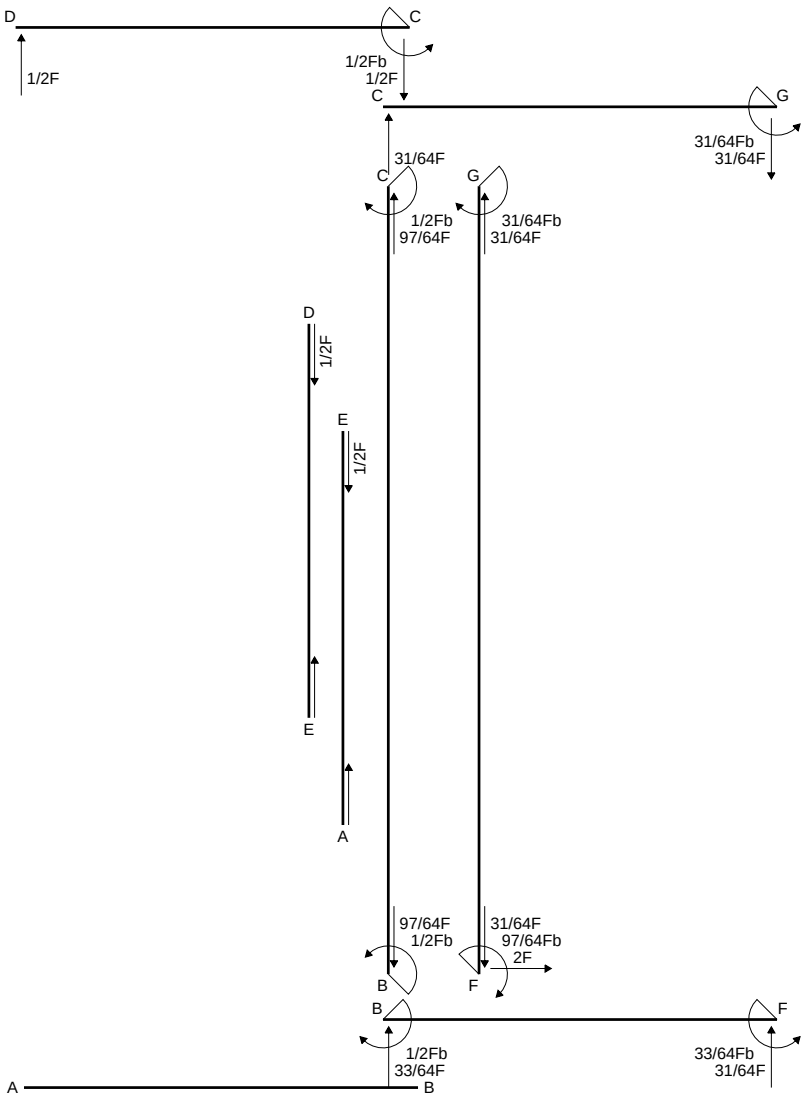
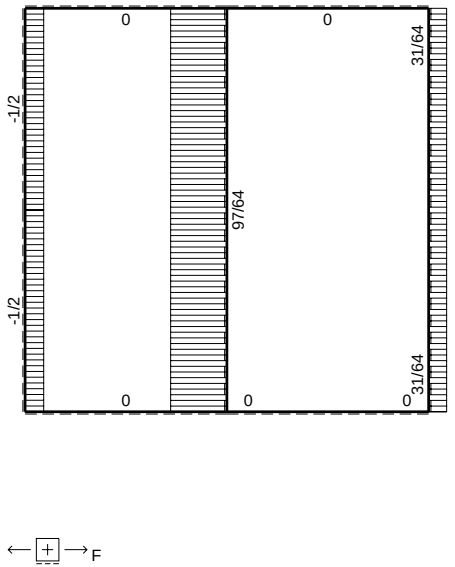
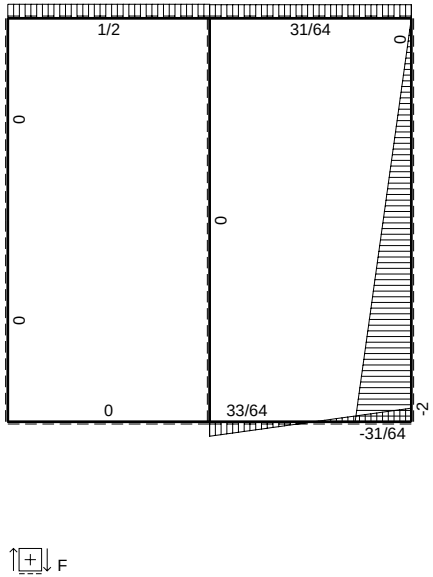
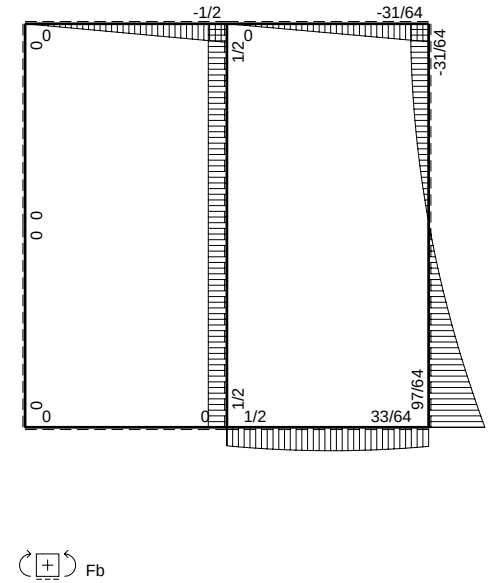

D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64
F


1/2
31/64
0
0
0
0
0
33/64
-2
-31/64
F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
0
1/2
33/64
97/64
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	FxEJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3XbEJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-FxEJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2XbEJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb²/EJ	8/3XbEJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 218726 CI08.xxxx.001

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

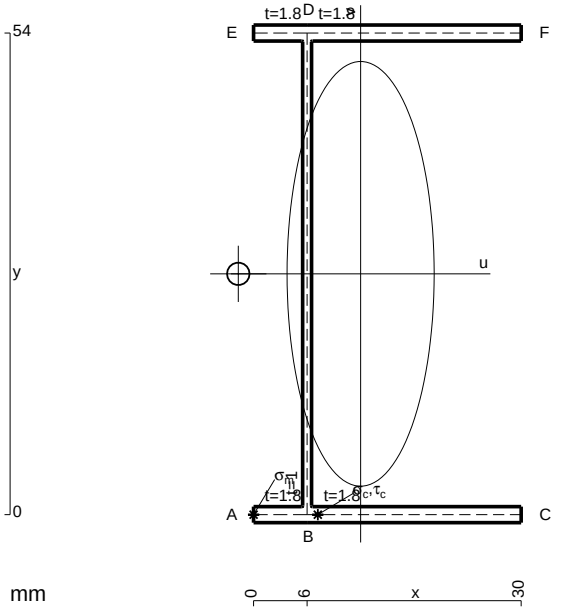
$= (-\ b +1/2\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b)\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

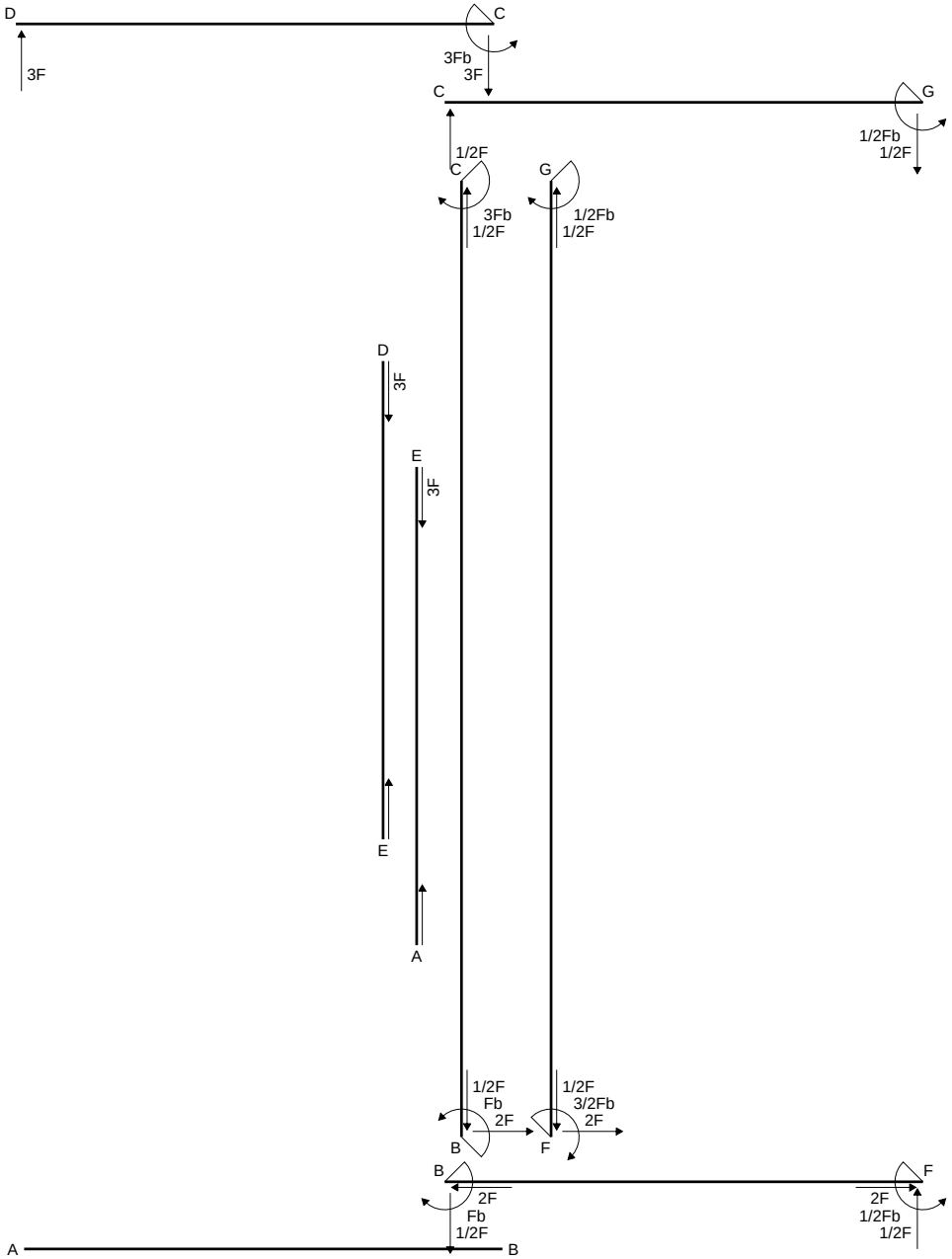
$= (-4/3\ b)\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

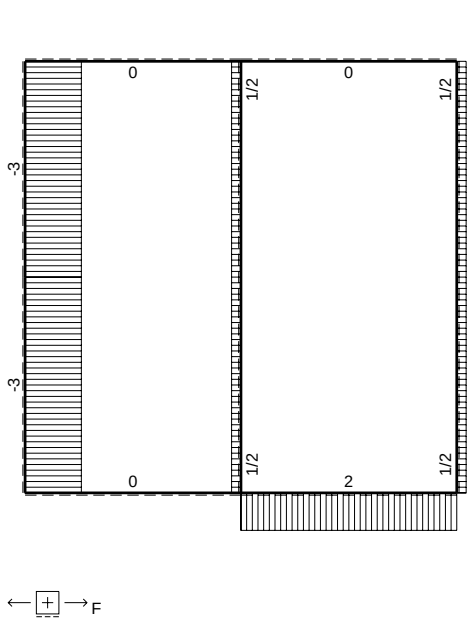


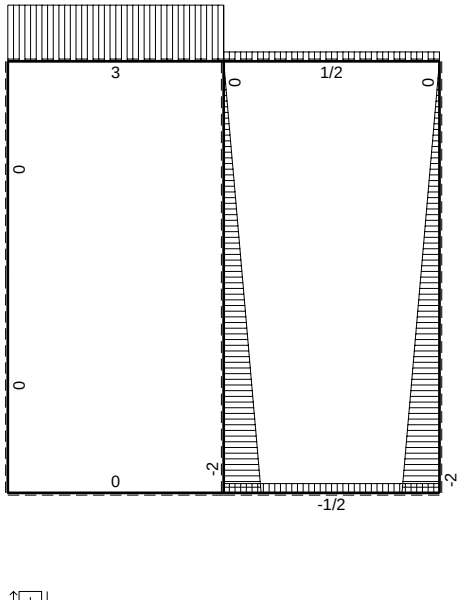
A = 162. mm^2
J_u = 91854. mm^4
J_v = 11016. mm^4
J_t = 134.6 mm^4
x_o = -13.71 mm
x_g = 12. mm
T_y = 1520. N
M_x = -714400. Nmm
u_m = -12. mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -210. N/mm^2
x_c = 6. mm
u_c = -6. mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -210. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 289.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.72 N/mm^2
$\tau_o = Tx_o t/J_t$ = 278.7 N/mm^2
t_c = 5472. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 543.5 N/mm^2

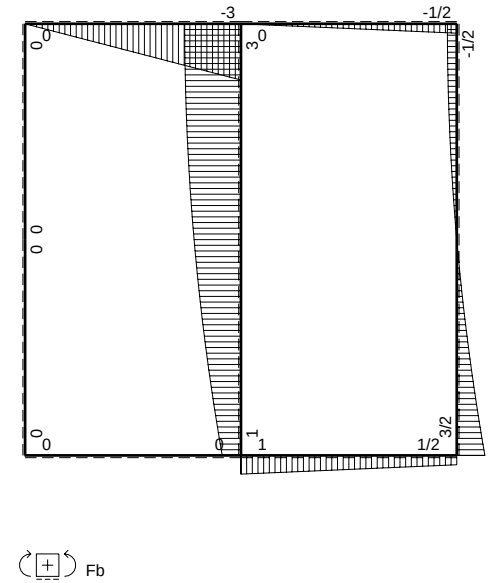
CI08.xxxx.001

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
1/2
0
1/2
-3
-3
0
1/2
2
1/2
F


3
0
1/2
0
0
0
0
-2
-2
-1/2
F


-3
-1/2
0
0
3
-1/2
0
0
0
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	3Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	-Fb-2Fx+1/2qx²	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 209802 CI08.xxxx.002

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

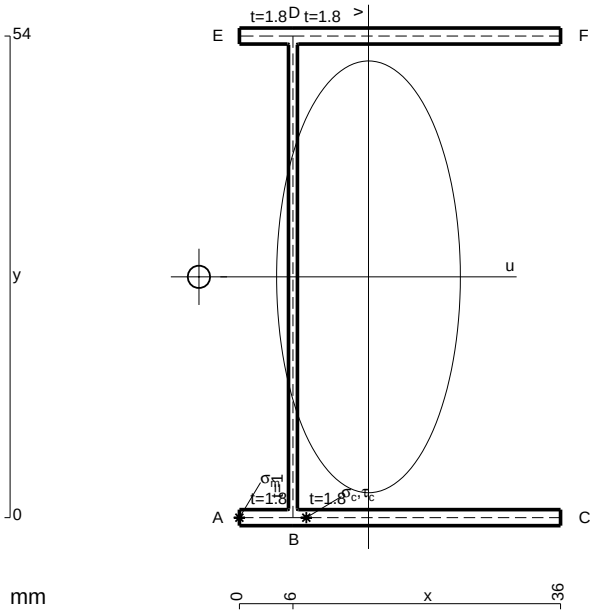
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

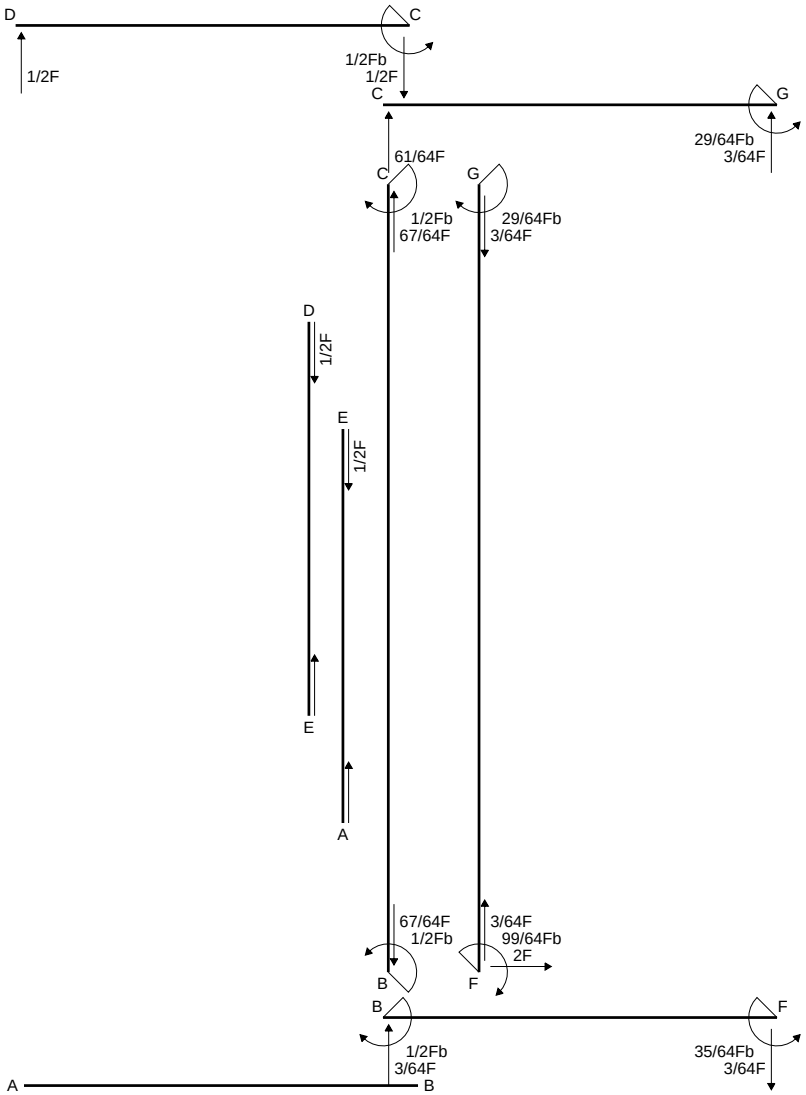
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

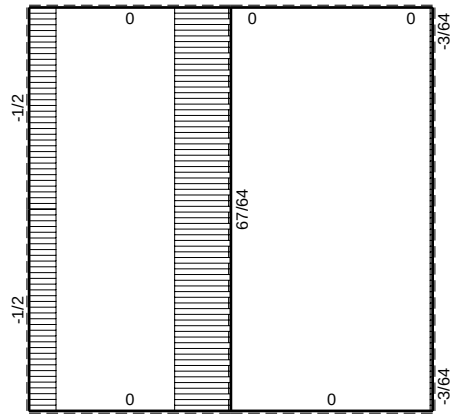


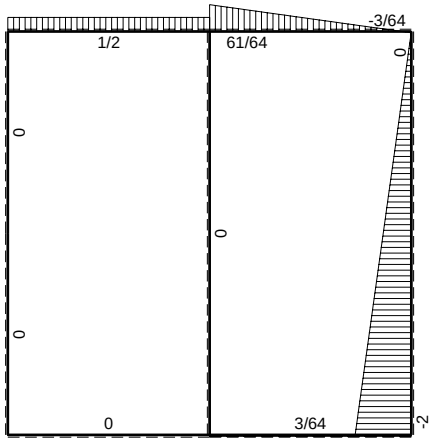
A = 183.6 mm^2
J_u = 107600. mm^4
J_v = 19486. mm^4
J_t = 158. mm^4
x_o = -19.01 mm
x_g = 14.47 mm
T_y = 1710. N
M_x = -872100. Nmm
u_m = -14.47 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -218.8 N/mm^2
x_c = 6. mm
u_c = -8.471 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -218.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 383.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.87 N/mm^2
$\tau_o = Tx_o t/J_t$ = 370.4 N/mm^2
t_c = 1026. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 698.9 N/mm^2

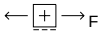
CI08.xxxx.002

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

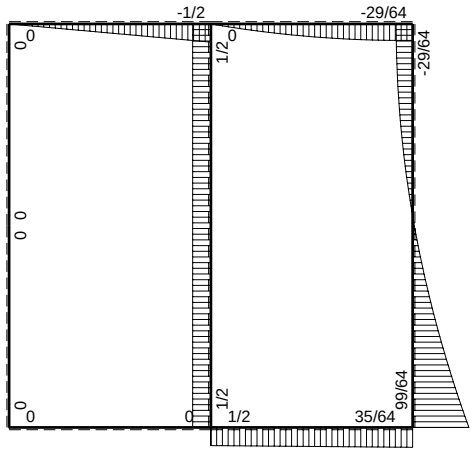

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64


-3/64
1/2
61/64
0
0
0
0
0
3/64
-2


F


F


-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
1/2
35/64
99/64


Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-29/24Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						29/64Fb	

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

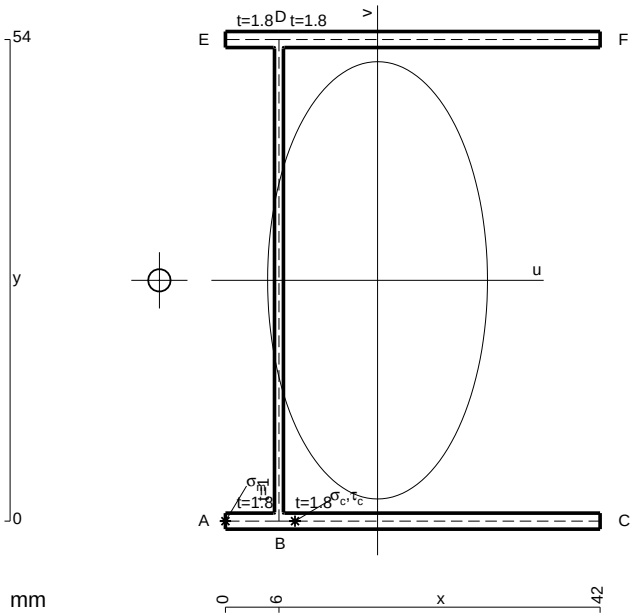
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

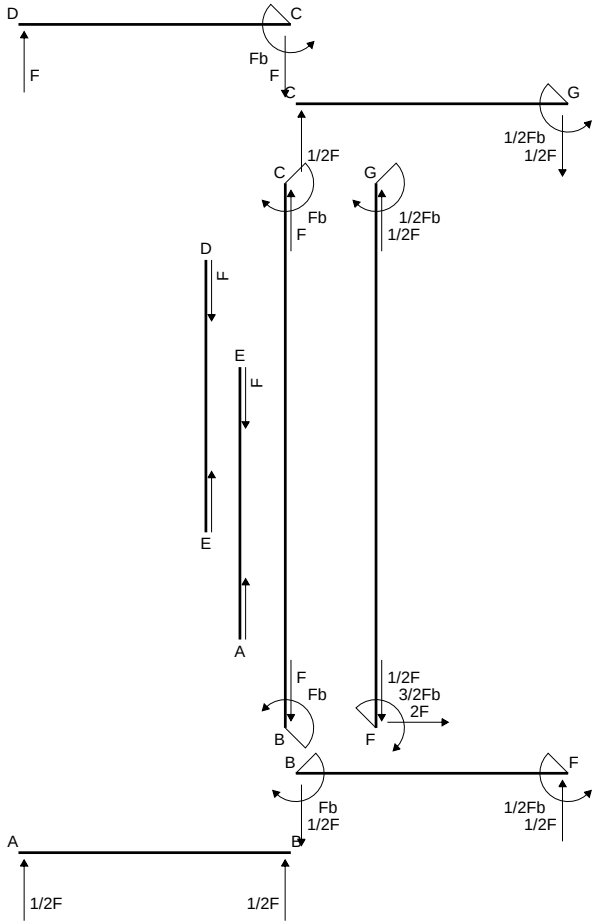
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

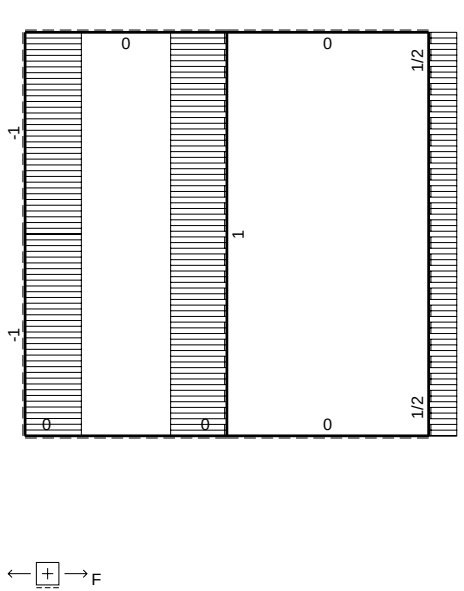


A = 205.2 mm^2
J_u = 123347. mm^4
J_v = 31179. mm^4
J_t = 181.3 mm^4
x_o = -24.46 mm
x_g = 17.05 mm
T_y = 1910. N
M_x = -1050500. Nmm
u_m = -17.05 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -229.9 N/mm^2
x_c = 6. mm
u_c = -11.05 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -229.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 478.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.05 N/mm^2
$\tau_o = Tx_o t/J_t$ = 463.8 N/mm^2
t_c = 6876. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 860.7 N/mm^2

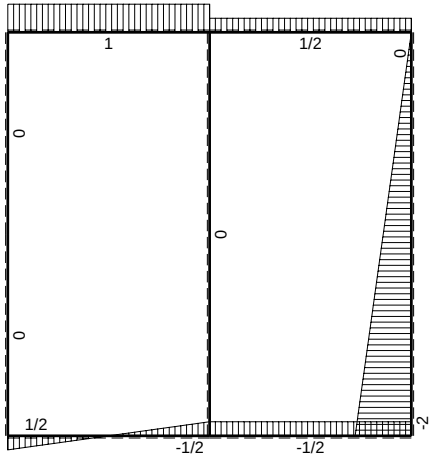
CI08.xxxx.003

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

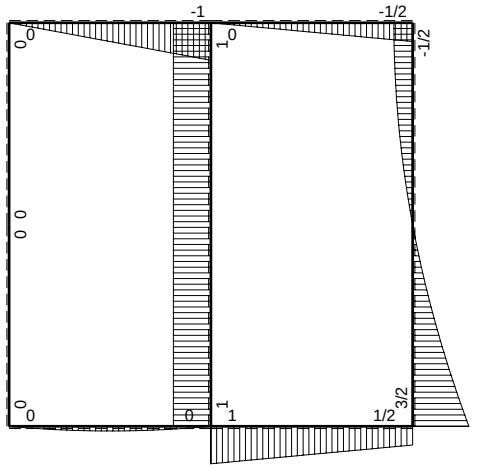

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

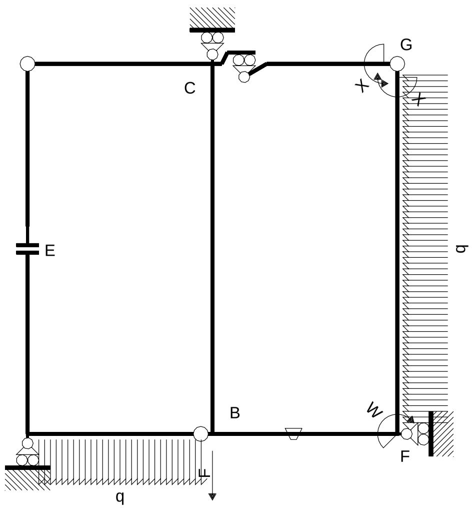
F

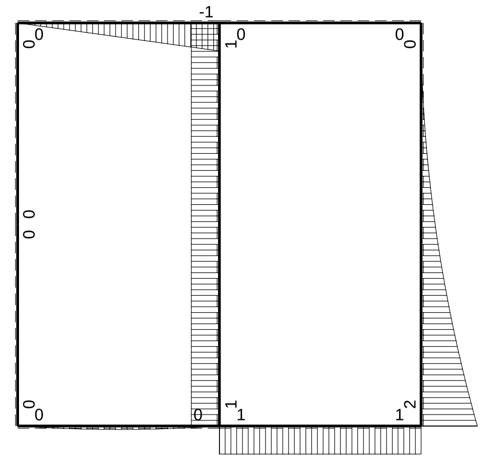

1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2

F

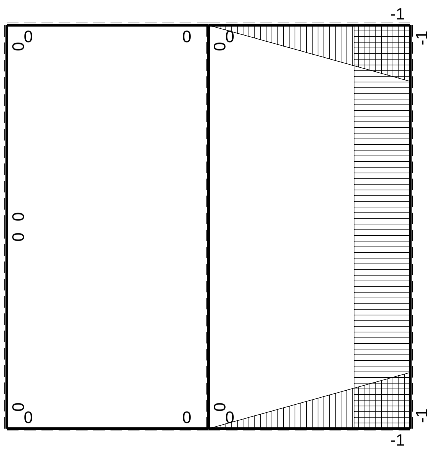

-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

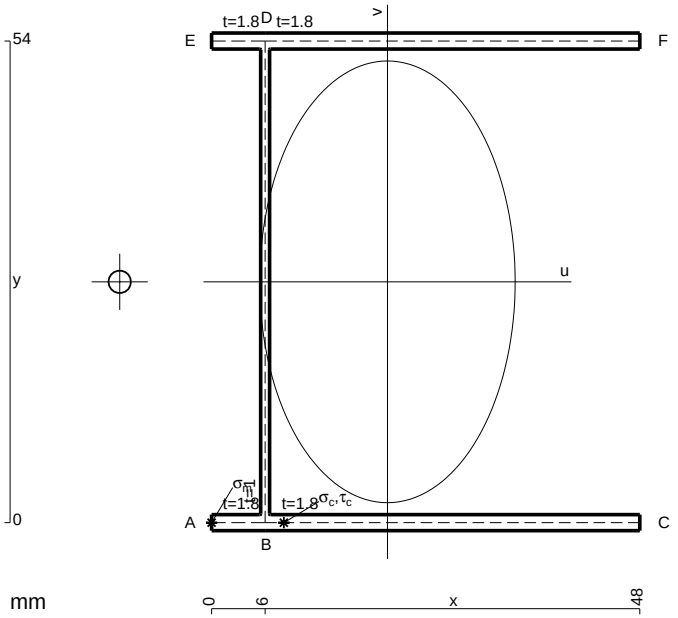
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

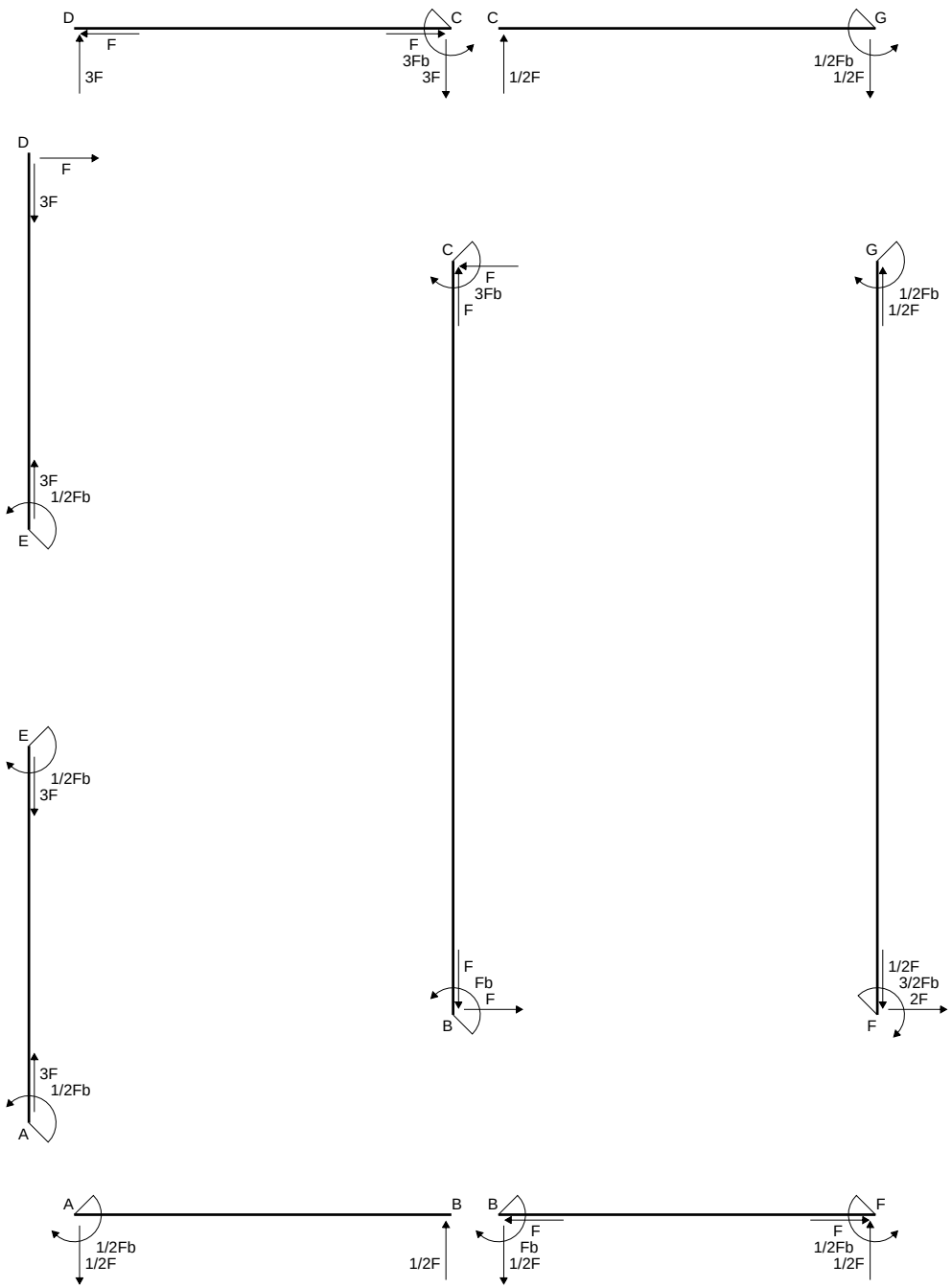
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

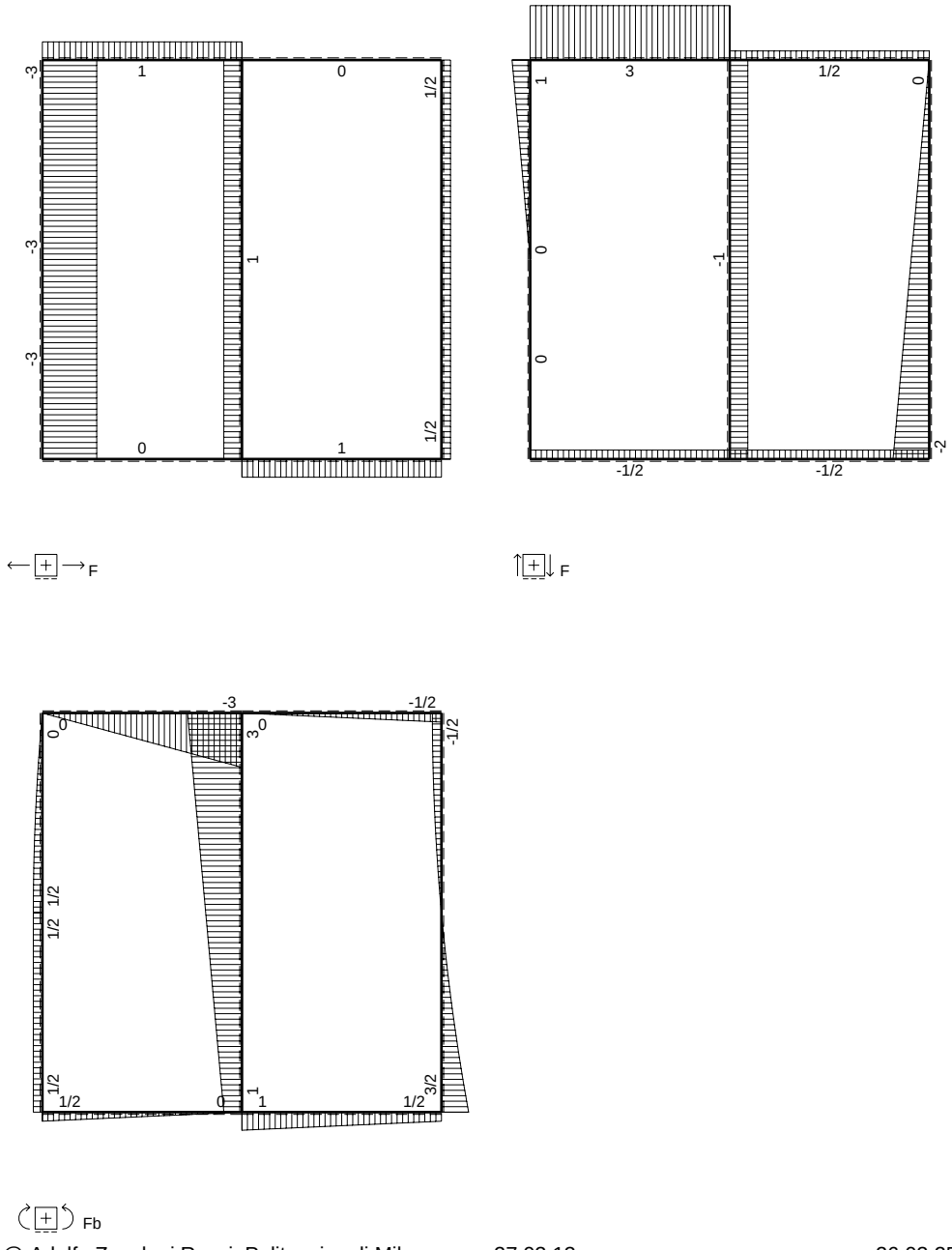


A = 226.8 mm^2
J_u = 139093. mm^4
J_v = 46508. mm^4
J_t = 204.6 mm^4
x_o = -30.02 mm
x_g = 19.71 mm
T_y = 2060. N
M_x = -1236000. Nmm
u_m = -19.71 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -239.9 N/mm^2
x_c = 6. mm
u_c = -13.71 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -239.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 560.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 16.79 N/mm^2
$\tau_o = Tx_o t/J_t$ = 543.9 N/mm^2
t_c = 3708. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1000. N/mm^2

CI08.xxxx.004

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
F
Fb
F
1/2F
3/2Fb
2F
B
F
3F
1/2Fb
A
A
B
B
F
F
Fb
F
1/2Fb
1/2F
1/2F
1/2Fb
1/2F


-3
1
0
1/2
-3
1
-3
0
1
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^{2}b^2	(-1/2+1/2)Fb^2/EJ	1/3XbEJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2XbEJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3FbEJ	8/3XbEJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 213350 CI08.xxxx.005

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (- x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [- x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

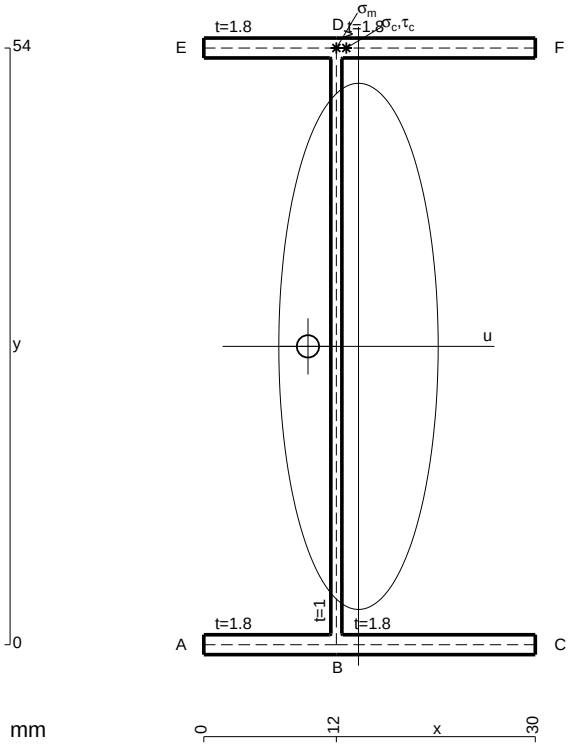
$= (- b +1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

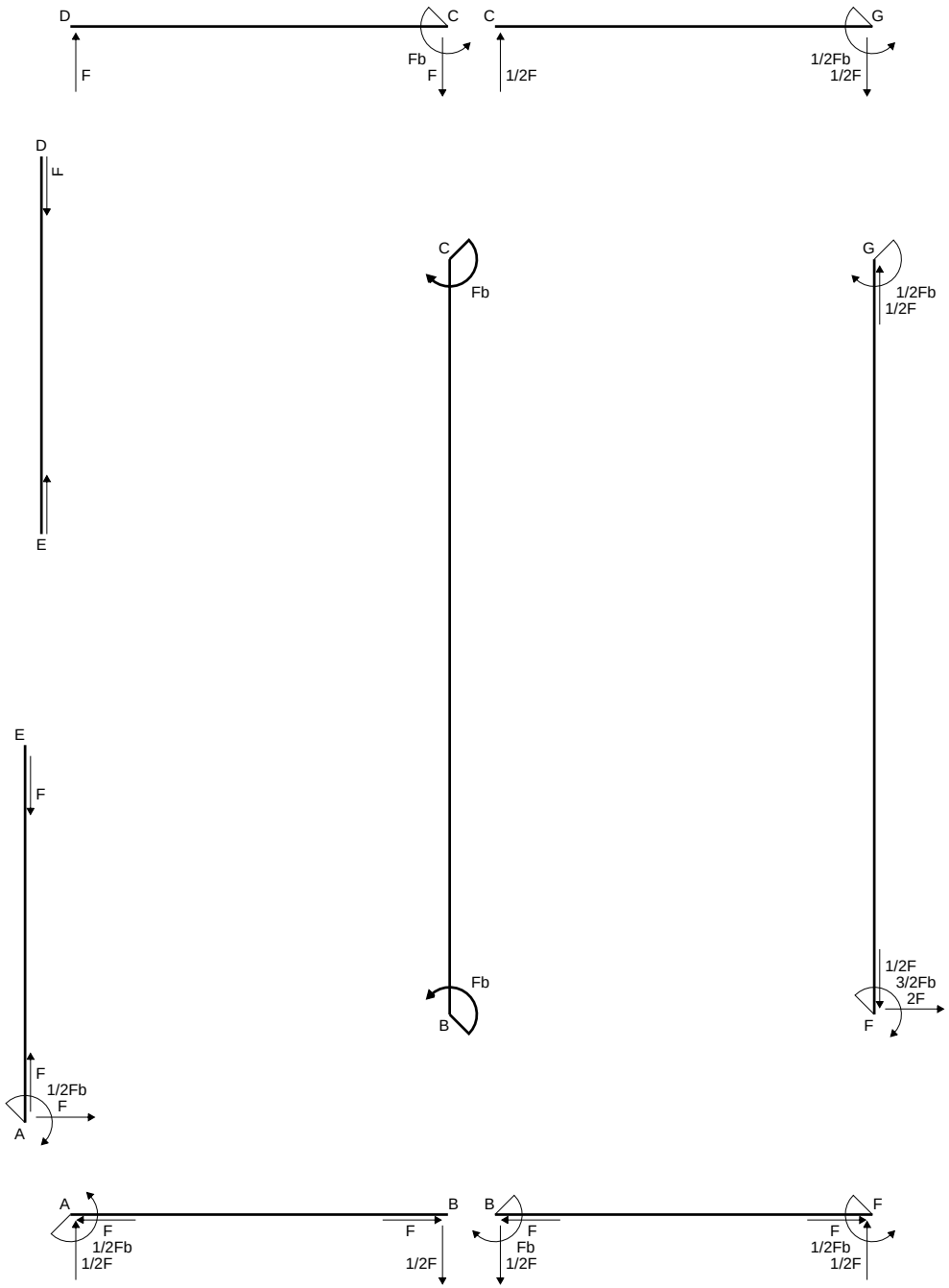
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

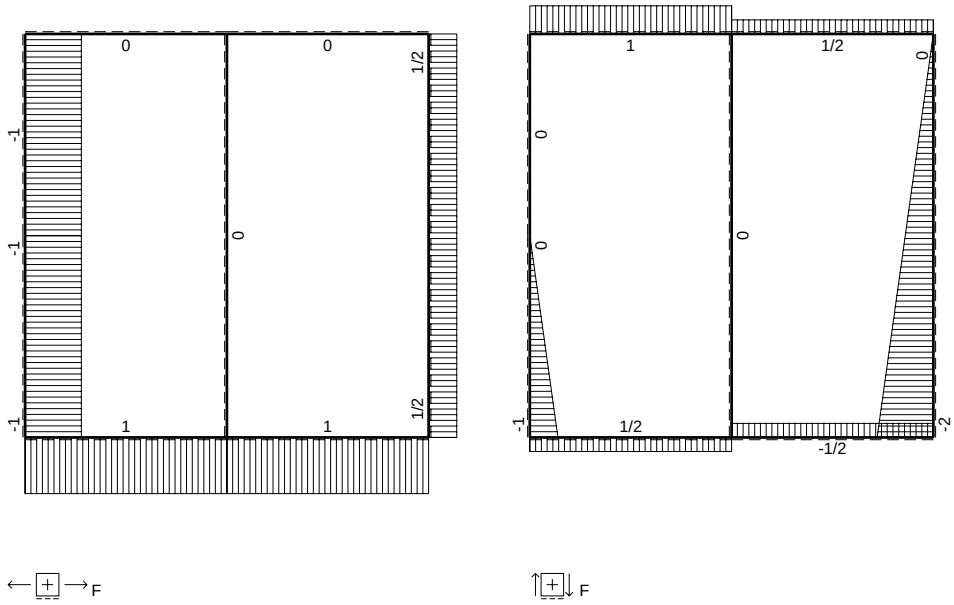


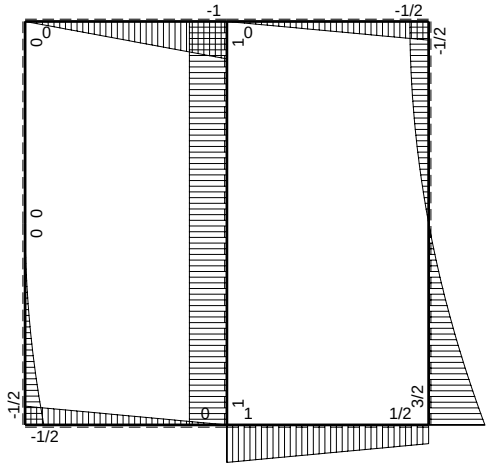
A = 162. mm^2
J_u = 91854. mm^4
J_v = 8424. mm^4
J_t = 134.6 mm^4
x_o = -4.571 mm
x_g = 14. mm
N = 350. N
T_y = 1050. N
M_x = -672000. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -2. mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 199.7 N/mm^2
x_c = 12. mm
y_c = 54. mm
u_c = -2. mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 199.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 69.73 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.556 N/mm^2
$\tau_o = Tx_o t/J_t$ = 64.17 N/mm^2
t_c = 630. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 233.4 N/mm^2

CI08.xxxx.005

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
G
1/2Fb
1/2F
Fb
B
1/2F
3/2Fb
2F
F
E
F
F
1/2Fb
F
A
A
B
B
F
F
F
F
1/2Fb
1/2F
1/2F
Fb
1/2F
1/2Fb
1/2F


0
0
1/2
-1
0
-1
1
1
1/2
-1
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F
F


-1
-1/2
0
1
0
-1/2
0 0
-1/2
0
1
1
1/2
3/2
-1/2


Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

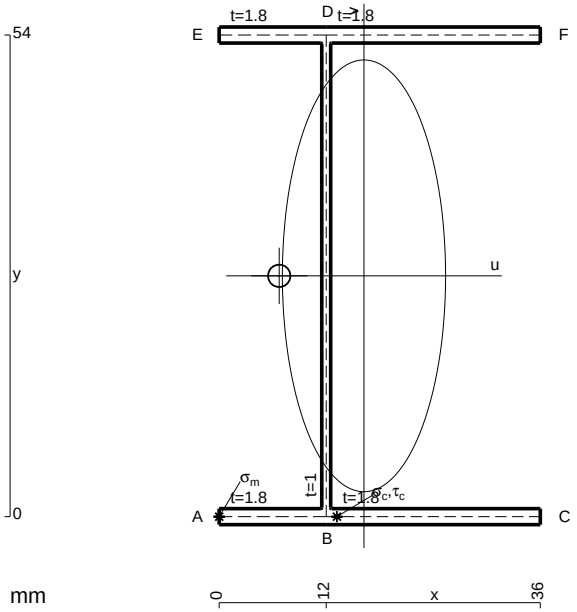
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

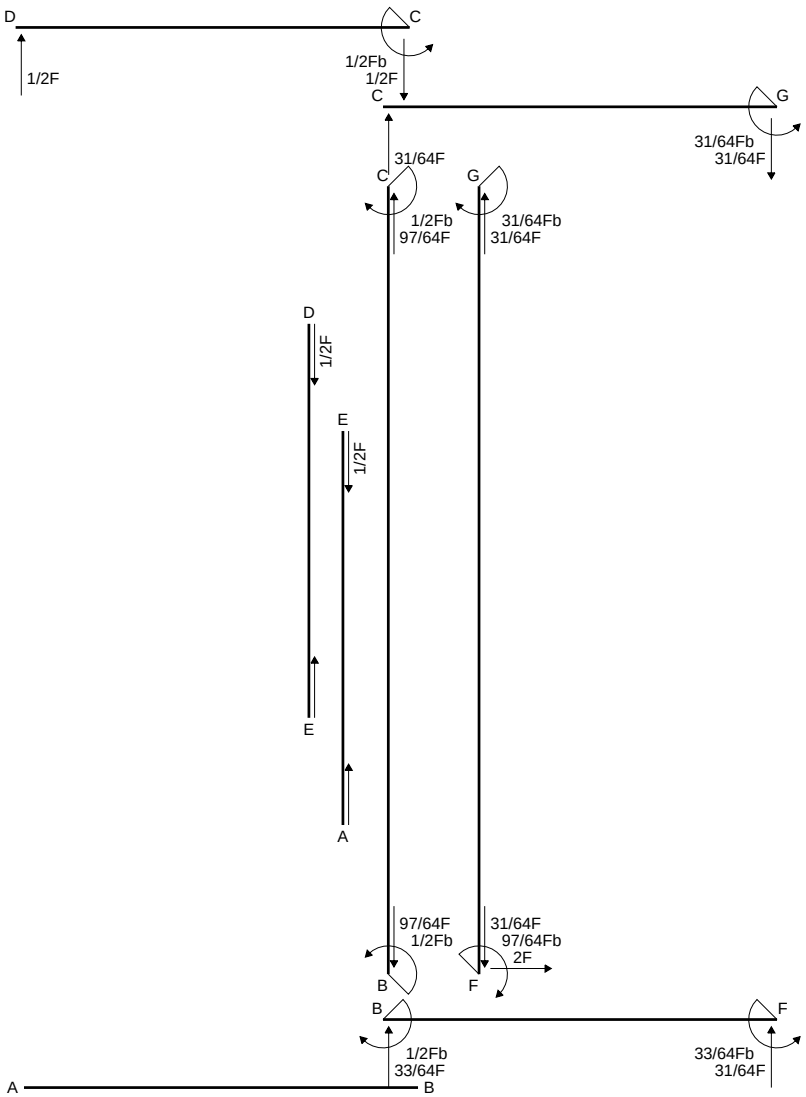
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

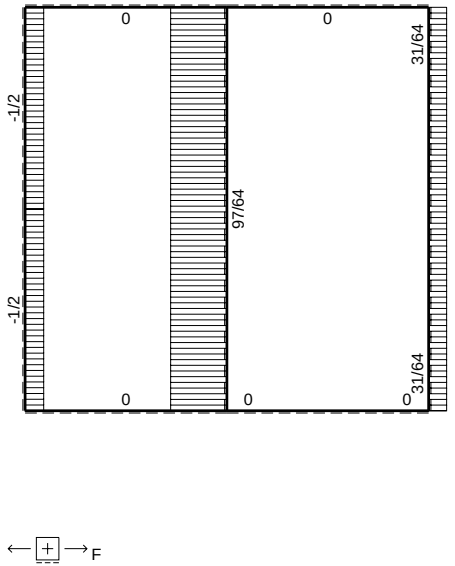


A = 183.6 mm^2
J_u = 107600. mm^4
J_v = 15369. mm^4
J_t = 158. mm^4
x_o = -9.504 mm
x_g = 16.24 mm
T_y = 1230. N
M_x = -836400. Nmm
u_m = -16.24 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -209.9 N/mm^2
x_c = 12. mm
u_c = -4.235 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -209.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 140.6 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.407 N/mm^2
$\tau_o = Tx_o t/J_t$ = 133.2 N/mm^2
t_c = 2214. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 321.5 N/mm^2

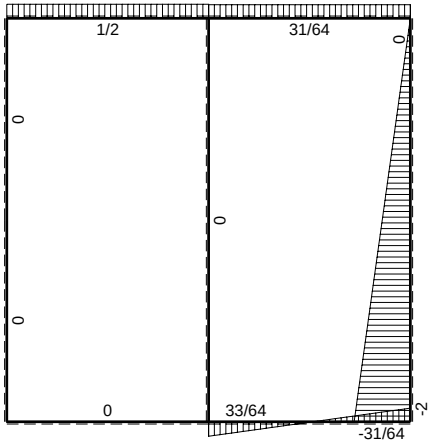
CI08.xxxx.006

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

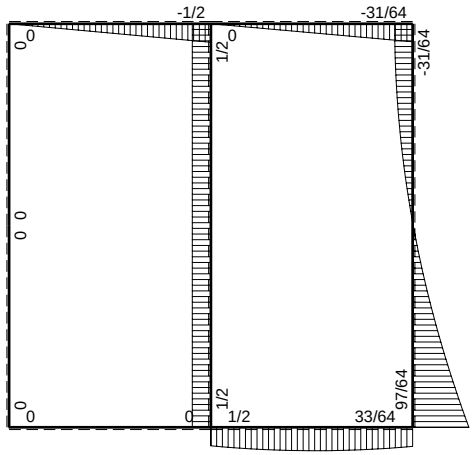

D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64

F


1/2
31/64
0
0
0
0
0
33/64
-2
-31/64

F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
0
0
1/2
33/64
97/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	FxEJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3XbEJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-FxEJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2XbEJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb²/EJ	8/3XbEJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 248355 CI08.xxxx.007

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

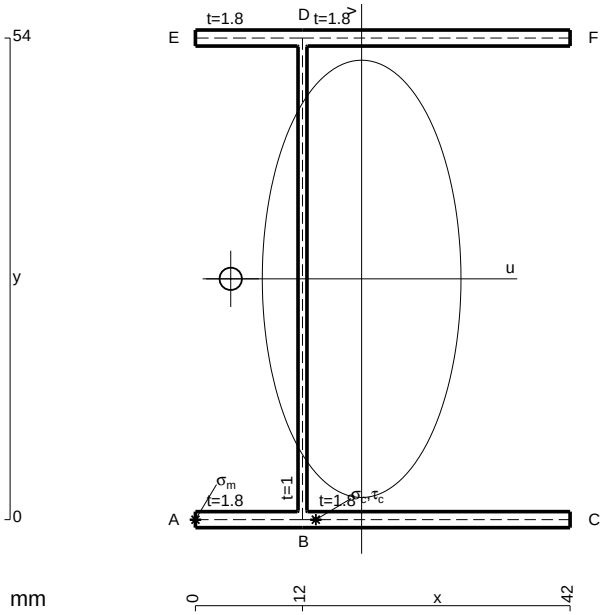
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

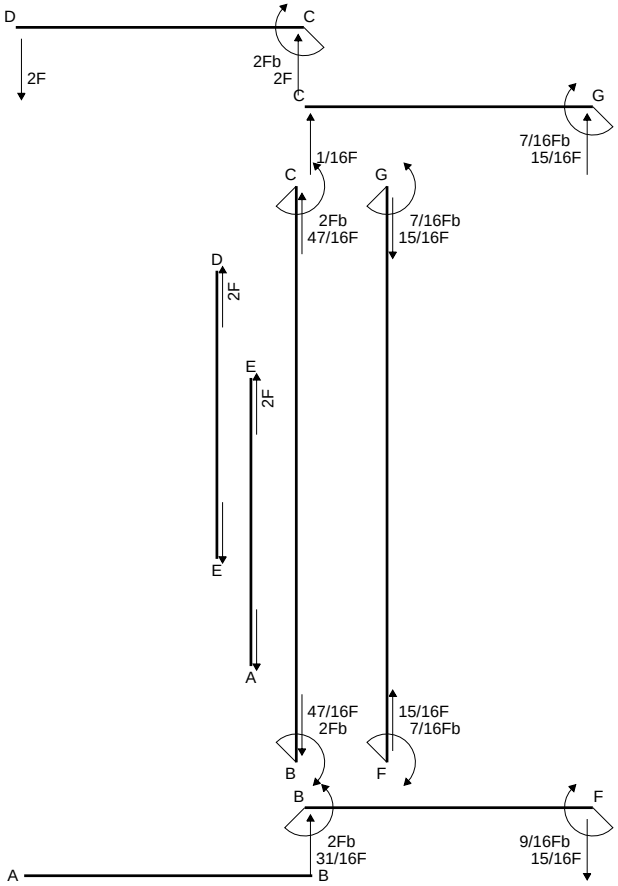
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

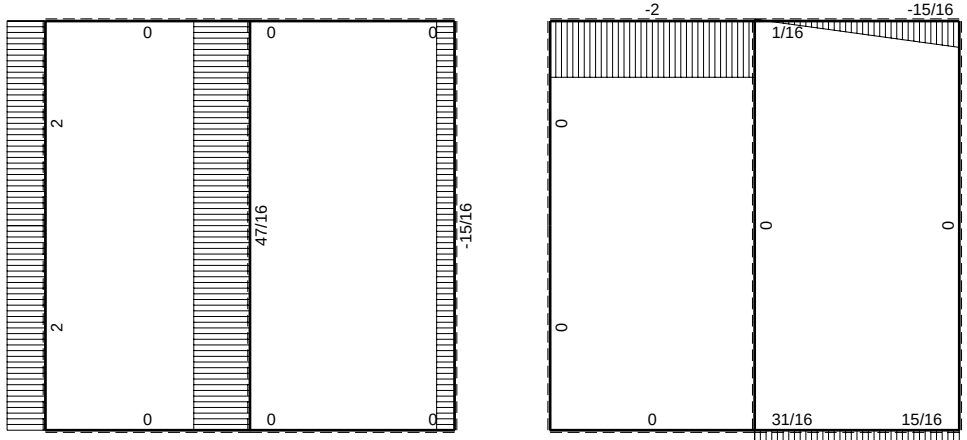


A = 205.2 mm^2
J_u = 123347. mm^4
J_v = 25449. mm^4
J_t = 181.3 mm^4
x_o = -14.67 mm
x_g = 18.63 mm
T_y = 1375. N
M_x = -1003750. Nmm
u_m = -18.63 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -219.7 N/mm^2
x_c = 12. mm
u_c = -6.632 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -219.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 209.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.029 N/mm^2
$\tau_o = Tx_o t/J_t$ = 200.3 N/mm^2
t_c = 4950. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 424. N/mm^2

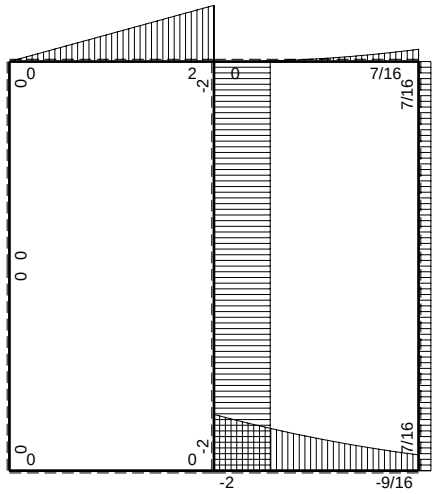
CI08.xxxx.007

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0
-2
-15/16
1/16
0
0
0
0
0
31/16
15/16

F F


0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
0
-2
7/16
-2
-9/16

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

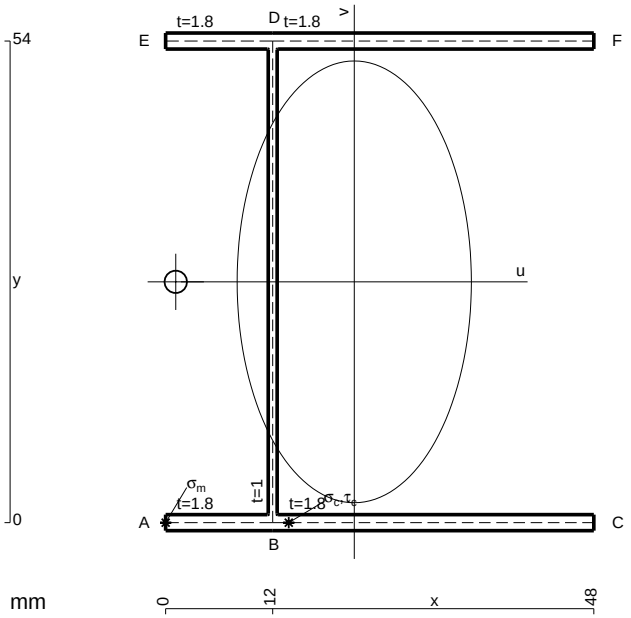
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

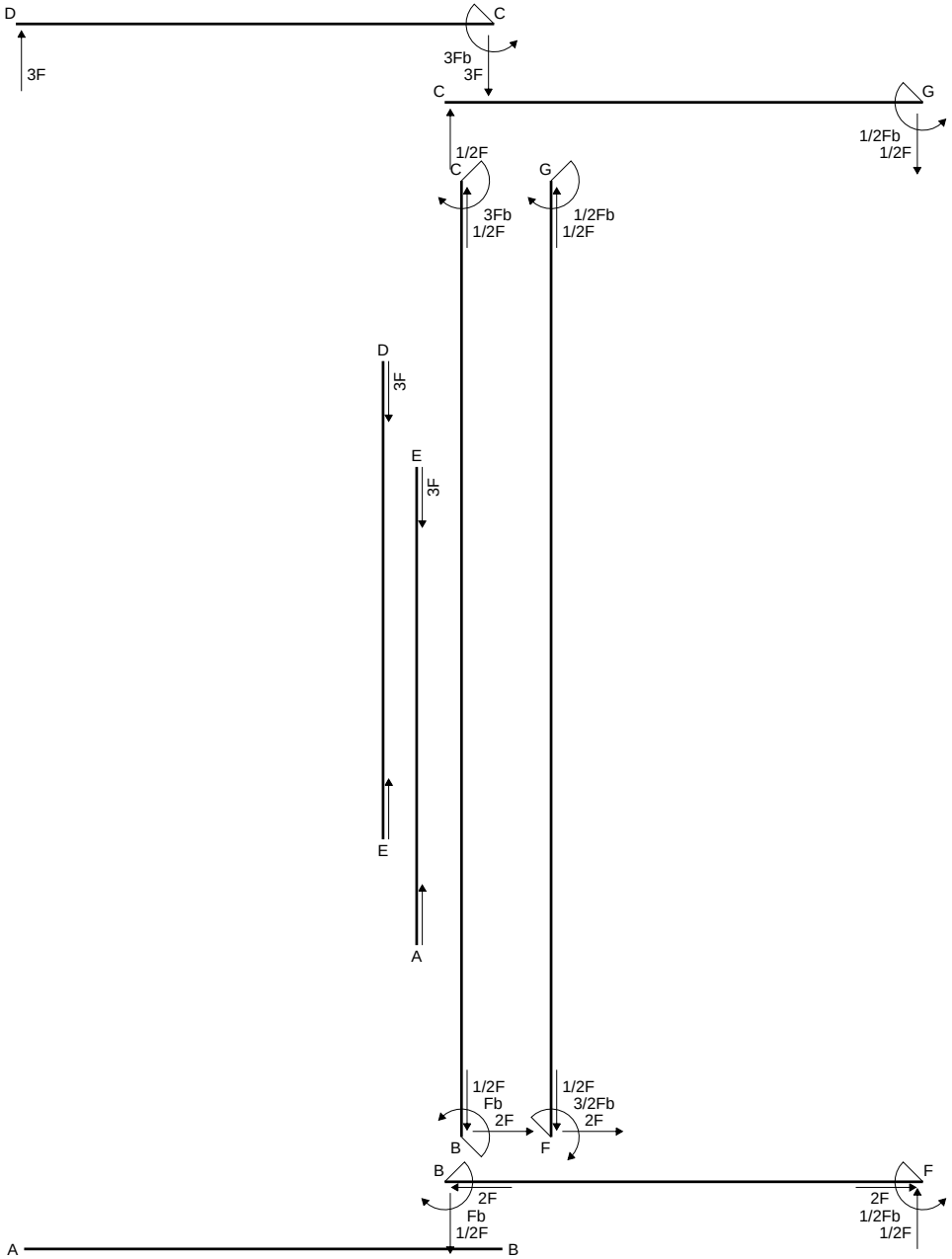
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

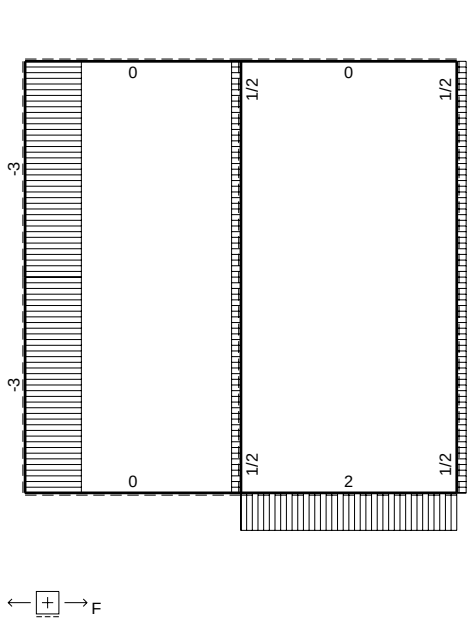


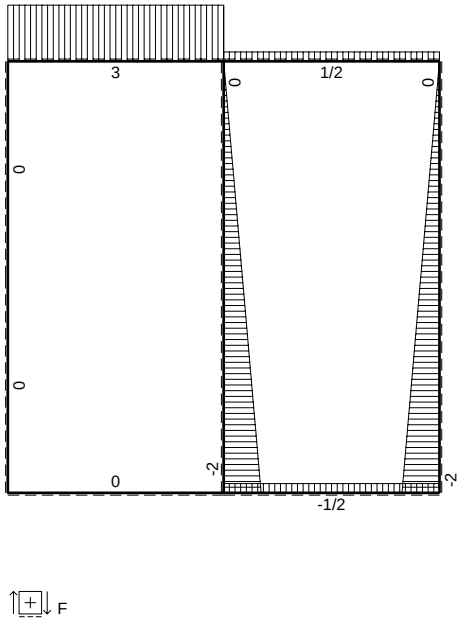
A = 226.8 mm^2
J_u = 139093. mm^4
J_v = 39102. mm^4
J_t = 204.6 mm^4
x_o = -20.01 mm
x_g = 21.14 mm
T_y = -1520. N
M_x = 1170400. Nmm
u_m = -21.14 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = 227.2 N/mm^2
x_c = 12. mm
u_c = -9.143 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = 227.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 278.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.62 N/mm^2
$\tau_o = Tx_o t/J_t$ = 267.6 N/mm^2
t_c = 1368. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 532.7 N/mm^2

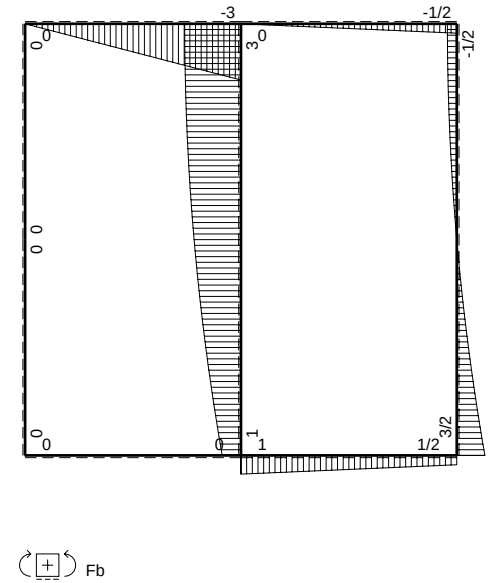
CI08.xxxx.008

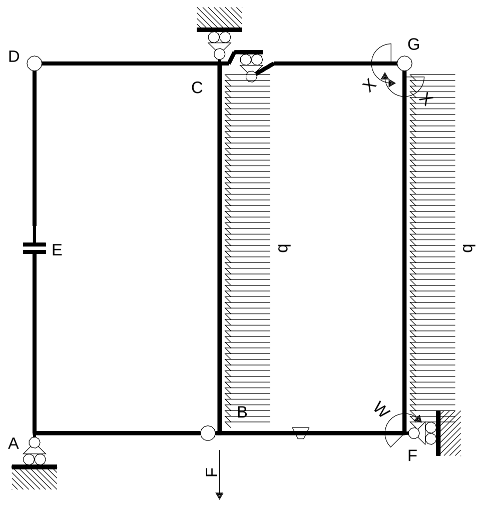
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

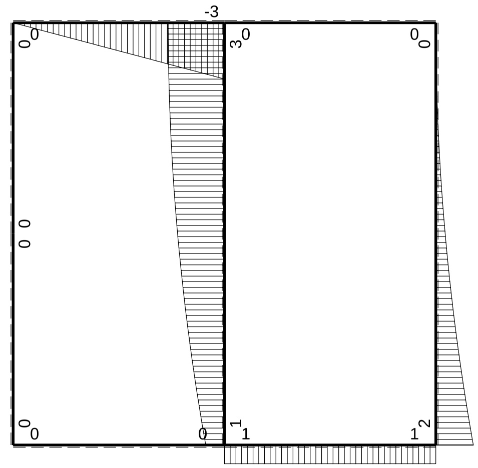

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
0
1/2
1/2
-3
-3
0
2
1/2
1/2
F

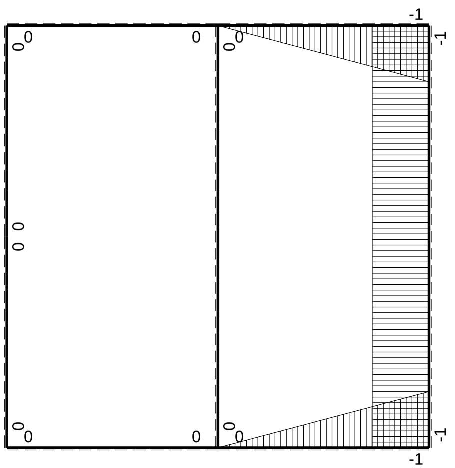

3
0
1/2
0
0
0
-2
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 937106 CI08.xxxx.009

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-3Fb+3Fx$	0	0	0	0	0+0	0
DC b	0	$3Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	Fb	$-Fb/EJ$	$-Fx$	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb$	Fb/EJ	$-Fb+Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	$8/3Xb/EJ$
iperstatica $X=W_{GC}$							$1/2Fb$	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (- x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [- x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

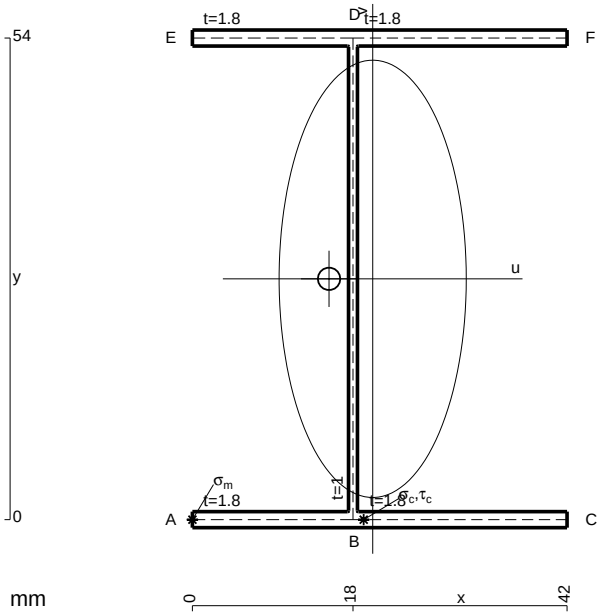
$= (- b +1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

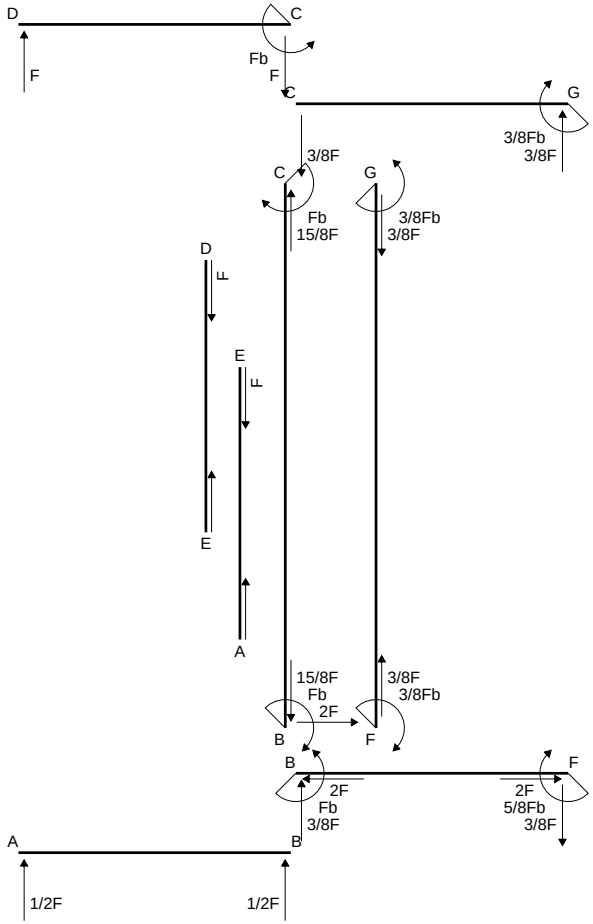
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

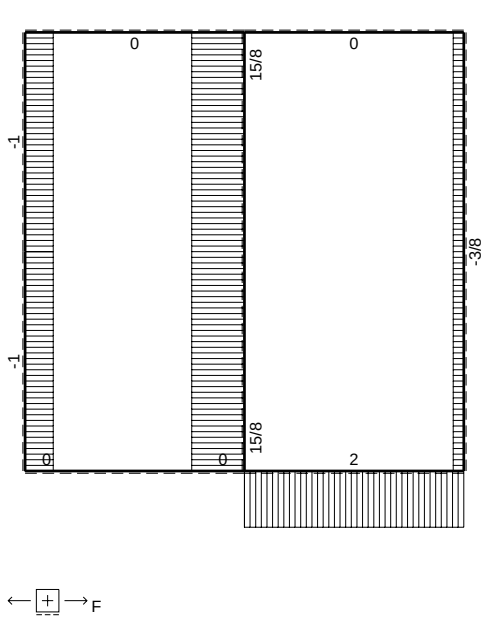


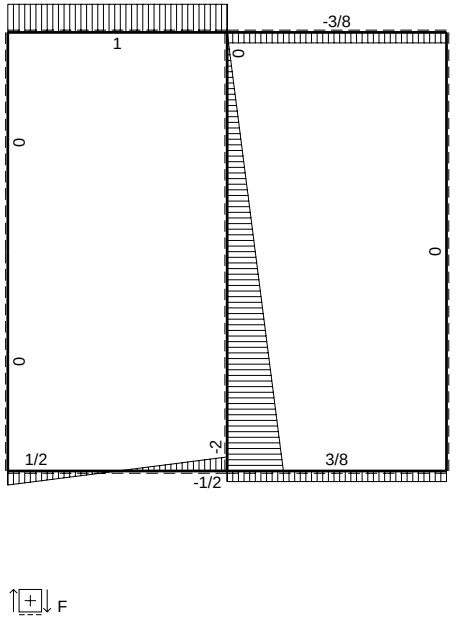
A = 205.2 mm^2
J_u = 123347. mm^4
J_v = 22585. mm^4
J_t = 181.3 mm^4
x_o = -4.891 mm
x_g = 20.21 mm
T_y = 1350. N
M_x = -1093500. Nmm
u_m = -20.21 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -239.4 N/mm^2
x_c = 18. mm
u_c = -2.211 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -239.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 72.65 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.092 N/mm^2
$\tau_o = Tx_o t/J_t$ = 65.56 N/mm^2
t_c = 810. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 270.4 N/mm^2

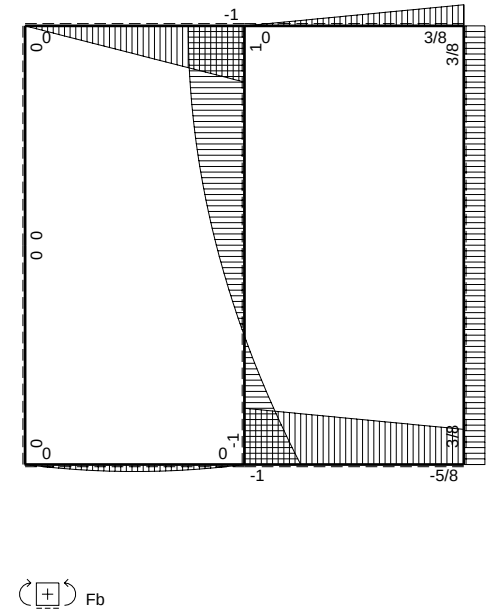
CI08.xxxx.009

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F


0
0
15/8
-1
-1
-3/8
0
0
15/8
0
0
2
F


-3/8
1
0
0
0
0
1/2
-2
3/8
-1/2
F


-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx^2	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Schema di calcolo iperstatico

PROCEDIMENTO E RISULTATI 214074 CI08.xxxx.010

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

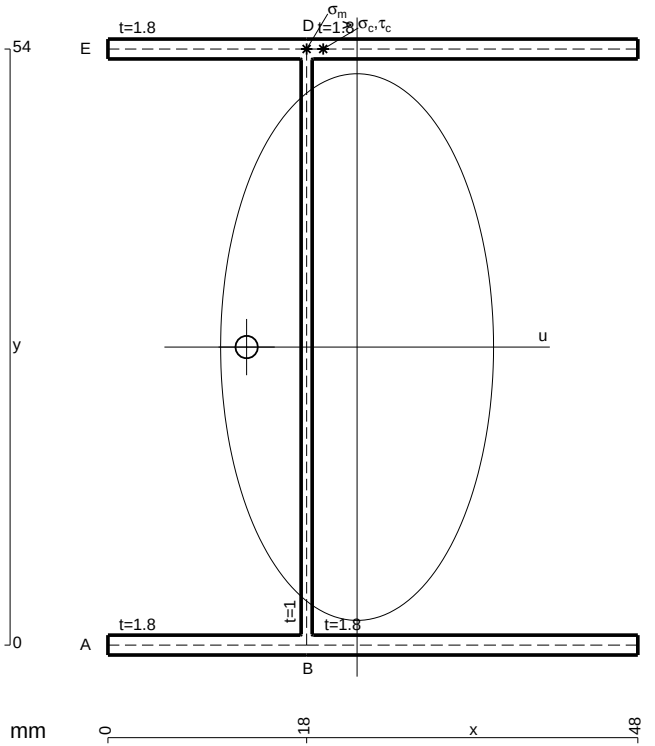
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

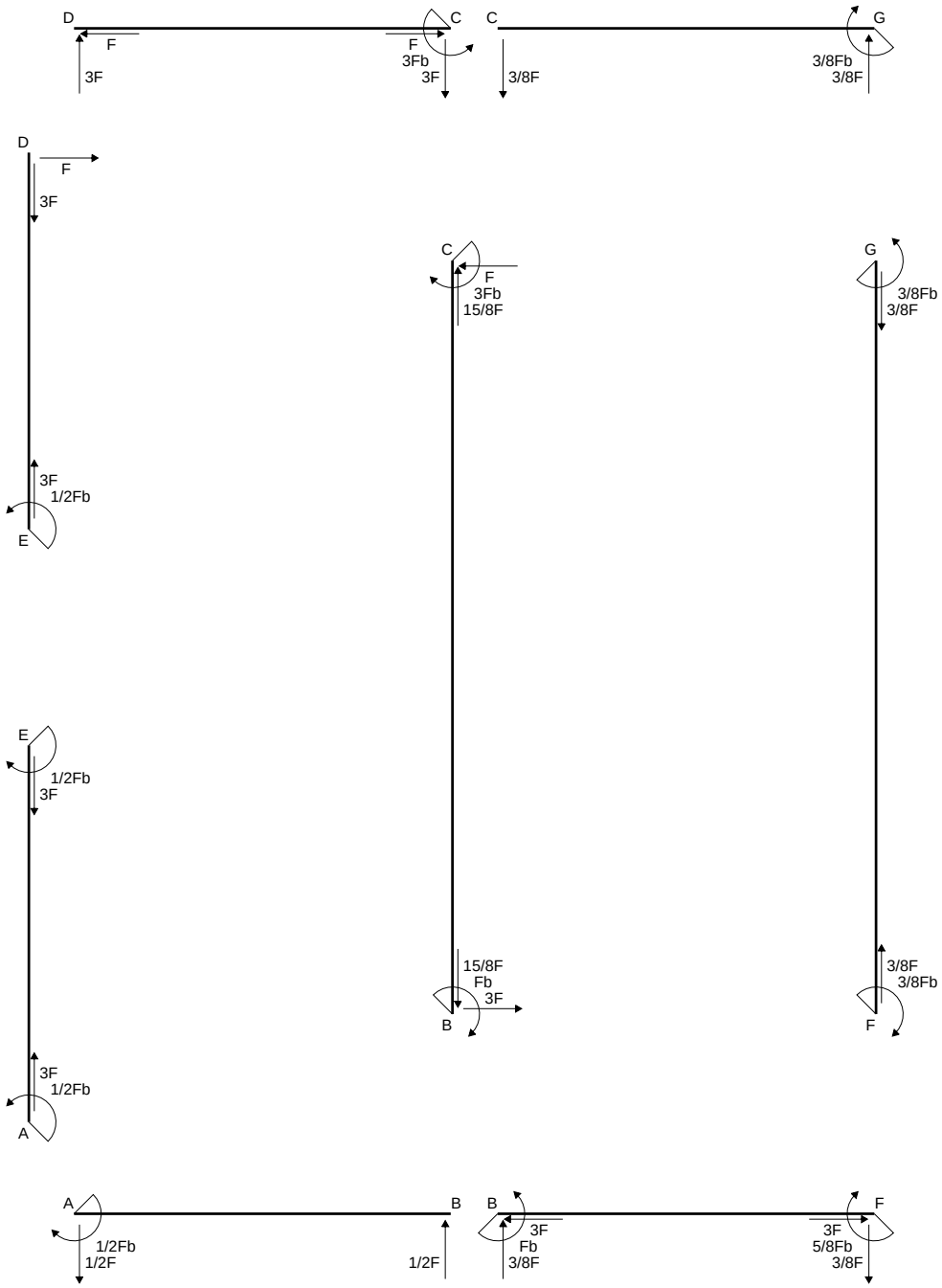
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

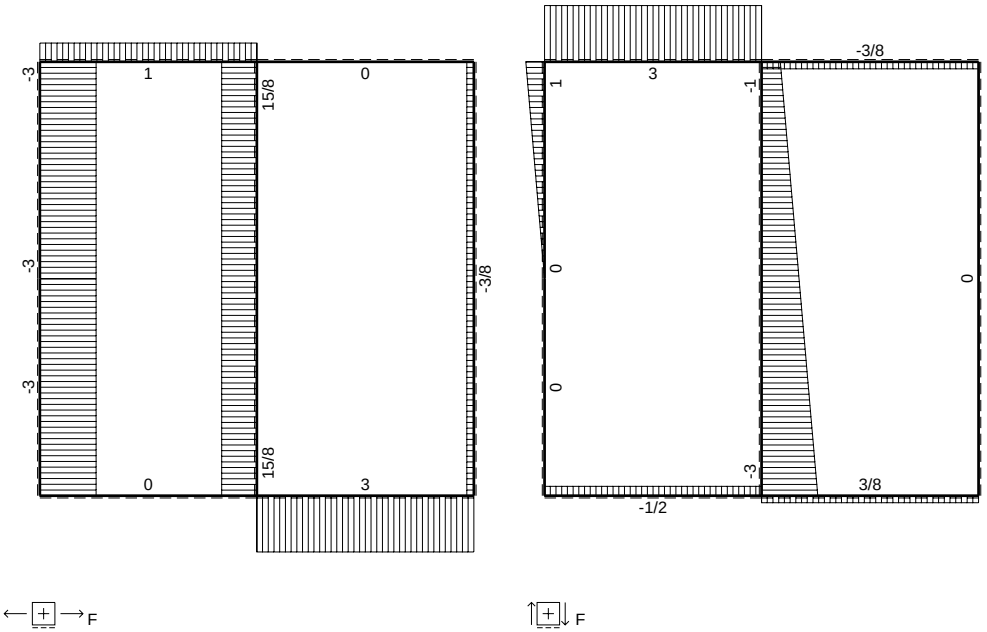


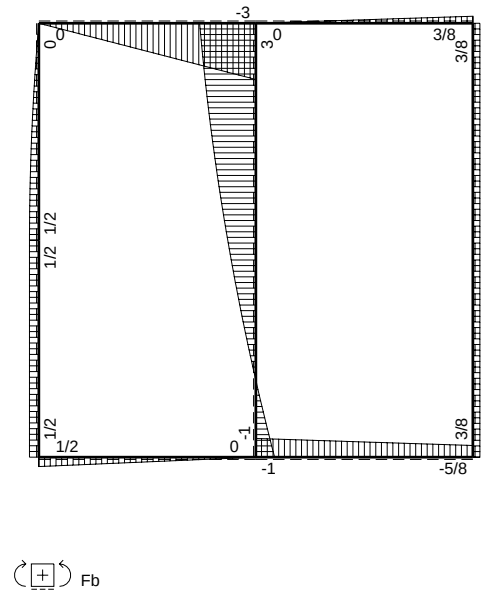
A = 226.8 mm^2
J_u = 139093. mm^4
J_v = 34659. mm^4
J_t = 204.6 mm^4
x_o = -10.01 mm
x_g = 22.57 mm
N = 4088. N
T_y = -4360. N
M_x = -937400. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = -4.571 mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 200. N/mm^2
x_c = 18. mm
y_c = 54. mm
u_c = -4.571 mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 200. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 409.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 25.39 N/mm^2
$\tau_o = Tx_o t/J_t$ = 383.7 N/mm^2
t_c = 3924. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 736.3 N/mm^2

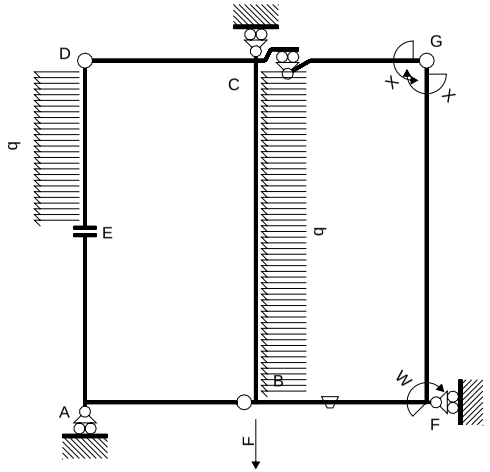
CI08.xxxx.010

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

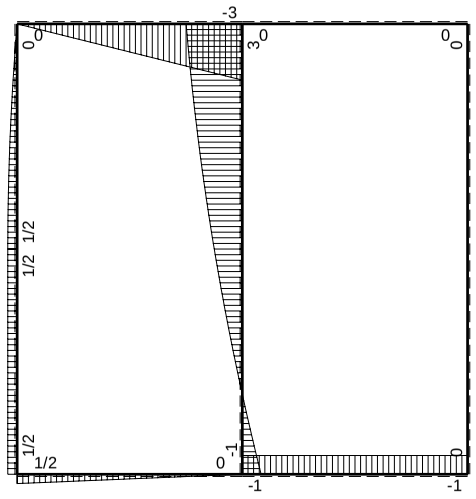

D
F
3F
C
F
3Fb
3F
C
3/8F
G
3/8Fb
3/8F
D
F
3F
3F
1/2Fb
E
C
F
3Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3F
15/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F


-3
1
0
15/8
-3
-3/8
-3
15/8
0
3
F
-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F

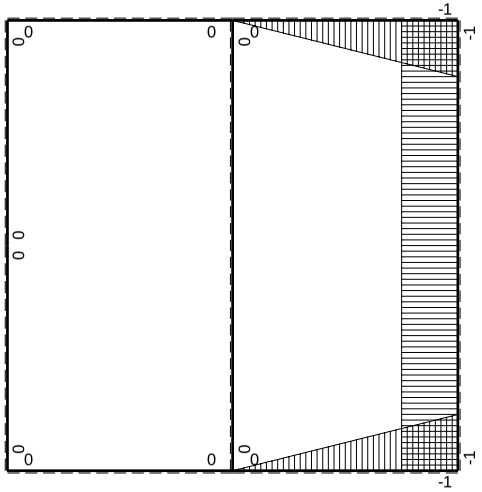

-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 240999 CI08.xxxx.011

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

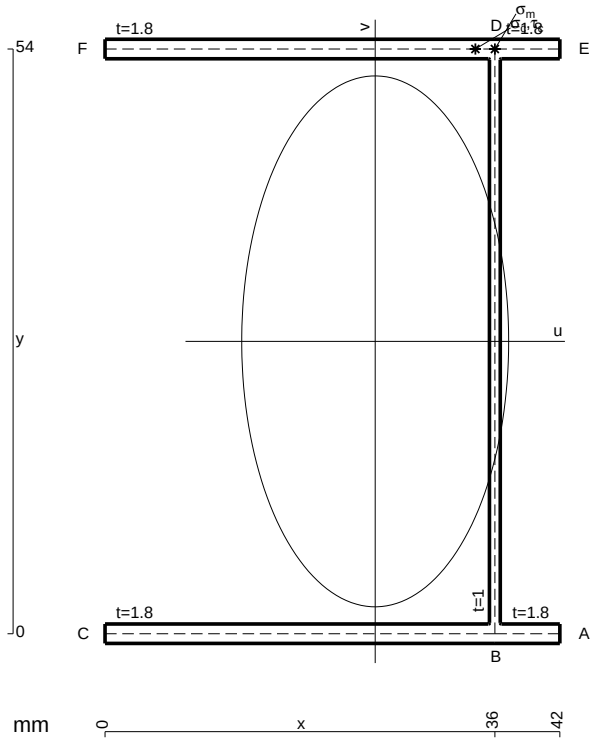
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

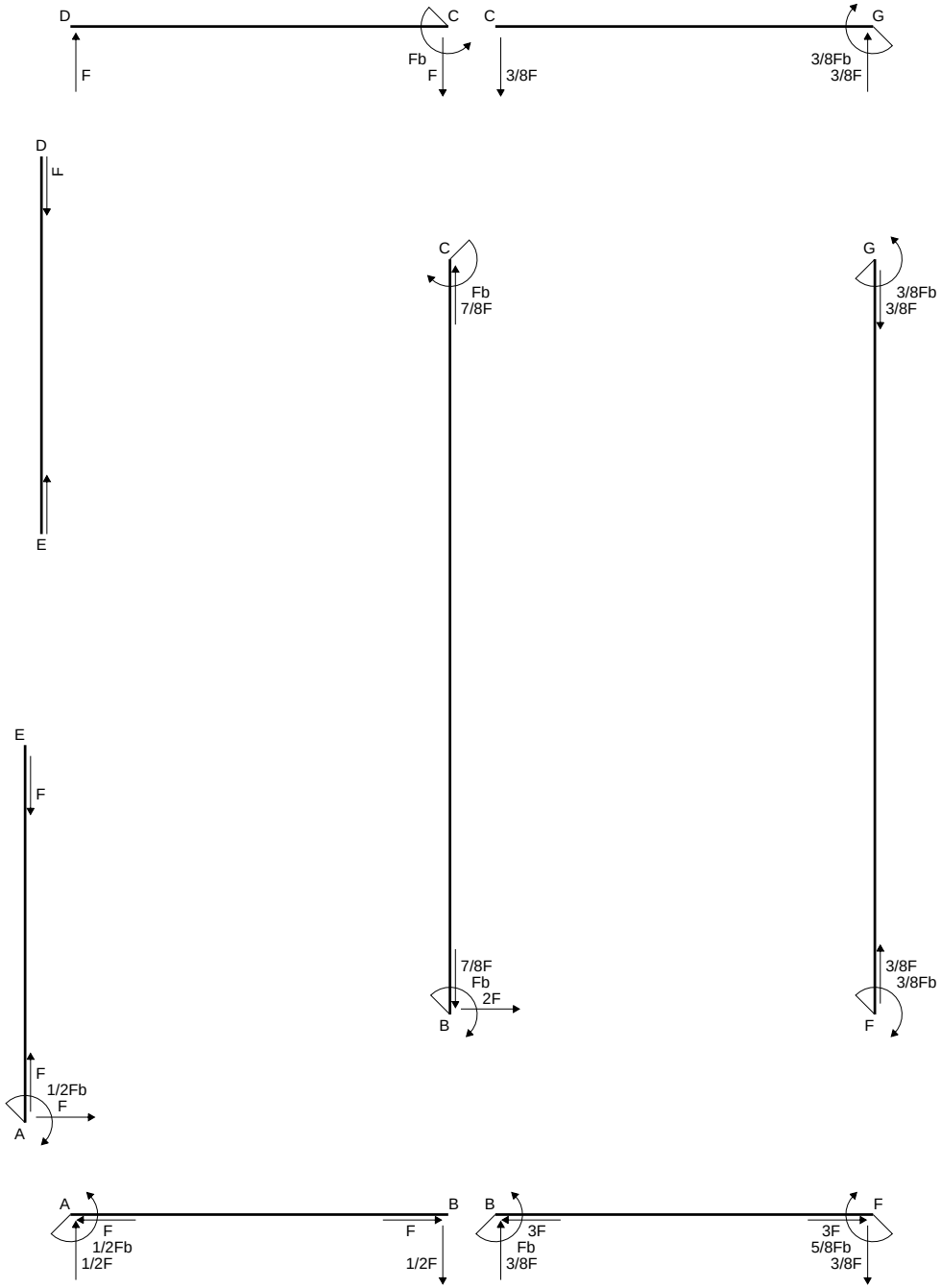
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

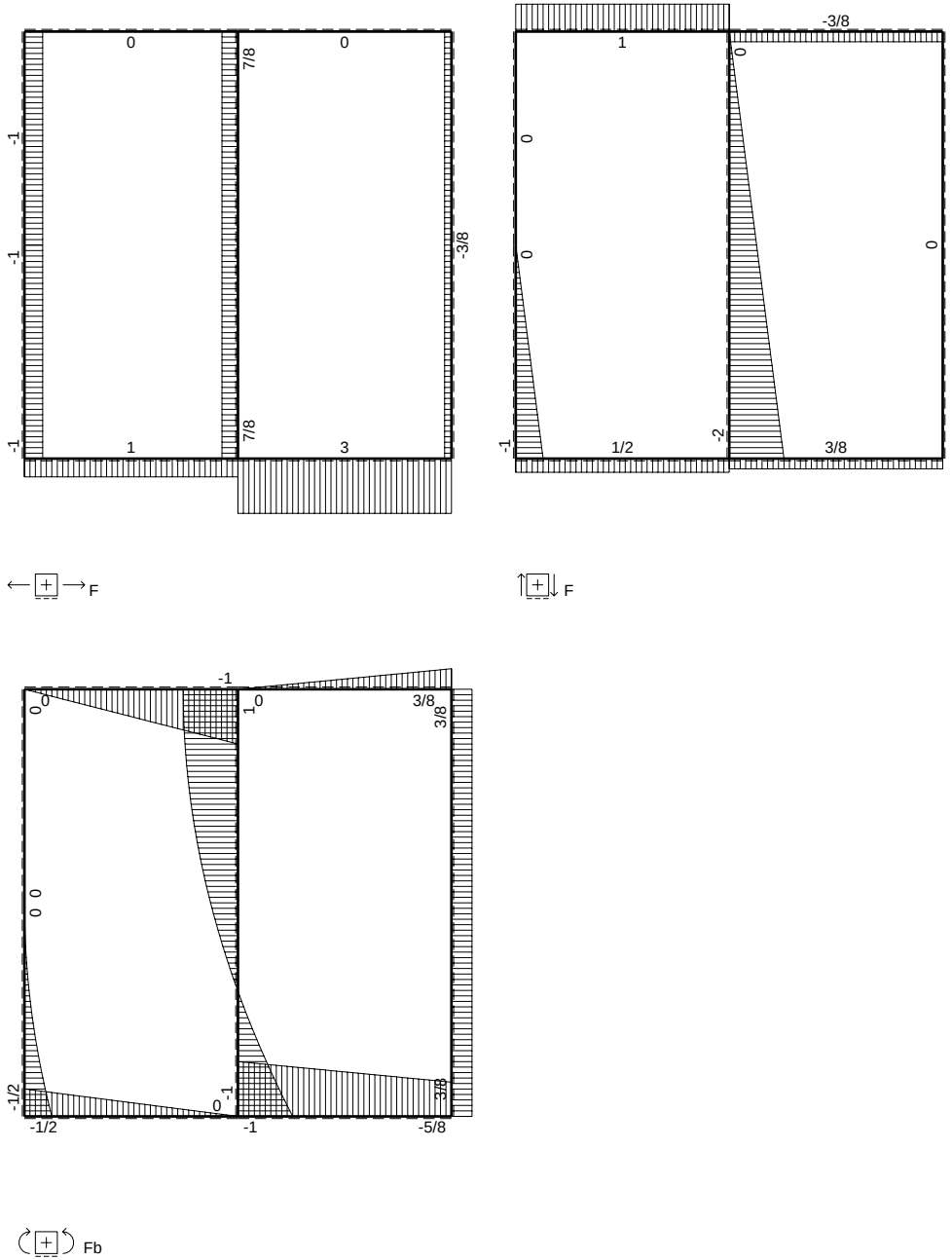


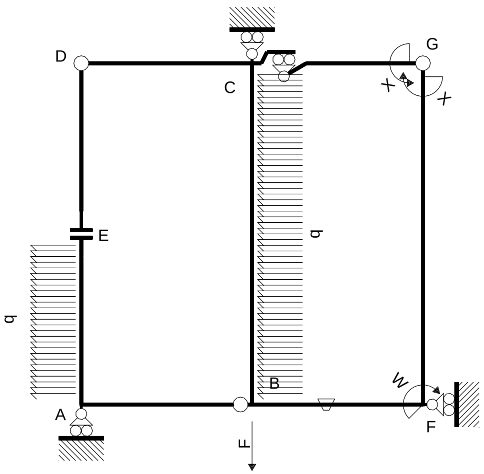
A = 205.2 mm^2
J_u = 123347. mm^4
J_v = 31179. mm^4
J_t = 181.3 mm^4
x_o = 24.46 mm
x_g = 24.95 mm
N = 660. N
T_y = 1980. N
M_x = -930600. Nmm
x_m = 36. mm
y_m = 54. mm
u_m = 11.05 mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 206.9 N/mm^2
x_c = 36. mm
y_c = 54. mm
u_c = 11.05 mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 206.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 496.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.6 N/mm^2
$\tau_o = Tx_o t/J_t$ = 480.8 N/mm^2
t_c = 1188. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 884.3 N/mm^2

CI08.xxxx.011

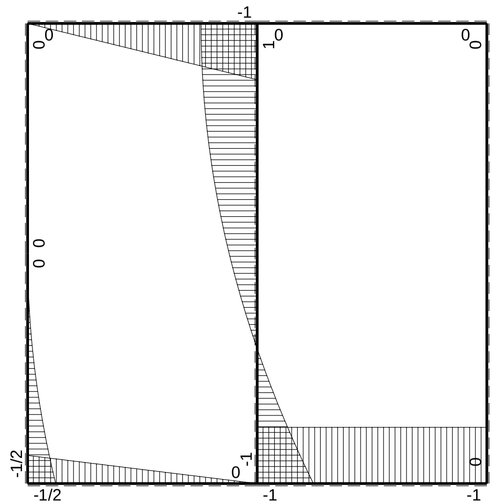
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
7/8F
G
3/8Fb
3/8F
E
F
F
1/2Fb
F
A
7/8F
Fb
2F
B
3/8F
3/8Fb
F
A
B
B
F
F
1/2Fb
1/2F
F
1/2F
3F
Fb
3/8F
3F
5/8Fb
3/8F

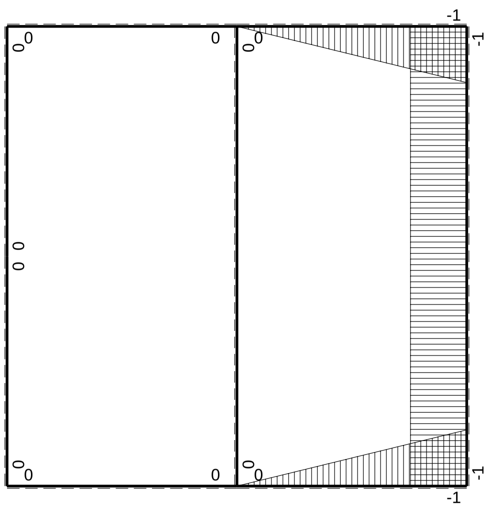

0
0
7/8
-1
-1
-3/8
-1
1
7/8
3
F
-3/8
1
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 280001 CI08.xxxx.012

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	$-x/b$	$-Fb$	$-Fb/EJ$	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	Fb	Fb/EJ	$Fb-Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	$8/3Xb/EJ$
iperstatica $X=W_{GC}$							$-3/8Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

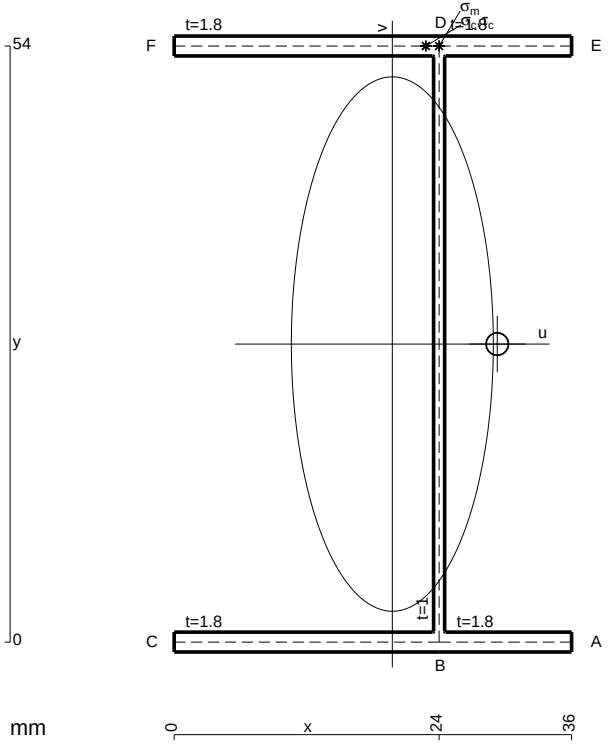
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

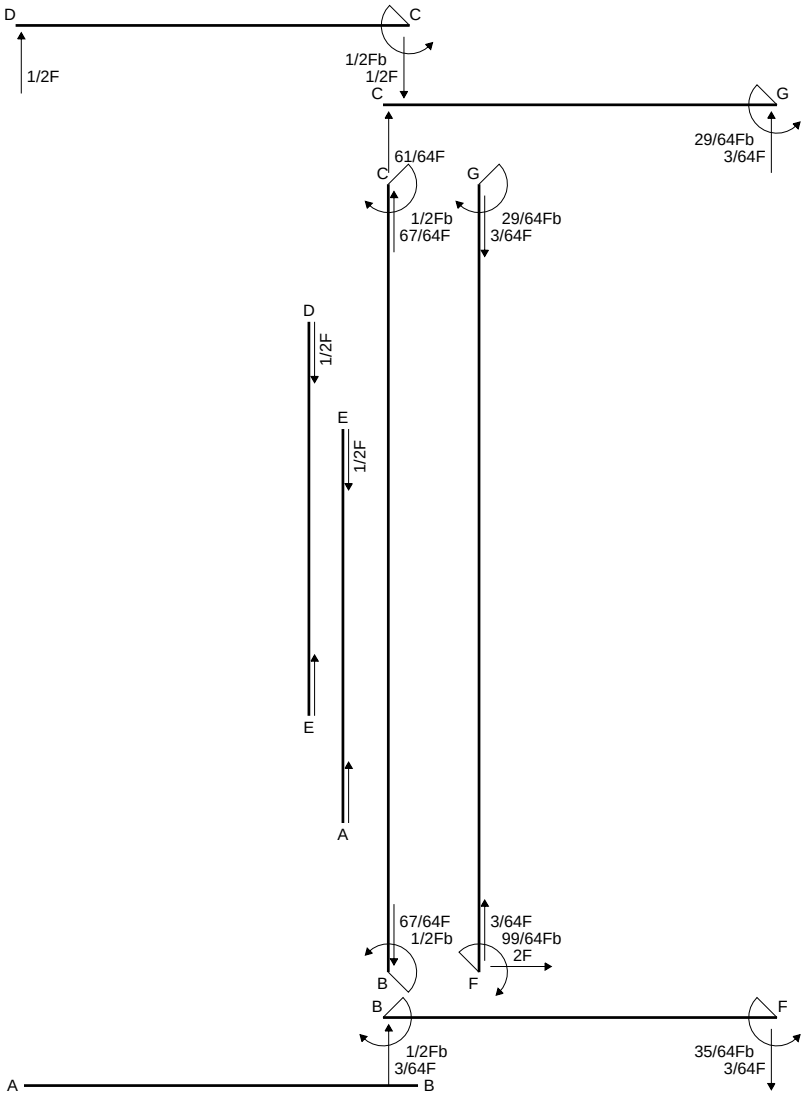
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

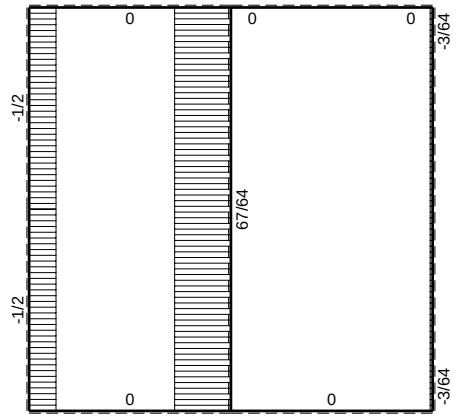


A = 183.6 mm^2
J_u = 107600. mm^4
J_v = 15369. mm^4
J_t = 158. mm^4
x_o = 9.504 mm
x_g = 19.76 mm
N = 1444. N
T_y = -3300. N
M_x = -841500. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 4.235 mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 219. N/mm^2
x_c = 24. mm
y_c = 54. mm
u_c = 4.235 mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 219. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 377.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 19.87 N/mm^2
$\tau_o = Tx_o t/J_t$ = 357.4 N/mm^2
t_c = 2970. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 689.1 N/mm^2

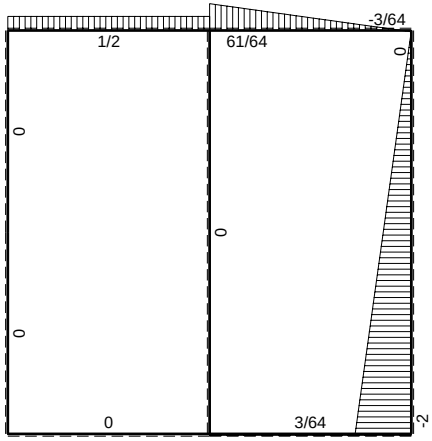
CI08.xxxx.012

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

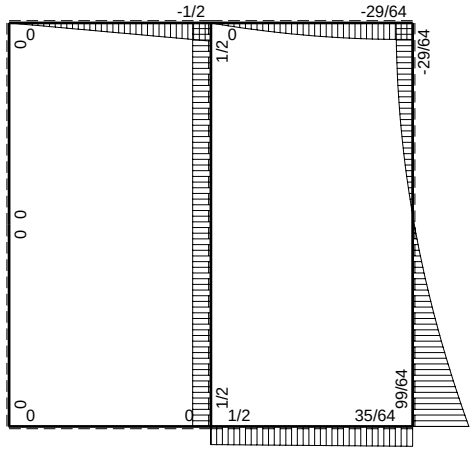

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64


F


-3/64
1/2
61/64
0
0
0
0
0
0
3/64
-2


F


-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
0
1/2
35/64
99/64


Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-29/24Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						29/64Fb	

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

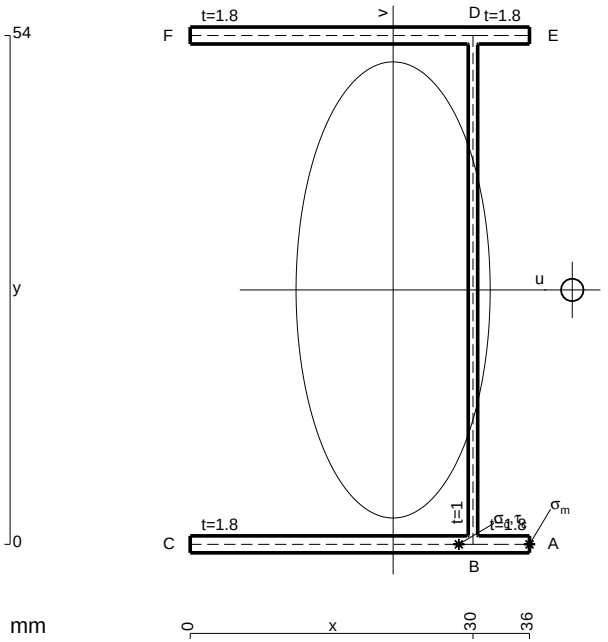
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

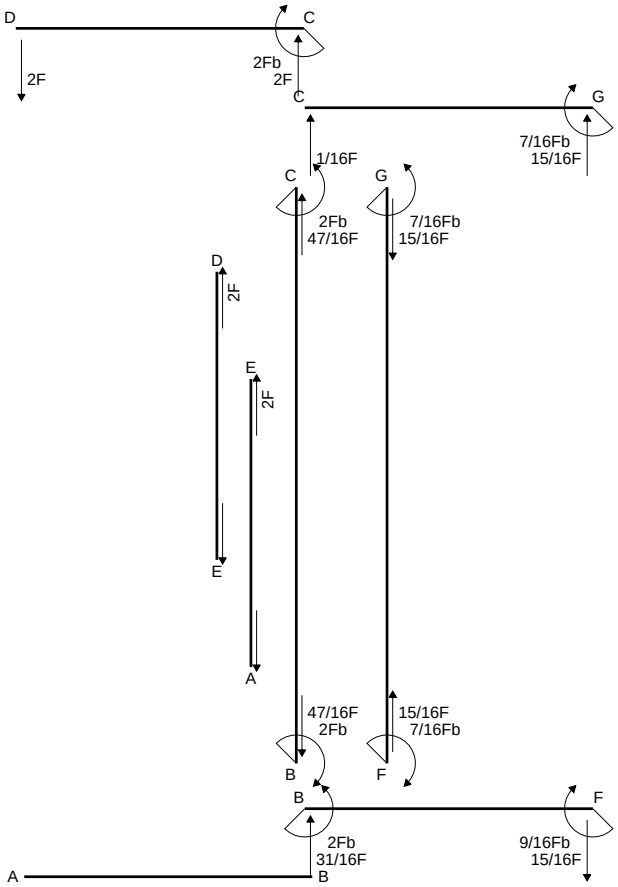
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

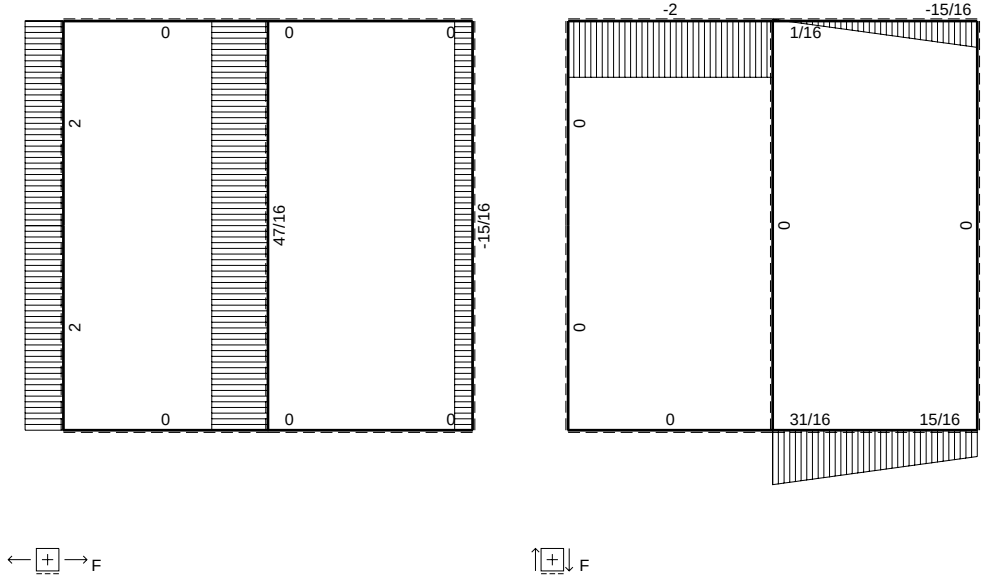


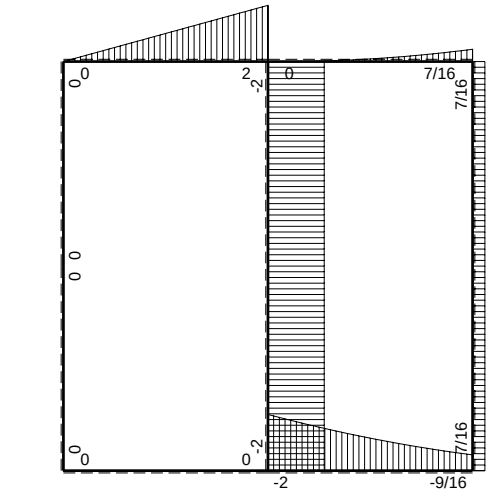
A = 183.6 mm^2
J_u = 107600. mm^4
J_v = 19486. mm^4
J_t = 158. mm^4
x_o = 19.01 mm
x_g = 21.53 mm
T_y = 1665. N
M_x = -915750. Nmm
x_m = 36. mm
u_m = 14.47 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -229.8 N/mm^2
x_c = 30. mm
u_c = 8.471 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -229.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 373.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.53 N/mm^2
$\tau_o = Tx_o t/J_t$ = 360.6 N/mm^2
t_c = 5994. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 685.9 N/mm^2

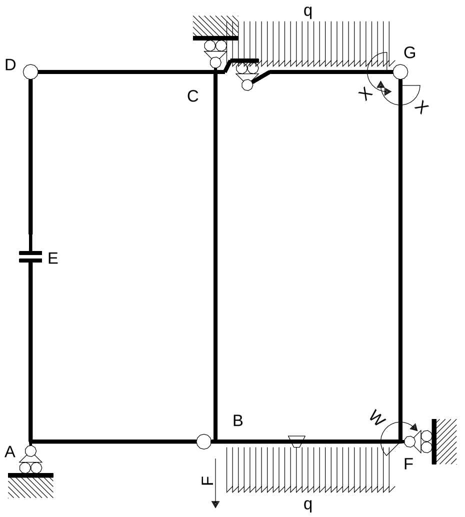
CI08.xxxx.013

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

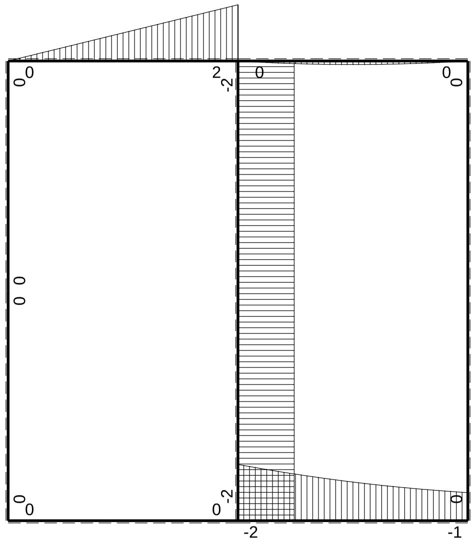

D
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0
-2
-15/16
1/16
0
0
0
0
0
31/16
15/16
F
F

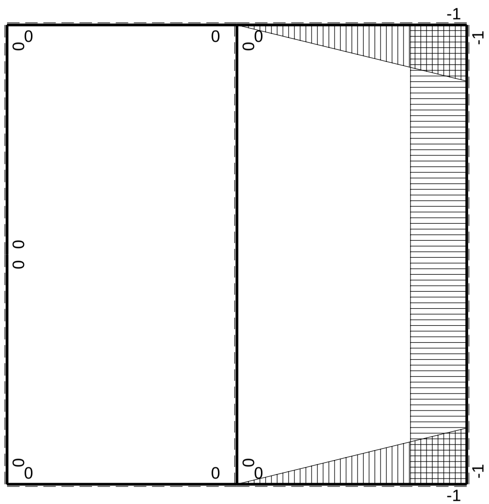

0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
0
7/16
-2
-9/16
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240995 CI08.xxxx.014

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
totali							$7/6Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

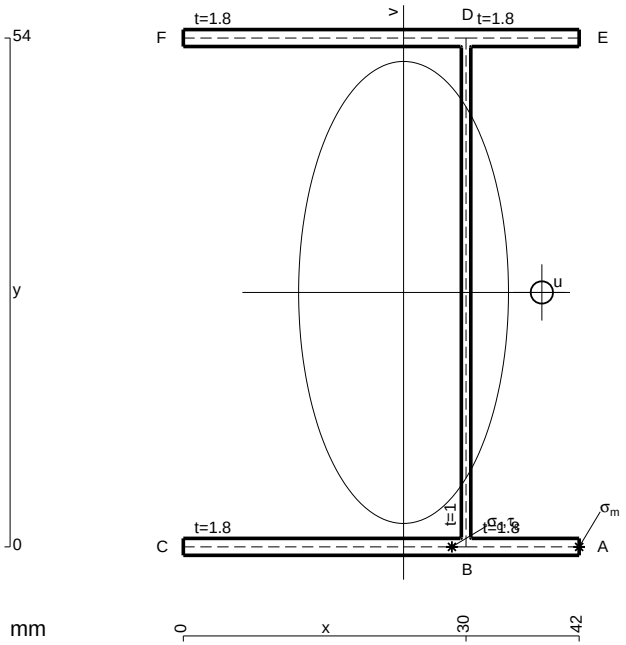
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

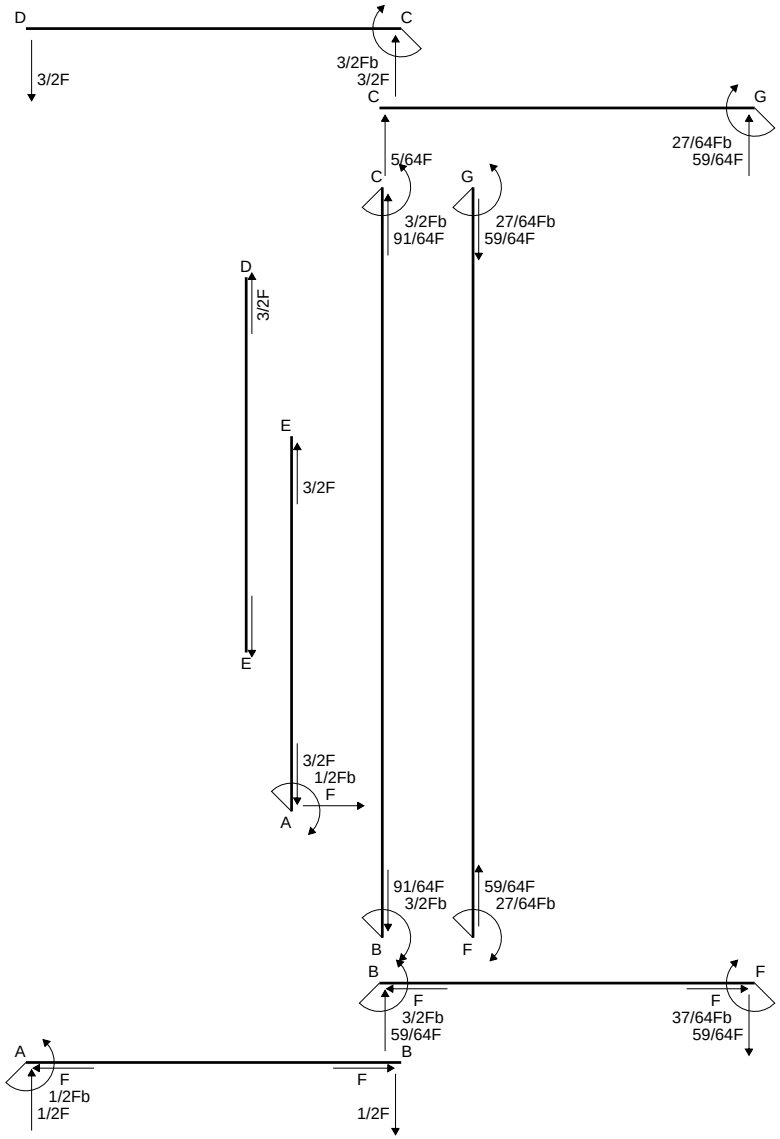
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

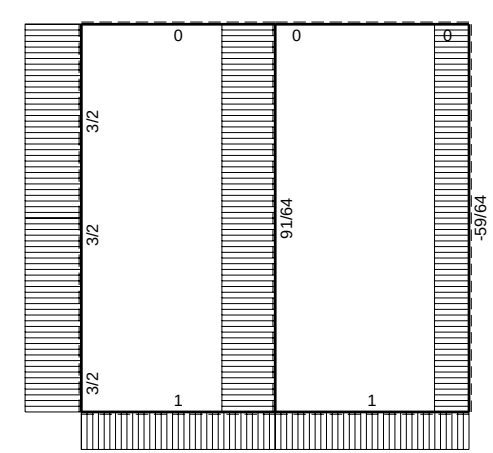


A = 205.2 mm^2
J_u = 123347. mm^4
J_v = 25449. mm^4
J_t = 181.3 mm^4
x_o = 14.67 mm
x_g = 23.37 mm
T_y = -1820. N
M_x = 1092000. Nmm
x_m = 42. mm
u_m = 18.63 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = 239. N/mm^2
x_c = 30. mm
u_c = 6.632 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = 239. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 277.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.95 N/mm^2
$\tau_o = Tx_o t/J_t$ = 265.2 N/mm^2
t_c = 1638. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 536.2 N/mm^2

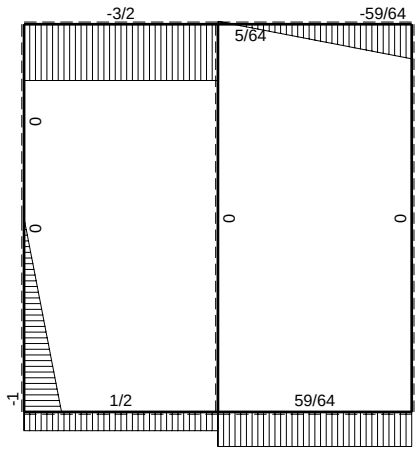
CI08.xxxx.014

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

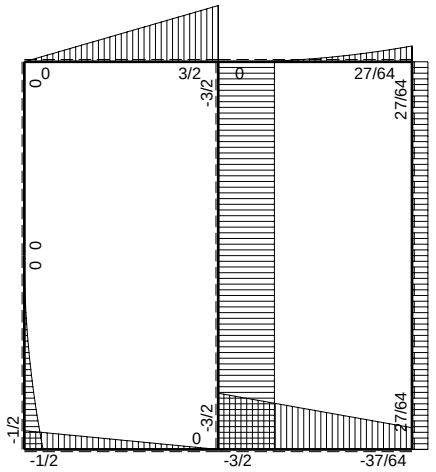

D
C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
3/2
91/64
-59/64
1
1

F


-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F


0
0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-1/2
-3/2
-37/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$9/8Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

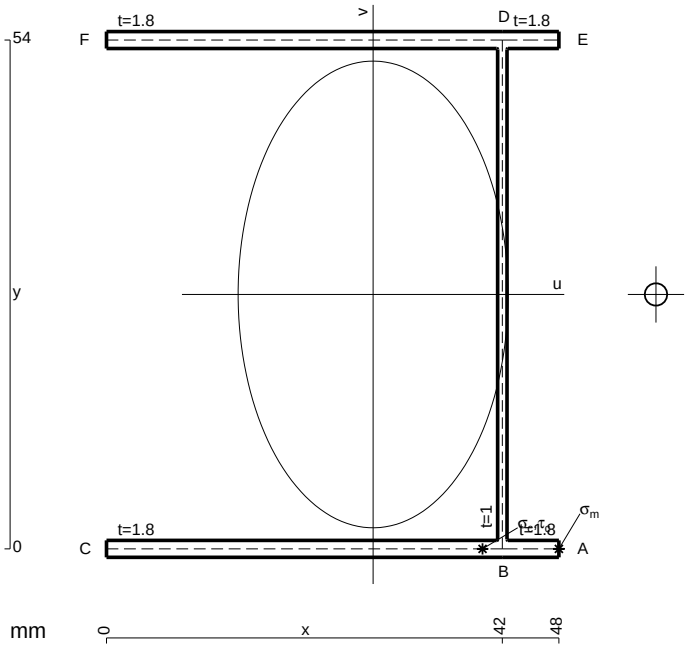
$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 984264 CI08.xxxx.015

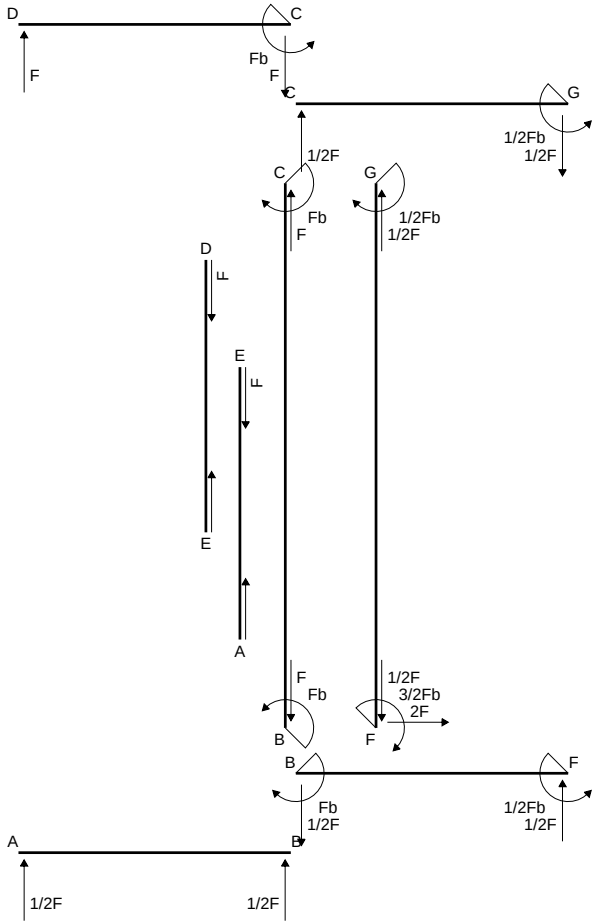


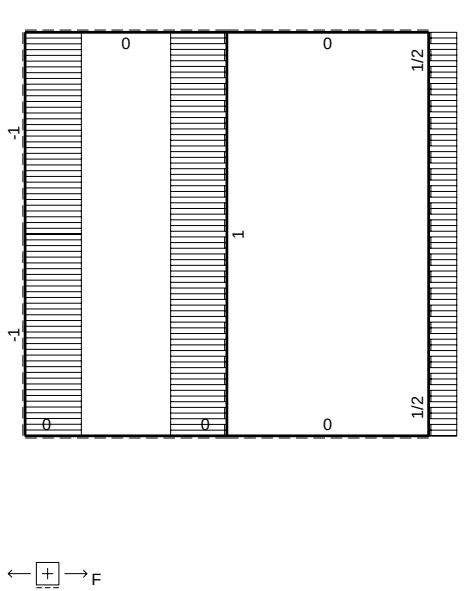
A = 226.8 mm^2
J_u = 139093. mm^4
J_v = 46508. mm^4
J_t = 204.6 mm^4
x_o = 30.02 mm
x_g = 28.29 mm
T_y = -1605. N
M_x = 1027200. Nmm
x_m = 48. mm
u_m = 19.71 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = 199.4 N/mm^2
x_c = 42. mm
u_c = 13.71 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = 199.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 436.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.09 N/mm^2
$\tau_o = Tx_ot/J_t$ = 423.8 N/mm^2
t_c = 1926. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 782.5 N/mm^2

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

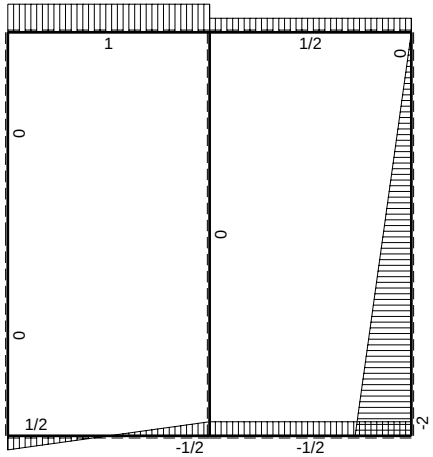
CI08.xxxx.015

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

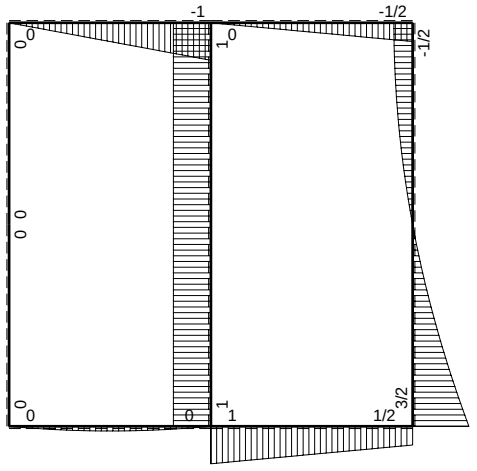

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

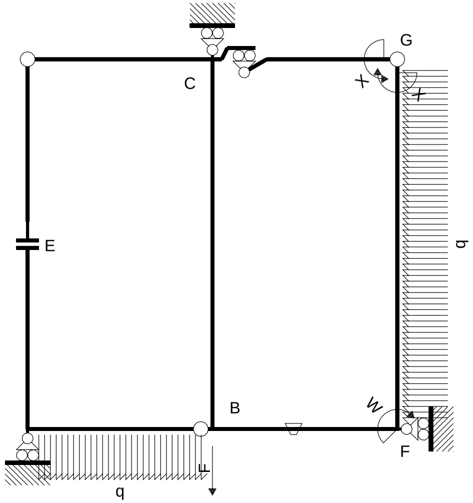
F

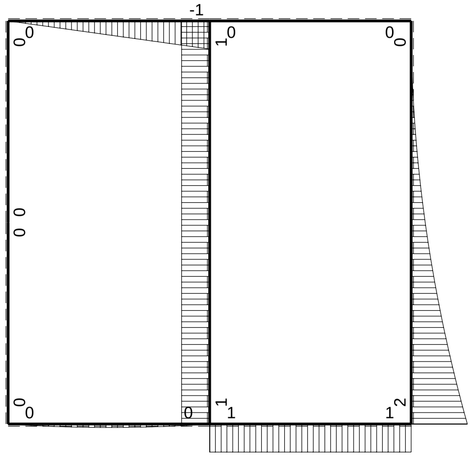

1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2

F

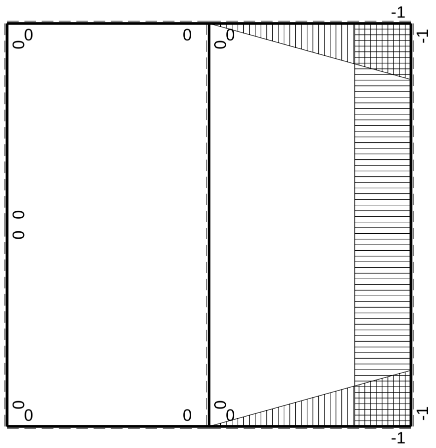

-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

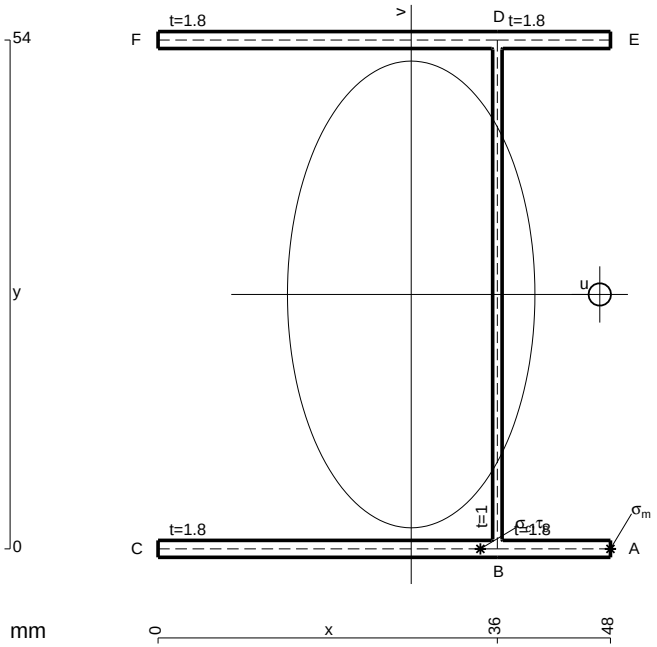
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

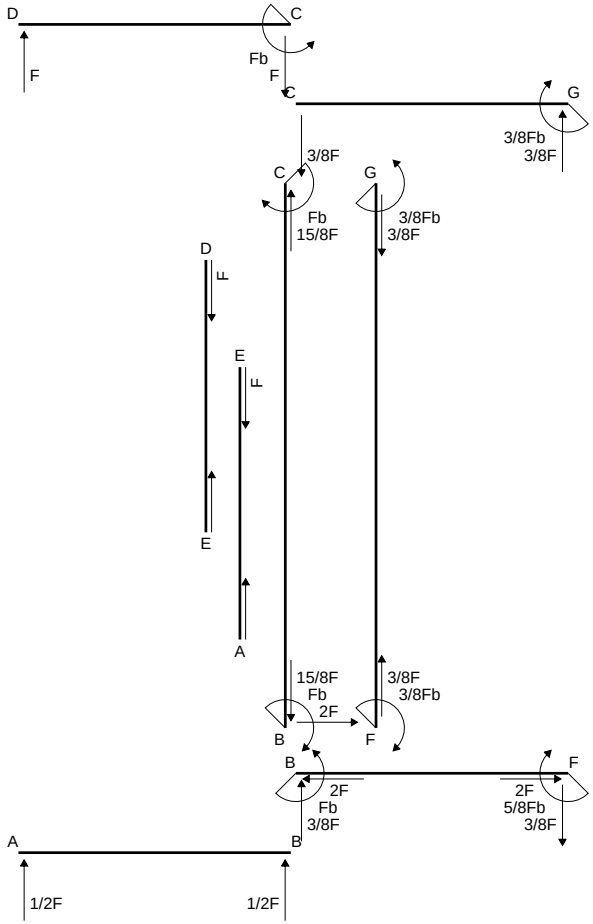
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

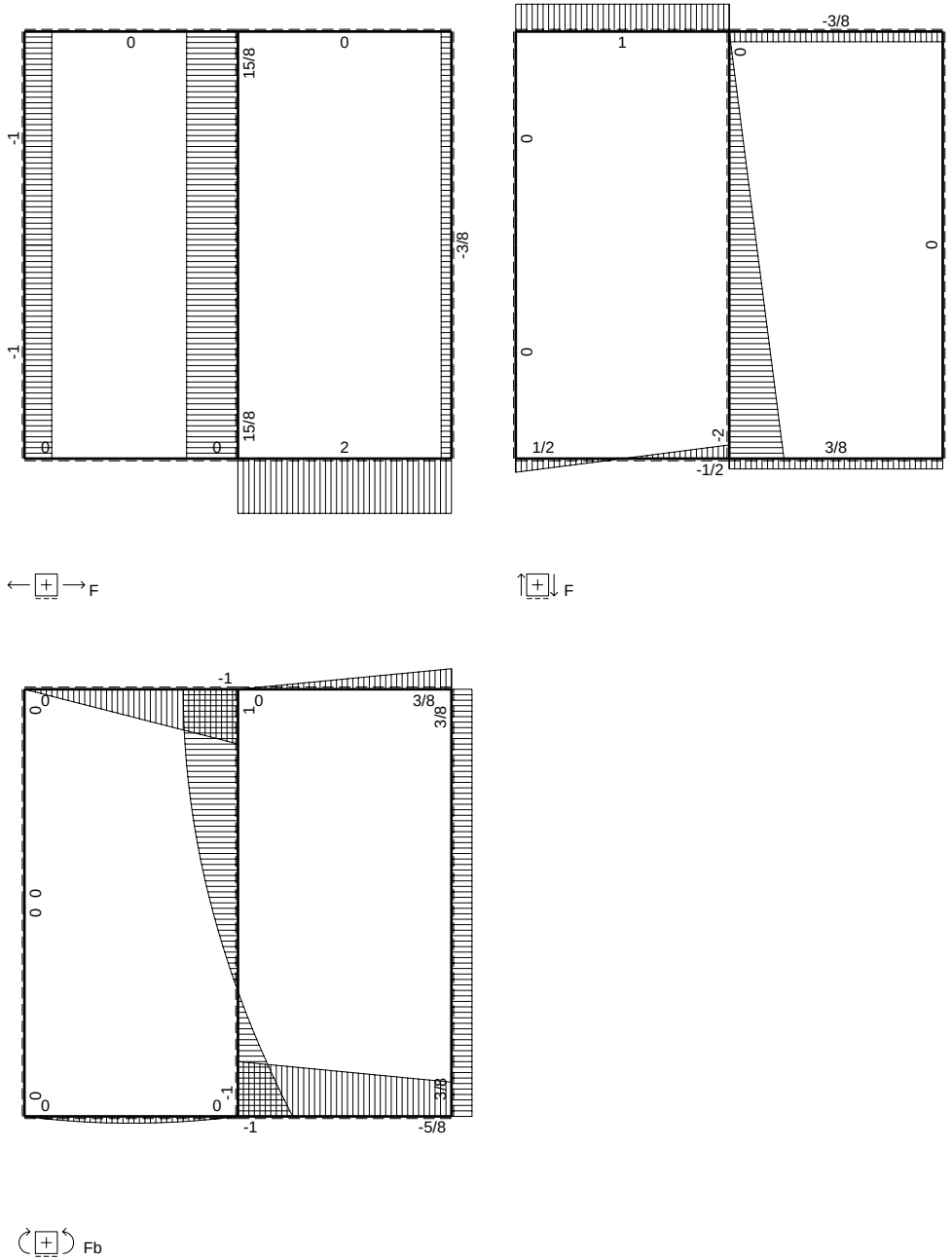


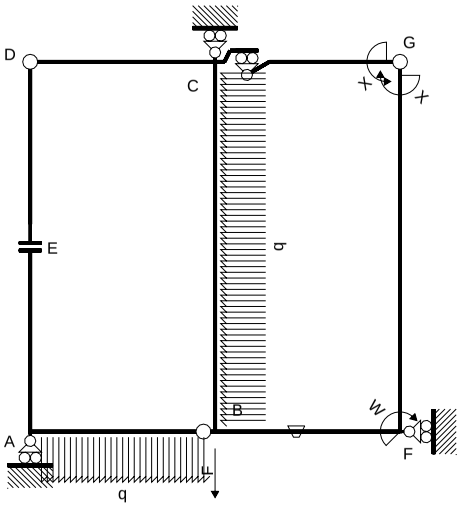
A = 226.8 mm^2
J_u = 139093. mm^4
J_v = 39102. mm^4
J_t = 204.6 mm^4
x_o = 20.01 mm
x_g = 26.86 mm
T_y = 1590. N
M_x = -1081200. Nmm
x_m = 48. mm
u_m = 21.14 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -209.9 N/mm^2
x_c = 36. mm
u_c = 9.143 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -209.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 291. N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.11 N/mm^2
$\tau_o = Tx_o t/J_t$ = 279.9 N/mm^2
t_c = 2862. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 546. N/mm^2

CI08.xxxx.016

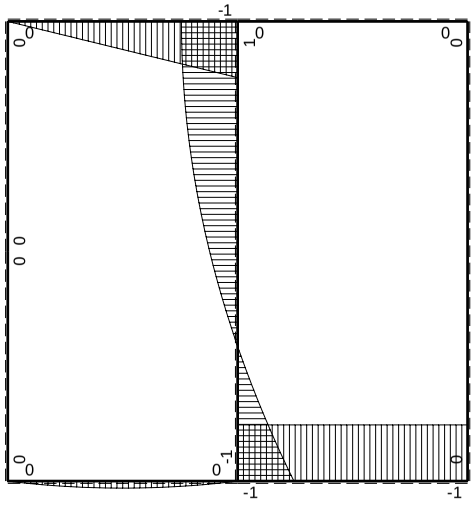
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F

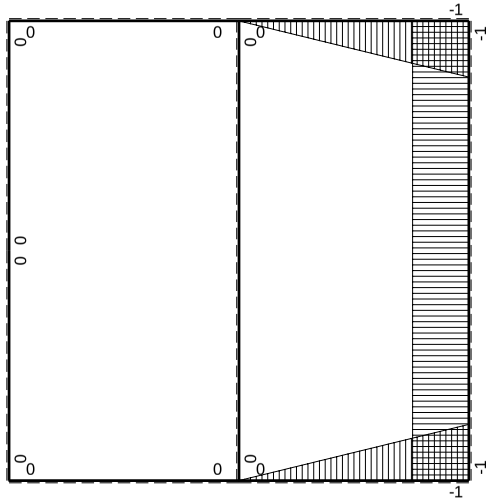

0
0
15/8
-1
-3/8
-1
15/8
0
0
2
F
1
-3/8
0
0
0
0
1/2
-2
3/8
-1/2
F
-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx²	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 240156 CI08.xxxx.017

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

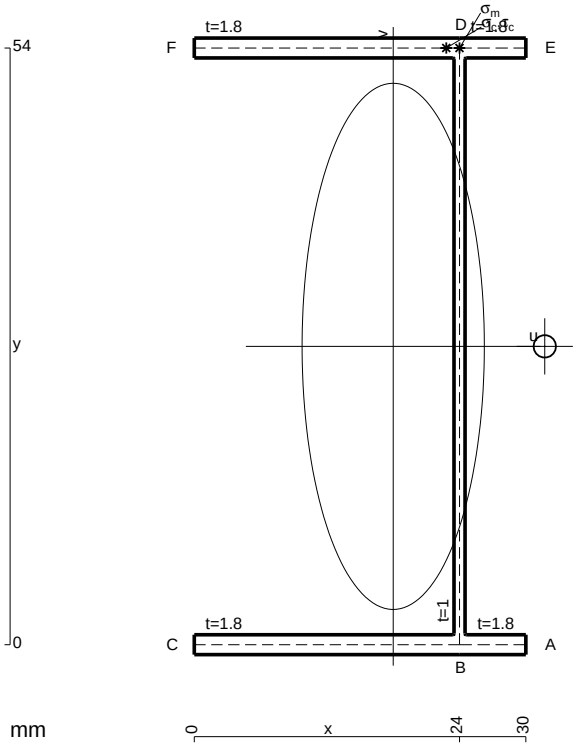
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

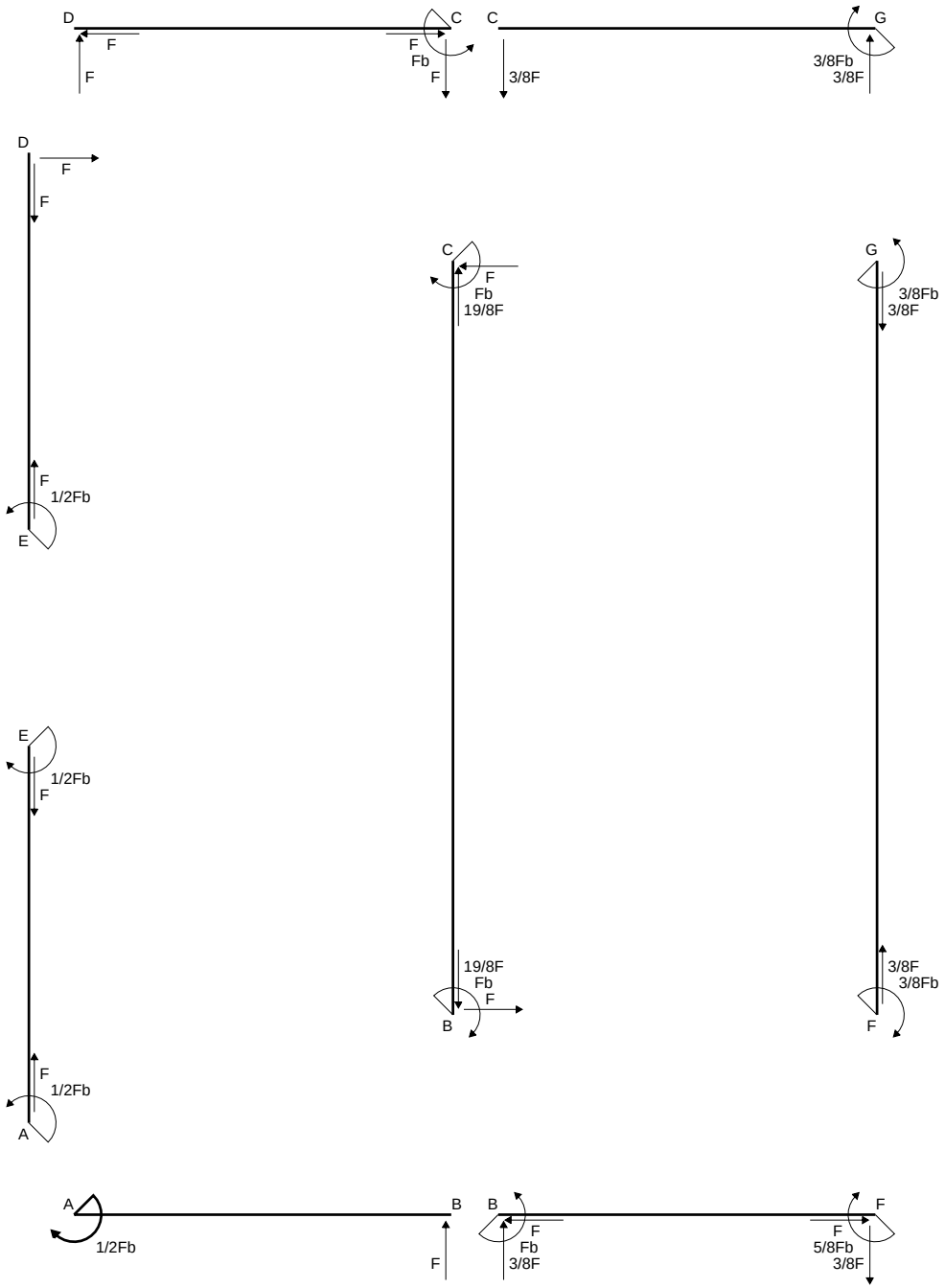
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

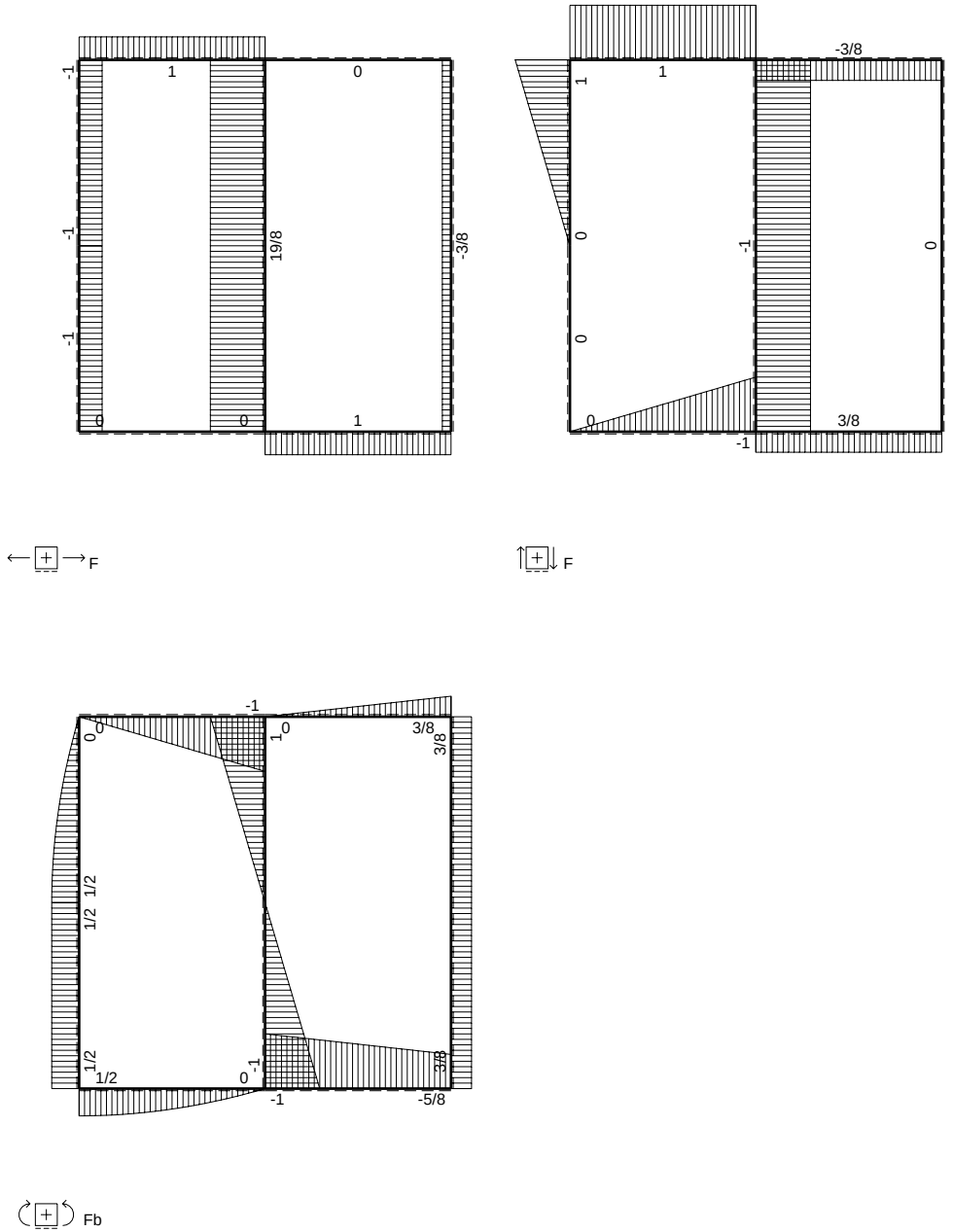


A = 162. mm^2
J_u = 91854. mm^4
J_v = 11016. mm^4
J_t = 134.6 mm^4
x_o = 13.71 mm
x_g = 18. mm
N = 1819. N
T_y = -1940. N
M_x = -708100. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 219.4 N/mm^2
x_c = 24. mm
y_c = 54. mm
u_c = 6. mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 219.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 369.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.69 N/mm^2
$\tau_o = Tx_o t/J_t$ = 355.7 N/mm^2
t_c = 1746. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 676.3 N/mm^2

CI08.xxxx.017

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
19/8F
G
3/8Fb
3/8F
E
1/2Fb
F
19/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F


-1
1
0
-1
19/8
-3/8
-1
0
0
1
F
-3/8
1
1
1
0
-1
0
0
0
3/8
-1
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
1/2
0
-1
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb$	0	0	0	0	0+0	0
AE b	0	$-1/2Fb$	0	0	0	0		
BF b	$-x/b$	$-Fb$	$-Fb/EJ$	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	Fb	Fb/EJ	$Fb-Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

PROCEDIMENTO E RISULTATI 236530 CI08.xxxx.018

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

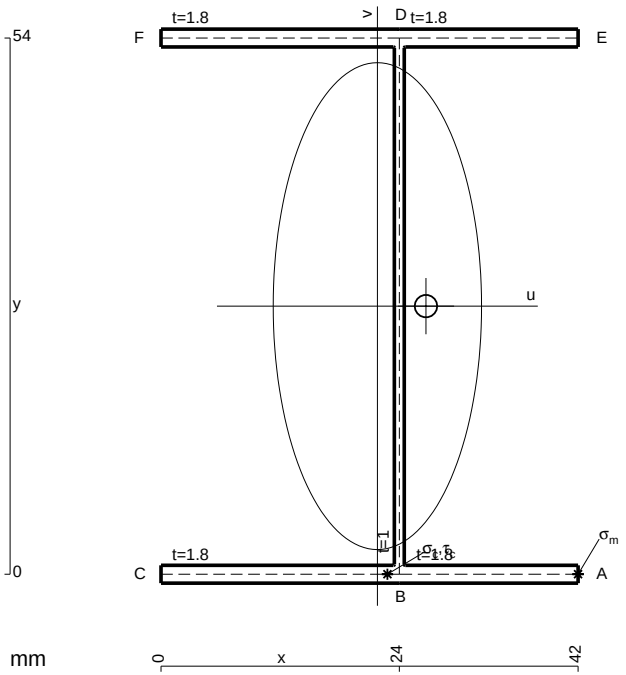
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

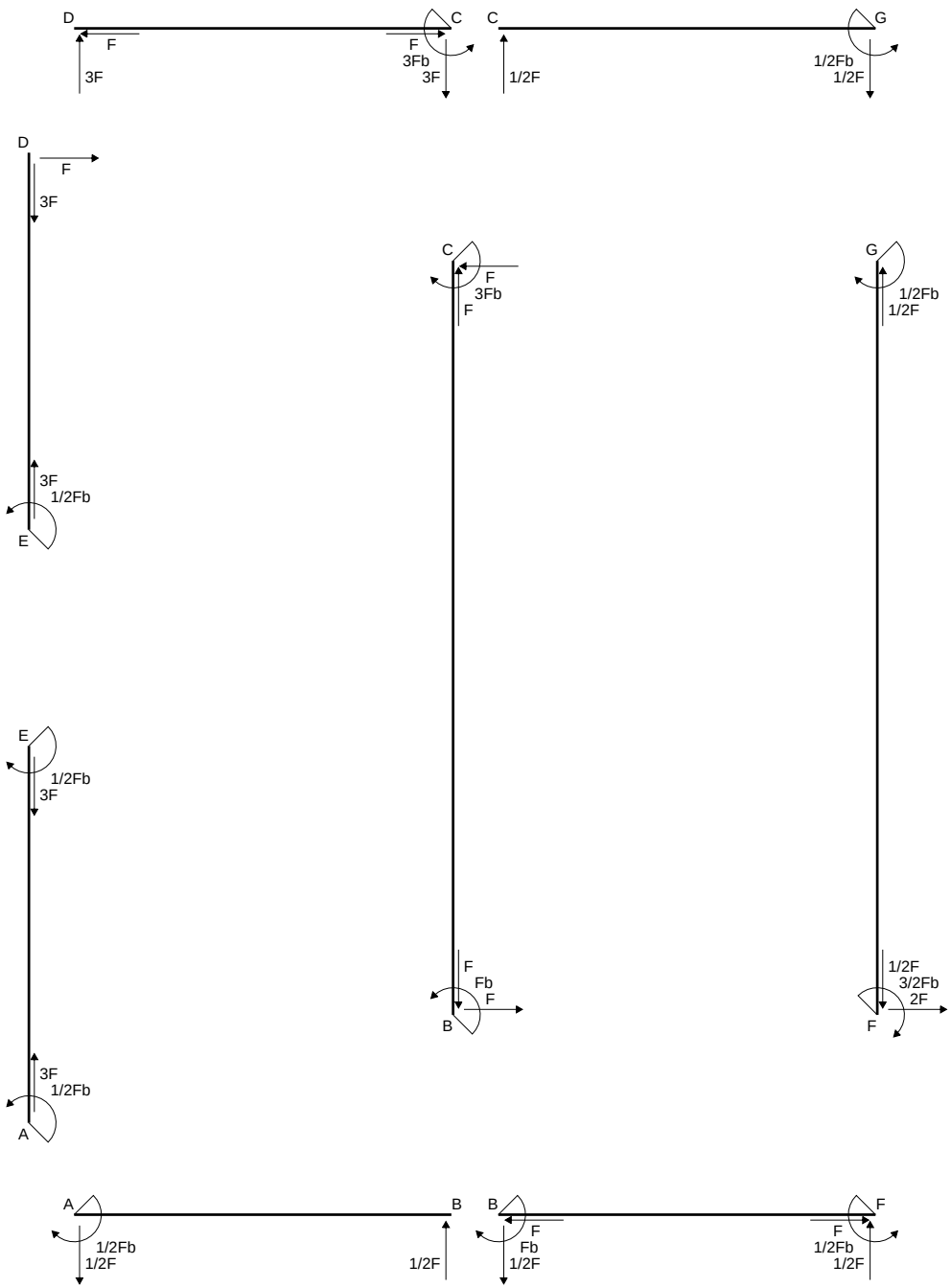
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

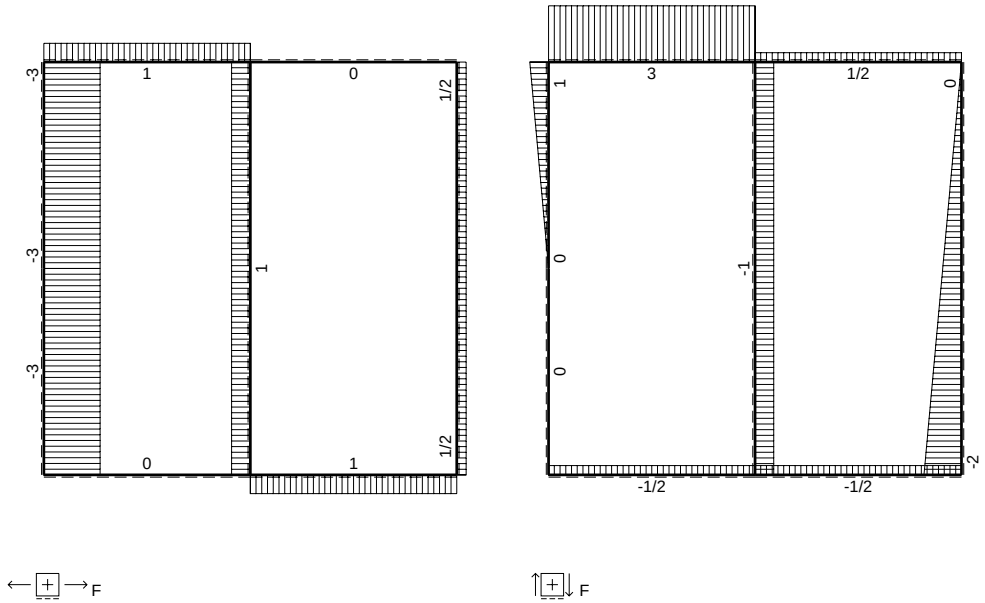


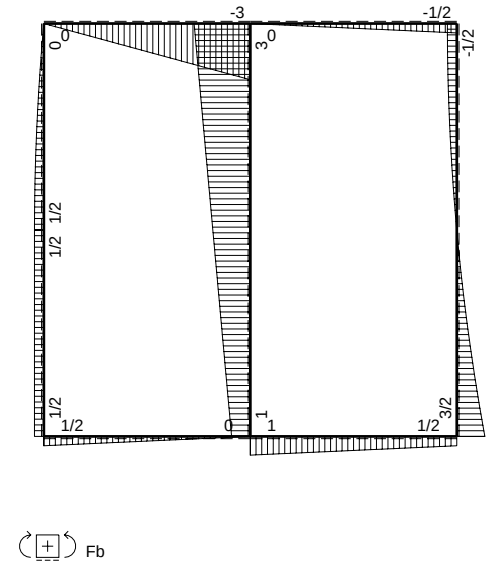
A = 205.2 mm^2
J_u = 123347. mm^4
J_v = 22585. mm^4
J_t = 181.3 mm^4
x_o = 4.891 mm
x_g = 21.79 mm
N = 3016. N
T_y = -1270. N
M_x = 977900. Nmm
x_m = 42. mm
u_m = 20.21 mm
v_m = -27. mm
$\sigma_m = N/A - Mv/J_u$ = 228.8 N/mm^2
x_c = 24. mm
u_c = 2.211 mm
v_c = -27. mm
$\sigma_c = N/A - Mv/J_u$ = 228.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 68.35 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.672 N/mm^2
$\tau_o = Tx_o t/J_t$ = 61.68 N/mm^2
t_c = 2286. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 257.6 N/mm^2

CI08.xxxx.018

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
F
Fb
F
B
1/2F
3/2Fb
2F
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


-3
1
0
1/2
-3
1
-3
0
1
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^{2}b^2	(-1/2+1/2)Fb^2/EJ	1/3XbEJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2XbEJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3FbEJ	8/3XbEJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 984261 CI08.xxxx.019

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (- x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [- x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

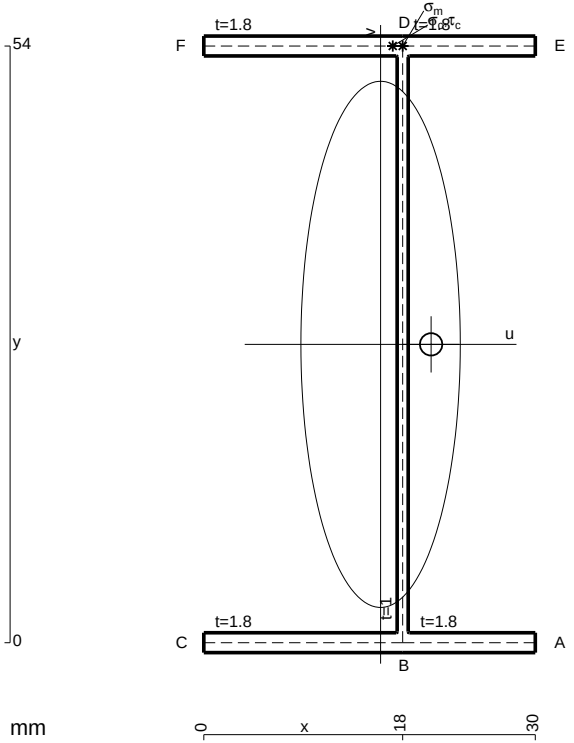
$= (- b +1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

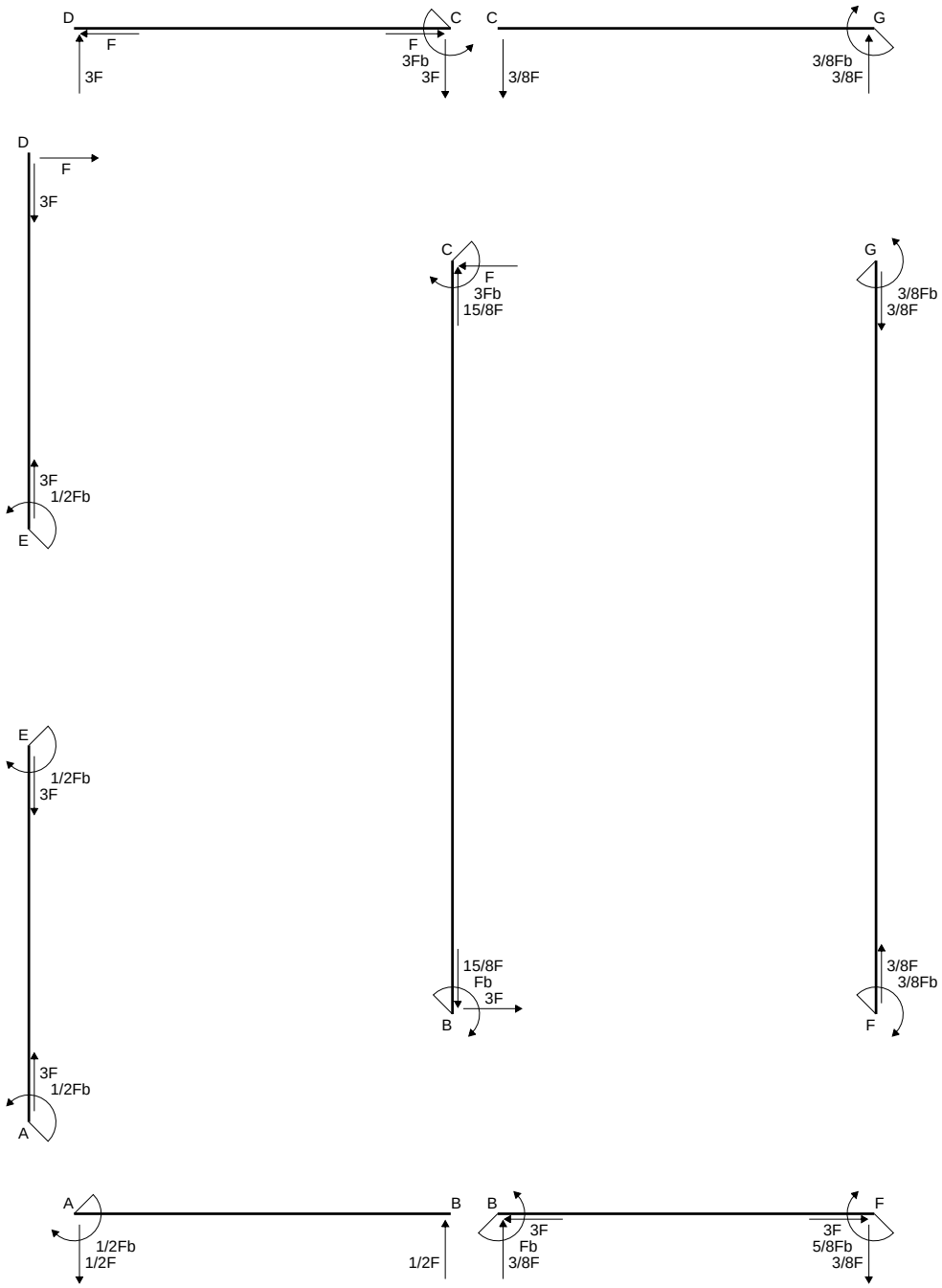
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

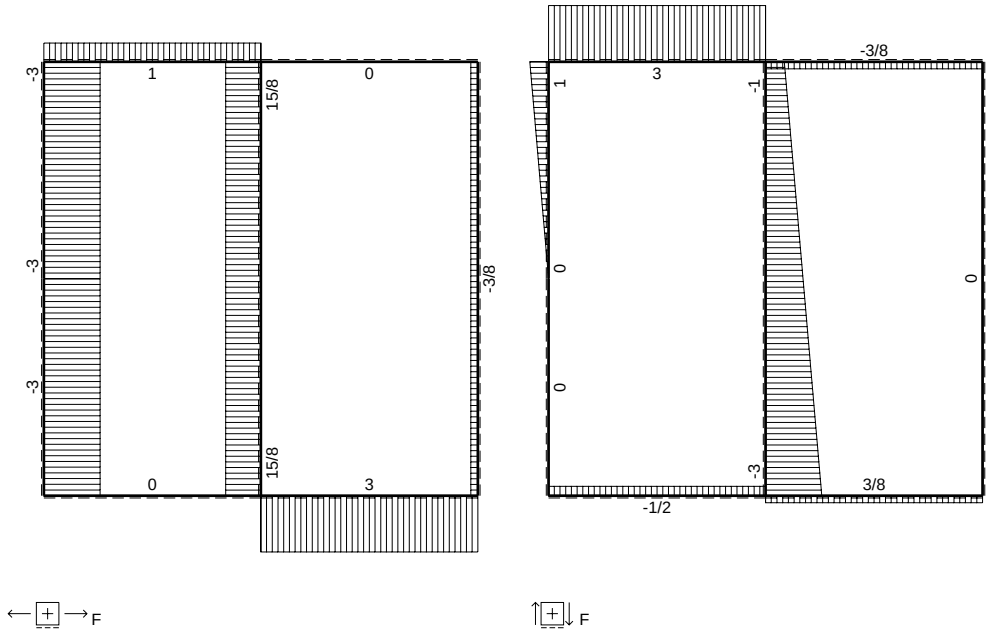


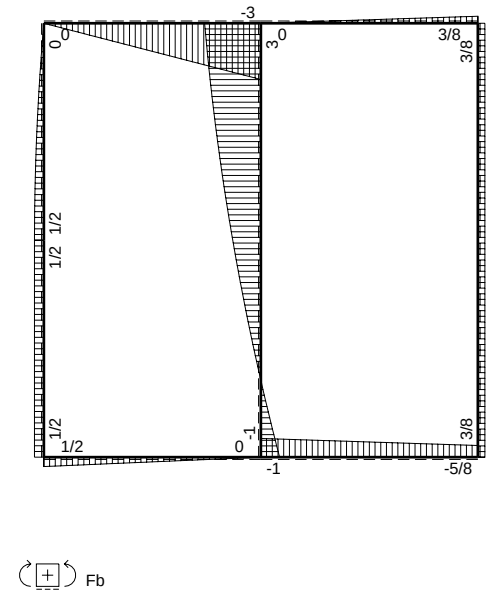
A = 162. mm^2
J_u = 91854. mm^4
J_v = 8424. mm^4
J_t = 134.6 mm^4
x_o = 4.571 mm
x_g = 16. mm
N = 330. N
T_y = 990. N
M_x = -801900. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 2. mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 237.8 N/mm^2
x_c = 18. mm
y_c = 54. mm
u_c = 2. mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 237.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 65.74 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.238 N/mm^2
$\tau_o = Tx_o t/J_t$ = 60.5 N/mm^2
t_c = 594. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 263.6 N/mm^2

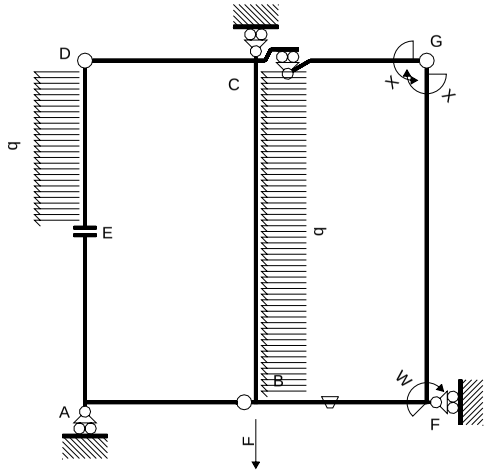
CI08.xxxx.019

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

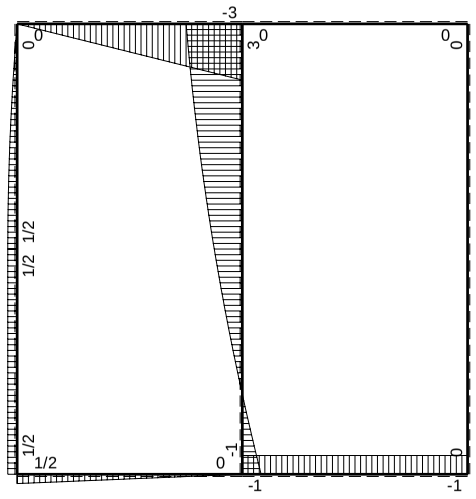

D
F
3F
C
F
3Fb
3F
C
3/8F
G
3/8Fb
3/8F
D
F
3F
3F
1/2Fb
E
C
F
3Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3F
15/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F


-3
1
0
15/8
-3
-3/8
-3
15/8
0
3
-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F
F

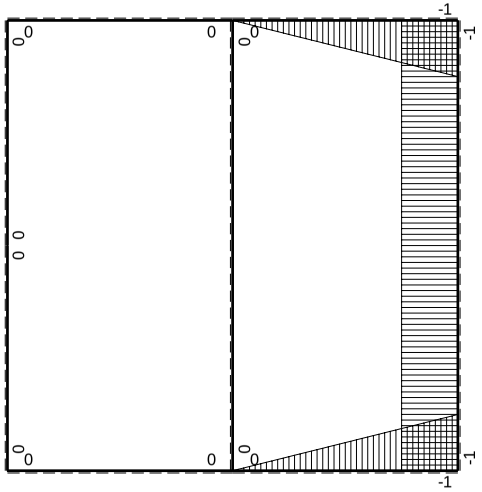

-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 248835 CI08.xxxx.020

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

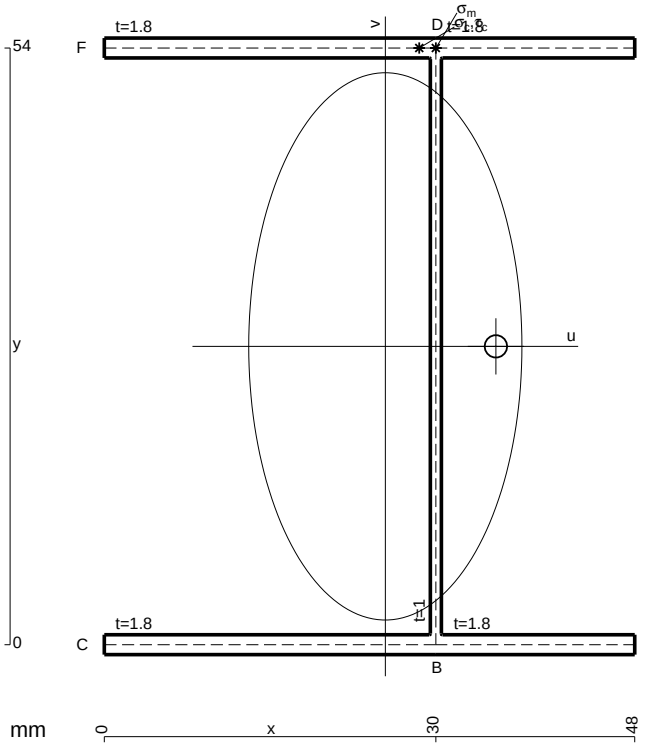
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

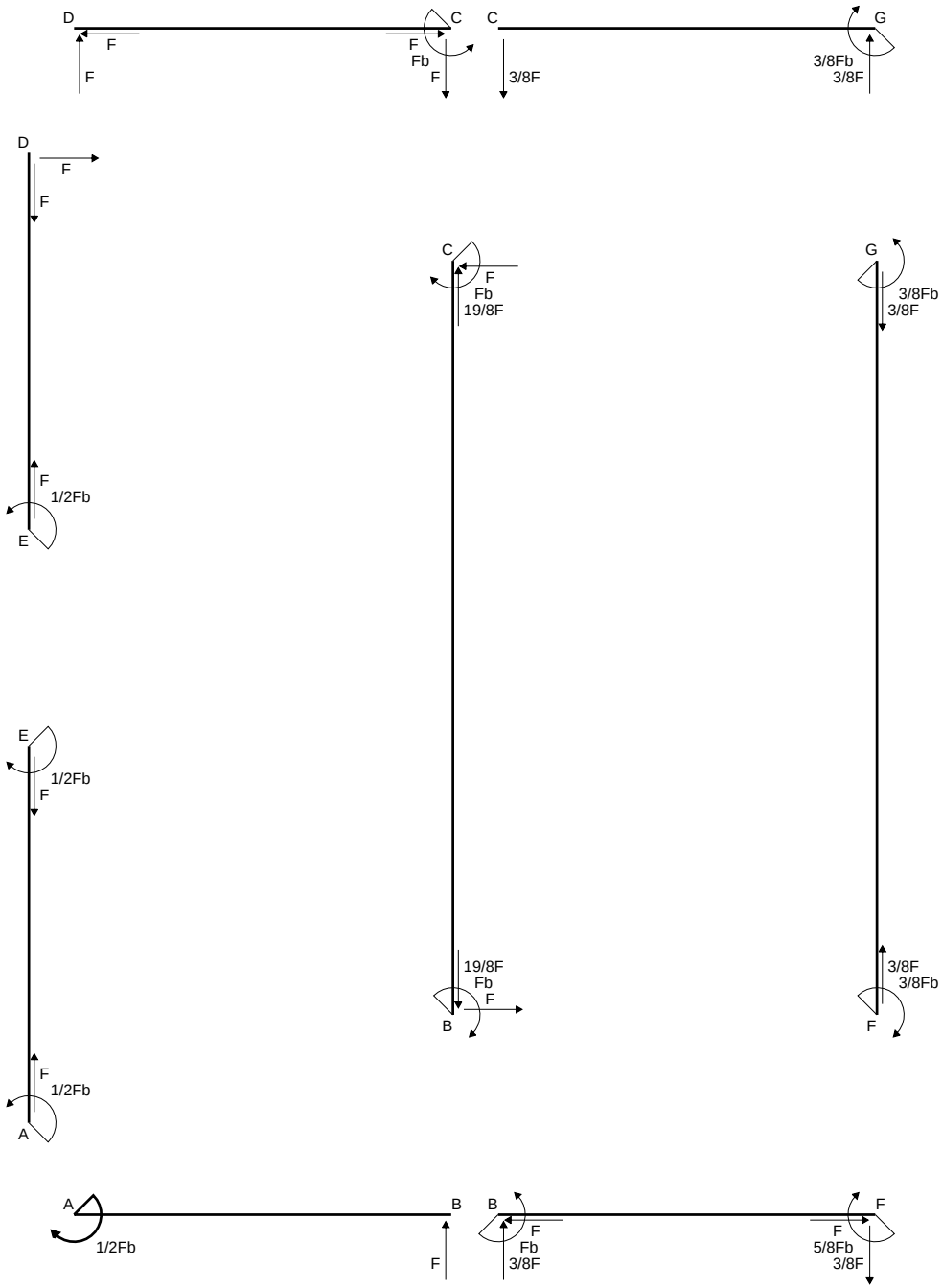
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

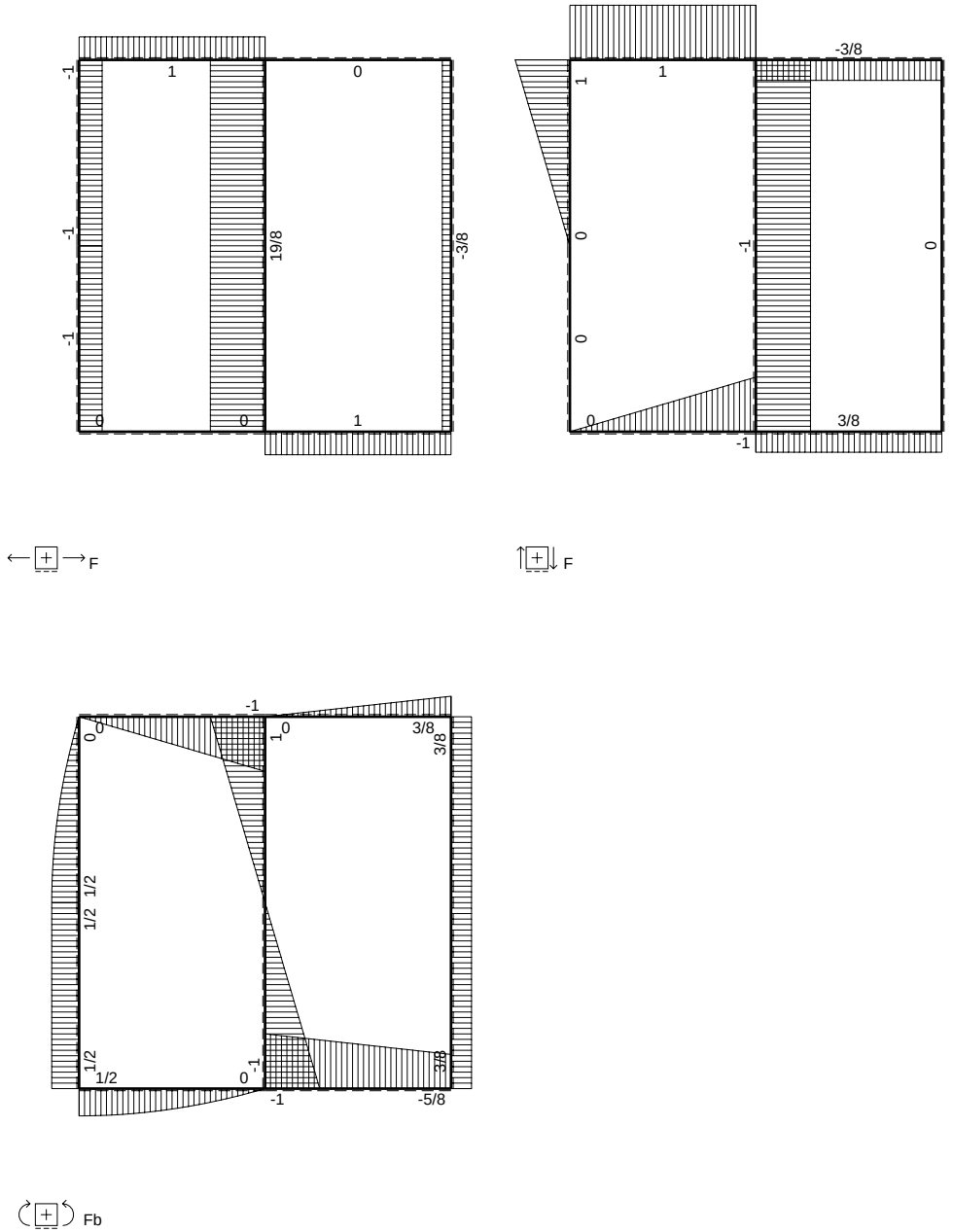


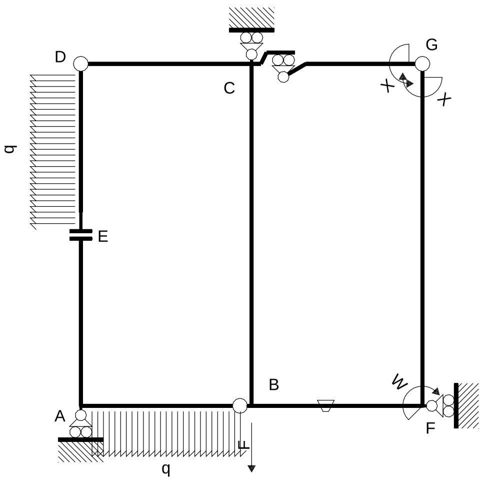
A = 226.8 mm^2
J_u = 139093. mm^4
J_v = 34659. mm^4
J_t = 204.6 mm^4
x_o = 10.01 mm
x_g = 25.43 mm
N = 780. N
T_y = 2340. N
M_x = -1006200. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 4.571 mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 198.8 N/mm^2
x_c = 30. mm
y_c = 54. mm
u_c = 4.571 mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 198.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 219.6 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.63 N/mm^2
$\tau_o = Tx_o t/J_t$ = 206. N/mm^2
t_c = 1404. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 429.1 N/mm^2

CI08.xxxx.020

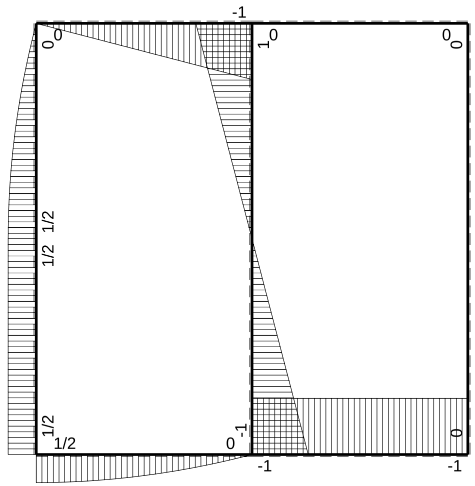
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
19/8F
G
3/8Fb
3/8F
E
1/2Fb
F
19/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F

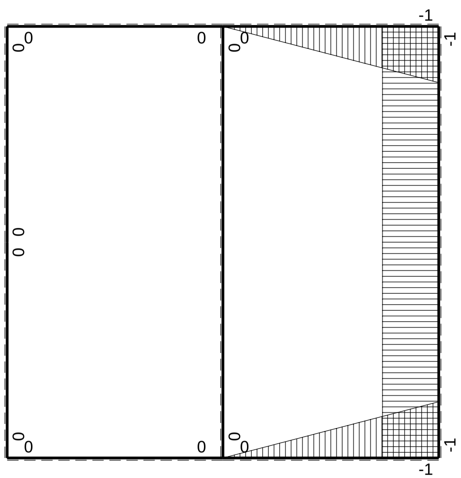

-1
1
0
-1
19/8
-3/8
-1
0
0
1
F
-3/8
1
1
0
-1
0
0
3/8
-1
F
-1
0
0
1
3/8
3/8
1/2
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238839 CI08.xxxx.021

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

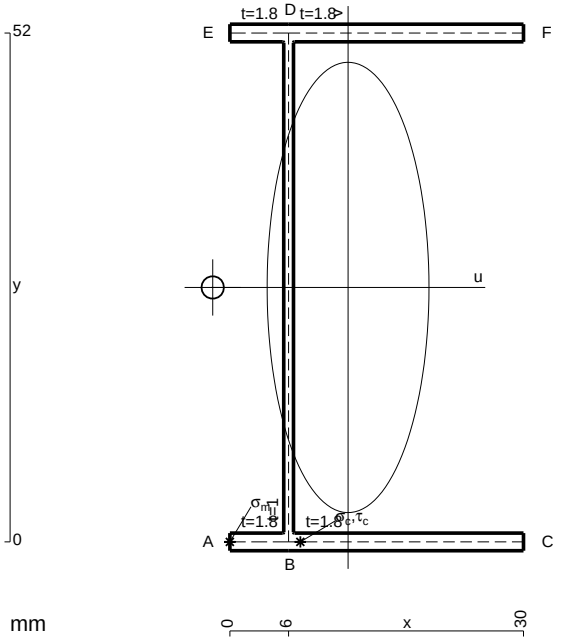
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

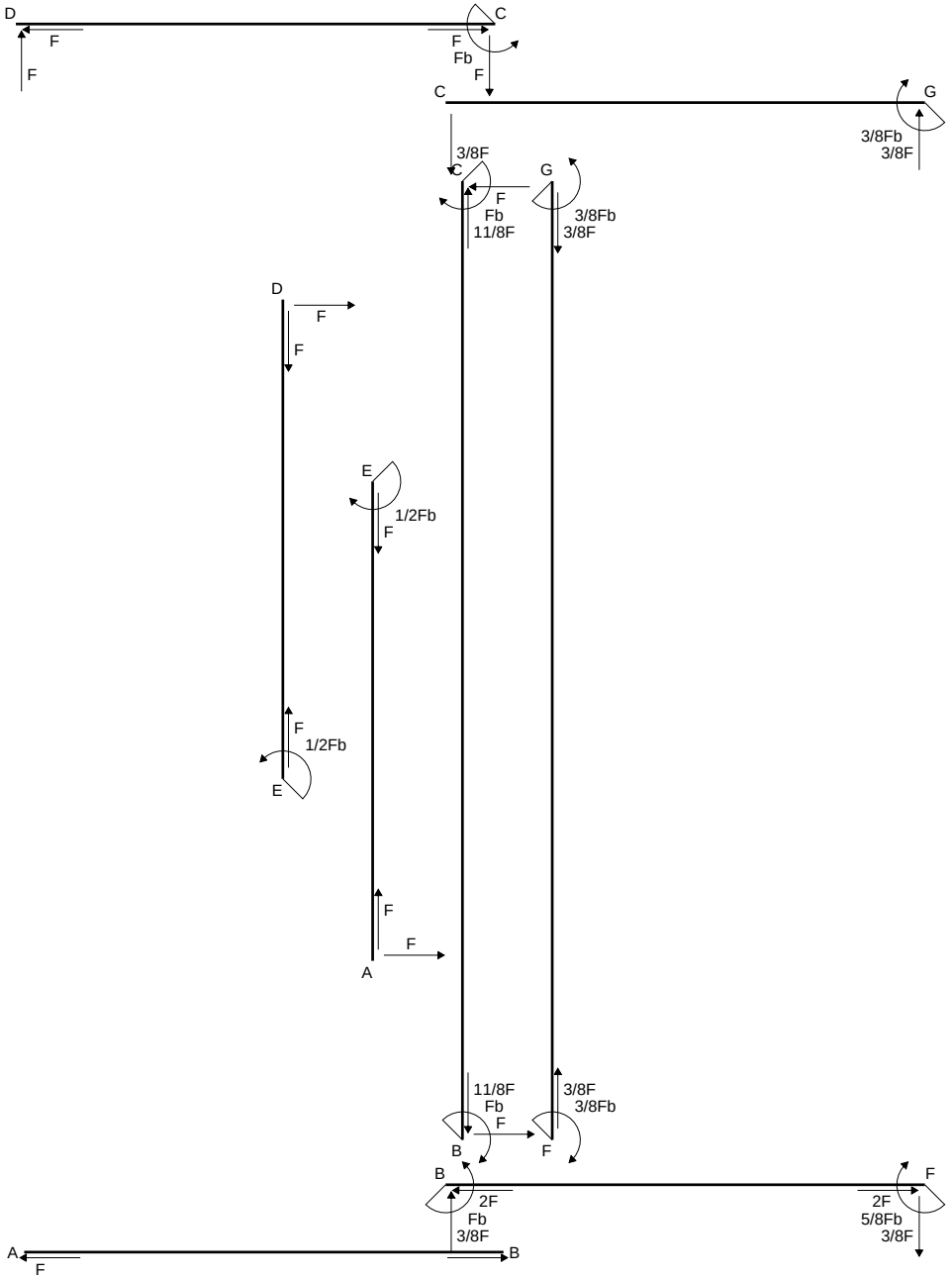
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

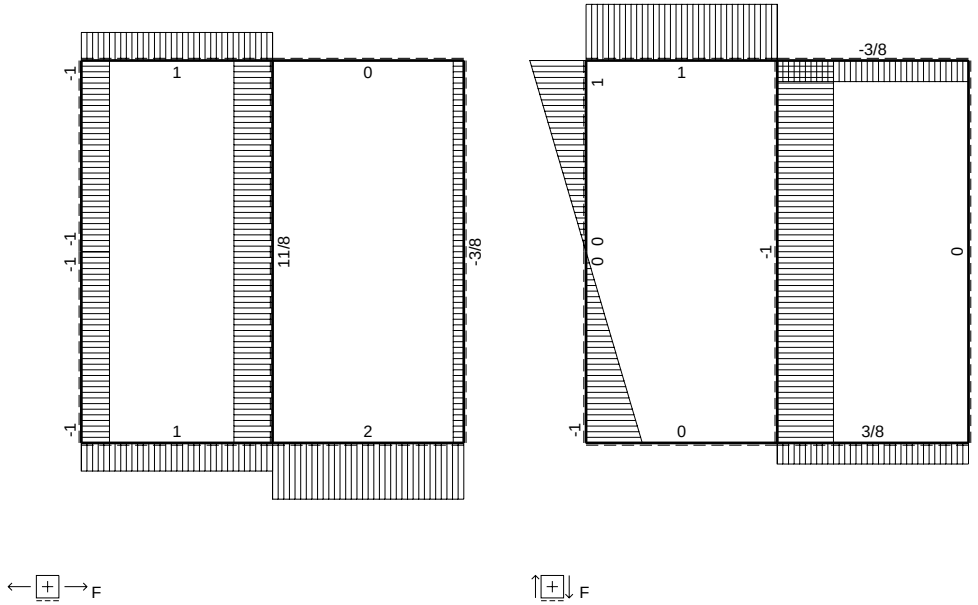


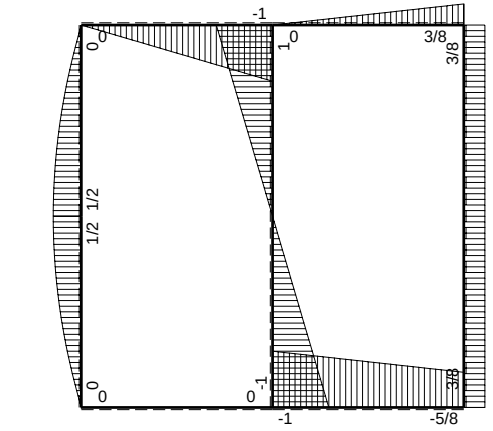
A = 160. mm^2
J_u = 84725. mm^4
J_v = 10943. mm^4
J_t = 134. mm^4
x_o = -13.83 mm
x_g = 12.07 mm
N = 3254. N
T_y = -1370. N
M_x = 616500. Nmm
u_m = -12.07 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = 209.5 N/mm^2
x_c = 6. mm
u_c = -6.075 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = 209.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 264.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.09 N/mm^2
$\tau_o = Tx_o t/J_t$ = 254.6 N/mm^2
t_c = 2466. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 504. N/mm^2

CI08.xxxx.021

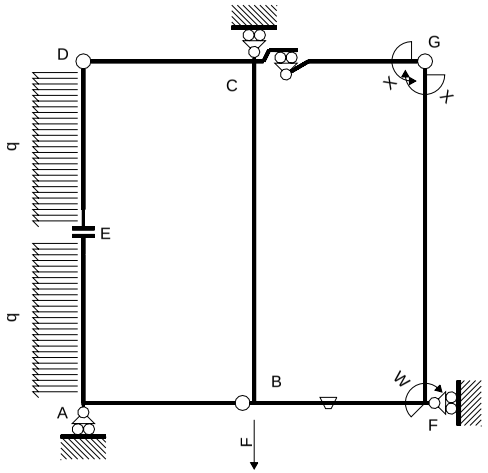
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
11/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
11/8F
Fb
F
3/8F
3/8Fb
F
B
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F

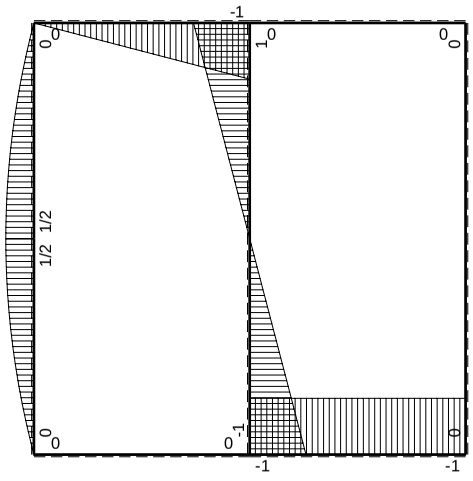

-1
1
0
-1
-1
11/8
-3/8
-1
1
2
F
-3/8
1
1
0 0
-1
0
-1
0
3/8
F


-1
0
0
1
0
3/8
3/8
1/2 1/2
0
0
0
-1
-1
3/8
-1
-5/8

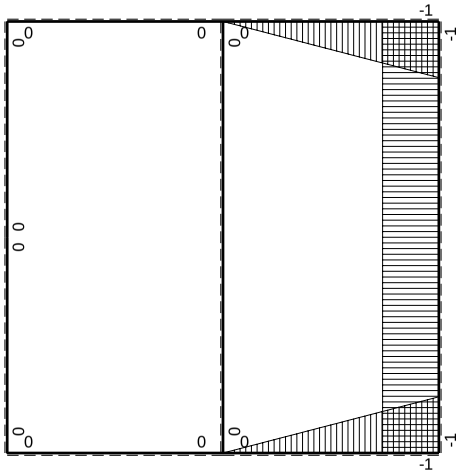

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 226747 CI08.xxxx.022

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

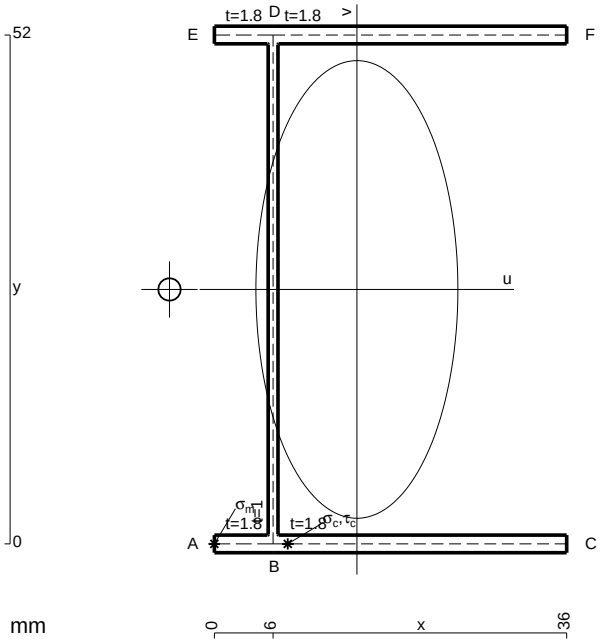
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

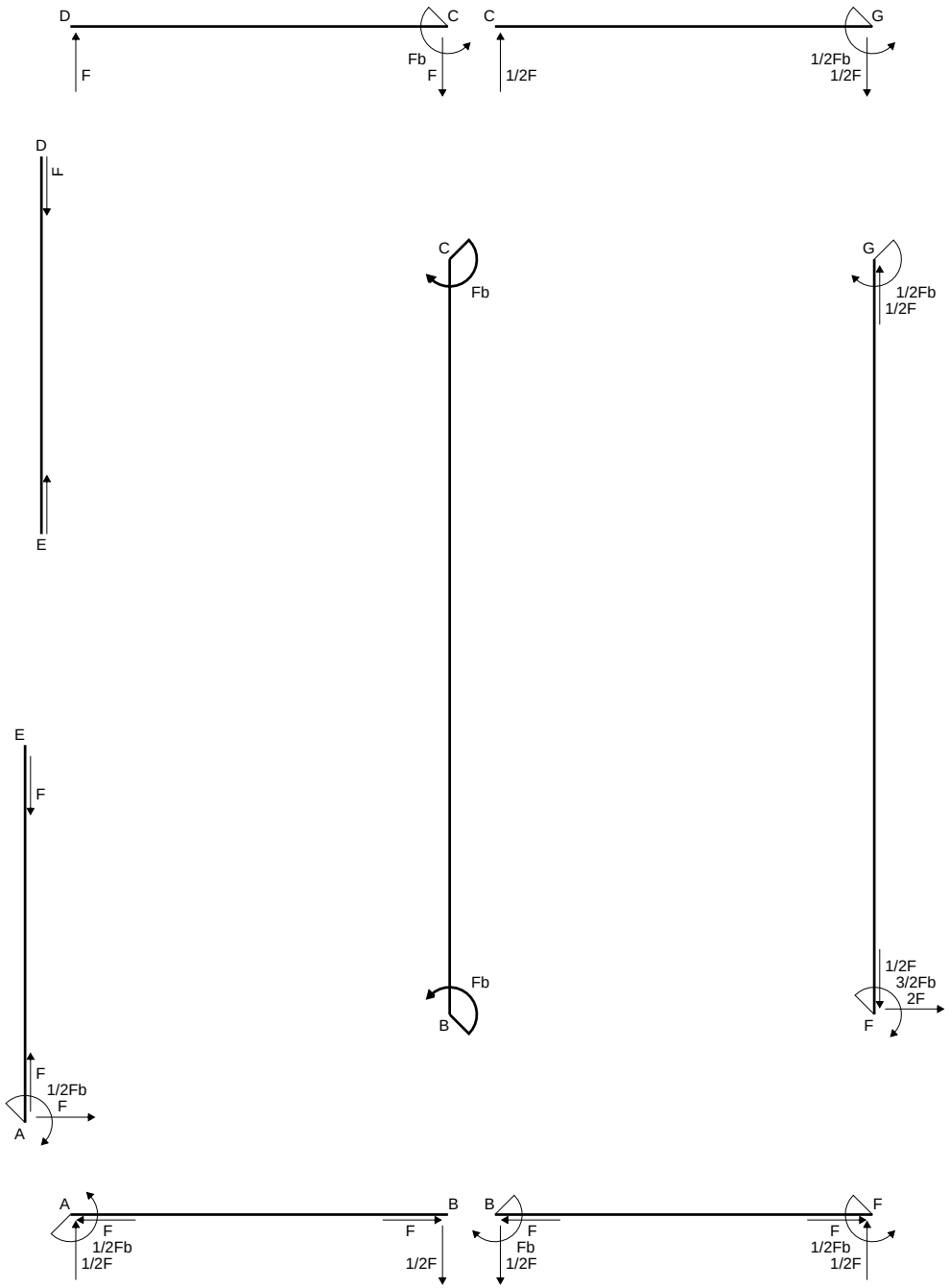
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

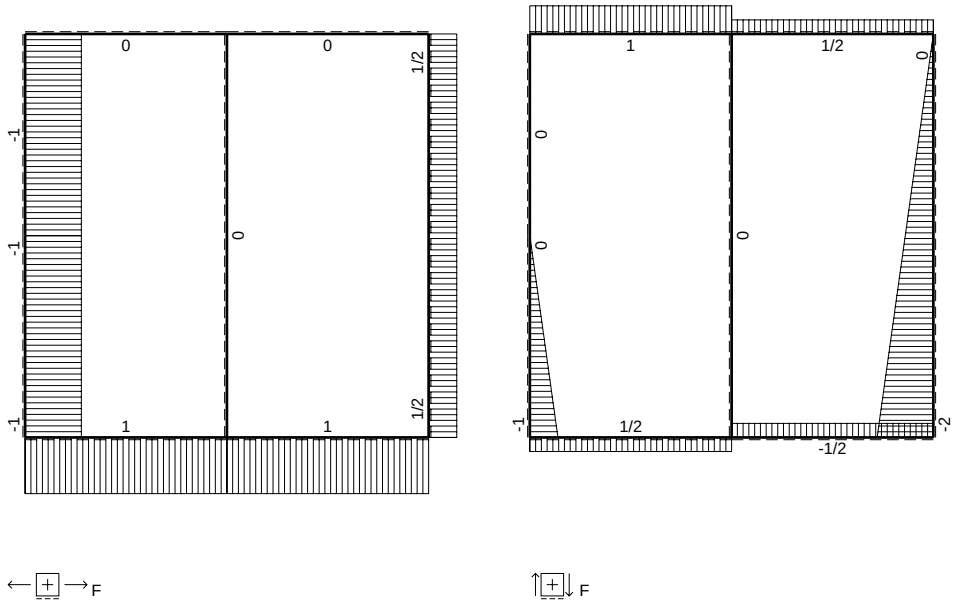


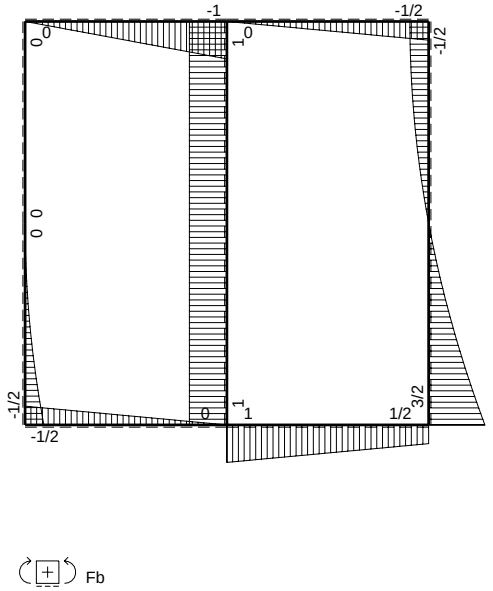
A = 181.6 mm^2
J_u = 99327. mm^4
J_v = 19341. mm^4
J_t = 157.3 mm^4
x_o = -19.15 mm
x_g = 14.56 mm
N = 2214. N
T_y = -1610. N
M_x = 788900. Nmm
u_m = -14.56 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = 218.7 N/mm^2
x_c = 6. mm
u_c = -8.564 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = 218.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 365.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.64 N/mm^2
$\tau_o = Tx_o t/J_t$ = 352.8 N/mm^2
t_c = 2898. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 669.6 N/mm^2

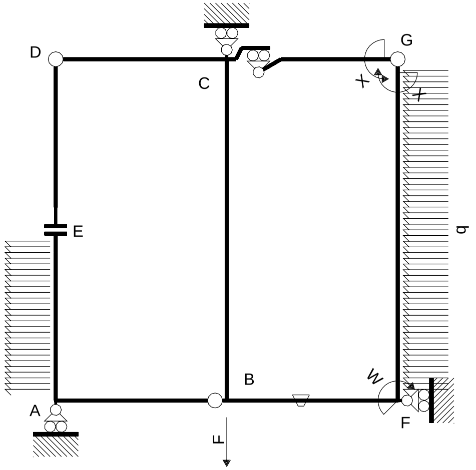
CI08.xxxx.022

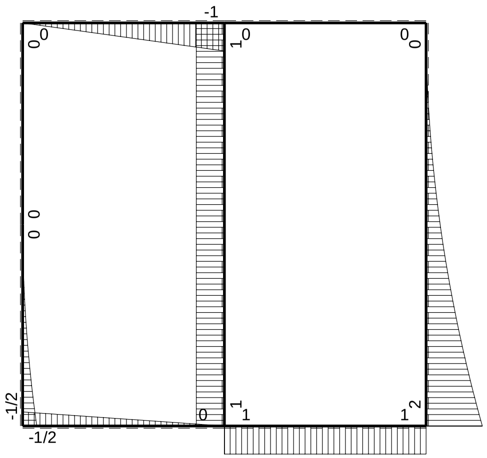
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
G
1/2Fb
1/2F
E
F
F
1/2Fb
F
A
Fb
B
1/2F
3/2Fb
2F
F
A
B
B
F
F
1/2Fb
1/2F
F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F

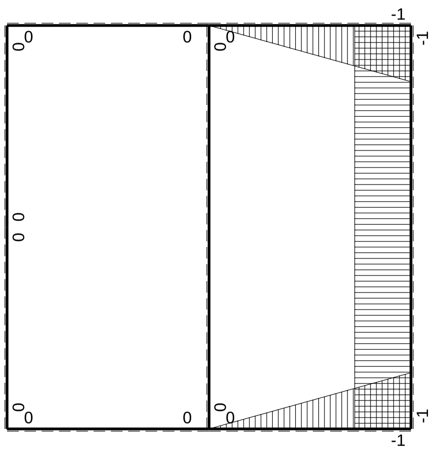

0
0
1/2
-1
0
-1
0
1
1
1/2
F
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F


-1
-1/2
0
1
0
-1/2
0 0
-1/2
0
1
1
1/2
3/2
-1/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	$-x/b$	Fb	$-Fb/EJ$	$-Fx$	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb$	Fb/EJ	$-Fb+Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	$-Fb$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	$8/3Xb/EJ$
iperstatica $X=W_{GC}$							$1/2Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

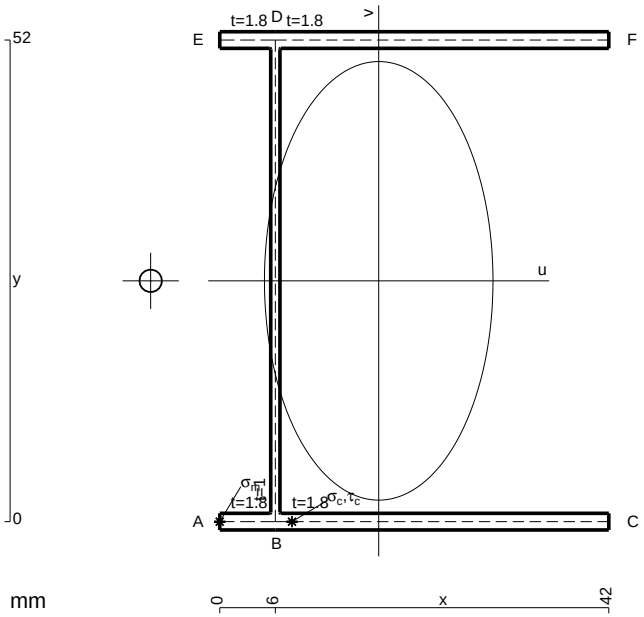
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

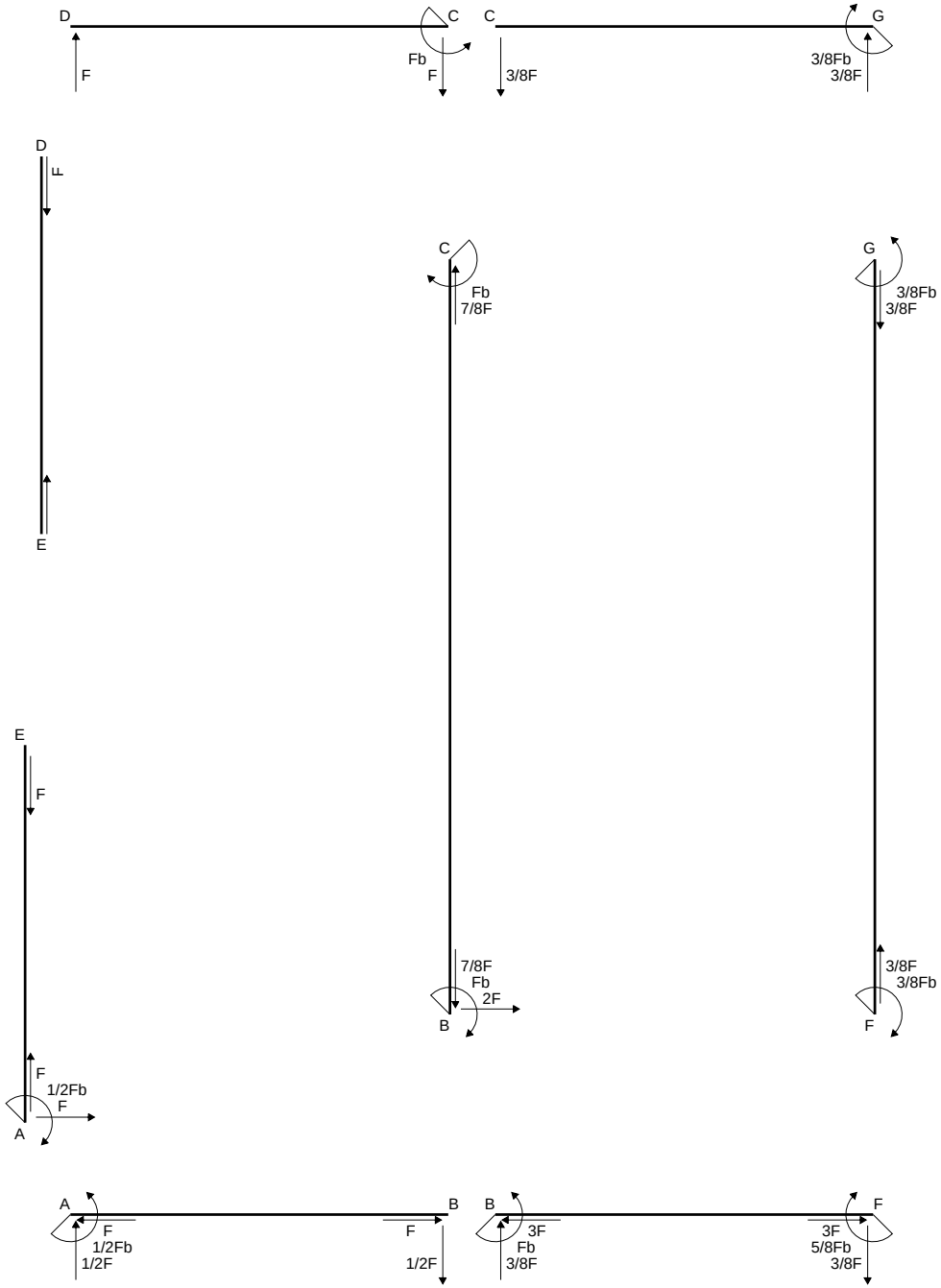
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

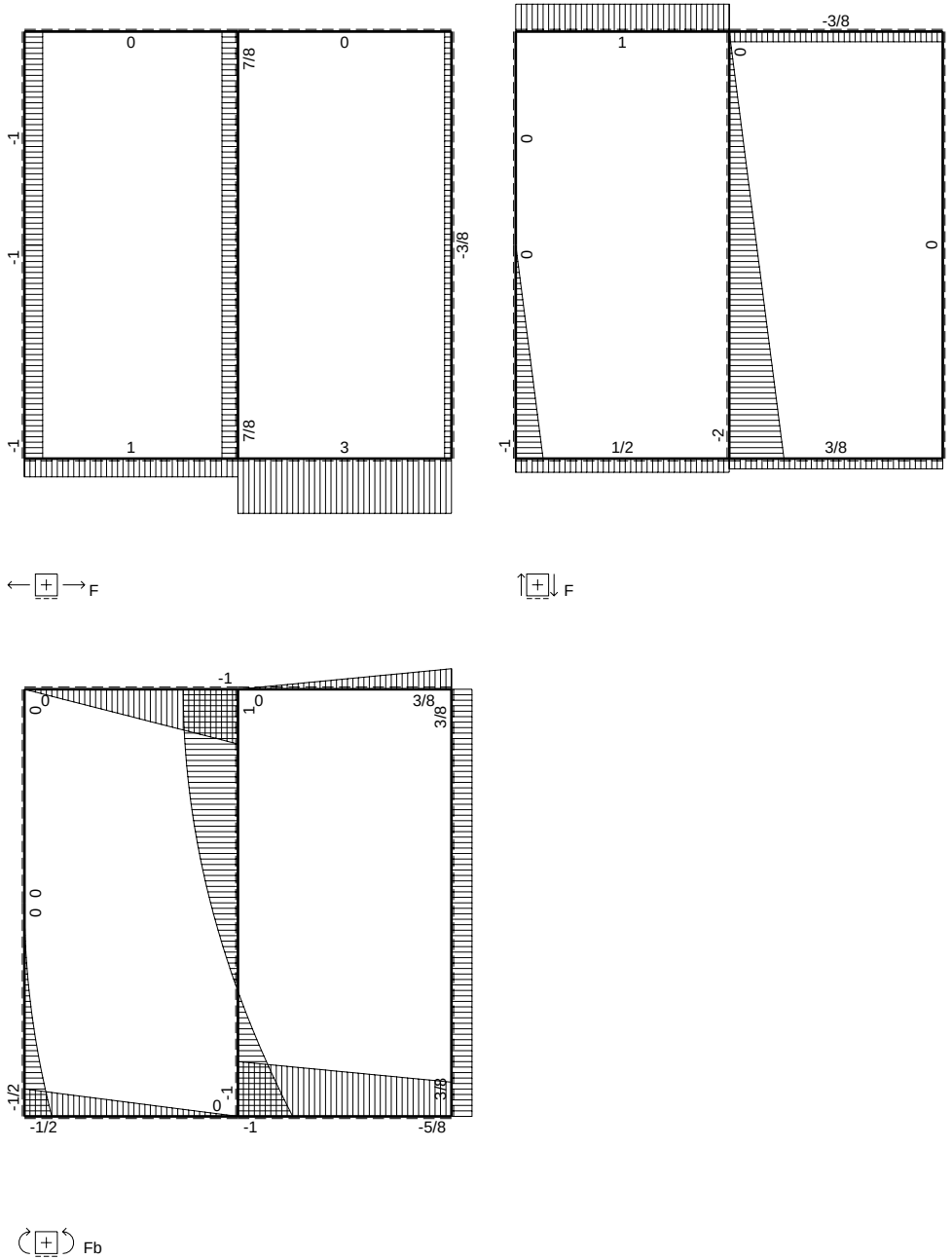


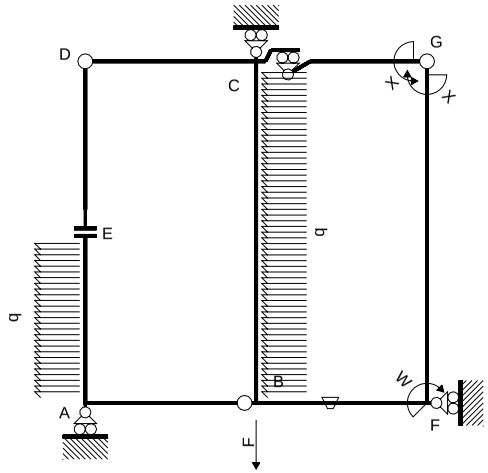
A = 203.2 mm^2
J_u = 113929. mm^4
J_v = 30932. mm^4
J_t = 180.6 mm^4
x_o = -24.62 mm
x_g = 17.16 mm
T_y = 1900. N
M_x = -1007000. Nmm
u_m = -17.16 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -229.8 N/mm^2
x_c = 6. mm
u_c = -11.16 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -229.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 481.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.61 N/mm^2
$\tau_o = Tx_o t/J_t$ = 466.1 N/mm^2
t_c = 3420. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 865.5 N/mm^2

CI08.xxxx.023

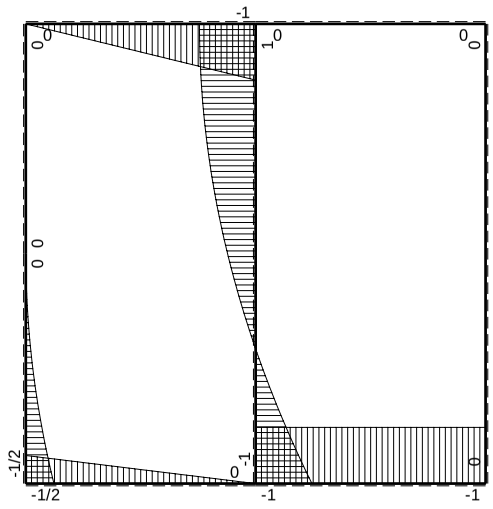
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
7/8F
G
3/8Fb
3/8F
E
F
7/8F
Fb
2F
B
3/8F
3/8Fb
F
F
1/2Fb
F
A
A
F
1/2Fb
1/2F
B
F
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F

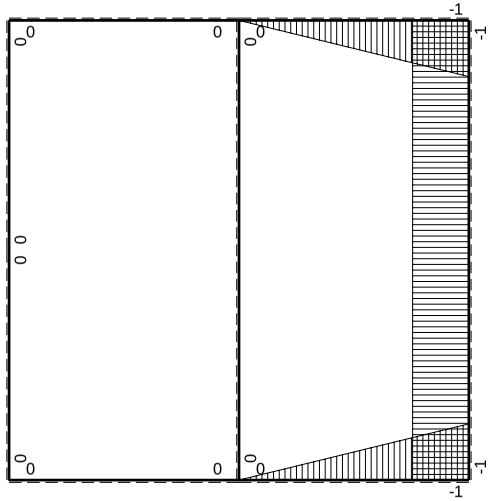

0
0
7/8
-1
-1
-3/8
-1
1
7/8
3
F
1
-3/8
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 917219 CI08.xxxx.024

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

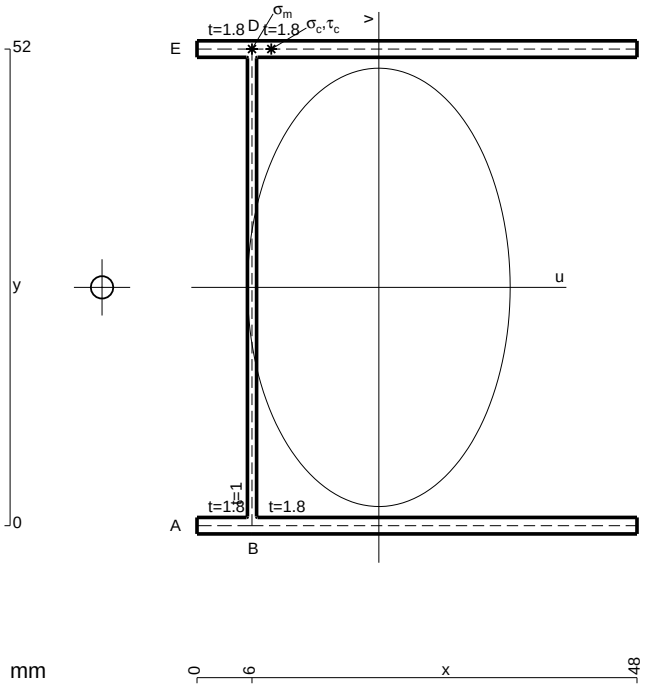
$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

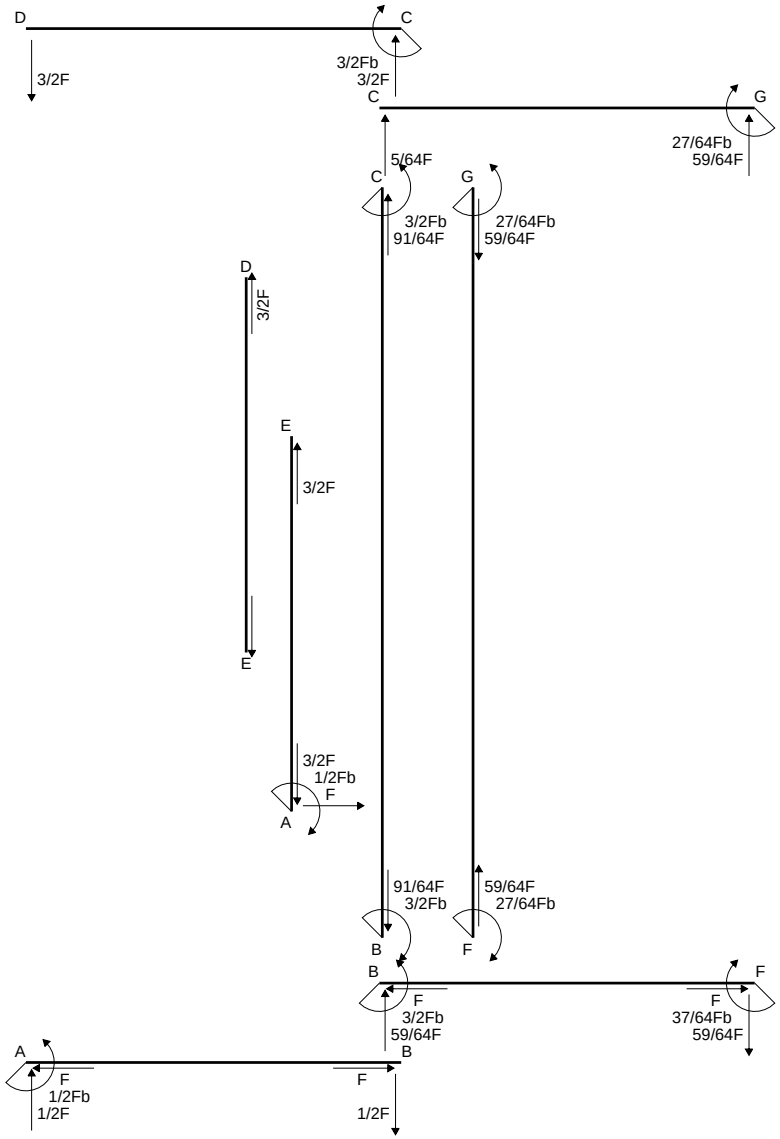
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

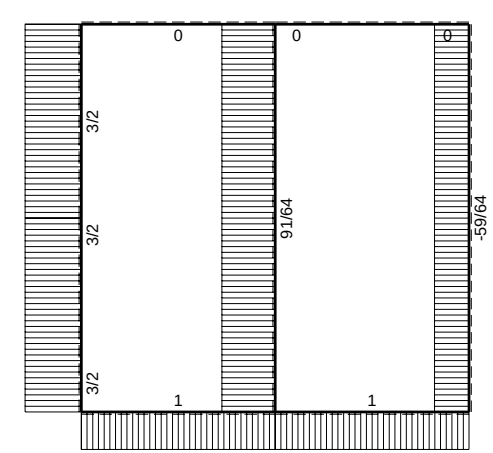


A = 224.8 mm^2
J_u = 128530. mm^4
J_v = 46128. mm^4
J_t = 204. mm^4
x_o = -30.2 mm
x_g = 19.84 mm
N = 1759. N
T_y = -4020. N
M_x = -1145700. Nmm
x_m = 6. mm
y_m = 52. mm
u_m = -13.84 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = 239.6 N/mm^2
x_c = 6. mm
y_c = 52. mm
u_c = -13.84 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = 239.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 1105. N/mm^2
$\tau_g = TS^*/tJ_u$ = 34.15 N/mm^2
$\tau_o = Tx_o t/J_t$ = 1071. N/mm^2
t_c = 3618. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1930. N/mm^2

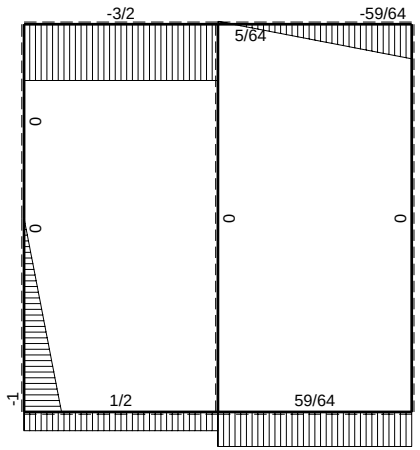
CI08.xxxx.024

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

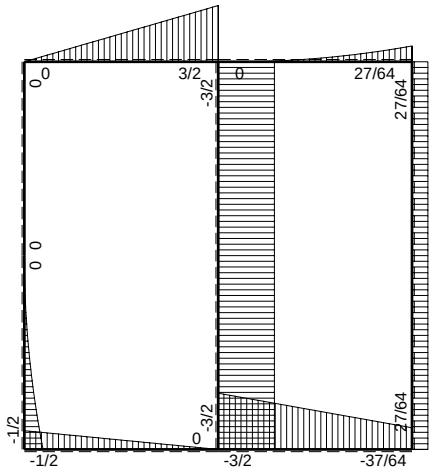

D
C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
3/2
91/64
-59/64
1
1

F


-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F


0
0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-1/2
-3/2
-37/64

Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	0	1/2Fx	0	0	0	0		
BA b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
	0	-3/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DC b	0	0	0	0	0	0	0+0	0
	0	-1/2qx^2	0	0	0	0		
DE b	0	1/2Fb-Fx+1/2qx^2	0	0	0	0	0+0	0
	0	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2		
EA b	-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2	(7/12+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
GC b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	-1	0	0	0	0	1	0+0	2Xb/EJ
FG 2b	1	0	0	0	0	1		
GF 2b	0	-3/2Fb	0	0	0	0	0+0	0
CB 2b	0	3/2Fb	0	0	0	0		
BC 2b	totali						9/8Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

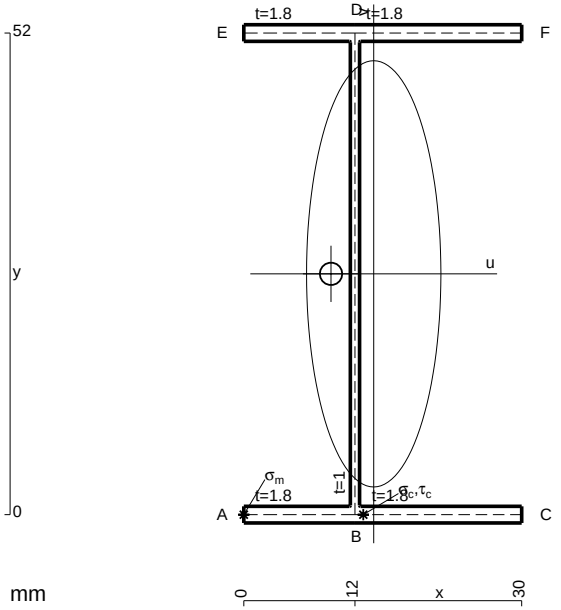
$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

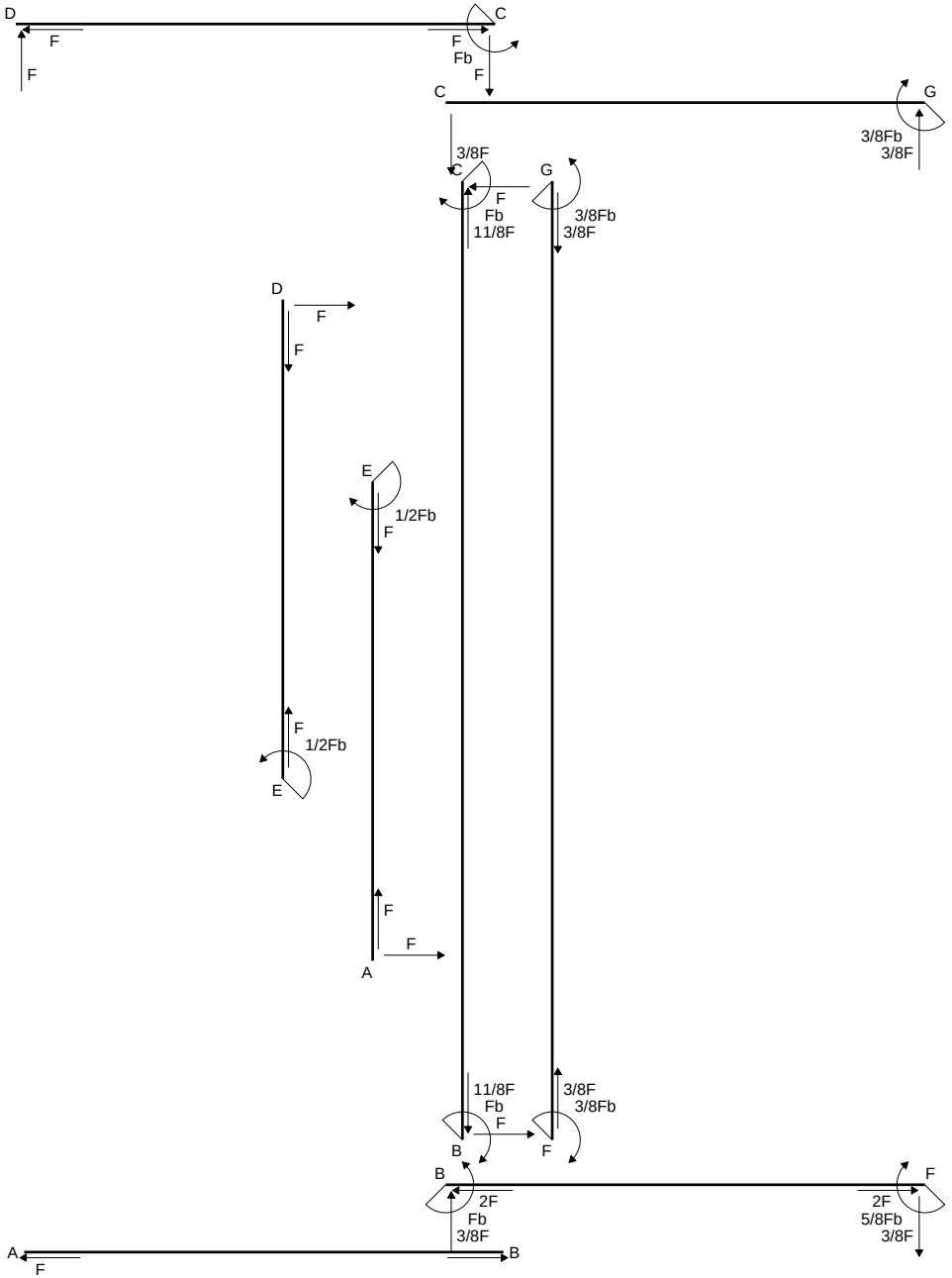
PROCEDIMENTO E RISULTATI 238759 CI08.xxxx.025

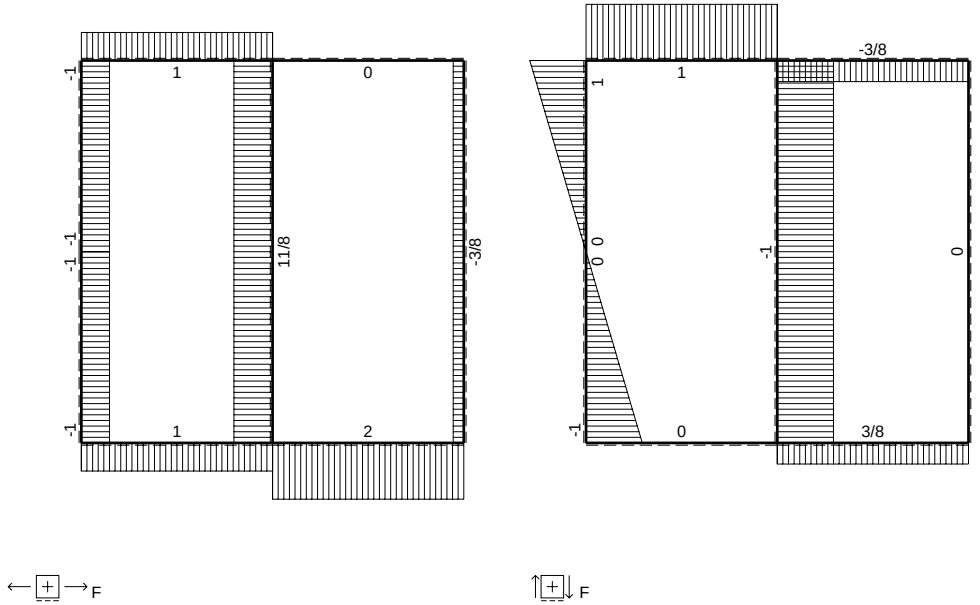


A = 160. mm^2
J_u = 84725. mm^4
J_v = 8416. mm^4
J_t = 134. mm^4
x_o = -4.61 mm
x_g = 14.02 mm
T_y = -1065. N
M_x = 649650. Nmm
u_m = -14.02 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = 199.4 N/mm^2
x_c = 12. mm
u_c = -2.025 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = 199.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 71.85 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.883 N/mm^2
$\tau_o = Tx_o t/J_t$ = 65.97 N/mm^2
t_c = 1278. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 235. N/mm^2

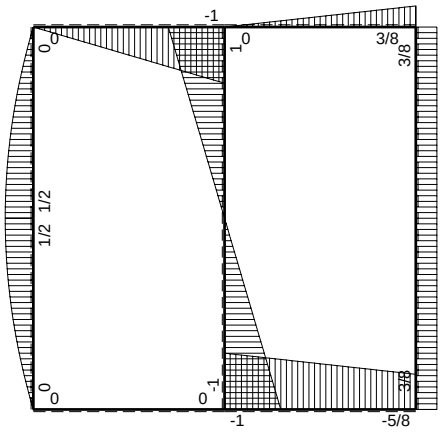
CI08.xxxx.025

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

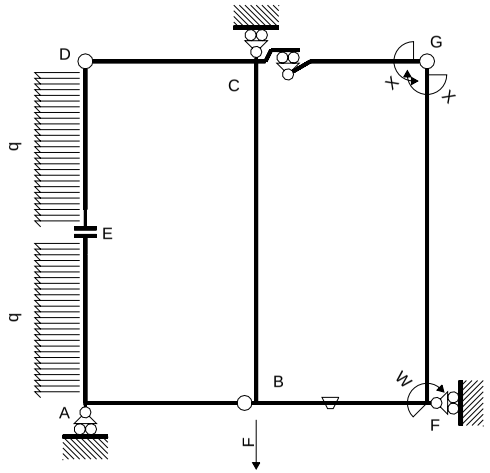

D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
11/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
11/8F
Fb
F
3/8F
3/8Fb
F
B
B
2F
Fb
3/8F
F
2F
5/8Fb
3/8F
A
F
B


-1
1
0
-1
11/8
-3/8
-1
1
2
-3/8
1
1
0
0
-1
0
-1
0
3/8

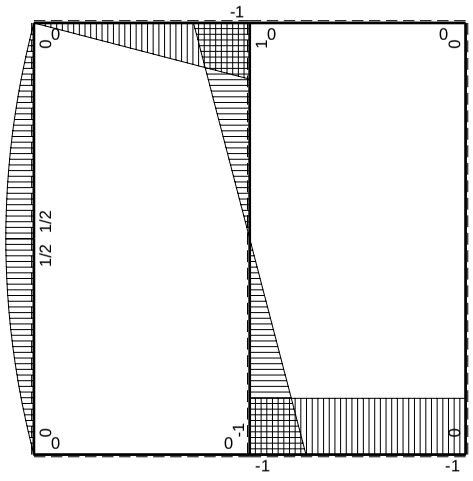
F F


-1
0
0
1
0
3/8
3/8
1/2
1/2
0
0
0
-1
-1
3/8
-1
-5/8

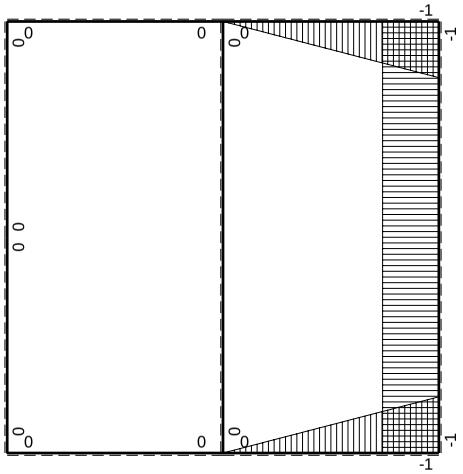
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb-1/2qx^2	0	0	0	0	0+0	0
AE b	0	-Fx+1/2qx^2	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238406 CI08.xxxx.026

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

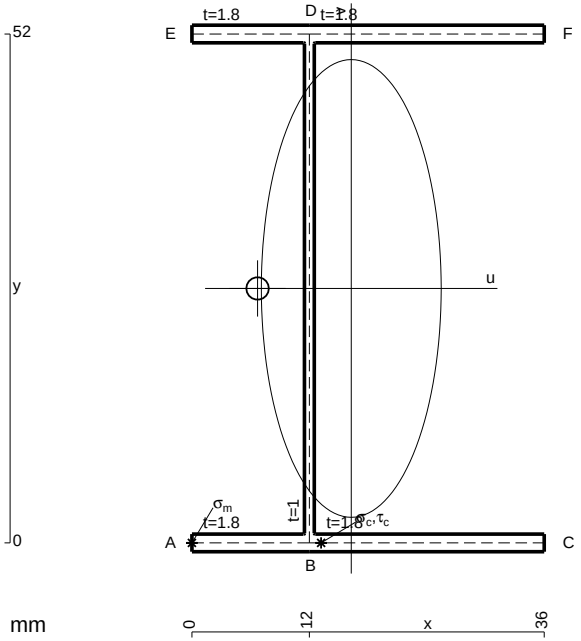
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

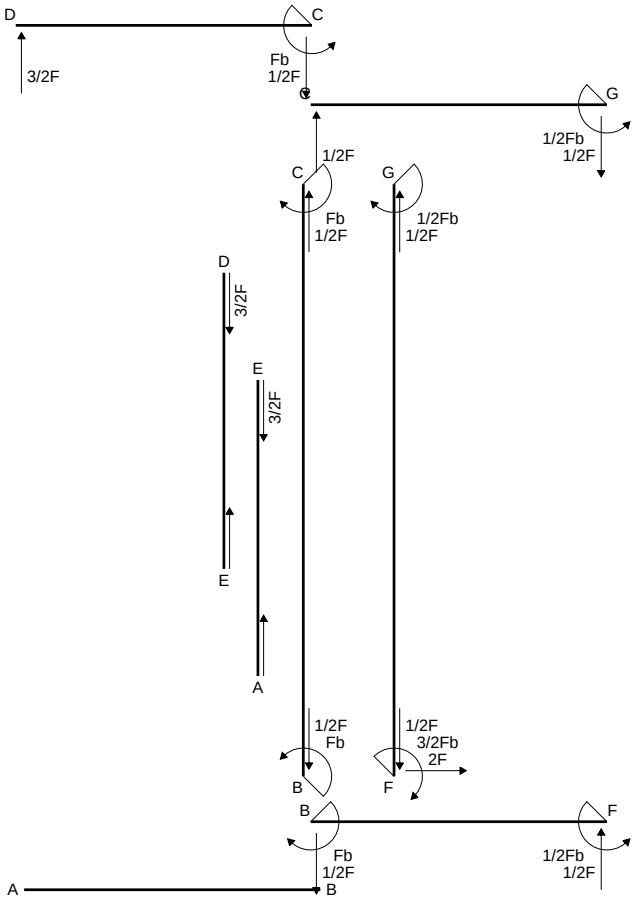
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

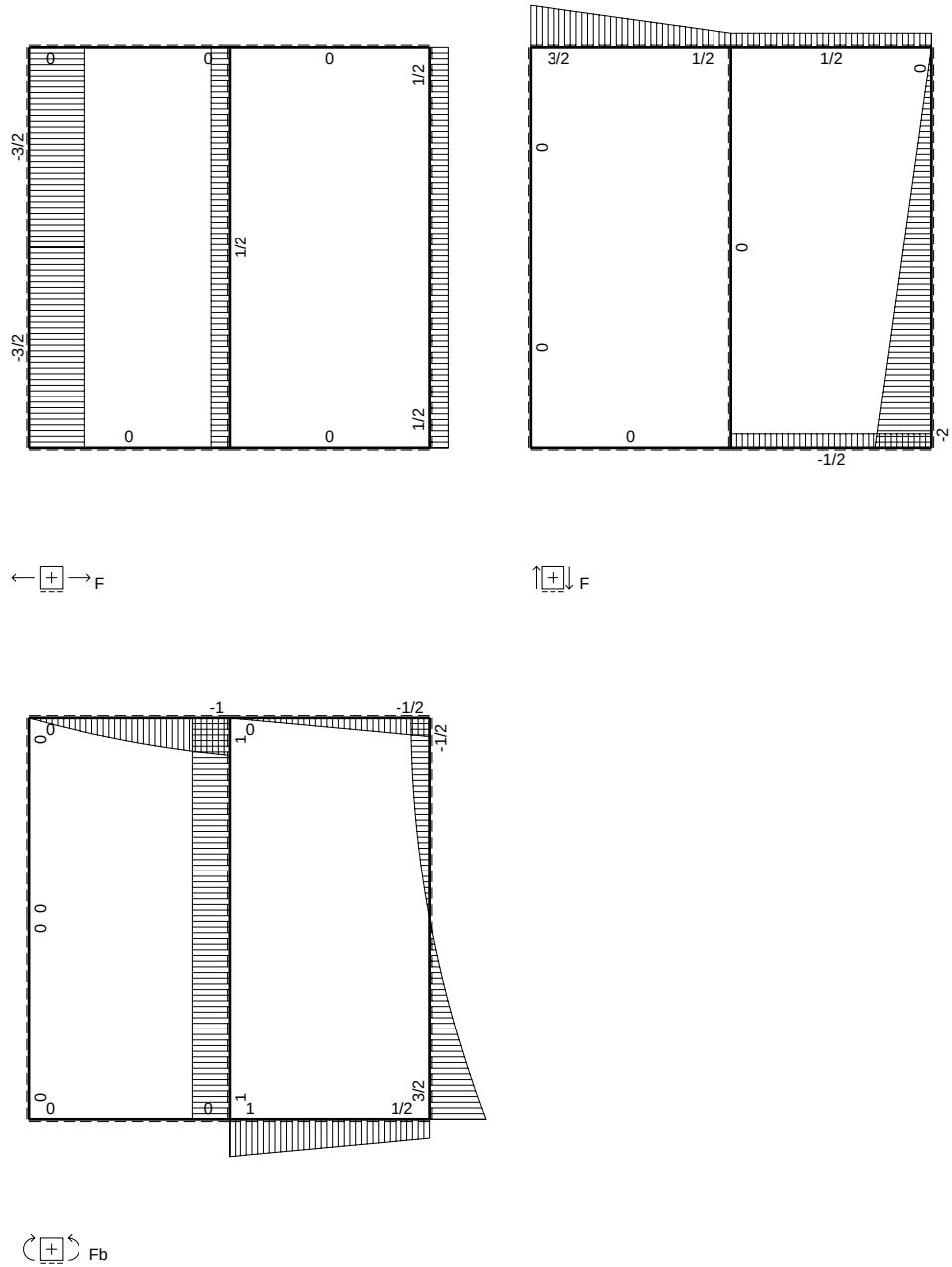


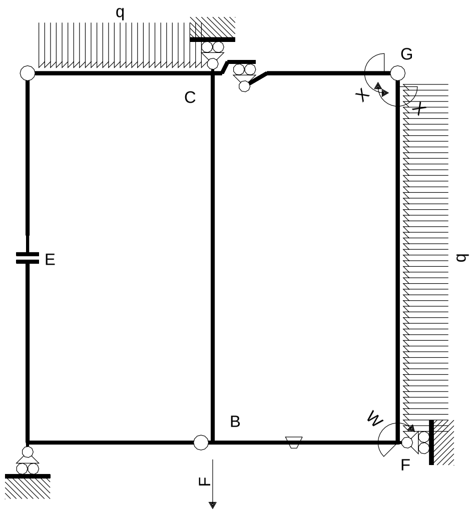
A = 181.6 mm^2
J_u = 99327. mm^4
J_v = 15333. mm^4
J_t = 157.3 mm^4
x_o = -9.574 mm
x_g = 16.28 mm
N = 1623. N
T_y = -1180. N
M_x = 767000. Nmm
u_m = -16.28 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = 209.7 N/mm^2
x_c = 12. mm
u_c = -4.282 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = 209.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 136.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.413 N/mm^2
$\tau_o = Tx_o t/J_t$ = 129.3 N/mm^2
t_c = 2124. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 316.3 N/mm^2

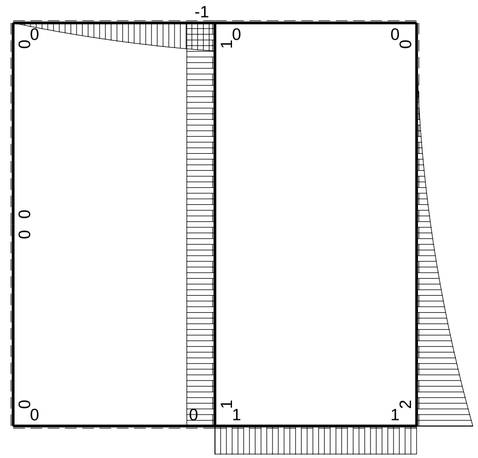
CI08.xxxx.026

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

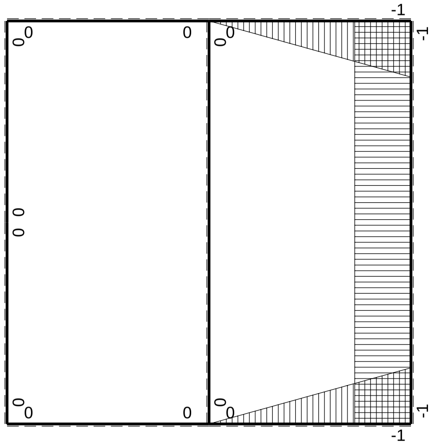

D
C
3/2F
Fb
1/2F
G
1/2Fb
1/2F
1/2F
C
G
Fb
1/2F
1/2Fb
1/2F
D
3/2F
E
3/2F
E
A
1/2F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B


0
0
0
1/2
-3/2
1/2
-3/2
0
0
1/2
F
3/2
1/2
1/2
0
0
0
0
-2
-1/2
F
-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211463 CI08.xxxx.027

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

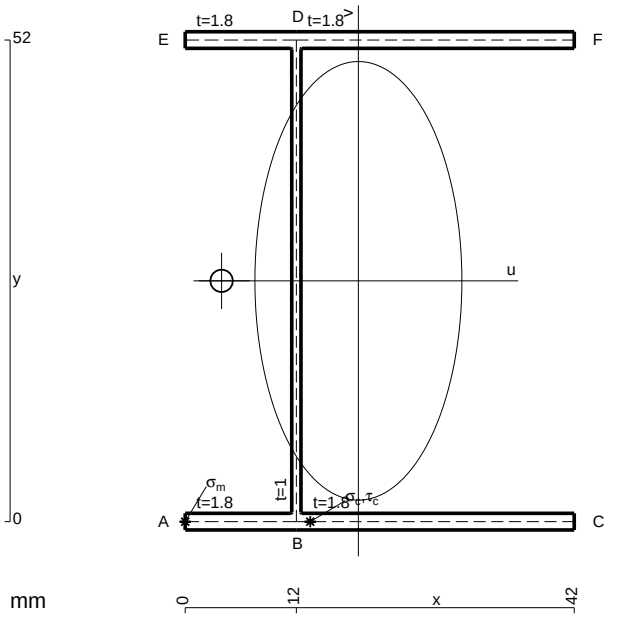
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

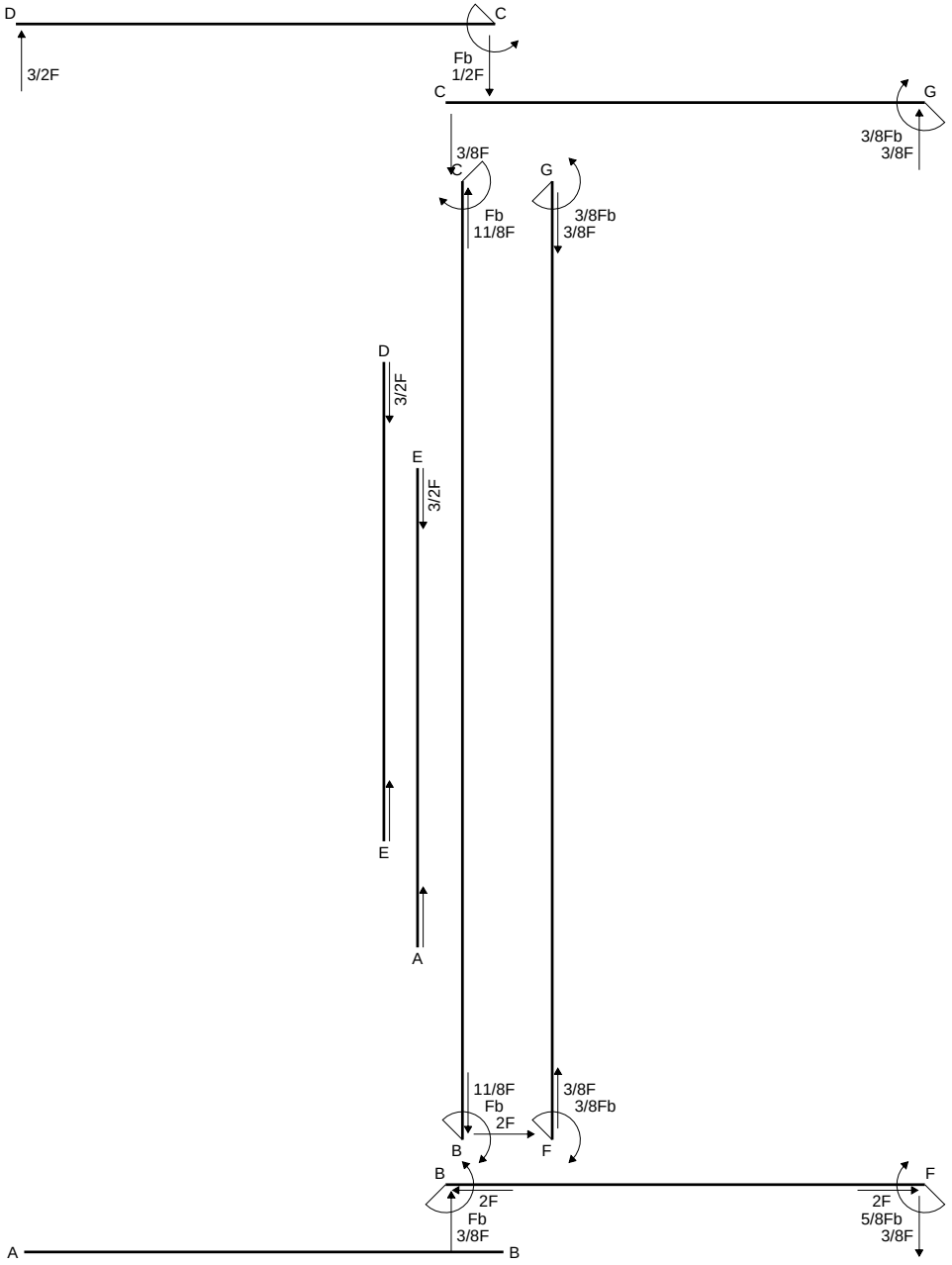
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

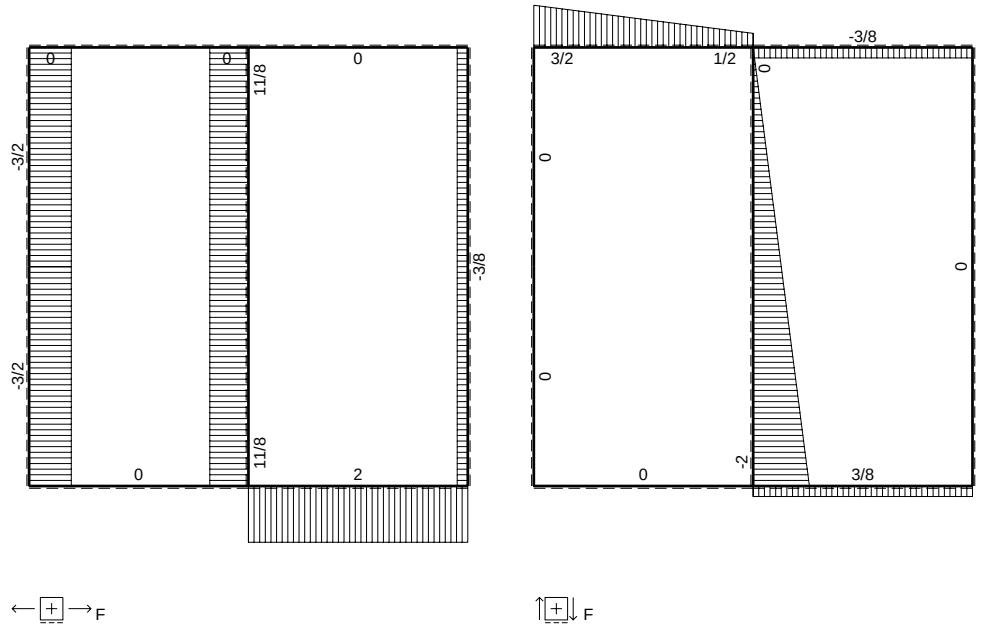


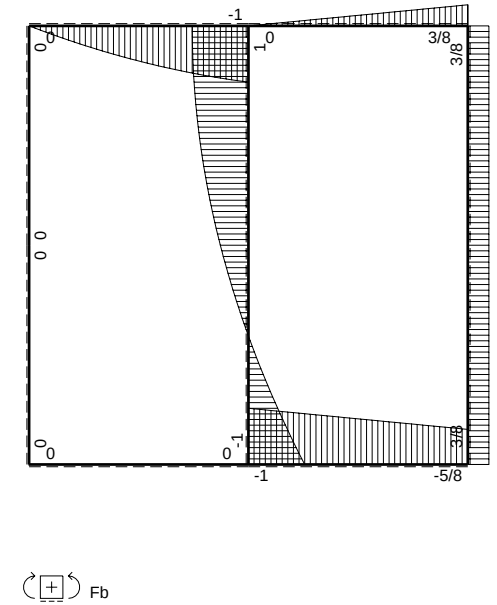
A = 203.2 mm^2
J_u = 113929. mm^4
J_v = 25361. mm^4
J_t = 180.6 mm^4
x_o = -14.77 mm
x_g = 18.7 mm
T_y = 2230. N
M_x = -961688. Nmm
u_m = -18.7 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -219.5 N/mm^2
x_c = 12. mm
u_c = -6.697 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -219.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 343.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.27 N/mm^2
$\tau_o = Tx_o t/J_t$ = 328.3 N/mm^2
t_c = 4014. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 634.2 N/mm^2

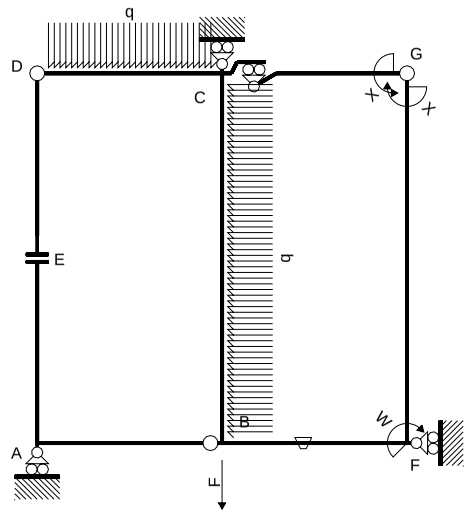
CI08.xxxx.027

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

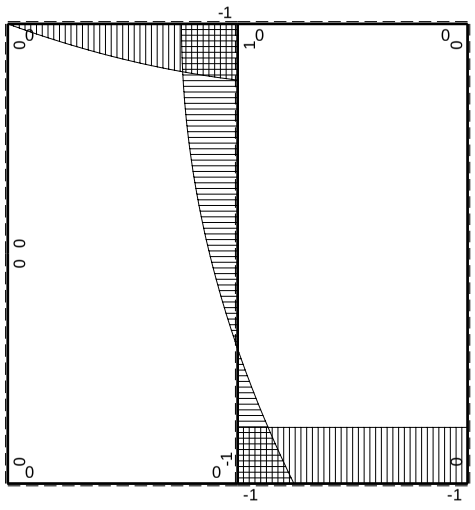

D
C
3/2F
Fb
1/2F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
11/8F
3/8Fb
3/8F
D
3/2F
E
3/2F
E
A
11/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B


0
0
0
11/8
-3/2
-3/8
-3/2
11/8
0
2
F
3/2
1/2
-3/8
0
0
0
0
-2
3/8
F

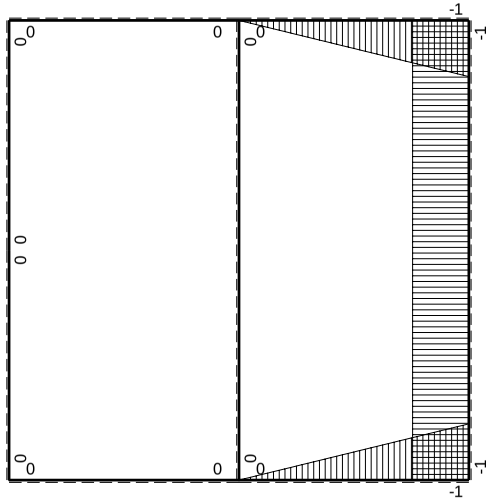

-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
						totali	Fb^2/EJ	8/3Xb/EJ
						iperstatica $X=W_{GC}$	-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 225153 CI08.xxxx.028

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

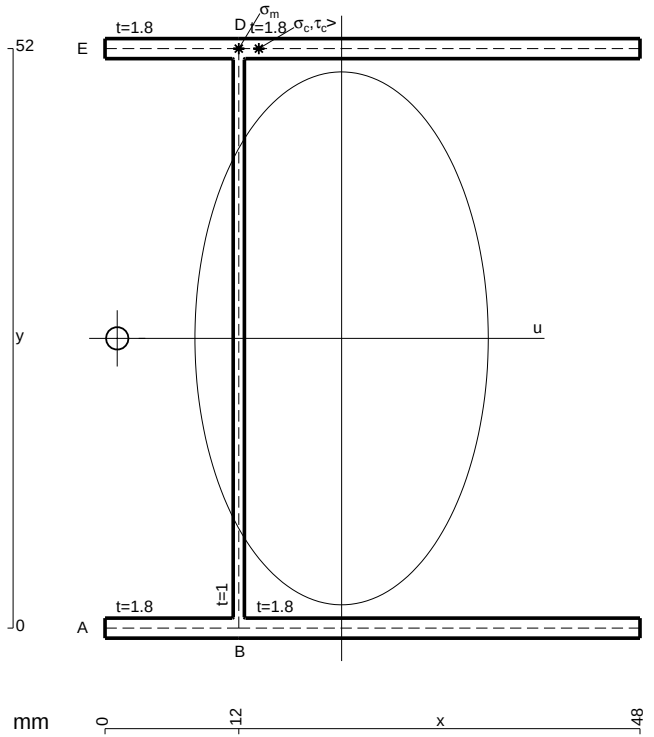
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

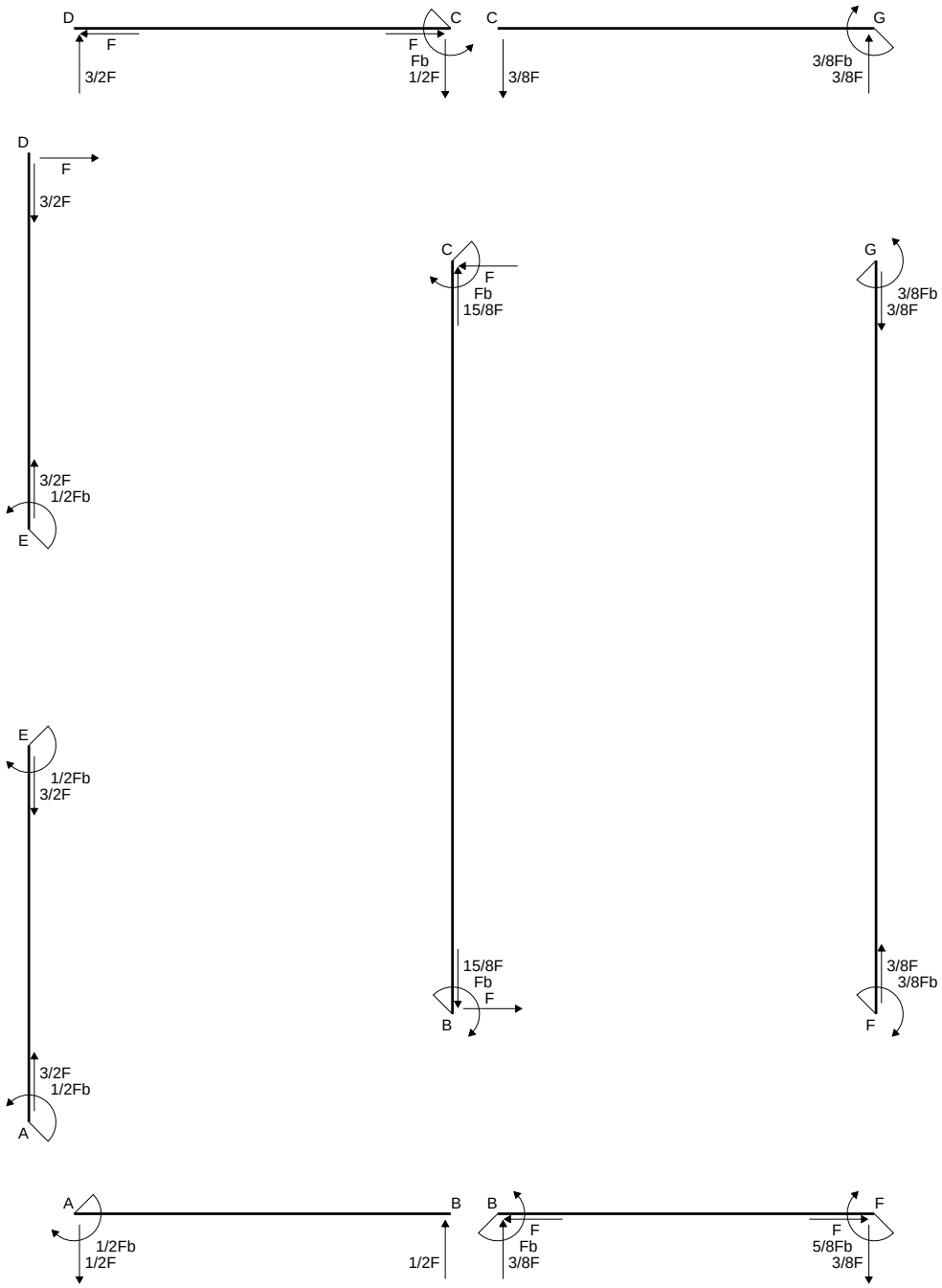
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

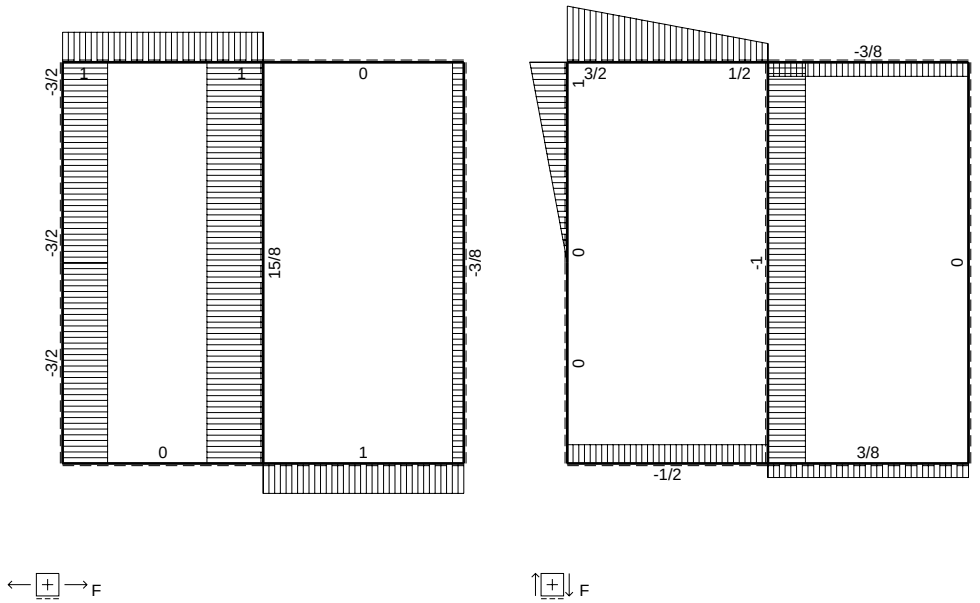


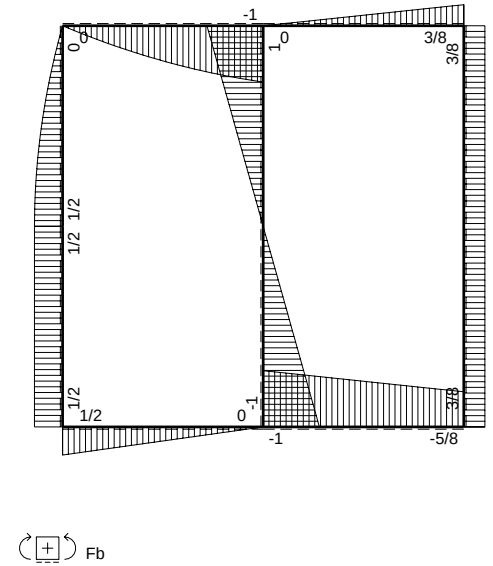
A = 224.8 mm^2
J_u = 128530. mm^4
J_v = 38934. mm^4
J_t = 204. mm^4
x_o = -20.13 mm
x_g = 21.22 mm
N = 2049. N
T_y = -2980. N
M_x = -1087700. Nmm
x_m = 12. mm
y_m = 52. mm
u_m = -9.224 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = 229.1 N/mm^2
x_c = 12. mm
y_c = 52. mm
u_c = -9.224 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = 229.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 551.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 21.7 N/mm^2
$\tau_o = Tx_o t/J_t$ = 529.4 N/mm^2
t_c = 2682. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 981.7 N/mm^2

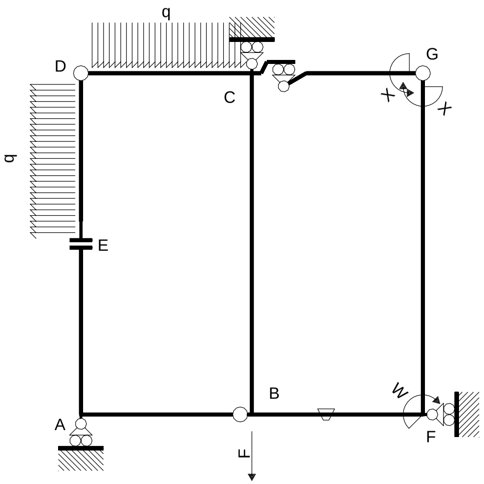
CI08.xxxx.028

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

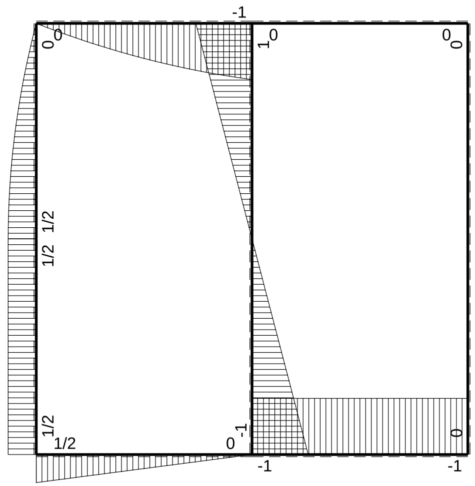

D
F
3/2F
C
C
F
Fb
1/2F
3/8F
G
3/8Fb
3/8F
D
F
3/2F
C
F
Fb
15/8F
G
3/8Fb
3/8F
3/2F
1/2Fb
E
E
1/2Fb
3/2F
15/8F
Fb
F
B
3/8F
3/8Fb
F
3/2F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F


-3/2
-3/2
-3/2
1
1
0
15/8
-3/8
0
1
F
-3/8
1
3/2
1/2
0
0
-1
0
3/8
-1/2
F

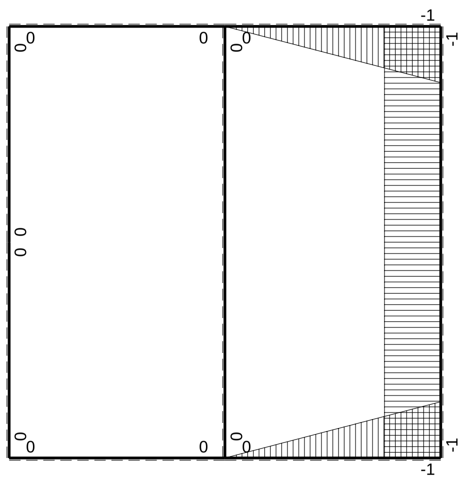

-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

↓	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

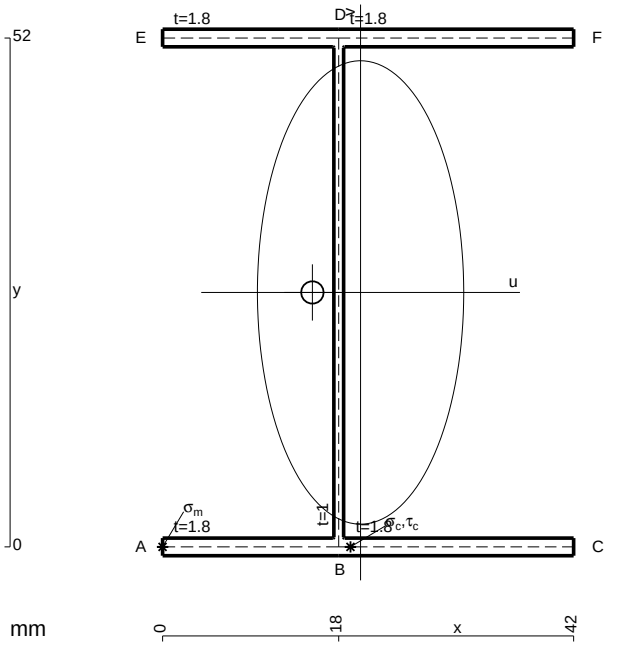
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

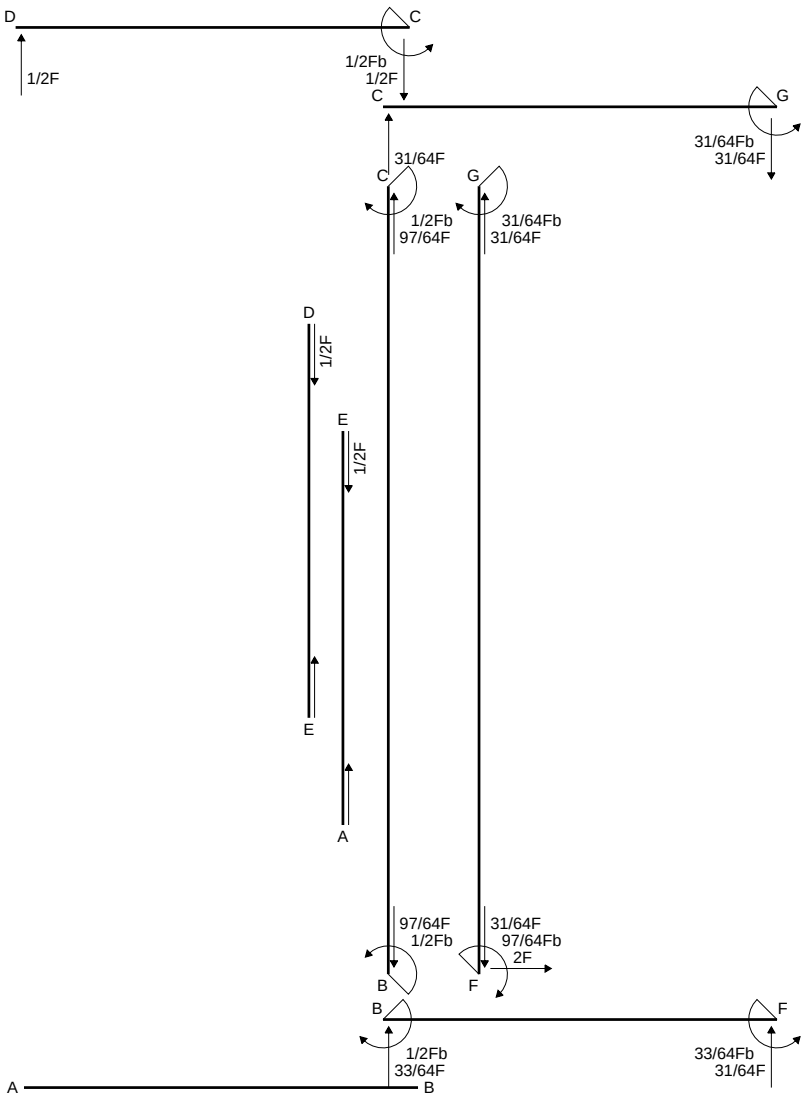
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

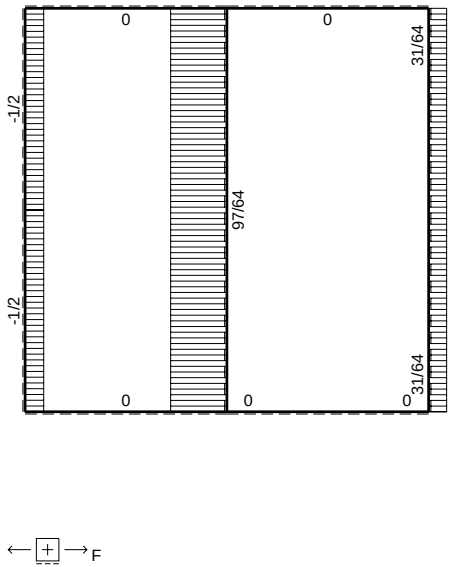


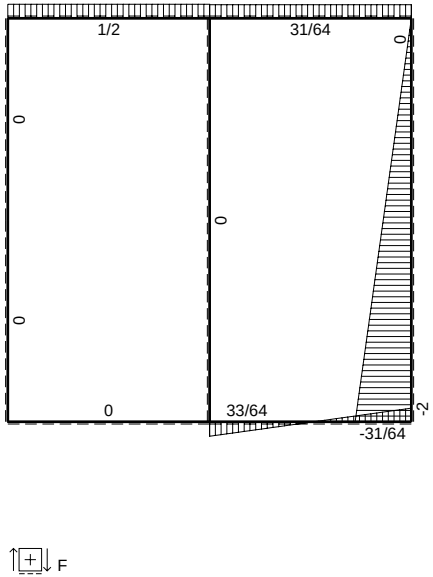
A = 203.2 mm^2
J_u = 113929. mm^4
J_v = 22575. mm^4
J_t = 180.6 mm^4
x_o = -4.924 mm
x_g = 20.23 mm
N = 2419. N
T_y = -1290. N
M_x = 993300. Nmm
u_m = -20.23 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = 238.6 N/mm^2
x_c = 18. mm
u_c = -2.232 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = 238.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 70.36 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.065 N/mm^2
$\tau_o = Tx_o t/J_t$ = 63.29 N/mm^2
t_c = 2322. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 267.9 N/mm^2

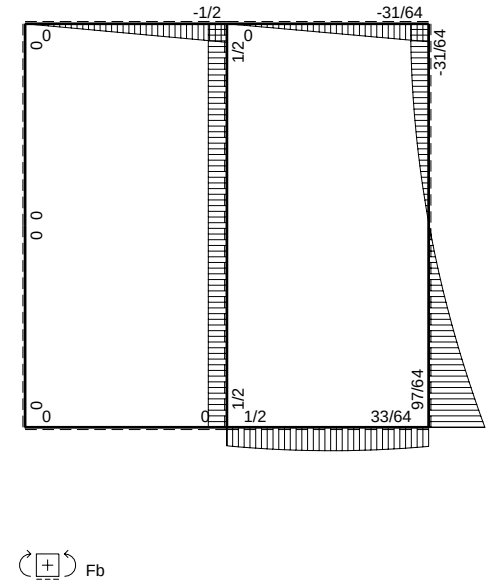
CI08.xxxx.029

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64
F


1/2
31/64
0
0
0
0
0
33/64
-2
-31/64
F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
1/2
33/64
97/64
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	FX/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-FX/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb²/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240664 CI08.xxxx.030

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b)\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 1/24\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

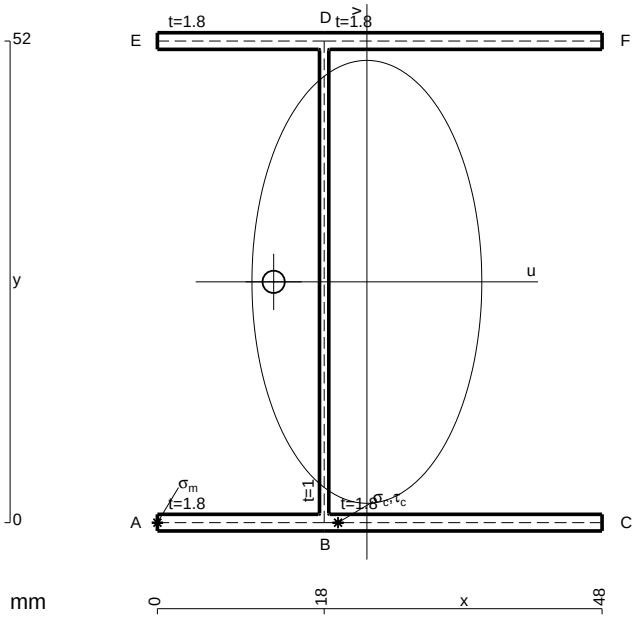
$= (-\ b +1/2\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 1/24\ Fb^2/EJ$

$L_{FG}^{Xo} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b)\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L_{GF}^{Xo} = \int_o^{2b} (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

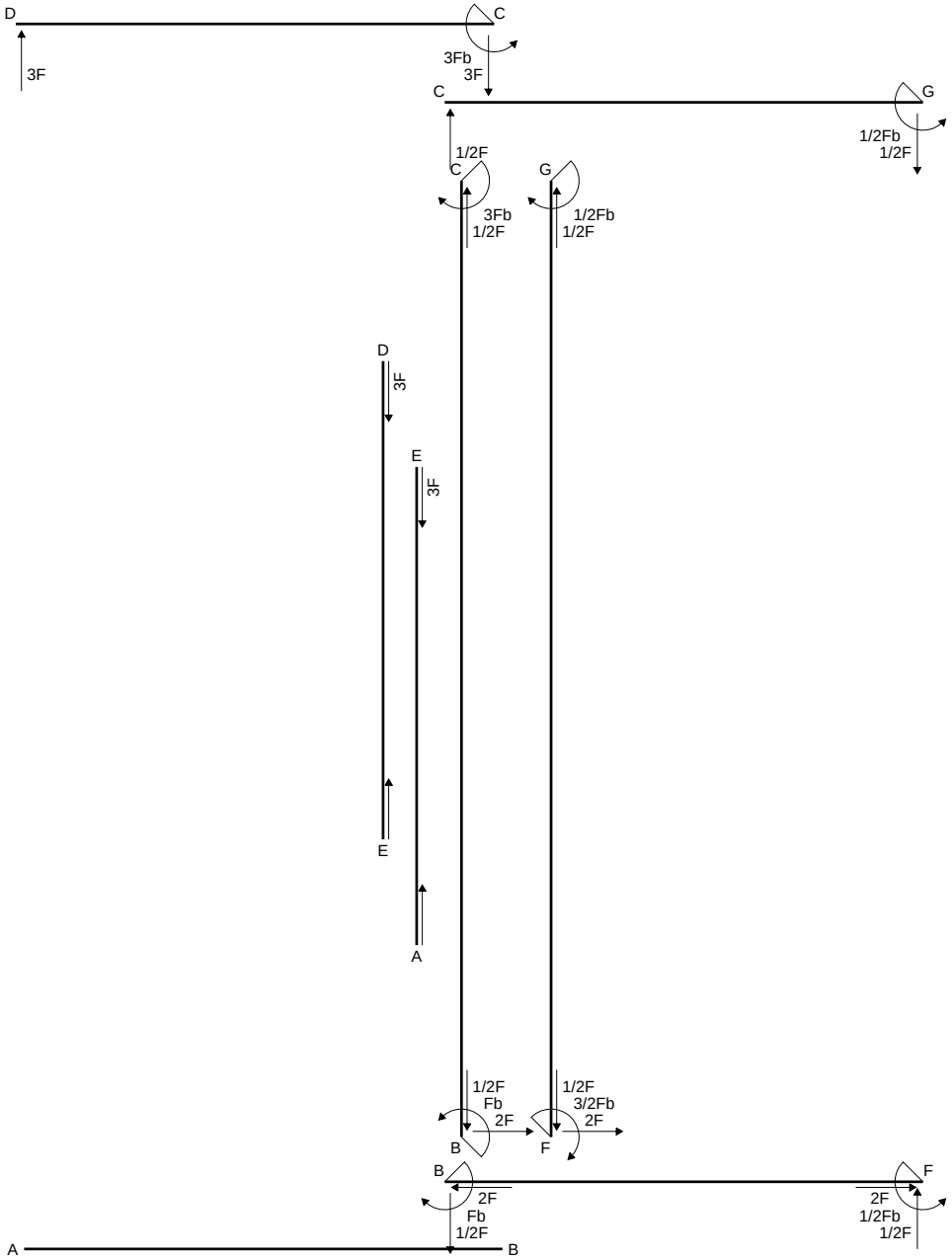
$= (-4/3\ b)\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

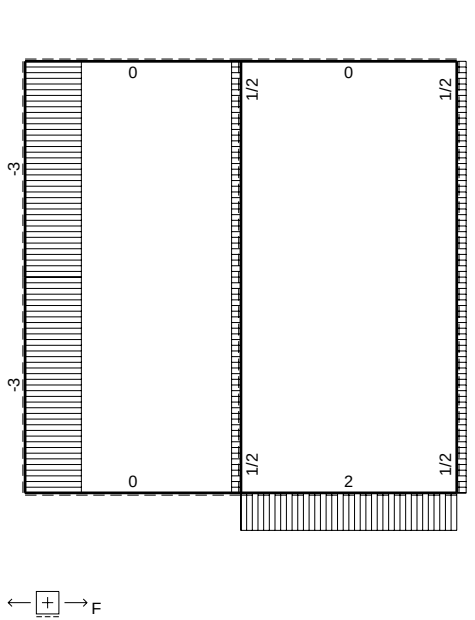


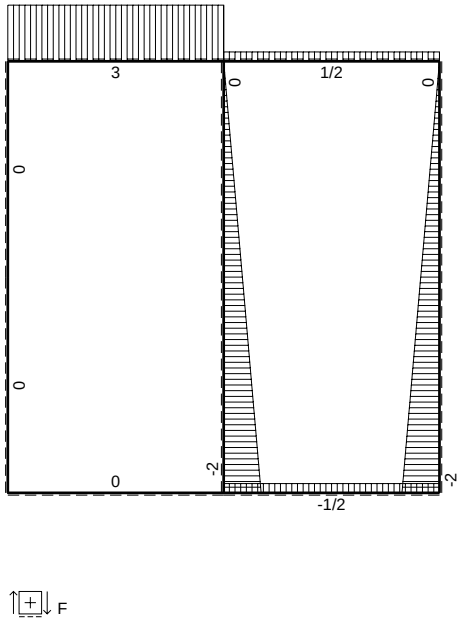
A = 224.8 mm^2
J_u = 128530. mm^4
J_v = 34617. mm^4
J_t = 204. mm^4
x_o = -10.07 mm
x_g = 22.61 mm
T_y = 2410. N
M_x = -988100. Nmm
u_m = -22.61 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -199.9 N/mm^2
x_c = 18. mm
u_c = -4.612 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -199.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 228.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 14.63 N/mm^2
$\tau_o = Tx_o t/J_t$ = 214.1 N/mm^2
t_c = 8676. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 443.7 N/mm^2

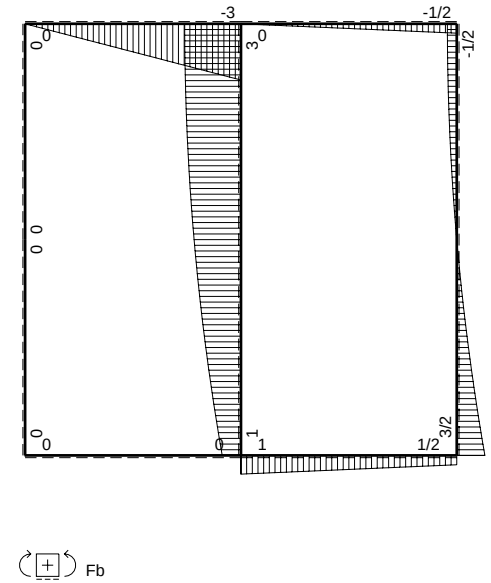
CI08.xxxx.030

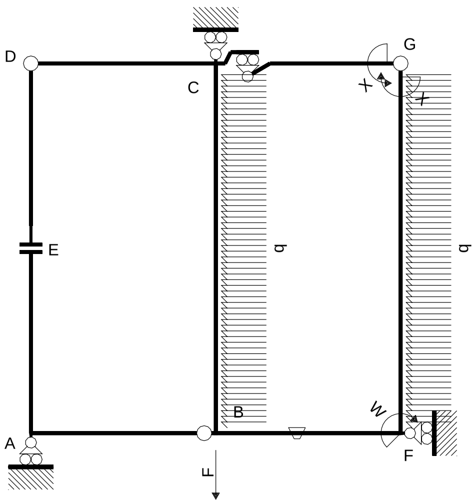
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

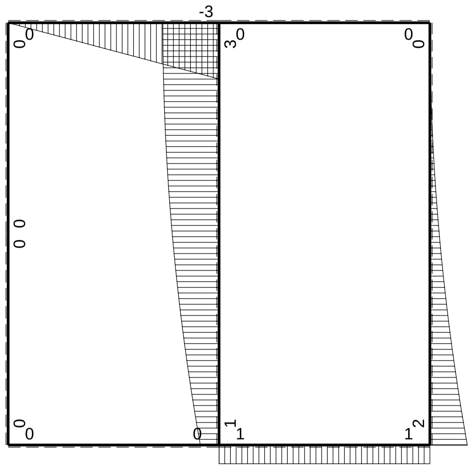

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
0
1/2
1/2
-3
-3
0
2
1/2
1/2
F

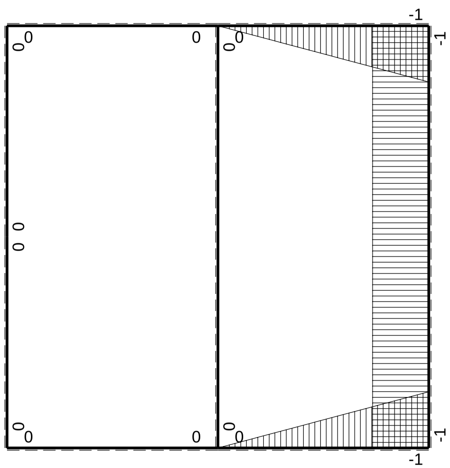

3
0
1/2
0
0
0
0
-2
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

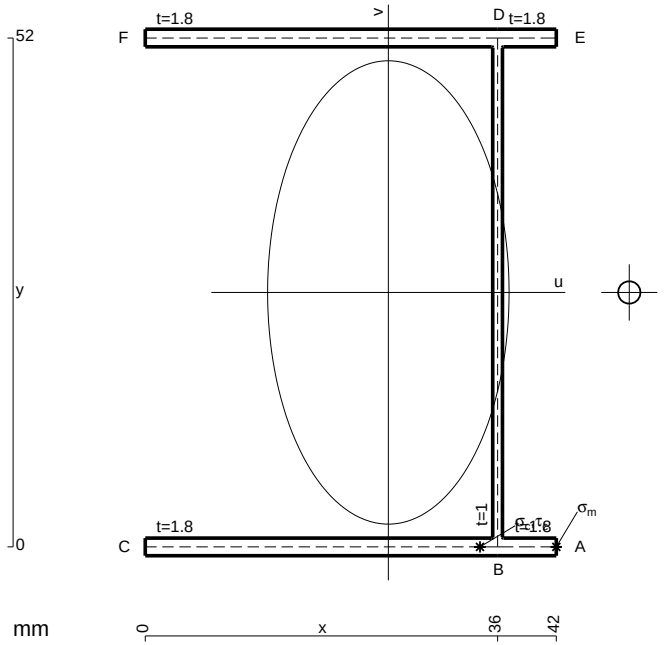
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

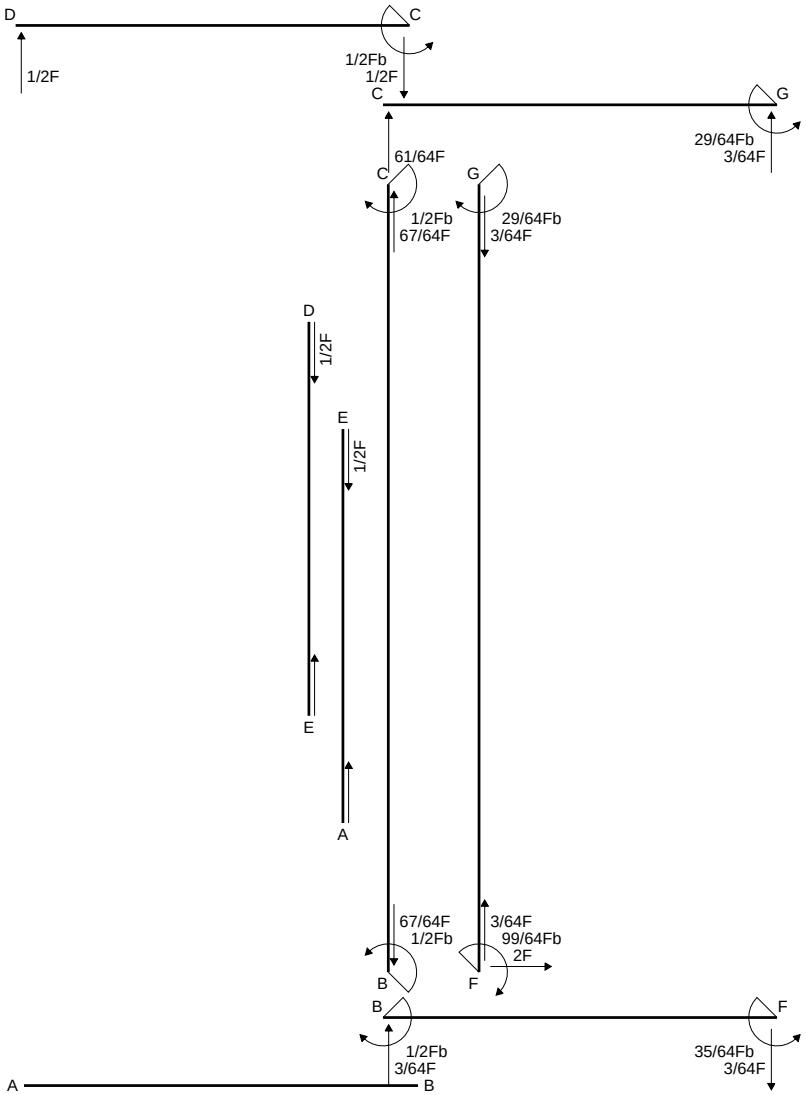
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

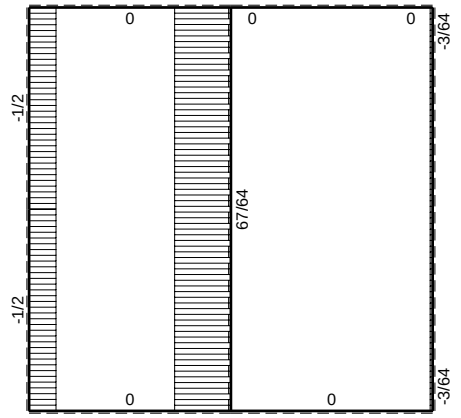


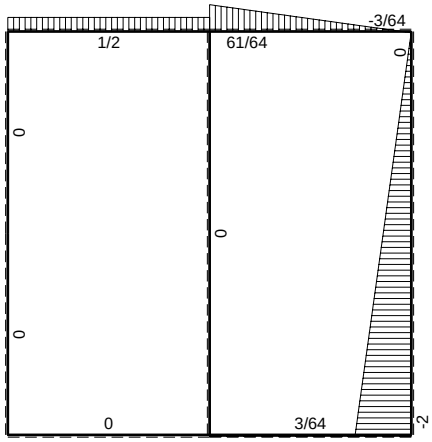
A = 203.2 mm^2
J_u = 113929. mm^4
J_v = 30932. mm^4
J_t = 180.6 mm^4
x_o = 24.62 mm
x_g = 24.84 mm
T_y = 2040. N
M_x = -918000. Nmm
x_m = 42. mm
u_m = 17.16 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -209.5 N/mm^2
x_c = 36. mm
u_c = 11.16 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -209.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 517.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 16.76 N/mm^2
$\tau_o = Tx_o t/J_t$ = 500.5 N/mm^2
t_c = 1224. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 920. N/mm^2

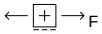
CI08.xxxx.031

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

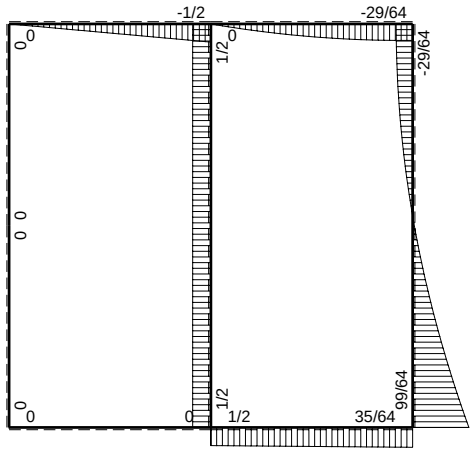

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64

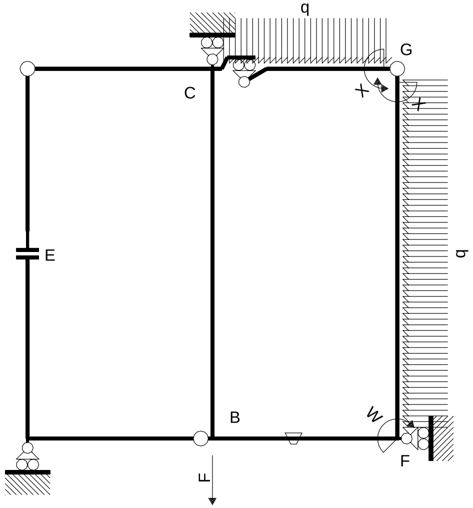

-3/64
1/2
61/64
0
0
0
0
0
3/64
-2

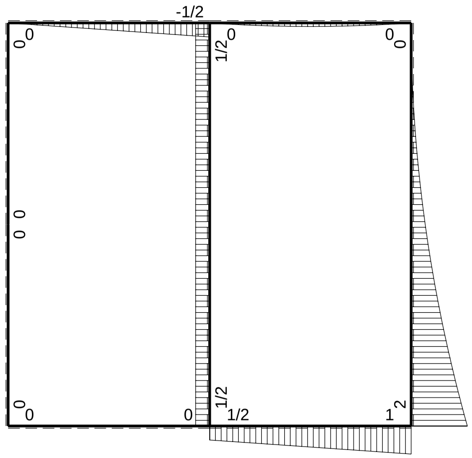

F


F

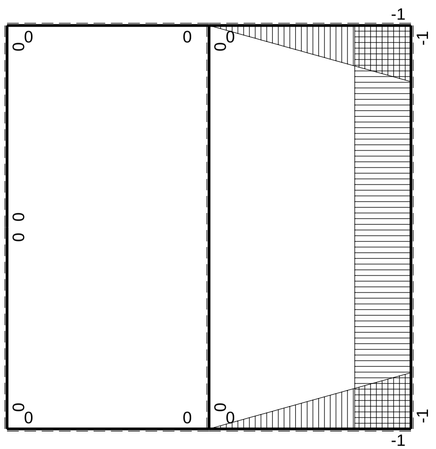

-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
1/2
35/64
99/64


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	$-1/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(-5/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	$-Fb+3/2Fx-1/2Fx^2/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
totali							$-29/24Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							29/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

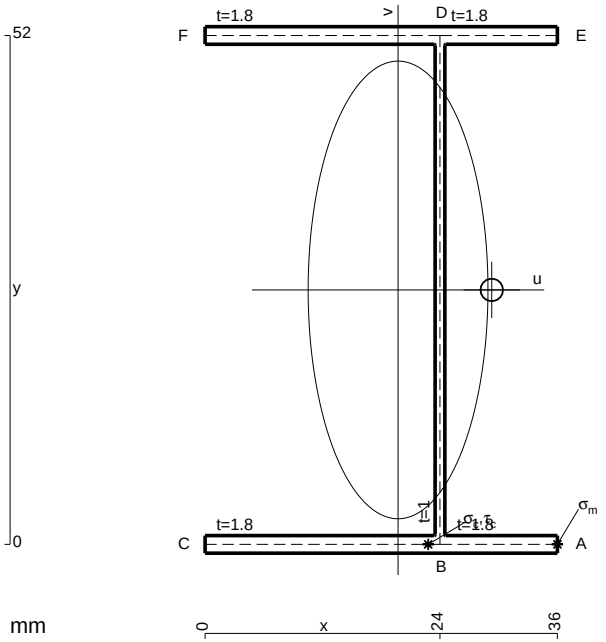
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

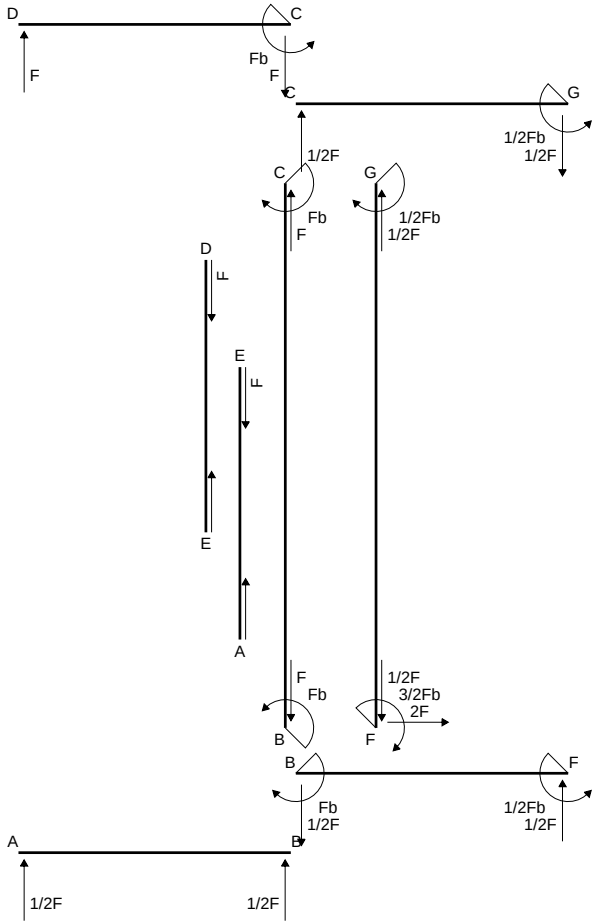
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

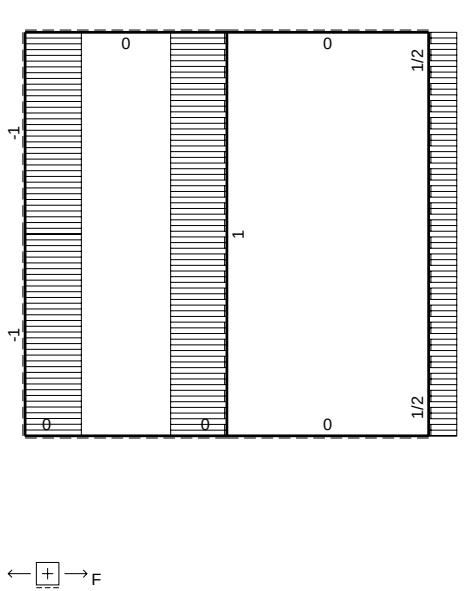


A = 181.6 mm^2
J_u = 99327. mm^4
J_v = 15333. mm^4
J_t = 157.3 mm^4
x_o = 9.574 mm
x_g = 19.72 mm
T_y = 1715. N
M_x = -840350. Nmm
x_m = 36. mm
u_m = 16.28 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -220. N/mm^2
x_c = 24. mm
u_c = 4.282 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -220. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 198.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.77 N/mm^2
$\tau_o = Tx_o t/J_t$ = 187.9 N/mm^2
t_c = 6174. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 408.4 N/mm^2

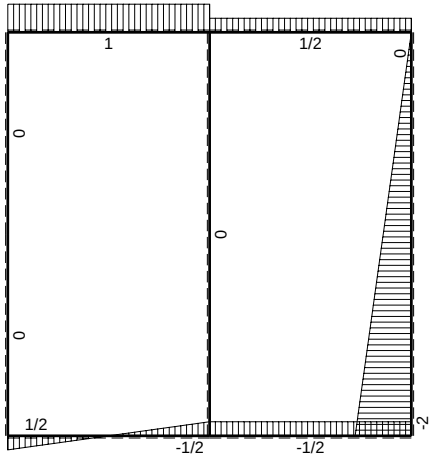
CI08.xxxx.032

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

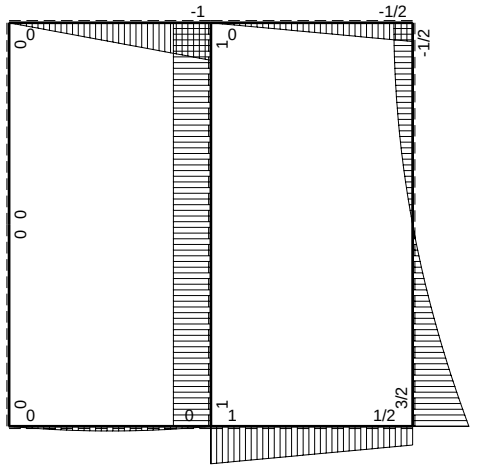

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

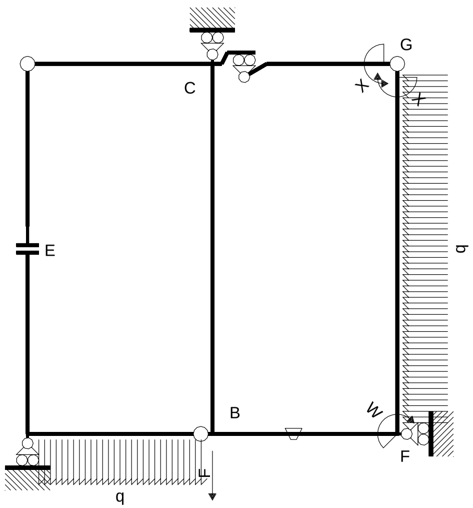
F

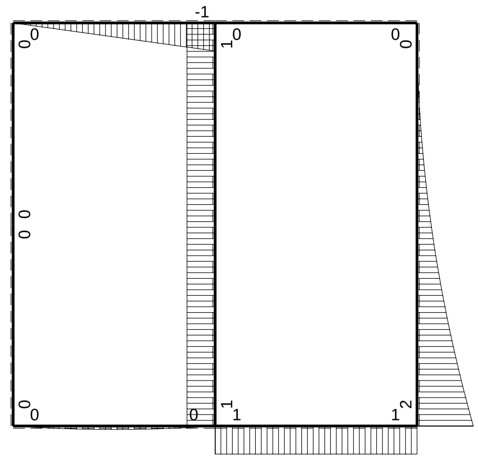

1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2

F

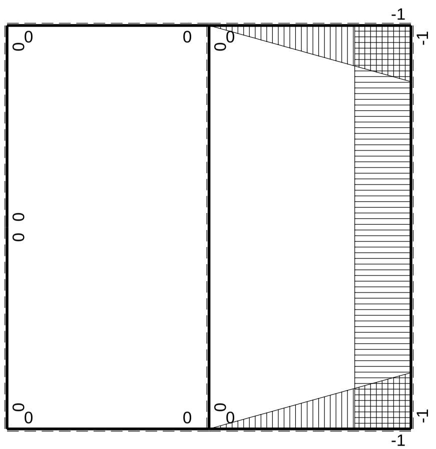

-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

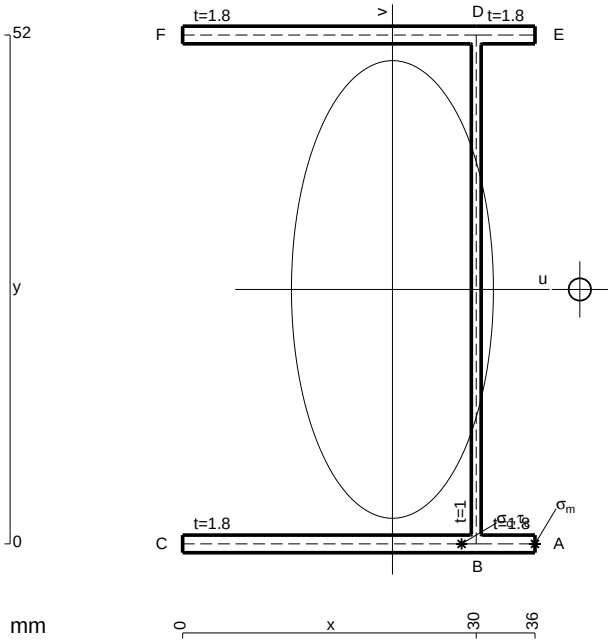
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 248492 CI08.xxxx.033

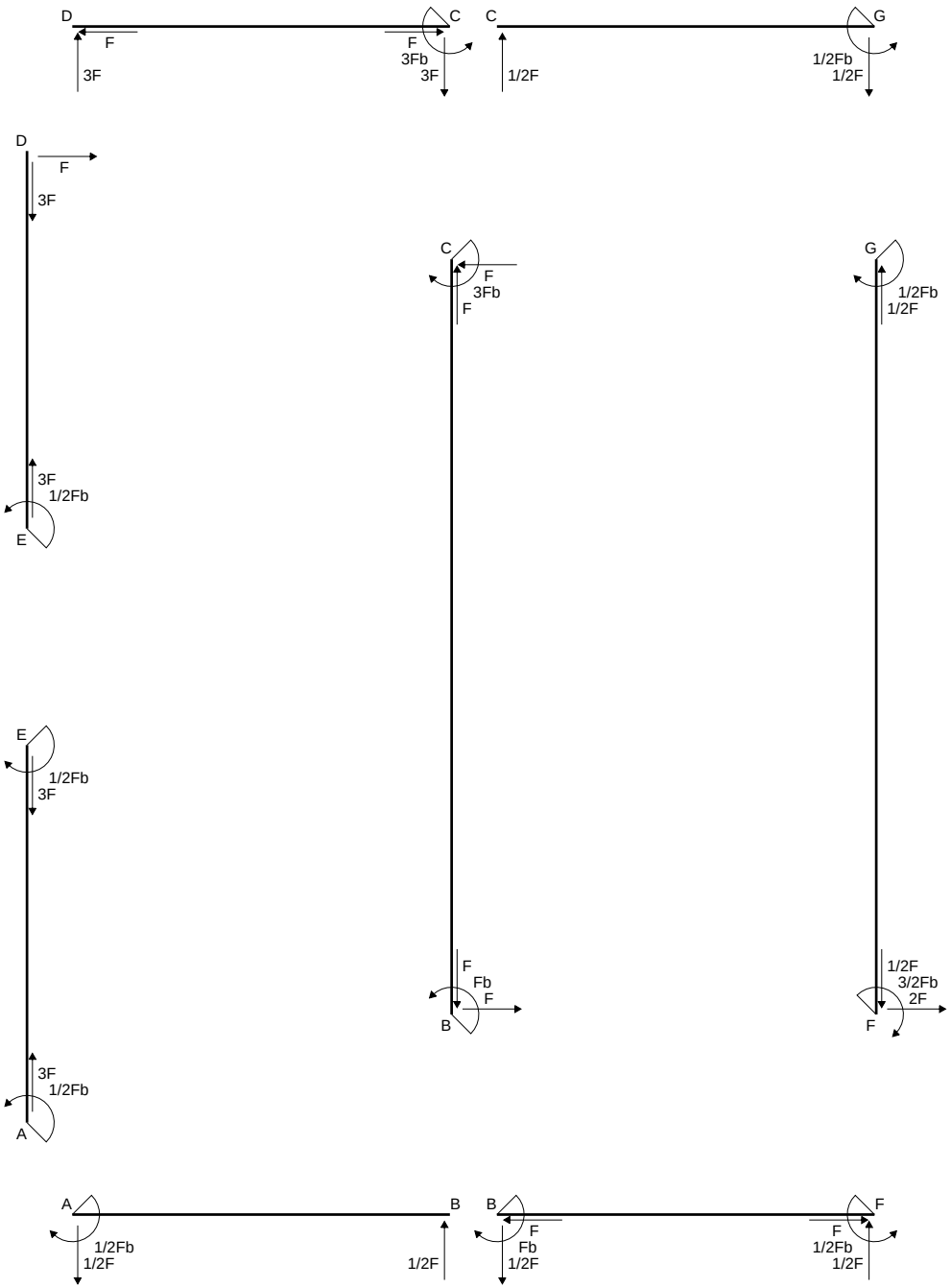


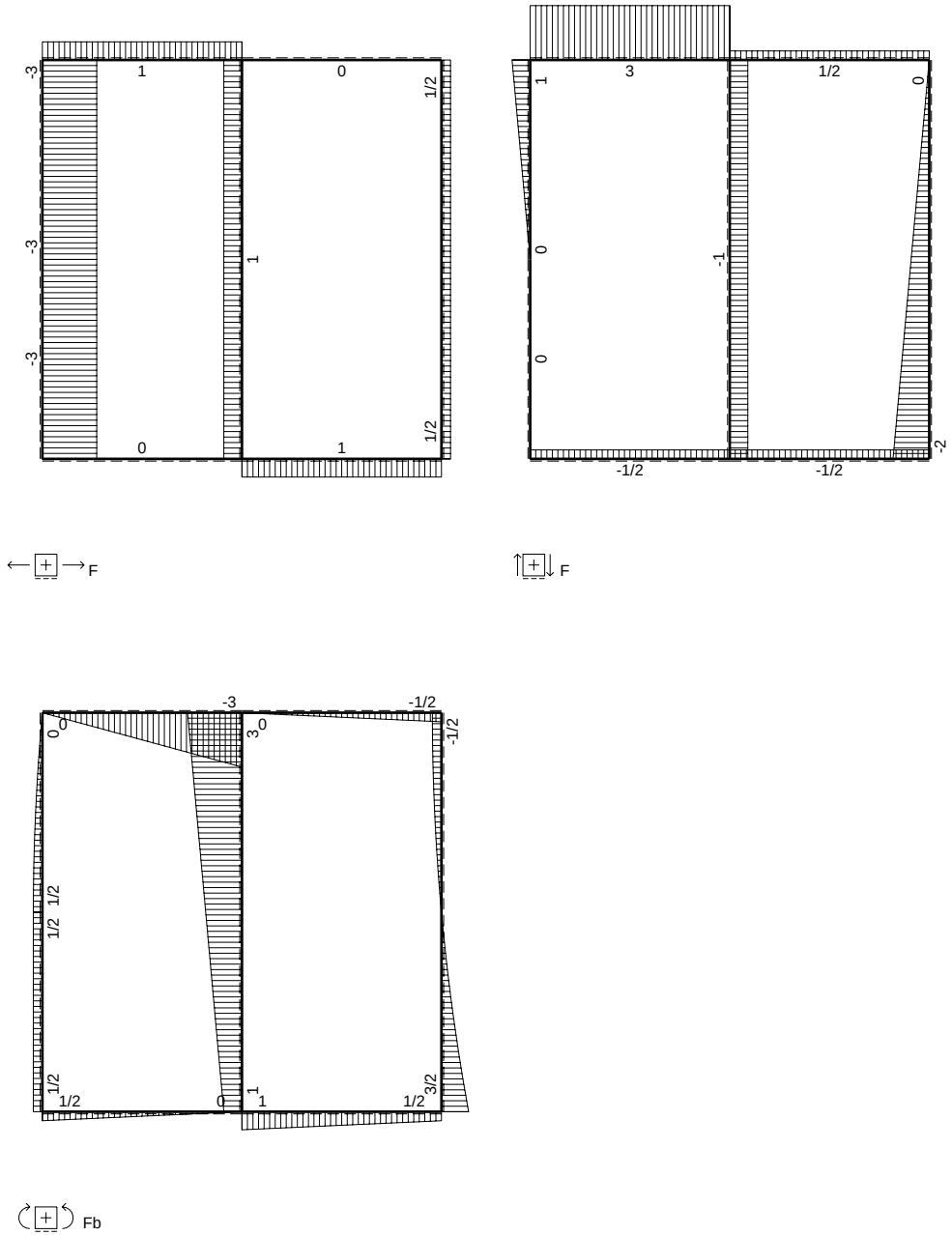
A = 181.6 mm^2
J_u = 99327. mm^4
J_v = 19341. mm^4
J_t = 157.3 mm^4
x_o = 19.15 mm
x_g = 21.44 mm
T_y = 1650. N
M_x = -874500. Nmm
x_m = 36. mm
u_m = 14.56 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -228.9 N/mm^2
x_c = 30. mm
u_c = 8.564 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -228.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 374.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.96 N/mm^2
$\tau_o = Tx_o t/J_t$ = 361.5 N/mm^2
t_c = 2970. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 687.9 N/mm^2

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

CI08.xxxx.033

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
F
Fb
F
1/2F
3/2Fb
2F
B
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


-3
1
0
1/2
-3
1
-3
0
1
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^{2}b^2	(-1/2+1/2)Fb^2/EJ	1/3XbEJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2XbEJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3Fb^2/EJ	8/3XbEJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

PROCEDIMENTO E RISULTATI 218343 CI08.xxxx.034

$$L^{XX}_{BF} = \int_o^b \left(x^2/b^2 \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2 \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b \right)\ 1/EJ\ = 1/3\ b/EJ$$

$$L^{XX}_{FB} = \int_o^b \left(1 -2\ x/b + x^2/b^2 \right)\ 1/EJ\ dx = \left[x - x^2/b +1/3\ x^3/b^2 \right]_o^b\ 1/EJ$$

$$= \left(b - b +1/3\ b \right)\ 1/EJ\ = 1/3\ b/EJ$$

$$L^{XX}_{GC} = \int_o^b \left(1 -2\ x/b + x^2/b^2 \right)\ 1/EJ\ dx = \left[x - x^2/b +1/3\ x^3/b^2 \right]_o^b\ 1/EJ$$

$$= \left(b - b +1/3\ b \right)\ 1/EJ\ = 1/3\ b/EJ$$

$$L^{XX}_{CG} = \int_o^b \left(x^2/b^2 \right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2 \right]_o^b\ 1/EJ$$

$$= \left(1/3\ b \right)\ 1/EJ\ = 1/3\ b/EJ$$

$$L^{XX}_{FG} = \int_o^{2b} \left(1 \right)\ 1/EJ\ dx = \left[x \right]_o^{2b}\ 1/EJ$$

$$= \left(2\ b \right)\ 1/EJ\ = 2\ b/EJ$$

$$L^{XX}_{GF} = \int_o^{2b} \left(1 \right)\ 1/EJ\ dx = \left[x \right]_o^{2b}\ 1/EJ$$

$$= \left(2\ b \right)\ 1/EJ\ = 2\ b/EJ$$

$$L^{Xo}_{BF} = \int_o^b \left(- x/b \right) Fb\ 1/EJ\ dx + \int_o^b \left(x/b \right) \theta\ dx = \left[-1/2\ x^2/b \right]_o^b Fb\ 1/EJ\ + \left[1/2\ x^2/b \right]_o^b \theta$$

$$= \left(-1/2\ b \right) Fb\ 1/EJ\ + \left(1/2\ b \right) \theta\ = 0$$

$$L^{Xo}_{FB} = \int_o^b \left(-1 + x/b \right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b \right) \theta\ dx$$

$$= \left[- x +1/2\ x^2/b \right]_o^b Fb\ 1/EJ\ + \left[- x +1/2\ x^2/b \right]_o^b \theta$$

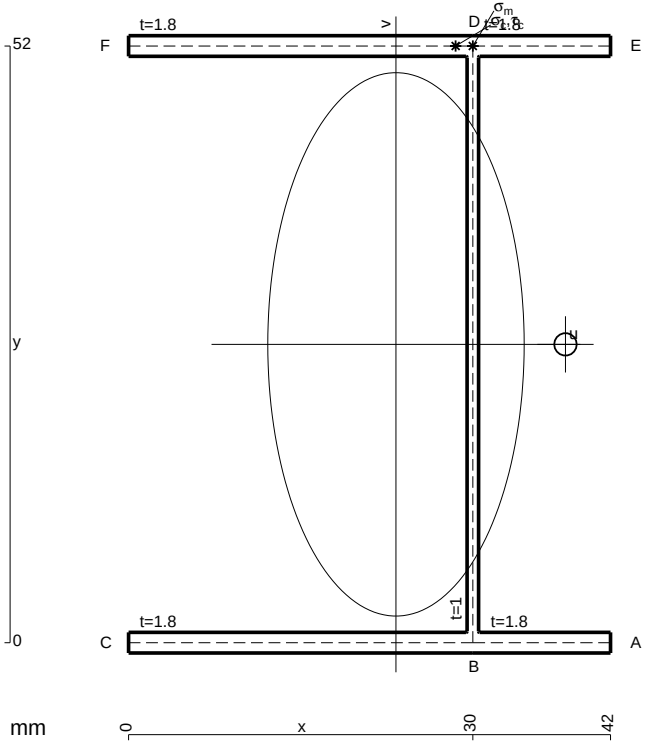
$$= \left(- b +1/2\ b \right) Fb\ 1/EJ\ + \left(- b +1/2\ b \right) \theta\ = 0$$

$$L^{Xo}_{FG} = \int_o^{2b} \left(-2 +2\ x/b -1/2\ x^2/b^2 \right) Fb\ 1/EJ\ dx = \left[-2\ x + x^2/b -1/6\ x^3/b^2 \right]_o^{2b} Fb\ 1/EJ$$

$$= \left(-4\ b +4\ b -4/3\ b \right) Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$$

$$L^{Xo}_{GF} = \int_o^{2b} \left(-1/2\ x^2/b^2 \right) Fb\ 1/EJ\ dx = \left[-1/6\ x^3/b^2 \right]_o^{2b} Fb\ 1/EJ$$

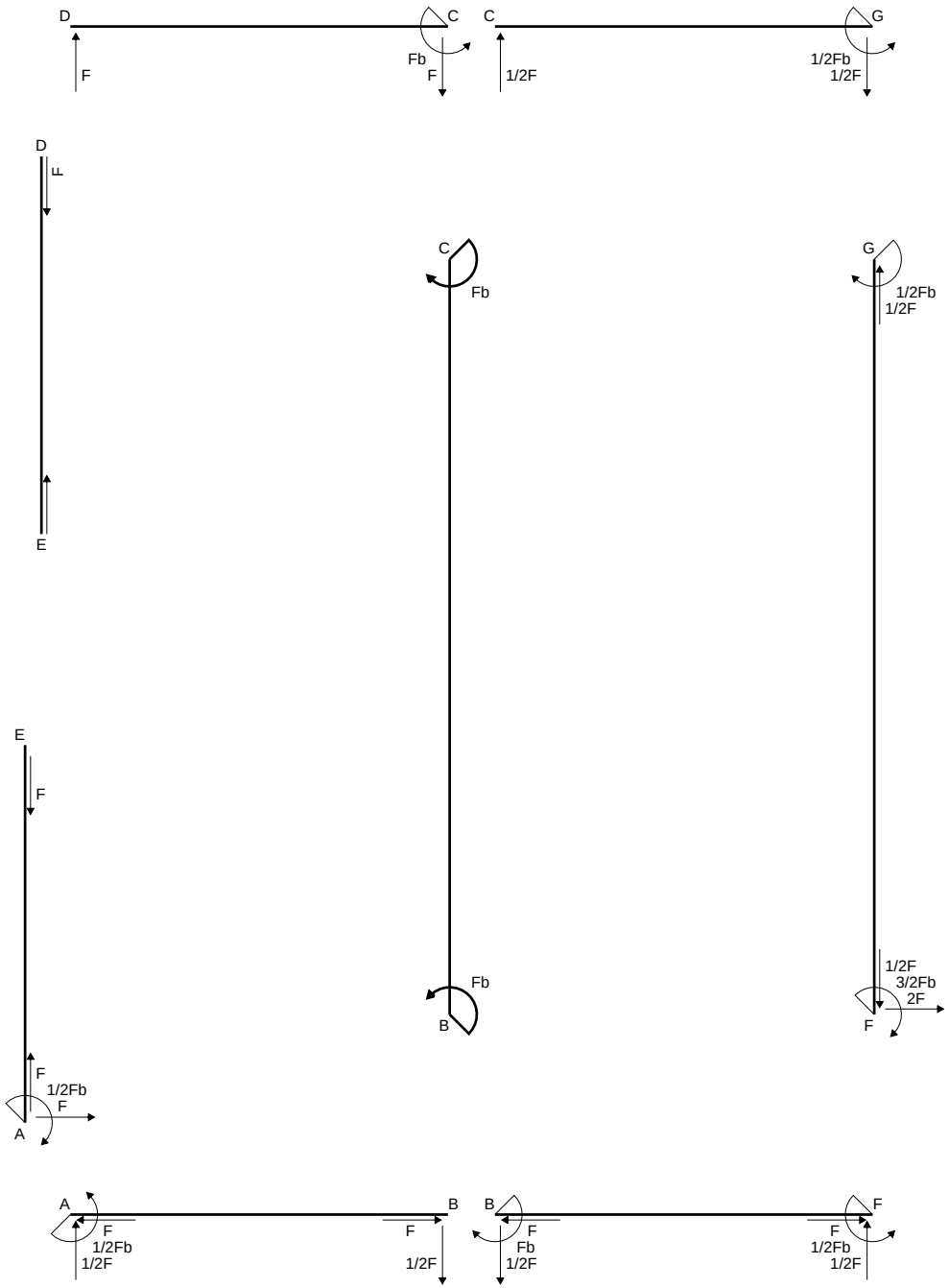
$$= \left(-4/3\ b \right) Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$$

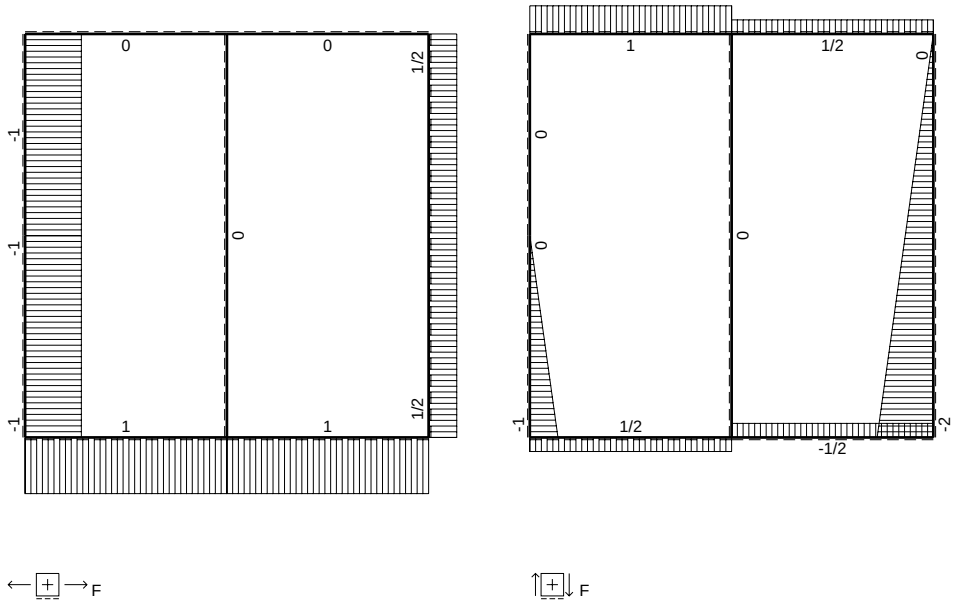


A = 203.2 mm^2
J_u = 113929. mm^4
J_v = 25361. mm^4
J_t = 180.6 mm^4
x_o = 14.77 mm
x_g = 23.3 mm
N = 600. N
T_y = 1800. N
M_x = -1026000. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 6.697 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = 237.1 N/mm^2
x_c = 30. mm
y_c = 52. mm
u_c = 6.697 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = 237.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 277.3 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.32 N/mm^2
$\tau_o = Tx_o t/J_t$ = 265. N/mm^2
t_c = 1080. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 535.6 N/mm^2

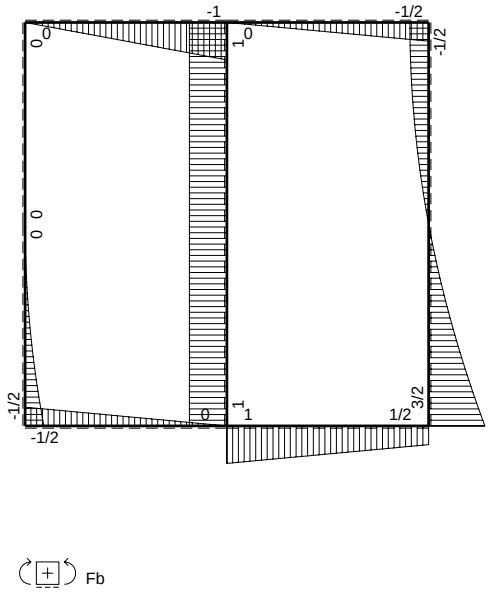
CI08.xxxx.034

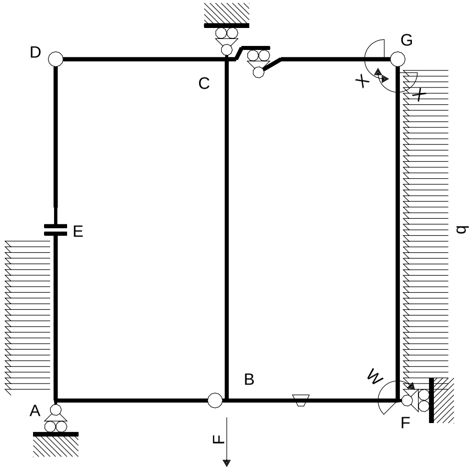
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

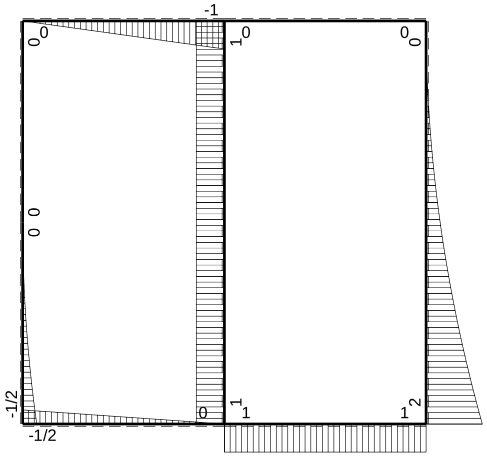

D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
G
1/2Fb
1/2F
Fb
B
1/2F
3/2Fb
2F
F
E
F
F
1/2Fb
F
A
A
B
B
F
F
F
F
1/2Fb
1/2F
1/2F
Fb
1/2F
1/2Fb
1/2F


0
0
1/2
-1
0
-1
1
1
1/2
F

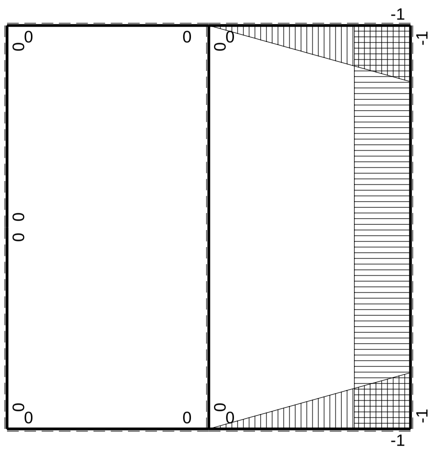
1
1/2
0
0
0
-1
1/2
-2
-1/2
F


-1
-1/2
0
1
0
-1/2
0
0
-1/2
0
1
1
1/2
3/2
-1/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 271093 CI08.xxxx.035

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

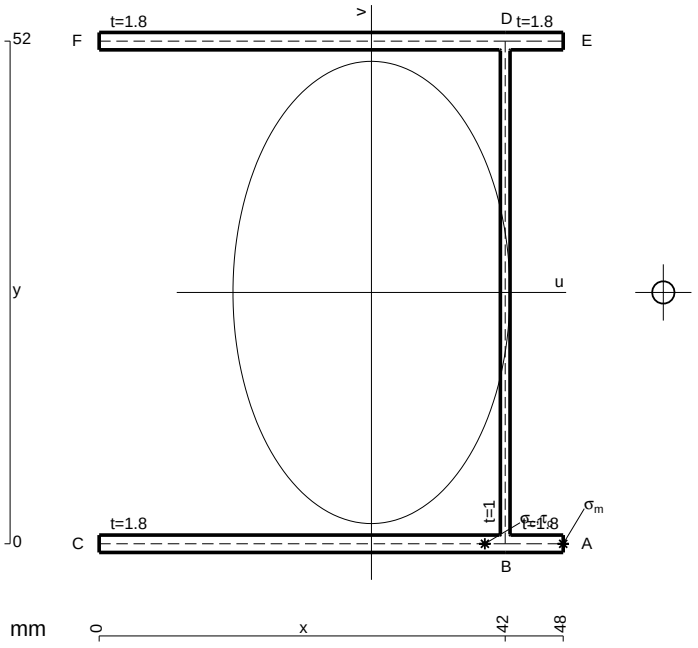
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

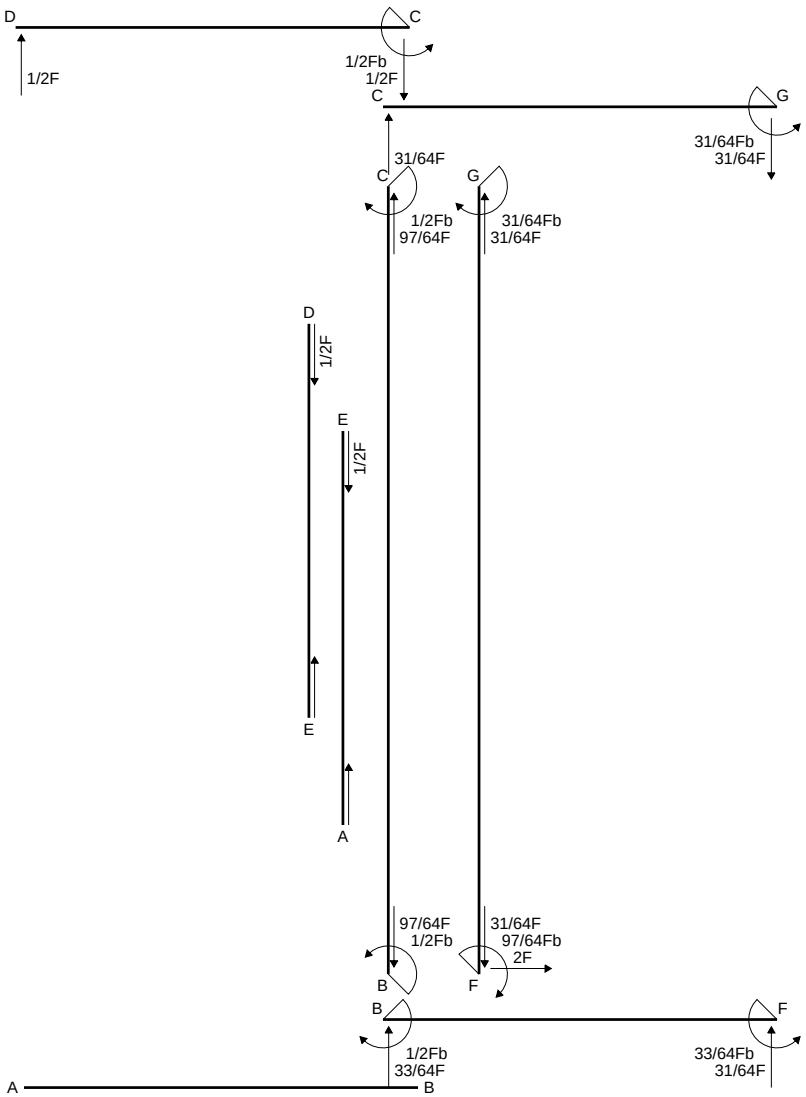
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

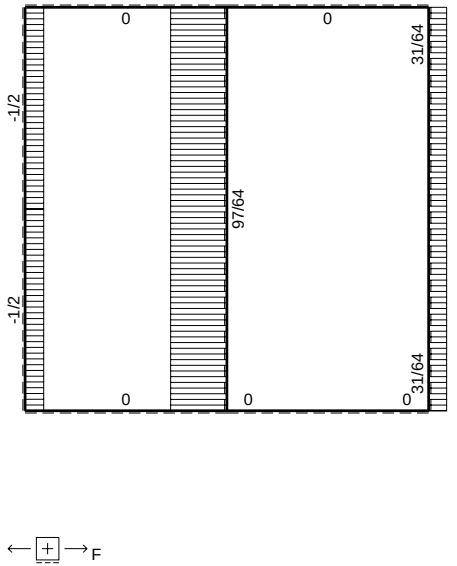


A = 224.8 mm^2
J_u = 128530. mm^4
J_v = 46128. mm^4
J_t = 204. mm^4
x_o = 30.2 mm
x_g = 28.16 mm
T_y = 1620. N
M_x = -988200. Nmm
x_m = 48. mm
u_m = 19.84 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -199.9 N/mm^2
x_c = 42. mm
u_c = 13.84 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -199.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 445.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.76 N/mm^2
$\tau_o = Tx_o t/J_t$ = 431.7 N/mm^2
t_c = 2916. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 797.1 N/mm^2

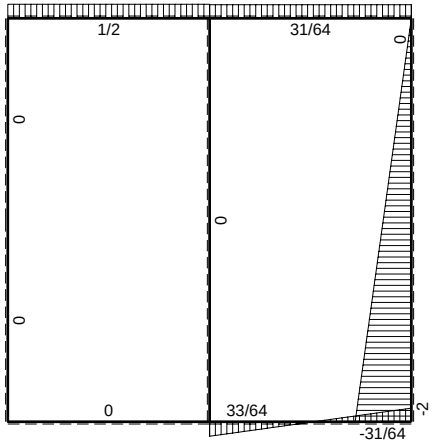
CI08.xxxx.035

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

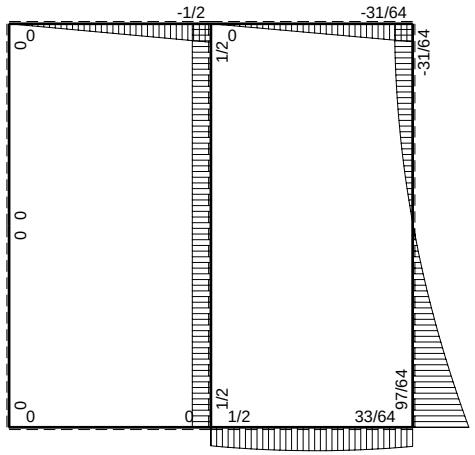

D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64

F


1/2
31/64
0
0
0
0
0
33/64
-2
-31/64

F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
0
0
1/2
33/64
97/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb²/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 216255 CI08.xxxx.036

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

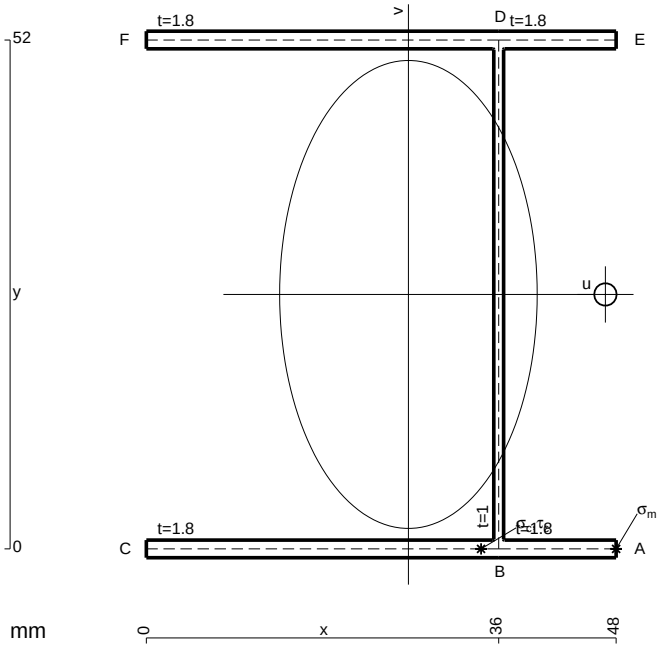
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

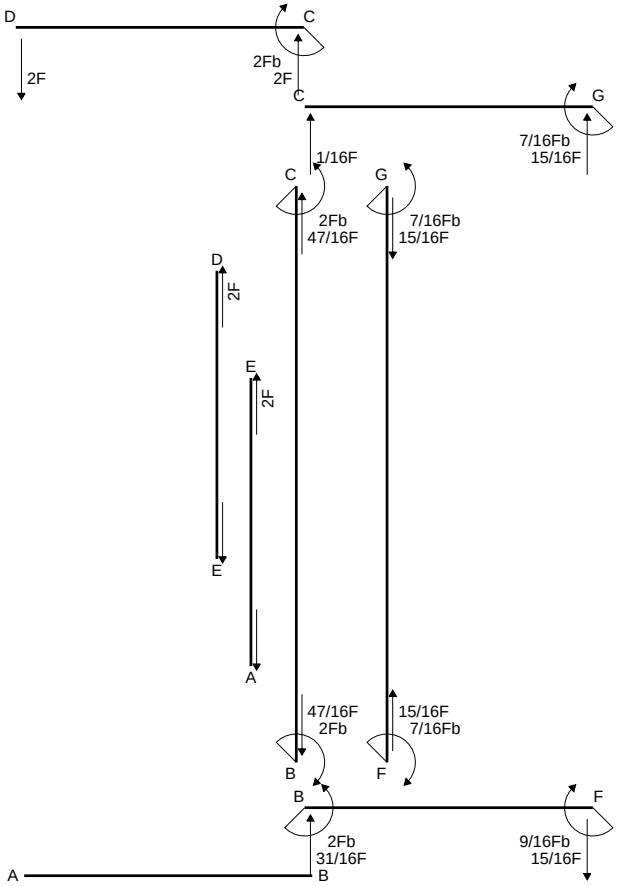
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

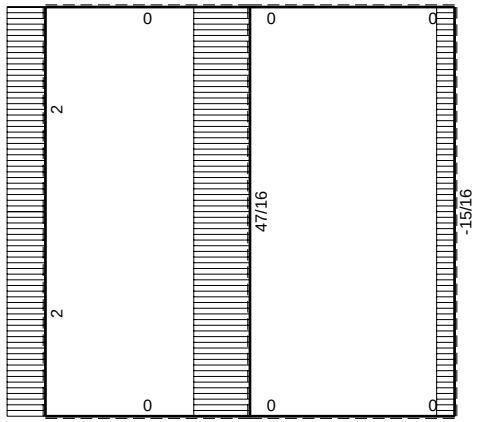


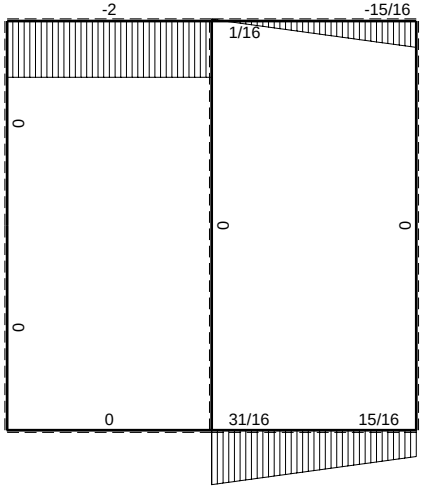
A = 224.8 mm^2
J_u = 128530. mm^4
J_v = 38934. mm^4
J_t = 204. mm^4
x_o = 20.13 mm
x_g = 26.78 mm
T_y = 1595. N
M_x = -1036750. Nmm
x_m = 48. mm
u_m = 21.22 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -209.7 N/mm^2
x_c = 36. mm
u_c = 9.224 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -209.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 295. N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.62 N/mm^2
$\tau_o = Tx_o t/J_t$ = 283.4 N/mm^2
t_c = 5742.
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 552.3 N/mm^2

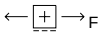
CI08.xxxx.036

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

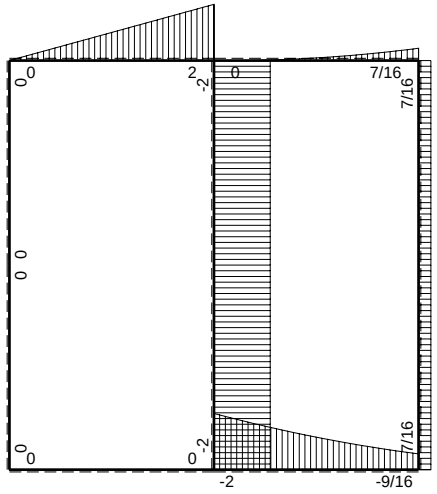

D
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0


-2
-15/16
1/16
0
0
0
0
0
31/16
15/16


F


F


0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
7/16
-2
-9/16


Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
totali							$7/6Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							-7/16Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

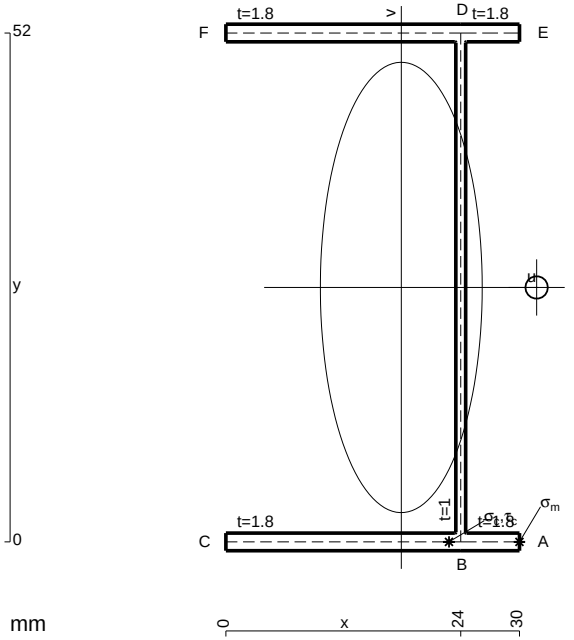
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

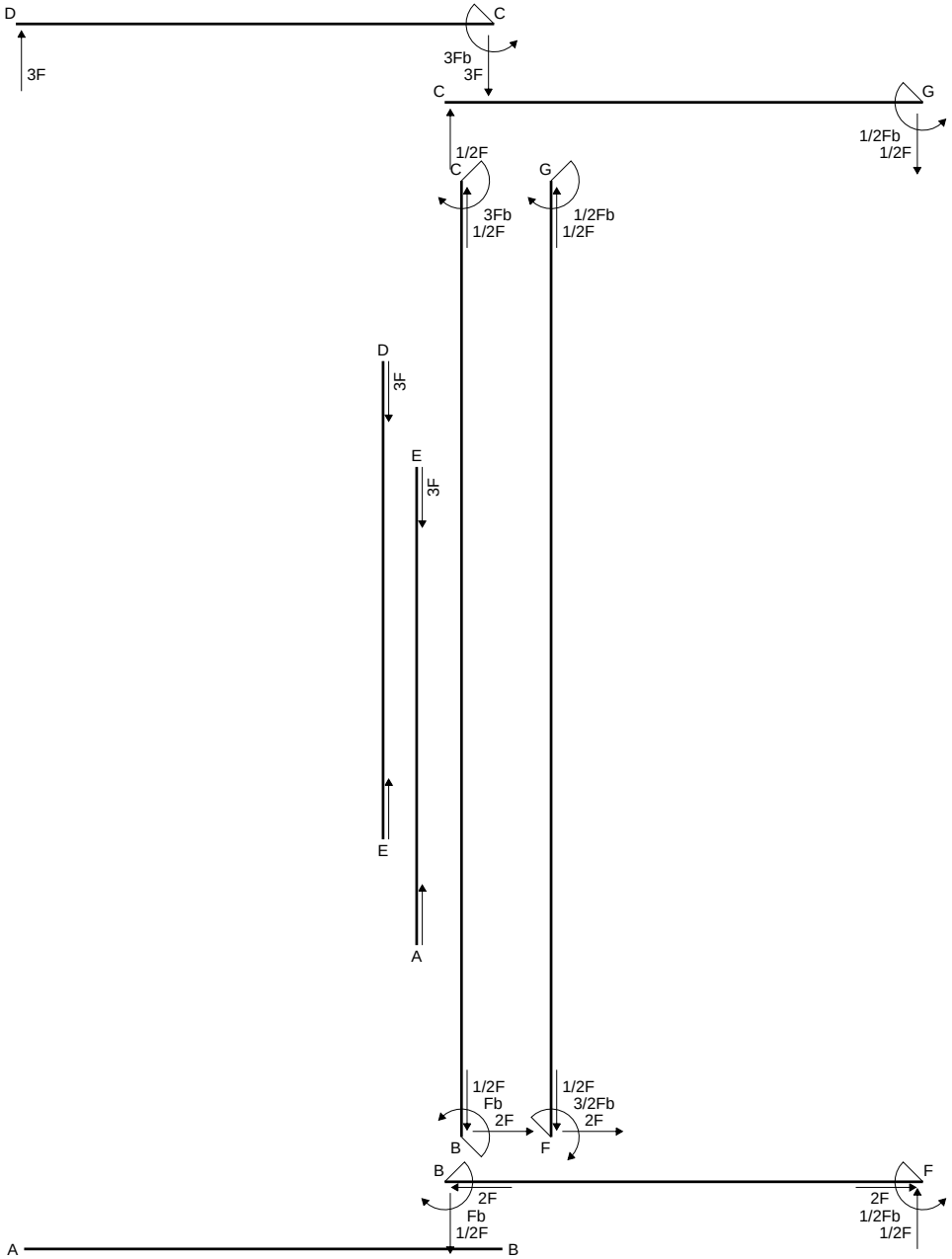
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

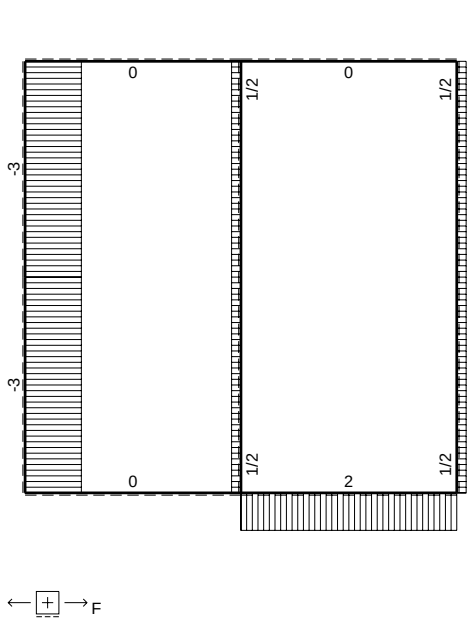


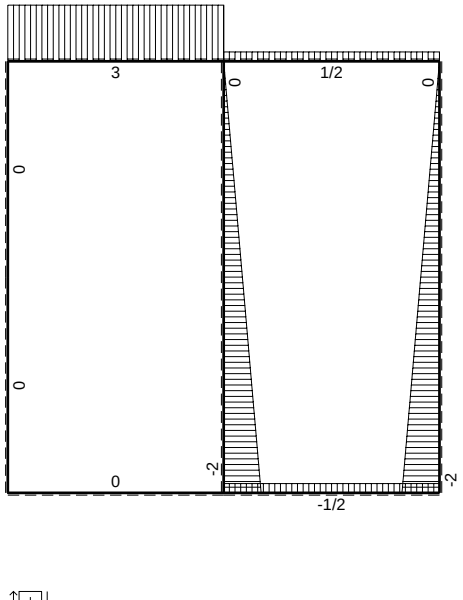
A = 160. mm^2
J_u = 84725. mm^4
J_v = 10943. mm^4
J_t = 134. mm^4
x_o = 13.83 mm
x_g = 17.92 mm
T_y = -1020. N
M_x = 703800. Nmm
x_m = 30. mm
u_m = 12.07 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = 216. N/mm^2
x_c = 24. mm
u_c = 6.075 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = 216. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 197. N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.512 N/mm^2
$\tau_o = Tx_o t/J_t$ = 189.5 N/mm^2
t_c = 918. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 403.9 N/mm^2

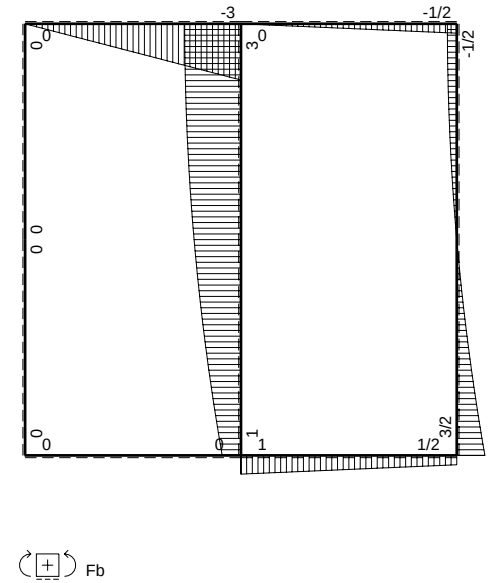
CI08.xxxx.037

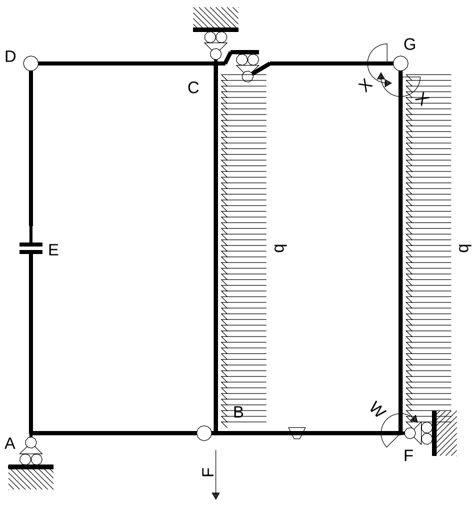
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

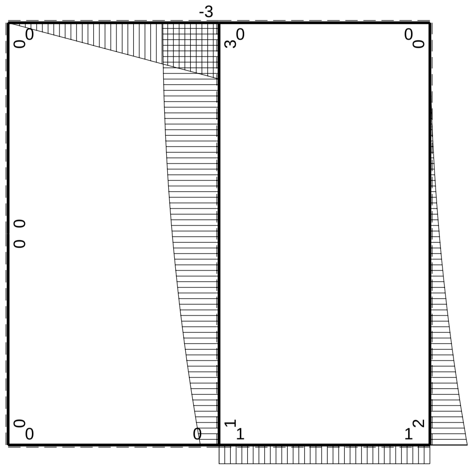

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
1/2
0
1/2
-3
-3
0
1/2
2
1/2
F

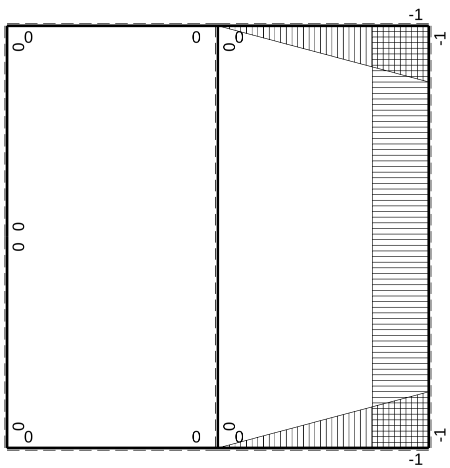

3
0
1/2
0
0
0
-2
0
-2
-1/2
F


-3
-1/2
0
0
3
-1/2
0
0
0
0
1
0
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 240450 CI08.xxxx.038

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

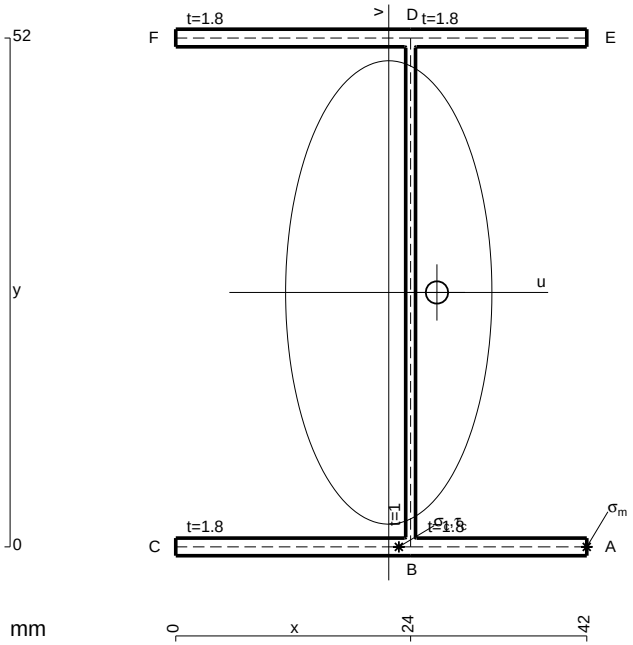
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

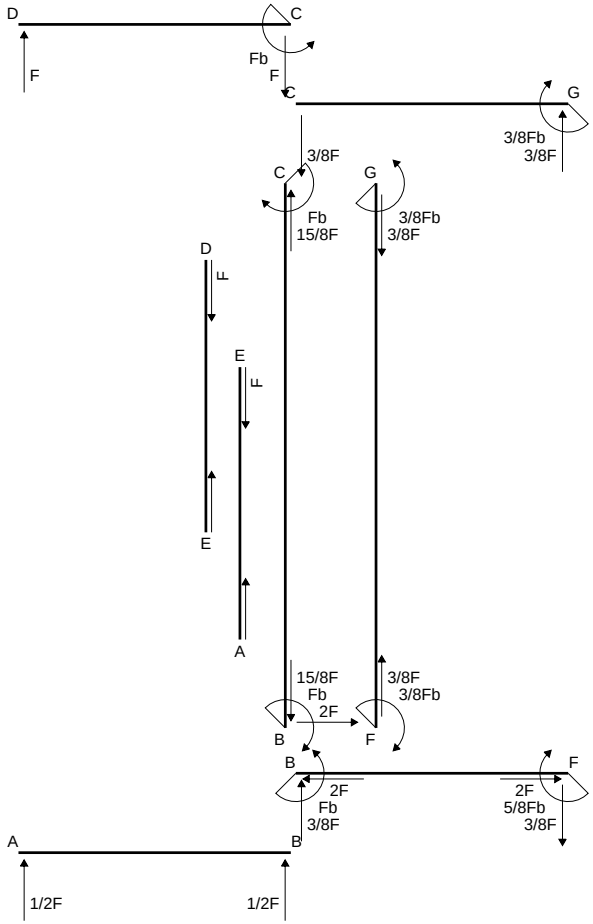
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

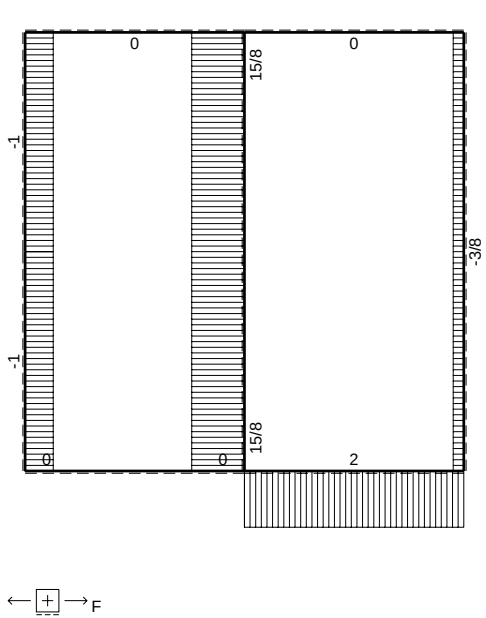


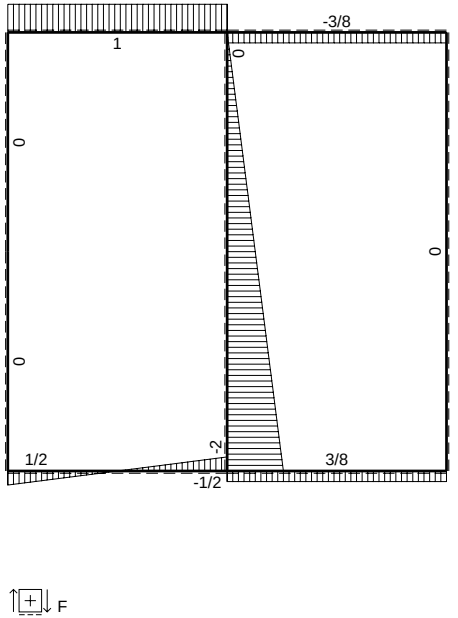
A = 203.2 mm^2
J_u = 113929. mm^4
J_v = 22575. mm^4
J_t = 180.6 mm^4
x_o = 4.924 mm
x_g = 21.77 mm
T_y = 1380. N
M_x = -1007400. Nmm
x_m = 42. mm
u_m = 20.23 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -229.9 N/mm^2
x_c = 24. mm
u_c = 2.232 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -229.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 75.27 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.558 N/mm^2
$\tau_o = Tx_o t/J_t$ = 67.71 N/mm^2
t_c = 828. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 264.3 N/mm^2

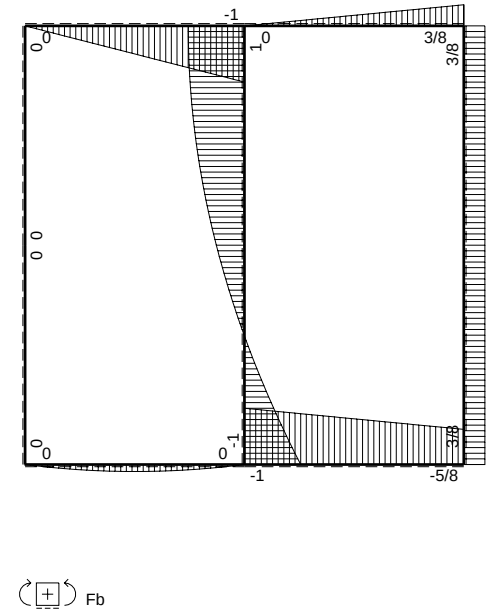
CI08.xxxx.038

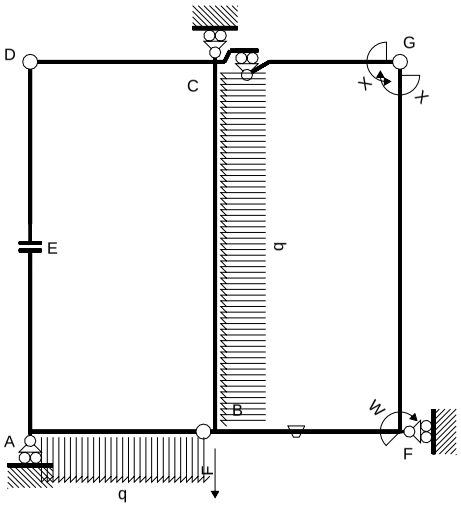
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F

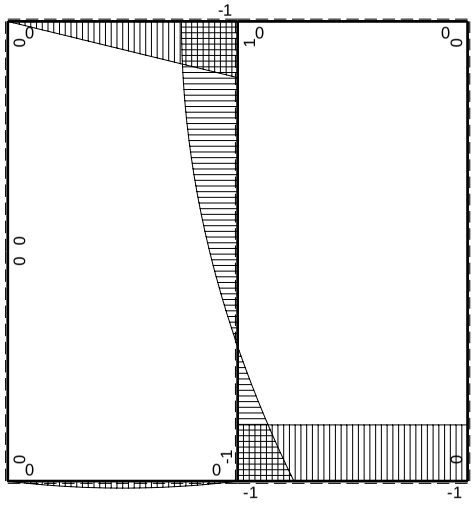

0
0
15/8
-1
-1
-3/8
0
0
15/8
0
0
2
F


-3/8
1
0
0
0
0
0
1/2
-2
3/8
-1/2
F

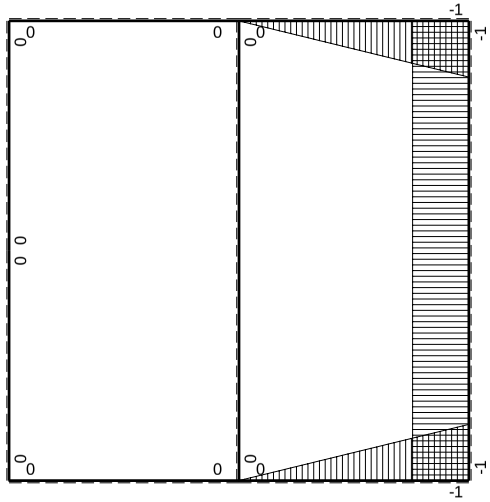

-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 237128 CI08.xxxx.039

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

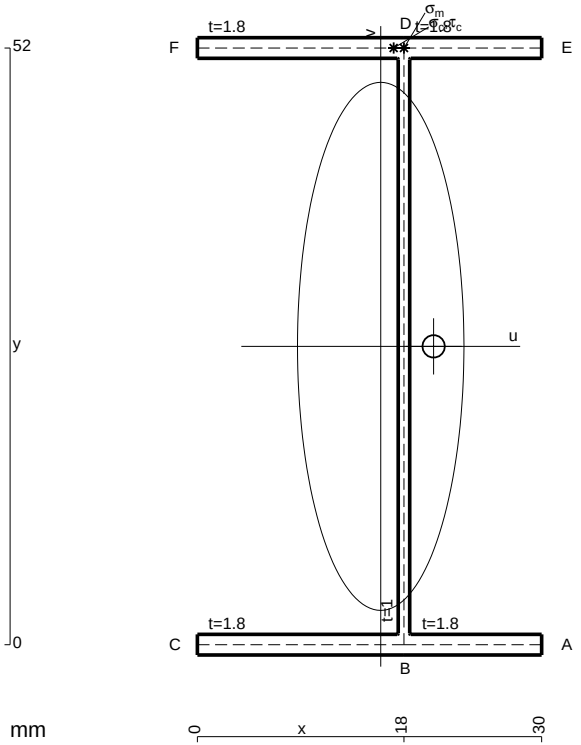
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

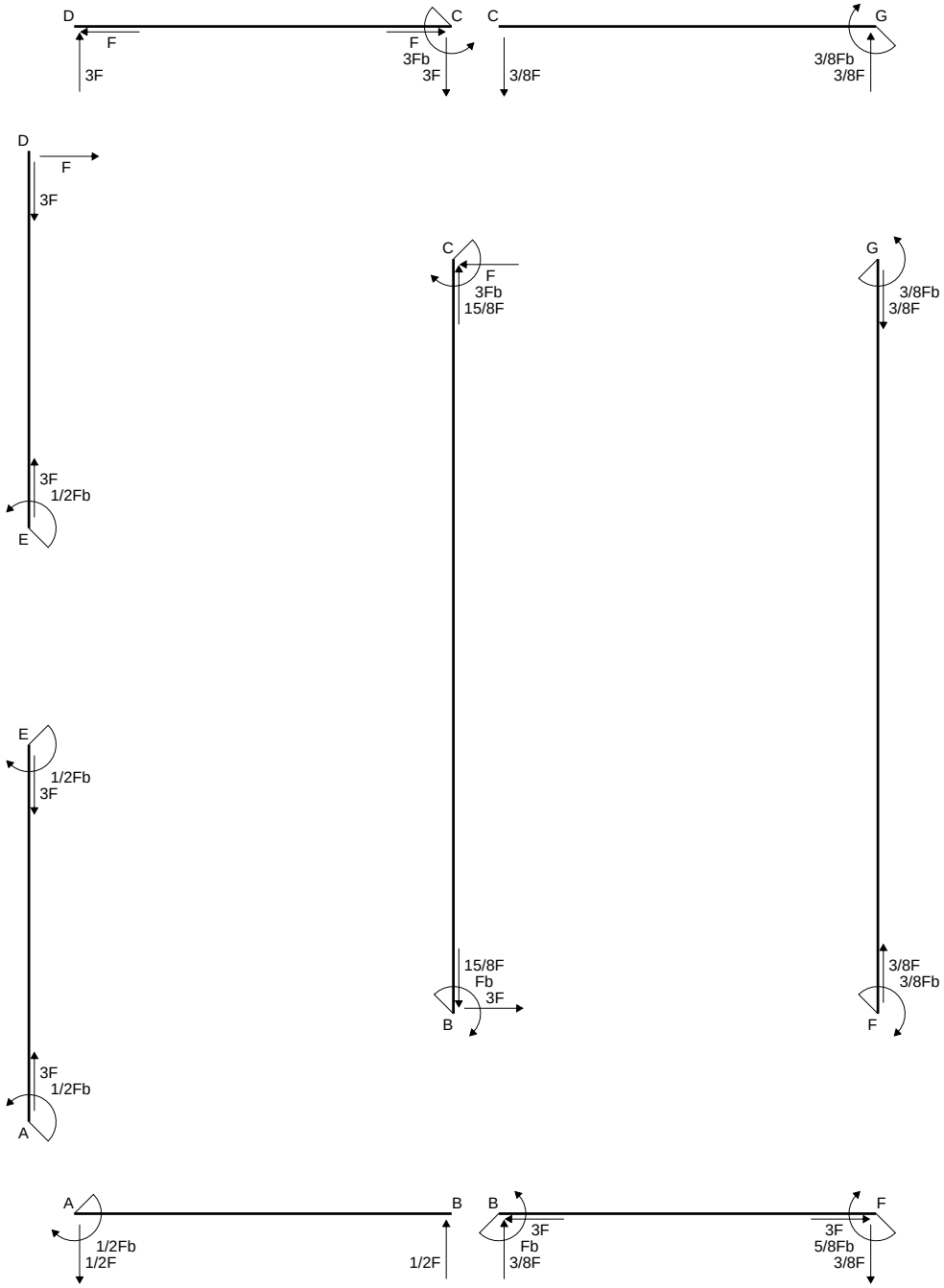
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

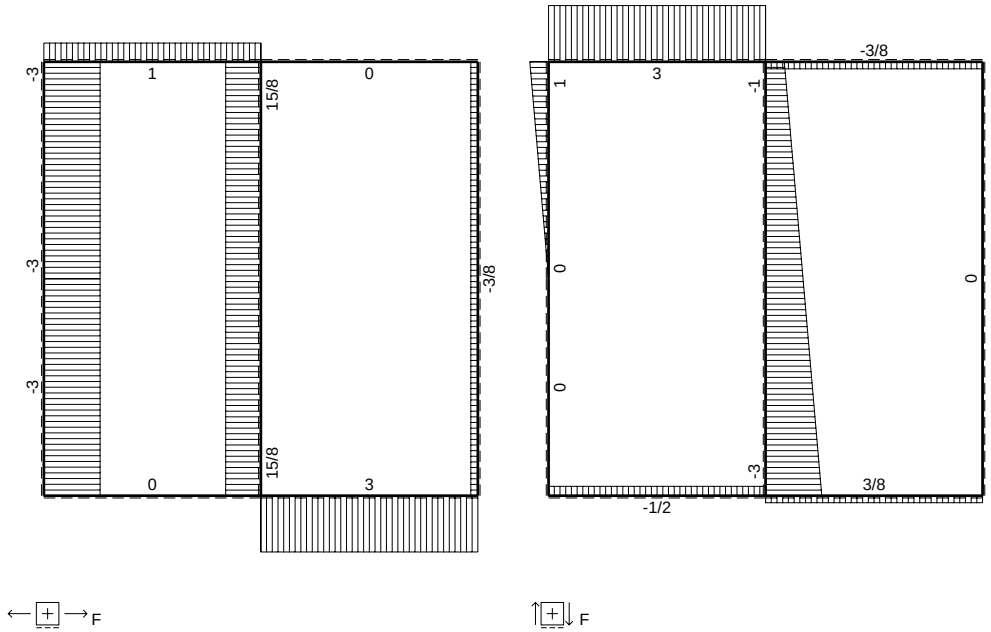


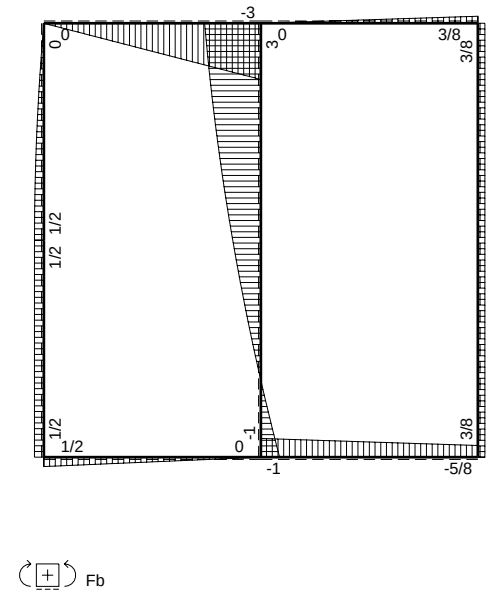
A = 160. mm^2
J_u = 84725. mm^4
J_v = 8416. mm^4
J_t = 134. mm^4
x_o = 4.61 mm
x_g = 15.98 mm
N = 1800. N
T_y = -1920. N
M_x = -739200. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 2.025 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = 238.1 N/mm^2
x_c = 18. mm
y_c = 52. mm
u_c = 2.025 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = 238.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 129.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.61 N/mm^2
$\tau_o = Tx_o t/J_t$ = 118.9 N/mm^2
t_c = 1728. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 327.1 N/mm^2

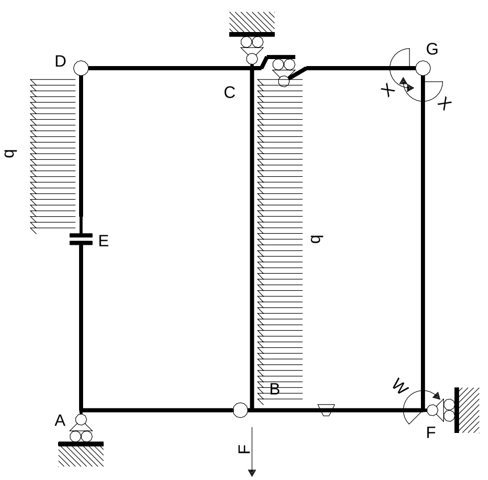
CI08.xxxx.039

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

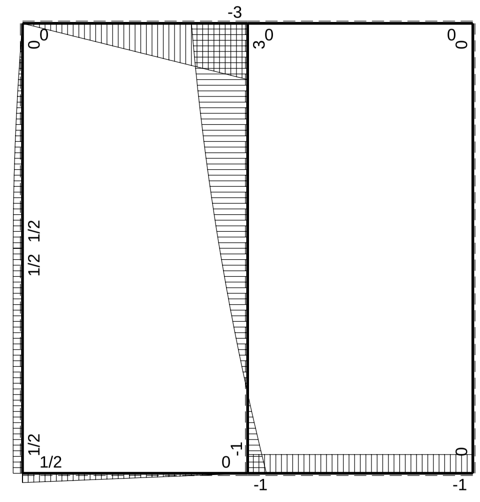

D
F
3F
C
F
3Fb
3F
C
3/8F
G
3/8Fb
3/8F
D
F
3F
C
F
3Fb
15/8F
G
3/8Fb
3/8F
3F
1/2Fb
E
15/8F
Fb
3F
B
3/8F
3/8Fb
F
E
1/2Fb
3F
3F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F


-3
1
15/8
0
-3
-3/8
-3
15/8
0
3
F
-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F

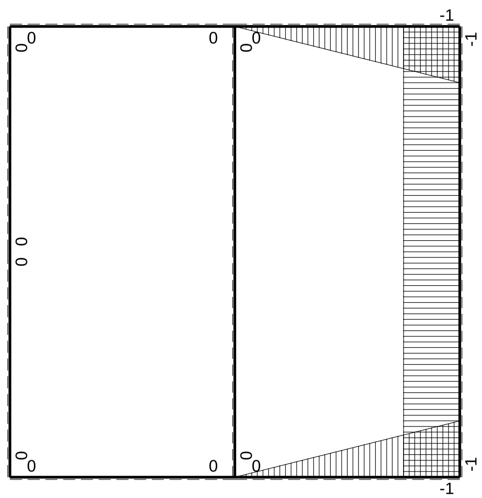

-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 248223 CI08.xxxx.040

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$3Fb-Fx-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-3Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

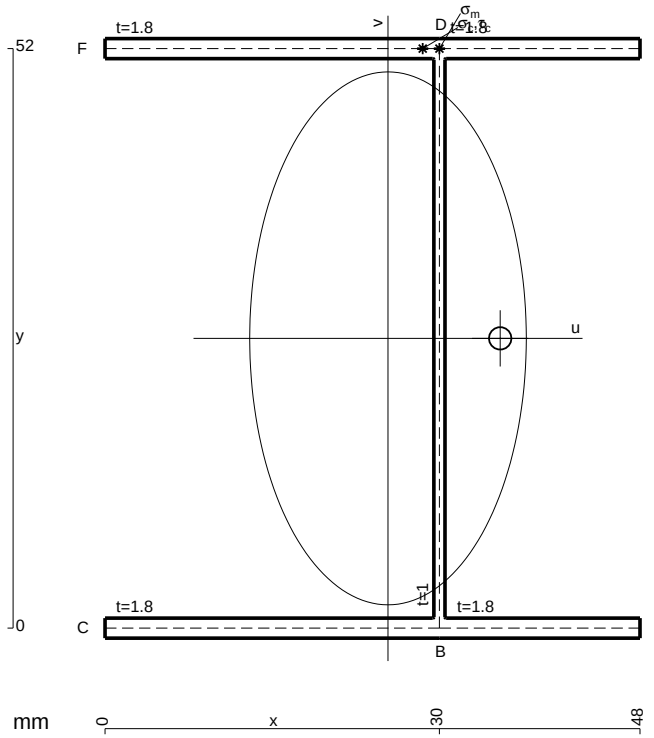
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

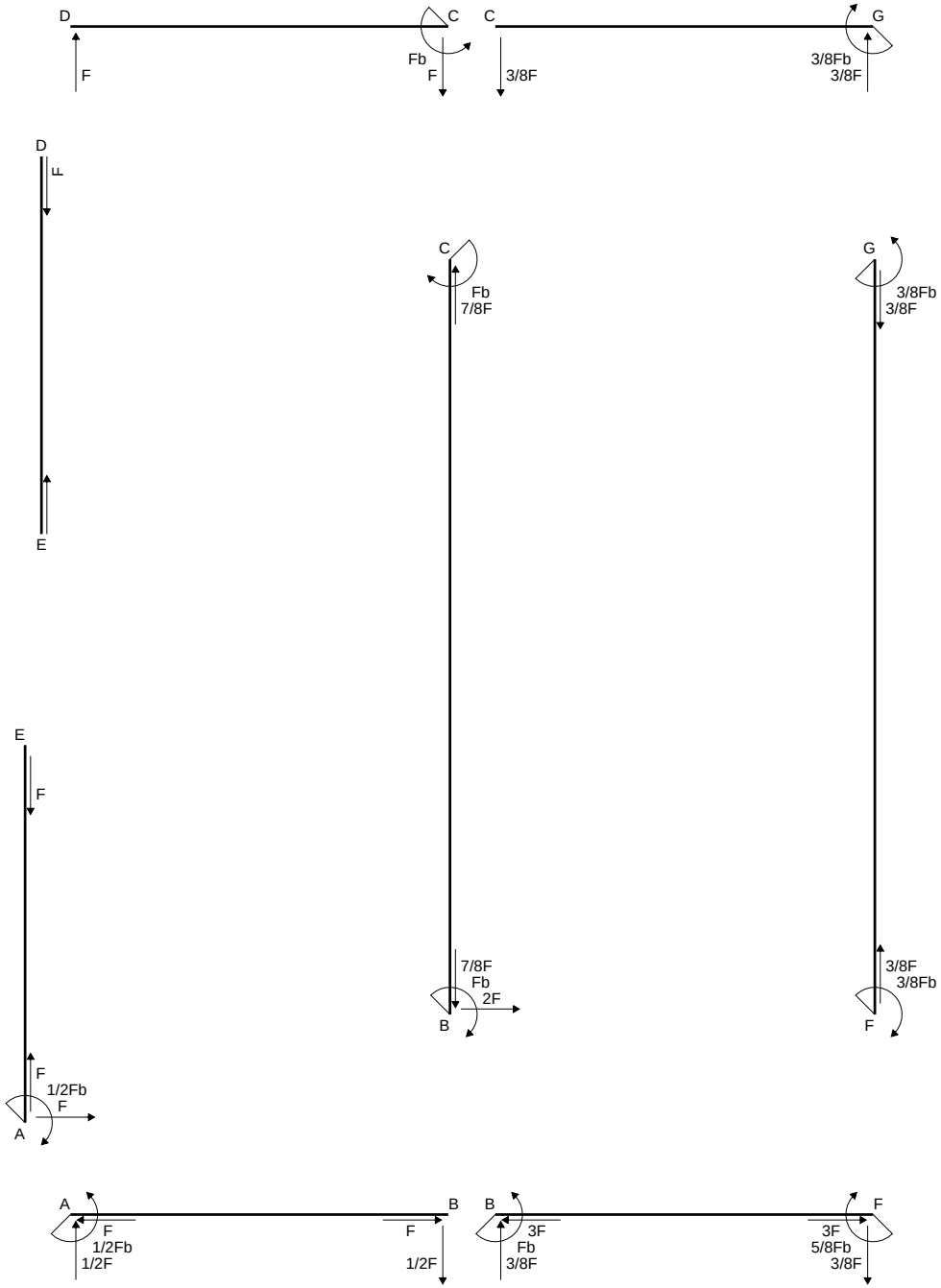
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

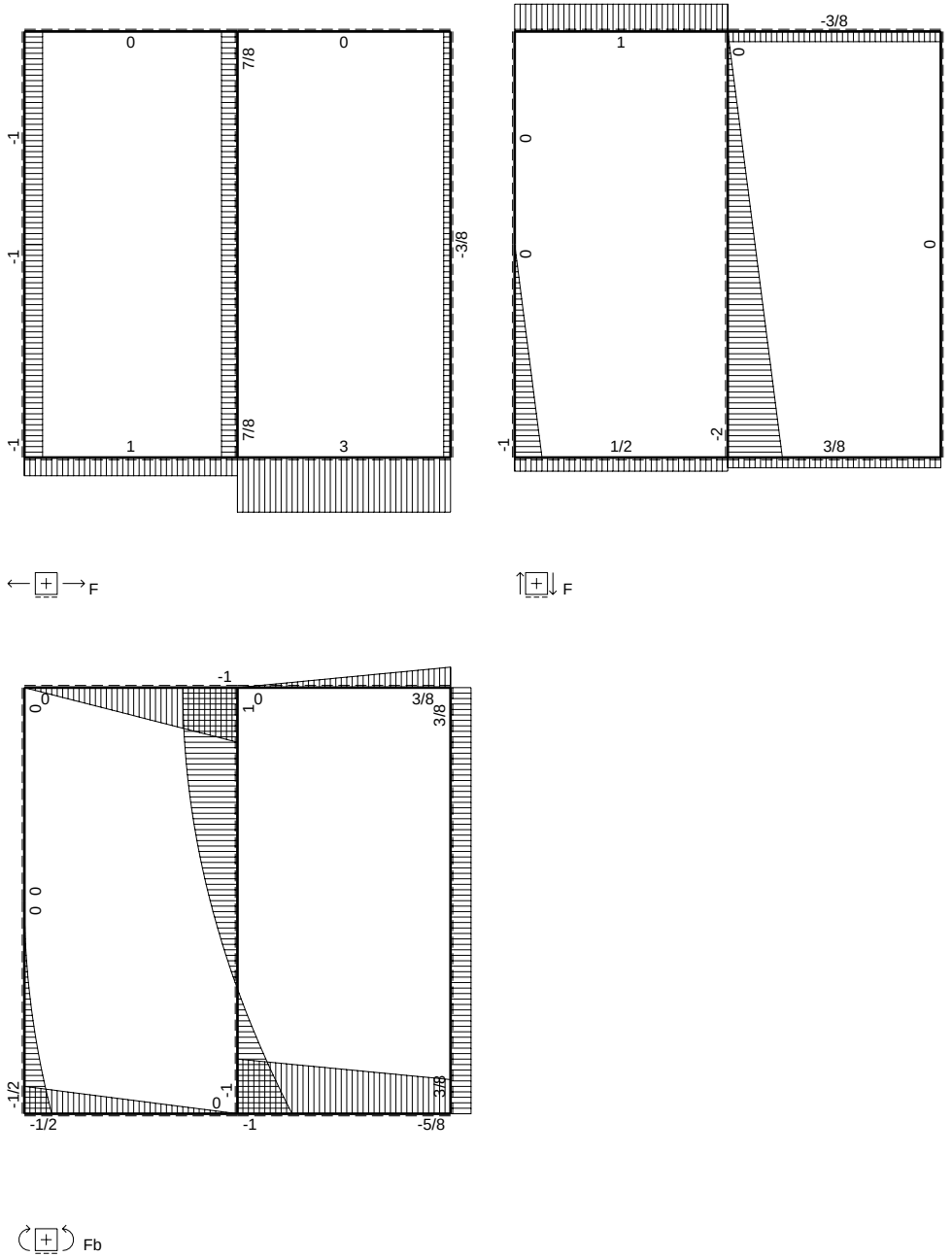


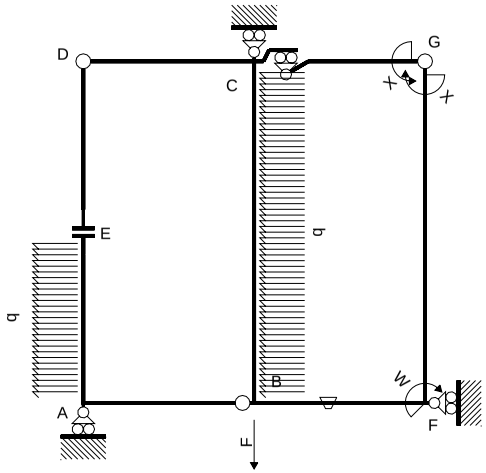
A = 224.8 mm^2
J_u = 128530. mm^4
J_v = 34617. mm^4
J_t = 204. mm^4
x_o = 10.07 mm
x_g = 25.39 mm
N = 780. N
T_y = 2340. N
M_x = -959400. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 4.612 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = 197.5 N/mm^2
x_c = 30. mm
y_c = 52. mm
u_c = 4.612 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = 197.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 222.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 14.2 N/mm^2
$\tau_o = Tx_o t/J_t$ = 207.9 N/mm^2
t_c = 1404. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 432.4 N/mm^2

CI08.xxxx.040

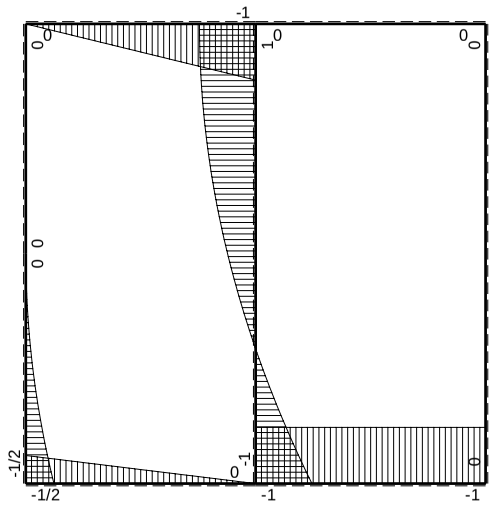
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
7/8F
G
3/8Fb
3/8F
E
F
F
1/2Fb
F
A
7/8F
Fb
2F
B
3/8F
3/8Fb
F
A
B
B
F
F
1/2Fb
1/2F
F
1/2F
3F
Fb
3/8F
3F
5/8Fb
3/8F

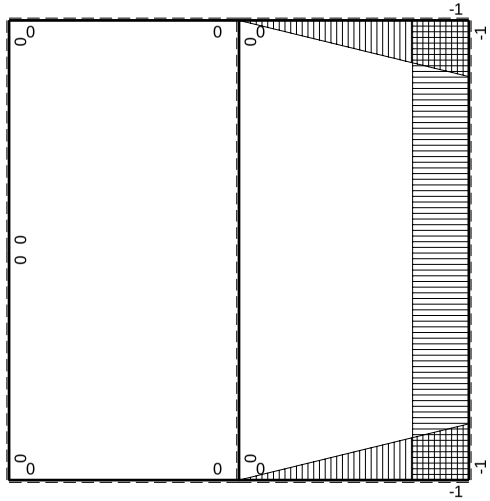

0
0
7/8
-1
-1
-3/8
-1
1
7/8
3
F
1
-3/8
0
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0
0
0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238290 CI08.xxxx.041

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

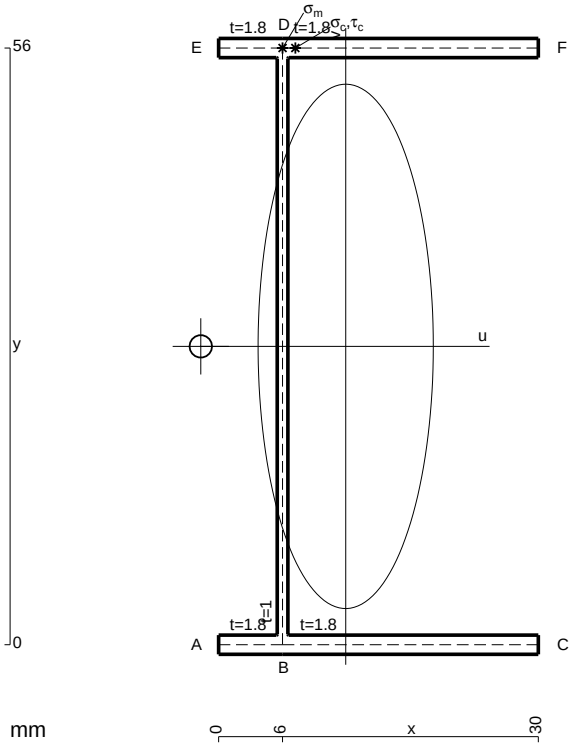
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

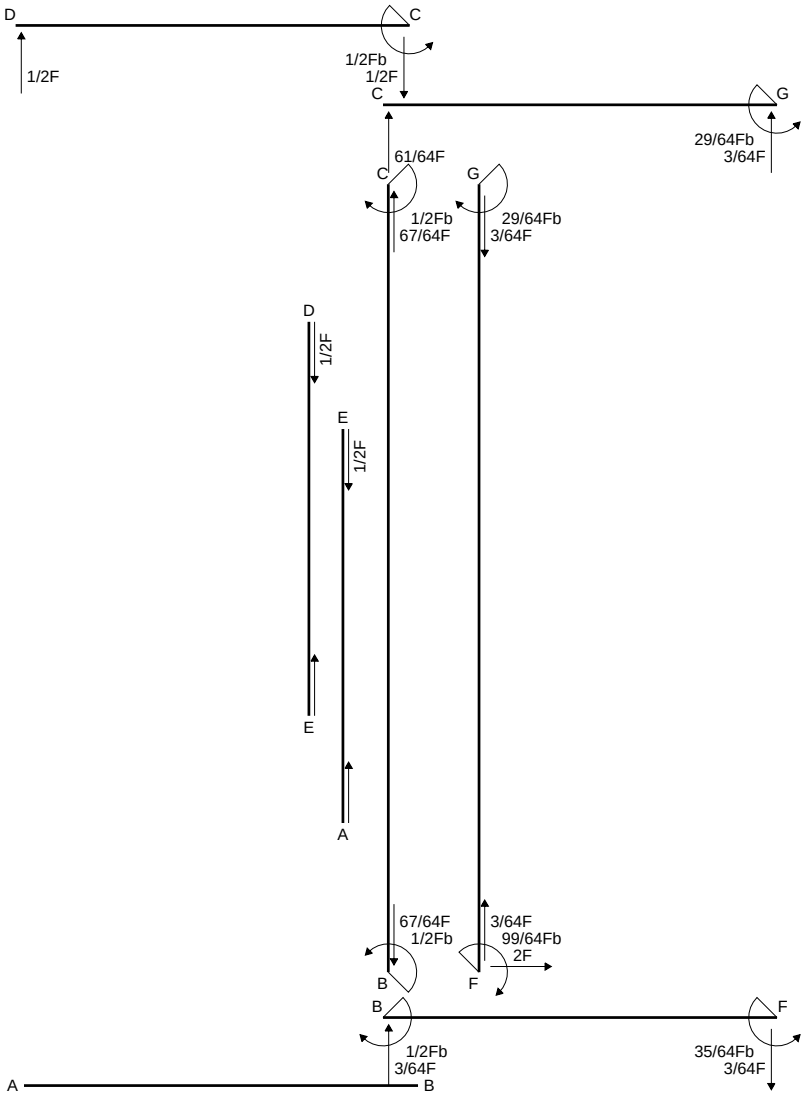
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

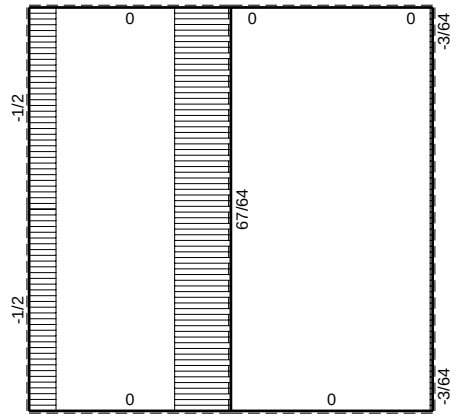


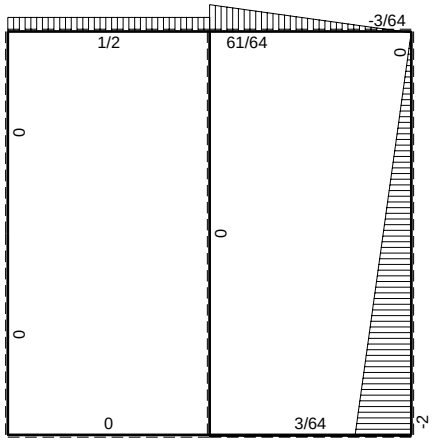
A = 164. mm^2
J_u = 99307. mm^4
J_v = 11087. mm^4
J_t = 135.3 mm^4
x_o = -13.6 mm
x_g = 11.93 mm
N = 1304. N
T_y = -2980. N
M_x = -715200. Nmm
x_m = 6. mm
y_m = 56. mm
u_m = -5.927 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 209.6 N/mm^2
x_c = 6. mm
y_c = 56. mm
u_c = -5.927 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 209.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 559.3 N/mm^2
$\tau_g = TS^*/tJ_u$ = 20.17 N/mm^2
$\tau_o = Tx_o t/J_t$ = 539.2 N/mm^2
t_c = 2682. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 991.2 N/mm^2

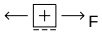
CI08.xxxx.041

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

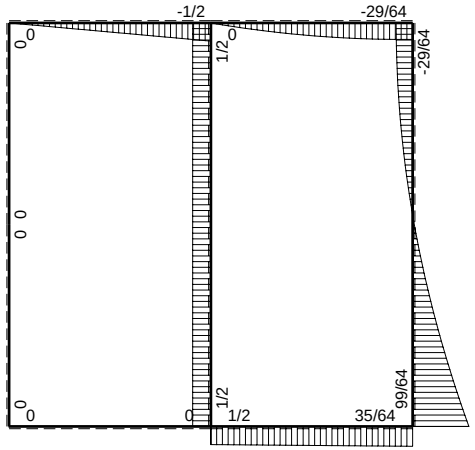

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64


-3/64
1/2
61/64
0
0
0
0
0
3/64
-2


F


F


-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
1/2
35/64
99/64


Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
totali							-29/24Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							29/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

PROCEDIMENTO E RISULTATI 286143 CI08.xxxx.042

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

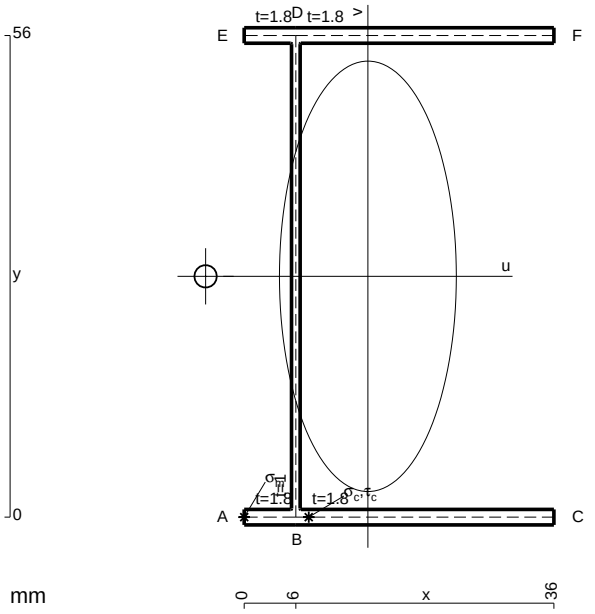
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

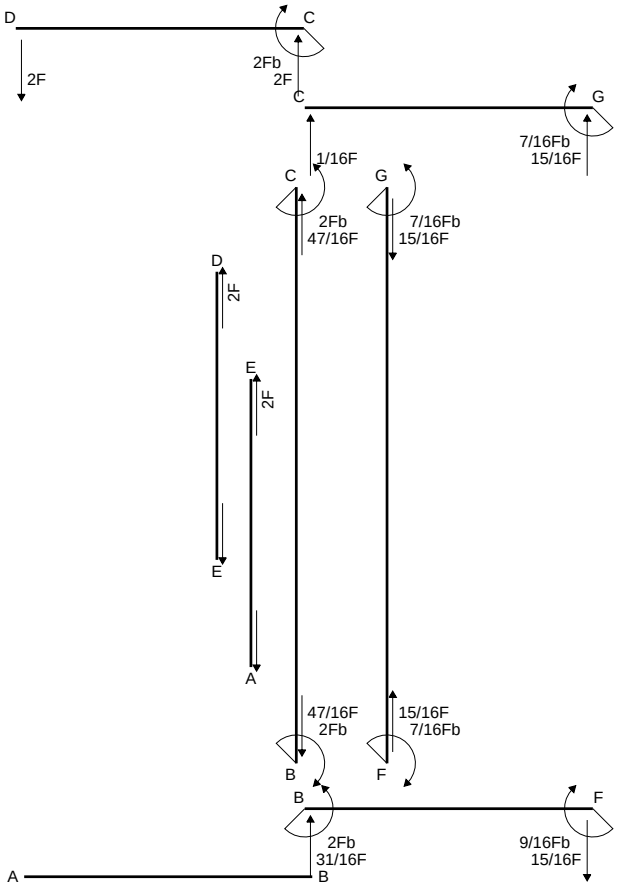
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

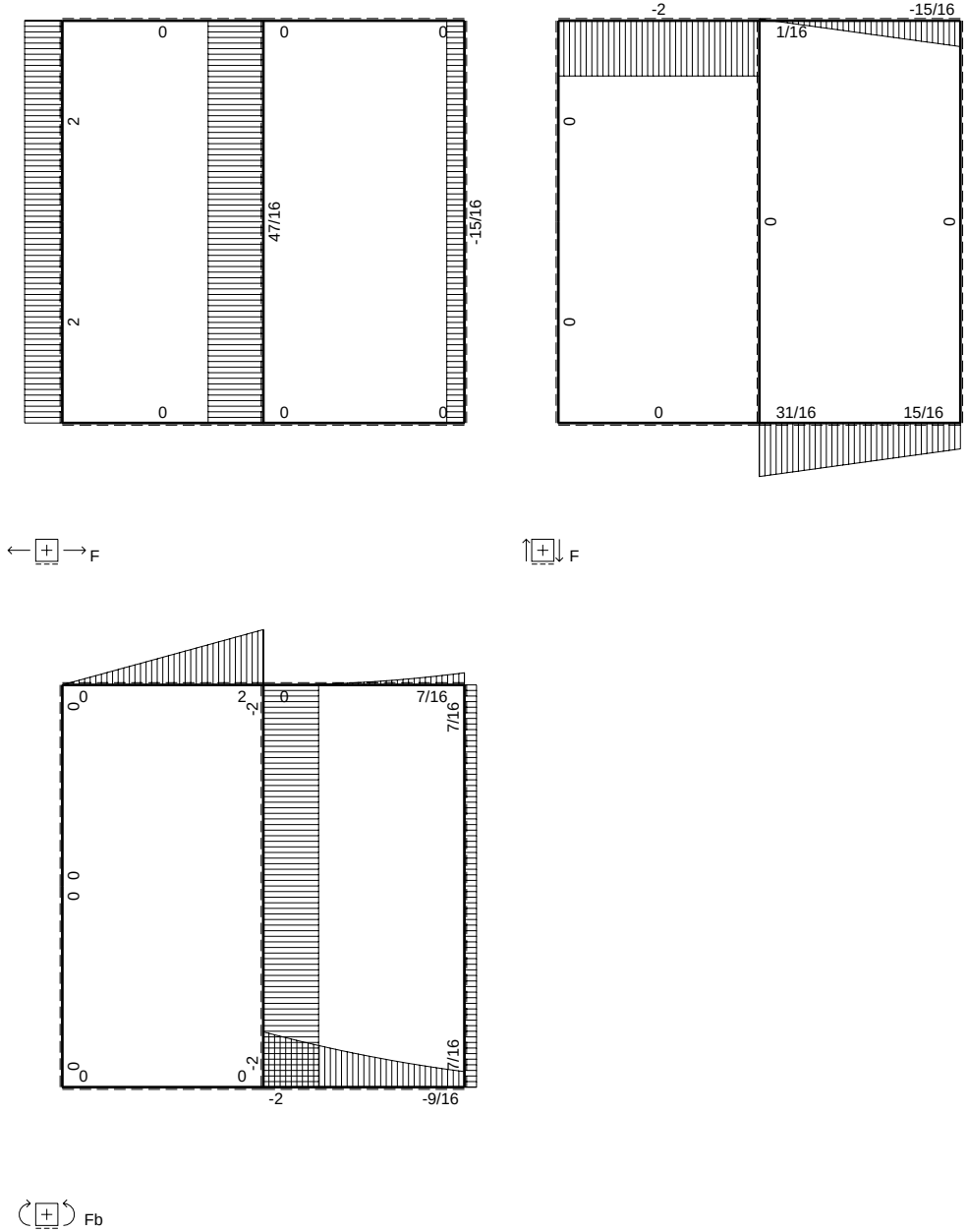


A = 185.6 mm^2
J_u = 116241. mm^4
J_v = 19628. mm^4
J_t = 158.6 mm^4
x_o = -18.87 mm
x_g = 14.38 mm
T_y = 1755. N
M_x = -912600. Nmm
u_m = -14.38 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -219.8 N/mm^2
x_c = 6. mm
u_c = -8.379 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -219.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 388.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.68 N/mm^2
$\tau_o = Tx_o t/J_t$ = 375.7 N/mm^2
t_c = 6318. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 707.8 N/mm^2

CI08.xxxx.042

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0
-2
-15/16
1/16
0
0
0
0
0
31/16
15/16
F
F
0
0
2
-2
0
7/16
7/16
0
0
0
0
0
-2
7/16
-2
-9/16
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
	0	-2Fx	0	0	0	0		
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
	1	0	0	0	0	1		
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
	0	2Fb	0	0	0	0		
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

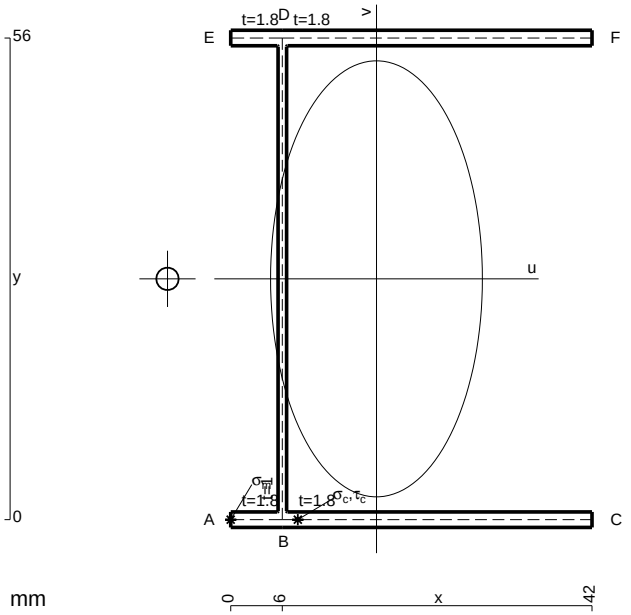
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

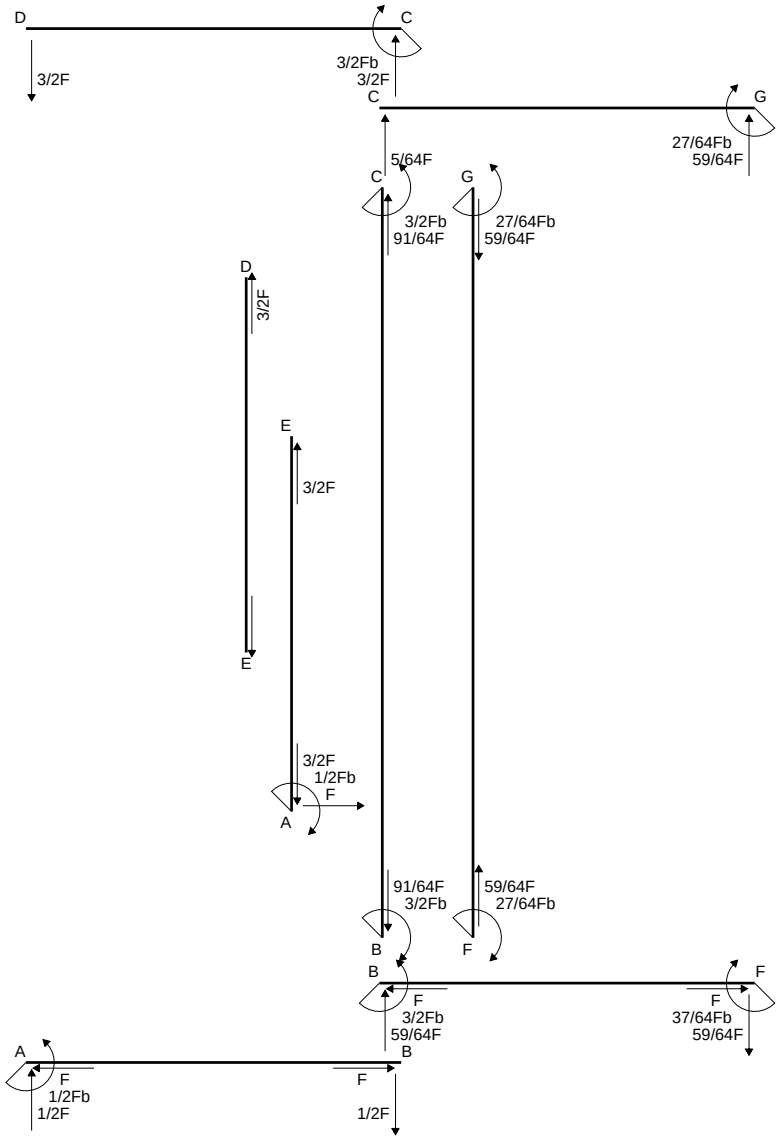
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

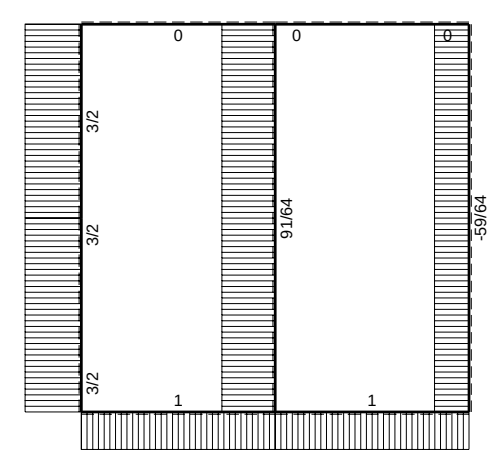


A = 207.2 mm^2
J_u = 133175. mm^4
J_v = 31421. mm^4
J_t = 182. mm^4
x_o = -24.3 mm
x_g = 16.95 mm
T_y = -1900. N
M_x = 1083000. Nmm
u_m = -16.95 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = 227.7 N/mm^2
x_c = 6. mm
u_c = -10.95 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = 227.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 471.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 14.38 N/mm^2
$\tau_o = Tx_o t/J_t$ = 456.7 N/mm^2
t_c = 1710. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 847.1 N/mm^2

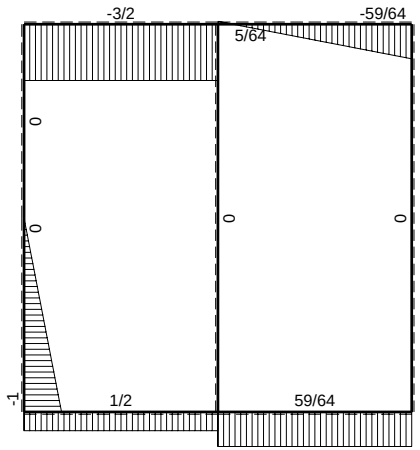
CI08.xxxx.043

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

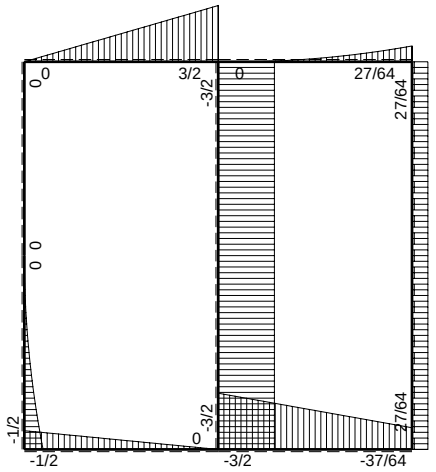

D
C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
3/2
91/64
-59/64
1
1

F


-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F


0
0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$9/8Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238791 CI08.xxxx.044

26.03.25 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

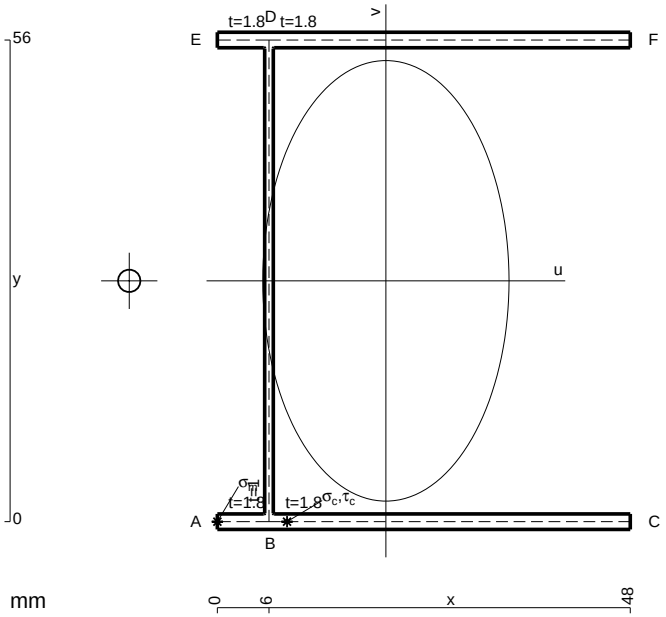
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

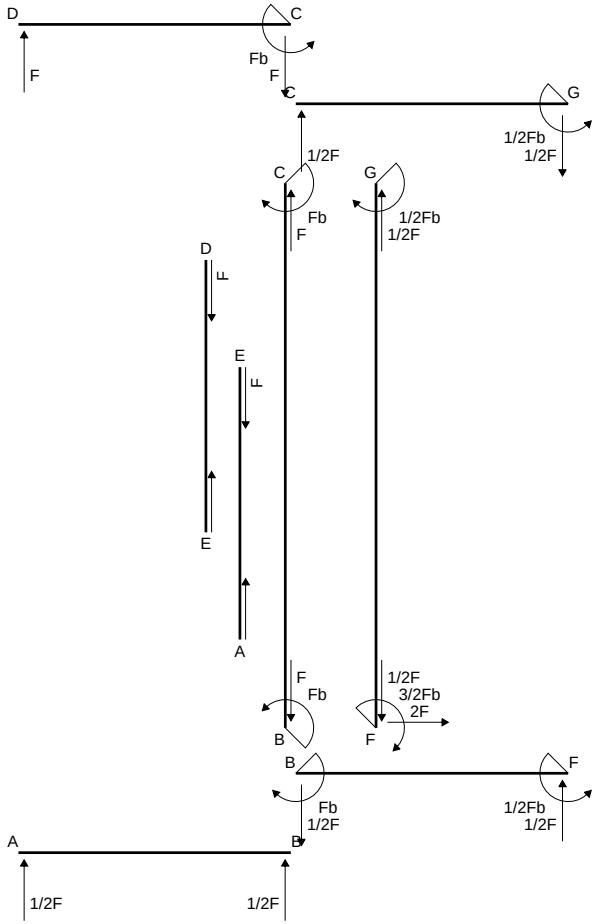
$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

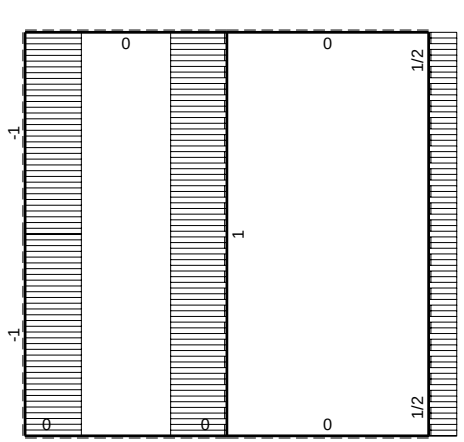


A = 228.8 mm^2
J_u = 150110. mm^4
J_v = 46881. mm^4
J_t = 205.3 mm^4
x_o = -29.84 mm
x_g = 19.59 mm
T_y = -2100. N
M_x = 1281000. Nmm
u_m = -19.59 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = 238.9 N/mm^2
x_c = 6. mm
u_c = -13.59 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = 238.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 565.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 16.45 N/mm^2
$\tau_o = Tx_o t/J_t$ = 549.4 N/mm^2
t_c = 2520. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1009. N/mm^2

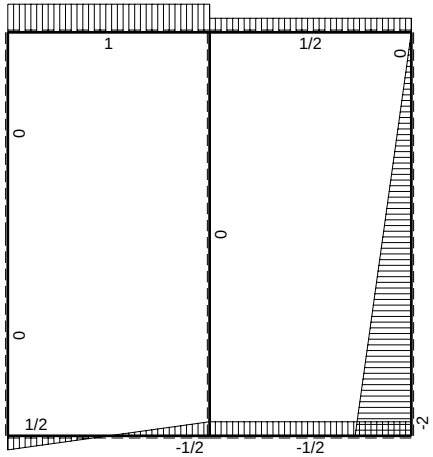
CI08.xxxx.044

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

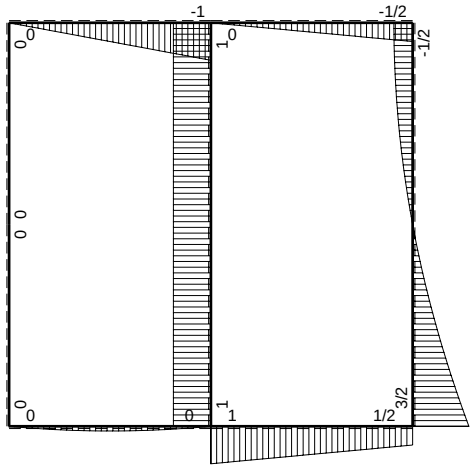

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

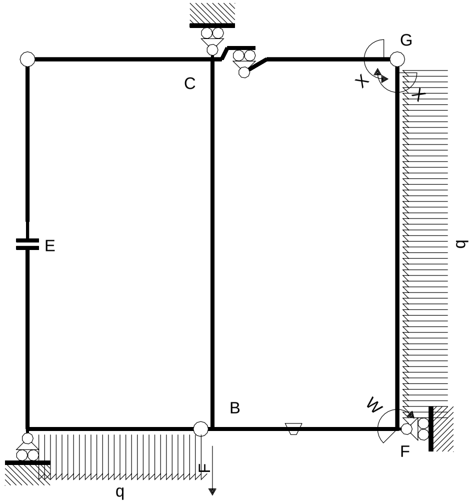

F

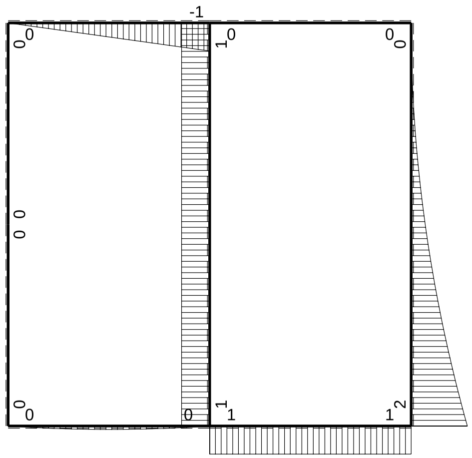

1
1/2
0
0
0
0
1/2
-2
-1/2
-1/2


F

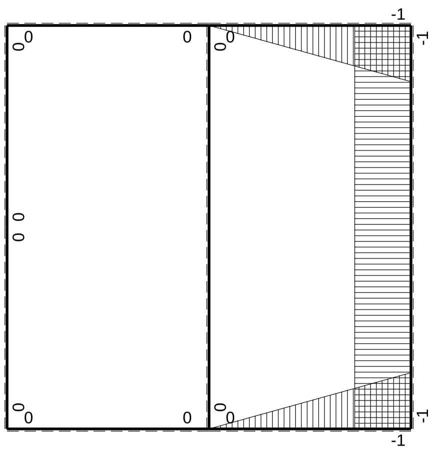

-1
-1/2
0
1
0
-1/2
0
0
0
0
1
1
1/2
3/2


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 238620 CI08.xxxx.045

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

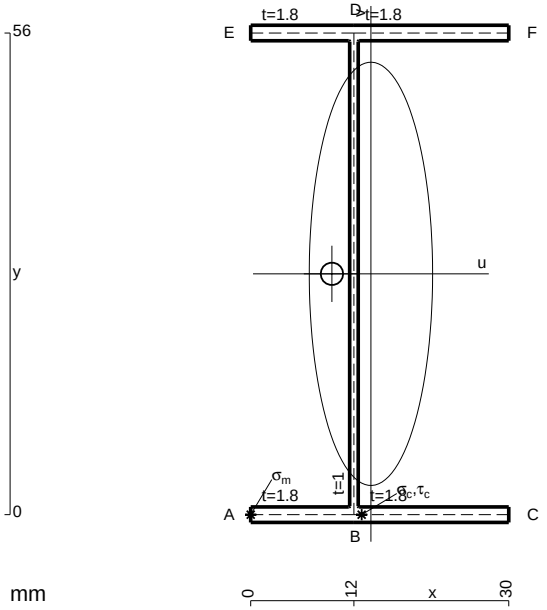
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

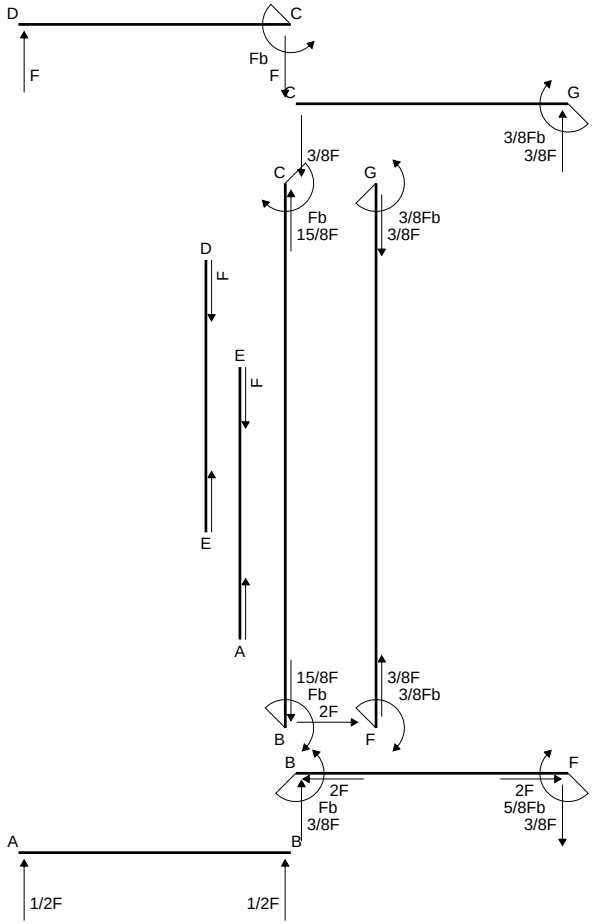
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

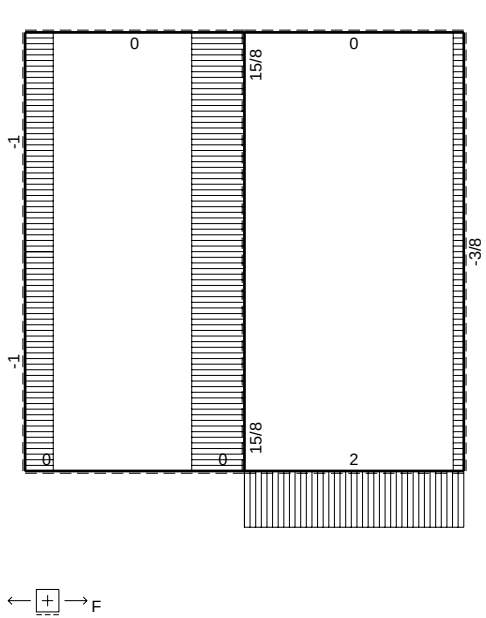


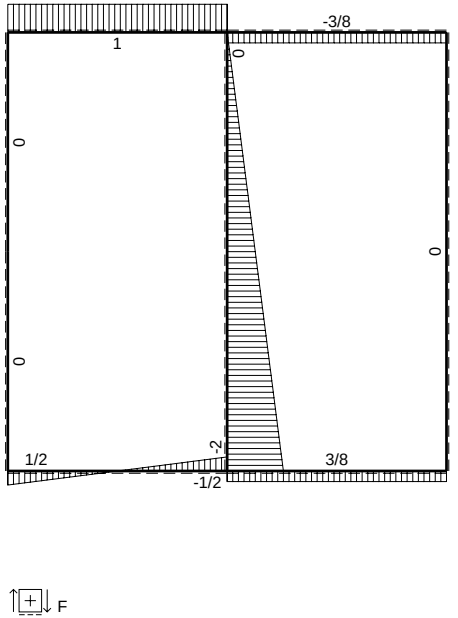
A = 164. mm^2
J_u = 99307. mm^4
J_v = 8432. mm^4
J_t = 135.3 mm^4
x_o = -4.534 mm
x_g = 13.98 mm
T_y = 1070. N
M_x = -706200. Nmm
u_m = -13.98 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -199.1 N/mm^2
x_c = 12. mm
u_c = -1.976 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -199.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 69.96 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.43 N/mm^2
$\tau_o = Tx_o t/J_t$ = 64.53 N/mm^2
t_c = 1926. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 233.1 N/mm^2

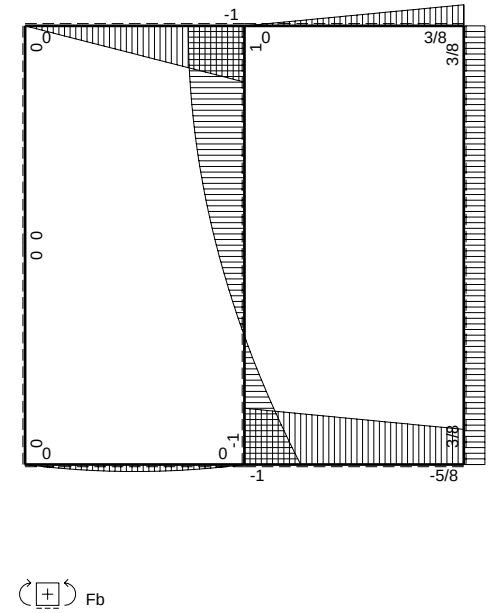
CI08.xxxx.045

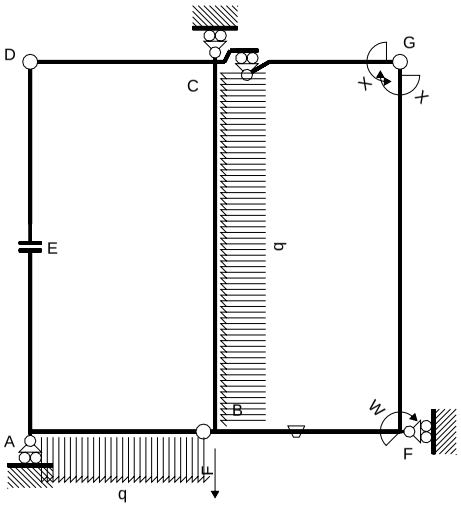
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F

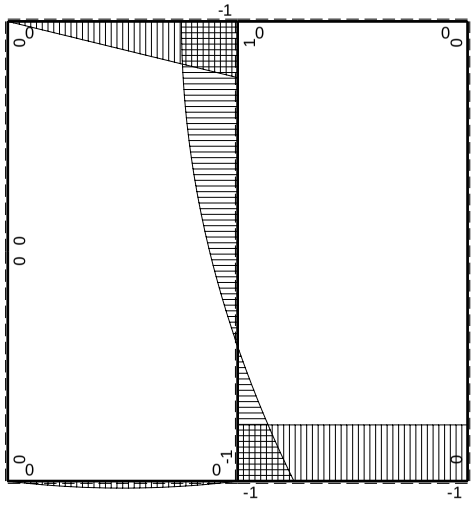

0
0
15/8
-1
-1
-3/8
0
0
15/8
0
0
2
F


-3/8
1
0
0
0
0
1/2
-2
3/8
-1/2
F

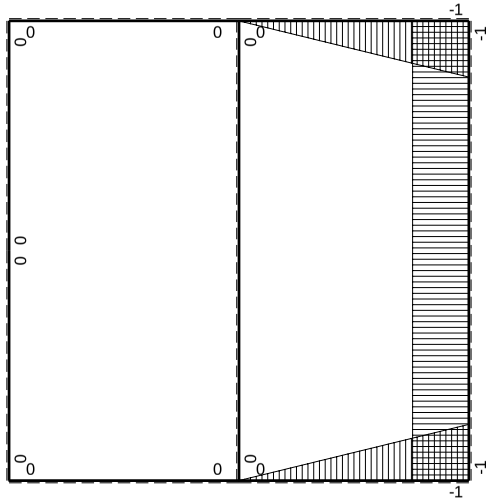

-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 939051 CI08.xxxx.046

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

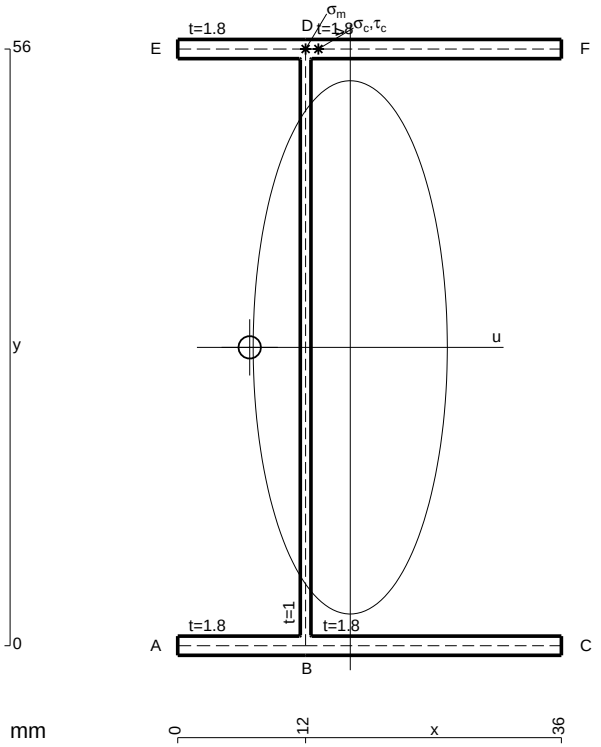
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 185.6 mm^2

J_u = 116241. mm^4

J_v = 15405. mm^4

J_t = 158.6 mm^4

x_o = -9.434 mm

x_g = 16.19 mm

N = 2194. N

T_y = -2340. N

M_x = -819000. Nmm

x_m = 12. mm

y_m = 56. mm

u_m = -4.19 mm

v_m = 28. mm

$\sigma_m = N/A - Mv/J_u$ = 209.1 N/mm^2

x_c = 12. mm

y_c = 56. mm

u_c = -4.19 mm

v_c = 28. mm

$\sigma_c = N/A - Mv/J_u$ = 209.1 N/mm^2

$\tau_c = \tau_g + \tau_{ou}$ = 264. N/mm^2

$\tau_g = TS^*/tJ_u$ = 13.53 N/mm^2

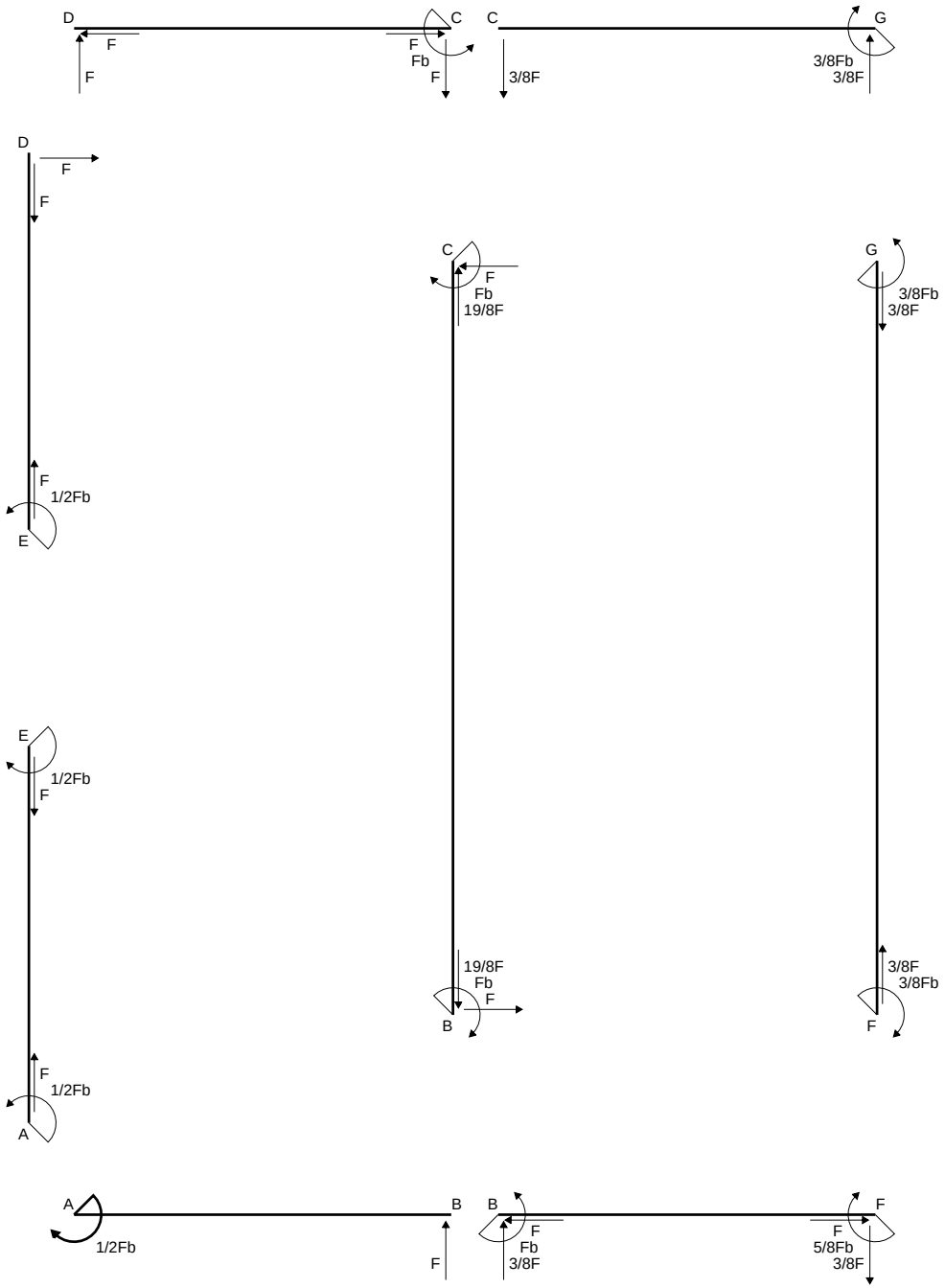
$\tau_o = Tx_o t/J_t$ = 250.5 N/mm^2

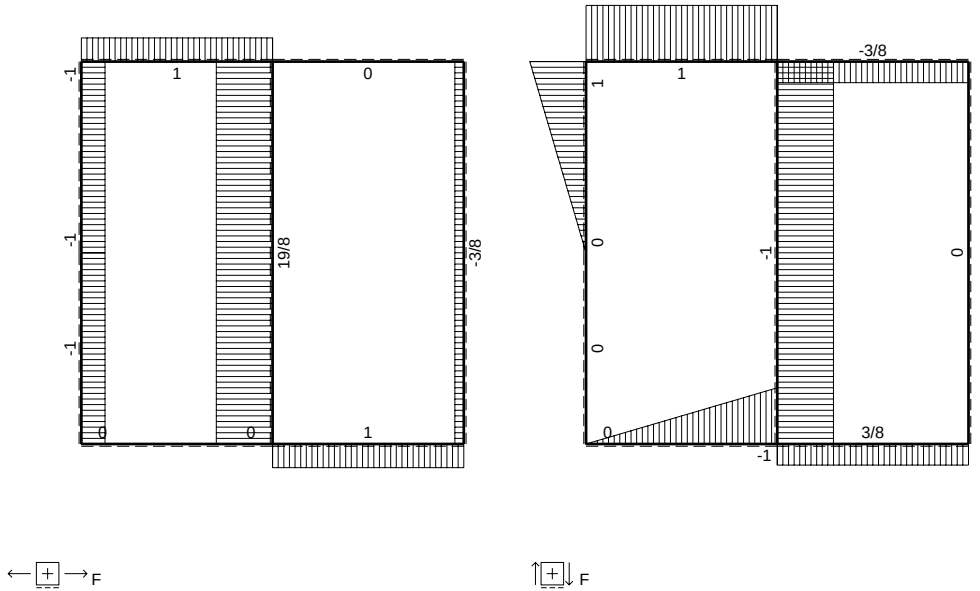
t_c = 2106. mm

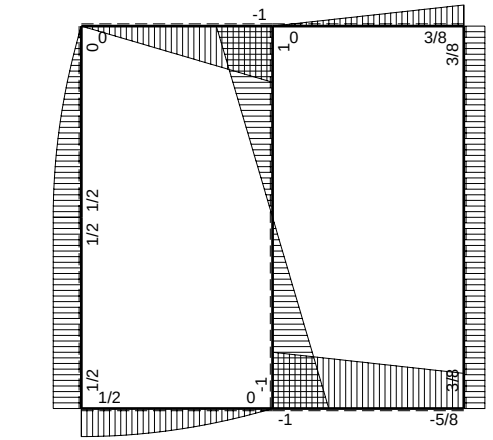
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 502.8 N/mm^2

CI08.xxxx.046

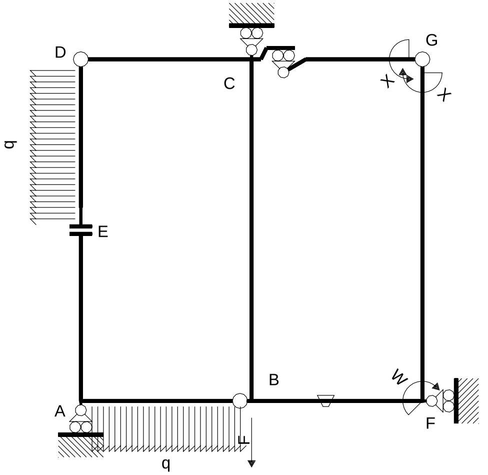
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
19/8F
G
3/8Fb
3/8F
E
1/2Fb
F
19/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F

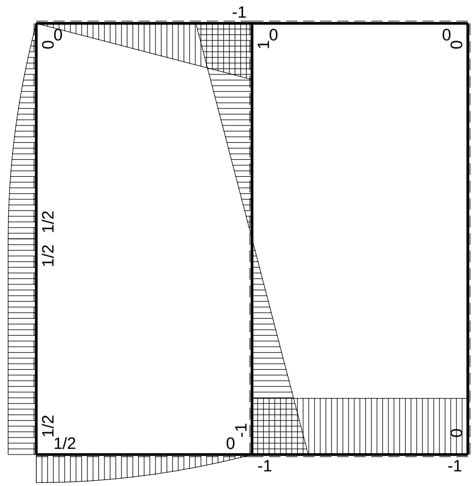

-1
1
0
-1
19/8
-3/8
-1
0
0
1
F
-3/8
1
1
1
0
-1
0
0
0
3/8
-1
F


-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8

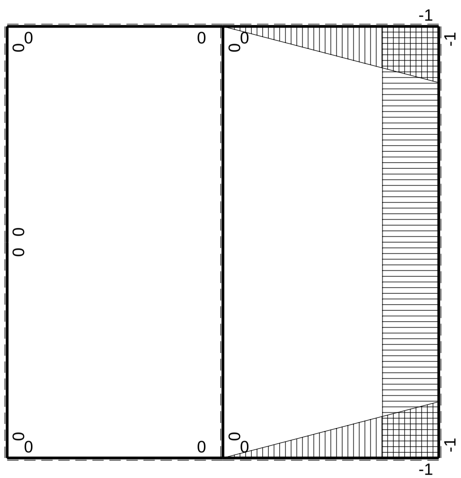

Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

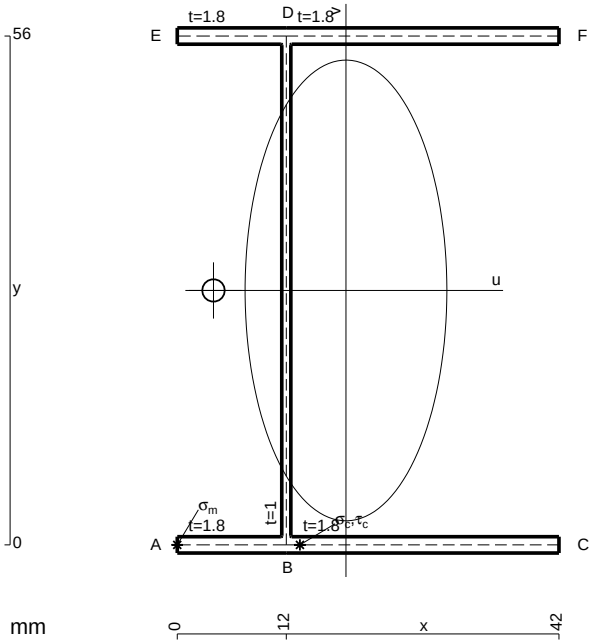
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

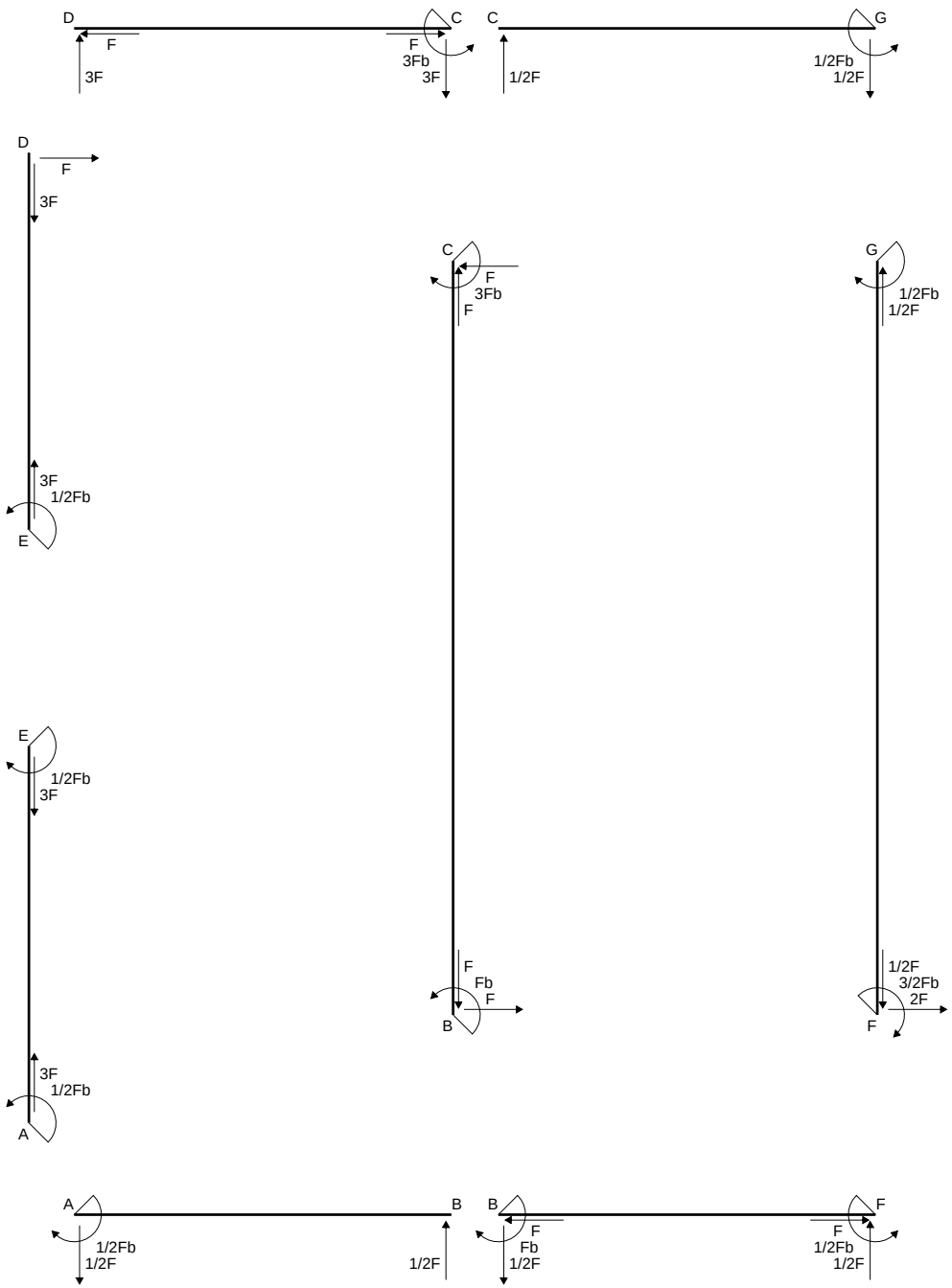
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

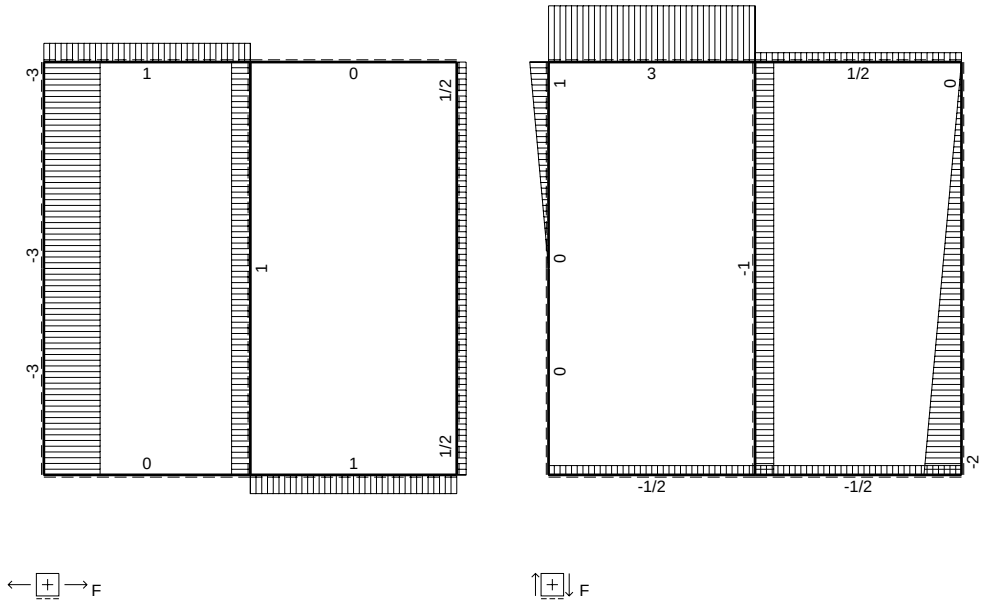


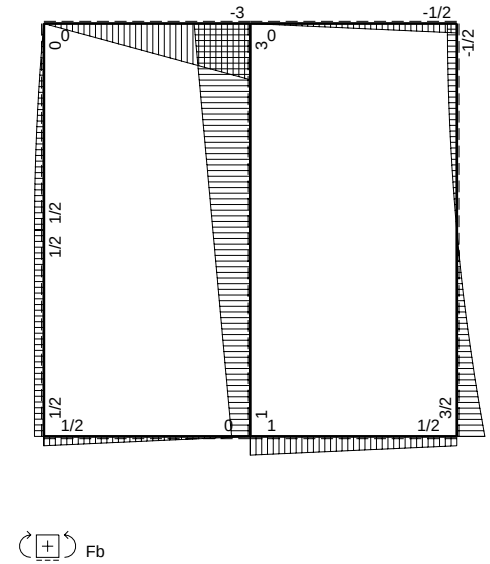
A = 207.2 mm^2
J_u = 133175. mm^4
J_v = 25537. mm^4
J_t = 182. mm^4
x_o = -14.58 mm
x_g = 18.57 mm
N = 3111. N
T_y = -1310. N
M_x = 969400. Nmm
u_m = -18.57 mm
v_m = -28. mm
$\sigma_m = N/A - Mv/J_u$ = 218.8 N/mm^2
x_c = 12. mm
u_c = -6.568 mm
v_c = -28. mm
$\sigma_c = N/A - Mv/J_u$ = 218.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 197.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 8.263 N/mm^2
$\tau_o = Tx_o t/J_t$ = 188.9 N/mm^2
t_c = 2358. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 405.6 N/mm^2

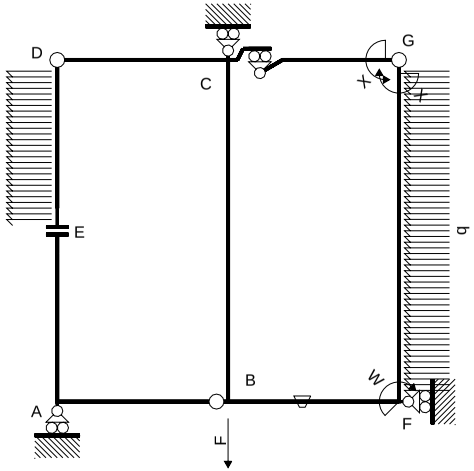
CI08.xxxx.047

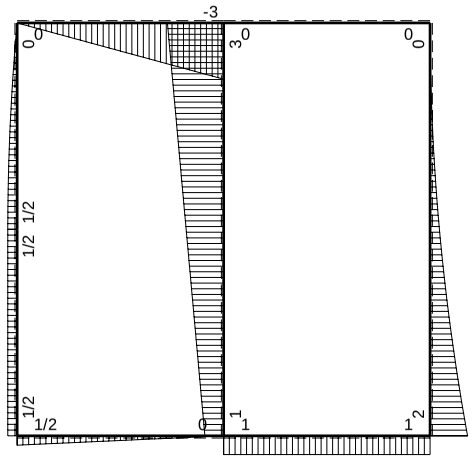
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
F
Fb
F
B
1/2F
3/2Fb
2F
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F

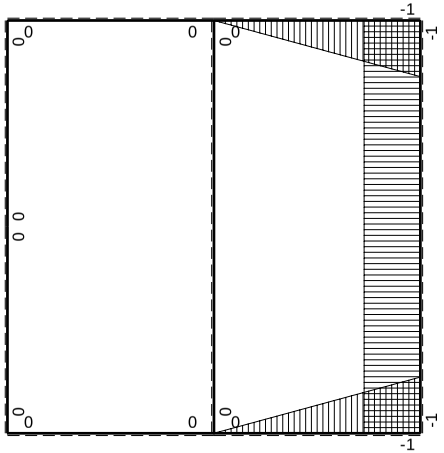

-3
1
0
1/2
-3
1
-3
0
1
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 236389 CI08.xxxx.048

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

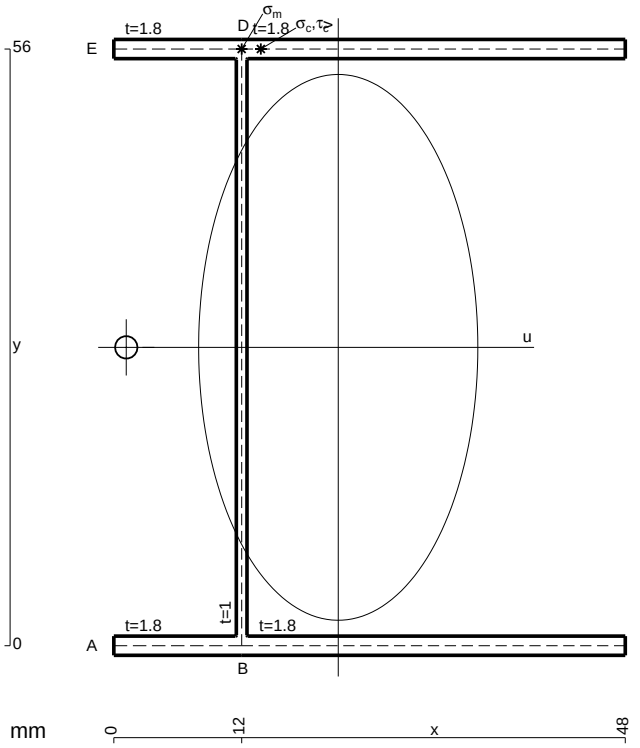
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

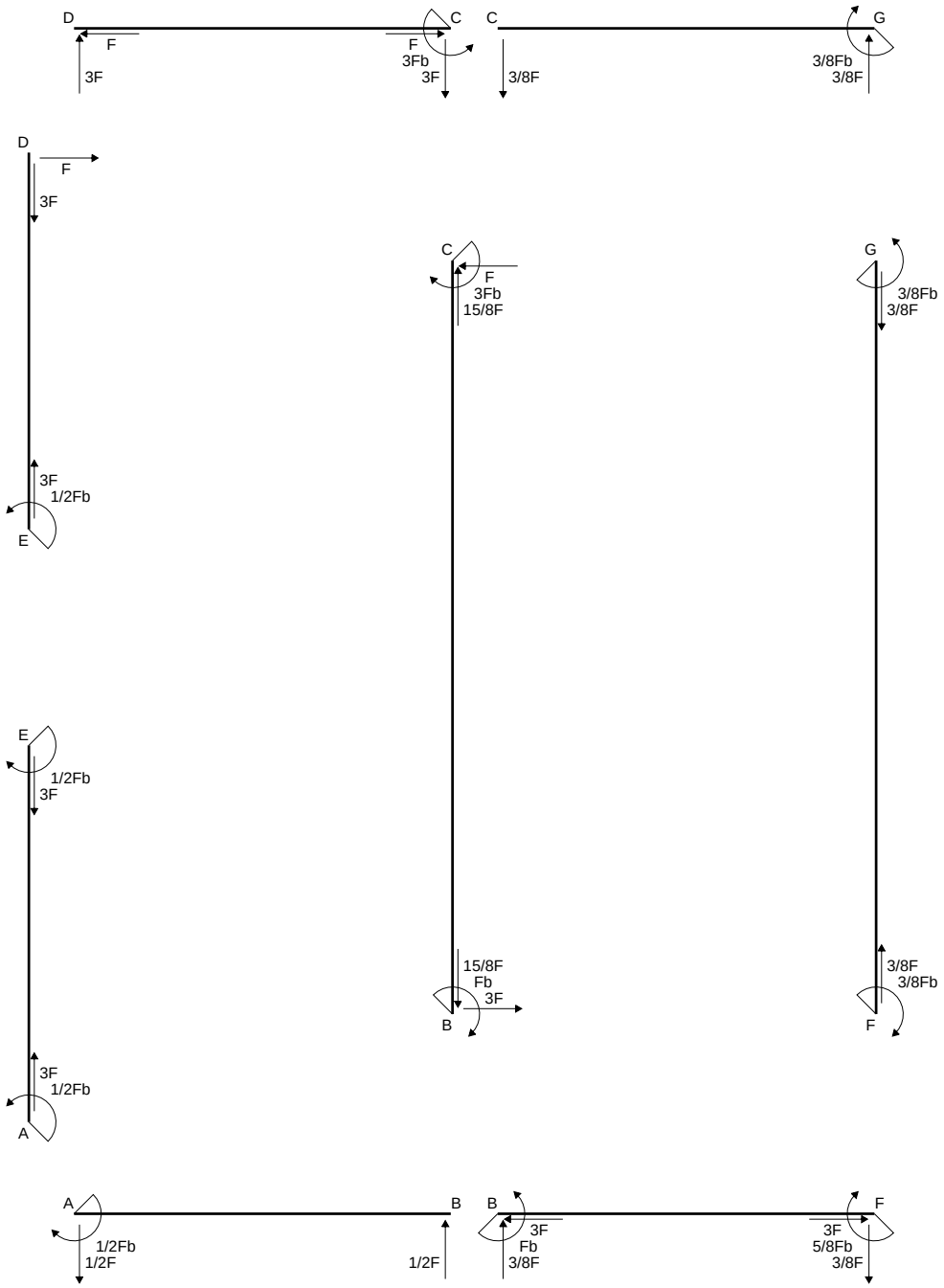
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

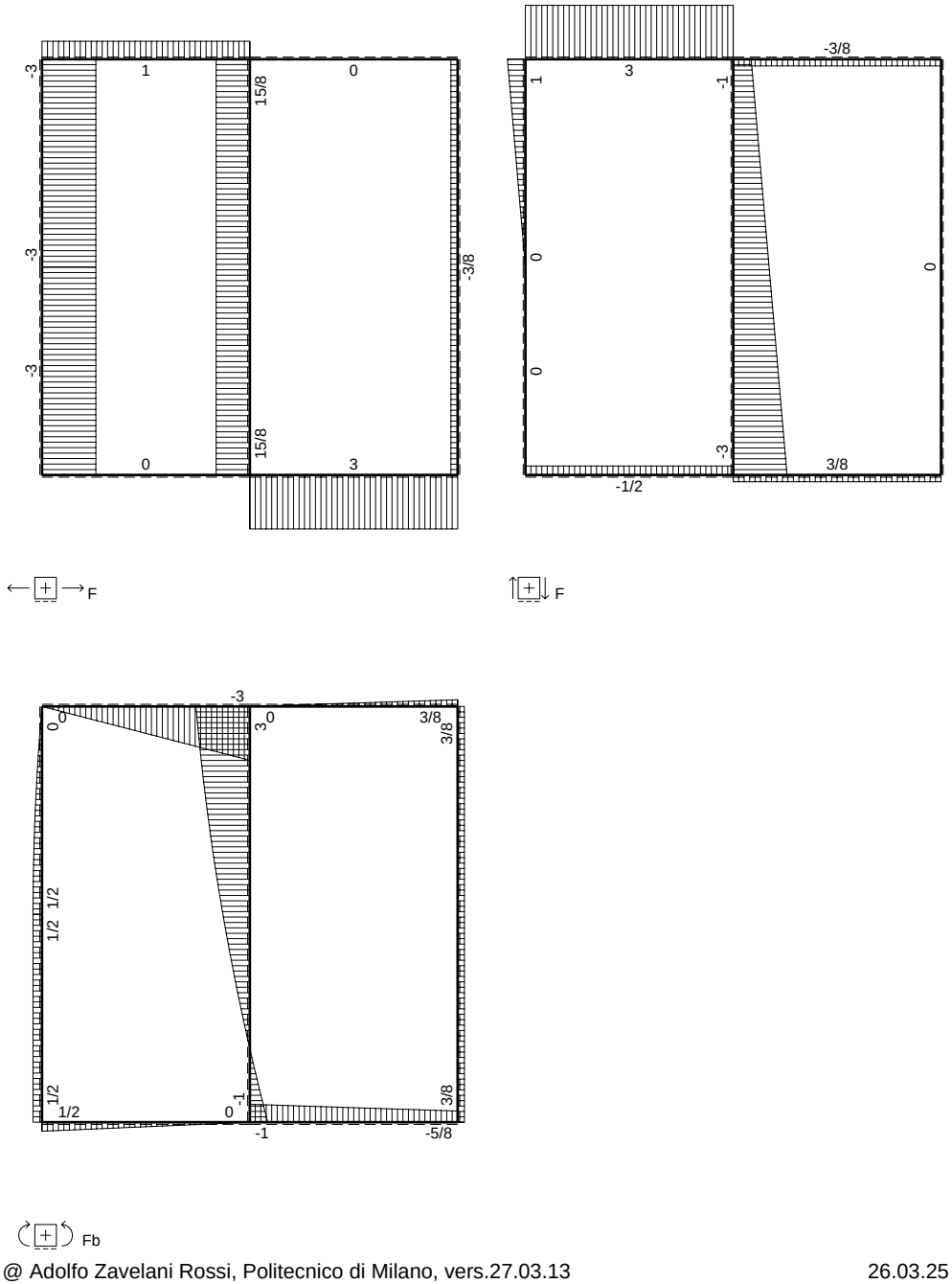


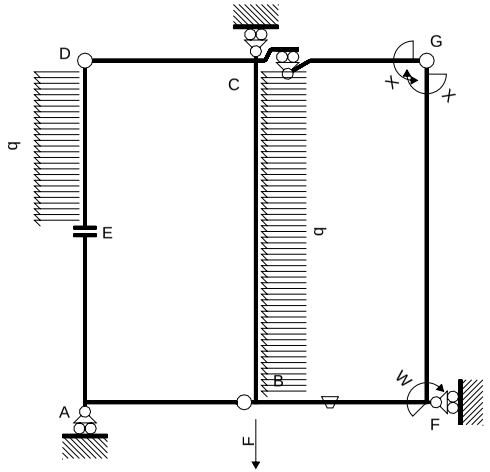
A = 228.8 mm^2
J_u = 150110. mm^4
J_v = 39268. mm^4
J_t = 205.3 mm^4
x_o = -19.89 mm
x_g = 21.06 mm
N = 510. N
T_y = 1530. N
M_x = -1208700. Nmm
x_m = 12. mm
y_m = 56. mm
u_m = -9.063 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 227.7 N/mm^2
x_c = 12. mm
y_c = 56. mm
u_c = -9.063 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 227.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 277.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.27 N/mm^2
$\tau_o = Tx_o t/J_t$ = 266.9 N/mm^2
t_c = 918. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 531.3 N/mm^2

CI08.xxxx.048

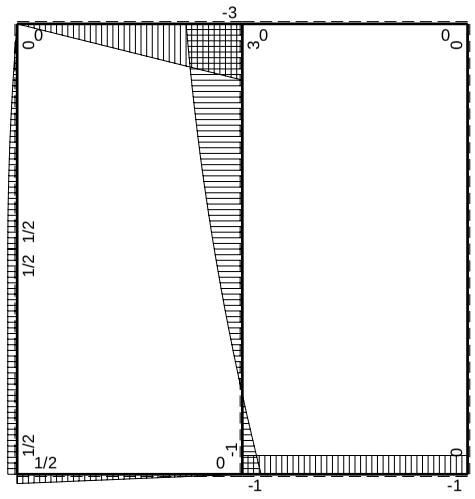
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
3F
C
F
3Fb
3F
C
3/8F
G
3/8Fb
3/8F
D
F
3F
3F
1/2Fb
E
C
F
3Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3F
15/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F

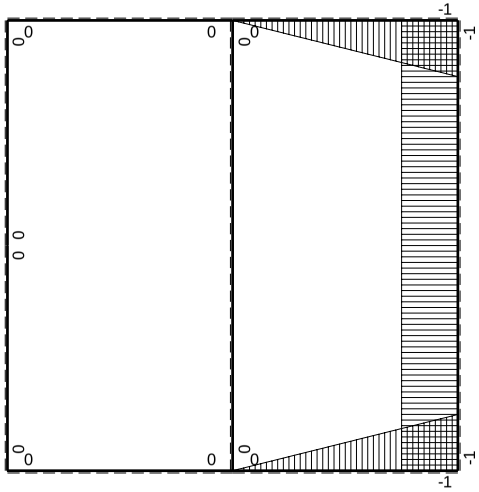

-3
1
15/8
0
-3/8
-3
-3
15/8
0
3
F
-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F
-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 987512 CI08.xxxx.049

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

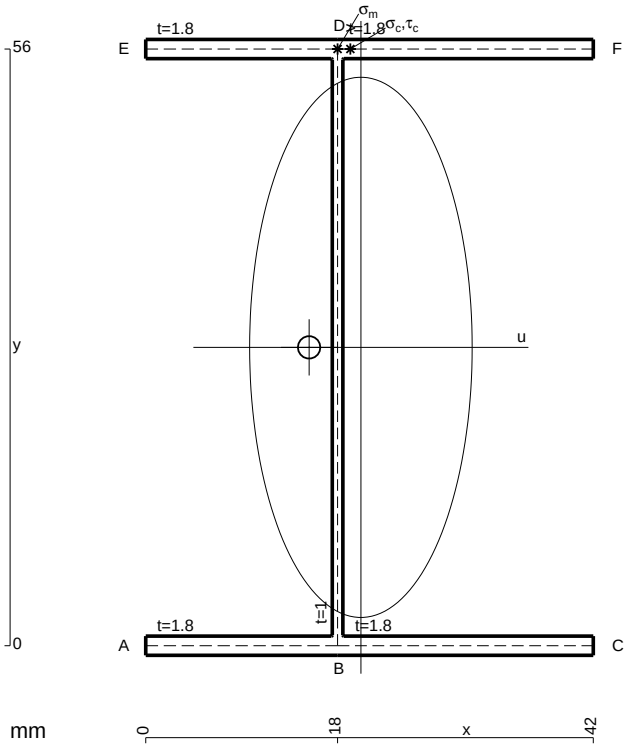
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

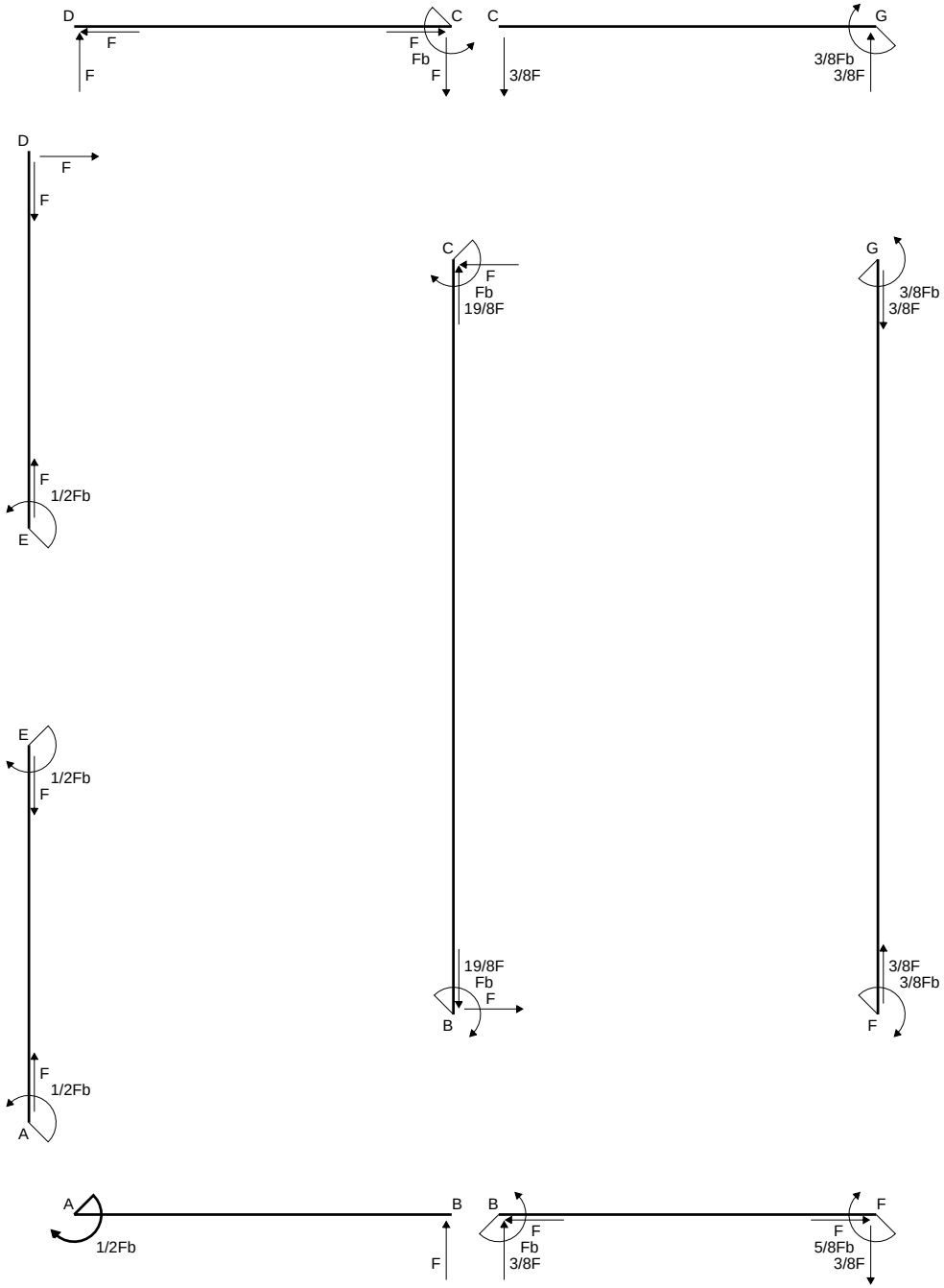
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

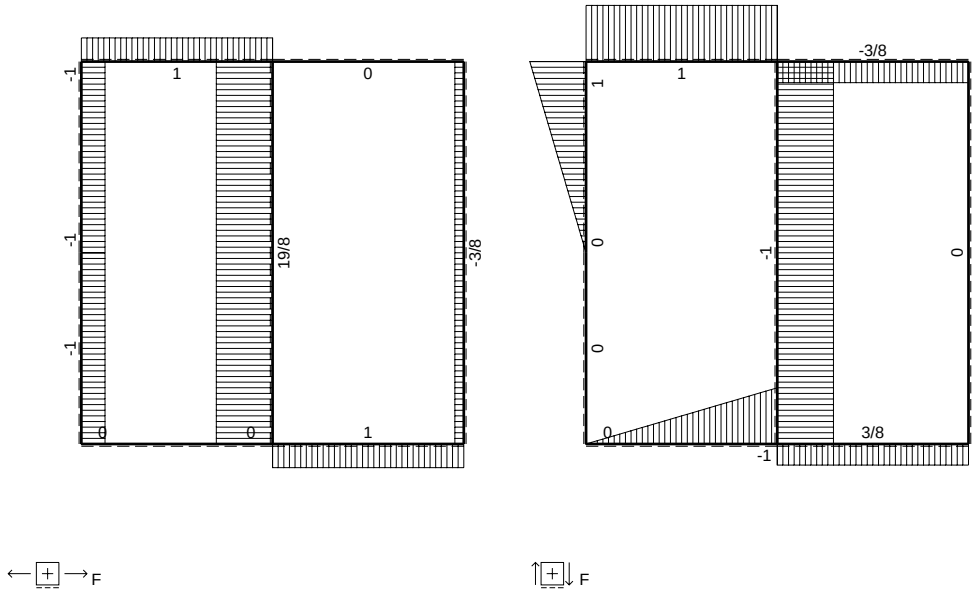


A = 207.2 mm^2
J_u = 133175. mm^4
J_v = 22594. mm^4
J_t = 182. mm^4
x_o = -4.86 mm
x_g = 20.19 mm
N = 450. N
T_y = 1350. N
M_x = -1120500. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = -2.189 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 237.8 N/mm^2
x_c = 18. mm
y_c = 56. mm
u_c = -2.189 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 237.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 71.71 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.812 N/mm^2
$\tau_o = Tx_o t/J_t$ = 64.9 N/mm^2
t_c = 810. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 268.2 N/mm^2

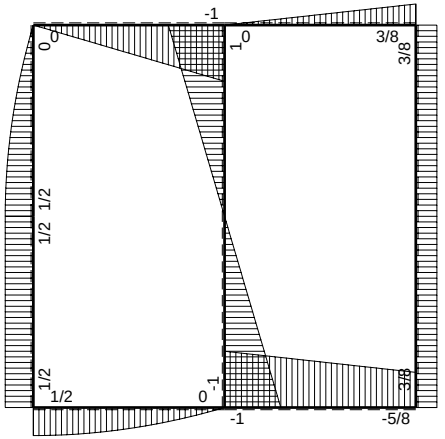
CI08.xxxx.049

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

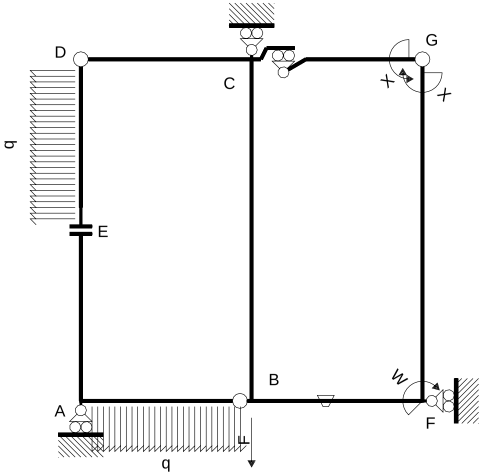

D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
19/8F
G
3/8Fb
3/8F
E
1/2Fb
F
19/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F


-1
1
0
-1
19/8
-3/8
-1
0
0
1
-3/8
1
1
0
-1
0
0
0
3/8
-1

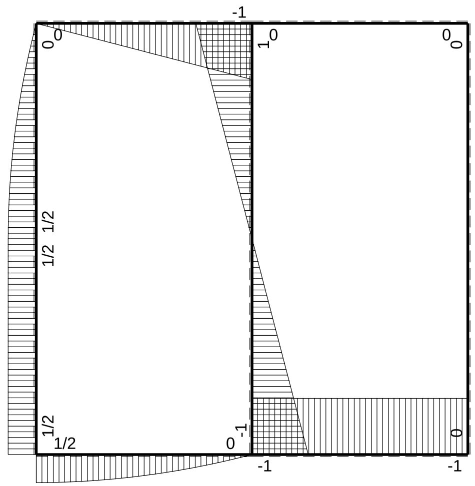
F F


-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8

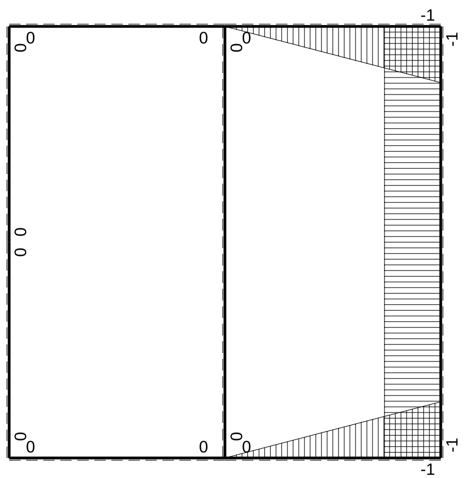
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

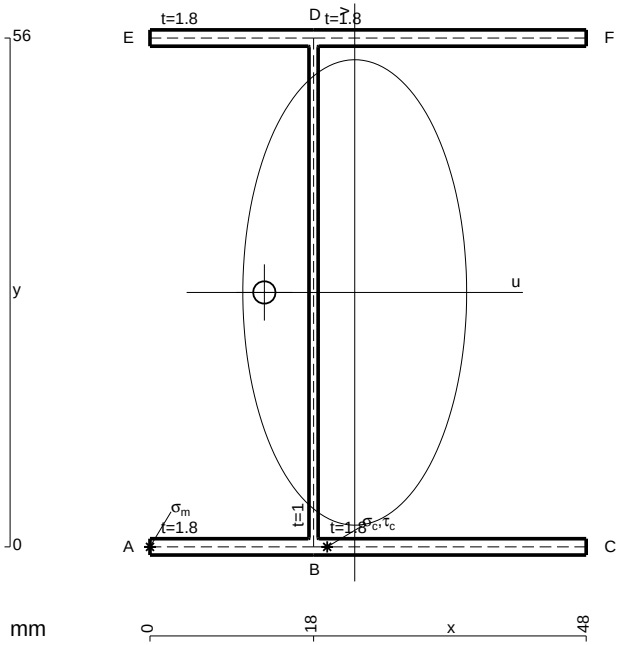
$L^{Xo}_{BF} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 238621 CI08.xxxx.050

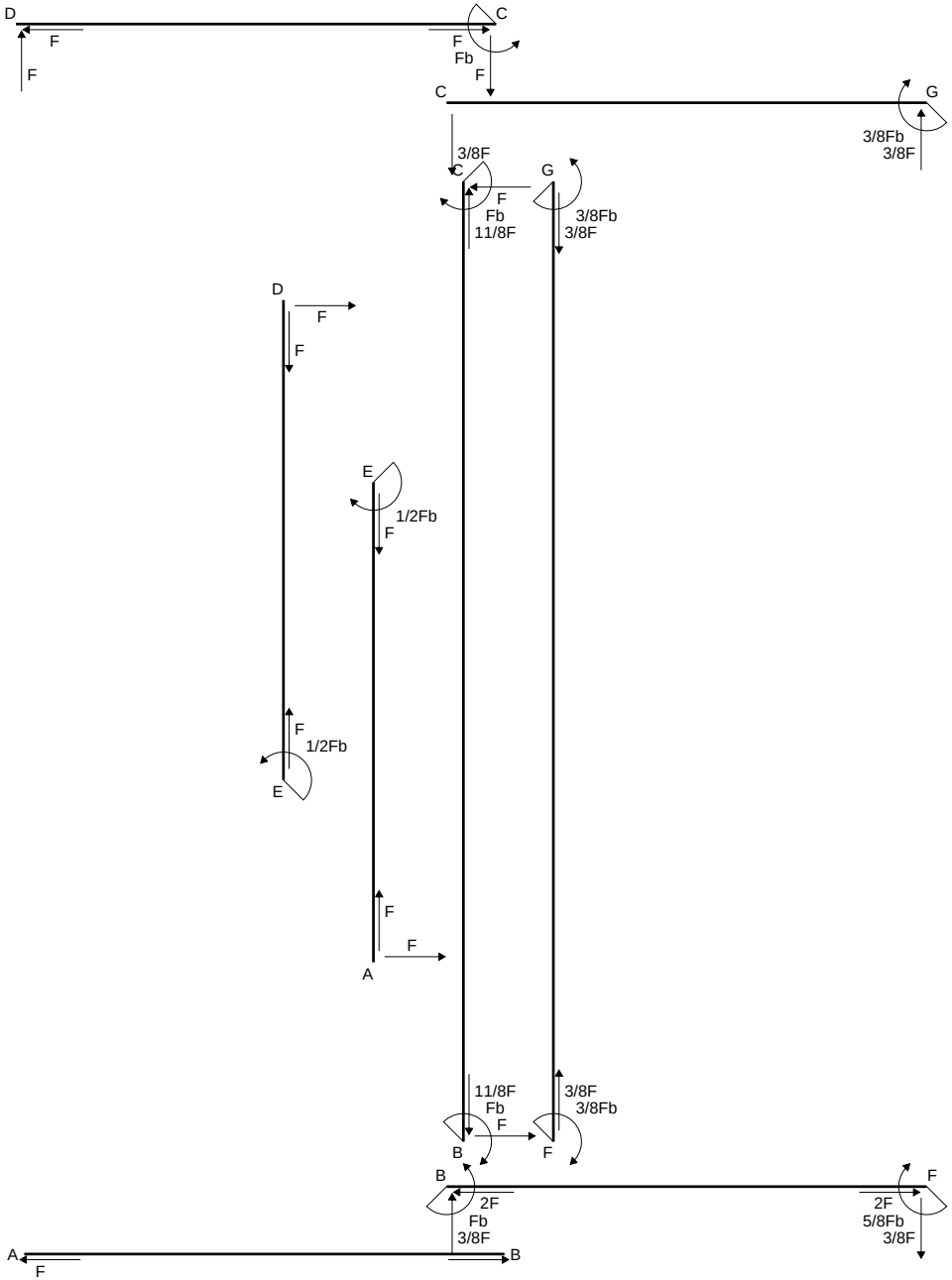


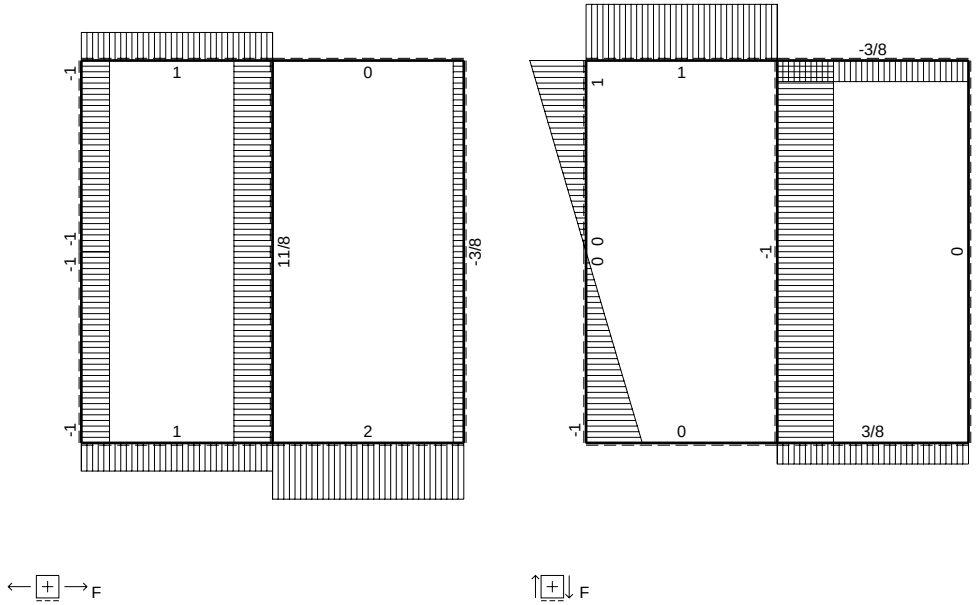
A = 228.8 mm^2
J_u = 150110. mm^4
J_v = 34700. mm^4
J_t = 205.3 mm^4
x_o = -9.947 mm
x_g = 22.53 mm
N = 5130. N
T_y = -2160. N
M_x = 950400. Nmm
u_m = -22.53 mm
v_m = -28. mm
$\sigma_m = N/A - Mv/J_u$ = 199.7 N/mm^2
x_c = 18. mm
u_c = -4.531 mm
v_c = -28. mm
$\sigma_c = N/A - Mv/J_u$ = 199.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 200.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.09 N/mm^2
$\tau_o = Tx_o t/J_t$ = 188.4 N/mm^2
t_c = 3888. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 400.5 N/mm^2

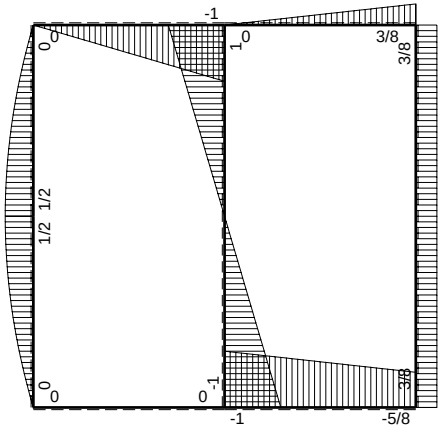
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

CI08.xxxx.050

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
11/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
11/8F
Fb
F
3/8F
3/8Fb
F
B
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F


-1
1
0
-1
-1
11/8
-3/8
-1
1
2
F
-3/8
1
1
1
0 0
-1
0
-1
0
3/8
F


-1
0
0
1
0
3/8
3/8
1/2 1/2
0
0
0
-1
-1
3/8
-1
-5/8

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3XbEJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbEJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3XbEJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

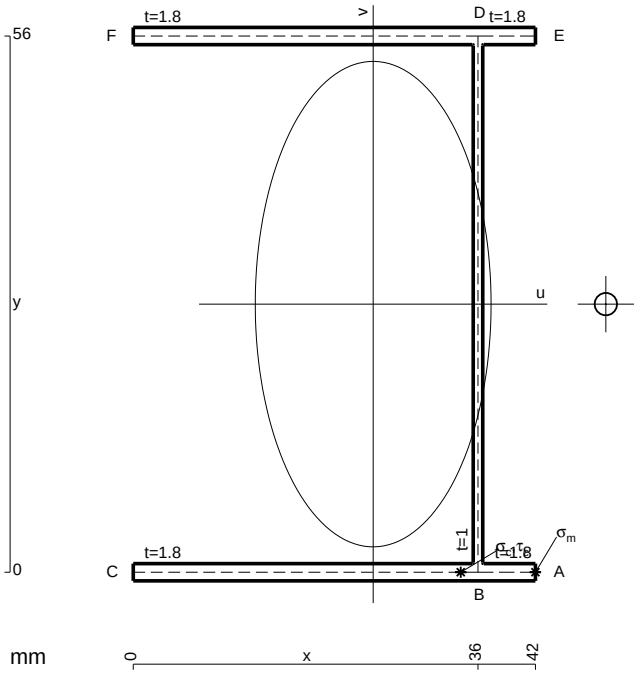
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

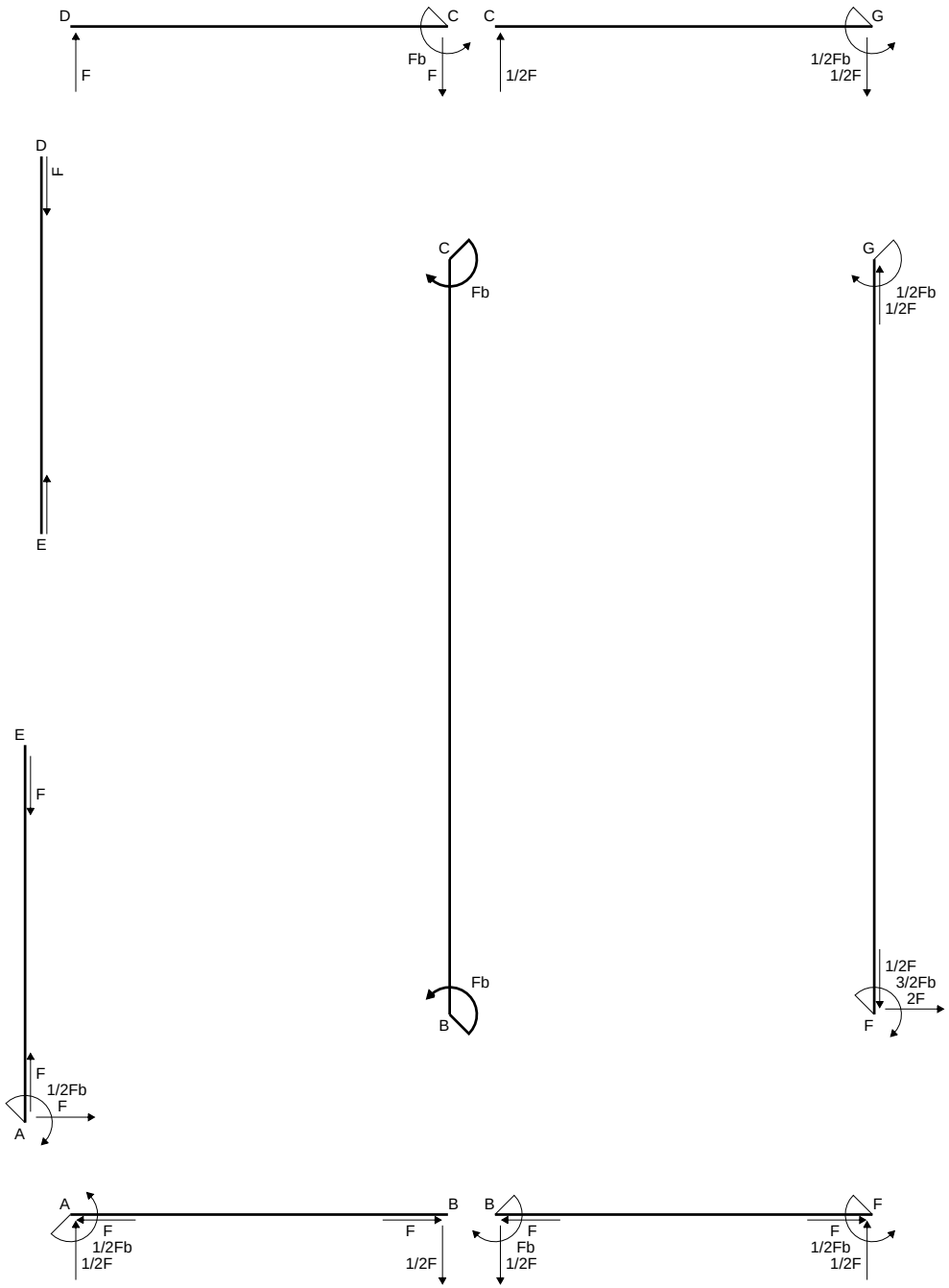
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

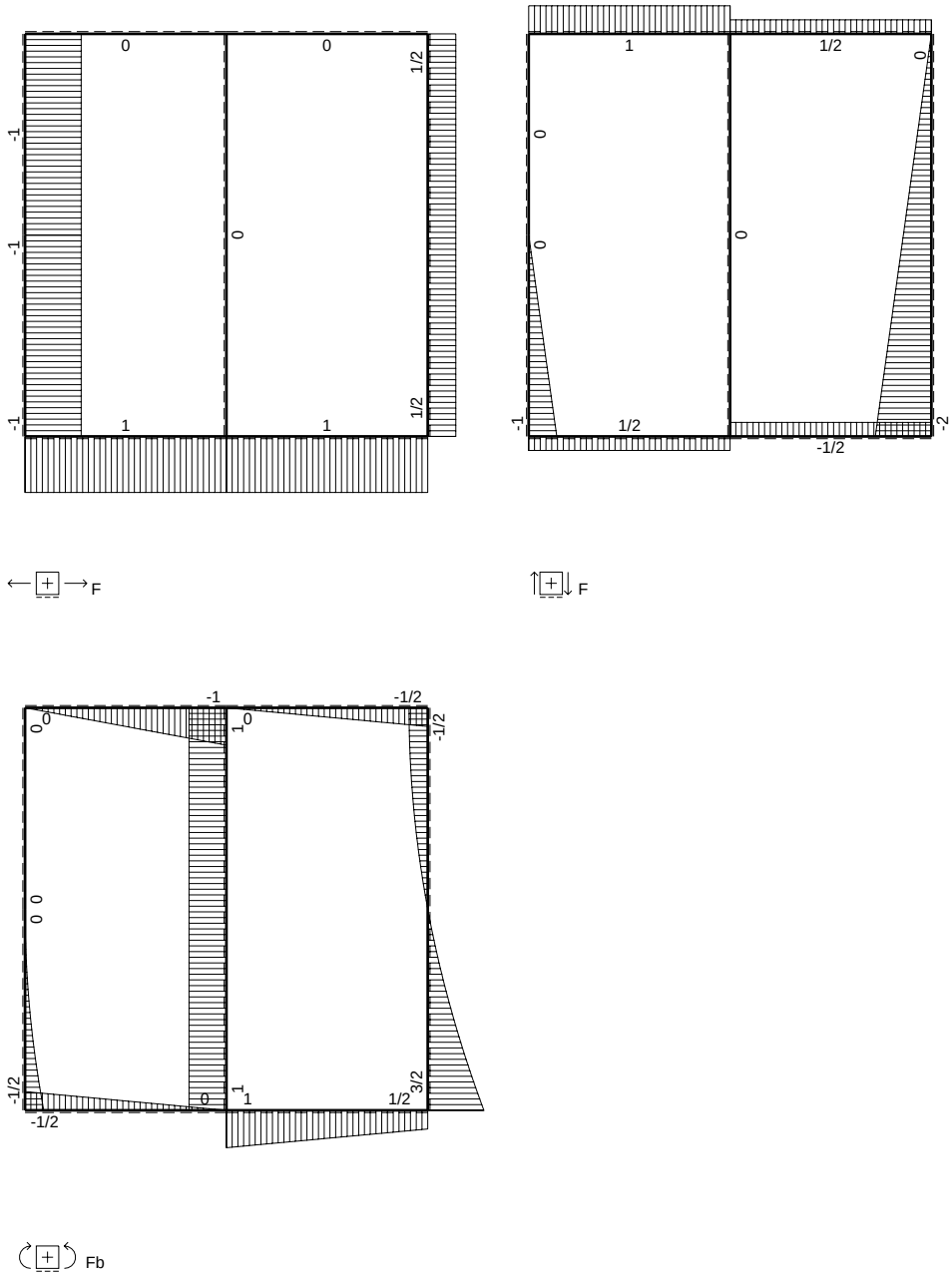


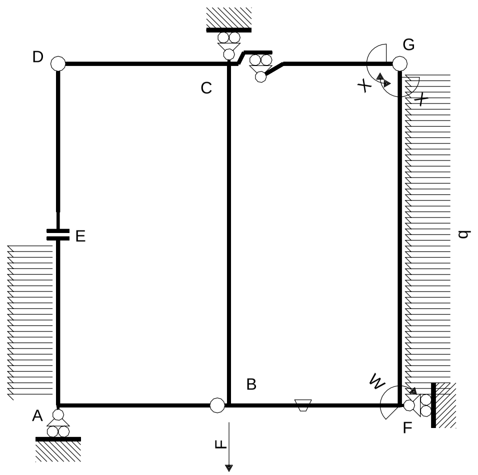
A = 207.2 mm^2
J_u = 133175. mm^4
J_v = 31421. mm^4
J_t = 182. mm^4
x_o = 24.3 mm
x_g = 25.05 mm
N = 2681. N
T_y = -1950. N
M_x = 936000. Nmm
x_m = 42. mm
u_m = 16.95 mm
v_m = -28. mm
$\sigma_m = N/A - Mv/J_u$ = 209.7 N/mm^2
x_c = 36. mm
u_c = 10.95 mm
v_c = -28. mm
$\sigma_c = N/A - Mv/J_u$ = 209.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 483.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 14.76 N/mm^2
$\tau_o = Tx_o t/J_t$ = 468.7 N/mm^2
t_c = 3510. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 863.2 N/mm^2

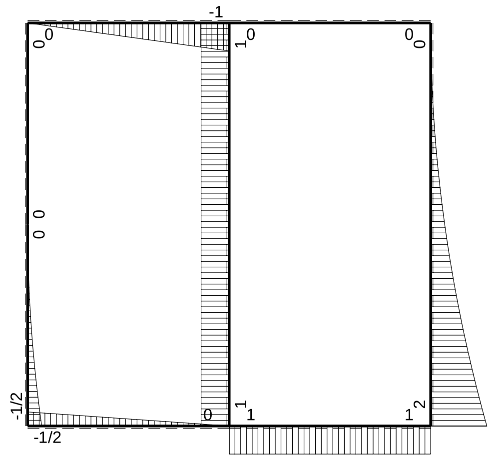
CI08.xxxx.051

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

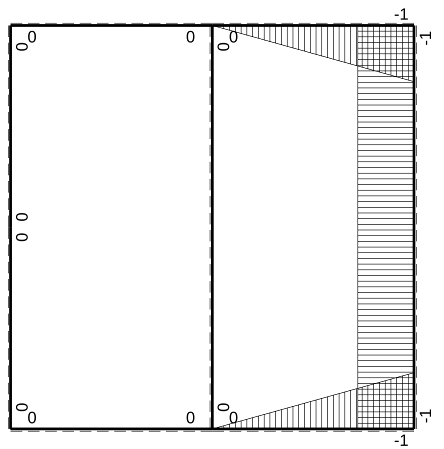

D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
G
1/2Fb
1/2F
E
F
F
1/2Fb
F
A
Fb
B
1/2F
3/2Fb
2F
F
A
B
B
F
F
1/2Fb
1/2F
F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


0
0
1/2
-1
0
-1
1
1
1/2
F
1
1/2
0
0
0
-1
1/2
-2
-1/2
F
-1
-1/2
0
0
1
0
-1/2
0 0
-1/2
0
1
1
1/2
3/2
-1/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (- x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [- x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

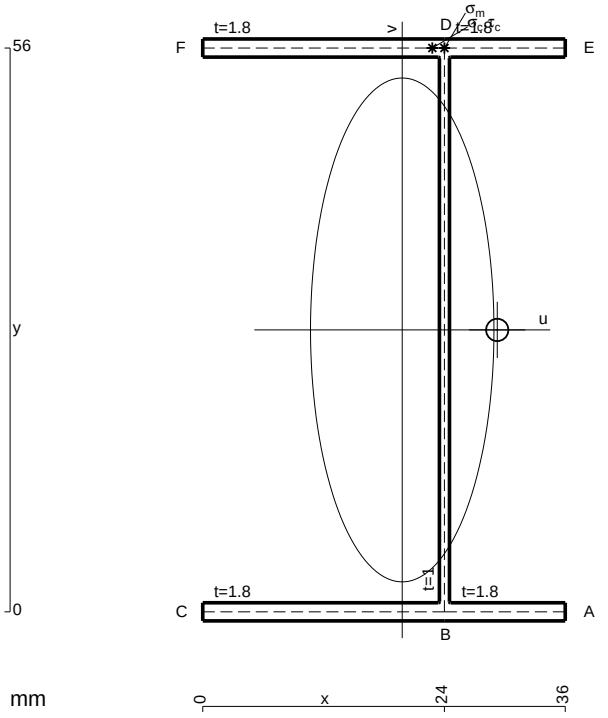
$= (- b +1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

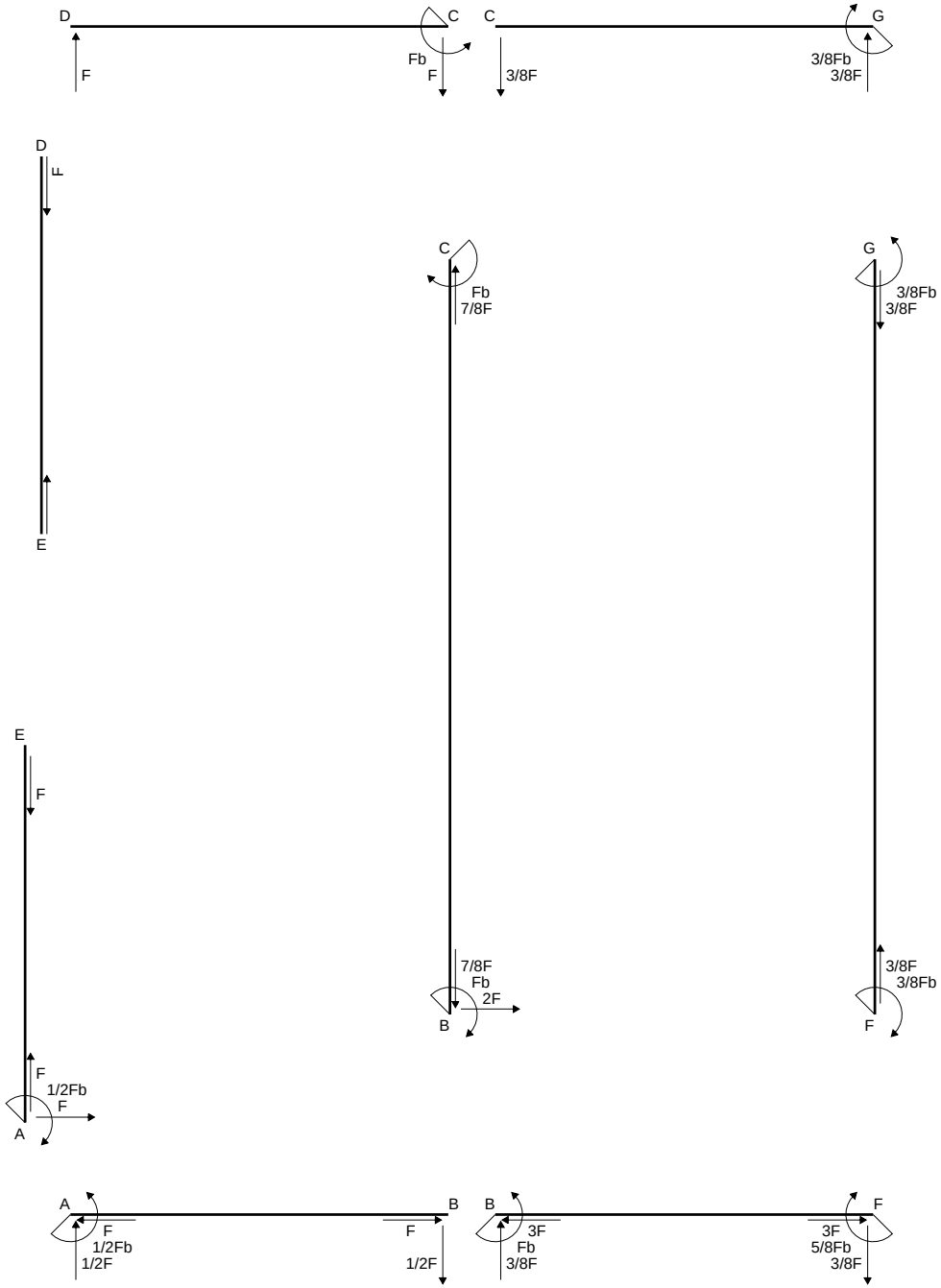
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

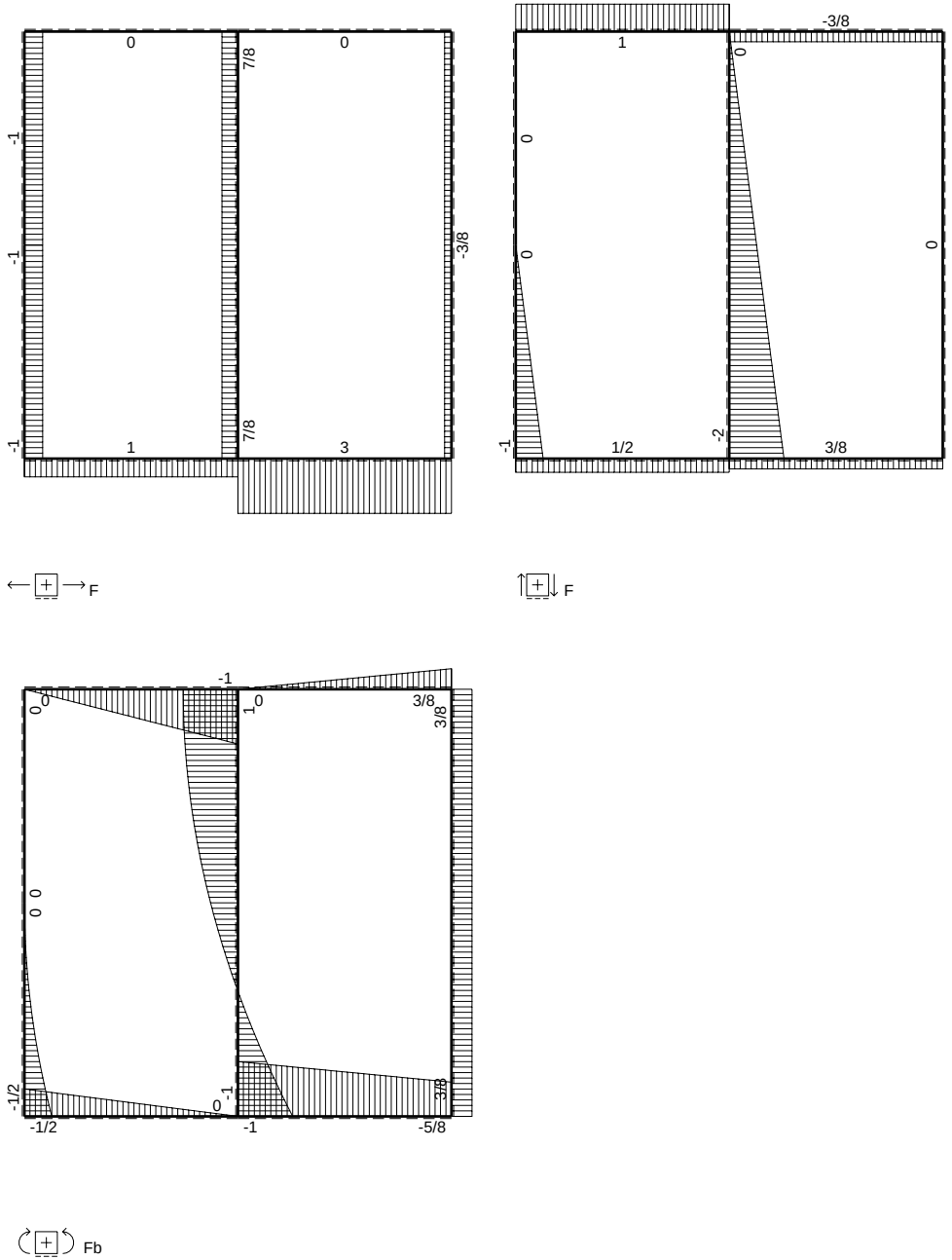


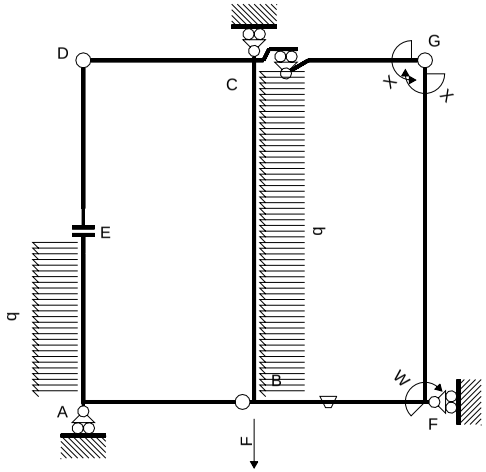
A = 185.6 mm^2
J_u = 116241. mm^4
J_v = 15405. mm^4
J_t = 158.6 mm^4
x_o = 9.434 mm
x_g = 19.81 mm
T_y = 1750. N
M_x = -910000. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 4.19 mm
v_m = 28. mm
$\sigma_m = -Mv/J_u$ = 219.2 N/mm^2
x_c = 24. mm
y_c = 56. mm
u_c = 4.19 mm
v_c = 28. mm
$\sigma_c = -Mv/J_u$ = 219.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 197.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.12 N/mm^2
$\tau_o = Tx_o t/J_t$ = 187.3 N/mm^2
t_c = 3150. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 406.2 N/mm^2

CI08.xxxx.052

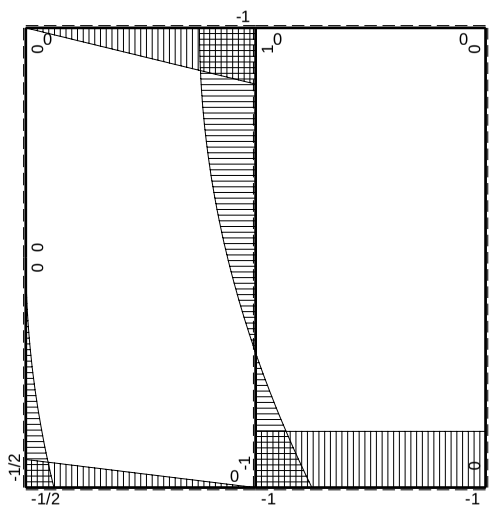
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
7/8F
G
3/8Fb
3/8F
E
F
7/8F
Fb
2F
B
3/8F
3/8Fb
F
F
1/2Fb
F
A
A
F
1/2Fb
1/2F
B
F
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F

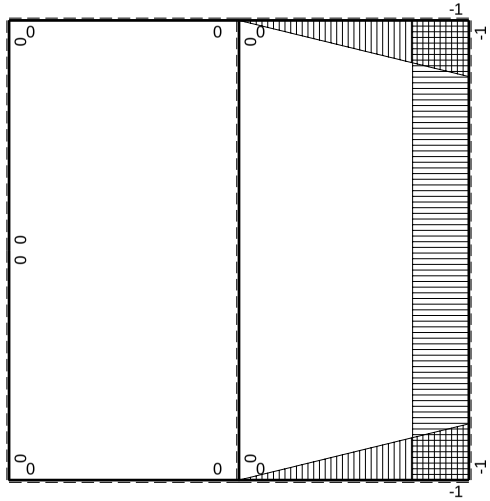

0
0
7/8
-1
-1
-3/8
-1
1
7/8
3
F
-3/8
1
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 215440 CI08.xxxx.053

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

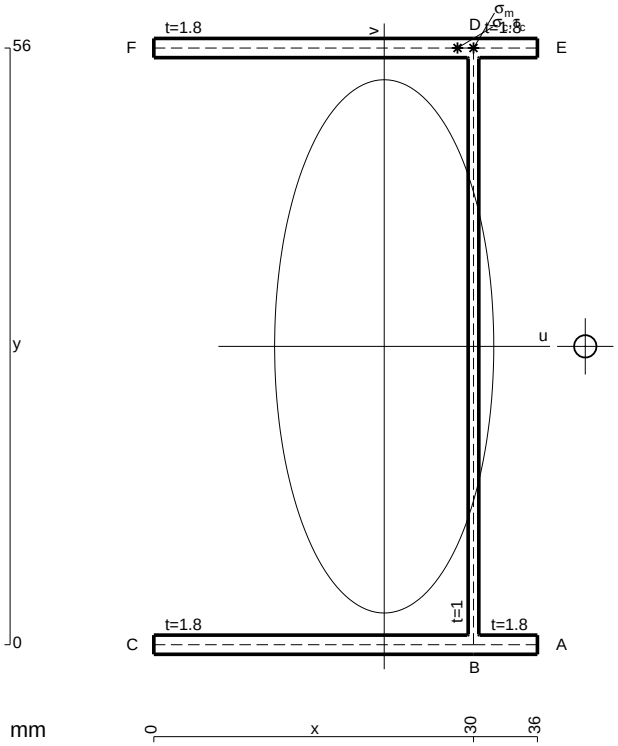
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

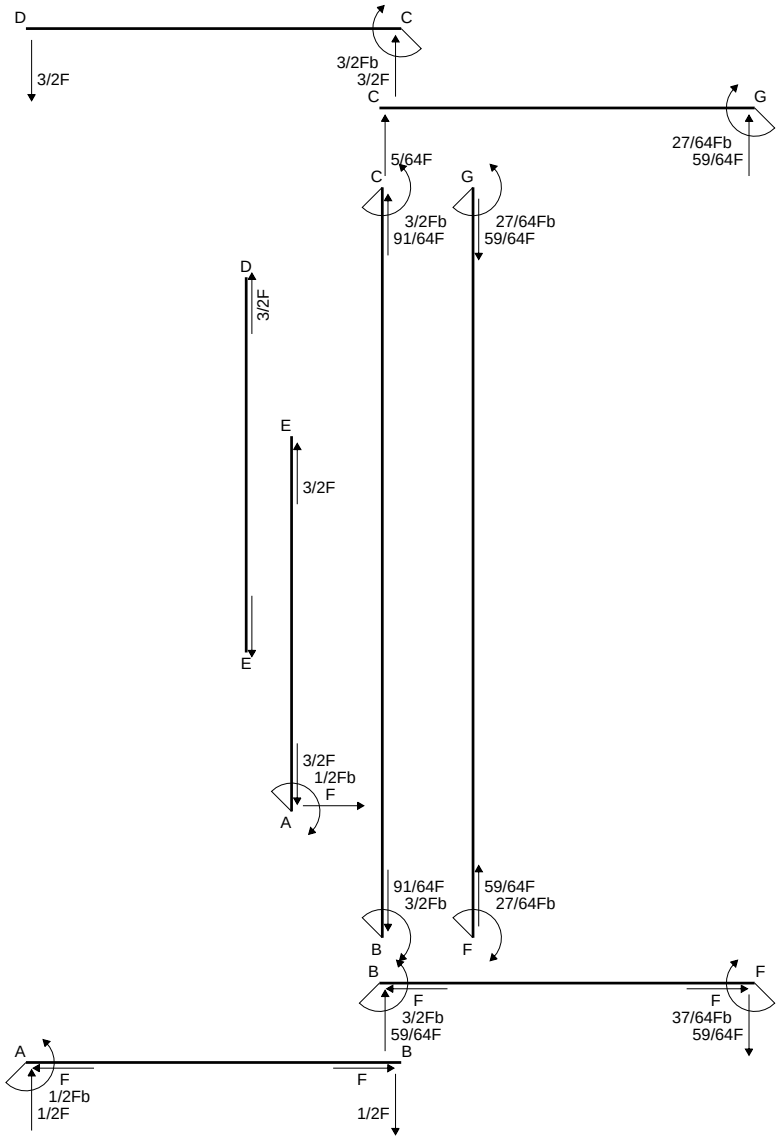
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

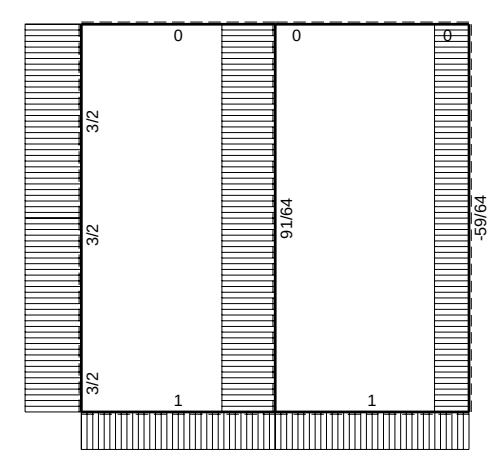


A = 185.6 mm^2
J_u = 116241. mm^4
J_v = 19628. mm^4
J_t = 158.6 mm^4
x_o = 18.87 mm
x_g = 21.62 mm
N = 1409. N
T_y = -3220. N
M_x = -917700. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 8.379 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 228.6 N/mm^2
x_c = 30. mm
y_c = 56. mm
u_c = 8.379 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 228.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 712.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 23.27 N/mm^2
$\tau_o = Tx_o t/J_t$ = 689.4 N/mm^2
t_c = 2898. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1255. N/mm^2

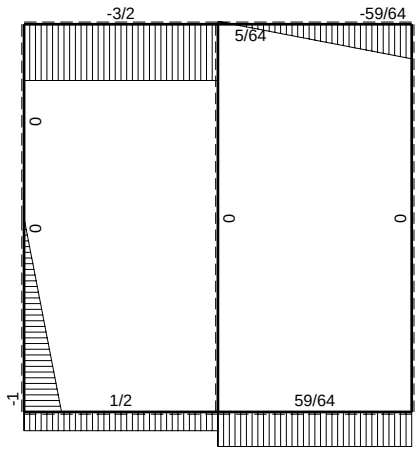
CI08.xxxx.053

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

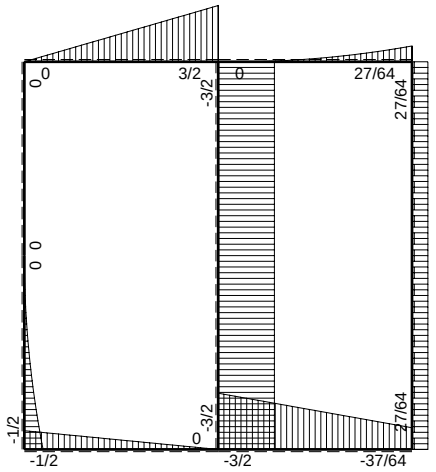

D
C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
3/2
91/64
-59/64
1
1

F


-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F


0
0
3/2
-3/2
0
27/64
27/64
0
0
-1/2
-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	-1/2qx^2	0	0	0	0	0+0	0
AE b	0	1/2Fb-Fx+1/2qx^2	0	0	0	0		
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(7/12+1/2)Fb^2/EJ	1/3XbJ/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3XbJ/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbJ/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						9/8Fb^2/EJ	8/3XbJ/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

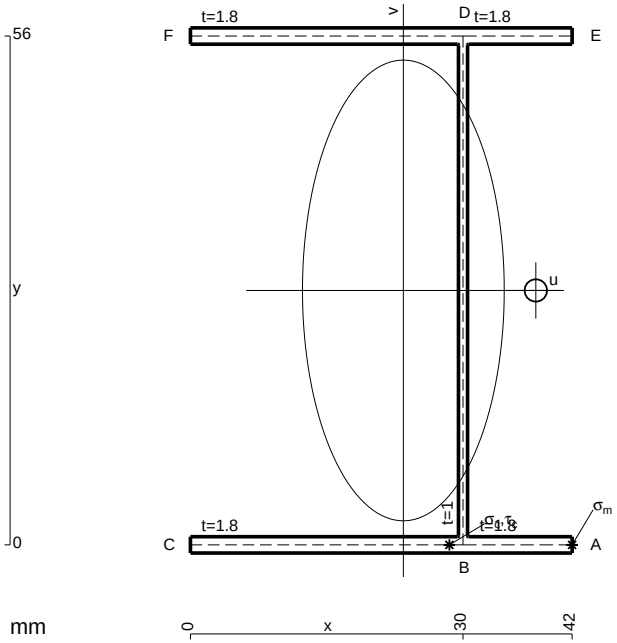
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

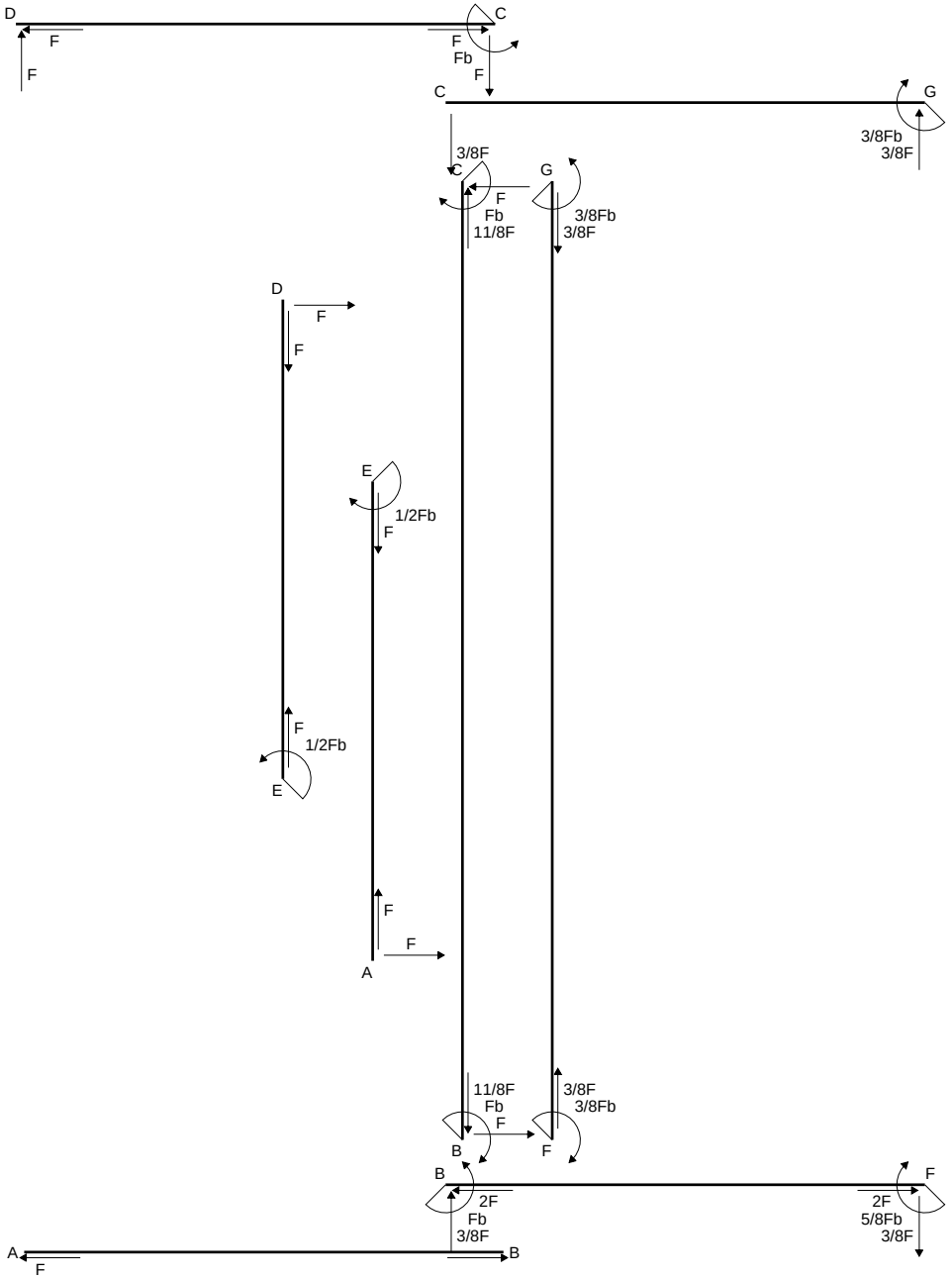
$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

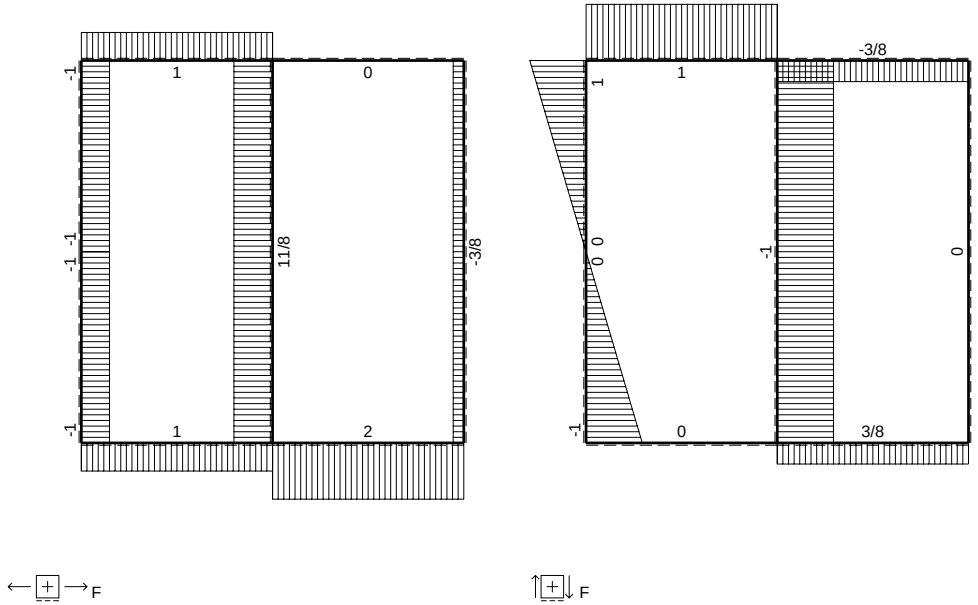


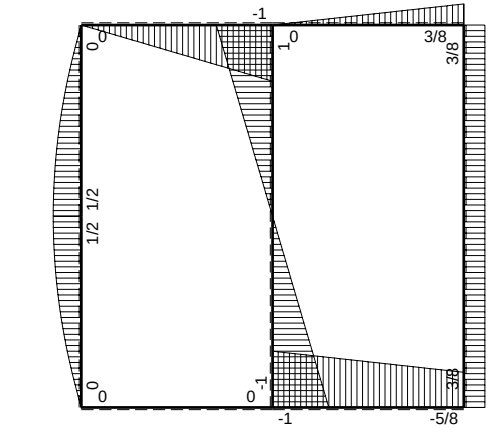
A = 207.2 mm^2
J_u = 133175. mm^4
J_v = 25537. mm^4
J_t = 182. mm^4
x_o = 14.58 mm
x_g = 23.43 mm
T_y = -1860. N
M_x = 1134600. Nmm
x_m = 42. mm
u_m = 18.57 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = 238.5 N/mm^2
x_c = 30. mm
u_c = 6.568 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = 238.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 280. N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.73 N/mm^2
$\tau_o = Tx_o t/J_t$ = 268.2 N/mm^2
t_c = 2232. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 540.4 N/mm^2

CI08.xxxx.054

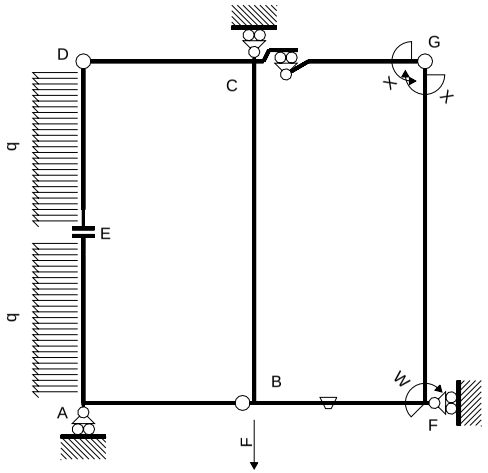
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
11/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
11/8F
Fb
F
3/8F
3/8Fb
F
B
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F

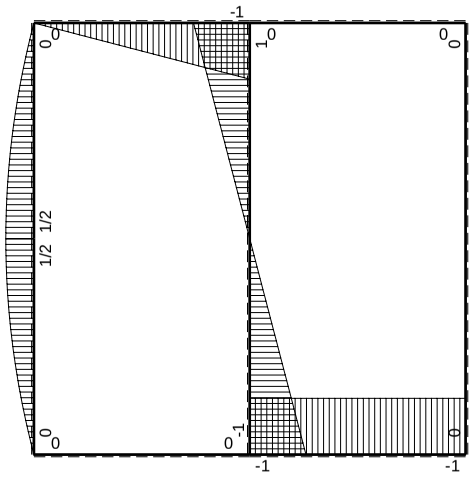

-1
1
0
-1
-1
11/8
-3/8
-1
1
2
F
-3/8
1
1
0 0
-1
0
-1
0
3/8
F


-1
0
0
1
0
3/8
3/8
1/2 1/2
0
0
0
-1
-1
3/8
-1
-5/8

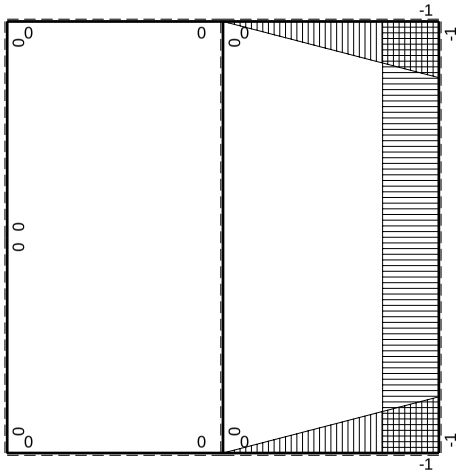

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
				totali			Fb^2/EJ	8/3Xb/EJ
				iperstatica $X=W_{GC}$			-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 215079 CI08.xxxx.055

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

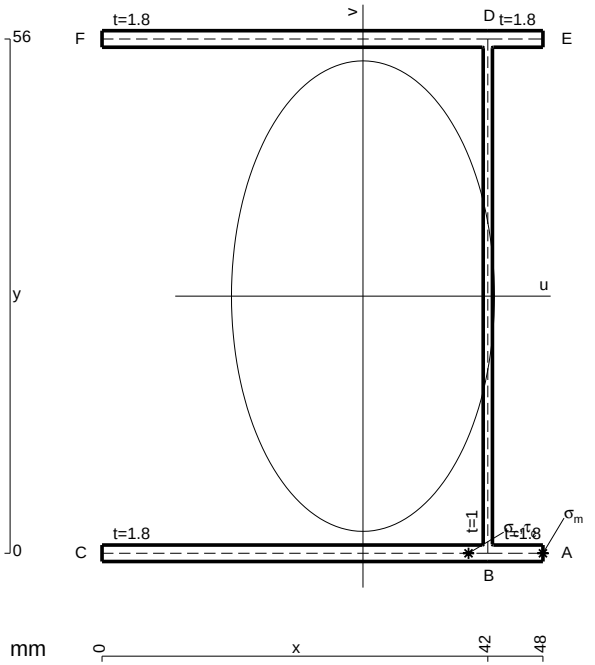
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

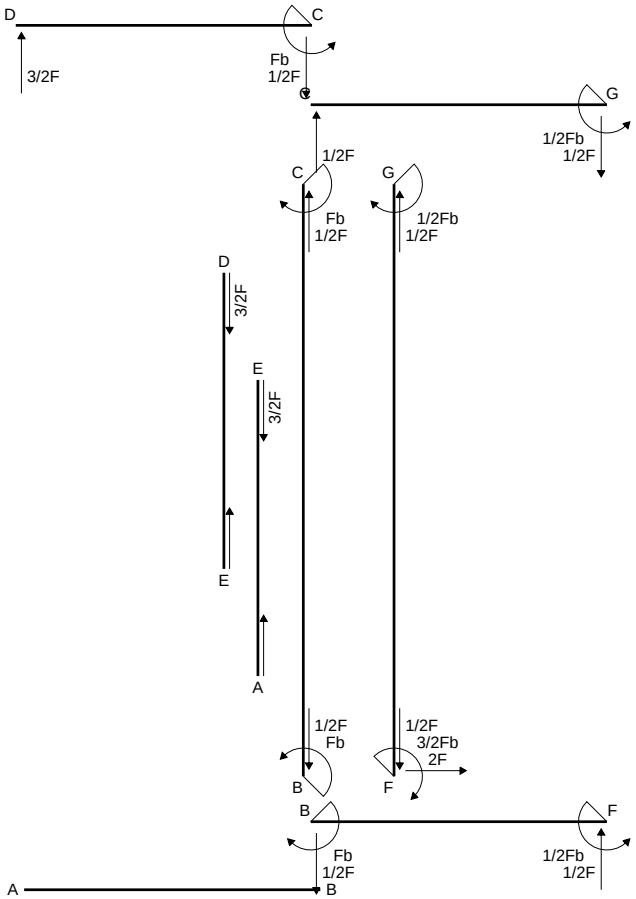
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

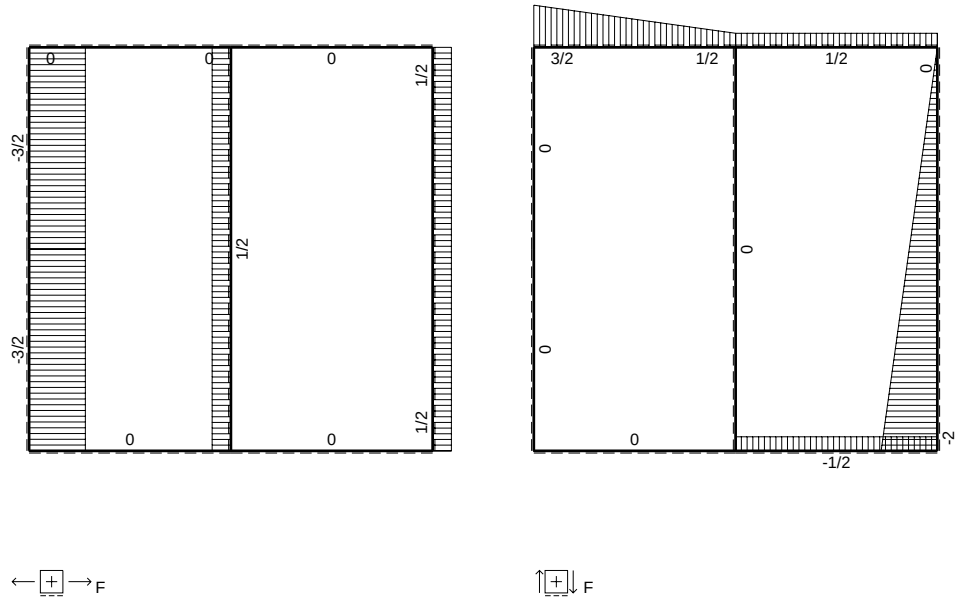


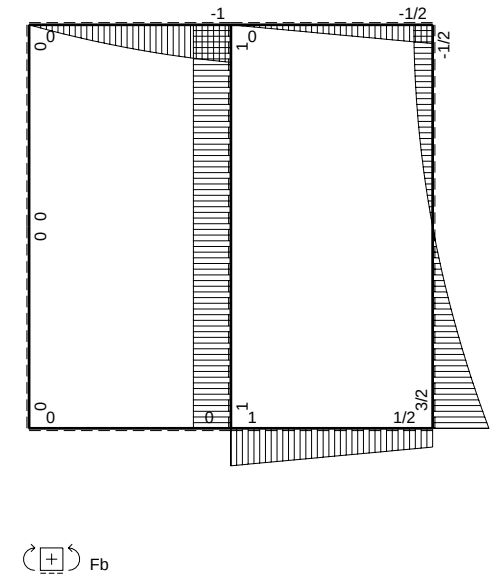
A = 228.8 mm^2
J_u = 150110. mm^4
J_v = 46881. mm^4
J_t = 205.3 mm^4
x_o = 29.84 mm
x_g = 28.41 mm
N = 2118. N
T_y = -1540. N
M_x = 1016400. Nmm
x_m = 48. mm
u_m = 19.59 mm
v_m = -28. mm
$\sigma_m = N/A - Mv/J_u$ = 198.8 N/mm^2
x_c = 42. mm
u_c = 13.59 mm
v_c = -28. mm
$\sigma_c = N/A - Mv/J_u$ = 198.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 415. N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.06 N/mm^2
$\tau_o = Tx_o t/J_t$ = 402.9 N/mm^2
t_c = 2772. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 745.8 N/mm^2

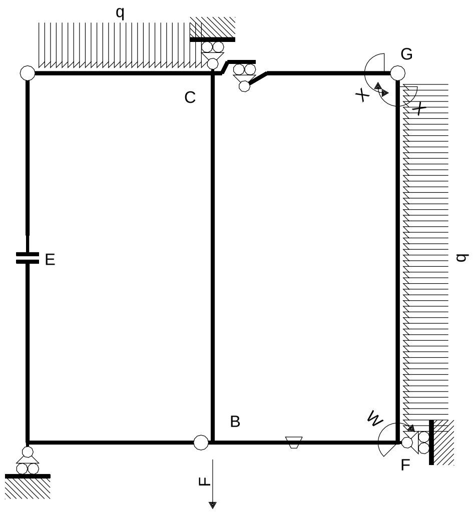
CI08.xxxx.055

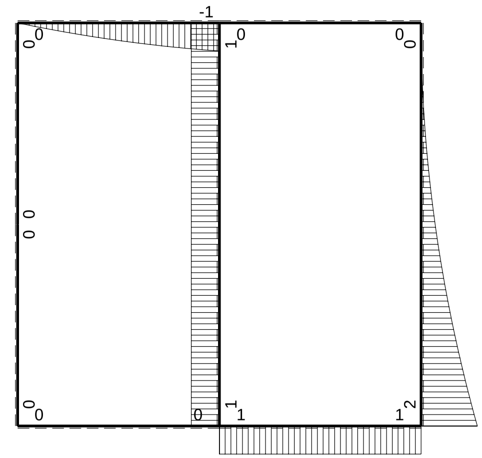
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3/2F
Fb
1/2F
G
1/2Fb
1/2F
1/2F
C
G
Fb
1/2F
1/2Fb
1/2F
D
3/2F
E
3/2F
E
A
1/2F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B

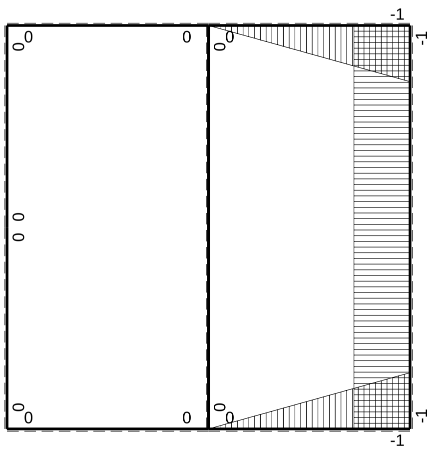

0
0
0
1/2
-3/2
1/2
-3/2
0
0
1/2
3/2
1/2
1/2
0
0
0
0
0
-2
-1/2
F
F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

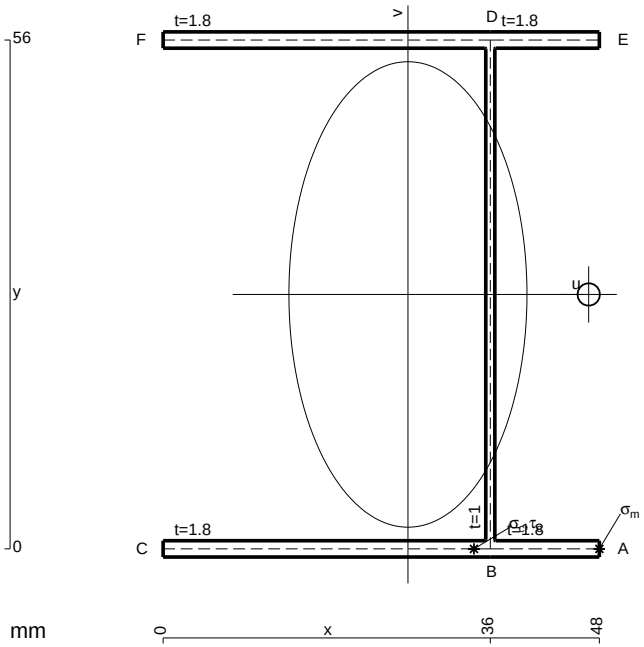
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

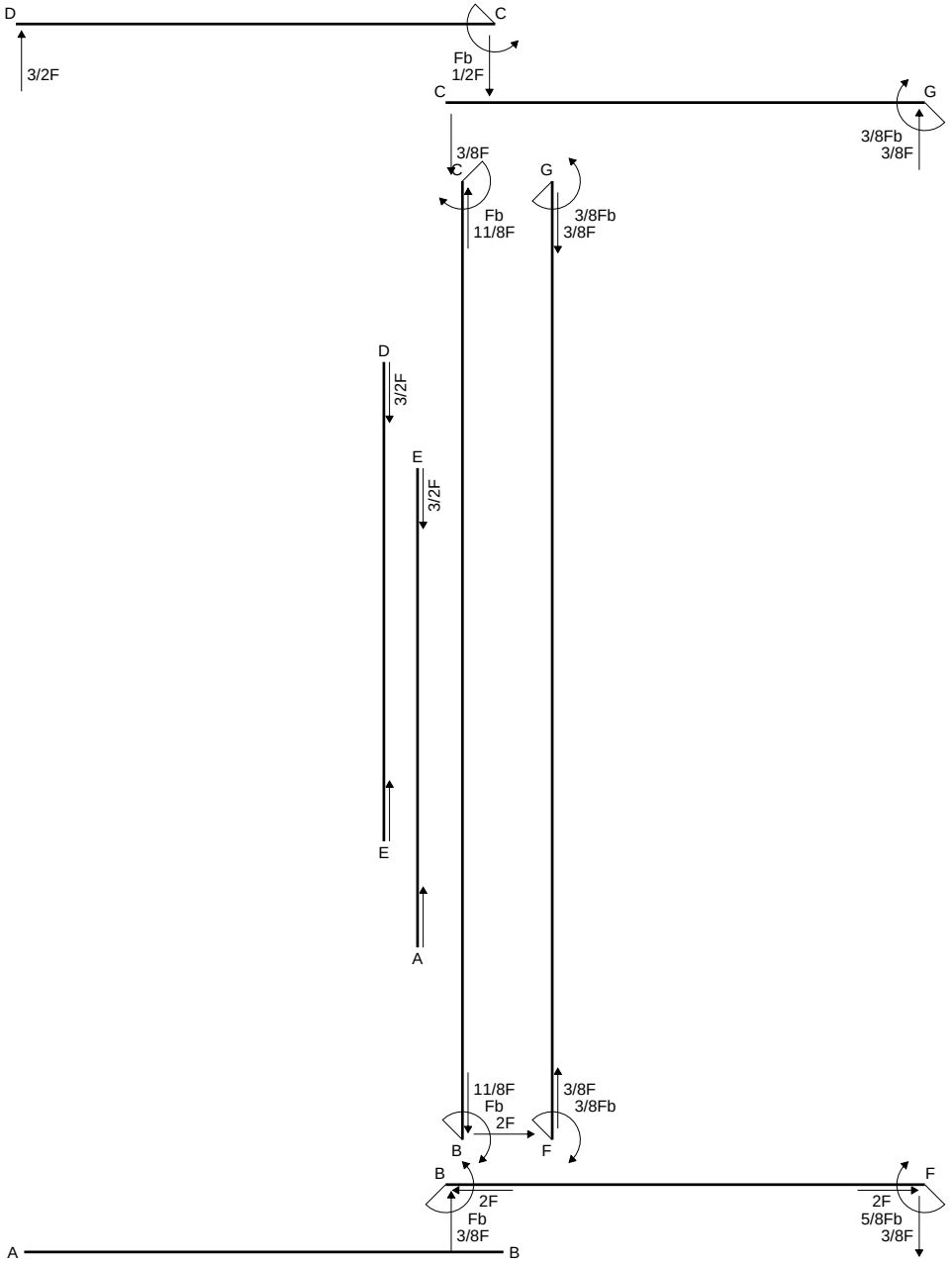
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

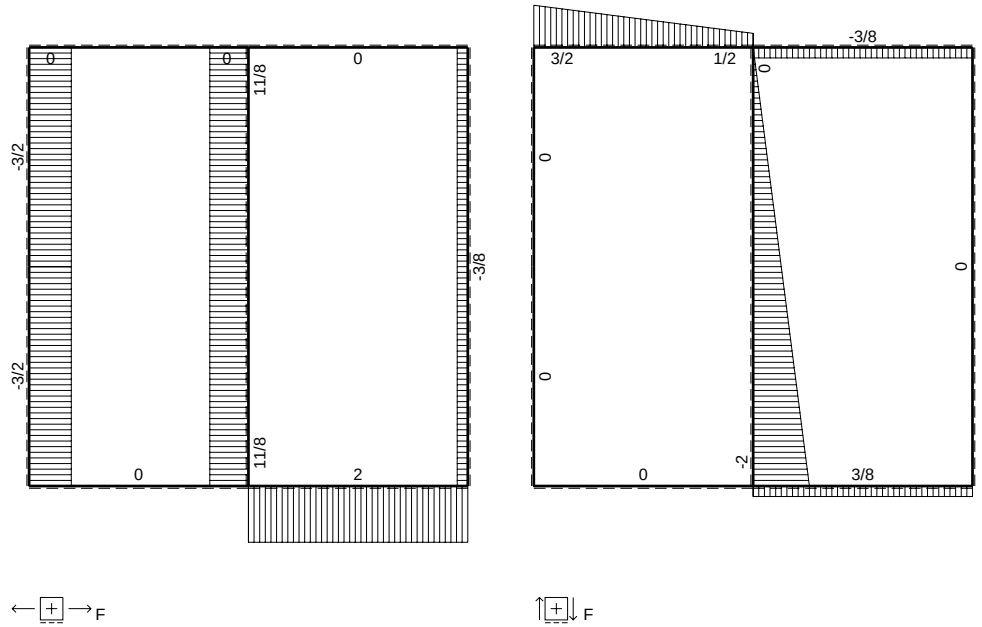


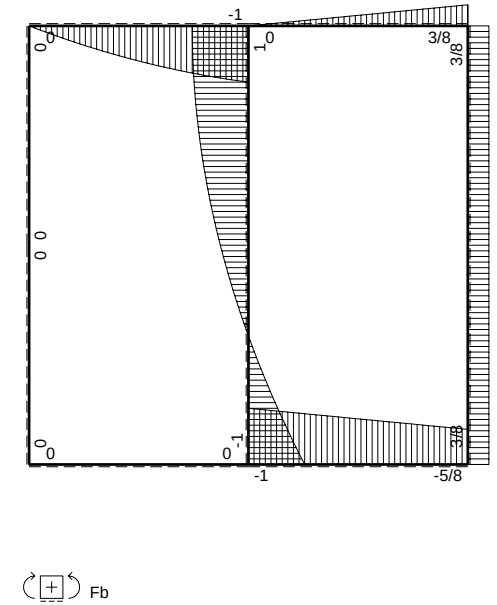
A = 228.8 mm^2
J_u = 150110. mm^4
J_v = 39268. mm^4
J_t = 205.3 mm^4
x_o = 19.89 mm
x_g = 26.94 mm
T_y = 2570. N
M_x = -1124380. Nmm
x_m = 48. mm
u_m = 21.06 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -209.7 N/mm^2
x_c = 36. mm
u_c = 9.063 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -209.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 465.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 17.26 N/mm^2
$\tau_o = Tx_o t/J_t$ = 448.3 N/mm^2
t_c = 4626. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 833.1 N/mm^2

CI08.xxxx.056

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3/2F
Fb
1/2F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
11/8F
3/8Fb
3/8F
D
3/2F
E
3/2F
E
A
11/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B


0
0
0
11/8
-3/2
-3/8
-3/2
11/8
0
2
F
3/2
1/2
-3/8
0
0
0
0
-2
3/8
F


-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_o flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 248788 CI08.xxxx.057

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

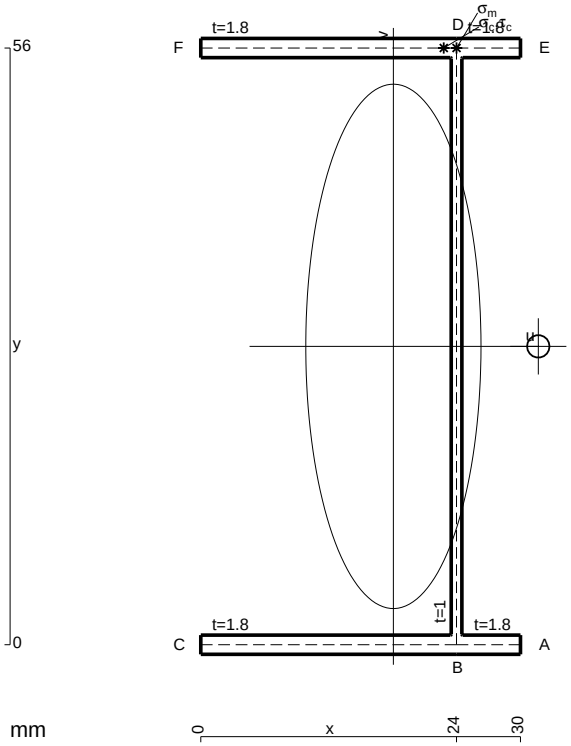
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

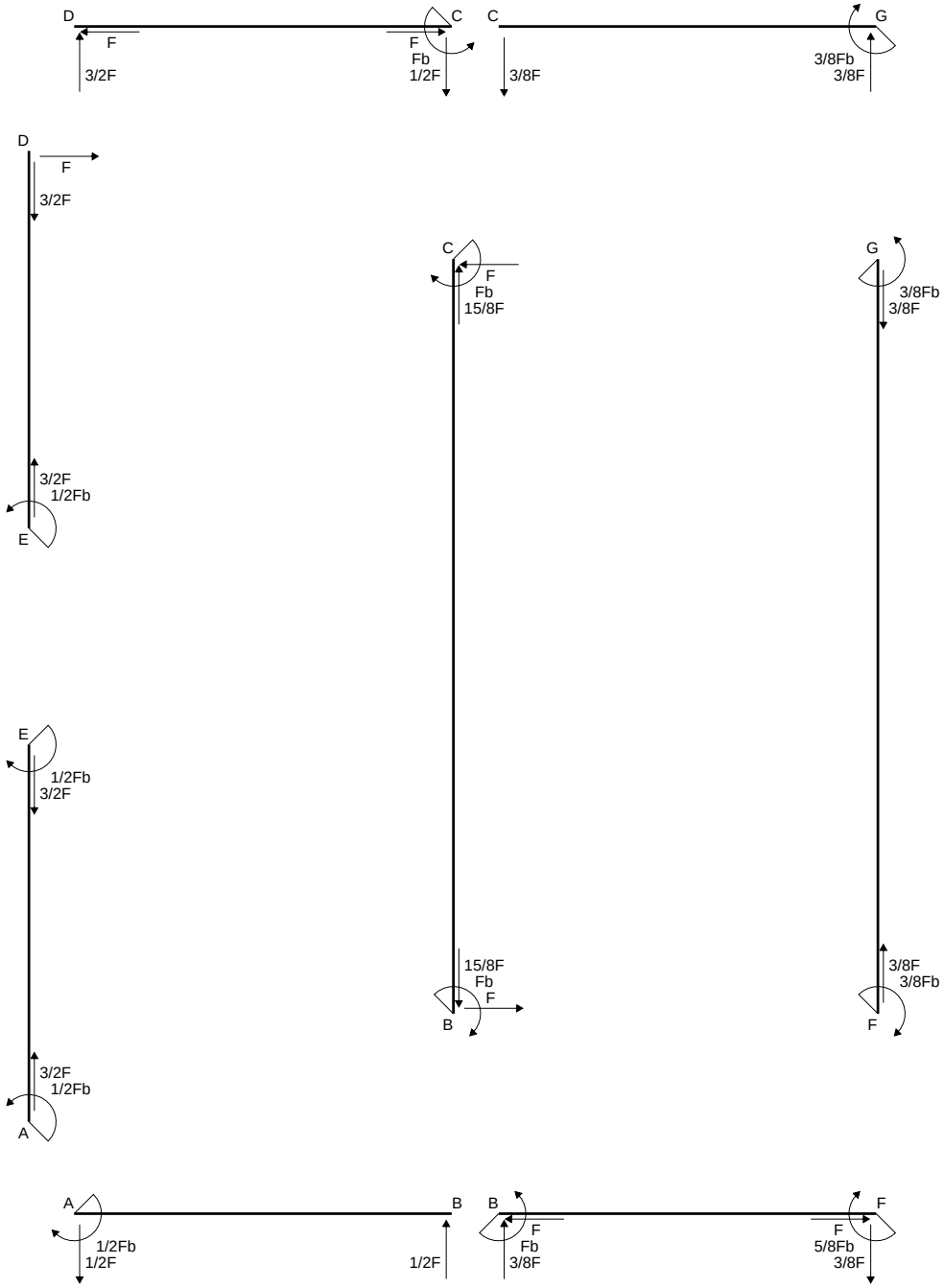
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

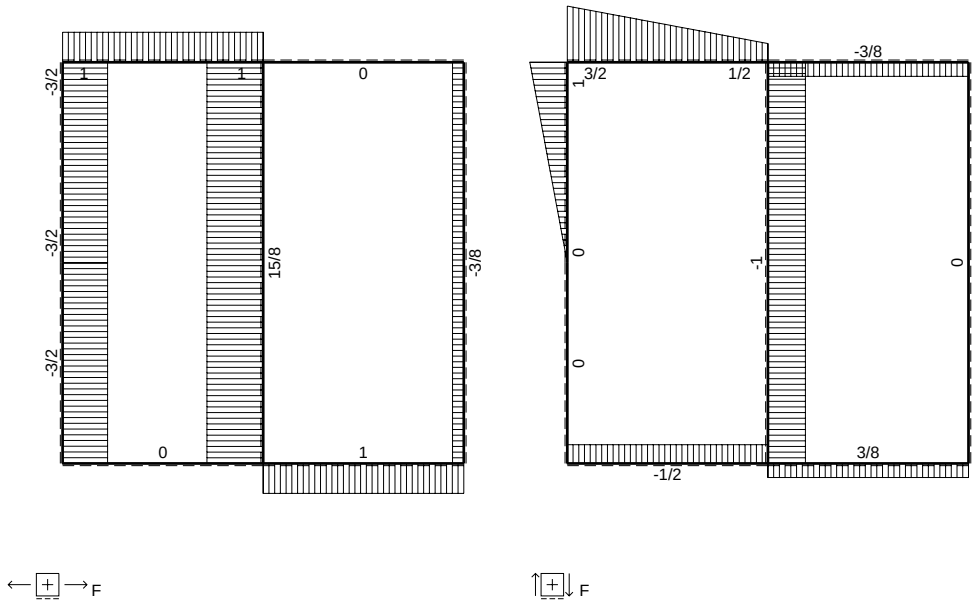


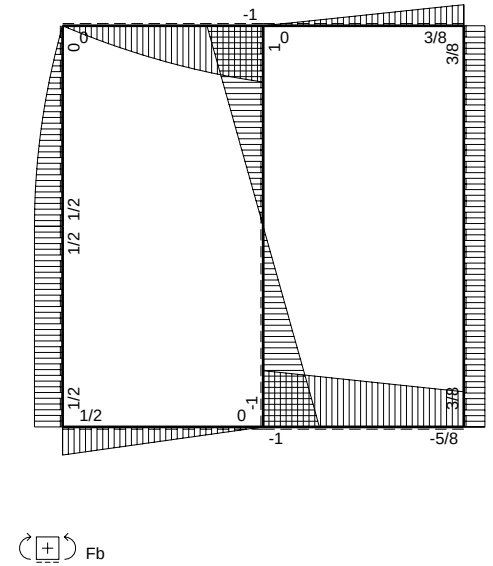
A = 164. mm^2
J_u = 99307. mm^4
J_v = 11087. mm^4
J_t = 135.3 mm^4
x_o = 13.6 mm
x_g = 18.07 mm
N = 1389. N
T_y = -2020. N
M_x = -747400. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 5.927 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 219.2 N/mm^2
x_c = 24. mm
y_c = 56. mm
u_c = 5.927 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 219.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 379.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.67 N/mm^2
$\tau_o = Tx_o t/J_t$ = 365.5 N/mm^2
t_c = 1818. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 692.3 N/mm^2

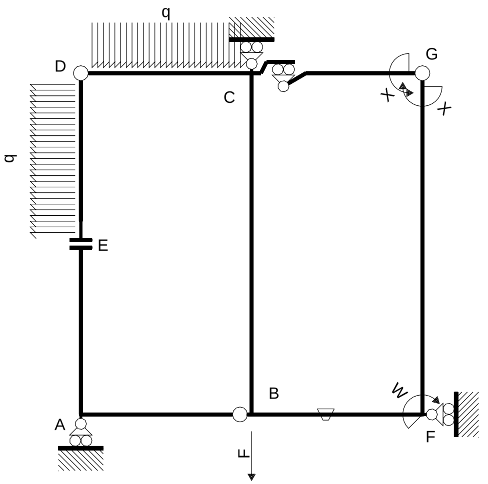
CI08.xxxx.057

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

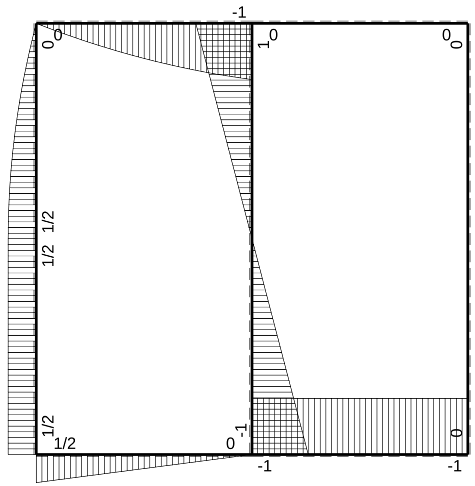

D
F
3/2F
C
C
F
Fb
1/2F
3/8F
G
3/8Fb
3/8F
D
F
3/2F
3/2F
1/2Fb
E
C
F
Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3/2F
15/8F
Fb
F
B
3/8F
3/8Fb
F
3/2F
1/2Fb
A
A
B
1/2Fb
1/2F
1/2F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F


-3/2
-3/2
-3/2
1
1
0
15/8
-3/8
0
1
F
-3/8
3/2
1/2
0
0
-1
0
3/8
-1/2
F

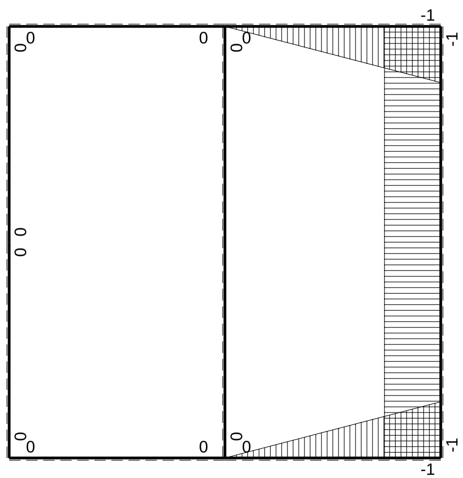

-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

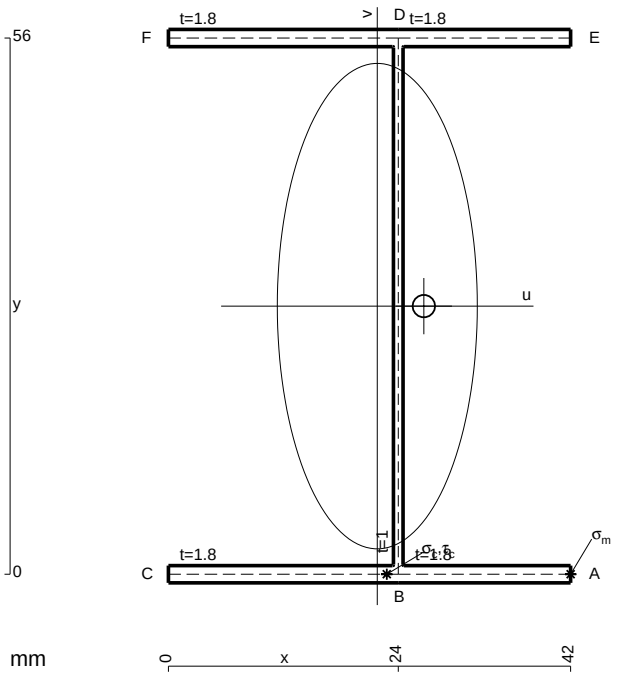
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

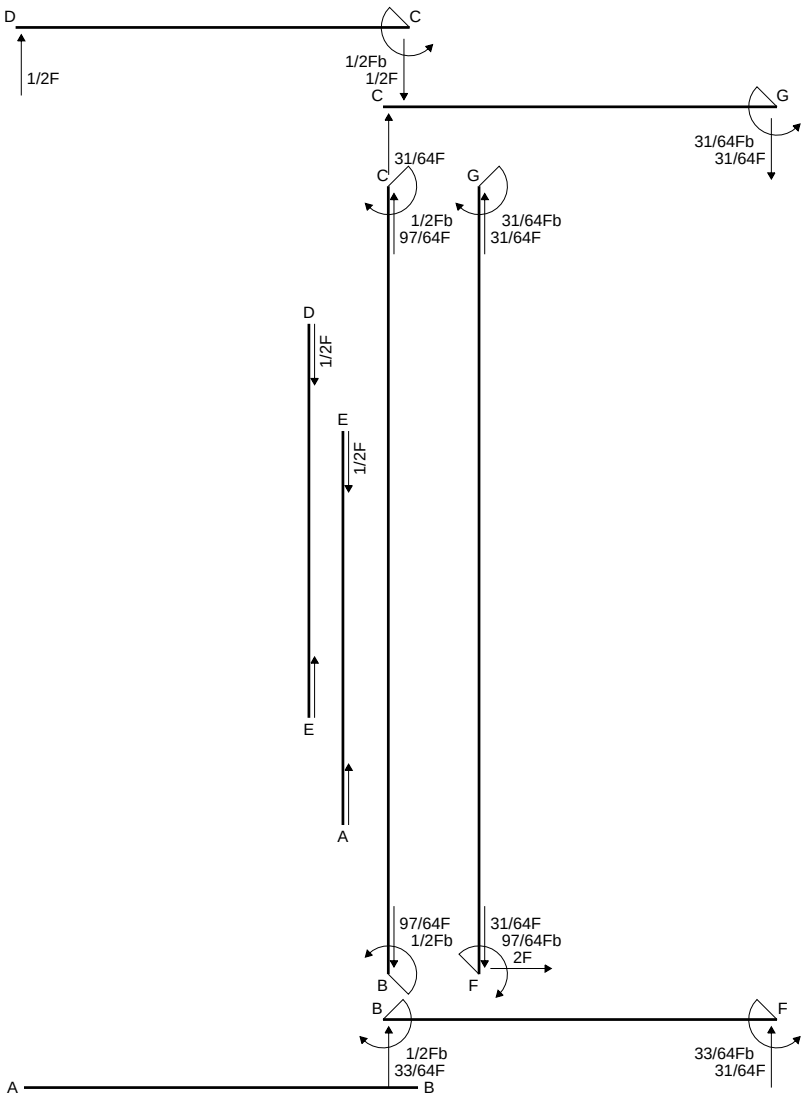
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

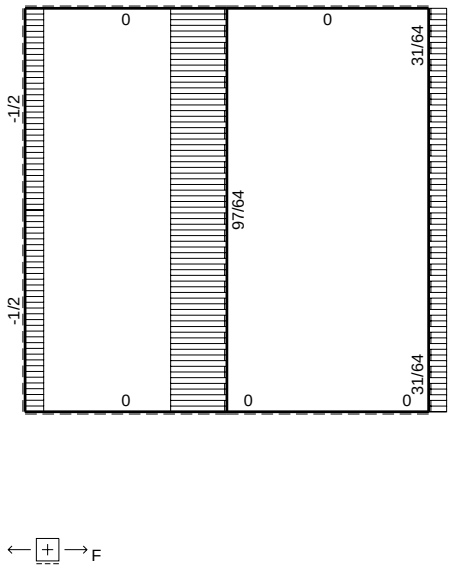


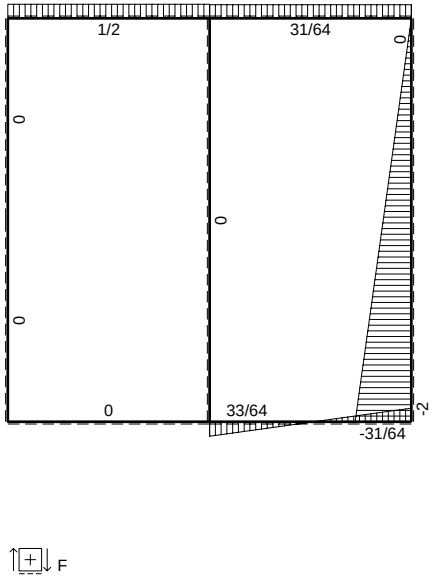
A = 207.2 mm^2
J_u = 133175. mm^4
J_v = 22594. mm^4
J_t = 182. mm^4
x_o = 4.86 mm
x_g = 21.81 mm
N = 2456. N
T_y = -1310. N
M_x = 1034900. Nmm
x_m = 42. mm
u_m = 20.19 mm
v_m = -28. mm
$\sigma_m = N/A - Mv/J_u$ = 229.4 N/mm^2
x_c = 24. mm
u_c = 2.189 mm
v_c = -28. mm
$\sigma_c = N/A - Mv/J_u$ = 229.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 69.58 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.61 N/mm^2
$\tau_o = Tx_o t/J_t$ = 62.97 N/mm^2
t_c = 2358. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 259.2 N/mm^2

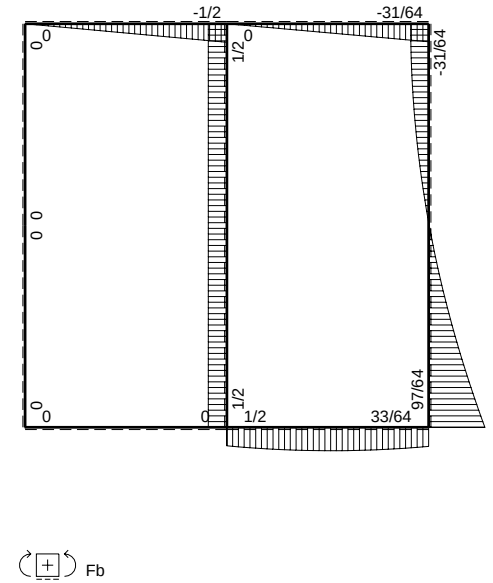
CI08.xxxx.058

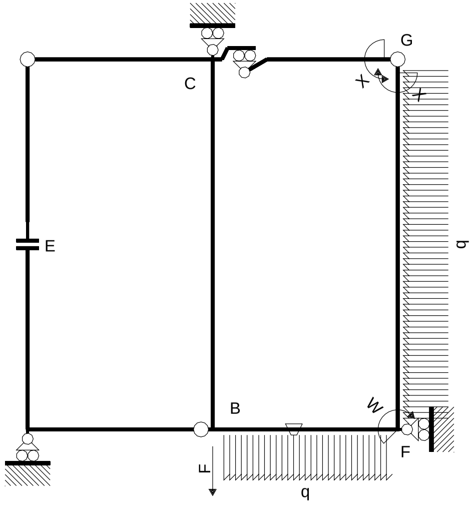
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

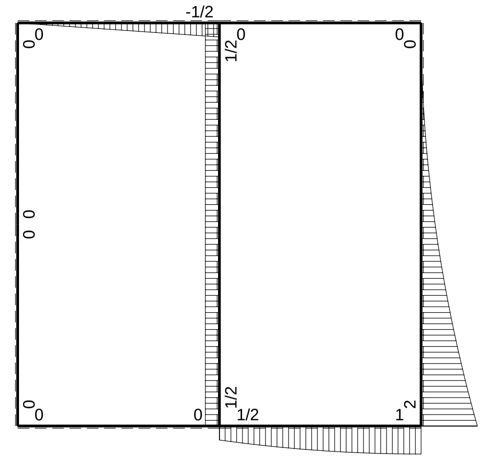

D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64
F

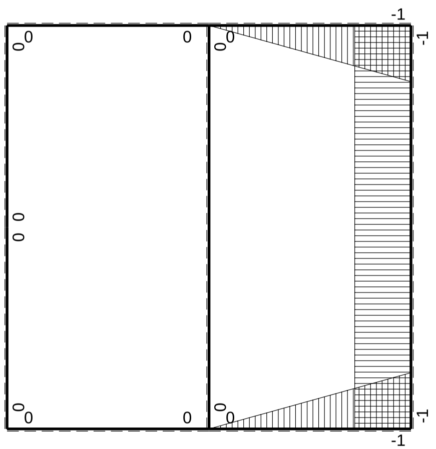

1/2
31/64
0
0
0
0
0
33/64
-2
-31/64
F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
1/2
33/64
97/64
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						$-31/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

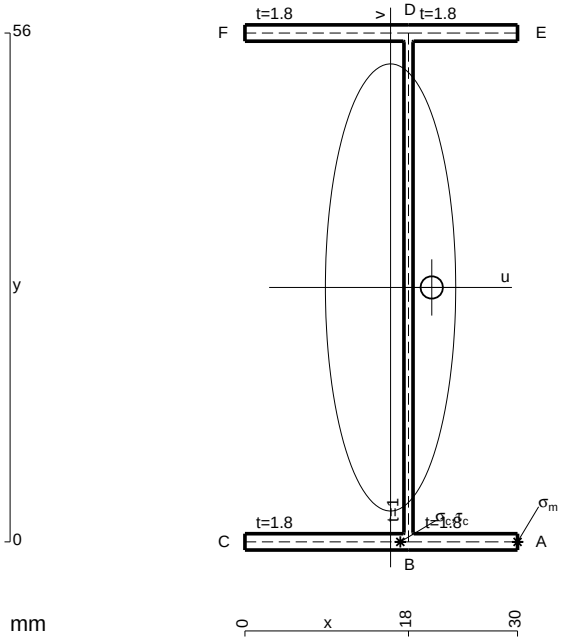
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L_{FG}^{Xo} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L_{GF}^{Xo} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

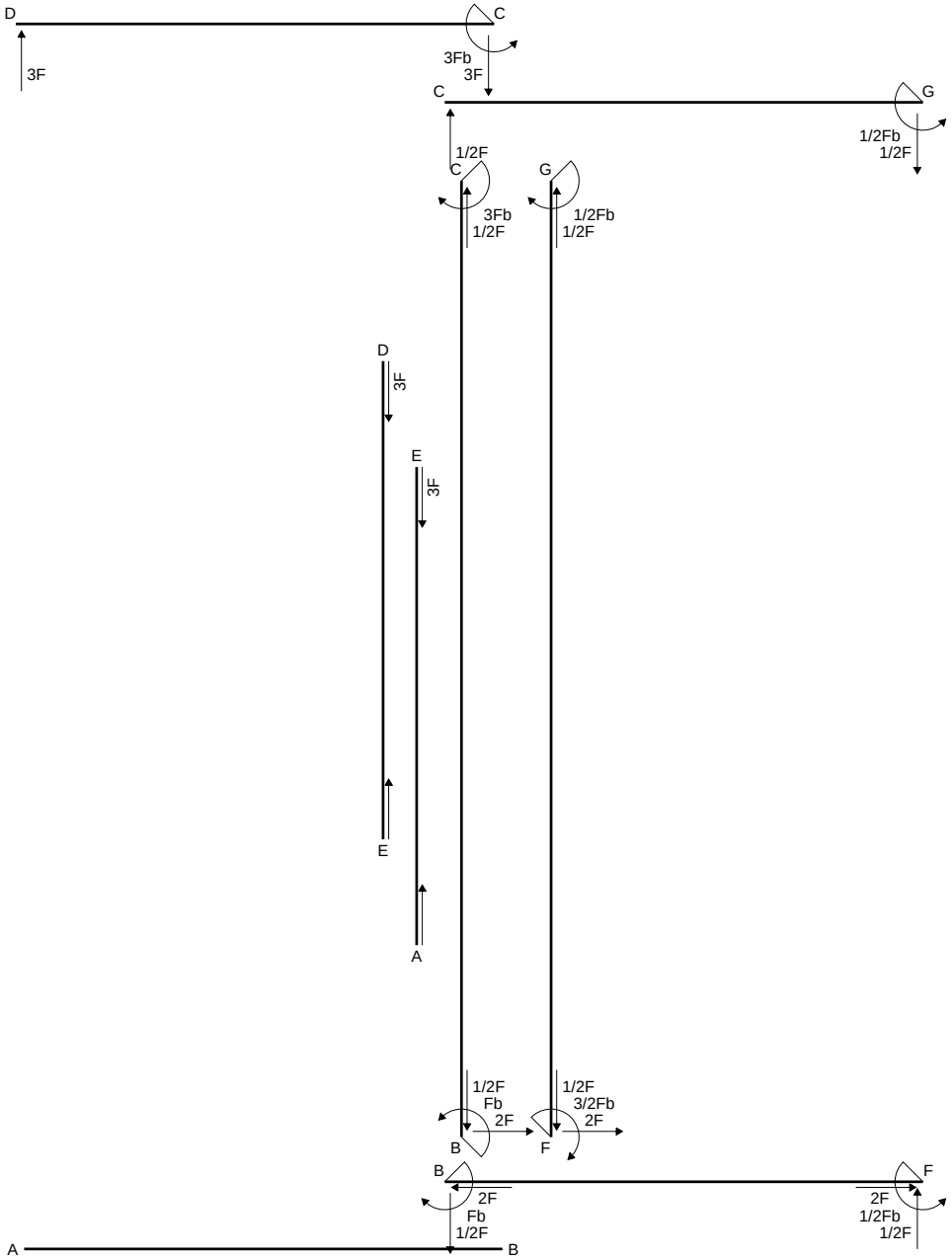
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

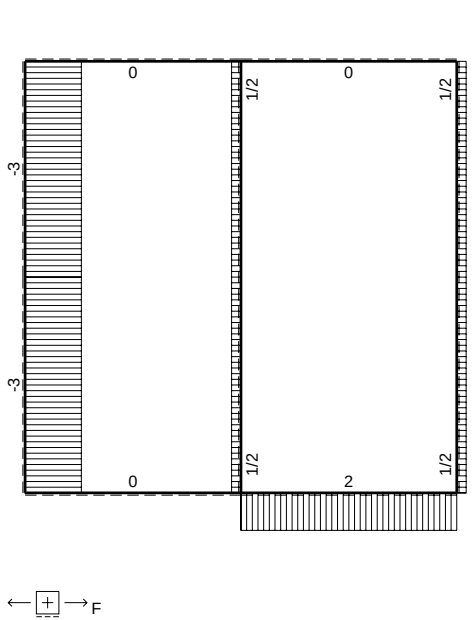


A = 164. mm^2
J_u = 99307. mm^4
J_v = 8432. mm^4
J_t = 135.3 mm^4
x_o = 4.534 mm
x_g = 16.02 mm
T_y = 1025. N
M_x = -850750. Nmm
x_m = 30. mm
u_m = 13.98 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -239.9 N/mm^2
x_c = 18. mm
u_c = 1.976 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -239.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 67.02 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.202 N/mm^2
$\tau_o = Tx_ot/J_t$ = 61.82 N/mm^2
t_c = 3690. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 266.5 N/mm^2

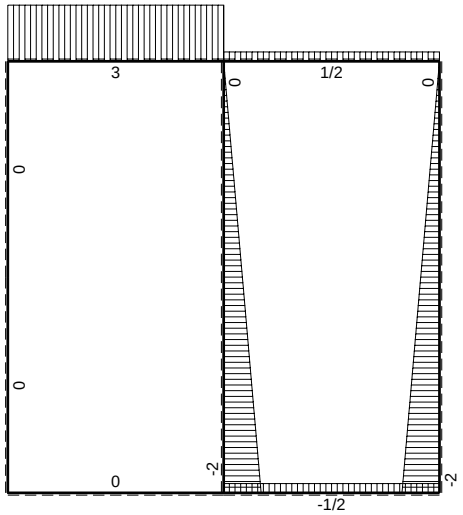
CI08.xxxx.059

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

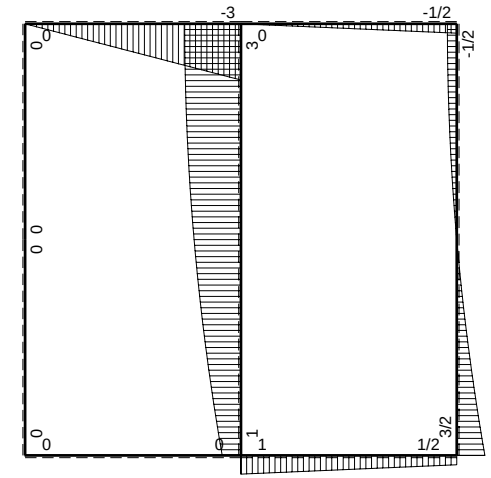

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
0
1/2
1/2
-3
-3
0
0
1/2
1/2
2

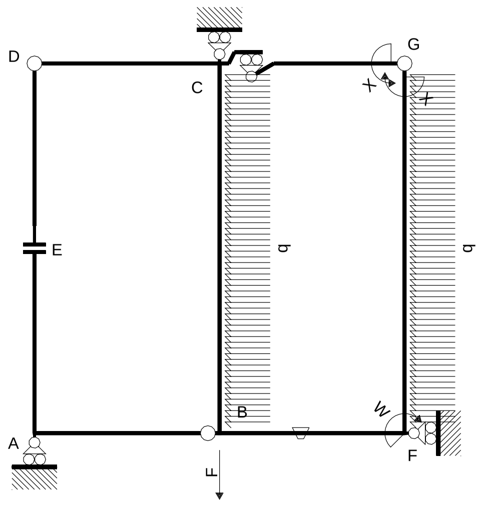
F

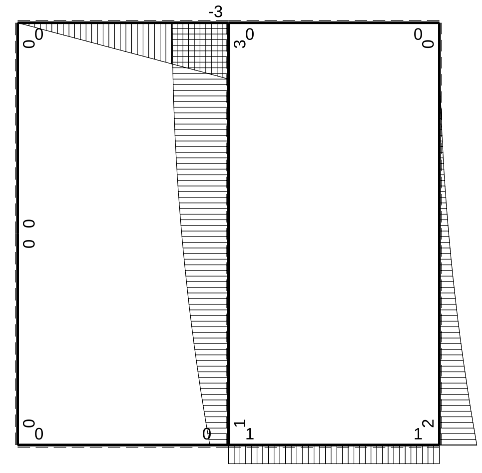

3
0
1/2
0
0
0
0
-2
-2
-1/2

F

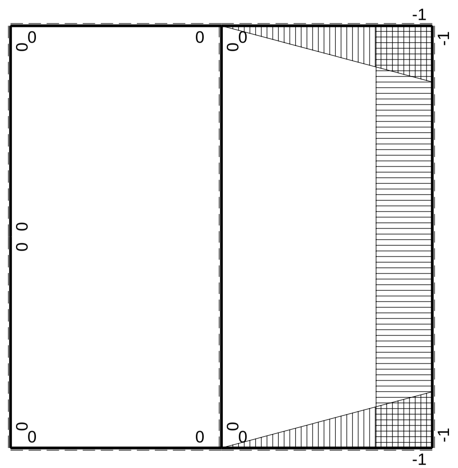

-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
0
1
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

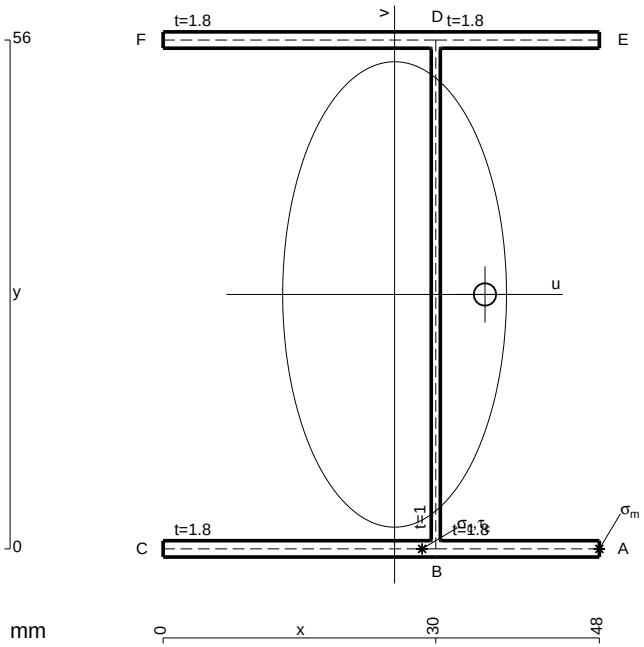
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

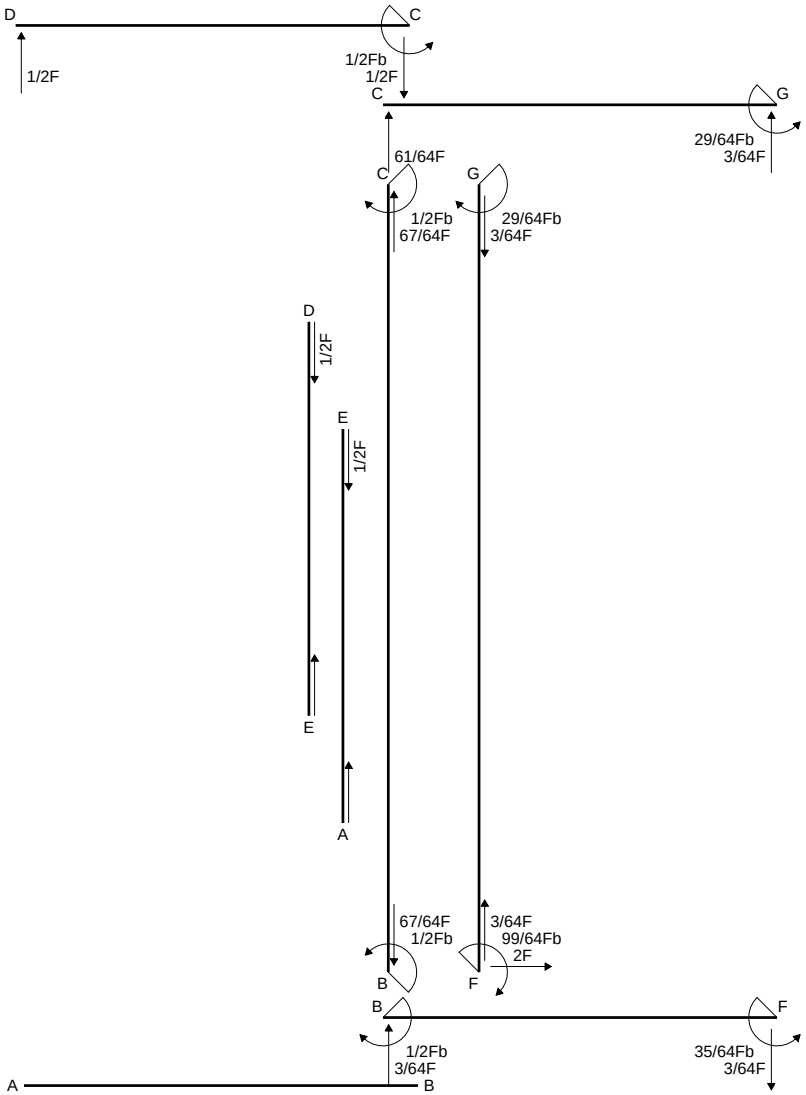
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

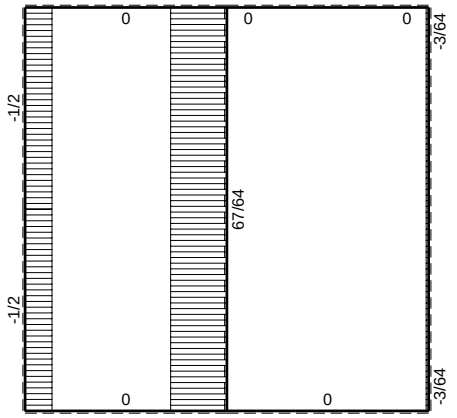


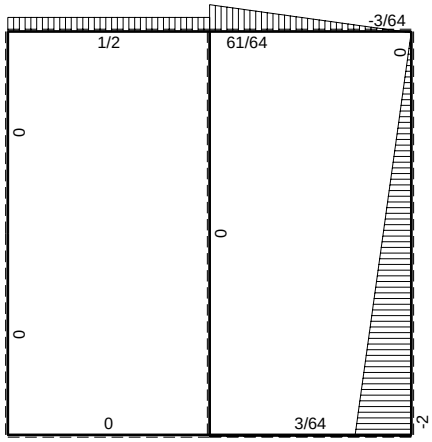
A = 228.8 mm^2
J_u = 150110. mm^4
J_v = 34700. mm^4
J_t = 205.3 mm^4
x_o = 9.947 mm
x_g = 25.47 mm
T_y = 2430. N
M_x = -1069200. Nmm
x_m = 48. mm
u_m = 22.53 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -199.4 N/mm^2
x_c = 30. mm
u_c = 4.531 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -199.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 225.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.6 N/mm^2
$\tau_o = Tx_o t/J_t$ = 211.9 N/mm^2
t_c = 1458. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 438.6 N/mm^2

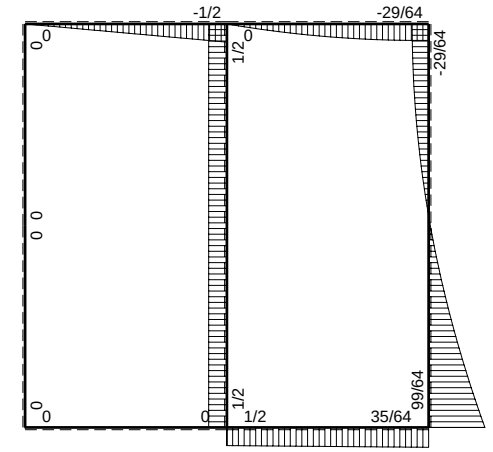
CI08.xxxx.060

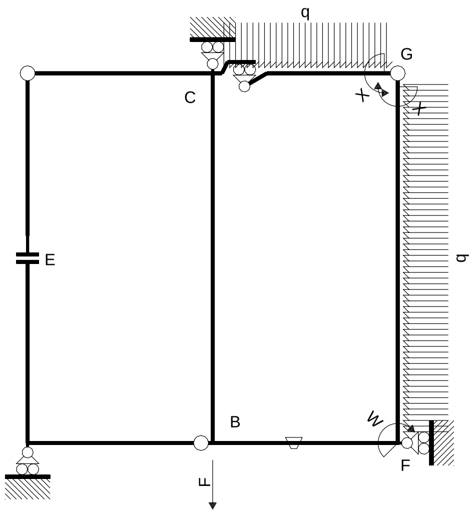
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

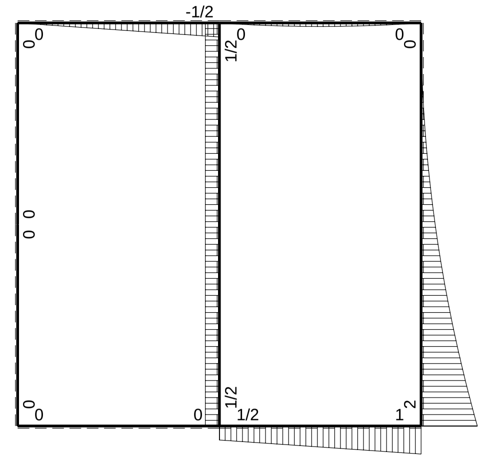

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64

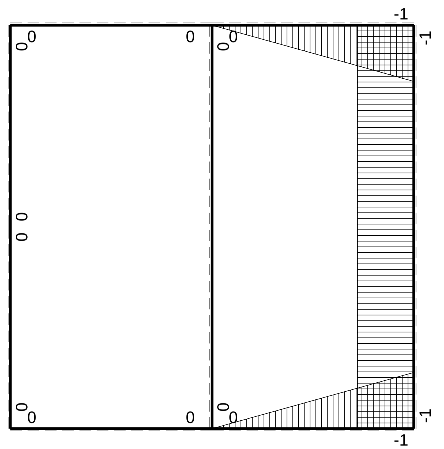

-3/64
1/2
61/64
0
0
0
0
0
3/64
-2


-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
0
1/2
35/64
99/64





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981229 CI08.xxxx.061

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	$-1/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(-5/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	$-Fb+3/2Fx-1/2Fx^2/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
totali							$-29/24Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							29/64Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

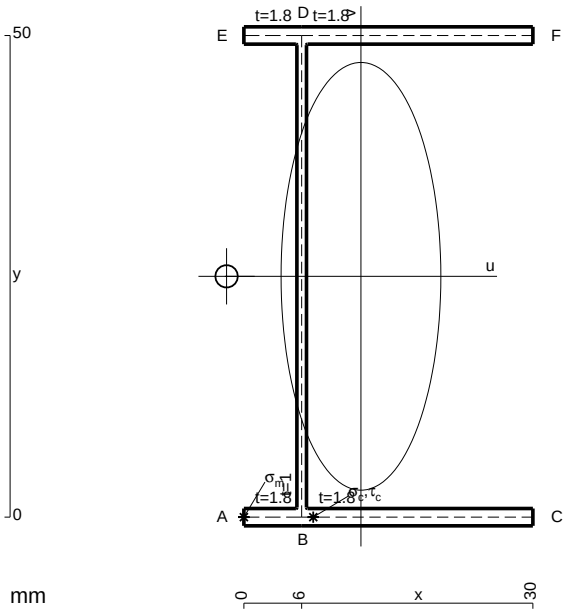
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

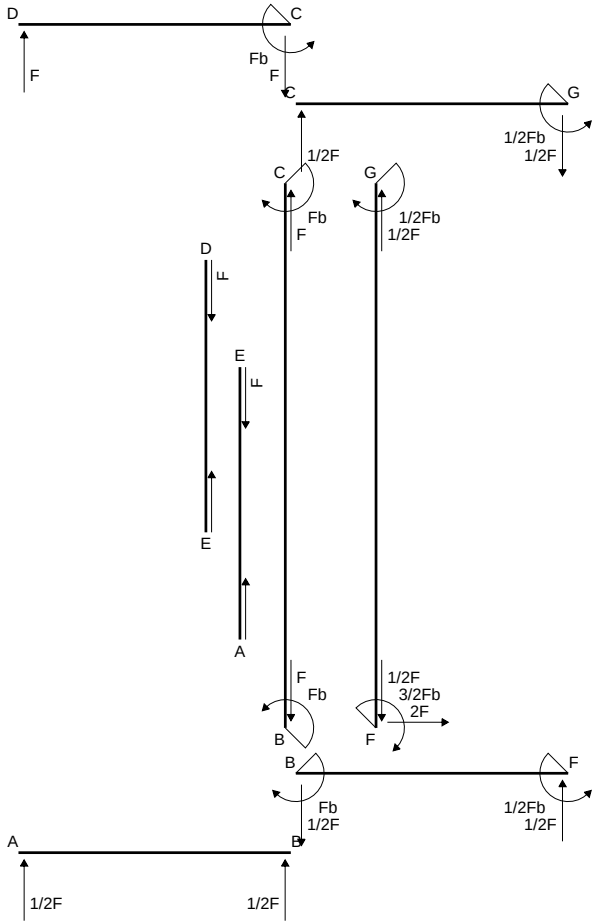
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

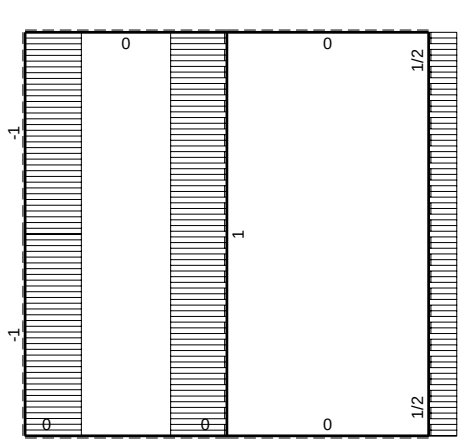


A = 158. mm^2
J_u = 77917. mm^4
J_v = 10868. mm^4
J_t = 133.3 mm^4
x_o = -13.95 mm
x_g = 12.15 mm
T_y = 1485. N
M_x = -653400. Nmm
u_m = -12.15 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = -209.6 N/mm^2
x_c = 6. mm
u_c = -6.152 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = -209.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 291.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.44 N/mm^2
$\tau_o = Tx_o t/J_t$ = 279.7 N/mm^2
t_c = 5346. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 546.1 N/mm^2

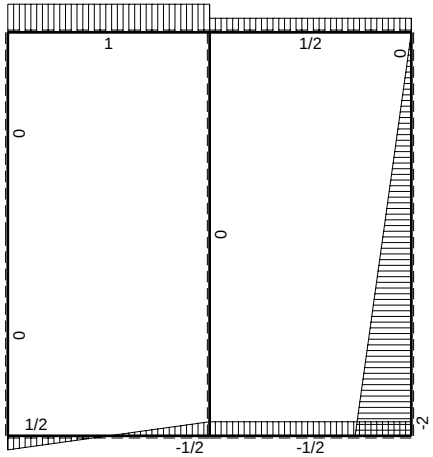
CI08.xxxx.061

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

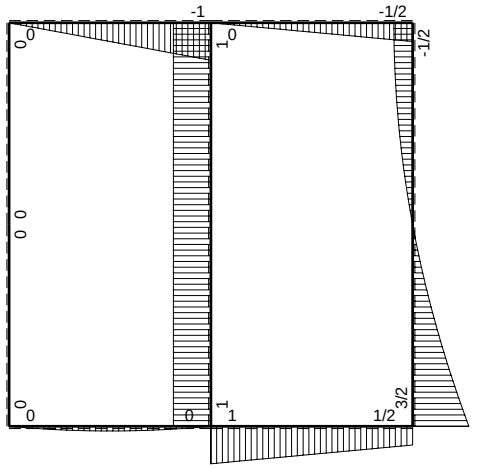

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

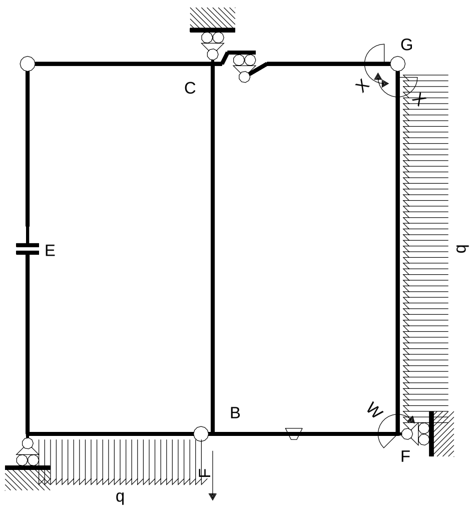
F

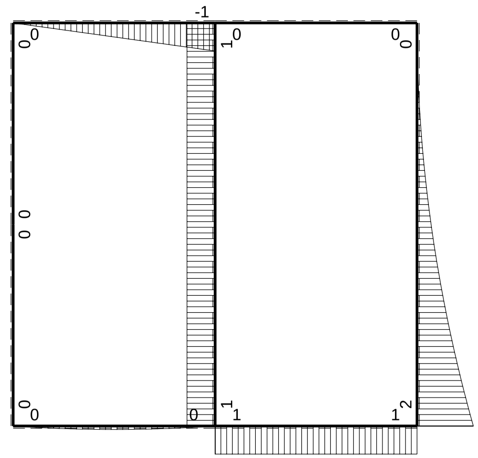

1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2

F

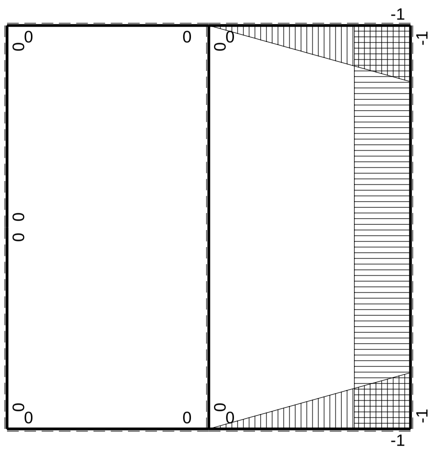

-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 239164 CI08.xxxx.062

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

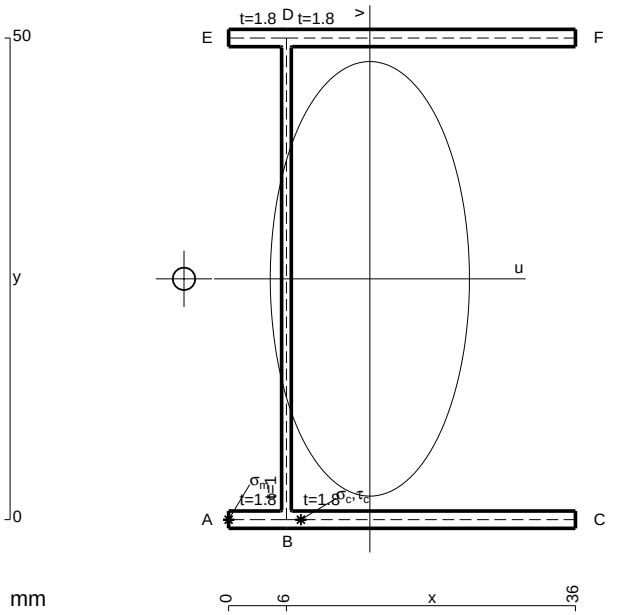
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

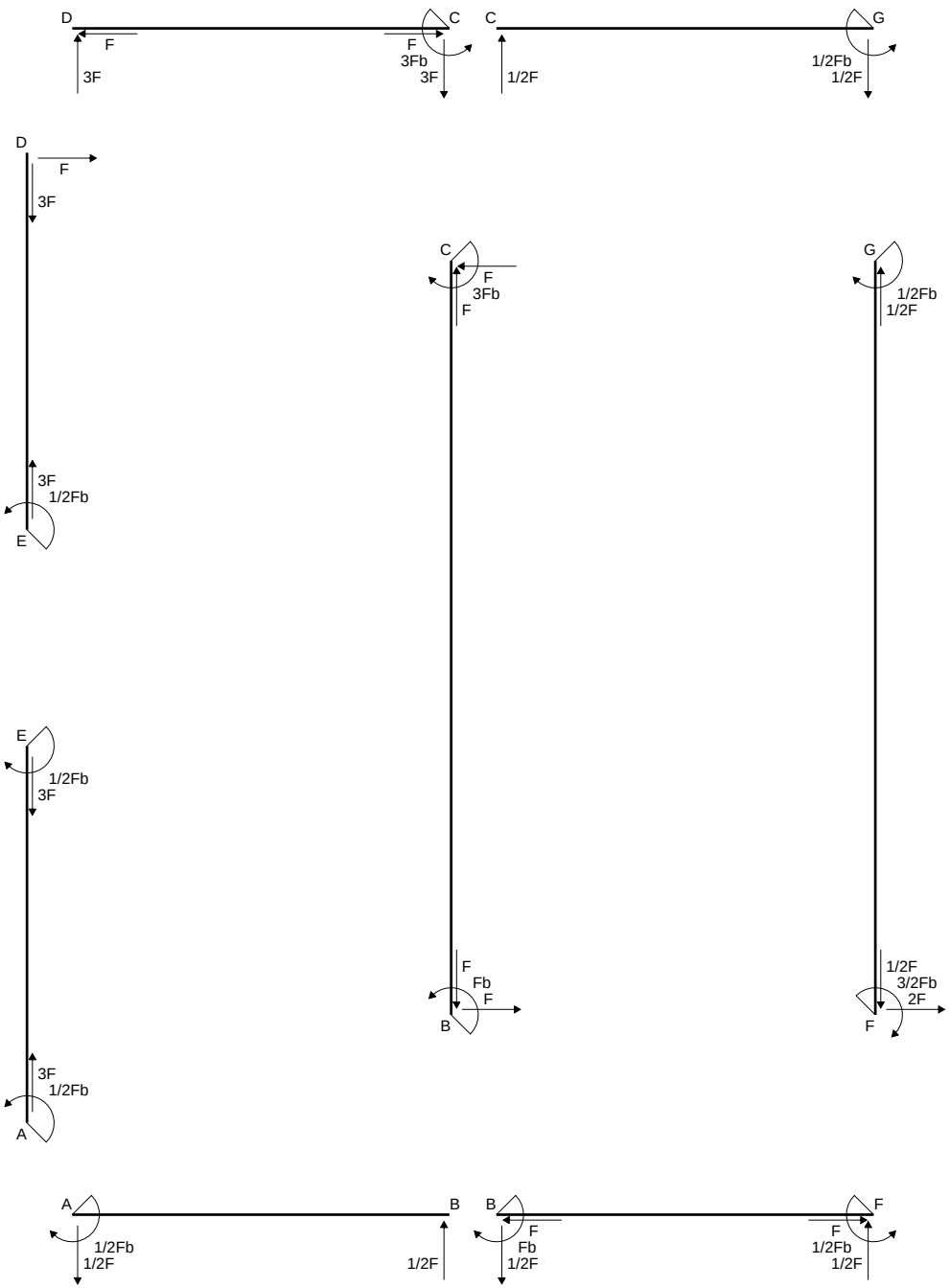
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

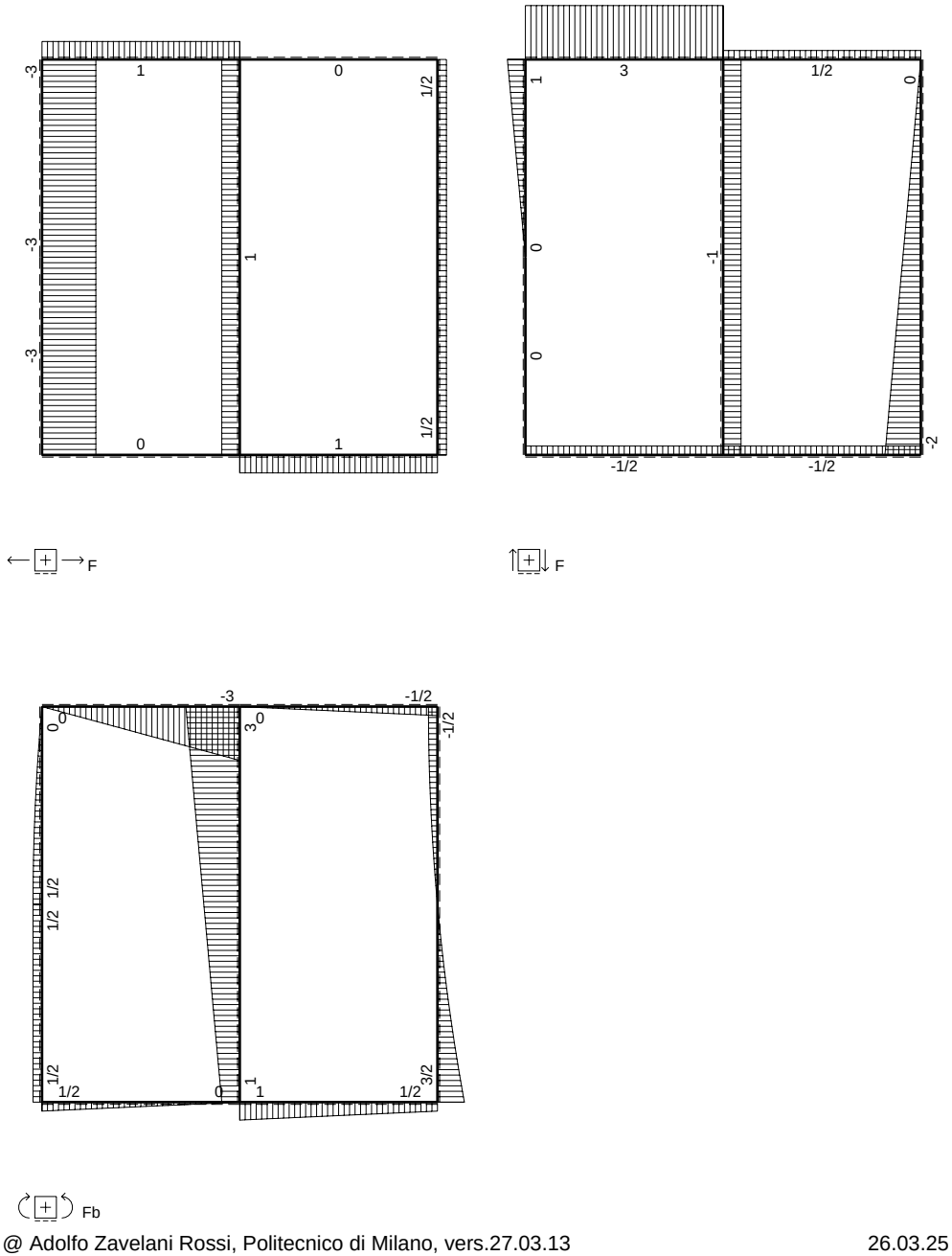


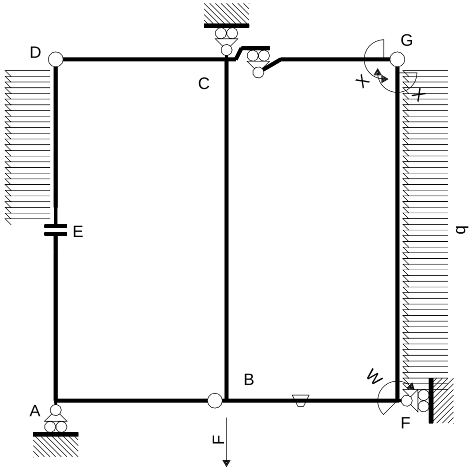
A = 179.6 mm^2
J_u = 91417. mm^4
J_v = 19192. mm^4
J_t = 156.6 mm^4
x_o = -19.29 mm
x_g = 14.66 mm
T_y = 1670. N
M_x = -801600. Nmm
u_m = -14.66 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = -219.2 N/mm^2
x_c = 6. mm
u_c = -8.659 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = -219.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 383.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.7 N/mm^2
$\tau_o = Tx_o t/J_t$ = 370.2 N/mm^2
t_c = 3006. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 700.2 N/mm^2

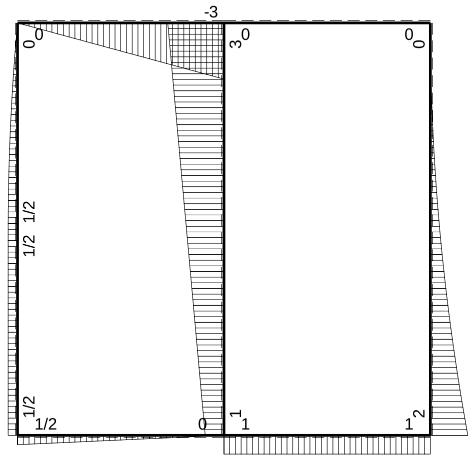
CI08.xxxx.062

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

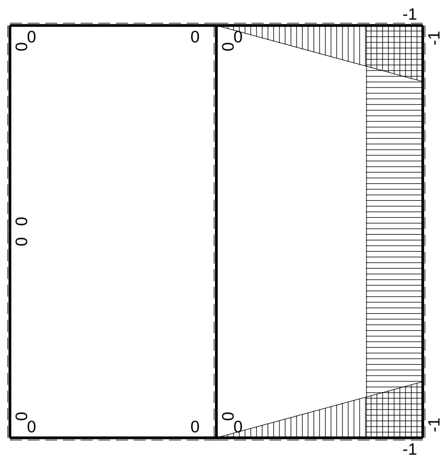

D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
F
Fb
F
1/2F
3/2Fb
2F
B
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


-3
1
0
1/2
-3
1
-3
0
1
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

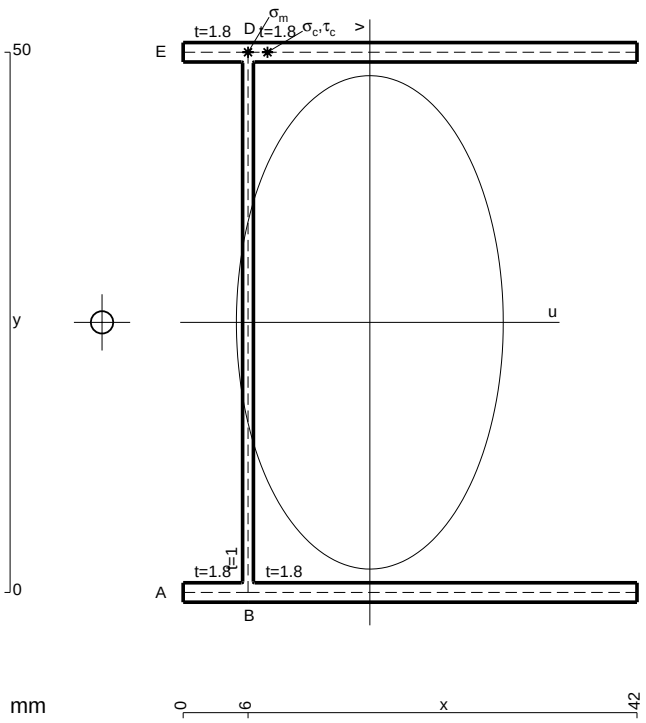
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

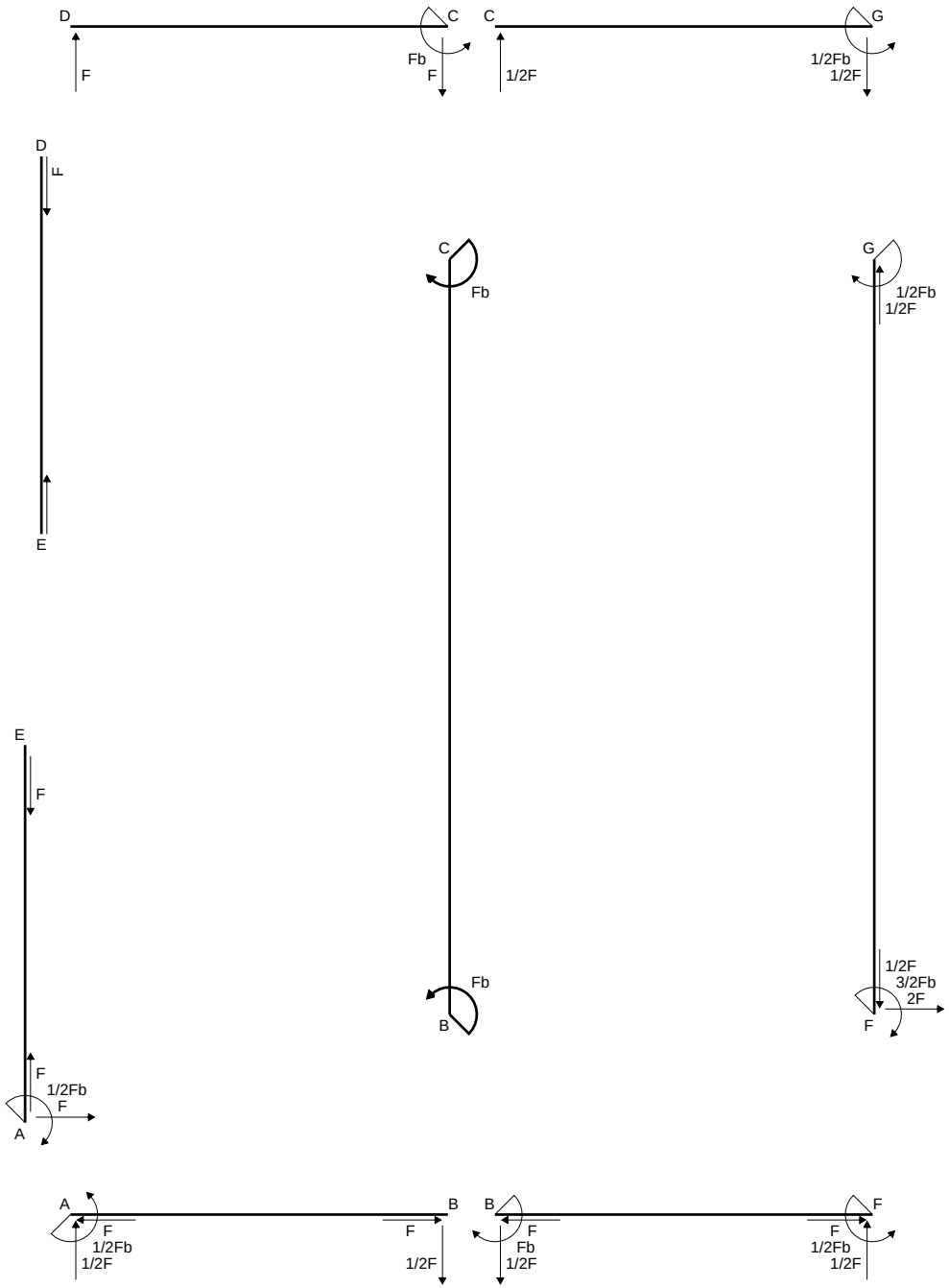
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

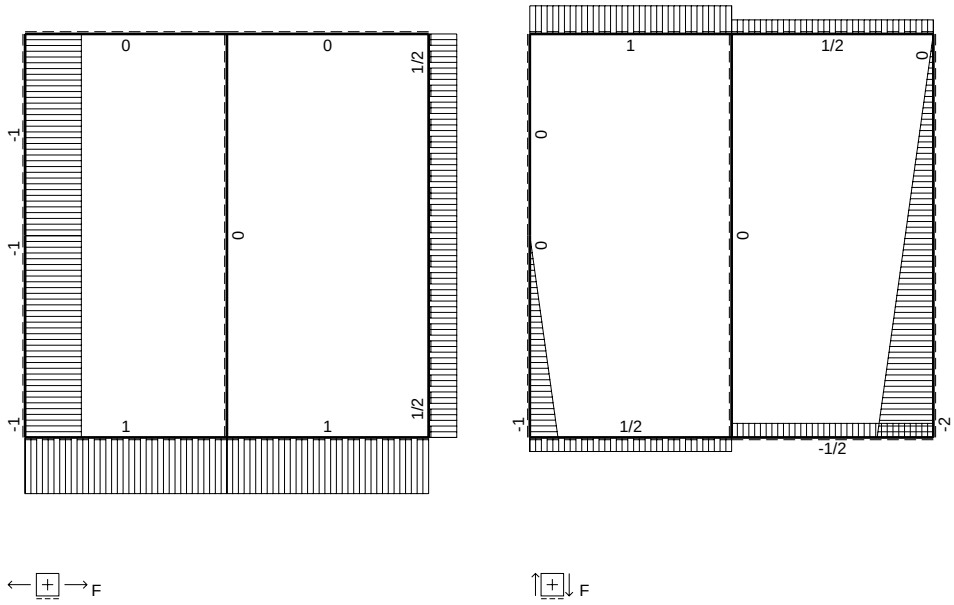


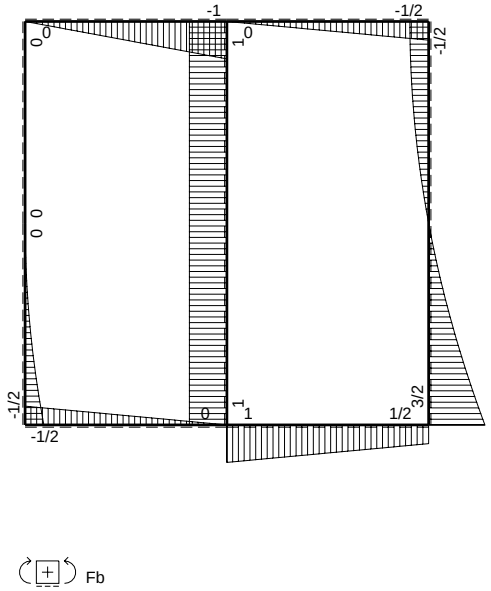
A = 201.2 mm^2
J_u = 104917. mm^4
J_v = 30681. mm^4
J_t = 180. mm^4
x_o = -24.78 mm
x_g = 17.27 mm
N = 610. N
T_y = 1830. N
M_x = -951600. Nmm
x_m = 6. mm
y_m = 50. mm
u_m = -11.27 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 229.8 N/mm^2
x_c = 6. mm
y_c = 50. mm
u_c = -11.27 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 229.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 469.3 N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.7 N/mm^2
$\tau_o = Tx_o t/J_t$ = 453.6 N/mm^2
t_c = 1098. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 844.7 N/mm^2

CI08.xxxx.063

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
G
1/2Fb
1/2F
Fb
B
1/2F
3/2Fb
2F
F
E
F
F
1/2Fb
F
A
A
B
B
F
F
F
F
1/2Fb
1/2F
1/2F
Fb
1/2F
1/2Fb
1/2F


0
0
1/2
-1
0
-1
0
-1
1
1
1/2
F
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F


-1
-1/2
0
1
0
-1/2
0
0
-1/2
0
1
1
1/2
3/2
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

PROCEDIMENTO E RISULTATI 241668 CI08.xxxx.064

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

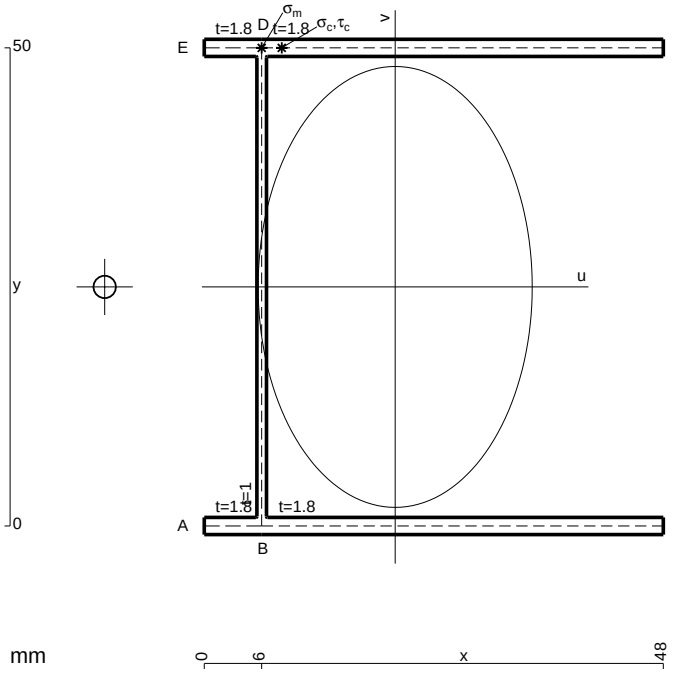
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

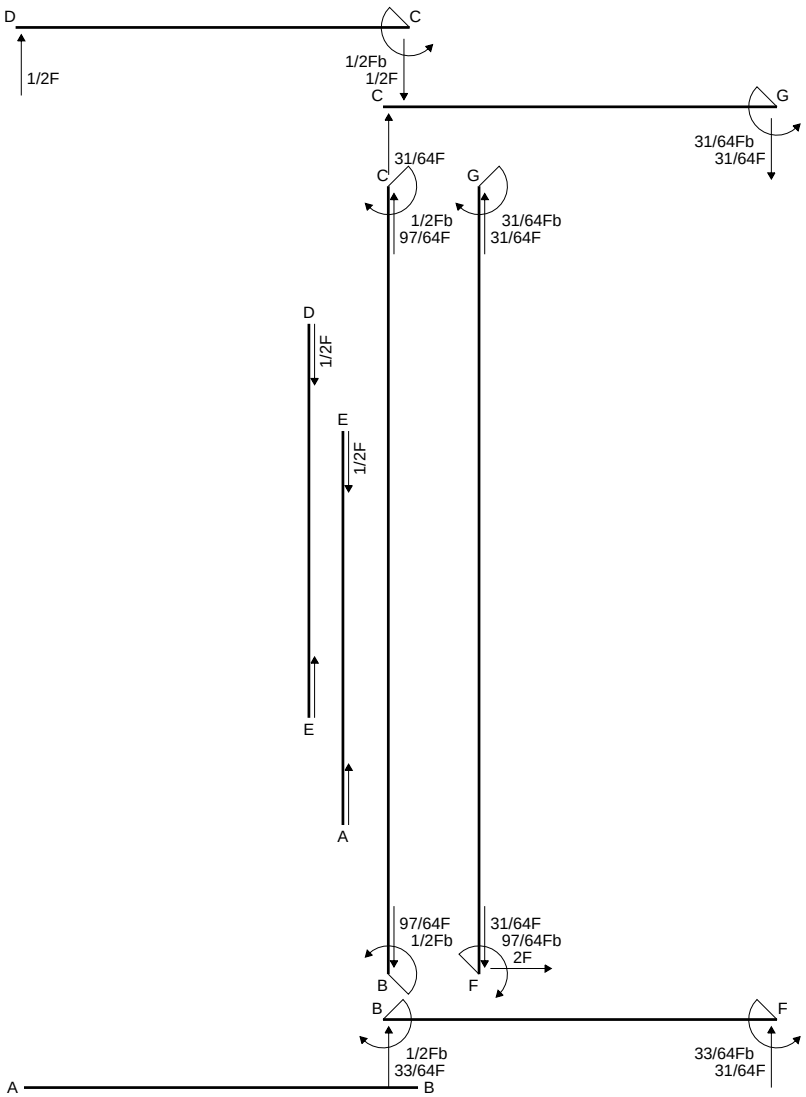
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

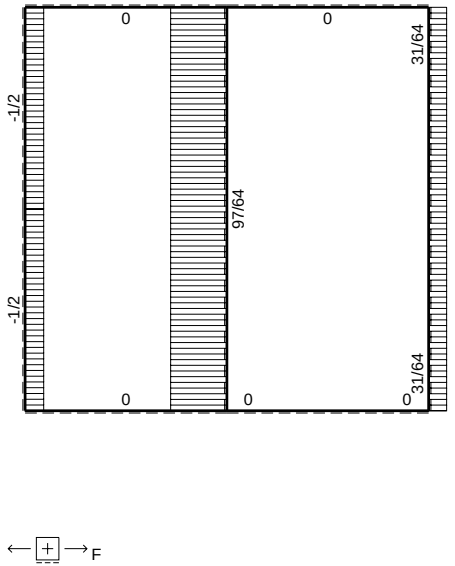


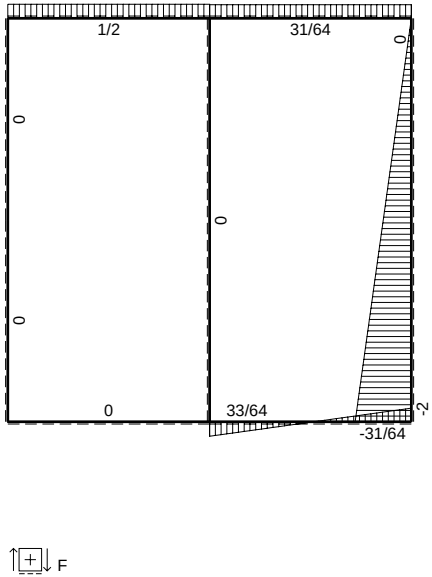
A = 222.8 mm^2
J_u = 118417. mm^4
J_v = 45742. mm^4
J_t = 203.3 mm^4
x_o = -30.38 mm
x_g = 19.96 mm
T_y = 2030. N
M_x = -1136800. Nmm
x_m = 6. mm
y_m = 50. mm
u_m = -13.96 mm
v_m = 25. mm
$\sigma_m = -Mv/J_u$ = 240. N/mm^2
x_c = 6. mm
y_c = 50. mm
u_c = -13.96 mm
v_c = 25. mm
$\sigma_c = -Mv/J_u$ = 240. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 564. N/mm^2
$\tau_g = TS^*/tJ_u$ = 18. N/mm^2
$\tau_o = Tx_o t/J_t$ = 546. N/mm^2
t_c = 3654. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1006. N/mm^2

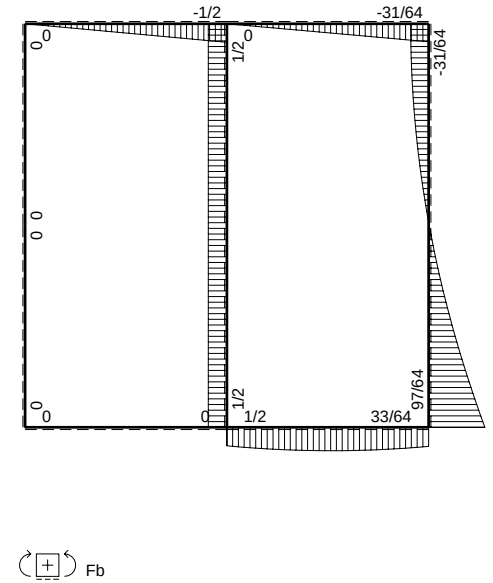
CI08.xxxx.064

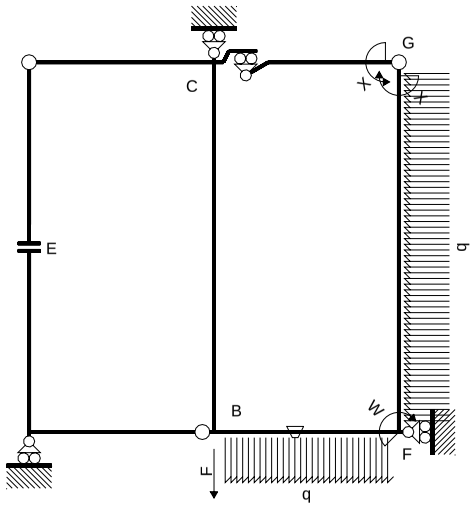
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

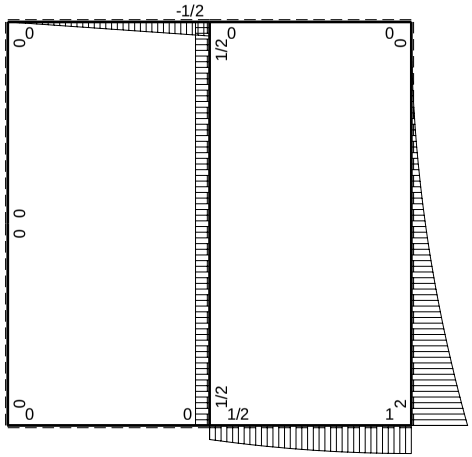

D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64
F

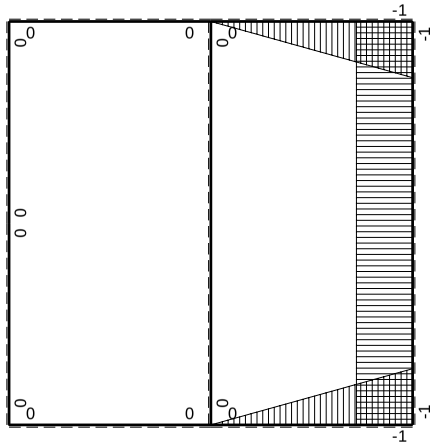

1/2
31/64
0
0
0
0
0
33/64
-2
-31/64
F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
0
1/2
33/64
97/64
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
		totali					$-31/24Fb^2/EJ$	8/3Xb/EJ
		iperstatica $X=W_{GC}$					31/64Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 959538 CI08.xxxx.065

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

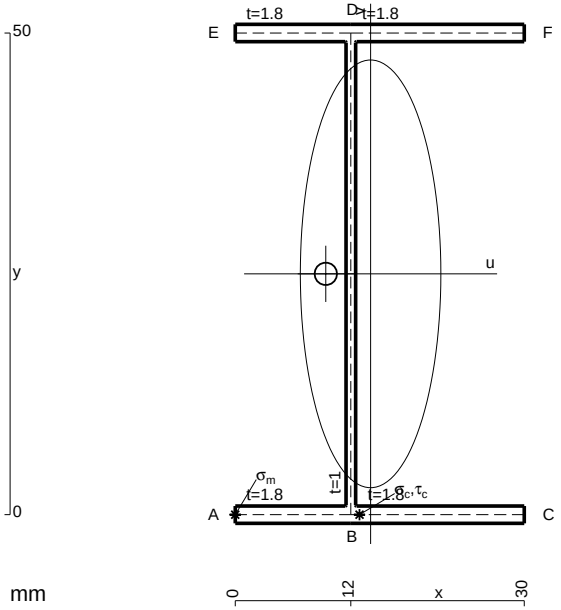
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

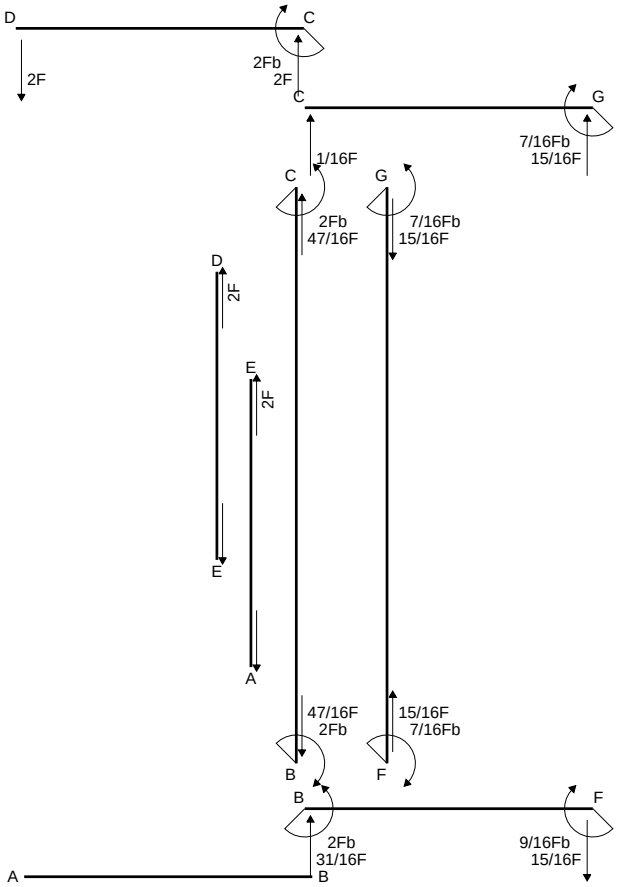
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

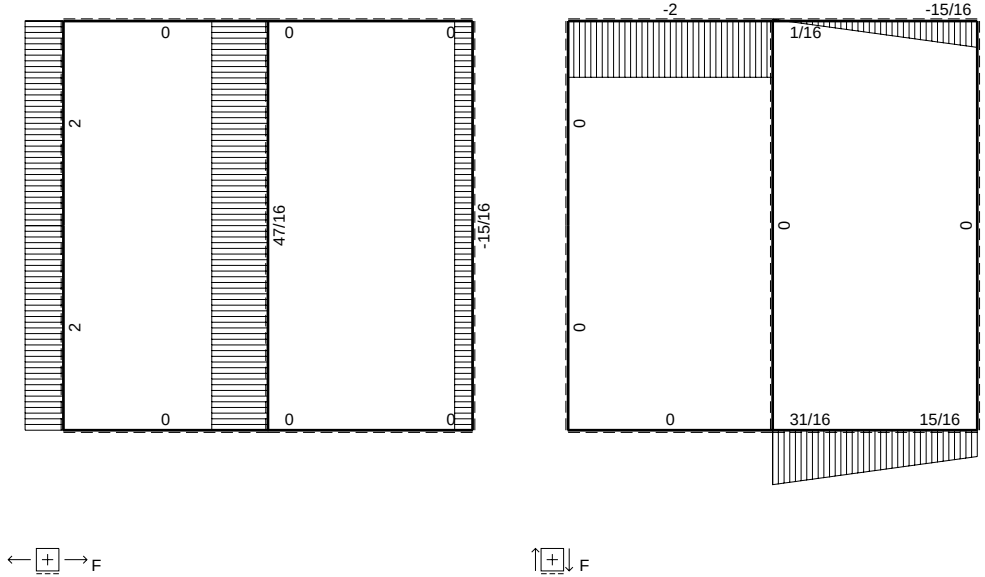


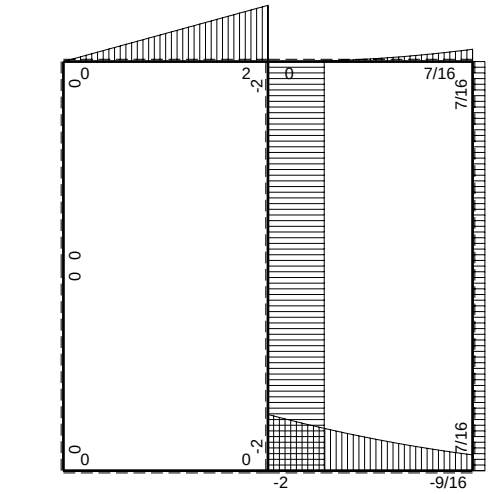
A = 158. mm^2
J_u = 77917. mm^4
J_v = 8408. mm^4
J_t = 133.3 mm^4
x_o = -4.65 mm
x_g = 14.05 mm
T_y = 1035. N
M_x = -621000. Nmm
u_m = -14.05 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = -199.3 N/mm^2
x_c = 12. mm
u_c = -2.051 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = -199.3 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 70.96 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.978 N/mm^2
$\tau_o = Tx_o t/J_t$ = 64.98 N/mm^2
t_c = 3726. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 234.1 N/mm^2

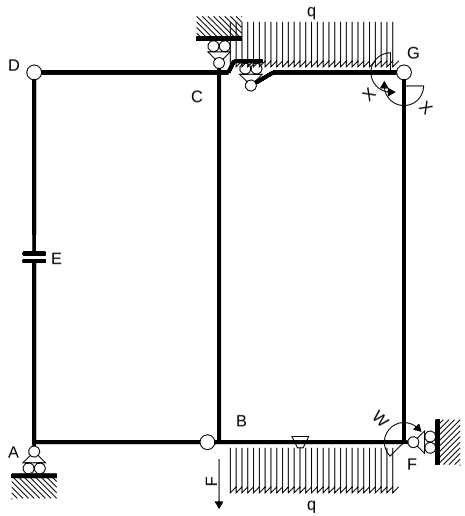
CI08.xxxx.065

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

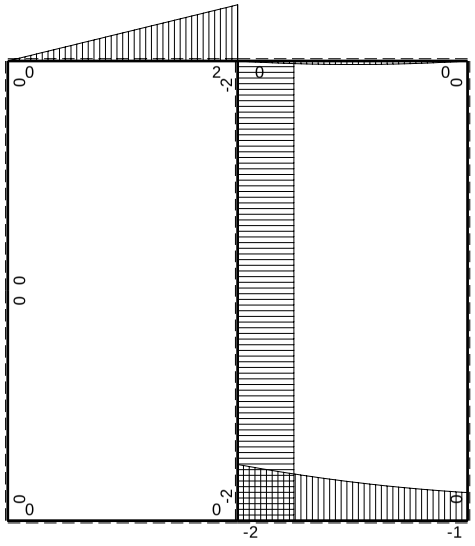

D
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0
-2
-15/16
1/16
0
0
0
0
0
31/16
15/16
F
F

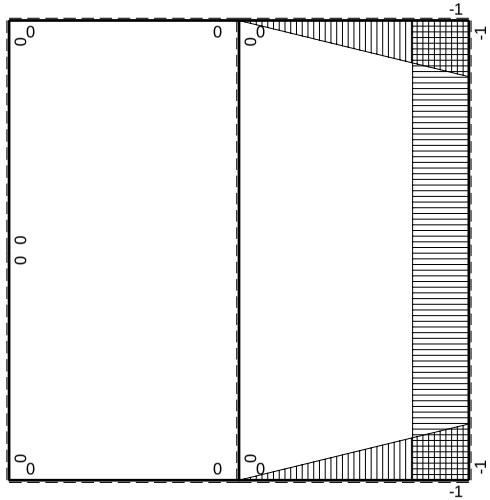

0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
0
-2
7/16
-2
-9/16
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 236900 CI08.xxxx.066

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

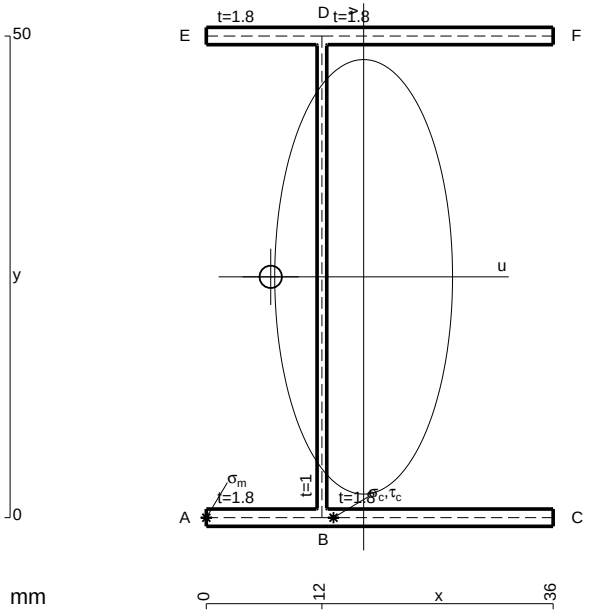
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

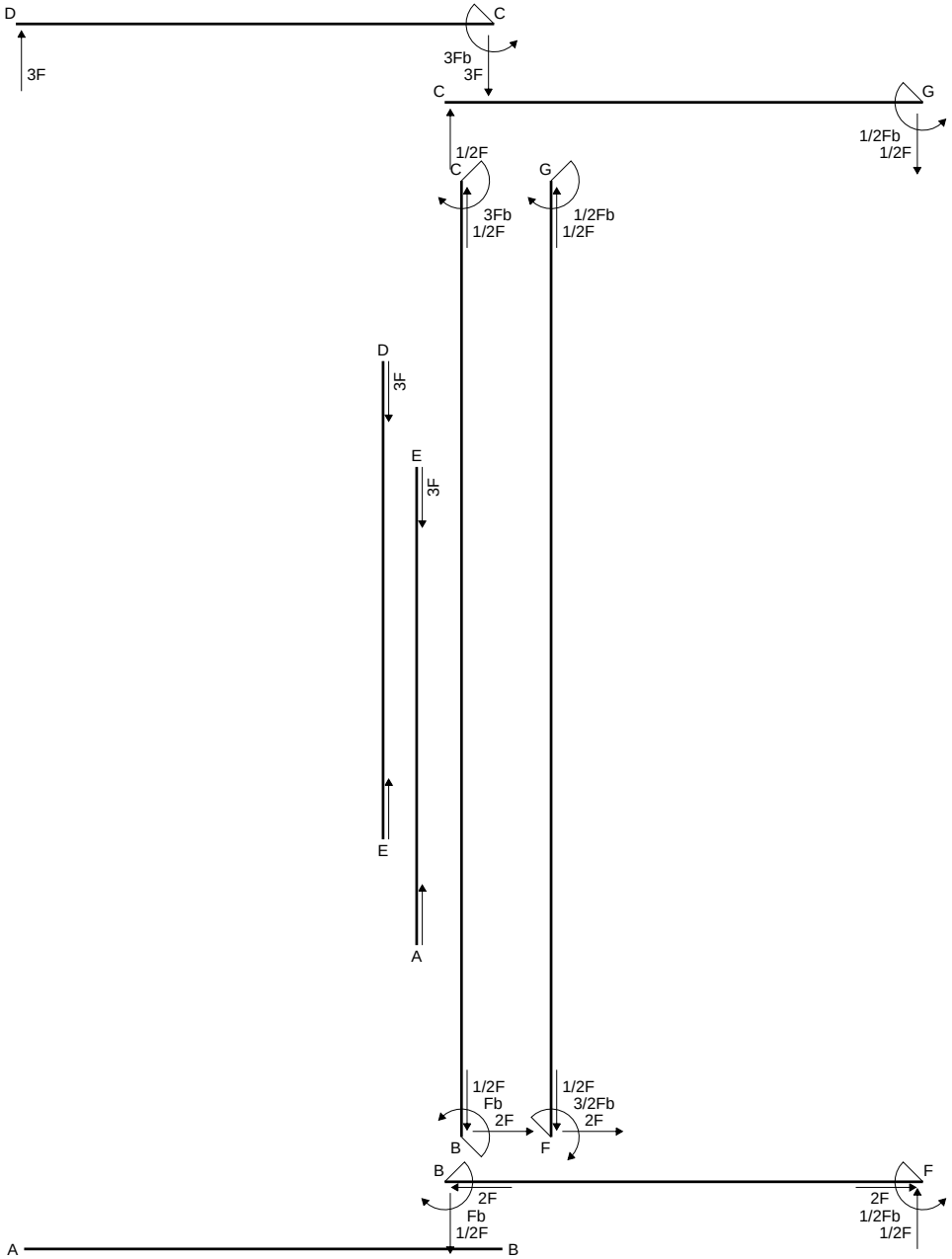
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

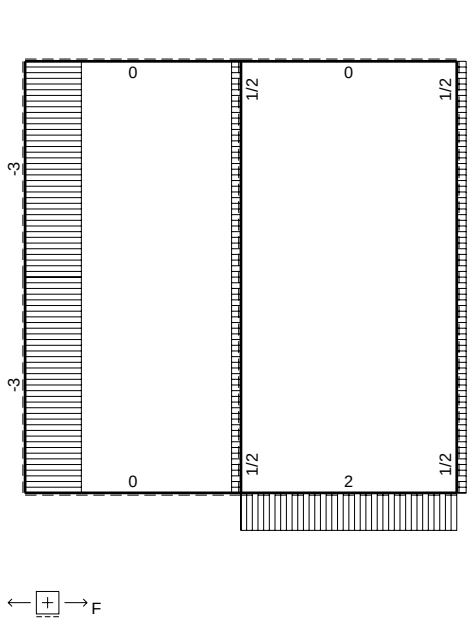


A = 179.6 mm^2
J_u = 91417. mm^4
J_v = 15296. mm^4
J_t = 156.6 mm^4
x_o = -9.646 mm
x_g = 16.33 mm
T_y = -1180. N
M_x = 755200. Nmm
u_m = -16.33 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = 206.5 N/mm^2
x_c = 12. mm
u_c = -4.33 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = 206.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 138.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.745 N/mm^2
$\tau_o = Tx_o t/J_t$ = 130.8 N/mm^2
t_c = 1062. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 316.6 N/mm^2

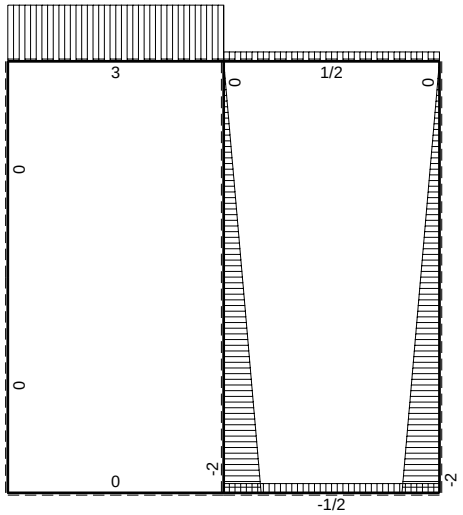
CI08.xxxx.066

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

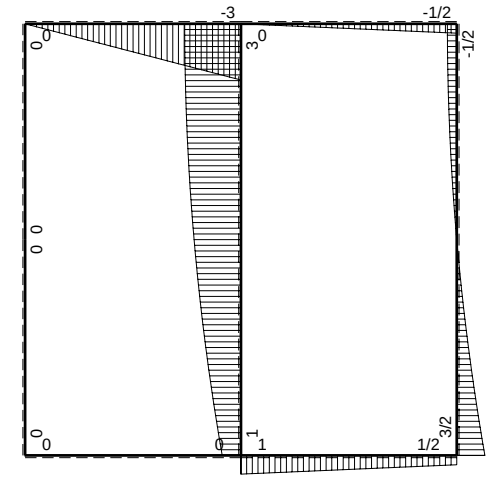

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
0
1/2
1/2
-3
-3
0
0
1/2
1/2
2

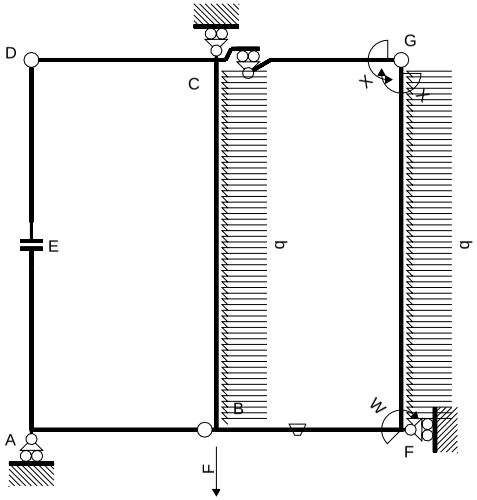
F

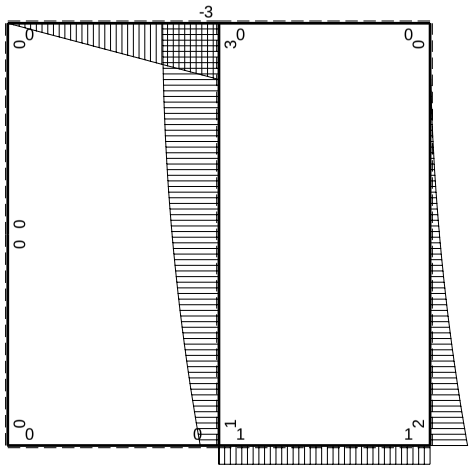

3
0
1/2
0
0
0
0
-2
-2
-1/2

F

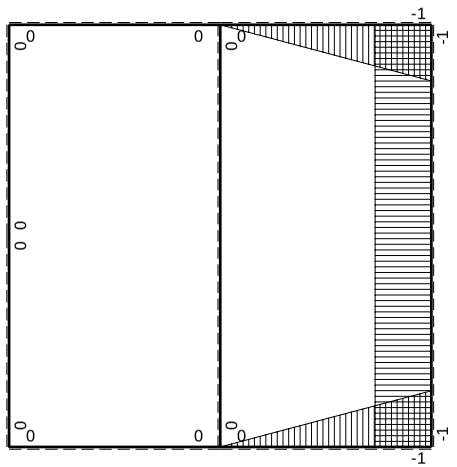

-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-4/3Fb^2$/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 239141 CI08.xxxx.067

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

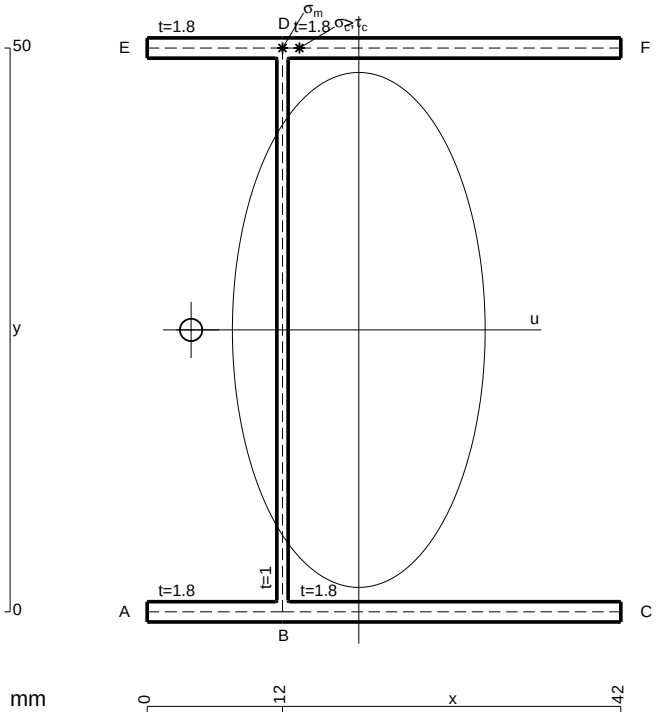
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

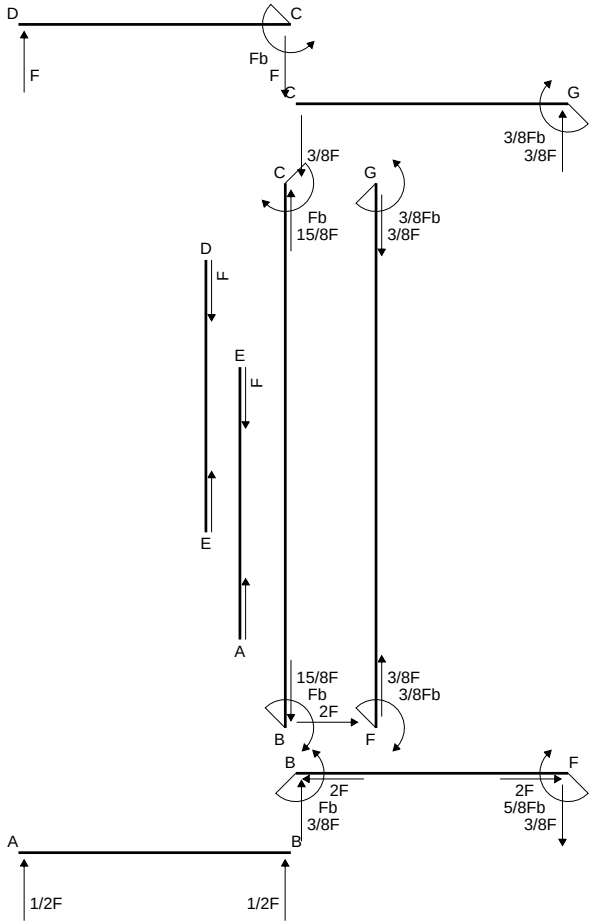
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

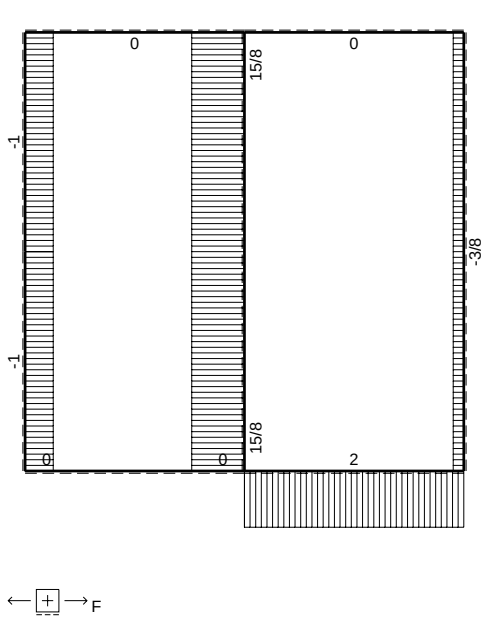


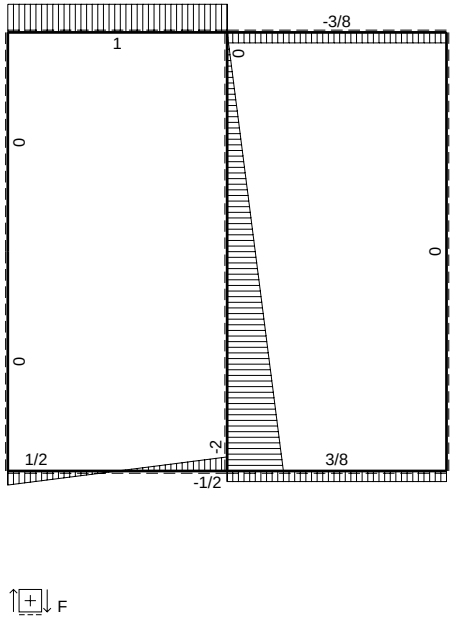
A = 201.2 mm^2
J_u = 104917. mm^4
J_v = 25270. mm^4
J_t = 180. mm^4
x_o = -14.87 mm
x_g = 18.76 mm
T_y = 1350. N
M_x = -918000. Nmm
x_m = 12. mm
y_m = 50. mm
u_m = -6.763 mm
v_m = 25. mm
$\sigma_m = -Mv/J_u$ = 218.7 N/mm^2
x_c = 12. mm
y_c = 50. mm
u_c = -6.763 mm
v_c = 25. mm
$\sigma_c = -Mv/J_u$ = 218.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 210.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.651 N/mm^2
$\tau_o = Tx_o t/J_t$ = 200.8 N/mm^2
t_c = 810. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 425.1 N/mm^2

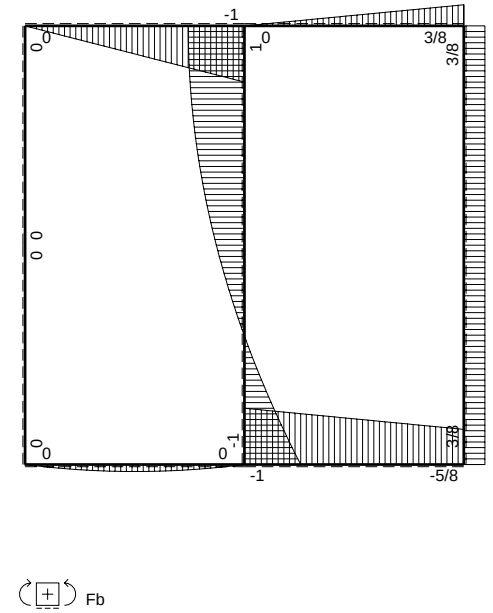
CI08.xxxx.067

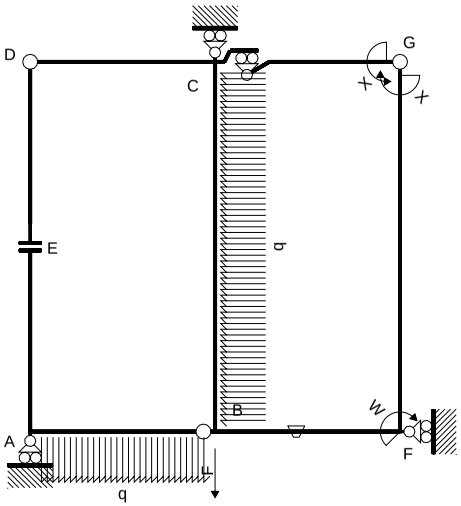
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F

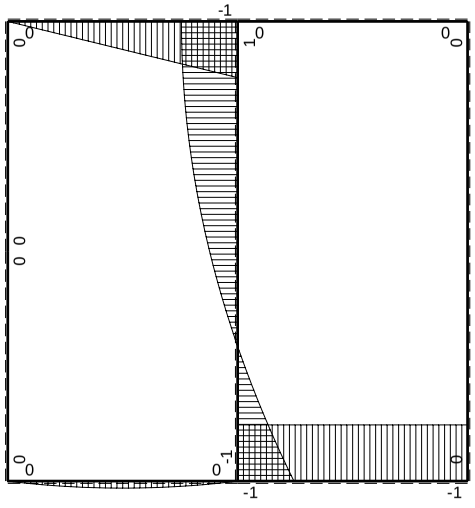

0
0
15/8
-1
-1
-3/8
0
0
15/8
0
0
2
F


-3/8
1
0
0
0
0
1/2
-2
3/8
-1/2
F

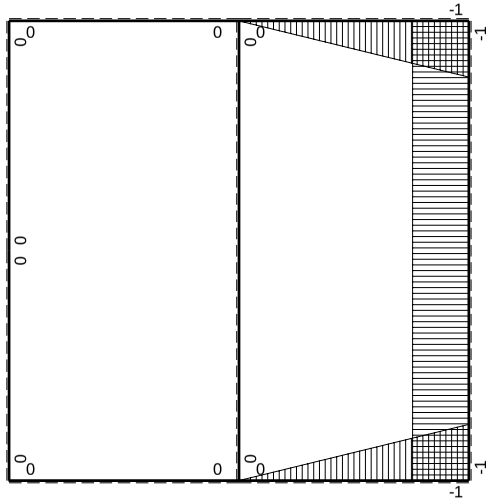

-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx²	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 247885 CI08.xxxx.068

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

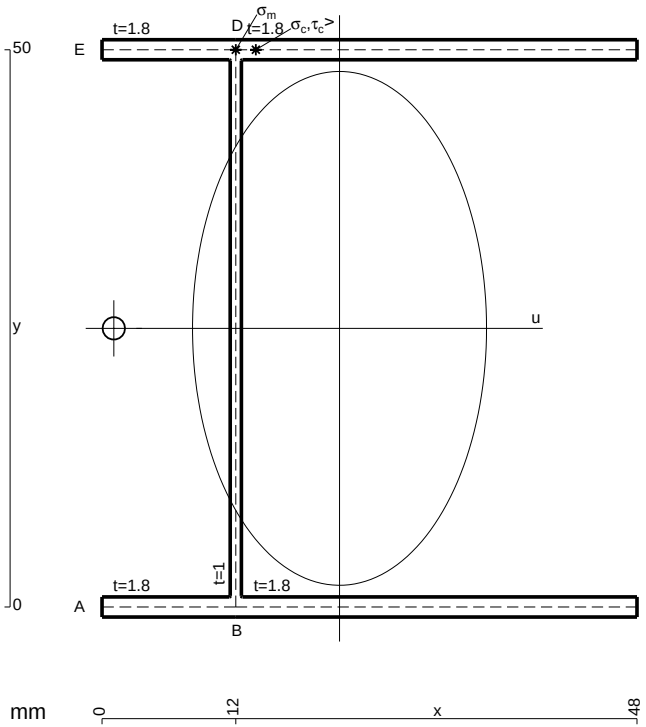
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

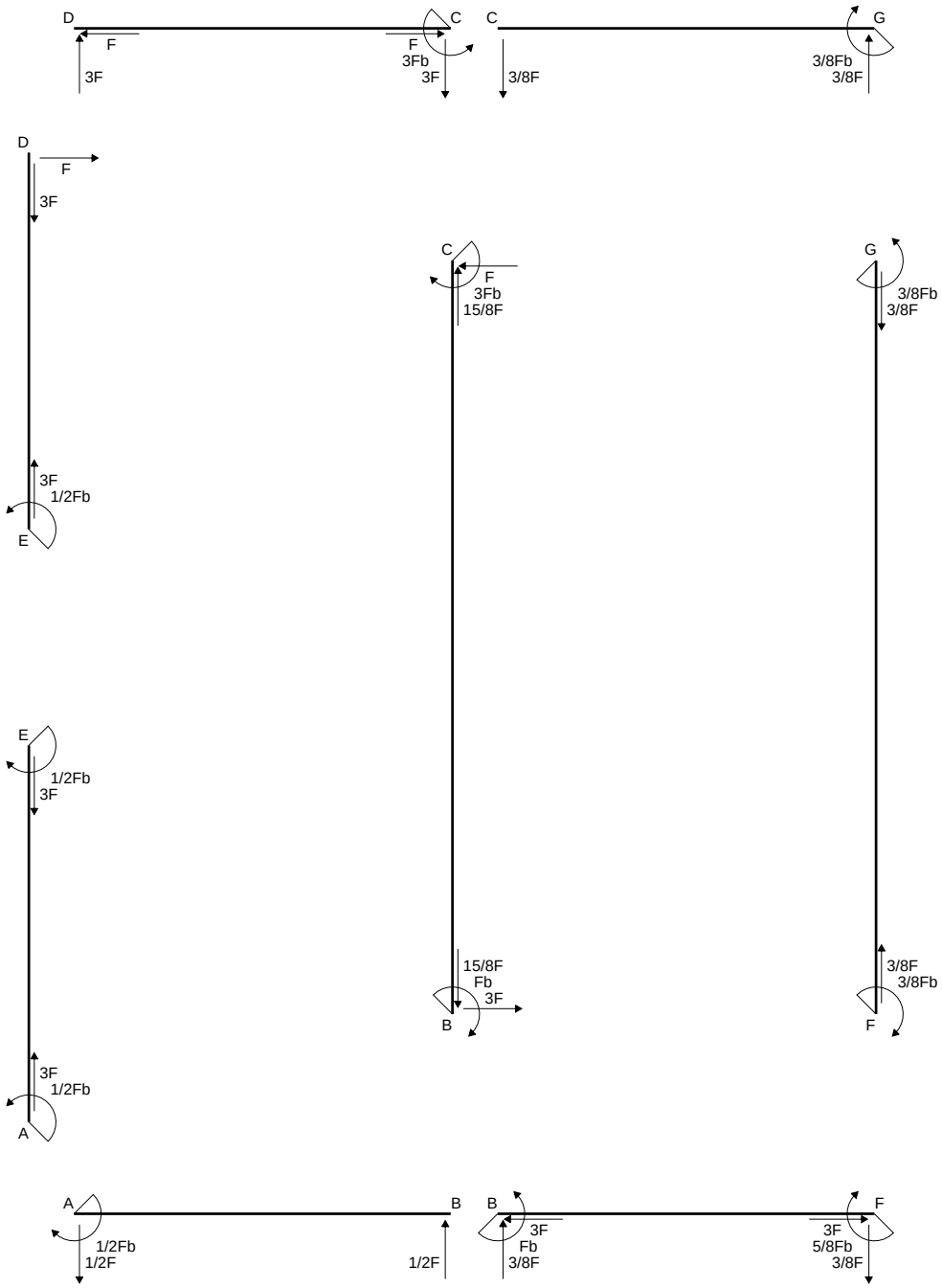
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

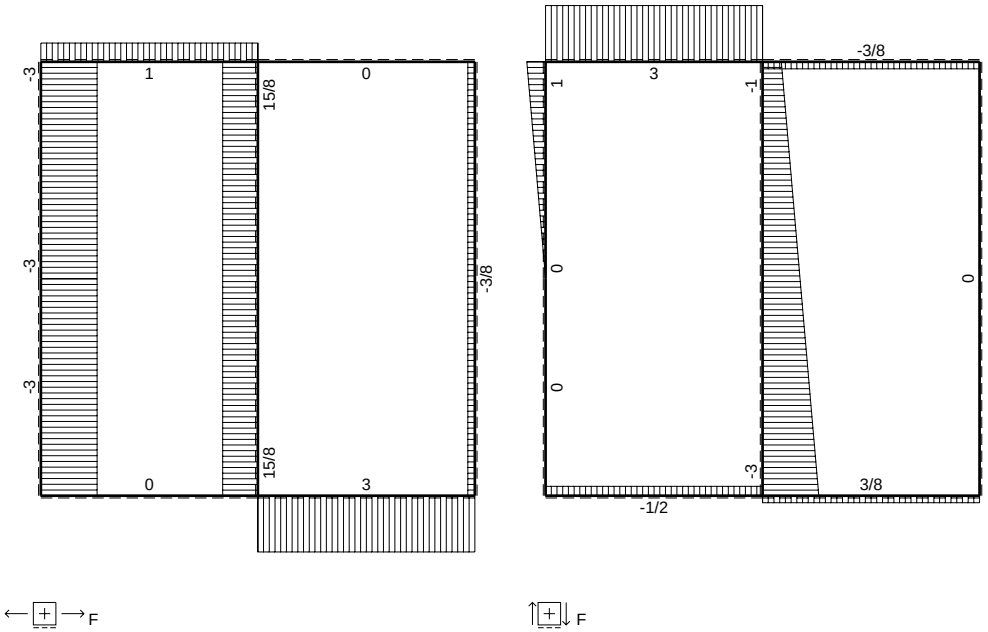


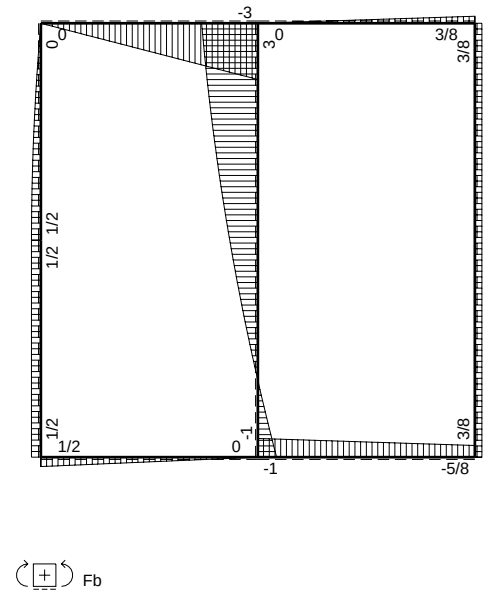
A = 222.8 mm^2
J_u = 118417. mm^4
J_v = 38762. mm^4
J_t = 203.3 mm^4
x_o = -20.25 mm
x_g = 21.31 mm
N = 2681. N
T_y = -2860. N
M_x = -1029600. Nmm
x_m = 12. mm
y_m = 50. mm
u_m = -9.307 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 229.4 N/mm^2
x_c = 12. mm
y_c = 50. mm
u_c = -9.307 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 229.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 534.6 N/mm^2
$\tau_g = TS^*/tJ_u$ = 21.74 N/mm^2
$\tau_o = Tx_o t/J_t$ = 512.8 N/mm^2
t_c = 2574. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 953.9 N/mm^2

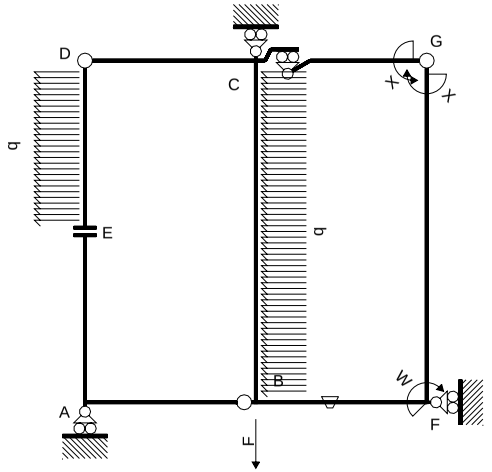
CI08.xxxx.068

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

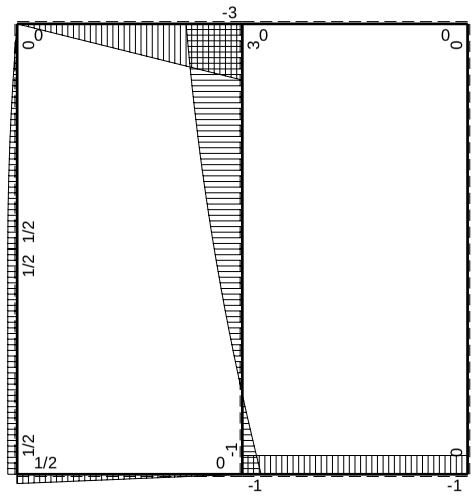

D
F
3F
C
F
3Fb
3F
C
3/8F
G
3/8Fb
3/8F
D
F
3F
3F
1/2Fb
E
C
F
3Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3F
15/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F


-3
-3
-3
1
0
15/8
15/8
0
3
-3/8
F
-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F

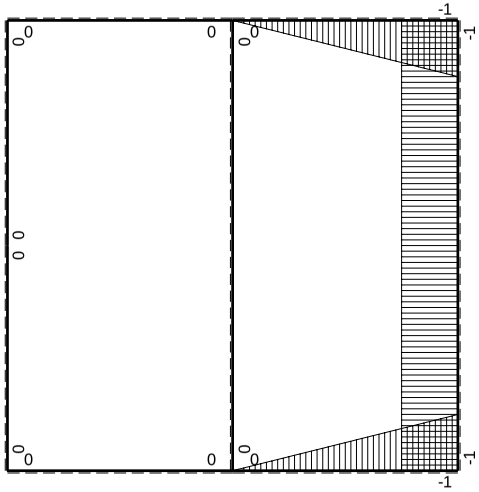

-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212041 CI08.xxxx.069

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

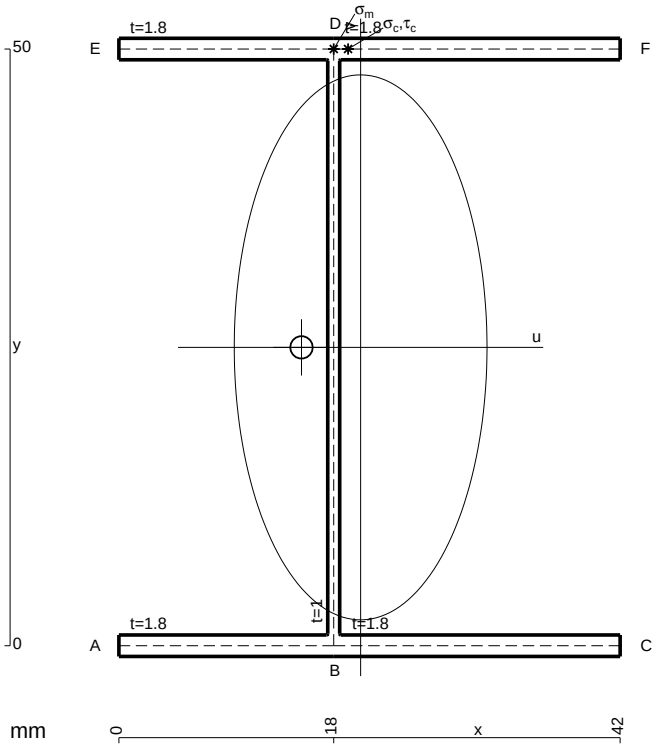
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

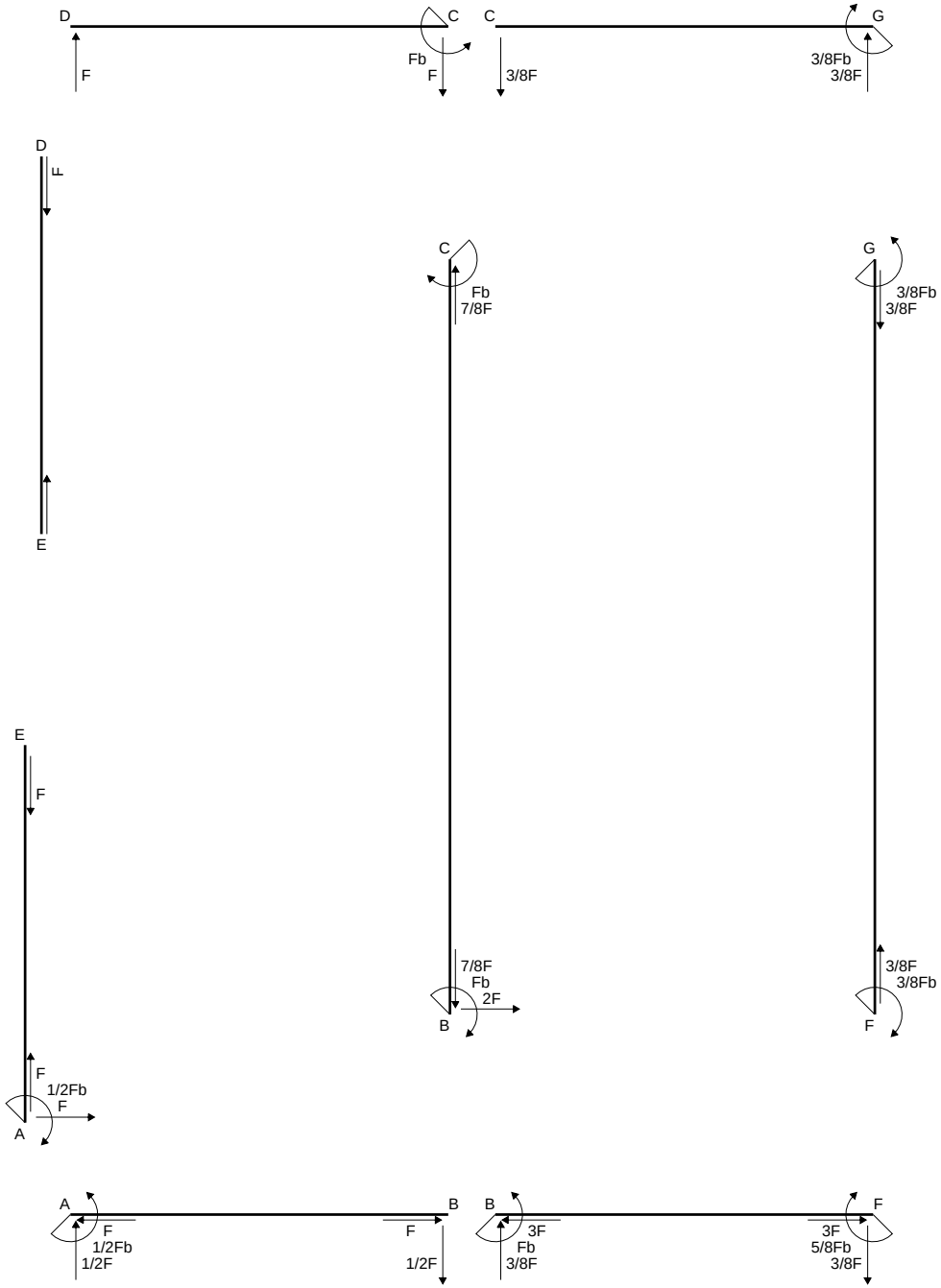
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

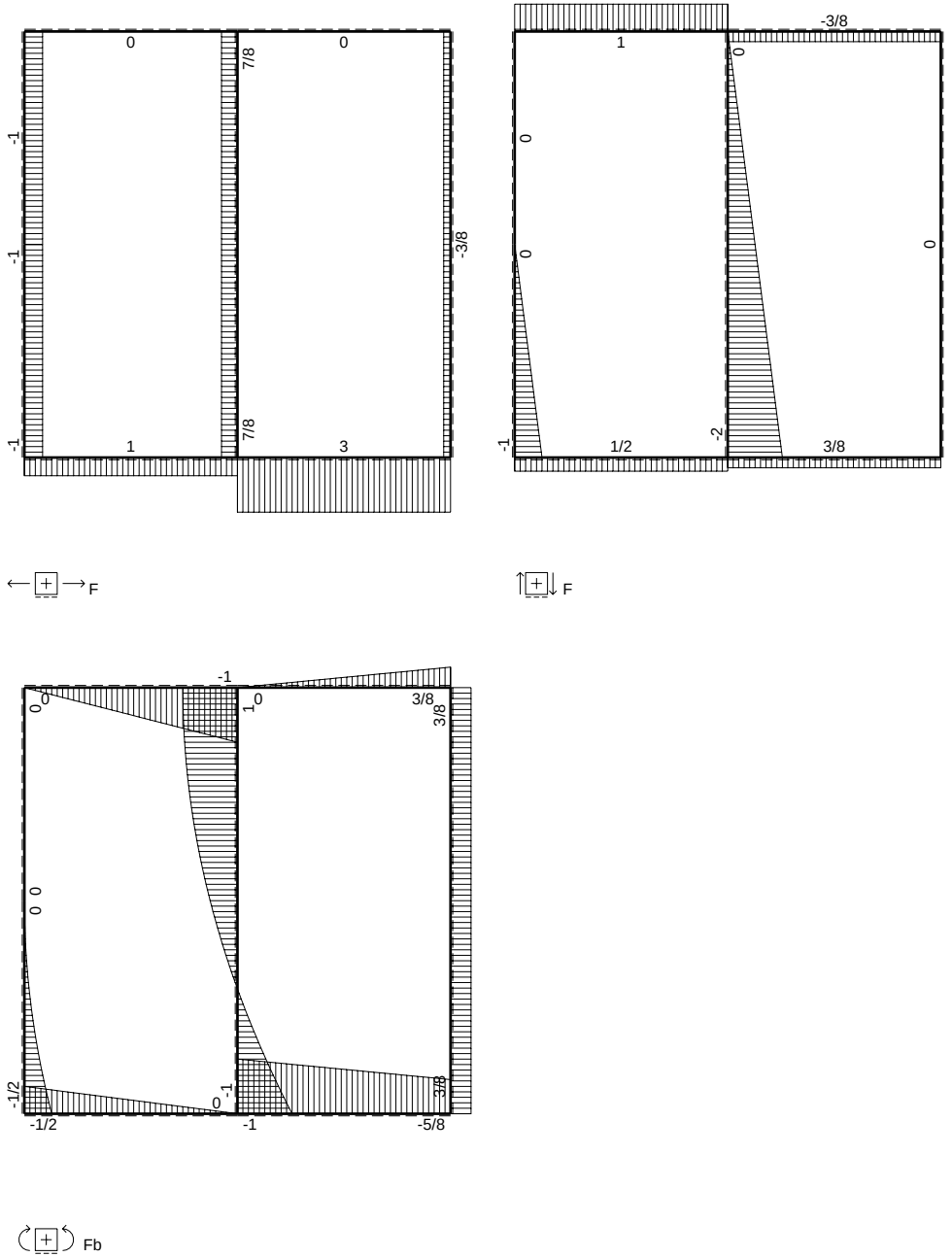


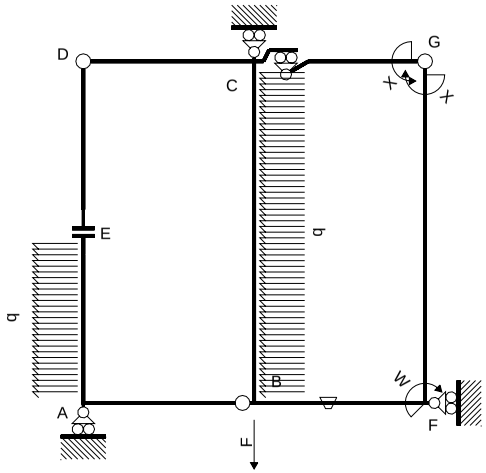
A = 201.2 mm^2
J_u = 104917. mm^4
J_v = 22565. mm^4
J_t = 180. mm^4
x_o = -4.957 mm
x_g = 20.25 mm
N = 430. N
T_y = 1290. N
M_x = -980400. Nmm
x_m = 18. mm
y_m = 50. mm
u_m = -2.254 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 235.8 N/mm^2
x_c = 18. mm
y_c = 50. mm
u_c = -2.254 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 235.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 71.33 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.377 N/mm^2
$\tau_o = Tx_o t/J_t$ = 63.95 N/mm^2
t_c = 774. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 266.2 N/mm^2

CI08.xxxx.069

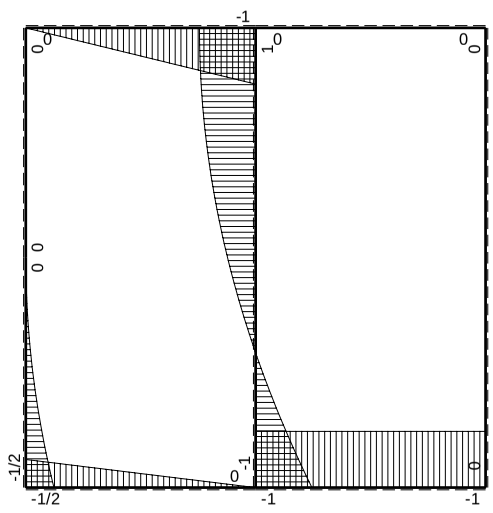
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
7/8F
G
3/8Fb
3/8F
E
F
7/8F
Fb
2F
B
3/8F
3/8Fb
F
F
1/2Fb
F
A
A
F
1/2Fb
1/2F
B
F
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F

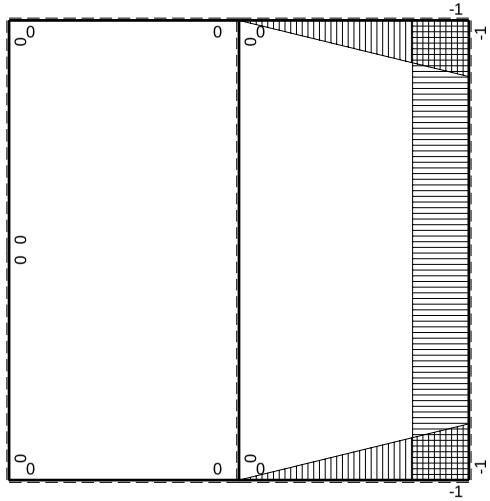

0
0
7/8
-1
-1
-3/8
-1
1
7/8
3
F
1
-3/8
0
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 216391 CI08.xxxx.070

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

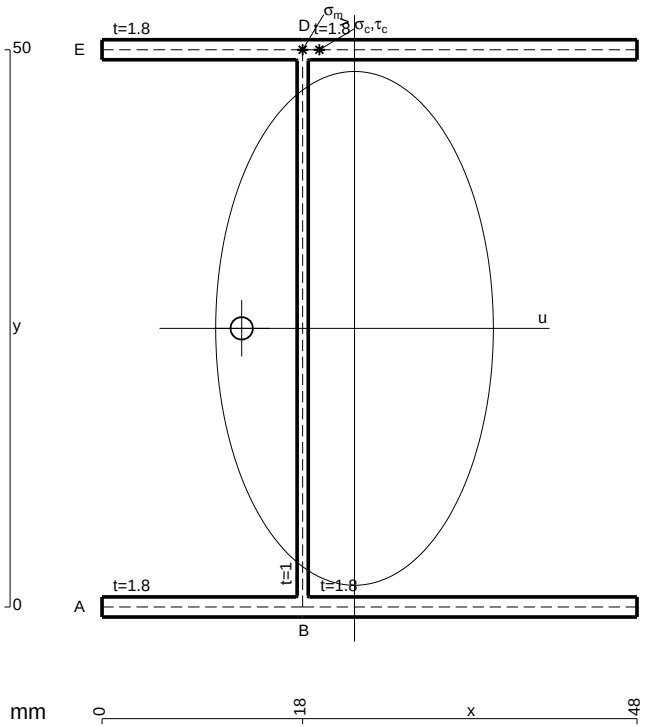
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

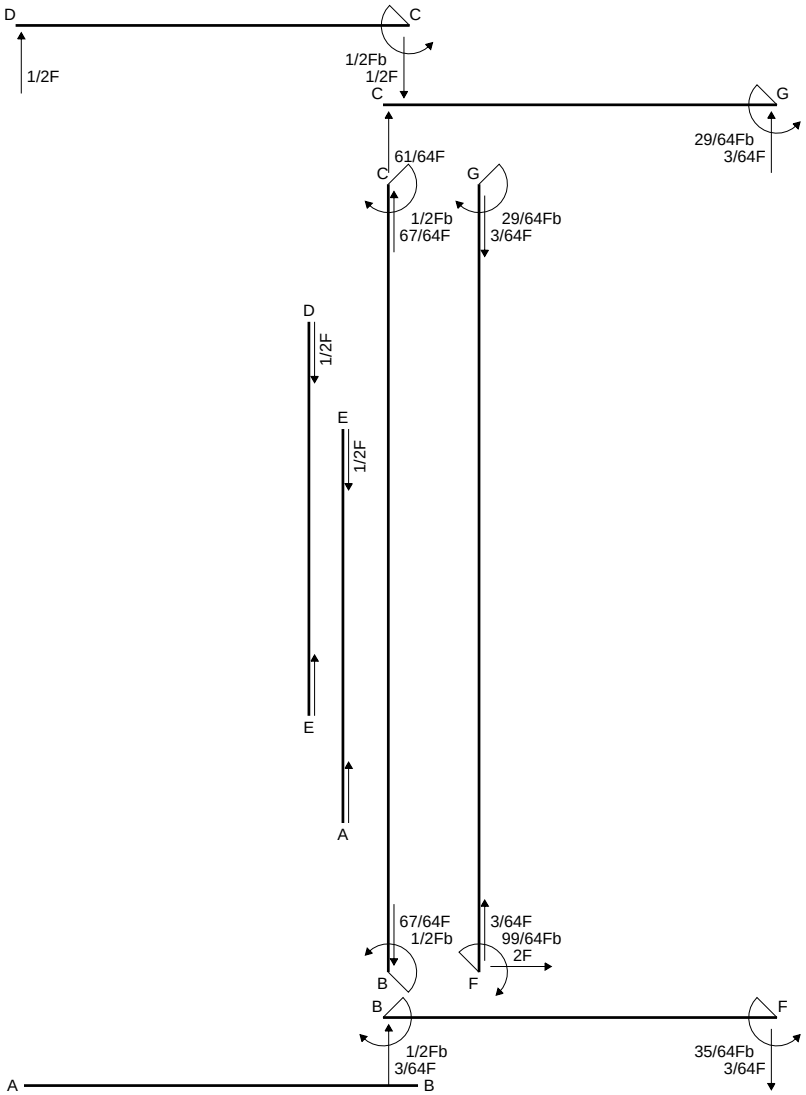
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

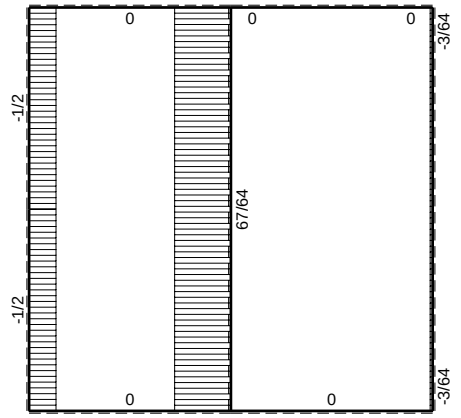


A = 222.8 mm^2
J_u = 118417. mm^4
J_v = 34574. mm^4
J_t = 203.3 mm^4
x_o = -10.13 mm
x_g = 22.65 mm
N = 1978. N
T_y = -4520. N
M_x = -904000. Nmm
x_m = 18. mm
y_m = 50. mm
u_m = -4.654 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 199.7 N/mm^2
x_c = 18. mm
y_c = 50. mm
u_c = -4.654 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 199.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 433.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 28.63 N/mm^2
$\tau_o = Tx_o t/J_t$ = 405.2 N/mm^2
t_c = 4068. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 777.6 N/mm^2

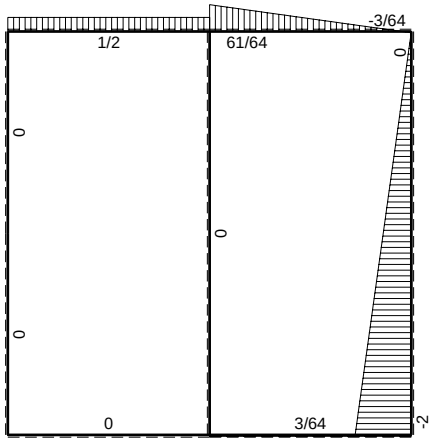
CI08.xxxx.070

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

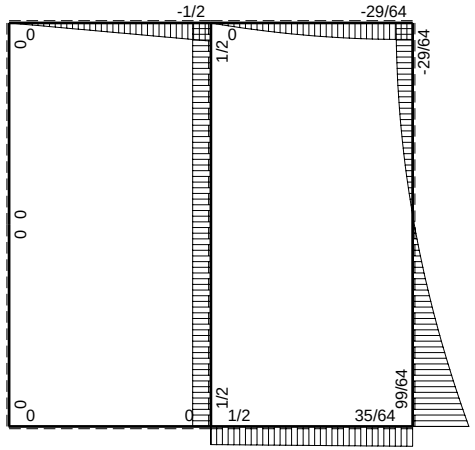

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64

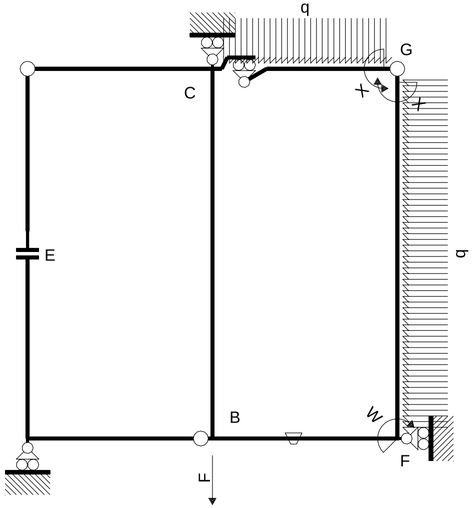
F

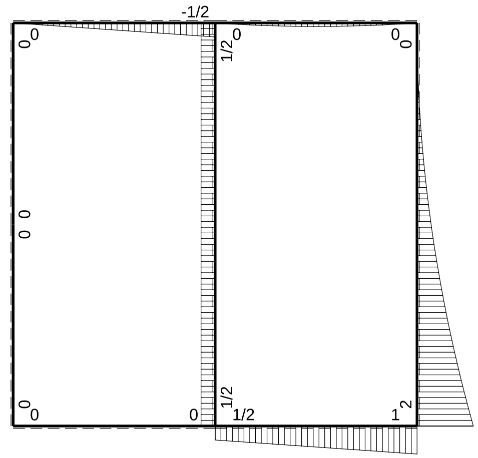

-3/64
1/2
61/64
0
0
0
0
0
3/64
-2

F

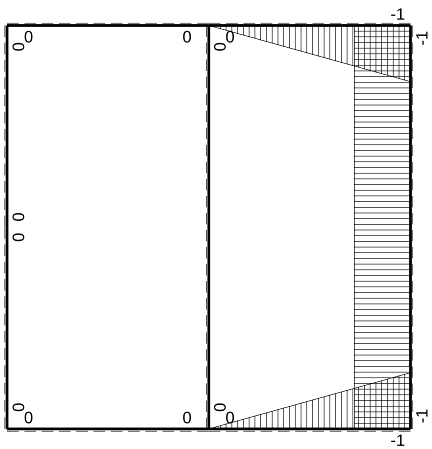

-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
1/2
35/64
99/64

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
DC b	0	$1/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	$1/2Fb+1/2Fx$	$-Fb/EJ$	$-1/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(-5/12+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb+1/2Fx$	Fb/EJ	$-Fb+3/2Fx-1/2Fx^2/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$1/2Fb$	0	0	0	0	0+0	0
BC 2b	0	$-1/2Fb$	0	0	0	0		
totali							$-29/24Fb^2/EJ$	$8/3Xb/EJ$
iperstatica $X=W_{GC}$							$29/64Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

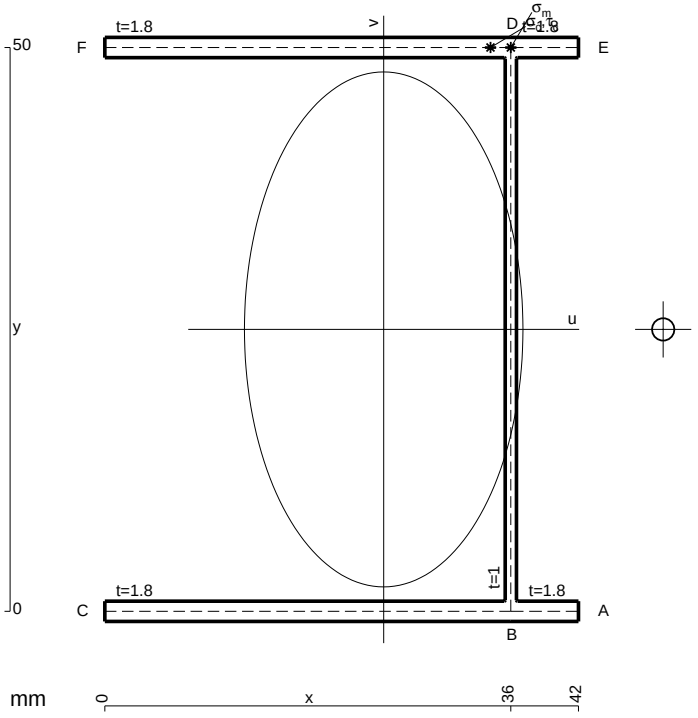
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

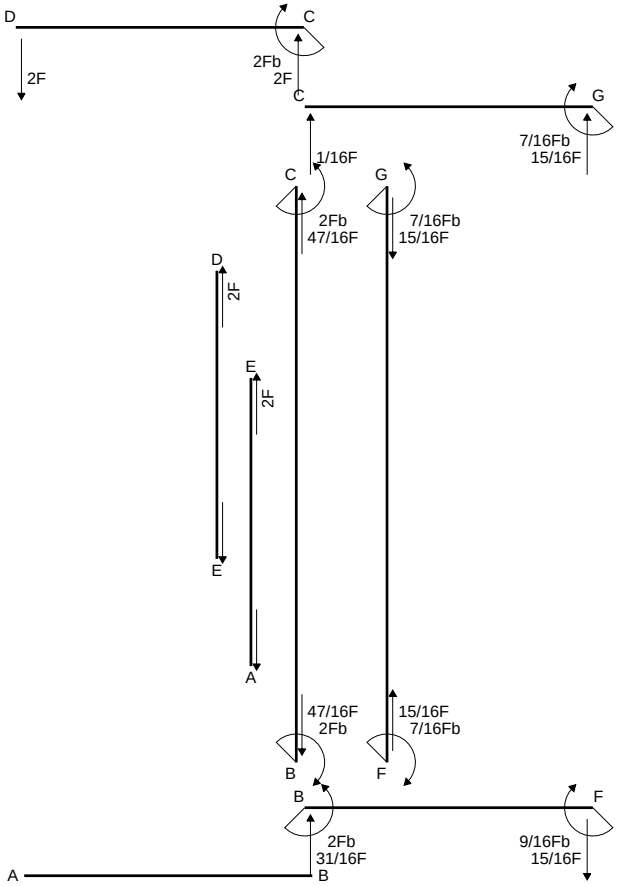
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

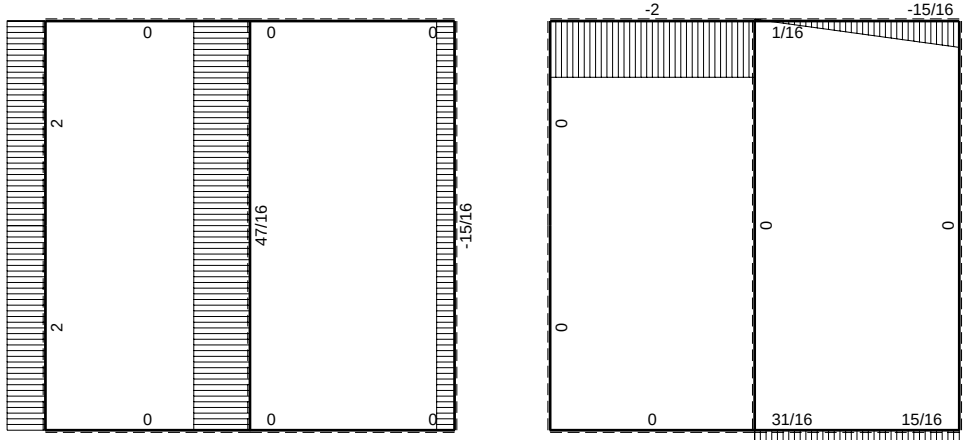


A = 201.2 mm^2
J_u = 104917. mm^4
J_v = 30681. mm^4
J_t = 180. mm^4
x_o = 24.78 mm
x_g = 24.73 mm
T_y = 2000. N
M_x = -880000. Nmm
x_m = 36. mm
y_m = 50. mm
u_m = 11.27 mm
v_m = 25. mm
$\sigma_m = -Mv/J_u$ = 209.7 N/mm^2
x_c = 36. mm
y_c = 50. mm
u_c = 11.27 mm
v_c = 25. mm
$\sigma_c = -Mv/J_u$ = 209.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 512.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 17.16 N/mm^2
$\tau_o = Tx_o t/J_t$ = 495.8 N/mm^2
t_c = 7200. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 912.8 N/mm^2

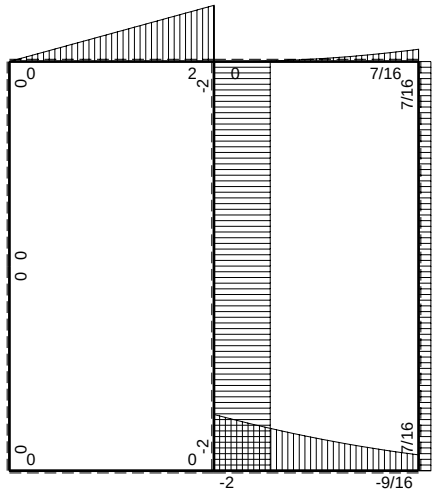
CI08.xxxx.071

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0
-2
-15/16
1/16
0
0
0
0
0
31/16
15/16

F F


0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
0
-2
7/16
-2
-9/16

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
	0	-2Fx	0	0	0	0		
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
	1	0	0	0	0	1		
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
	0	2Fb	0	0	0	0		
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

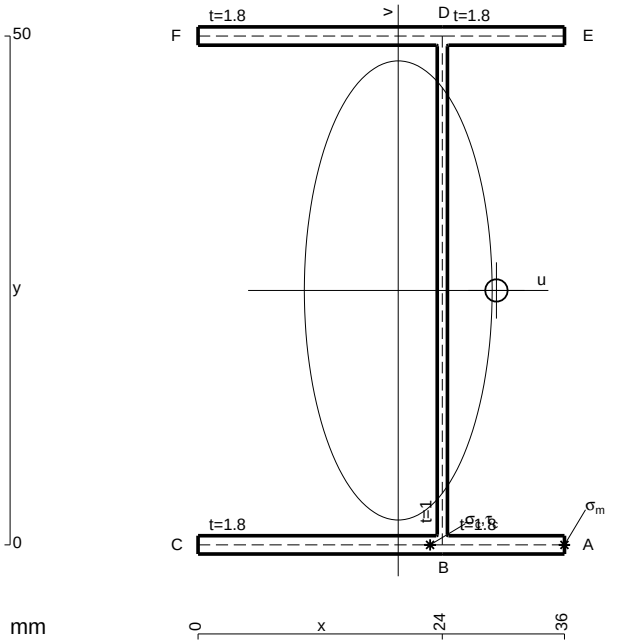
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

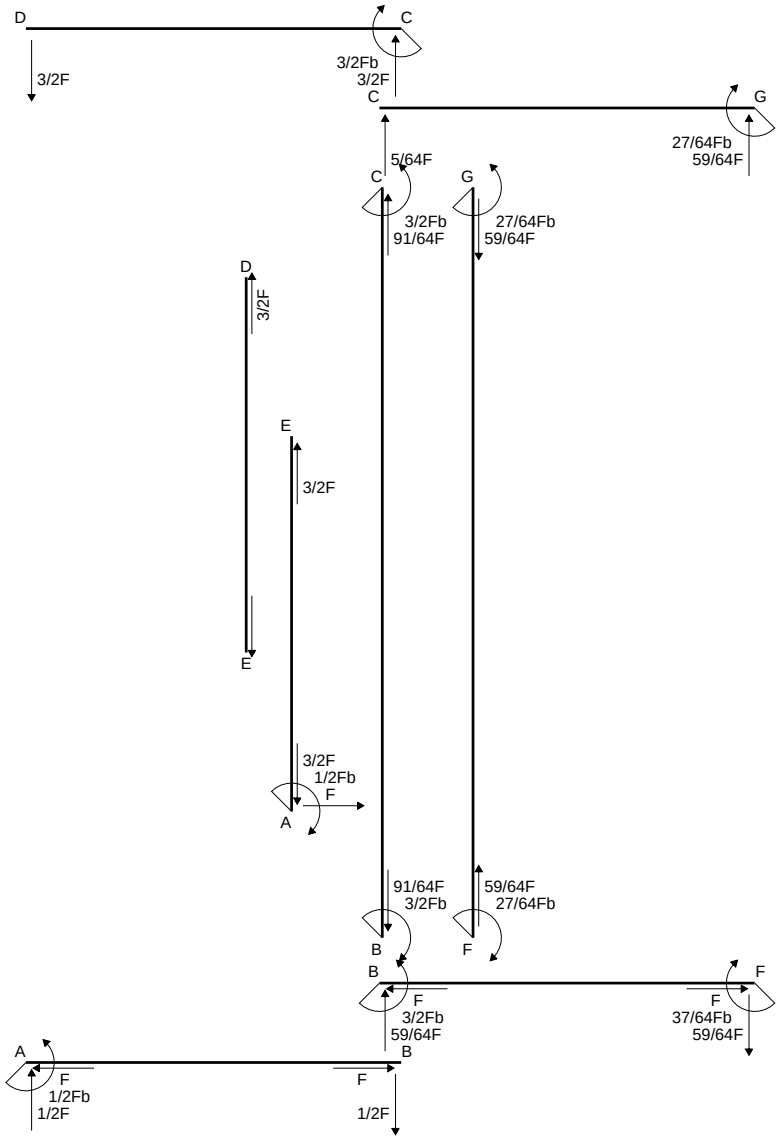
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

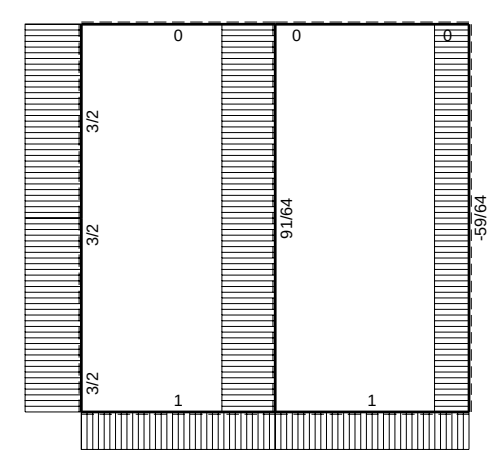


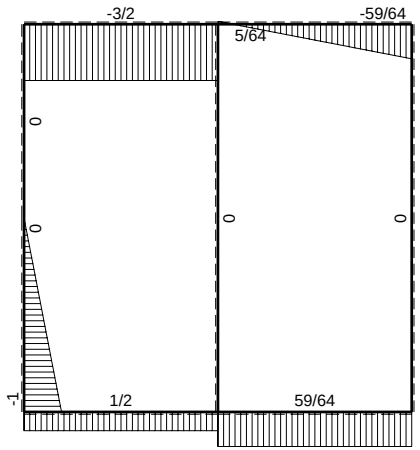
A = 179.6 mm^2
J_u = 91417. mm^4
J_v = 15296. mm^4
J_t = 156.6 mm^4
x_o = 9.646 mm
x_g = 19.67 mm
T_y = -1660. N
M_x = 796800. Nmm
x_m = 36. mm
u_m = 16.33 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = 217.9 N/mm^2
x_c = 24. mm
u_c = 4.33 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = 217.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 194.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.9 N/mm^2
$\tau_o = Tx_o t/J_t$ = 184. N/mm^2
t_c = 1494. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 401.8 N/mm^2

CI08.xxxx.072

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

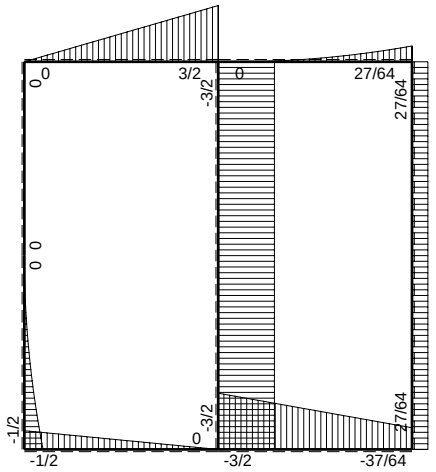

D
C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
C
5/64F
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
3/2
91/64
-59/64
1
1


-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F

F


0
0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	0	1/2Fx	0	0	0	0		
BA b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
	0	-3/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DC b	0	0	0	0	0	0	0+0	0
	0	-1/2qx^2	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	1/2Fb-Fx+1/2qx^2	0	0	0	0	0+0	0
	0	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(7/12+1/2)Fb^2/EJ	1/3Xb/EJ
AE b	0	1/2Fb-Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
	-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
EA b	0	-1/2qx^2	0	0	0	0	0+0	0
	0	0	0	0	0	0		
BF b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
totali							9/8Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-27/64Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

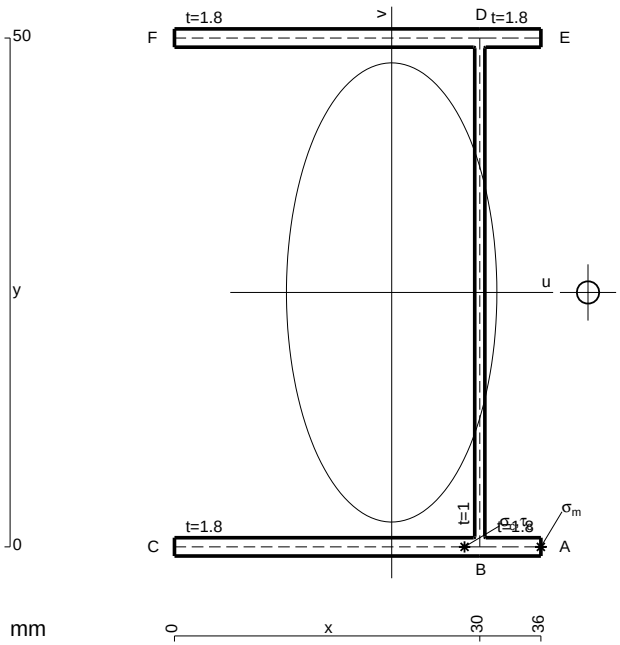
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

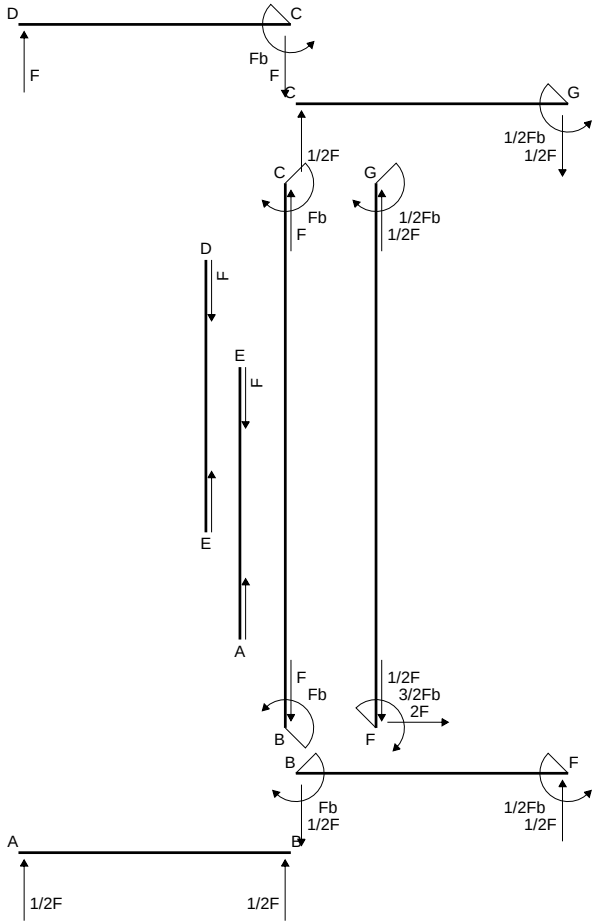
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

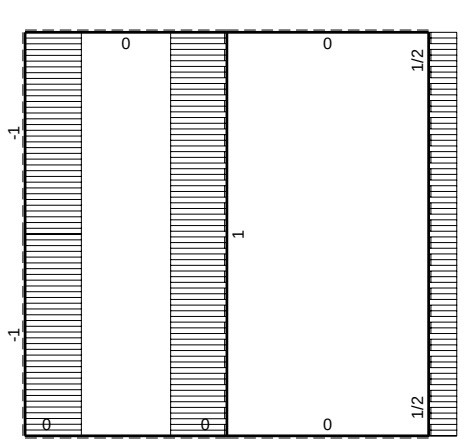


A = 179.6 mm^2
J_u = 91417. mm^4
J_v = 19192. mm^4
J_t = 156.6 mm^4
x_o = 19.29 mm
x_g = 21.34 mm
T_y = -1605. N
M_x = 834600. Nmm
x_m = 36. mm
u_m = 14.66 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = 228.2 N/mm^2
x_c = 30. mm
u_c = 8.659 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = 228.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 369. N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.17 N/mm^2
$\tau_o = Tx_ot/J_t$ = 355.8 N/mm^2
t_c = 1926. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 678.6 N/mm^2

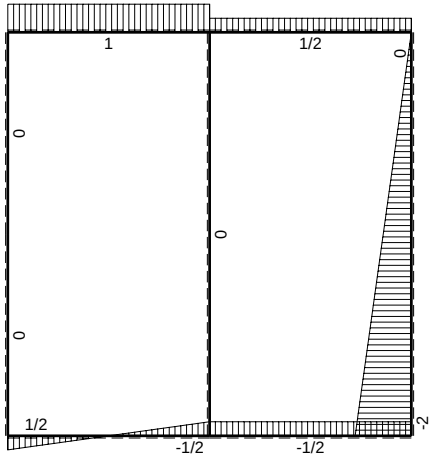
CI08.xxxx.073

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

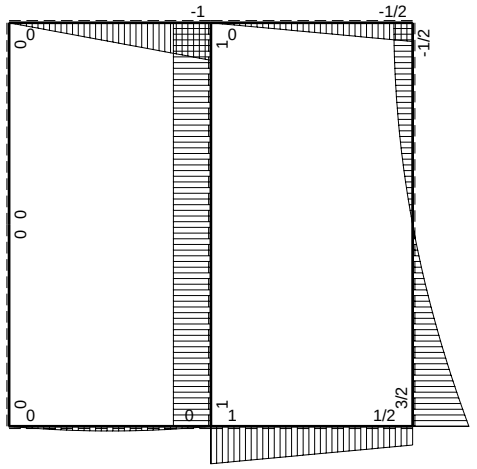

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

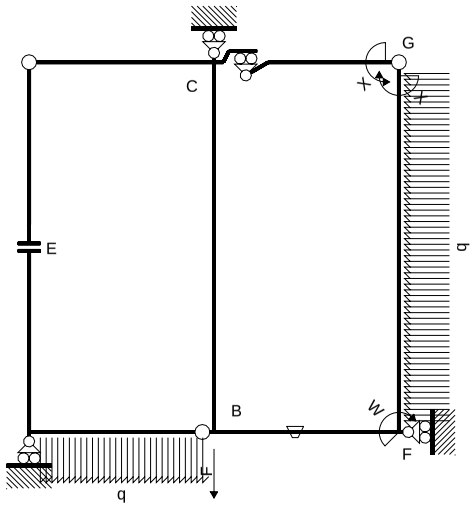
F

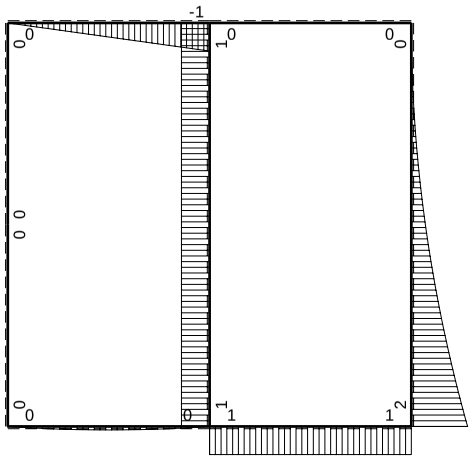

1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2

F

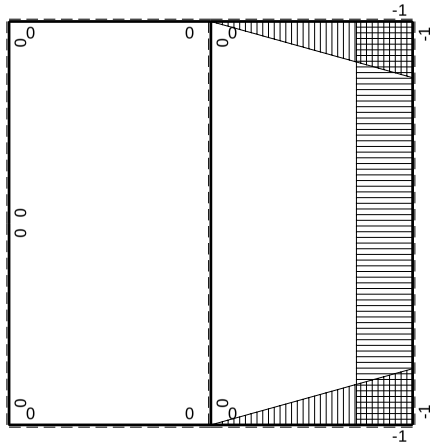

-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
1
1/2
3/2

Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
BA b	0	-1/2Fx+1/2qx^2	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
		totali					-4/3Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

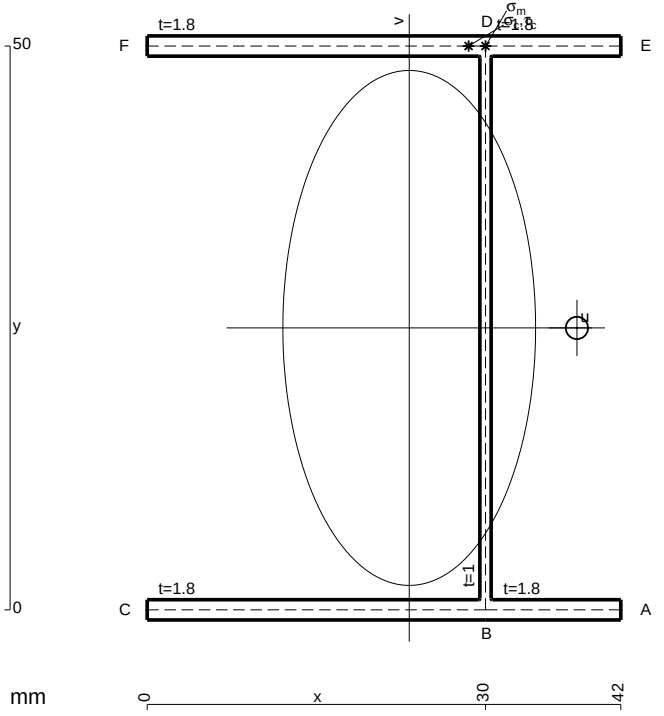
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

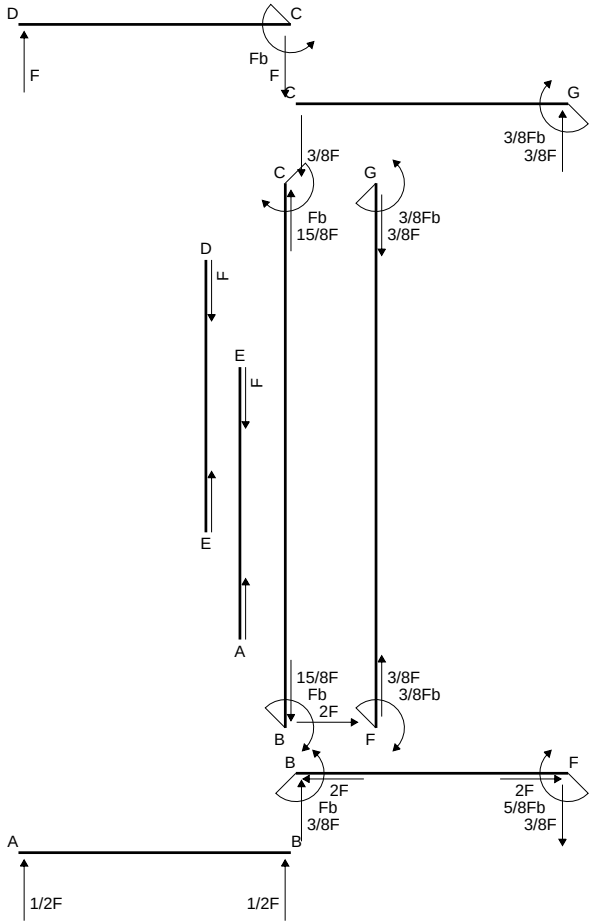
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

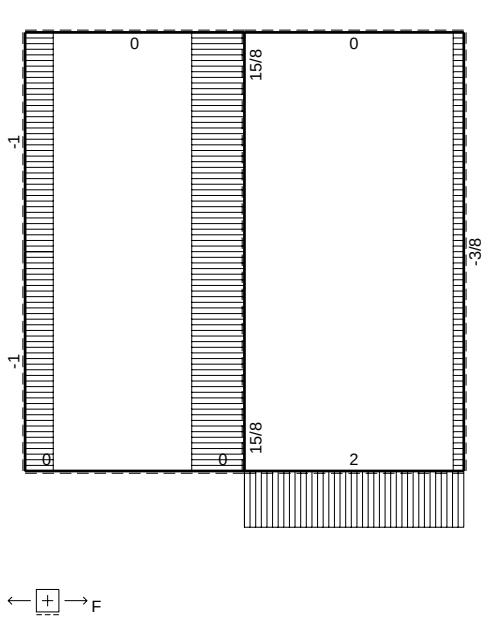


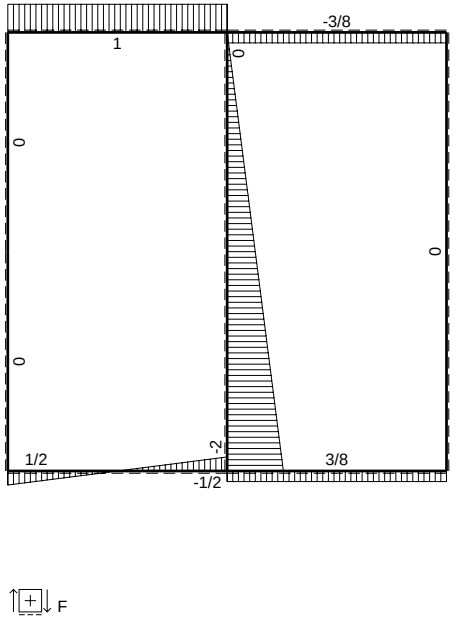
A = 201.2 mm^2
J_u = 104917. mm^4
J_v = 25270. mm^4
J_t = 180. mm^4
x_o = 14.87 mm
x_g = 23.24 mm
T_y = 1790. N
M_x = -1002400. Nmm
x_m = 30. mm
y_m = 50. mm
u_m = 6.763 mm
v_m = 25. mm
$\sigma_m = -Mv/J_u$ = 238.9 N/mm^2
x_c = 30. mm
y_c = 50. mm
u_c = 6.763 mm
v_c = 25. mm
$\sigma_c = -Mv/J_u$ = 238.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 279. N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.8 N/mm^2
$\tau_o = Tx_o t/J_t$ = 266.2 N/mm^2
t_c = 3222. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 539.1 N/mm^2

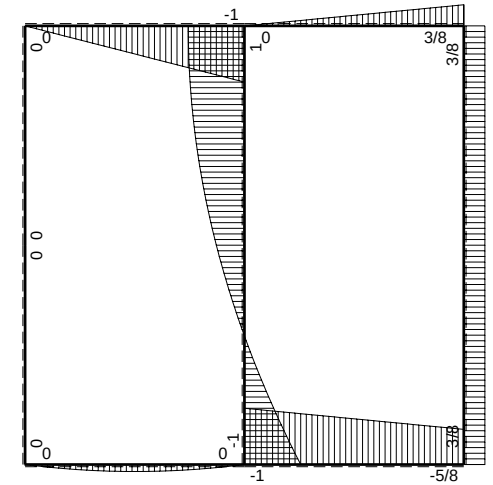
CI08.xxxx.074

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F


0
0
15/8
-1
-1
-3/8
0
0
15/8
0
0
2
F


-3/8
1
0
0
0
0
1/2
-2
3/8
-1/2
F


-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Schema di calcolo iperstatico

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

PROCEDIMENTO E RISULTATI 216487 CI08.xxxx.075

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

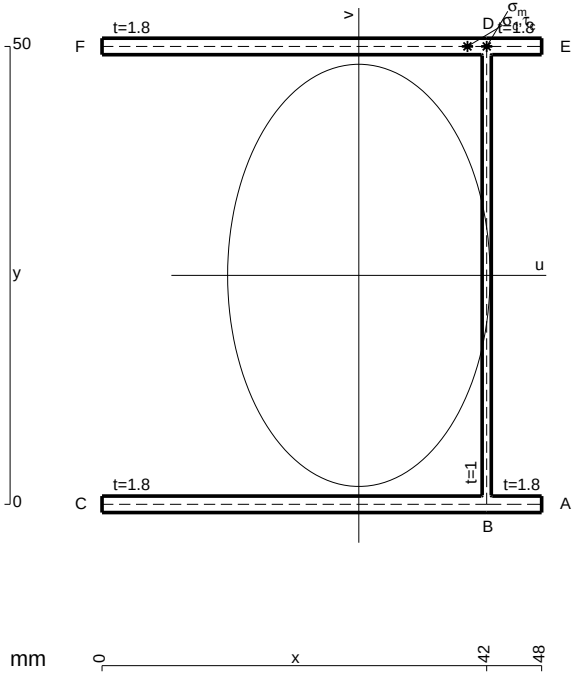
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

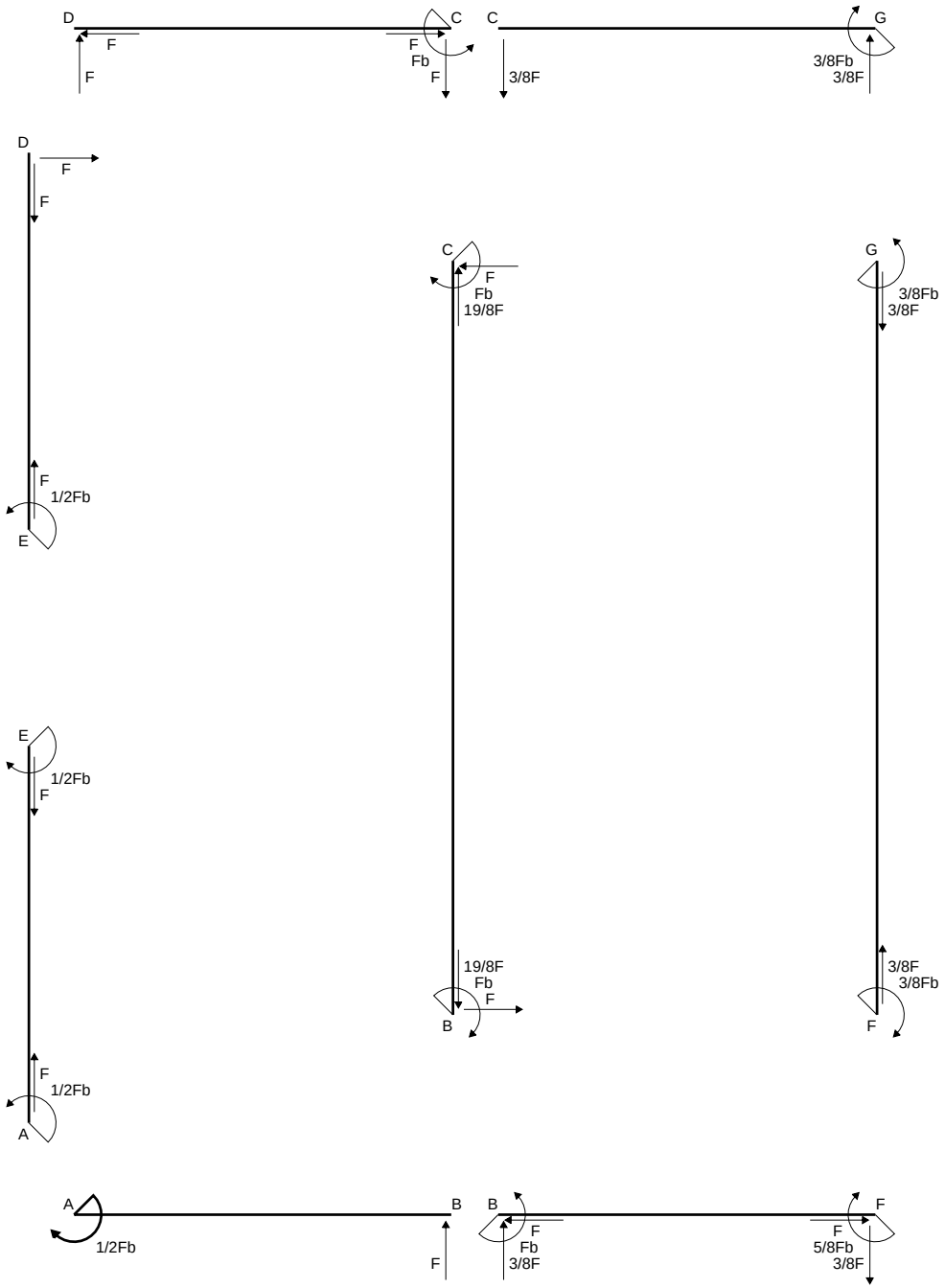
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

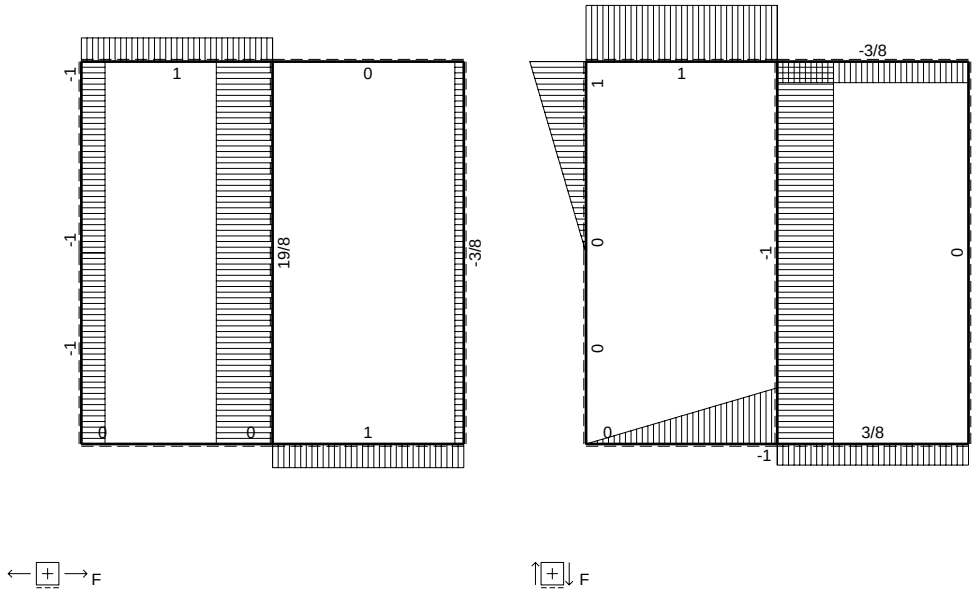


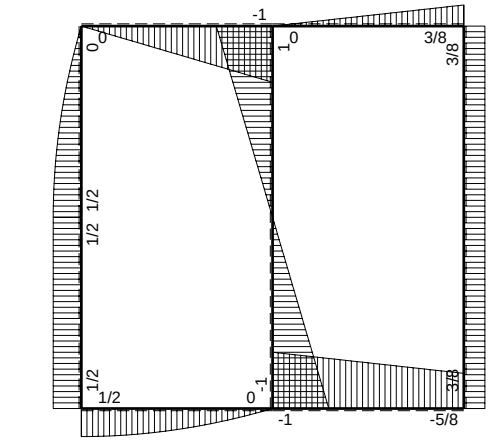
A = 222.8 mm^2
J_u = 118417. mm^4
J_v = 45742. mm^4
J_t = 203.3 mm^4
x_o = 30.38 mm
x_g = 28.04 mm
N = 2775. N
T_y = -2960. N
M_x = -888000. Nmm
x_m = 42. mm
y_m = 50. mm
u_m = 13.96 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 199.9 N/mm^2
x_c = 42. mm
y_c = 50. mm
u_c = 13.96 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 199.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 822.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 26.25 N/mm^2
$\tau_o = Tx_o t/J_t$ = 796.1 N/mm^2
t_c = 2664. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1438. N/mm^2

CI08.xxxx.075

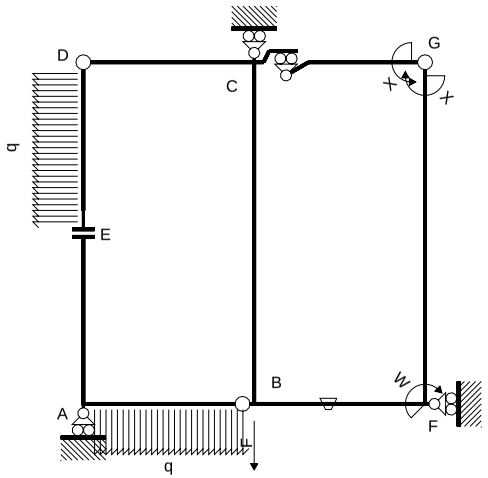
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
19/8F
G
3/8Fb
3/8F
E
1/2Fb
F
F
1/2Fb
A
19/8F
Fb
F
B
3/8F
3/8Fb
F
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F

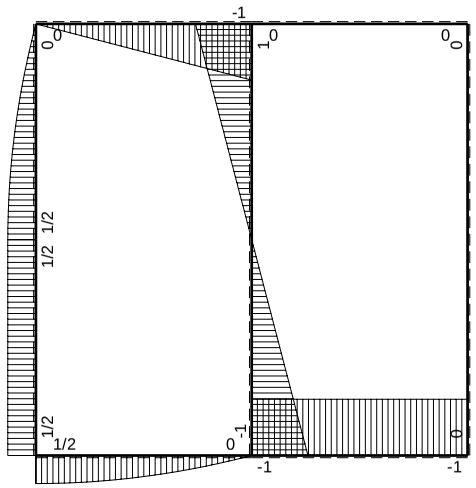

-1
1
0
-1
19/8
-3/8
-1
0
0
1
F
-3/8
1
1
1
0
-1
0
0
0
3/8
-1
F


-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8

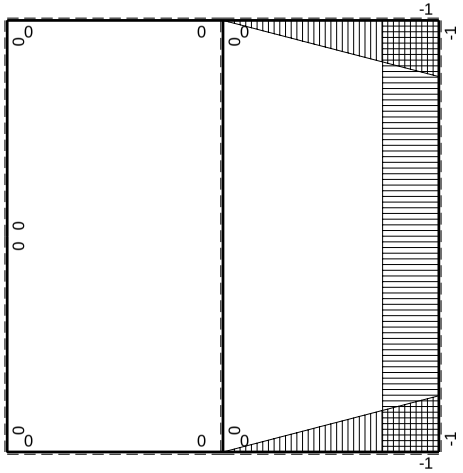

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 963079 CI08.xxxx.076

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

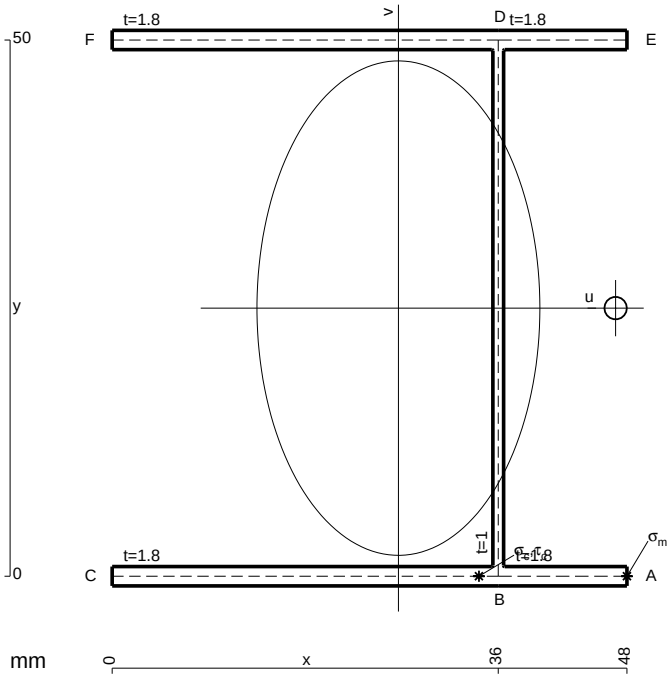
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

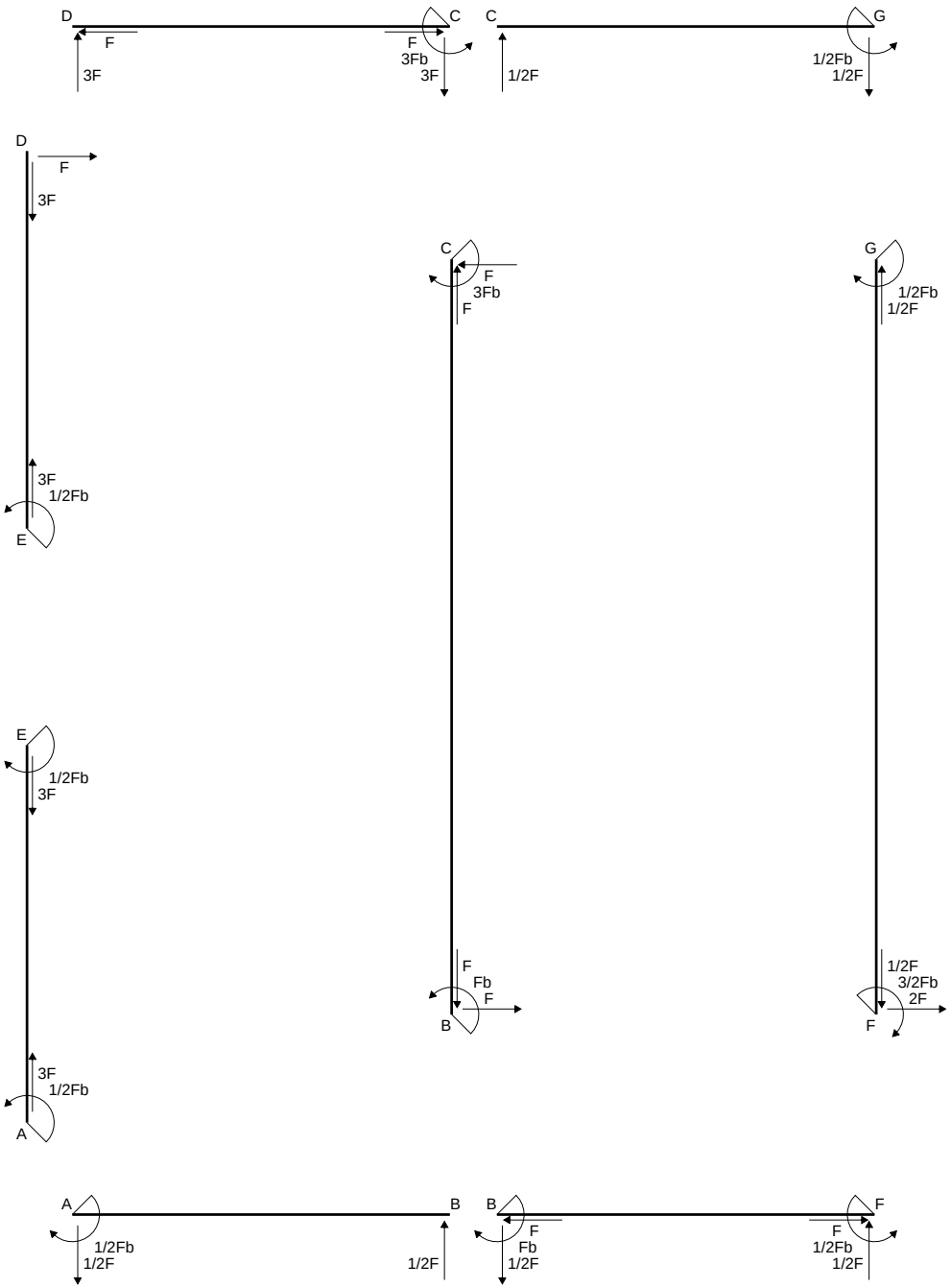
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

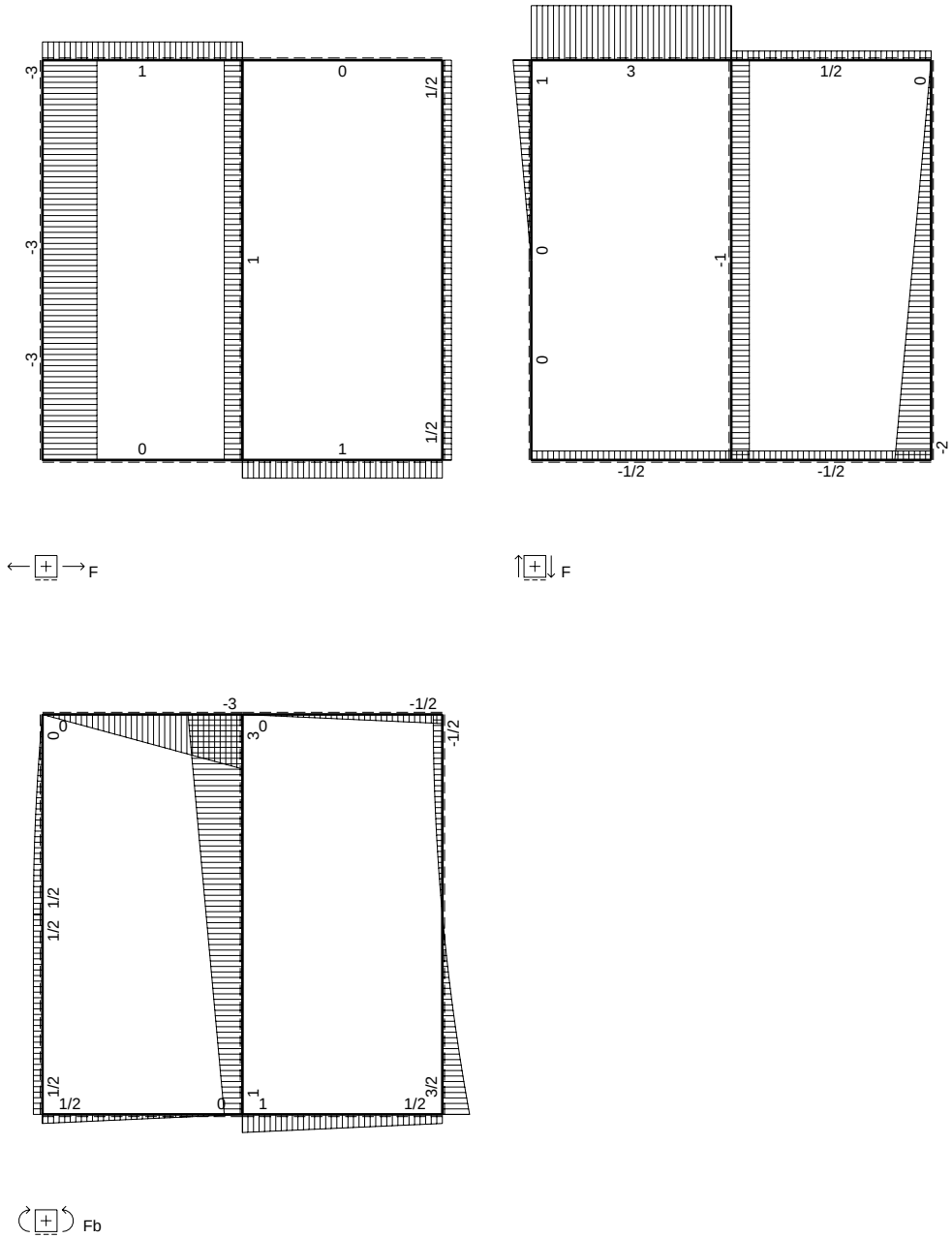


A = 222.8 mm^2
J_u = 118417. mm^4
J_v = 38762. mm^4
J_t = 203.3 mm^4
x_o = 20.25 mm
x_g = 26.69 mm
N = 3420. N
T_y = -1440. N
M_x = 921600. Nmm
x_m = 48. mm
u_m = 21.31 mm
v_m = -25. mm
$\sigma_m = N/A - Mv/J_u$ = 209.9 N/mm^2
x_c = 36. mm
u_c = 9.307 mm
v_c = -25. mm
$\sigma_c = N/A - Mv/J_u$ = 209.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 269.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.94 N/mm^2
$\tau_o = Tx_o t/J_t$ = 258.2 N/mm^2
t_c = 2592.
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 511.3 N/mm^2

CI08.xxxx.076

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
F
Fb
F
1/2F
3/2Fb
2F
B
F
3F
1/2Fb
A
A
B
B
F
F
Fb
F
1/2Fb
1/2F
1/2F
1/2Fb
1/2F


-3
1
0
1/2
-3
1
-3
0
1
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3XbJ/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3XbJ/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2XbJ/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3Fb^2/EJ	8/3XbJ/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

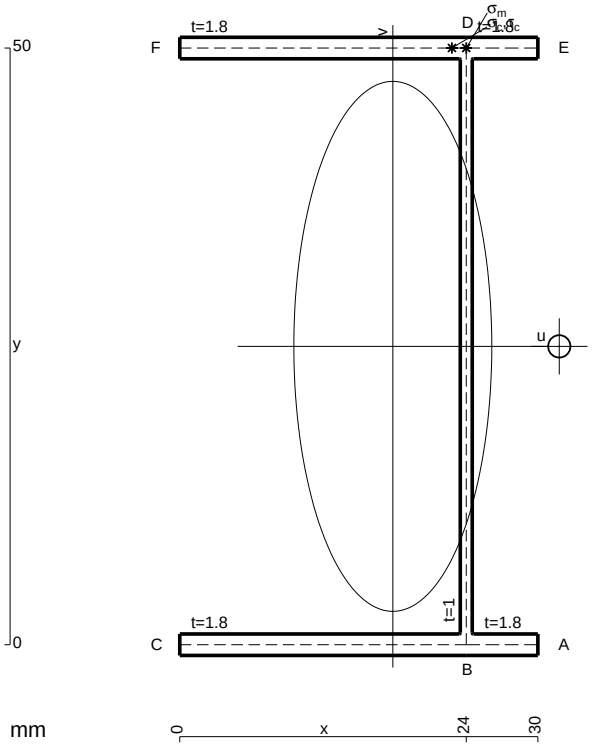
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

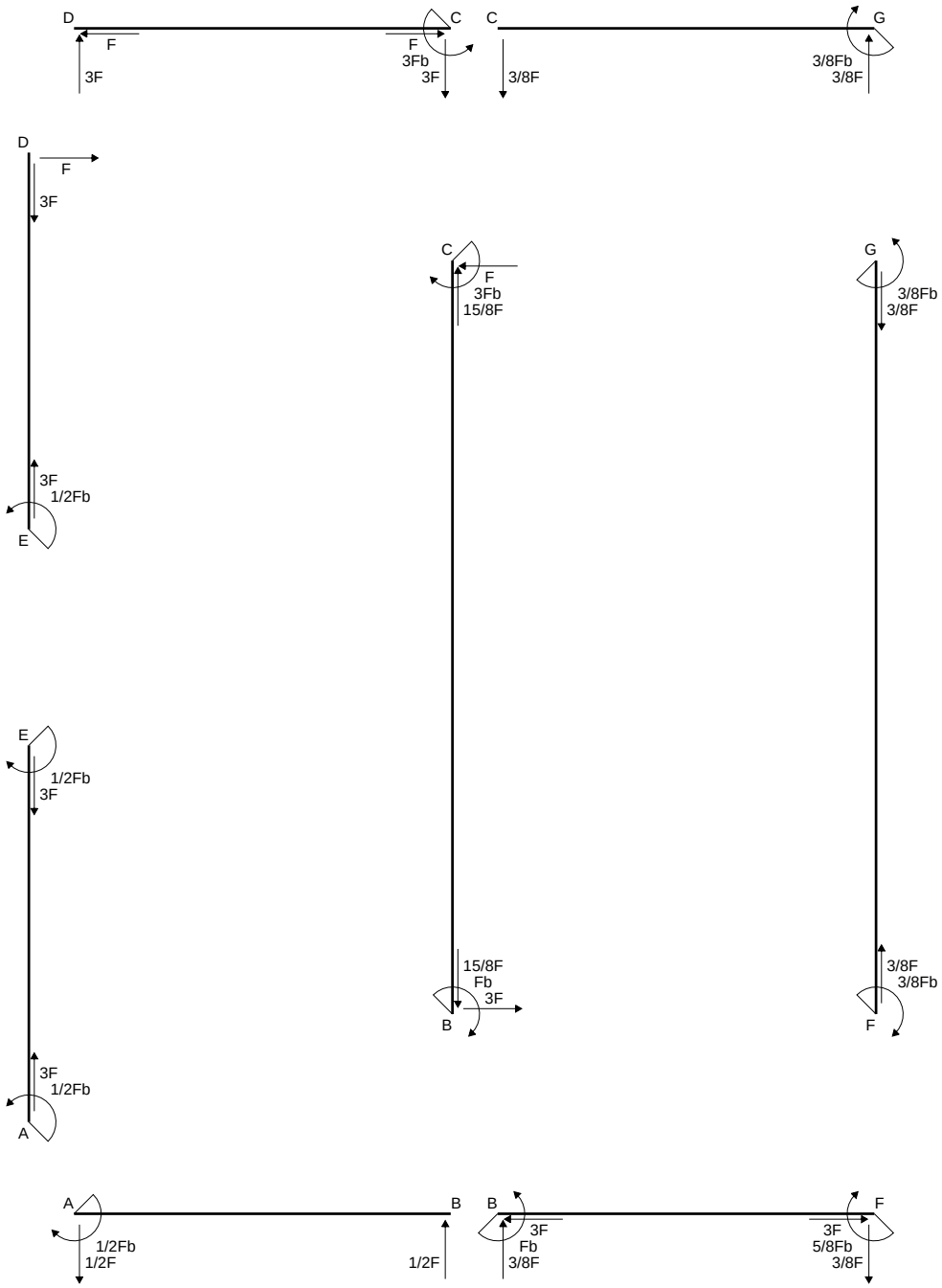
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

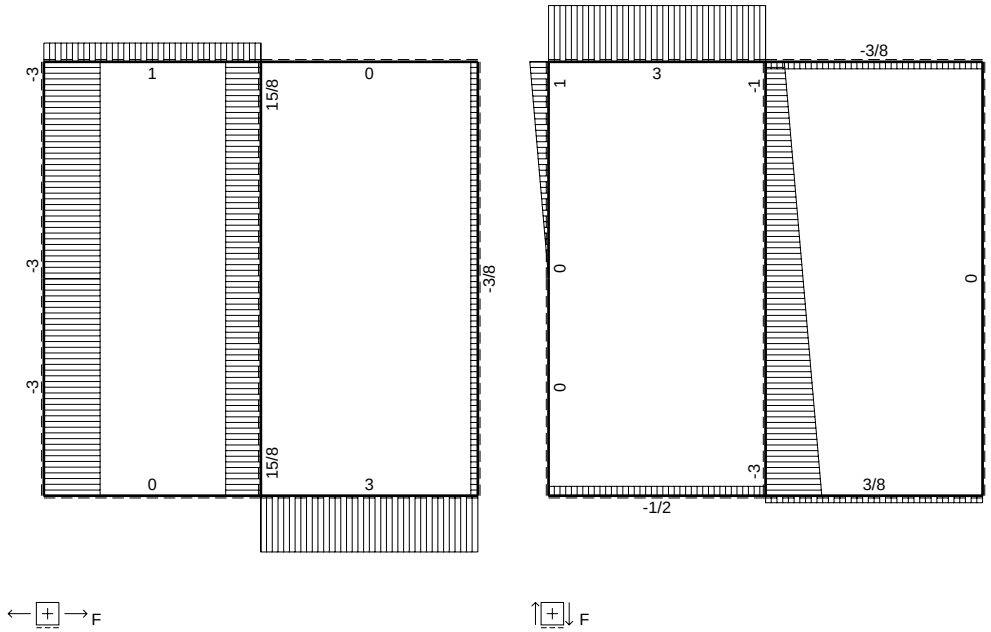


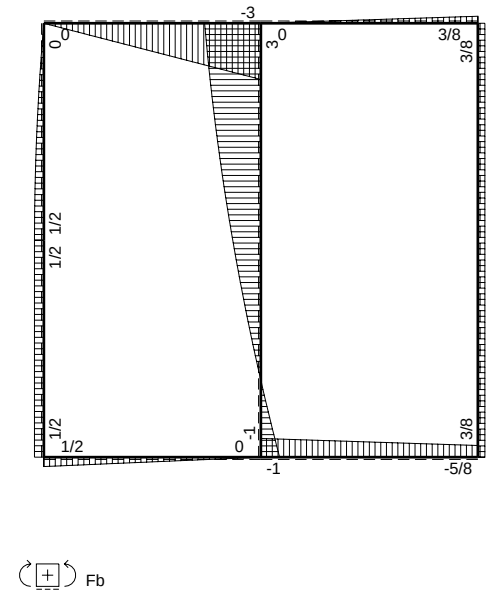
A = 158. mm^2
J_u = 77917. mm^4
J_v = 10868. mm^4
J_t = 133.3 mm^4
x_o = 13.95 mm
x_g = 17.85 mm
N = 330. N
T_y = 990. N
M_x = -673200. Nmm
x_m = 24. mm
y_m = 50. mm
u_m = 6.152 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 218.1 N/mm^2
x_c = 24. mm
y_c = 50. mm
u_c = 6.152 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 218.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 194.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.624 N/mm^2
$\tau_o = Tx_o t/J_t$ = 186.5 N/mm^2
t_c = 594. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 400.7 N/mm^2

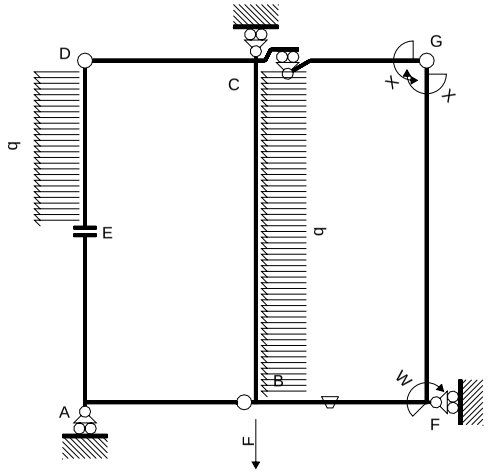
CI08.xxxx.077

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

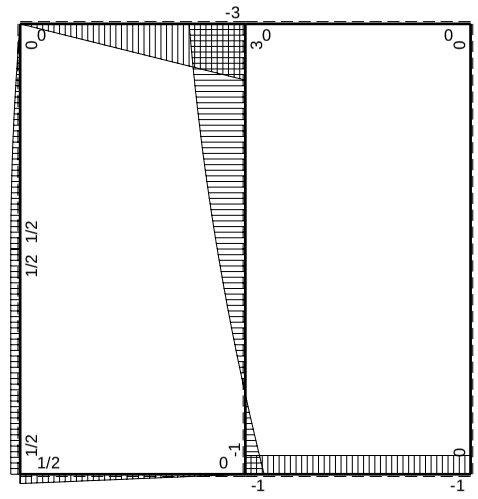

D
F
3F
C
F
3Fb
3F
C
3/8F
G
3/8Fb
3/8F
D
F
3F
3F
1/2Fb
E
C
F
3Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3F
15/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F


-3
1
0
15/8
-3
-3/8
-3
15/8
0
3
F
-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F

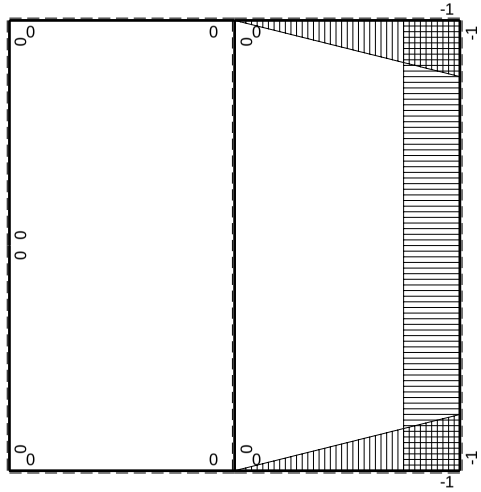

-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 988248 CI08.xxxx.078

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

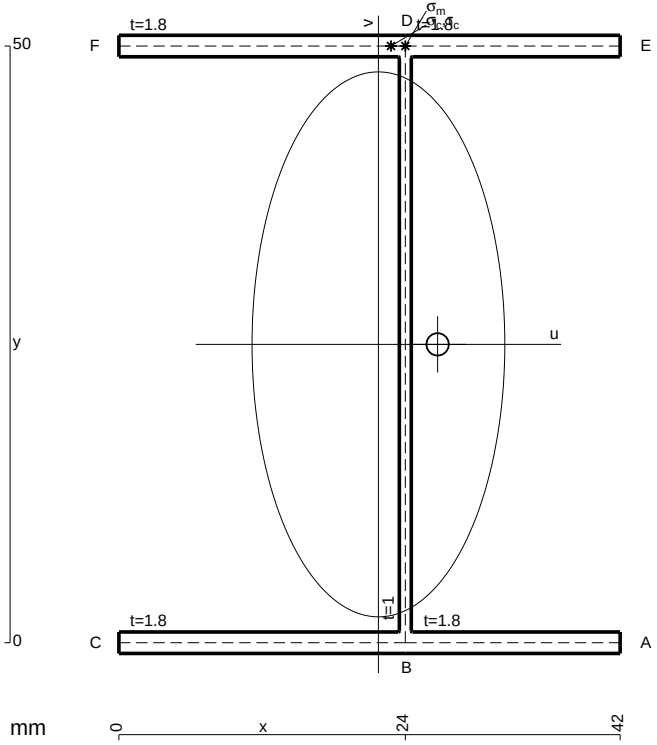
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

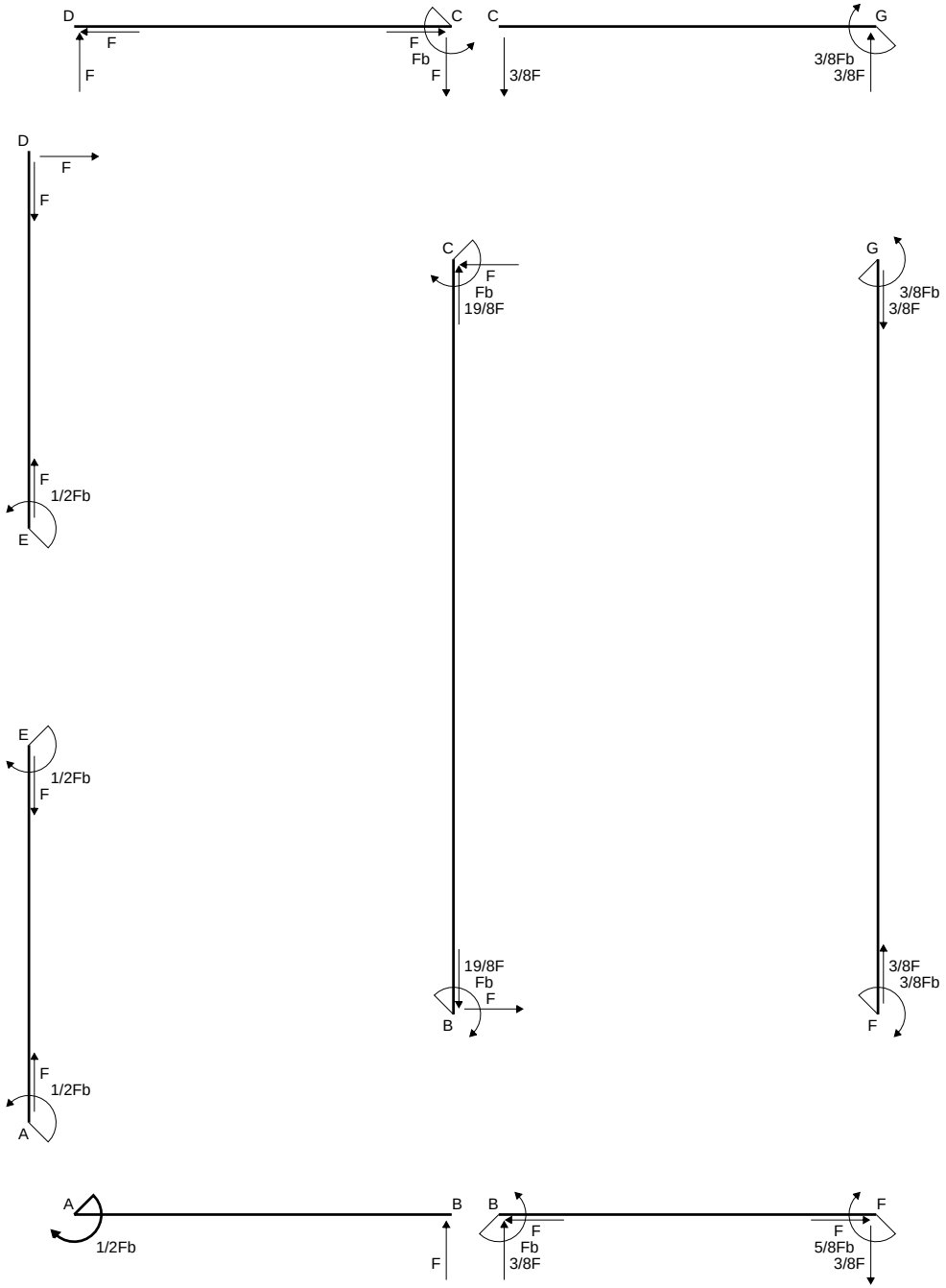
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

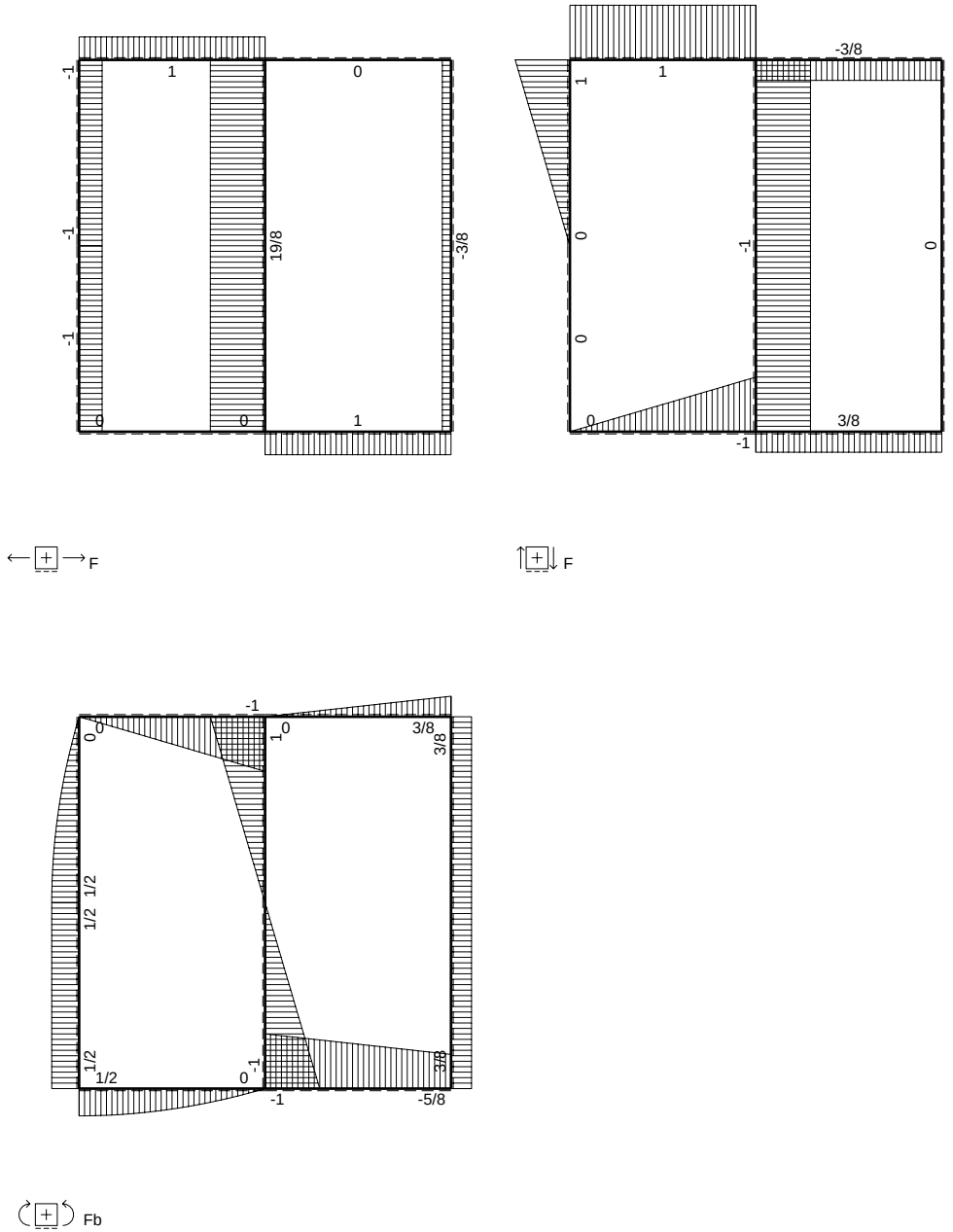


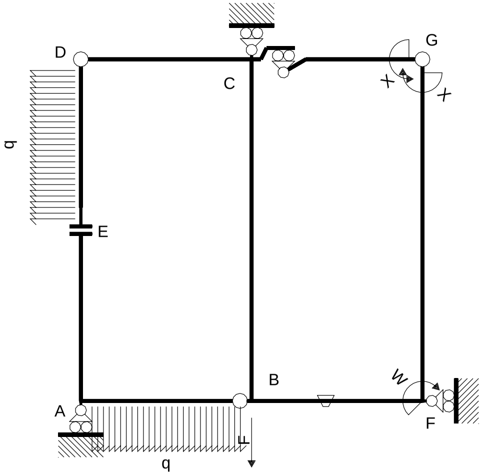
A = 201.2 mm^2
J_u = 104917. mm^4
J_v = 22565. mm^4
J_t = 180. mm^4
x_o = 4.957 mm
x_g = 21.75 mm
N = 440. N
T_y = 1320. N
M_x = -950400. Nmm
x_m = 24. mm
y_m = 50. mm
u_m = 2.254 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 228.7 N/mm^2
x_c = 24. mm
y_c = 50. mm
u_c = 2.254 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 228.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 72.99 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.549 N/mm^2
$\tau_o = Tx_o t/J_t$ = 65.44 N/mm^2
t_c = 792. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 261.3 N/mm^2

CI08.xxxx.078

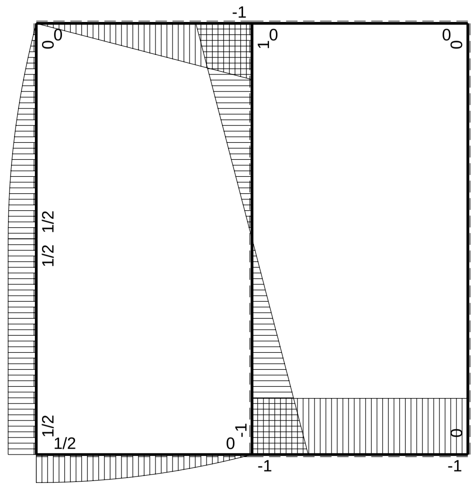
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
19/8F
G
3/8Fb
3/8F
E
1/2Fb
F
19/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F

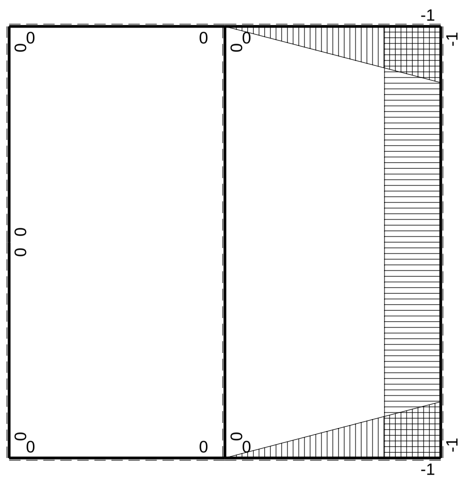

-1
1
0
-1
19/8
-3/8
-1
0
0
1
F
-3/8
1
1
1
0
-1
0
0
0
3/8
-1
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

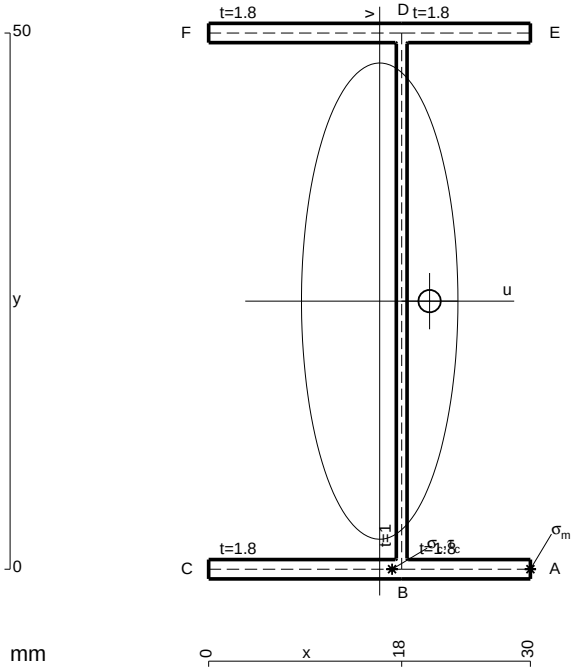
$L^{Xo}_{BF} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 218577 CI08.xxxx.079

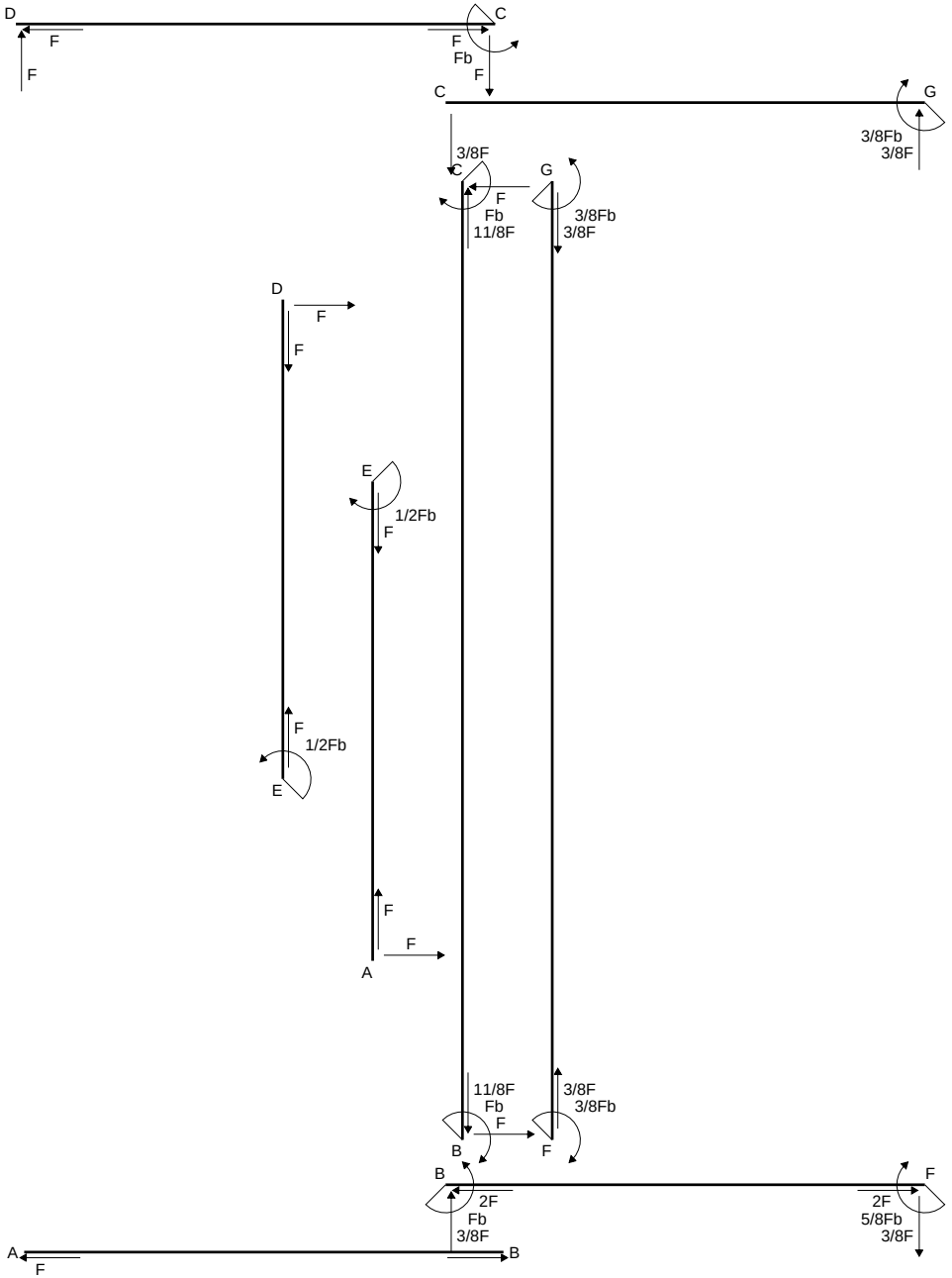


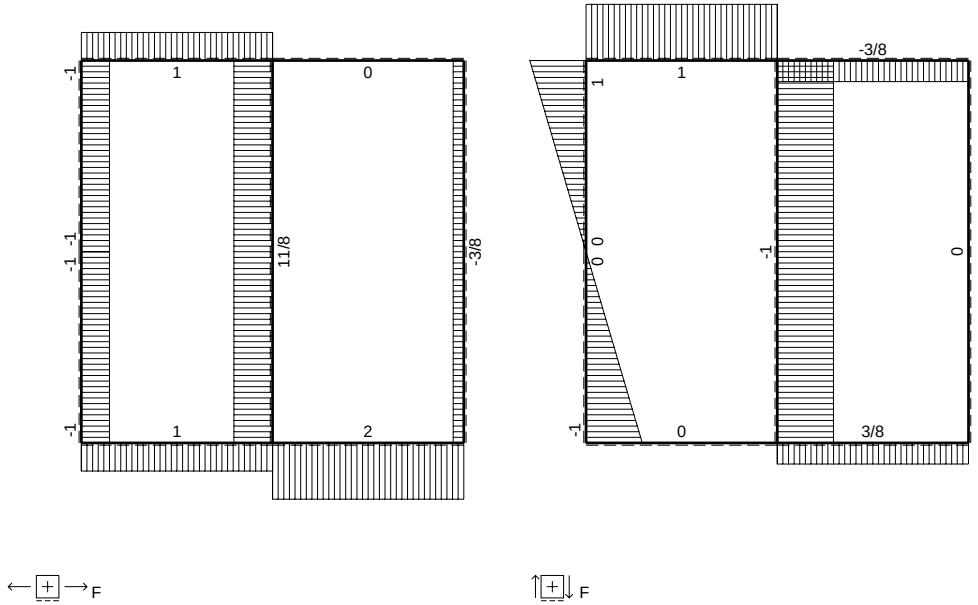
A = 158. mm^2
J_u = 77917. mm^4
J_v = 8408. mm^4
J_t = 133.3 mm^4
x_o = 4.65 mm
x_g = 15.95 mm
N = 2185. N
T_y = -920. N
M_x = 699200. Nmm
x_m = 30. mm
u_m = 14.05 mm
v_m = -25. mm
$\sigma_m = N/A - Mv/J_u$ = 238.2 N/mm^2
x_c = 18. mm
u_c = 2.051 mm
v_c = -25. mm
$\sigma_c = N/A - Mv/J_u$ = 238.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 63.07 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.313 N/mm^2
$\tau_o = Tx_o t/J_t$ = 57.76 N/mm^2
t_c = 1656. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 262. N/mm^2

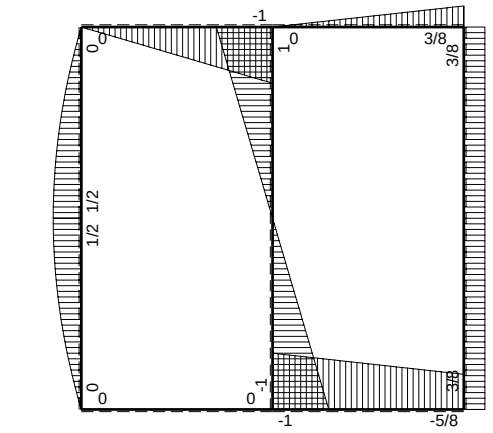
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

CI08.xxxx.079

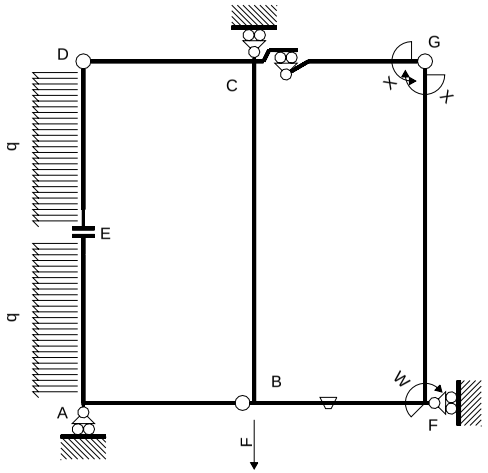
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
11/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
11/8F
Fb
F
3/8F
3/8Fb
F
B
B
2F
Fb
3/8F
F
2F
5/8Fb
3/8F
A
F
B

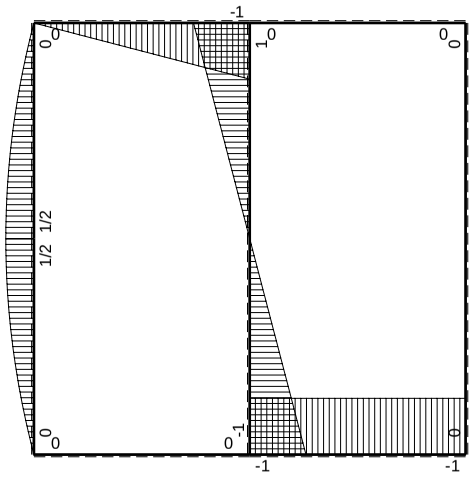

-1
1
0
-1
-1
11/8
-3/8
-1
1
2
F
-3/8
1
1
0 0
-1
0
-1
0
3/8
F


-1
0
0
1
0
3/8
3/8
1/2 1/2
0
0
0
-1
-1
3/8
-1
-5/8

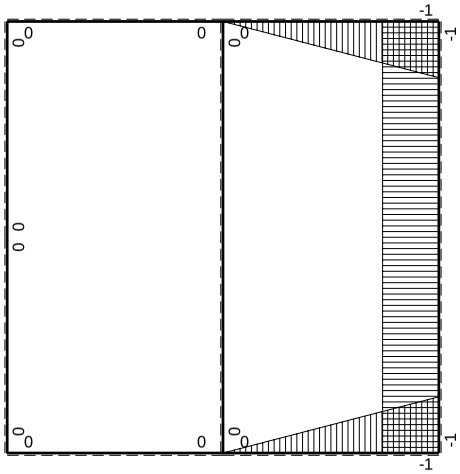

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 986307 CI08.xxxx.080

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

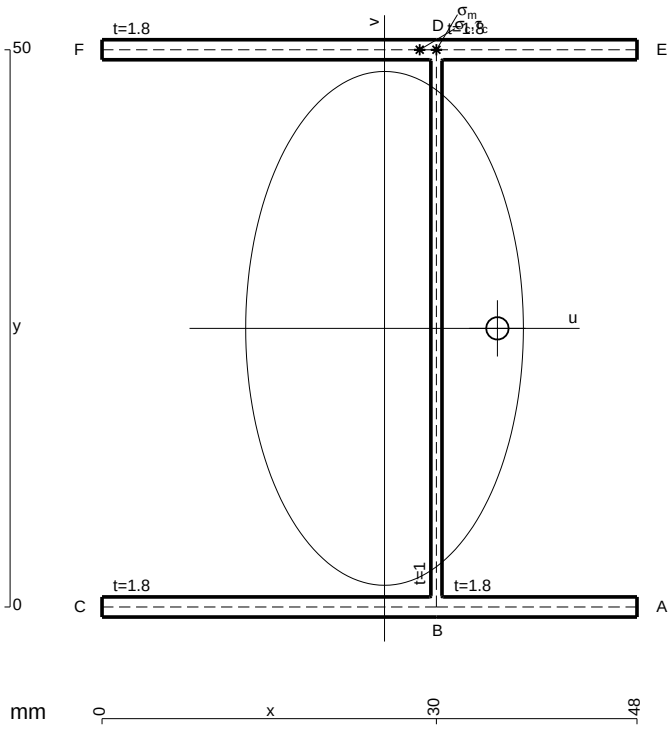
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

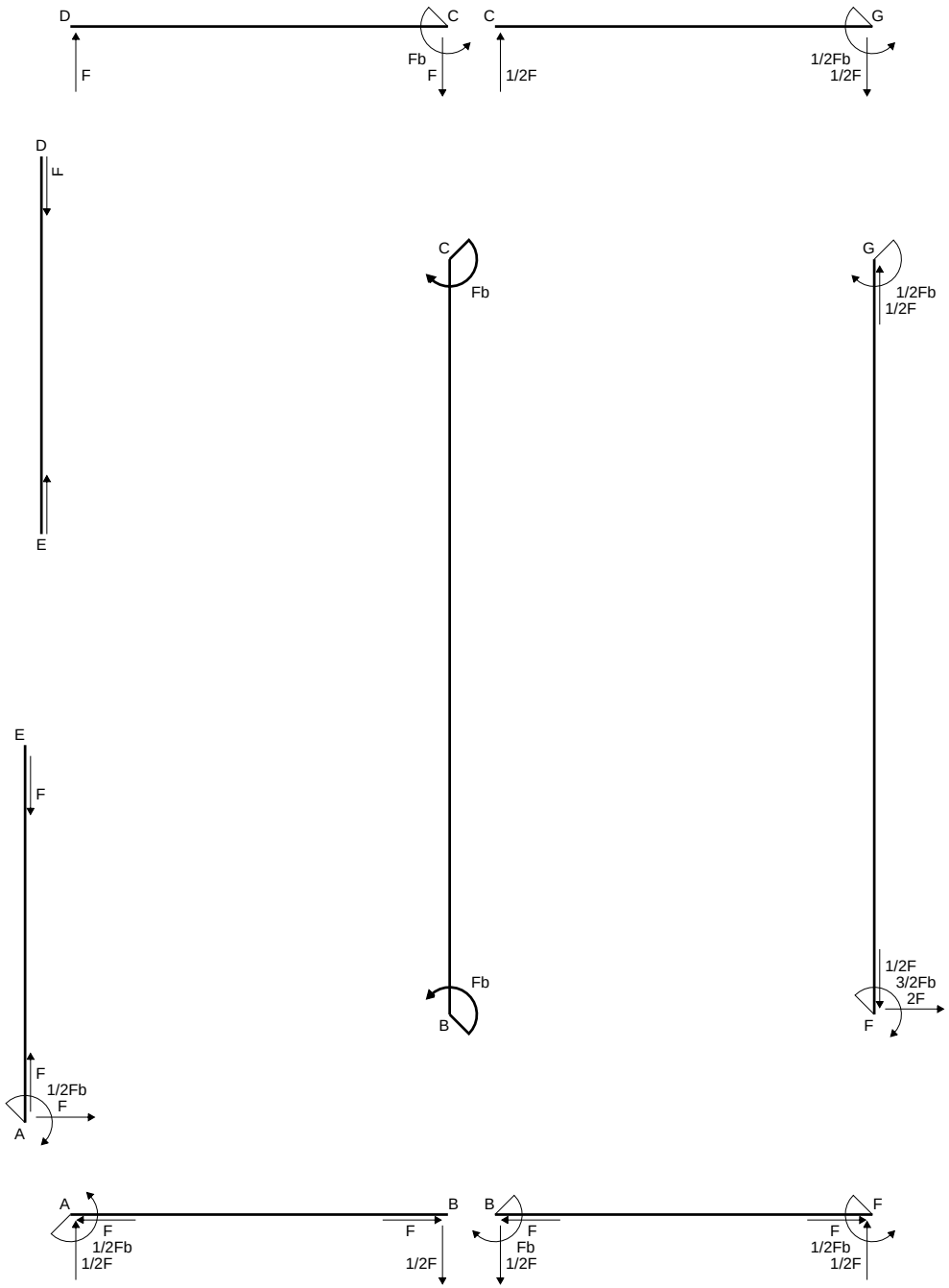
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

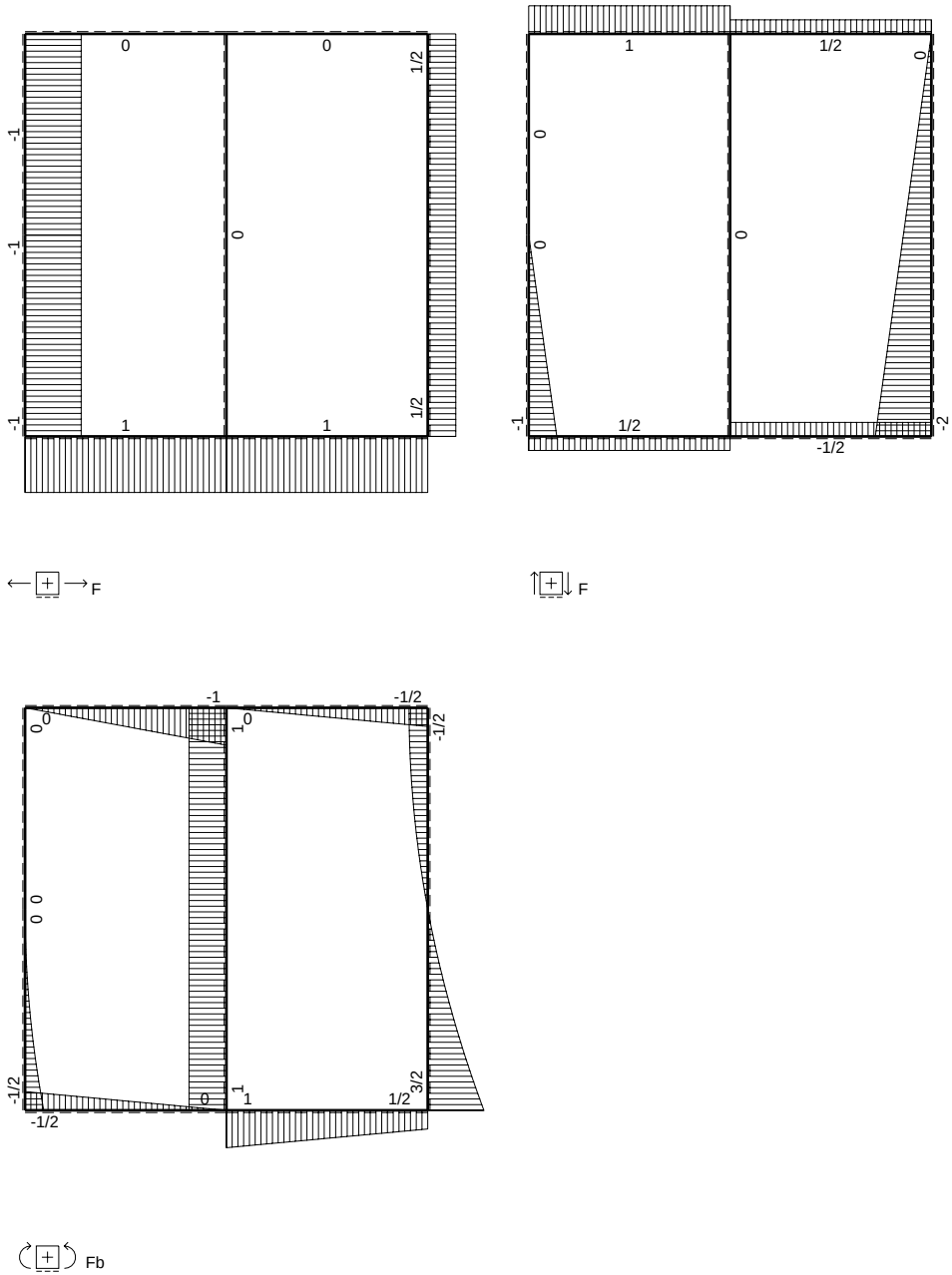


A = 222.8 mm^2
J_u = 118417. mm^4
J_v = 34574. mm^4
J_t = 203.3 mm^4
x_o = 10.13 mm
x_g = 25.35 mm
N = 3025. N
T_y = -2200. N
M_x = -880000. Nmm
x_m = 30. mm
y_m = 50. mm
u_m = 4.654 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 199.4 N/mm^2
x_c = 30. mm
y_c = 50. mm
u_c = 4.654 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 199.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 211.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.93 N/mm^2
$\tau_o = Tx_o t/J_t$ = 197.2 N/mm^2
t_c = 3960. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 416.6 N/mm^2

CI08.xxxx.080

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
G
1/2Fb
1/2F
Fb
B
1/2F
3/2Fb
2F
F
E
F
F
1/2Fb
F
A
A
B
B
F
F
F
F
1/2Fb
1/2F
1/2F
Fb
1/2F
F
1/2Fb
1/2F


0
0
1/2
-1
0
-1
0
-1
1
1
1/2
F
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F
-1
-1/2
0
0
1
0
-1/2
0 0
-1/2
0
1
1
1/2
3/2
-1/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

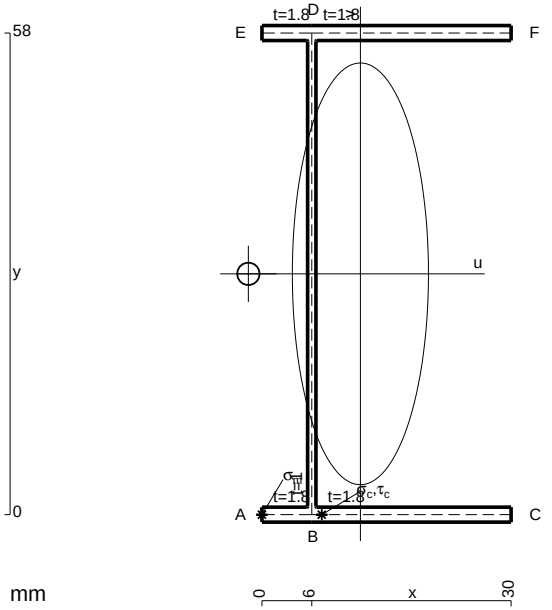
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

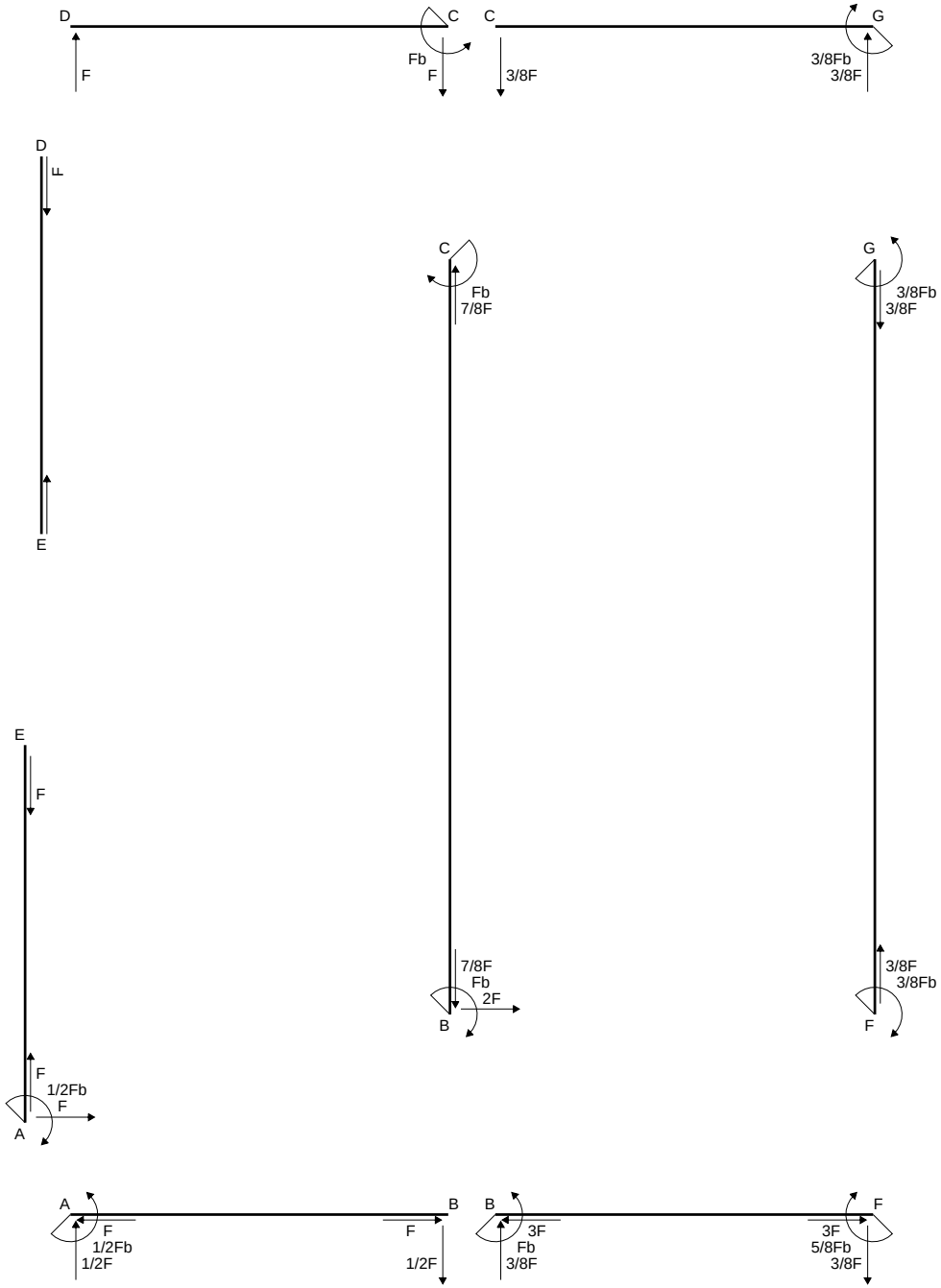
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

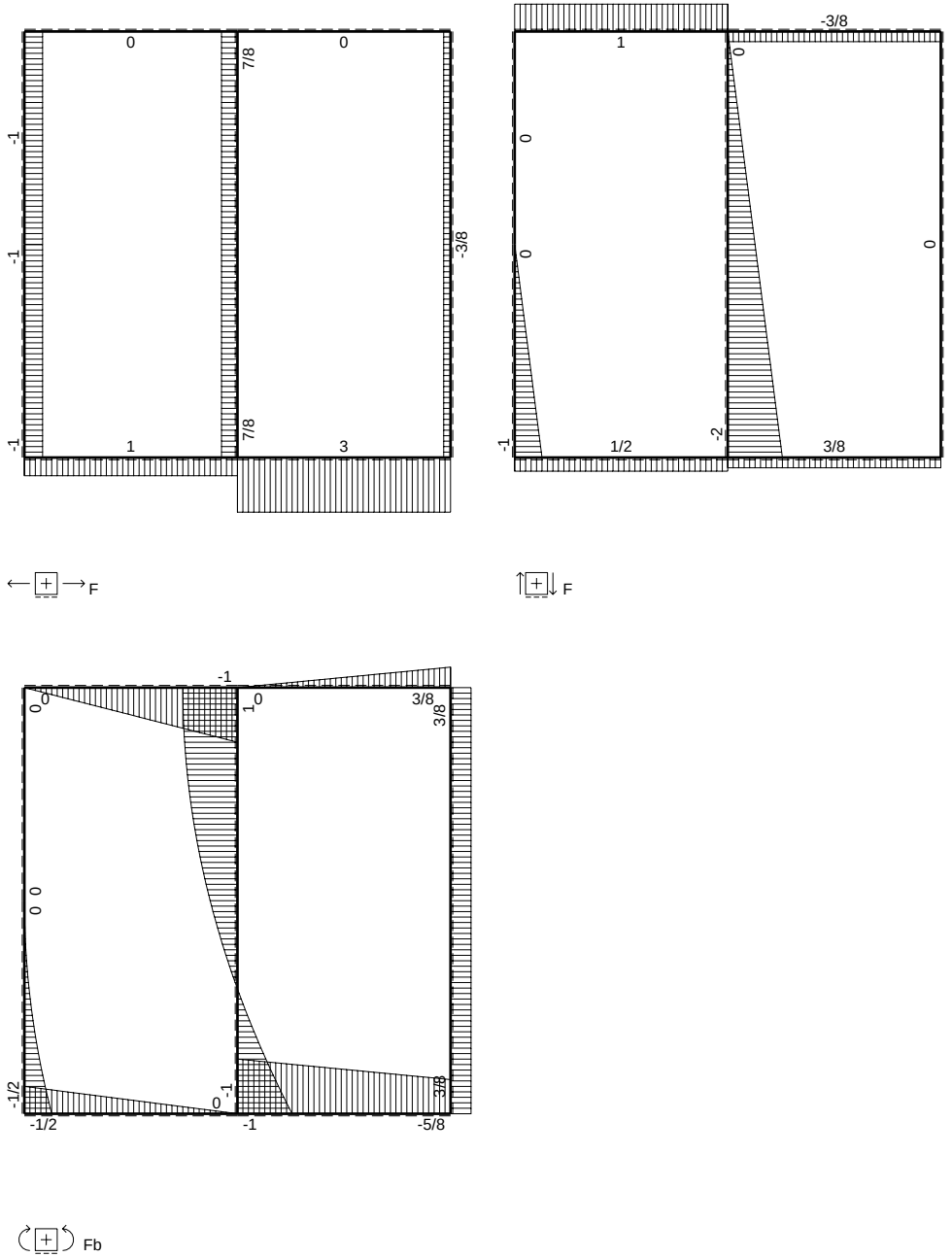


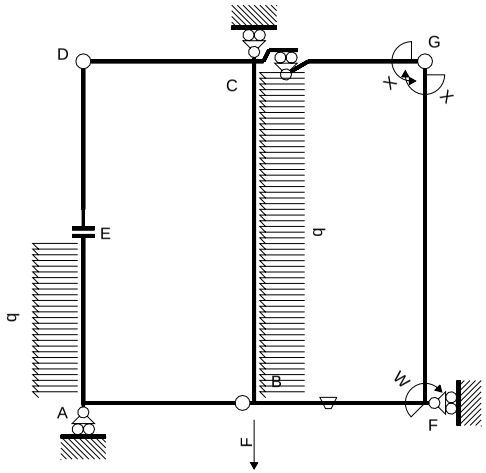
A = 166. mm^2
J_u = 107087. mm^4
J_v = 11157. mm^4
J_t = 136. mm^4
x_o = -13.49 mm
x_g = 11.86 mm
T_y = 1550. N
M_x = -775000. Nmm
u_m = -11.86 mm
v_m = -29. mm
$\sigma_m = -Mv/J_u$ = -209.9 N/mm^2
x_c = 6. mm
u_c = -5.855 mm
v_c = -29. mm
$\sigma_c = -Mv/J_u$ = -209.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 286.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.07 N/mm^2
$\tau_o = Tx_o t/J_t$ = 276.8 N/mm^2
t_c = 2790. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 539.3 N/mm^2

CI08.xxxx.081

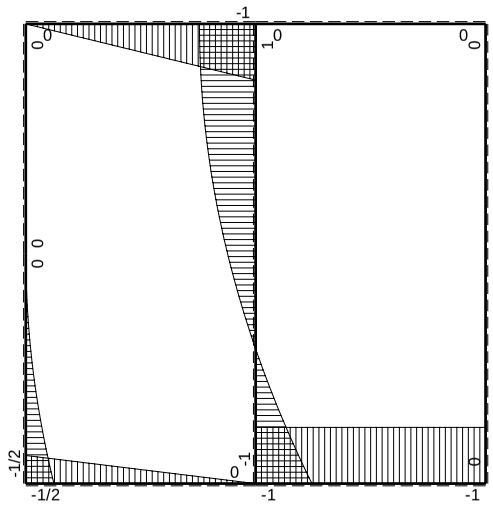
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
7/8F
G
3/8Fb
3/8F
E
F
F
1/2Fb
F
A
7/8F
Fb
2F
B
3/8F
3/8Fb
F
A
B
B
F
F
1/2Fb
1/2F
F
1/2F
3F
Fb
3/8F
3F
5/8Fb
3/8F

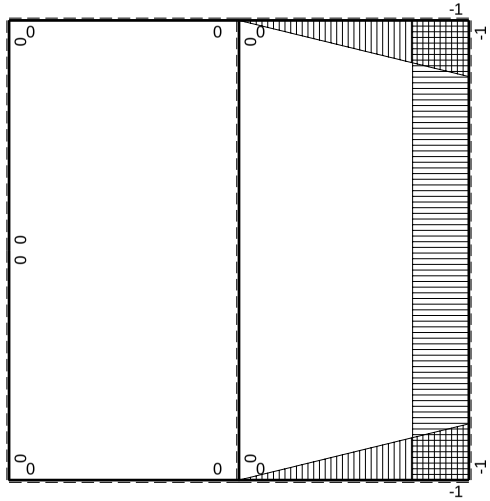

0
0
7/8
-1
-1
-1
-3/8
1
7/8
3
F
-3/8
1
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 238903 CI08.xxxx.082

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

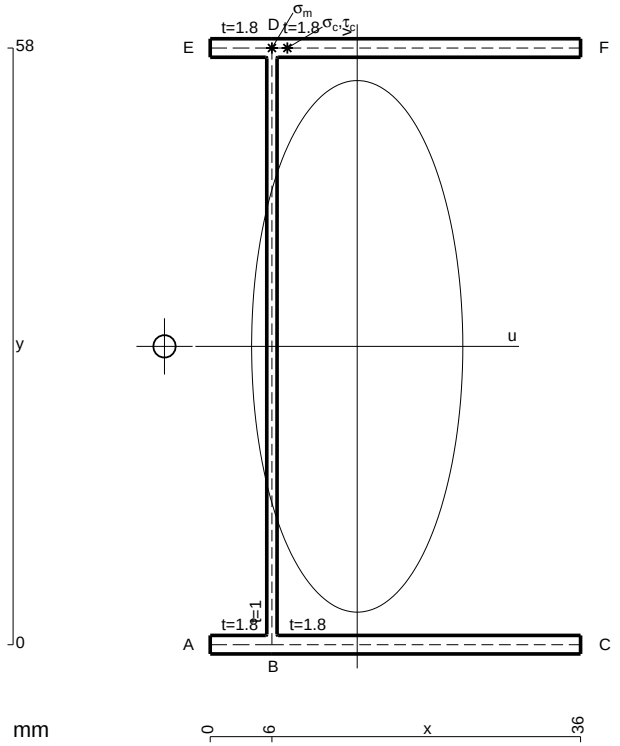
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

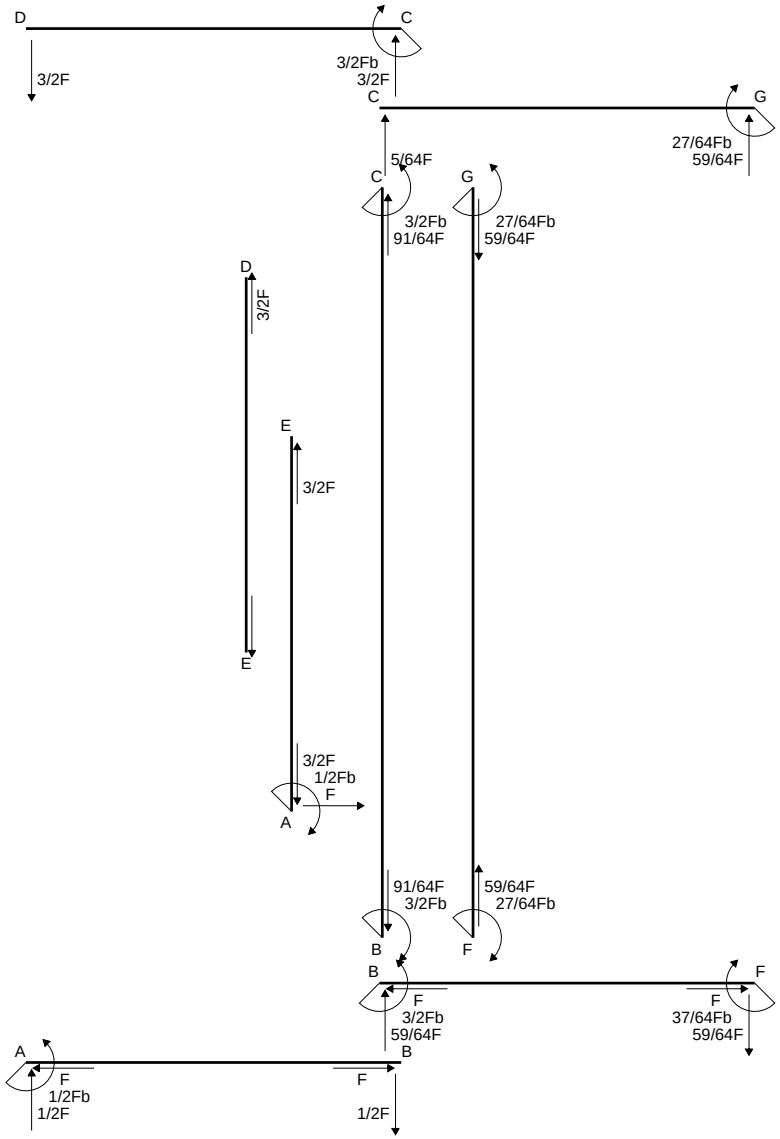
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

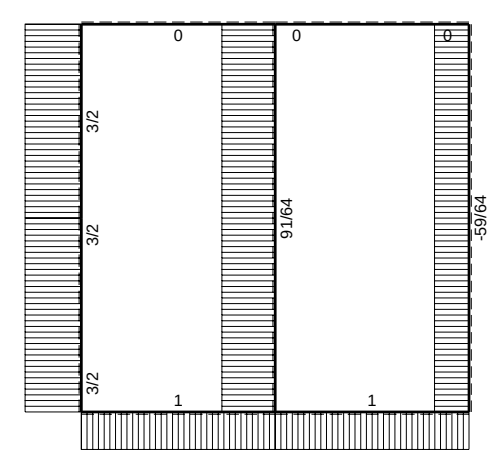


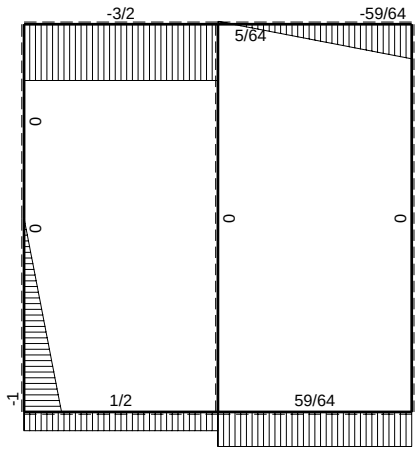
A = 187.6 mm^2
J_u = 125253. mm^4
J_v = 19767. mm^4
J_t = 159.3 mm^4
x_o = -18.73 mm
x_g = 14.29 mm
N = 1453. N
T_y = -3320. N
M_x = -913000. Nmm
x_m = 6. mm
y_m = 58. mm
u_m = -8.29 mm
v_m = 29. mm
$\sigma_m = N/A - Mv/J_u$ = 219.1 N/mm^2
x_c = 6. mm
y_c = 58. mm
u_c = -8.29 mm
v_c = 29. mm
$\sigma_c = N/A - Mv/J_u$ = 219.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 725.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 23.06 N/mm^2
$\tau_o = Tx_o t/J_t$ = 702.7 N/mm^2
t_c = 2988. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1276. N/mm^2

CI08.xxxx.082

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

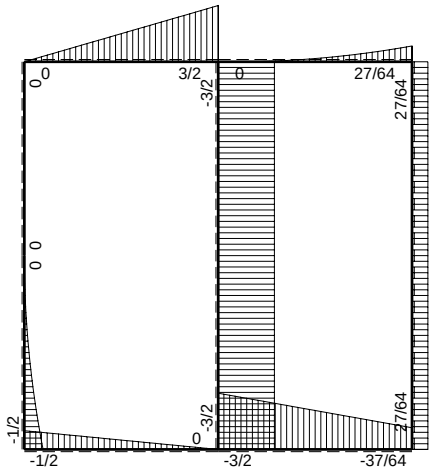

D
C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
C
5/64F
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
3/2
91/64
-59/64
1
1


-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F

F


0
0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-1/2
-3/2
-37/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	-1/2qx^2	0	0	0	0	0+0	0
AE b	0	1/2Fb-Fx+1/2qx^2	0	0	0	0		
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(7/12+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						9/8Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

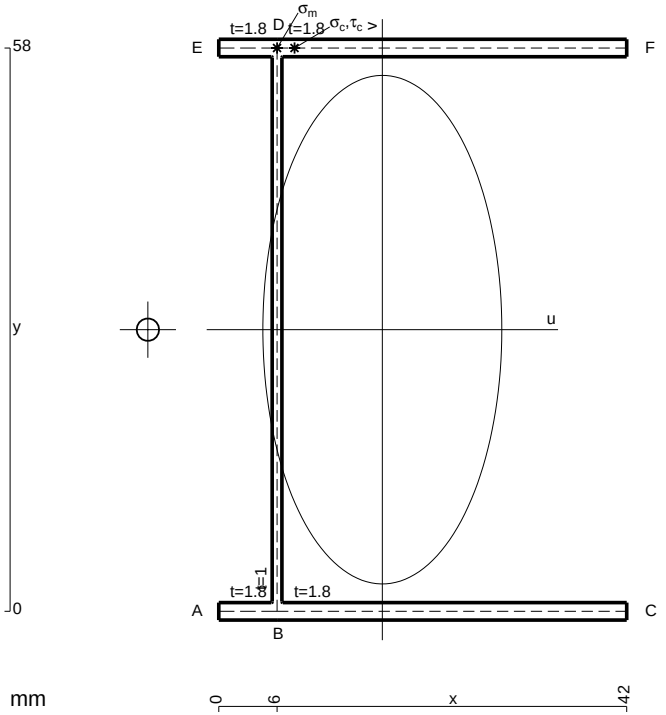
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L_{GC}^{Xo} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L_{CG}^{Xo} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

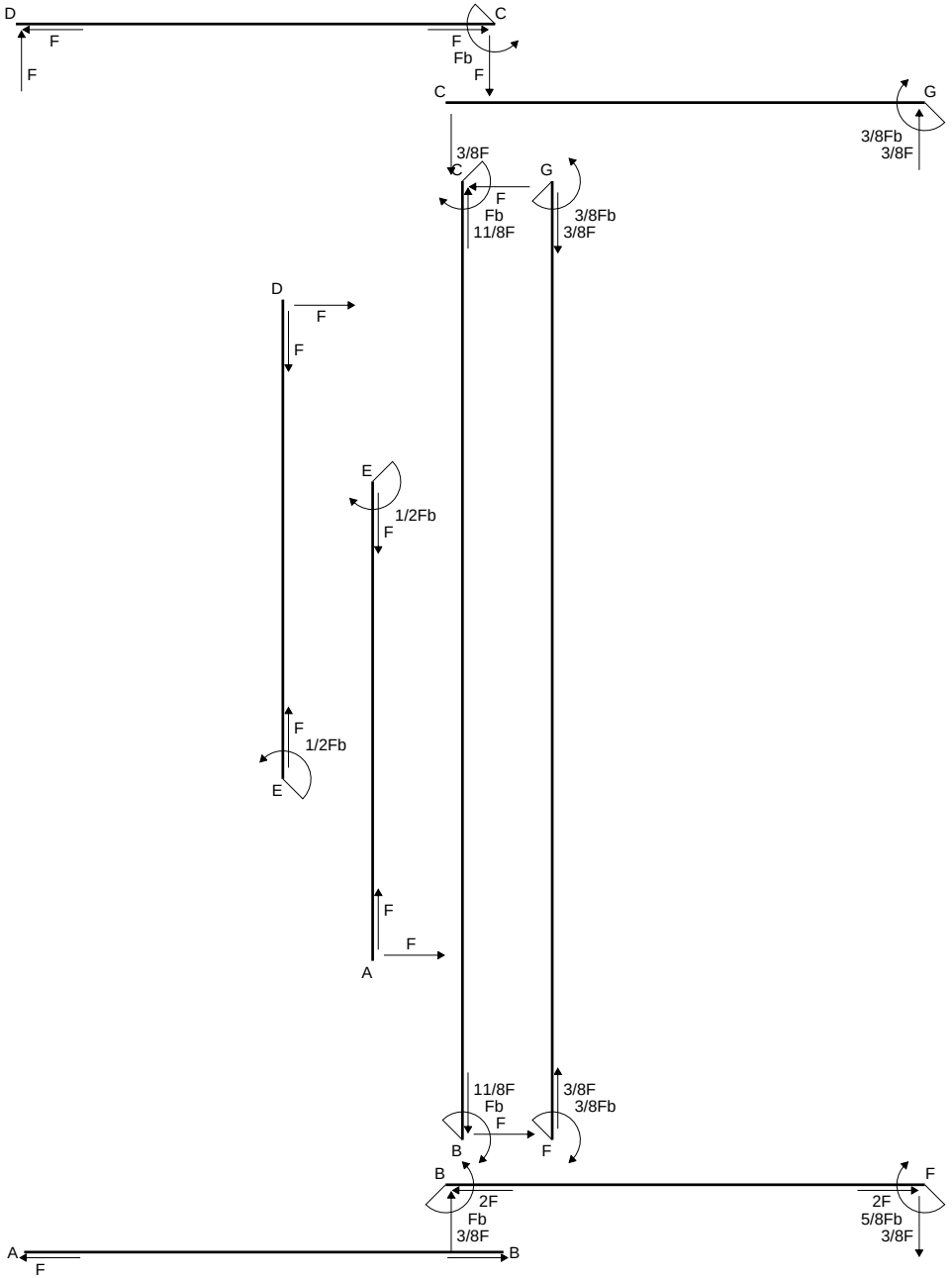
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

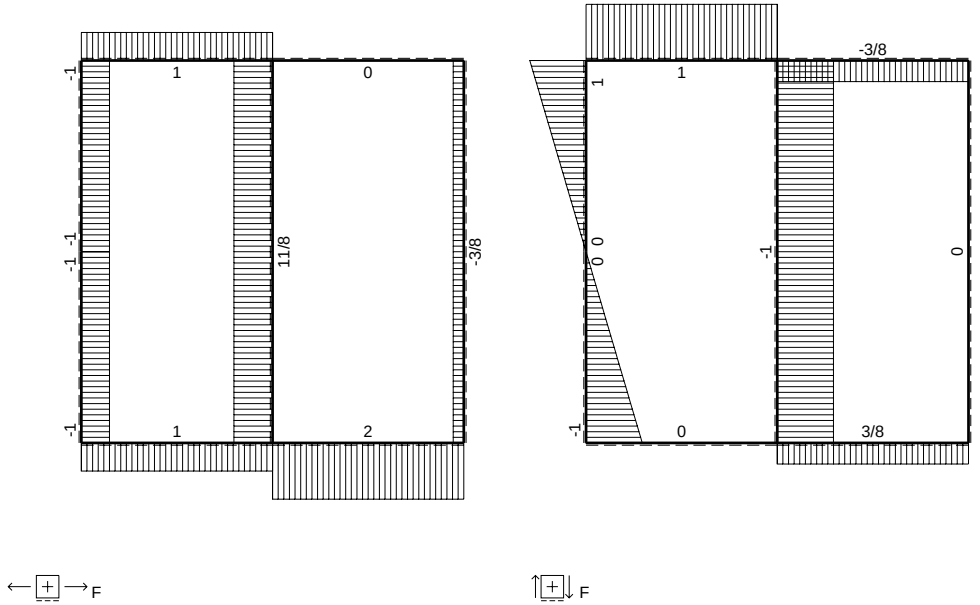


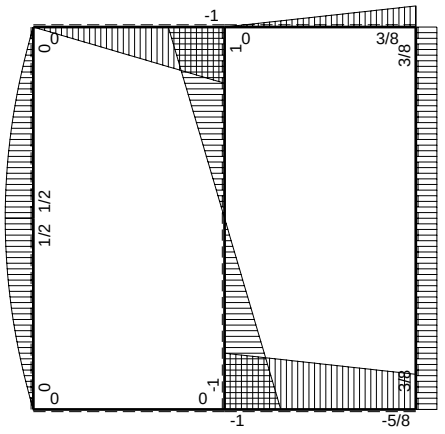
A = 209.2 mm^2
J_u = 143419. mm^4
J_v = 31658. mm^4
J_t = 182.6 mm^4
x_o = -24.14 mm
x_g = 16.84 mm
T_y = -1920. N
M_x = 1132800. Nmm
x_m = 6. mm
y_m = 58. mm
u_m = -10.84 mm
v_m = 29. mm
$\sigma_m = -Mv/J_u$ = -229.1 N/mm^2
x_c = 6. mm
y_c = 58. mm
u_c = -10.84 mm
v_c = 29. mm
$\sigma_c = -Mv/J_u$ = -229.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 470.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.98 N/mm^2
$\tau_o = Tx_o t/J_t$ = 456.8 N/mm^2
t_c = 2304. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 847. N/mm^2

CI08.xxxx.083

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
11/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
11/8F
Fb
F
3/8F
3/8Fb
F
B
B
2F
Fb
3/8F
F
2F
5/8Fb
3/8F
A
F
B


-1
1
0
-1
-1
11/8
-3/8
-1
1
2
F
-3/8
1
1
0 0
-1
0
-1
0
3/8
F


-1
0
0
1
0
3/8
3/8
1/2 1/2
0
0
0
-1
-1
3/8
-1
-5/8


Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3XbEJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbEJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3XbEJ
	iperstatica $X=W_{GC}$							-3/8Fb

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

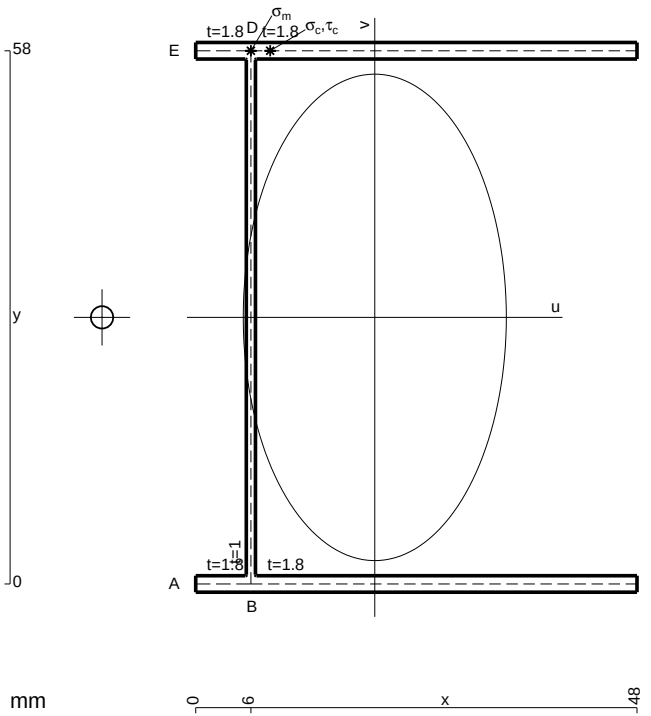
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

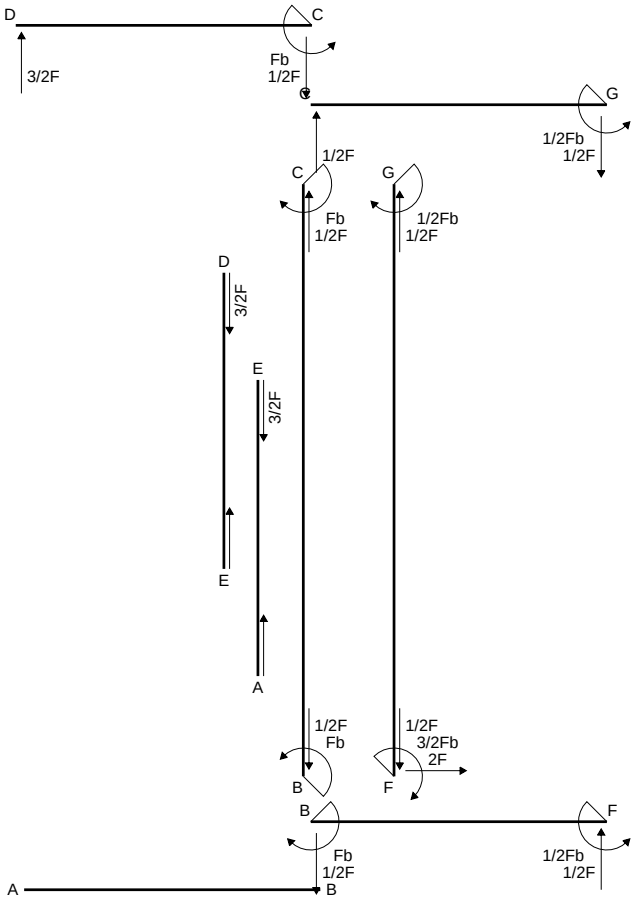
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

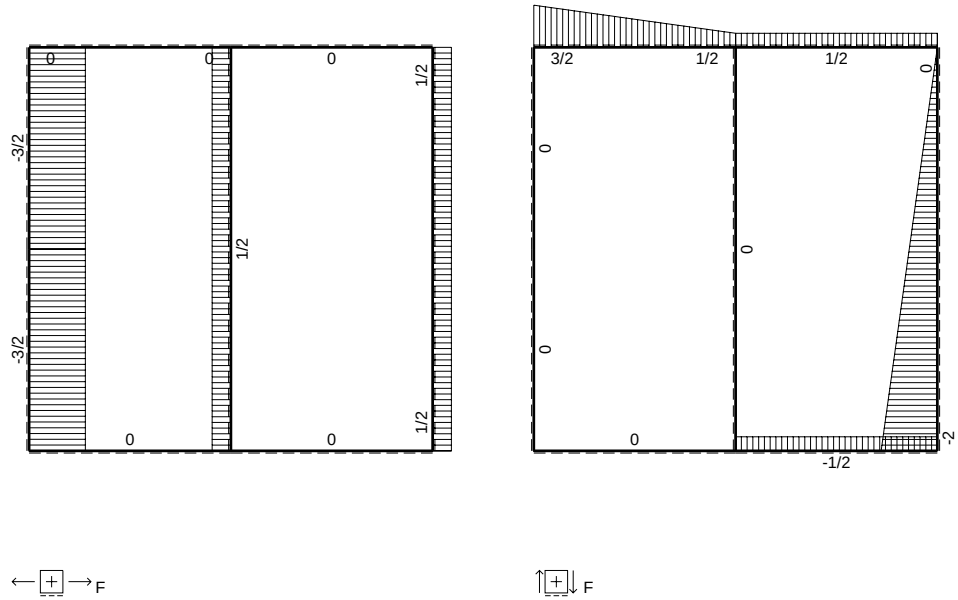


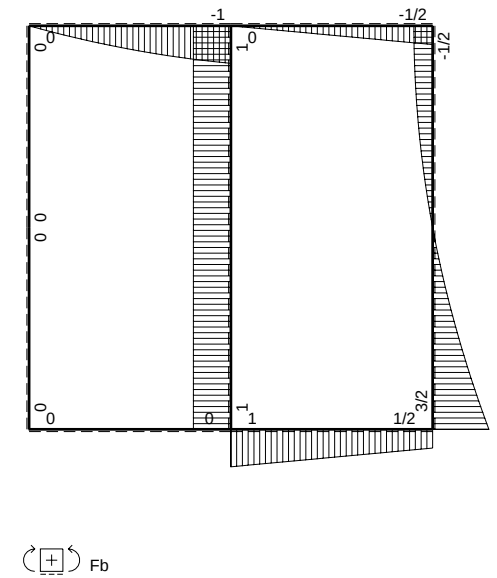
A = 230.8 mm^2
J_u = 161584. mm^4
J_v = 47247. mm^4
J_t = 206. mm^4
x_o = -29.67 mm
x_g = 19.48 mm
N = 2723. N
T_y = -1980. N
M_x = -1267200. Nmm
x_m = 6. mm
y_m = 58. mm
u_m = -13.48 mm
v_m = 29. mm
$\sigma_m = N/A - Mv/J_u$ = 239.2 N/mm^2
x_c = 6. mm
y_c = 58. mm
u_c = -13.48 mm
v_c = 29. mm
$\sigma_c = N/A - Mv/J_u$ = 239.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 528.3 N/mm^2
$\tau_g = TS^*/tJ_u$ = 14.93 N/mm^2
$\tau_o = Tx_o t/J_t$ = 513.3 N/mm^2
t_c = 3564. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 945.7 N/mm^2

CI08.xxxx.084

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3/2F
Fb
1/2F
G
1/2Fb
1/2F
1/2F
C
G
Fb
1/2F
1/2Fb
1/2F
D
3/2F
E
3/2F
E
A
1/2F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B


0
0
0
1/2
-3/2
1/2
-3/2
0
0
1/2
F
3/2
1/2
1/2
0
0
0
0
-2
-1/2
F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
1
1/2
3/2
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+1/2Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	3/2Fx-1/2qx^2	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							-4/3Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

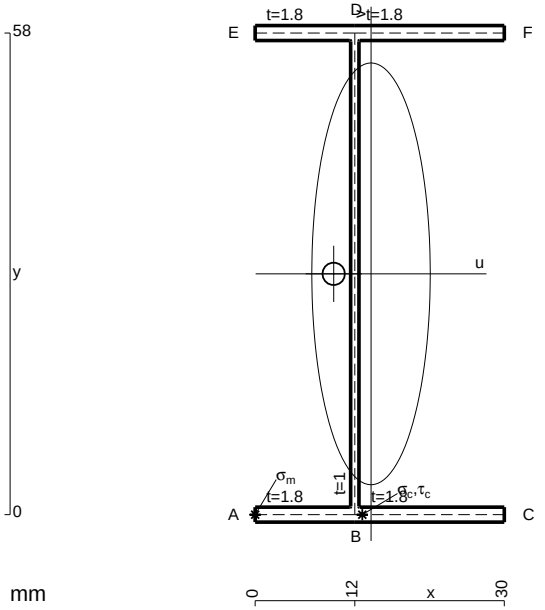
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

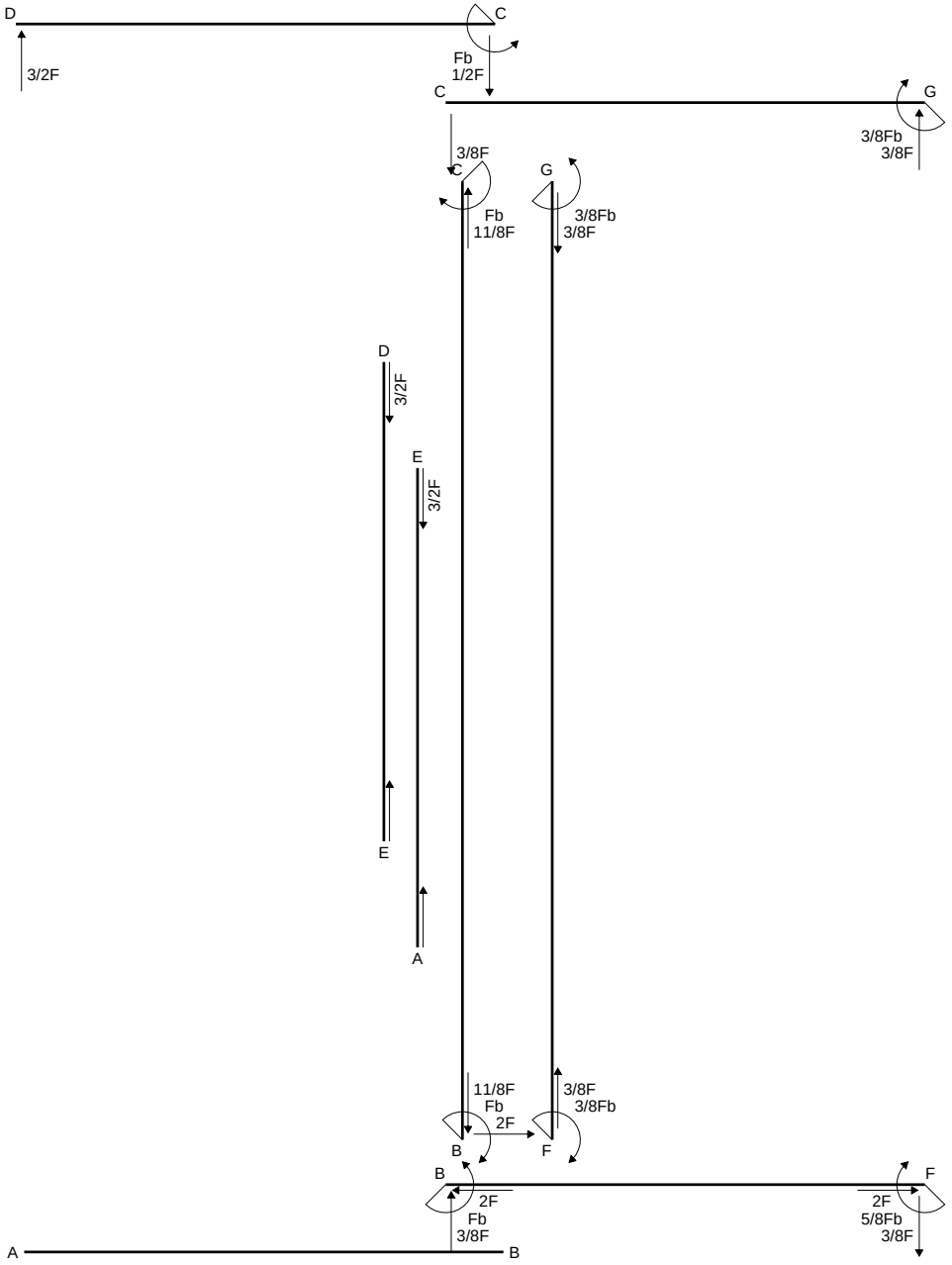
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

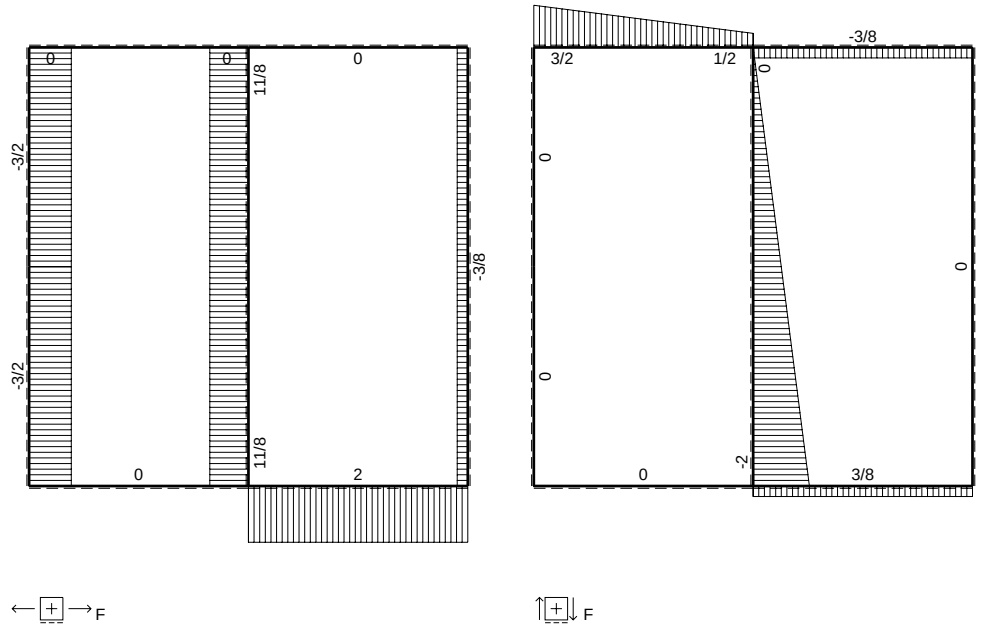


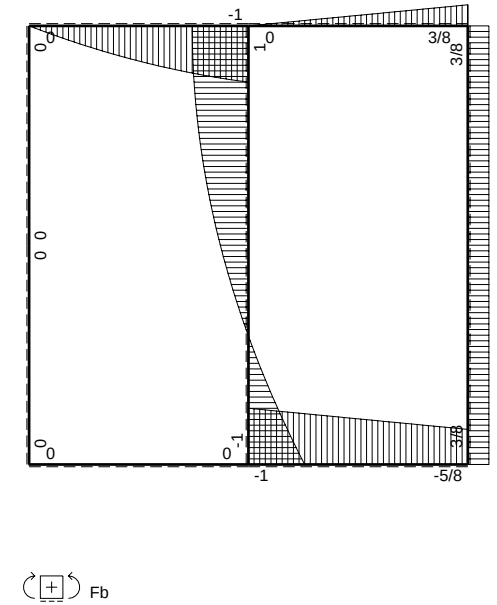
A = 166. mm^2
J_u = 107087. mm^4
J_v = 8440. mm^4
J_t = 136. mm^4
x_o = -4.496 mm
x_g = 13.95 mm
T_y = 1710. N
M_x = -737438. Nmm
u_m = -13.95 mm
v_m = -29. mm
$\sigma_m = -Mv/J_u$ = -199.7 N/mm^2
x_c = 12. mm
u_c = -1.952 mm
v_c = -29. mm
$\sigma_c = -Mv/J_u$ = -199.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 110.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 8.335 N/mm^2
$\tau_o = Tx_o t/J_t$ = 101.8 N/mm^2
t_c = 3078. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 276.1 N/mm^2

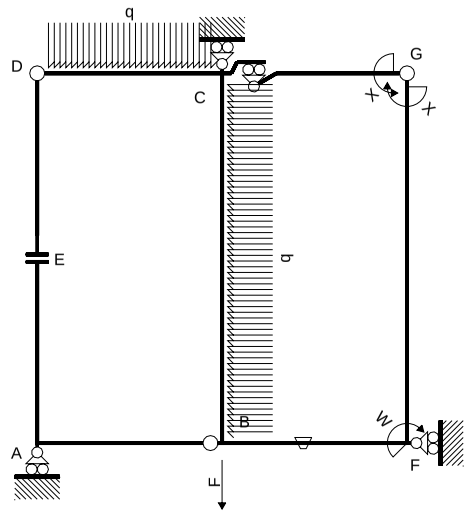
CI08.xxxx.085

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

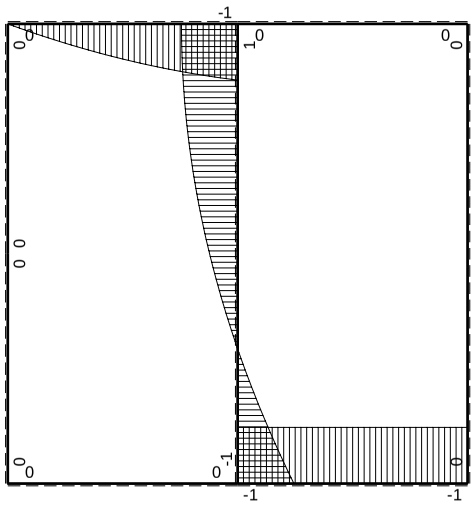

D
C
3/2F
Fb
1/2F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
11/8F
3/8Fb
3/8F
D
3/2F
E
3/2F
E
A
11/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B


0
0
0
11/8
-3/2
-3/8
-3/2
11/8
0
2
F
3/2
1/2
-3/8
0
0
0
0
-2
3/8
F

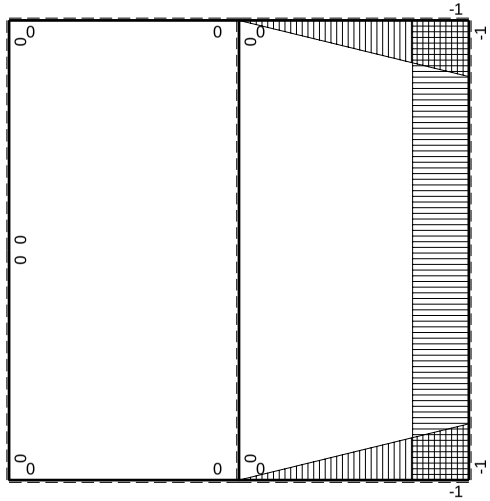

-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+1/2Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	3/2Fx-1/2qx^2	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx^2	0	0	0	0		
						totali	Fb^2/EJ	8/3Xb/EJ
						iperstatica $X=W_{GC}$	-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 248282 CI08.xxxx.086

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

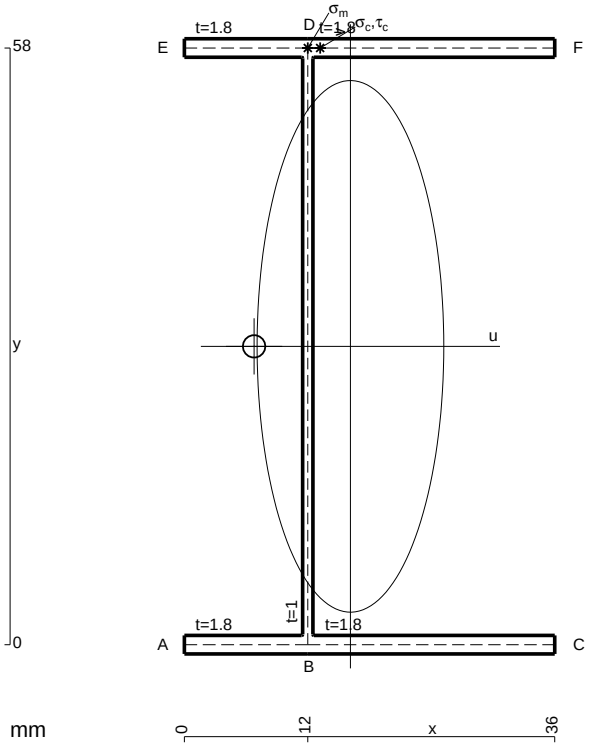
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

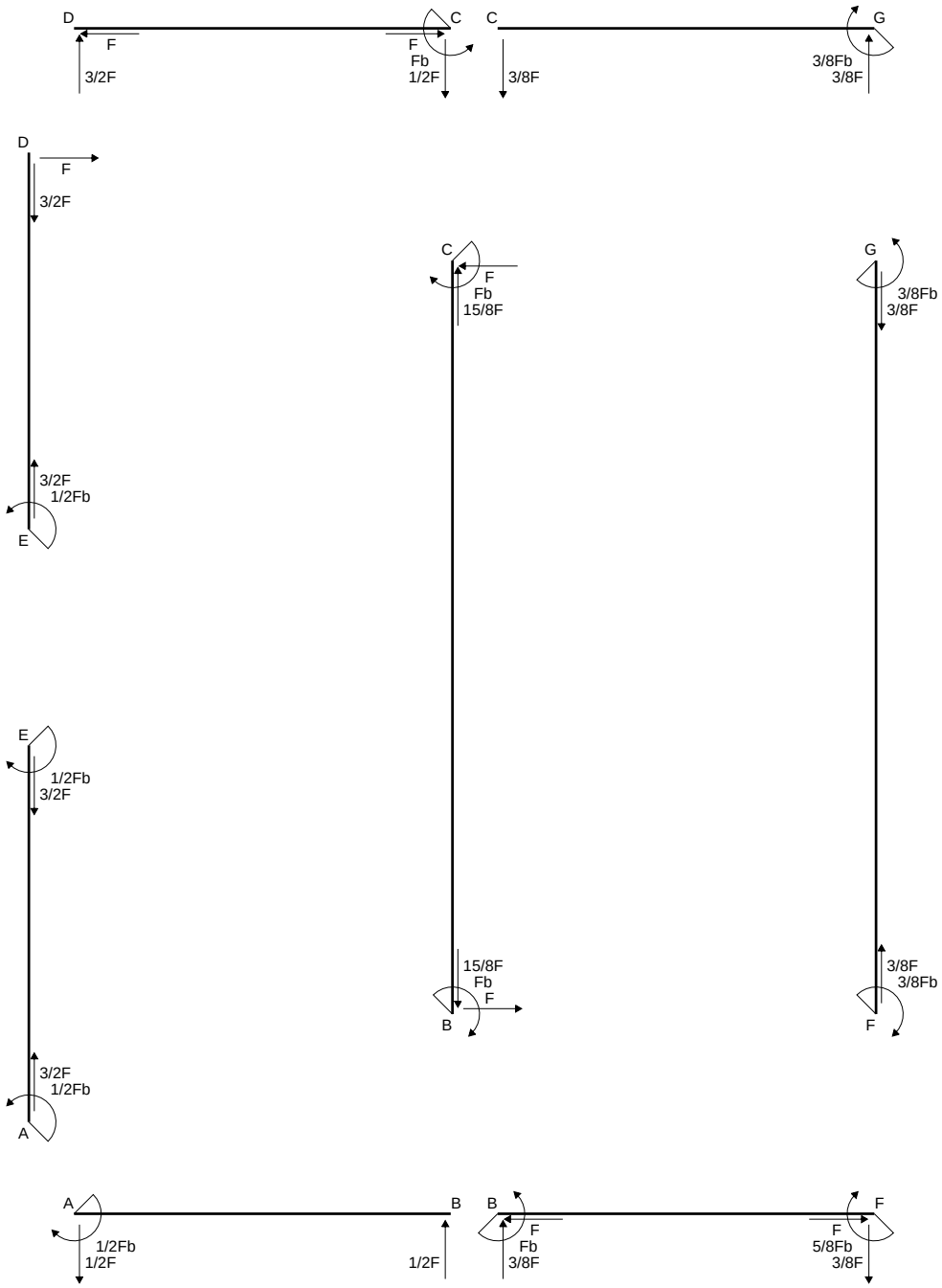
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

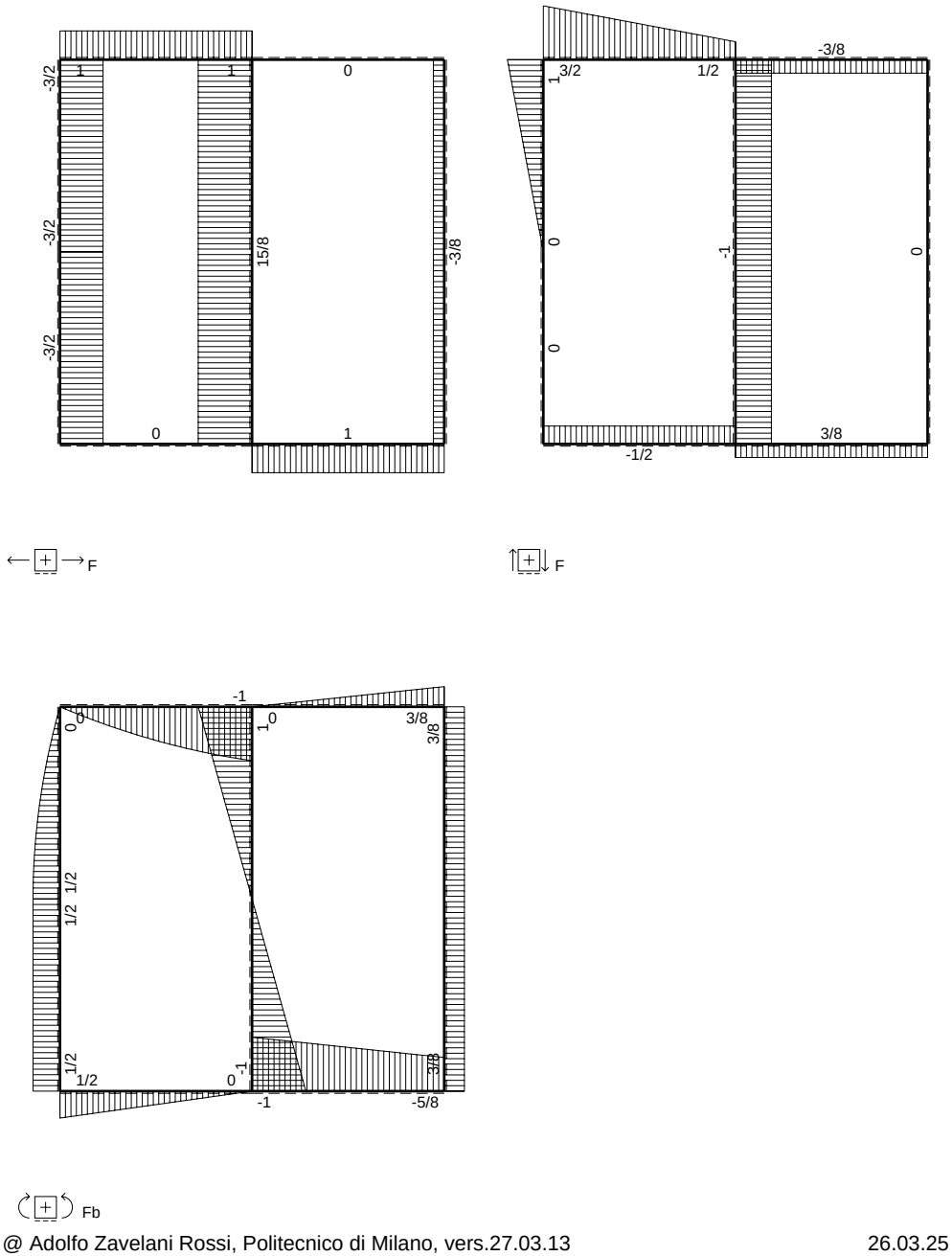


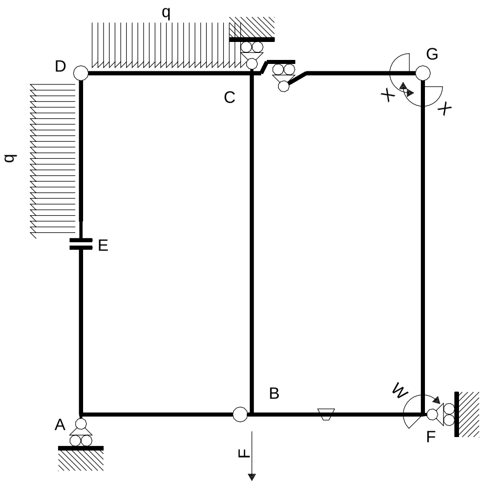
A = 187.6 mm^2
J_u = 125253. mm^4
J_v = 15439. mm^4
J_t = 159.3 mm^4
x_o = -9.366 mm
x_g = 16.15 mm
N = 1636. N
T_y = -2380. N
M_x = -868700. Nmm
x_m = 12. mm
y_m = 58. mm
u_m = -4.145 mm
v_m = 29. mm
$\sigma_m = N/A - Mv/J_u$ = 209.9 N/mm^2
x_c = 12. mm
y_c = 58. mm
u_c = -4.145 mm
v_c = 29. mm
$\sigma_c = N/A - Mv/J_u$ = 209.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 265.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.23 N/mm^2
$\tau_o = Tx_o t/J_t$ = 251.9 N/mm^2
t_c = 2142. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 504.9 N/mm^2

CI08.xxxx.086

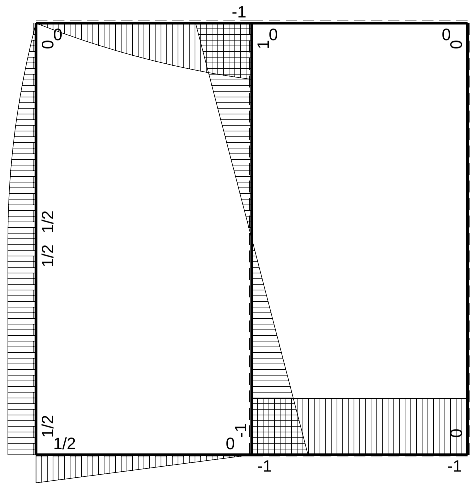
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3/2F
F
Fb
1/2F
3/8F
3/8Fb
3/8F
D
F
3/2F
C
F
Fb
15/8F
G
3/8Fb
3/8F
3/2F
1/2Fb
E
E
1/2Fb
3/2F
15/8F
Fb
F
B
3/8F
3/8Fb
F
3/2F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
3/8F
F
5/8Fb
3/8F

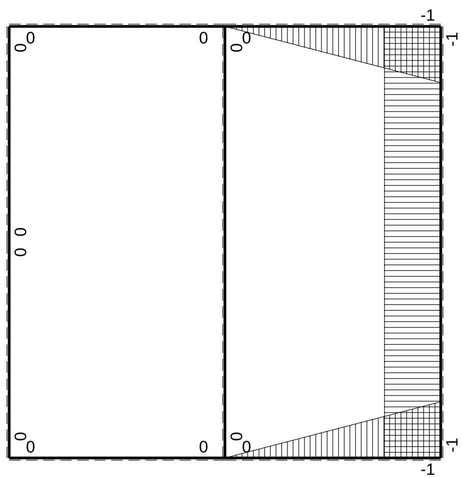

-3/2
-3/2
-3/2
1
1
0
15/8
-3/8
0
1
F
-3/8
1
3/2
1/2
0
0
-1
0
3/8
-1/2
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

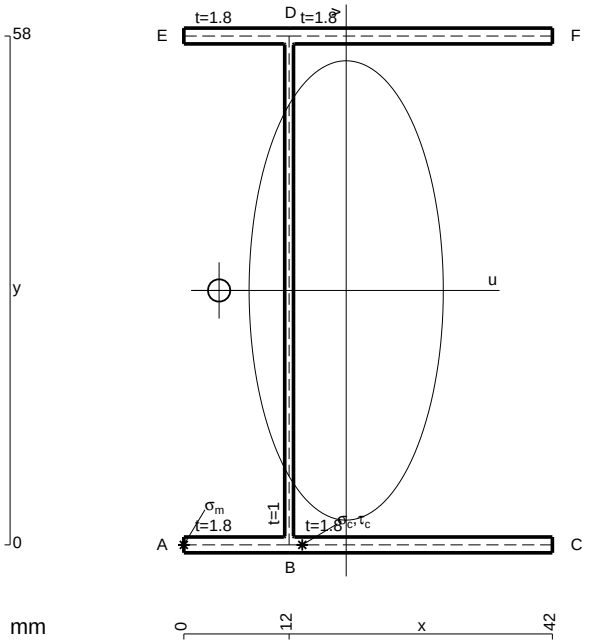
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

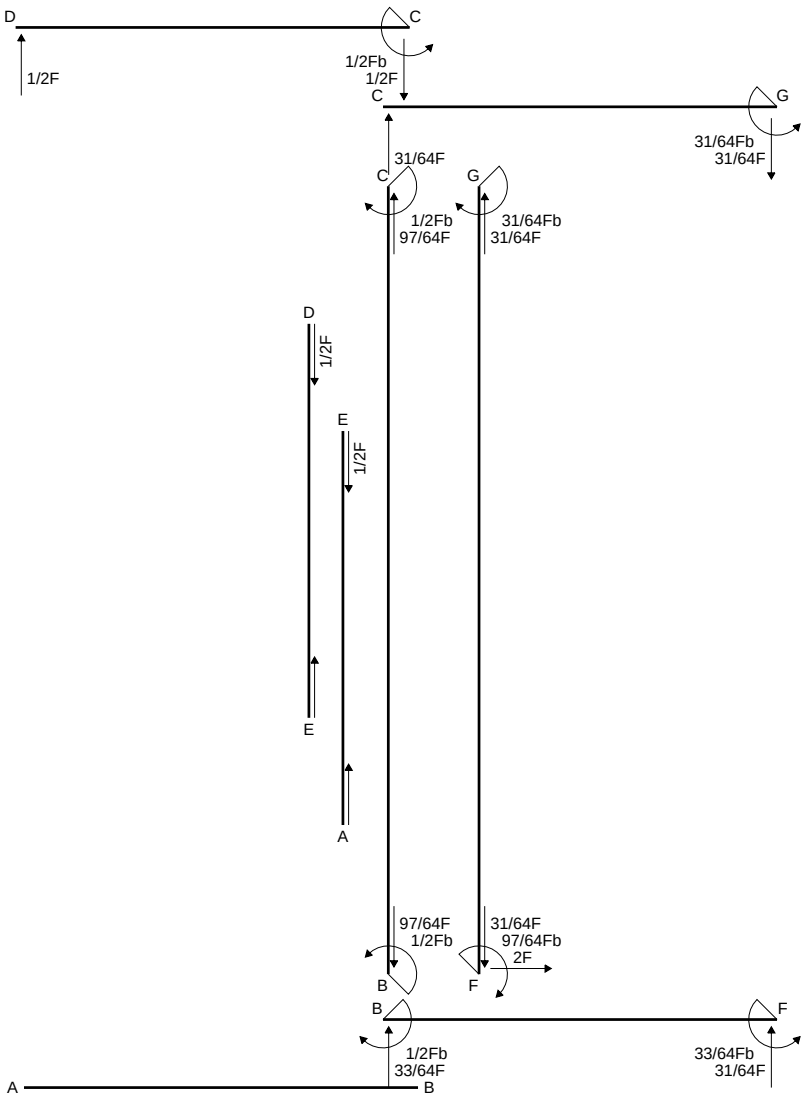
$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

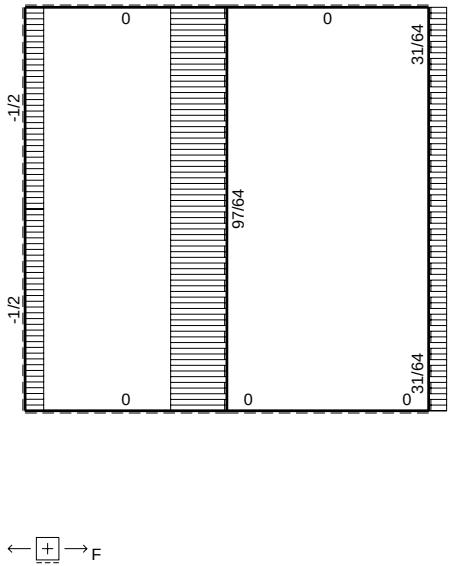


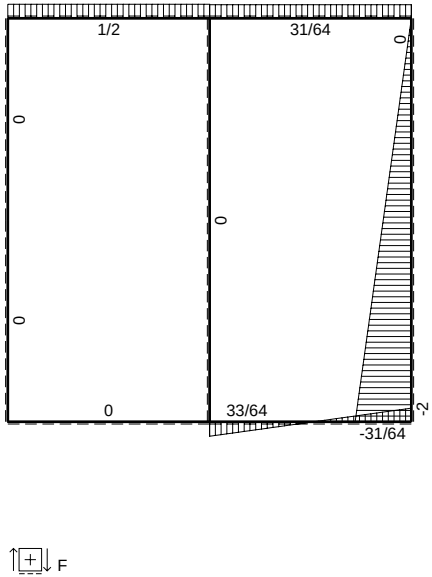
A = 209.2 mm^2
J_u = 143419. mm^4
J_v = 25622. mm^4
J_t = 182.6 mm^4
x_o = -14.48 mm
x_g = 18.5 mm
N = 2456. N
T_y = -1310. N
M_x = 1021800. Nmm
u_m = -18.5 mm
v_m = -29. mm
$\sigma_m = N/A - Mv/J_u$ = 218.4 N/mm^2
x_c = 12. mm
u_c = -6.505 mm
v_c = -29. mm
$\sigma_c = N/A - Mv/J_u$ = 218.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 195. N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.947 N/mm^2
$\tau_o = Tx_o t/J_t$ = 187. N/mm^2
t_c = 2358. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 402.1 N/mm^2

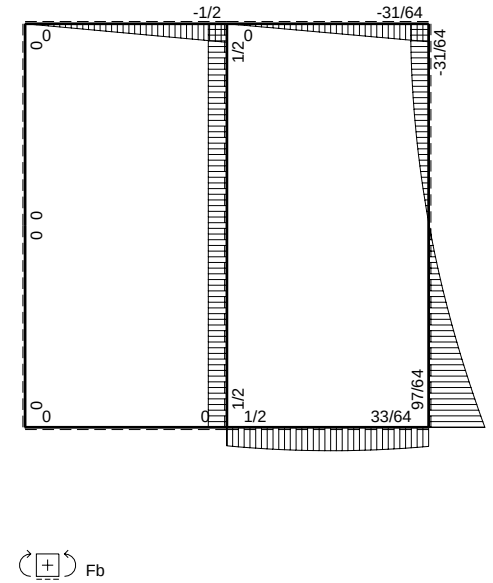
CI08.xxxx.087

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64
F


1/2
31/64
0
0
0
0
0
33/64
-2
-31/64
F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
0
1/2
33/64
97/64
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	FX/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-FX/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb²/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 241349 CI08.xxxx.088

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

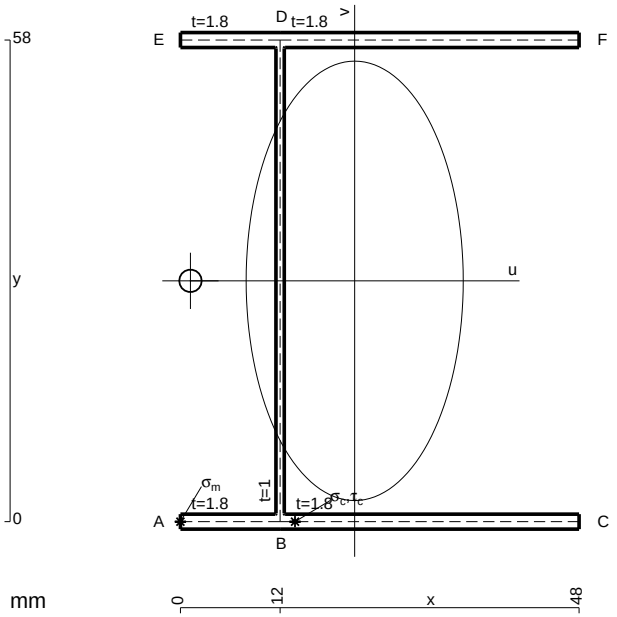
$= (-\ b +1/2\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b)\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

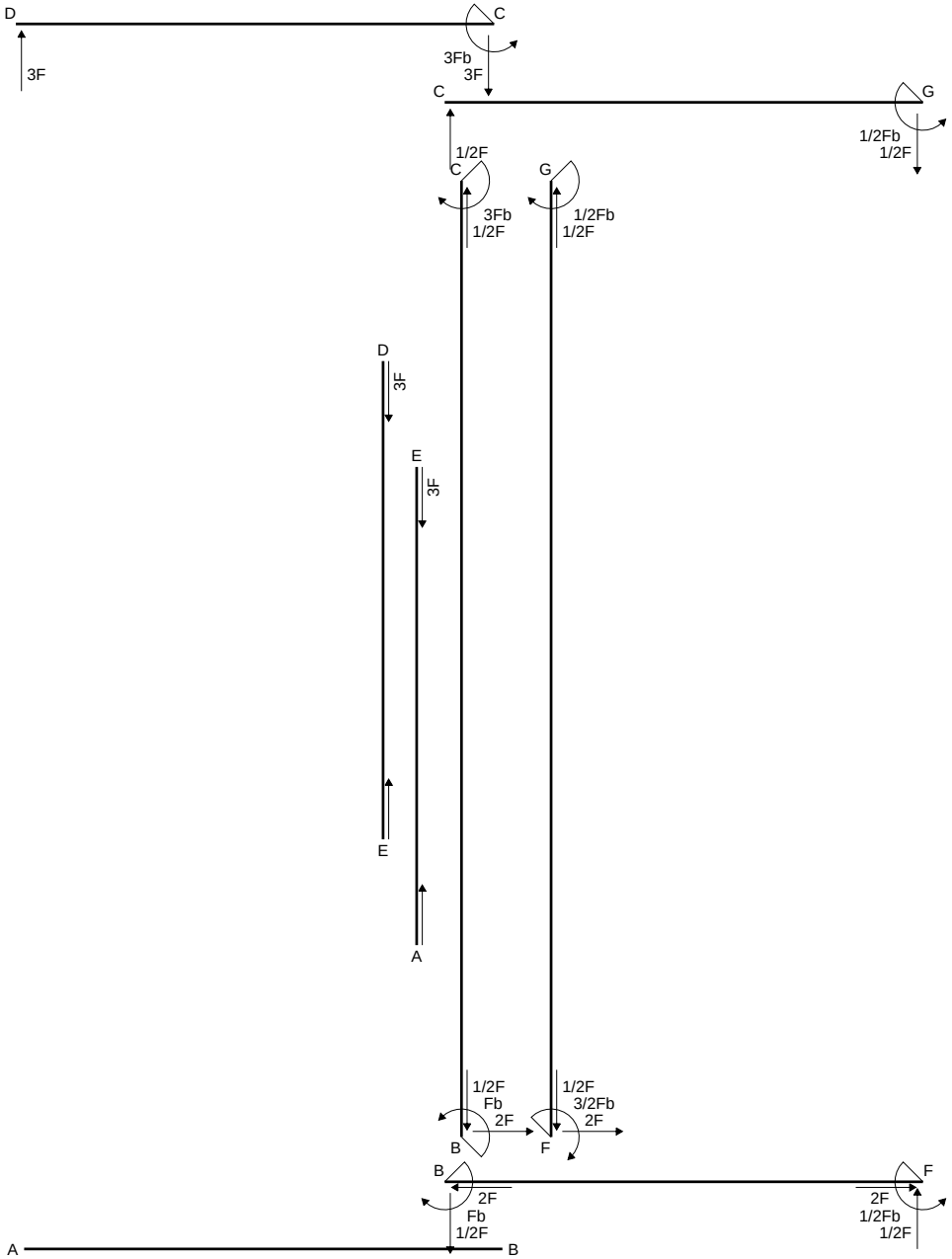
$= (-4/3\ b)\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

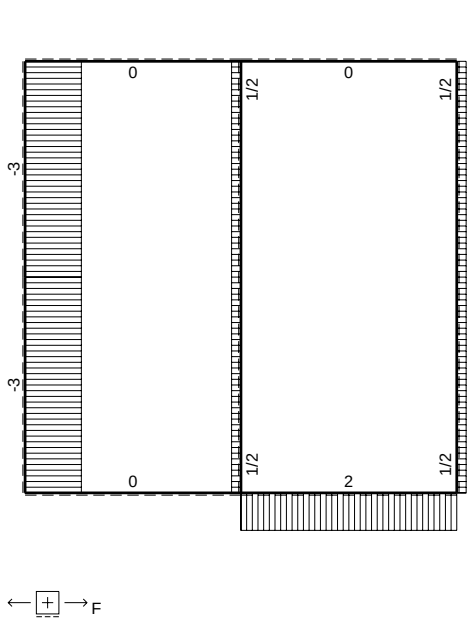


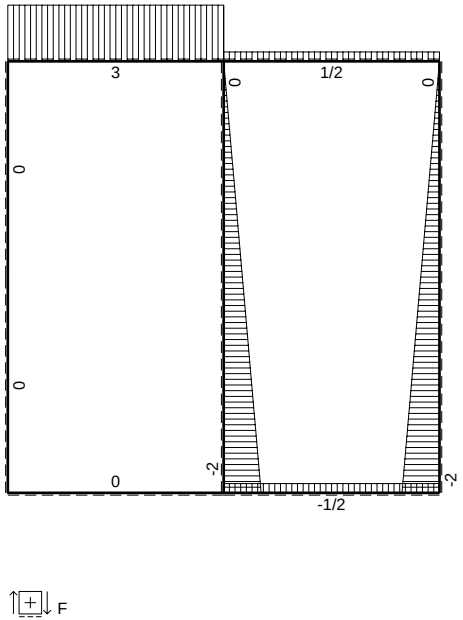
A = 230.8 mm^2
J_u = 161584. mm^4
J_v = 39431. mm^4
J_t = 206. mm^4
x_o = -19.78 mm
x_g = 20.98 mm
T_y = 1560. N
M_x = -1279200. Nmm
u_m = -20.98 mm
v_m = -29. mm
$\sigma_m = -Mv/J_u$ = -229.6 N/mm^2
x_c = 12. mm
u_c = -8.984 mm
v_c = -29. mm
$\sigma_c = -Mv/J_u$ = -229.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 279.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.08 N/mm^2
$\tau_o = Tx_o t/J_t$ = 269.6 N/mm^2
t_c = 5616. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 536.1 N/mm^2

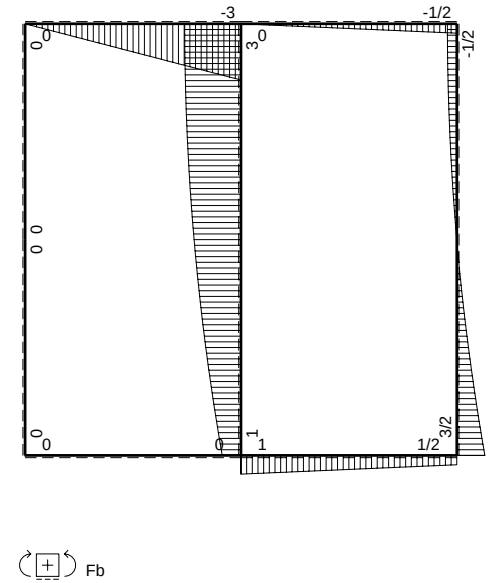
CI08.xxxx.088

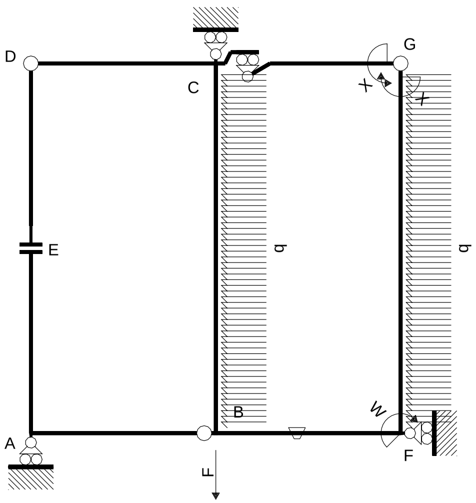
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

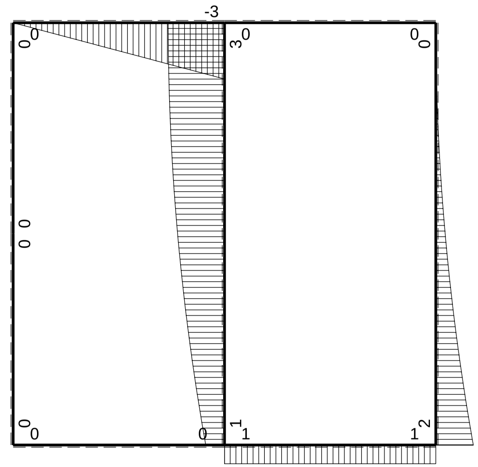

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
0
1/2
1/2
-3
-3
0
2
1/2
1/2
F

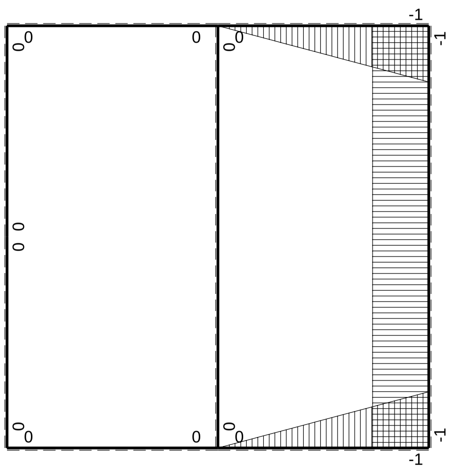

3
0
1/2
0
0
0
0
-2
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

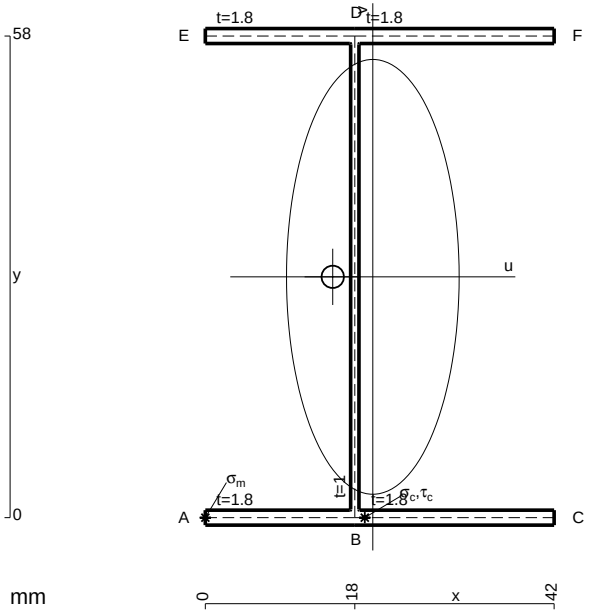
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

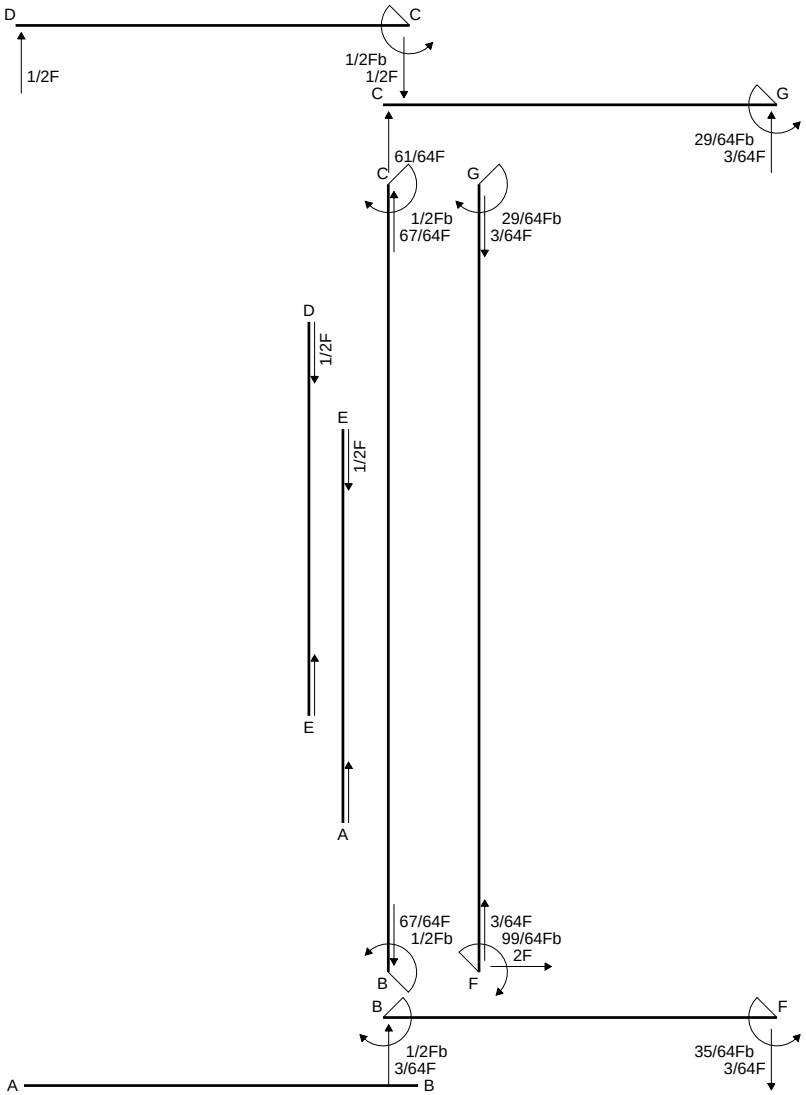
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

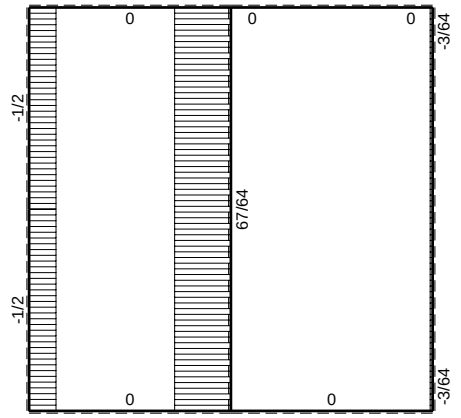


A = 209.2 mm^2
J_u = 143419. mm^4
J_v = 22604. mm^4
J_t = 182.6 mm^4
x_o = -4.828 mm
x_g = 20.17 mm
T_y = 1350. N
M_x = -1174500. Nmm
u_m = -20.17 mm
v_m = -29. mm
$\sigma_m = -Mv/J_u$ = -237.5 N/mm^2
x_c = 18. mm
u_c = -2.168 mm
v_c = -29. mm
$\sigma_c = -Mv/J_u$ = -237.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 70.79 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.551 N/mm^2
$\tau_o = Tx_o t/J_t$ = 64.24 N/mm^2
t_c = 810. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 267.3 N/mm^2

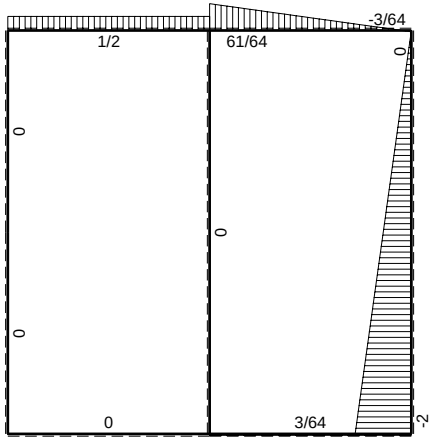
CI08.xxxx.089

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

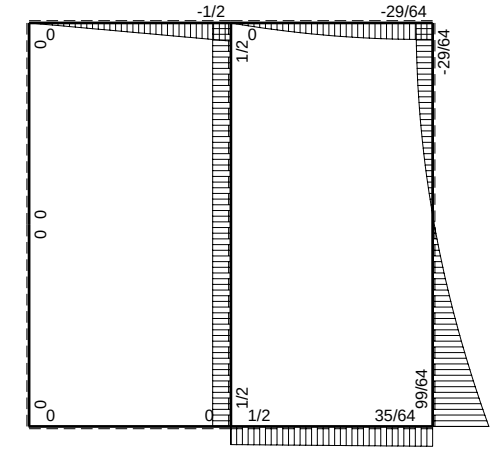

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64

F


-3/64
1/2
61/64
0
0
0
0
0
3/64
-2

F


-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
0
1/2
1/2
35/64
99/64

Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-29/24Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						29/64Fb	

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

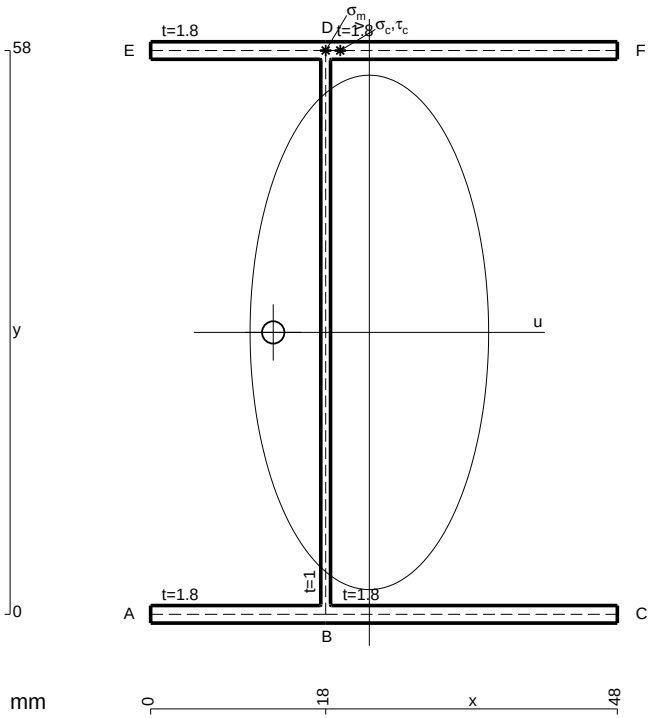
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

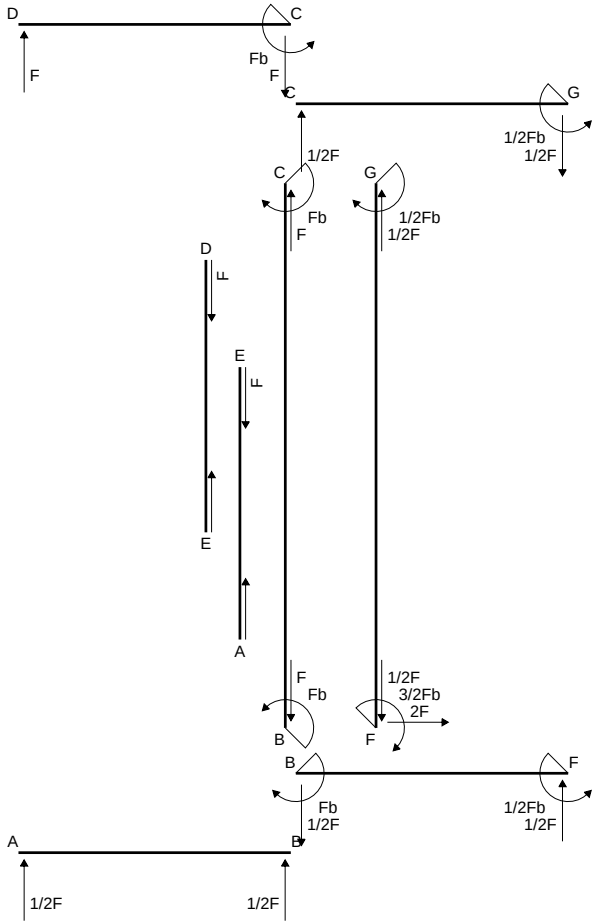
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

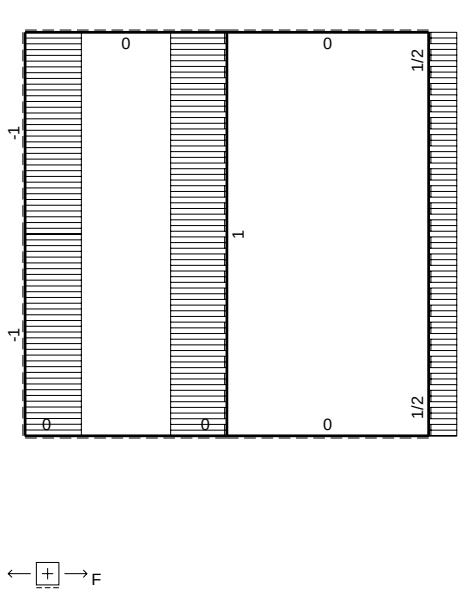


A = 230.8 mm^2
J_u = 161584. mm^4
J_v = 34741. mm^4
J_t = 206. mm^4
x_o = -9.888 mm
x_g = 22.49 mm
T_y = 2420. N
M_x = -1113200. Nmm
x_m = 18. mm
y_m = 58. mm
u_m = -4.492 mm
v_m = 29. mm
$\sigma_m = -Mv/J_u$ = 199.8 N/mm^2
x_c = 18. mm
y_c = 58. mm
u_c = -4.492 mm
v_c = 29. mm
$\sigma_c = -Mv/J_u$ = 199.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 222.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.03 N/mm^2
$\tau_o = Tx_o t/J_t$ = 209.1 N/mm^2
t_c = 8712. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 433.6 N/mm^2

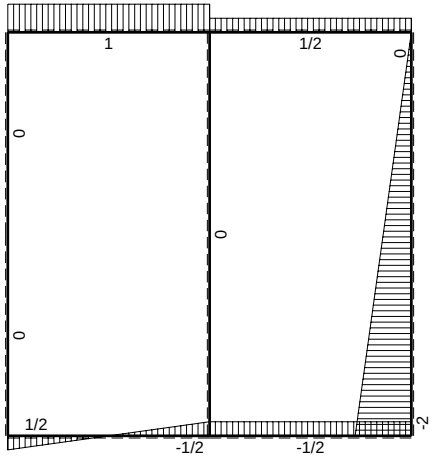
CI08.xxxx.090

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

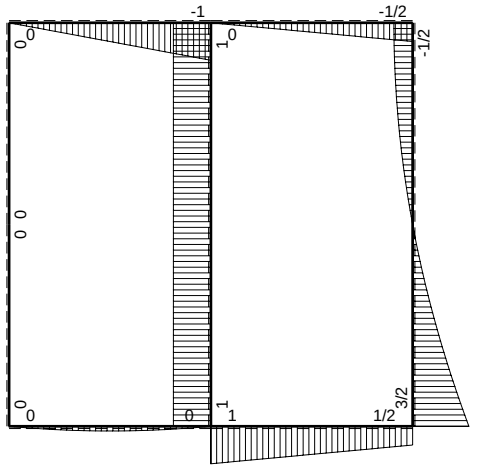

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

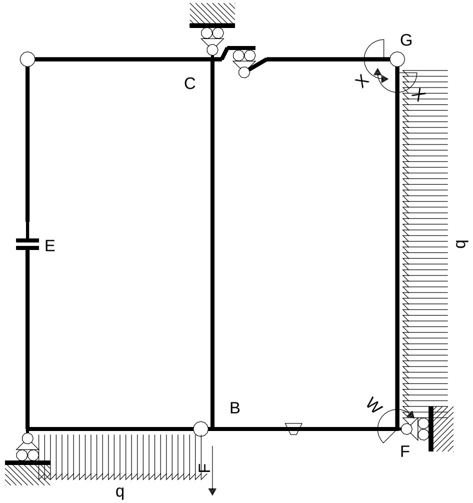
F

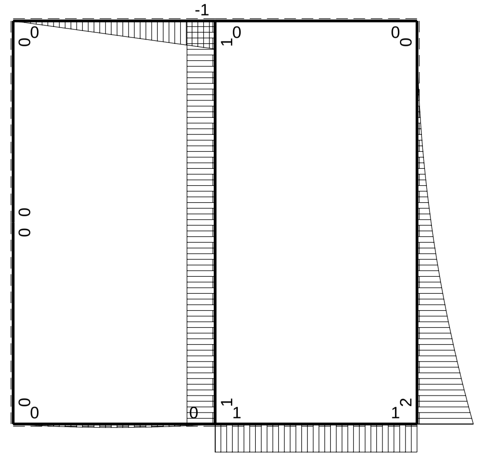

1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2

F

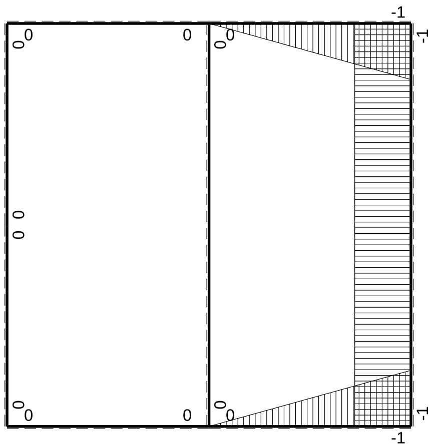

-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

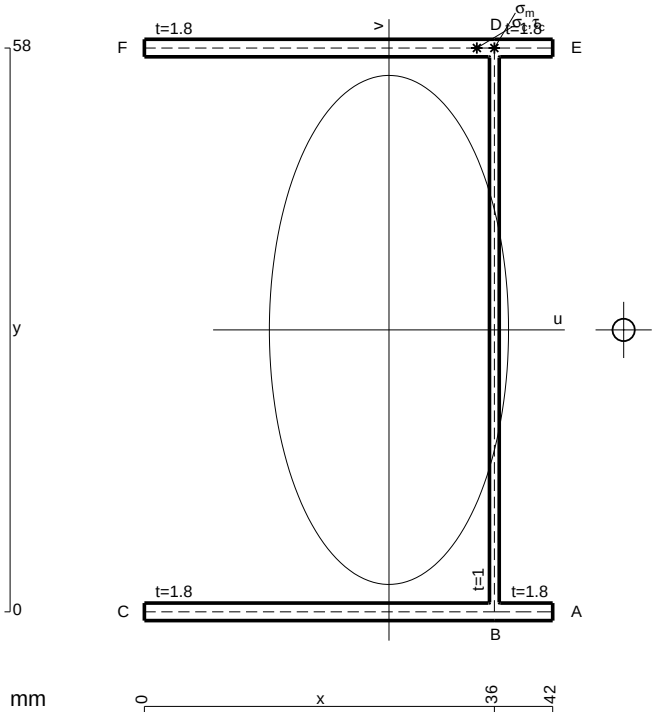
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

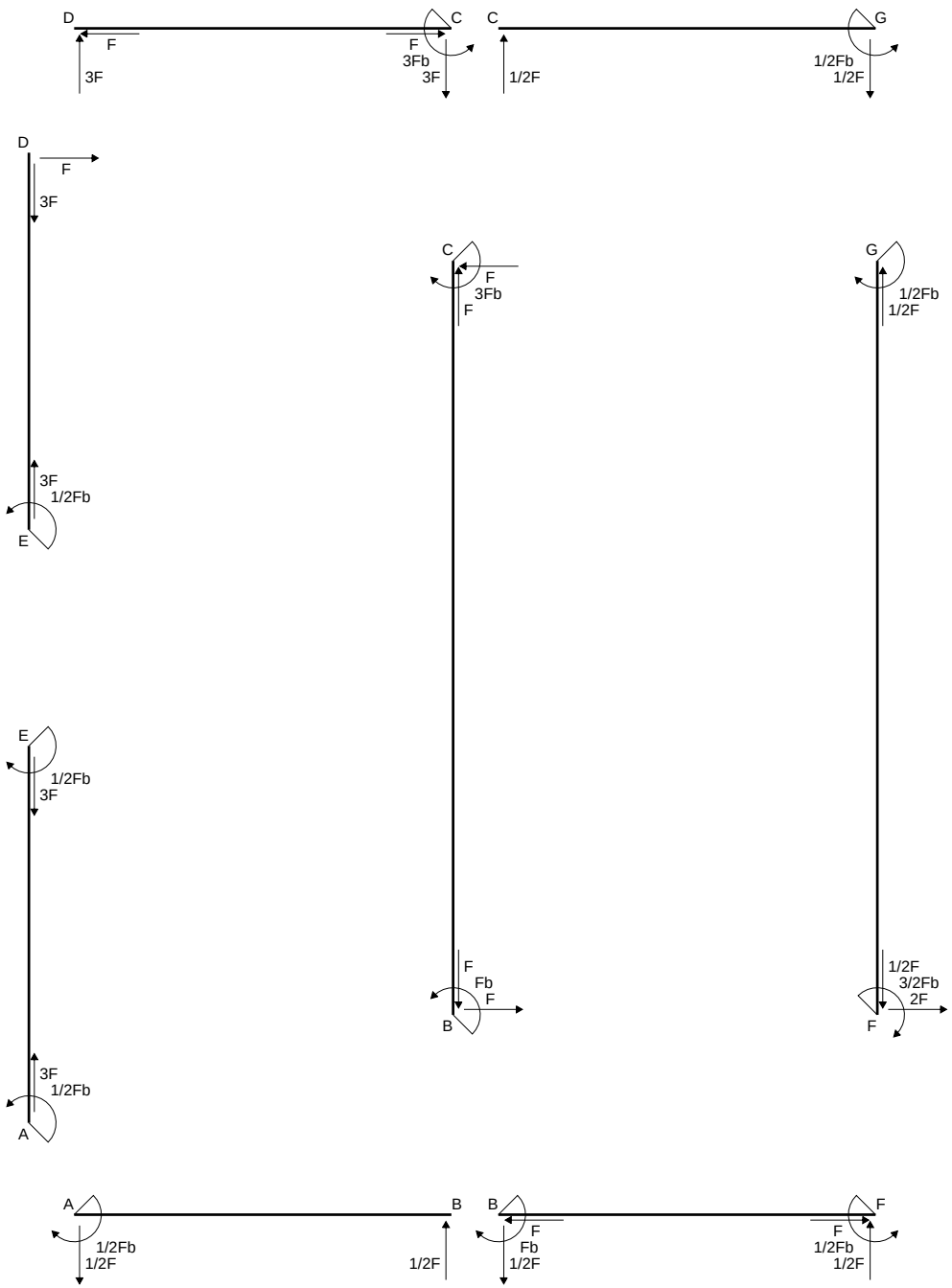
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

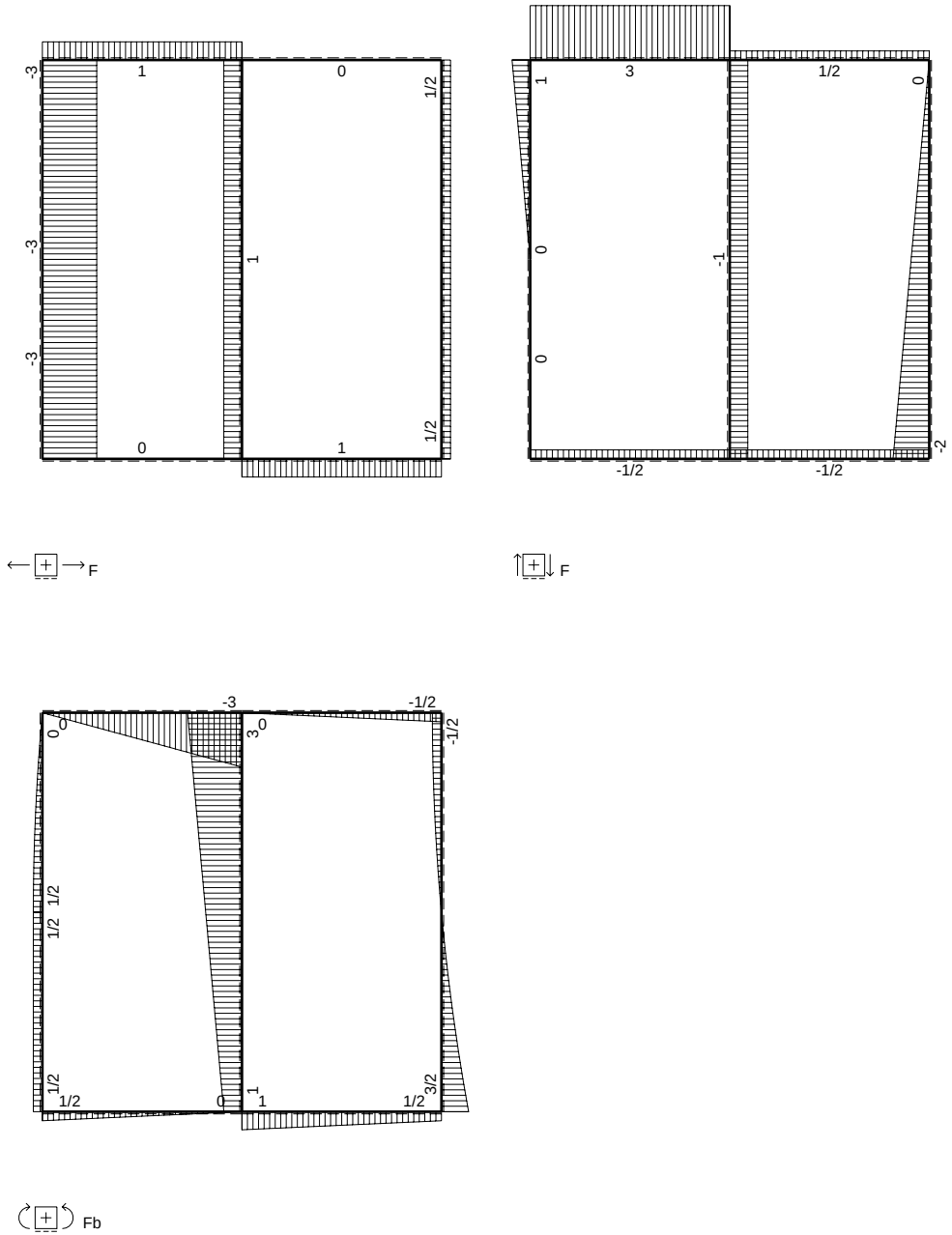


A = 209.2 mm^2
J_u = 143419. mm^4
J_v = 31658. mm^4
J_t = 182.6 mm^4
x_o = 24.14 mm
x_g = 25.16 mm
T_y = 2070. N
M_x = -1035000. Nmm
x_m = 36. mm
y_m = 58. mm
u_m = 10.84 mm
v_m = 29. mm
$\sigma_m = -Mv/J_u$ = 209.3 N/mm^2
x_c = 36. mm
y_c = 58. mm
u_c = 10.84 mm
v_c = 29. mm
$\sigma_c = -Mv/J_u$ = 209.3 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 507.6 N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.07 N/mm^2
$\tau_o = Tx_o t/J_t$ = 492.5 N/mm^2
t_c = 3726. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 903.7 N/mm^2

CI08.xxxx.091

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
F
Fb
F
1/2F
3/2Fb
2F
B
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


-3
1
0
1/2
-3
1
-3
0
1
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^{2}b^2	(-1/2+1/2)Fb^2/EJ	1/3XbEJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2XbEJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3FbEJ	8/3XbEJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

PROCEDIMENTO E RISULTATI 939352 CI08.xxxx.092

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

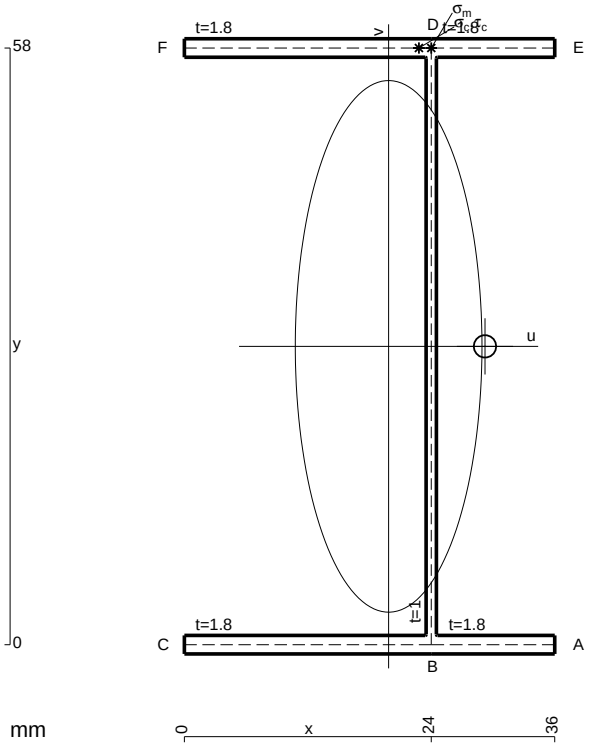
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

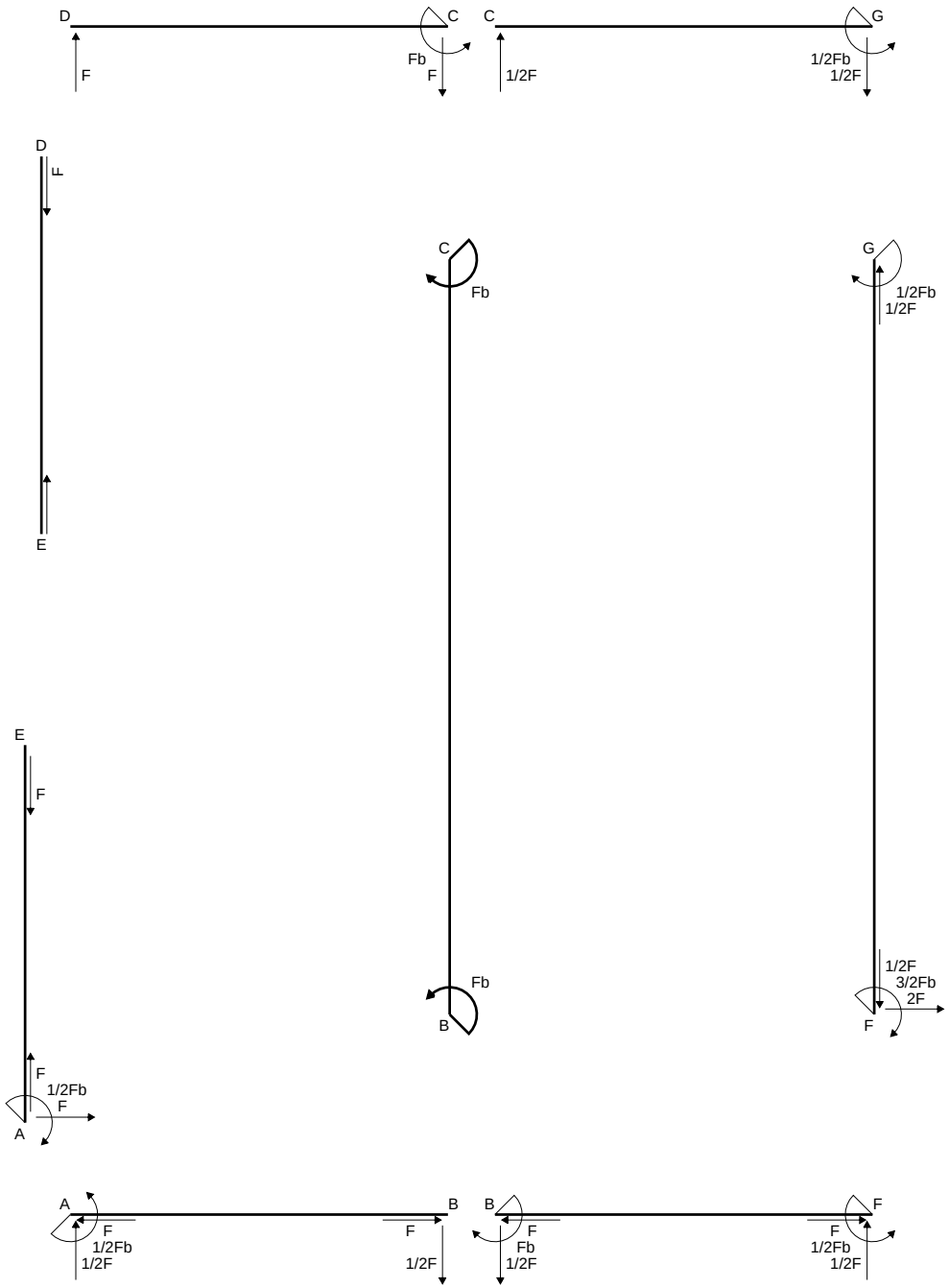
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

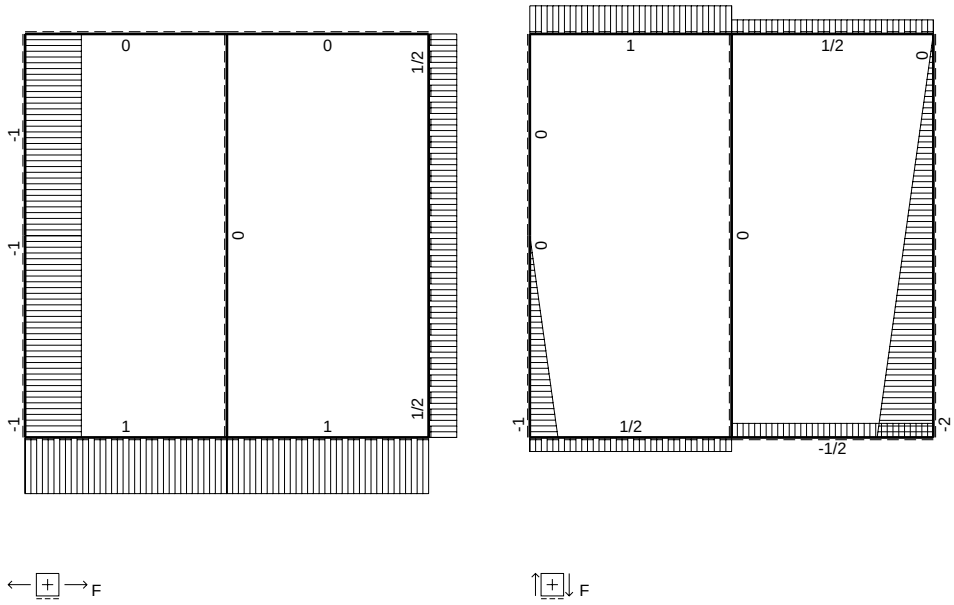


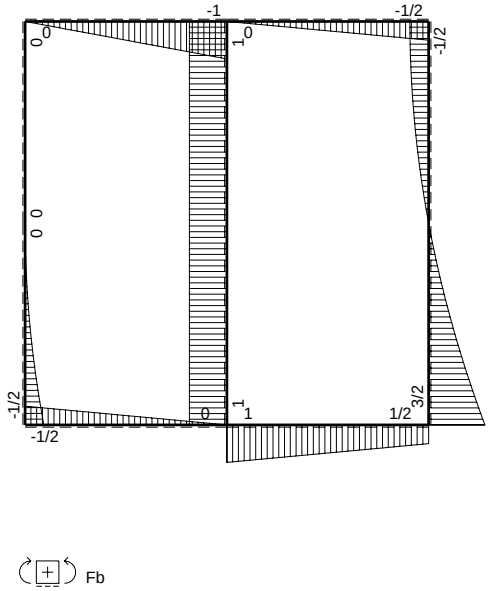
A = 187.6 mm^2
J_u = 125253. mm^4
J_v = 15439. mm^4
J_t = 159.3 mm^4
x_o = 9.366 mm
x_g = 19.85 mm
N = 560. N
T_y = 1680. N
M_x = -924000. Nmm
x_m = 24. mm
y_m = 58. mm
u_m = 4.145 mm
v_m = 29. mm
$\sigma_m = N/A - Mv/J_u$ = 216.9 N/mm^2
x_c = 24. mm
y_c = 58. mm
u_c = 4.145 mm
v_c = 29. mm
$\sigma_c = N/A - Mv/J_u$ = 216.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 187.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.335 N/mm^2
$\tau_o = Tx_o t/J_t$ = 177.8 N/mm^2
t_c = 1008. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 390. N/mm^2

CI08.xxxx.092

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
G
1/2Fb
1/2F
Fb
B
1/2F
3/2Fb
2F
F
E
F
F
1/2Fb
F
A
A
B
B
F
F
F
F
1/2Fb
1/2F
1/2F
Fb
1/2F
1/2Fb
1/2F


0
0
1/2
-1
0
-1
0
-1
1
1
1/2
F
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F


-1
-1/2
0
1
0
-1/2
0
0
-1/2
0
1
1
1/2
3/2
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L_{FB}^{Xo} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

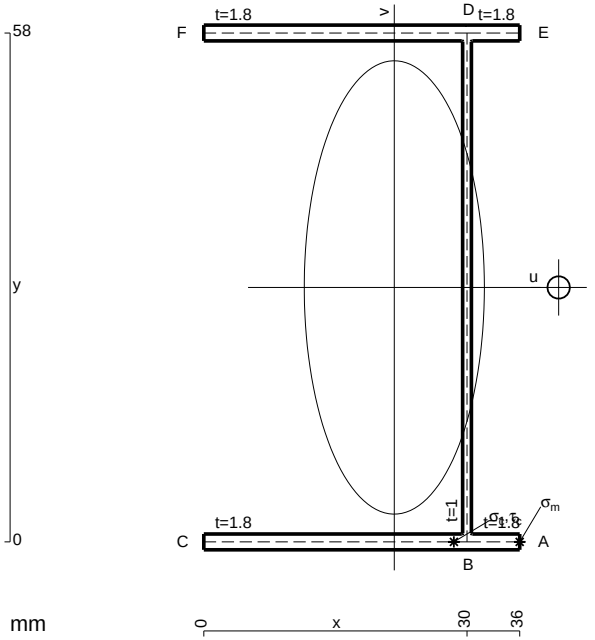
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L_{FG}^{Xo} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L_{GF}^{Xo} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

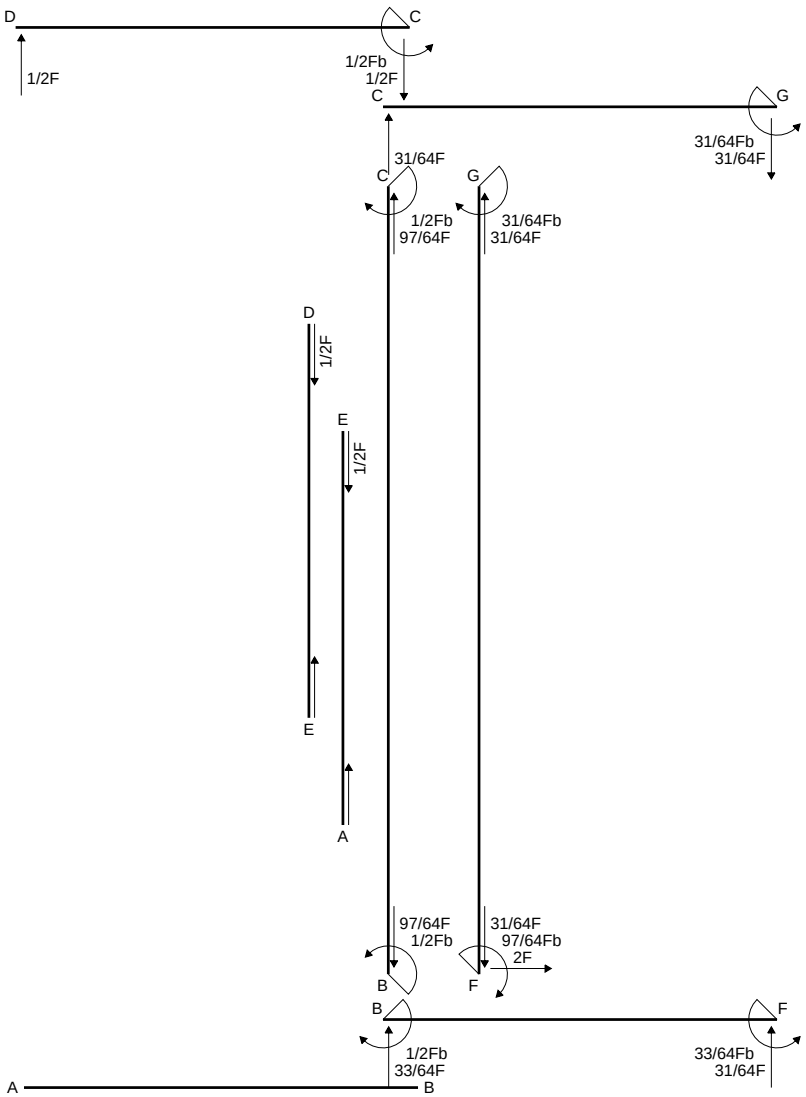
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

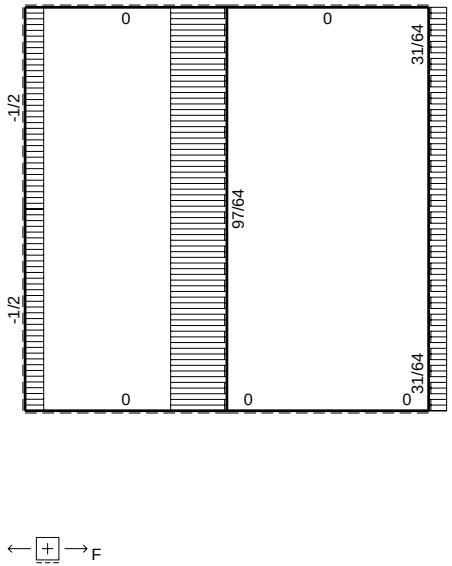


A = 187.6 mm^2
J_u = 125253. mm^4
J_v = 19767. mm^4
J_t = 159.3 mm^4
x_o = 18.73 mm
x_g = 21.71 mm
T_y = 1680. N
M_x = -991200. Nmm
x_m = 36. mm
u_m = 14.29 mm
v_m = -29. mm
$\sigma_m = -Mv/J_u$ = -229.5 N/mm^2
x_c = 30. mm
u_c = 8.29 mm
v_c = -29. mm
$\sigma_c = -Mv/J_u$ = -229.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 367.3 N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.67 N/mm^2
$\tau_o = Tx_o t/J_t$ = 355.6 N/mm^2
t_c = 3024. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 676.2 N/mm^2

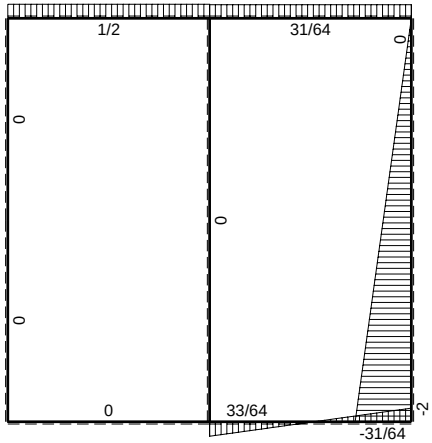
CI08.xxxx.093

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

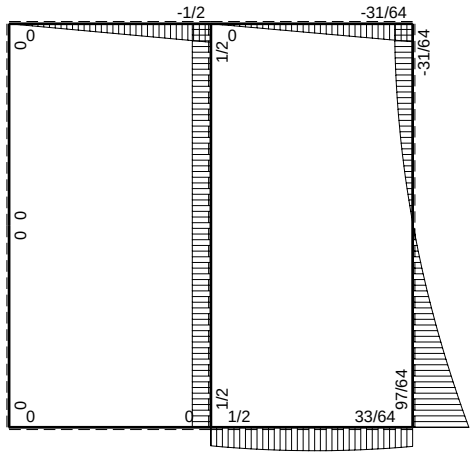

D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64

F


1/2
31/64
0
0
0
0
0
33/64
-2
-31/64

F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
0
1/2
1/2
33/64
97/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+Fx-1/2qx^2	-Fb/EJ	-1/2Fx-Fx^2/b+1/2qx^3/b	FxEJ	x^2/b^2	(-11/24+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2qx^2	FbEJ	-Fb+Fx+1/2Fx^2/b-1/2qx^3/b	FbEJ-FxEJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

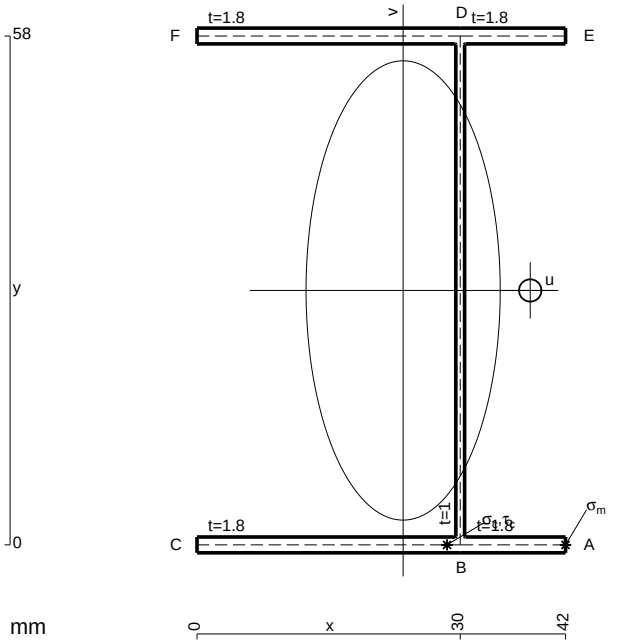
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

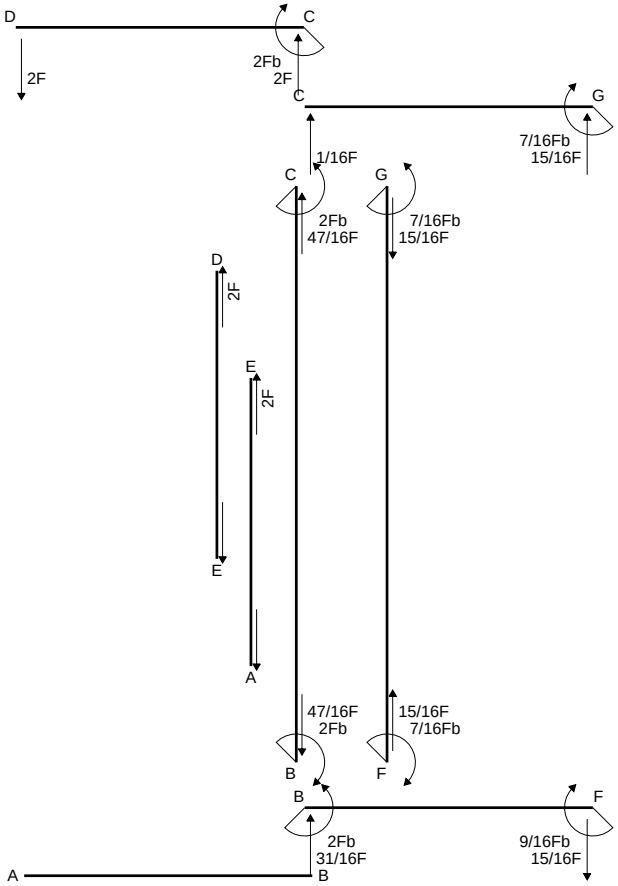
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

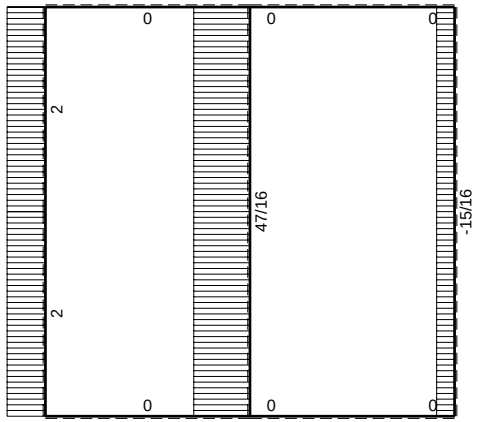


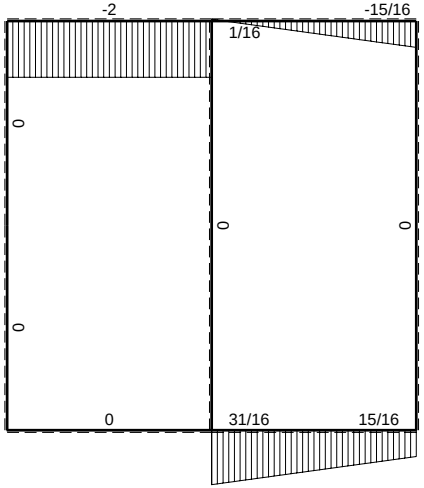
A = 209.2 mm^2
J_u = 143419. mm^4
J_v = 25622. mm^4
J_t = 182.6 mm^4
x_o = 14.48 mm
x_g = 23.5 mm
T_y = 1850. N
M_x = -1184000. Nmm
x_m = 42. mm
u_m = 18.5 mm
v_m = -29. mm
$\sigma_m = -Mv/J_u$ = -239.4 N/mm^2
x_c = 30. mm
u_c = 6.505 mm
v_c = -29. mm
$\sigma_c = -Mv/J_u$ = -239.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 275.3 N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.22 N/mm^2
$\tau_o = Tx_ot/J_t$ = 264.1 N/mm^2
t_c = 6660. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 533.6 N/mm^2

CI08.xxxx.094

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

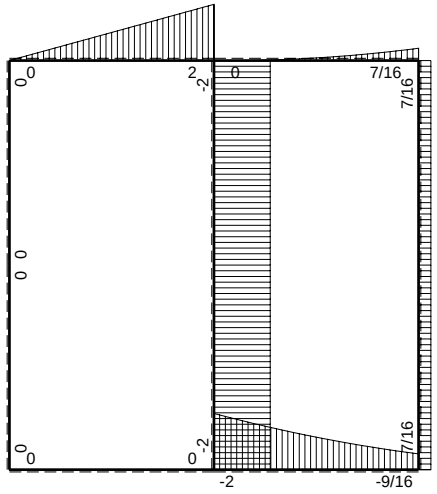

D
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0


-2
-15/16
1/16
0
0
0
0
0
31/16
15/16

F

F


0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
0
-2
7/16
-2
-9/16

Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
	0	-2Fx	0	0	0	0		
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
AE b	0	0	0	0	0	0		
BF b	-x/b	-2Fb+3/2Fx-1/2qx^2	-Fb/EJ	2Fx-3/2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/8+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
	1	0	0	0	0	1		
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
	0	2Fb	0	0	0	0		
BC 2b	0	2Fb	0	0	0	0		
	totali						7/6Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

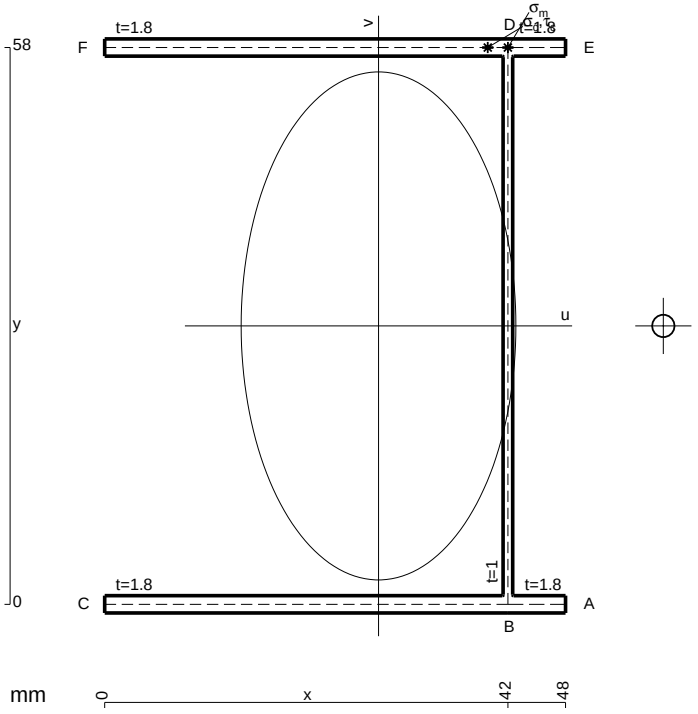
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

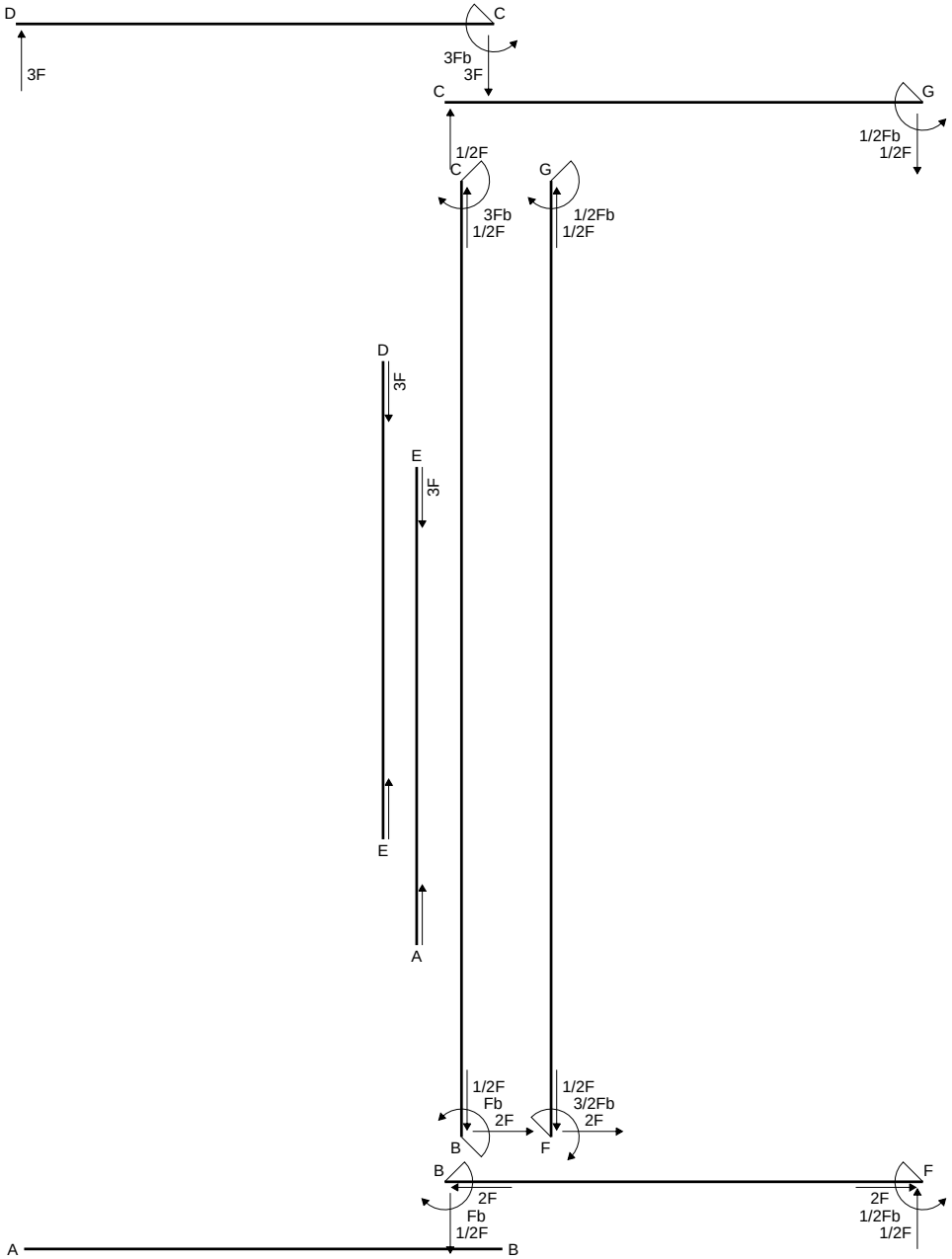
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

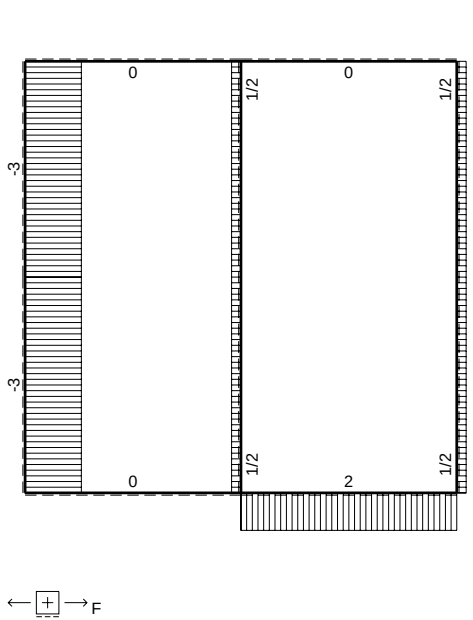


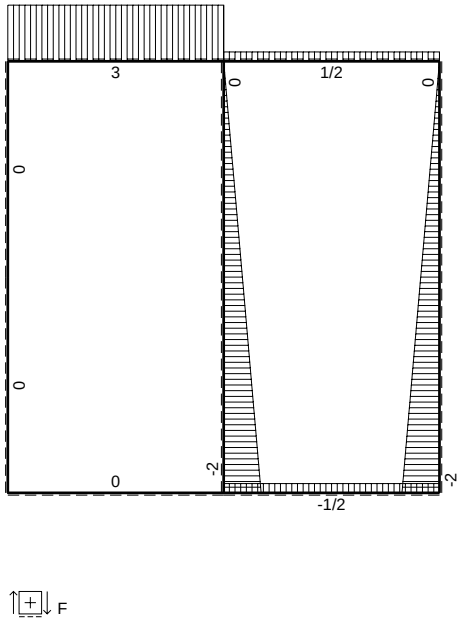
A = 230.8 mm^2
J_u = 161584. mm^4
J_v = 47247. mm^4
J_t = 206. mm^4
x_o = 29.67 mm
x_g = 28.52 mm
T_y = -1600. N
M_x = 1104000. Nmm
x_m = 42. mm
y_m = 58. mm
u_m = 13.48 mm
v_m = 29. mm
$\sigma_m = -Mv/J_u$ = -198.1 N/mm^2
x_c = 42. mm
y_c = 58. mm
u_c = 13.48 mm
v_c = 29. mm
$\sigma_c = -Mv/J_u$ = -198.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 426.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.06 N/mm^2
$\tau_o = Tx_o t/J_t$ = 414.8 N/mm^2
t_c = 1440. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 765.5 N/mm^2

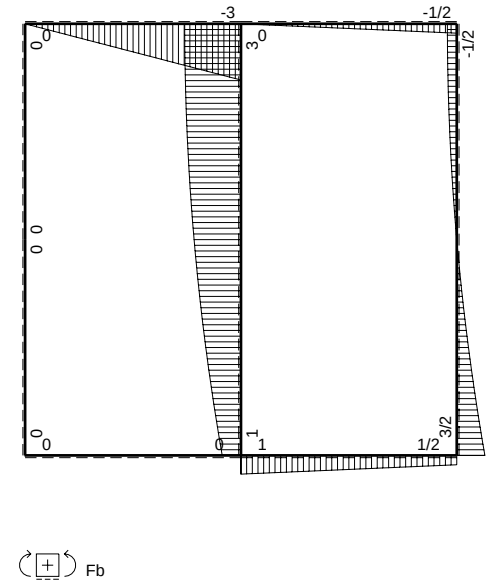
CI08.xxxx.095

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
1/2
0
1/2
-3
-3
0
1/2
2
1/2
F


3
0
1/2
0
0
0
0
-2
-2
-1/2
F


-3
-1/2
0
0
3
-1/2
0
0
0
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	3Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	-Fb-2Fx+1/2qx²	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

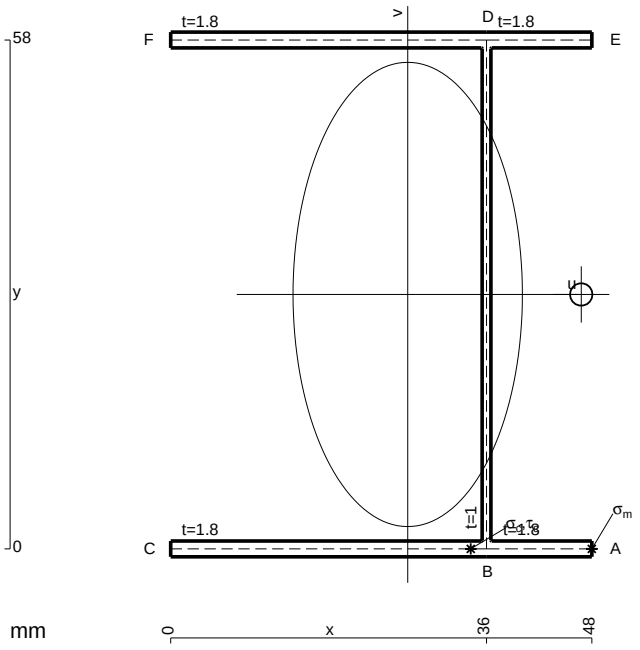
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

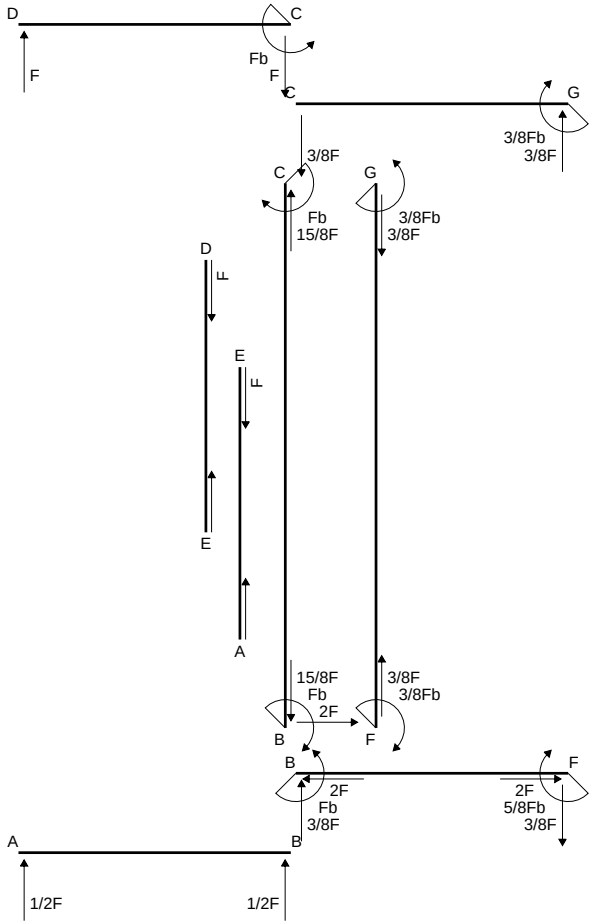
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

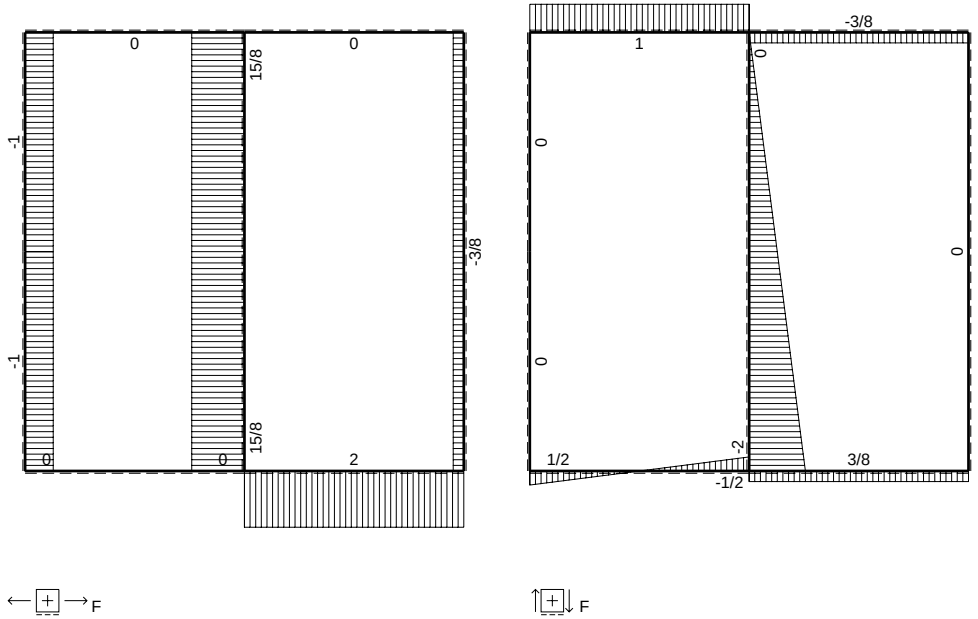


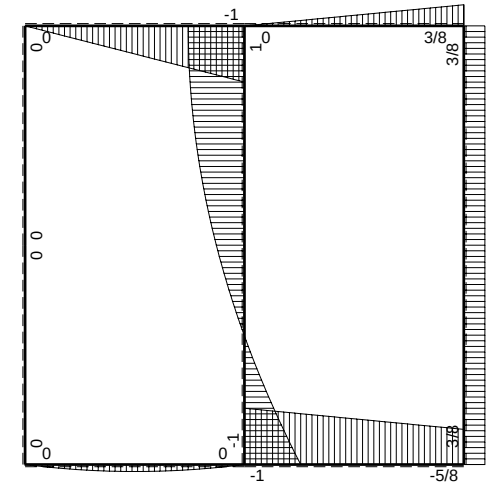
A = 230.8 mm^2
J_u = 161584. mm^4
J_v = 39431. mm^4
J_t = 206. mm^4
x_o = 19.78 mm
x_g = 27.02 mm
T_y = 1590. N
M_x = -1160700. Nmm
x_m = 48. mm
u_m = 20.98 mm
v_m = -29. mm
$\sigma_m = -Mv/J_u$ = -208.3 N/mm^2
x_c = 36. mm
u_c = 8.984 mm
v_c = -29. mm
$\sigma_c = -Mv/J_u$ = -208.3 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 285.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.27 N/mm^2
$\tau_o = Tx_o t/J_t$ = 274.8 N/mm^2
t_c = 954. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 535.9 N/mm^2

CI08.xxxx.096

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F


0
0
15/8
-1
-1
-3/8
0
0
15/8
0
0
2
F
-3/8
1
0
0
0
0
1/2
-2
3/8
-1/2
F


-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx^2	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Schema di calcolo iperstatico

PROCEDIMENTO E RISULTATI 239831 CI08.xxxx.097

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

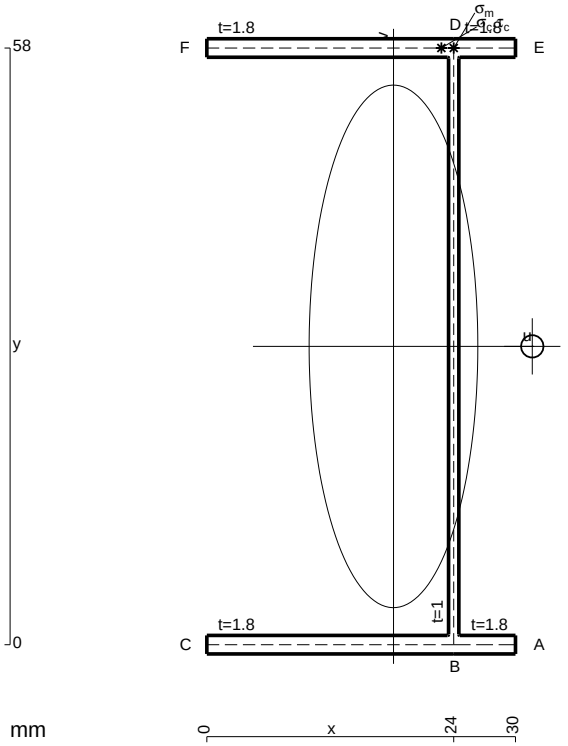
$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

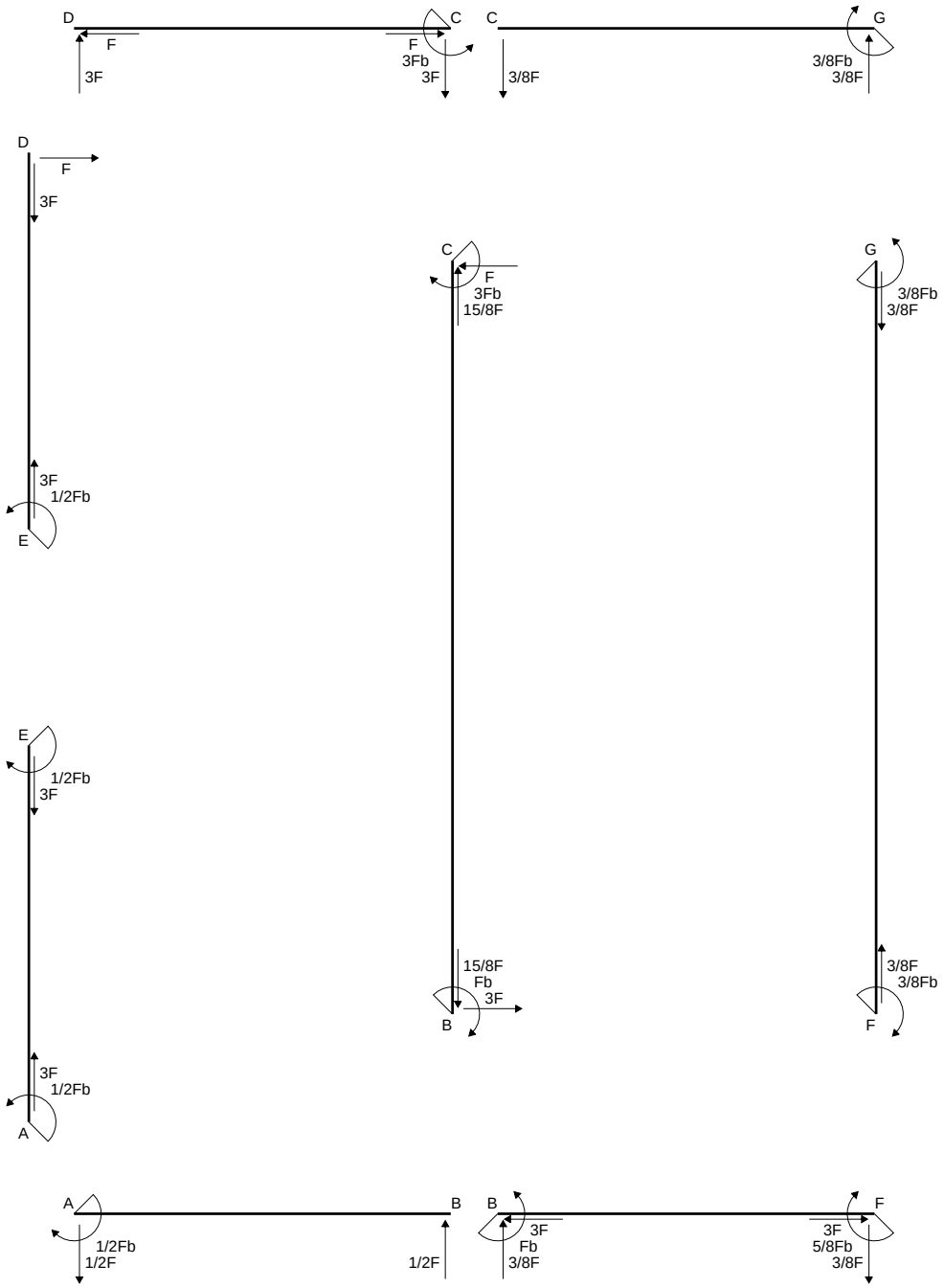
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

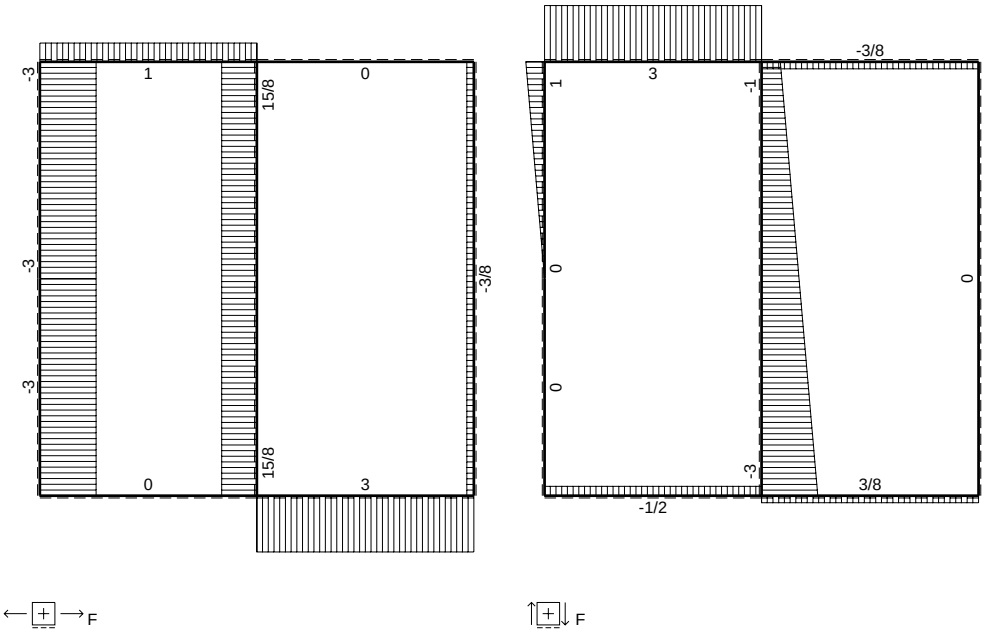


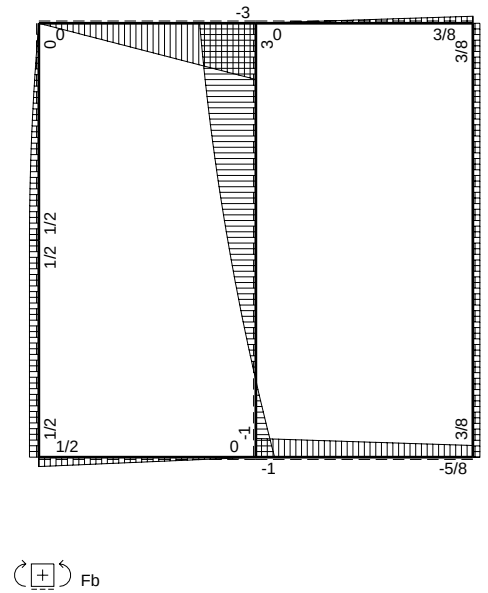
A = 166. mm^2
J_u = 107087. mm^4
J_v = 11157. mm^4
J_t = 136. mm^4
x_o = 13.49 mm
x_g = 18.14 mm
N = 1838. N
T_y = -1960. N
M_x = -764400. Nmm
x_m = 24. mm
y_m = 58. mm
u_m = 5.855 mm
v_m = 29. mm
$\sigma_m = N/A - Mv/J_u$ = 218.1 N/mm^2
x_c = 24. mm
y_c = 58. mm
u_c = 5.855 mm
v_c = 29. mm
$\sigma_c = N/A - Mv/J_u$ = 218.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 362.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.74 N/mm^2
$\tau_o = Tx_o t/J_t$ = 350. N/mm^2
t_c = 1764. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 665. N/mm^2

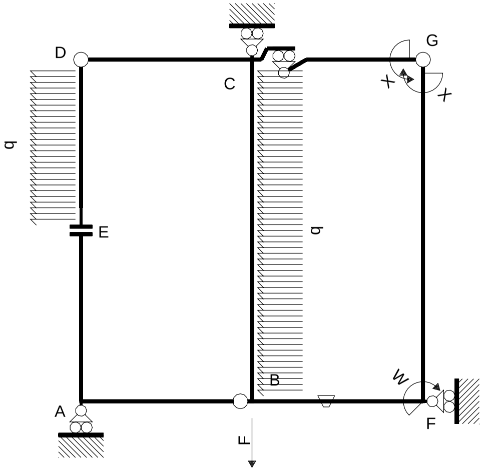
CI08.xxxx.097

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

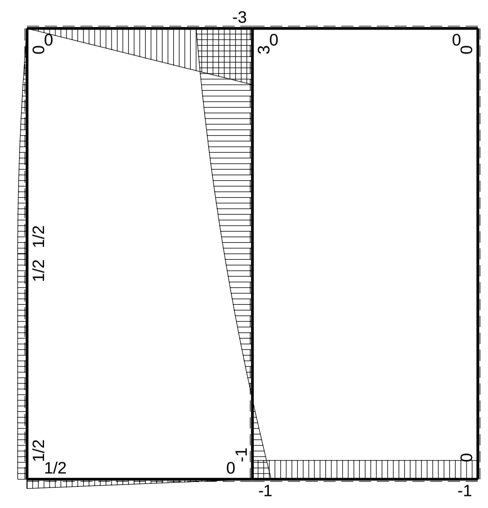

D
F
3F
C
F
3Fb
3F
C
3/8F
G
3/8Fb
3/8F
D
F
3F
3F
1/2Fb
E
C
F
3Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3F
15/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F


-3
1
0
15/8
-3
-3/8
-3
15/8
0
3
F
-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F

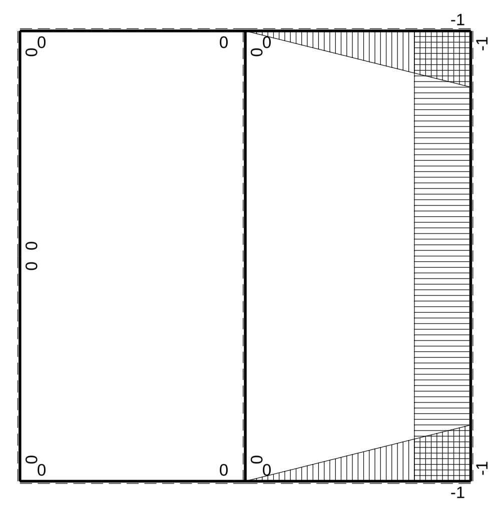

-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2Fx$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx$	0	0	0	0		
CD b	0	$-3Fb+3Fx$	0	0	0	0	0+0	0
DC b	0	$3Fx$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb$	0	0	0	0	0+0	0
AE b	0	$-1/2Fb$	0	0	0	0		
BF b	$-x/b$	$-Fb$	$-Fb/EJ$	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	Fb	Fb/EJ	$Fb-Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$3Fb-Fx-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-3Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$-3/8Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

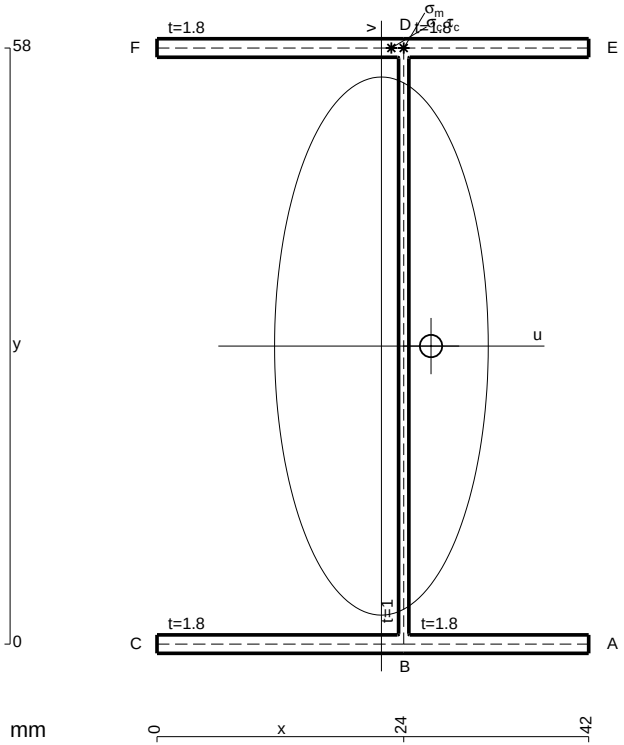
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

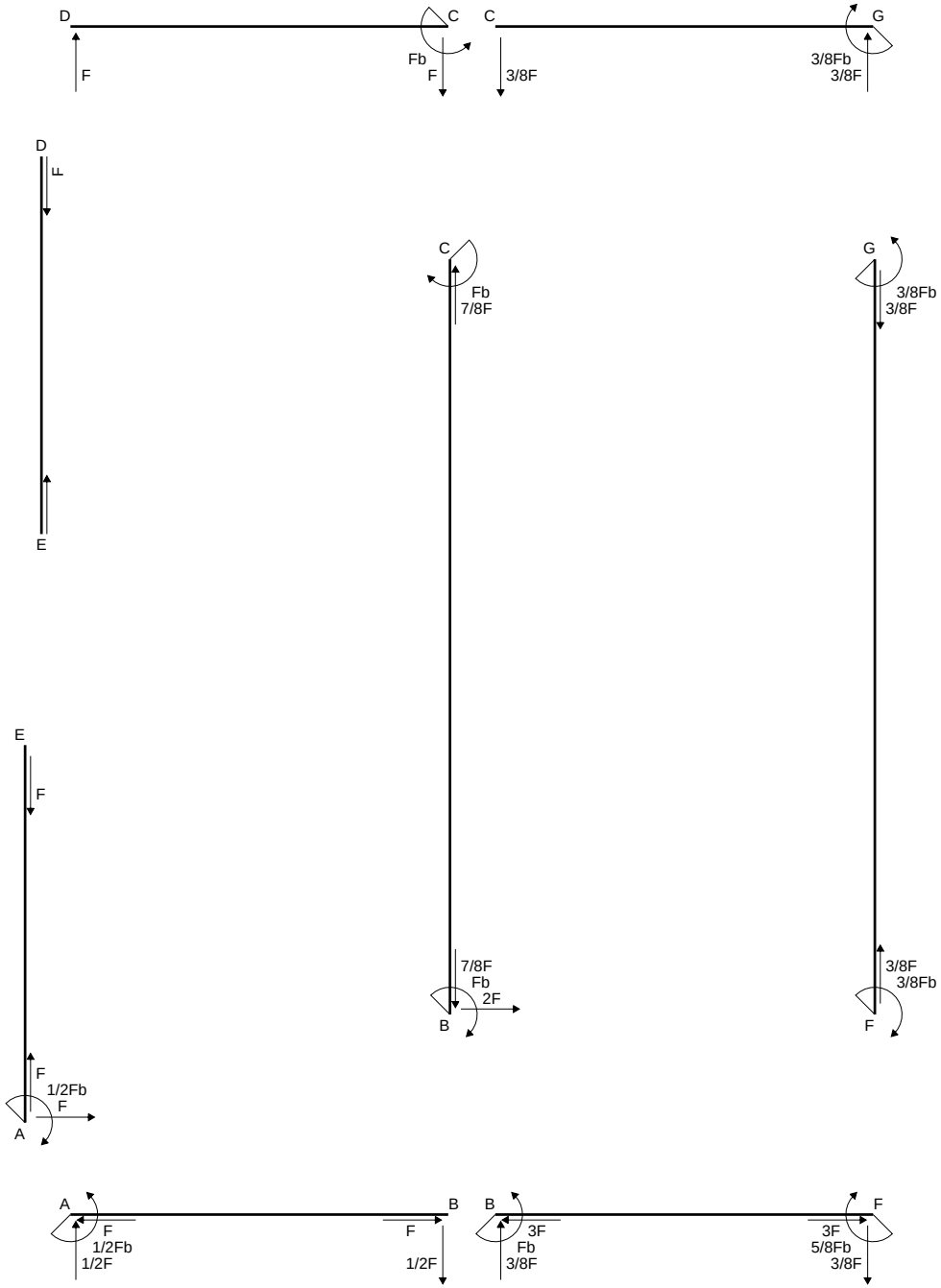
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

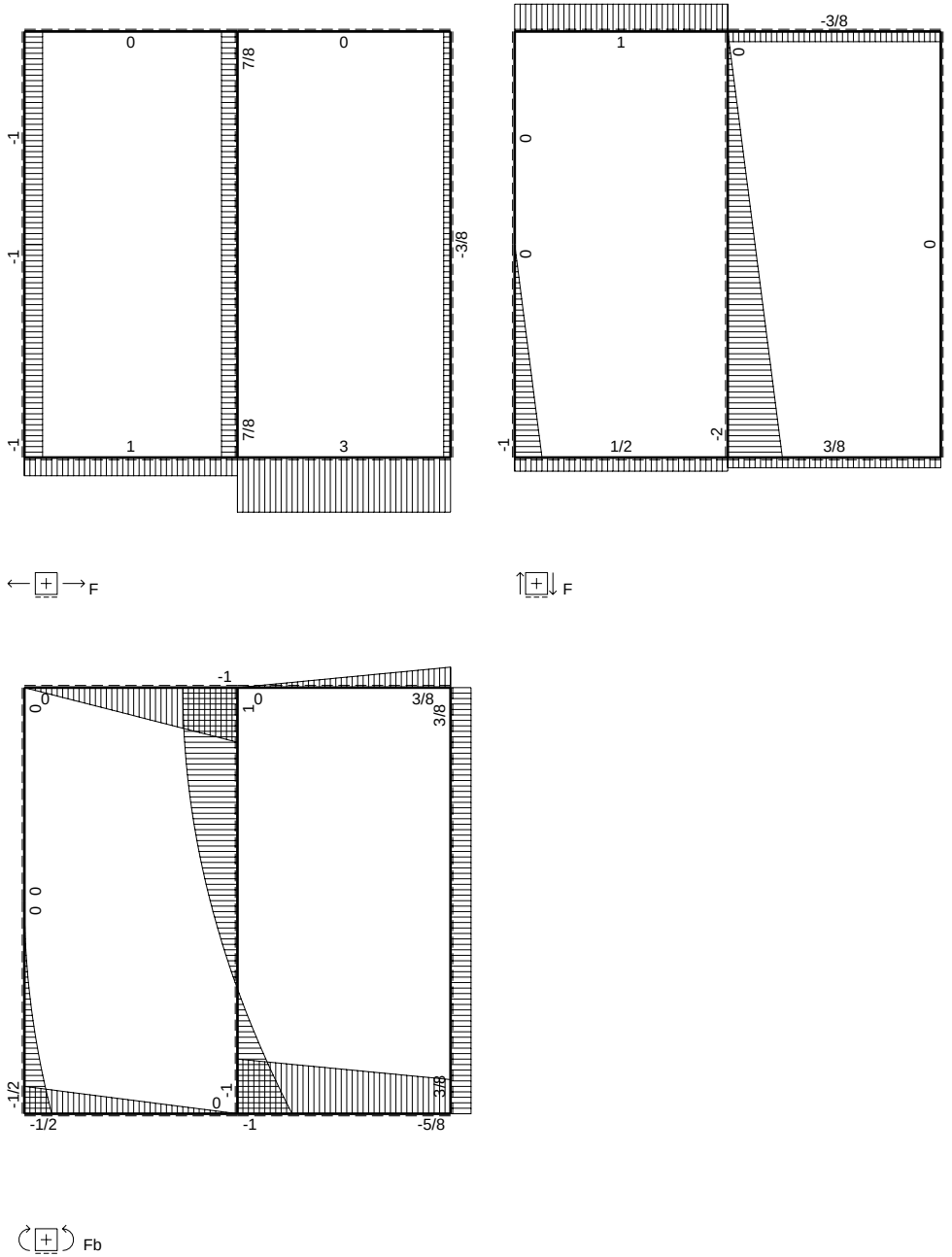


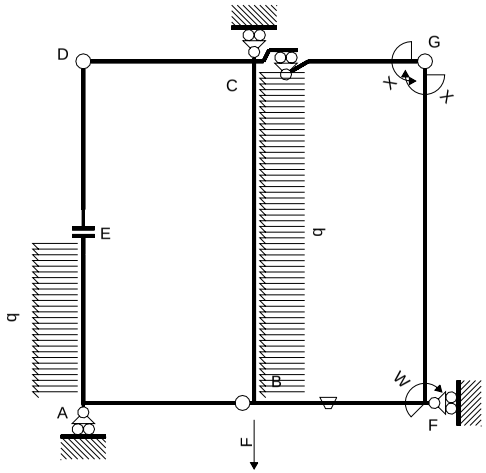
A = 209.2 mm^2
J_u = 143419. mm^4
J_v = 22604. mm^4
J_t = 182.6 mm^4
x_o = 4.828 mm
x_g = 21.83 mm
N = 450. N
T_y = 1350. N
M_x = -1107000. Nmm
x_m = 24. mm
y_m = 58. mm
u_m = 2.168 mm
v_m = 29. mm
$\sigma_m = N/A - Mv/J_u$ = 226. N/mm^2
x_c = 24. mm
y_c = 58. mm
u_c = 2.168 mm
v_c = 29. mm
$\sigma_c = N/A - Mv/J_u$ = 226. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 70.79 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.551 N/mm^2
$\tau_o = Tx_o t/J_t$ = 64.24 N/mm^2
t_c = 810. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 257.1 N/mm^2

CI08.xxxx.098

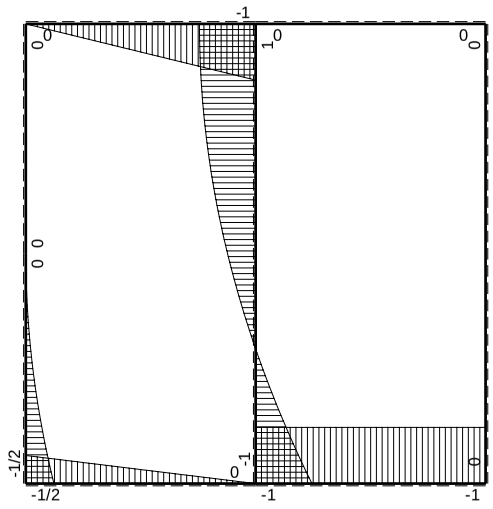
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
7/8F
G
3/8Fb
3/8F
E
F
7/8F
Fb
2F
B
3/8F
3/8Fb
F
F
1/2Fb
F
A
A
F
1/2Fb
1/2F
B
F
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F

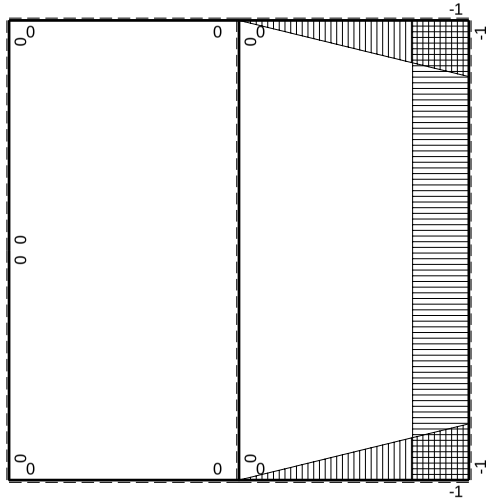

0
0
7/8
-1
-1
-3/8
-1
1
7/8
3
F
1
-3/8
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213451 CI08.xxxx.099

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

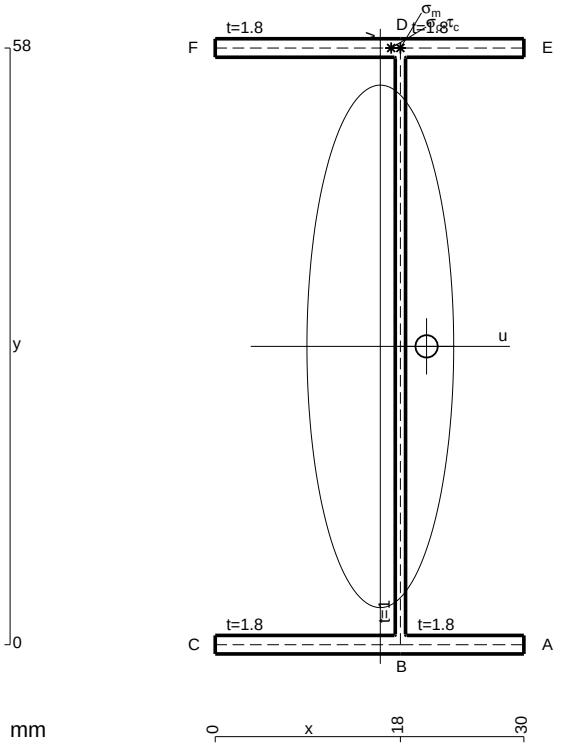
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

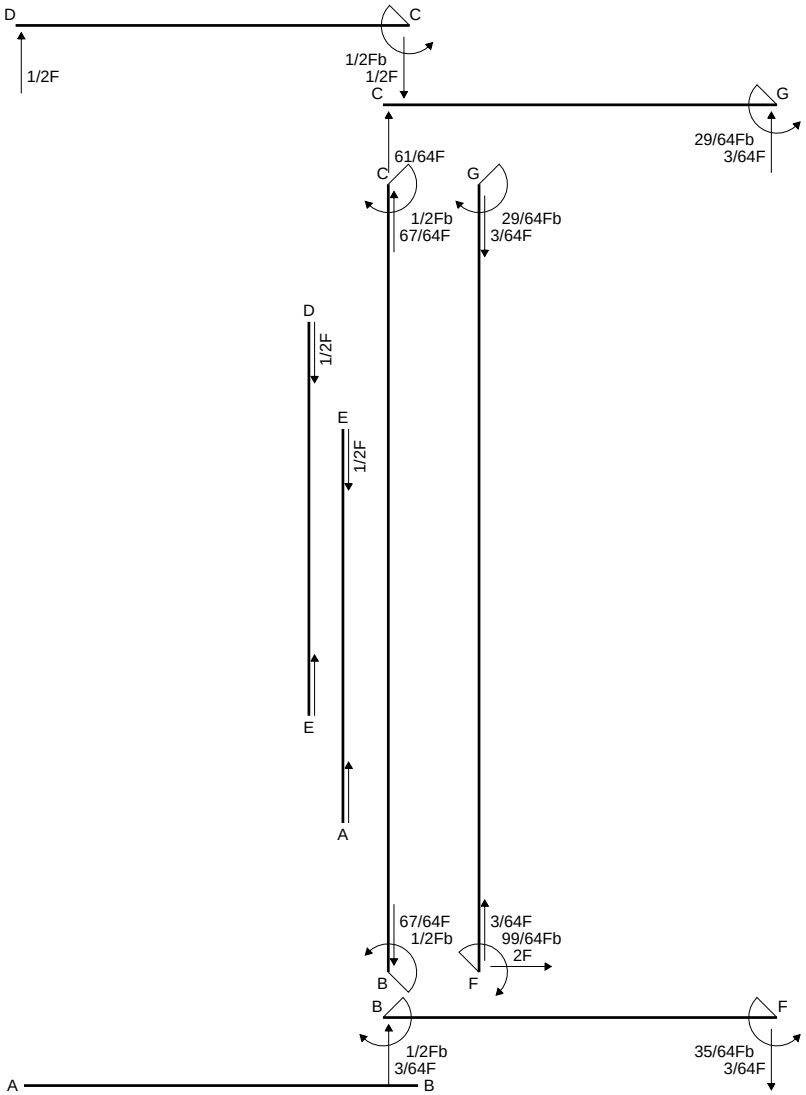
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

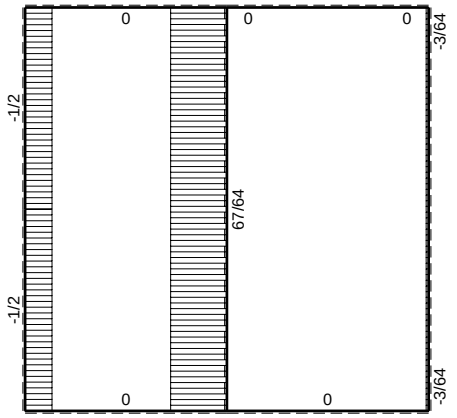


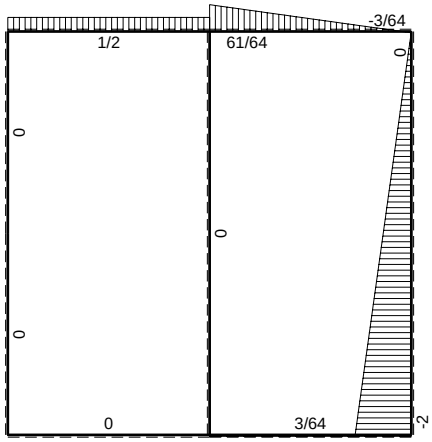
A = 166. mm^2
J_u = 107087. mm^4
J_v = 8440. mm^4
J_t = 136. mm^4
x_o = 4.496 mm
x_g = 16.05 mm
N = 866.3 N
T_y = -1980. N
M_x = -861300. Nmm
x_m = 18. mm
y_m = 58. mm
u_m = 1.952 mm
v_m = 29. mm
$\sigma_m = N/A - Mv/J_u$ = 238.5 N/mm^2
x_c = 18. mm
y_c = 58. mm
u_c = 1.952 mm
v_c = 29. mm
$\sigma_c = N/A - Mv/J_u$ = 238.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 127.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.652 N/mm^2
$\tau_o = Tx_o t/J_t$ = 117.9 N/mm^2
t_c = 1782. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 325. N/mm^2

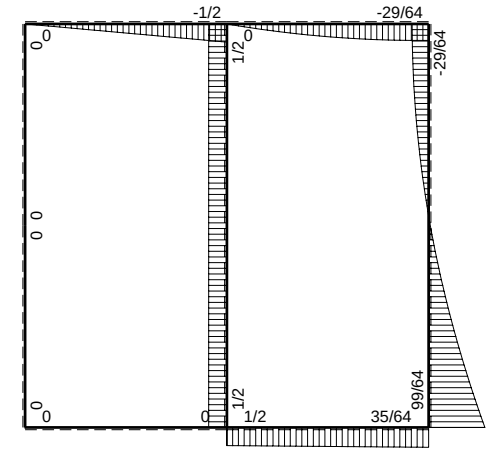
CI08.xxxx.099

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64


-3/64
1/2
61/64
0
0
0
0
0
3/64
-2


-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
0
1/2
35/64
99/64

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-29/24Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						29/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

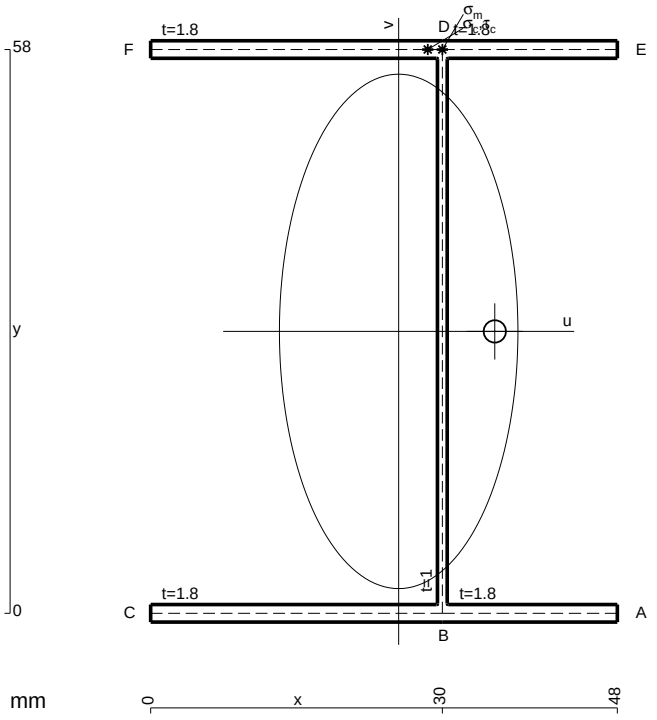
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

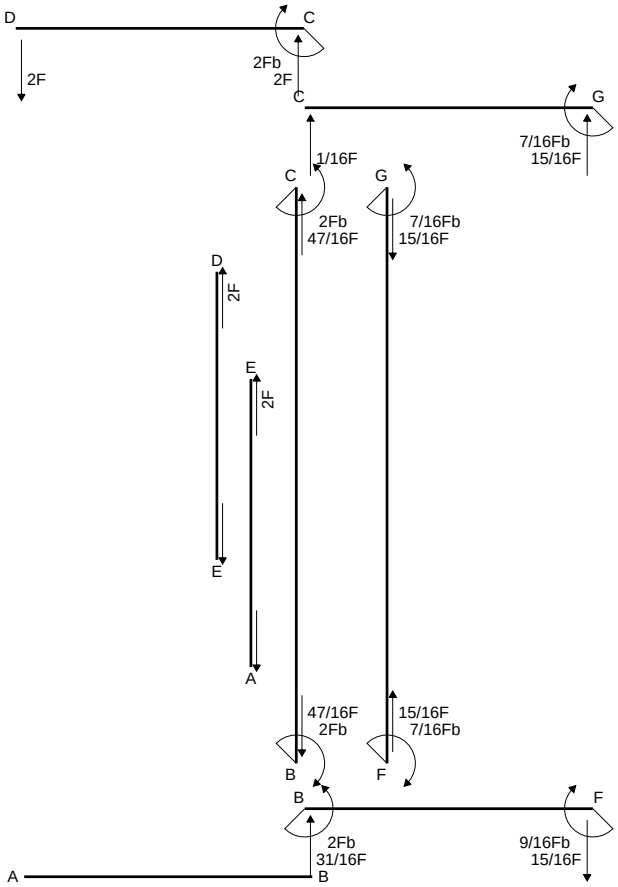
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

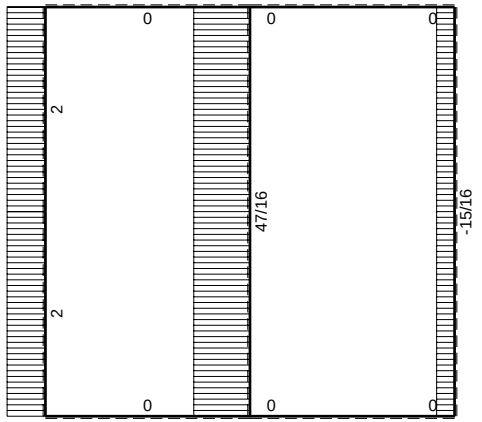


A = 230.8 mm^2
J_u = 161584. mm^4
J_v = 34741. mm^4
J_t = 206. mm^4
x_o = 9.888 mm
x_g = 25.51 mm
T_y = 2420. N
M_x = -1113200. Nmm
x_m = 30. mm
y_m = 58. mm
u_m = 4.492 mm
v_m = 29. mm
$\sigma_m = -Mv/J_u$ = 199.8 N/mm^2
x_c = 30. mm
y_c = 58. mm
u_c = 4.492 mm
v_c = 29. mm
$\sigma_c = -Mv/J_u$ = 199.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 222.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.03 N/mm^2
$\tau_o = Tx_o t/J_t$ = 209.1 N/mm^2
t_c = 8712. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 433.6 N/mm^2

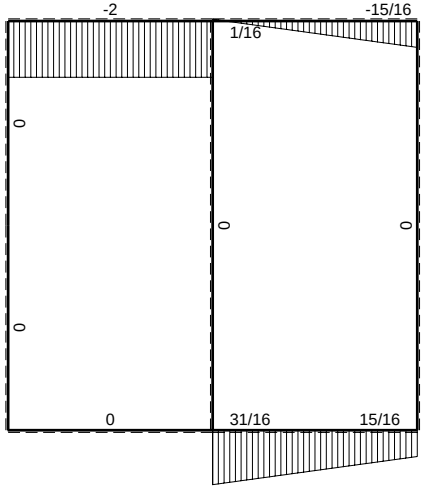
CI08.xxxx.100

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

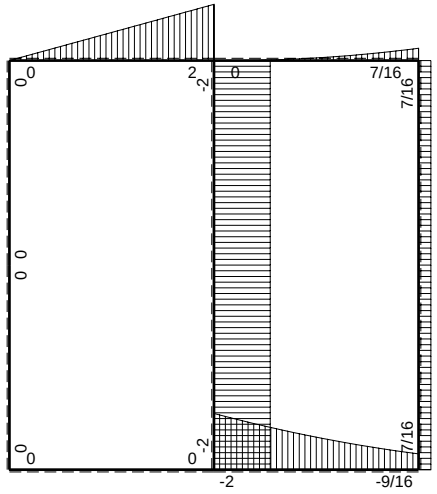

D
C
2Fb
2F
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0

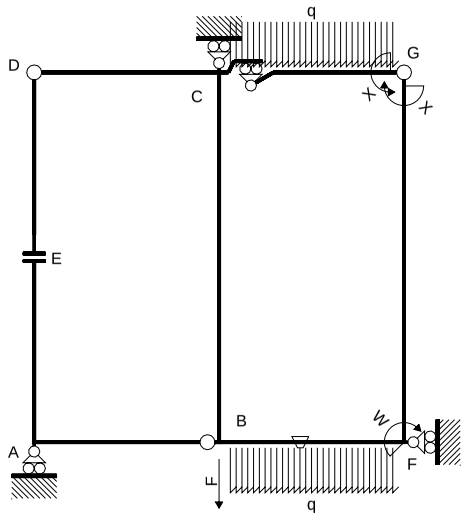
F


-2
-15/16
1/16
0
0
0
0
0
31/16
15/16

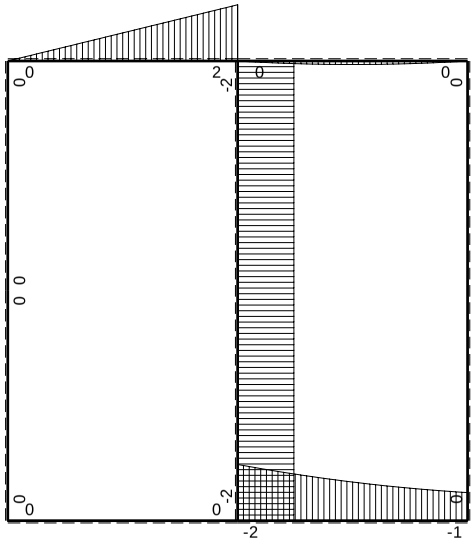
F


0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
0
-2
7/16
-2
-9/16

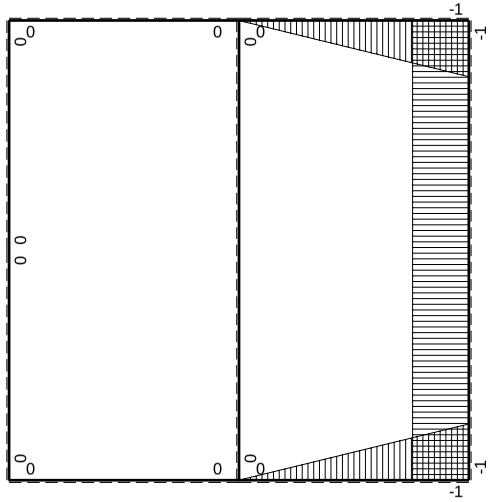
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 939153 CI08.xxxx.101

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

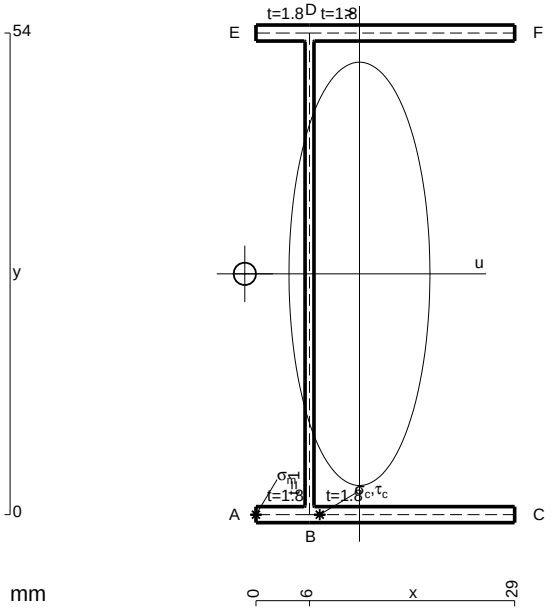
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

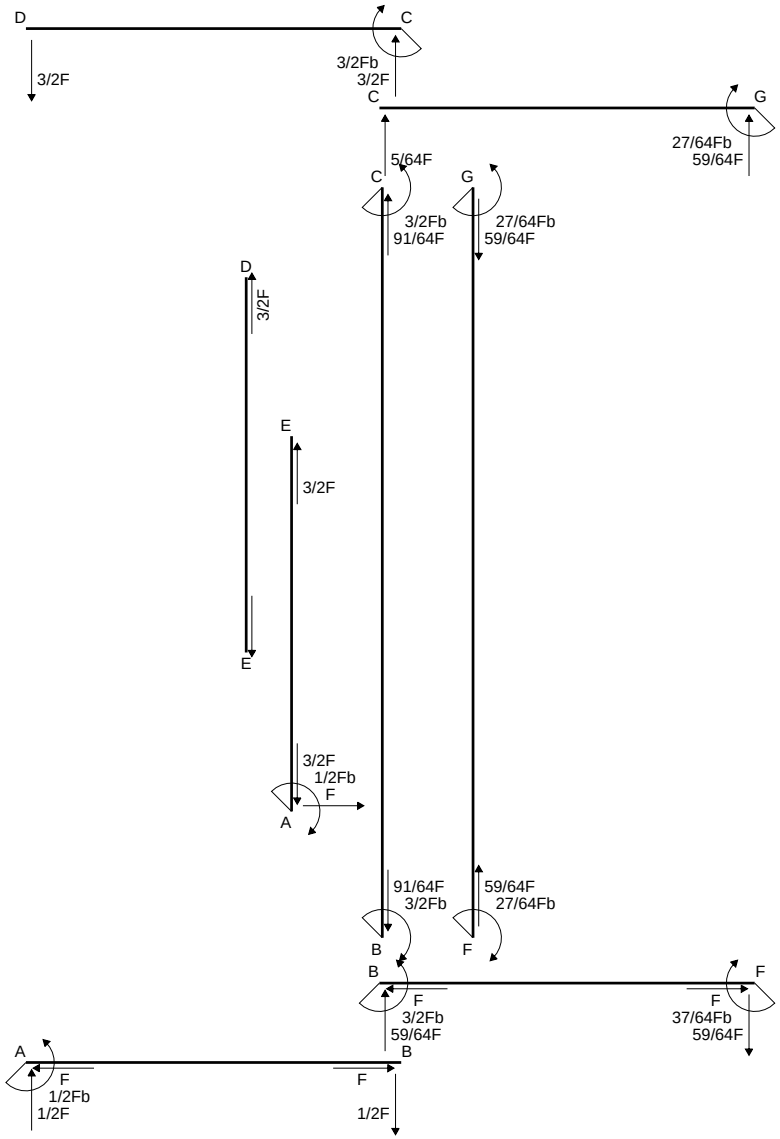
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

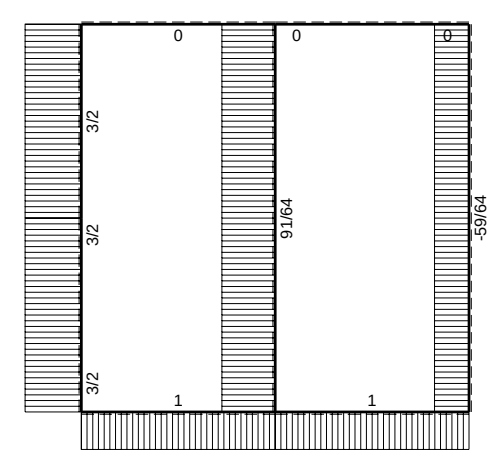


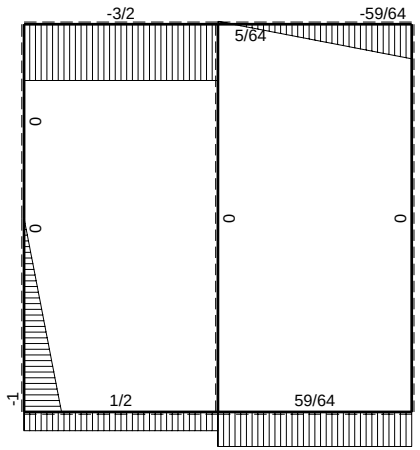
A = 158.4 mm^2
J_u = 89230. mm^4
J_v = 9888. mm^4
J_t = 130.8 mm^4
x_o = -12.85 mm
x_g = 11.6 mm
T_y = -1460. N
M_x = 686200. Nmm
u_m = -11.6 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = 207.6 N/mm^2
x_c = 6. mm
u_c = -5.602 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = 207.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 268.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.16 N/mm^2
$\tau_o = Tx_o t/J_t$ = 258.3 N/mm^2
t_c = 1314. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 509.3 N/mm^2

CI08.xxxx.101

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

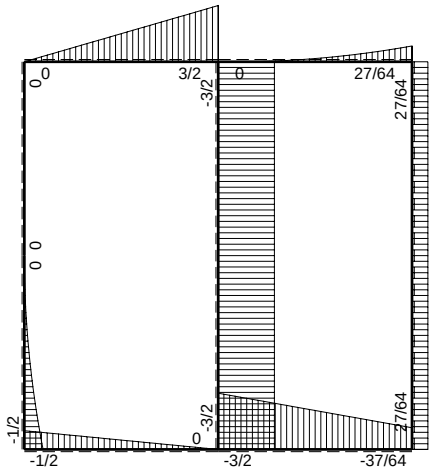

D
C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
3/2
91/64
-59/64
1
1


-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F

F


0
0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-1/2
-3/2
-37/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	0	1/2Fx	0	0	0	0		
BA b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
	0	-3/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DC b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
EA b	0	-1/2qx^2	0	0	0	0	0+0	0
	0	1/2Fb-Fx+1/2qx^2	0	0	0	0		
AE b	0	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(7/12+1/2)Fb^2/EJ	1/3Xb/EJ
	1-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
BF b	-x/b	-1/2Fb+Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
	-1	0	0	0	0	1	0+0	2Xb/EJ
FG 2b	-1	0	0	0	0	1		
	1	0	0	0	0	1		
GF 2b	1	0	0	0	0	1	0+0	2Xb/EJ
	0	-3/2Fb	0	0	0	0		
CB 2b	0	3/2Fb	0	0	0	0	0+0	0
	0	3/2Fb	0	0	0	0		
BC 2b	0	-3/2Fb	0	0	0	0	0+0	0
	0	-1/2Fb+1/2Fx	0	0	0	0		
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	totali						9/8Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

Sviluppi di calcolo iperstatica

M_o flessione da carichi assegnati

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

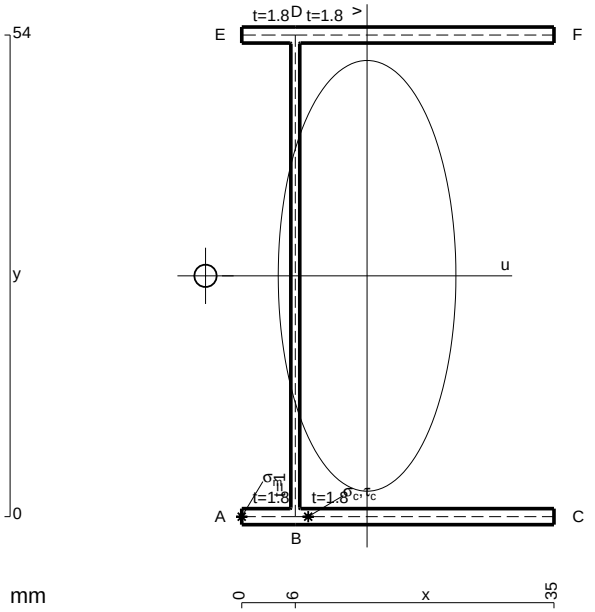
$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 249282 CI08.xxxx.102

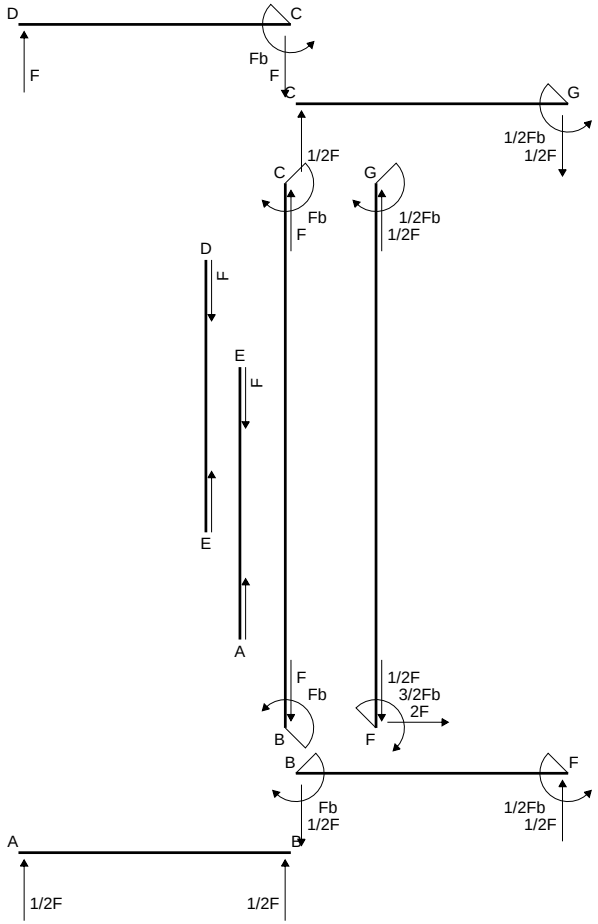


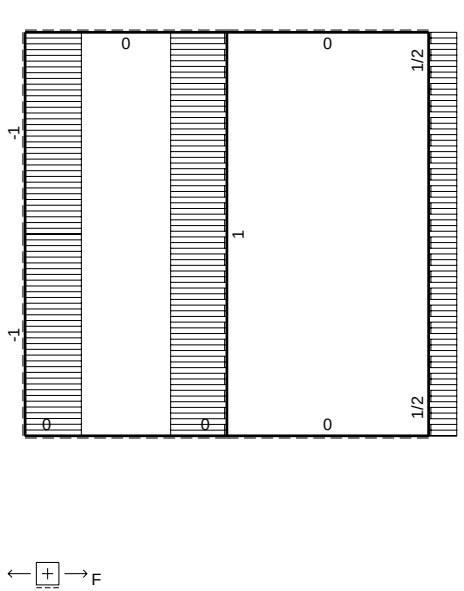
A = 180. mm^2
J_u = 104976. mm^4
J_v = 17862. mm^4
J_t = 154.1 mm^4
x_o = -18.11 mm
x_g = 14.05 mm
T_y = -1665. N
M_x = 849150. Nmm
u_m = -14.05 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = 218.4 N/mm^2
x_c = 6. mm
u_c = -8.05 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = 218.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 364.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.42 N/mm^2
$\tau_o = Tx_o t/J_t$ = 352.3 N/mm^2
t_c = 1998. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 668.4 N/mm^2

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

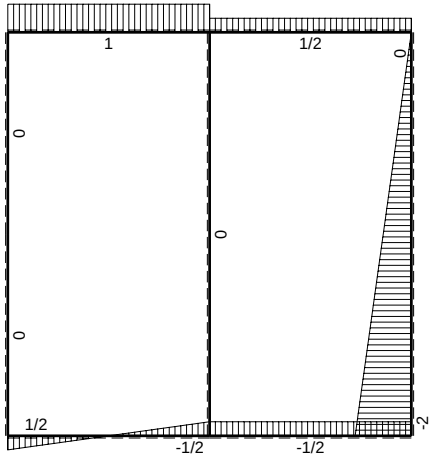
CI08.xxxx.102

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

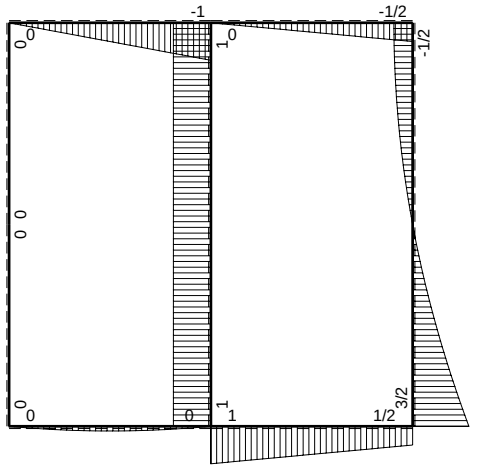

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

F


1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2

F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

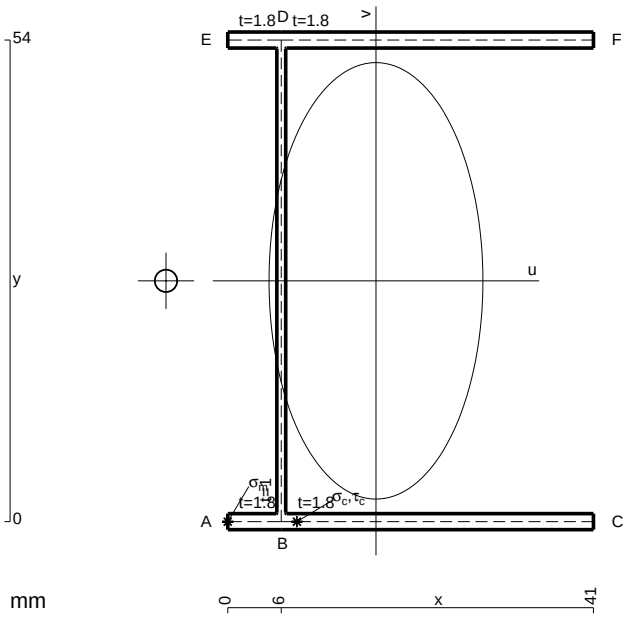
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

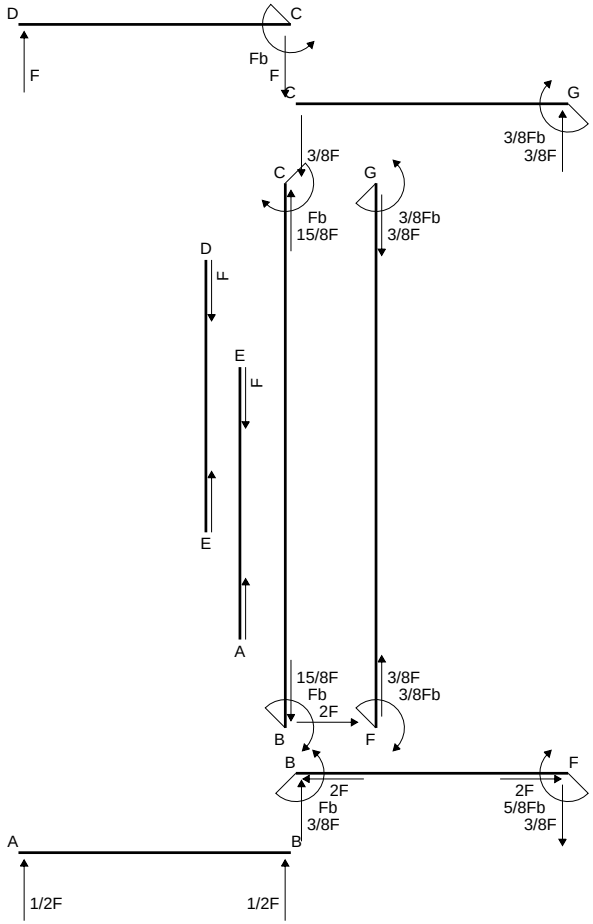
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

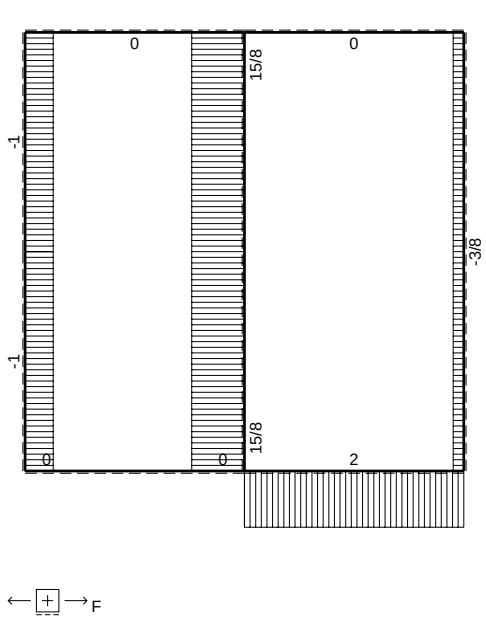


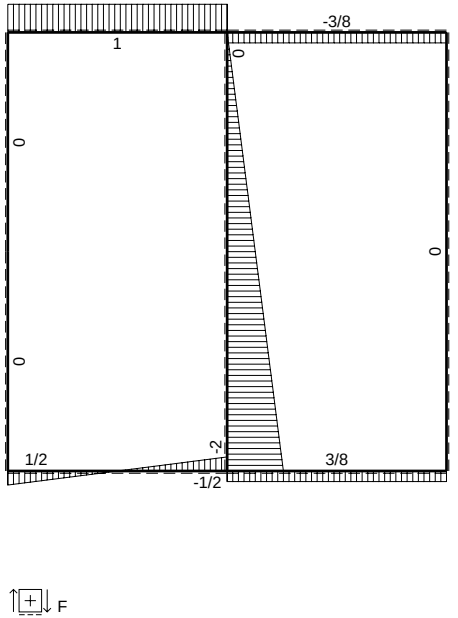
A = 201.6 mm^2
J_u = 120722. mm^4
J_v = 28989. mm^4
J_t = 177.4 mm^4
x_o = -23.54 mm
x_g = 16.62 mm
T_y = 1860. N
M_x = -1023000. Nmm
u_m = -16.62 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -228.8 N/mm^2
x_c = 6. mm
u_c = -10.62 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -228.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 458.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 14.56 N/mm^2
$\tau_o = Tx_o t/J_t$ = 444.2 N/mm^2
t_c = 3348. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 826.9 N/mm^2

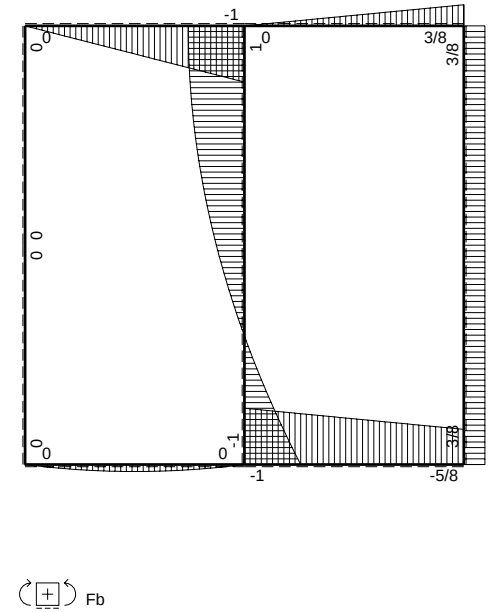
CI08.xxxx.103

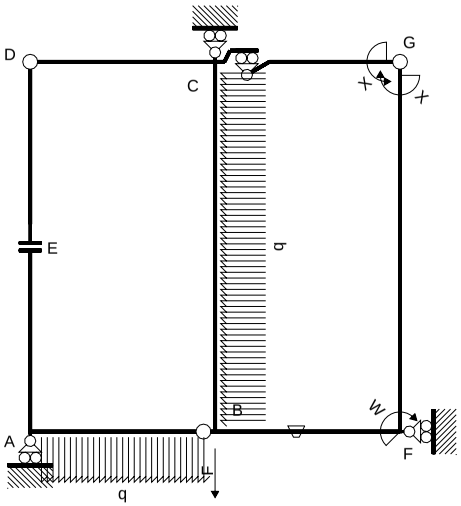
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F

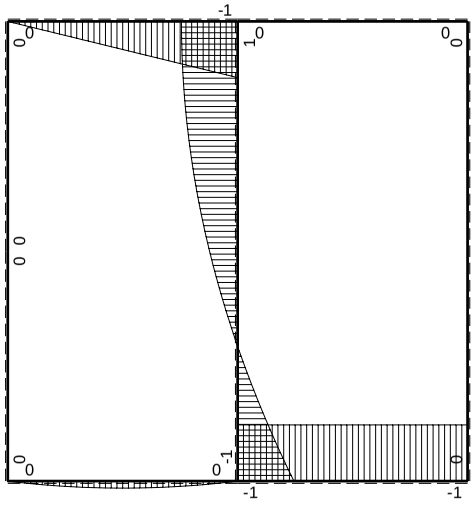

0
0
15/8
-1
-1
-3/8
0
0
15/8
0
0
2
F


-3/8
1
0
0
0
0
1/2
-2
3/8
-1/2
F

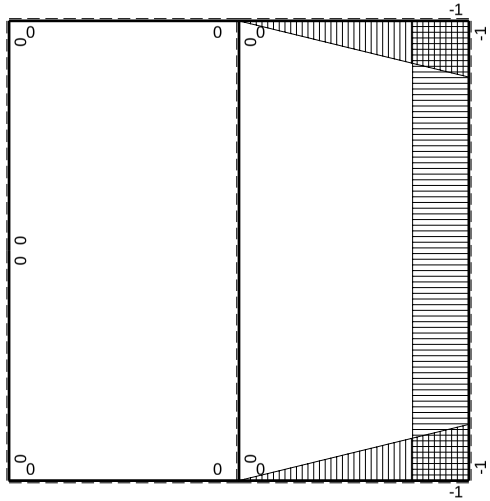

-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx²	0	0	0	0		
		totali					Fb²/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 237065 CI08.xxxx.104

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

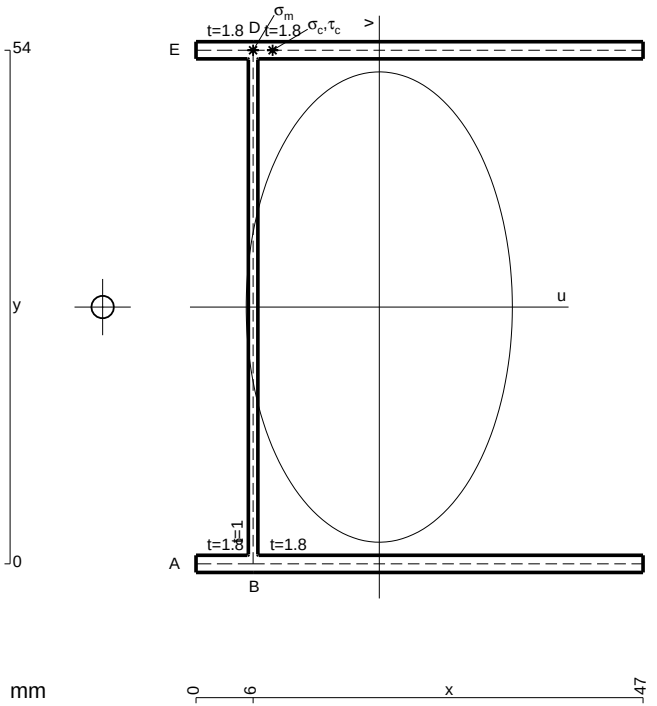
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

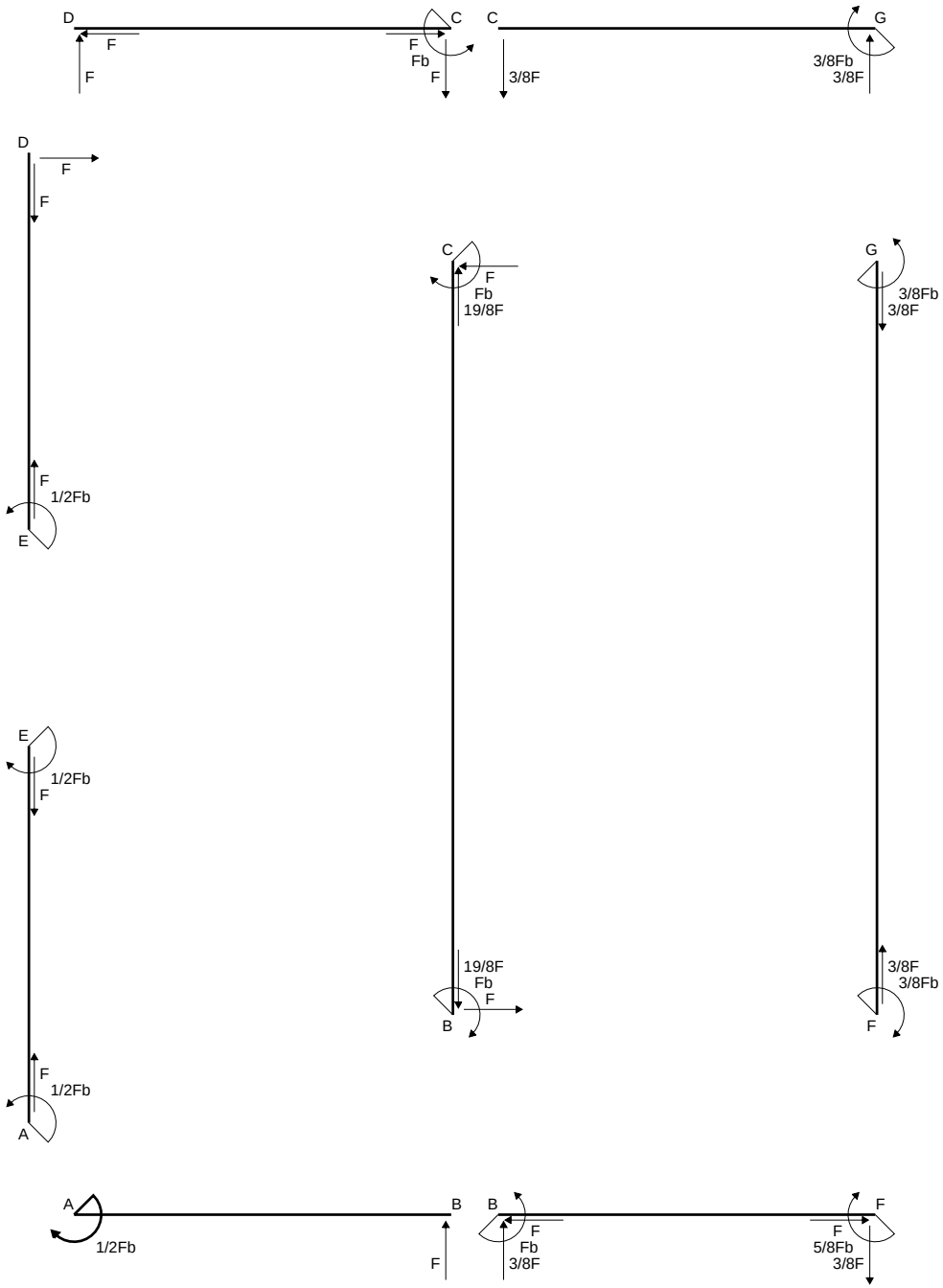
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

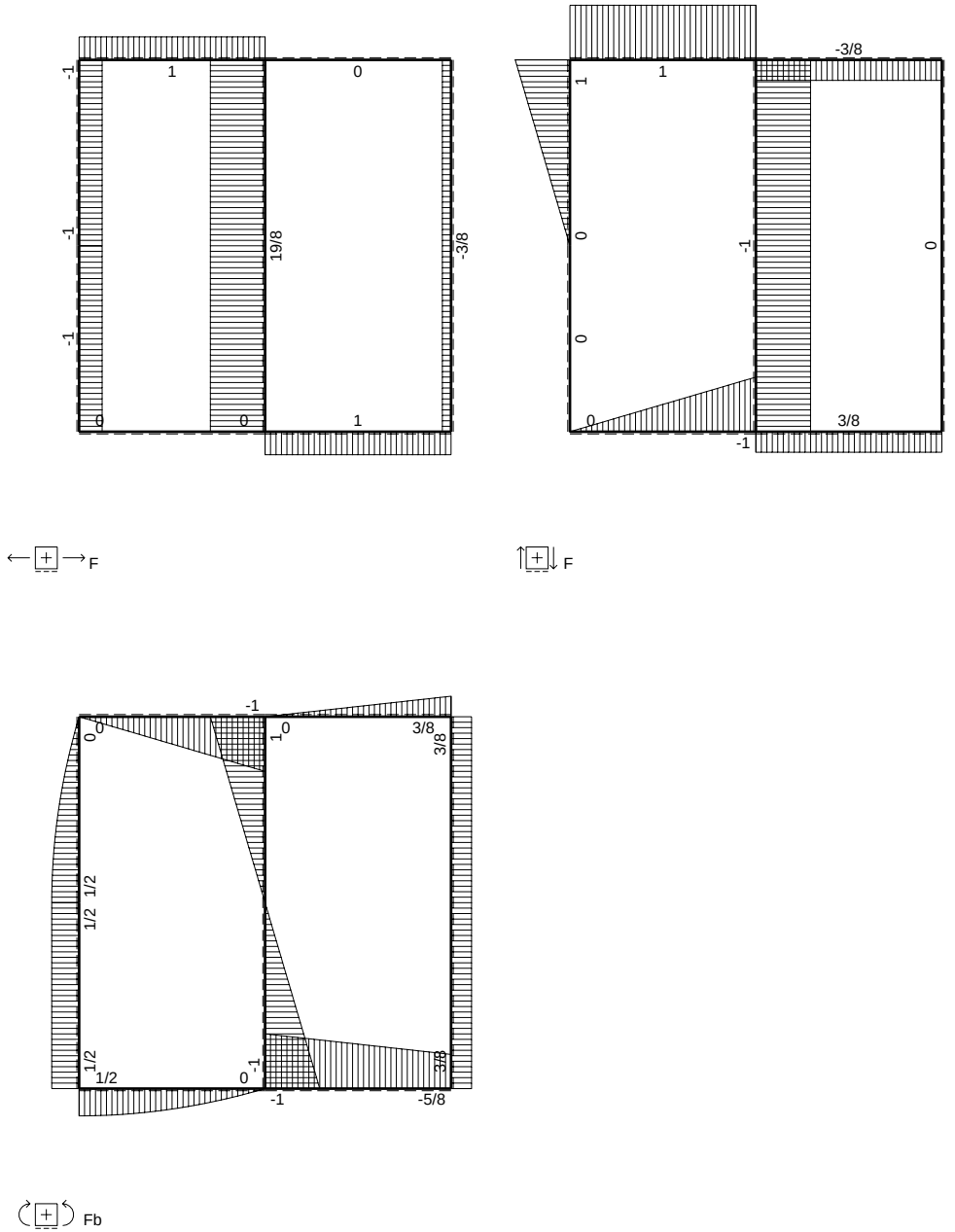


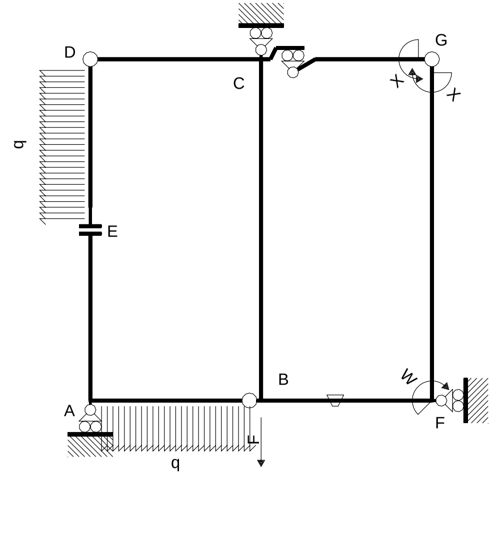
A = 223.2 mm^2
J_u = 136469. mm^4
J_v = 43683. mm^4
J_t = 200.7 mm^4
x_o = -29.08 mm
x_g = 19.27 mm
N = 3525. N
T_y = -3760. N
M_x = -1128000. Nmm
x_m = 6. mm
y_m = 54. mm
u_m = -13.27 mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 239. N/mm^2
x_c = 6. mm
y_c = 54. mm
u_c = -13.27 mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 239. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 1011. N/mm^2
$\tau_g = TS^*/tJ_u$ = 30.5 N/mm^2
$\tau_o = Tx_o t/J_t$ = 980.6 N/mm^2
t_c = 3384. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1767. N/mm^2

CI08.xxxx.104

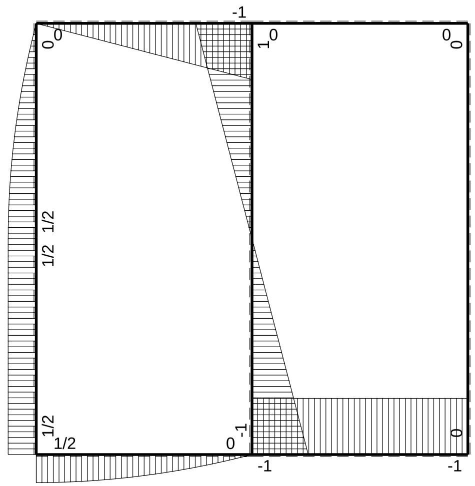
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
19/8F
G
3/8Fb
3/8F
E
1/2Fb
F
19/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F

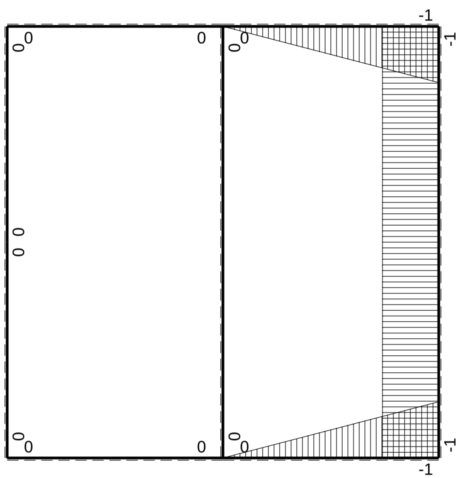

-1
1
0
-1
19/8
-3/8
-1
0
1
F
-3/8
1
1
1
0
-1
0
0
0
3/8
-1
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

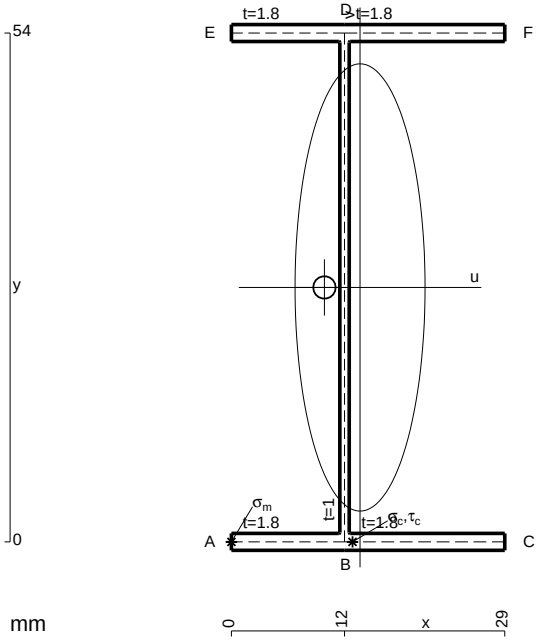
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 247861 CI08.xxxx.105

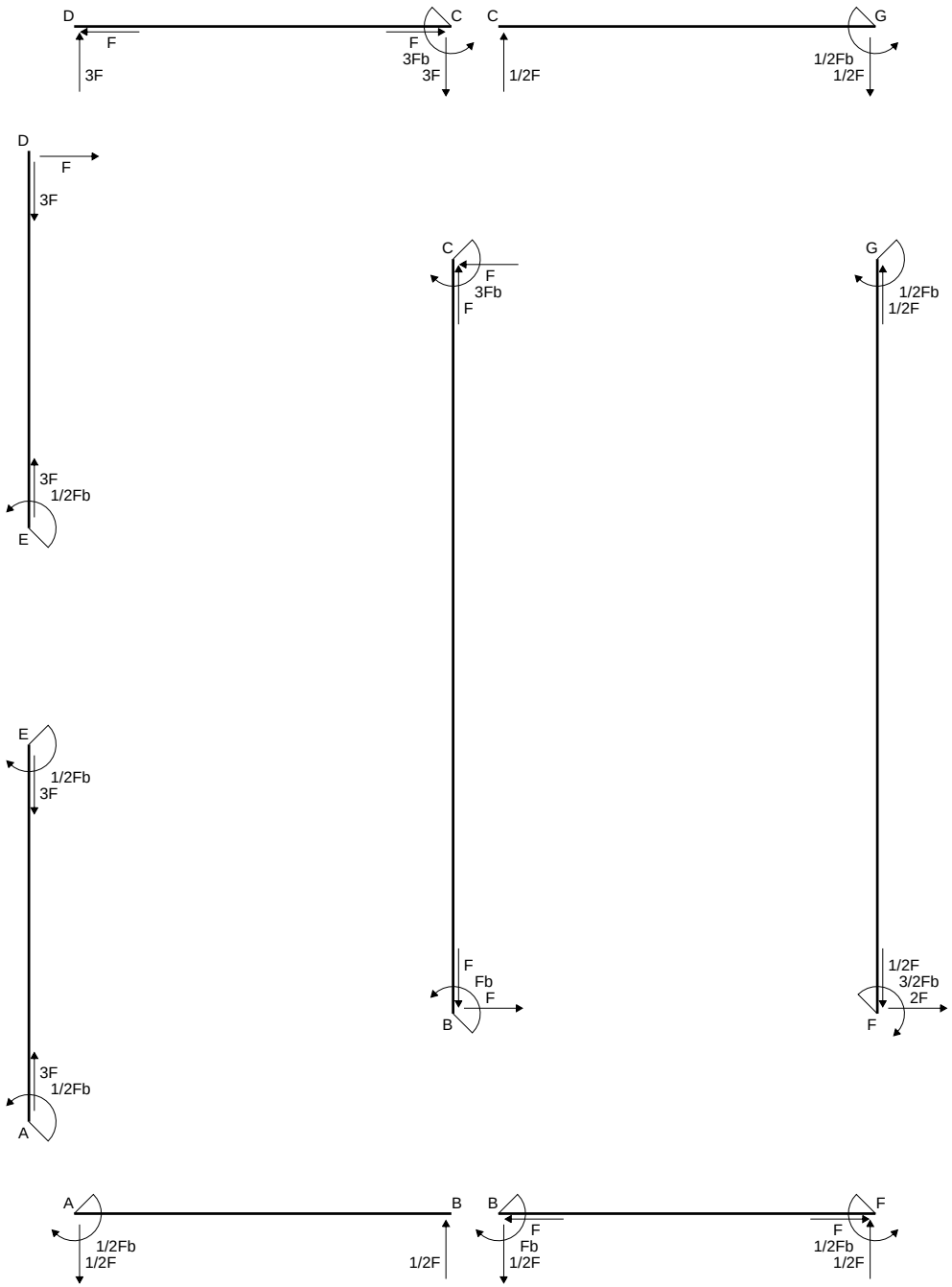


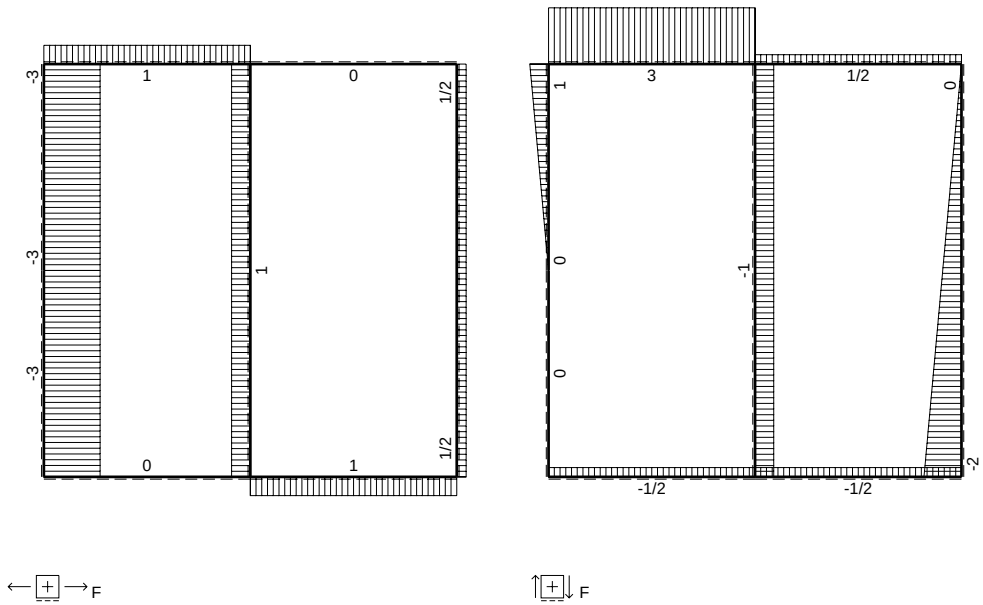
A = 158.4 mm^2
J_u = 89230. mm^4
J_v = 7539. mm^4
J_t = 130.8 mm^4
x_o = -3.78 mm
x_g = 13.65 mm
N = 2256. N
T_y = -950. N
M_x = 608000. Nmm
u_m = -13.65 mm
v_m = -27. mm
$\sigma_m = N/A - Mv/J_u$ = 198.2 N/mm^2
x_c = 12. mm
u_c = -1.648 mm
v_c = -27. mm
$\sigma_c = N/A - Mv/J_u$ = 198.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 54.32 N/mm^2
$\tau_g = TS^*/tJ_u$ = 4.887 N/mm^2
$\tau_o = Tx_o t/J_t$ = 49.44 N/mm^2
t_c = 1710. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 219.4 N/mm^2

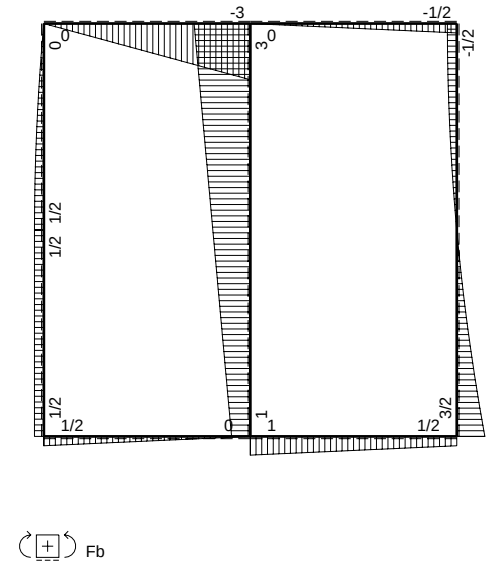
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

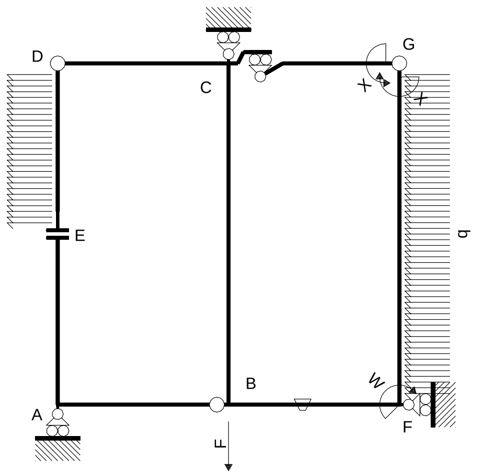
CI08.xxxx.105

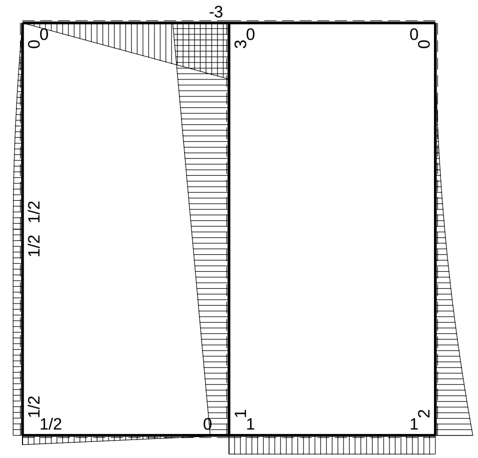
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
F
Fb
F
1/2F
3/2Fb
2F
B
F
3F
1/2Fb
A
A
B
B
F
F
Fb
F
1/2Fb
1/2F
1/2F
1/2Fb
1/2F

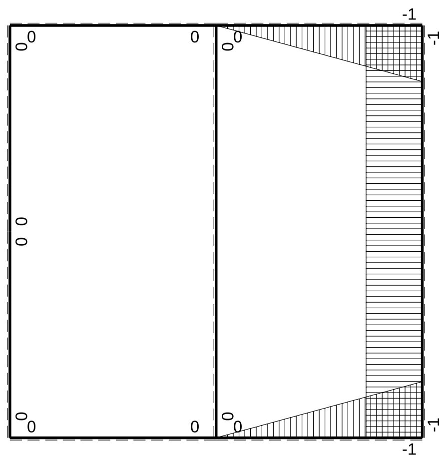

-3
-3
-3
1
0
1
0
1
1/2
1/2
1
3
1/2
0
0
0
-1
-2
-1/2
-1/2
F
F


-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237766 CI08.xxxx.106

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

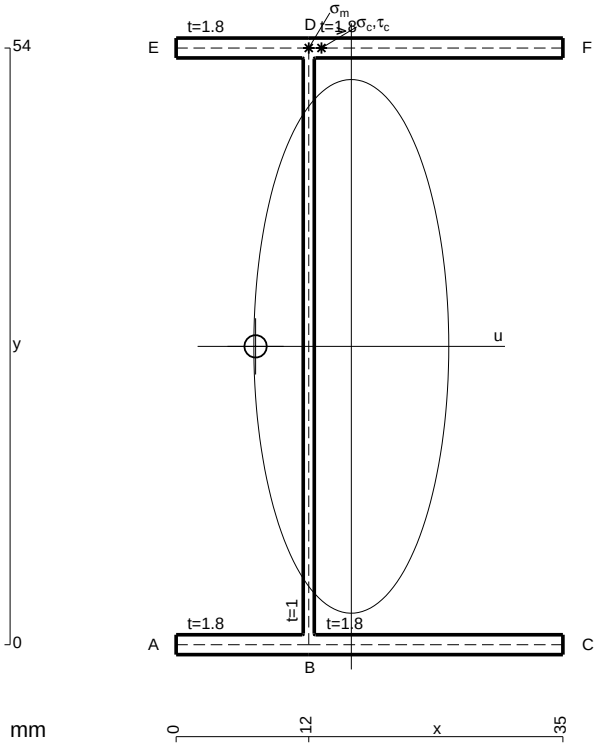
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

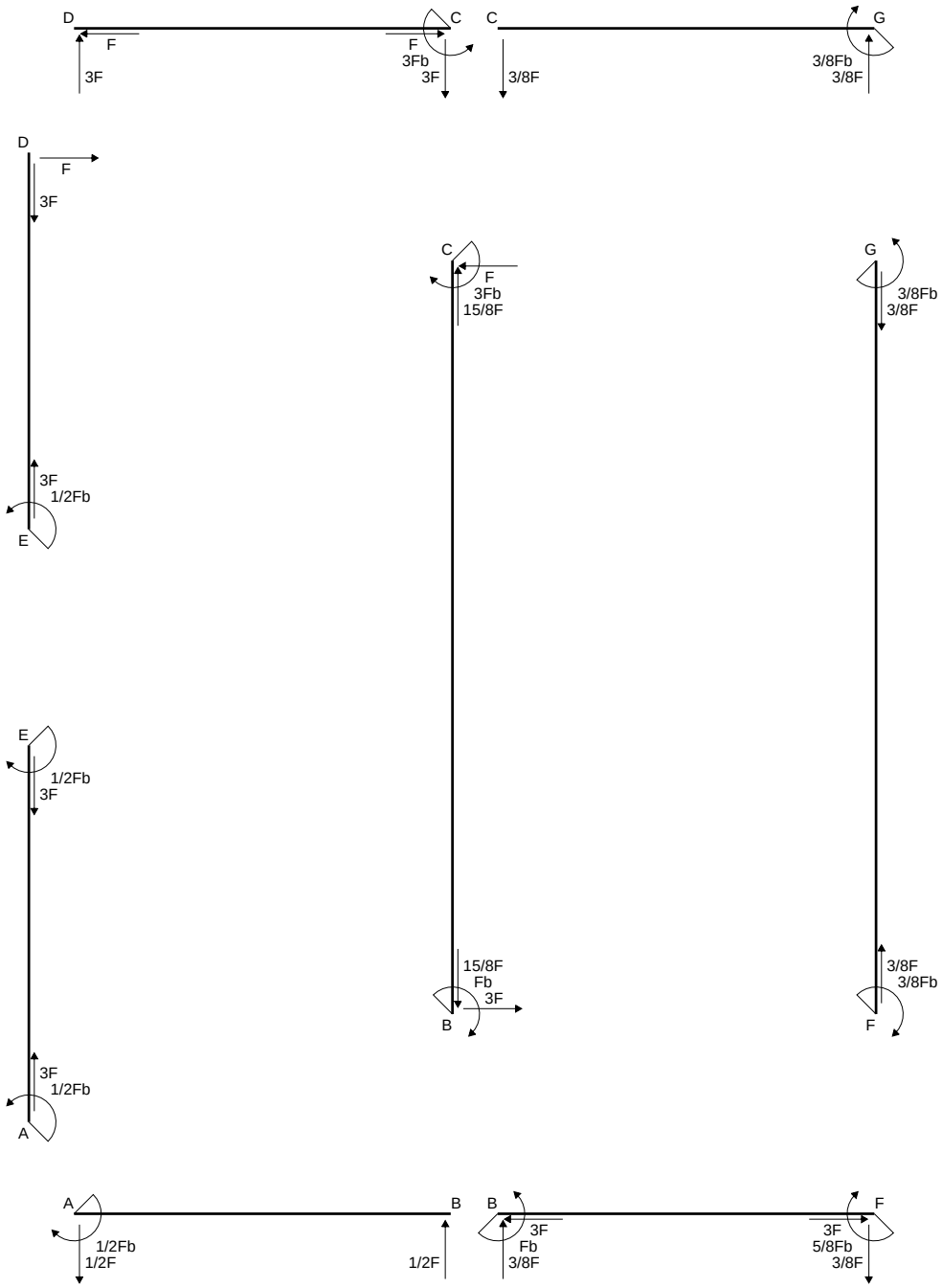
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

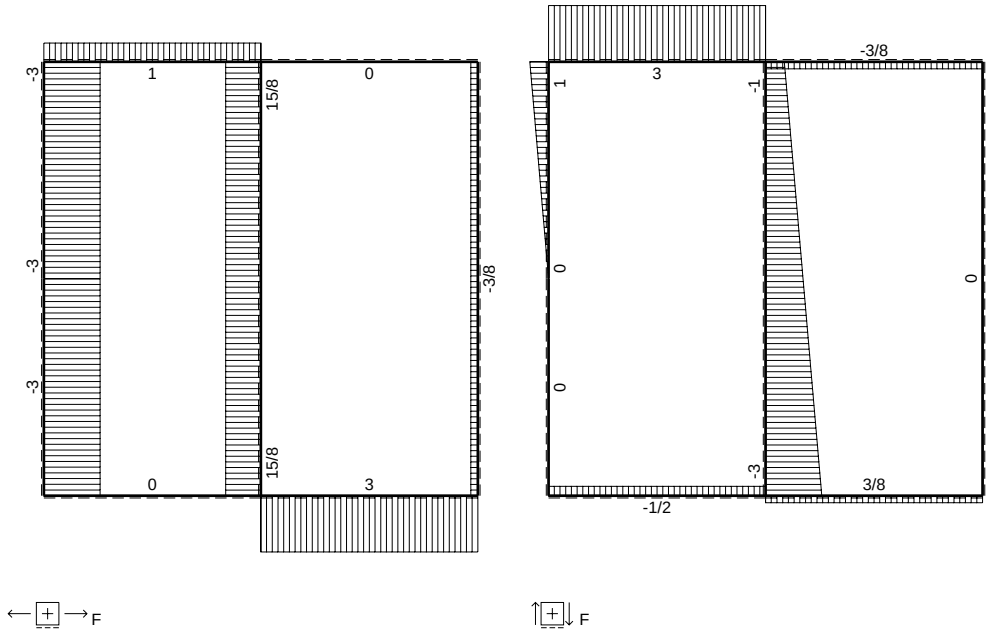


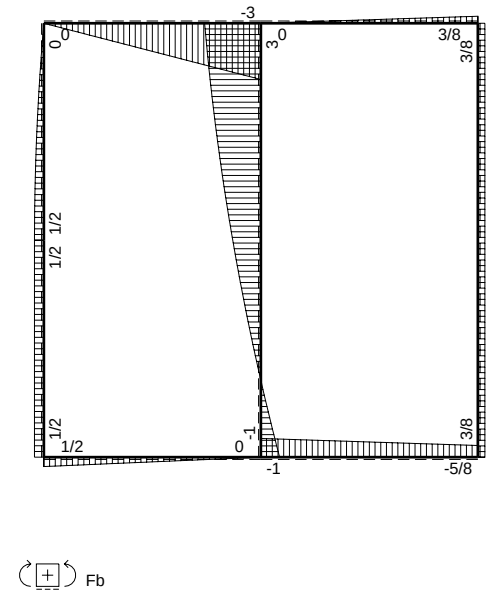
A = 180. mm^2
J_u = 104976. mm^4
J_v = 14006. mm^4
J_t = 154.1 mm^4
x_o = -8.663 mm
x_g = 15.85 mm
N = 390. N
T_y = 1170. N
M_x = -795600. Nmm
x_m = 12. mm
y_m = 54. mm
u_m = -3.85 mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 206.8 N/mm^2
x_c = 12. mm
y_c = 54. mm
u_c = -3.85 mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 206.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 125.3 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.921 N/mm^2
$\tau_o = Tx_o t/J_t$ = 118.4 N/mm^2
t_c = 702. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 299.8 N/mm^2

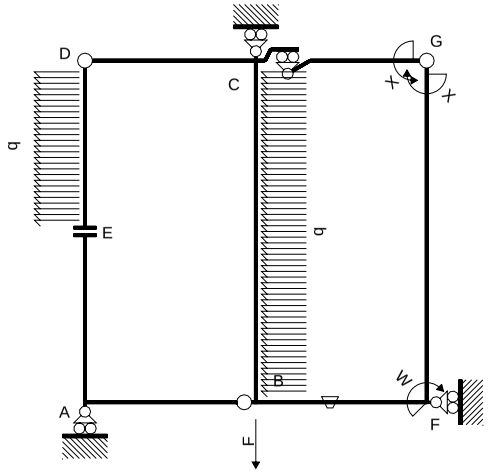
CI08.xxxx.106

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

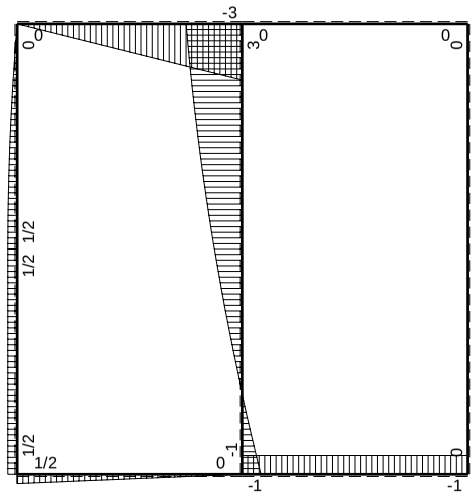

D
F
3F
C
F
3Fb
3F
C
3/8F
G
3/8Fb
3/8F
D
F
3F
3F
1/2Fb
E
C
F
3Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3F
15/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F


-3
1
15/8
0
-3/8
-3
-3
15/8
0
3
1
3
-1
-3/8
0
0
0
-3
3/8
-1/2
F
F

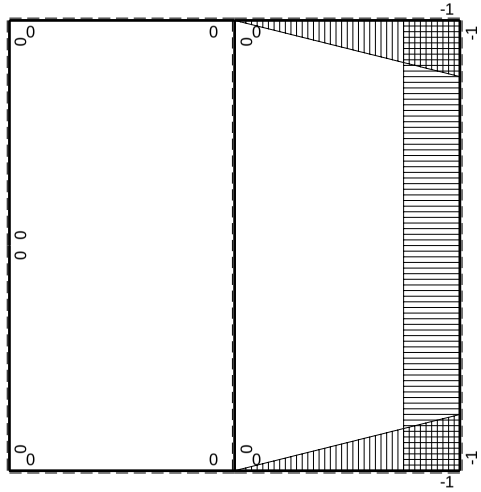

-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 988213 CI08.xxxx.107

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

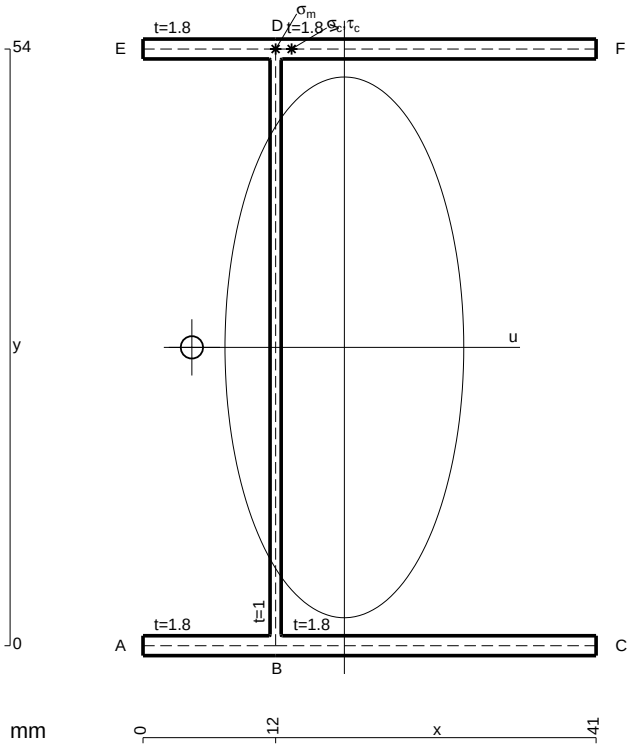
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



$A = 201.6\ mm^2$

$J_u = 120722.\ mm^4$

$J_v = 23533.\ mm^4$

$J_t = 177.4\ mm^4$

$x_o = -13.8\ mm$

$x_g = 18.22\ mm$

$N = 440.\ N$

$T_y = 1320.\ N$

$M_x = -963600.\ Nmm$

$x_m = 12.\ mm$

$y_m = 54.\ mm$

$u_m = -6.223\ mm$

$v_m = 27.\ mm$

$\sigma_m = N/A - Mv/J_u = 217.7\ N/mm^2$

$x_c = 12.\ mm$

$y_c = 54.\ mm$

$u_c = -6.223\ mm$

$v_c = 27.\ mm$

$\sigma_c = N/A - Mv/J_u = 217.7\ N/mm^2$

$\tau_c = \tau_g + \tau_{ou} = 193.4\ N/mm^2$

$\tau_g = TS^*/tJ_u = 8.561\ N/mm^2$

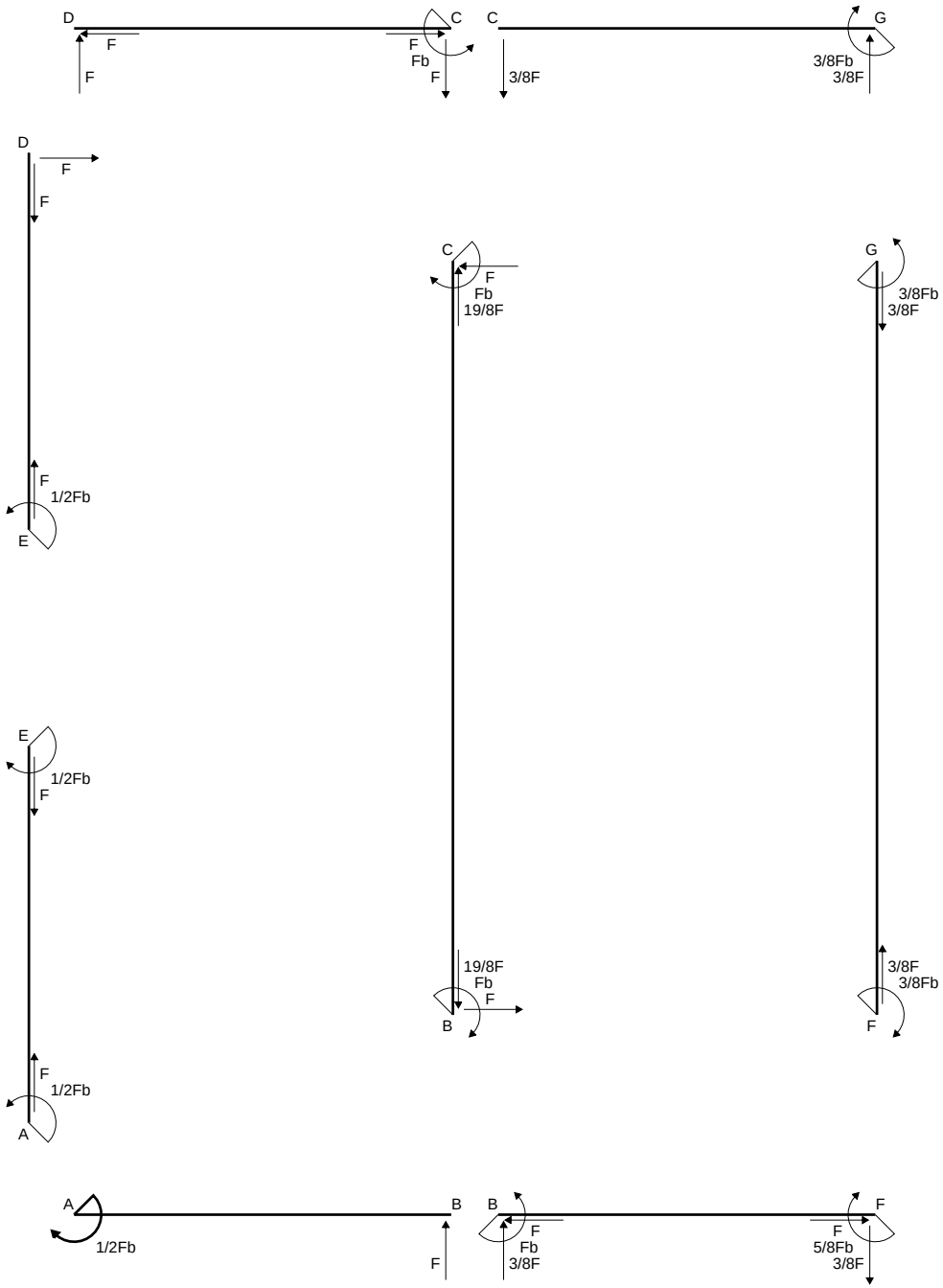
$\tau_o = Tx_o t/J_t = 184.8\ N/mm^2$

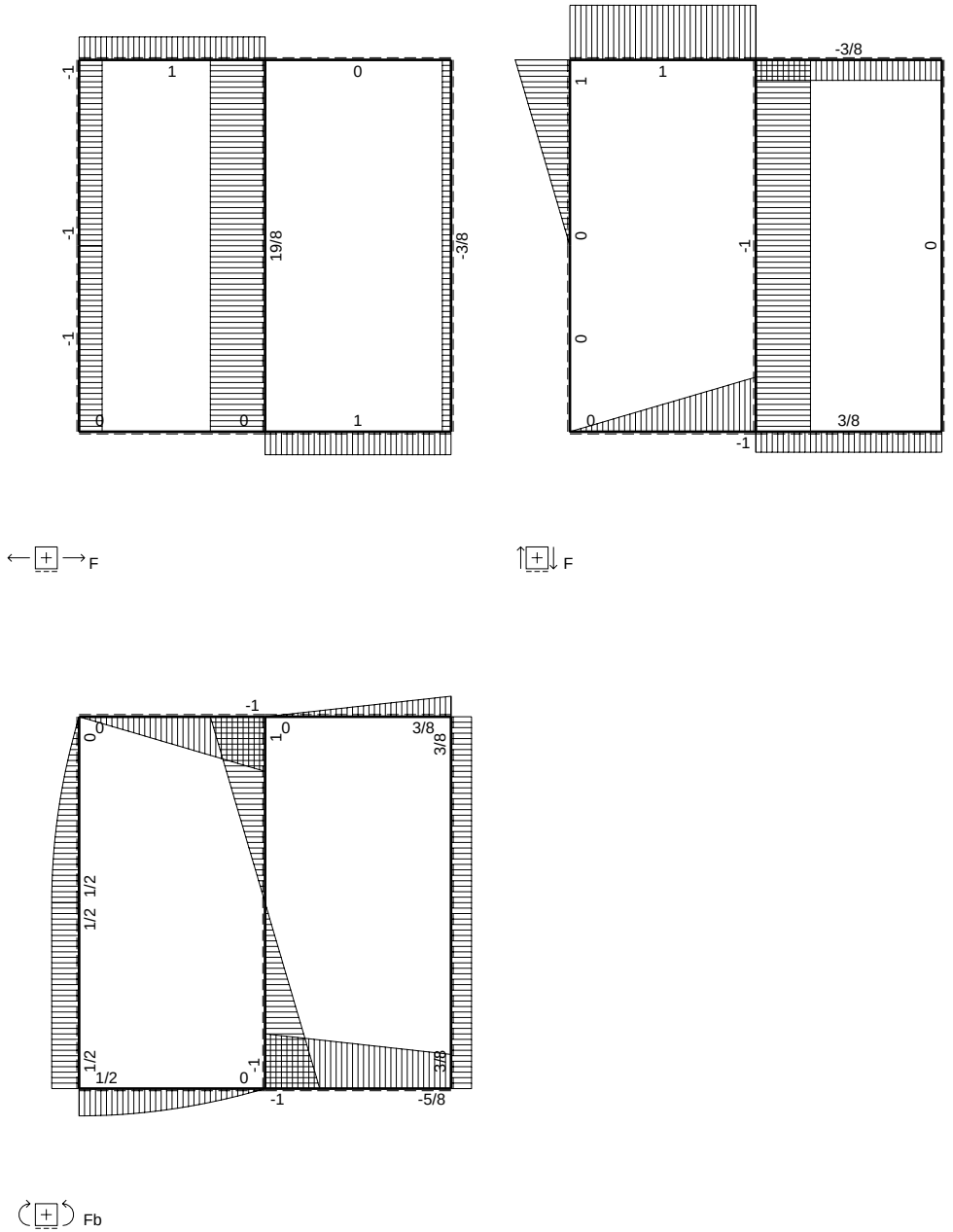
$t_c = 792.\ mm$

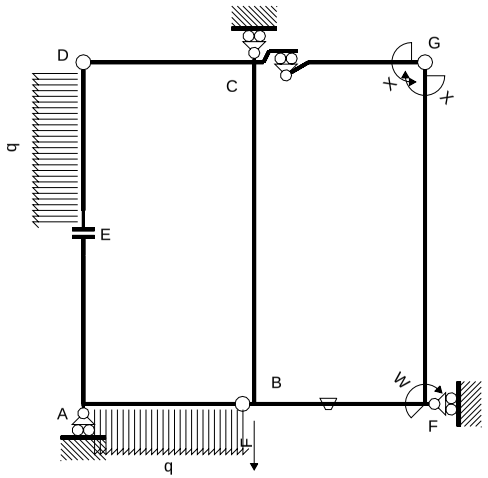
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 399.5\ N/mm^2$

CI08.xxxx.107

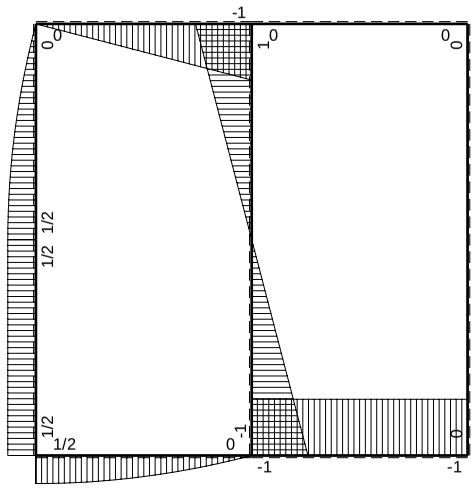
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
19/8F
G
3/8Fb
3/8F
E
1/2Fb
F
19/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F

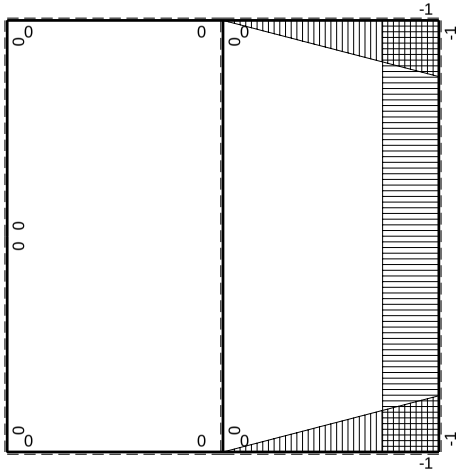

-1
1
0
-1
19/8
-3/8
-1
0
0
1
F
-3/8
1
1
0
-1
0
0
3/8
-1
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 236654 CI08.xxxx.108

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

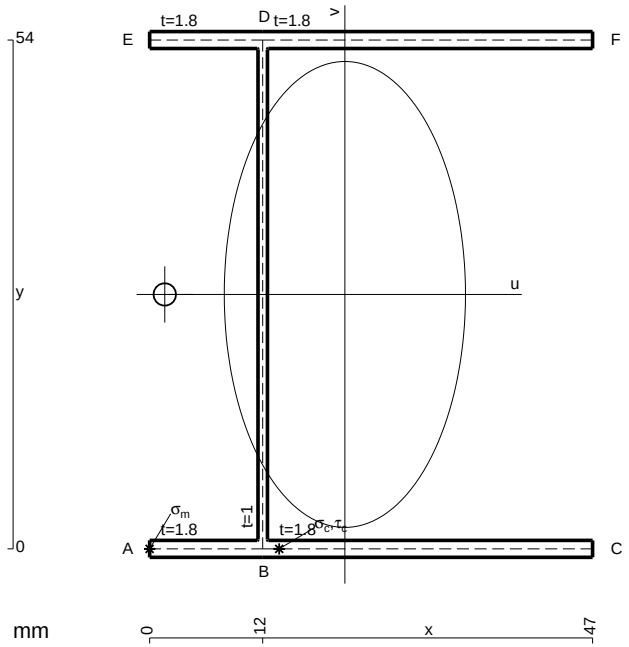
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

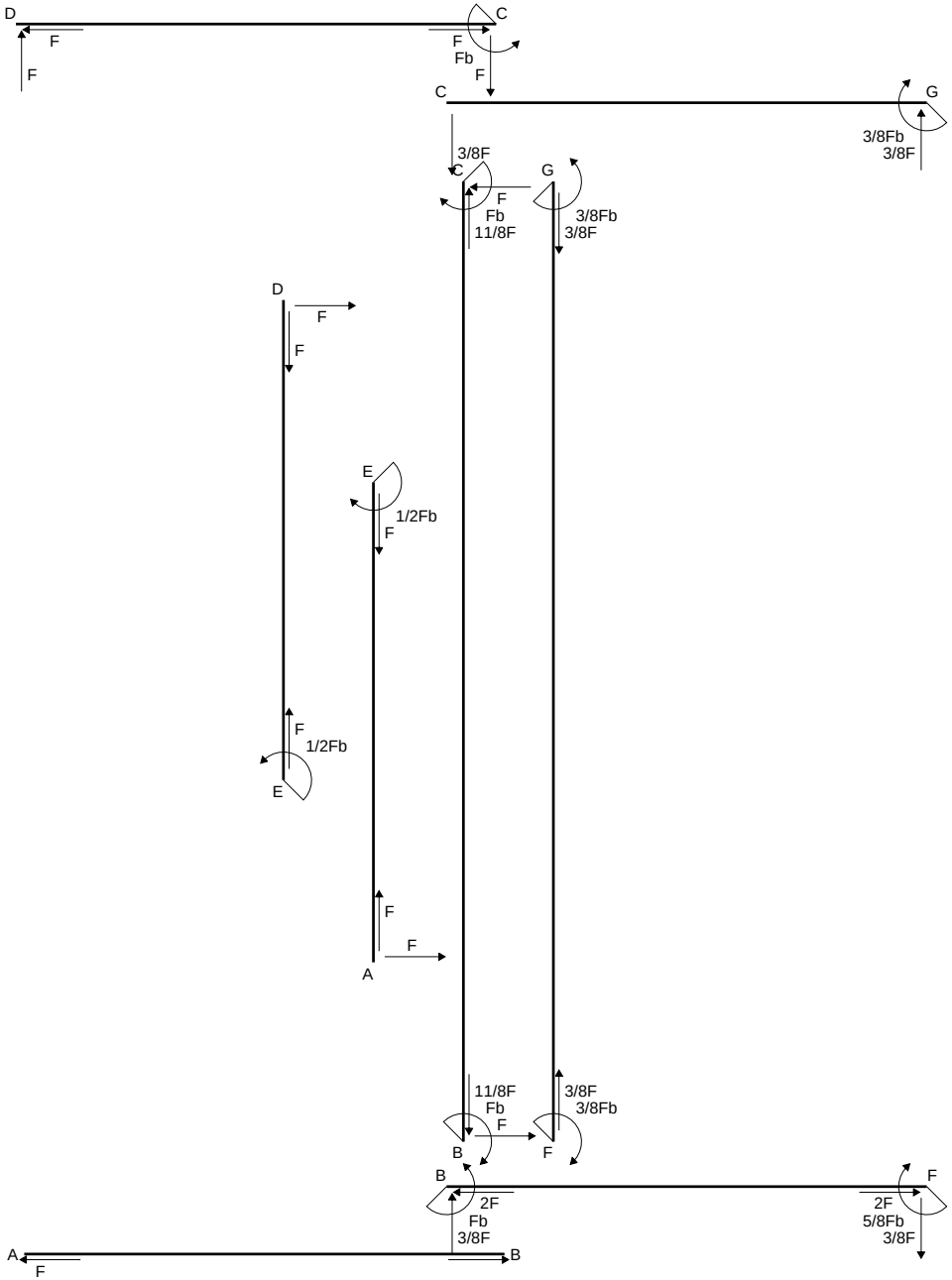
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

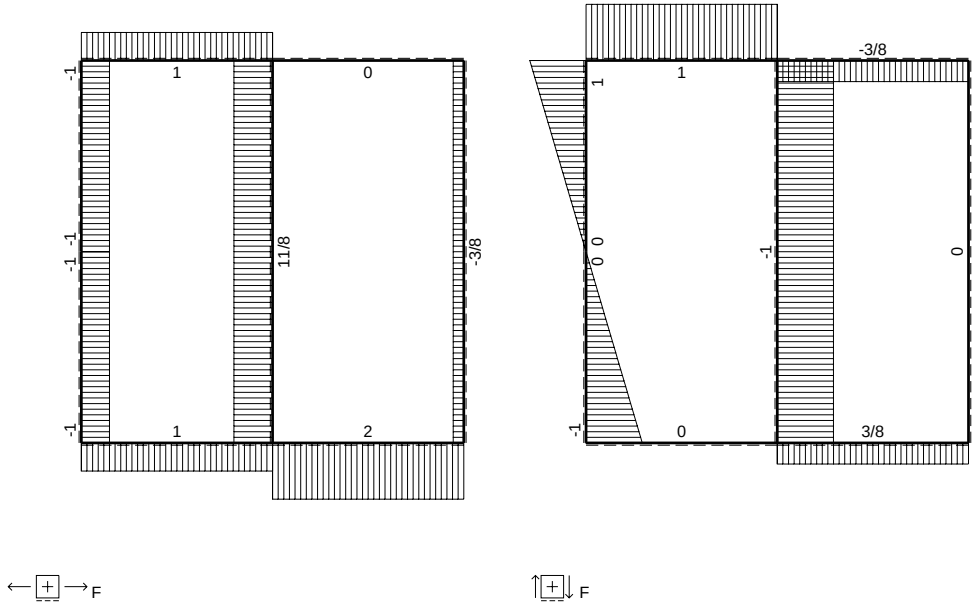


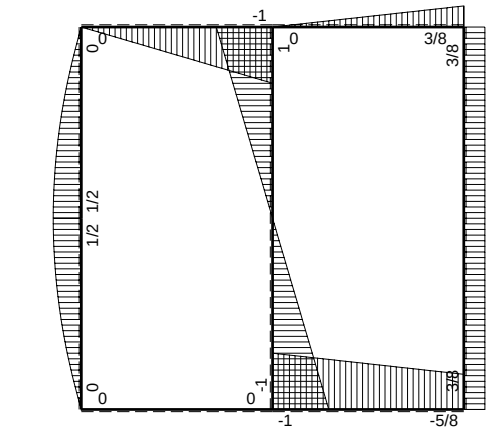
A = 223.2 mm^2
J_u = 136469. mm^4
J_v = 36561. mm^4
J_t = 200.7 mm^4
x_o = -19.11 mm
x_g = 20.72 mm
N = 3349. N
T_y = -1410. N
M_x = 1085700. Nmm
u_m = -20.72 mm
v_m = -27. mm
$\sigma_m = N/A - Mv/J_u$ = 229.8 N/mm^2
x_c = 12. mm
u_c = -8.718 mm
v_c = -27. mm
$\sigma_c = N/A - Mv/J_u$ = 229.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 251.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.764 N/mm^2
$\tau_o = Tx_o t/J_t$ = 241.6 N/mm^2
t_c = 2538. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 492.4 N/mm^2

CI08.xxxx.108

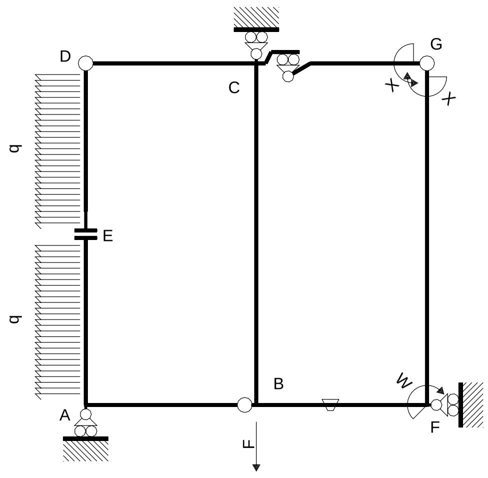
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
11/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
11/8F
Fb
F
3/8F
3/8Fb
F
B
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
F
B

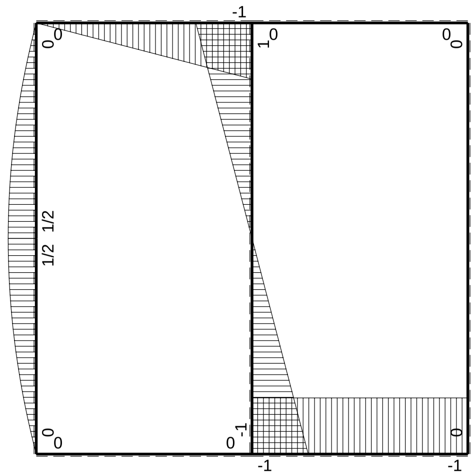

-1
1
0
-1
-1
11/8
-3/8
-1
1
2
F
-3/8
1
1
0 0
-1
0
-1
0
3/8
F


-1
0
0
1
0
3/8
3/8
1/2 1/2
0
0
0
-1
-1
3/8
-1
-5/8

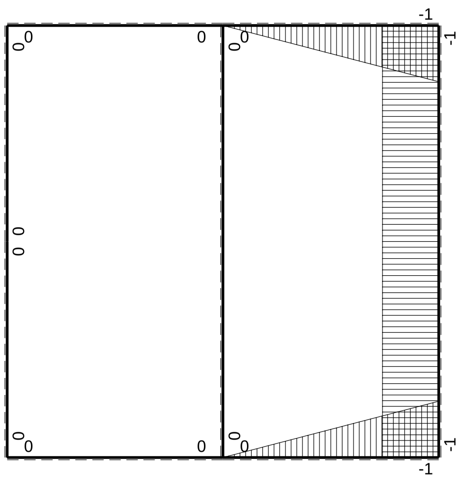

Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

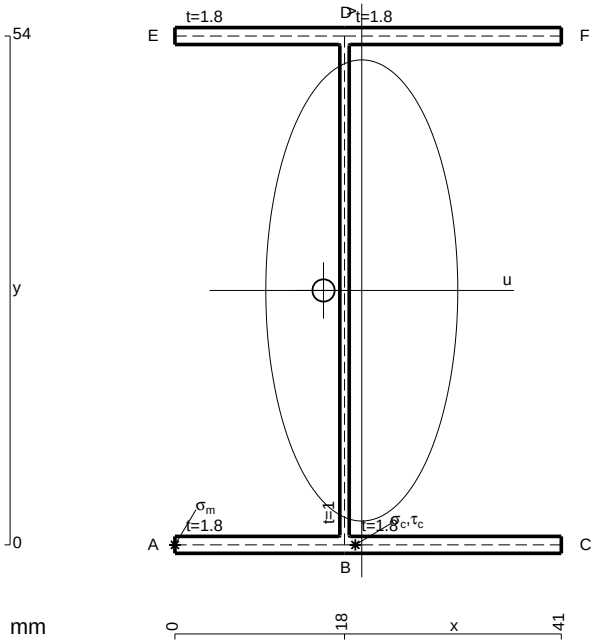
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

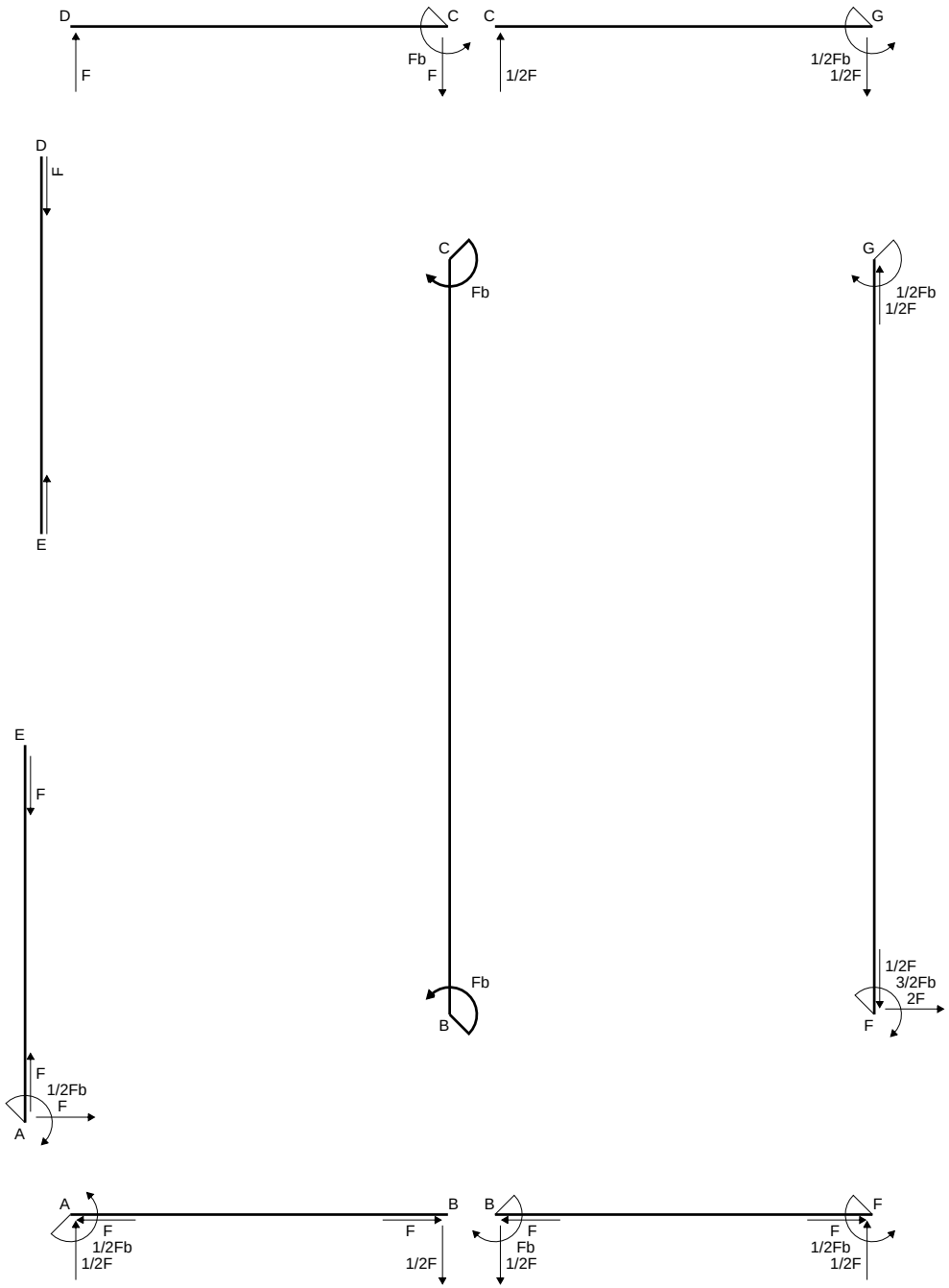
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

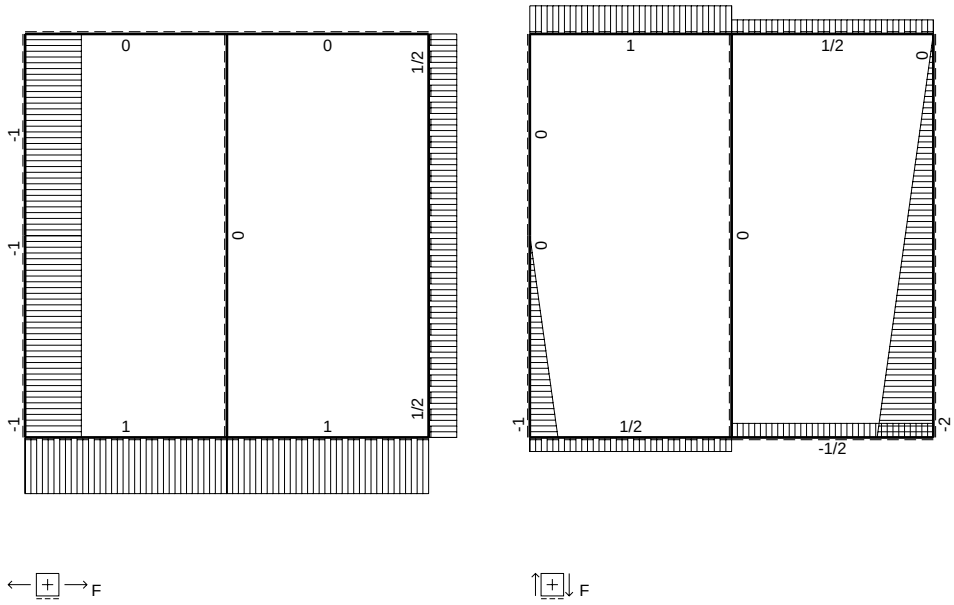


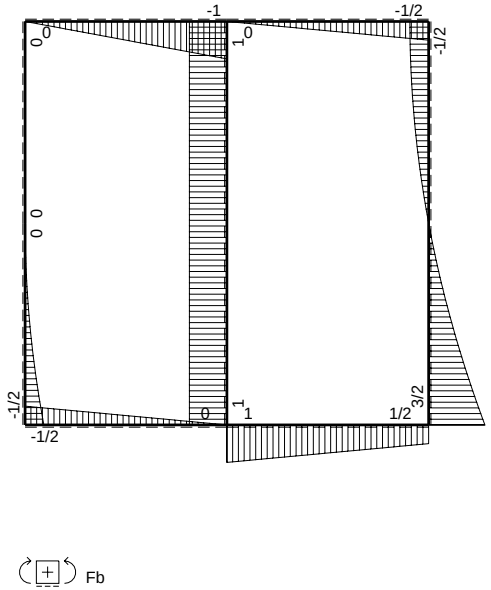
A = 201.6 mm^2
J_u = 120722. mm^4
J_v = 20923. mm^4
J_t = 177.4 mm^4
x_o = -4.059 mm
x_g = 19.83 mm
N = 1746. N
T_y = -1270. N
M_x = 1028700. Nmm
u_m = -19.83 mm
v_m = -27. mm
$\sigma_m = N/A - Mv/J_u$ = 238.7 N/mm^2
x_c = 18. mm
u_c = -1.83 mm
v_c = -27. mm
$\sigma_c = N/A - Mv/J_u$ = 238.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 58.83 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.533 N/mm^2
$\tau_o = Tx_o t/J_t$ = 52.3 N/mm^2
t_c = 2286. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 259.6 N/mm^2

CI08.xxxx.109

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
G
1/2Fb
1/2F
E
F
F
1/2Fb
F
A
Fb
B
1/2F
3/2Fb
2F
F
A
B
B
F
F
1/2Fb
1/2F
F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


0
0
1/2
-1
0
-1
0
1
1
1/2
F
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F


-1
-1/2
0
0
1
0
-1/2
0 0
-1/2
0
1
1
1/2
3/2
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

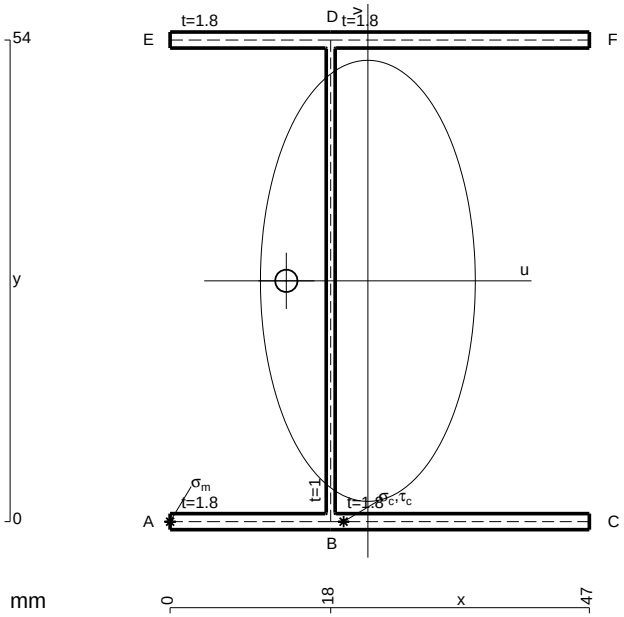
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

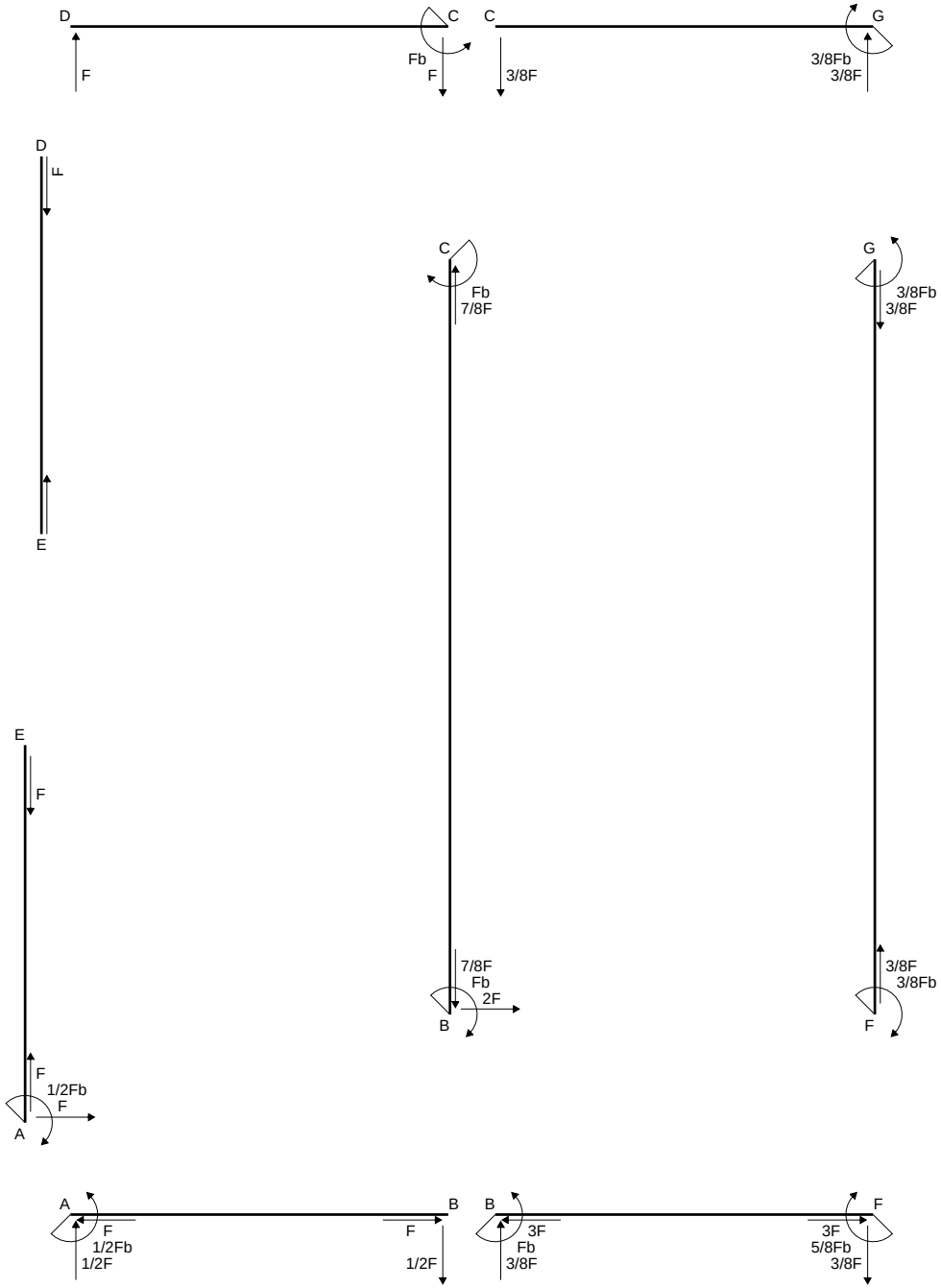
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

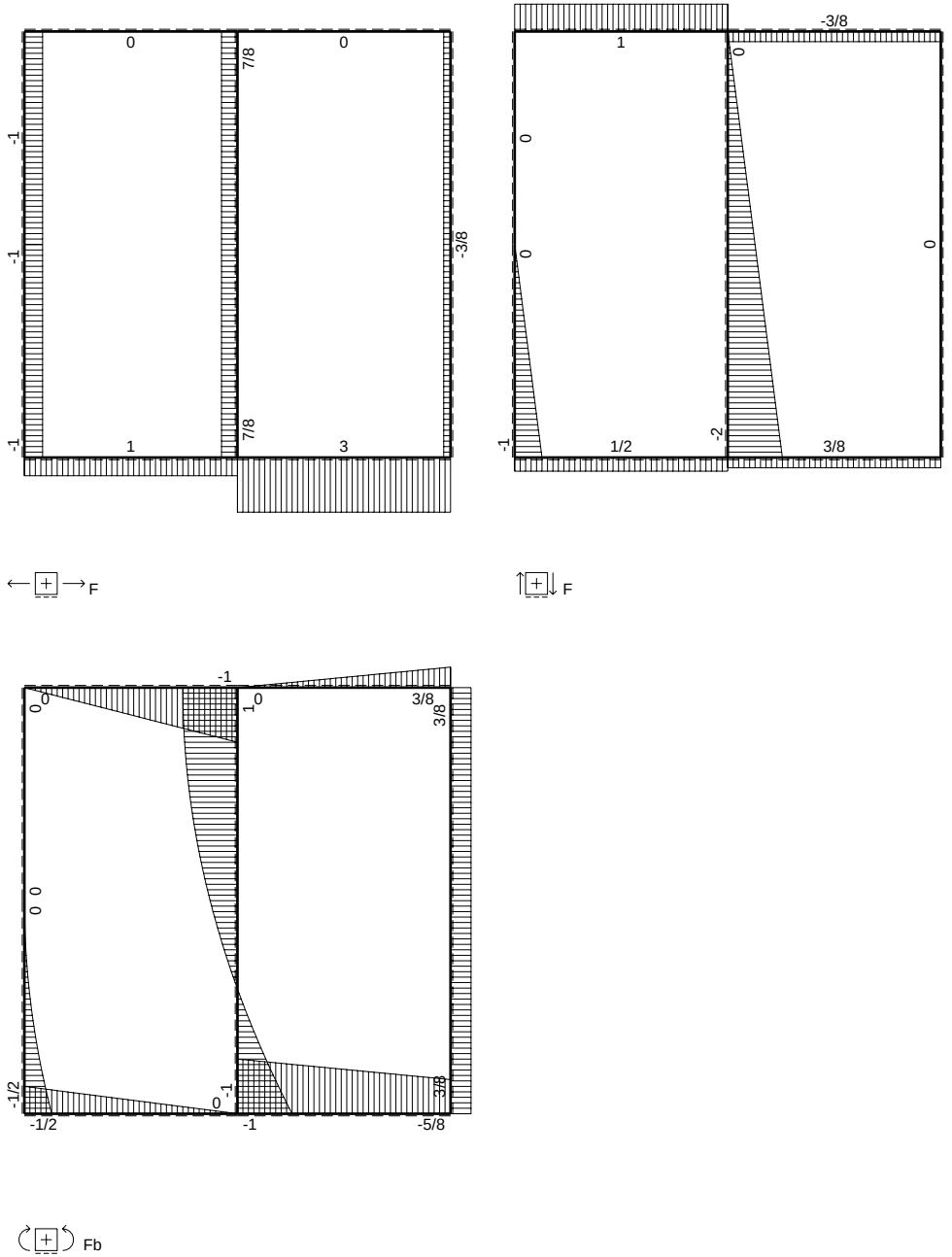


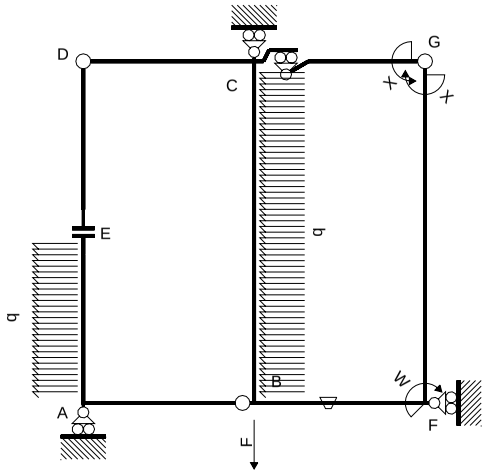
A = 223.2 mm^2
J_u = 136469. mm^4
J_v = 32385. mm^4
J_t = 200.7 mm^4
x_o = -9.141 mm
x_g = 22.17 mm
T_y = 2350. N
M_x = -1010500. Nmm
u_m = -22.17 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -199.9 N/mm^2
x_c = 18. mm
u_c = -4.169 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -199.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 206.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.48 N/mm^2
$\tau_o = Tx_o t/J_t$ = 192.6 N/mm^2
t_c = 4230. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 409.1 N/mm^2

CI08.xxxx.110

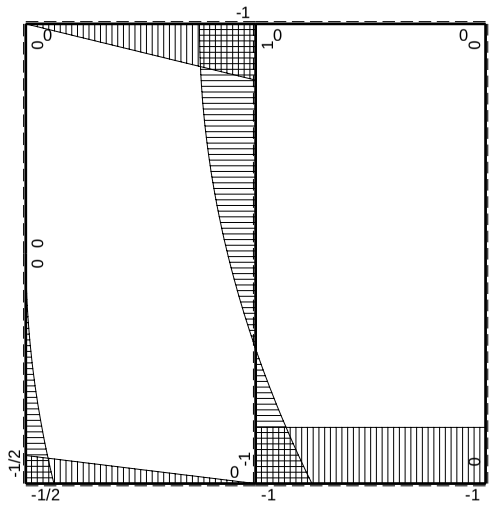
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
7/8F
G
3/8Fb
3/8F
E
F
F
1/2Fb
F
A
7/8F
Fb
2F
B
3/8F
3/8Fb
F
A
B
B
F
F
1/2Fb
1/2F
F
1/2F
3F
Fb
3/8F
3F
5/8Fb
3/8F

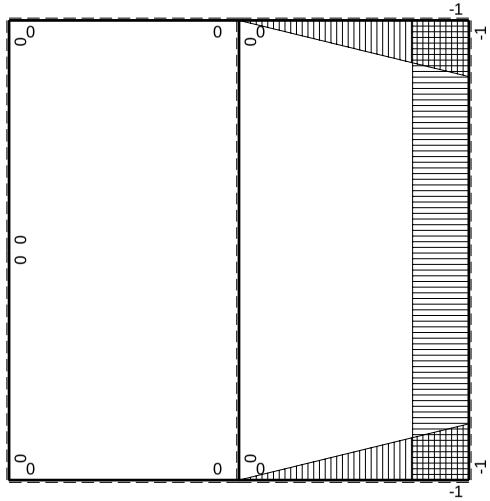

0
0
7/8
-1
-1
-3/8
-1
1
7/8
3
F
1
-3/8
0
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0
0
0
0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 240962 CI08.xxxx.111

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

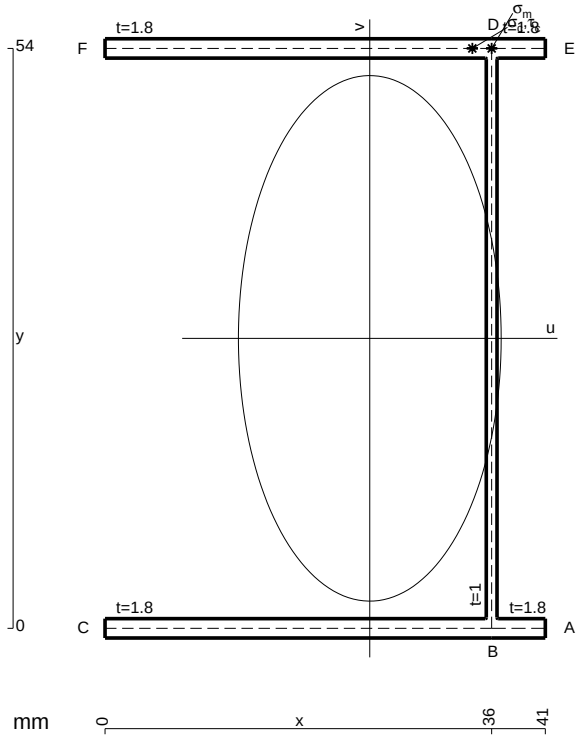
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b]_o^b\ \theta$

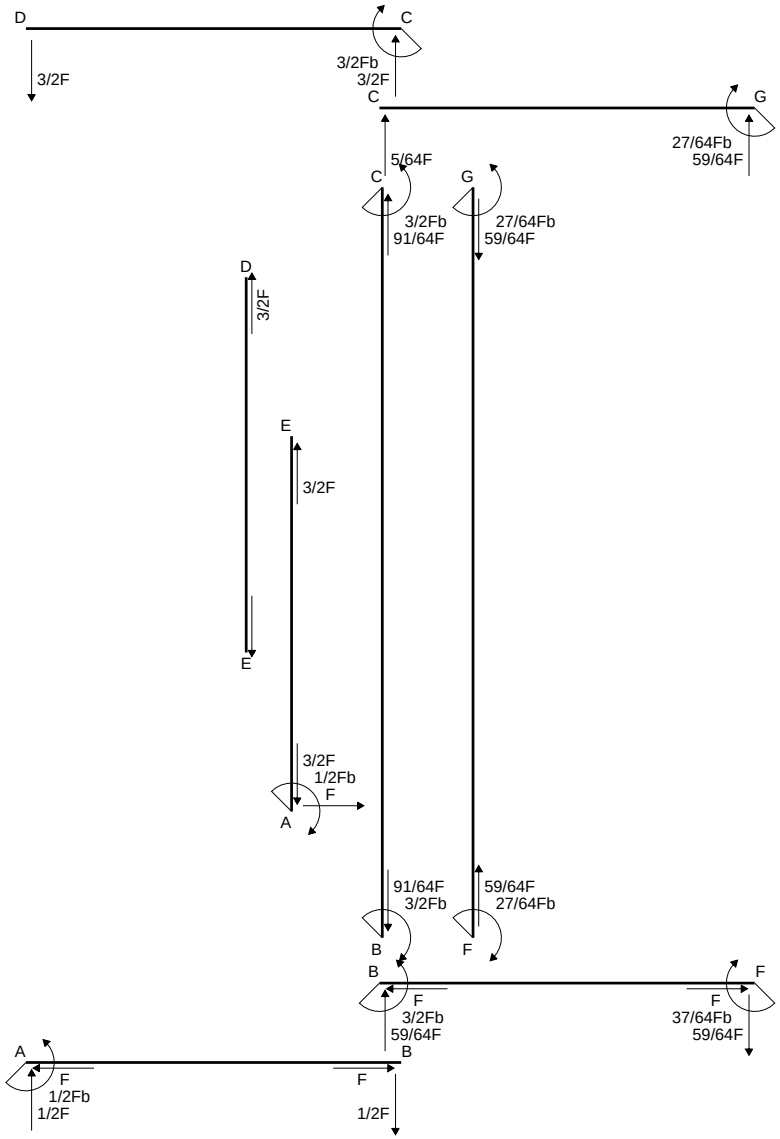
$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ =\ Fb^2/EJ$

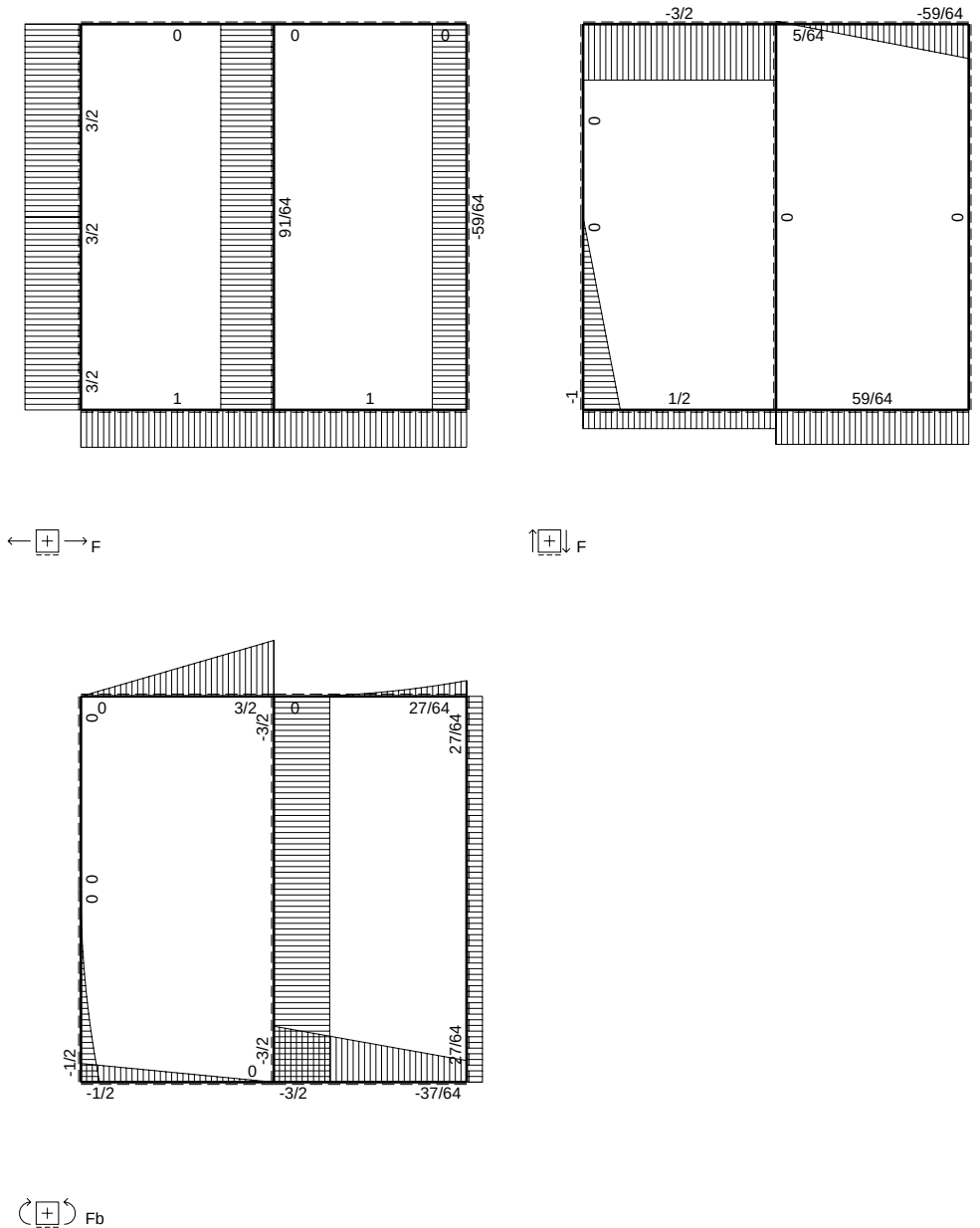


A = 201.6 mm^2
J_u = 120722. mm^4
J_v = 30175. mm^4
J_t = 177.4 mm^4
x_o = 25.16 mm
x_g = 24.65 mm
N = 1671. N
T_y = -3820. N
M_x = -897700. Nmm
x_m = 36. mm
y_m = 54. mm
u_m = 11.35 mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 209.1 N/mm^2
x_c = 36. mm
y_c = 54. mm
u_c = 11.35 mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 209.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 1006. N/mm^2
$\tau_g = TS^*/tJ_u$ = 30.76 N/mm^2
$\tau_o = Tx_o t/J_t$ = 975.3 N/mm^2
t_c = 3438. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1755. N/mm^2

CI08.xxxx.111

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
3/2
91/64
-59/64
1
1
F
-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64
F
0 0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	0	1/2Fx	0	0	0	0		
BA b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
	0	-3/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DC b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
EA b	0	-1/2qx^2	0	0	0	0	0+0	0
	0	1/2Fb-Fx+1/2qx^2	0	0	0	0		
AE b	0	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(7/12+1/2)Fb^2/EJ	1/3Xb/EJ
BF b	-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
FB b	1-x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
GC b	-1+x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
CG b	x/b	0	0	0	0	1	0+0	2Xb/EJ
FG 2b	-1	0	0	0	0	1		
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						9/8Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 961156 CI08.xxxx.112

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

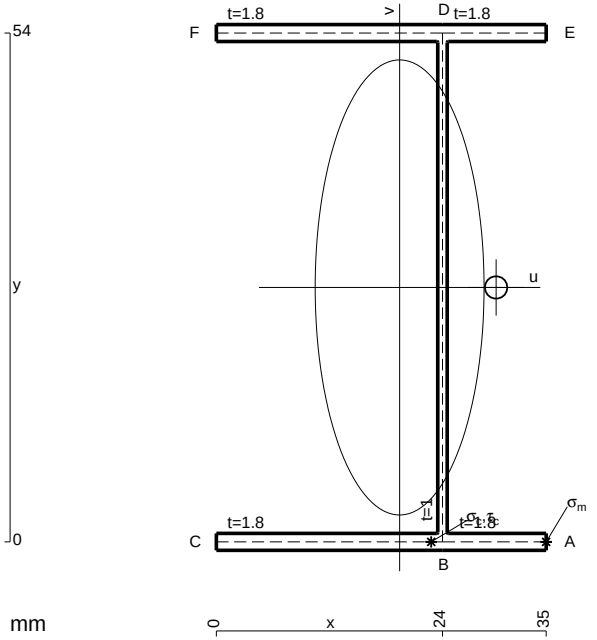
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

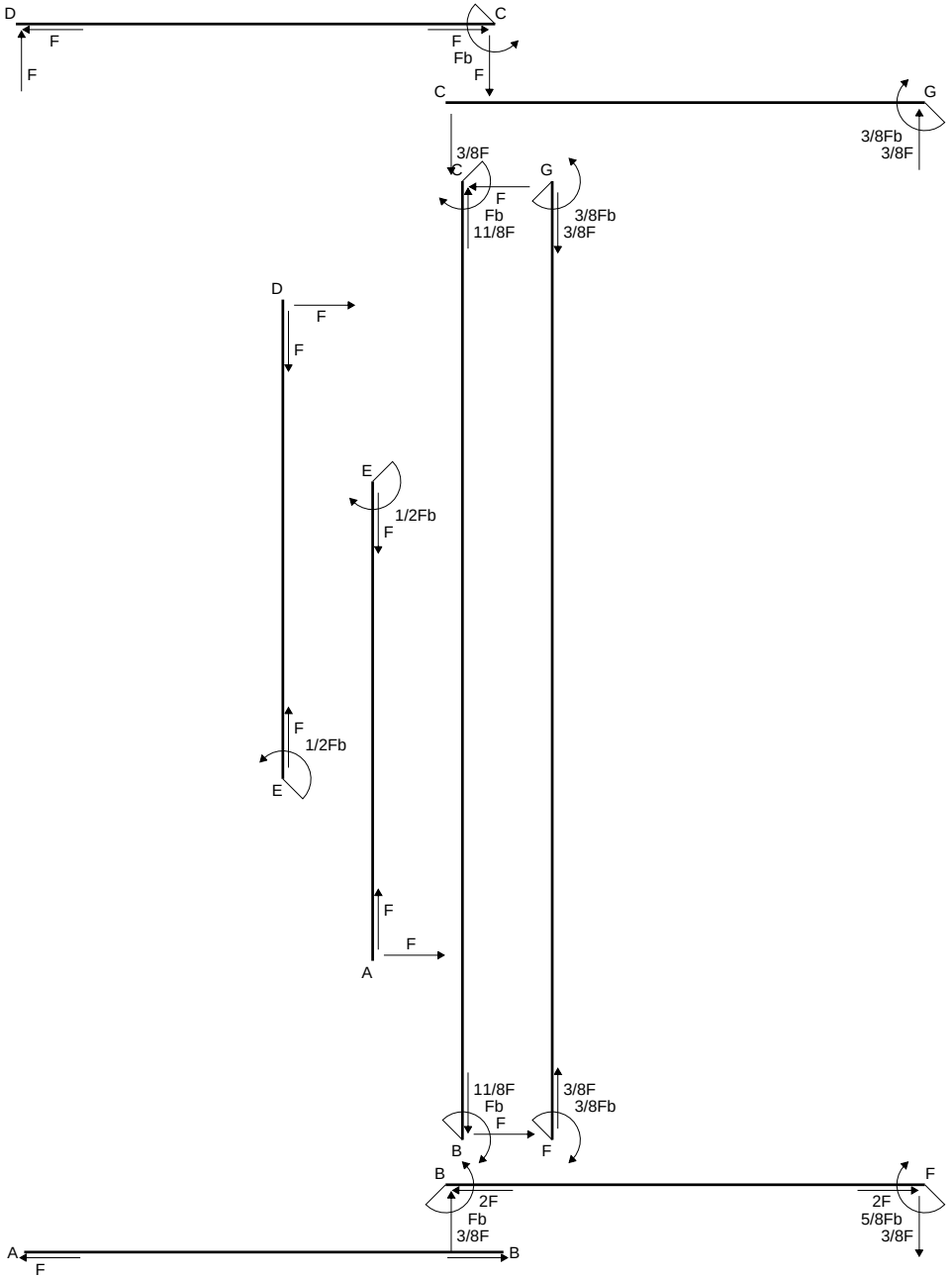
$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

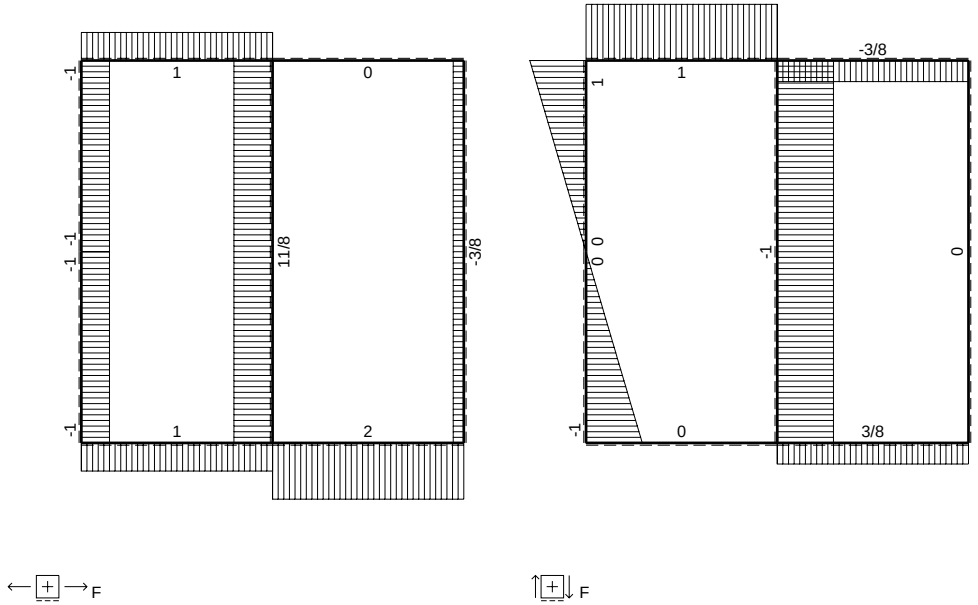


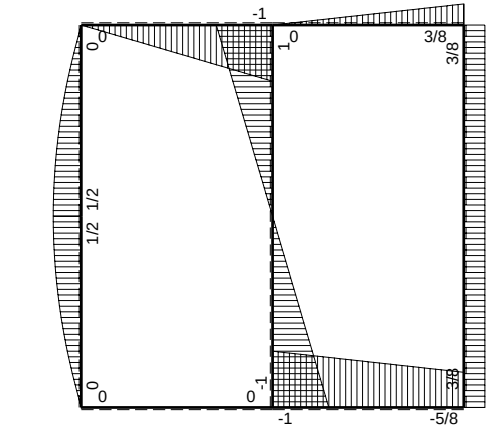
A = 180. mm^2
J_u = 104976. mm^4
J_v = 14460. mm^4
J_t = 154.1 mm^4
x_o = 10.24 mm
x_g = 19.45 mm
T_y = -1665. N
M_x = 849150. Nmm
x_m = 35. mm
u_m = 15.55 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = 218.4 N/mm^2
x_c = 24. mm
u_c = 4.55 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = 218.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 209.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.28 N/mm^2
$\tau_o = Tx_o t/J_t$ = 199.1 N/mm^2
t_c = 1998. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 423.4 N/mm^2

CI08.xxxx.112

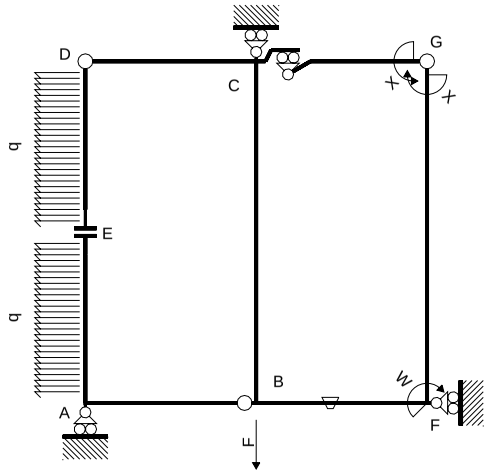
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
11/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
11/8F
Fb
F
3/8F
3/8Fb
F
B
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F

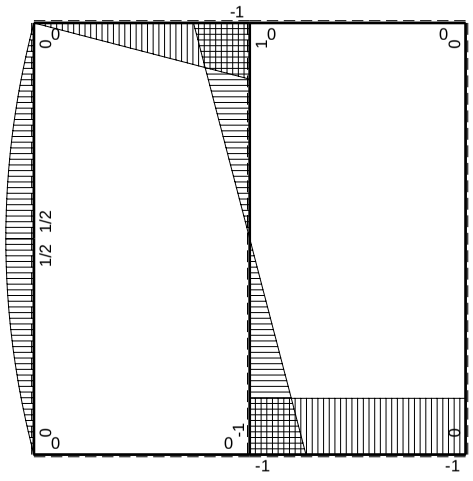

-1
1
0
-1
-1
11/8
-3/8
-1
1
2
F
-3/8
1
1
0 0
-1
0
-1
0
3/8
F


-1
0
0
1
0
3/8
3/8
1/2 1/2
0
0
0
-1
-1
3/8
-1
-5/8

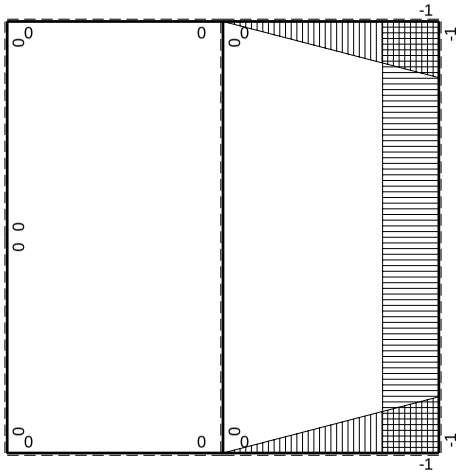

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb-1/2qx^2	0	0	0	0	0+0	0
AE b	0	-Fx+1/2qx^2	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 959354 CI08.xxxx.113

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

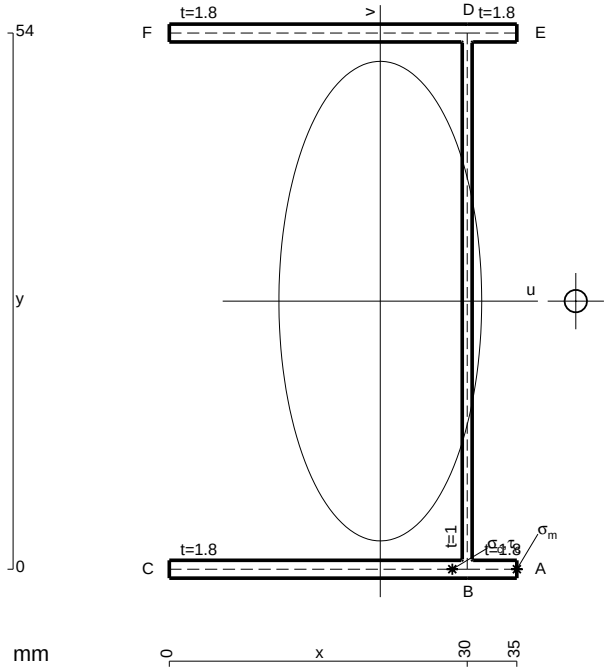
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

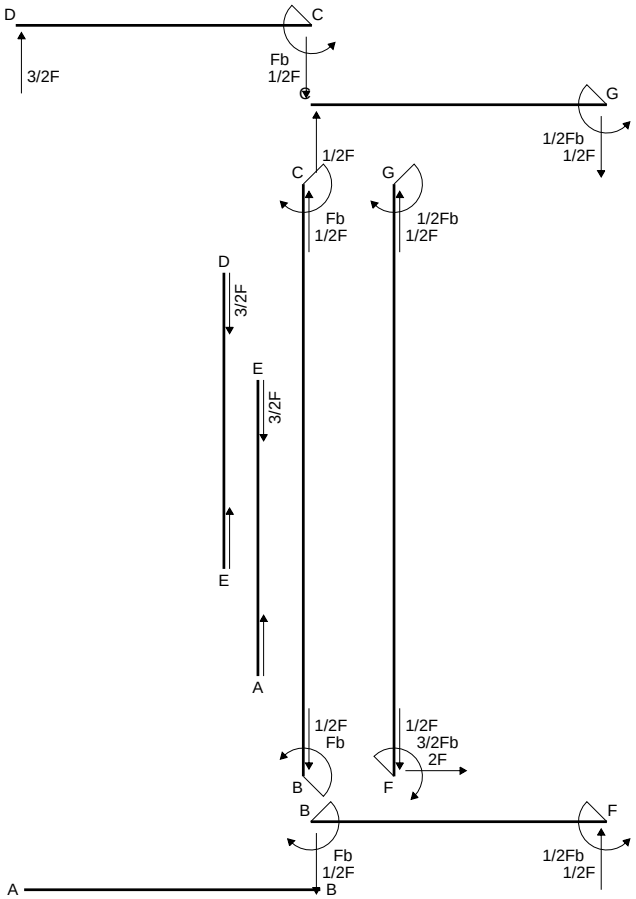
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

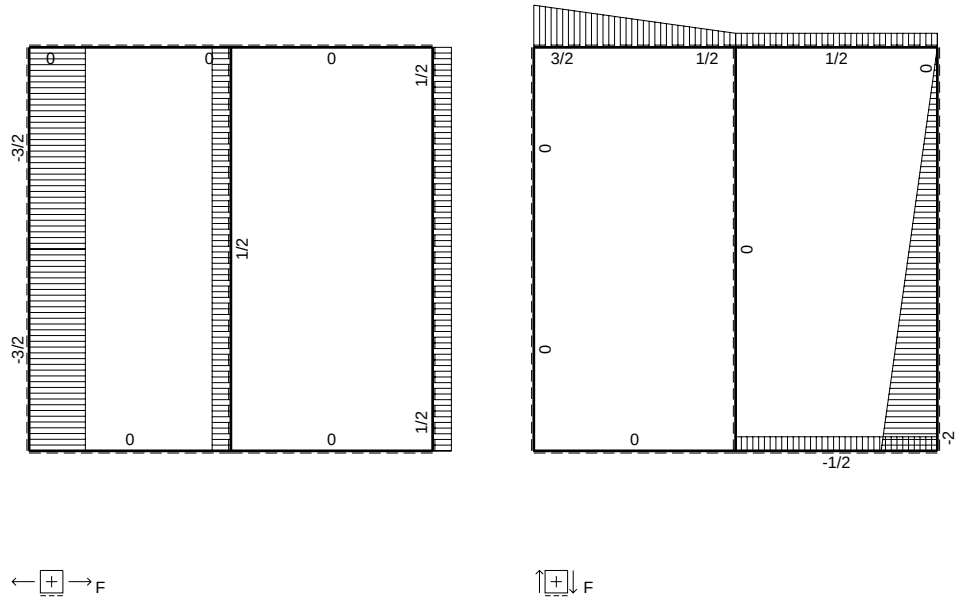


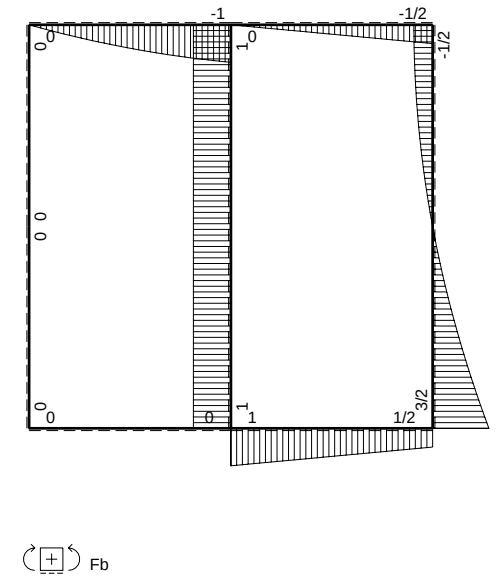
A = 180. mm^2
J_u = 104976. mm^4
J_v = 18769. mm^4
J_t = 154.1 mm^4
x_o = 19.69 mm
x_g = 21.25 mm
N = 2118. N
T_y = -1540. N
M_x = 847000. Nmm
x_m = 35. mm
u_m = 13.75 mm
v_m = -27. mm
$\sigma_m = N/A - Mv/J_u$ = 229.6 N/mm^2
x_c = 30. mm
u_c = 8.75 mm
v_c = -27. mm
$\sigma_c = N/A - Mv/J_u$ = 229.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 366.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.88 N/mm^2
$\tau_o = Tx_o t/J_t$ = 354.2 N/mm^2
t_c = 2772. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 674.4 N/mm^2

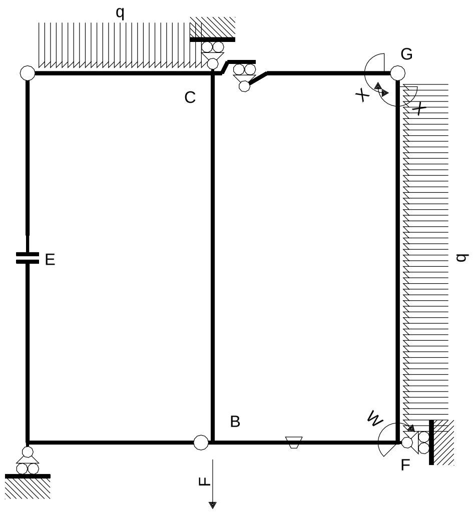
CI08.xxxx.113

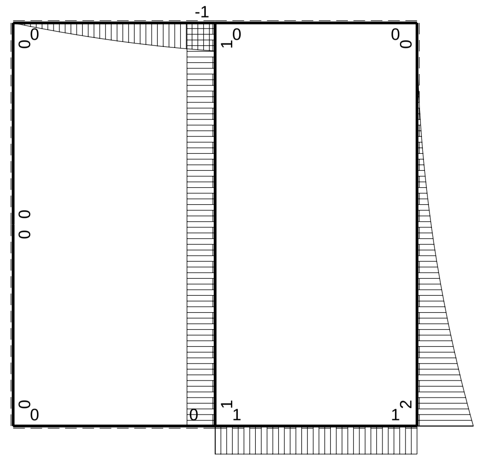
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3/2F
Fb
1/2F
G
1/2Fb
1/2F
1/2F
C
G
Fb
1/2F
1/2Fb
1/2F
D
3/2F
E
3/2F
E
A
1/2F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B

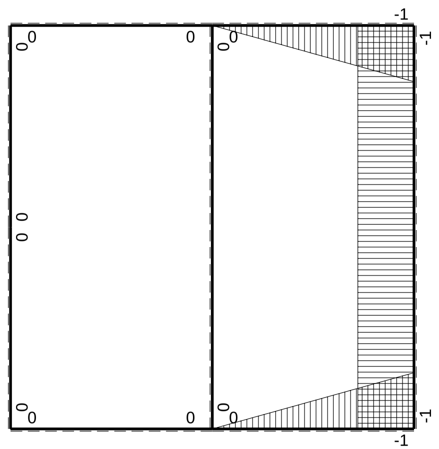

0
0
0
1/2
-3/2
1/2
-3/2
0
0
1/2
3/2
1/2
1/2
0
0
0
0
0
-2
-1/2
F
F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	$1-x/b$	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [-1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [-\ x +1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b]_o^b\ \theta$

$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b +1/2\ b)\ \theta\ = 0$

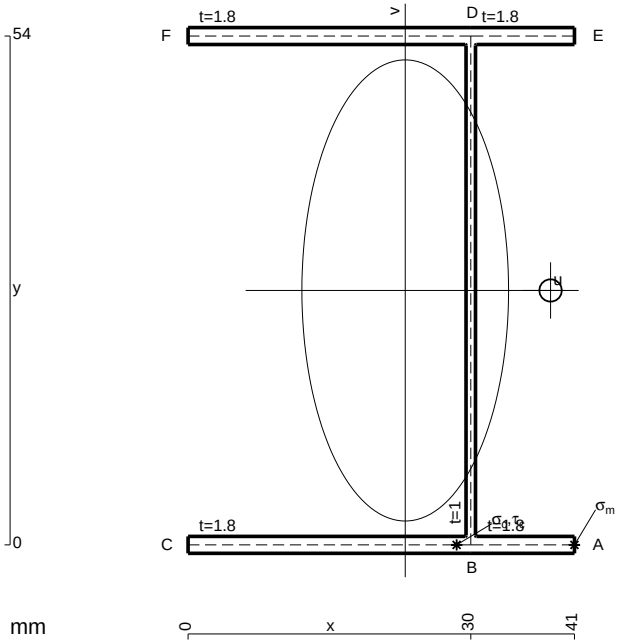
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b)\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b)\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 241822 CI08.xxxx.114

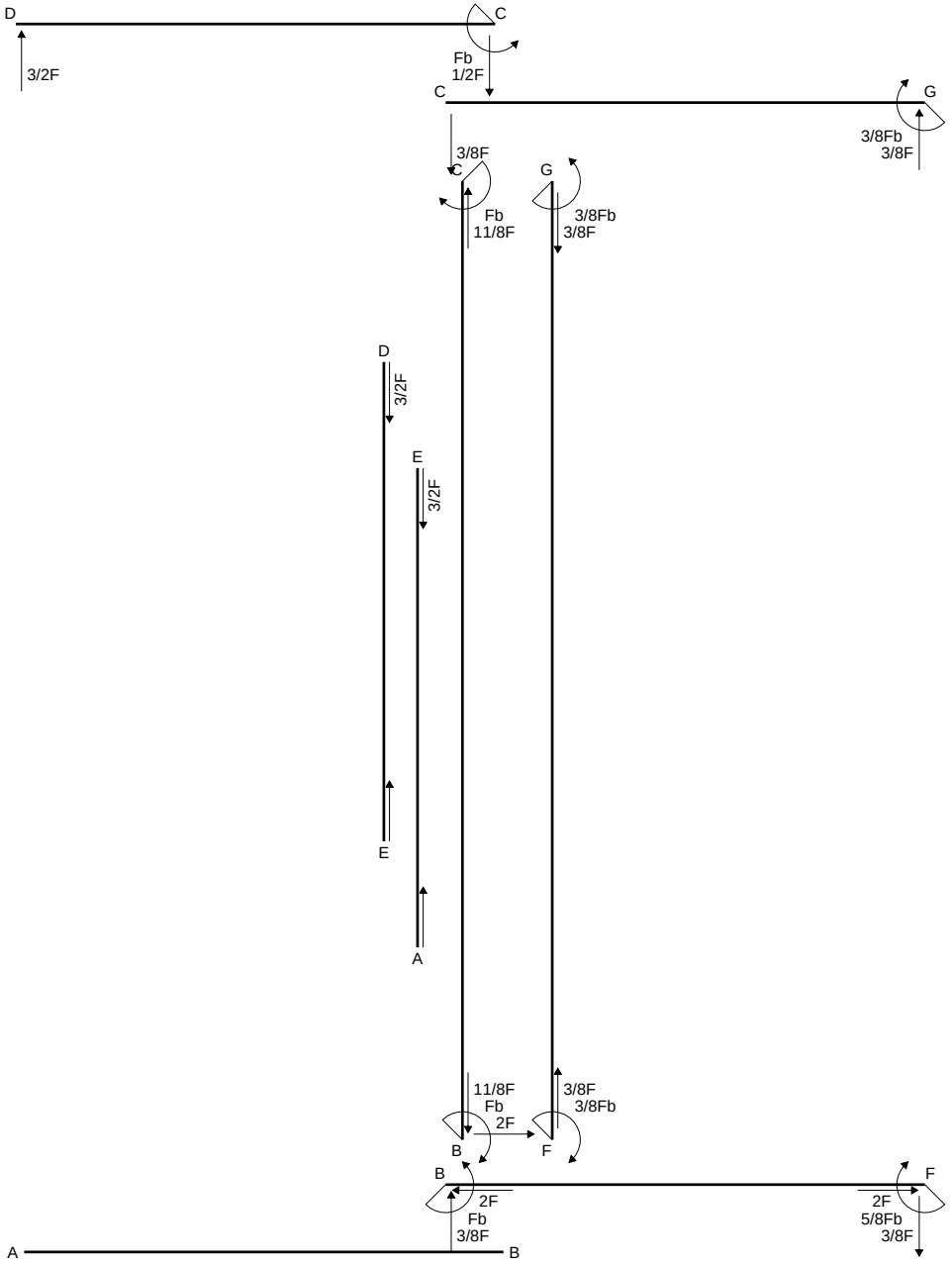


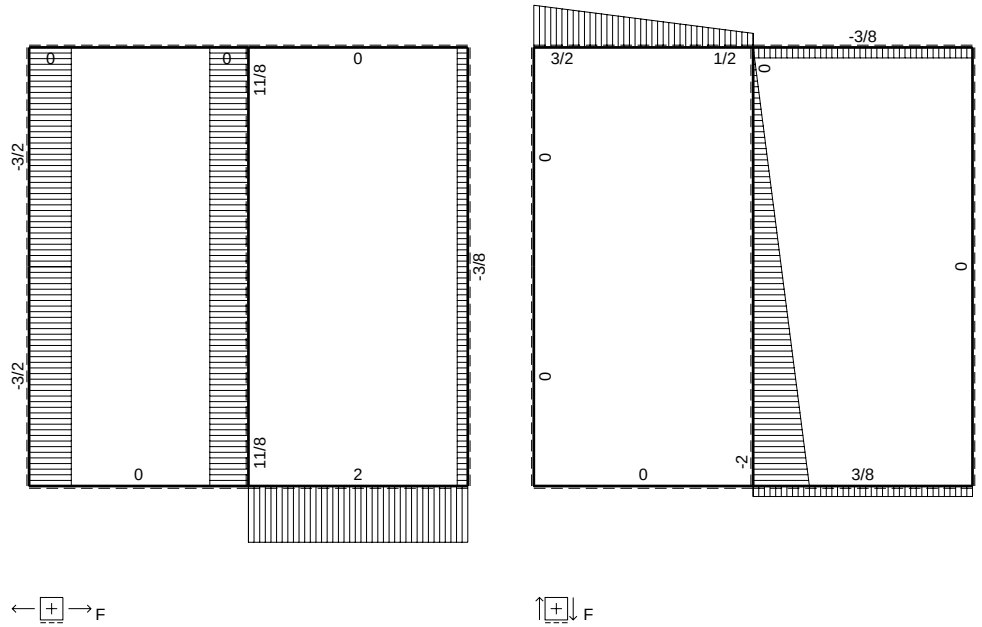
A = 201.6 mm^2
J_u = 120722. mm^4
J_v = 24244. mm^4
J_t = 177.4 mm^4
x_o = 15.42 mm
x_g = 23.04 mm
T_y = 2860. N
M_x = -1072500. Nmm
x_m = 41. mm
u_m = 17.96 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -239.9 N/mm^2
x_c = 30. mm
u_c = 6.955 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -239.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 466.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 19.19 N/mm^2
$\tau_o = Tx_ot/J_t$ = 447.5 N/mm^2
t_c = 5148. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 843.2 N/mm^2

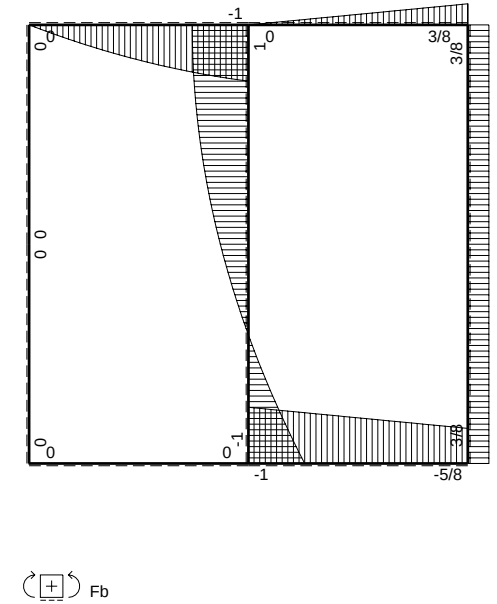
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

CI08.xxxx.114

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3/2F
Fb
1/2F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
11/8F
3/8Fb
3/8F
D
3/2F
E
3/2F
E
A
11/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B


0
0
0
11/8
-3/2
-3/8
-3/2
11/8
0
2
F
3/2
1/2
-3/8
0
0
0
0
-2
3/8
F


-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
0
-1
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_o flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 241748 CI08.xxxx.115

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

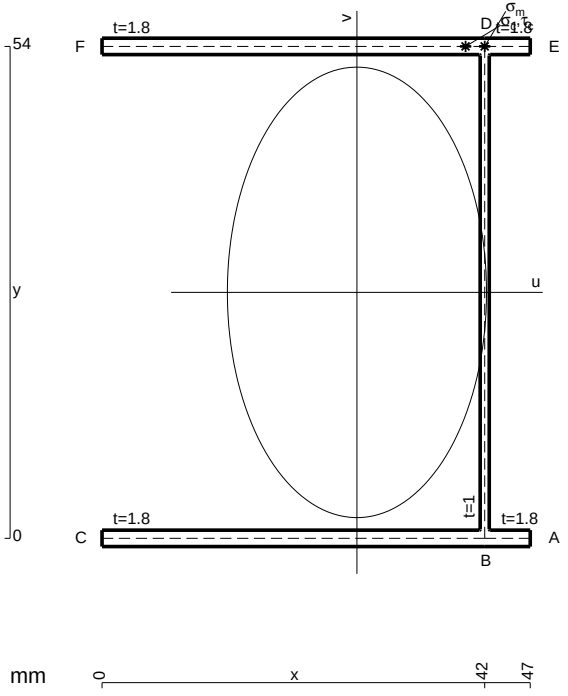
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

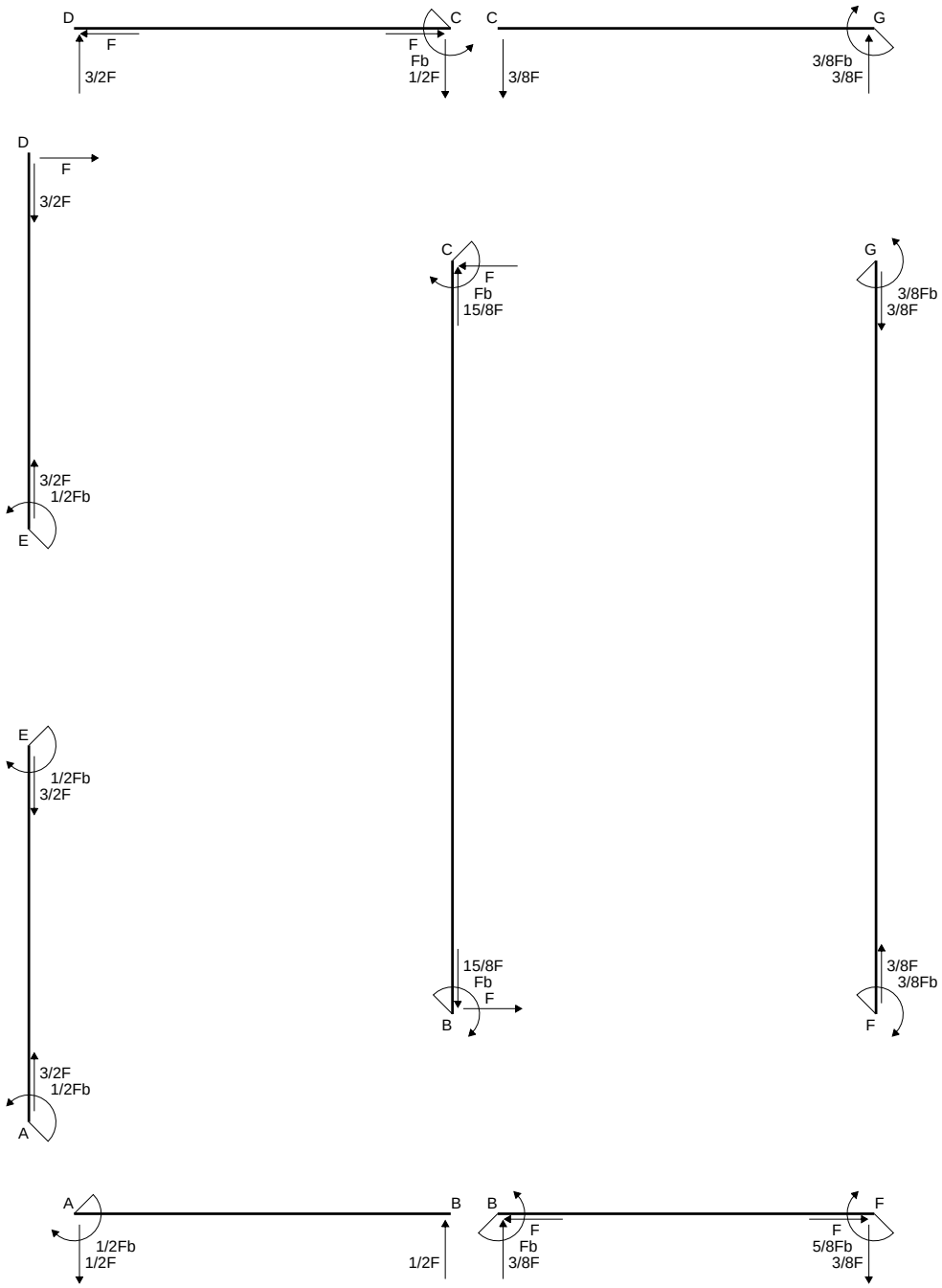
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

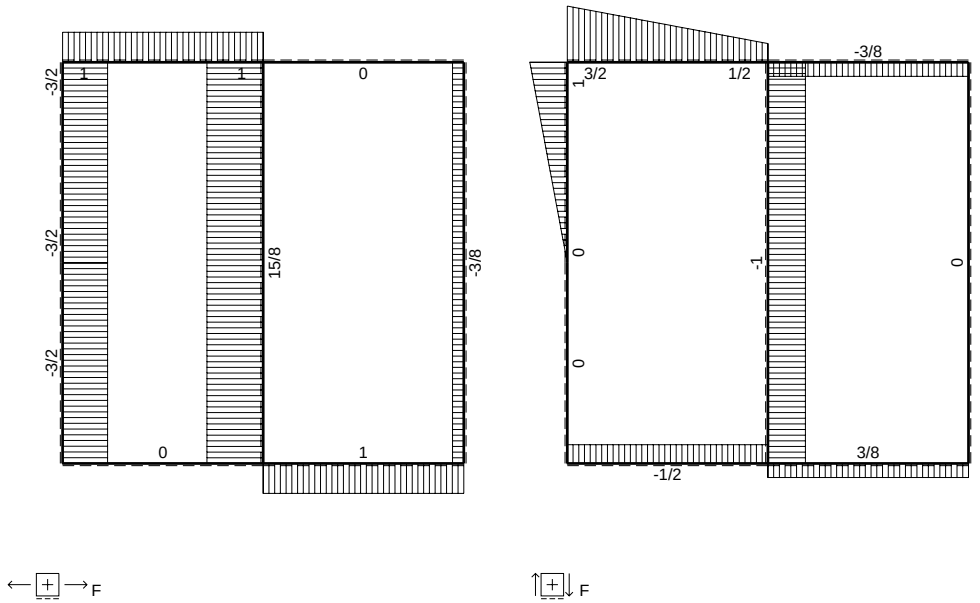


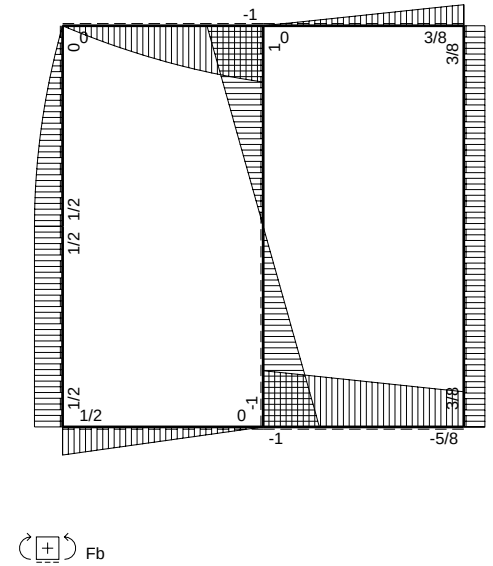
A = 223.2 mm^2
J_u = 136469. mm^4
J_v = 45157. mm^4
J_t = 200.7 mm^4
x_o = 30.75 mm
x_g = 27.98 mm
N = 2063. N
T_y = -3000. N
M_x = -960000. Nmm
x_m = 42. mm
y_m = 54. mm
u_m = 14.02 mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 199.2 N/mm^2
x_c = 42. mm
y_c = 54. mm
u_c = 14.02 mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 199.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 852. N/mm^2
$\tau_g = TS^*/tJ_u$ = 24.93 N/mm^2
$\tau_o = Tx_o t/J_t$ = 827.1 N/mm^2
t_c = 2700. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1489. N/mm^2

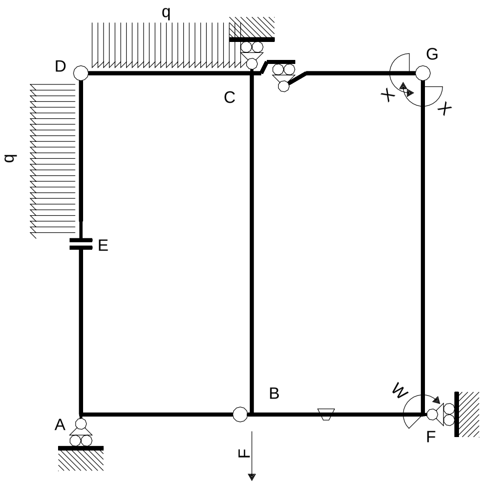
CI08.xxxx.115

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

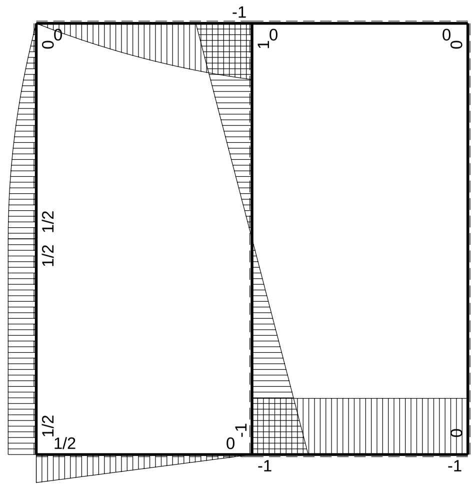

D
F
3/2F
C
F
Fb
1/2F
C
3/8F
G
3/8Fb
3/8F
D
F
3/2F
3/2F
1/2Fb
E
C
F
Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3/2F
15/8F
Fb
F
B
3/8F
3/8Fb
F
3/2F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F


-3/2
-3/2
-3/2
1
1
0
15/8
-3/8
0
1
F
-3/8
1
3/2
1/2
0
0
-1
0
3/8
-1/2
F

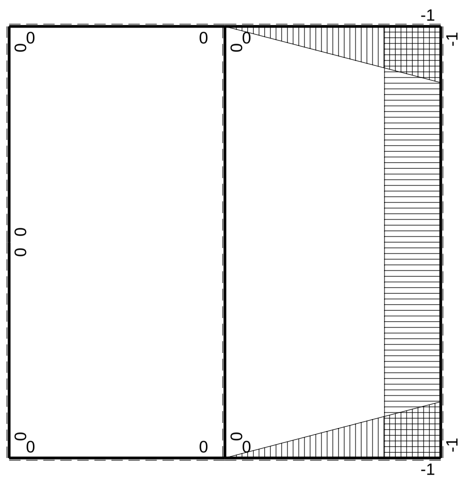

-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

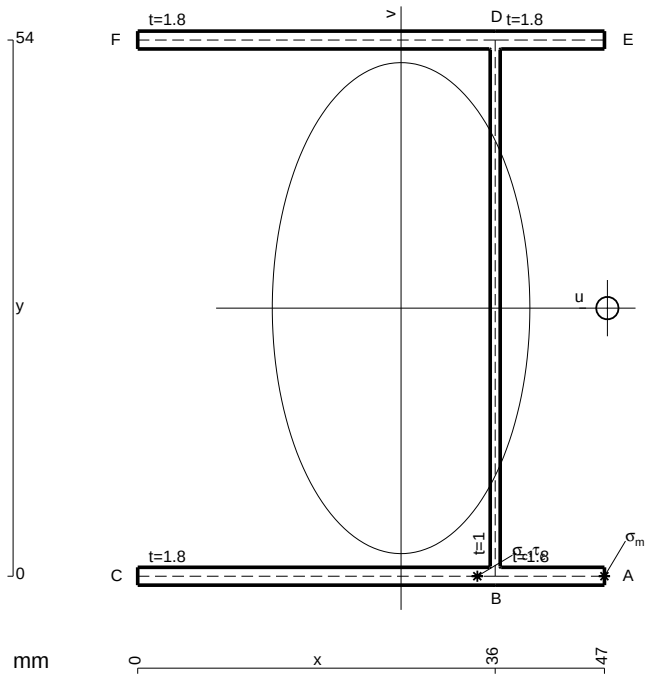
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

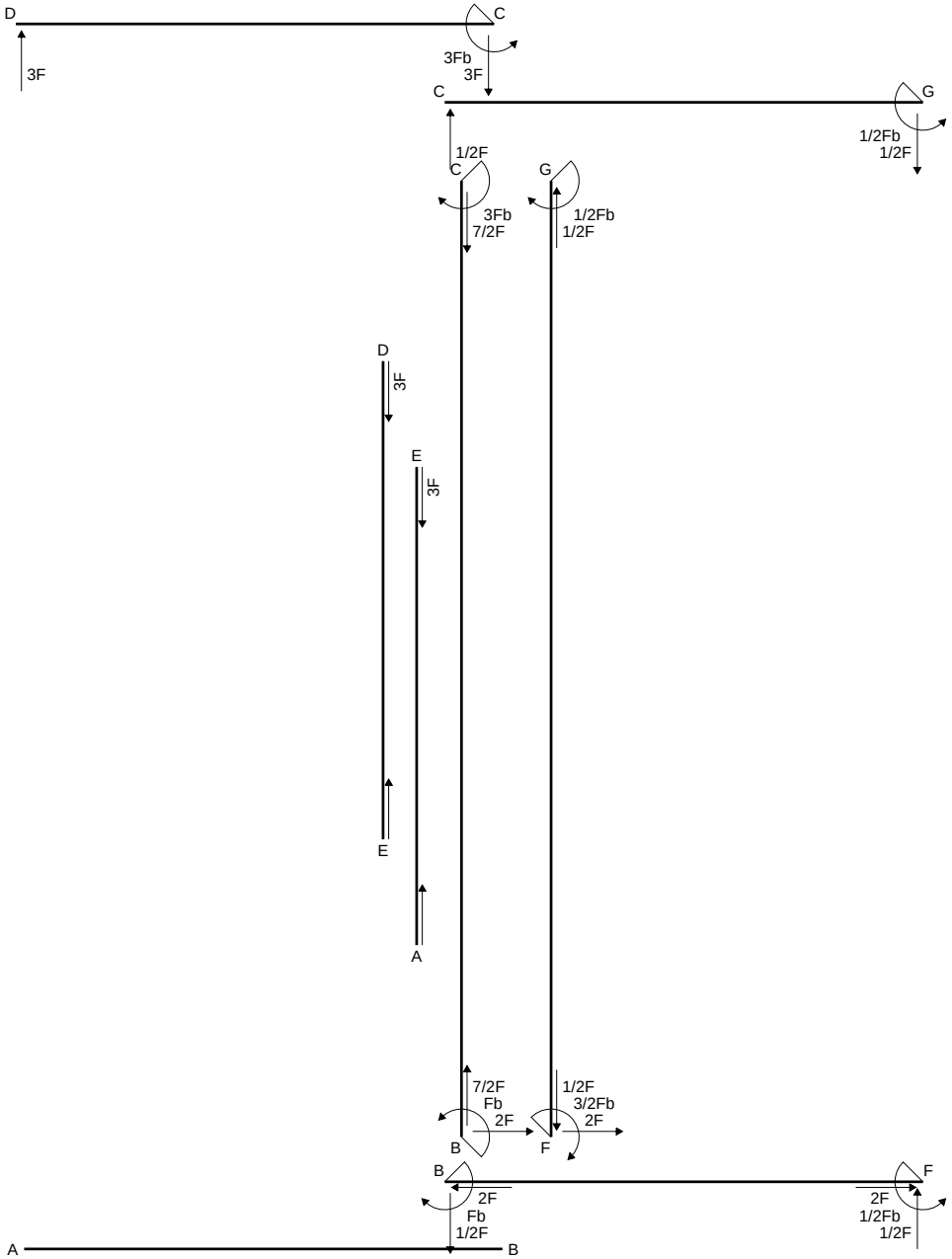
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

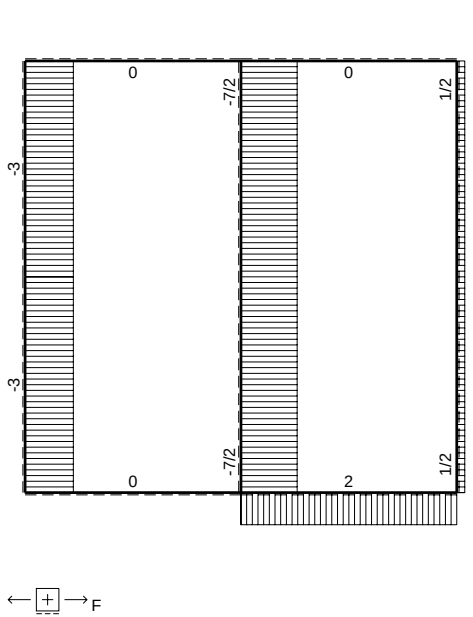


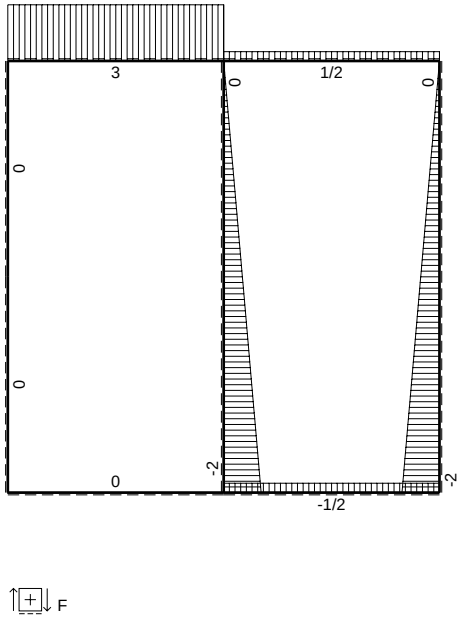
A = 223.2 mm^2
J_u = 136469. mm^4
J_v = 37543. mm^4
J_t = 200.7 mm^4
x_o = 20.77 mm
x_g = 26.52 mm
N = 2738. N
T_y = -1460. N
M_x = 992800. Nmm
x_m = 47. mm
u_m = 20.48 mm
v_m = -27. mm
$\sigma_m = N/A - Mv/J_u$ = 208.7 N/mm^2
x_c = 36. mm
u_c = 9.476 mm
v_c = -27. mm
$\sigma_c = N/A - Mv/J_u$ = 208.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 282.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.4 N/mm^2
$\tau_o = Tx_o t/J_t$ = 272. N/mm^2
t_c = 2628. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 531.7 N/mm^2

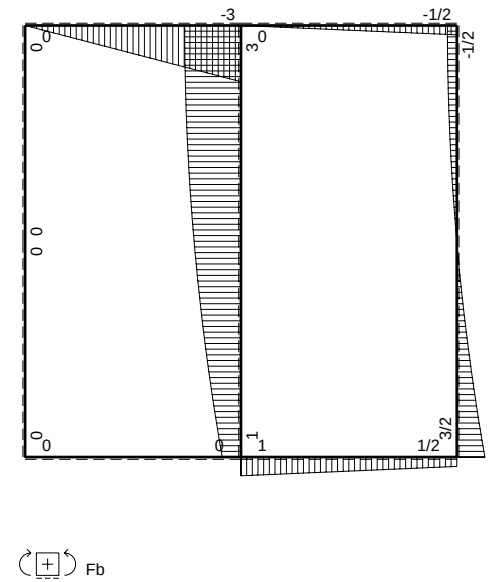
CI08.xxxx.116

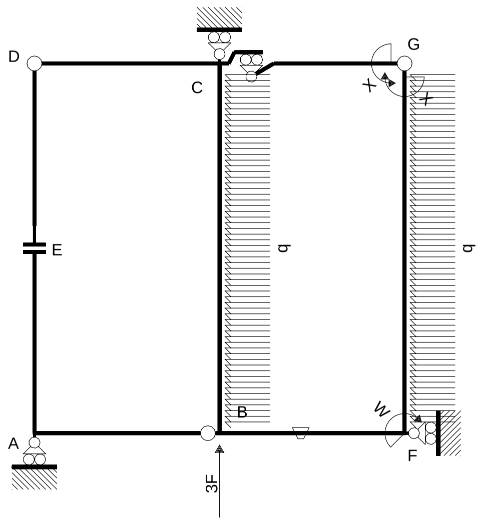
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

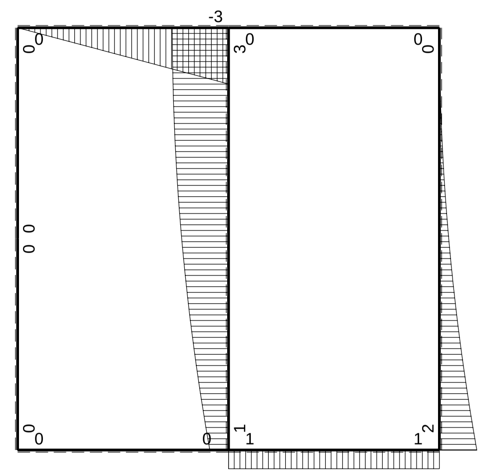

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
7/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
7/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
-7/2
0
1/2
-3
-3
0
-7/2
2
1/2
F

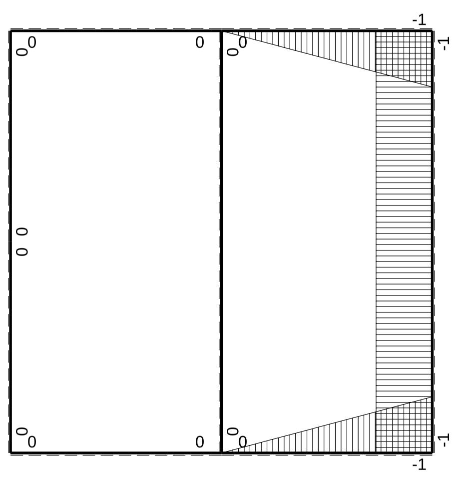

3
0
1/2
0
0
0
-2
0
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 984797 CI08.xxxx.117

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

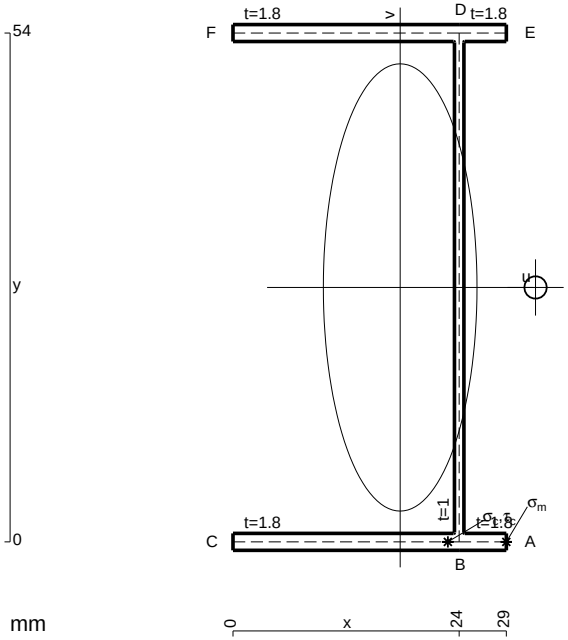
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 984797 CI08.xxxx.117

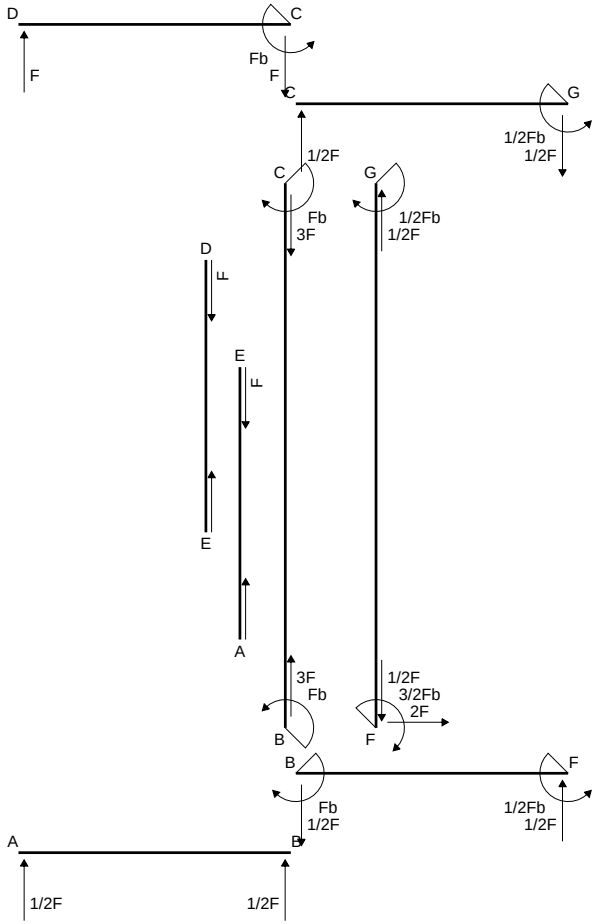


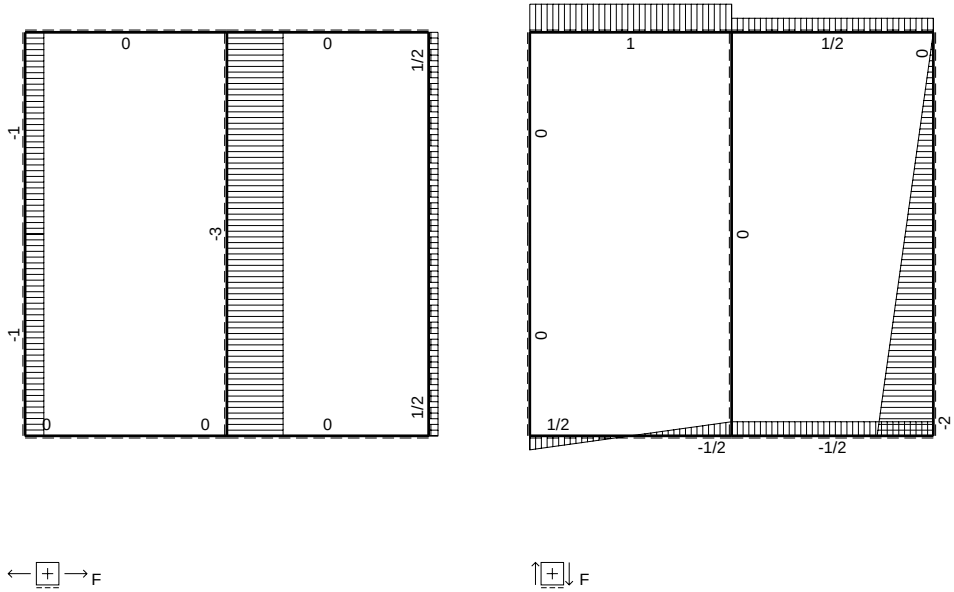
A = 158.4 mm^2
J_u = 89230. mm^4
J_v = 10529. mm^4
J_t = 130.8 mm^4
x_o = 14.36 mm
x_g = 17.74 mm
T_y = 990. N
M_x = -722700. Nmm
x_m = 29. mm
u_m = 11.26 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -218.7 N/mm^2
x_c = 24. mm
u_c = 6.261 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -218.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 203. N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.19 N/mm^2
$\tau_o = Tx_o t/J_t$ = 195.8 N/mm^2
t_c = 594. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 414. N/mm^2

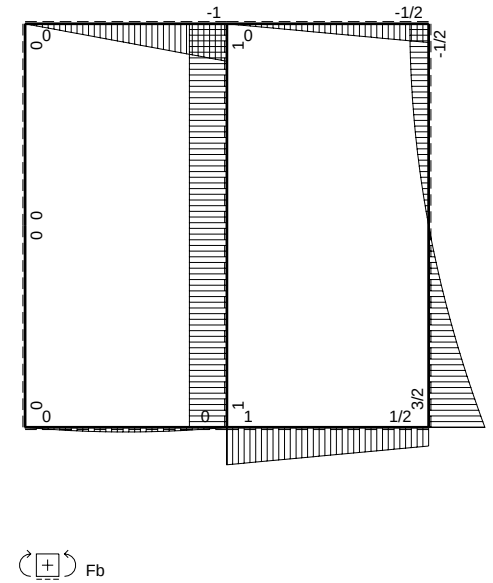
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

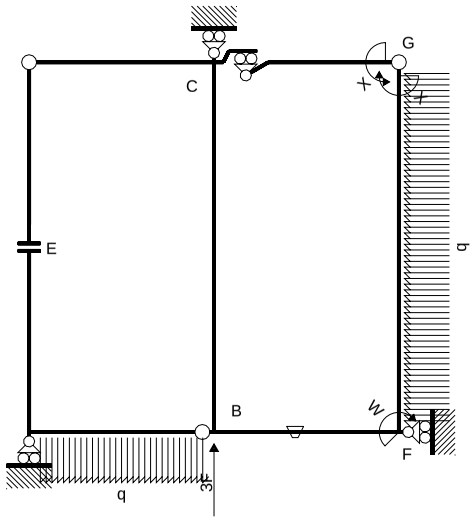
CI08.xxxx.117

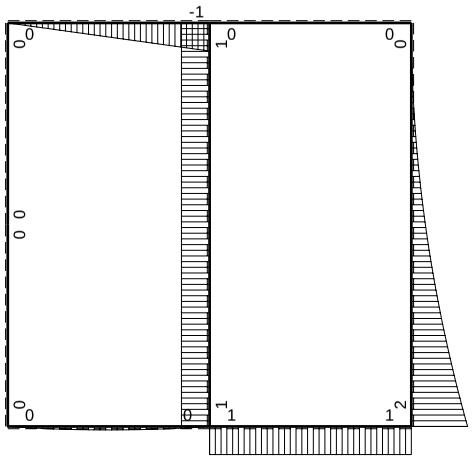
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
3F
1/2Fb
1/2F
D
F
E
F
E
A
3F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F

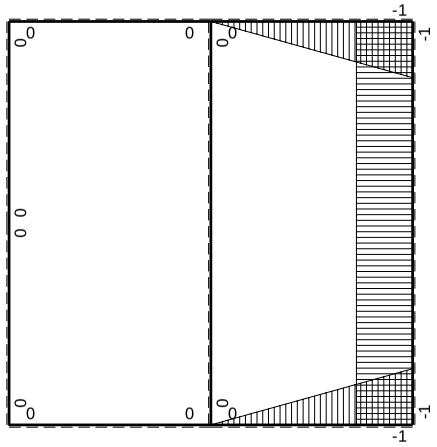

0
0
1/2
-1
-3
-1
0
0
0
1/2
F
1
1/2
0
0
0
1/2
-2
-1/2
-1/2
F


-1
-1/2
0
1
0
-1/2
0
0
0
0
1
3/2
0
1
1/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 280859 CI08.xxxx.118

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

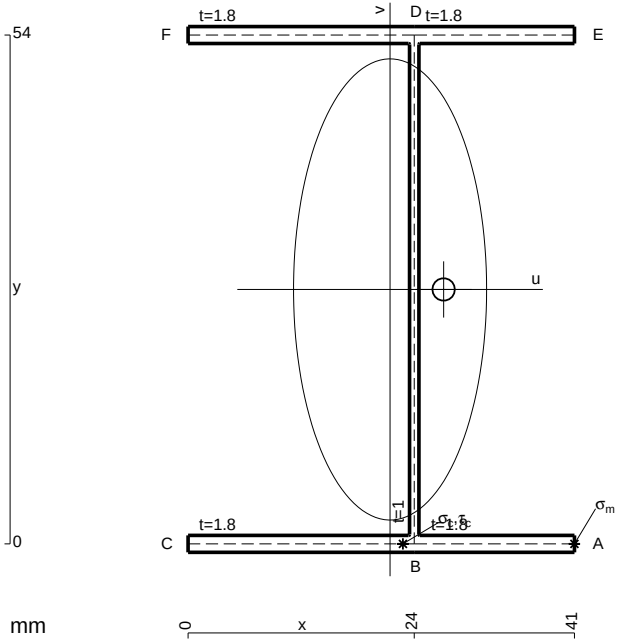
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

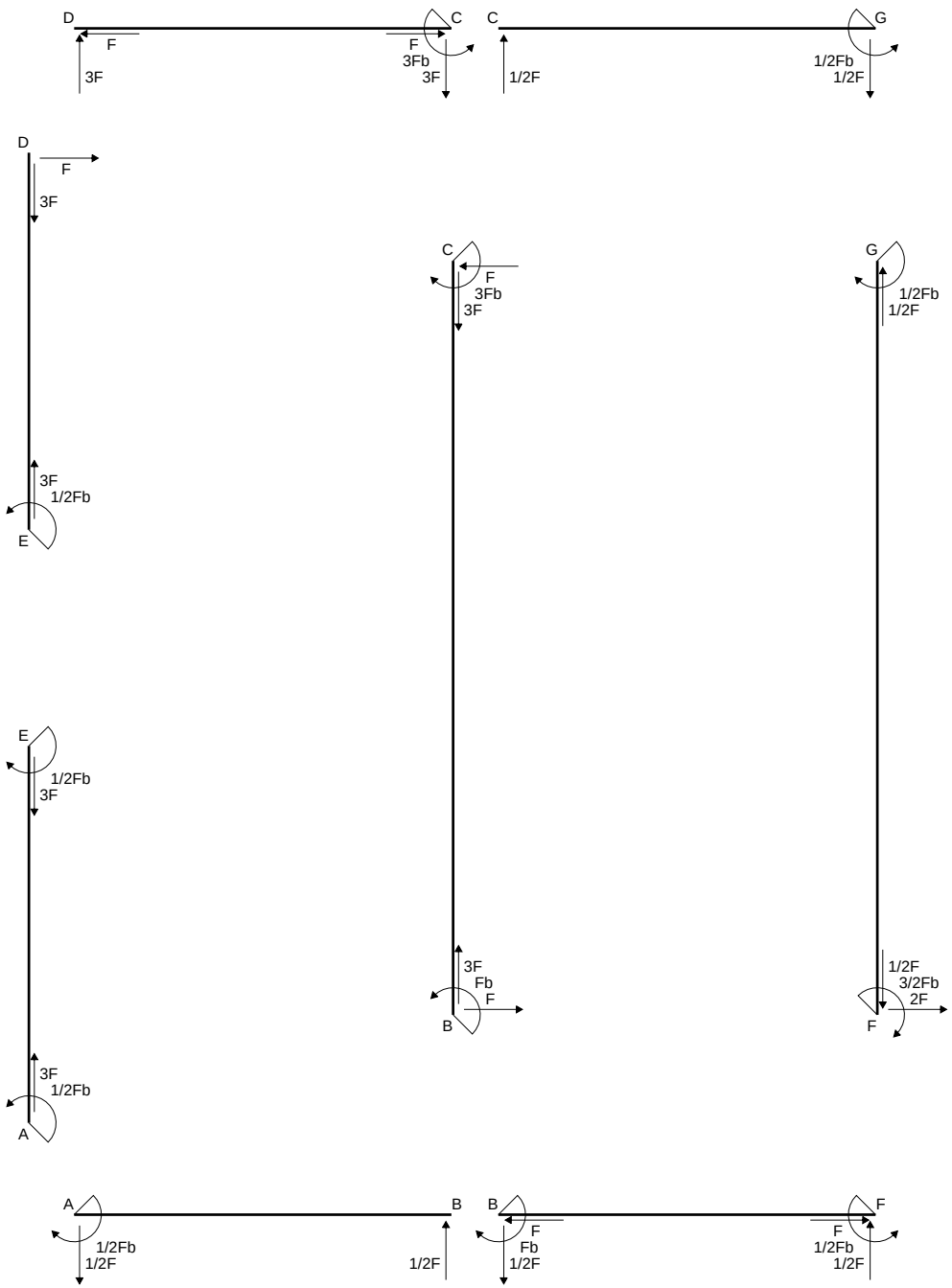
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

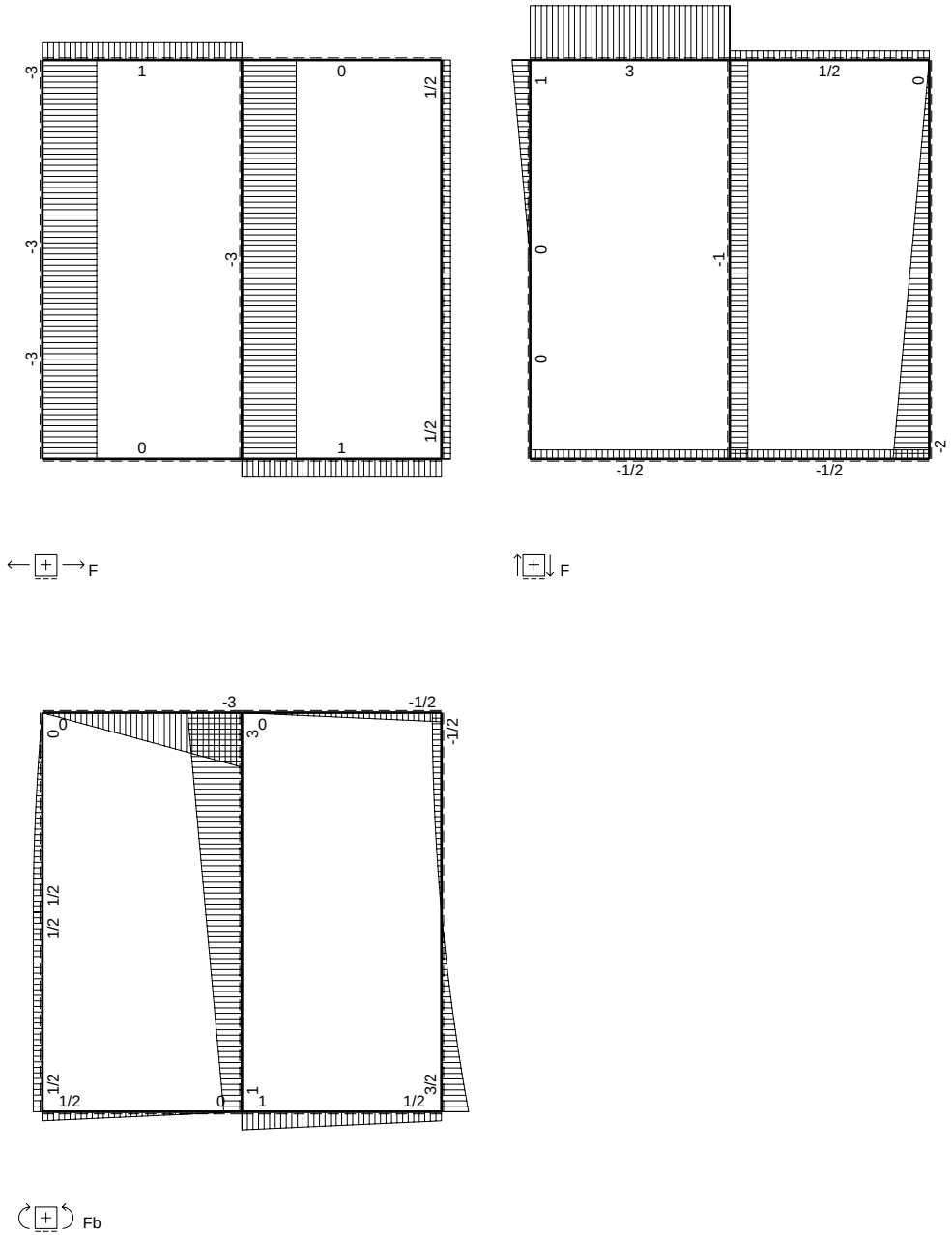


A = 201.6 mm^2
J_u = 120722. mm^4
J_v = 21161. mm^4
J_t = 177.4 mm^4
x_o = 5.682 mm
x_g = 21.44 mm
T_y = 1330. N
M_x = -1024100. Nmm
x_m = 41. mm
u_m = 19.56 mm
v_m = -27. mm
$\sigma_m = -Mv/J_u$ = -229. N/mm^2
x_c = 24. mm
u_c = 2.563 mm
v_c = -27. mm
$\sigma_c = -Mv/J_u$ = -229. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 83.81 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.139 N/mm^2
$\tau_o = Tx_o t/J_t$ = 76.68 N/mm^2
t_c = 2394. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 271.2 N/mm^2

CI08.xxxx.118

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
3F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
3F
Fb
F
B
1/2F
3/2Fb
2F
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


-3
1
0
1/2
-3
-3
-3
0
1
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^{2}b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
	totali						-4/3Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 241731 CI08.xxxx.119

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (- x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [- x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

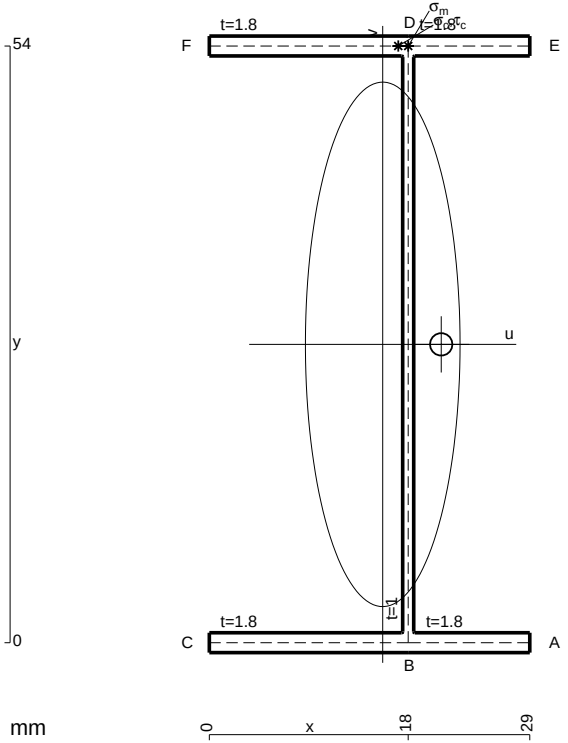
$= (- b +1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

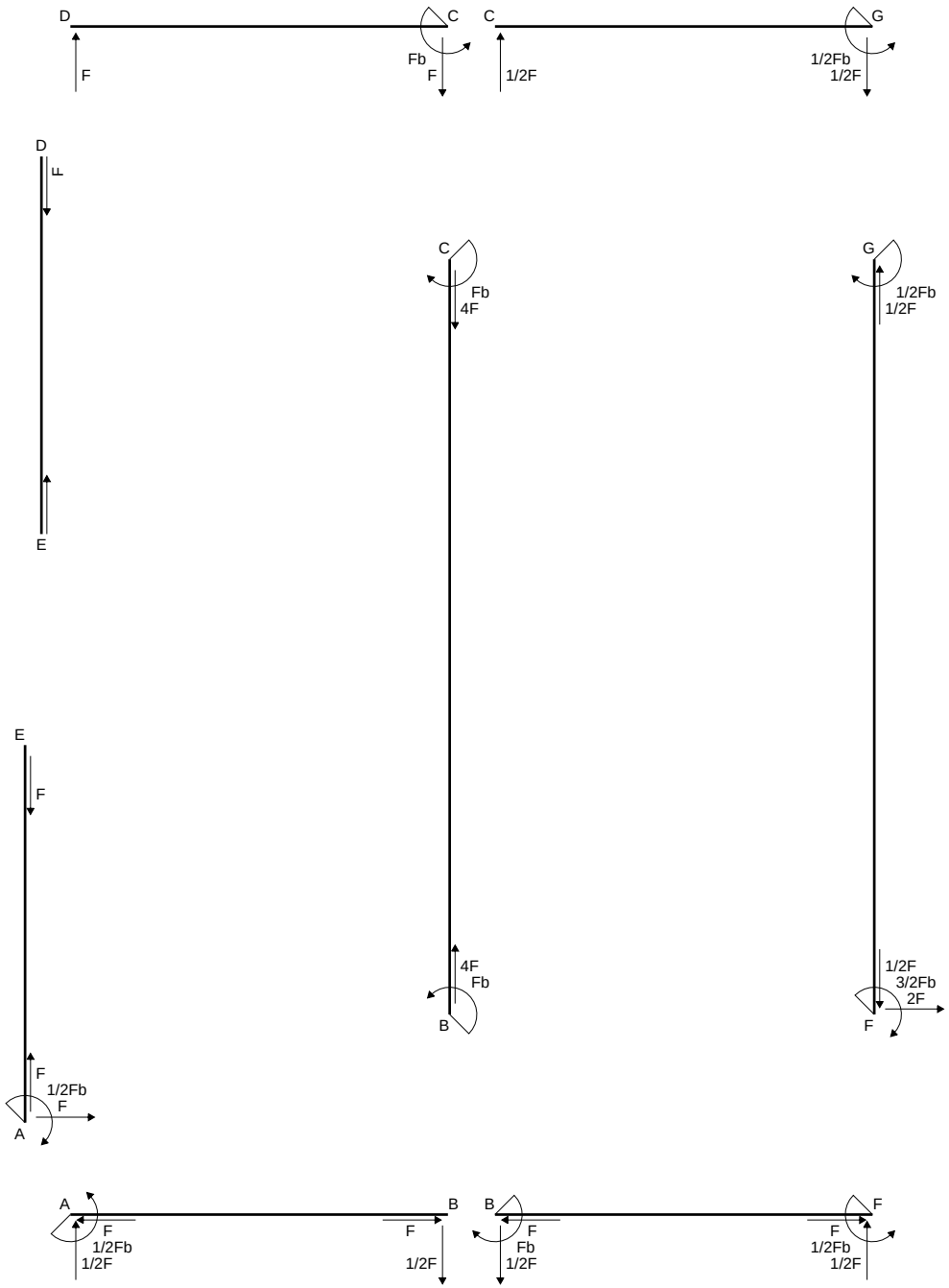
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

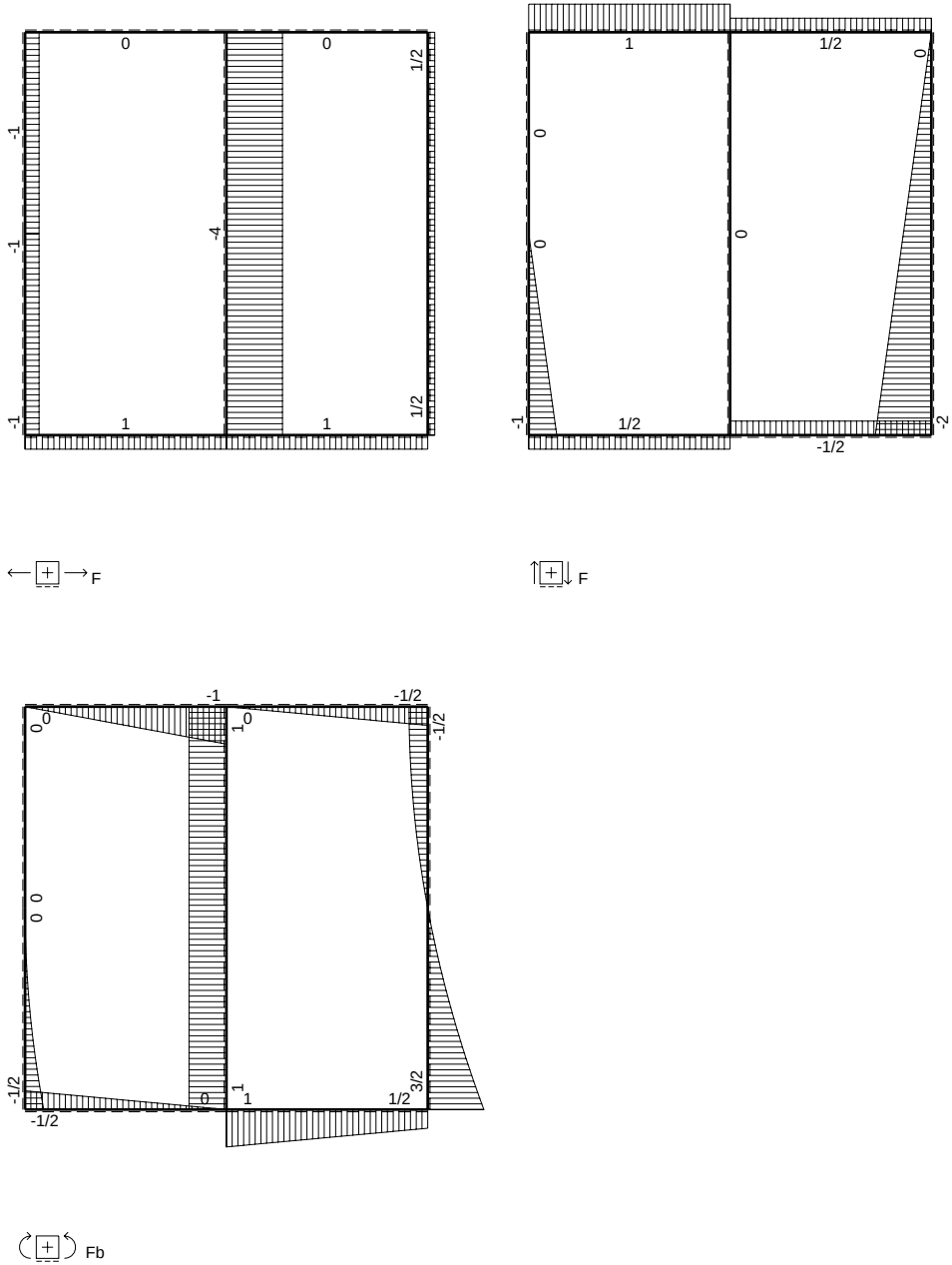


A = 158.4 mm^2
J_u = 89230. mm^4
J_v = 7753. mm^4
J_t = 130.8 mm^4
x_o = 5.292 mm
x_g = 15.69 mm
N = 320. N
T_y = 960. N
M_x = -777600. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 2.307 mm
v_m = 27. mm
$\sigma_m = N/A - Mv/J_u$ = 237.3 N/mm^2
x_c = 18. mm
y_c = 54. mm
u_c = 2.307 mm
v_c = 27. mm
$\sigma_c = N/A - Mv/J_u$ = 237.3 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 75.17 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.229 N/mm^2
$\tau_o = Tx_o t/J_t$ = 69.94 N/mm^2
t_c = 576. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 270.7 N/mm^2

CI08.xxxx.119

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
4F
G
1/2Fb
1/2F
4F
Fb
B
1/2F
3/2Fb
2F
F
E
F
F
1/2Fb
F
A
A
B
B
F
F
F
F
1/2Fb
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


0
0
1/2
-1
-1
-4
-1
1
1
1/2
F
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F
-1
-1/2
0
0
1
0
-1/2
0 0
0 0
-1/2
0
1
1
1/2
3/2
-1/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

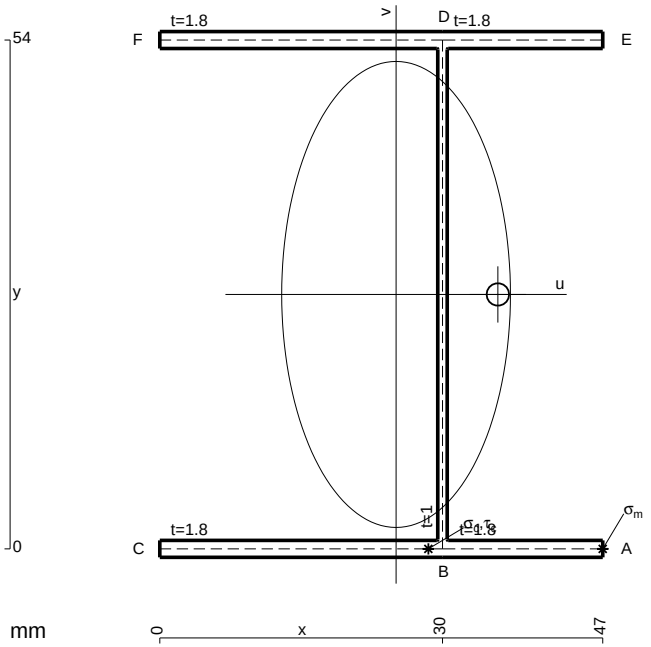
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

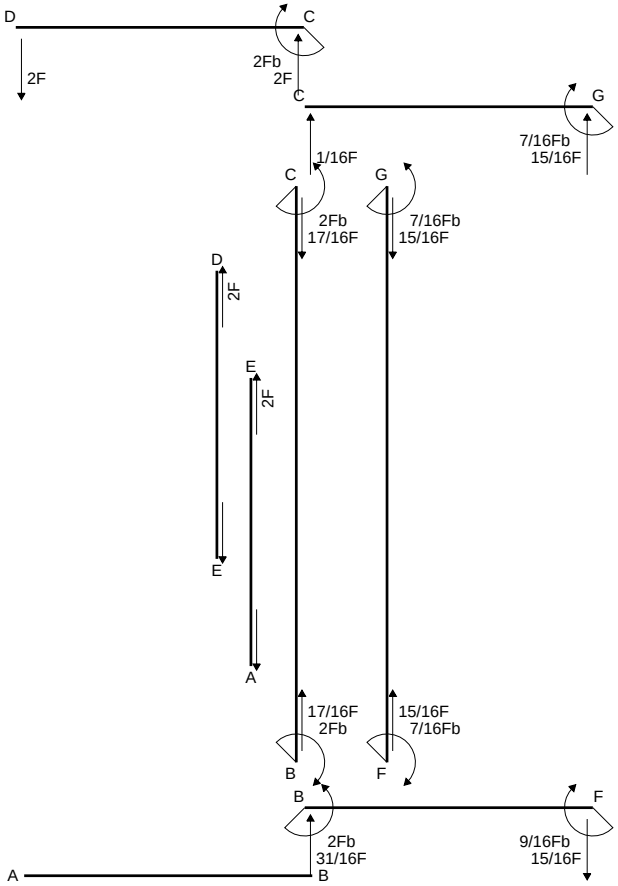
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

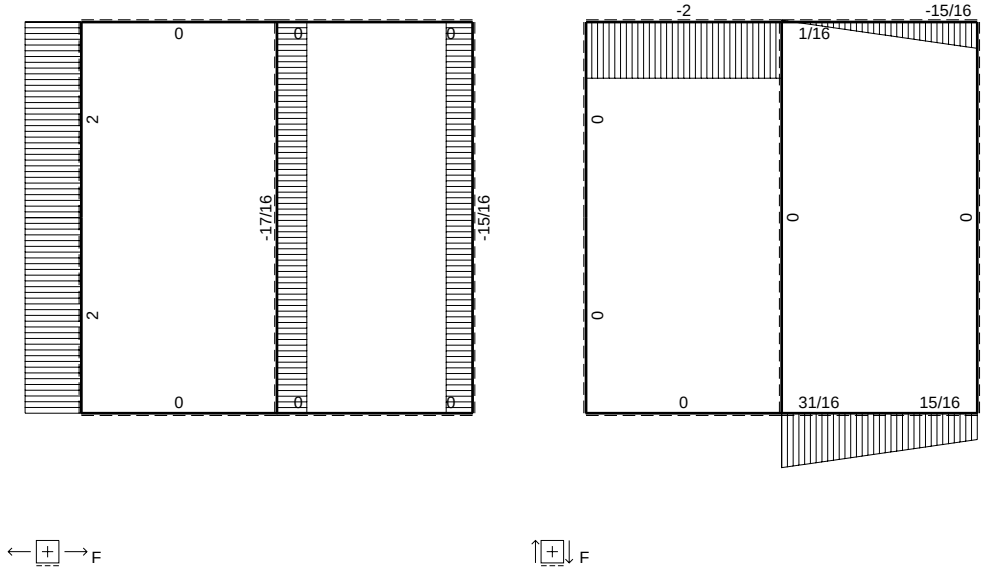


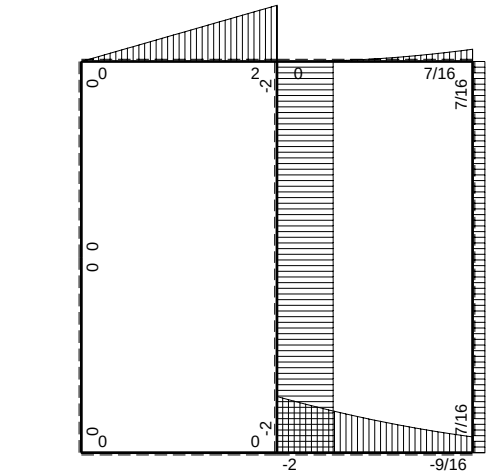
$A = 223.2\ mm^2$
$J_u = 136469.\ mm^4$
$J_v = 32876.\ mm^4$
$J_t = 200.7\ mm^4$
$x_o = 10.8\ mm$
$x_g = 25.07\ mm$
$T_y = 2350.\ N$
$M_x = -1010500.\ Nmm$
$x_m = 47.\ mm$
$u_m = 21.93\ mm$
$v_m = -27.\ mm$
$\sigma_m = -Mv/J_u = -199.9\ N/mm^2$
$x_c = 30.\ mm$
$u_c = 4.927\ mm$
$v_c = -27.\ mm$
$\sigma_c = -Mv/J_u = -199.9\ N/mm^2$
$\tau_c = \tau_g + \tau_{ou} = 241.6\ N/mm^2$
$\tau_g = TS^*/tJ_u = 13.95\ N/mm^2$
$\tau_o = Tx_o t/J_t = 227.6\ N/mm^2$
$t_c = 4230.\ mm$
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 463.7\ N/mm^2$

CI08.xxxx.120

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
17/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
17/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
-17/16
-15/16
2
0
0
0
-2
-15/16
1/16
0
0
0
0
31/16
15/16
F
F


0
2
0
7/16
0
-2
7/16
0
0
0
0
-2
7/16
-2
-9/16

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	FxEJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3XbEJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-FxEJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3XbEJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbEJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
totali							$7/6Fb^2/EJ$	8/3XbEJ
iperstatica $X=W_{GC}$							-7/16Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211288 CI08.xxxx.121

PROCEDIMENTO E RISULTATI 211288 CI08.xxxx.121

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

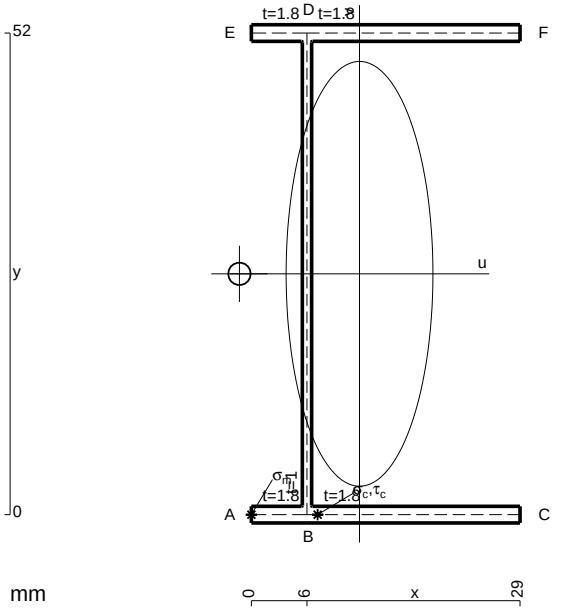
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

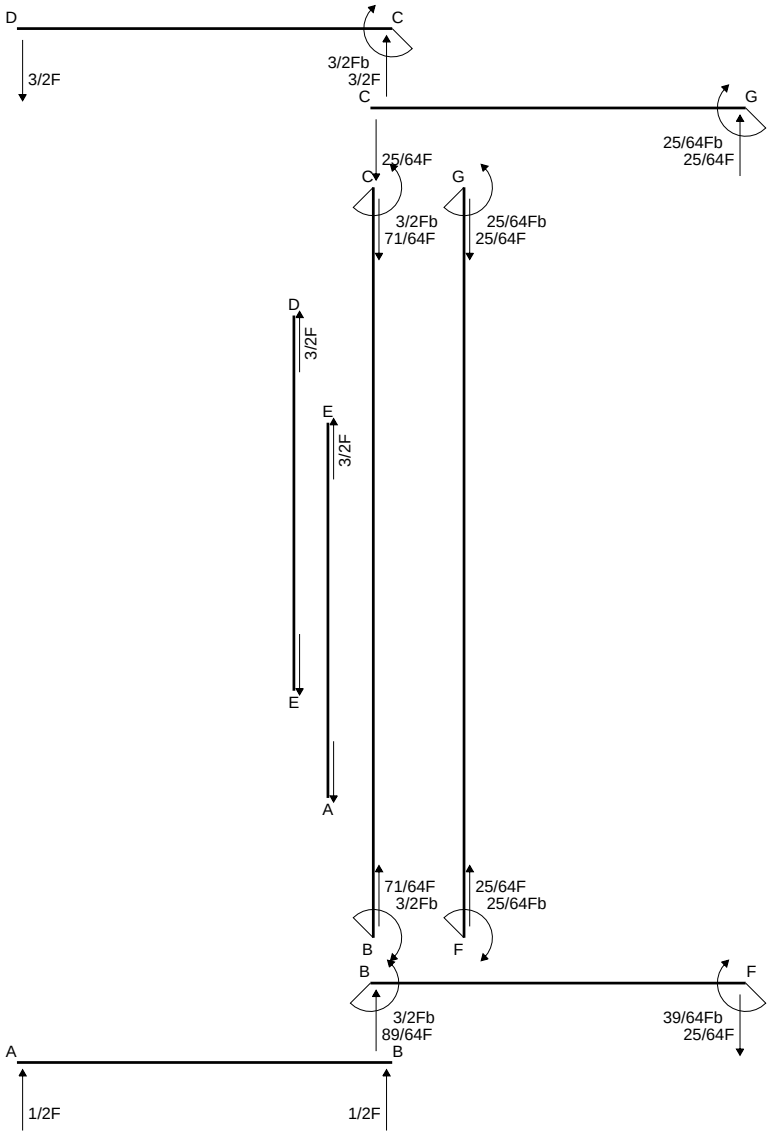
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

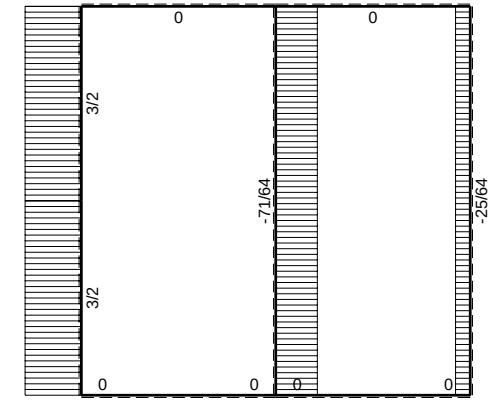


A = 156.4 mm^2
J_u = 82292. mm^4
J_v = 9825. mm^4
J_t = 130.1 mm^4
x_o = -12.96 mm
x_g = 11.67 mm
T_y = -1460. N
M_x = 657000. Nmm
u_m = -11.67 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = 207.6 N/mm^2
x_c = 6. mm
u_c = -5.674 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = 207.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 272.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.61 N/mm^2
$\tau_o = Tx_o t/J_t$ = 261.9 N/mm^2
t_c = 1314. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 515.6 N/mm^2

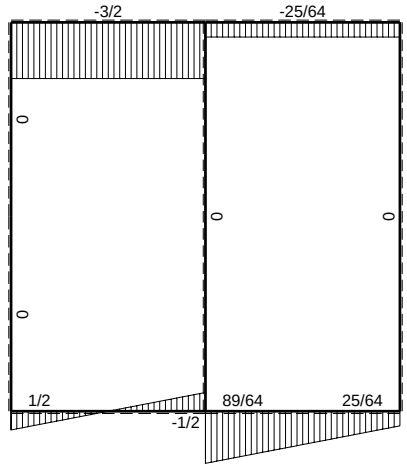
CI08.xxxx.121

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

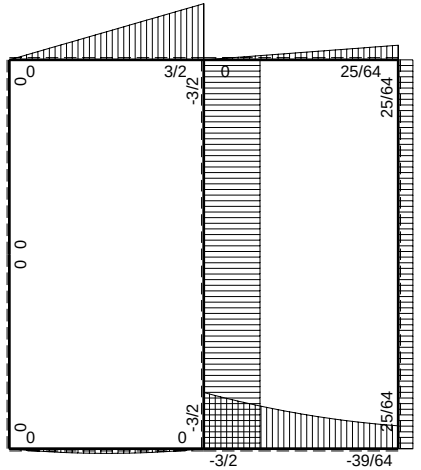

D
C
3/2F
3/2Fb
3/2F
C
G
25/64F
25/64Fb
25/64F
C
G
3/2Fb
71/64F
25/64Fb
25/64F
D
3/2F
E
3/2F
E
A
71/64F
3/2Fb
25/64F
25/64Fb
B
F
B
F
3/2Fb
89/64F
39/64Fb
25/64F
B
A
1/2F
1/2F


0
0
3/2
-71/64
-25/64
3/2
0
0
0
0

F


-3/2
-25/64
0
0
0
0
0
1/2
89/64
25/64
-1/2

F


0
0
3/2
-3/2
0
25/64
25/64
0
0
0
0
-3/2
25/64
-3/2
-39/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
BA b	0	-1/2Fx+1/2qx^2	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-3/2Fb+Fx-1/2qx^2	-Fb/EJ	3/2Fx-Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(13/24+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2qx^2	Fb/EJ	Fb-Fx+1/2Fx^2/b-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
totali							25/24Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-25/64Fb	

Sviluppi di calcolo iperstatica

M_o flessione da carichi assegnati

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

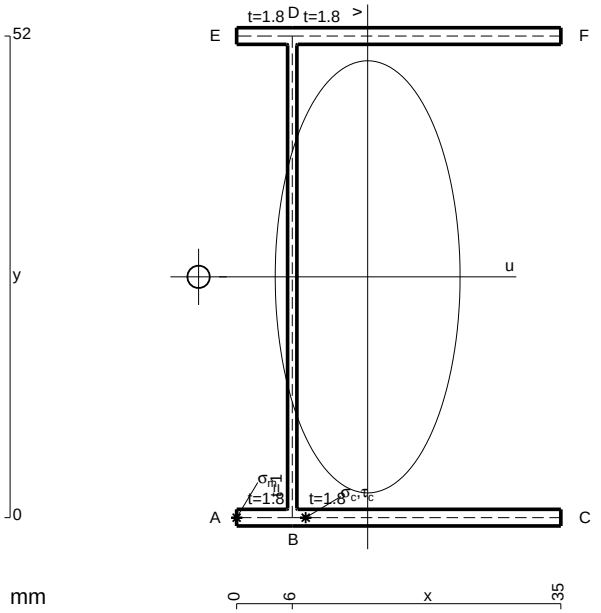
$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

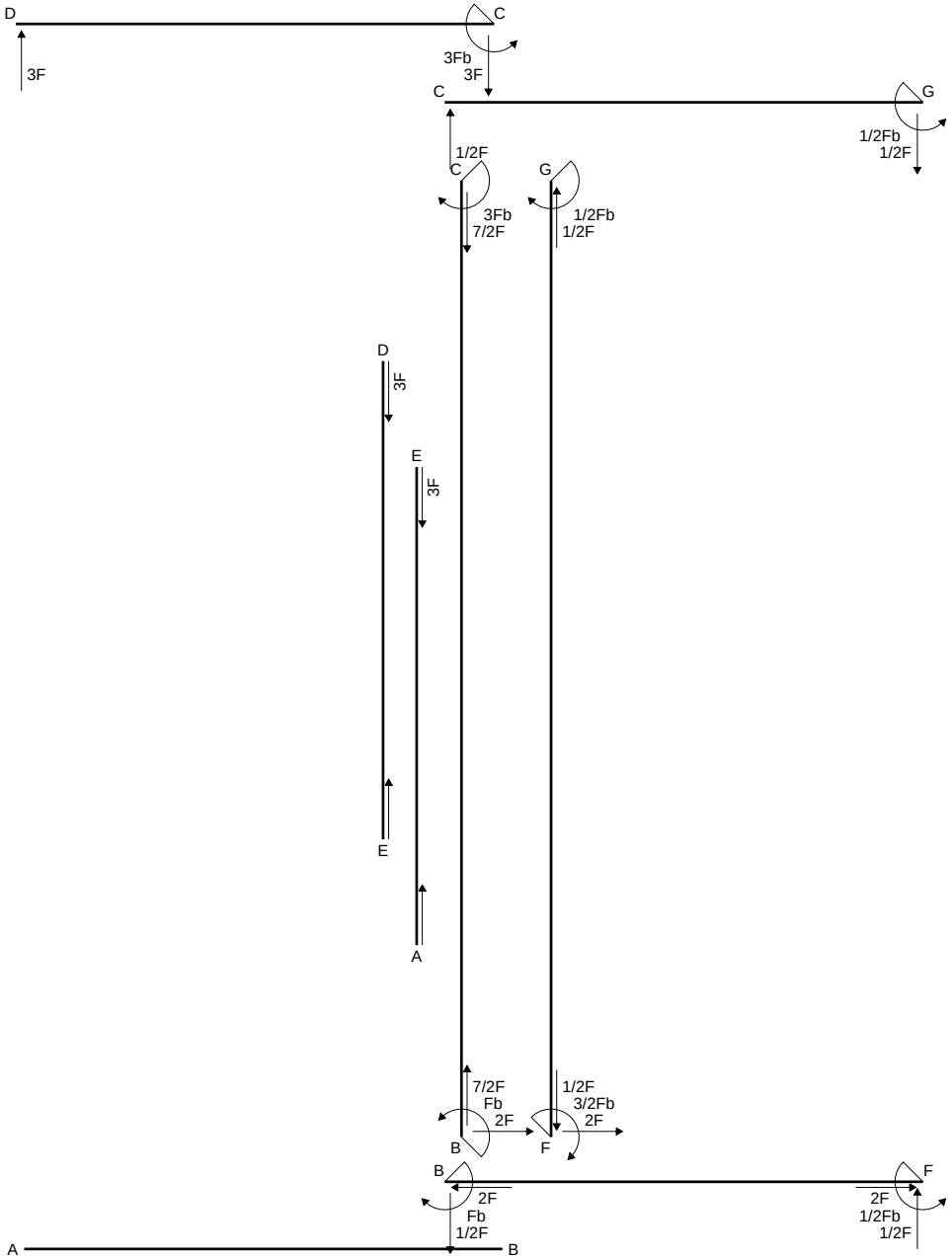
$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

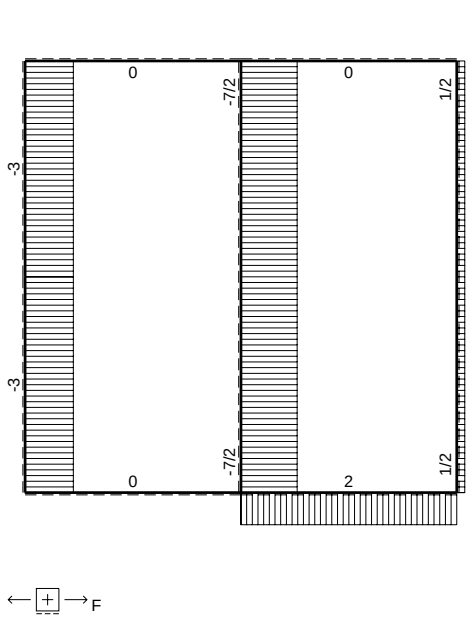


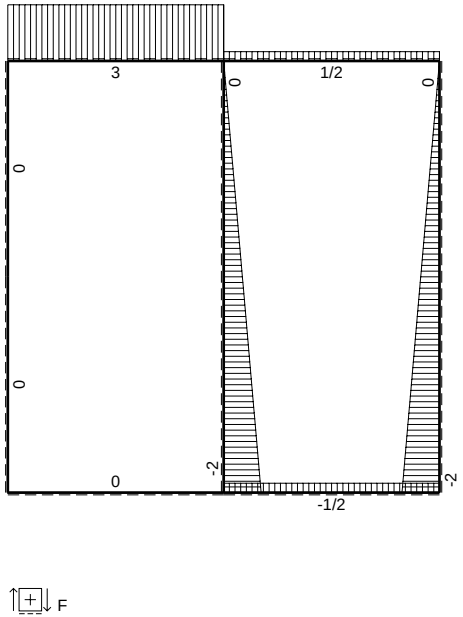
A = 178. mm^2
J_u = 96893. mm^4
J_v = 17731. mm^4
J_t = 153.4 mm^4
x_o = -18.25 mm
x_g = 14.14 mm
T_y = -1665. N
M_x = 815850. Nmm
u_m = -14.14 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = 218.9 N/mm^2
x_c = 6. mm
u_c = -8.14 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = 218.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 369.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.96 N/mm^2
$\tau_o = Tx_o t/J_t$ = 356.5 N/mm^2
t_c = 1998. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 676.4 N/mm^2

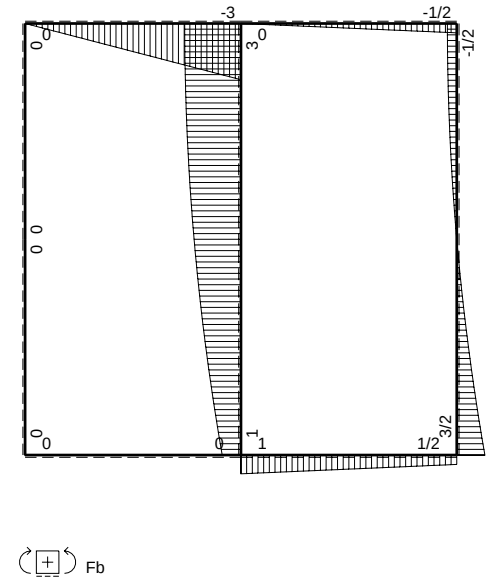
CI08.xxxx.122

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
7/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
7/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
-7/2
0
1/2
-3
-3
0
-7/2
2
1/2
F


3
0
1/2
0
0
0
0
-2
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	3Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	-Fb-2Fx+1/2qx²	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 248692 CI08.xxxx.123

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

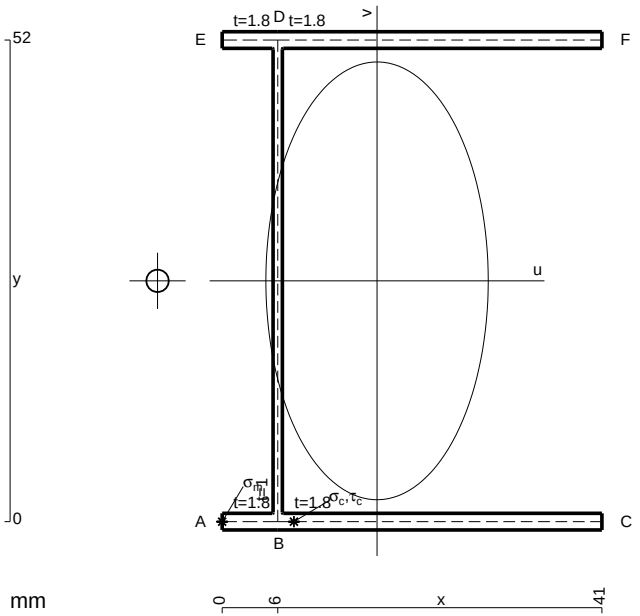
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

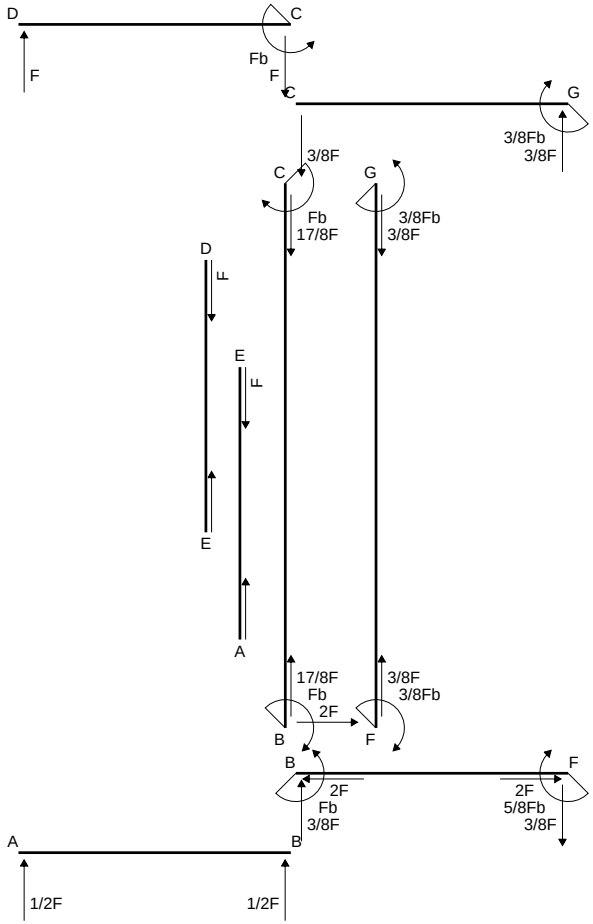
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

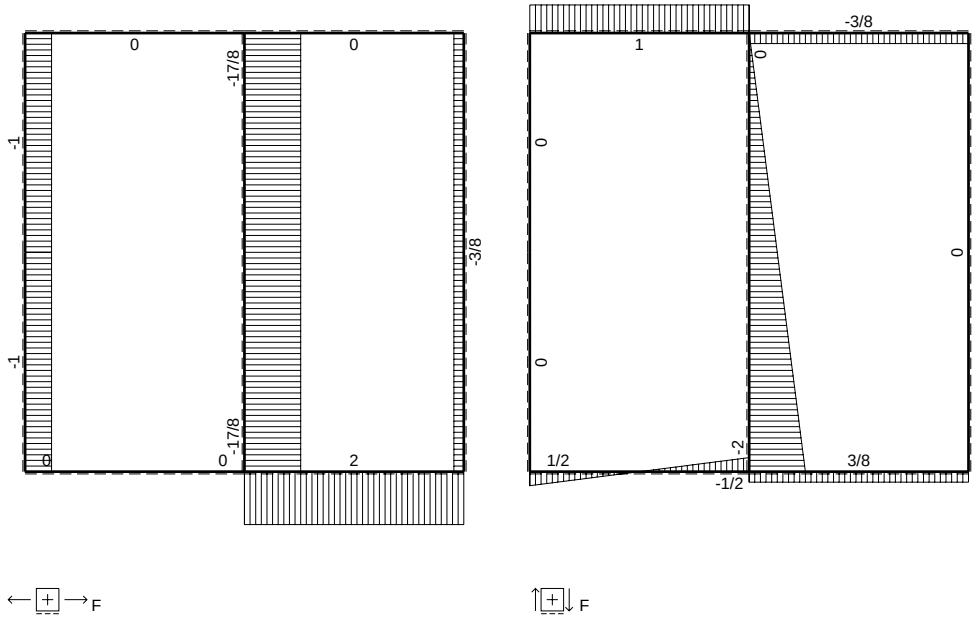


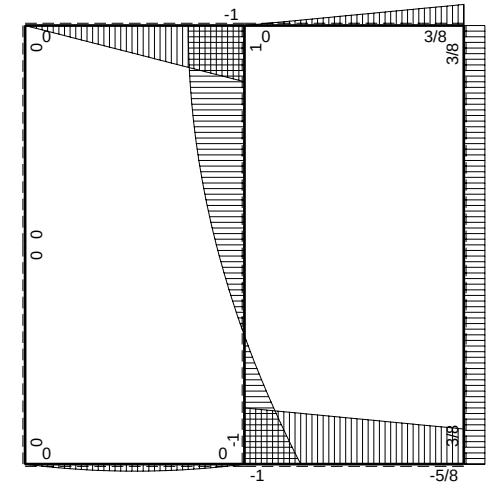
A = 199.6 mm^2
J_u = 111495. mm^4
J_v = 28761. mm^4
J_t = 176.7 mm^4
x_o = -23.7 mm
x_g = 16.72 mm
T_y = 1860. N
M_x = -985800. Nmm
u_m = -16.72 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -229.9 N/mm^2
x_c = 6. mm
u_c = -10.72 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -229.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 464.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.18 N/mm^2
$\tau_o = Tx_o t/J_t$ = 448.9 N/mm^2
t_c = 1116. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 836.1 N/mm^2

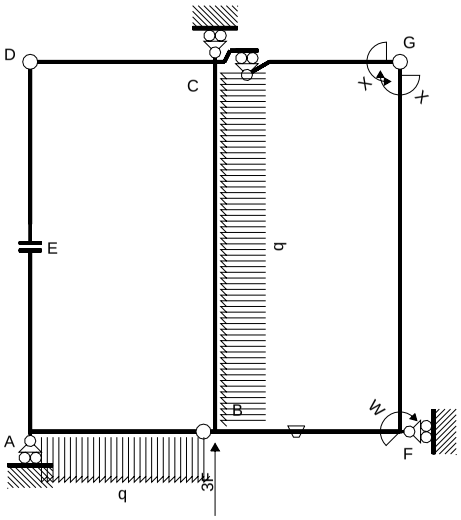
CI08.xxxx.123

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

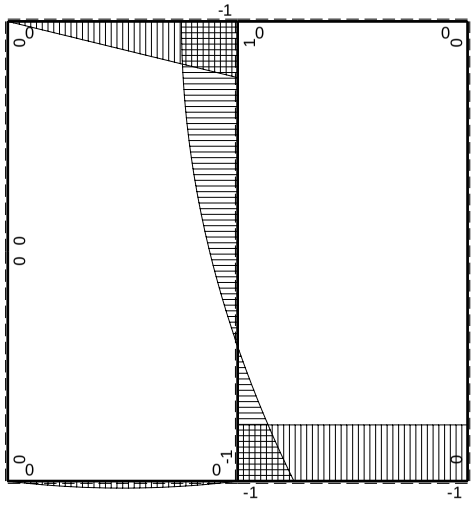

D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
17/8F
3/8Fb
3/8F
D
F
E
F
E
A
17/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F


0
0
-17/8
-1
-1
-3/8
0
0
-17/8
0
2
F
-3/8
1
0
0
0
0
1/2
-2
3/8
-1/2
F

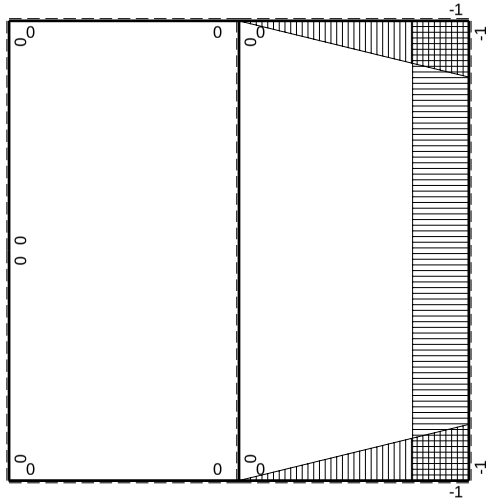

-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 237373 CI08.xxxx.124

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

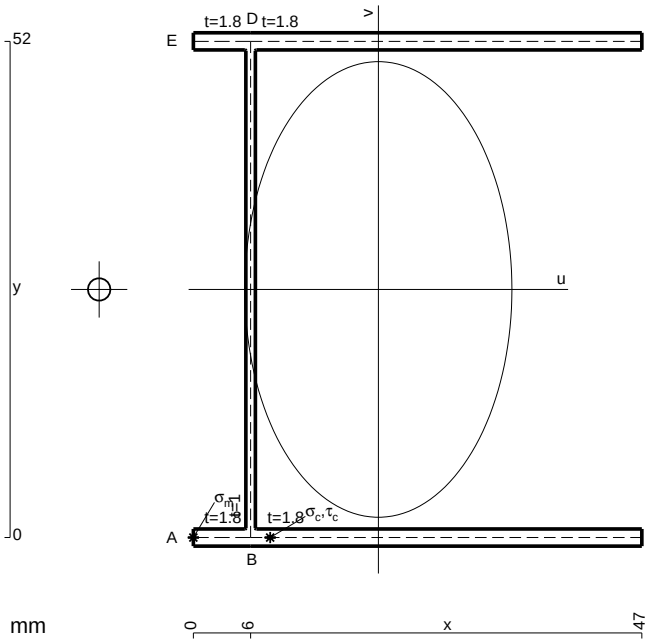
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

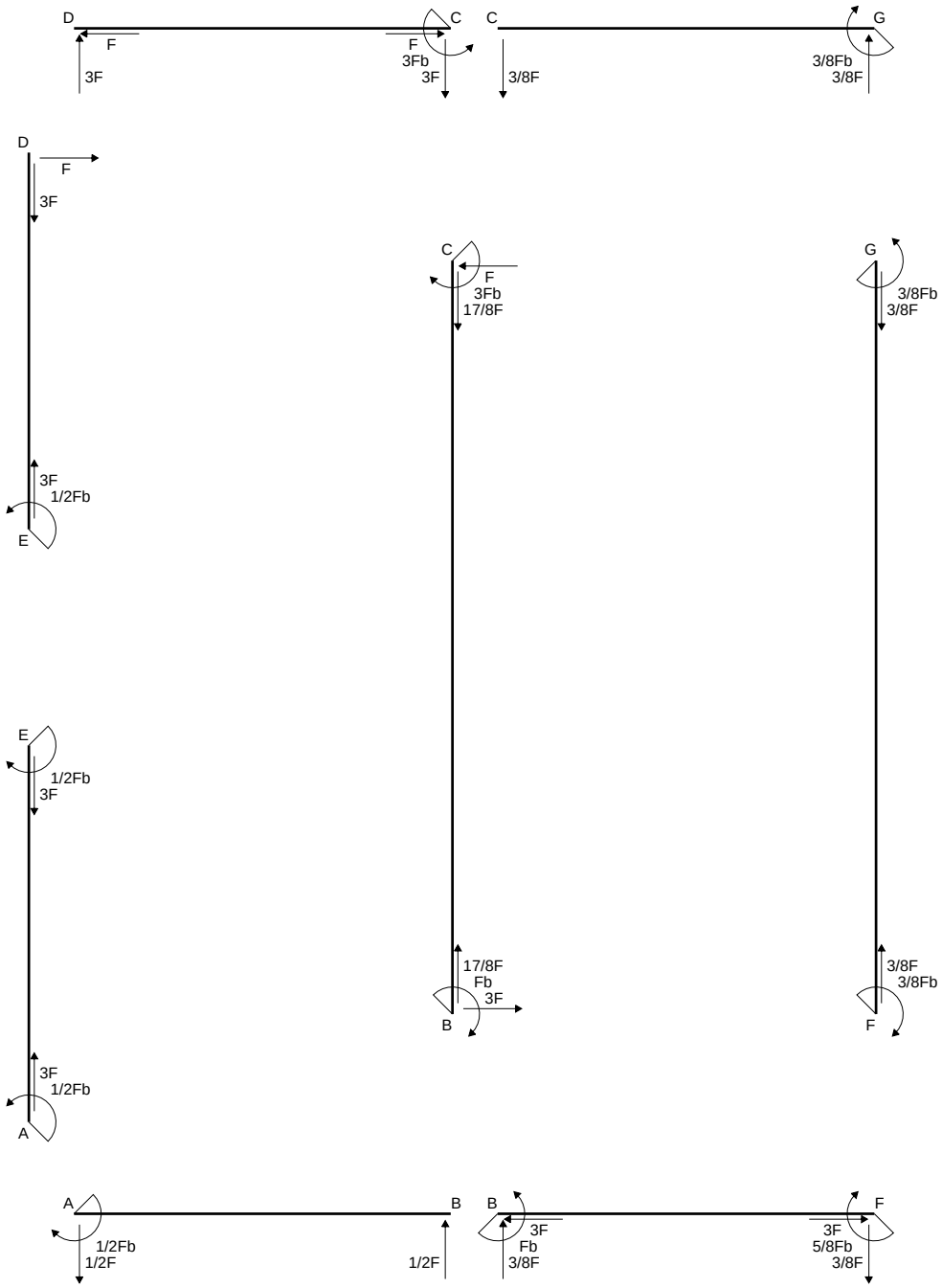
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

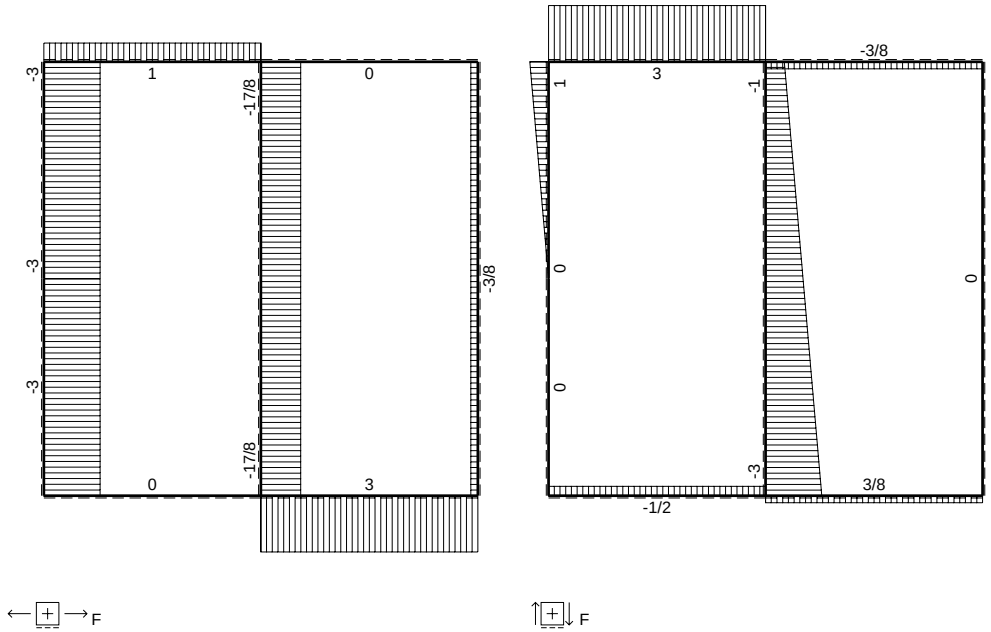


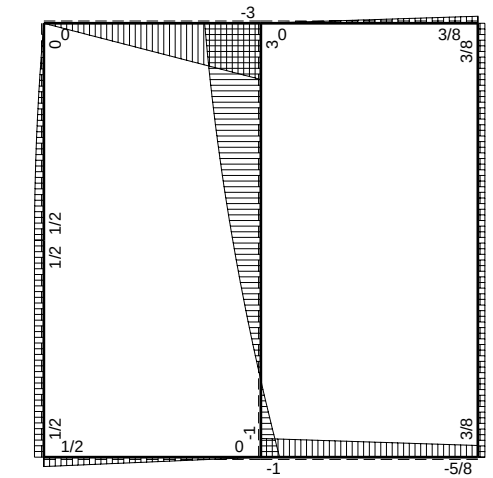
A = 221.2 mm^2
J_u = 126097. mm^4
J_v = 43328. mm^4
J_t = 200.1 mm^4
x_o = -29.26 mm
x_g = 19.39 mm
N = -3995. N
T_y = -3760. N
M_x = -1071600. Nmm
u_m = -19.39 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = -239. N/mm^2
x_c = 6. mm
u_c = -13.39 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = -239. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 1022. N/mm^2
$\tau_g = TS^*/tJ_u$ = 31.79 N/mm^2
$\tau_o = Tx_o t/J_t$ = 989.8 N/mm^2
t_c = 3384. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1786. N/mm^2

CI08.xxxx.124

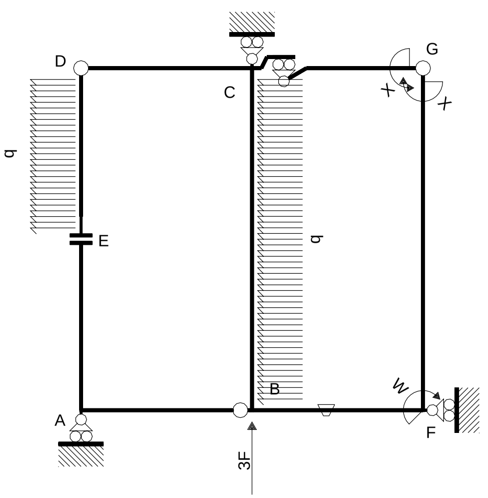
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
3/8F
3/8Fb
3/8F
D
F
3F
C
F
3Fb
17/8F
G
3/8Fb
3/8F
3F
1/2Fb
E
E
1/2Fb
3F
17/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
3F
Fb
3/8F
3F
5/8Fb
3/8F

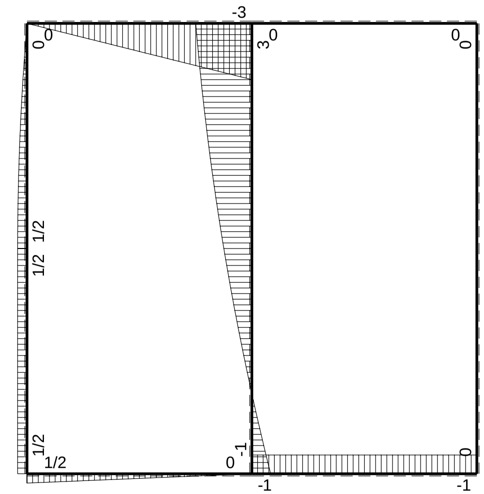

-3
1
0
-17/8
-3
-3/8
-3
-17/8
0
3
F
-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F


-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8

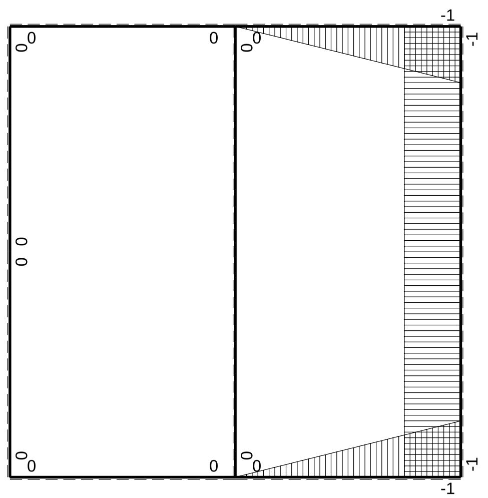

Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$3Fb-Fx-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-3Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

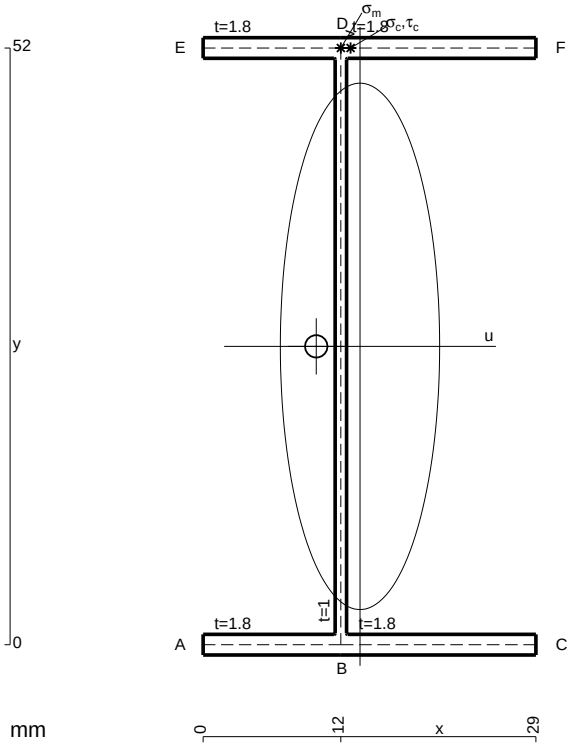
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

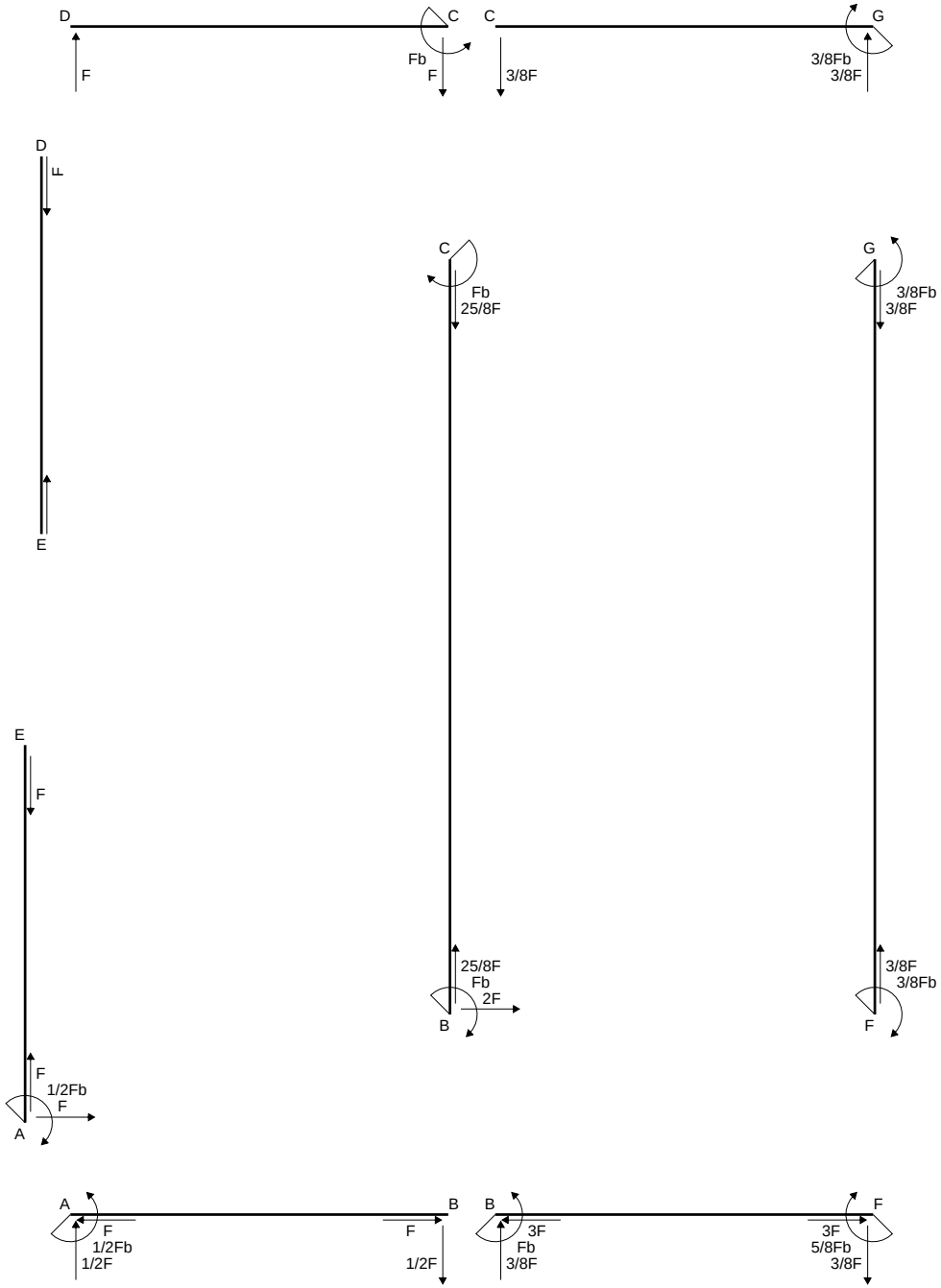
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

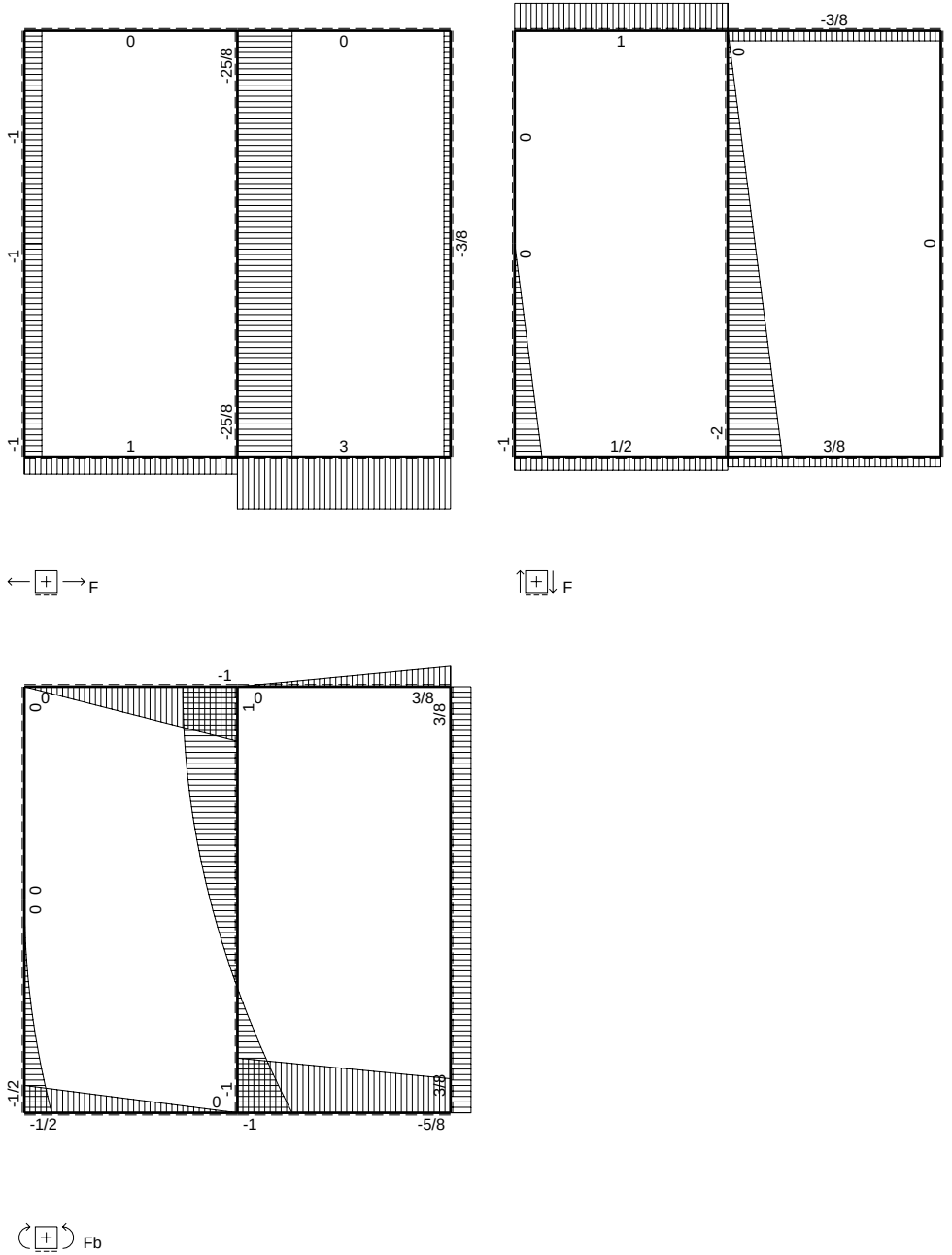


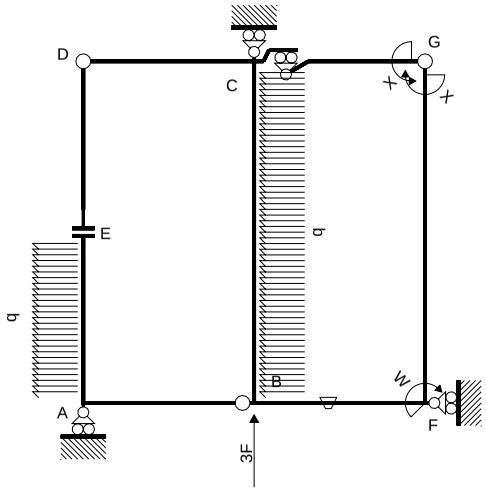
A = 156.4 mm^2
J_u = 82292. mm^4
J_v = 7534. mm^4
J_t = 130.1 mm^4
x_o = -3.813 mm
x_g = 13.67 mm
N = 340. N
T_y = 1020. N
M_x = -622200. Nmm
x_m = 12. mm
y_m = 52. mm
u_m = -1.669 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = 198.8 N/mm^2
x_c = 12. mm
y_c = 52. mm
u_c = -1.669 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = 198.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 59.29 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.479 N/mm^2
$\tau_o = Tx_o t/J_t$ = 53.81 N/mm^2
t_c = 612. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 223.7 N/mm^2

CI08.xxxx.125

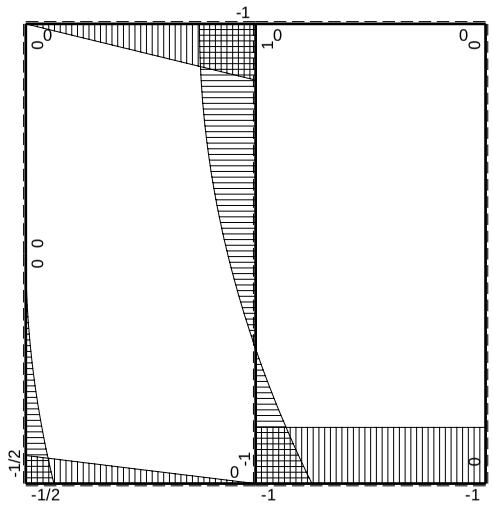
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
25/8F
G
3/8Fb
3/8F
E
F
25/8F
Fb
2F
B
3/8F
3/8Fb
F
F
1/2Fb
F
A
A
F
1/2Fb
1/2F
B
F
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F

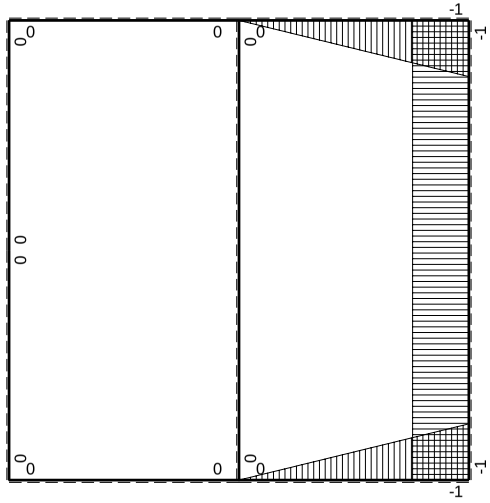

0
-25/8
0
-1
-1
-3/8
-1
1
-25/8
3
F
-3/8
1
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

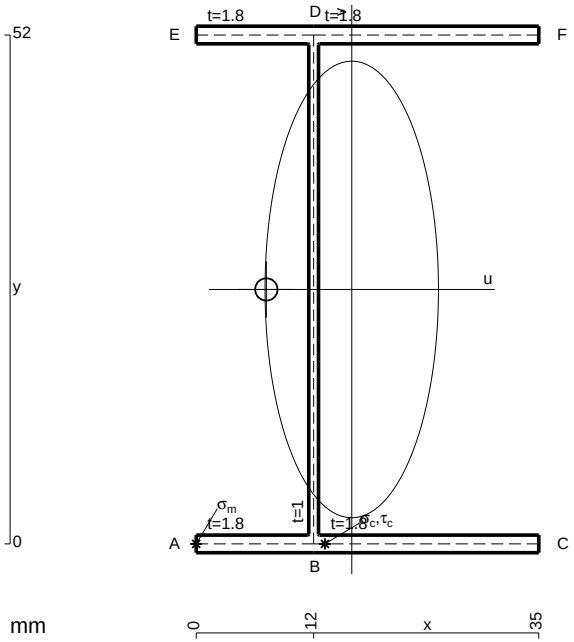
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

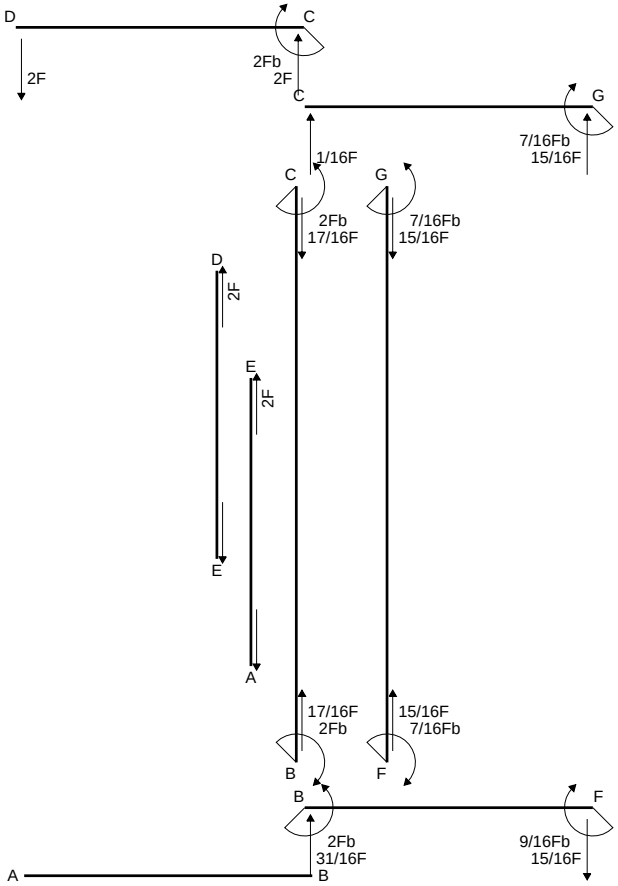
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

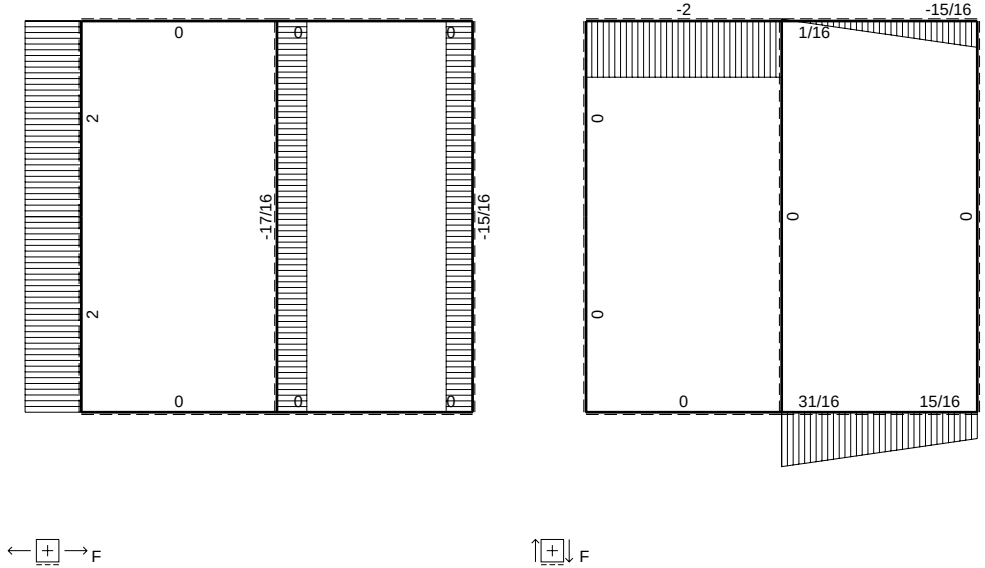


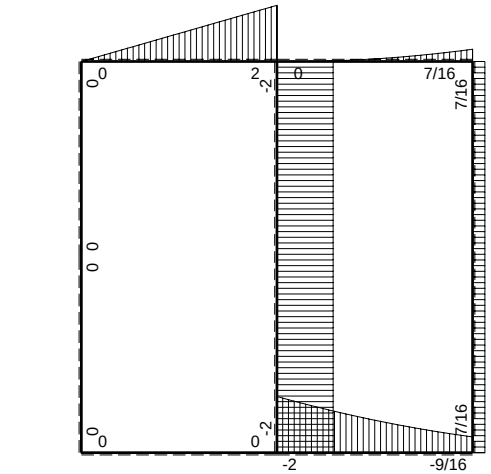
A = 178. mm^2
J_u = 96893. mm^4
J_v = 13976. mm^4
J_t = 153.4 mm^4
x_o = -8.728 mm
x_g = 15.89 mm
N = -3406. N
T_y = -2180. N
M_x = -708500. Nmm
u_m = -15.89 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = -209.3 N/mm^2
x_c = 12. mm
u_c = -3.893 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = -209.3 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 236.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.45 N/mm^2
$\tau_o = Tx_o t/J_t$ = 223.2 N/mm^2
t_c = 1962. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 460.3 N/mm^2

CI08.xxxx.126

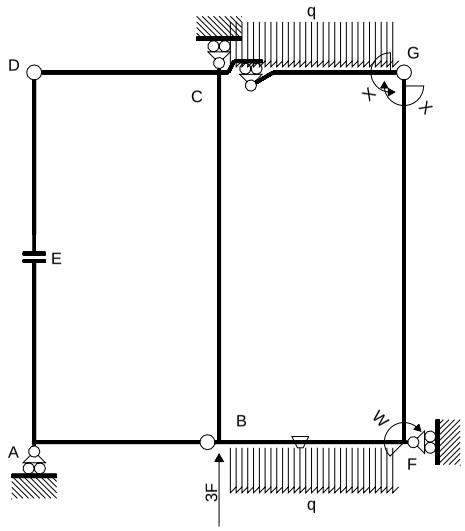
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
17/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
17/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B

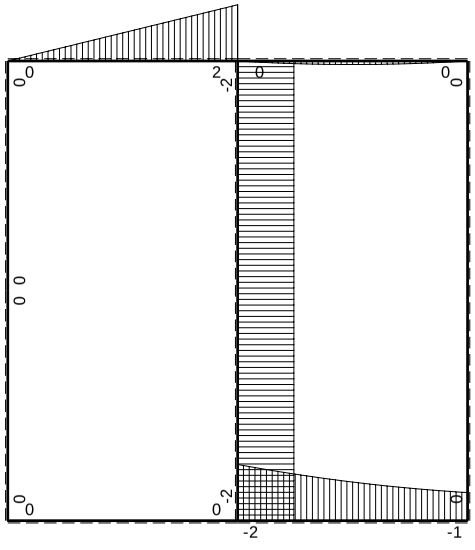

0
0
0
2
-17/16
-15/16
2
0
0
0
-2
-15/16
1/16
0
0
0
0
31/16
15/16
F
F


0
2
0
7/16
0
-2
7/16
0
0
0
0
-2
7/16
-2
-9/16

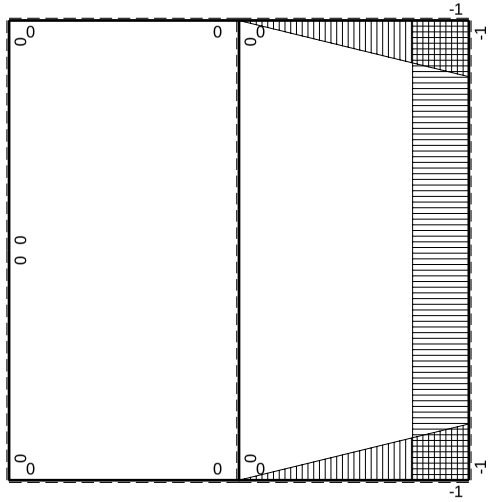
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 917806 CI08.xxxx.127

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

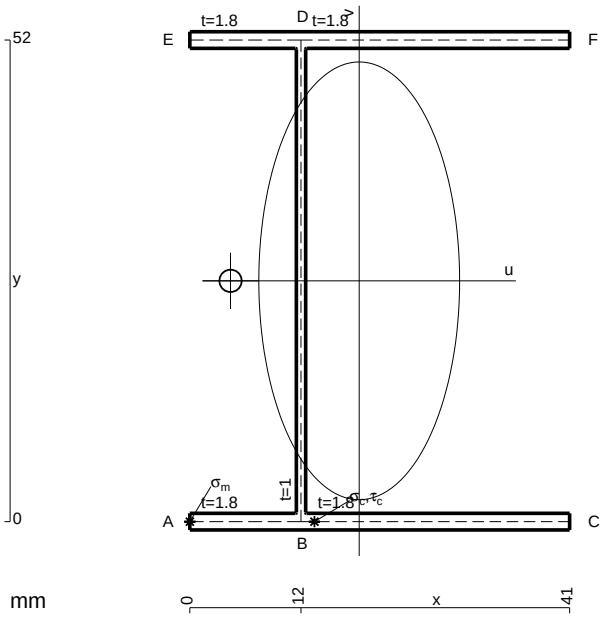
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

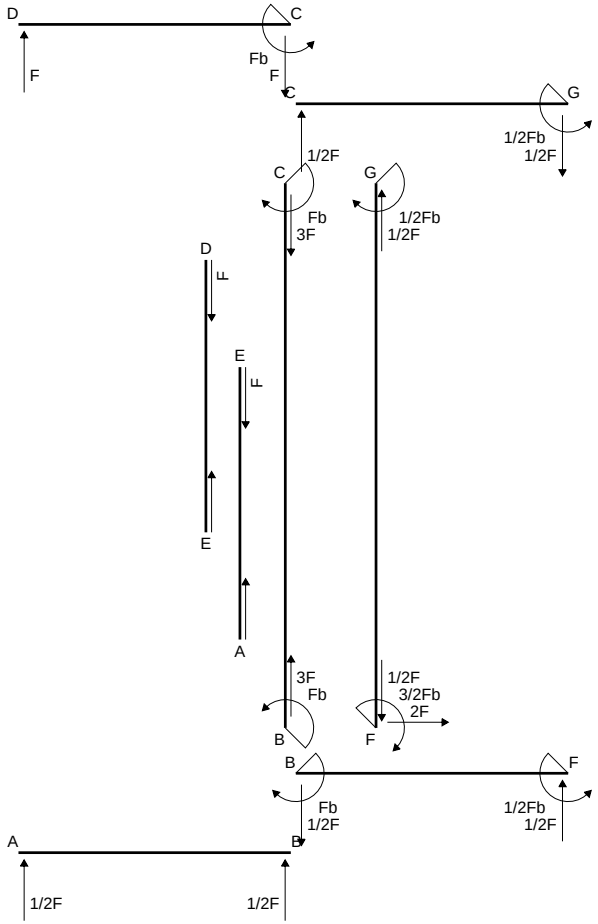
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

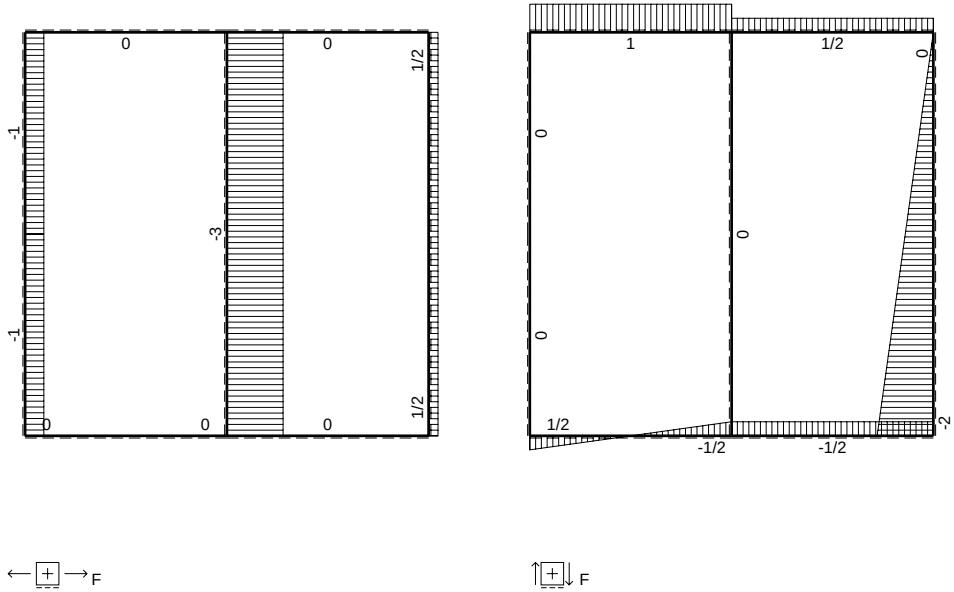


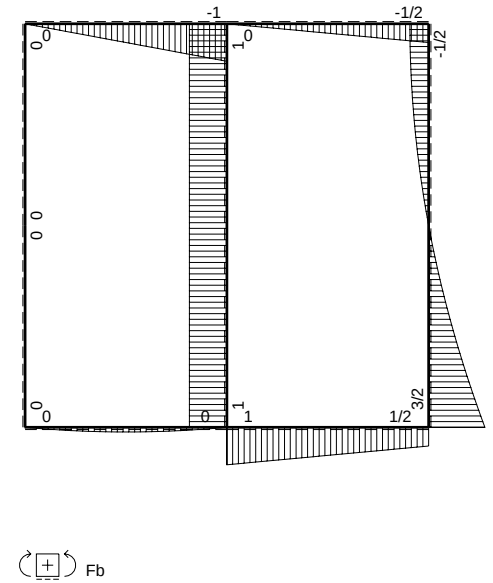
A = 199.6 mm^2
J_u = 111495. mm^4
J_v = 23455. mm^4
J_t = 176.7 mm^4
x_o = -13.89 mm
x_g = 18.29 mm
T_y = -1360. N
M_x = 938400. Nmm
u_m = -18.29 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = 218.8 N/mm^2
x_c = 12. mm
u_c = -6.286 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = 218.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 201.6 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.197 N/mm^2
$\tau_o = Tx_o t/J_t$ = 192.4 N/mm^2
t_c = 1224. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 412.1 N/mm^2

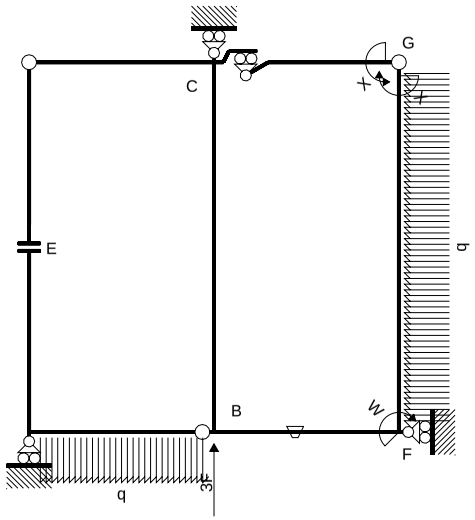
CI08.xxxx.127

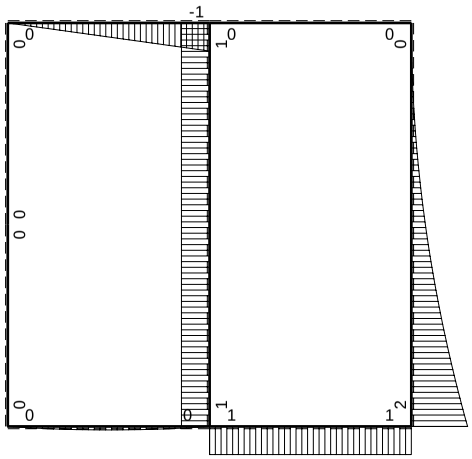
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
3F
1/2Fb
1/2F
D
F
E
F
E
A
3F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F

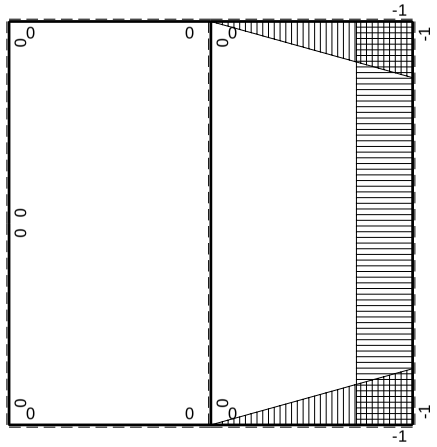

0
0
1/2
-1
-3
-1
0
0
0
1/2
F
1
1/2
0
0
0
1/2
-2
-1/2
-1/2
F


-1
-1/2
0
1
0
-1/2
0
0
0
0
1
1
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 237731 CI08.xxxx.128

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

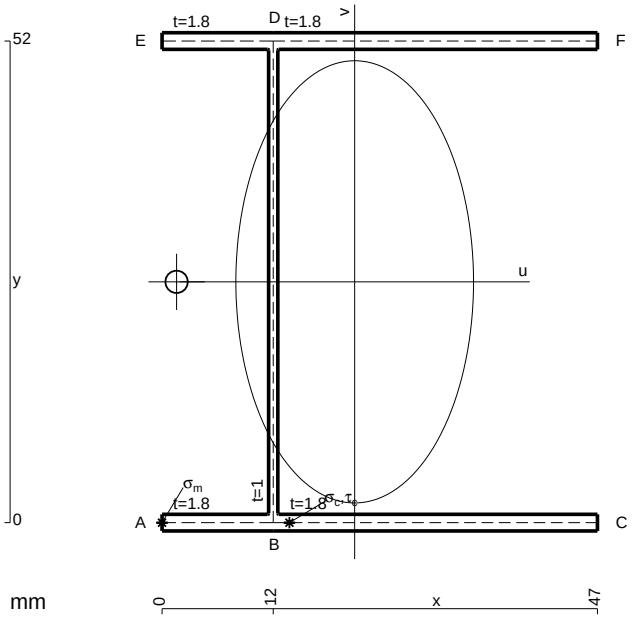
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

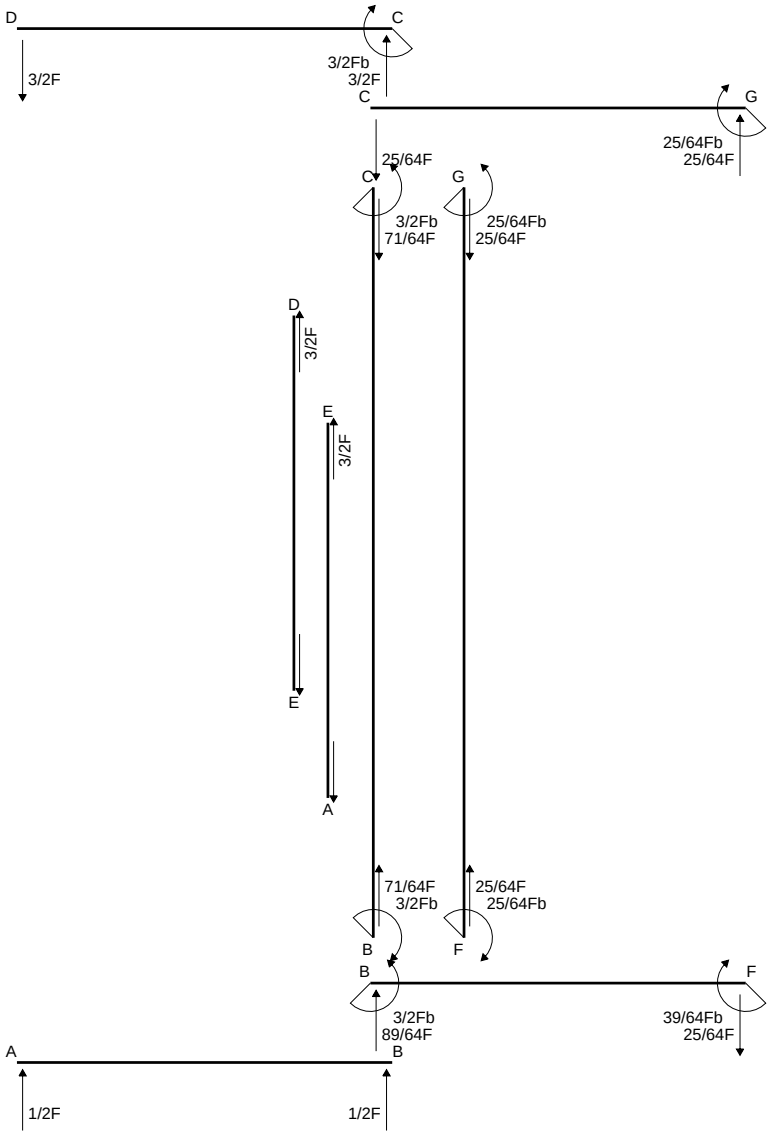
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

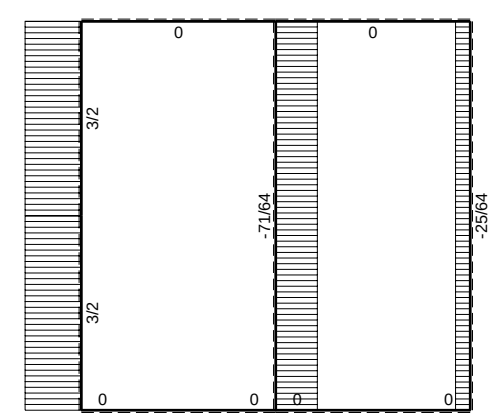


A = 221.2 mm^2
J_u = 126097. mm^4
J_v = 36407. mm^4
J_t = 200.1 mm^4
x_o = -19.23 mm
x_g = 20.8 mm
T_y = 1520. N
M_x = -1109600. Nmm
u_m = -20.8 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -228.8 N/mm^2
x_c = 12. mm
u_c = -8.797 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -228.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 273.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.97 N/mm^2
$\tau_o = Tx_o t/J_t$ = 262.9 N/mm^2
t_c = 2736. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 526.7 N/mm^2

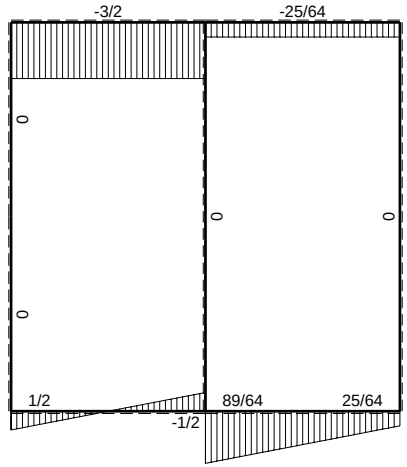
CI08.xxxx.128

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

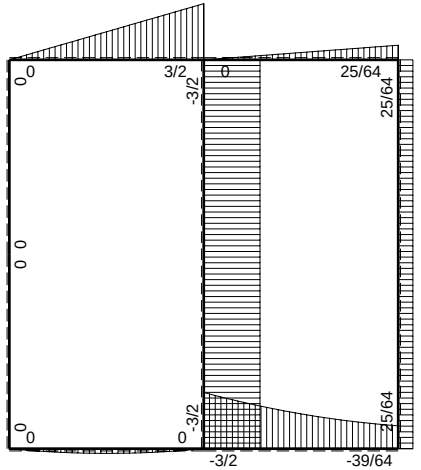

D
C
3/2F
3/2Fb
3/2F
C
G
25/64F
25/64Fb
25/64F
C
G
3/2Fb
71/64F
25/64Fb
25/64F
D
3/2F
E
3/2F
E
A
71/64F
3/2Fb
25/64F
25/64Fb
B
F
B
F
3/2Fb
89/64F
39/64Fb
25/64F
B
A
1/2F
1/2F


0
0
3/2
-71/64
-25/64
3/2
0
0
0
0

F


-3/2
-25/64
0
0
0
0
1/2
89/64
25/64
-1/2

F


0
0
3/2
-3/2
0
25/64
25/64
0
0
0
0
-3/2
25/64
-3/2
-39/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	$3/2Fb-3/2Fx$	0	0	0	0	0+0	0
DC b	0	$-3/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-3/2Fb+Fx-1/2qx^2$	-Fb/EJ	$3/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(13/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2qx^2$	Fb/EJ	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
totali							$25/24Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							-25/64Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_o flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

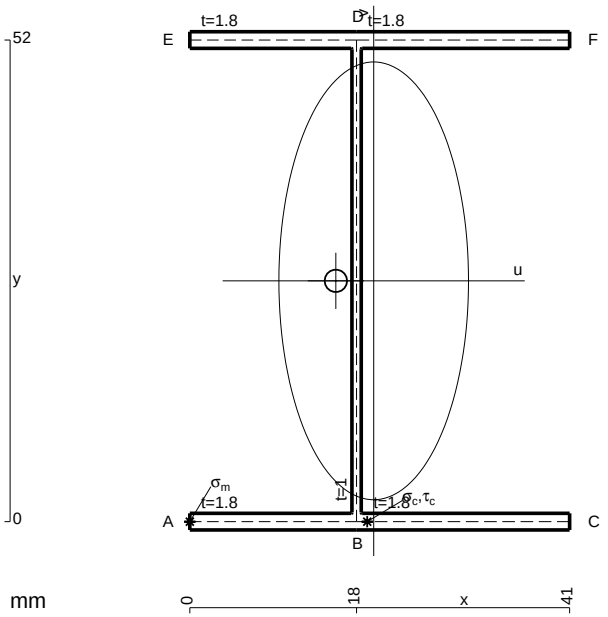
$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

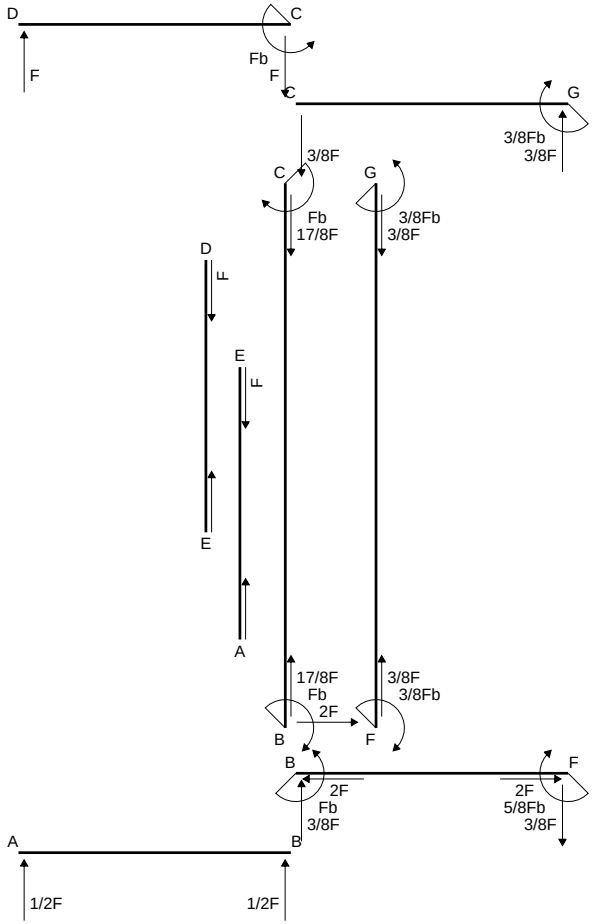
$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

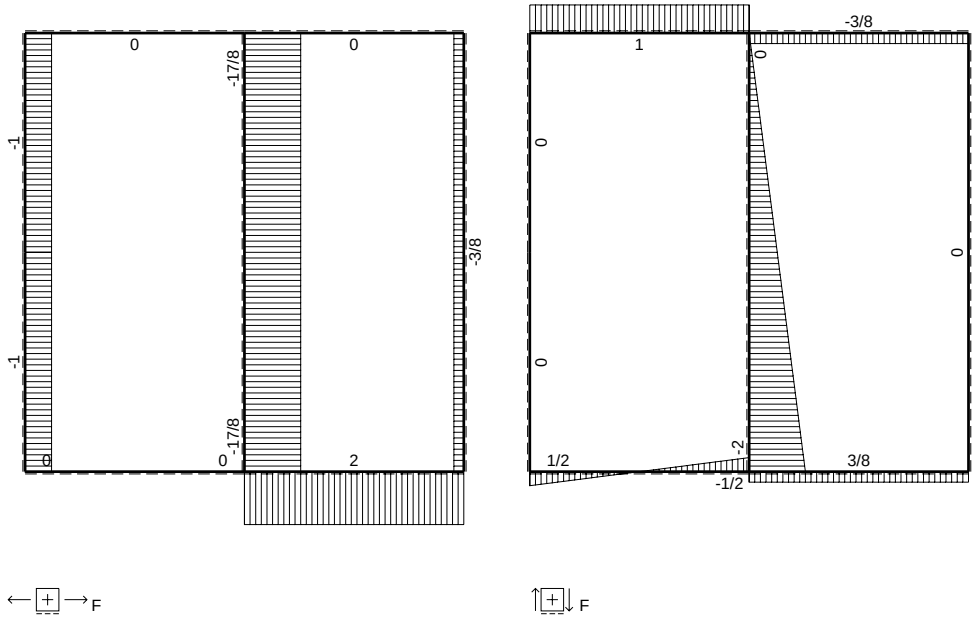


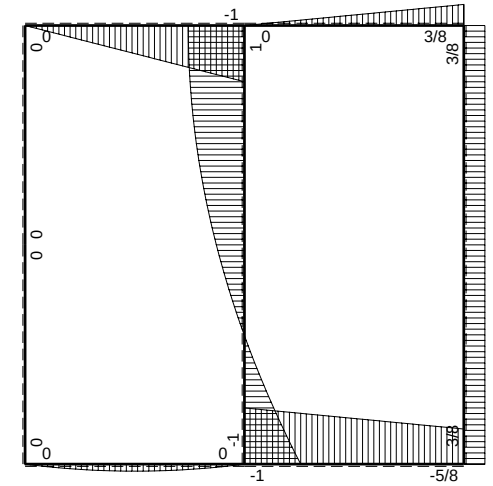
A = 199.6 mm^2
J_u = 111495. mm^4
J_v = 20917. mm^4
J_t = 176.7 mm^4
x_o = -4.086 mm
x_g = 19.85 mm
T_y = -1335. N
M_x = 1027950. Nmm
u_m = -19.85 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = 239.7 N/mm^2
x_c = 18. mm
u_c = -1.849 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = 239.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 62.71 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.16 N/mm^2
$\tau_o = Tx_o t/J_t$ = 55.55 N/mm^2
t_c = 1602. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 263.2 N/mm^2

CI08.xxxx.129

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
17/8F
3/8Fb
3/8F
D
F
E
F
E
A
17/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F


0
0
-17/8
-1
-3/8
-1
0
-17/8
0
2
1
-3/8
0
0
0
0
1/2
-2
3/8
-1/2
F
F


-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	$1/3XLb/EJ$
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3XLb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2XLb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	$8/3XLb/EJ$
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_o flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 218352 CI08.xxxx.130

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

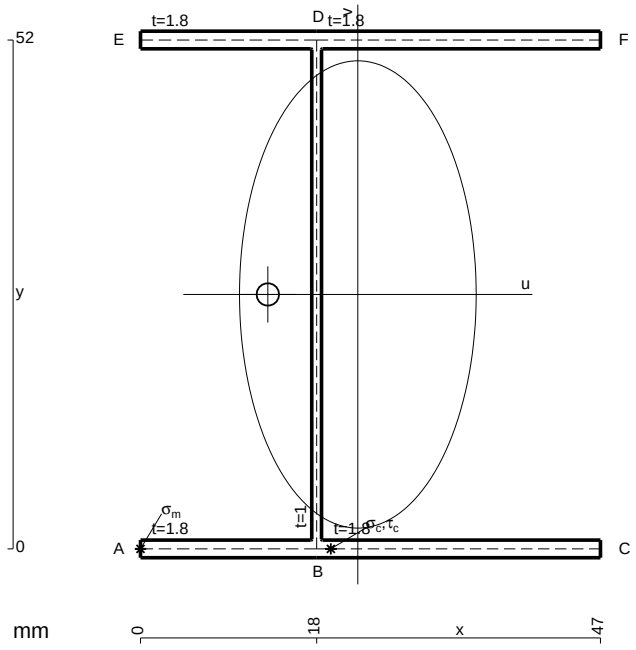
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

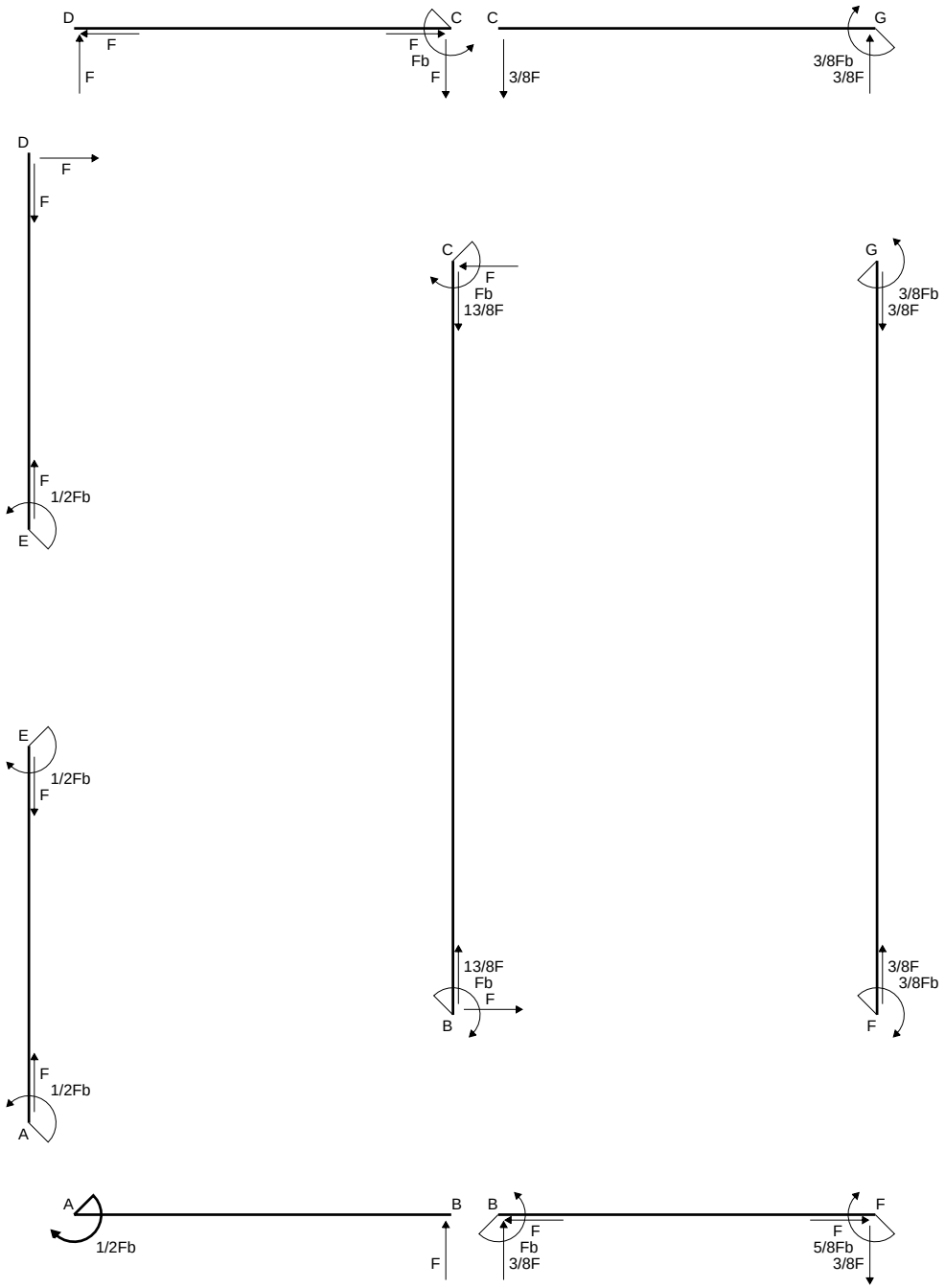
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

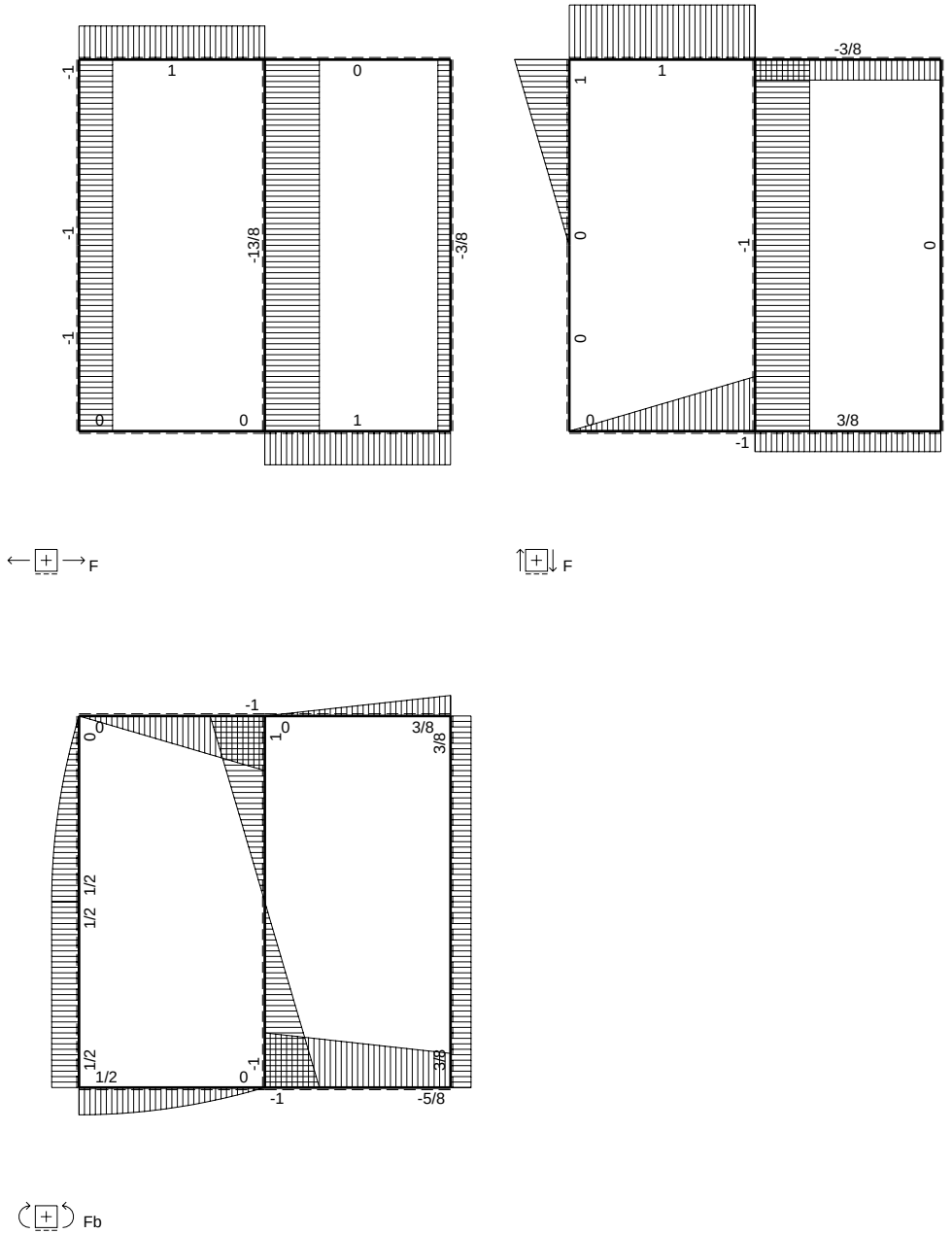


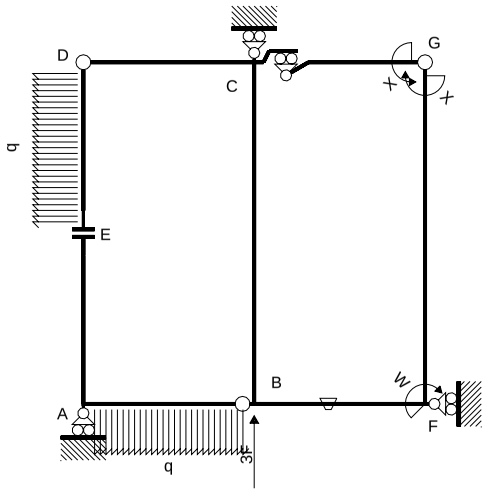
A = 221.2 mm^2
J_u = 126097. mm^4
J_v = 32350. mm^4
J_t = 200.1 mm^4
x_o = -9.196 mm
x_g = 22.21 mm
N = -4505. N
T_y = -4240. N
M_x = -869200. Nmm
u_m = -22.21 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = -199.6 N/mm^2
x_c = 18. mm
u_c = -4.207 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = -199.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 376.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 25.35 N/mm^2
$\tau_o = Tx_o t/J_t$ = 350.8 N/mm^2
t_c = 3816. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 681.4 N/mm^2

CI08.xxxx.130

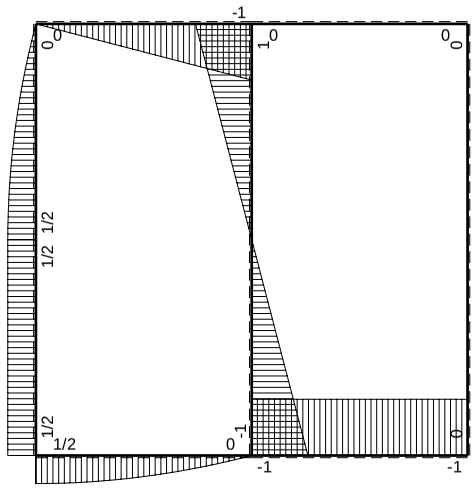
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
13/8F
G
3/8Fb
3/8F
E
1/2Fb
F
13/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F

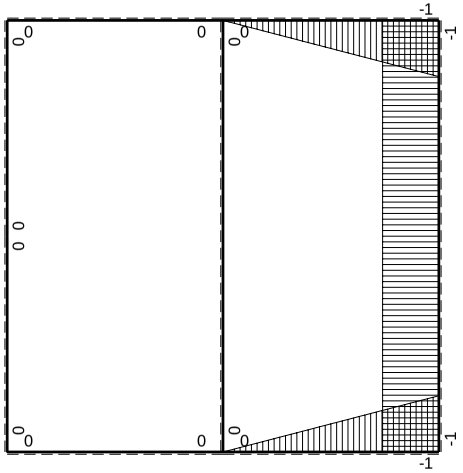

-1
1
0
-1
-13/8
-3/8
-1
0
0
1
F
-3/8
1
1
1
0
-1
0
0
0
3/8
-1
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 218194 CI08.xxxx.131

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

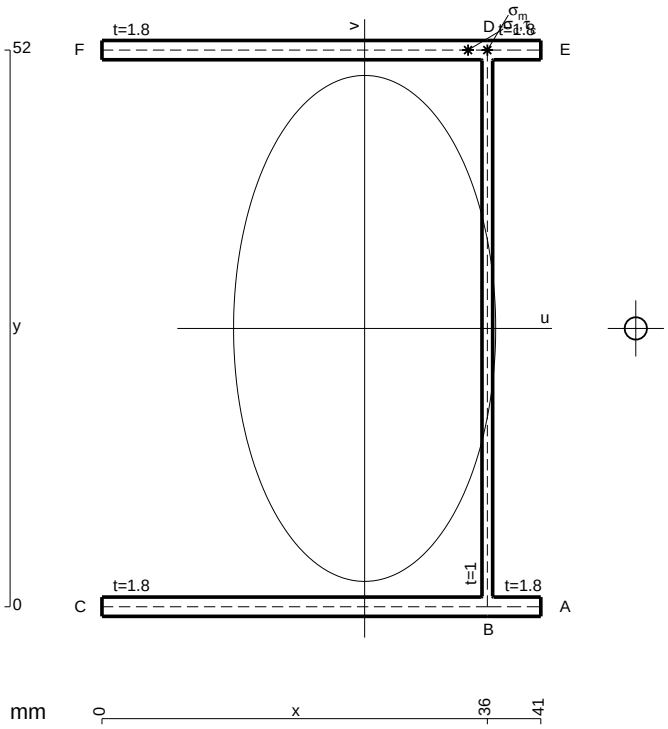
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

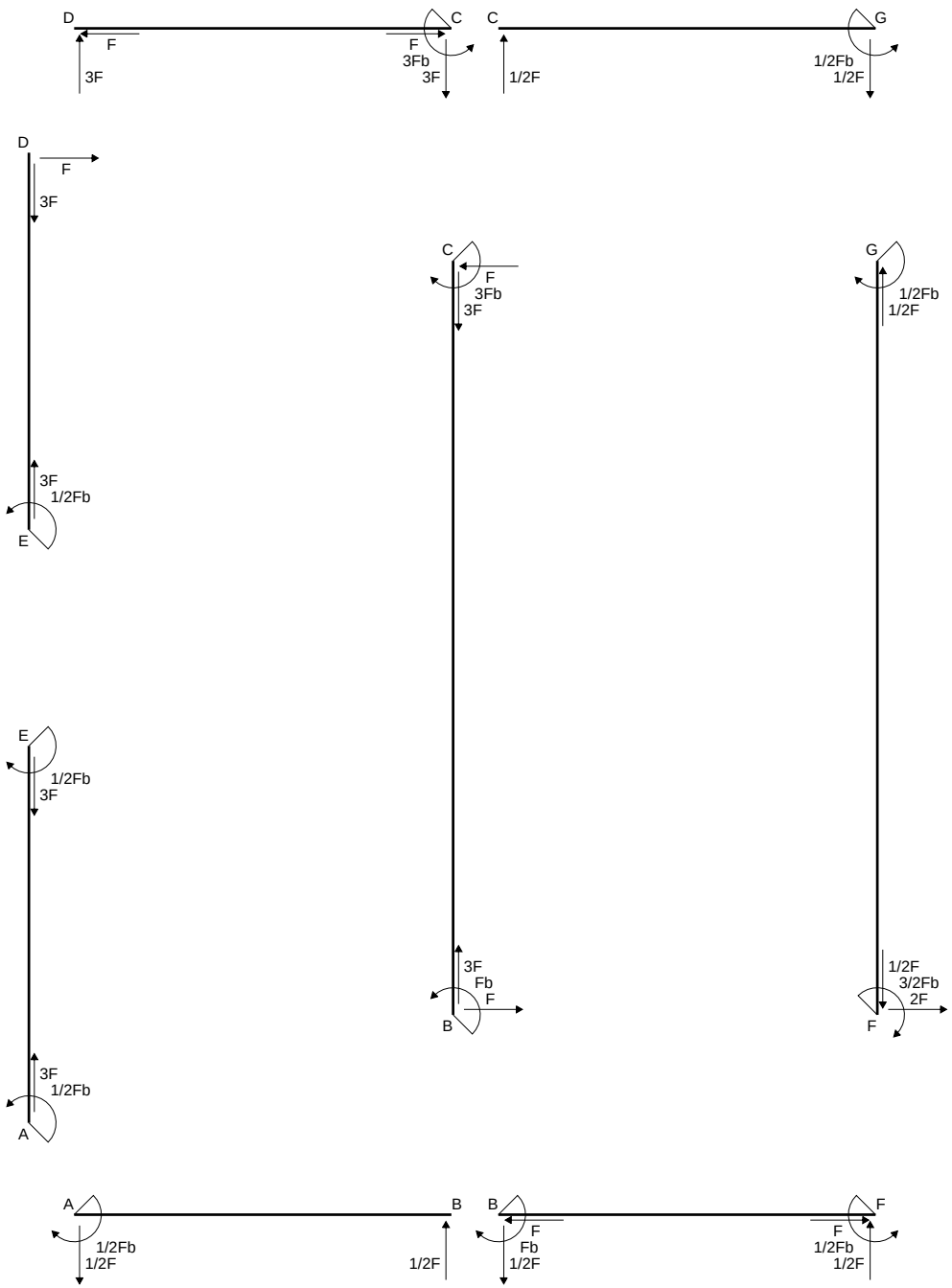
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

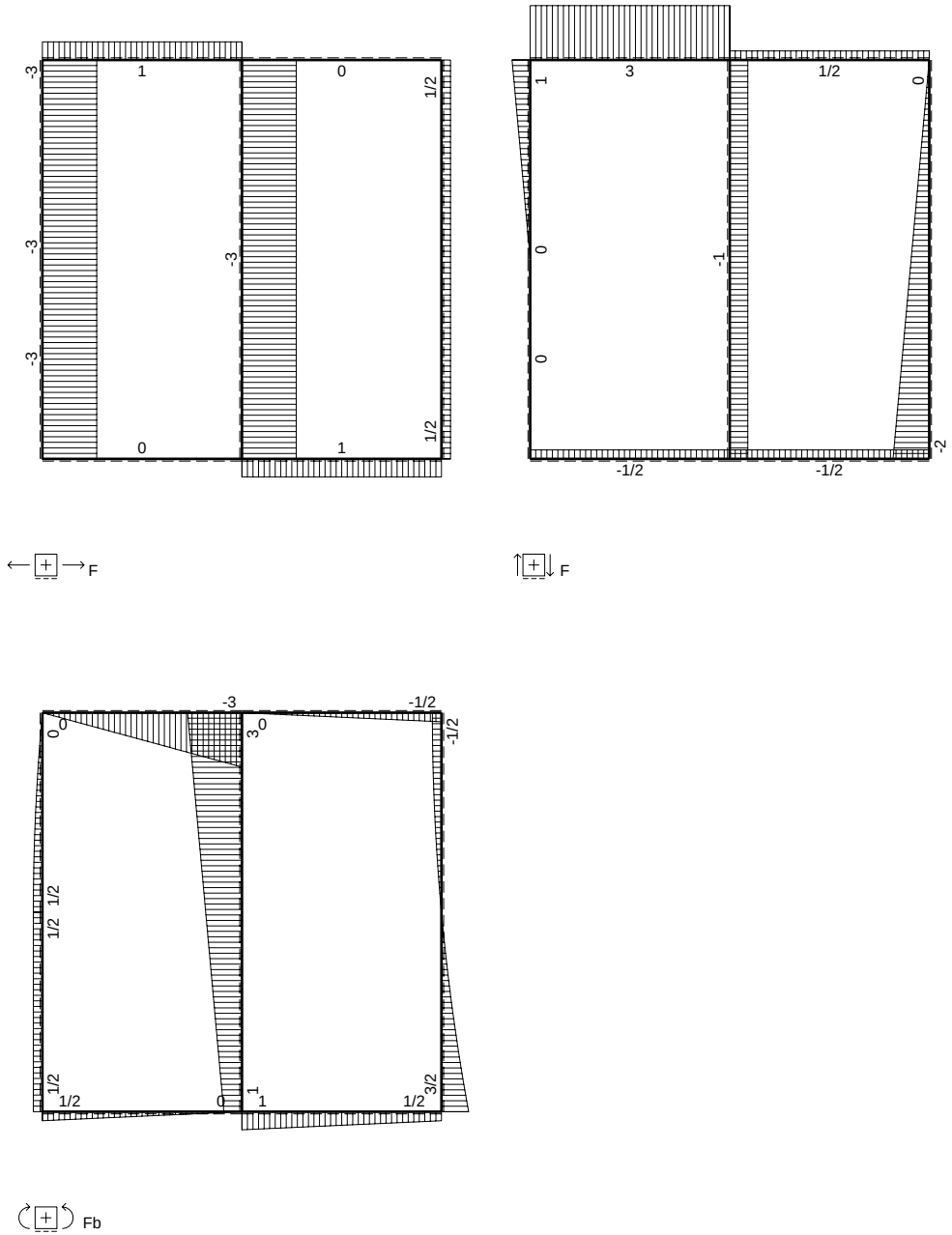


A = 199.6 mm^2
J_u = 111495. mm^4
J_v = 29915. mm^4
J_t = 176.7 mm^4
x_o = 25.33 mm
x_g = 24.54 mm
N = -3006. N
T_y = -1850. N
M_x = 832500. Nmm
x_m = 36. mm
y_m = 52. mm
u_m = 11.46 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = -209.2 N/mm^2
x_c = 36. mm
y_c = 52. mm
u_c = 11.46 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = -209.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 492.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.53 N/mm^2
$\tau_o = Tx_o t/J_t$ = 477.3 N/mm^2
t_c = 3330. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 878.9 N/mm^2

CI08.xxxx.131

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
3F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
3F
Fb
F
B
1/2F
3/2Fb
2F
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


-3
1
0
1/2
-3
-3
0
1
1/2
F
1
3
1/2
0
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^{2}b^2	(-1/2+1/2)Fb^2/EJ	1/3XbEJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2XbEJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3FbEJ	8/3XbEJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

PROCEDIMENTO E RISULTATI 218622 CI08.xxxx.132

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

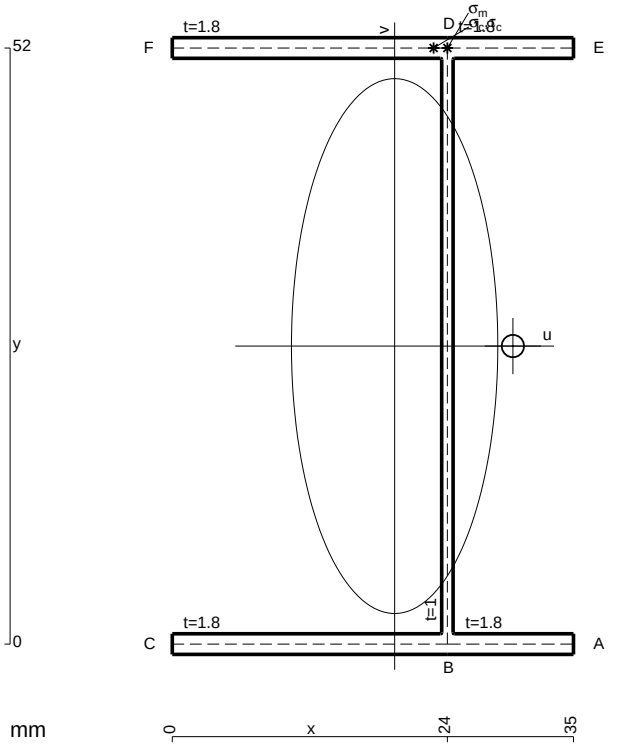
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

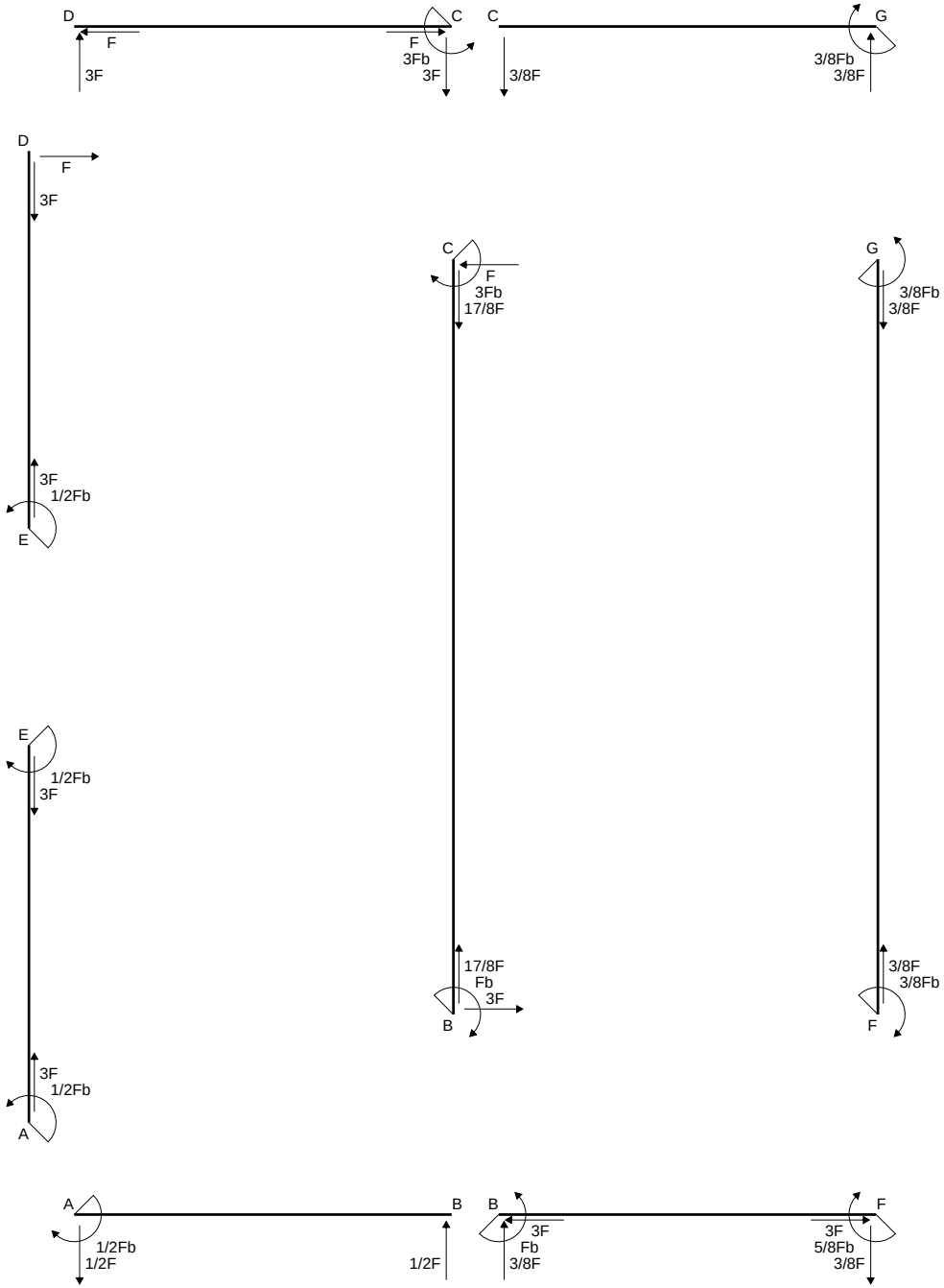
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

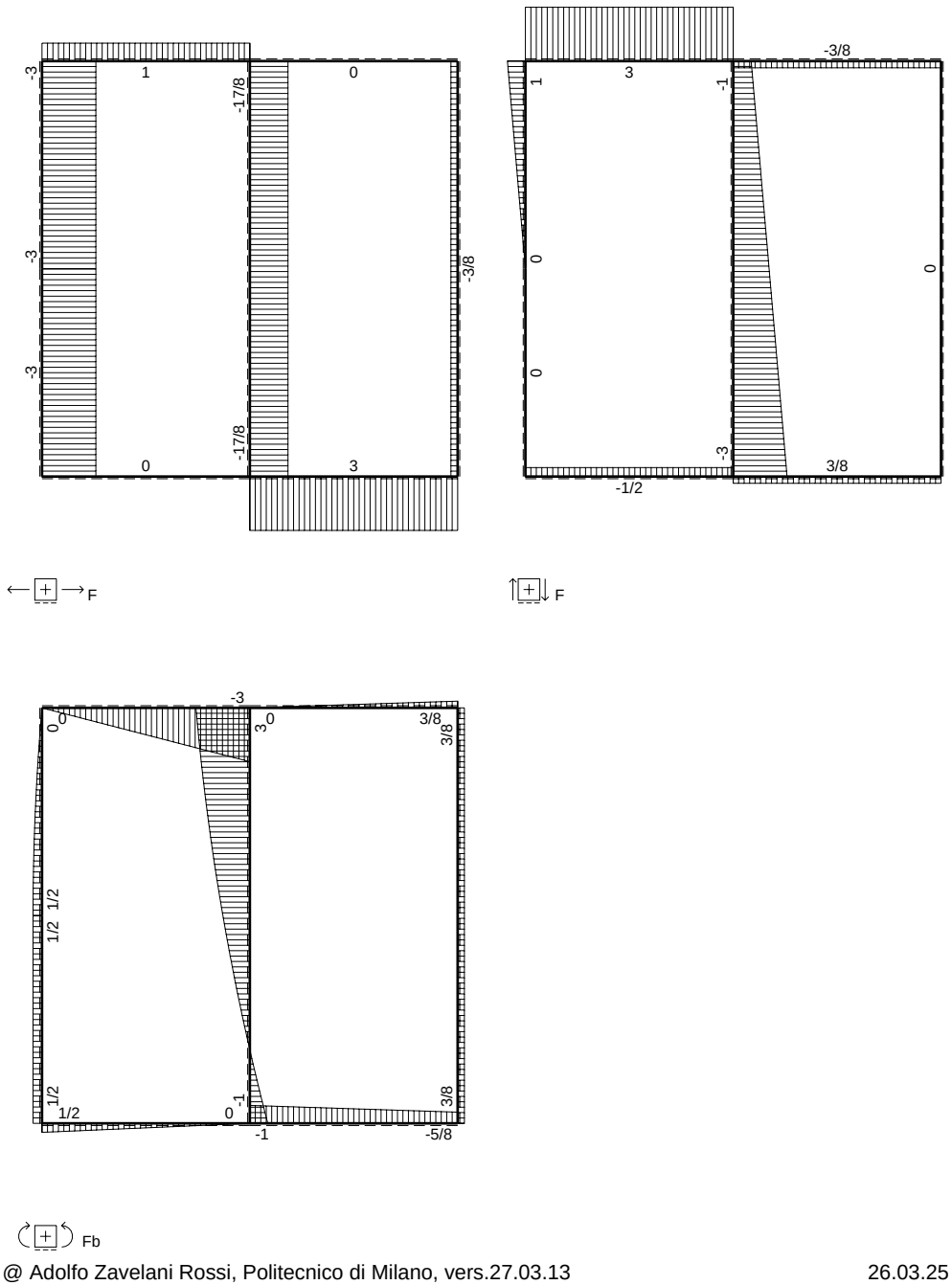


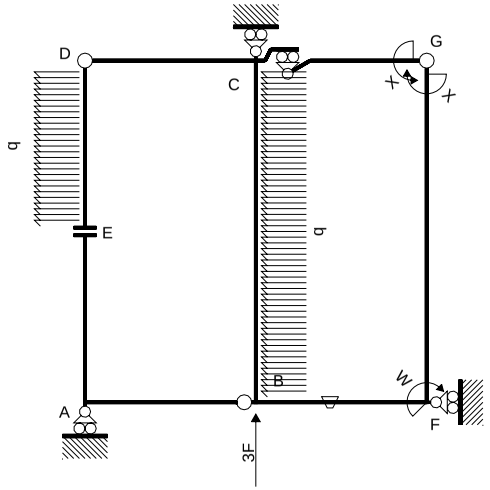
A = 178. mm^2
J_u = 96893. mm^4
J_v = 14418. mm^4
J_t = 153.4 mm^4
x_o = 10.32 mm
x_g = 19.4 mm
N = 540. N
T_y = 1620. N
M_x = -793800. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 4.601 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = 216. N/mm^2
x_c = 24. mm
y_c = 52. mm
u_c = 4.601 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = 216. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 206.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.43 N/mm^2
$\tau_o = Tx_ot/J_t$ = 196.1 N/mm^2
t_c = 972. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 417.8 N/mm^2

CI08.xxxx.132

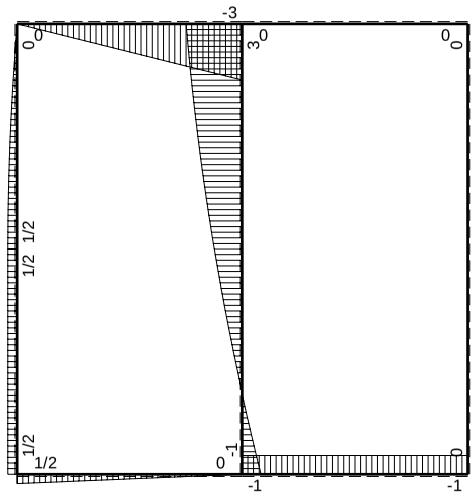
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
3/8F
3/8Fb
3/8F
D
F
3F
C
F
3Fb
17/8F
G
3/8Fb
3/8F
3F
1/2Fb
E
E
1/2Fb
3F
17/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
3F
Fb
3/8F
3F
5/8Fb
3/8F

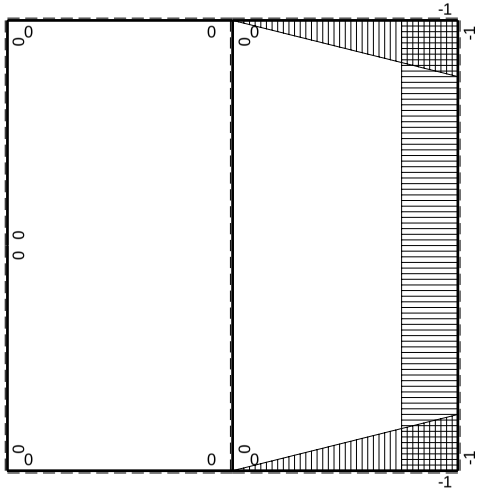

-3
1
0
-17/8
-3
-3/8
-3
-17/8
0
3
F
-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F
-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 214029 CI08.xxxx.133

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

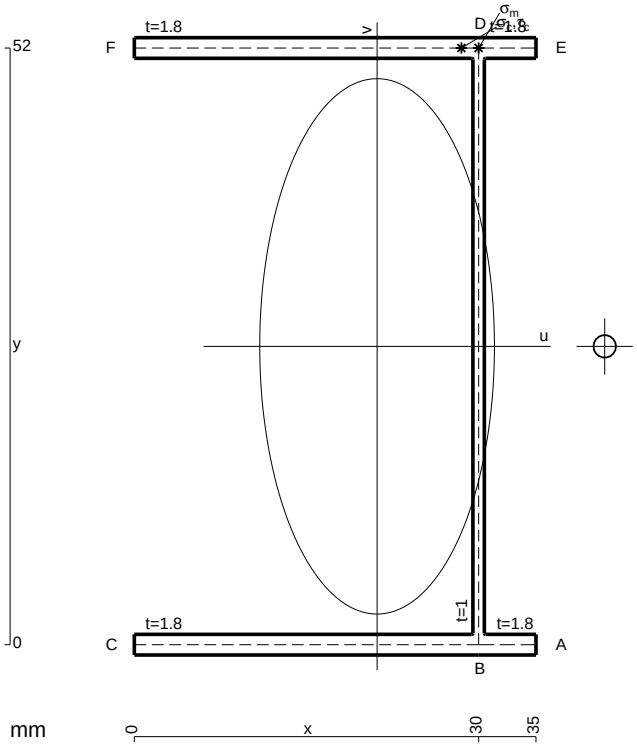
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

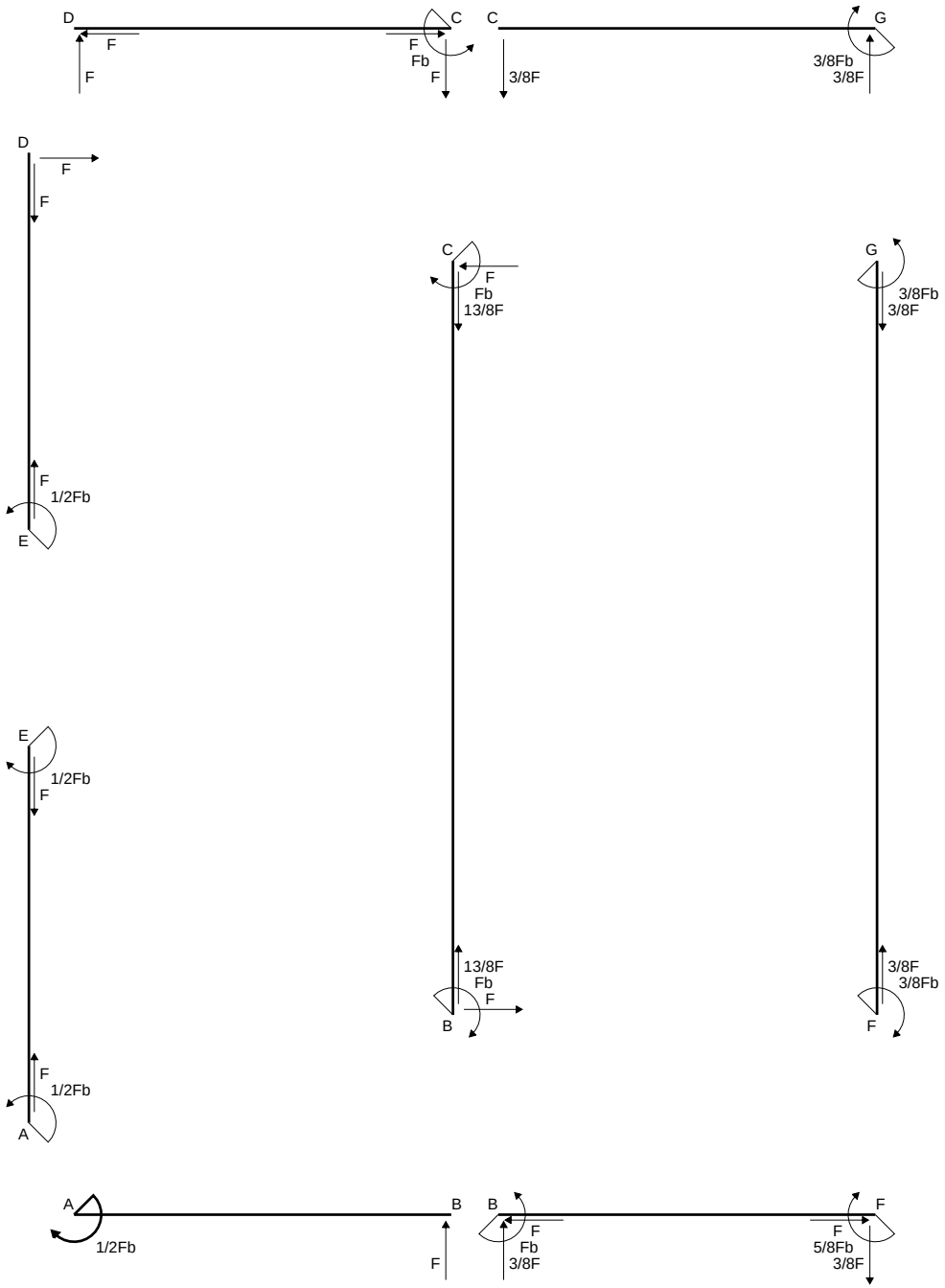
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

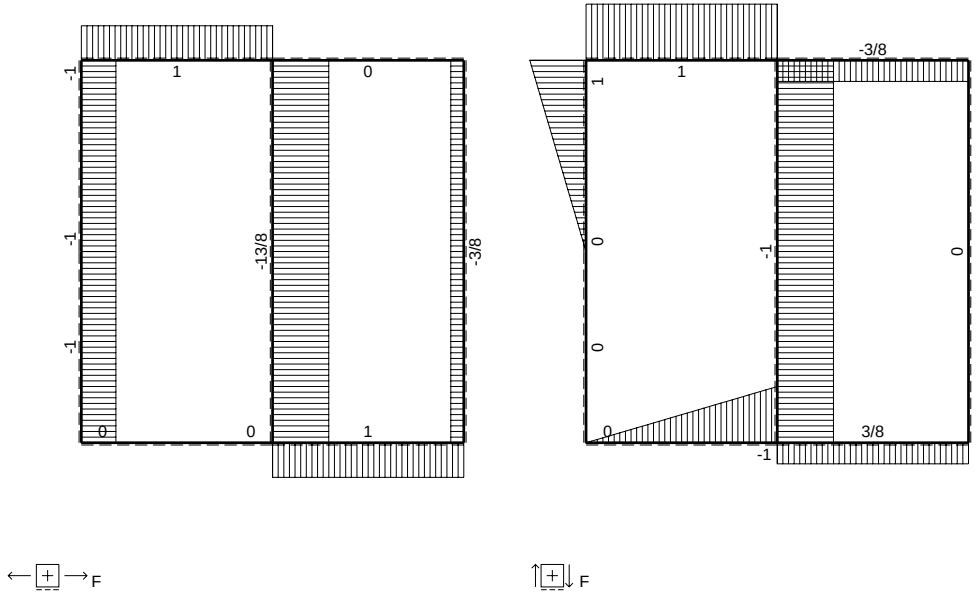


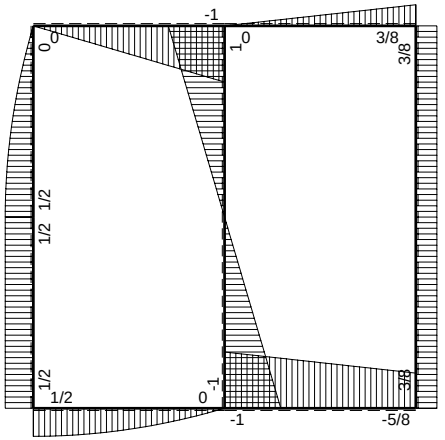
A = 178. mm^2
J_u = 96893. mm^4
J_v = 18614. mm^4
J_t = 153.4 mm^4
x_o = 19.84 mm
x_g = 21.15 mm
N = 530. N
T_y = 1590. N
M_x = -842700. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 8.848 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = 229.1 N/mm^2
x_c = 30. mm
y_c = 52. mm
u_c = 8.848 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = 229.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 382.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.8 N/mm^2
$\tau_o = Tx_o t/J_t$ = 370.1 N/mm^2
t_c = 954. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 701.6 N/mm^2

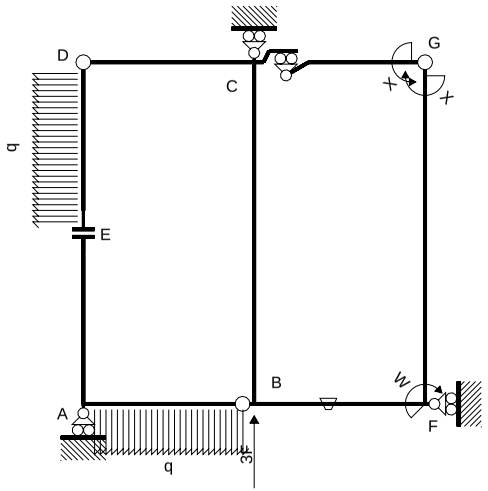
CI08.xxxx.133

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

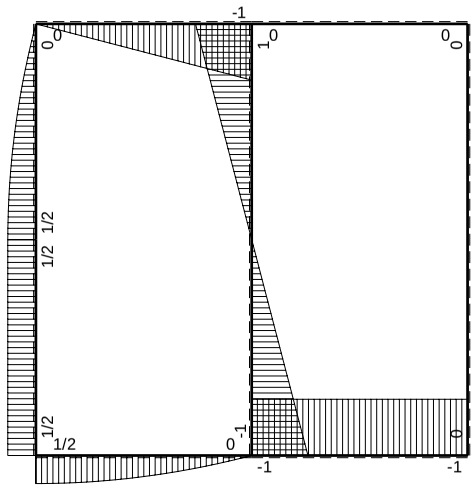

D
C
C
G
F
F
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
F
C
F
Fb
13/8F
G
3/8Fb
3/8F
F
1/2Fb
E
E
1/2Fb
F
13/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
B
B
F
1/2Fb
F
F
Fb
3/8F
F
5/8Fb
3/8F


-1
1
0
-1
-13/8
-3/8
-1
0
0
1
-3/8
1
1
0
-1
0
0
3/8
-1
F

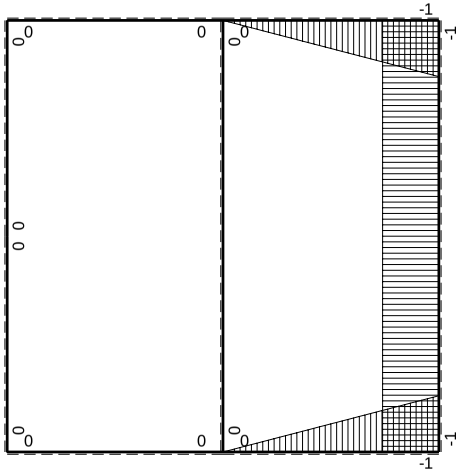

-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209692 CI08.xxxx.134

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

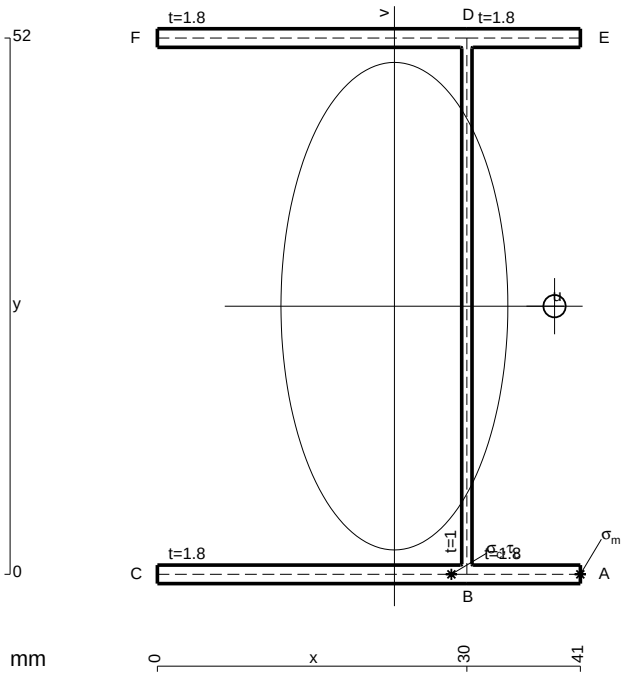
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

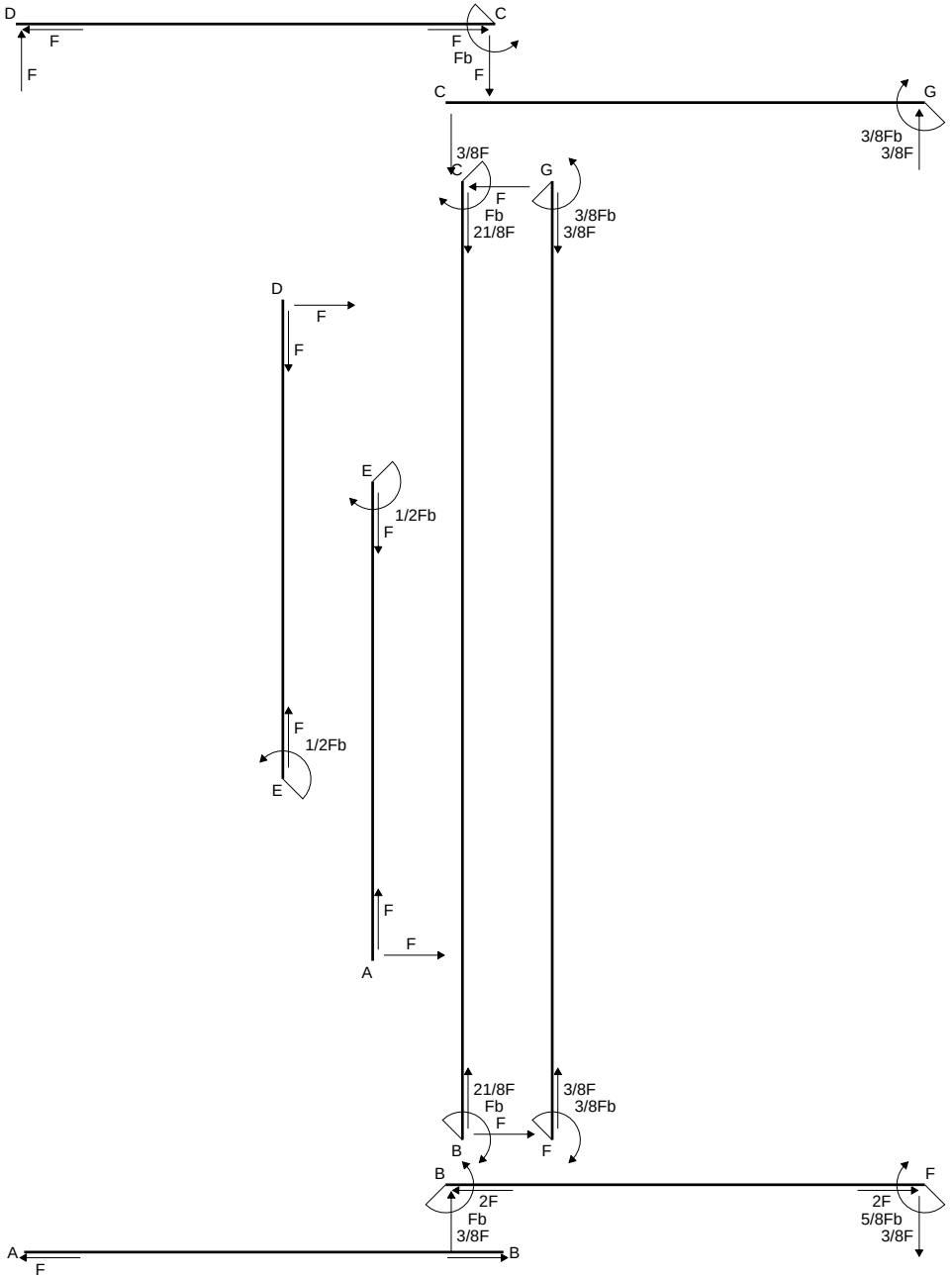
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

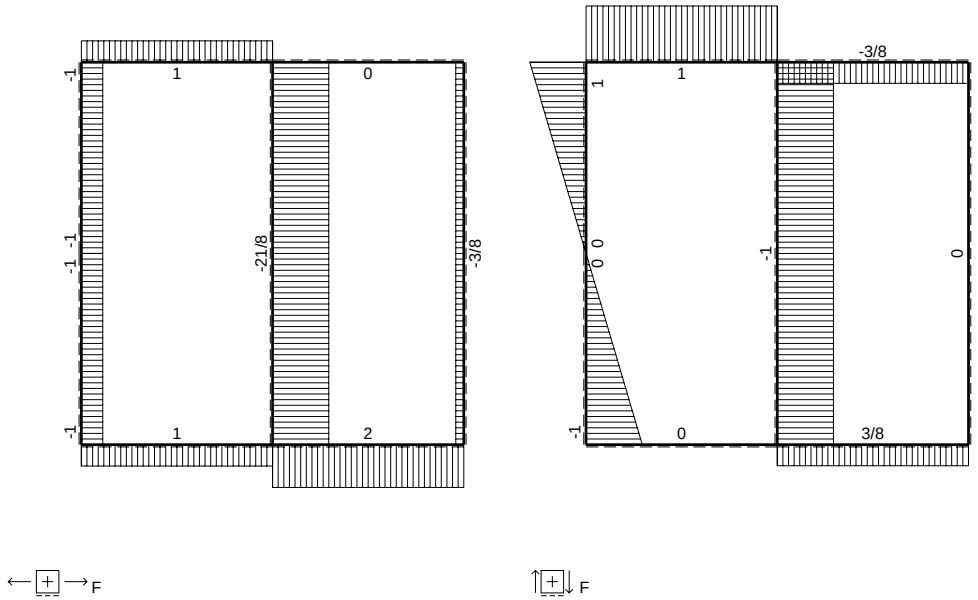


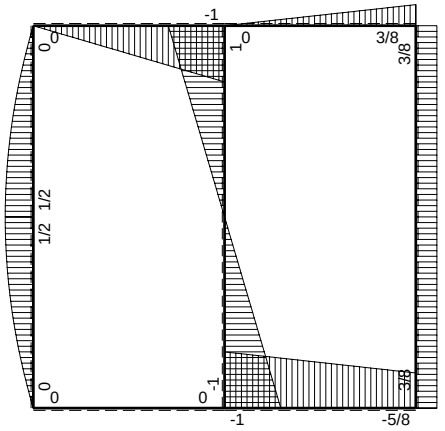
A = 199.6 mm^2
J_u = 111495. mm^4
J_v = 24147. mm^4
J_t = 176.7 mm^4
x_o = 15.53 mm
x_g = 22.98 mm
N = -2763. N
T_y = -1700. N
M_x = -969000. Nmm
x_m = 41. mm
u_m = 18.02 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = -239.8 N/mm^2
x_c = 30. mm
u_c = 7.025 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = -239.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 280.7 N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.89 N/mm^2
$\tau_o = Tx_o t/J_t$ = 268.8 N/mm^2
t_c = 3060. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 542.1 N/mm^2

CI08.xxxx.134

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
21/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
21/8F
Fb
F
3/8F
3/8Fb
F
B
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F


-1
1
0
-1
-1
-21/8
-3/8
-1
1
2
-3/8
1
1
0
0
-1
0
-1
0
3/8
F


-1
0
0
1
0
3/8
3/8
1/2
1/2
0
0
0
-1
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3XbEJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbEJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3XbEJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

M_o flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 218087 CI08.xxxx.135

Schema di calcolo iperstatico

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

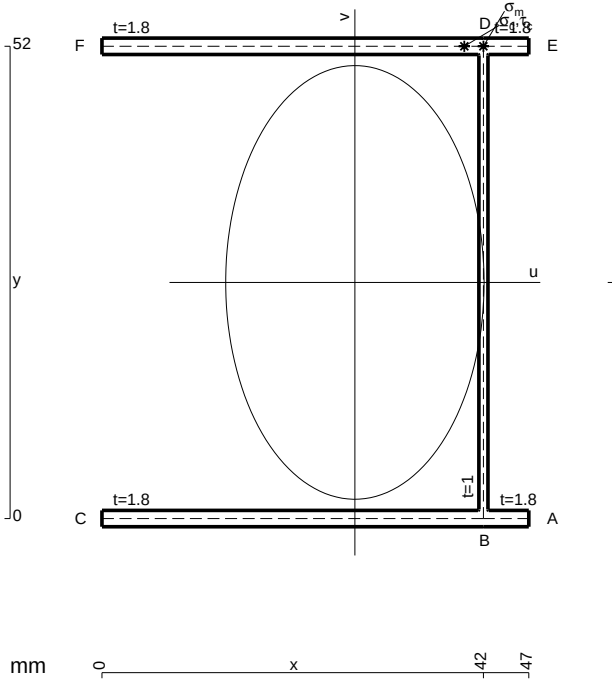
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

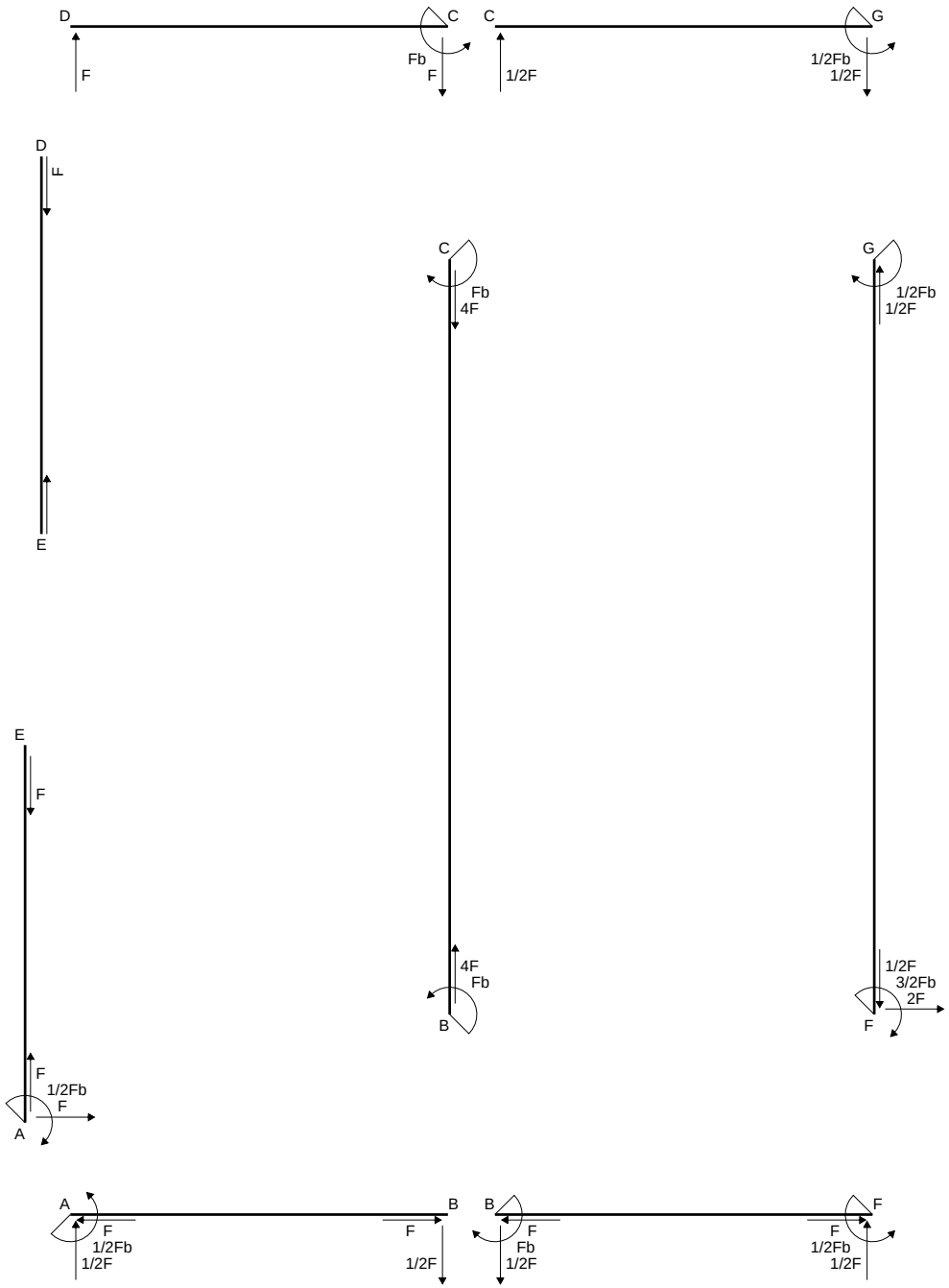
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

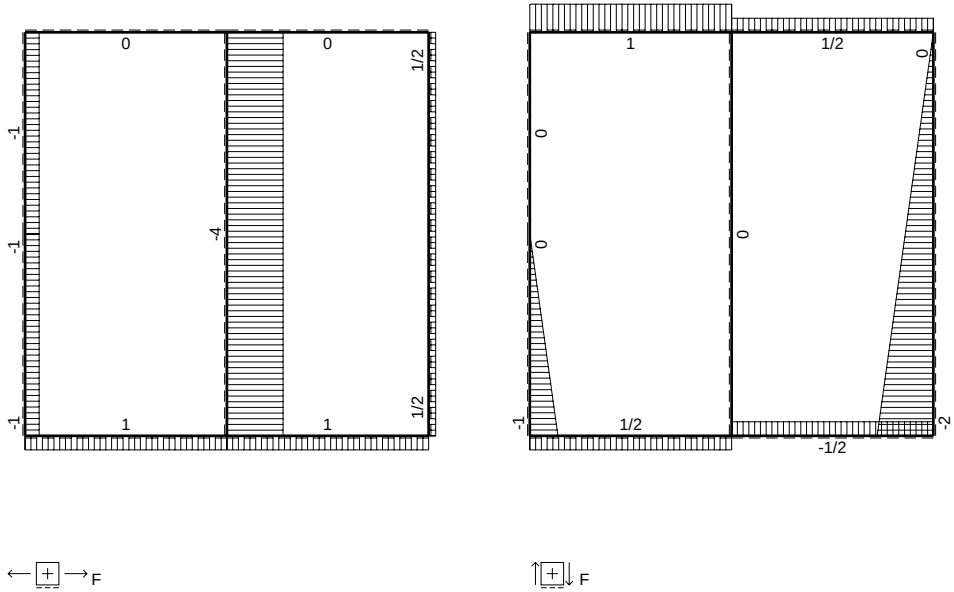


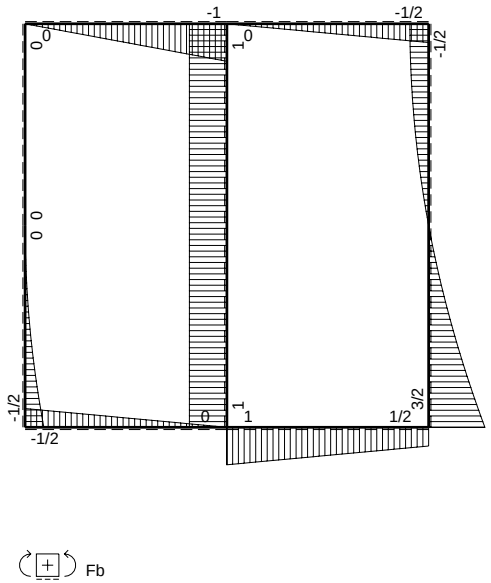
A = 221.2 mm^2
J_u = 126097. mm^4
J_v = 44760. mm^4
J_t = 200.1 mm^4
x_o = 30.93 mm
x_g = 27.85 mm
N = -3806. N
T_y = -1450. N
M_x = 884500. Nmm
x_m = 42. mm
y_m = 52. mm
u_m = 14.15 mm
v_m = 26. mm
$\sigma_m = N/A - Mv/J_u$ = -199.6 N/mm^2
x_c = 42. mm
y_c = 52. mm
u_c = 14.15 mm
v_c = 26. mm
$\sigma_c = N/A - Mv/J_u$ = -199.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 416.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.56 N/mm^2
$\tau_o = Tx_o t/J_t$ = 403.5 N/mm^2
t_c = 2610. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 747.8 N/mm^2

CI08.xxxx.135

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
4F
G
1/2Fb
1/2F
E
F
F
1/2Fb
F
A
4F
Fb
B
1/2F
3/2Fb
2F
F
A
B
B
F
F
1/2Fb
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


0
0
1/2
-1
-1
-4
-1
1
1
1/2
F
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F


-1
-1/2
0
0
1
0
-1/2
0 0
0
-1/2
1
1
1/2
3/2
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$1/2Fb$	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

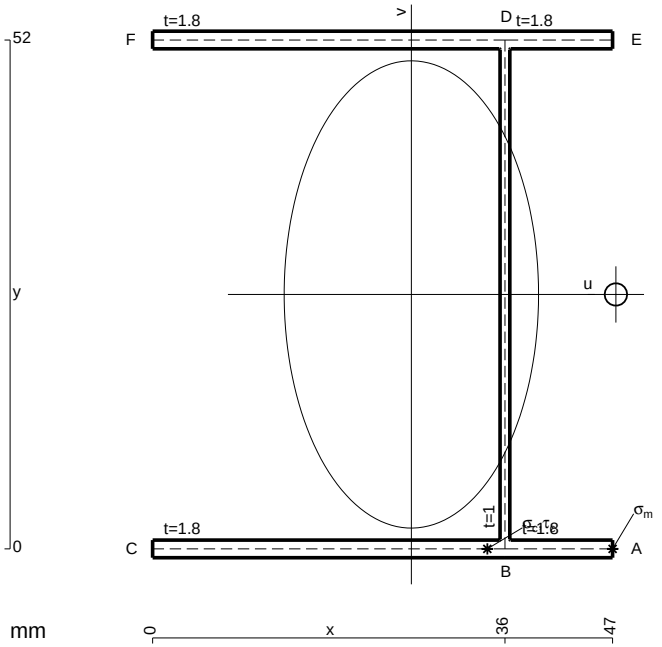
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

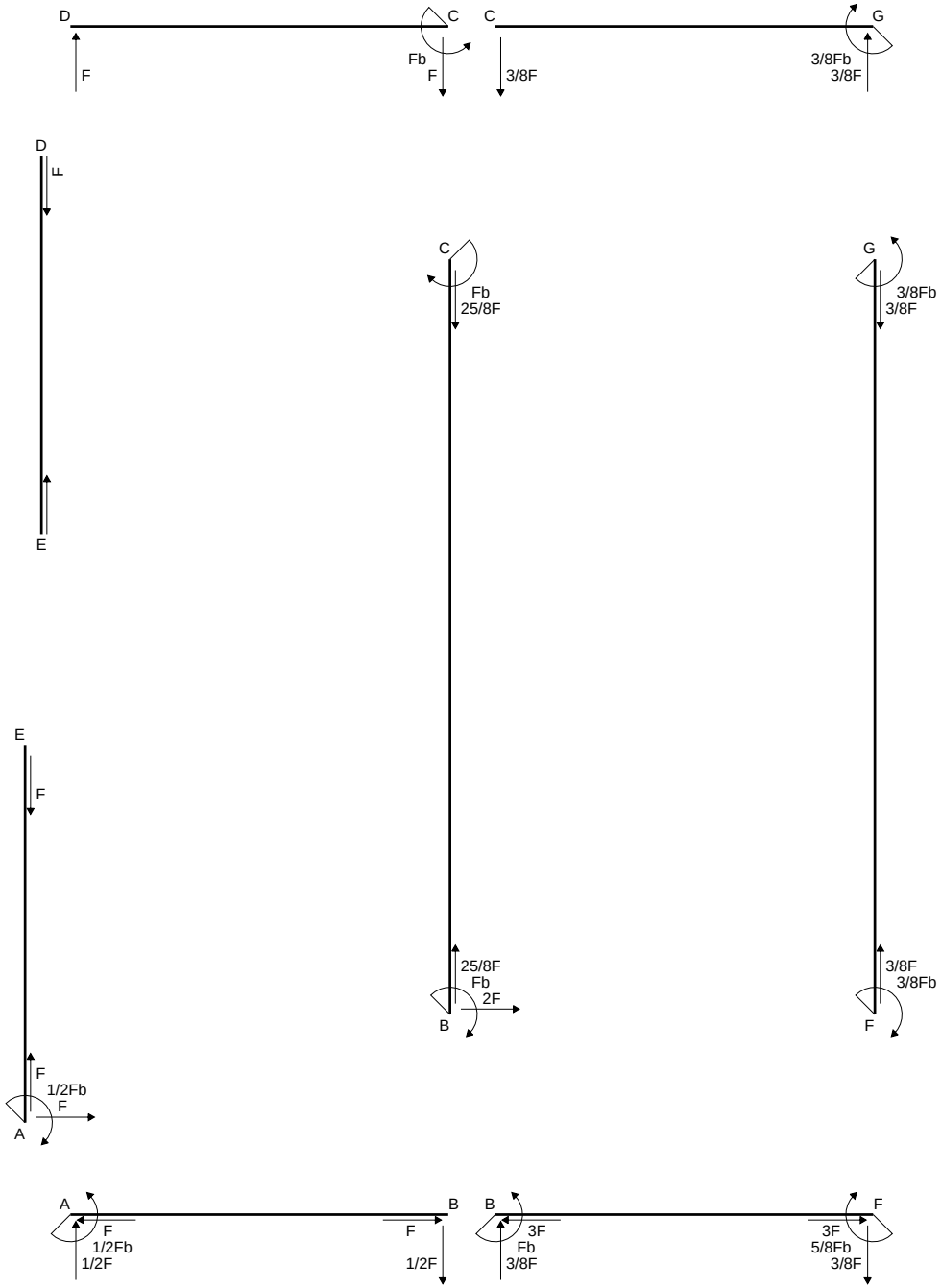
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

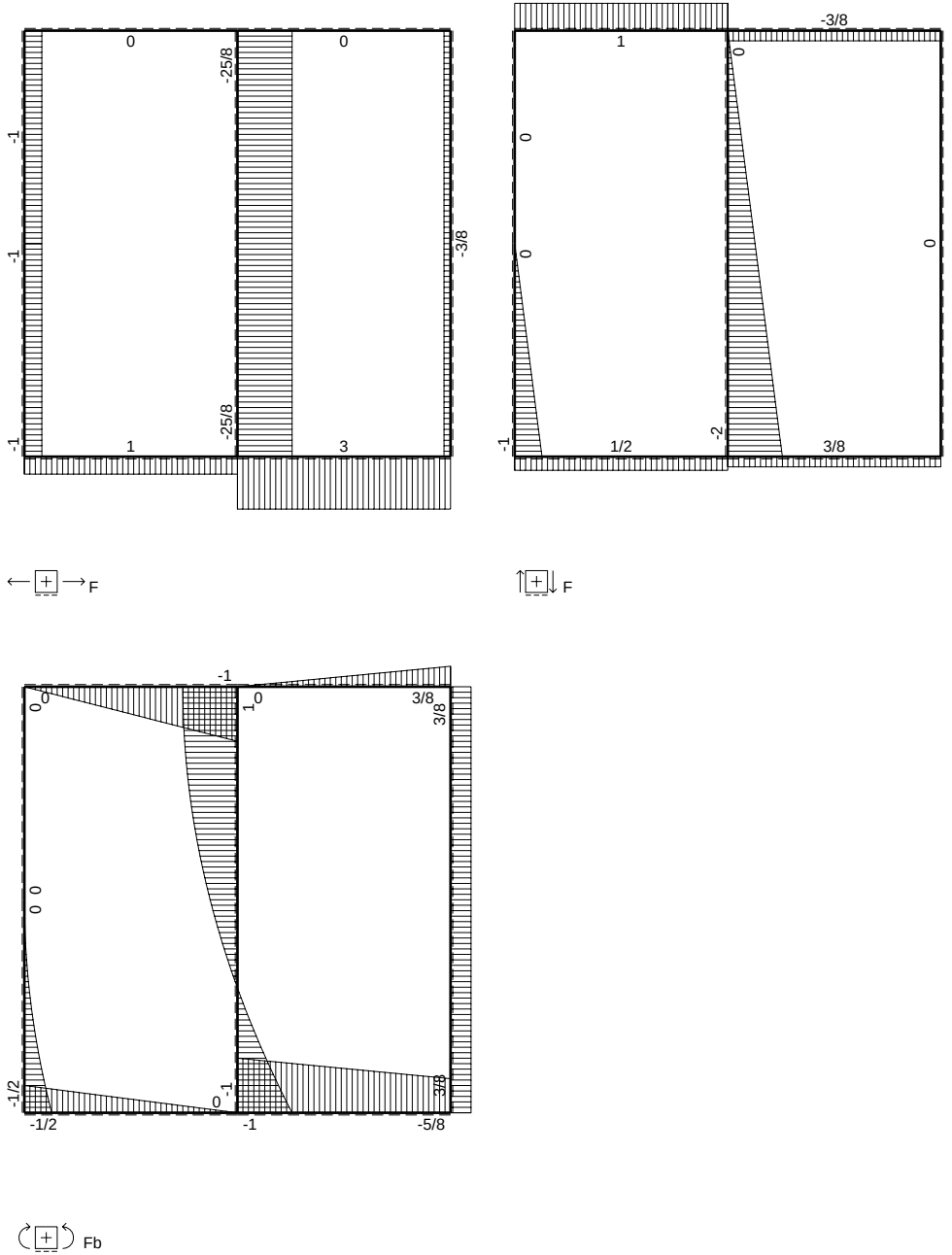


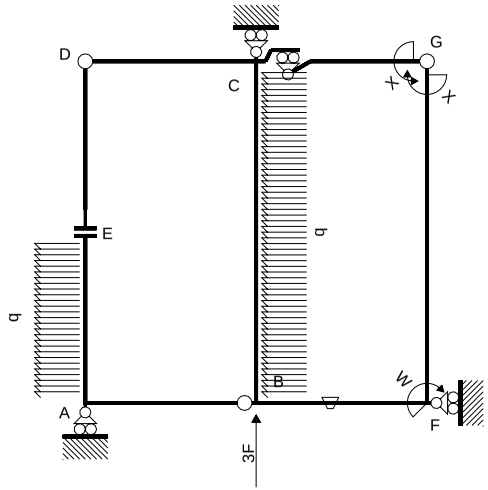
A = 221.2 mm^2
J_u = 126097. mm^4
J_v = 37362. mm^4
J_t = 200.1 mm^4
x_o = 20.9 mm
x_g = 26.44 mm
T_y = 1560. N
M_x = -1014000. Nmm
x_m = 47. mm
u_m = 20.56 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -209.1 N/mm^2
x_c = 36. mm
u_c = 9.561 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -209.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 304.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 11.58 N/mm^2
$\tau_o = Tx_o t/J_t$ = 293.3 N/mm^2
t_c = 2808. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 568. N/mm^2

CI08.xxxx.136

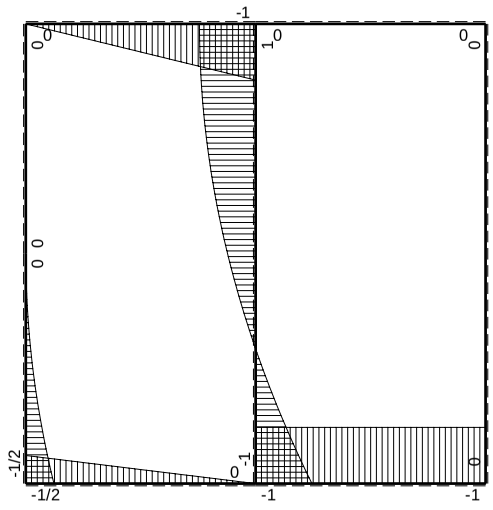
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
25/8F
G
3/8Fb
3/8F
E
F
25/8F
Fb
2F
B
3/8F
3/8Fb
F
F
1/2Fb
F
A
A
F
1/2Fb
1/2F
B
F
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F

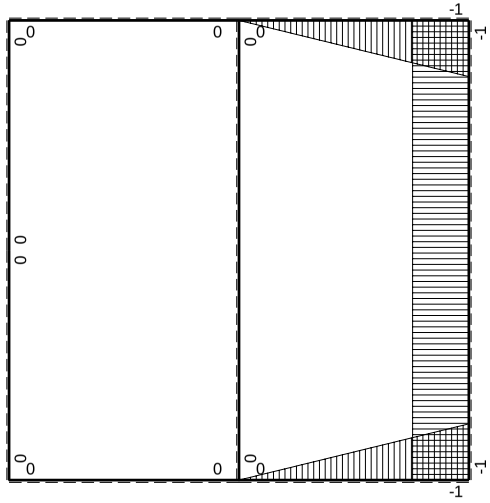

0
-25/8
0
-1
-1
-3/8
-1
-25/8
1
3
F
-3/8
1
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 241556 CI08.xxxx.137

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

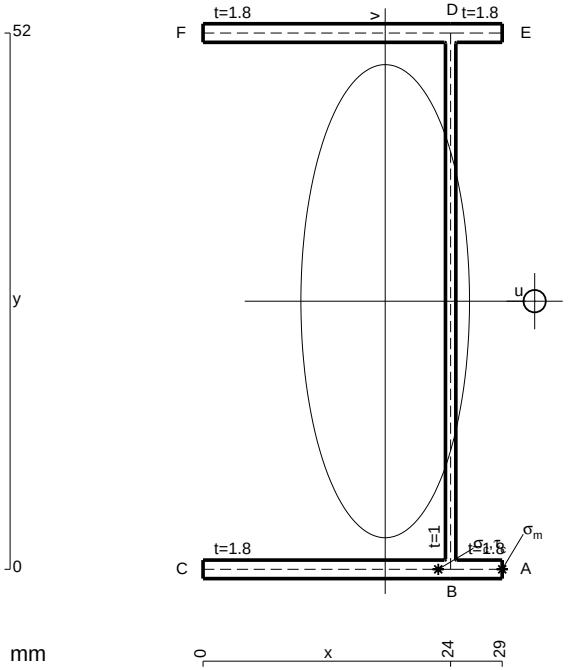
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

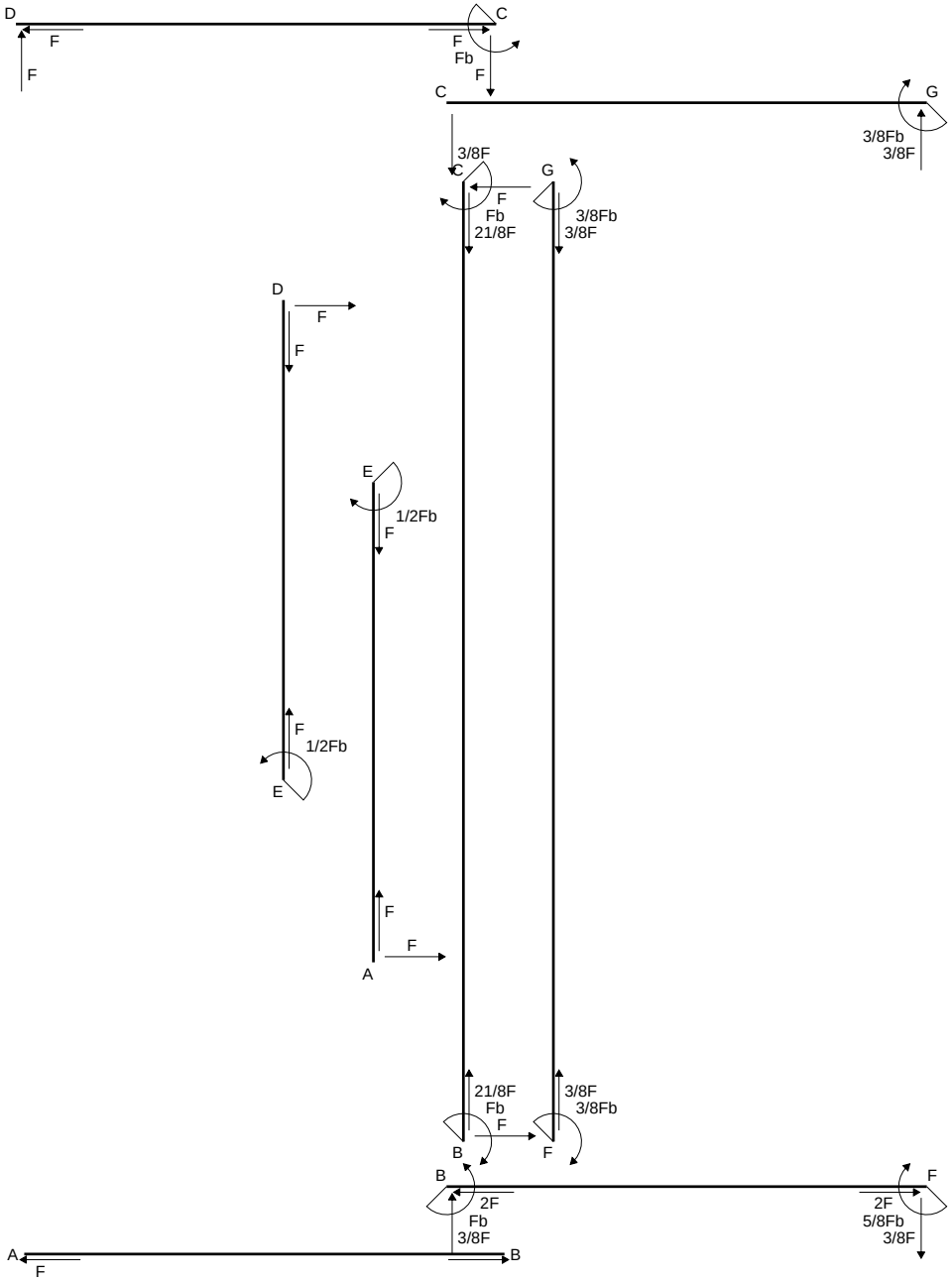
$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

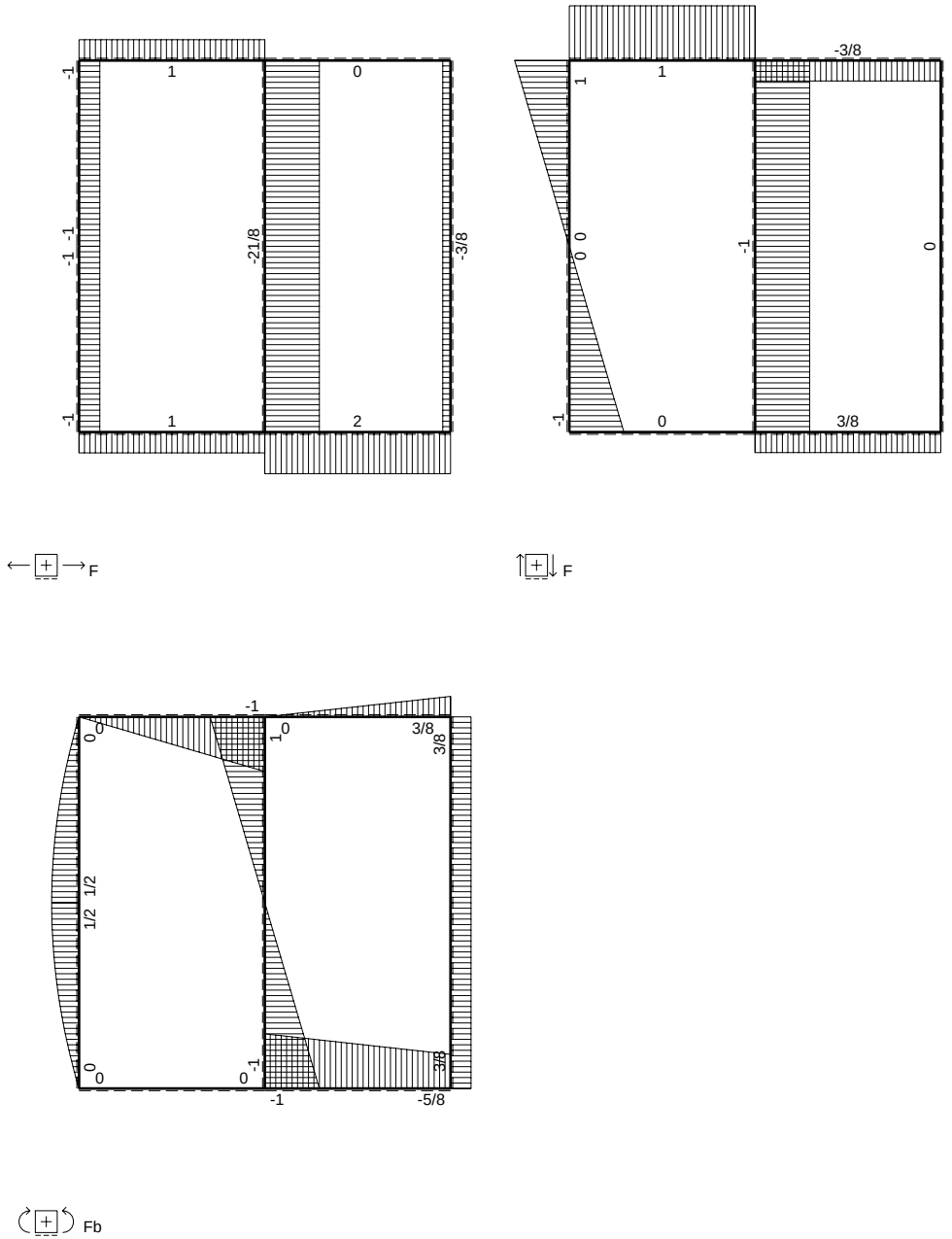


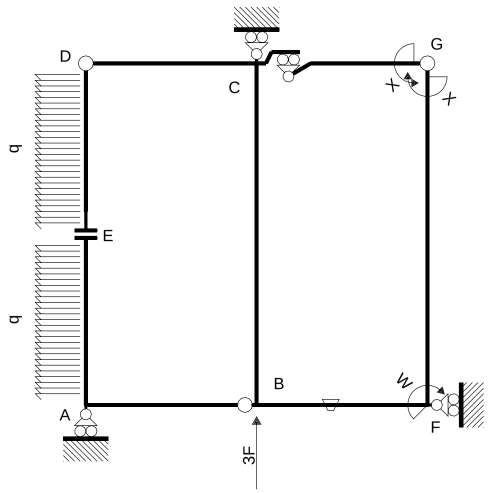
A = 156.4 mm^2
J_u = 82292. mm^4
J_v = 10449. mm^4
J_t = 130.1 mm^4
x_o = 14.49 mm
x_g = 17.66 mm
N = -2875. N
T_y = -1840. N
M_x = -634800. Nmm
x_m = 29. mm
u_m = 11.34 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = -218.9 N/mm^2
x_c = 24. mm
u_c = 6.341 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = -218.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 382.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 13.95 N/mm^2
$\tau_o = Tx_o t/J_t$ = 368.9 N/mm^2
t_c = 1656. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 698.3 N/mm^2

CI08.xxxx.137

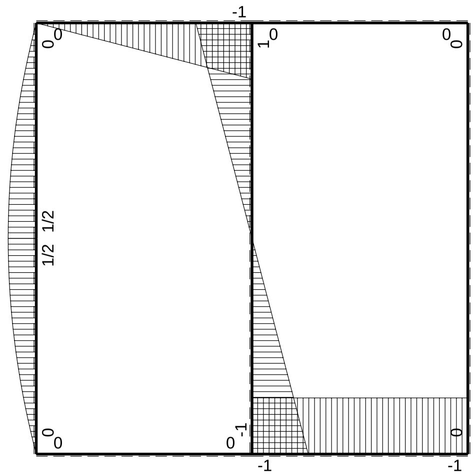
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
21/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
21/8F
Fb
F
3/8F
3/8Fb
F
B
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F

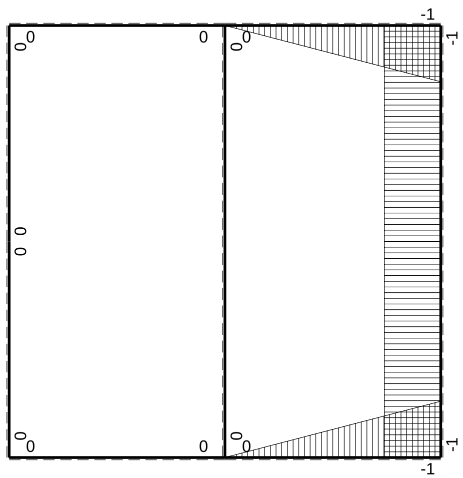

-1
1
0
-1
-1
-21/8
-3/8
-1
1
2
F
-3/8
1
1
1
0
0
-1
0
-1
0
3/8
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
0
0
0
-1
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

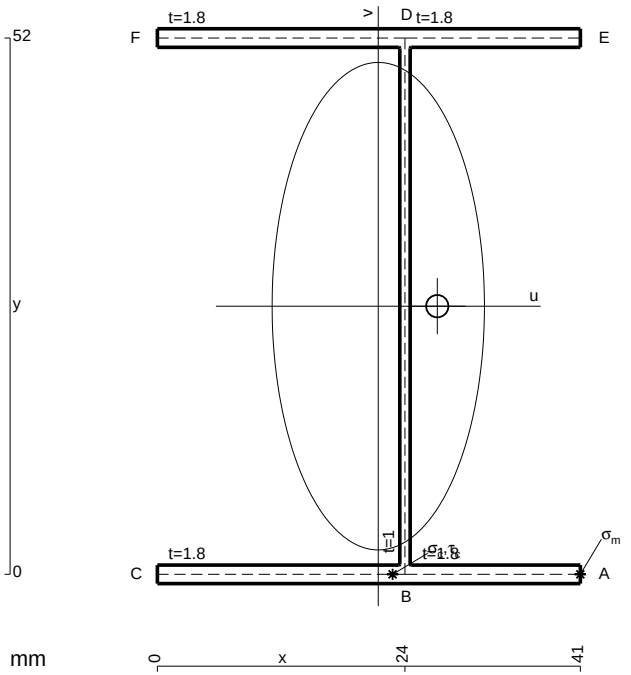
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

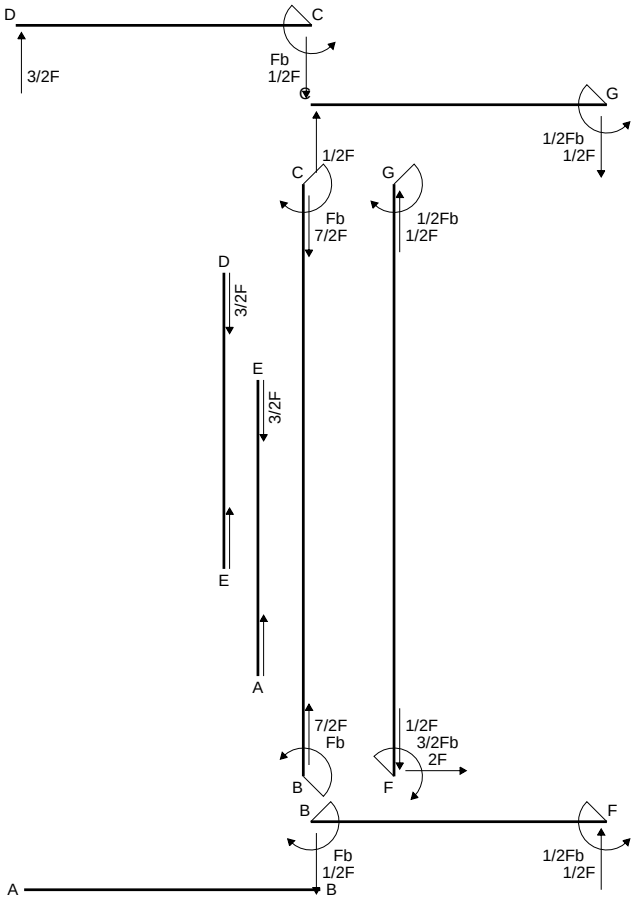
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

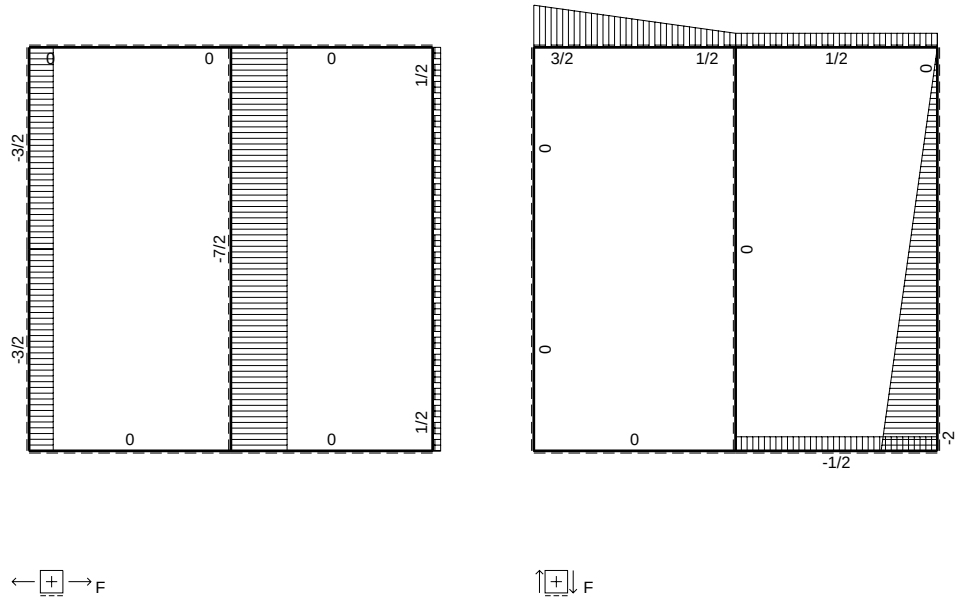


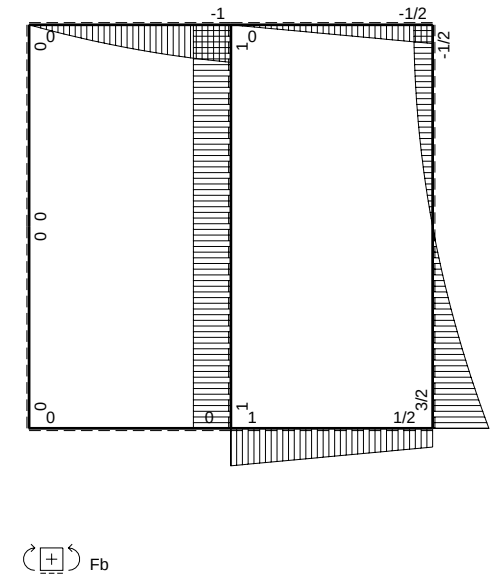
A = 199.6 mm^2
J_u = 111495. mm^4
J_v = 21147. mm^4
J_t = 176.7 mm^4
x_o = 5.72 mm
x_g = 21.41 mm
N = -3281. N
T_y = -1250. N
M_x = -912500. Nmm
x_m = 41. mm
u_m = 19.59 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u = -229.2\ N/mm^2$
x_c = 24. mm
u_c = 2.588 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u = -229.2\ N/mm^2$
$\tau_c = \tau_g + \tau_{ou} = 79.82\ N/mm^2$
$\tau_g = TS^*/tJ_u = 6.996\ N/mm^2$
$\tau_o = Tx_o t/J_t = 72.82\ N/mm^2$
t_c = 2250. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 267.7\ N/mm^2$

CI08.xxxx.138

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3/2F
Fb
1/2F
G
1/2Fb
1/2F
1/2F
C
G
Fb
7/2F
1/2Fb
1/2F
D
3/2F
E
3/2F
E
A
7/2F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B


0
0
0
1/2
-3/2
-7/2
-3/2
1/2
0
0
0
3/2
1/2
1/2
0
0
0
0
-2
-1/2
F
F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+1/2Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	3/2Fx-1/2qx^2	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							-4/3Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

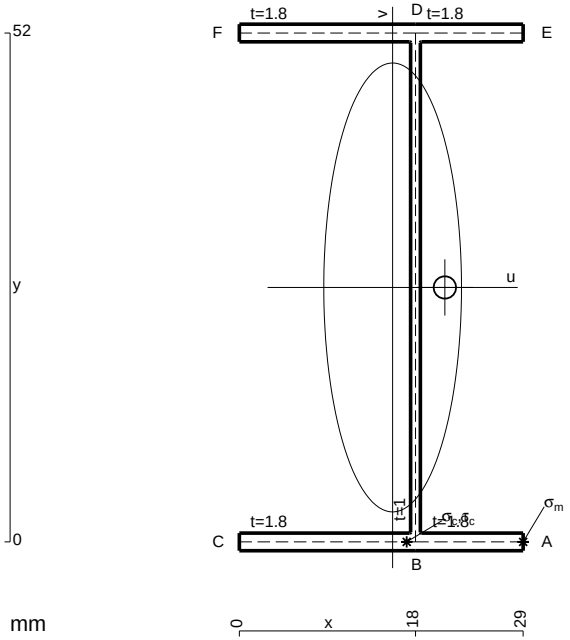
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

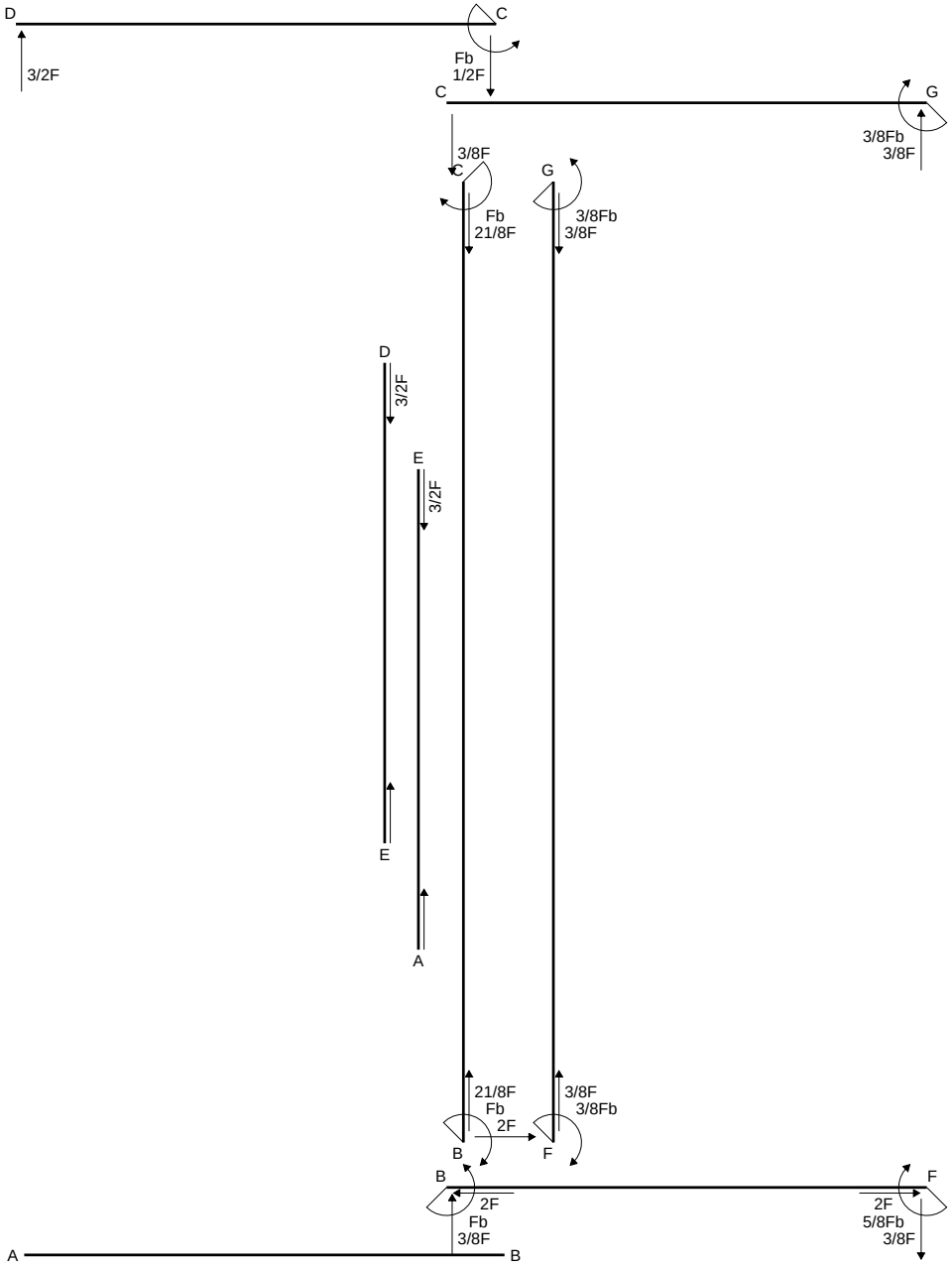
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

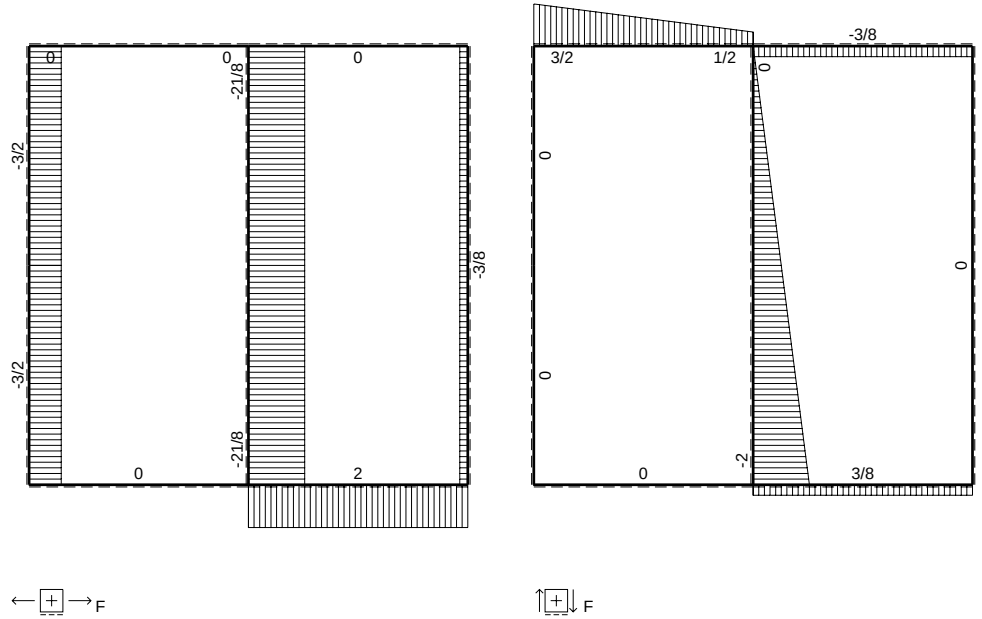


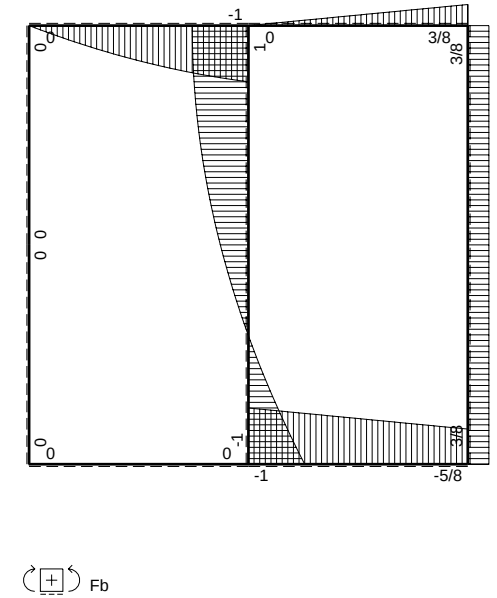
A = 156.4 mm^2
J_u = 82292. mm^4
J_v = 7742. mm^4
J_t = 130.1 mm^4
x_o = 5.338 mm
x_g = 15.66 mm
T_y = 1570. N
M_x = -755563. Nmm
x_m = 29. mm
u_m = 13.34 mm
v_m = -26. mm
$\sigma_m = -Mv/J_u$ = -238.7 N/mm^2
x_c = 18. mm
u_c = 2.336 mm
v_c = -26. mm
$\sigma_c = -Mv/J_u$ = -238.7 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 124.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 8.929 N/mm^2
$\tau_o = Tx_o t/J_t$ = 116. N/mm^2
t_c = 2826.
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 322.1 N/mm^2

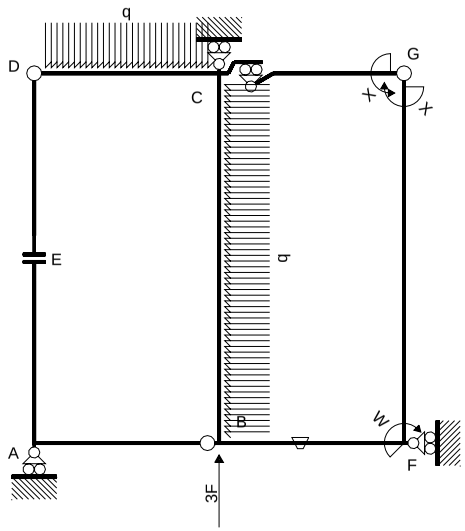
CI08.xxxx.139

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

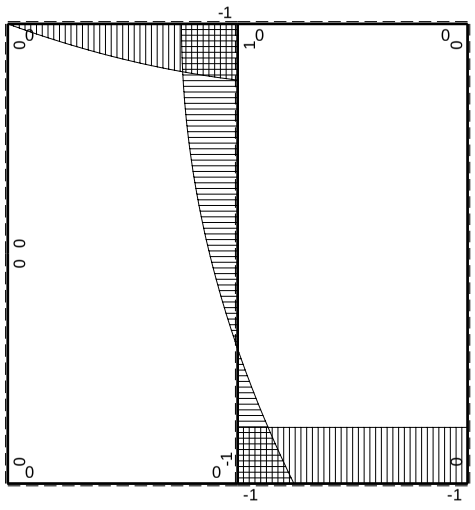

D
C
3/2F
Fb
1/2F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
21/8F
3/8Fb
3/8F
D
3/2F
E
3/2F
E
A
21/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B


0
0
-21/8
0
-3/2
-3/8
-3/2
-21/8
0
2
3/2
1/2
-3/8
0
0
0
0
-2
3/8
F
F

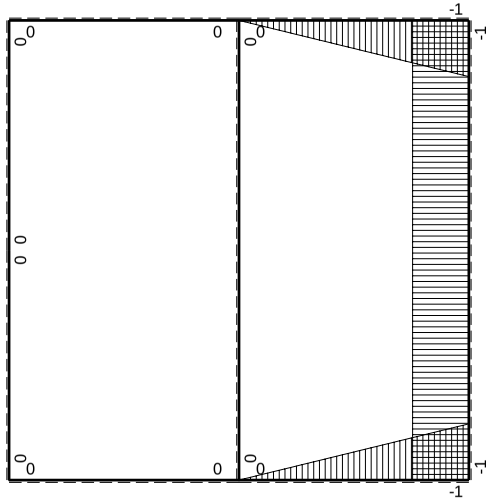

-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+1/2Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	3/2Fx-1/2qx^2	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx^2	0	0	0	0		
						totali	Fb^2/EJ	8/3Xb/EJ
						iperstatica $X=W_{GC}$	-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 960451 CI08.xxxx.140

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

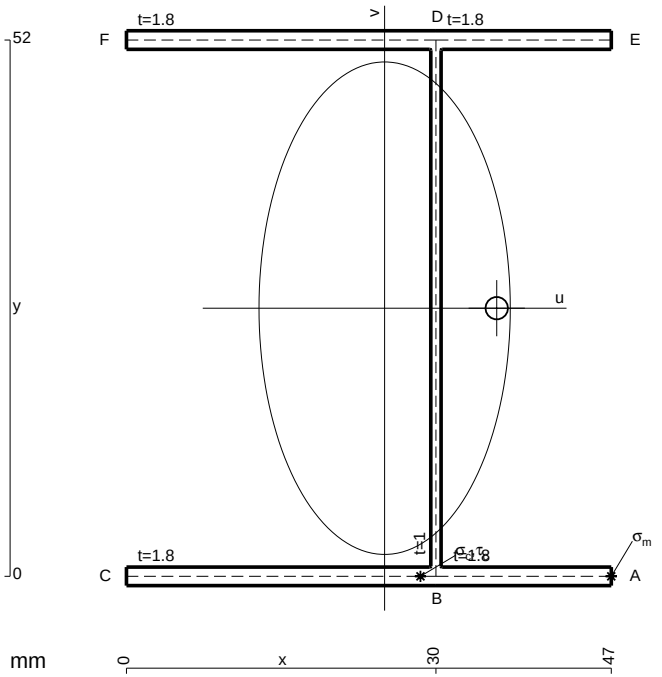
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

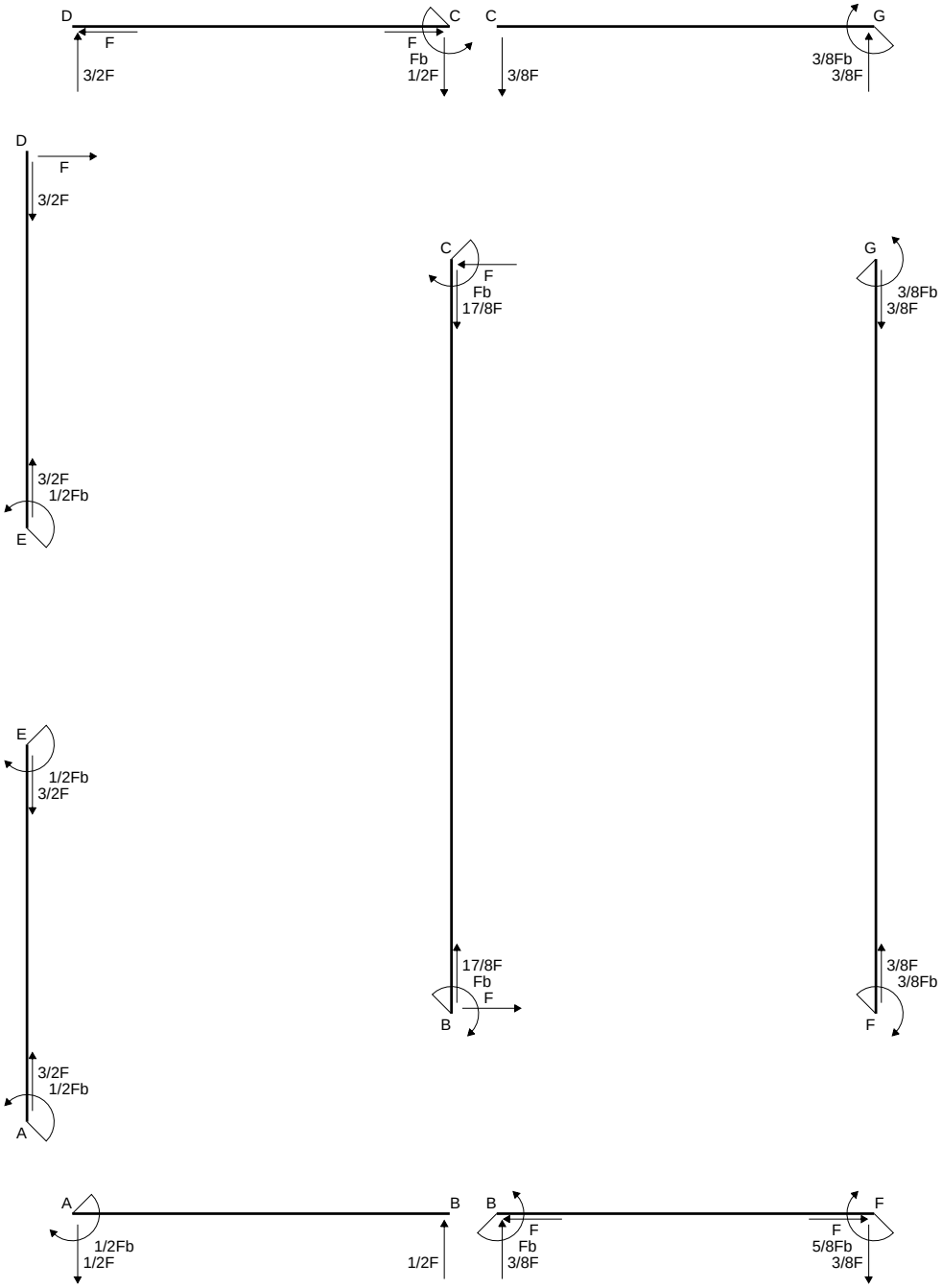
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

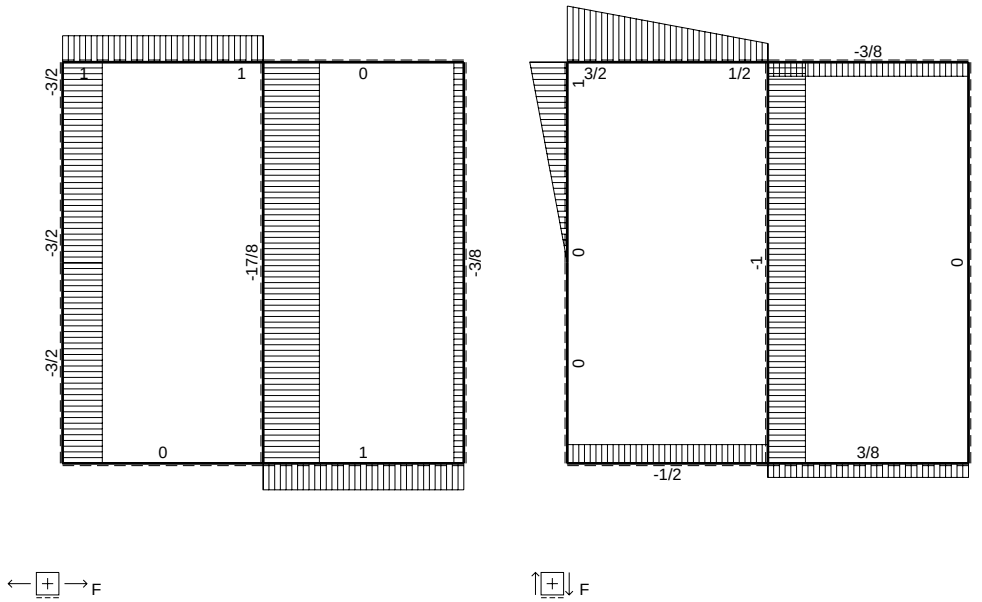


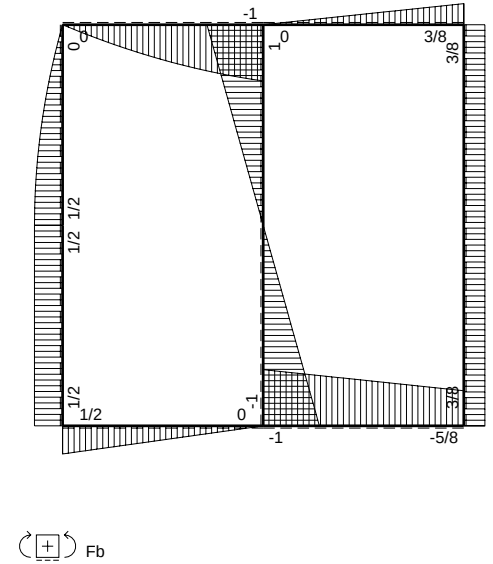
A = 221.2 mm^2
J_u = 126097. mm^4
J_v = 32827. mm^4
J_t = 200.1 mm^4
x_o = 10.87 mm
x_g = 25.03 mm
N = -5434. N
T_y = -4140. N
M_x = -848700. Nmm
x_m = 47. mm
u_m = 21.97 mm
v_m = -26. mm
$\sigma_m = N/A - Mv/J_u$ = -199.6 N/mm^2
x_c = 30. mm
u_c = 4.972 mm
v_c = -26. mm
$\sigma_c = N/A - Mv/J_u$ = -199.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 430.4 N/mm^2
$\tau_g = TS^*/tJ_u$ = 25.61 N/mm^2
$\tau_o = Tx_o t/J_t$ = 404.8 N/mm^2
t_c = 3726. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 771.7 N/mm^2

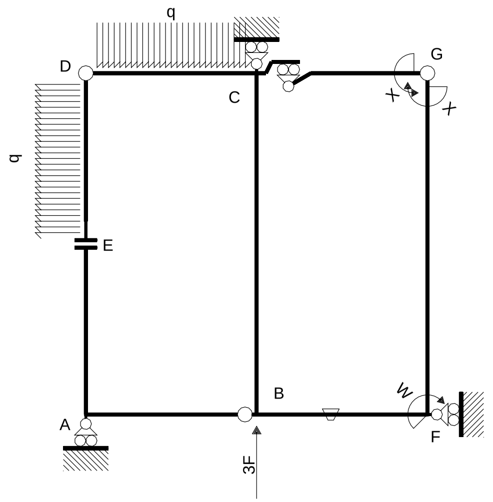
CI08.xxxx.140

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

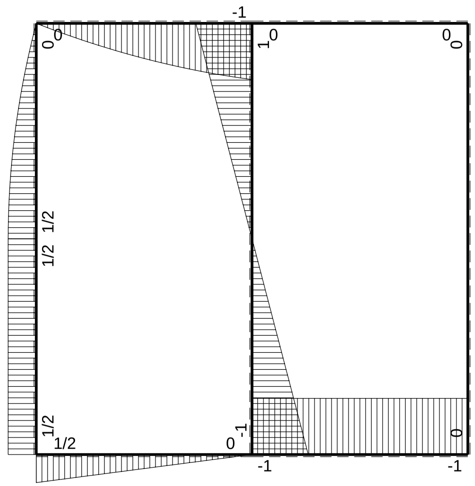

D
F
3/2F
C
F
Fb
1/2F
C
3/8F
G
3/8Fb
3/8F
D
F
3/2F
3/2F
1/2Fb
E
C
F
Fb
17/8F
G
3/8Fb
3/8F
E
1/2Fb
3/2F
17/8F
Fb
F
B
3/8F
3/8Fb
F
3/2F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F


-3/2
-3/2
-3/2
1
1
0
-17/8
-3/8
0
1
F
-3/8
1
3/2
1/2
0
0
-1
0
-1/2
3/8
F

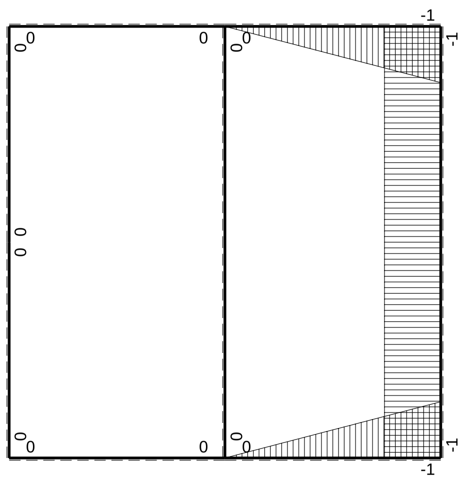

-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
-1
3/8
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

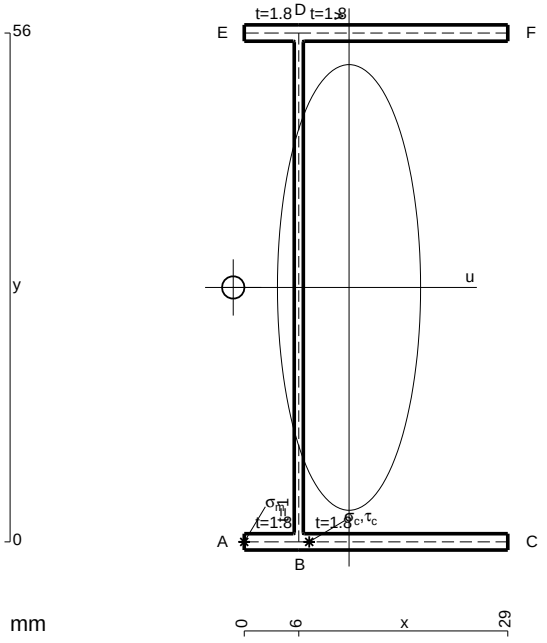
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

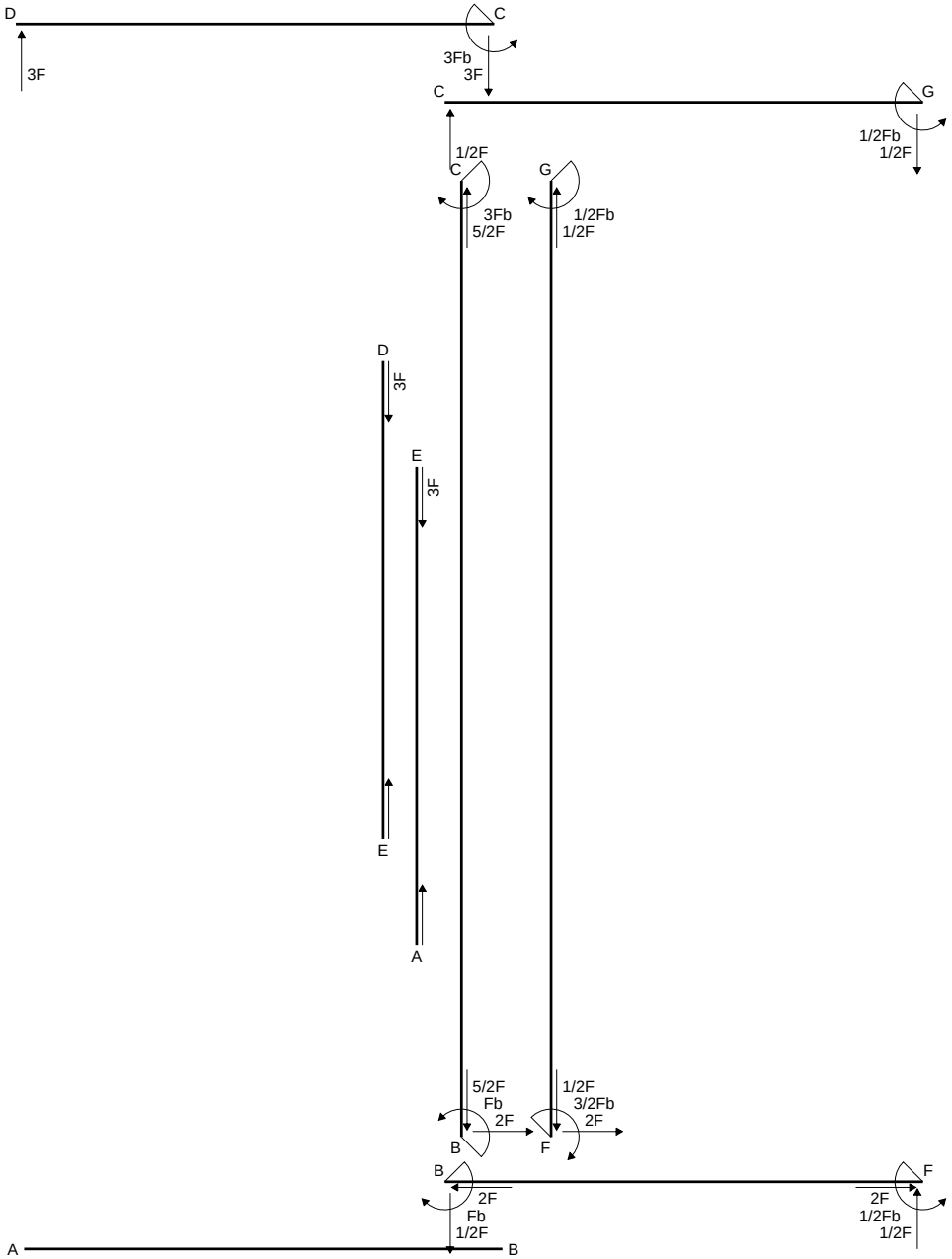
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

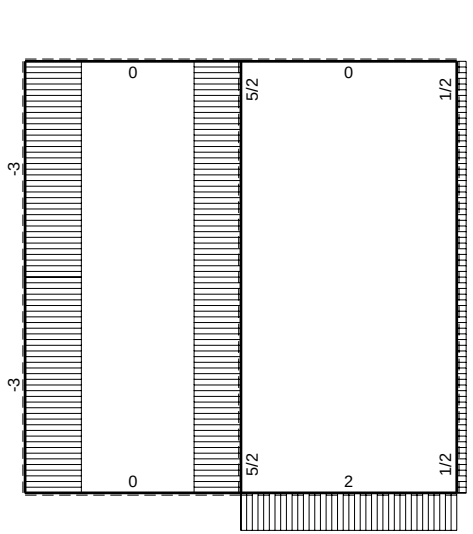


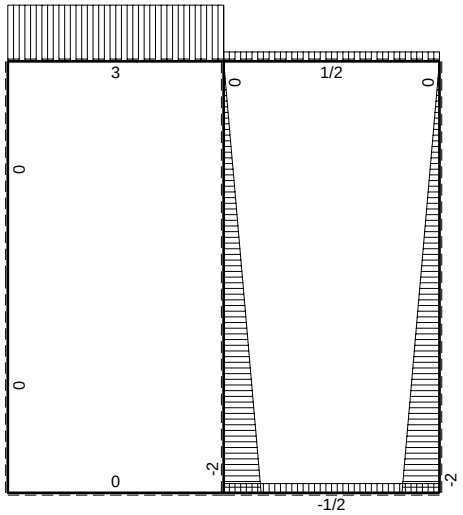
A = 160.4 mm^2
J_u = 96484. mm^4
J_v = 9950. mm^4
J_t = 131.4 mm^4
x_o = -12.74 mm
x_g = 11.53 mm
N = -2911. N
T_y = -1370. N
M_x = -657600. Nmm
u_m = -11.53 mm
v_m = -28. mm
$\sigma_m = N/A - Mv/J_u$ = -209. N/mm^2
x_c = 6. mm
u_c = -5.532 mm
v_c = -28. mm
$\sigma_c = N/A - Mv/J_u$ = -209. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 248.3 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.144 N/mm^2
$\tau_o = Tx_o t/J_t$ = 239.1 N/mm^2
t_c = 2466. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 478.1 N/mm^2

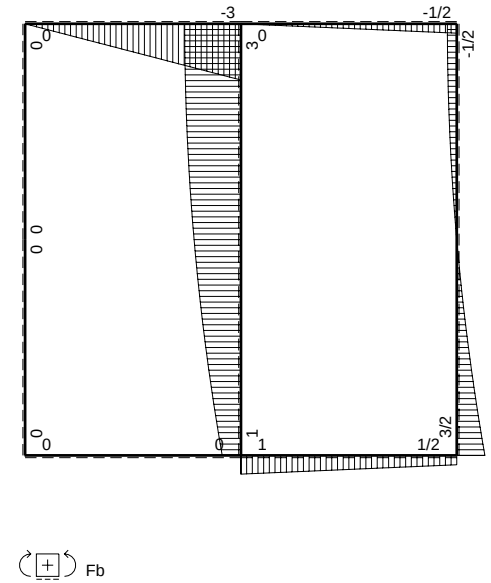
CI08.xxxx.141

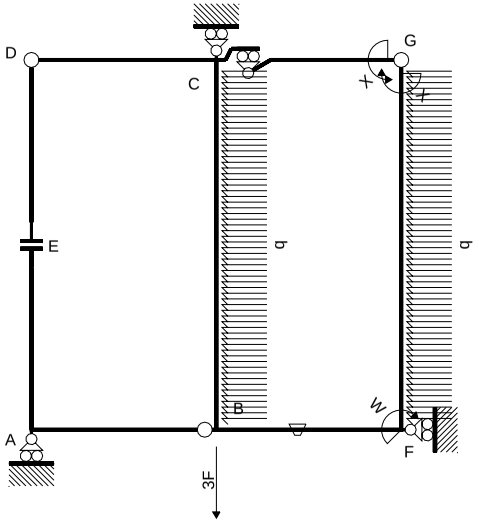
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

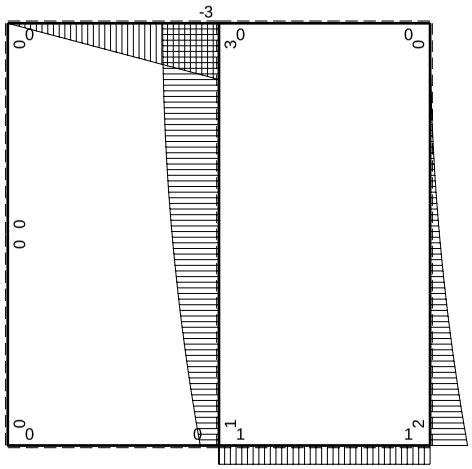

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
5/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
5/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
5/2
0
1/2
-3
-3
0
5/2
2
1/2
F

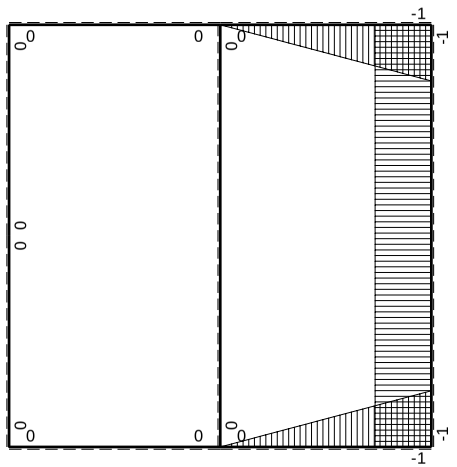

3
0
1/2
0
0
0
-2
0
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-4/3Fb^2$/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 939660 CI08.xxxx.142

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

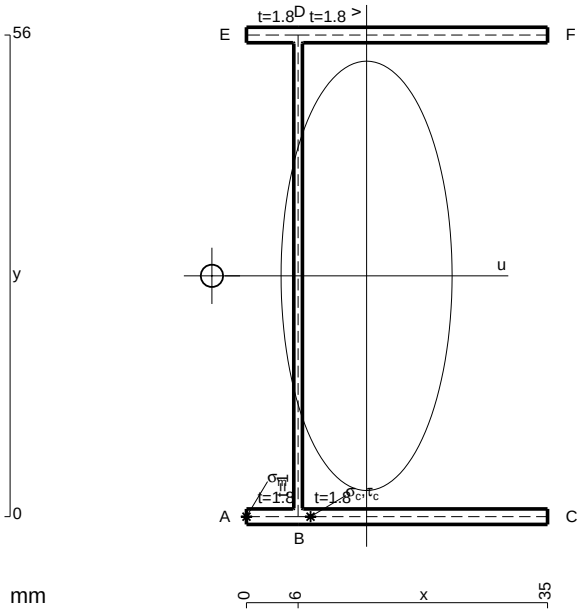
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

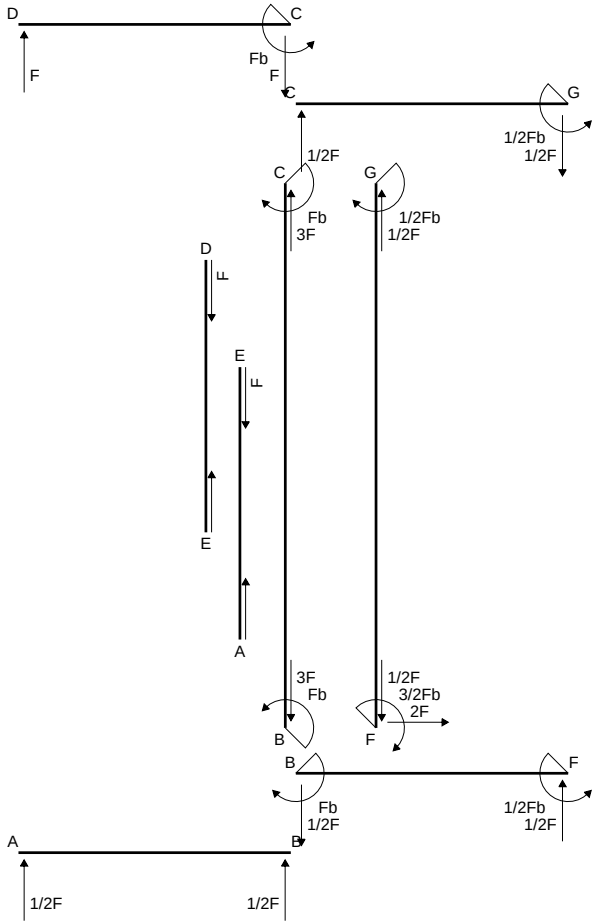
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

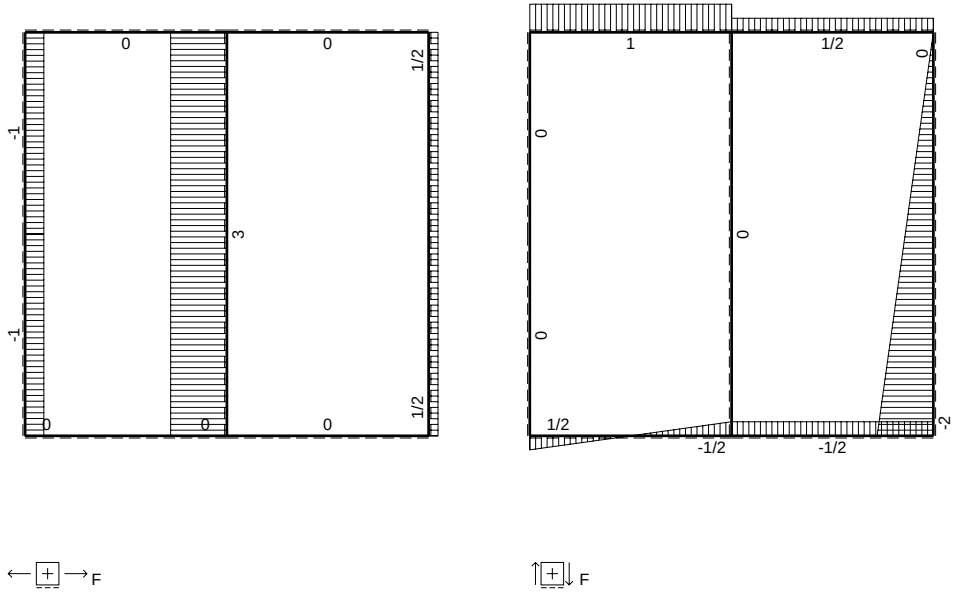


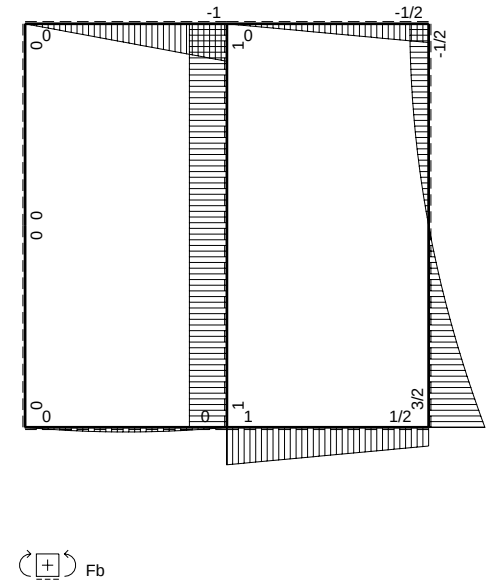
A = 182. mm^2
J_u = 113419. mm^4
J_v = 17990. mm^4
J_t = 154.7 mm^4
x_o = -17.98 mm
x_g = 13.96 mm
T_y = 1710. N
M_x = -889200. Nmm
u_m = -13.96 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -219.5 N/mm^2
x_c = 6. mm
u_c = -7.962 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -219.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 369.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.24 N/mm^2
$\tau_o = Tx_o t/J_t$ = 357.6 N/mm^2
t_c = 1026. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 677.1 N/mm^2

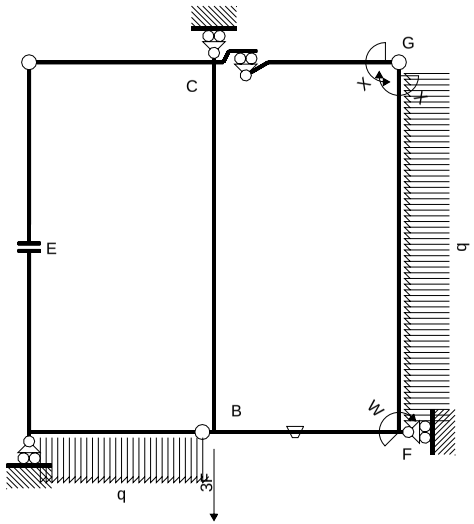
CI08.xxxx.142

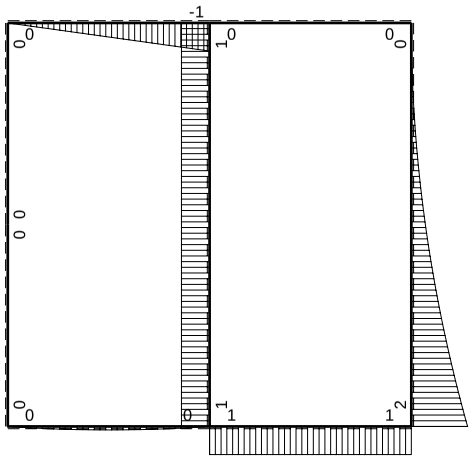
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
3F
1/2Fb
1/2F
D
F
E
F
E
A
3F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F

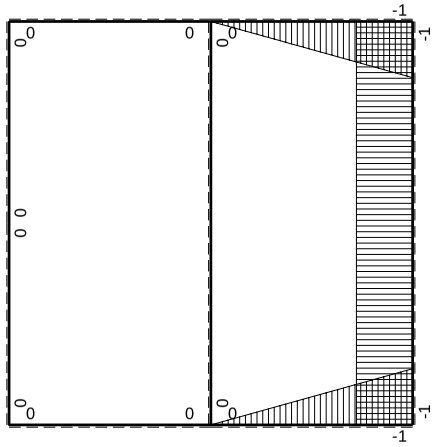

0
0
1/2
-1
3
-1
0
0
0
1/2
F
1
1/2
0
0
0
1/2
-2
-1/2
-1/2
F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 986254 CI08.xxxx.143

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

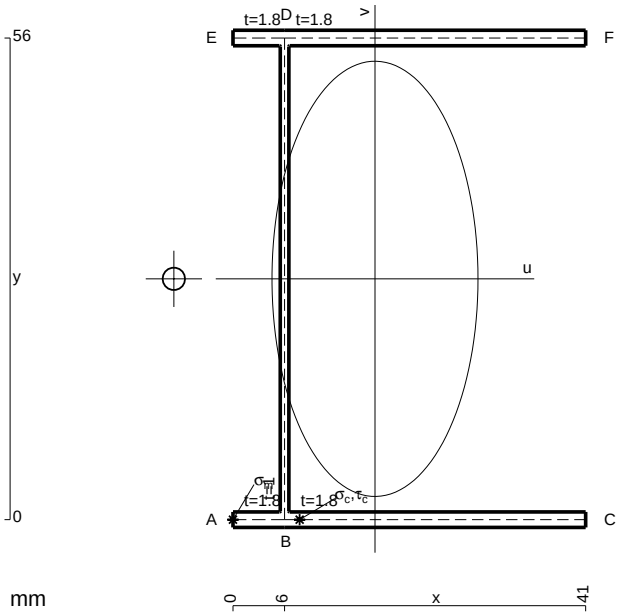
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

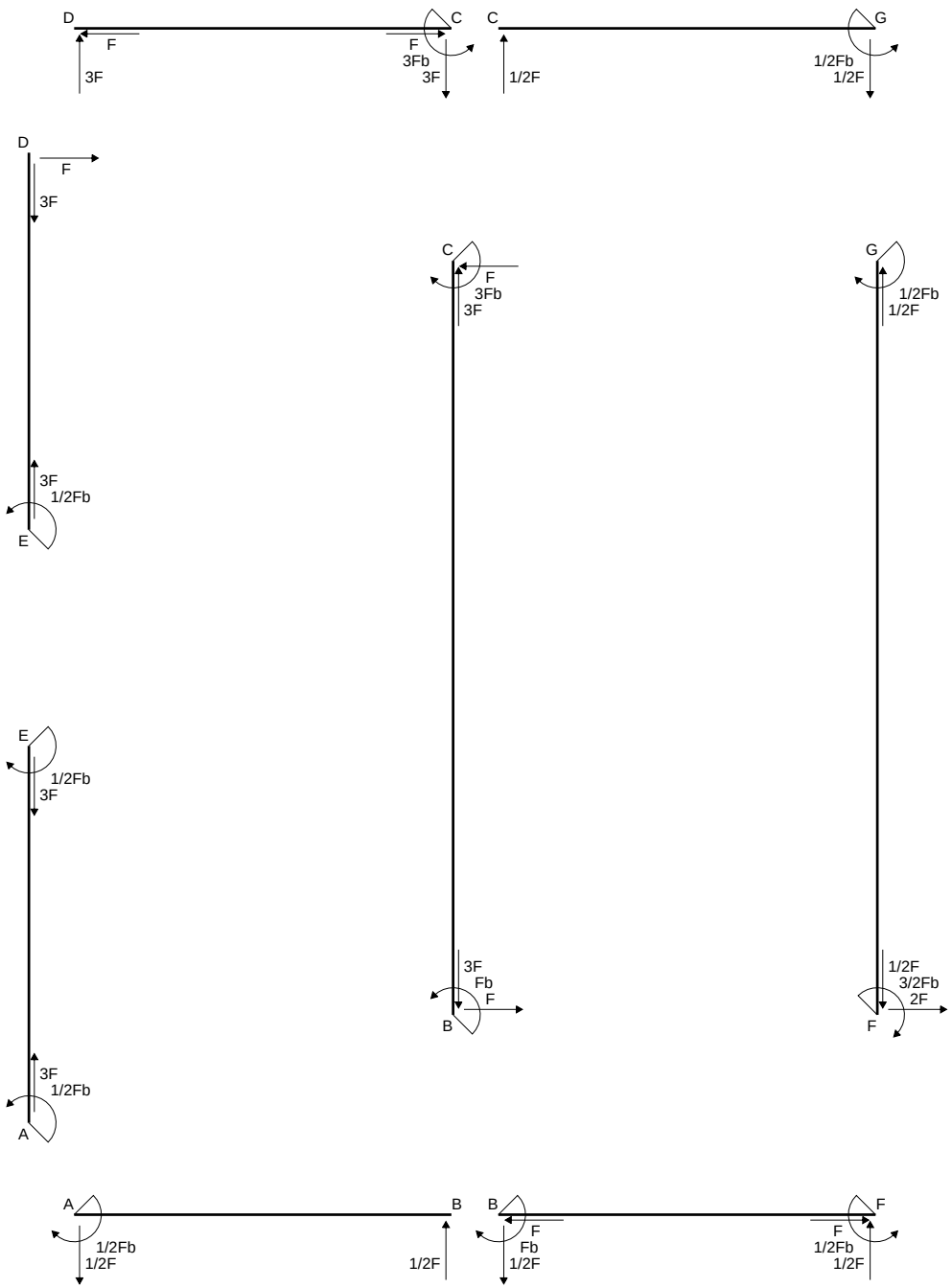
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

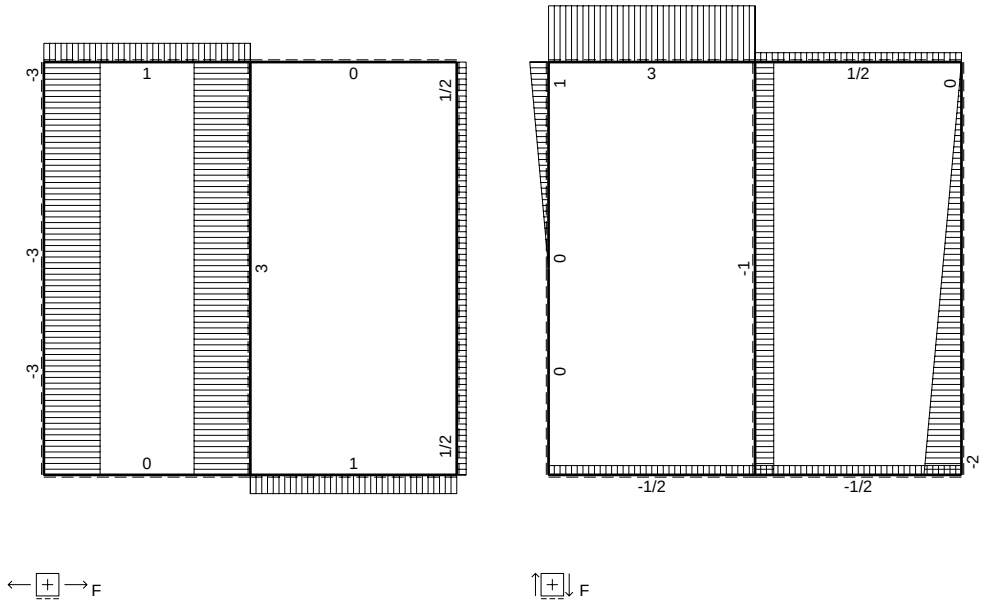


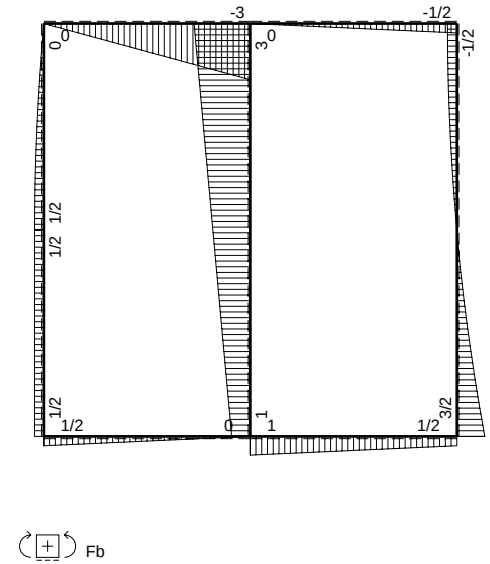
A = 203.6 mm^2
J_u = 130353. mm^4
J_v = 29212. mm^4
J_t = 178.1 mm^4
x_o = -23.38 mm
x_g = 16.51 mm
T_y = 1870. N
M_x = -1065900. Nmm
u_m = -16.51 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -229. N/mm^2
x_c = 6. mm
u_c = -10.51 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -229. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 456.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 14.06 N/mm^2
$\tau_o = Tx_o t/J_t$ = 442. N/mm^2
t_c = 3366. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 822.4 N/mm^2

CI08.xxxx.143

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
3F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
3F
Fb
F
1/2F
3/2Fb
2F
B
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


-3
1
0
1/2
-3
3
-3
0
1
1/2
F
1
3
1/2
0
0
0
-1
-2
-1/2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

PROCEDIMENTO E RISULTATI 214984 CI08.xxxx.144

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

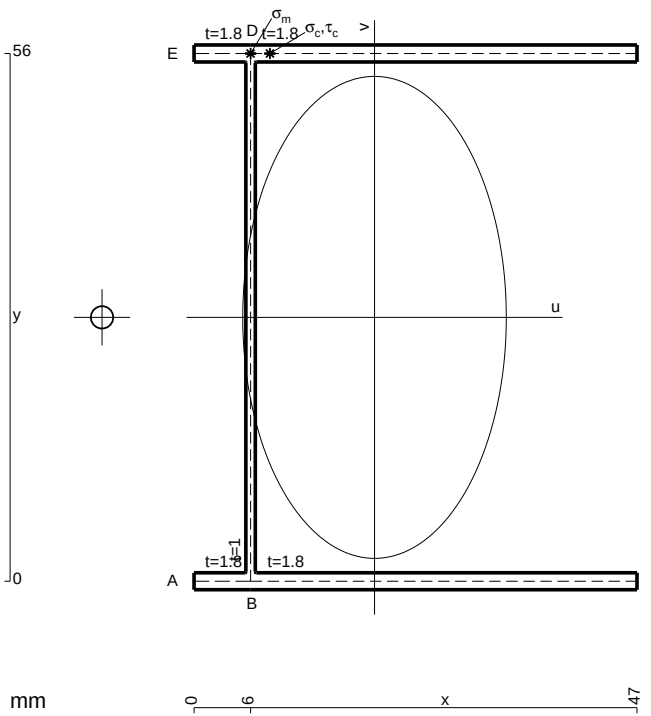
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

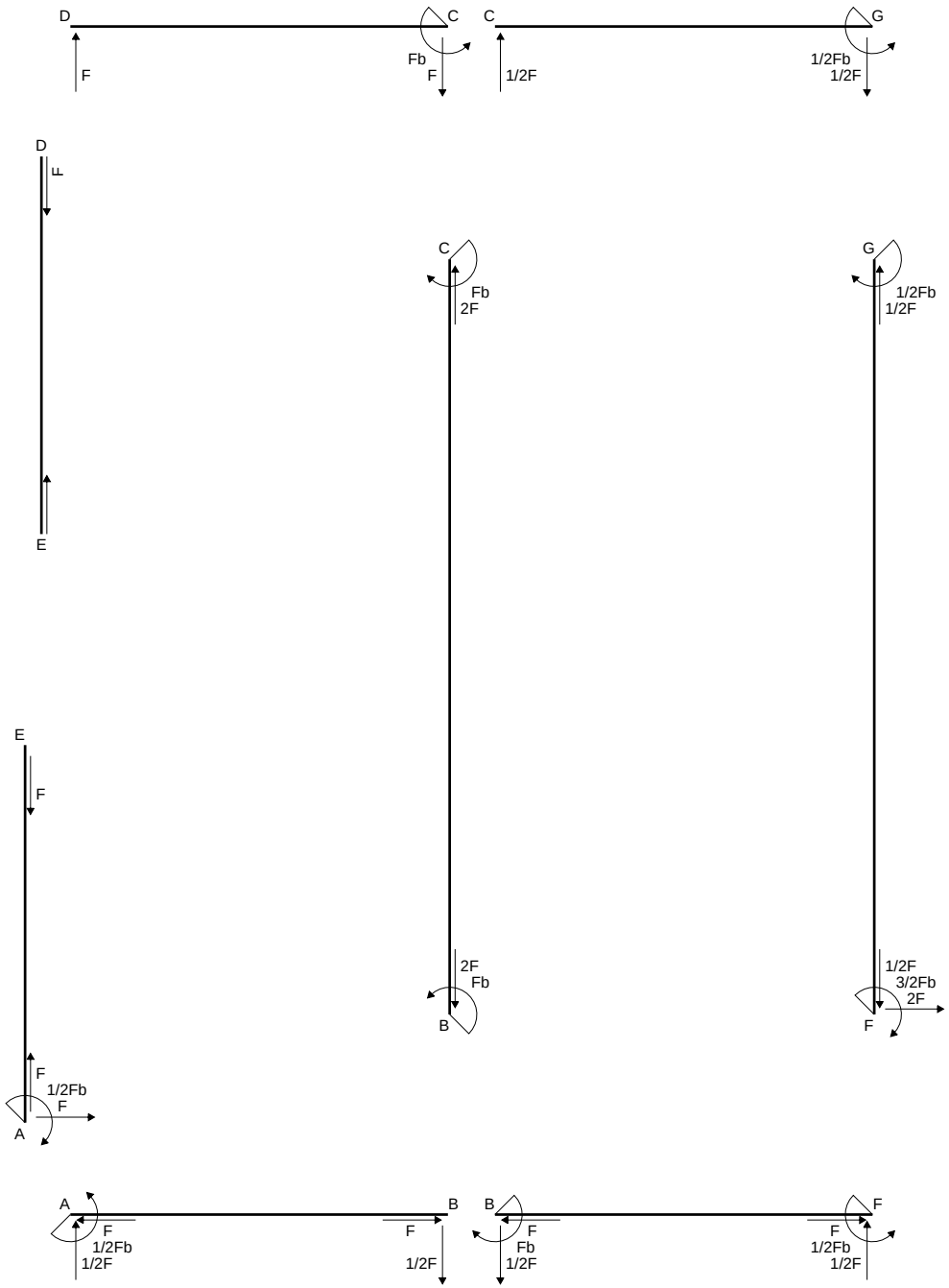
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

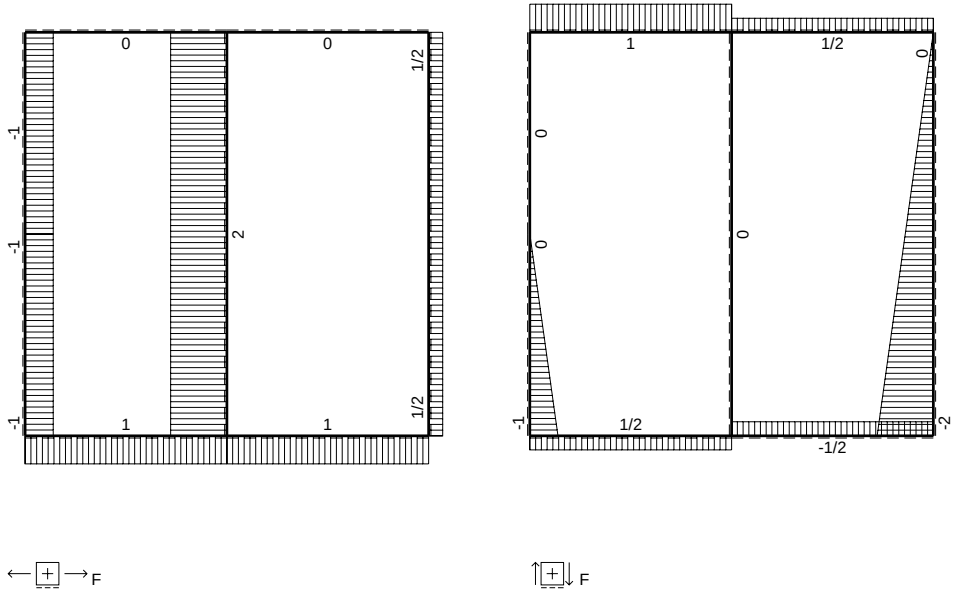


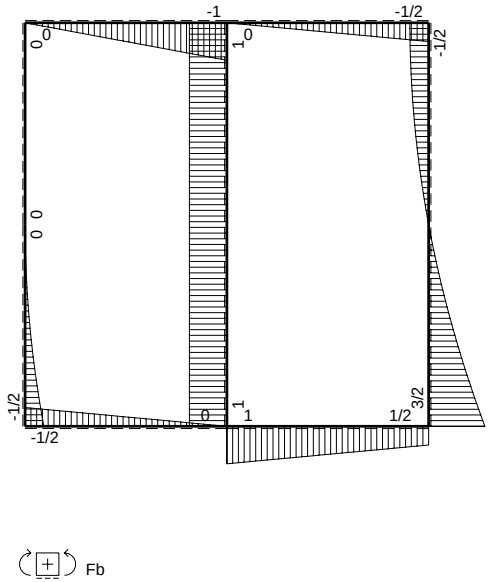
A = 225.2 mm^2
J_u = 147287. mm^4
J_v = 44032. mm^4
J_t = 201.4 mm^4
x_o = -28.91 mm
x_g = 19.15 mm
N = 680. N
T_y = 2040. N
M_x = -1244400. Nmm
x_m = 6. mm
y_m = 56. mm
u_m = -13.15 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 239.6 N/mm^2
x_c = 6. mm
y_c = 56. mm
u_c = -13.15 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 239.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 543. N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.9 N/mm^2
$\tau_o = Tx_o t/J_t$ = 527.1 N/mm^2
t_c = 1224. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 970.5 N/mm^2

CI08.xxxx.144

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
2F
G
1/2Fb
1/2F
2F
Fb
B
1/2F
3/2Fb
2F
F
E
F
F
1/2Fb
F
A
A
B
B
F
F
F
F
1/2Fb
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


0
0
1/2
-1
2
-1
-1
1
1
1/2
F
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F


-1
-1/2
0
0
1
0
-1/2
0 0
0 0
-1/2
0
1
1
1/2
3/2
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

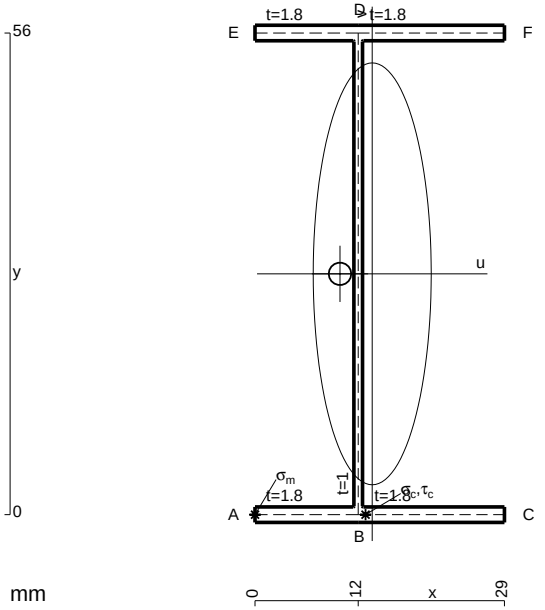
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

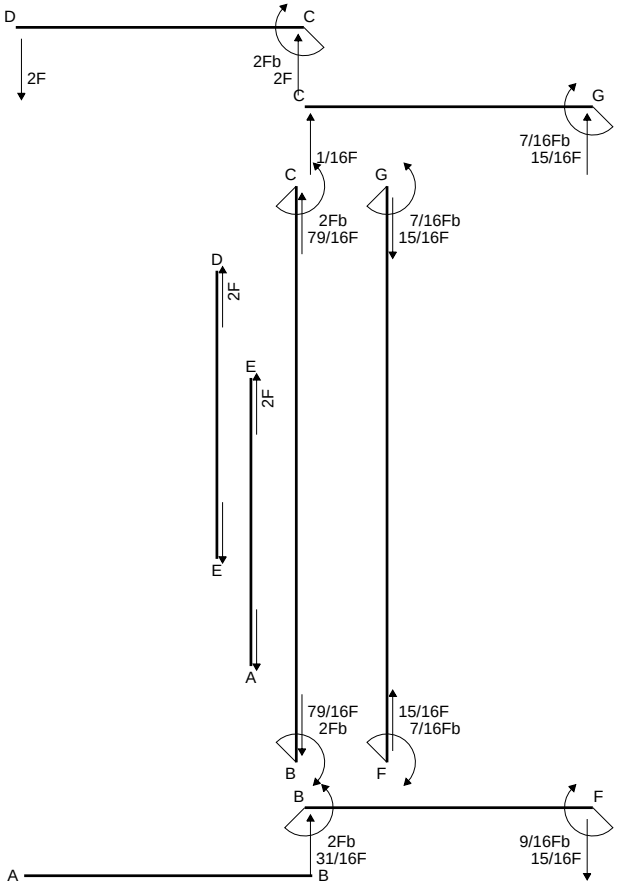
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

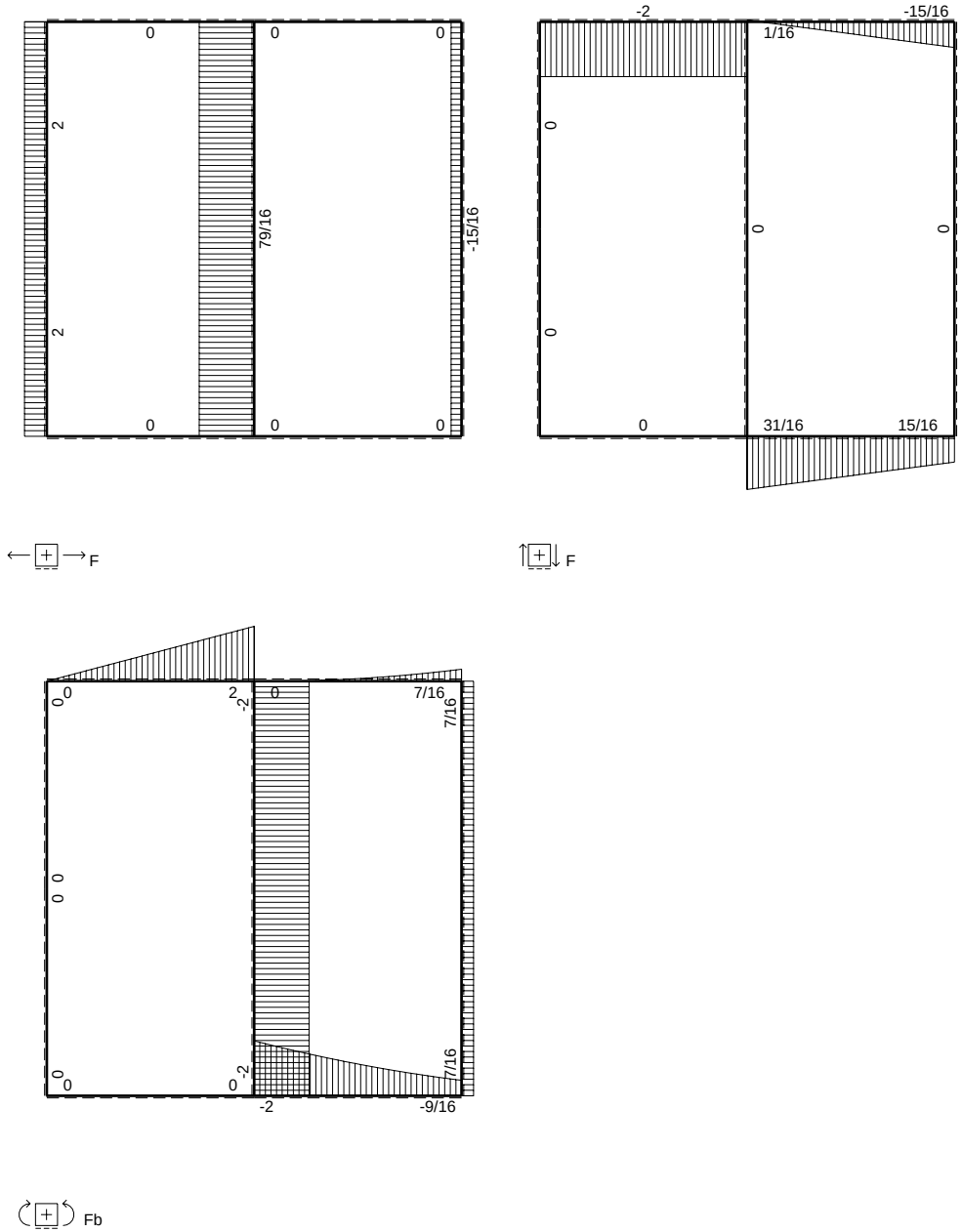


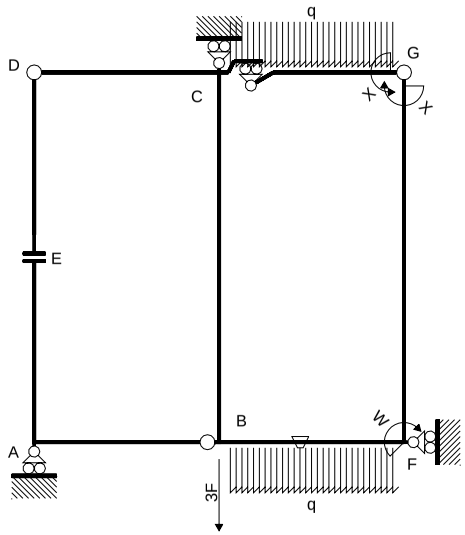
A = 160.4 mm^2
J_u = 96484. mm^4
J_v = 7545. mm^4
J_t = 131.4 mm^4
x_o = -3.748 mm
x_g = 13.63 mm
T_y = 1040. N
M_x = -686400. Nmm
u_m = -13.63 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -199.2 N/mm^2
x_c = 12. mm
u_c = -1.627 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -199.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 58.52 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.131 N/mm^2
$\tau_o = Tx_o t/J_t$ = 53.39 N/mm^2
t_c = 1872. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 223.5 N/mm^2

CI08.xxxx.145

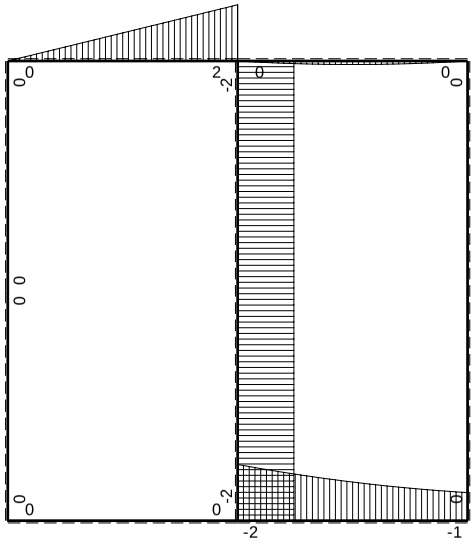
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
2Fb
2F
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
79/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
79/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B

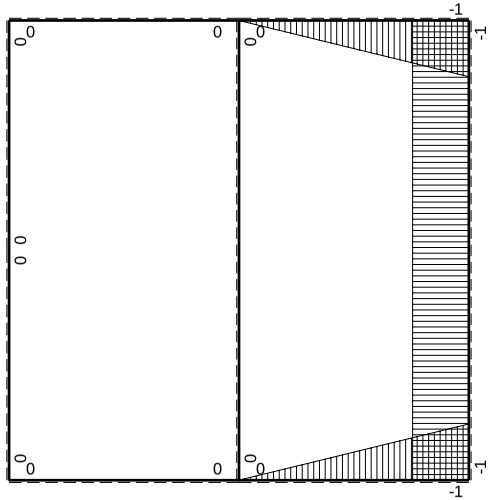

0
0
0
2
79/16
-15/16
2
0
0
0
F
-2
-15/16
1/16
0
0
0
0
31/16
15/16
F
0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
7/16
-2
-9/16
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212930 CI08.xxxx.146

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

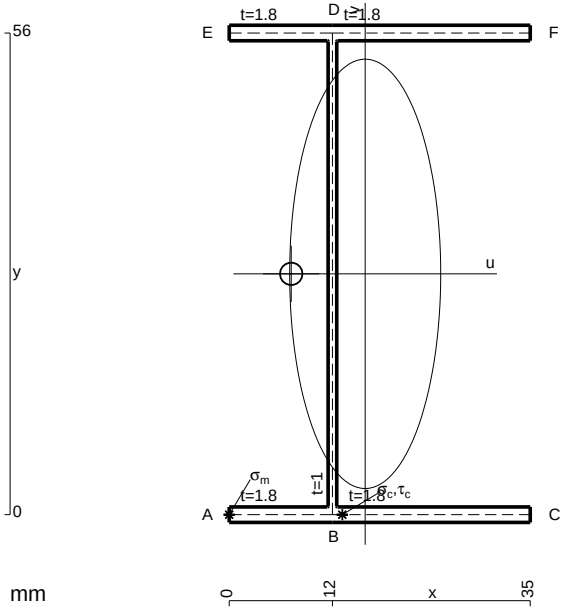
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

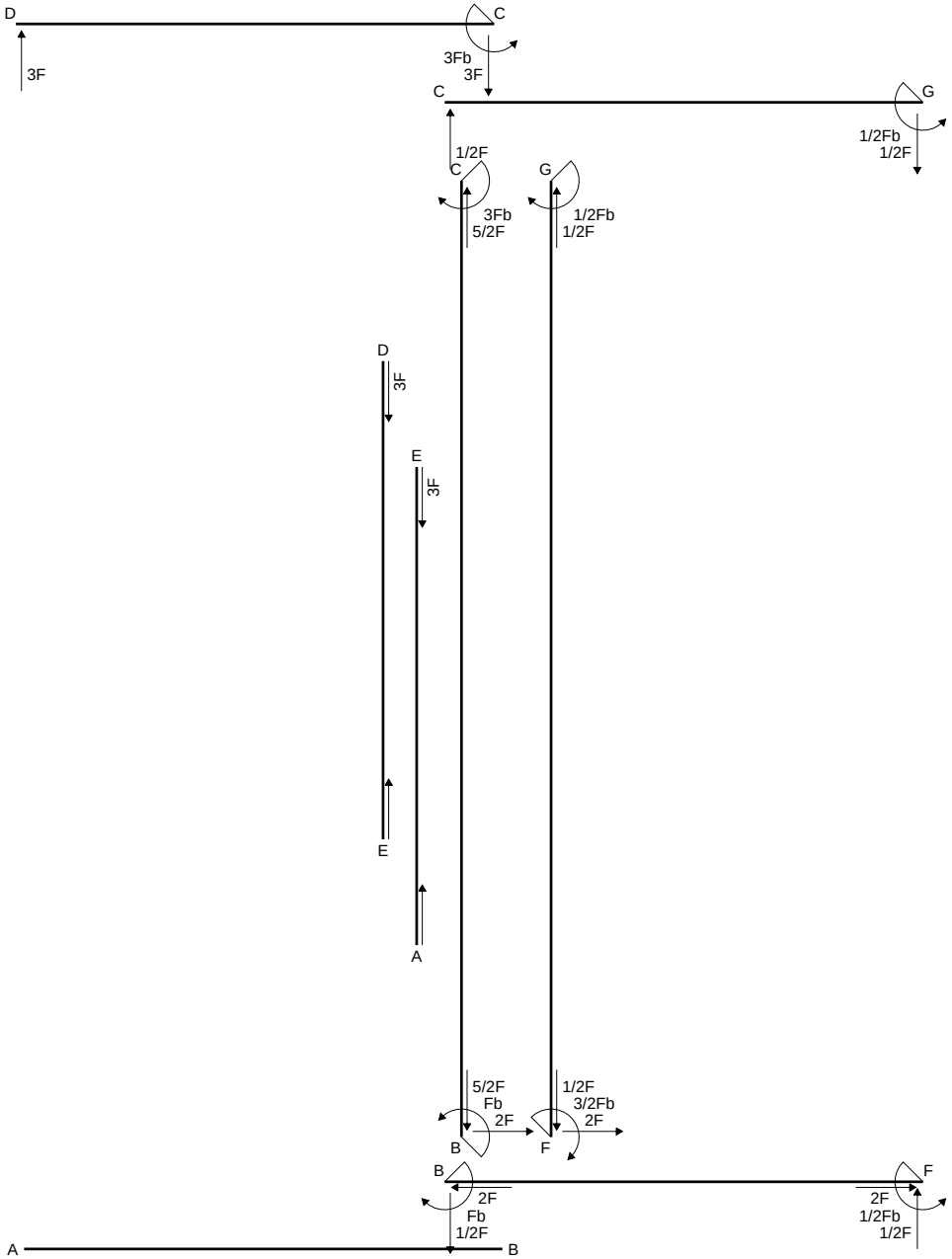
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

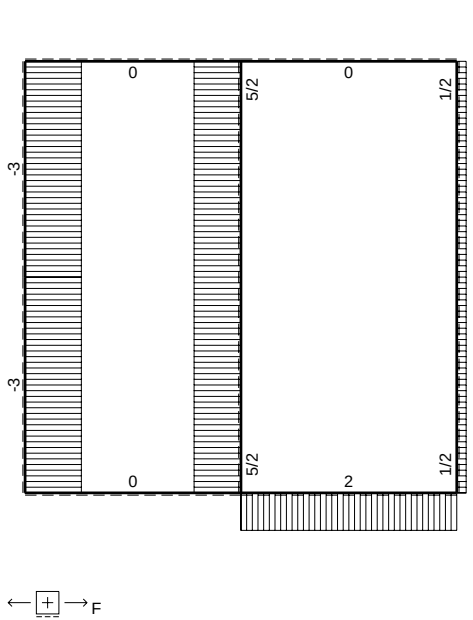


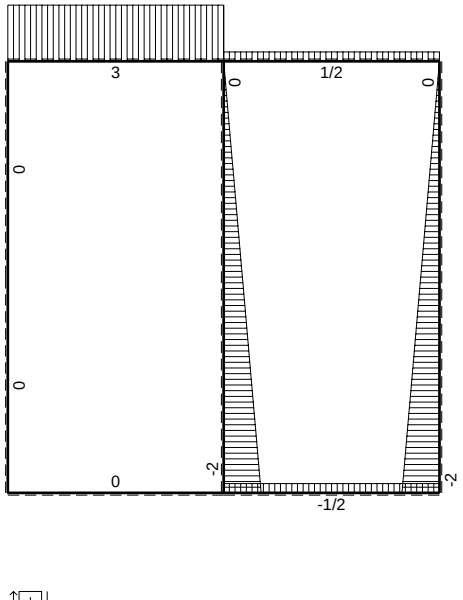
A = 182. mm^2
J_u = 113419. mm^4
J_v = 14035. mm^4
J_t = 154.7 mm^4
x_o = -8.598 mm
x_g = 15.81 mm
T_y = -1200. N
M_x = 840000. Nmm
u_m = -15.81 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = 207.4 N/mm^2
x_c = 12. mm
u_c = -3.808 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = 207.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 126.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.814 N/mm^2
$\tau_o = Tx_o t/J_t$ = 120. N/mm^2
t_c = 1080. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 302.1 N/mm^2

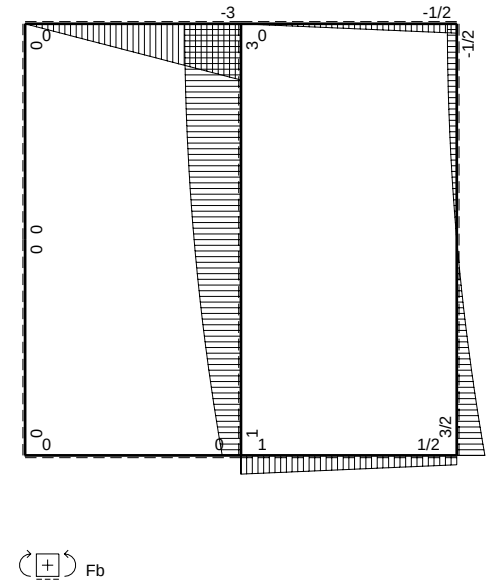
CI08.xxxx.146

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
5/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
5/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
5/2
0
1/2
-3
-3
0
5/2
2
1/2
F


3
0
1/2
0
0
0
-2
0
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	$8/3Xb/EJ$
iperstatica $X=W_{GC}$							1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

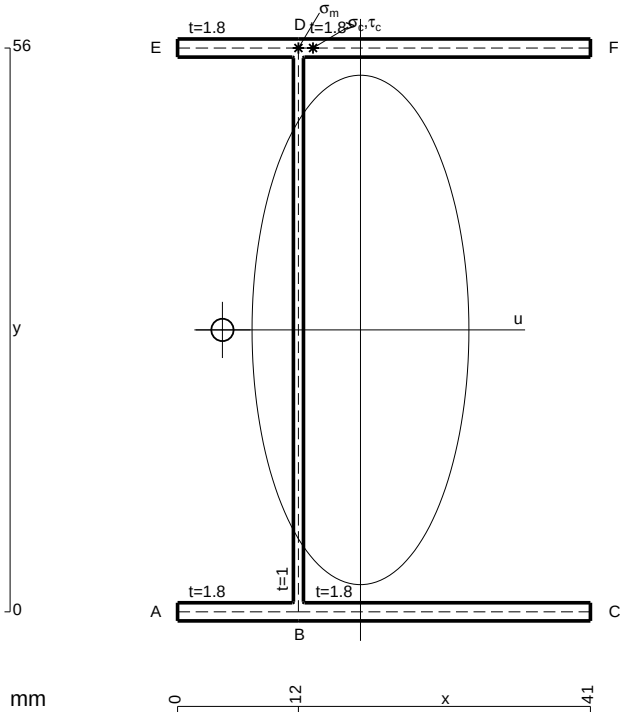
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

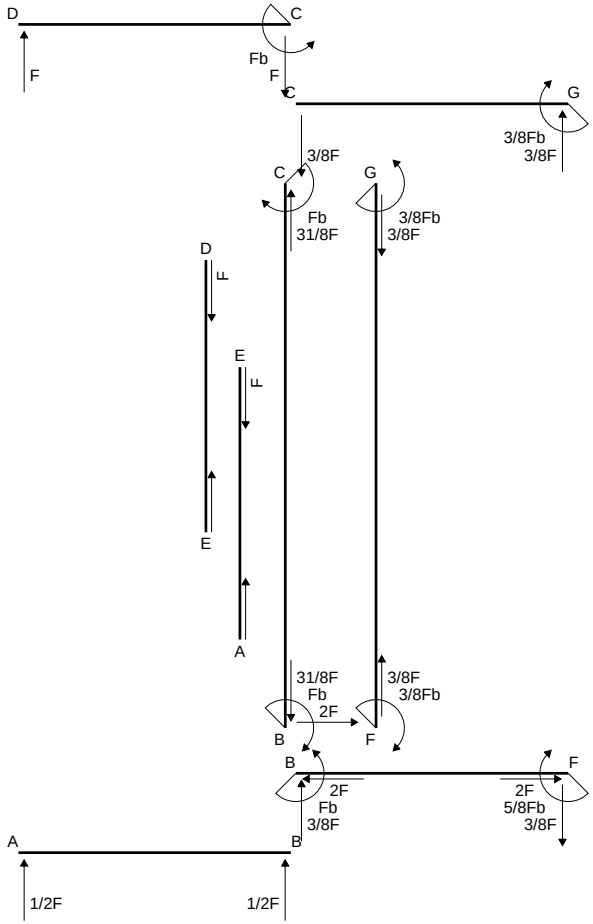
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

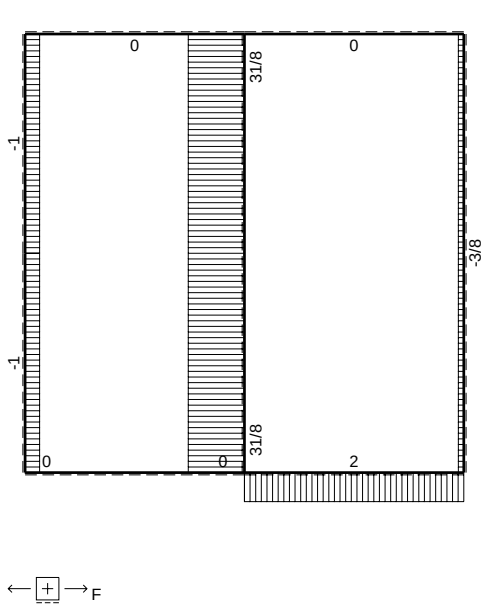


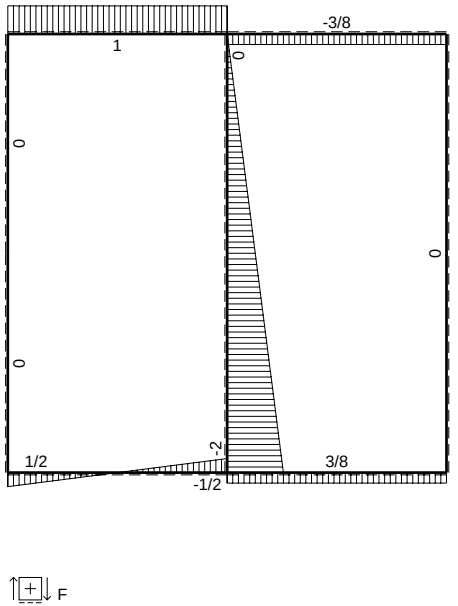
A = 203.6 mm^2
J_u = 130353. mm^4
J_v = 23610. mm^4
J_t = 178.1 mm^4
x_o = -13.71 mm
x_g = 18.16 mm
T_y = 1380. N
M_x = -1021200. Nmm
x_m = 12. mm
y_m = 56. mm
u_m = -6.162 mm
v_m = 28. mm
$\sigma_m = -Mv/J_u$ = 219.4 N/mm^2
x_c = 12. mm
y_c = 56. mm
u_c = -6.162 mm
v_c = 28. mm
$\sigma_c = -Mv/J_u$ = 219.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 199.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 8.596 N/mm^2
$\tau_o = Tx_ot/J_t$ = 191.2 N/mm^2
t_c = 828. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 409.7 N/mm^2

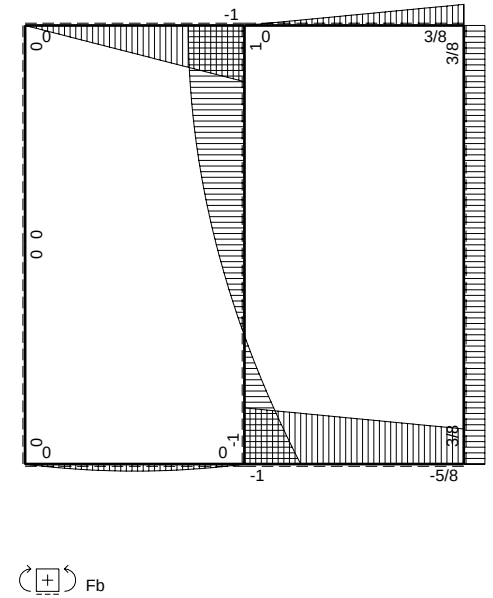
CI08.xxxx.147

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
31/8F
3/8Fb
3/8F
D
F
E
F
E
A
31/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F


0
0
31/8
-1
-1
-3/8
0
31/8
0
0
2
F


-3/8
1
0
0
0
0
1/2
-2
3/8
-1/2
F


-1
0
0
1
0
3/8
3/8
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

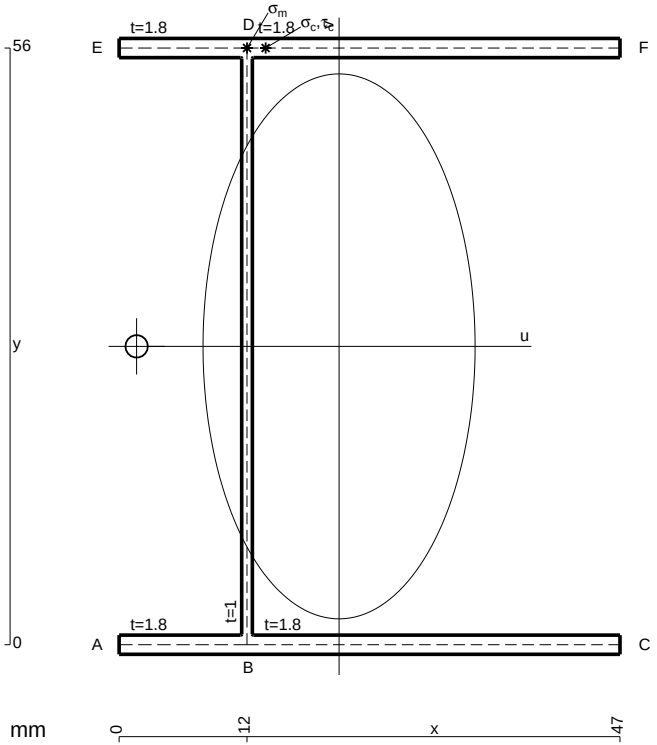
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

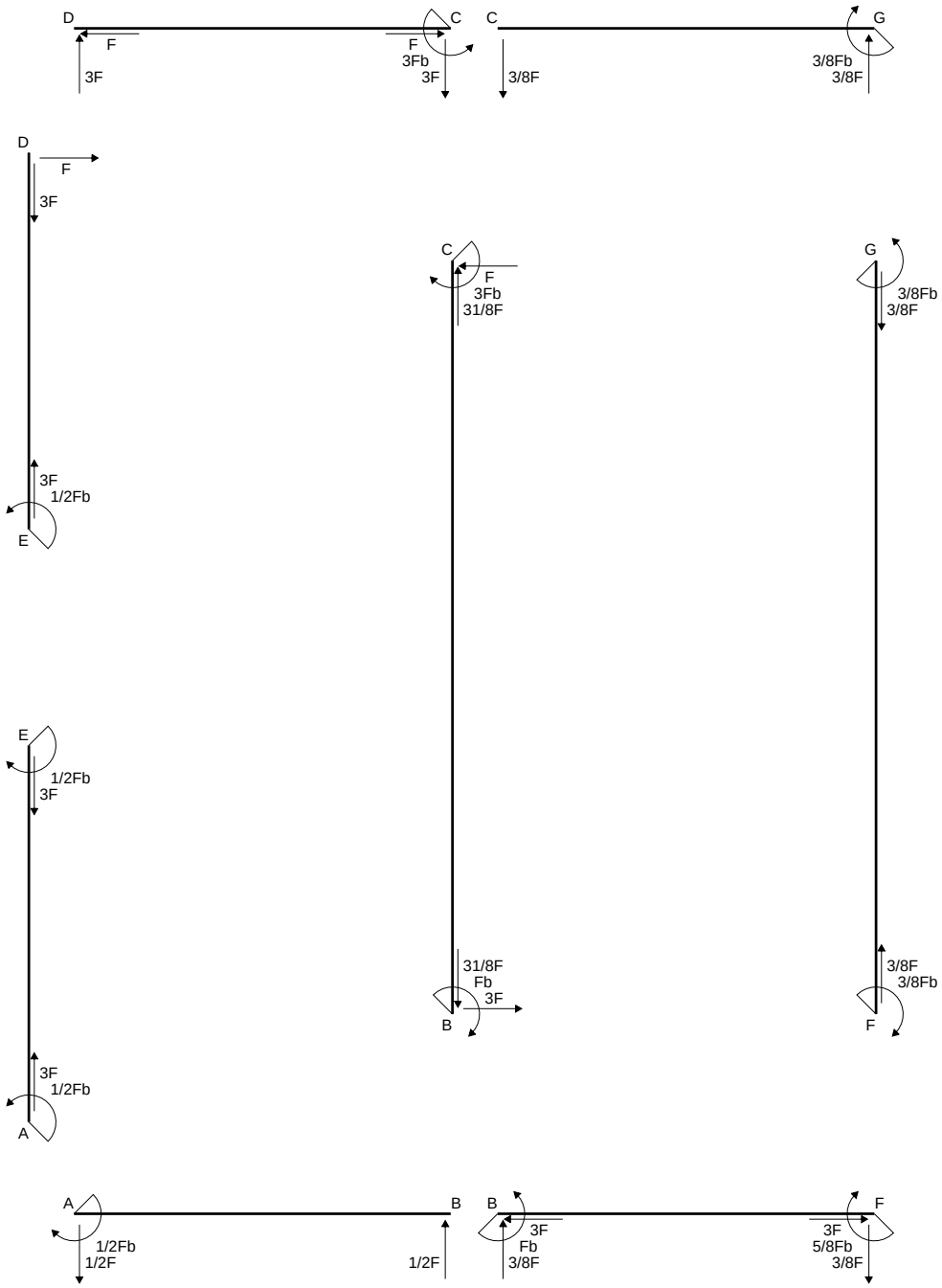
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

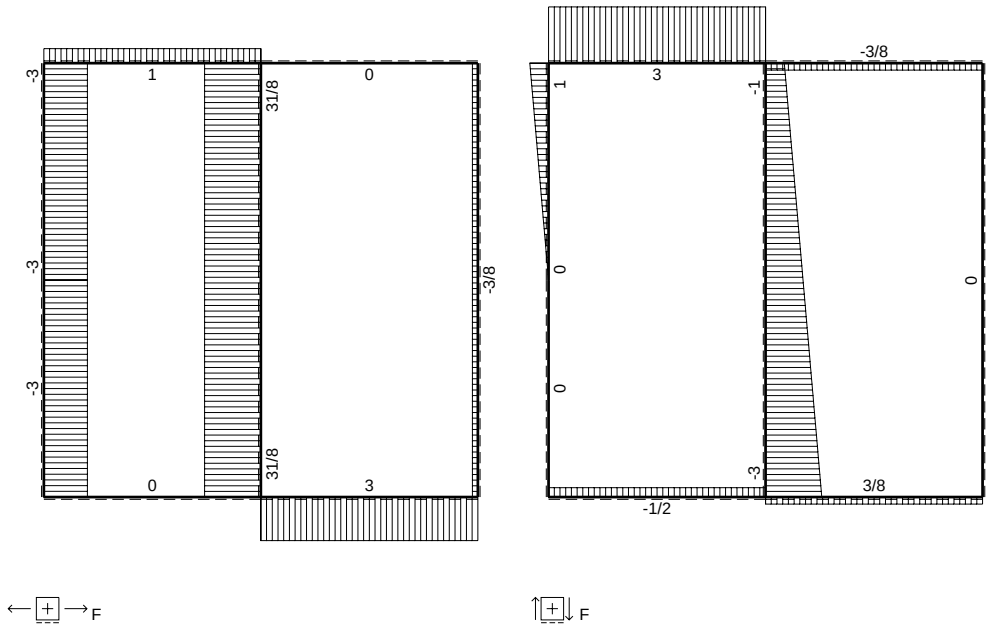


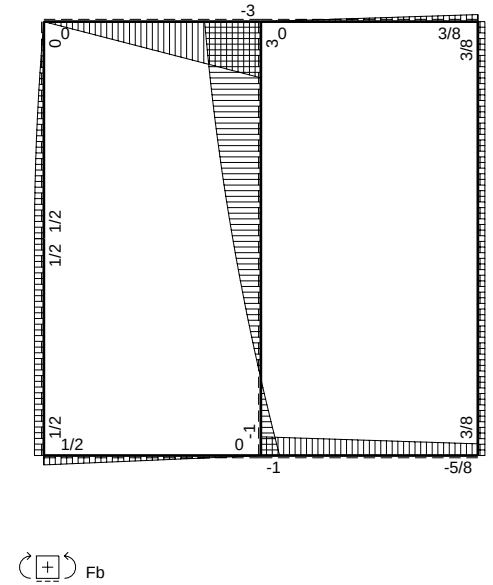
A = 225.2 mm^2
J_u = 147287. mm^4
J_v = 36711. mm^4
J_t = 201.4 mm^4
x_o = -19. mm
x_g = 20.64 mm
N = 5309. N
T_y = -2740. N
M_x = -1082300. Nmm
x_m = 12. mm
y_m = 56. mm
u_m = -8.64 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 229.3 N/mm^2
x_c = 12. mm
y_c = 56. mm
u_c = -8.64 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 229.3 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 483.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 18.23 N/mm^2
$\tau_o = Tx_o t/J_t$ = 465.2 N/mm^2
t_c = 2466. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 868.2 N/mm^2

CI08.xxxx.148

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
3/8F
3/8Fb
3/8F
D
F
3F
C
F
3Fb
31/8F
G
3/8Fb
3/8F
3F
1/2Fb
E
E
1/2Fb
3F
31/8F
Fb
3F
3/8F
3/8Fb
B
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
3F
Fb
3/8F
3F
5/8Fb
3/8F


-3
1
0
31/8
-3/8
-3
-3
31/8
0
3
1
3
-1
-3/8
0
0
0
-3
3/8
-1/2
F
F


-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 237095 CI08.xxxx.149

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

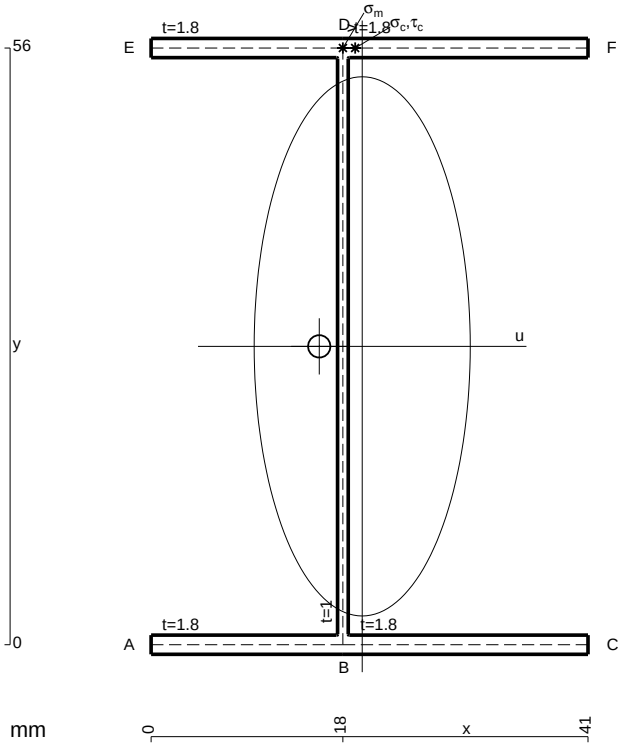
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

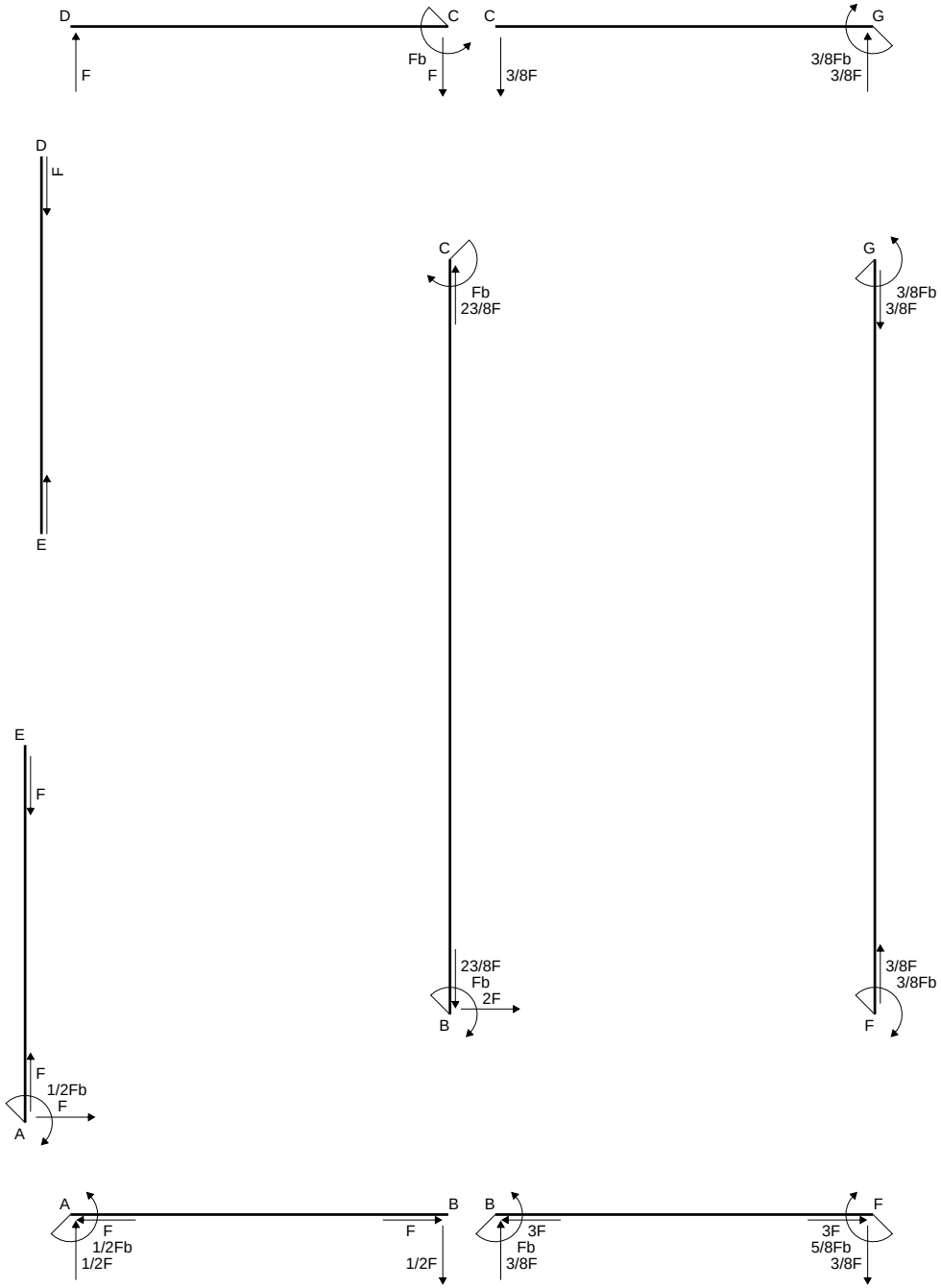
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

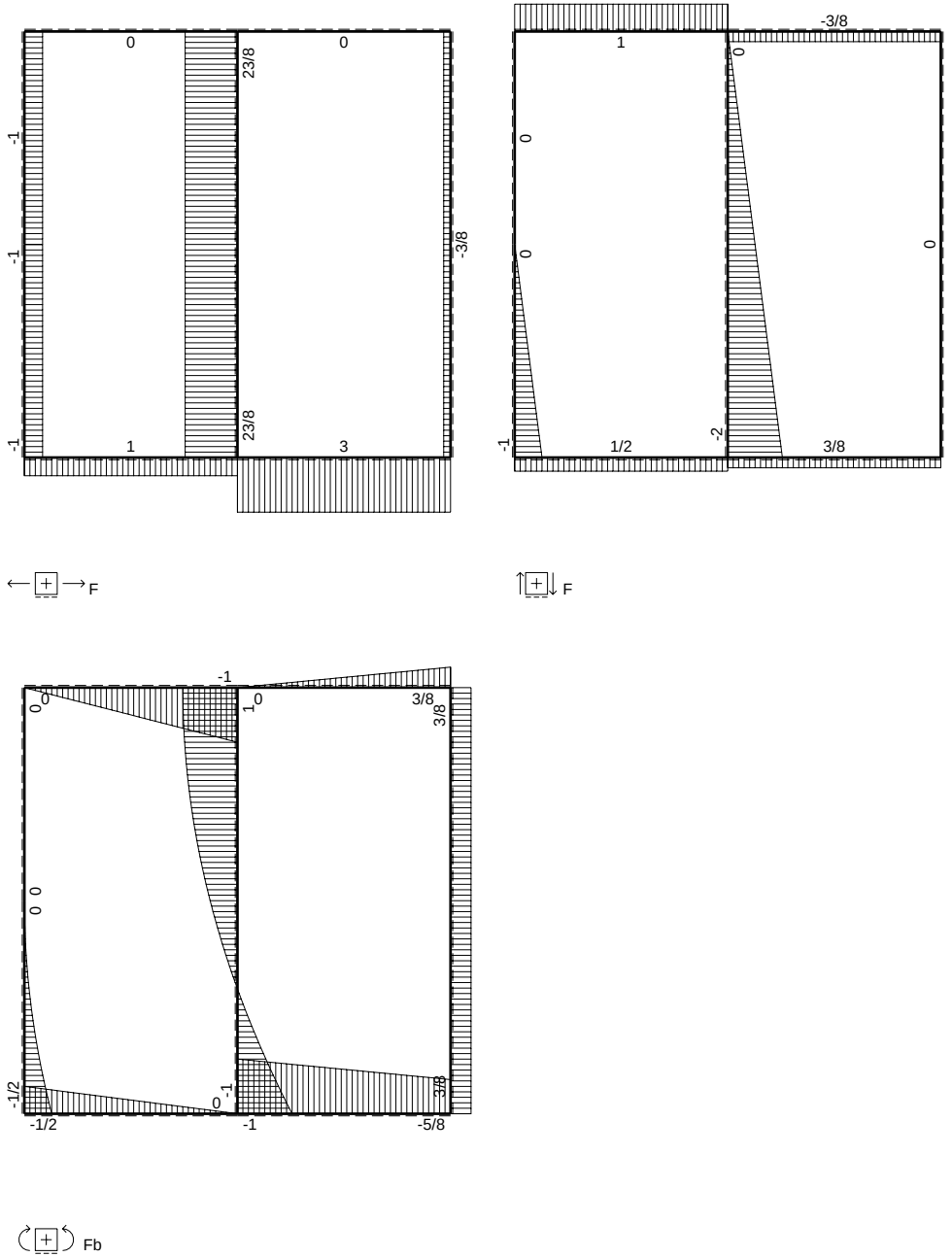


A = 203.6 mm^2
J_u = 130353. mm^4
J_v = 20930. mm^4
J_t = 178.1 mm^4
x_o = -4.032 mm
x_g = 19.81 mm
N = 440. N
T_y = 1320. N
M_x = -1095600. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = -1.812 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 237.5 N/mm^2
x_c = 18. mm
y_c = 56. mm
u_c = -1.812 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 237.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 60.32 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.521 N/mm^2
$\tau_o = Tx_o t/J_t$ = 53.79 N/mm^2
t_c = 792. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 259.5 N/mm^2

CI08.xxxx.149

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
23/8F
G
3/8Fb
3/8F
E
F
23/8F
Fb
2F
B
3/8F
3/8Fb
F
F
1/2Fb
F
A
A
F
1/2Fb
1/2F
B
F
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F


0
0
23/8
-1
-1
-1
-3/8
1
23/8
3
F
1
-3/8
0
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
0
-1
3/8
-1/2
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Schema di calcolo iperstatico

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

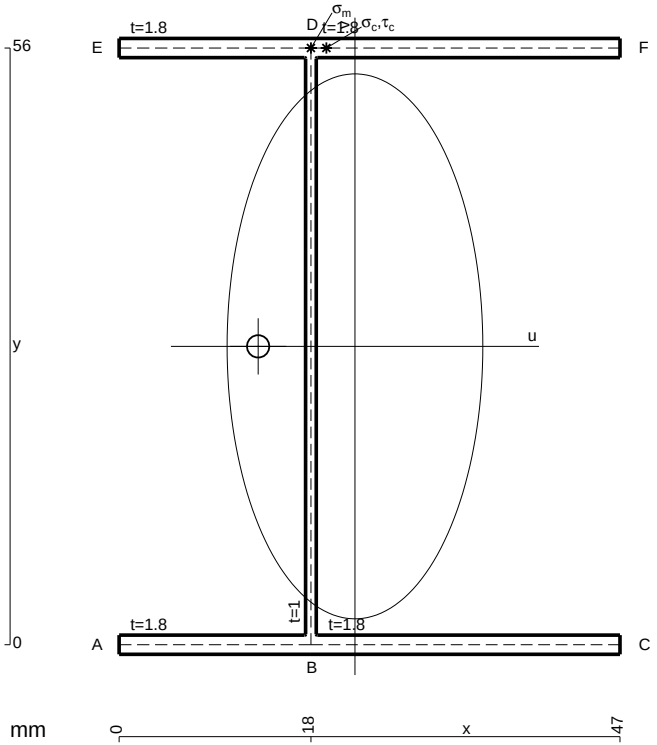
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

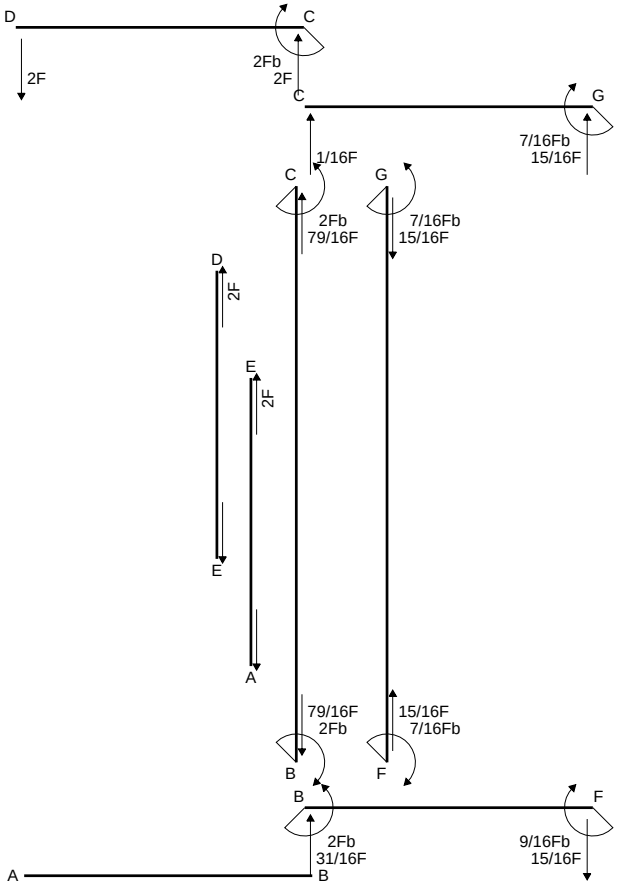
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

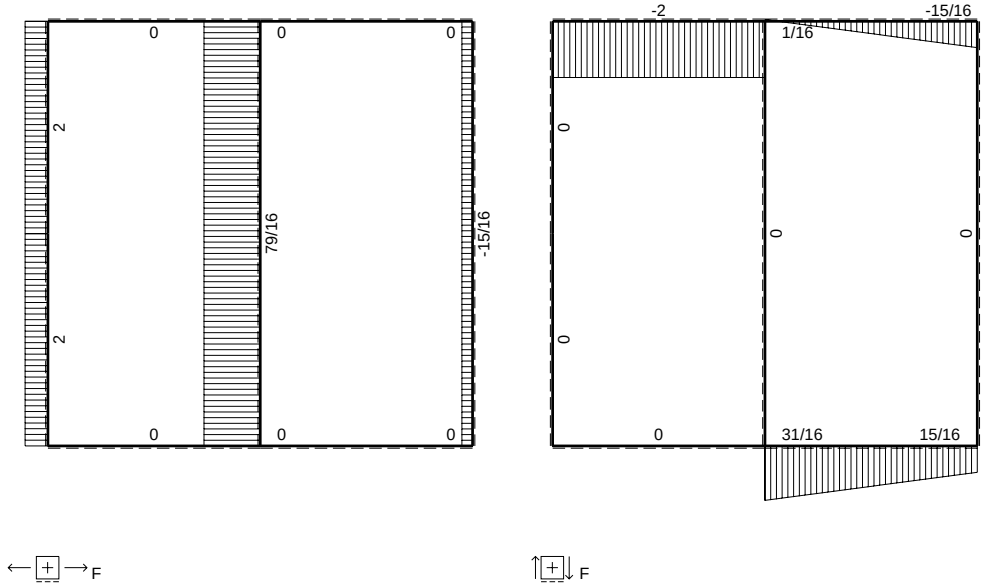


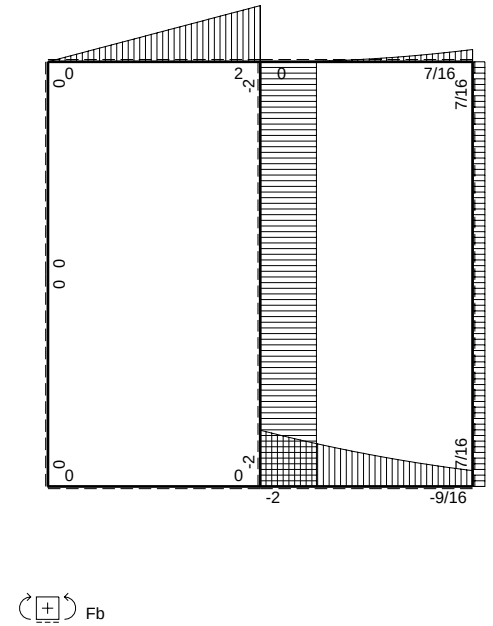
A = 225.2 mm^2
J_u = 147287. mm^4
J_v = 32420. mm^4
J_t = 201.4 mm^4
x_o = -9.086 mm
x_g = 22.13 mm
N = 5951. N
T_y = -4140. N
M_x = -910800. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = -4.132 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 199.6 N/mm^2
x_c = 18. mm
y_c = 56. mm
u_c = -4.132 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 199.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 359. N/mm^2
$\tau_g = TS^*/tJ_u$ = 22.82 N/mm^2
$\tau_o = Tx_o t/J_t$ = 336.2 N/mm^2
t_c = 3726. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 653.1 N/mm^2

CI08.xxxx.150

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
2Fb
2F
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
79/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
79/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
79/16
-15/16
2
0
0
0
-2
-15/16
1/16
0
0
0
0
31/16
15/16
F
F


0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
7/16
-2
-9/16
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
	0	-2Fx	0	0	0	0		
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
AE b	0	0	0	0	0	0		
BF b	-x/b	-2Fb+3/2Fx-1/2qx^2	-Fb/EJ	2Fx-3/2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/8+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
	1	0	0	0	0	1		
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
	0	2Fb	0	0	0	0		
BC 2b	0	2Fb	0	0	0	0		
	totali						7/6Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

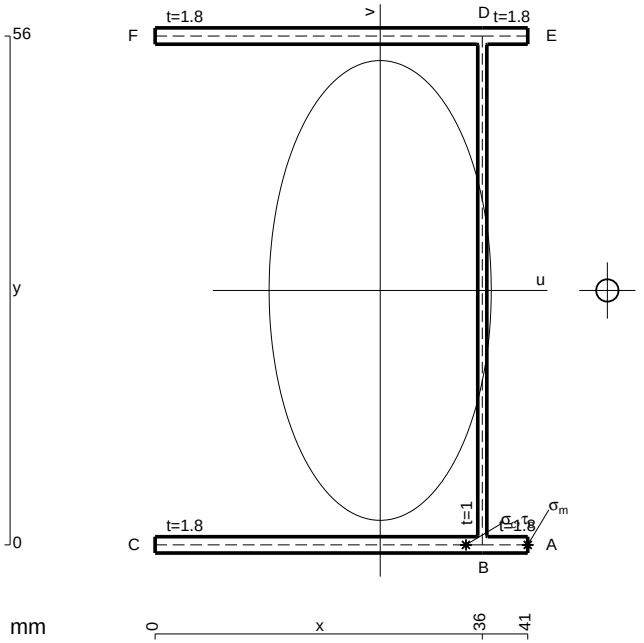
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

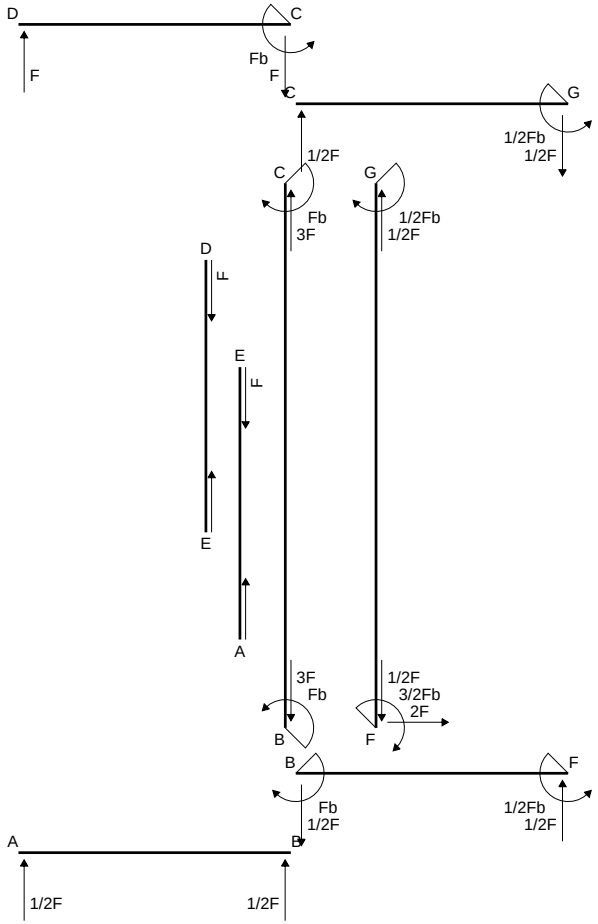
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

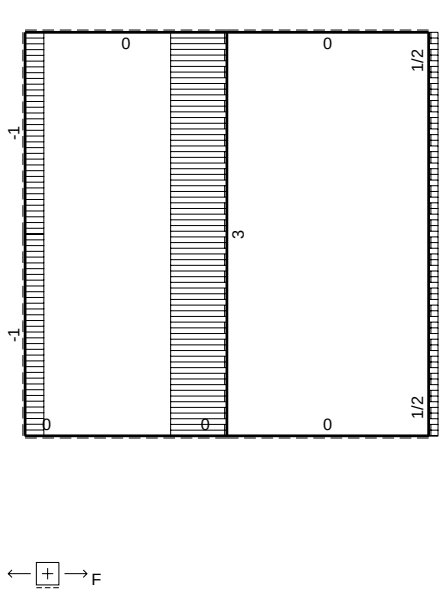


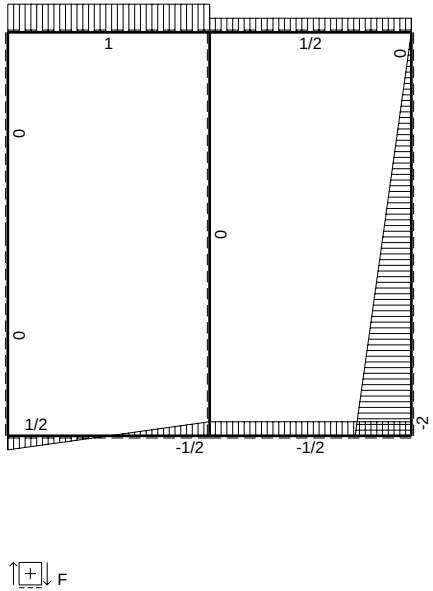
A = 203.6 mm^2
J_u = 130353. mm^4
J_v = 30430. mm^4
J_t = 178.1 mm^4
x_o = 25. mm
x_g = 24.76 mm
T_y = -2020. N
M_x = 969600. Nmm
x_m = 41. mm
u_m = 16.24 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = 208.3 N/mm^2
x_c = 36. mm
u_c = 11.24 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = 208.3 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 526. N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.62 N/mm^2
$\tau_o = Tx_o t/J_t$ = 510.4 N/mm^2
t_c = 1818. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 934.6 N/mm^2

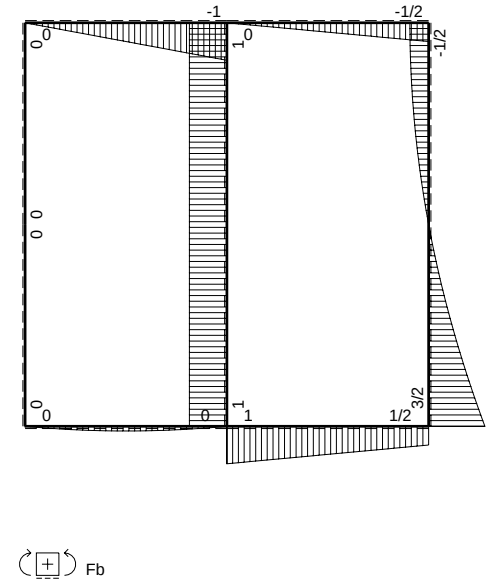
CI08.xxxx.151

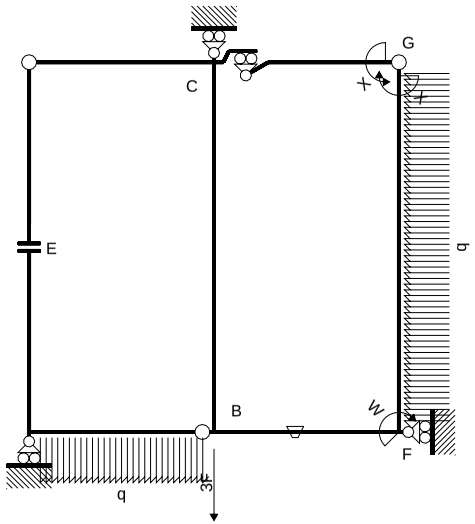
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

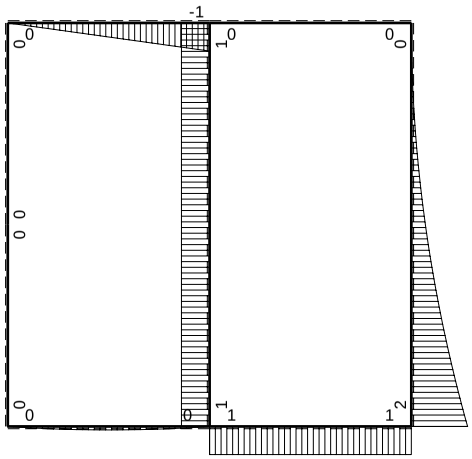

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
3F
1/2Fb
1/2F
D
F
E
F
E
A
3F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
3
-1
0
0
0
1/2
F

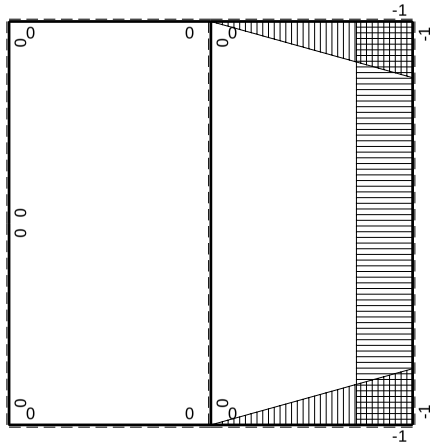

1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2
F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 248640 CI08.xxxx.152

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L_{FB}^{Xo} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

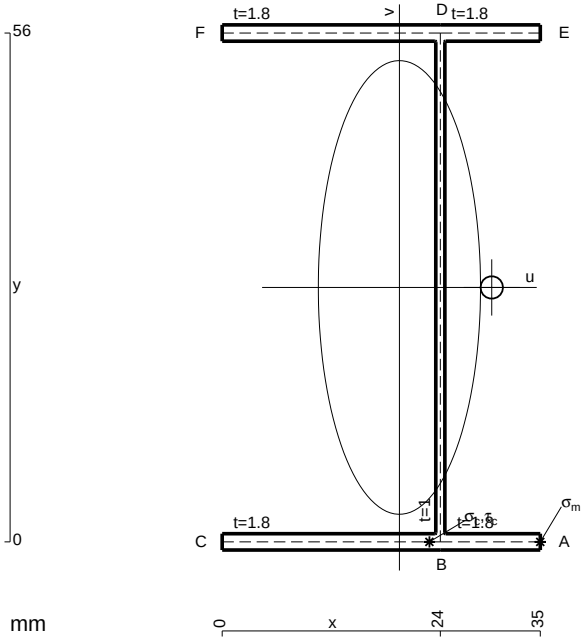
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L_{FG}^{Xo} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L_{GF}^{Xo} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

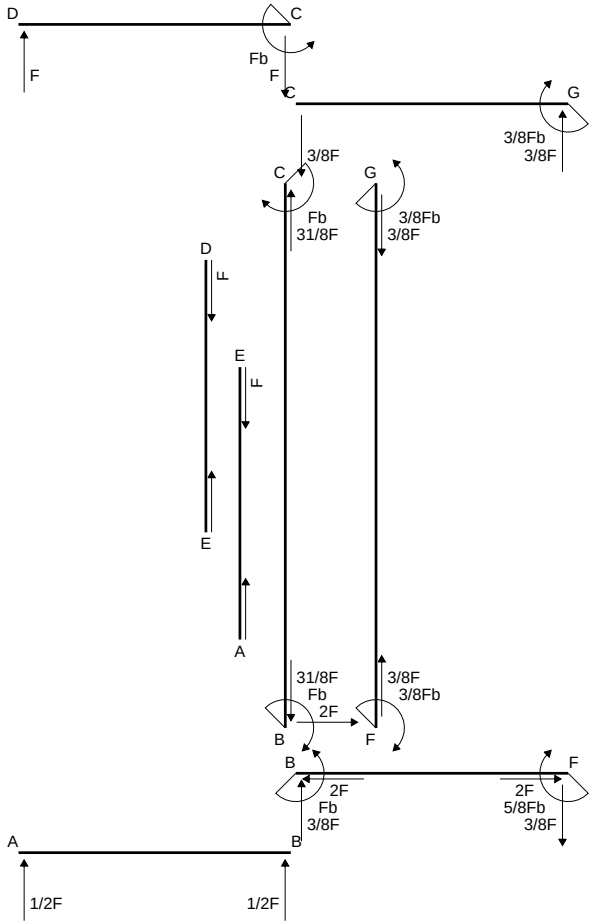
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

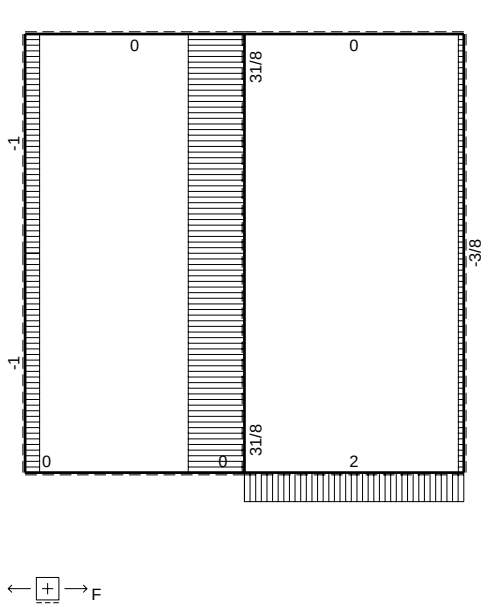


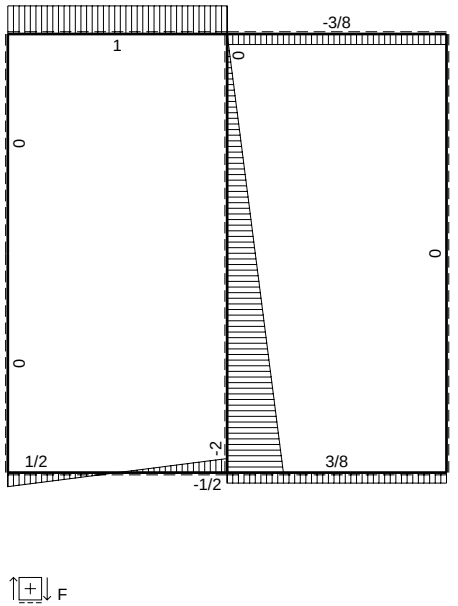
A = 182. mm^2
J_u = 113419. mm^4
J_v = 14501. mm^4
J_t = 154.7 mm^4
x_o = 10.16 mm
x_g = 19.5 mm
T_y = 1710. N
M_x = -889200. Nmm
x_m = 35. mm
u_m = 15.5 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -219.5 N/mm^2
x_c = 24. mm
u_c = 4.5 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -219.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 212.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.13 N/mm^2
$\tau_o = Tx_o t/J_t$ = 202.1 N/mm^2
t_c = 3078. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 428.2 N/mm^2

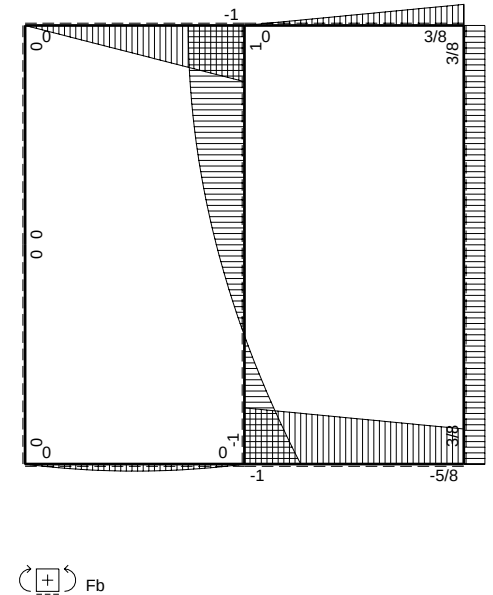
CI08.xxxx.152

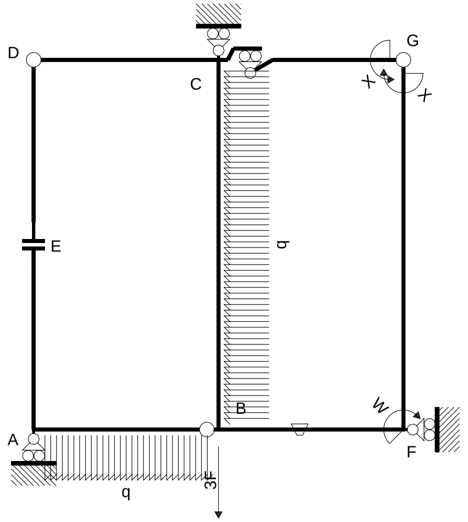
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
31/8F
3/8Fb
3/8F
D
F
E
F
E
A
31/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F

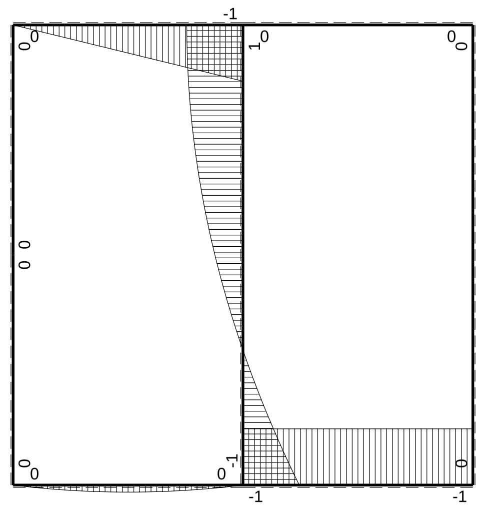

0
0
31/8
-1
-1
-3/8
0
31/8
0
0
2
F


-3/8
1
0
0
0
0
1/2
-2
3/8
-1/2
F

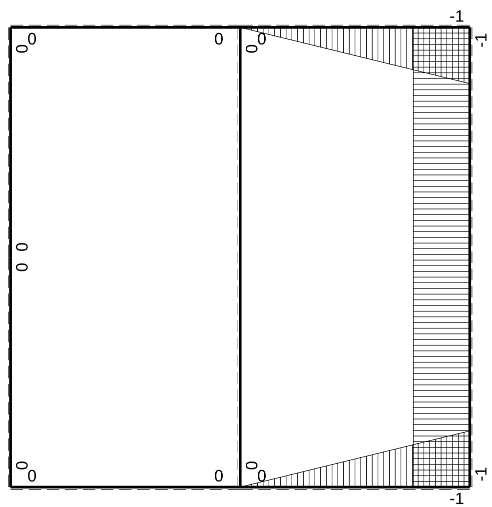

-1
0
0
1
0
3/8
3/8
0
0
0
0
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

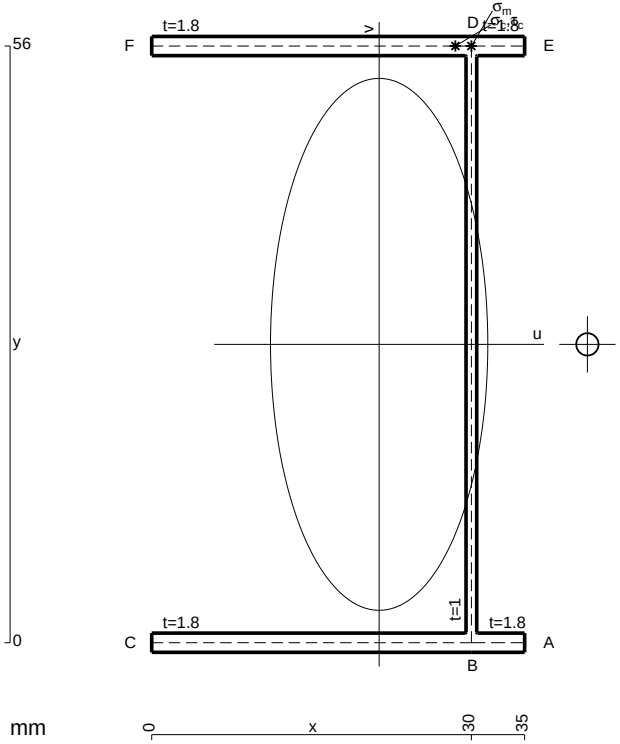
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

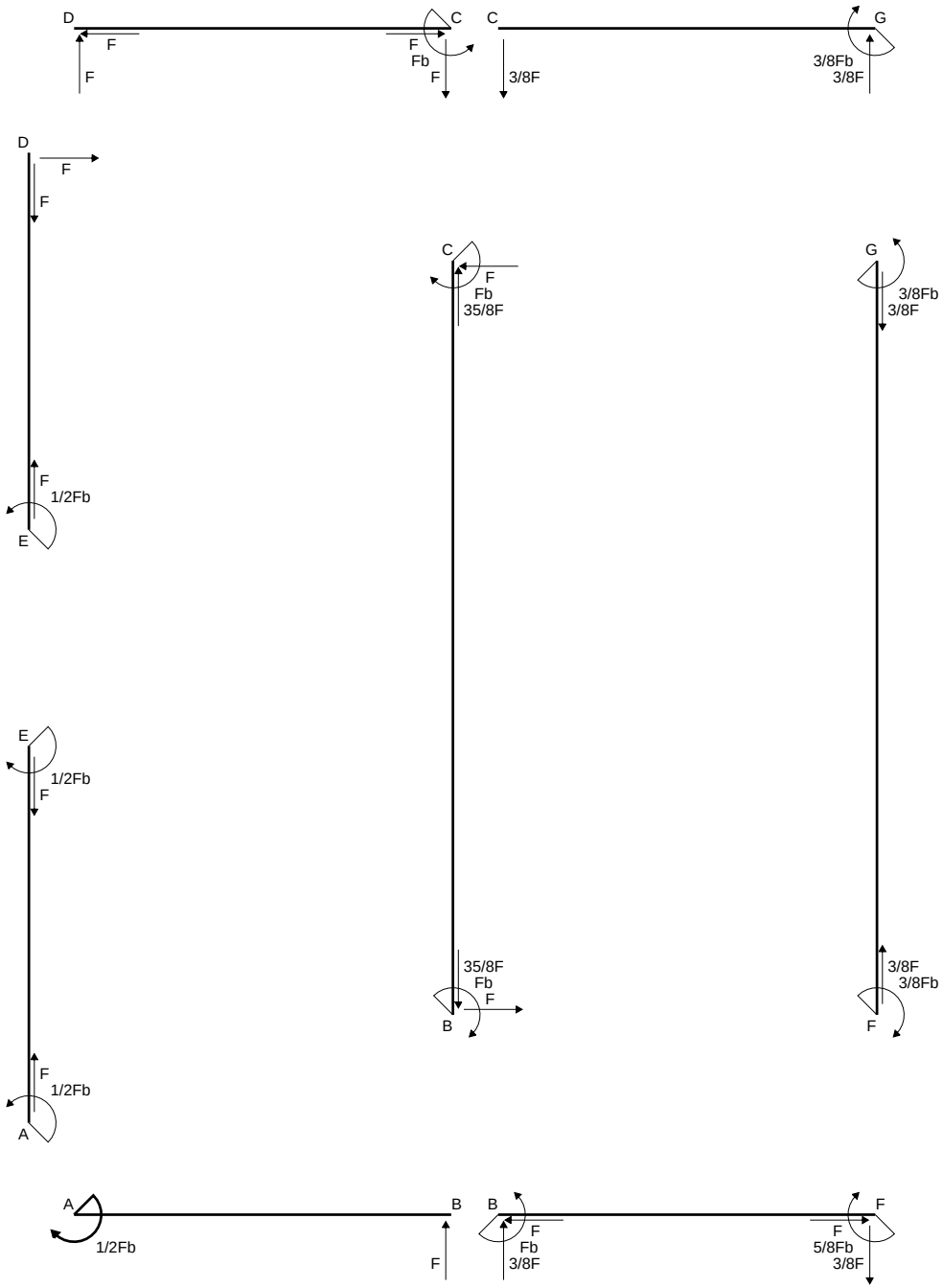
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

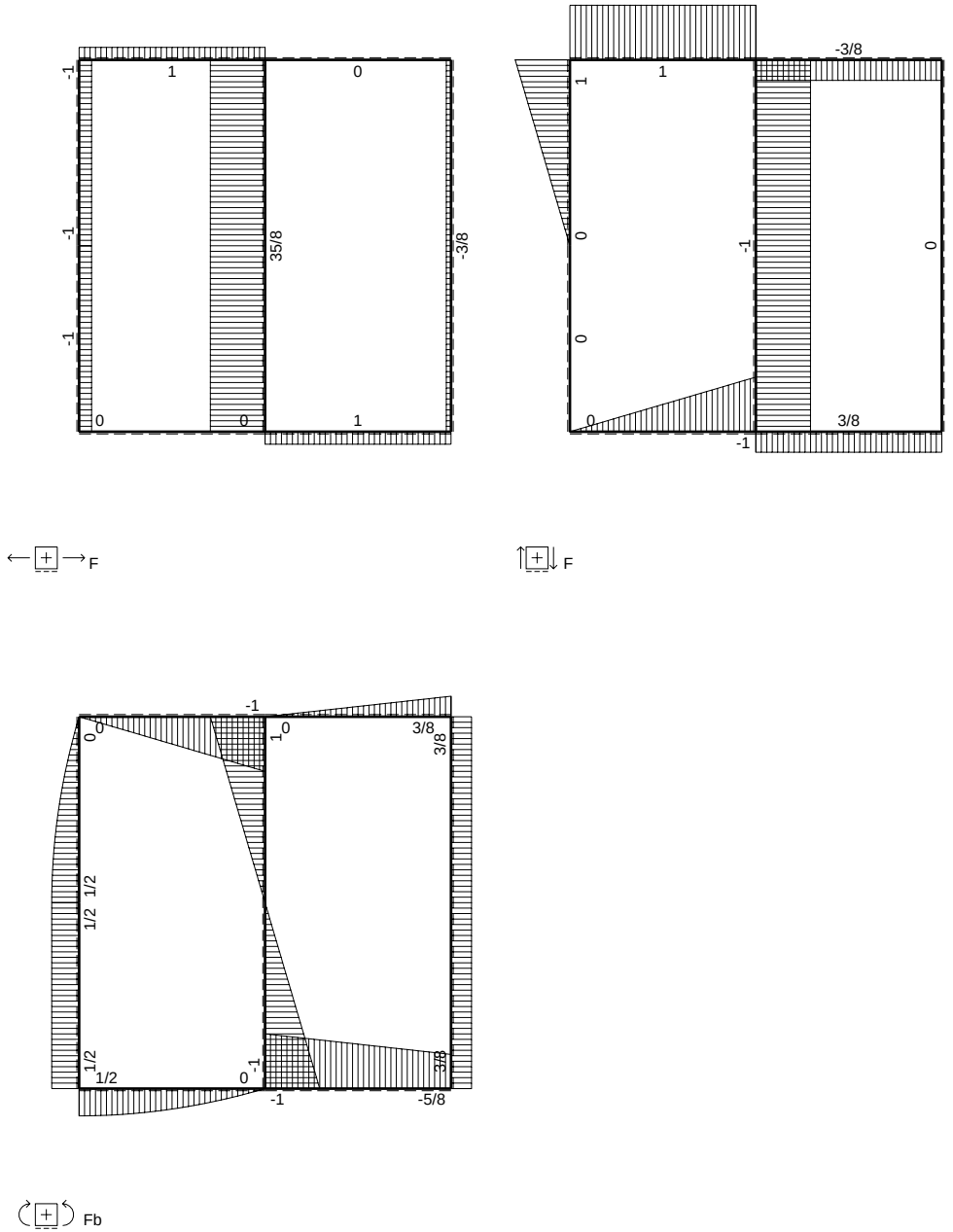


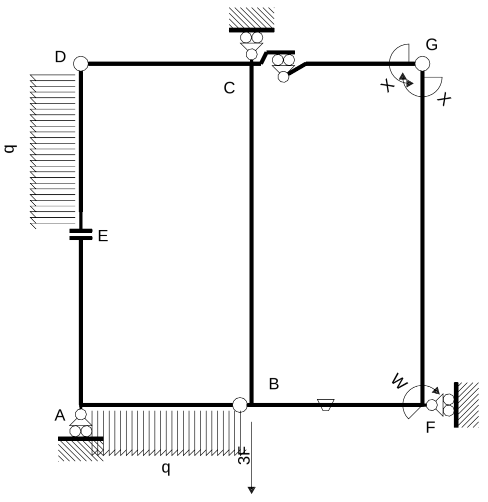
A = 182. mm^2
J_u = 113419. mm^4
J_v = 18920. mm^4
J_t = 154.7 mm^4
x_o = 19.54 mm
x_g = 21.35 mm
N = 5464. N
T_y = -2820. N
M_x = -803700. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 8.654 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 228.4 N/mm^2
x_c = 30. mm
y_c = 56. mm
u_c = 8.654 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 228.4 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 661.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 20.89 N/mm^2
$\tau_o = Tx_o t/J_t$ = 641. N/mm^2
t_c = 2538. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1169. N/mm^2

CI08.xxxx.153

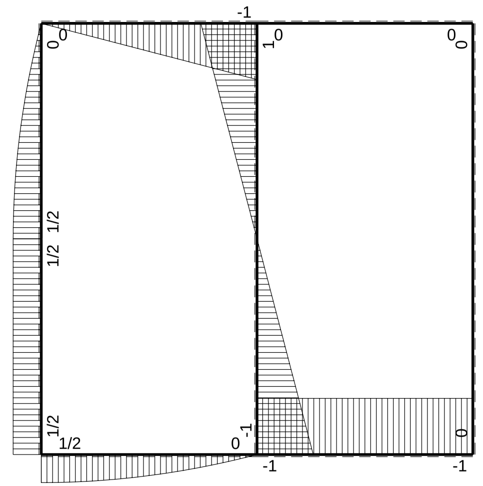
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
C
F
Fb
35/8F
G
3/8Fb
3/8F
F
1/2Fb
E
E
1/2Fb
F
35/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F

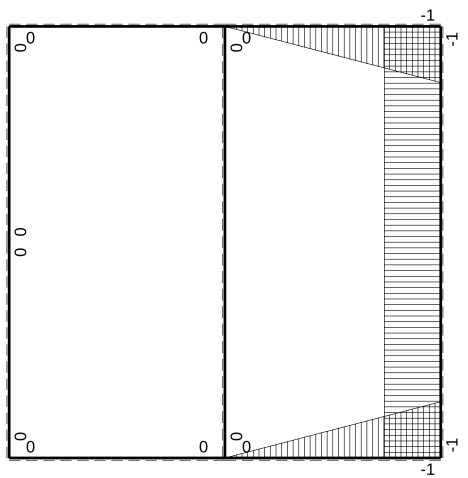

-1
1
0
-1
35/8
-3/8
-1
0
0
1
F
-3/8
1
1
0
-1
0
0
0
3/8
-1
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

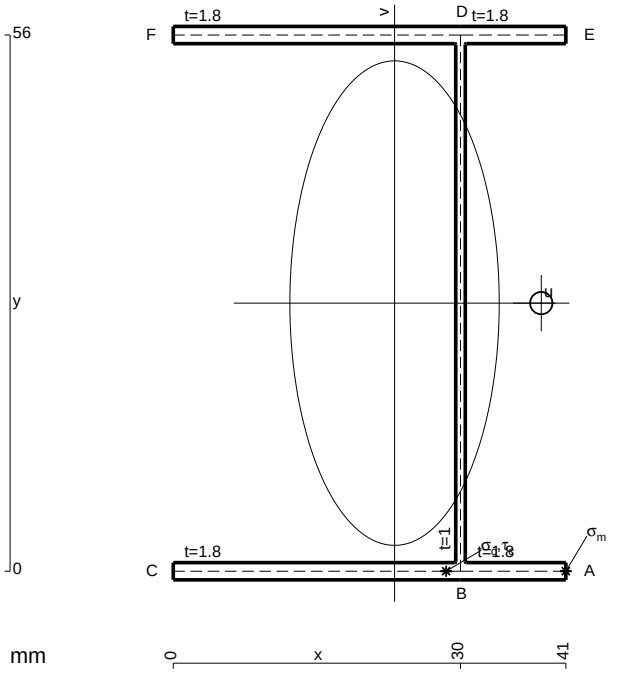
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

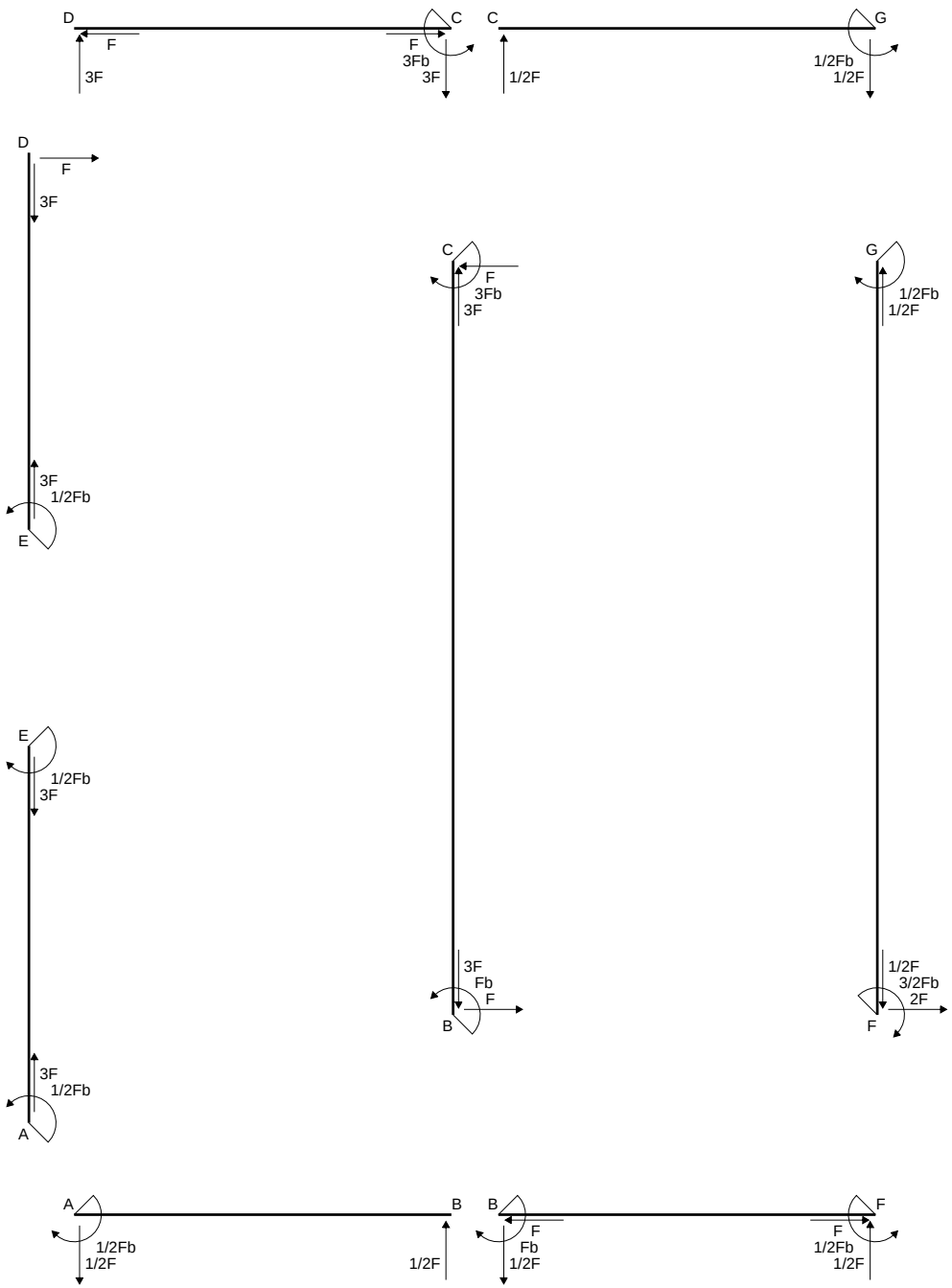
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

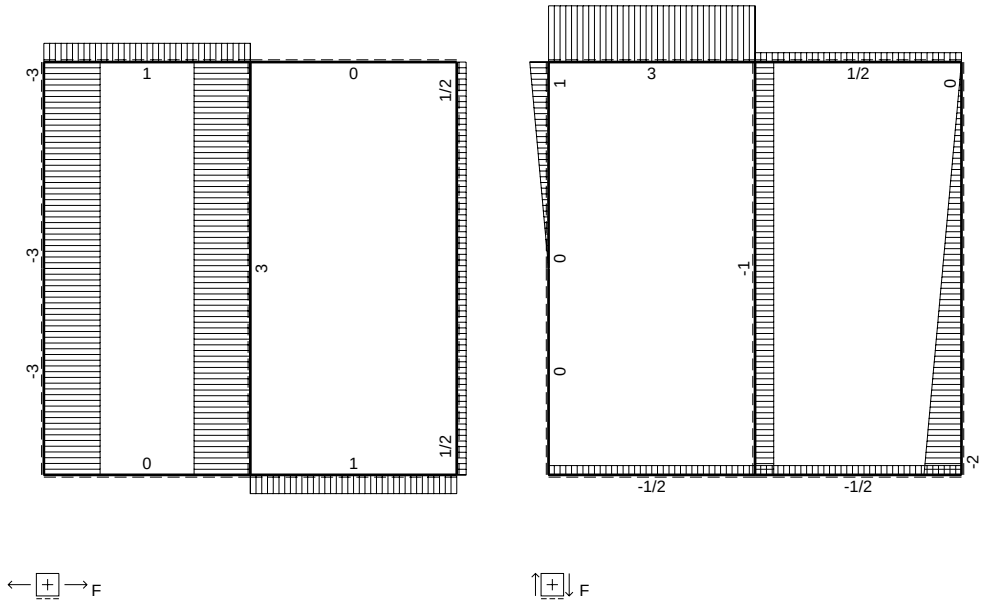


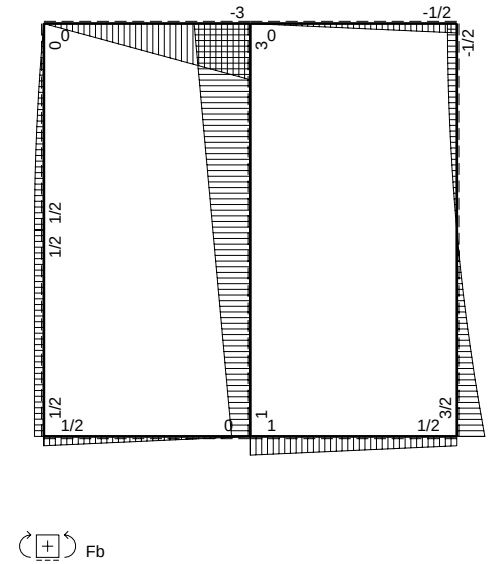
A = 203.6 mm^2
J_u = 130353. mm^4
J_v = 24340. mm^4
J_t = 178.1 mm^4
x_o = 15.32 mm
x_g = 23.11 mm
N = 6869. N
T_y = -1570. N
M_x = 957700. Nmm
x_m = 41. mm
u_m = 17.89 mm
v_m = -28. mm
$\sigma_m = N/A - Mv/J_u$ = 239.5 N/mm^2
x_c = 30. mm
u_c = 6.887 mm
v_c = -28. mm
$\sigma_c = N/A - Mv/J_u$ = 239.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 253.2 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.12 N/mm^2
$\tau_o = Tx_o t/J_t$ = 243.1 N/mm^2
t_c = 2826. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 499.7 N/mm^2

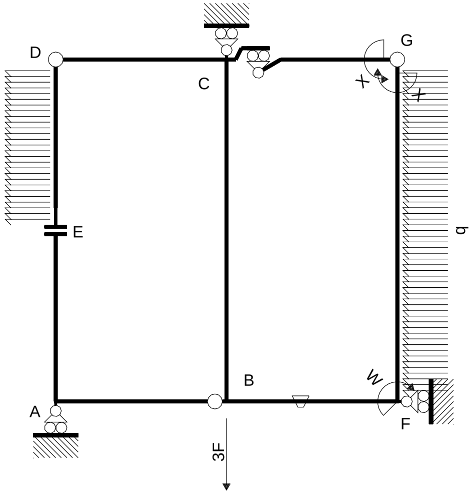
CI08.xxxx.154

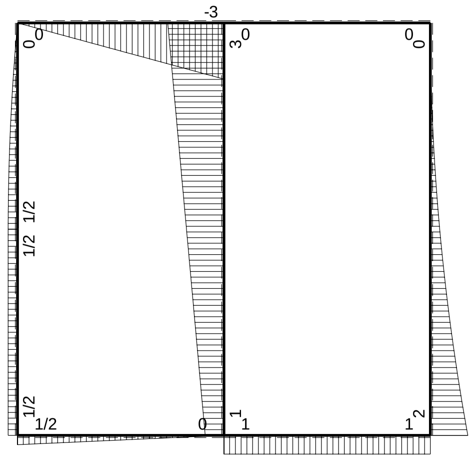
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
3F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
3F
Fb
F
B
1/2F
3/2Fb
2F
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F

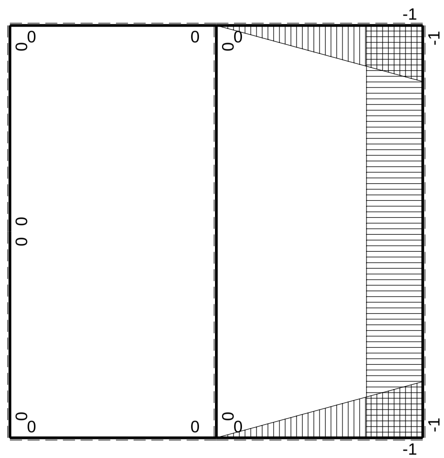

-3
1
0
1/2
-3
3
-3
0
1
1/2
1
3
1/2
0
0
-1
0
-2
-1/2
-1/2
F
F


-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L_{FB}^{Xo} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

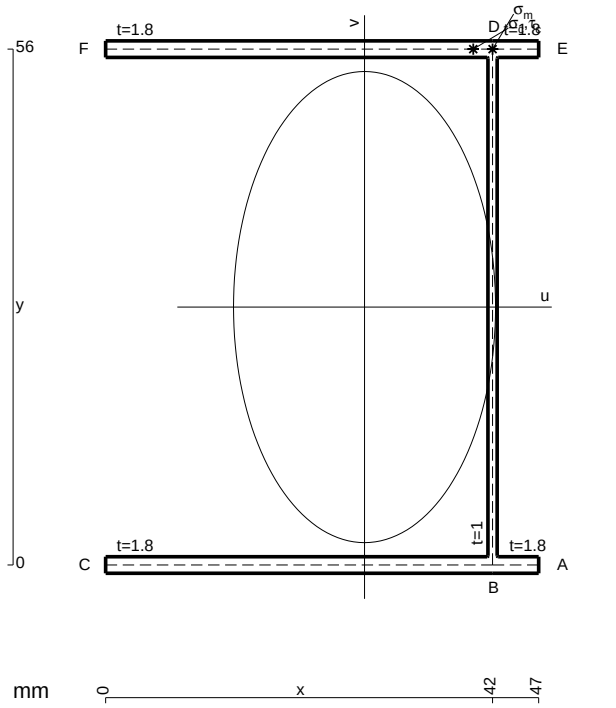
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L_{FG}^{Xo} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L_{GF}^{Xo} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

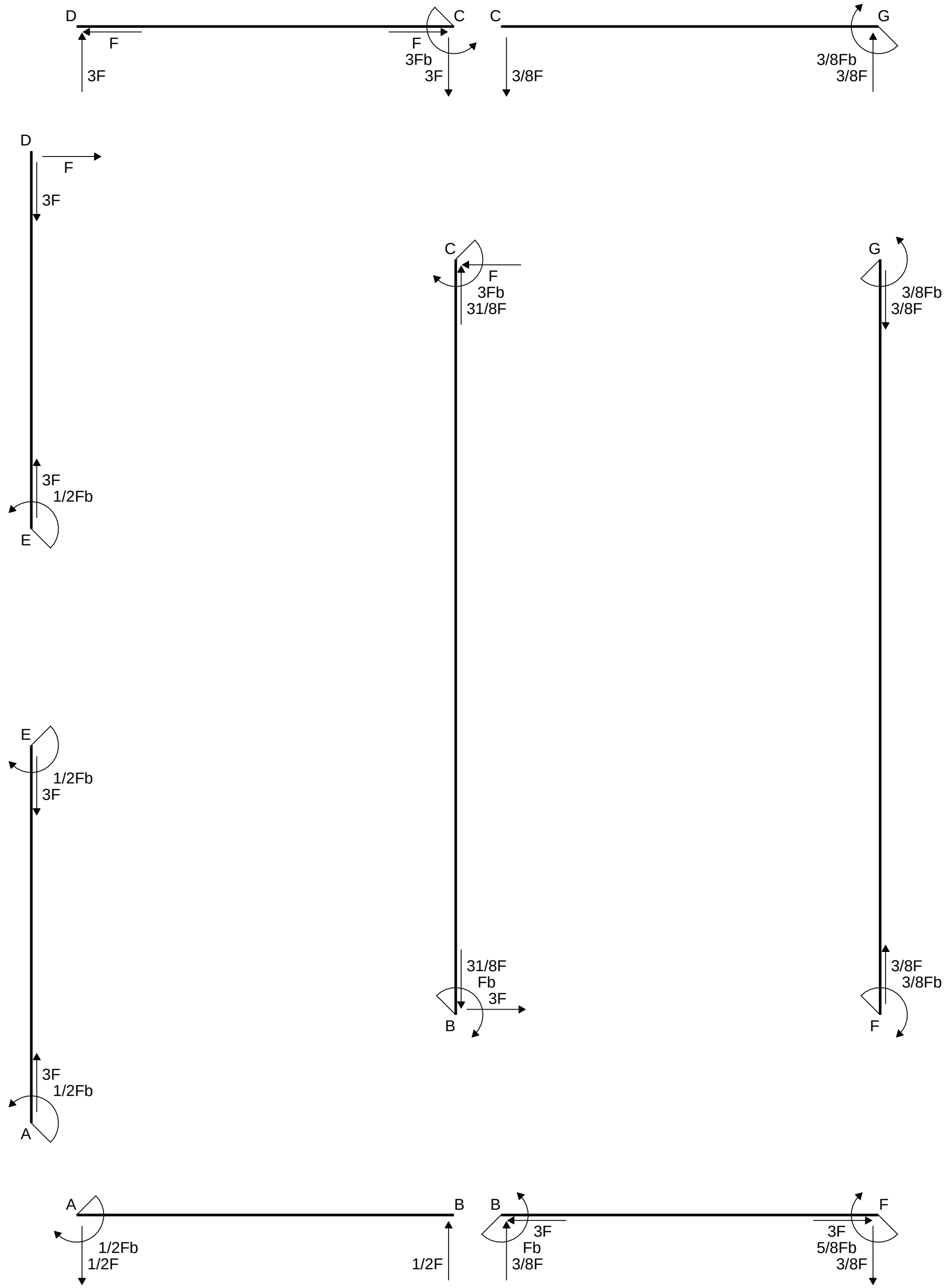
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

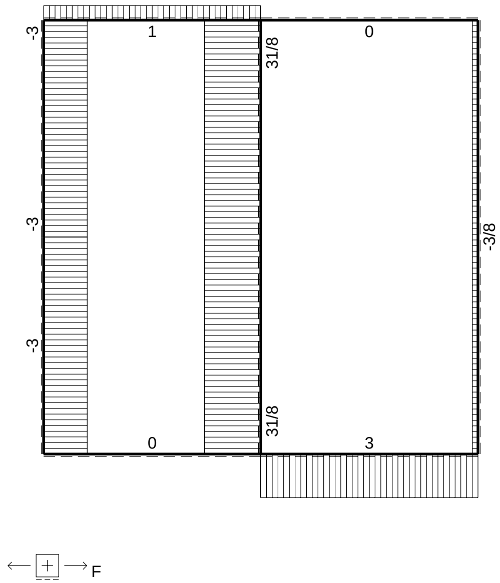


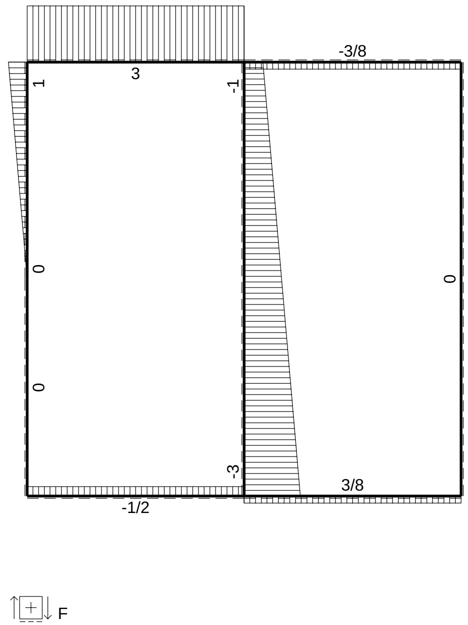
A = 225.2 mm^2
J_u = 147287. mm^4
J_v = 45547. mm^4
J_t = 201.4 mm^4
x_o = 30.56 mm
x_g = 28.1 mm
N = 520. N
T_y = 1560. N
M_x = -1029600. Nmm
x_m = 42. mm
y_m = 56. mm
u_m = 13.9 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 198. N/mm^2
x_c = 42. mm
y_c = 56. mm
u_c = 13.9 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 198. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 438.5 N/mm^2
$\tau_g = TS^*/tJ_u$ = 12.46 N/mm^2
$\tau_o = Tx_o t/J_t$ = 426.1 N/mm^2
t_c = 936. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 785. N/mm^2

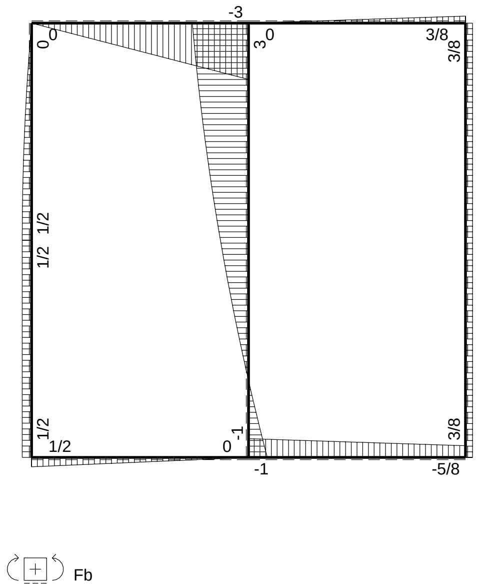
CI08.xxxx.155

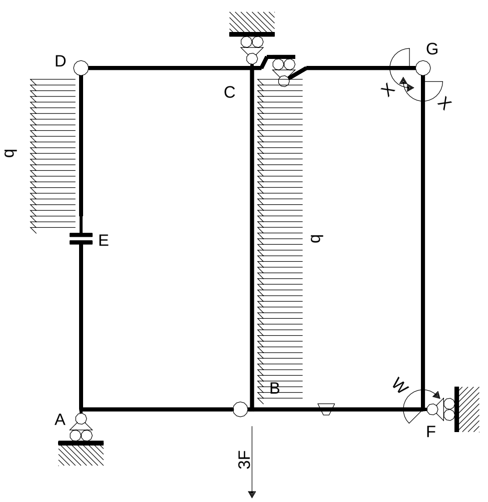
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
3Fb
3F
C
G
3/8F
3/8Fb
3/8F
D
F
3F
3F
1/2Fb
E
C
F
3Fb
31/8F
G
3/8Fb
3/8F
E
1/2Fb
3F
31/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
B
1/2Fb
1/2F
1/2F
B
F
3F
Fb
3/8F
3F
5/8Fb
3/8F

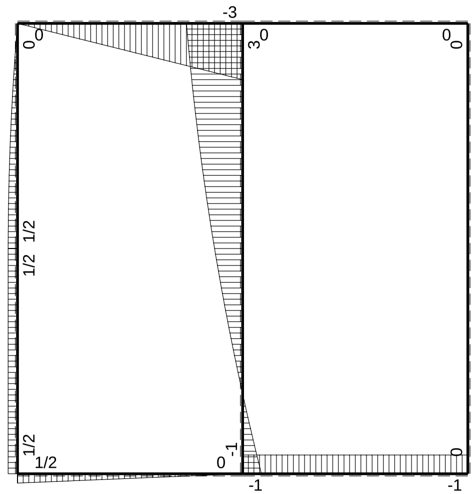

-3
1
0
31/8
-3
-3/8
-3
0
31/8
3
F


-3/8
1
3
-1
0
0
0
-3
3/8
-1/2
F

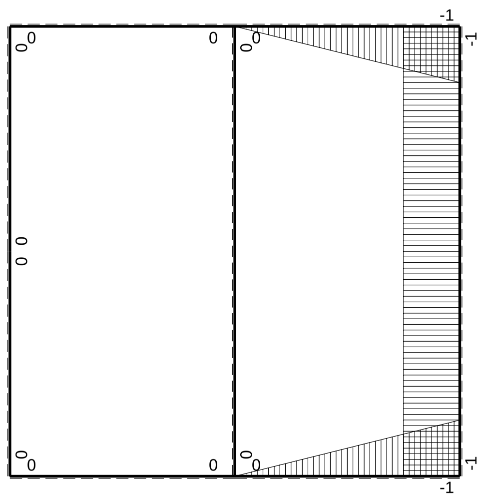

-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$3Fb-Fx-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-3Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

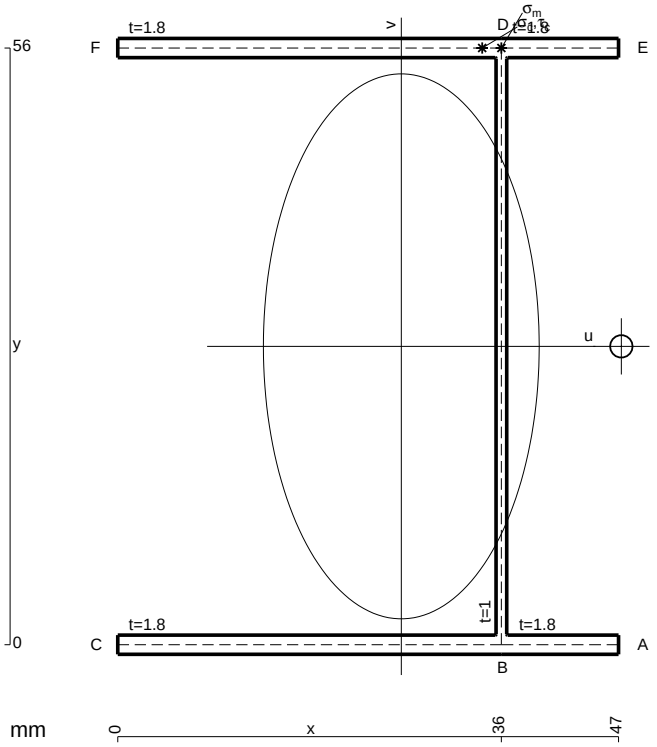
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

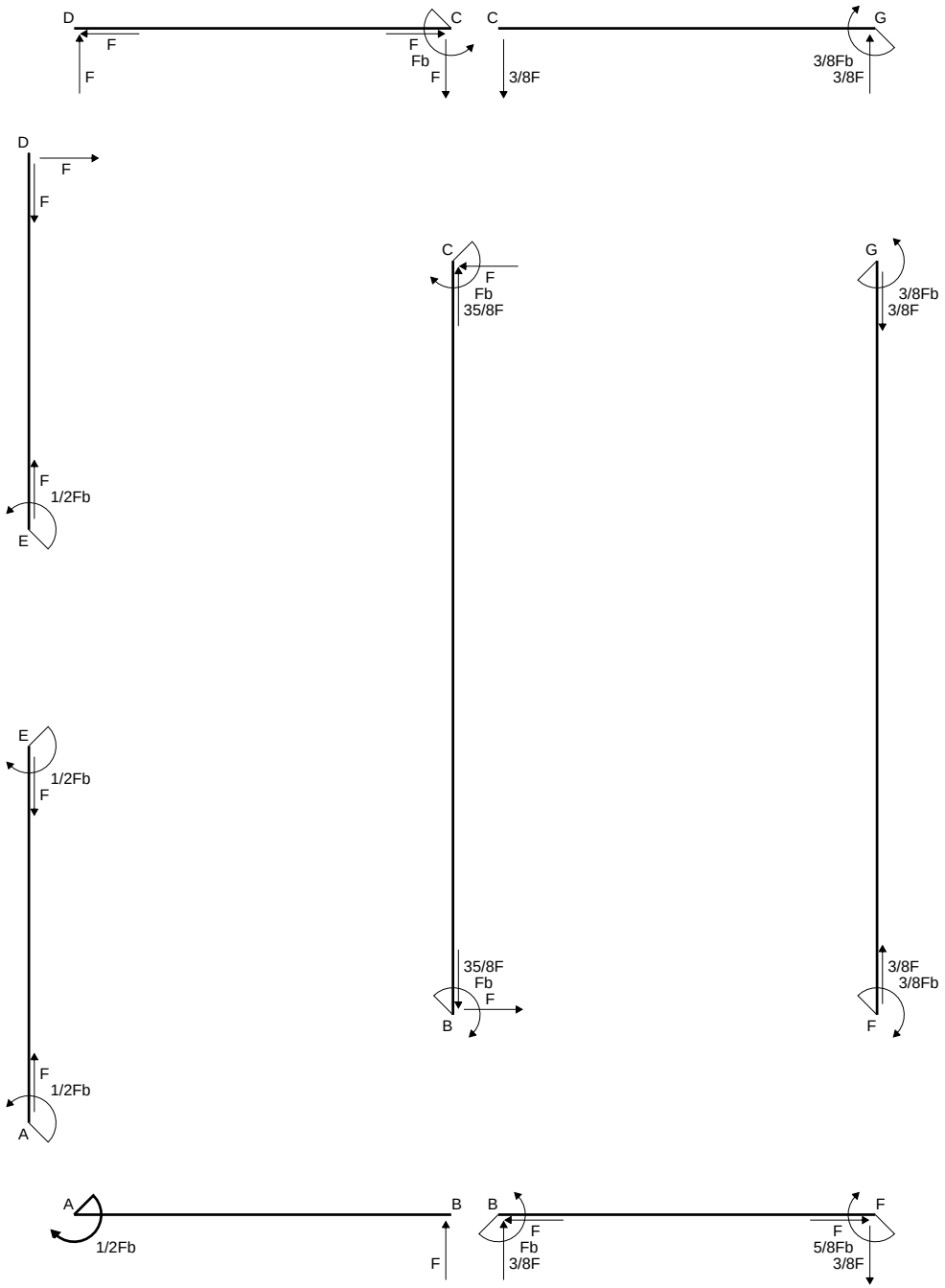
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

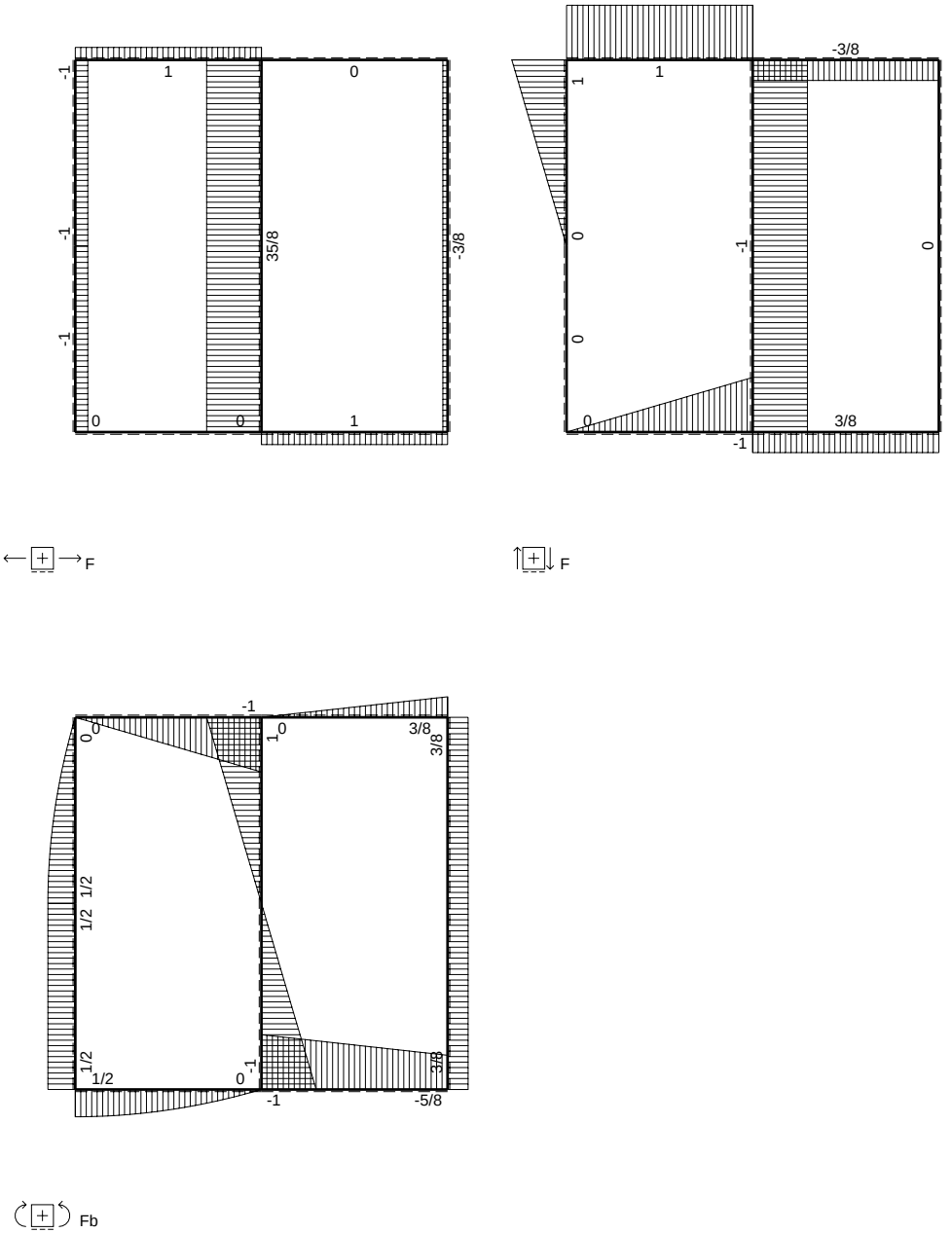


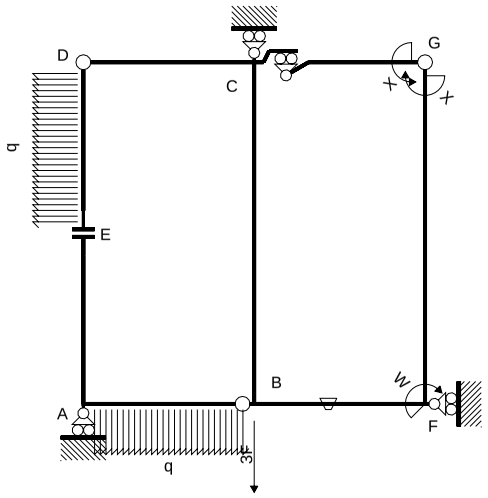
A = 225.2 mm^2
J_u = 147287. mm^4
J_v = 37721. mm^4
J_t = 201.4 mm^4
x_o = 20.65 mm
x_g = 26.61 mm
N = 520. N
T_y = 1560. N
M_x = -1092000. Nmm
x_m = 36. mm
y_m = 56. mm
u_m = 9.392 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 209.9 N/mm^2
x_c = 36. mm
y_c = 56. mm
u_c = 9.392 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 209.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 298.6 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.68 N/mm^2
$\tau_o = Tx_o t/J_t$ = 287.9 N/mm^2
t_c = 936. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 558.1 N/mm^2

CI08.xxxx.156

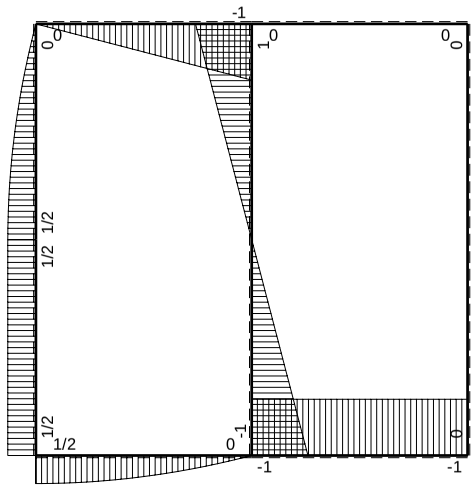
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
F
F
C
C
F
Fb
F
3/8F
G
3/8Fb
3/8F
D
F
F
F
1/2Fb
E
C
F
Fb
35/8F
G
3/8Fb
3/8F
E
1/2Fb
F
35/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
A
A
1/2Fb
B
F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F

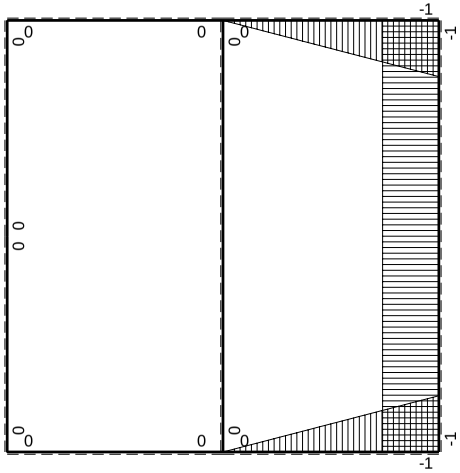

-1
1
0
-1
35/8
-3/8
-1
0
0
1
F
-3/8
1
1
0
-1
0
0
0
3/8
-1
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 239858 CI08.xxxx.157

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

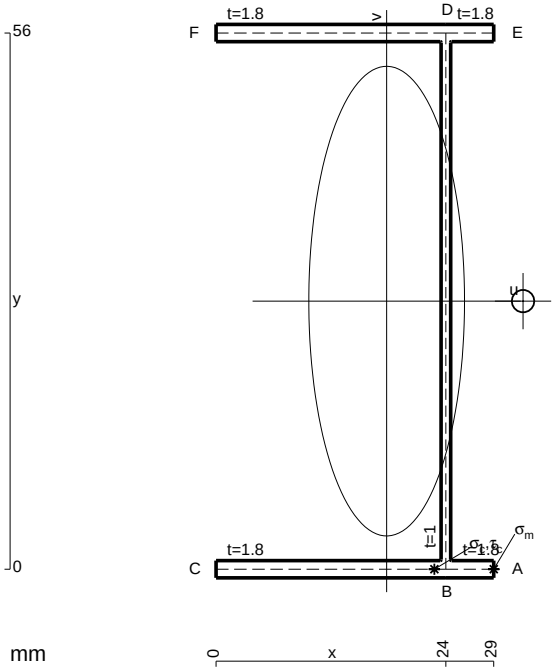
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

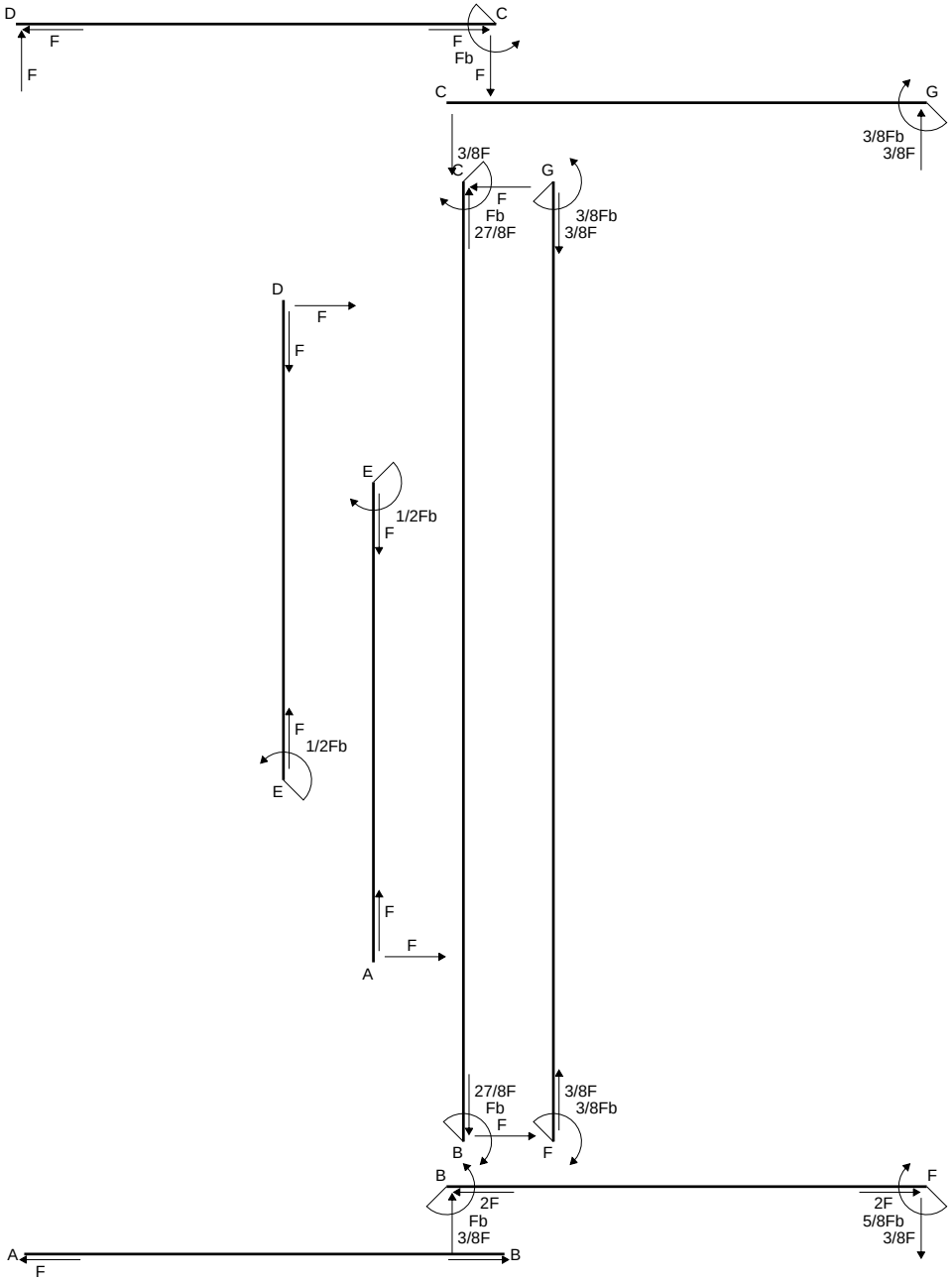
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

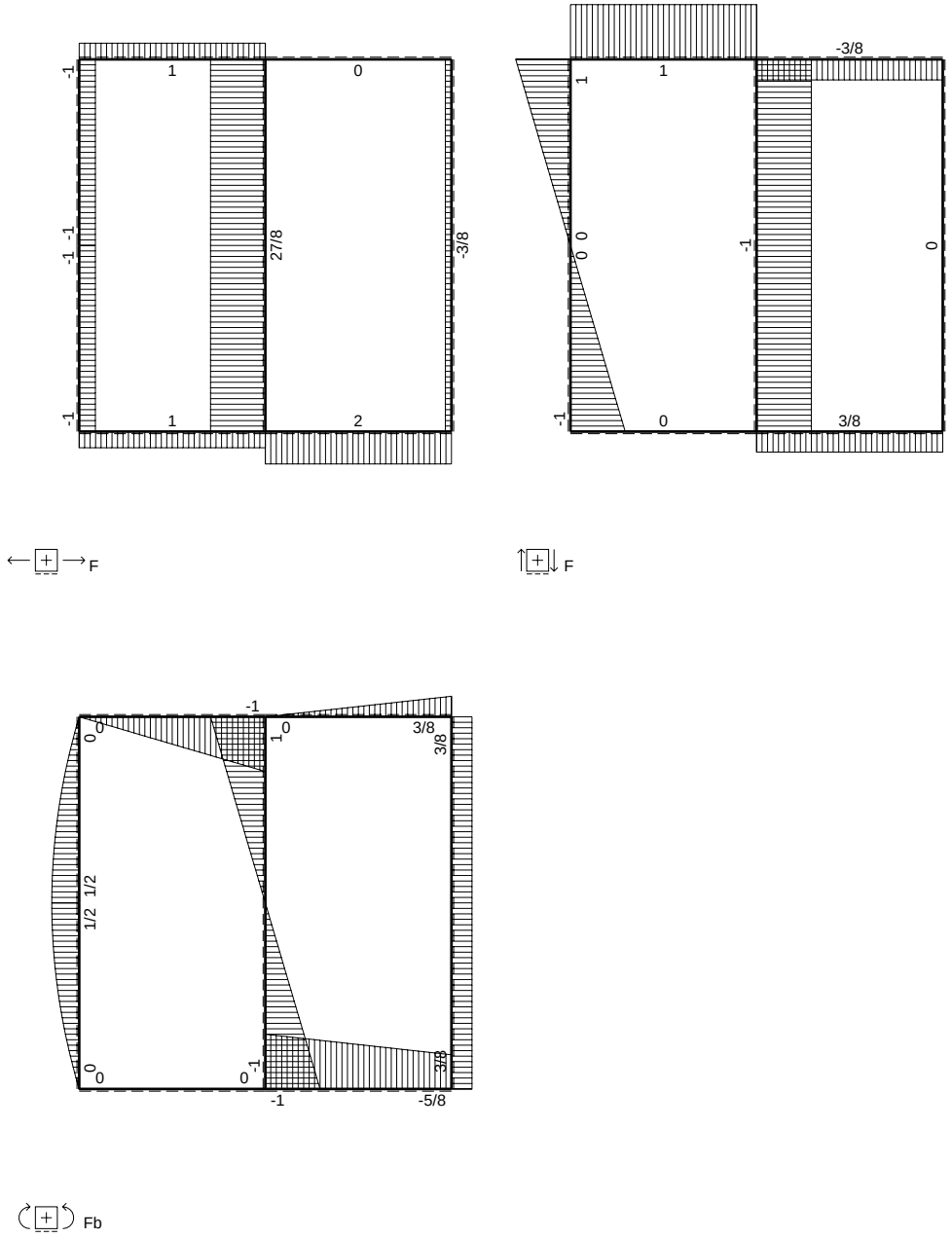


A = 160.4 mm^2
J_u = 96484. mm^4
J_v = 10606. mm^4
J_t = 131.4 mm^4
x_o = 14.24 mm
x_g = 17.82 mm
N = 3938. N
T_y = -900. N
M_x = 666000. Nmm
x_m = 29. mm
u_m = 11.18 mm
v_m = -28. mm
$\sigma_m = N/A - Mv/J_u$ = 217.8 N/mm^2
x_c = 24. mm
u_c = 6.183 mm
v_c = -28. mm
$\sigma_c = N/A - Mv/J_u$ = 217.8 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 181.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.268 N/mm^2
$\tau_o = Tx_o t/J_t$ = 175.6 N/mm^2
t_c = 1620. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 382.9 N/mm^2

CI08.xxxx.157

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
27/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
27/8F
Fb
F
3/8F
3/8Fb
B
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F


-1
1
0
-1
-1
27/8
-3/8
-1
1
2
F
-3/8
1
1
0
0
-1
0
-1
0
3/8
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
0
0
0
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3XbJ/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbJ/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbJ/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb²/EJ	8/3XbJ/EJ
	iperstatica $X=W_{GC}$							-3/8Fb

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

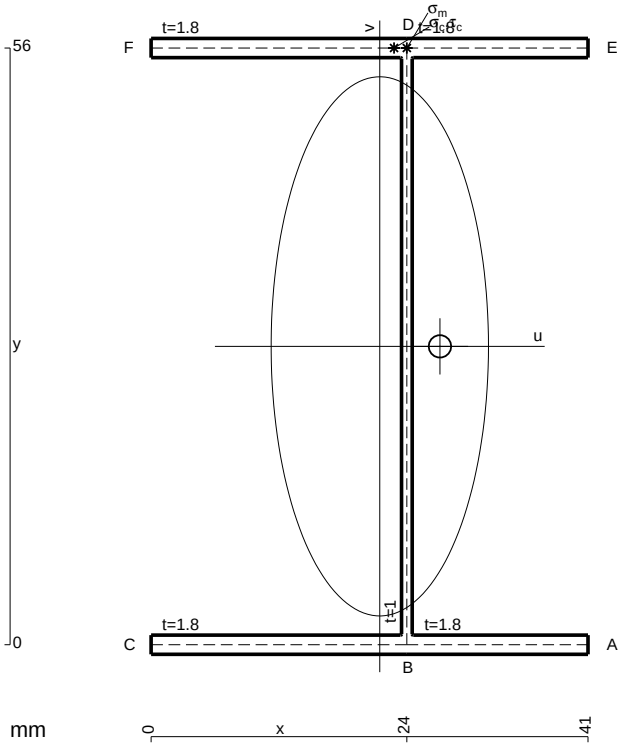
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

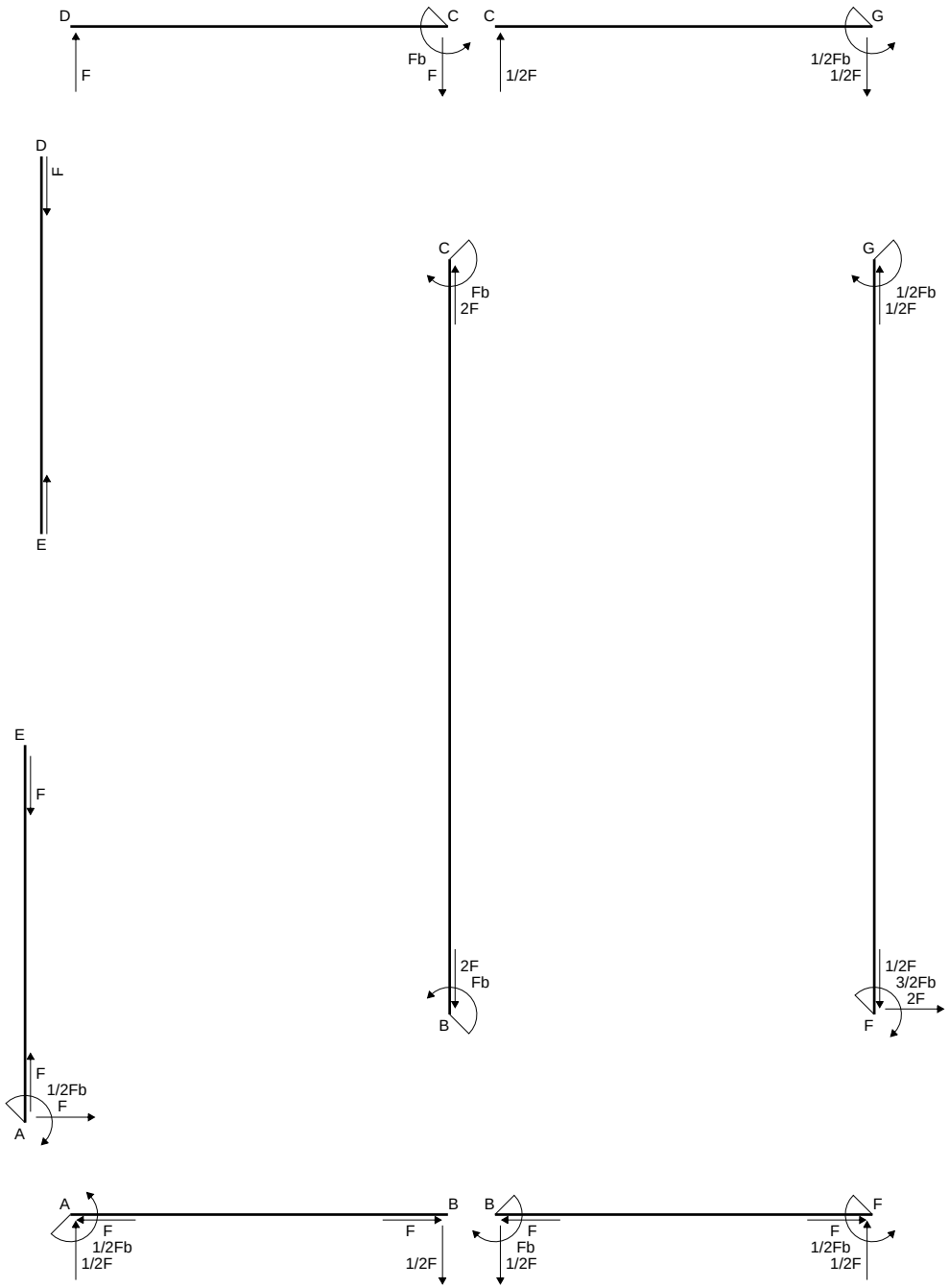
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

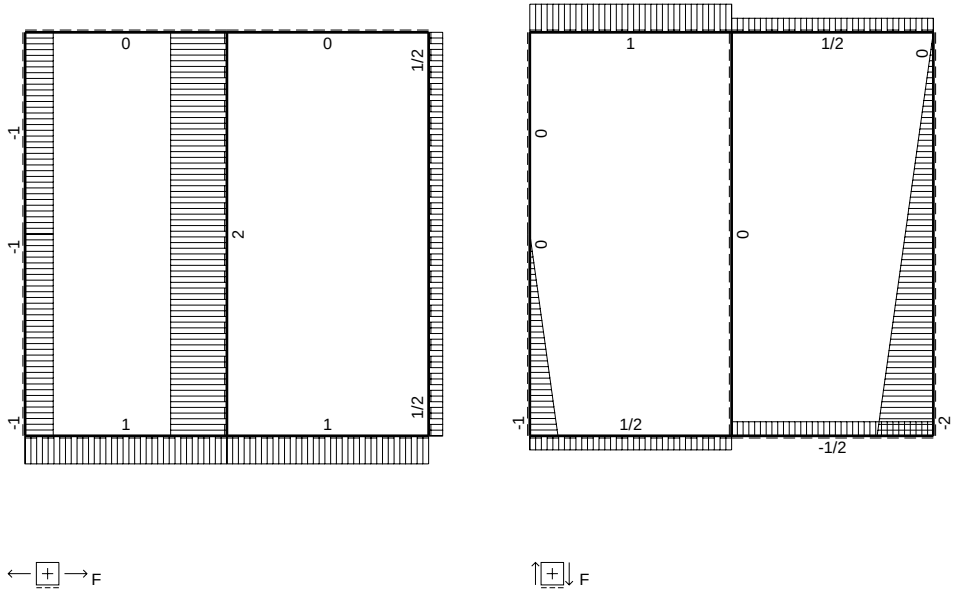


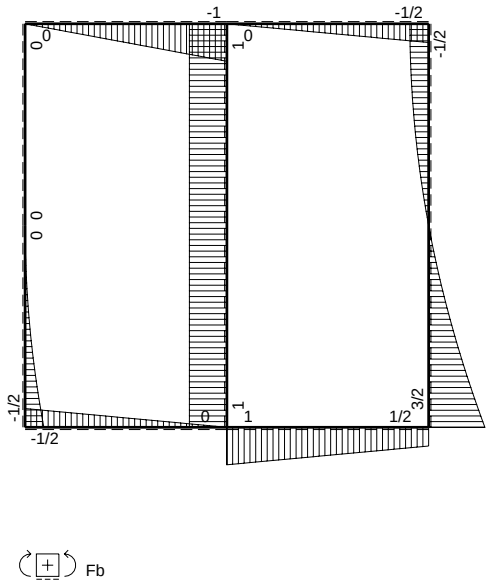
A = 203.6 mm^2
J_u = 130353. mm^4
J_v = 21174. mm^4
J_t = 178.1 mm^4
x_o = 5.644 mm
x_g = 21.46 mm
N = 4151. N
T_y = -1230. N
M_x = -971700. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 2.537 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 229.1 N/mm^2
x_c = 24. mm
y_c = 56. mm
u_c = 2.537 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 229.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 76.52 N/mm^2
$\tau_g = TS^*/tJ_u$ = 6.341 N/mm^2
$\tau_o = Tx_o t/J_t$ = 70.18 N/mm^2
t_c = 2214. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 264.7 N/mm^2

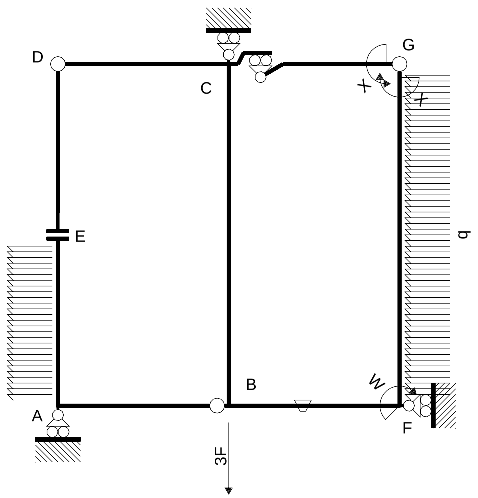
CI08.xxxx.158

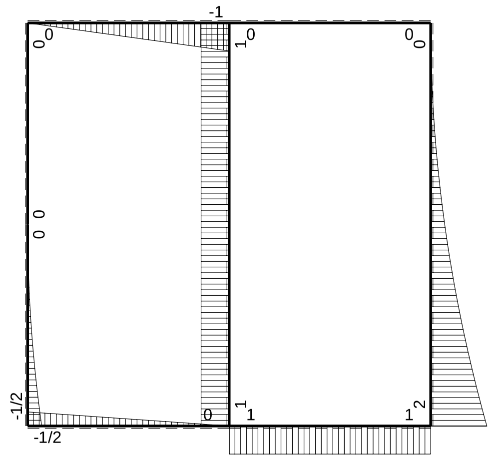
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
1/2F
1/2Fb
1/2F
D
F
E
C
Fb
2F
G
1/2Fb
1/2F
E
F
F
1/2Fb
F
A
2F
Fb
B
1/2F
3/2Fb
2F
F
A
B
B
F
F
F
1/2Fb
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F

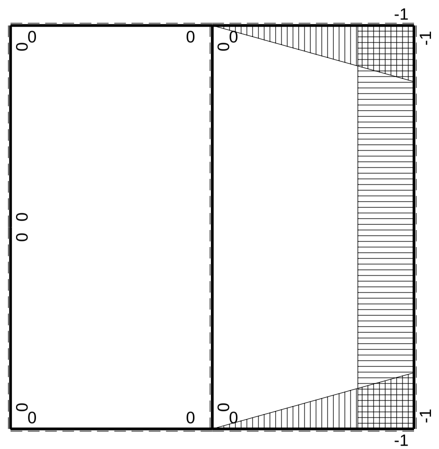

0
0
1/2
-1
-1
2
-1
1
1
1/2
F
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F


-1
-1/2
0
0
1
0
-1/2
0 0
0 0
-1/2
0
1
1
1/2
3/2
-1/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

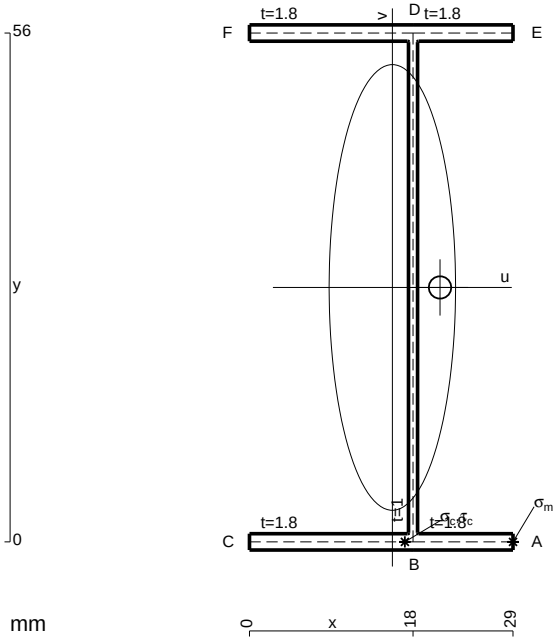
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

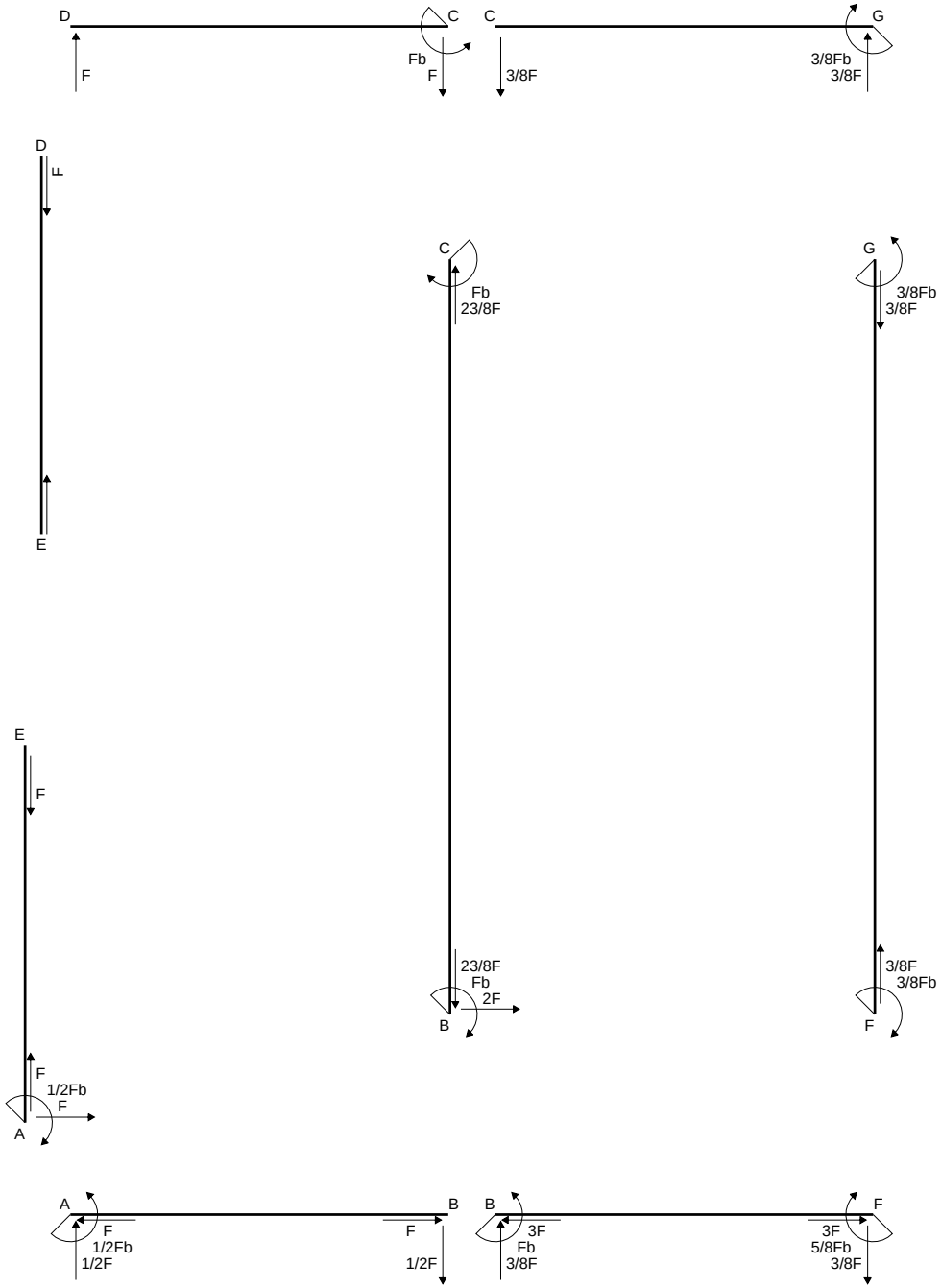
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

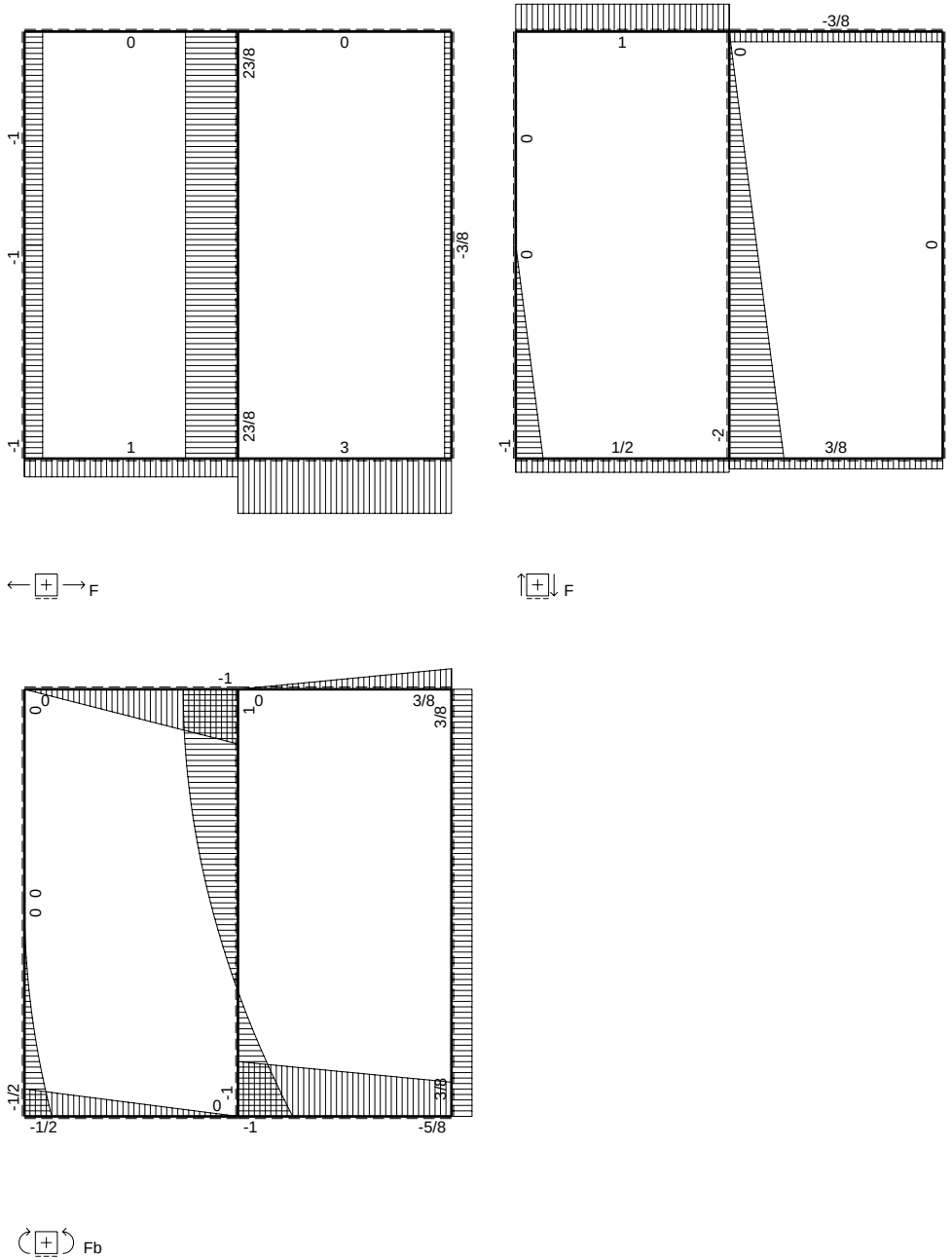


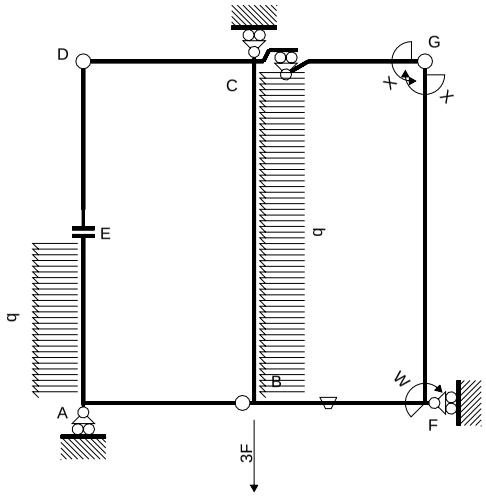
A = 160.4 mm^2
J_u = 96484. mm^4
J_v = 7763. mm^4
J_t = 131.4 mm^4
x_o = 5.247 mm
x_g = 15.72 mm
T_y = 990. N
M_x = -821700. Nmm
x_m = 29. mm
u_m = 13.28 mm
v_m = -28. mm
$\sigma_m = -Mv/J_u$ = -238.5 N/mm^2
x_c = 18. mm
u_c = 2.278 mm
v_c = -28. mm
$\sigma_c = -Mv/J_u$ = -238.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 76.32 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.171 N/mm^2
$\tau_o = Tx_o t/J_t$ = 71.15 N/mm^2
t_c = 1782. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 272.6 N/mm^2

CI08.xxxx.159

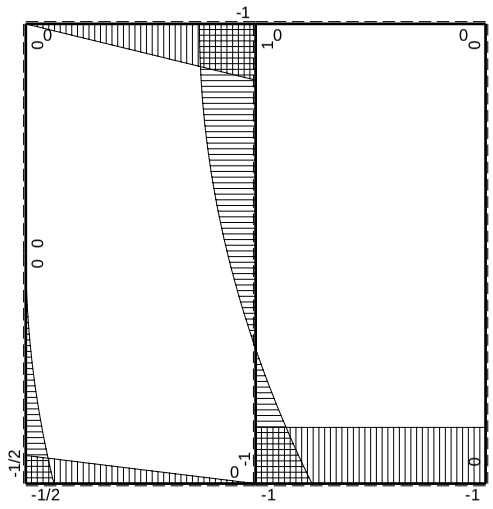
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
23/8F
G
3/8Fb
3/8F
E
F
23/8F
Fb
2F
B
3/8F
3/8Fb
F
F
1/2Fb
F
A
A
F
1/2Fb
1/2F
B
F
1/2F
B
3F
Fb
3/8F
F
3F
5/8Fb
3/8F

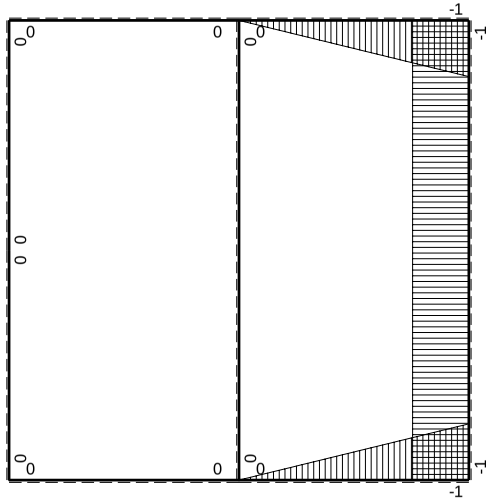

0
0
23/8
-1
-1
-3/8
-1
1
23/8
3
F
1
-3/8
0
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212582 CI08.xxxx.160

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

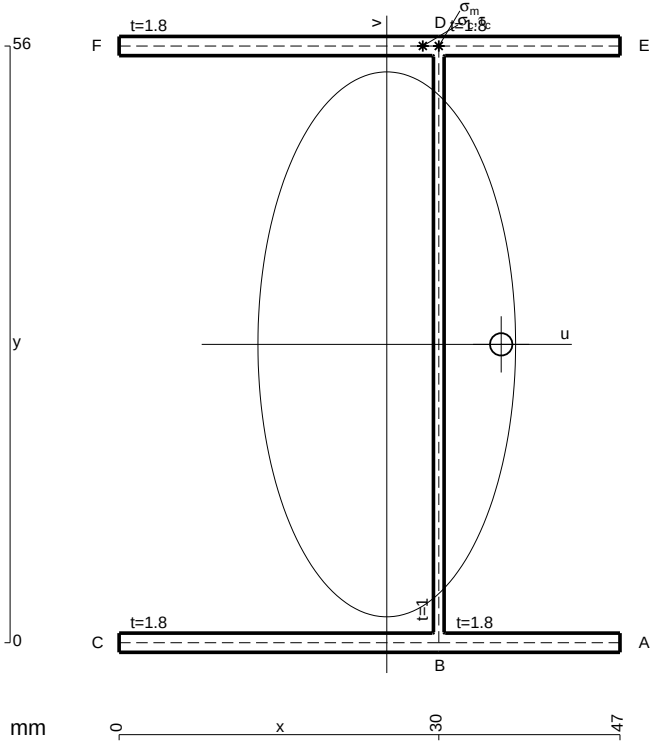
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

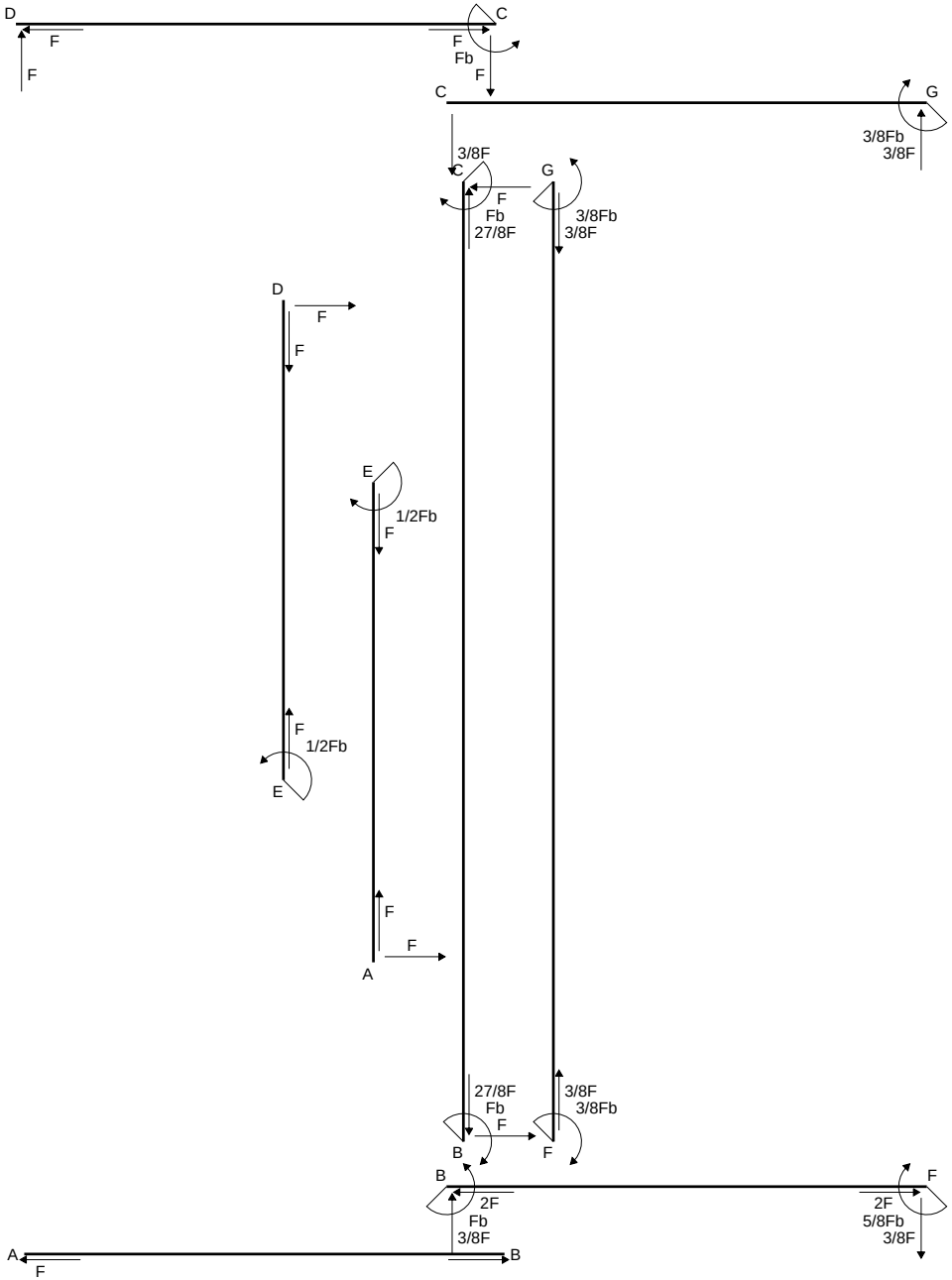
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

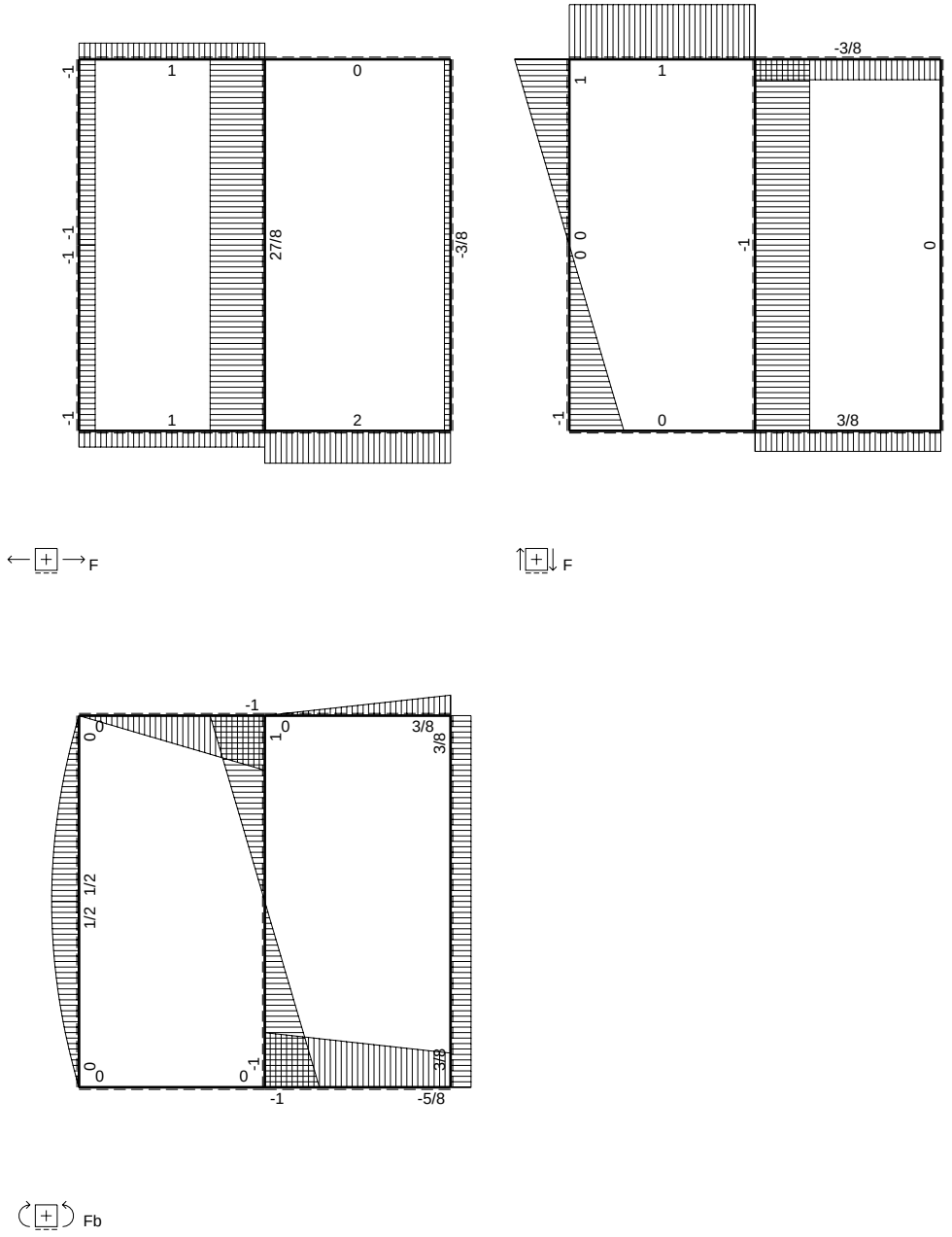


A = 225.2 mm^2
J_u = 147287. mm^4
J_v = 32925. mm^4
J_t = 201.4 mm^4
x_o = 10.74 mm
x_g = 25.12 mm
N = 5951. N
T_y = -4140. N
M_x = -910800. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 4.884 mm
v_m = 28. mm
$\sigma_m = N/A - Mv/J_u$ = 199.6 N/mm^2
x_c = 30. mm
y_c = 56. mm
u_c = 4.884 mm
v_c = 28. mm
$\sigma_c = N/A - Mv/J_u$ = 199.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 420.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 23.61 N/mm^2
$\tau_o = Tx_o t/J_t$ = 397.3 N/mm^2
t_c = 3726. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 755.9 N/mm^2

CI08.xxxx.160

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
F
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
F
Fb
27/8F
3/8Fb
3/8F
D
F
F
E
1/2Fb
F
F
1/2Fb
E
F
F
A
27/8F
Fb
F
3/8F
3/8Fb
B
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F


-1
1
0
-1
-1
27/8
-3/8
-1
1
2
F
-3/8
1
1
0
0
-1
0
-1
0
3/8
F
-1
0
0
1
0
3/8
3/8
1/2
1/2
0
0
0
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

PROCEDIMENTO E RISULTATI 239499 CI08.xxxx.161

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

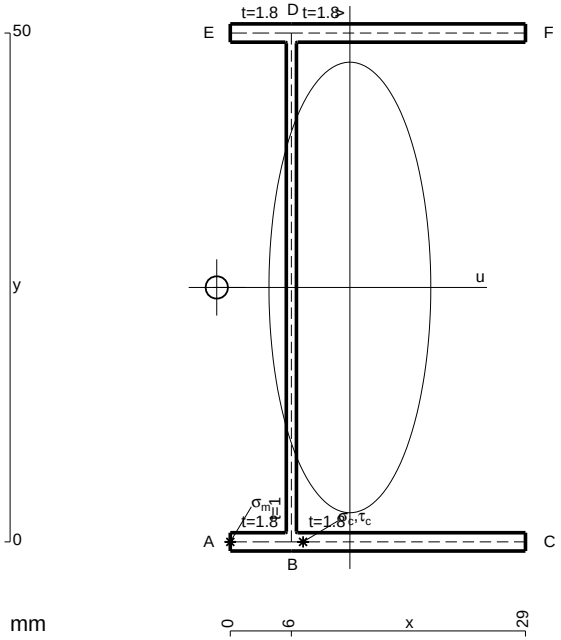
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b]_o^b\ \theta$

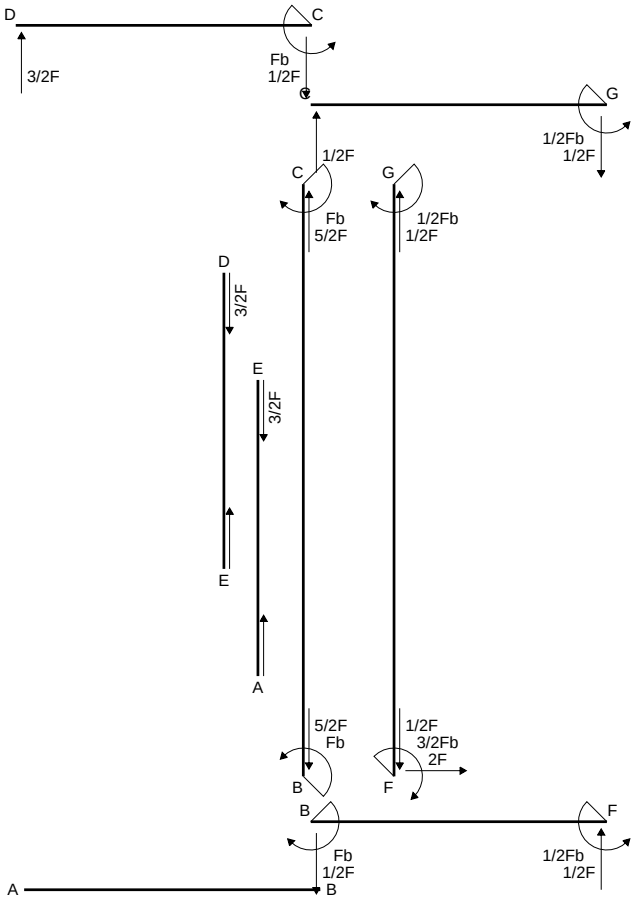
$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ =\ Fb^2/EJ$

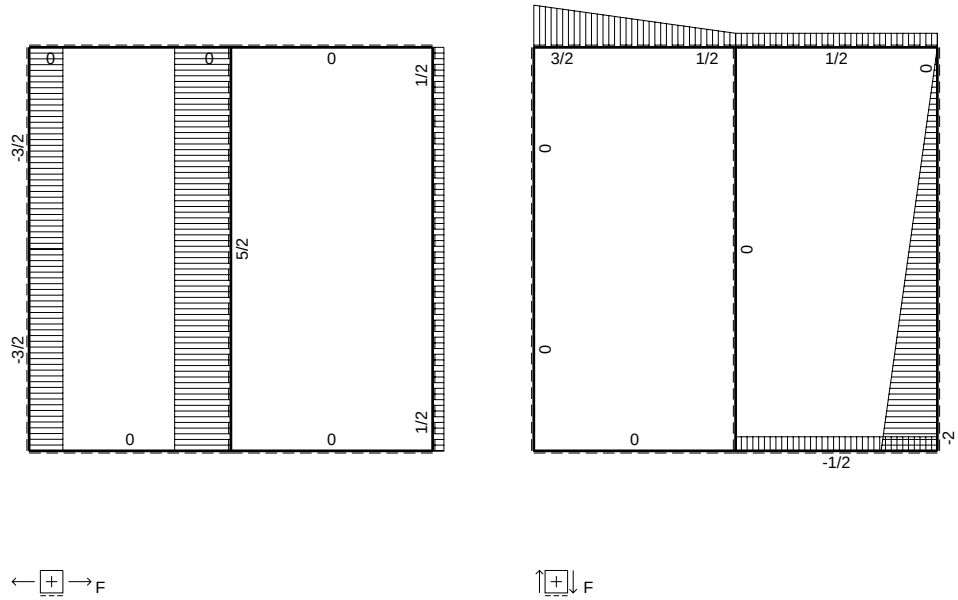


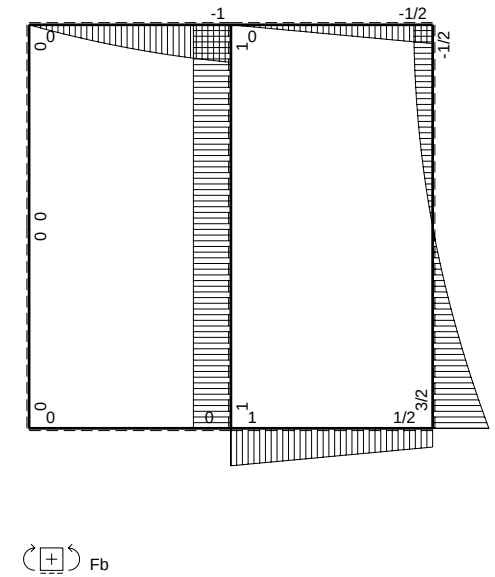
A = 154.4 mm^2
J_u = 75667. mm^4
J_v = 9759. mm^4
J_t = 129.4 mm^4
x_o = -13.08 mm
x_g = 11.75 mm
N = 4219. N
T_y = -1250. N
M_x = 550000. Nmm
u_m = -11.75 mm
v_m = -25. mm
$\sigma_m = N/A - Mv/J_u$ = 209. N/mm^2
x_c = 6. mm
u_c = -5.747 mm
v_c = -25. mm
$\sigma_c = N/A - Mv/J_u$ = 209. N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 236.9 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.499 N/mm^2
$\tau_o = Tx_o t/J_t$ = 227.4 N/mm^2
t_c = 2250. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 460.4 N/mm^2

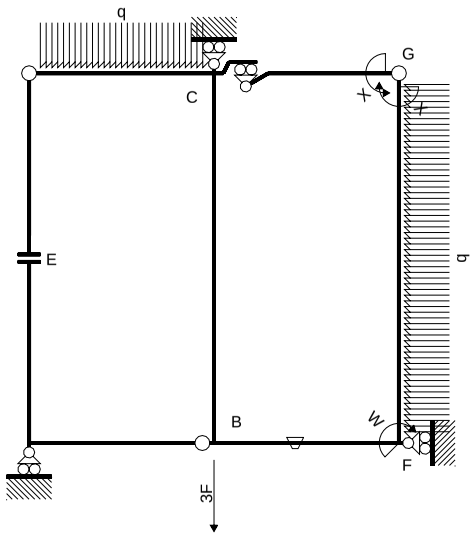
CI08.xxxx.161

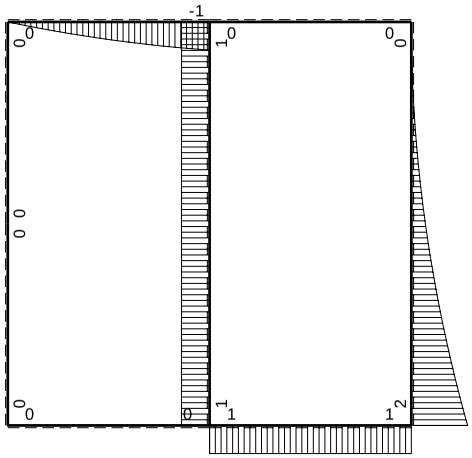
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3/2F
Fb
1/2F
G
1/2Fb
1/2F
1/2F
C
G
Fb
5/2F
1/2Fb
1/2F
D
3/2F
E
3/2F
E
A
5/2F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B

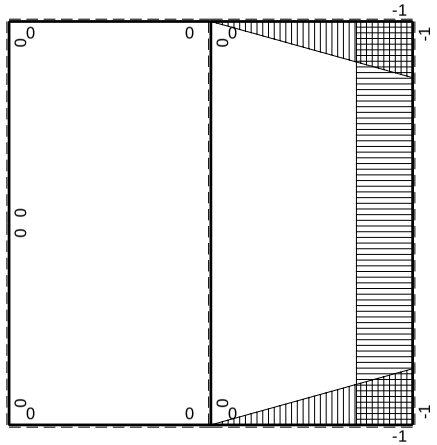

0
0
0
1/2
-3/2
5/2
-3/2
0
0
1/2
3/2
1/2
1/2
0
0
0
0
-2
-1/2
F
F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
1
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+1/2Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	3/2Fx-1/2qx^2	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						-4/3Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 240788 CI08.xxxx.162

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

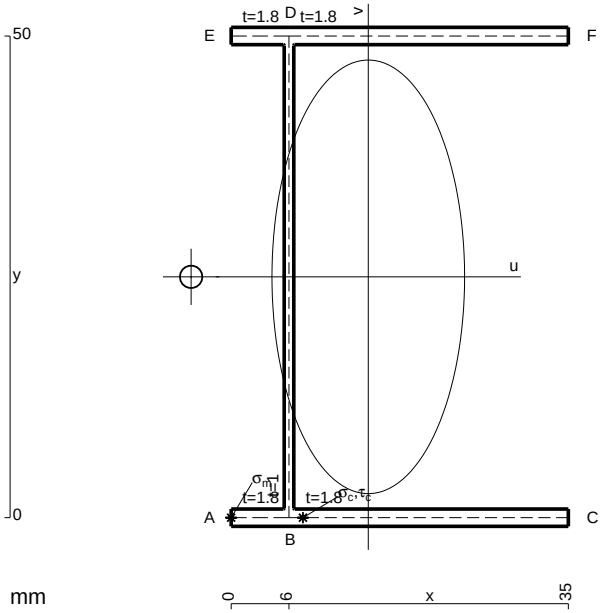
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

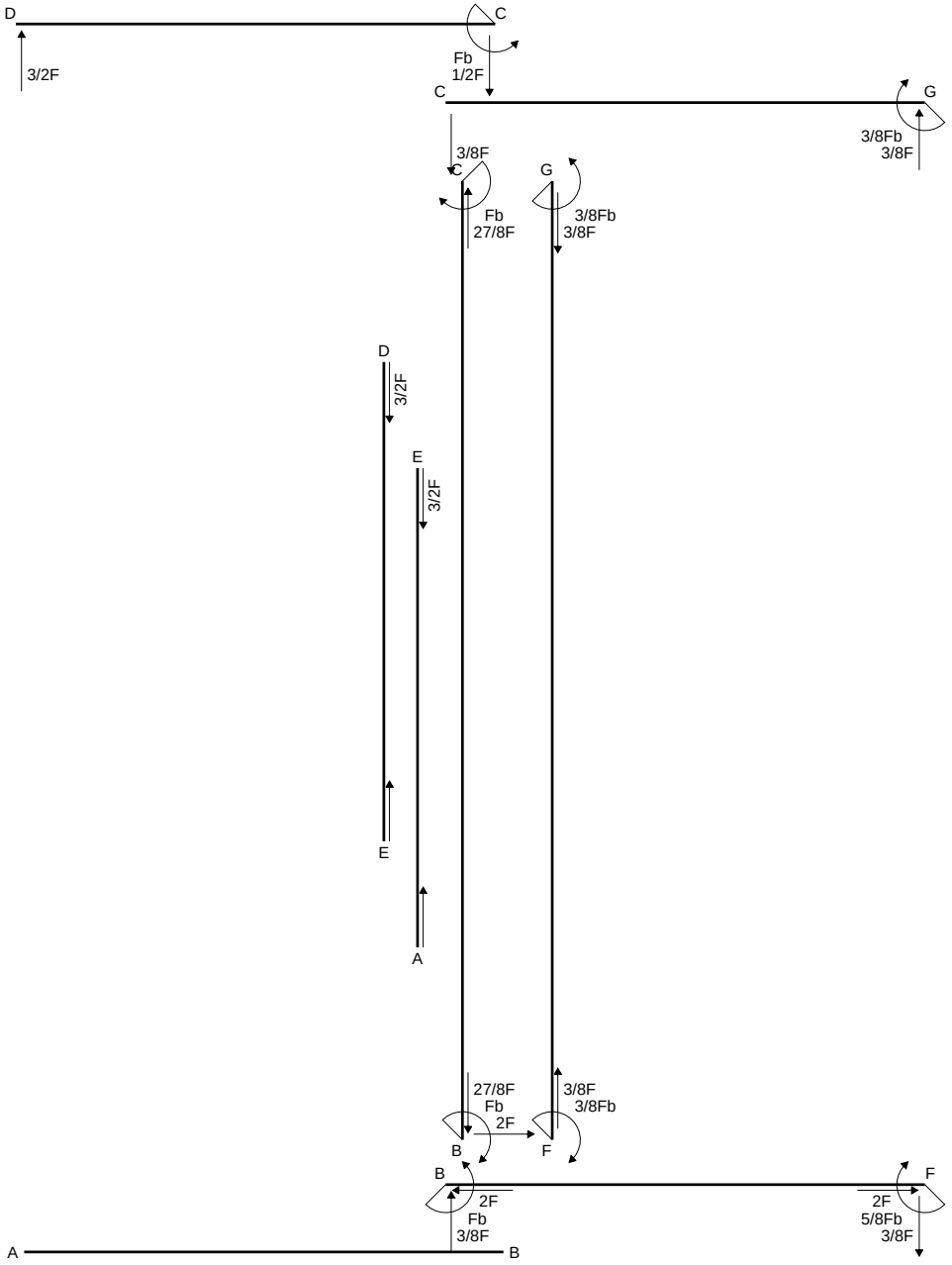
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

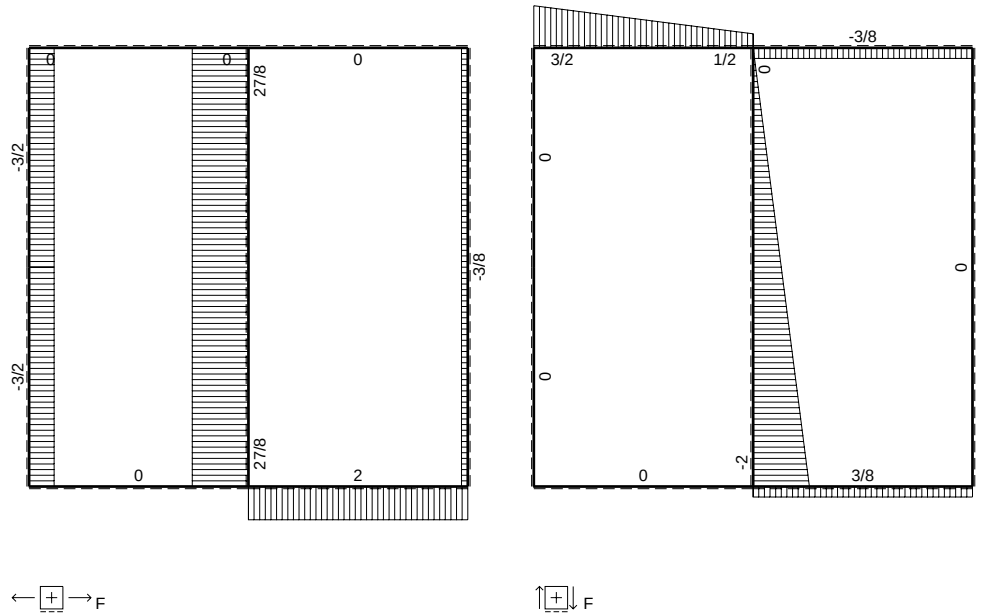


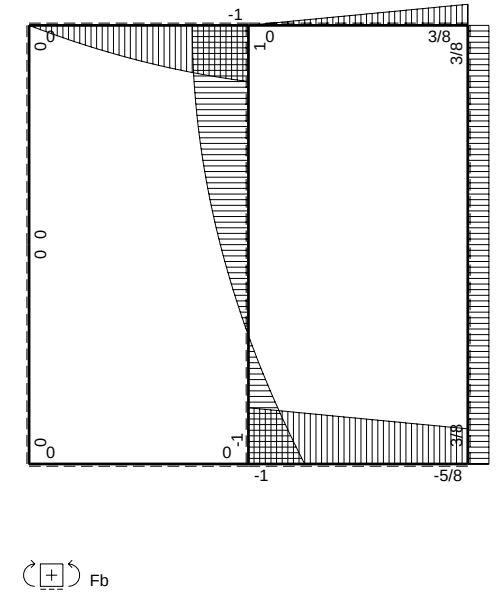
A = 176. mm^2
J_u = 89167. mm^4
J_v = 17596. mm^4
J_t = 152.7 mm^4
x_o = -18.39 mm
x_g = 14.23 mm
T_y = 2610. N
M_x = -783000. Nmm
u_m = -14.23 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = -219.5 N/mm^2
x_c = 6. mm
u_c = -8.233 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = -219.5 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 586.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 21.22 N/mm^2
$\tau_o = Tx_o t/J_t$ = 565.6 N/mm^2
t_c = 4698. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1040. N/mm^2

CI08.xxxx.162

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
3/2F
Fb
1/2F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
27/8F
3/8Fb
3/8F
D
3/2F
E
3/2F
E
A
27/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B


0
0
0
27/8
-3/2
-3/8
-3/2
27/8
0
2
F
-3/8
3/2
1/2
0
0
0
-2
0
3/8
F


-1
0
0
1
0
3/8
3/8
0
0
0
0
0
-1
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_o flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 215365 CI08.xxxx.163

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

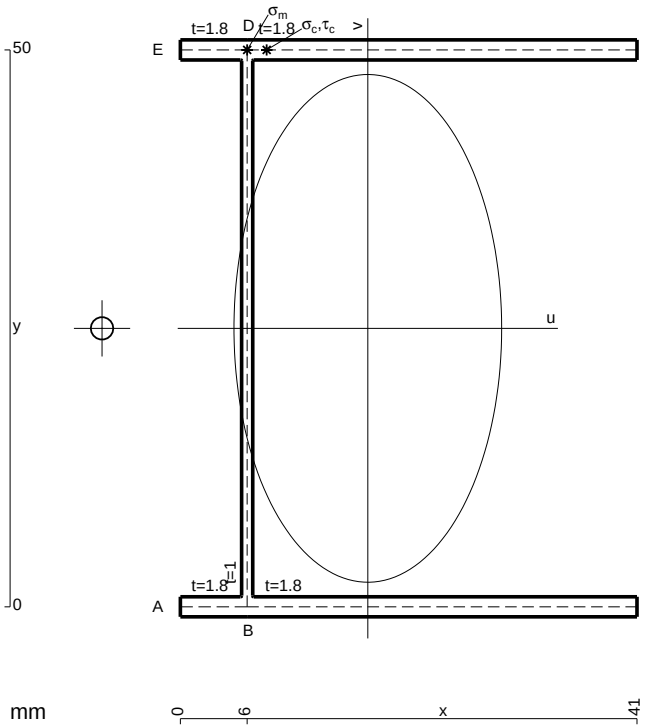
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

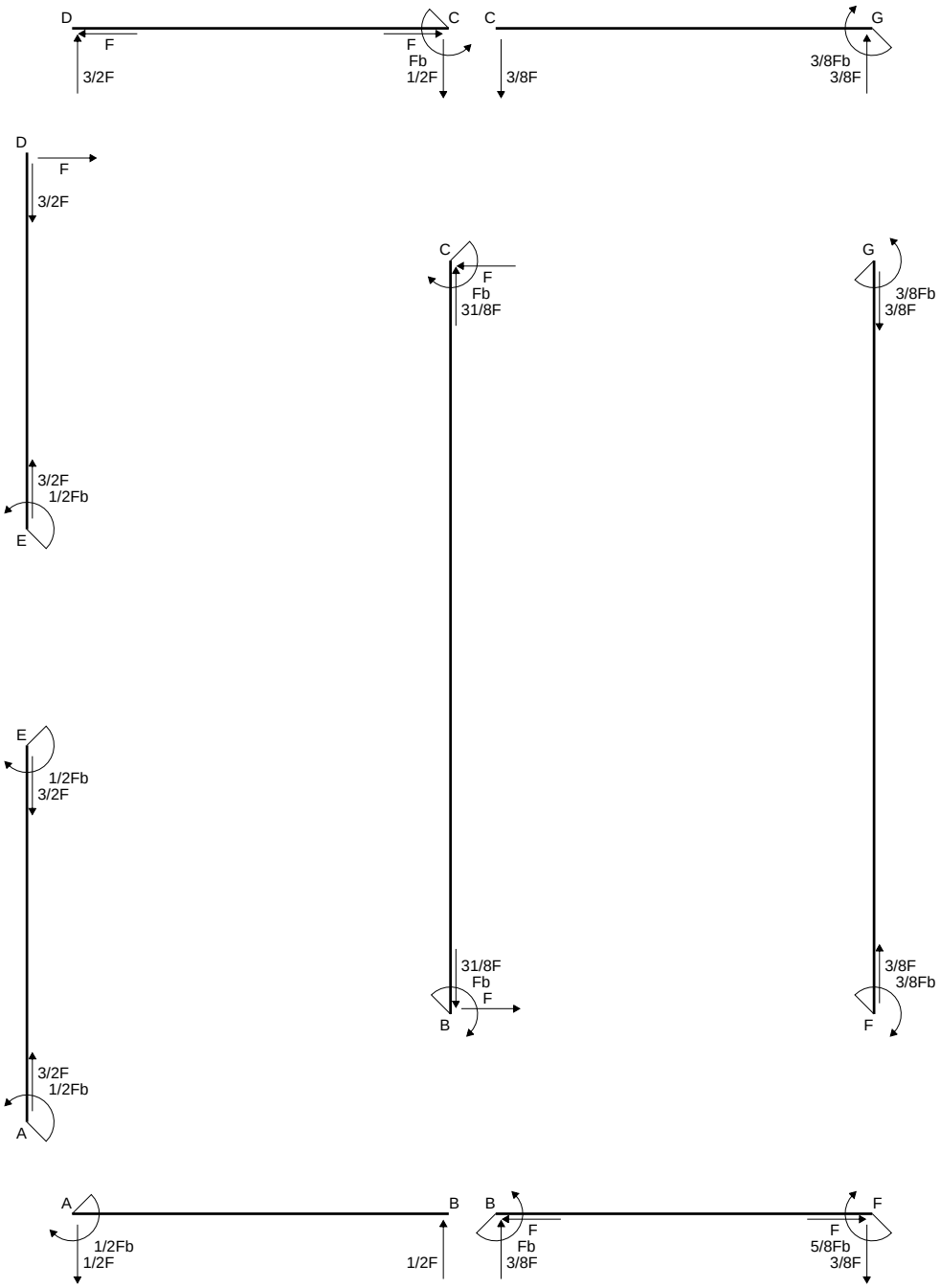
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

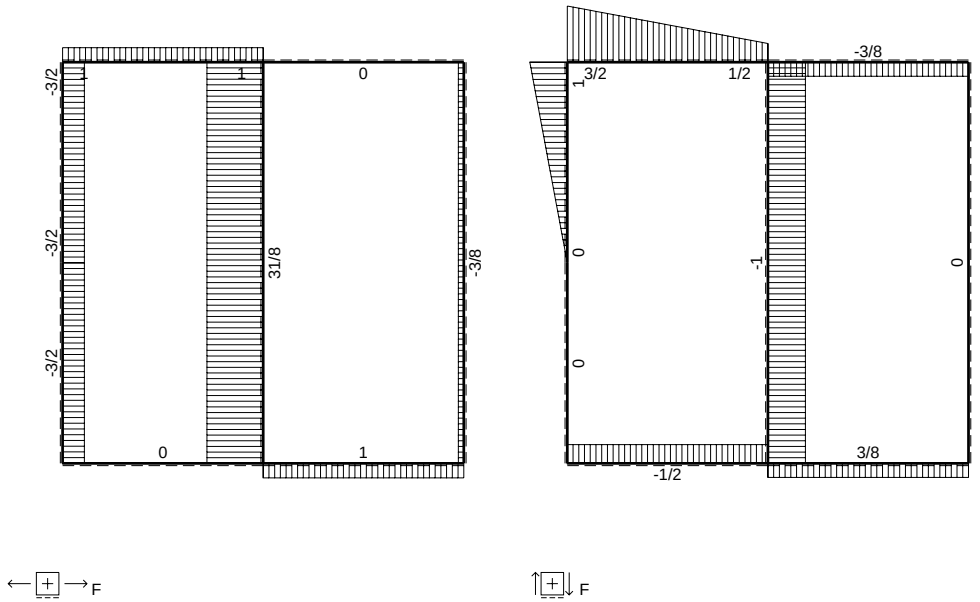


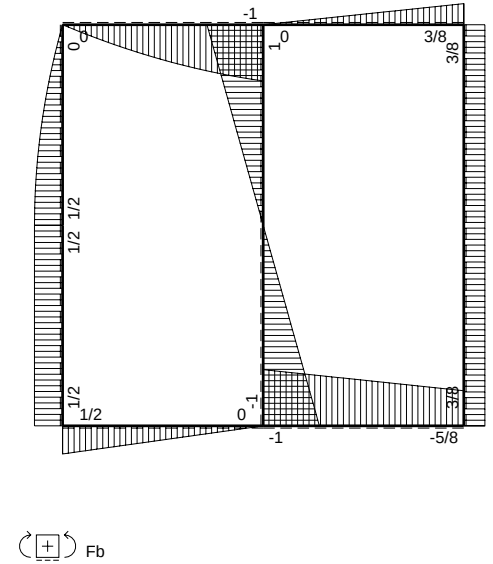
A = 197.6 mm^2
J_u = 102667. mm^4
J_v = 28529. mm^4
J_t = 176.1 mm^4
x_o = -23.86 mm
x_g = 16.83 mm
N = 5400. N
T_y = -3200. N
M_x = -832000. Nmm
x_m = 6. mm
y_m = 50. mm
u_m = -10.83 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 229.9 N/mm^2
x_c = 6. mm
y_c = 50. mm
u_c = -10.83 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 229.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 807.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 27.27 N/mm^2
$\tau_o = Tx_o t/J_t$ = 780.5 N/mm^2
t_c = 2880. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 1418. N/mm^2

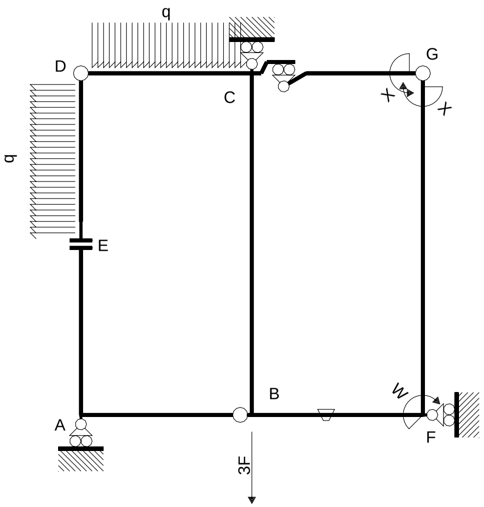
CI08.xxxx.163

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

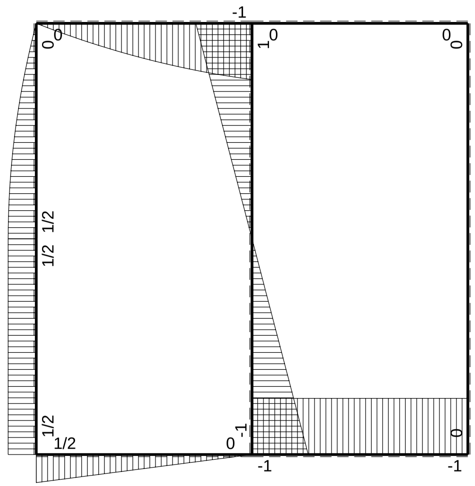

D
F
3/2F
C
F
Fb
1/2F
C
3/8F
G
3/8Fb
3/8F
D
F
3/2F
3/2F
1/2Fb
E
C
F
Fb
31/8F
G
3/8Fb
3/8F
E
1/2Fb
3/2F
31/8F
Fb
F
B
3/8F
3/8Fb
F
3/2F
1/2Fb
A
A
1/2Fb
1/2F
B
1/2F
B
F
Fb
3/8F
F
F
5/8Fb
3/8F


-3/2
-3/2
-3/2
1
1
0
31/8
-3/8
0
1
F
-3/8
1
3/2
1/2
0
0
-1
0
3/8
-1/2
F

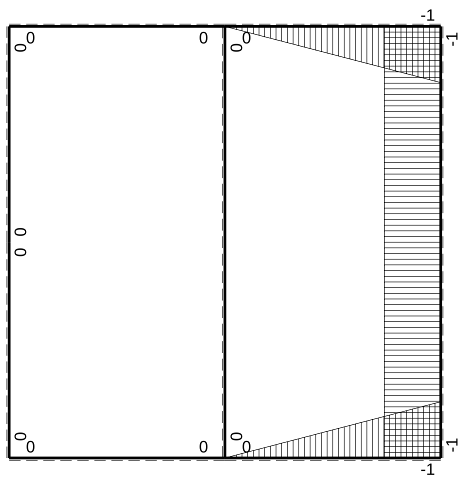

-1
0
0
1
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
-1
-1
3/8
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

↓	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

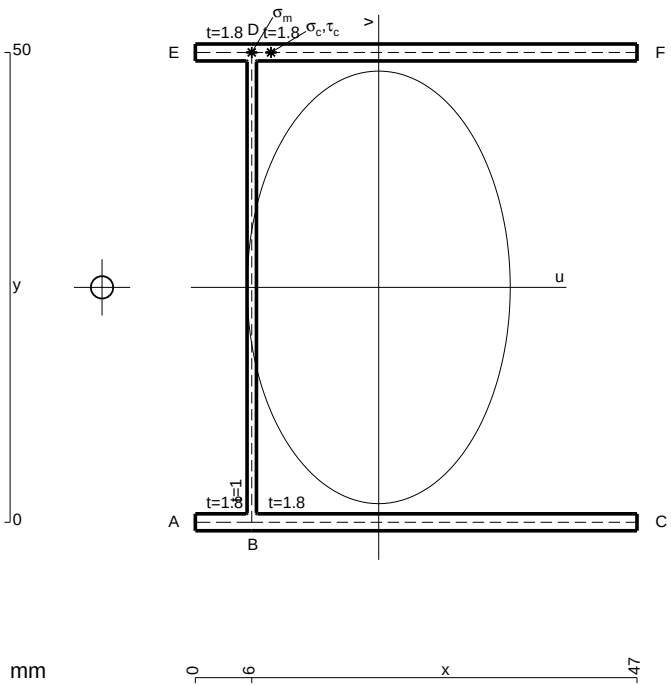
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

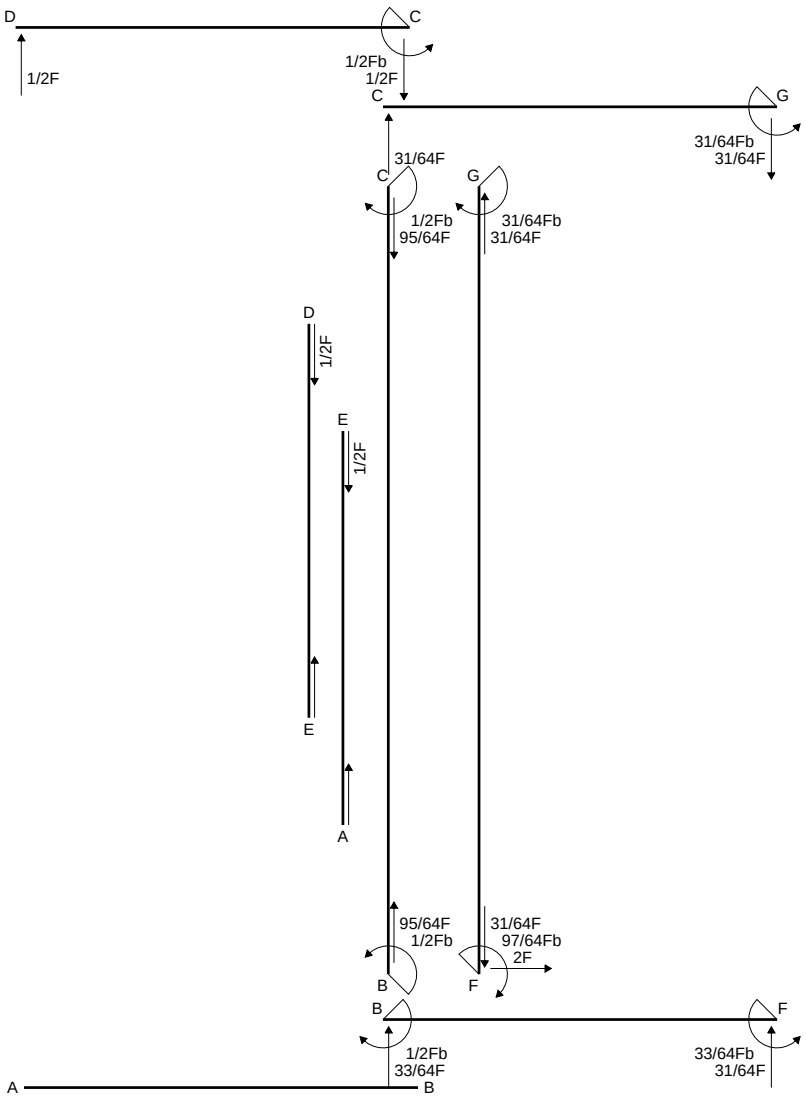
$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

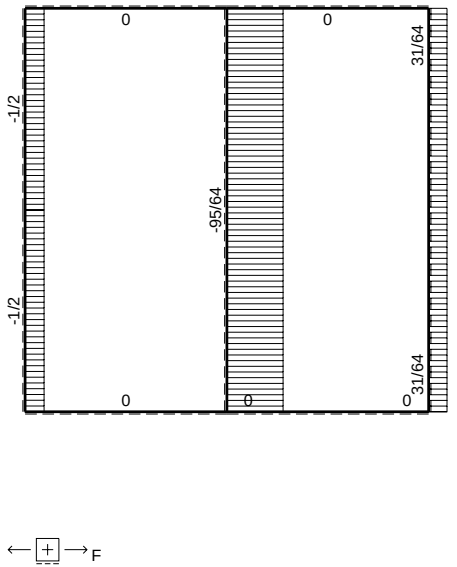


A = 219.2 mm^2
J_u = 116167. mm^4
J_v = 42967. mm^4
J_t = 199.4 mm^4
x_o = -29.44 mm
x_g = 19.51 mm
N = 6704. N
T_y = -1730. N
M_x = -968800. Nmm
x_m = 6. mm
y_m = 50. mm
u_m = -13.51 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 239.1 N/mm^2
x_c = 6. mm
y_c = 50. mm
u_c = -13.51 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 239.1 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 475. N/mm^2
$\tau_g = TS^*/tJ_u$ = 15.26 N/mm^2
$\tau_o = Tx_o t/J_t$ = 459.7 N/mm^2
t_c = 3114. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 856.8 N/mm^2

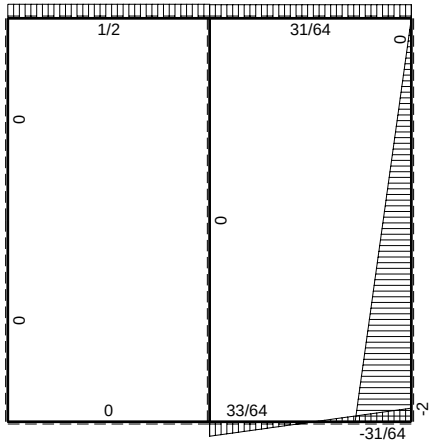
CI08.xxxx.164

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

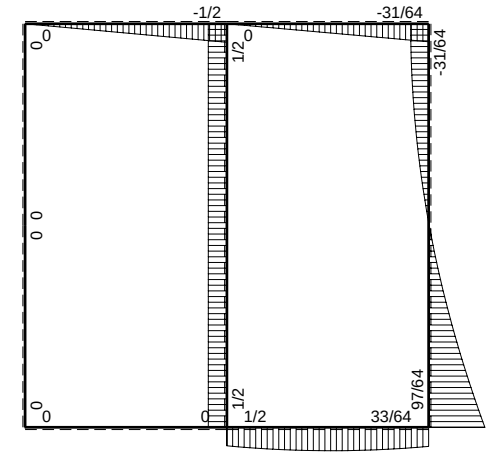

D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
95/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
95/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
-95/64
-1/2
0
0
0
31/64

F


1/2
31/64
0
0
0
0
0
33/64
-2
-31/64

F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
0
1/2
33/64
97/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+Fx-1/2qx^2	-Fb/EJ	-1/2Fx-Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(-11/24+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2qx^2	Fb/EJ	-Fb+Fx+1/2Fx^2/b-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 215513 CI08.xxxx.165

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

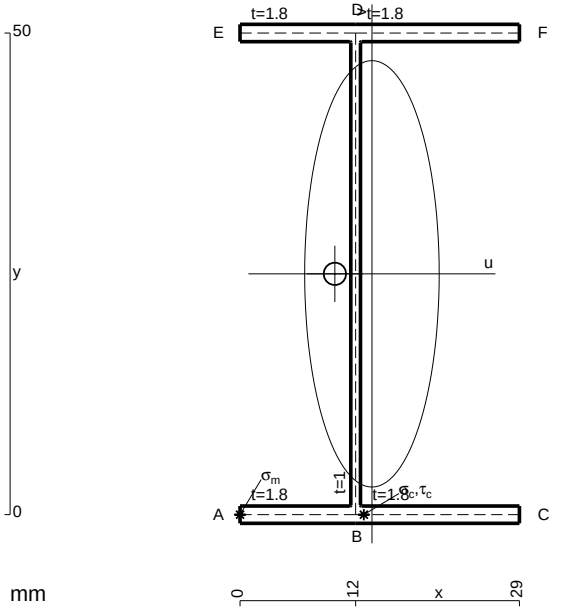
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

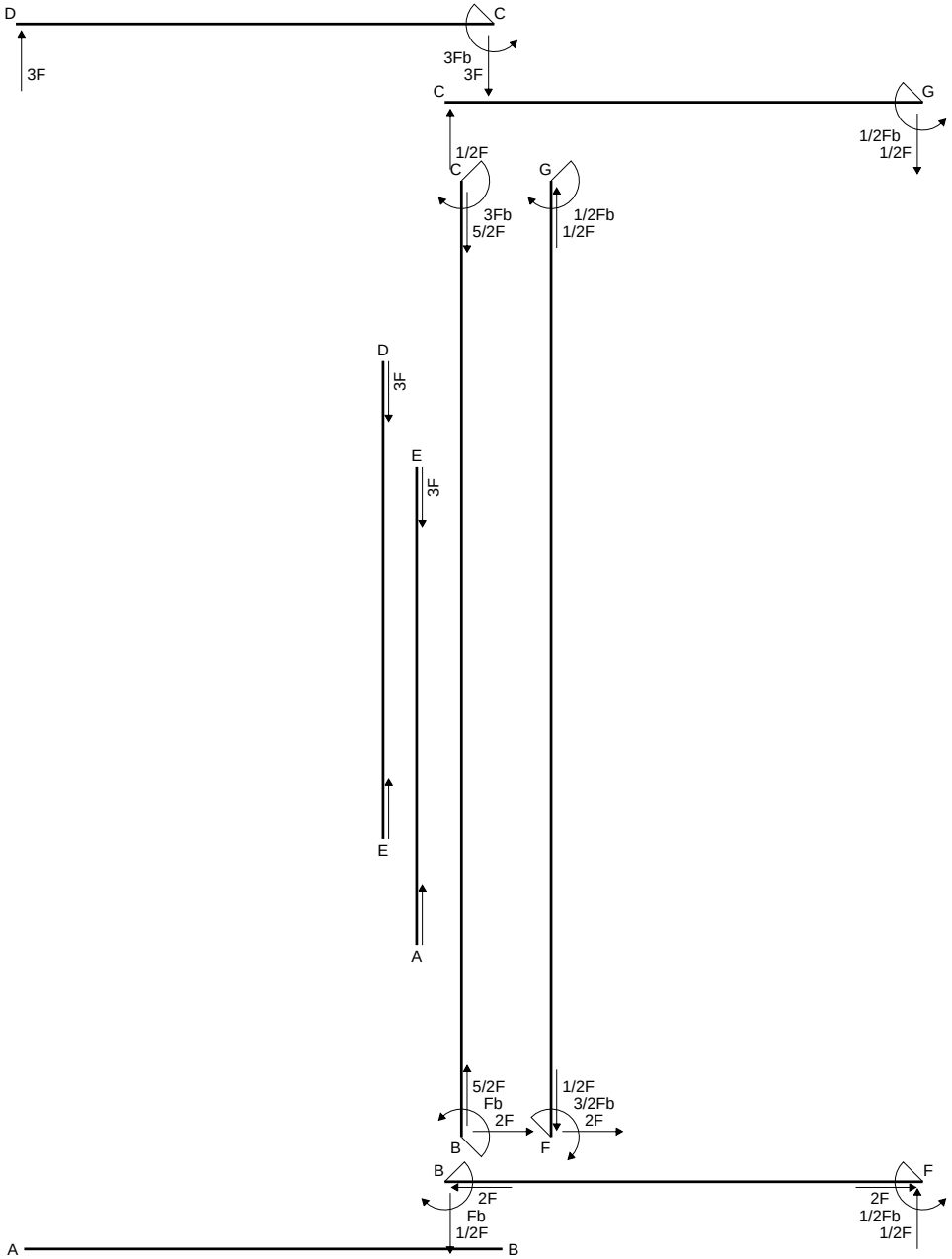
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

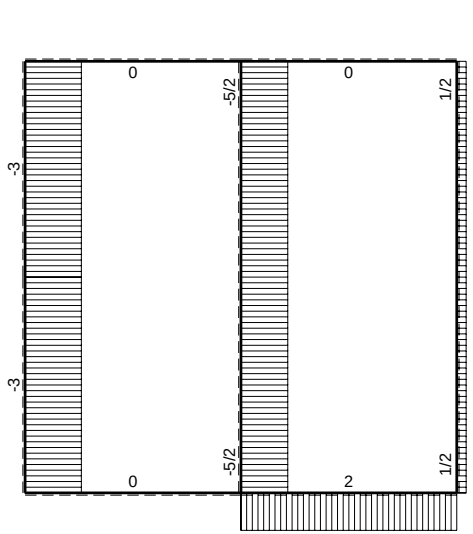


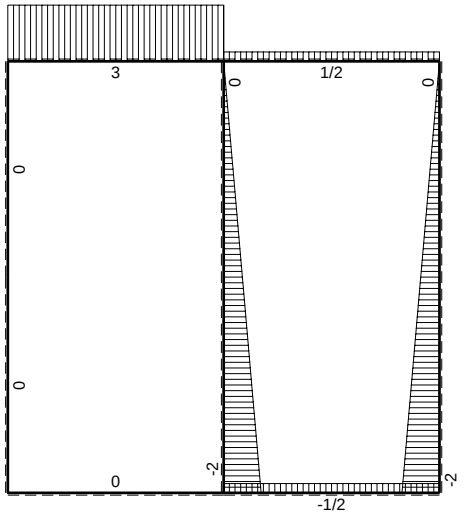
A = 154.4 mm^2
J_u = 75667. mm^4
J_v = 7528. mm^4
J_t = 129.4 mm^4
x_o = -3.846 mm
x_g = 13.69 mm
T_y = 1005. N
M_x = -603000. Nmm
u_m = -13.69 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = -199.2 N/mm^2
x_c = 12. mm
u_c = -1.69 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = -199.2 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 59.41 N/mm^2
$\tau_g = TS^*/tJ_u$ = 5.645 N/mm^2
$\tau_o = Tx_o t/J_t$ = 53.76 N/mm^2
t_c = 3618. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 224.2 N/mm^2

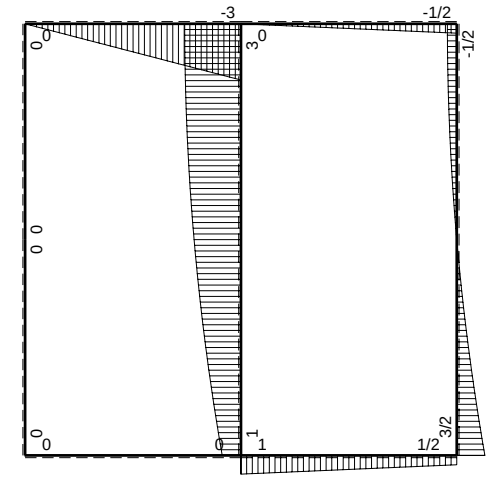
CI08.xxxx.165

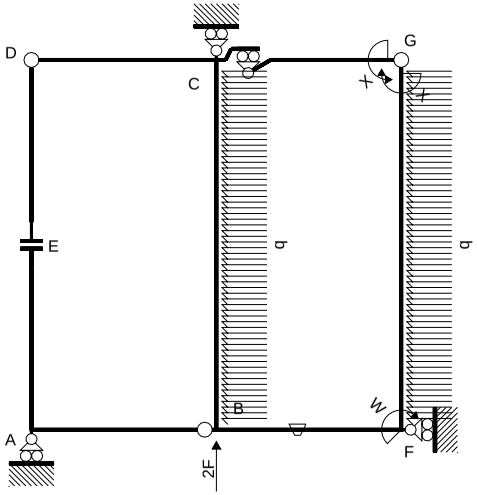
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

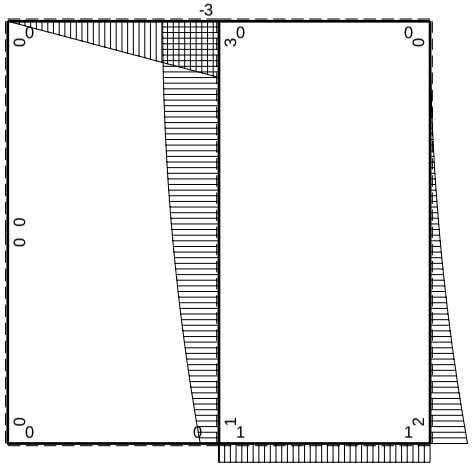

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
5/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
5/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
-5/2
0
1/2
-3
-3
0
-5/2
2
1/2
F

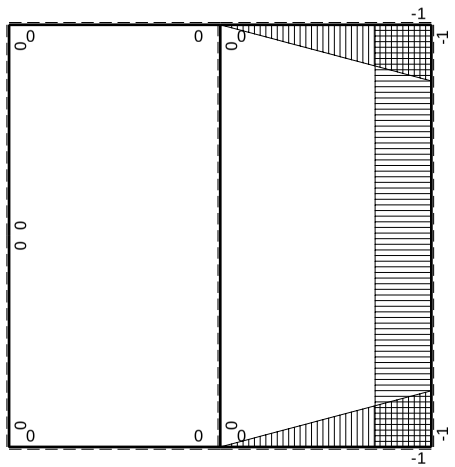

3
0
1/2
0
0
0
-2
0
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-4/3Fb^2$/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 986216 CI08.xxxx.166

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (- x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

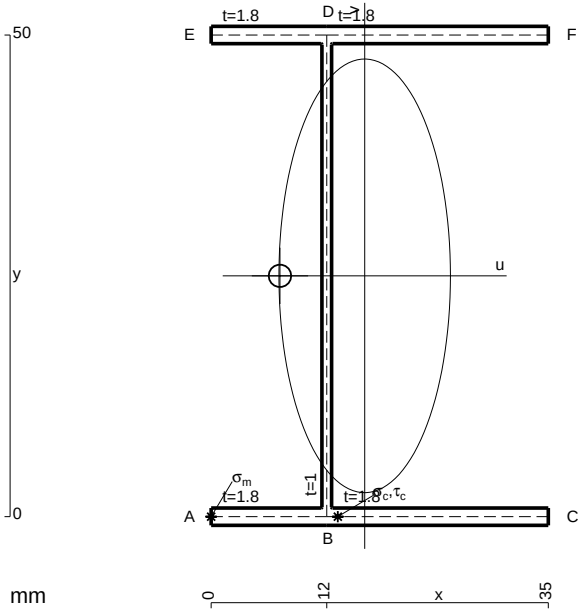
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

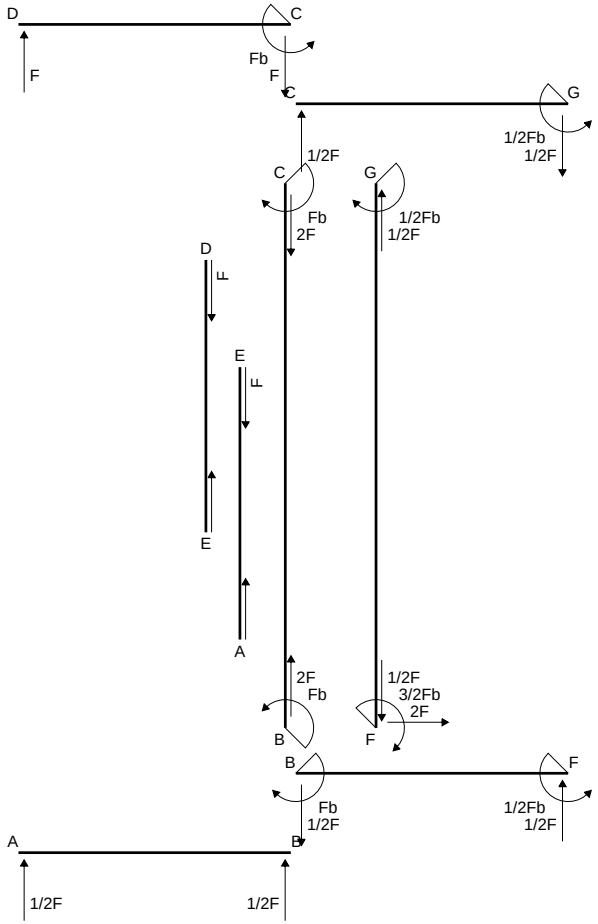
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

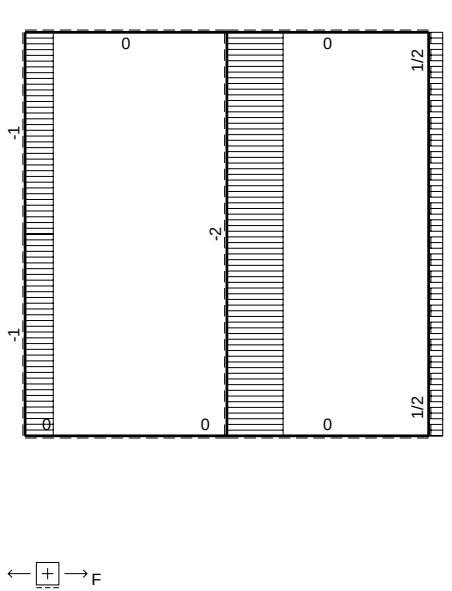


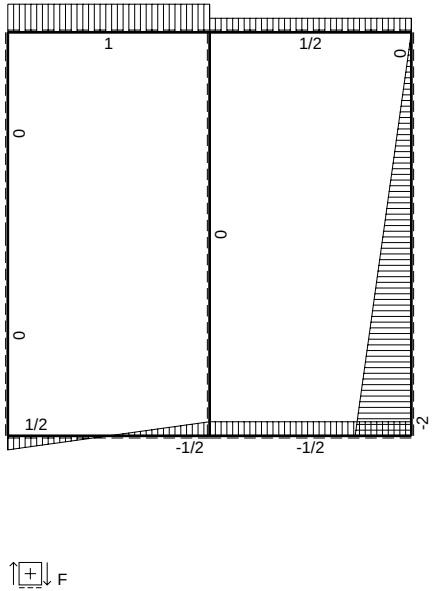
A = 176. mm^2
J_u = 89167. mm^4
J_v = 13945. mm^4
J_t = 152.7 mm^4
x_o = -8.795 mm
x_g = 15.94 mm
T_y = 1170. N
M_x = -748800. Nmm
u_m = -15.94 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = -209.9 N/mm^2
x_c = 12. mm
u_c = -3.938 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = -209.9 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 128.8 N/mm^2
$\tau_g = TS^*/tJ_u$ = 7.545 N/mm^2
$\tau_o = Tx_o t/J_t$ = 121.3 N/mm^2
t_c = 702. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 306.3 N/mm^2

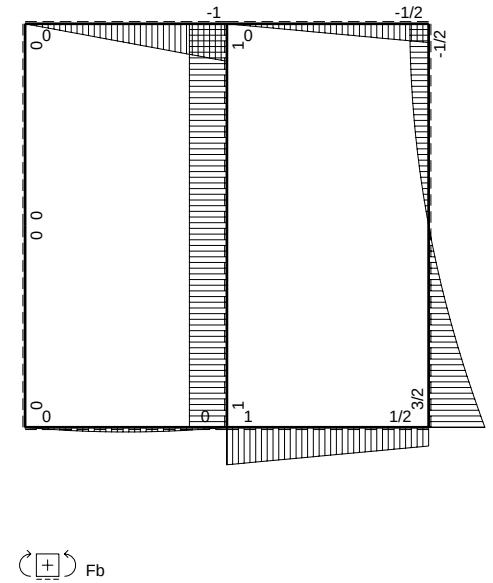
CI08.xxxx.166

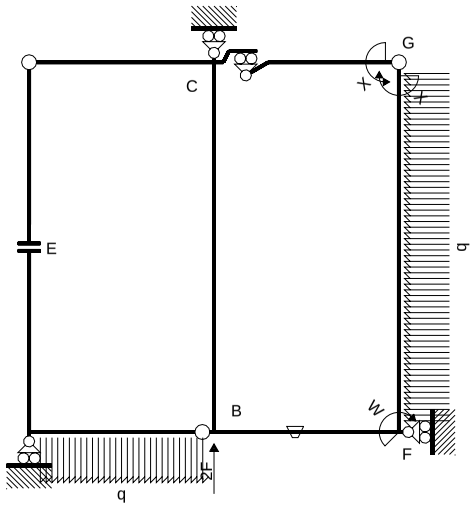
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

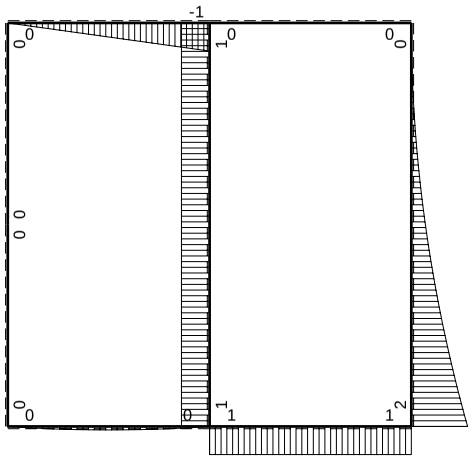

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
2F
1/2Fb
1/2F
D
F
E
F
E
A
2F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
-2
-1
0
0
0
1/2
F

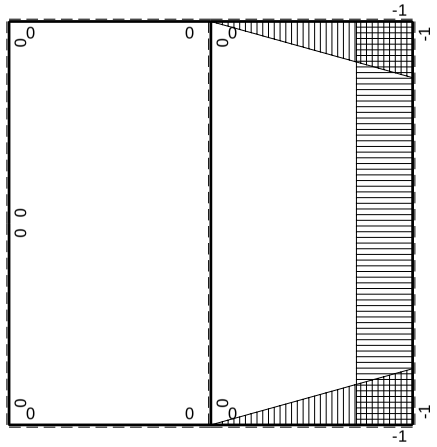

1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2
F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 242053 CI08.xxxx.167

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

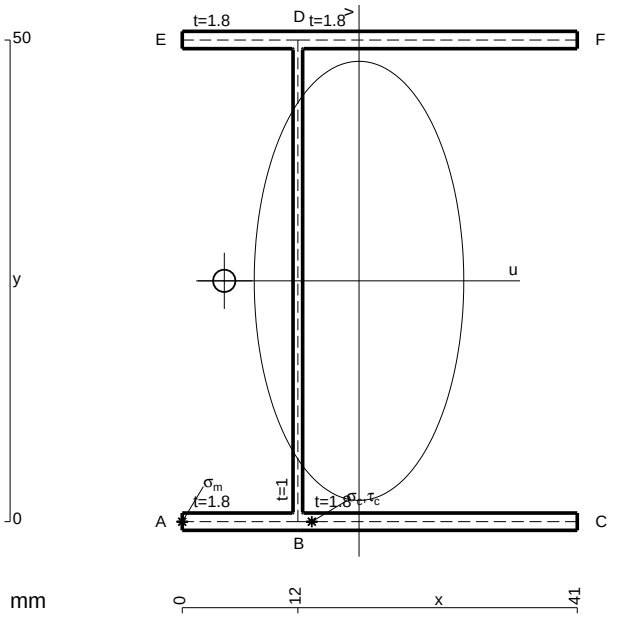
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

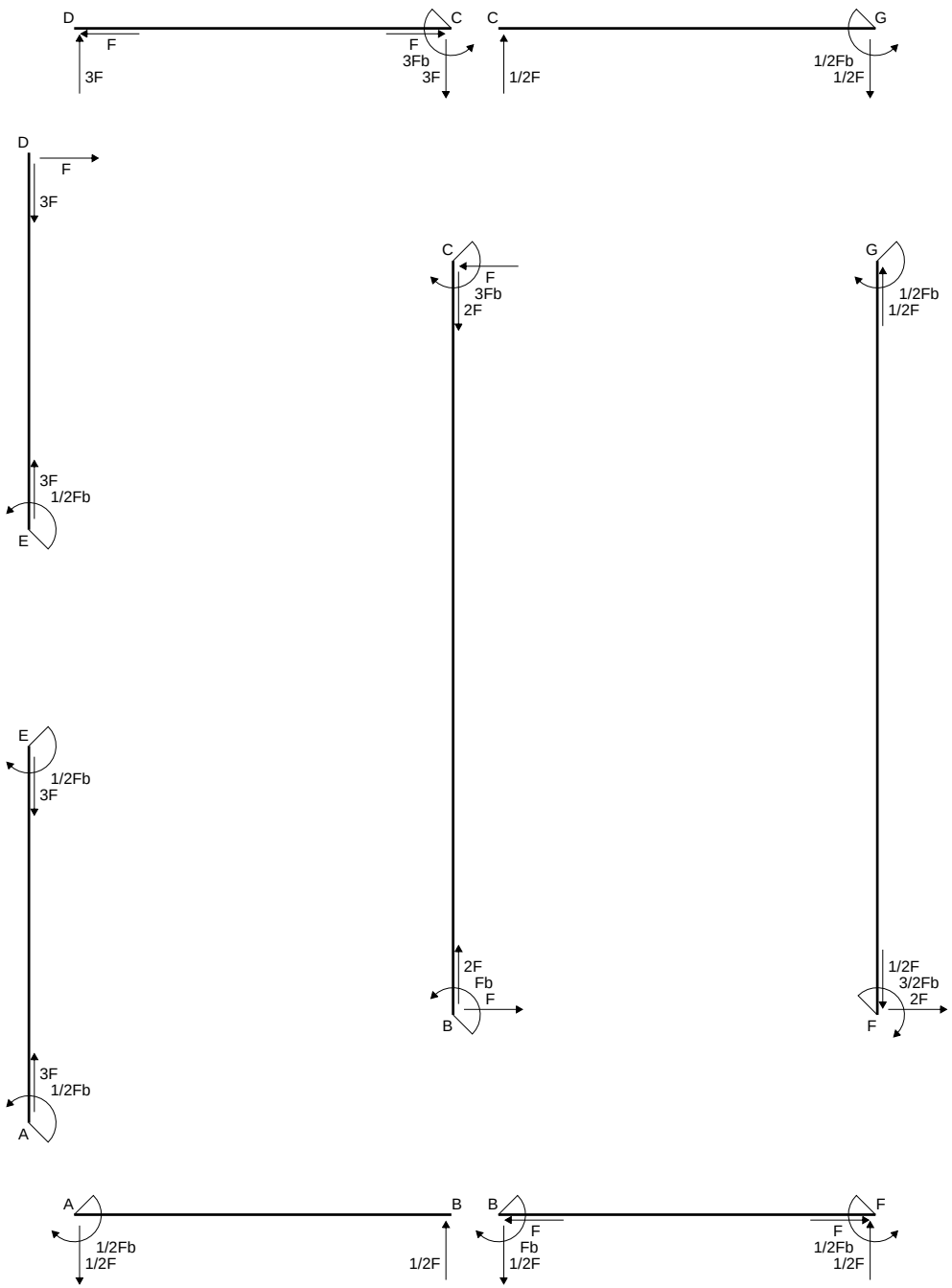
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

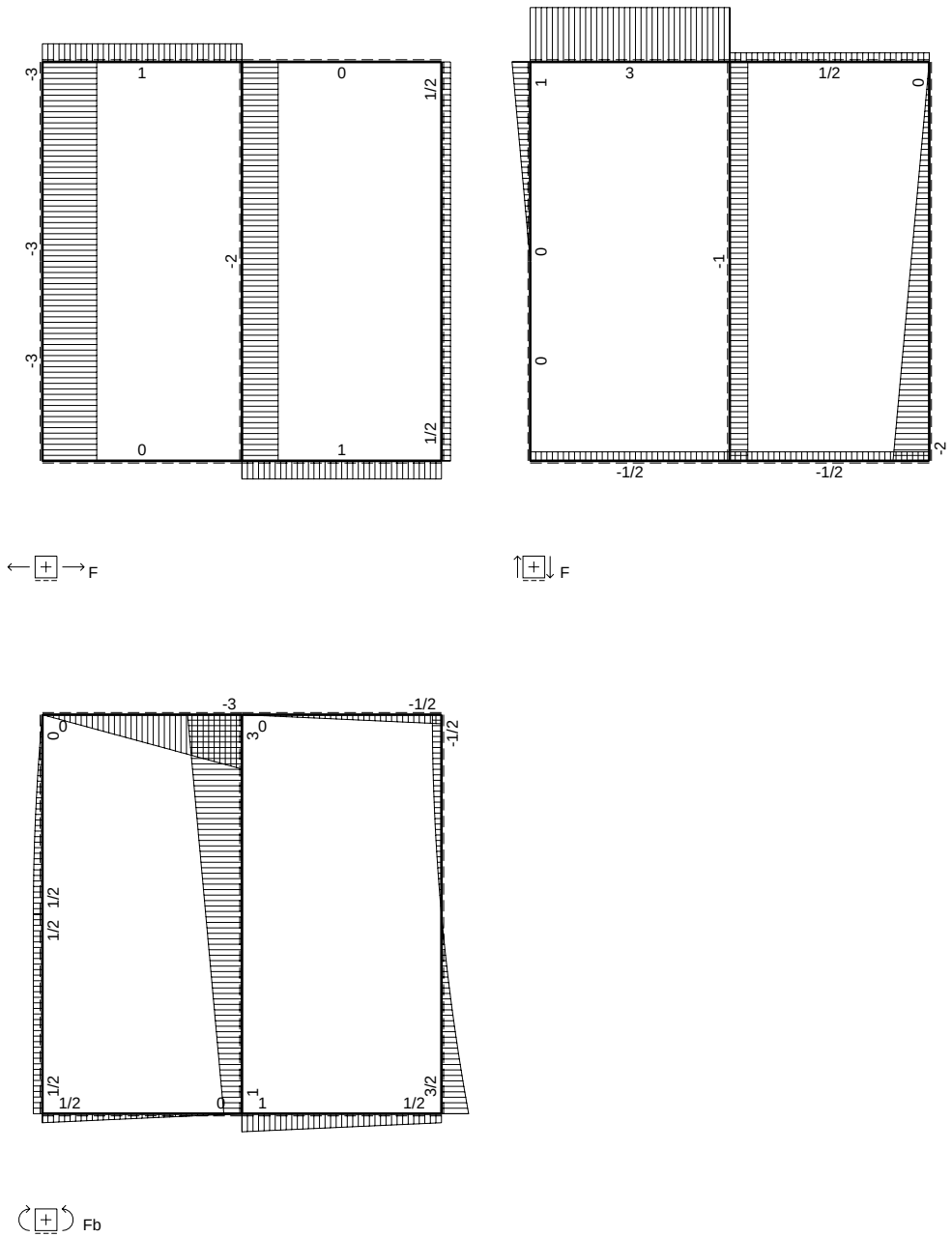


A = 197.6 mm^2
J_u = 102667. mm^4
J_v = 23375. mm^4
J_t = 176.1 mm^4
x_o = -13.99 mm
x_g = 18.35 mm
T_y = 1320. N
M_x = -897600. Nmm
u_m = -18.35 mm
v_m = -25. mm
$\sigma_m = -Mv/J_u$ = -218.6 N/mm^2
x_c = 12. mm
u_c = -6.349 mm
v_c = -25. mm
$\sigma_c = -Mv/J_u$ = -218.6 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 198.1 N/mm^2
$\tau_g = TS^*/tJ_u$ = 9.321 N/mm^2
$\tau_o = Tx_o t/J_t$ = 188.7 N/mm^2
t_c = 2376. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 406.8 N/mm^2

CI08.xxxx.167

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25


D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
3F
C
F
3Fb
2F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
2F
Fb
F
B
1/2F
3/2Fb
2F
F
3F
1/2Fb
A
A
B
B
F
1/2Fb
1/2F
F
Fb
1/2F
F
1/2Fb
1/2F


-3
-3
-3
1
0
-2
0
1
1/2
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 247954 CI08.xxxx.168

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

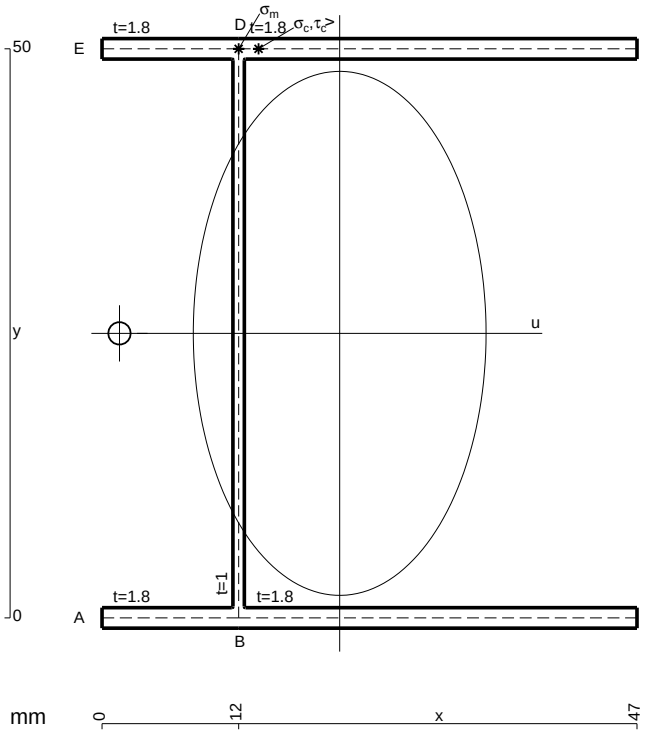
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 219.2 mm^2
J_u = 116167. mm^4
J_v = 36251. mm^4
J_t = 199.4 mm^4
x_o = -19.35 mm
x_g = 20.88 mm
N = 480. N
T_y = 1440. N
M_x = -1036800. Nmm
x_m = 12. mm
y_m = 50. mm
u_m = -8.877 mm
v_m = 25. mm
$\sigma_m = N/A - Mv/J_u$ = 225.3 N/mm^2
x_c = 12. mm
y_c = 50. mm
u_c = -8.877 mm
v_c = 25. mm
$\sigma_c = N/A - Mv/J_u$ = 225.3 N/mm^2
$\tau_c = \tau_g + \tau_{ou}$ = 262.3 N/mm^2
$\tau_g = TS^*/tJ_u$ = 10.85 N/mm^2
$\tau_o = Tx_o t/J_t$ = 251.5 N/mm^2
t_c = 864. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 507.1 N/mm^2

CI08.xxxx.168

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 26.03.25