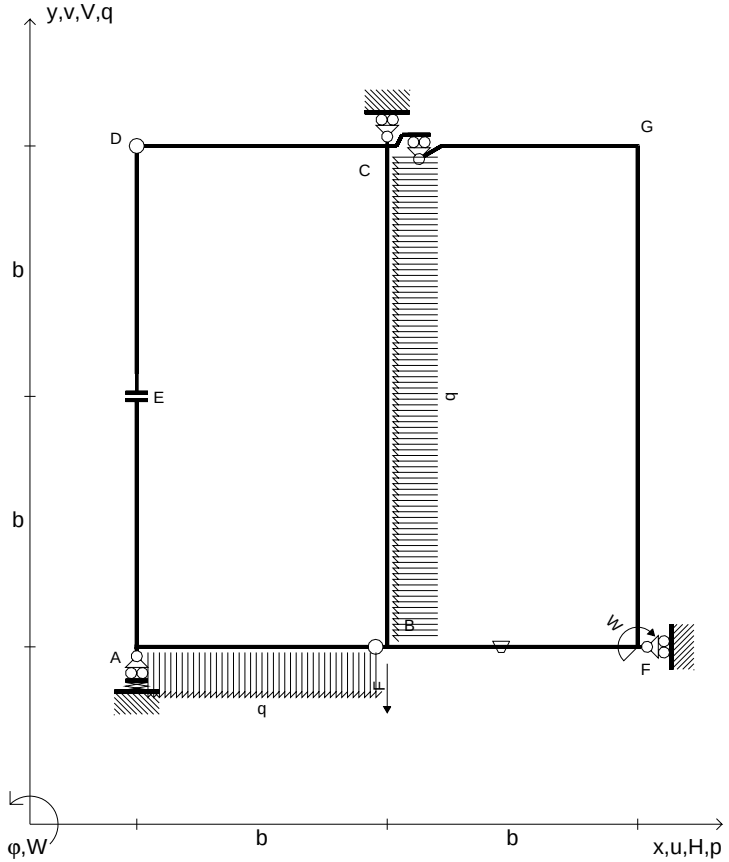
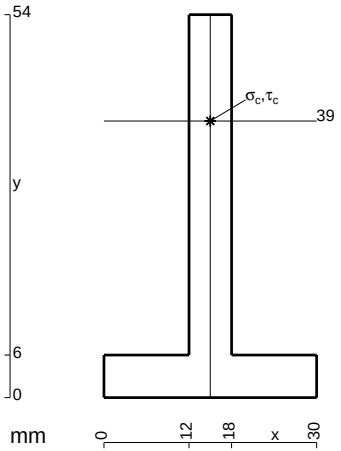
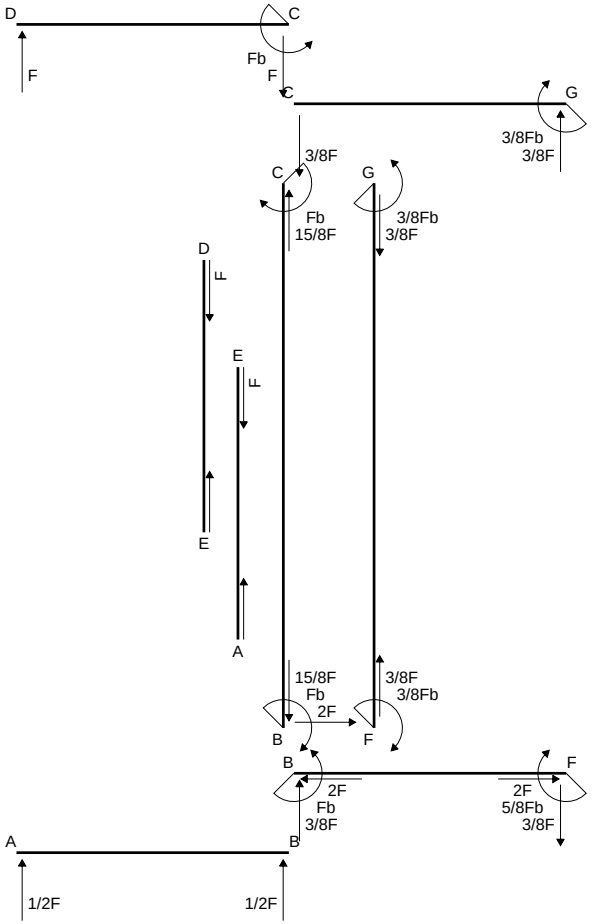
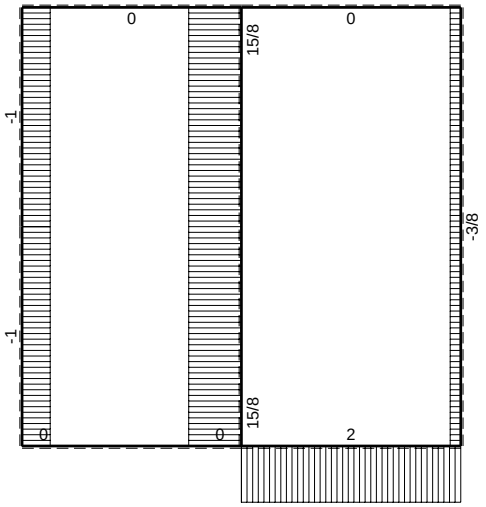
$V_B = -F$
$W_F = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{CB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



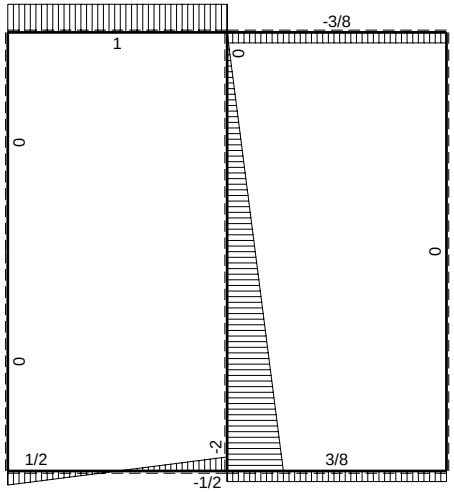


Reazioni iperstatiche in soluzione: $X=W_{GC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 430 mm, F = 1780 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

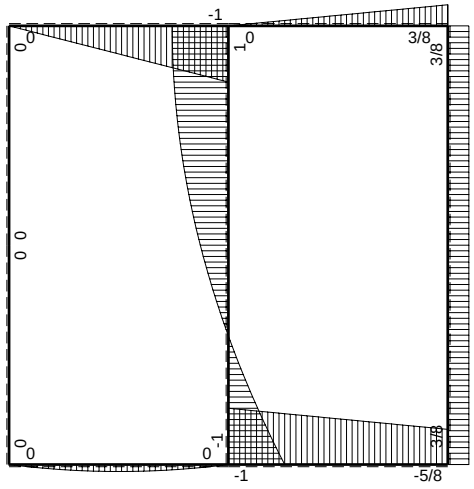




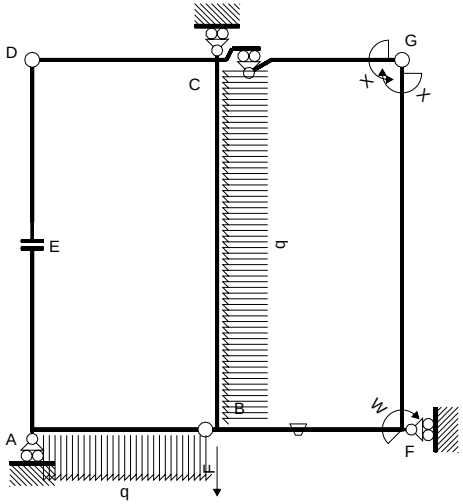
←[+]→ F



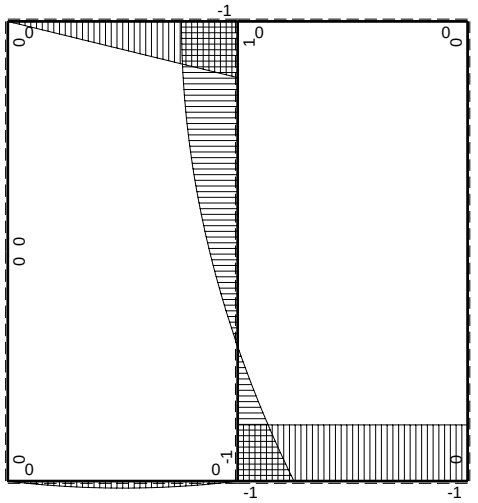
↑[+]↓ F



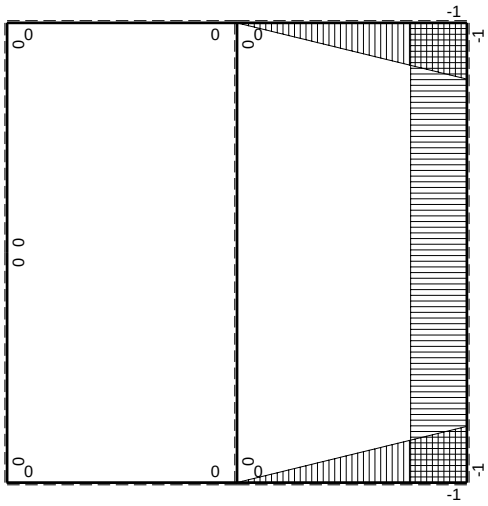
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x\theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx^2	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

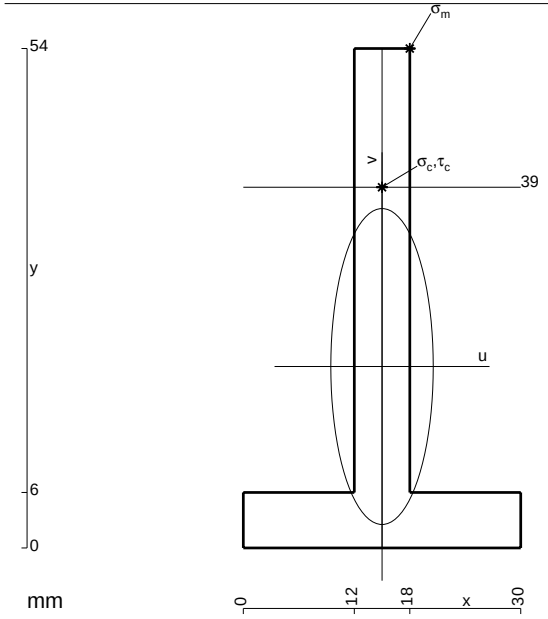
$= (2\ b)\ 1/EJ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ + [1/2\ x^2/b]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ + (1/2\ b)\ \theta = Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ + [- x +1/2\ x^2/b]_o^b\ \theta$

$= (\ b -1/2\ b)\ Fb\ 1/EJ + (- b +1/2\ b)\ \theta = Fb^2/EJ$



A = 468. mm^2
J_u = 136587. mm^4
J_v = 14364. mm^4
y_g = 19.62 mm
N = 3338. N
T_y = -3560. N
M_x = -765400. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 34.38 mm
$\sigma_m = N/A - Mv/J_u$ = 199.8 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 19.38 mm
$\sigma_c = N/A - Mv/J_u$ = 115.8 N/mm^2
τ_c = 10.51 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.2 N/mm^2
S^* = 2420. mm^3

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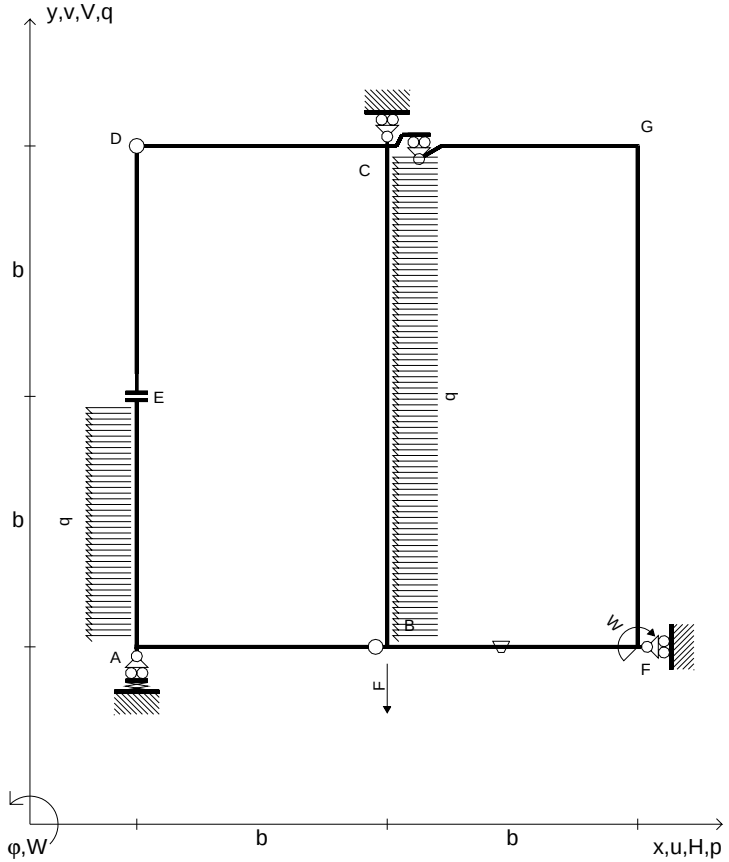
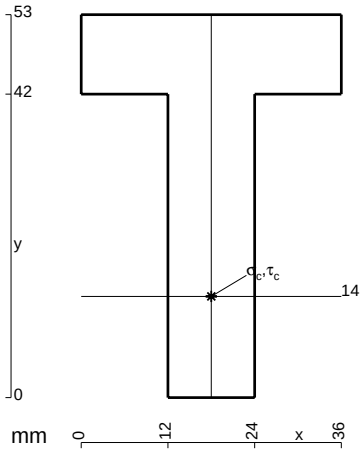
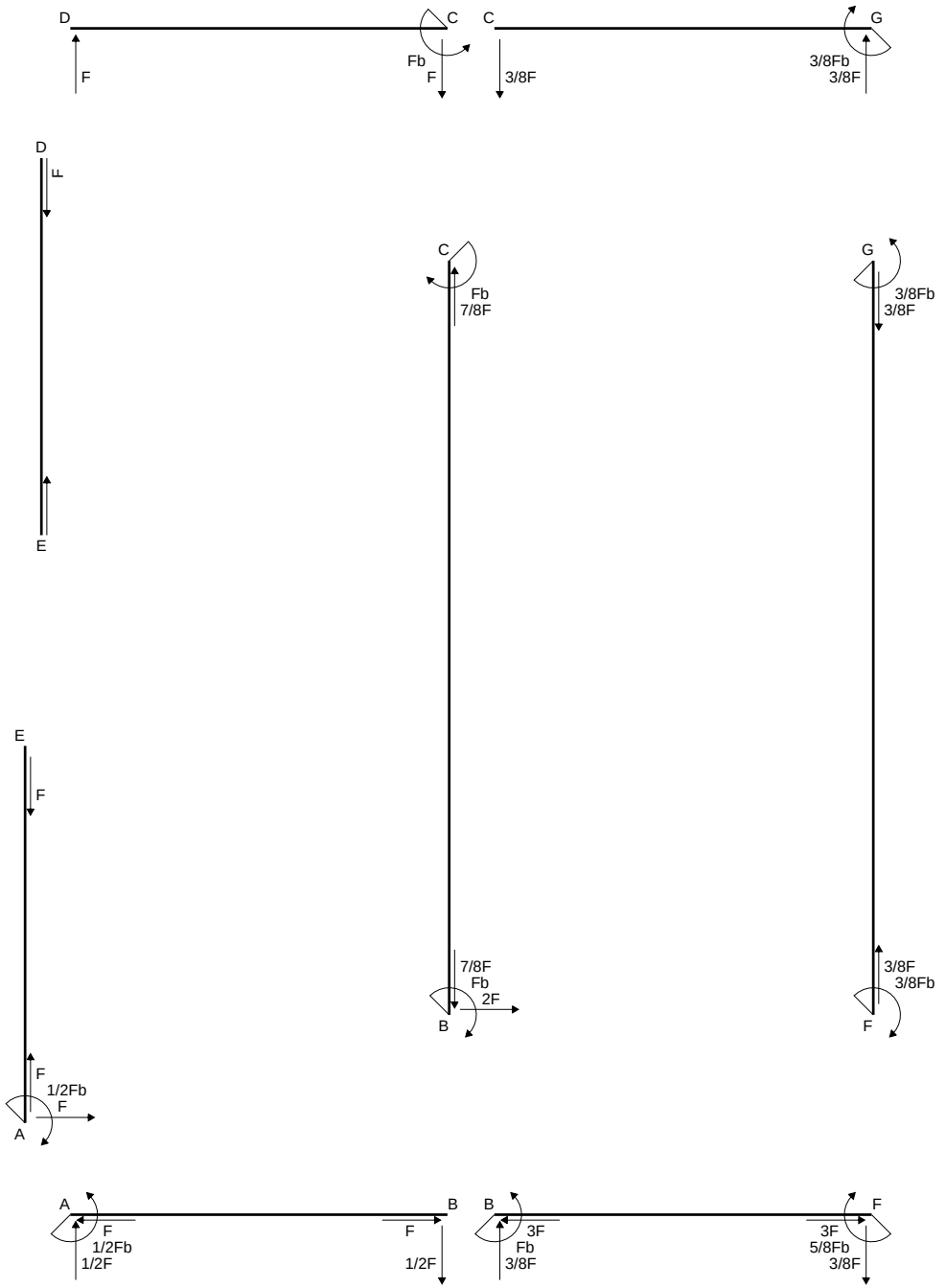
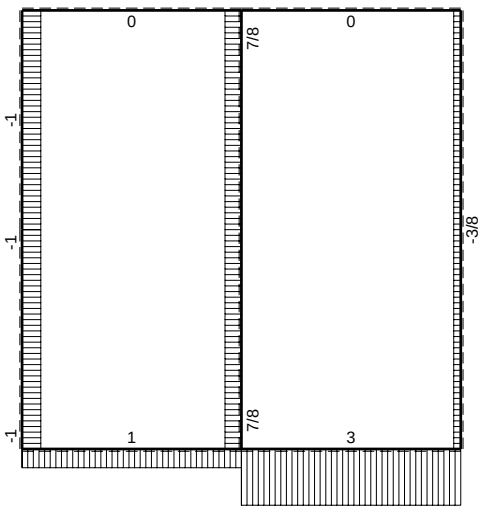
$V_B = -F$
$W_F = -W = -Fb$
$p_{EA} = -q = -F/b$
$p_{CB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$

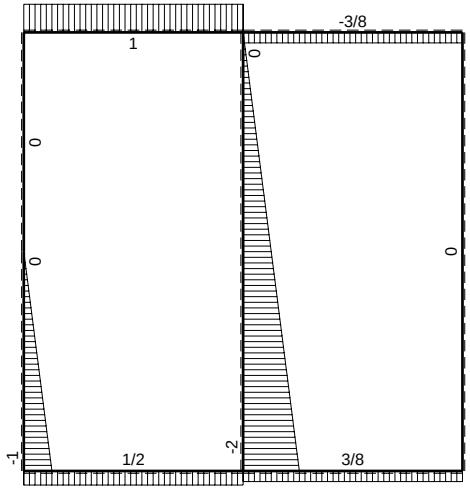




Reazioni iperstatiche in soluzione: $X=W_{GC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 580 mm, F = 2990 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

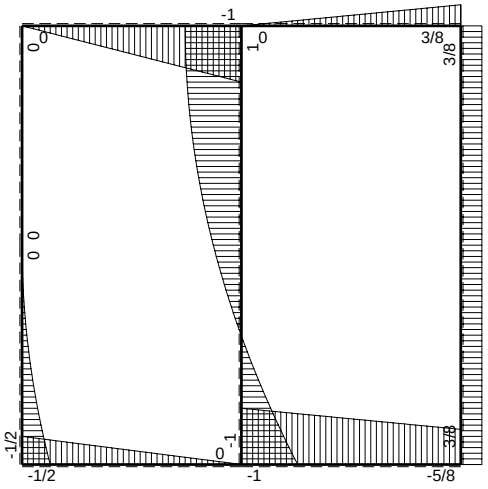




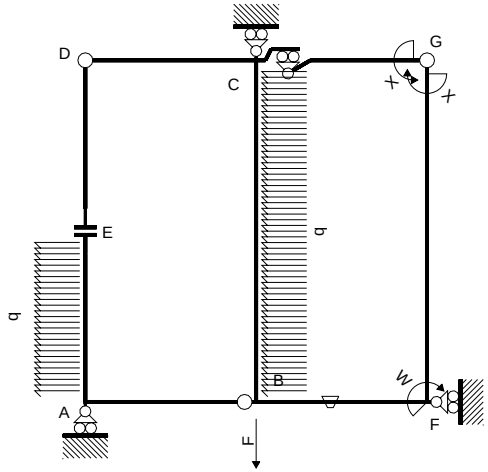


←□→ F

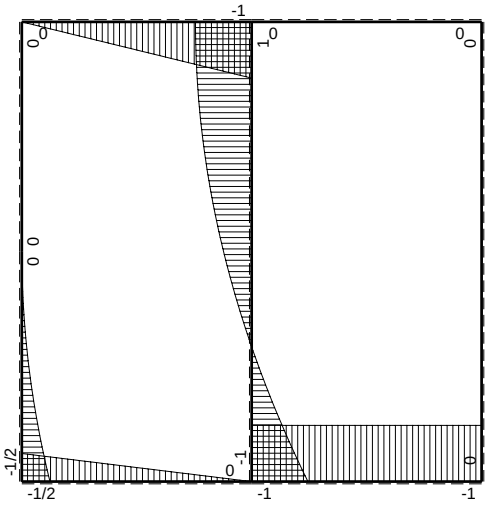
↑□↓ F



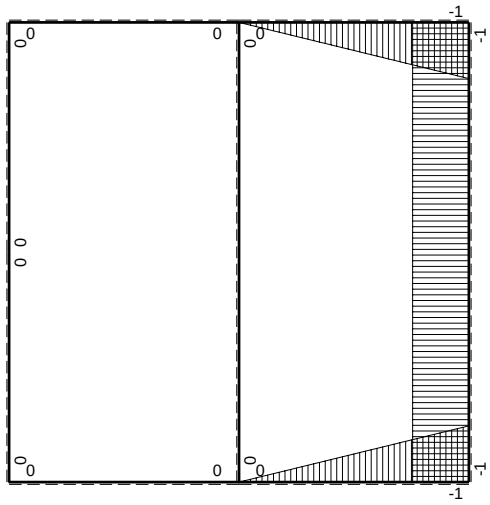
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_o$	$M_x\theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211309 ESΣ08.xxxx.024

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

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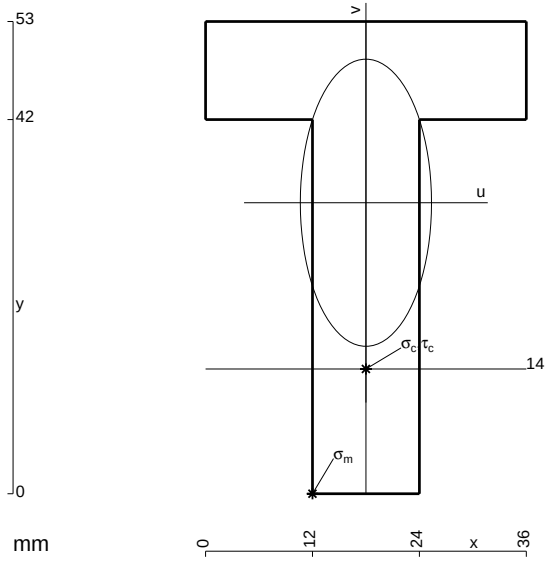

A = 900. mm^2
J_u = 233812. mm^4
J_v = 48816. mm^4
y_g = 32.66 mm
N = 2616. N
T_y = -5980. N
M_x = -1734200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.66 mm
$\sigma_m = N/A - Mv/J_u$ = -239.3 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.66 mm
$\sigma_c = N/A - Mv/J_u$ = -135.5 N/mm^2
τ_c = 9.188 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.4 N/mm^2
S^* = 4311. mm^3

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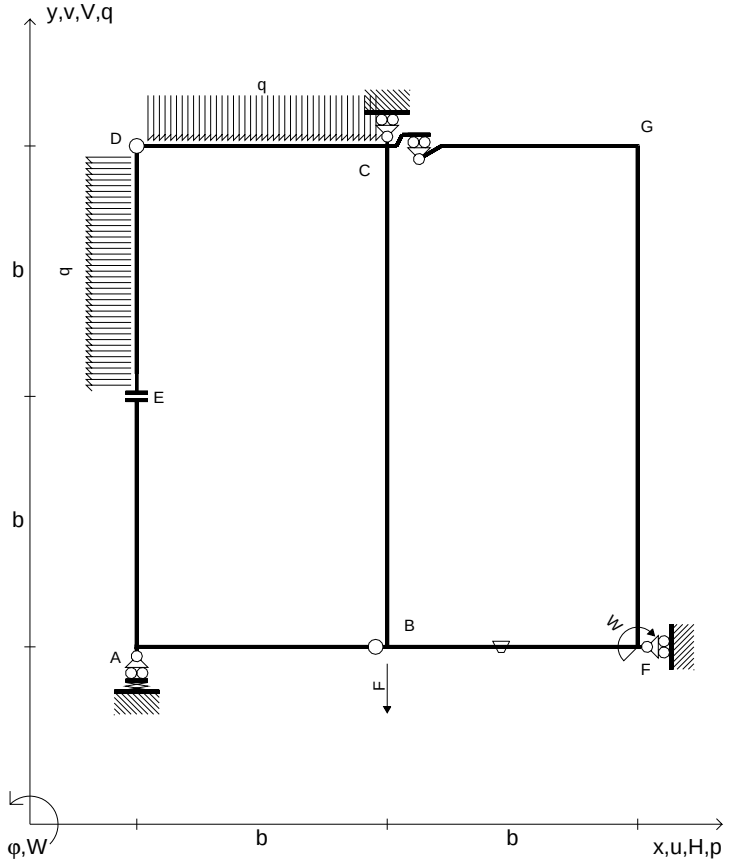
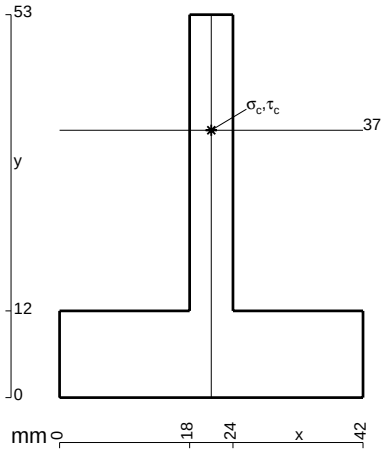
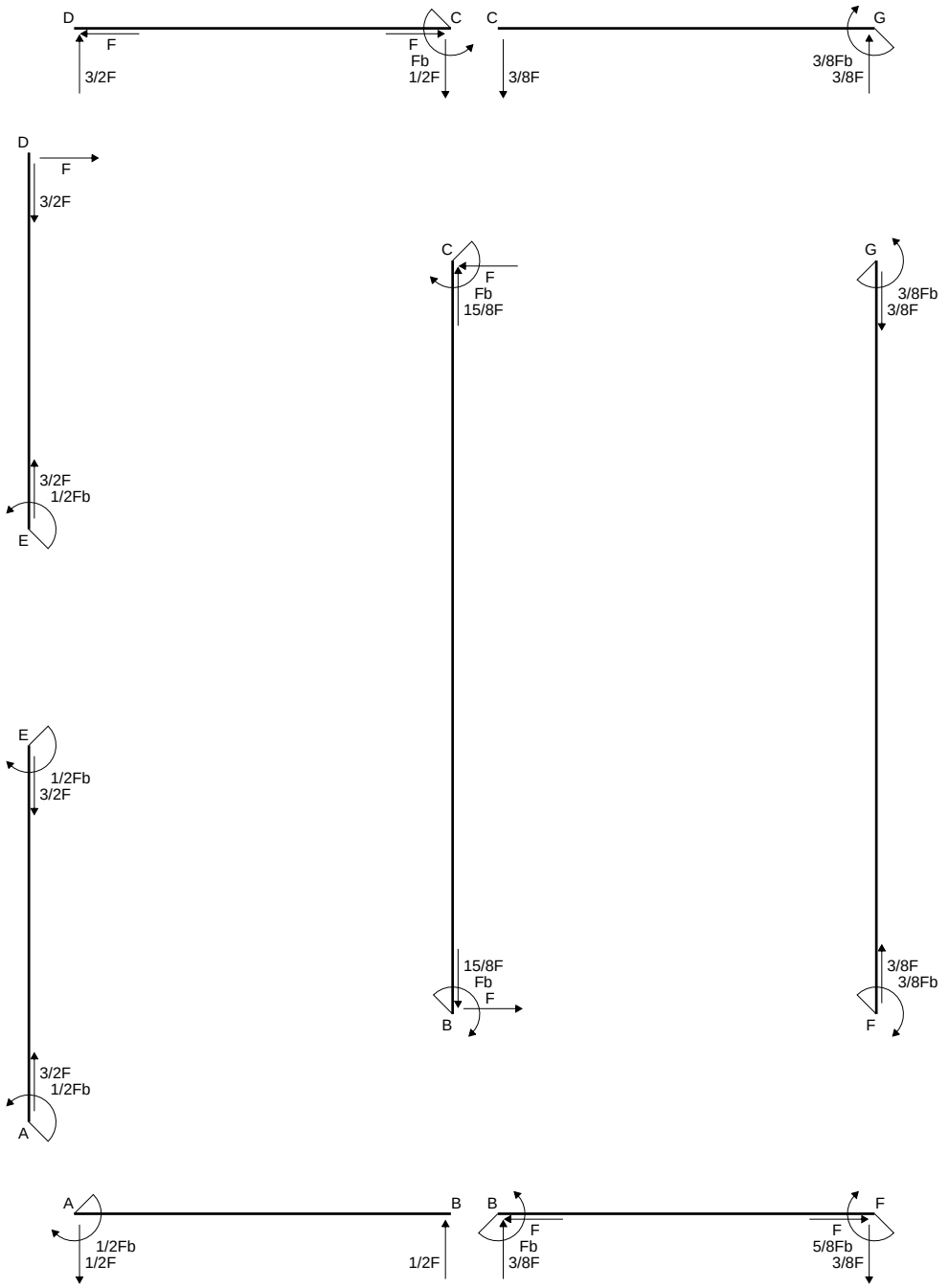
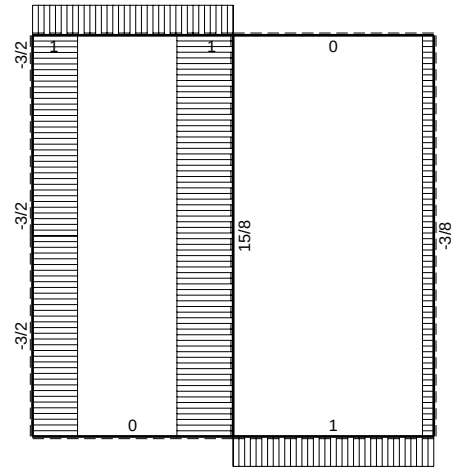
$V_B = -F$
$W_F = -W = -Fb$
$q_{CD} = -q = -F/b$
$p_{DE} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



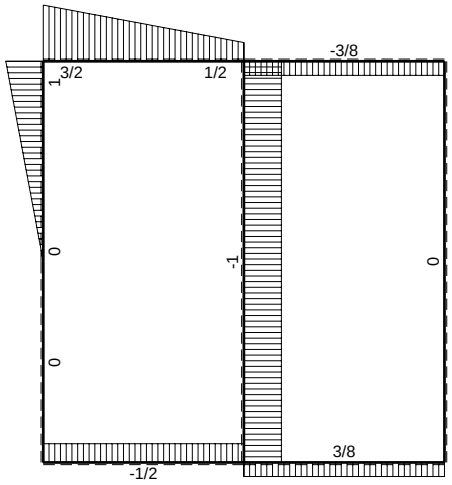


Reazioni iperstatiche in soluzione: $X=W_{GC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 790 mm, F = 1220 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

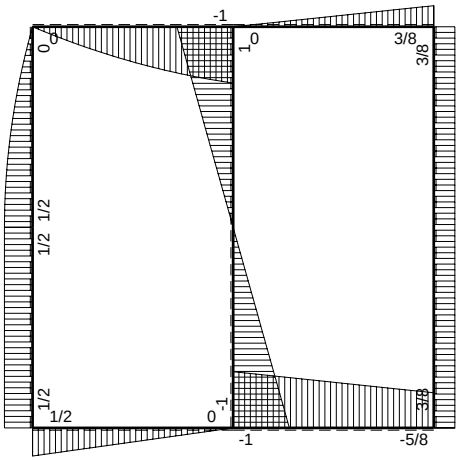




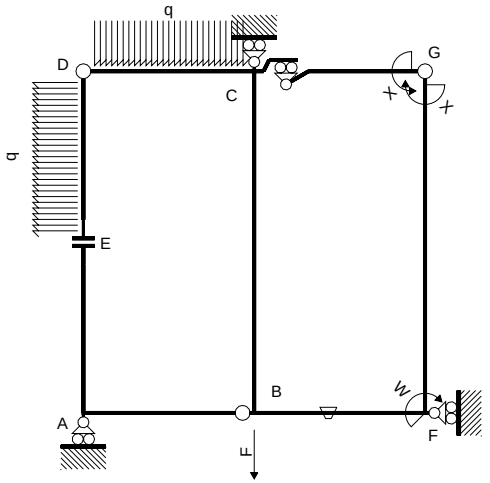
$\leftarrow \boxplus \rightarrow F$



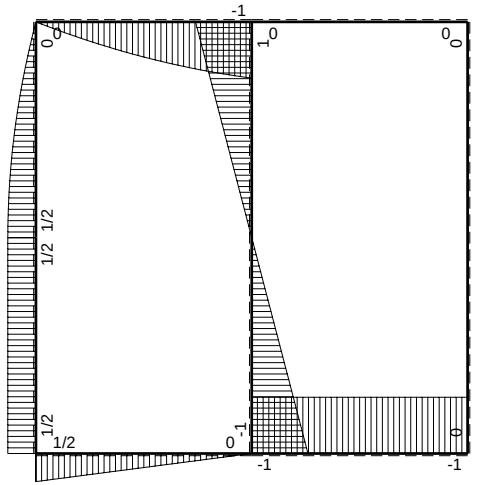
$\uparrow \boxplus \downarrow F$



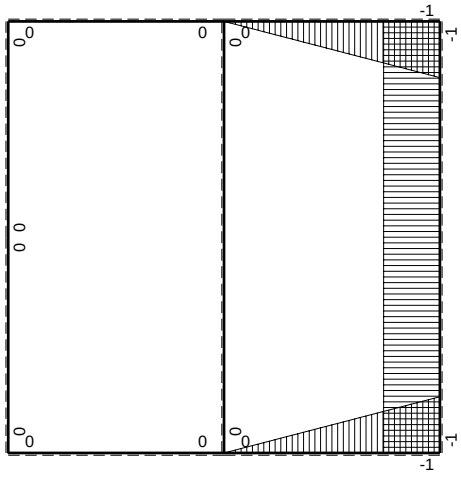
$\circlearrowleft \boxplus \circlearrowright$ Fb



Schema di calcolo iperstatico



$\circlearrowleft \boxplus \circlearrowright$ M_o flessione da carichi assegnati



$\circlearrowleft \boxplus \circlearrowright$ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-Fb+1/2Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	3/2Fx-1/2qx^2	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b]_o^b\ \theta$

$= (\ b -1/2\ b)\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ =\ Fb^2/EJ$

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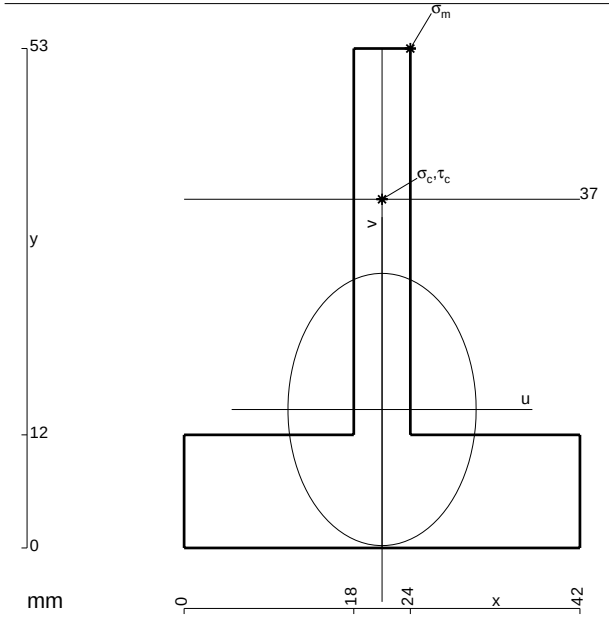

A = 750. mm^2
J_u = 156599. mm^4
J_v = 74826. mm^4
y_g = 14.69 mm
N = 2288. N
T_y = -1220. N
M_x = -963800. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 38.31 mm
$\sigma_m = N/A - Mv/J_u$ = 238.8 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22.31 mm
$\sigma_c = N/A - Mv/J_u$ = 140.3 N/mm^2
τ_c = 3.778 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.5 N/mm^2
S^* = 2910. mm^3

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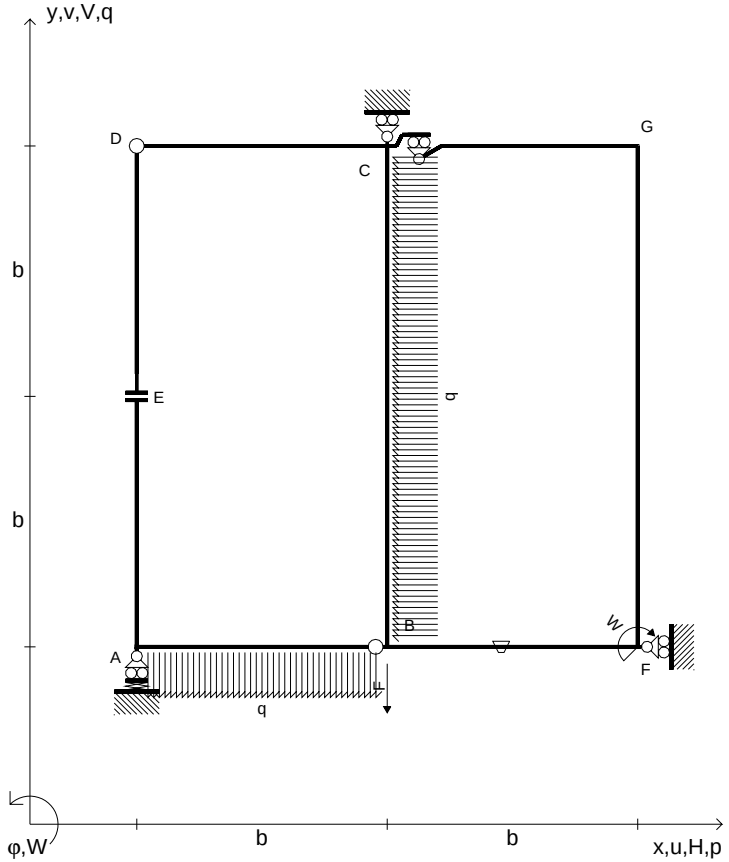
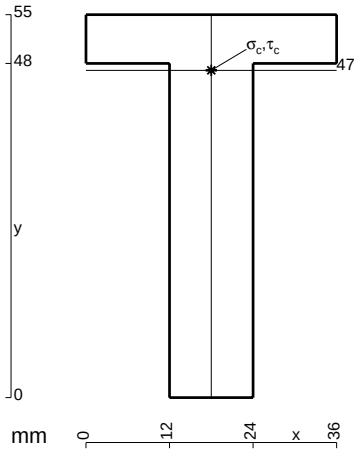
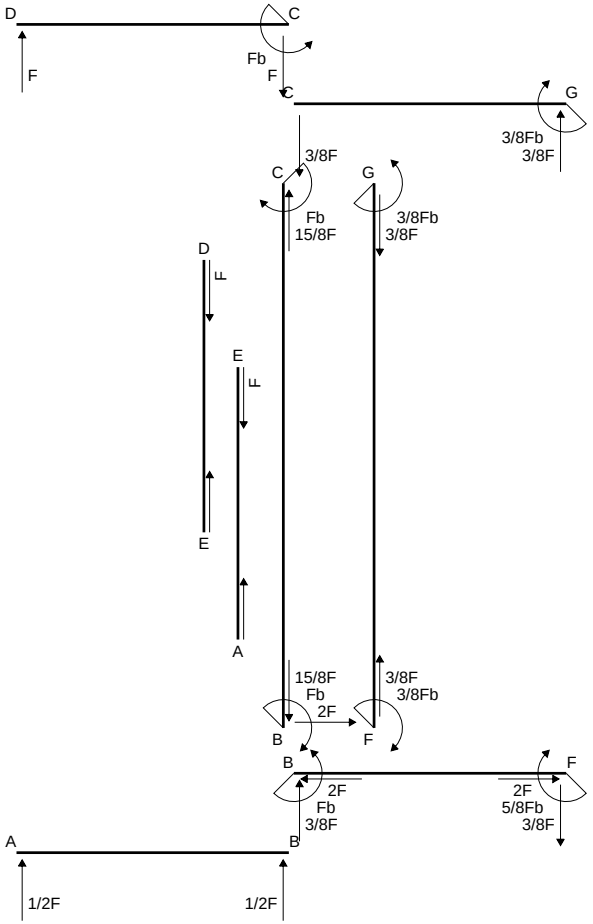
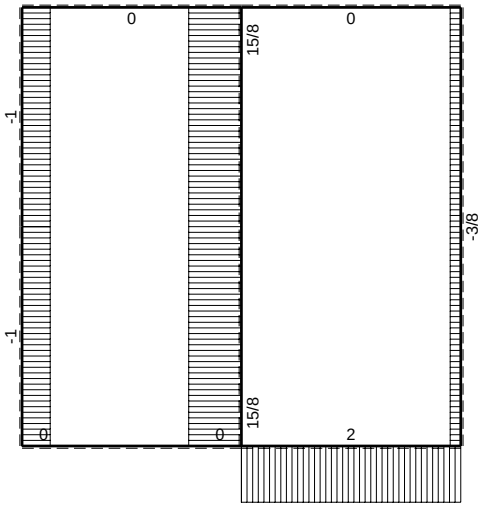
$V_B = -F$
$W_F = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{CB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 3EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



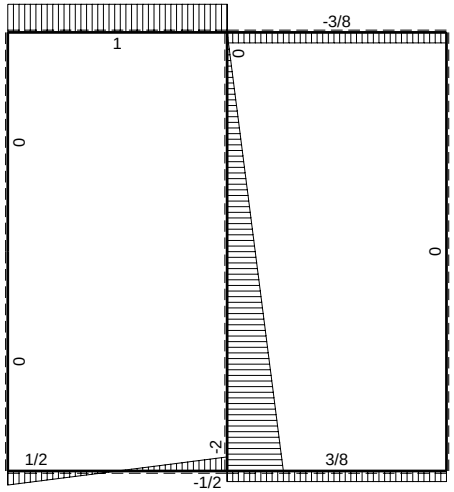


Reazioni iperstatiche in soluzione: $X=W_{GF}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 830 mm, F = 2220 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

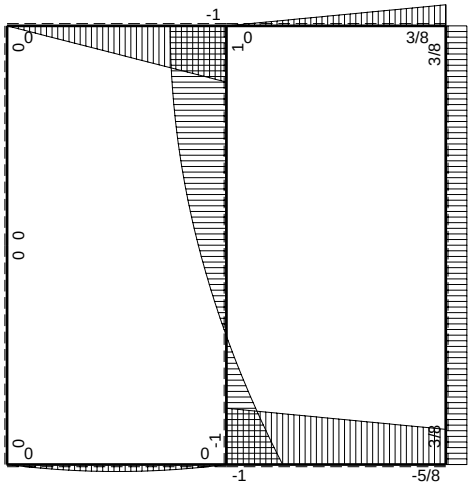




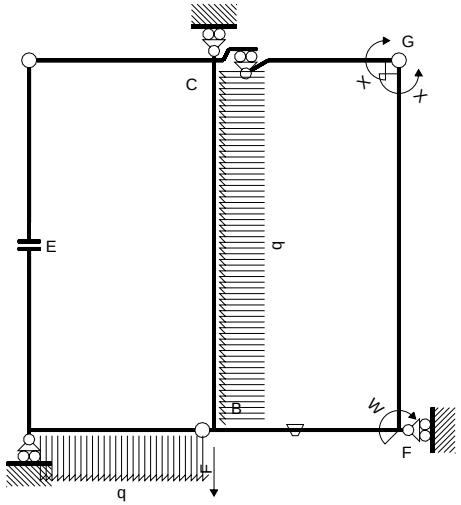
←⊞→ F

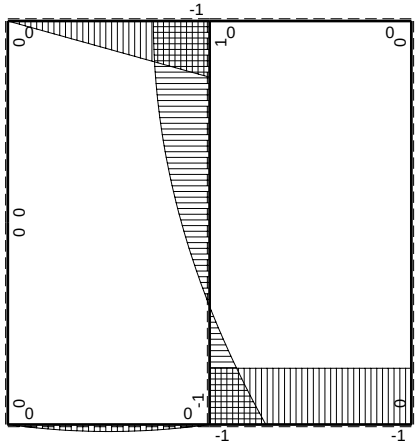


↑⊞↓ F

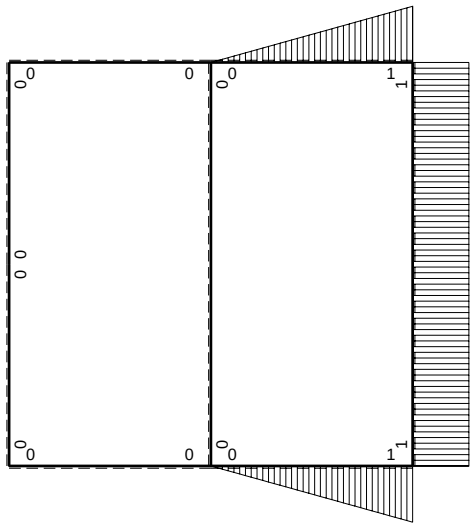


↻⊞↺ Fb





↻⊞↺ M_o flessione da carichi assegnati



↻⊞↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	x/b	-Fb	-Fb/EJ	-Fx	-Fx/EJ	x^2/b^2	$(-1/2-1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	-1+x/b	Fb	Fb/EJ	-Fb+Fx	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	-x/b	0	0	0	0	x^2/b^2		
FG 2b	1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	-1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GF}$						3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

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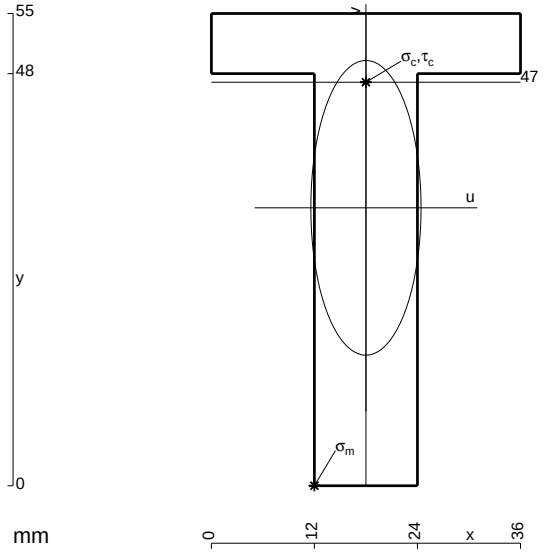

A = 828. mm^2
J_u = 244195. mm^4
J_v = 34128. mm^4
y_g = 32.37 mm
N = 4163. N
T_y = -4440. N
M_x = -1842600. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.37 mm
$\sigma_m = N/A - Mv/J_u$ = -239.2 N/mm^2
x_c = 18. mm
y_c = 47. mm
v_c = 14.63 mm
$\sigma_c = N/A - Mv/J_u$ = 115.4 N/mm^2
τ_c = 7.58 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.2 N/mm^2
S^* = 5002. mm^3

ESΣ08.xxxx.039

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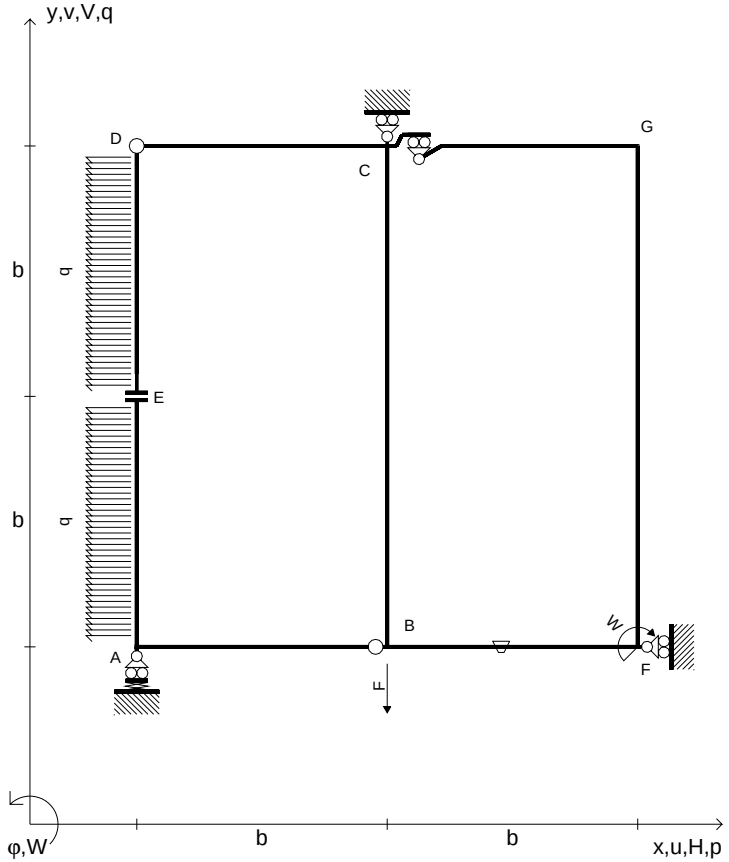
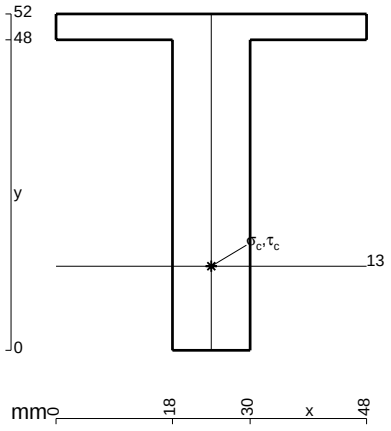
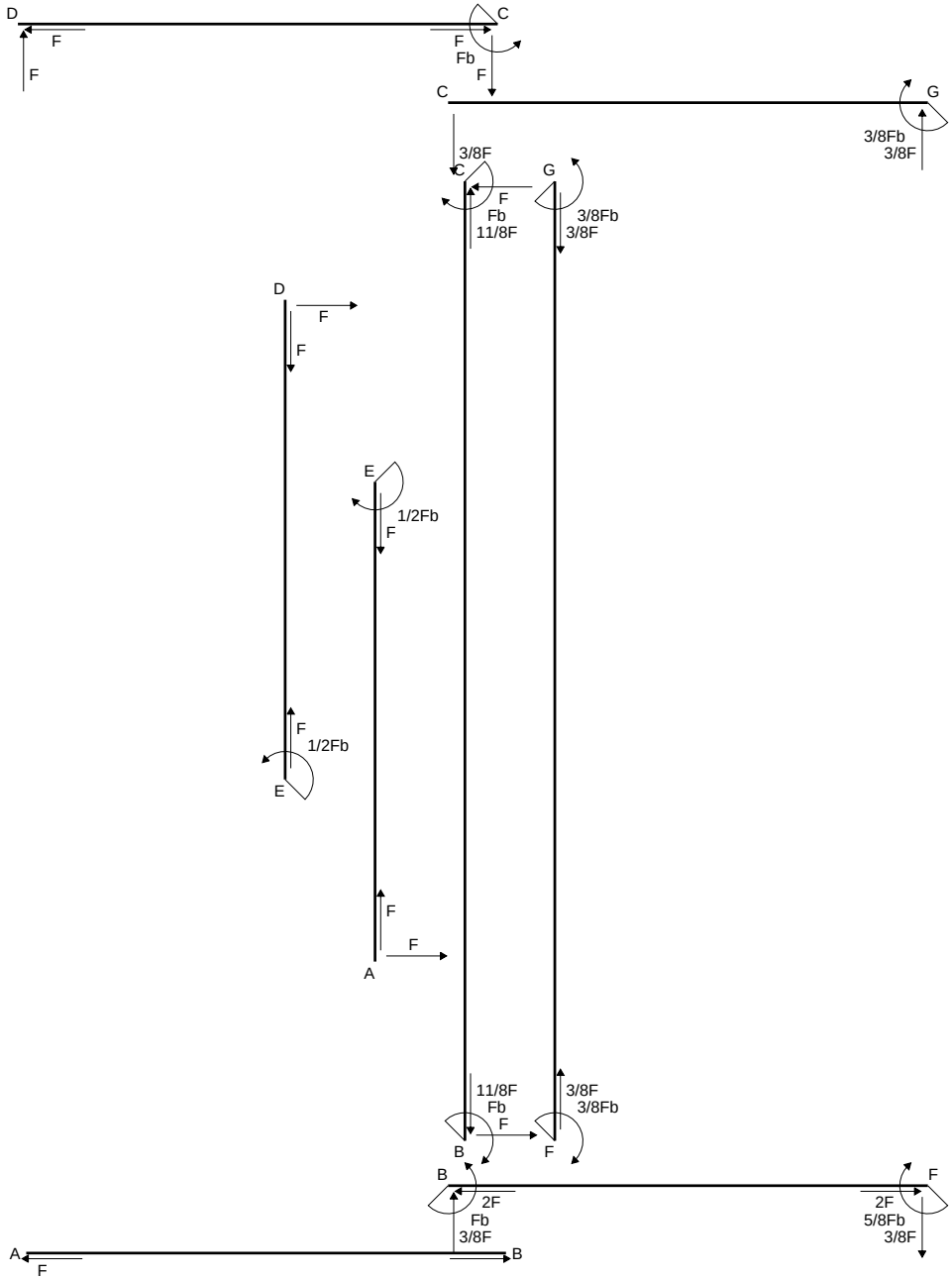
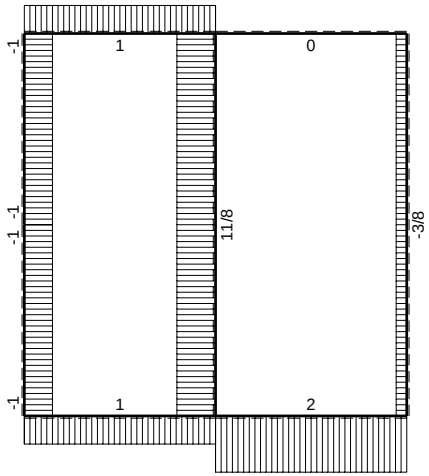
$V_B = -F$
$W_F = -W = -Fb$
$p_{DE} = -q = -F/b$
$p_{EA} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 3EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



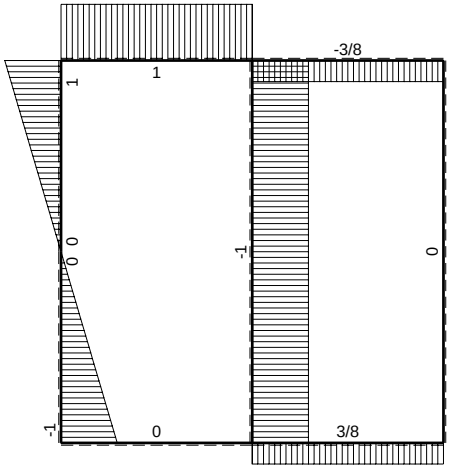


Reazioni iperstatiche in soluzione: $X=W_{GF}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 450 mm, F = 3100 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

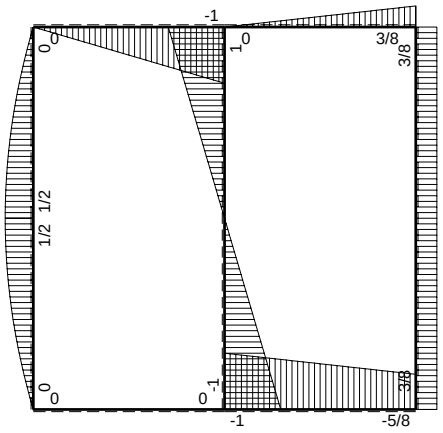




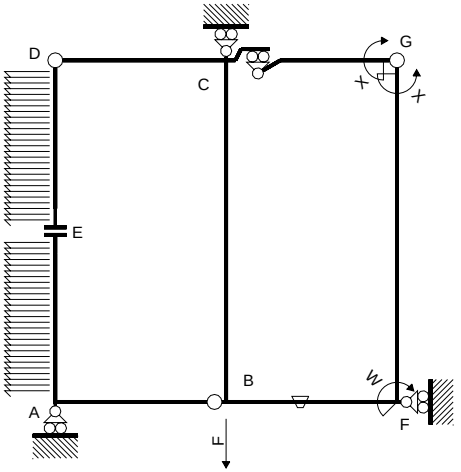
←□→ F

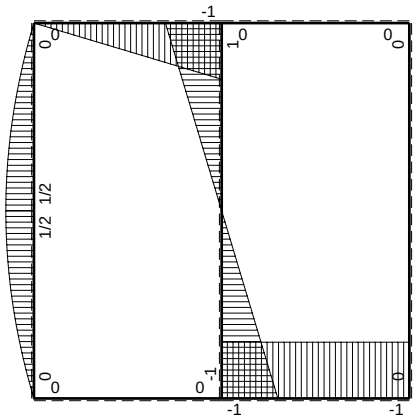


↑□↓ F

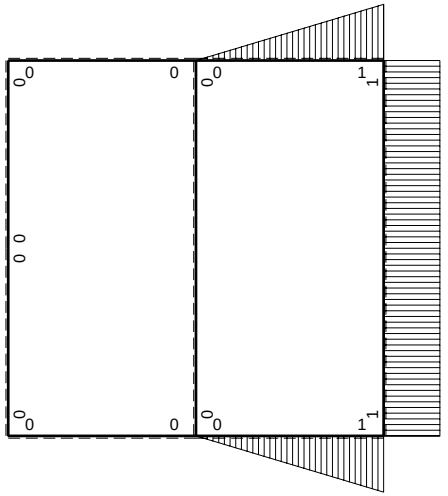


□ Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	x/b	-Fb	-Fb/EJ	-Fx	-Fx/EJ	x^2/b^2	$(-1/2-1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	-1+x/b	Fb	Fb/EJ	-Fb+Fx	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	-x/b	0	0	0	0	x^2/b^2		
FG 2b	1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	-1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						$-Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GF}$						3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 218425 ESΣ08.xxxx.051

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

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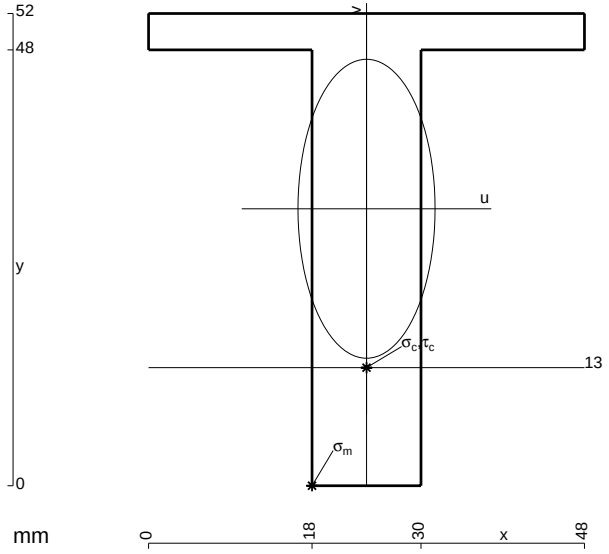

A = 768. mm^2
J_u = 208192. mm^4
J_v = 43776. mm^4
y_g = 30.5 mm
N = 4263. N
T_y = -3100. N
M_x = 1395000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -30.5 mm
$\sigma_m = N/A - Mv/J_u$ = 209.9 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -17.5 mm
$\sigma_c = N/A - Mv/J_u$ = 122.8 N/mm^2
τ_c = 4.646 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.1 N/mm^2
S^* = 3744. mm^3

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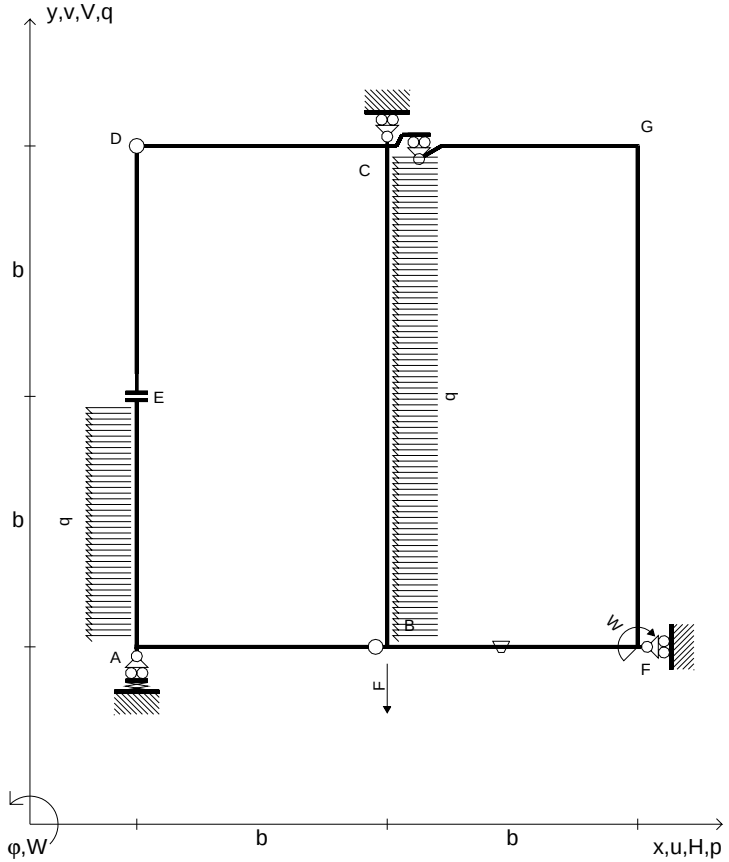
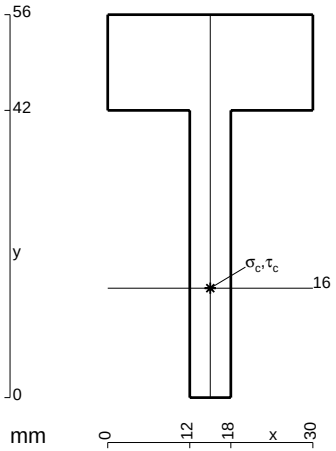
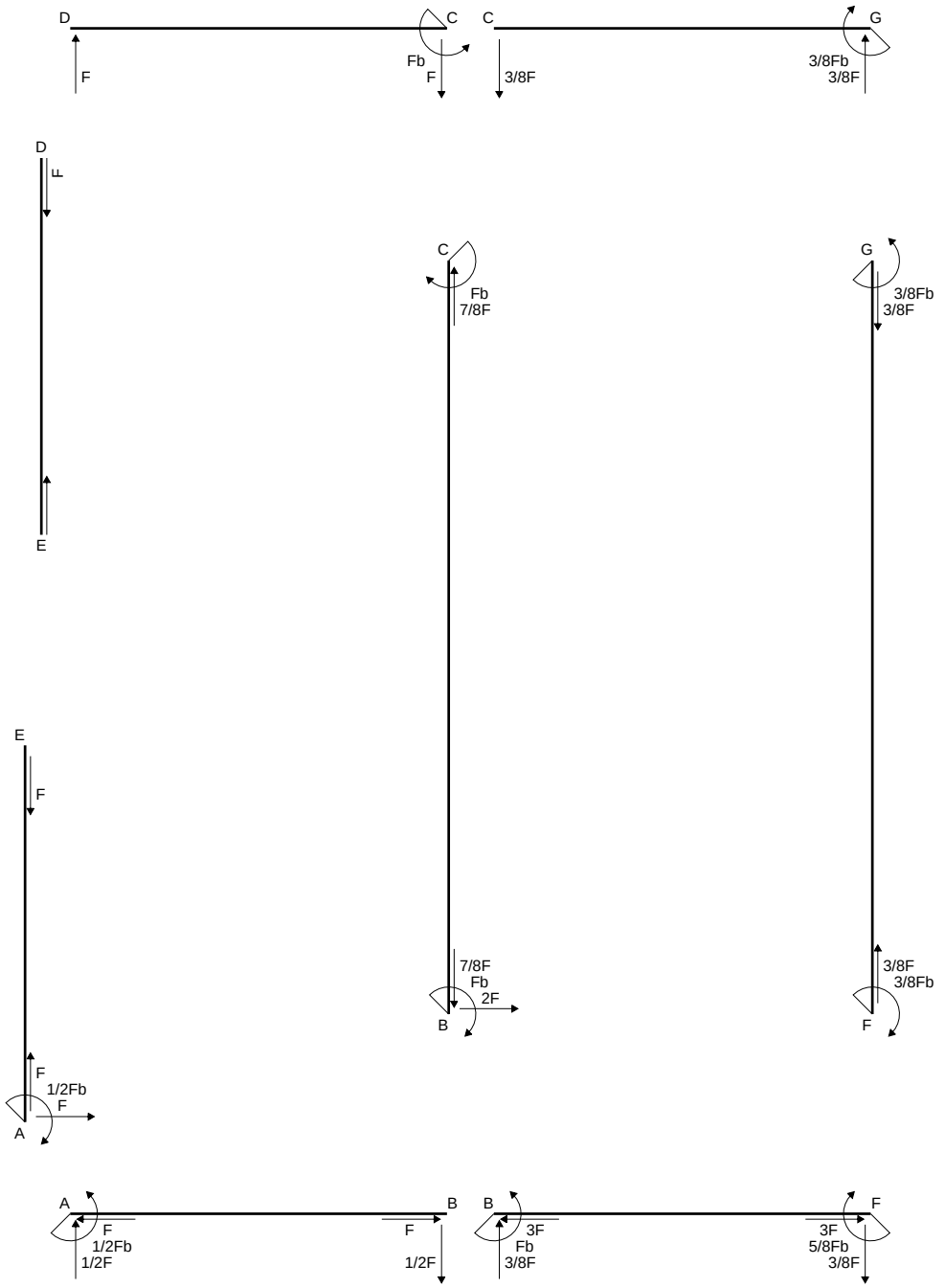
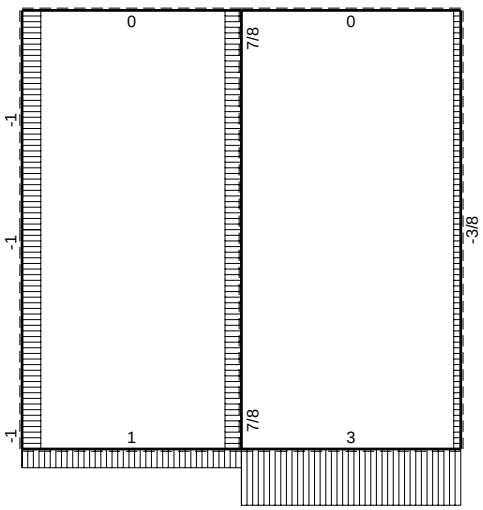
$V_B = -F$
$W_F = -W = -Fb$
$p_{EA} = -q = -F/b$
$p_{CB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 2EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



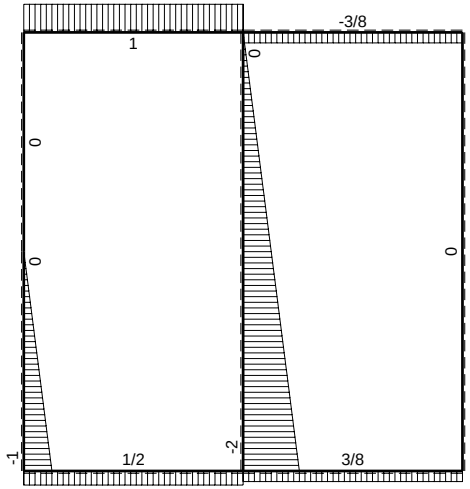


Reazioni iperstatiche in soluzione: $X=W_{GF}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 440 mm, F = 2000 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

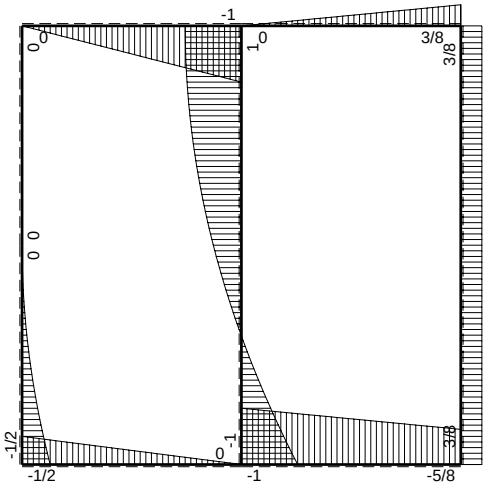




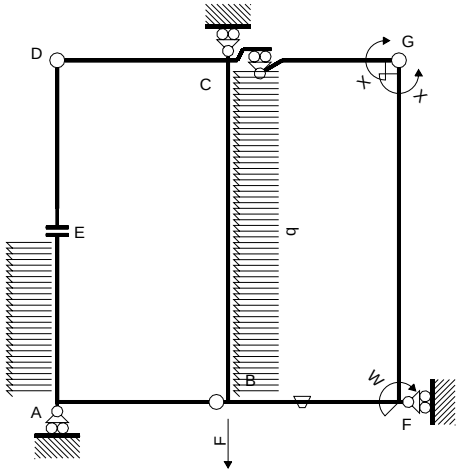
←□→ F

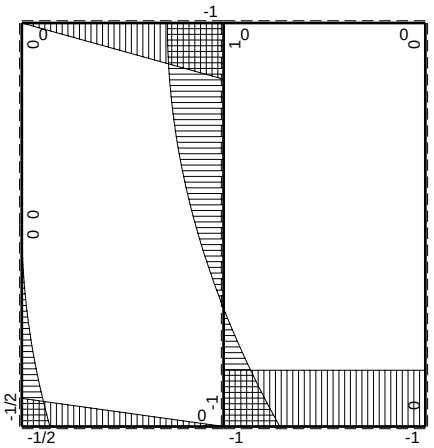


↑□↓ F

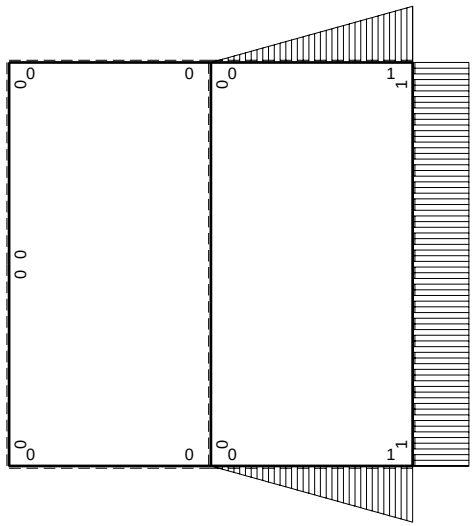


□ Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	x/b	-Fb	-Fb/EJ	-Fx	-Fx/EJ	x^2/b^2	$(-1/2-1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	-1+x/b	Fb	Fb/EJ	-Fb+Fx	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	-x/b	0	0	0	0	x^2/b^2		
FG 2b	1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	-1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
		totali					$-Fb^2/EJ$	8/3Xb/EJ
		iperstatica $X=W_{GF}$					3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

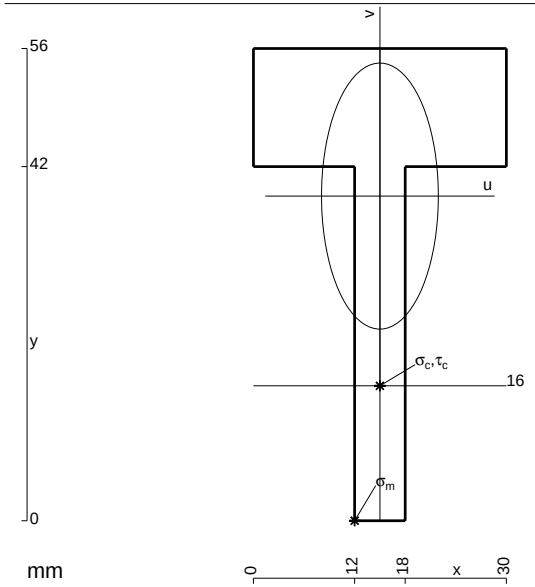
$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$



A = 672. mm^2
J_u = 167384. mm^4
J_v = 32256. mm^4
y_g = 38.5 mm
N = 1750. N
T_y = -4000. N
M_x = -880000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -38.5 mm
$\sigma_m = N/A - Mv/J_u$ = -199.8 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -22.5 mm
$\sigma_c = N/A - Mv/J_u$ = -115.7 N/mm^2
τ_c = 11.66 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.4 N/mm^2
S^* = 2928. mm^3

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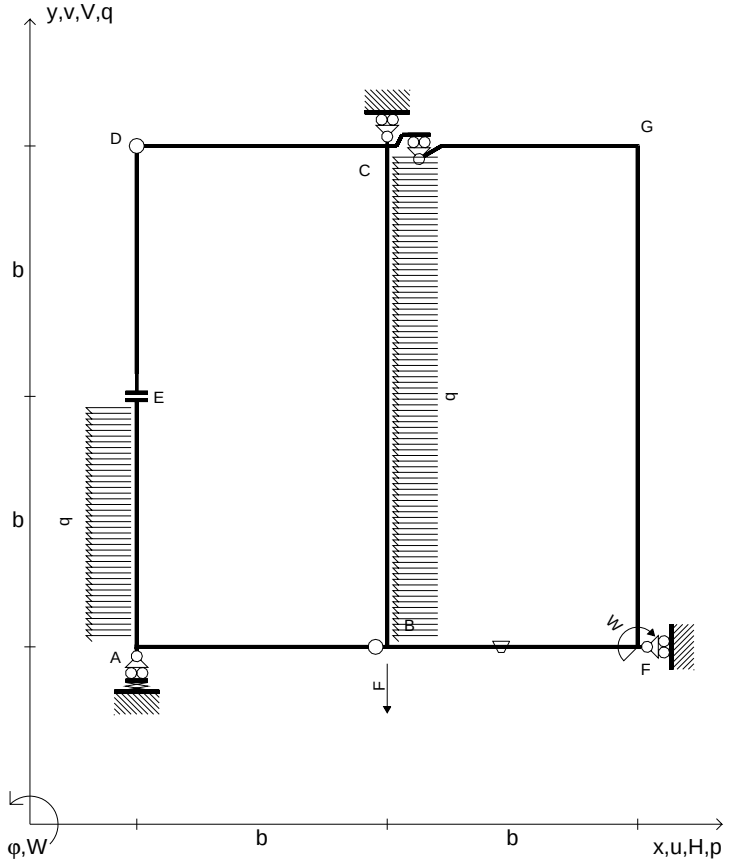
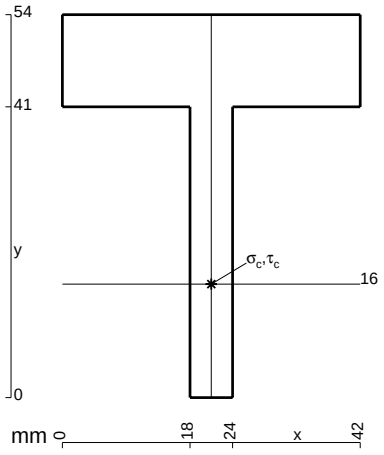
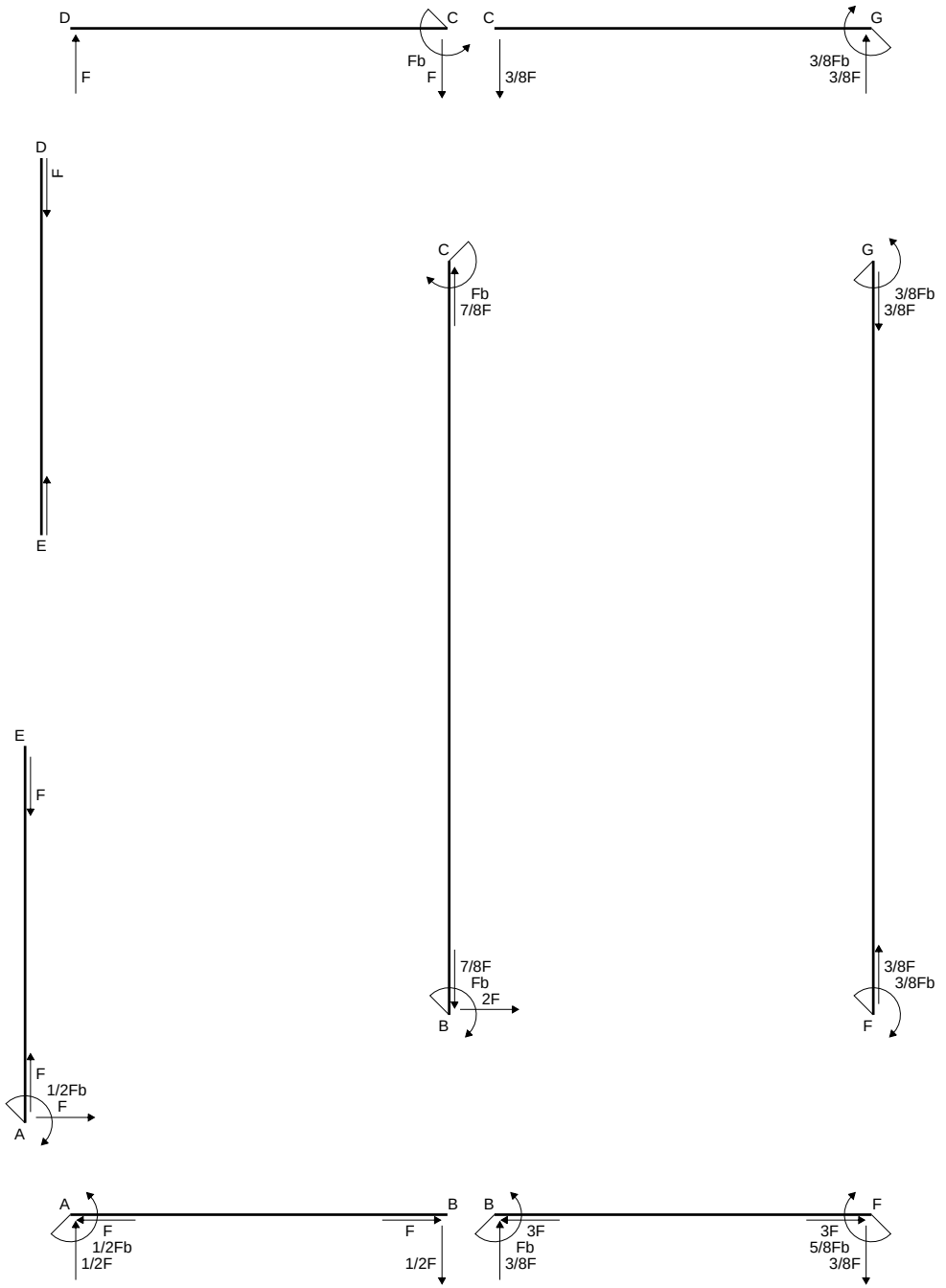
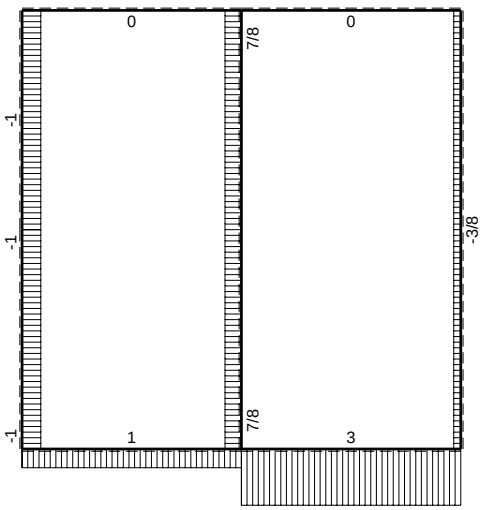
$V_B = -F$
$W_F = -W = -Fb$
$p_{EA} = -q = -F/b$
$p_{CB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 2EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



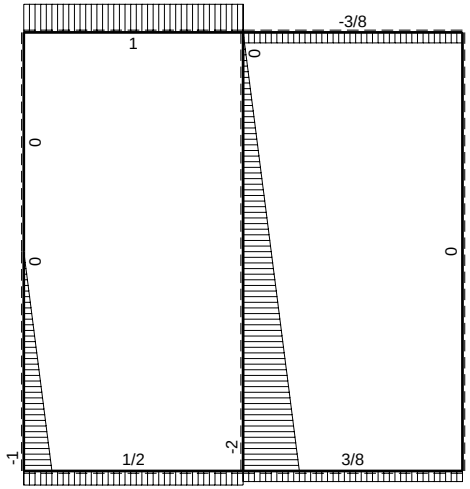


Reazioni iperstatiche in soluzione: $X=W_{GF}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 510 mm, F = 1840 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

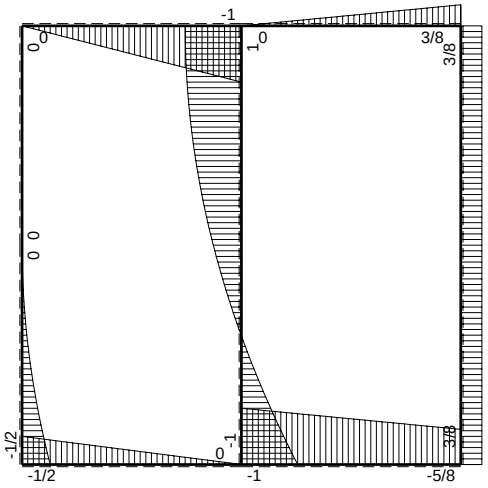




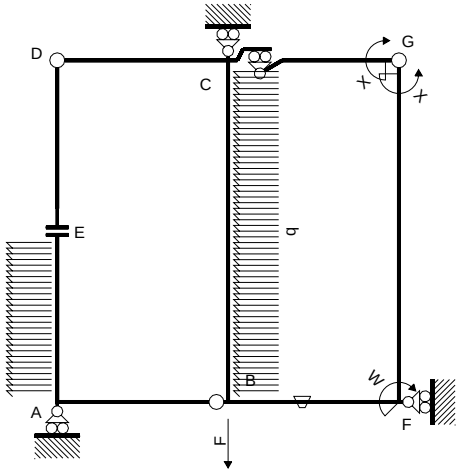
←□→ F

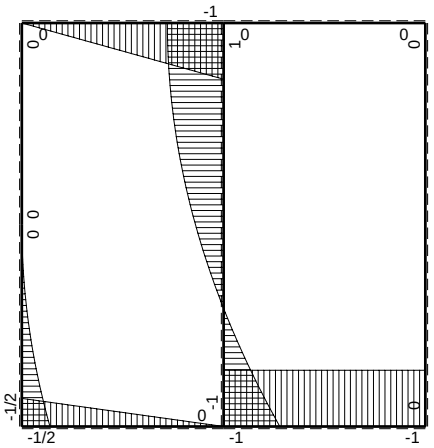


↑□↓ F

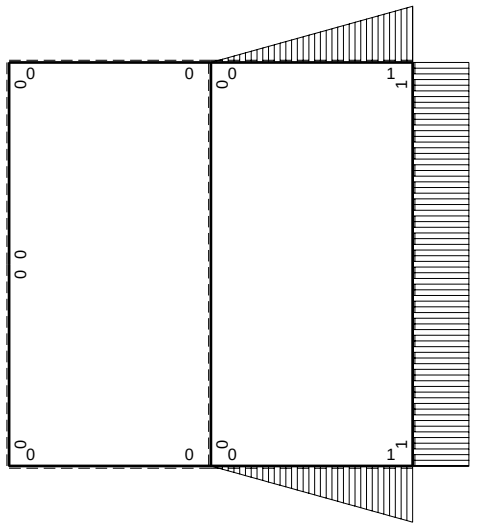


□ Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	x/b	-Fb	-Fb/EJ	-Fx	-Fx/EJ	x^2/b^2	$(-1/2-1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	-1+x/b	Fb	Fb/EJ	-Fb+Fx	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	-x/b	0	0	0	0	x^2/b^2		
FG 2b	1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	-1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
		totali					$-Fb^2/EJ$	8/3Xb/EJ
		iperstatica $X=W_{GF}$					3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

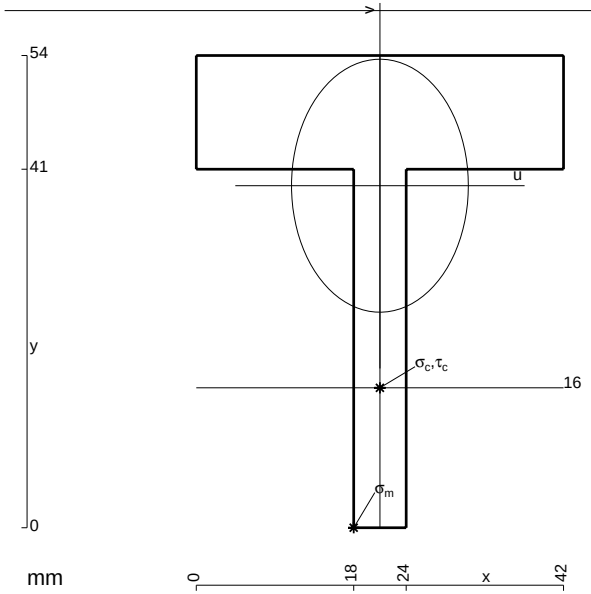
$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$



A = 792. mm^2
J_u = 165782. mm^4
J_v = 81000. mm^4
y_g = 39.11 mm
N = 1610. N
T_y = -3680. N
M_x = -938400. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.11 mm
$\sigma_m = N/A - Mv/J_u$ = -219.4 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -23.11 mm
$\sigma_c = N/A - Mv/J_u$ = -128.8 N/mm^2
τ_c = 11.05 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.2 N/mm^2
S^* = 2987. mm^3

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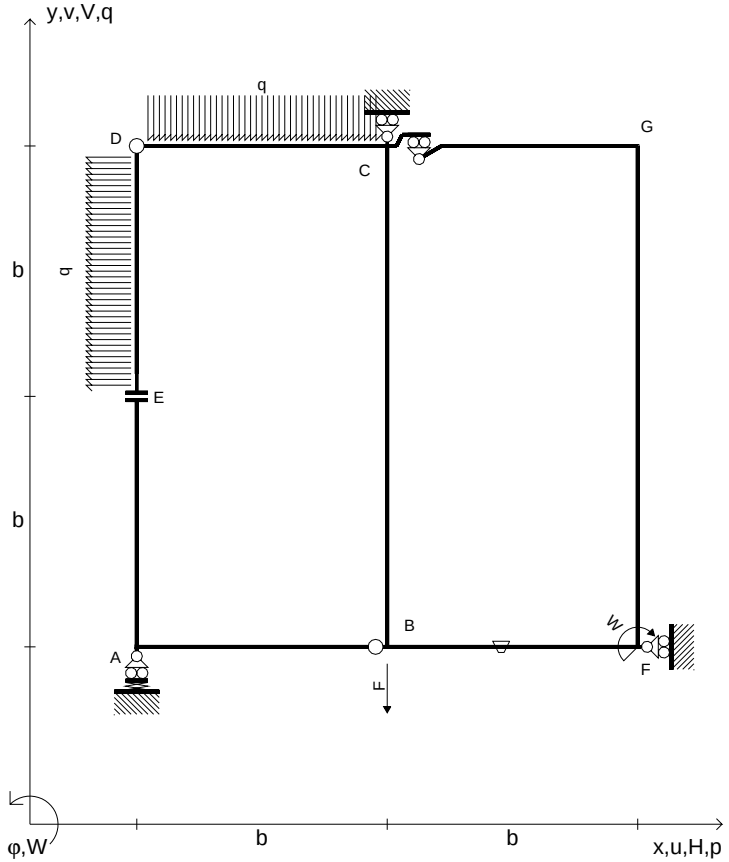
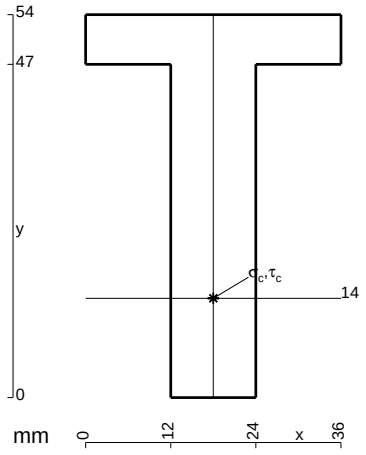
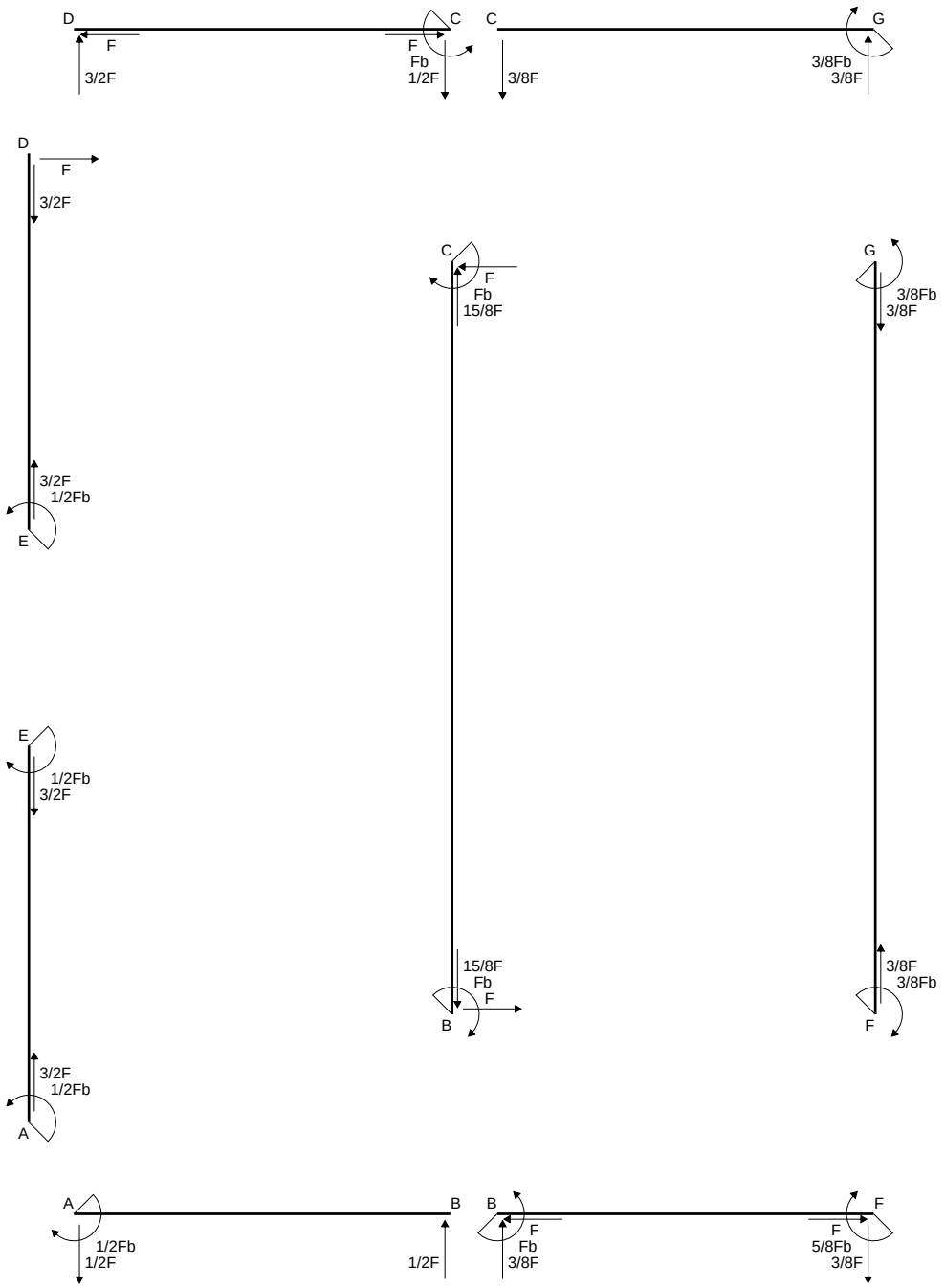
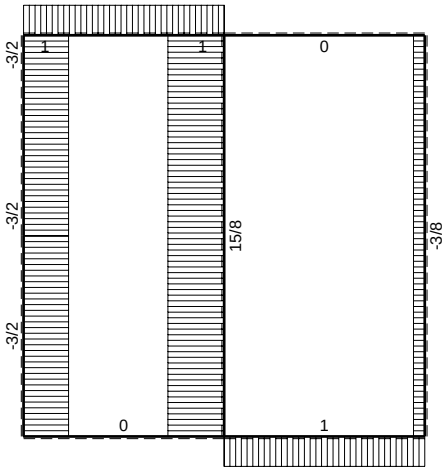
$V_B = -F$
$W_F = -W = -Fb$
$q_{CD} = -q = -F/b$
$p_{DE} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 2EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



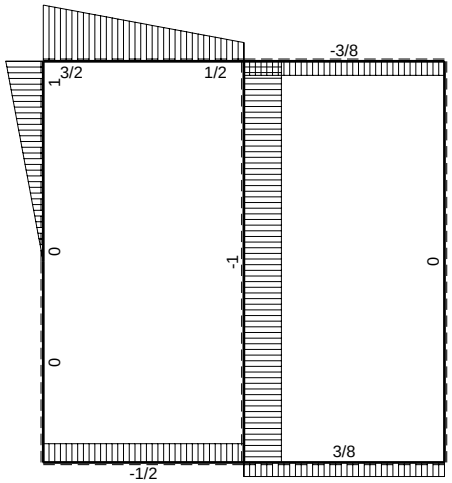


Reazioni iperstatiche in soluzione: $X=W_{GF}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 730 mm, F = 2140 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

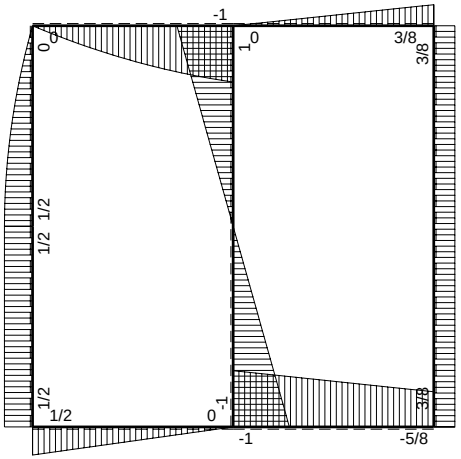




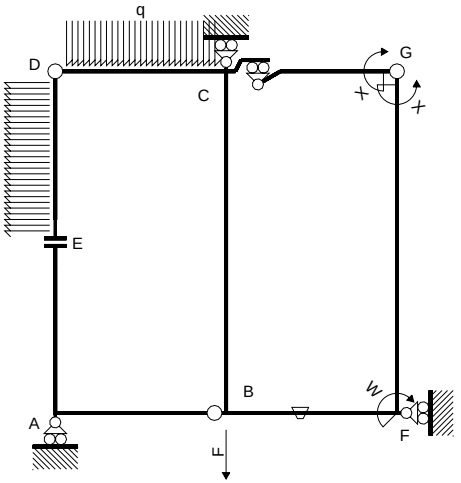
←□→ F

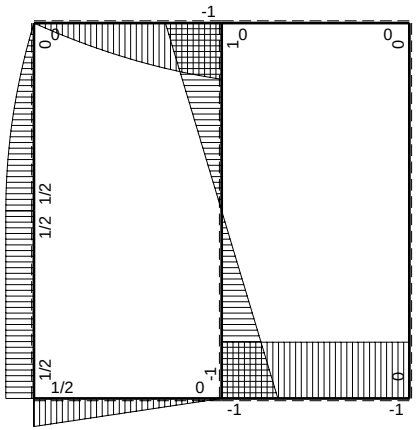


↑□↓ F

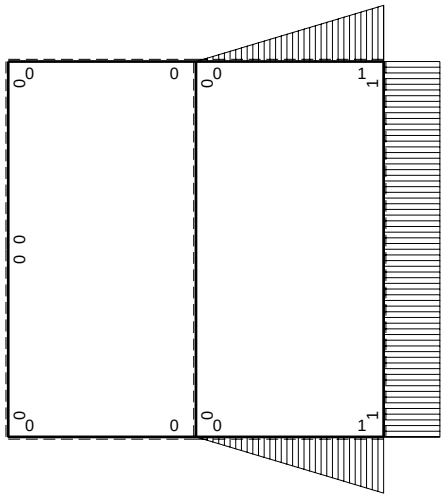


□ Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	x/b	-Fb	-Fb/EJ	-Fx	-Fx/EJ	x^2/b^2	$(-1/2-1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	-1+x/b	Fb	Fb/EJ	-Fb+Fx	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	-x/b	0	0	0	0	x^2/b^2		
FG 2b	1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	-1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						$-Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GF}$						3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213267 ESΣ08.xxxx.087

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

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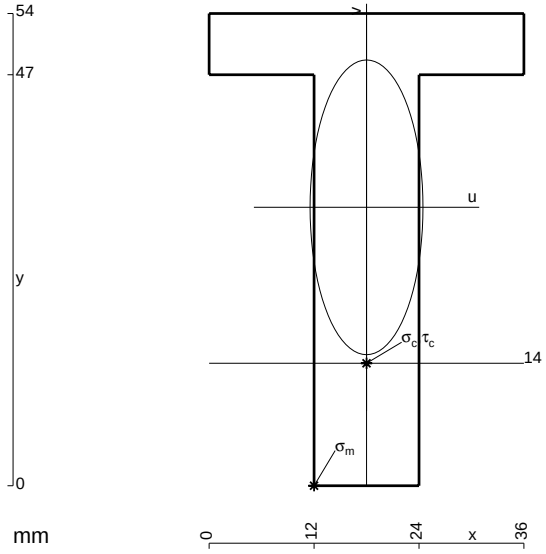

A = 816. mm^2
J_u = 231827. mm^4
J_v = 33984. mm^4
y_g = 31.84 mm
N = 4013. N
T_y = -2140. N
M_x = 1562200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.84 mm
$\sigma_m = N/A - Mv/J_u$ = 219.5 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -17.84 mm
$\sigma_c = N/A - Mv/J_u$ = 125.1 N/mm^2
τ_c = 3.21 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.2 N/mm^2
S^* = 4173. mm^3

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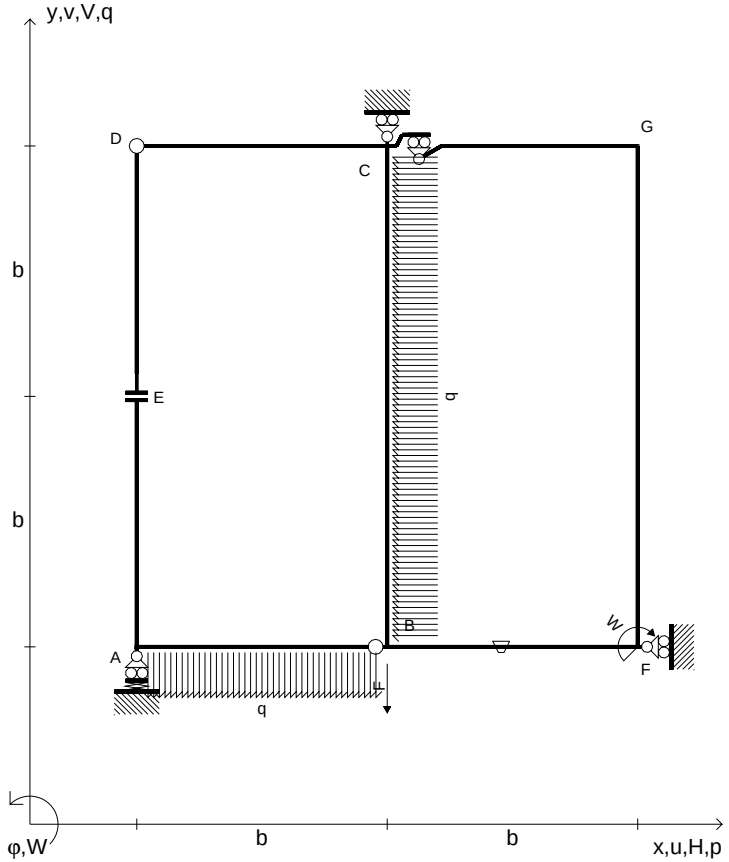
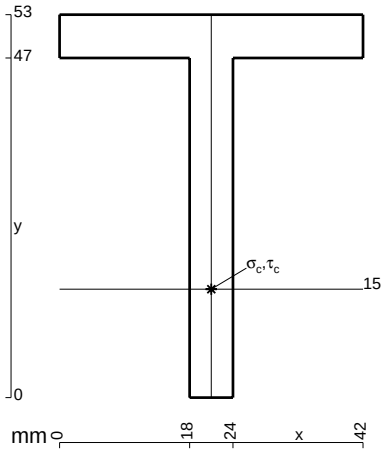
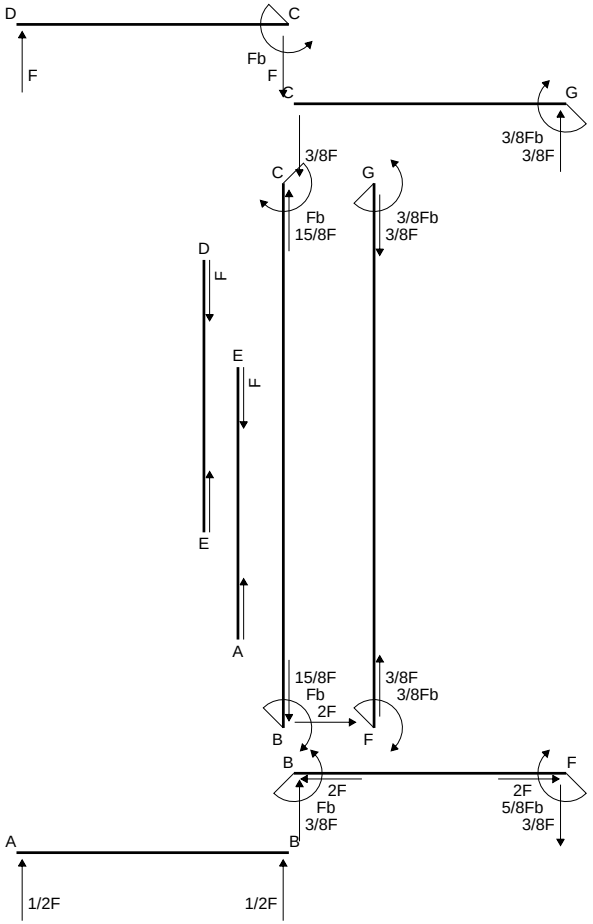
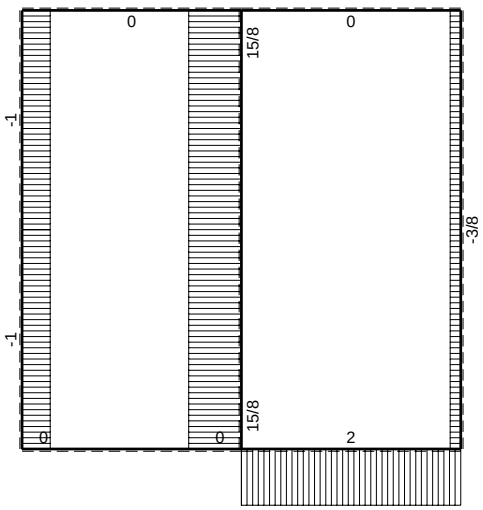
$V_B = -F$
$W_F = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{CB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



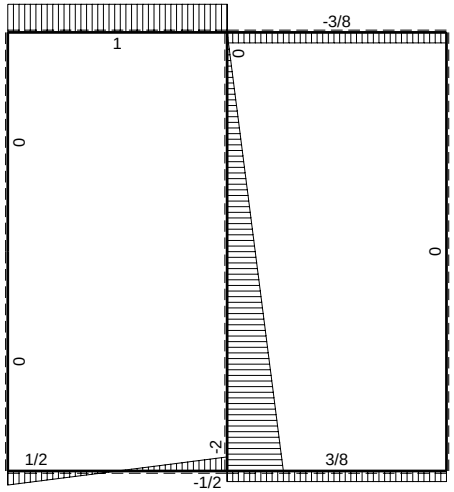


Reazioni iperstatiche in soluzione: $X=W_{GF}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 710 mm, F = 1280 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

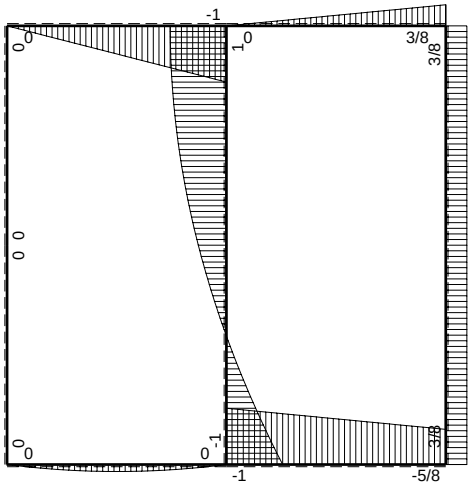




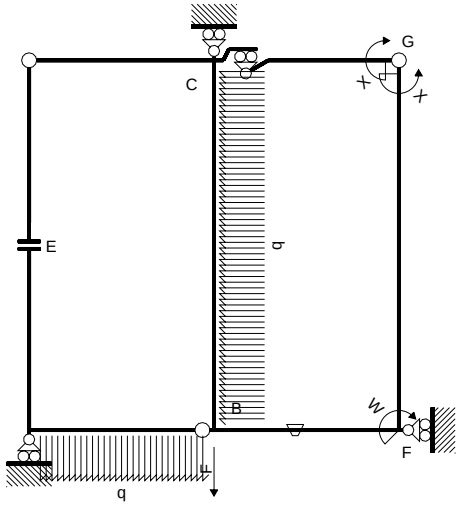
←⊞→ F

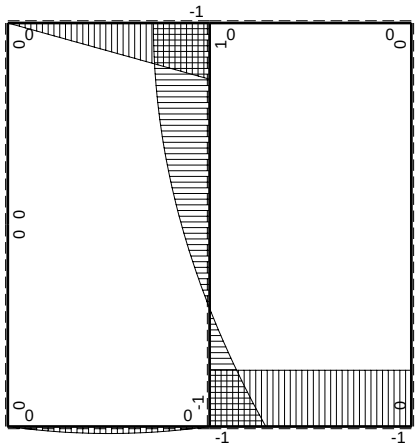


↑⊞↓ F

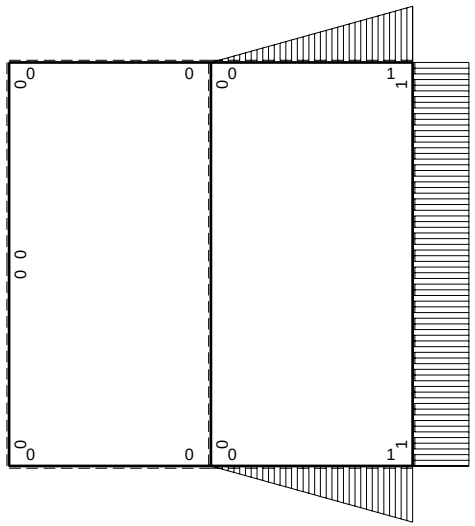


↻⊞↺ Fb





↻⊞↺ M_o flessione da carichi assegnati



↻⊞↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	x/b	-Fb	-Fb/EJ	-Fx	-Fx/EJ	x^2/b^2	$(-1/2-1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	-1+x/b	Fb	Fb/EJ	-Fb+Fx	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	-x/b	0	0	0	0	x^2/b^2		
FG 2b	1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	-1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GF}$						3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 209775 ESΣ08.xxxx.097

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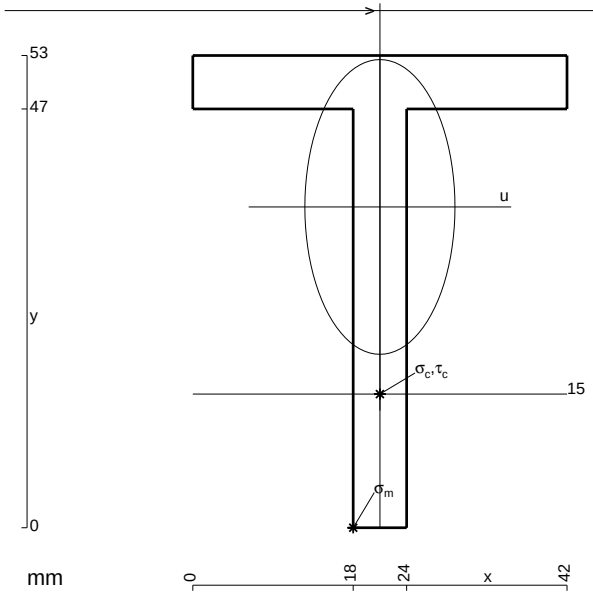

A = 534. mm^2
J_u = 146122. mm^4
J_v = 37890. mm^4
y_g = 36.01 mm
N = 2400. N
T_y = -2560. N
M_x = -908800. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -36.01 mm
$\sigma_m = N/A - Mv/J_u$ = -219.4 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -21.01 mm
$\sigma_c = N/A - Mv/J_u$ = -126.1 N/mm^2
τ_c = 7.491 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.8 N/mm^2
S^* = 2566. mm^3

ESΣ08.xxxx.097

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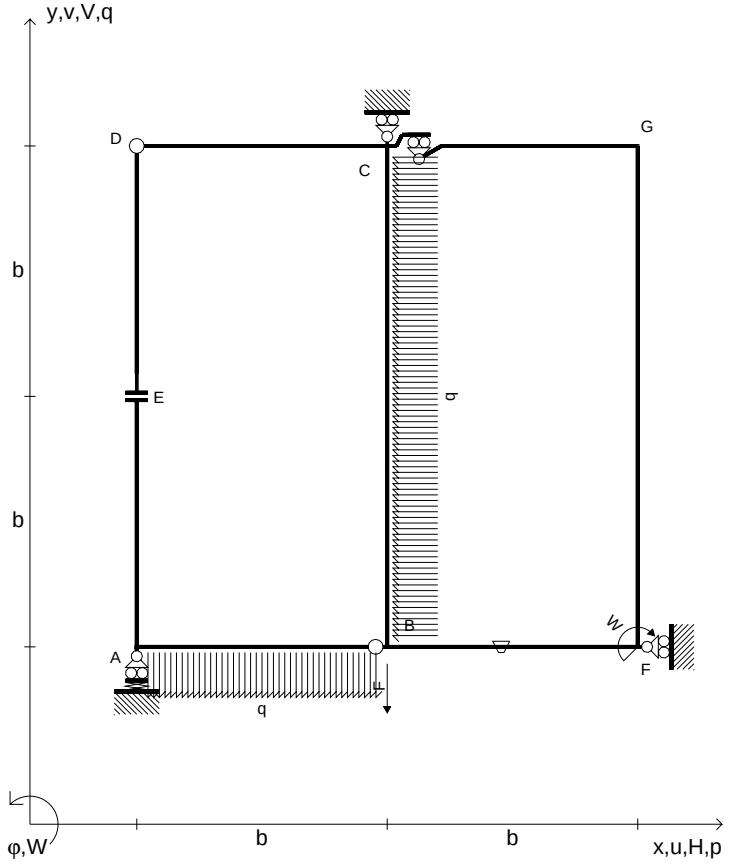
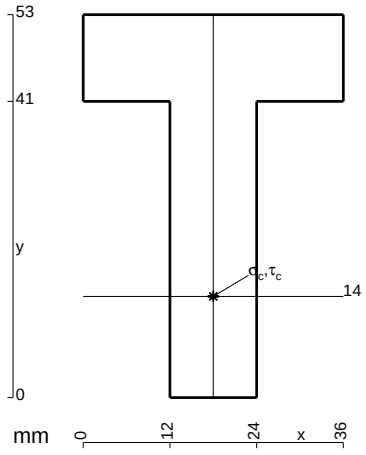
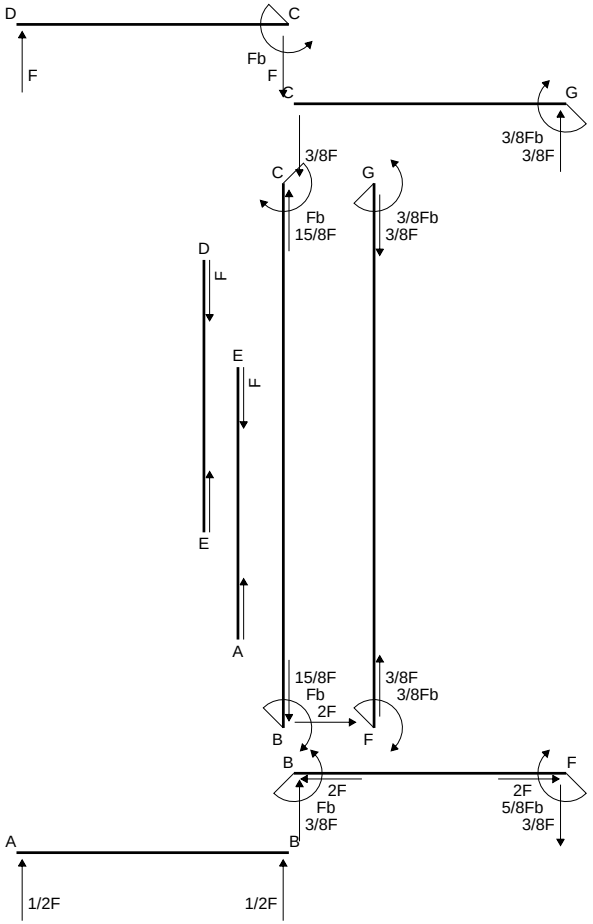
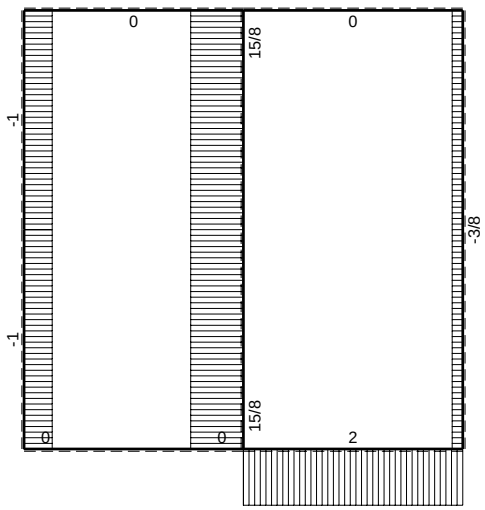
$V_B = -F$
$W_F = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{CB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



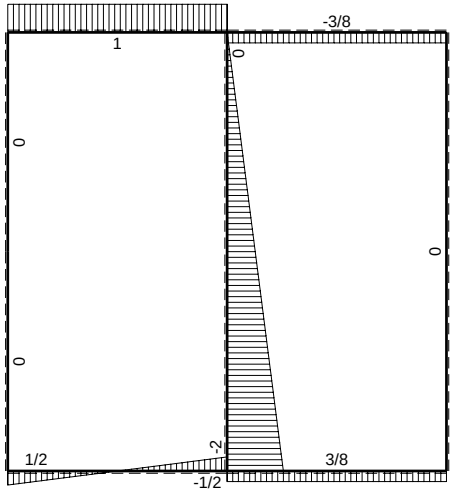


Reazioni iperstatiche in soluzione: $X=W_{GF}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 580 mm, F = 3040 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

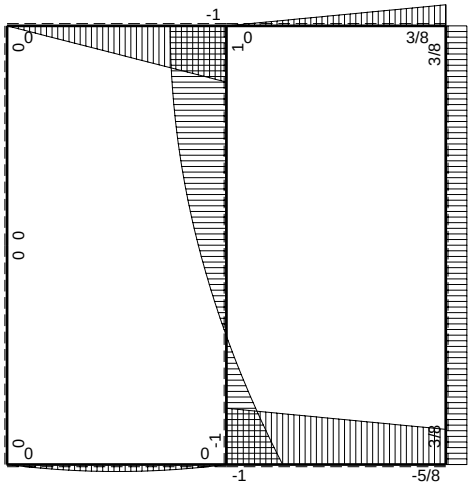




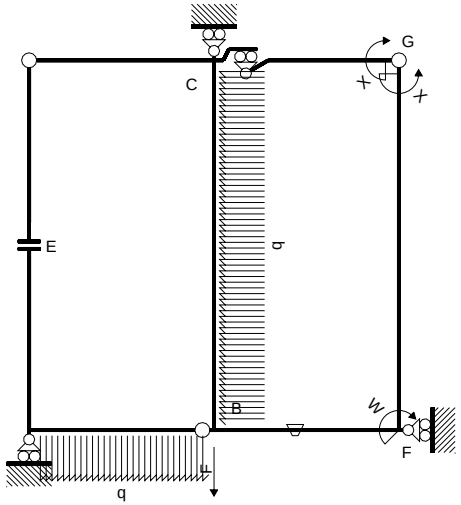
←⊞→ F

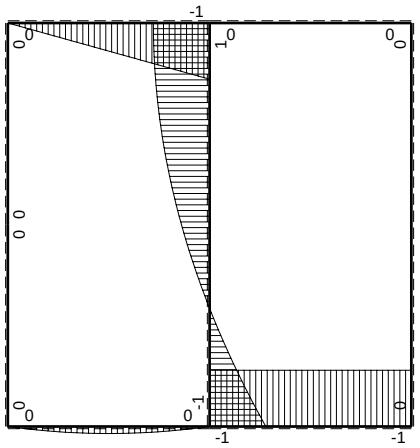


↑⊞↓ F

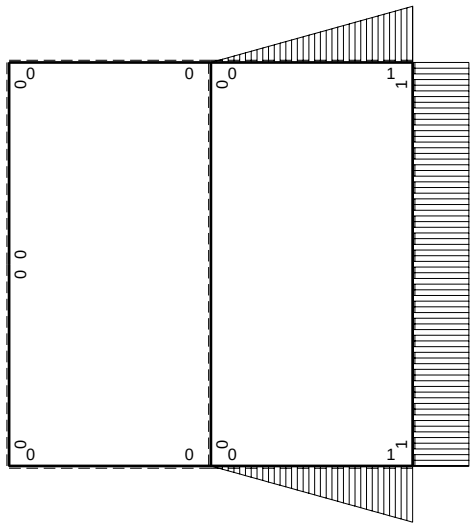


⟲⊞⟳ Fb





⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	x/b	-Fb	-Fb/EJ	-Fx	-Fx/EJ	x^2/b^2	$(-1/2-1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	-1+x/b	Fb	Fb/EJ	-Fb+Fx	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	-x/b	0	0	0	0	x^2/b^2		
FG 2b	1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	-1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GF}$						3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 210089 ESΣ08.xxxx.104

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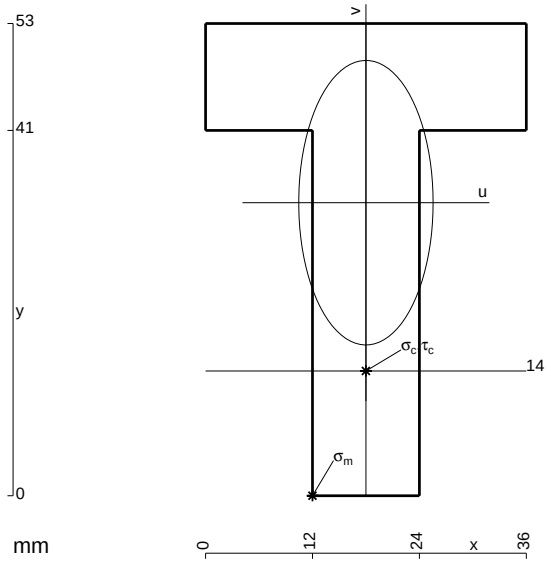
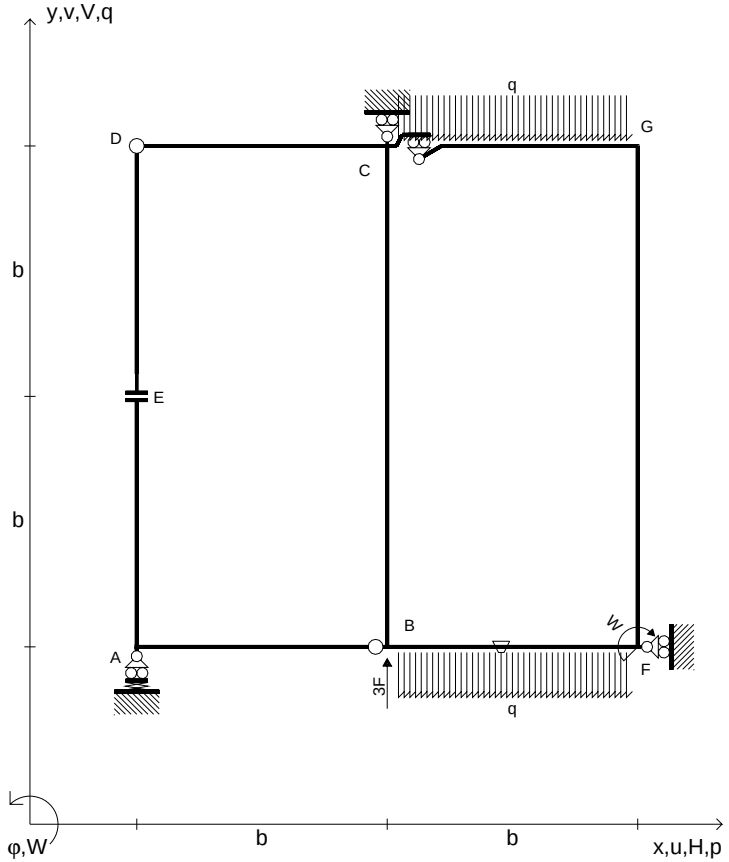
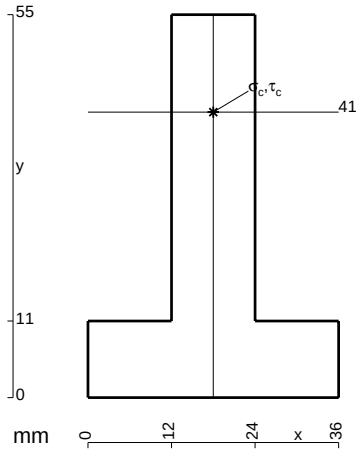


A = 924. mm^2
J_u = 235641. mm^4
J_v = 52560. mm^4
y_g = 32.89 mm
N = 5700. N
T_y = -6080. N
M_x = -1763200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.89 mm
$\sigma_m = N/A - Mv/J_u$ = -239.9 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.89 mm
$\sigma_c = N/A - Mv/J_u$ = -135.2 N/mm^2
τ_c = 9.352 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.1 N/mm^2
S^* = 4349. mm^3

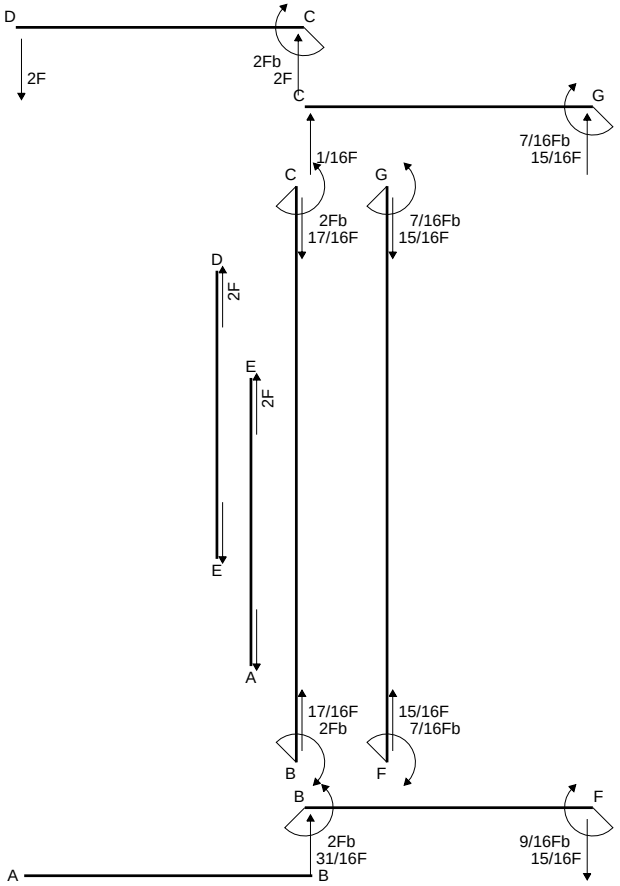
ESΣ08.xxxx.104
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

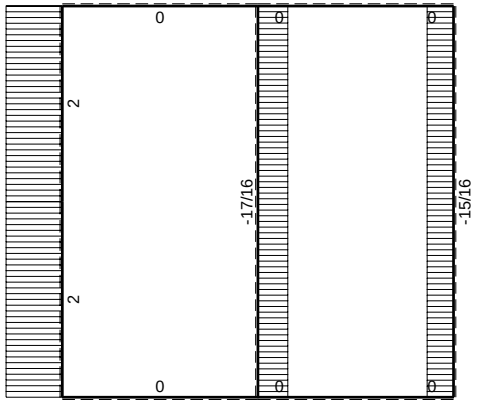
$V_B = 3F$
$W_F = -W = -Fb$
$q_{BF} = -q = -F/b$
$q_{GC} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



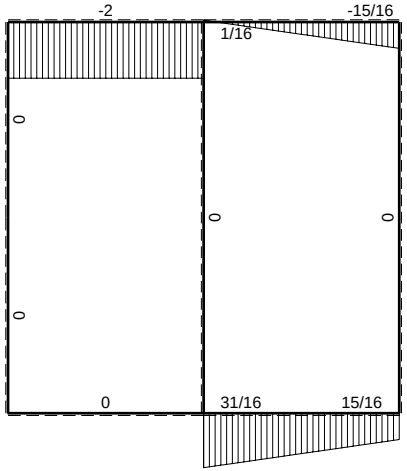


Reazioni iperstatiche in soluzione: $X=W_{GC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 480 mm, F = 1680 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Curvatura θ asta BF positiva se convessa a destra con inizio B.

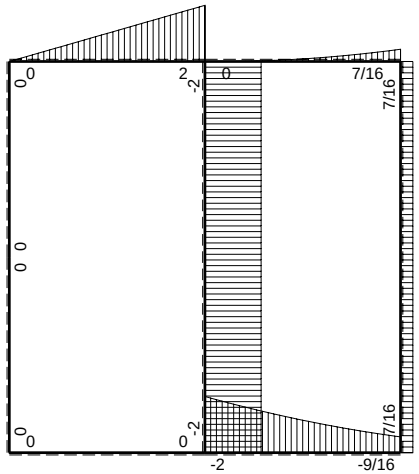




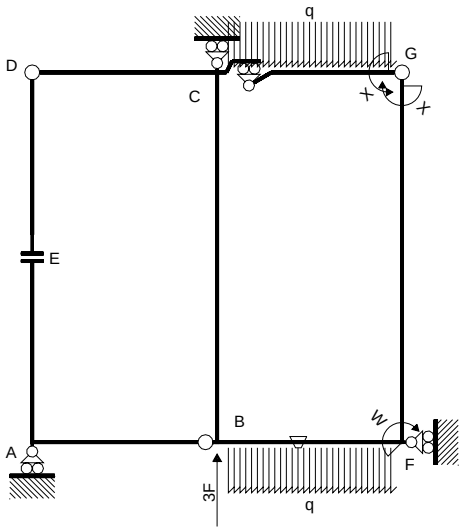
←[+]→ F



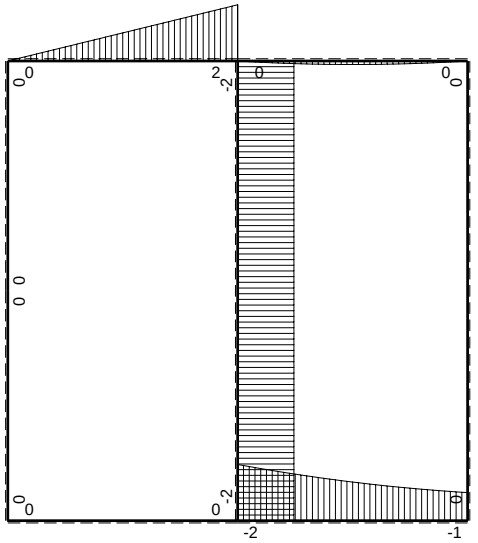
↑[+]↓ F



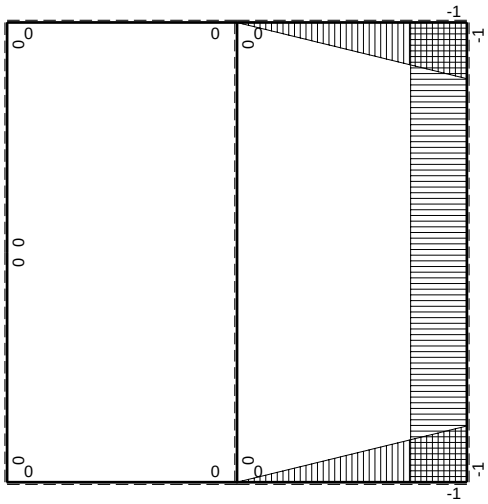
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-2Fb+3/2Fx-1/2qx^2	-Fb/EJ	2Fx-3/2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/8+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
				totali			7/6Fb^2/EJ	8/3Xb/EJ
				iperstatica $X=W_{GC}$			-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (b - b +1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (b - b +1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ + [1/2\ x^2/b]_o^b\ \theta$

$= (b -1/2\ b +1/8\ b)\ Fb\ 1/EJ + (1/2\ b)\ \theta = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [x -1/4\ x^2/b -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ + [- x +1/2\ x^2/b]_o^b\ \theta$

$= (b -1/4\ b -1/8\ b)\ Fb\ 1/EJ + (- b +1/2\ b)\ \theta = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ = 1/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 211210 ESΣ08.xxxx.121

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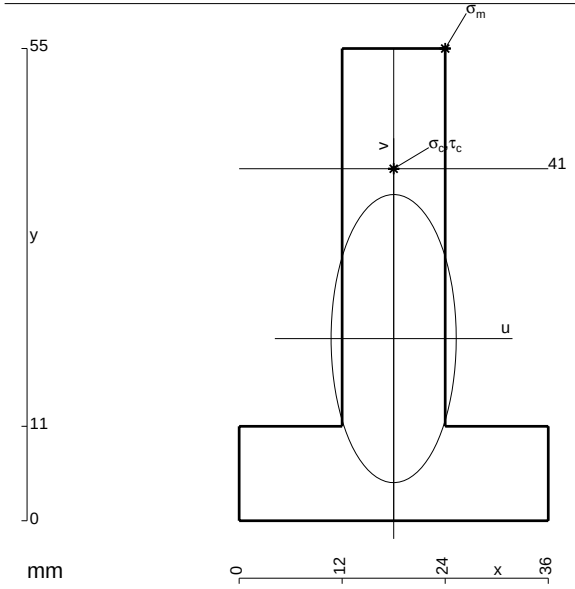
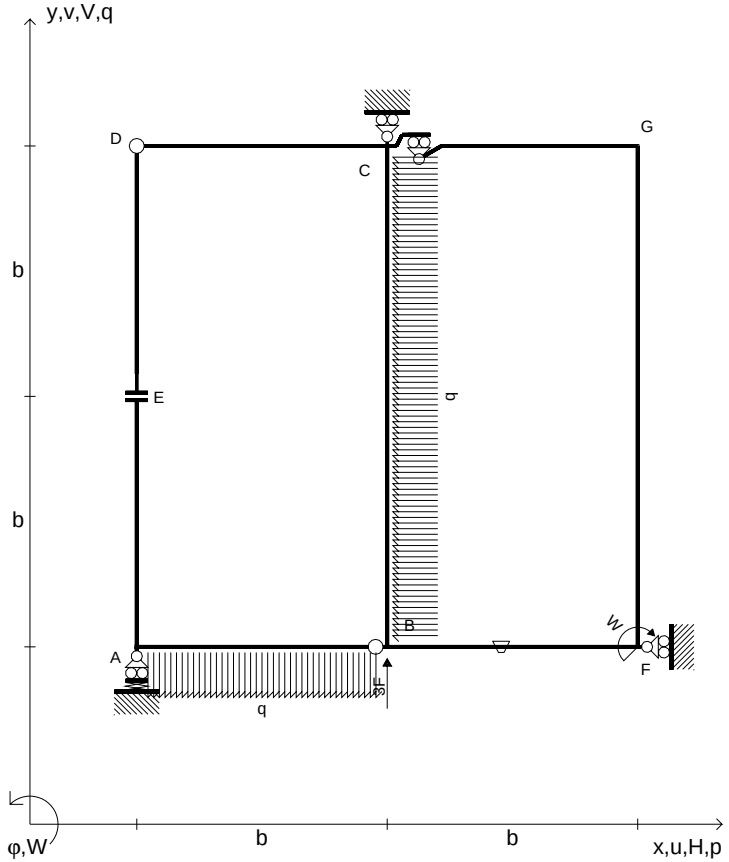
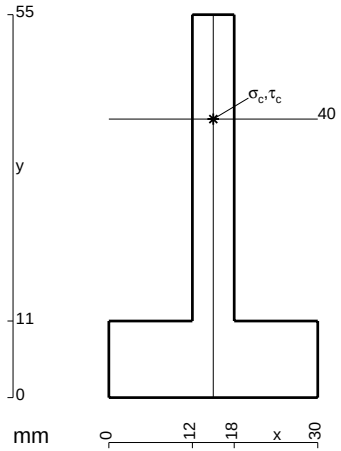


A = 924. mm^2
J_u = 260306. mm^4
J_v = 49104. mm^4
y_g = 21.21 mm
T_y = -3360. N
M_x = 1612800. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 33.79 mm
$\sigma_m = -Mv/J_u$ = -209.3 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 19.79 mm
$\sigma_c = -Mv/J_u$ = -122.6 N/mm^2
τ_c = 4.84 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.9 N/mm^2
S^* = 4500. mm^3

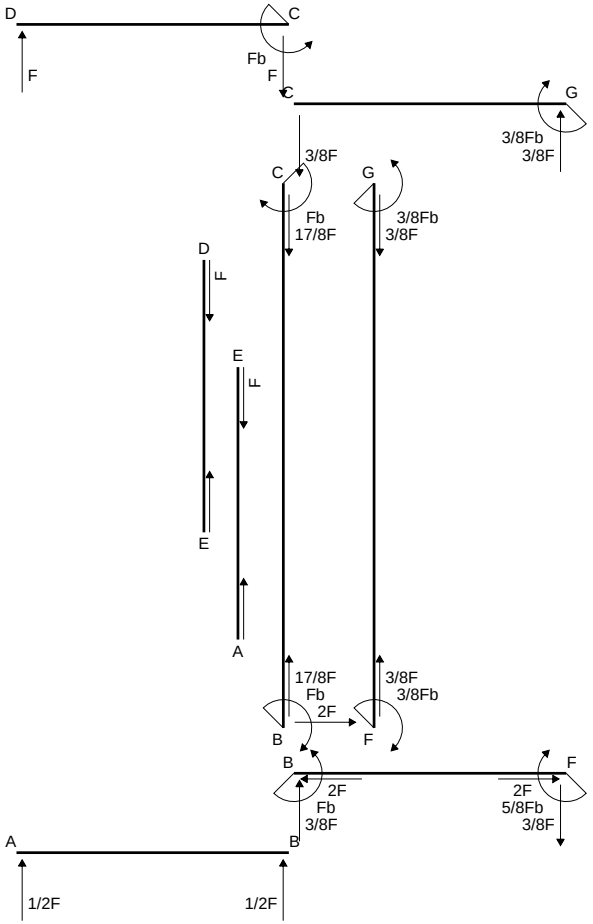
ESΣ08.xxxx.121
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

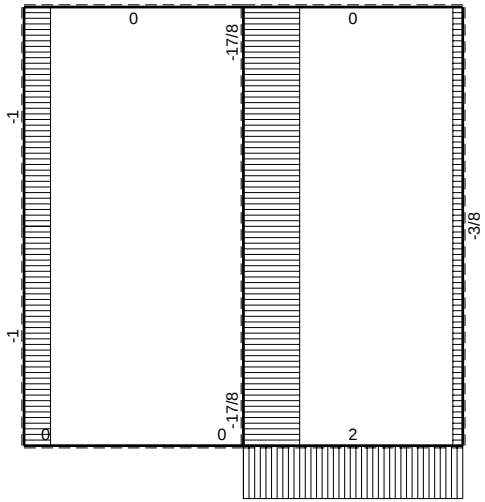
$V_B = 3F$
$W_F = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{CB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



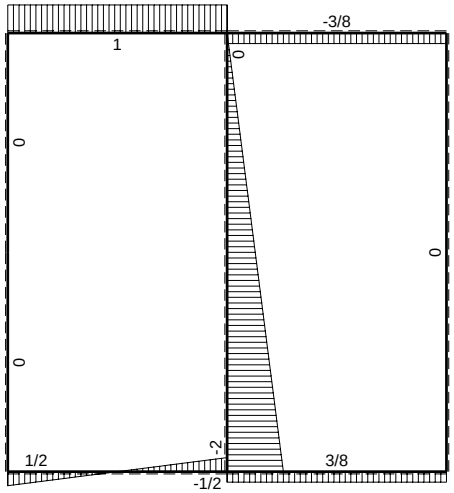


Reazioni iperstatiche in soluzione: $X=W_{GF}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CB ha la sezione riportata e dimensioni in mm, con:
b = 610 mm, F = 1690 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a B
Curvatura θ asta BF positiva se convessa a destra con inizio B.

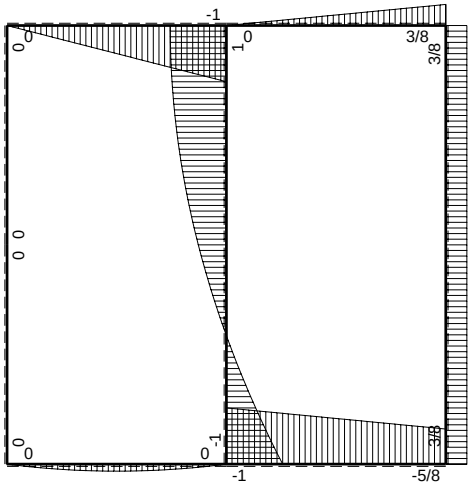




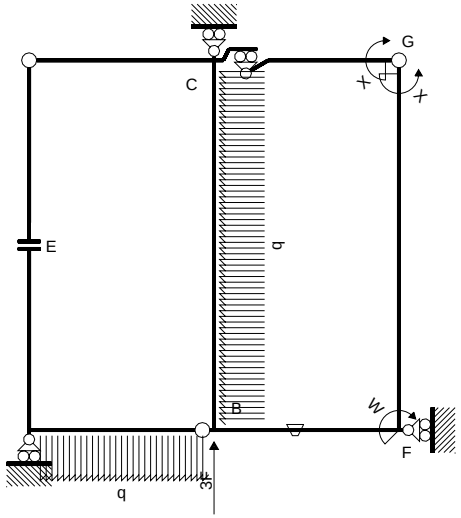
←⊞→ F

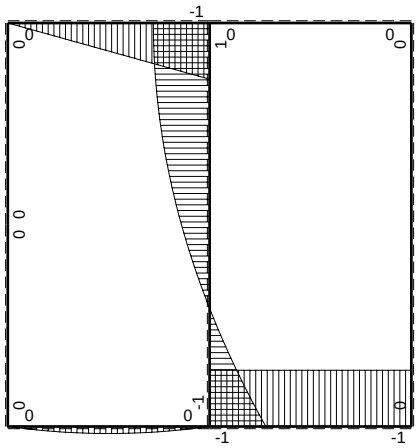


↑⊞↓ F

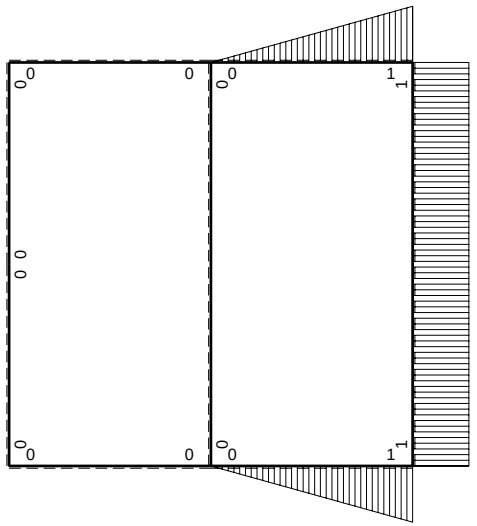


↻⊞↺ Fb





↻⊞↺ M_o flessione da carichi assegnati



↻⊞↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	x/b	-Fb	-Fb/EJ	-Fx	-Fx/EJ	x^2/b^2	$(-1/2-1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	-1+x/b	Fb	Fb/EJ	-Fb+Fx	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	-x/b	0	0	0	0	x^2/b^2		
FG 2b	1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	-1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GF}$						3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 213131 ESΣ08.xxxx.124

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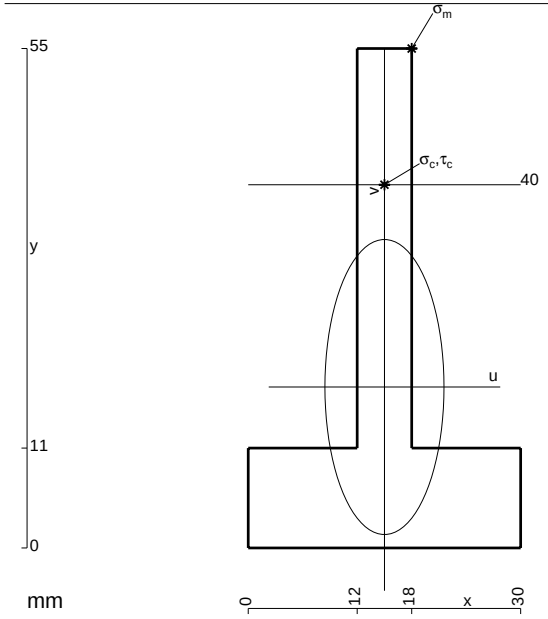

$A = 594.\ mm^2$
$J_u = 156836.\ mm^4$
$J_v = 25542.\ mm^4$
$y_g = 17.72$ mm
$N = -3591.$ N
$T_y = -3380.$ N
$M_x = -1030900.$ Nmm
$x_m = 18.$ mm
$y_m = 55.$ mm
$u_m = 3.$ mm
$v_m = 37.28$ mm
$\sigma_m = N/A - Mv/J_u = 239.\ N/mm^2$
$x_c = 15.$ mm
$y_c = 40.$ mm
$v_c = 22.28$ mm
$\sigma_c = N/A - Mv/J_u = 140.4\ N/mm^2$
$\tau_c = 9.626\ N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 141.4\ N/mm^2$
$S^* = 2680.\ mm^3$

ESΣ08.xxxx.124

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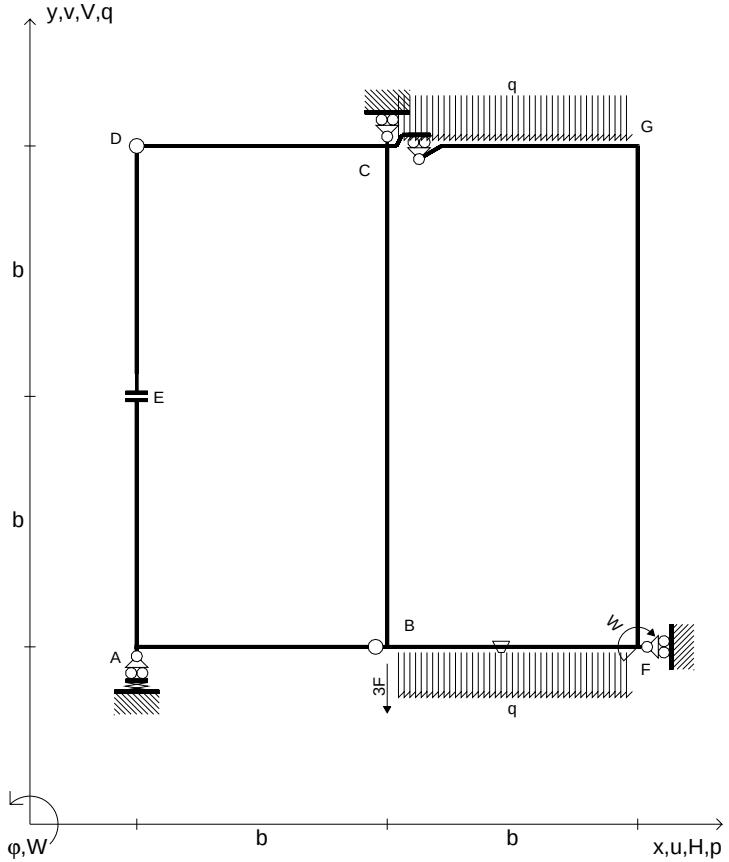
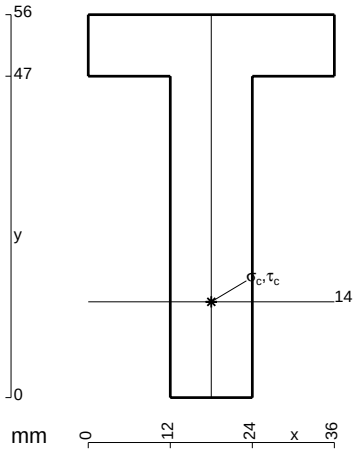
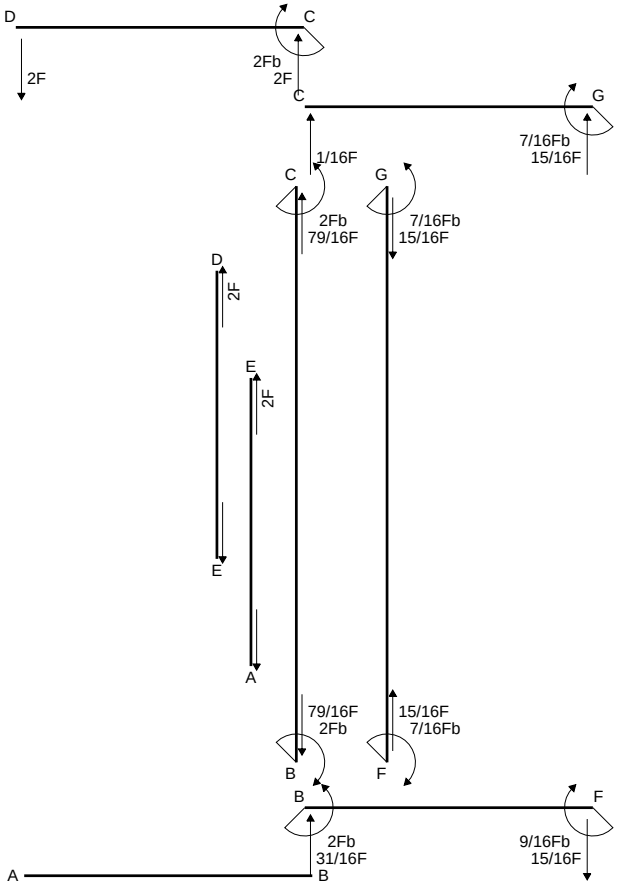
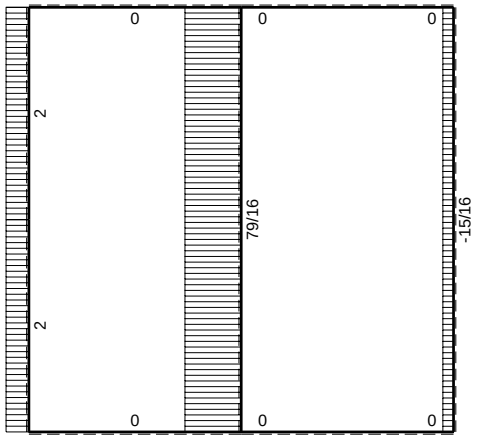
$V_B = -3F$
$W_F = -W = -Fb$
$q_{GC} = -q = -F/b$
$q_{BF} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



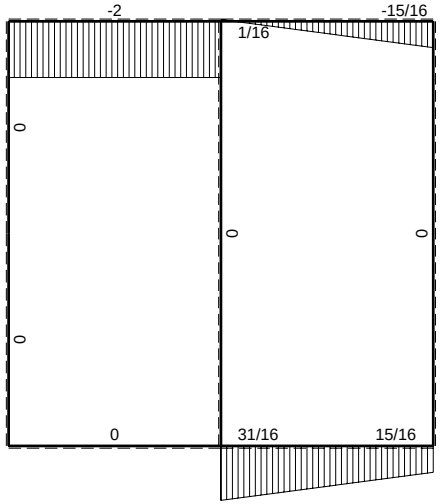


Reazioni iperstatiche in soluzione: $X=W_{GC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 480 mm, F = 1730 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Curvatura θ asta BF positiva se convessa a destra con inizio B.

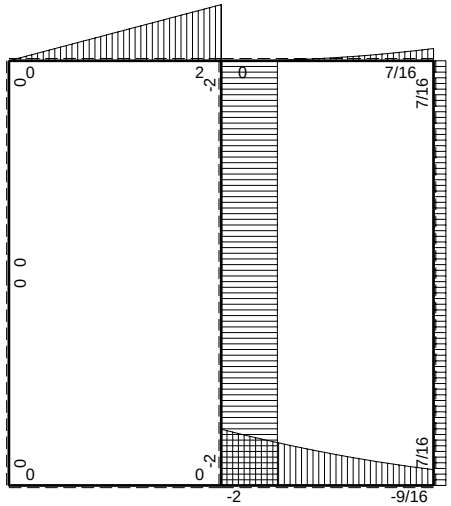




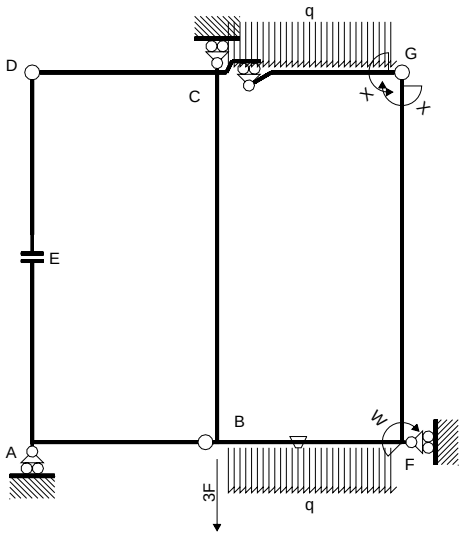
←□→ F



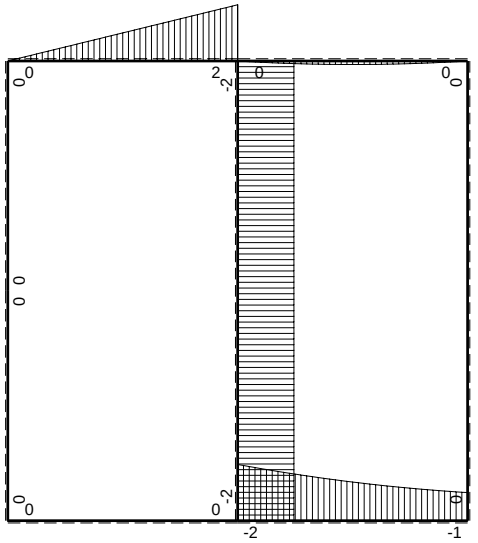
↑□↓ F



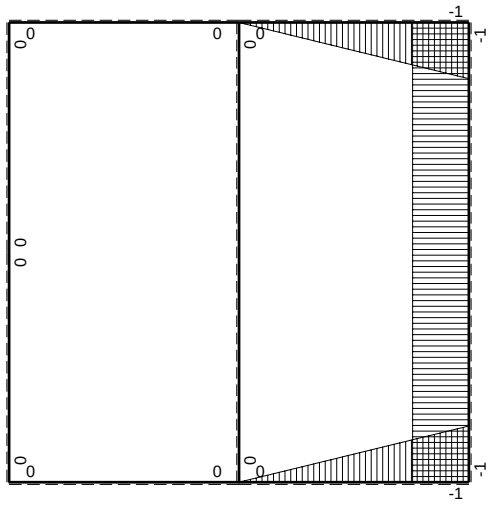
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b]_o^b\ \theta$

$= (\ b -1/4\ b -1/8\ b)\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 212409 ESΣ08.xxxx.151

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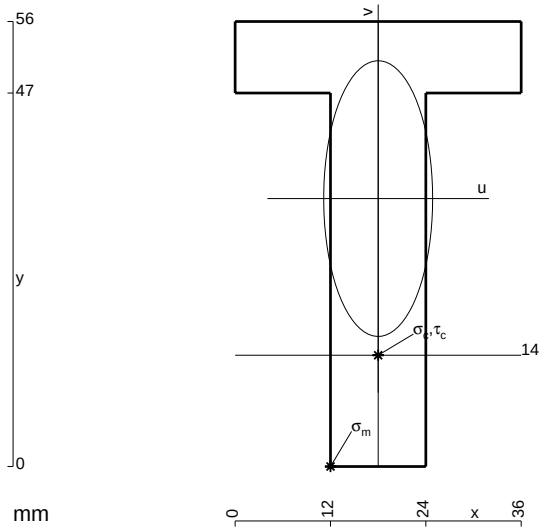

A = 888. mm^2
J_u = 267344. mm^4
J_v = 41760. mm^4
y_g = 33.72 mm
T_y = -3460. N
M_x = 1660800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.72 mm
$\sigma_m = -Mv/J_u$ = 209.5 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.72 mm
$\sigma_c = -Mv/J_u$ = 122.5 N/mm^2
τ_c = 4.841 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.8 N/mm^2
S^* = 4488. mm^3

ESΣ08.xxxx.151

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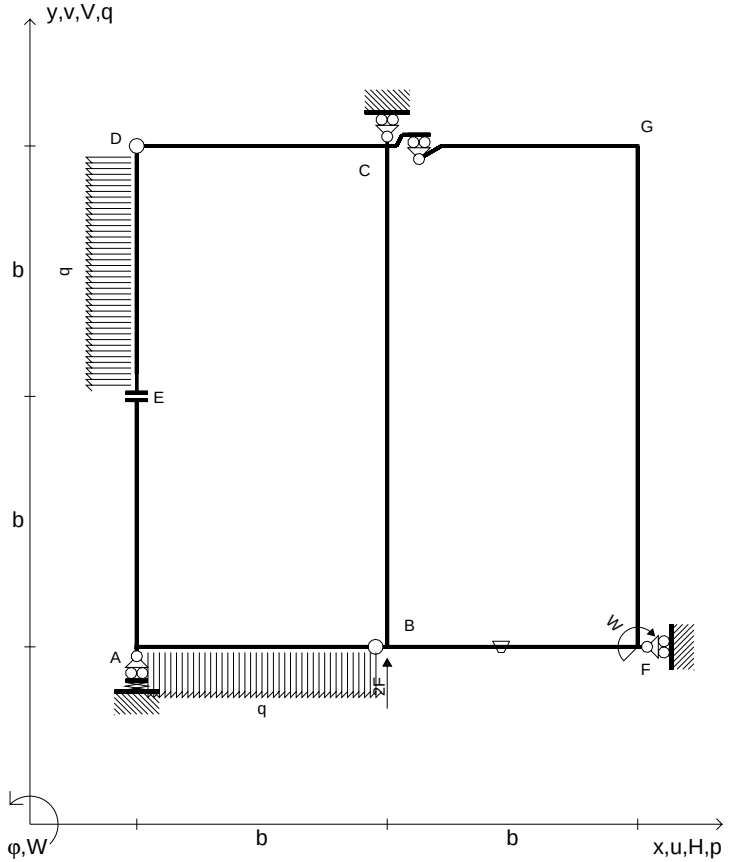
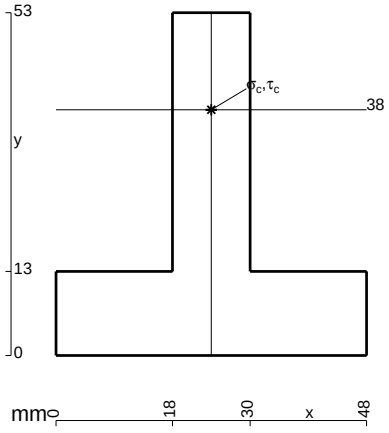
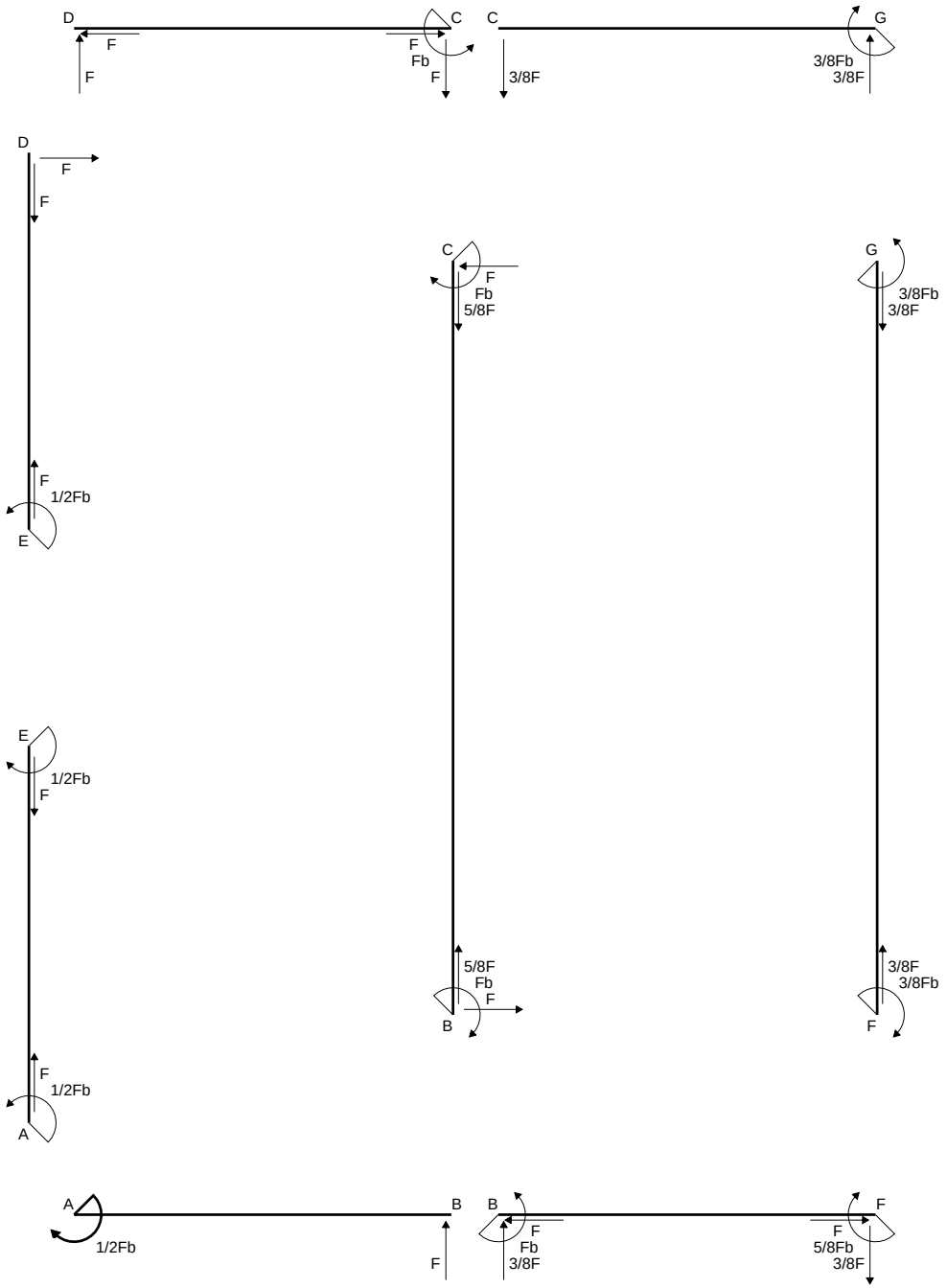
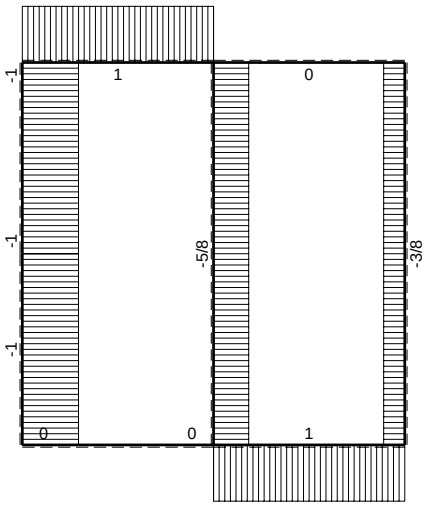
$V_B = 2F$
$W_F = -W = -Fb$
$p_{DE} = -q = -F/b$
$q_{AB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



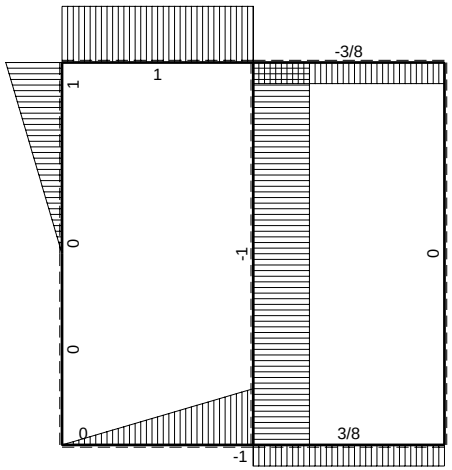


Reazioni iperstatiche in soluzione: $X=W_{GC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 460 mm, F = 3380 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Curvatura θ asta BF positiva se convessa a destra con inizio B.

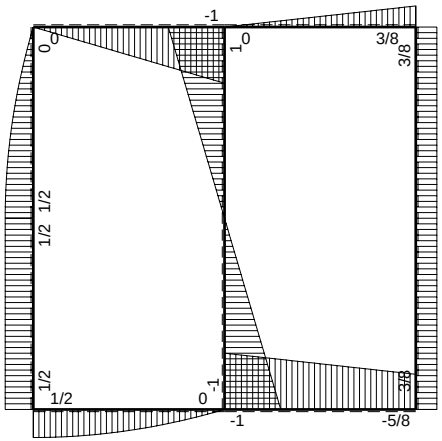




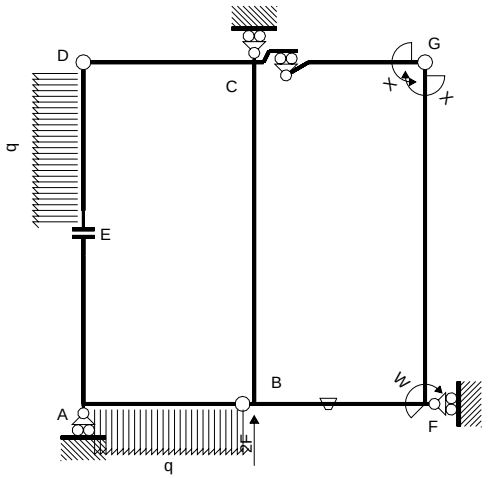
←⊞→ F



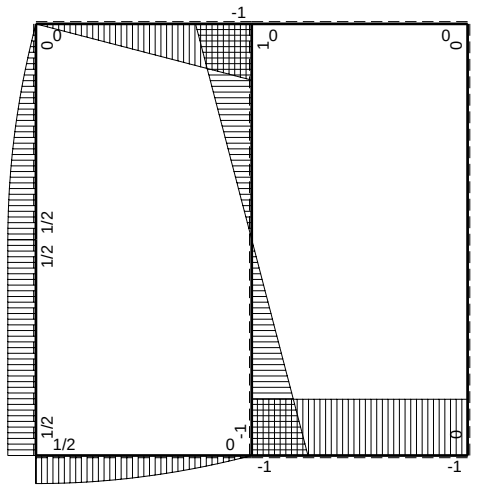
↑⊞↓ F



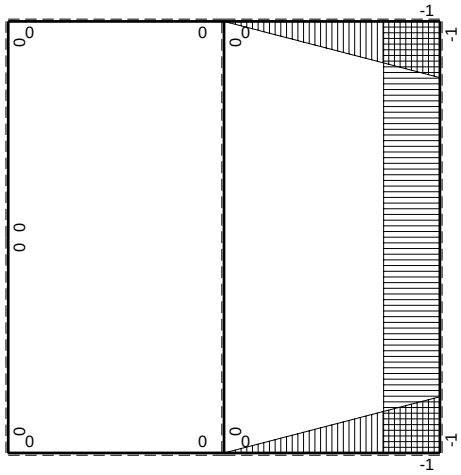
↻⊞↺ Fb



Schema di calcolo iperstatico



↻⊞↺ M_o flessione da carichi assegnati



↻⊞↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

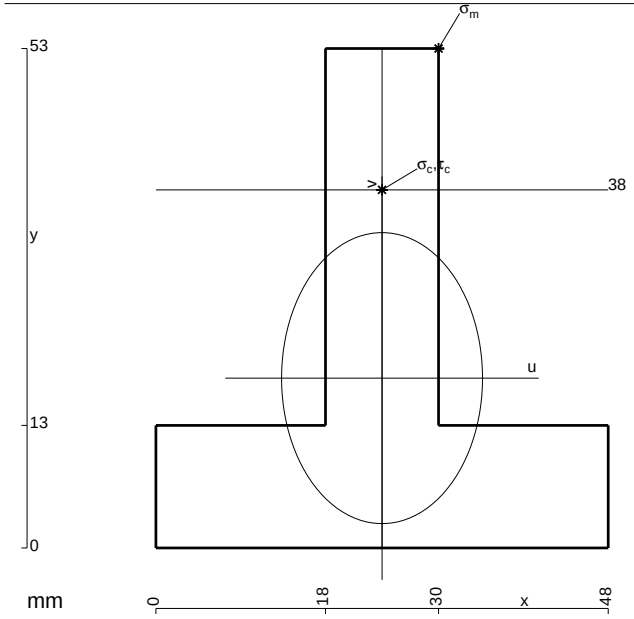
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 1104. mm^2
J_u = 263311. mm^4
J_v = 125568. mm^4
y_g = 18.02 mm
N = 3380. N
T_y = 3380. N
M_x = -1554800. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 34.98 mm
$\sigma_m = N/A - Mv/J_u$ = 209.6 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.98 mm
$\sigma_c = N/A - Mv/J_u$ = 121. N/mm^2
τ_c = 5.291 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 121.4 N/mm^2
S^* = 4946. mm^3

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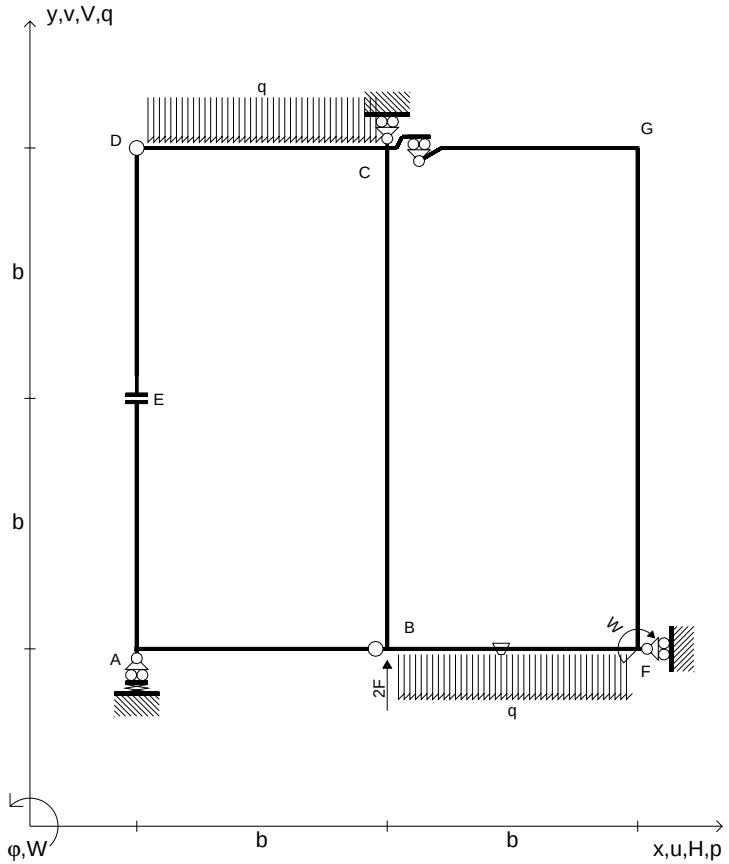
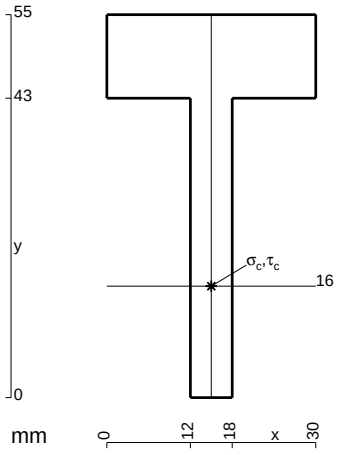
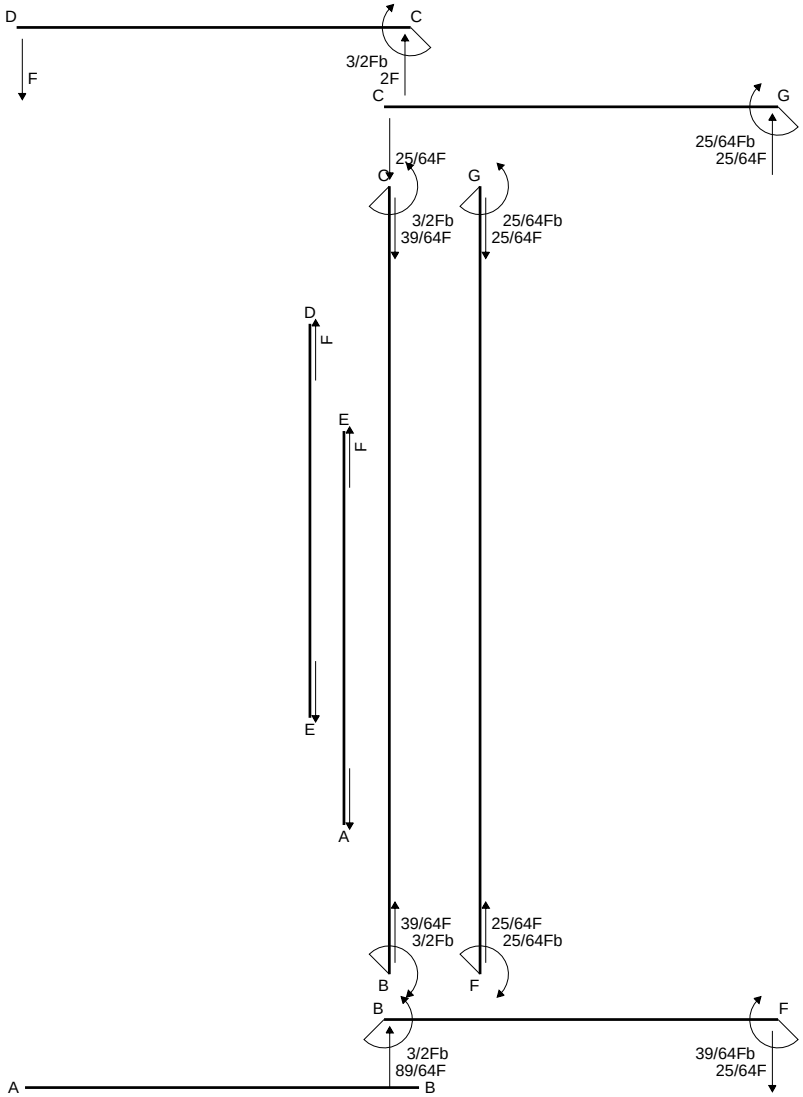
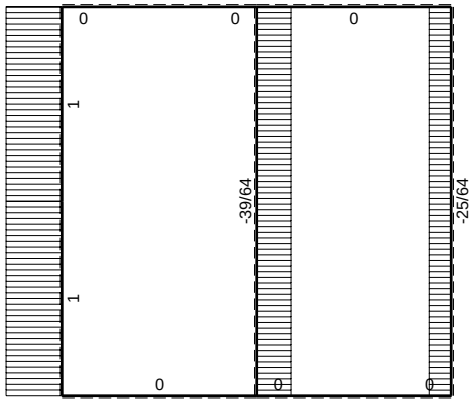
$V_B = 2F$
$W_F = -W = -Fb$
$q_{CD} = -q = -F/b$
$q_{BF} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



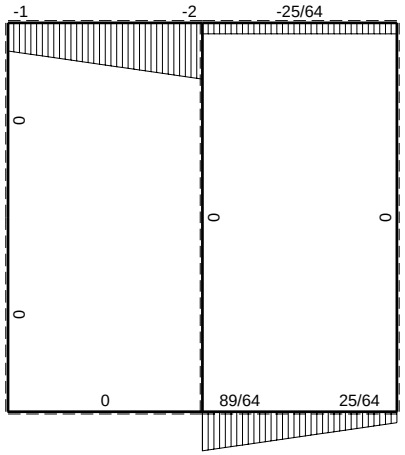


Reazioni iperstatiche in soluzione: $X=W_{GC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 790 mm, F = 810 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Curvatura θ asta BF positiva se convessa a destra con inizio B.

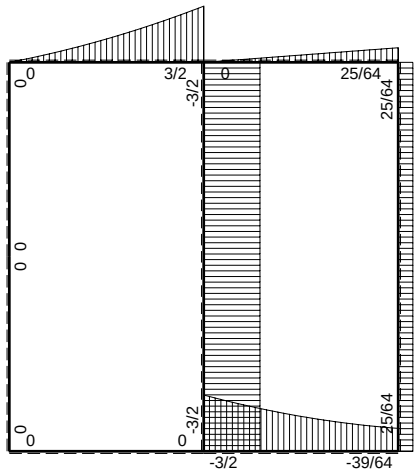




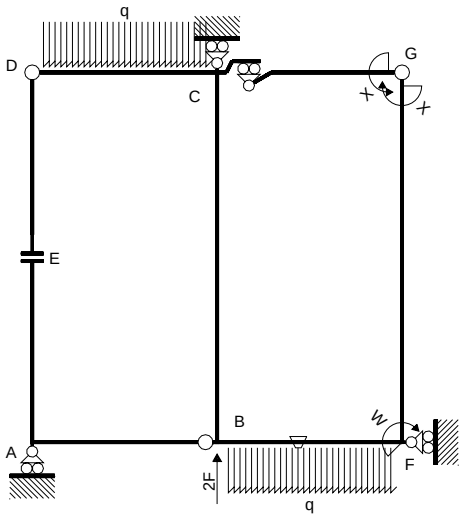
←[+]→ F



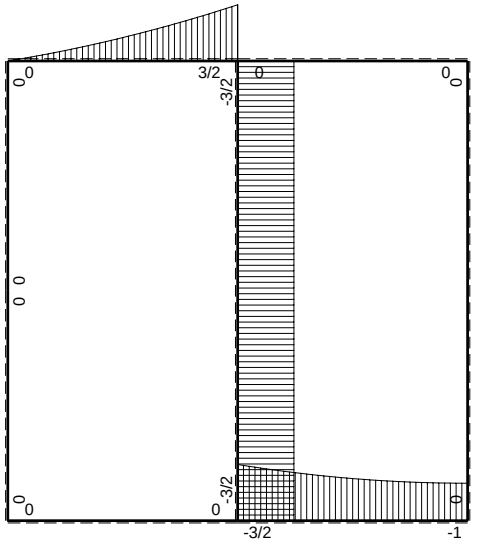
↑[+]↓ F



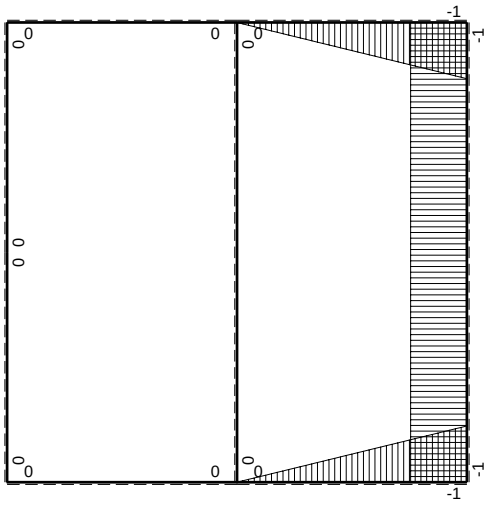
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	3/2Fb-2Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	-Fx-1/2qx^2	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-3/2Fb+Fx-1/2qx^2	-Fb/EJ	3/2Fx-Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(13/24+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2qx^2	Fb/EJ	Fb-Fx+1/2Fx^2/b-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						25/24Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-25/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 25/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 280320 ESΣ08.xxxx.198

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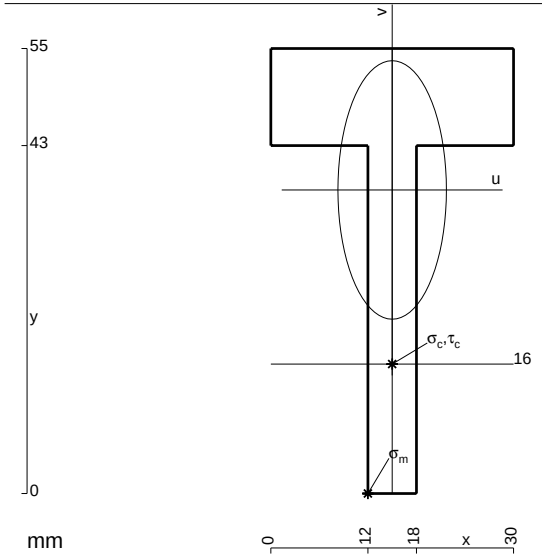

A = 618. mm^2
J_u = 157731. mm^4
J_v = 27774. mm^4
y_g = 37.52 mm
T_y = -1620. N
M_x = 959850. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.52 mm
$\sigma_m = -Mv/J_u$ = 228.3 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.52 mm
$\sigma_c = -Mv/J_u$ = 131. N/mm^2
τ_c = 4.851 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.2 N/mm^2
S^* = 2834. mm^3

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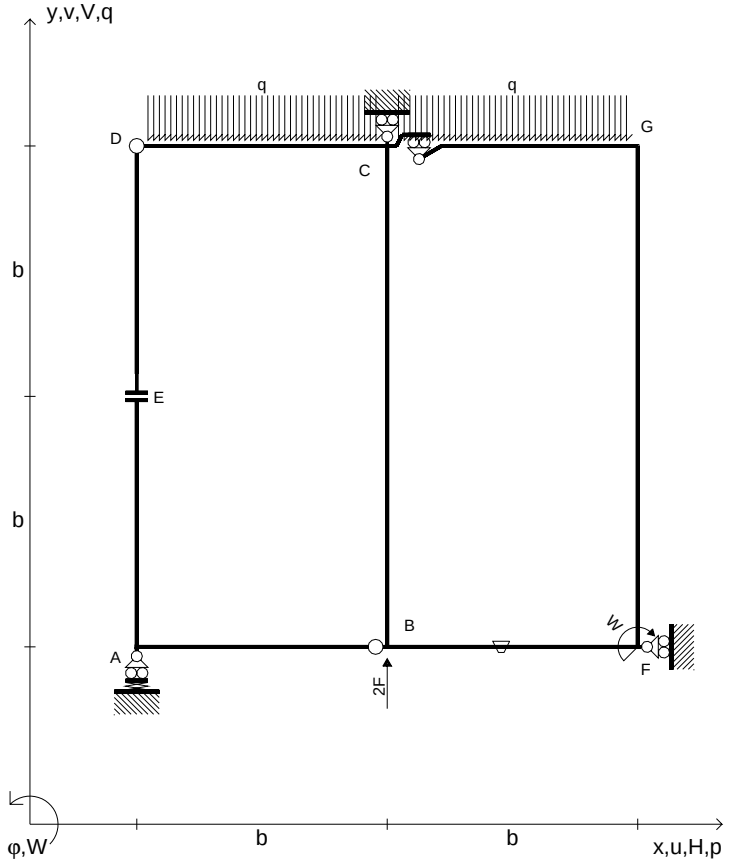
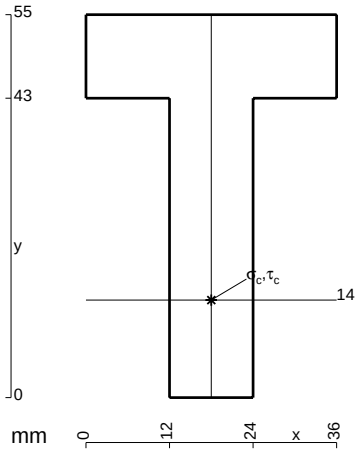
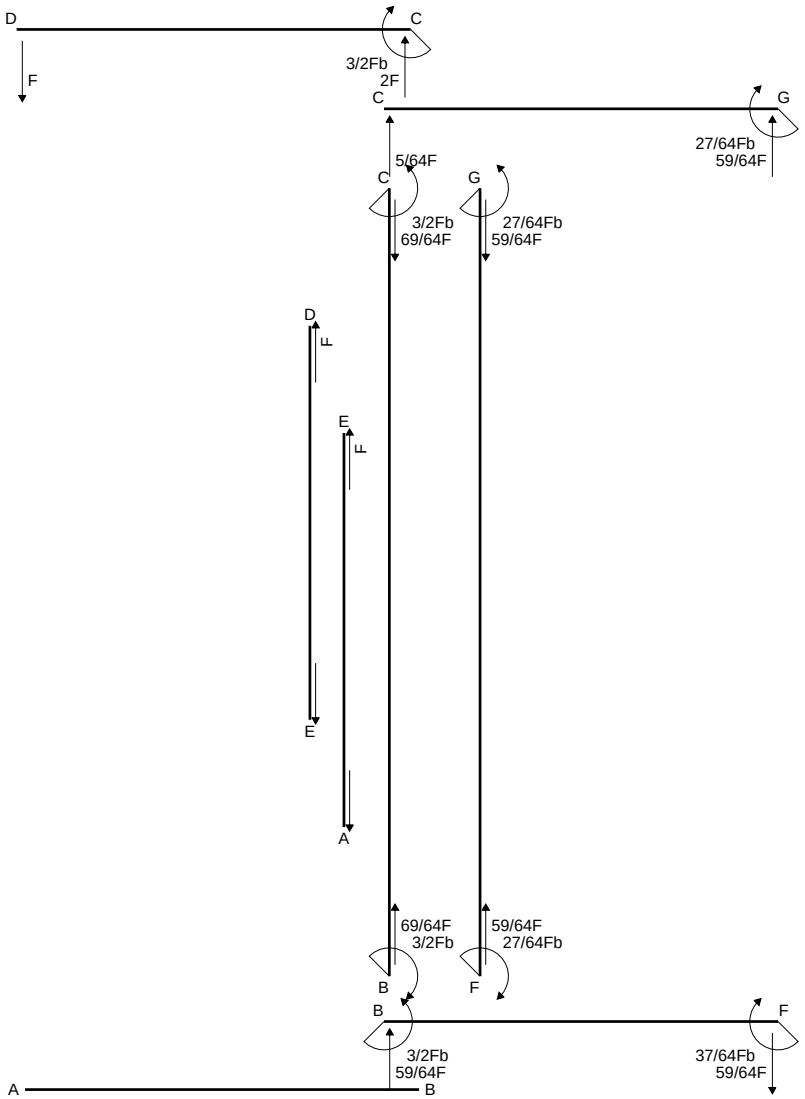
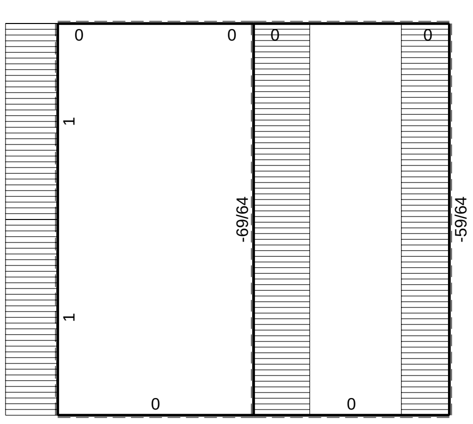
$V_B = 2F$
$W_F = -W = -Fb$
$q_{CD} = -q = -F/b$
$q_{GC} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



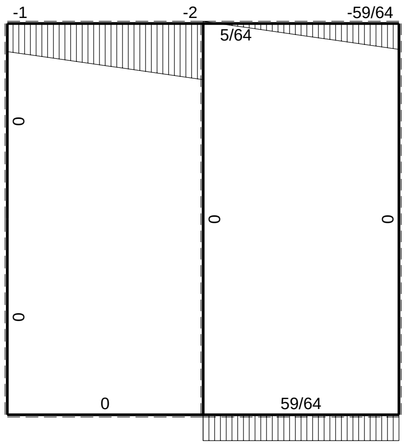


Reazioni iperstatiche in soluzione: $X=W_{GC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 440 mm, F = 2330 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Curvatura θ asta BF positiva se convessa a destra con inizio B.

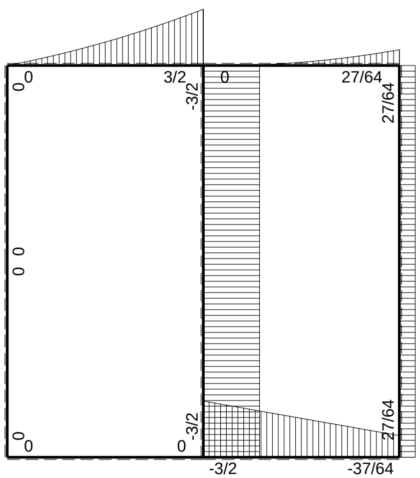




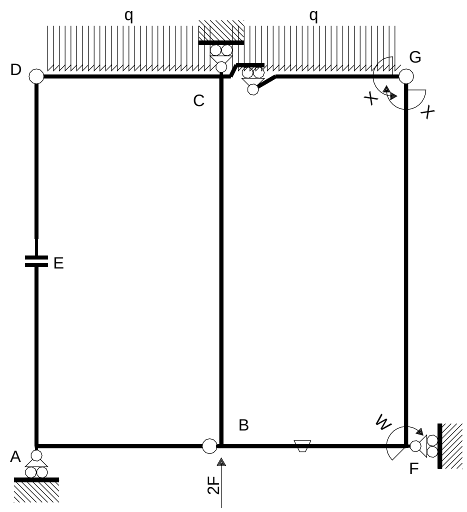
F



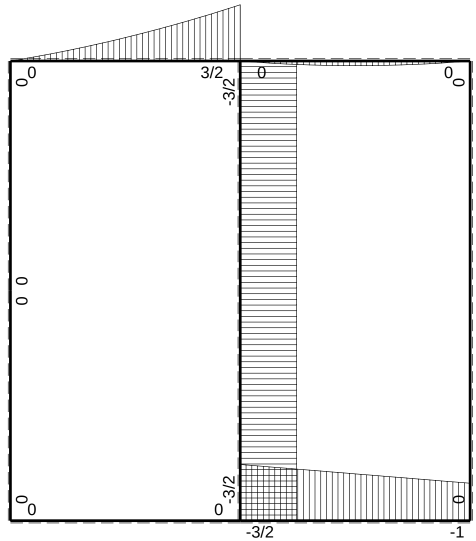
F



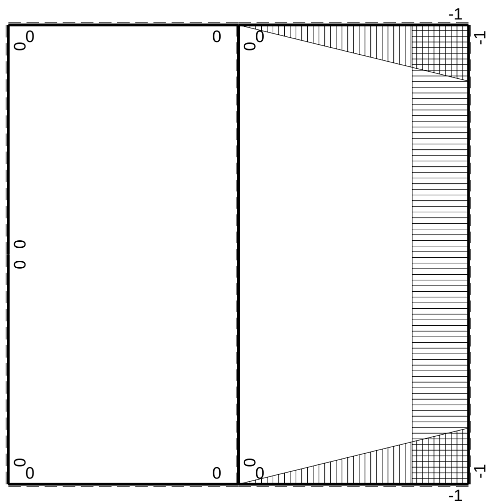
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-3/2Fb+1/2Fx$	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx$	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
				totali			9/8Fb²/EJ	8/3Xb/EJ
				iperstatica $X=W_{GC}$			-27/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ + [1/2\ x^2/b]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ + (1/2\ b)\ \theta = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ + [- x +1/2\ x^2/b]_o^b\ \theta$

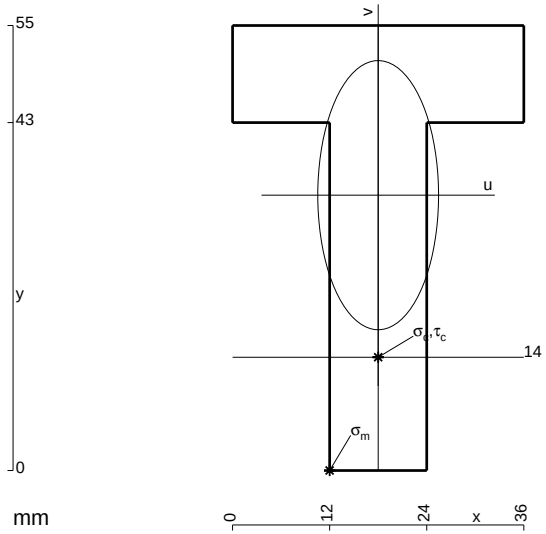
$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ + (- b +1/2\ b)\ \theta = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ = 1/24\ Fb^2/EJ$



A = 948. mm^2
J_u = 262515. mm^4
J_v = 52848. mm^4
y_g = 34.03 mm
T_y = -4660. N
M_x = 1537800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -34.03 mm
$\sigma_m = -Mv/J_u$ = 199.4 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -20.03 mm
$\sigma_c = -Mv/J_u$ = 117.3 N/mm^2
τ_c = 6.718 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.9 N/mm^2
S^* = 4541. mm^3

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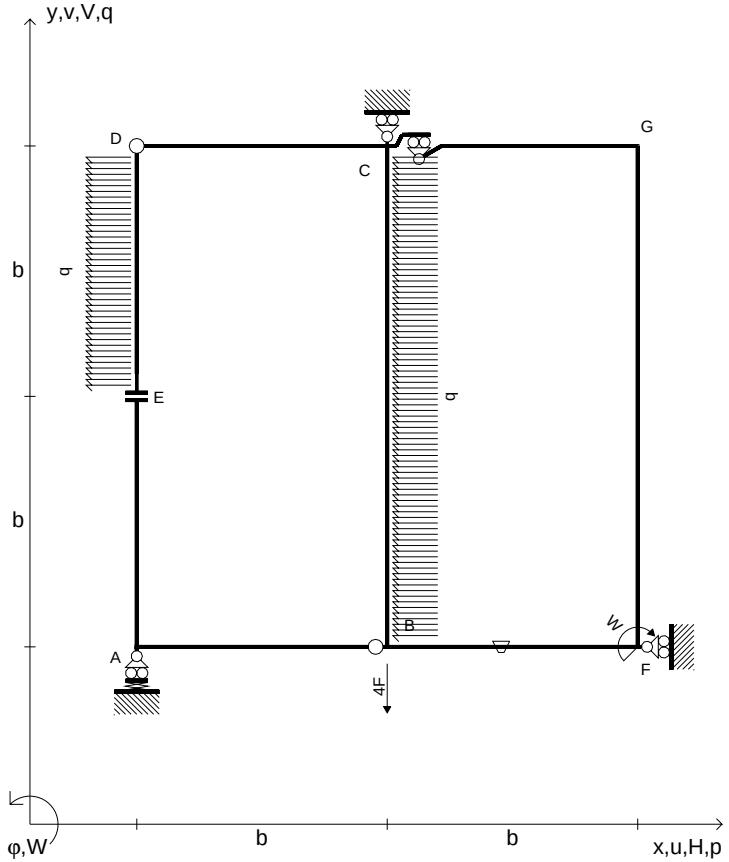
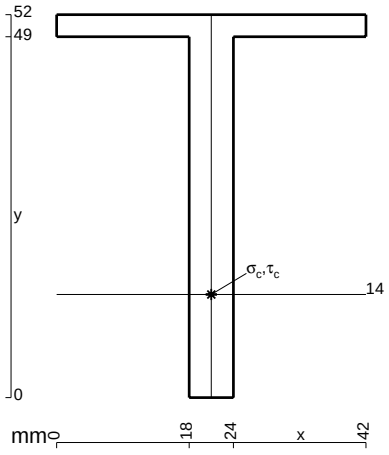
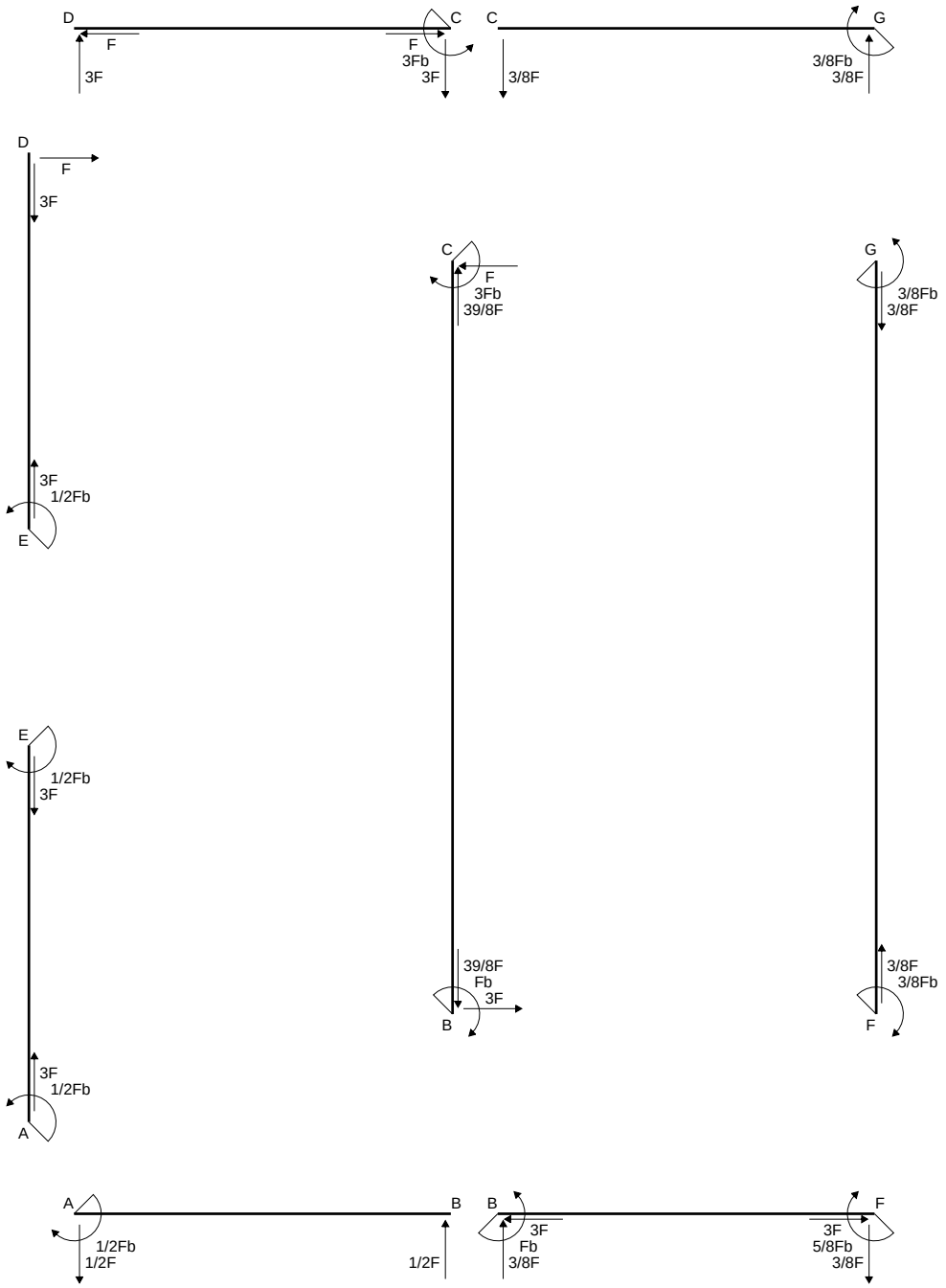
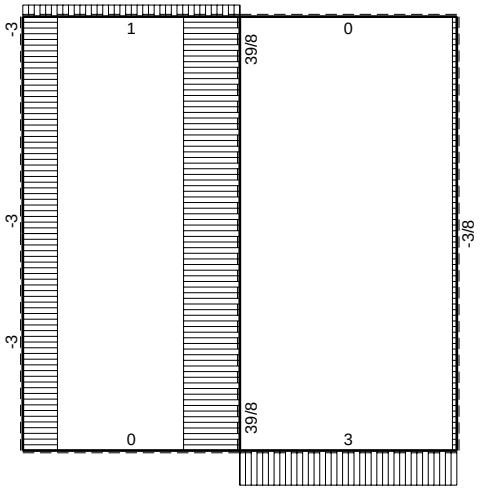
$V_B = -4F$
$W_F = -W = -Fb$
$p_{DE} = -q = -F/b$
$p_{CB} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



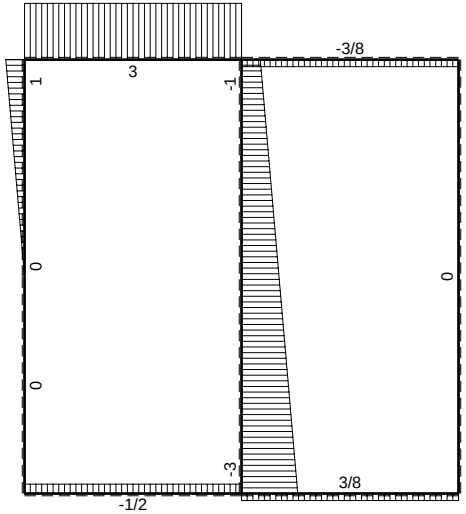


Reazioni iperstatiche in soluzione: $X=W_{GC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 770 mm, F = 380 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Curvatura θ asta BF positiva se convessa a destra con inizio B.

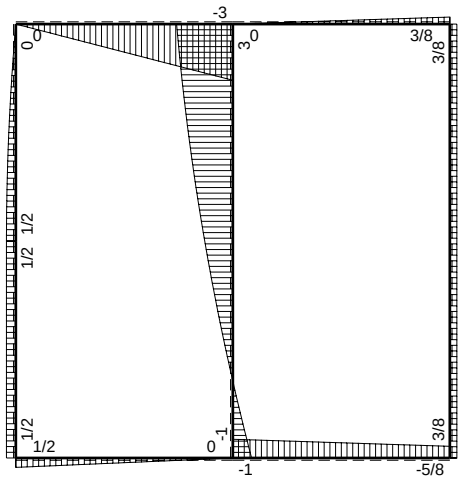




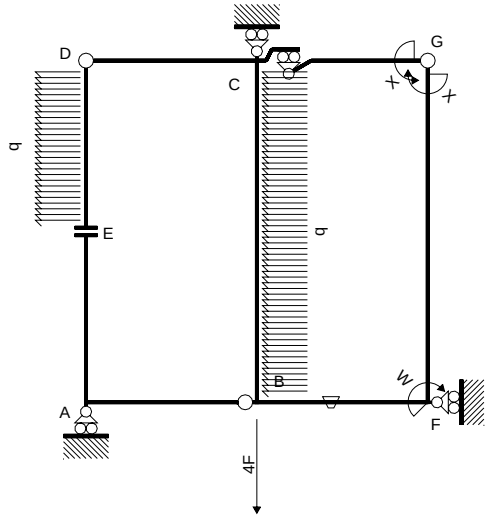
←□→ F



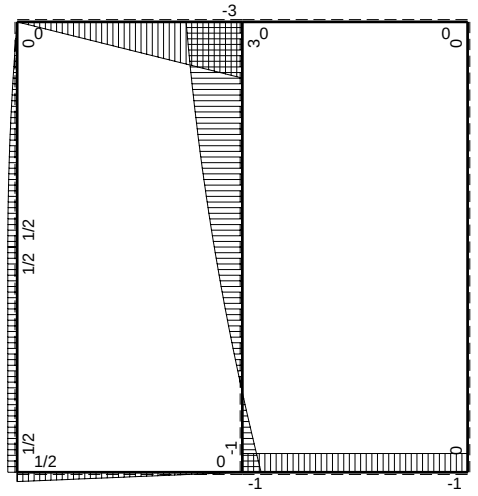
↑□↓ F



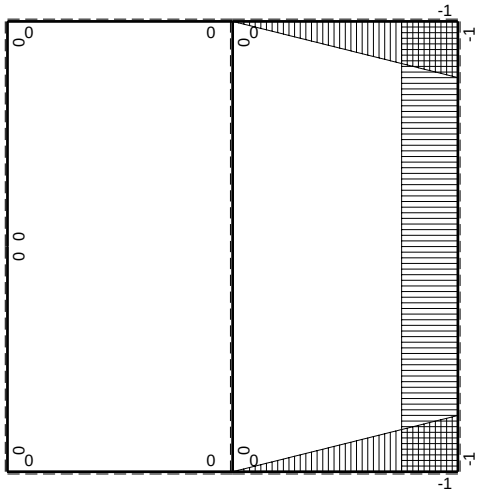
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x\theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$3Fb-Fx-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-3Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982827 ESΣ08.xxxx.209

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (x/b)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

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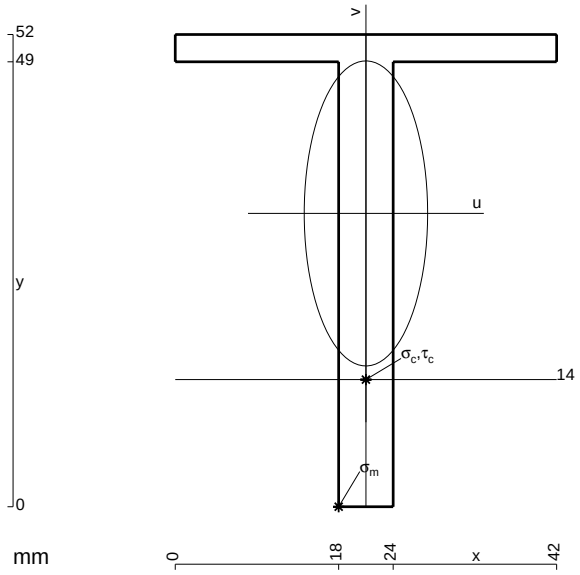

$A = 420.\ mm^2$
$J_u = 118542.\ mm^4$
$J_v = 19404.\ mm^4$
$y_g = 32.3\ mm$
$N = 380.\ N$
$T_y = 1140.\ N$
$M_x = -877800.\ Nmm$
$x_m = 18.\ mm$
$u_m = -3.\ mm$
$v_m = -32.3\ mm$
$\sigma_m = N/A - Mv/J_u = -238.3\ N/mm^2$
$x_c = 21.\ mm$
$y_c = 14.\ mm$
$v_c = -18.3\ mm$
$\sigma_c = N/A - Mv/J_u = -134.6\ N/mm^2$
$\tau_c = 3.406\ N/mm^2$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 134.7\ N/mm^2$
$S^* = 2125.\ mm^3$

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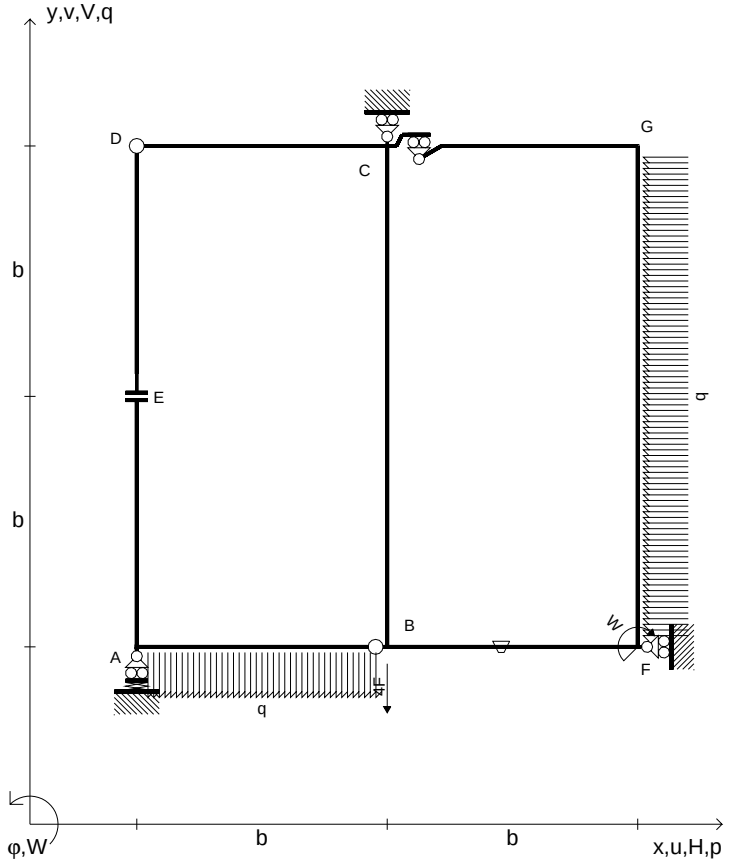
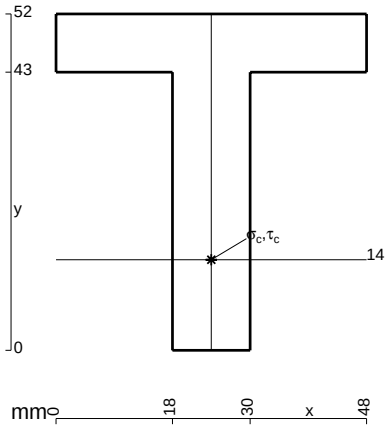
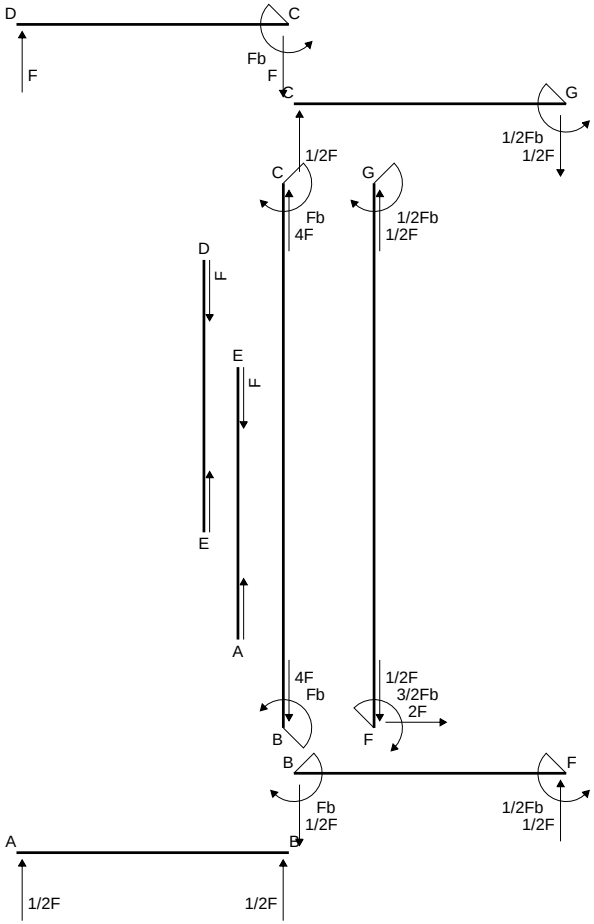
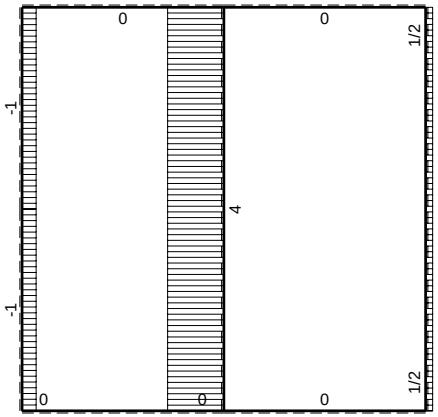
$V_B = -4F$
$W_F = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{BF} = -\theta = -\alpha T/b = -bF/EJ$
$k_A = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$
$EJ_{BF} = EJ$
$EJ_{GC} = EJ$
$EJ_{FG} = EJ$
$EJ_{CB} = EJ$



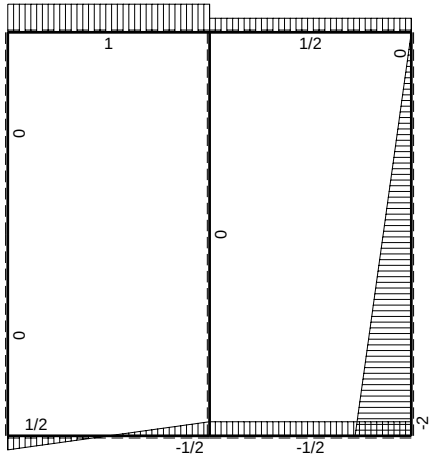


Reazioni iperstatiche in soluzione: $X=V_{CG}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 490 mm, F = 3240 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Curvatura θ asta BF positiva se convessa a destra con inizio B.

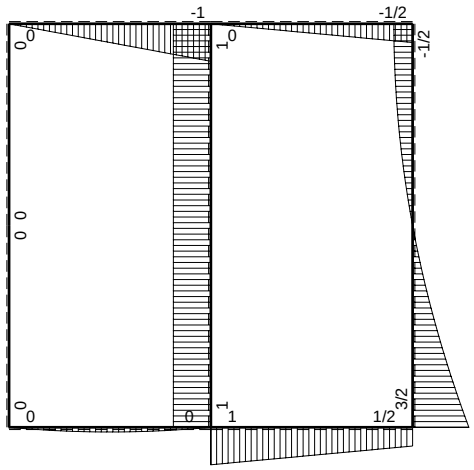




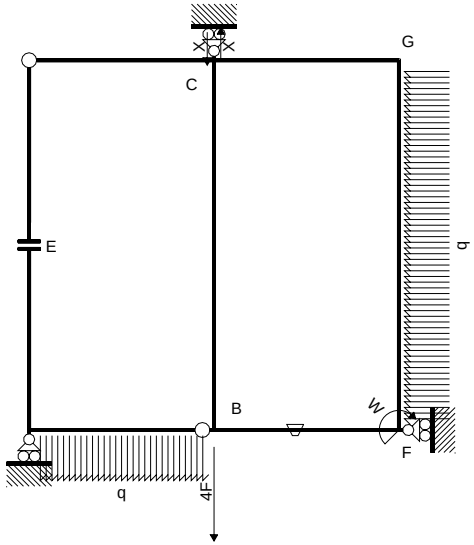
←[+]→ F

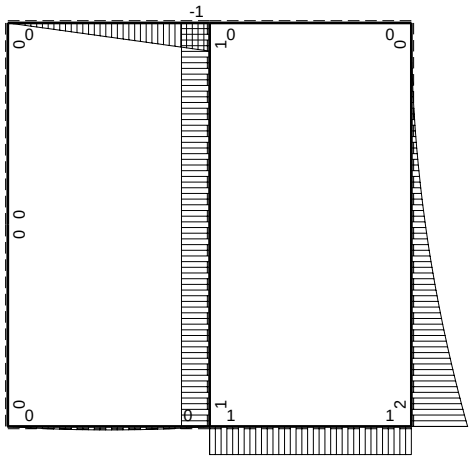


↑[+]↓ F

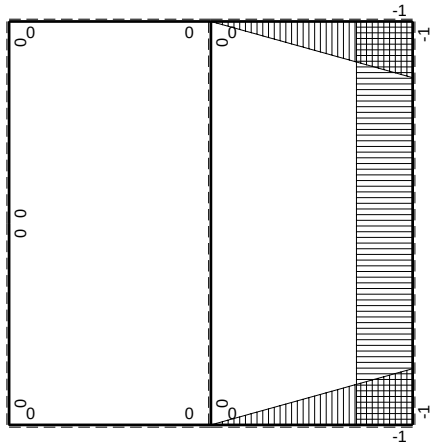


[+] Fb





[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_{CG}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x	Fb	-Fb/EJ	-Fbx	Fxb/EJ	x^2	$(-1/2+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
FB b	b-x	-Fb	Fb/EJ	$-Fb^2+Fbx$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CG b	x	0	0	0	0	x^2		
FG 2b	-b	$2Fb-2Fx+1/2qx^2$	0	$-2Fb^2+2Fbx-1/2Fx^2$	0	b^2	$(-4/3+0)Fb^3/EJ$	$2Xb^3/EJ$
GF 2b	b	$-1/2qx^2$	0	$-1/2Fx^2$	0	b^2		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^3/EJ$	$8/3Xb^3/EJ$
	iperstatica $X=V_{CG}$						1/2F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984497 ESΣ08.xxxx.212

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b\,)\, b^2\, 1/EJ\ = 1/3\ b^3/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\, x/b + x^2/b^2\,)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b\,)\, b^2\, 1/EJ\ = 1/3\ b^3/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\, x/b + x^2/b^2\,)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b\,)\, b^2\, 1/EJ\ = 1/3\ b^3/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2\,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b\,)\, b^2\, 1/EJ\ = 1/3\ b^3/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1\,)\, b^2\, 1/EJ\, dx = [\, x\,]_o^{2b}\, b^2\, 1/EJ$

$= (2\, b\,)\, b^2\, 1/EJ\ = 2\ b^3/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1\,)\, b^2\, 1/EJ\, dx = [\, x\,]_o^{2b}\, b^2\, 1/EJ$

$= (2\, b\,)\, b^2\, 1/EJ\ = 2\ b^3/EJ$

$L_{BF}^{Xo} = \int_o^b (-\, x/b\,)\, Fb^2\, 1/EJ\, dx + \int_o^b (\, x/b\,)\, \theta\ dx = [-1/2\, x^2/b\,]_o^b\, Fb^2\, 1/EJ\ + [1/2\, x^2/b\,]_o^b\, \theta$

$= (-1/2\, b\,)\, Fb^2\, 1/EJ\ + (1/2\, b\,)\, \theta\ = 0$

$L_{FB}^{Xo} = \int_o^b (-1 + x/b\,)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 + x/b\,)\, \theta\ dx$

$= [-\, x +1/2\, x^2/b\,]_o^b\, Fb^2\, 1/EJ\ + [-\, x +1/2\, x^2/b\,]_o^b\, \theta$

$= (-\, b +1/2\, b\,)\, Fb^2\, 1/EJ\ + (-\, b +1/2\, b\,)\, \theta\ = 0$

$L_{FG}^{Xo} = \int_o^{2b} (-2 +2\, x/b -1/2\, x^2/b^2\,)\, Fb^2\, 1/EJ\, dx = [-2\, x + x^2/b -1/6\, x^3/b^2\,]_o^{2b}\, Fb^2\, 1/EJ$

$= (-4\, b +4\, b -4/3\, b\,)\, Fb^2\, 1/EJ\ = -4/3\ Fb^3/EJ$

$L_{GF}^{Xo} = \int_o^{2b} (-1/2\, x^2/b^2\,)\, Fb^2\, 1/EJ\, dx = [-1/6\, x^3/b^2\,]_o^{2b}\, Fb^2\, 1/EJ$

$= (-4/3\, b\,)\, Fb^2\, 1/EJ\ = -4/3\ Fb^3/EJ$

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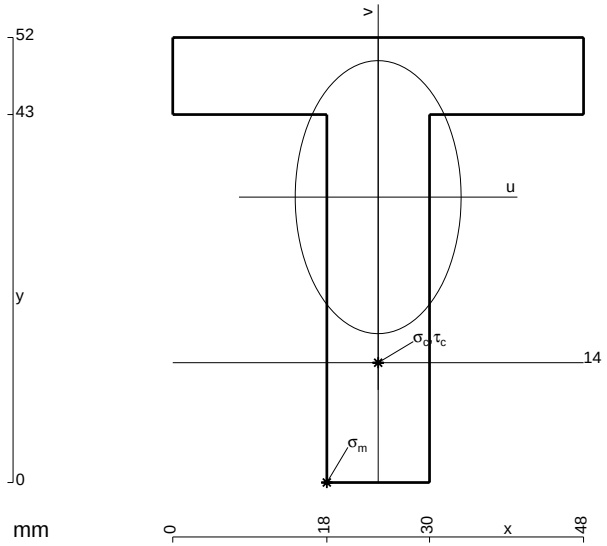

A = 948. mm^2
J_u = 241377. mm^4
J_v = 89136. mm^4
y_g = 33.35 mm
T_y = 3240. N
M_x = -1587600. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.35 mm
$\sigma_m = -Mv/J_u$ = -219.3 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19.35 mm
$\sigma_c = -Mv/J_u$ = -127.3 N/mm^2
τ_c = 4.951 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.5 N/mm^2
S^* = 4426. mm^3

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