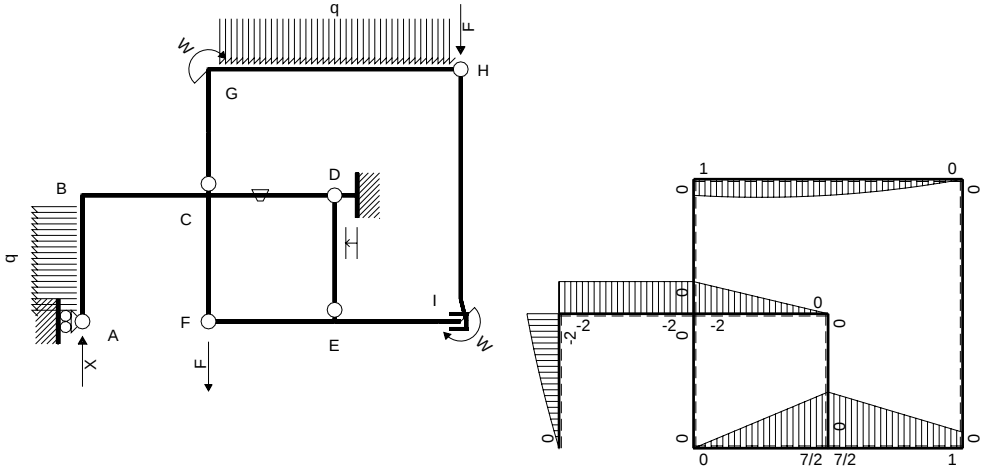
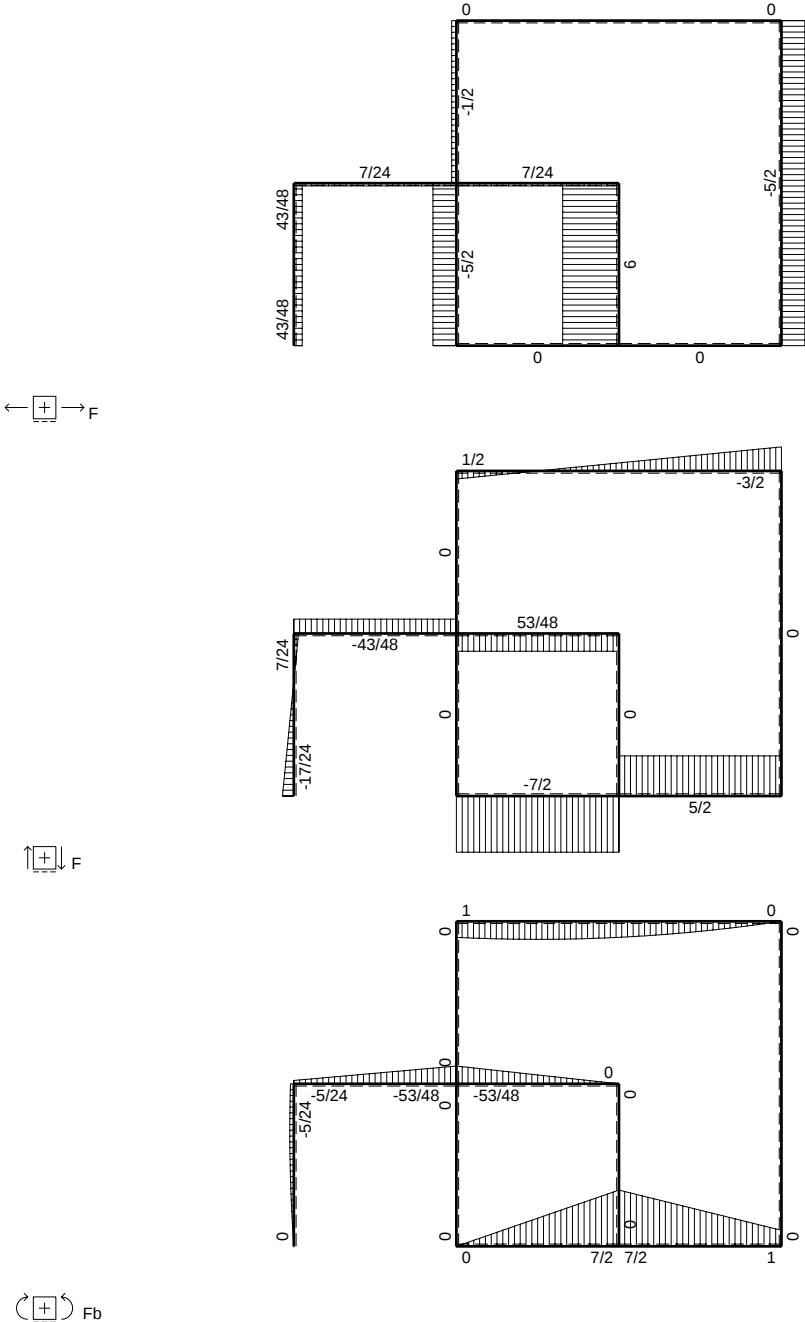
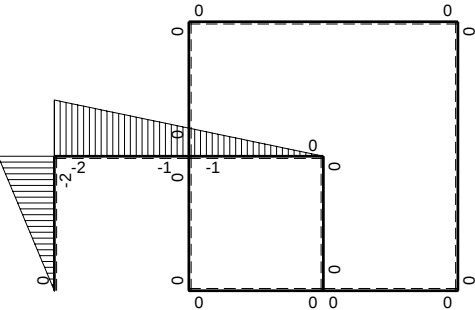




Schema di calcolo iperstatico

M_o flessione da carichi assegnati



M_x flessione da iperstatica $X=1$

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-2x	-5/2Fx+1/2qx ²	0	5Fx ² -qx ³	0	4x ²	(17/12+0)Fb ³ /EJ	4/3Xb ³ /EJ
BA b	2b-2x	2Fb-3/2Fx-1/2qx ²	0	4Fb ² -7Fbx+2Fx ² +qx ³	0	4b ² -8bx+4x ²		
BC b	-2b+x	-2Fb	0	4Fb ² -2Fbx	0	4b ² -4bx+x ²	(3+0)Fb ³ /EJ	7/3Xb ³ /EJ
CB b	b+x	2Fb	0	2Fb ² +2Fbx	0	b ² +2bx+x ²		
CD b	-b+x	-2Fb+2Fx	-Fb/EJ	2Fb ² -4Fbx+2Fx ²	Fb ² /EJ-Fxb/EJ	b ² -2bx+x ²	(2/3+1/2)Fb ³ /EJ	1/3Xb ³ /EJ
DC b	x	2Fx	Fb/EJ	2Fx ²	Fxb/EJ	x ²		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-7/2Fx	0	0	0	0	0+0	0
FE b	0	-7/2Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb+1/2Fx-1/2qx ²	0	0	0	0	0+0	0
HG 2b	0	-3/2Fx+1/2qx ²	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+5/2Fx	0	0	0	0		
D	cedimento nodo -H _{1D} u _D						-2Fb ³ /EJ	
	totali						43/12Fb ³ /EJ	4Xb ³ /EJ
	iperstatica $X=V_A$						-43/48F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = \left[4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = \left[4 x - 4 x^2/b + 4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = \left[4 x - 2 x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = \left[x + x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = \left[1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (5 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = \left[5/3 x^3/b^2 - 1/4 x^4/b^3 \right]_0^b Fb^2 1/EJ$$

$$= (5/3 b - 1/4 b) Fb^2 1/EJ = 17/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (4 - 7 x/b + 2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = \left[4 x - 7/2 x^2/b + 2/3 x^3/b^2 + 1/4 x^4/b^3 \right]_0^b Fb^2 1/EJ$$

$$= (4 b - 7/2 b + 2/3 b + 1/4 b) Fb^2 1/EJ = 17/12 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (4 - 2 x/b) Fb^2 1/EJ dx = \left[4 x - x^2/b \right]_0^b Fb^2 1/EJ$$

$$= (4 b - b) Fb^2 1/EJ = 3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (2 + 2 x/b) Fb^2 1/EJ dx = \left[2 x + x^2/b \right]_0^b Fb^2 1/EJ$$

$$= (2 b + b) Fb^2 1/EJ = 3 Fb^3/EJ$$

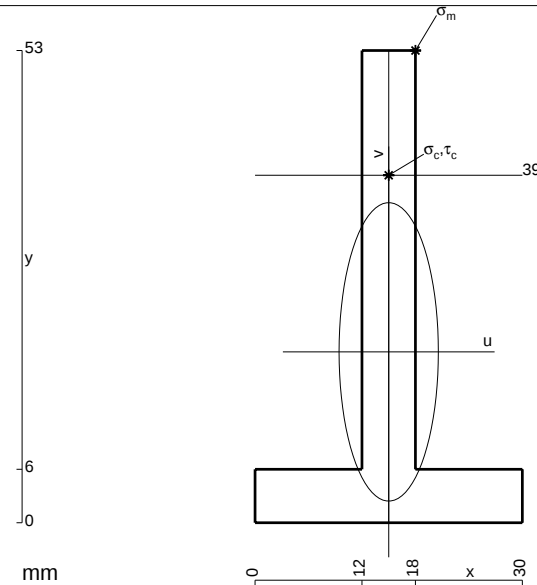
$$L_{CD}^{xo} = \int_0^b (2 - 4 x/b + 2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

$$= \left[2 x - 2 x^2/b + 2/3 x^3/b^2 \right]_0^b Fb^2 1/EJ + \left[x - 1/2 x^2/b \right]_0^b \theta$$

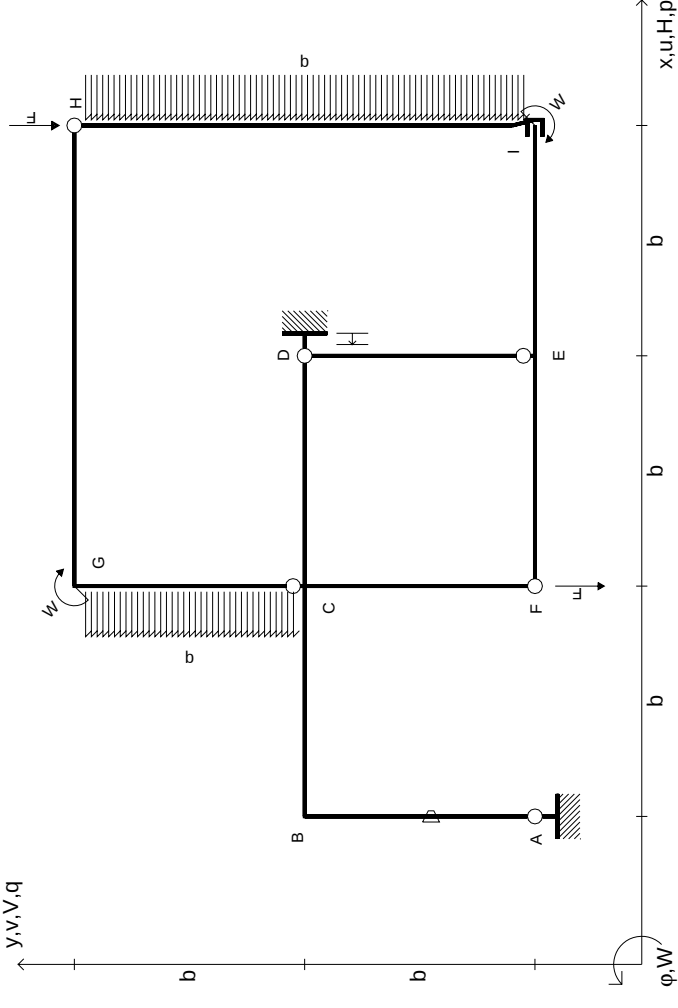
$$= (2 b - 2 b + 2/3 b) Fb^2 1/EJ + (b - 1/2 b) \theta = 7/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx = \left[2/3 x^3/b^2 \right]_0^b Fb^2 1/EJ + \left[-1/2 x^2/b \right]_0^b \theta$$

$$= (2/3 b) Fb^2 1/EJ + (-1/2 b) \theta = 7/6 Fb^3/EJ$$



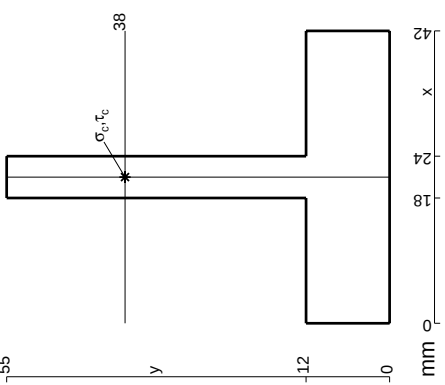
$$\begin{aligned} A &= 462. \text{ mm}^2 \\ J_u &= 129608. \text{ mm}^4 \\ J_v &= 14346. \text{ mm}^4 \\ y_g &= 19.18 \text{ mm} \\ T_y &= -1120. \text{ N} \\ M_x &= 784000. \text{ Nmm} \\ x_m &= 18. \text{ mm} \\ y_m &= 53. \text{ mm} \\ u_m &= 3. \text{ mm} \\ v_m &= 33.82 \text{ mm} \\ \sigma_m &= -Mv/J_u = -204.6 \text{ N/mm}^2 \\ x_c &= 15. \text{ mm} \\ y_c &= 39. \text{ mm} \\ v_c &= 19.82 \text{ mm} \\ \sigma_c &= -Mv/J_u = -119.9 \text{ N/mm}^2 \\ \tau_c &= 3.245 \text{ N/mm}^2 \\ \sigma_o &= \sqrt{\sigma^2 + 3\tau^2} = 120.1 \text{ N/mm}^2 \\ S &= 2253. \text{ mm}^3 \end{aligned}$$



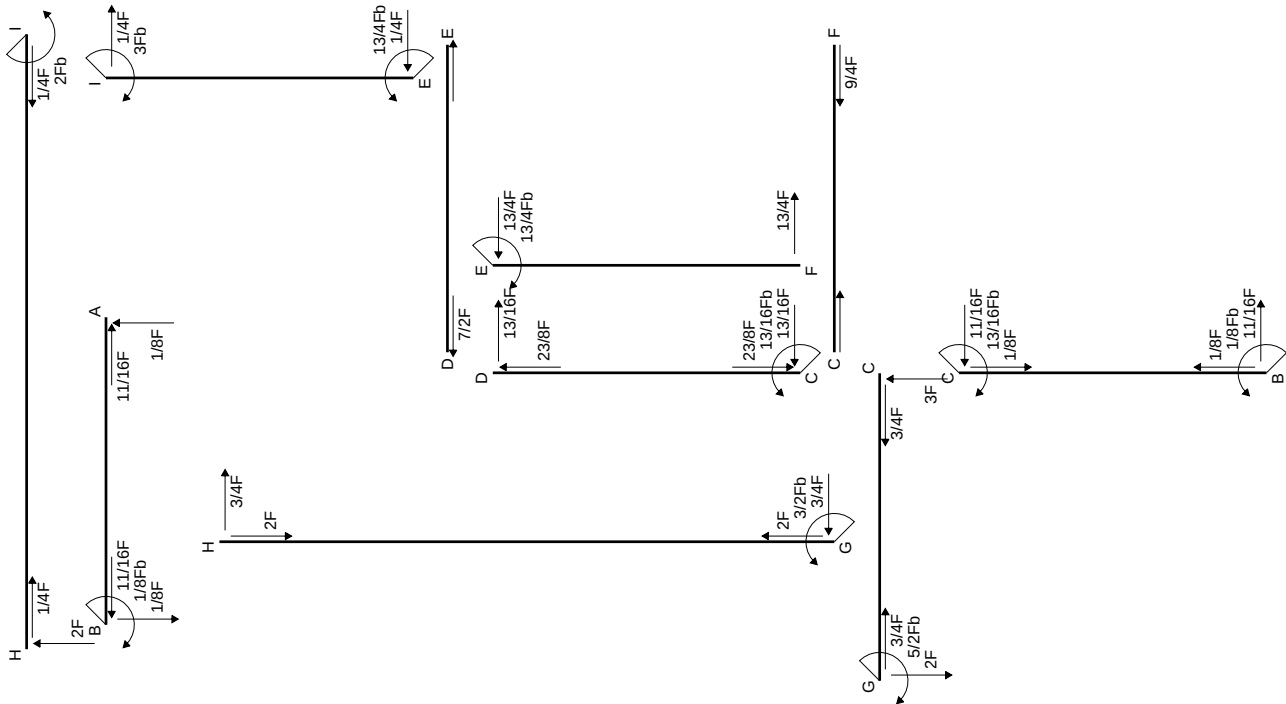
$$V_A = -F$$
$$V_E = -F$$
$$W_I = -W = -Fb$$
$$W_G = -W = -Fb$$
$$p_{HI} = -q = -F/b$$
$$p_{CG} = -q = -F/b$$

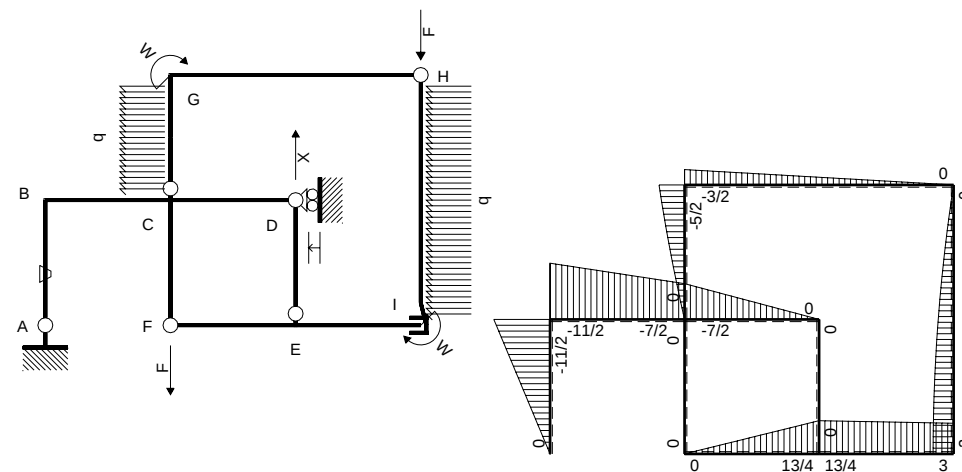
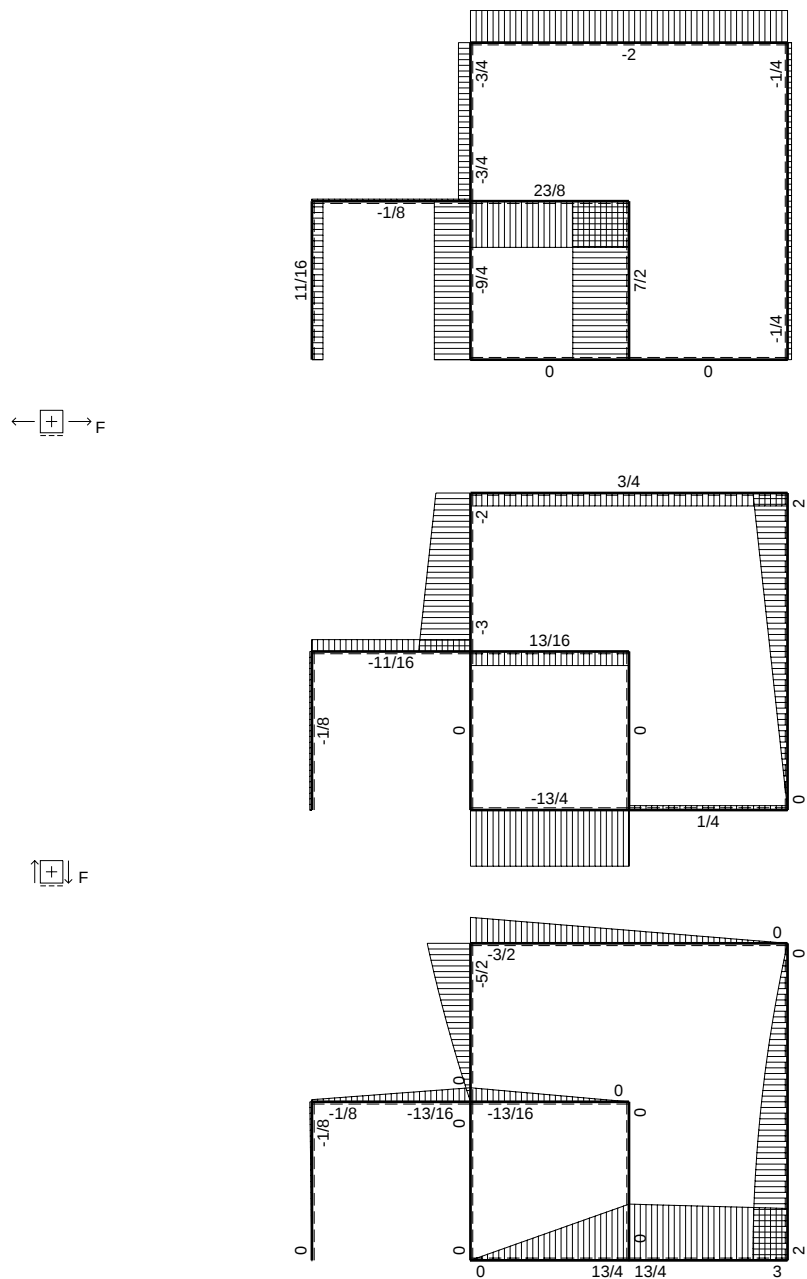
$$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$$
$$u_D = -\delta = -b^3 F/EJ$$
$$EJ_{AB} = EJ$$
$$EJ_{BC} = EJ$$
$$EJ_{CD} = EJ$$
$$EJ_{DE} = EJ$$

$$EJ_{EF} = EJ$$
$$EJ_{FC} = EJ$$
$$EJ_{CG} = EJ$$
$$EJ_{GH} = EJ$$
$$EJ_{HI} = EJ$$
$$EJ_{IE} = EJ$$



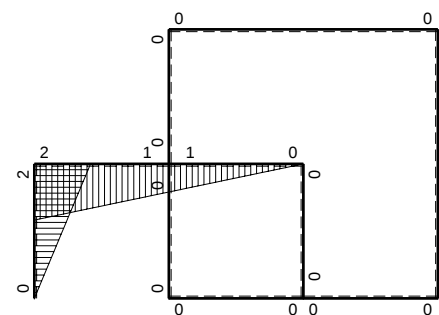
Reazioni iperstatiche in soluzione: $X=V_D$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{yz} - x_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
 $b = 670 \text{ mm}$, $F = 400 \text{ N}$
Calcolare sulla sezione E la massima tensione normale σ_m .
Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
Lembo inferiore sezione su traveggio trave, a destra da E a F
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.





Schema di calcolo iperstatico

 M_0 flessione da carichi assegnati



 M_x flessione da iperstatica $X=1$

Quadro contributi PLV per iperstatica $X=V_D$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int x M_x M_x/EJ dx$
AB b	2x	-11/2Fx	-Fb/EJ	-11Fx ²	-2Fxb/EJ	4x ²	$(-11/3-1)Fb^3/EJ$	$4/3xb^3/EJ$
BA b	-2b+2x	11/2Fb-11/2Fx	Fb/EJ	$-11Fb^2+22Fbx-11Fx^2$	$-2Fb^2/EJ+2Fxb/EJ$	$4b^2-8bx+4x^2$		
BC b	2b-x	-11/2Fb+2Fx	0	$-11Fb^2+19/2Fbx-2Fx^2$	0	$4b^2-4bx+x^2$	$(-83/12+0)Fb^3/EJ$	$7/3xb^3/EJ$
CB b	-b-x	7/2Fb+2Fx	0	$-7/2Fb^2-11/2Fbx-2Fx^2$	0	$b^2+2bx+x^2$		
CD b	b-x	-7/2Fb+7/2Fx	0	$-7/2Fb^2+7Fbx-7/2Fx^2$	0	$b^2-2bx+x^2$	$(-7/6+0)Fb^3/EJ$	$1/3xb^3/EJ$
DC b	-x	7/2Fx	0	$-7/2Fx^2$	0	x ²		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	13/4Fb-13/4Fx	0	0	0	0	0+0	0
FE b	0	-13/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-3Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$5/2Fb-2Fx-1/2qx^2$	0	0	0	0		
GH 2b	0	$-3/2Fb+3/4Fx$	0	0	0	0	0+0	0
HG 2b	0	3/4Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+1/4Fx	0	0	0	0	0+0	0
EI b	0	-13/4Fb+1/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						2Fb ³ /EJ	
	totali						-43/4Fb ³ /EJ	4xb ³ /EJ
	iperstatica $X=V_D$						43/16F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = [4 x - 4 x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = [4 x - 2 x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-11 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-2 x/b) \theta dx = [-11/3 x^3/b^2]_0^b Fb^2 1/EJ + [-x^2/b]_0^b \theta$$

$$= (-11/3 b) Fb^2 1/EJ + (-b) \theta = -14/3 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-11 + 22 x/b - 11 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (2 - 2 x/b) \theta dx$$

$$= [-11 x + 11 x^2/b - 11/3 x^3/b^2]_0^b Fb^2 1/EJ + [2 x - x^2/b]_0^b \theta$$

$$= (-11 b + 11 b - 11/3 b) Fb^2 1/EJ + (2 b - b) \theta = -14/3 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-11 + 19/2 x/b - 2 x^2/b^2) Fb^2 1/EJ dx = [-11 x + 19/4 x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-11 b + 19/4 b - 2/3 b) Fb^2 1/EJ = -83/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-7/2 - 11/2 x/b - 2 x^2/b^2) Fb^2 1/EJ dx = [-7/2 x - 11/4 x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

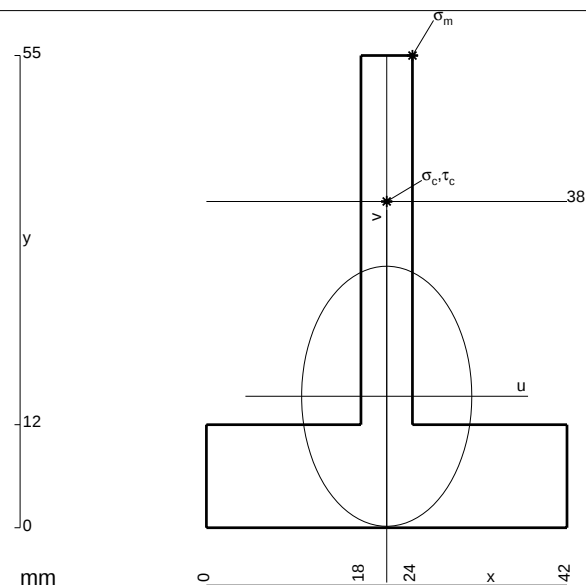
$$= (-7/2 b - 11/4 b - 2/3 b) Fb^2 1/EJ = -83/12 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (-7/2 + 7 x/b - 7/2 x^2/b^2) Fb^2 1/EJ dx = [-7/2 x + 7/2 x^2/b - 7/6 x^3/b^2]_0^b Fb^2 1/EJ$$

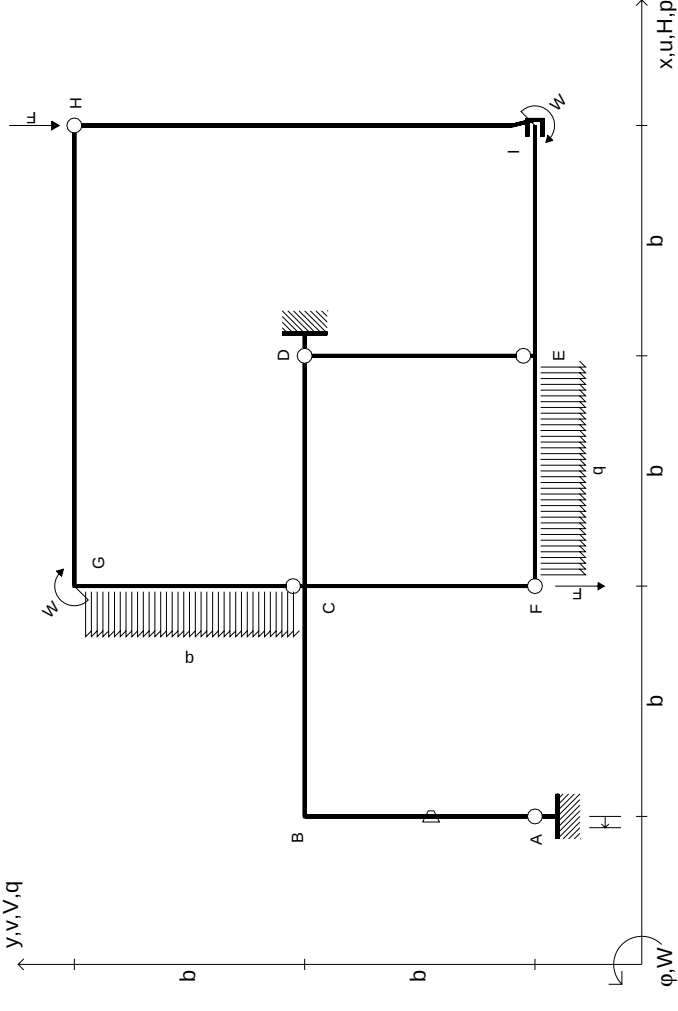
$$= (-7/2 b + 7/2 b - 7/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (-7/2 x^2/b^2) Fb^2 1/EJ dx = [-7/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-7/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$



$$\begin{aligned} A &= 762. \text{ mm}^2 \\ J_u &= 174852. \text{ mm}^4 \\ J_v &= 74862. \text{ mm}^4 \\ y_g &= 15.31 \text{ mm} \\ T_y &= -1300. \text{ N} \\ M_x &= 871000. \text{ Nmm} \\ x_m &= 24. \text{ mm} \\ y_m &= 55. \text{ mm} \\ u_m &= 3. \text{ mm} \\ v_m &= 39.69 \text{ mm} \\ \sigma_m &= -Mv/J_u = -197.7 \text{ N/mm}^2 \\ x_c &= 21. \text{ mm} \\ y_c &= 38. \text{ mm} \\ v_c &= 22.69 \text{ mm} \\ \sigma_c &= -Mv/J_u = -113. \text{ N/mm}^2 \\ \tau_c &= 3.942 \text{ N/mm}^2 \\ \sigma_o &= \sqrt{\sigma^2 + 3\tau^2} = 113.2 \text{ N/mm}^2 \\ S &= 3181. \text{ mm}^3 \end{aligned}$$



$V_A = -F$
 $V_E = -F$
 $W_I = -W = -Fb$
 $W_G = -W = -Fb$
 $p_{CG} = -q = -F/b$
 $q_{EF} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
 $u_A = -\delta = -b^3 F/EJ$
 $EJ_{AB} = EJ$
 $EJ_{BC} = EJ$
 $EJ_{CD} = EJ$
 $EJ_{DE} = EJ$

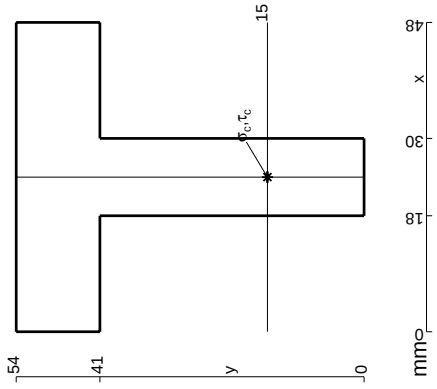
$EJ_{EF} = EJ$
 $EJ_{FC} = EJ$
 $EJ_{CG} = EJ$
 $EJ_{GH} = EJ$
 $EJ_{HI} = EJ$
 $EJ_{IE} = EJ$

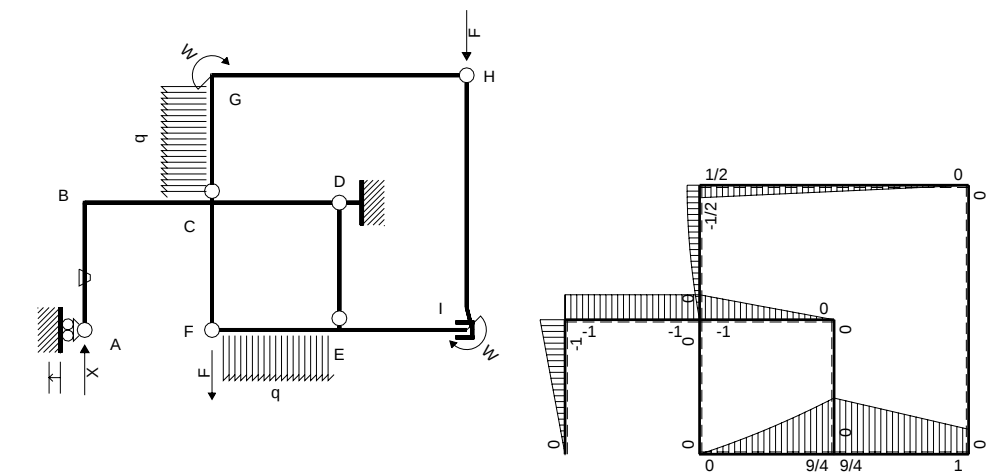
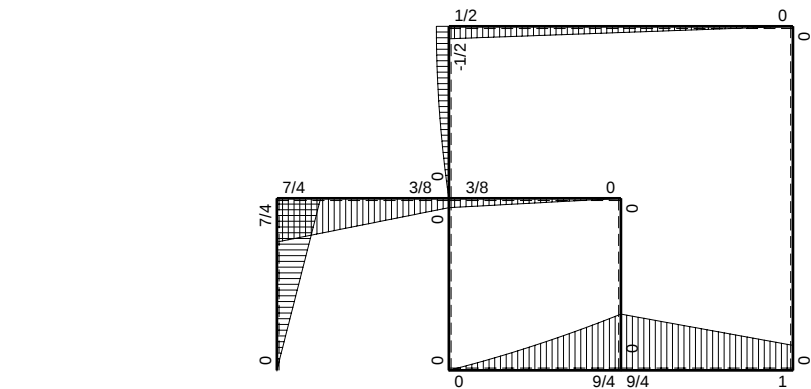
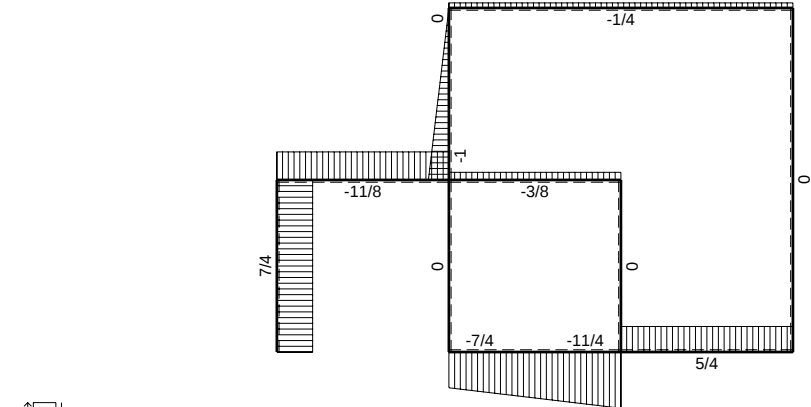
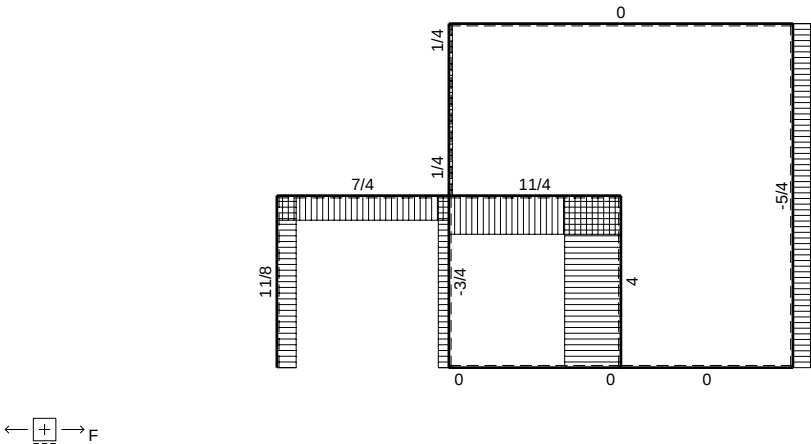
Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{yz} - x_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.

La trave EF ha la sezione riportata e dimensioni in mm, con:
 $b = 630 \text{ mm}, F = 1320 \text{ N}$

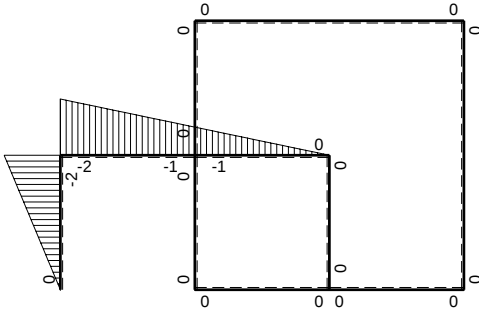
Calcolare sulla sezione E la massima tensione normale σ_m .
Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.





Schema di calcolo iperstatico

M_0 flessione da carichi assegnati



Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-2x	-Fx	-Fb/EJ	$2Fx^2$	$2Fxb/EJ$	$4x^2$	$(2/3+1)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	2b-2x	Fb-Fx	Fb/EJ	$2Fb^2-4Fbx+2Fx^2$	$2Fb^2/EJ-2Fxb/EJ$	$4b^2-8bx+4x^2$		
BC b	-2b+x	-Fb	0	$2Fb^2-Fbx$	0	$4b^2-4bx+x^2$	$(3/2+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	b+x	Fb	0	Fb^2+Fbx	0	$b^2+2bx+x^2$		
CD b	-b+x	-Fb+Fx	0	$Fb^2-2Fbx+Fx^2$	0	$b^2-2bx+x^2$	$(1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	x	Fx	0	Fx^2	0	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$9/4Fb-11/4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-7/4Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/4Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx$	0	0	0	0	0+0	0
EI b	0	$-9/4Fb+5/4Fx$	0	0	0	0		
A	cedimento nodo -H _{1A} u _A						$2Fb^3/EJ$	
	totali						$11/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_A$						-11/8F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = [4 x - 4 x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = [4 x - 2 x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{x_0} = \int_0^b (2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (2 x/b) \theta dx = [2/3 x^3/b^2]_0^b Fb^2 1/EJ + [x^2/b]_0^b \theta$$

$$= (2/3 b) Fb^2 1/EJ + (b) \theta = 5/3 Fb^3/EJ$$

$$L_{BA}^{x_0} = \int_0^b (2 - 4 x/b + 2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-2 + 2 x/b) \theta dx$$

$$= [2 x - 2 x^2/b + 2/3 x^3/b^2]_0^b Fb^2 1/EJ + [-2 x + x^2/b]_0^b \theta$$

$$= (2 b - 2 b + 2/3 b) Fb^2 1/EJ + (-2 b + b) \theta = 5/3 Fb^3/EJ$$

$$L_{BC}^{x_0} = \int_0^b (2 - x/b) Fb^2 1/EJ dx = [2 x - 1/2 x^2/b]_0^b Fb^2 1/EJ$$

$$= (2 b - 1/2 b) Fb^2 1/EJ = 3/2 Fb^3/EJ$$

$$L_{CB}^{x_0} = \int_0^b (1 + x/b) Fb^2 1/EJ dx = [x + 1/2 x^2/b]_0^b Fb^2 1/EJ$$

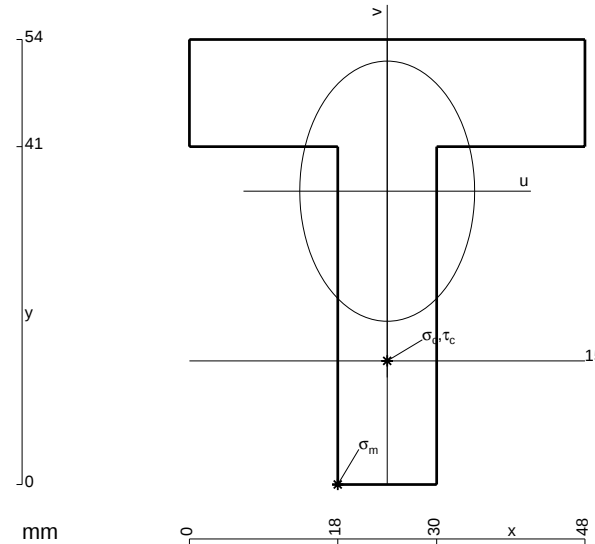
$$= (b + 1/2 b) Fb^2 1/EJ = 3/2 Fb^3/EJ$$

$$L_{CD}^{x_0} = \int_0^b (1 - 2 x/b + x^2/b^2) Fb^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

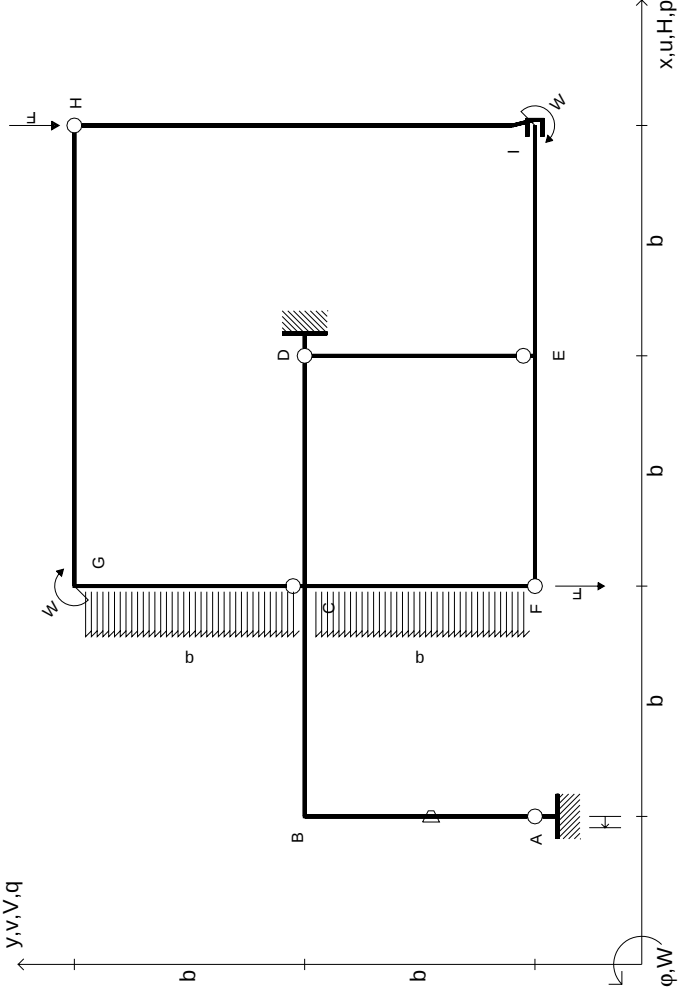
$$= (b - b + 1/3 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{DC}^{x_0} = \int_0^b (x^2/b^2) Fb^2 1/EJ dx = [1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/3 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$



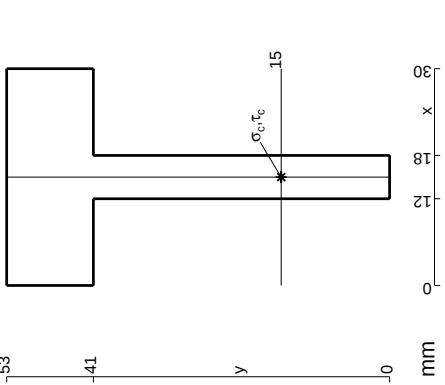
$$\begin{aligned} A &= 1116. \text{ mm}^2 \\ J_u &= 278254. \text{ mm}^4 \\ J_v &= 125712. \text{ mm}^4 \\ y_g &= 35.6 \text{ mm} \\ T_y &= -3630. \text{ N} \\ M_x &= 1871100. \text{ Nmm} \\ x_m &= 18. \text{ mm} \\ u_m &= -6. \text{ mm} \\ v_m &= -35.6 \text{ mm} \\ \sigma_m &= -Mv/J_u = 239.4 \text{ N/mm}^2 \\ x_c &= 24. \text{ mm} \\ y_c &= 15. \text{ mm} \\ v_c &= -20.6 \text{ mm} \\ \sigma_c &= -Mv/J_u = 138.5 \text{ N/mm}^2 \\ \tau_c &= 5.498 \text{ N/mm}^2 \\ \sigma_0 &= \sqrt{\sigma^2 + 3\tau^2} = 138.8 \text{ N/mm}^2 \\ S &= 5057. \text{ mm}^3 \end{aligned}$$



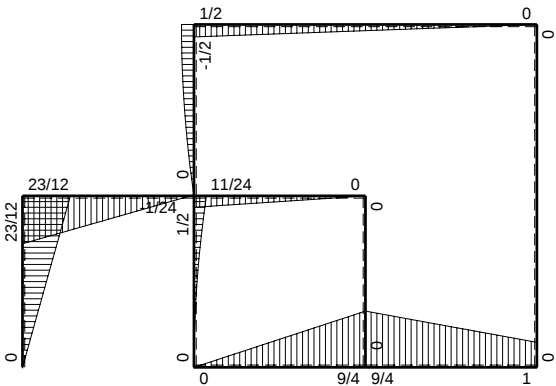
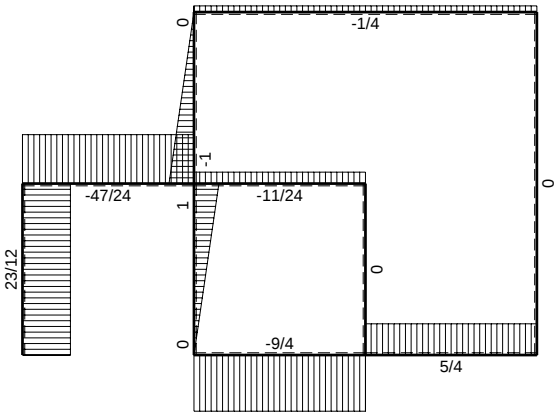
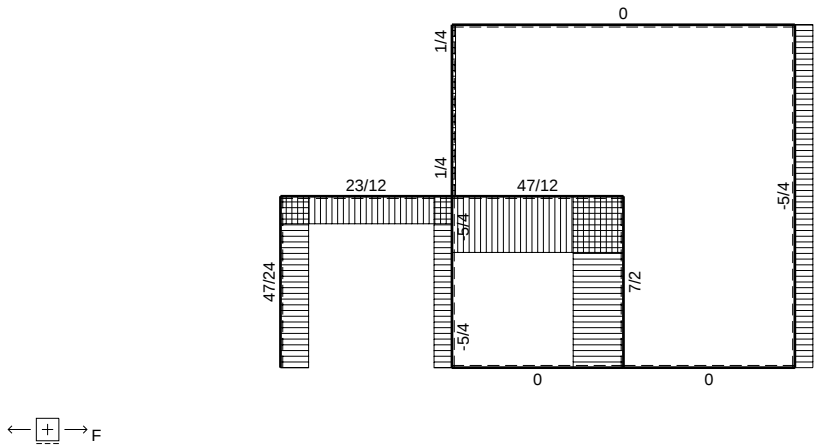
$V_A = -F$
 $V_E = -F$
 $W_I = -W = -Fb$
 $W_G = -W = -Fb$
 $p_{CG} = -q = -F/b$
 $p_{FC} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
 $u_A = -\delta = -b^3 F/EJ$
 $EJ_{AB} = EJ$
 $EJ_{BC} = EJ$
 $EJ_{CD} = EJ$
 $EJ_{DE} = EJ$

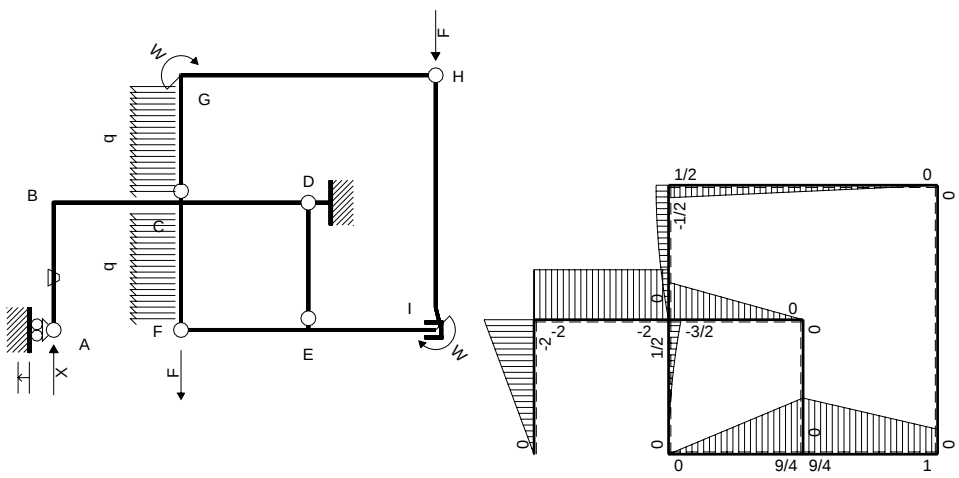
$EJ_{EF} = EJ$
 $EJ_{FC} = EJ$
 $EJ_{CG} = EJ$
 $EJ_{GH} = EJ$
 $EJ_{HI} = EJ$
 $EJ_{IE} = EJ$



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y .
 $J_{yz} - x_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y .
La trave EF ha la sezione riportata e dimensioni in mm, con:
 $b = 520 \text{ mm}$, $F = 730 \text{ N}$
Calcolare sulla sezione E la massima tensione normale σ_m .
Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
Lembo inferiore sezione su traveggio trave, a destra da E a F
Curvatura θ asta AB positiva se convessa a destra con inizio A .
Spostamento orizzontale assoluto u imposto al nodo A .

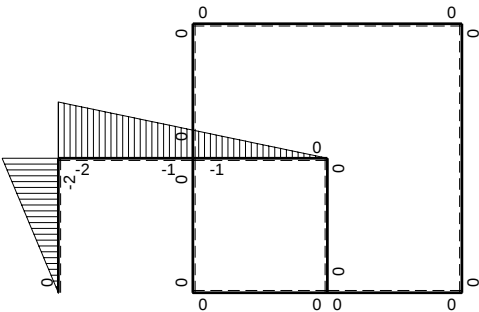


$\left(\begin{smallmatrix} + \\ \curvearrowright \end{smallmatrix} \right) F_b$



Schema di calcolo iperstatico

$\left(\begin{smallmatrix} + \\ \curvearrowright \end{smallmatrix} \right) M_0$ flessione da carichi assegnati



$\left(\begin{smallmatrix} + \\ \curvearrowright \end{smallmatrix} \right) M_x$ flessione da iperstatica $X=1$

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-2x	-2Fx	-Fb/EJ	$4Fx^2$	$2Fxb/EJ$	$4x^2$	$(4/3+1)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	2b-2x	2Fb-2Fx	Fb/EJ	$4Fb^2-8Fbx+4Fx^2$	$2Fb^2/EJ-2Fxb/EJ$	$4b^2-8bx+4x^2$		
BC b	-2b+x	-2Fb	0	$4Fb^2-2Fbx$	0	$4b^2-4bx+x^2$	$(3+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	b+x	2Fb	0	$2Fb^2+2Fbx$	0	$b^2+2bx+x^2$		
CD b	-b+x	-3/2Fb+3/2Fx	0	$3/2Fb^2-3Fbx+3/2Fx^2$	0	$b^2-2bx+x^2$	$(1/2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	x	3/2Fx	0	$3/2Fx^2$	0	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$9/4Fb-9/4Fx$	0	0	0	0	0+0	0
FE b	0	$-9/4Fx$	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/4Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx$	0	0	0	0	0+0	0
EI b	0	$-9/4Fb+5/4Fx$	0	0	0	0		
A	cedimento nodo - $H_{1A}u_A$						$2Fb^3/EJ$	
	totali						$47/6Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_A$						$-47/24F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = \left[4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = \left[4 x - 4 x^2/b + 4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = \left[4 x - 2 x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = \left[x + x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = \left[1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{x_0} = \int_0^b (4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (2 x/b) \theta dx = \left[4/3 x^3/b^2 \right]_0^b Fb^2 1/EJ + \left[x^2/b \right]_0^b \theta$$

$$= (4/3 b) Fb^2 1/EJ + (b) \theta = 7/3 Fb^3/EJ$$

$$L_{BA}^{x_0} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-2 + 2 x/b) \theta dx$$

$$= \left[4 x - 4 x^2/b + 4/3 x^3/b^2 \right]_0^b Fb^2 1/EJ + \left[-2 x + x^2/b \right]_0^b \theta$$

$$= (4 b - 4 b + 4/3 b) Fb^2 1/EJ + (-2 b + b) \theta = 7/3 Fb^3/EJ$$

$$L_{BC}^{x_0} = \int_0^b (4 - 2 x/b) Fb^2 1/EJ dx = \left[4 x - x^2/b \right]_0^b Fb^2 1/EJ$$

$$= (4 b - b) Fb^2 1/EJ = 3 Fb^3/EJ$$

$$L_{CB}^{x_0} = \int_0^b (2 + 2 x/b) Fb^2 1/EJ dx = \left[2 x + x^2/b \right]_0^b Fb^2 1/EJ$$

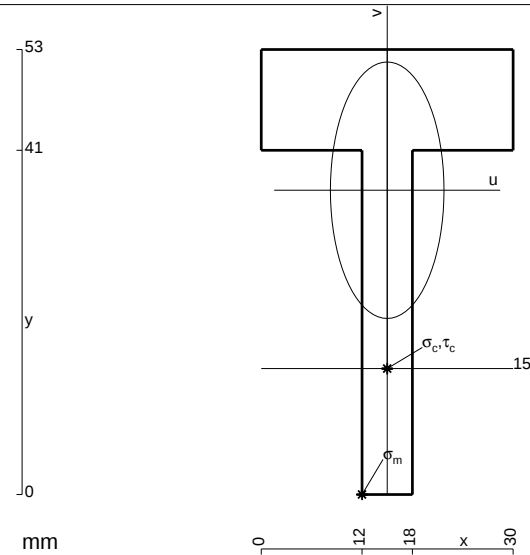
$$= (2 b + b) Fb^2 1/EJ = 3 Fb^3/EJ$$

$$L_{CD}^{x_0} = \int_0^b (3/2 - 3 x/b + 3/2 x^2/b^2) Fb^2 1/EJ dx = \left[3/2 x - 3/2 x^2/b + 1/2 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

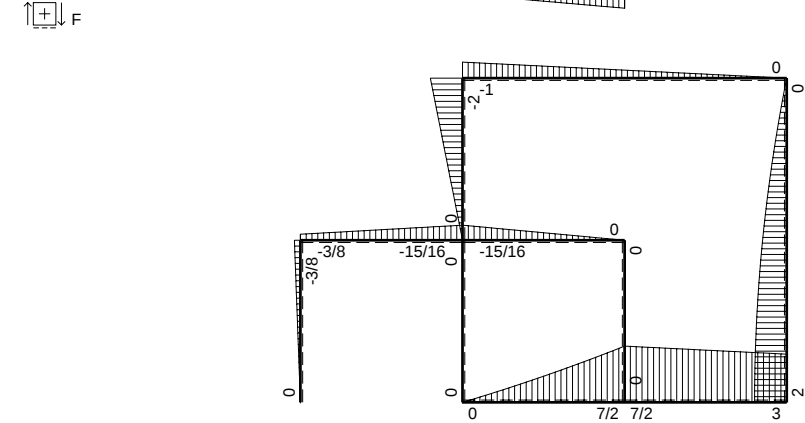
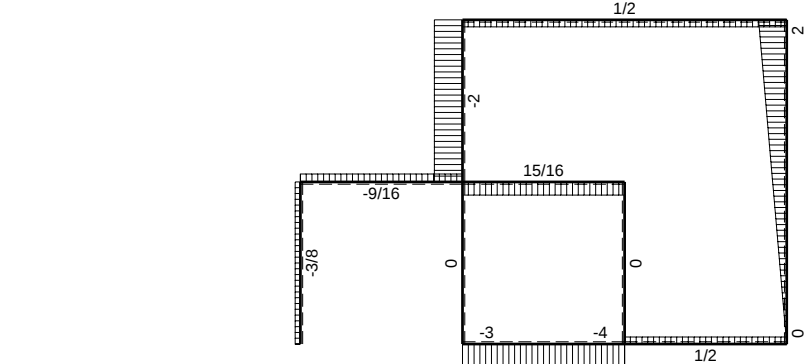
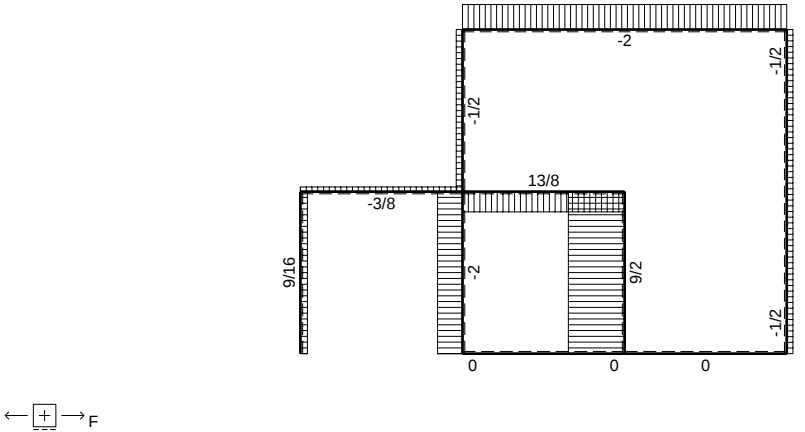
$$= (3/2 b - 3/2 b + 1/2 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

$$L_{DC}^{x_0} = \int_0^b (3/2 x^2/b^2) Fb^2 1/EJ dx = \left[1/2 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

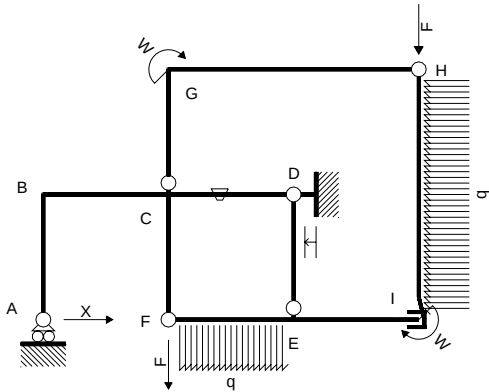
$$= (1/2 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$



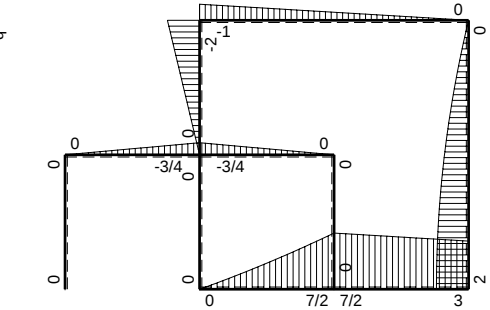
$$\begin{aligned} A &= 606. \text{ mm}^2 \\ J_u &= 141406. \text{ mm}^4 \\ J_v &= 27738. \text{ mm}^4 \\ y_g &= 36.24 \text{ mm} \\ T_y &= -1643. \text{ N} \\ M_x &= 854100. \text{ Nmm} \\ x_m &= 12. \text{ mm} \\ u_m &= -3. \text{ mm} \\ v_m &= -36.24 \text{ mm} \\ \sigma_m &= -Mv/J_u = 218.9 \text{ N/mm}^2 \\ x_c &= 15. \text{ mm} \\ y_c &= 15. \text{ mm} \\ v_c &= -21.24 \text{ mm} \\ \sigma_c &= -Mv/J_u = 128.3 \text{ N/mm}^2 \\ \tau_c &= 5.008 \text{ N/mm}^2 \\ \sigma_0 &= \sqrt{\sigma^2 + 3\tau^2} = 128.6 \text{ N/mm}^2 \\ S &= 2587. \text{ mm}^3 \end{aligned}$$



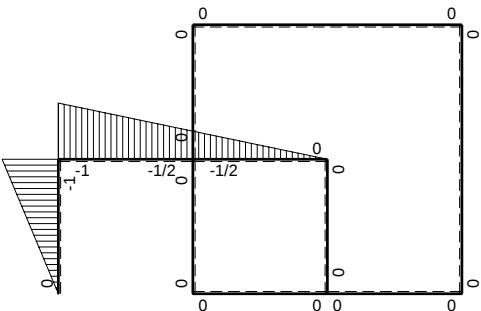
$\left(\begin{array}{c} + \\ \square \end{array} \right) F_b$



Schema di calcolo iperstatico



$\left(\begin{array}{c} + \\ \square \end{array} \right) M_o$ flessione da carichi assegnati



$\left(\begin{array}{c} + \\ \square \end{array} \right) M_x$ flessione da iperstatica $X=1$

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3 X b^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-3/4Fx	0	$3/4Fbx-3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/4+0)Fb^3/EJ$	$7/12 X b^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	0	$3/8Fb^2-3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	$-Fb/EJ$	$3/8Fb^2-3/4Fbx+3/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12 X b^3/EJ$
DC b	$1/2x$	$3/4Fx$	Fb/EJ	$3/8Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-3/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						$3/8F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left(\frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left(1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left(1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left(\frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left(\frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left(\frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[\frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left(\frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{x_0} = \int_0^b \left(\frac{3}{4} \frac{x}{b} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{3}{8} \frac{x^2}{b} - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{4} Fb^3/EJ$$

$$L_{CB}^{x_0} = \int_0^b \left(\frac{3}{8} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[\frac{3}{8} x - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left(\frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{4} Fb^3/EJ$$

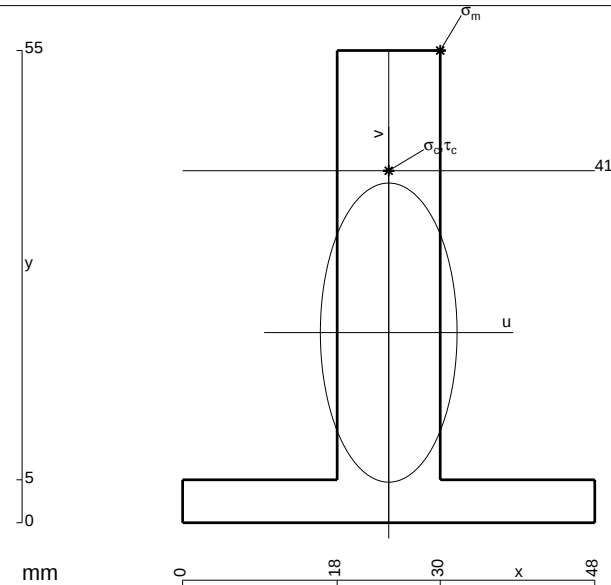
$$L_{CD}^{x_0} = \int_0^b \left(\frac{3}{8} - \frac{3}{4} \frac{x}{b} + \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[\frac{3}{8} x - \frac{3}{8} \frac{x^2}{b} + \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{3}{8} b - \frac{3}{8} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left(\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{3}{8} Fb^3/EJ$$

$$L_{DC}^{x_0} = \int_0^b \left(\frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left(-\frac{1}{2} \frac{x}{b} \right) \theta dx = \left[\frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[-\frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left(\frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left(-\frac{1}{4} b \right) \theta = \frac{3}{8} Fb^3/EJ$$



$$\begin{aligned} A &= 840. \text{ mm}^2 \\ J_u &= 255143. \text{ mm}^4 \\ J_v &= 53280. \text{ mm}^4 \\ y_g &= 22.14 \text{ mm} \\ T_y &= -3480. \text{ N} \\ M_x &= 1766100. \text{ Nmm} \\ x_m &= 30. \text{ mm} \\ y_m &= 55. \text{ mm} \\ u_m &= 6. \text{ mm} \\ v_m &= 32.86 \text{ mm} \\ \sigma_m &= -Mv/J_u = -227.4 \text{ N/mm}^2 \\ x_c &= 24. \text{ mm} \\ y_c &= 41. \text{ mm} \\ v_c &= 18.86 \text{ mm} \\ \sigma_c &= -Mv/J_u = -130.5 \text{ N/mm}^2 \\ \tau_c &= 4.937 \text{ N/mm}^2 \\ \sigma_o &= \sqrt{\sigma^2 + 3\tau^2} = 130.8 \text{ N/mm}^2 \\ S &= 4344. \text{ mm}^3 \end{aligned}$$