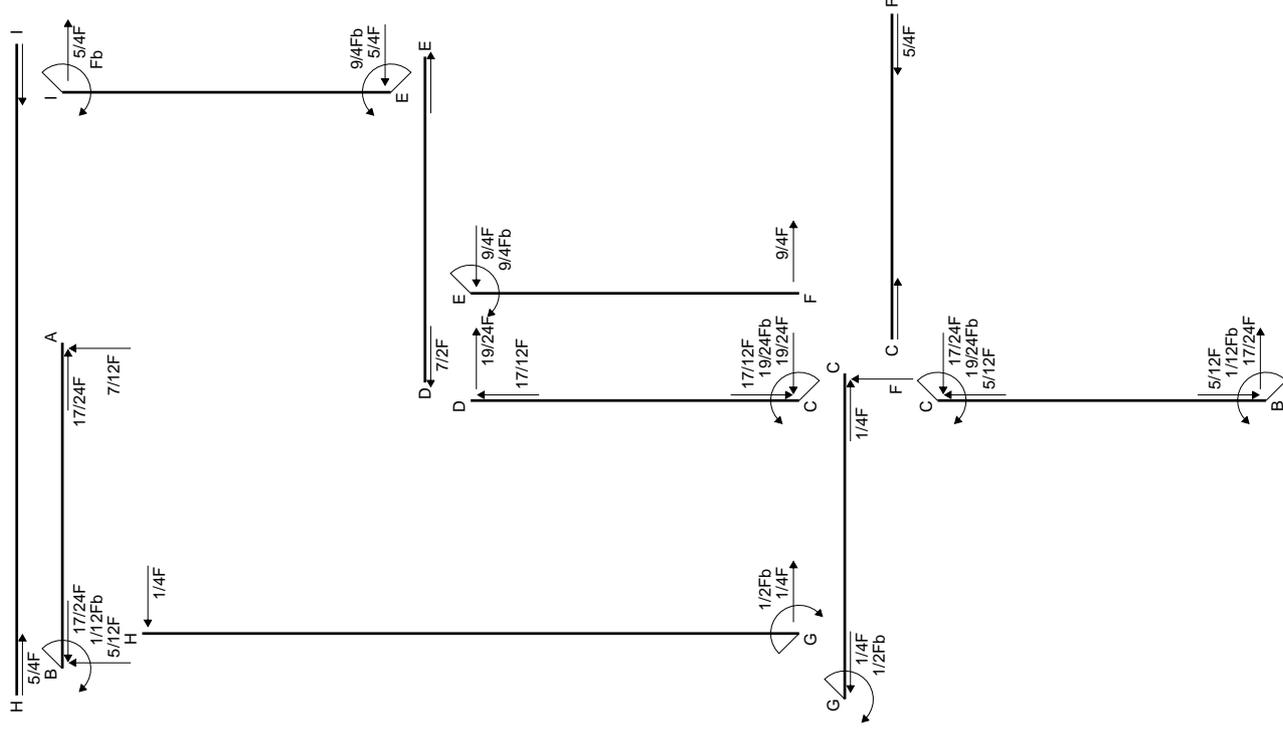


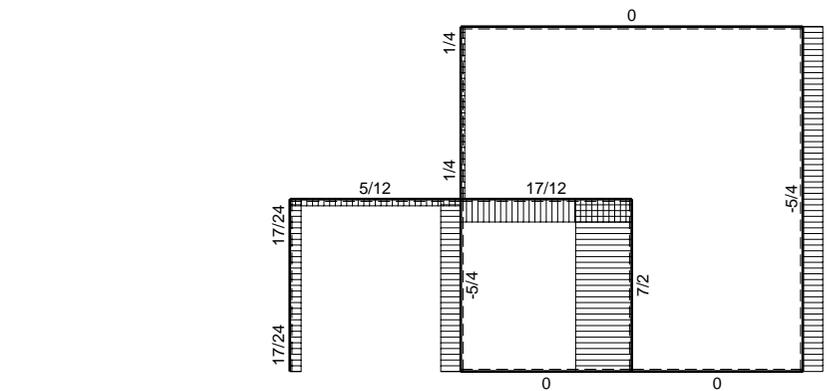
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{AB} &= -q = +F/b \\
 P_{CG} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{AB} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

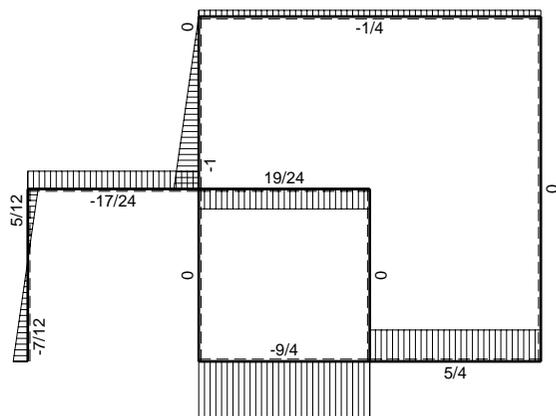
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=V_D$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y .
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta AB positiva se convessa a destra con inizio A.
 Spostamento orizzontale assoluto u imposto al nodo D.

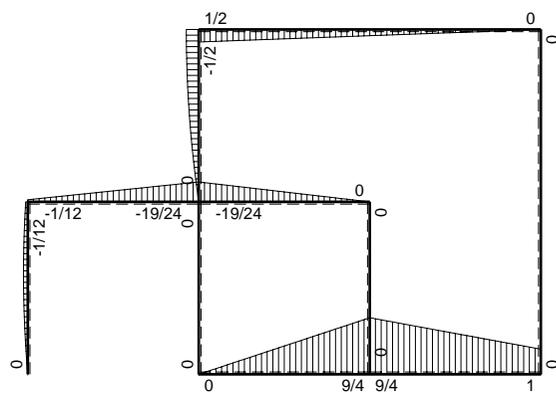




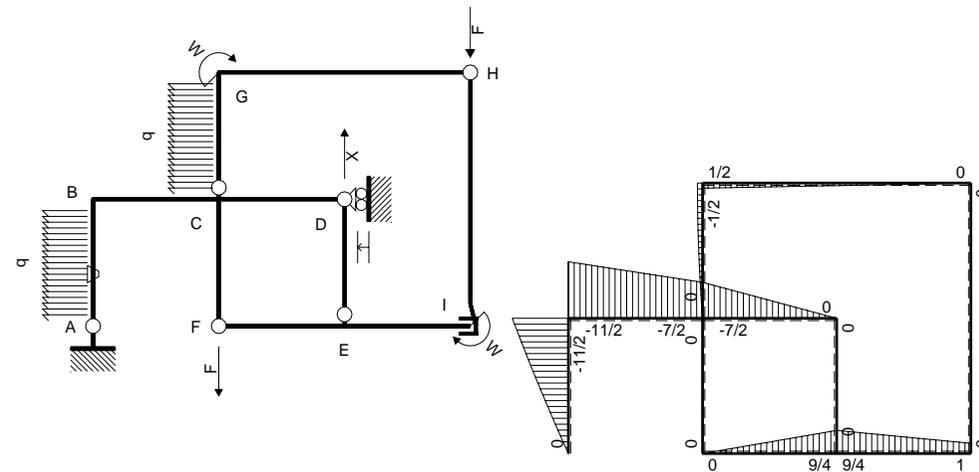
← (+) → F



↑ (+) ↓ F

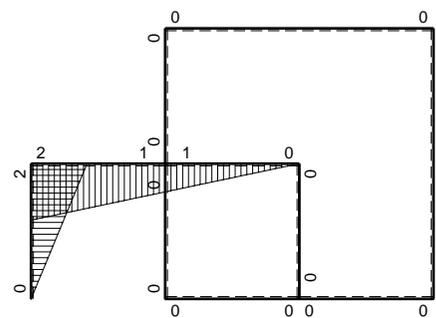


⤵ (+) ⤴ F_b



Schema di calcolo iperstatico

⤵ (+) ⤴ M_o flessione da carichi assegnati



⤵ (+) ⤴ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_D$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	2x	-6Fx+1/2qx ²	-Fb/EJ	-12Fx ² +qx ³	-2Fxb/EJ	4x ²	(-15/4-1)Fb ³ /EJ	4/3Xb ³ /EJ	
BA b	-2b+2x	11/2Fb-5Fx-1/2qx ²	Fb/EJ	-11Fb ² +21Fbx-9Fx ² -qx ³	-2Fb ² /EJ+2Fxb/EJ	4b ² -8bx+4x ²			
BC b	2b-x	-11/2Fb+2Fx	0	-11Fb ² +19/2Fbx-2Fx ²	0	4b ² -4bx+x ²	(-83/12+0)Fb ³ /EJ	7/3Xb ³ /EJ	
CB b	-b-x	7/2Fb+2Fx	0	-7/2Fb ² -11/2Fbx-2Fx ²	0	b ² +2bx+x ²			
CD b	b-x	-7/2Fb+7/2Fx	0	-7/2Fb ² +7Fbx-7/2Fx ²	0	b ² -2bx+x ²	(-7/6+0)Fb ³ /EJ	1/3Xb ³ /EJ	
DC b	-x	7/2Fx	0	-7/2Fx ²	0	x ²			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0	
FE b	0	-9/4Fx	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	-Fx+1/2qx ²	0	0	0	0	0+0	0	
GC b	0	1/2Fb-1/2qx ²	0	0	0	0			
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0	
HG 2b	0	-1/4Fx	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0	
EI b	0	-9/4Fb+5/4Fx	0	0	0	0			
D	cedimento nodo -H _{1D} u _D							2Fb ³ /EJ	
	totali							-65/6Fb ³ /EJ	4Xb ³ /EJ
	iperstatica X=V _D							65/24F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = [4 x - 4 x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = [4 x - 2 x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-12 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-2 x/b) \theta dx$$

$$= [-4 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ + [-x^2/b]_0^b \theta$$

$$= (-4 b + 1/4 b) Fb^2 1/EJ + (-b) \theta = -19/4 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-11 + 21 x/b - 9 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx + \int_0^b (2 - 2 x/b) \theta dx$$

$$= [-11 x + 21/2 x^2/b - 3 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ + [2 x - x^2/b]_0^b \theta$$

$$= (-11 b + 21/2 b - 3 b - 1/4 b) Fb^2 1/EJ + (2 b - b) \theta = -19/4 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-11 + 19/2 x/b - 2 x^2/b^2) Fb^2 1/EJ dx = [-11 x + 19/4 x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-11 b + 19/4 b - 2/3 b) Fb^2 1/EJ = -83/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-7/2 - 11/2 x/b - 2 x^2/b^2) Fb^2 1/EJ dx = [-7/2 x - 11/4 x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

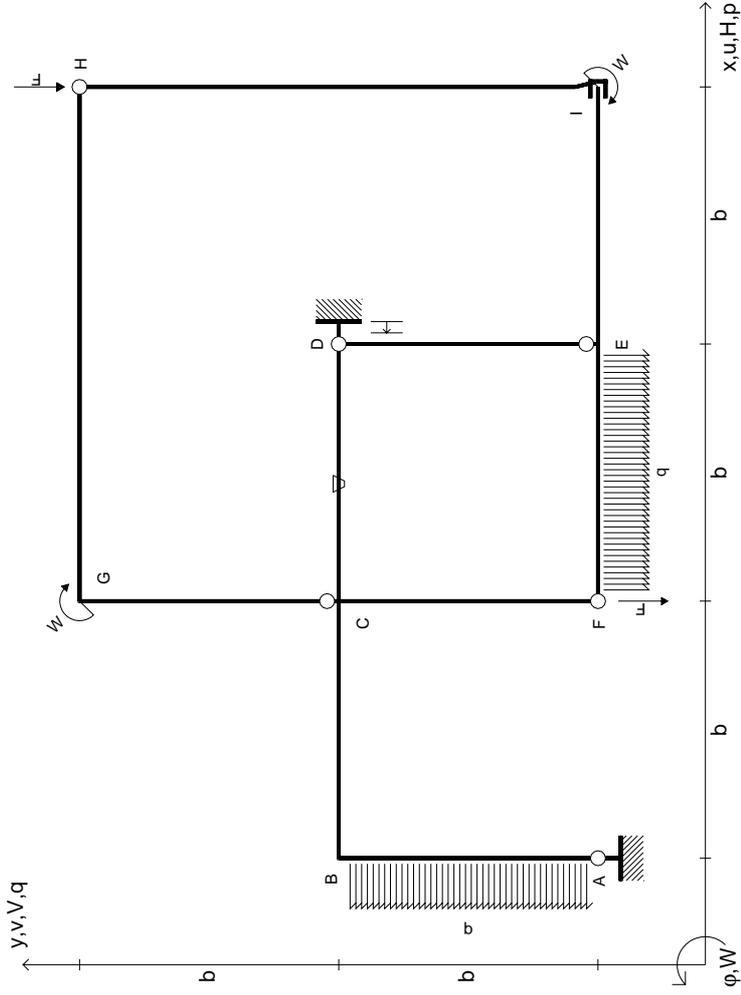
$$= (-7/2 b - 11/4 b - 2/3 b) Fb^2 1/EJ = -83/12 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (-7/2 + 7 x/b - 7/2 x^2/b^2) Fb^2 1/EJ dx = [-7/2 x + 7/2 x^2/b - 7/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-7/2 b + 7/2 b - 7/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (-7/2 x^2/b^2) Fb^2 1/EJ dx = [-7/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-7/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$

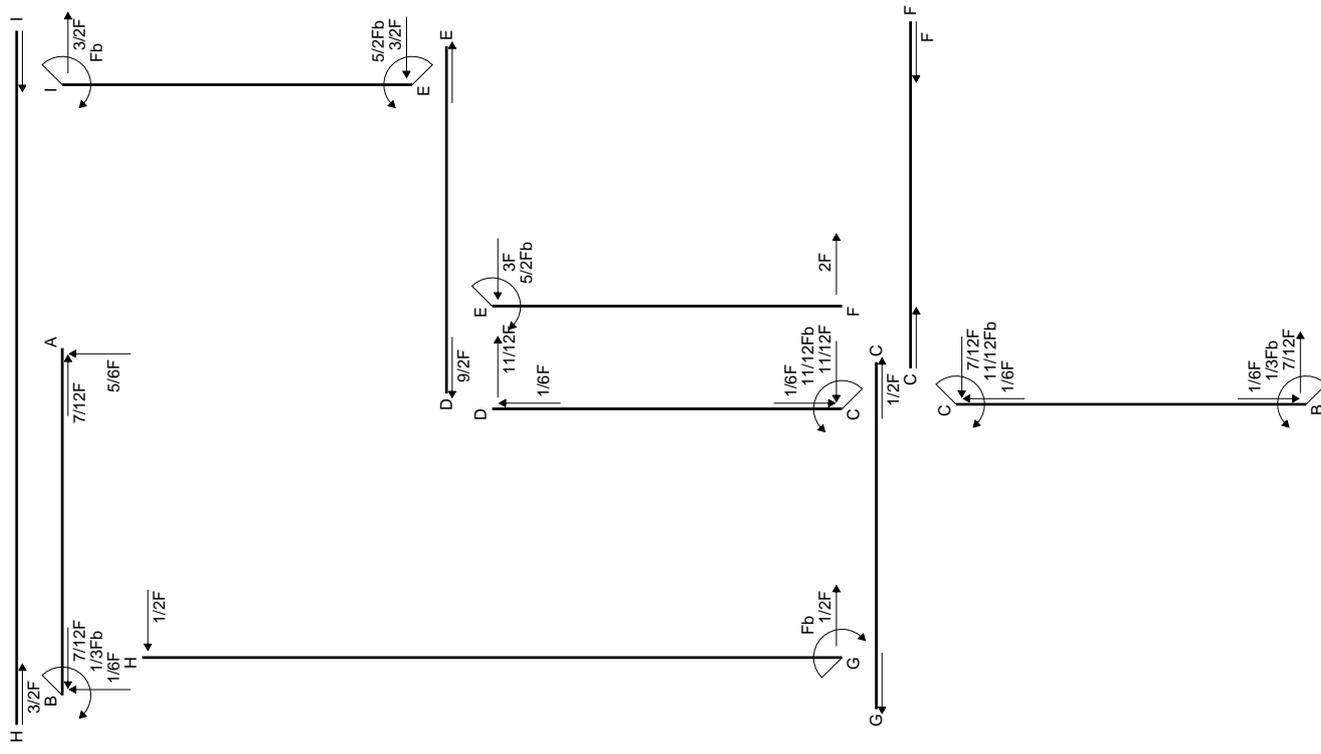


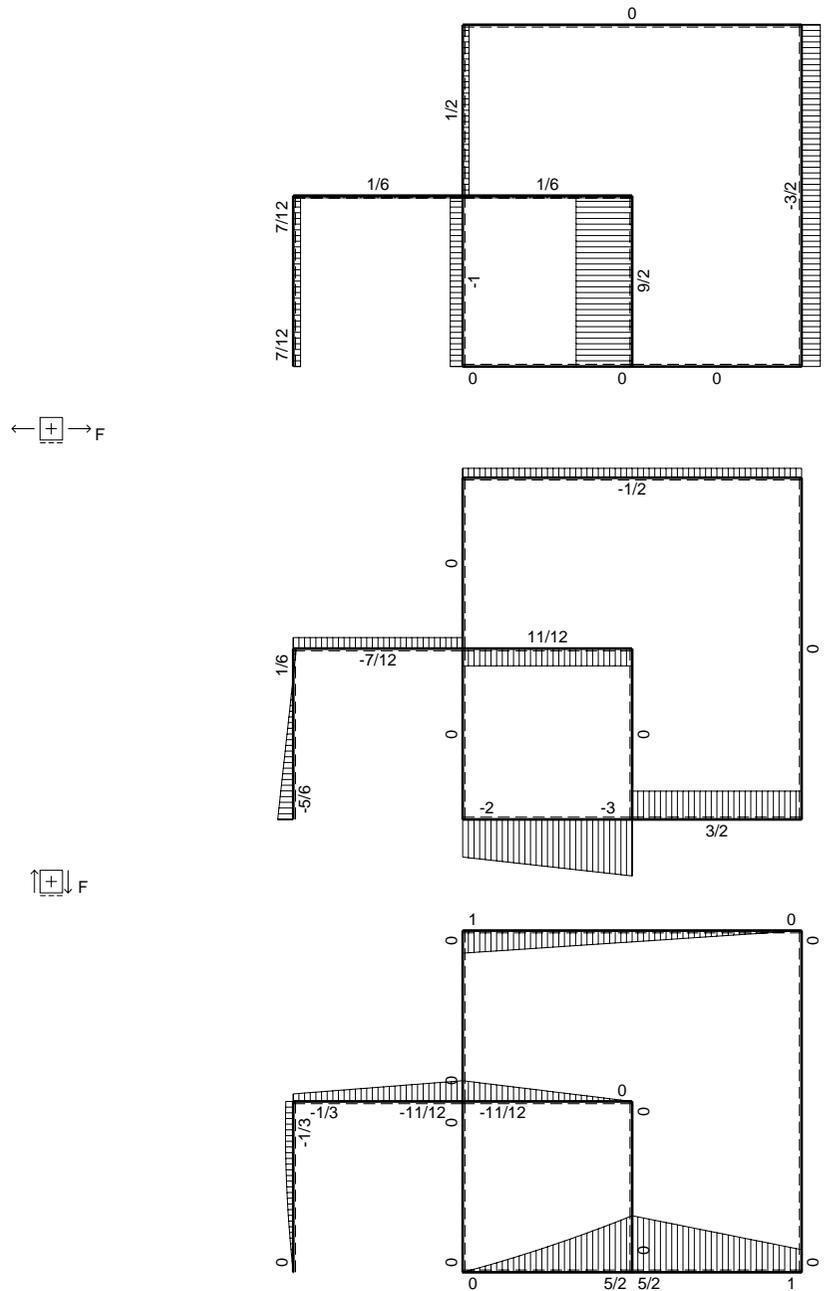
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{AB} &= -q = -F/b \\
 q_{EF} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{CD} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E J_{AB} &= EJ \\
 E J_{BC} &= EJ \\
 E J_{CD} &= EJ \\
 E J_{DE} &= EJ
 \end{aligned}$$

$$\begin{aligned}
 E J_{EF} &= EJ \\
 E J_{FC} &= EJ \\
 E J_{CG} &= EJ \\
 E J_{GH} &= EJ \\
 E J_{HI} &= EJ \\
 E J_{IE} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta CD positiva se convessa a destra con inizio C.
 Spostamento orizzontale assoluto u imposto al nodo D.

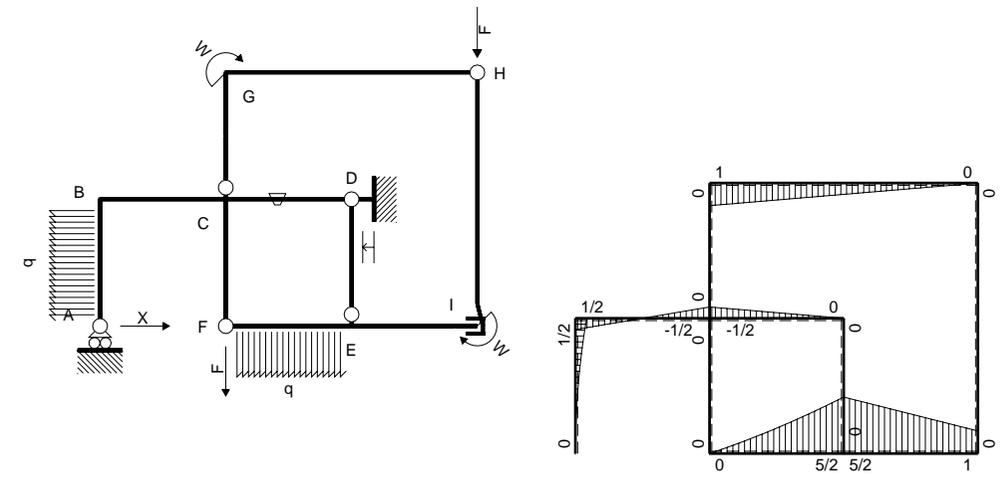




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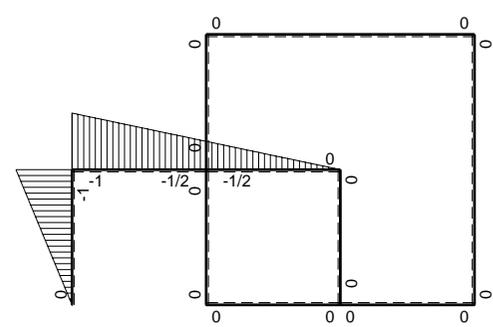
↑ (+) ↓ F

⊙ (+) ⊙ F_b



Schema di calcolo iperstatico

⊙ (+) ⊙ M_o flessione da carichi assegnati



⊙ (+) ⊙ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$			
BC b	-b+1/2x	$1/2Fb-Fx$	0	$-1/2Fb^2+5/4Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(-1/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
CB b	$1/2b+1/2x$	$1/2Fb-Fx$	0	$1/4Fb^2-1/4Fbx-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	$-Fb/EJ$	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$	
DC b	$1/2x$	$1/2Fx$	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	0	0	0	0	0	0+0	0	
GC b	0	0	0	0	0	0			
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0	
HG 2b	0	$-1/2Fx$	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0	
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0			
D	cedimento nodo $-H_{1D}u_D$							$-Fb^3/EJ$	
	totali							$-5/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$							$5/6F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-1/2 x^3/b^3) Fb^2 1/EJ dx = [-1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [-1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + 5/4 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 5/8 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 5/8 b - 1/6 b) Fb^2 1/EJ = -1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/8 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

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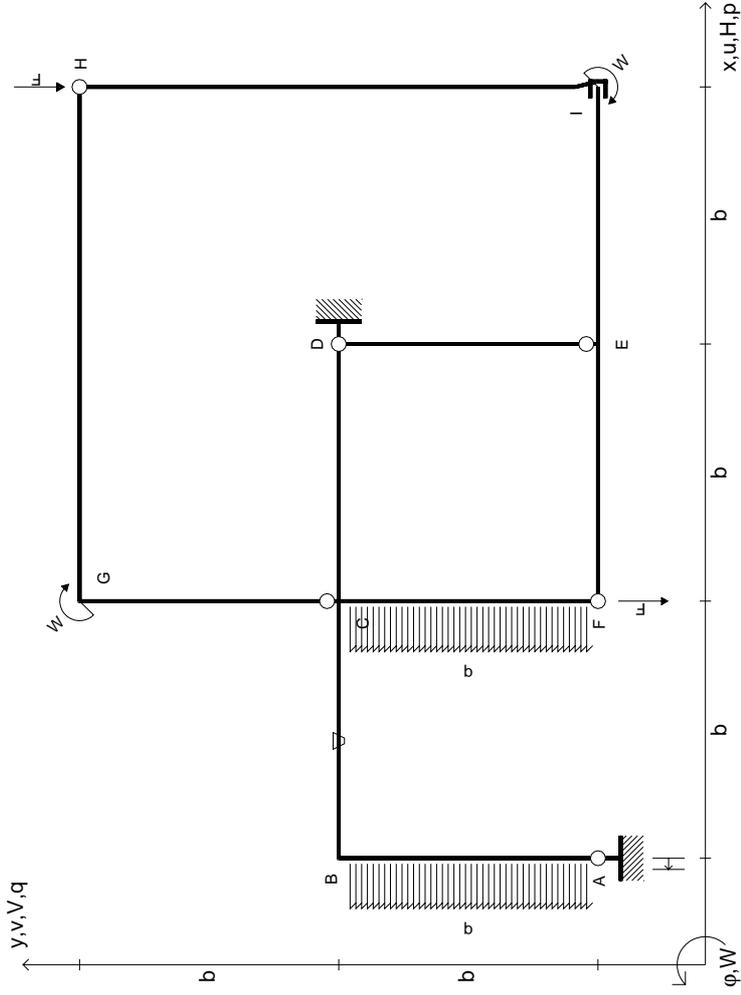
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb^2 1/EJ + (-1/4 b) \theta = 1/3 Fb^3/EJ$$

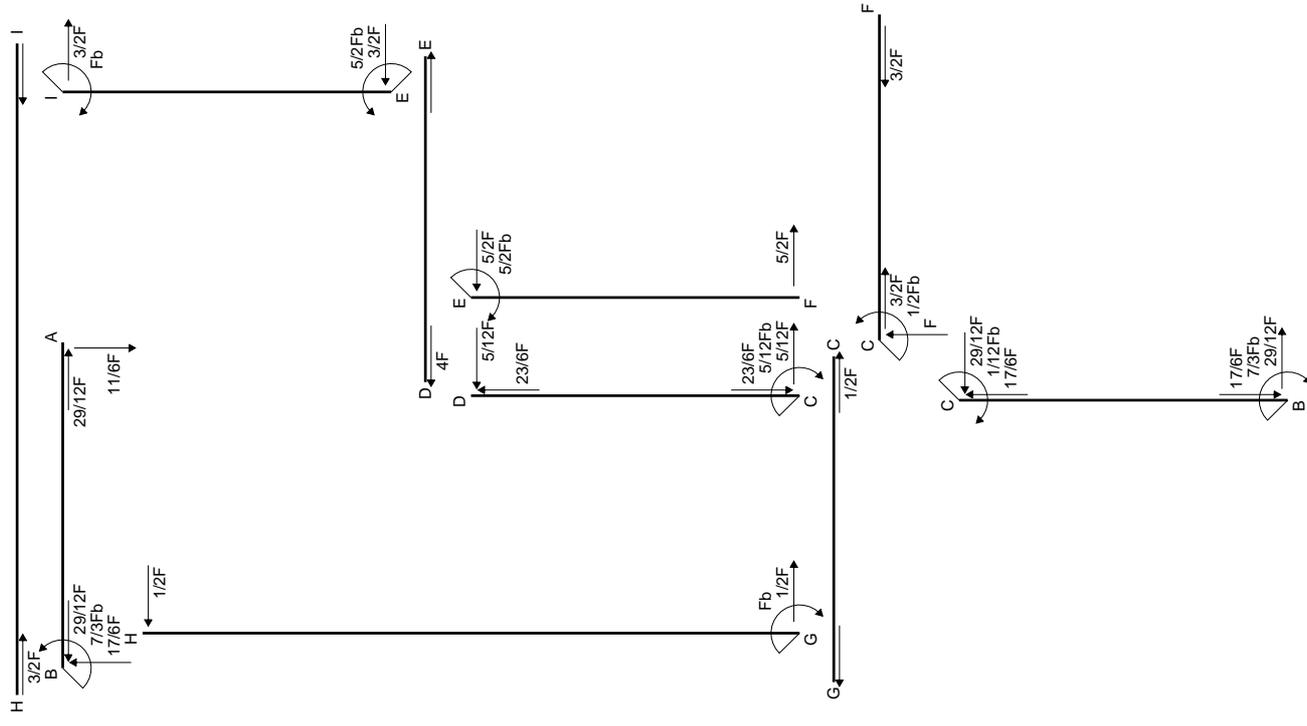


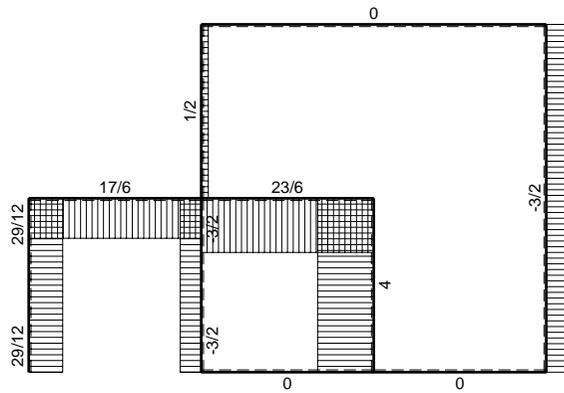
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 P_{FC} &= -q = -F/b
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$$\begin{aligned}
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 u_A &= -\delta = -b^3F/EJ \\
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 E_{J_{BC}} &= EJ \\
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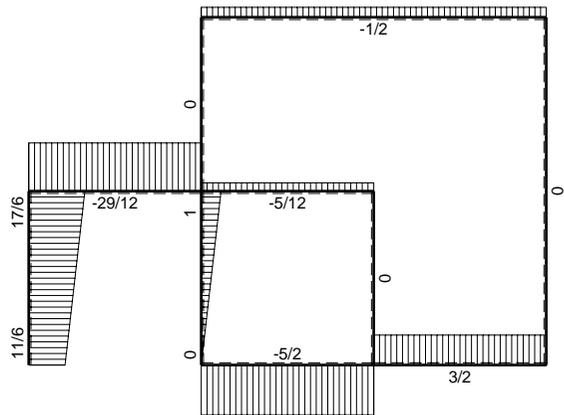
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 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo A.

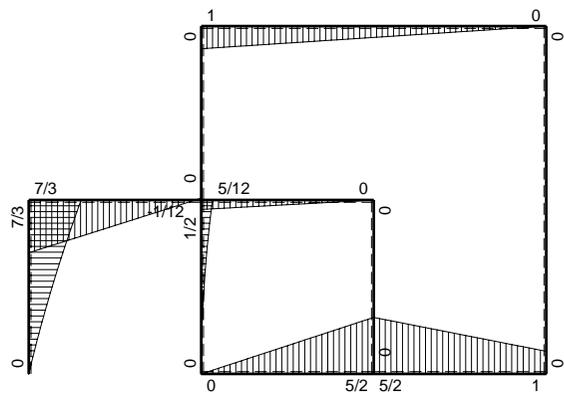




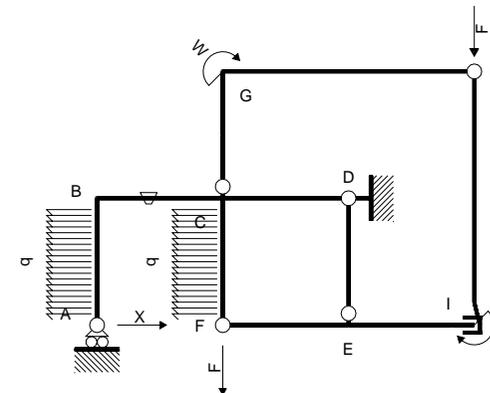
← (+) → F



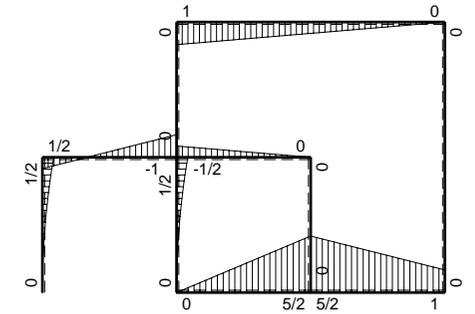
↑ (+) ↓ F



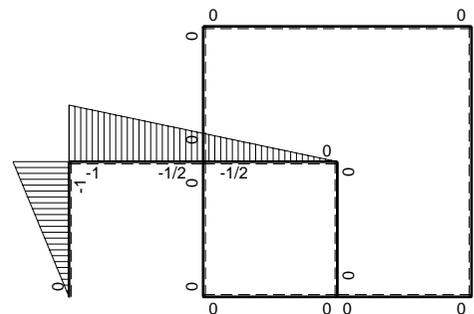
⌚ (+) ⌚ F_b



Schema di calcolo iperstatico



⌚ (+) ⌚ M_o flessione da carichi assegnati



⌚ (+) ⌚ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_A

→	M _x (x)	M ₀ (x)	θ	M _x M ₀	M _x θ	M _x M _x	∫M _x (M ₀ /EJ+θ)dx	∫XM _x M _x /EJdx
AB b	-x	1/2qx ²	0	-1/2qx ³	0	x ²	(-1/8+0)Fb ³ /EJ	1/3Xb ³ /EJ
BA b	b-x	-1/2Fb+Fx-1/2qx ²	0	-1/2Fb ² +3/2Fbx-3/2Fx ² +1/2qx ³	0	b ² -2bx+x ²		
BC b	-b+1/2x	1/2Fb-3/2Fx	-Fb/EJ	-1/2Fb ² +7/4Fbx-3/4Fx ²	Fb ² /EJ-1/2Fxb/EJ	b ² -bx+1/4x ²	(1/8+3/4)Fb ³ /EJ	7/12Xb ³ /EJ
CB b	1/2b+1/2x	Fb-3/2Fx	Fb/EJ	1/2Fb ² -1/4Fbx-3/4Fx ²	1/2Fb ² /EJ+1/2Fxb/EJ	1/4b ² +1/2bx+1/4x ²		
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb ² -1/2Fbx+1/4Fx ²	0	1/4b ² -1/2bx+1/4x ²	(1/12+0)Fb ³ /EJ	1/12Xb ³ /EJ
DC b	1/2x	1/2Fx	0	1/4Fx ²	0	1/4x ²		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	5/2Fb-5/2Fx	0	0	0	0	0+0	0
FE b	0	-5/2Fx	0	0	0	0		
FC b	0	1/2qx ²	0	0	0	0	0+0	0
CF b	0	-1/2Fb+Fx-1/2qx ²	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-5/2Fb+3/2Fx	0	0	0	0		
A	cedimento nodo -H _{1A} u _A						Fb ³ /EJ	
	totali						11/6Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _A						-11/6F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(x^2/b^2 \right) b^2 1/EJ dx = \left[1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 1/EJ dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + 1/4 x^2/b^2 \right) b^2 1/EJ dx = \left[x - 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(1/4 + 1/2 x/b + 1/4 x^2/b^2 \right) b^2 1/EJ dx = \left[1/4 x + 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(1/4 - 1/2 x/b + 1/4 x^2/b^2 \right) b^2 1/EJ dx = \left[1/4 x - 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(1/4 x^2/b^2 \right) b^2 1/EJ dx = \left[1/12 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left(-1/2 x^3/b^3 \right) Fb^2 1/EJ dx = \left[-1/8 x^4/b^3 \right]_0^b Fb^2 1/EJ$$

$$= (-1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left(-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3 \right) Fb^2 1/EJ dx$$

$$= \left[-1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3 \right]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(-1/2 + 7/4 x/b - 3/4 x^2/b^2 \right) Fb^2 1/EJ dx + \int_0^b \left(1 - 1/2 x/b \right) \theta dx$$

$$= \left[-1/2 x + 7/8 x^2/b - 1/4 x^3/b^2 \right]_0^b Fb^2 1/EJ + \left[x - 1/4 x^2/b \right]_0^b \theta$$

$$= (-1/2 b + 7/8 b - 1/4 b) Fb^2 1/EJ + (b - 1/4 b) \theta = 7/8 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(1/2 - 1/4 x/b - 3/4 x^2/b^2 \right) Fb^2 1/EJ dx + \int_0^b \left(-1/2 - 1/2 x/b \right) \theta dx$$

$$= \left[1/2 x - 1/8 x^2/b - 1/4 x^3/b^2 \right]_0^b Fb^2 1/EJ + \left[-1/2 x - 1/4 x^2/b \right]_0^b \theta$$

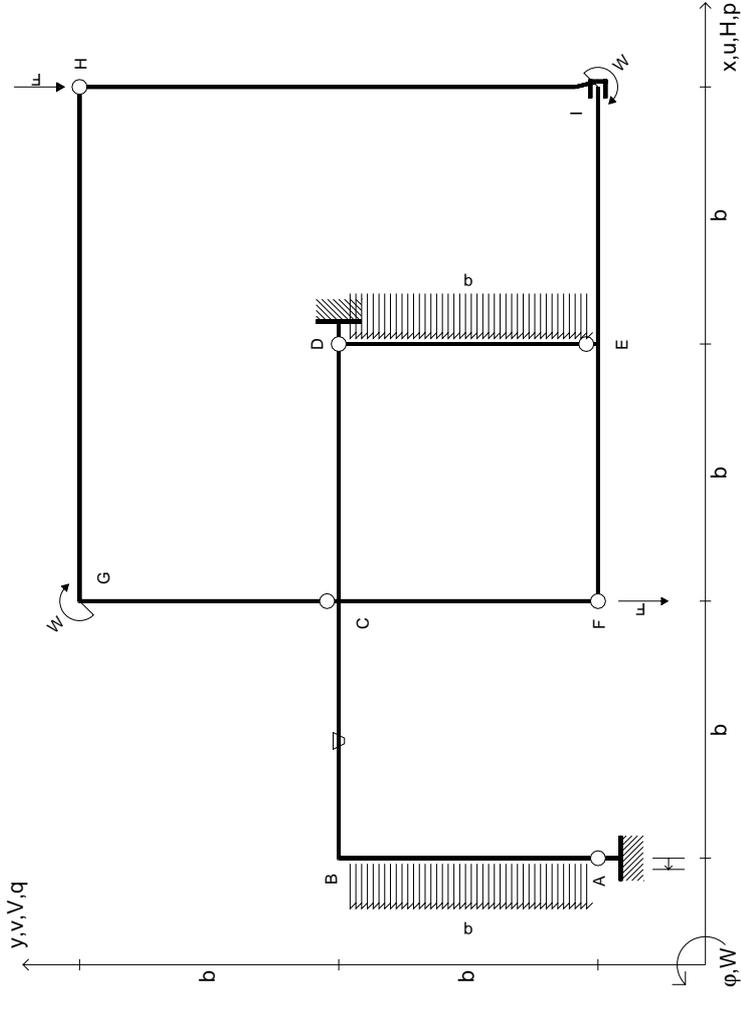
$$= (1/2 b - 1/8 b - 1/4 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = 7/8 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left(1/4 - 1/2 x/b + 1/4 x^2/b^2 \right) Fb^2 1/EJ dx = \left[1/4 x - 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(1/4 x^2/b^2 \right) Fb^2 1/EJ dx = \left[1/12 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

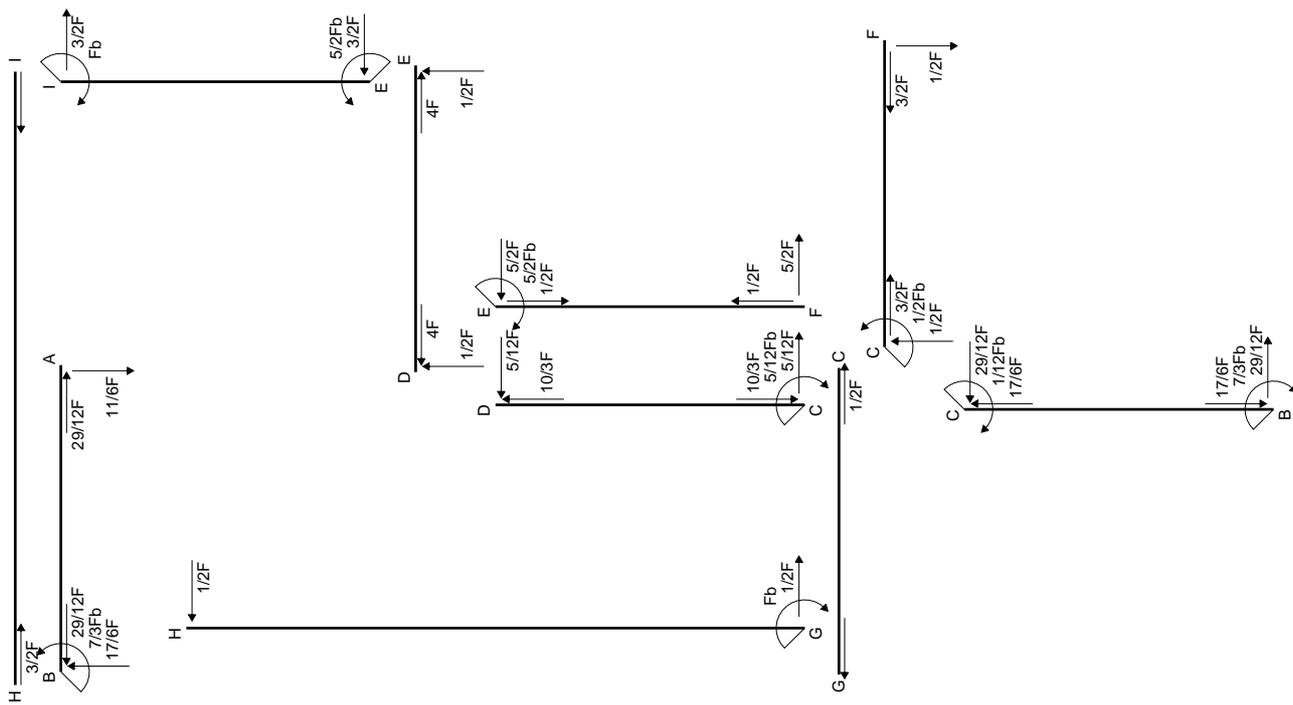


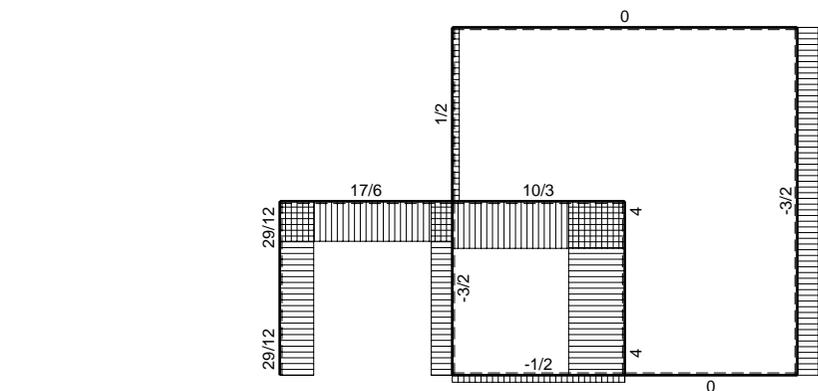
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_1 &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{AB} &= -q = +F/b \\
 P_{DE} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_A &= -\delta = -b^3 F/EJ \\
 EJ_{AB} &= EJ \\
 EJ_{BC} &= EJ \\
 EJ_{CD} &= EJ \\
 EJ_{DE} &= EJ
 \end{aligned}$$

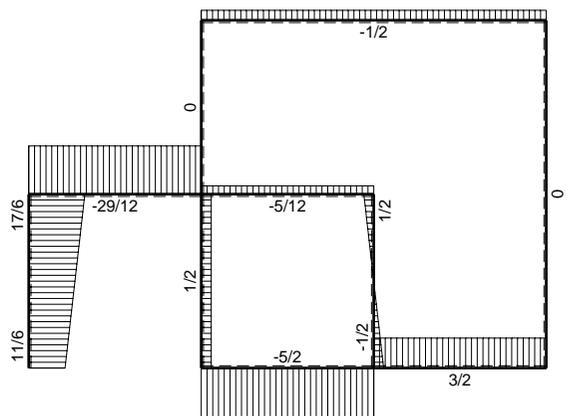
$$\begin{aligned}
 EJ_{EF} &= EJ \\
 EJ_{FC} &= EJ \\
 EJ_{CG} &= EJ \\
 EJ_{GH} &= EJ \\
 EJ_{HI} &= EJ \\
 EJ_{IE} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=W_{BC}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo A.

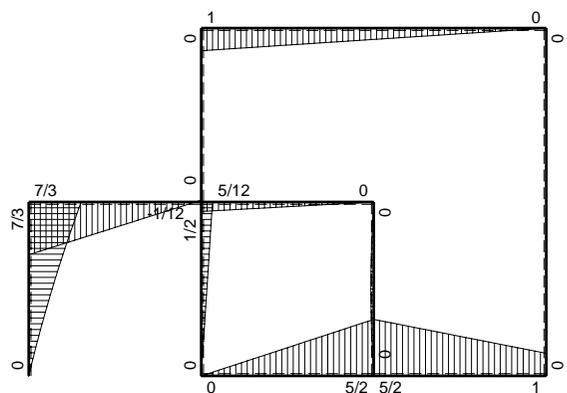




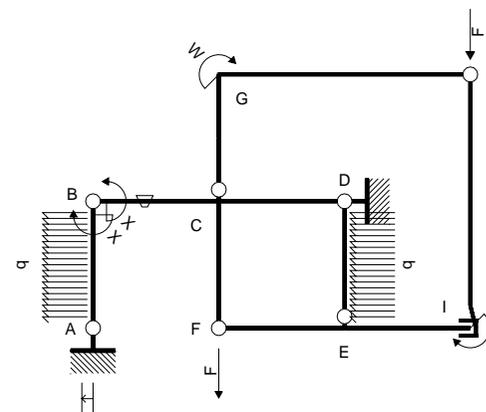
← (+) → F



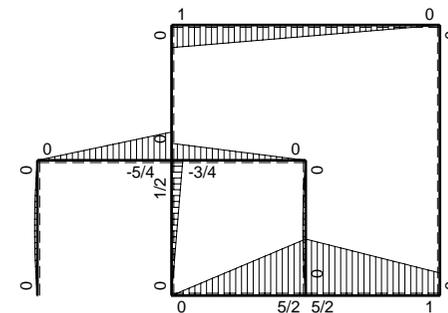
↑ (+) ↓ F



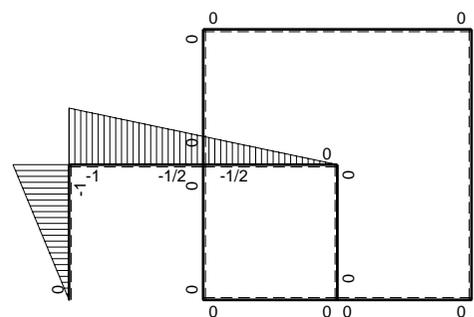
⤵ (+) ⤴ F_b



Schema di calcolo iperstatico



⤵ (+) ⤴ M₀ flessione da carichi assegnati



⤵ (+) ⤴ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$
AB b	$-x/b$	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	$1-x/b$	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
BC b	$-1+1/2x/b$	$-5/4Fx$	$-Fb/EJ$	$5/4Fx-5/8Fx^2/b$	$Fb/EJ-1/2Fx/EJ$	$1-x/b+1/4x^2/b^2$	$(5/12+3/4)Fb^2/EJ$	$7/12Xb/EJ$
CB b	$1/2+1/2x/b$	$5/4Fb-5/4Fx$	Fb/EJ	$5/8Fb-5/8Fx^2/b$	$1/2Fb/EJ+1/2Fx/EJ$	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	$-1/2+1/2x/b$	$-3/4Fb+3/4Fx$	0	$3/8Fb-3/4Fx+3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(1/8+0)Fb^2/EJ$	$1/12Xb/EJ$
DC b	$1/2x/b$	$3/4Fx$	0	$3/8Fx^2/b$	0	$1/4x^2/b^2$		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	$5/2Fb-5/2Fx$	0	0	0	0	0+0	0
FE b	0	$-5/2Fx$	0	0	0	0		
FC b	0	$1/2Fx$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^2/EJ	
	totali						$7/3Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						$-7/3Fb$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

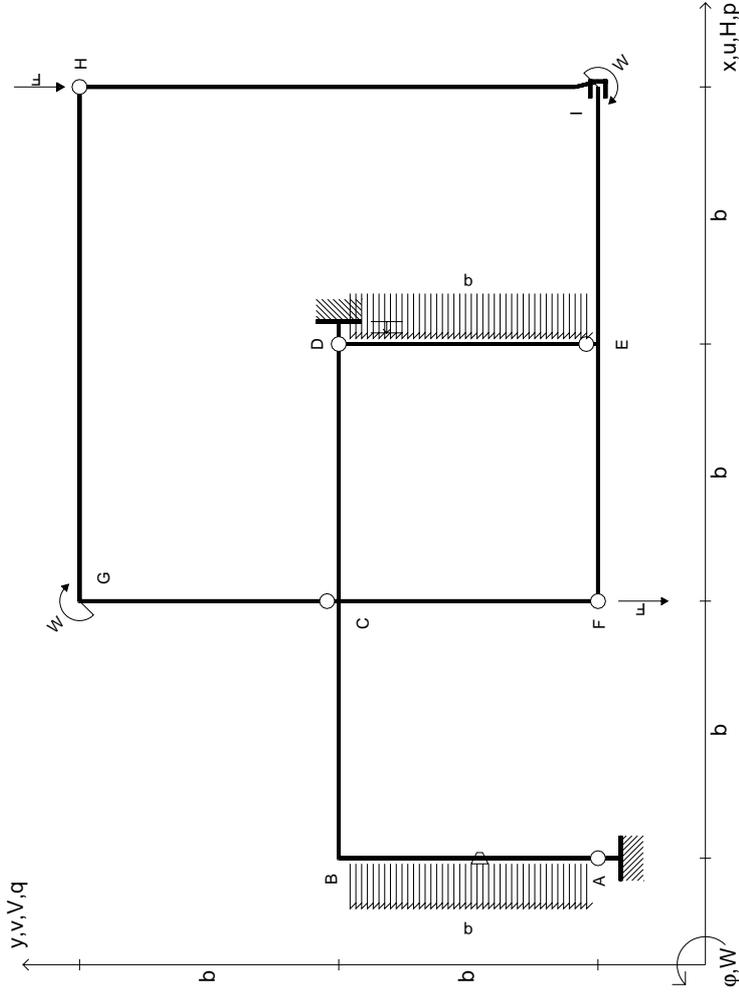
$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

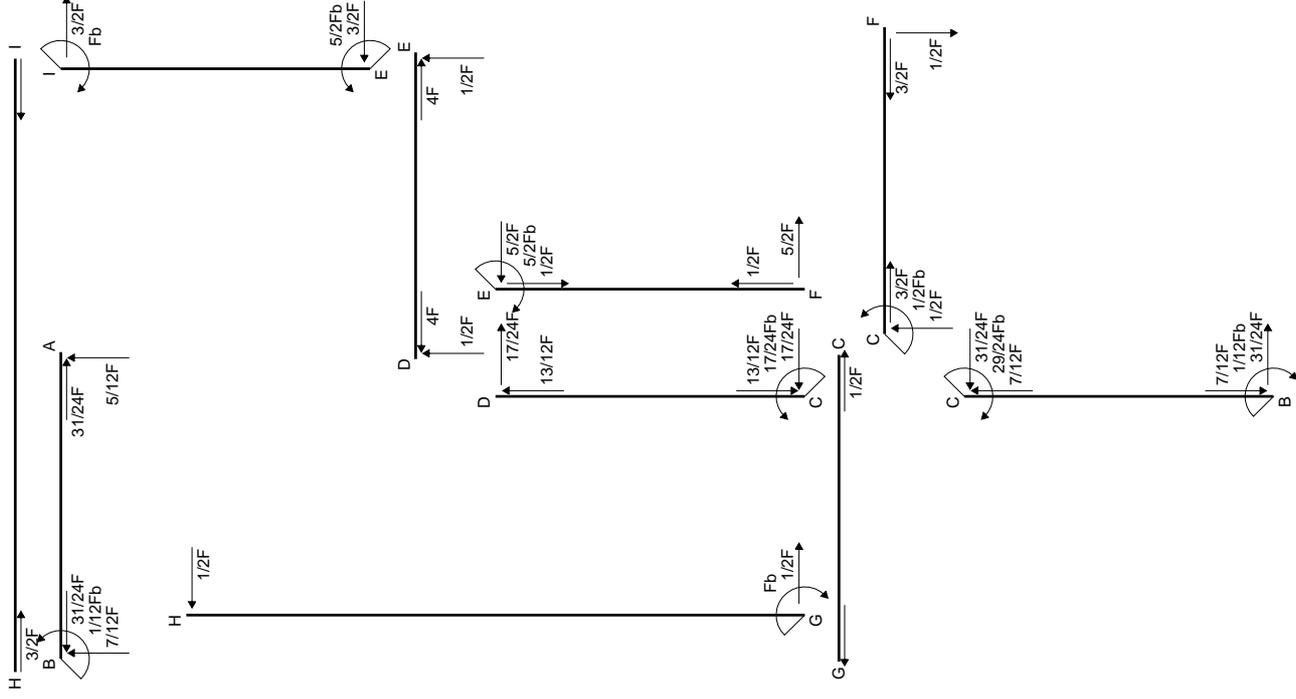


$$\begin{aligned}
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 V_F &= -F \\
 W_1 &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{AB} &= -q = +F/b \\
 P_{DE} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{AB} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3F/EJ \\
 EJ_{AB} &= EJ \\
 EJ_{BC} &= EJ \\
 EJ_{CD} &= EJ \\
 EJ_{DE} &= EJ
 \end{aligned}$$

$$\begin{aligned}
 EJ_{EF} &= EJ \\
 EJ_{FC} &= EJ \\
 EJ_{CG} &= EJ \\
 EJ_{GH} &= EJ \\
 EJ_{HI} &= EJ \\
 EJ_{IE} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste misurati in proiezione sugli assi x,y.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta AB positiva se convessa a destra con inizio A.
 Spostamento orizzontale assoluto u imposto al nodo D.



Quadro contributi PLV per iperstatica X=H_A

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	-x	1/2qx ²	-Fb/EJ	-1/2qx ³	Fxb/EJ	x ²	(-1/8+1/2)Fb ³ /EJ	1/3Xb ³ /EJ	
BA b	b-x	-1/2Fb+Fx-1/2qx ²	Fb/EJ	-1/2Fb ² +3/2Fbx-3/2Fx ² +1/2qx ³	Fb ² /EJ-Fxb/EJ	b ² -2bx+x ²			
BC b	-b+1/2x	1/2Fb-3/2Fx	0	-1/2Fb ² +7/4Fbx-3/4Fx ²	0	b ² -bx+1/4x ²	(1/8+0)Fb ³ /EJ	7/12Xb ³ /EJ	
CB b	1/2b+1/2x	Fb-3/2Fx	0	1/2Fb ² -1/4Fbx-3/4Fx ²	0	1/4b ² +1/2bx+1/4x ²			
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb ² -1/2Fbx+1/4Fx ²	0	1/4b ² -1/2bx+1/4x ²	(1/12+0)Fb ³ /EJ	1/12Xb ³ /EJ	
DC b	1/2x	1/2Fx	0	1/4Fx ²	0	1/4x ²			
DE b	0	1/2Fx-1/2qx ²	0	0	0	0	0+0	0	
ED b	0	-1/2Fx+1/2qx ²	0	0	0	0			
EF b	0	5/2Fb-5/2Fx	0	0	0	0	0+0	0	
FE b	0	-5/2Fx	0	0	0	0			
FC b	0	1/2Fx	0	0	0	0	0+0	0	
CF b	0	-1/2Fb+1/2Fx	0	0	0	0			
CG b	0	0	0	0	0	0	0+0	0	
GC b	0	0	0	0	0	0			
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0	
HG 2b	0	-1/2Fx	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0	
EI b	0	-5/2Fb+3/2Fx	0	0	0	0			
D	cedimento nodo -H _{1D} u _D							-Fb ³ /EJ	
	totali							-5/12Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _A							5/12F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (x/b) \theta dx = [-1/8 x^4/b^3]_0^b Fb^2 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (-1/8 b) Fb^2 1/EJ + (1/2 b) \theta = 3/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [-1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ + (-b + 1/2 b) \theta = 3/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + 7/4 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 7/8 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 7/8 b - 1/4 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/4 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/8 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

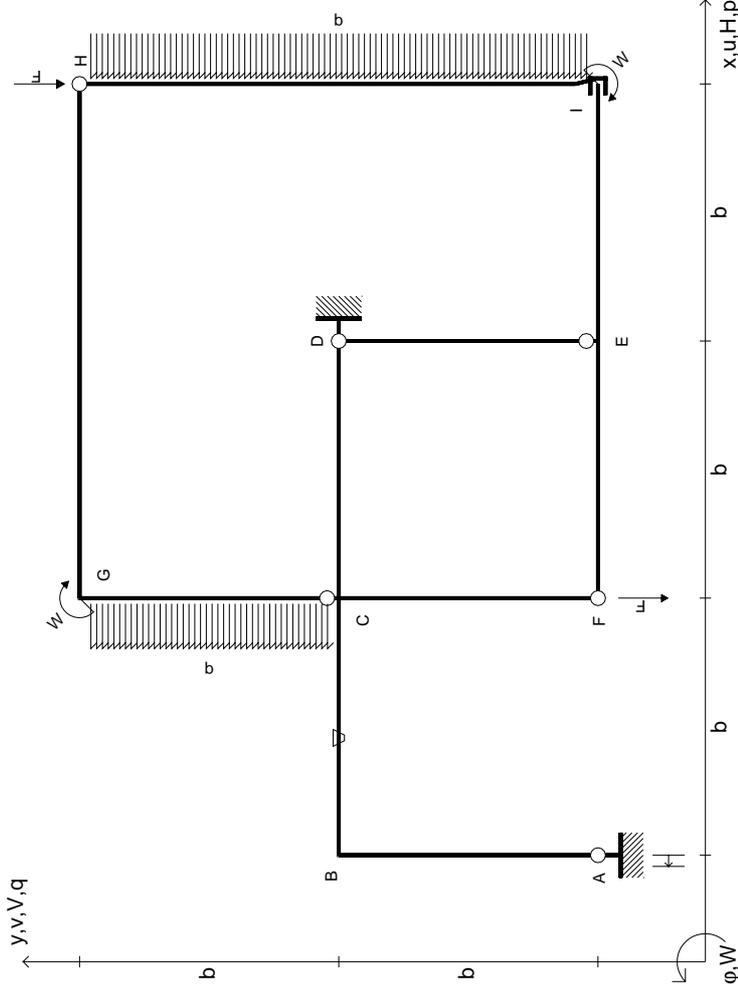
$$= (1/2 b - 1/8 b - 1/4 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

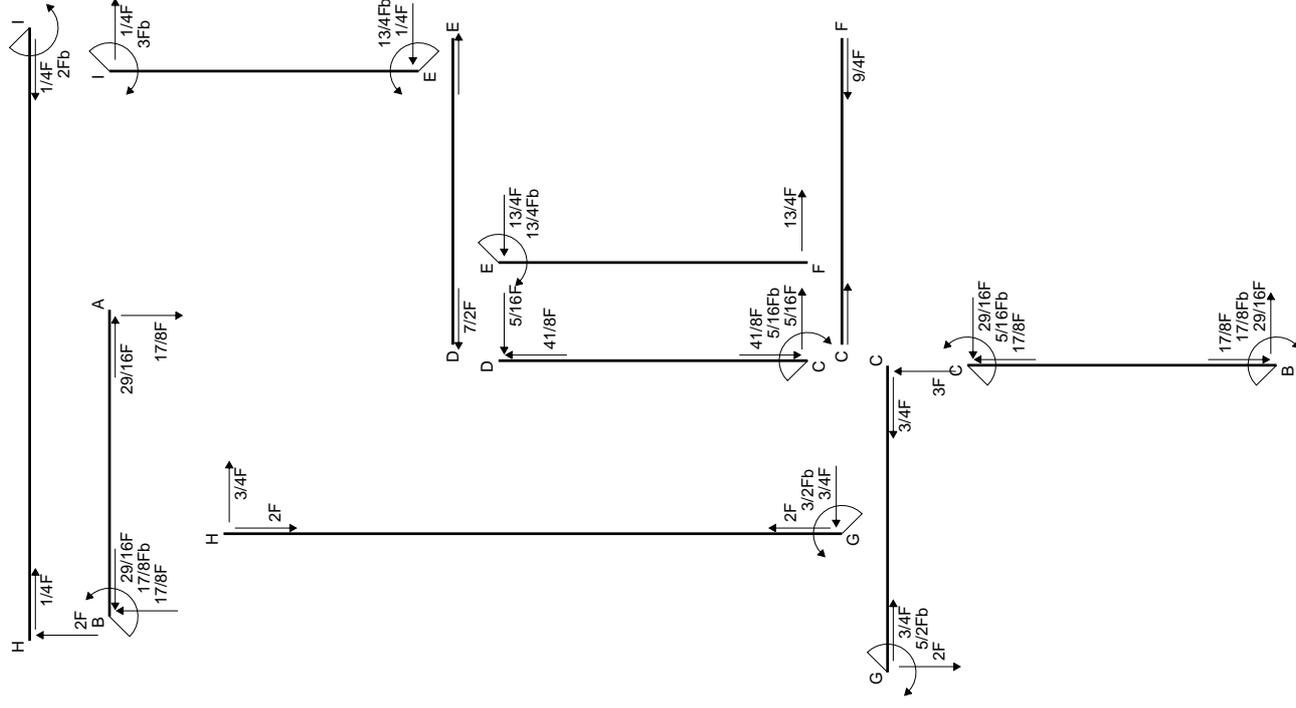


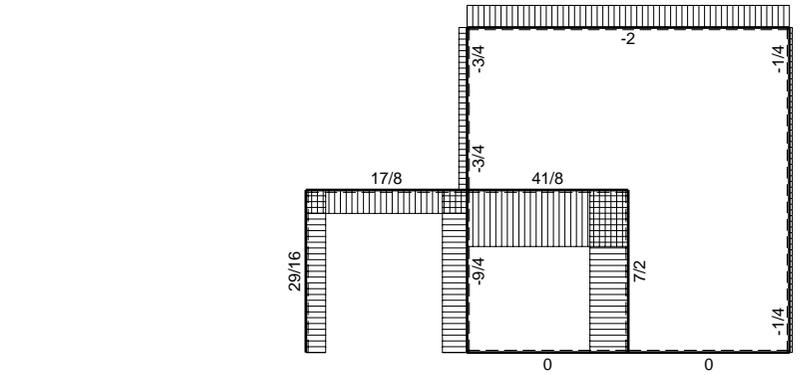
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_1 &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{HI} &= -q = -F/b \\
 P_{CG} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_A &= -\delta = -b^3F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

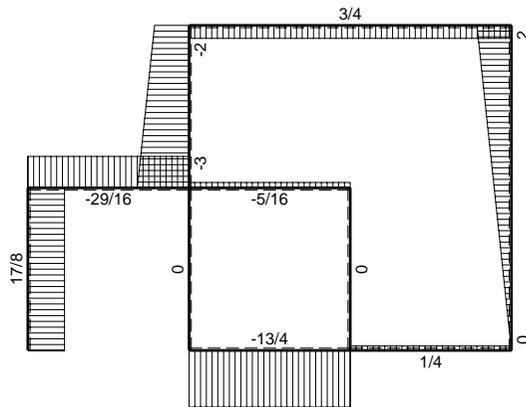
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y .
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo A.

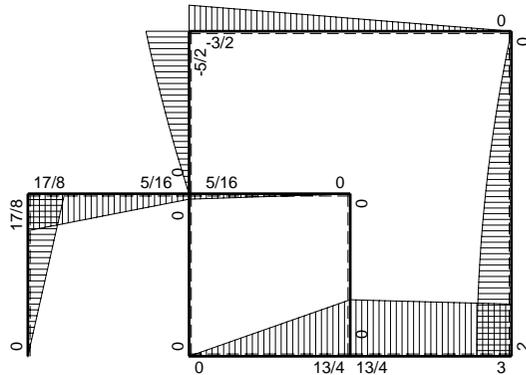




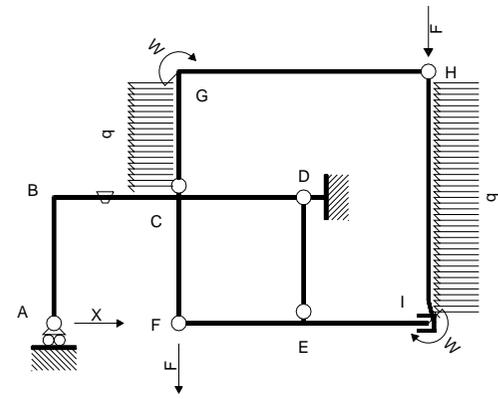
← (+) → F



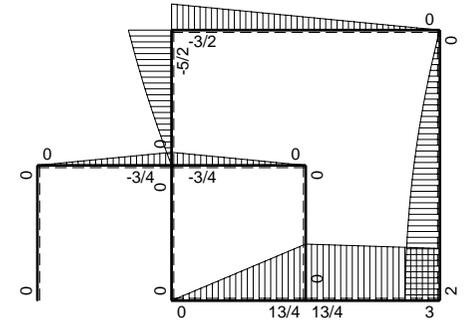
↑ (+) ↓ F



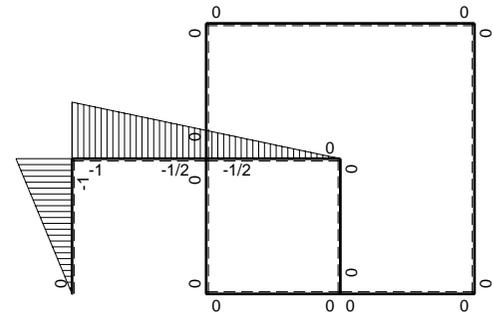
⊕ (+) ⊖ F_b



Schema di calcolo iperstatico



⊕ (+) ⊖ M_o flessione da carichi assegnati



⊕ (+) ⊖ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3 X b^3 / EJ$
BA b	b-x	0	0	0	0	$b^2 - 2bx + x^2$		
BC b	-b+1/2x	-3/4Fx	-Fb/EJ	$3/4Fbx - 3/8Fx^2$	$Fb^2/EJ - 1/2Fxb/EJ$	$b^2 - bx + 1/4x^2$	$(1/4 + 3/4)Fb^3/EJ$	$7/12 X b^3 / EJ$
CB b	1/2b+1/2x	3/4Fb-3/4Fx	Fb/EJ	$3/8Fb^2 - 3/8Fx^2$	$1/2Fb^2/EJ + 1/2Fxb/EJ$	$1/4b^2 + 1/2bx + 1/4x^2$		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	$3/8Fb^2 - 3/4Fbx + 3/8Fx^2$	0	$1/4b^2 - 1/2bx + 1/4x^2$	$(1/8 + 0)Fb^3/EJ$	$1/12 X b^3 / EJ$
DC b	1/2x	3/4Fx	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$13/4Fb - 13/4Fx$	0	0	0	0	0+0	0
FE b	0	$-13/4Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-3Fx + 1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$5/2Fb - 2Fx - 1/2qx^2$	0	0	0	0		
GH 2b	0	$-3/2Fb + 3/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$3/4Fx$	0	0	0	0		
HI 2b	0	$2Fx - 1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb + 1/2qx^2$	0	0	0	0		
IE b	0	$3Fb + 1/4Fx$	0	0	0	0	0+0	0
EI b	0	$-13/4Fb + 1/4Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A} u_A$						Fb^3/EJ	
	totali						$17/8 Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						$-17/8F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ + (b - 1/4 b) \theta = Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

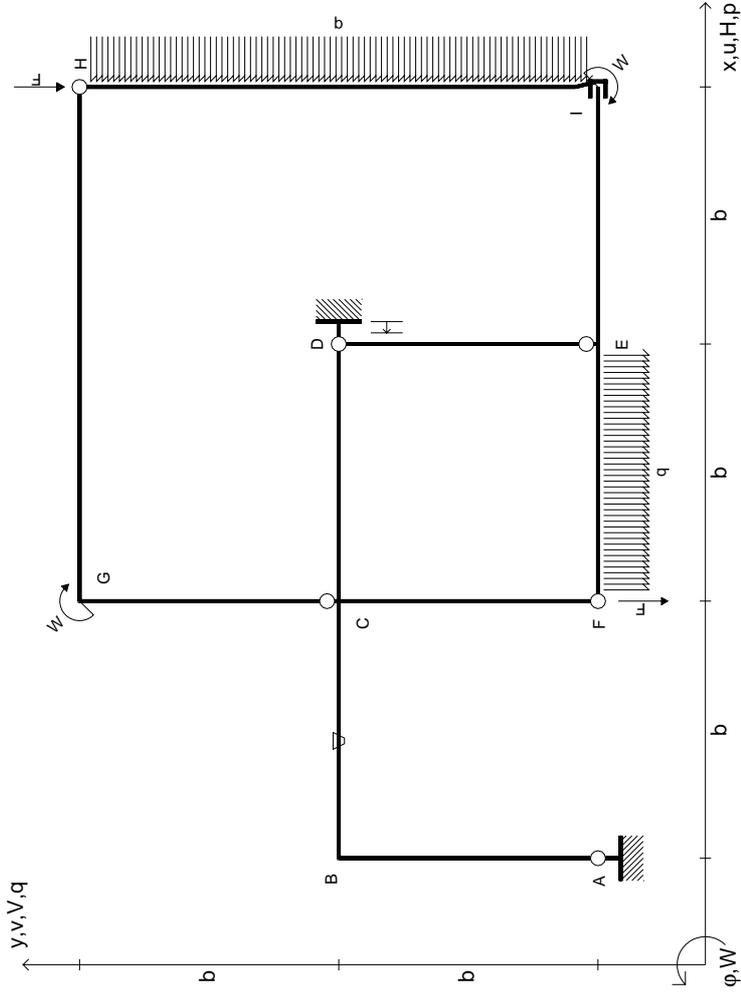
$$= (3/8 b - 1/8 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

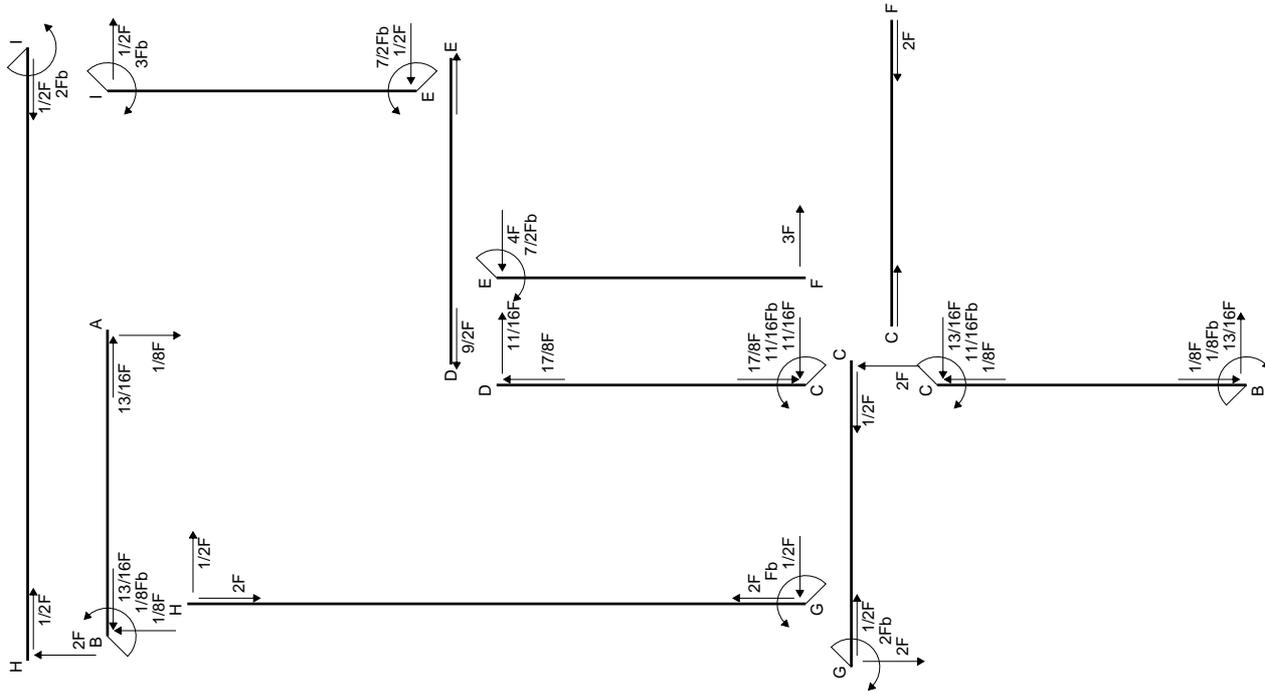


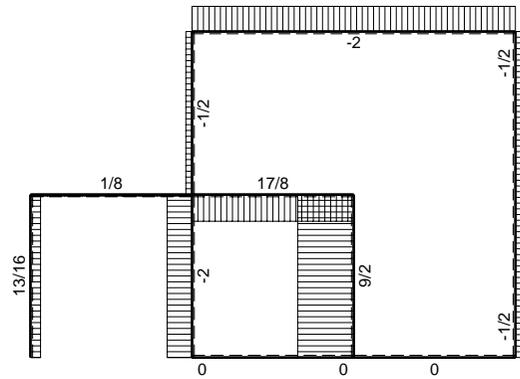
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = -Fb \\
 W_G &= -W = -Fb \\
 P_{HI} &= -q = -F/b \\
 q_{EF} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

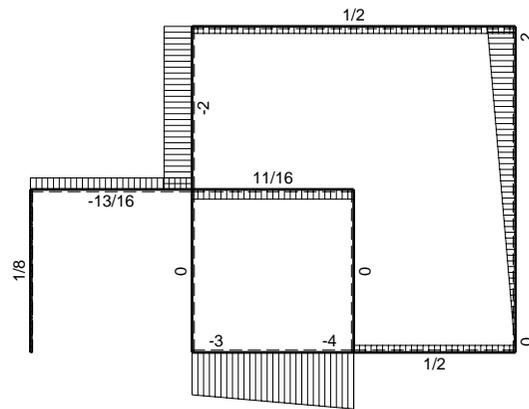
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=W_{BC}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo D.

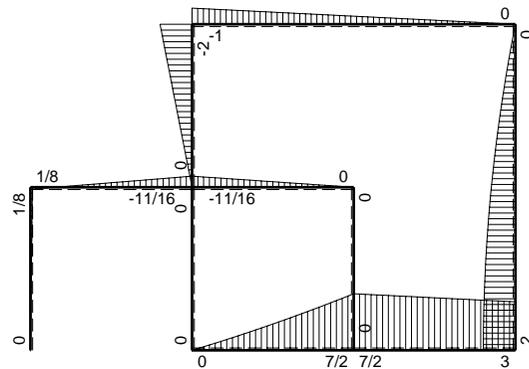




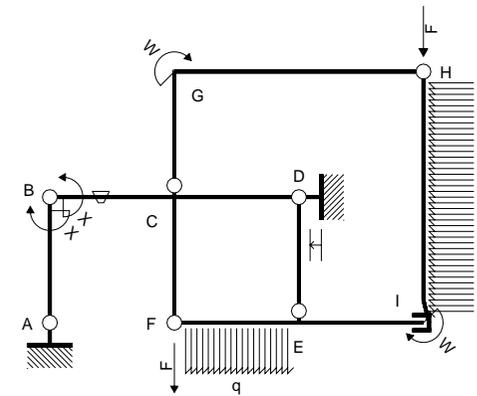
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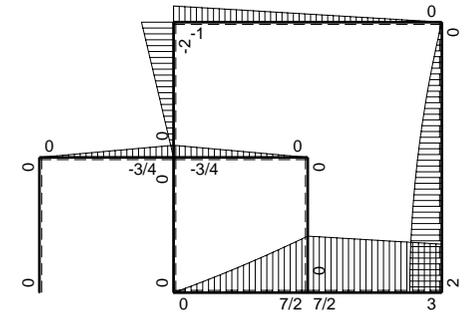
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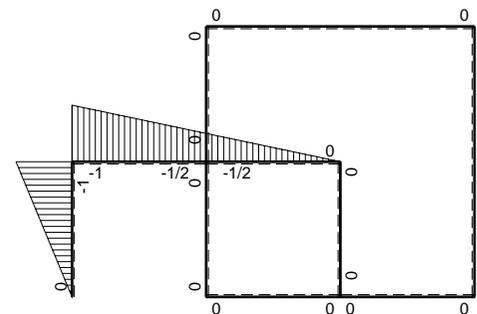
⤵ (+) ⤴ F_b



Schema di calcolo iperstatico



⤵ (+) ⤴ M_o flessione da carichi assegnati



⤵ (+) ⤴ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-x/b$	0	0	0	0	x^2/b^2	0+0	$1/3Xb/EJ$	
BA b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$			
BC b	$-1+1/2x/b$	$-3/4Fx$	$-Fb/EJ$	$3/4Fx-3/8Fx^2/b$	$Fb/EJ-1/2Fx/EJ$	$1-x/b+1/4x^2/b^2$	$(1/4+3/4)Fb^2/EJ$	$7/12Xb/EJ$	
CB b	$1/2+1/2x/b$	$3/4Fb-3/4Fx$	Fb/EJ	$3/8Fb-3/8Fx^2/b$	$1/2Fb/EJ+1/2Fx/EJ$	$1/4+1/2x/b+1/4x^2/b^2$			
CD b	$-1/2+1/2x/b$	$-3/4Fb+3/4Fx$	0	$3/8Fb-3/4Fx+3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(1/8+0)Fb^2/EJ$	$1/12Xb/EJ$	
DC b	$1/2x/b$	$3/4Fx$	0	$3/8Fx^2/b$	0	$1/4x^2/b^2$			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	$-2Fx$	0	0	0	0	0+0	0	
GC b	0	$2Fb-2Fx$	0	0	0	0			
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0	
HG 2b	0	$1/2Fx$	0	0	0	0			
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0	
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0			
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0	
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0			
D	cedimento nodo $-H_{1D}u_D$							$-Fb^2/EJ$	
	totali							$1/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$							$-1/8Fb$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

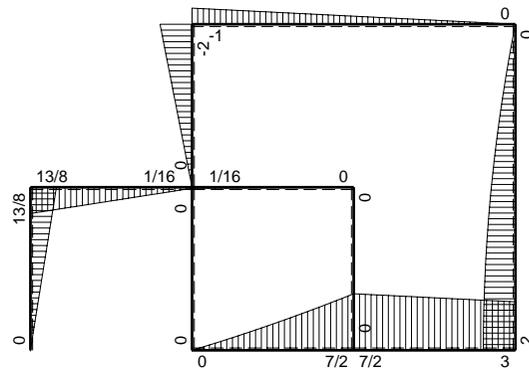
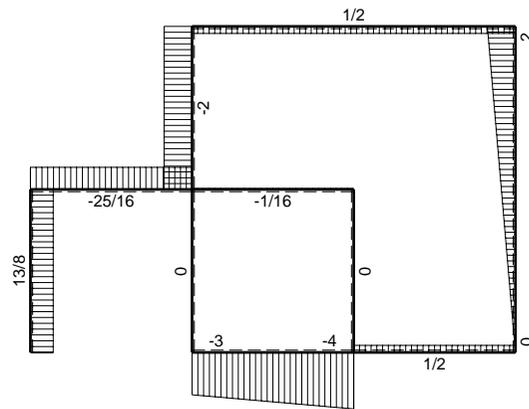
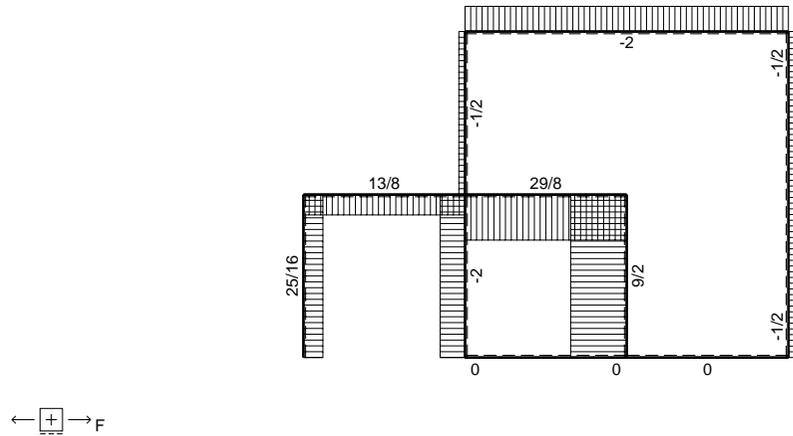
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

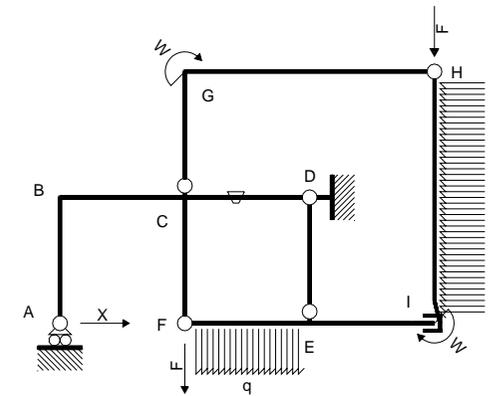
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

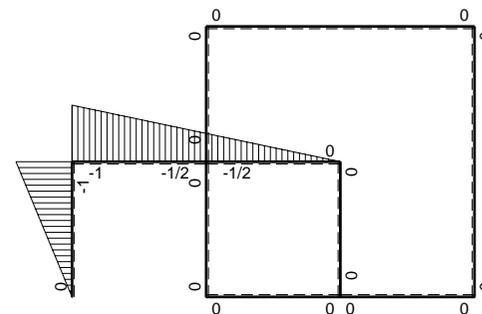
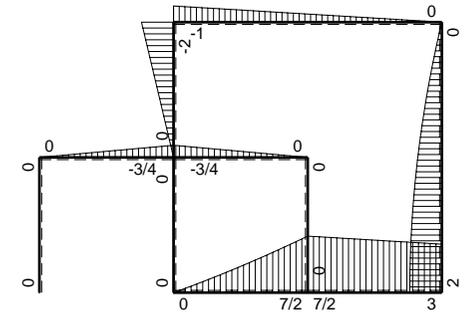


$\left[\oplus \right] F_b$



Schema di calcolo iperstatico

$\left[\oplus \right] M_o$ flessione da carichi assegnati



$\left[\oplus \right] M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3 X b^3 / EJ$
BA b	b-x	0	0	0	0	$b^2 - 2bx + x^2$		
BC b	-b+1/2x	-3/4Fx	0	$3/4 Fbx - 3/8 Fx^2$	0	$b^2 - bx + 1/4 x^2$	$(1/4+0)Fb^3/EJ$	$7/12 X b^3 / EJ$
CB b	1/2b+1/2x	3/4Fb-3/4Fx	0	$3/8 Fb^2 - 3/8 Fx^2$	0	$1/4 b^2 + 1/2 bx + 1/4 x^2$		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	-Fb/EJ	$3/8 Fb^2 - 3/4 Fbx + 3/8 Fx^2$	$1/2 Fb^2 / EJ - 1/2 Fxb / EJ$	$1/4 b^2 - 1/2 bx + 1/4 x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12 X b^3 / EJ$
DC b	1/2x	3/4Fx	Fb/EJ	$3/8 Fx^2$	$1/2 Fxb / EJ$	$1/4 x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2 Fb - 4Fx + 1/2 qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx - 1/2 qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	$2Fx - 1/2 qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb + 1/2 qx^2$	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A} u_A$						Fb^3/EJ	
	totali						$13/8 Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-13/8F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

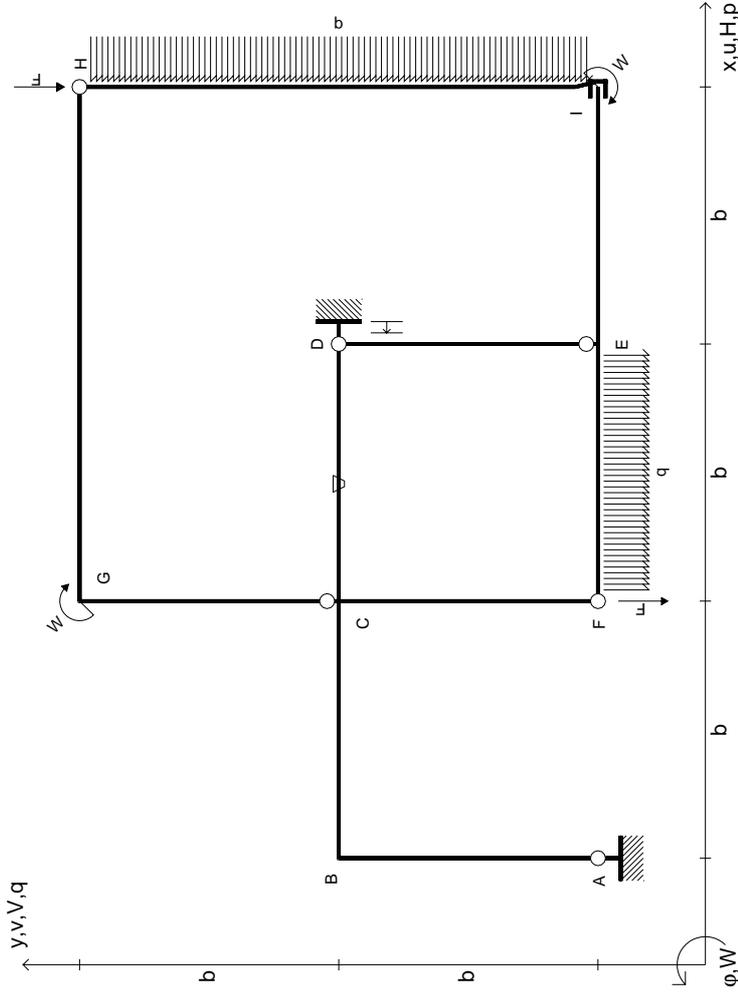
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb^2 1/EJ + (-1/4 b) \theta = 3/8 Fb^3/EJ$$

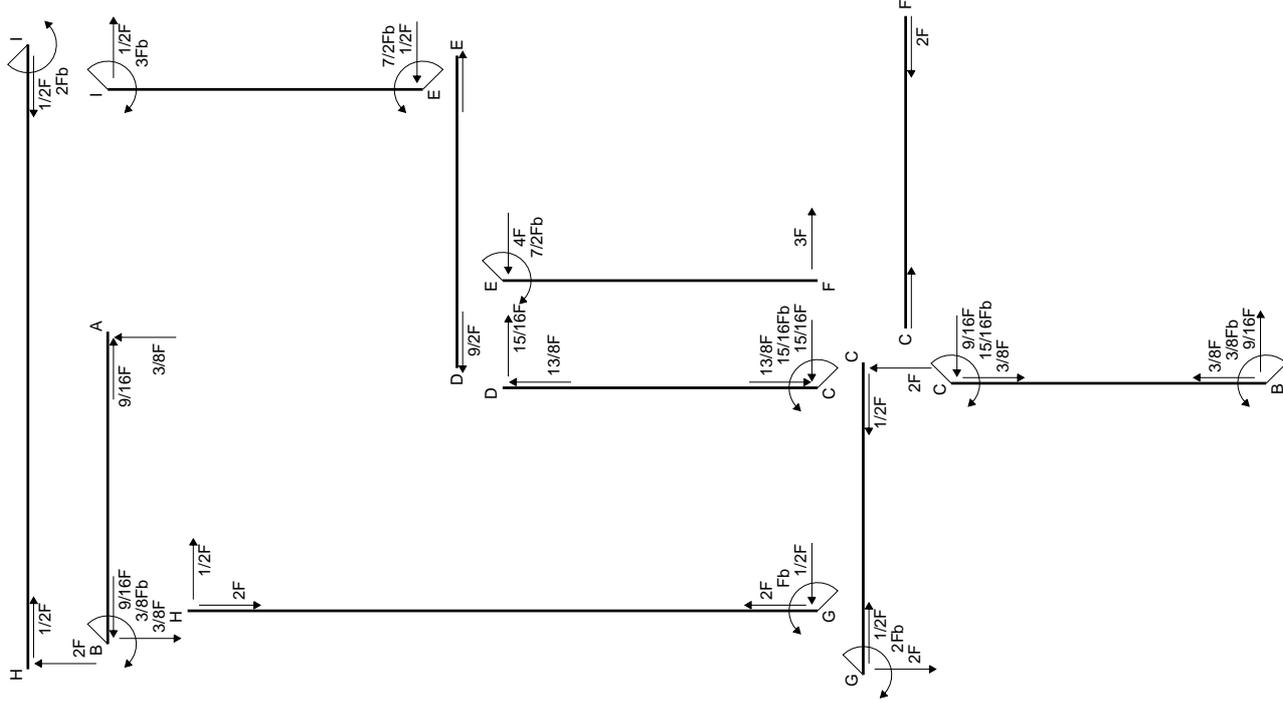


$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_1 &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{HI} &= -q = -F/b \\
 q_{EF} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{CD} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=V_D$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta CD positiva se convessa a destra con inizio C.
 Spostamento orizzontale assoluto u imposto al nodo D.
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Quadro contributi PLV per iperstatica $X=V_D$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	2x	-15/2Fx	0	-15Fx ²	0	4x ²	(-5+0)Fb ³ /EJ	4/3Xb ³ /EJ
BA b	-2b+2x	15/2Fb-15/2Fx	0	-15Fb ² +30Fbx-15Fx ²	0	4b ² -8bx+4x ²		
BC b	2b-x	-15/2Fb+3Fx	0	-15Fb ² +27/2Fbx-3Fx ²	0	4b ² -4bx+x ²	(-37/4+0)Fb ³ /EJ	7/3Xb ³ /EJ
CB b	-b-x	9/2Fb+3Fx	0	-9/2Fb ² -15/2Fbx-3Fx ²	0	b ² +2bx+x ²		
CD b	b-x	-9/2Fb+9/2Fx	-Fb/EJ	-9/2Fb ² +9Fbx-9/2Fx ²	-Fb ² /EJ+Fxb/EJ	b ² -2bx+x ²	(-3/2-1/2)Fb ³ /EJ	1/3Xb ³ /EJ
DC b	-x	9/2Fx	Fb/EJ	-9/2Fx ²	-Fxb/EJ	x ²		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-4Fx+1/2qx ²	0	0	0	0	0+0	0
FE b	0	-3Fx-1/2qx ²	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	2Fx-1/2qx ²	0	0	0	0	0+0	0
IH 2b	0	-2Fb+1/2qx ²	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
D	cedimento nodo -H _{1D} u _D						2Fb ³ /EJ	
	totali						-57/4Fb ³ /EJ	4Xb ³ /EJ
	iperstatica X=V _D						57/16F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = [4 x - 4 x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = [4 x - 2 x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-15 x^2/b^2) Fb^2 1/EJ dx = [-5 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-5 b) Fb^2 1/EJ = -5 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-15 + 30 x/b - 15 x^2/b^2) Fb^2 1/EJ dx = [-15 x + 15 x^2/b - 5 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-15 b + 15 b - 5 b) Fb^2 1/EJ = -5 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-15 + 27/2 x/b - 3 x^2/b^2) Fb^2 1/EJ dx = [-15 x + 27/4 x^2/b - x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-15 b + 27/4 b - b) Fb^2 1/EJ = -37/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-9/2 - 15/2 x/b - 3 x^2/b^2) Fb^2 1/EJ dx = [-9/2 x - 15/4 x^2/b - x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-9/2 b - 15/4 b - b) Fb^2 1/EJ = -37/4 Fb^3/EJ$$

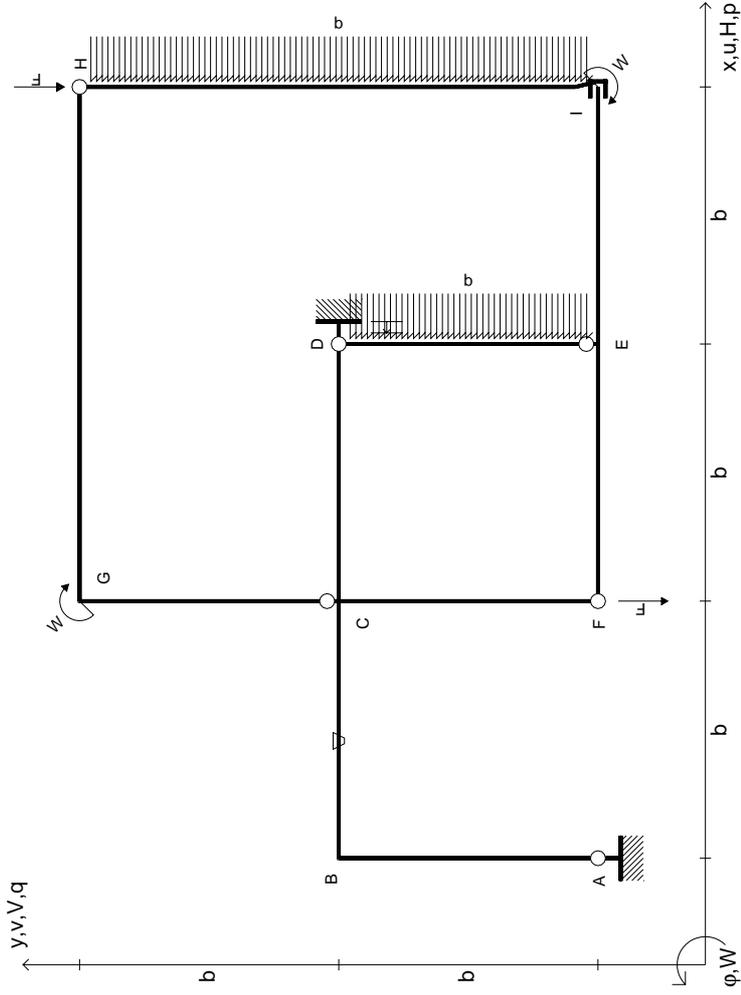
$$L_{CD}^{xo} = \int_0^b (-9/2 + 9 x/b - 9/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [-9/2 x + 9/2 x^2/b - 3/2 x^3/b^2]_0^b Fb^2 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-9/2 b + 9/2 b - 3/2 b) Fb^2 1/EJ + (-b + 1/2 b) \theta = -2 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (-9/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (x/b) \theta dx = [-3/2 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (-3/2 b) Fb^2 1/EJ + (1/2 b) \theta = -2 Fb^3/EJ$$

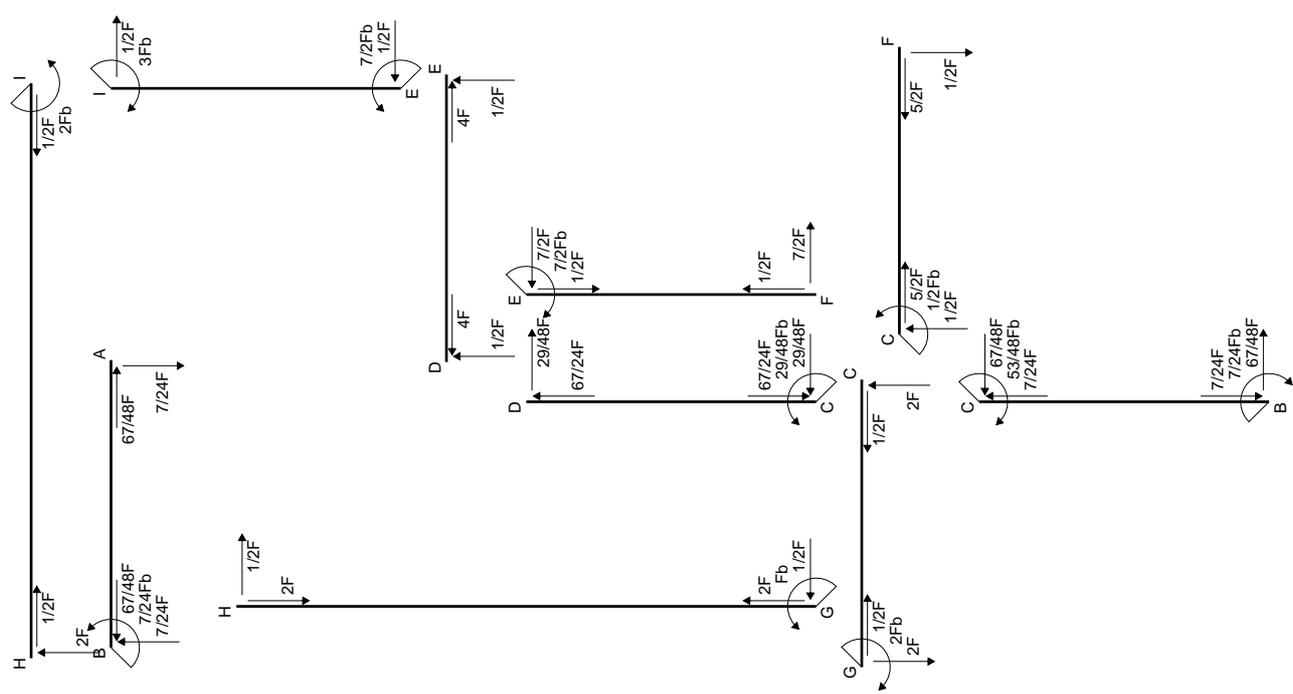


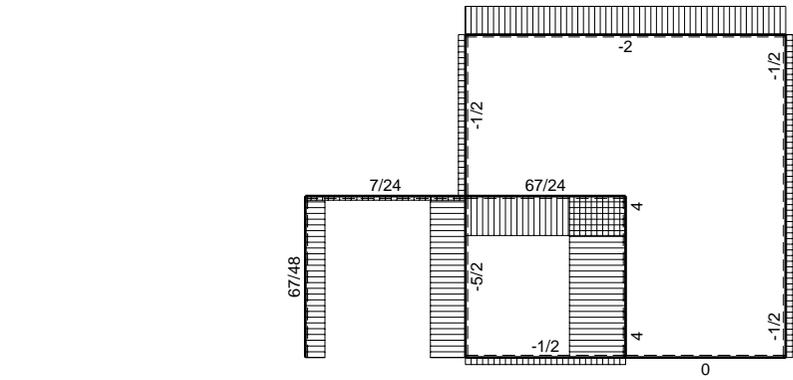
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_1 &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{HI} &= -q = -F/b \\
 P_{DE} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

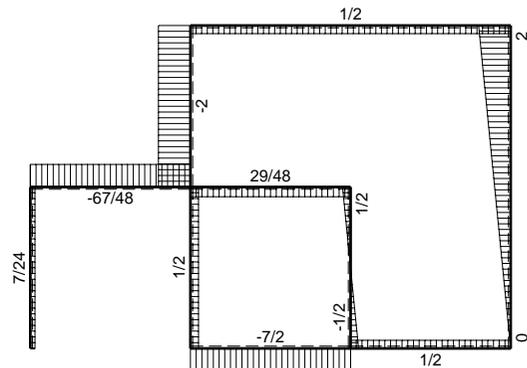
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
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 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x, y .
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo D.

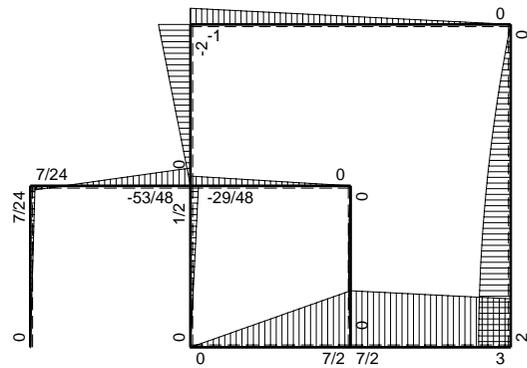




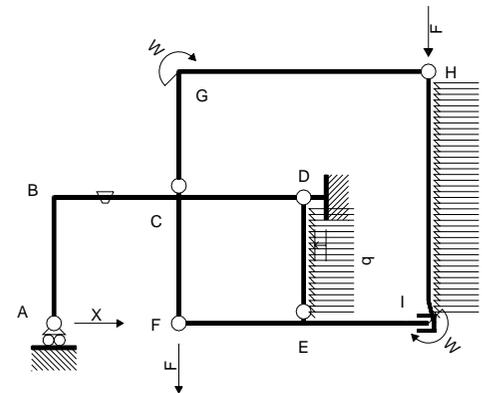
← (+) → F



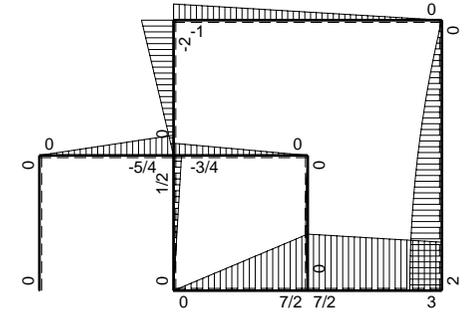
↑ (+) ↓ F



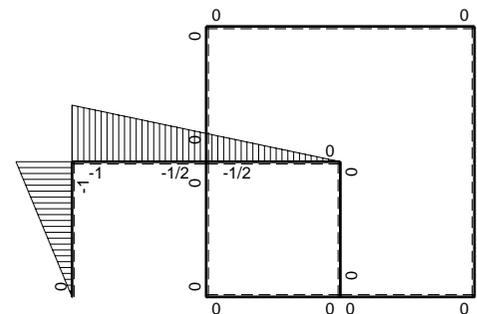
⌚ (+) ↻ F_b



Schema di calcolo iperstatico



⌚ (+) ↻ M₀ flessione da carichi assegnati



⌚ (+) ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_A

→	M _x (x)	M ₀ (x)	θ	M _x M ₀	M _x θ	M _x M _x	∫M _x (M ₀ /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	-x	0	0	0	0	x ²	0+0	1/3Xb ³ /EJ	
BA b	b-x	0	0	0	0	b ² -2bx+x ²			
BC b	-b+1/2x	-5/4Fx	-Fb/EJ	5/4Fbx-5/8Fx ²	Fb ² /EJ-1/2Fxb/EJ	b ² -bx+1/4x ²	(5/12+3/4)Fb ³ /EJ	7/12Xb ³ /EJ	
CB b	1/2b+1/2x	5/4Fb-5/4Fx	Fb/EJ	5/8Fb ² -5/8Fx ²	1/2Fb ² /EJ+1/2Fxb/EJ	1/4b ² +1/2bx+1/4x ²			
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	3/8Fb ² -3/4Fbx+3/8Fx ²	0	1/4b ² -1/2bx+1/4x ²	(1/8+0)Fb ³ /EJ	1/12Xb ³ /EJ	
DC b	1/2x	3/4Fx	0	3/8Fx ²	0	1/4x ²			
DE b	0	1/2Fx-1/2qx ²	0	0	0	0	0+0	0	
ED b	0	-1/2Fx+1/2qx ²	0	0	0	0			
EF b	0	7/2Fb-7/2Fx	0	0	0	0	0+0	0	
FE b	0	-7/2Fx	0	0	0	0			
FC b	0	1/2Fx	0	0	0	0	0+0	0	
CF b	0	-1/2Fb+1/2Fx	0	0	0	0			
CG b	0	-2Fx	0	0	0	0	0+0	0	
GC b	0	2Fb-2Fx	0	0	0	0			
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0	
HG 2b	0	1/2Fx	0	0	0	0			
HI 2b	0	2Fx-1/2qx ²	0	0	0	0	0+0	0	
IH 2b	0	-2Fb+1/2qx ²	0	0	0	0			
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0	
EI b	0	-7/2Fb+1/2Fx	0	0	0	0			
D	cedimento nodo -H _{1D} u _D							-Fb ³ /EJ	
	totali							7/24Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _A							-7/24F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(\frac{5}{4} \frac{x}{b} - \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{5}{8} \frac{x^2}{b} - \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{5}{8} b - \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} + \left(b - \frac{1}{4} b \right) \theta = \frac{7}{6} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(\frac{5}{8} - \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(-\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{5}{8} x - \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[-\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

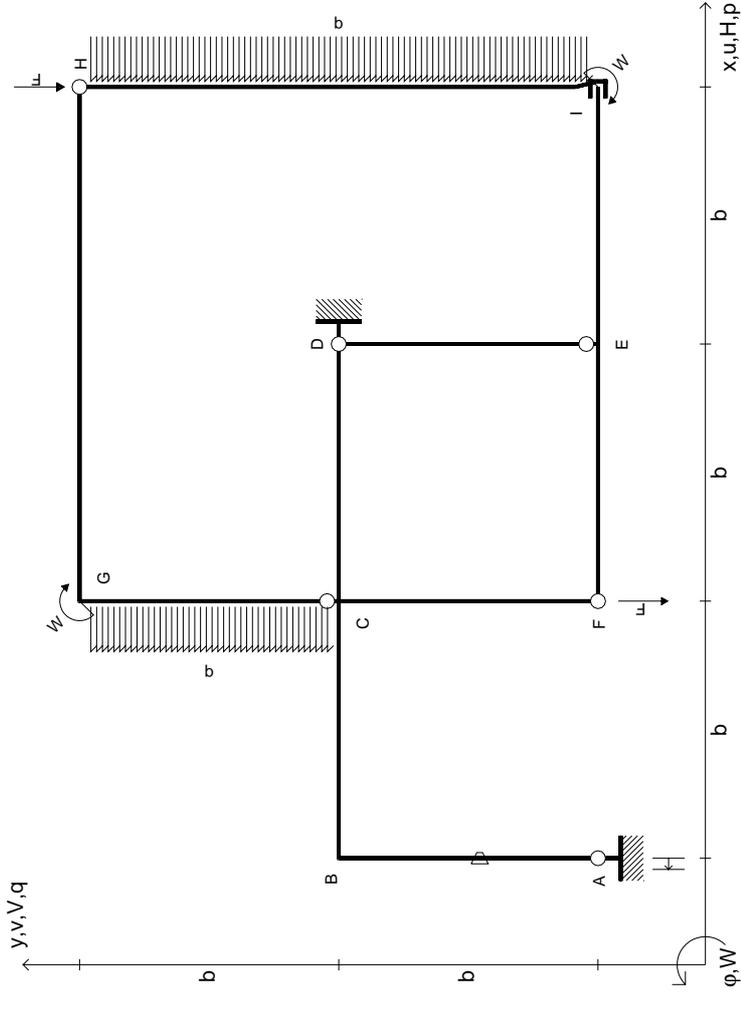
$$= \left(\frac{5}{8} b - \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} + \left(-\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{7}{6} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left(\frac{3}{8} - \frac{3}{4} \frac{x}{b} + \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{3}{8} x - \frac{3}{8} \frac{x^2}{b} + \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{3}{8} b - \frac{3}{8} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(\frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

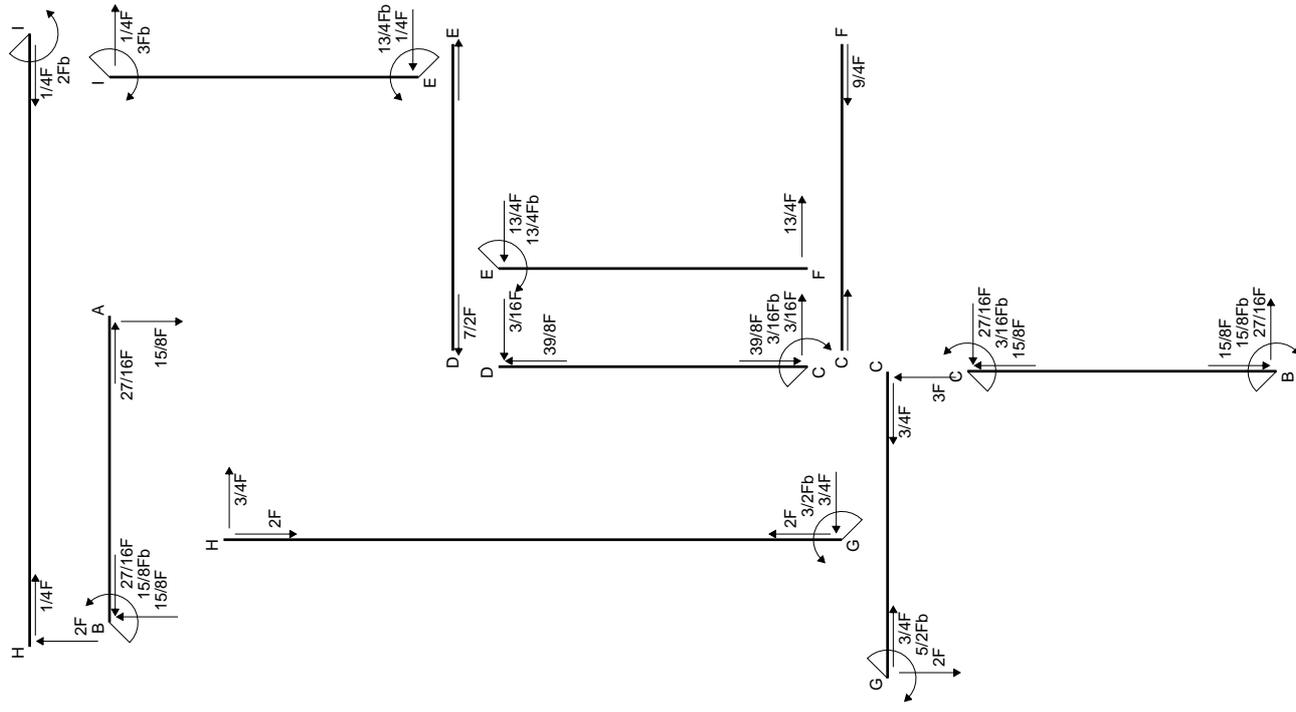


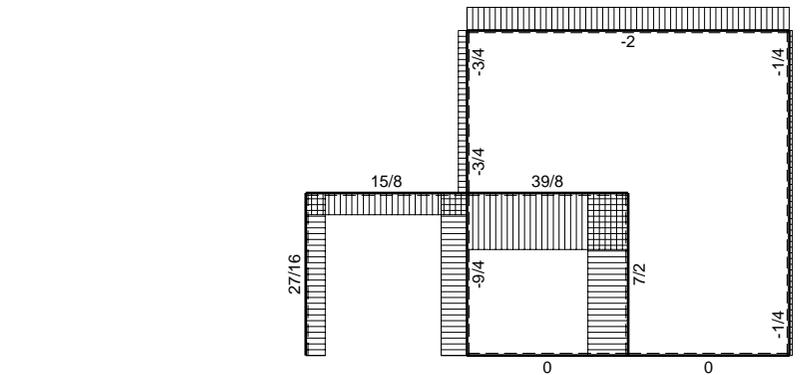
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_1 &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{CG} &= -q = -F/b \\
 P_{HI} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{AB} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_A &= -\delta = -b^3F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

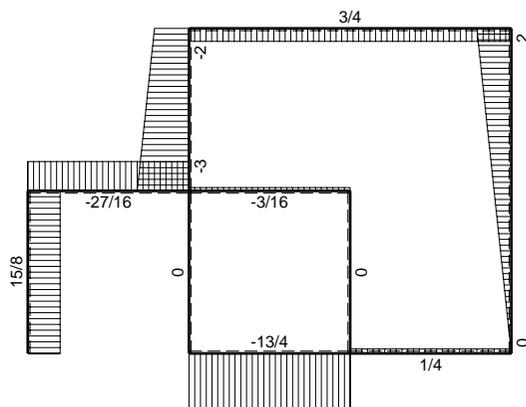
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta AB positiva se convessa a destra con inizio A.
 Spostamento orizzontale assoluto u imposto al nodo A.

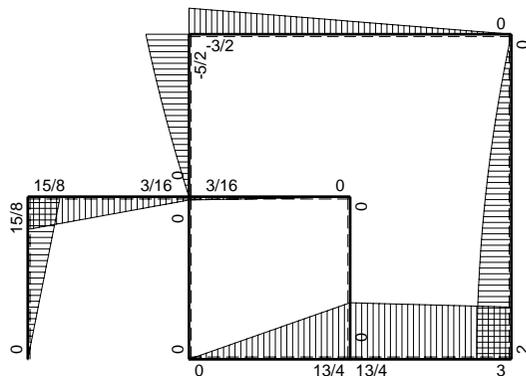




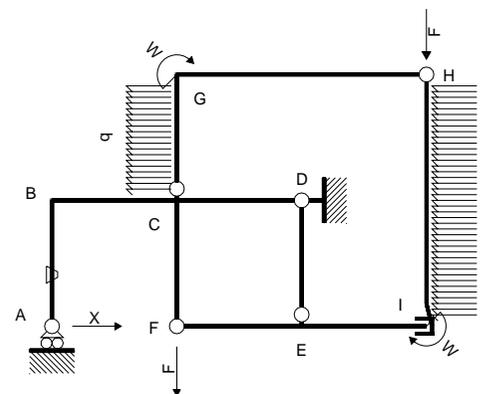
← (+) → F



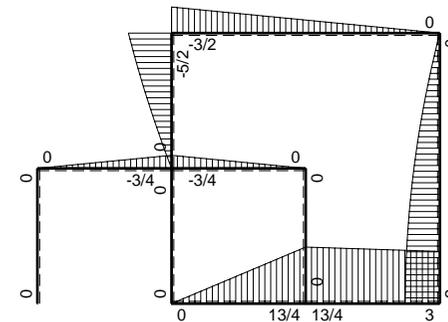
↑ (+) ↓ F



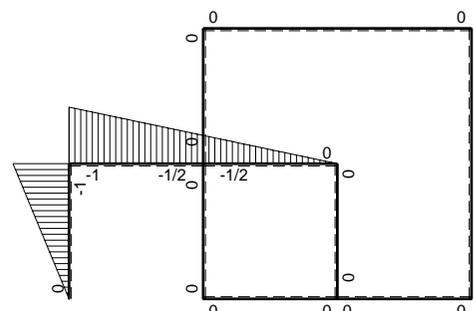
⊕ (+) ⊖ F_b



Schema di calcolo iperstatico



⊕ (+) ⊖ M₀ flessione da carichi assegnati



⊕ (+) ⊖ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	-b+1/2x	-3/4Fx	0	$3/4Fbx-3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/4+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	3/4Fb-3/4Fx	0	$3/8Fb^2-3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	3/4Fx	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	13/4Fb-13/4Fx	0	0	0	0	0+0	0
FE b	0	-13/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-3Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$5/2Fb-2Fx-1/2qx^2$	0	0	0	0		
GH 2b	0	$-3/2Fb+3/4Fx$	0	0	0	0	0+0	0
HG 2b	0	3/4Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+1/4Fx	0	0	0	0	0+0	0
EI b	0	-13/4Fb+1/4Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$15/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-15/8F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

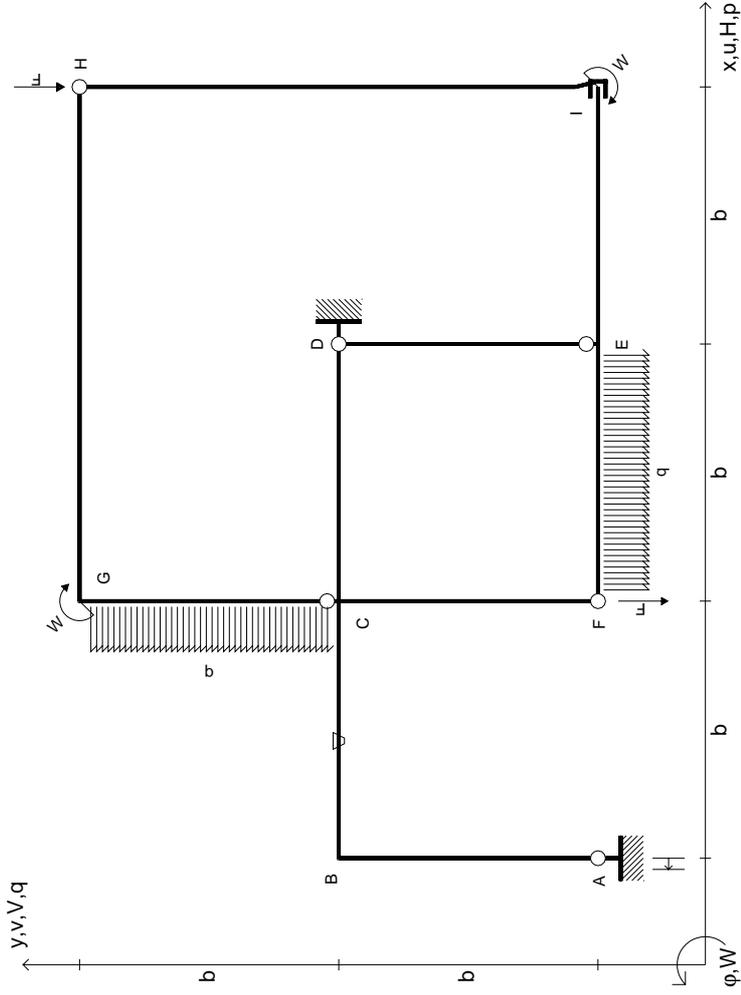
$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

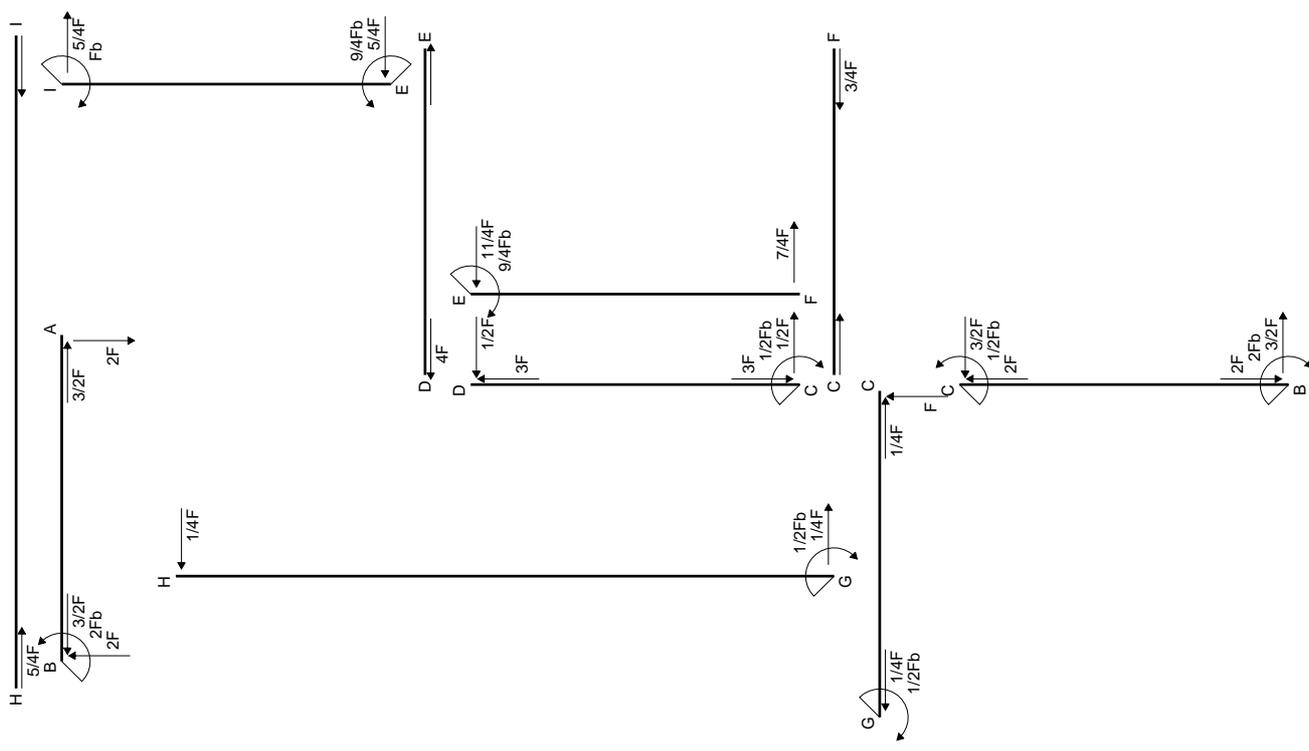


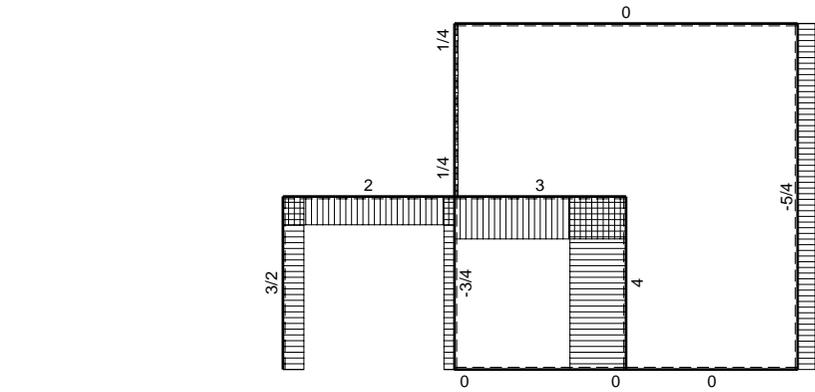
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{CG} &= -q = -F/b \\
 q_{EF} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 U_A &= -\delta = -b^3F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

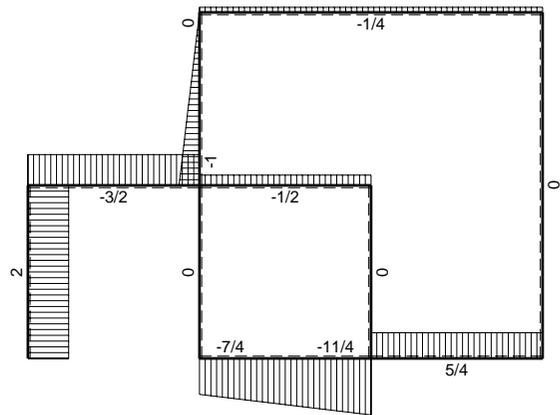
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

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 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo A.

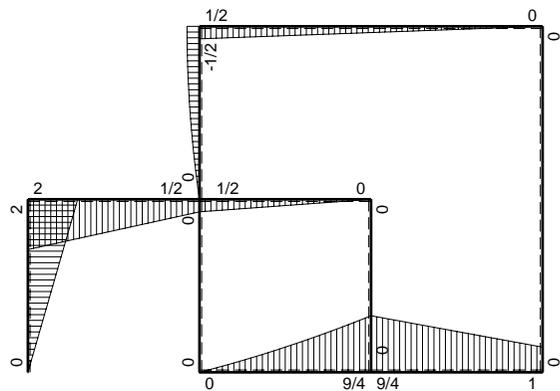




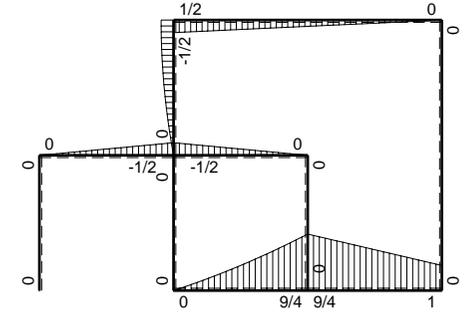
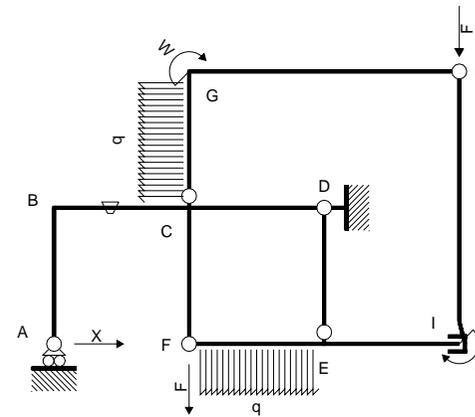
← (+) → F



↑ (+) ↓ F

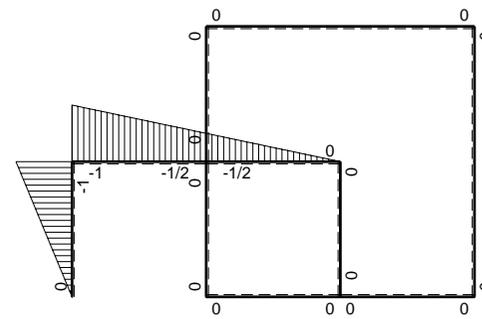


⊕ (+) ⊖ F_b



Schema di calcolo iperstatico

⊕ (+) ⊖ M₀ flessione da carichi assegnati



⊕ (+) ⊖ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-1/2Fx	-Fb/EJ	$1/2Fbx-1/4Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/6+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$1/2Fb-1/2Fx$	Fb/EJ	$1/4Fb^2-1/4Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$9/4Fb-11/4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-7/4Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/4Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx$	0	0	0	0	0+0	0
EI b	0	$-9/4Fb+5/4Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$2Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-2F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(\frac{1}{2} \frac{x}{b} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{1}{4} \frac{x^2}{b} - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{1}{4} b - \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left(b - \frac{1}{4} b \right) \theta = \frac{11}{12} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(-\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{1}{4} x - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[-\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

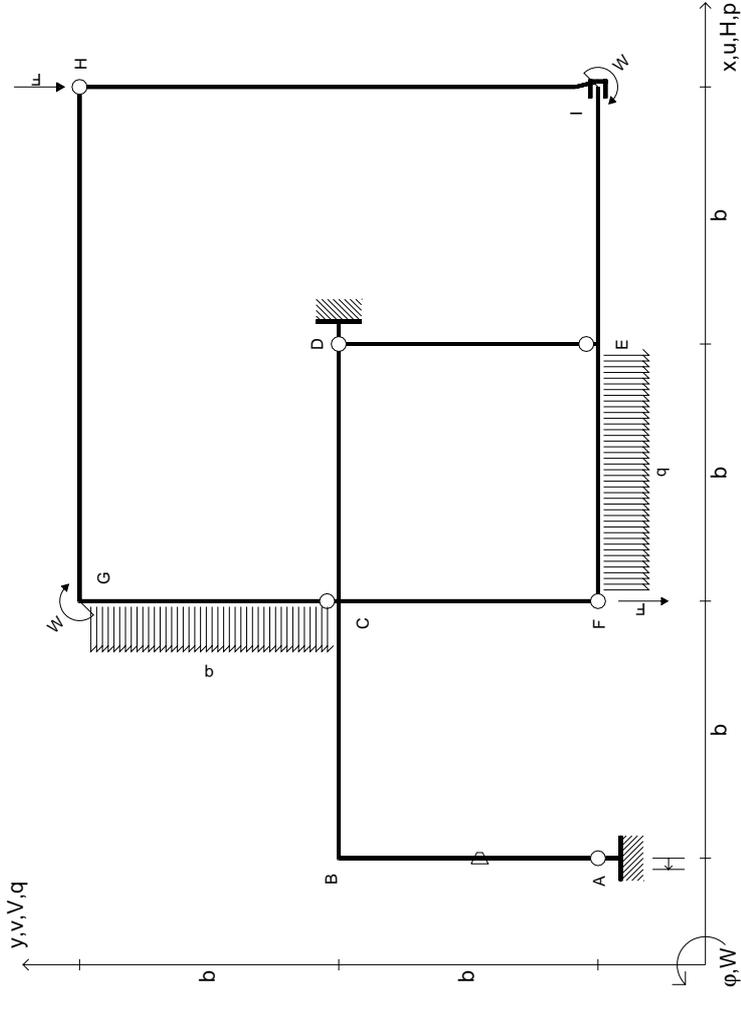
$$= \left(\frac{1}{4} b - \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left(-\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{11}{12} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

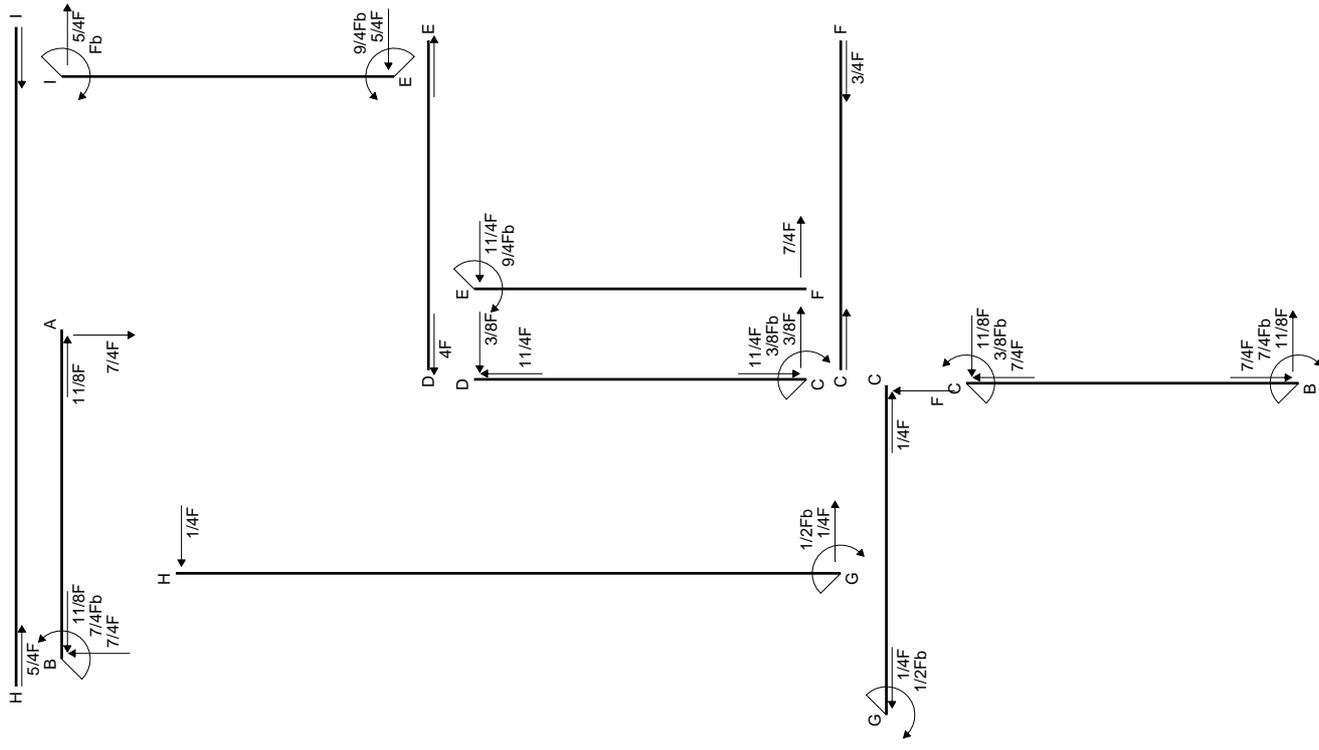


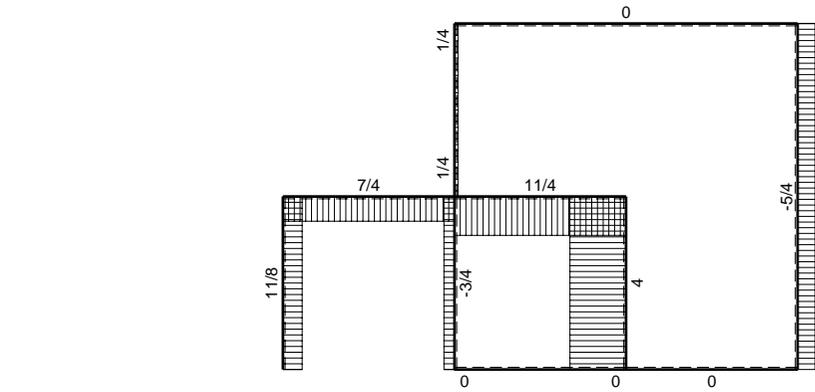
$$\begin{aligned} V_H &= -F \\ V_F &= -F \\ W_I &= -W = +Fb \\ W_G &= -W = -Fb \\ P_{CG} &= -q = -F/b \\ q_{EF} &= -q = -F/b \end{aligned}$$

$$\begin{aligned} \theta_{AB} &= -\theta = -\alpha T/b = -bF/EJ \\ U_A &= -\delta = -b^3 F/EJ \\ E_{J_{AB}} &= EJ \\ E_{J_{BC}} &= EJ \\ E_{J_{CD}} &= EJ \\ E_{J_{DE}} &= EJ \end{aligned}$$

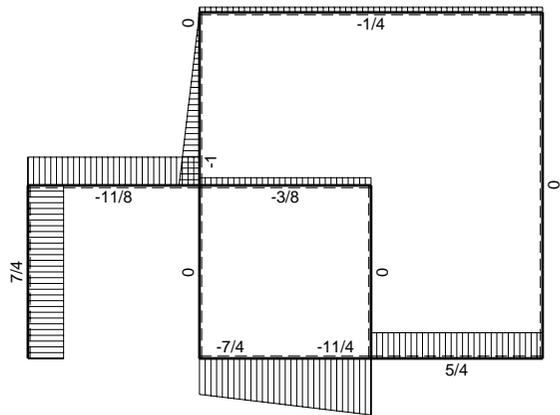
$$\begin{aligned} E_{J_{EF}} &= EJ \\ E_{J_{FC}} &= EJ \\ E_{J_{CG}} &= EJ \\ E_{J_{GH}} &= EJ \\ E_{J_{HI}} &= EJ \\ E_{J_{IE}} &= EJ \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
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 Carichi di aste curve misurati in proiezione sugli assi x,y .
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta AB positiva se convessa a destra con inizio A.
 Spostamento orizzontale assoluto u imposto al nodo A.

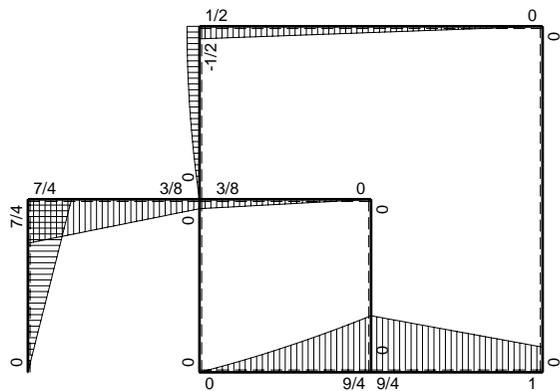




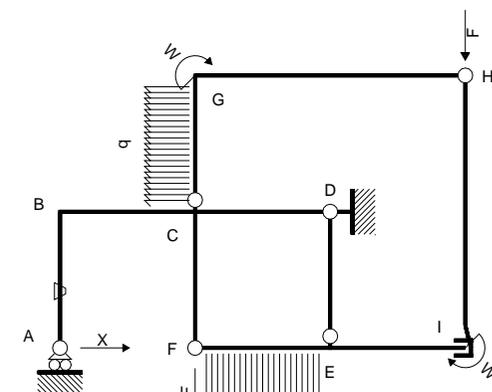
← (+) → F



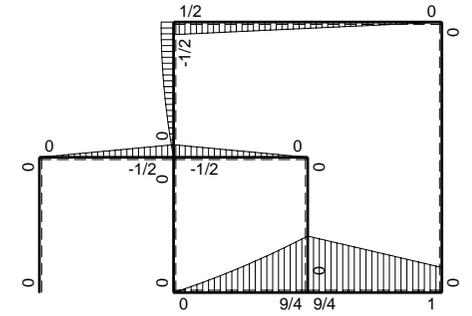
↑ (+) ↓ F



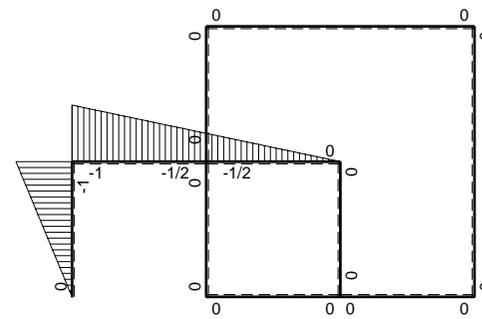
⊕ (+) ⊖ F_b



Schema di calcolo iperstatico



⊕ (+) ⊖ M_o flessione da carichi assegnati



⊕ (+) ⊖ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	-b+1/2x	-1/2Fx	0	$1/2Fbx-1/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$1/2Fb-1/2Fx$	0	$1/4Fb^2-1/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$9/4Fb-11/4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-7/4Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/4Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx$	0	0	0	0	0+0	0
EI b	0	$-9/4Fb+5/4Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$7/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						$-7/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left(\frac{x}{b} \right) \theta dx = \left[\frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left(-1 + x/b \right) \theta dx = \left[-x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(-b + \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(\frac{1}{2} \frac{x}{b} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{4} \frac{x^2}{b} - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

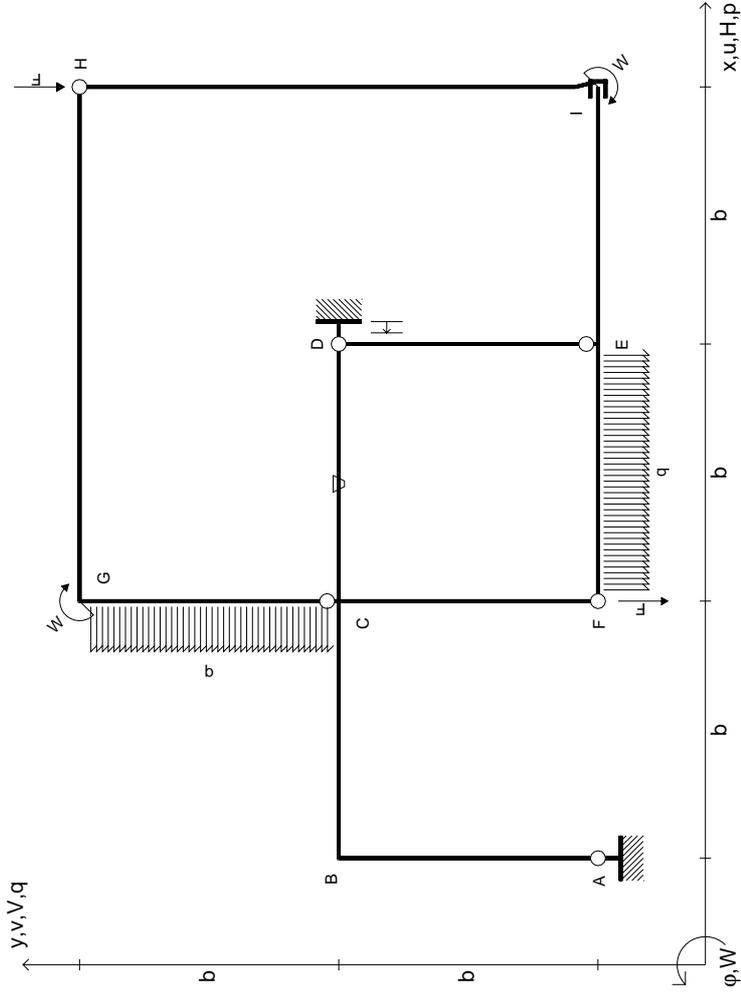
$$= \left(\frac{1}{4} b - \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

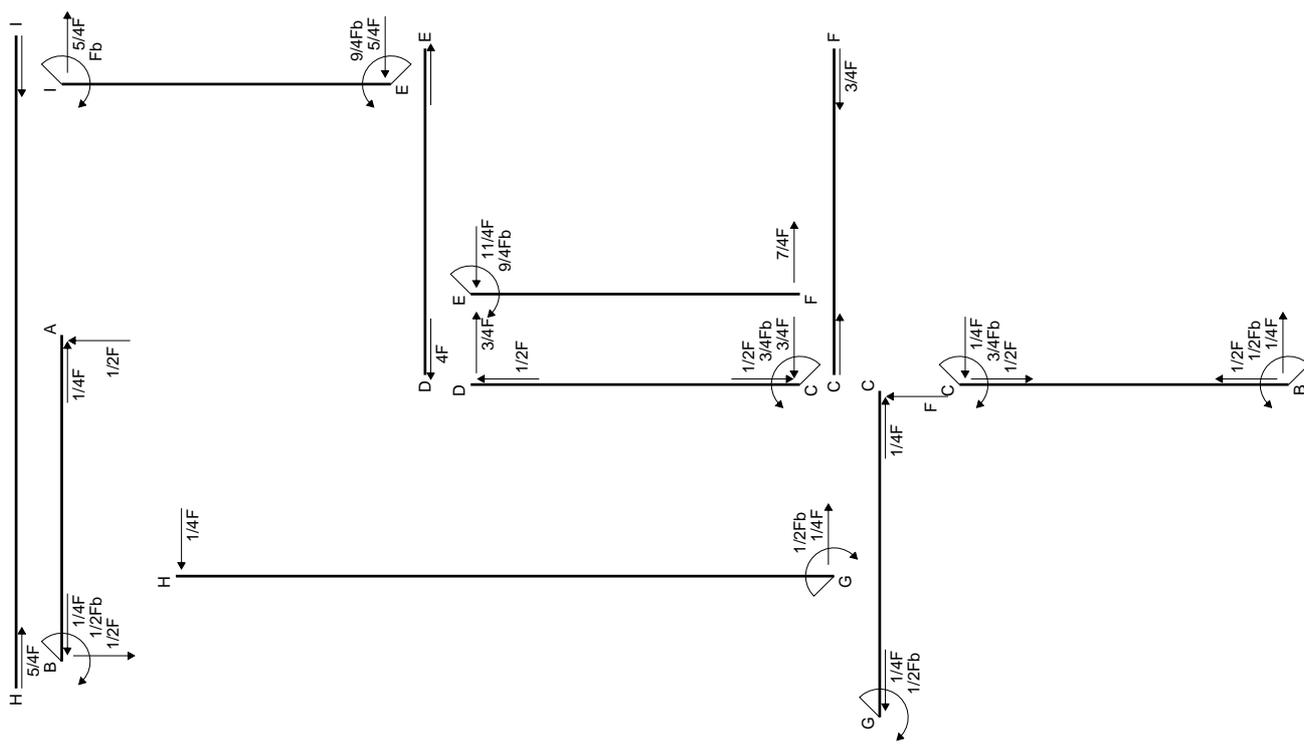


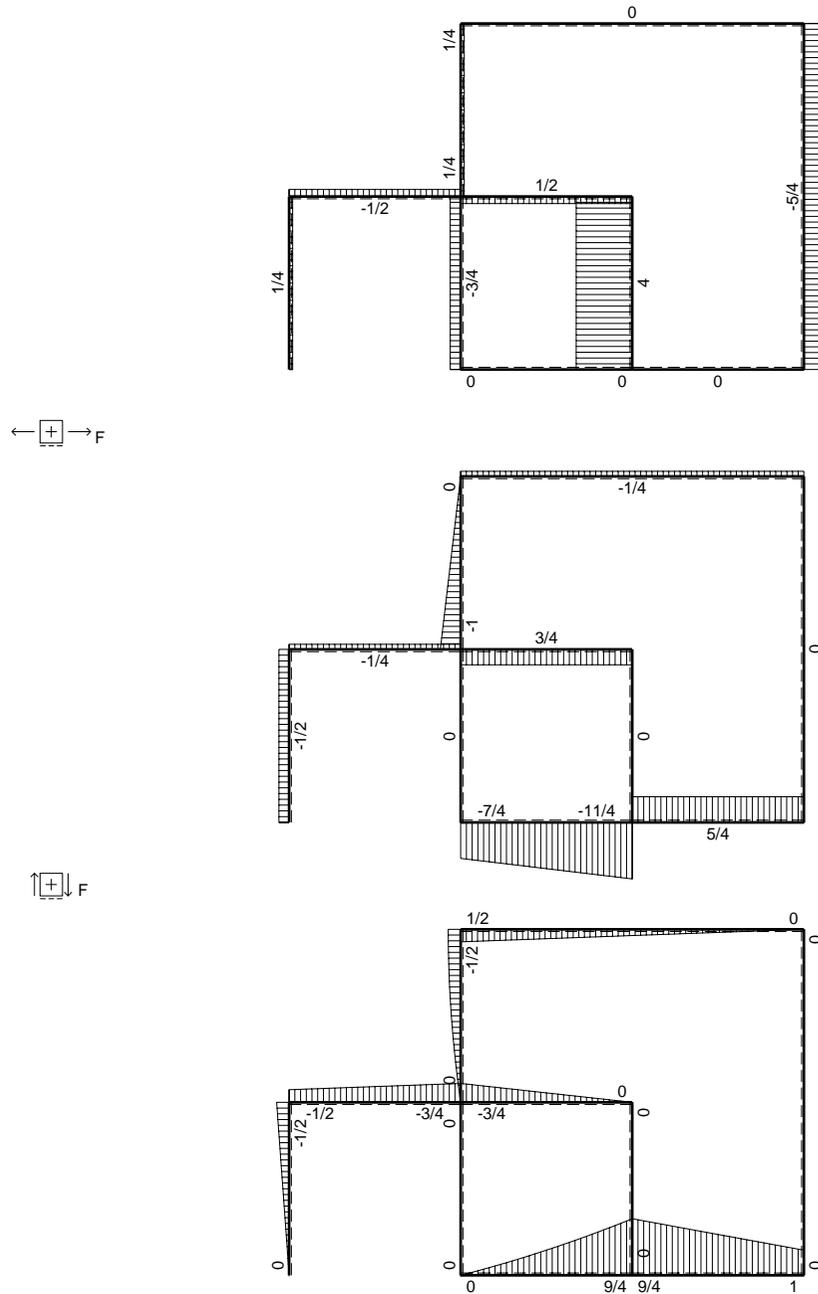
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{CG} &= -q = -F/b \\
 q_{EF} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{CD} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
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 \end{aligned}$$

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 Curvatura θ asta CD positiva se convessa a destra con inizio C.
 Spostamento orizzontale assoluto u imposto al nodo D.

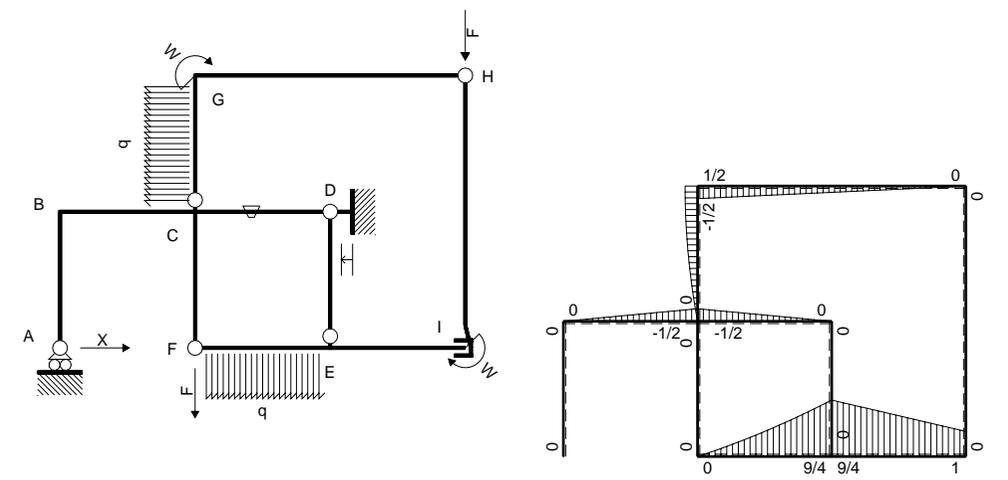




← (+) → F

↑ (+) ↓ F

⊕ (+) ⊖ (-) F_b



Schema di calcolo iperstatico

⊕ (+) ⊖ (-) M₀ flessione da carichi assegnati

⊕ (+) ⊖ (-) M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$	
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$			
BC b	-b+1/2x	-1/2Fx	0	$1/2Fbx-1/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
CB b	$1/2b+1/2x$	$1/2Fb-1/2Fx$	0	$1/4Fb^2-1/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	$-Fb/EJ$	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$	
DC b	$1/2x$	$1/2Fx$	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	$9/4Fb-11/4Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-7/4Fx-1/2qx^2$	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0	
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0			
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0	
HG 2b	0	$-1/4Fx$	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	$Fb+5/4Fx$	0	0	0	0	0+0	0	
EI b	0	$-9/4Fb+5/4Fx$	0	0	0	0			
D	cedimento nodo $-H_{1D}u_D$							$-Fb^3/EJ$	
	totali							$-1/2Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$							$1/2F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

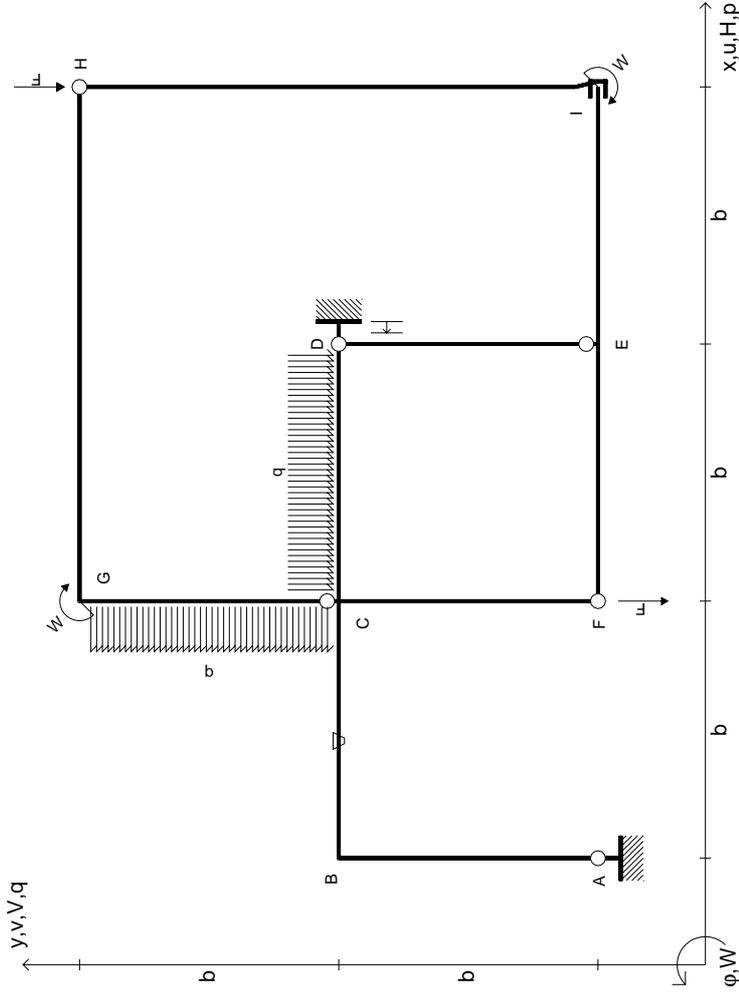
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb^2 1/EJ + (-1/4 b) \theta = 1/3 Fb^3/EJ$$

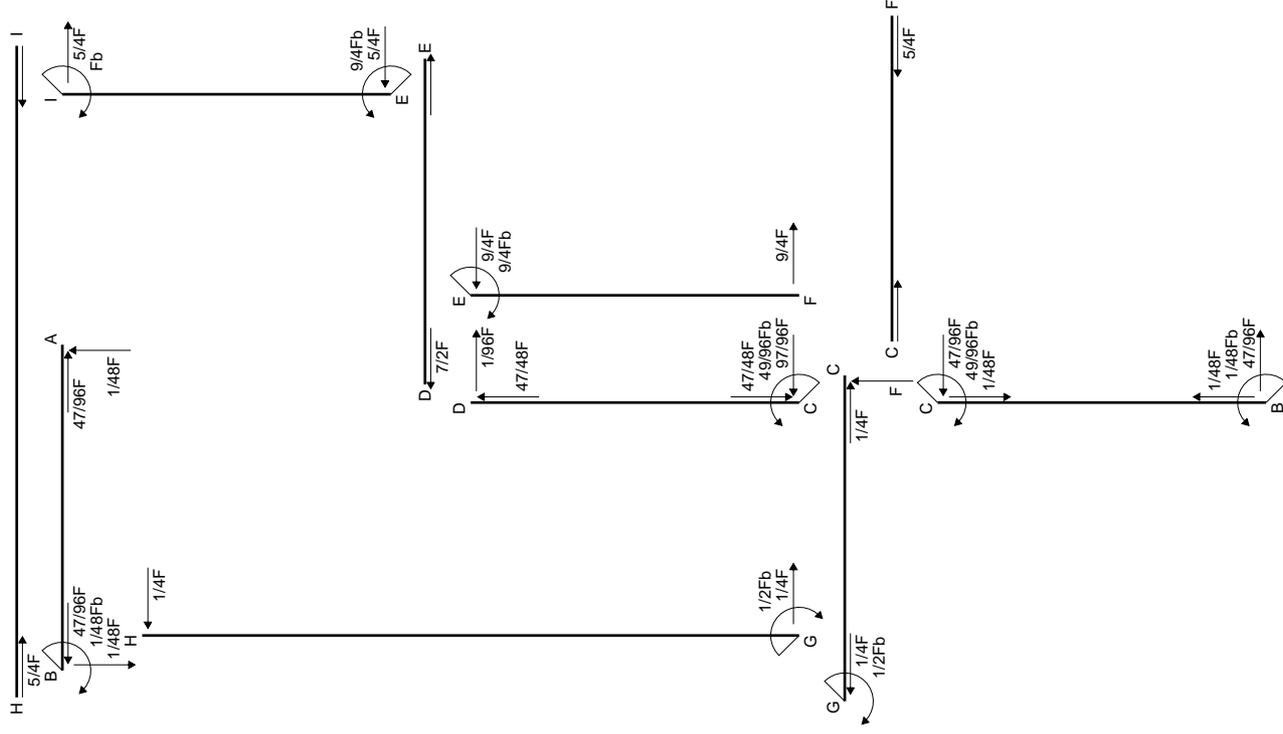


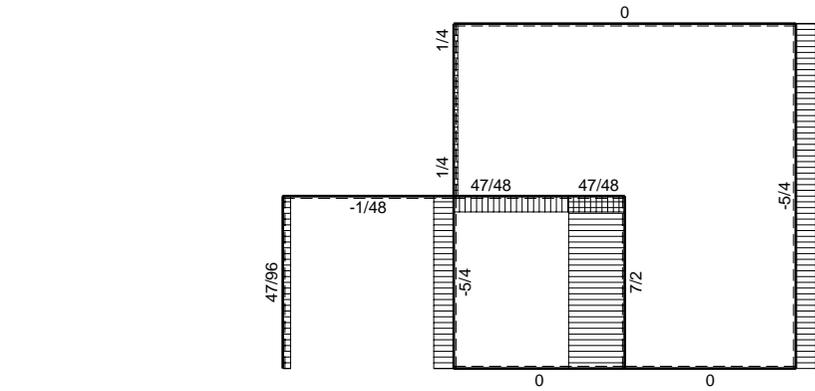
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_1 &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{CG} &= -q = -F/b \\
 q_{CD} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

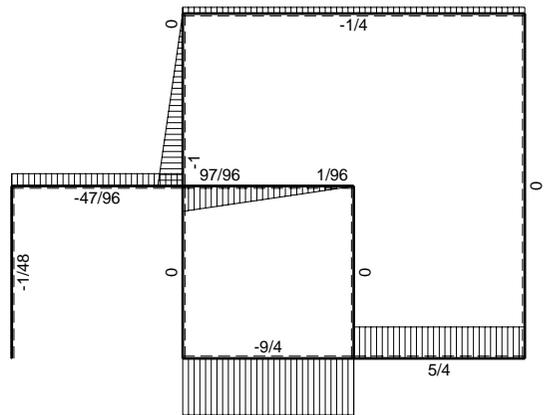
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
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 Spostamento orizzontale assoluto u imposto al nodo D.

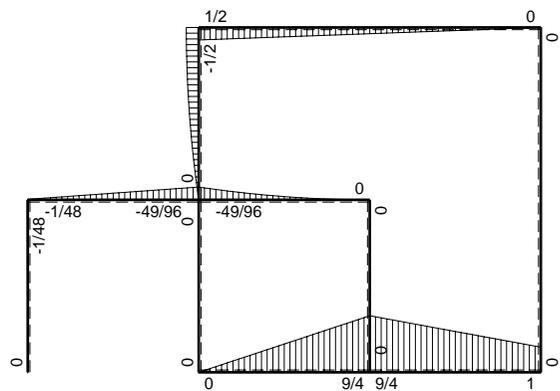




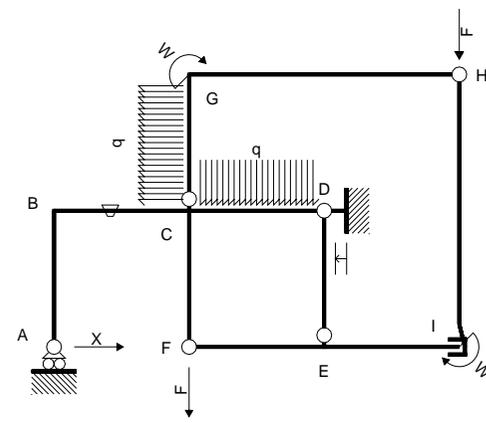
← (+) → F



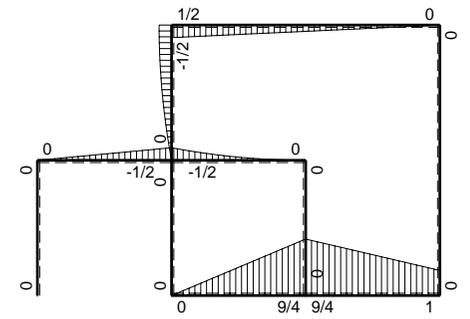
↑ (+) ↓ F



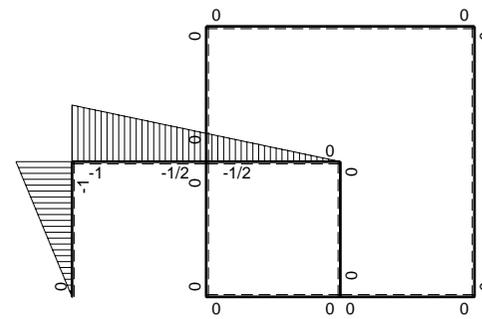
⤵ (+) ⤴ F_b



Schema di calcolo iperstatico



⤵ (+) ⤴ M_o flessione da carichi assegnati



⤵ (+) ⤴ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_A

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	-x	0	0	0	0	x ²	0+0	1/3Xb ³ /EJ	
BA b	b-x	0	0	0	0	b ² -2bx+x ²			
BC b	-b+1/2x	-1/2Fx	-Fb/EJ	1/2Fbx-1/4Fx ²	Fb ² /EJ-1/2Fxb/EJ	b ² -bx+1/4x ²	(1/6+3/4)Fb ³ /EJ	7/12Xb ³ /EJ	
CB b	1/2b+1/2x	1/2Fb-1/2Fx	Fb/EJ	1/4Fb ² -1/4Fx ²	1/2Fb ² /EJ+1/2Fxb/EJ	1/4b ² +1/2bx+1/4x ²			
CD b	-1/2b+1/2x	-1/2Fb+Fx-1/2qx ²	0	1/4Fb ² -3/4Fbx+3/4Fx ² -1/4qx ³	0	1/4b ² -1/2bx+1/4x ²	(1/16+0)Fb ³ /EJ	1/12Xb ³ /EJ	
DC b	1/2x	1/2qx ²	0	1/4qx ³	0	1/4x ²			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0	
FE b	0	-9/4Fx	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	-Fx+1/2qx ²	0	0	0	0	0+0	0	
GC b	0	1/2Fb-1/2qx ²	0	0	0	0			
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0	
HG 2b	0	-1/4Fx	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0	
EI b	0	-9/4Fb+5/4Fx	0	0	0	0			
D	cedimento nodo -H _{1D} u _D							-Fb ³ /EJ	
	totali							-1/48Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _A							1/48F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ + (b - 1/4 b) \theta = 11/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = 11/12 Fb^3/EJ$$

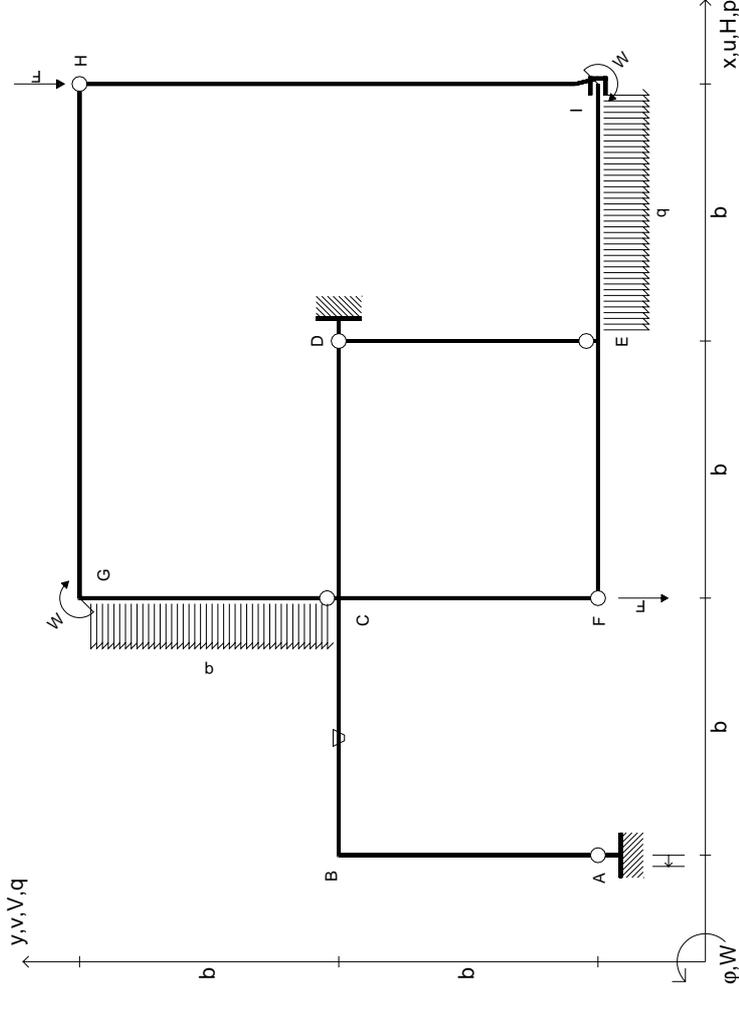
$$L_{CD}^{xo} = \int_0^b (1/4 - 3/4 x/b + 3/4 x^2/b^2 - 1/4 x^3/b^3) Fb^2 1/EJ dx$$

$$= [1/4 x - 3/8 x^2/b + 1/4 x^3/b^2 - 1/16 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 3/8 b + 1/4 b - 1/16 b) Fb^2 1/EJ = 1/16 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^3/b^3) Fb^2 1/EJ dx = [1/16 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (1/16 b) Fb^2 1/EJ = 1/16 Fb^3/EJ$$

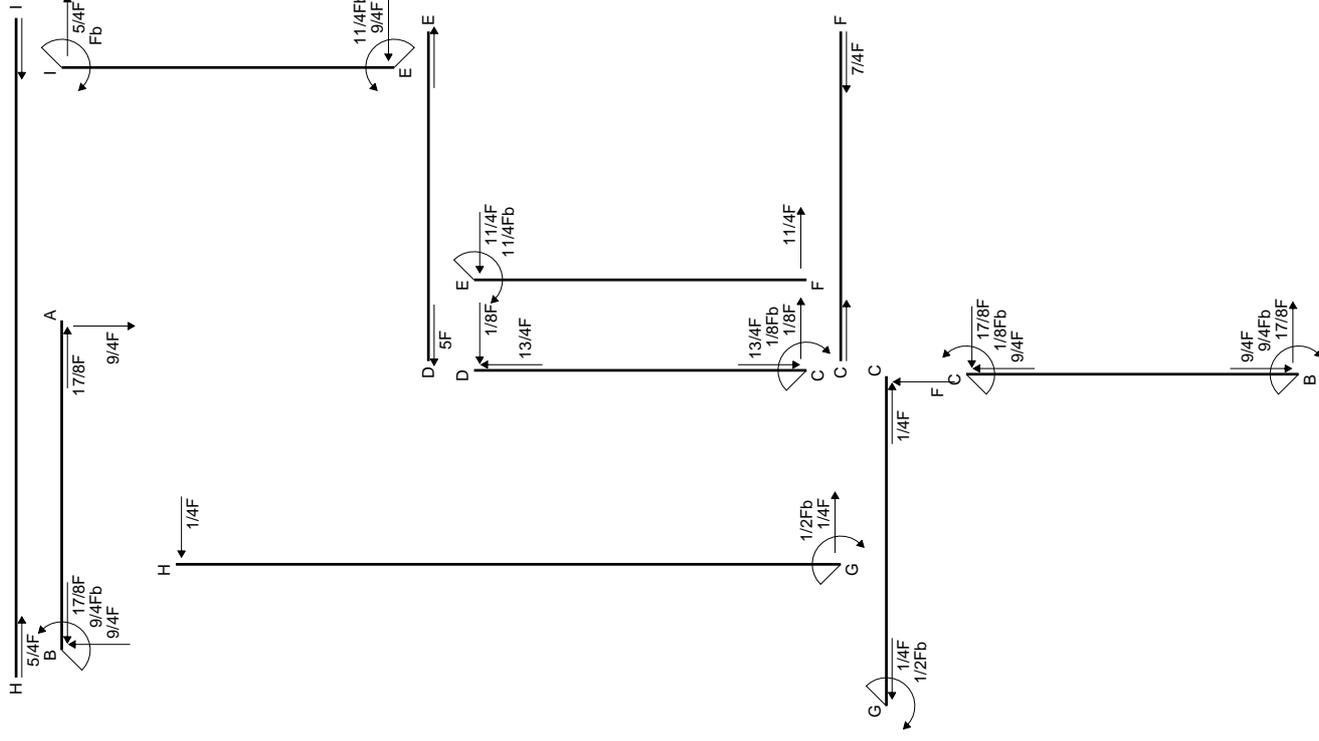


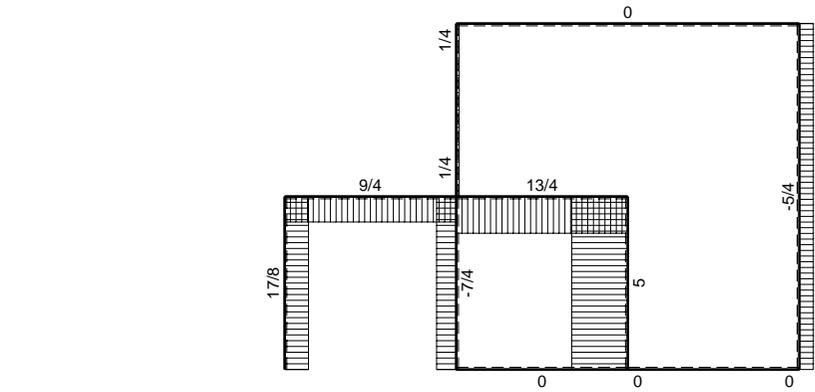
$$\begin{aligned}
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 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{CG} &= -q = -F/b \\
 q_{IE} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 U_A &= -\delta = -b^3F/EJ \\
 EJ_{AB} &= EJ \\
 EJ_{BC} &= EJ \\
 EJ_{CD} &= EJ \\
 EJ_{DE} &= EJ
 \end{aligned}$$

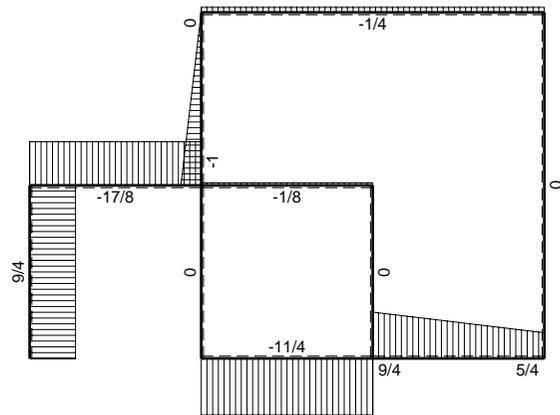
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 EJ_{EF} &= EJ \\
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Reazioni iperstatiche in soluzione: $X=H_A$
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 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo A.

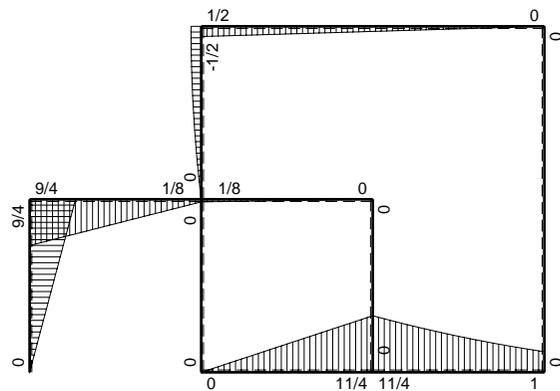




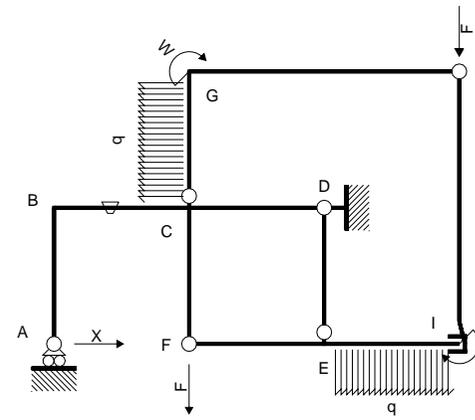
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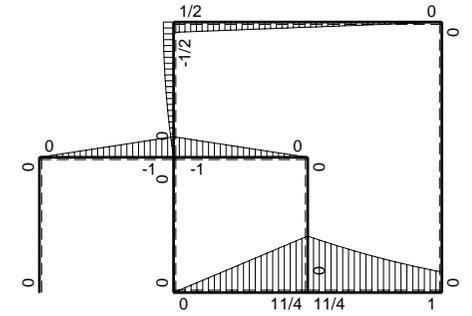
↑ (+) ↓ F



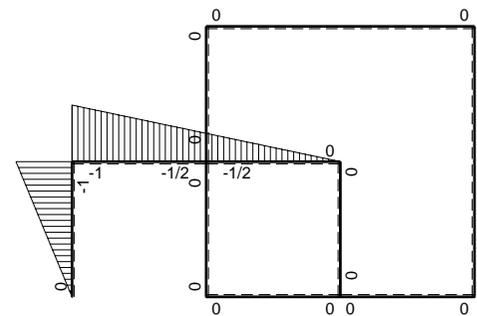
⊕ (+) ⊖ F_b



Schema di calcolo iperstatico



⊕ (+) ⊖ M_o flessione da carichi assegnati



⊕ (+) ⊖ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3 X b^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-Fx	-Fb/EJ	$Fbx-1/2Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/3+3/4)Fb^3/EJ$	$7/12 X b^3/EJ$
CB b	$1/2b+1/2x$	Fb-Fx	Fb/EJ	$1/2Fb^2-1/2Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	-Fb+Fx	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12 X b^3/EJ$
DC b	$1/2x$	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$11/4Fb-11/4Fx$	0	0	0	0	0+0	0
FE b	0	$-11/4Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/4Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-11/4Fb+9/4Fx-1/2qx^2$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$9/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						$-9/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

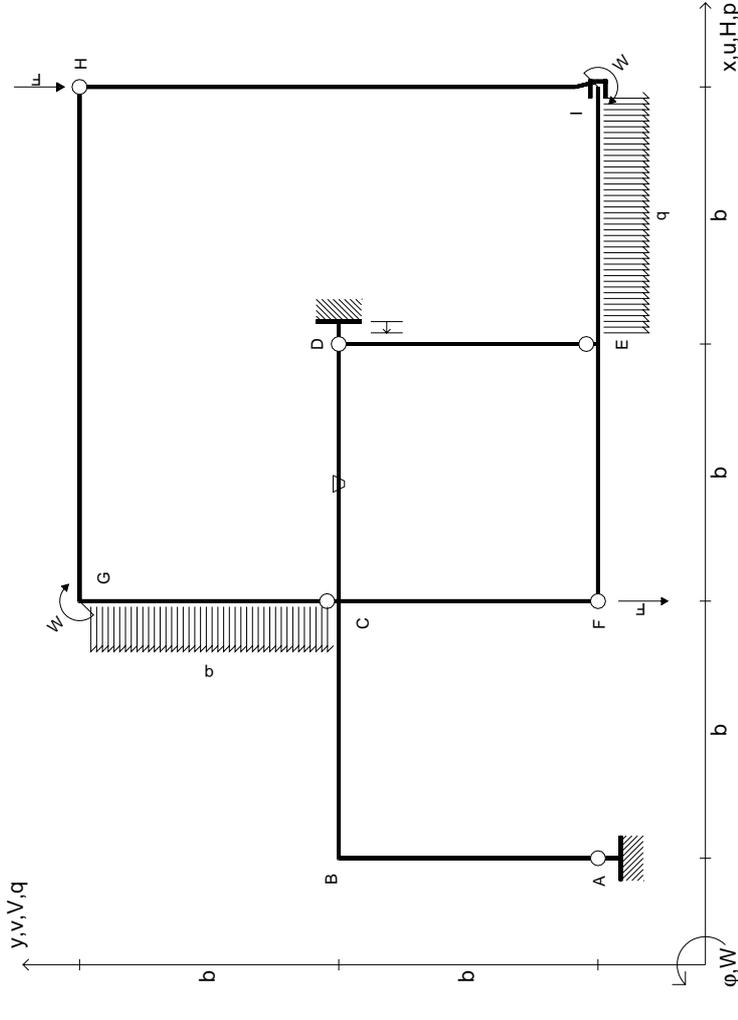
$$= (1/2 b - 1/6 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

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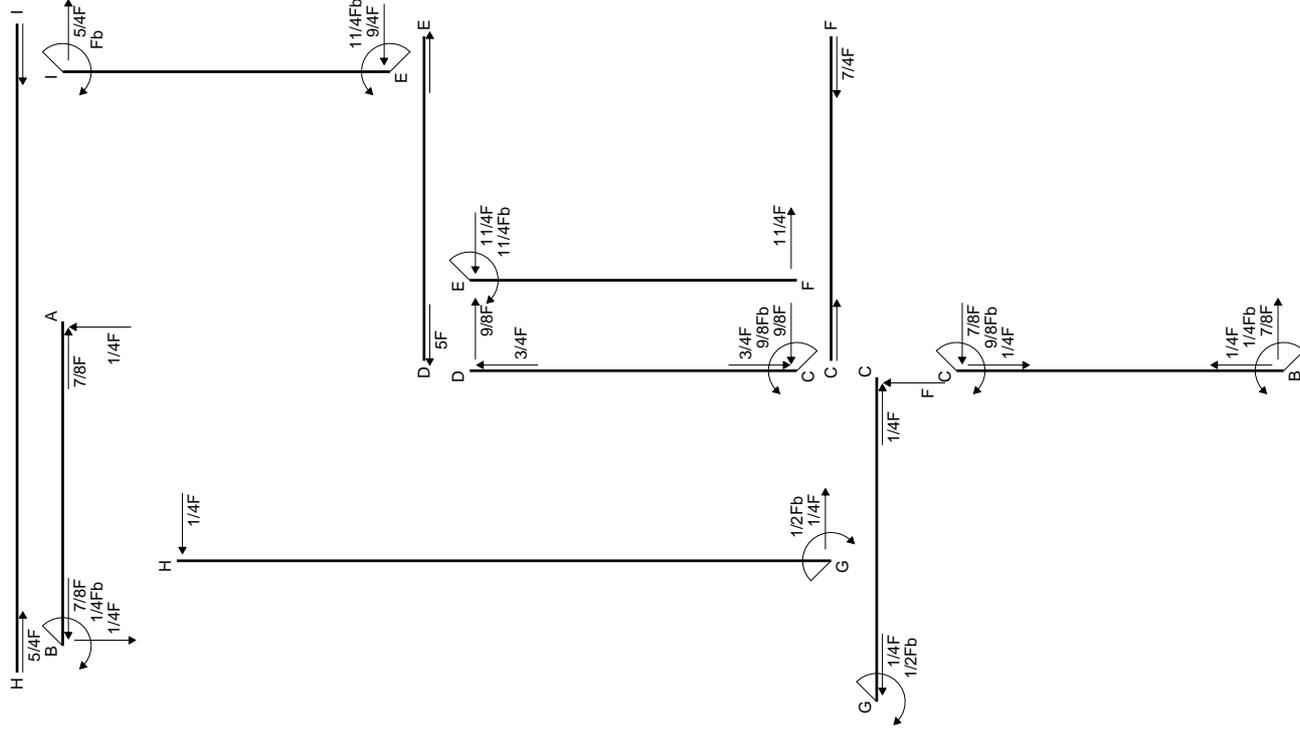


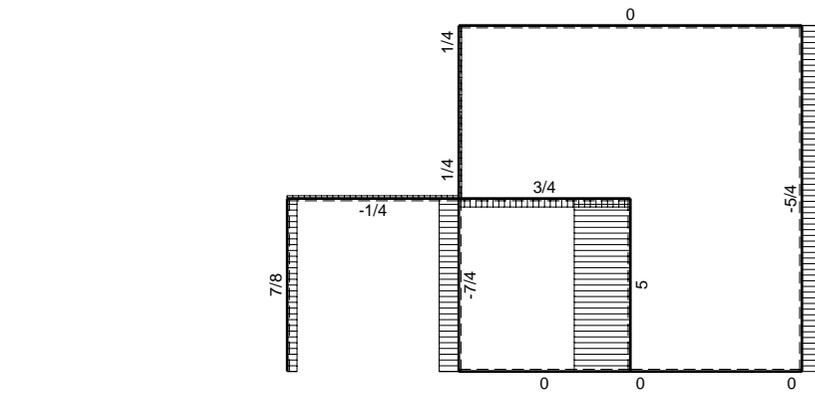
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 u_D &= -\delta = -b^3 F/EJ \\
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 E_{J_{DE}} &= EJ
 \end{aligned}$$

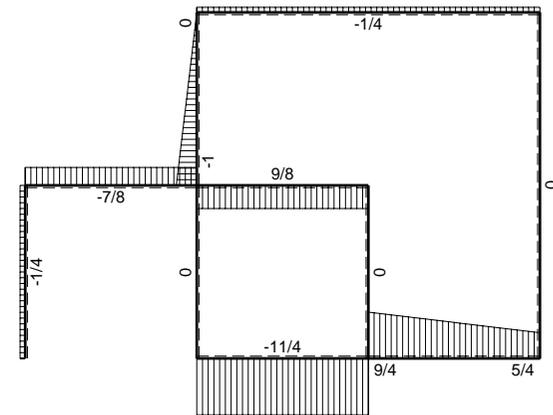
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 Curvatura θ asta CD positiva se convessa a destra con inizio C.
 Spostamento orizzontale assoluto u imposto al nodo D.

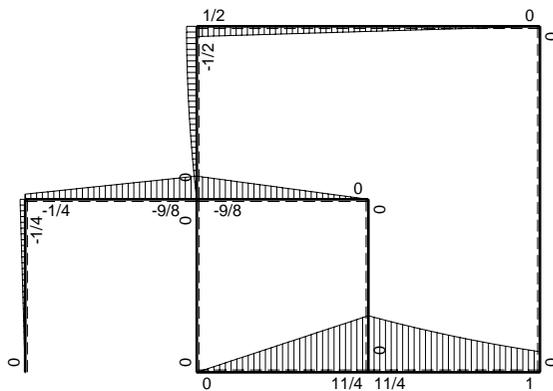




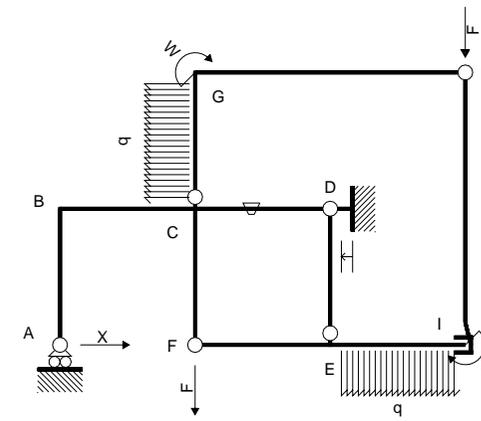
← ⊕ → F



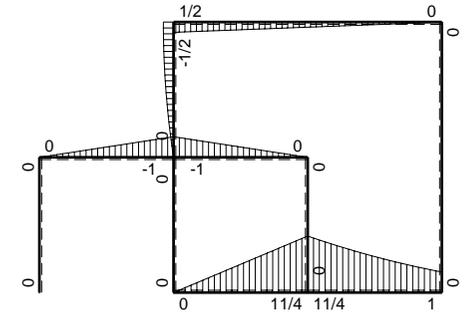
↑ ⊕ ↓ F



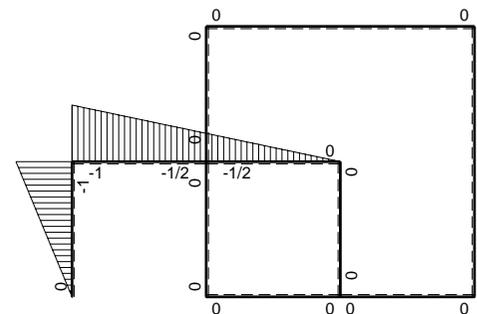
⊕ ↻ F_b



Schema di calcolo iperstatico



⊕ ↻ M_o flessione da carichi assegnati



⊕ ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	-x	0	0	0	0	x^2	0+0	$1/3 X b^3/EJ$	
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$			
BC b	-b+1/2x	-Fx	0	$Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(1/3+0)Fb^3/EJ$	$7/12 X b^3/EJ$	
CB b	$1/2b+1/2x$	$Fb-Fx$	0	$1/2Fb^2-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
CD b	$-1/2b+1/2x$	$-Fb+Fx$	$-Fb/EJ$	$1/2Fb^2-Fbx+1/2Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/6+1/4)Fb^3/EJ$	$1/12 X b^3/EJ$	
DC b	$1/2x$	Fx	Fb/EJ	$1/2Fx^2$	$1/2Fxb/EJ$	$1/4x^2$			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	$11/4Fb-11/4Fx$	0	0	0	0	0+0	0	
FE b	0	$-11/4Fx$	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0	
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0			
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0	
HG 2b	0	$-1/4Fx$	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	$Fb+5/4Fx+1/2qx^2$	0	0	0	0	0+0	0	
EI b	0	$-11/4Fb+9/4Fx-1/2qx^2$	0	0	0	0			
D	cedimento nodo $-H_{1D}u_D$							$-Fb^3/EJ$	
	totali							$-1/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$							$1/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(\frac{x}{b} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{2} \frac{x^2}{b} - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{3} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(\frac{1}{2} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{2} x - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{3} Fb^3/EJ$$

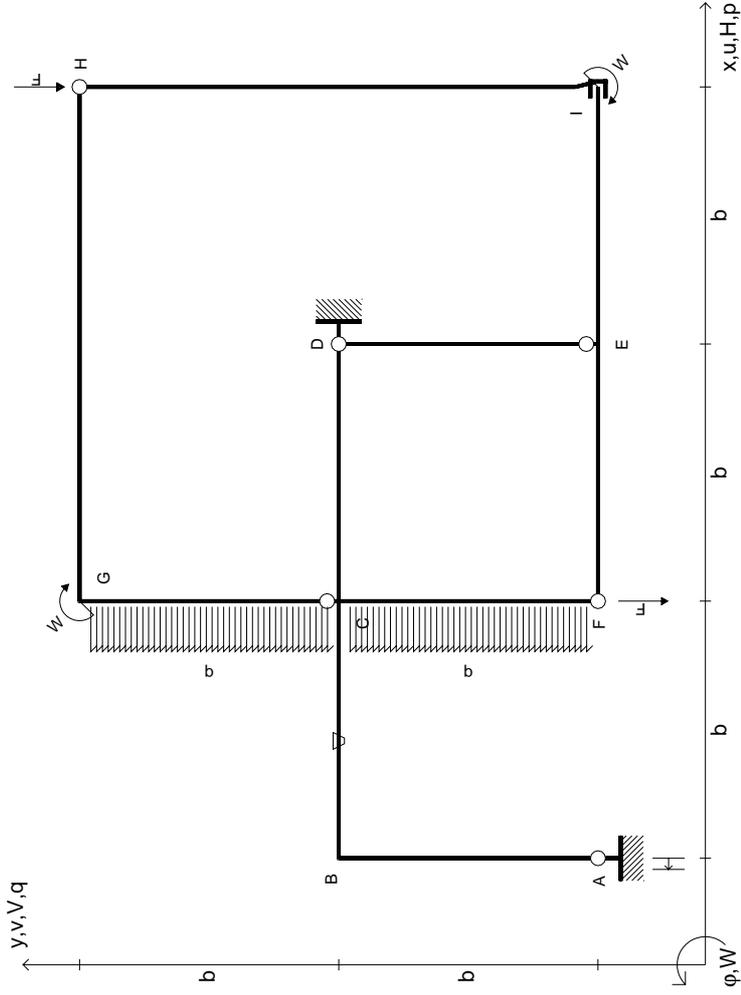
$$L_{CD}^{xo} = \int_0^b \left(\frac{1}{2} - \frac{x}{b} + \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{1}{2} x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{1}{2} b - \frac{1}{2} b + \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} + \left(\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{5}{12} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(\frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(-\frac{1}{2} \frac{x}{b} \right) \theta dx = \left[\frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[-\frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{1}{6} b \right) Fb^2 \frac{1}{EJ} + \left(-\frac{1}{4} b \right) \theta = \frac{5}{12} Fb^3/EJ$$

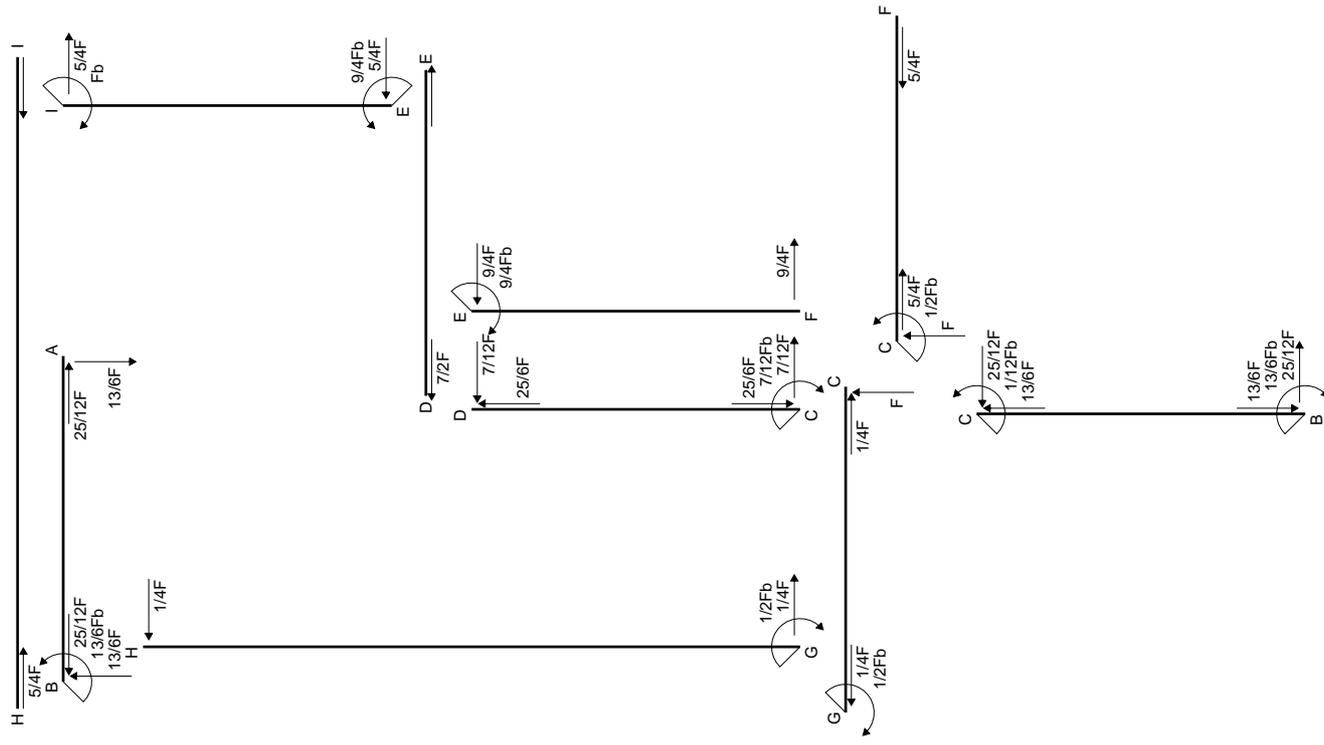


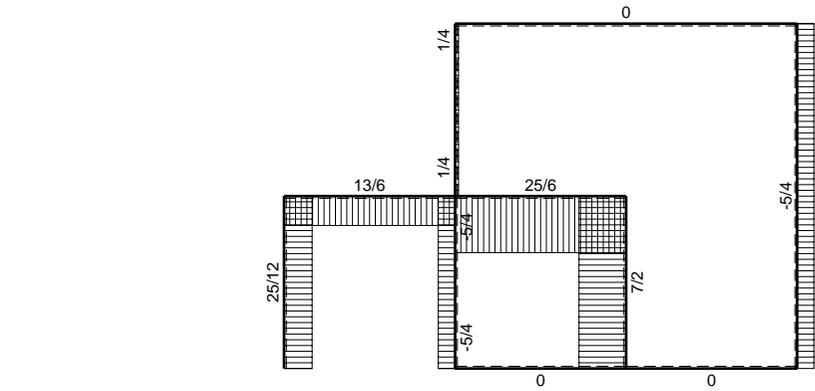
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 P_{CG} &= -q = -F/b \\
 P_{FC} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 U_A &= -\delta = -b^3F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

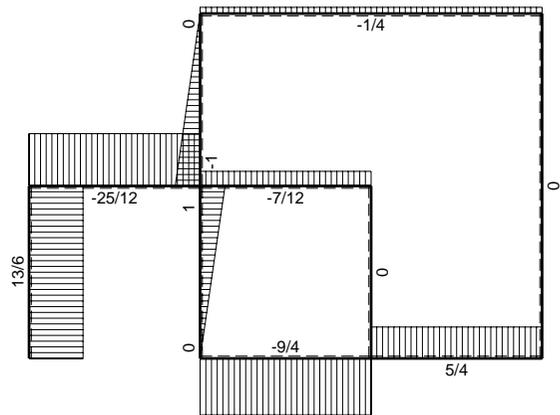
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=V_D$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y .
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo A.

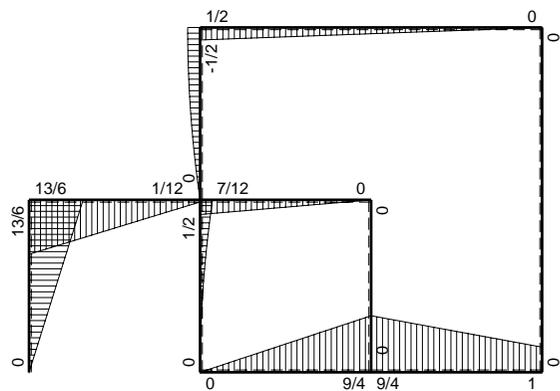




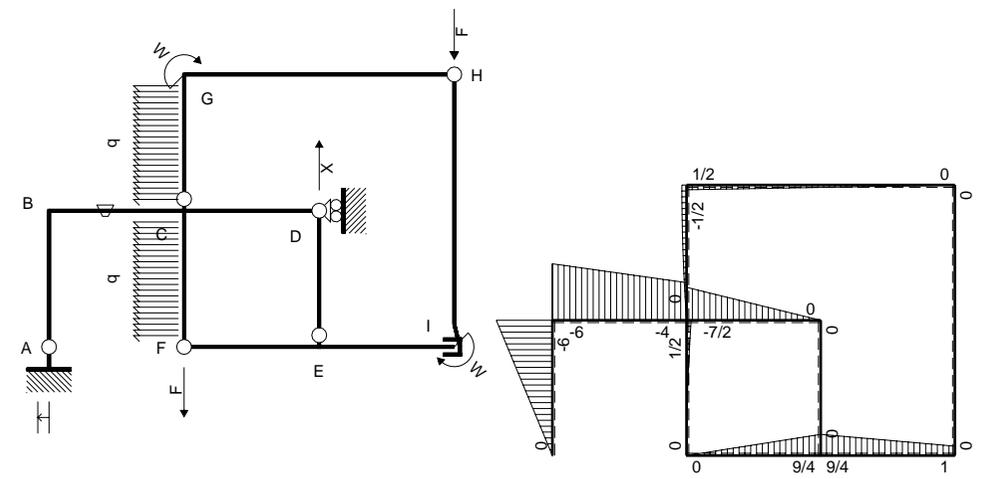
← (+) → F



↑ (+) ↓ F

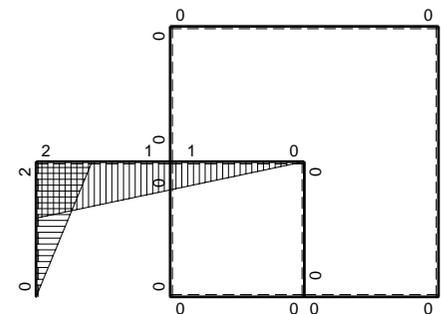


⊕ F_b



Schema di calcolo iperstatico

⊕ M₀ flessione da carichi assegnati



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_D$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	2x	-6Fx	0	-12Fx ²	0	4x ²	(-4+0)Fb ³ /EJ	4/3Xb ³ /EJ	
BA b	-2b+2x	6Fb-6Fx	0	-12Fb ² +24Fbx-12Fx ²	0	4b ² -8bx+4x ²			
BC b	2b-x	-6Fb+2Fx	-Fb/EJ	-12Fb ² +10Fbx-2Fx ²	-2Fb ² /EJ+Fxb/EJ	4b ² -4bx+x ²	(-23/3-3/2)Fb ³ /EJ	7/3Xb ³ /EJ	
CB b	-b-x	4Fb+2Fx	Fb/EJ	-4Fb ² -6Fbx-2Fx ²	-Fb ² /EJ-Fxb/EJ	b ² +2bx+x ²			
CD b	b-x	-7/2Fb+7/2Fx	0	-7/2Fb ² +7Fbx-7/2Fx ²	0	b ² -2bx+x ²	(-7/6+0)Fb ³ /EJ	1/3Xb ³ /EJ	
DC b	-x	7/2Fx	0	-7/2Fx ²	0	x ²			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0	
FE b	0	-9/4Fx	0	0	0	0			
FC b	0	1/2qx ²	0	0	0	0	0+0	0	
CF b	0	-1/2Fb+Fx-1/2qx ²	0	0	0	0			
CG b	0	-Fx+1/2qx ²	0	0	0	0	0+0	0	
GC b	0	1/2Fb-1/2qx ²	0	0	0	0			
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0	
HG 2b	0	-1/4Fx	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0	
EI b	0	-9/4Fb+5/4Fx	0	0	0	0			
A	cedimento nodo -H _{1A} u _A							-2Fb ³ /EJ	
	totali							-49/3Fb ³ /EJ	4Xb ³ /EJ
	iperstatica X=V _D							49/12F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = [4 x - 4 x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = [4 x - 2 x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-12 x^2/b^2) Fb^2 1/EJ dx = [-4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-4 b) Fb^2 1/EJ = -4 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-12 + 24 x/b - 12 x^2/b^2) Fb^2 1/EJ dx = [-12 x + 12 x^2/b - 4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-12 b + 12 b - 4 b) Fb^2 1/EJ = -4 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-12 + 10 x/b - 2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-2 + x/b) \theta dx$$

$$= [-12 x + 5 x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ + [-2 x + 1/2 x^2/b]_0^b \theta$$

$$= (-12 b + 5 b - 2/3 b) Fb^2 1/EJ + (-2 b + 1/2 b) \theta = -55/6 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-4 - 6 x/b - 2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 + x/b) \theta dx$$

$$= [-4 x - 3 x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ + [x + 1/2 x^2/b]_0^b \theta$$

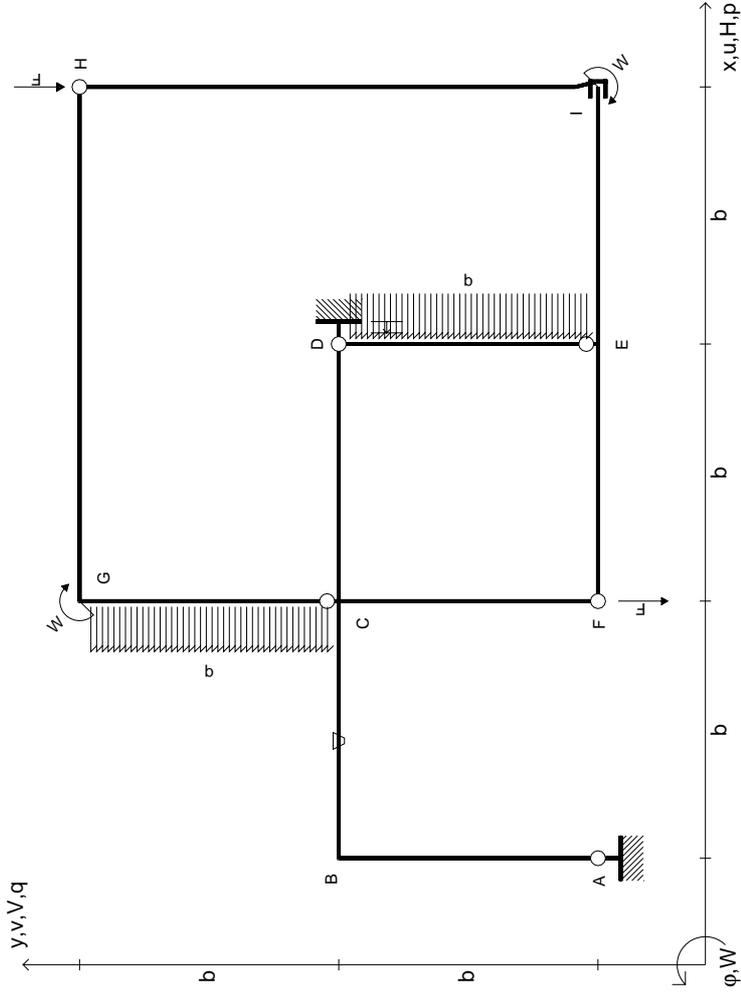
$$= (-4 b - 3 b - 2/3 b) Fb^2 1/EJ + (b + 1/2 b) \theta = -55/6 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (-7/2 + 7 x/b - 7/2 x^2/b^2) Fb^2 1/EJ dx = [-7/2 x + 7/2 x^2/b - 7/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-7/2 b + 7/2 b - 7/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (-7/2 x^2/b^2) Fb^2 1/EJ dx = [-7/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-7/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$



$$V_H = -F$$

$$V_F = -F$$

$$W_I = -W = +Fb$$

$$W_G = -W = -Fb$$

$$P_{CG} = -q = -F/b$$

$$P_{DE} = -q = -F/b$$

$$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$$

$$u_D = -\delta = -b^3 F/EJ$$

$$EJ_{AB} = EJ$$

$$EJ_{BC} = EJ$$

$$EJ_{CD} = EJ$$

$$EJ_{DE} = EJ$$

$$EJ_{EF} = EJ$$

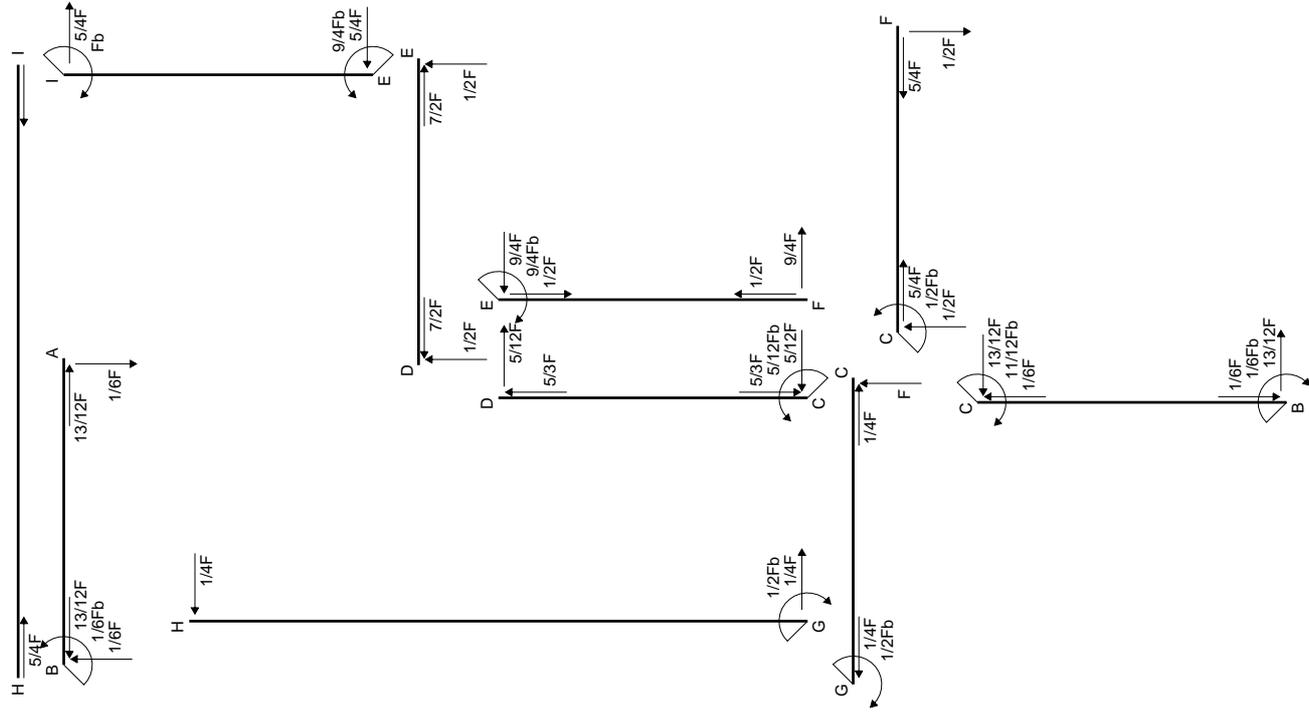
$$EJ_{FC} = EJ$$

$$EJ_{CG} = EJ$$

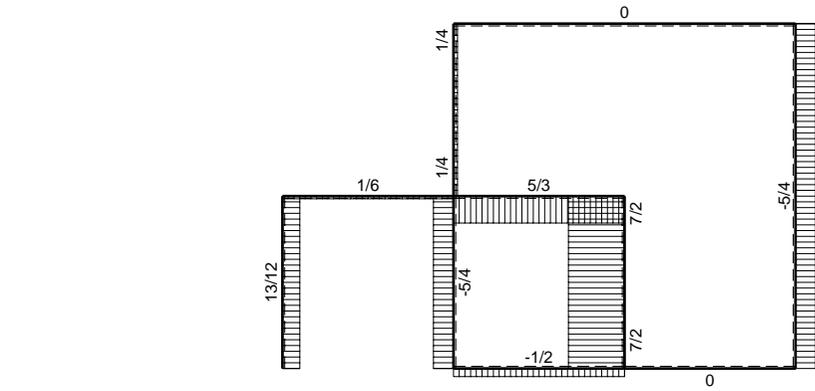
$$EJ_{GH} = EJ$$

$$EJ_{HI} = EJ$$

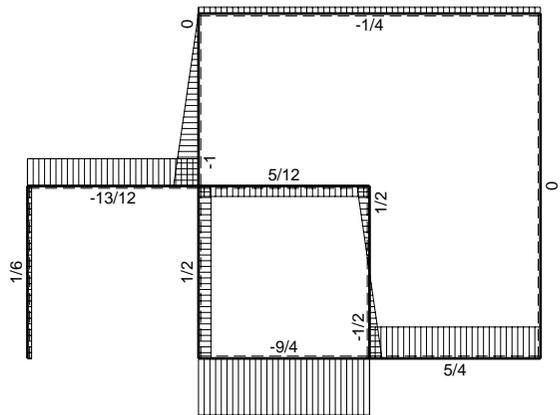
$$EJ_{IE} = EJ$$



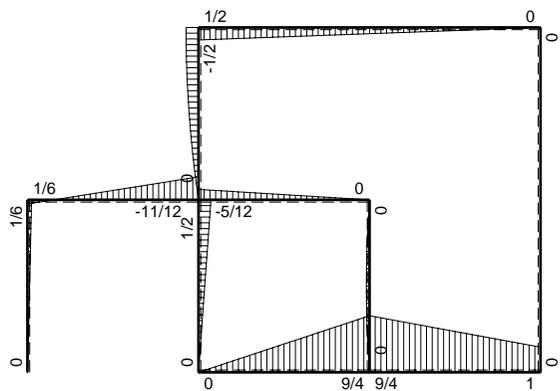
Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
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 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo D.



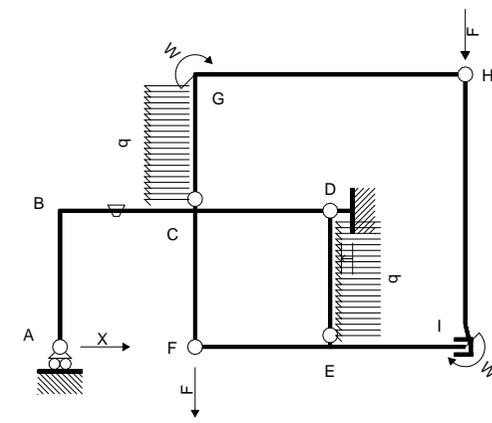
← (+) → F



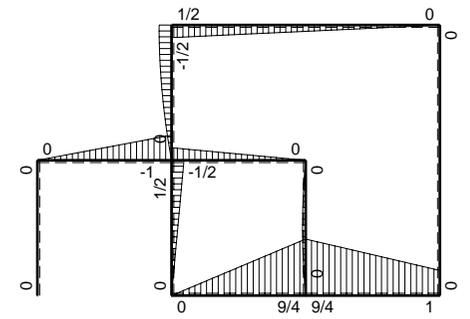
↑ (+) ↓ F



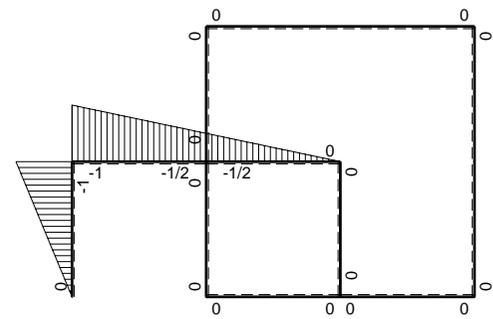
⤵ (+) ⤴ F_b



Schema di calcolo iperstatico



⤵ (+) ⤴ M_o flessione da carichi assegnati



⤵ (+) ⤴ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3 X b^3 / EJ$
BA b	b-x	0	0	0	0	$b^2 - 2bx + x^2$		
BC b	-b+1/2x	-Fx	-Fb/EJ	$Fbx - 1/2Fx^2$	$Fb^2/EJ - 1/2Fxb/EJ$	$b^2 - bx + 1/4x^2$	$(1/3+3/4)Fb^3/EJ$	$7/12 X b^3 / EJ$
CB b	$1/2b + 1/2x$	Fb-Fx	Fb/EJ	$1/2Fb^2 - 1/2Fx^2$	$1/2Fb^2/EJ + 1/2Fxb/EJ$	$1/4b^2 + 1/2bx + 1/4x^2$		
CD b	$-1/2b + 1/2x$	$-1/2Fb + 1/2Fx$	0	$1/4Fb^2 - 1/2Fbx + 1/4Fx^2$	0	$1/4b^2 - 1/2bx + 1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12 X b^3 / EJ$
DC b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	$1/2Fx - 1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx + 1/2qx^2$	0	0	0	0		
EF b	0	$9/4Fb - 9/4Fx$	0	0	0	0	0+0	0
FE b	0	$-9/4Fx$	0	0	0	0		
FC b	0	$1/2Fx$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb + 1/2Fx$	0	0	0	0		
CG b	0	$-Fx + 1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb - 1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb - 1/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/4Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb + 5/4Fx$	0	0	0	0	0+0	0
EI b	0	$-9/4Fb + 5/4Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						$-1/6F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(\frac{x}{b} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{1}{2} \frac{x^2}{b} - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} + \left(b - \frac{1}{4} b \right) \theta = \frac{13}{12} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(\frac{1}{2} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(-\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{1}{2} x - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[-\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

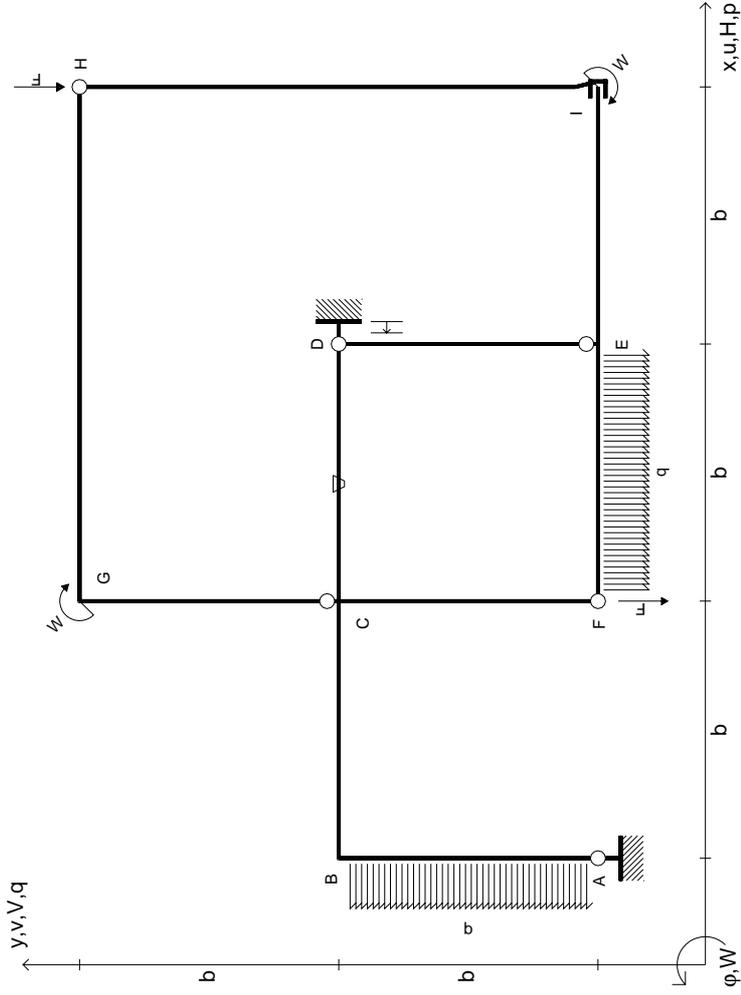
$$= \left(\frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} + \left(-\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{13}{12} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

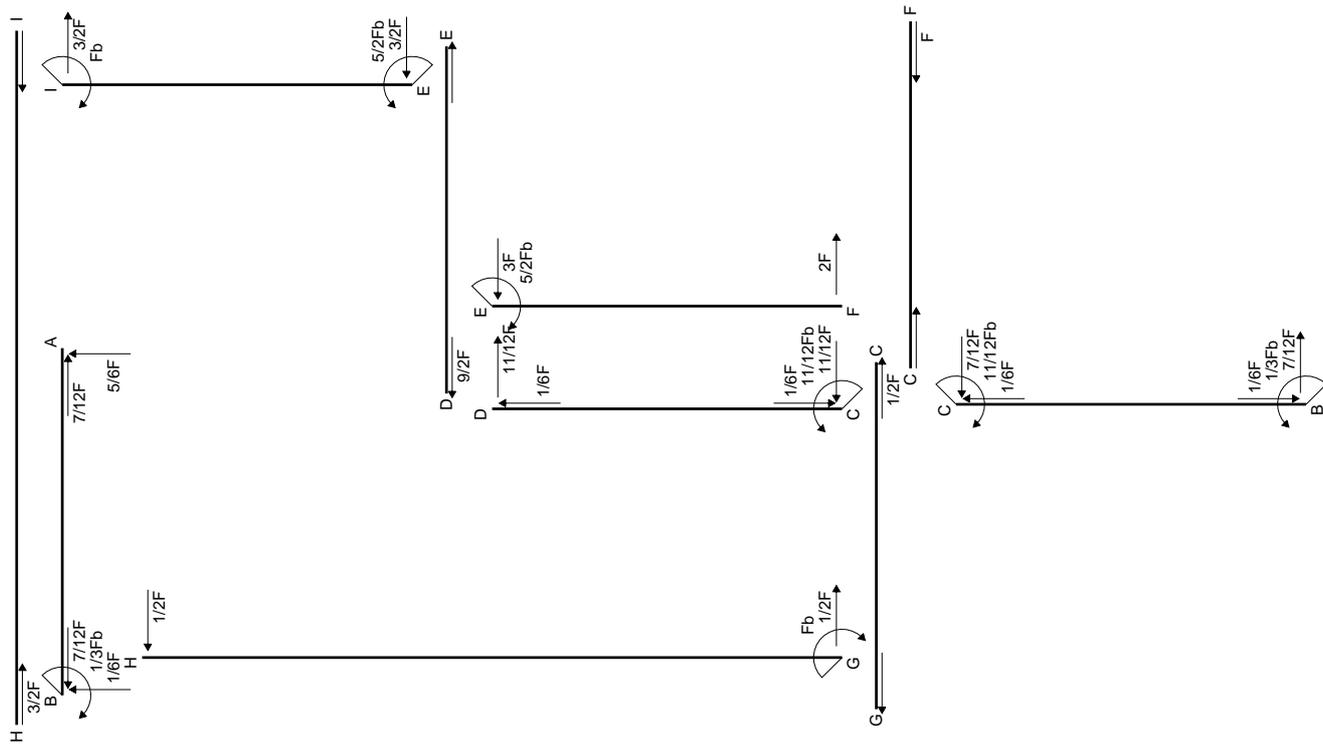


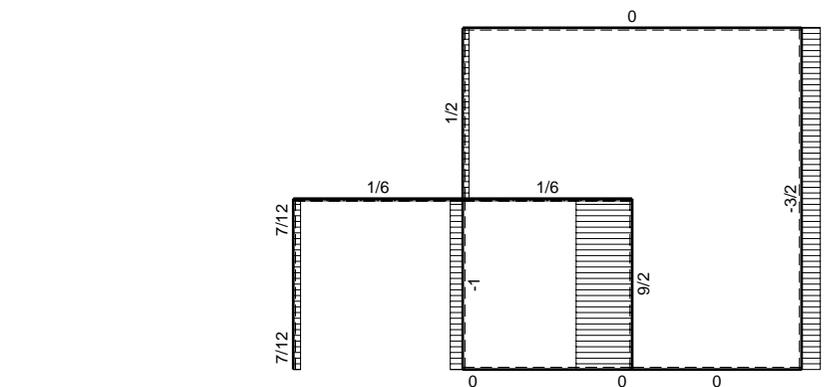
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_1 &= -W = +Fb \\
 W_G &= -W = -Fb \\
 q_{EF} &= -q = +F/b \\
 P_{AB} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{CD} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

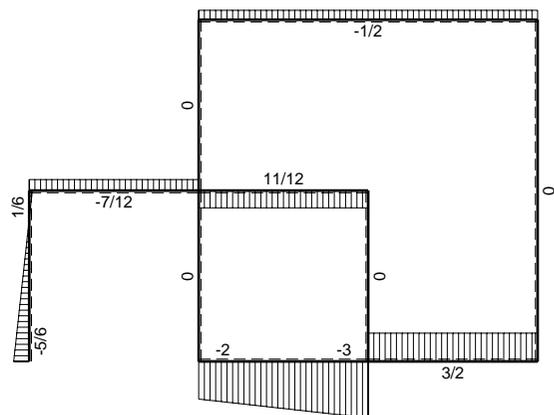
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta CD positiva se convessa a destra con inizio C.
 Spostamento orizzontale assoluto u imposto al nodo D.

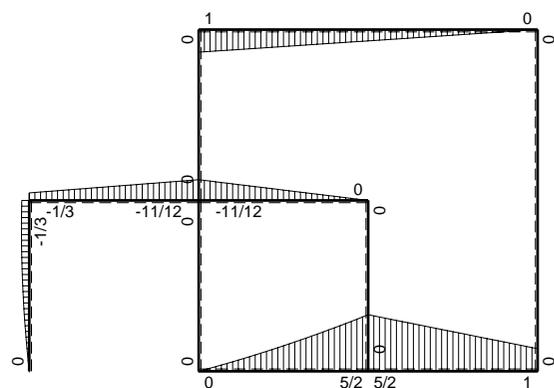




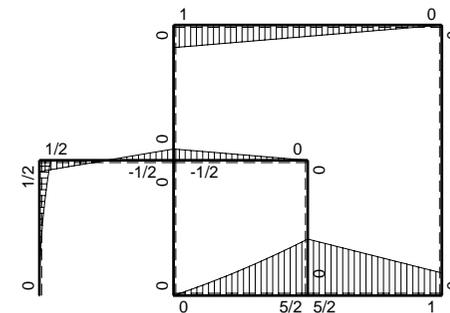
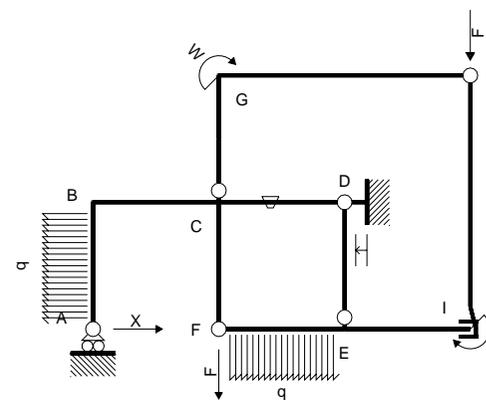
← (+) → F



↑ (+) ↓ F

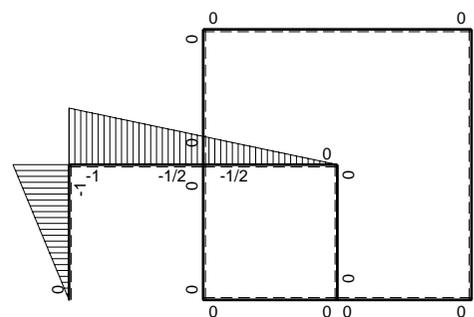


⊙ (+) ⊙ F_b



Schema di calcolo iperstatico

⊙ (+) ⊙ M_o flessione da carichi assegnati



⊙ (+) ⊙ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	$1/2Fb-Fx$	0	$-1/2Fb^2+5/4Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(-1/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$1/2Fb-Fx$	0	$1/4Fb^2-1/4Fbx-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	$-Fb/EJ$	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-5/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						$5/6F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-1/2 x^3/b^3) Fb^2 1/EJ dx = [-1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [-1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + 5/4 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 5/8 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 5/8 b - 1/6 b) Fb^2 1/EJ = -1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/8 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/8 b - 1/6 b) Fb^2 1/EJ = -1/24 Fb^3/EJ$$

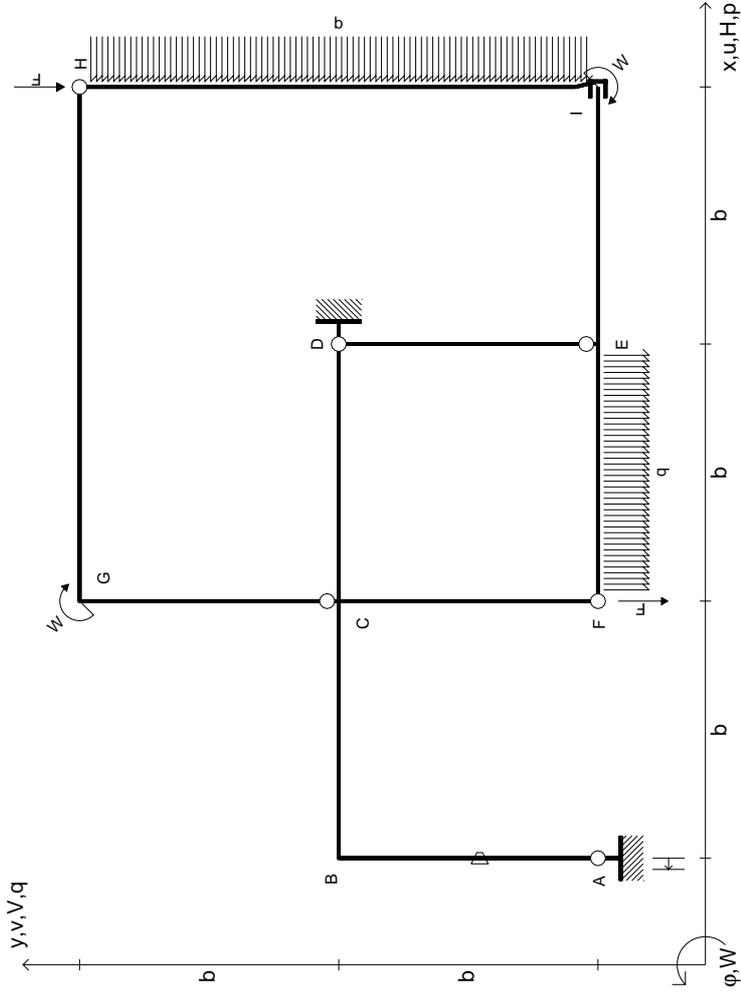
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb^2 1/EJ + (-1/4 b) \theta = 1/3 Fb^3/EJ$$

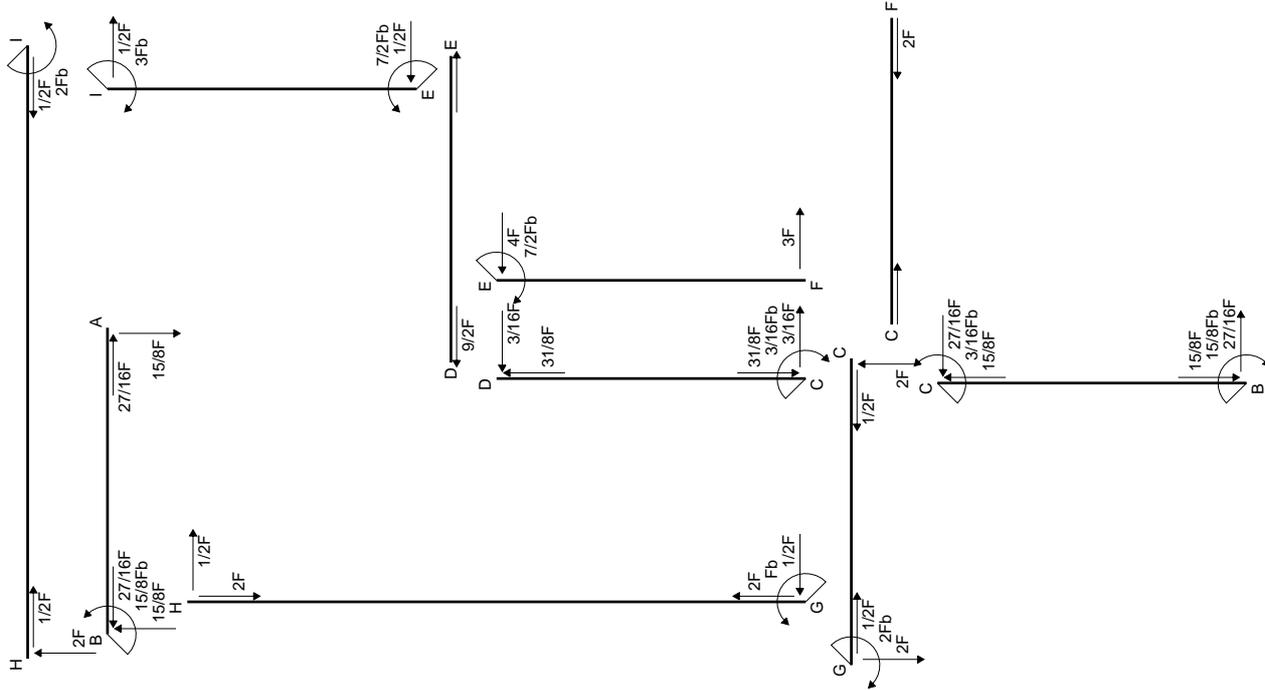


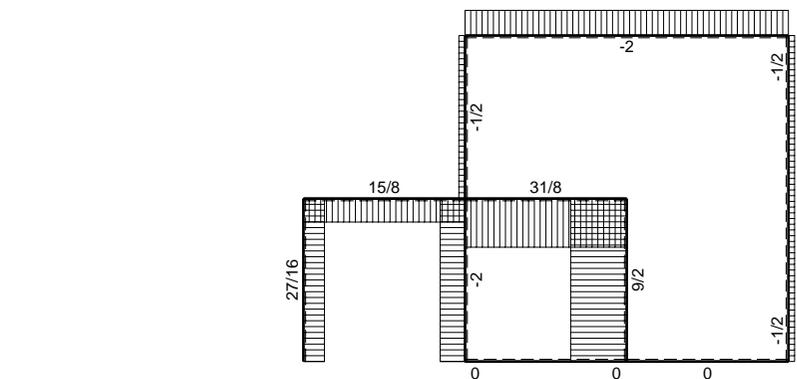
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 q_{EF} &= -q = +F/b \\
 P_{HI} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{AB} &= -\theta = -\alpha T/b = -bF/EJ \\
 U_A &= -\delta = -b^3 F/EJ \\
 EJ_{AB} &= EJ \\
 EJ_{BC} &= EJ \\
 EJ_{CD} &= EJ \\
 EJ_{DE} &= EJ
 \end{aligned}$$

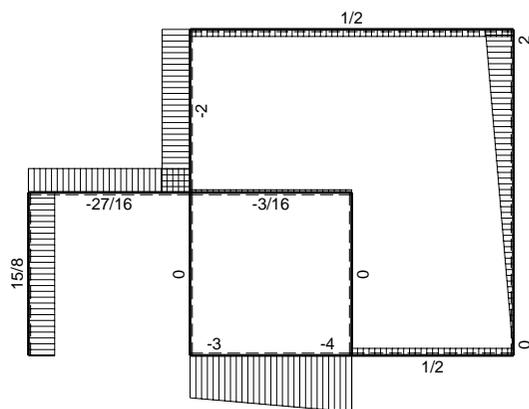
$$\begin{aligned}
 EJ_{EF} &= EJ \\
 EJ_{FC} &= EJ \\
 EJ_{CG} &= EJ \\
 EJ_{GH} &= EJ \\
 EJ_{HI} &= EJ \\
 EJ_{IE} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
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 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta AB positiva se convessa a destra con inizio A.
 Spostamento orizzontale assoluto u imposto al nodo A.

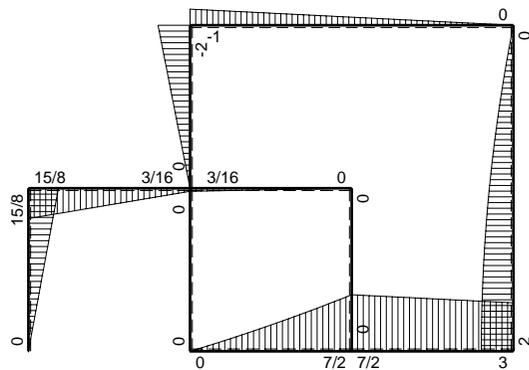




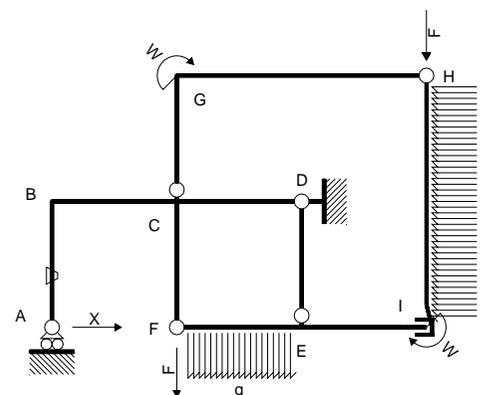
← (+) → F



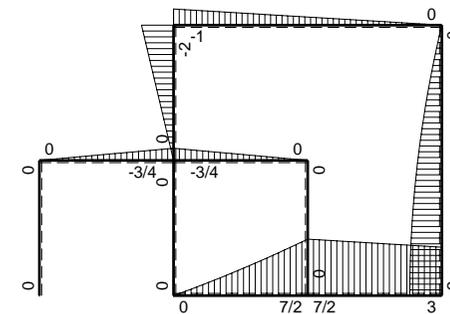
↑ (+) ↓ F



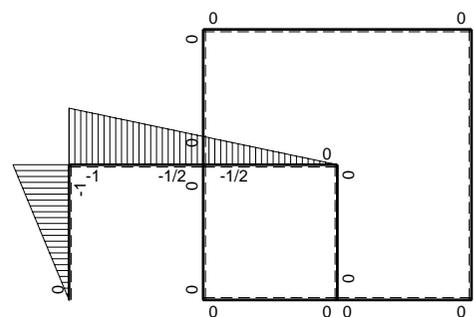
⊙ (+) ⊙ F_b



Schema di calcolo iperstatico



⊙ (+) ⊙ M_o flessione da carichi assegnati



⊙ (+) ⊙ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	-b+1/2x	-3/4Fx	0	$3/4Fbx-3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/4+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	0	$3/8Fb^2-3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$15/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-15/8F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left(\frac{x}{b} \right) \theta dx = \left[\frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left(-1 + x/b \right) \theta dx = \left[-x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(-b + \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(\frac{3}{4} \frac{x}{b} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{3}{8} \frac{x^2}{b} - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{4} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(\frac{3}{8} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{3}{8} x - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

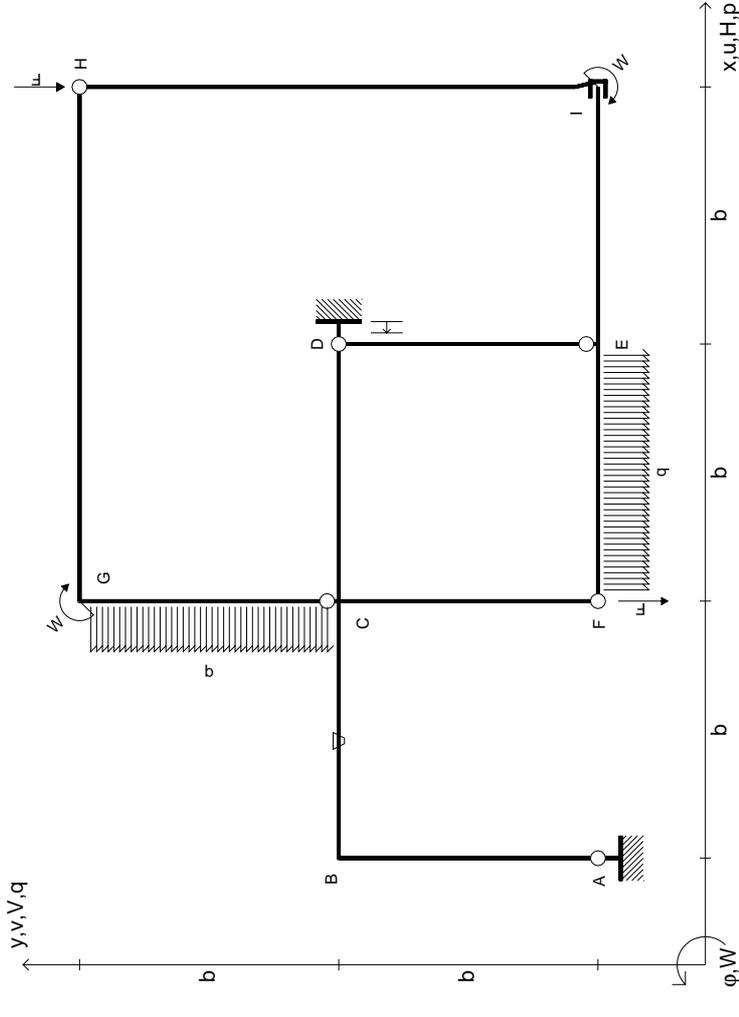
$$= \left(\frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{4} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left(\frac{3}{8} - \frac{3}{4} \frac{x}{b} + \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{3}{8} x - \frac{3}{8} \frac{x^2}{b} + \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{3}{8} b - \frac{3}{8} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(\frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

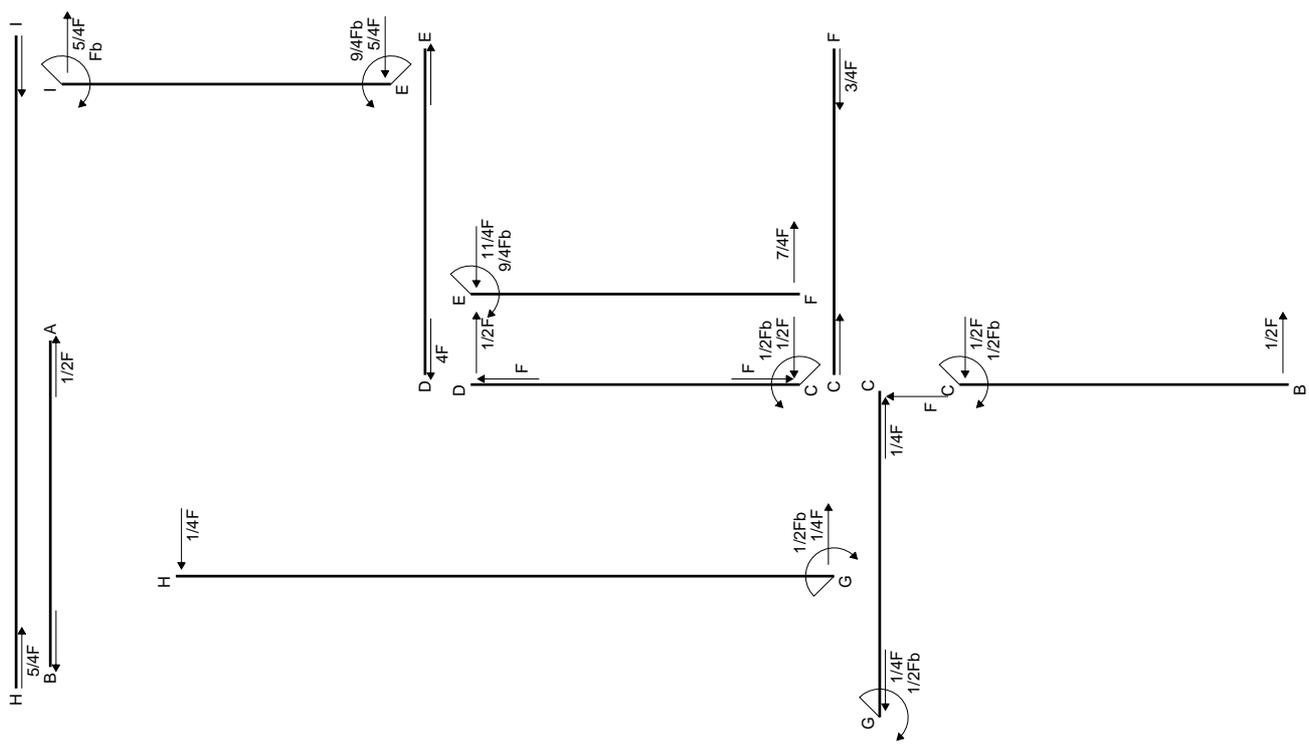


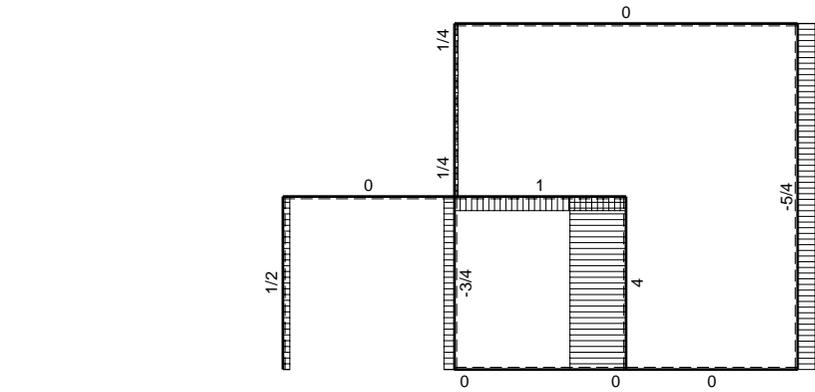
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 q_{EF} &= -q = +F/b \\
 P_{CG} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

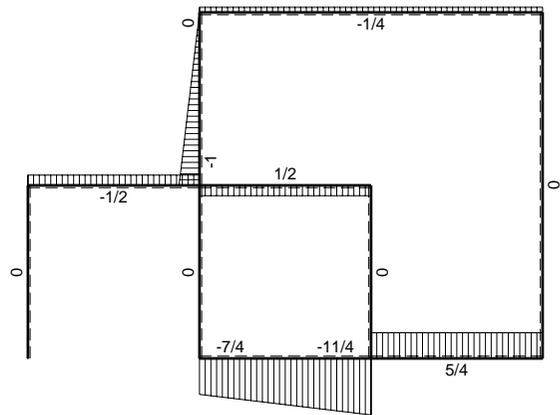
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

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 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y .
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo D.

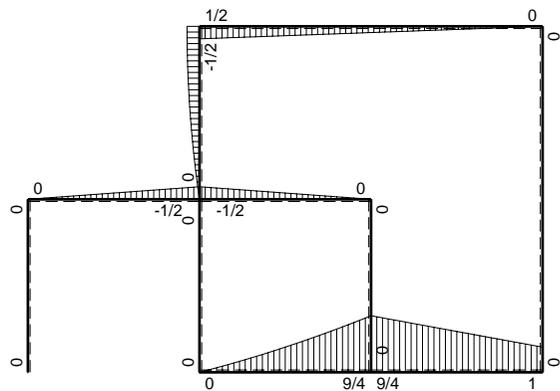




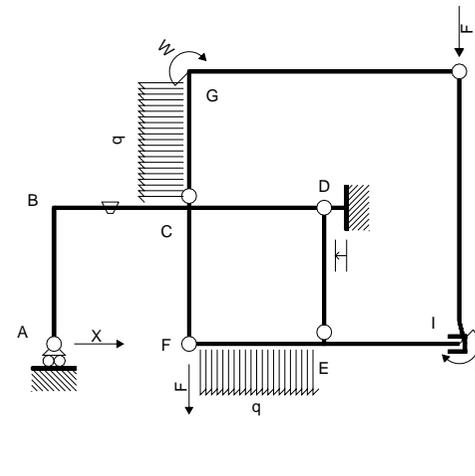
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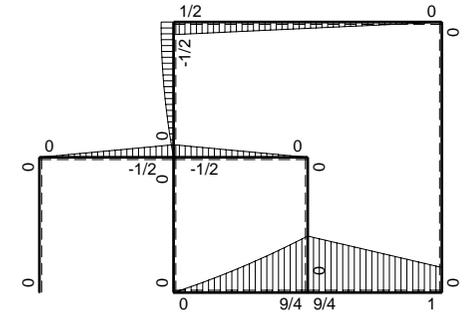
↑ (+) ↓ F



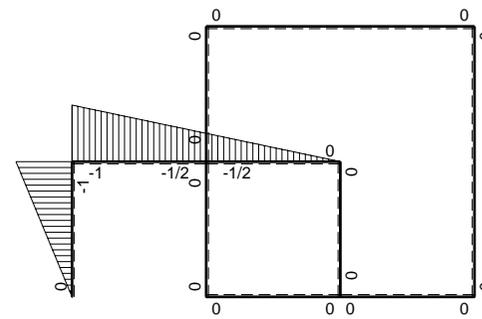
⤵ (+) ⤴ F_b



Schema di calcolo iperstatico



⤵ (+) ⤴ M_o flessione da carichi assegnati



⤵ (+) ⤴ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_A

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	-x	0	0	0	0	x ²	0+0	1/3Xb ³ /EJ	
BA b	b-x	0	0	0	0	b ² -2bx+x ²			
BC b	-b+1/2x	-1/2Fx	-Fb/EJ	1/2Fbx-1/4Fx ²	Fb ² /EJ-1/2Fxb/EJ	b ² -bx+1/4x ²	(1/6+3/4)Fb ³ /EJ	7/12Xb ³ /EJ	
CB b	1/2b+1/2x	1/2Fb-1/2Fx	Fb/EJ	1/4Fb ² -1/4Fx ²	1/2Fb ² /EJ+1/2Fxb/EJ	1/4b ² +1/2bx+1/4x ²			
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb ² -1/2Fbx+1/4Fx ²	0	1/4b ² -1/2bx+1/4x ²	(1/12+0)Fb ³ /EJ	1/12Xb ³ /EJ	
DC b	1/2x	1/2Fx	0	1/4Fx ²	0	1/4x ²			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	9/4Fb-11/4Fx+1/2qx ²	0	0	0	0	0+0	0	
FE b	0	-7/4Fx-1/2qx ²	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	-Fx+1/2qx ²	0	0	0	0	0+0	0	
GC b	0	1/2Fb-1/2qx ²	0	0	0	0			
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0	
HG 2b	0	-1/4Fx	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0	
EI b	0	-9/4Fb+5/4Fx	0	0	0	0			
D	cedimento nodo -H _{1D} u _D							-Fb ³ /EJ	
	totali							0	Xb ³ /EJ
	iperstatica X=H _A							0	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(\frac{1}{2} \frac{x}{b} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{1}{4} \frac{x^2}{b} - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{1}{4} b - \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left(b - \frac{1}{4} b \right) \theta = \frac{11}{12} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(-\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{1}{4} x - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[-\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

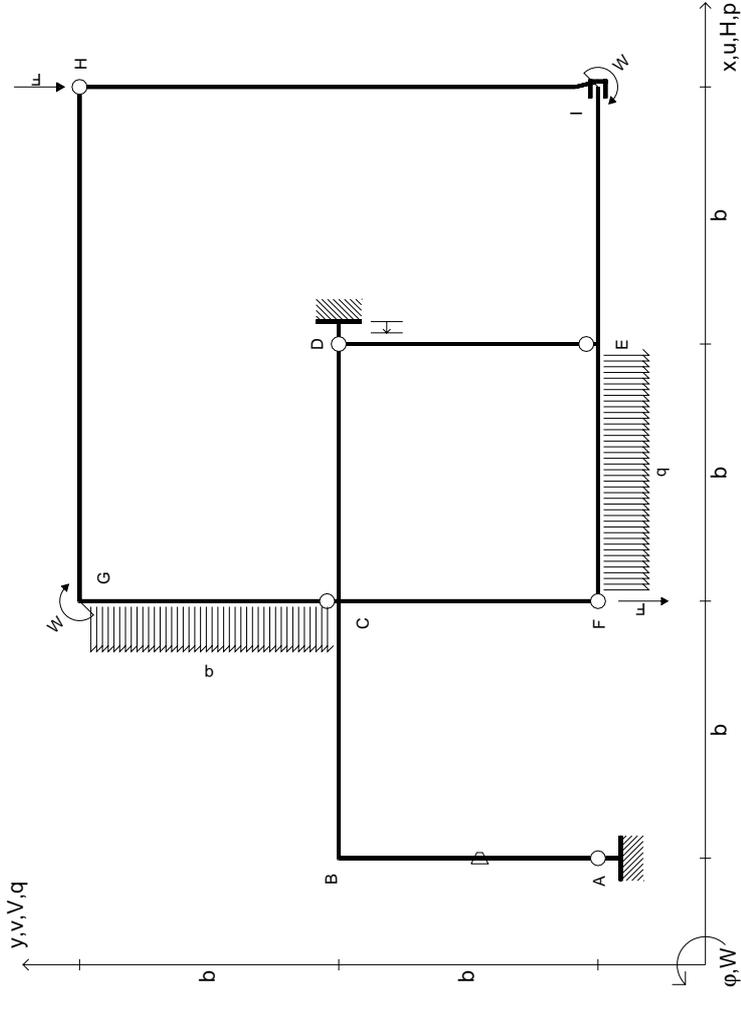
$$= \left(\frac{1}{4} b - \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left(-\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{11}{12} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

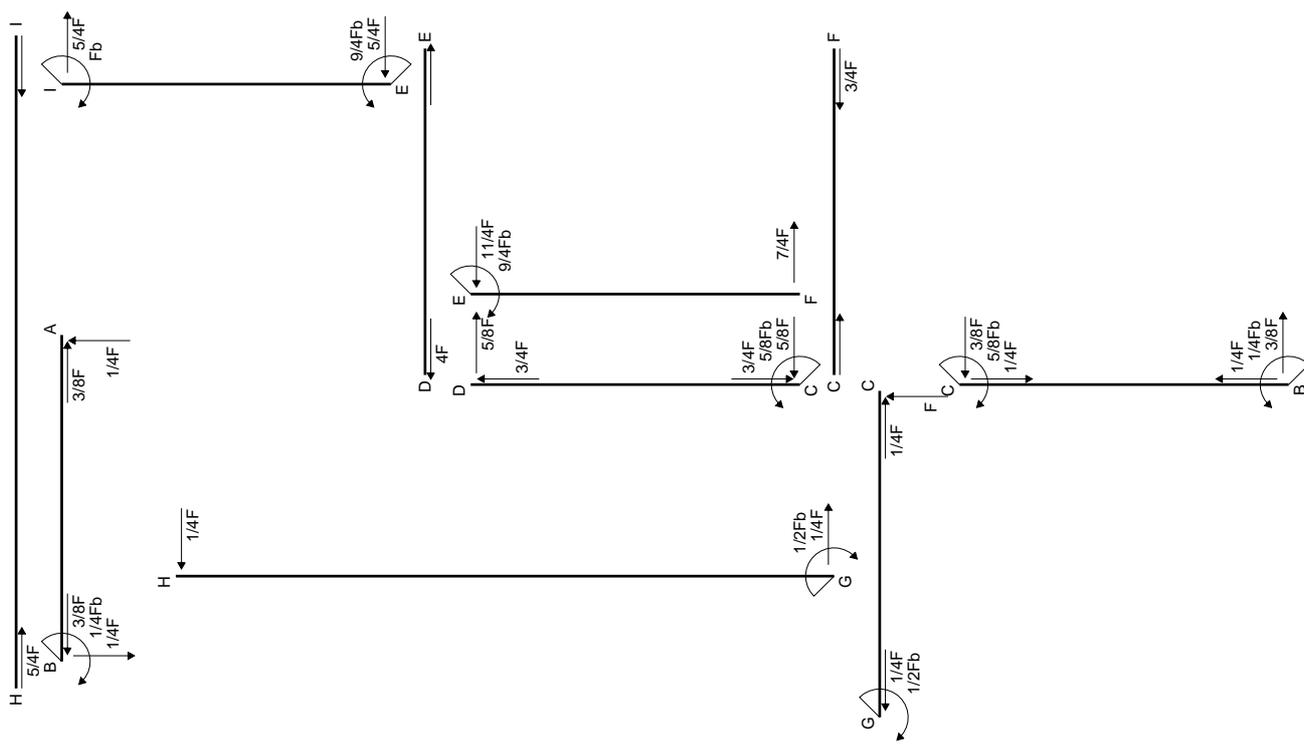


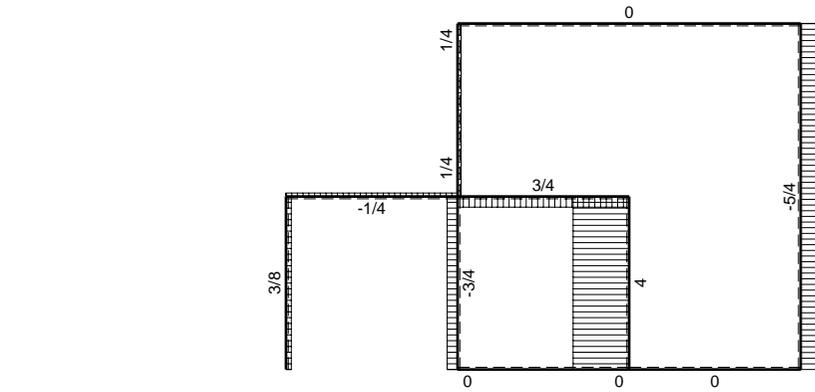
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
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 q_{EF} &= -q = +F/b \\
 P_{CG} &= -q = -F/b
 \end{aligned}$$

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 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

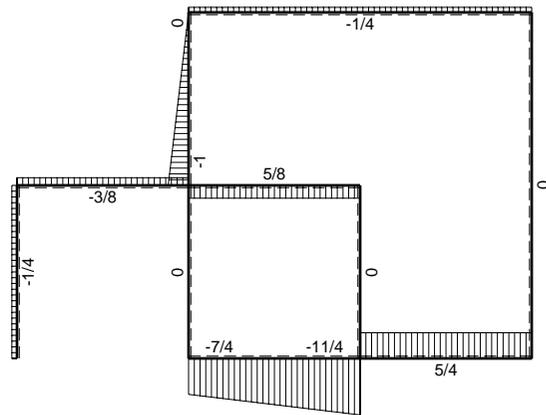
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
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 Curvatura θ asta AB positiva se convessa a destra con inizio A.
 Spostamento orizzontale assoluto u imposto al nodo D.

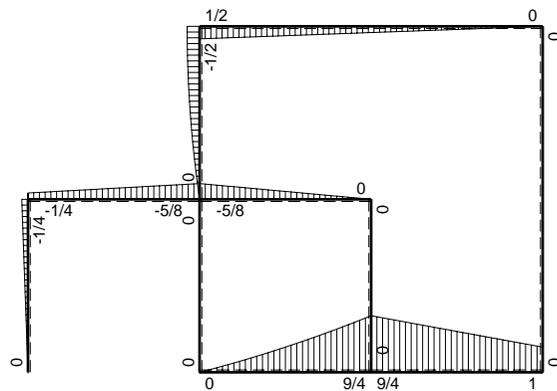




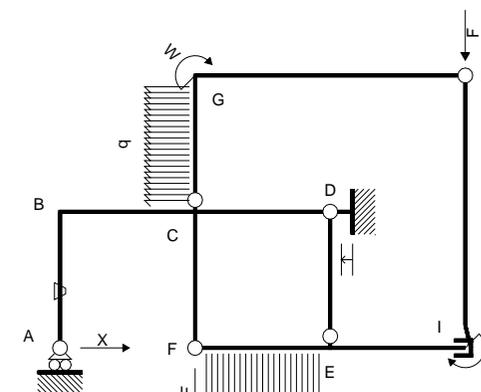
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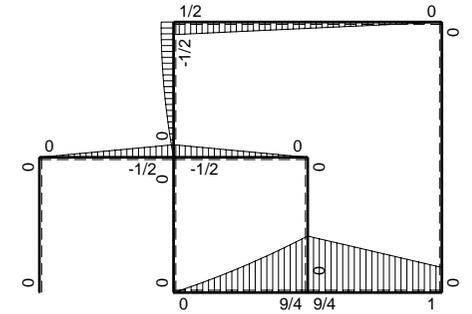
↑ (+) ↓ F



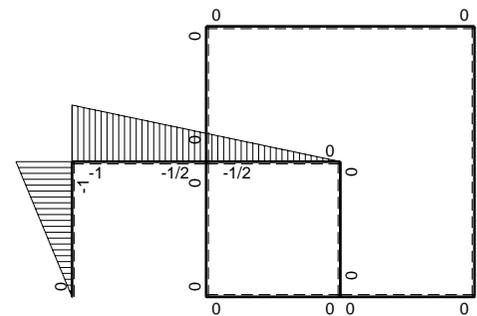
⊙ (+) ⊙ F_b



Schema di calcolo iperstatico



⊙ (+) ⊙ M_o flessione da carichi assegnati



⊙ (+) ⊙ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_A

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x ²	(0+1/2)Fb ³ /EJ	1/3Xb ³ /EJ	
BA b	b-x	0	Fb/EJ	0	Fb ² /EJ-Fxb/EJ	b ² -2bx+x ²			
BC b	-b+1/2x	-1/2Fx	0	1/2Fbx-1/4Fx ²	0	b ² -bx+1/4x ²	(1/6+0)Fb ³ /EJ	7/12Xb ³ /EJ	
CB b	1/2b+1/2x	1/2Fb-1/2Fx	0	1/4Fb ² -1/4Fx ²	0	1/4b ² +1/2bx+1/4x ²			
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb ² -1/2Fbx+1/4Fx ²	0	1/4b ² -1/2bx+1/4x ²	(1/12+0)Fb ³ /EJ	1/12Xb ³ /EJ	
DC b	1/2x	1/2Fx	0	1/4Fx ²	0	1/4x ²			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	9/4Fb-11/4Fx+1/2qx ²	0	0	0	0	0+0	0	
FE b	0	-7/4Fx-1/2qx ²	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	-Fx+1/2qx ²	0	0	0	0	0+0	0	
GC b	0	1/2Fb-1/2qx ²	0	0	0	0			
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0	
HG 2b	0	-1/4Fx	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0	
EI b	0	-9/4Fb+5/4Fx	0	0	0	0			
D	cedimento nodo -H _{1D} u _D							-Fb ³ /EJ	
	totali							-1/4Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _A							1/4F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left(\frac{x}{b} \right) \theta dx = \left[\frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left(-1 + x/b \right) \theta dx = \left[-x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(-b + \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(\frac{1}{2} \frac{x}{b} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{4} \frac{x^2}{b} - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

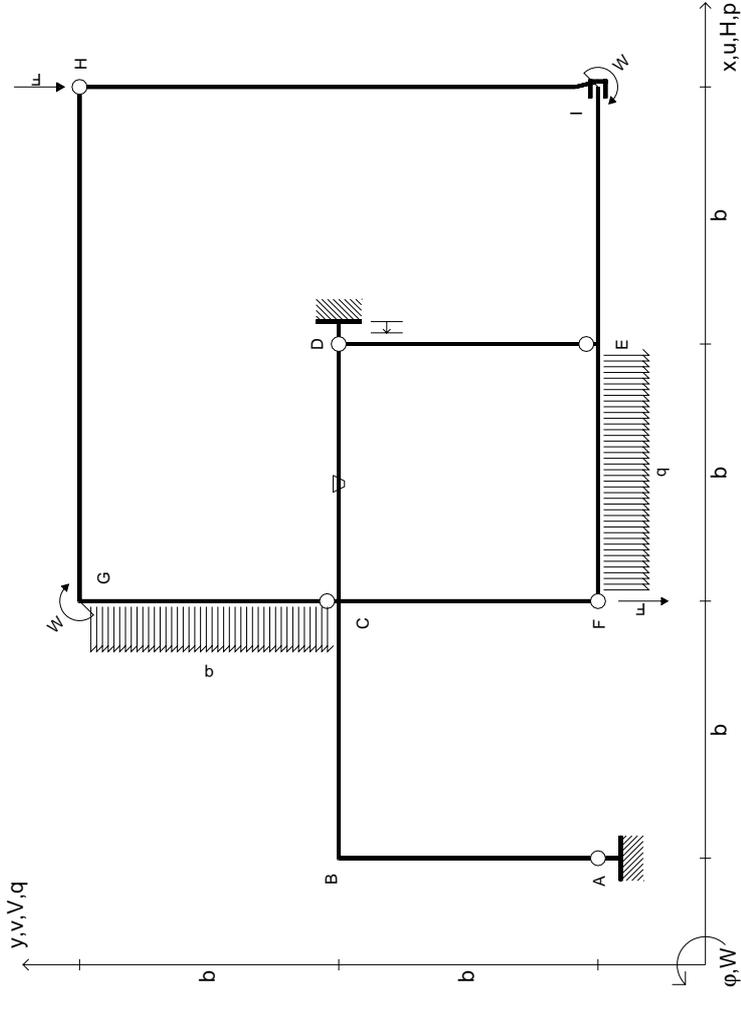
$$= \left(\frac{1}{4} b - \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

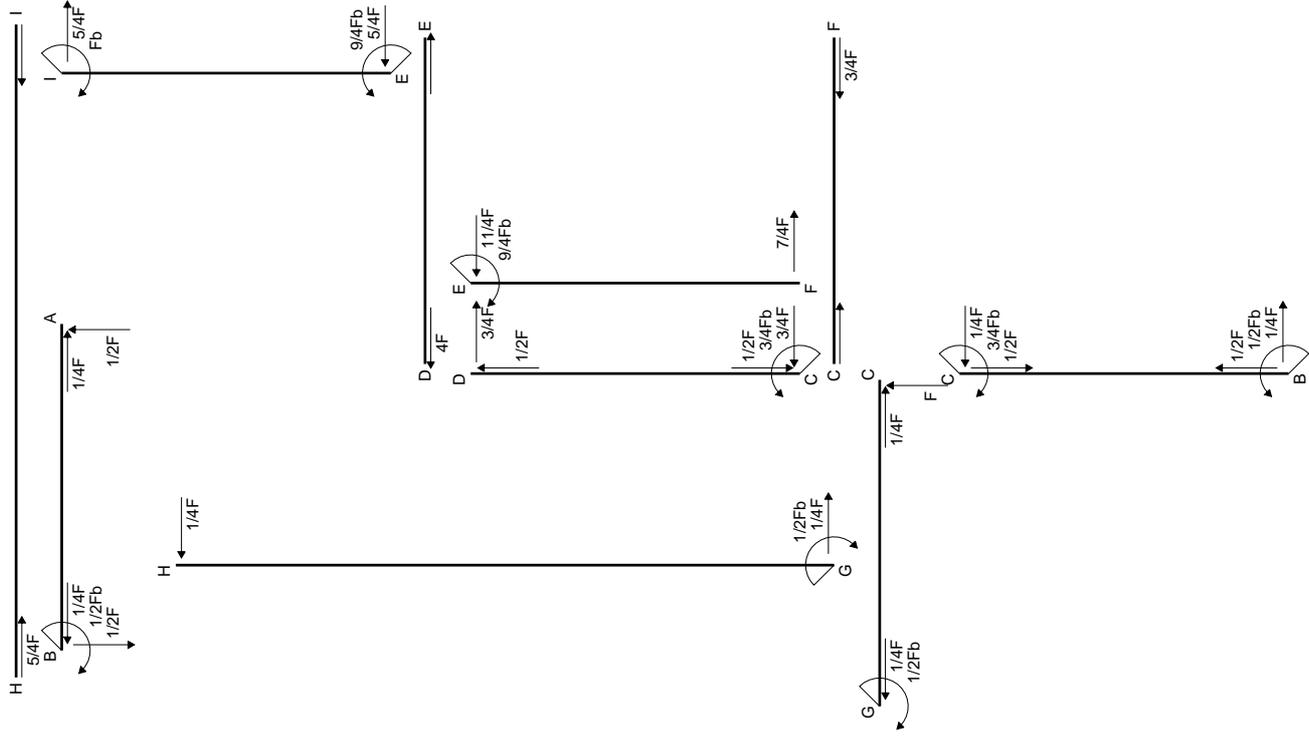


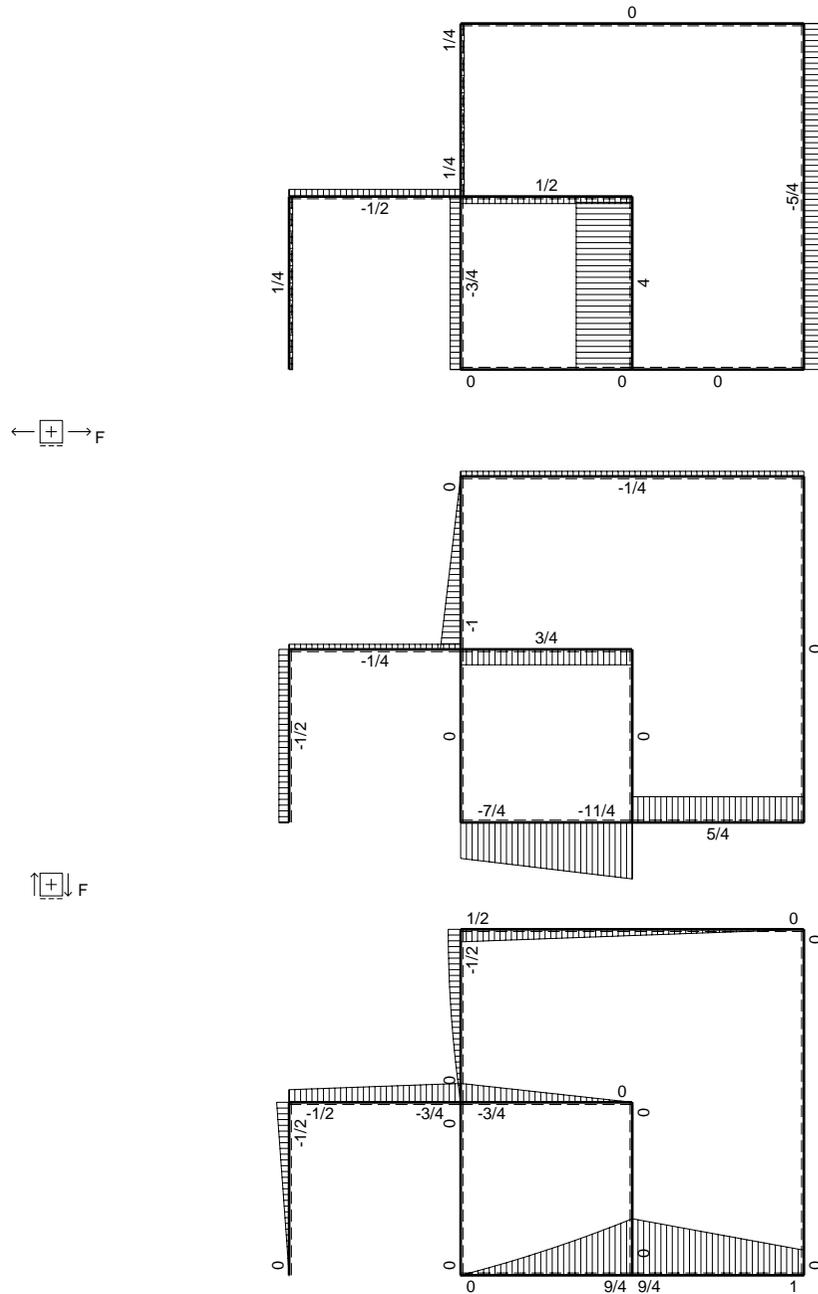
$$\begin{aligned} V_H &= -F \\ V_F &= -F \\ W_I &= -W = +Fb \\ W_G &= -W = -Fb \\ q_{EF} &= -q = +F/b \\ P_{CG} &= -q = -F/b \end{aligned}$$

$$\begin{aligned} \theta_{CD} &= -\theta = -\alpha T/b = -bF/EJ \\ u_D &= -\delta = -b^3 F/EJ \\ E_{J_{AB}} &= EJ \\ E_{J_{BC}} &= EJ \\ E_{J_{CD}} &= EJ \\ E_{J_{DE}} &= EJ \end{aligned}$$

$$\begin{aligned} E_{J_{EF}} &= EJ \\ E_{J_{FC}} &= EJ \\ E_{J_{CG}} &= EJ \\ E_{J_{GH}} &= EJ \\ E_{J_{HI}} &= EJ \\ E_{J_{IE}} &= EJ \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y .
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta CD positiva se convessa a destra con inizio C.
 Spostamento orizzontale assoluto u imposto al nodo D.

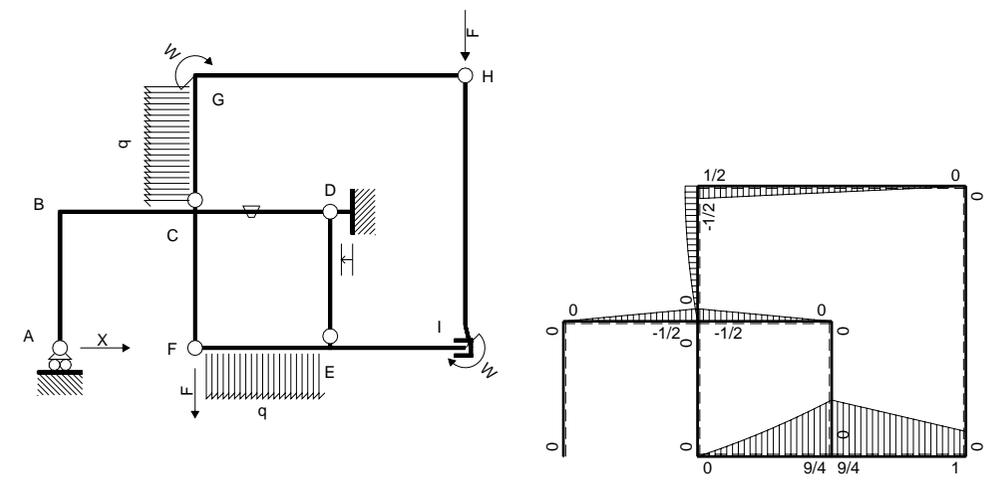




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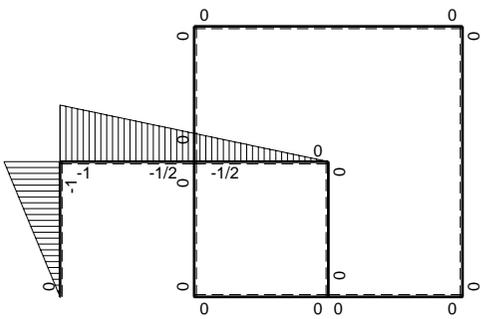
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⊕ (+) ⊖ (-) F_b



Schema di calcolo iperstatico

⊕ (+) ⊖ (-) M₀ flessione da carichi assegnati



⊕ (+) ⊖ (-) M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$	
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$			
BC b	-b+1/2x	-1/2Fx	0	$1/2Fbx-1/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
CB b	$1/2b+1/2x$	$1/2Fb-1/2Fx$	0	$1/4Fb^2-1/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	$-Fb/EJ$	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$	
DC b	$1/2x$	$1/2Fx$	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	$9/4Fb-11/4Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-7/4Fx-1/2qx^2$	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0	
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0			
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0	
HG 2b	0	$-1/4Fx$	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	$Fb+5/4Fx$	0	0	0	0	0+0	0	
EI b	0	$-9/4Fb+5/4Fx$	0	0	0	0			
D	cedimento nodo $-H_{1D}u_D$							$-Fb^3/EJ$	
	totali							$-1/2Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$							$1/2F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

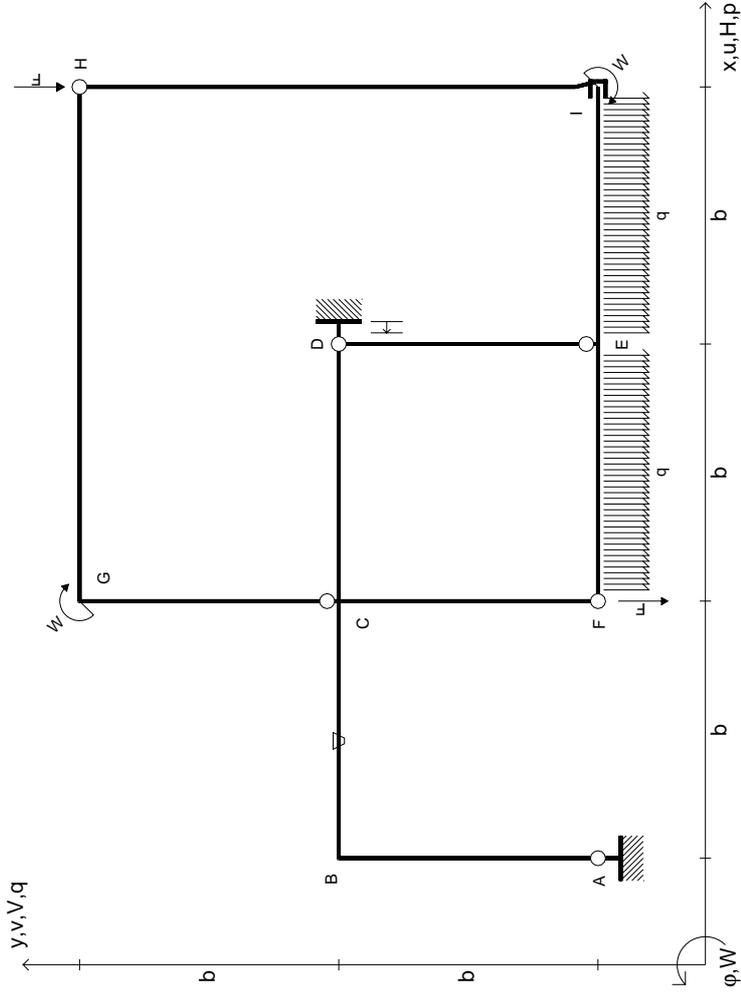
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb^2 1/EJ + (-1/4 b) \theta = 1/3 Fb^3/EJ$$

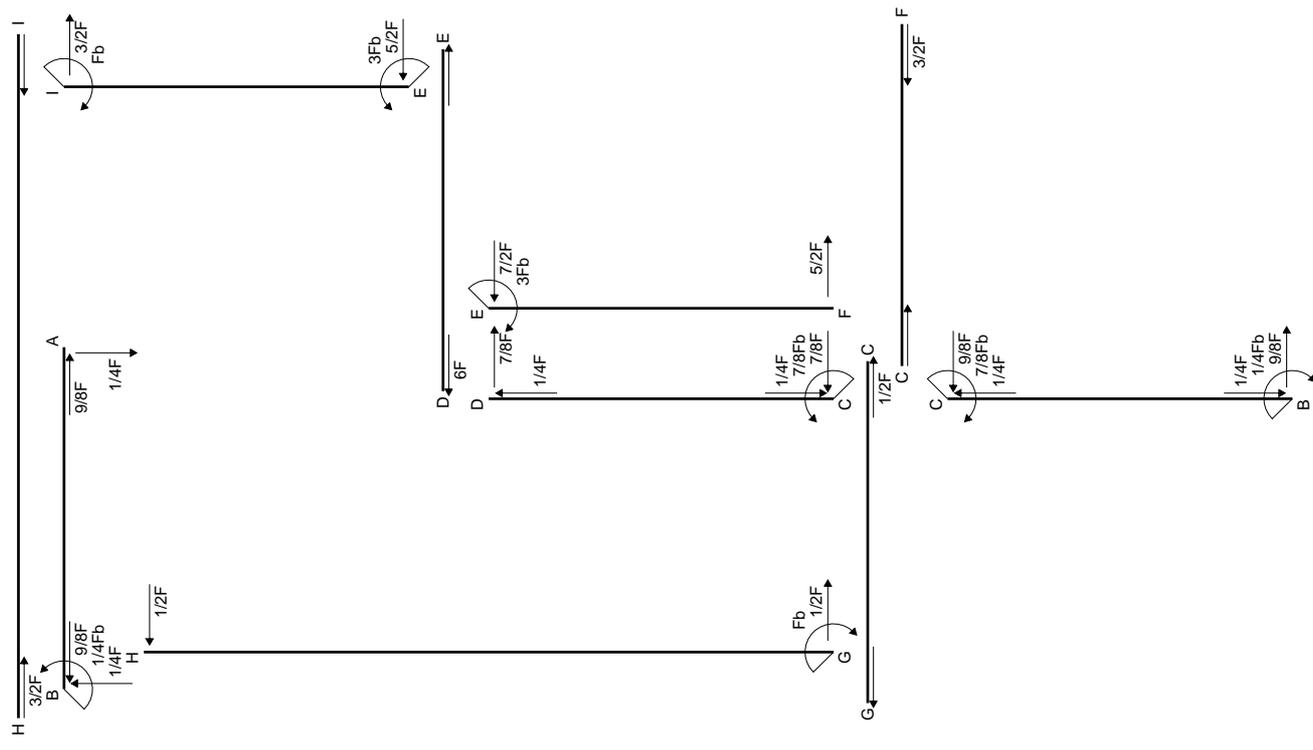


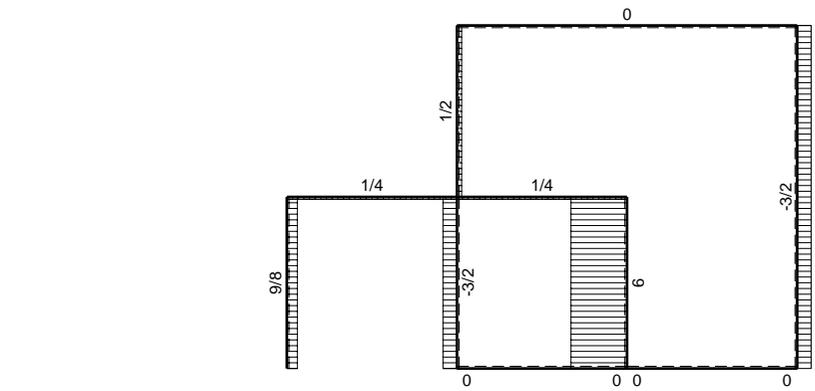
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 q_{EF} &= -q = +F/b \\
 q_{IE} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{BC} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

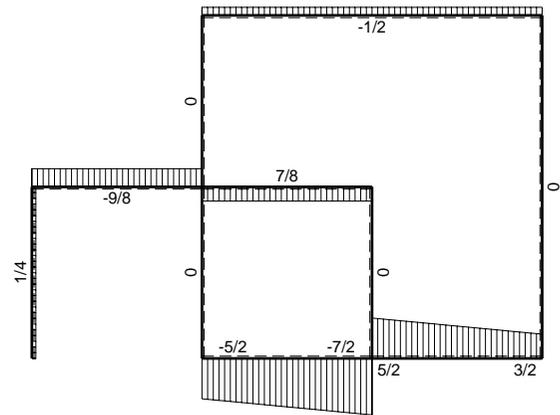
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_b$
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 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y .
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo D.

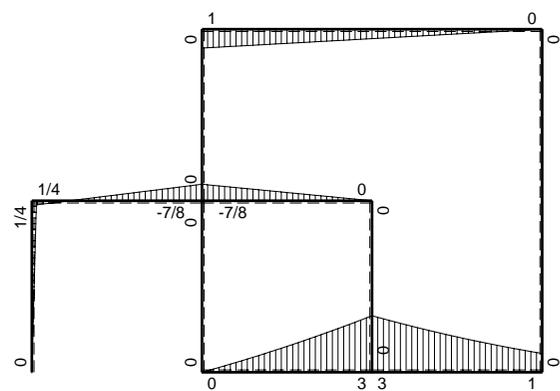




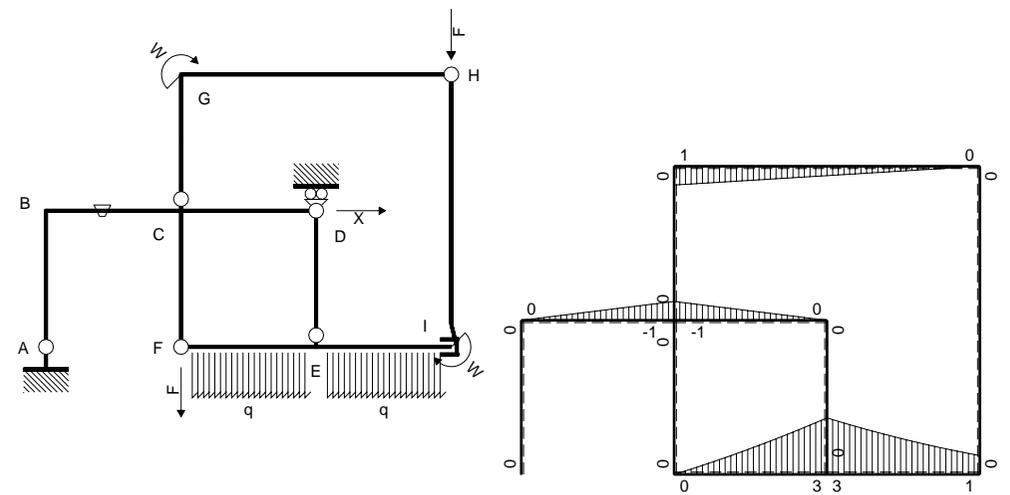
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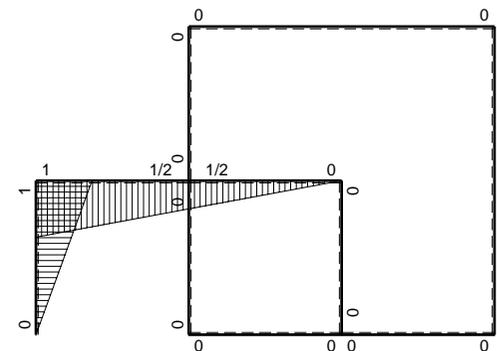


⌚ (+) ⌚ F_b



Schema di calcolo iperstatico

⌚ (+) ⌚ M₀ flessione da carichi assegnati



⌚ (+) ⌚ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_D$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$	
BA b	-b+x	0	0	0	0	$b^2-2bx+x^2$			
BC b	b-1/2x	-Fx	-Fb/EJ	$-Fbx+1/2Fx^2$	$-Fb^2/EJ+1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(-1/3-3/4)Fb^3/EJ$	$7/12Xb^3/EJ$	
CB b	-1/2b-1/2x	Fb-Fx	Fb/EJ	$-1/2Fb^2+1/2Fx^2$	$-1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$			
CD b	1/2b-1/2x	-Fb+Fx	0	$-1/2Fb^2+Fbx-1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
DC b	-1/2x	Fx	0	$-1/2Fx^2$	0	$1/4x^2$			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	$3Fb-7/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-5/2Fx-1/2qx^2$	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	0	0	0	0	0	0+0	0	
GC b	0	0	0	0	0	0			
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0	
HG 2b	0	-1/2Fx	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0			
D	cedimento nodo $-H_{1D}u_D$						Fb^3/EJ		
	totali						$-1/4Fb^3/EJ$	Xb^3/EJ	
	iperstatica $X=H_D$						$1/4F$		

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1 + 1/2 x/b) \theta dx$$

$$= [-1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [-x + 1/4 x^2/b]_0^b \theta$$

$$= (-1/2 b + 1/6 b) Fb^2 1/EJ + (-b + 1/4 b) \theta = -13/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1/2 + 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 + 1/2 x/b) \theta dx$$

$$= [-1/2 x + 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x + 1/4 x^2/b]_0^b \theta$$

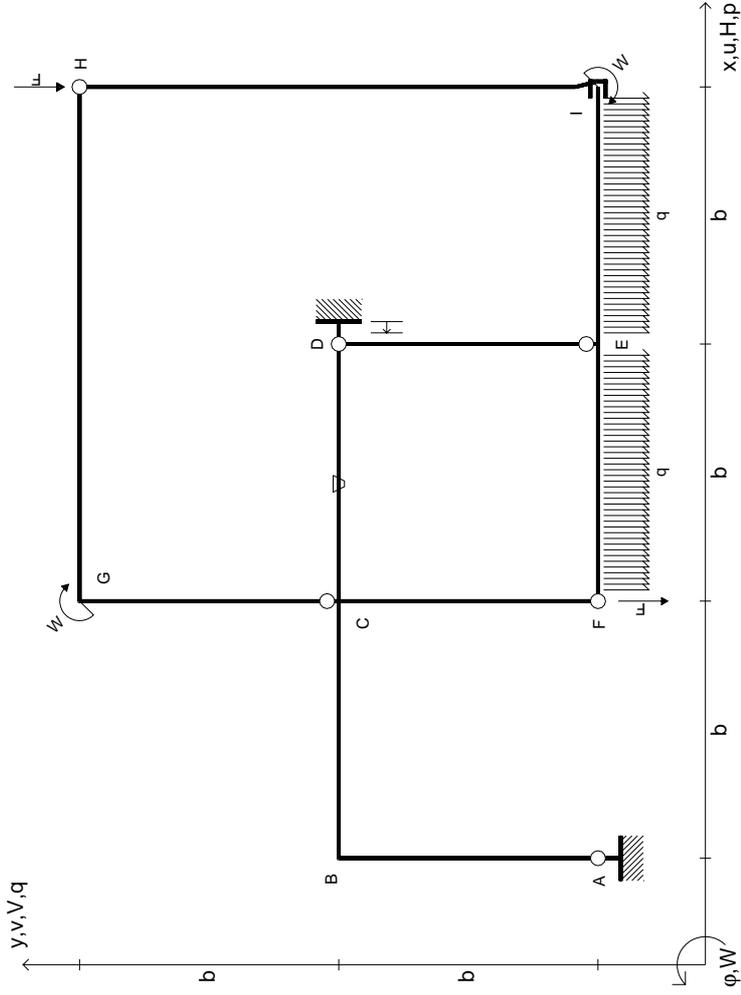
$$= (-1/2 b + 1/6 b) Fb^2 1/EJ + (1/2 b + 1/4 b) \theta = -13/12 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (-1/2 + x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 1/2 b - 1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (-1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

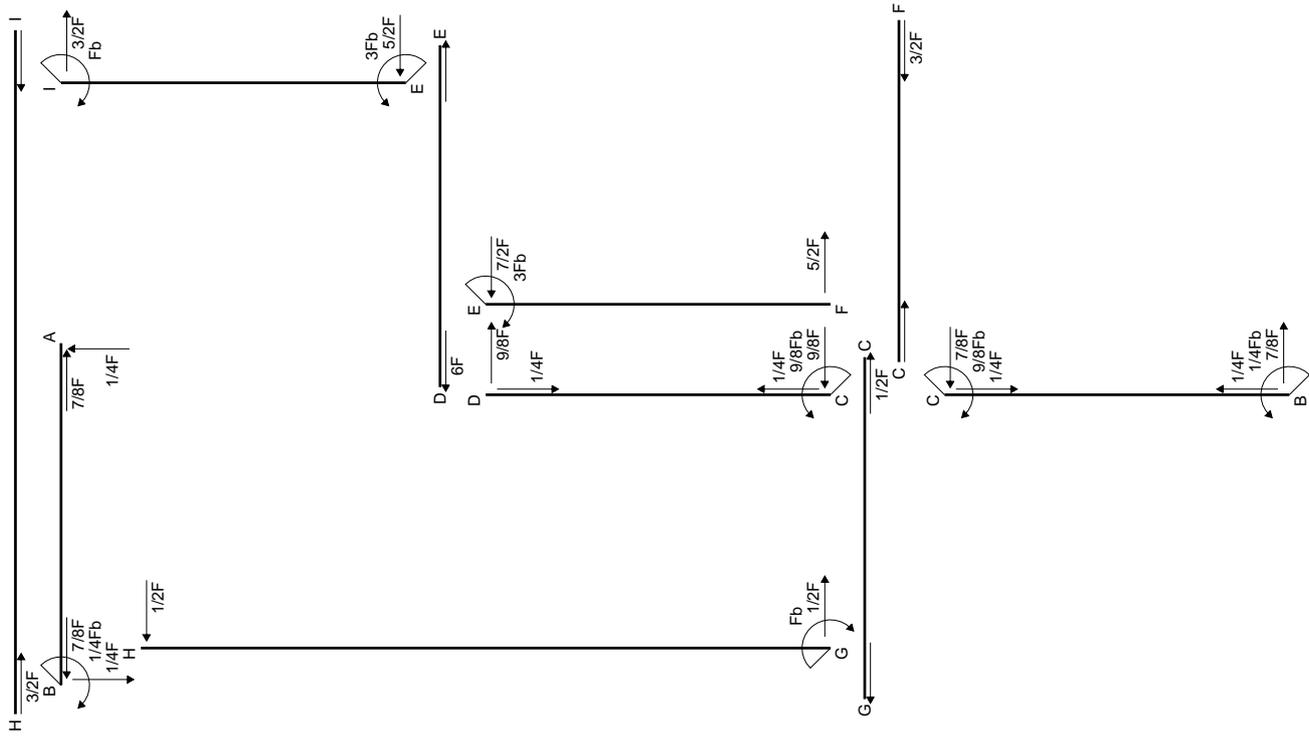


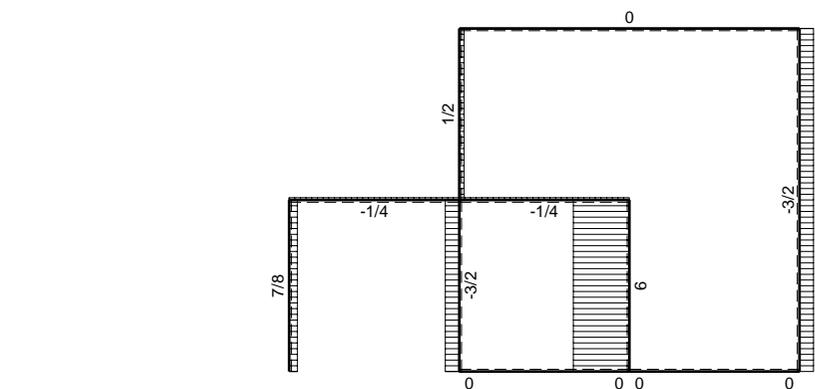
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 q_{EF} &= -q = +F/b \\
 q_{IE} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{CD} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

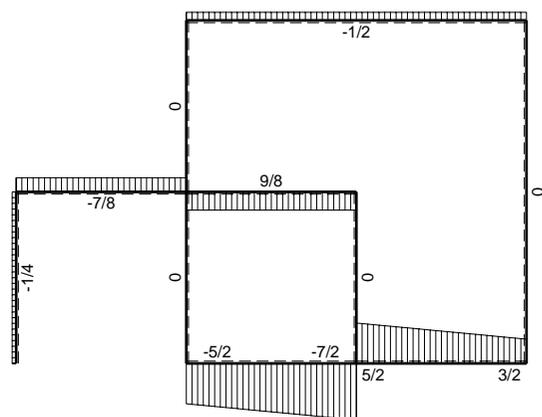
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{yz} - X_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta CD positiva se convessa a destra con inizio C.
 Spostamento orizzontale assoluto u imposto al nodo D.

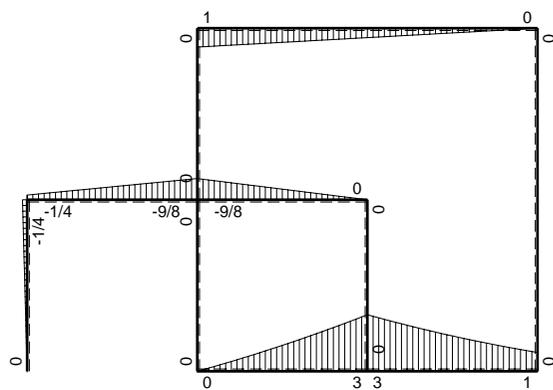




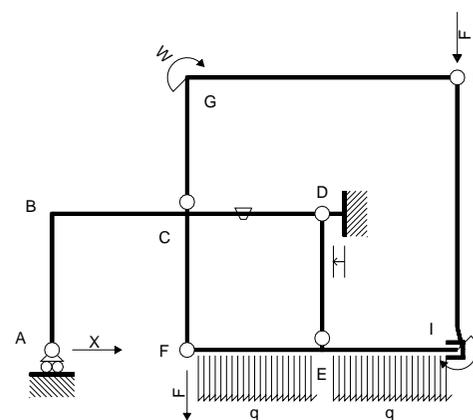
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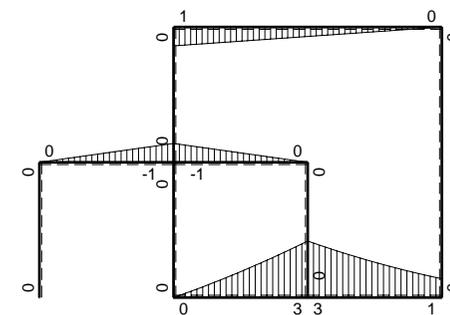
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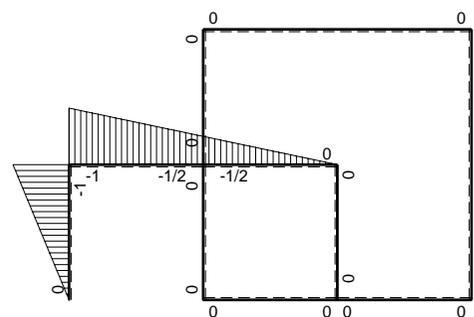
⤵ (+) ⤴ F_b



Schema di calcolo iperstatico



⤵ (+) ⤴ M_o flessione da carichi assegnati



⤵ (+) ⤴ M_x flessione da iperstatica $X=1$

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	-x	0	0	0	0	x^2	0+0	$1/3 X b^3/EJ$	
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$			
BC b	-b+1/2x	-Fx	0	$Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(1/3+0)Fb^3/EJ$	$7/12 X b^3/EJ$	
CB b	$1/2b+1/2x$	$Fb-Fx$	0	$1/2Fb^2-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
CD b	$-1/2b+1/2x$	$-Fb+Fx$	$-Fb/EJ$	$1/2Fb^2-Fbx+1/2Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/6+1/4)Fb^3/EJ$	$1/12 X b^3/EJ$	
DC b	$1/2x$	Fx	Fb/EJ	$1/2Fx^2$	$1/2Fxb/EJ$	$1/4x^2$			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	$3Fb-7/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-5/2Fx-1/2qx^2$	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	0	0	0	0	0	0+0	0	
GC b	0	0	0	0	0	0			
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0	
HG 2b	0	$-1/2Fx$	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0			
D	cedimento nodo $-H_{1D}u_D$							$-Fb^3/EJ$	
	totali							$-1/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$							$1/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

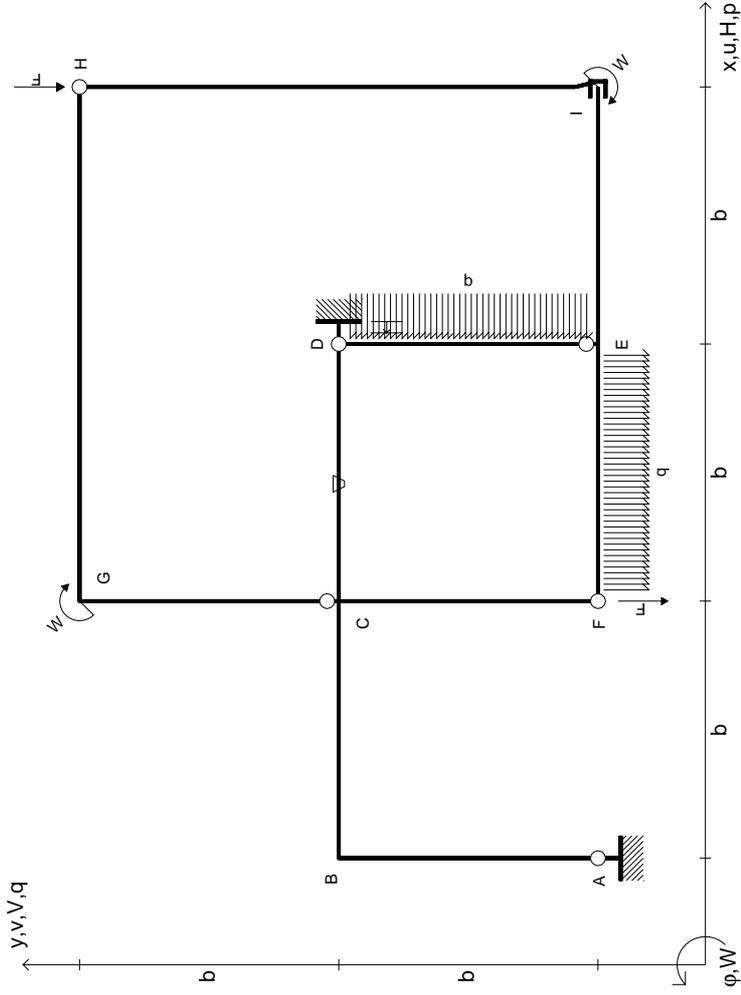
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb^2 1/EJ + (-1/4 b) \theta = 5/12 Fb^3/EJ$$

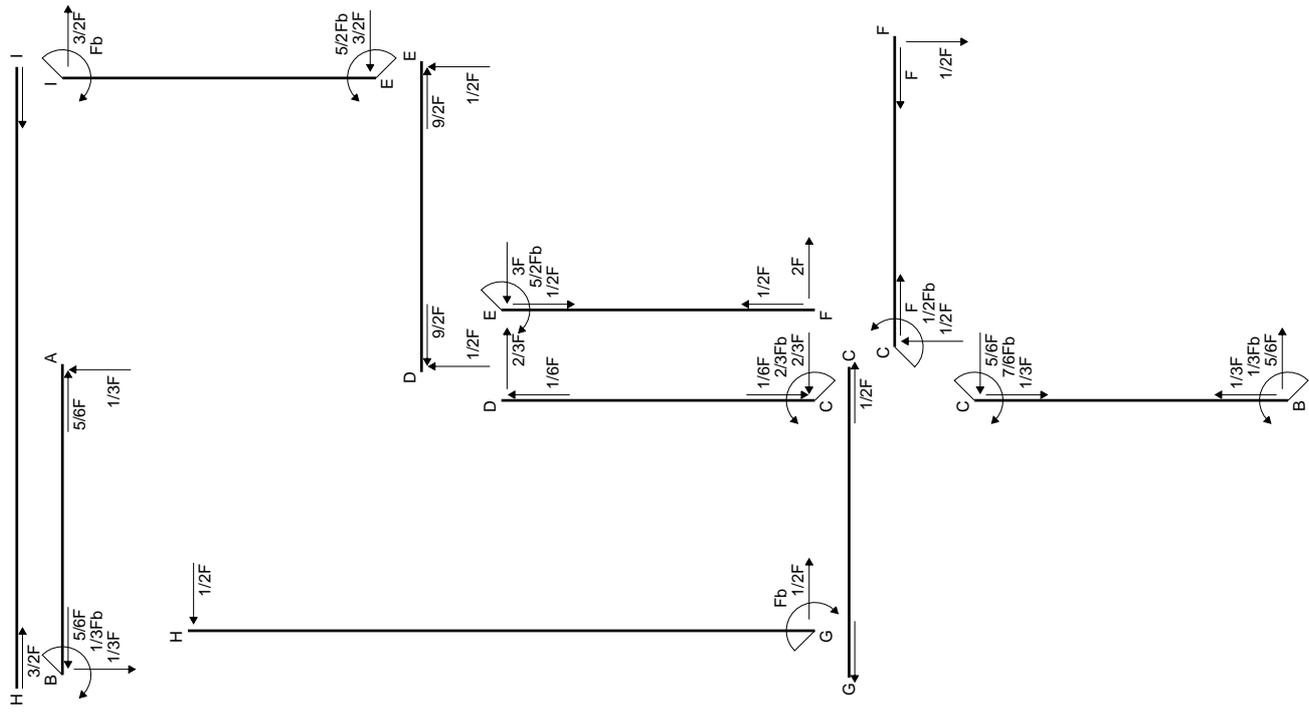


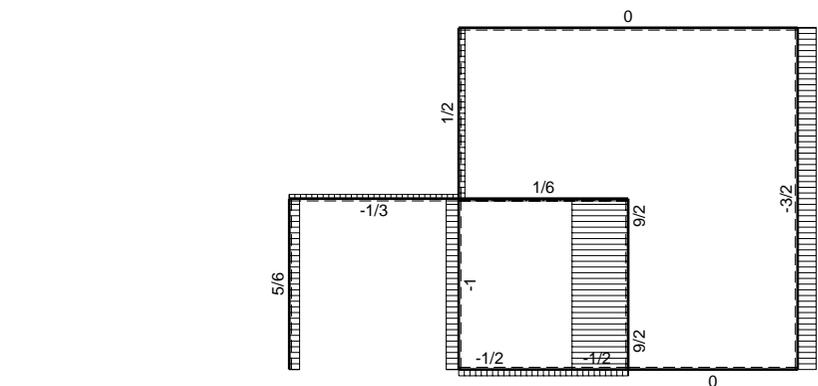
$$\begin{aligned}
 V_H &= -F \\
 V_F &= -F \\
 W_I &= -W = +Fb \\
 W_G &= -W = -Fb \\
 q_{EF} &= -q = +F/b \\
 P_{DE} &= -q = -F/b
 \end{aligned}$$

$$\begin{aligned}
 \theta_{CD} &= -\theta = -\alpha T/b = -bF/EJ \\
 u_D &= -\delta = -b^3 F/EJ \\
 E_{J_{AB}} &= EJ \\
 E_{J_{BC}} &= EJ \\
 E_{J_{CD}} &= EJ \\
 E_{J_{DE}} &= EJ
 \end{aligned}$$

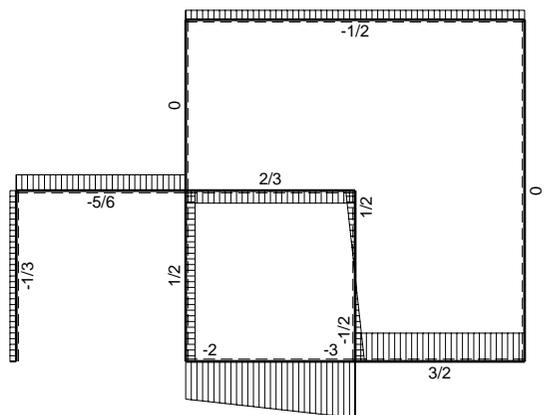
$$\begin{aligned}
 E_{J_{EF}} &= EJ \\
 E_{J_{FC}} &= EJ \\
 E_{J_{CG}} &= EJ \\
 E_{J_{GH}} &= EJ \\
 E_{J_{HI}} &= EJ \\
 E_{J_{IE}} &= EJ
 \end{aligned}$$

Reazioni iperstatiche in soluzione: $X=H_A$
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 Spostamento orizzontale assoluto u imposto al nodo D.

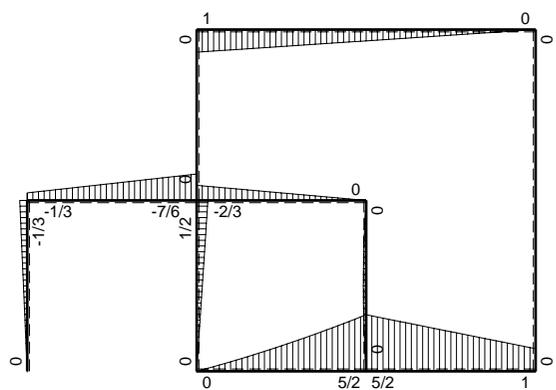




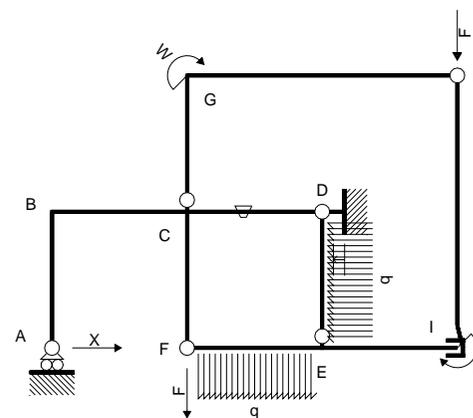
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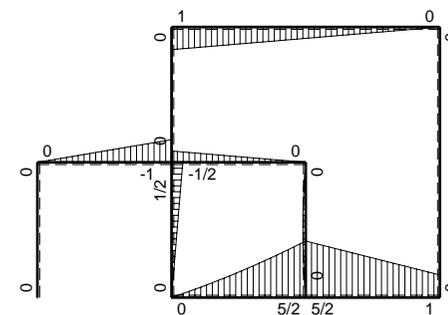
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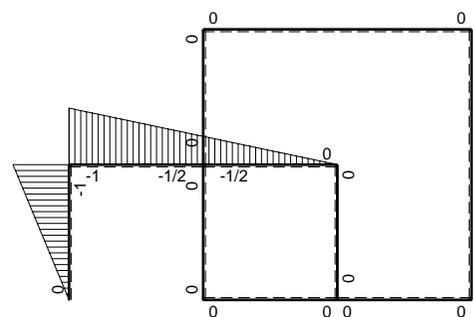
⊙ (+) ⊙ F_b



Schema di calcolo iperstatico



⊙ (+) ⊙ M_o flessione da carichi assegnati



⊙ (+) ⊙ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	-x	0	0	0	0	x^2	0+0	$1/3 X b^3/EJ$	
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$			
BC b	-b+1/2x	-Fx	0	$Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(1/3+0)Fb^3/EJ$	$7/12 X b^3/EJ$	
CB b	1/2b+1/2x	Fb-Fx	0	$1/2Fb^2-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	-Fb/EJ	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12 X b^3/EJ$	
DC b	1/2x	1/2Fx	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$			
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0	
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0			
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0			
FC b	0	1/2Fx	0	0	0	0	0+0	0	
CF b	0	-1/2Fb+1/2Fx	0	0	0	0			
CG b	0	0	0	0	0	0	0+0	0	
GC b	0	0	0	0	0	0			
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0	
HG 2b	0	-1/2Fx	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0	
EI b	0	-5/2Fb+3/2Fx	0	0	0	0			
D	cedimento nodo $-H_{1D}u_D$							$-Fb^3/EJ$	
	totali							$-1/3Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$							1/3F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left(\frac{x}{b} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{2} \frac{x^2}{b} - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{3} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left(\frac{1}{2} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{1}{2} x - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{3} Fb^3/EJ$$

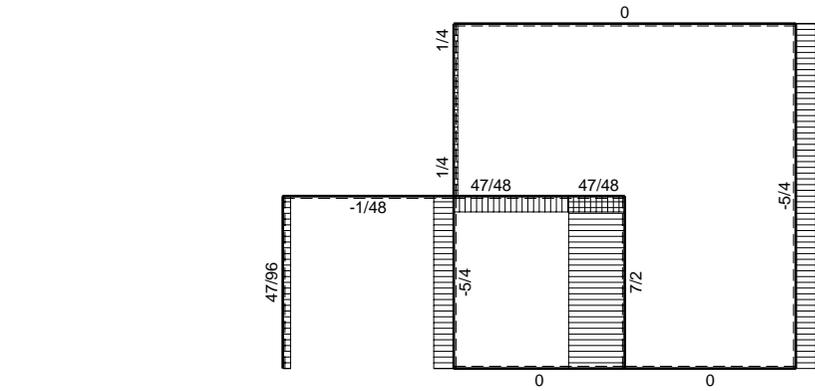
$$L_{CD}^{xo} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

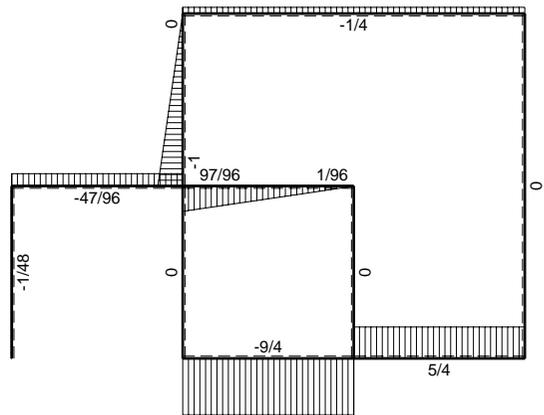
$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left(\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{1}{3} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(-\frac{1}{2} \frac{x}{b} \right) \theta dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[-\frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

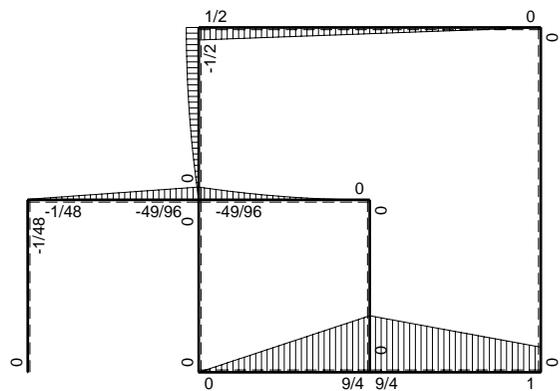
$$= \left(\frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left(-\frac{1}{4} b \right) \theta = \frac{1}{3} Fb^3/EJ$$



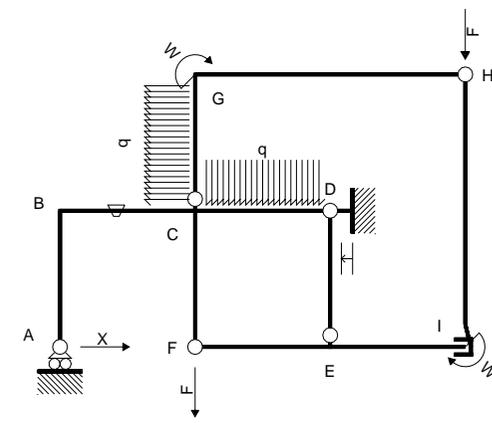
← (+) → F



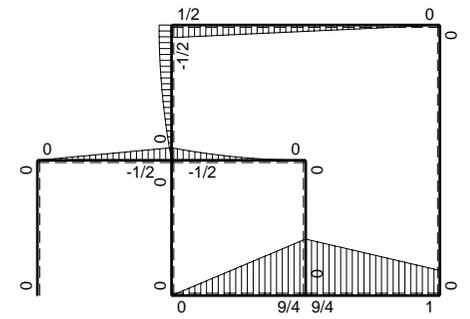
↑ (+) ↓ F



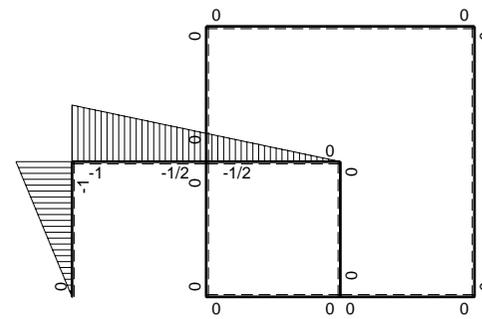
⤵ (+) ⤴ F_b



Schema di calcolo iperstatico



⤵ (+) ⤴ M_o flessione da carichi assegnati



⤵ (+) ⤴ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$	
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$			
BC b	-b+1/2x	-1/2Fx	-Fb/EJ	$1/2Fbx-1/4Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/6+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$	
CB b	$1/2b+1/2x$	$1/2Fb-1/2Fx$	Fb/EJ	$1/4Fb^2-1/4Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$			
CD b	$-1/2b+1/2x$	$-1/2Fb+Fx-1/2qx^2$	0	$1/4Fb^2-3/4Fbx+3/4Fx^2-1/4qx^3$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/16+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
DC b	$1/2x$	$1/2qx^2$	0	$1/4qx^3$	0	$1/4x^2$			
DE b	0	0	0	0	0	0	0+0	0	
ED b	0	0	0	0	0	0			
EF b	0	$9/4Fb-9/4Fx$	0	0	0	0	0+0	0	
FE b	0	$-9/4Fx$	0	0	0	0			
FC b	0	0	0	0	0	0	0+0	0	
CF b	0	0	0	0	0	0			
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0	
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0			
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0	
HG 2b	0	$-1/4Fx$	0	0	0	0			
HI 2b	0	0	0	0	0	0	0+0	0	
IH 2b	0	0	0	0	0	0			
IE b	0	$Fb+5/4Fx$	0	0	0	0	0+0	0	
EI b	0	$-9/4Fb+5/4Fx$	0	0	0	0			
D	cedimento nodo $-H_{1D}u_D$							$-Fb^3/EJ$	
	totali							$-1/48Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$							$1/48F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ + (b - 1/4 b) \theta = 11/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = 11/12 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 3/4 x/b + 3/4 x^2/b^2 - 1/4 x^3/b^3) Fb^2 1/EJ dx$$

$$= [1/4 x - 3/8 x^2/b + 1/4 x^3/b^2 - 1/16 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 3/8 b + 1/4 b - 1/16 b) Fb^2 1/EJ = 1/16 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^3/b^3) Fb^2 1/EJ dx = [1/16 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (1/16 b) Fb^2 1/EJ = 1/16 Fb^3/EJ$$