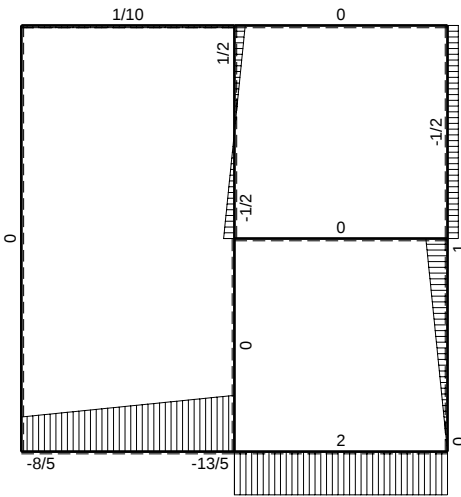
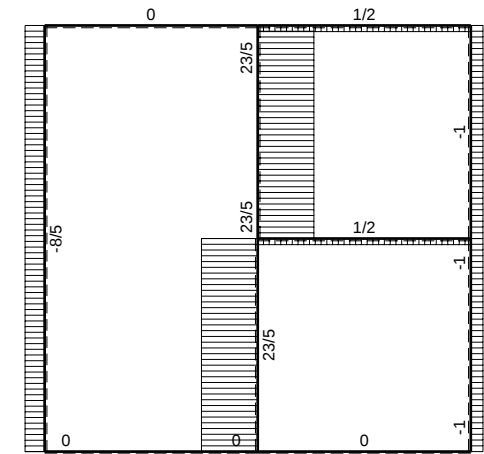
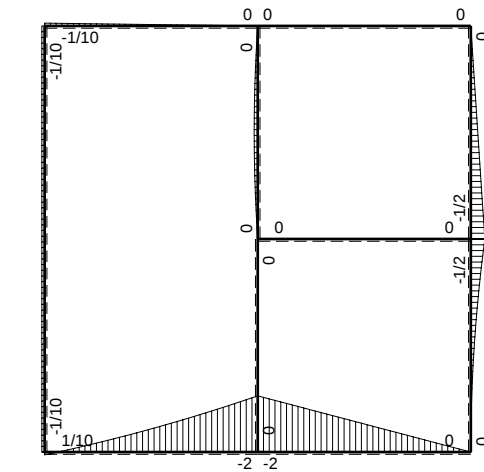


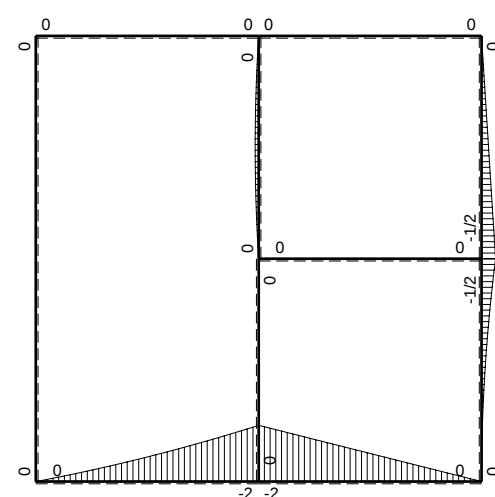
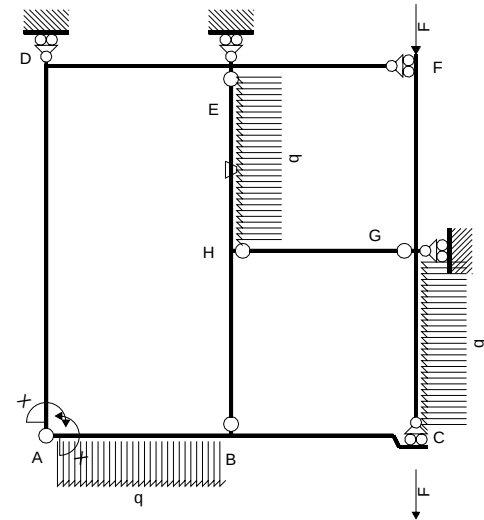


← ⊕ → F

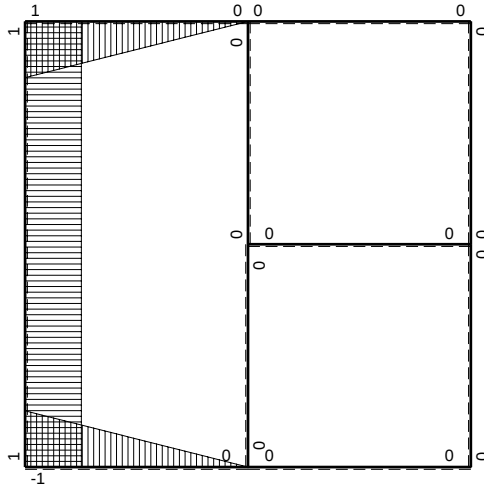
↑ ⊕ ↓ F



⊕ F_b



⊕ M₀ flessione da carichi assegnati



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	$-1+x/b$	$-3/2Fx-1/2qx^2$	0	$3/2Fx-Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	x/b	$2Fb-5/2Fx+1/2qx^2$	0	$2Fx-5/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	1	0	0	0	0	1	0+0	$2Xb/EJ$
DA 2b	-1	0	0	0	0	1		
DE b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	0	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	$-1/2Fx$	0	0	0	0	0+0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0		
GC b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0	0+0	0
CG b	0	$1/2qx^2$	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	0	0	0	0	0	0+0	0
BH b	0	0	0	0	0	0		
HE b	0	$-1/2Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{0AB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AB}$						$-1/10Fb$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \frac{1}{EJ} dx + 1 \cdot \frac{1}{4} \frac{b}{EJ} = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b \frac{1}{EJ} + 1 \cdot \frac{1}{4} \frac{b}{EJ}$$

$$= (b - b + 1/3 b) \frac{1}{EJ} + 1 \cdot \frac{1}{4} \frac{b}{EJ} = 7/12 \frac{b}{EJ}$$

$$L_{BA}^{xx} = \int_0^b (x^2/b^2) \frac{1}{EJ} dx + 1 \cdot \frac{1}{4} \frac{b}{EJ} = \left[1/3 x^3/b^2 \right]_0^b \frac{1}{EJ} + 1 \cdot \frac{1}{4} \frac{b}{EJ}$$

$$= (1/3 b) \frac{1}{EJ} + 1 \cdot \frac{1}{4} \frac{b}{EJ} = 7/12 \frac{b}{EJ}$$

$$L_{AD}^{xx} = \int_0^{2b} (1) \frac{1}{EJ} dx = \left[x \right]_0^{2b} \frac{1}{EJ}$$

$$= (2b) \frac{1}{EJ} = 2 \frac{b}{EJ}$$

$$L_{DA}^{xx} = \int_0^{2b} (1) \frac{1}{EJ} dx = \left[x \right]_0^{2b} \frac{1}{EJ}$$

$$= (2b) \frac{1}{EJ} = 2 \frac{b}{EJ}$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \frac{1}{EJ} dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b \frac{1}{EJ}$$

$$= (b - b + 1/3 b) \frac{1}{EJ} = 1/3 \frac{b}{EJ}$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) \frac{1}{EJ} dx = \left[1/3 x^3/b^2 \right]_0^b \frac{1}{EJ}$$

$$= (1/3 b) \frac{1}{EJ} = 1/3 \frac{b}{EJ}$$

$$L_{AB}^{x_0} = \int_0^b (3/2 x/b - x^2/b^2 - 1/2 x^3/b^3) Fb \frac{1}{EJ} dx + 1 \cdot 0 \frac{1}{4} \frac{Fb^2}{EJ}$$

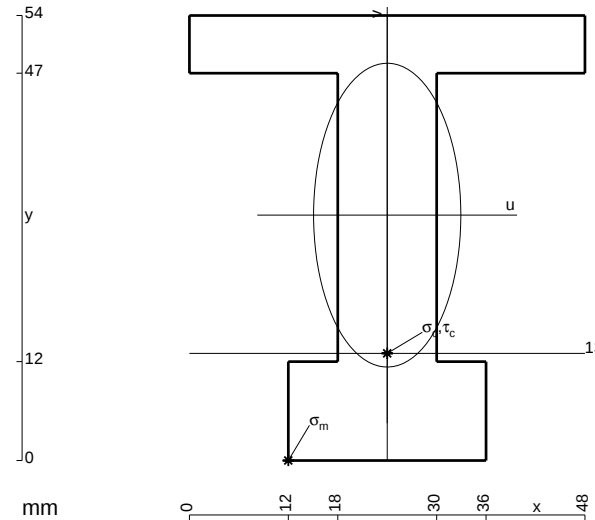
$$= \left[3/4 x^2/b - 1/3 x^3/b^2 - 1/8 x^4/b^3 \right]_0^b Fb \frac{1}{EJ} + 1 \cdot 0 \frac{1}{4} \frac{Fb^2}{EJ}$$

$$= (3/4 b - 1/3 b - 1/8 b) Fb \frac{1}{EJ} + 1 \cdot 0 \frac{1}{4} \frac{Fb^2}{EJ} = 7/24 \frac{Fb^2}{EJ}$$

$$L_{BA}^{x_0} = \int_0^b (2x/b - 5/2 x^2/b^2 + 1/2 x^3/b^3) Fb \frac{1}{EJ} dx + 1 \cdot 0 \frac{1}{4} \frac{Fb^2}{EJ}$$

$$= \left[x^2/b - 5/6 x^3/b^2 + 1/8 x^4/b^3 \right]_0^b Fb \frac{1}{EJ} + 1 \cdot 0 \frac{1}{4} \frac{Fb^2}{EJ}$$

$$= (b - 5/6 b + 1/8 b) Fb \frac{1}{EJ} + 1 \cdot 0 \frac{1}{4} \frac{Fb^2}{EJ} = 7/24 \frac{Fb^2}{EJ}$$



$$A = 1044. \text{ mm}^2$$

$$J_u = 354848. \text{ mm}^4$$

$$J_v = 83376. \text{ mm}^4$$

$$y_g = 29.78 \text{ mm}$$

$$T_y = 3260. \text{ N}$$

$$M_x = -2379800. \text{ Nmm}$$

$$x_m = 12. \text{ mm}$$

$$u_m = -12. \text{ mm}$$

$$v_m = -29.78 \text{ mm}$$

$$\sigma_m = -Mv/J_u = -199.7 \text{ N/mm}^2$$

$$x_c = 24. \text{ mm}$$

$$y_c = 13. \text{ mm}$$

$$v_c = -16.78 \text{ mm}$$

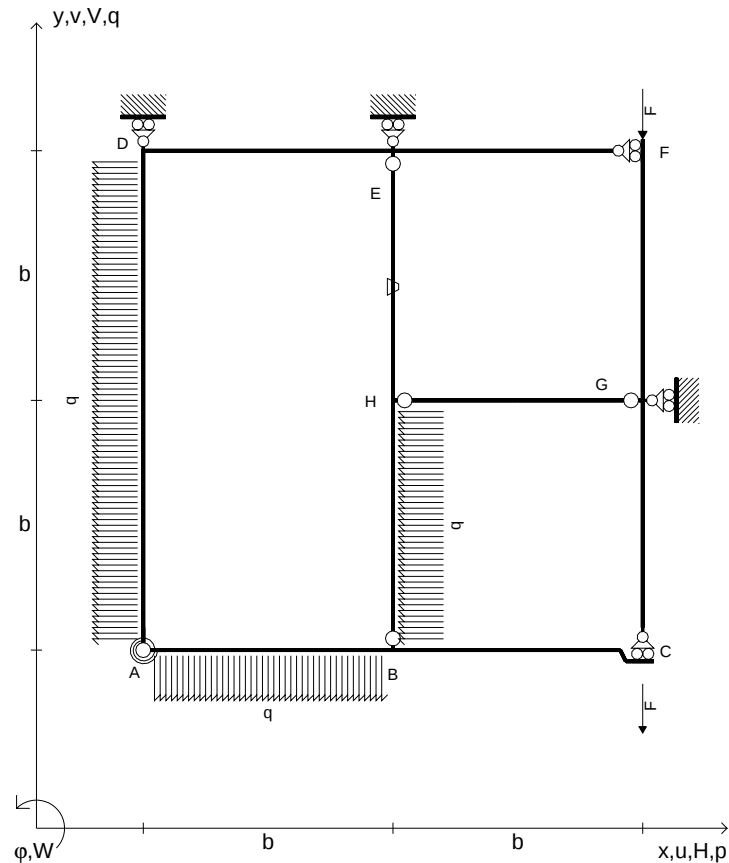
$$\sigma_c = -Mv/J_u = -112.5 \text{ N/mm}^2$$

$$\tau_c = 5.401 \text{ N/mm}^2$$

$$\sigma_0 = \sqrt{\sigma^2 + 3\tau^2} = 112.9 \text{ N/mm}^2$$

$$S = 7055. \text{ mm}^3$$

$$\begin{aligned}
 V_{FG} &= -F \\
 V_{CB} &= -F \\
 q_{AB} &= -q = -F/b \\
 p_{AD} &= -q = -F/b \\
 p_{HB} &= -q = -F/b \\
 \theta_{HE} &= -\theta = -\alpha T/b = -bF/EJ \\
 k_{AB} &= 4EJ/b \\
 EJ_{AB} &= EJ \\
 EJ_{BC} &= EJ \\
 EJ_{AD} &= EJ \\
 EJ_{DE} &= EJ \\
 EJ_{EF} &= EJ \\
 EJ_{FG} &= EJ \\
 EJ_{GC} &= EJ \\
 EJ_{HG} &= EJ \\
 EJ_{HB} &= EJ \\
 EJ_{HE} &= EJ
 \end{aligned}$$



Reazioni iperstatiche in soluzione: $X=W_{AD}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

$J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave BC ha la sezione riportata e dimensioni in mm, con:

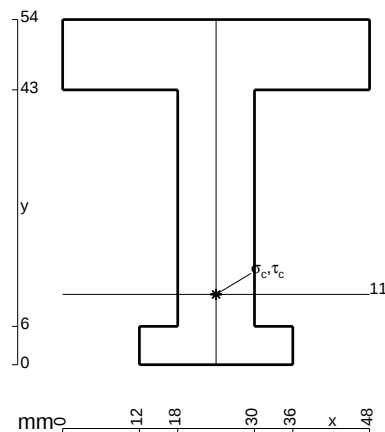
$b = 490 \text{ mm}$, $F = 2120 \text{ N}$

Calcolare sulla sezione B la massima tensione normale σ_m .

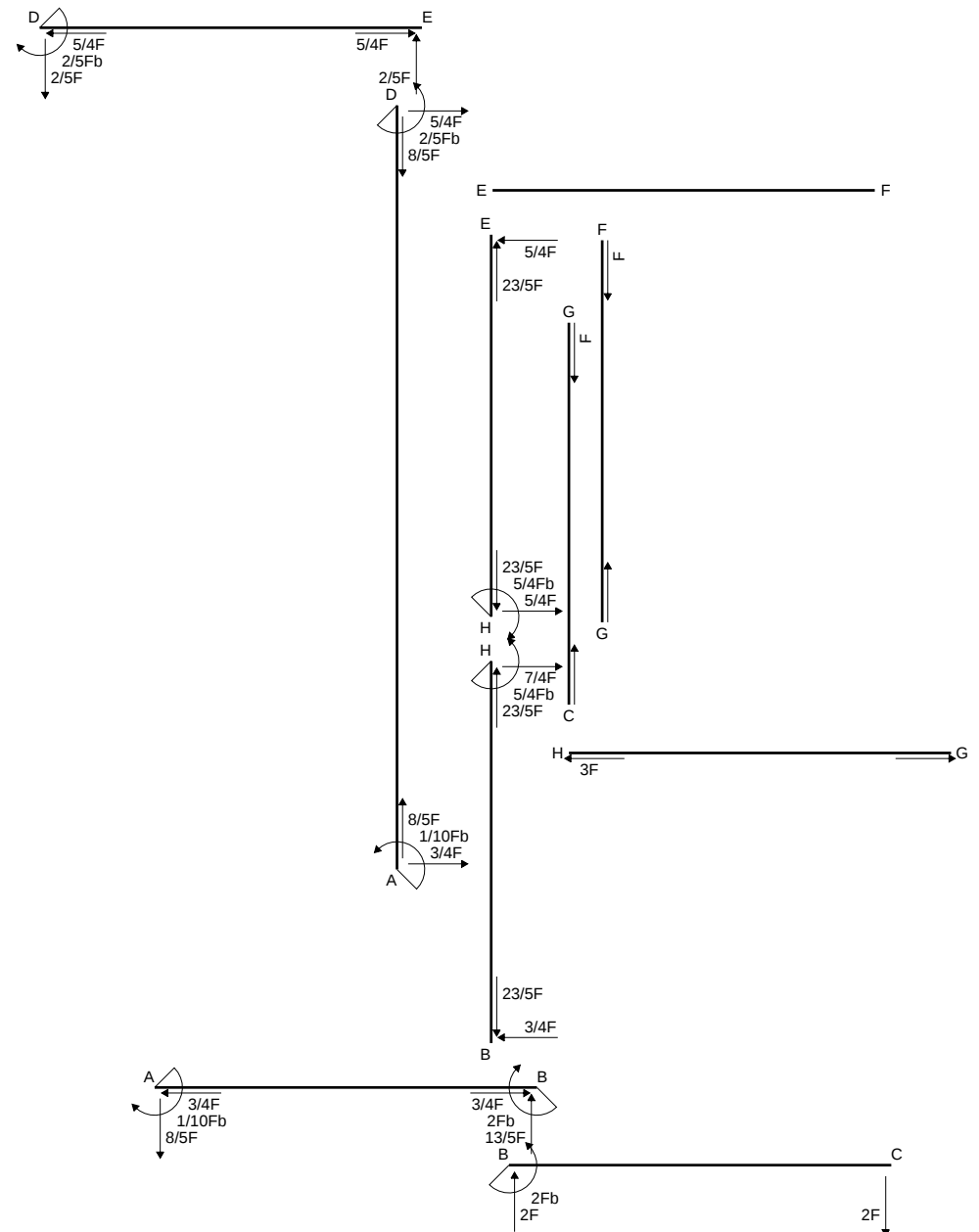
Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.

Lembo inferiore sezione su tratteggio trave, a destra da B a C
Curvatura θ asta HE positiva se convessa a destra con inizio H.

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

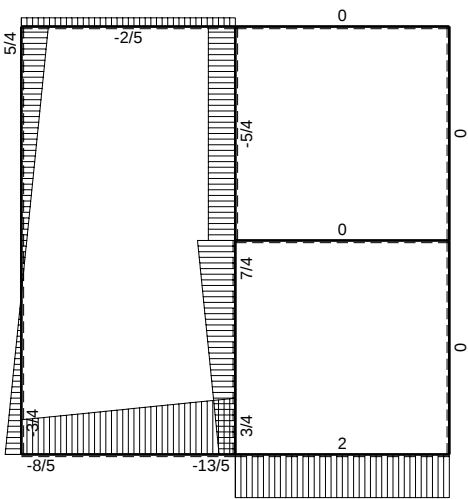
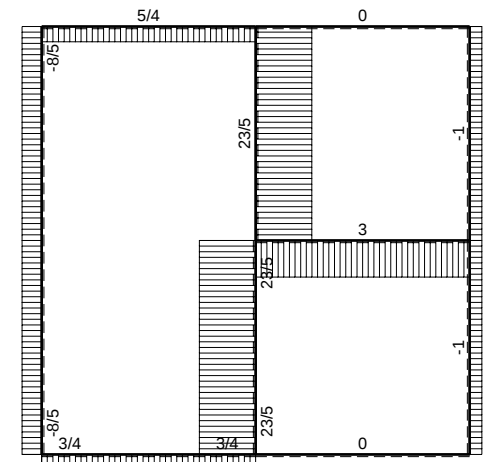


14.05.25



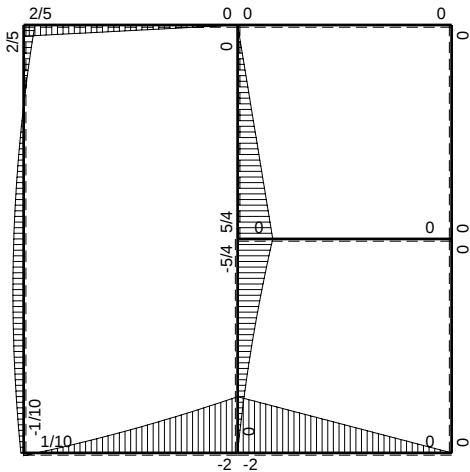
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

14.05.25

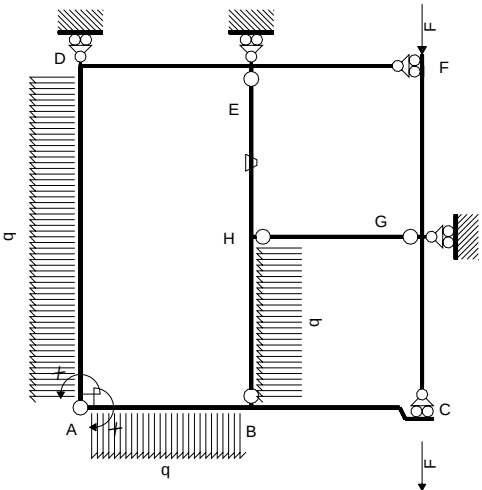


←  → F

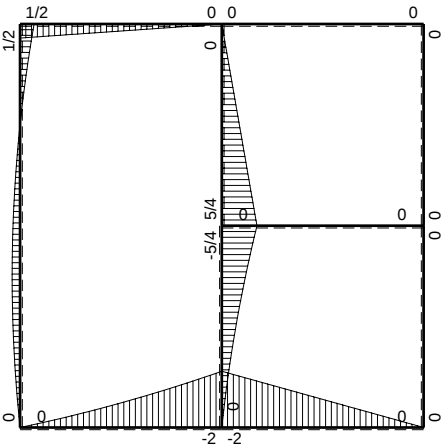
↑  ↓ F



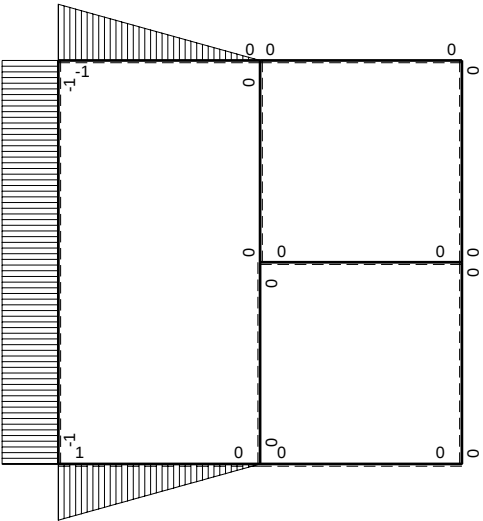
↺  ↻ F_b



Schema di calcolo iperstatico



↺  ↻ M₀ flessione da carichi assegnati



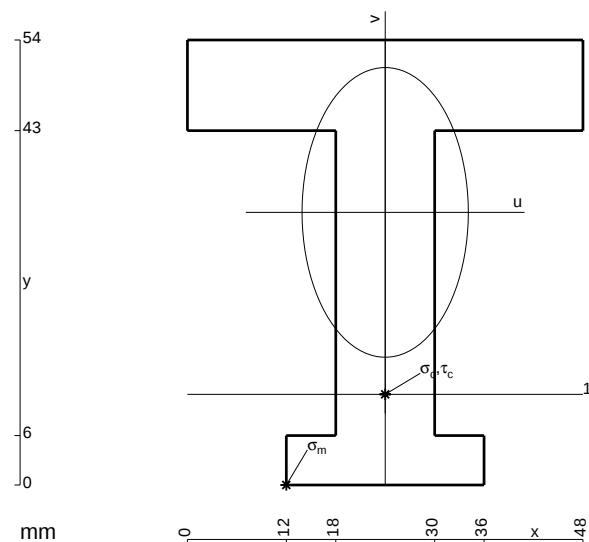
↺  ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AD}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	$1-x/b$	$-3/2Fx-1/2qx^2$	0	$-3/2Fx+Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-7/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	$-x/b$	$2Fb-5/2Fx+1/2qx^2$	0	$-2Fx+5/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
BC b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
CB b	0	$2Fx$	0	0	0	0		
AD 2b	-1	$-3/4Fx+1/2qx^2$	0	$3/4Fx-1/2Fx^2/b$	0	1	$(1/6+0)Fb^2/EJ$	$2Xb/EJ$
DA 2b	1	$-1/2Fb+5/4Fx-1/2qx^2$	0	$-1/2Fb+5/4Fx-1/2Fx^2/b$	0	1		
DE b	$-1+x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
EF b	0	0	0	0	0	0	0+0	0
FE b	0	0	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GC b	0	0	0	0	0	0	0+0	0
CG b	0	0	0	0	0	0		
HG b	0	0	0	0	0	0	0+0	0
GH b	0	0	0	0	0	0		
HB b	0	$-5/4Fb+7/4Fx-1/2qx^2$	0	0	0	0	0+0	0
BH b	0	$3/4Fx+1/2qx^2$	0	0	0	0		
HE b	0	$5/4Fb-5/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
EH b	0	$-5/4Fx$	Fb/EJ	0	0	0		
AB	molla asta $-W_{1AB}(W_{0AB}+XW_{1AB})/k_{AB}$							$1/4Xb/EJ$
	totali						$-7/24Fb^2/EJ$	$35/12Xb/EJ$
	iperstatica $X=W_{AD}$						$1/10Fb$	

Sviluppi di calcolo iperstatica

$$\begin{aligned}
L_{AB}^{xx} &= \int_0^b (1 - 2x/b + x^2/b^2) \frac{1}{EJ} dx - 1 \cdot (-1) \frac{1}{4} \frac{b}{EJ} = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b \frac{1}{EJ} - 1 \cdot (-1) \frac{1}{4} \frac{b}{EJ} \\
&= (b - b + 1/3 b) \frac{1}{EJ} - 1 \cdot (-1) \frac{1}{4} \frac{b}{EJ} = 7/12 \frac{b}{EJ} \\
L_{BA}^{xx} &= \int_0^b (x^2/b^2) \frac{1}{EJ} dx - 1 \cdot (-1) \frac{1}{4} \frac{b}{EJ} = \left[1/3 x^3/b^2 \right]_0^b \frac{1}{EJ} - 1 \cdot (-1) \frac{1}{4} \frac{b}{EJ} \\
&= (1/3 b) \frac{1}{EJ} - 1 \cdot (-1) \frac{1}{4} \frac{b}{EJ} = 7/12 \frac{b}{EJ} \\
L_{AD}^{xx} &= \int_0^{2b} (1) \frac{1}{EJ} dx = \left[x \right]_0^{2b} \frac{1}{EJ} \\
&= (2b) \frac{1}{EJ} = 2 \frac{b}{EJ} \\
L_{DA}^{xx} &= \int_0^{2b} (1) \frac{1}{EJ} dx = \left[x \right]_0^{2b} \frac{1}{EJ} \\
&= (2b) \frac{1}{EJ} = 2 \frac{b}{EJ} \\
L_{DE}^{xx} &= \int_0^b (1 - 2x/b + x^2/b^2) \frac{1}{EJ} dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b \frac{1}{EJ} \\
&= (b - b + 1/3 b) \frac{1}{EJ} = 1/3 \frac{b}{EJ} \\
L_{ED}^{xx} &= \int_0^b (x^2/b^2) \frac{1}{EJ} dx = \left[1/3 x^3/b^2 \right]_0^b \frac{1}{EJ} \\
&= (1/3 b) \frac{1}{EJ} = 1/3 \frac{b}{EJ} \\
L_{AB}^{x_0} &= \int_0^b (-3/2 x/b + x^2/b^2 + 1/2 x^3/b^3) Fb \frac{1}{EJ} dx - 1 \cdot 0 \frac{1}{4} Fb^2/EJ \\
&= \left[-3/4 x^2/b + 1/3 x^3/b^2 + 1/8 x^4/b^3 \right]_0^b Fb \frac{1}{EJ} - 1 \cdot 0 \frac{1}{4} Fb^2/EJ \\
&= (-3/4 b + 1/3 b + 1/8 b) Fb \frac{1}{EJ} - 1 \cdot 0 \frac{1}{4} Fb^2/EJ = -7/24 Fb^2/EJ \\
L_{BA}^{x_0} &= \int_0^b (-2 x/b + 5/2 x^2/b^2 - 1/2 x^3/b^3) Fb \frac{1}{EJ} dx - 1 \cdot 0 \frac{1}{4} Fb^2/EJ \\
&= \left[-x^2/b + 5/6 x^3/b^2 - 1/8 x^4/b^3 \right]_0^b Fb \frac{1}{EJ} - 1 \cdot 0 \frac{1}{4} Fb^2/EJ \\
&= (-b + 5/6 b - 1/8 b) Fb \frac{1}{EJ} - 1 \cdot 0 \frac{1}{4} Fb^2/EJ = -7/24 Fb^2/EJ \\
L_{AD}^{x_0} &= \int_0^{2b} (3/4 x/b - 1/2 x^2/b^2) Fb \frac{1}{EJ} dx = \left[3/8 x^2/b - 1/6 x^3/b^2 \right]_0^{2b} Fb \frac{1}{EJ} \\
&= (3/2 b - 4/3 b) Fb \frac{1}{EJ} = 1/6 Fb^2/EJ \\
L_{DA}^{x_0} &= \int_0^{2b} (-1/2 + 5/4 x/b - 1/2 x^2/b^2) Fb \frac{1}{EJ} dx = \left[-1/2 x + 5/8 x^2/b - 1/6 x^3/b^2 \right]_0^{2b} Fb \frac{1}{EJ} \\
&= (-b + 5/2 b - 4/3 b) Fb \frac{1}{EJ} = 1/6 Fb^2/EJ \\
L_{DE}^{x_0} &= \int_0^b (-$$



$$\begin{aligned} A &= 1116. \text{ mm}^2 \\ J_u &= 344933. \text{ mm}^4 \\ J_v &= 113616. \text{ mm}^4 \\ y_g &= 33.08 \text{ mm} \\ T_y &= 4240. \text{ N} \\ M_x &= -2077600. \text{ Nmm} \\ x_m &= 12. \text{ mm} \\ u_m &= -12. \text{ mm} \\ v_m &= -33.08 \text{ mm} \\ \sigma_m &= -Mv/J_u = -199.3 \text{ N/mm}^2 \\ x_c &= 24. \text{ mm} \\ y_c &= 11. \text{ mm} \\ v_c &= -22.08 \text{ mm} \\ \sigma_c &= -Mv/J_u = -133. \text{ N/mm}^2 \\ \tau_c &= 5.948 \text{ N/mm}^2 \\ \sigma_o &= \sqrt{\sigma^2 + 3\tau^2} = 133.4 \text{ N/mm}^2 \\ S &= 5806. \text{ mm}^3 \end{aligned}$$