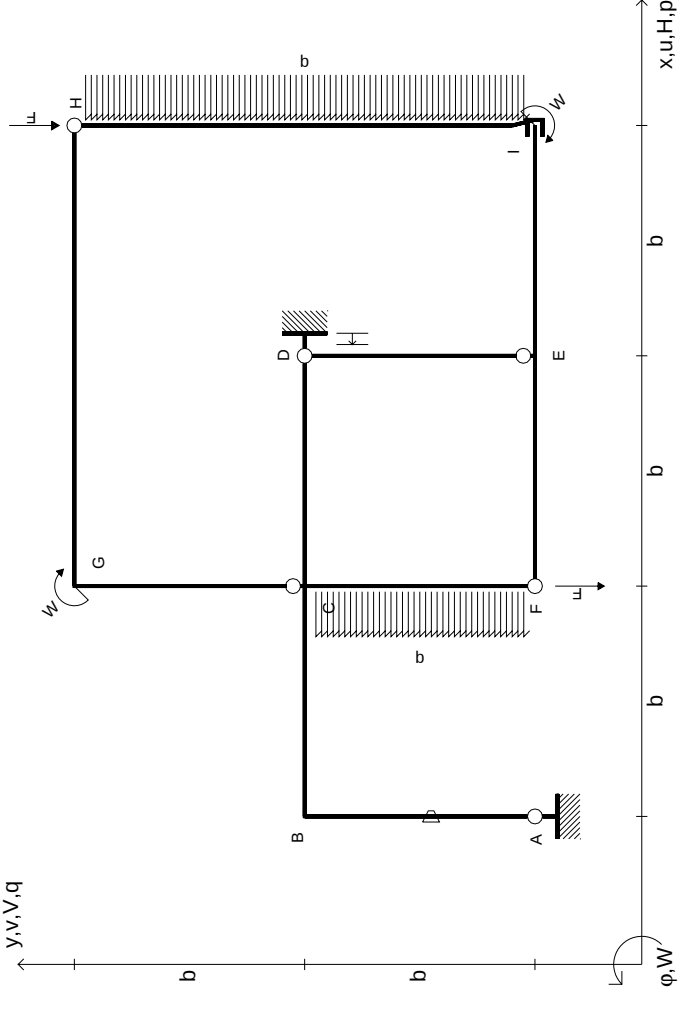




$$V_A = -F$$
$$V_E = -F$$
$$W_B = -W = -Fb$$
$$W_G = -W = -Fb$$
$$p_{HI} = -q = -F/b$$
$$p_{FC} = -q = -F/b$$

$$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$$
$$u_D = -\delta = -b^3 F/EJ$$
$$EJ_{AB} = EJ$$
$$EJ_{BC} = EJ$$
$$EJ_{CD} = EJ$$
$$EJ_{DE} = EJ$$

$$EJ_{EF} = EJ$$
$$EJ_{FC} = EJ$$
$$EJ_{CG} = EJ$$
$$EJ_{GH} = EJ$$
$$EJ_{HI} = EJ$$
$$EJ_{IE} = EJ$$

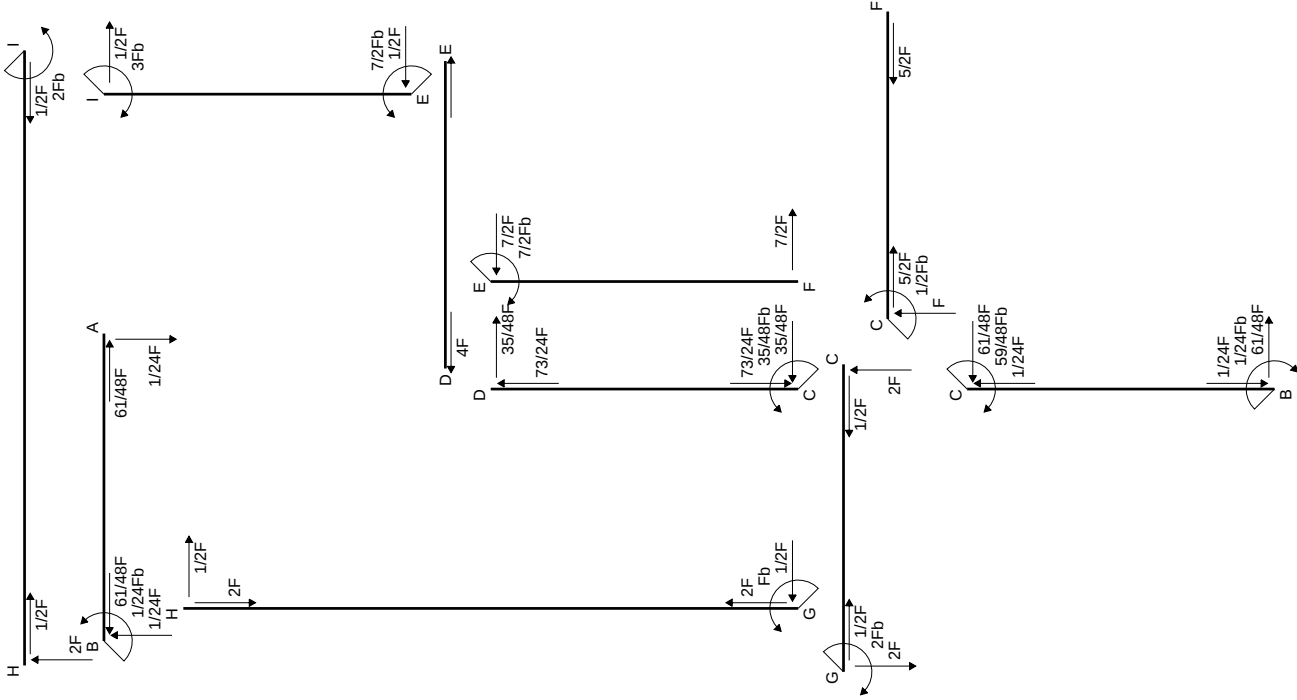
Reazioni iperstatiche in soluzione: $X=V_D$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.

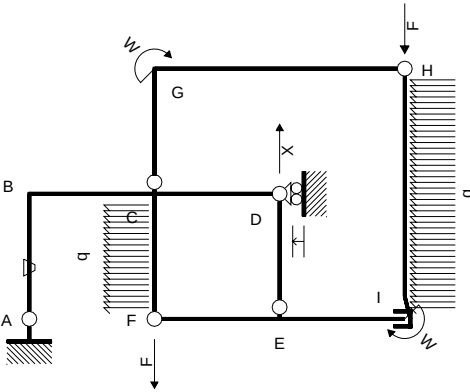
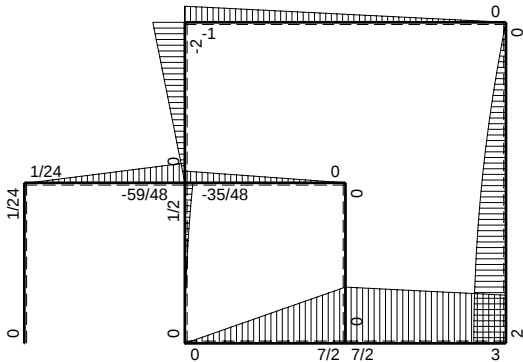
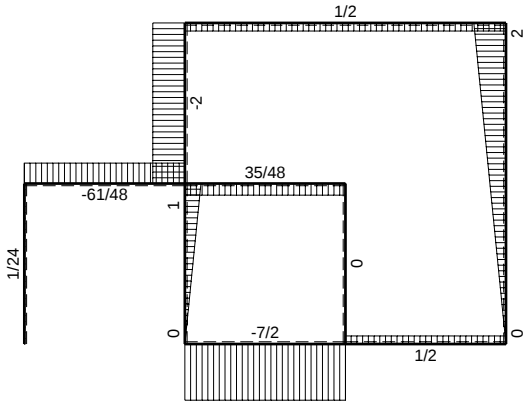
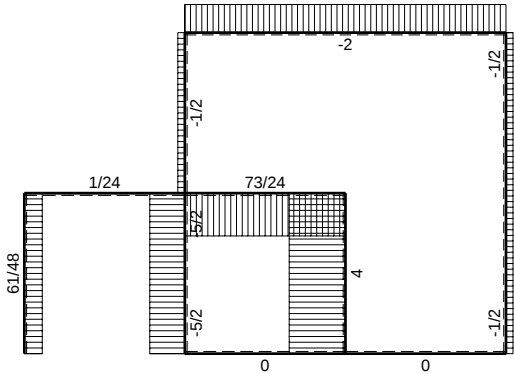
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{yz} - x_z - \theta_{yz}$ riferimento locale asta YZ con origine in Y.

La trave EF ha la sezione riportata e dimensioni in mm, con:
 $b = 730 \text{ mm}$, $F = 370 \text{ N}$

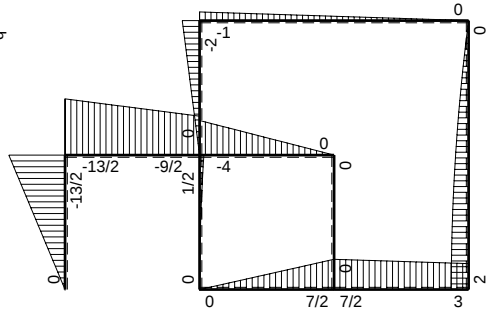
Calcolare sulla sezione E la massima tensione normale σ_m .
Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.

Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

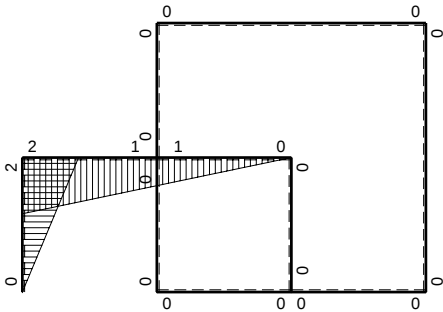




Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_D$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	2x	-13/2Fx	-Fb/EJ	-13Fx ²	-2Fxb/EJ	4x ²	$(-13/3-1)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	-2b+2x	13/2Fb-13/2Fx	Fb/EJ	$-13Fb^2+26Fbx-13Fx^2$	$-2Fb^2/EJ+2Fxb/EJ$	$4b^2-8bx+4x^2$		
BC b	2b-x	-13/2Fb+2Fx	0	$-13Fb^2+21/2Fbx-2Fx^2$	0	$4b^2-4bx+x^2$	$(-101/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	-b-x	9/2Fb+2Fx	0	$-9/2Fb^2-13/2Fbx-2Fx^2$	0	$b^2+2bx+x^2$		
CD b	b-x	-4Fb+4Fx	0	$-4Fb^2+8Fbx-4Fx^2$	0	$b^2-2bx+x^2$	$(-4/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	-x	4Fx	0	$-4Fx^2$	0	x ²		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-7/2Fx	0	0	0	0	0+0	0
FE b	0	-7/2Fx	0	0	0	0		
FC b	0	1/2qx ²	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
D	cedimento nodo -H _{1D} u _D						2Fb ³ /EJ	
	totali						-157/12Fb ³ /EJ	4Xb ³ /EJ
	iperstatica $X=V_D$						157/48F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = \left[4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = \left[4 x - 4 x^2/b + 4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = \left[4 x - 2 x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = \left[x + x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = \left[1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-13 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-2 x/b) \theta dx = \left[-13/3 x^3/b^2 \right]_0^b Fb^2 1/EJ + \left[-x^2/b \right]_0^b \theta$$

$$= (-13/3 b) Fb^2 1/EJ + (-b) \theta = -16/3 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-13 + 26 x/b - 13 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (2 - 2 x/b) \theta dx$$

$$= \left[-13 x + 13 x^2/b - 13/3 x^3/b^2 \right]_0^b Fb^2 1/EJ + \left[2 x - x^2/b \right]_0^b \theta$$

$$= (-13 b + 13 b - 13/3 b) Fb^2 1/EJ + (2 b - b) \theta = -16/3 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-13 + 21/2 x/b - 2 x^2/b^2) Fb^2 1/EJ dx = \left[-13 x + 21/4 x^2/b - 2/3 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

$$= (-13 b + 21/4 b - 2/3 b) Fb^2 1/EJ = -101/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-9/2 - 13/2 x/b - 2 x^2/b^2) Fb^2 1/EJ dx = \left[-9/2 x - 13/4 x^2/b - 2/3 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

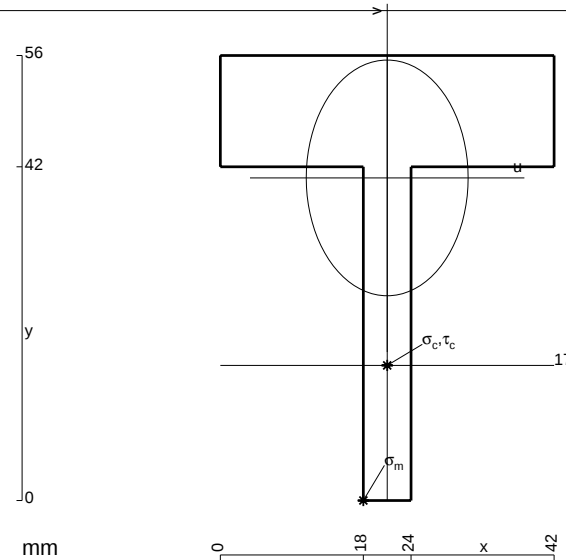
$$= (-9/2 b - 13/4 b - 2/3 b) Fb^2 1/EJ = -101/12 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (-4 + 8 x/b - 4 x^2/b^2) Fb^2 1/EJ dx = \left[-4 x + 4 x^2/b - 4/3 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

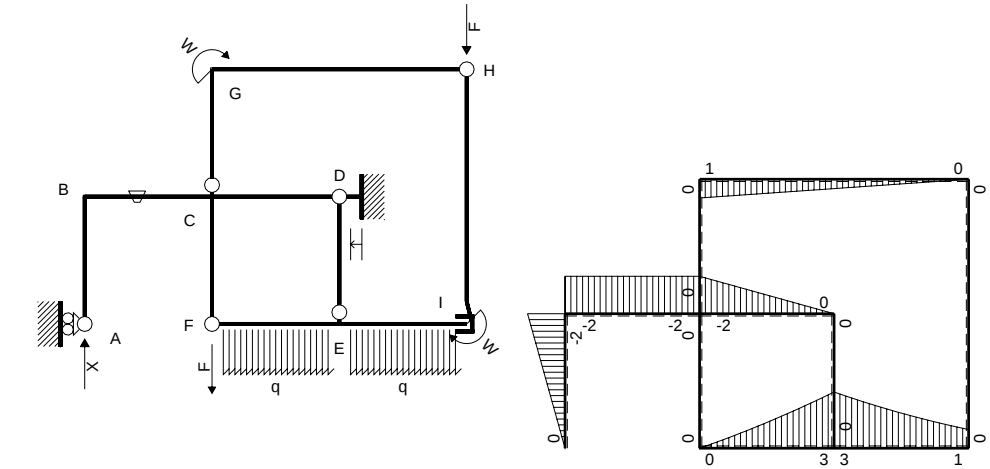
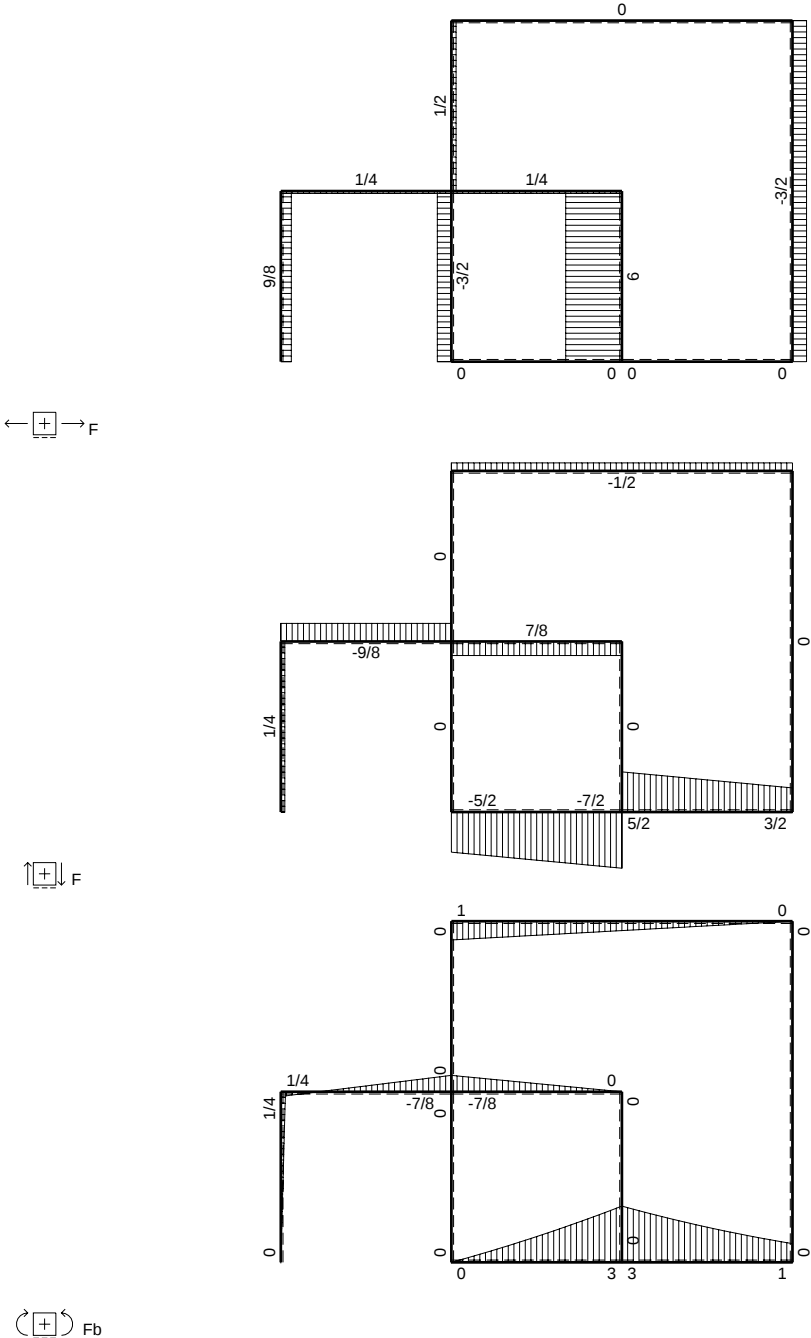
$$= (-4 b + 4 b - 4/3 b) Fb^2 1/EJ = -4/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (-4 x^2/b^2) Fb^2 1/EJ dx = \left[-4/3 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

$$= (-4/3 b) Fb^2 1/EJ = -4/3 Fb^3/EJ$$

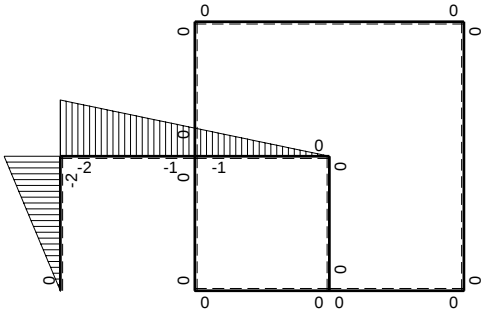


$$\begin{aligned} A &= 840. \text{ mm}^2 \\ J_u &= 184946. \text{ mm}^4 \\ J_v &= 87192. \text{ mm}^4 \\ y_g &= 40.6 \text{ mm} \\ T_y &= -1295. \text{ N} \\ M_x &= 945350. \text{ Nmm} \\ x_m &= 18. \text{ mm} \\ u_m &= -3. \text{ mm} \\ v_m &= -40.6 \text{ mm} \\ \sigma_m &= -Mv/J_u = 207.5 \text{ N/mm}^2 \\ x_c &= 21. \text{ mm} \\ y_c &= 17. \text{ mm} \\ v_c &= -23.6 \text{ mm} \\ \sigma_c &= -Mv/J_u = 120.6 \text{ N/mm}^2 \\ \tau_c &= 3.821 \text{ N/mm}^2 \\ \sigma_0 &= \sqrt{\sigma^2 + 3\tau^2} = 120.8 \text{ N/mm}^2 \\ S &= 3274. \text{ mm}^3 \end{aligned}$$



Schema di calcolo iperstatico

M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	-2x	-2Fx	0	$4Fx^2$	0	$4x^2$	$(4/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	2b-2x	2Fb-2Fx	0	$4Fb^2-8Fbx+4Fx^2$	0	$4b^2-8bx+4x^2$		
BC b	-2b+x	-2Fb	-Fb/EJ	$4Fb^2-2Fbx$	$2Fb^2/EJ-Fxb/EJ$	$4b^2-4bx+x^2$	$(3+3/2)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	b+x	2Fb	Fb/EJ	$2Fb^2+2Fbx$	$Fb^2/EJ+Fxb/EJ$	$b^2+2bx+x^2$		
CD b	-b+x	-2Fb+2Fx	0	$2Fb^2-4Fbx+2Fx^2$	0	$b^2-2bx+x^2$	$(2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	x	2Fx	0	$2Fx^2$	0	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$3Fb-7/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-5/2Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-2Fb^3/EJ$	
	totali						$9/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_A$						$-9/8F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = \left[4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = \left[4 x - 4 x^2/b + 4/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = \left[4 x - 2 x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = \left[x + x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = \left[1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (4 x^2/b^2) Fb^2 1/EJ dx = \left[4/3 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

$$= (4/3 b) Fb^2 1/EJ = 4/3 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) Fb^2 1/EJ dx = \left[4 x - 4 x^2/b + 4/3 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) Fb^2 1/EJ = 4/3 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (4 - 2 x/b) Fb^2 1/EJ dx + \int_0^b (2 - x/b) \theta dx = \left[4 x - x^2/b \right]_0^b Fb^2 1/EJ + \left[2 x - 1/2 x^2/b \right]_0^b \theta$$

$$= (4 b - b) Fb^2 1/EJ + (2 b - 1/2 b) \theta = 9/2 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (2 + 2 x/b) Fb^2 1/EJ dx + \int_0^b (-1 - x/b) \theta dx = \left[2 x + x^2/b \right]_0^b Fb^2 1/EJ + \left[-x - 1/2 x^2/b \right]_0^b \theta$$

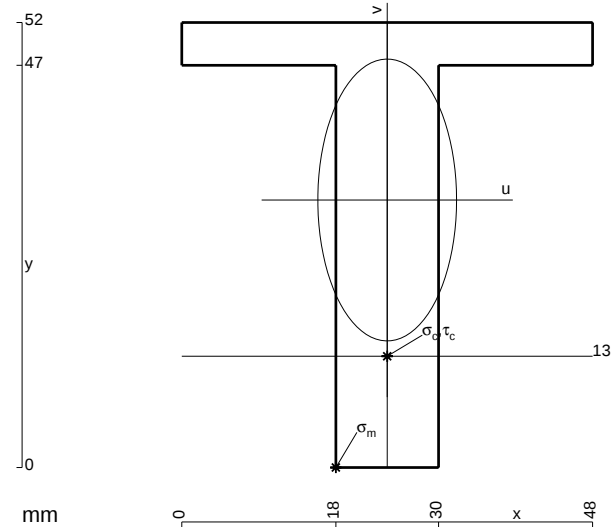
$$= (2 b + b) Fb^2 1/EJ + (-b - 1/2 b) \theta = 9/2 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (2 - 4 x/b + 2 x^2/b^2) Fb^2 1/EJ dx = \left[2 x - 2 x^2/b + 2/3 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

$$= (2 b - 2 b + 2/3 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (2 x^2/b^2) Fb^2 1/EJ dx = \left[2/3 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

$$= (2/3 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$



$$\begin{aligned} A &= 804. \text{ mm}^2 \\ J_u &= 218133. \text{ mm}^4 \\ J_v &= 52848. \text{ mm}^4 \\ y_g &= 31.26 \text{ mm} \\ T_y &= -3605. \text{ N} \\ M_x &= 1452300. \text{ Nmm} \\ x_m &= 18. \text{ mm} \\ u_m &= -6. \text{ mm} \\ v_m &= -31.26 \text{ mm} \\ \sigma_m &= -Mv/J_u = 208.1 \text{ N/mm}^2 \\ x_c &= 24. \text{ mm} \\ y_c &= 13. \text{ mm} \\ v_c &= -18.26 \text{ mm} \\ \sigma_c &= -Mv/J_u = 121.6 \text{ N/mm}^2 \\ \tau_c &= 5.32 \text{ N/mm}^2 \\ \sigma_o &= \sqrt{\sigma^2 + 3\tau^2} = 121.9 \text{ N/mm}^2 \\ S &= 3863. \text{ mm}^3 \end{aligned}$$