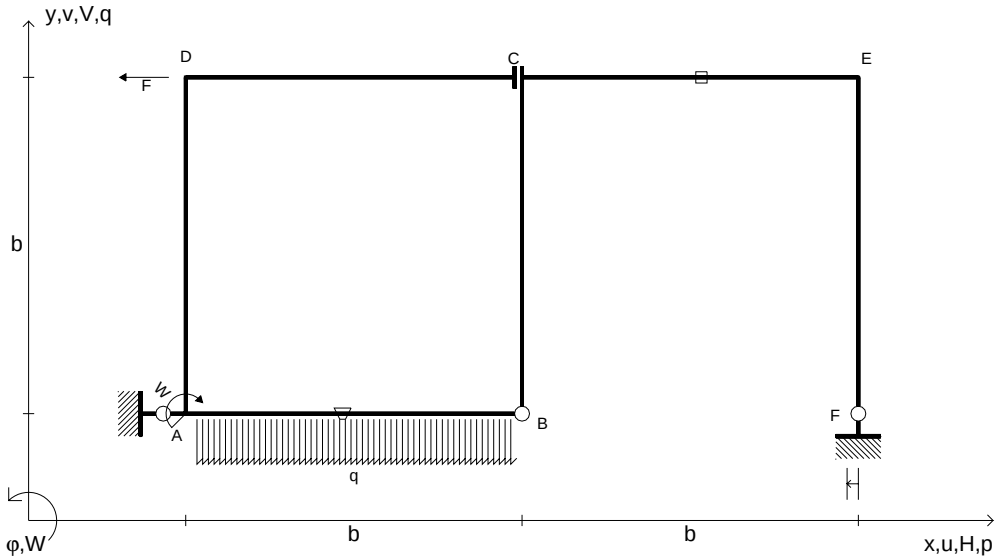


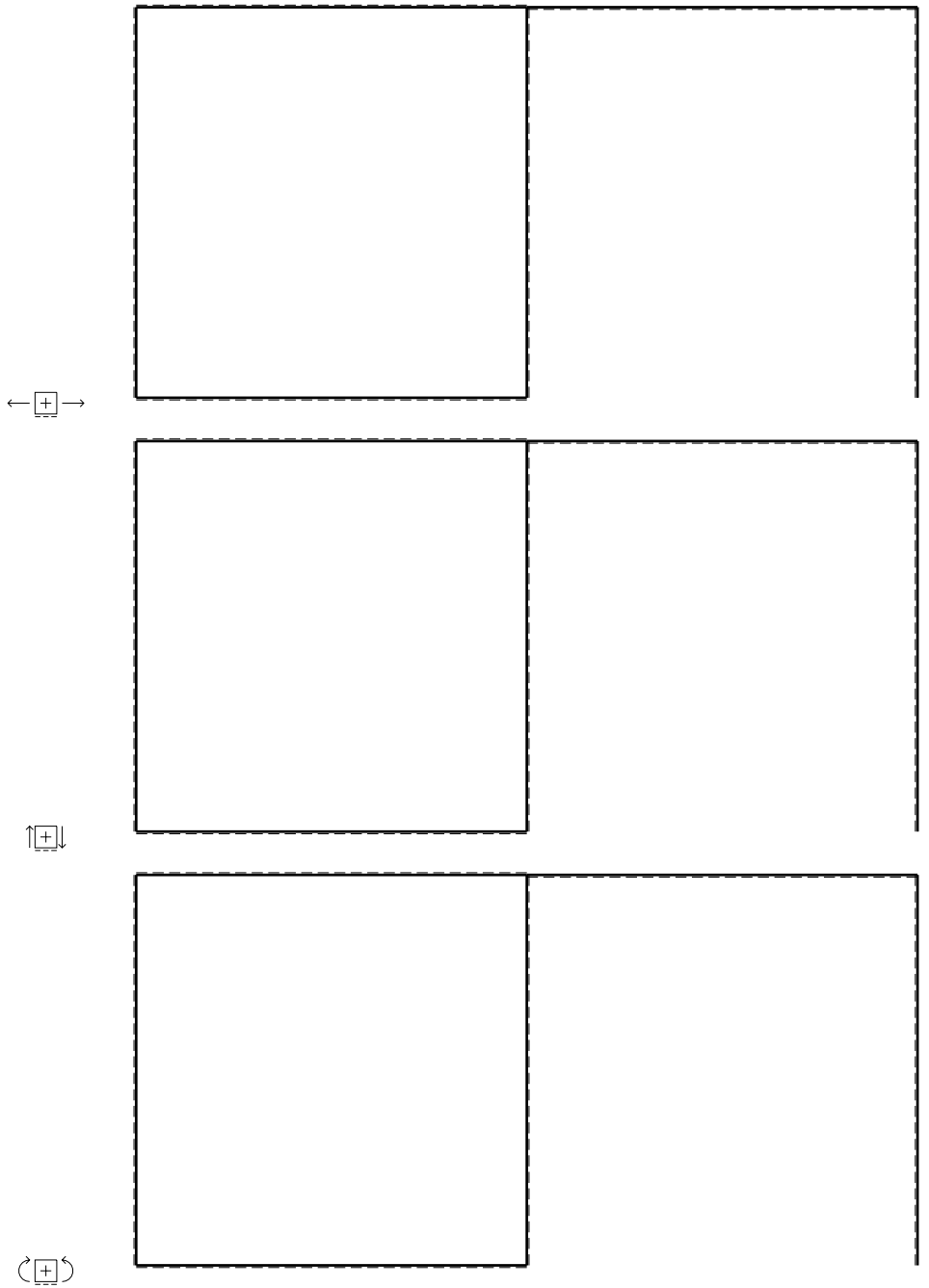
$H_D = -F$
$W_A = -W = -Fb$
$q_{AB} = -q = -F/b$
$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$

$\varepsilon_{CE} = -\alpha T = -b^2F/EJ$
$u_F = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$

$EJ_{CD} = EJ$
$EJ_{DA} = EJ$
$EJ_{CE} = EJ$
$EJ_{EF} = EJ$

ANALISI STRUTTURALE CON PLV
1) Analisi cinematica
2) Declassamento con indicazione delle iperstatiche scelte
3) Diagrammi del momento M0, M*, M**
4) Espressione del PLV
5) Valori numerici delle iperstatiche
6) Diagrammi finali delle azioni interne

Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Elongazione termica specifica ε assegnata su asta CE.
Spostamento orizzontale assoluto u imposto al nodo F.



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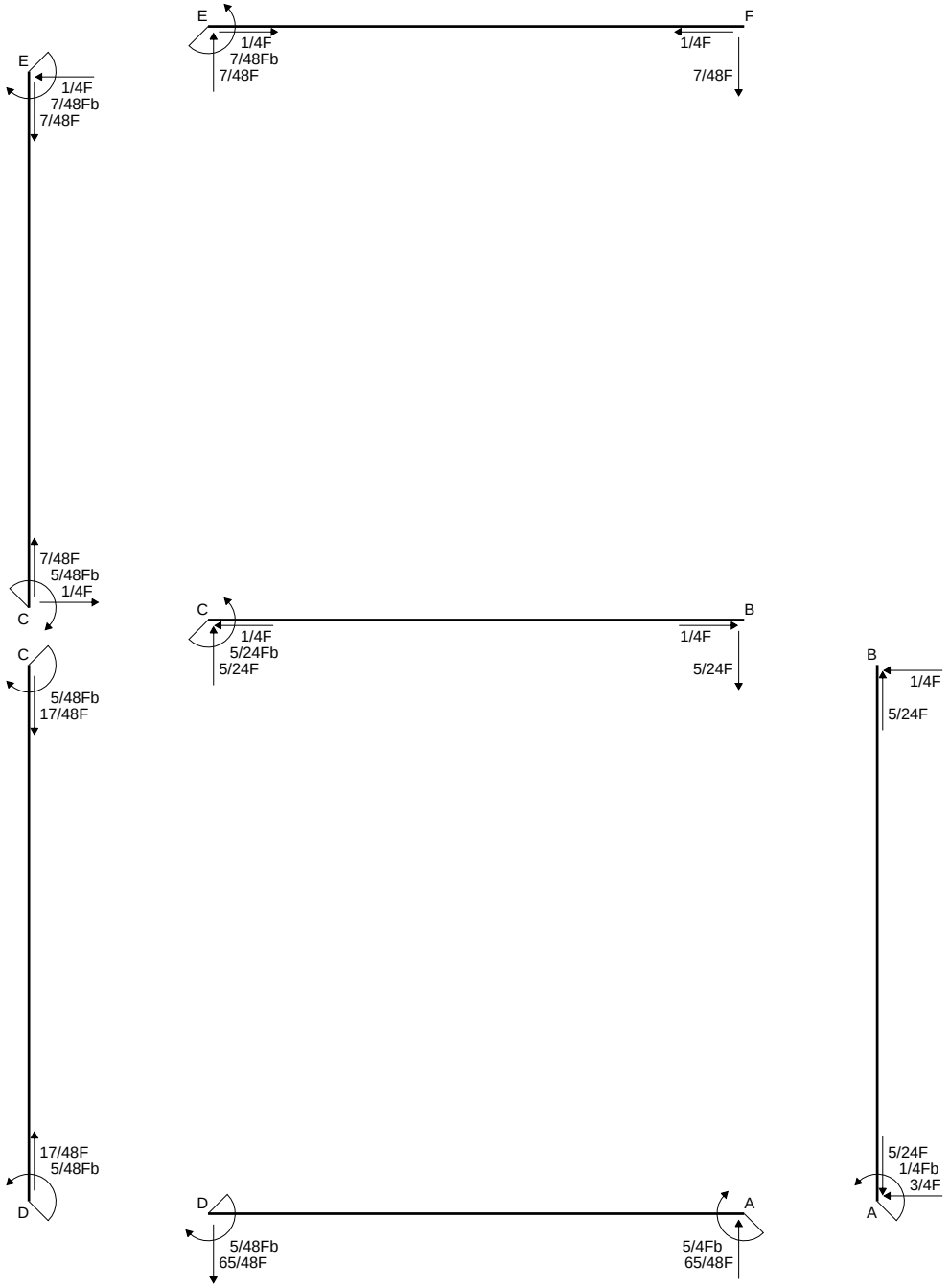
E
F
1/4F
7/48Fb
7/48F
1/4F
7/48F
E
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7/48Fb
7/48F
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5/48Fb
1/4F
C
C
B
1/4F
5/24Fb
5/24F
1/4F
5/24F
C
5/48Fb
17/48F
B
1/4F
5/24F
17/48F
5/48Fb
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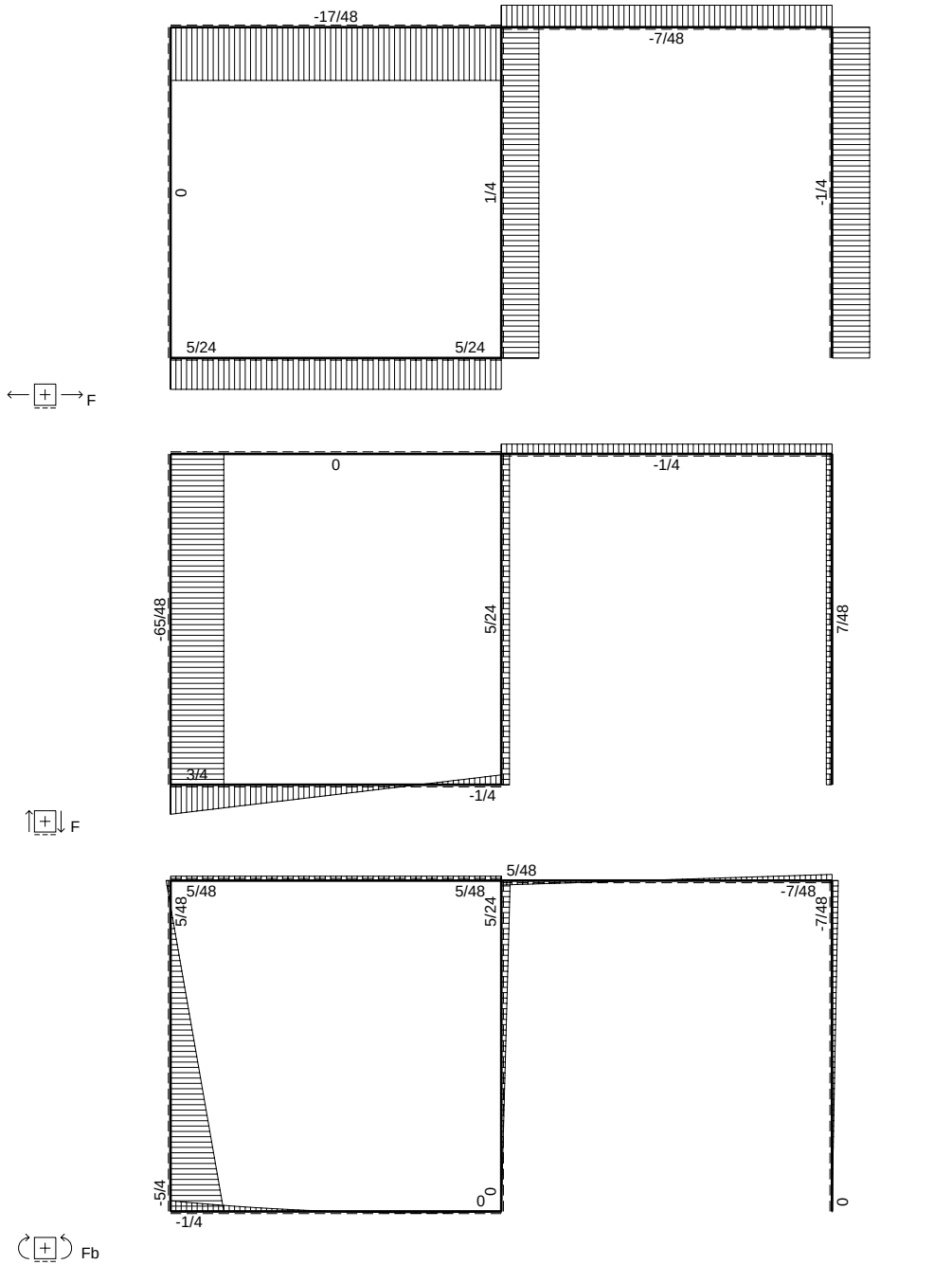
3/4F
A
D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-1/4
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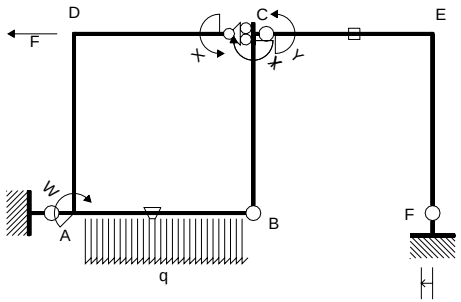
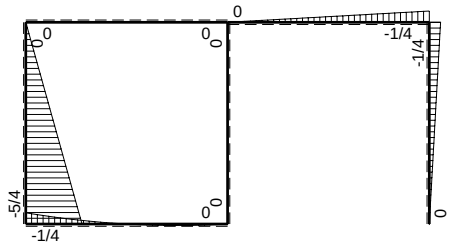
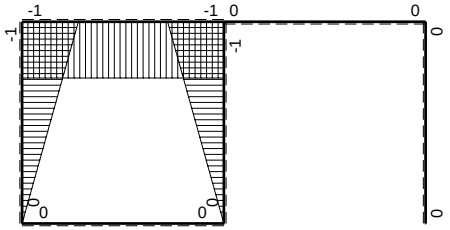
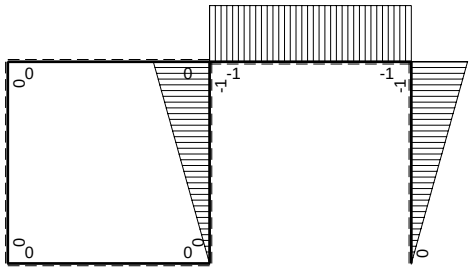
0
0
-1/4
Fb





M_o flessione da carichi assegnati





M_x flessione da iperstatica X=1

M_y flessione da iperstatica Y=1

CALCOLO REAZIONI IPERSTATICHE X = W_{CD} Y = W_{CE}

Azioni di calcolo di iperstatiche	Azioni di calcolo di iperstatiche	Azioni interne di base
$M^X_{AB}=(0)$	$M^Y_{AB}=(0)$	$M^o_{AB}=(-1/4+3/4\ x/b-1/2\ x^2/b^2)Fb$
$M^X_{BA}=(0)$	$M^Y_{BA}=(0)$	$M^o_{BA}=(-1/4\ x/b+1/2\ x^2/b^2)Fb$
$M^X_{BC}=(-\ x/b)$	$M^Y_{BC}=(-\ x/b)$	$M^o_{BC}=(0)Fb$
$M^X_{CB}=(1-\ x/b)$	$M^Y_{CB}=(1-\ x/b)$	$M^o_{CB}=(0)Fb$
$M^X_{CD}=(-1)$	$M^Y_{CD}=(0)$	$M^o_{CD}=(0)Fb$
$M^X_{DC}=(1)$	$M^Y_{DC}=(0)$	$M^o_{DC}=(0)Fb$
$M^X_{DA}=(-1+\ x/b)$	$M^Y_{DA}=(0)$	$M^o_{DA}=(-5/4\ x/b)Fb$
$M^X_{AD}=(\ x/b)$	$M^Y_{AD}=(0)$	$M^o_{AD}=(5/4-5/4\ x/b)Fb$
$M^X_{CE}=(0)$	$M^Y_{CE}=(-1)$	$M^o_{CE}=(-1/4\ x/b)Fb$
$M^X_{EC}=(0)$	$M^Y_{EC}=(1)$	$M^o_{EC}=(1/4-1/4\ x/b)Fb$
$M^X_{EF}=(0)$	$M^Y_{EF}=(-1+\ x/b)$	$M^o_{EF}=(-1/4+1/4\ x/b)Fb$
$M^X_{FE}=(0)$	$M^Y_{FE}=(\ x/b)$	$M^o_{FE}=(1/4\ x/b)Fb$

Lavoro da forza unitaria X per deformazione da forza unitaria X

$$L^{XX}_{BC} = \int_o^b M^X_{BC}\ M^X_{BC}\ 1/EJ_{BC}\ dx = \int_o^b (-\ x/b)^2\ 1/EJ_{BC}\ dx$$
$$= \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$
$$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L^{XX}_{CB} = \int_o^b M^X_{CB}\ M^X_{CB}\ 1/EJ_{CB}\ dx = \int_o^b (1-\ x/b)^2\ 1/EJ_{CB}\ dx$$
$$= \int_o^b (1\ -2\ x/b\ +\ x^2/b^2\ )\ 1/EJ\ dx = \left[\ x\ -\ x^2/b\ +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$
$$= (\ b\ -\ b\ +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L^{XX}_{CD} = \int_o^b M^X_{CD}\ M^X_{CD}\ 1/EJ_{CD}\ dx = \int_o^b (-1)^2\ 1/EJ_{CD}\ dx$$
$$= \int_o^b (1\ )\ 1/EJ\ dx = \left[\ x\ \right]_o^b\ 1/EJ$$
$$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$$

$$L^{XX}_{DC} = \int_o^b M^X_{DC}\ M^X_{DC}\ 1/EJ_{DC}\ dx = \int_o^b (1)^2\ 1/EJ_{DC}\ dx$$
$$= \int_o^b (1\ )\ 1/EJ\ dx = \left[\ x\ \right]_o^b\ 1/EJ$$
$$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$$

$$L^{XX}_{DA} = \int_o^b M^X_{DA}\ M^X_{DA}\ 1/EJ_{DA}\ dx = \int_o^b (-1+\ x/b)^2\ 1/EJ_{DA}\ dx$$
$$= \int_o^b (1\ -2\ x/b\ +\ x^2/b^2\ )\ 1/EJ\ dx = \left[\ x\ -\ x^2/b\ +1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$
$$= (\ b\ -\ b\ +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$$

$$L^{XX}_{AD} = \int_o^b M^X_{AD}\ M^X_{AD}\ 1/EJ_{AD}\ dx = \int_o^b (\ x/b)^2\ 1/EJ_{AD}\ dx$$
$$= \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$$
$$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$$

Lavoro da forza unitaria X per deformazione da forza unitaria Y

$L_{BC}^{XY} = \int_o^b M_{BC}^X M_{BC}^Y 1/EJ_{BC}\, dx = \int_o^b (-x/b)(-x/b)\ 1/EJ_{BC}\, dx$

$= \int_o^b (x^2/b^2)\ 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\ 1/EJ$

$= (1/3\, b)\ 1/EJ = 1/3\ b/EJ$

$L_{CB}^{XY} = \int_o^b M_{CB}^X M_{CB}^Y 1/EJ_{CB}\, dx = \int_o^b (1-x/b)(1-x/b)\ 1/EJ_{CB}\, dx$

$= \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2\,]_o^b\ 1/EJ$

$= (\, b - b + 1/3\, b\,)\ 1/EJ = 1/3\ b/EJ$

Lavoro da forza unitaria X per deformazione da azioni assegnate

$L_{DA}^{Xo} = \int_o^b M_{DA}^X M_{DA}^o 1/EJ_{DA}\, dx = \int_o^b (-1 + x/b)(-5/4\, x/b)\ Fb\ 1/EJ_{DA}\, dx$

$= \int_o^b (5/4\, x/b - 5/4\, x^2/b^2)\ Fb\ 1/EJ\, dx = [5/8\, x^2/b - 5/12\, x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (5/8\, b - 5/12\, b)\ Fb\ 1/EJ = 5/24\ Fb^2/EJ$

$L_{AD}^{Xo} = \int_o^b M_{AD}^X M_{AD}^o 1/EJ_{AD}\, dx = \int_o^b (x/b)(5/4 - 5/4\, x/b)\ Fb\ 1/EJ_{AD}\, dx$

$= \int_o^b (5/4\, x/b - 5/4\, x^2/b^2)\ Fb\ 1/EJ\, dx = [5/8\, x^2/b - 5/12\, x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (5/8\, b - 5/12\, b)\ Fb\ 1/EJ = 5/24\ Fb^2/EJ$

$L_F^{Xo} = H_F^X u_F = 0\ (-1)\ Fb^2/EJ = 0$

Lavoro da forza unitaria Y per deformazione da forza unitaria Y

$L_{BC}^{YY} = \int_o^b M_{BC}^Y M_{BC}^Y 1/EJ_{BC}\, dx = \int_o^b (-x/b)^2\ 1/EJ_{BC}\, dx$

$= \int_o^b (x^2/b^2)\ 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\ 1/EJ$

$= (1/3\, b)\ 1/EJ = 1/3\ b/EJ$

$L_{CB}^{YY} = \int_o^b M_{CB}^Y M_{CB}^Y 1/EJ_{CB}\, dx = \int_o^b (1-x/b)^2\ 1/EJ_{CB}\, dx$

$= \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2\,]_o^b\ 1/EJ$

$= (\, b - b + 1/3\, b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{CE}^{YY} = \int_o^b M_{CE}^Y M_{CE}^Y 1/EJ_{CE}\, dx = \int_o^b (-1)^2\ 1/EJ_{CE}\, dx$

$= \int_o^b (1)\ 1/EJ\, dx = [\, x\,]_o^b\ 1/EJ$

$= (\, b\,)\ 1/EJ = b/EJ$

$L_{EC}^{YY} = \int_o^b M_{EC}^Y M_{EC}^Y 1/EJ_{EC}\, dx = \int_o^b (1)^2\ 1/EJ_{EC}\, dx$

$= \int_o^b (1)\ 1/EJ\, dx = [\, x\,]_o^b\ 1/EJ$

$= (\, b\,)\ 1/EJ = b/EJ$

$L_{EF}^{YY} = \int_o^b M_{EF}^Y M_{EF}^Y 1/EJ_{EF}\, dx = \int_o^b (-1 + x/b)^2\ 1/EJ_{EF}\, dx$

$= \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2\,]_o^b\ 1/EJ$

$= (\, b - b + 1/3\, b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{FE}^{YY} = \int_o^b M_{FE}^Y M_{FE}^Y 1/EJ_{FE}\, dx = \int_o^b (x/b)^2\ 1/EJ_{FE}\, dx$

$= \int_o^b (x^2/b^2)\ 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\ 1/EJ$

$= (1/3\, b)\ 1/EJ = 1/3\ b/EJ$

Lavoro da forza unitaria Y per deformazione da azioni assegnate

$L_{CE}^{Yo} = \int_o^b M_{CE}^Y M_{CE}^o 1/EJ_{CE}\, dx + H_{CE}^Y \varepsilon_{CE} (x_E - x_C) = \int_o^b (-1)(-1/4\, x/b)\ Fb\ 1/EJ_{CE}\, dx - 1\ \ (-1)\ 1\ Fb^2/EJ$

$= \int_o^b (1/4\, x/b)\ Fb\ 1/EJ\, dx - 1\ \ (-1)\ 1\ Fb^2/EJ = [1/8\, x^2/b]_o^b\ Fb\ 1/EJ\ -1\ \ (-1)\ 1\ Fb^2/EJ$

$= (1/8\, b)\ Fb\ 1/EJ\ -1\ \ (-1)\ 1\ Fb^2/EJ = 9/8\ Fb^2/EJ$

$L_{EC}^{Yo} = \int_o^b M_{EC}^Y M_{EC}^o 1/EJ_{EC}\, dx + H_{EC}^Y \varepsilon_{EC} (x_C - x_E) = \int_o^b (1)(1/4 - 1/4\, x/b)\ Fb\ 1/EJ_{EC}\, dx - 1\ \ (-1)\ 1\ Fb^2/EJ$

$= \int_o^b (1/4 - 1/4\, x/b)\ Fb\ 1/EJ\, dx - 1\ \ (-1)\ 1\ Fb^2/EJ = [1/4\, x - 1/8\, x^2/b]_o^b\ Fb\ 1/EJ\ -1\ \ (-1)\ 1\ Fb^2/EJ$

$= (1/4\, b - 1/8\, b)\ Fb\ 1/EJ\ -1\ \ (-1)\ 1\ Fb^2/EJ = 9/8\ Fb^2/EJ$

$L_{EF}^{Yo} = \int_o^b M_{EF}^Y M_{EF}^o 1/EJ_{EF}\, dx = \int_o^b (-1 + x/b)(-1/4 + 1/4\, x/b)\ Fb\ 1/EJ_{EF}\, dx$

$= \int_o^b (1/4 - 1/2\, x/b + 1/4\, x^2/b^2)\ Fb\ 1/EJ\, dx = [1/4\, x - 1/4\, x^2/b + 1/12\, x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/4\, b - 1/4\, b + 1/12\, b)\ Fb\ 1/EJ = 1/12\ Fb^2/EJ$

$L_{FE}^{Yo} = \int_o^b M_{FE}^Y M_{FE}^o 1/EJ_{FE}\, dx = \int_o^b (x/b)(1/4\, x/b)\ Fb\ 1/EJ_{FE}\, dx$

$= \int_o^b (1/4\, x^2/b^2)\ Fb\ 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/12\, b)\ Fb\ 1/EJ = 1/12\ Fb^2/EJ$

$L_F^{Yo} = H_F^Y u_F = 1\ (-1)\ Fb^2/EJ = -\ Fb^2/EJ$

Contributi nulli elementi

$L_{AB}^{XX}\ L_{CE}^{XX}\ L_{EF}^{XX}\ L_{AB}^{XY}\ L_{CD}^{XY}\ L_{DA}^{XY}\ L_{CE}^{XY}\ L_{EF}^{XY}\ L_{AB}^{Xo}\ L_{BC}^{Xo}\ L_{CD}^{Xo}\ L_{CE}^{Xo}\ L_{EF}^{Xo}\ L_{AB}^{YY}\ L_{CD}^{YY}\ L_{DA}^{YY}$
$L_{AB}^{Yo}\ L_{BC}^{Yo}\ L_{CD}^{Yo}\ L_{DA}^{Yo}$

Contributi nulli nodi vincolati

$L_A^{XX}\ L_F^{XX}\ L_A^{XY}\ L_F^{XY}\ L_A^{Xo}\ L_F^{Xo}\ L_A^{YY}\ L_F^{YY}\ L_A^{Yo}$

Sistema risolvente

$(L_{BC}^{XX} + L_{CD}^{XX} + L_{DA}^{XX})\ X + (L_{BC}^{XY})\ Y = -(L_{DA}^{Xo})$
$(L_{BC}^{YX})\ X + (L_{BC}^{YY} + L_{CE}^{YY} + L_{EF}^{YY})\ Y = -(L_{CE}^{Yo} + L_{EF}^{Yo} + L_F^{Yo})$

Sistema risolvente

$(1/3 + 1 + 1/3)\ X\, b + (1/3)\ Y\, b = (-5/24)\ Fb^2$
$(1/3)\ X\, b + (1/3 + 1 + 1/3)\ Y\, b = (-9/8 - 1/12 + 1)\ Fb^2$

Sistema risolvente

$5/3\ X_b + 1/3\ Y_b = -5/24\ Fb^2$

$1/3\ X_b + 5/3\ Y_b = -5/24\ Fb^2$

Soluzione

$X = -5/48\ Fb \quad Y = -5/48\ Fb$

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