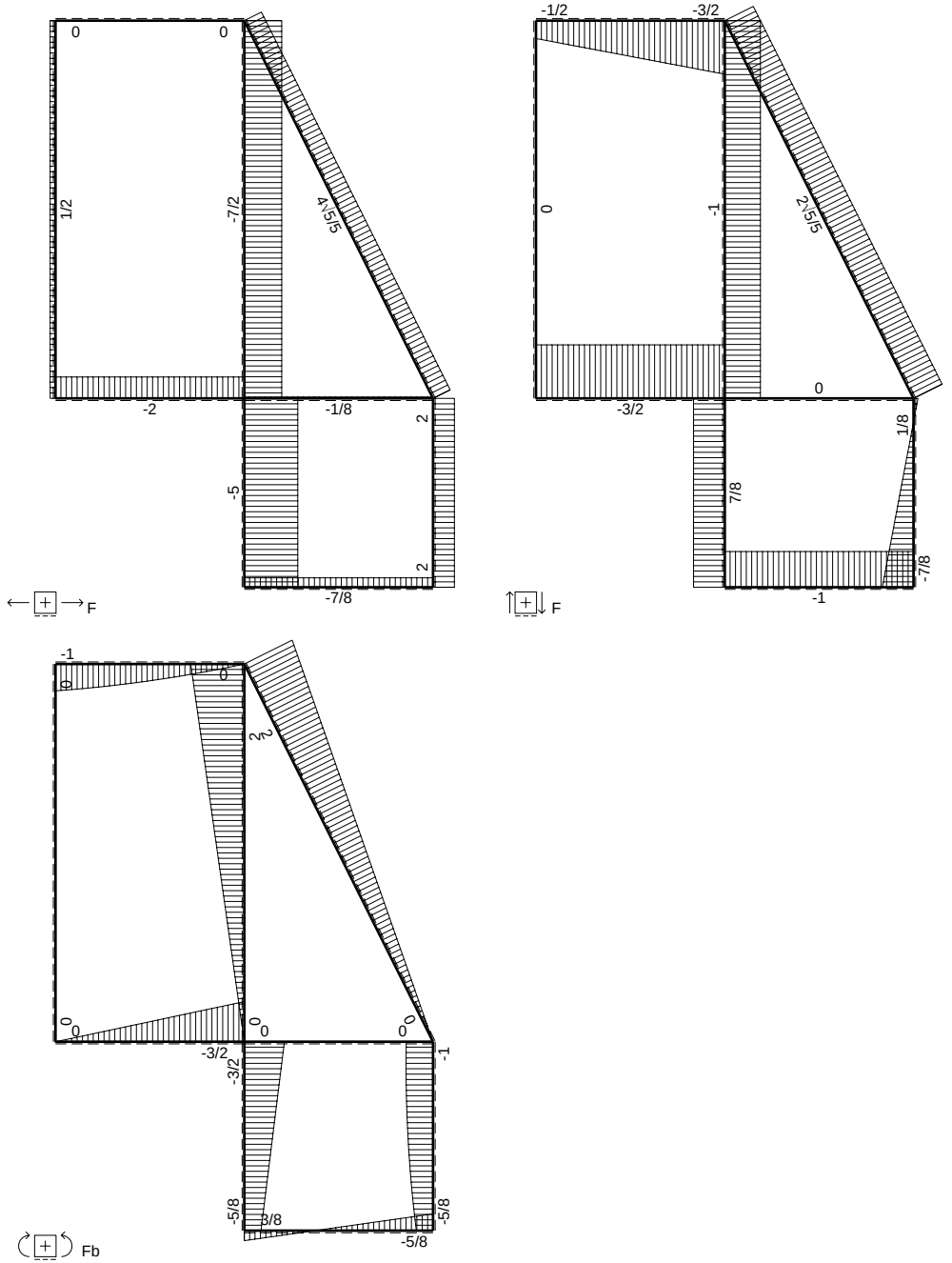
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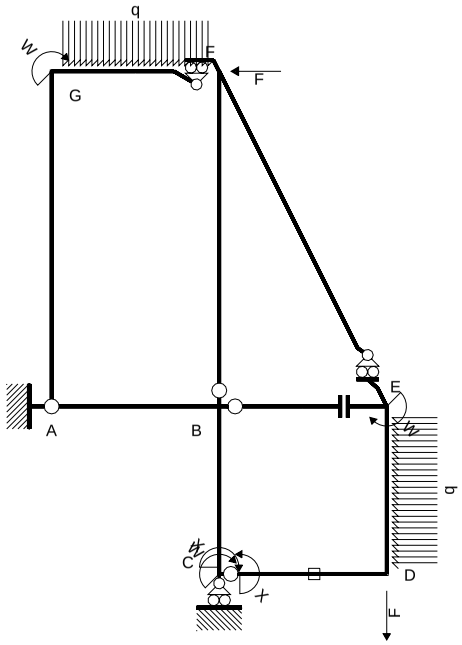
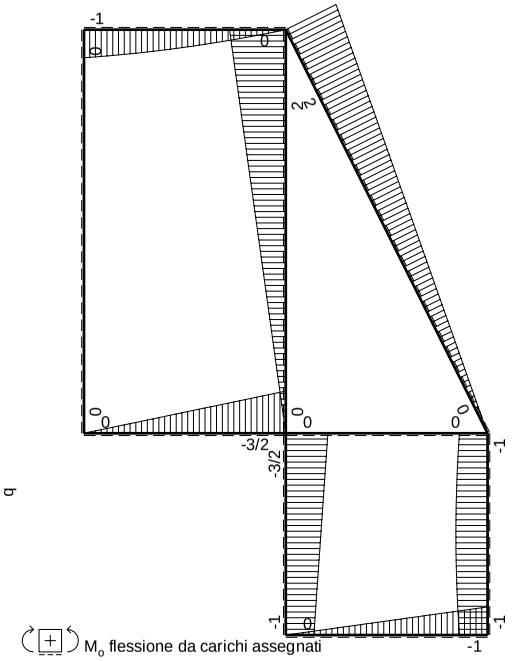
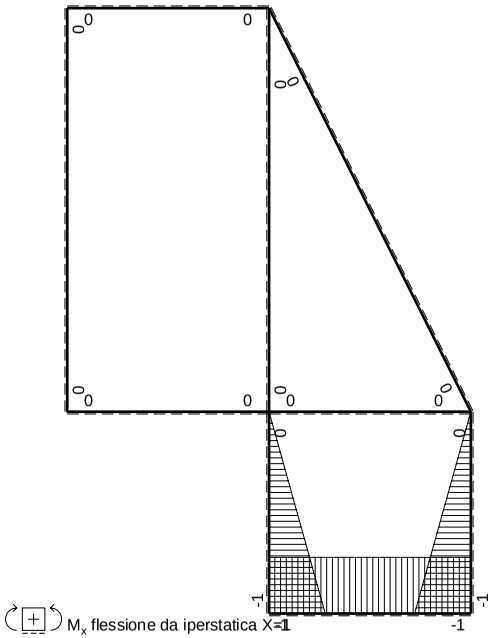


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	$3/2Fx-1/2Fx^2/b$	x^2/b^2	$7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

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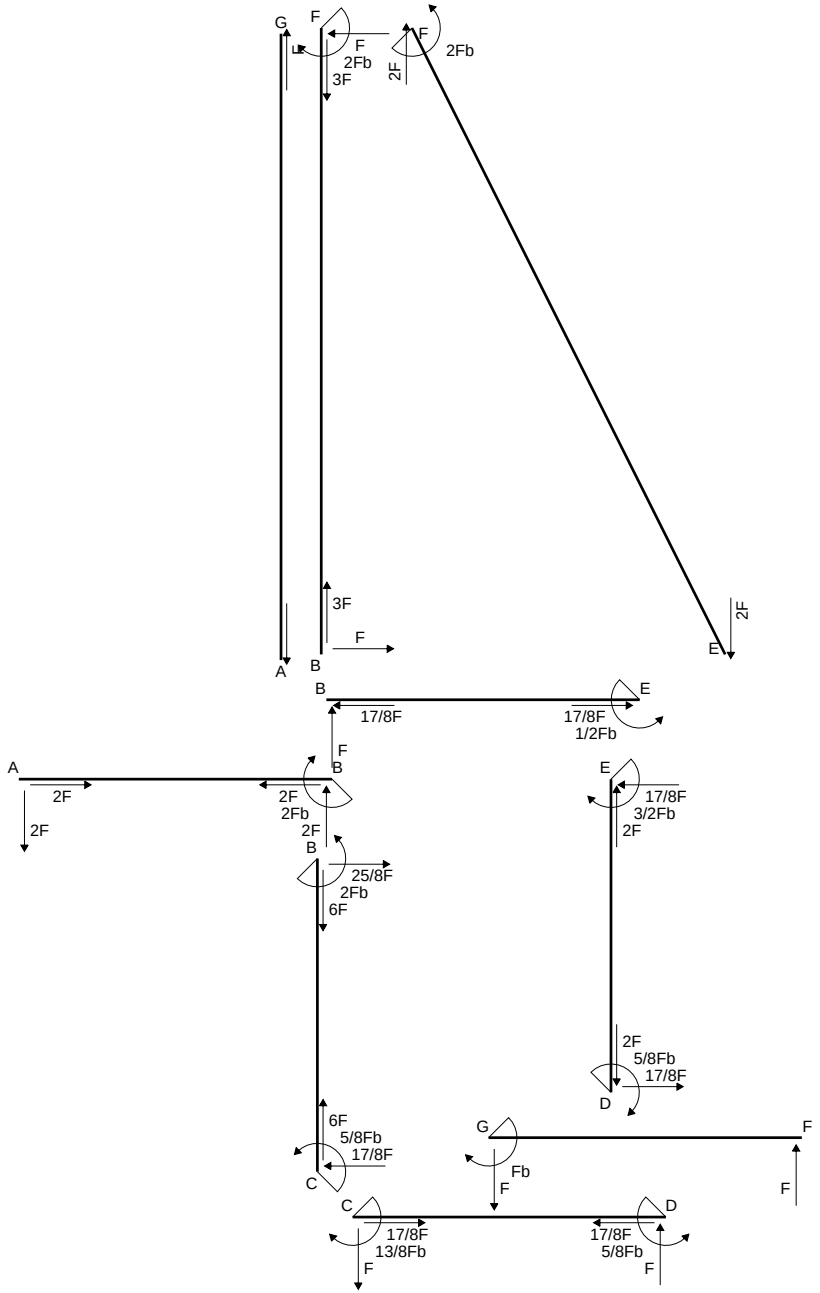
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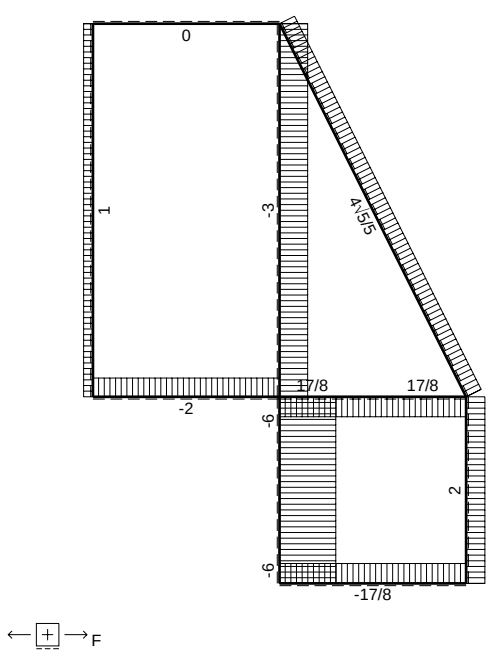
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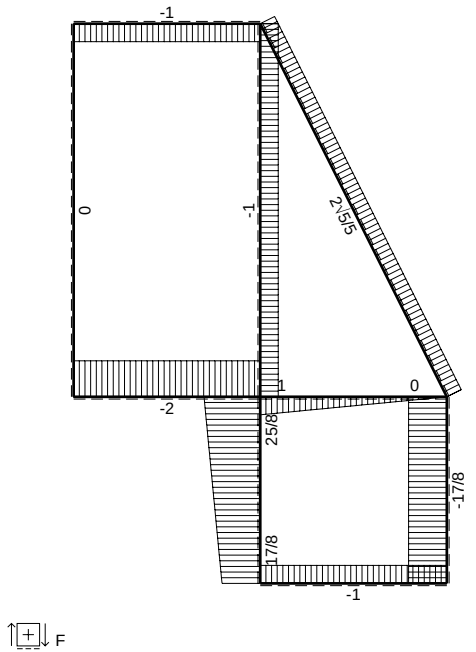
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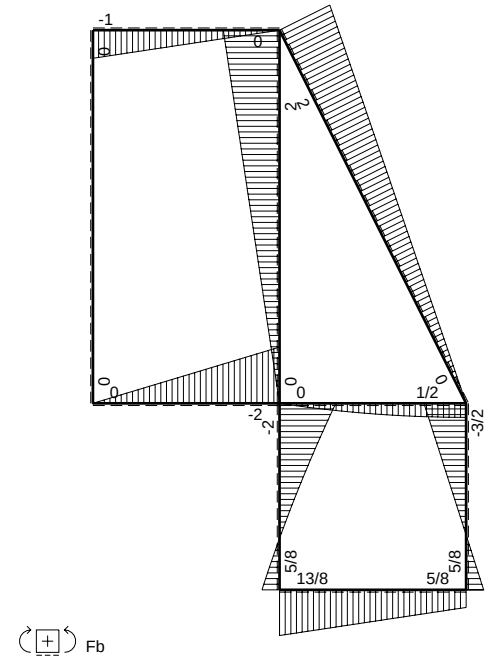
-1
F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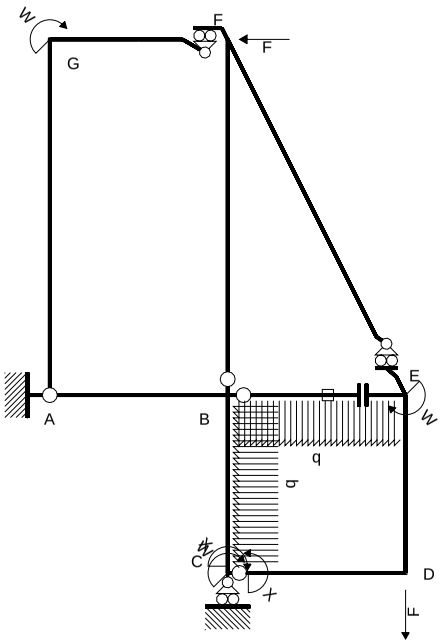
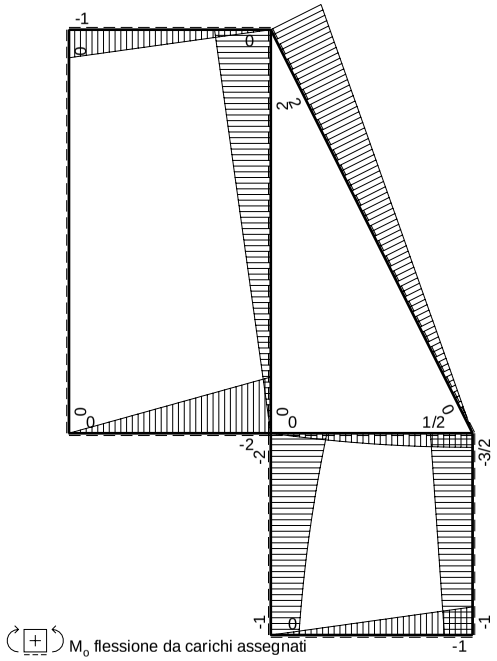
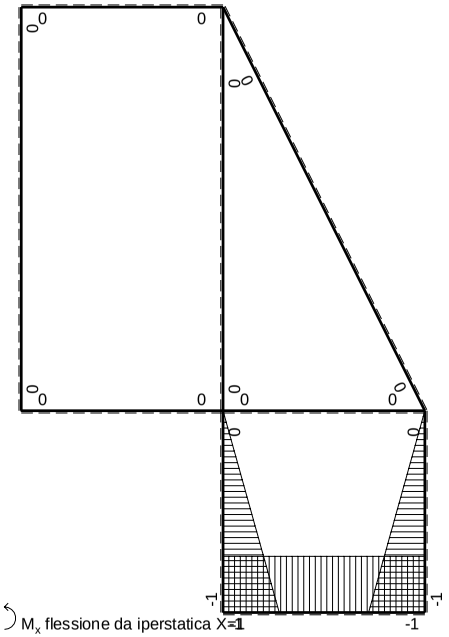


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	$2Fx-3/2Fx^2/b+1/2qx^3/b$	x^2/b^2	$5/8Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	$Fb-1/2Fx-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$3/2Fb-1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$65/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

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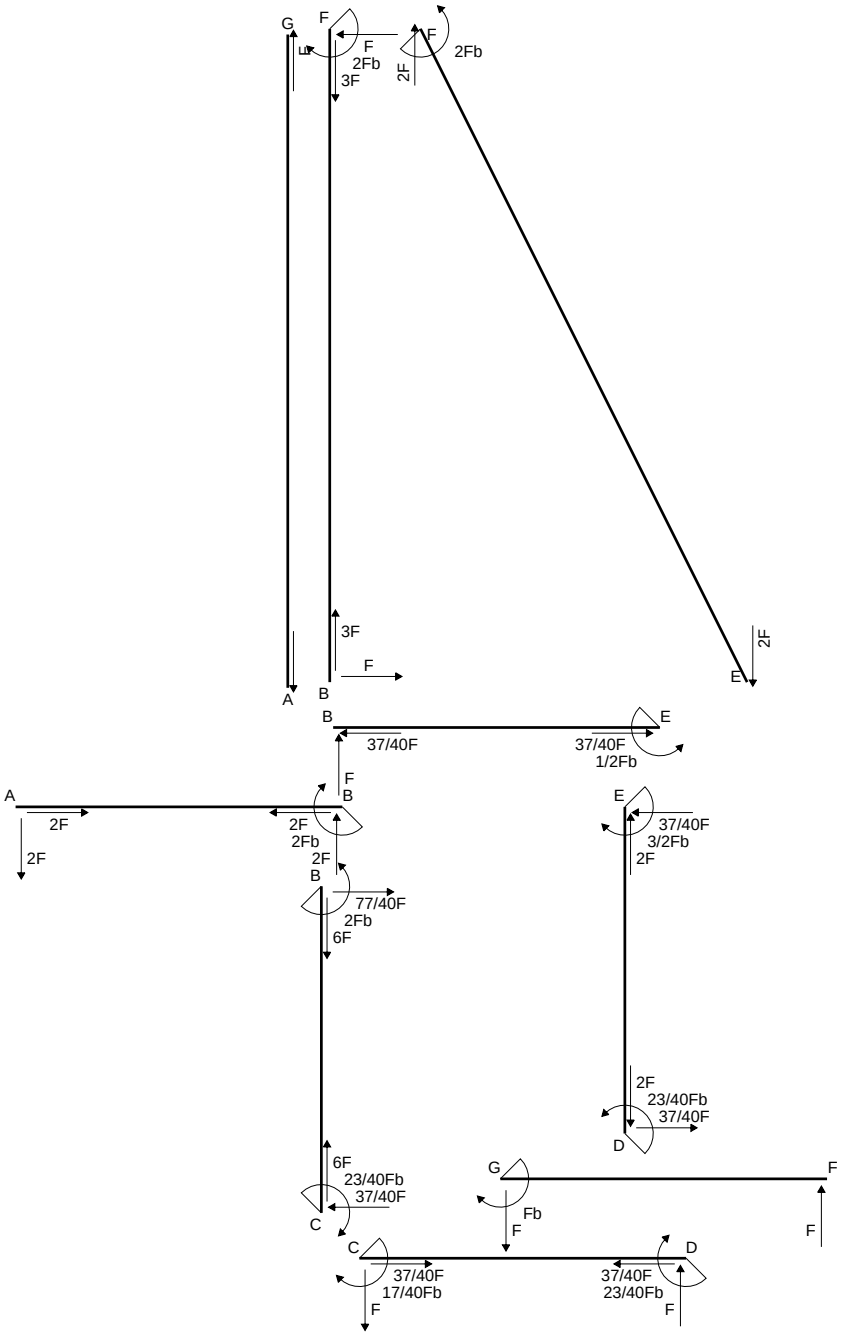
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B
77/40F
2Fb
6F
E
37/40F
3/2Fb
2F
2F
23/40Fb
37/40F
D
6F
23/40Fb
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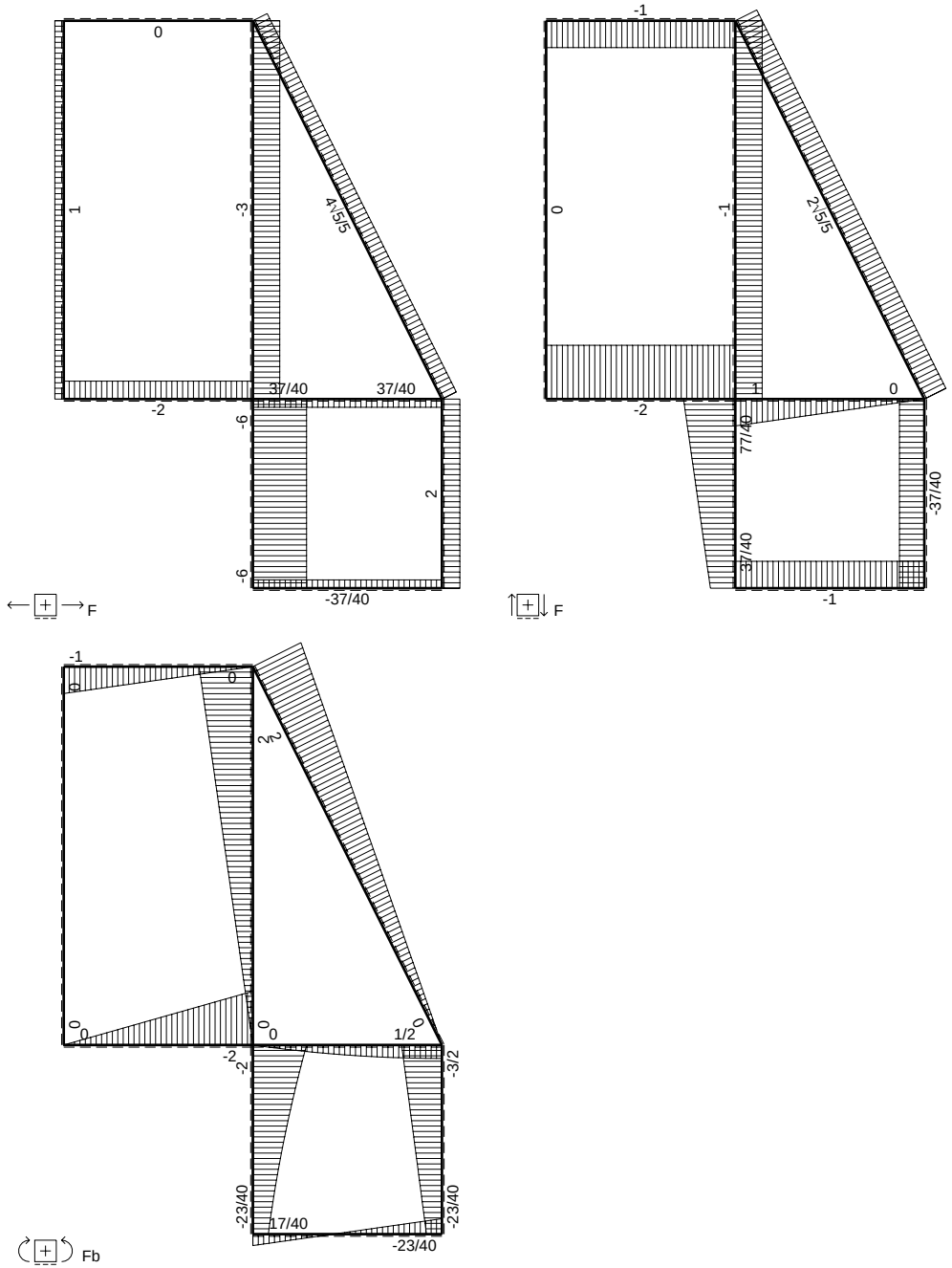
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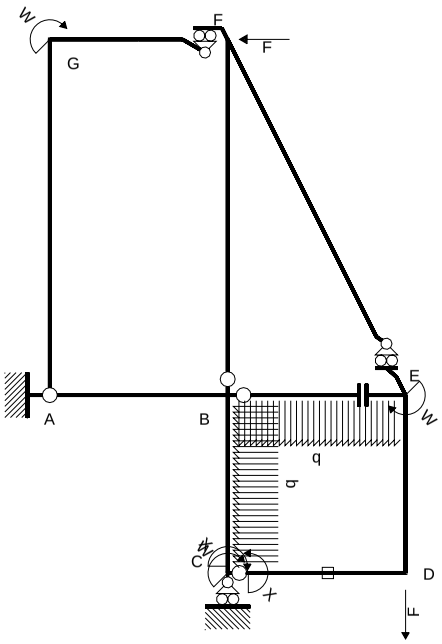
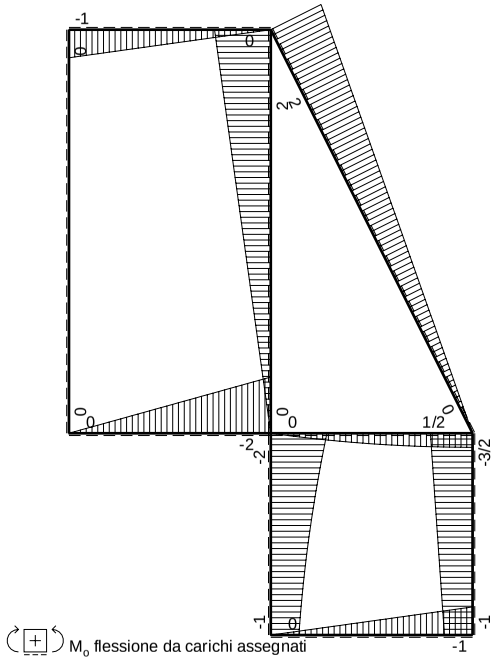
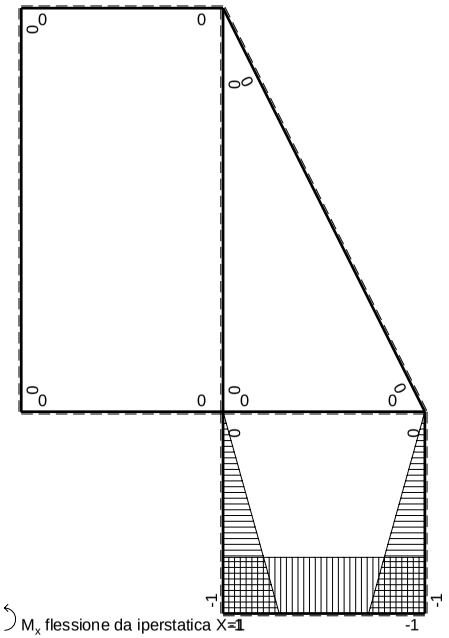


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	$2Fx-3/2Fx^2/b+1/2qx^3/b$	x^2/b^2	$5/8Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	$Fb-1/2Fx-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$7/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$3/2Fb-1/2Fx$	$3/2Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$17/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-17/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ \ Fb^2/EJ$

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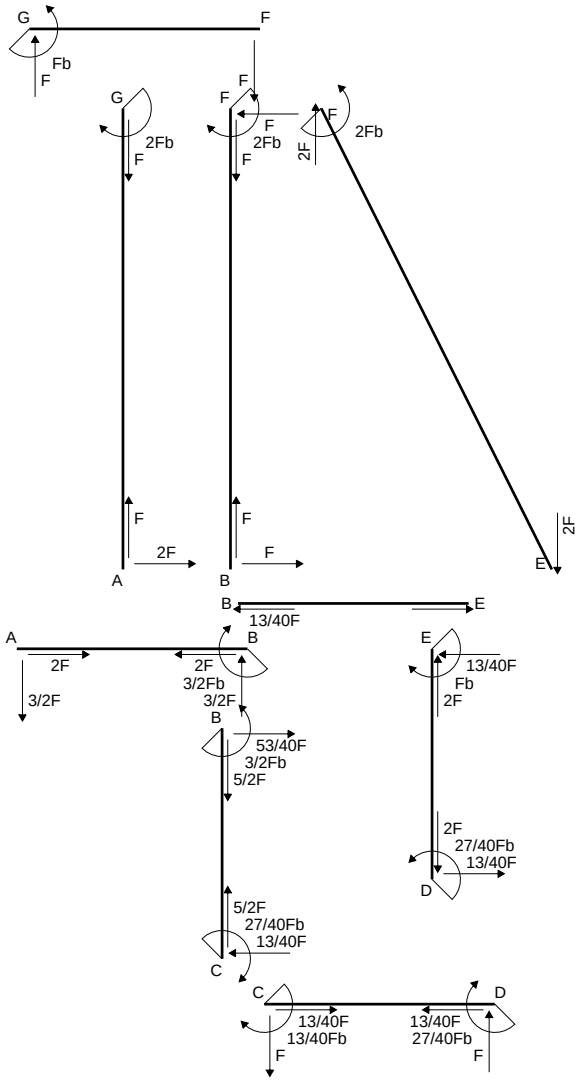
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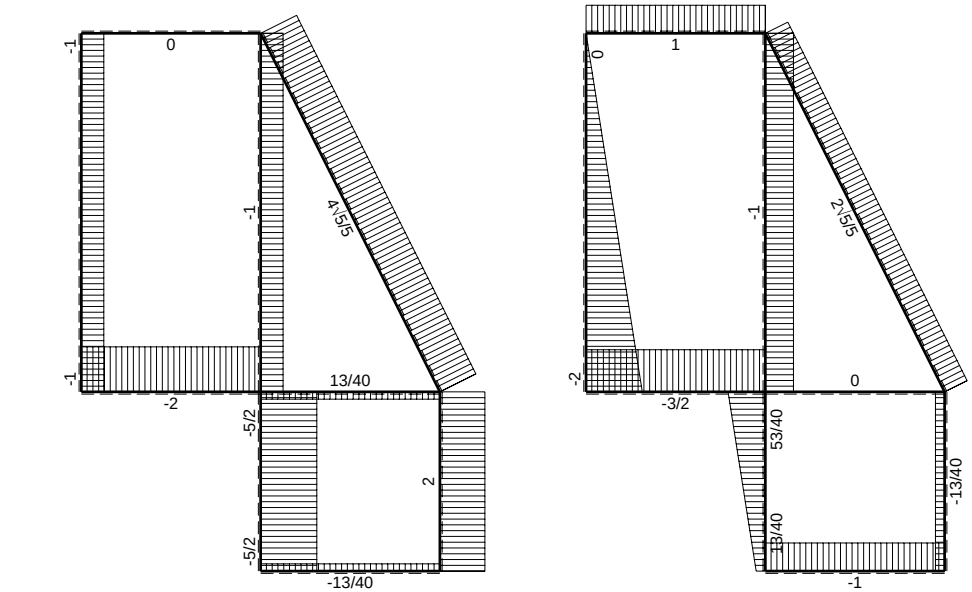
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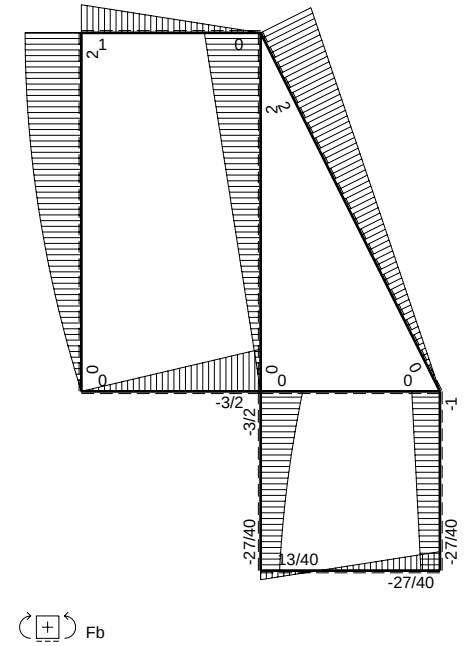
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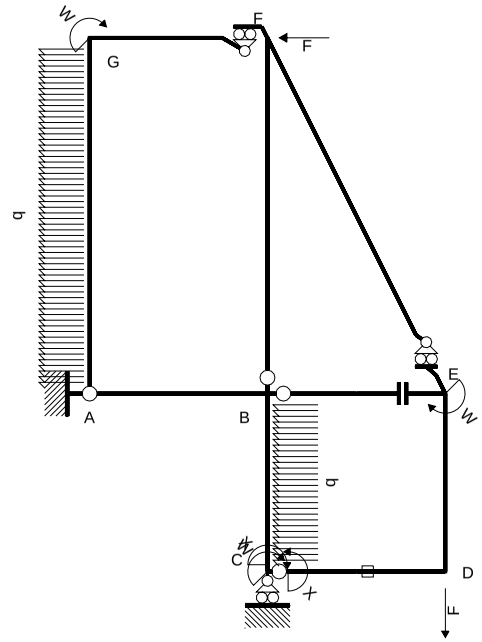
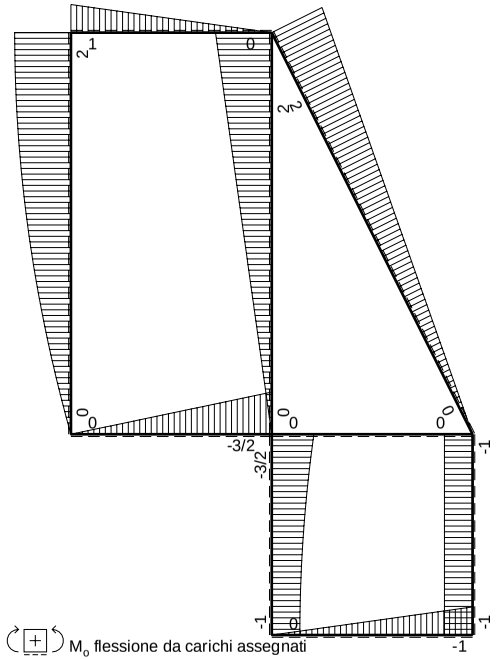
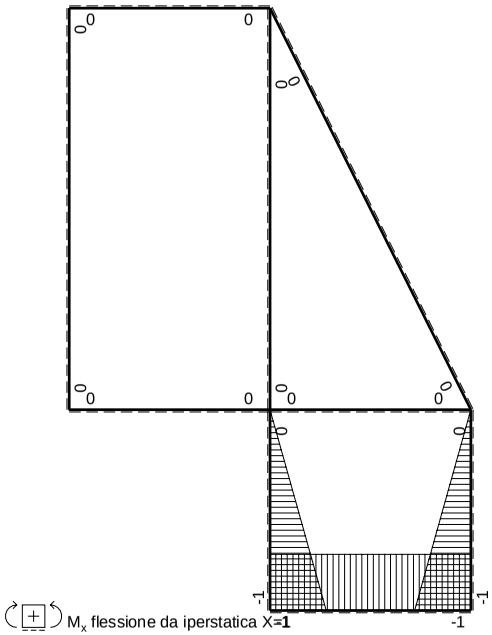
-27/40
Fb



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	$-3/2Fb+Fx-1/2qx^2$	$3/2Fx-Fx^2/b+1/2qx^3/b$	x^2/b^2	$13/24Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2qx^2$	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-13/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

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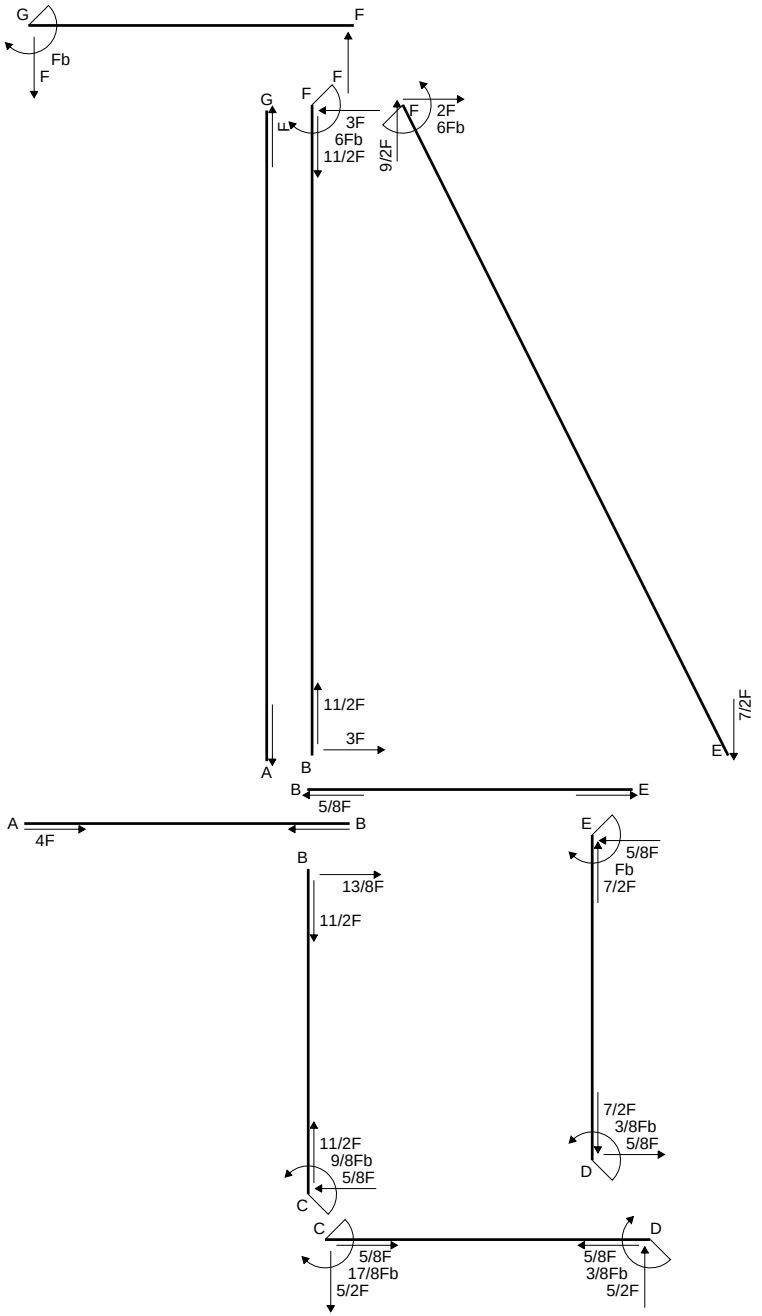
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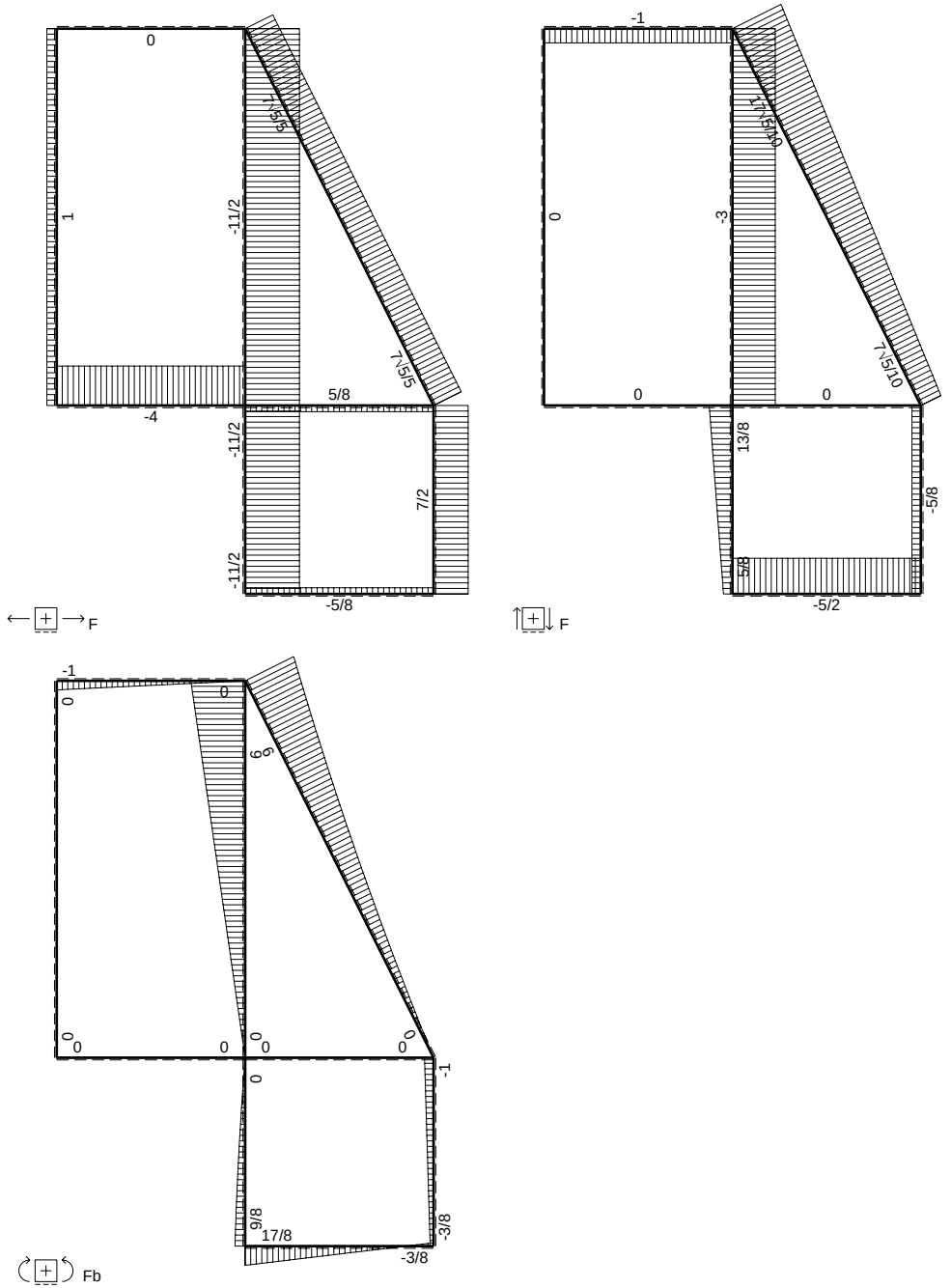
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0
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5/8
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F
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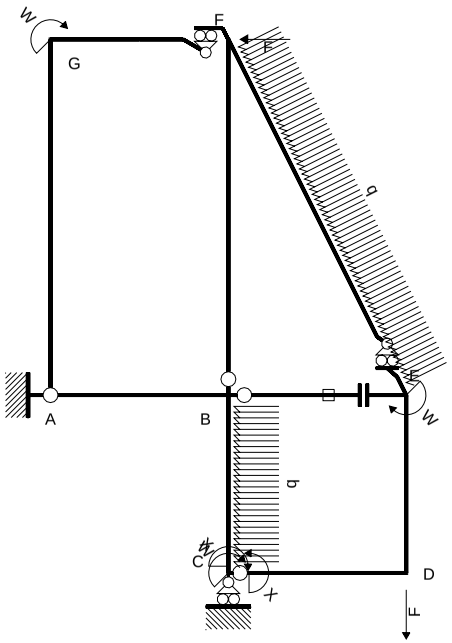
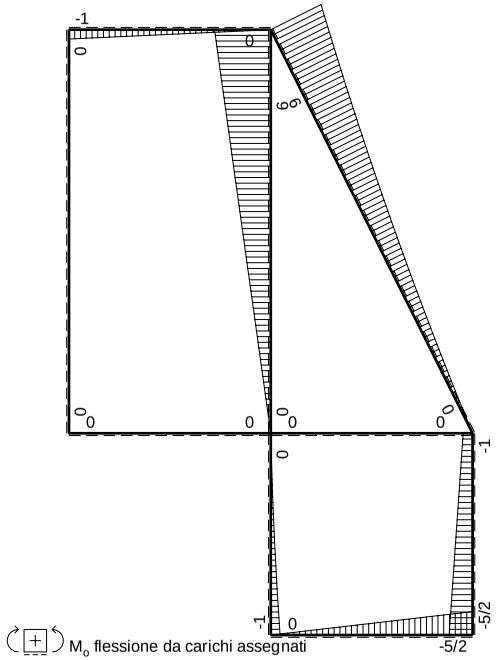
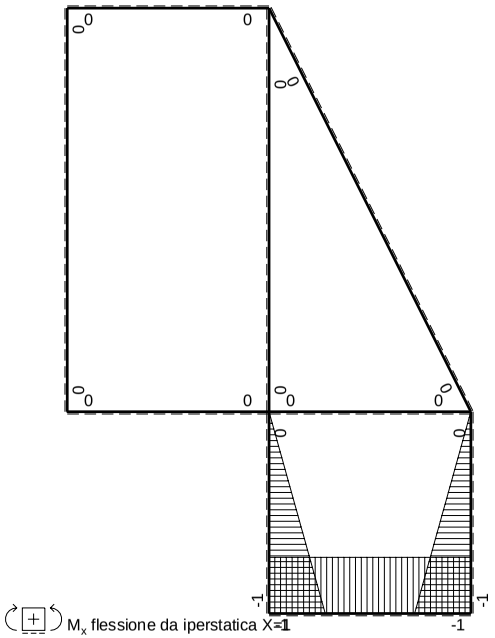
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	$-1/2Fx-1/2qx^2$	$1/2Fx^2/b+1/2qx^3/b$	x^2/b^2	$7/24Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	$Fb-3/2Fx+1/2qx^2$	$Fb-5/2Fx+2Fx^2/b-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+3/2Fx	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	Fb+3/2Fx	$Fx+3/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$85/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-17/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b +1/8\ b)\ Fb\ 1/EJ\ = 7/24\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +2/3\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +2/3\ b -1/8\ b)\ Fb\ 1/EJ\ = 7/24\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/4\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b)\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b)\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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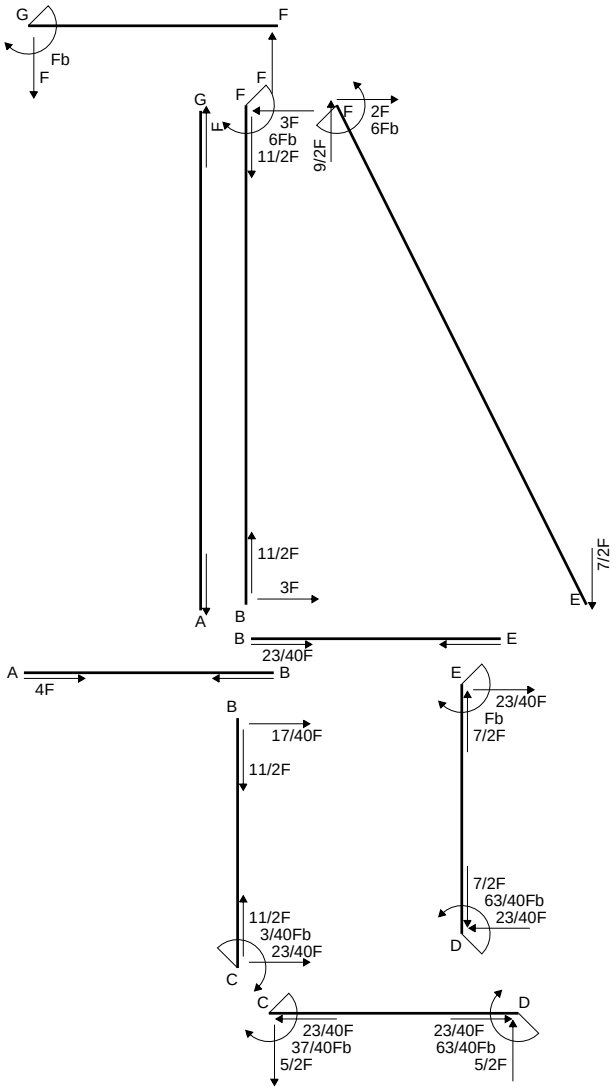
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B
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B
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B
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E
23/40F
Fb
7/2F
B
17/40F
11/2F
7/2F
63/40Fb
23/40F
D
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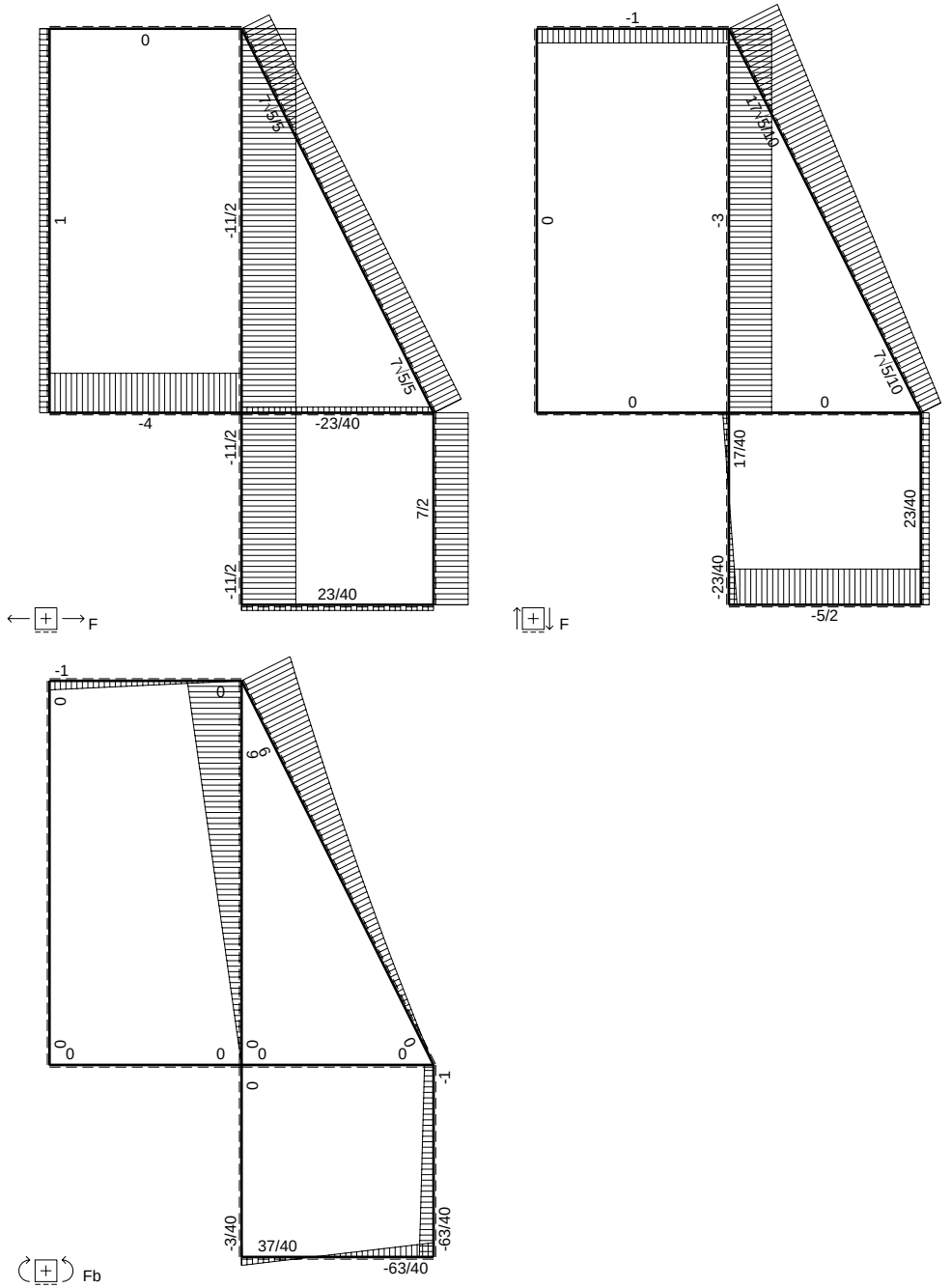
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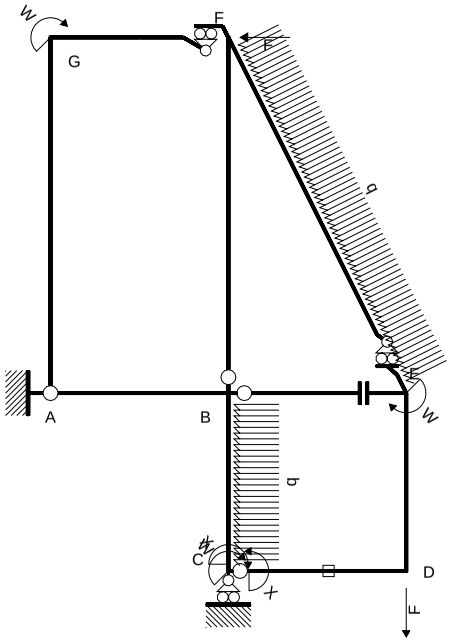
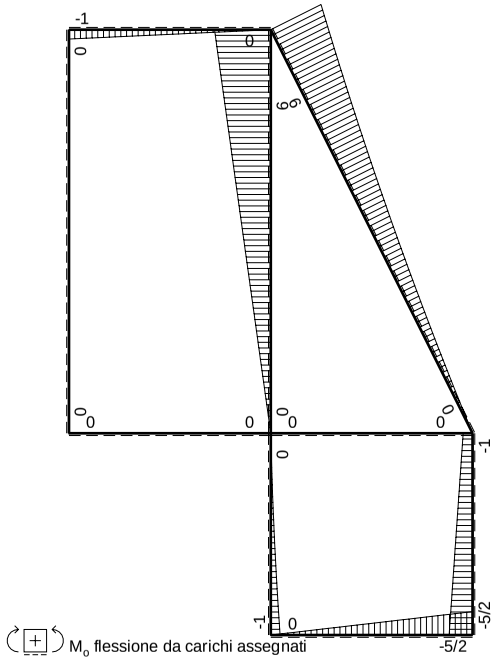
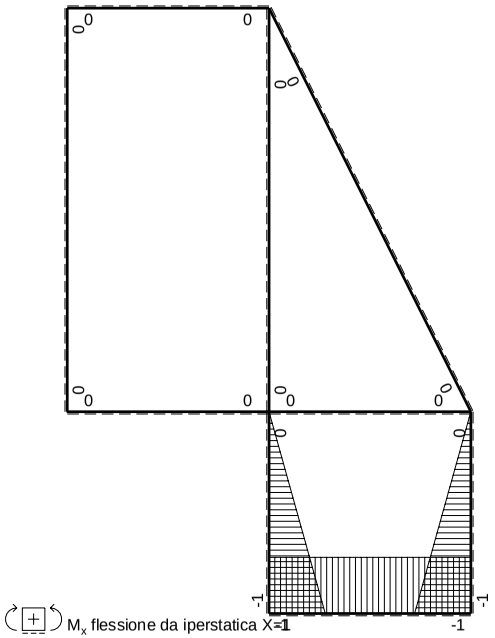
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	$-1/2Fx-1/2qx^2$	$1/2Fx^2/b+1/2qx^3/b$	x^2/b^2	$7/24Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	$Fb-3/2Fx+1/2qx^2$	$Fb-5/2Fx+2Fx^2/b-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	$-5/2Fb+3/2Fx$	$5/2Fb-4Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	$1/3Xb/EJ$
ED b	x/b	$Fb+3/2Fx$	$Fx+3/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$37/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-37/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b +1/8\ b)\ Fb\ 1/EJ\ = 7/24\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -5/2\ x/b +2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -5/4\ x^2/b +2/3\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (\ b -5/4\ b +2/3\ b -1/8\ b)\ Fb\ 1/EJ\ = 7/24\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -4\ x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -2\ x^2/b +1/2\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -2\ b +1/2\ b)\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/2\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/2\ b)\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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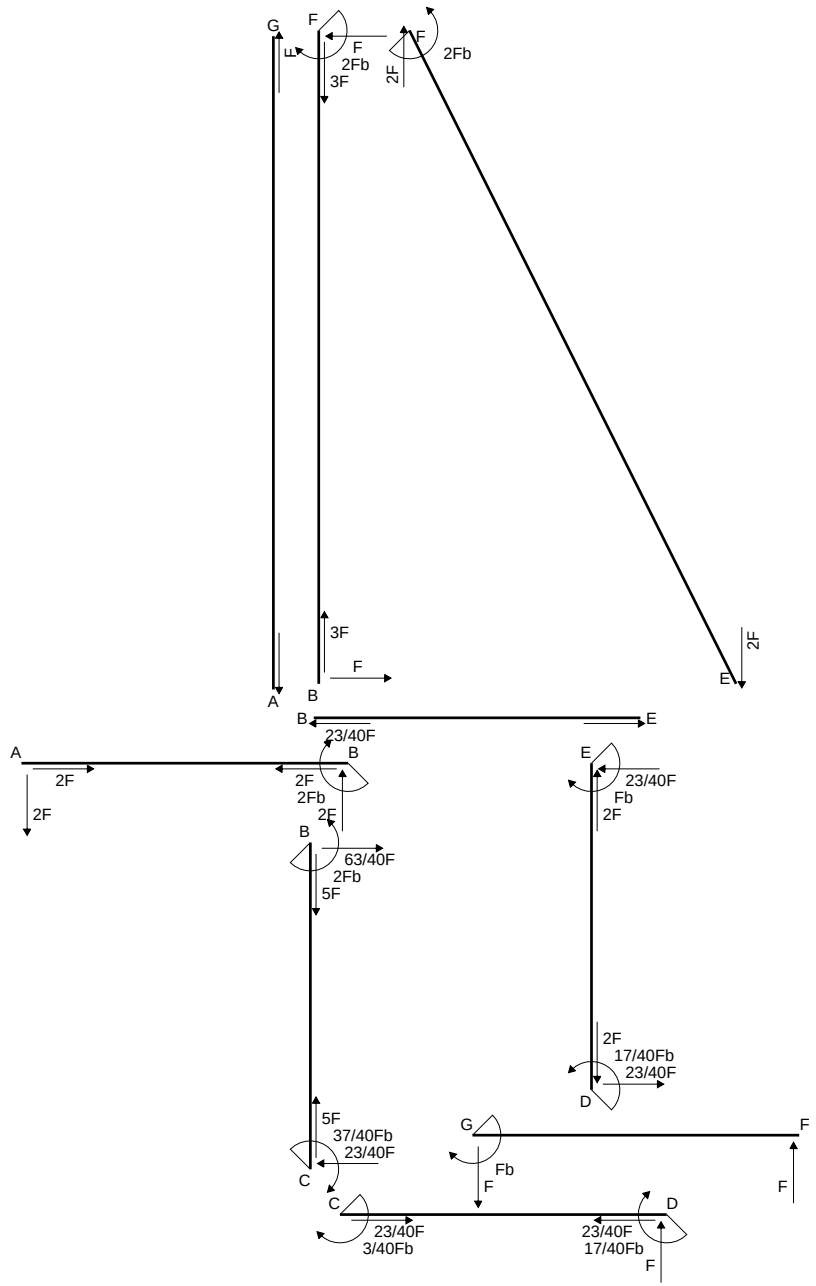
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2F
B
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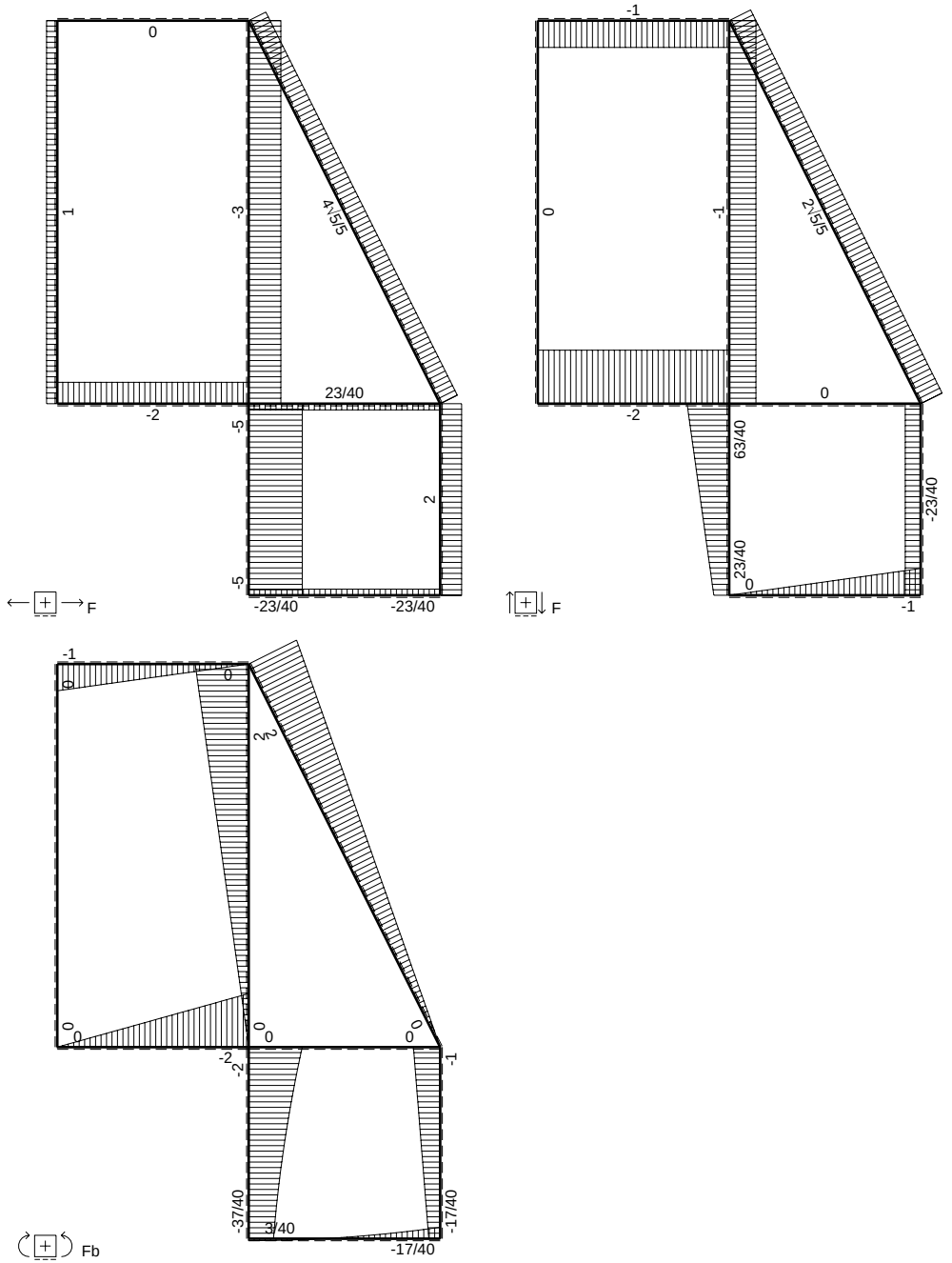
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0
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0
63/40
-23/40
23/40
0
-1
F
-1
0
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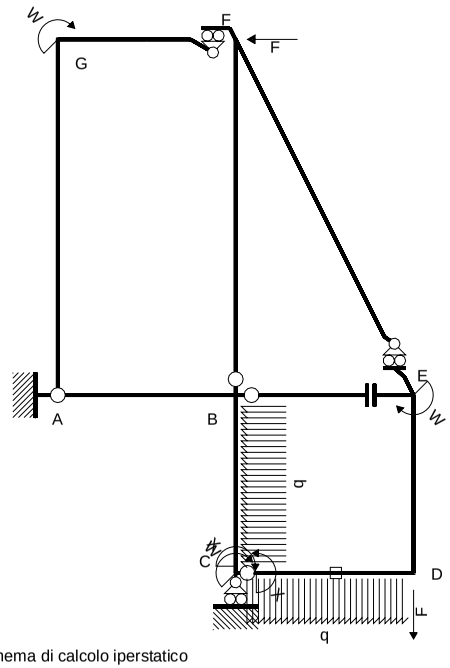
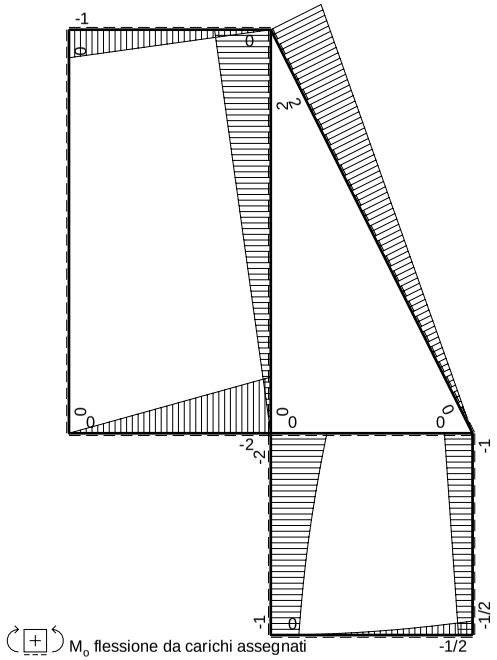
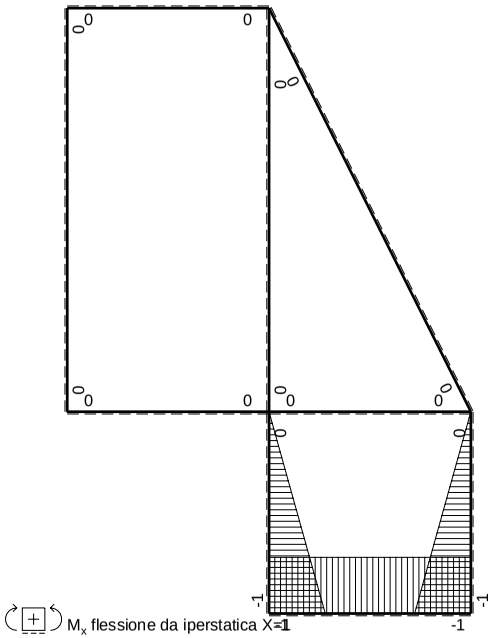
-17/40
-17/40
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	$2Fx-3/2Fx^2/b+1/2qx^3/b$	x^2/b^2	$5/8Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	$Fb-1/2Fx-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb-1/2Fx$	$1/2Fb-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb-1/2Fx$	$Fx-1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/8Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-3/40Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

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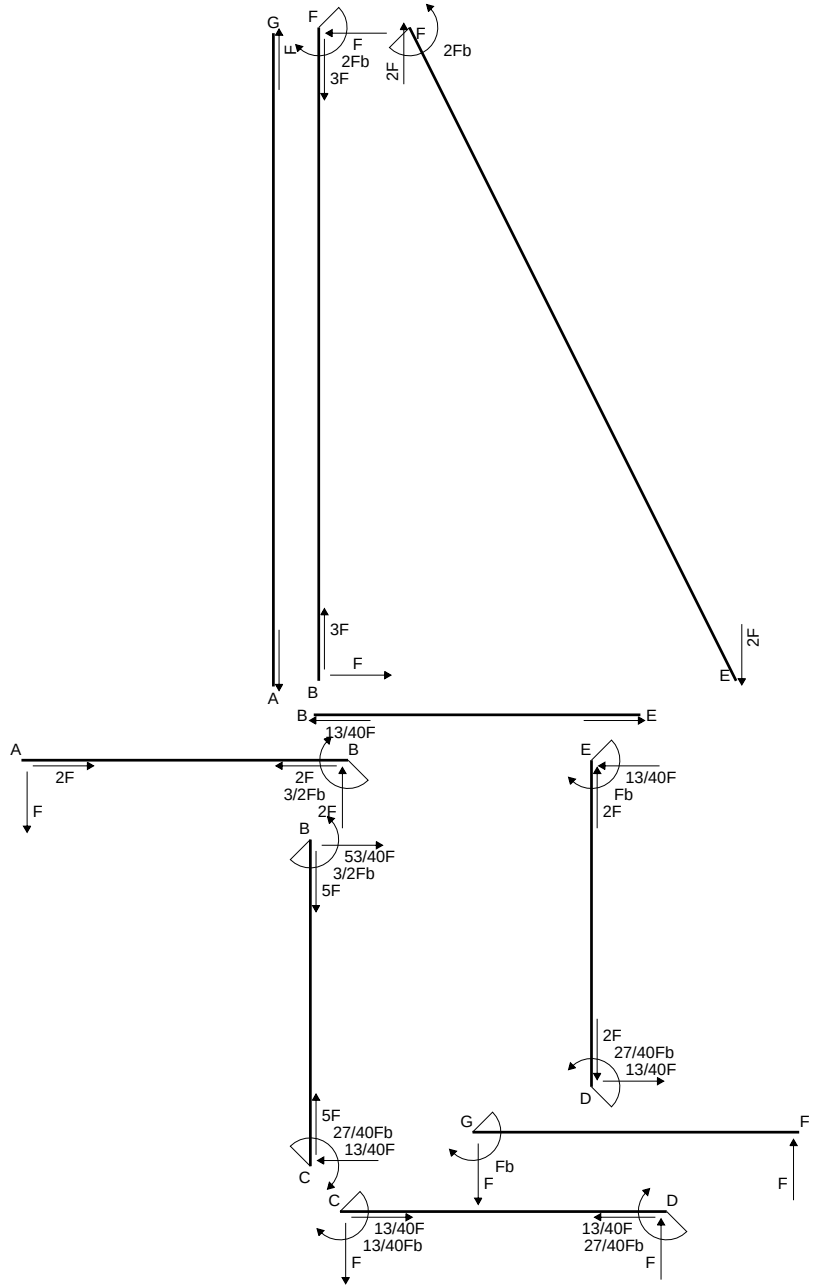
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2F
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B
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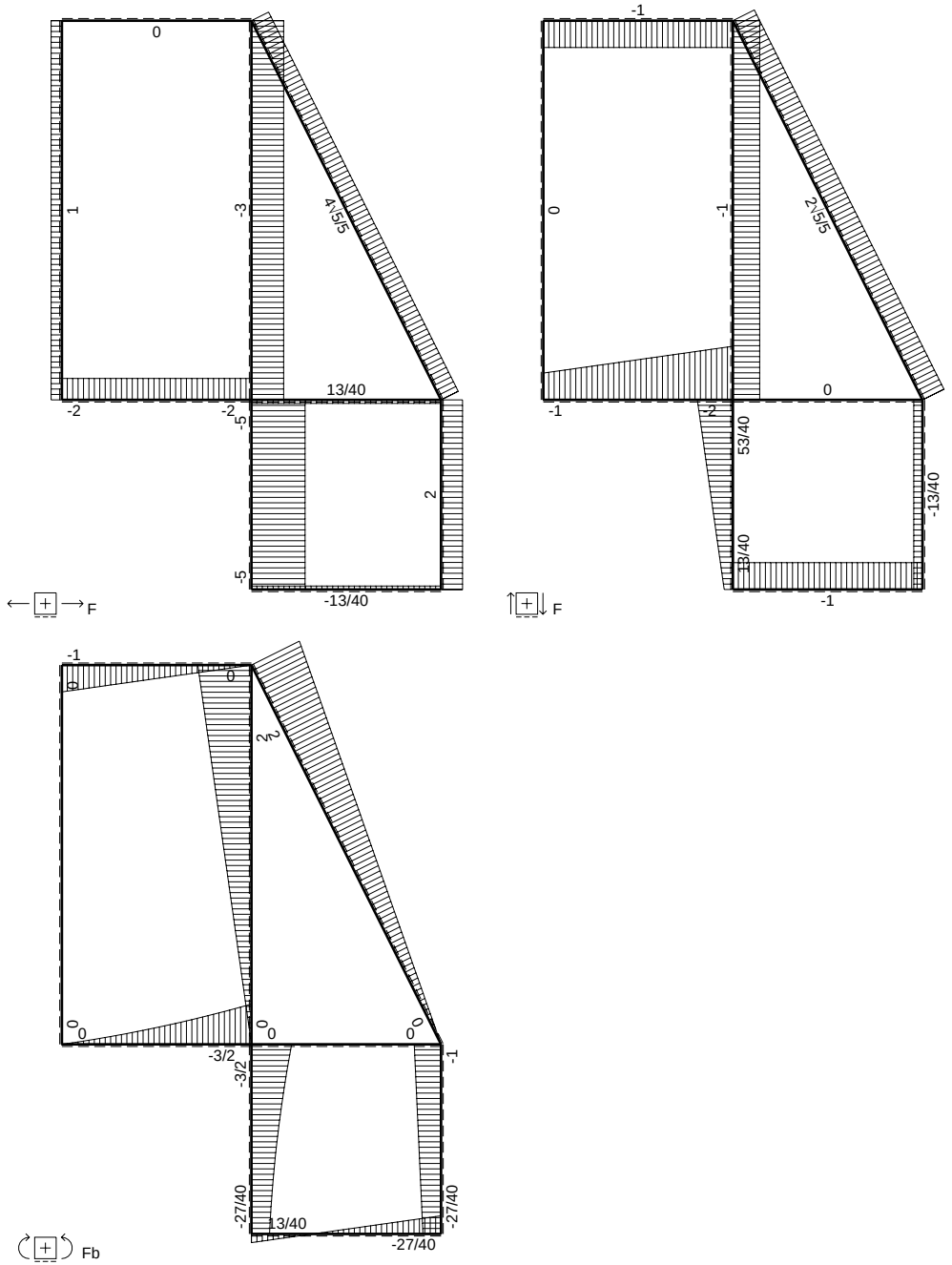
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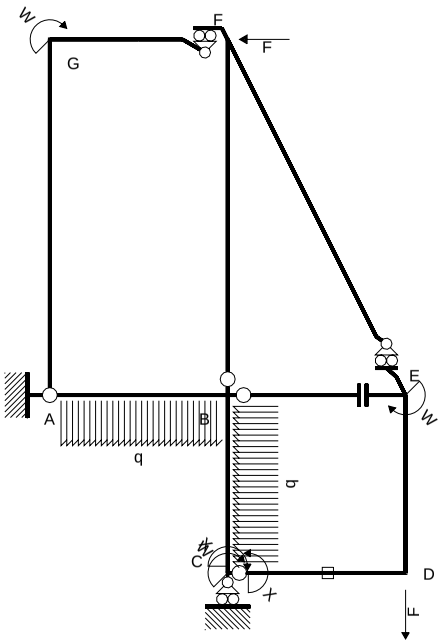
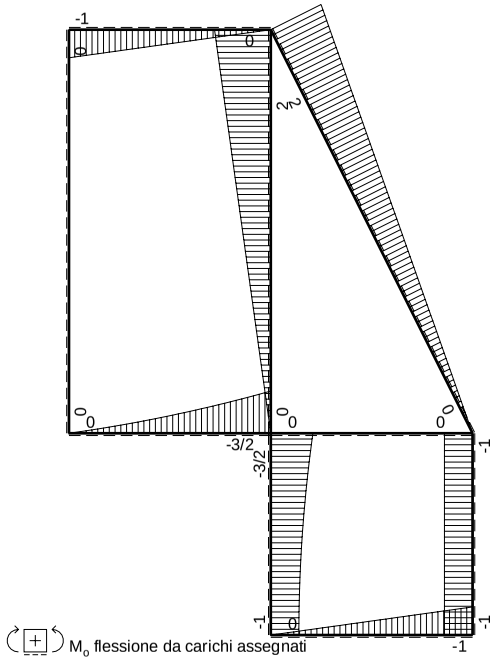
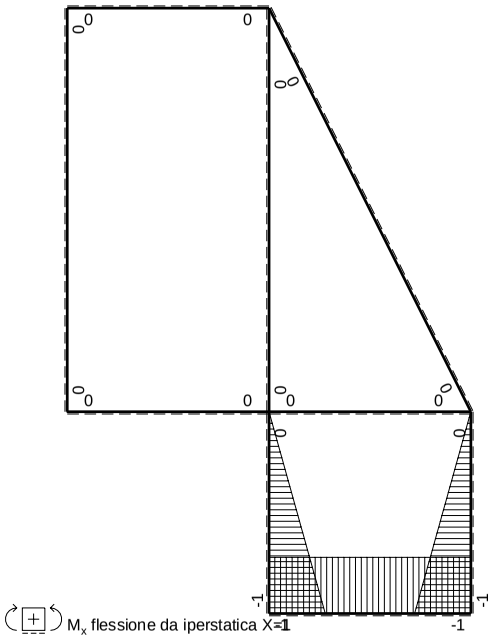


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-Fx-1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb-2Fx+1/2qx^2$	0	0		
BC b	$-x/b$	$-3/2Fb+Fx-1/2qx^2$	$3/2Fx-Fx^2/b+1/2qx^3/b$	x^2/b^2	$13/24Fb^2/EJ$	$1/3Xb/EJ$
CB b	$1-x/b$	$Fb+1/2qx^2$	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	$-1+x/b$	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb	Fx	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$13/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

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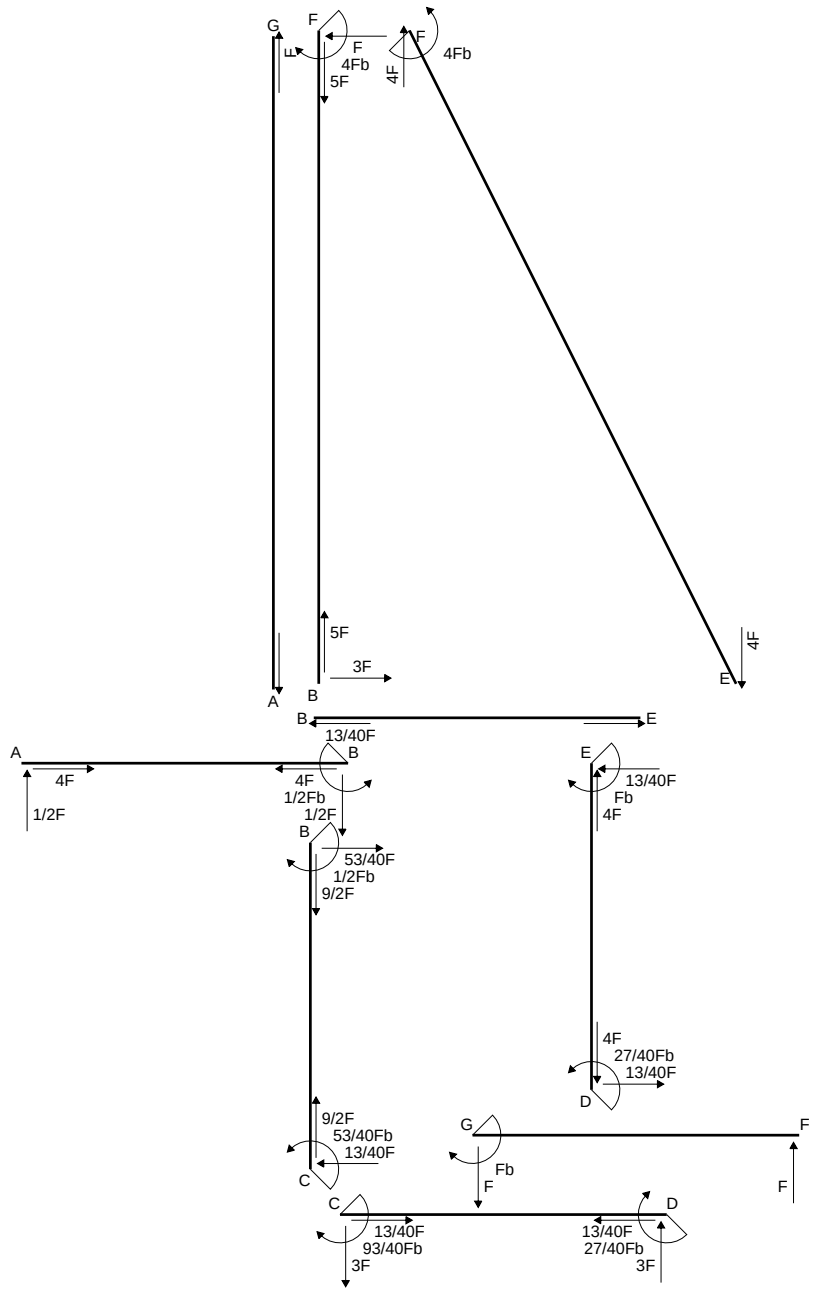
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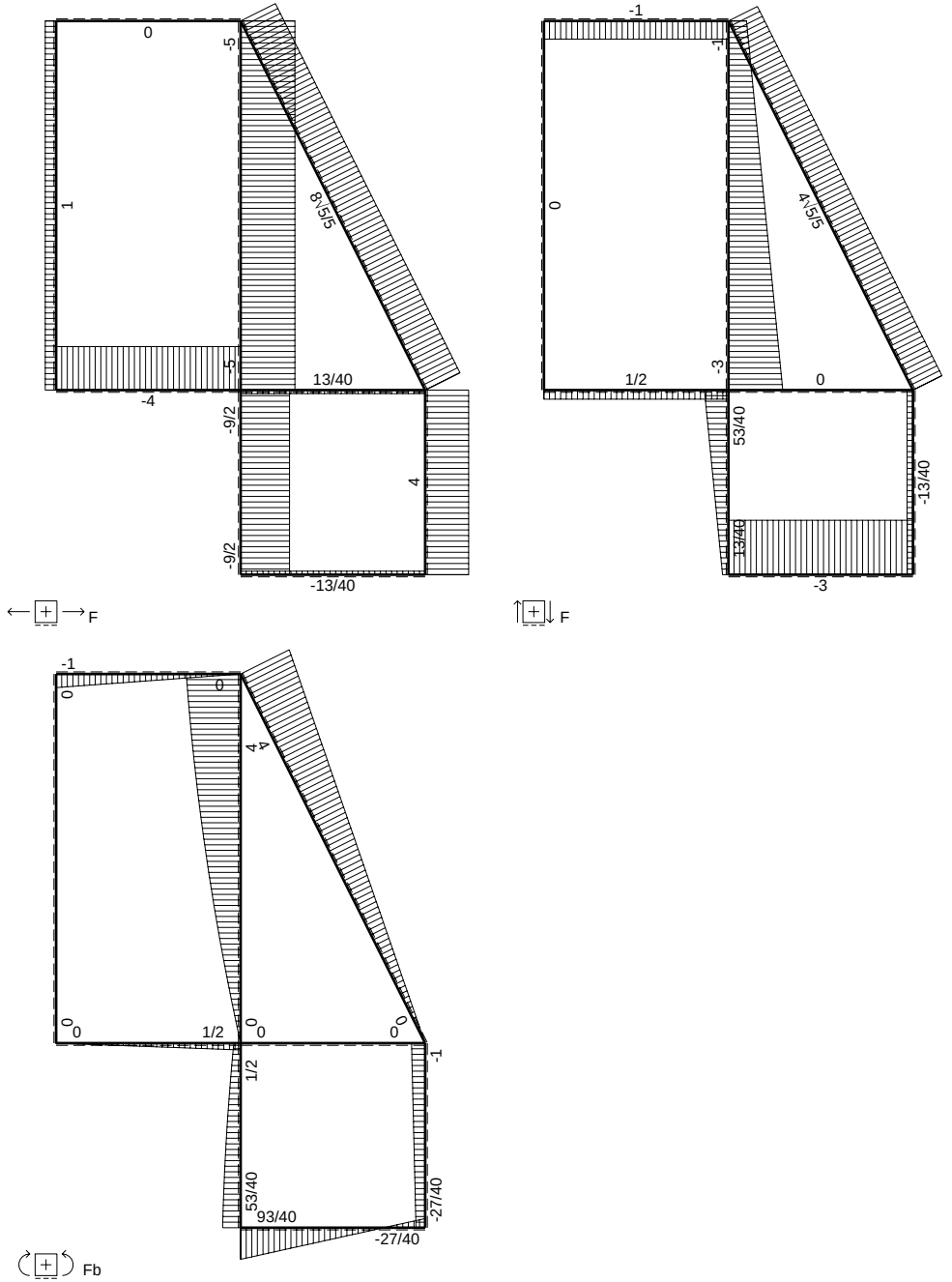
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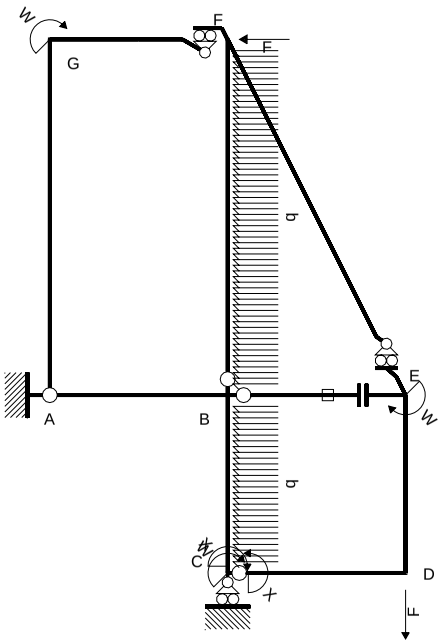
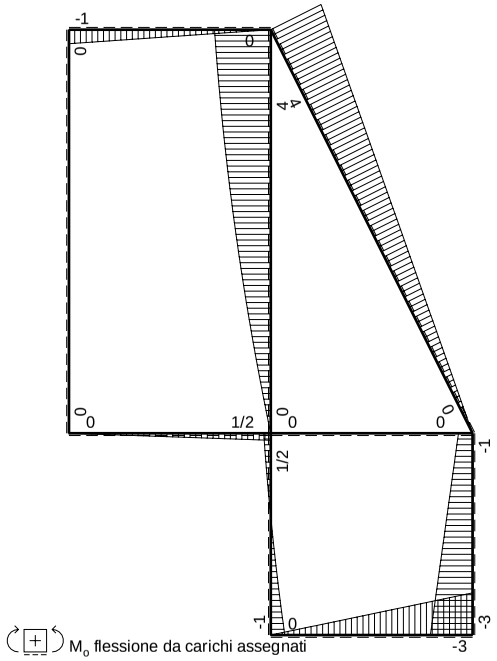
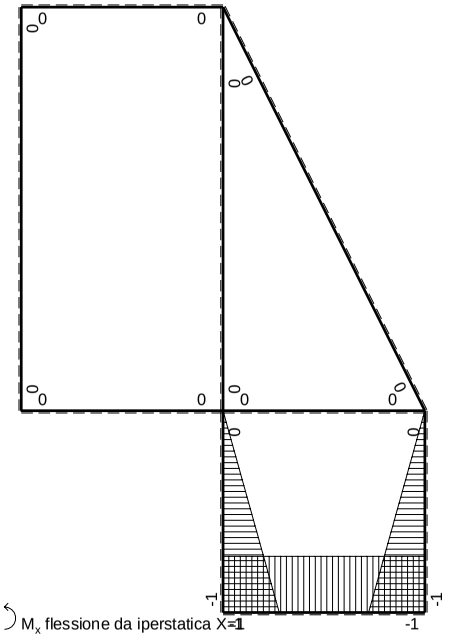
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-Fx-1/2qx^2	-1/2Fx+Fx^2/b+1/2qx^3/b	x^2/b^2	5/24Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-2Fx+1/2qx^2	Fb-3Fx+5/2Fx^2/b-1/2qx^3/b	1-2x/b+x^2/b^2		
CD b	-1	-3Fx	3Fx	1	3/2Fb^2/EJ	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	3Fb-5Fx+2Fx^2/b	1-2x/b+x^2/b^2	7/6Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+2Fx	Fx+2Fx^2/b	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	4Fb-Fx-1/2qx^2	0	0	0	0
BF 2b	0	-3Fx+1/2qx^2	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				31/8Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-93/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b + x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +5/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +5/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +5/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b)\ Fb\ 1/EJ\ dx = [3/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (3/2\ b)\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b)\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b)\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

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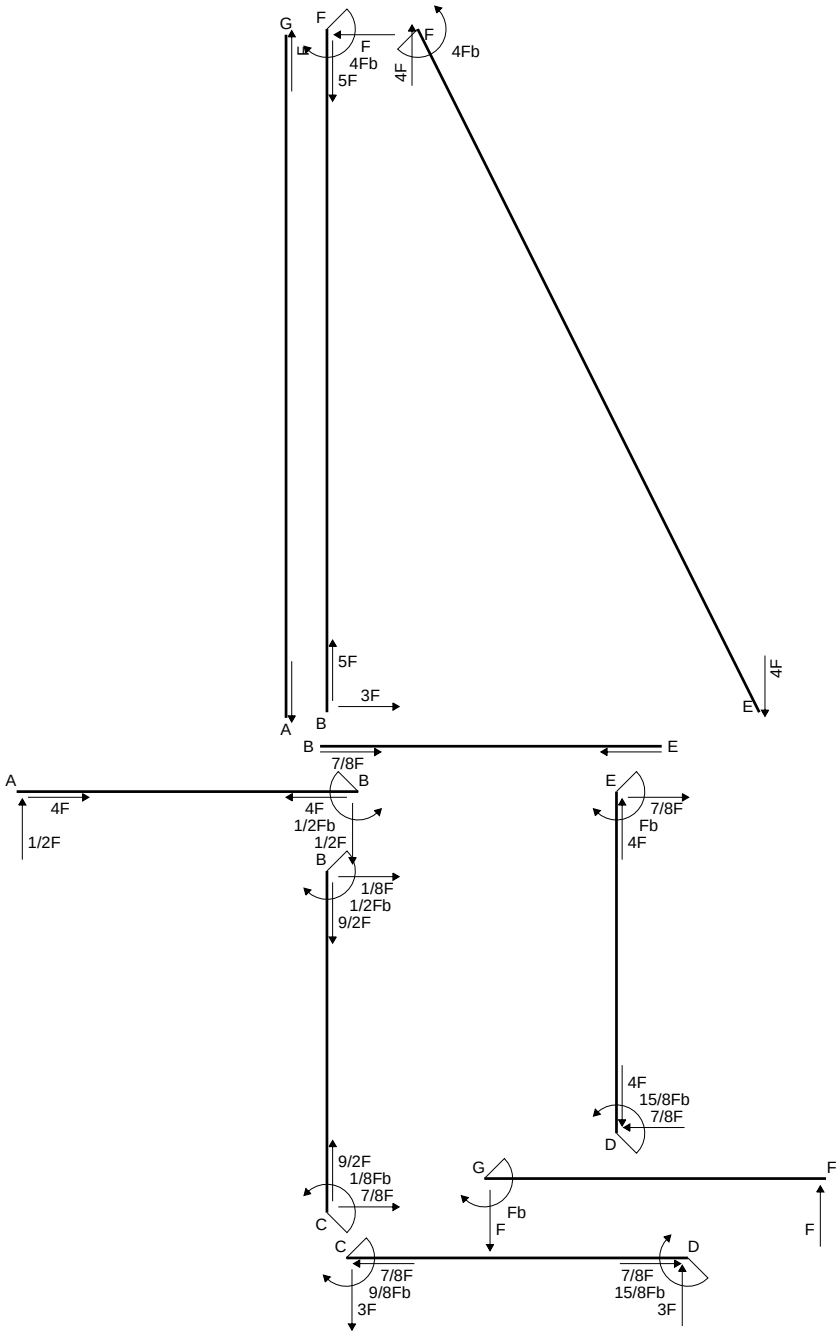
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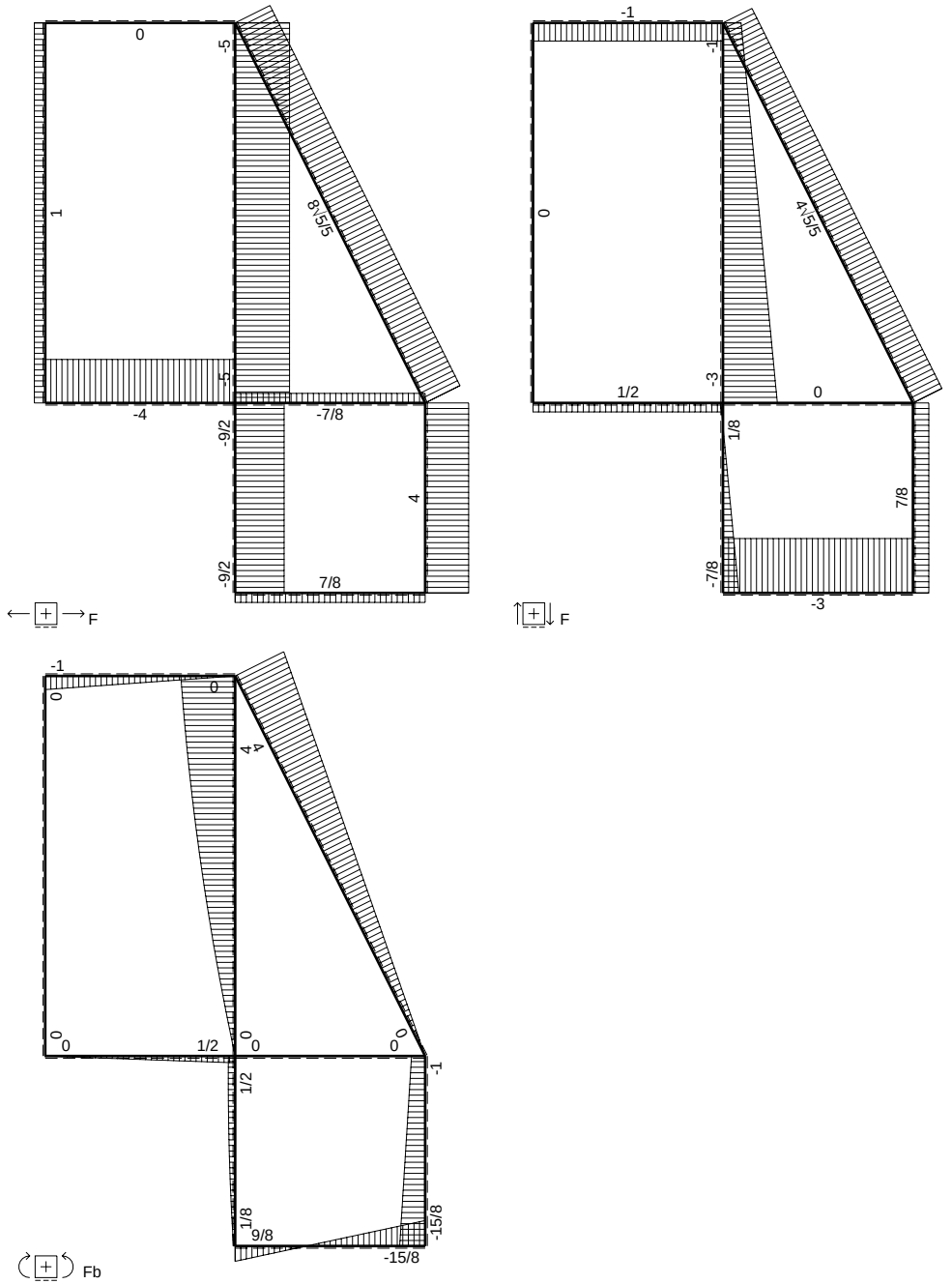
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4√2
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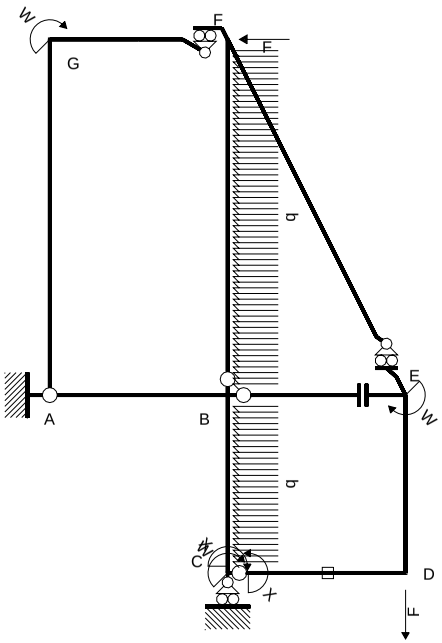
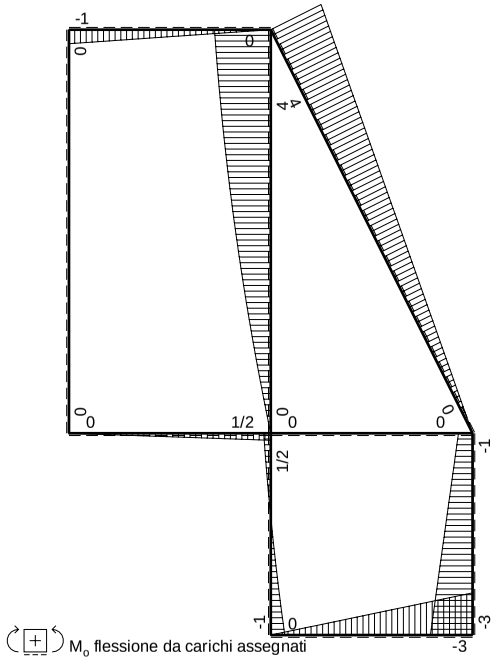
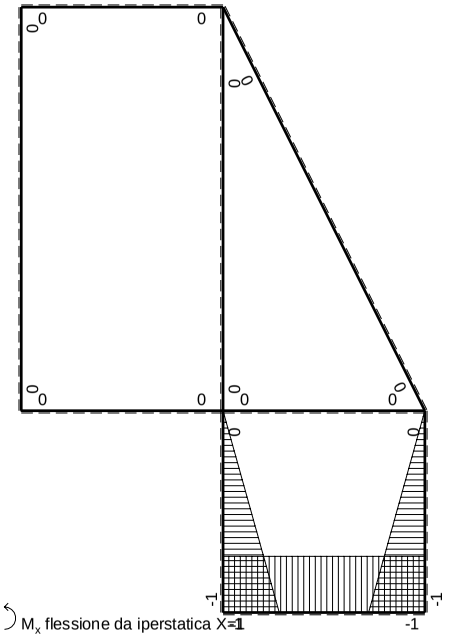
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb-Fx-1/2qx^2	-1/2Fx+Fx^2/b+1/2qx^3/b	x^2/b^2	5/24Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-2Fx+1/2qx^2	Fb-3Fx+5/2Fx^2/b-1/2qx^3/b	1-2x/b+x^2/b^2		
CD b	-1	-3Fx	3Fx	1	3/2Fb^2/EJ	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+2Fx	3Fb-5Fx+2Fx^2/b	1-2x/b+x^2/b^2	7/6Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+2Fx	Fx+2Fx^2/b	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	4Fb-Fx-1/2qx^2	0	0	0	0
BF 2b	0	-3Fx+1/2qx^2	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				15/8Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b + x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +5/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +5/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +5/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -5\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -5/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -5/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 7/6\ \ Fb^2/EJ$

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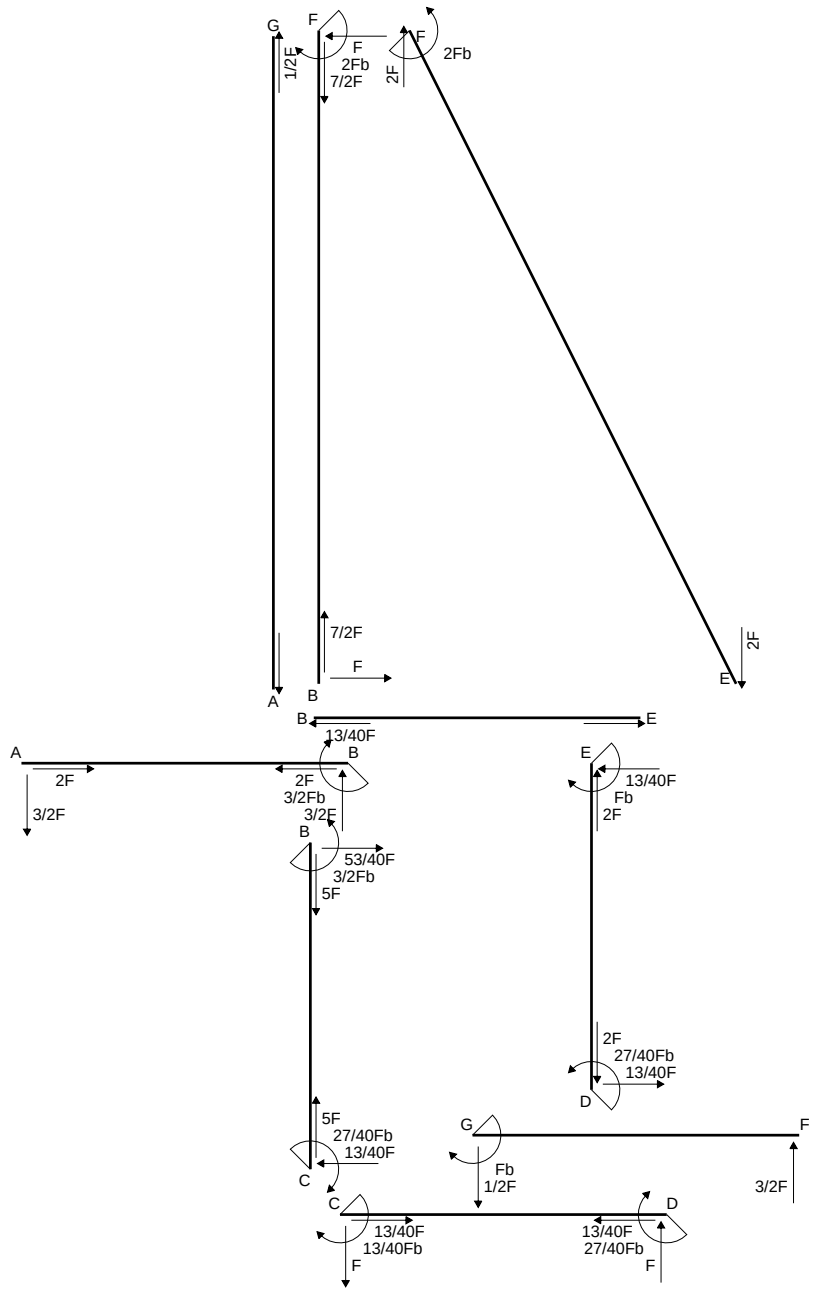
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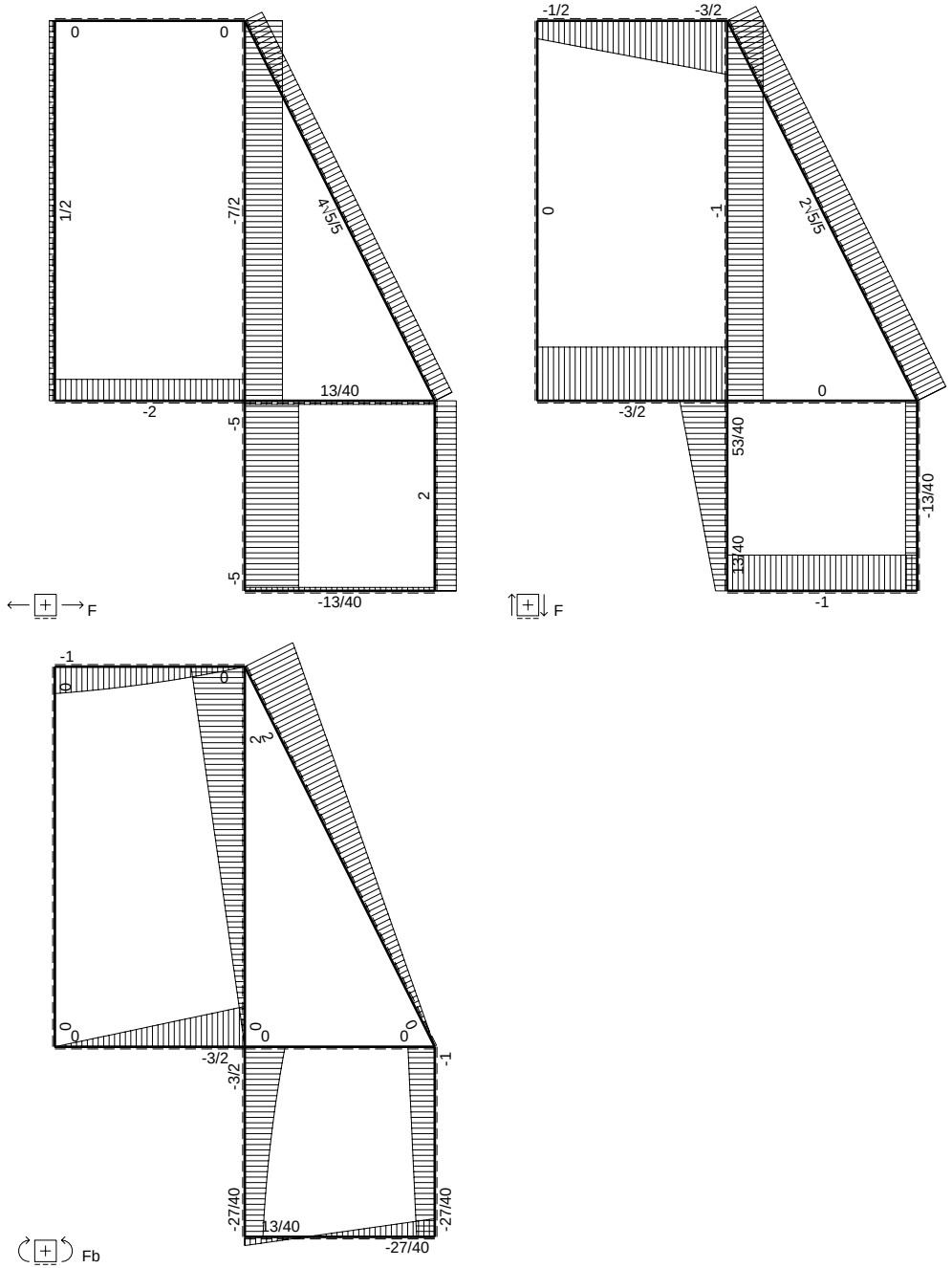
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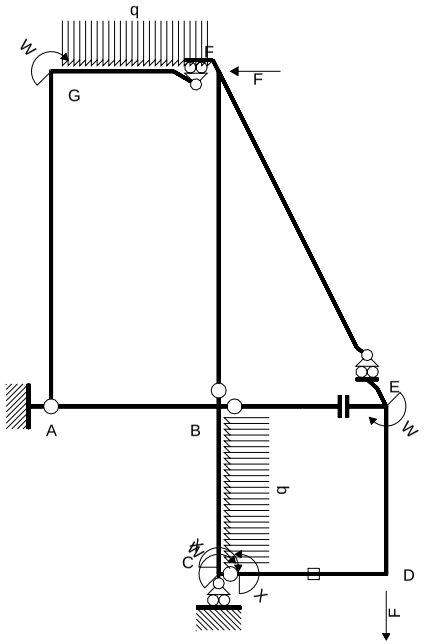
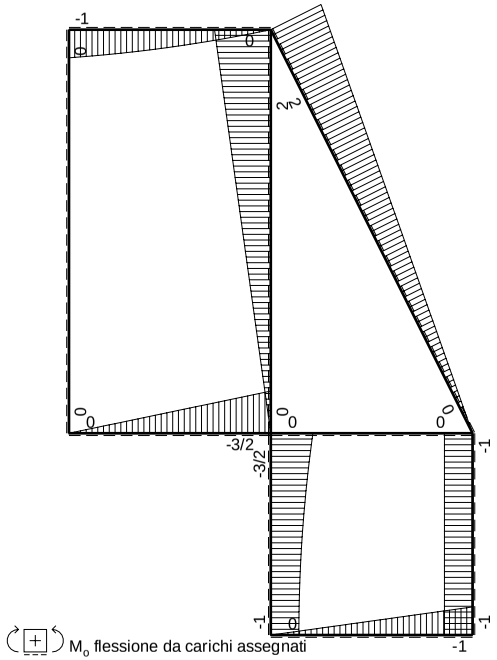
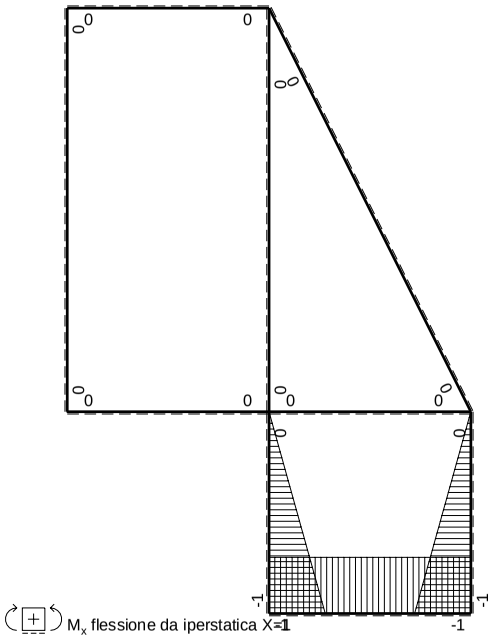


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	$-3/2Fb+Fx-1/2qx^2$	$3/2Fx-Fx^2/b+1/2qx^3/b$	x^2/b^2	$13/24Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	$Fb+1/2qx^2$	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb	Fb-Fx	$1-2x/b+x^2/b^2$	$1/2Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb	Fx	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-13/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ \ Fb^2/EJ$

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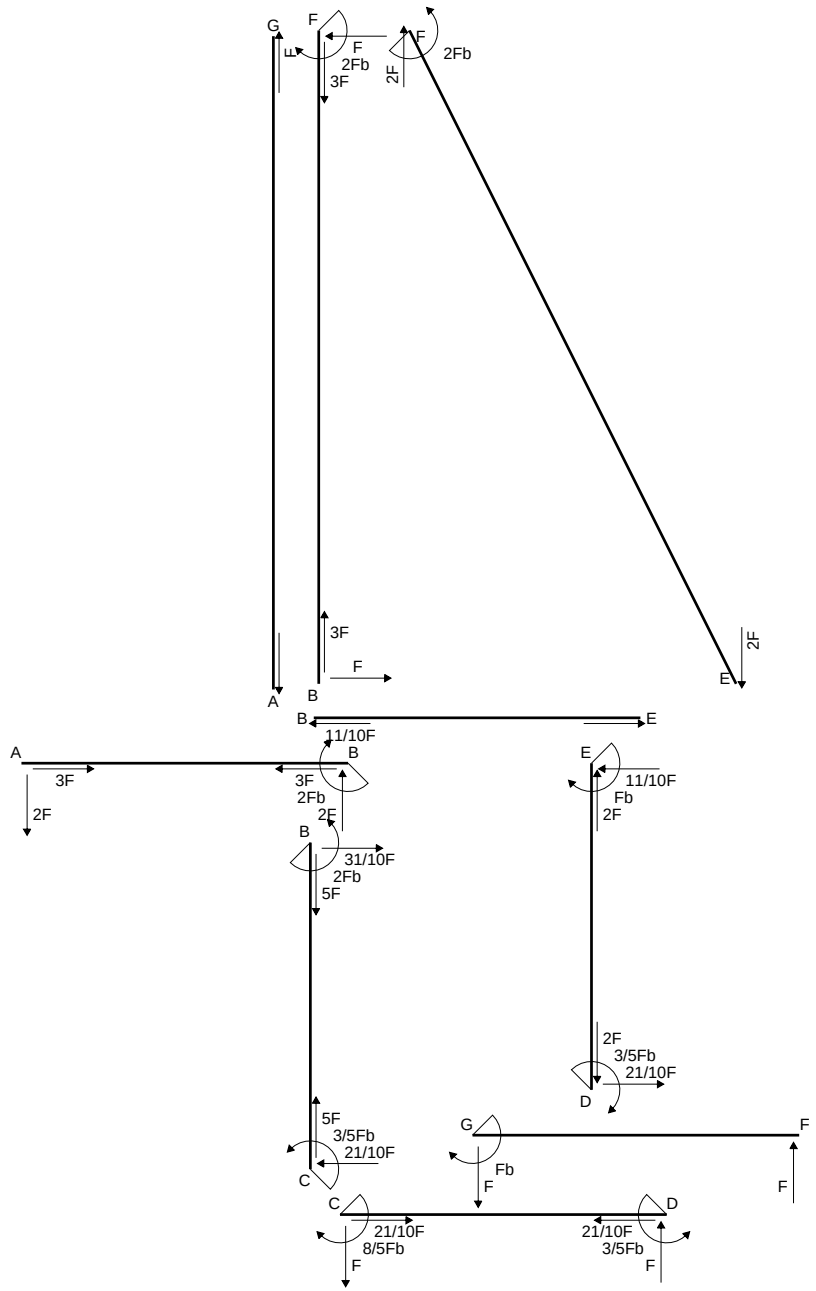
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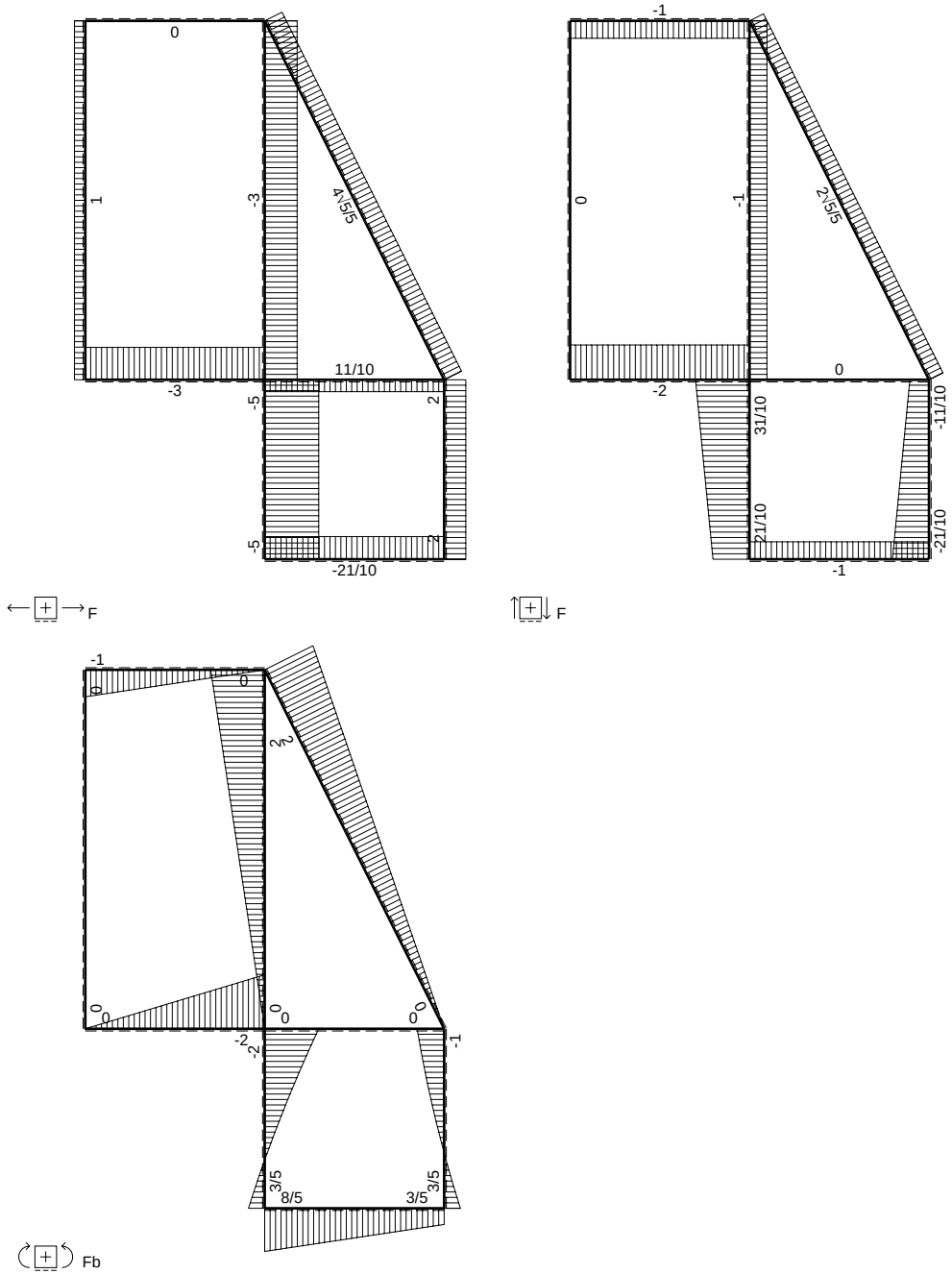
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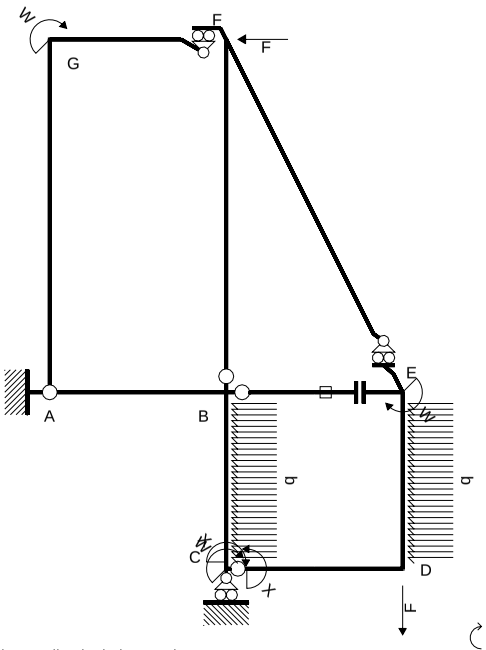
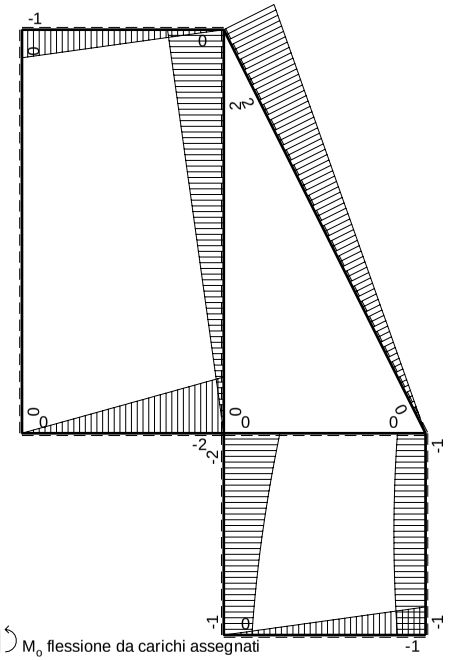
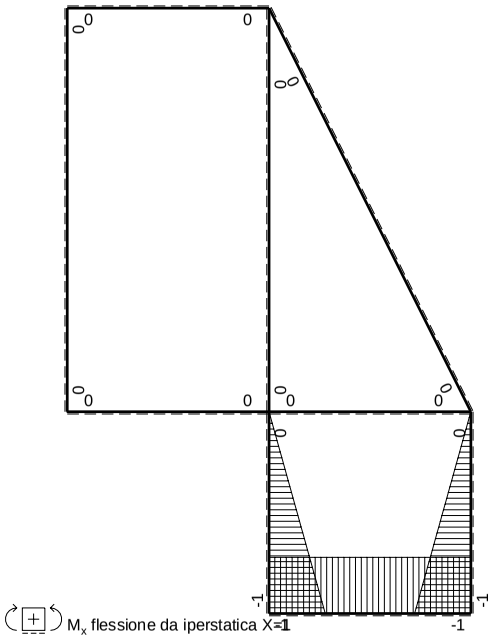


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	$2Fx-3/2Fx^2/b+1/2qx^3/b$	x^2/b^2	$5/8Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	$Fb-1/2Fx-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$8/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-8/5Fb	

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/8\ b)\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

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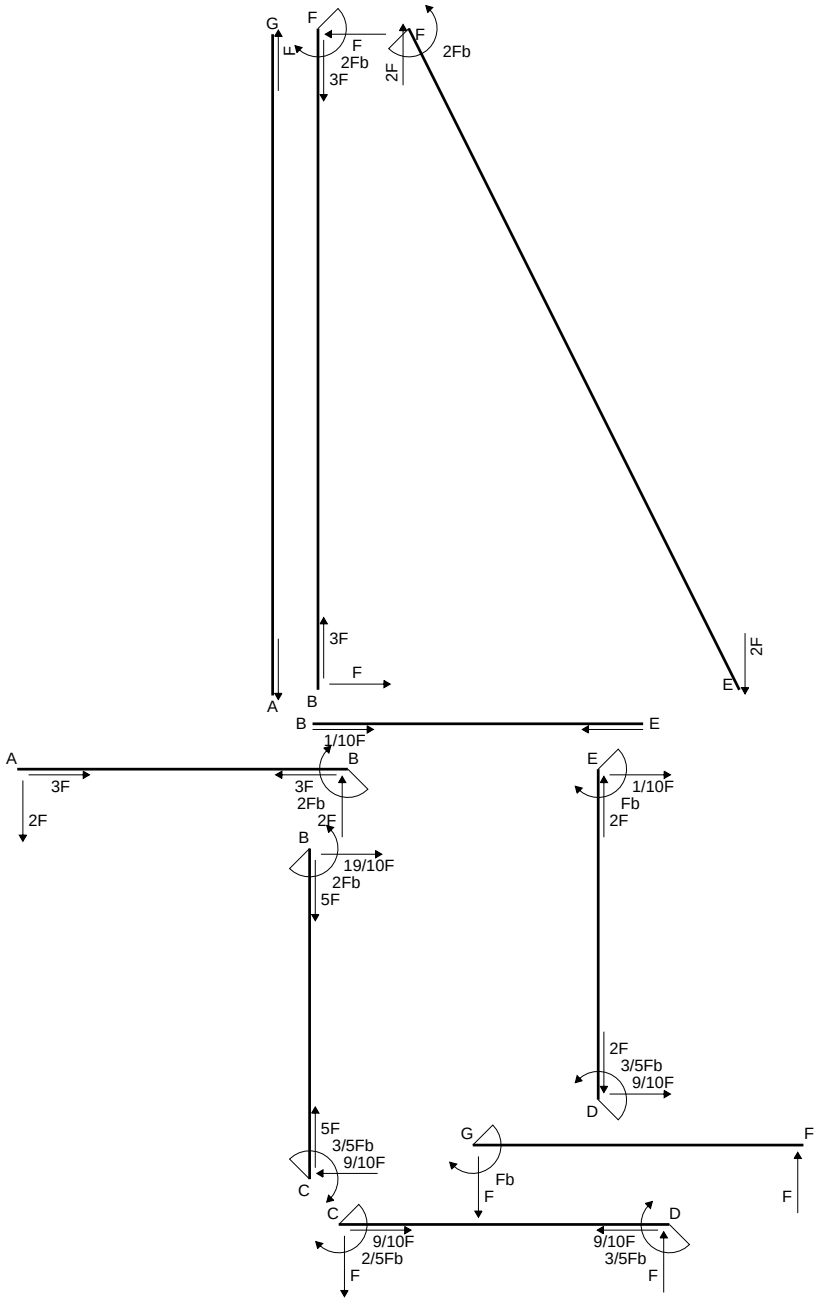
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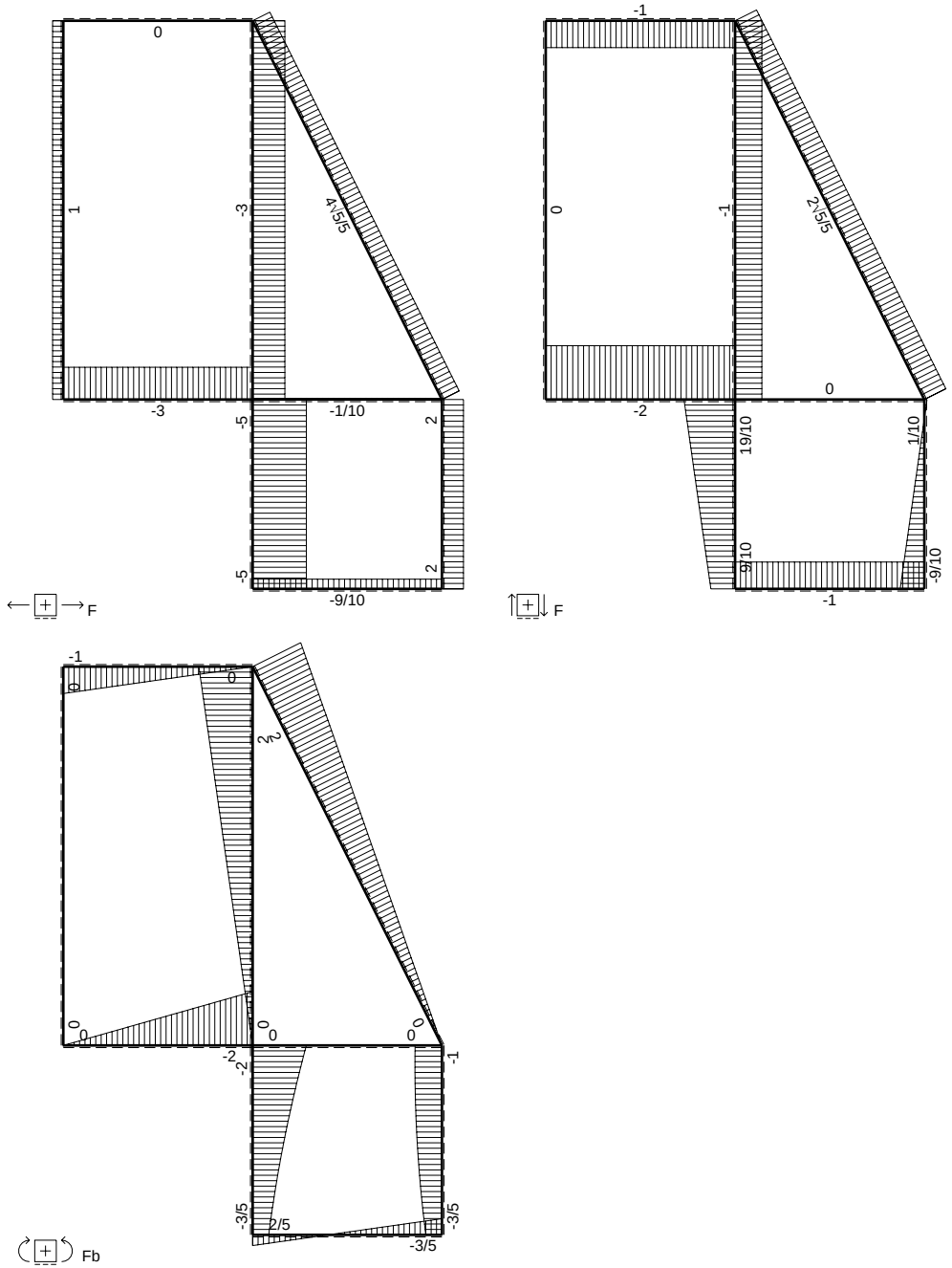
3/5Fb
F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-3
4√5/5
-3
-5
-1/10
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-5
2
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F
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-1
2√5/5
0
-2
0
19/10
1/10
9/10
-9/10
-1
F
-1
0
2
0
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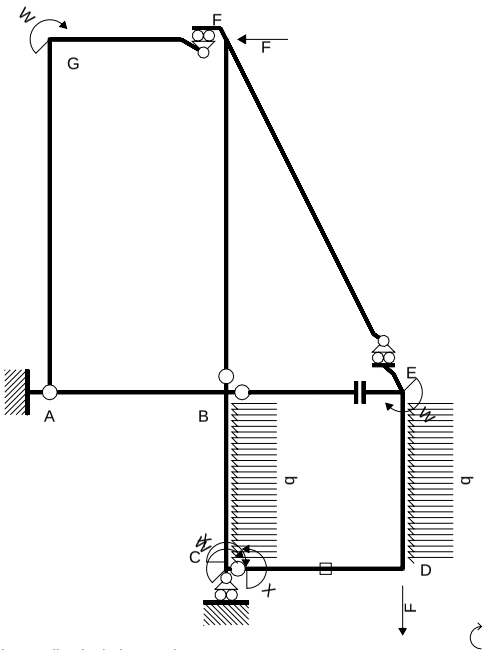
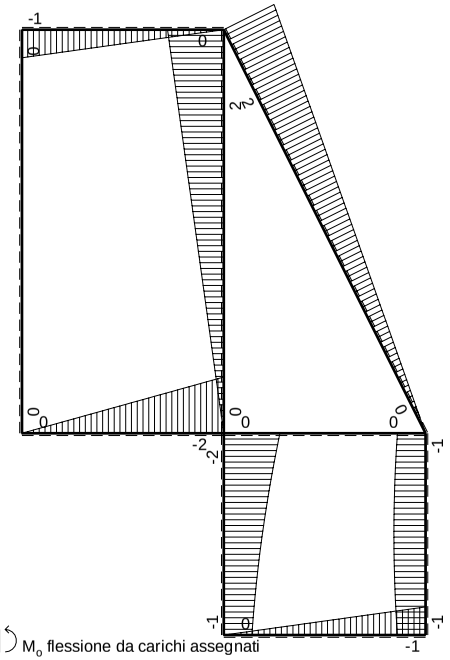
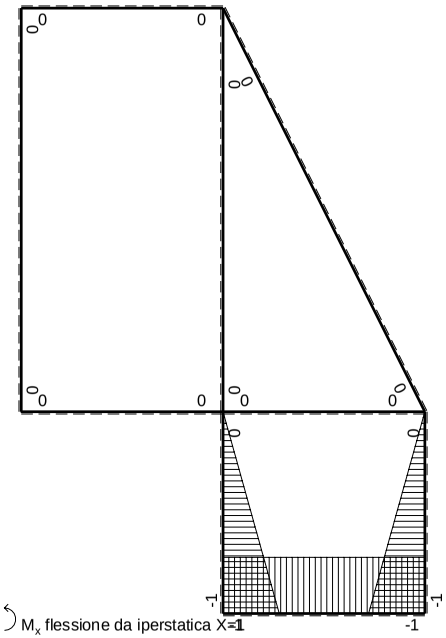
0
0
-2
-2
-1
-3/5
2/5
-3/5
-3/5
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	$2Fx-3/2Fx^2/b+1/2qx^3/b$	x^2/b^2	$5/8Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	$Fb-1/2Fx-1/2qx^3/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb-1/2Fx+1/2qx^2$	$Fb-1/2Fx-Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$13/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb+1/2Fx-1/2qx^2$	$Fx+1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$2/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-2/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 5/8\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ \ Fb^2/EJ$

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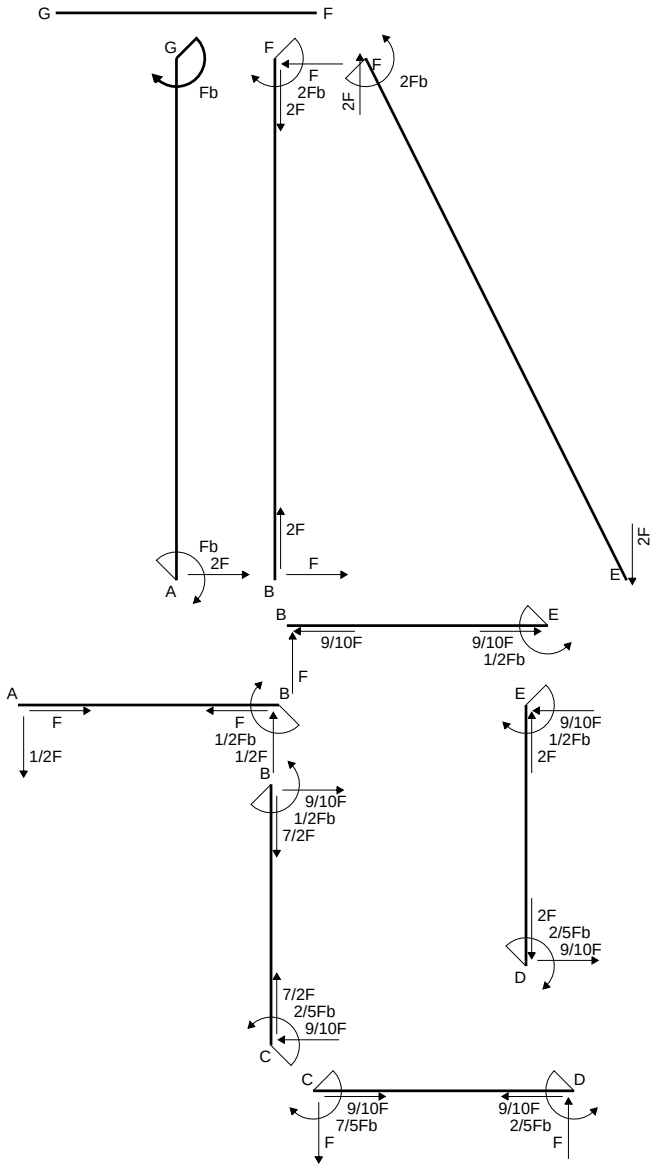
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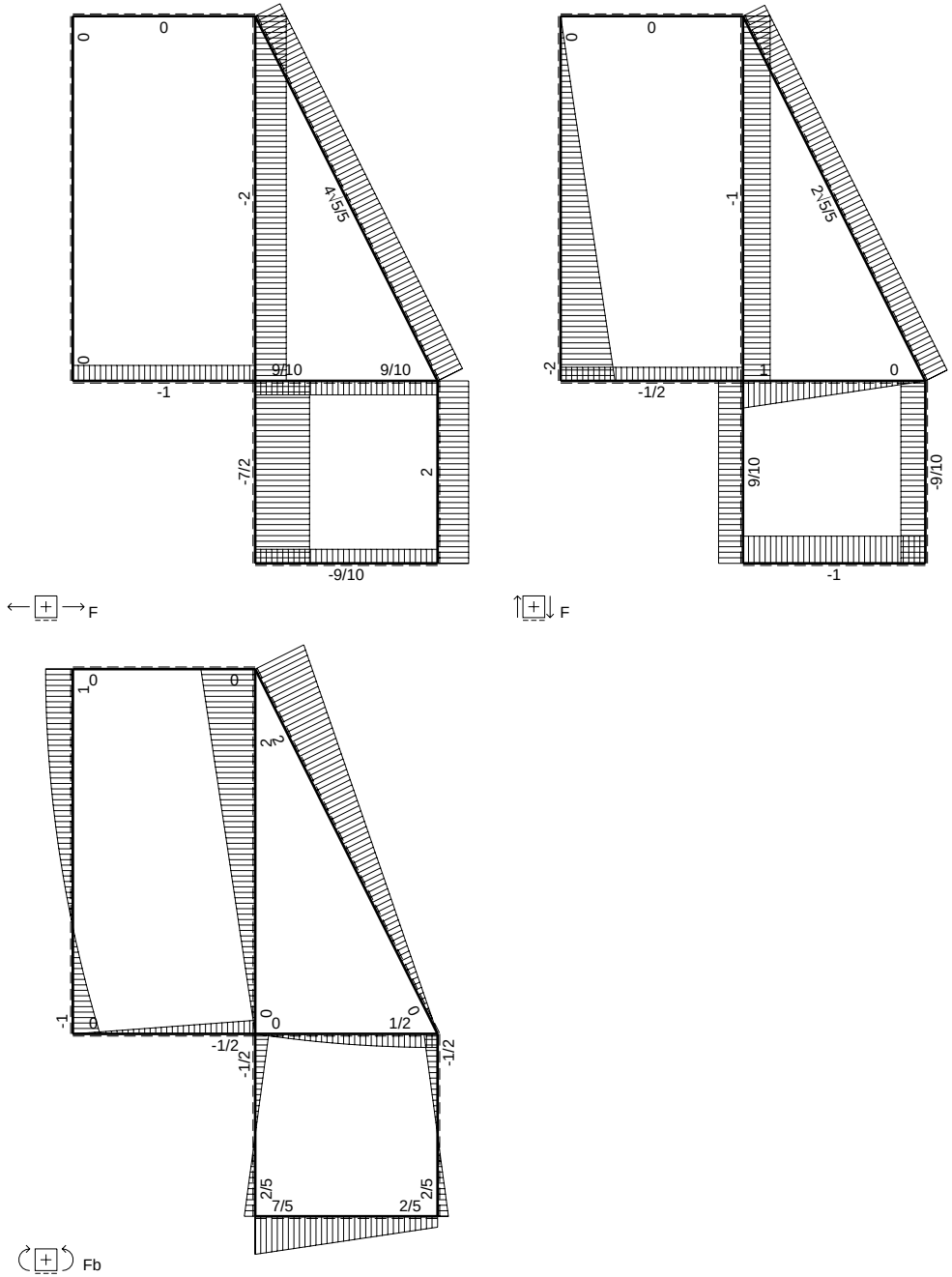
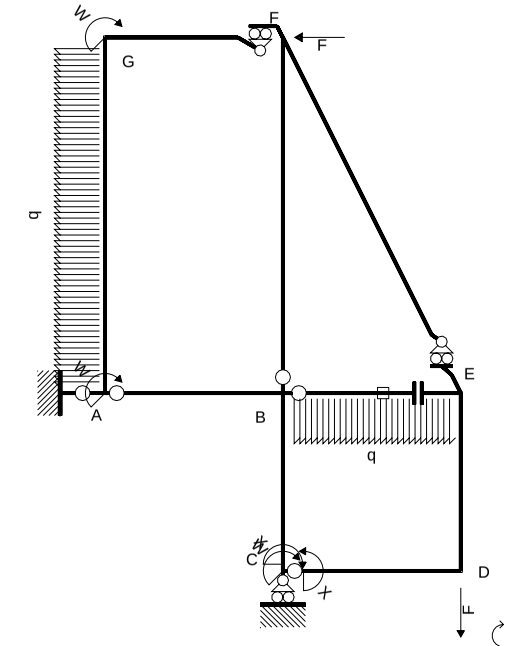
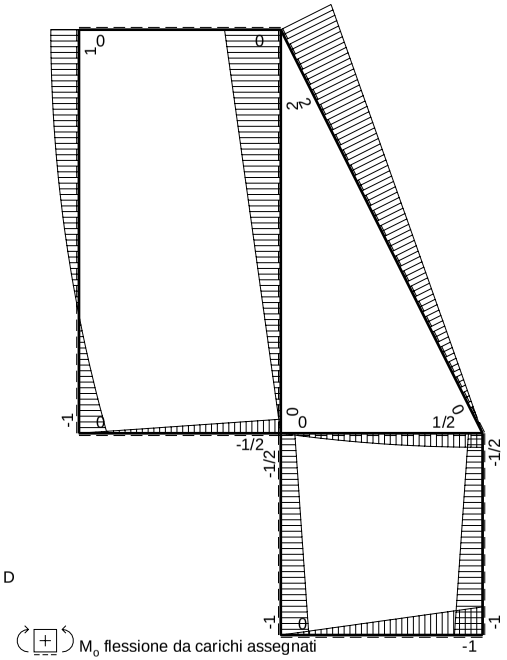
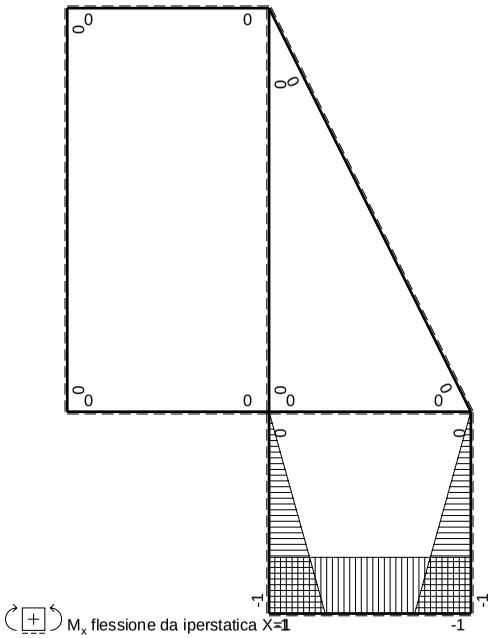
F
Fb
2Fb
2F
A
B
E
9/10F
1/2Fb
1/2F
7/2F
2/5Fb
7/5Fb
C
D


F
F
Fb



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb+1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-7/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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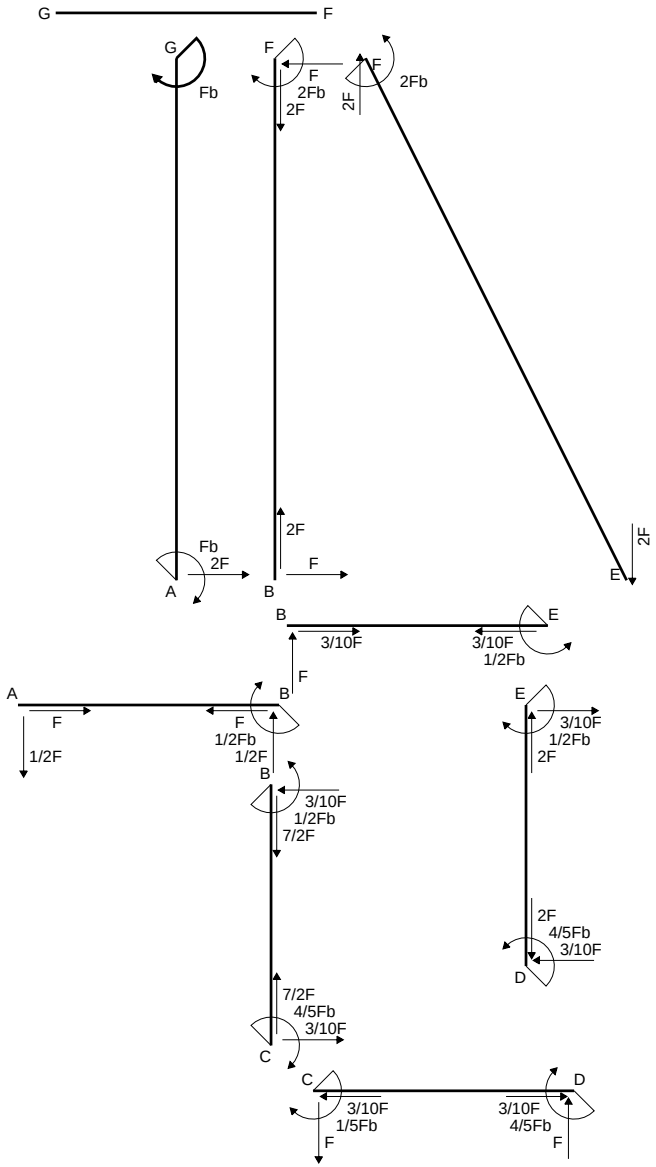
G
F
Fb
2Fb
2F
A
B
E
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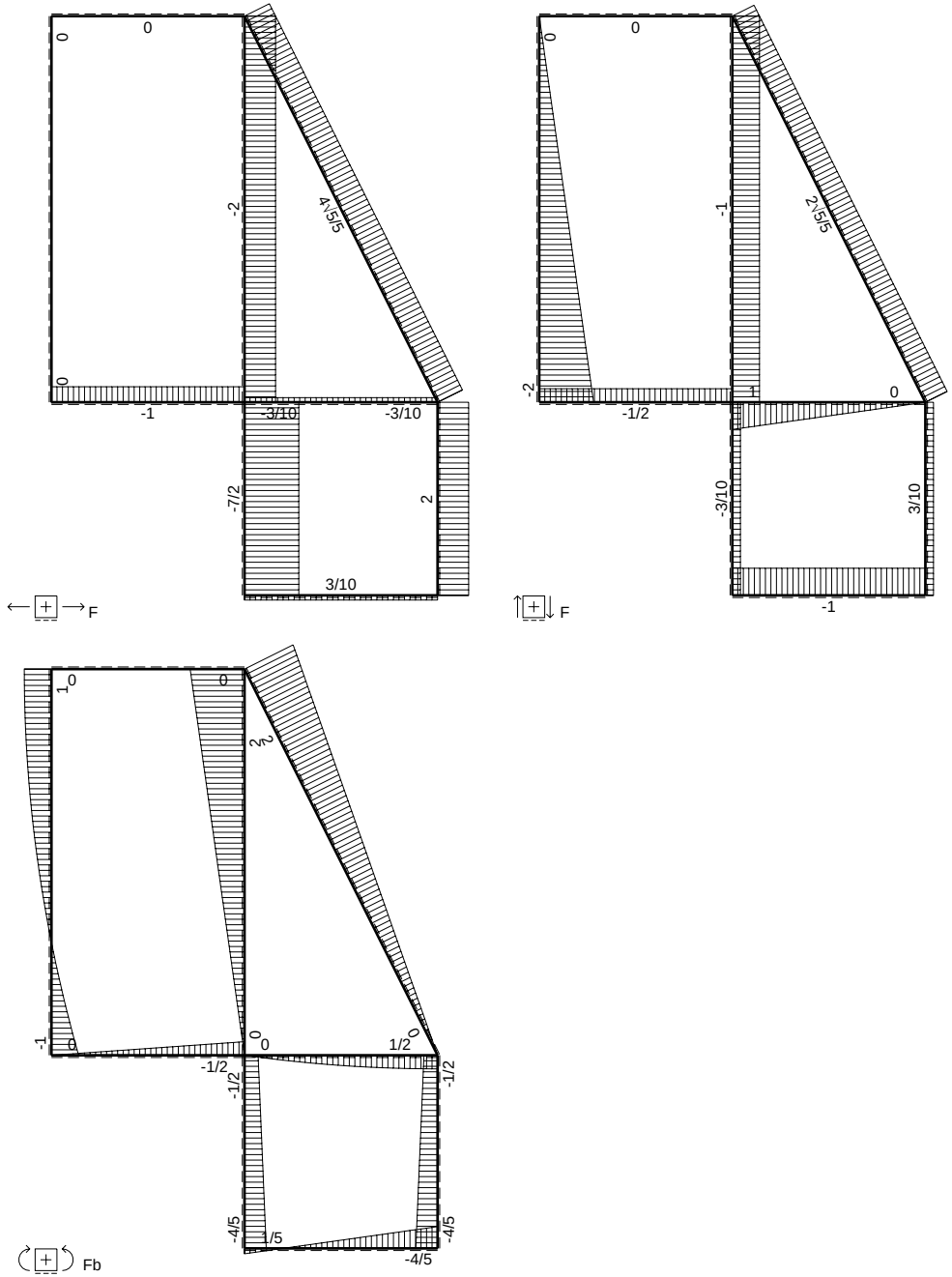
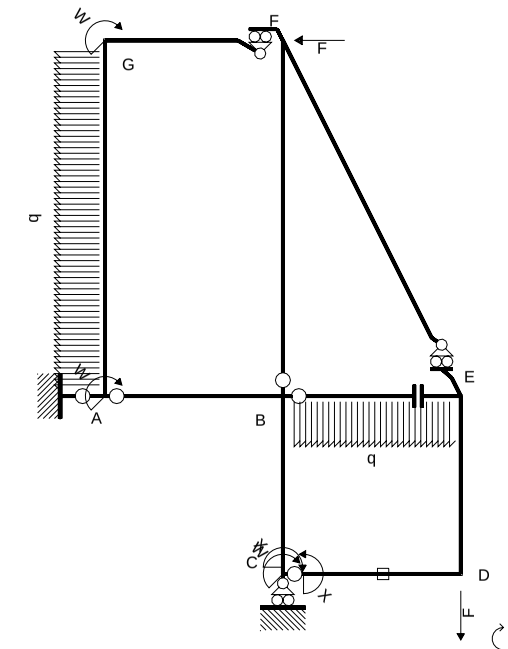
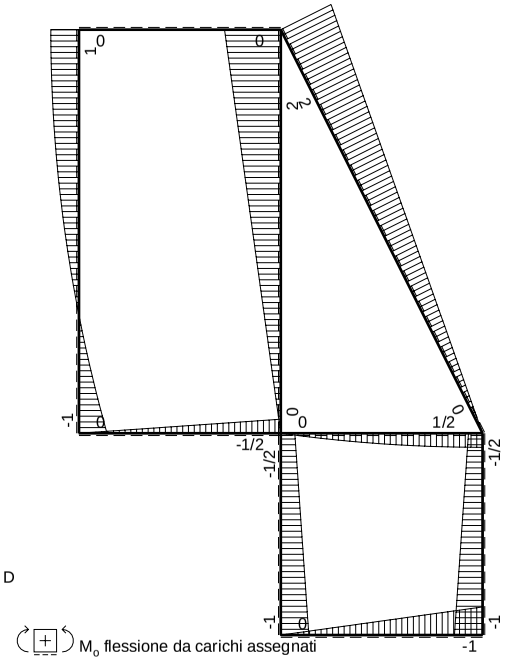
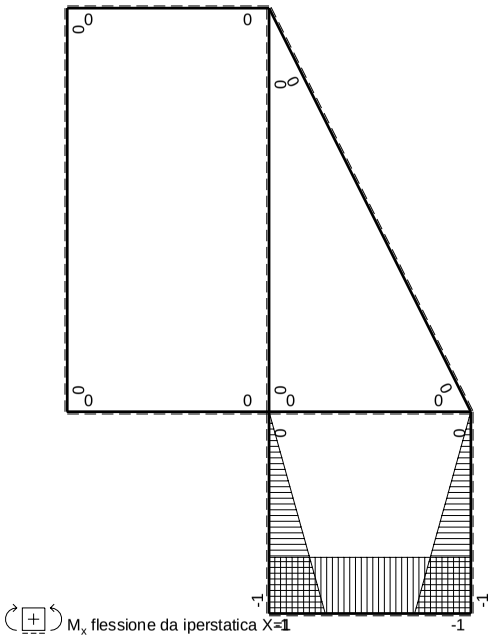
1/2Fb
1/2F
7/2F
4/5Fb
C
D
1/5Fb


F
-2
4/5
-1
-3/10
-7/2
2
3/10
-1/2
-3/10
Fb
1/2
-4/5
1/5



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb+1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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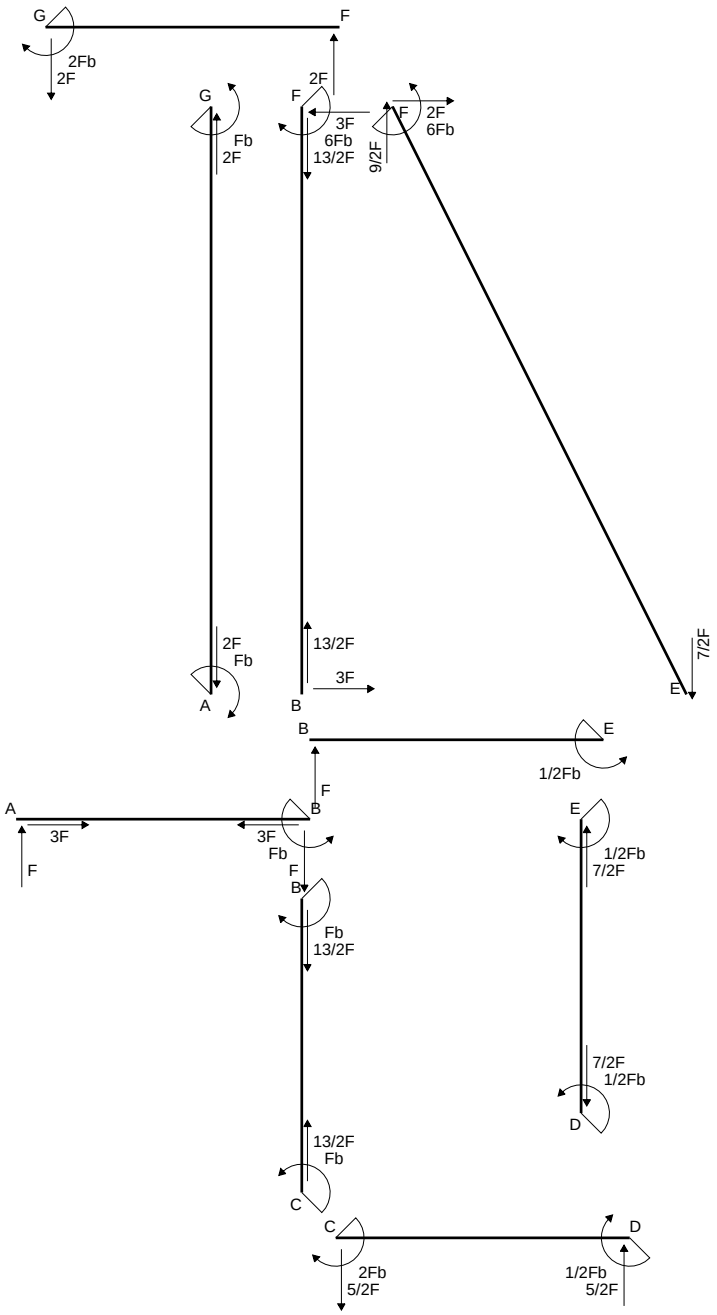
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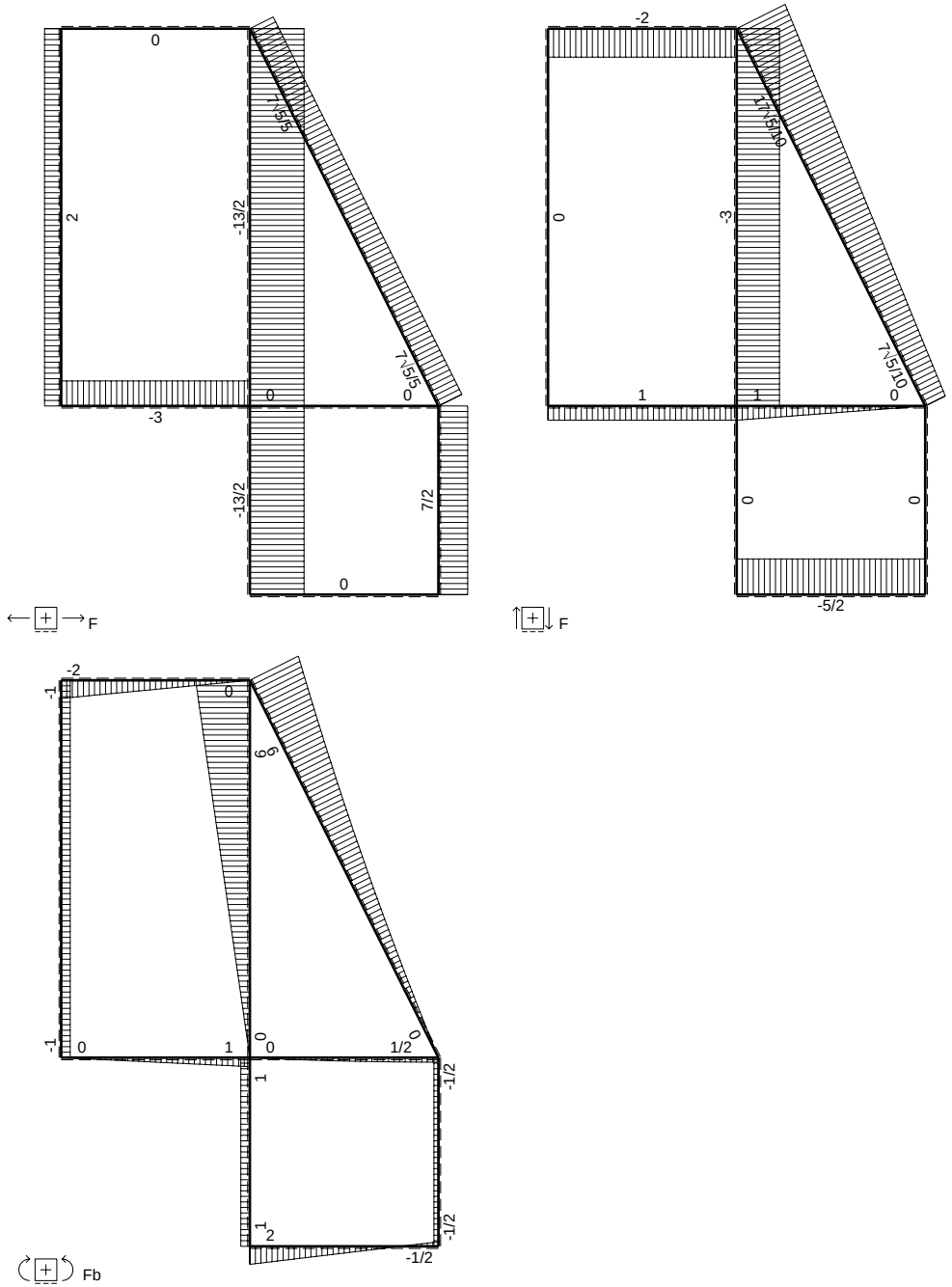
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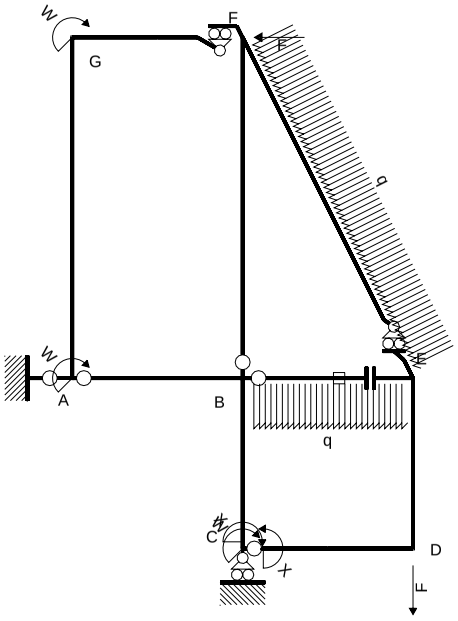
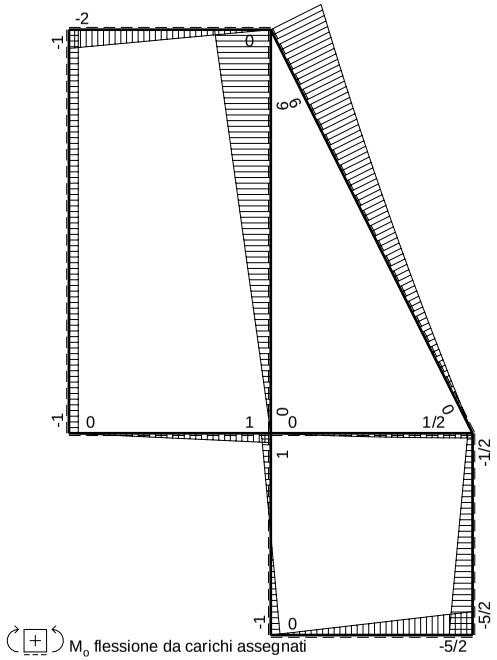
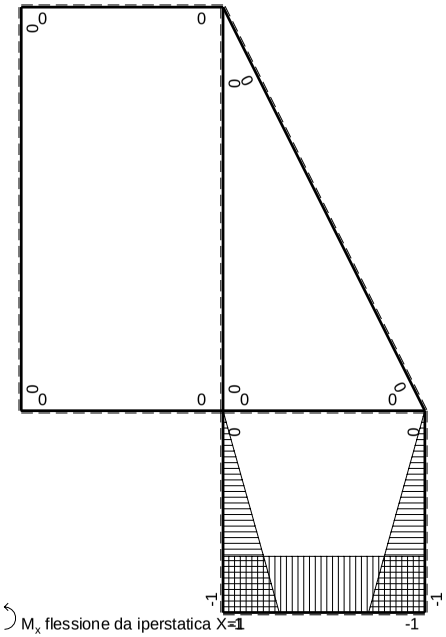


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+2Fx	$5/2Fb-9/2Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$11/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb+2Fx	$1/2Fx+2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$10/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-2Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -9/2\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -9/4\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -9/4\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

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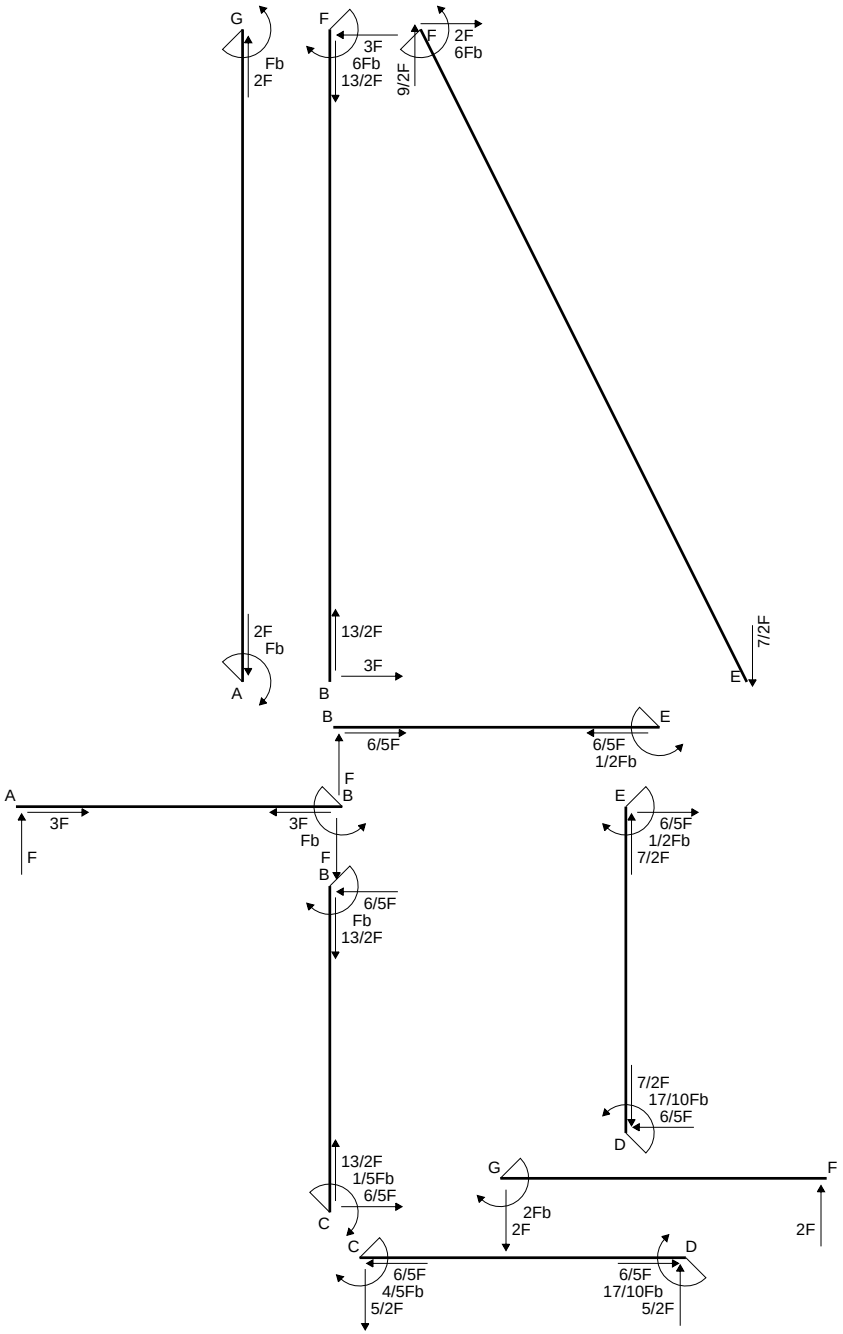
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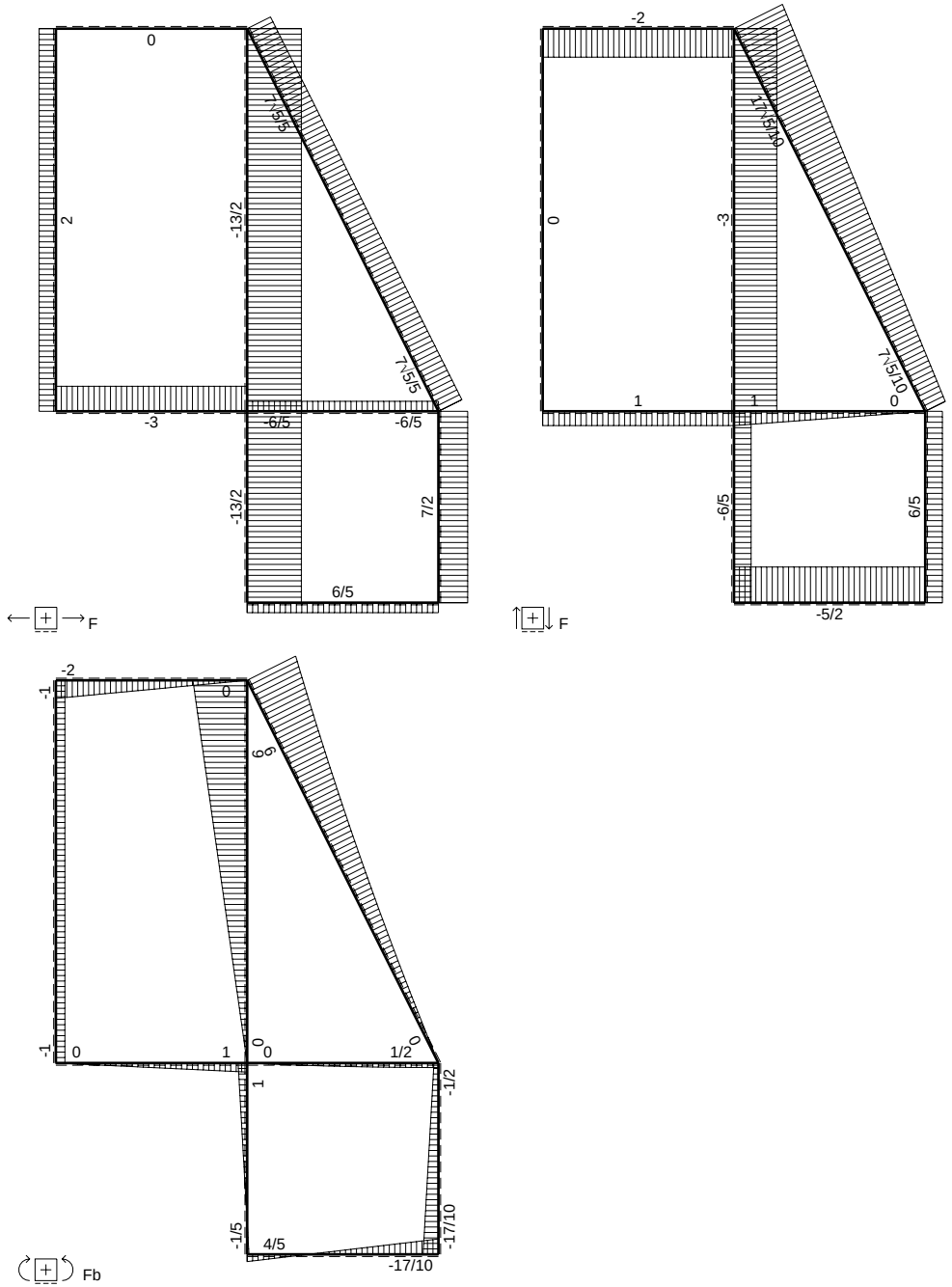
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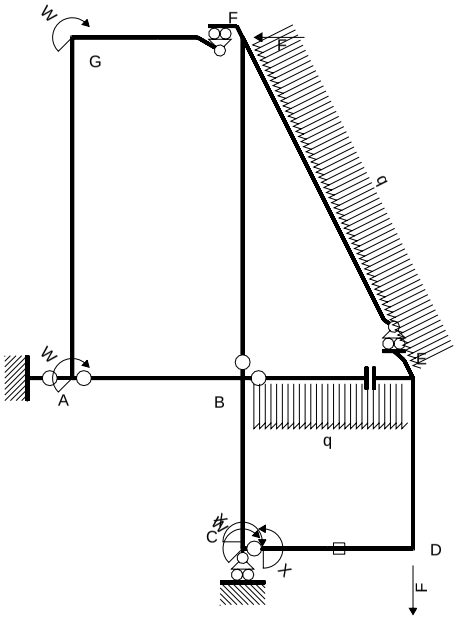
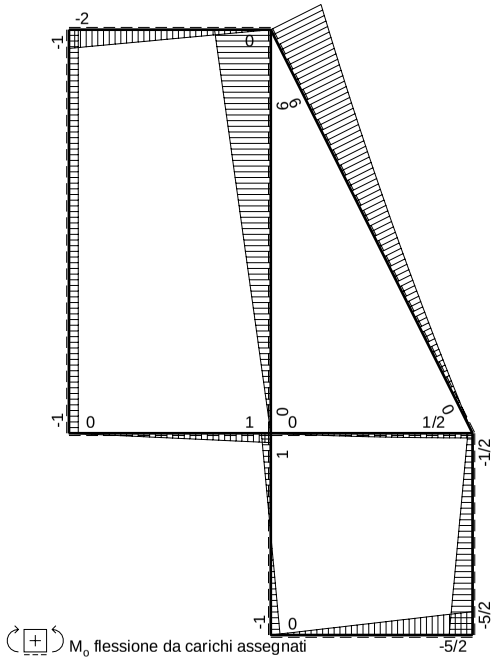
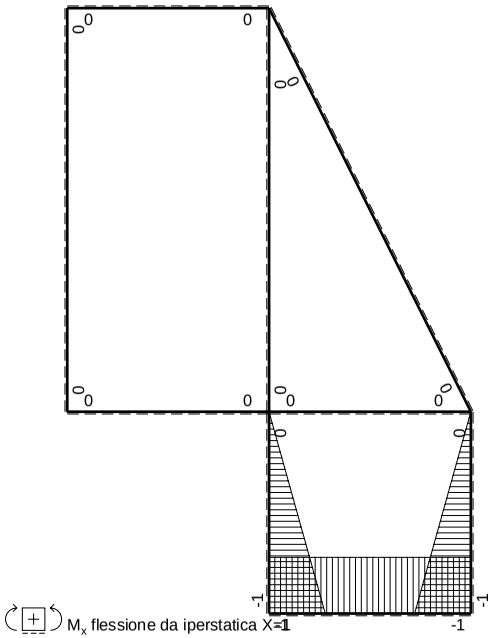


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+2Fx	$5/2Fb-9/2Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$11/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb+2Fx	$1/2Fx+2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$4/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-4/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -9/2\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -9/4\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -9/4\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 11/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 11/12\ \ Fb^2/EJ$

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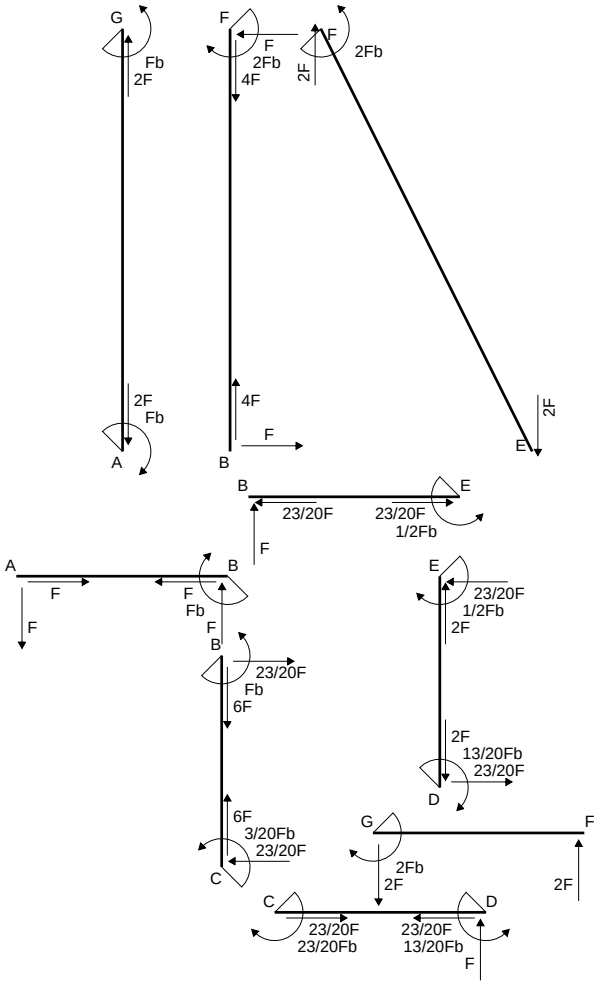
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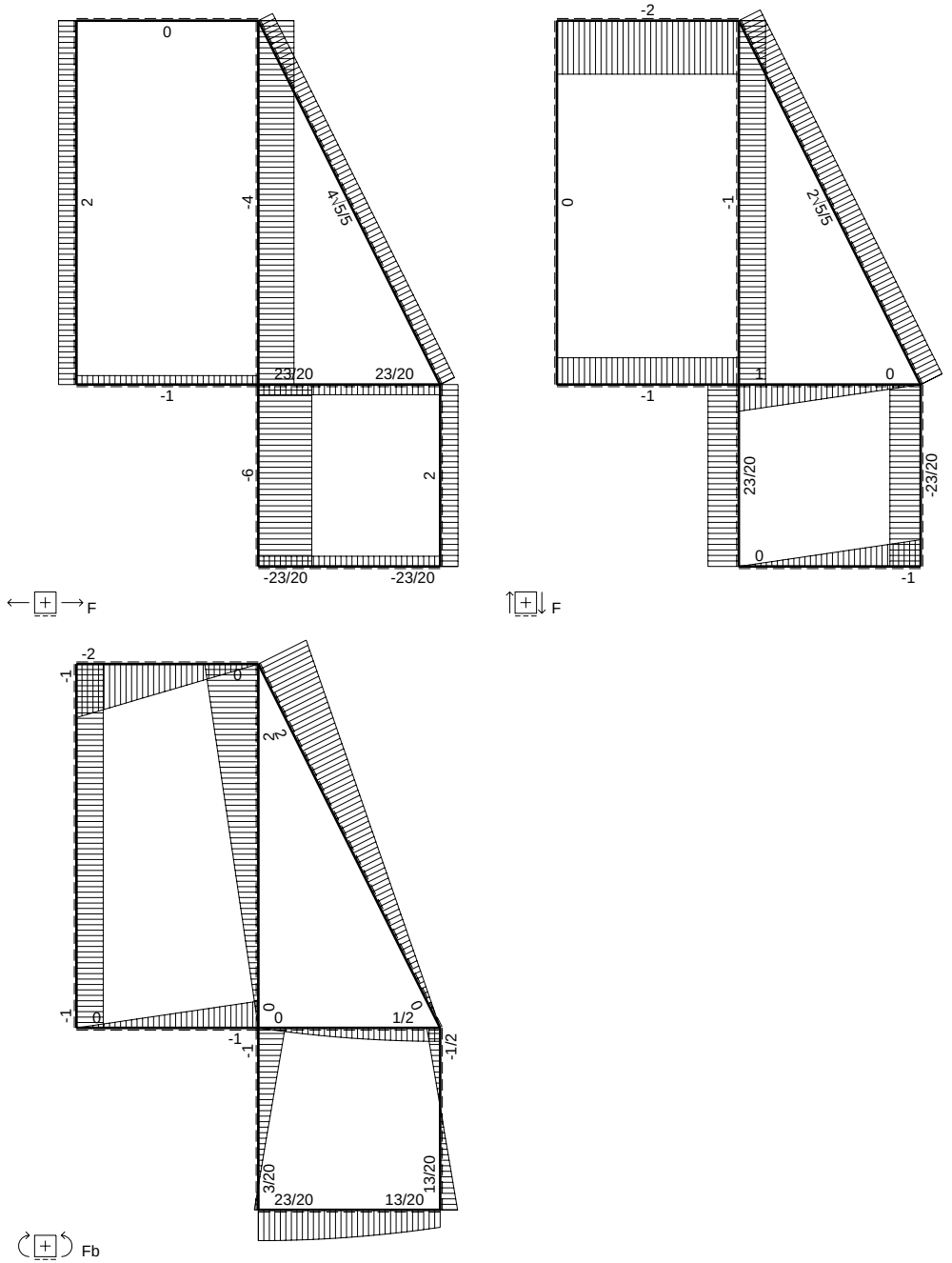
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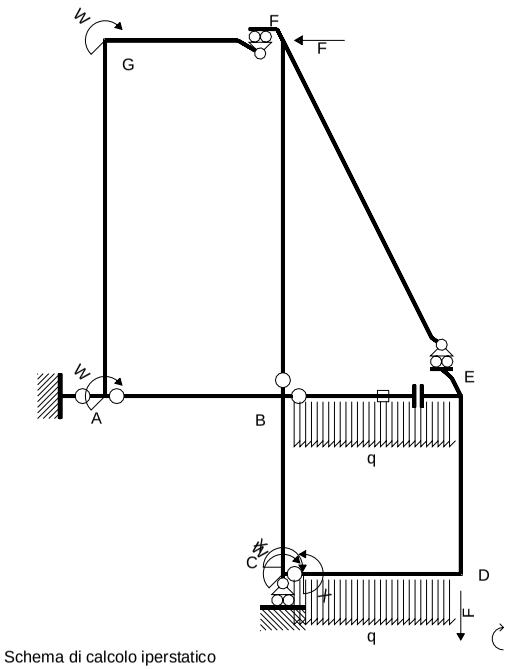
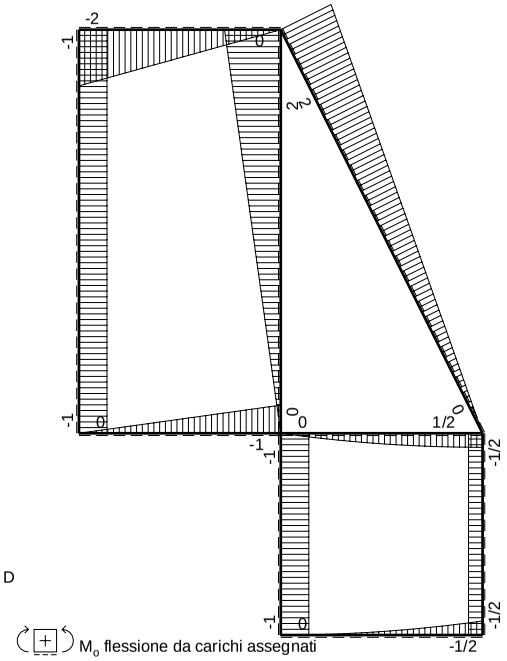
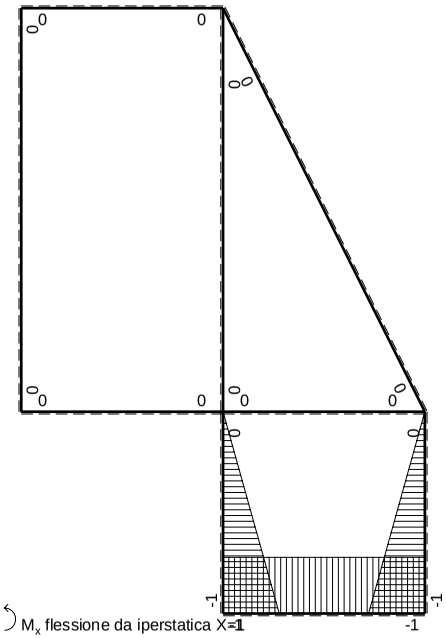


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb	1/2Fb-1/2Fx	$1-2x/b+x^2/b^2$	$1/4Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb	1/2Fx	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$23/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-23/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x/b)\ Fb\ 1/EJ\ dx = [1/2\ x -1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/4\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

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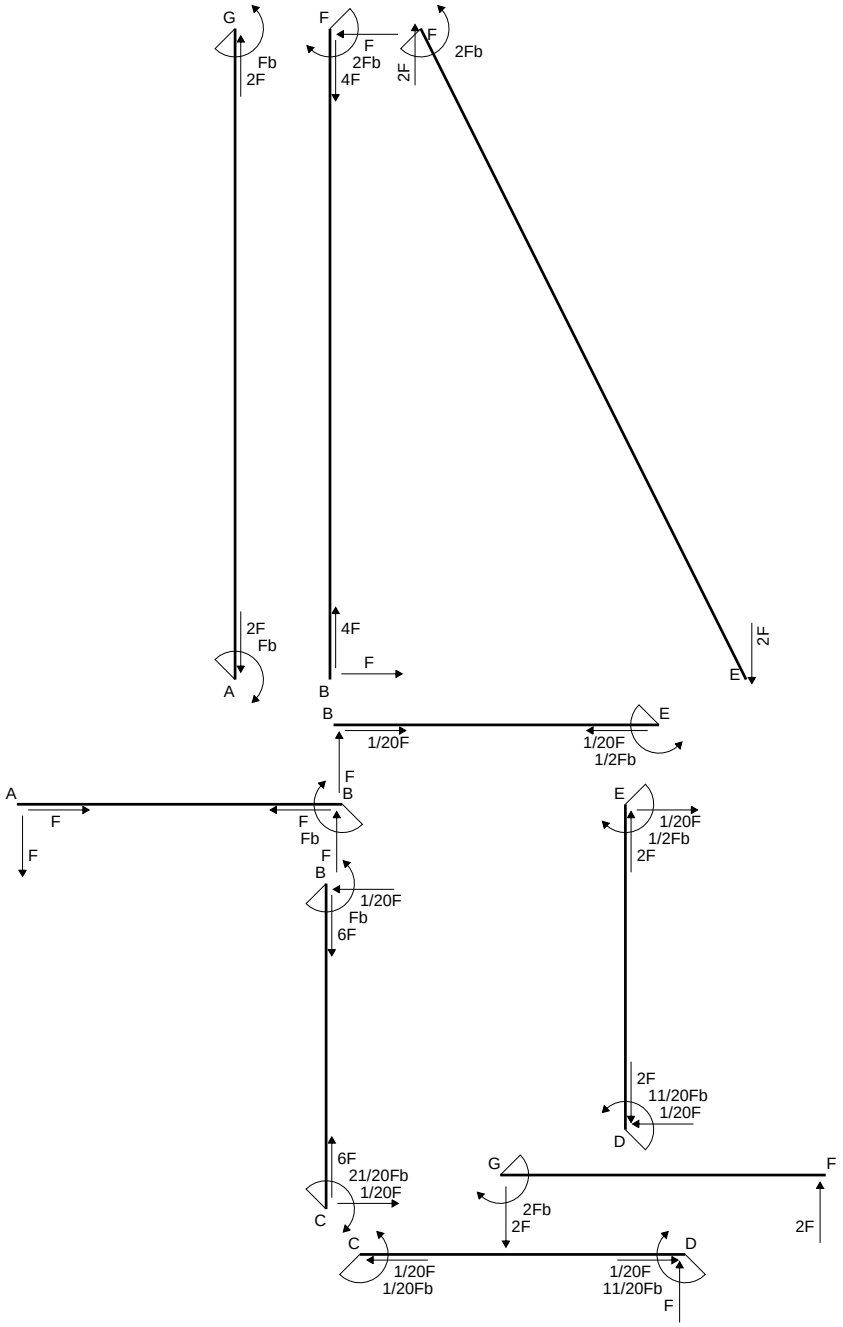
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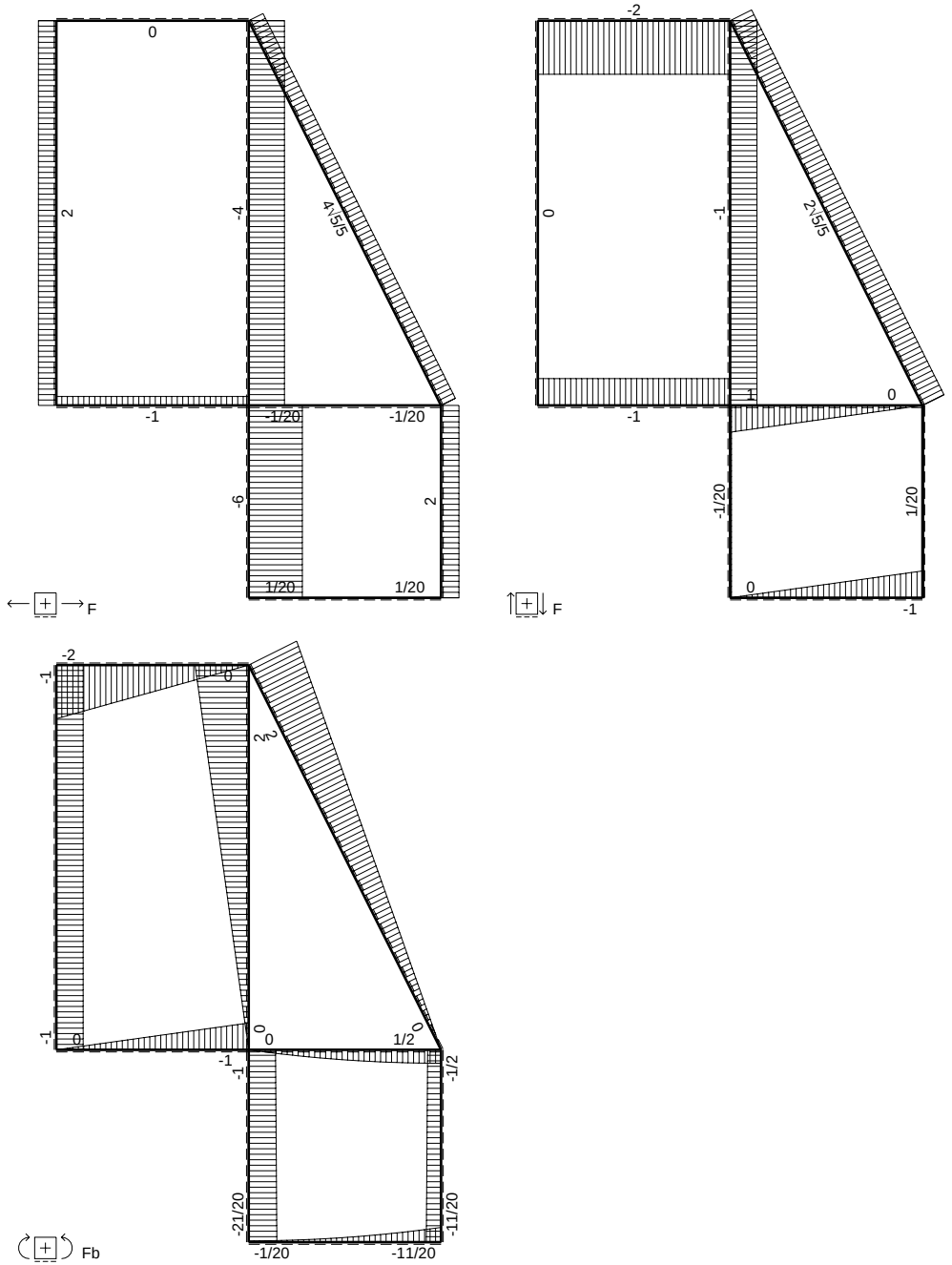
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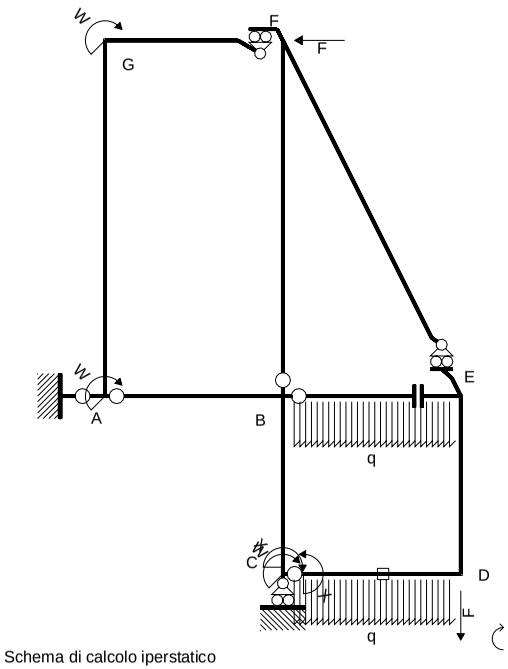
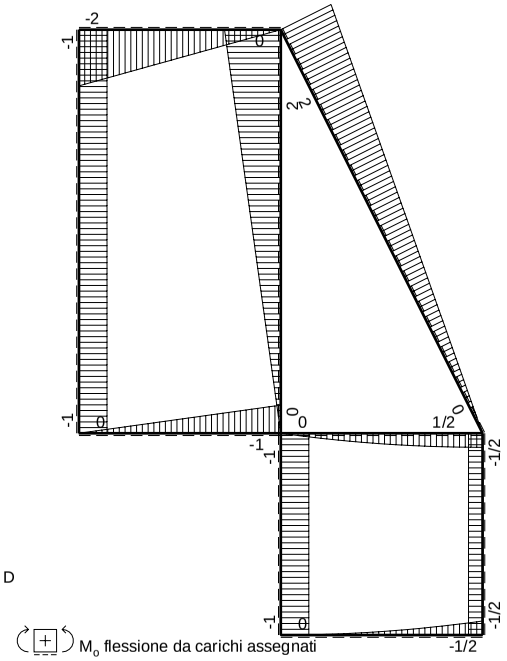
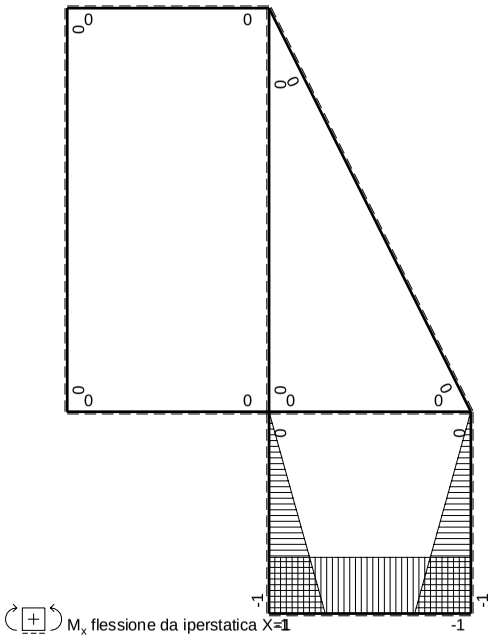
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb	1/2Fb-1/2Fx	$1-2x/b+x^2/b^2$	$1/4Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb	1/2Fx	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-1/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$1/20Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/4\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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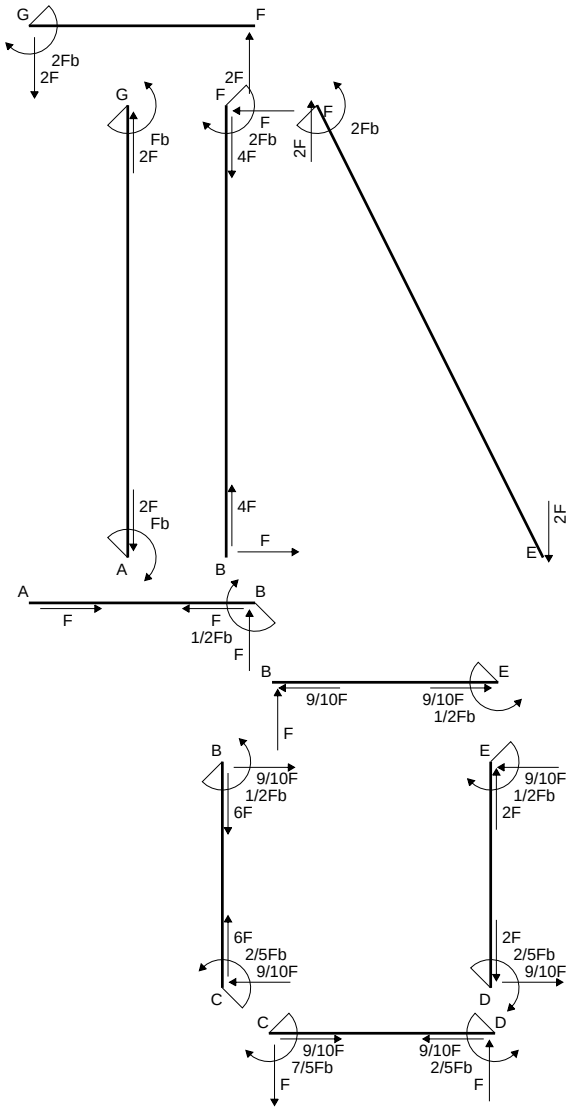
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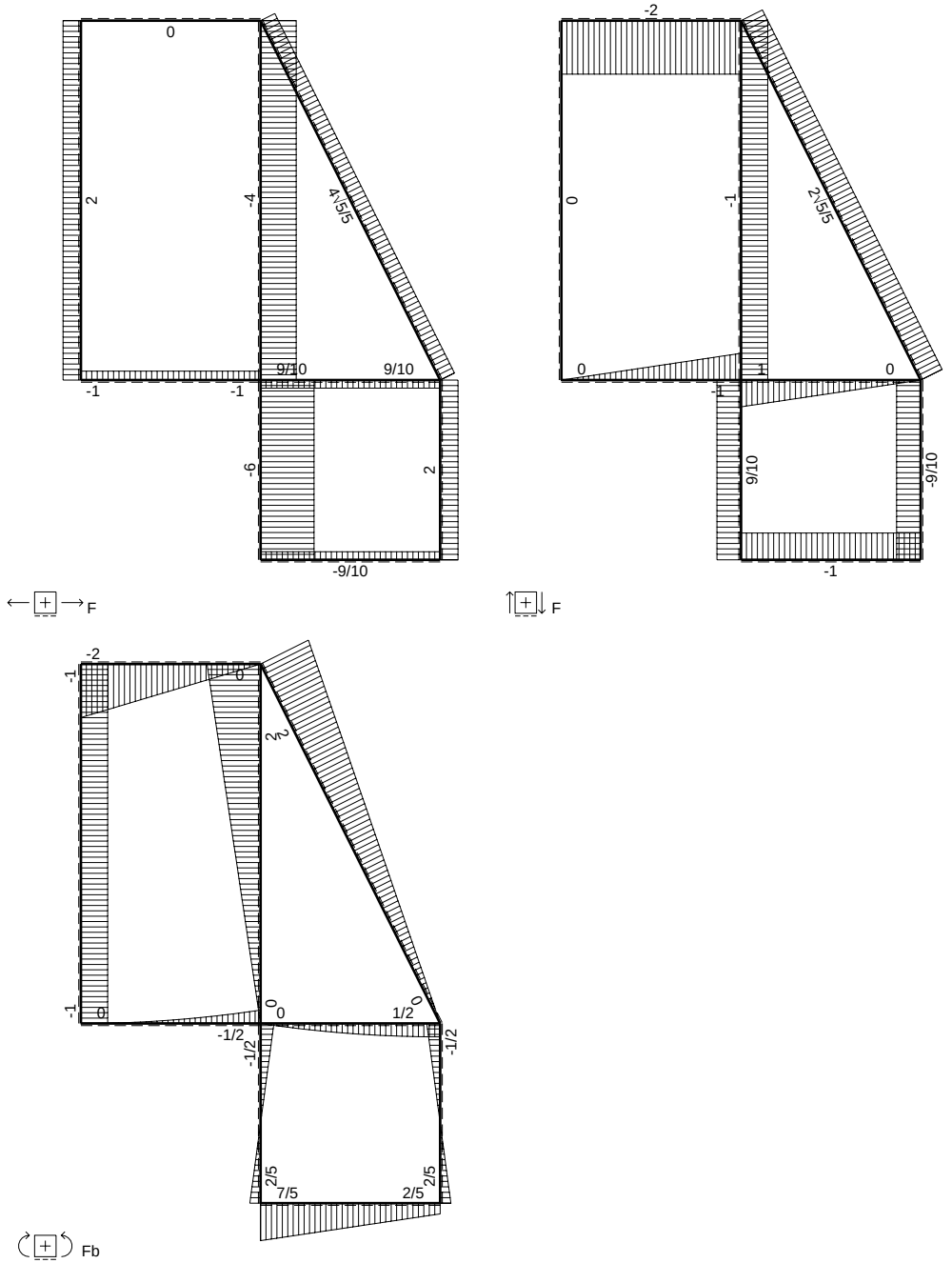
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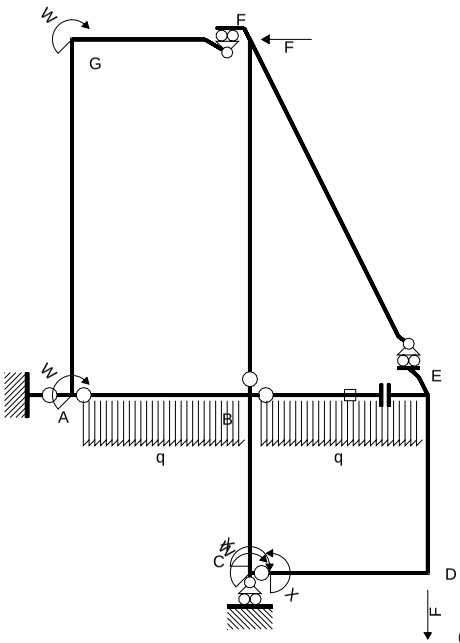
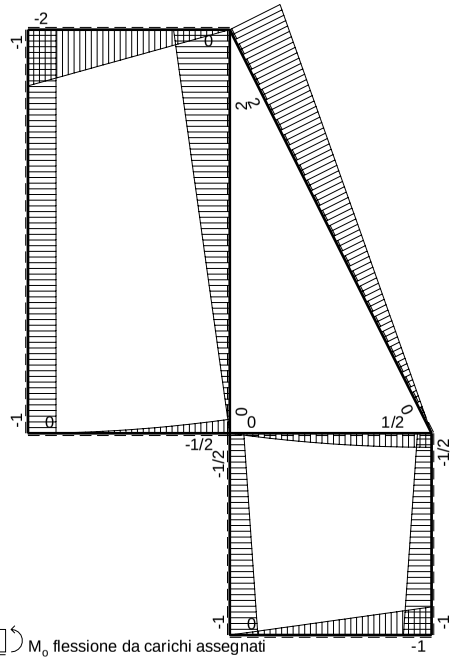
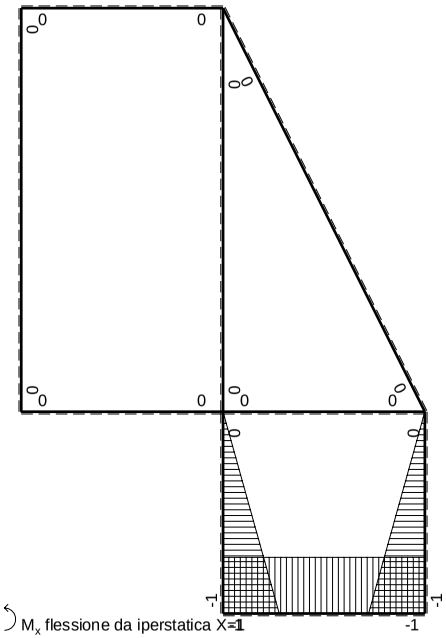


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fb+1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-7/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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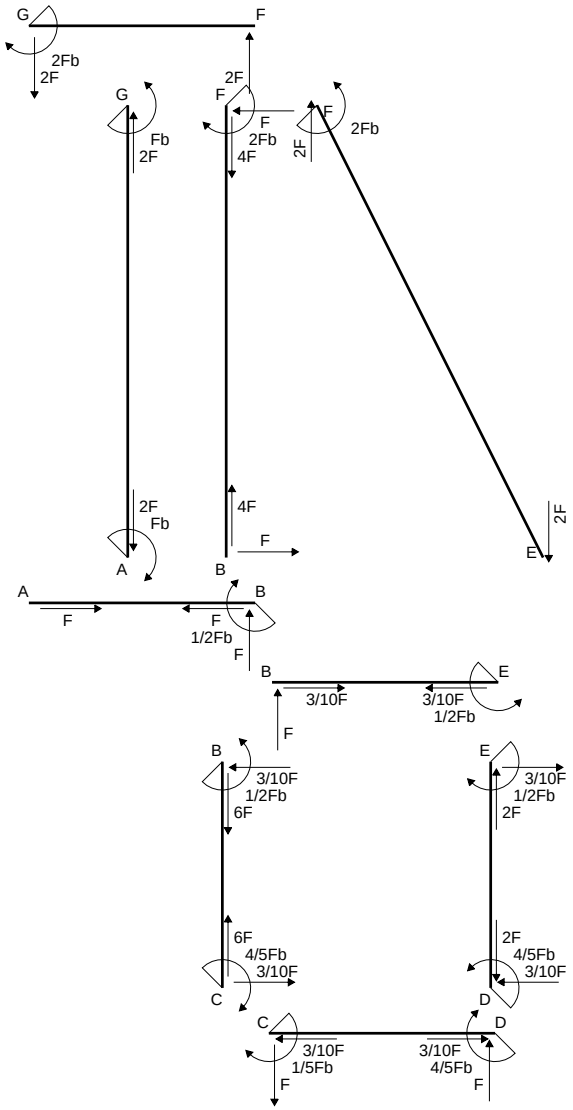
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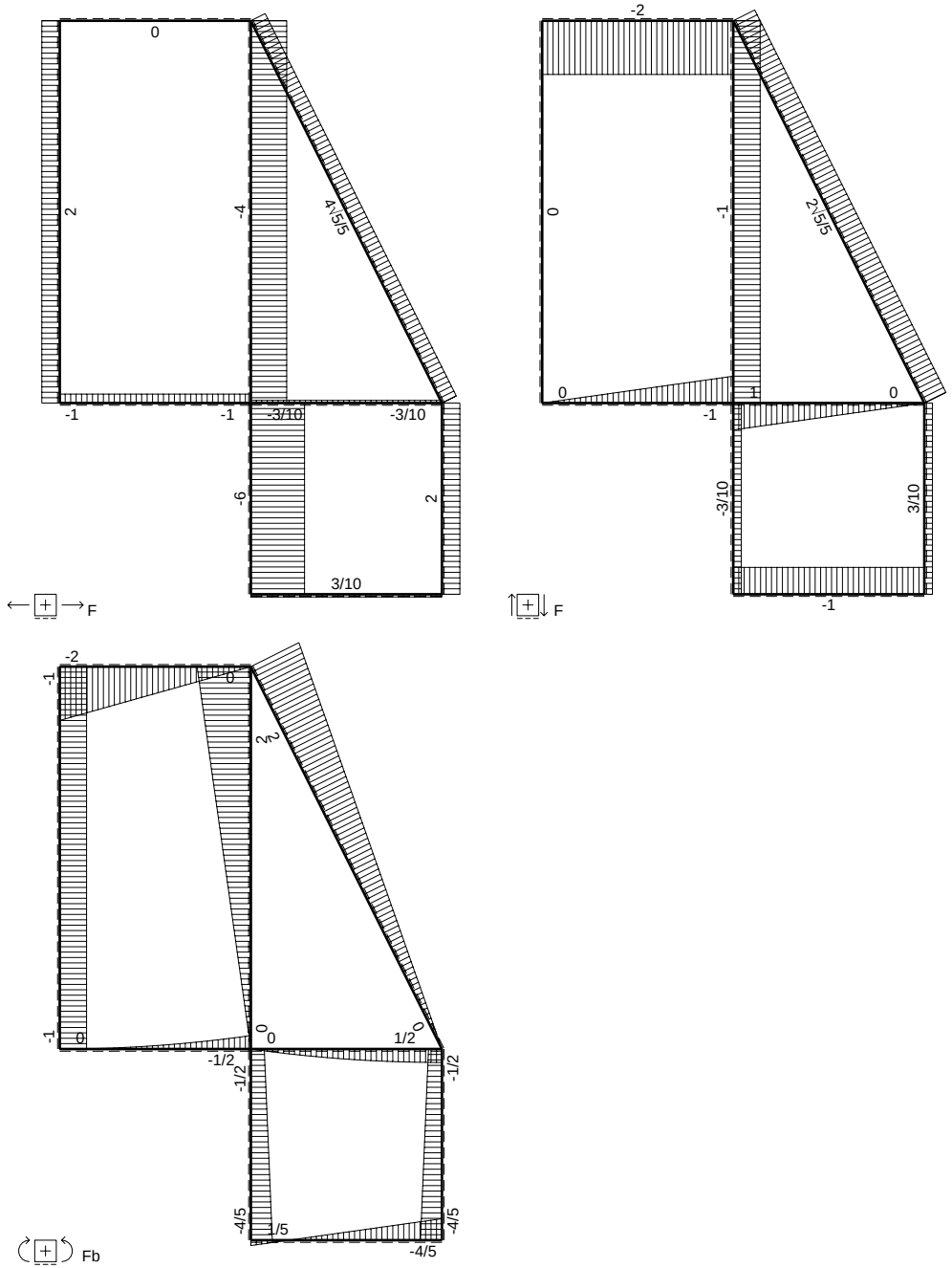
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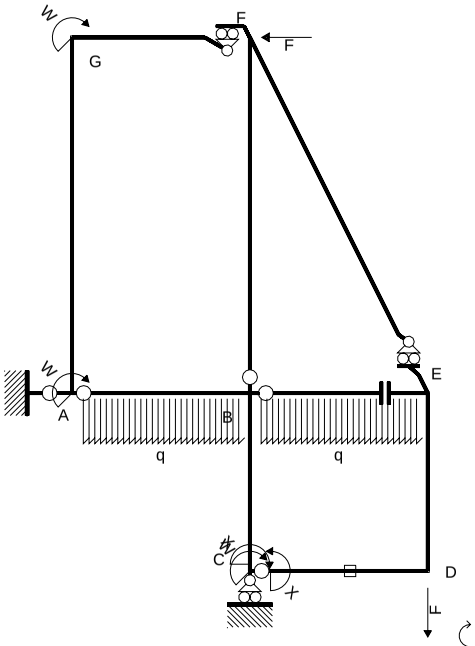
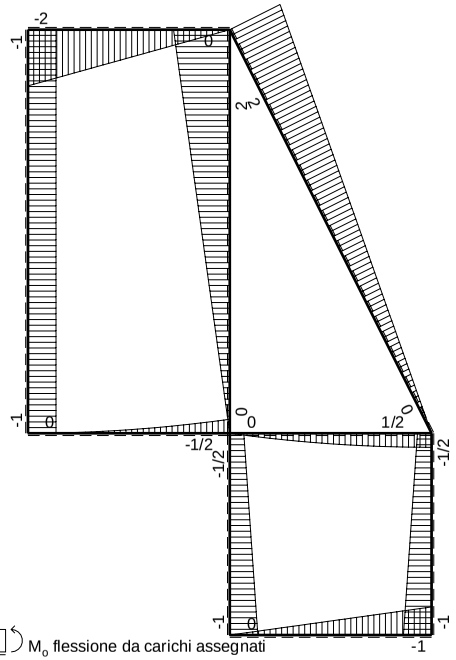
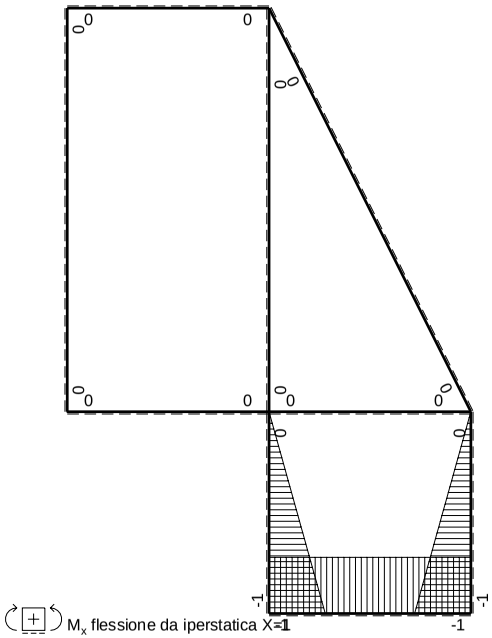


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fb+1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-1/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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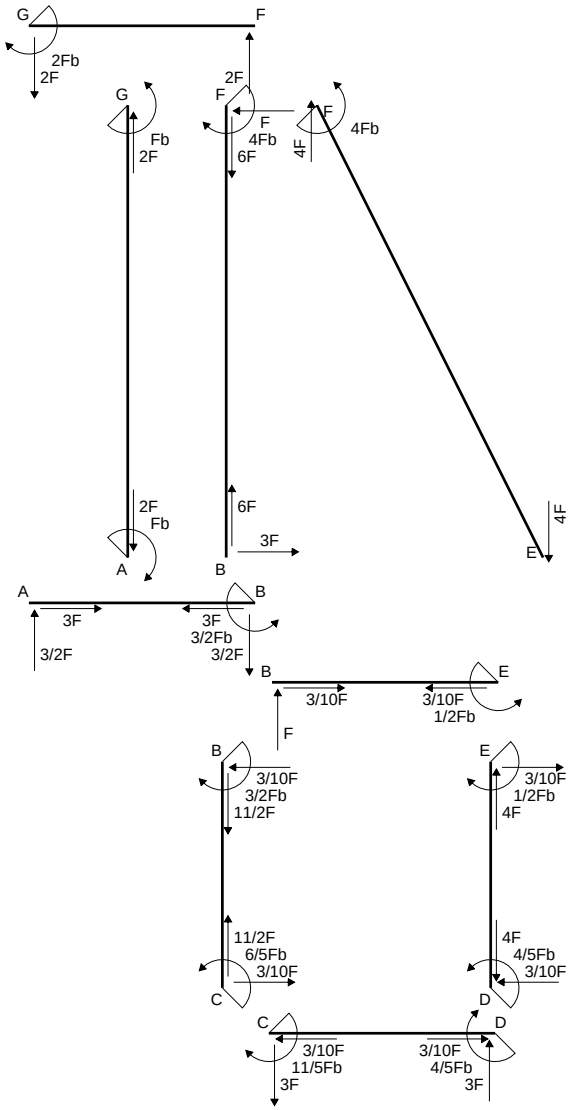
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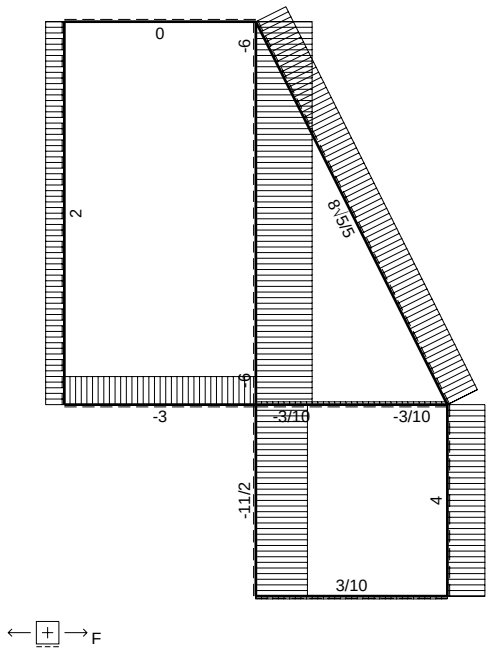
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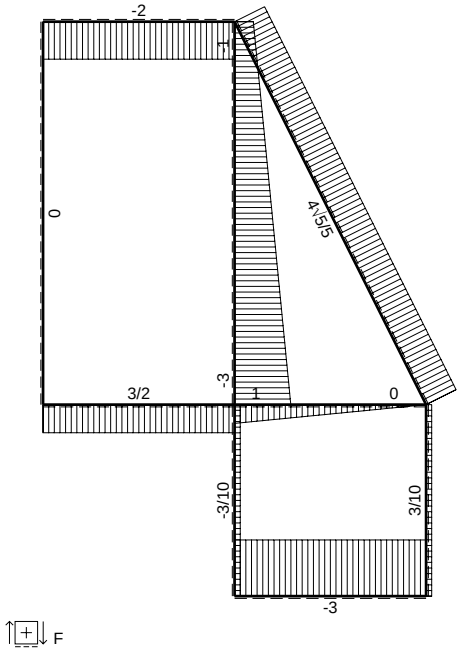
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-3
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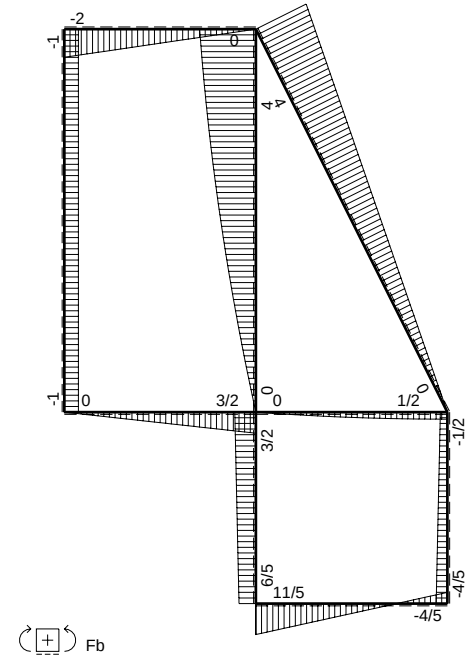
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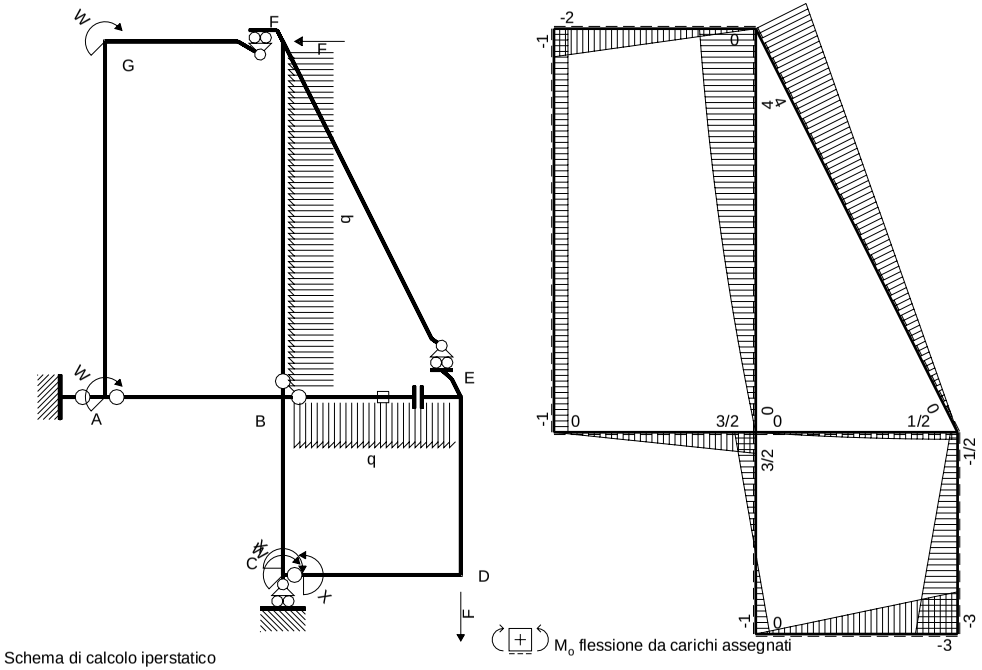
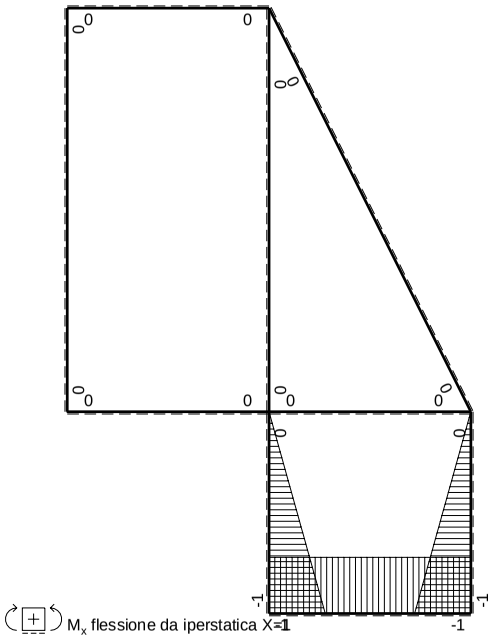


Schema di calcolo iperstatico

M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+5/2Fx	$3Fb-11/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$13/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fb+5/2Fx	$1/2Fx+5/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$11/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-11/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -11/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -11/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -11/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 13/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 13/12\ Fb^2/EJ$

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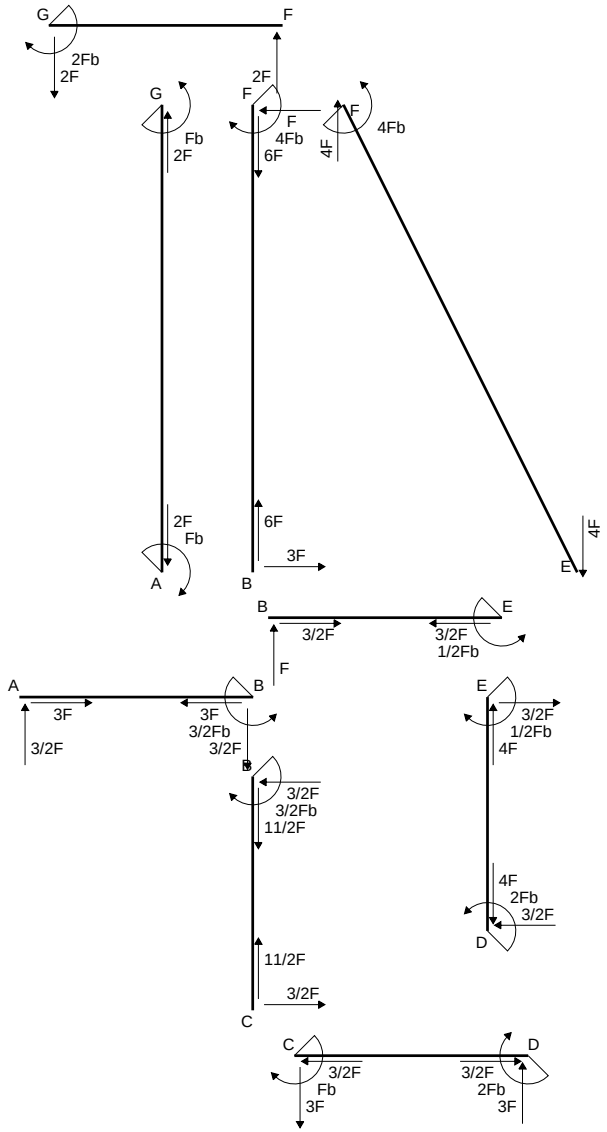
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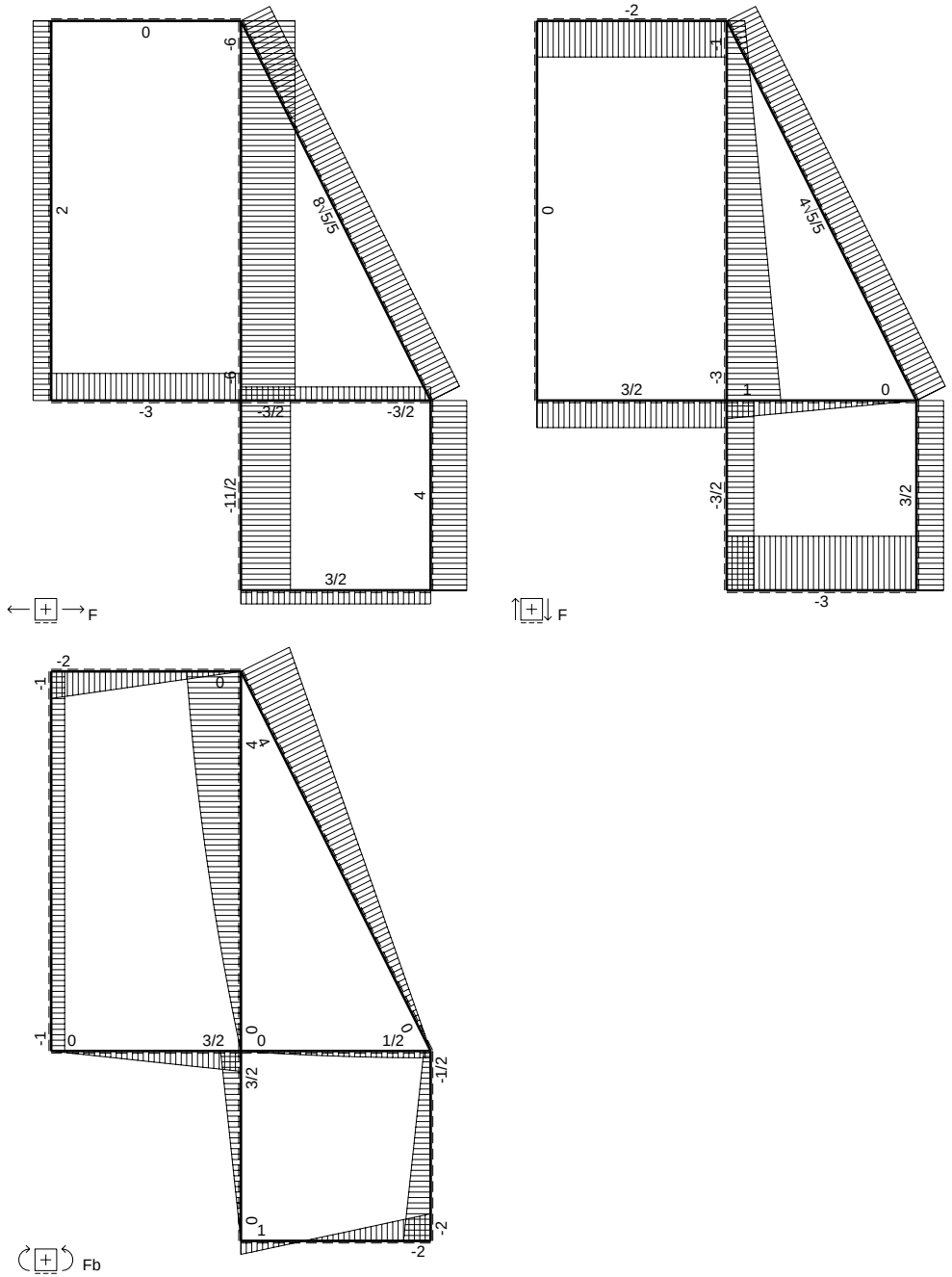
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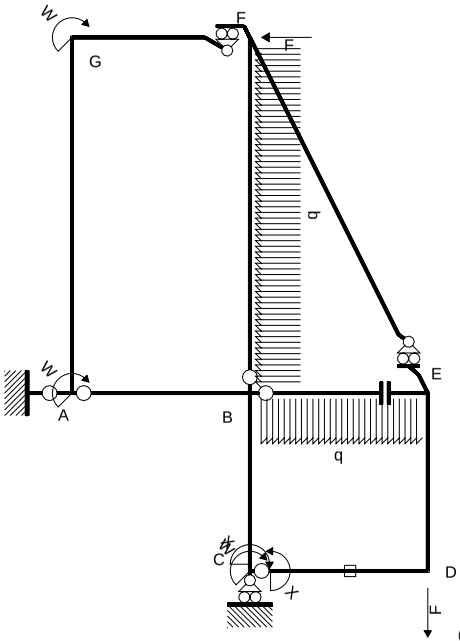
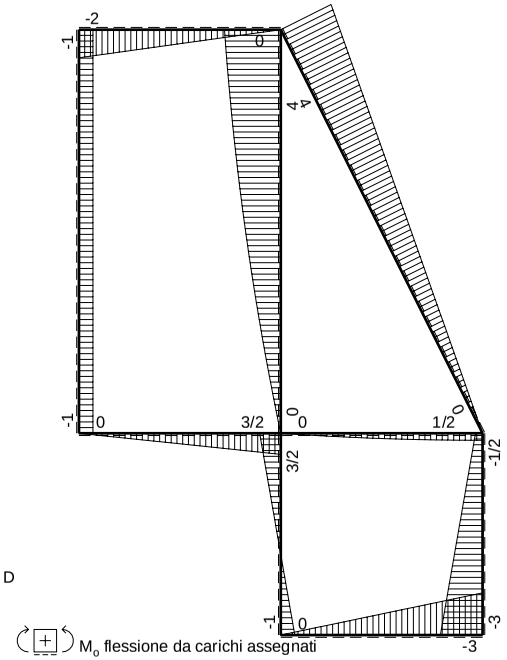
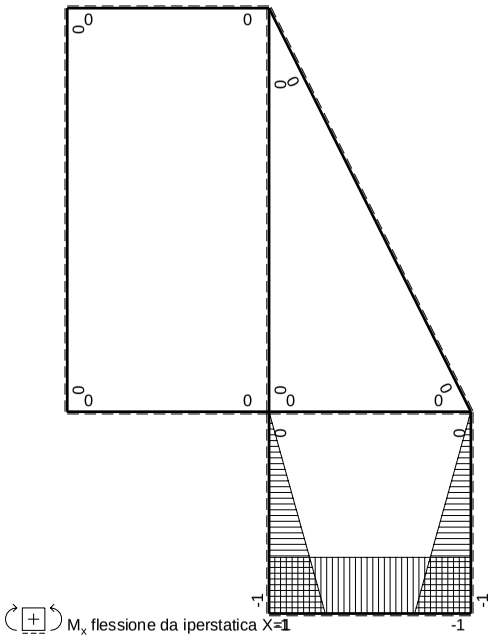


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+5/2Fx	$3Fb-11/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$13/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fb+5/2Fx	$1/2Fx+5/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -11/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -11/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -11/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 13/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 13/12\ \ Fb^2/EJ$

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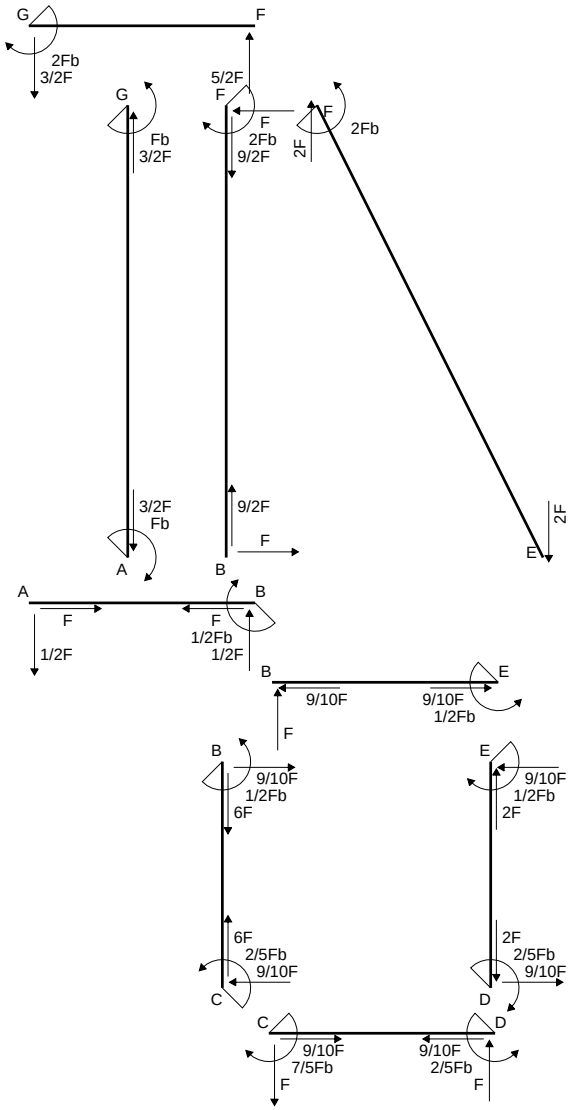
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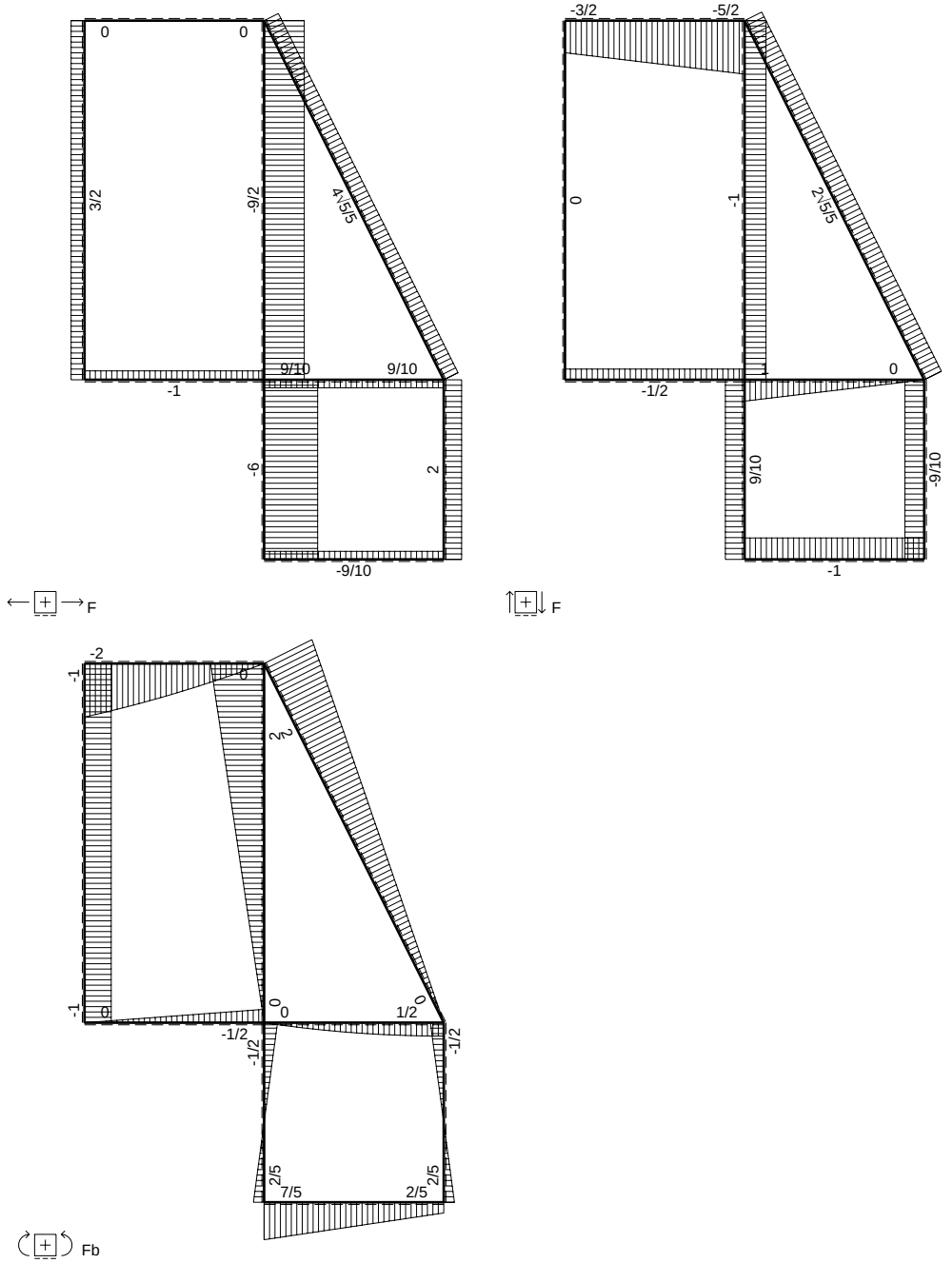
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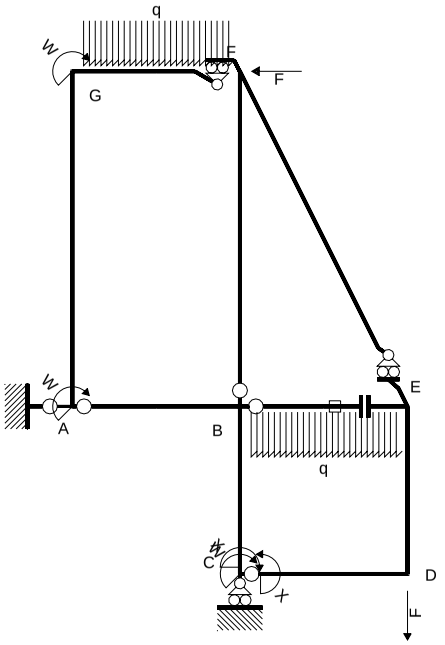
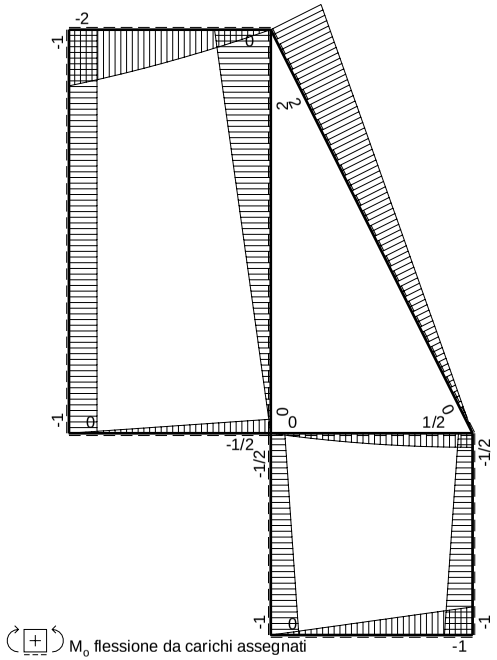
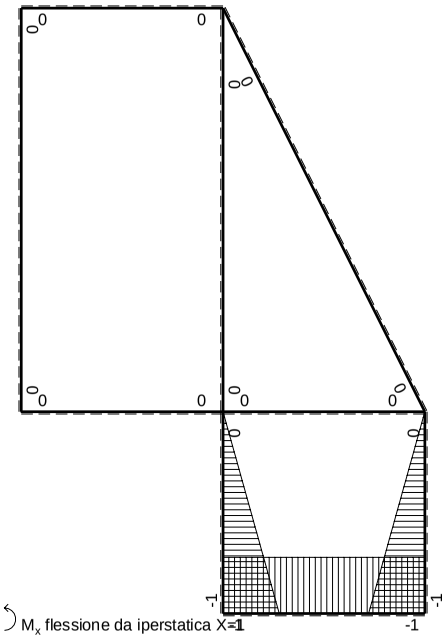


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	1/2Fb+1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-5/2Fx+1/2qx^2	0	0	0	0
GF b	0	2Fb-3/2Fx-1/2qx^2	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				7/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-7/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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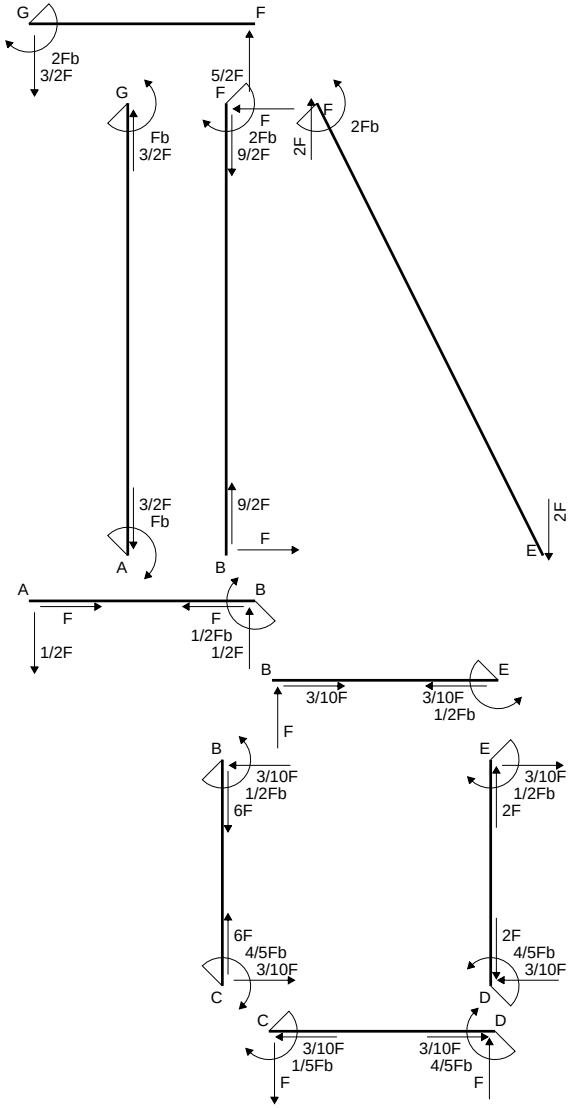
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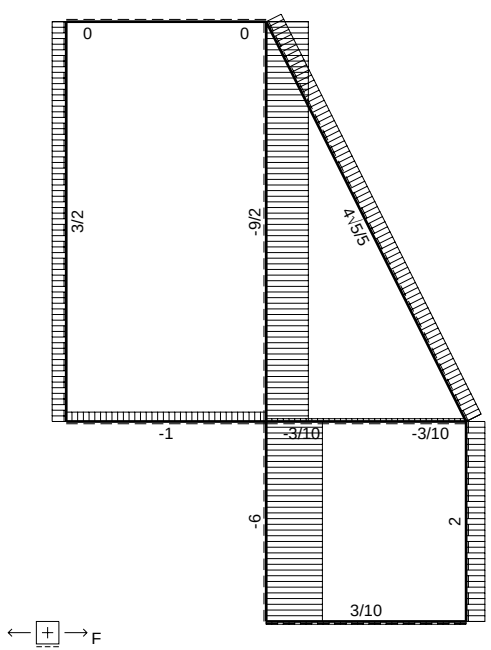
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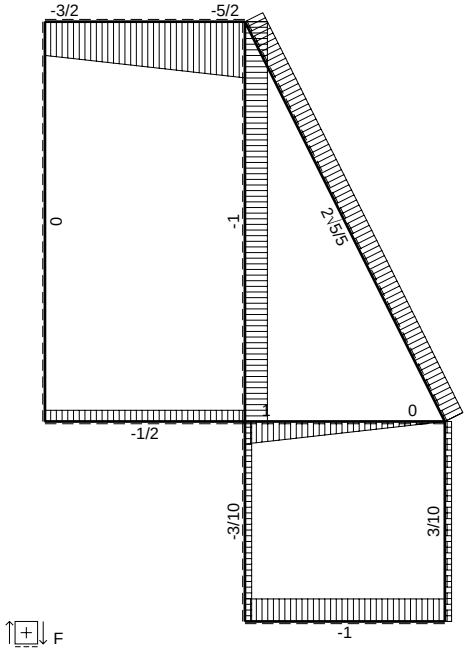
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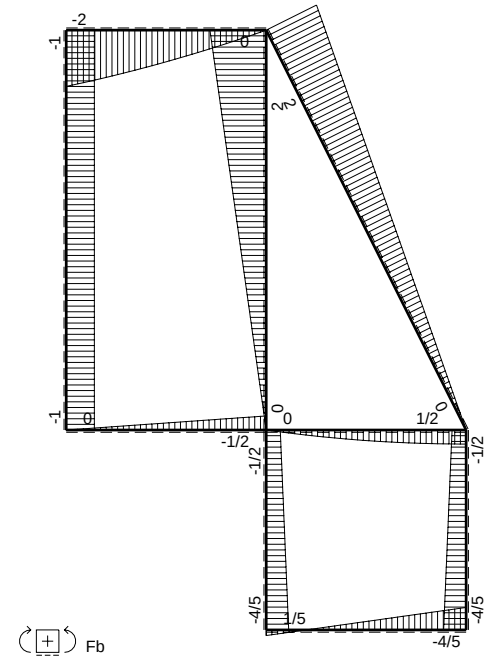
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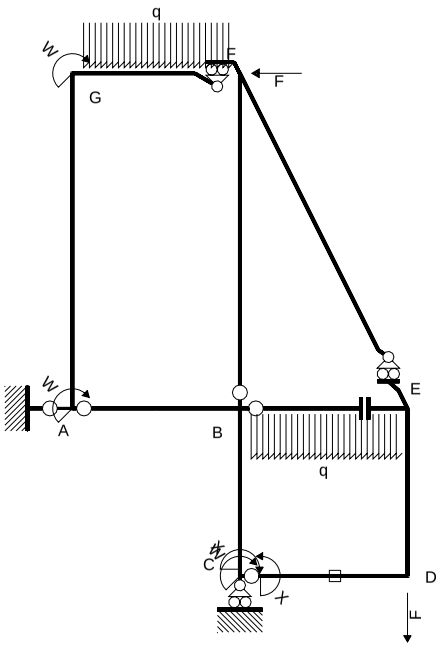
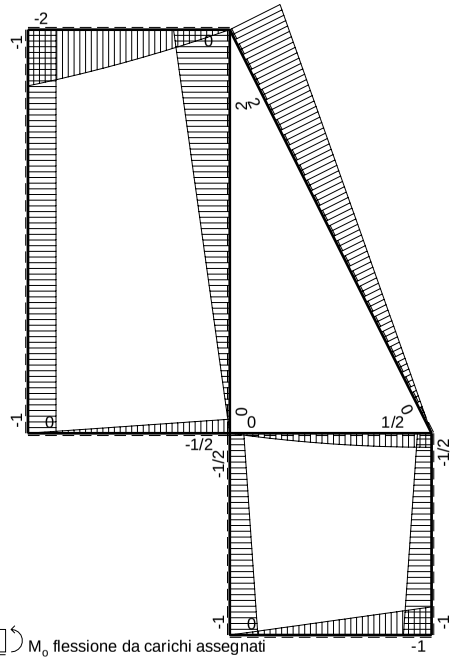
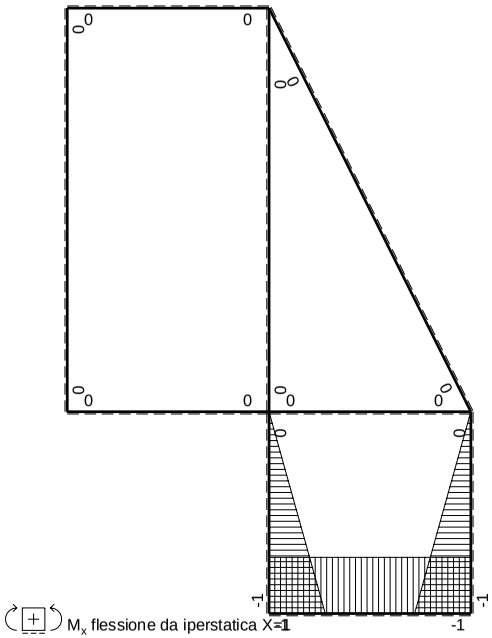


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	1/2Fb+1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-5/2Fx+1/2qx^2	0	0	0	0
GF b	0	2Fb-3/2Fx-1/2qx^2	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				1/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-1/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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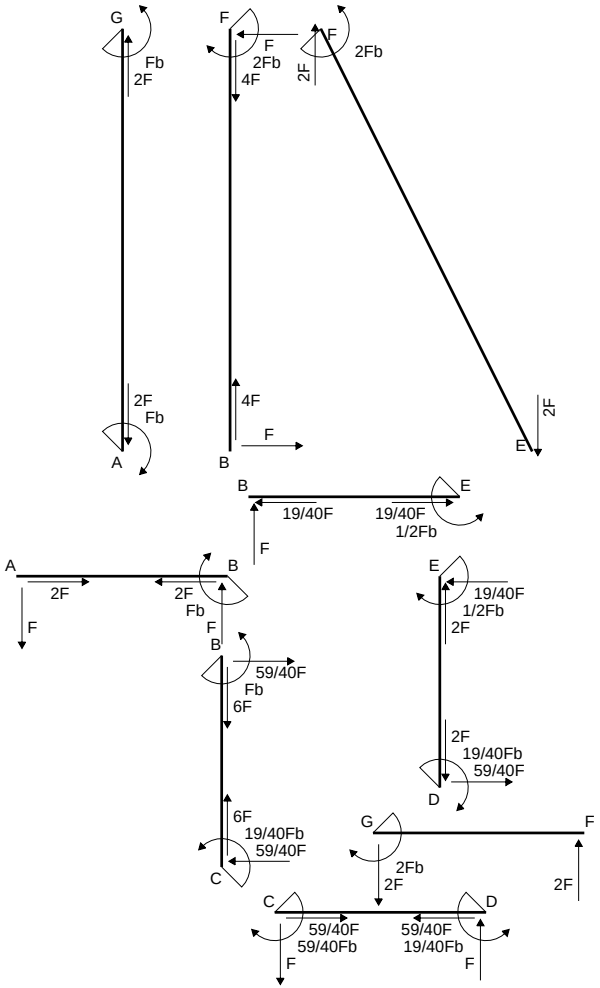
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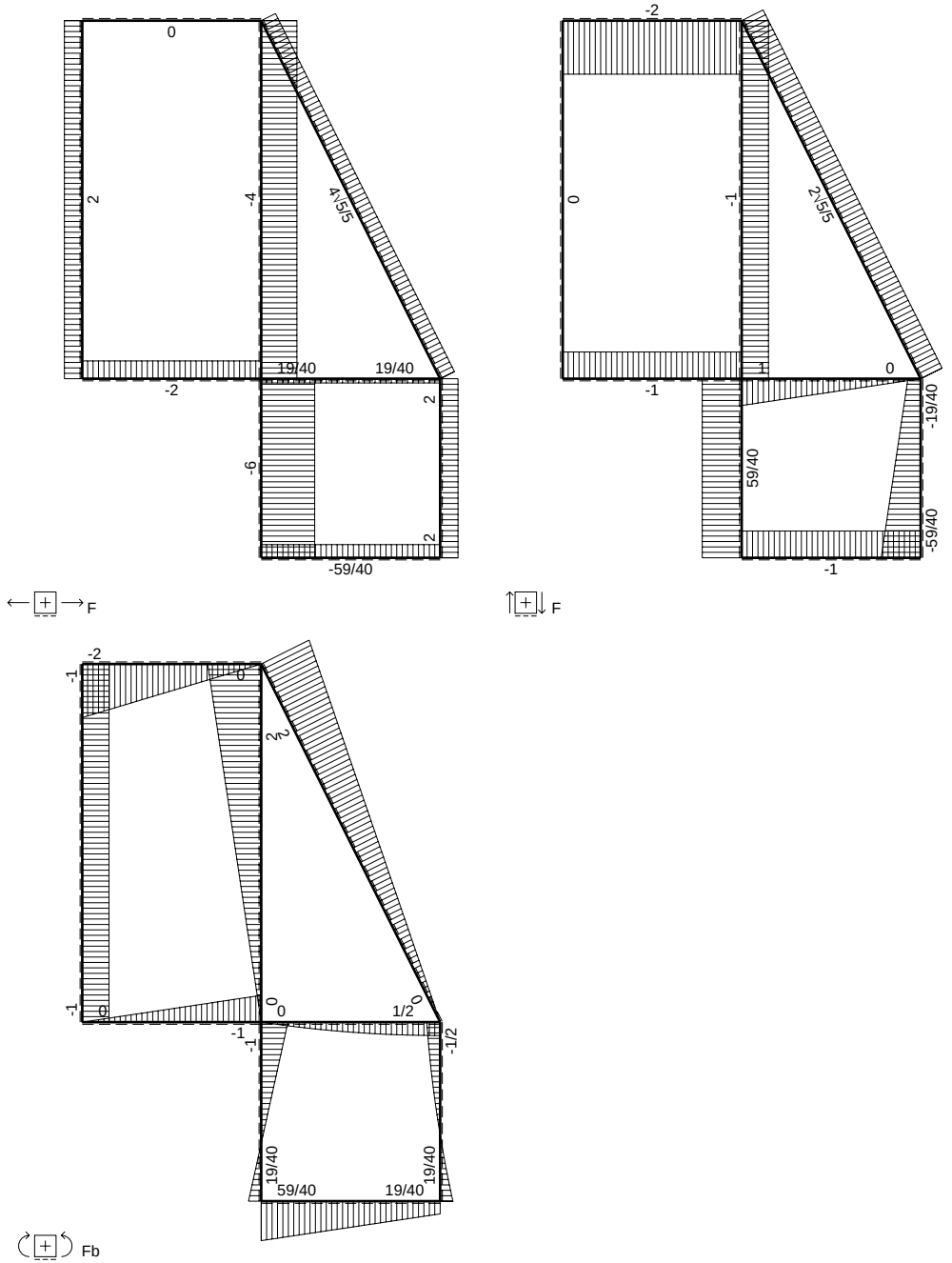
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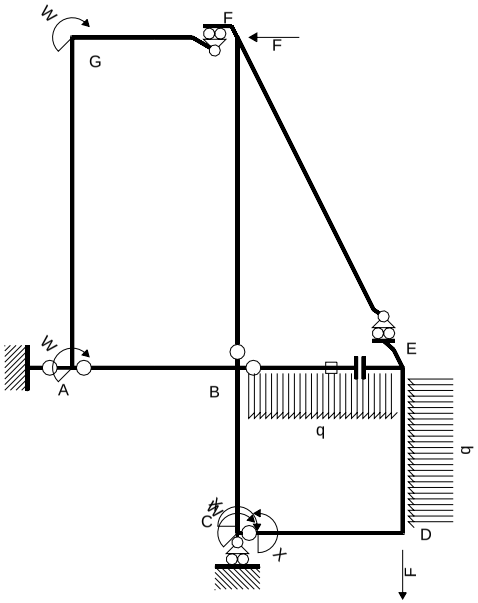
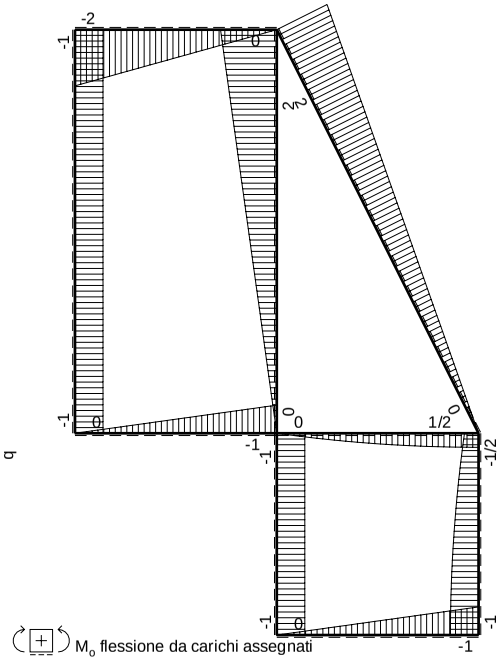
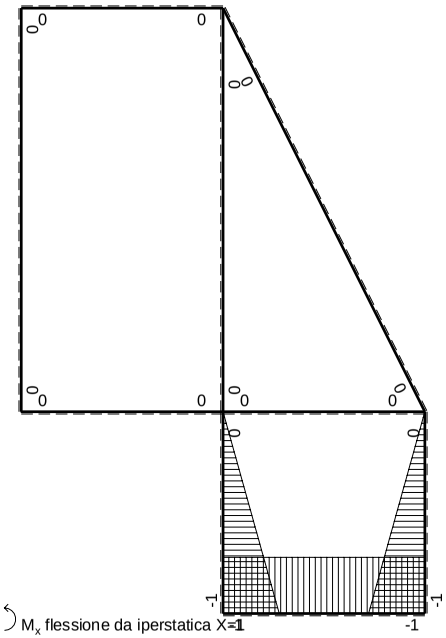


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb+1/2qx^2$	$Fb-Fx-1/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$11/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$1/2Fb+Fx-1/2qx^2$	$1/2Fx+Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$59/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-59/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 - x/b -1/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b -1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b -1/6\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 11/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b + x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/3\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 11/24\ Fb^2/EJ$

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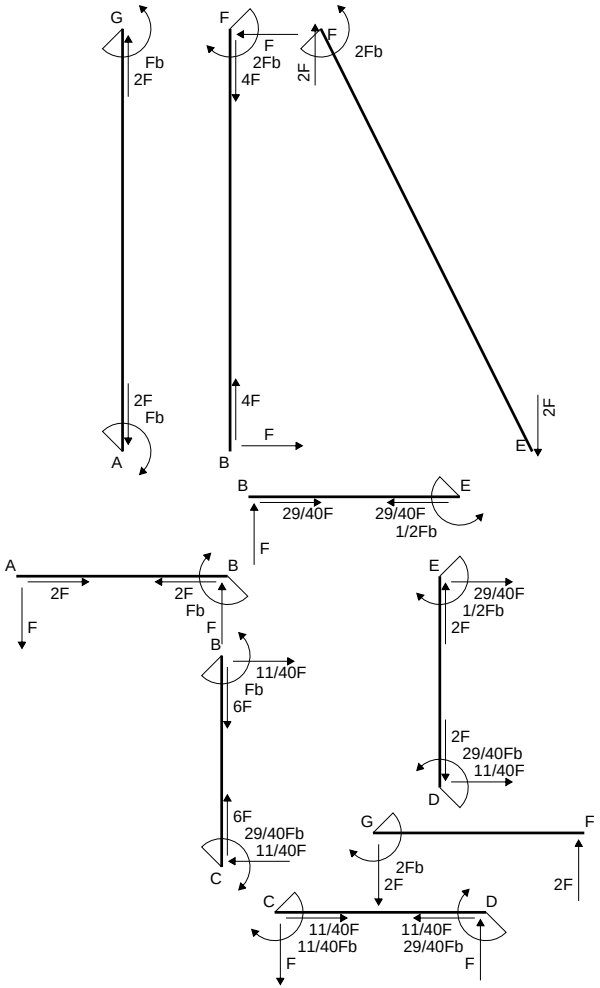
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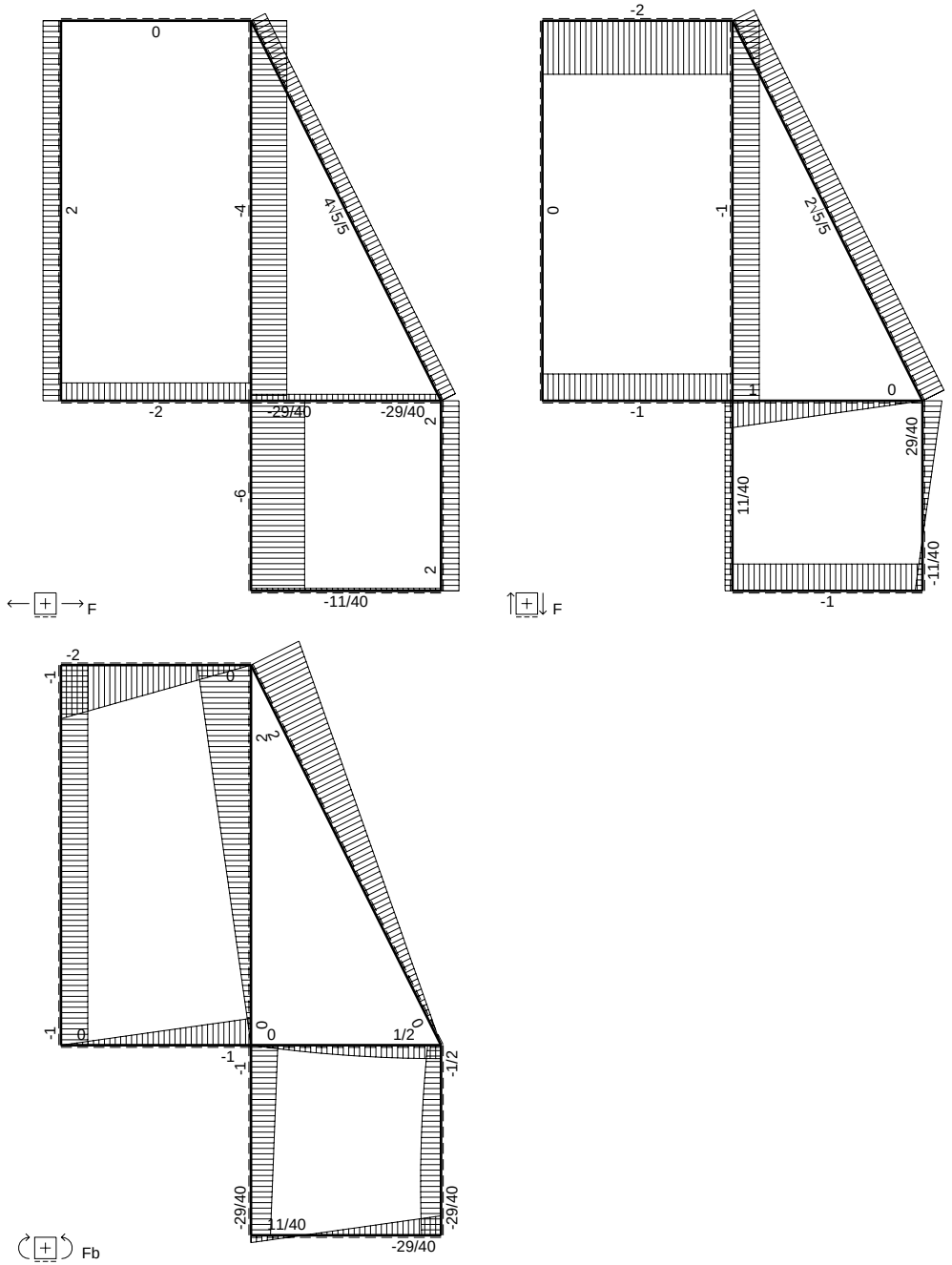
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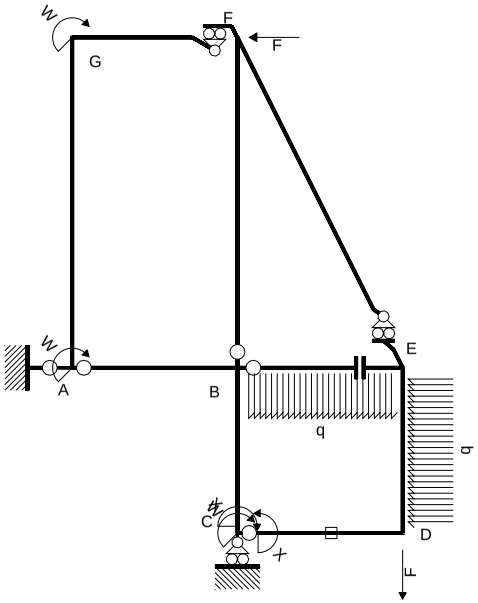
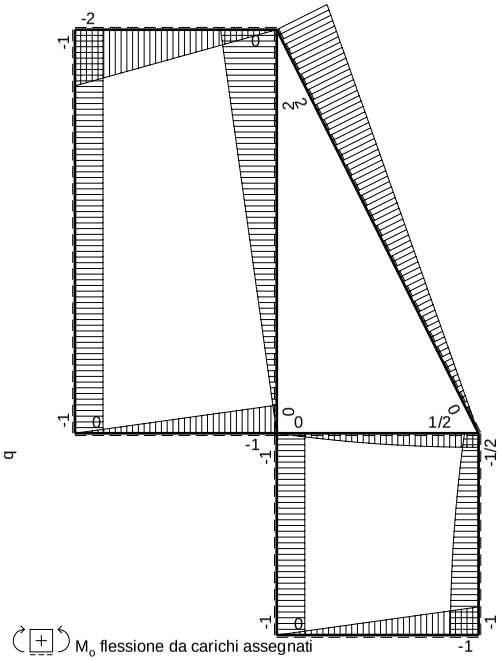
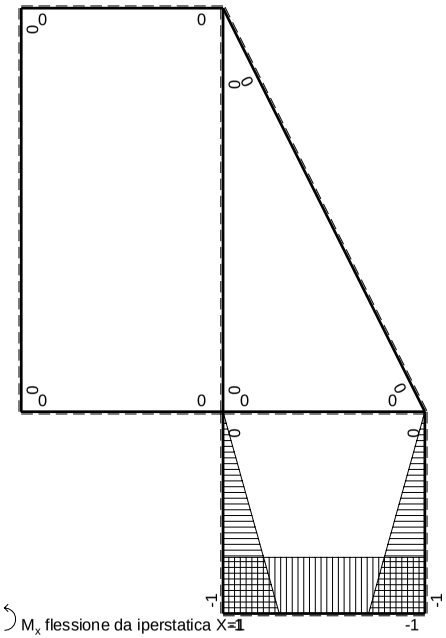


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb+1/2qx^2$	$Fb-Fx-1/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$11/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$1/2Fb+Fx-1/2qx^2$	$1/2Fx+Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$11/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-11/40Fb$	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 - x/b -1/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b -1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b -1/6\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b + x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/3\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 11/24\ \ Fb^2/EJ$

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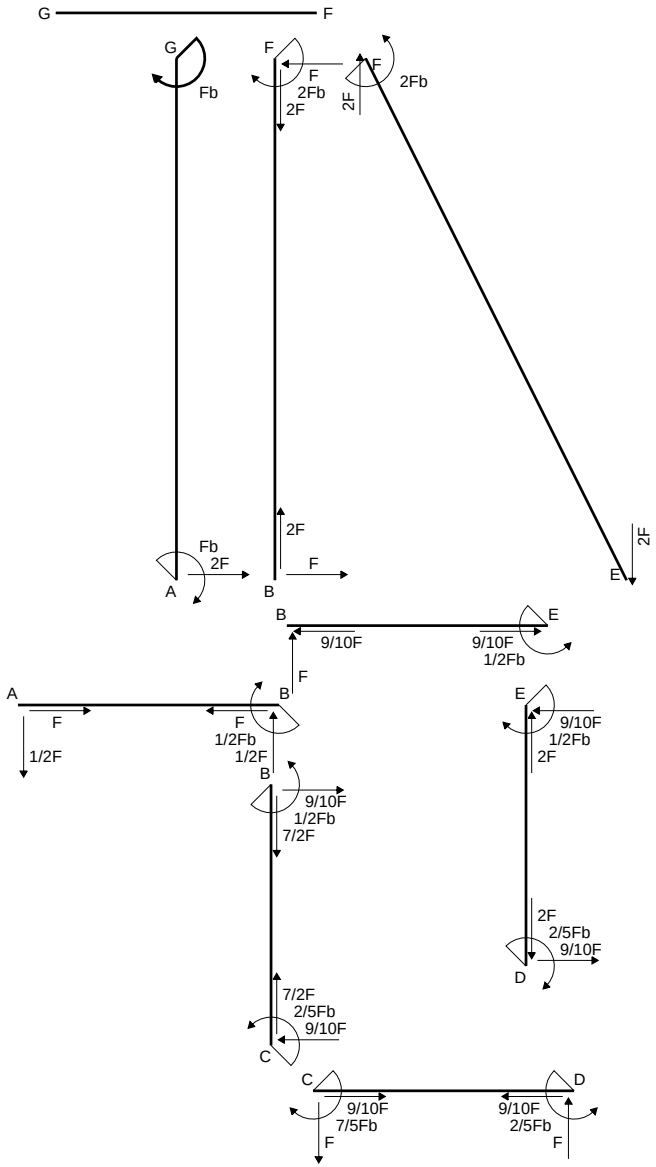
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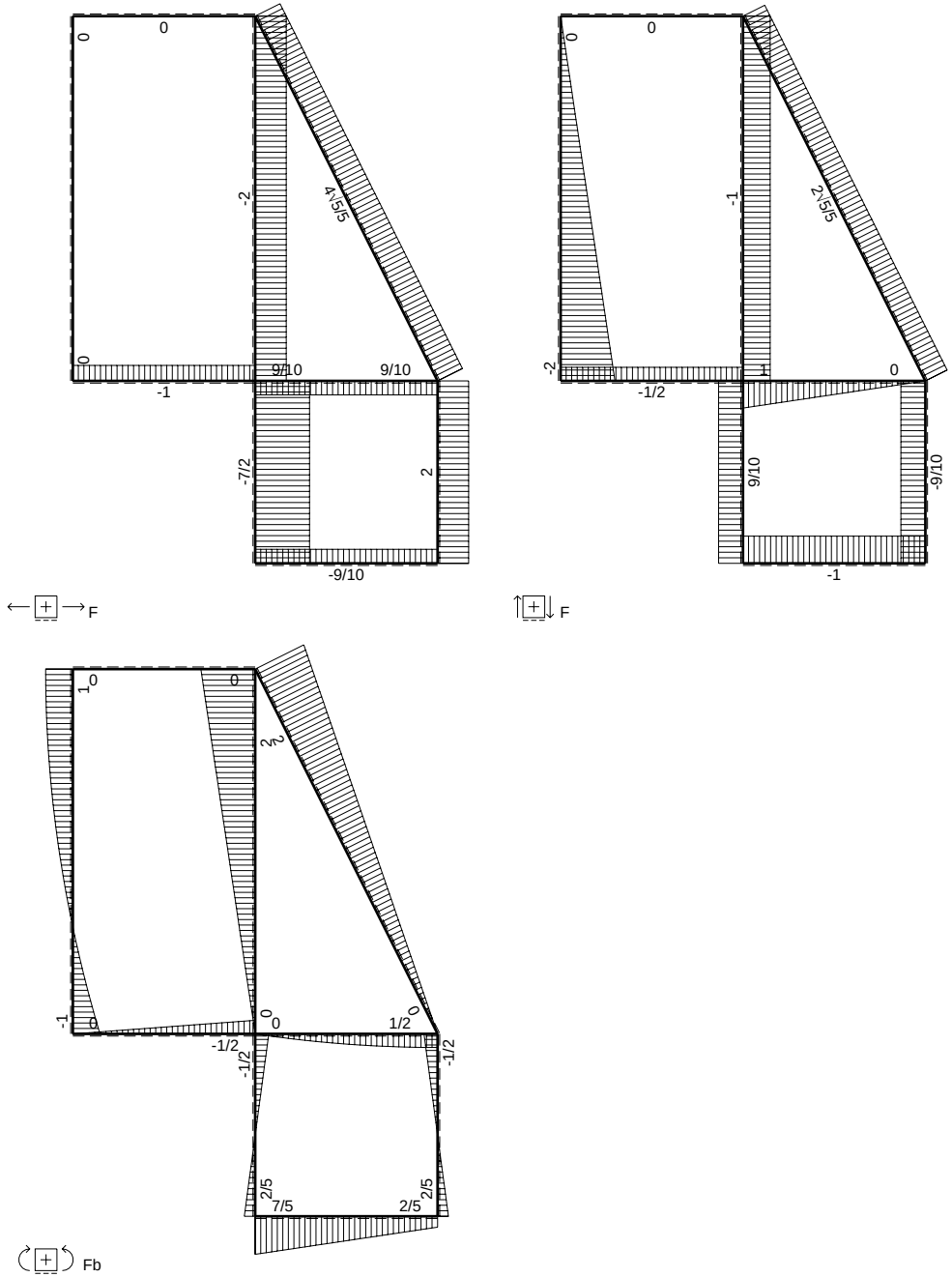
F
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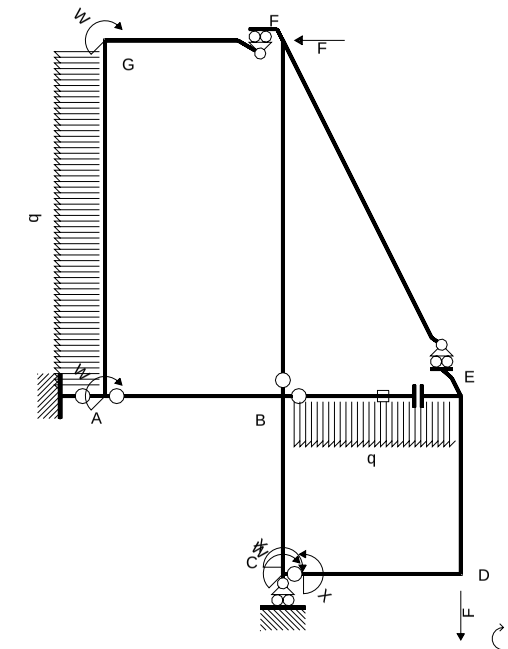
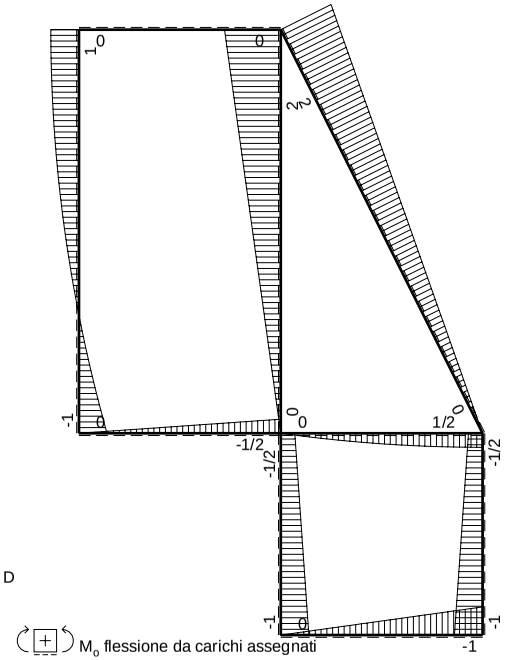
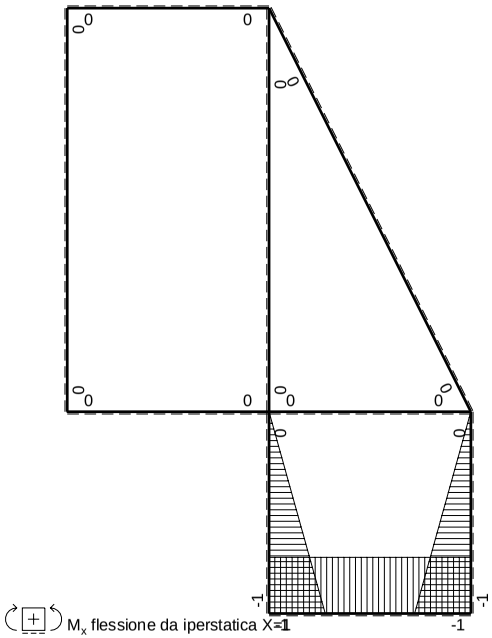


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	1/2Fb+1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	Fb-1/2qx^2	0	0	0	0
AG 2b	0	Fb-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				7/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-7/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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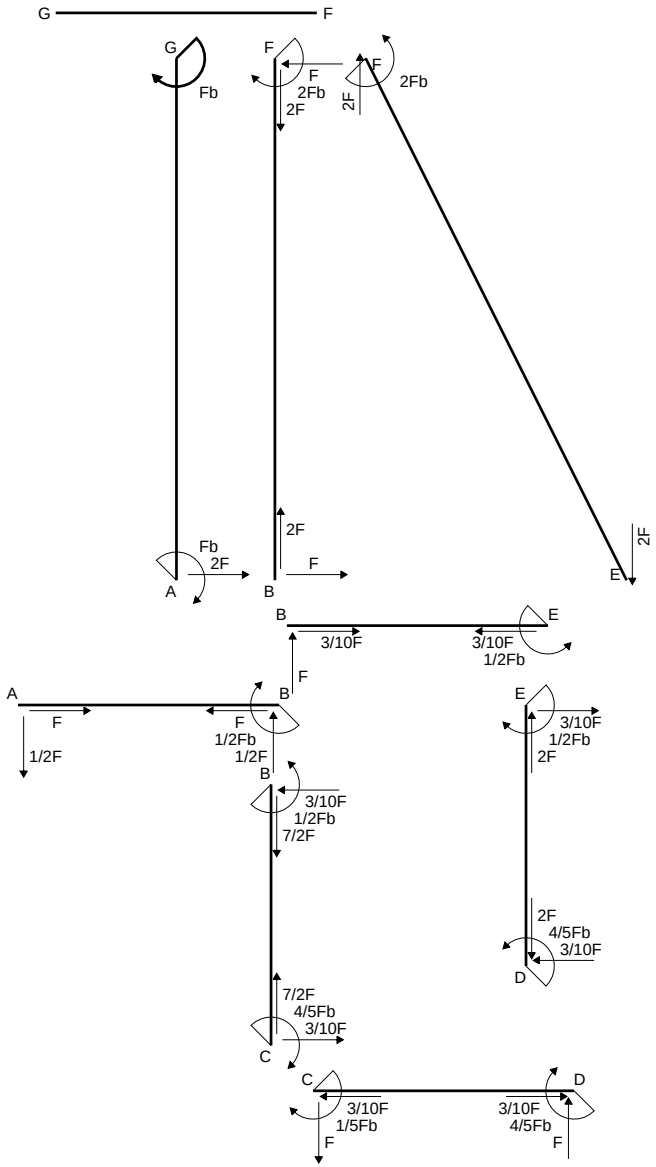
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B
E
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1/2F
7/2F
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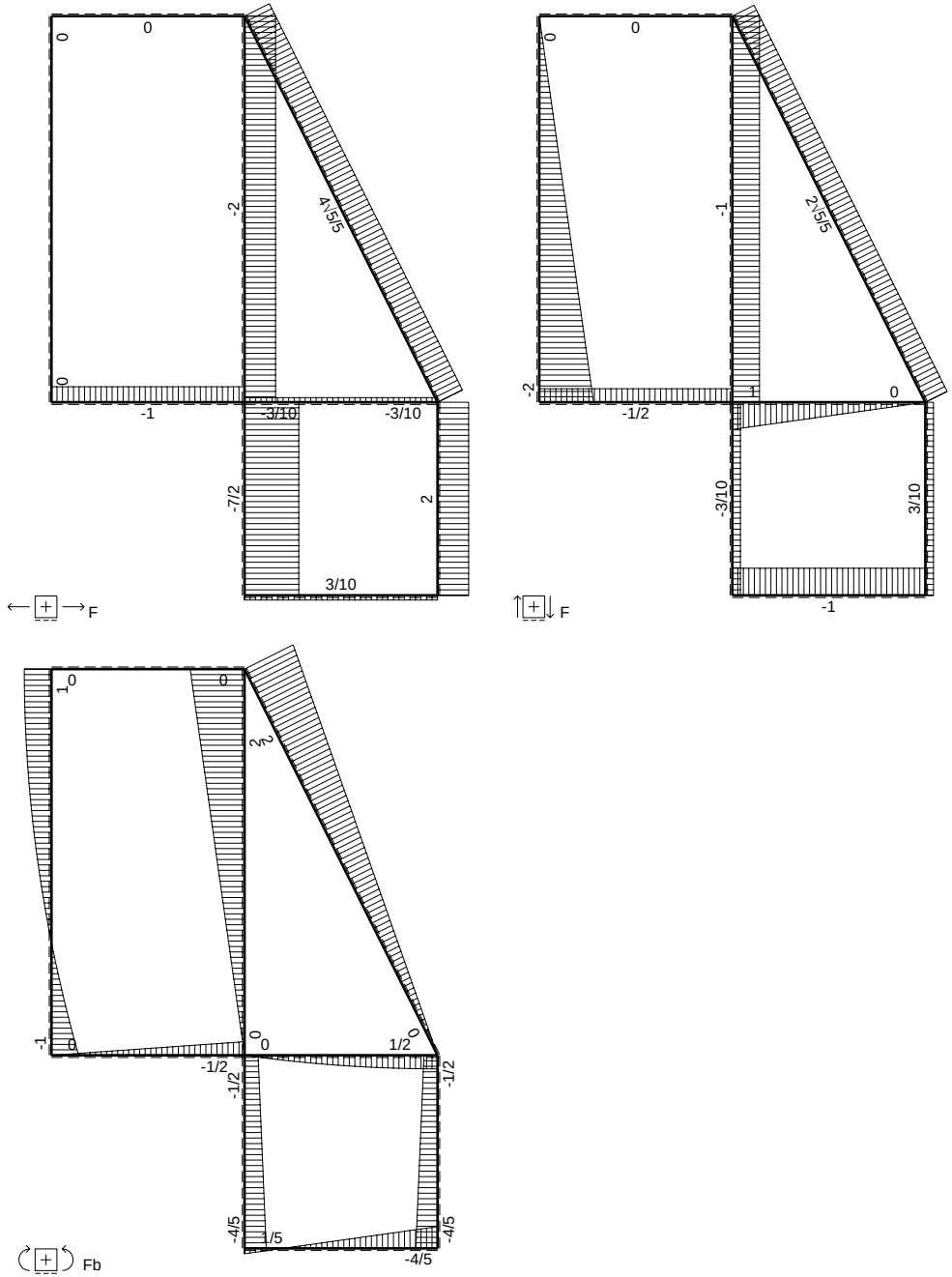
C
D
1/5Fb


F
Fb
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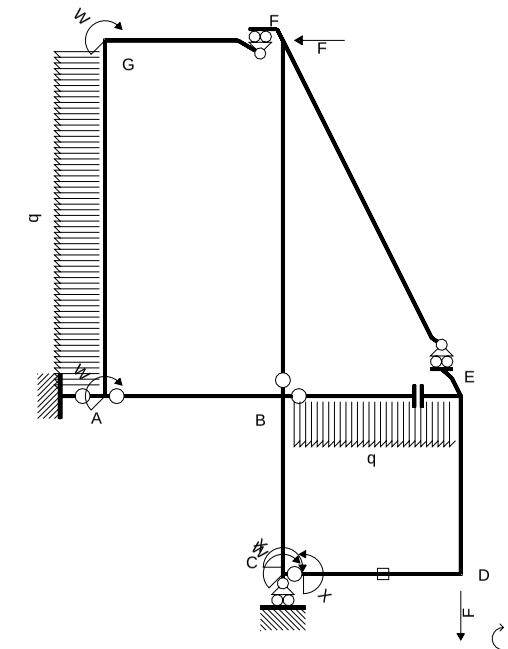
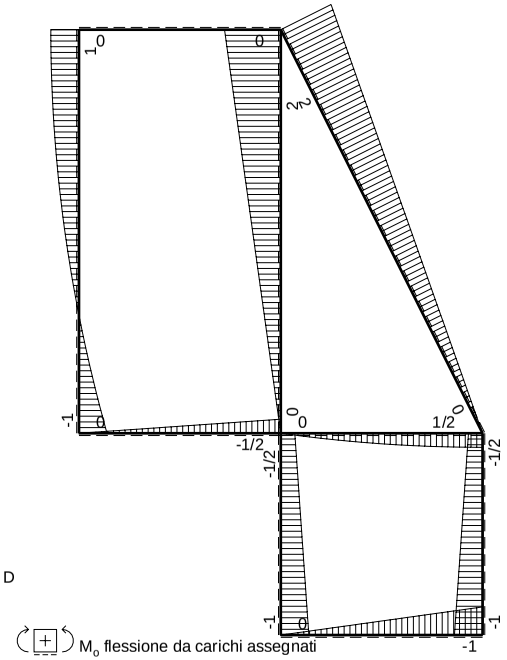
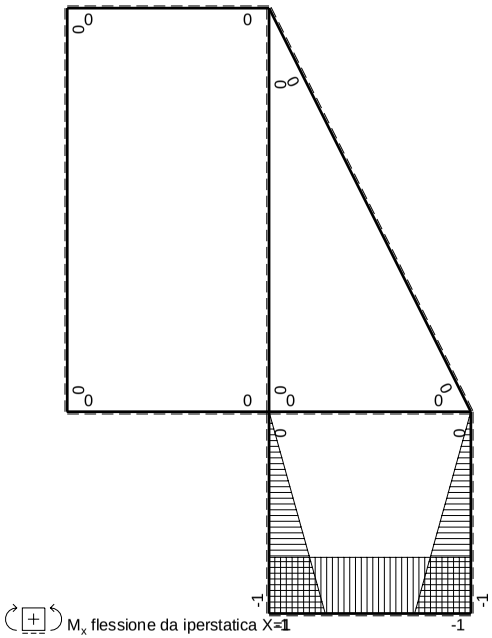
1/5
1/2



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb+1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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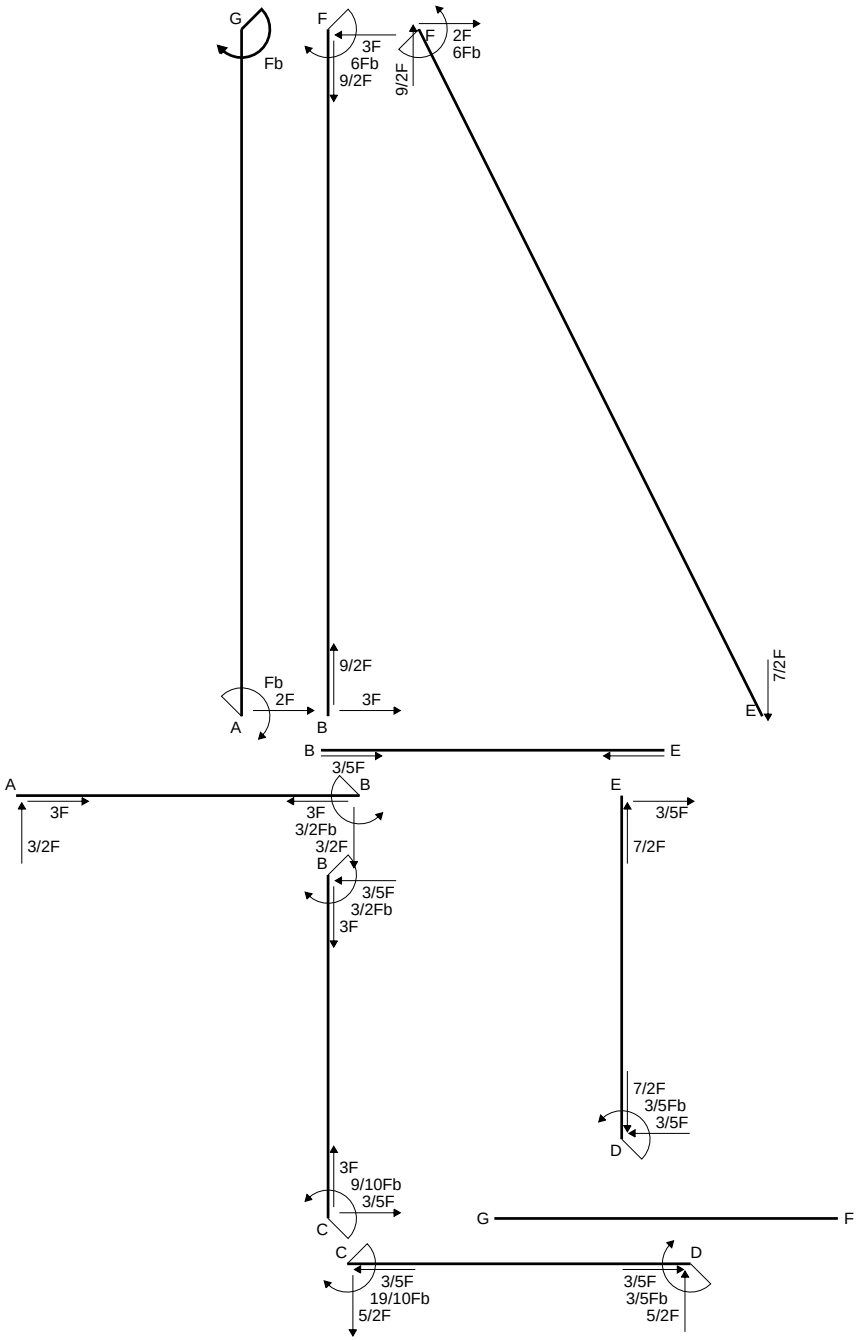
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Fb
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6Fb
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9/2F
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A
B
3F
B
E
3/5F
A
B
3F
3/2F
3F
3/2Fb
3/2F
E
3/5F
7/2F
B
3/5F
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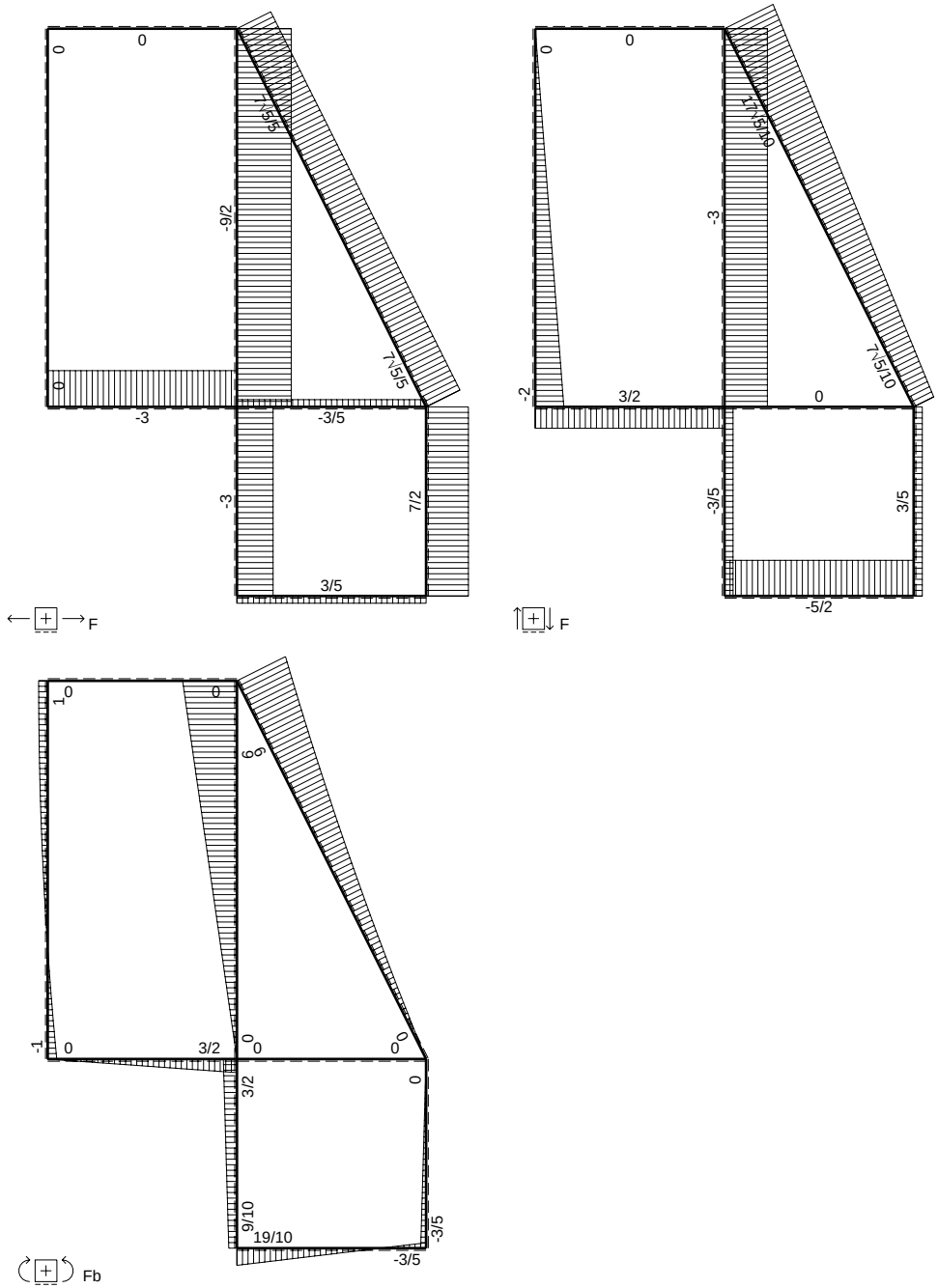
5/2F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-2
3/2
0
-3/5
3/5
-5/2
F
1
0
0
9
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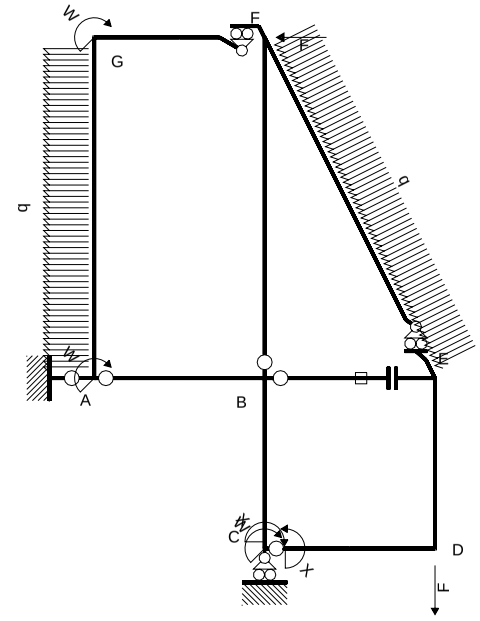
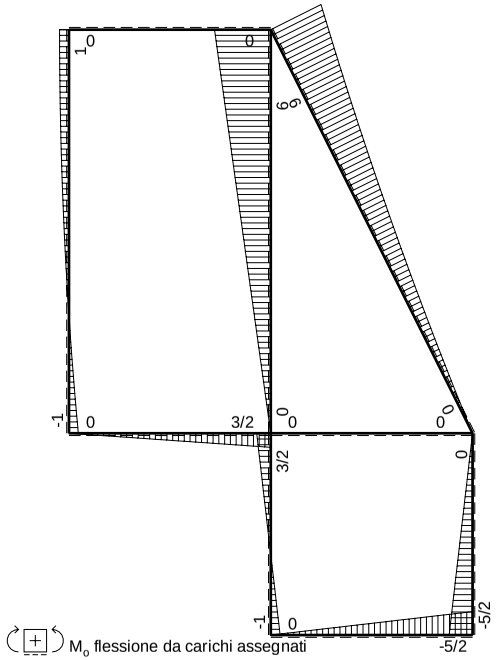
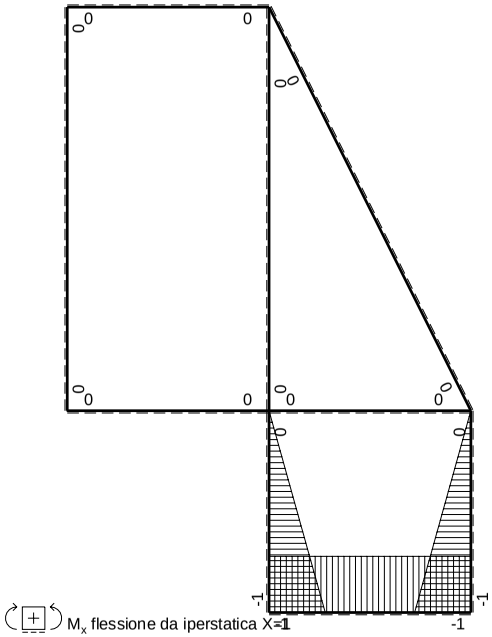
3/2
0
0
0
3/2
0
9/10
19/10
-3/5
-3/5
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$19/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-19/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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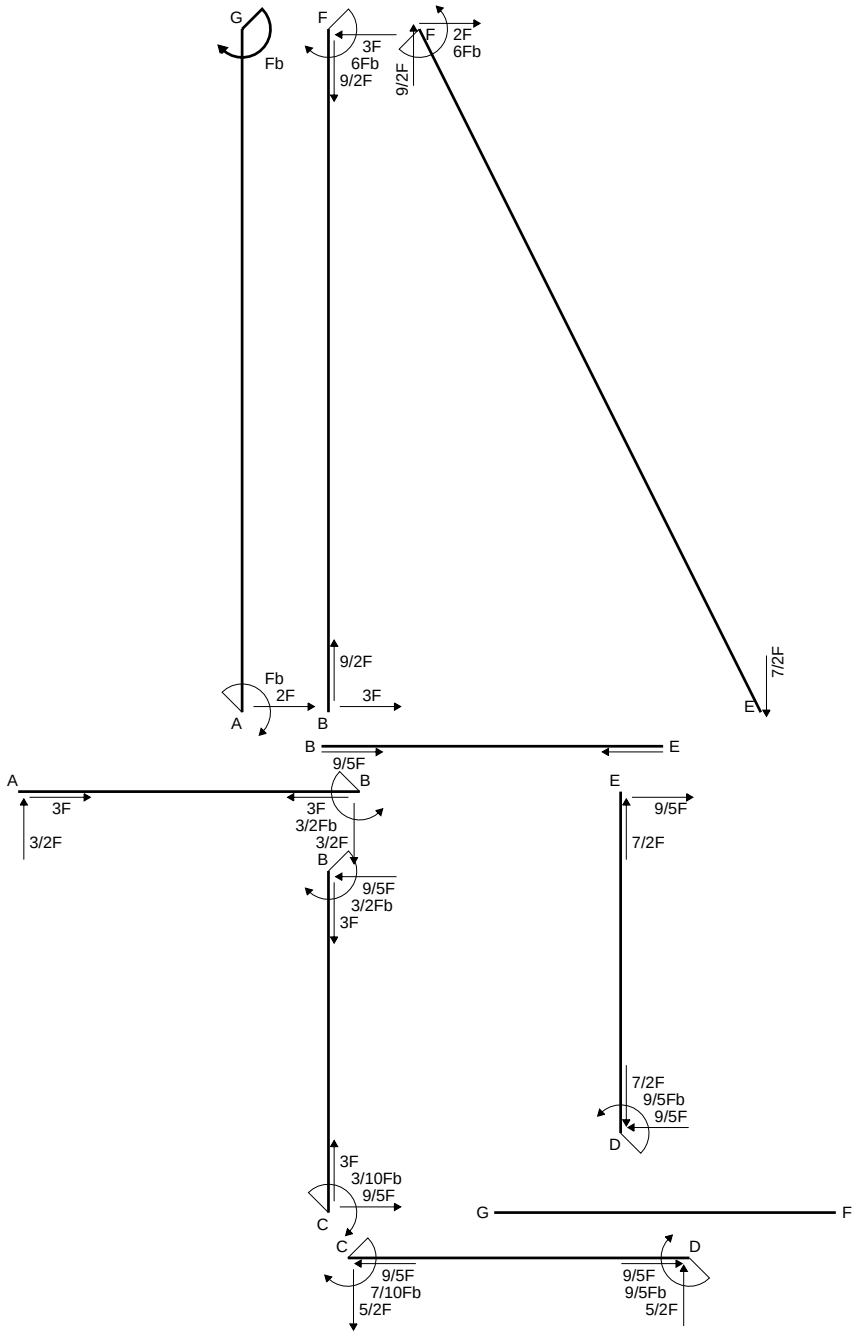
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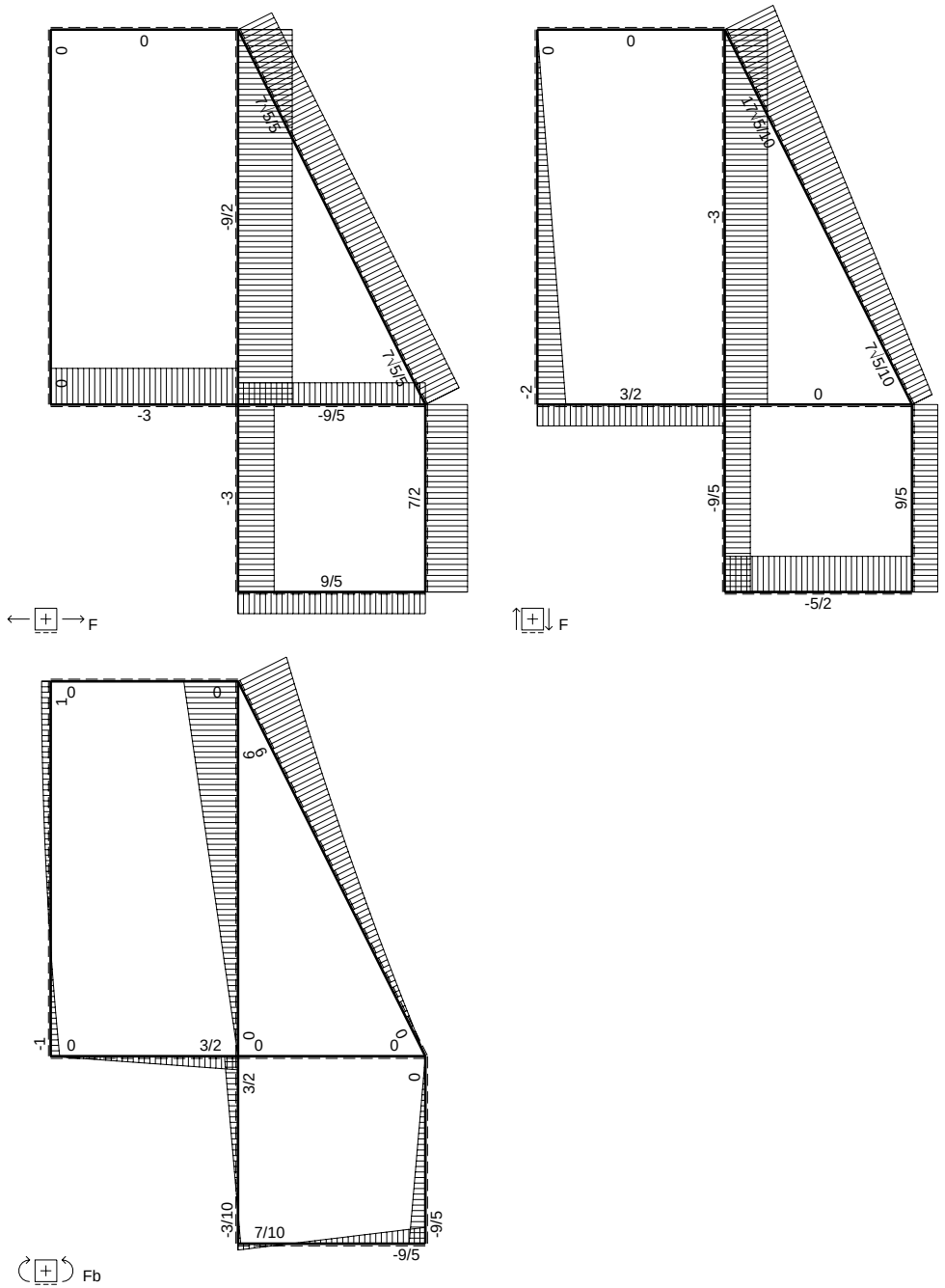
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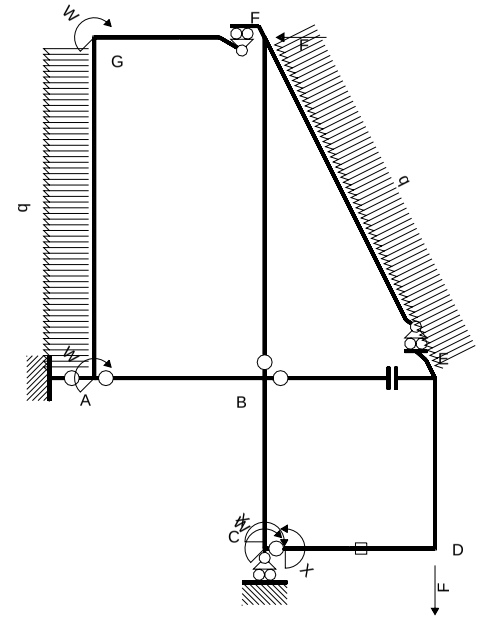
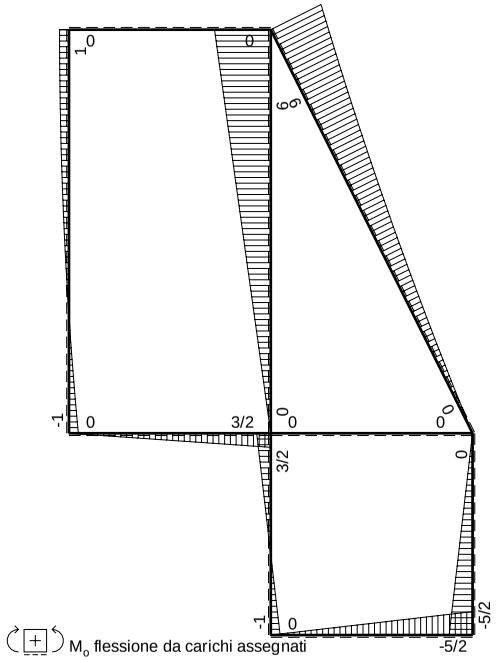
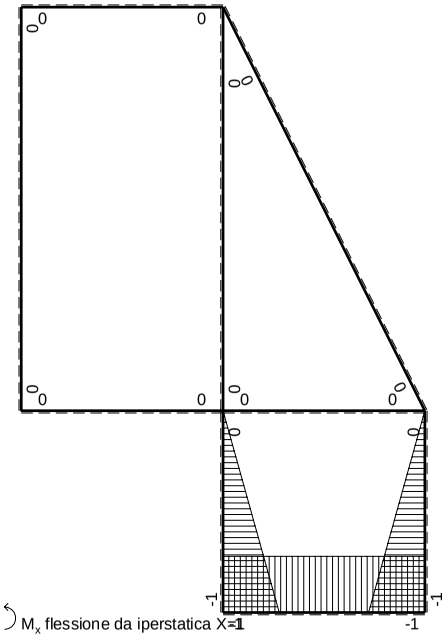


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$7/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-7/10Fb$	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

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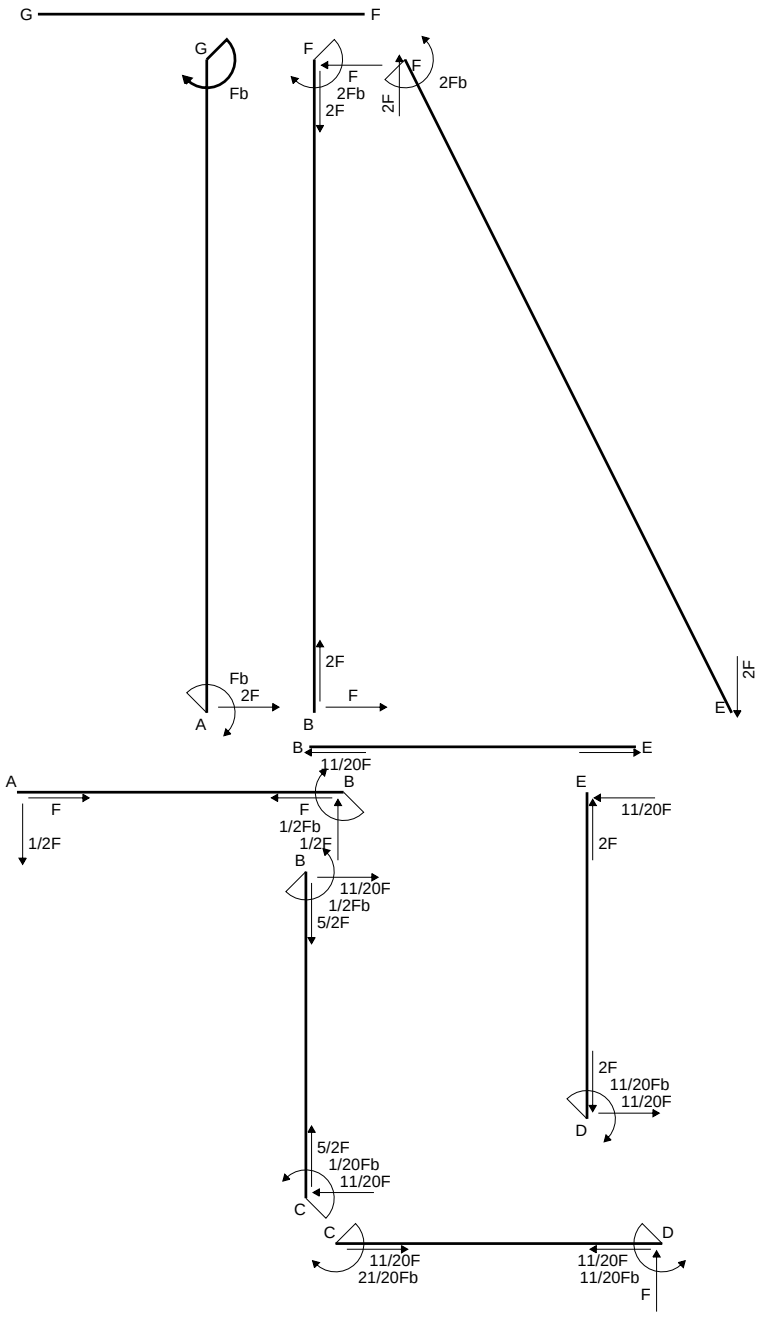
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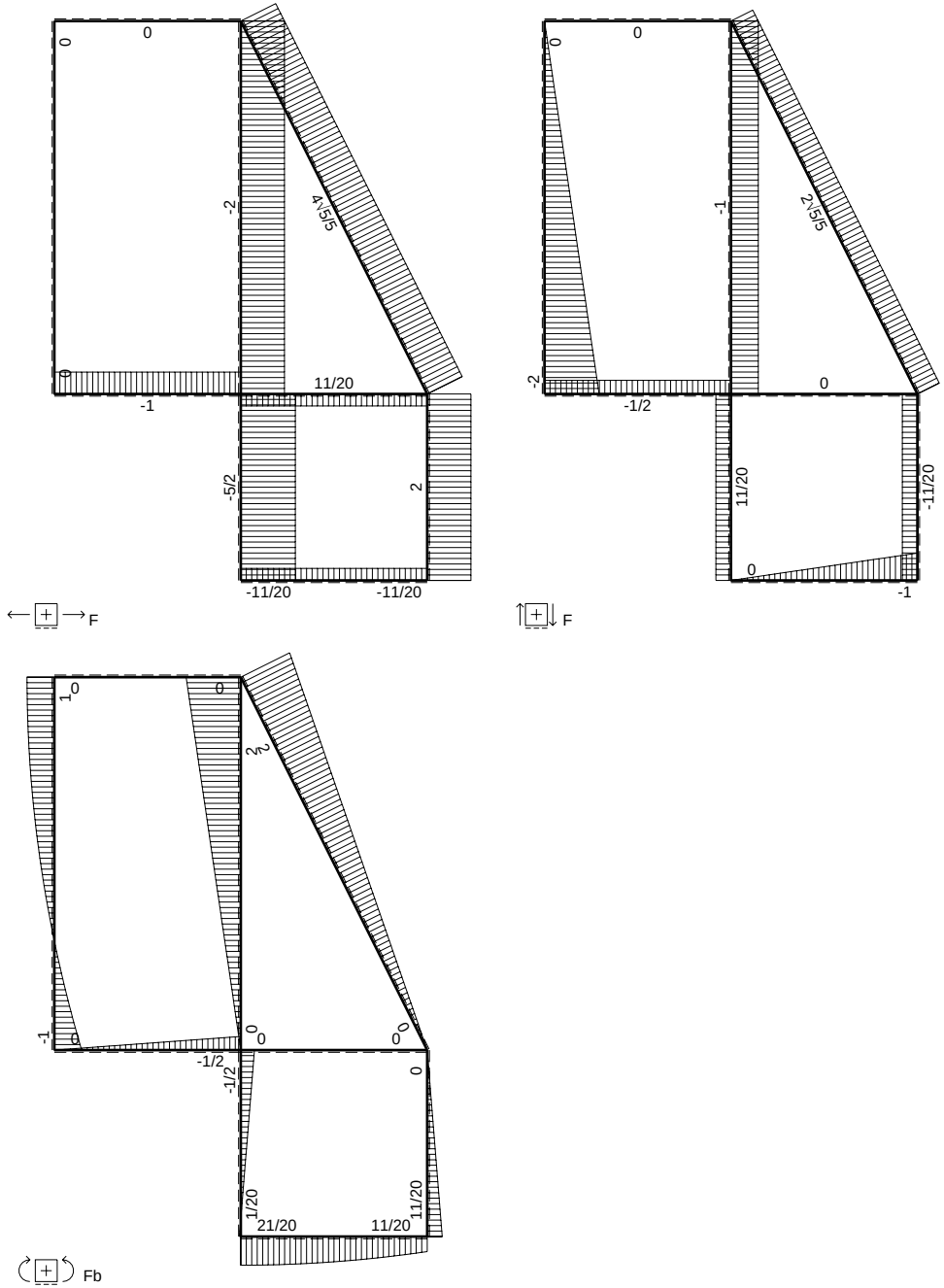
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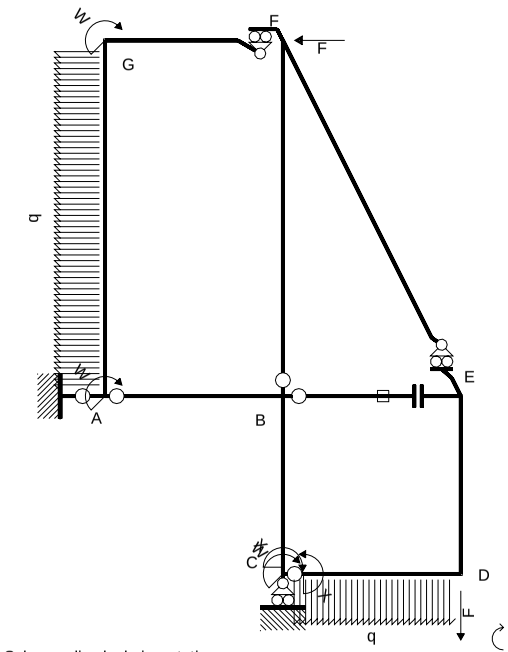
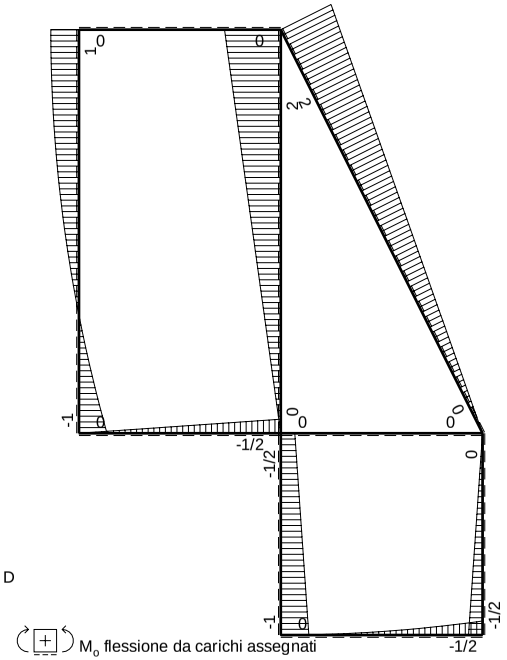
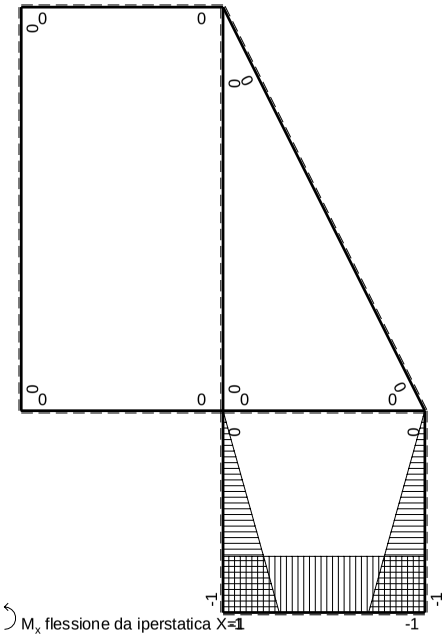


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-21/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

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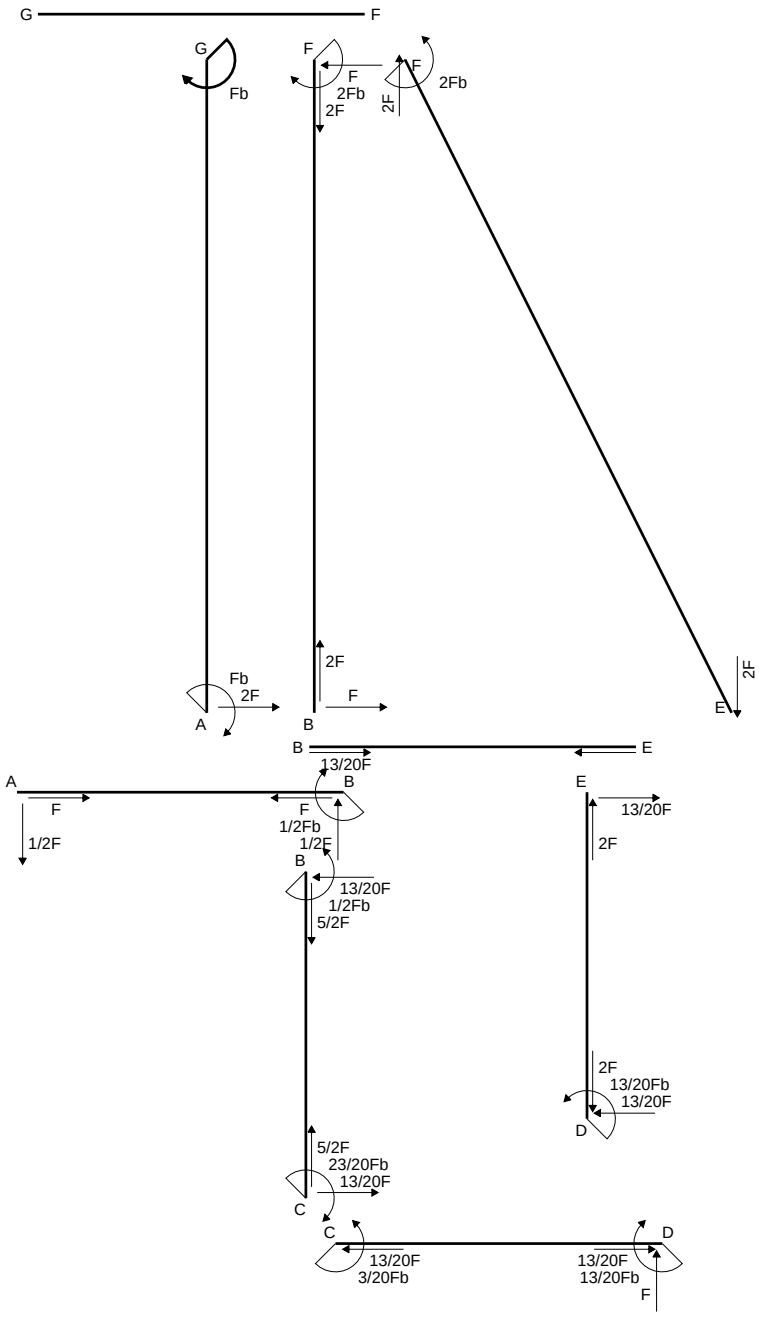
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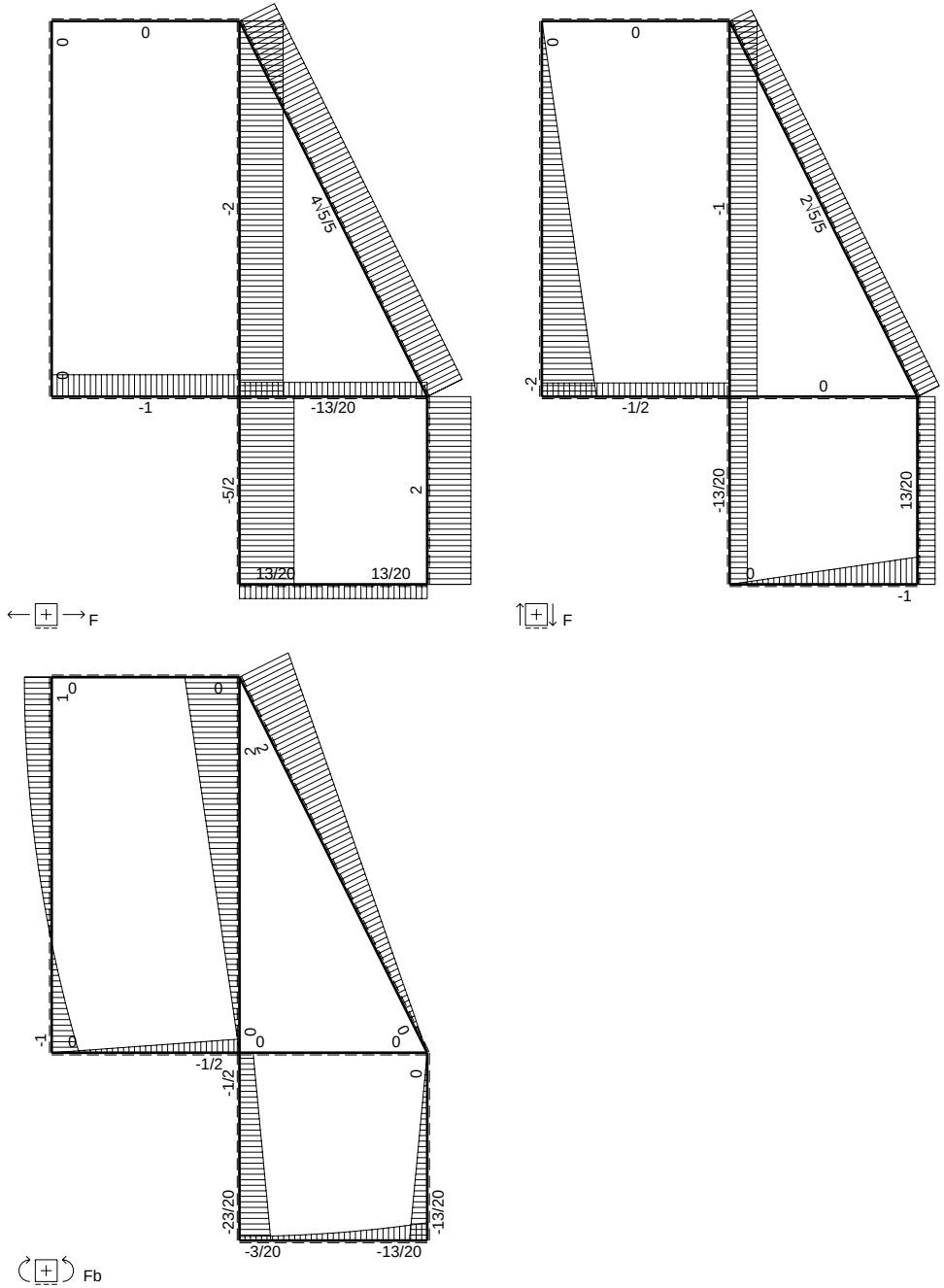
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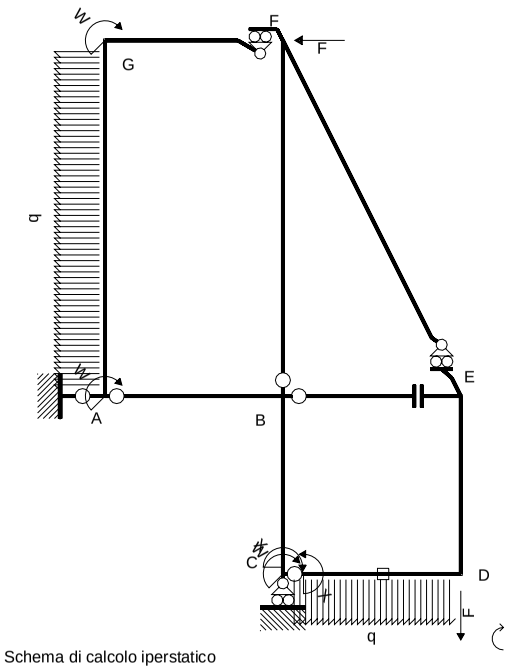
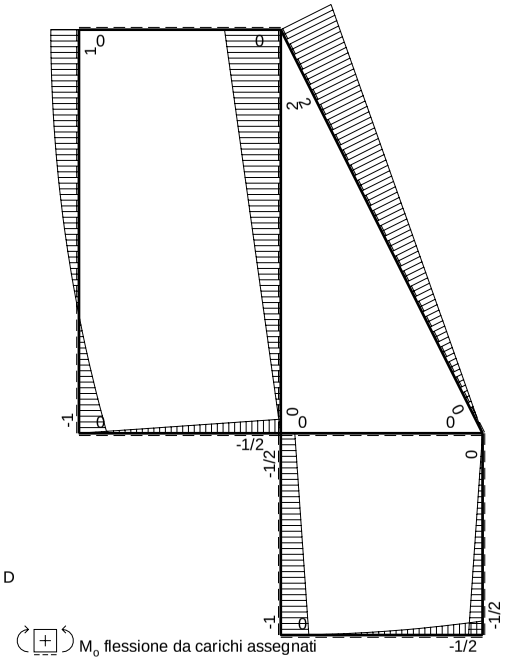
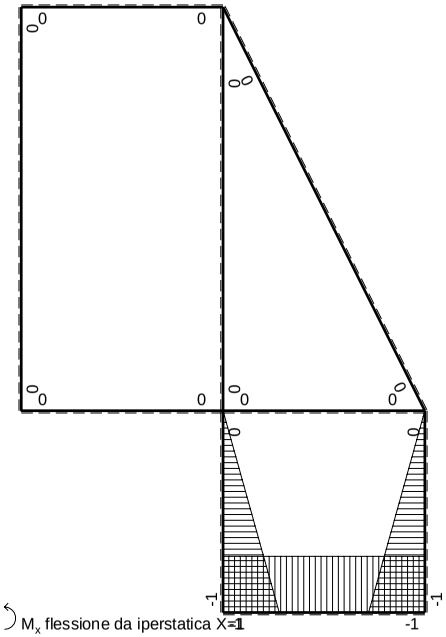


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-1/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				3/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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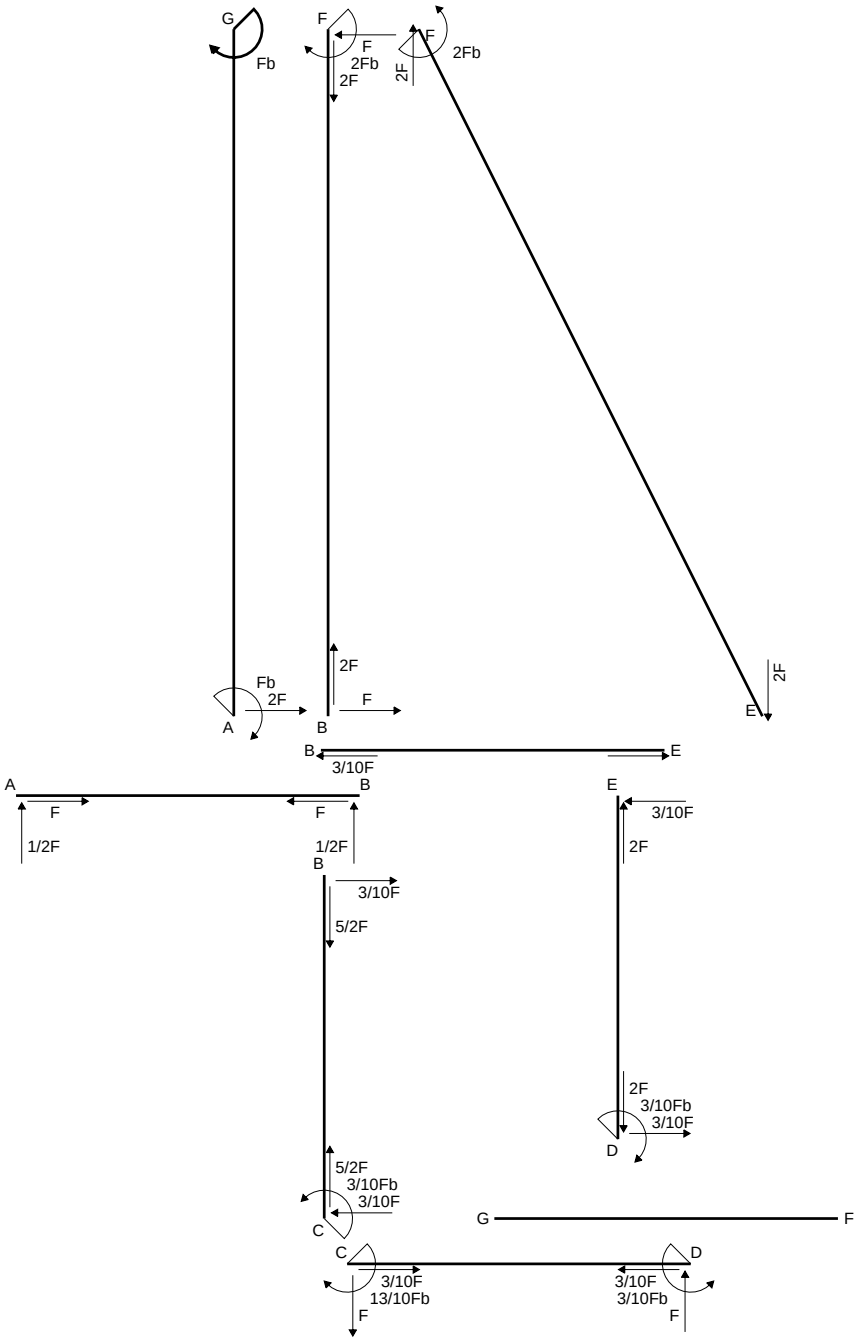
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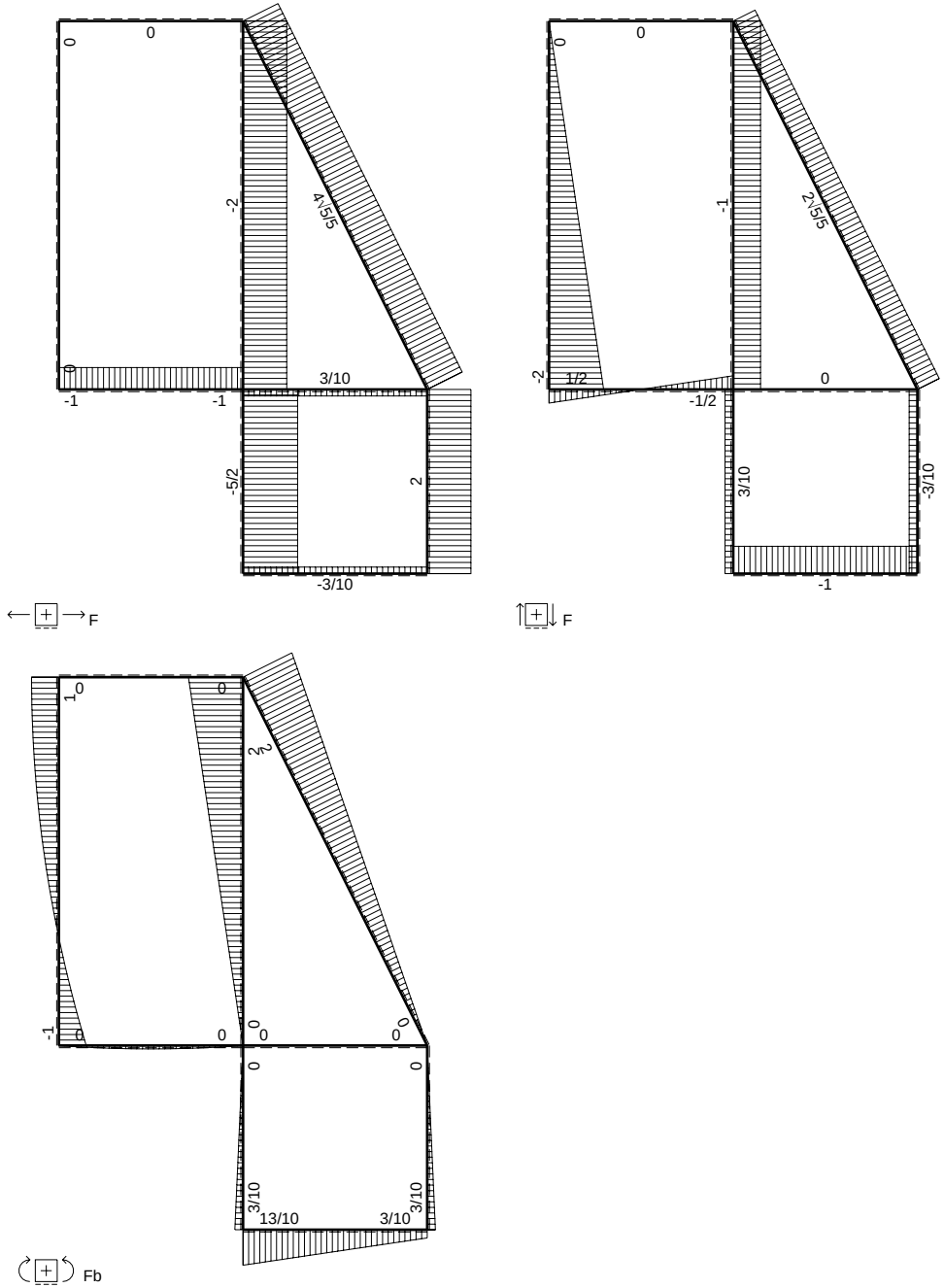
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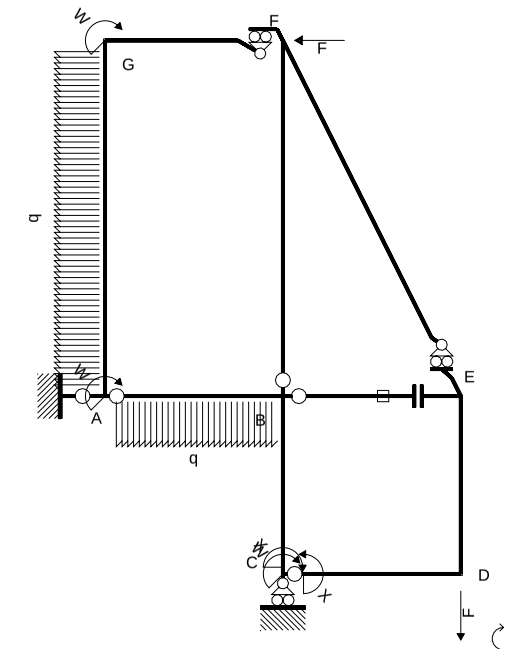
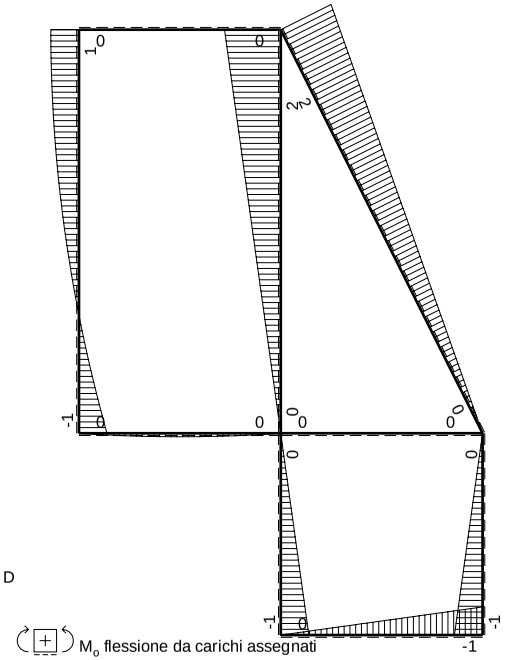
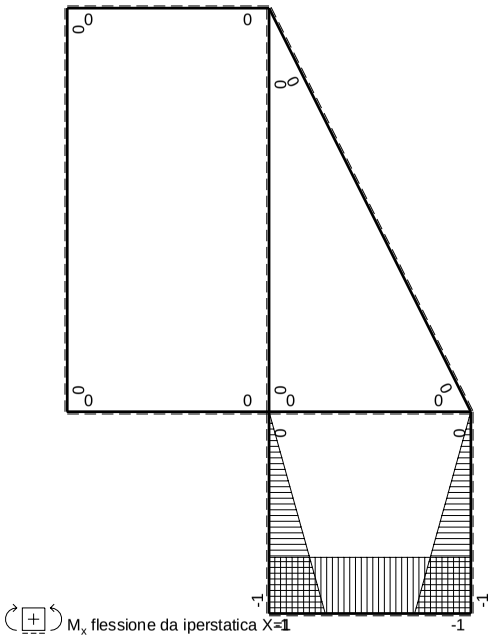


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$13/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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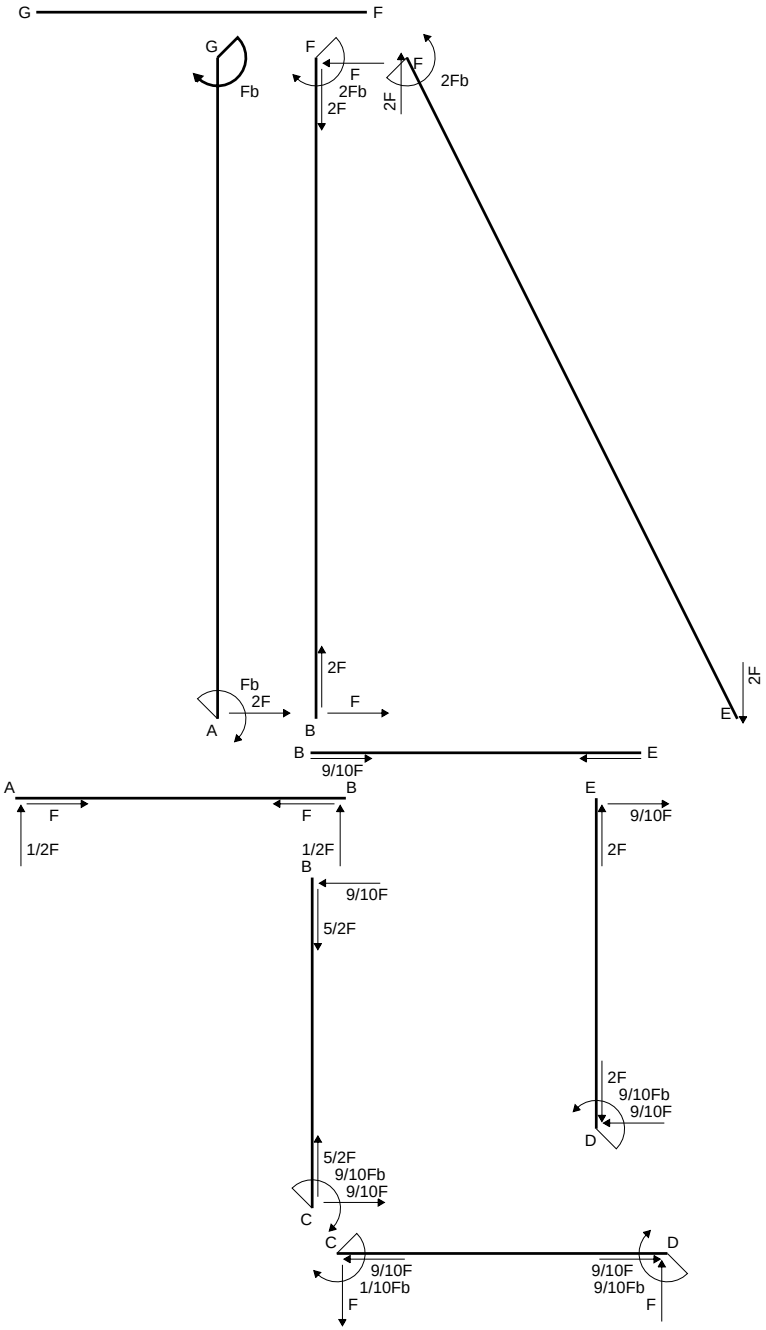
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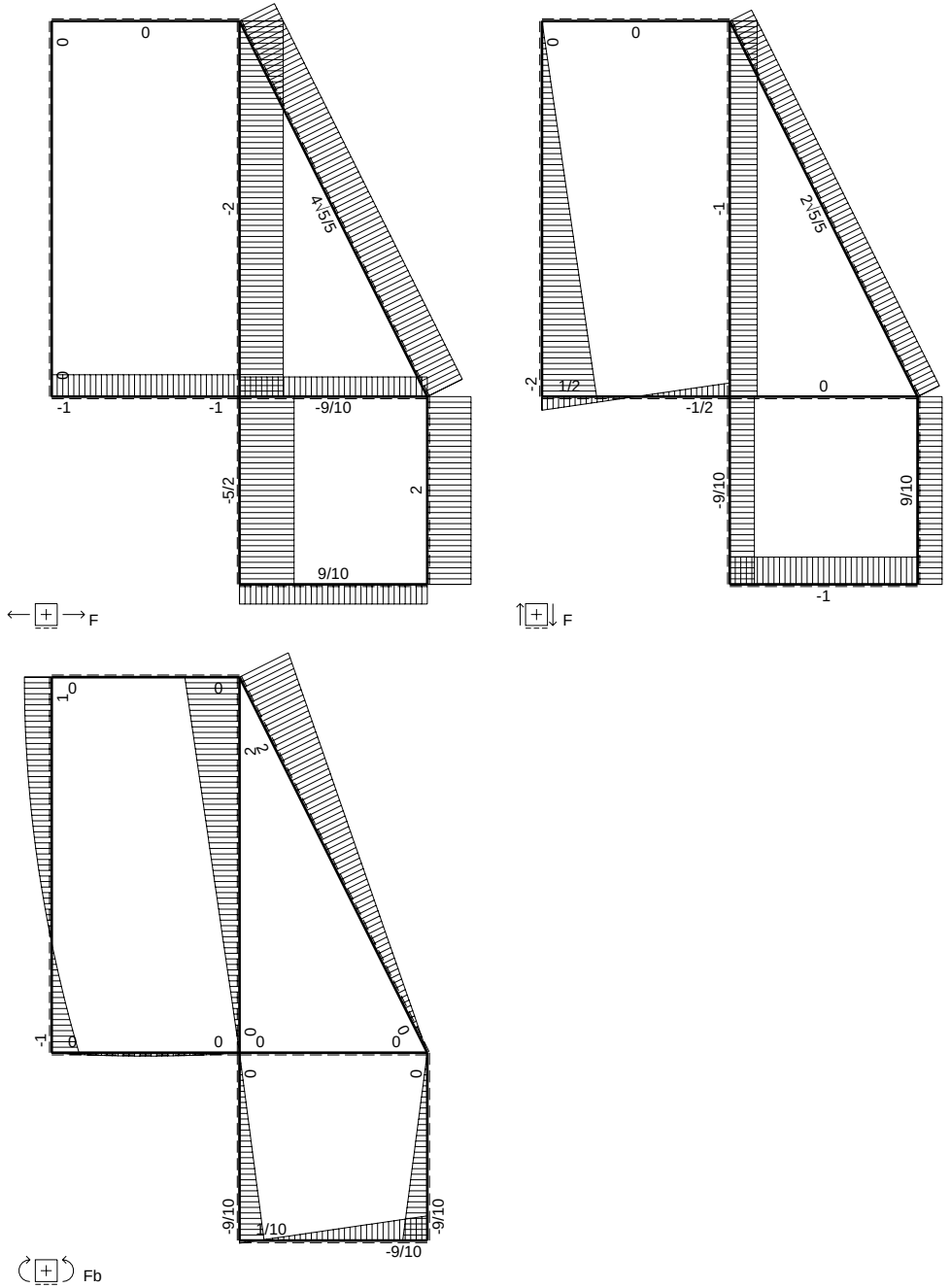
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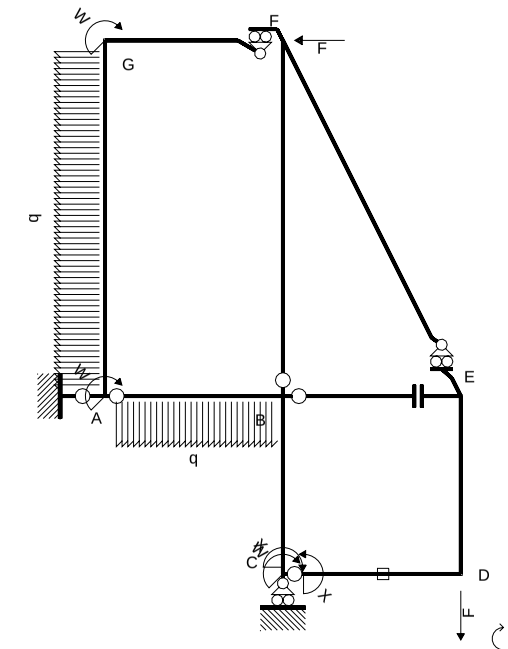
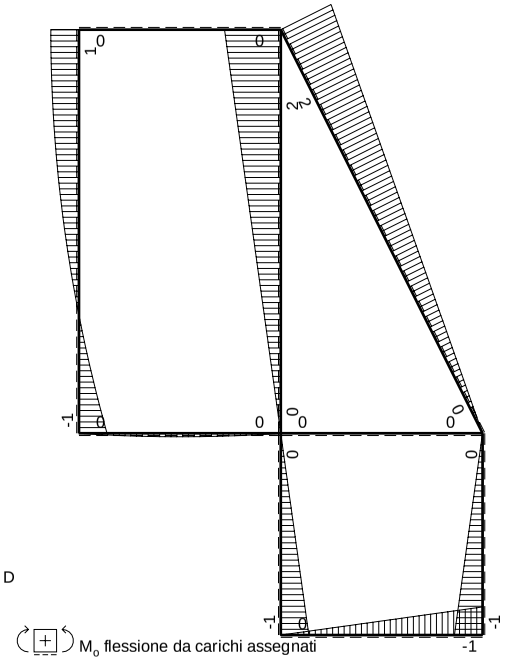
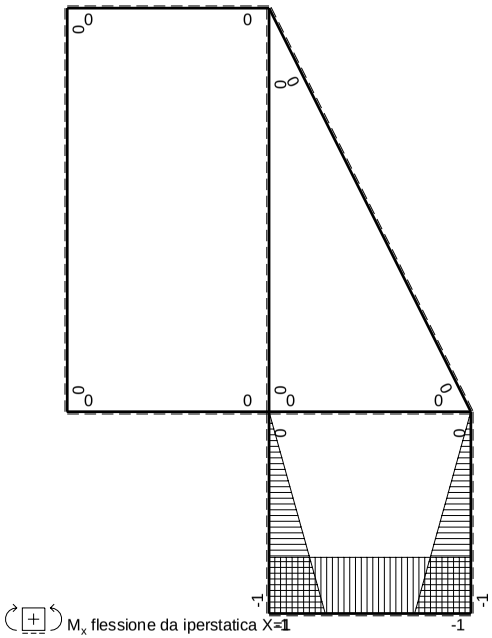


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/10Fb	

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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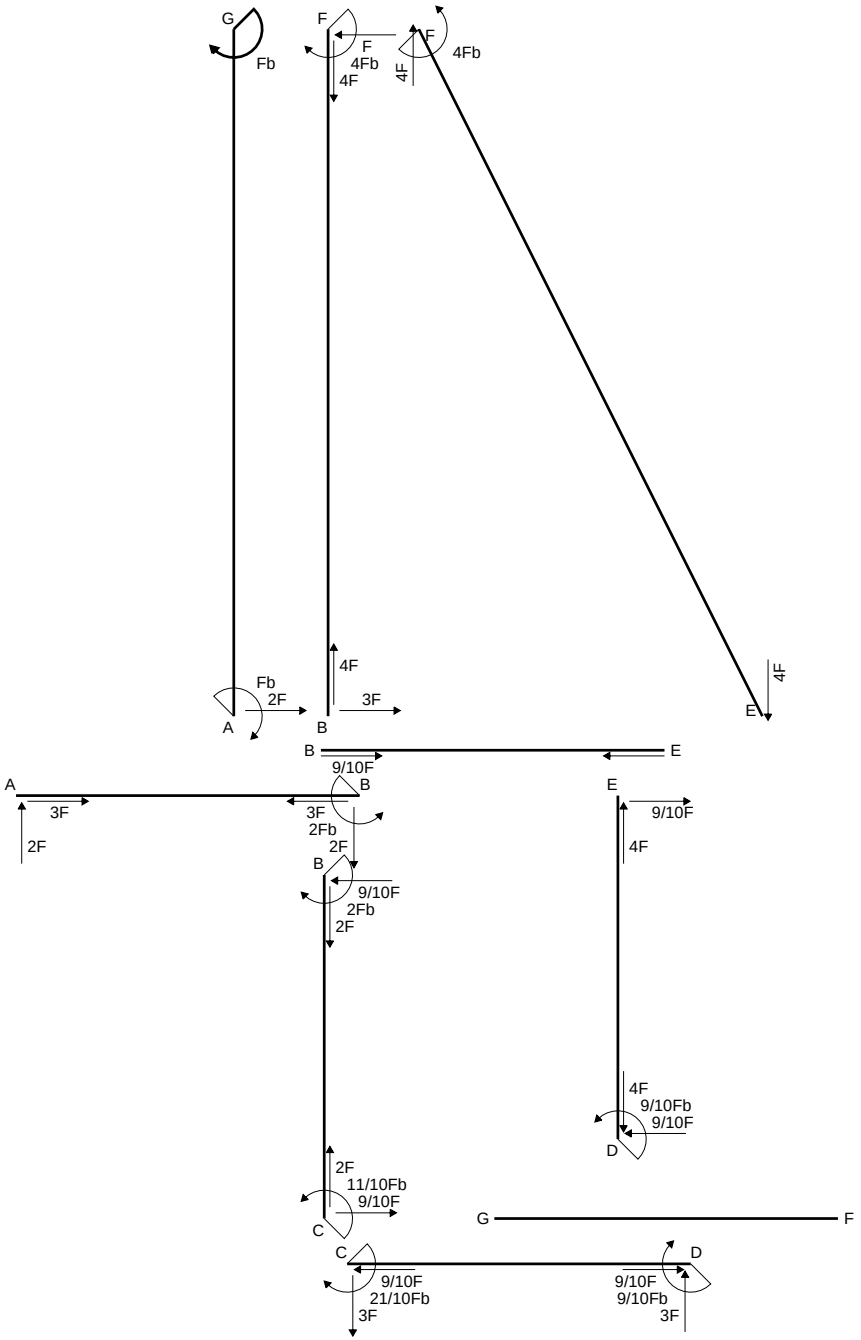
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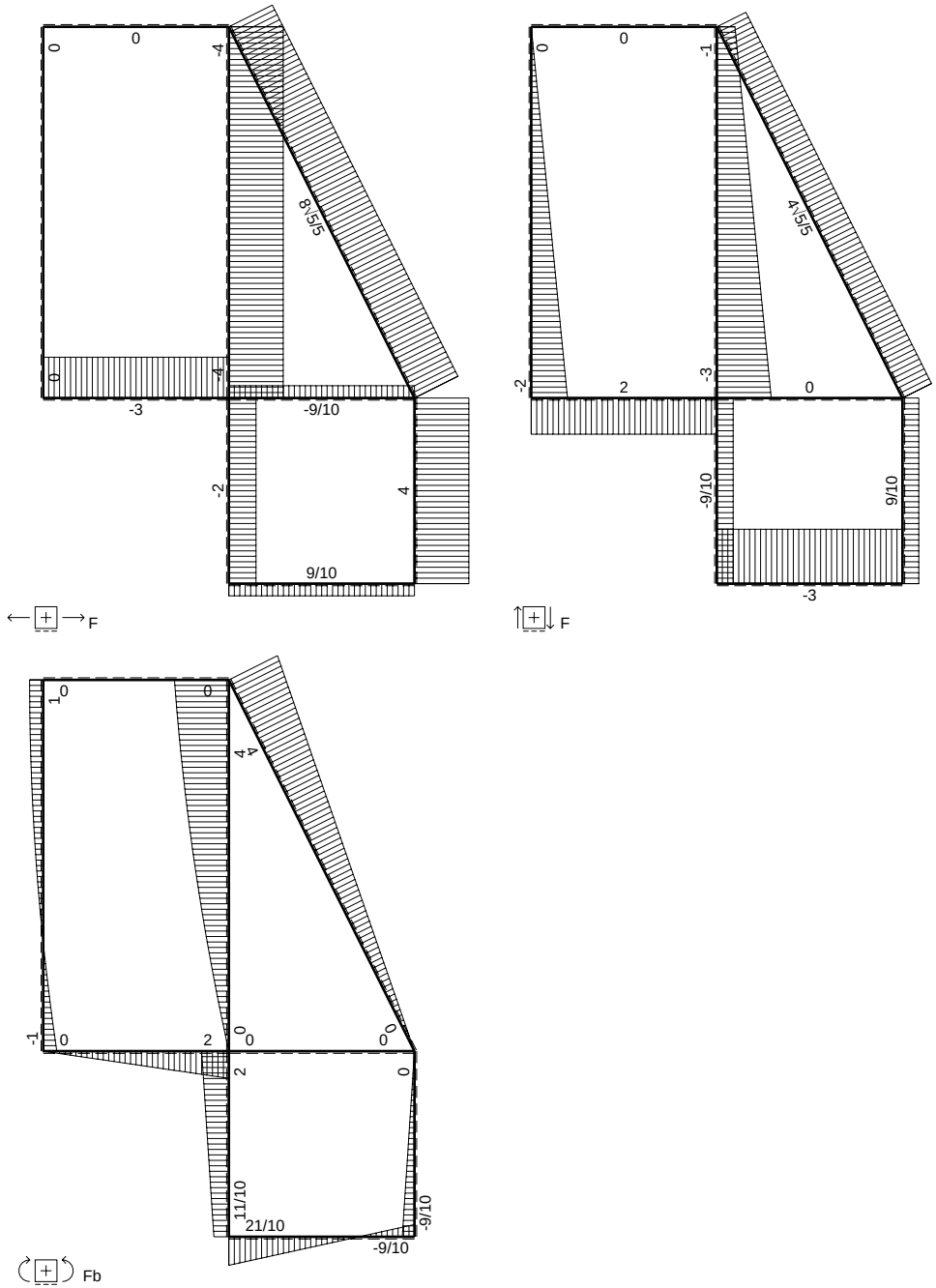
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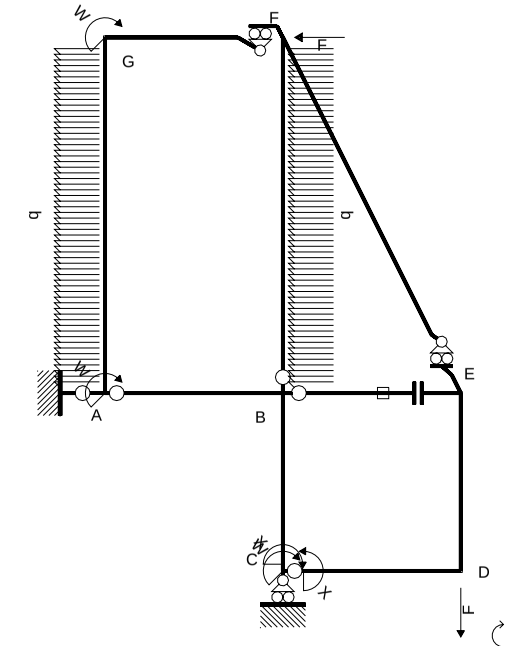
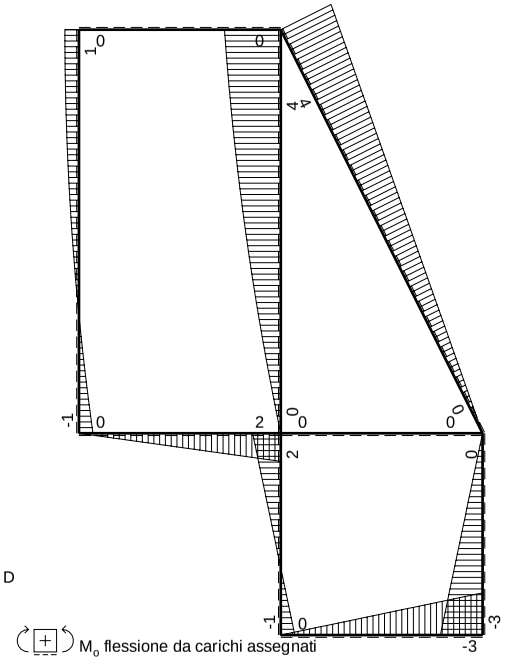
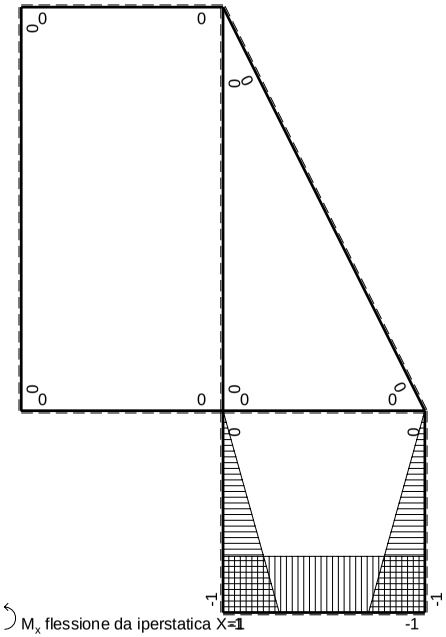


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	2Fx	0	0	0	0
BA b	0	-2Fb+2Fx	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-2\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L_{CB}^{Xo} = \int_o^b (1 -4\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -6\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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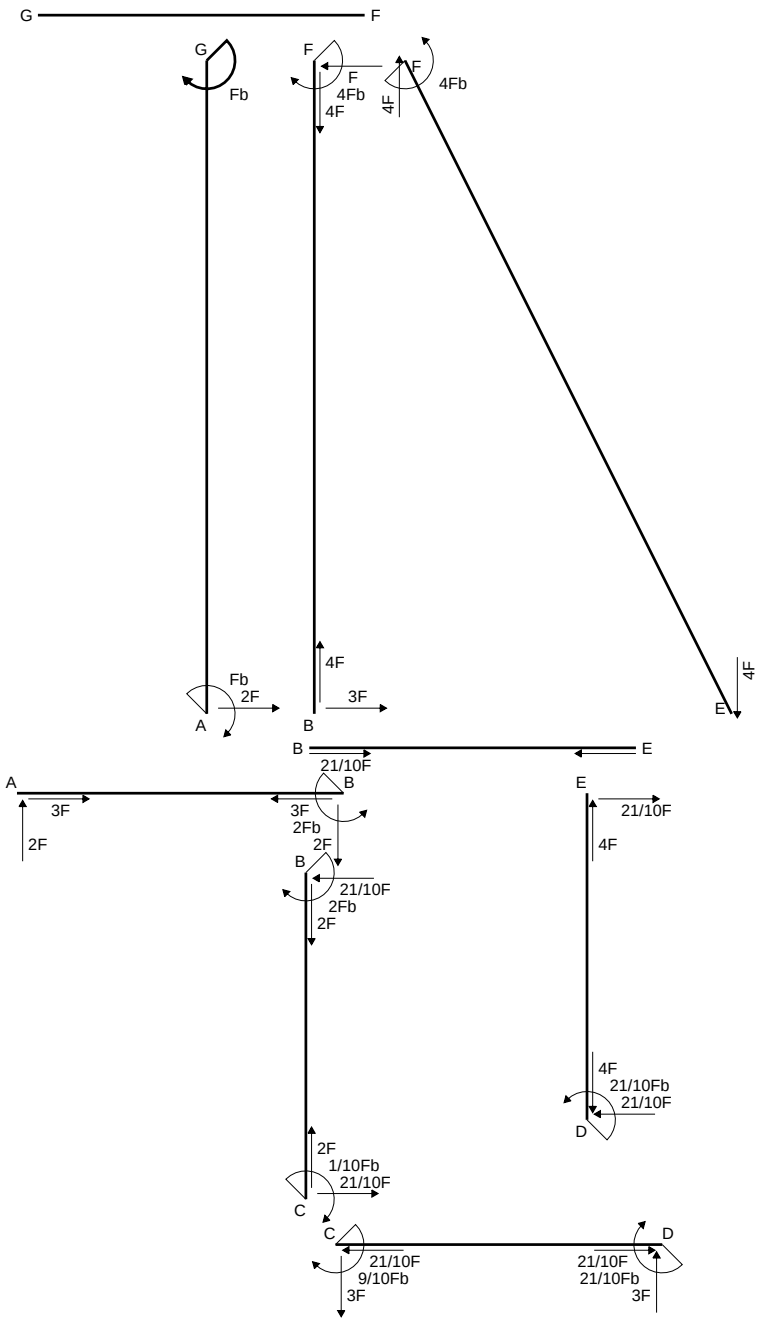
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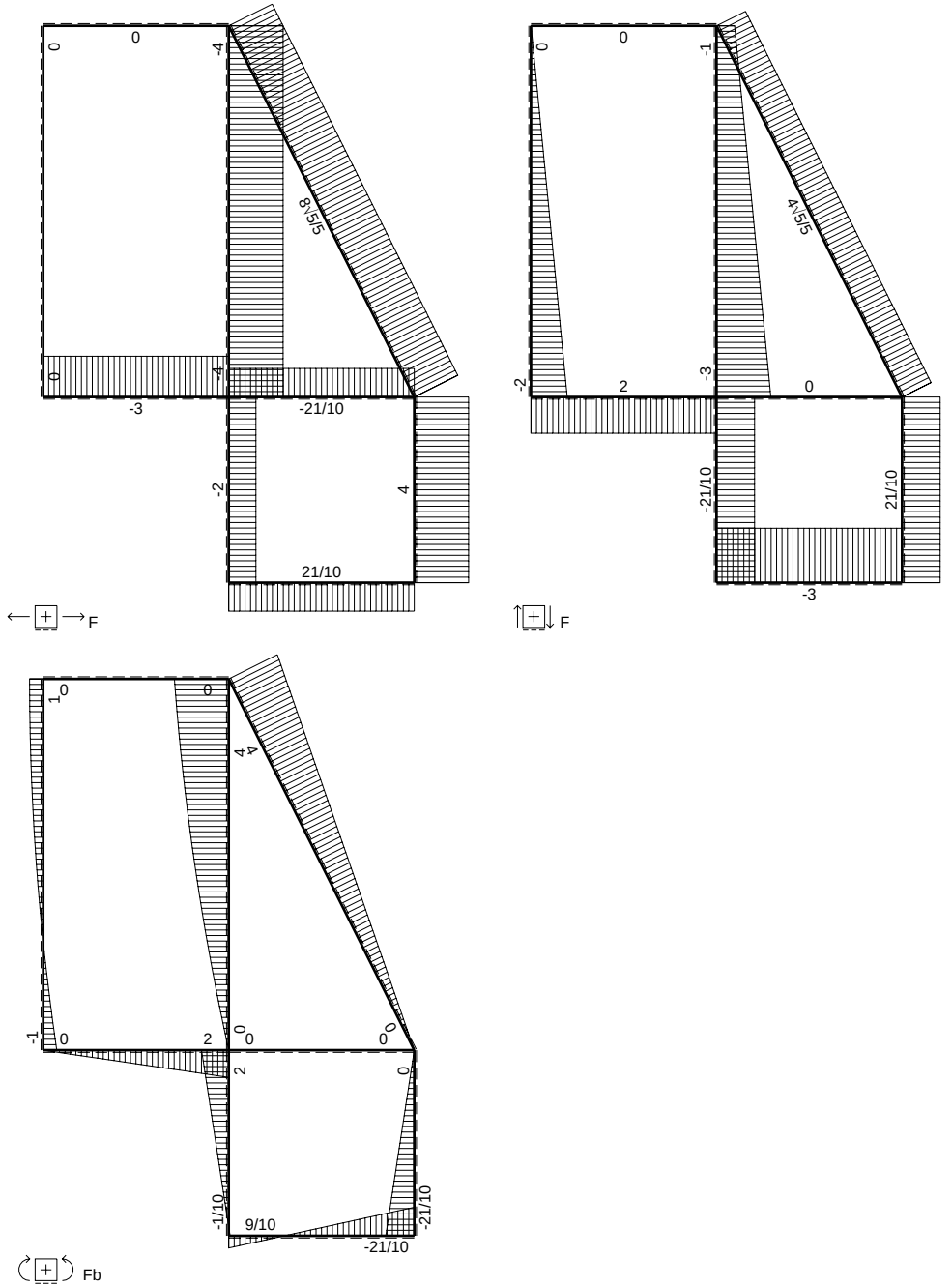
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21/10
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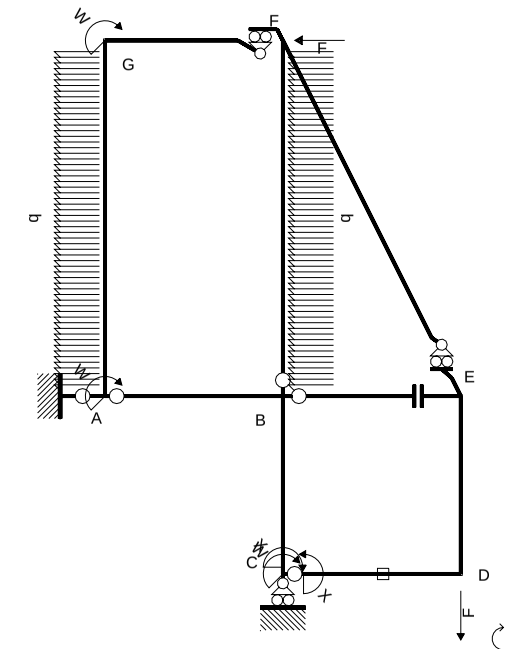
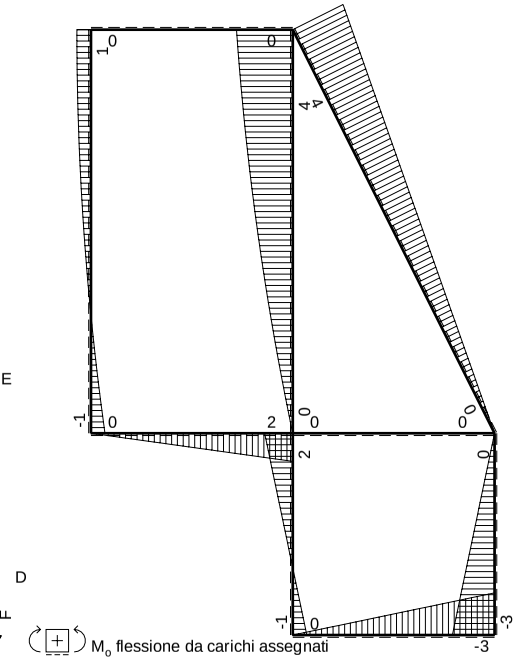
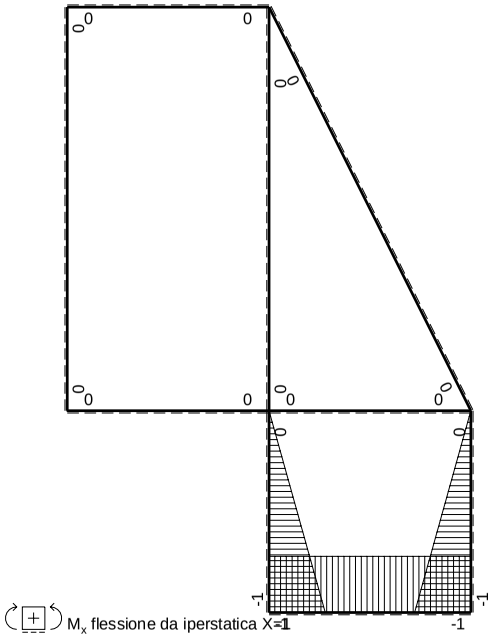
Fb



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	2Fx	0	0	0	0
BA b	0	-2Fb+2Fx	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (- b + b)\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b)\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b)\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b)\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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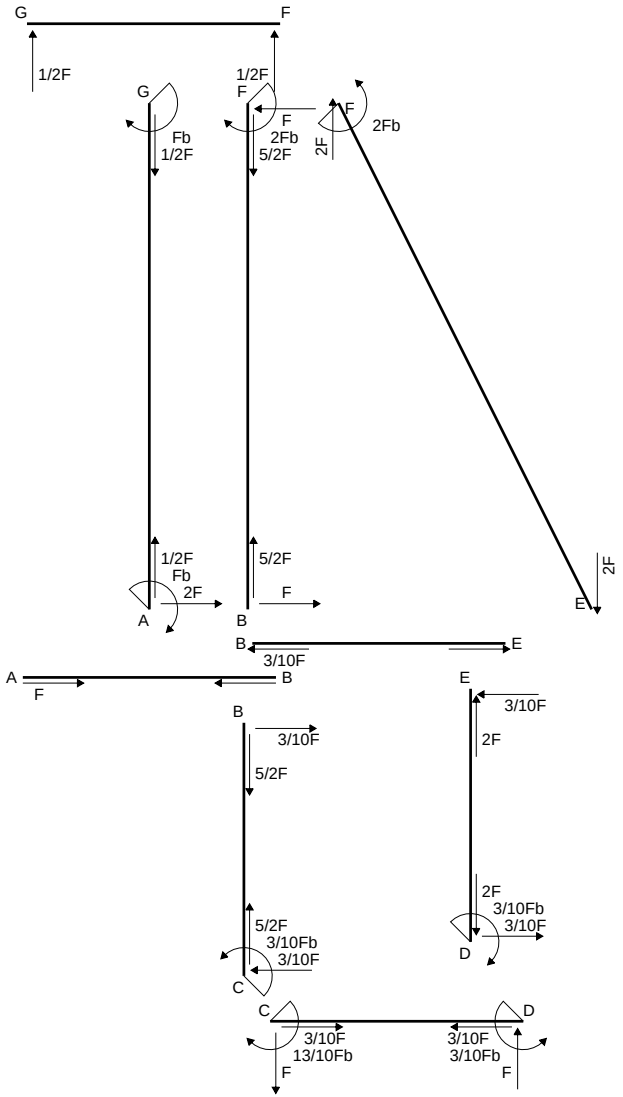
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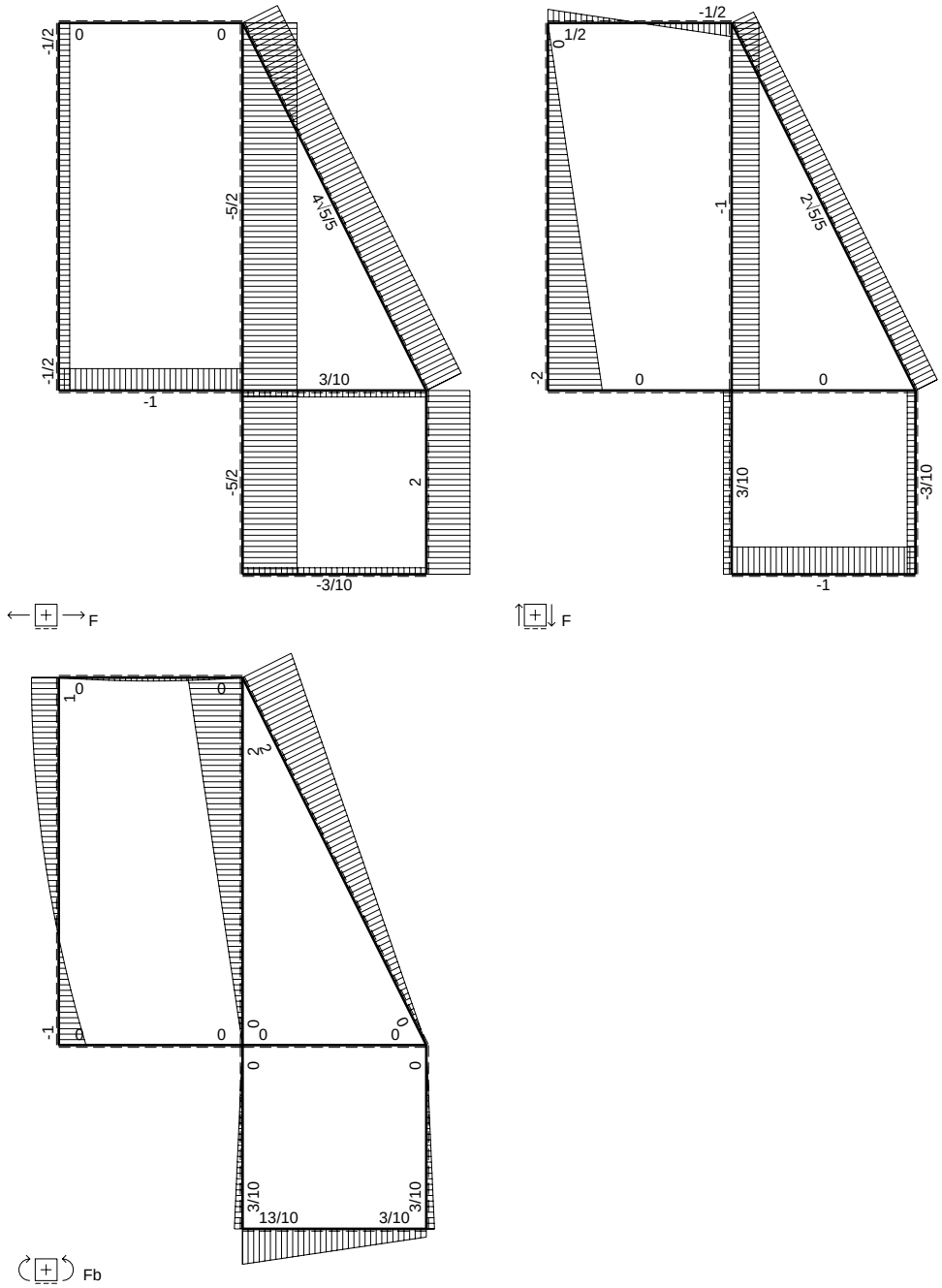
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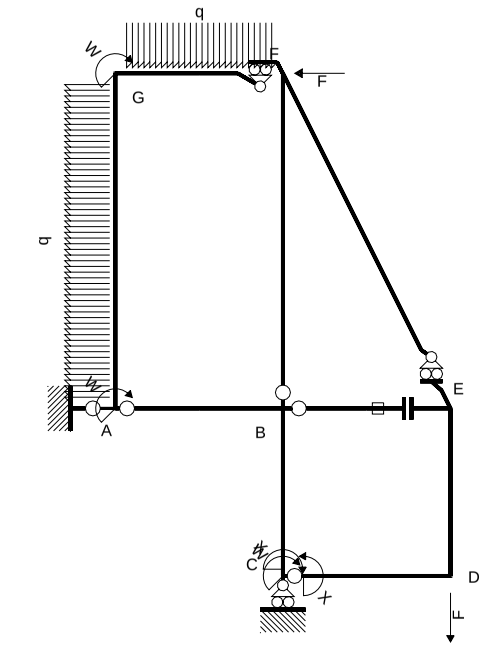
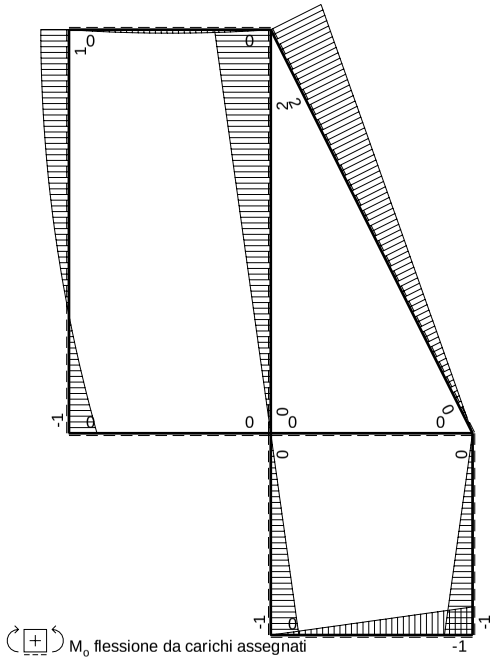
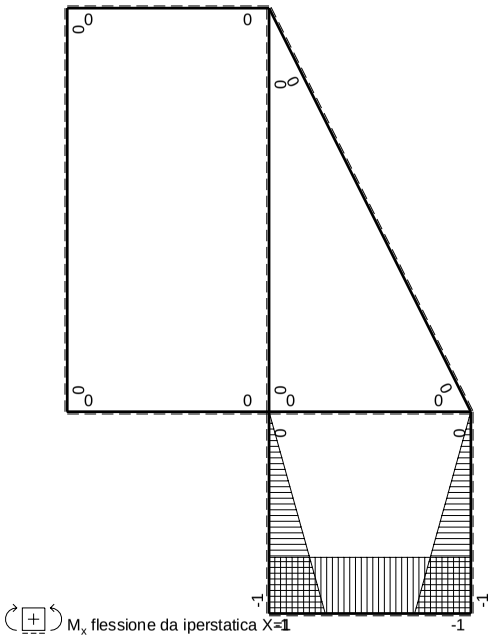


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$13/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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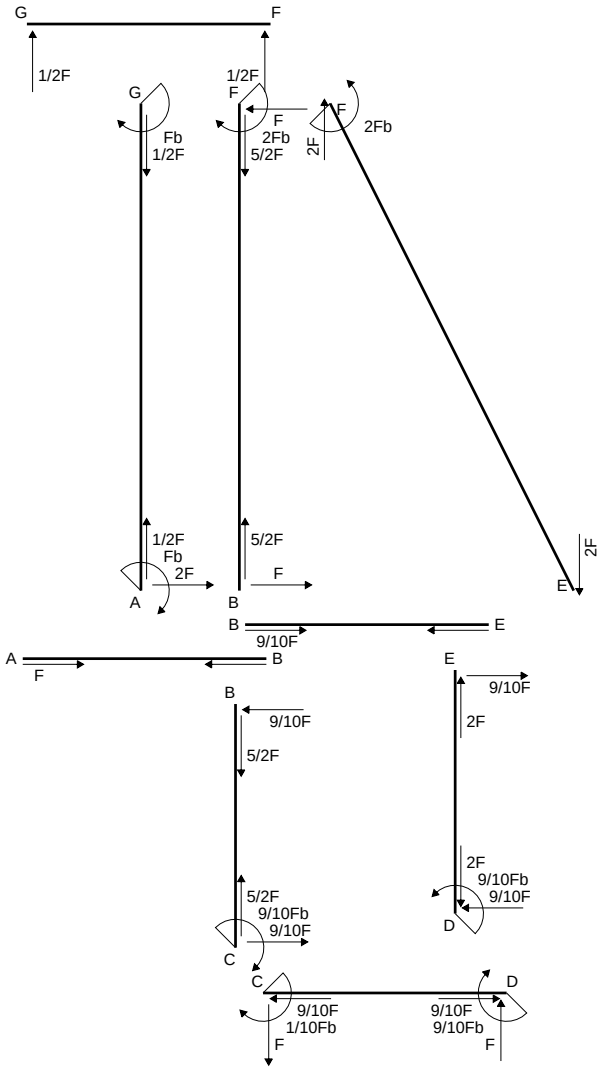
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9/10F
B
9/10F
5/2F
2F
2F
9/10Fb
9/10F
D
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9/10F
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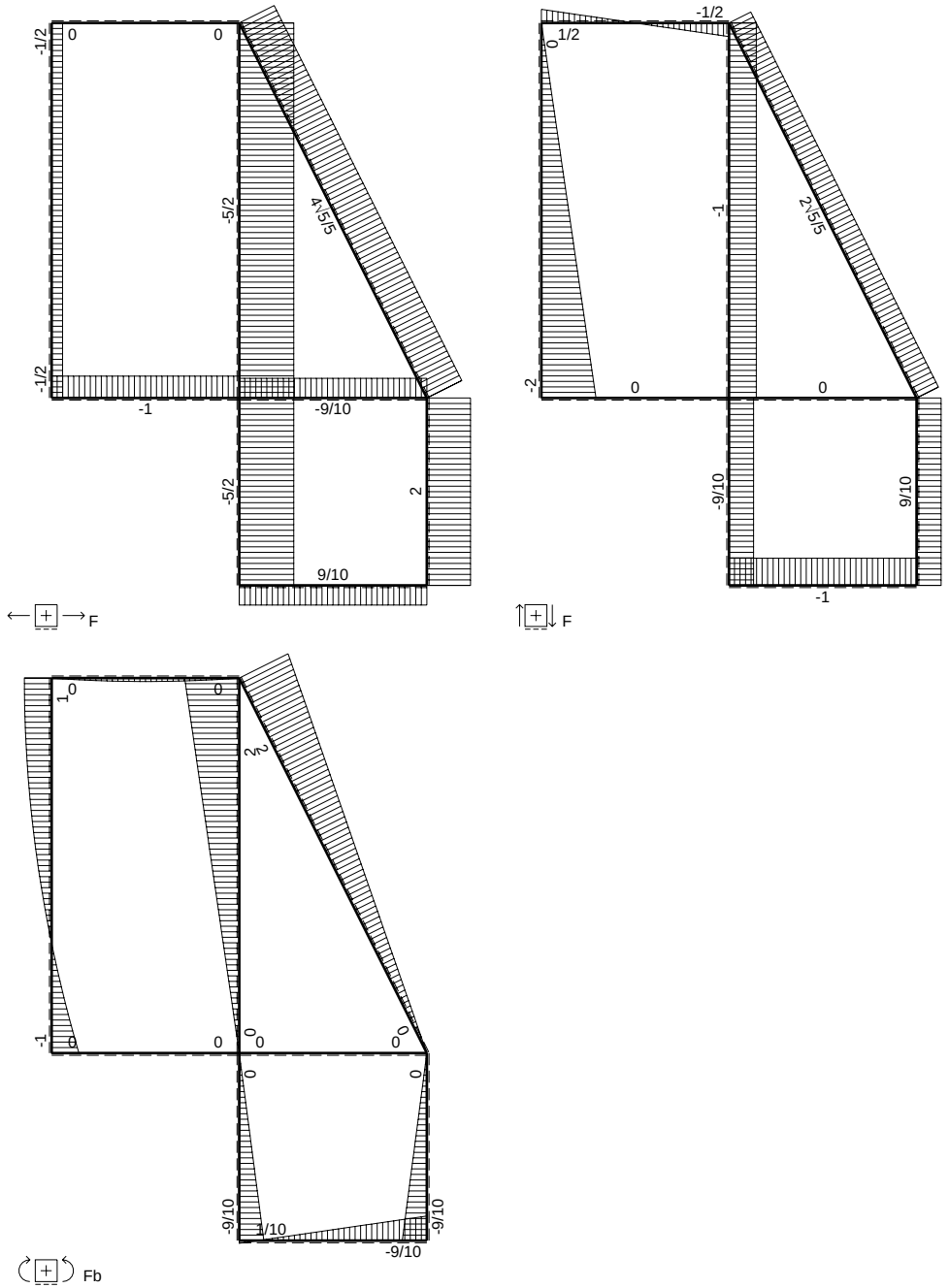
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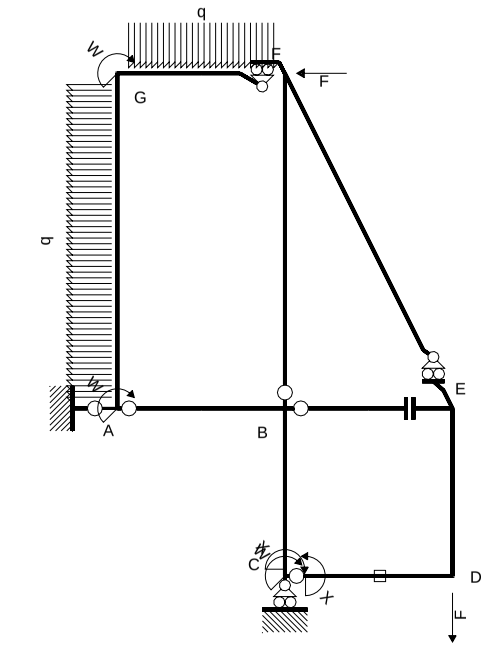
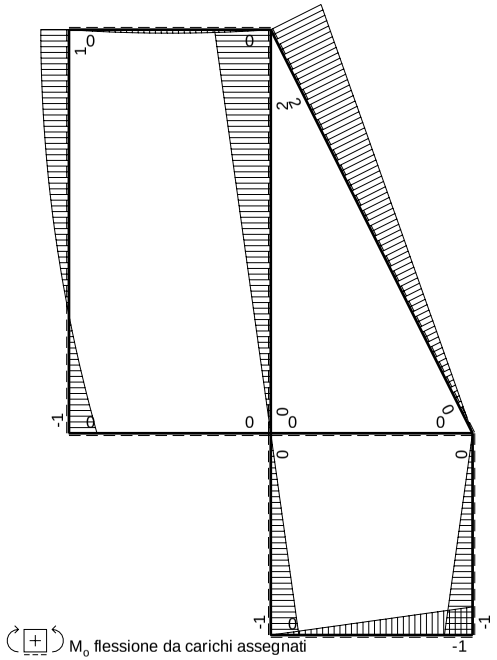
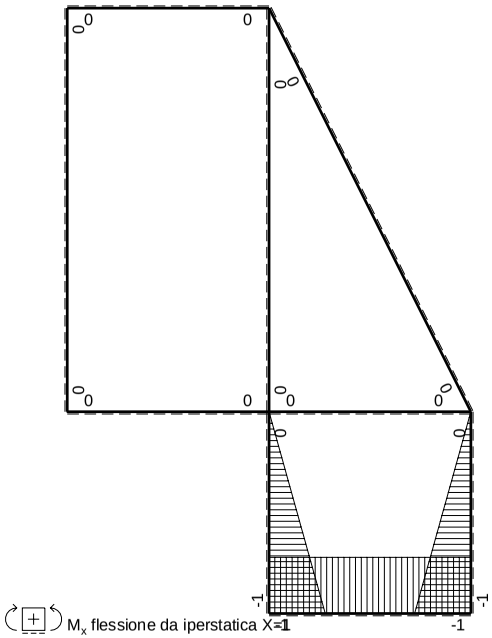


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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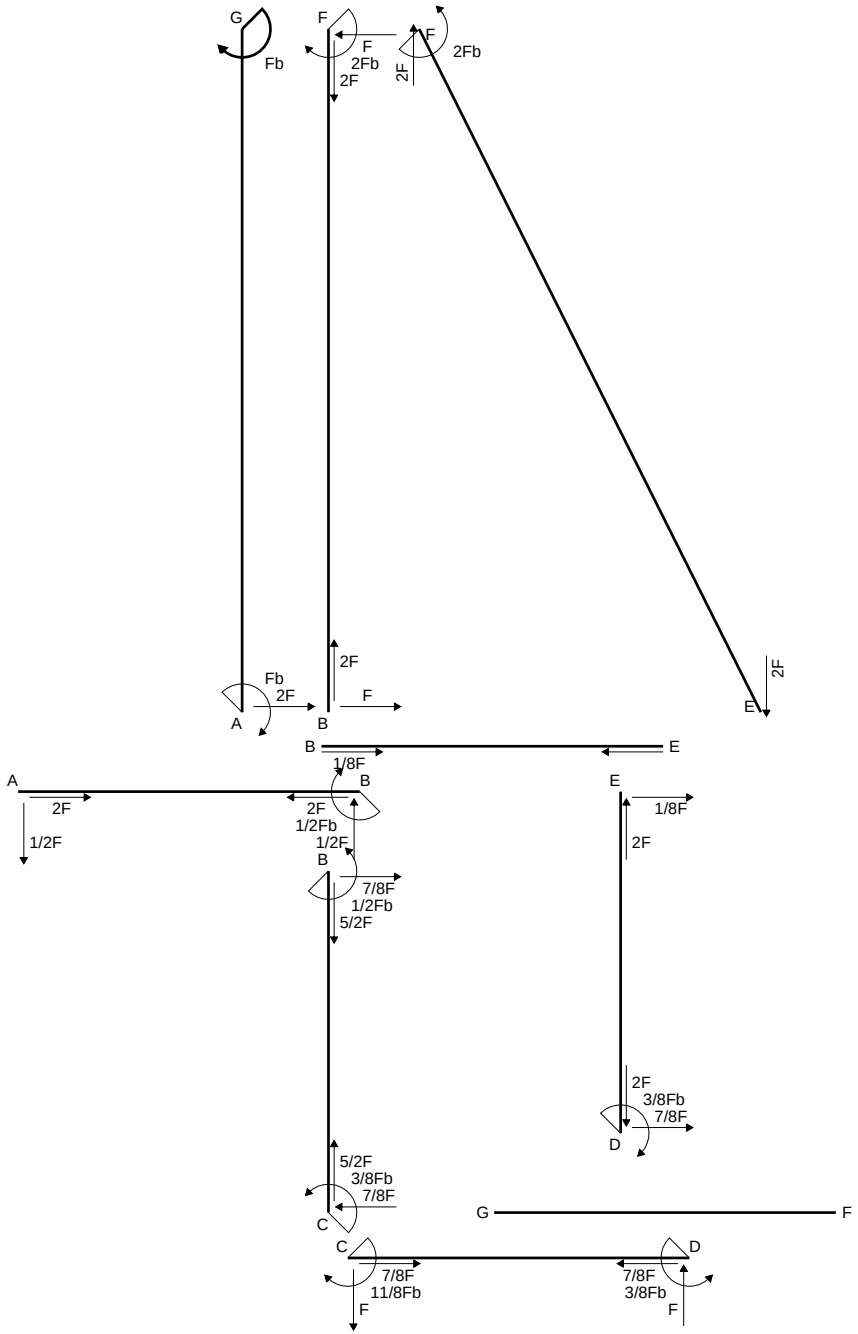
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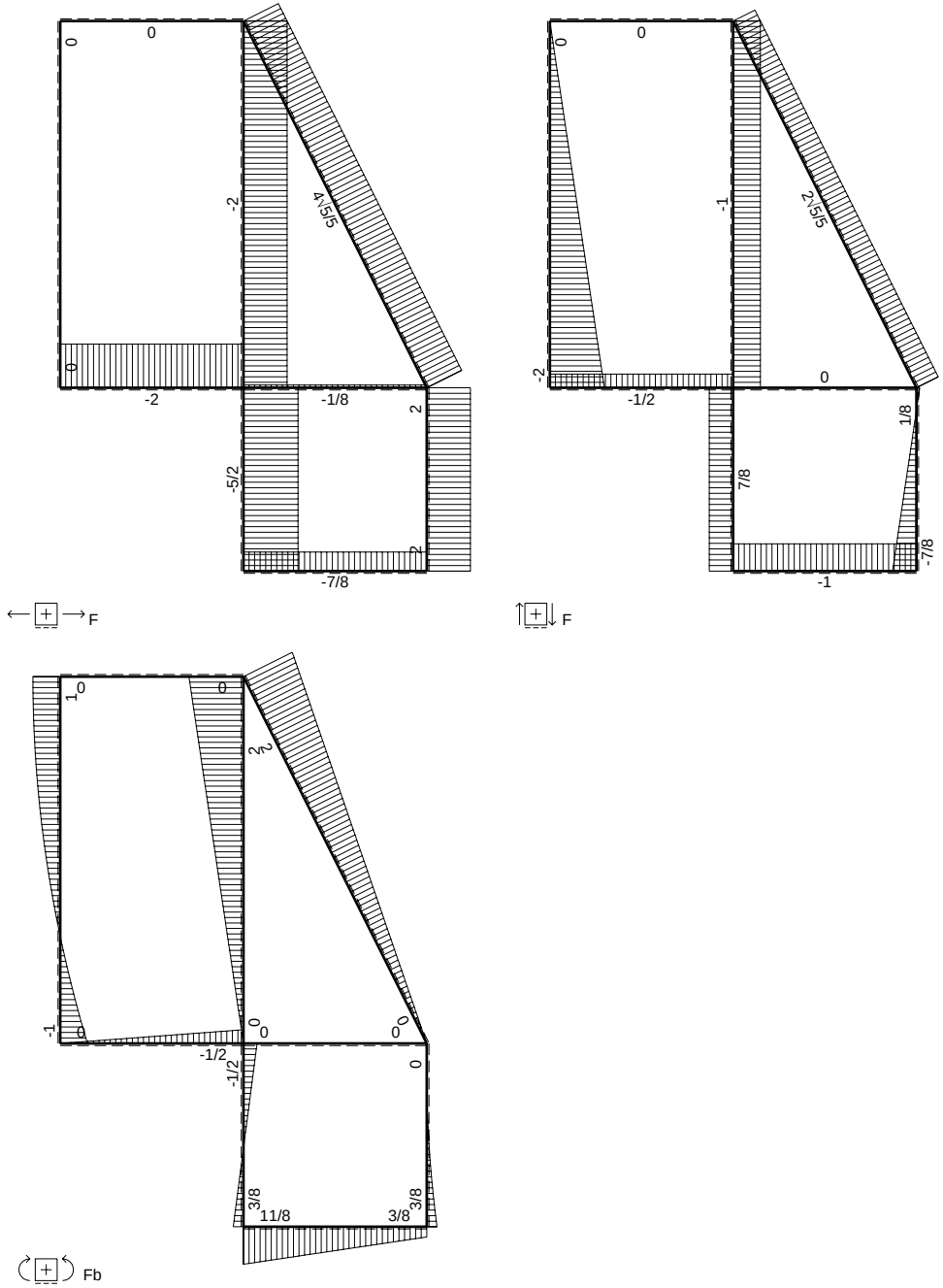
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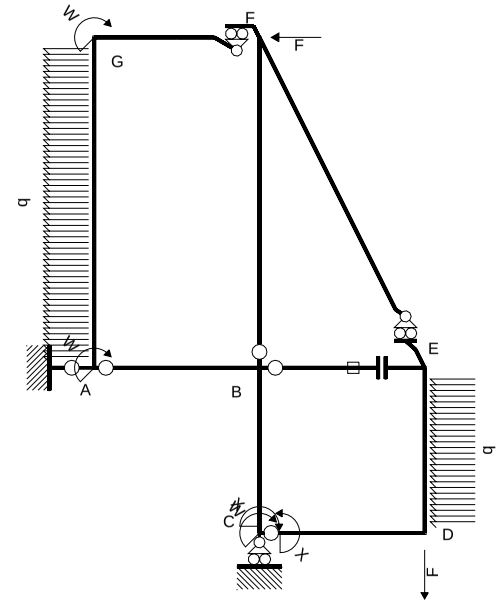
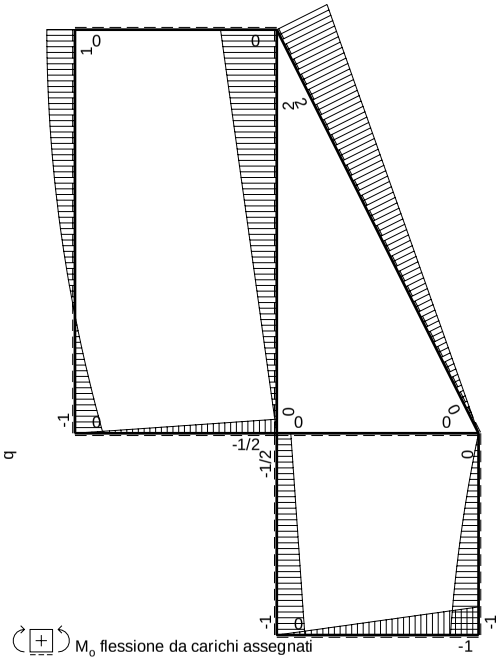
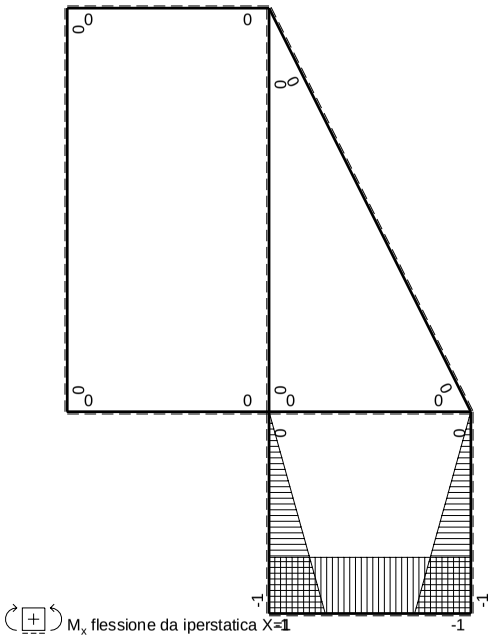
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx+1/2qx^2	Fb-3/2Fx+1/2qx^3/b	1-2x/b+x^2/b^2	3/8Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fx-1/2qx^2	3/2Fx^2/b-1/2qx^3/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	Fb-1/2qx^2	0	0	0	0
AG 2b	0	Fb-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				55/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-11/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -3/2\ x/b +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

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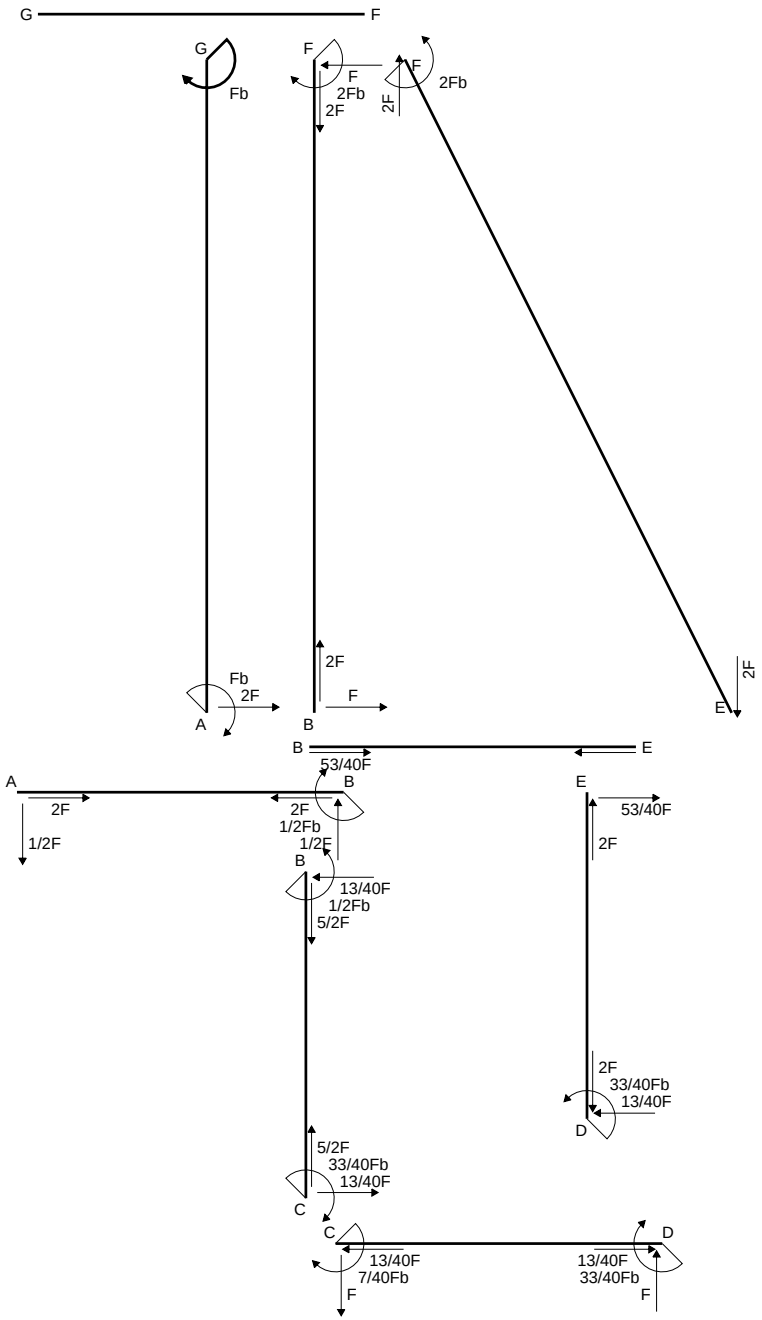
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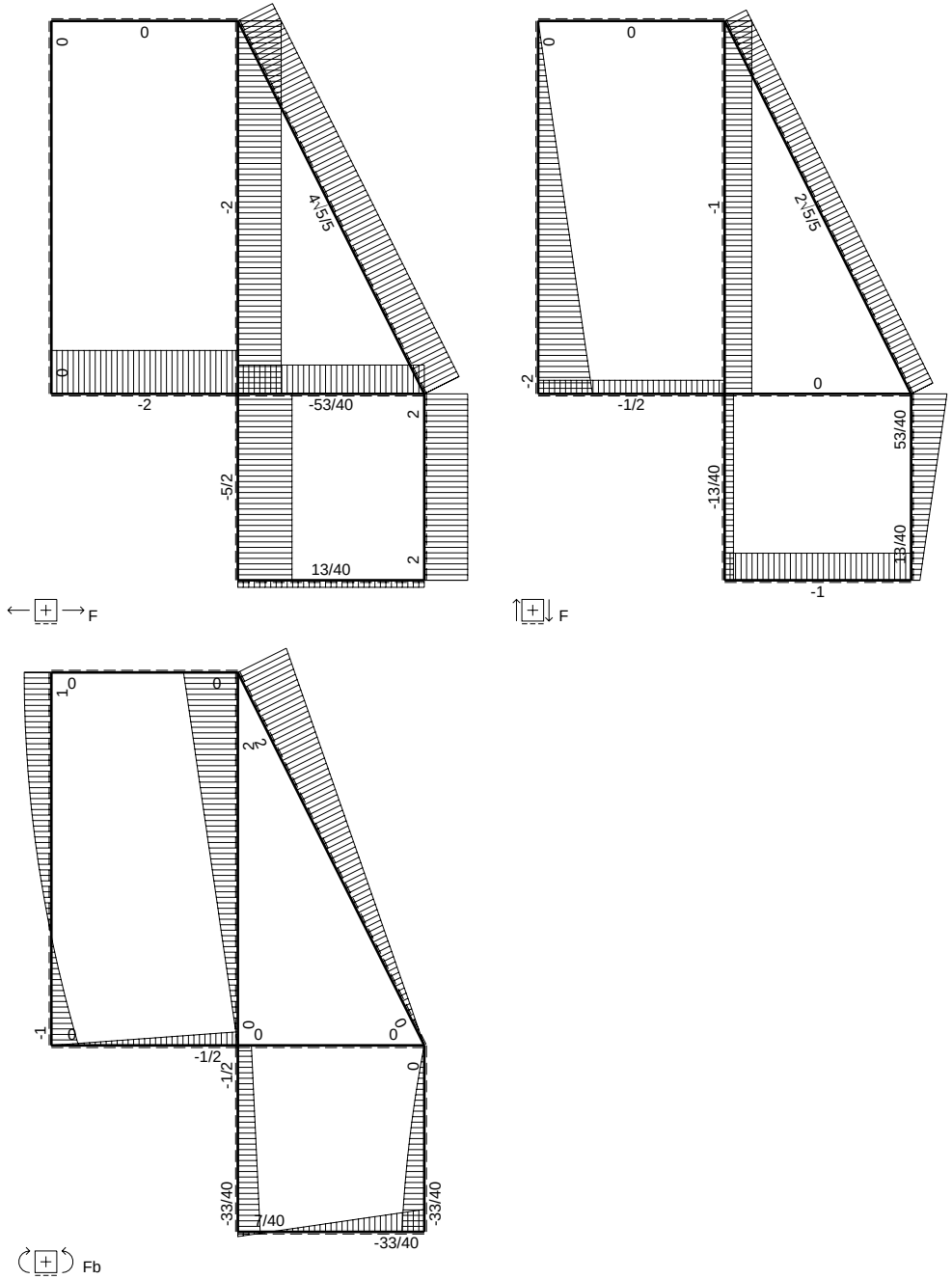
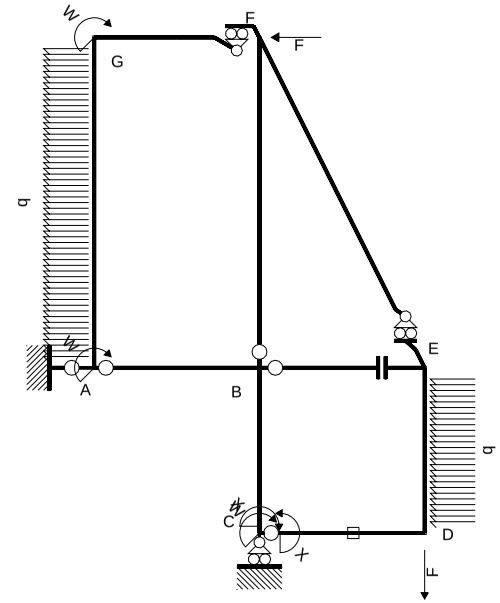
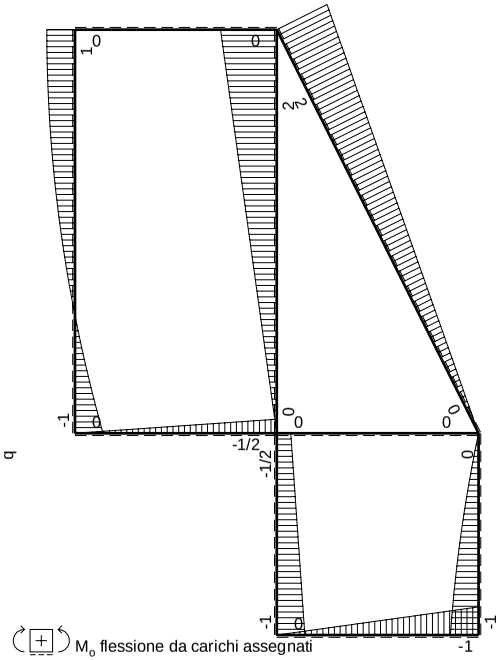
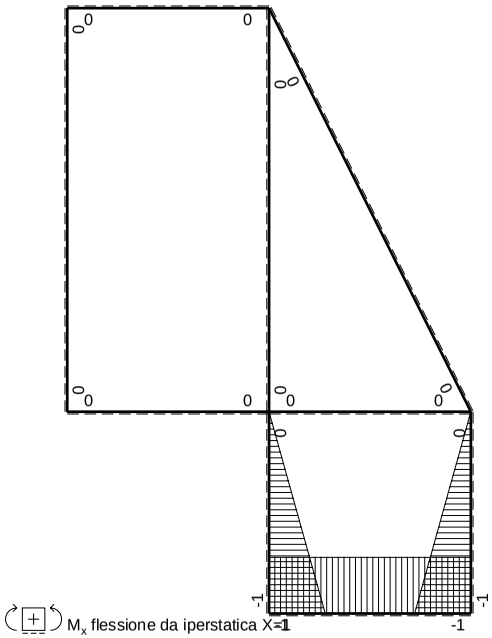

F
F
Fb



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx+1/2qx^2	Fb-3/2Fx+1/2qx^3/b	1-2x/b+x^2/b^2	3/8Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fx-1/2qx^2	3/2Fx^2/b-1/2qx^3/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	Fb-1/2qx^2	0	0	0	0
AG 2b	0	Fb-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				7/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-7/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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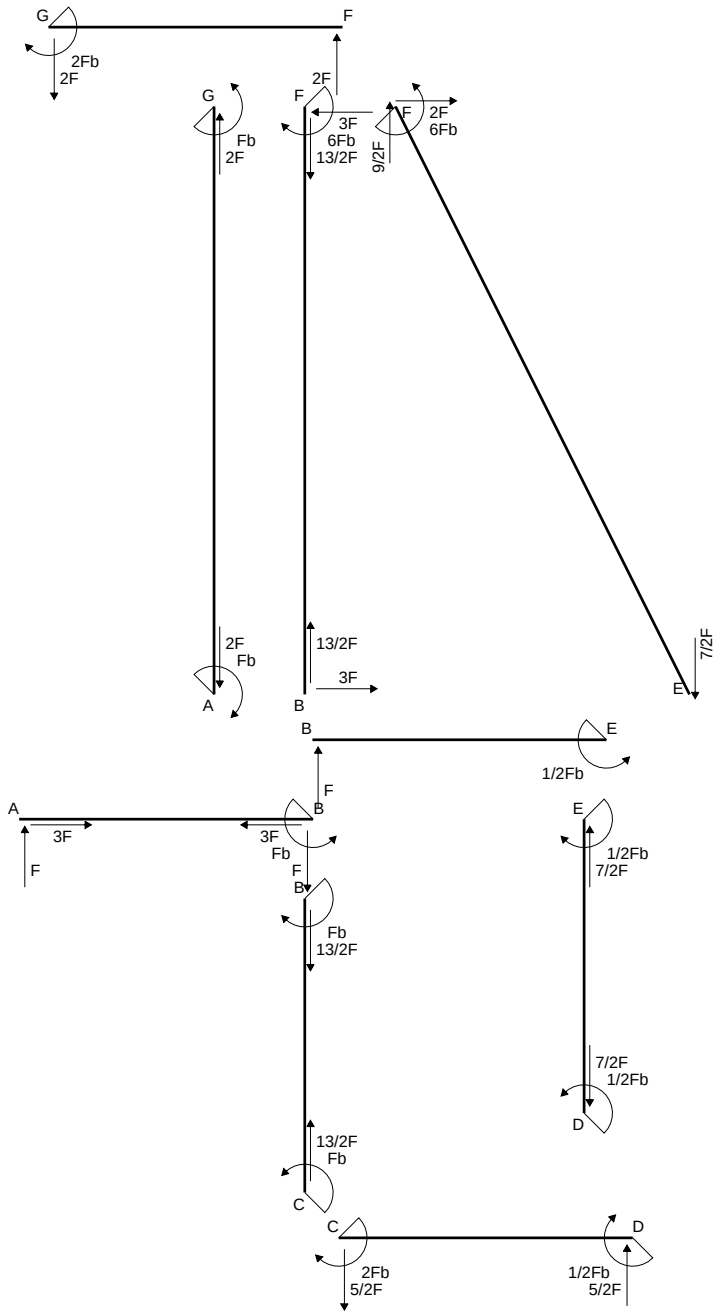
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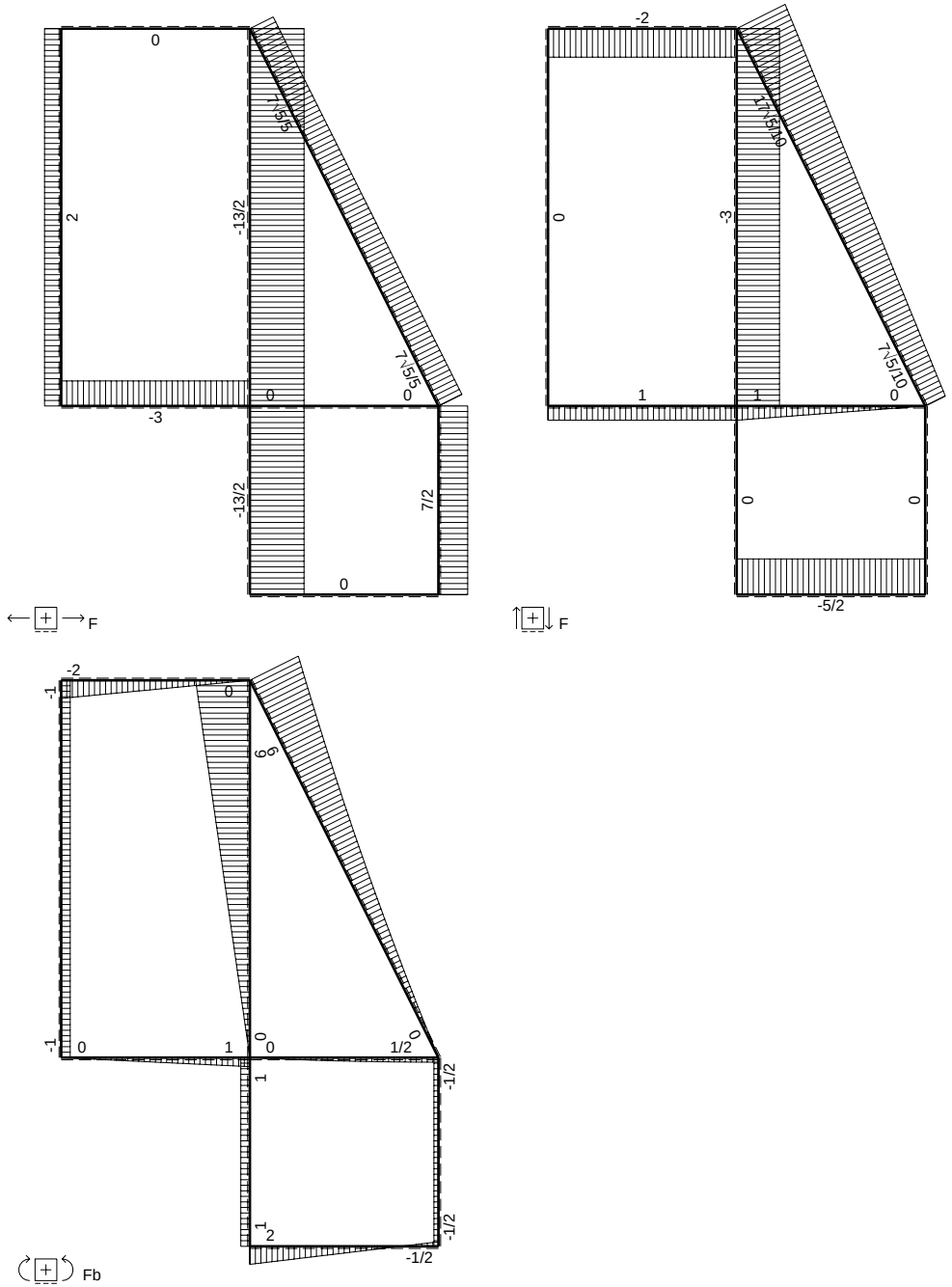
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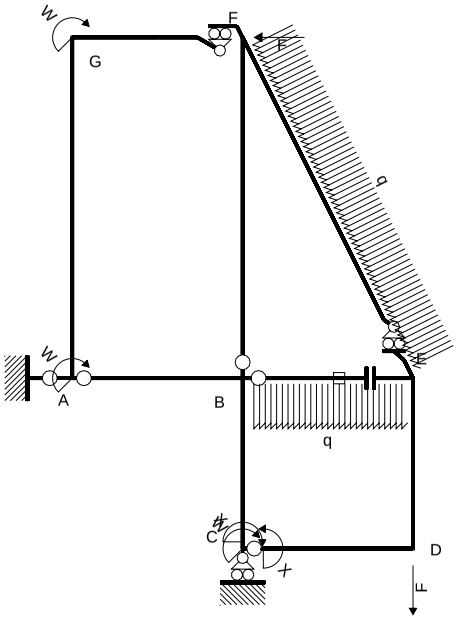
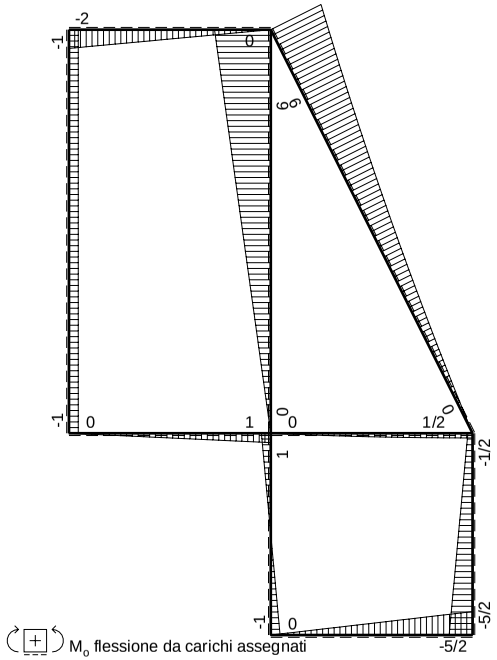
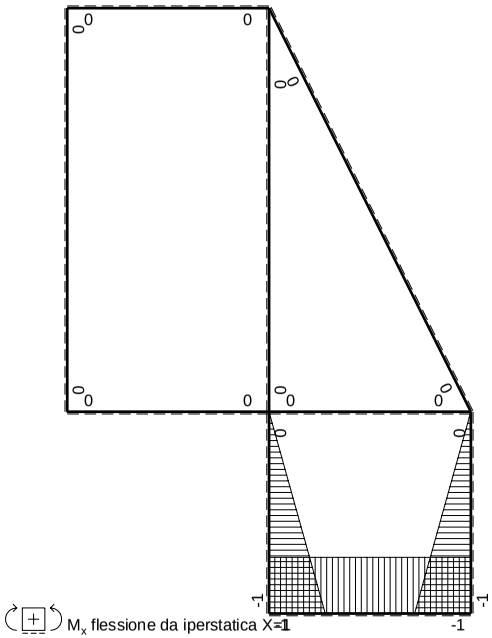


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+2Fx	$5/2Fb-9/2Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$11/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb+2Fx	$1/2Fx+2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$10/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -9/2\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -9/4\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -9/4\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

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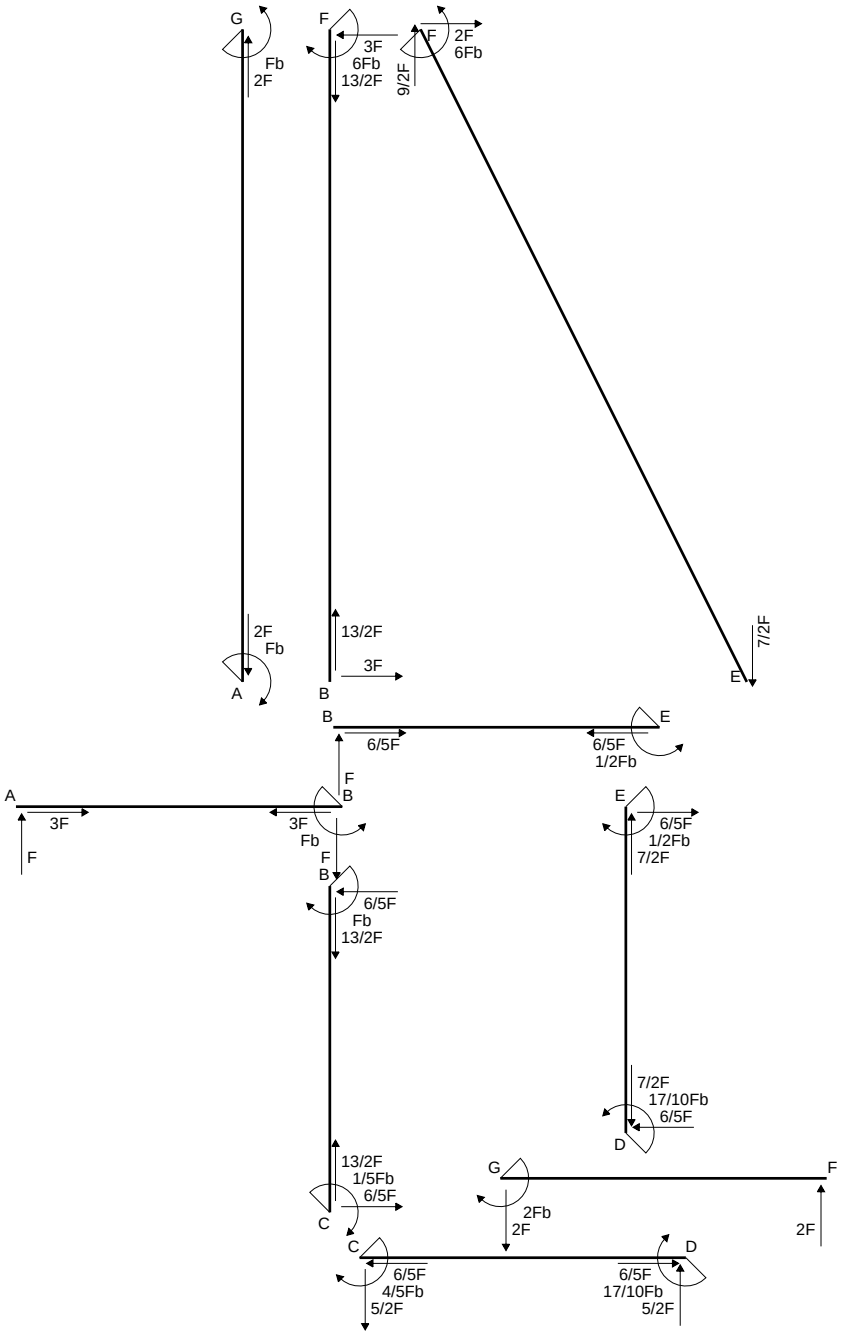
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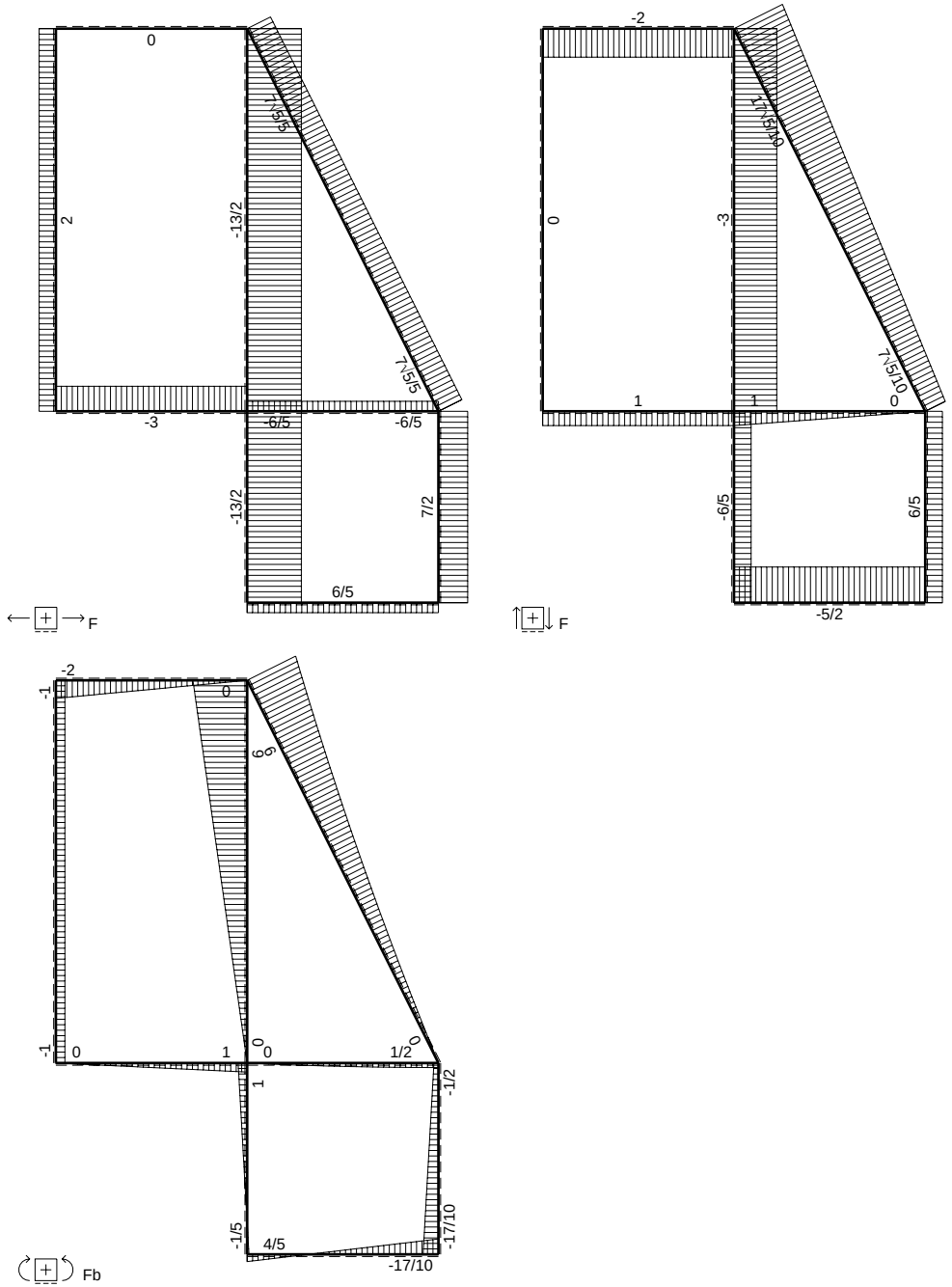
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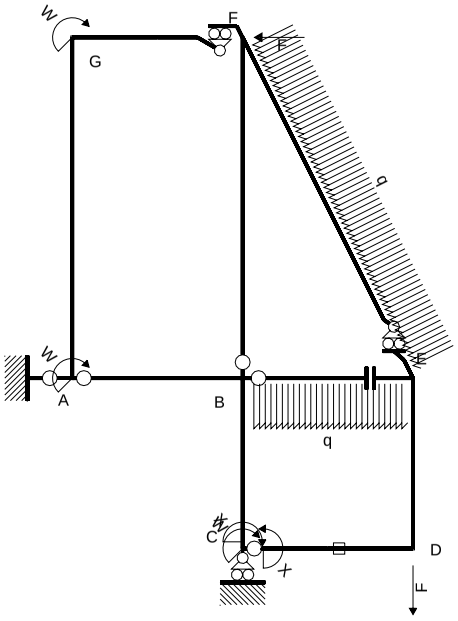
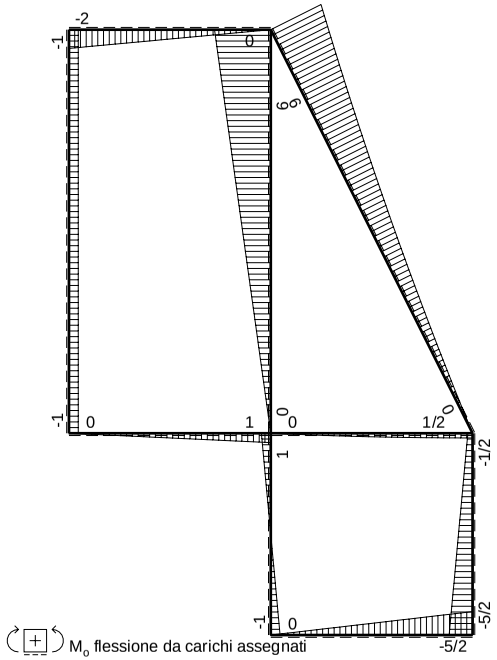
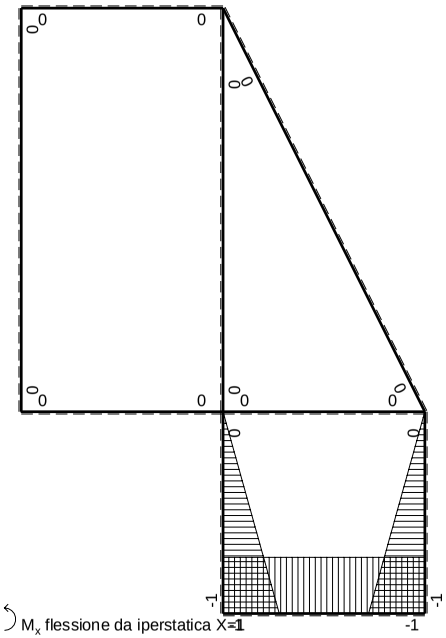


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+2Fx	$5/2Fb-9/2Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$11/12Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb+2Fx	$1/2Fx+2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$4/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-4/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -9/2\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -9/4\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -9/4\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 11/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 11/12\ \ Fb^2/EJ$

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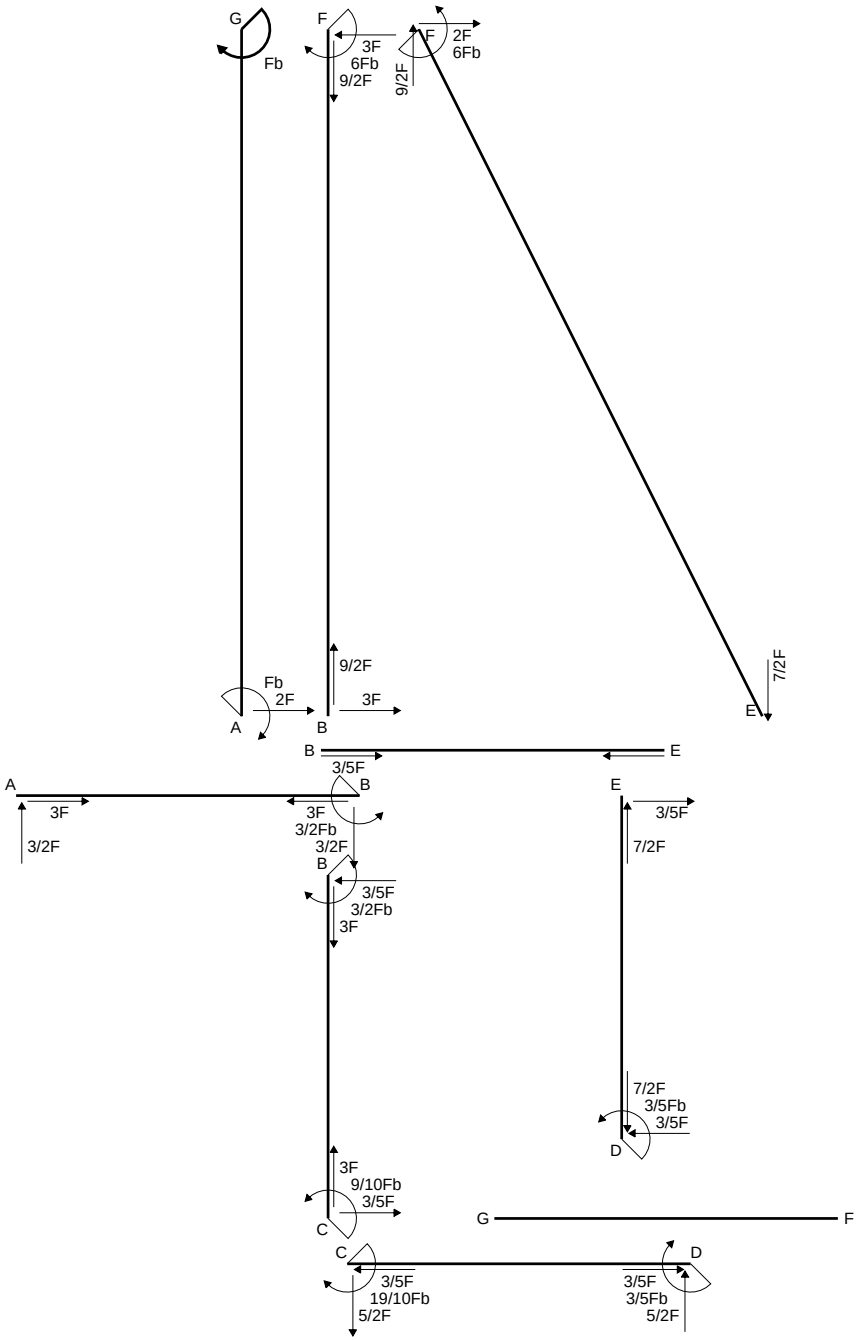
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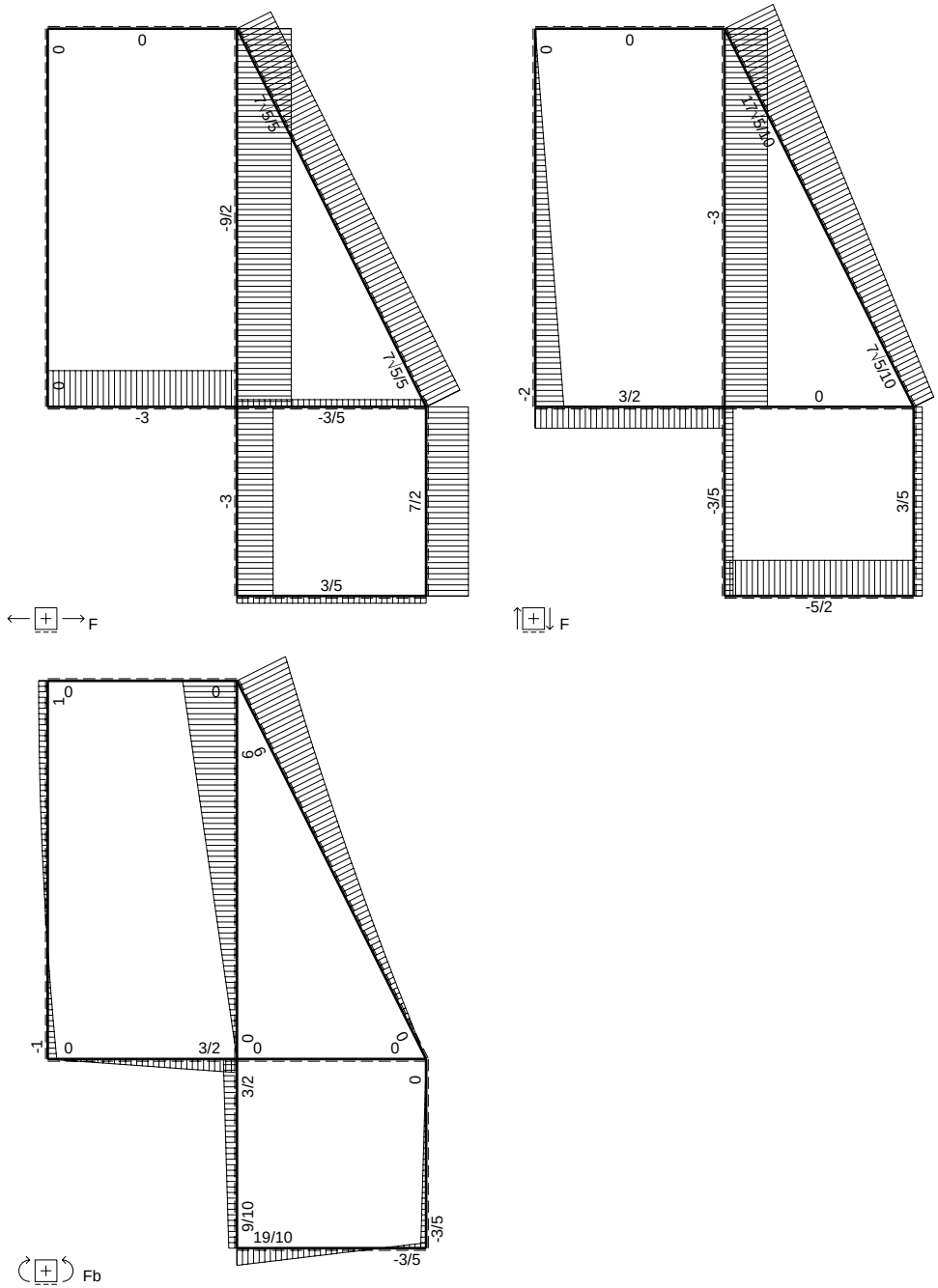
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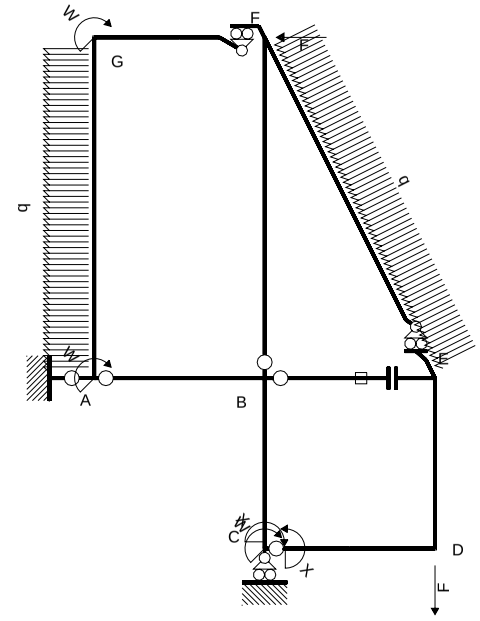
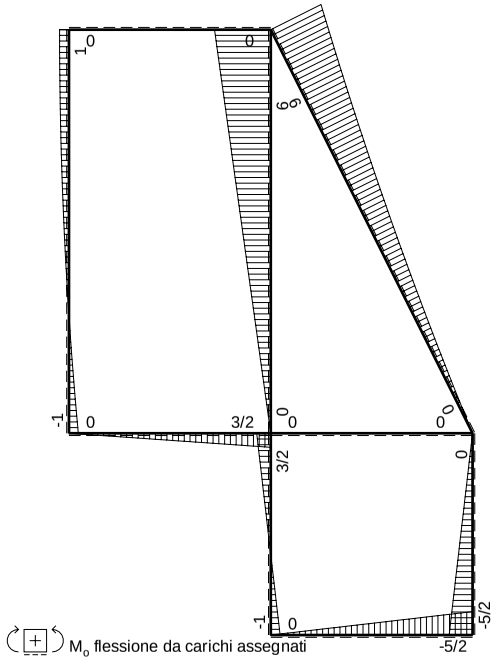
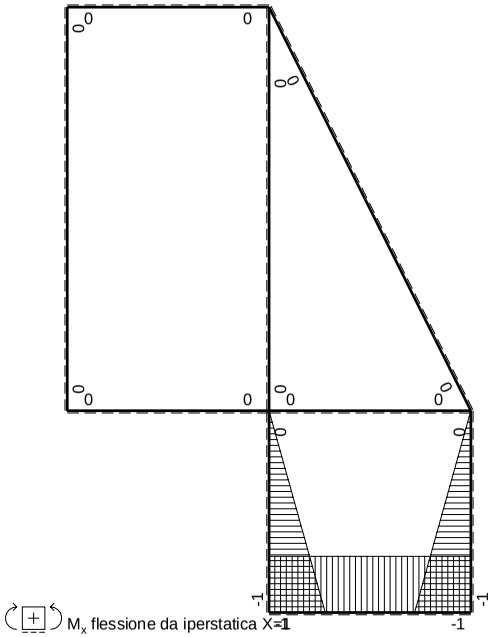


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$19/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-19/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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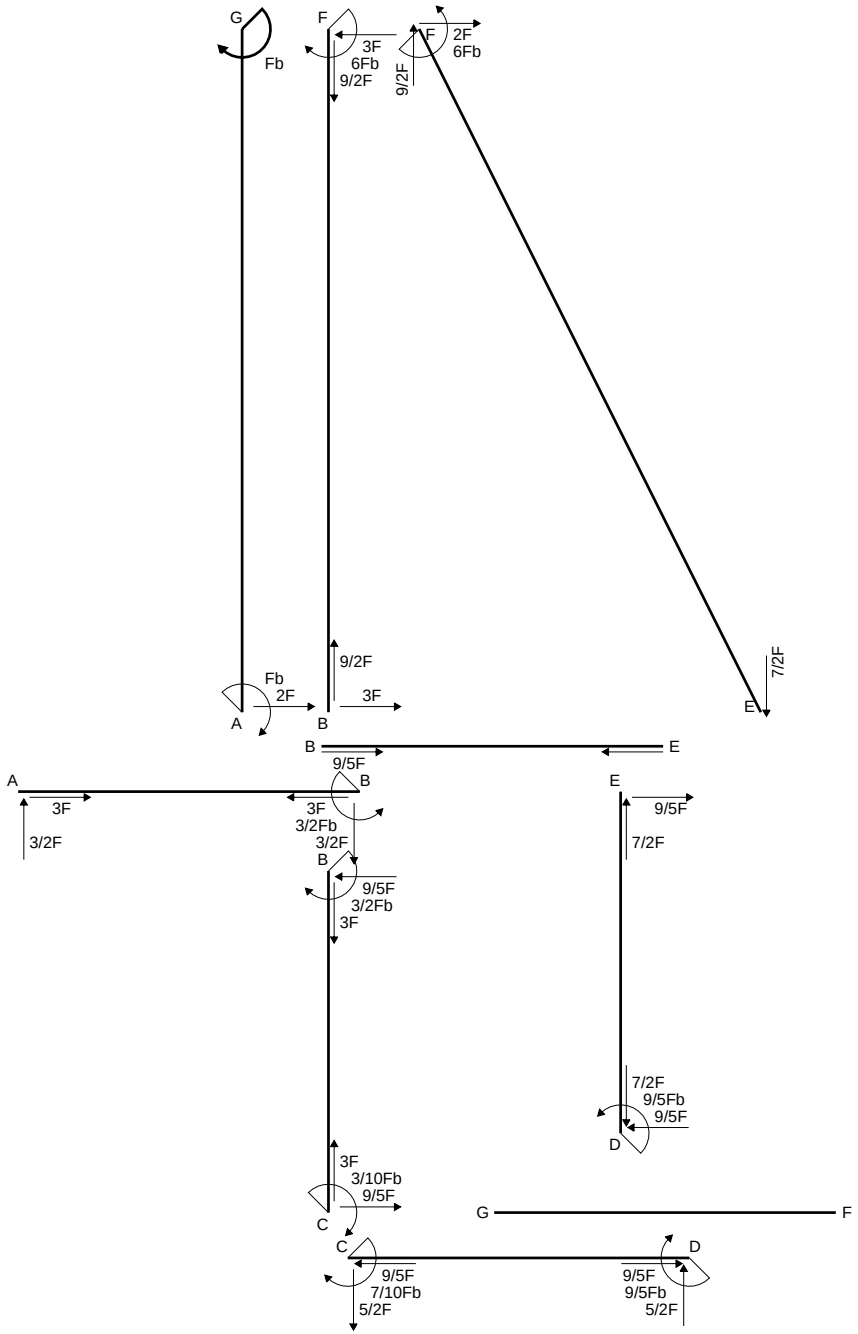
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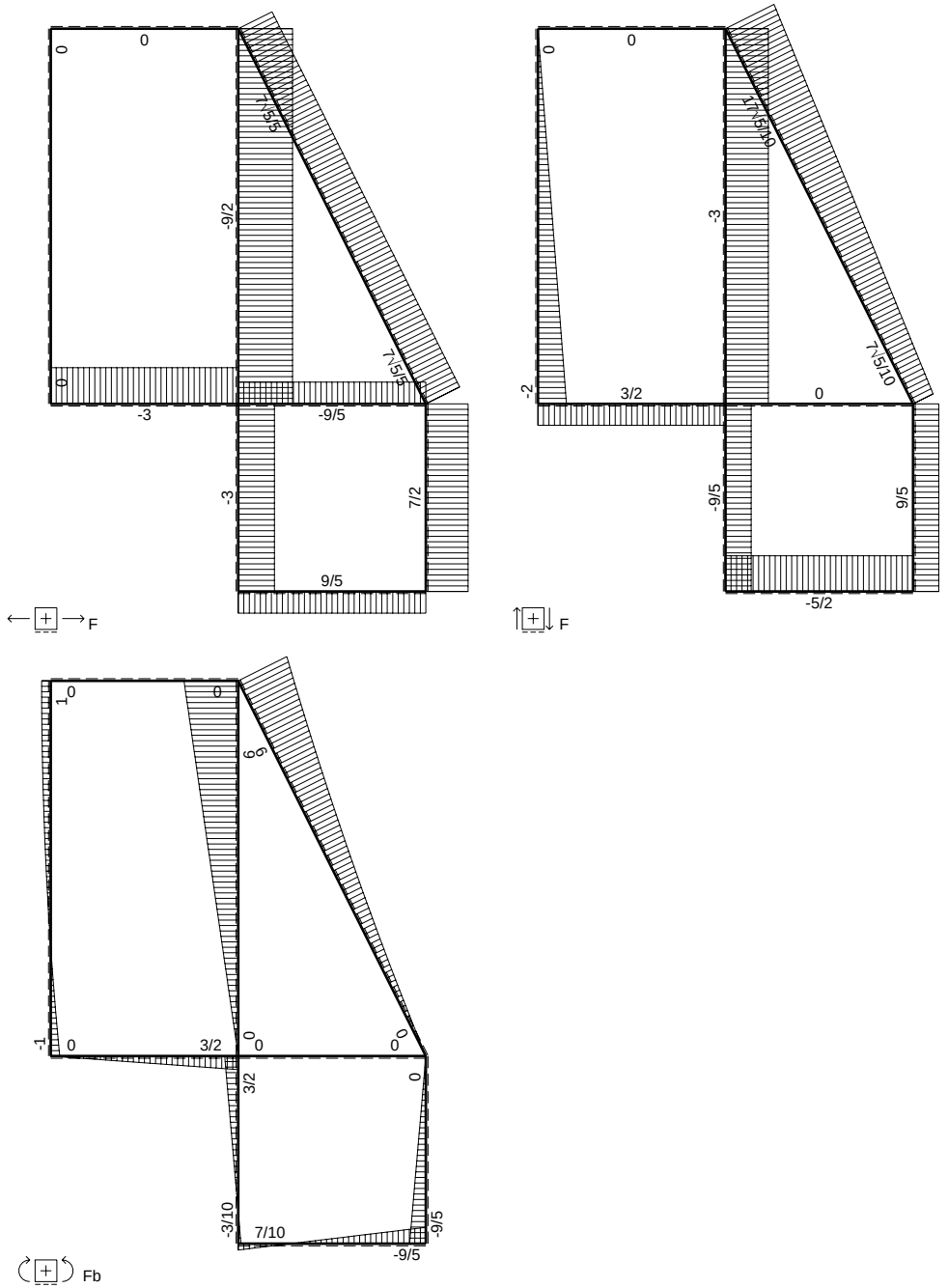
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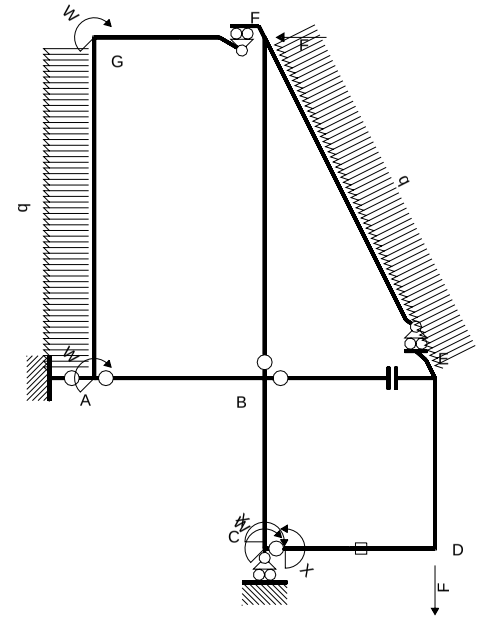
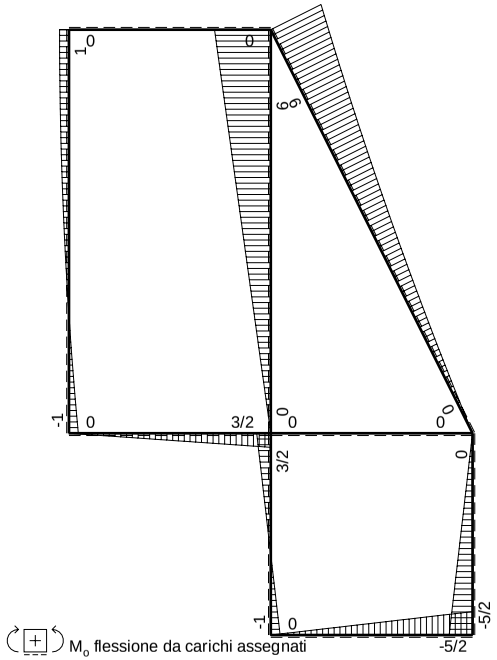
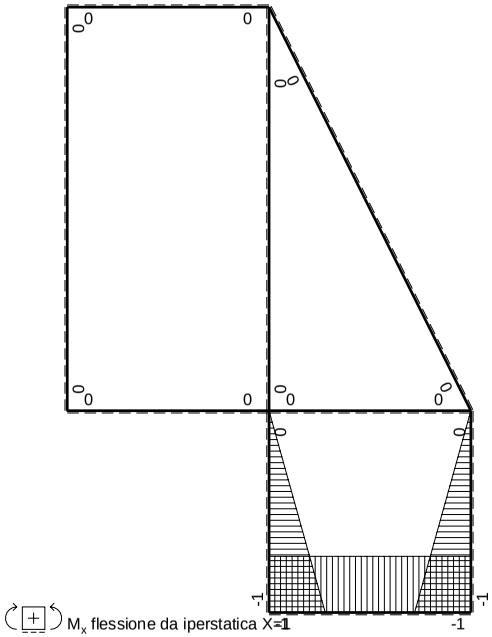
-9/5
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$7/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-7/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

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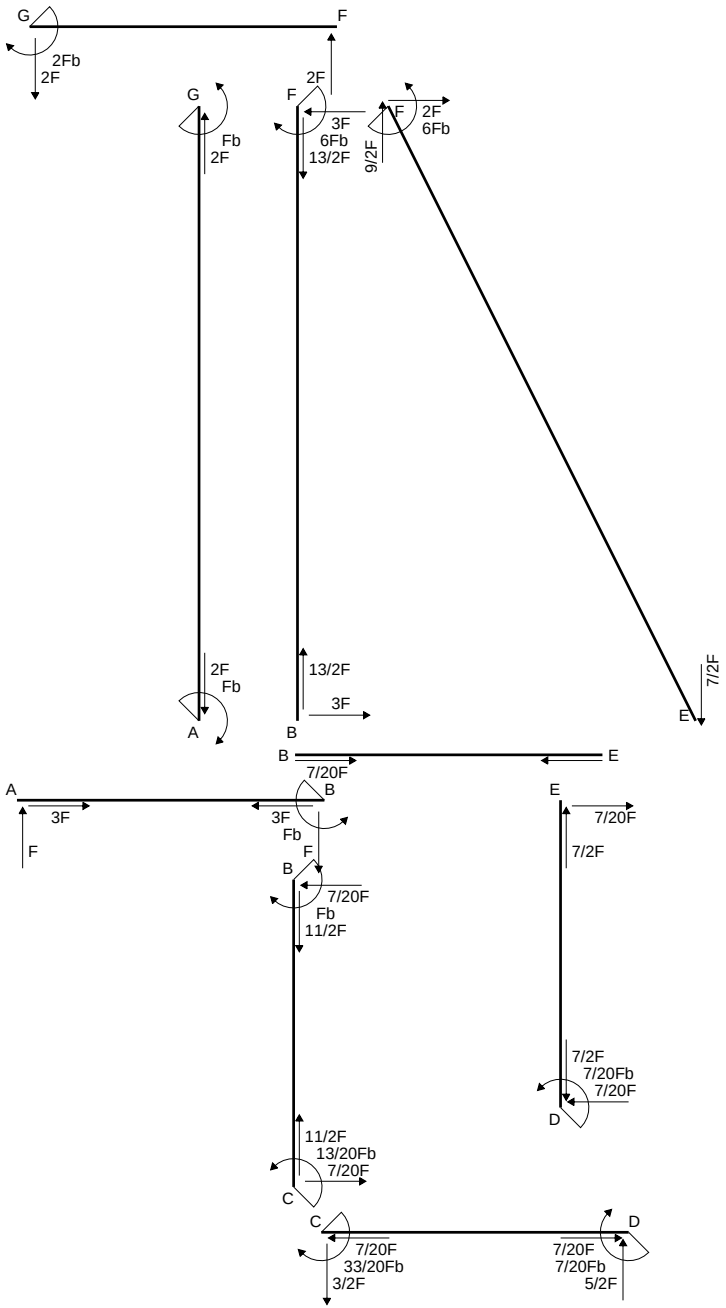
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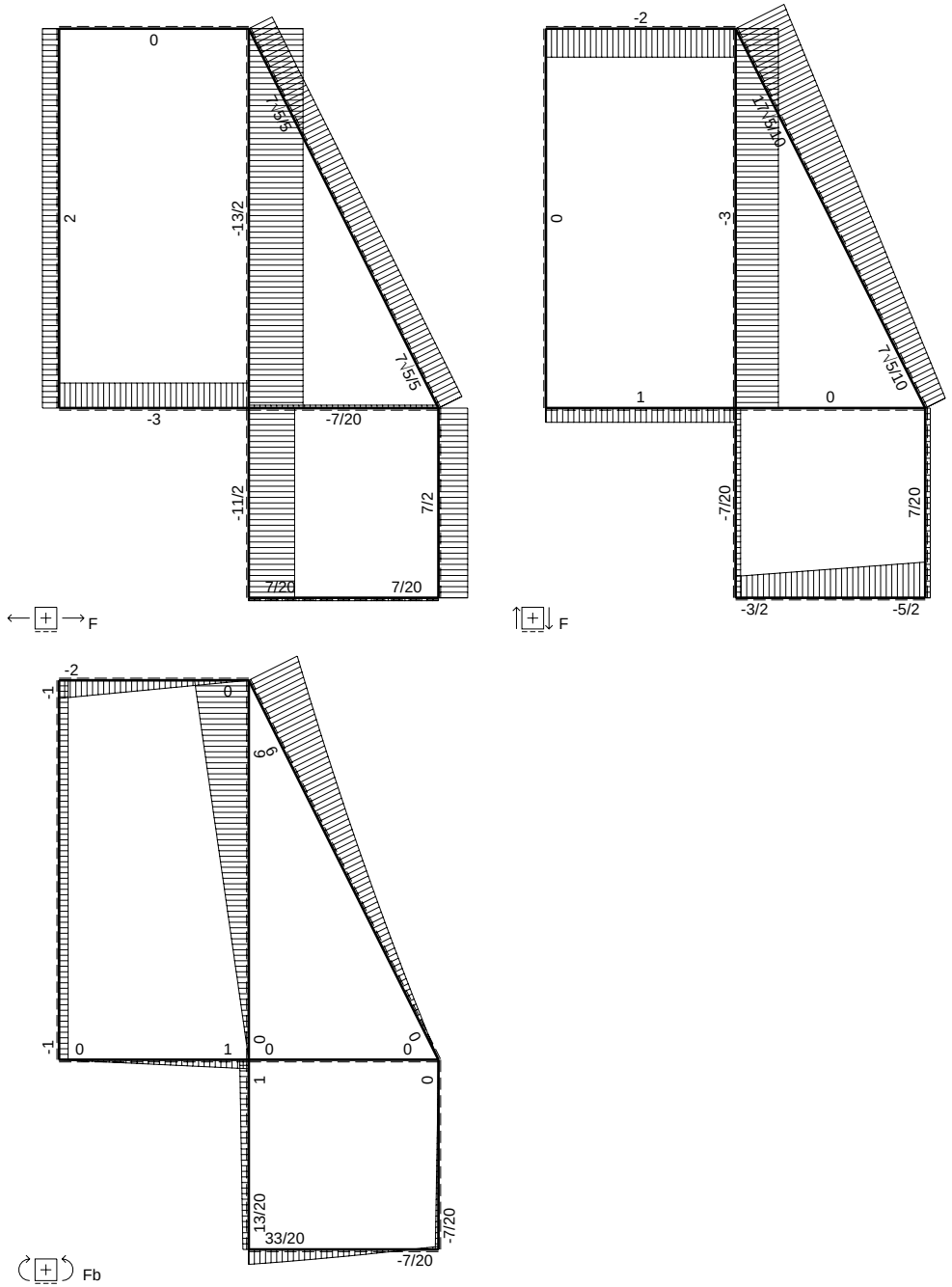
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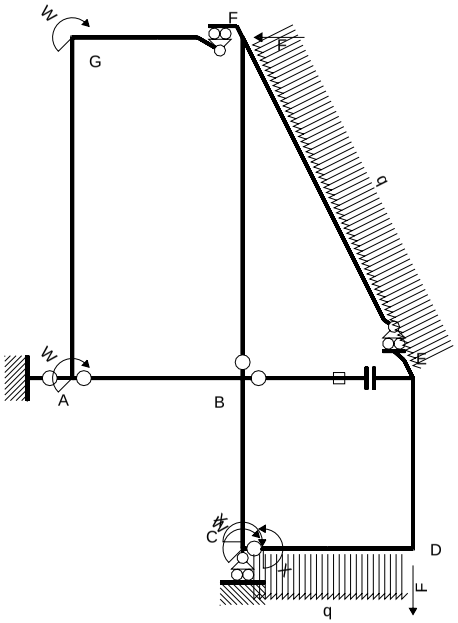
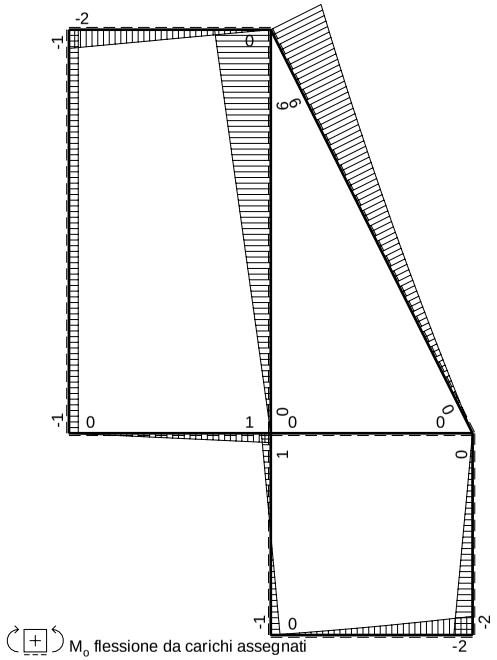
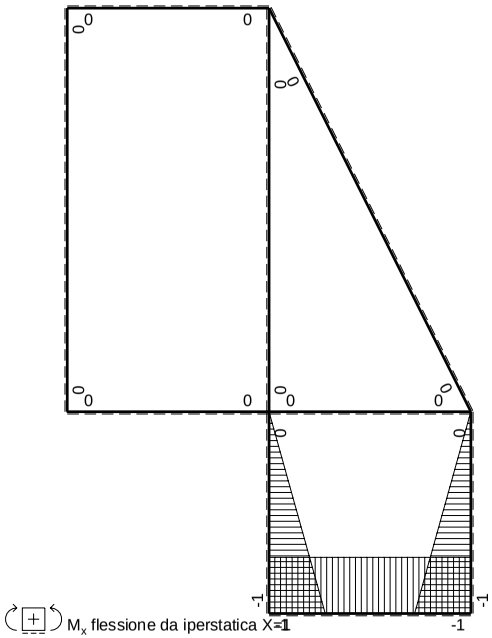


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-3/2Fx-1/2qx^2$	$3/2Fx+1/2Fx^2/b$	1	$11/12Fb^2/EJ$	Xb/EJ
DC b	1	$2Fb-5/2Fx+1/2qx^2$	$2Fb-5/2Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-2Fb+2Fx	$2Fb-4Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$2/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	2Fx	$2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$11/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-33/20Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (2 -5/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [2\ x -5/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -5/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (2 -4\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [2\ x -2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

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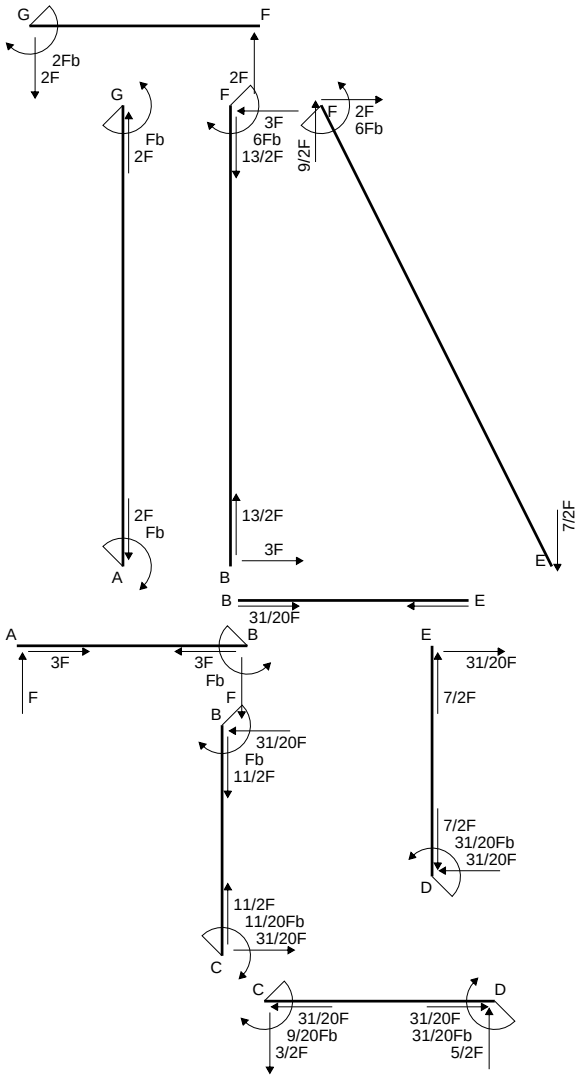
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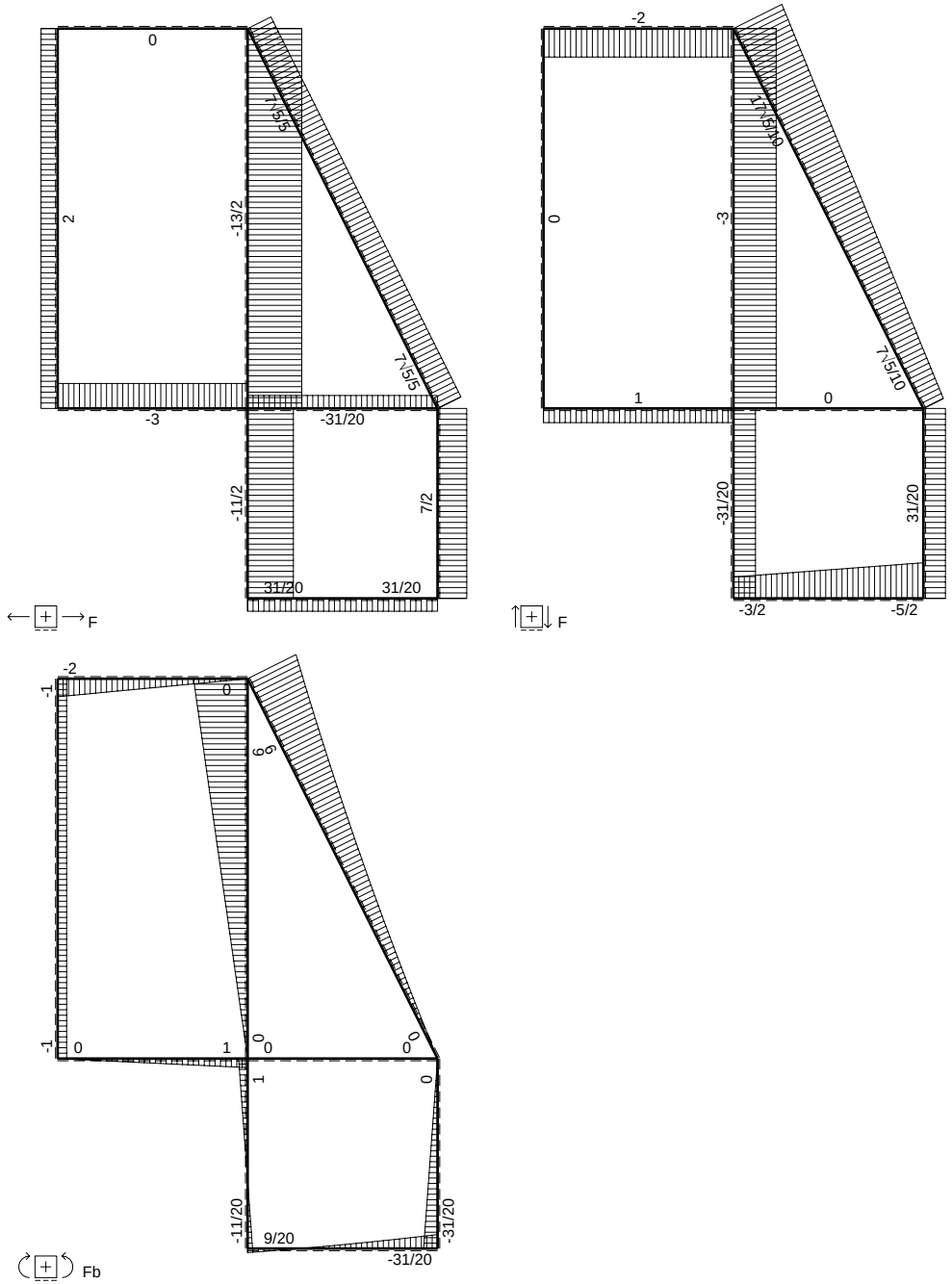
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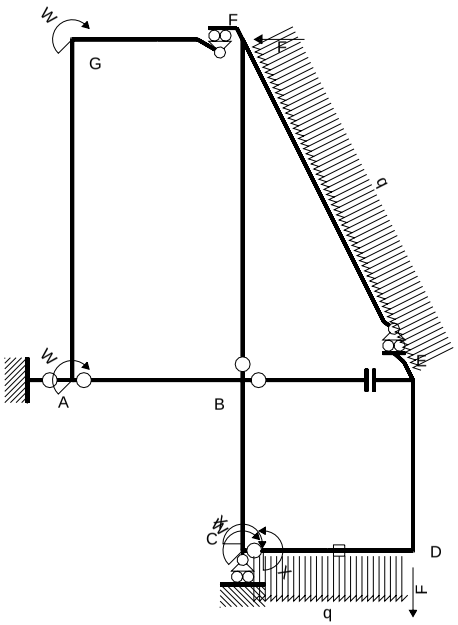
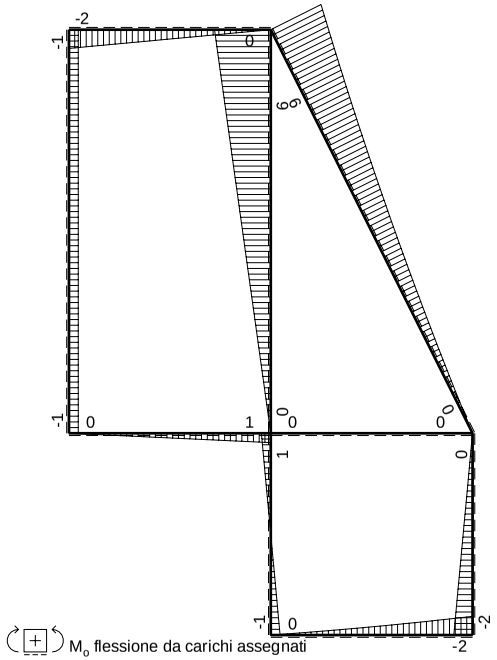
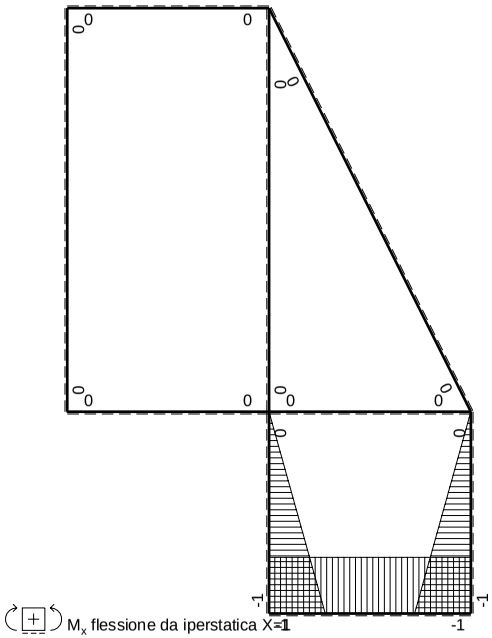


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-3/2Fx-1/2qx^2$	$3/2Fx+1/2Fx^2/b$	1	$11/12Fb^2/EJ$	Xb/EJ
DC b	1	$2Fb-5/2Fx+1/2qx^2$	$2Fb-5/2Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-2Fb+2Fx	$2Fb-4Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$2/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	2Fx	$2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-9/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [3/4\ x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/4\ b +1/6\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/12\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (2 -5/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [2\ x -5/4\ x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (2\ b -5/4\ b +1/6\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/12\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (2 -4\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [2\ x -2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (2\ b -2\ b +2/3\ b)\ Fb\ 1/EJ\ = 2/3\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (2/3\ b)\ Fb\ 1/EJ\ = 2/3\ \ Fb^2/EJ$

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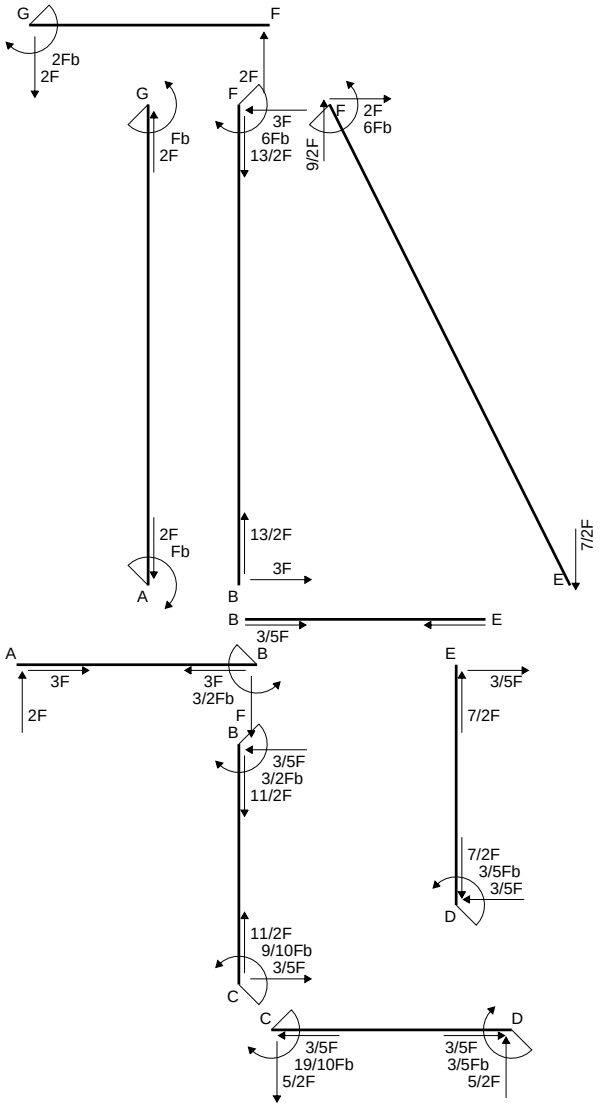
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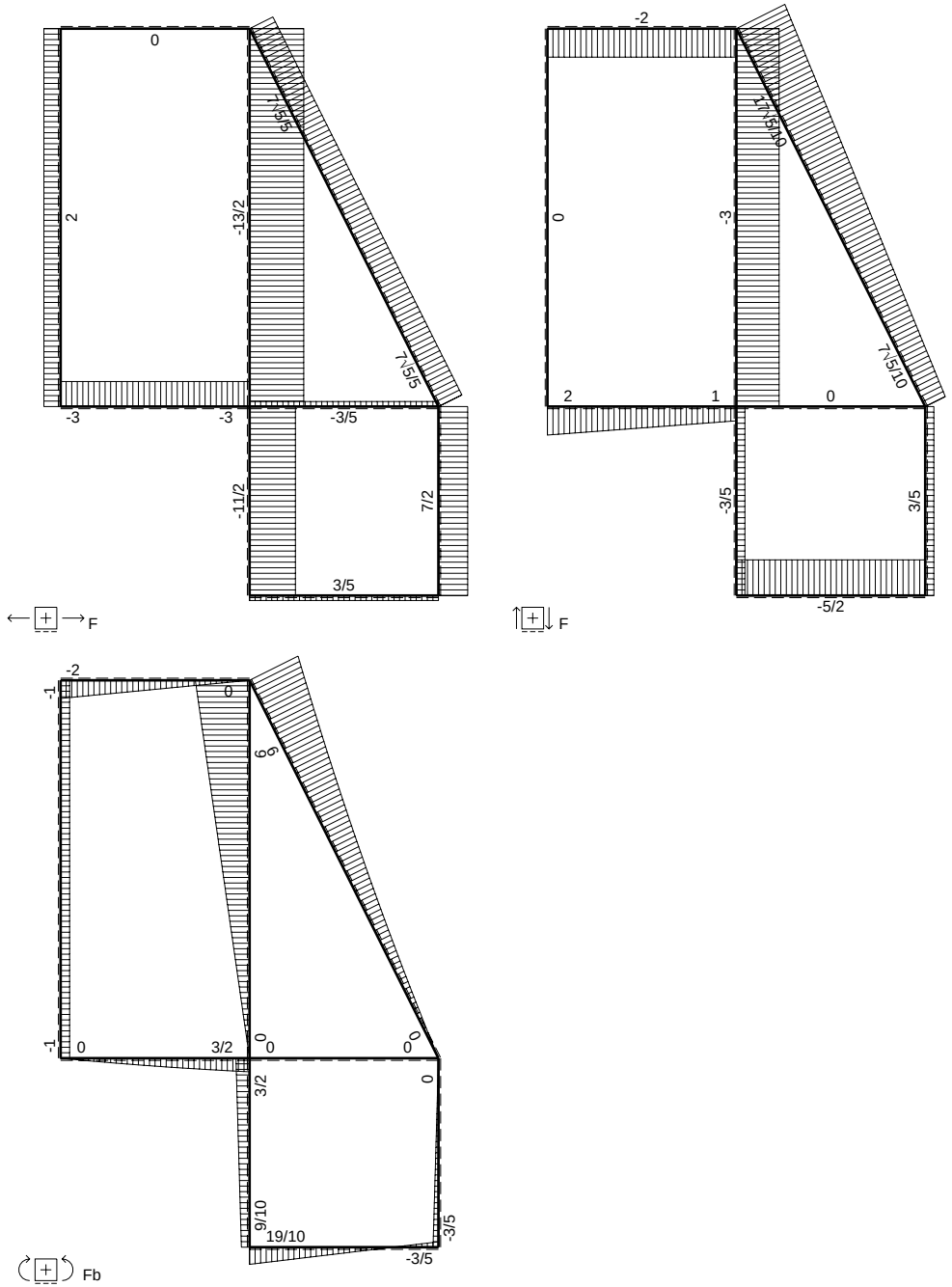
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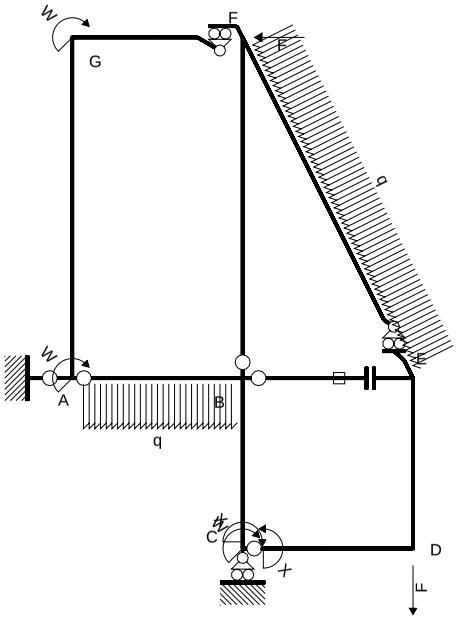
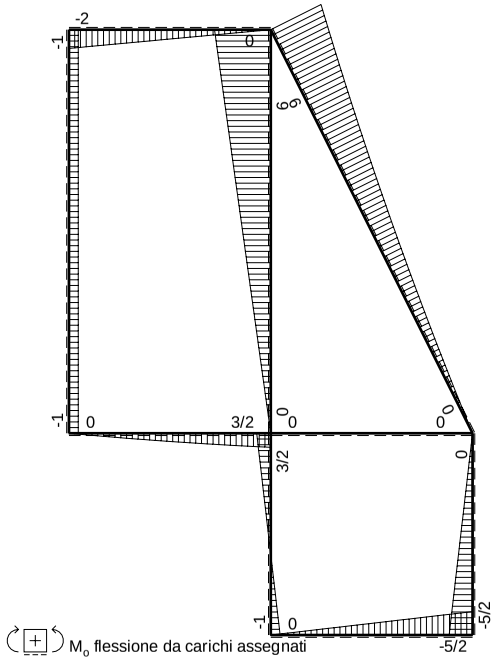
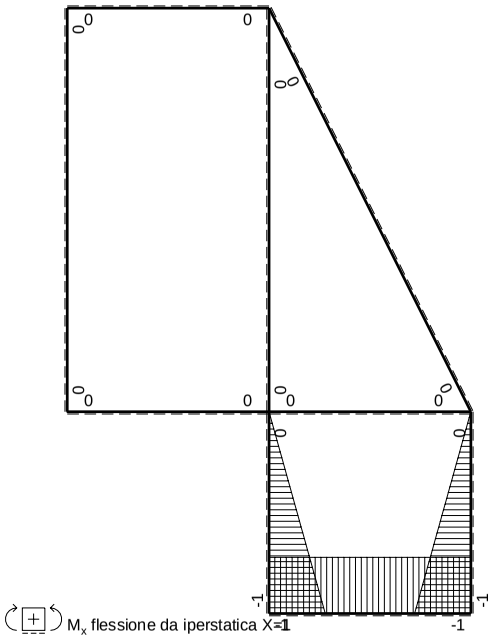
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-3/2Fb+Fx+1/2qx^2$	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$19/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-19/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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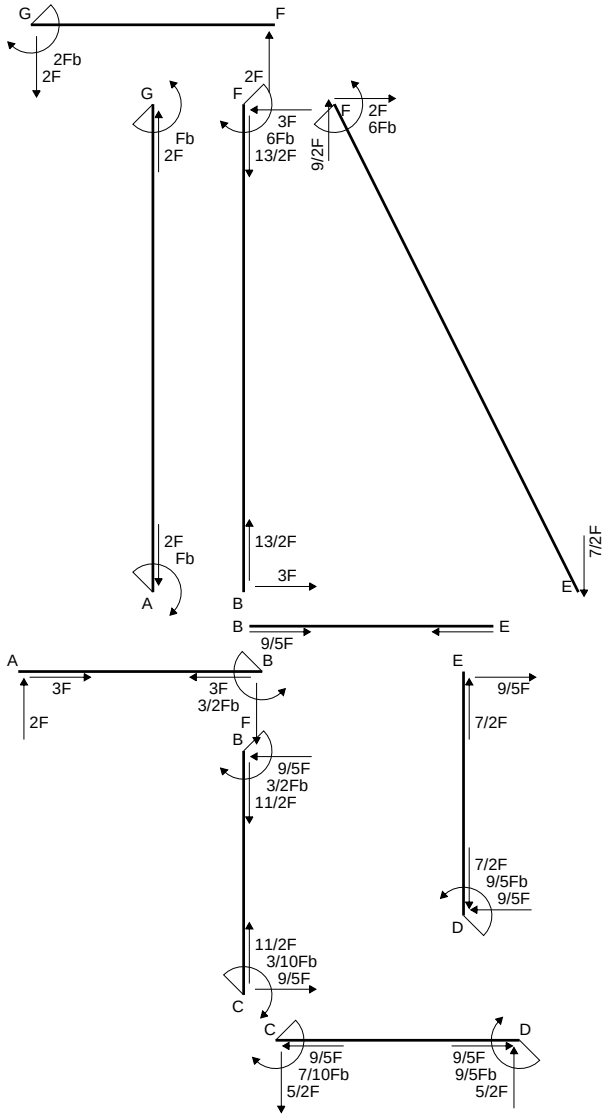
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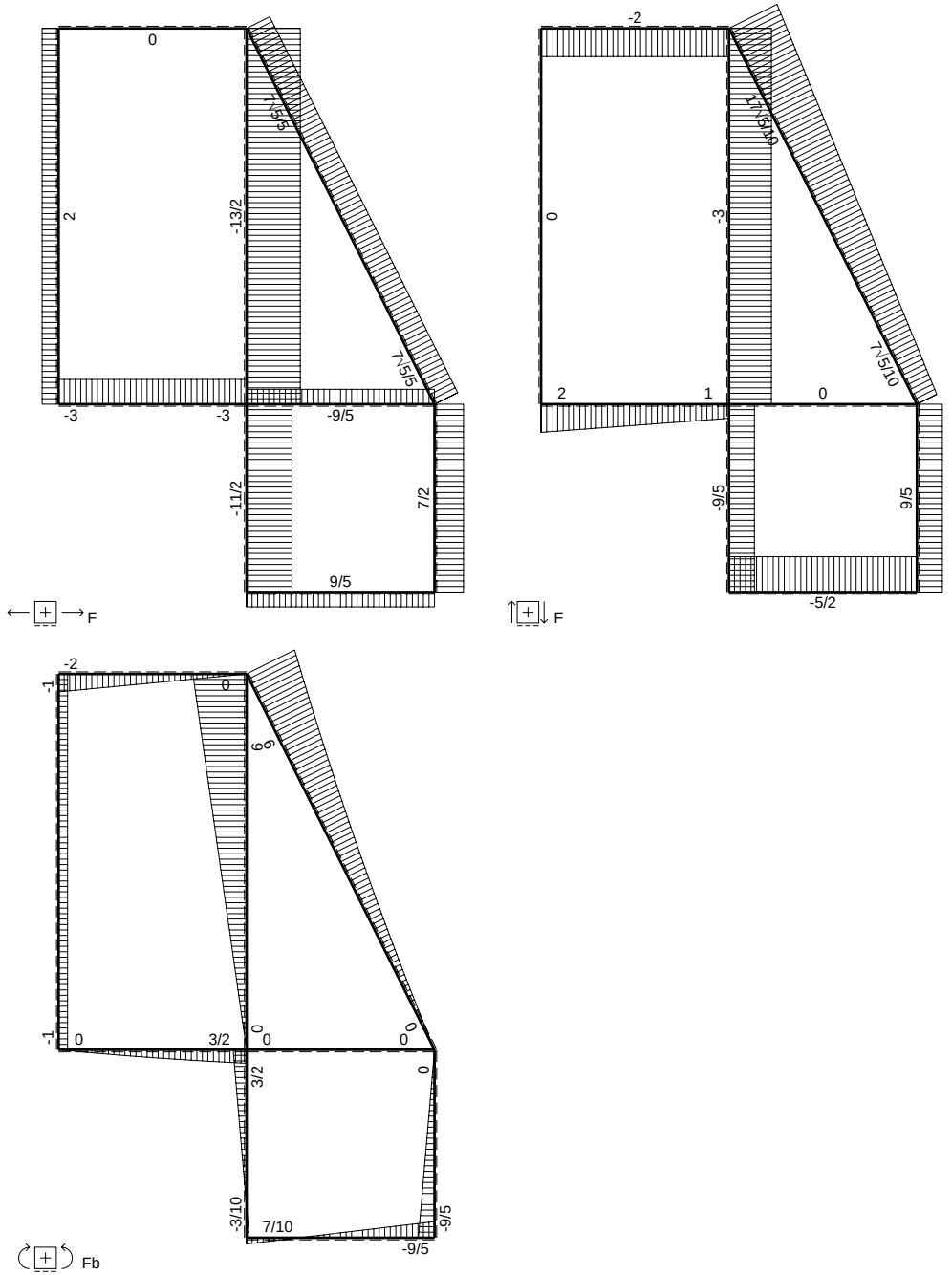
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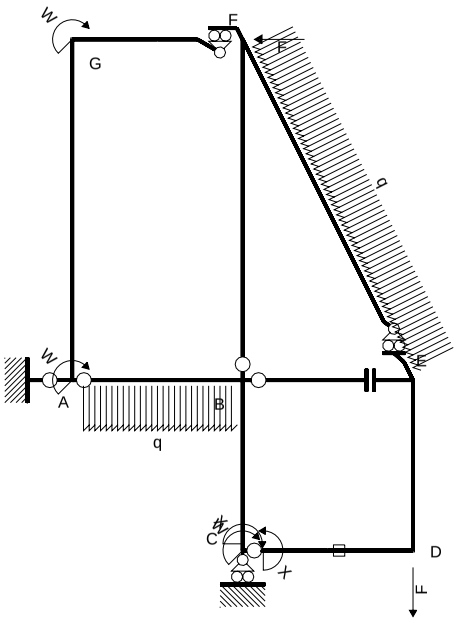
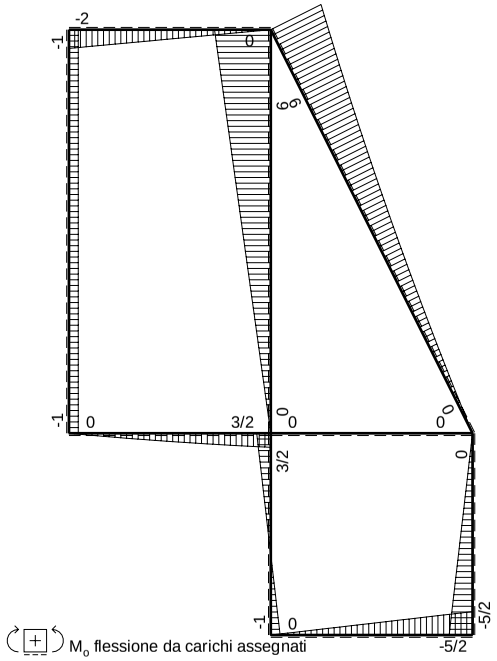
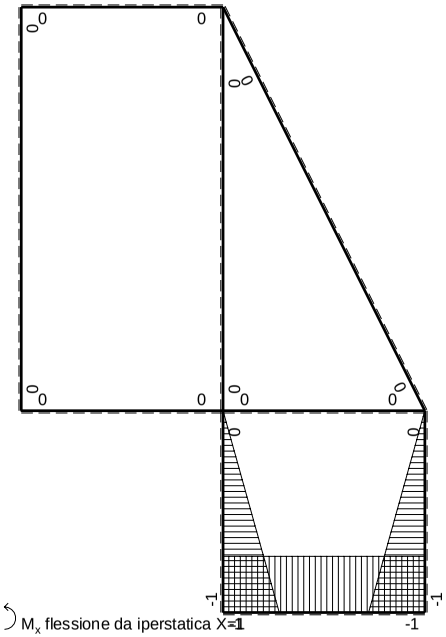
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-3/2Fb+Fx+1/2qx^2$	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$7/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-7/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

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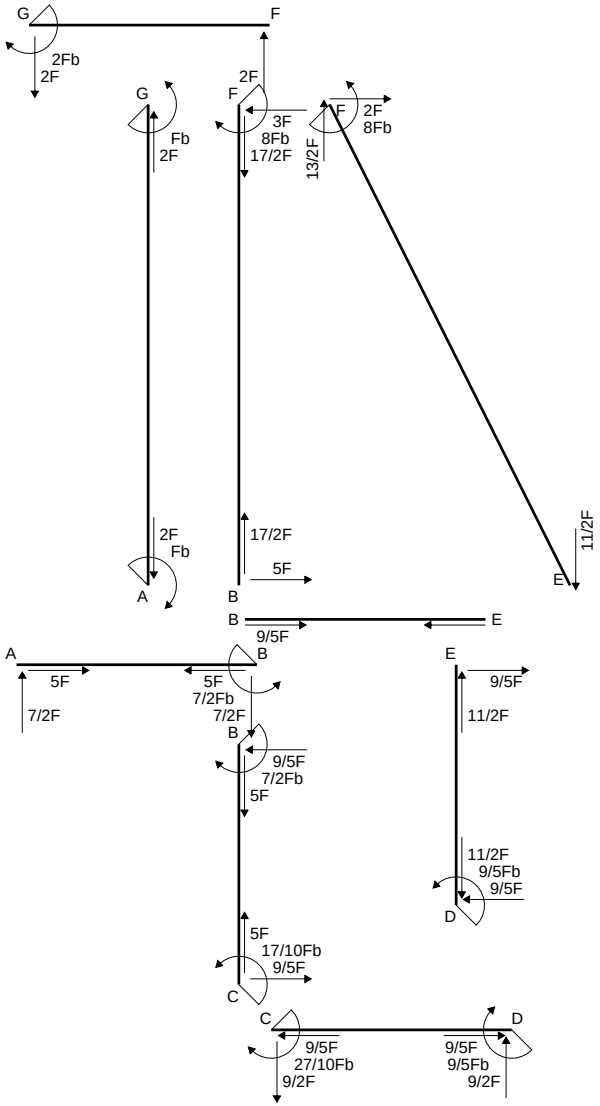
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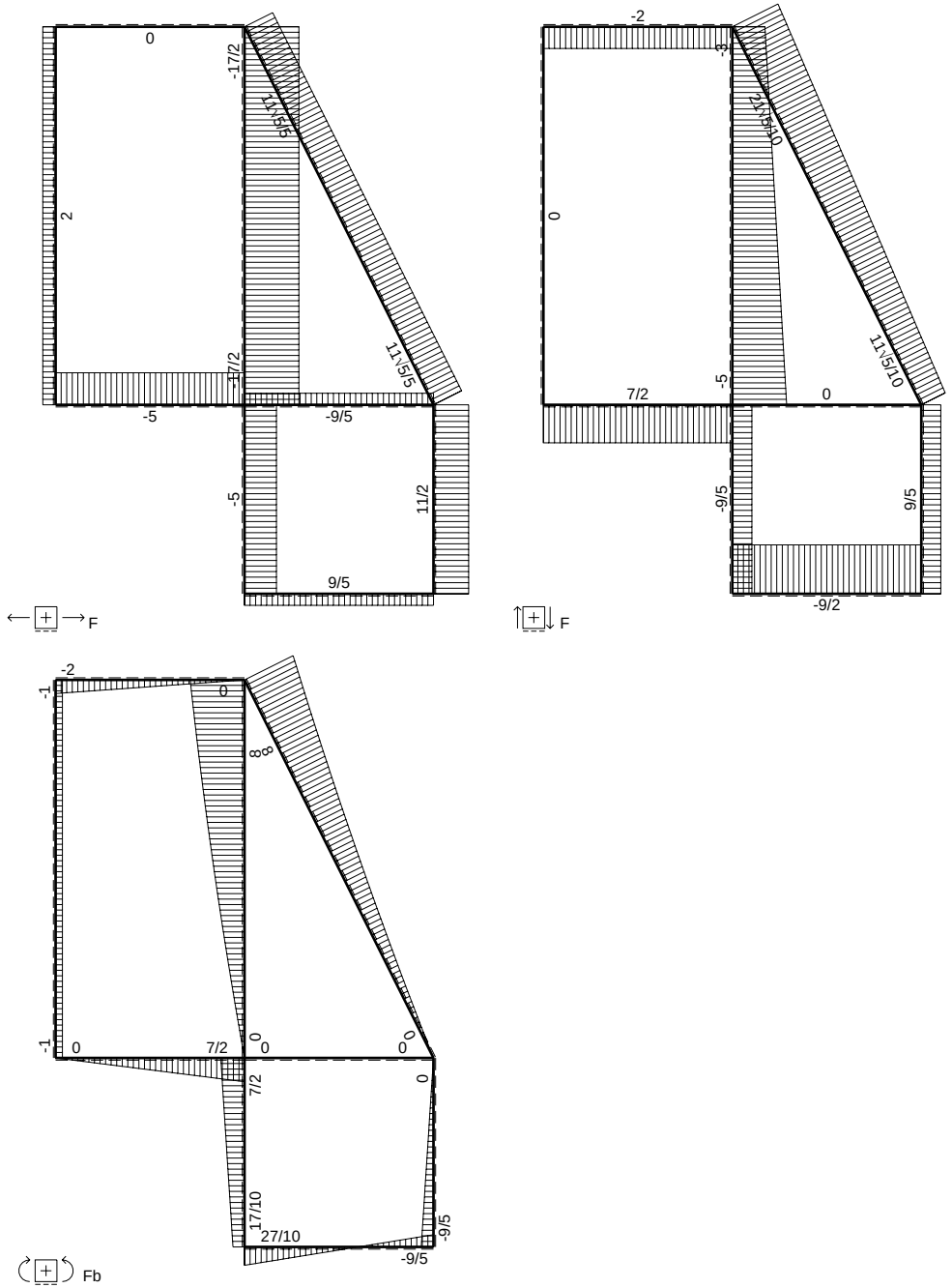
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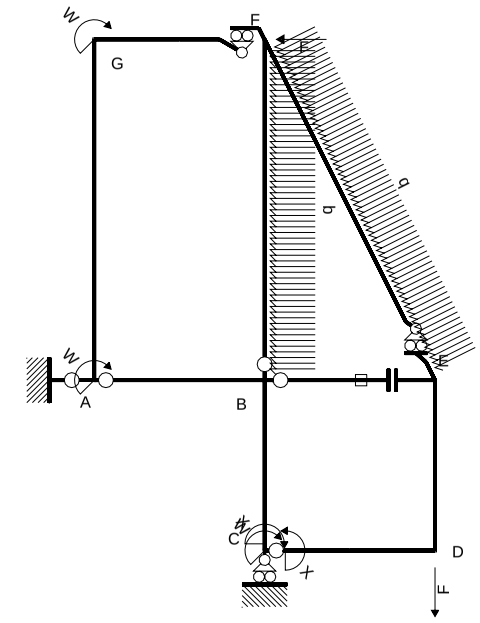
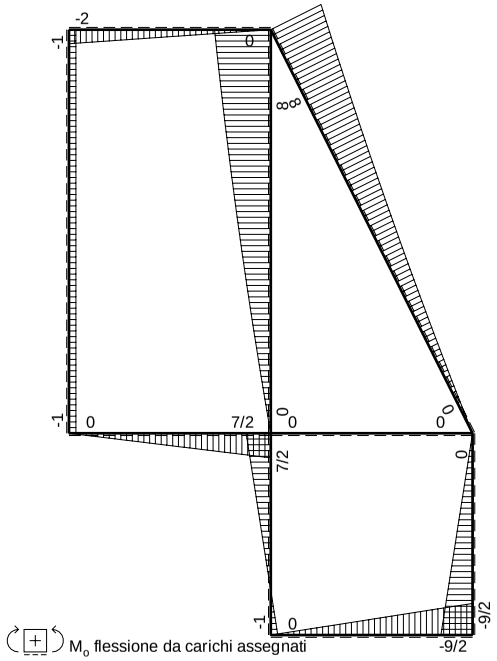
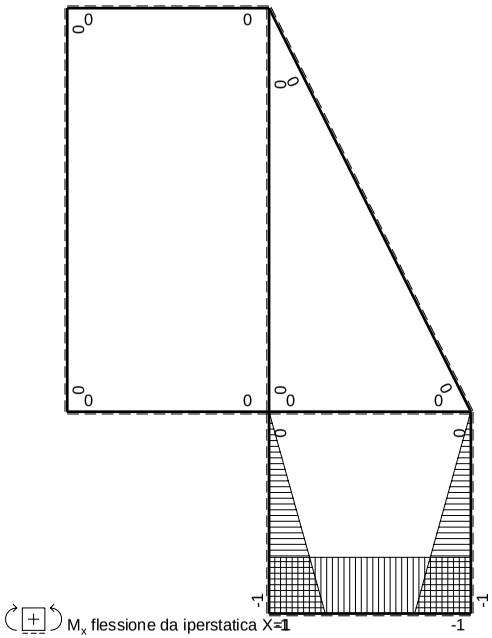
-9/5
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	7/2Fx	0	0	0	0
BA b	0	-7/2Fb+7/2Fx	0	0		
BC b	-x/b	7/2Fb-9/2Fx	$-7/2Fx+9/2Fx^2/b$	x^2/b^2	$-1/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-9/2Fx	$Fb-11/2Fx+9/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-9/2Fx	9/2Fx	1	$9/4Fb^2/EJ$	Xb/EJ
DC b	1	9/2Fb-9/2Fx	9/2Fb-9/2Fx	1		
DE b	-1+x/b	$-9/2Fb+9/2Fx$	$9/2Fb-9Fx+9/2Fx^2/b$	$1-2x/b+x^2/b^2$	$3/2Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	9/2Fx	$9/2Fx^2/b$	x^2/b^2		
EF √5b	0	$11\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$8Fb-3Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-5Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$9/2Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-27/10Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-7/2\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-7/4\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-7/4\ b +3/2\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -11/2\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -11/4\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -11/4\ b +3/2\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (9/2\ x/b\ )\ Fb\ 1/EJ\ dx = [9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (9/4\ b\ )\ Fb\ 1/EJ\ = 9/4\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (9/2 -9/2\ x/b\ )\ Fb\ 1/EJ\ dx = [9/2\ x -9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -9/4\ b\ )\ Fb\ 1/EJ\ = 9/4\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (9/2 -9\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [9/2\ x -9/2\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -9/2\ b +3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

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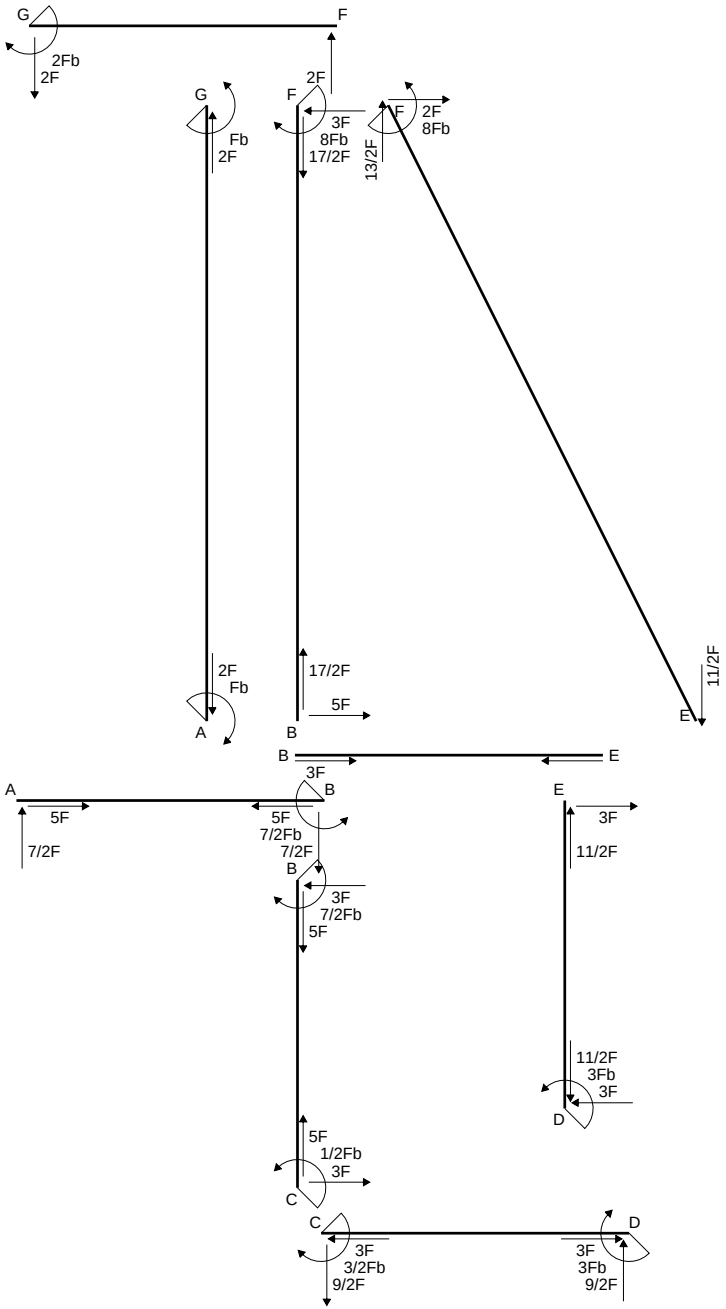
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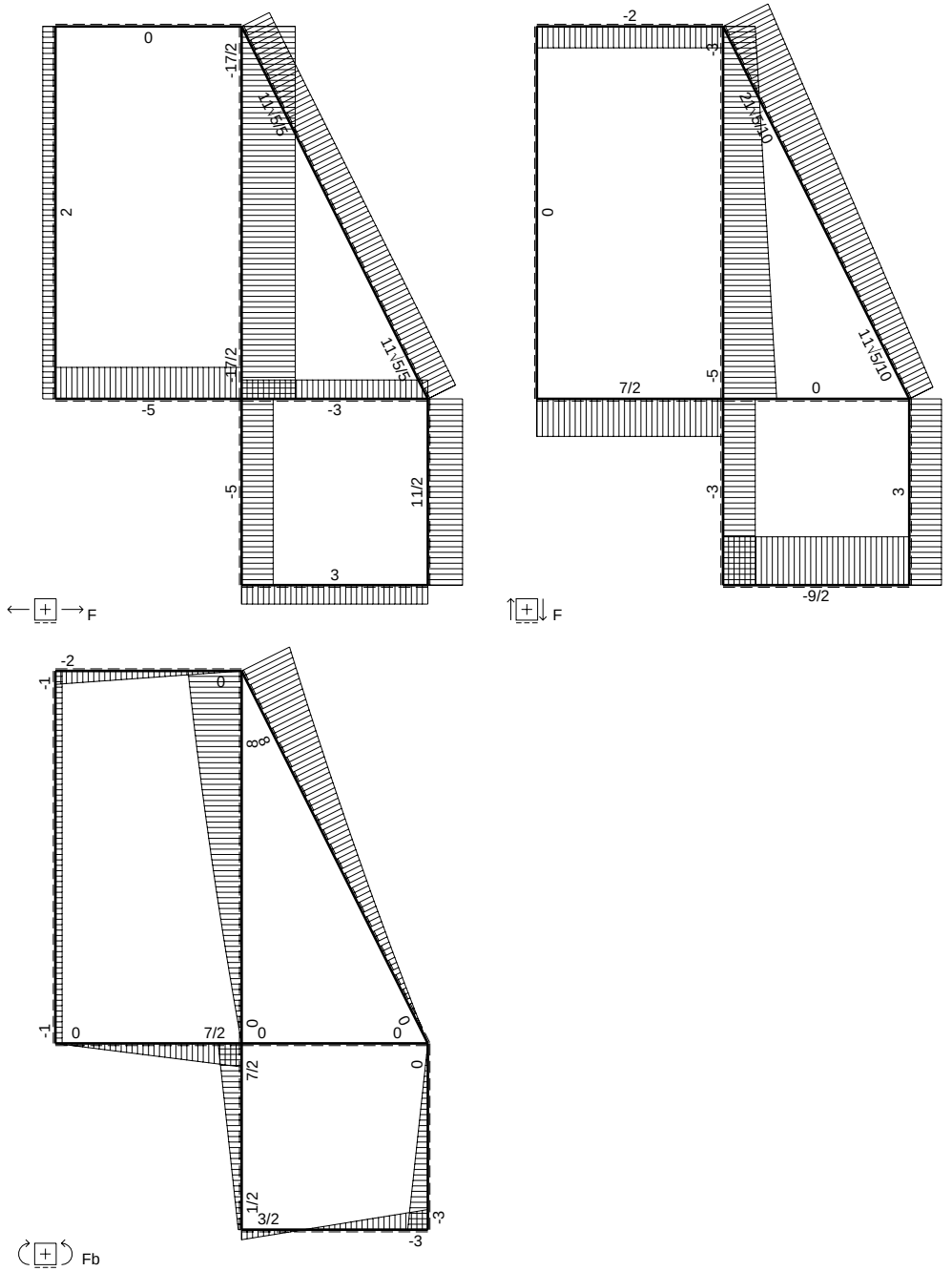
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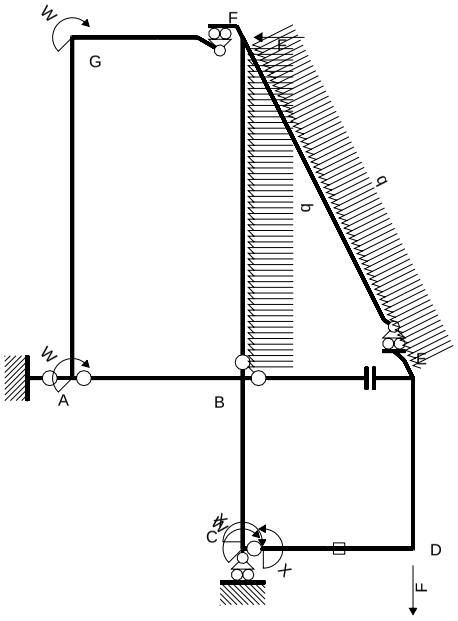
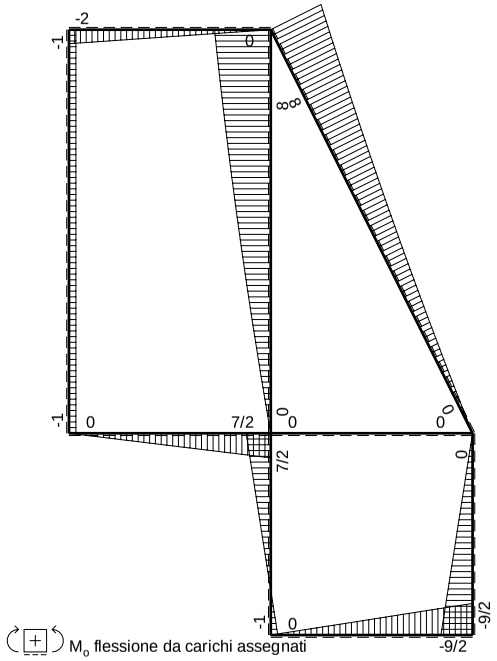
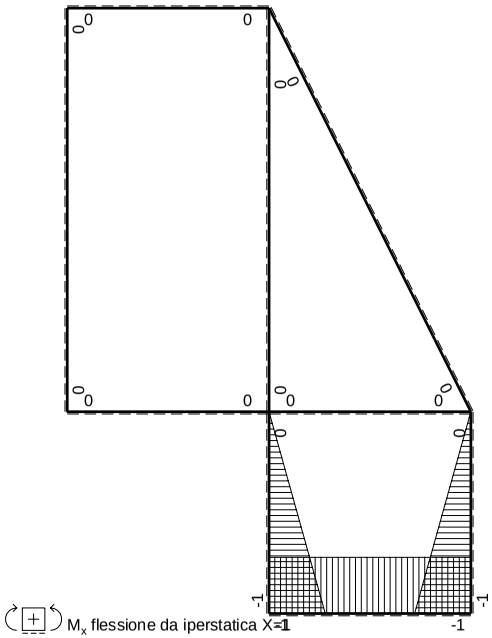


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	7/2Fx	0	0	0	0
BA b	0	-7/2Fb+7/2Fx	0	0		
BC b	-x/b	7/2Fb-9/2Fx	-7/2Fx+9/2Fx^2/b	x^2/b^2	-1/4Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-9/2Fx	Fb-11/2Fx+9/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-9/2Fx	9/2Fx	1	9/4Fb^2/EJ	Xb/EJ
DC b	1	9/2Fb-9/2Fx	9/2Fb-9/2Fx	1		
DE b	-1+x/b	-9/2Fb+9/2Fx	9/2Fb-9Fx+9/2Fx^2/b	1-2x/b+x^2/b^2	3/2Fb^2/EJ	1/3Xb/EJ
ED b	x/b	9/2Fx	9/2Fx^2/b	x^2/b^2		
EF √5b	0	11√5/10Fx+1/2qx^2	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	8Fb-3Fx-1/2qx^2	0	0	0	0
BF 2b	0	-5Fx+1/2qx^2	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				5/2Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-3/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-7/2\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-7/4\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-7/4\ b +3/2\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -11/2\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -11/4\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -11/4\ b +3/2\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (9/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (9/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 5/4\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (9/2 -9/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [9/2\ x -9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (9/2\ b -9/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 5/4\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (9/2 -9\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [9/2\ x -9/2\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -9/2\ b +3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

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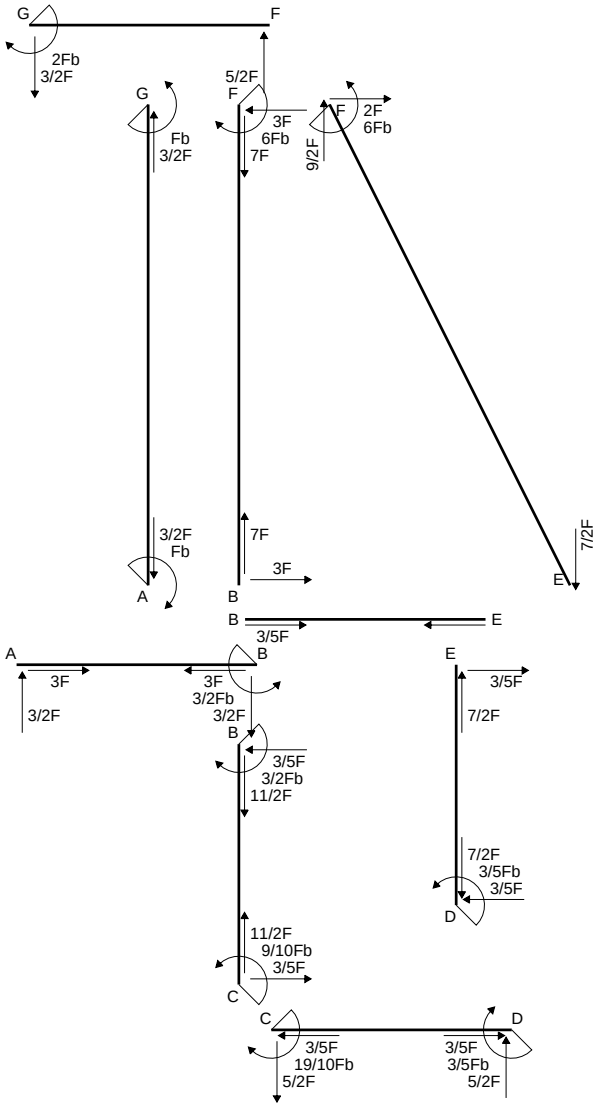
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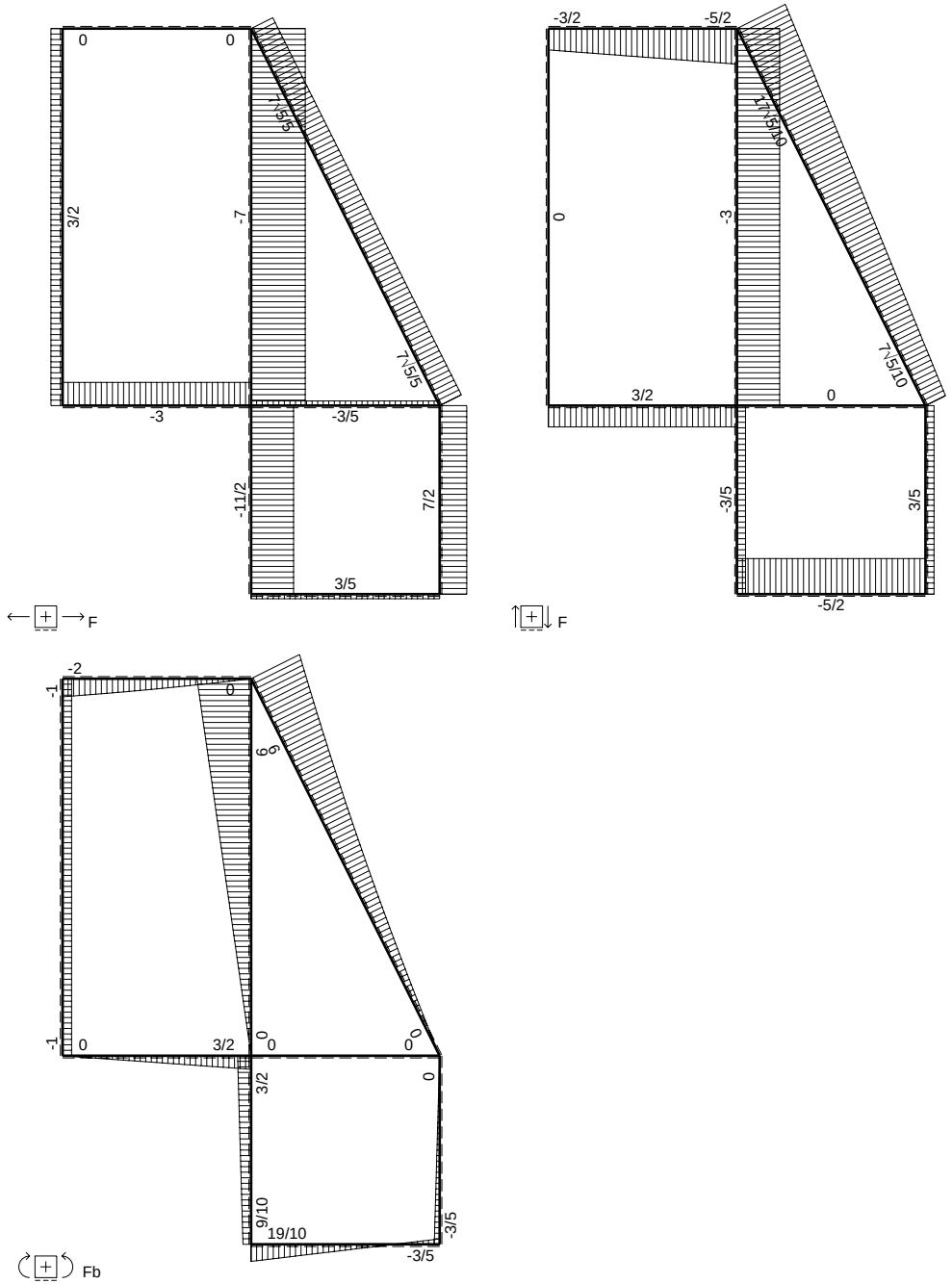
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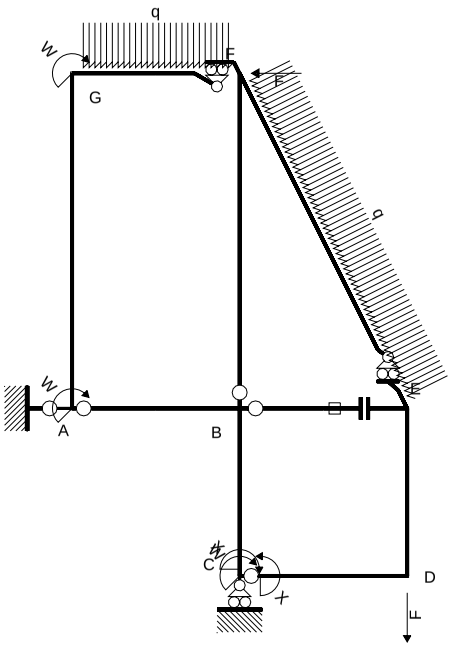
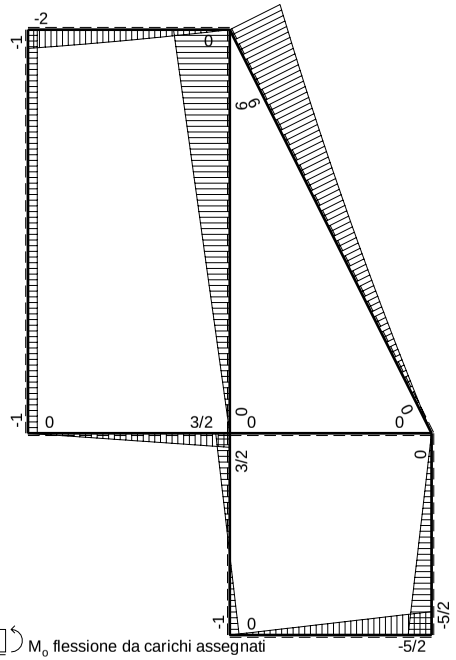
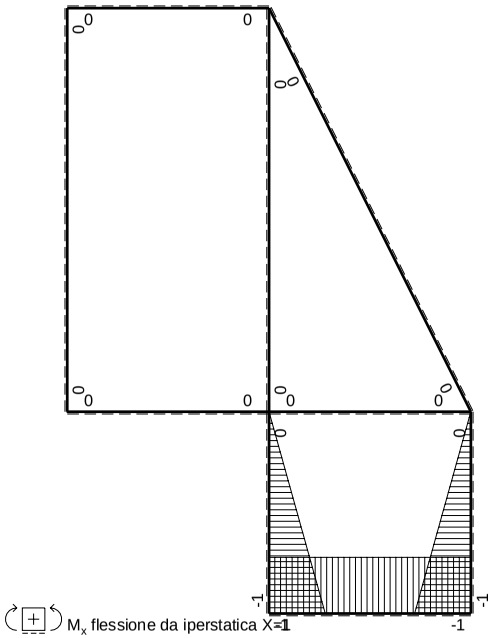


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	-3/2Fx+5/2Fx^2/b	x^2/b^2	1/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	Fb-7/2Fx+5/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-5/2Fx	5/2Fx	1	5/4Fb^2/EJ	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	5/2Fb-5Fx+5/2Fx^2/b	1-2x/b+x^2/b^2	5/6Fb^2/EJ	1/3Xb/EJ
ED b	x/b	5/2Fx	5/2Fx^2/b	x^2/b^2		
EF √5b	0	7√5/10Fx+1/2qx^2	0	0	0	0
FG b	0	-5/2Fx+1/2qx^2	0	0	0	0
GF b	0	2Fb-3/2Fx-1/2qx^2	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-19/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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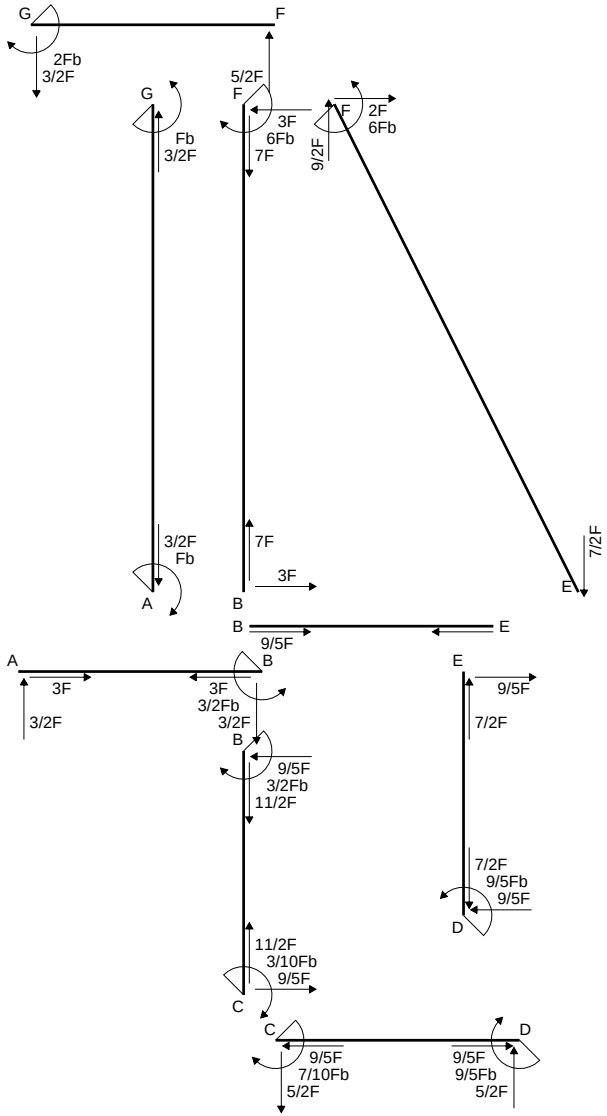
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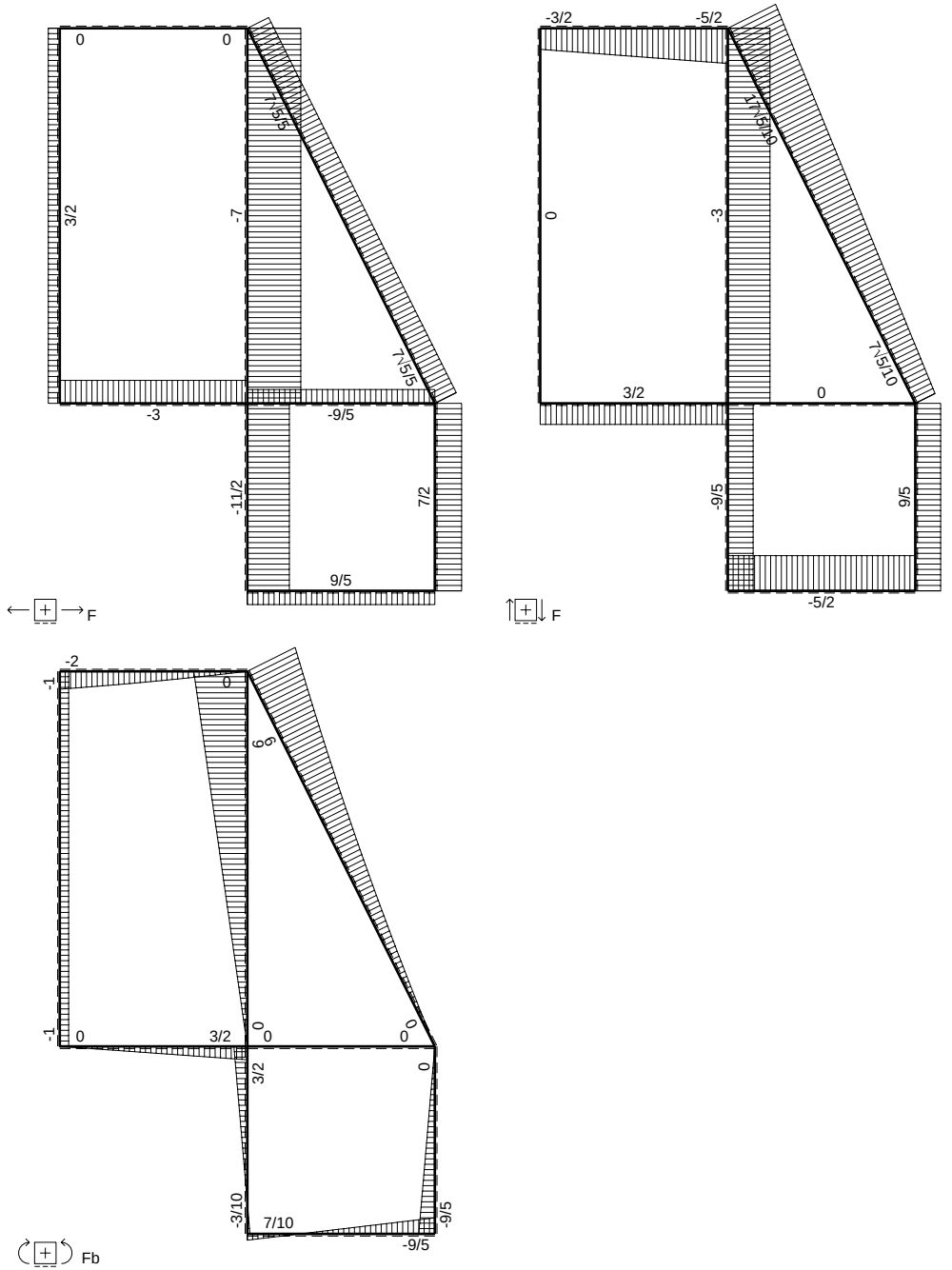
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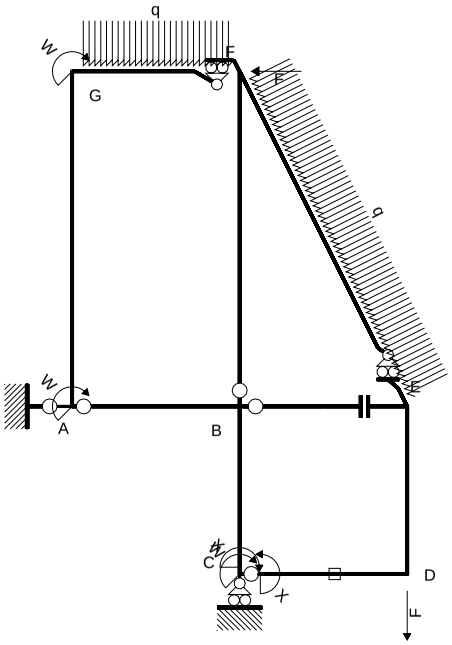
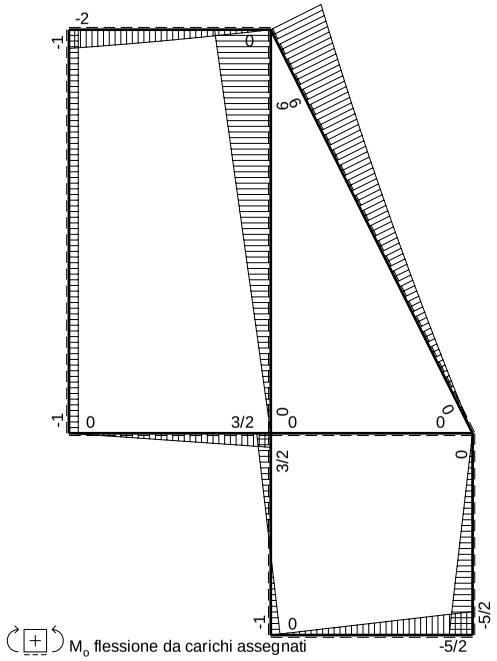
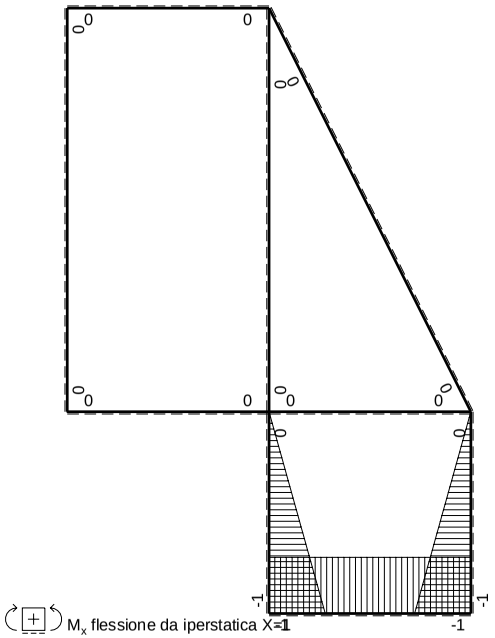


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$7/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-7/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

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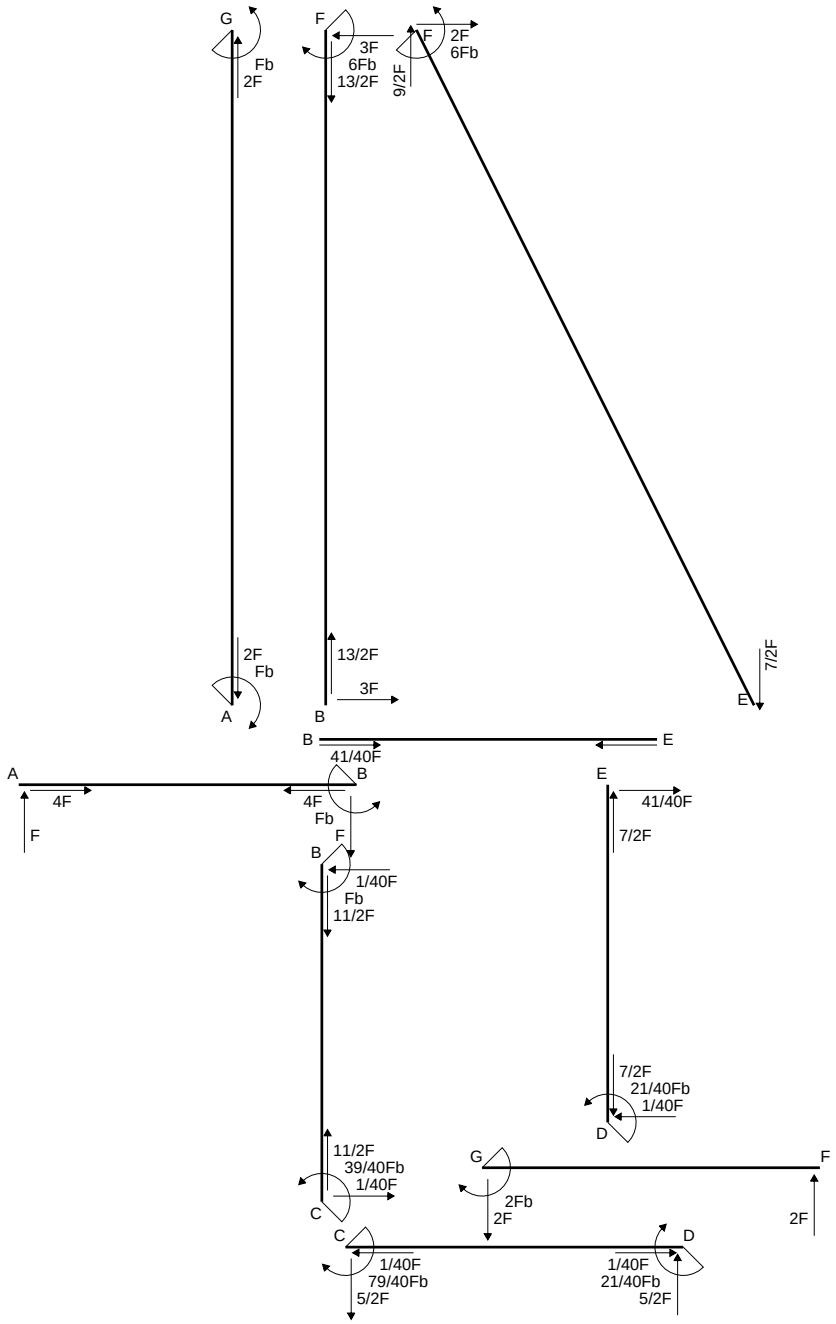
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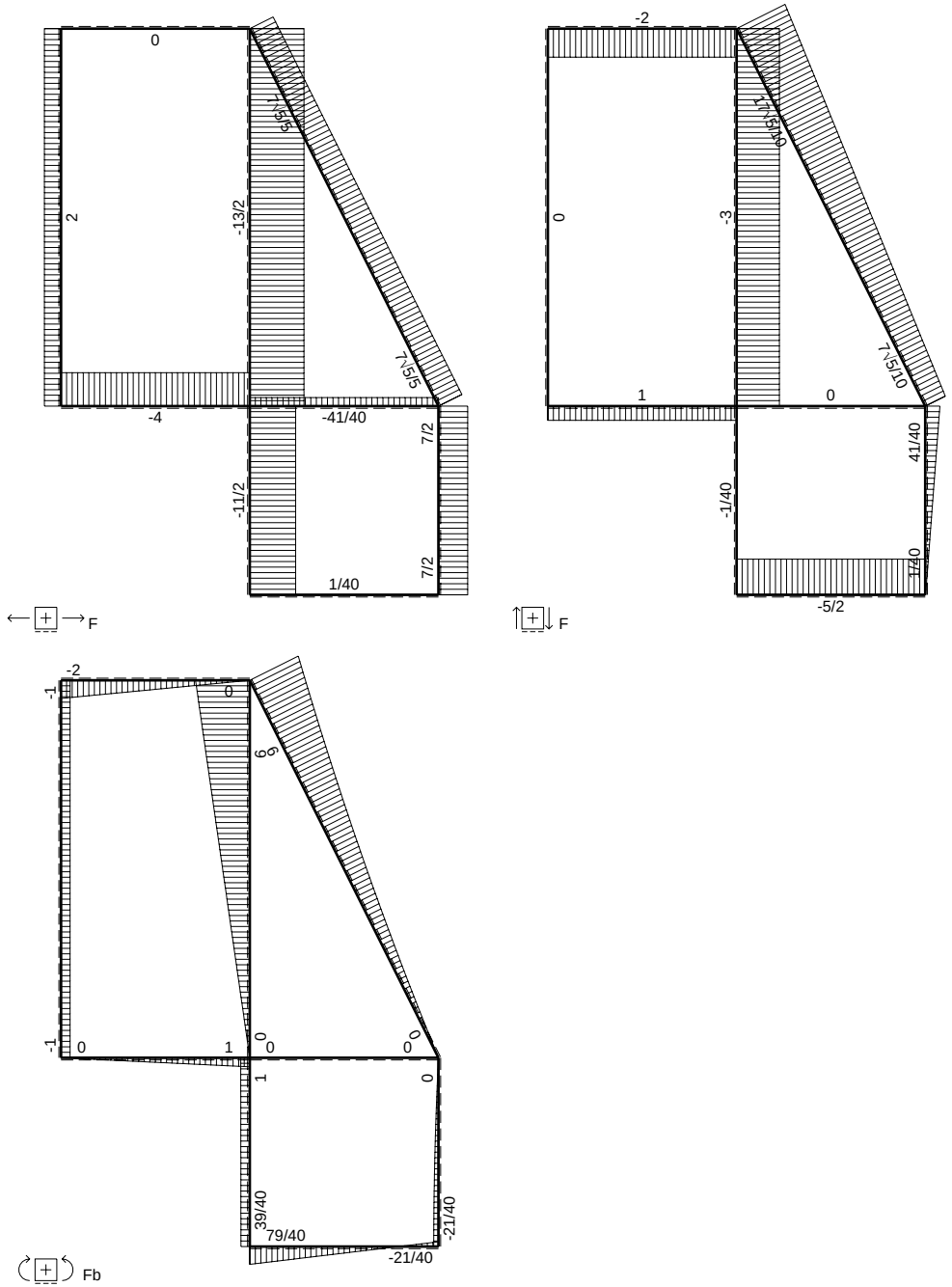
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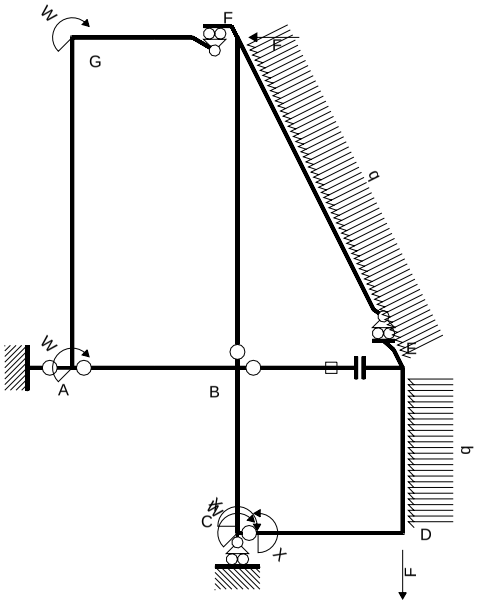
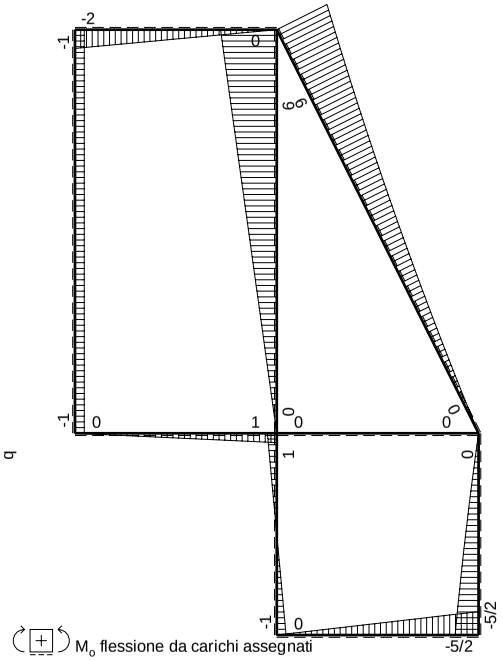
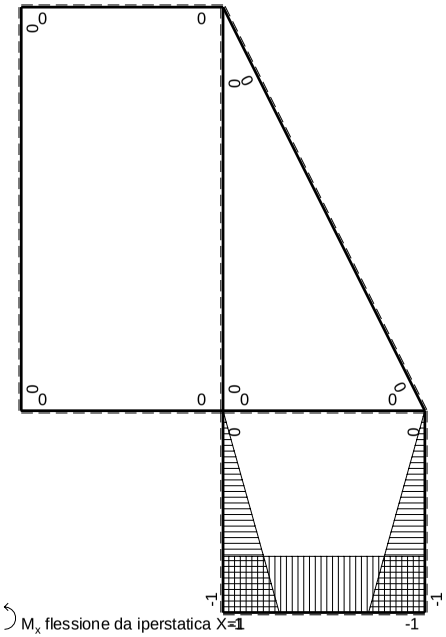
-21/40
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 213077 ESΣ02.xxxx.163

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	$-5/2Fb+2Fx+1/2qx^2$	$5/2Fb-9/2Fx+3/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$7/8Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$3Fx-1/2qx^2$	$3Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$79/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-79/40Fb	

PROCEDIMENTO E RISULTATI 213077 ESΣ02.xxxx.163

Sviluppi di calcolo iperstatica

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$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (- x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/4\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -9/2\ x/b +3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [5/2\ x -9/4\ x^2/b +1/2\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -9/4\ b +1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = 7/8\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (\ b -1/8\ b)\ Fb\ 1/EJ\ = 7/8\ Fb^2/EJ$

ESΣ02.xxxx.163

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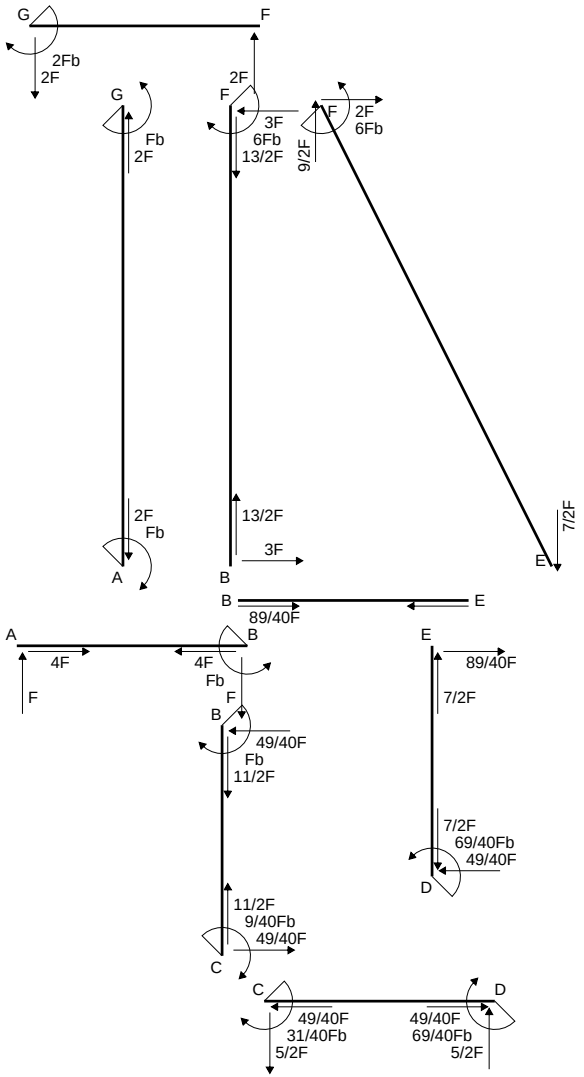
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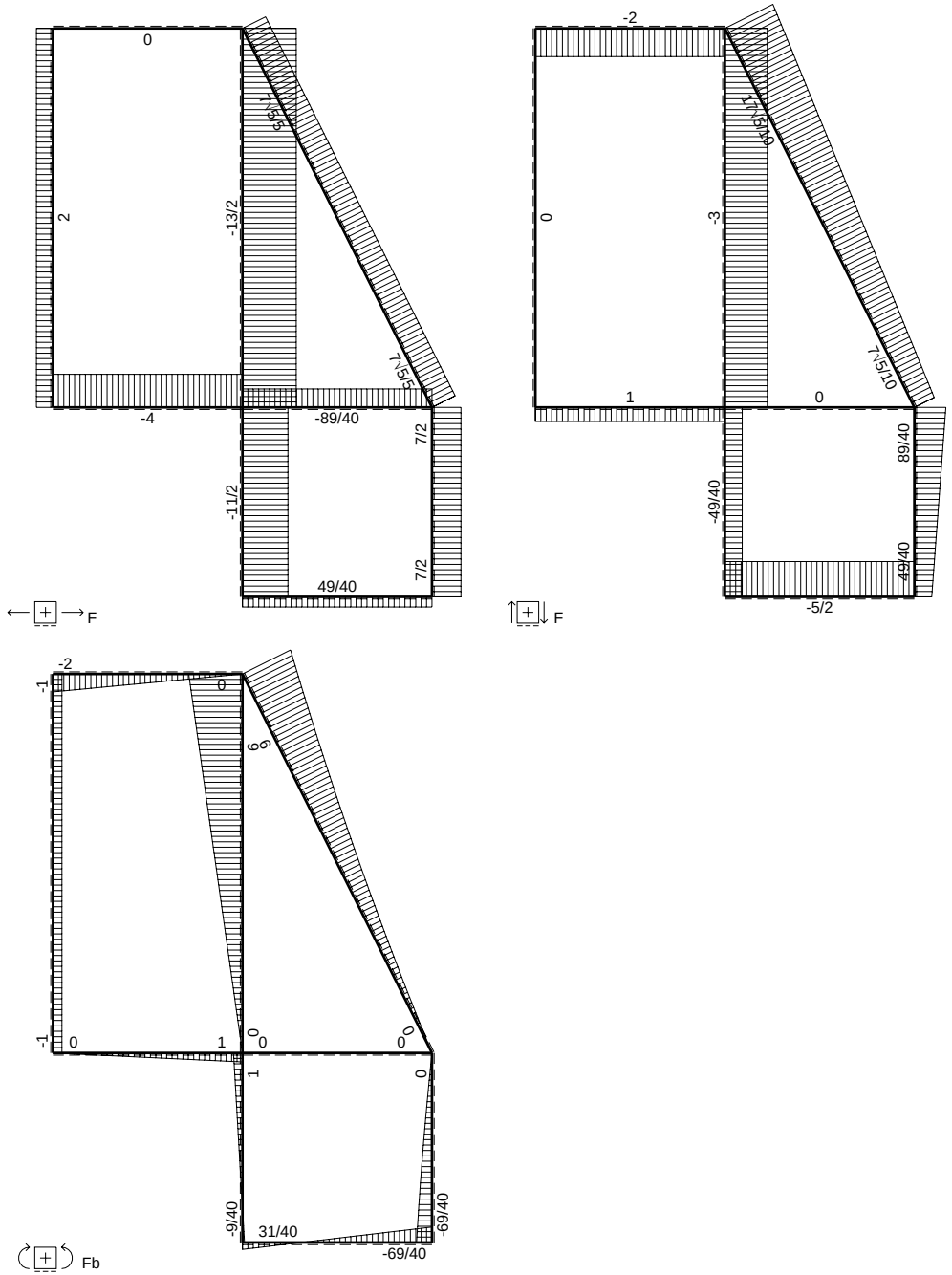
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49/40
-5/2
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-1
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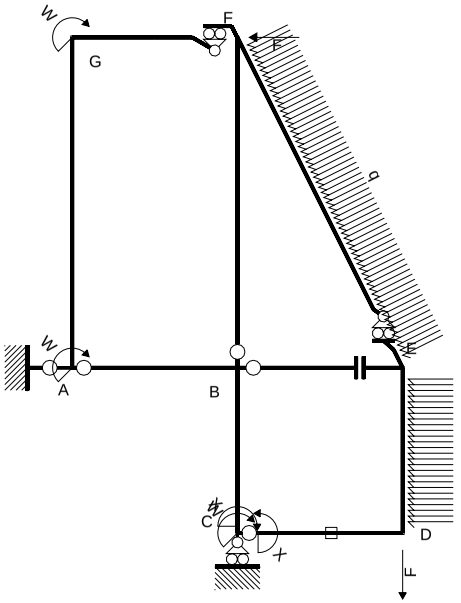
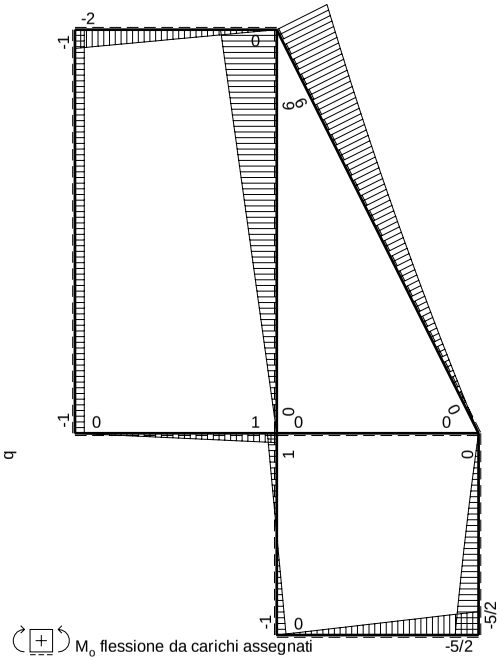
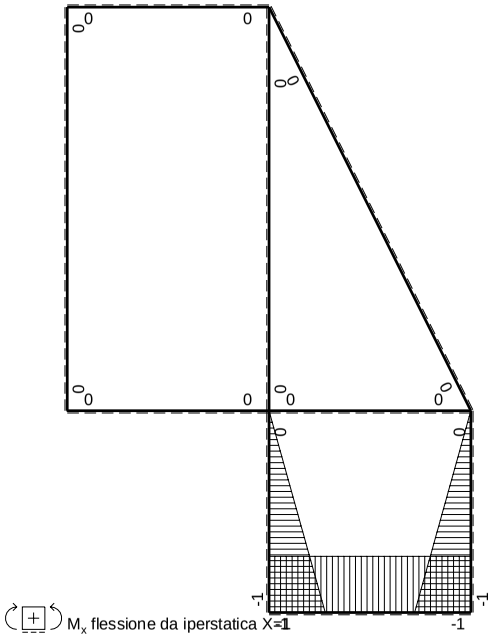
31/40
-69/40
-69/40
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239868 ESΣ02.xxxx.164

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	$-5/2Fb+2Fx+1/2qx^2$	$5/2Fb-9/2Fx+3/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$7/8Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$3Fx-1/2qx^2$	$3Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$31/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-31/40Fb	

PROCEDIMENTO E RISULTATI 239868 ESΣ02.xxxx.164

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -9/2\ x/b +3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [5/2\ x -9/4\ x^2/b +1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -9/4\ b +1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 7/8\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 7/8\ \ Fb^2/EJ$

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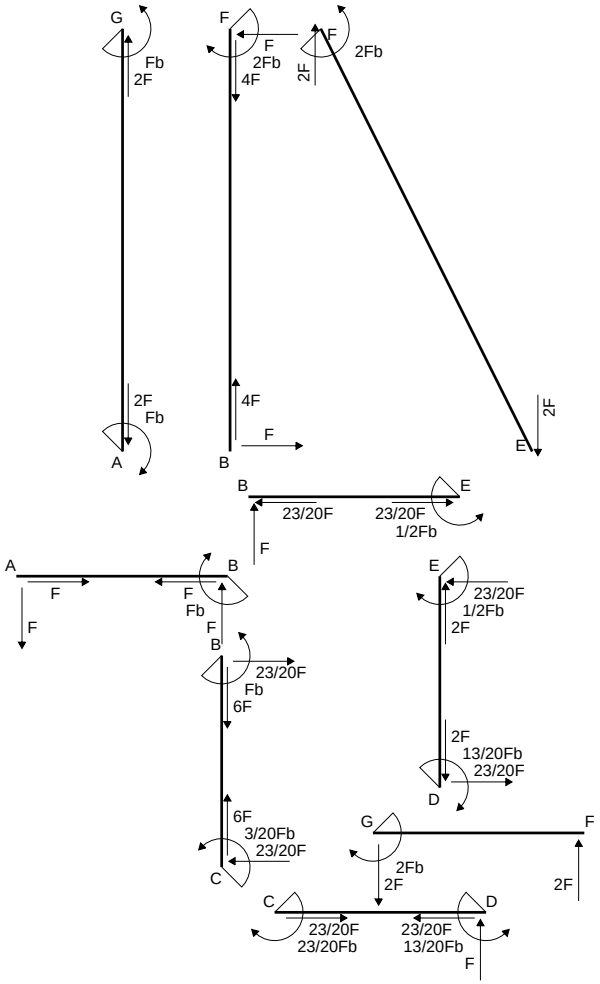
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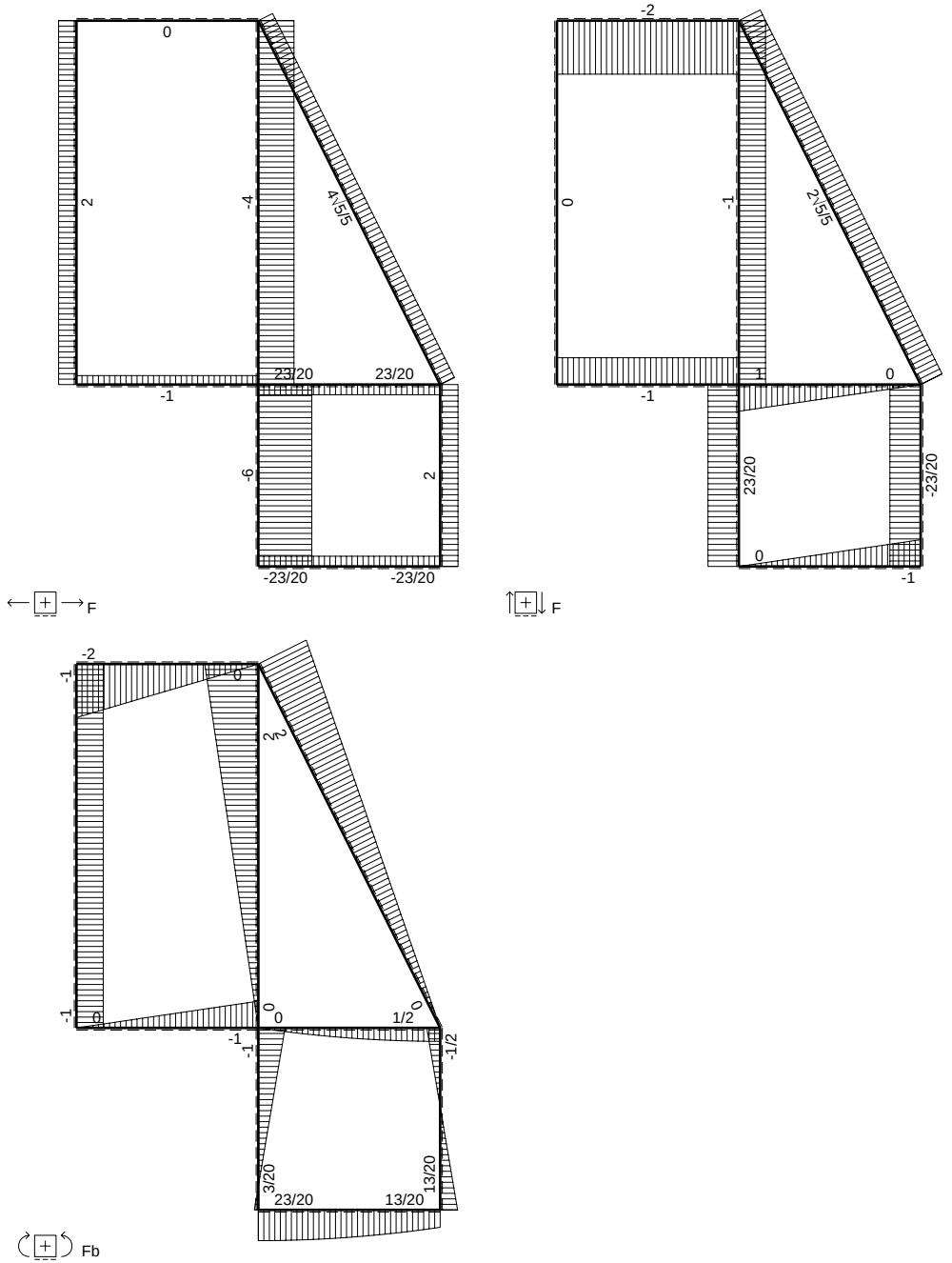
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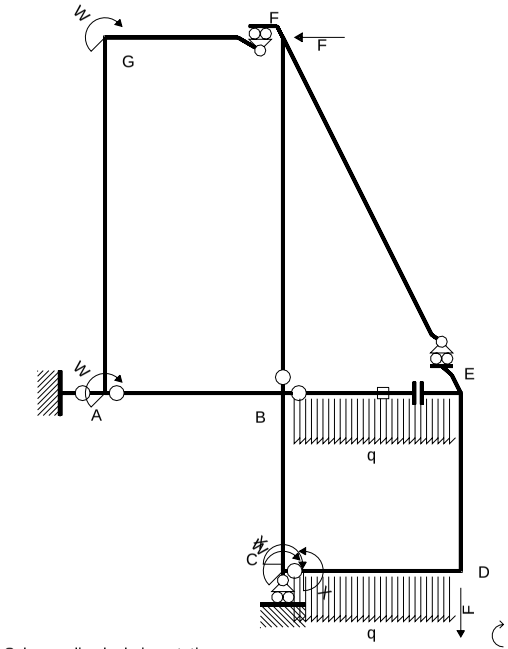
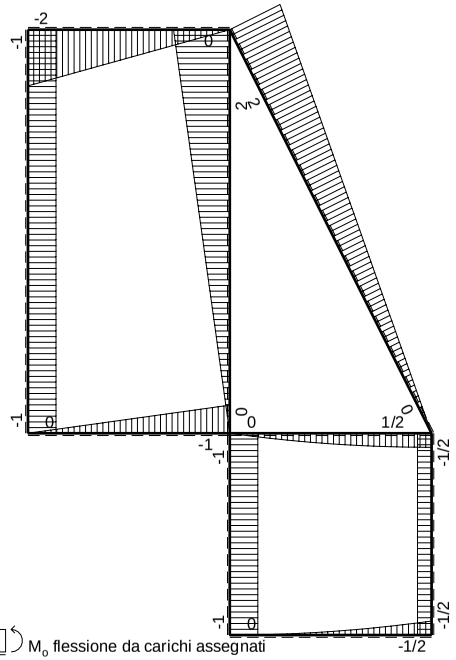
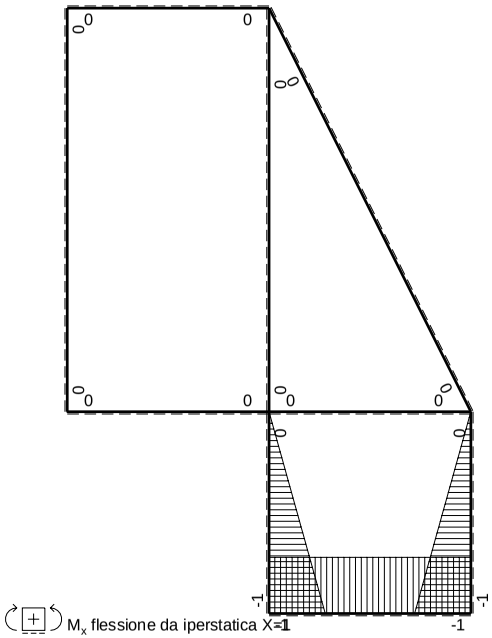
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb	1/2Fb-1/2Fx	$1-2x/b+x^2/b^2$	$1/4Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb	1/2Fx	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$23/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-23/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 -1/2\ x/b)\ Fb\ 1/EJ\ dx = [1/2\ x -1/4\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/4\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/4\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

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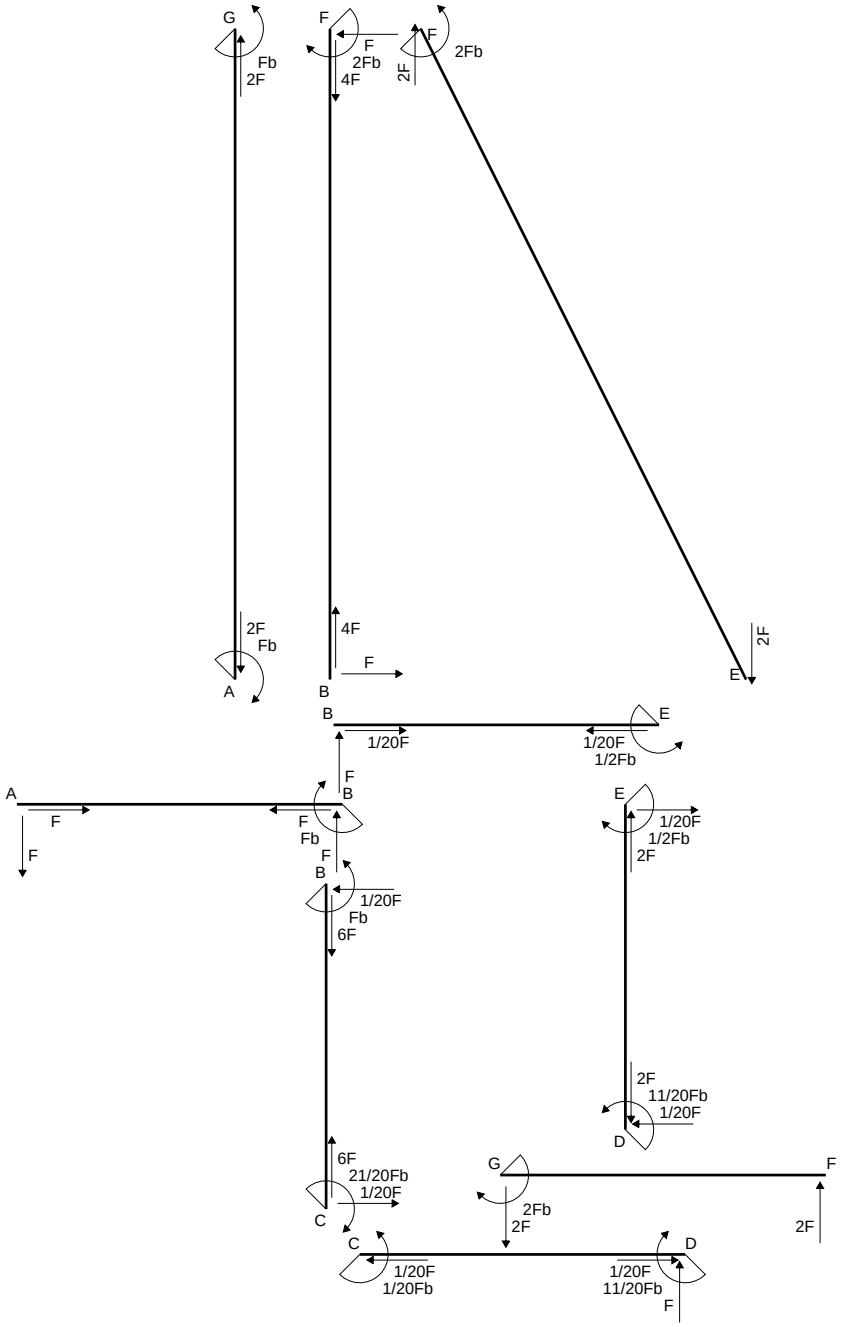
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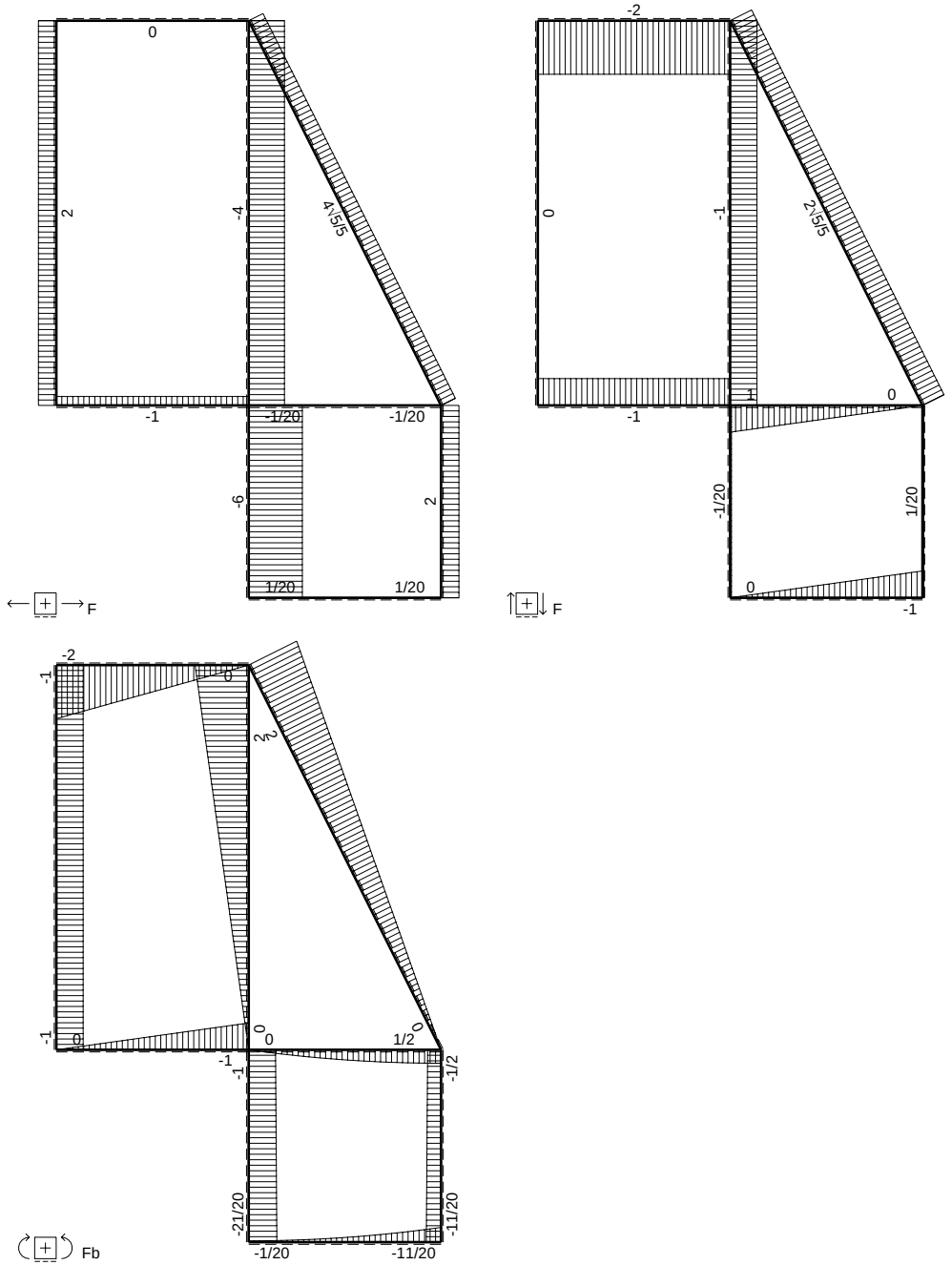
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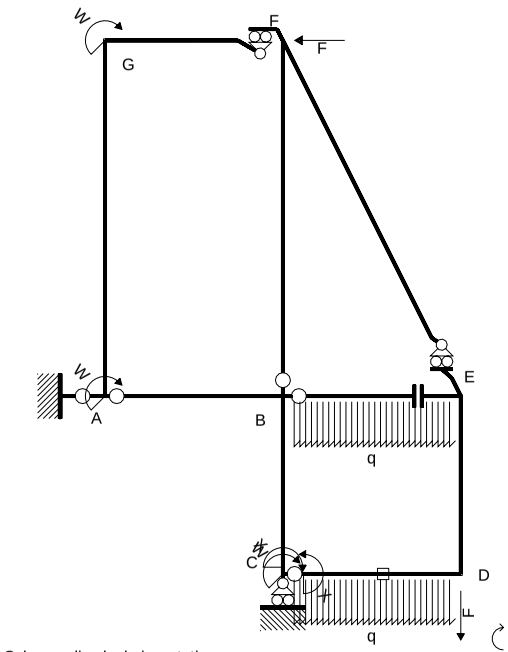
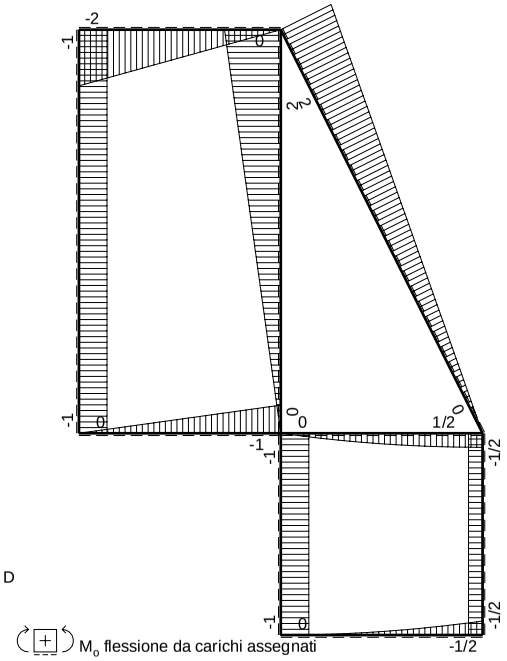
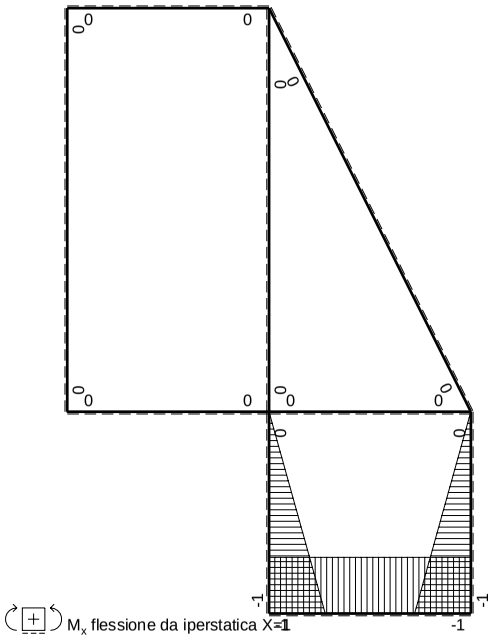


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb	1/2Fb-1/2Fx	$1-2x/b+x^2/b^2$	$1/4Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fb	1/2Fx	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-1/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$1/20Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/4\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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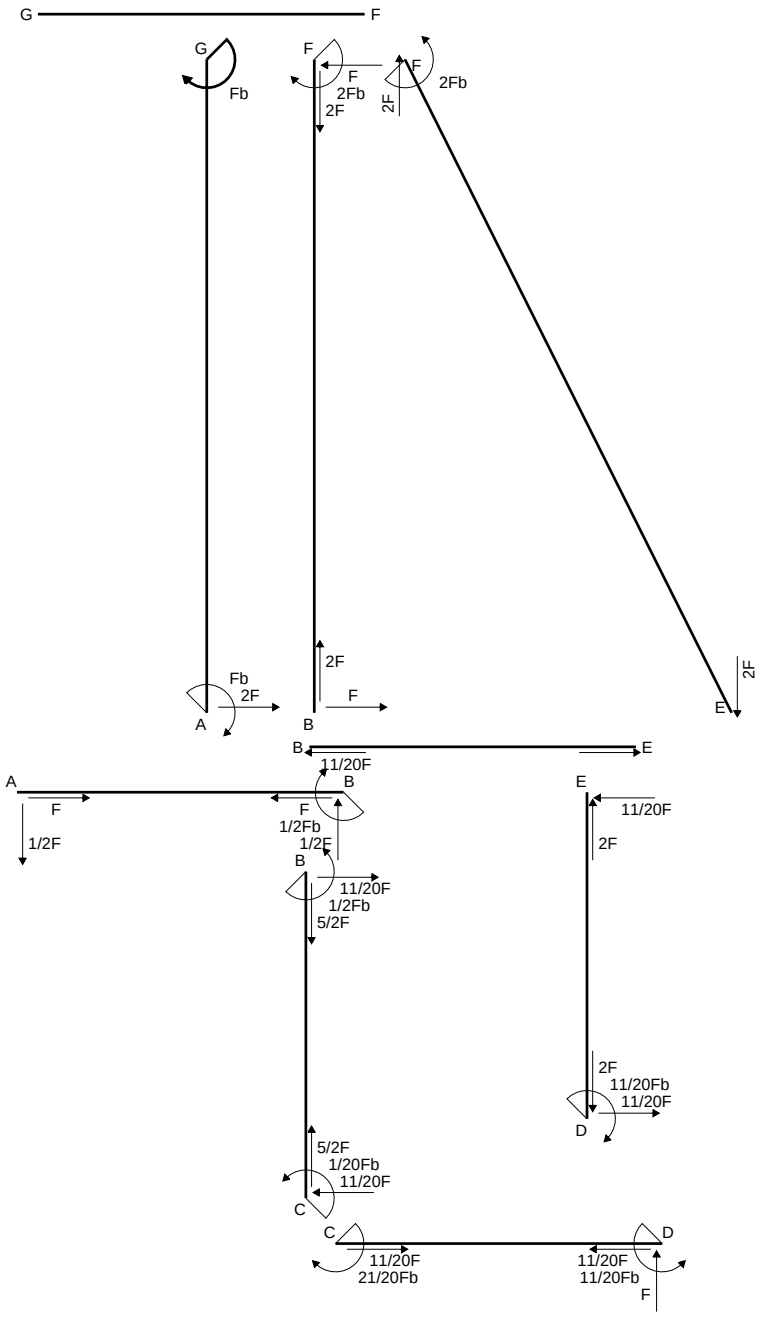
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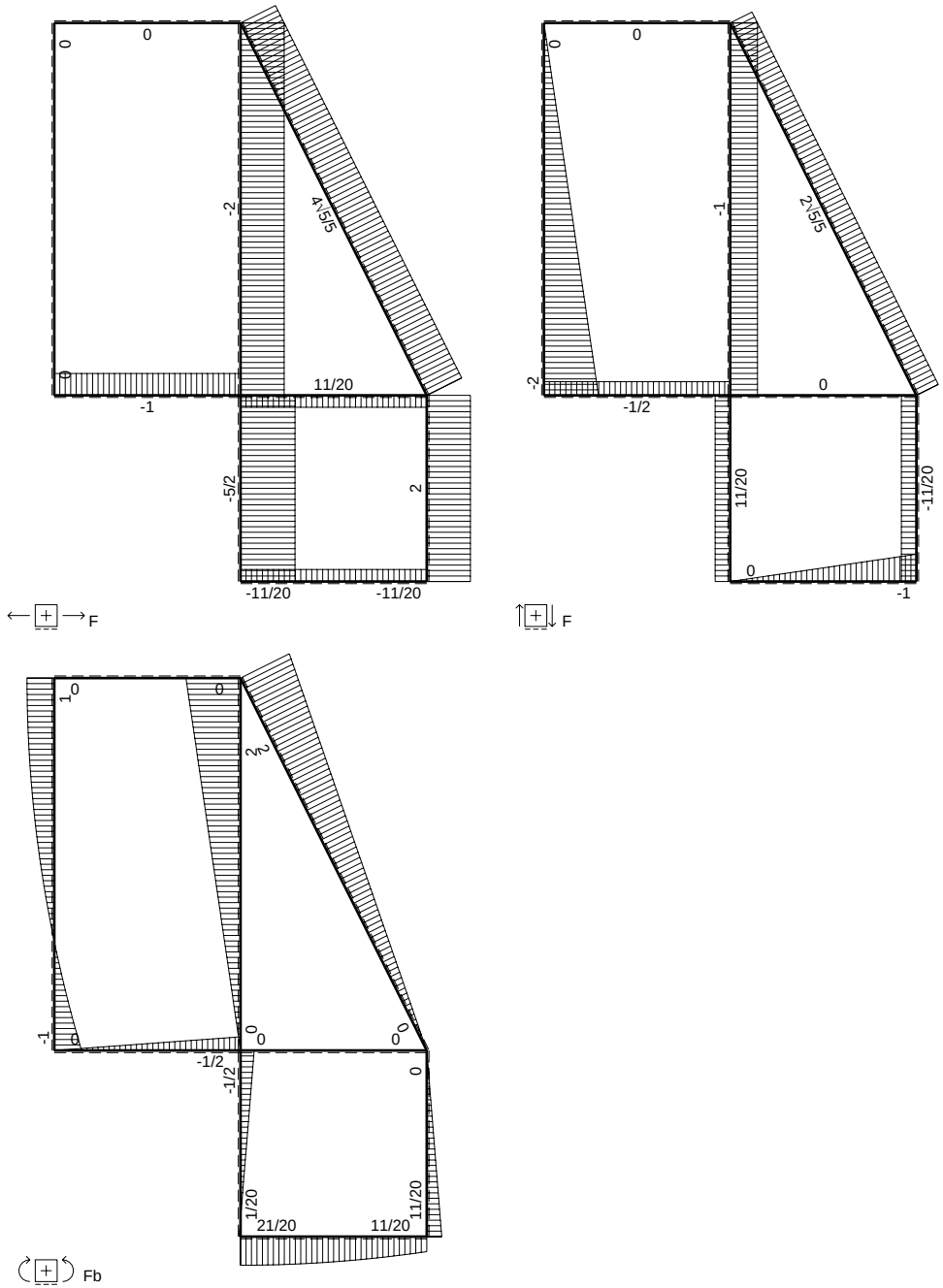
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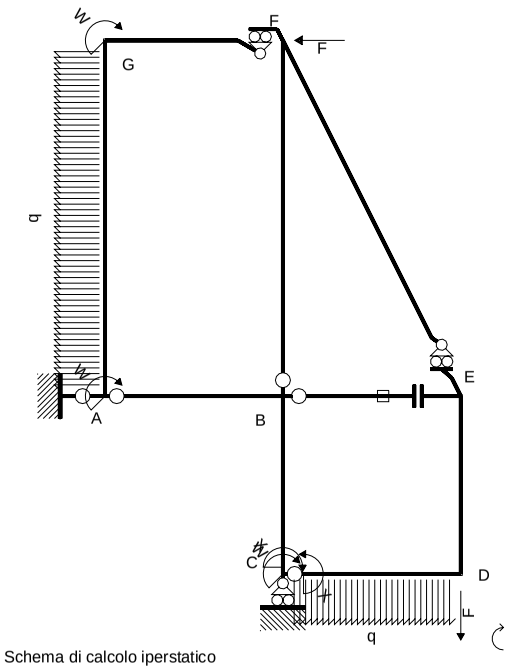
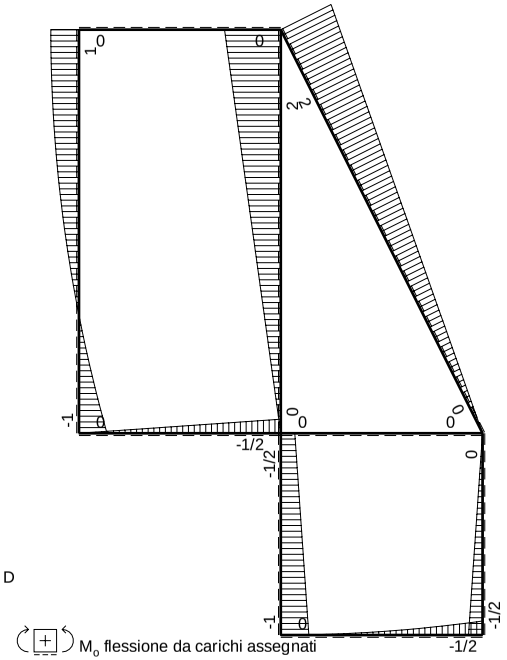
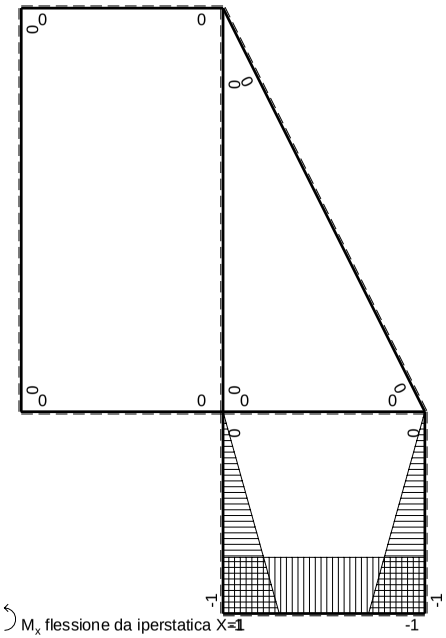


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-21/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

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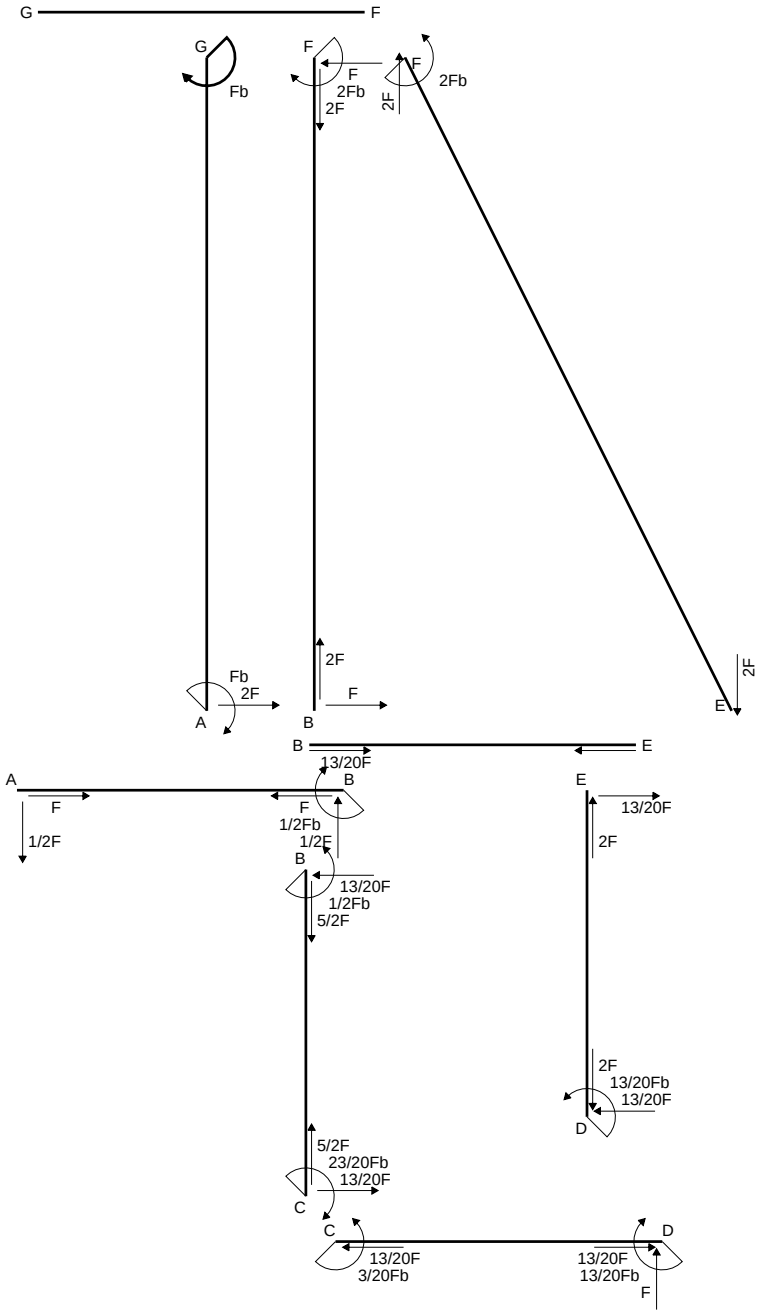
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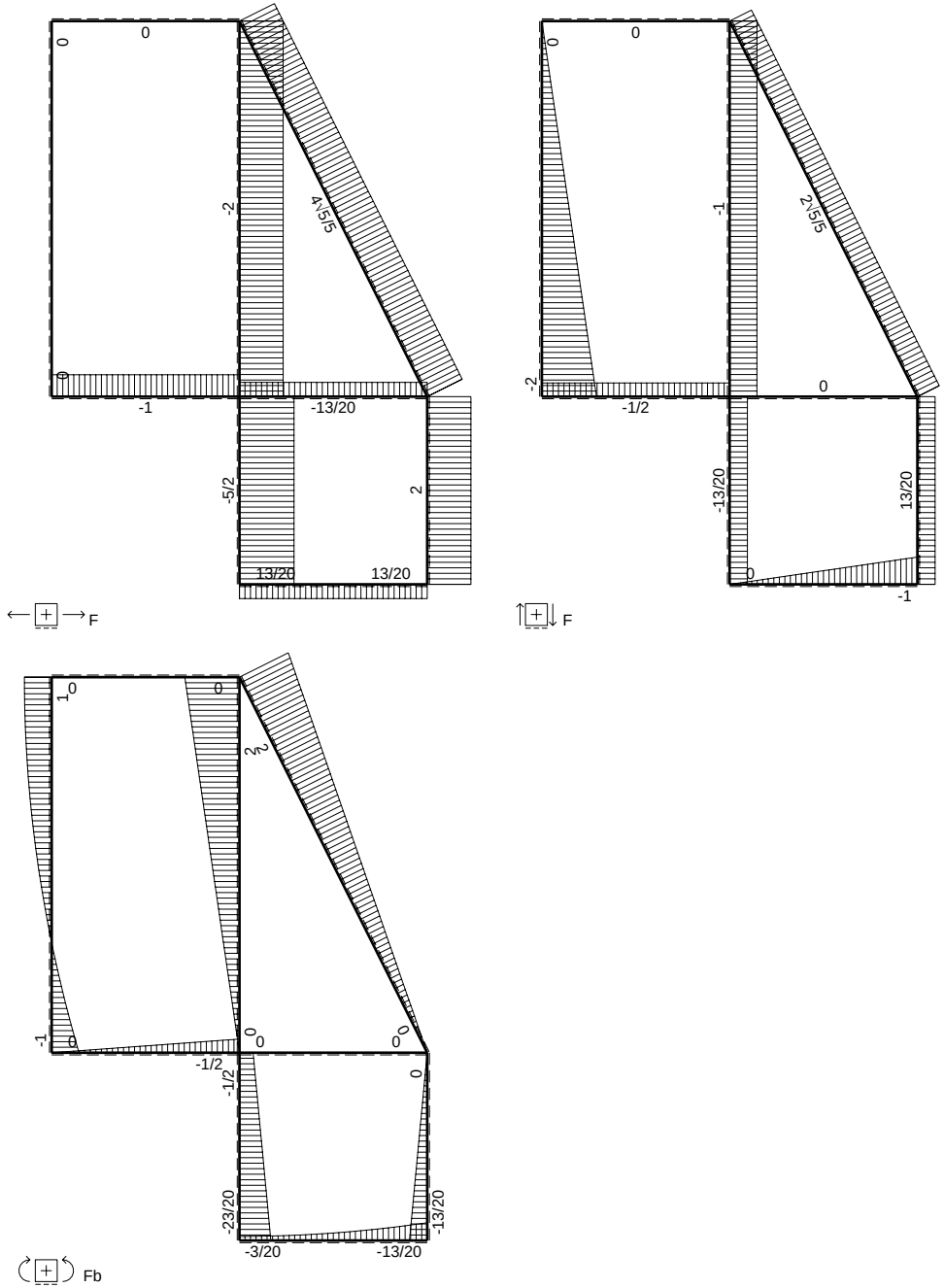
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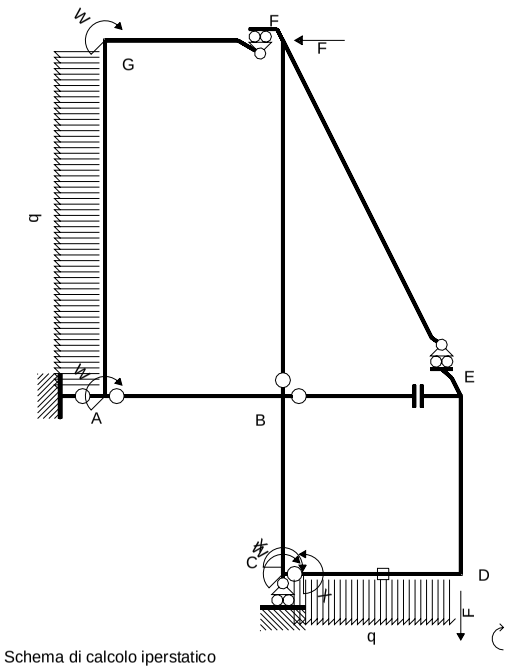
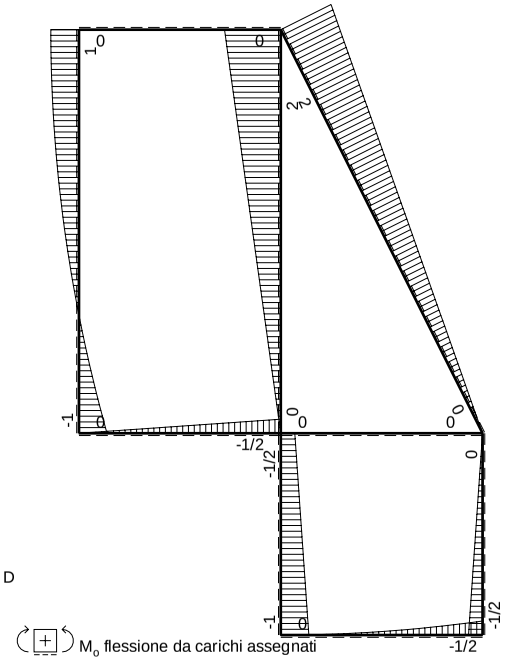
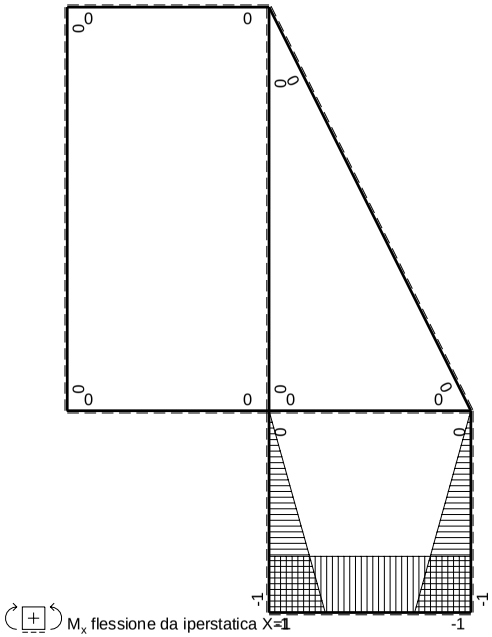


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	1/2Fx+1/2Fx²/b	x^2/b^2	5/12Fb²/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	Fb-3/2Fx+1/2Fx²/b	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	1/6Fb²/EJ	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	1/6Fb²/EJ	1/3Xb/EJ
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-1/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				3/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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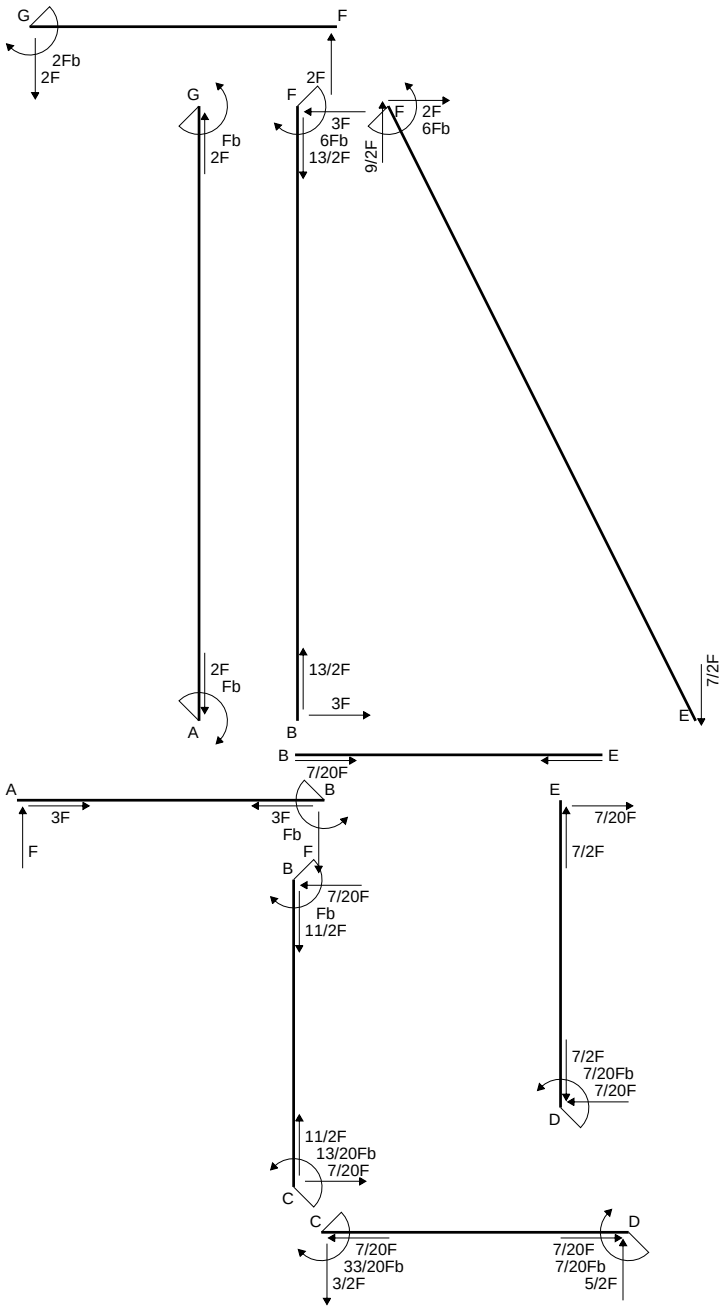
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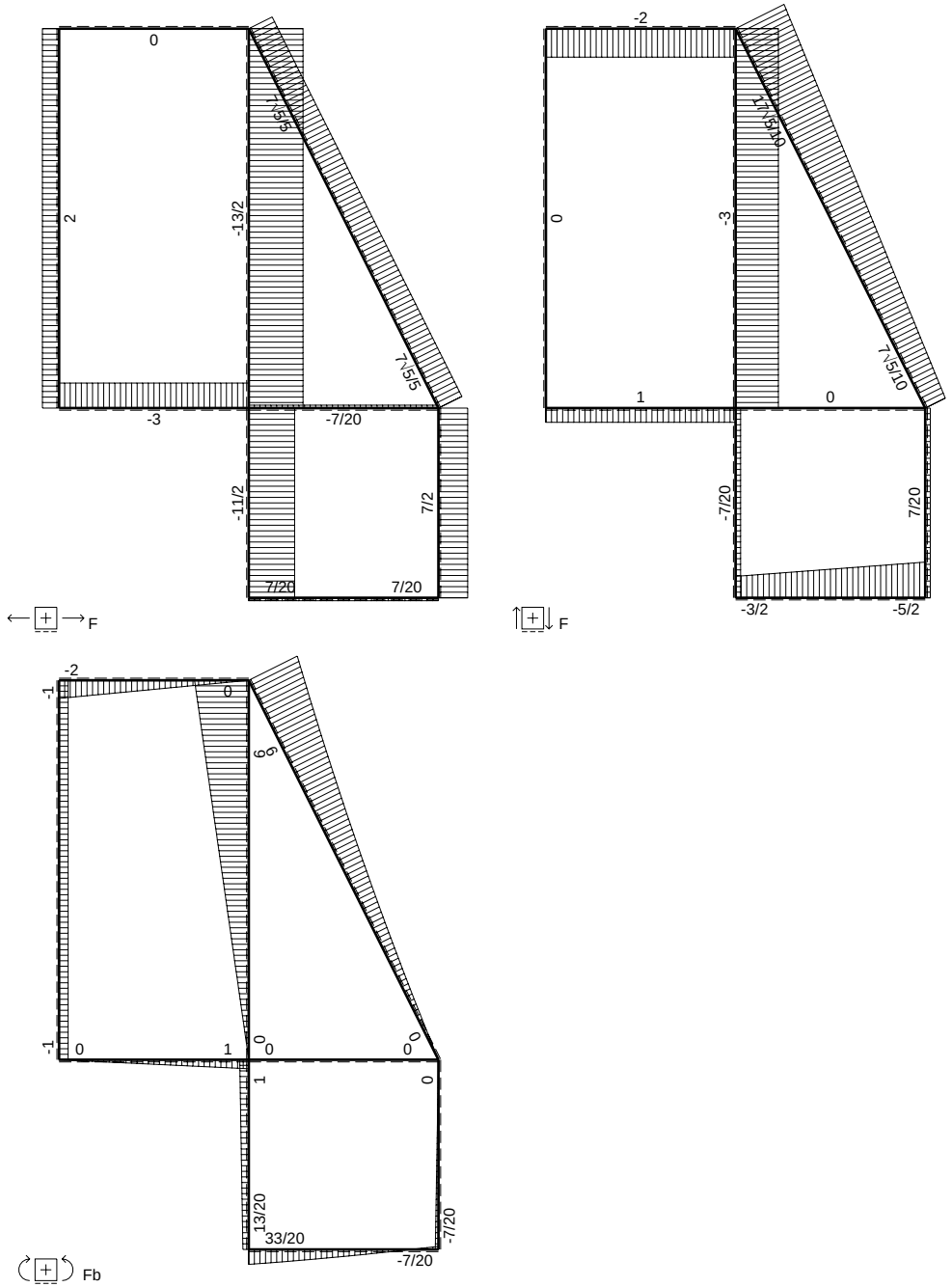
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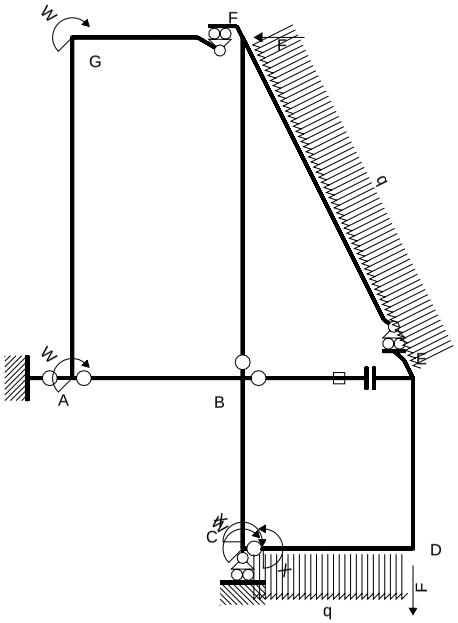
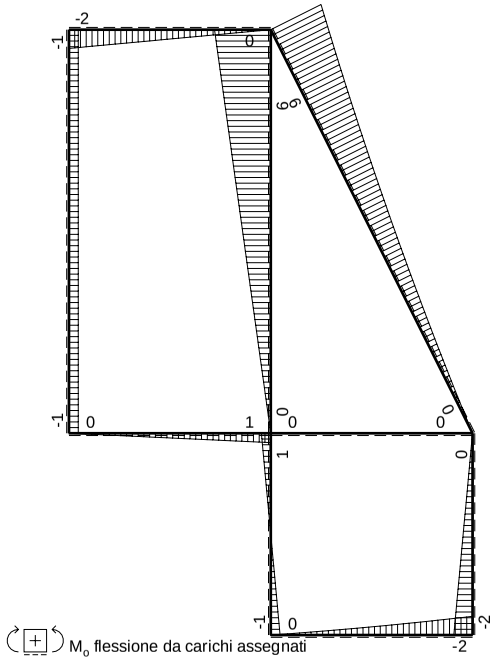
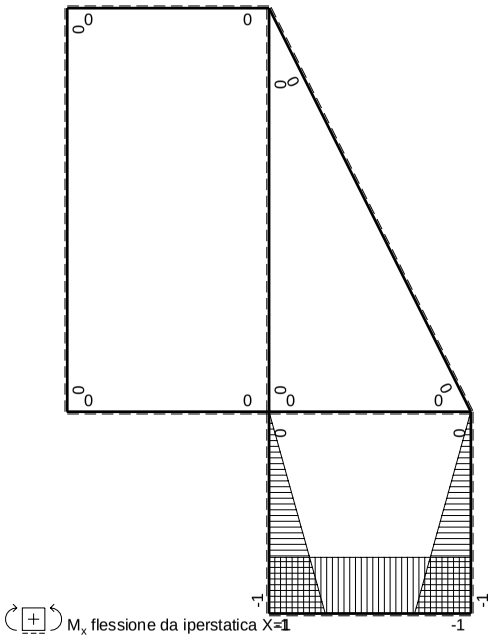


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-3/2Fx-1/2qx^2$	$3/2Fx+1/2Fx^2/b$	1	$11/12Fb^2/EJ$	Xb/EJ
DC b	1	$2Fb-5/2Fx+1/2qx^2$	$2Fb-5/2Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-2Fb+2Fx	$2Fb-4Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$2/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	2Fx	$2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$11/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-33/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (- x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (2 -5/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [2\ x -5/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -5/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 11/12\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (2 -4\ x/b +2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [2\ x -2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

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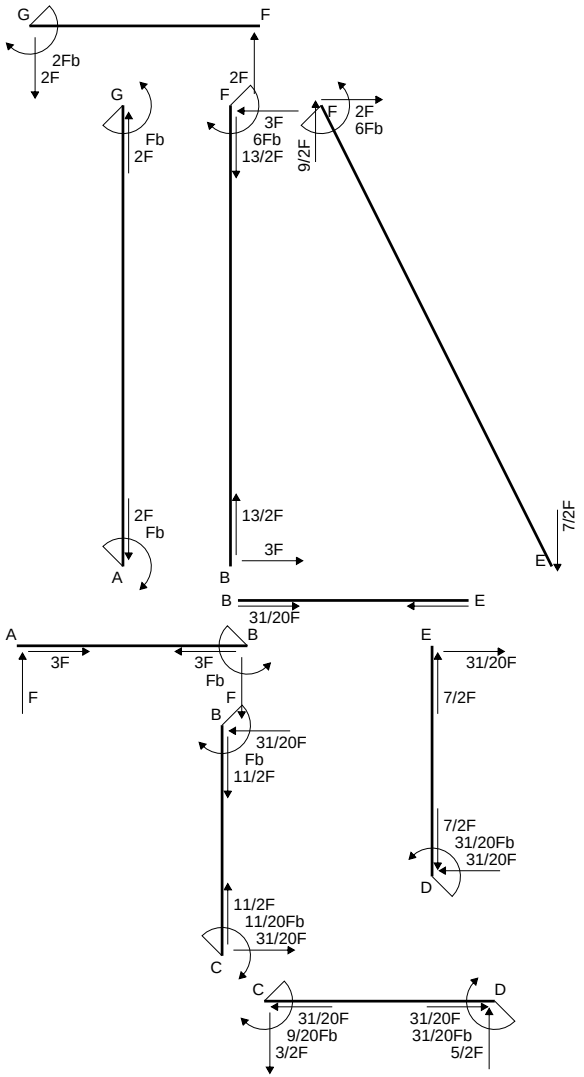
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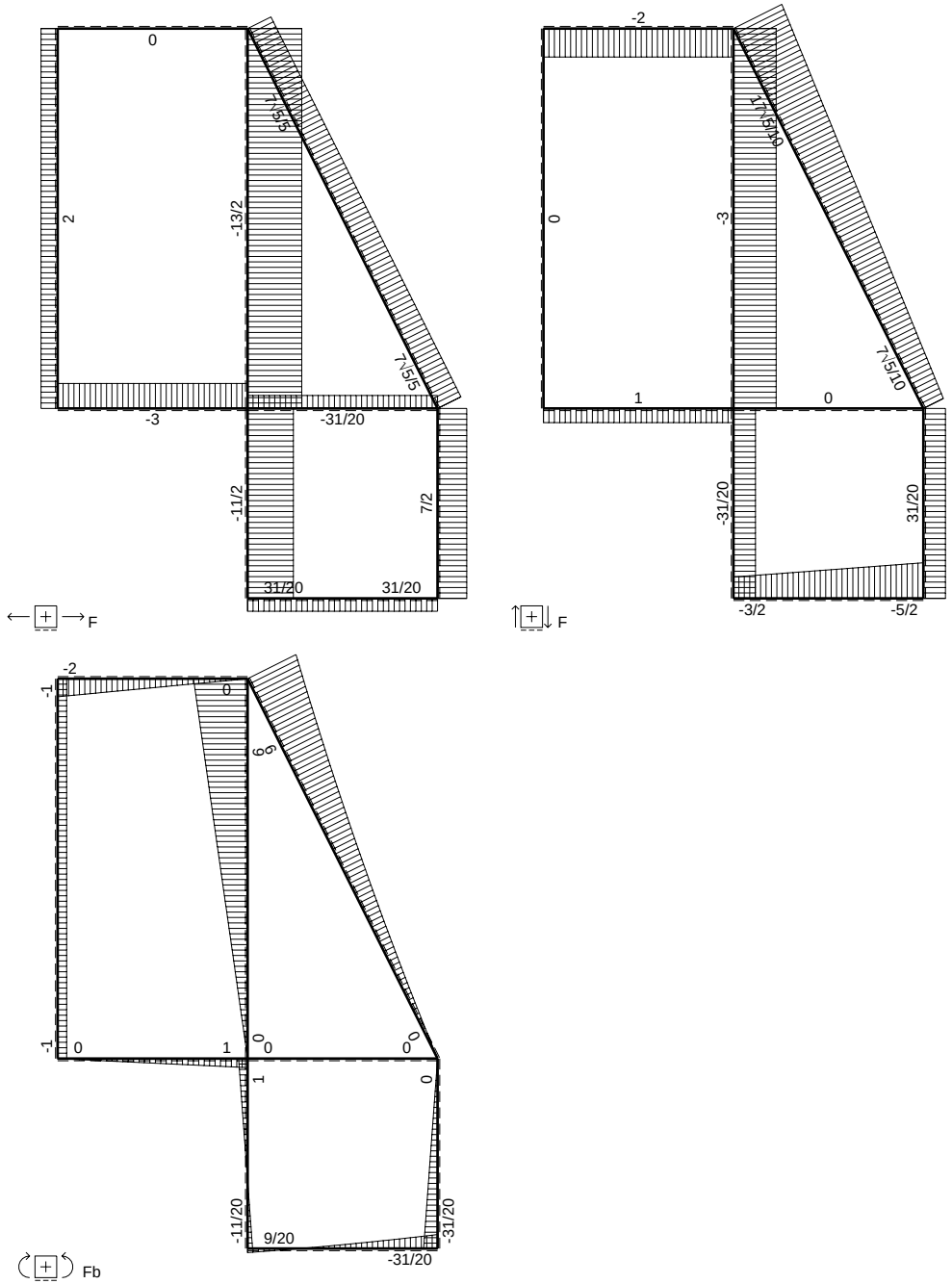
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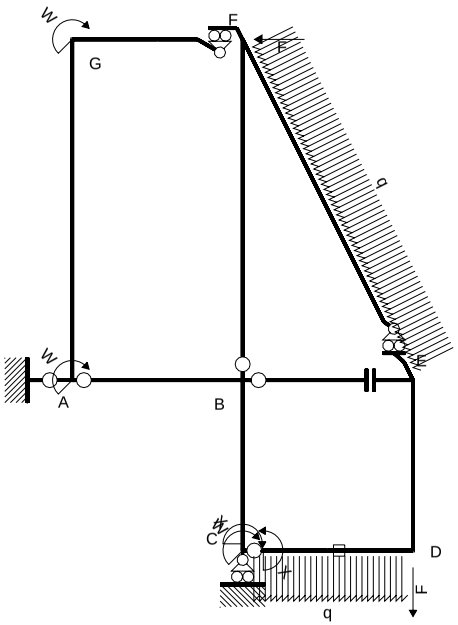
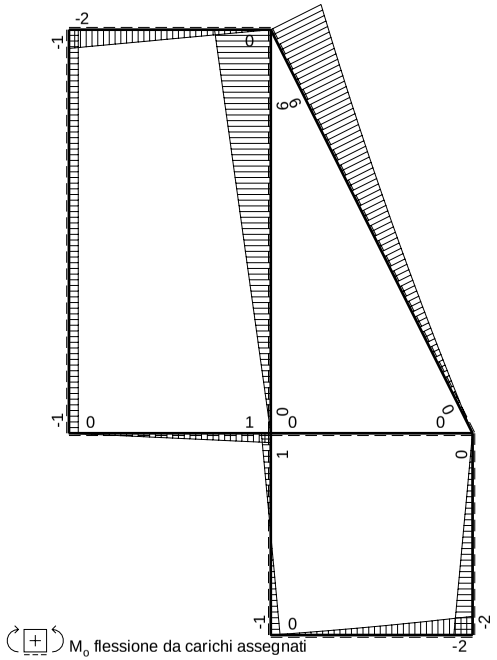
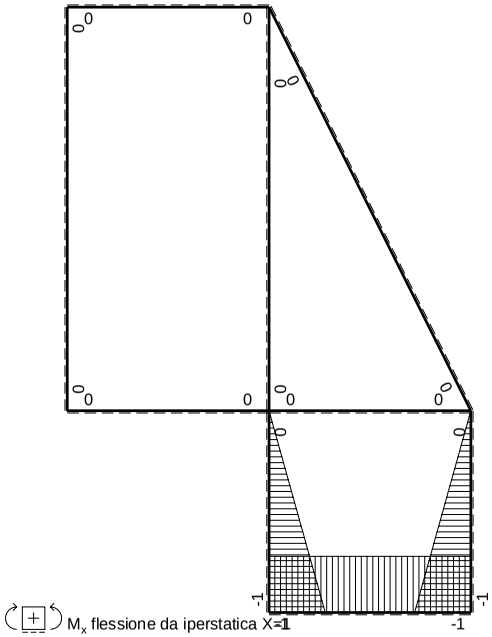
-31/20
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb-2Fx	$-Fx+2Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-2Fx	$Fb-3Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-3/2Fx-1/2qx^2$	$3/2Fx+1/2Fx^2/b$	1	$11/12Fb^2/EJ$	Xb/EJ
DC b	1	$2Fb-5/2Fx+1/2qx^2$	$2Fb-5/2Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-2Fb+2Fx	$2Fb-4Fx+2Fx^2/b$	$1-2x/b+x^2/b^2$	$2/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	2Fx	$2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-9/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (- x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/12\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (2 -5/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [2\ x -5/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (2\ b -5/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/12\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (2 -4\ x/b +2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [2\ x -2\ x^2/b +2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b -2\ b +2/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2/3\ b\ )\ Fb\ 1/EJ\ = 2/3\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 955642 ESΣ02.xxxx.170

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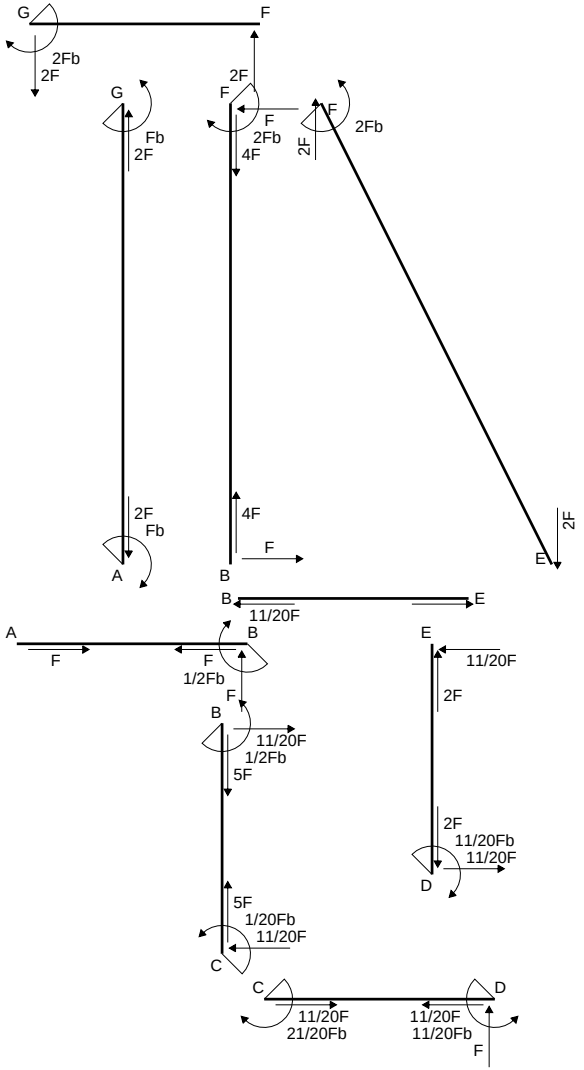
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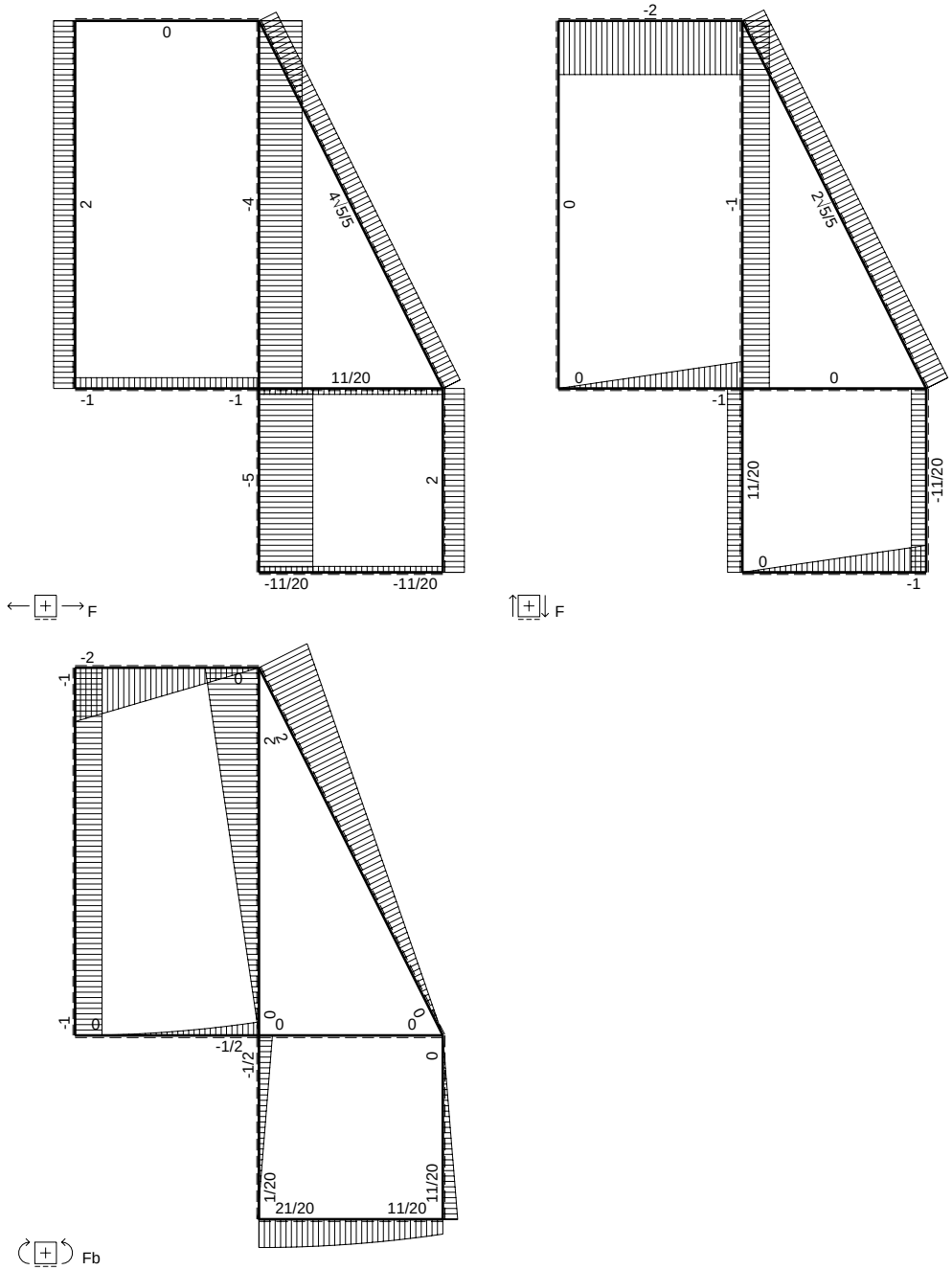
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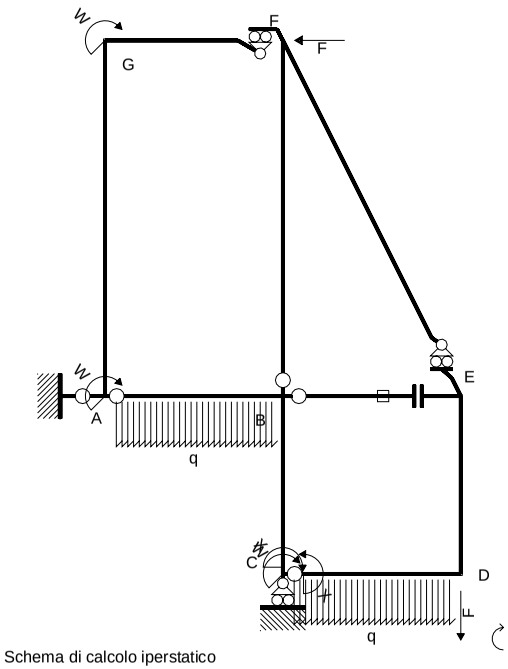
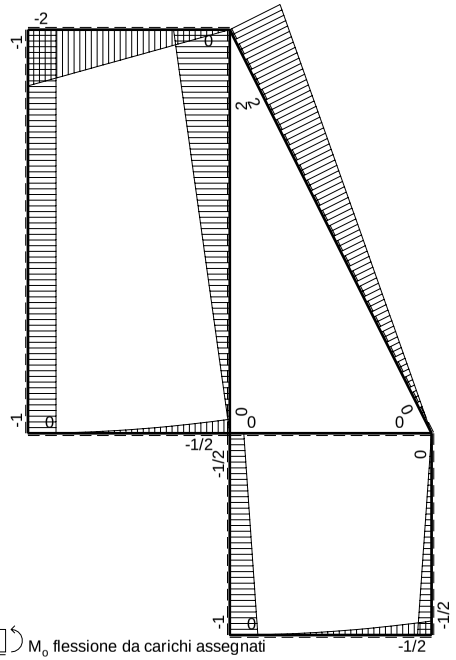
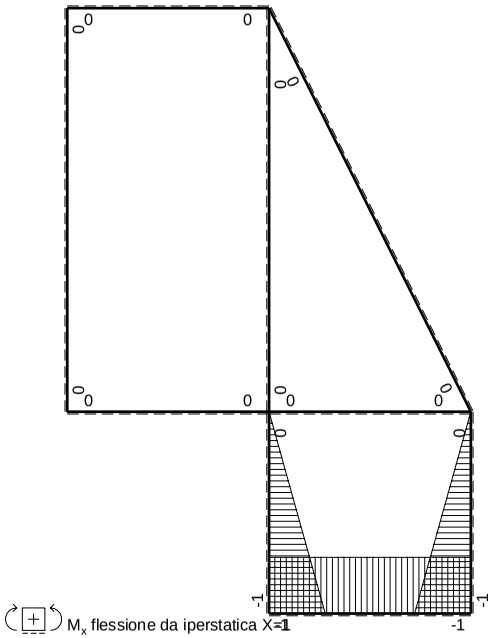


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	$-x/b$	$-1/2Fb-1/2Fx$	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	$1-x/b$	$Fb-1/2Fx$	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	$-1+x/b$	$-1/2Fb+1/2Fx$	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$1/2Fx$	$1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-2Fx$	0	0	0	0
GF b	0	$2Fb-2Fx$	0	0		
GA 2b	0	$-Fb$	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$2Fb-Fx$	0	0	0	0
BF 2b	0	$-Fx$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-21/20Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

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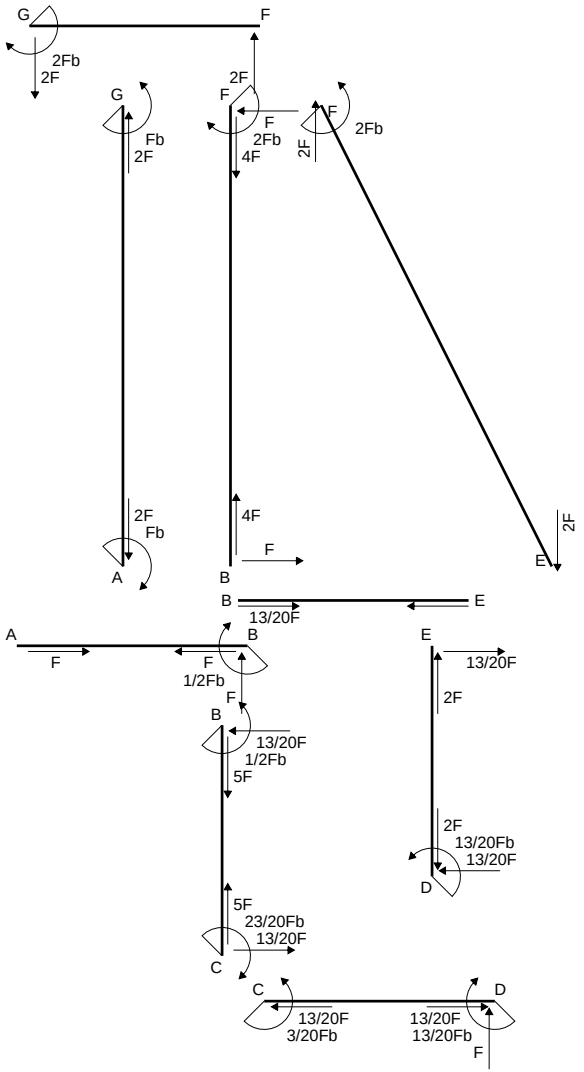
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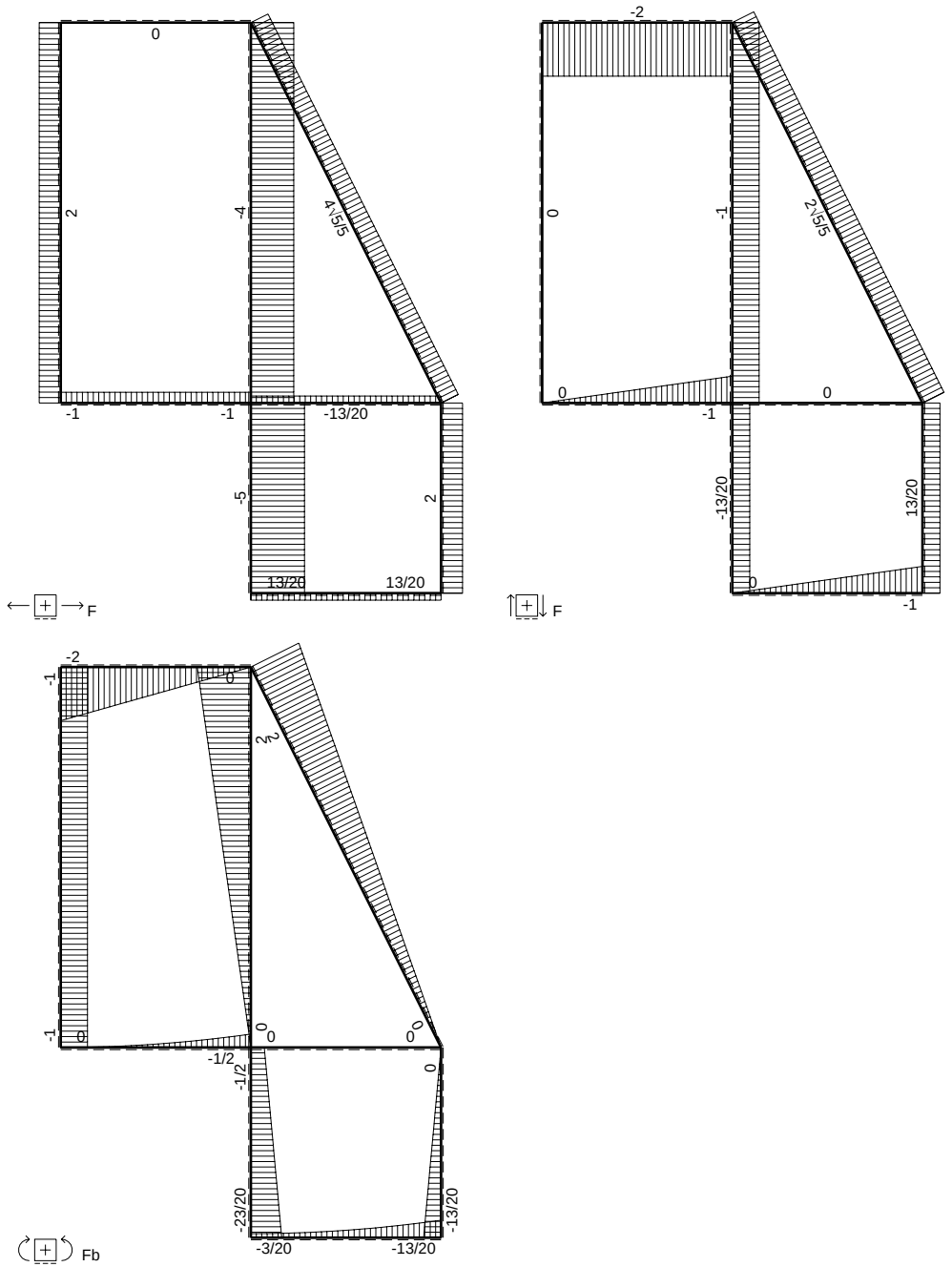
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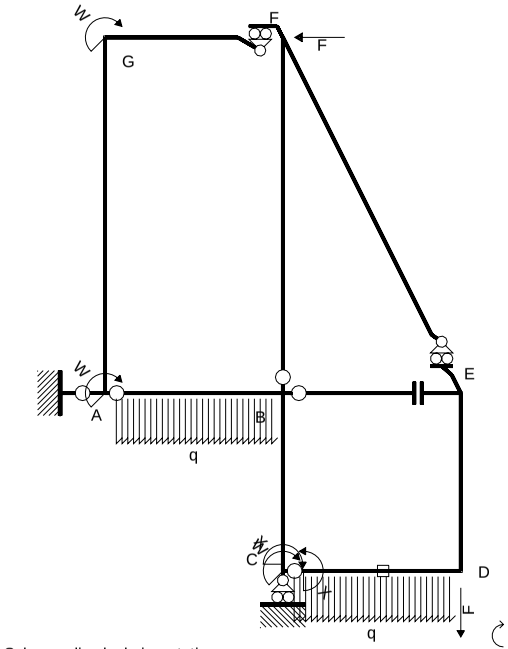
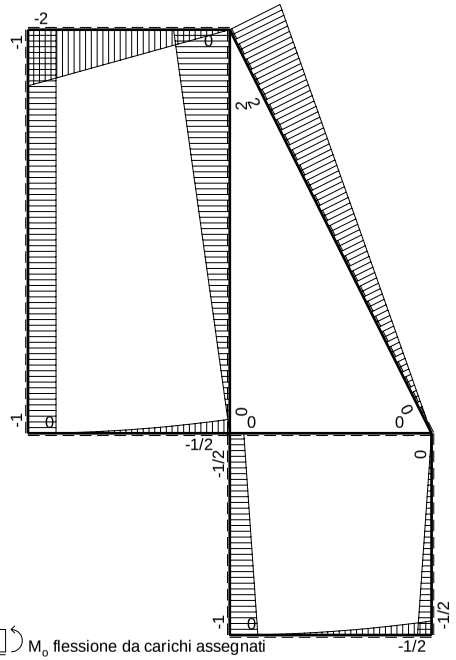
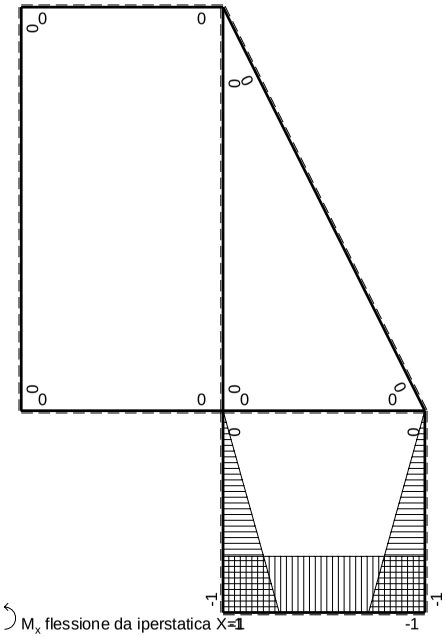


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	$-x/b$	$-1/2Fb-1/2Fx$	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	$1-x/b$	$Fb-1/2Fx$	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	$-1+x/b$	$-1/2Fb+1/2Fx$	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$1/2Fx$	$1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-1/4Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				3/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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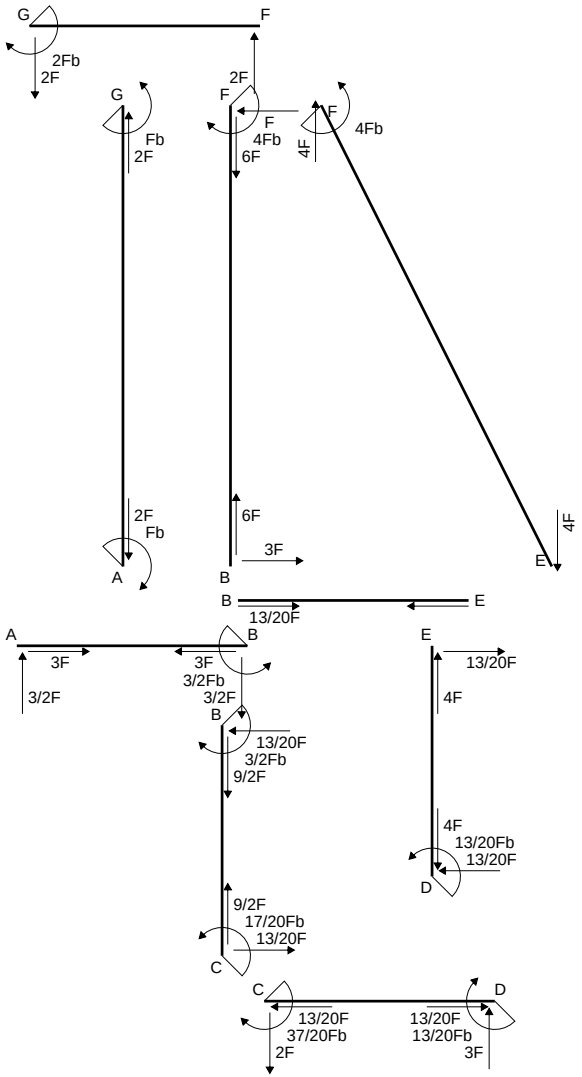
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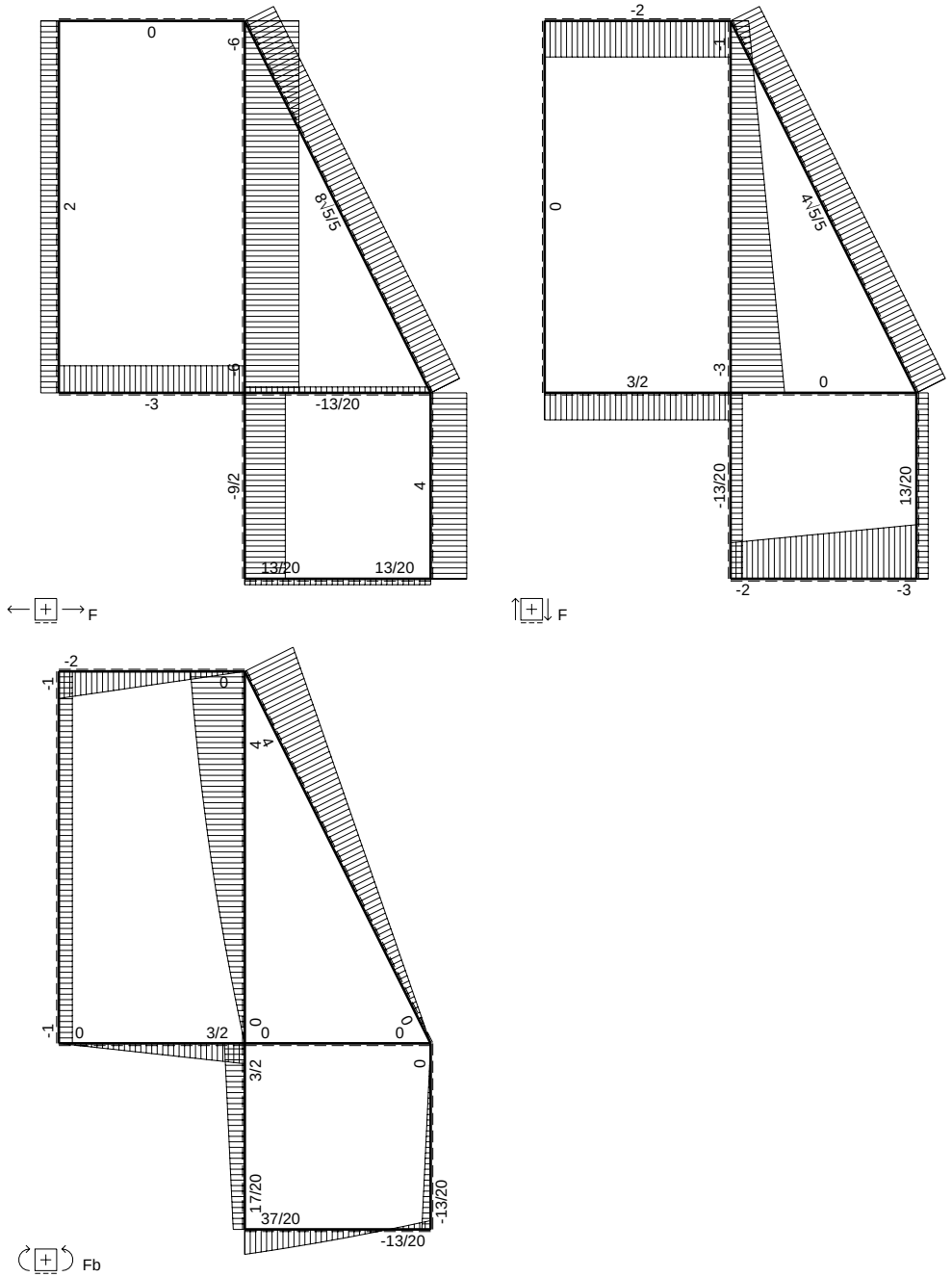
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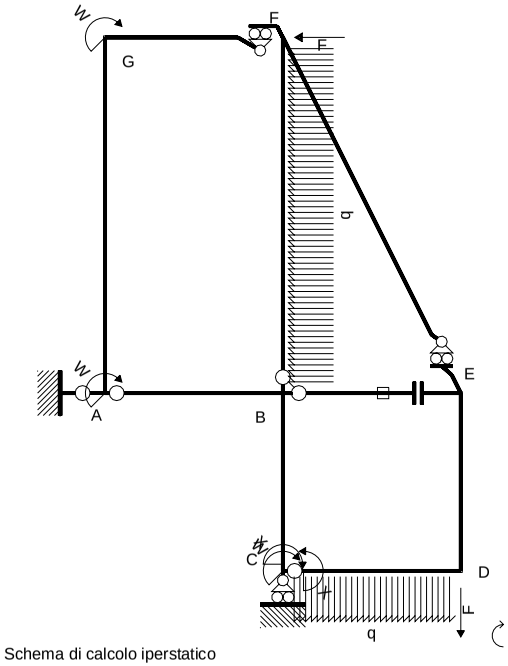
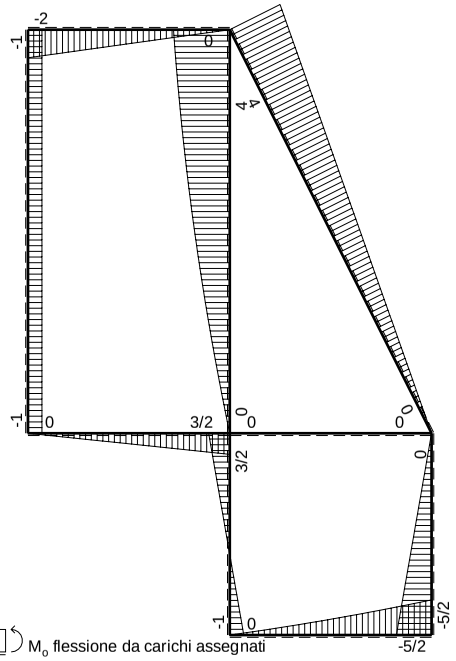
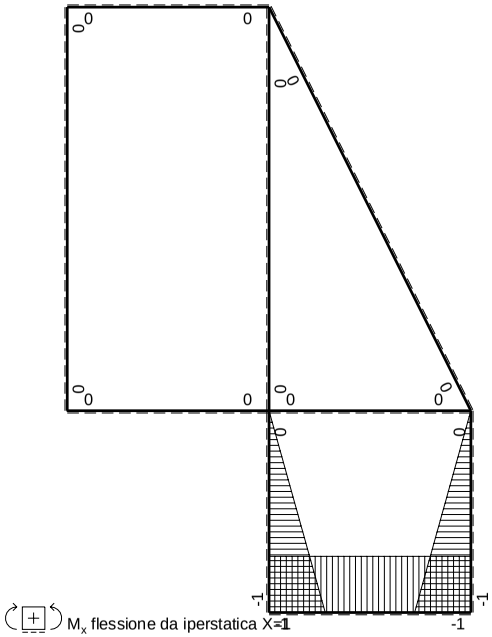


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-2Fx-1/2qx^2$	$2Fx+1/2Fx^2/b$	1	$7/6Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-3Fx+1/2qx^2$	$5/2Fb-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$37/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-37/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -3\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -3/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -3/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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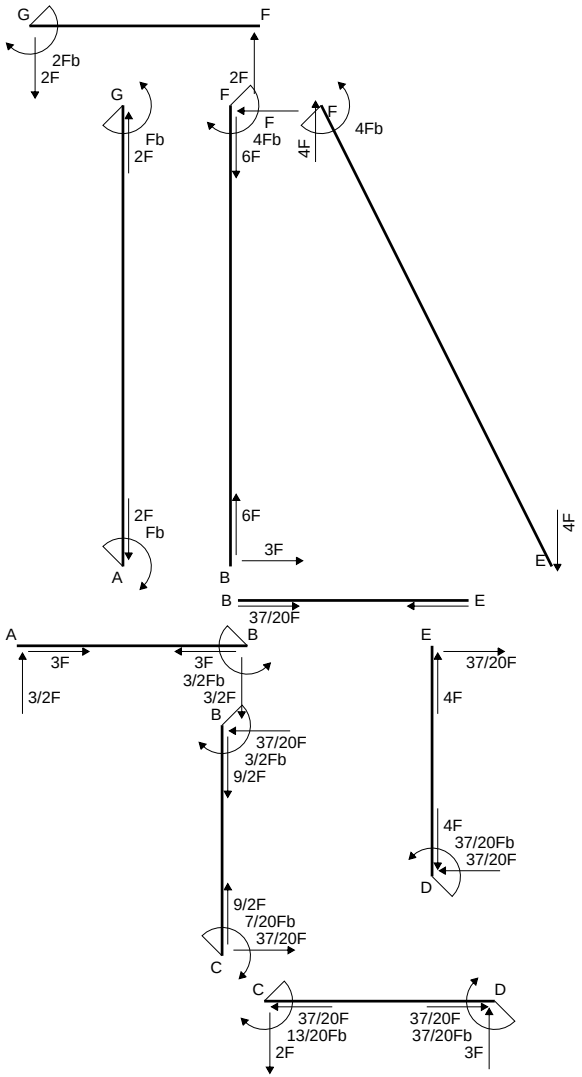
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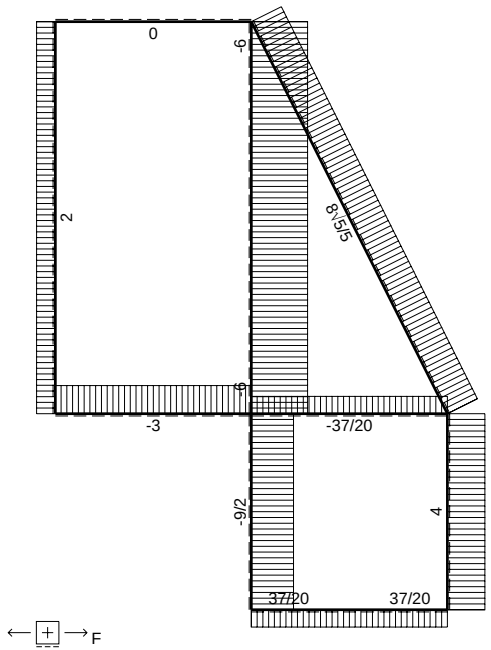
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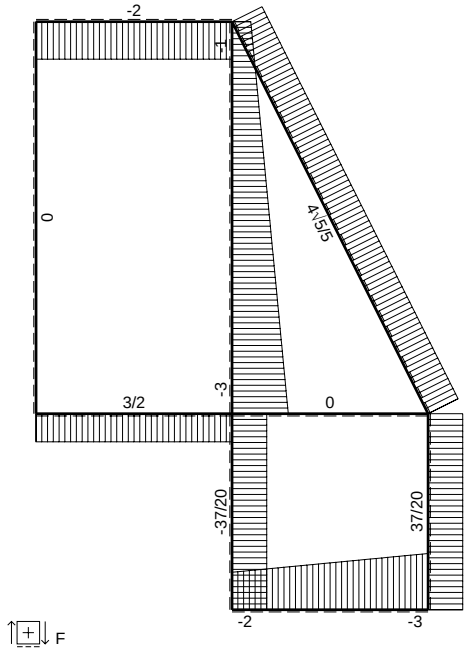
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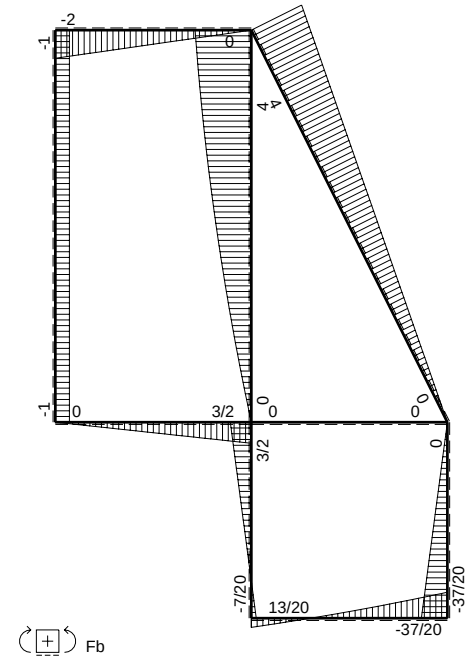
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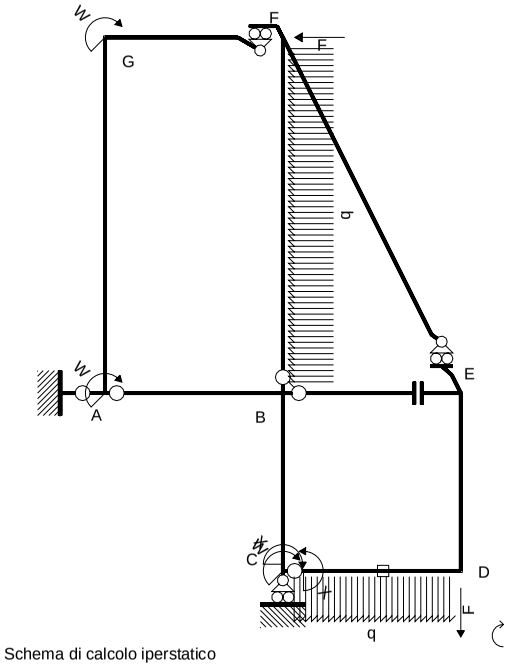
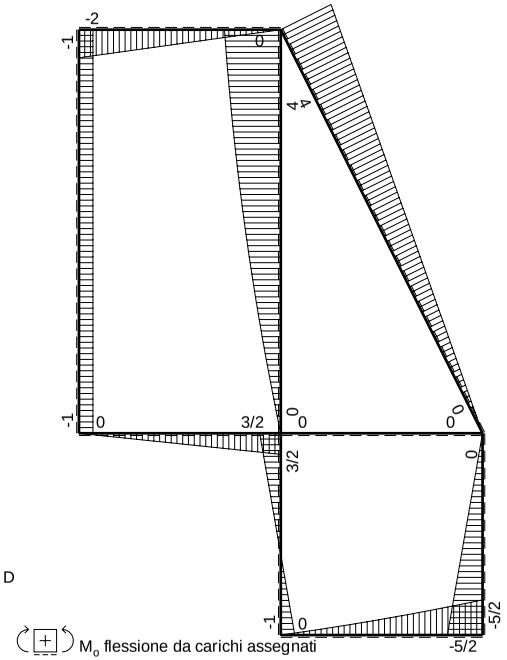
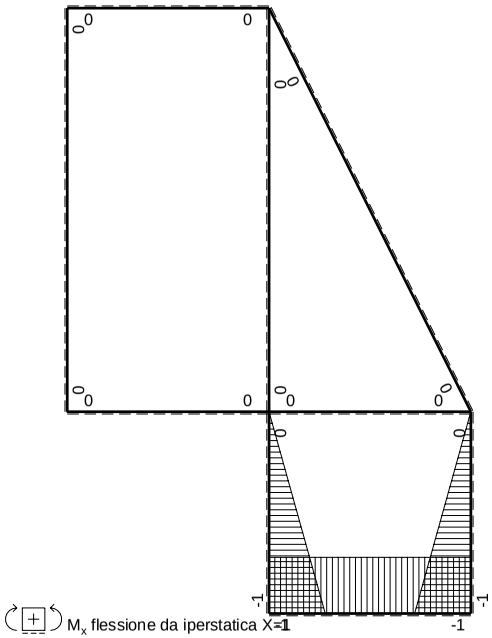


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-2Fx-1/2qx^2$	$2Fx+1/2Fx^2/b$	1	$7/6Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-3Fx+1/2qx^2$	$5/2Fb-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$13/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -3\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [5/2\ x -3/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/2\ b -3/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

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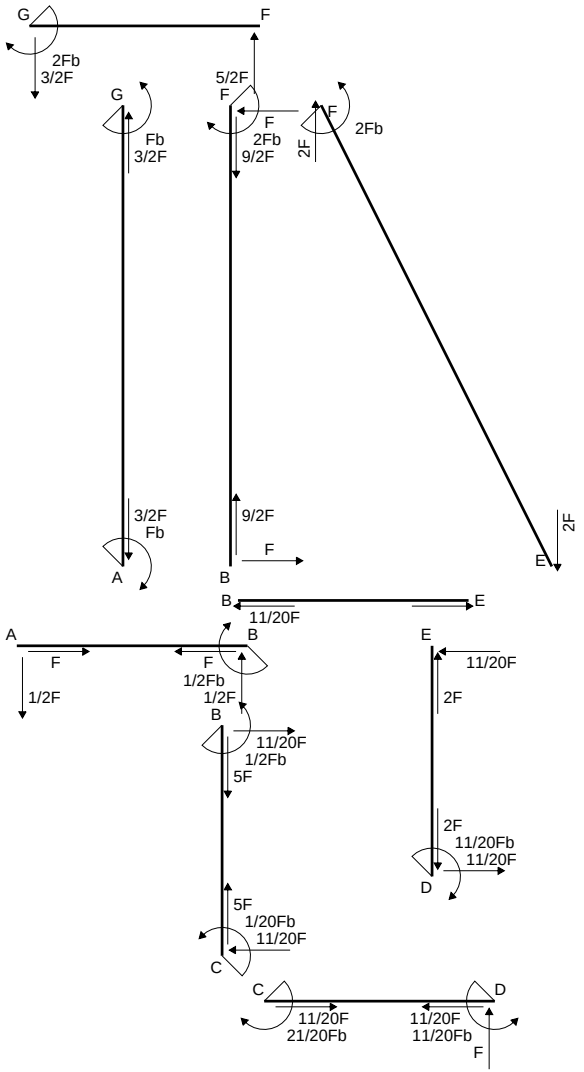
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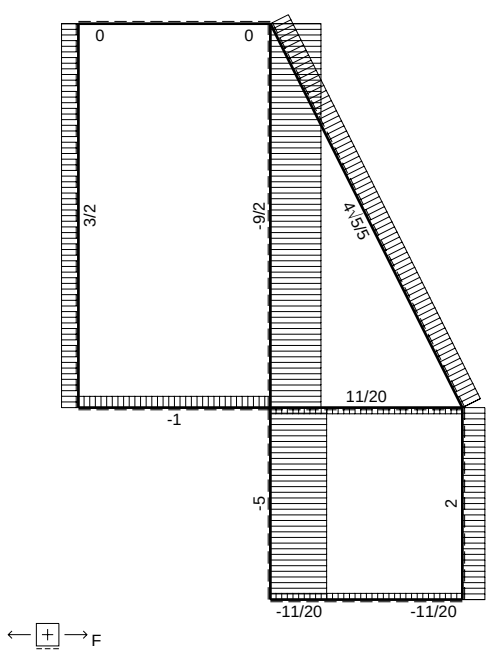
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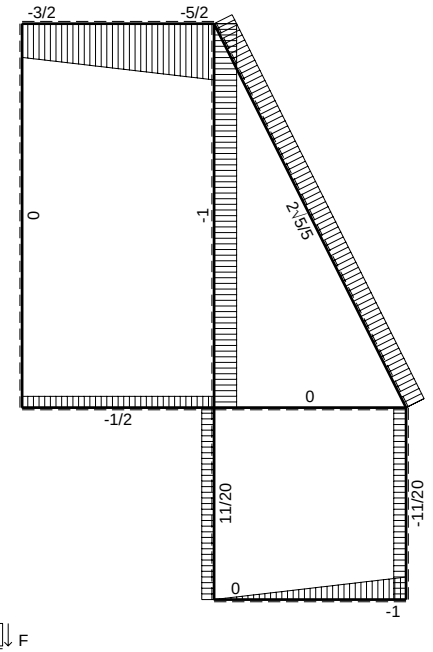
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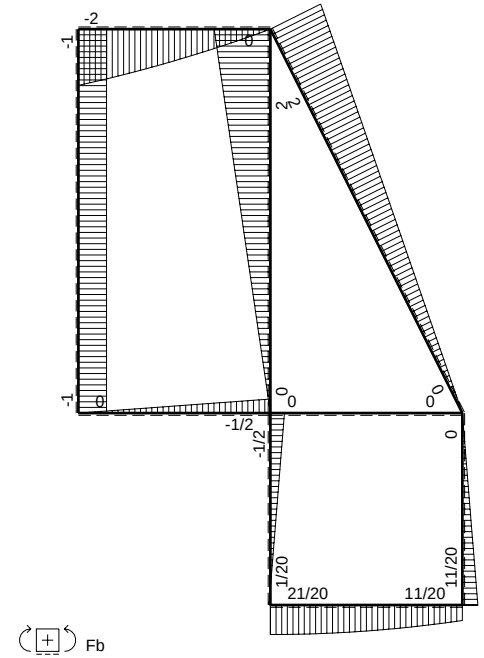
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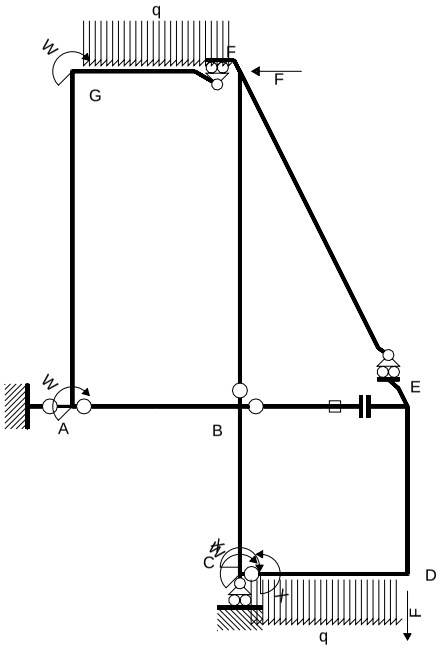
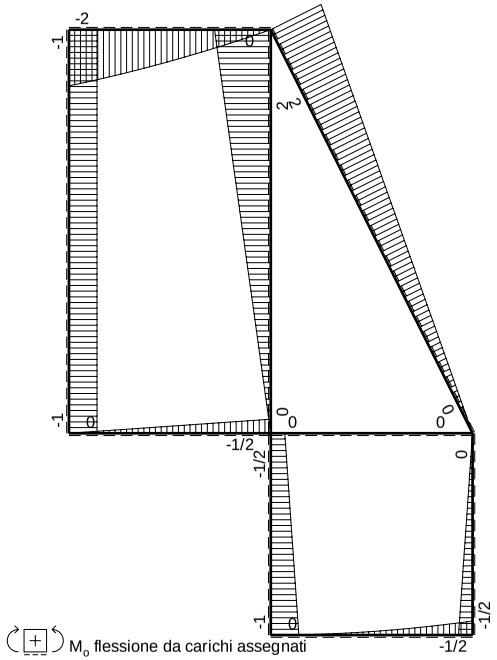
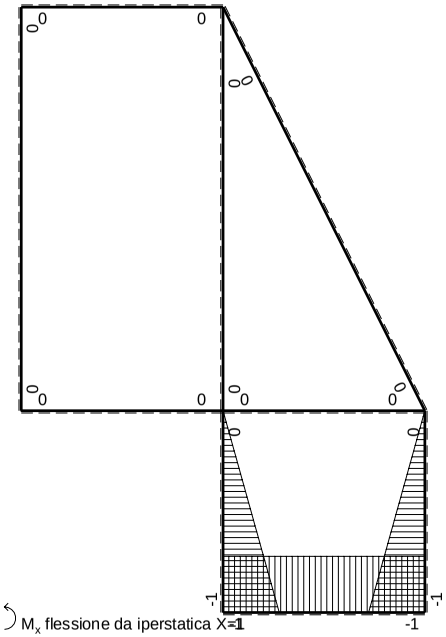


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

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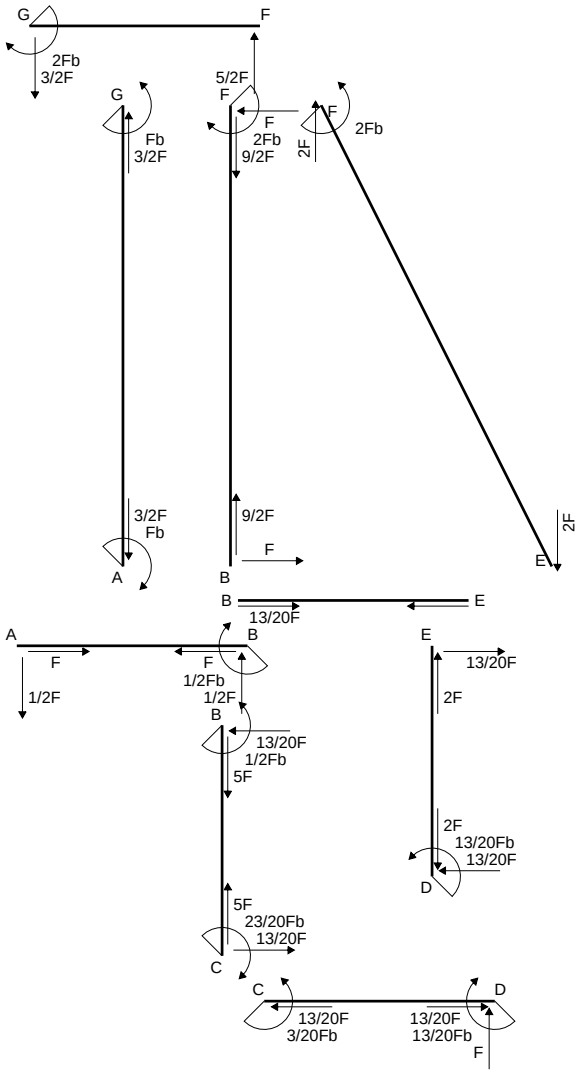
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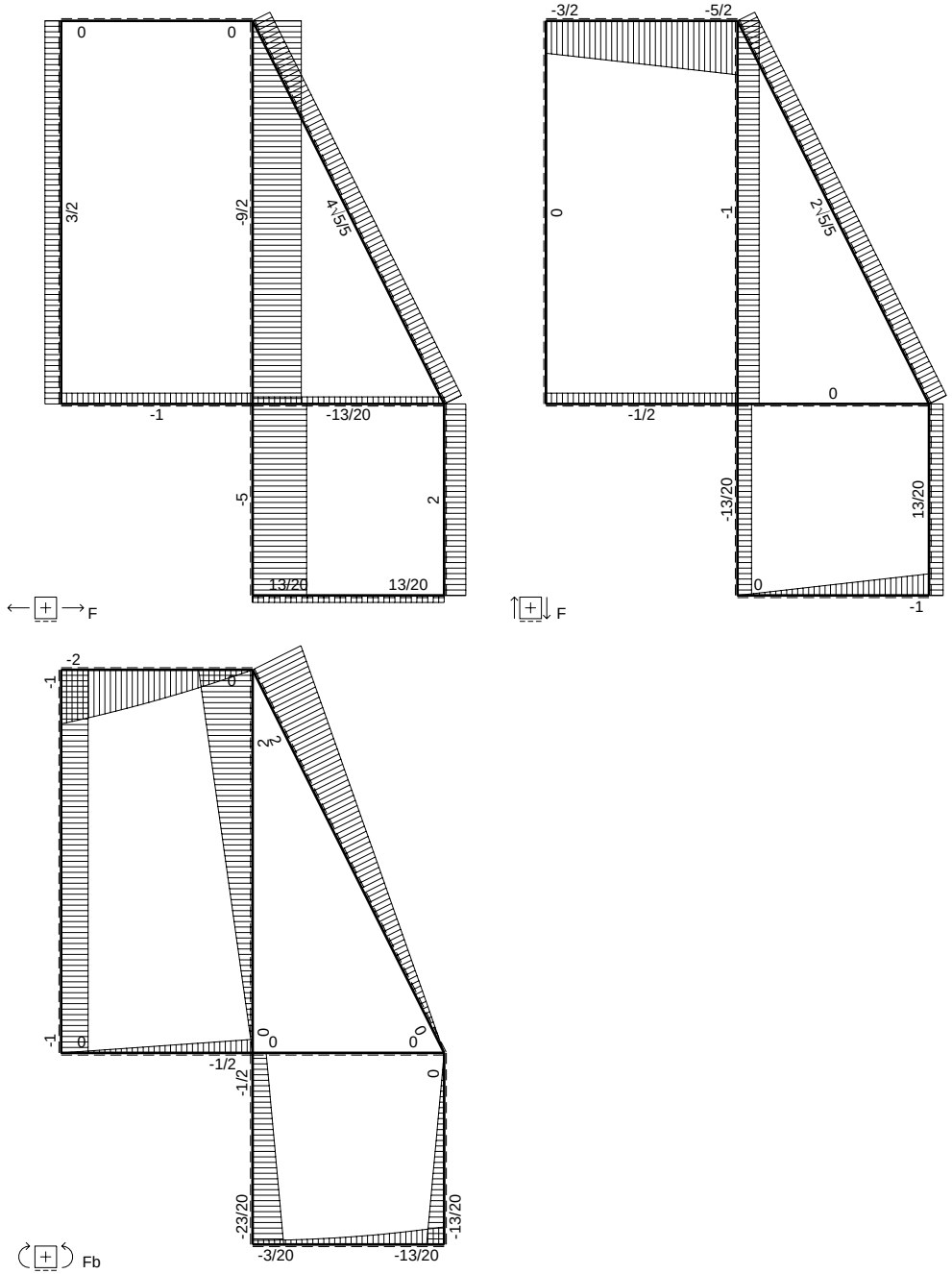
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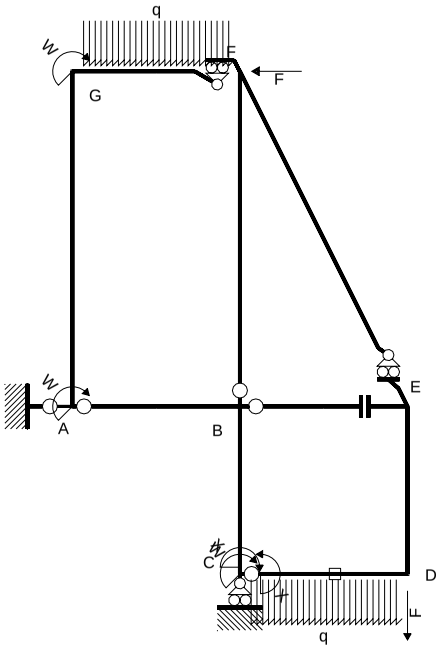
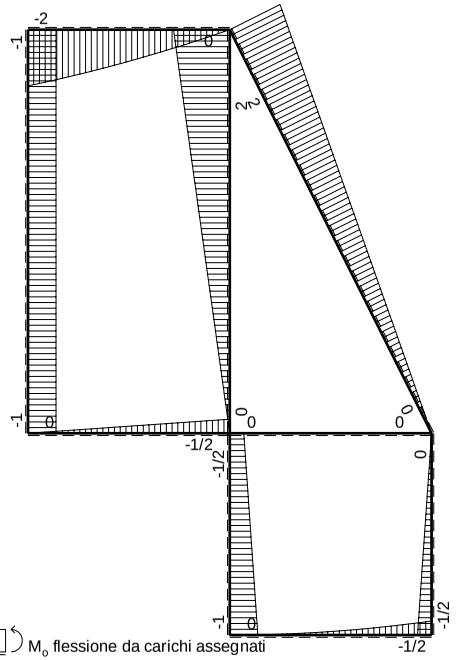
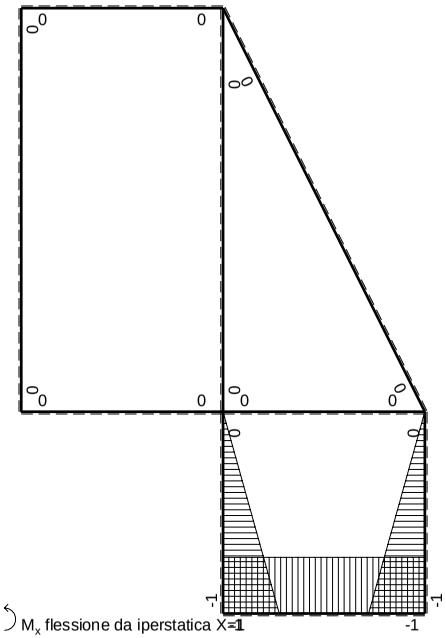


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-1/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				3/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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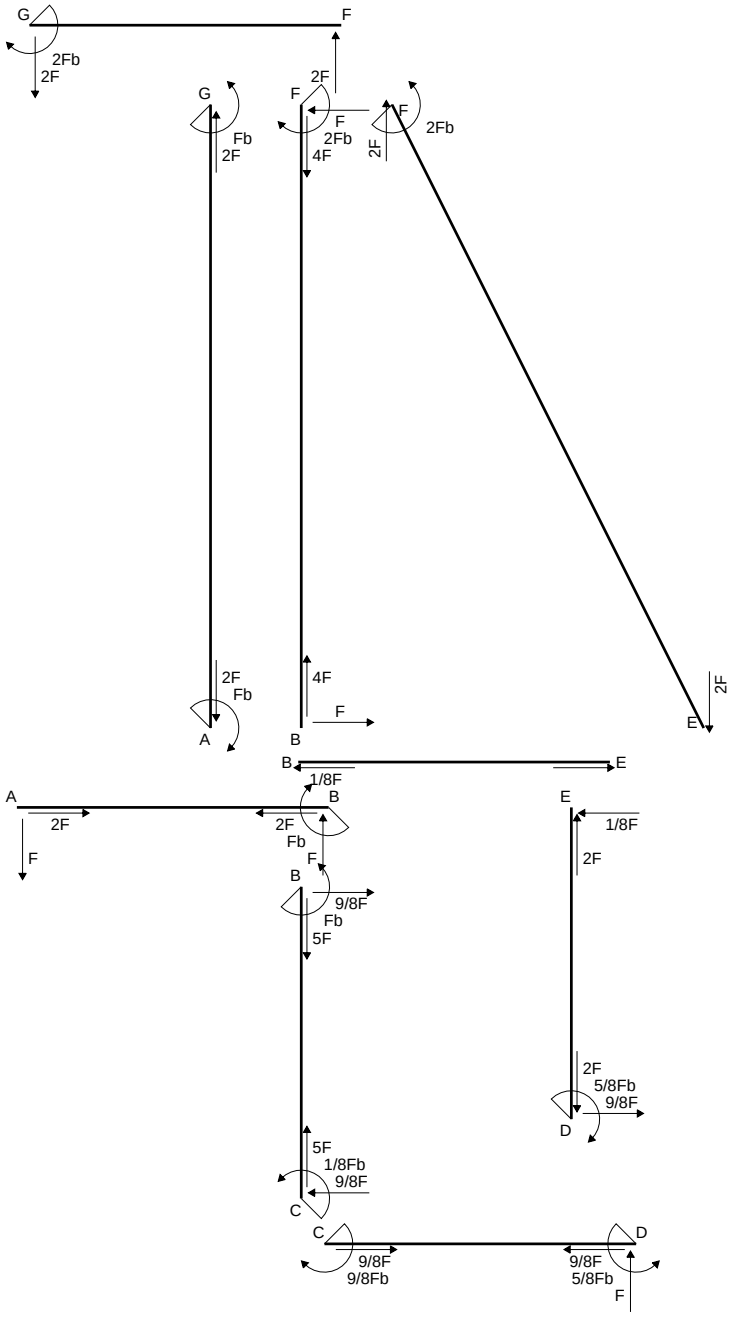
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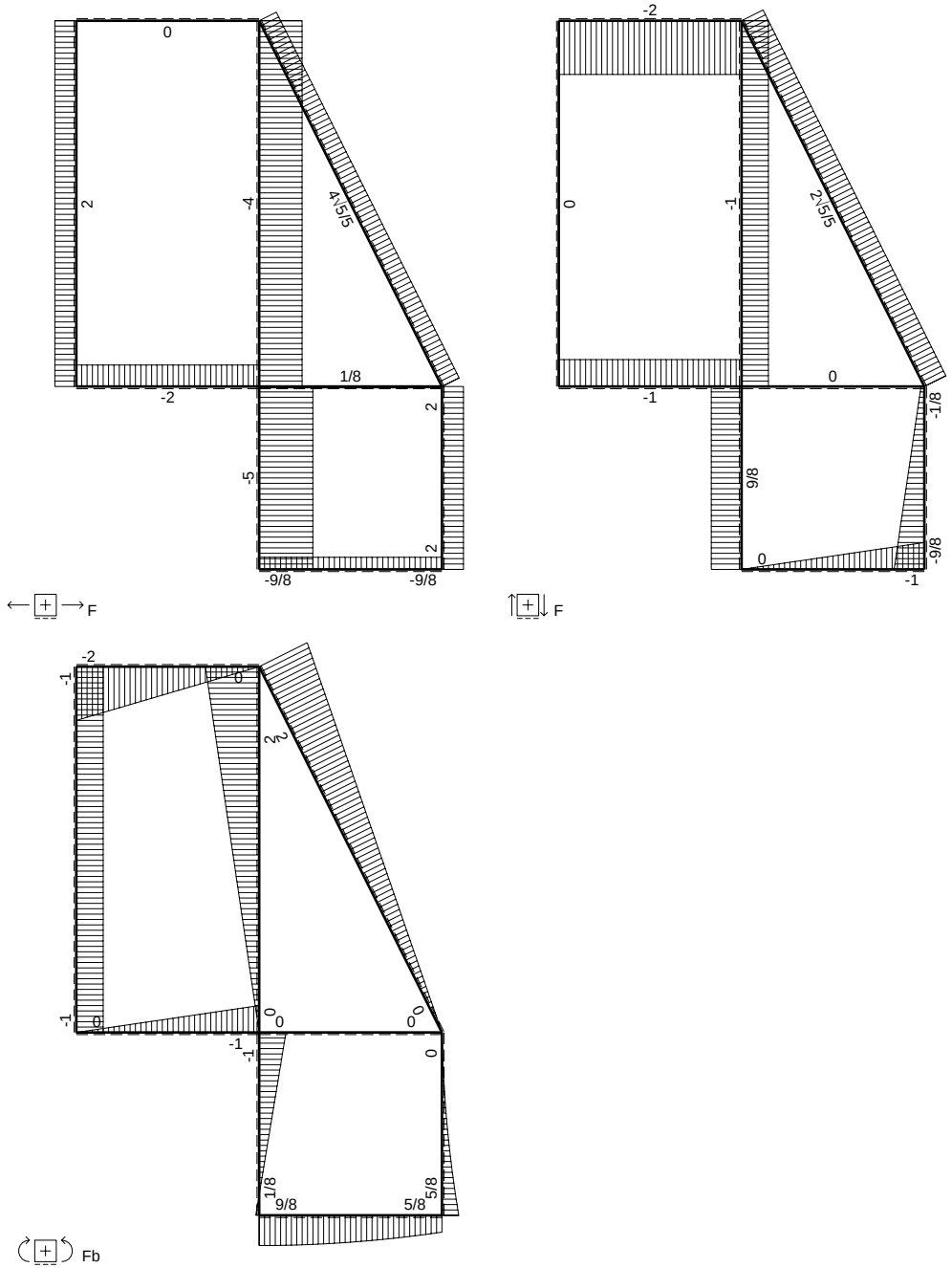
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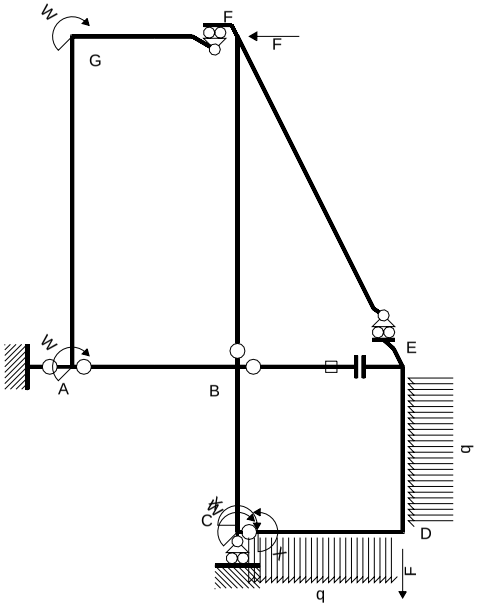
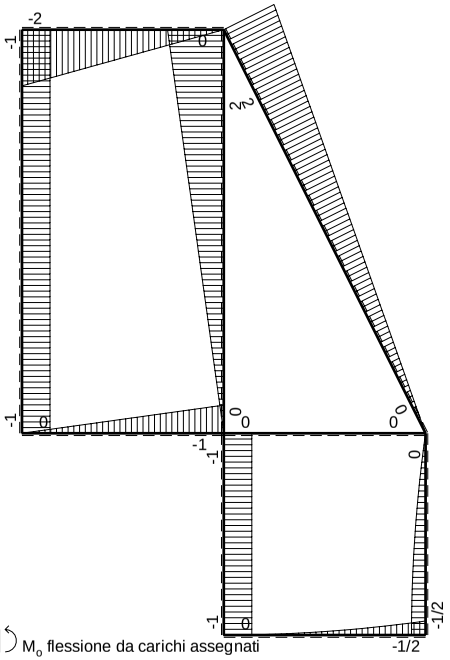
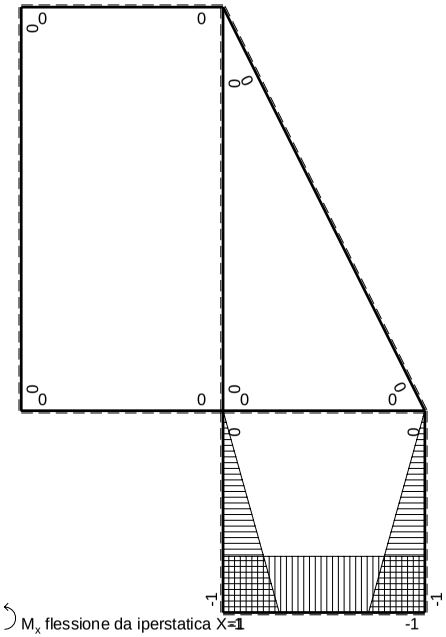


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ02.xxxx.177

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb+1/2qx^2$	$1/2Fb-1/2Fx-1/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$5/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fx-1/2qx^2$	$Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$15/8Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-9/8Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ02.xxxx.177

Sviluppi di calcolo iperstatica

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$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 -1/2\ x/b -1/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [1/2\ x -1/4\ x^2/b -1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/4\ b -1/6\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

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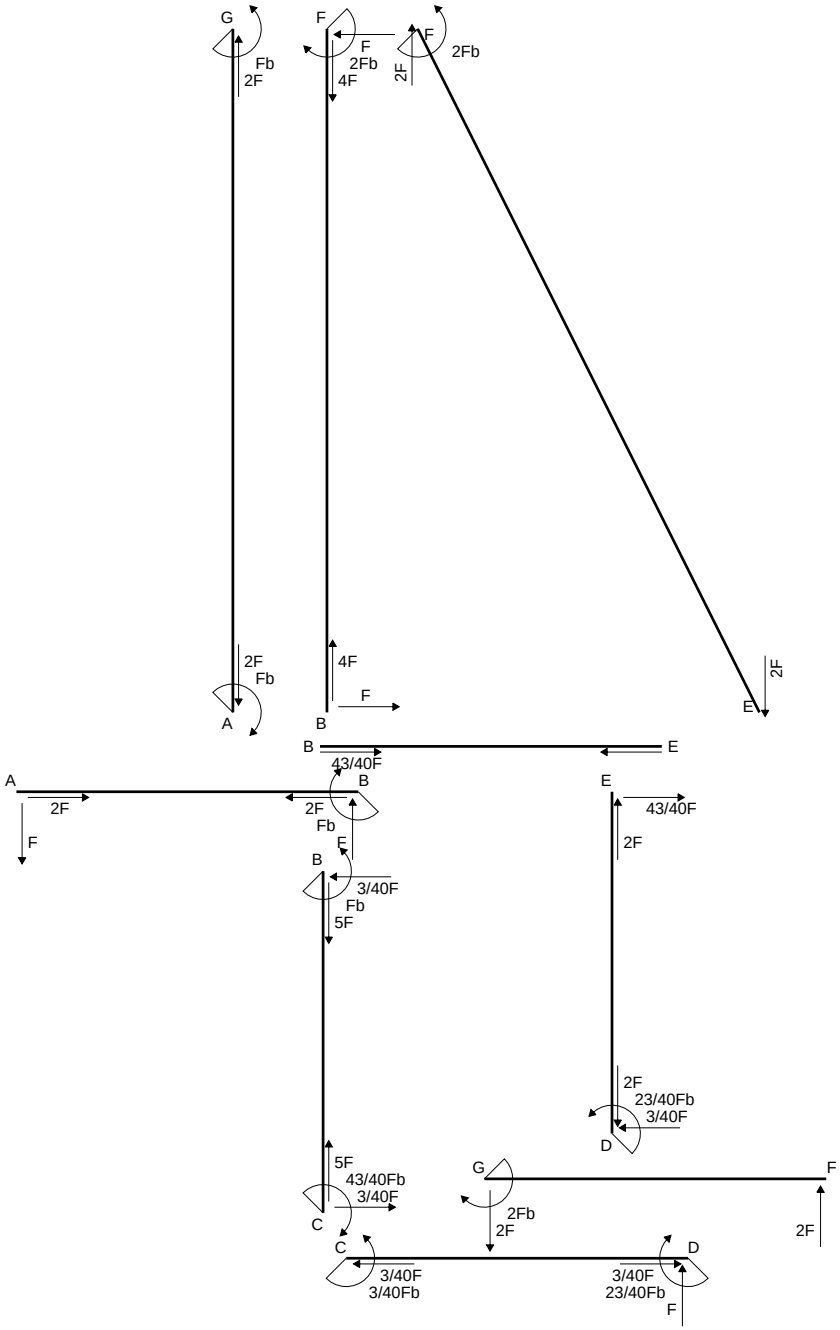
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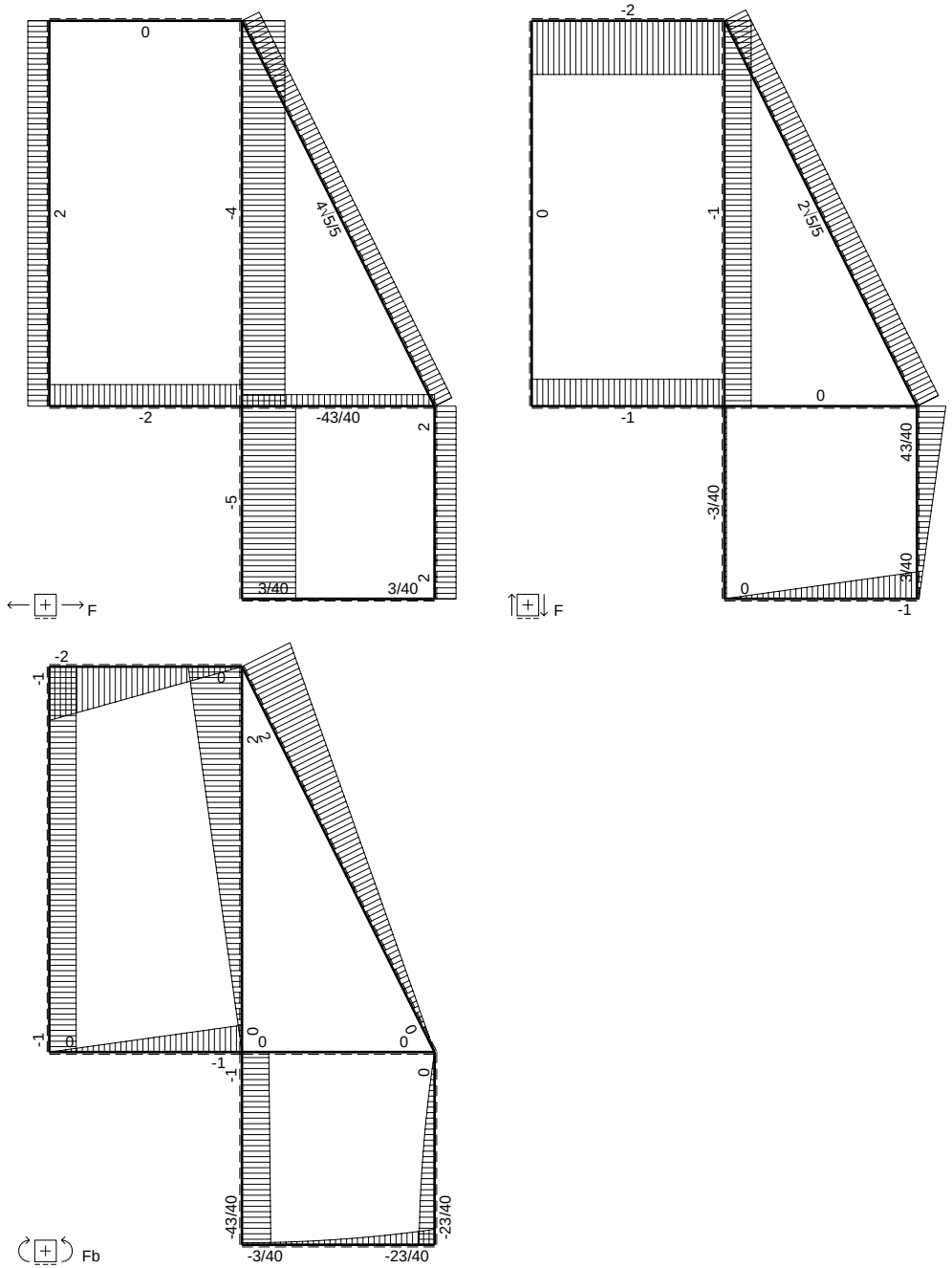
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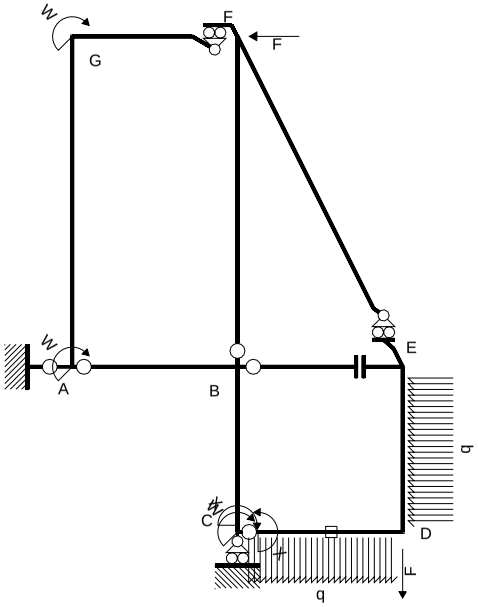
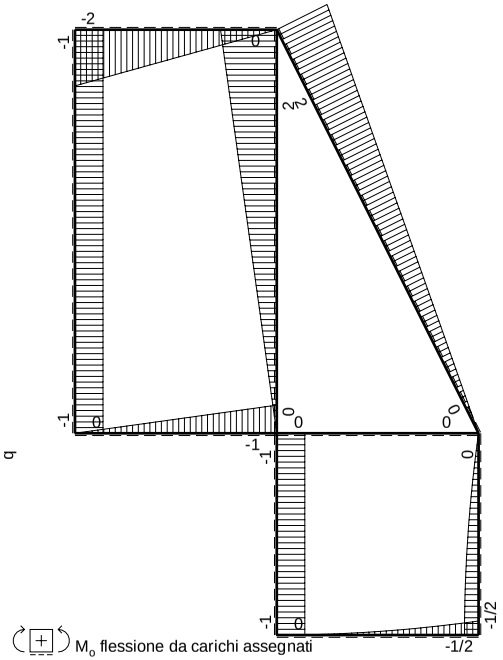
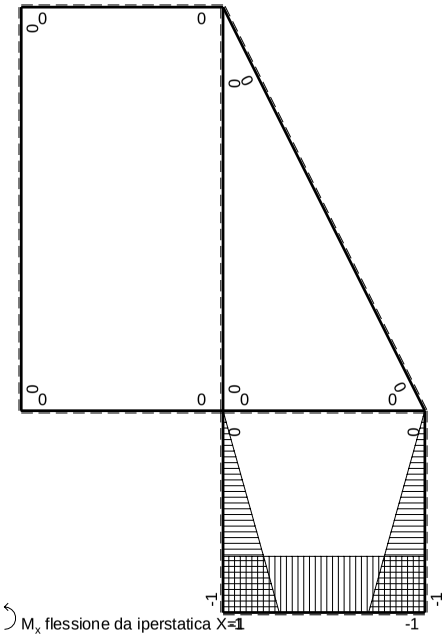


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ02.xxxx.178

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb	Fx	x^2/b^2	$1/2Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb	Fb-Fx	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	$-1/2Fb+1/2qx^2$	$1/2Fb-1/2Fx-1/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$5/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fx-1/2qx^2$	$Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-1/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				3/40Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ02.xxxx.178

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ = [1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -5/6\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ 1\ Fb^2/EJ = -5/6\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 -1/2\ x/b -1/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [1/2\ x -1/4\ x^2/b -1/6\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/4\ b -1/6\ b +1/8\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/3\ b -1/8\ b)\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ02.xxxx.178

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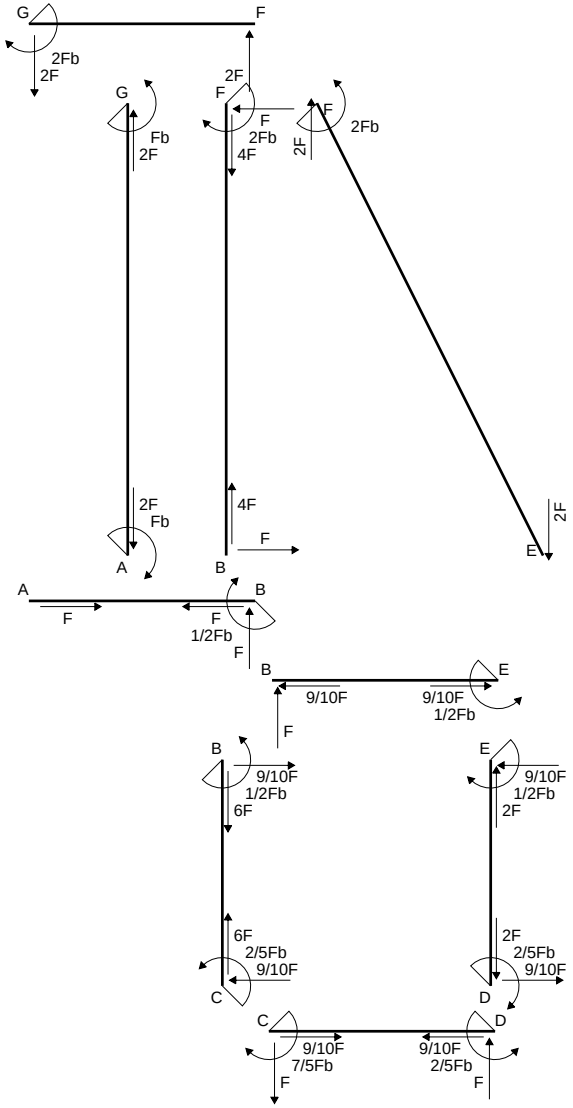
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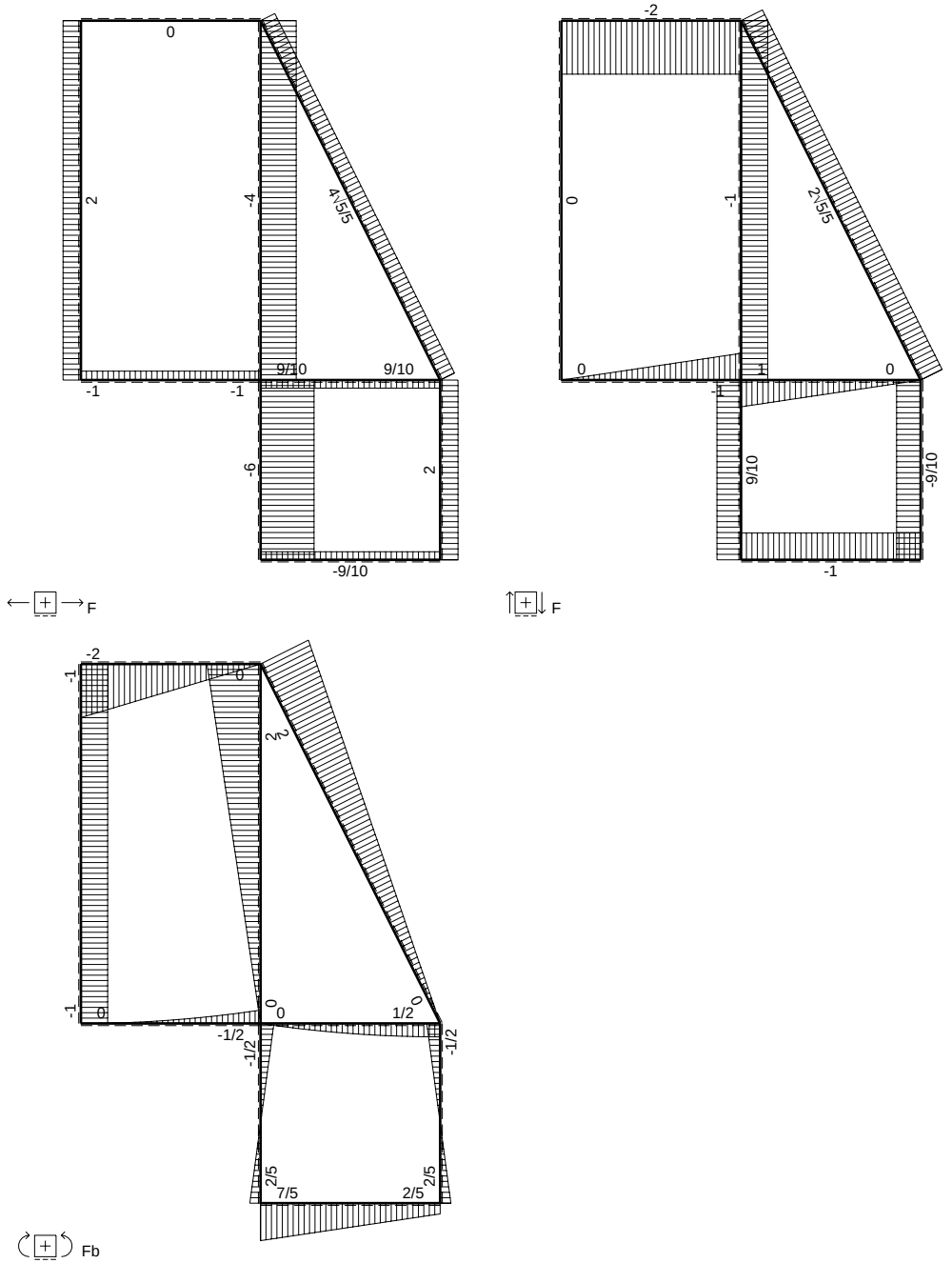
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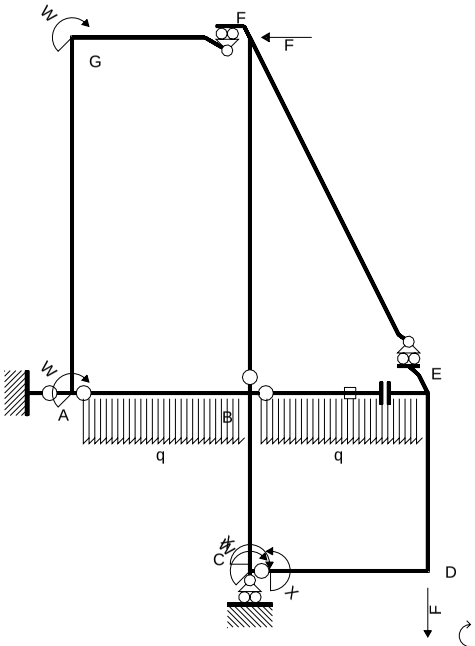
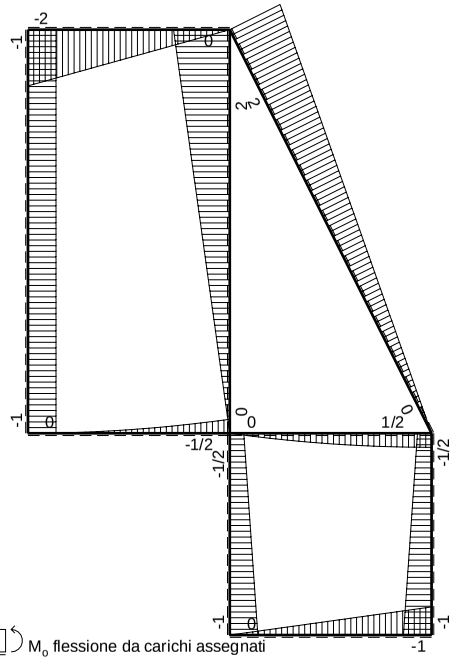
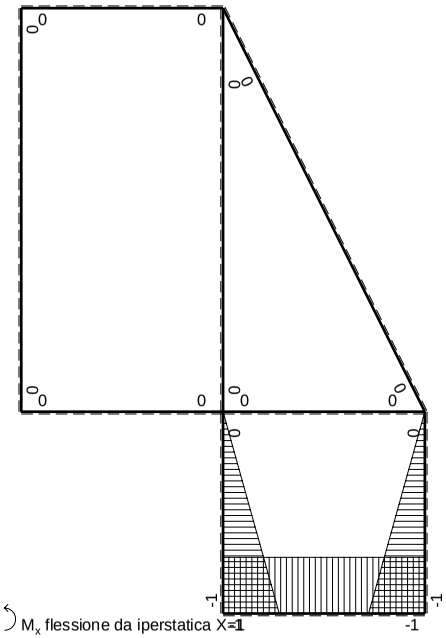


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fb+1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx²	0	0	0	0
EB b	0	-1/2Fb+1/2qx²	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/3Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-7/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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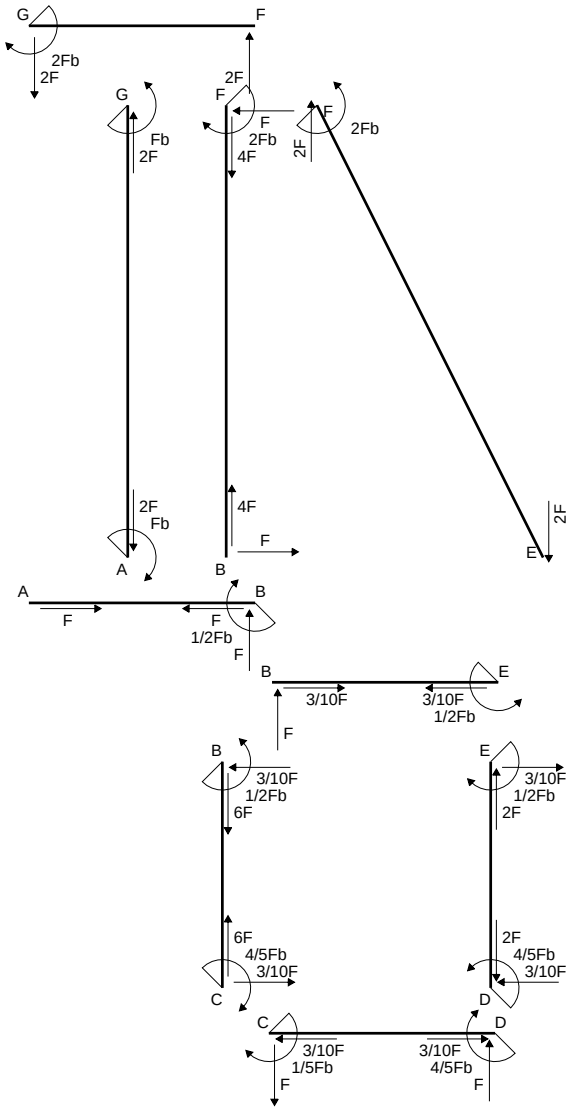
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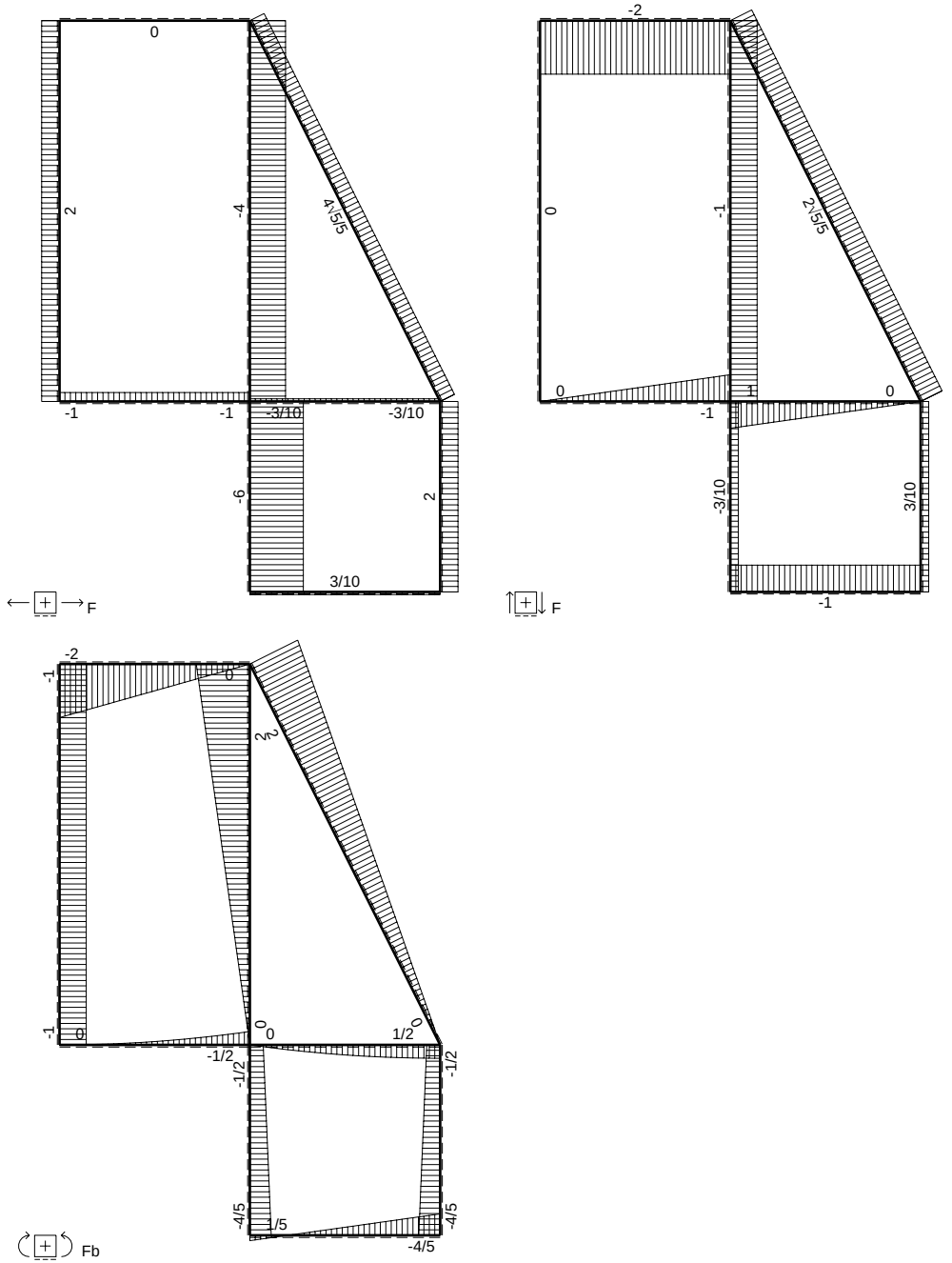
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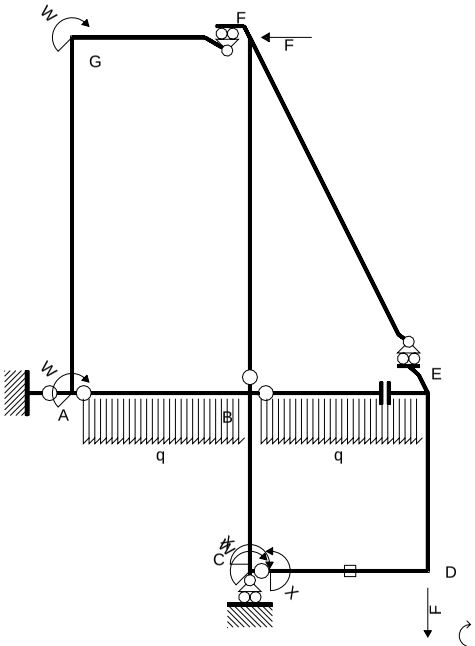
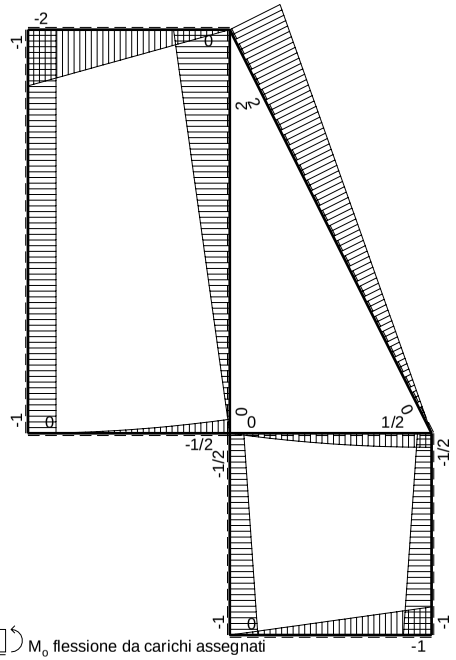
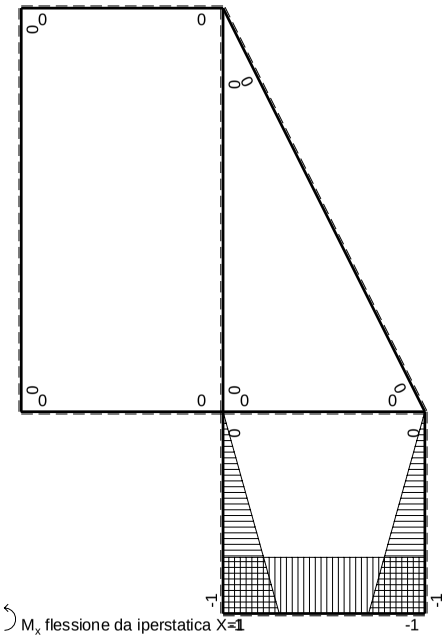


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fb+1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-1/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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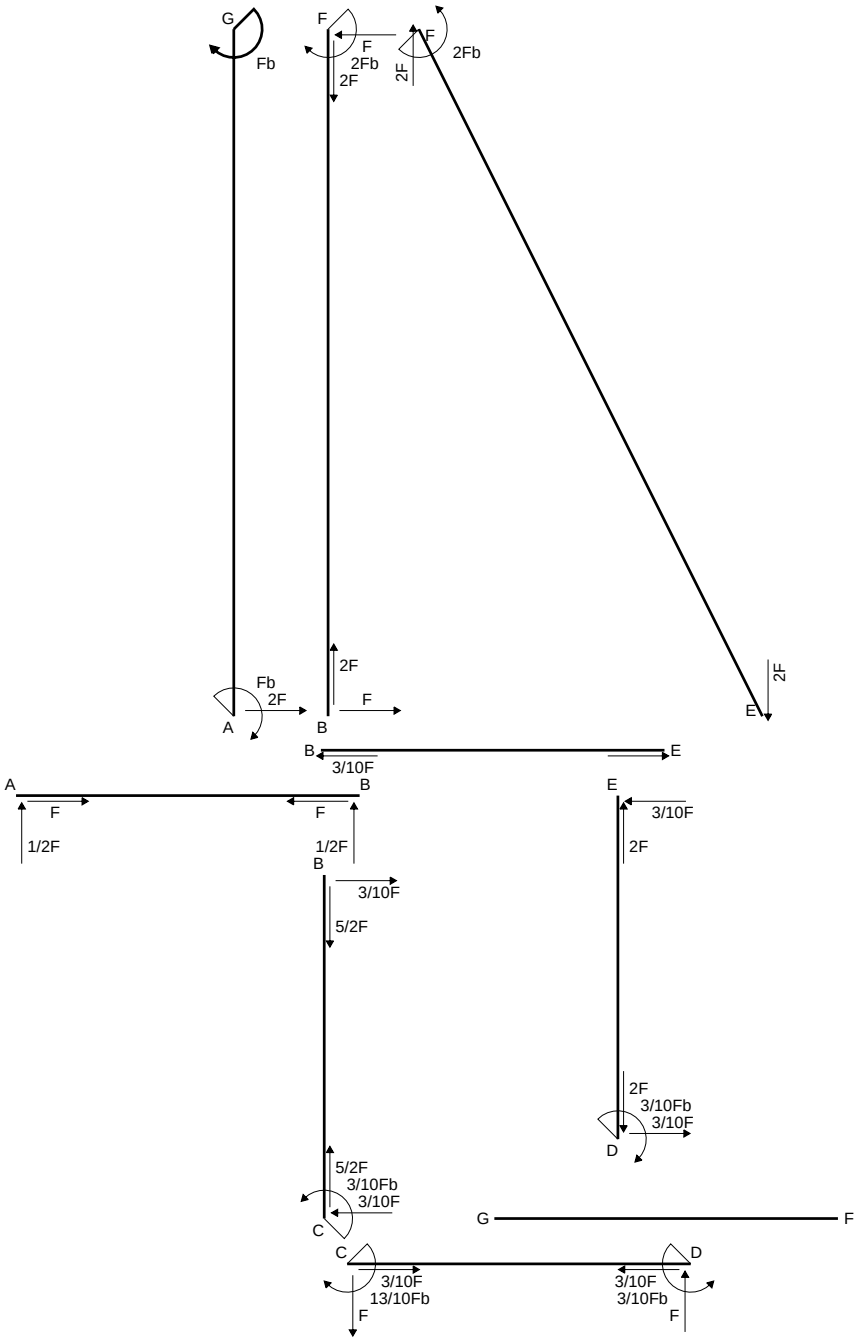
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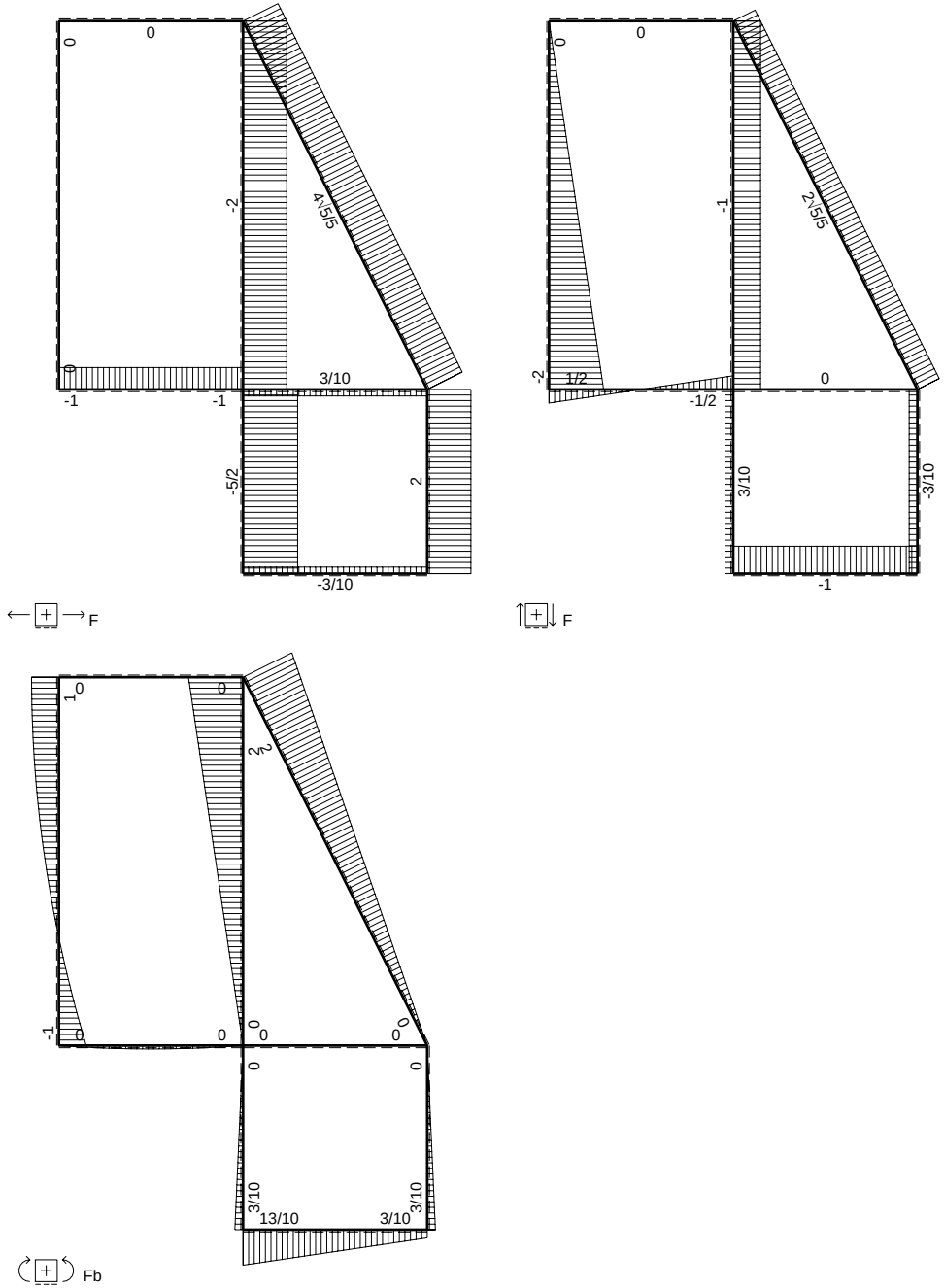
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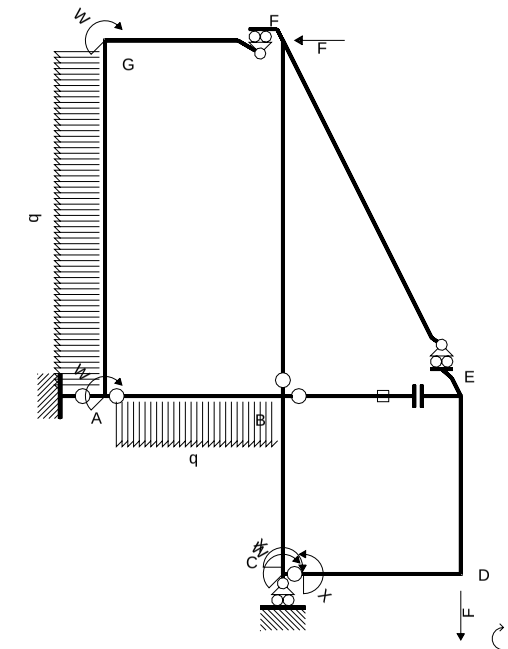
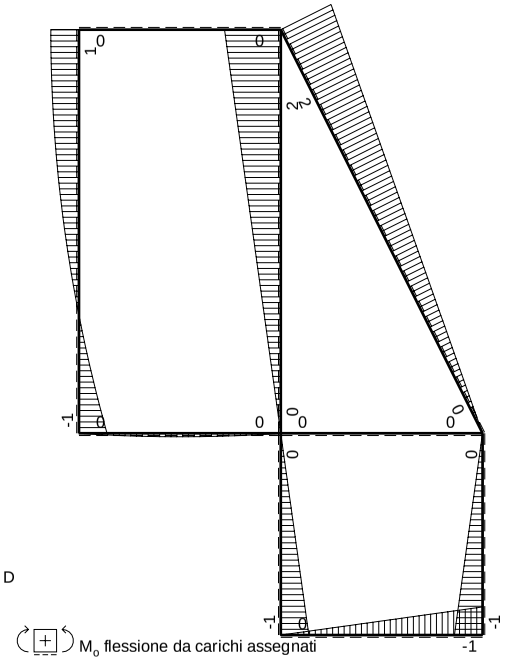
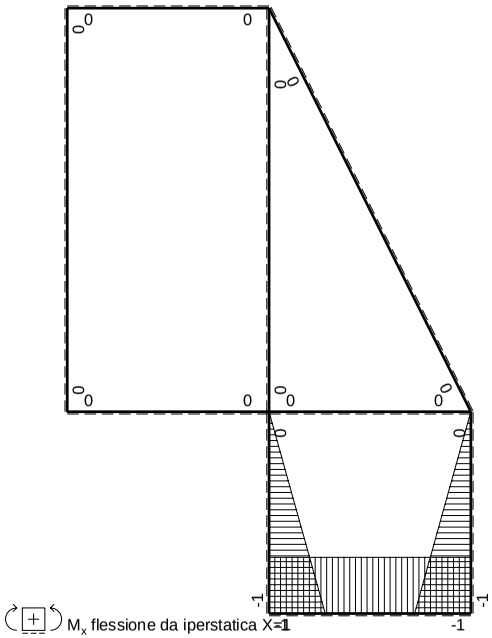


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$13/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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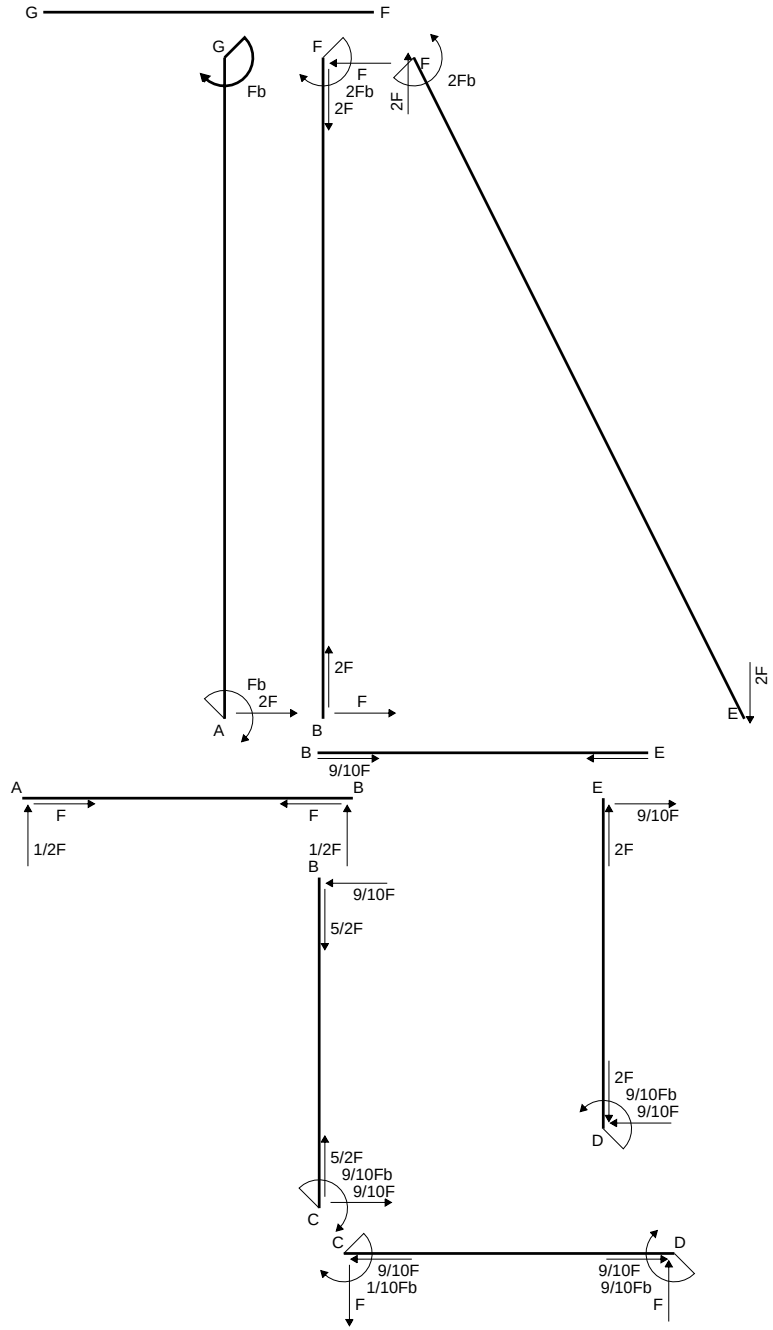
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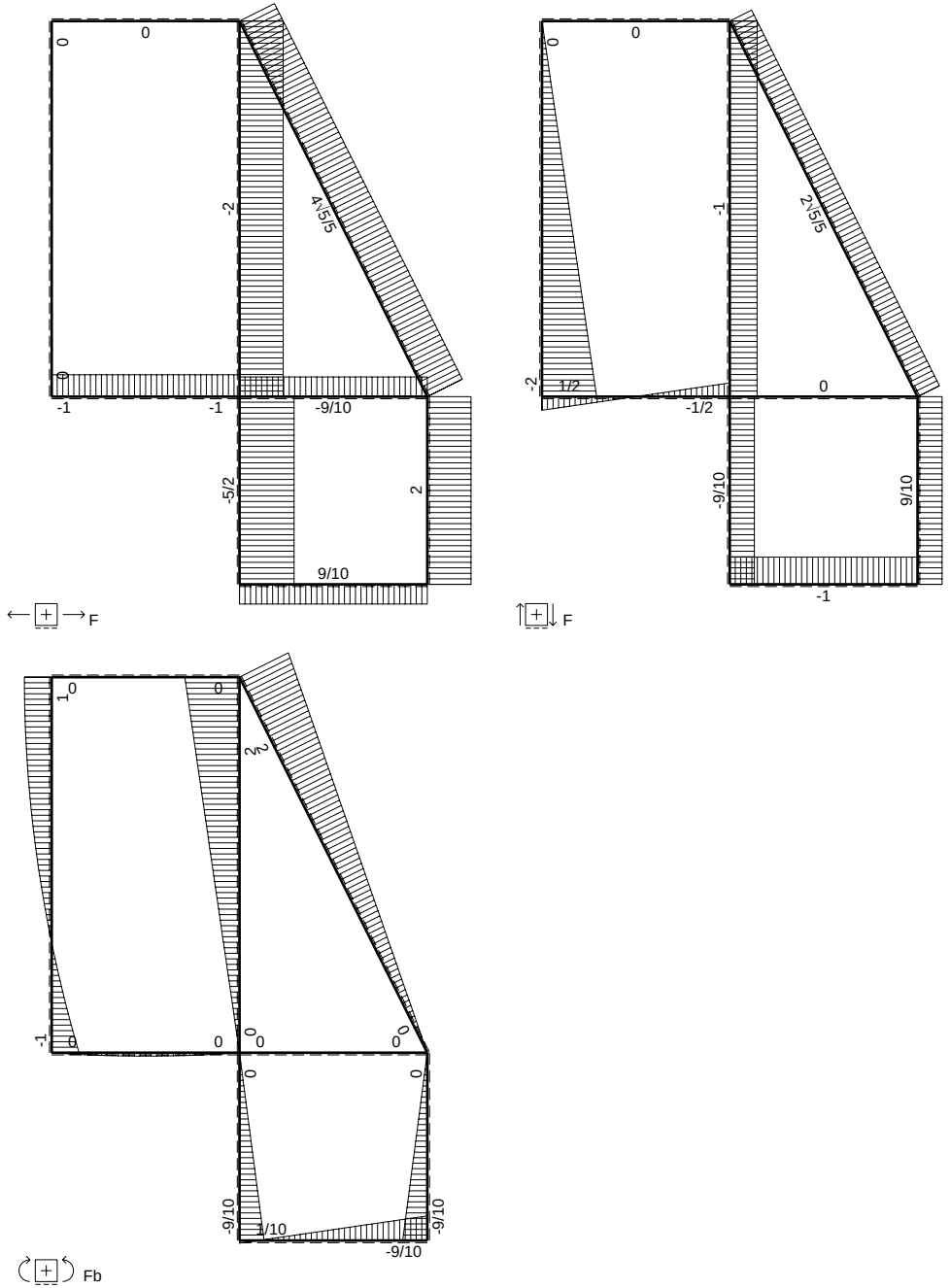
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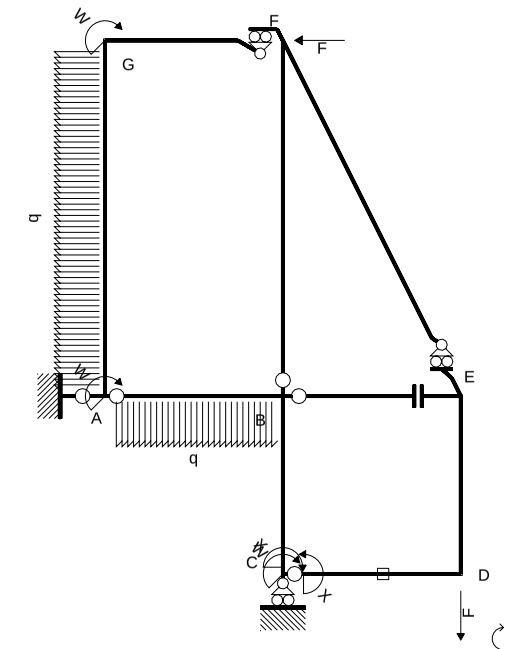
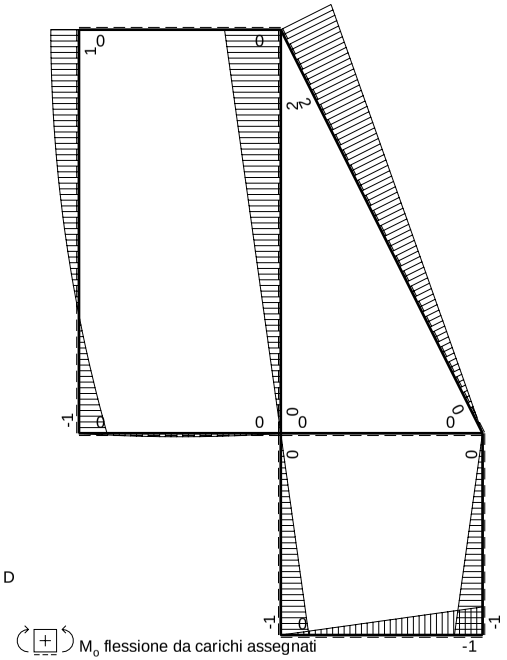
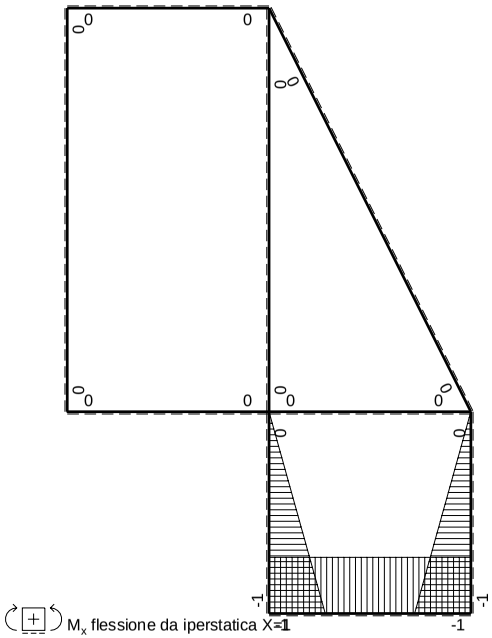


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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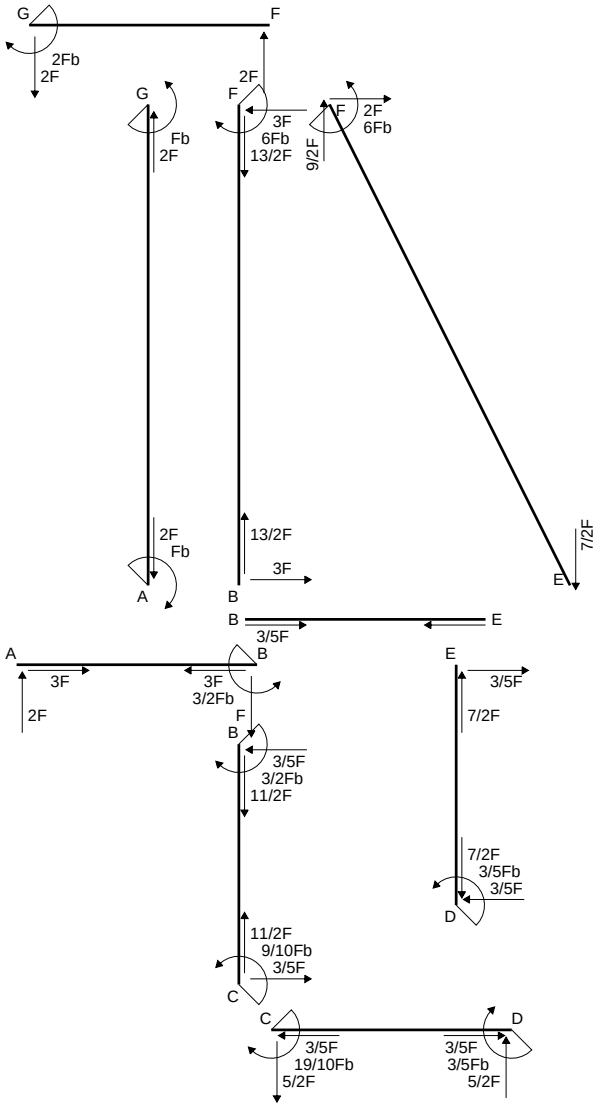
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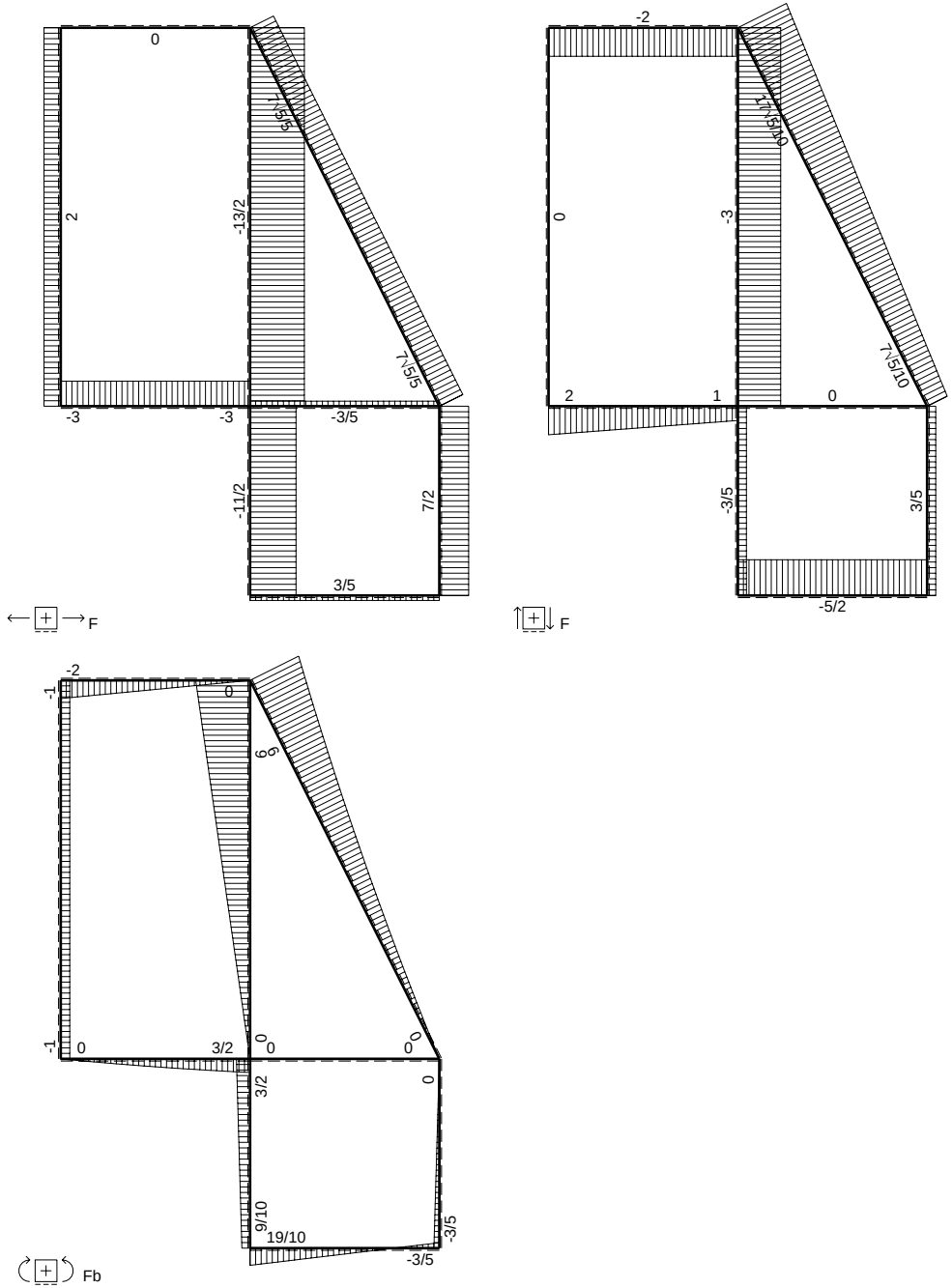
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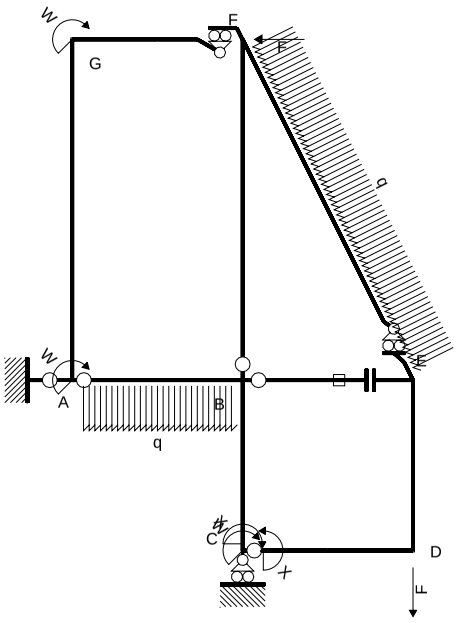
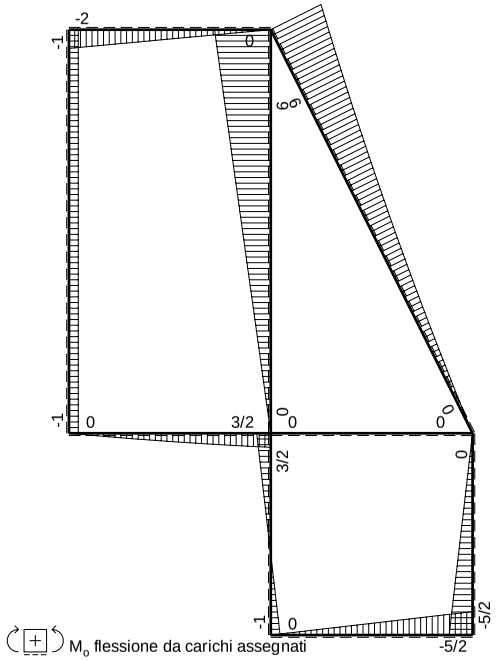
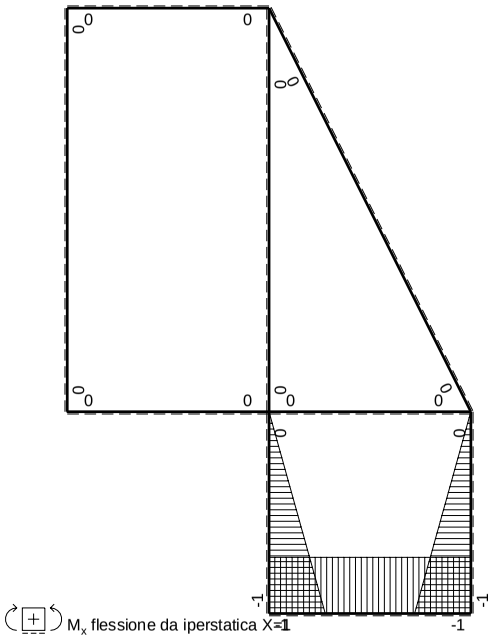


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-3/2Fb+Fx+1/2qx^2$	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$19/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-19/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b)\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b)\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b)\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b)\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b)\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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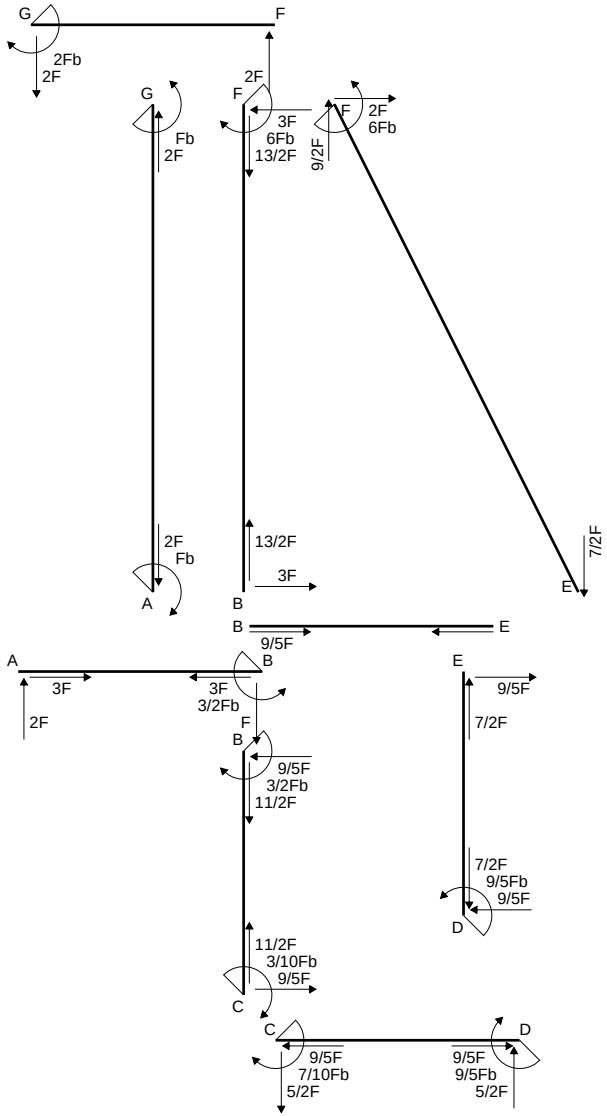
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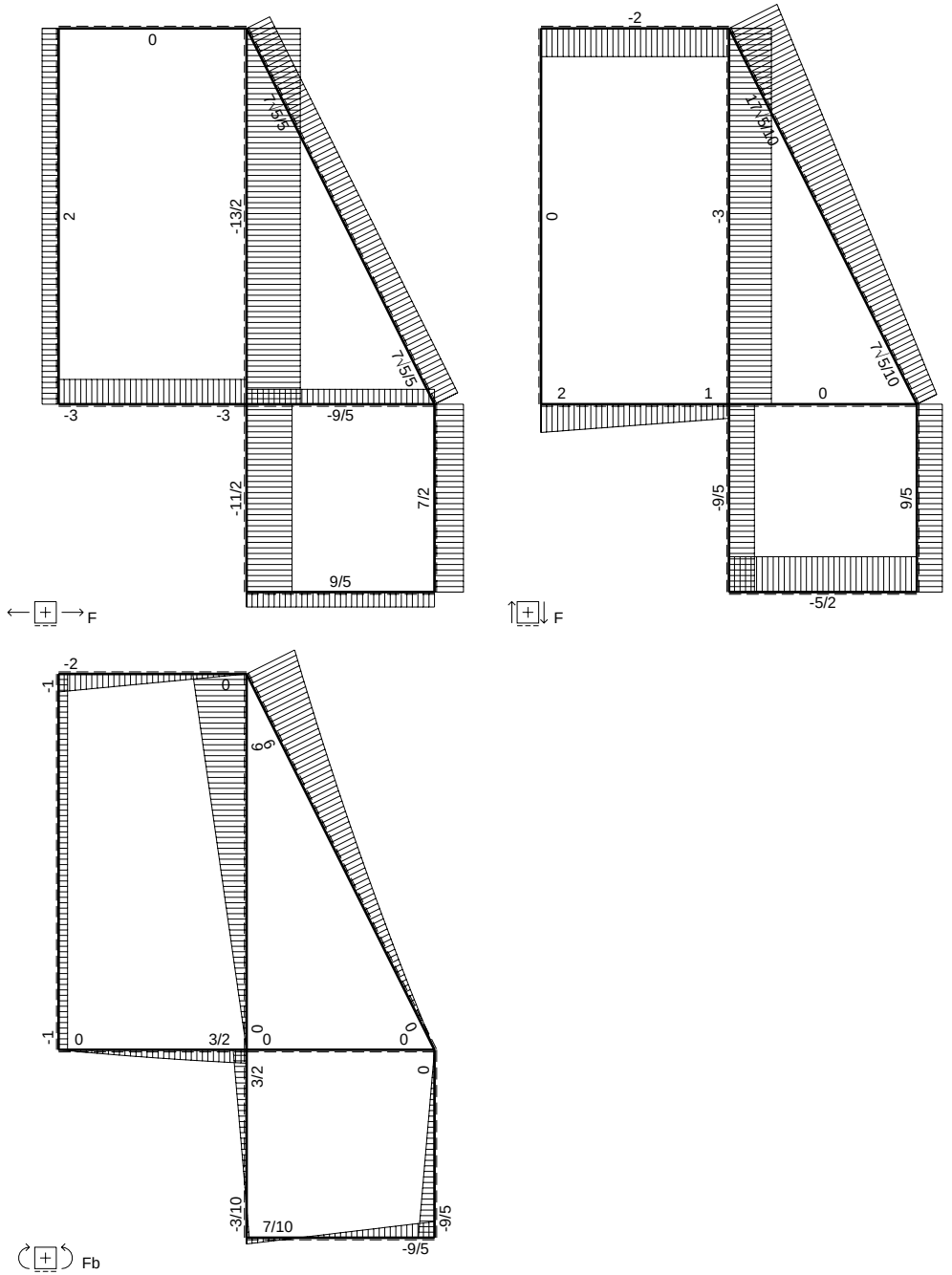
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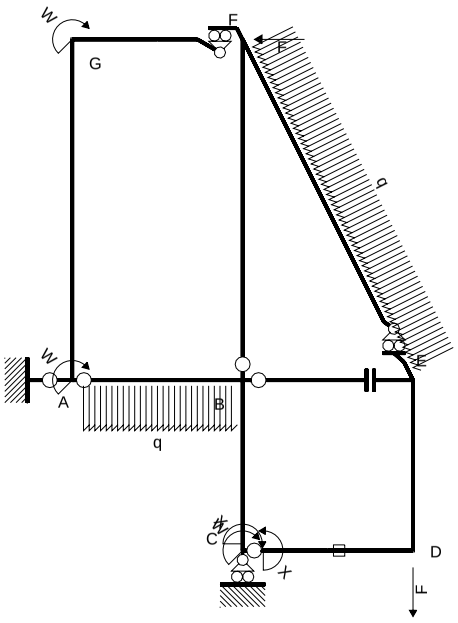
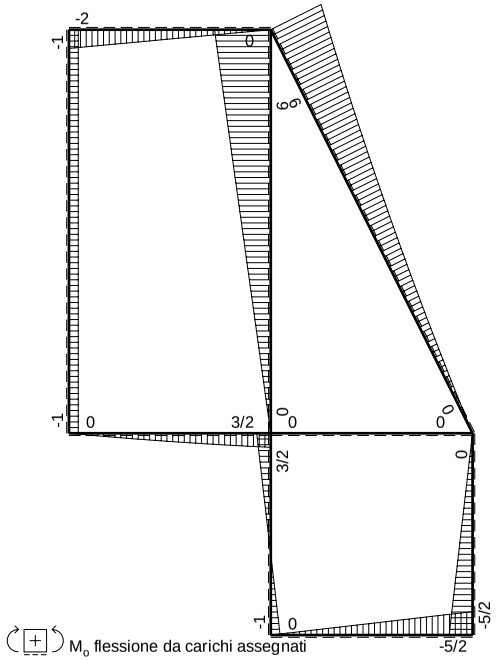
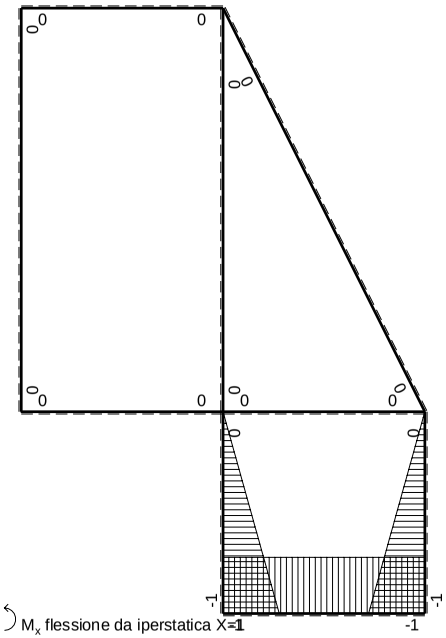
-9/5
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-3/2Fb+Fx+1/2qx^2$	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-5/2Fx	5/2Fx	1	$5/4Fb^2/EJ$	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$7/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-7/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

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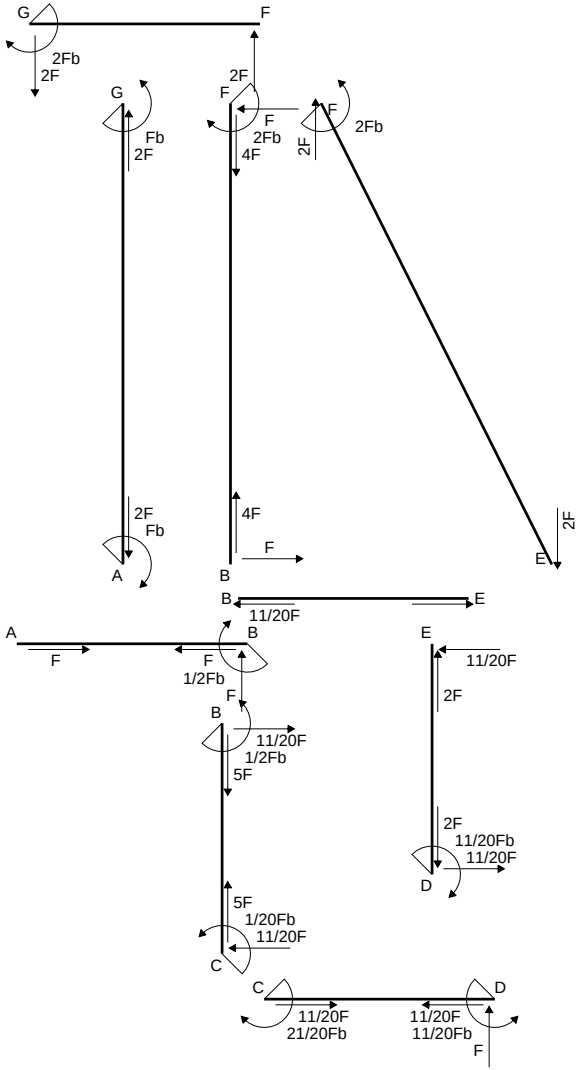
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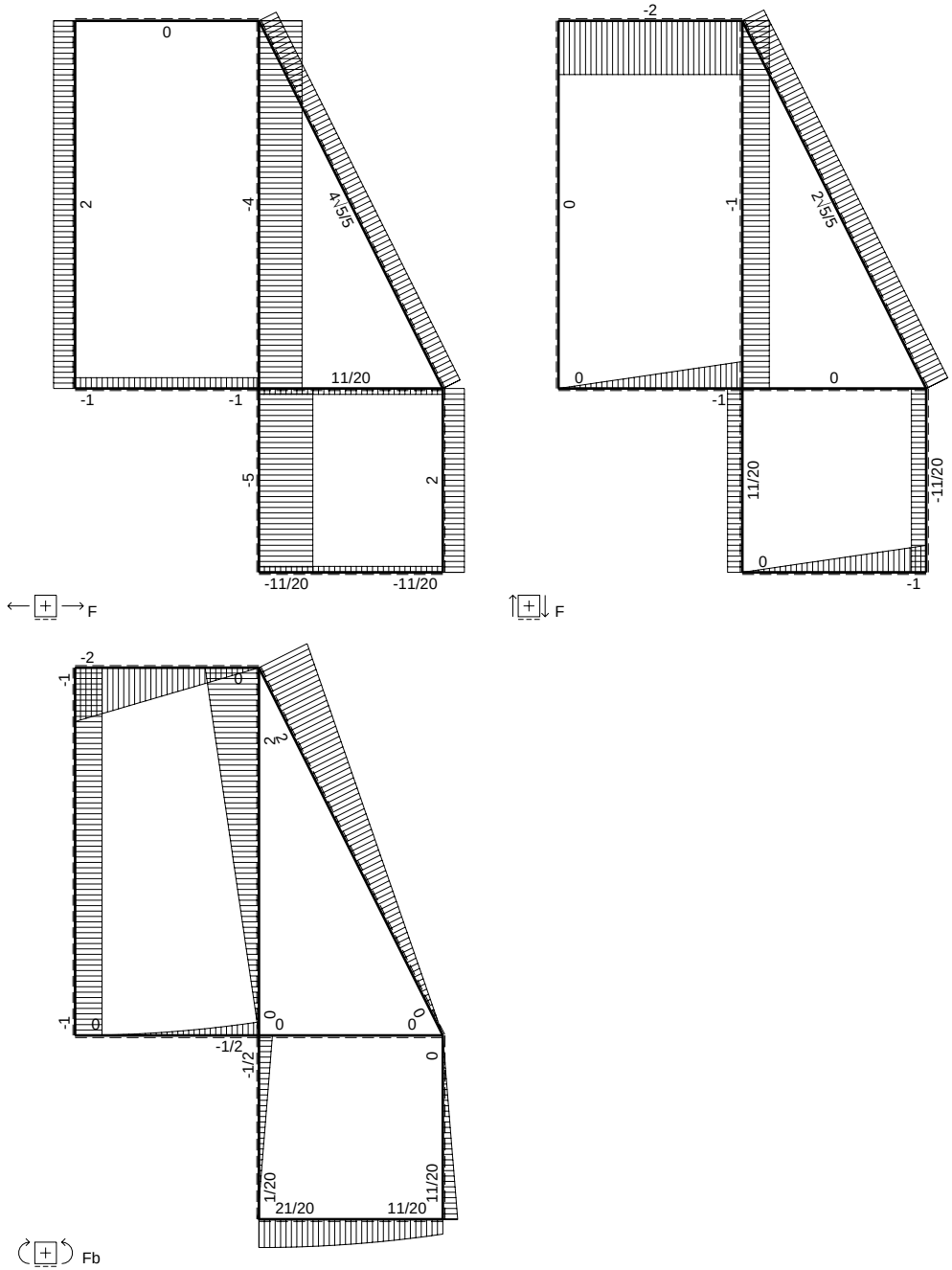
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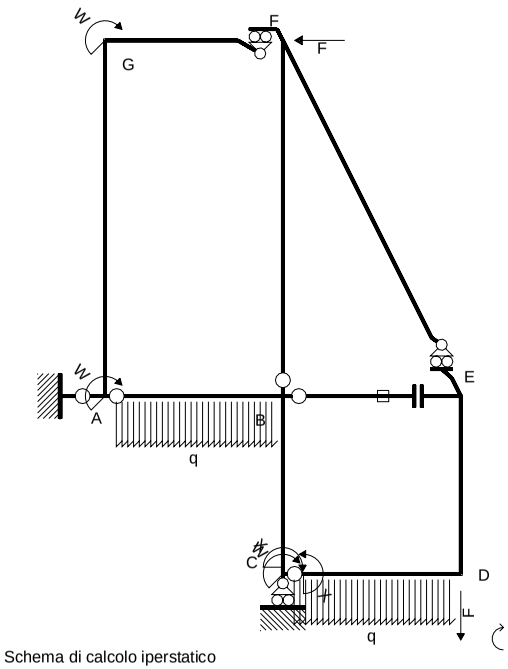
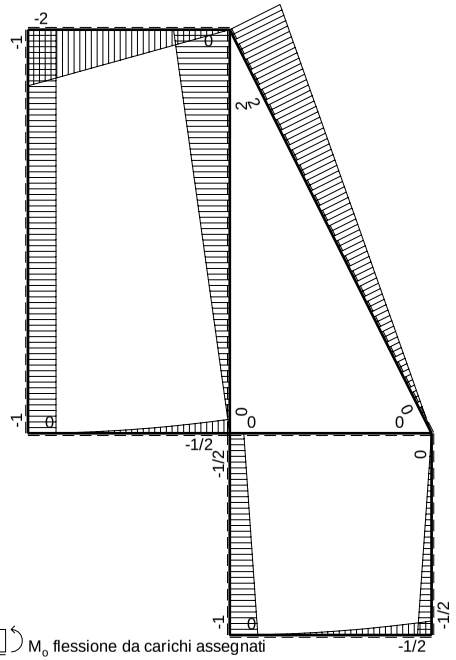
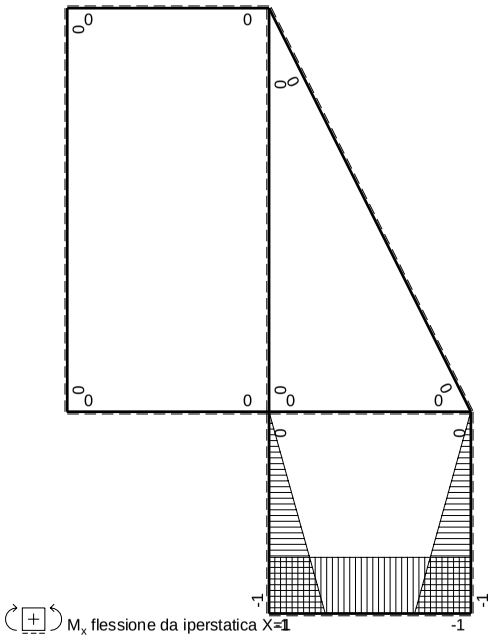


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

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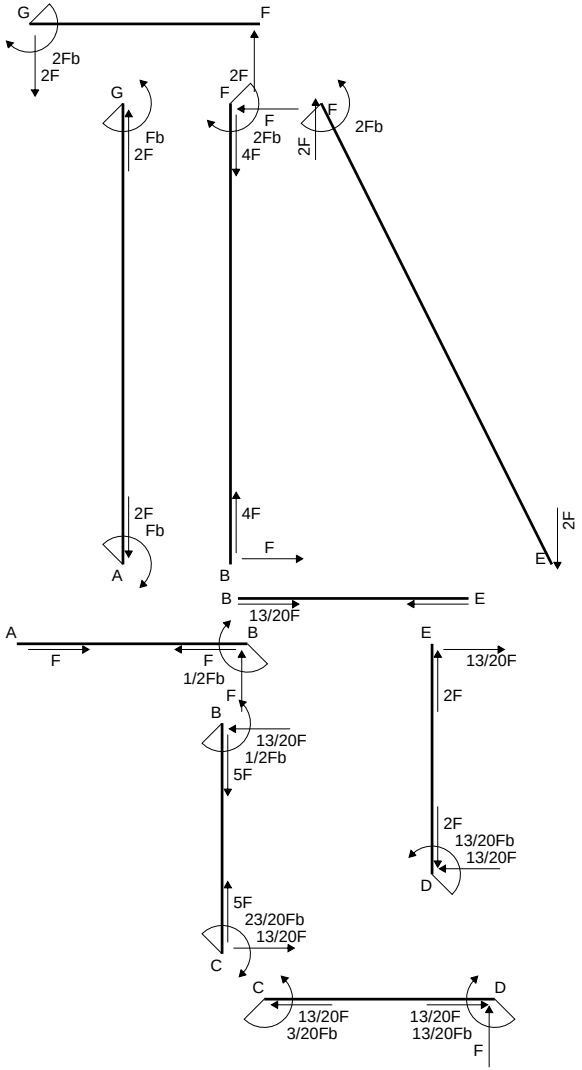
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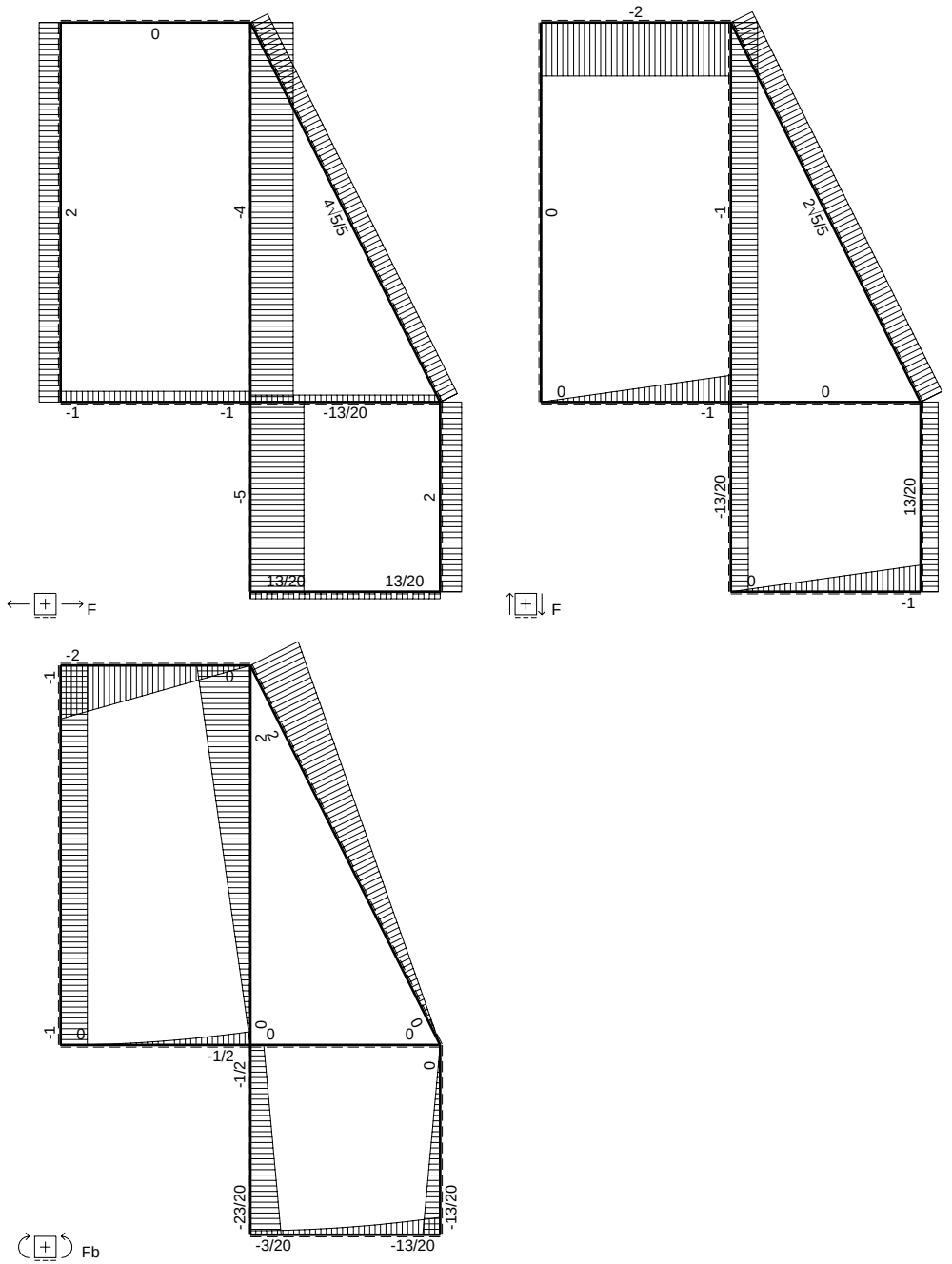
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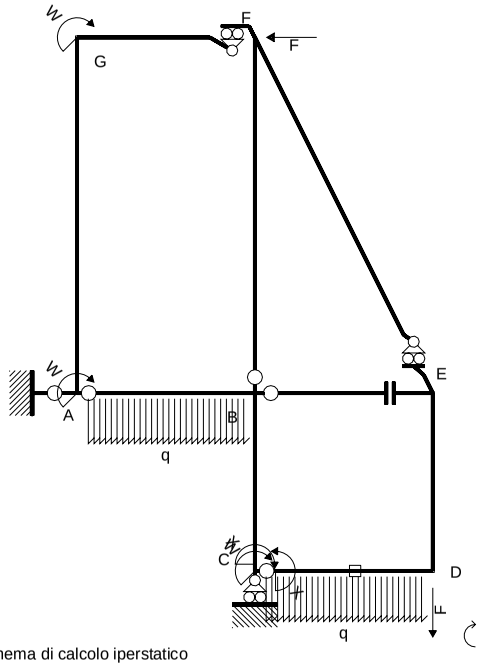
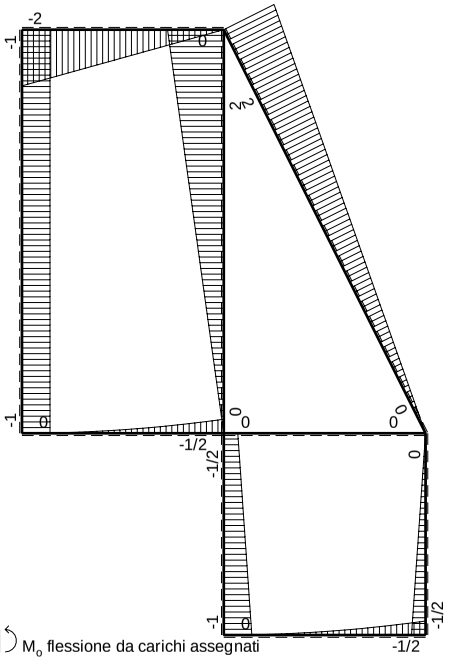
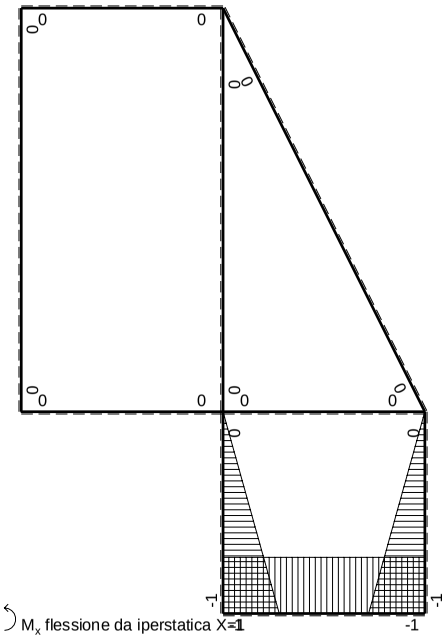


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-1/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				3/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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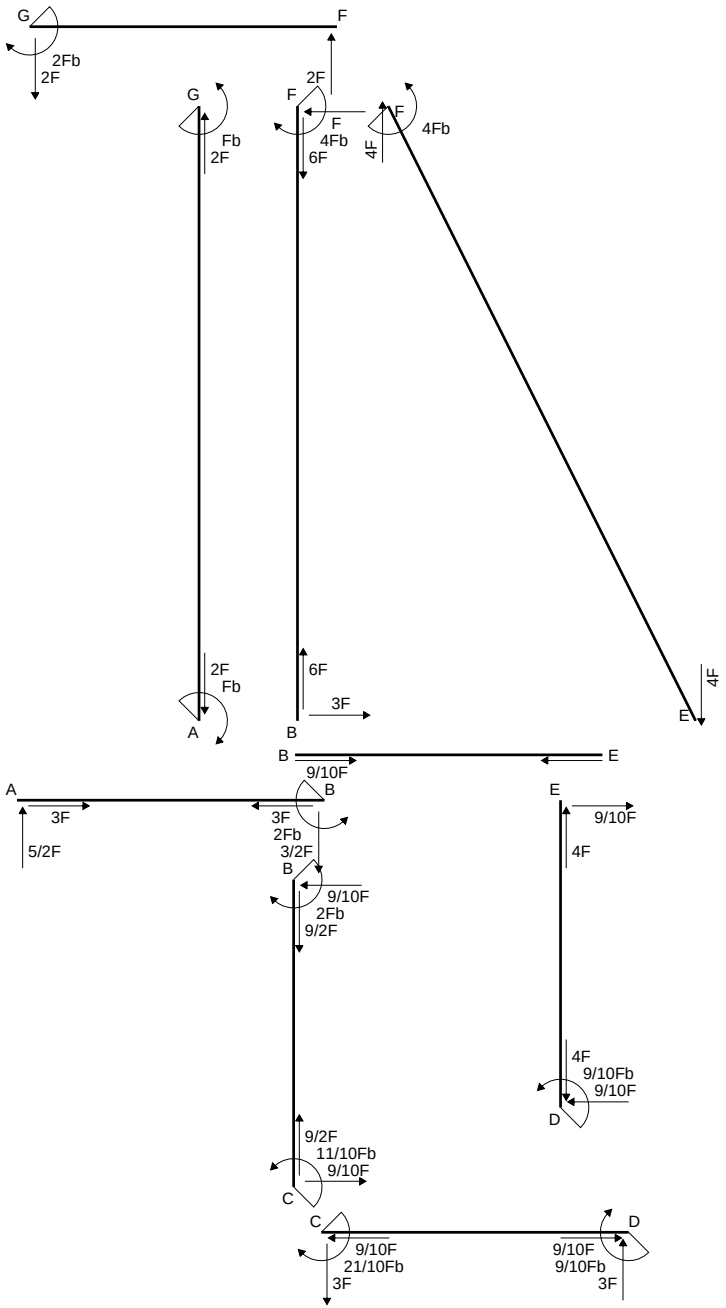
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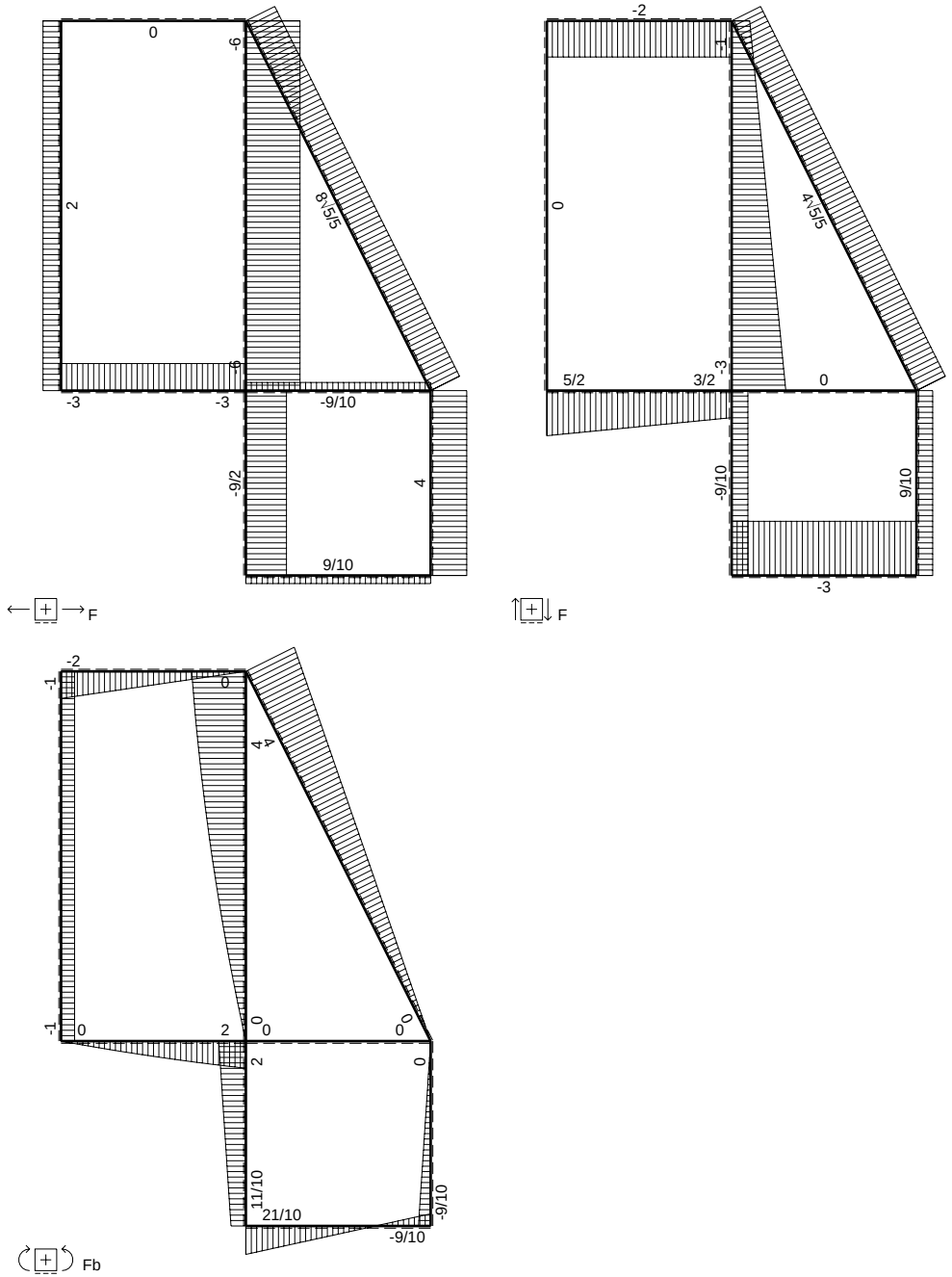
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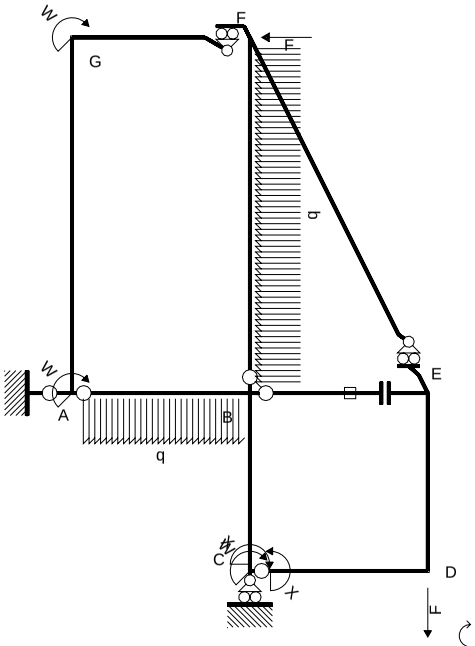
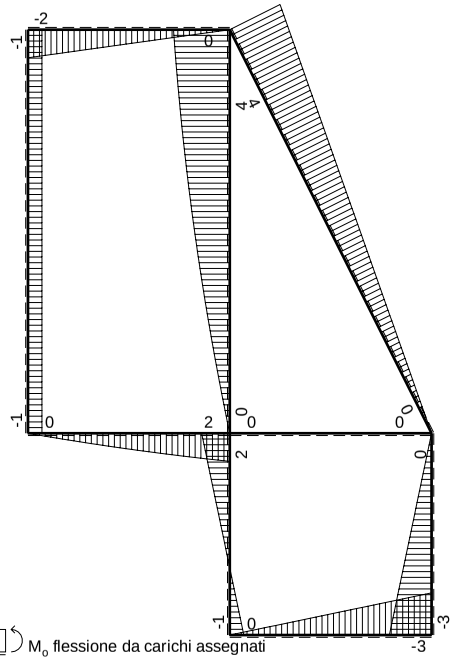
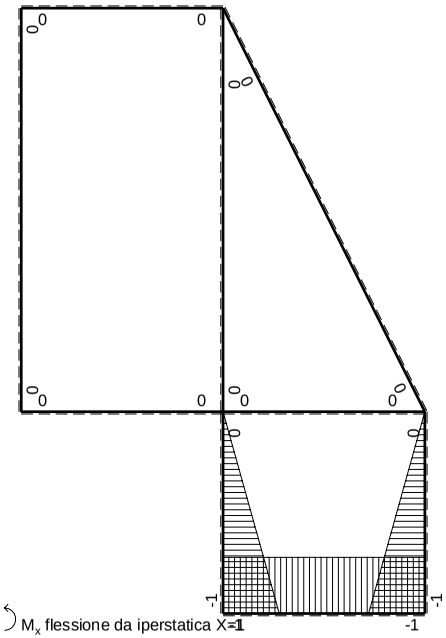


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$5/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-2Fb+3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (- b + b)\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

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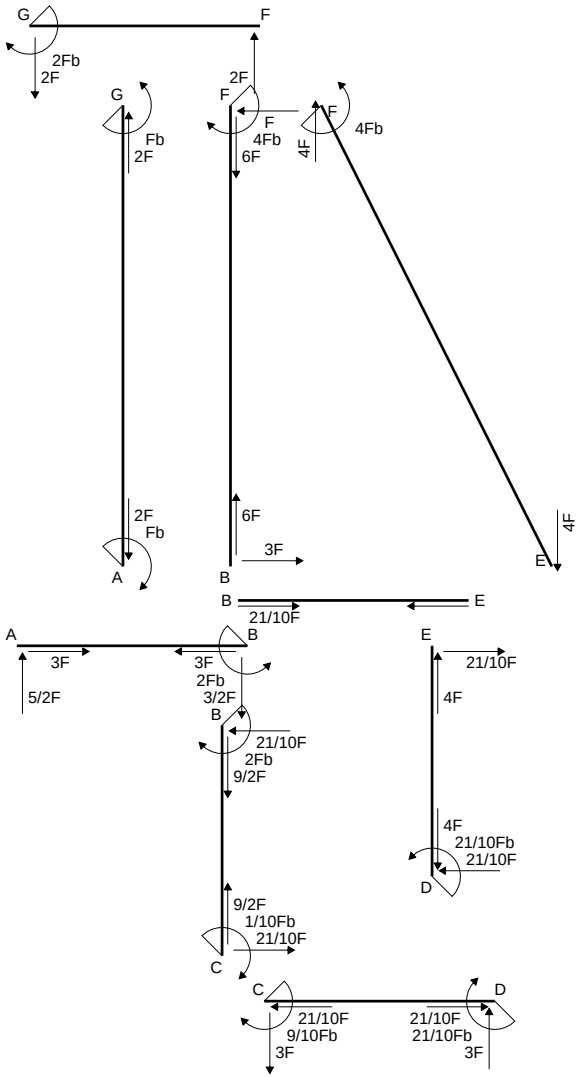
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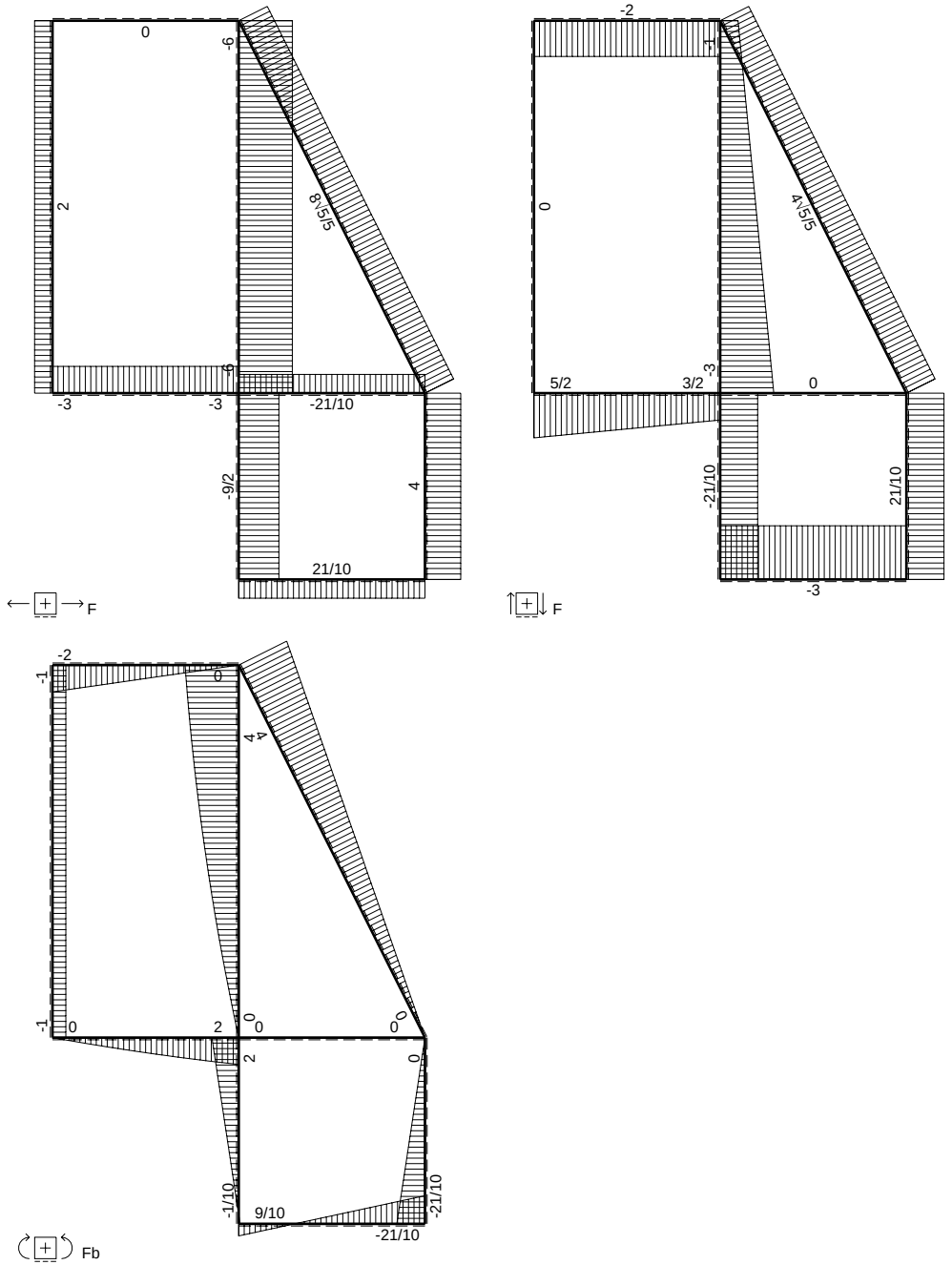
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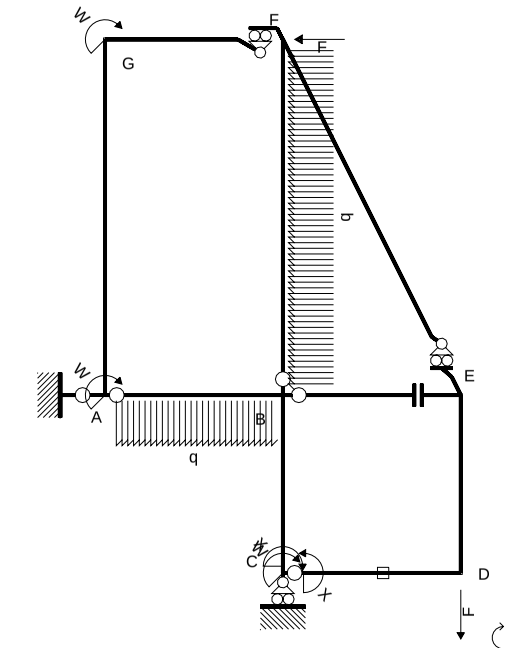
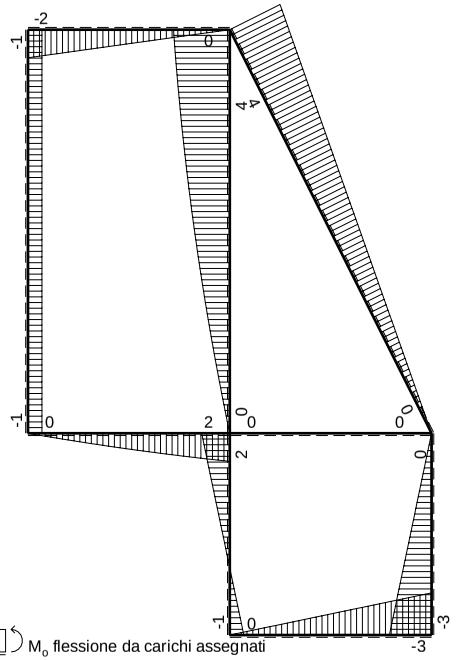
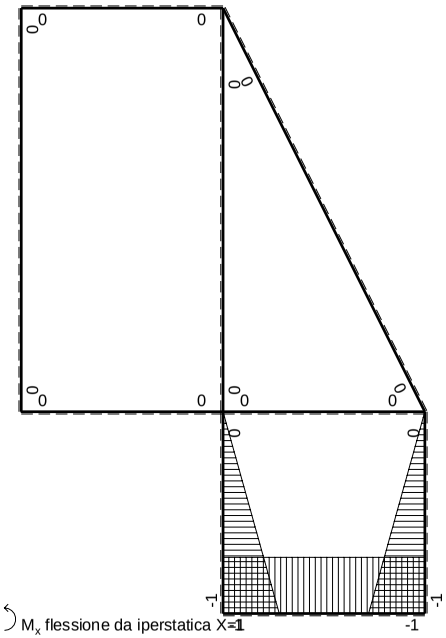


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$5/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-2Fb+3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (- b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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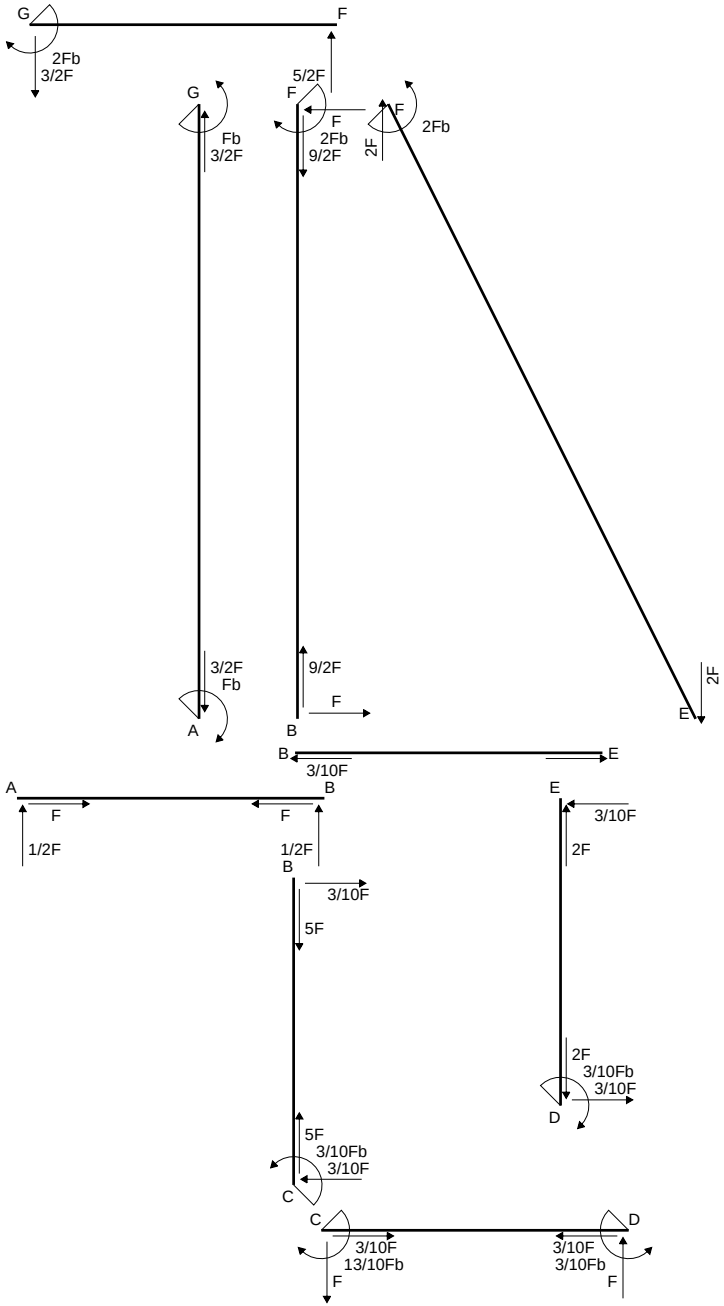
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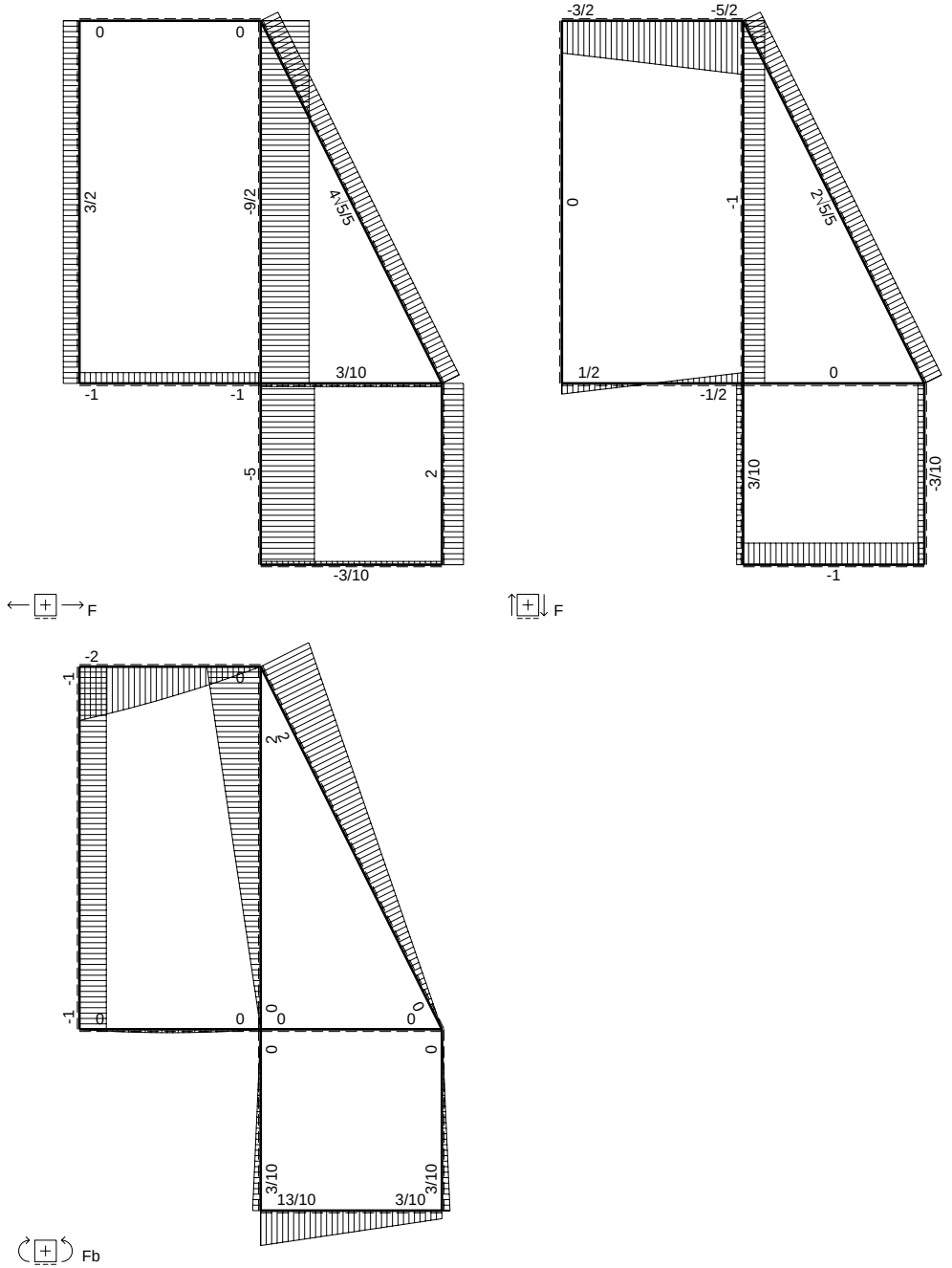
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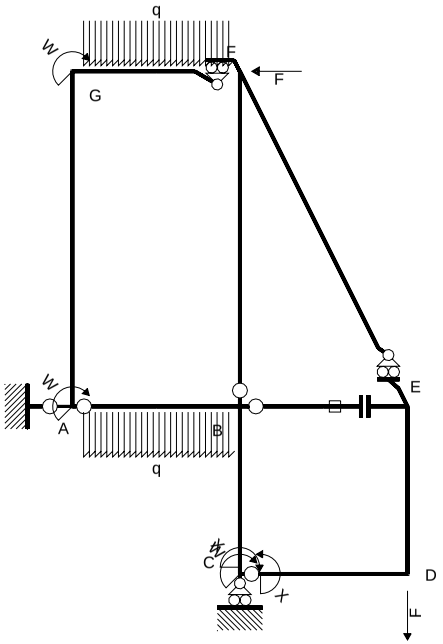
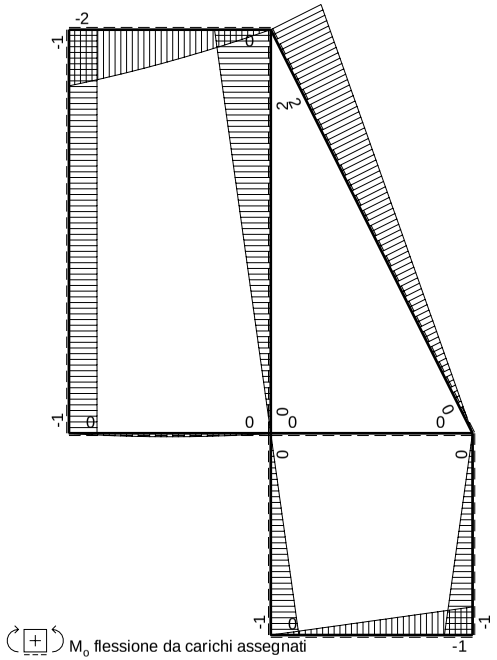
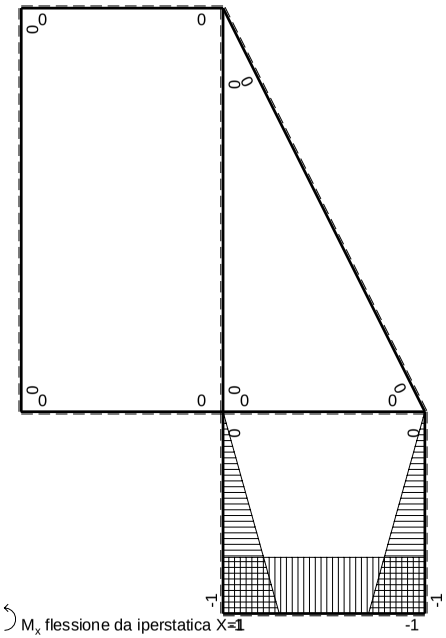


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$13/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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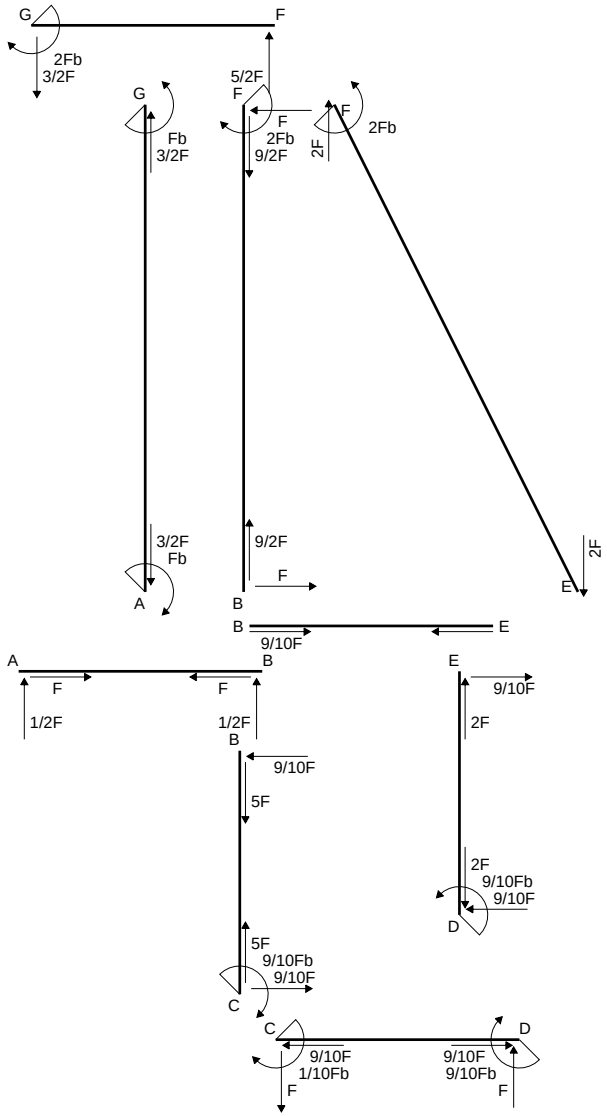
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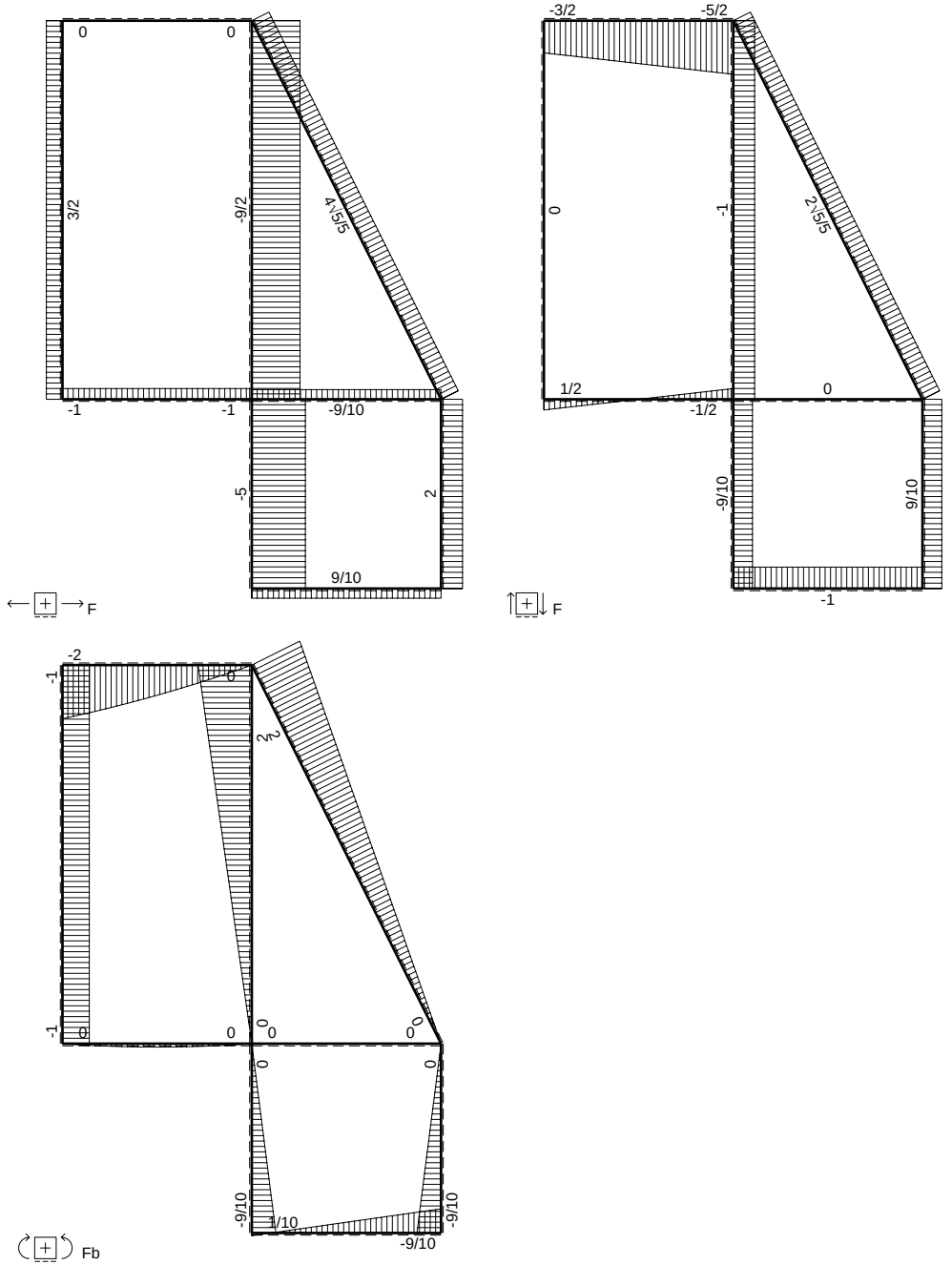
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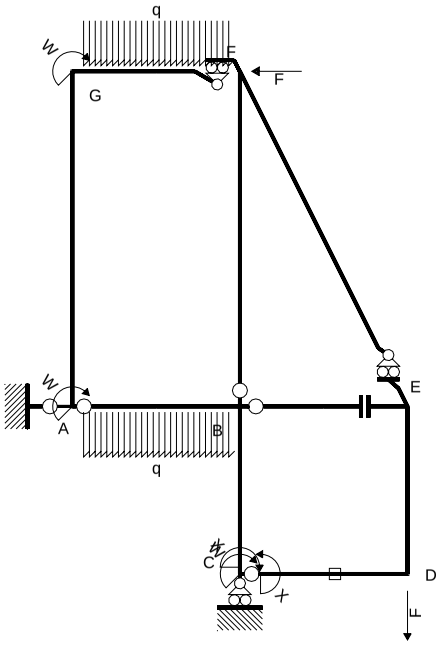
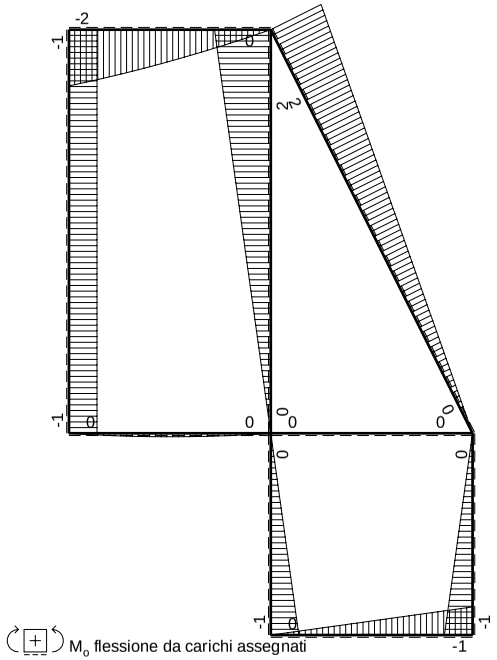
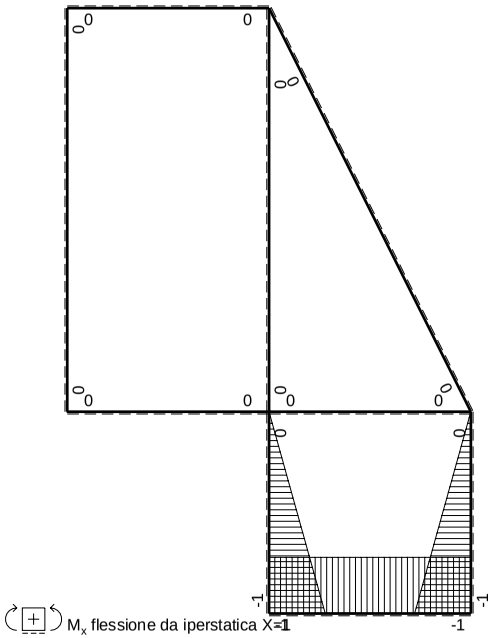


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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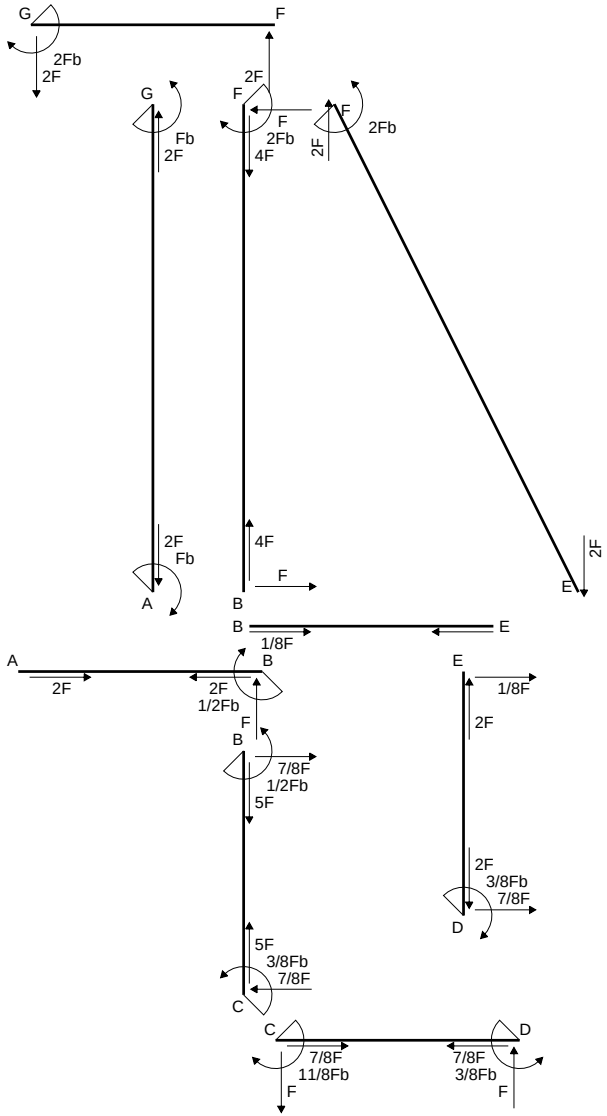
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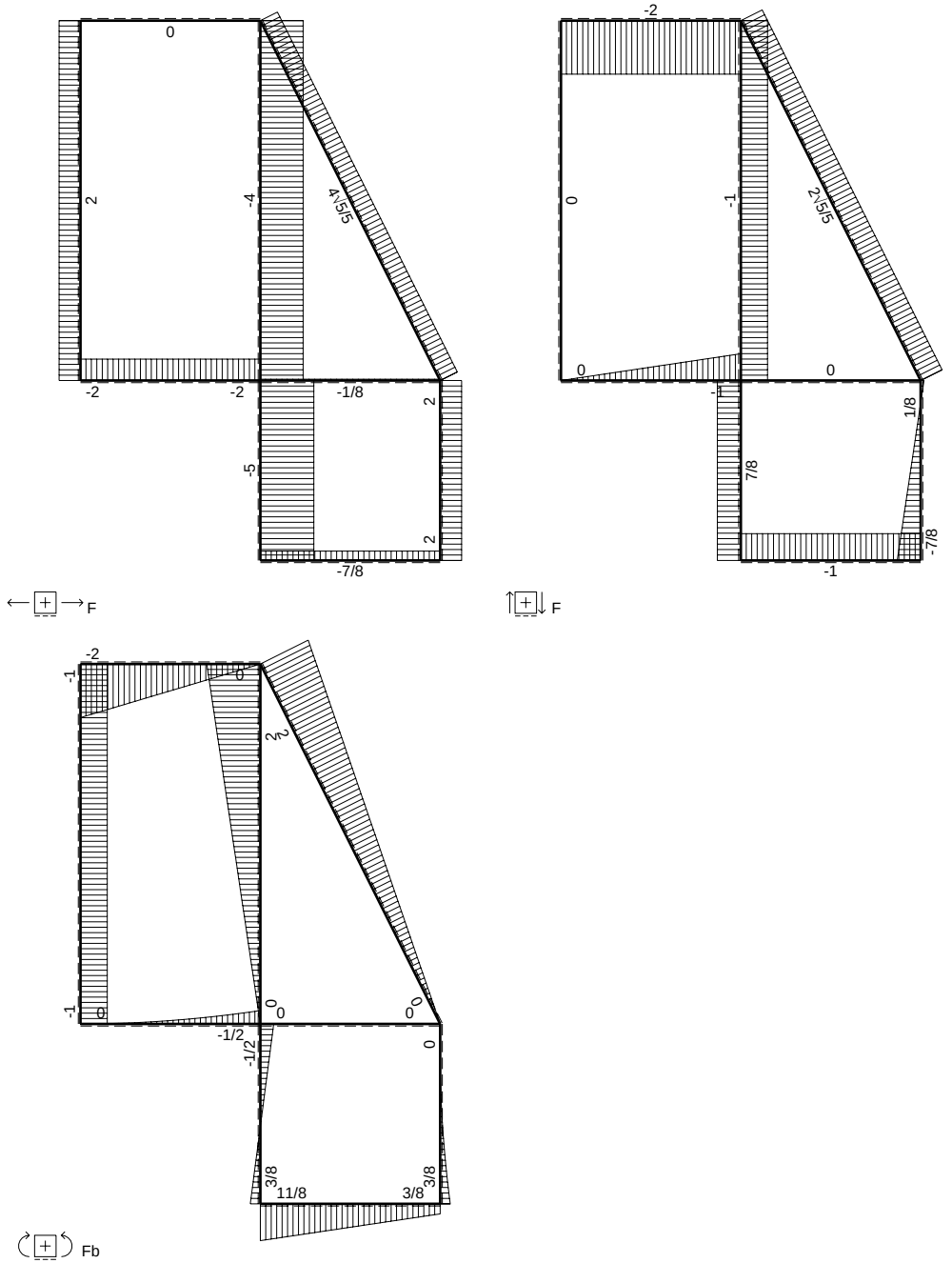
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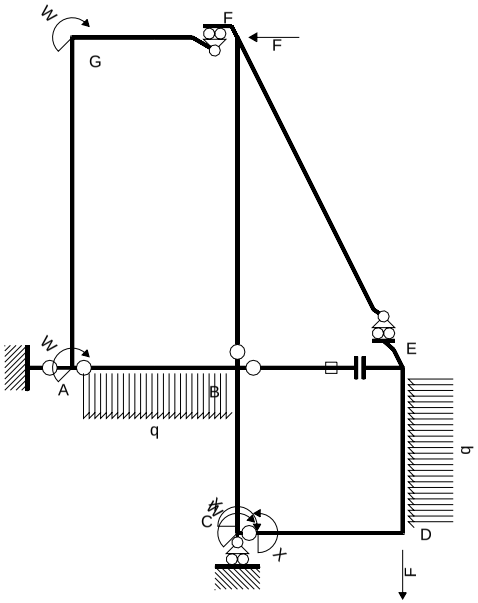
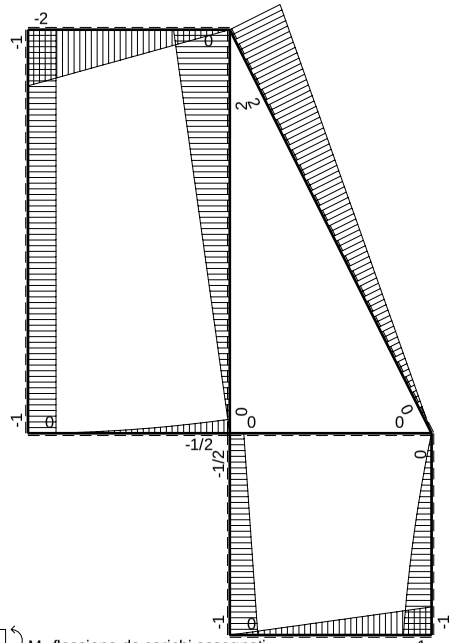
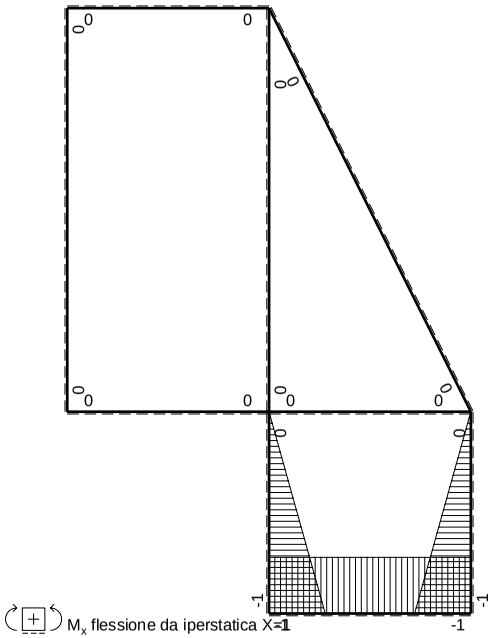


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb+1/2Fx+1/2qx^2$	$Fb-3/2Fx+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$3/8Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$3/2Fx-1/2qx^2$	$3/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$55/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-11/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

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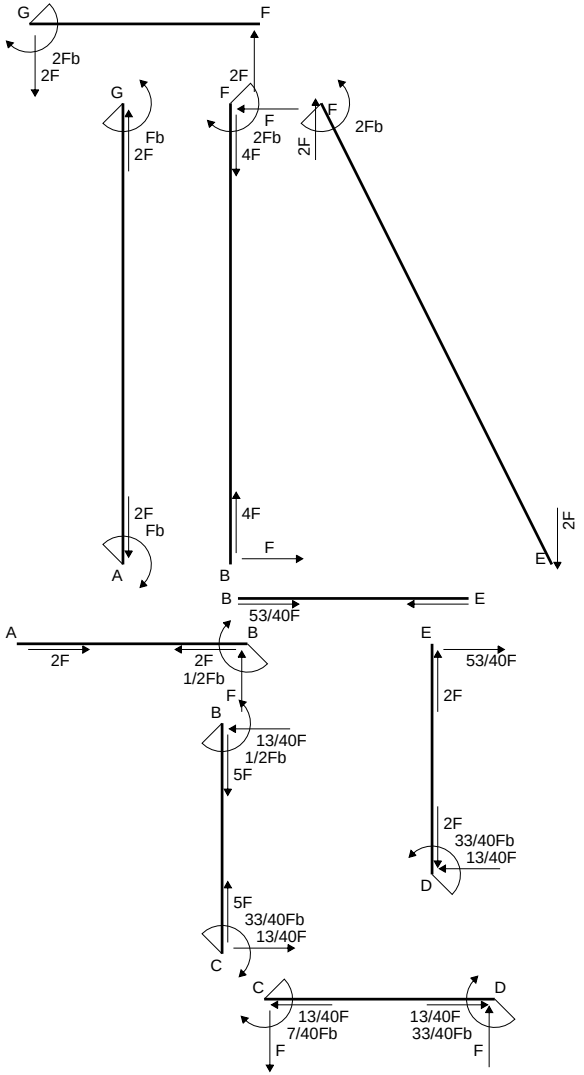
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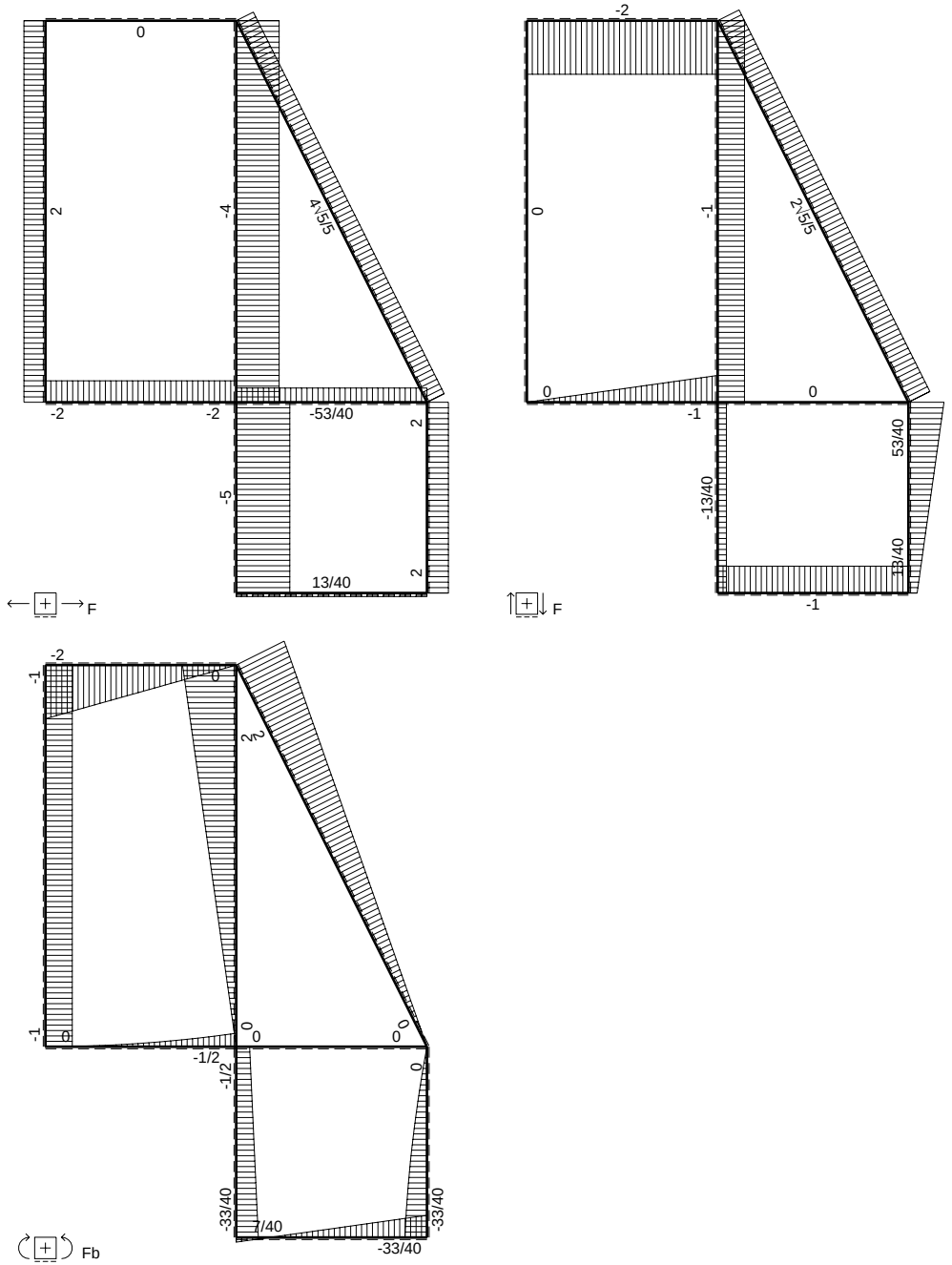
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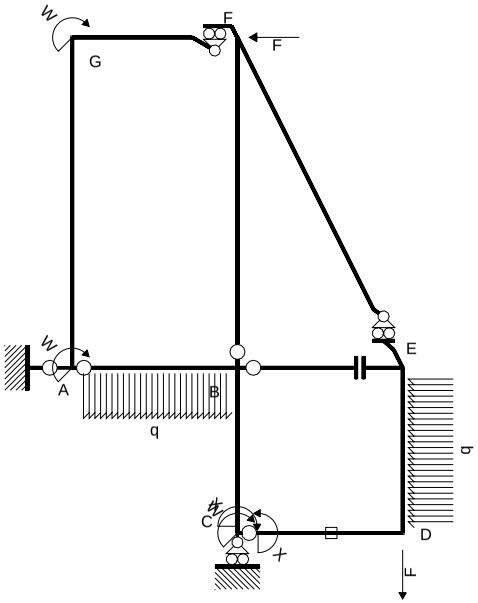
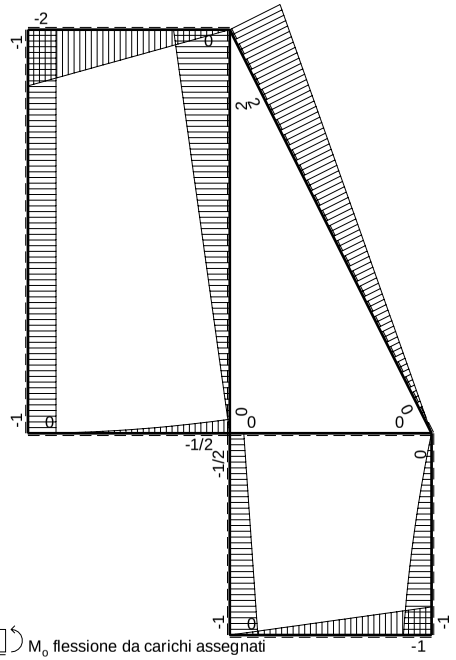
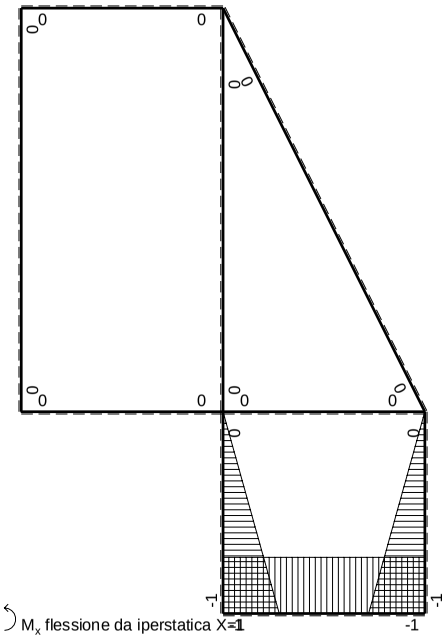


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	$-Fb+1/2Fx+1/2qx^2$	$Fb-3/2Fx+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$3/8Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$3/2Fx-1/2qx^2$	$3/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$7/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-7/40Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

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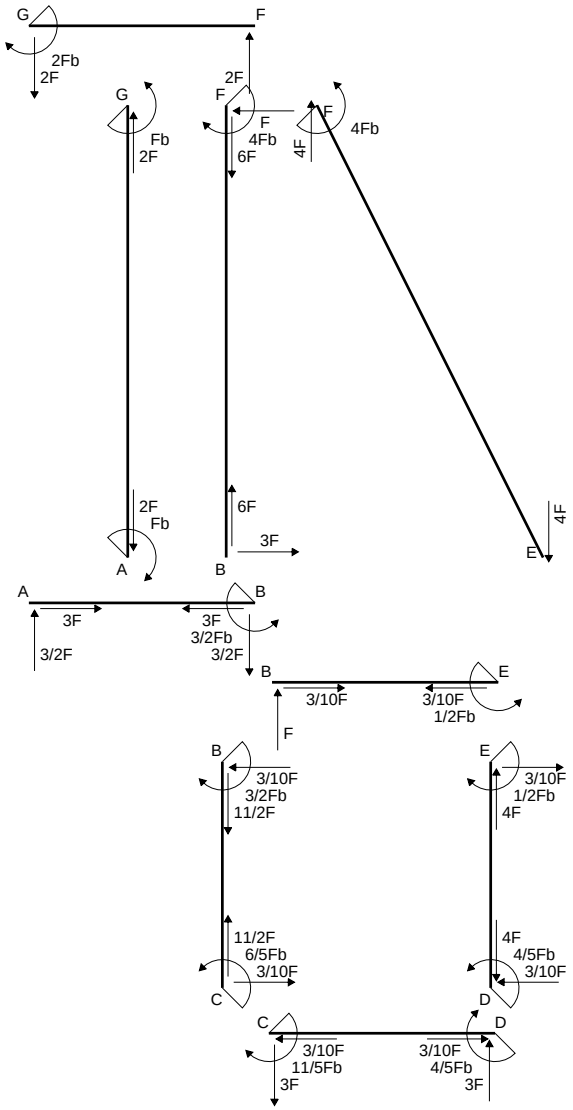
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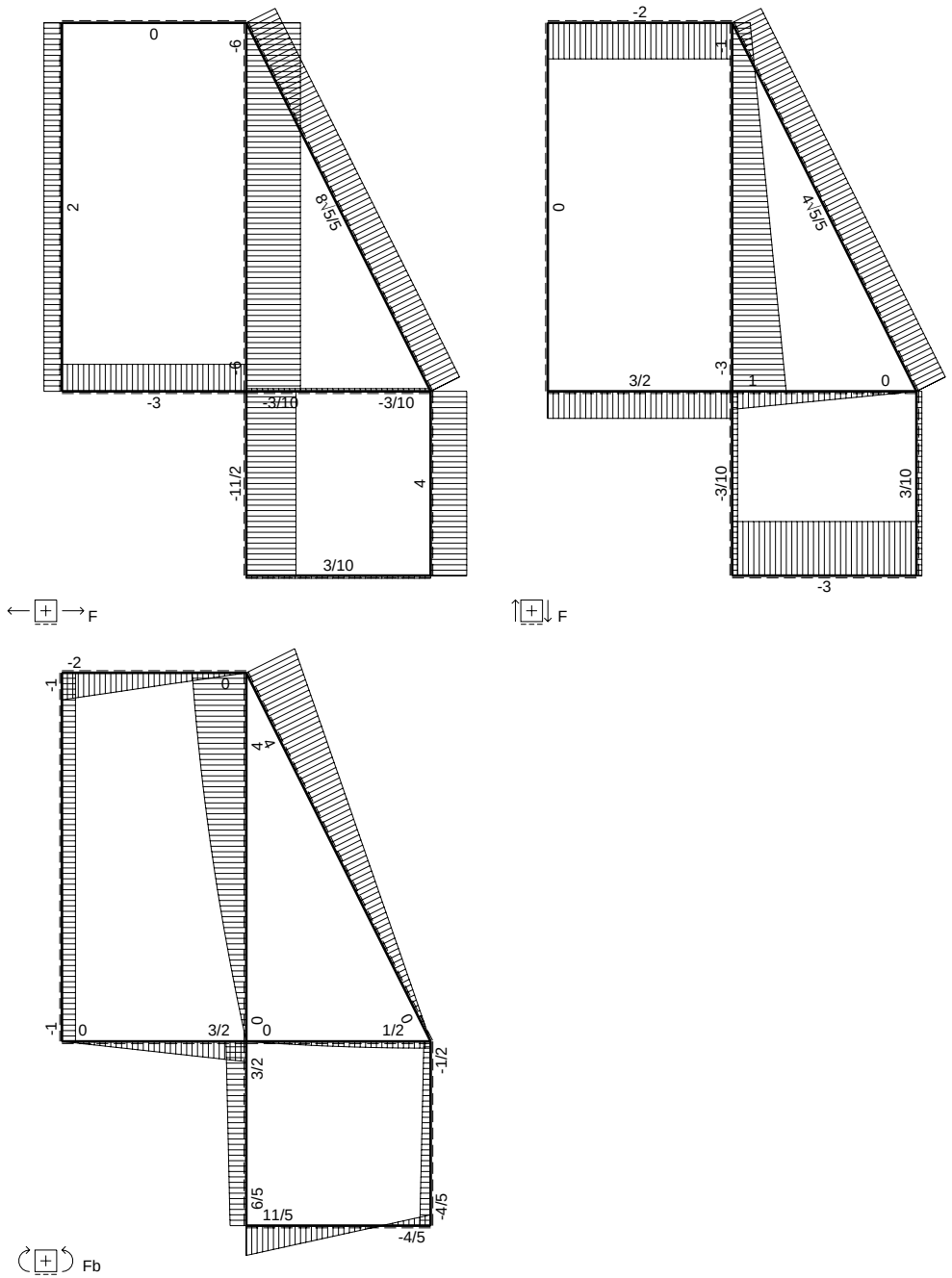
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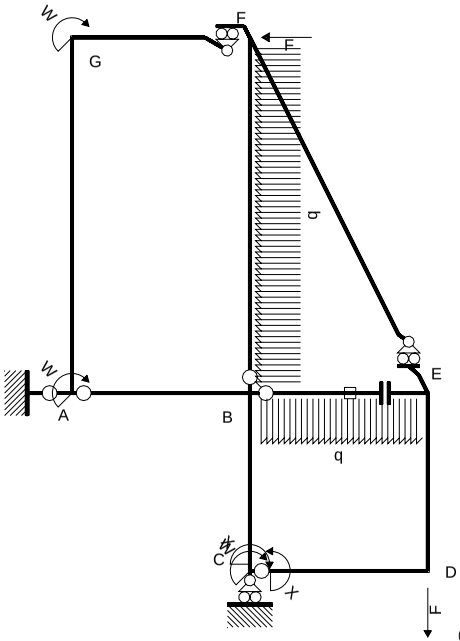
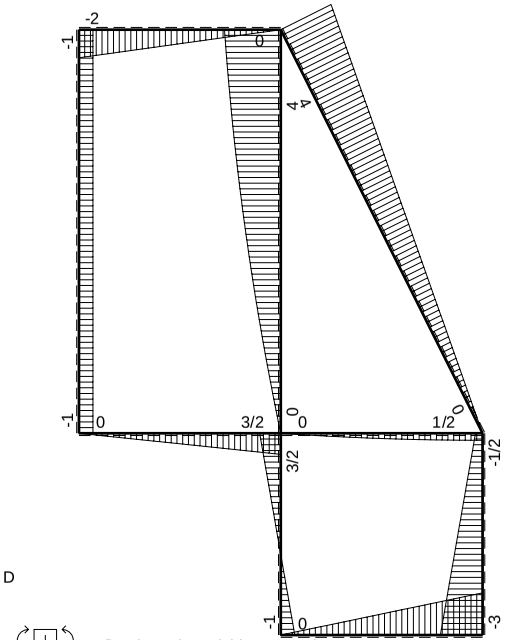
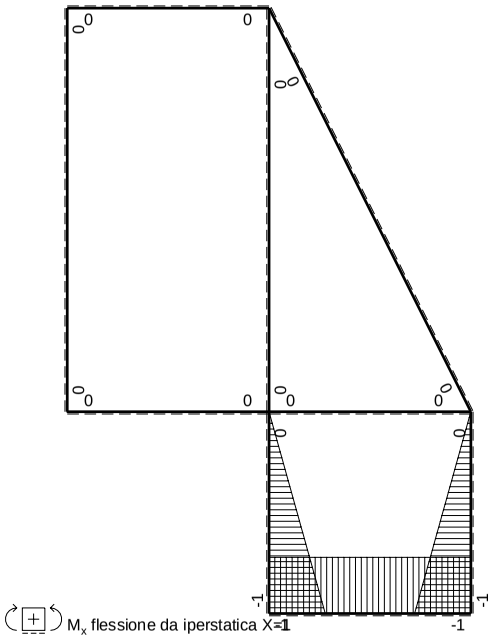


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+5/2Fx	$3Fb-11/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$13/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fb+5/2Fx	$1/2Fx+5/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$11/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-11/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -11/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -11/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -11/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 13/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 13/12\ Fb^2/EJ$

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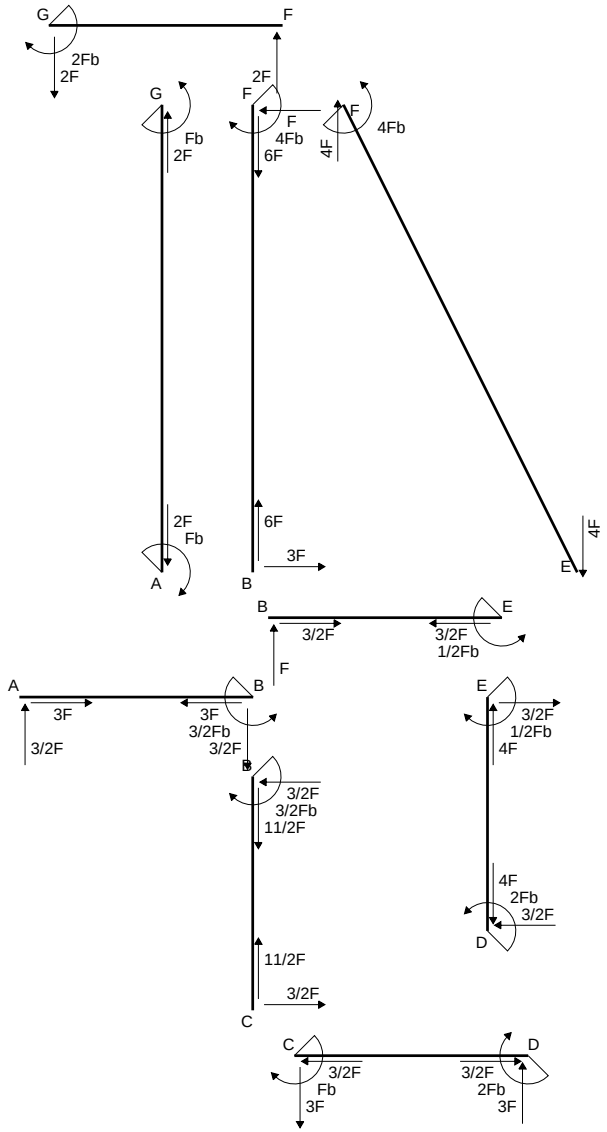
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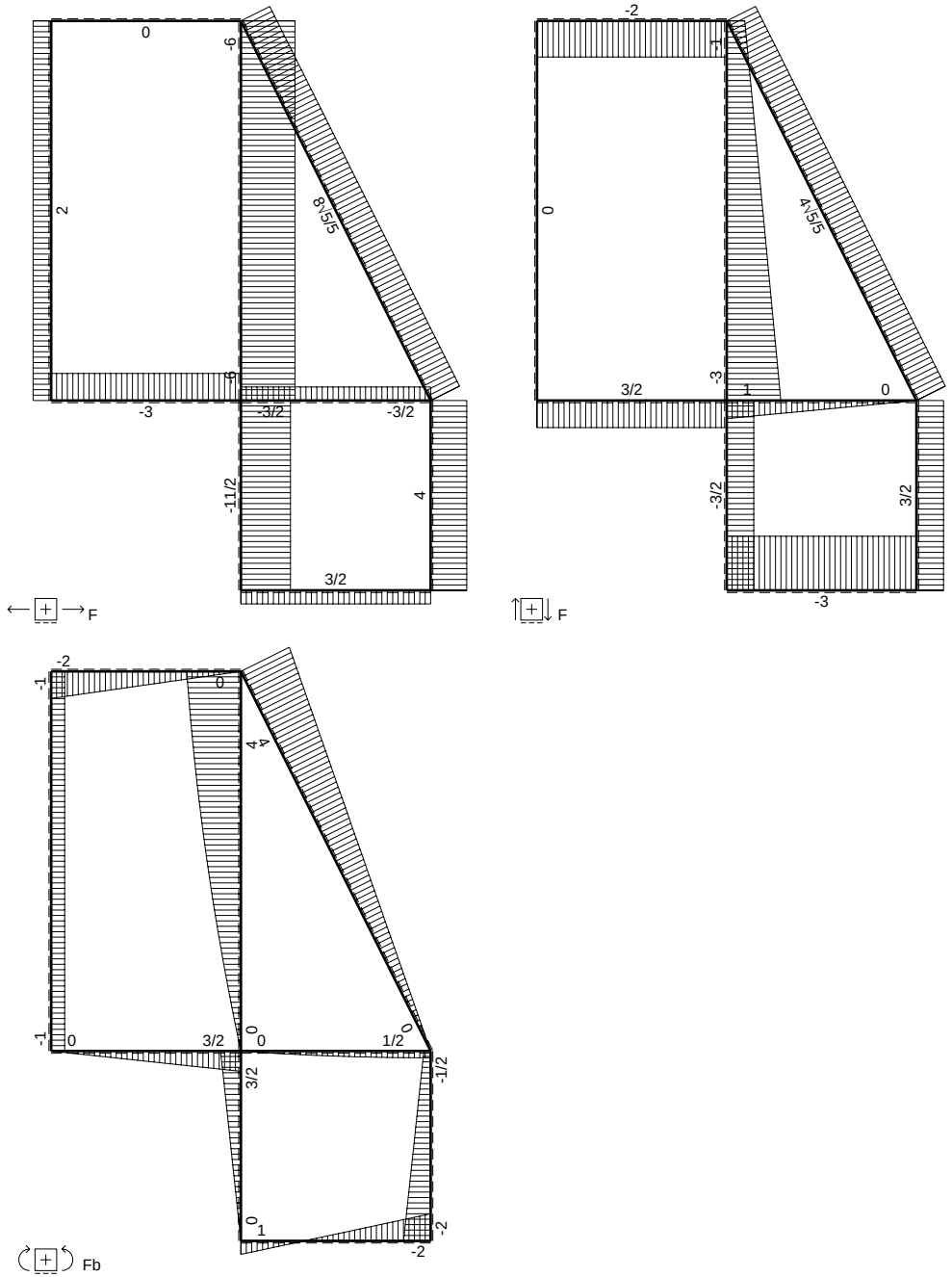
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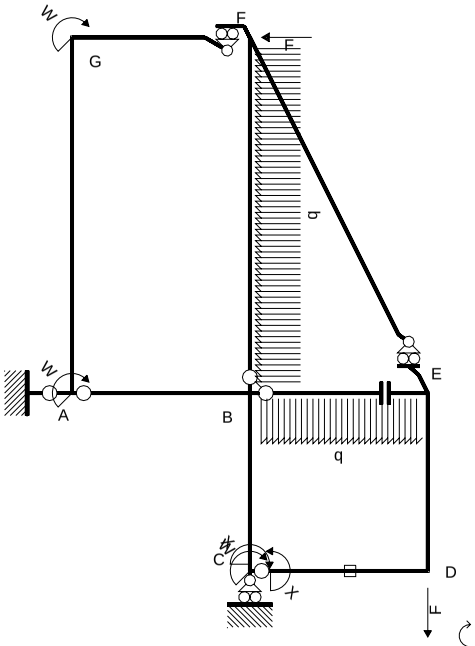
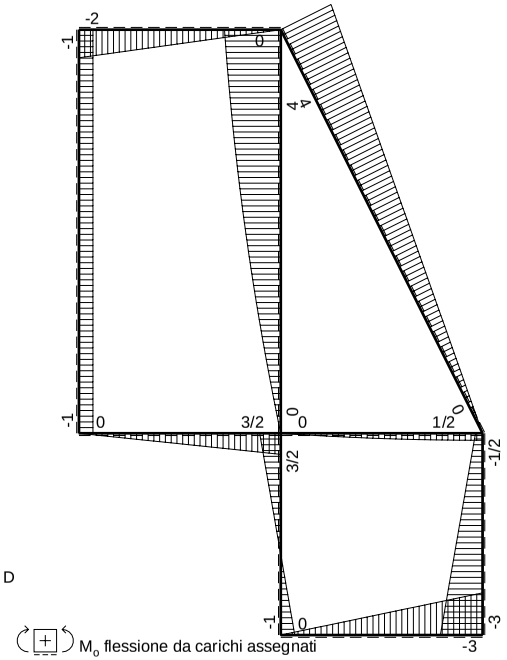
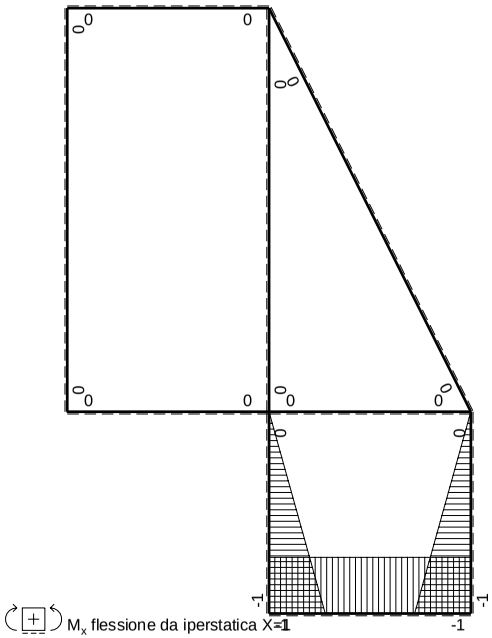


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+5/2Fx	$3Fb-11/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$13/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fb+5/2Fx	$1/2Fx+5/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$5/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -11/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -11/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -11/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 13/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 13/12\ \ Fb^2/EJ$

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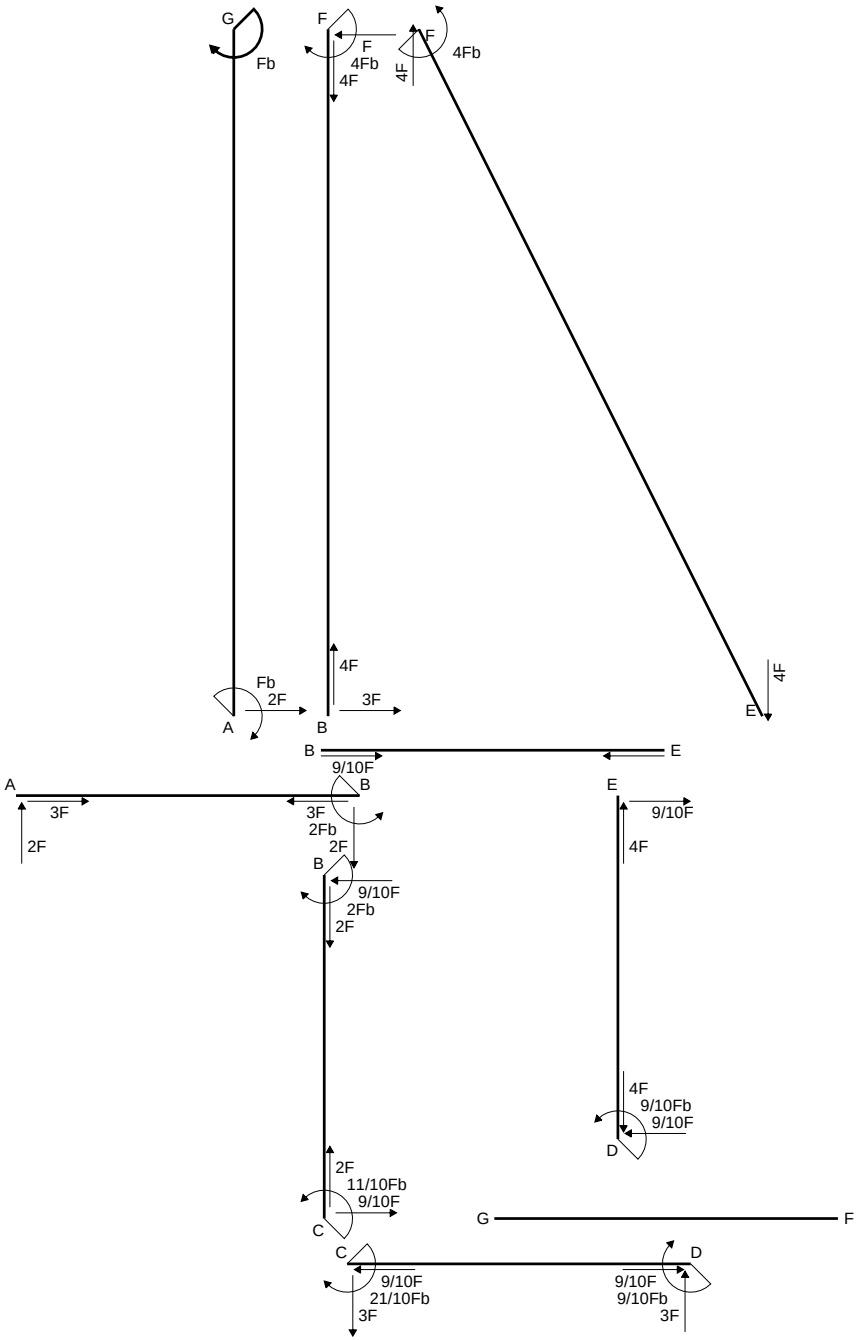
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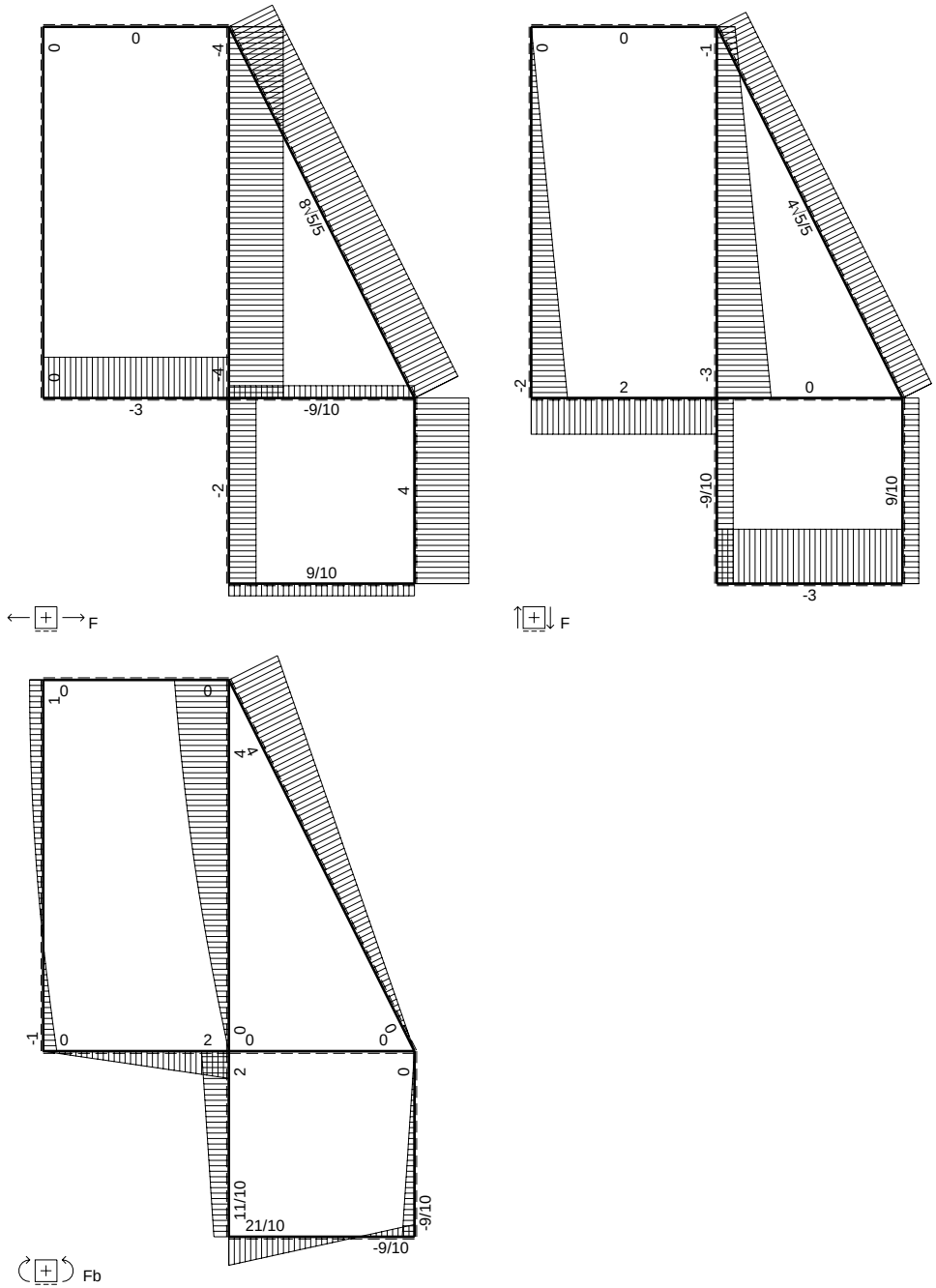
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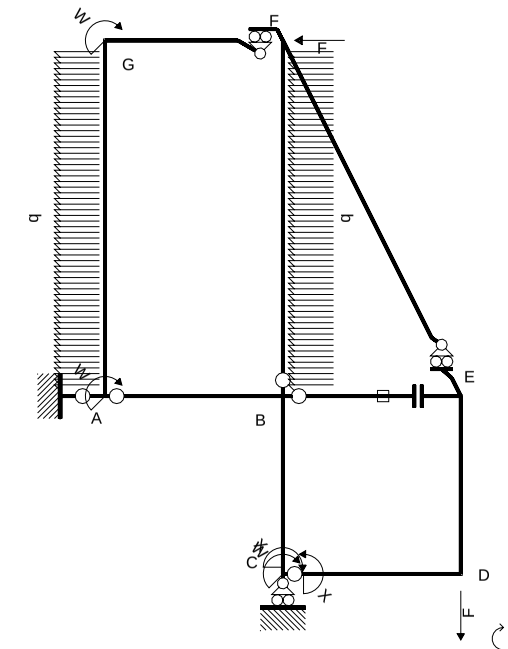
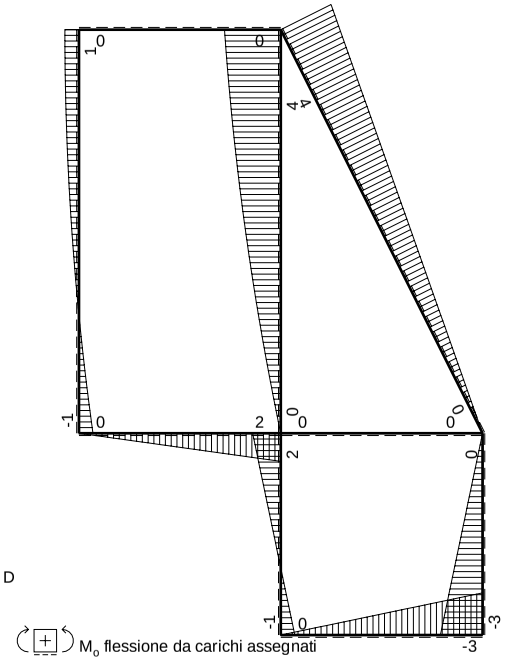
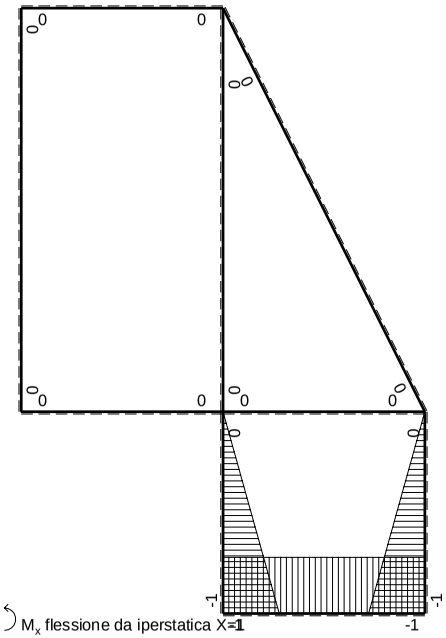


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	2Fx	0	0	0	0
BA b	0	-2Fb+2Fx	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b + b)\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

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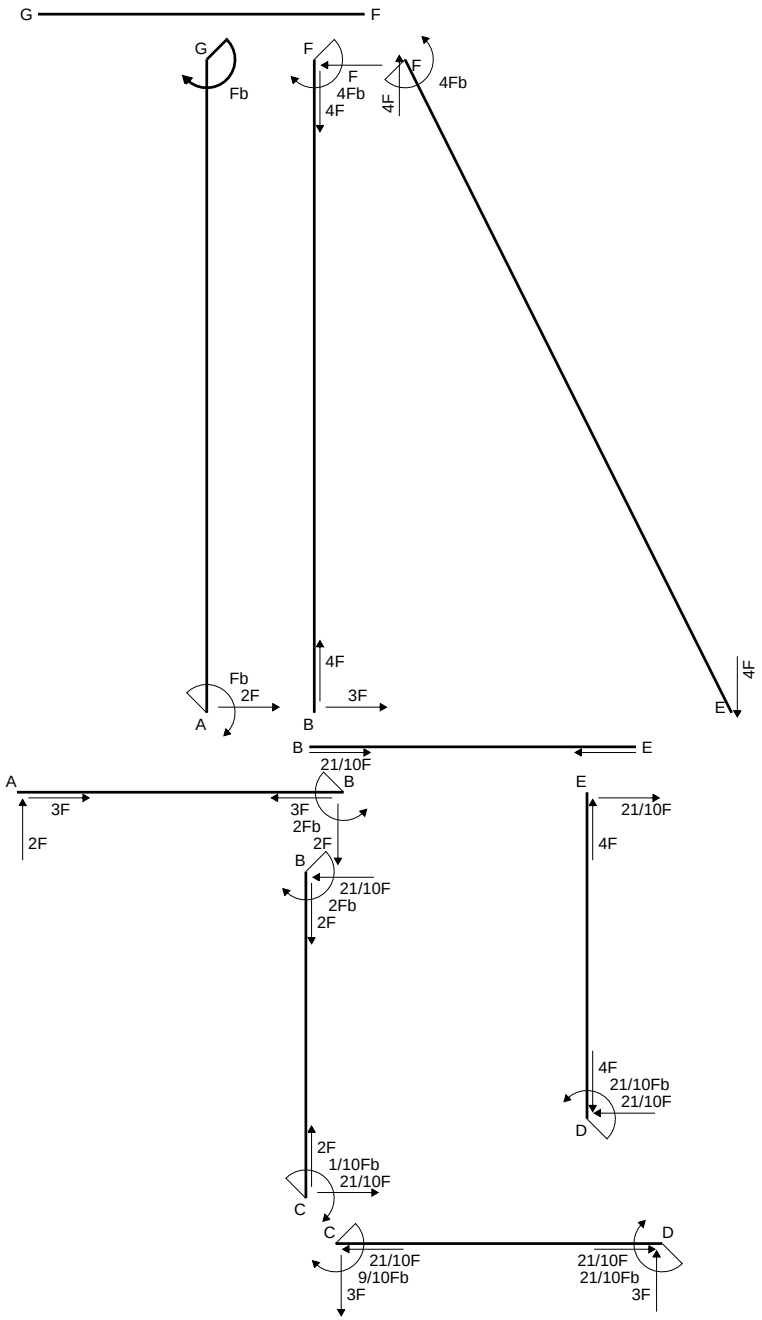
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E
3F
2F
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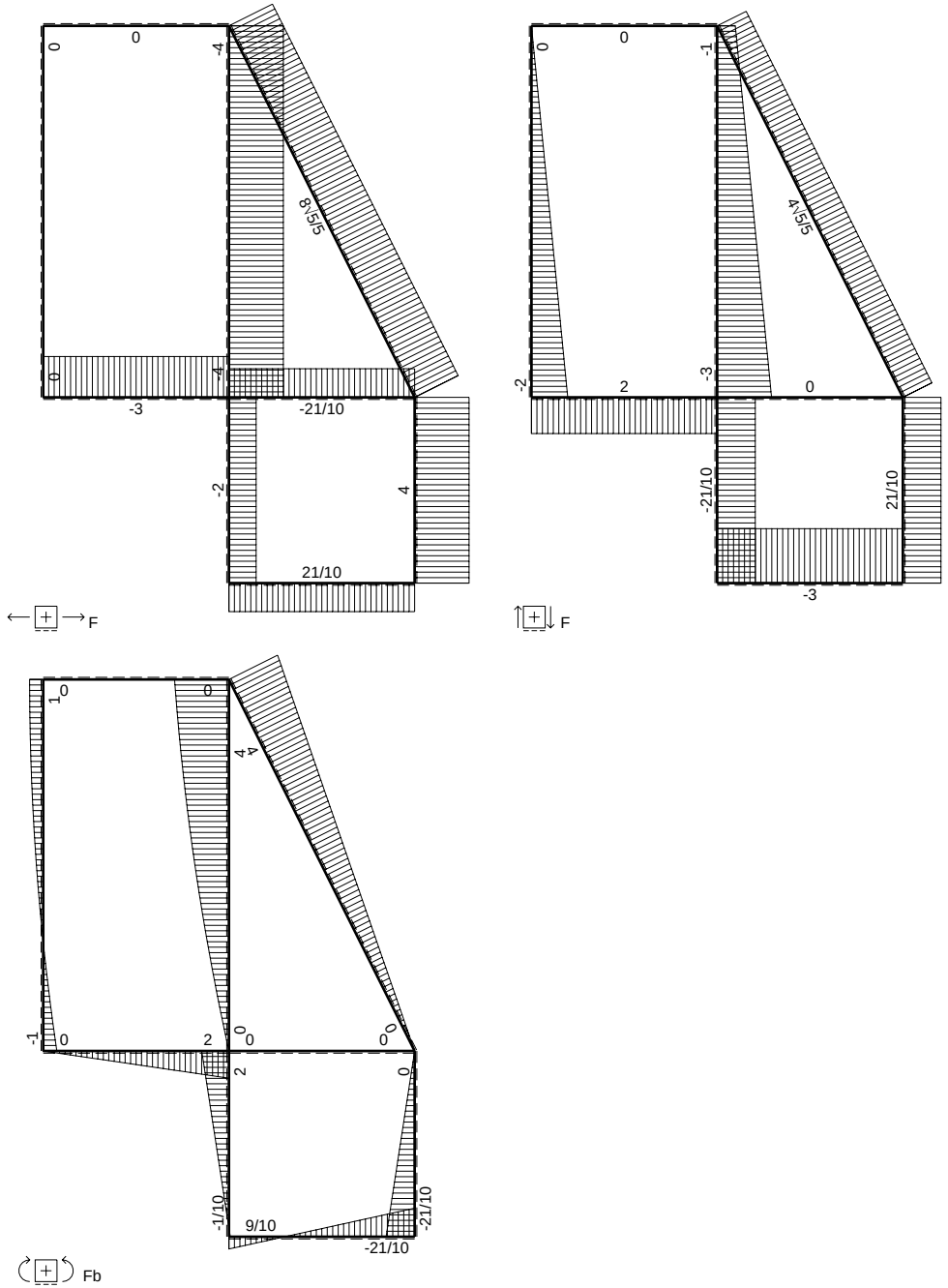
2Fb
21/10Fb
1/10Fb
9/10Fb
C
D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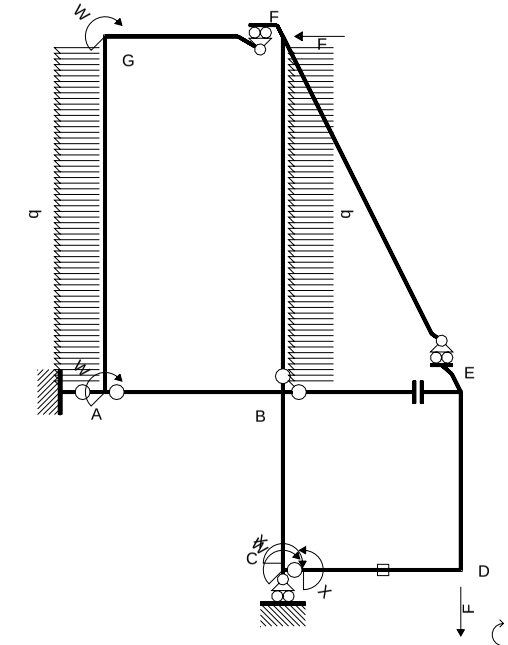
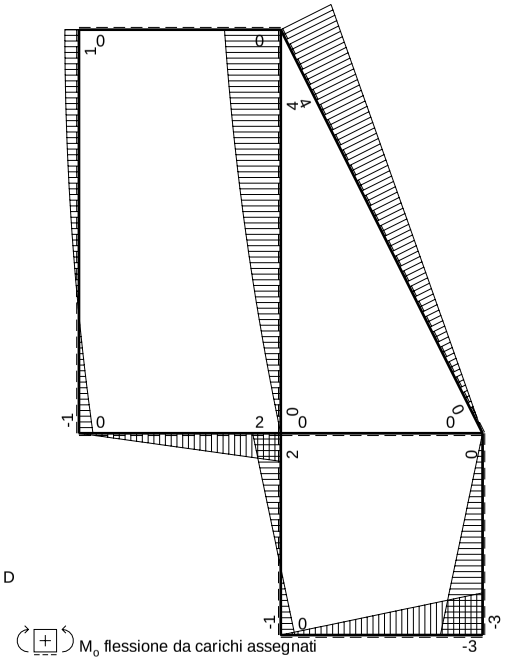
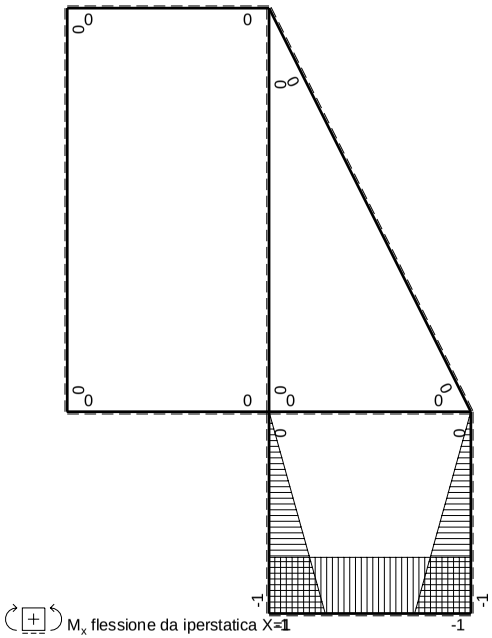


Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	2Fx	0	0	0	0
BA b	0	-2Fb+2Fx	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-2\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L_{CB}^{Xo} = \int_o^b (1 -4\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -6\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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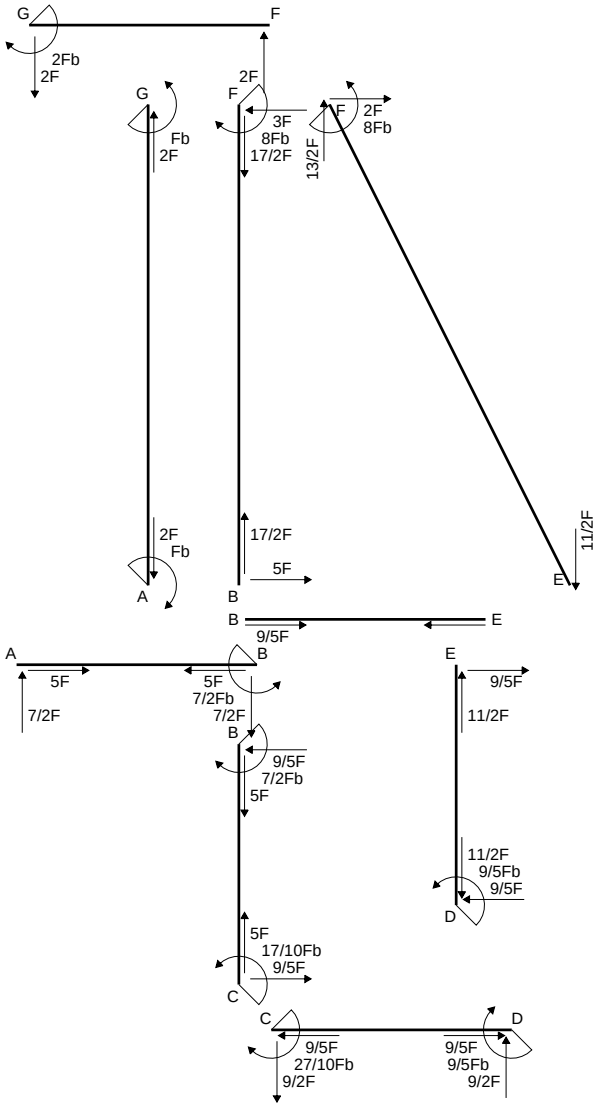
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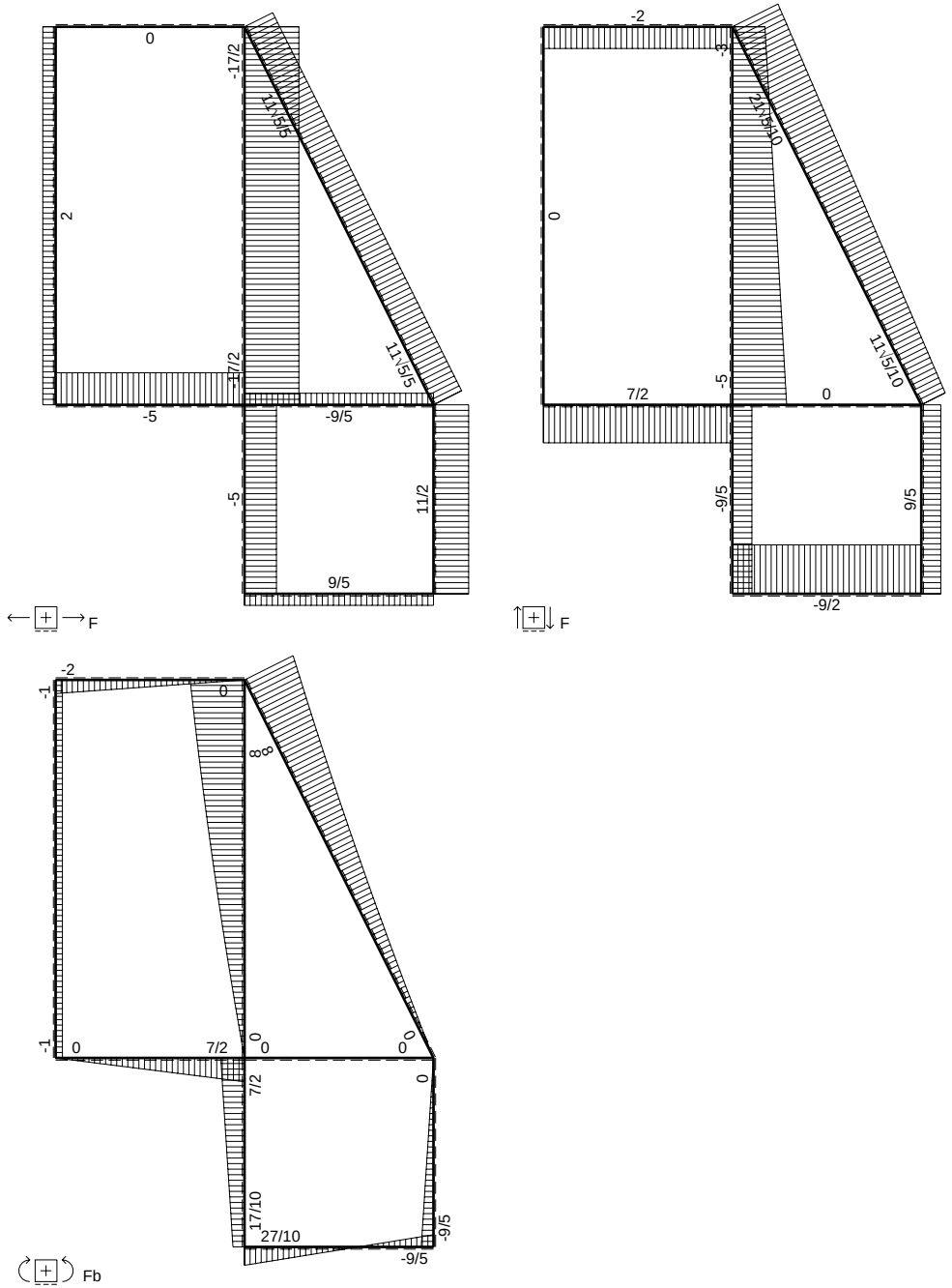
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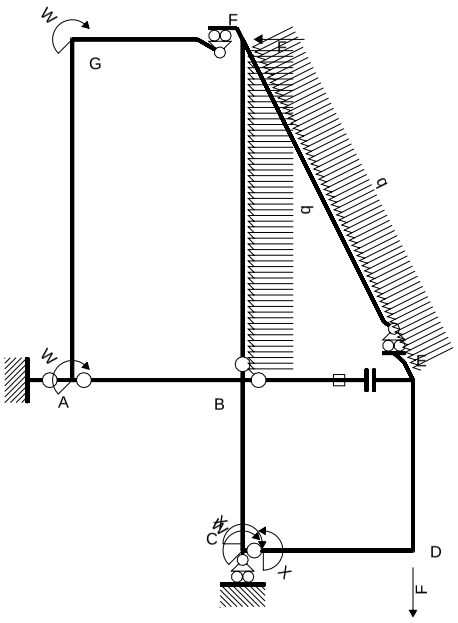
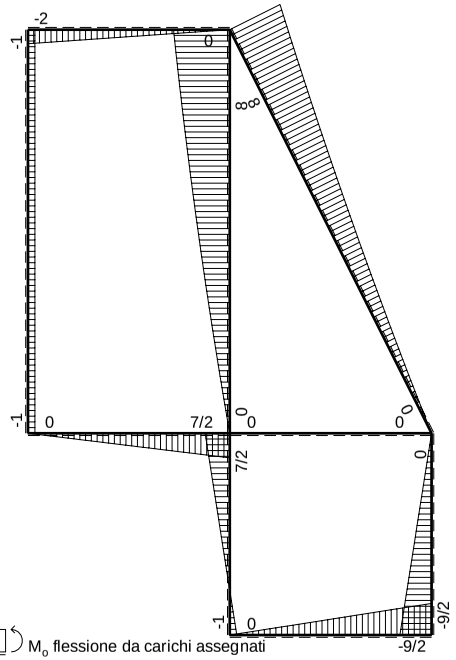
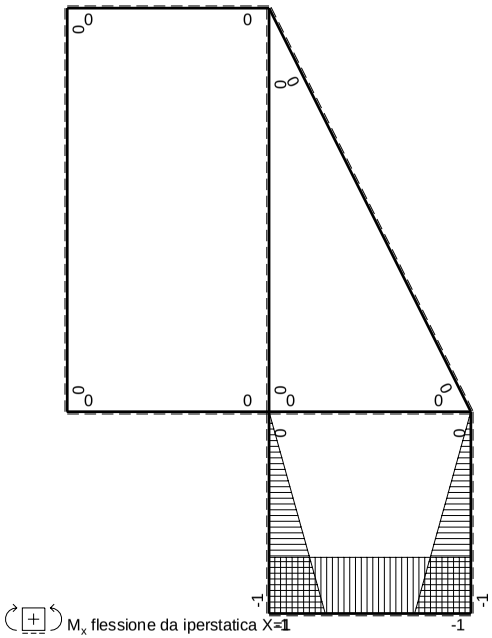
-9/5
-9/5
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	7/2Fx	0	0	0	0
BA b	0	-7/2Fb+7/2Fx	0	0		
BC b	-x/b	7/2Fb-9/2Fx	-7/2Fx+9/2Fx^2/b	x^2/b^2	-1/4Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-9/2Fx	Fb-11/2Fx+9/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-9/2Fx	9/2Fx	1	9/4Fb^2/EJ	Xb/EJ
DC b	1	9/2Fb-9/2Fx	9/2Fb-9/2Fx	1		
DE b	-1+x/b	-9/2Fb+9/2Fx	9/2Fb-9Fx+9/2Fx^2/b	1-2x/b+x^2/b^2	3/2Fb^2/EJ	1/3Xb/EJ
ED b	x/b	9/2Fx	9/2Fx^2/b	x^2/b^2		
EF √5b	0	11√5/10Fx+1/2qx^2	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	8Fb-3Fx-1/2qx^2	0	0	0	0
BF 2b	0	-5Fx+1/2qx^2	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				9/2Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-27/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-7/2\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-7/4\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-7/4\ b +3/2\ b\ )\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -11/2\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -11/4\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -11/4\ b +3/2\ b\ )\ Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (9/2\ x/b\ )\ Fb\ 1/EJ\ dx = [9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (9/4\ b\ )\ Fb\ 1/EJ\ = 9/4\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (9/2 -9/2\ x/b\ )\ Fb\ 1/EJ\ dx = [9/2\ x -9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -9/4\ b\ )\ Fb\ 1/EJ\ = 9/4\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (9/2 -9\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [9/2\ x -9/2\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -9/2\ b +3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

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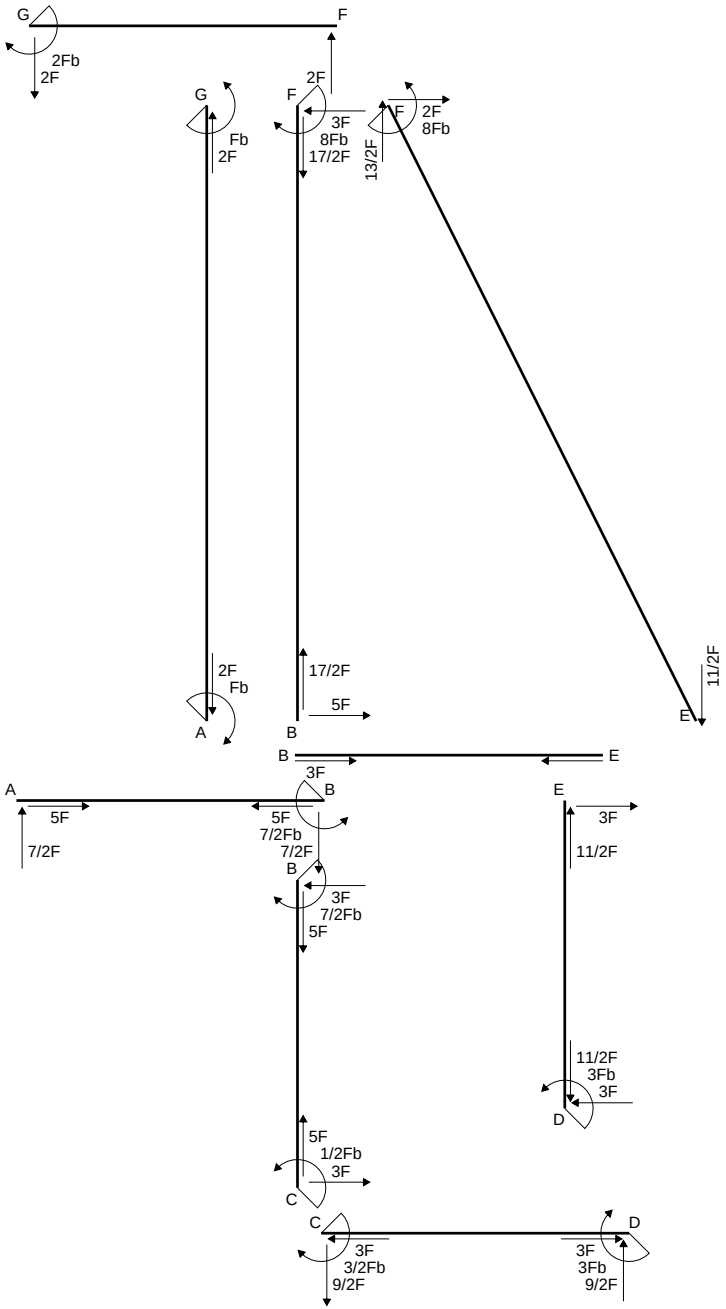
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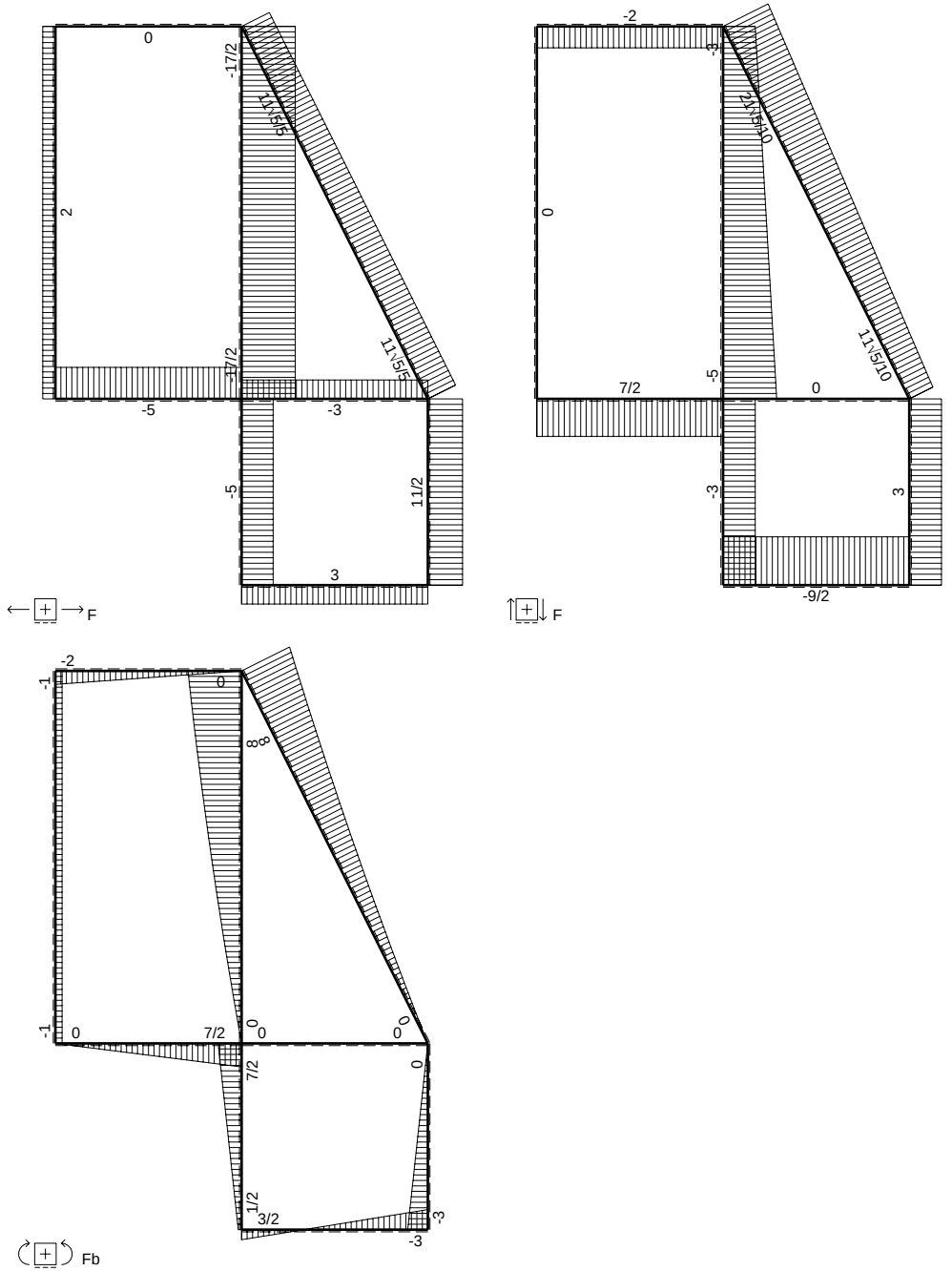
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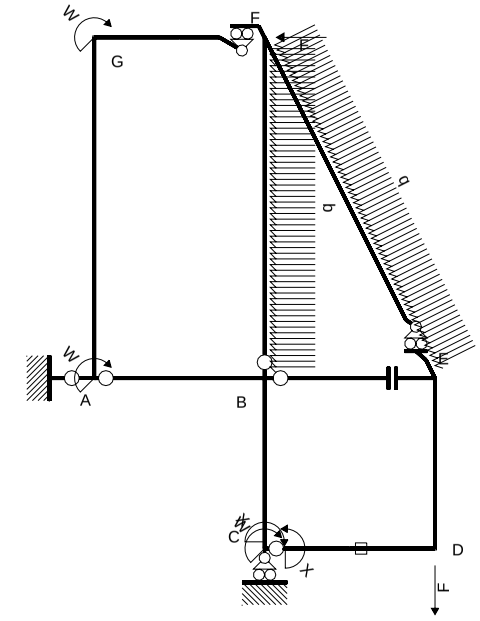
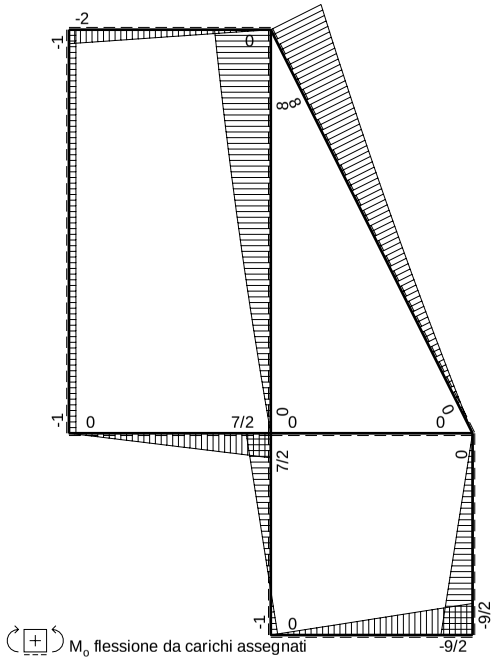
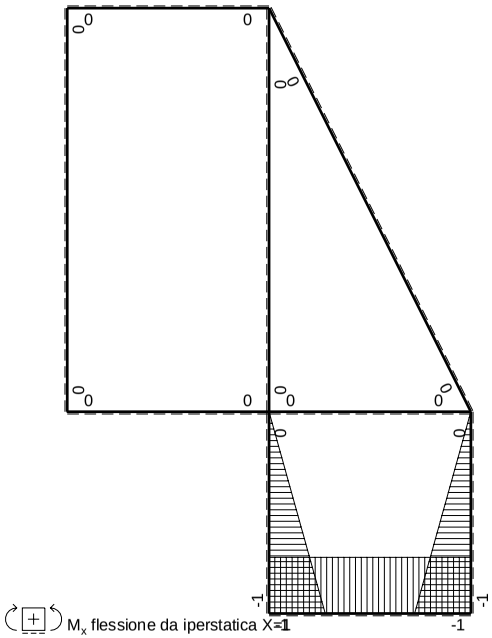
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	7/2Fx	0	0	0	0
BA b	0	-7/2Fb+7/2Fx	0	0		
BC b	-x/b	7/2Fb-9/2Fx	-7/2Fx+9/2Fx^2/b	x^2/b^2	-1/4Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-9/2Fx	Fb-11/2Fx+9/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-9/2Fx	9/2Fx	1	9/4Fb^2/EJ	Xb/EJ
DC b	1	9/2Fb-9/2Fx	9/2Fb-9/2Fx	1		
DE b	-1+x/b	-9/2Fb+9/2Fx	9/2Fb-9Fx+9/2Fx^2/b	1-2x/b+x^2/b^2	3/2Fb^2/EJ	1/3Xb/EJ
ED b	x/b	9/2Fx	9/2Fx^2/b	x^2/b^2		
EF √5b	0	11√5/10Fx+1/2qx^2	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	8Fb-3Fx-1/2qx^2	0	0	0	0
BF 2b	0	-5Fx+1/2qx^2	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				5/2Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-3/2Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-7/2\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-7/4\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-7/4\ b +3/2\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -11/2\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -11/4\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -11/4\ b +3/2\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (9/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (9/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 5/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (9/2 -9/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [9/2\ x -9/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (9/2\ b -9/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 5/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (9/2 -9\ x/b +9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [9/2\ x -9/2\ x^2/b +3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (9/2\ b -9/2\ b +3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (9/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

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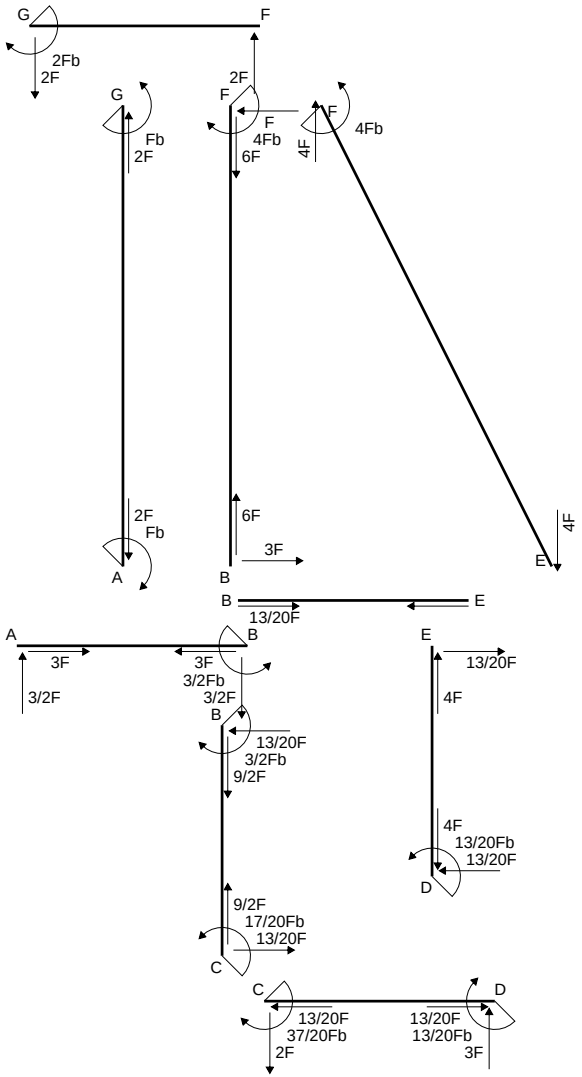
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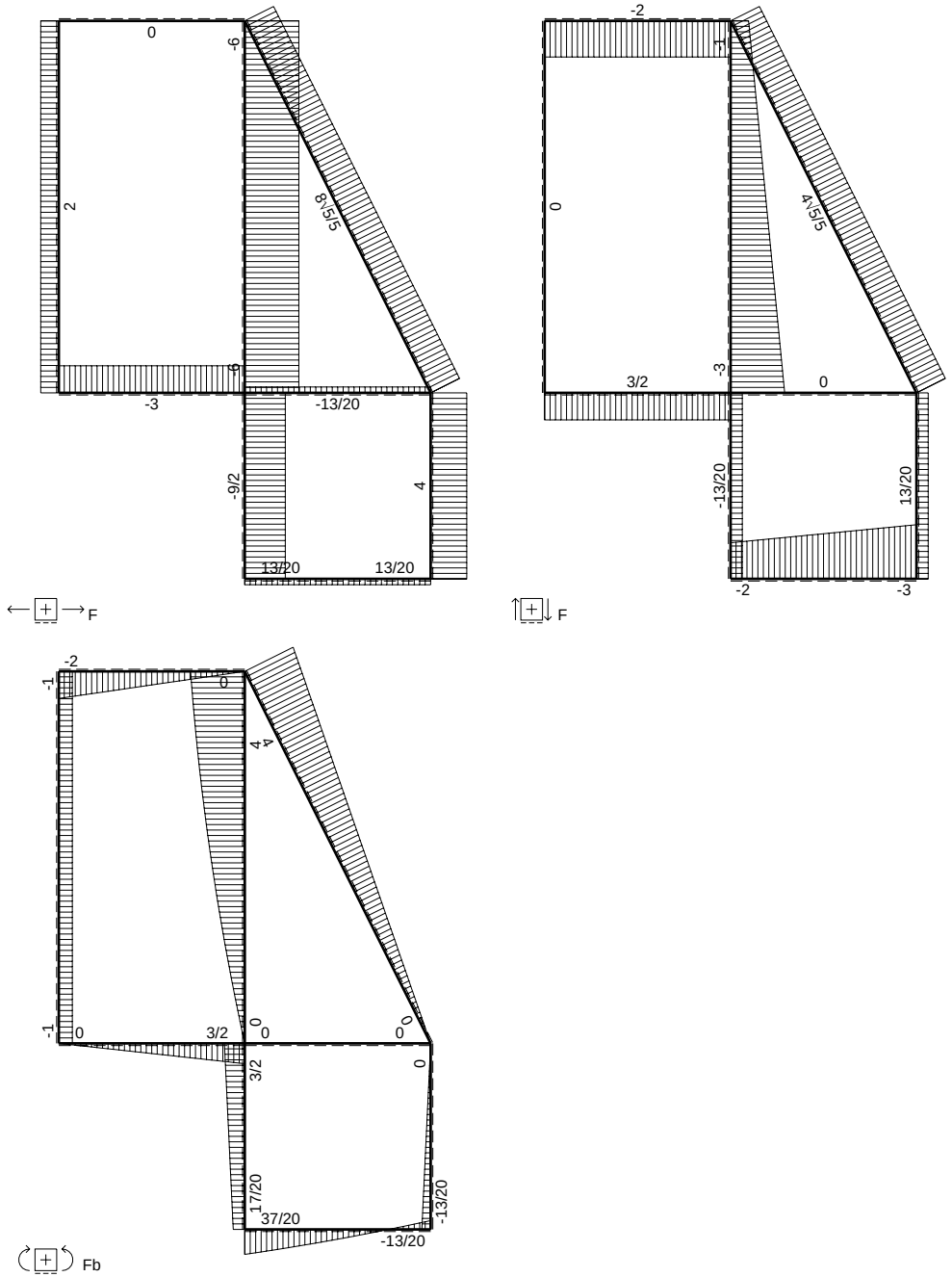
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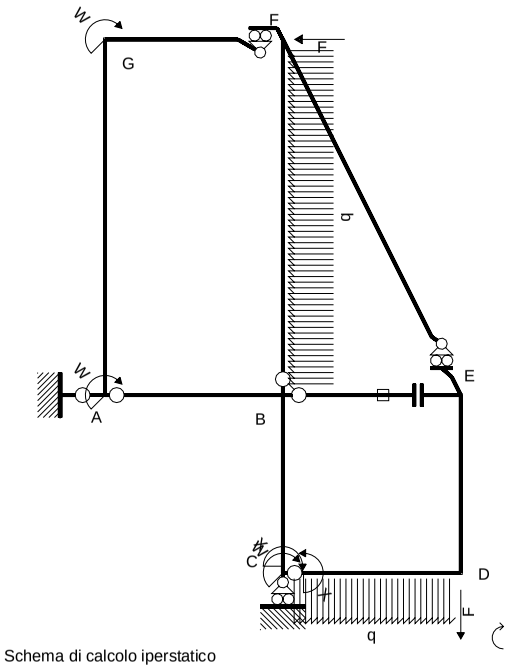
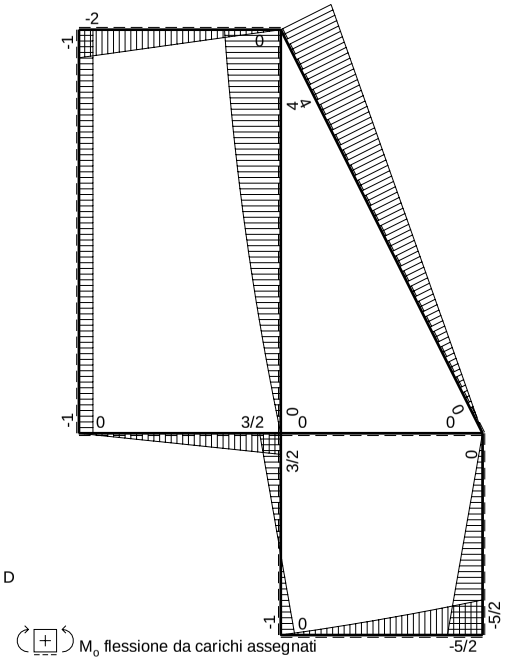
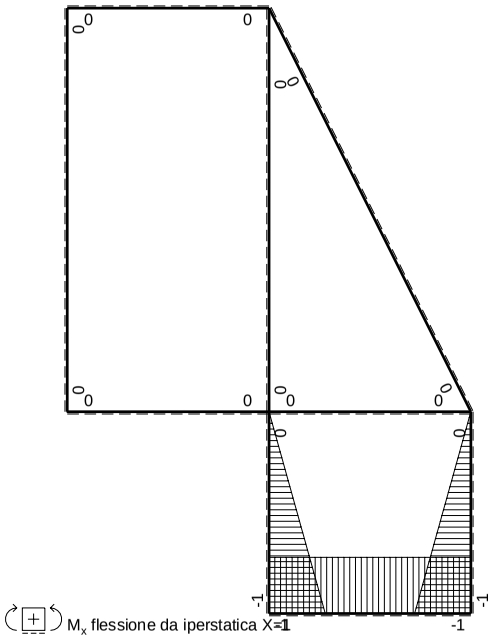


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-2Fx-1/2qx^2$	$2Fx+1/2Fx^2/b$	1	$7/6Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-3Fx+1/2qx^2$	$5/2Fb-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$37/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$-37/20Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (5/2 -3\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -3/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -3/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 7/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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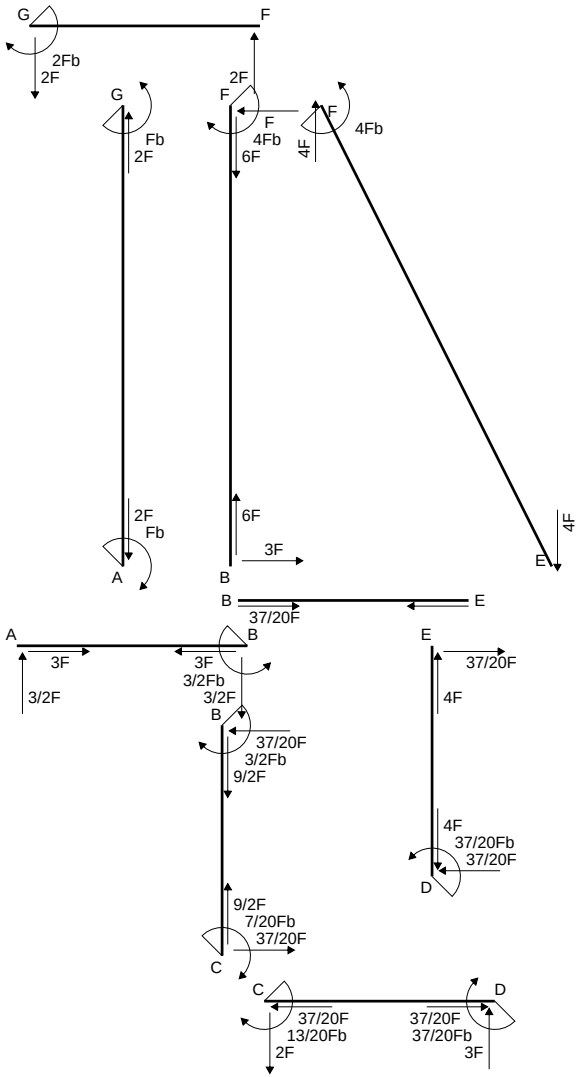
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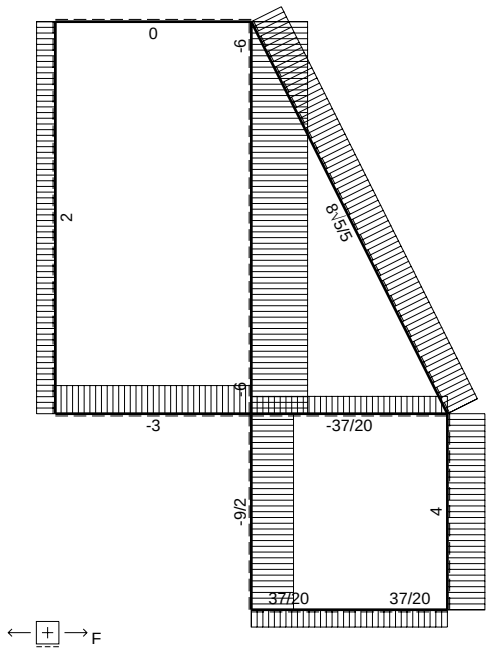
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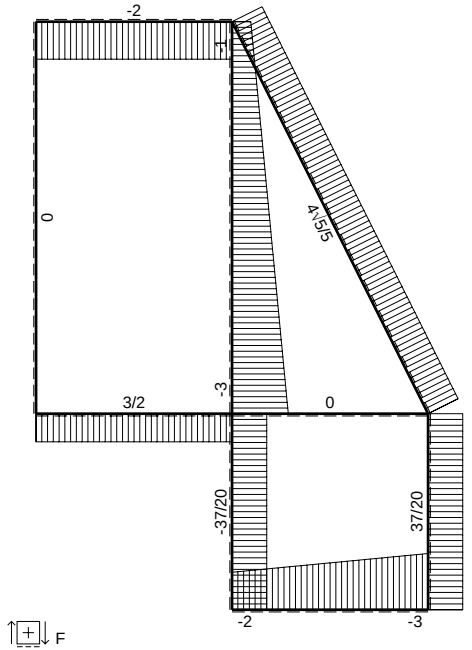
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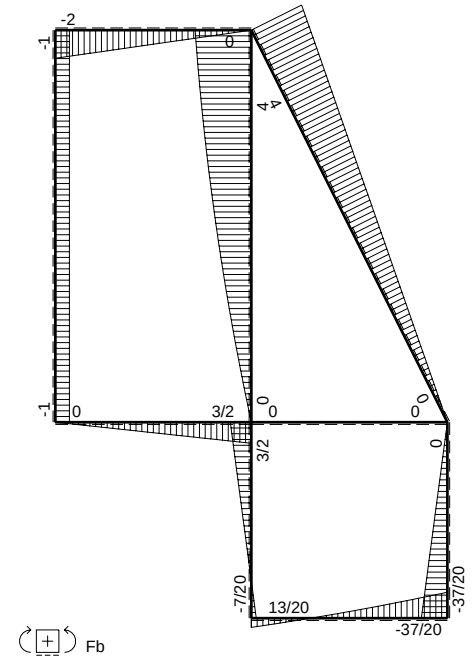
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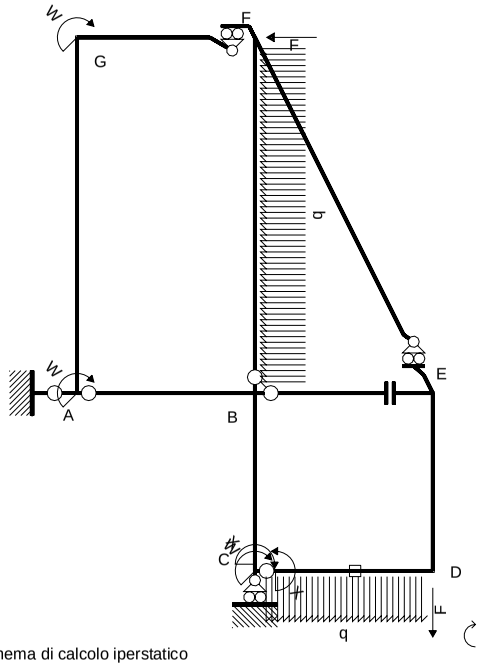
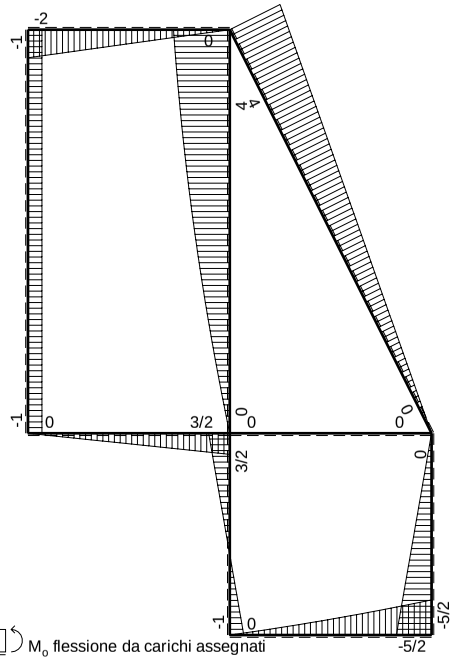
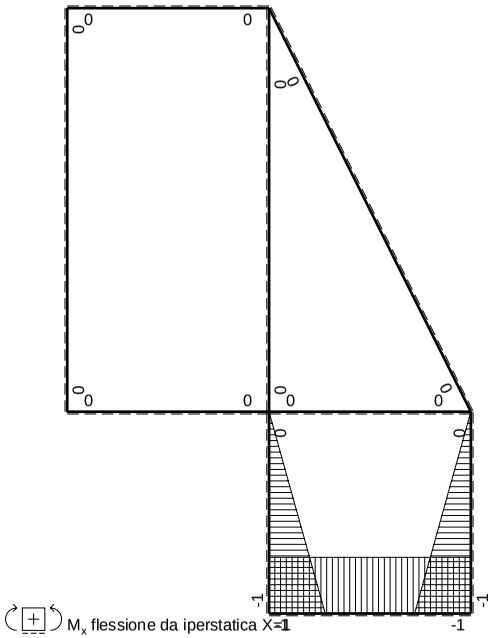


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-2Fx-1/2qx^2$	$2Fx+1/2Fx^2/b$	1	$7/6Fb^2/EJ$	Xb/EJ
DC b	1	$5/2Fb-3Fx+1/2qx^2$	$5/2Fb-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	$5/2Fb-5Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	5/2Fx	$5/2Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$13/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -3\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [5/2\ x -3/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/2\ b -3/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ02.xxxx.200

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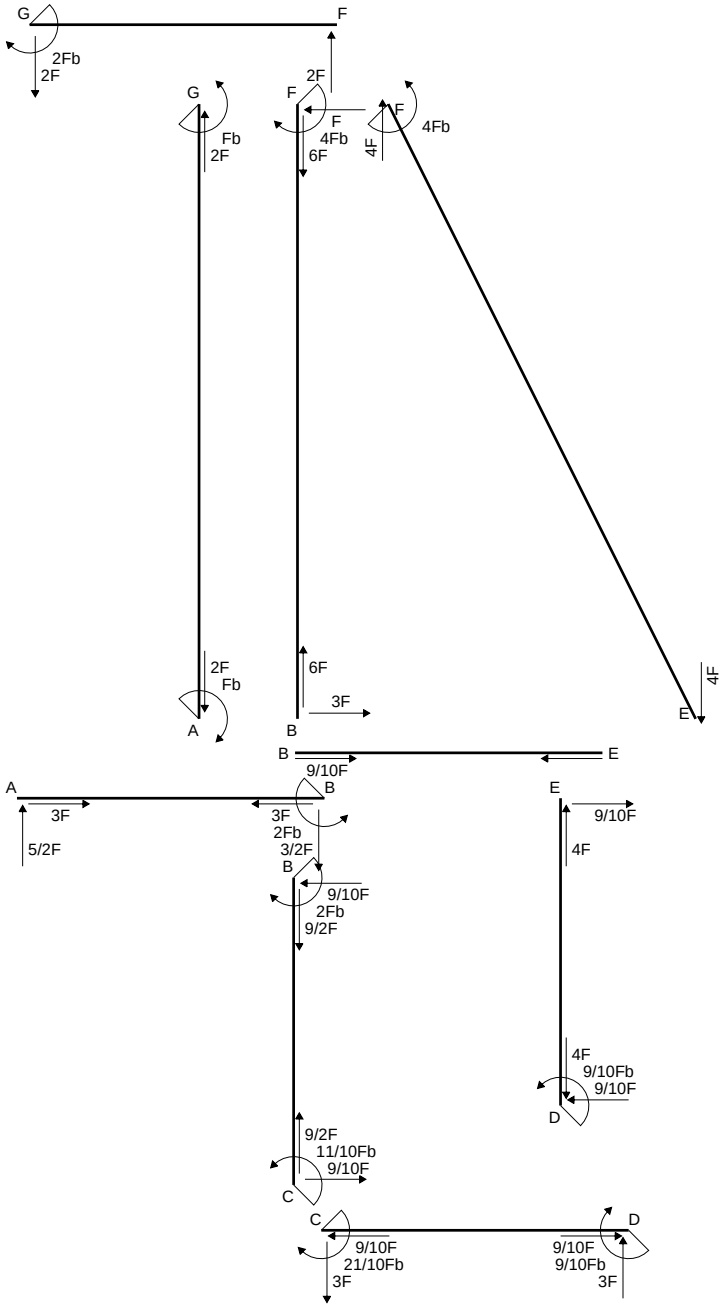
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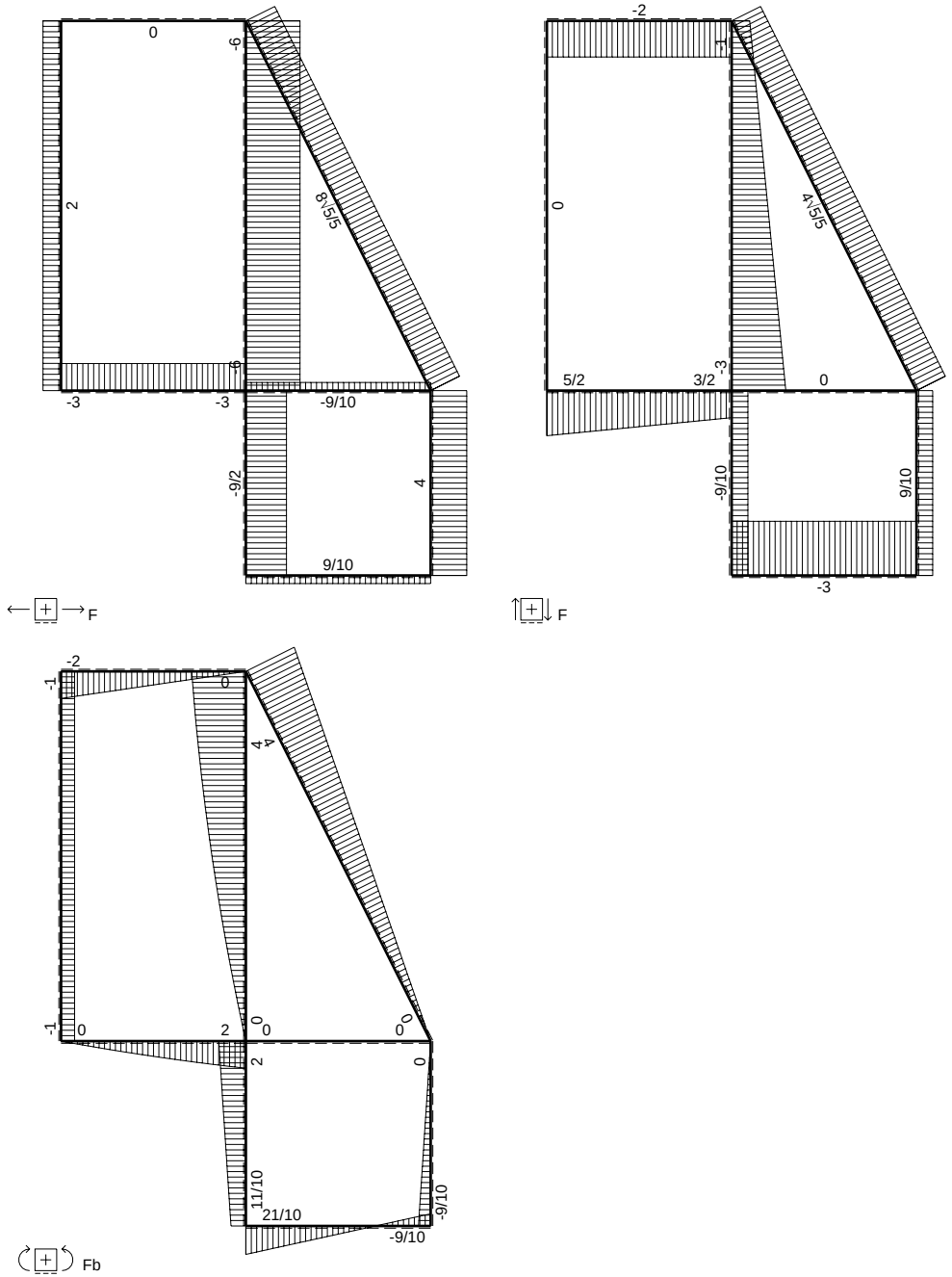
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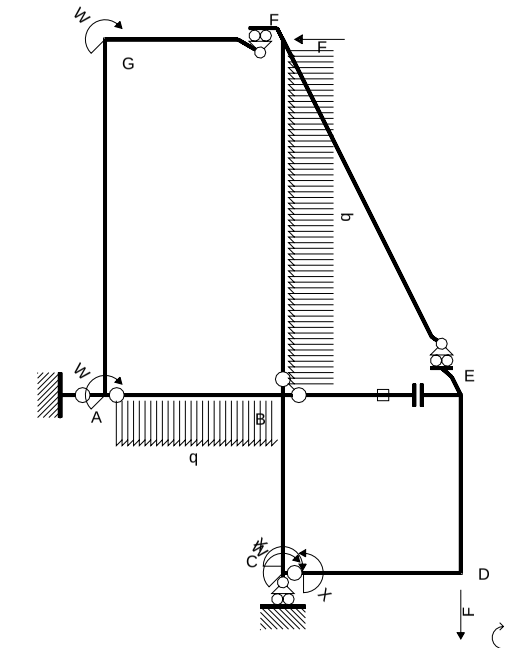
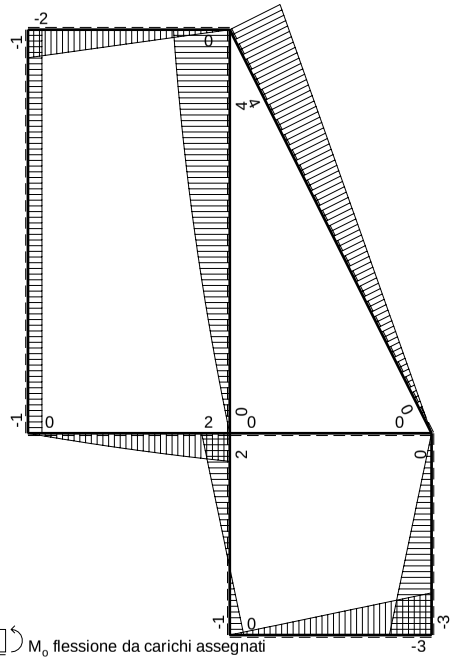
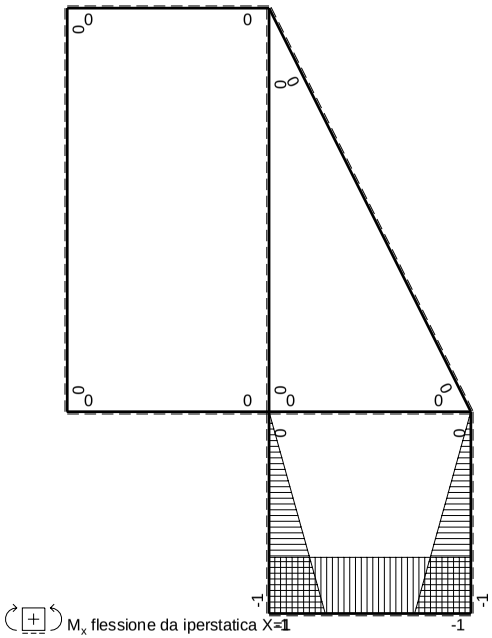


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$5/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-2Fb+3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (- b + b)\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

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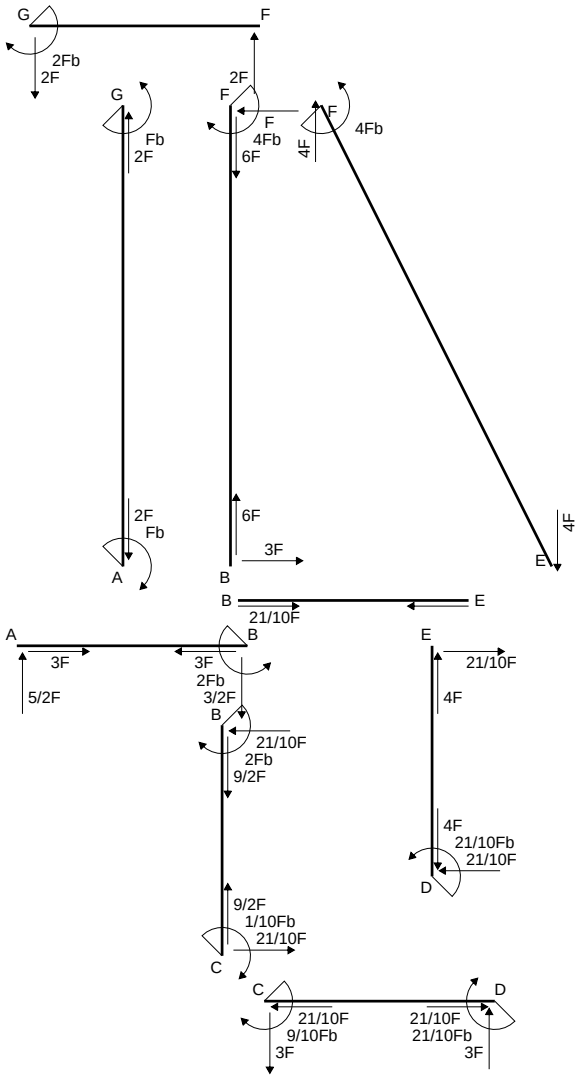
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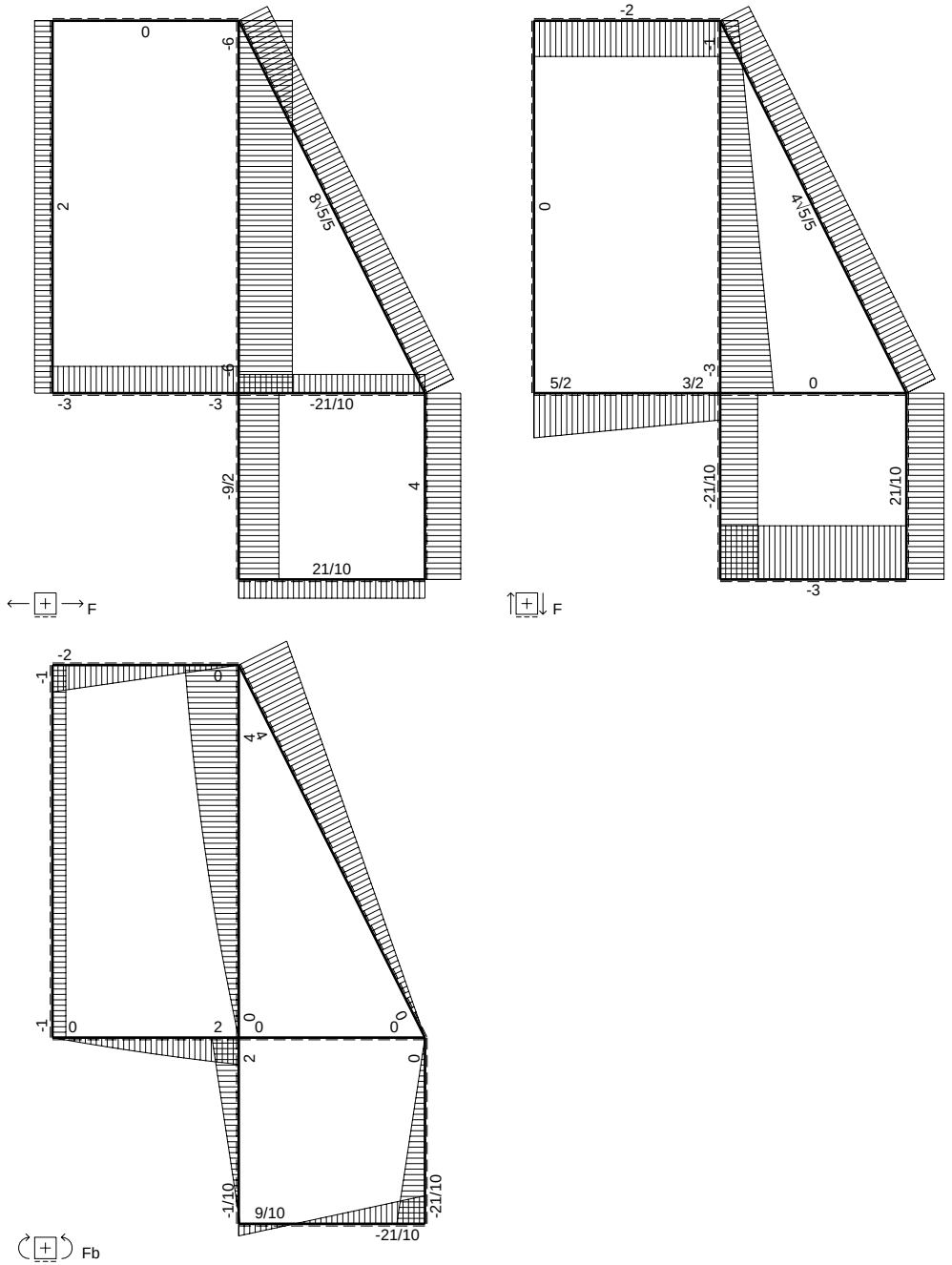
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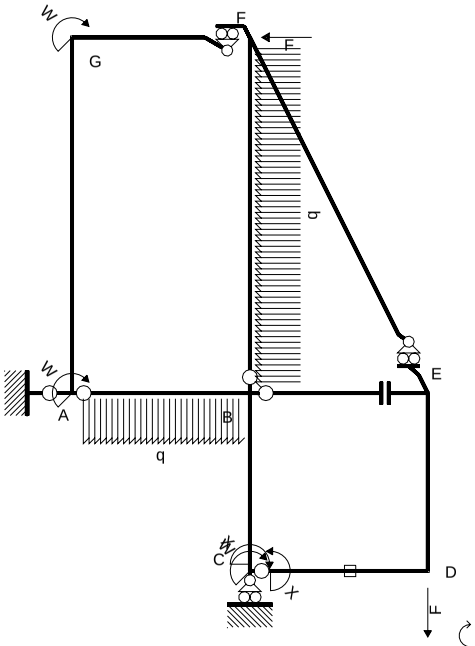
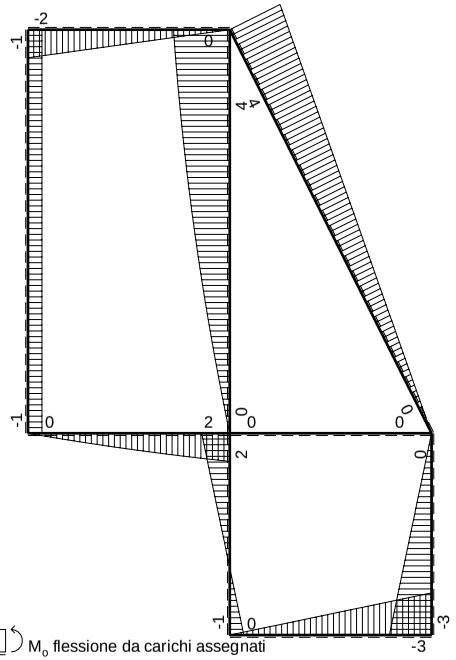
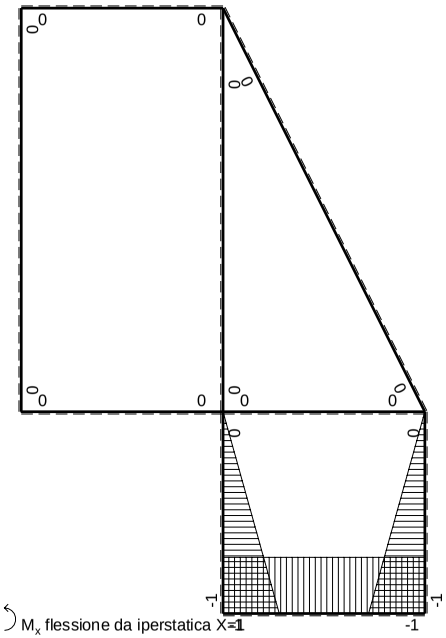


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$5/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-2Fb+3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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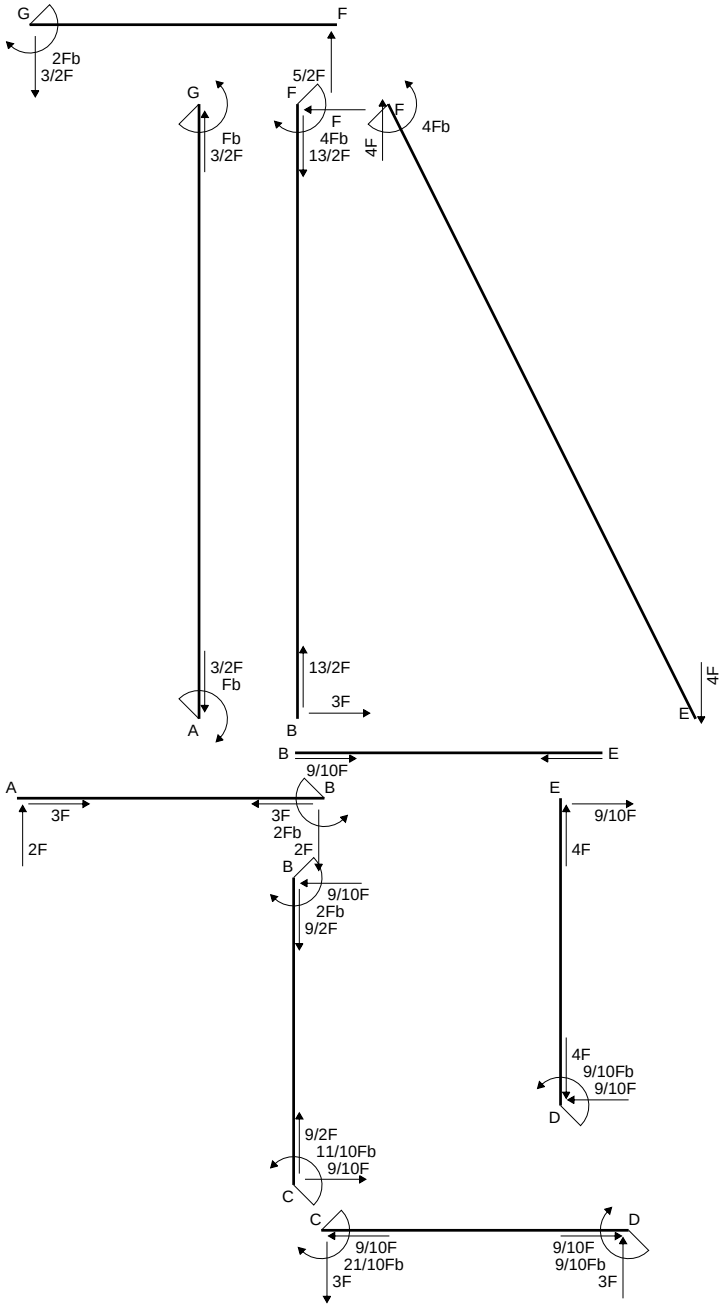
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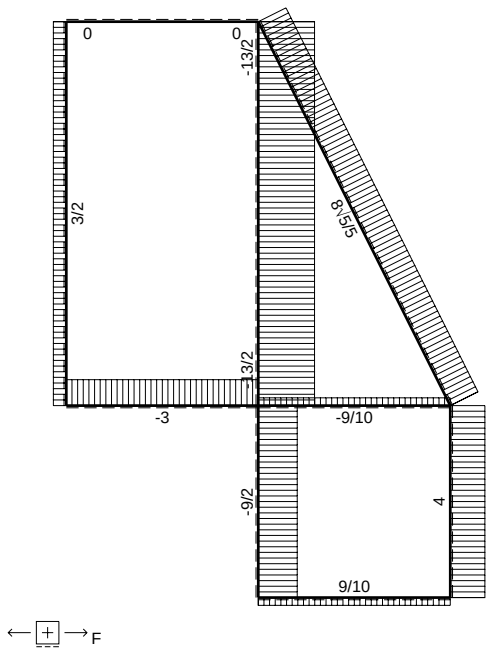
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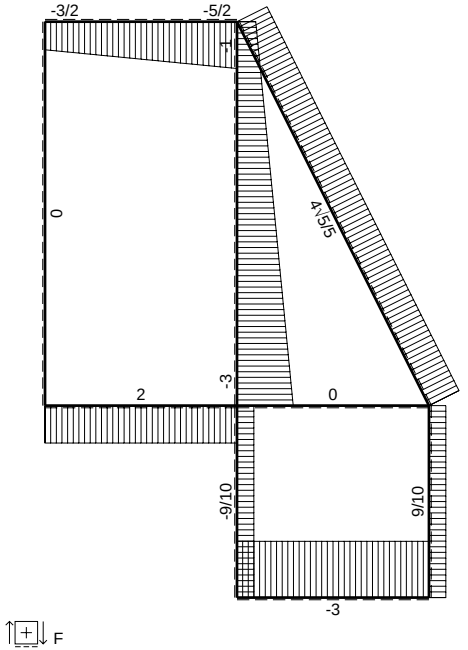
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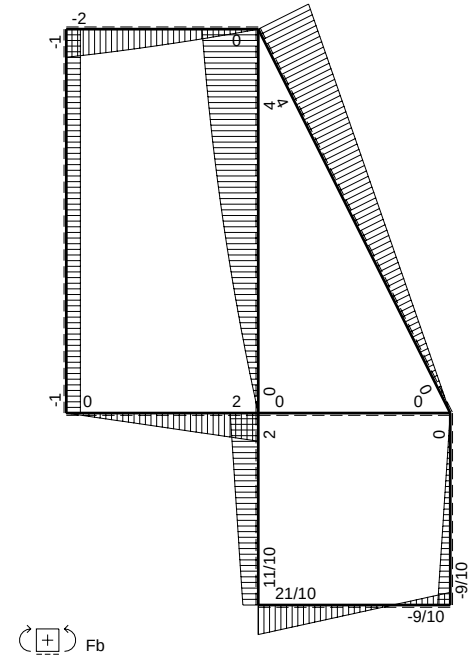
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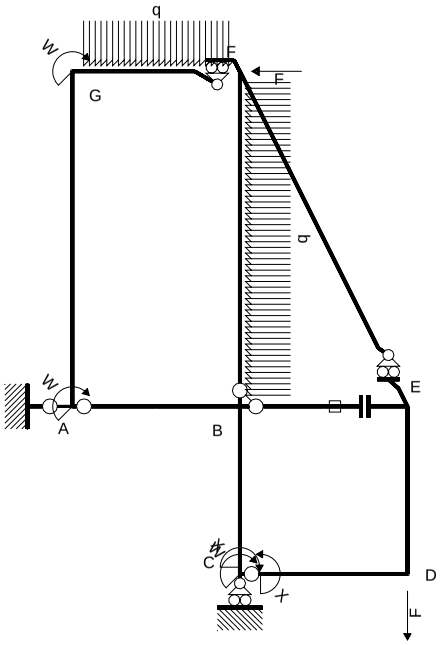
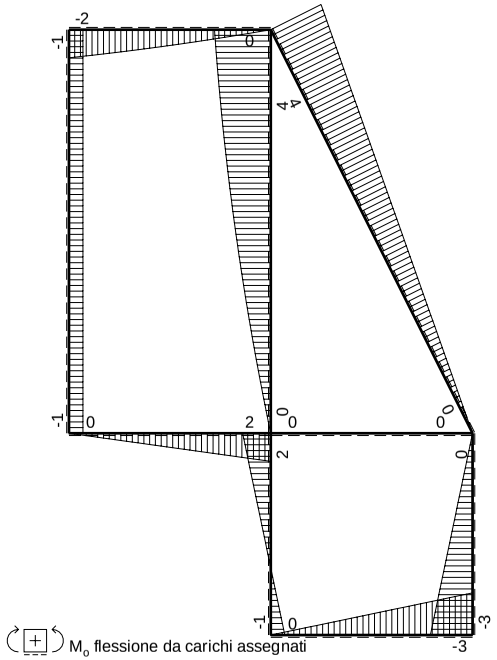
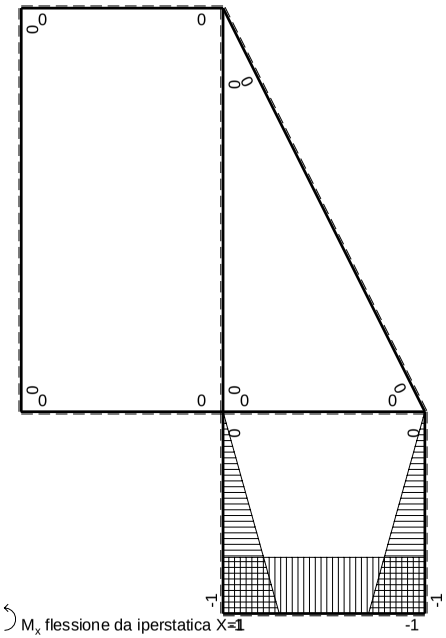


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	2Fx	0	0	0	0
BA b	0	-2Fb+2Fx	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF √5b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b + b)\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ \ Fb^2/EJ$

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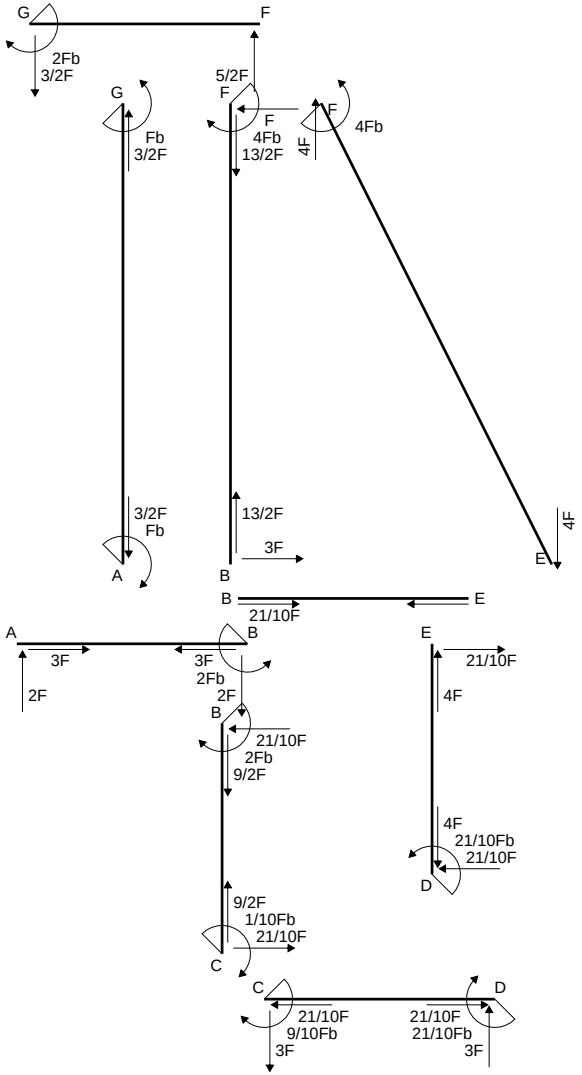
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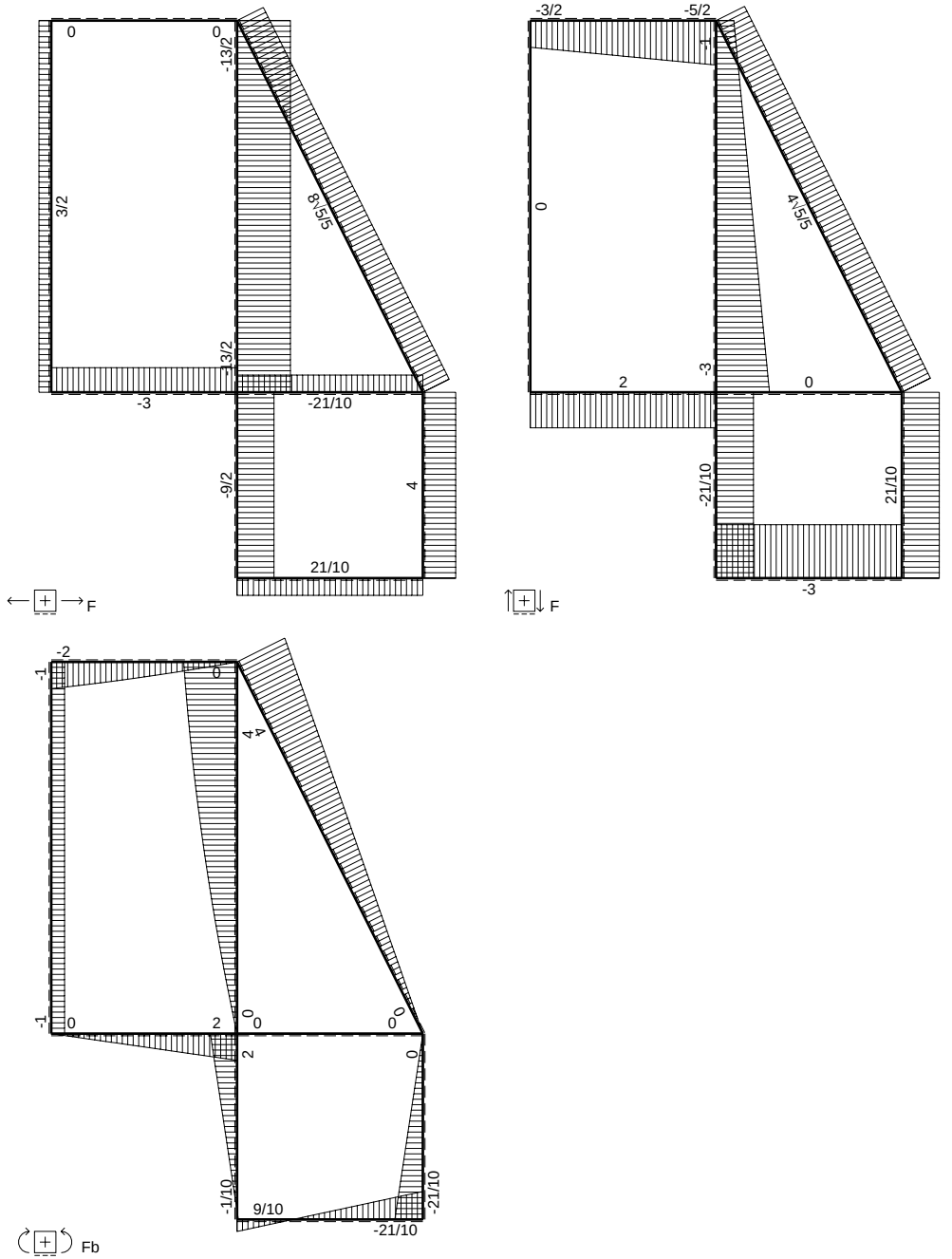
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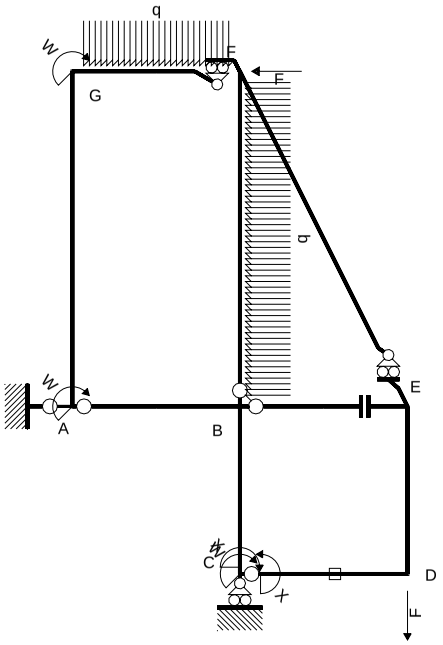
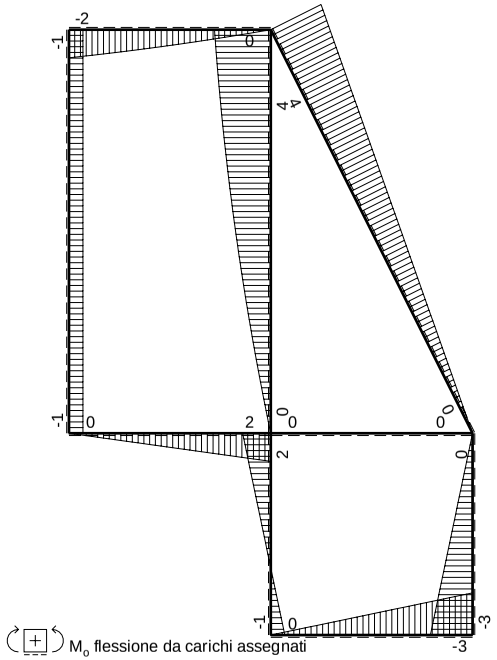
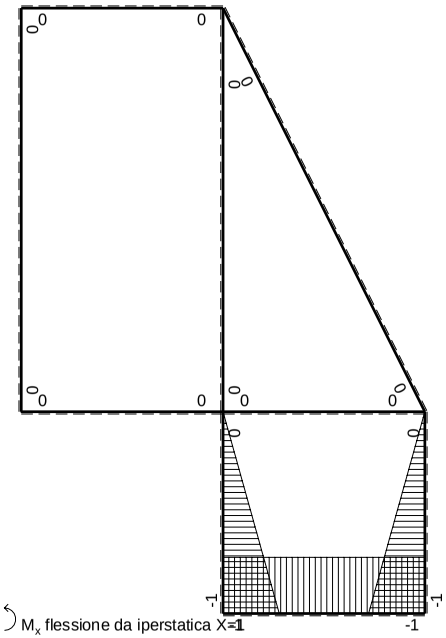


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	2Fx	0	0	0	0
BA b	0	-2Fb+2Fx	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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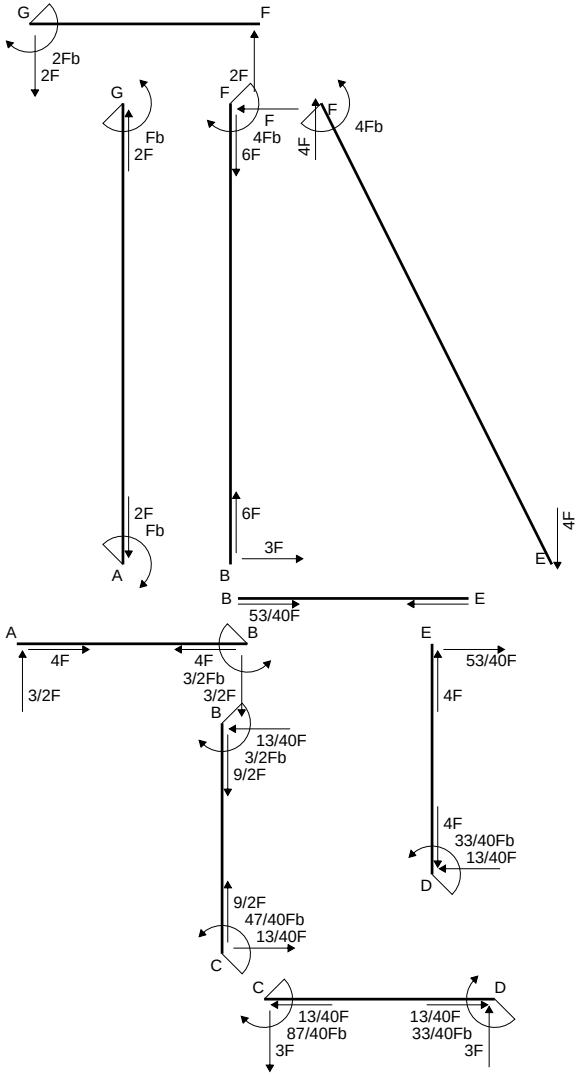
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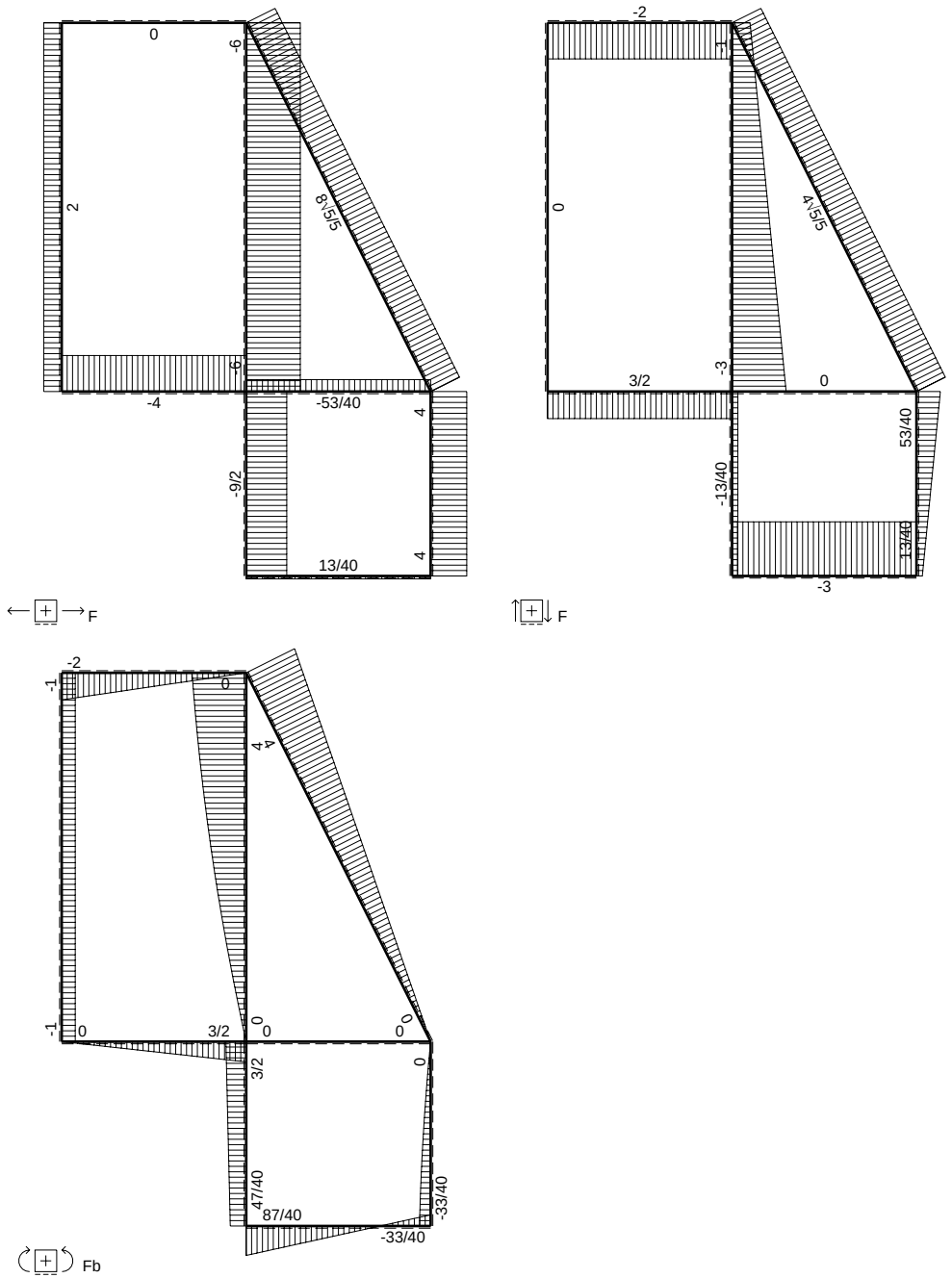
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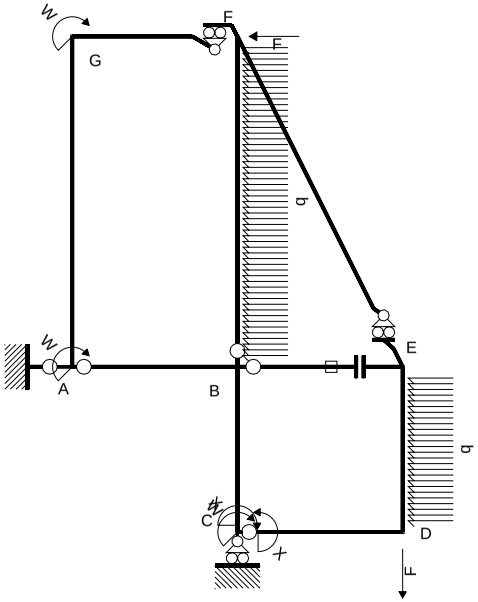
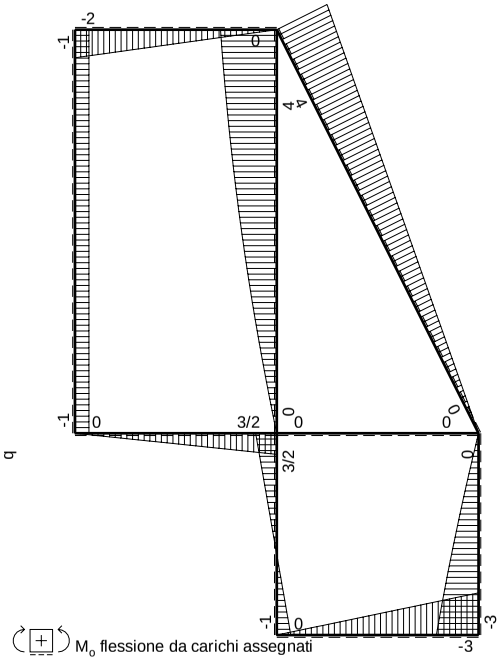
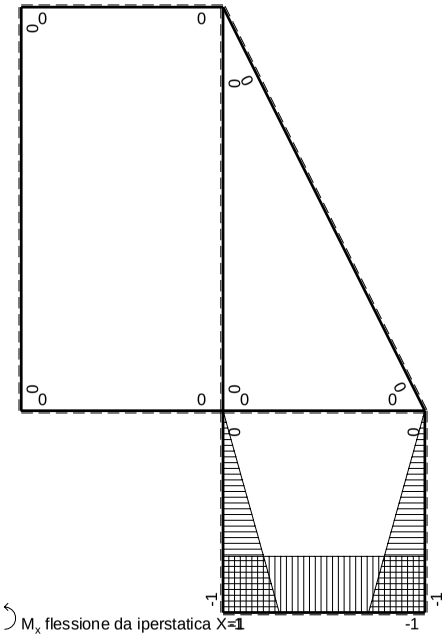
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ02.xxxx.205

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Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	$-3/2Fx+5/2Fx^2/b$	x^2/b^2	$1/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	$Fb-7/2Fx+5/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	$-3Fb+5/2Fx+1/2qx^2$	$3Fb-11/2Fx+2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$25/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$7/2Fx-1/2qx^2$	$7/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$29/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-87/40Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ02.xxxx.205

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3 -11/2\ x/b +2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [3\ x -11/4\ x^2/b +2/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -11/4\ b +2/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 25/24\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (7/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [7/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (7/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 25/24\ Fb^2/EJ$

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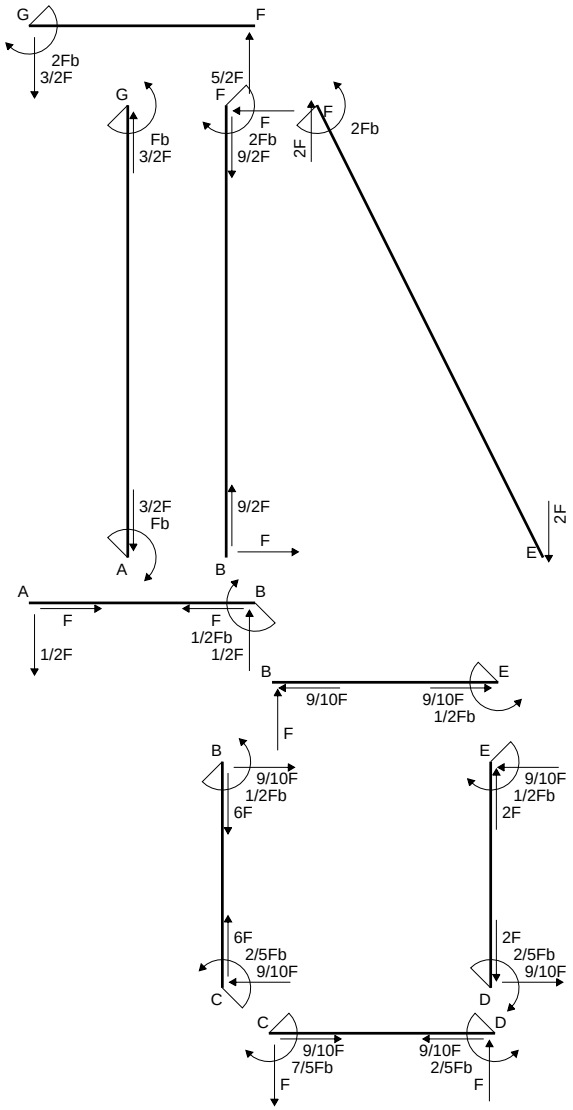
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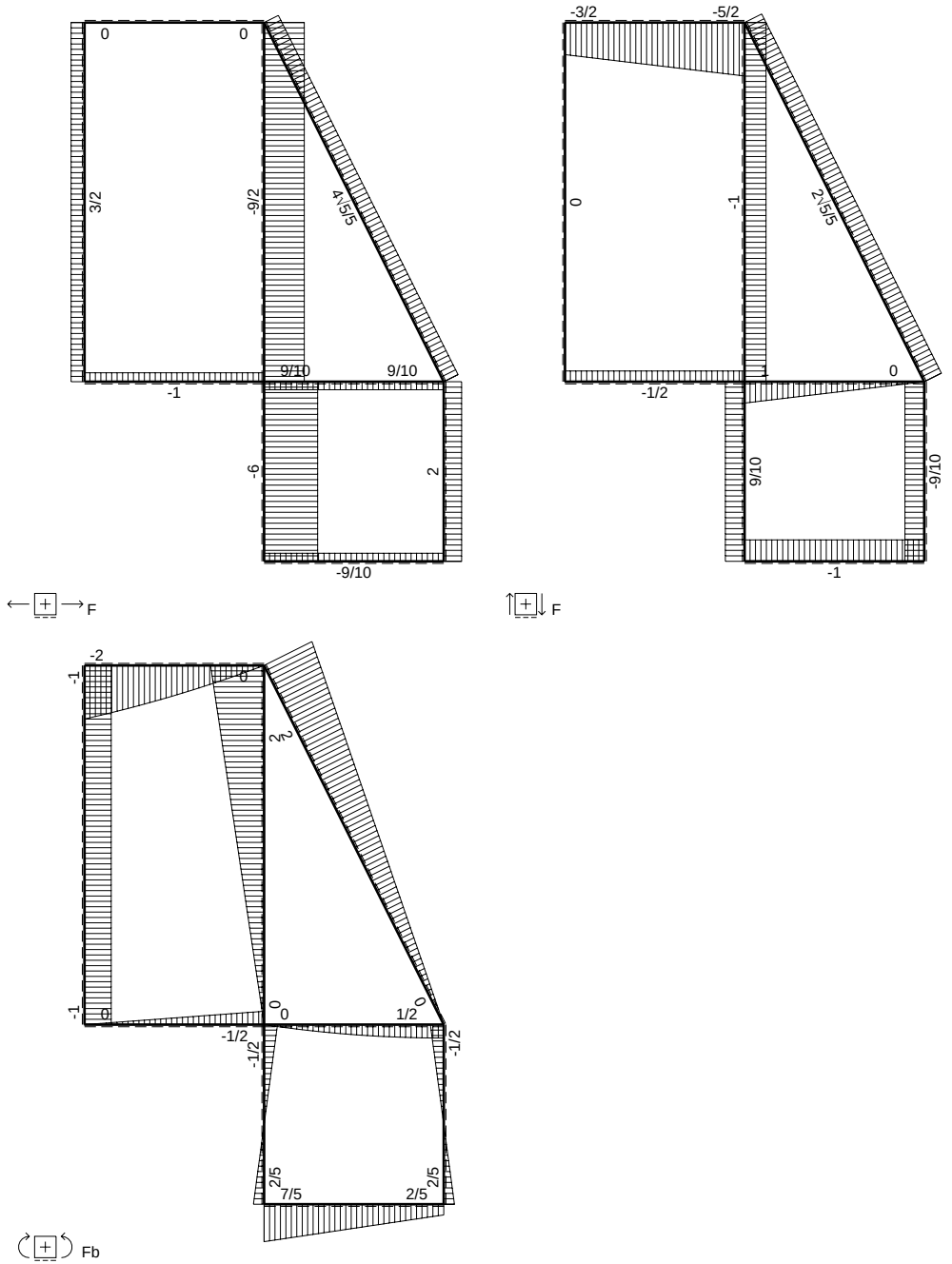
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1
0
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9/10
-9/10
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F
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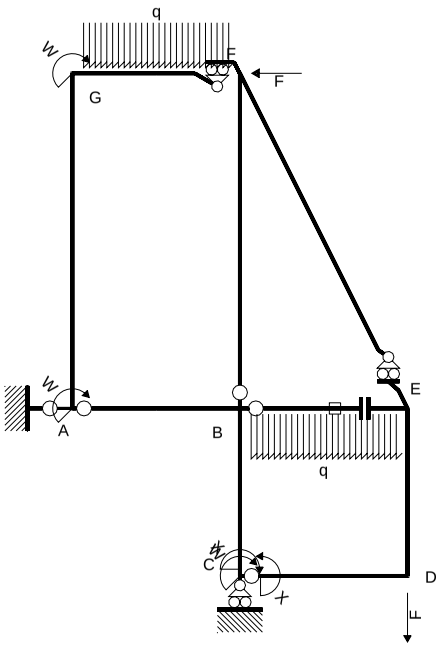
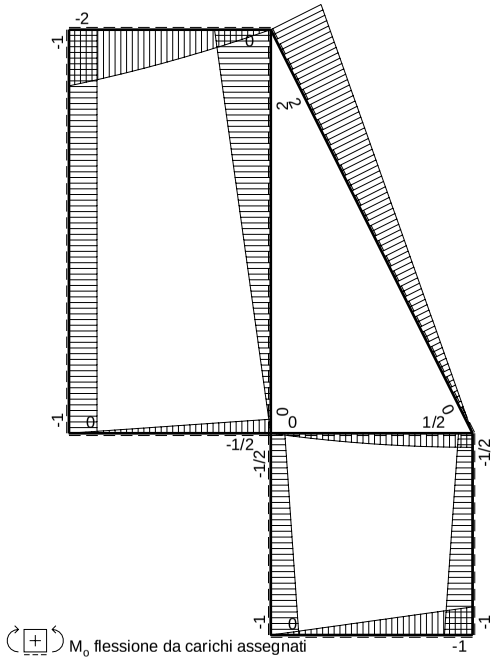
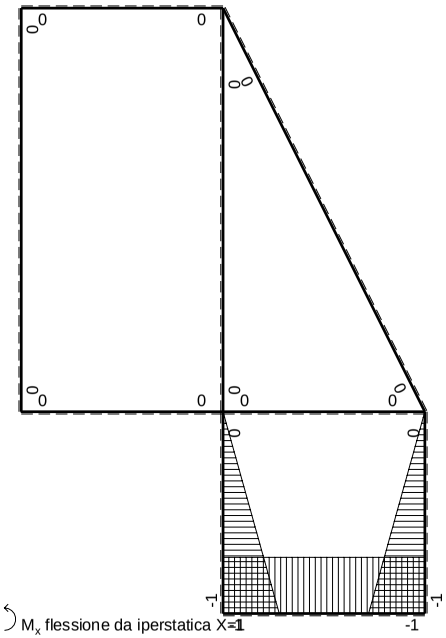


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	1/2Fb+1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-5/2Fx+1/2qx^2	0	0	0	0
GF b	0	2Fb-3/2Fx-1/2qx^2	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				7/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-7/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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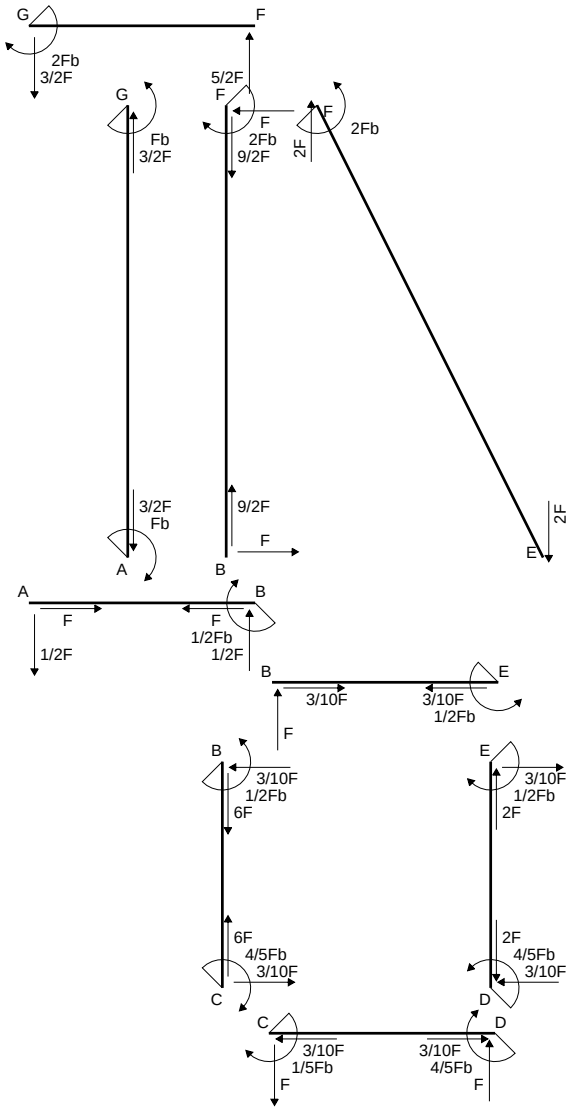
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3/10F
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F
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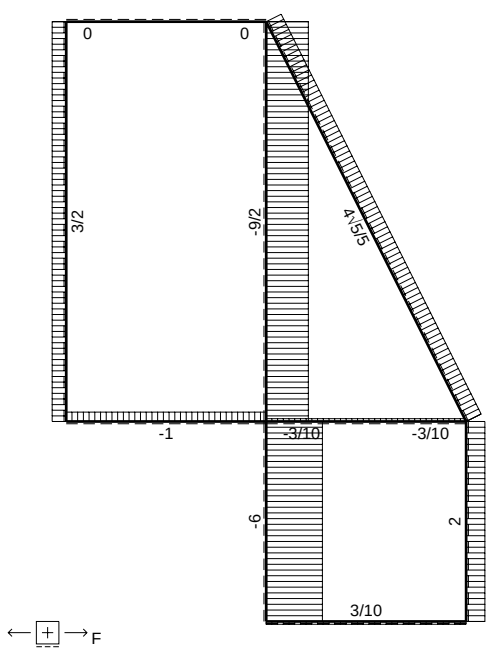
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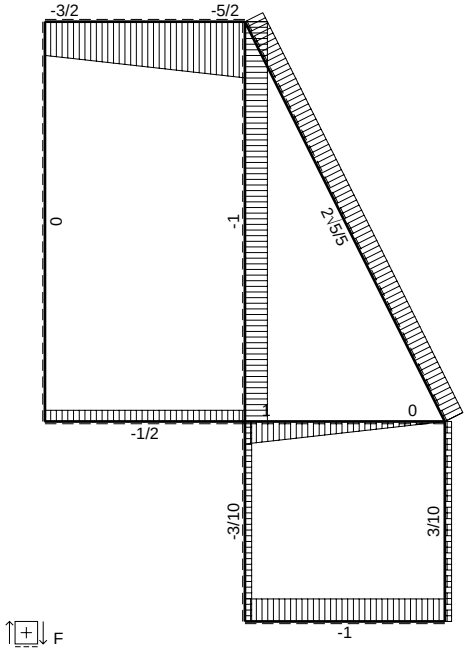
-6
2
3/10
F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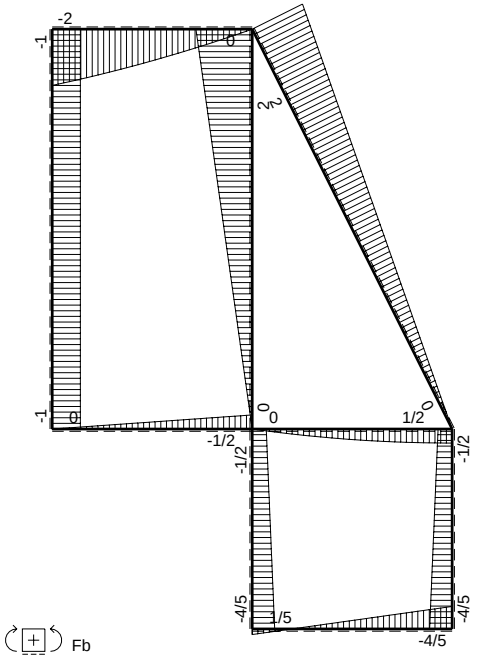
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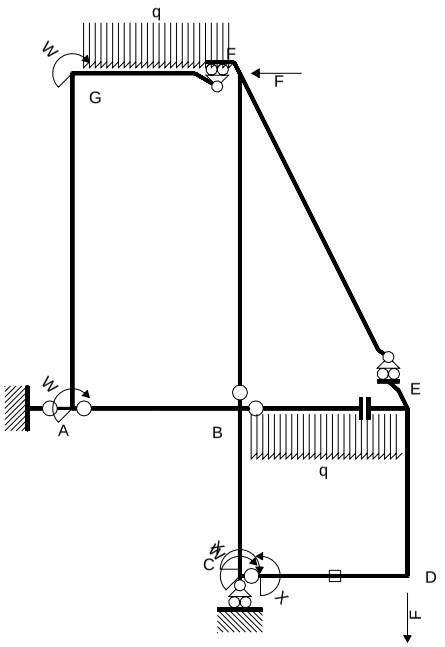
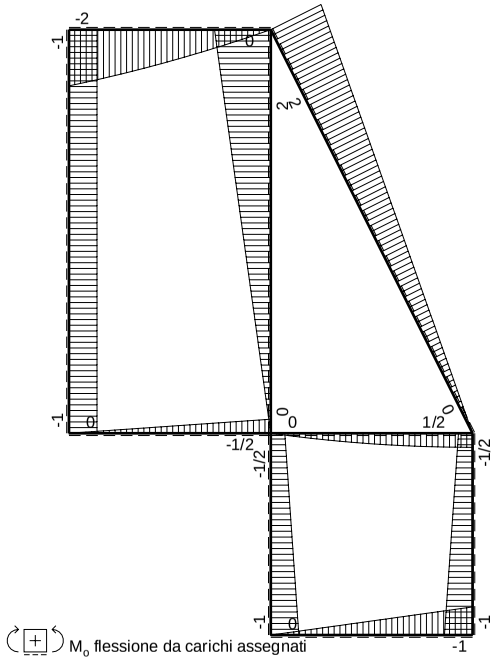
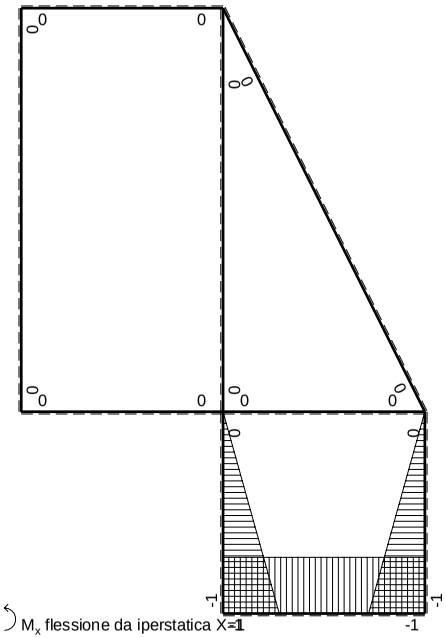
-4/5
-4/5
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+1/2Fx	Fb-3/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2	5/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	1/2Fb+1/2Fx	1/2Fx+1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-5/2Fx+1/2qx^2	0	0	0	0
GF b	0	2Fb-3/2Fx-1/2qx^2	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				1/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-1/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

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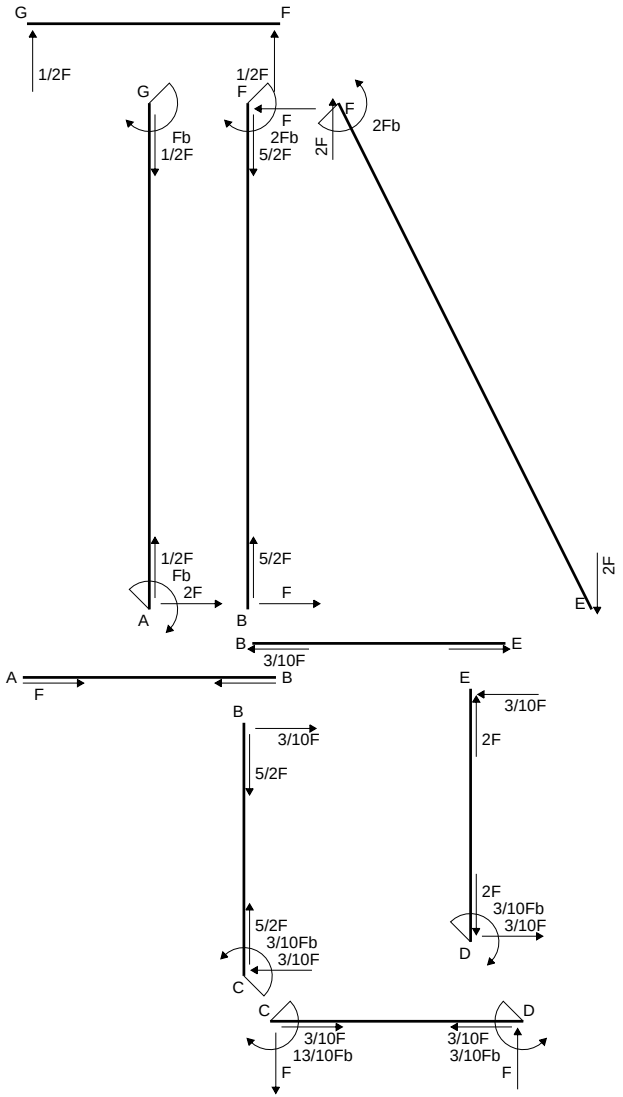
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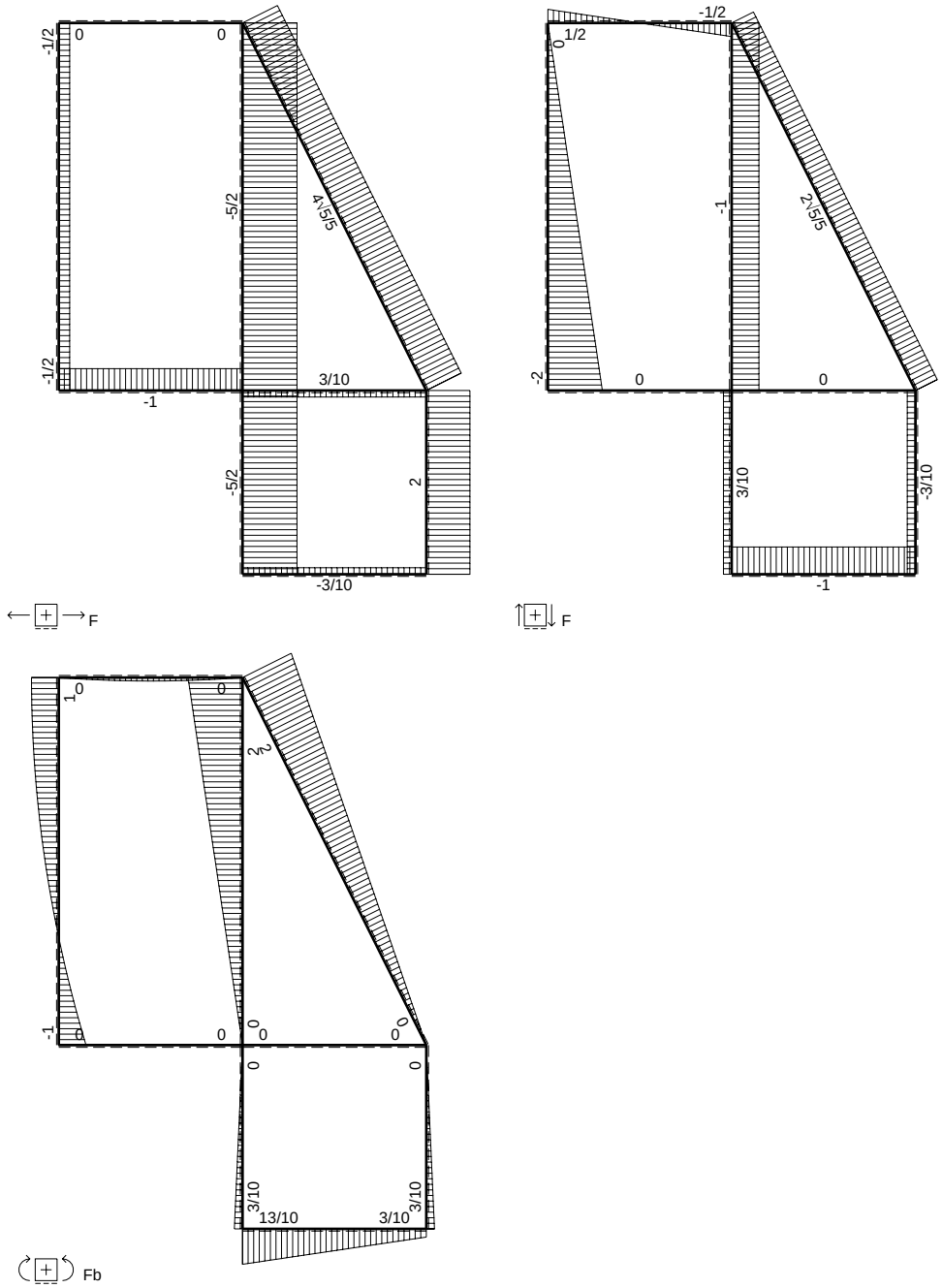
F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1/2
0
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2√5/5
-2
0
0
3/10
-3/10
-1
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1
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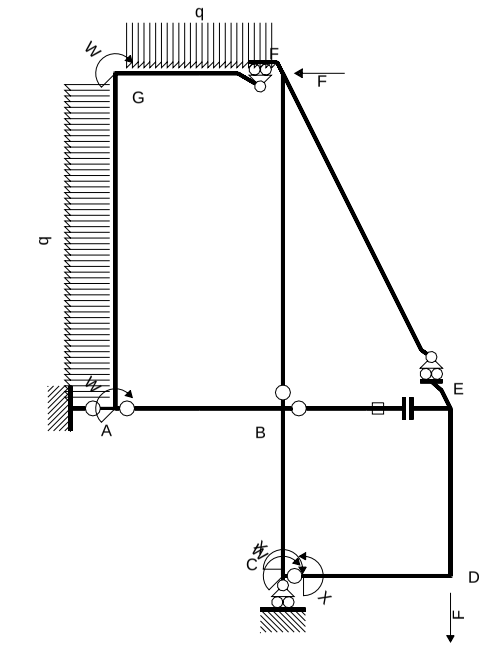
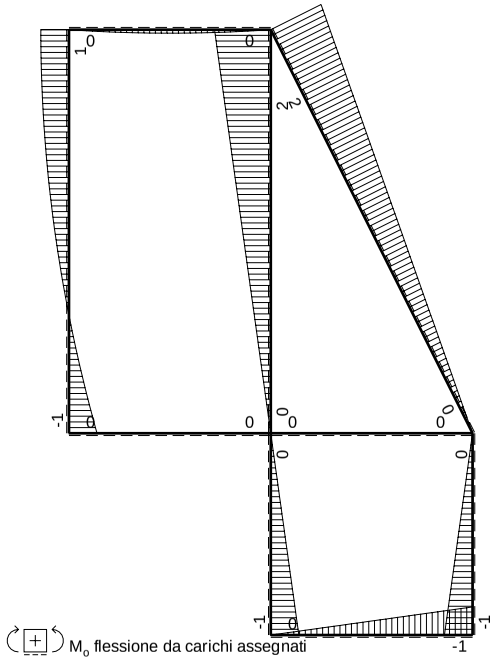
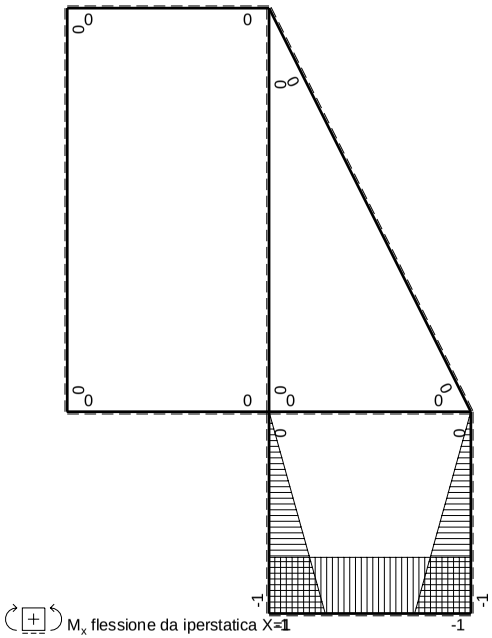


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$13/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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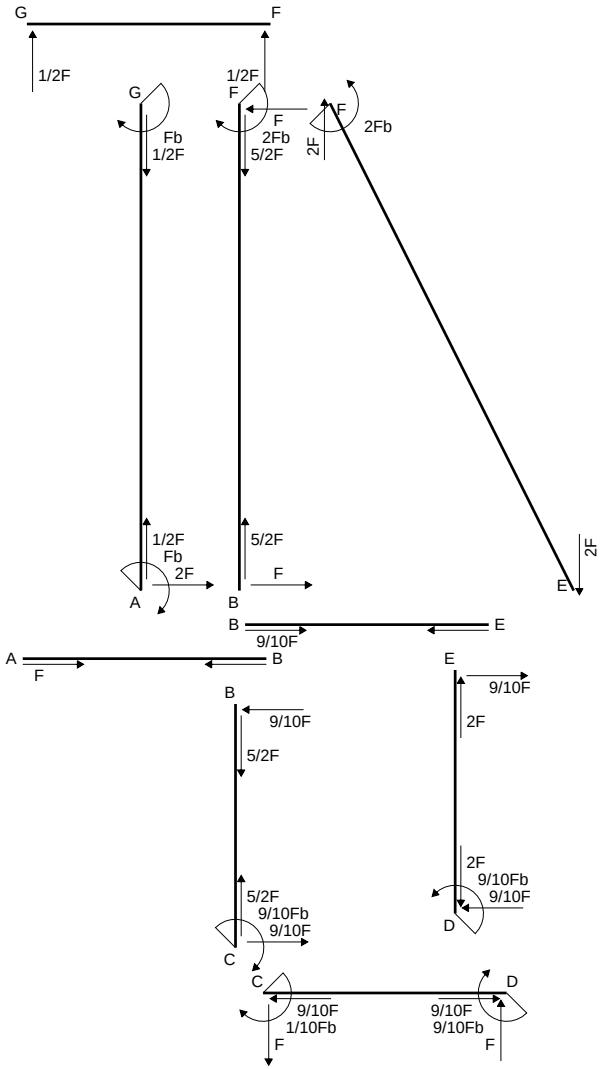
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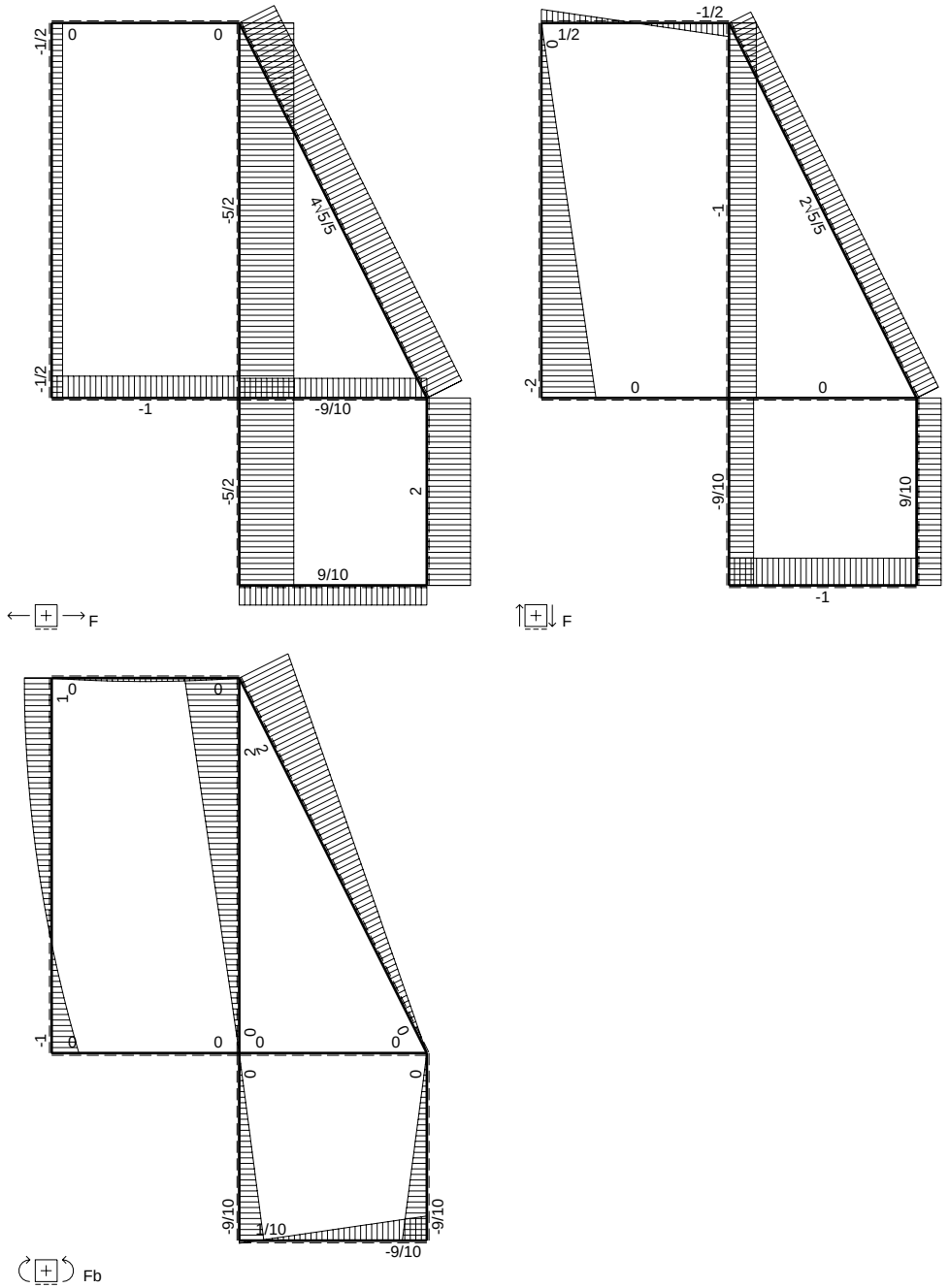
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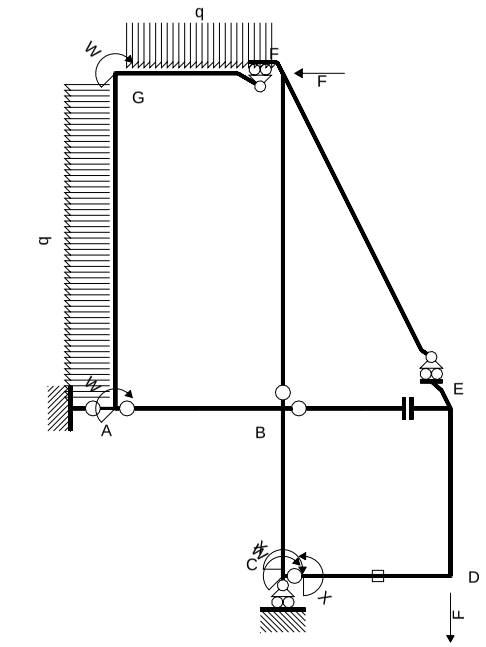
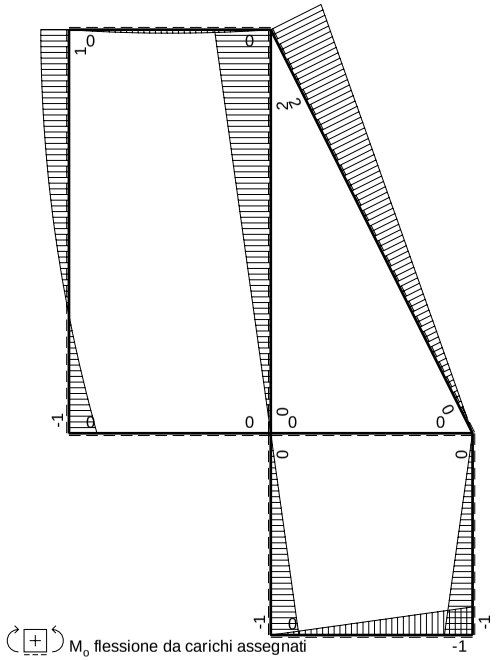
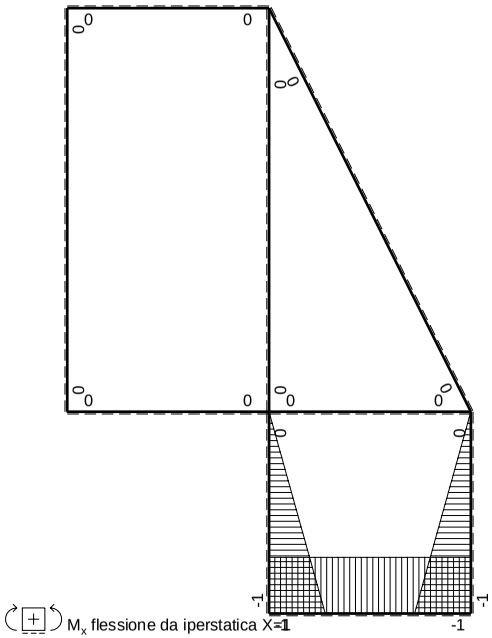


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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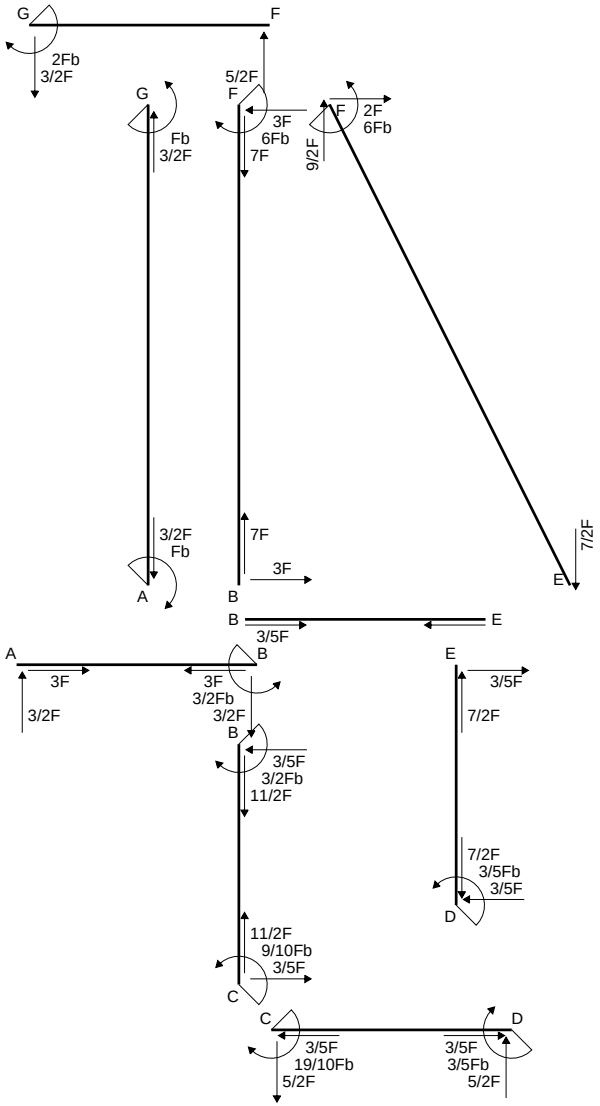
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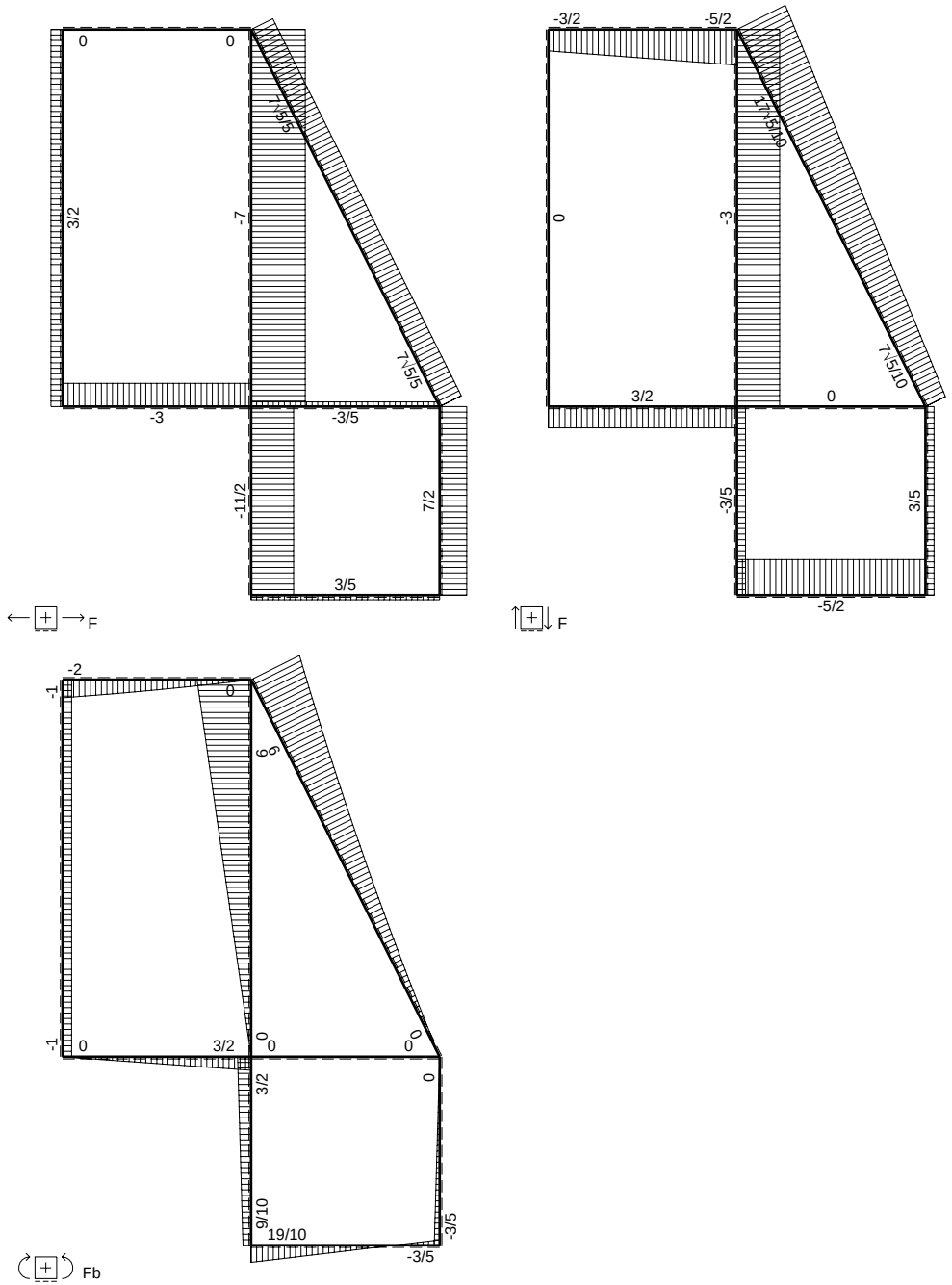
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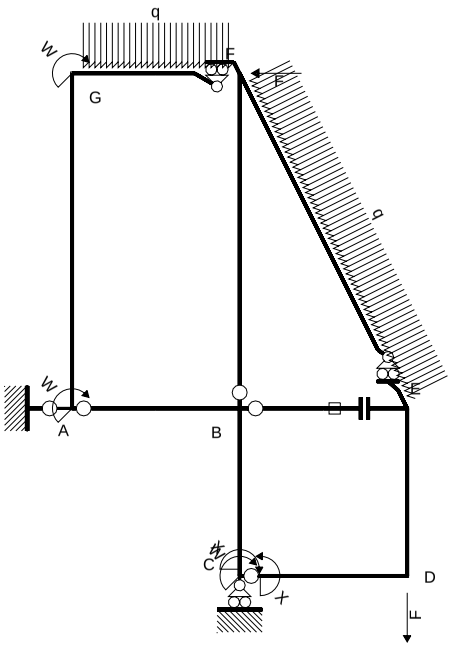
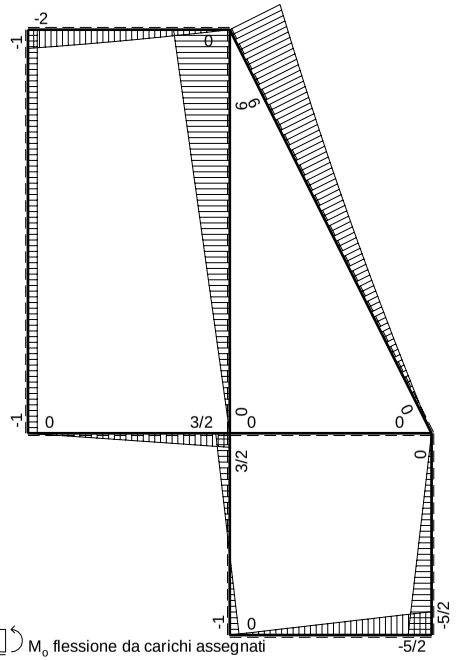
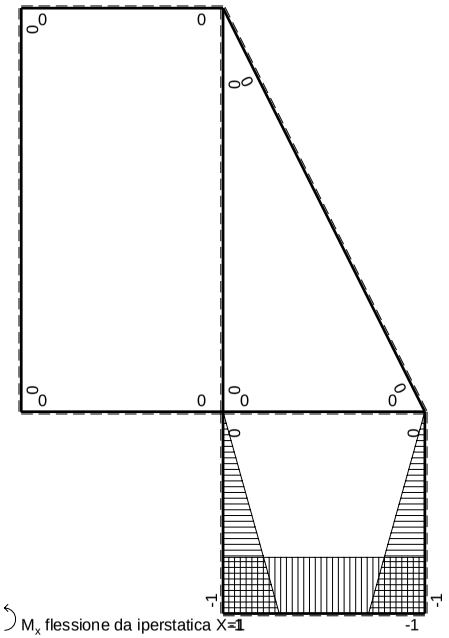


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	-3/2Fx+5/2Fx^2/b	x^2/b^2	1/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	Fb-7/2Fx+5/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-5/2Fx	5/2Fx	1	5/4Fb^2/EJ	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	5/2Fb-5Fx+5/2Fx^2/b	1-2x/b+x^2/b^2	5/6Fb^2/EJ	1/3Xb/EJ
ED b	x/b	5/2Fx	5/2Fx^2/b	x^2/b^2		
EF √5b	0	7√5/10Fx+1/2qx^2	0	0	0	0
FG b	0	-5/2Fx+1/2qx^2	0	0	0	0
GF b	0	2Fb-3/2Fx-1/2qx^2	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-19/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ = 5/4\ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ Fb^2/EJ$

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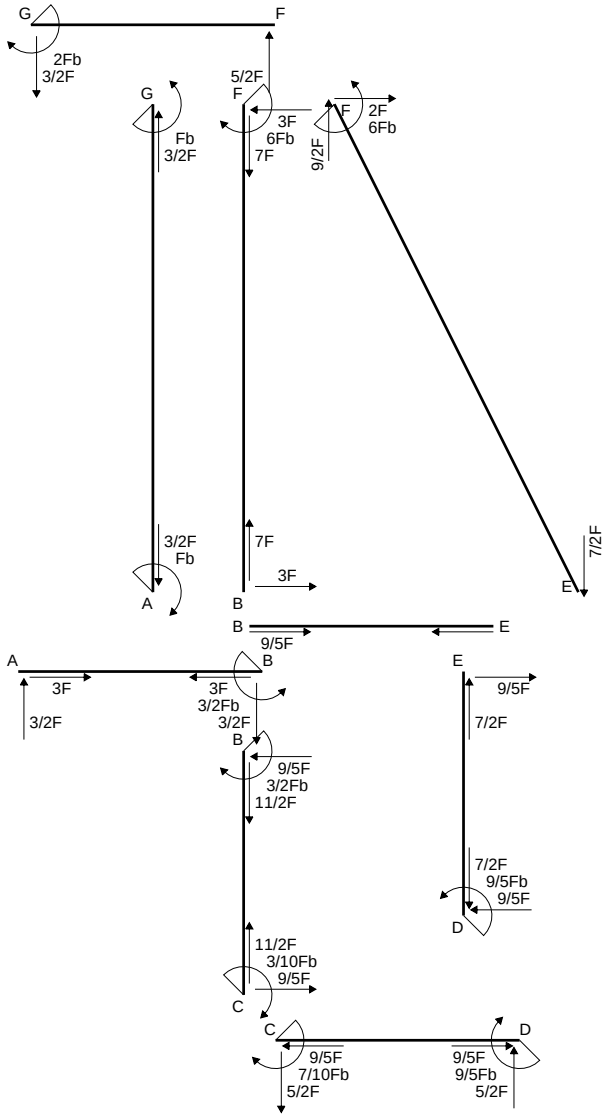
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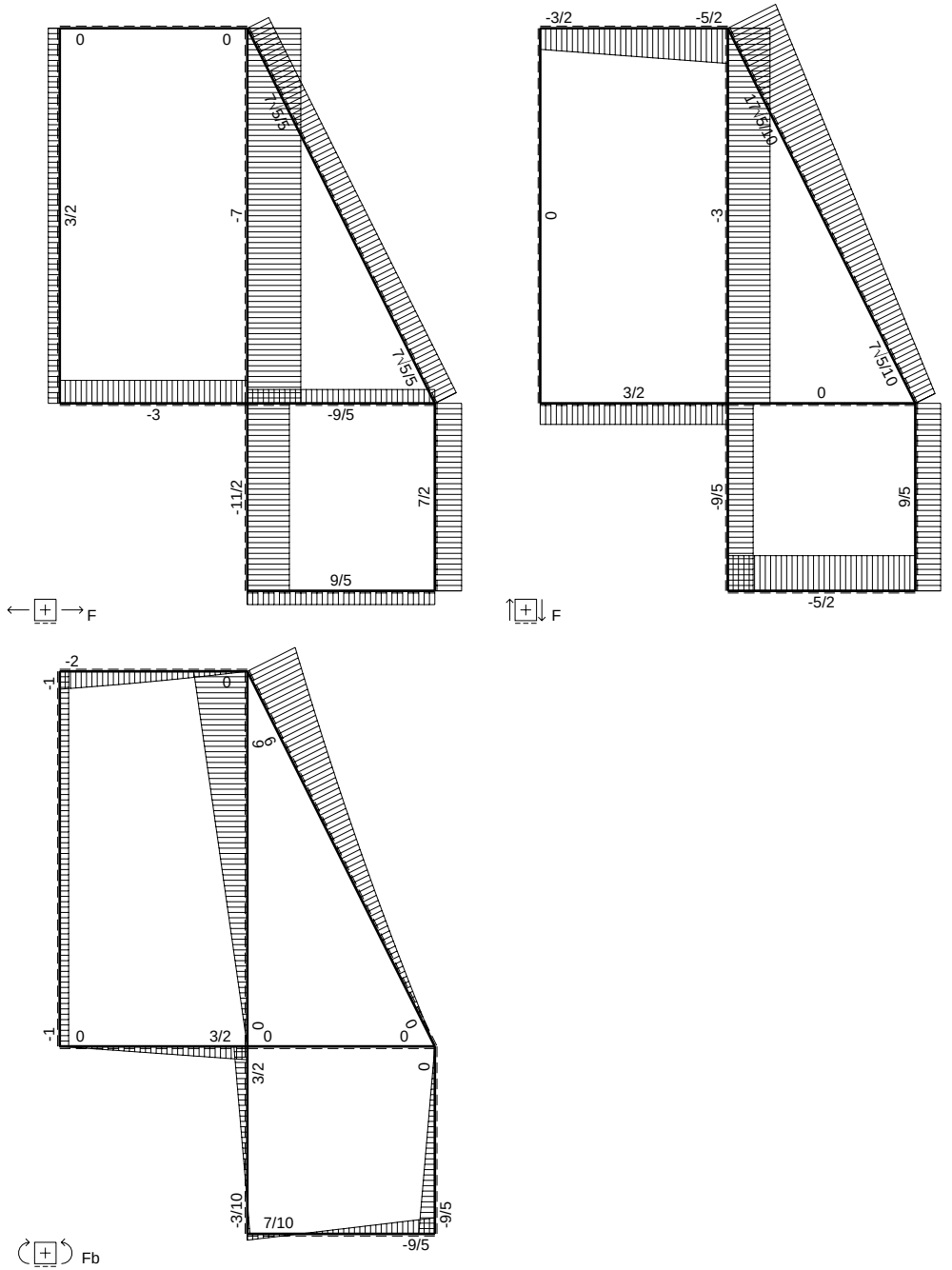
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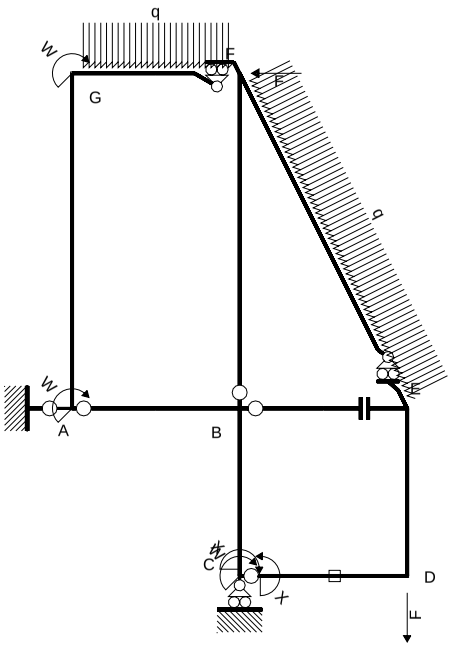
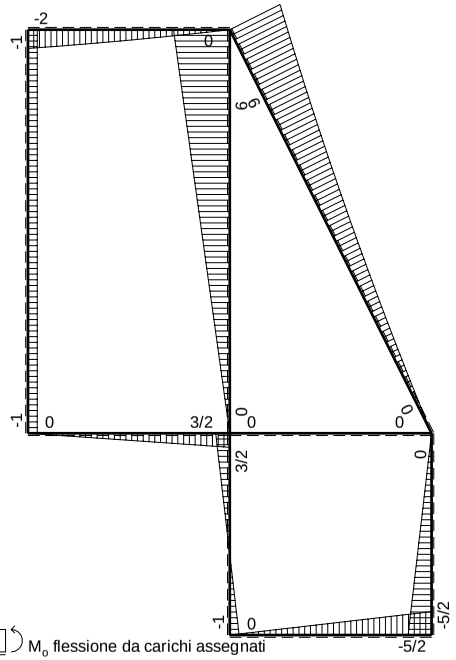
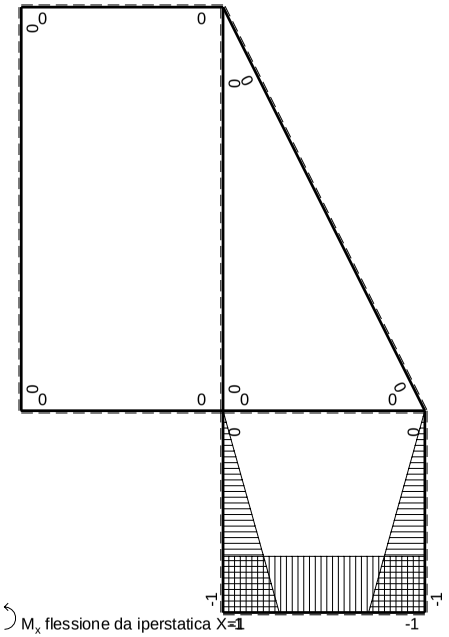
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb-5/2Fx	-3/2Fx+5/2Fx^2/b	x^2/b^2	1/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb-5/2Fx	Fb-7/2Fx+5/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-5/2Fx	5/2Fx	1	5/4Fb^2/EJ	Xb/EJ
DC b	1	5/2Fb-5/2Fx	5/2Fb-5/2Fx	1		
DE b	-1+x/b	-5/2Fb+5/2Fx	5/2Fb-5Fx+5/2Fx^2/b	1-2x/b+x^2/b^2	5/6Fb^2/EJ	1/3Xb/EJ
ED b	x/b	5/2Fx	5/2Fx^2/b	x^2/b^2		
EF √5b	0	7√5/10Fx+1/2qx^2	0	0	0	0
FG b	0	-5/2Fx+1/2qx^2	0	0	0	0
GF b	0	2Fb-3/2Fx-1/2qx^2	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				-Fb^2/EJ	
	totali				7/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-7/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -7/2\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -7/4\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -7/4\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/2 -5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [5/2\ x -5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/E$

$= (5/2\ b -5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (5/6\ b\ )\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

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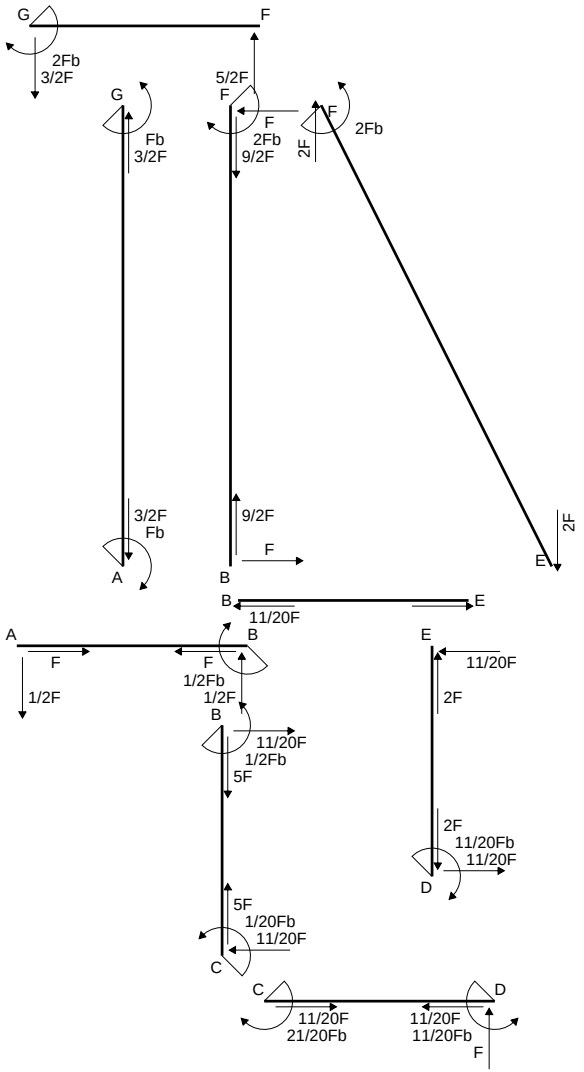
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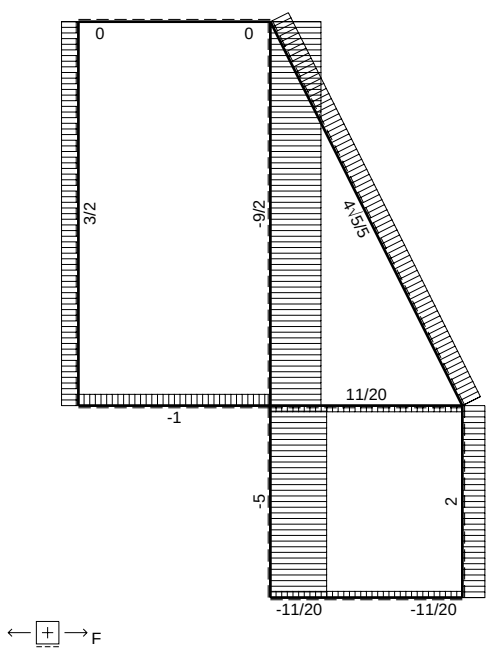
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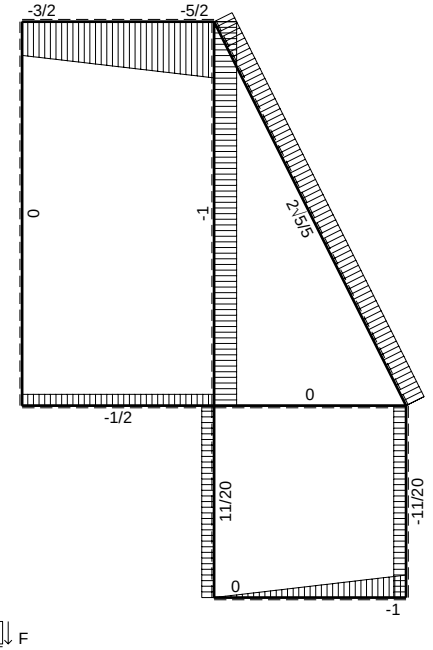
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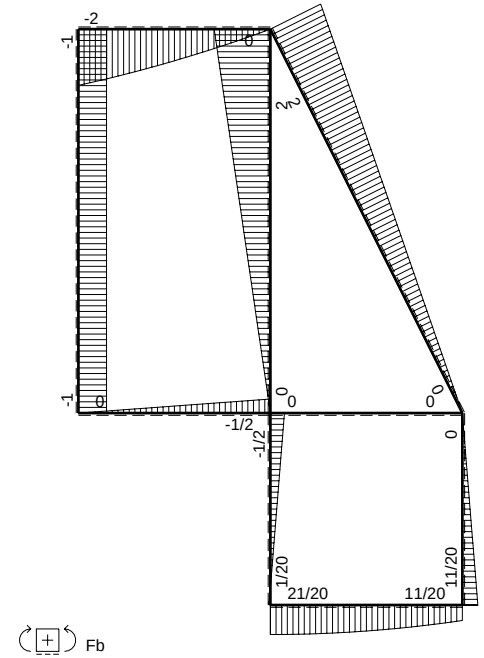
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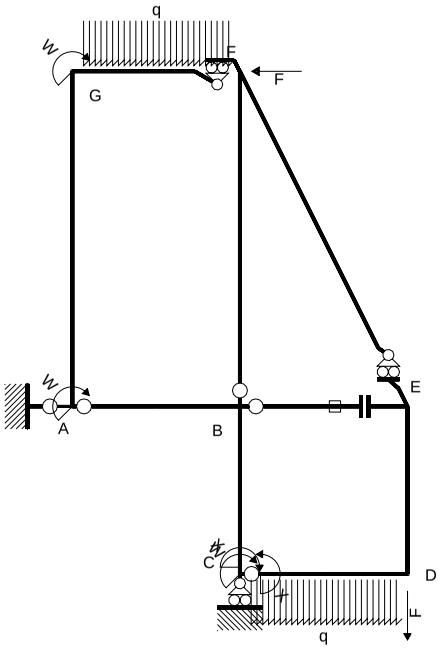
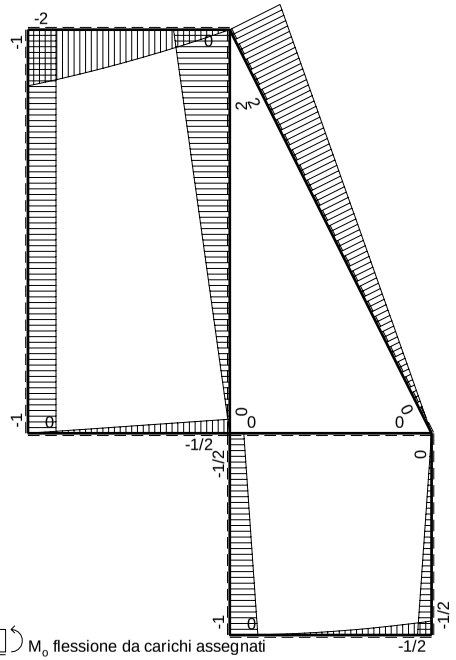
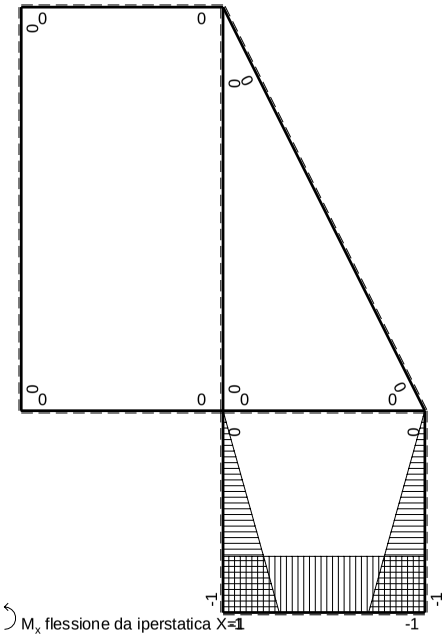
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

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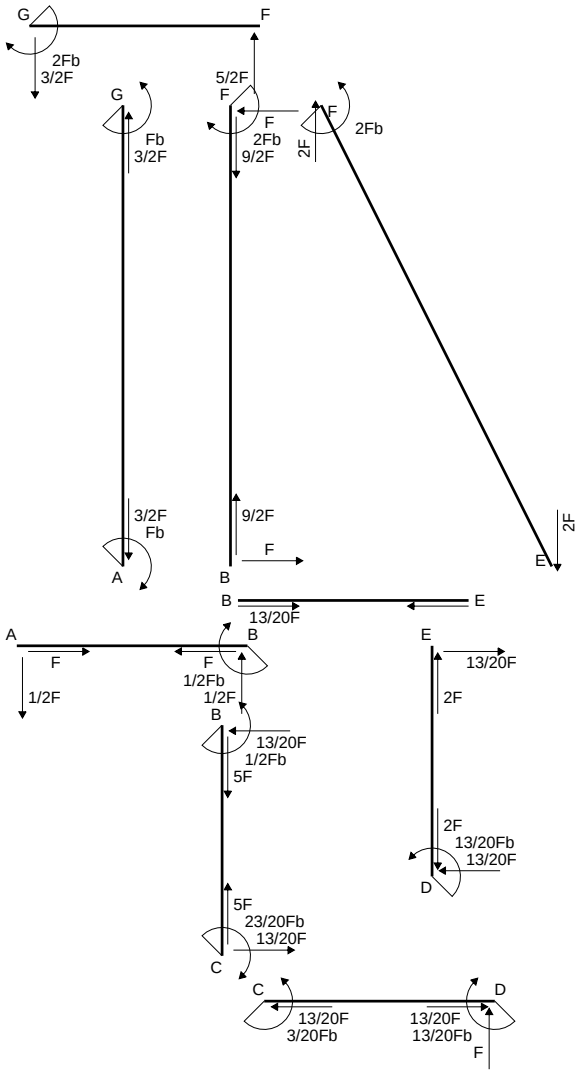
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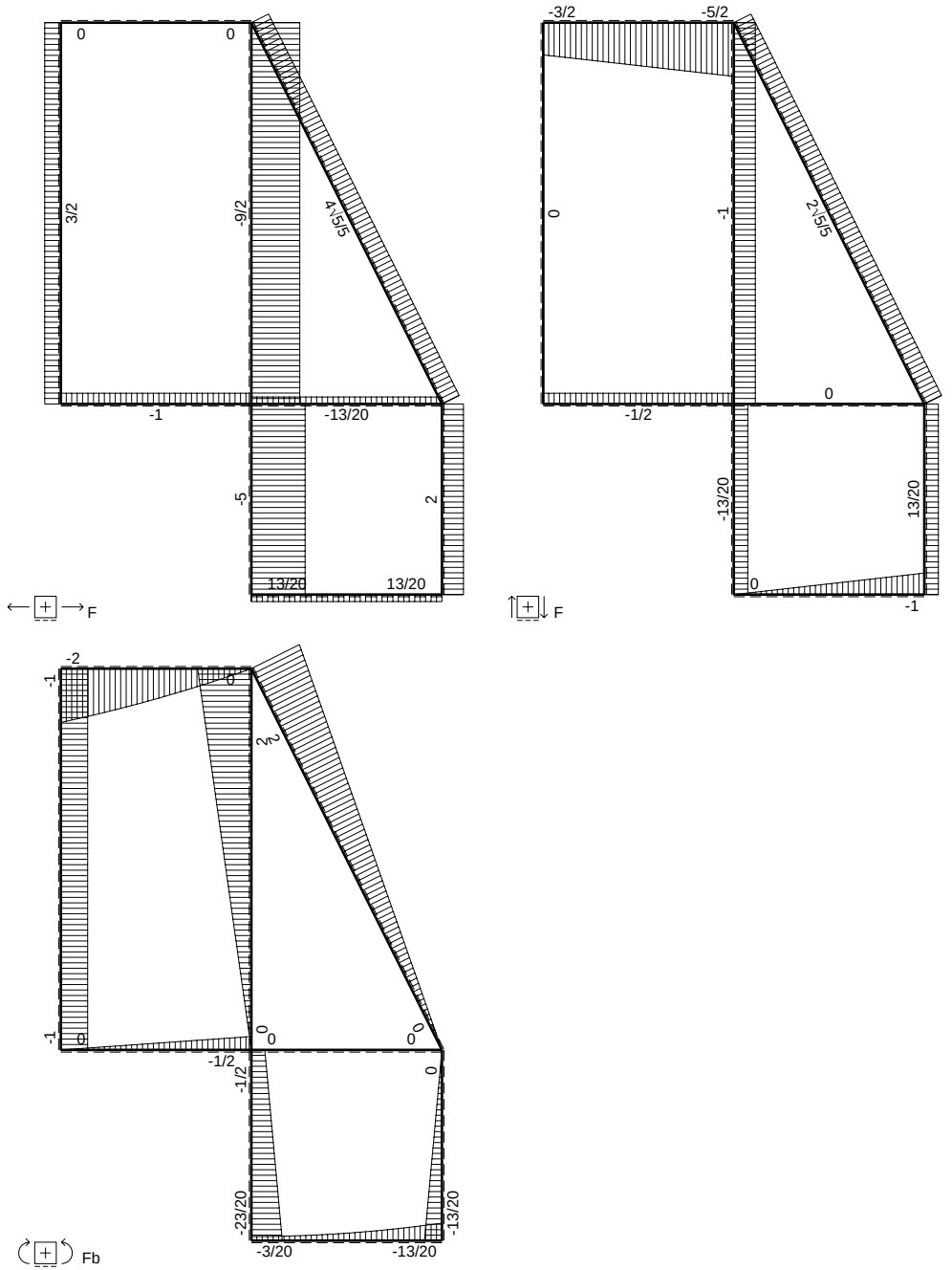
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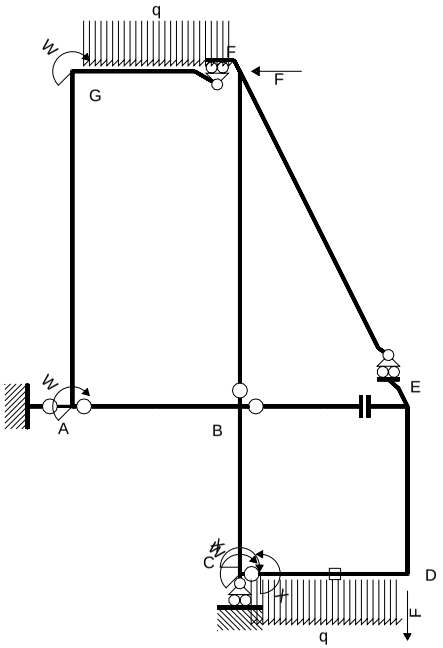
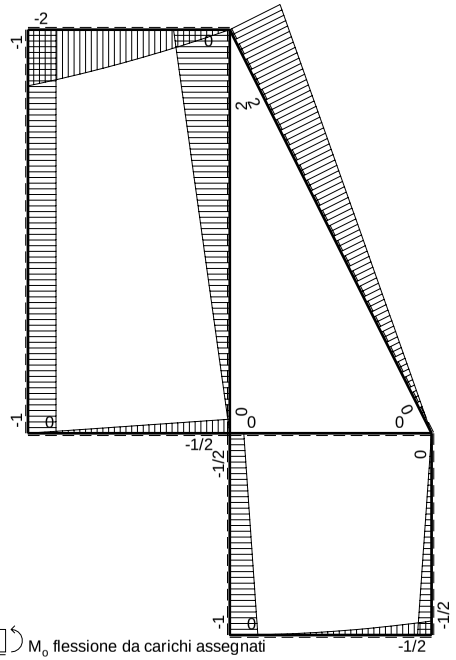
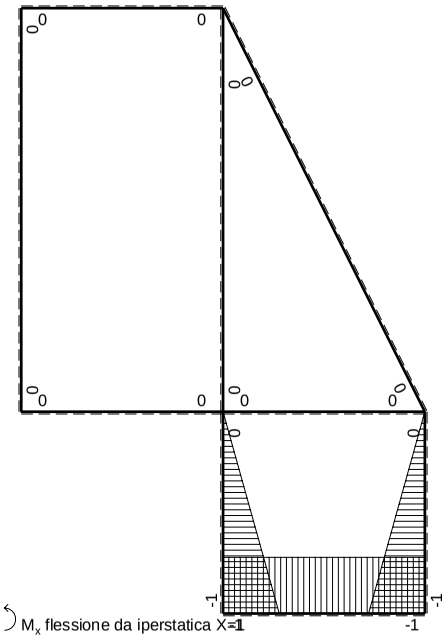


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb-1/2Fx	$1/2Fx+1/2Fx^2/b$	x^2/b^2	$5/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fb-1/2Fx	$Fb-3/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$-1/2qx^2$	$1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
DC b	1	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-1/2Fb+1/2Fx	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fx	$1/2Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-1/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				3/20Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -3/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -3/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -3/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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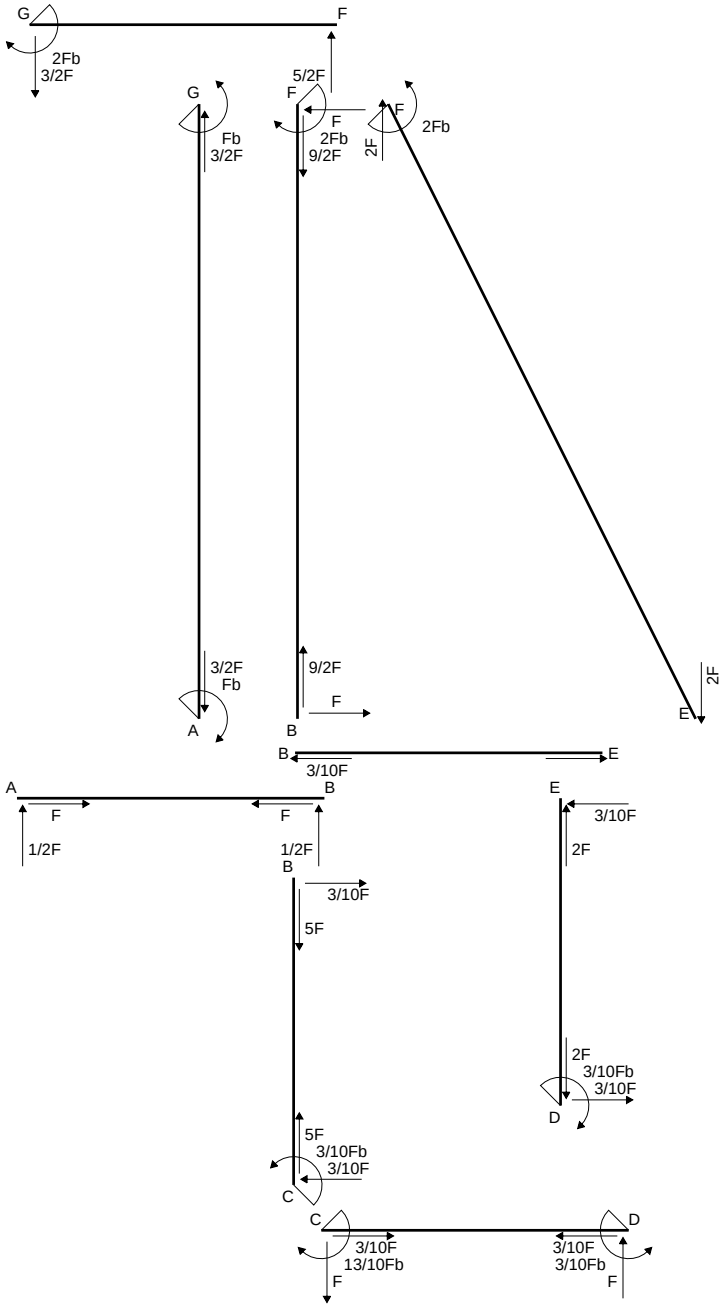
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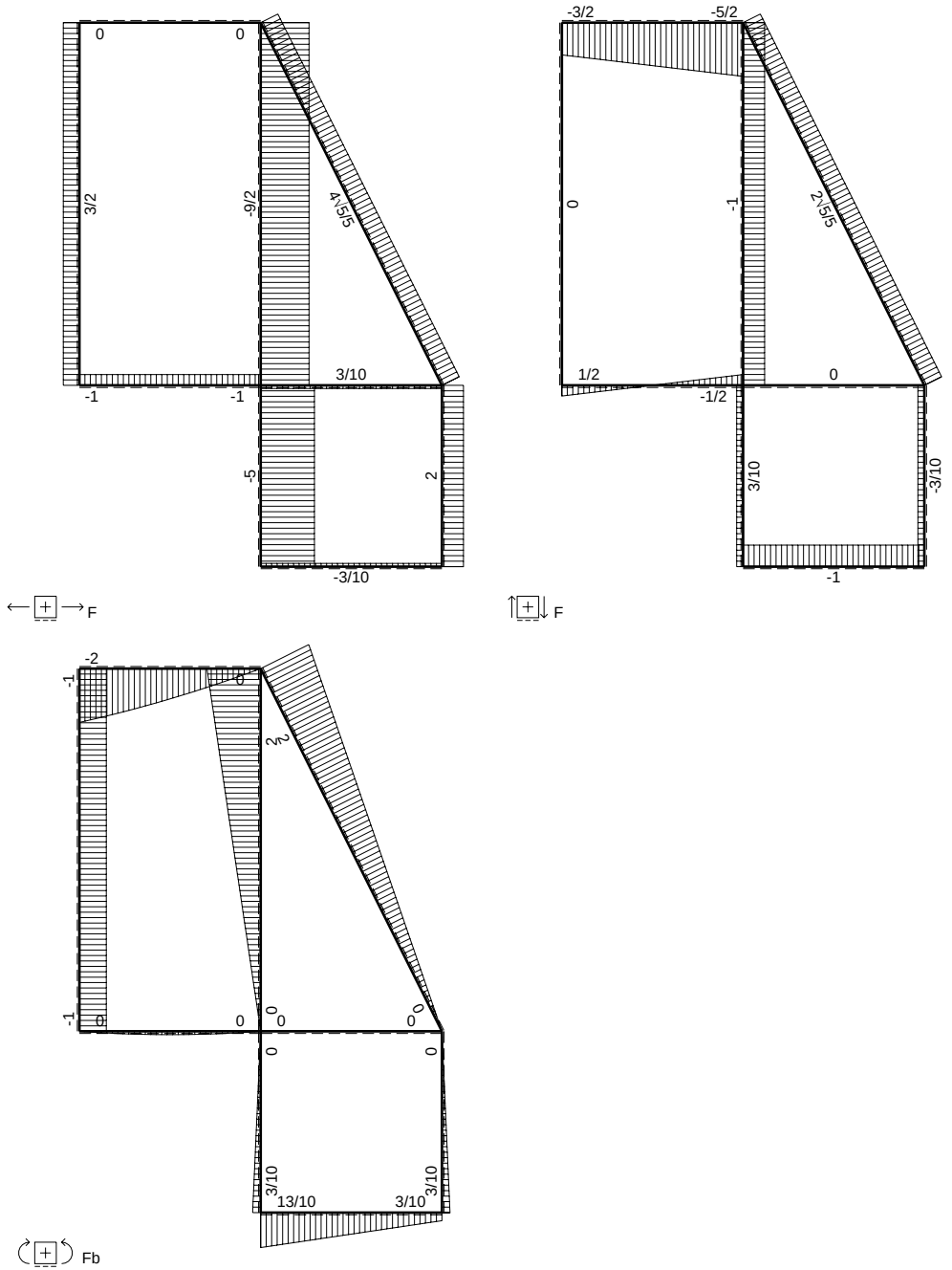
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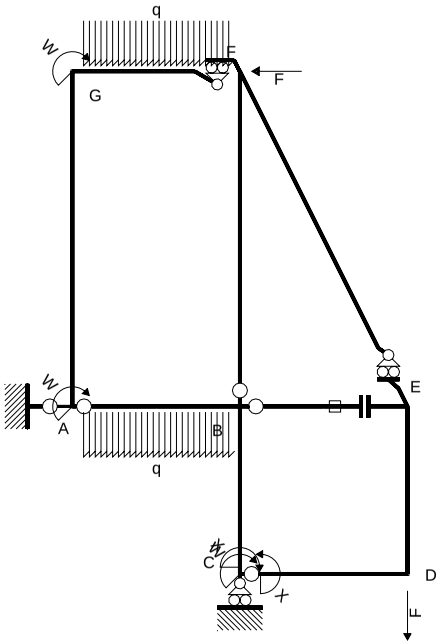
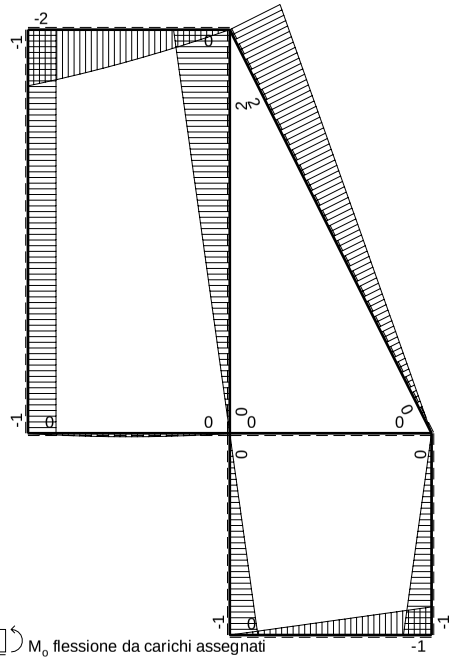
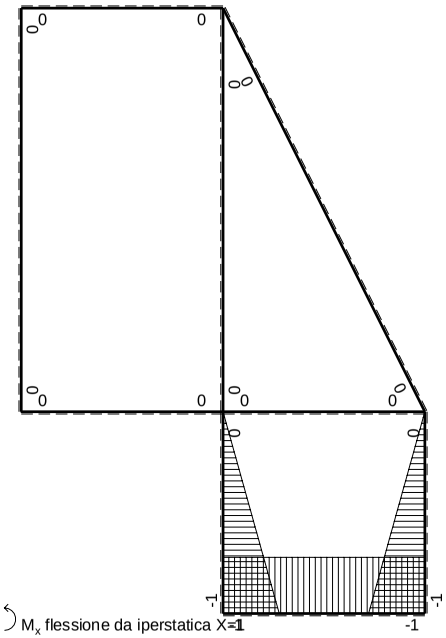


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$13/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-13/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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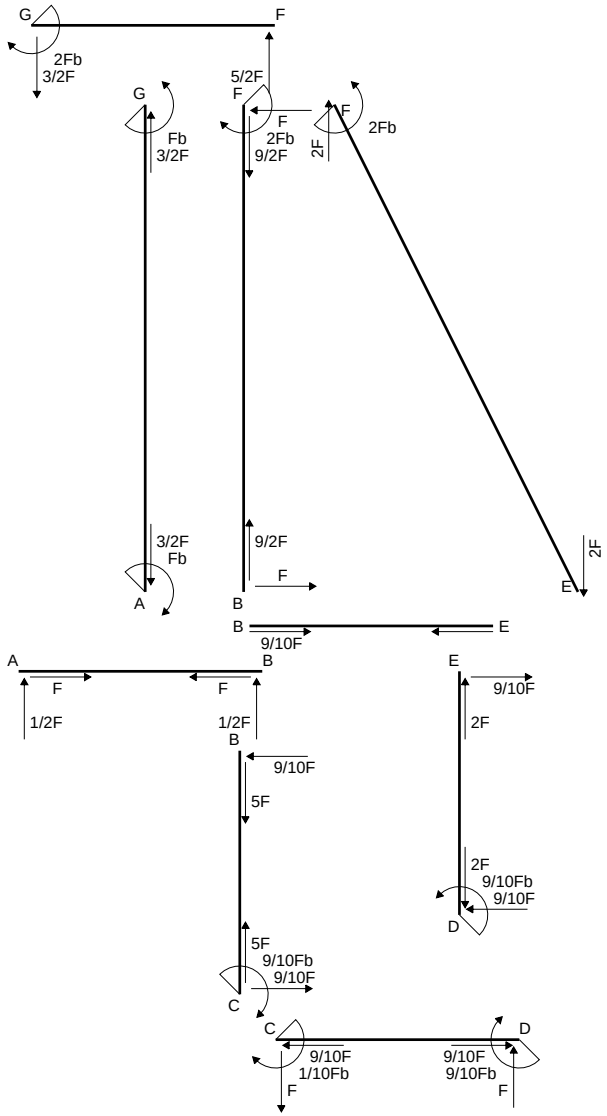
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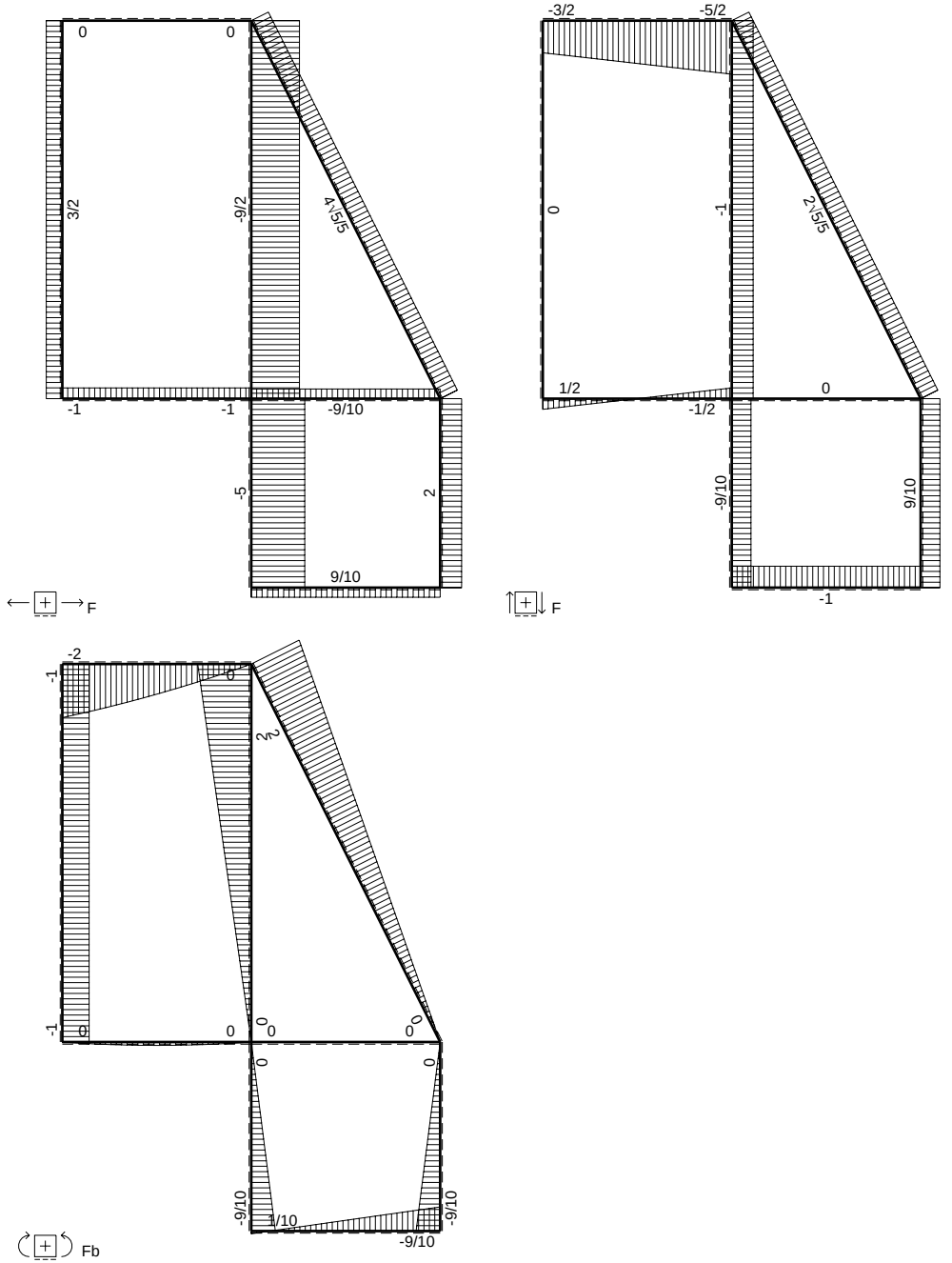
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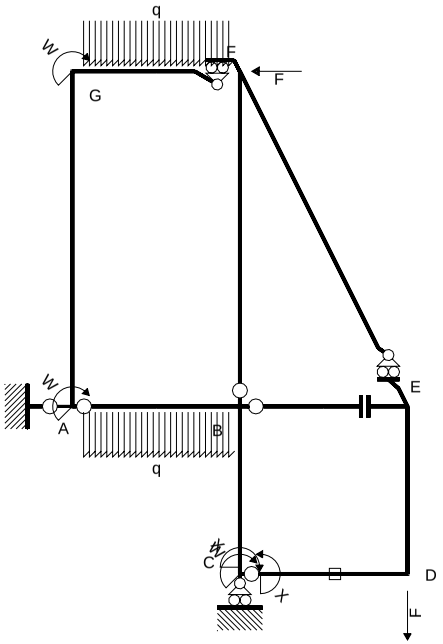
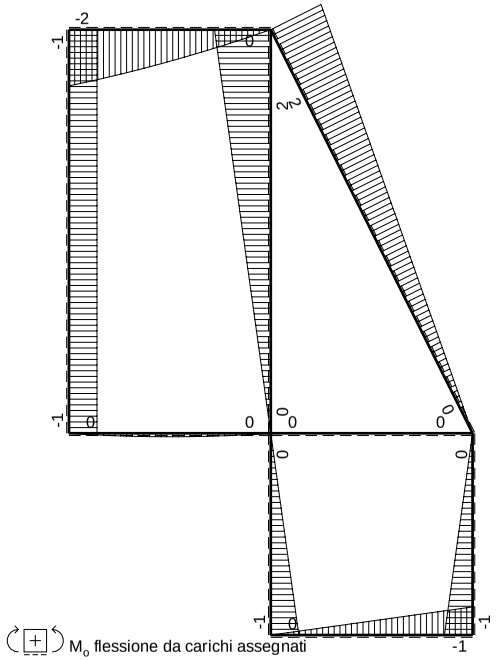
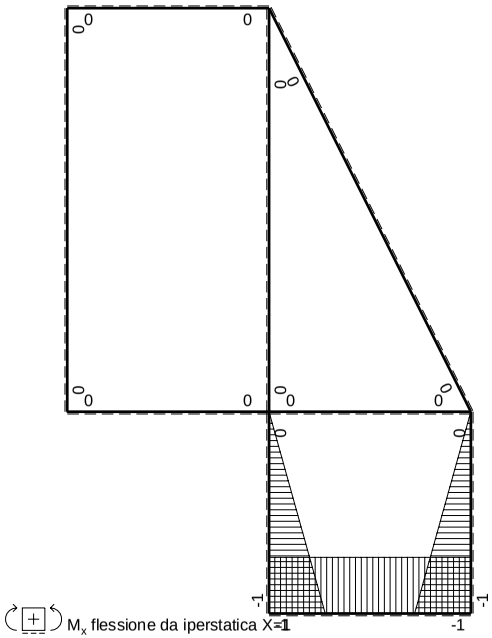


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-Fx	Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb+Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fx	Fx^2/b	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$1/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				-1/10Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1 -2\ x/b + x^2/b^2)\ Fb\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (\ b - b +1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x^2/b^2)\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/3\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

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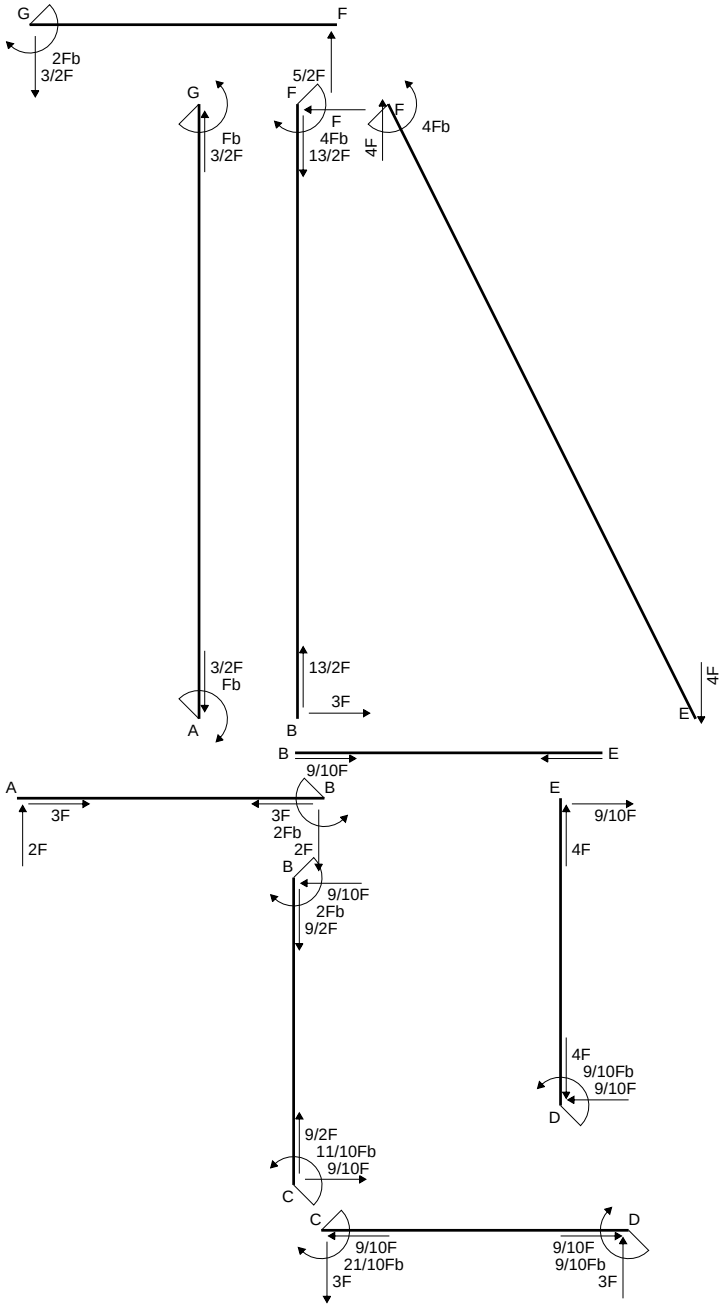
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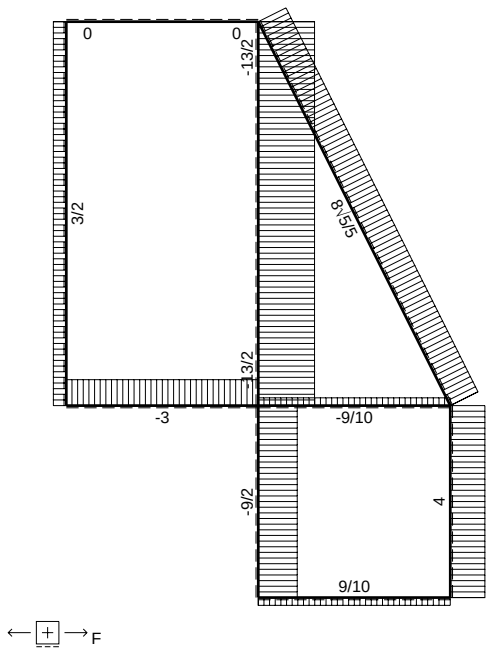
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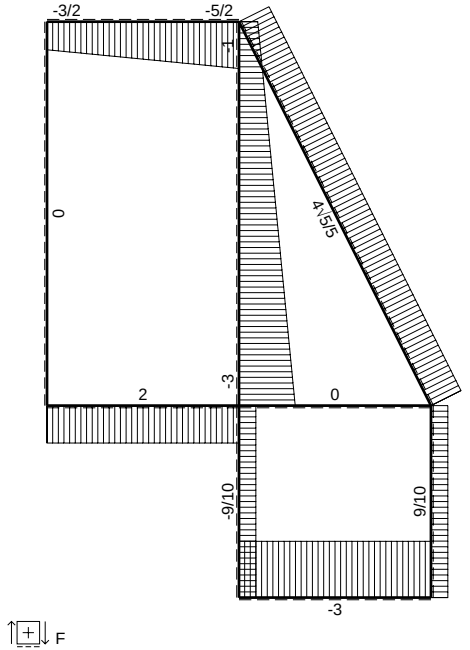
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4\/5/5
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-3
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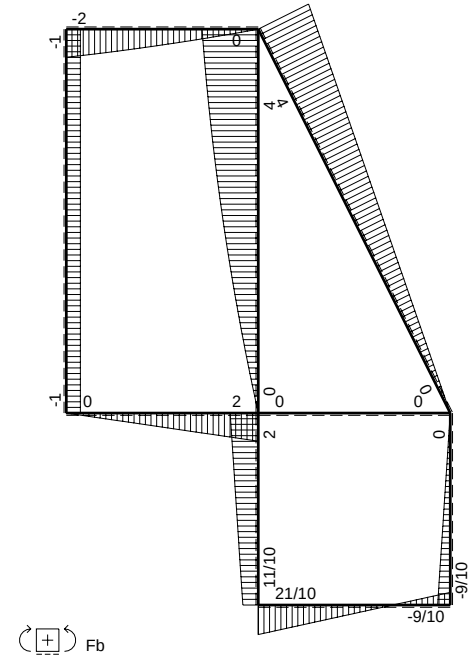
9/10
-3
F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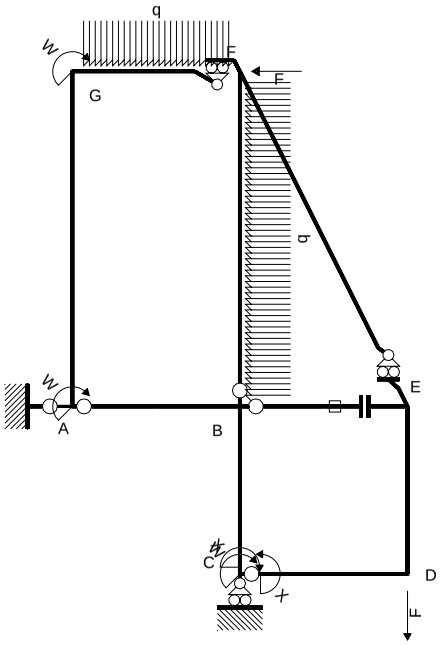
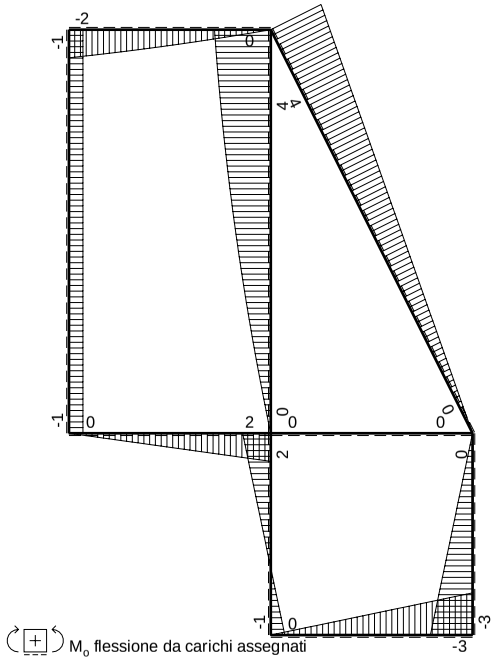
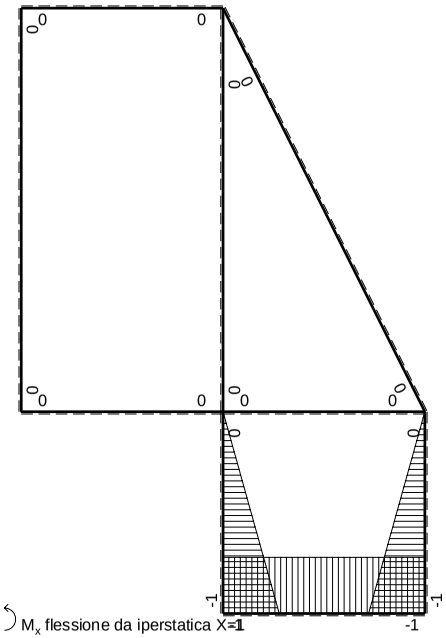
21/10
-9/10
-9/10
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	2Fx	0	0	0	0
BA b	0	-2Fb+2Fx	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$7/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-21/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (- b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ = 3/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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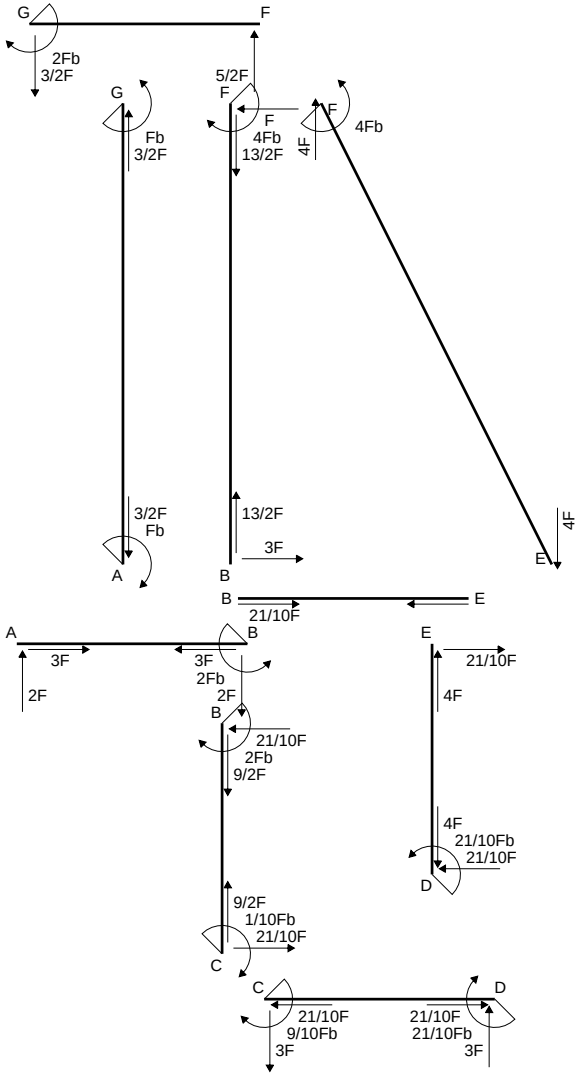
G
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4Fb
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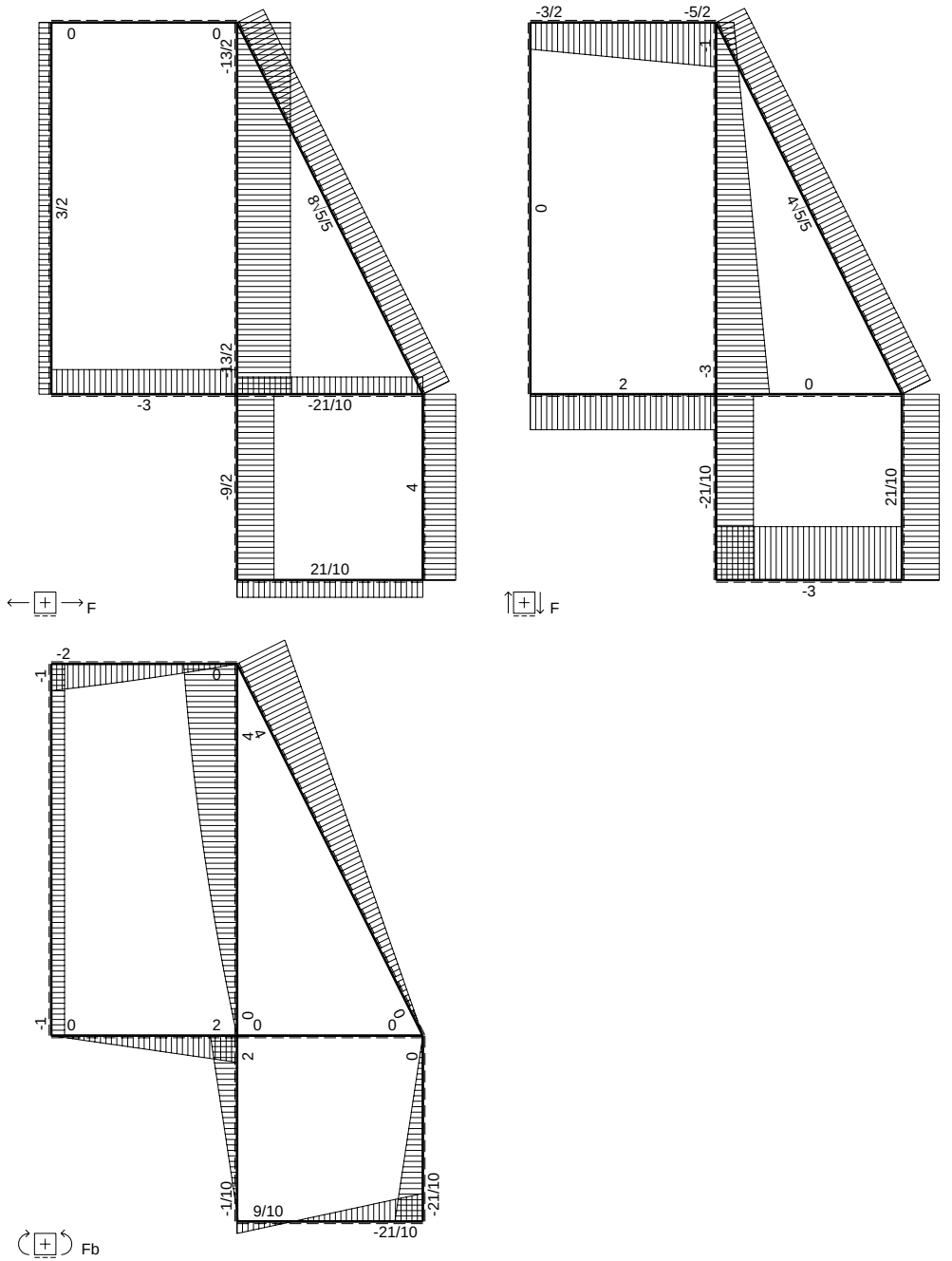
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0
4√5/5
2
-3
0
-21/10
21/10
-3
F
-2
-1
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-1/10
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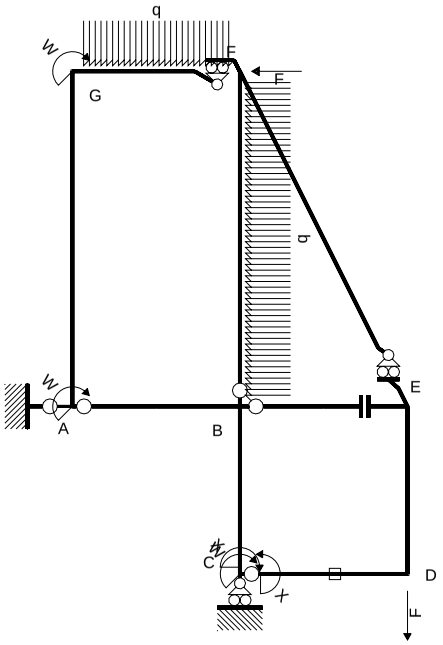
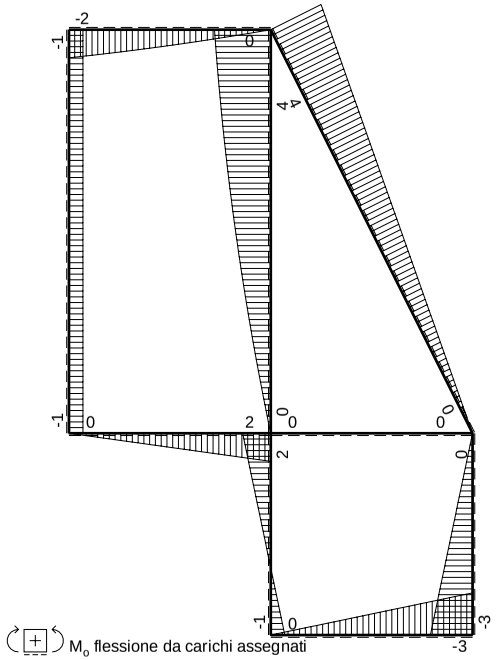
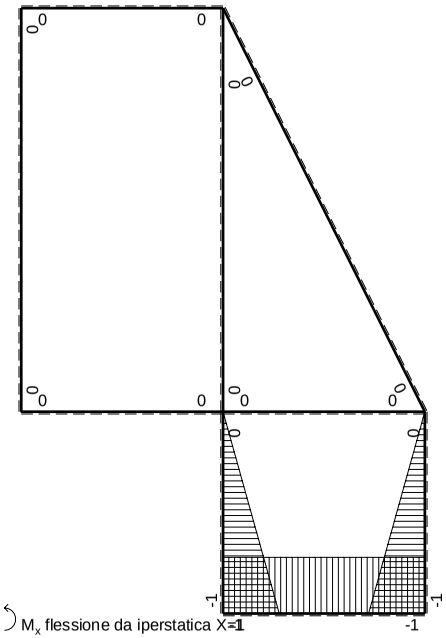
-21/10
-21/10
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_0(x)$	M_xM_0	M_xM_x	$\int M_xM_0/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	2Fx	0	0	0	0
BA b	0	-2Fb+2Fx	0	0		
BC b	-x/b	2Fb-3Fx	$-2Fx+3Fx^2/b$	x^2/b^2	0	1/3Xb/EJ
CB b	1-x/b	Fb-3Fx	$Fb-4Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	-3Fx	3Fx	1	$3/2Fb^2/EJ$	Xb/EJ
DC b	1	3Fb-3Fx	3Fb-3Fx	1		
DE b	-1+x/b	-3Fb+3Fx	$3Fb-6Fx+3Fx^2/b$	$1-2x/b+x^2/b^2$	Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3Fx	$3Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-9/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CB} = \int_o^b (1 -4\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{CD} = \int_o^b (3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3 -3\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [3\ x -3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3\ b -3/2\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (3 -6\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3\ x -3\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3\ b -3\ b + b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ )\ Fb\ 1/EJ\ =\ Fb^2/EJ$

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