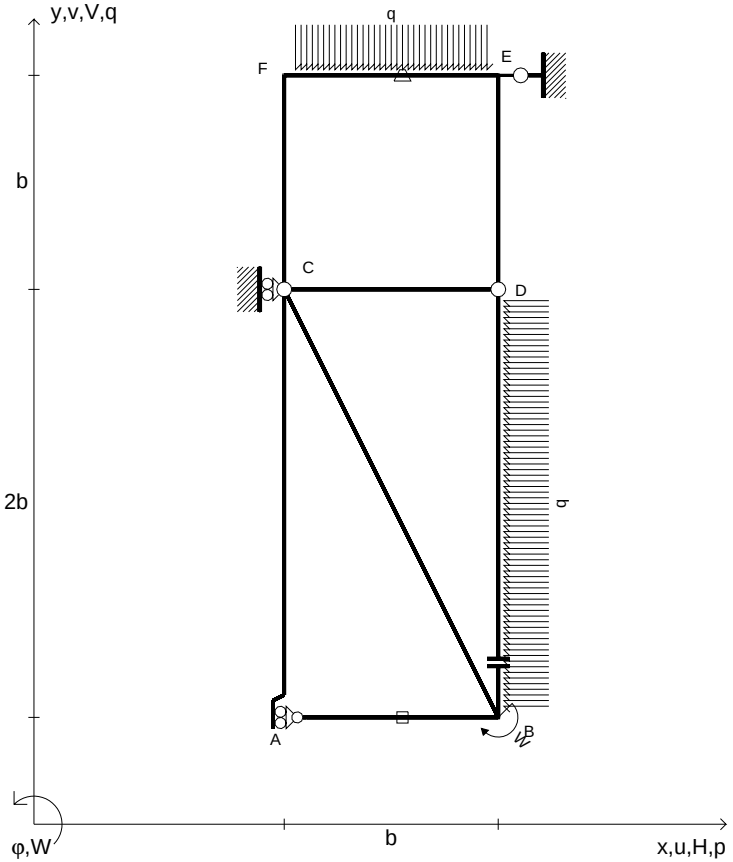
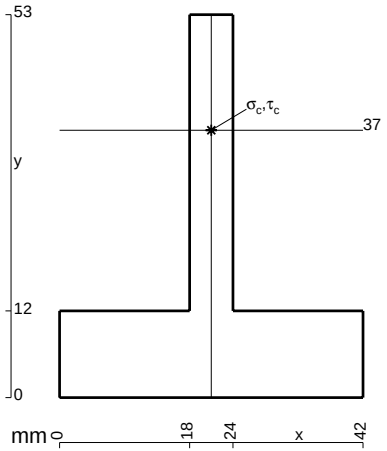
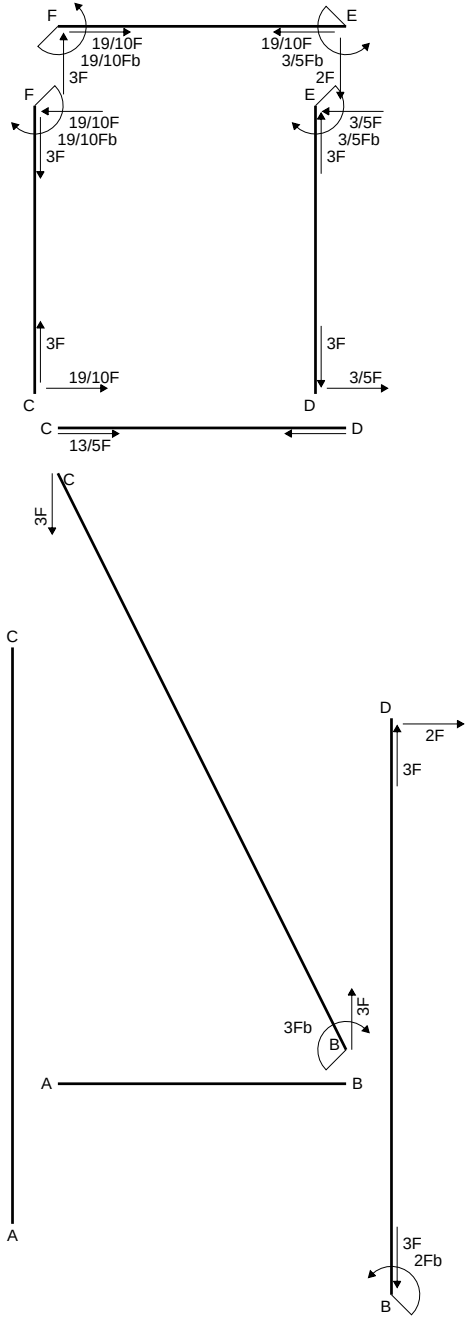
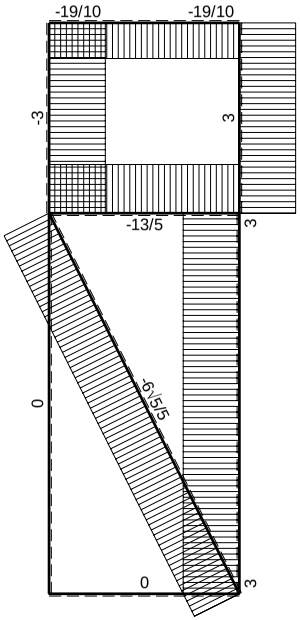
$W_B = -W = -Fb$
$p_{DB} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\varepsilon_{AB} = -\alpha T = -b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$

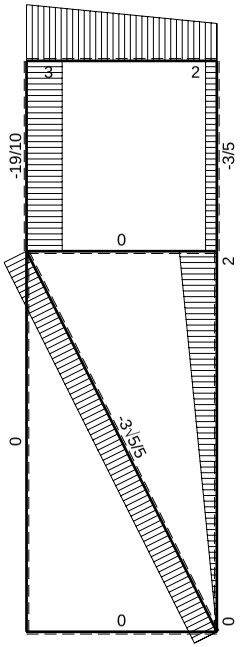


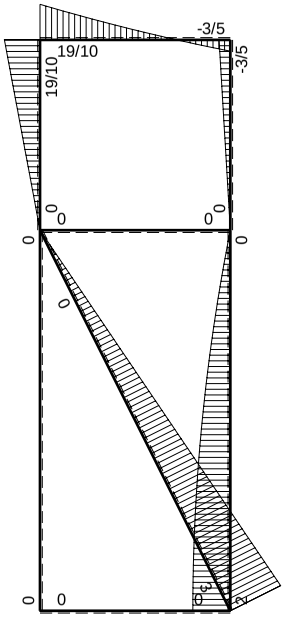


Reazioni iperstatiche in soluzione: $X=W_{FC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave BC ha la sezione riportata e dimensioni in mm, con:
b = 700 mm, F = 460 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da B a C
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.

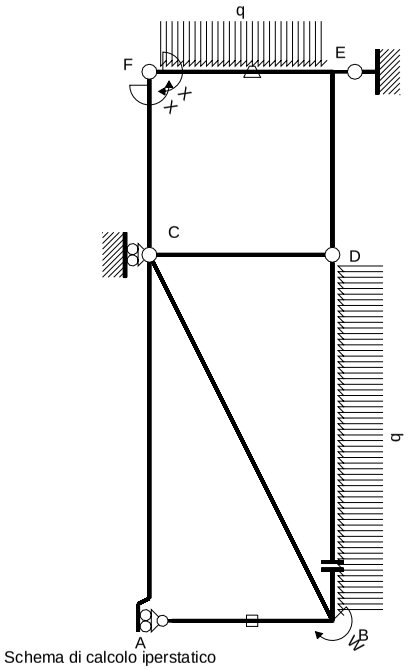




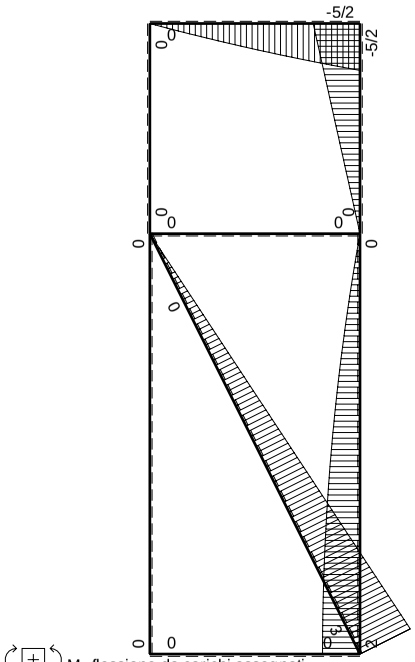




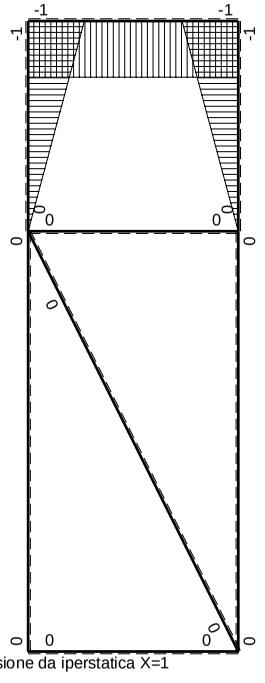
←⊞→ F ↑⊞↓ F ↻⊞↺ Fb



Schema di calcolo iperstatico



↻⊞↺ M_0 flessione da carichi assegnati



↻⊞↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{FC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	3Fb-3$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	-5/2Fx	0	5/2Fx^2/b	0	x^2/b^2	(5/6+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	5/2Fb-5/2Fx	0	5/2Fb-5Fx+5/2Fx^2/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	-5/2Fb+2Fx+1/2qx^2	-Fb/EJ	5/2Fb-2Fx-1/2Fx^2/b	Fb/EJ	1	(4/3+1)Fb^2/EJ	Xb/EJ
FE b	1	3Fx-1/2qx^2	Fb/EJ	3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{FC}$						-19/10Fb	

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{DE}^{Xo} = \int_o^b (5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (5/6\ b)\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (5/2 -5\ x/b +5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/2\ x -5/2\ x^2/b +5/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (5/2\ b -5/2\ b +5/6\ b)\ Fb\ 1/EJ\ = 5/6\ \ Fb^2/EJ$

$L_{EF}^{Xo} = \int_o^b (5/2 -2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx$

$= [5/2\ x - x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (5/2\ b - b -1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 7/3\ \ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [3/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

$= (3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 7/3\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 211463 ESΣ09.xxxx.029

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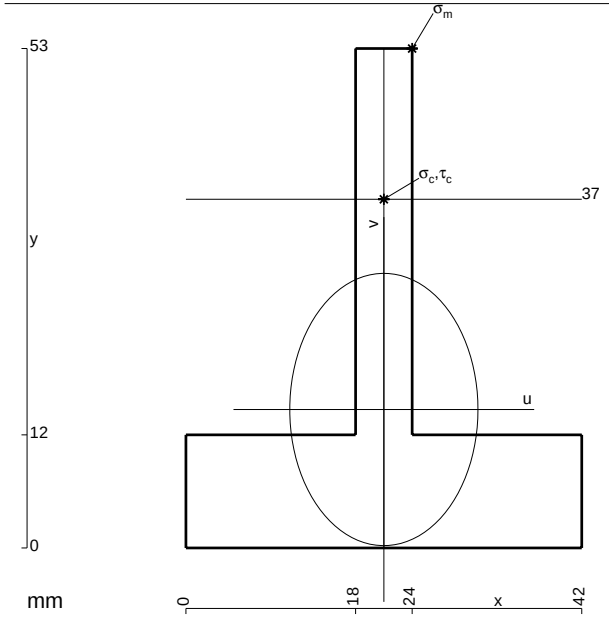
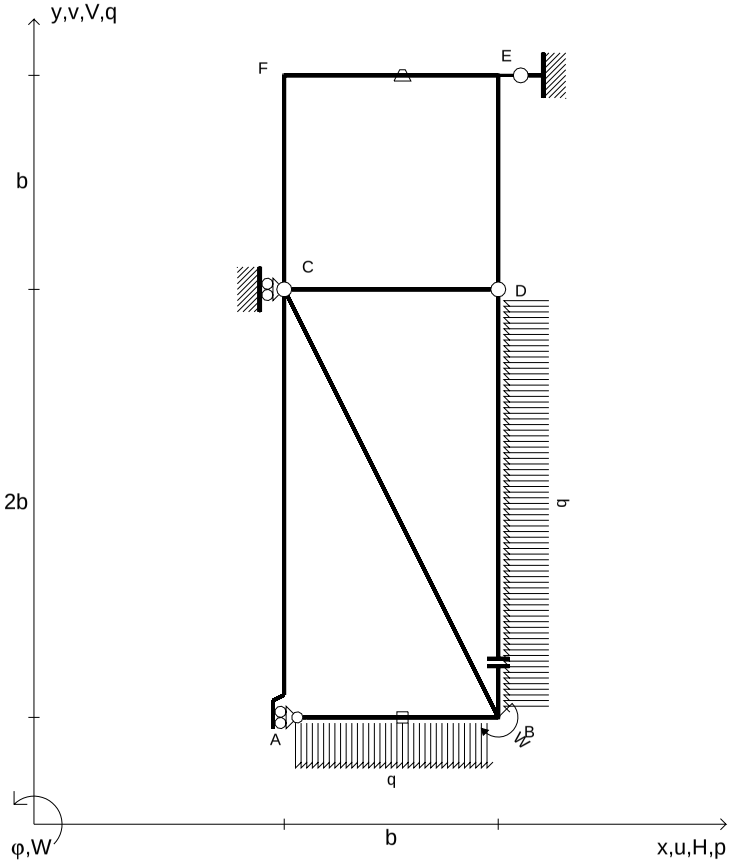


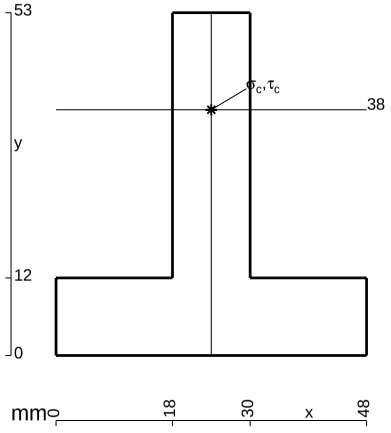
A = 750. mm^2
J_u = 156599. mm^4
J_v = 74826. mm^4
y_g = 14.69 mm
N = -1234. N
T_y = -617.2 N
M_x = 966000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 38.31 mm
$\sigma_m = N/A - Mv/J_u$ = -238. N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22.31 mm
$\sigma_c = N/A - Mv/J_u$ = -139.3 N/mm^2
τ_c = 1.911 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.3 N/mm^2
S^* = 2910. mm^3

ESΣ09.xxxx.029

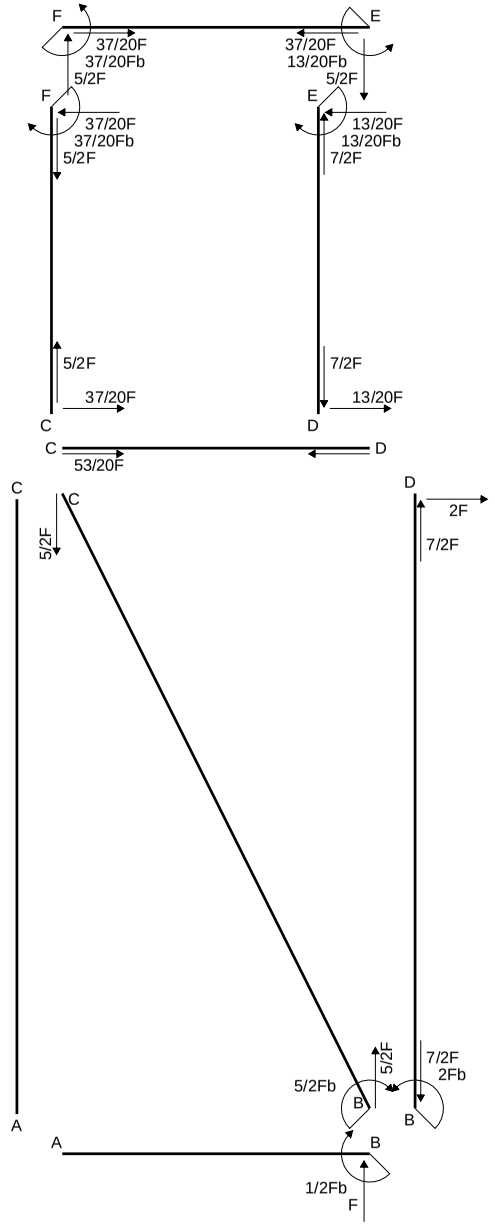
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

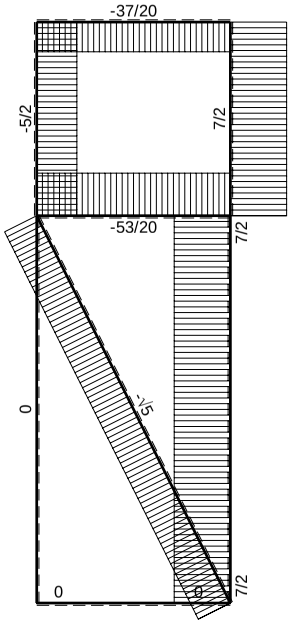
$W_B = -W = -Fb$
$p_{DB} = -q = -F/b$
$q_{AB} = -q = -F/b$
$\varepsilon_{AB} = -\alpha T = -b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$

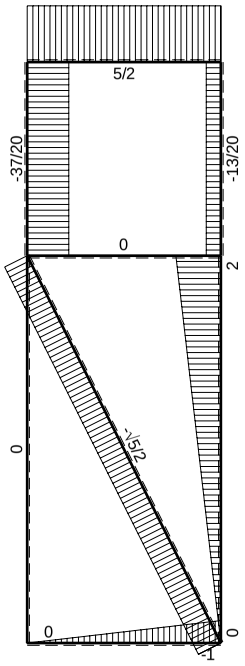


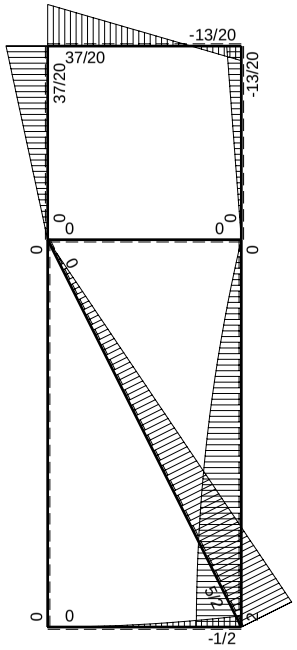


Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave BC ha la sezione riportata e dimensioni in mm, con:
b = 400 mm, F = 1550 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da B a C
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.

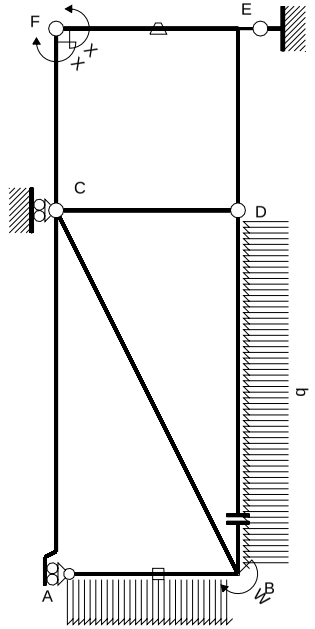




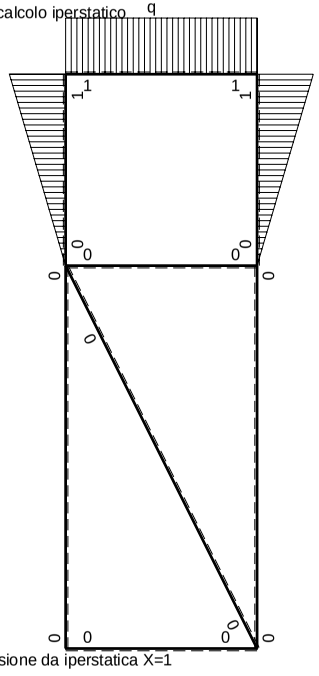




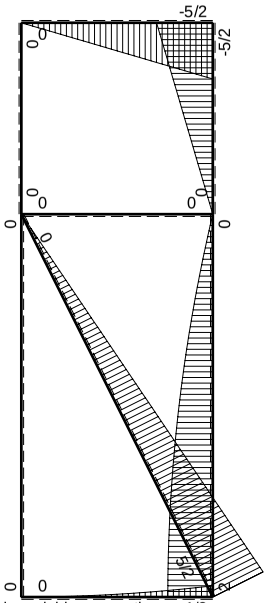
$\leftarrow \boxplus \rightarrow F$ $\uparrow \boxplus \downarrow F$ $\circlearrowleft \boxplus \circlearrowright Fb$



Schema di calcolo iperstatico



$\circlearrowleft \boxplus \circlearrowright M_x$ flessione da iperstatica X=1



$\circlearrowleft \boxplus \circlearrowright M_0$ flessione da carichi assegnati

Quadro contributi PLV per iperstatica $X=W_{FE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$5/2Fb-\sqrt{5}/2Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	1	-5/2Fb+5/2Fx	-Fb/EJ	-5/2Fb+5/2Fx	-Fb/EJ	1	$(-5/4-1)Fb^2/EJ$	Xb/EJ
FE b	-1	5/2Fx	Fb/EJ	-5/2Fx	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x^2/b^2		
	totali						$-37/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{FE}$						37/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -9/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -9/4\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 225153 ESΣ09.xxxx.031

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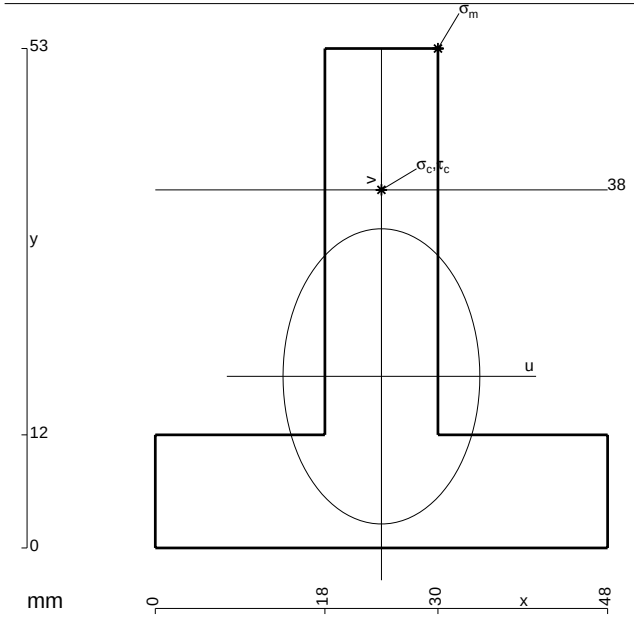
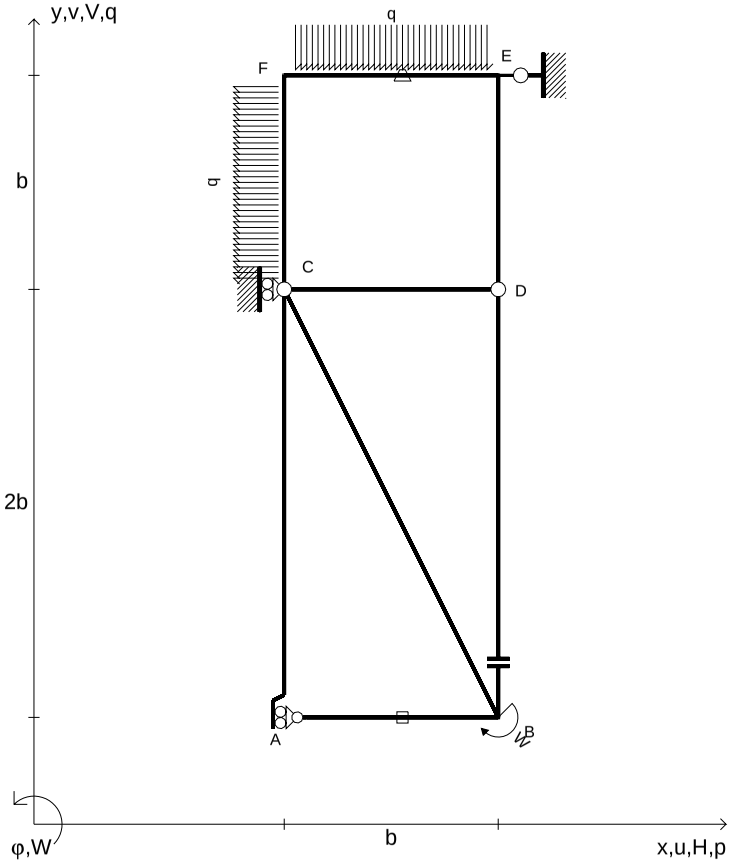


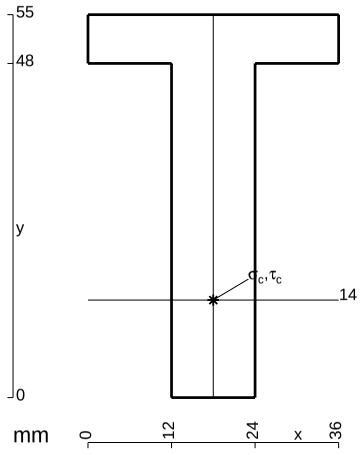
A = 1068. mm^2
J_u = 262174. mm^4
J_v = 116496. mm^4
y_g = 18.21 mm
N = -3466. N
T_y = -1733. N
M_x = 1550000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 34.79 mm
$\sigma_m = N/A - Mv/J_u$ = -208.9 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.79 mm
$\sigma_c = N/A - Mv/J_u$ = -120.3 N/mm^2
τ_c = 2.706 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 120.3 N/mm^2
S^* = 4913. mm^3

ESΣ09.xxxx.031

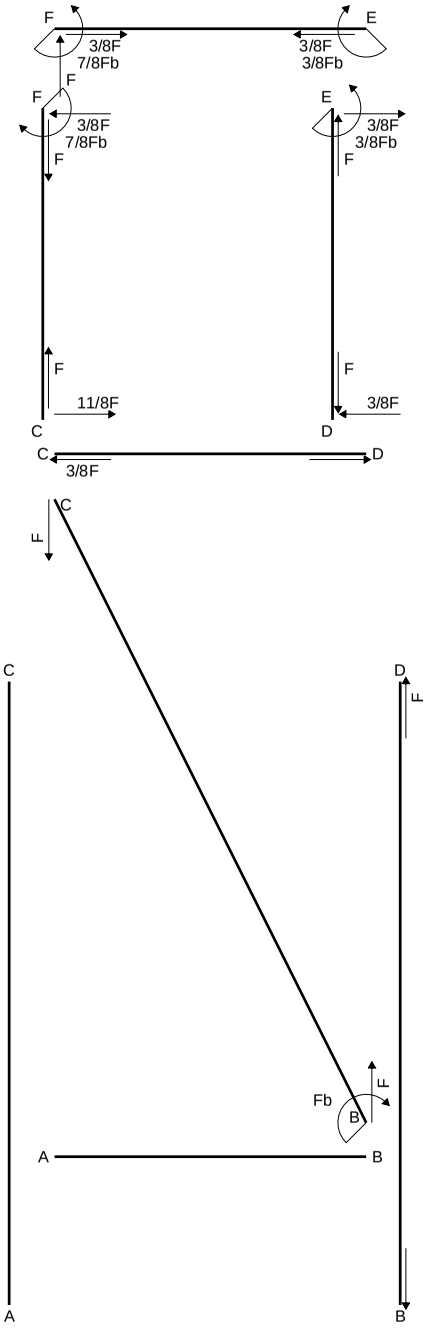
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

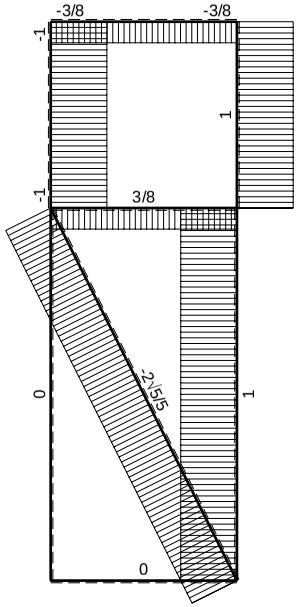
$W_B = -W = -Fb$
$p_{FC} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\varepsilon_{AB} = 4\alpha T = 4b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$



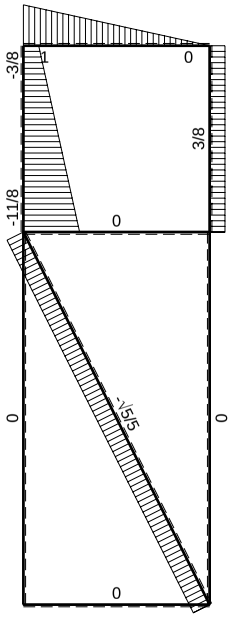


Reazioni iperstatiche in soluzione: $X=W_{FC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave BC ha la sezione riportata e dimensioni in mm, con:
b = 740 mm, F = 2470 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da B a C
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.

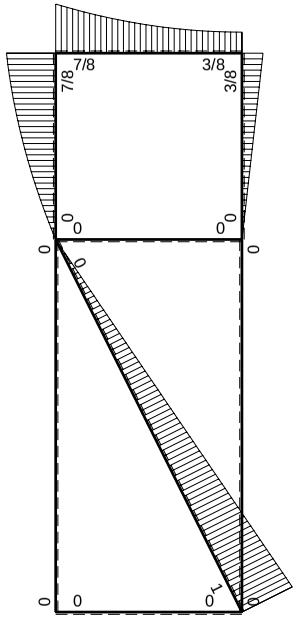




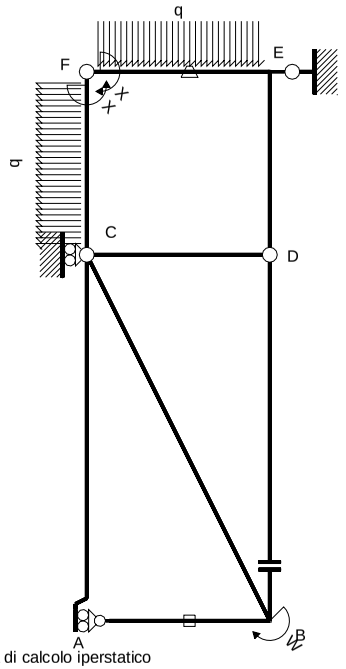
←□→ F



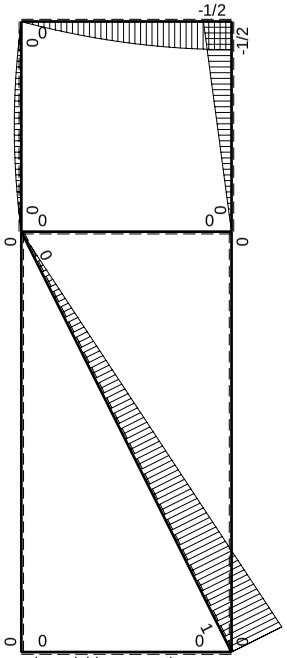
↑□↓ F



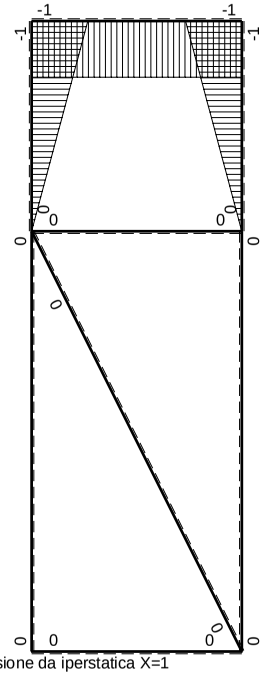
↺□↻ Fb



Schema di calcolo iperstatico



↺□↻ M_0 flessione da carichi assegnati



↺□↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{FC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx	0	$1/2Fx^2/b$	0	x^2/b^2	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	1/2Fb-1/2Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	-1/2Fb+1/2qx^2	-Fb/EJ	$1/2Fb-1/2Fx^2/b$	Fb/EJ	1	$(1/3+1)Fb^2/EJ$	Xb/EJ
FE b	1	Fx-1/2qx^2	Fb/EJ	$Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	1/2Fx-1/2qx^2	0	$-1/2Fx+Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-1/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx+1/2qx^2	0	$-1/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
		totali					$35/24Fb^2/EJ$	5/3Xb/EJ
		iperstatica $X=W_{FC}$					-7/8Fb	

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{EF}^{Xo} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/2\ x -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 4/3\ \ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = 4/3\ \ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (-1/2\ x/b + x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/3\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

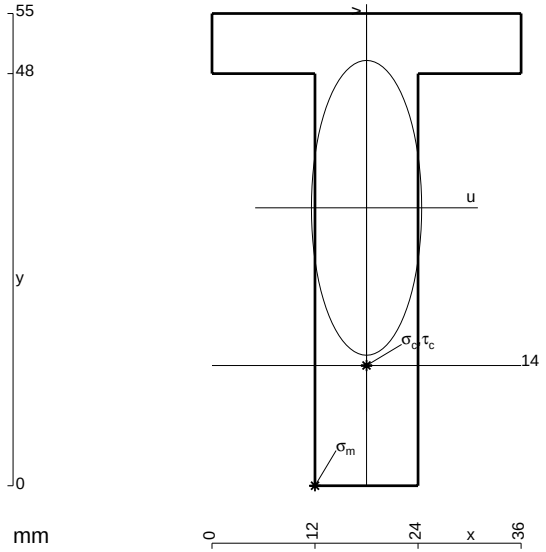
$= (-1/4\ b +1/3\ b -1/8\ b)\ Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (-1/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b +1/8\ b)\ Fb\ 1/EJ\ = -1/24\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 954987 ESΣ09.xxxx.039

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



A = 828. mm^2
J_u = 244195. mm^4
J_v = 34128. mm^4
y_g = 32.37 mm
N = -2209. N
T_y = -1105. N
M_x = 1827800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.37 mm
$\sigma_m = N/A - Mv/J_u$ = 239.6 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.37 mm
$\sigma_c = N/A - Mv/J_u$ = 134.8 N/mm^2
τ_c = 1.607 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.9 N/mm^2
S^* = 4262. mm^3

ESΣ09.xxxx.039

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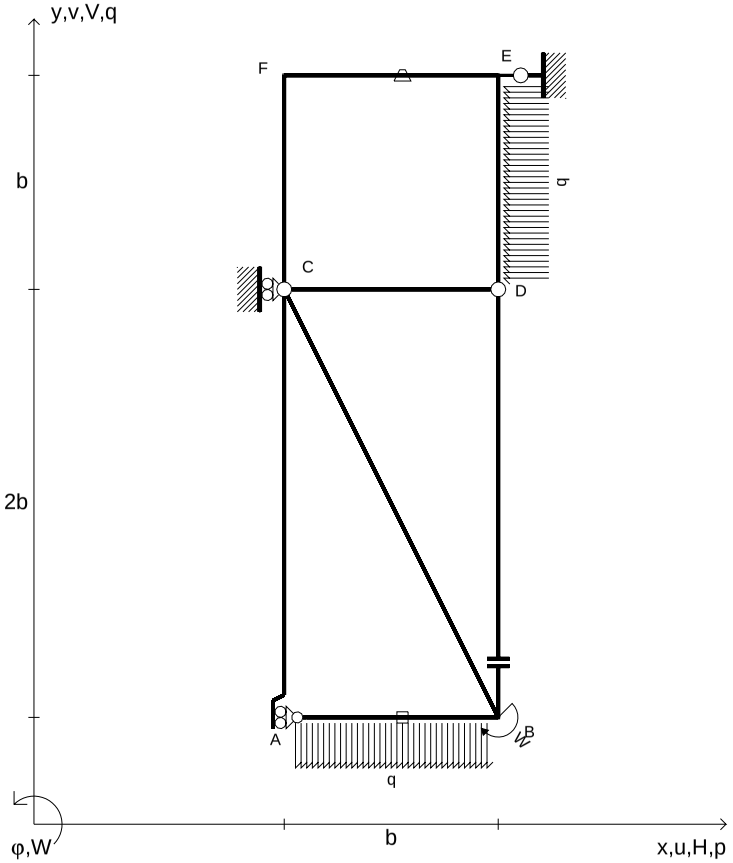
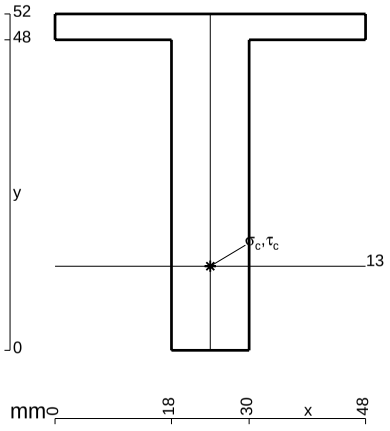
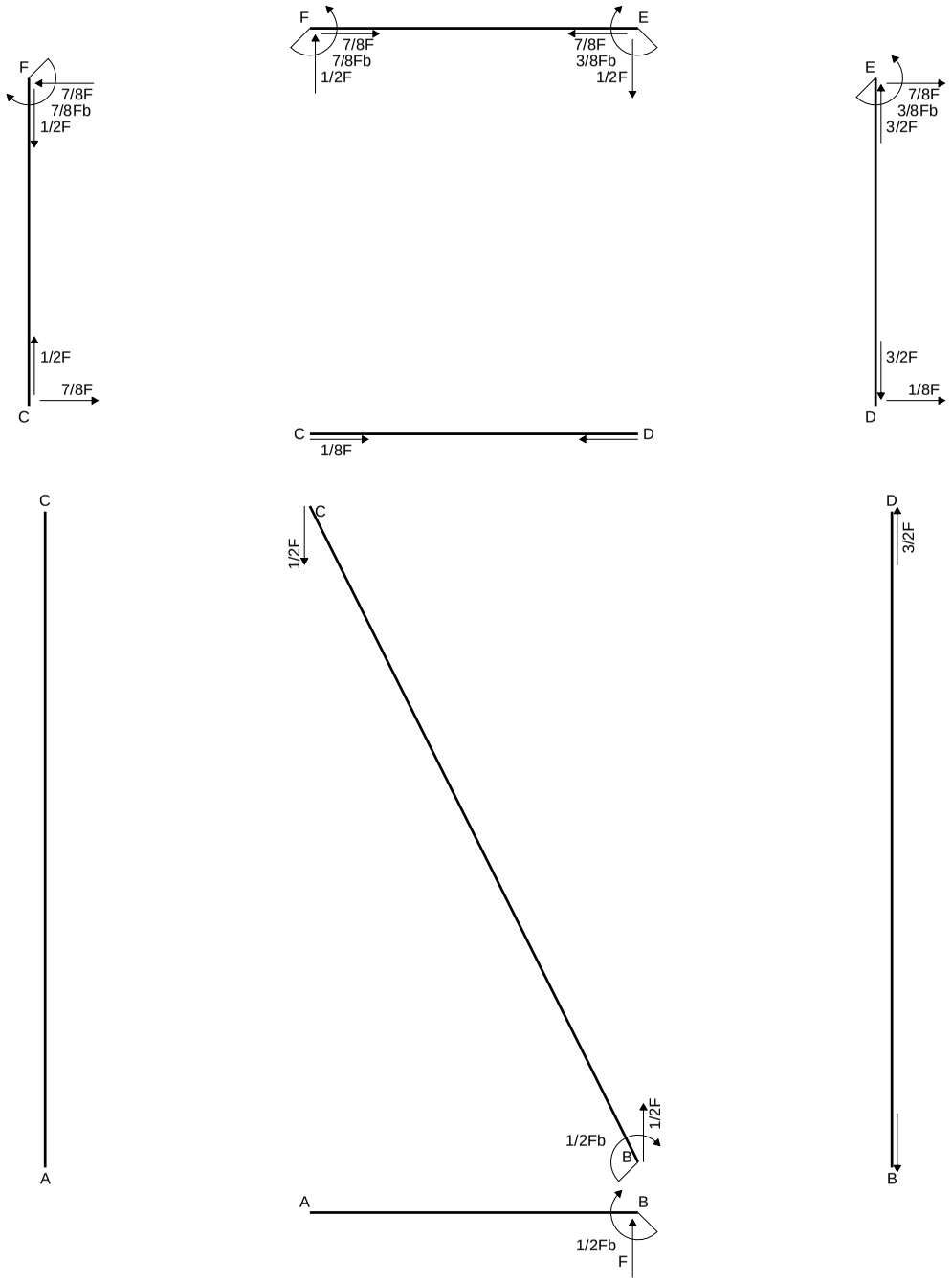
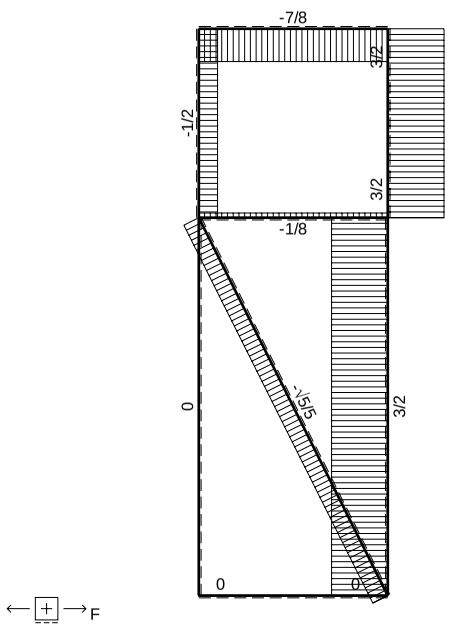
$W_B = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{DE} = -q = -F/b$
$\varepsilon_{AB} = 4\alpha T = 4b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$

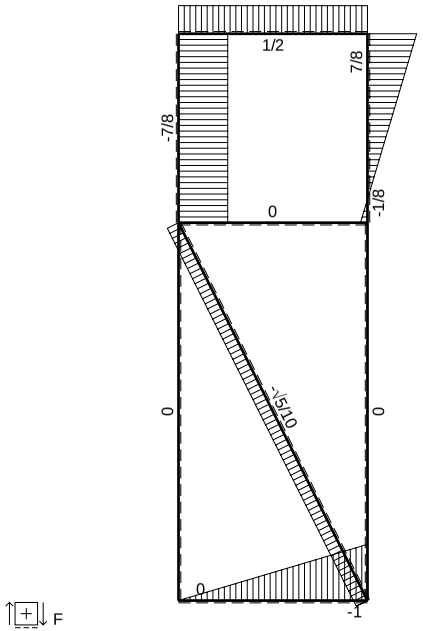


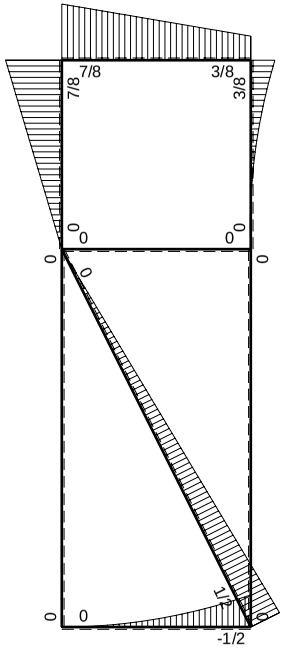


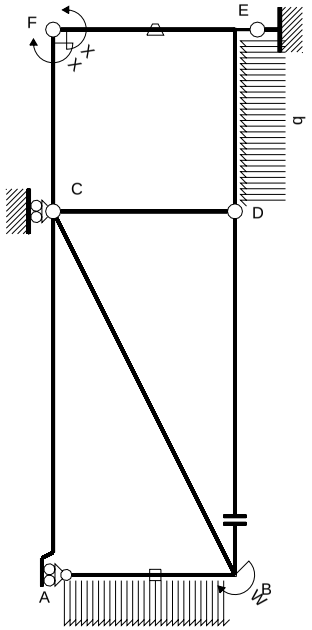
Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 400 mm, F = 7160 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.



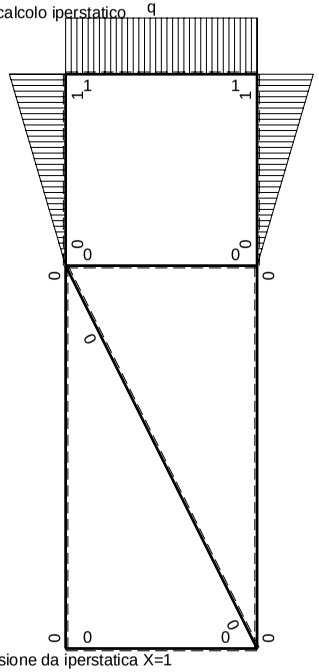


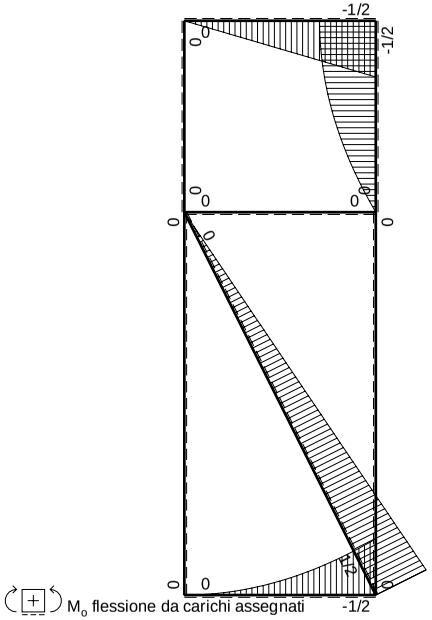






Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica X=W_{FE}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x/b	-Fx+1/2qx²	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-1+x/b	1/2Fb-1/2qx²	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	1	-1/2Fb+1/2Fx	-Fb/EJ	-1/2Fb+1/2Fx	-Fb/EJ	1	$(-1/4-1)Fb^2/EJ$	Xb/EJ
FE b	-1	1/2Fx	Fb/EJ	-1/2Fx	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x^2/b^2		
	totali						$-35/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica X=W_{FE}						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

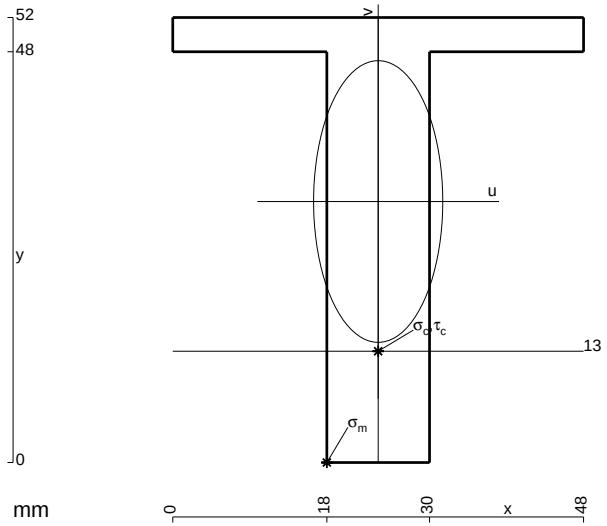
$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -5/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -5/4\ Fb^2/EJ$



A = 768. mm^2
J_u = 208192. mm^4
J_v = 43776. mm^4
y_g = 30.5 mm
T_y = -7160. N
M_x = -1432000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -30.5 mm
$\sigma_m = -Mv/J_u$ = -209.8 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -17.5 mm
$\sigma_c = -Mv/J_u$ = -120.4 N/mm^2
τ_c = 10.73 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.8 N/mm^2
S^* = 3744. mm^3

ESΣ09.xxxx.051

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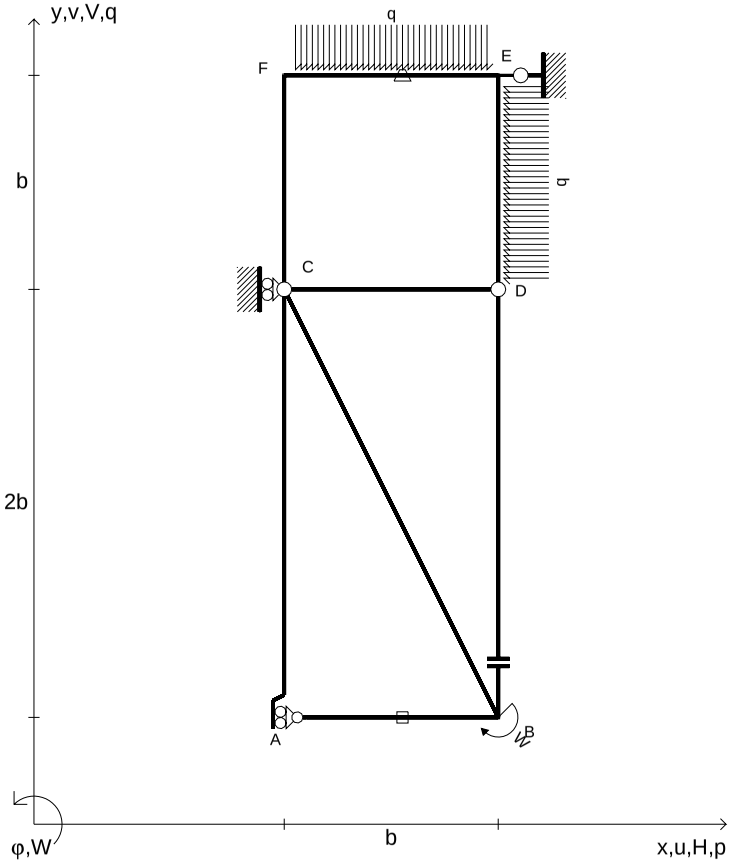
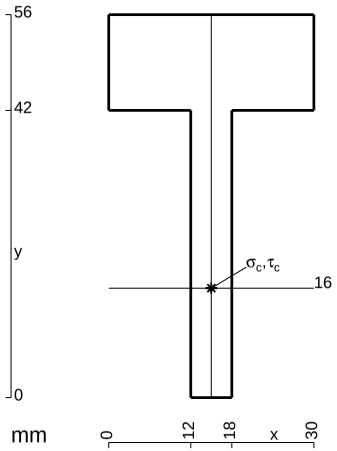
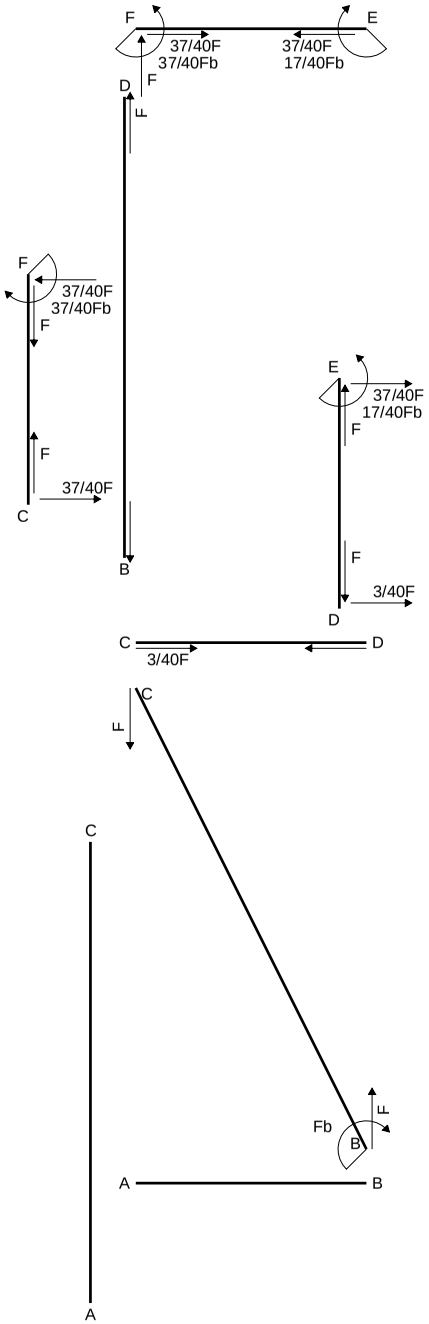
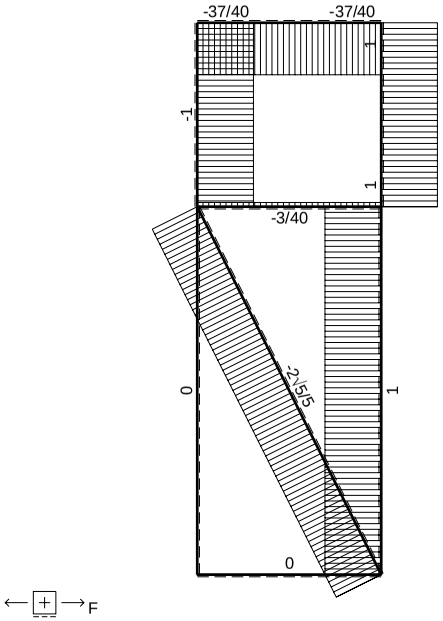
$W_B = -W = -Fb$
$p_{DE} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\varepsilon_{AB} = -2\alpha T = -2b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$



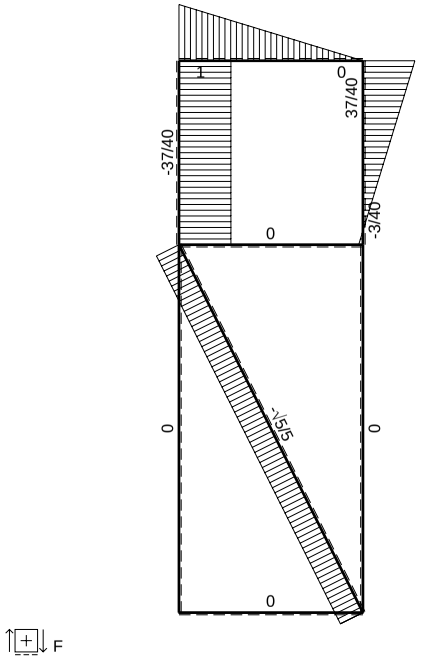


Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave BC ha la sezione riportata e dimensioni in mm, con:
b = 390 mm, F = 2260 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da B a C
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.

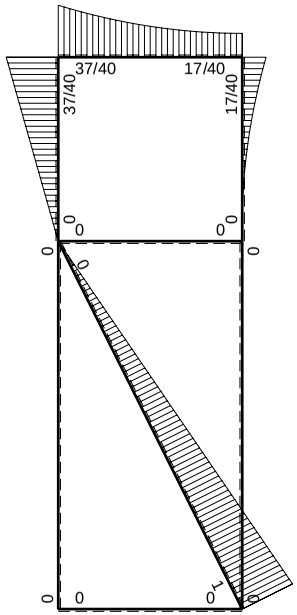




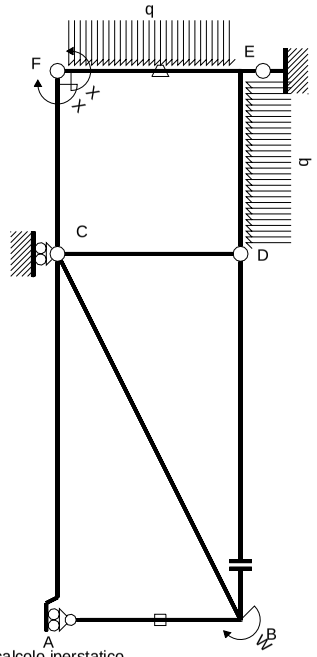
← F →



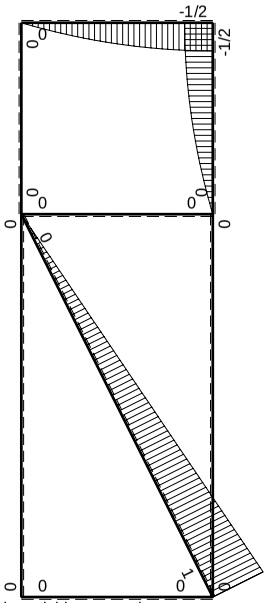
↑ F ↓



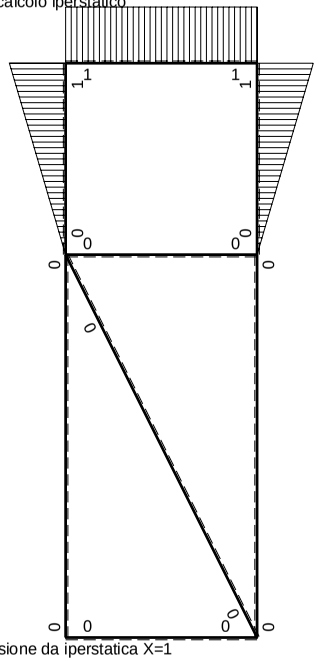
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{FE}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x/b	-Fx+1/2qx²	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-1+x/b	1/2Fb-1/2qx²	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	1	-1/2Fb+1/2qx²	-Fb/EJ	$-1/2Fb+1/2Fx^2/b$	-Fb/EJ	1	$(-1/3-1)Fb^2/EJ$	Xb/EJ
FE b	-1	Fx-1/2qx²	Fb/EJ	$-Fx+1/2Fx^2/b$	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x^2/b^2		
	totali						$-37/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica X=W_{FE}						37/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2 +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

$= (-1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -4/3\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (- x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -4/3\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 212301 ESΣ09.xxxx.070

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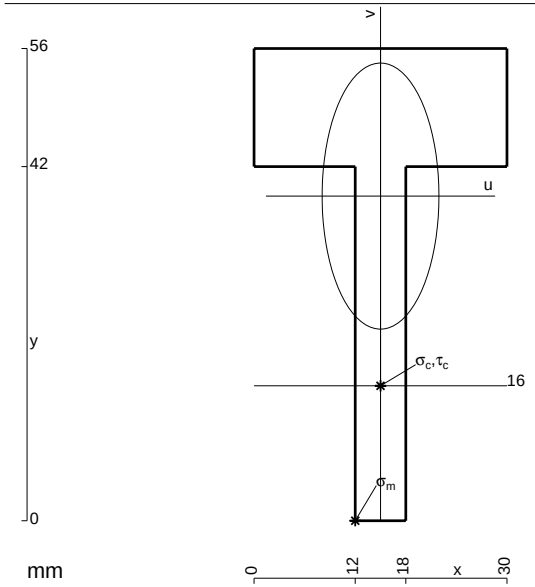

A = 672. mm^2
J_u = 167384. mm^4
J_v = 32256. mm^4
y_g = 38.5 mm
N = -2021. N
T_y = -1011. N
M_x = 881400. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -38.5 mm
$\sigma_m = N/A - Mv/J_u$ = 199.7 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -22.5 mm
$\sigma_c = N/A - Mv/J_u$ = 115.5 N/mm^2
τ_c = 2.947 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.6 N/mm^2
S^* = 2928. mm^3

ESΣ09.xxxx.070

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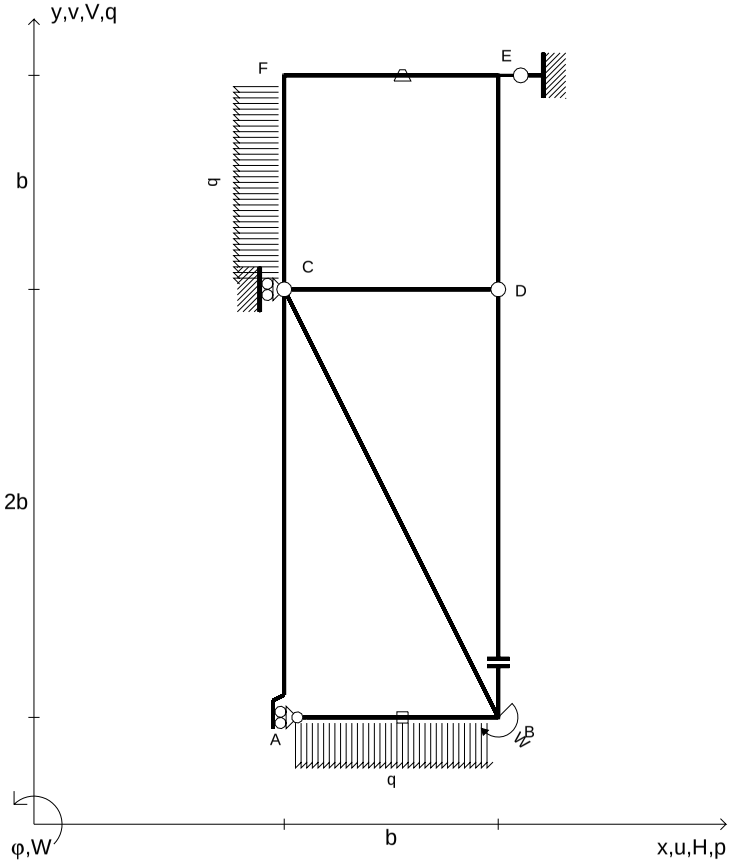
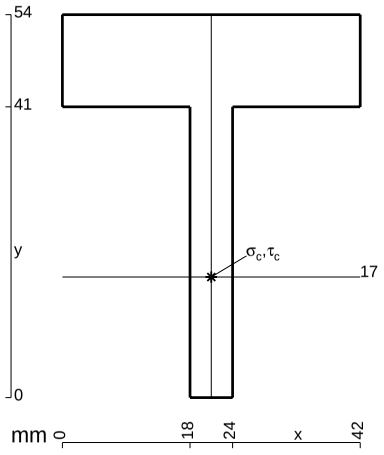
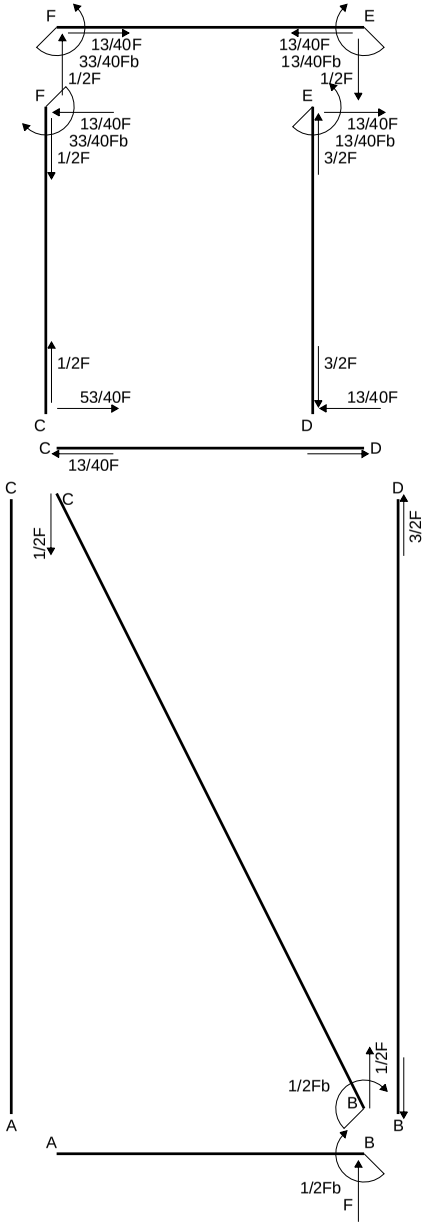
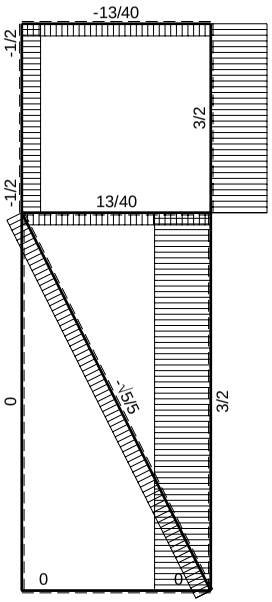
$W_B = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{FC} = -q = -F/b$
$\varepsilon_{AB} = -2\alpha T = -2b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$



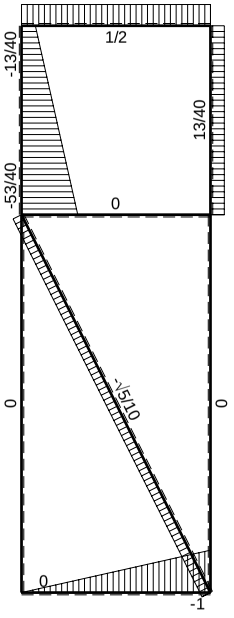


Reazioni iperstatiche in soluzione: $X=H_{DC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 450 mm, F = 4140 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.

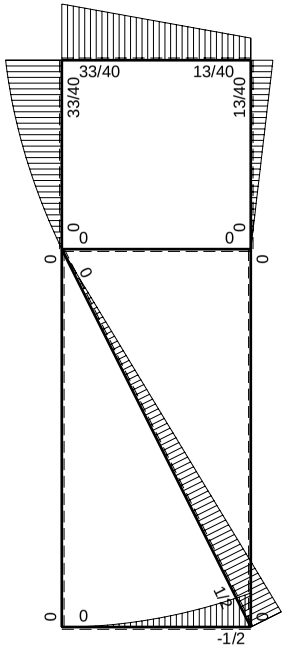




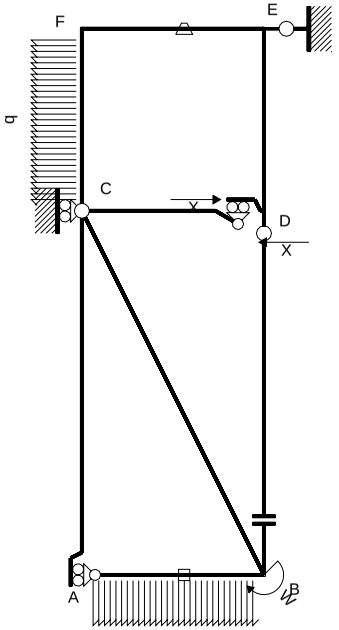
F



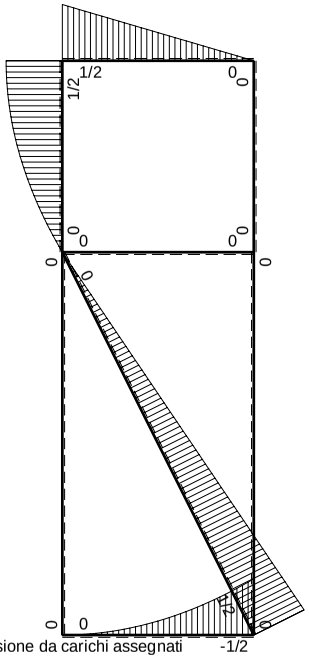
F



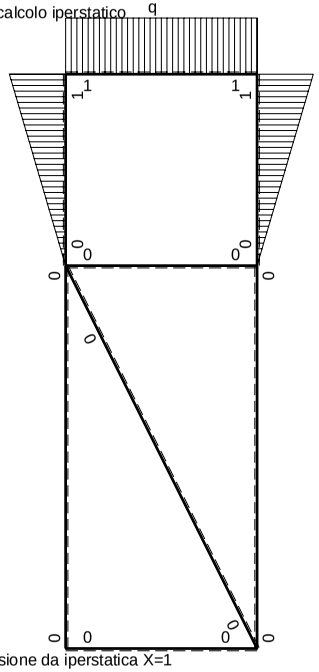
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_{DC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
ED b	-b+x	0	0	0	0	$b^2-2bx+x^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	b	1/2Fx	-Fb/EJ	1/2Fbx	$-Fb^2/EJ$	b^2	$(1/4-1)Fb^3/EJ$	Xb^3/EJ
FE b	-b	-1/2Fb+1/2Fx	Fb/EJ	$1/2Fb^2-1/2Fbx$	$-Fb^2/EJ$	b^2		
FC b	b-x	$1/2Fb-1/2qx^2$	0	$1/2Fb^2-1/2Fbx-1/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(5/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CF b	-x	$-Fx+1/2qx^2$	0	$Fx^2-1/2qx^3$	0	x^2		
	totali						$-13/24Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica X=H_{DC}						13/40F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 986591 ESΣ09.xxxx.082

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{EF} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x]_o^b\, b^2\, 1/EJ$

$= (\, b)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{FE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x]_o^b\, b^2\, 1/EJ$

$= (\, b)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx = [1/4\, x^2/b]_o^b\, Fb^2\, 1/EJ + [- x]_o^b\, \theta$

$= (1/4\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = -3/4\ Fb^3/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2 -1/2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx = [1/2\, x -1/4\, x^2/b]_o^b\, Fb^2\, 1/EJ + [\, x]_o^b\, \theta$

$= (1/2\, b -1/4\, b)\, Fb^2\, 1/EJ + (\, b)\, \theta = -3/4\ Fb^3/EJ$

$L^{Xo}_{FC} = \int_o^b (1/2 -1/2\, x/b -1/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [1/2\, x -1/4\, x^2/b -1/6\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/4\, b -1/6\, b +1/8\, b)\, Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$

$L^{Xo}_{CF} = \int_o^b (x^2/b^2 -1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [1/3\, x^3/b^2 -1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (1/3\, b -1/8\, b)\, Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$

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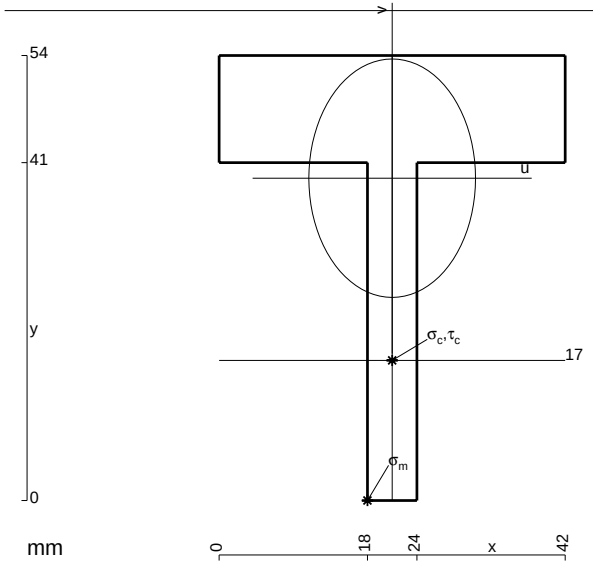

A = 792. mm^2
J_u = 165782. mm^4
J_v = 81000. mm^4
y_g = 39.11 mm
T_y = -4140. N
M_x = -931500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.11 mm
$\sigma_m = -Mv/J_u$ = -219.8 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.11 mm
$\sigma_c = -Mv/J_u$ = -124.3 N/mm^2
τ_c = 13. N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.3 N/mm^2
S^* = 3123. mm^3

ESΣ09.xxxx.082

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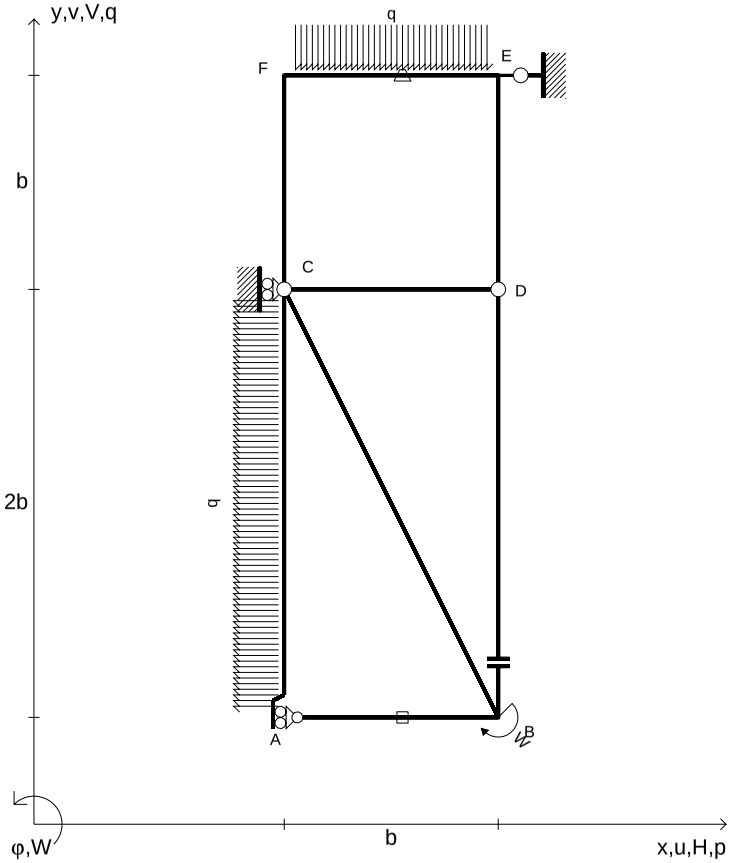
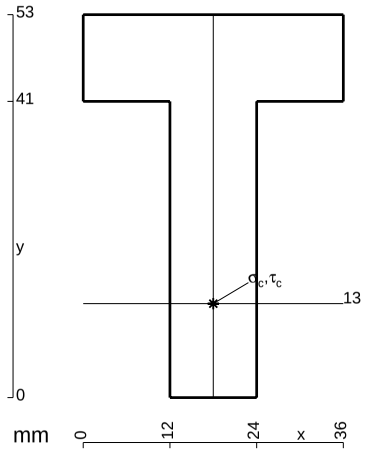
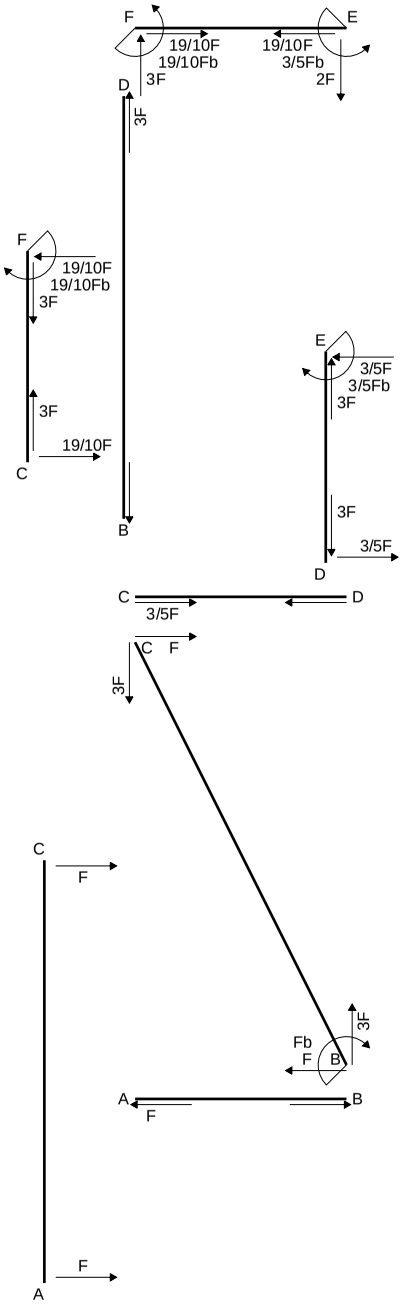
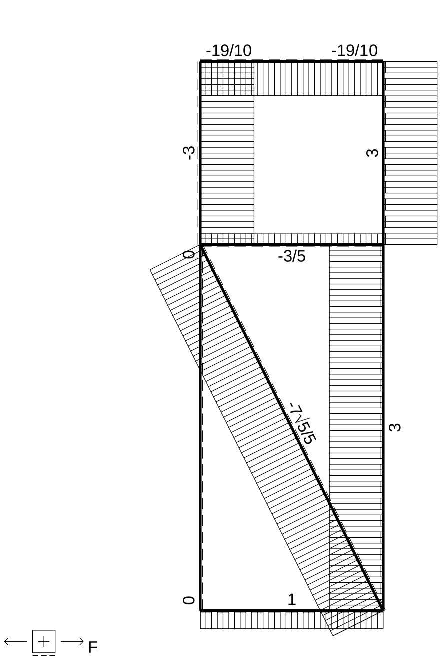
$W_B = -W = -Fb$
$p_{AC} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\varepsilon_{AB} = 3\alpha T = 3b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$

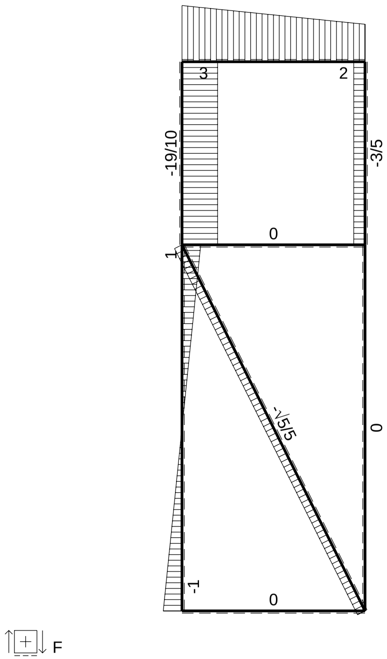


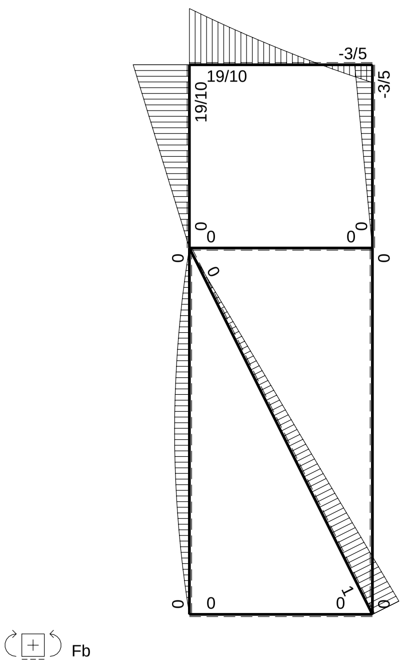


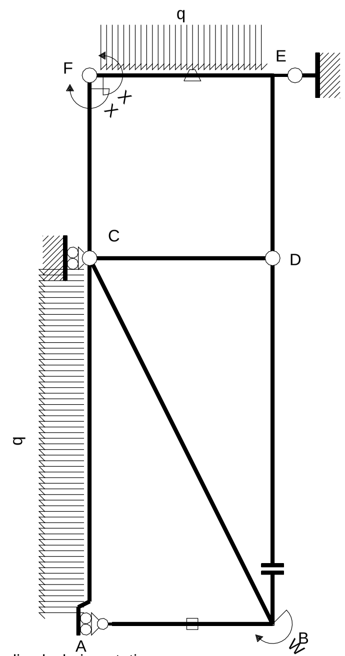
Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave BC ha la sezione riportata e dimensioni in mm, con:
b = 510 mm, F = 3540 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da B a C
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.



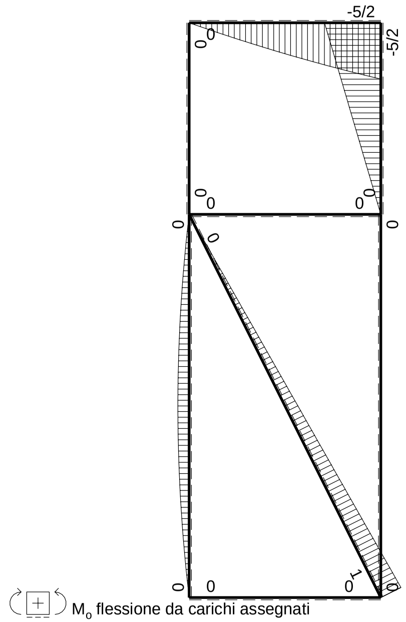




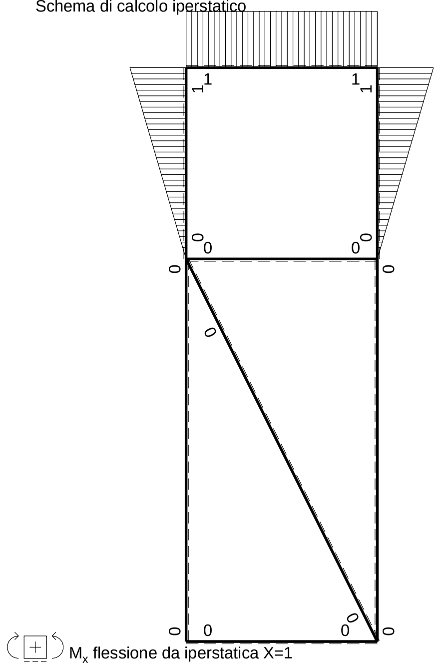




Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{FE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
ED b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	1	-5/2Fb+2Fx+1/2qx^2	-Fb/EJ	-5/2Fb+2Fx+1/2Fx^2/b	-Fb/EJ	1	(-4/3-1)Fb^2/EJ	Xb/EJ
FE b	-1	3Fx-1/2qx^2	Fb/EJ	-3Fx+1/2Fx^2/b	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x^2/b^2		
	totali						-19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{FE}$						19/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 218005 ESΣ09.xxxx.104

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2 +2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x + x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

$= (-5/2\ b + b +1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -7/3\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-3/2\ b +1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -7/3\ \ Fb^2/EJ$

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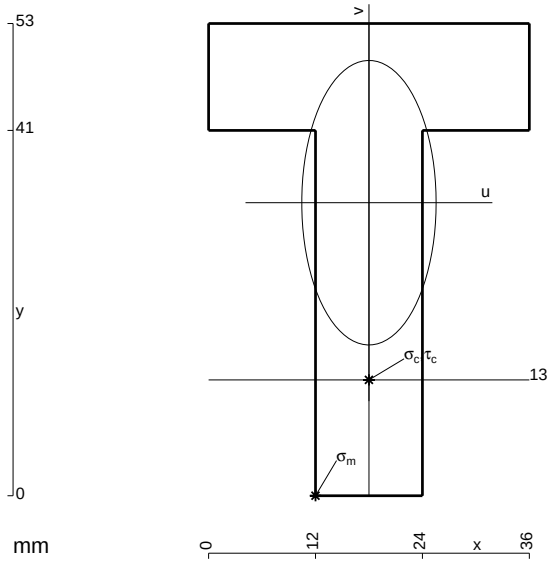

$A = 924.\ mm^2$
$J_u = 235641.\ mm^4$
$J_v = 52560.\ mm^4$
$y_g = 32.89$ mm
$N = -11082.$ N
$T_y = -1583.$ N
$M_x = 1805400.$ Nmm
$x_m = 12.$ mm
$u_m = -6.$ mm
$v_m = -32.89$ mm
$\sigma_m = N/A - Mv/J_u = 240.\ N/mm^2$
$x_c = 18.$ mm
$y_c = 13.$ mm
$v_c = -19.89$ mm
$\sigma_c = N/A - Mv/J_u = 140.4\ N/mm^2$
$\tau_c = 2.305\ N/mm^2$
$\sigma_0 = \sqrt{\sigma^2+3\tau^2} = 140.5\ N/mm^2$
$S^* = 4117.\ mm^3$

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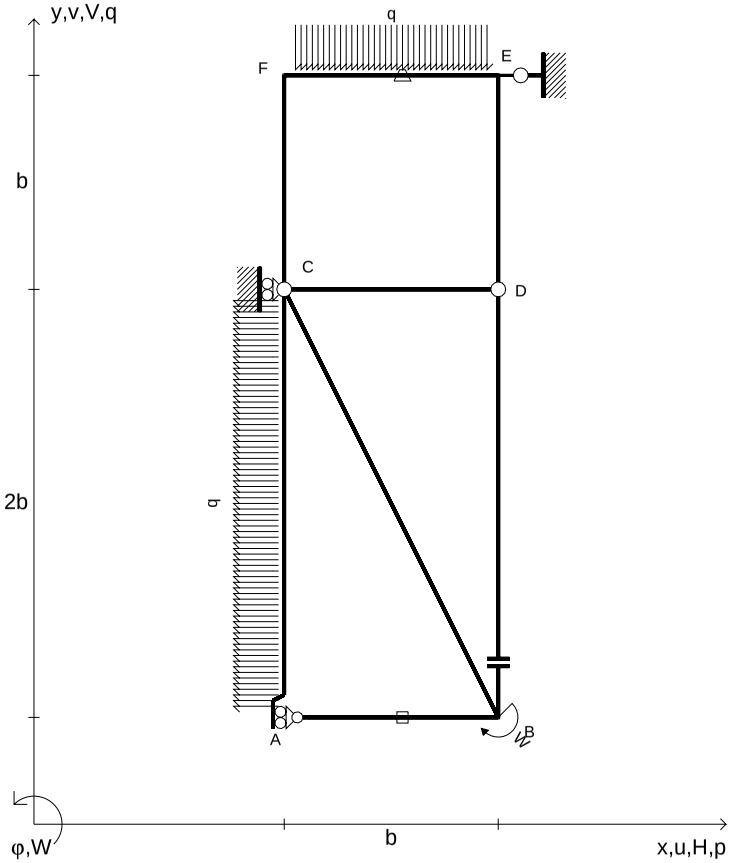
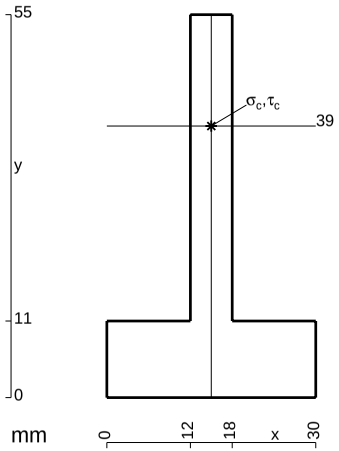
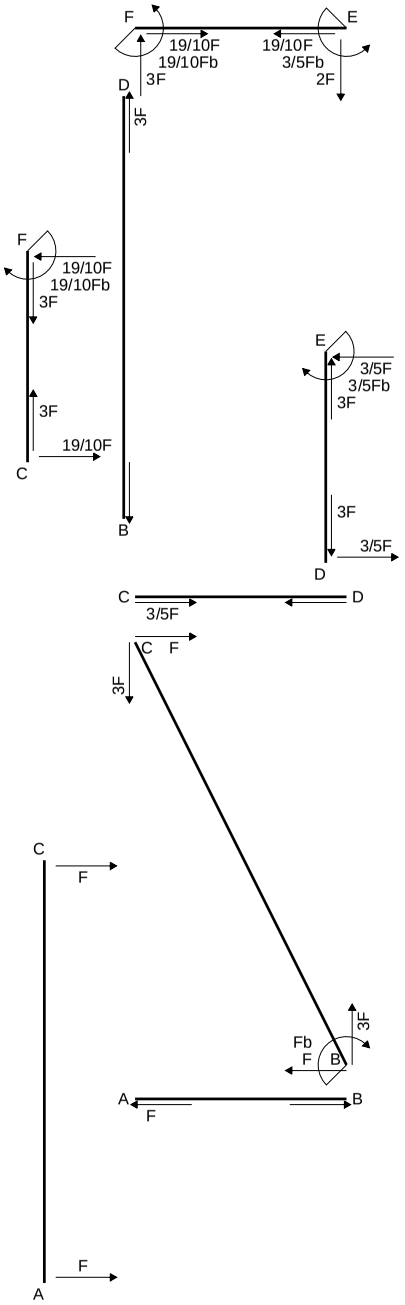
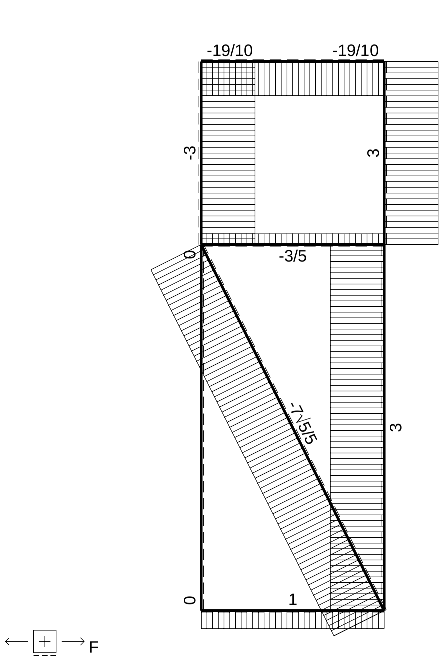
$W_B = -W = -Fb$
$p_{AC} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\varepsilon_{AB} = 3\alpha T = 3b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$

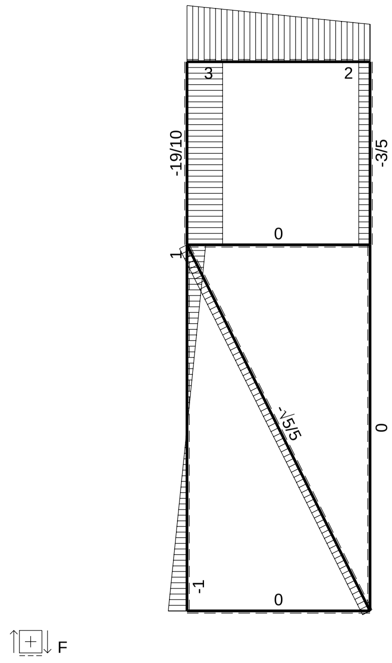


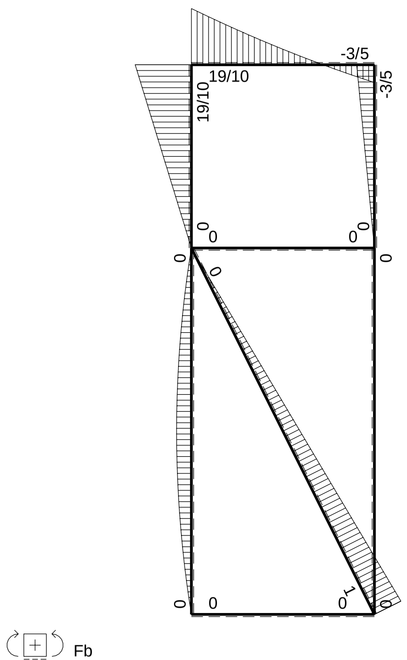


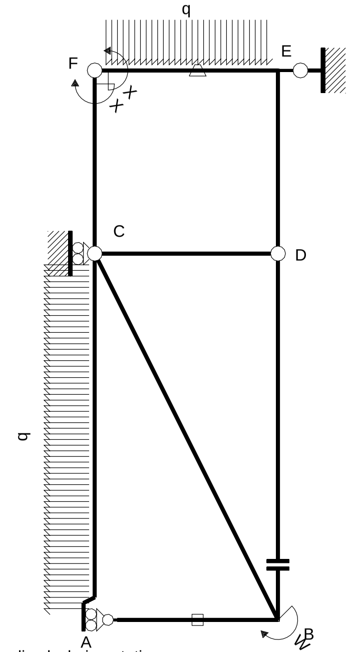
Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave BC ha la sezione riportata e dimensioni in mm, con:
b = 540 mm, F = 1790 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da B a C
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.



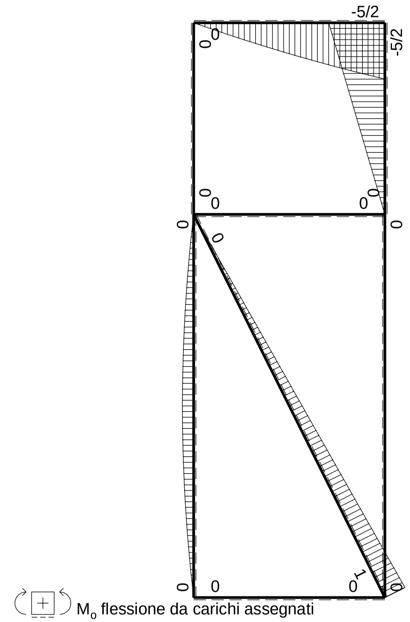


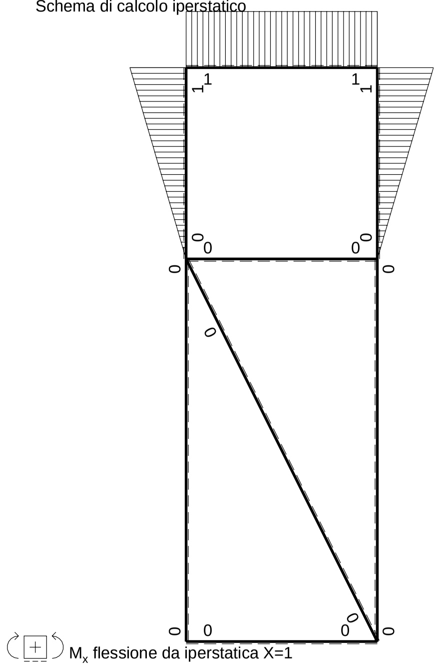






Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{FE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
ED b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	1	-5/2Fb+2Fx+1/2qx^2	-Fb/EJ	-5/2Fb+2Fx+1/2Fx^2/b	-Fb/EJ	1	(-4/3-1)Fb^2/EJ	Xb/EJ
FE b	-1	3Fx-1/2qx^2	Fb/EJ	-3Fx+1/2Fx^2/b	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x^2/b^2		
	totali						-19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{FE}$						19/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (b - b +1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [x]_o^b\ 1/EJ$

$= (b)\ 1/EJ = b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [x]_o^b\ 1/EJ$

$= (b)\ 1/EJ = b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (b - b +1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b)\ Fb\ 1/EJ = -5/6\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ = -5/6\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2 +2\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x + x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ + [- x]_o^b\ \theta$

$= (-5/2\ b + b +1/6\ b)\ Fb\ 1/EJ + (- b)\ \theta = -7/3\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3\ x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b +1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ + [x]_o^b\ \theta$

$= (-3/2\ b +1/6\ b)\ Fb\ 1/EJ + (b)\ \theta = -7/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 957847 ESΣ09.xxxx.124

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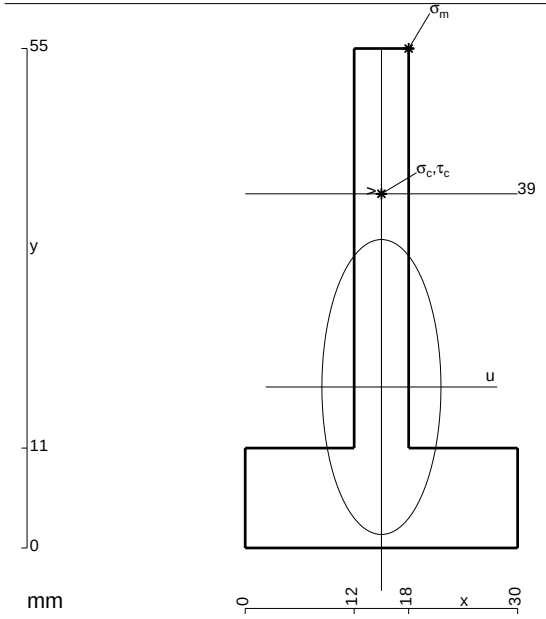

A = 594. mm^2
J_u = 156836. mm^4
J_v = 25542. mm^4
y_g = 17.72 mm
N = -5604. N
T_y = -800.5 N
M_x = 966600. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 37.28 mm
$\sigma_m = N/A - Mv/J_u$ = -239.2 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 21.28 mm
$\sigma_c = N/A - Mv/J_u$ = -140.6 N/mm^2
τ_c = 2.391 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 140.6 N/mm^2
S^* = 2811. mm^3

ESΣ09.xxxx.124
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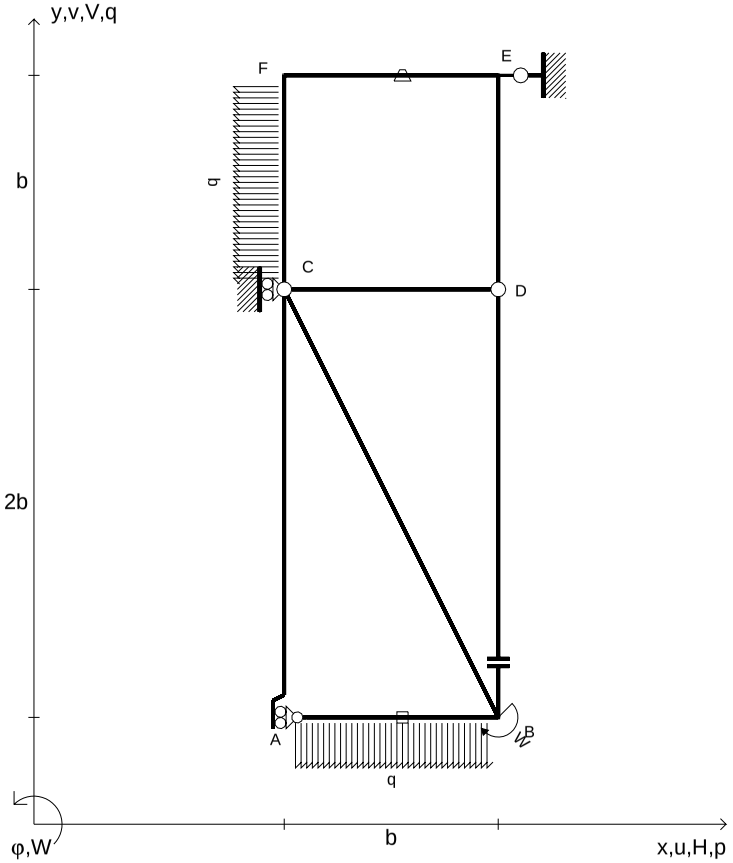
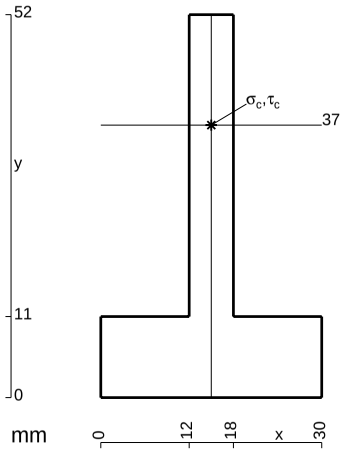
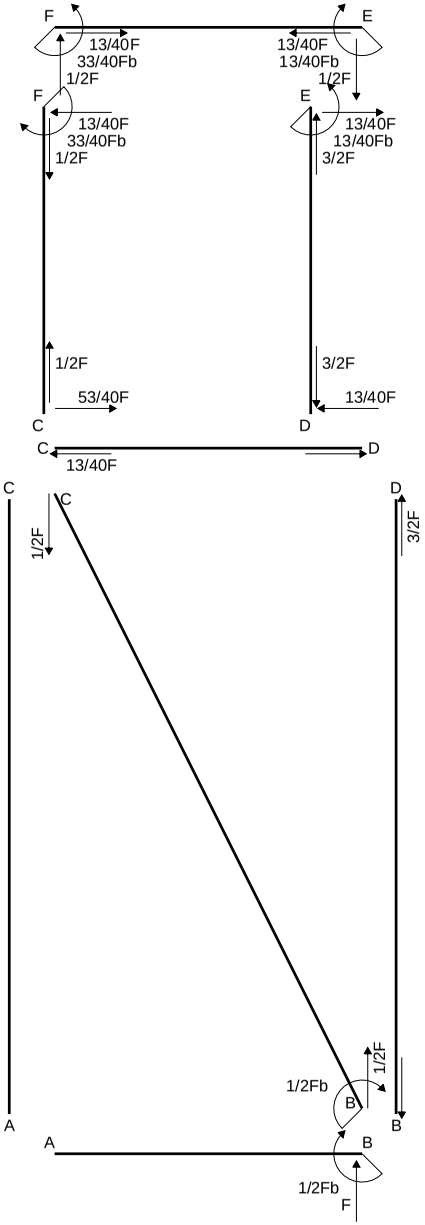
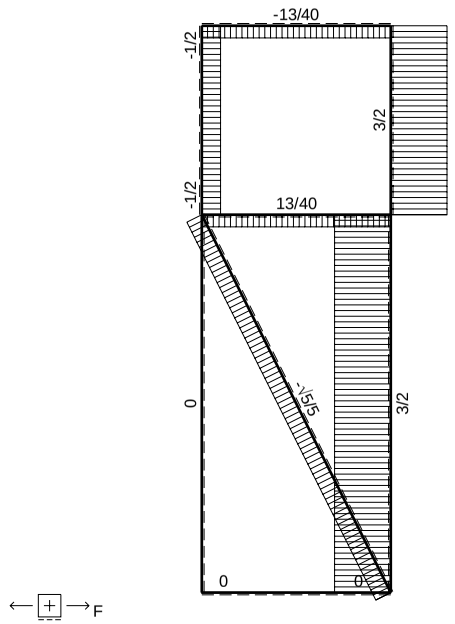
$W_B = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{FC} = -q = -F/b$
$\varepsilon_{AB} = -3\alpha T = -3b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$

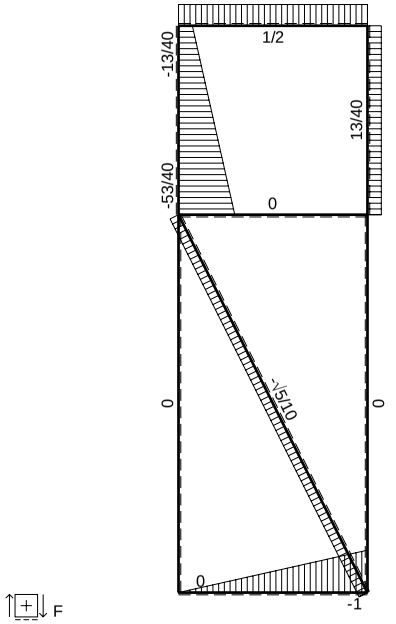


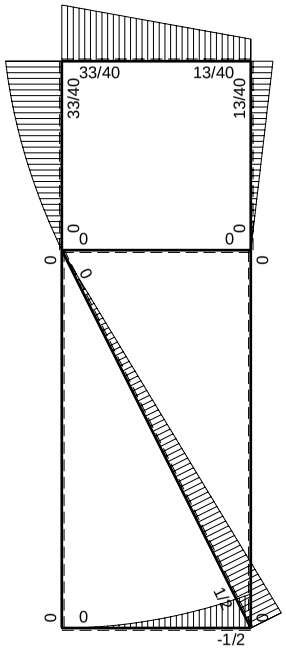


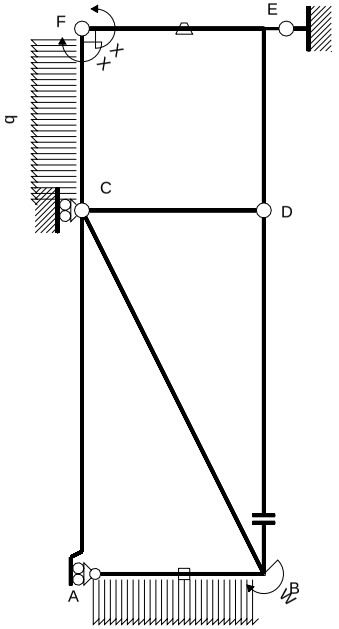
Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 370 mm, F = 4060 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.



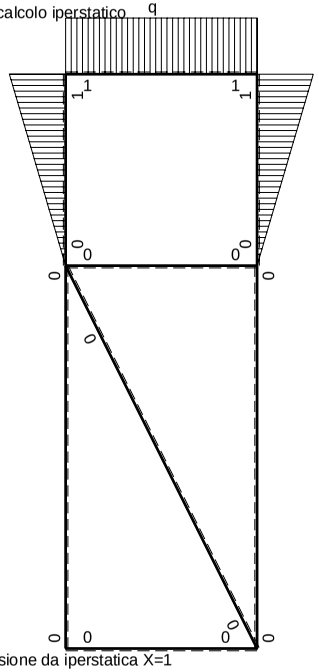


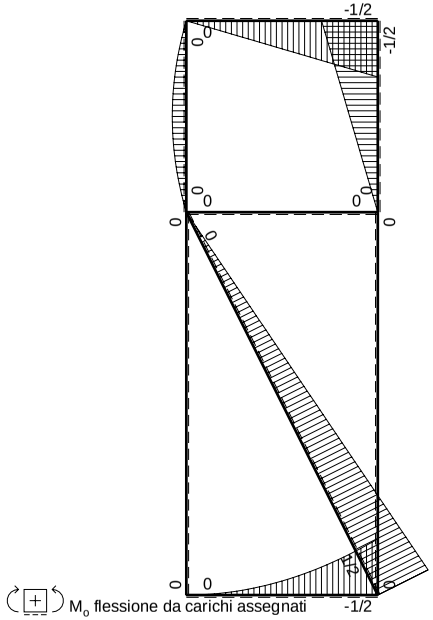






Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{FE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
ED b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	1	-1/2Fb+1/2Fx	-Fb/EJ	$-1/2Fb+1/2Fx$	-Fb/EJ	1	(-1/4-1)Fb^2/EJ	Xb/EJ
FE b	-1	1/2Fx	Fb/EJ	-1/2Fx	-Fb/EJ	1		
FC b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
		totali					-11/8Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{FE}$					33/40Fb	

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{EF}^{Xo} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -5/4\ \ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -5/4\ \ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 998772 ESΣ09.xxxx.140

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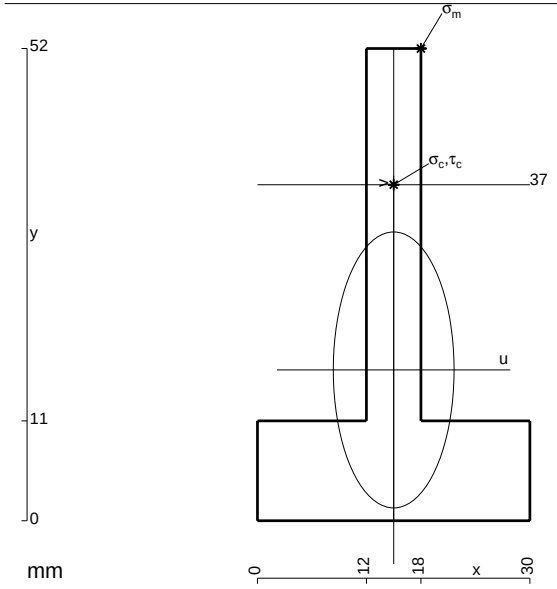

A = 576. mm^2
J_u = 133062. mm^4
J_v = 25488. mm^4
y_g = 16.6 mm
T_y = -4060. N
M_x = -751100. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 3. mm
v_m = 35.4 mm
$\sigma_m = -Mv/J_u$ = 199.8 N/mm^2
x_c = 15. mm
y_c = 37. mm
v_c = 20.4 mm
$\sigma_c = -Mv/J_u$ = 115.1 N/mm^2
τ_c = 12.77 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.2 N/mm^2
S^* = 2511. mm^3

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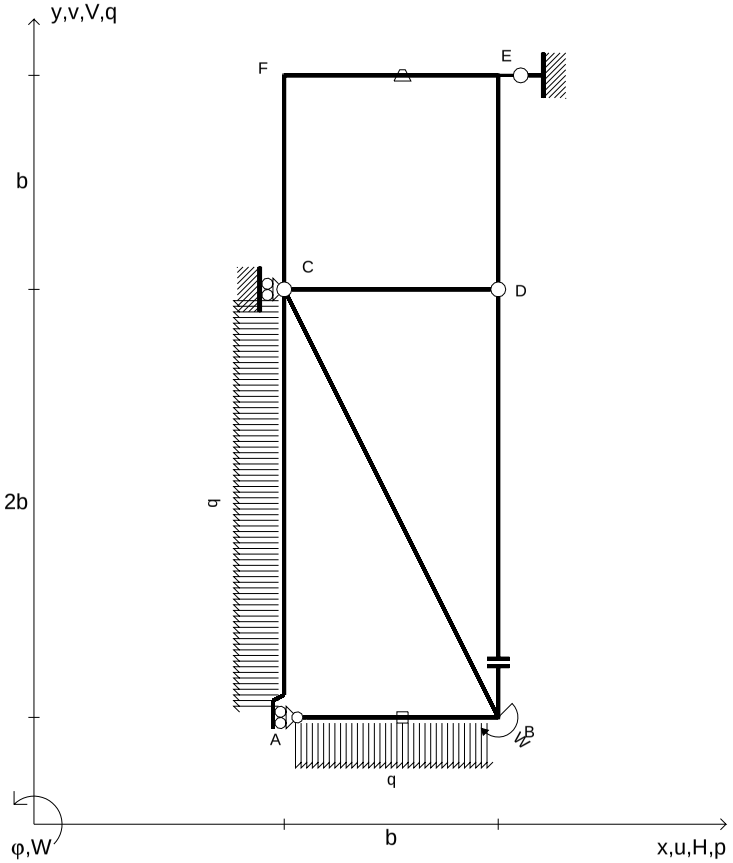
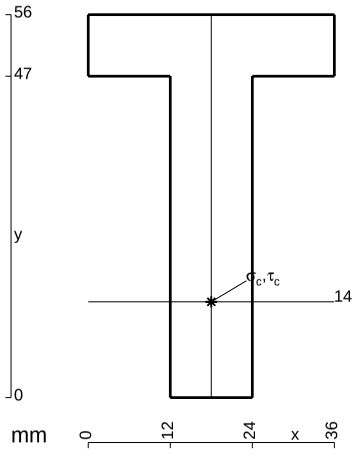
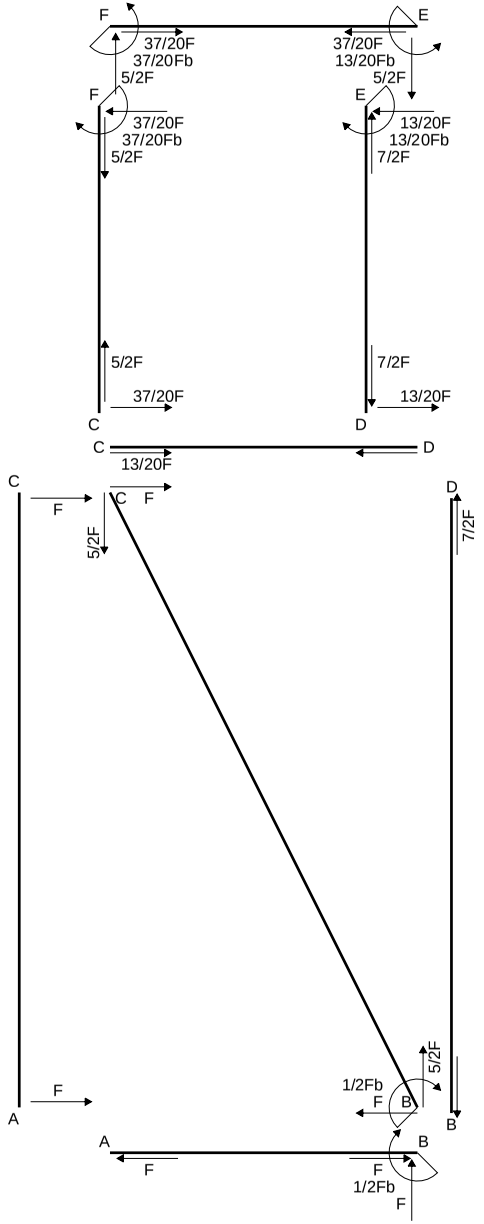
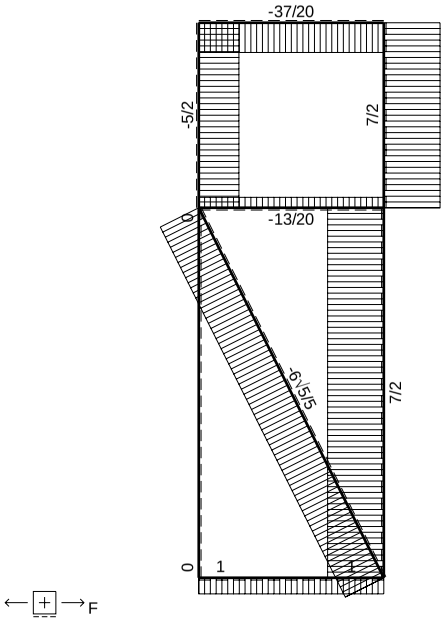
$W_B = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{AC} = -q = -F/b$
$\varepsilon_{AB} = -3\alpha T = -3b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$

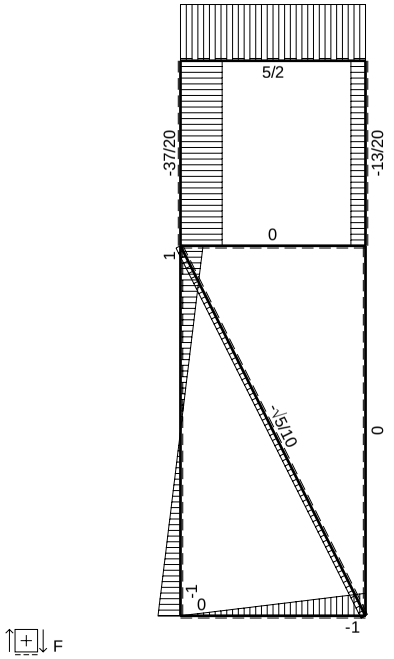


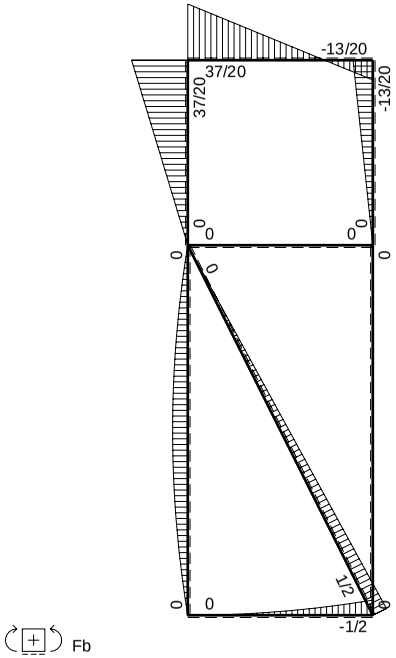


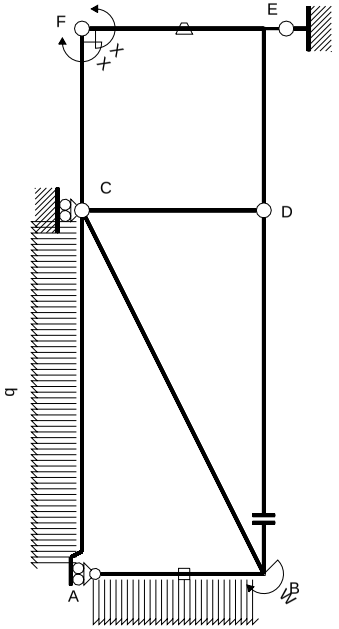
Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 420 mm, F = 8280 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.



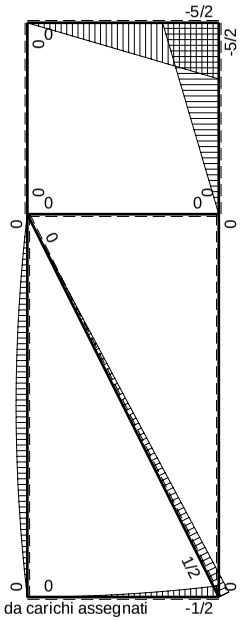


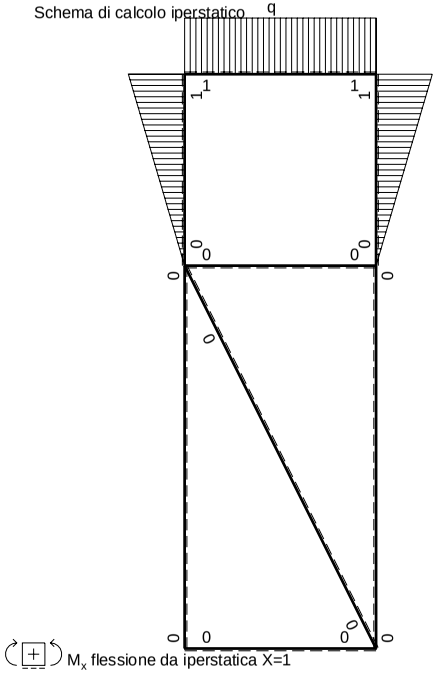






Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{FE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2qx^2	0	0	0	0	0+0	0
BA b	0	1/2Fb-Fx+1/2qx^2	0	0	0	0		
BC $\sqrt{5}$b	0	1/2Fb-$\sqrt{5}$/10Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
ED b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	1	-5/2Fb+5/2Fx	-Fb/EJ	-5/2Fb+5/2Fx	-Fb/EJ	1	(-5/4-1)Fb^2/EJ	Xb/EJ
FE b	-1	5/2Fx	Fb/EJ	-5/2Fx	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x^2/b^2		
	totali						-37/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{FE}$						37/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 994726 ESΣ09.xxxx.151

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ \ = -9/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = -9/4\ \ Fb^2/EJ$

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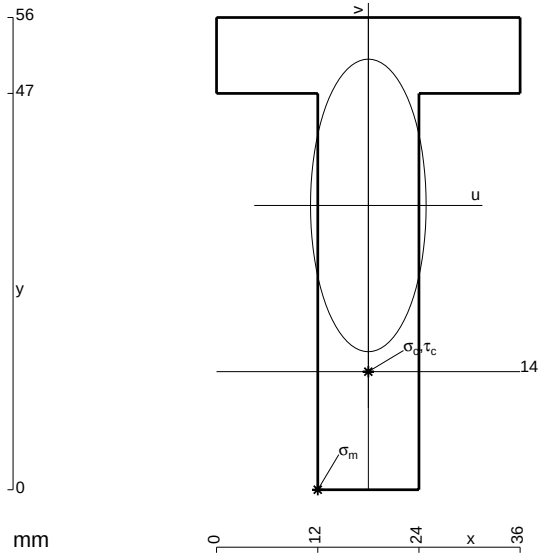

$A = 888.\ mm^2$
$J_u = 267344.\ mm^4$
$J_v = 41760.\ mm^4$
$y_g = 33.72\ mm$
$N = 8280.\ N$
$T_y = -8280.\ N$
$M_x = -1738800.\ Nmm$
$x_m = 12.\ mm$
$u_m = -6.\ mm$
$v_m = -33.72\ mm$
$\sigma_m = N/A - Mv/J_u = -210.\ N/mm^2$
$x_c = 18.\ mm$
$y_c = 14.\ mm$
$v_c = -19.72\ mm$
$\sigma_c = N/A - Mv/J_u = -118.9\ N/mm^2$
$\tau_c = 11.58\ N/mm^2$
$\sigma_{id} = \sqrt{\sigma^2+3\tau^2} = 120.6\ N/mm^2$
$S^* = 4488.\ mm^3$

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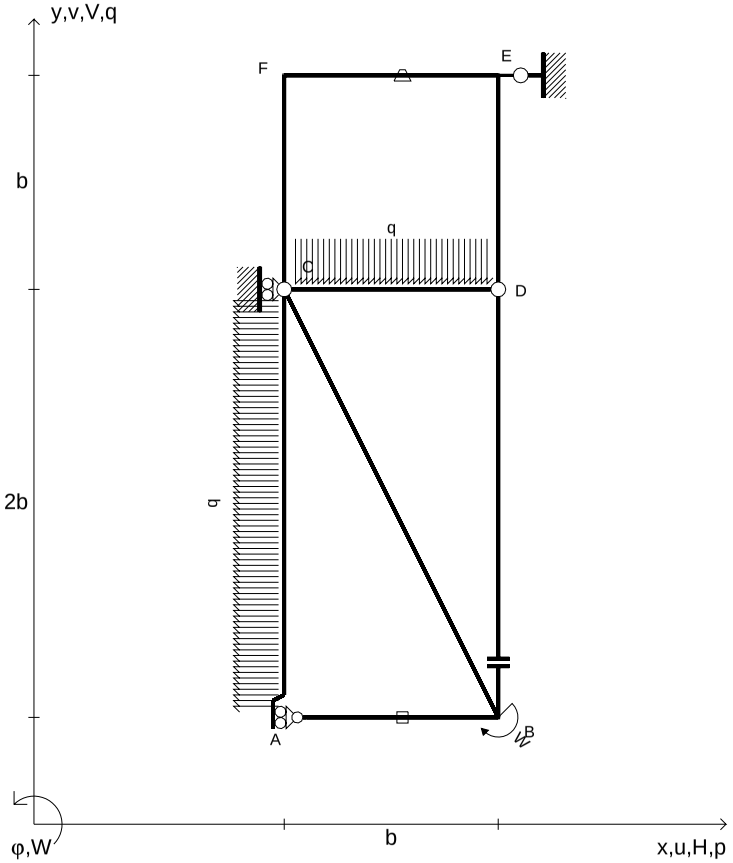
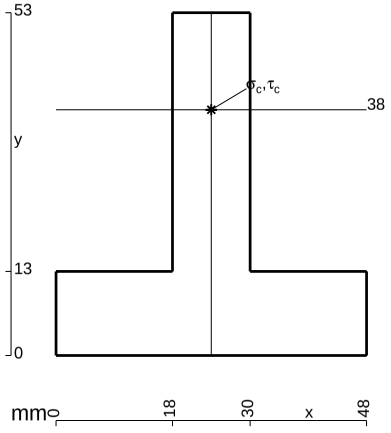
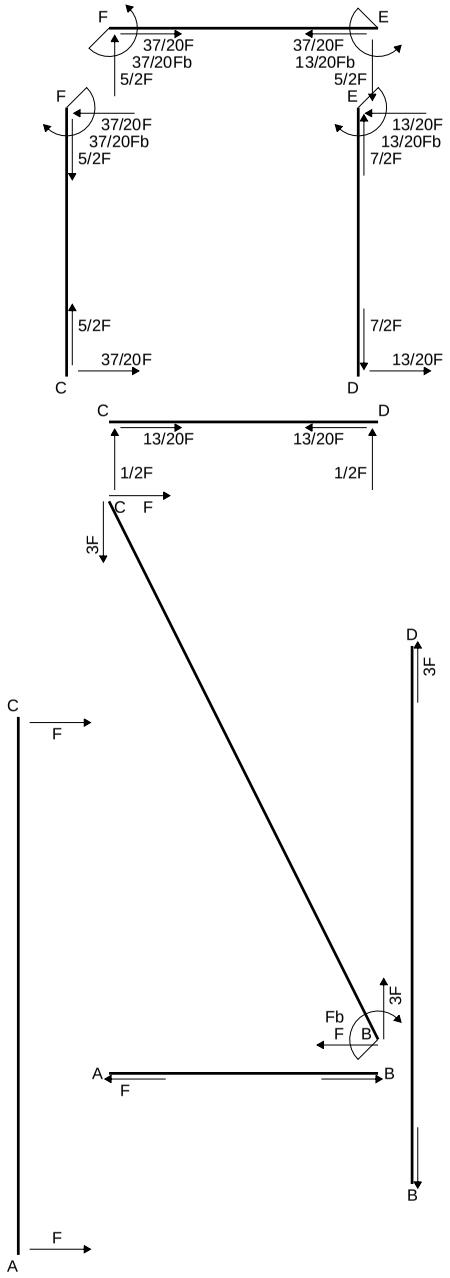
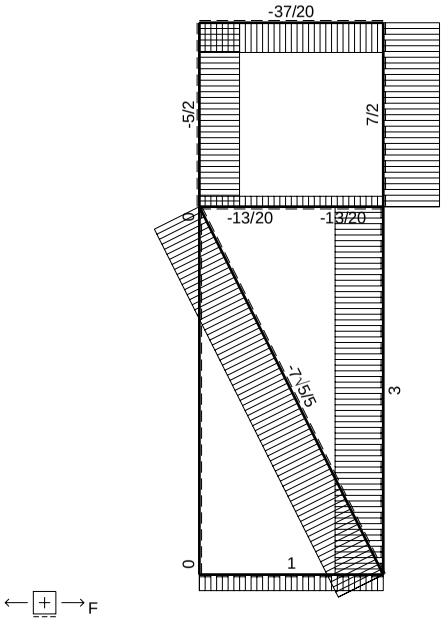
$W_B = -W = -Fb$
$p_{AC} = -q = -F/b$
$q_{CD} = -q = -F/b$
$\varepsilon_{AB} = 2\alpha T = 2b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$





Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave BC ha la sezione riportata e dimensioni in mm, con:
b = 400 mm, F = 3750 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da B a C
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.





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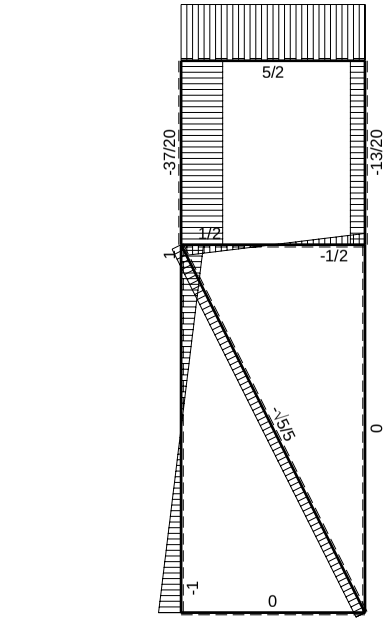
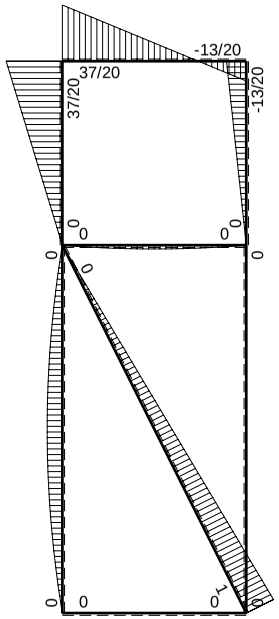
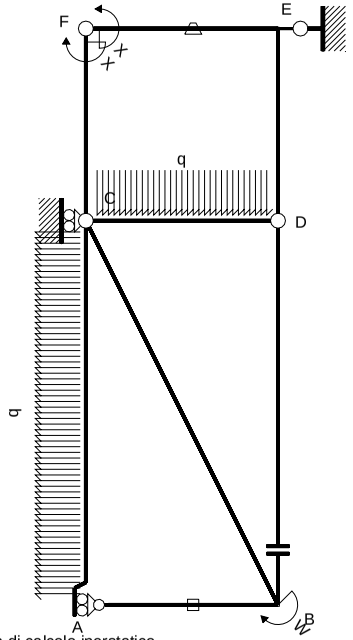


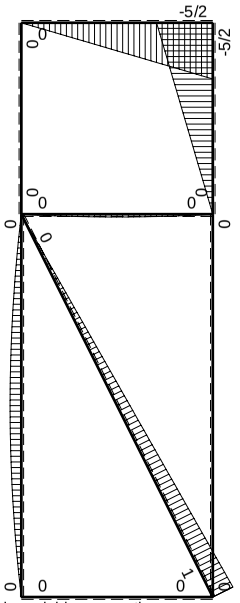
F



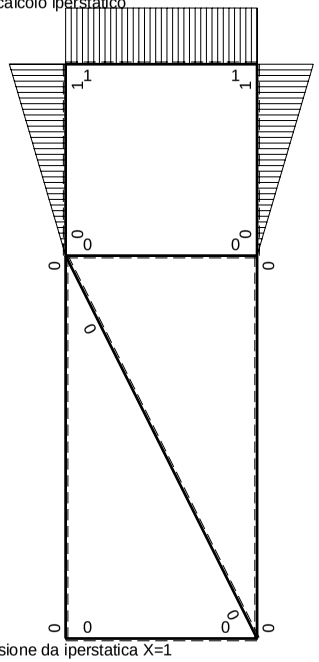
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{FE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
ED b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	1	-5/2Fb+5/2Fx	-Fb/EJ	-5/2Fb+5/2Fx	-Fb/EJ	1	(-5/4-1)Fb^2/EJ	Xb/EJ
FE b	-1	5/2Fx	Fb/EJ	-5/2Fx	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x^2/b^2		
	totali						-37/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{FE}$						37/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ \ = -9/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = -9/4\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ09.xxxx.191

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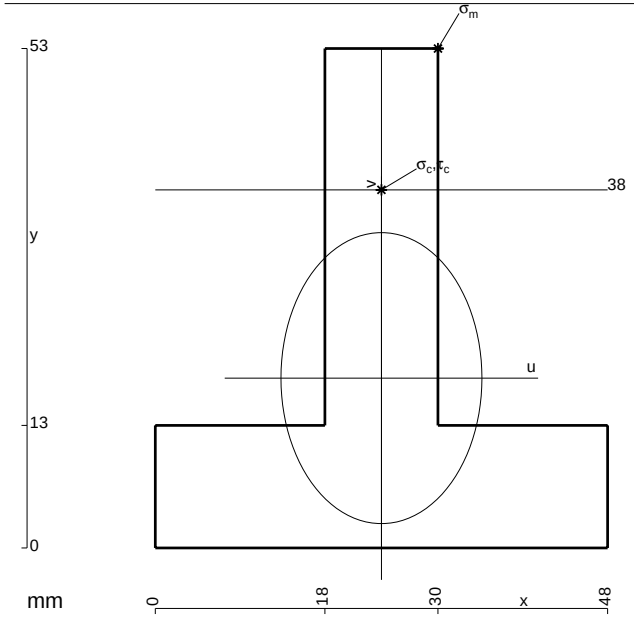

A = 1104. mm^2
J_u = 263311. mm^4
J_v = 125568. mm^4
y_g = 18.02 mm
N = -11739. N
T_y = -1677. N
M_x = 1500000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 34.98 mm
$\sigma_m = N/A - Mv/J_u$ = -209.9 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.98 mm
$\sigma_c = N/A - Mv/J_u$ = -124.4 N/mm^2
τ_c = 2.625 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 124.5 N/mm^2
S^* = 4946. mm^3

ESΣ09.xxxx.191

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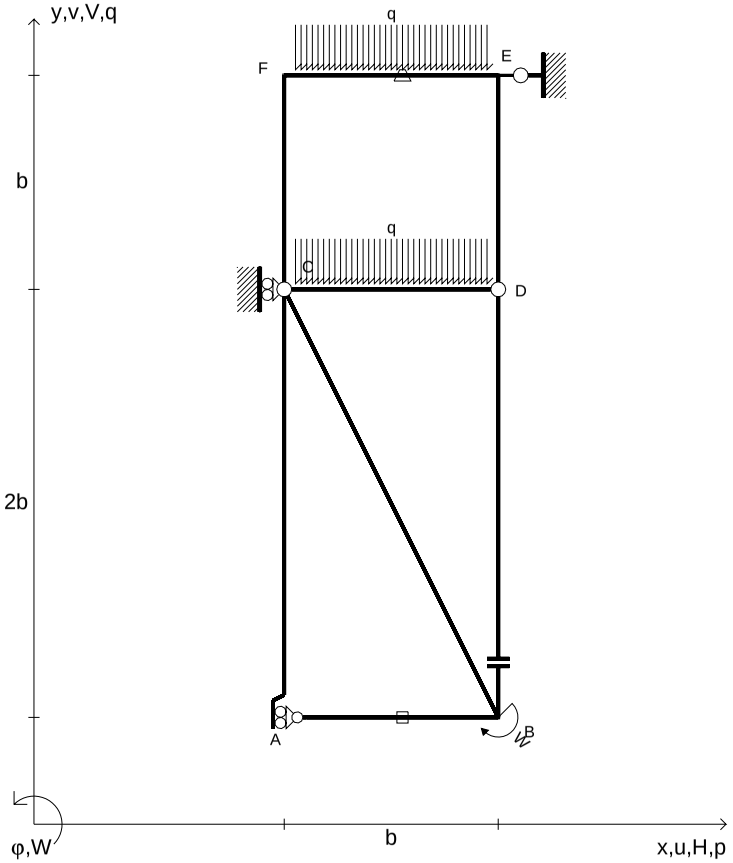
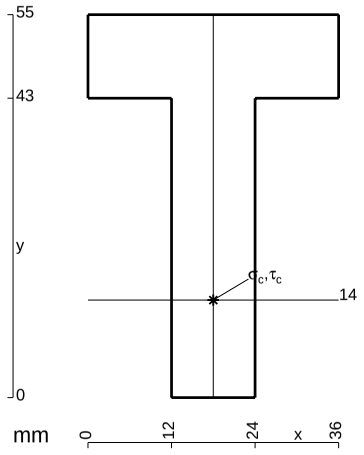
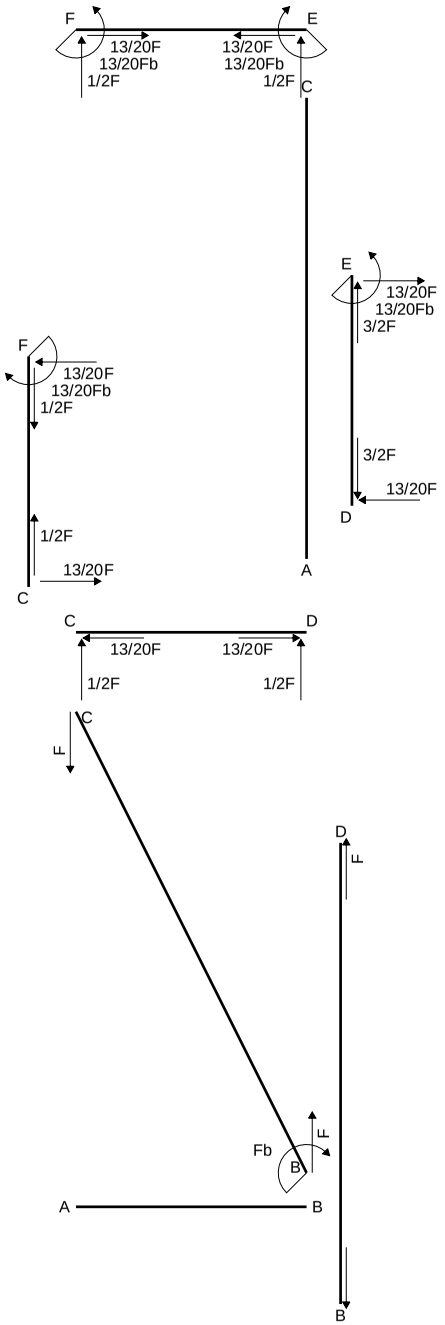
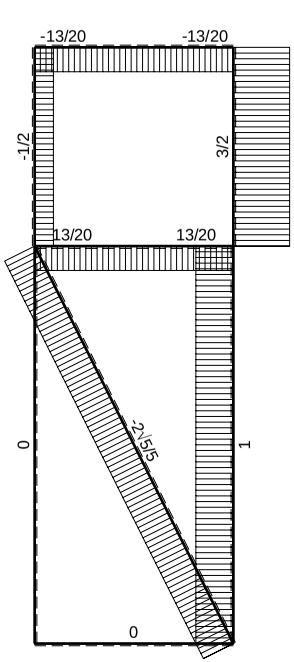
$W_B = -W = -Fb$
$q_{EF} = -q = -F/b$
$q_{CD} = -q = -F/b$
$\varepsilon_{AB} = -4\alpha T = -4b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$

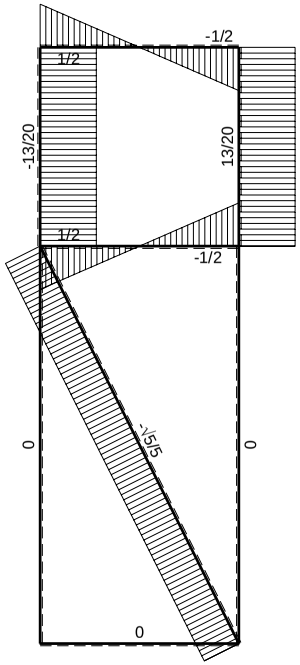


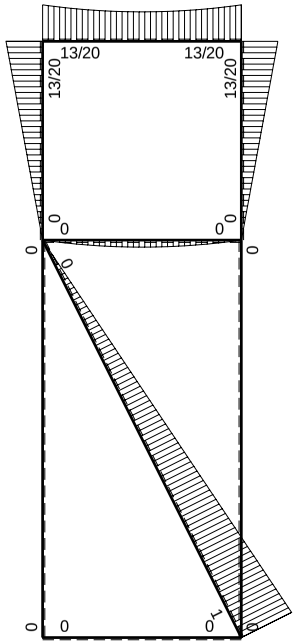


Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave BC ha la sezione riportata e dimensioni in mm, con:
b = 390 mm, F = 4030 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da B a C
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.

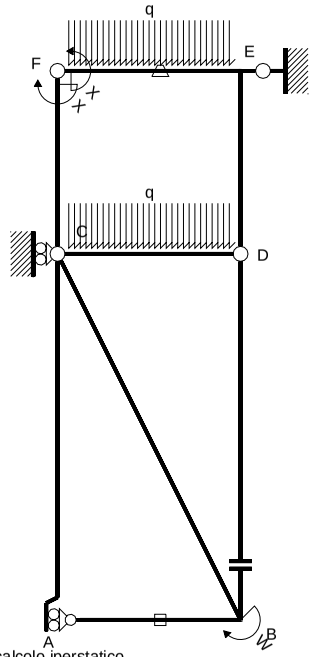




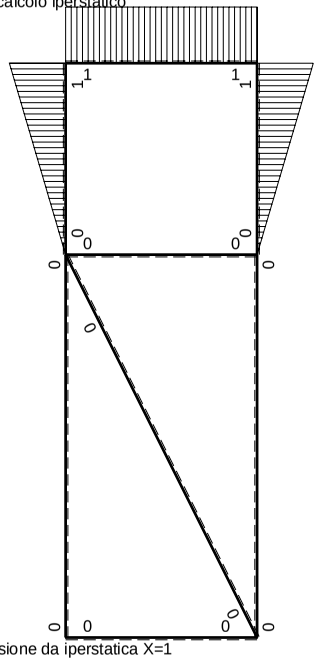




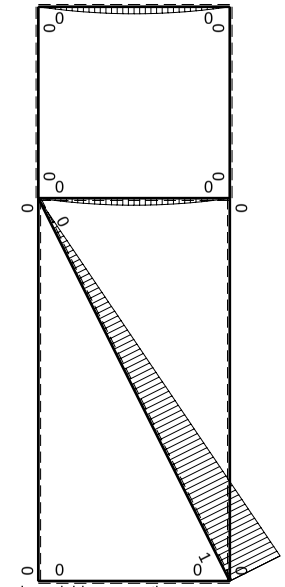
←⊞→ F ↑⊞↓ F ↻⊞↺ Fb



Schema di calcolo iperstatico



↻⊞↺ M_x flessione da iperstatica X=1



↻⊞↺ M_0 flessione da carichi assegnati

Quadro contributi PLV per iperstatica $X=W_{FE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	1	-1/2Fx+1/2qx^2	-Fb/EJ	-1/2Fx+1/2Fx^2/b	-Fb/EJ	1	(-1/12-1)Fb^2/EJ	Xb/EJ
FE b	-1	1/2Fx-1/2qx^2	Fb/EJ	-1/2Fx+1/2Fx^2/b	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x^2/b^2		
	totali						-13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{FE}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ09.xxxx.200

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx$

$= [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx$

$= [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -13/12\ Fb^2/EJ$

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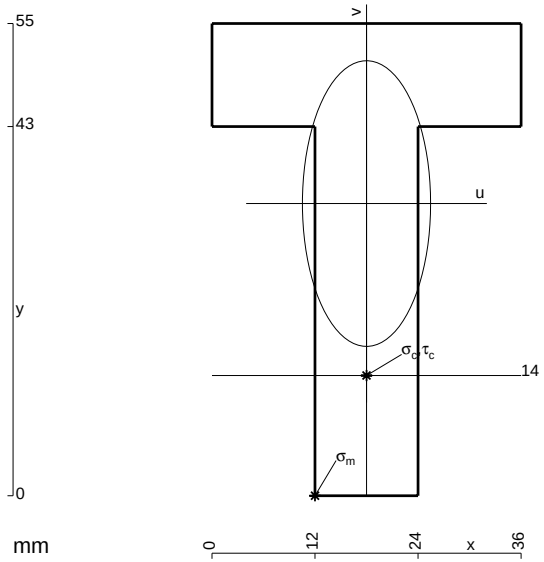

A = 948. mm^2
J_u = 262515. mm^4
J_v = 52848. mm^4
y_g = 34.03 mm
N = -3605. N
T_y = -1802. N
M_x = 1571700. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -34.03 mm
$\sigma_m = N/A - Mv/J_u$ = 199.9 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -20.03 mm
$\sigma_c = N/A - Mv/J_u$ = 116.1 N/mm^2
τ_c = 2.598 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.2 N/mm^2
S^* = 4541. mm^3

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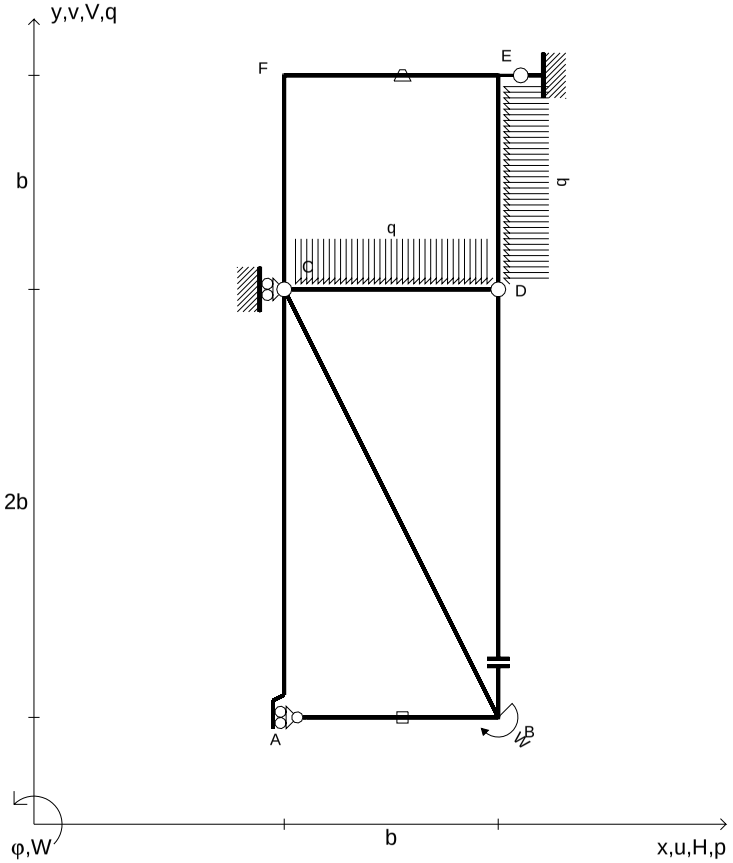
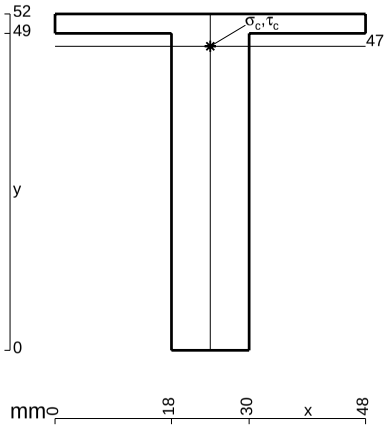
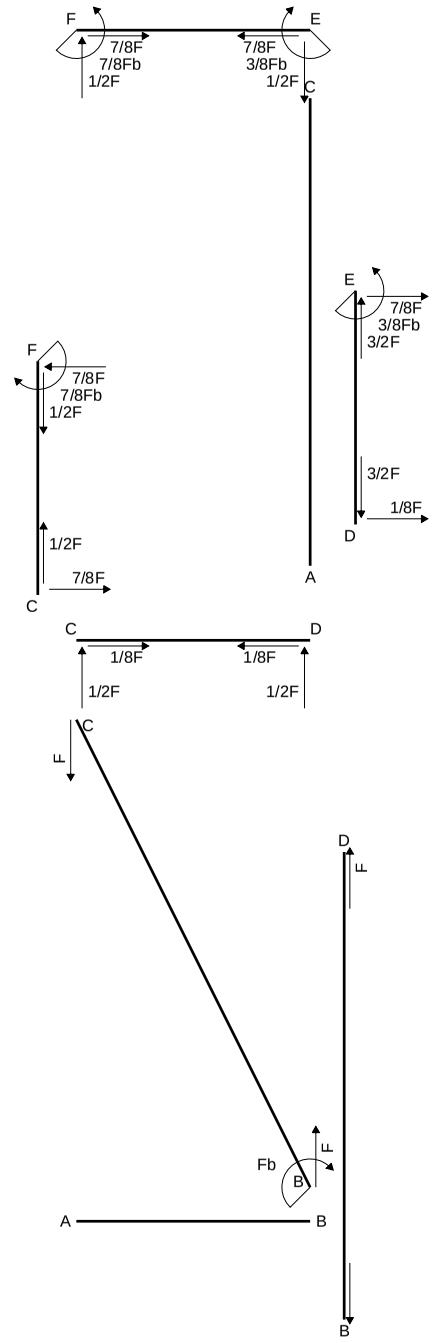
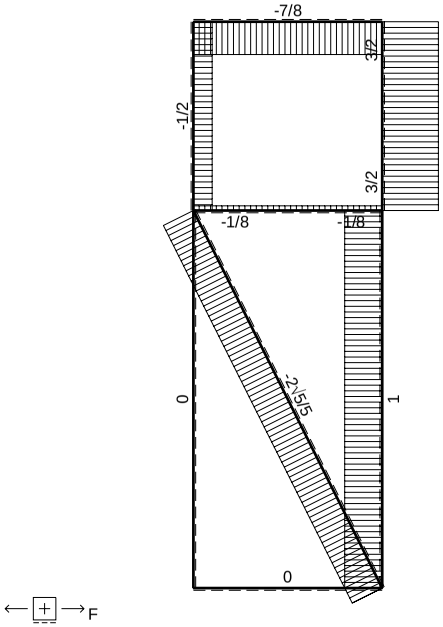
$W_B = -W = -Fb$
$p_{DE} = -q = -F/b$
$q_{CD} = -q = -F/b$
$\varepsilon_{AB} = -4\alpha T = -4b^2F/EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{AC} = EJ$
$EJ_{DB} = EJ$
$EJ_{DE} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$

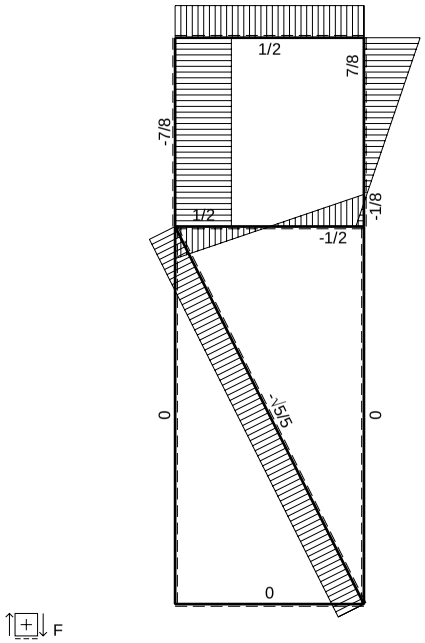


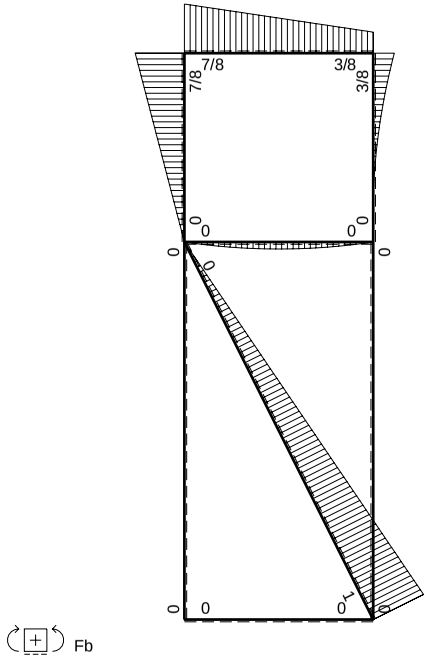


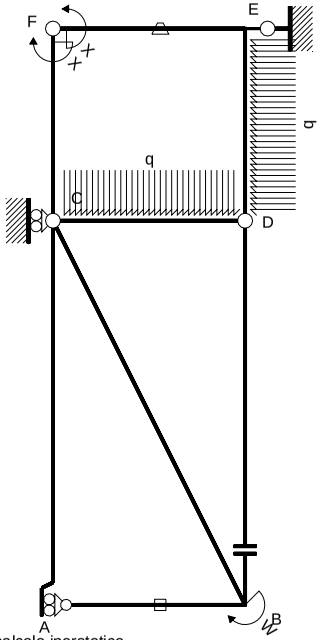
Reazioni iperstatiche in soluzione: $X=W_{FE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave BC ha la sezione riportata e dimensioni in mm, con:
b = 400 mm, F = 3540 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da B a C
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta EF positiva se convessa a destra con inizio E.



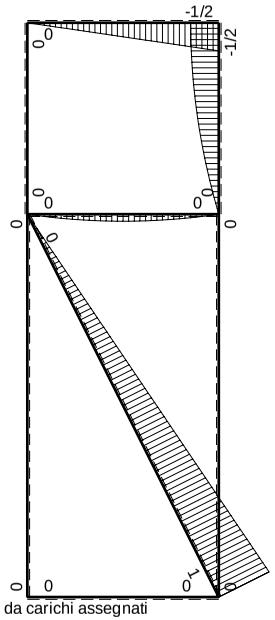


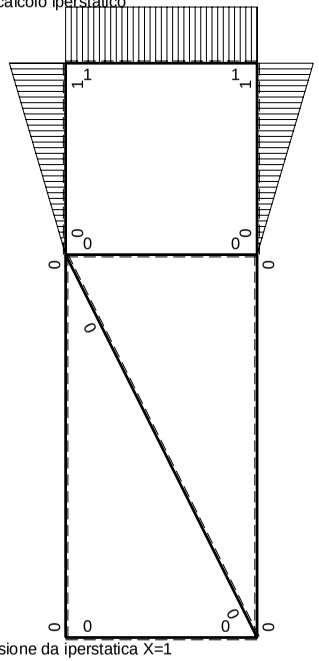






Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica X=W_{FE}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	x/b	-Fx+1/2qx²	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	-1+x/b	1/2Fb-1/2qx²	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	1/2Fx-1/2qx²	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx²	0	0	0	0		
EF b	1	-1/2Fb+1/2Fx	-Fb/EJ	-1/2Fb+1/2Fx	-Fb/EJ	1	$(-1/4-1)Fb^2/EJ$	Xb/EJ
FE b	-1	1/2Fx	Fb/EJ	-1/2Fx	-Fb/EJ	1		
FC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	-x/b	0	0	0	0	x^2/b^2		
	totali						$-35/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica X=W_{FE}						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -5/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -5/4\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ09.xxxx.211

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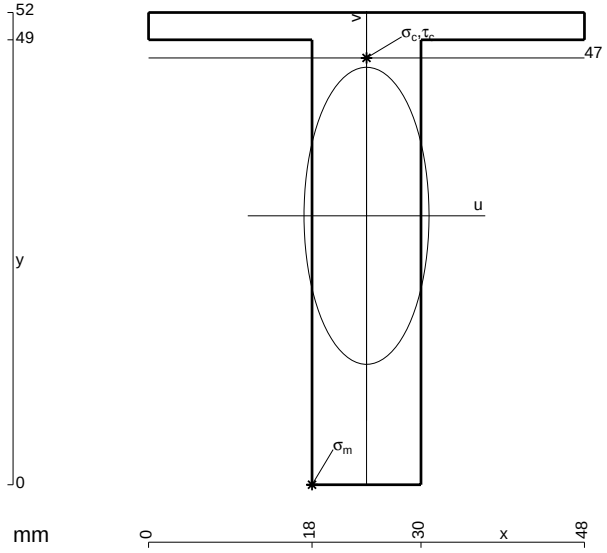

A = 732. mm^2
J_u = 195951. mm^4
J_v = 34704. mm^4
y_g = 29.61 mm
N = -3166. N
T_y = -1583. N
M_x = 1416000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -29.61 mm
$\sigma_m = N/A - Mv/J_u$ = 209.7 N/mm^2
x_c = 24. mm
y_c = 47. mm
v_c = 17.39 mm
$\sigma_c = N/A - Mv/J_u$ = -130. N/mm^2
τ_c = 2.322 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130. N/mm^2
S^* = 3449. mm^3

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