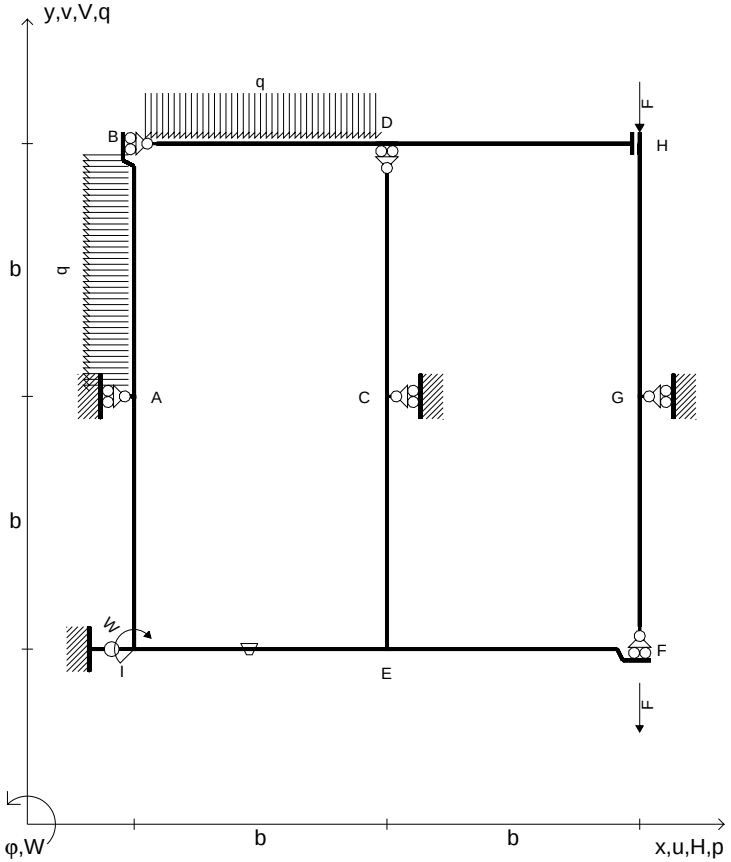
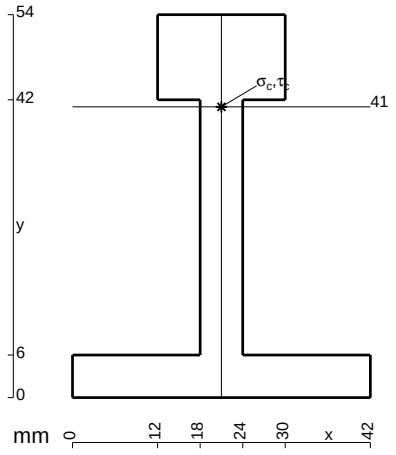
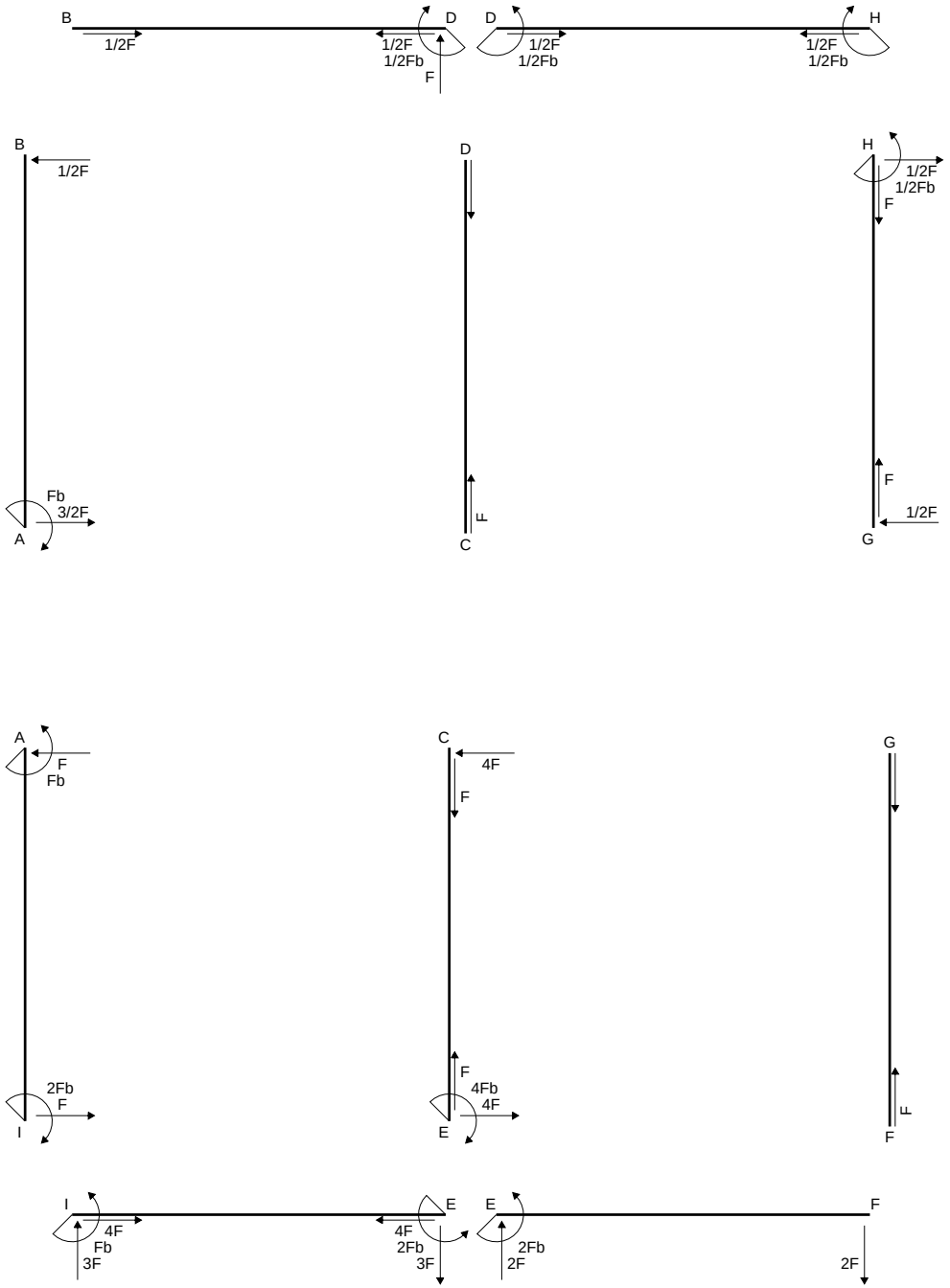
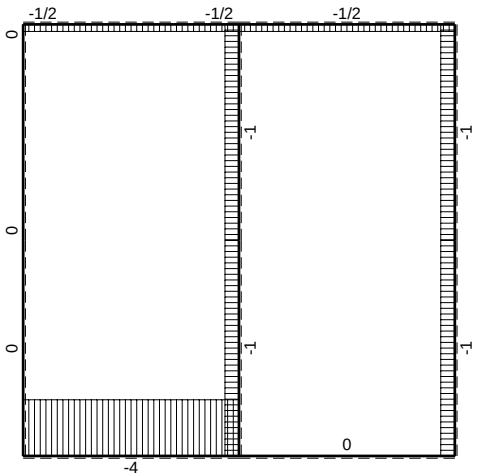
$V_{HG} = -F$
$V_{FE} = -F$
$W_I = -W = -Fb$
$q_{DB} = -q = -F/b$
$p_{AB} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



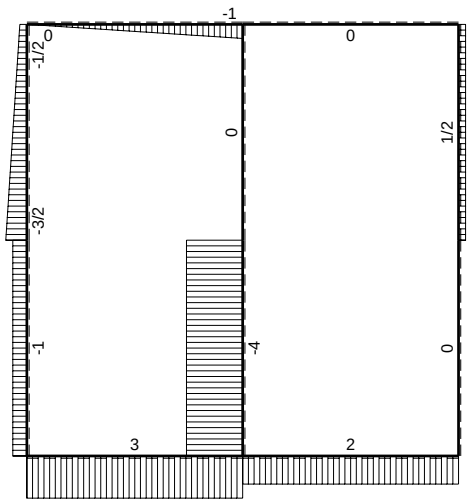


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 540 mm, F = 1610 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

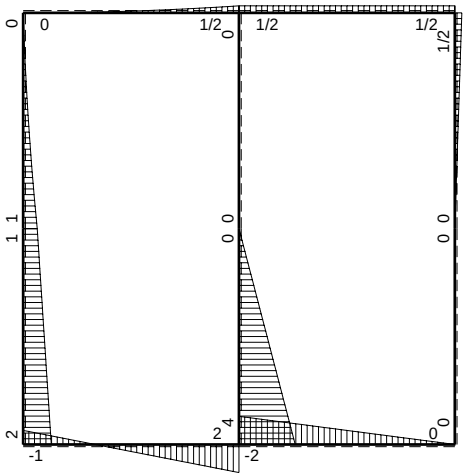




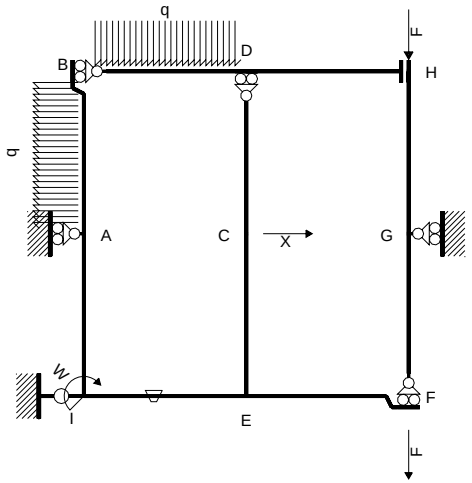
←□→ F



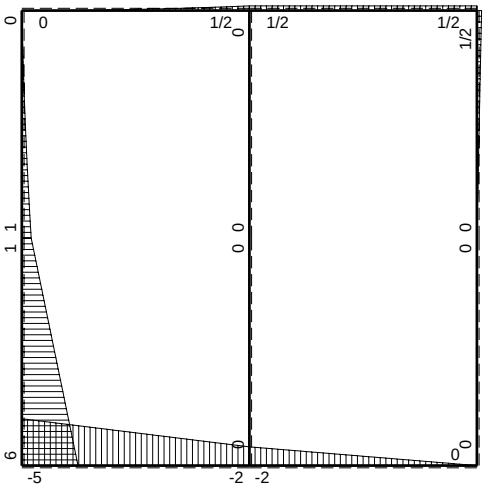
↑□↓ F



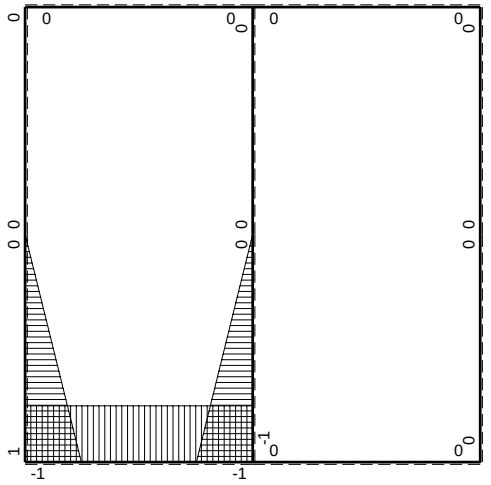
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$Fb-3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx-1/2qx^2$	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
FE b	0	$2Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$1/2Fx$	0	0	0	0	0+0	0
HG b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
HD b	0	$1/2Fb$	0	0	0	0	0+0	0
DH b	0	$-1/2Fb$	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	-b	$-5Fb+3Fx$	$-Fb/EJ$	$5Fb^2-3Fbx$	Fb^2/EJ	b^2	$(7/2+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$2Fb+3Fx$	Fb/EJ	$2Fb^2+3Fbx$	Fb^2/EJ	b^2		
EC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CE b	x	0	0	0	0	x^2		
IA b	b-x	$6Fb-5Fx$	0	$6Fb^2-11Fbx+5Fx^2$	0	$b^2-2bx+x^2$	$(13/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	$-Fb-5Fx$	0	$Fbx+5Fx^2$	0	x^2		
	totali						$20/3Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-4F	

PROCEDIMENTO E RISULTATI 957901 ESΣ07.xxxx.010

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (5 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [5\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (5\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 9/2\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (2 + 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [2\, x + 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

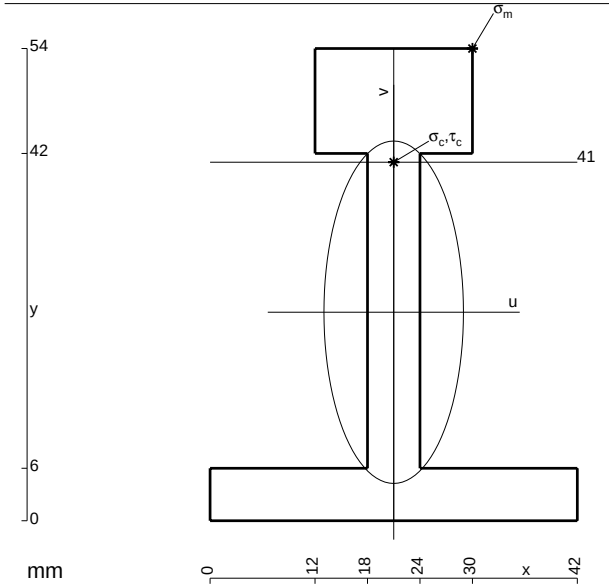
$= (2\, b + 3/2\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = 9/2\ Fb^3/EJ$

$L^{Xo}_{IA} = \int_o^b (6 - 11\, x/b + 5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [6\, x - 11/2\, x^2/b + 5/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (6\, b - 11/2\, b + 5/3\, b)\, Fb^2\, 1/EJ = 13/6\ Fb^3/EJ$

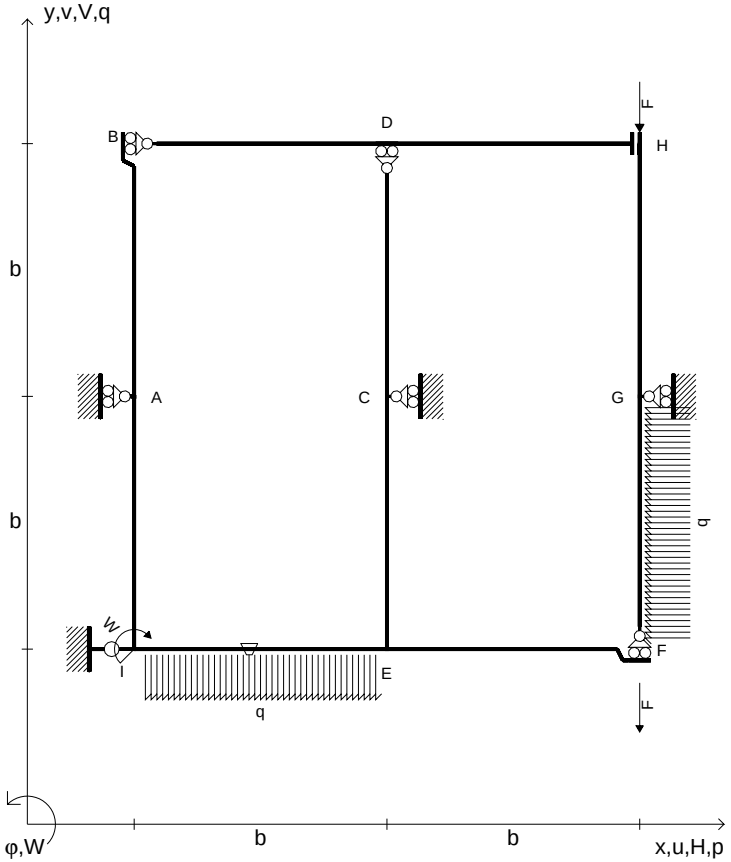
$L^{Xo}_{AI} = \int_o^b (\, x/b + 5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b + 5/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

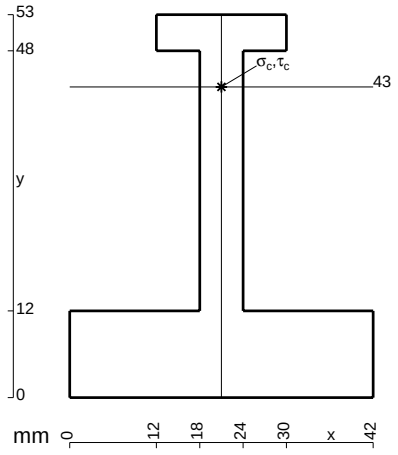
$= (1/2\, b + 5/3\, b)\, Fb^2\, 1/EJ = 13/6\ Fb^3/EJ$



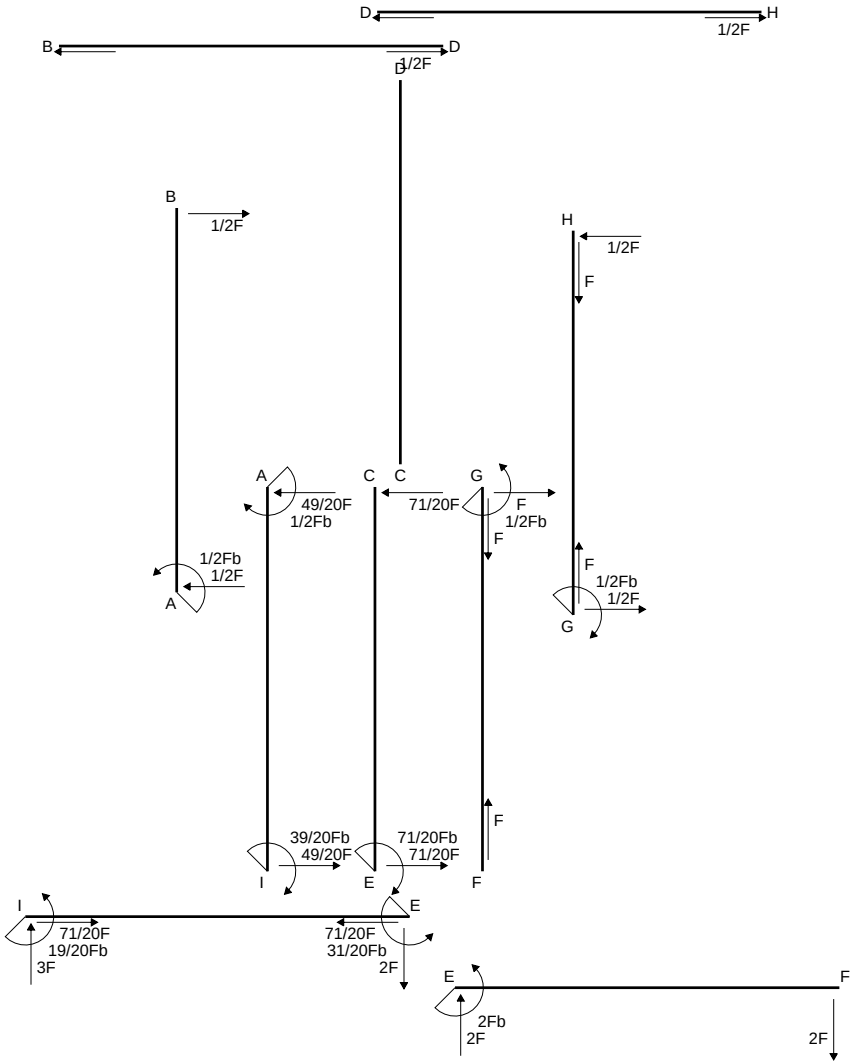
A = 684. mm^2
J_u = 262207. mm^4
J_v = 43524. mm^4
y_g = 23.84 mm
T_y = 3220. N
M_x = -1738800. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 9. mm
v_m = 30.16 mm
$\sigma_m = -Mv/J_u$ = 200. N/mm^2
x_c = 21. mm
y_c = 41. mm
v_c = 17.16 mm
$\sigma_c = -Mv/J_u$ = 113.8 N/mm^2
τ_c = 10.9 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 115.3 N/mm^2
S^* = 5324. mm^3

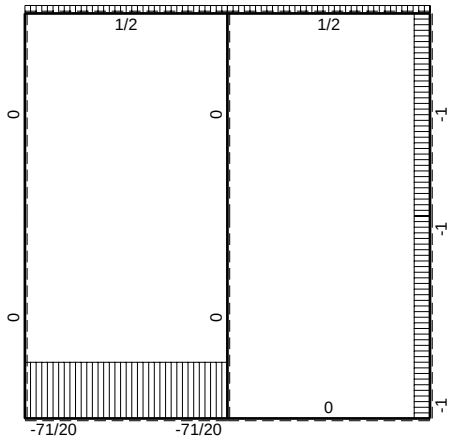
$V_{HG} = -F$
$V_{FE} = -F$
$W_I = -W = -Fb$
$p_{FG} = -q = -F/b$
$q_{IE} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



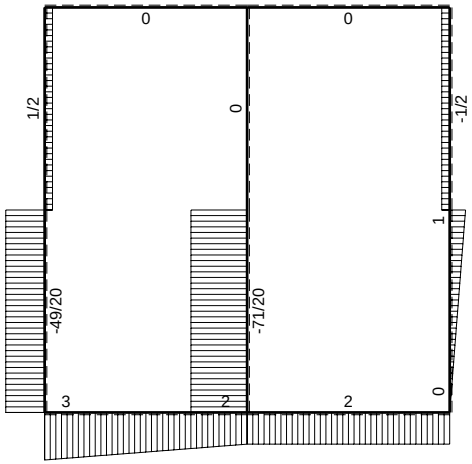


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 1000 mm, F = 760 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

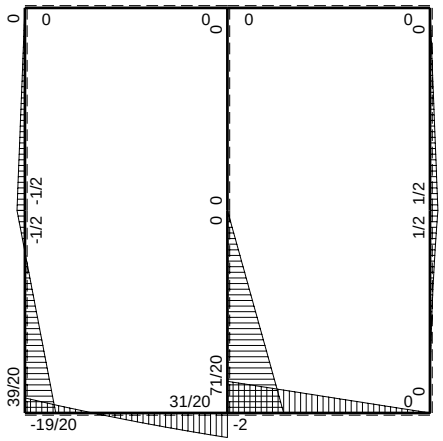




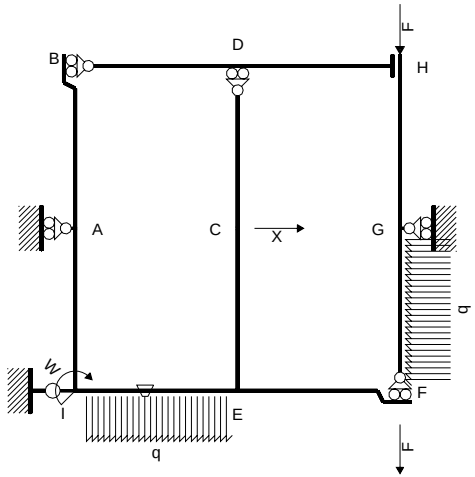
←□→ F



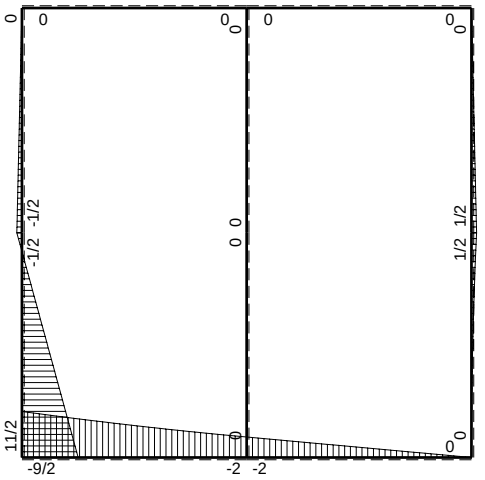
↑□↓ F



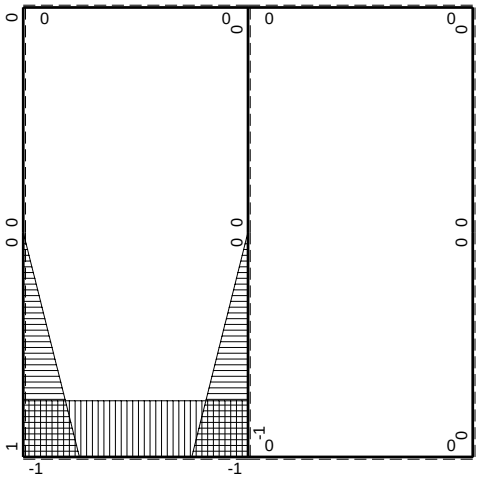
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210099 ESΣ07.xxxx.029

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fx	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-b	$-9/2Fb+3Fx-1/2qx^2$	-Fb/EJ	$9/2Fb^2-3Fbx+1/2Fx^2$	Fb^2/EJ	b^2	$(19/6+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$2Fb+2Fx+1/2qx^2$	Fb/EJ	$2Fb^2+2Fbx+1/2Fx^2$	Fb^2/EJ	b^2		
EC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CE b	x	0	0	0	0	x^2		
IA b	b-x	11/2Fb-6Fx	0	$11/2Fb^2-23/2Fbx+6Fx^2$	0	$b^2-2bx+x^2$	$(7/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	1/2Fb-6Fx	0	$-1/2Fbx+6Fx^2$	0	x^2		
	totali						$71/12Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-71/20F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (9/2 - 3\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx$

$= [9/2\, x - 3/2\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (9/2\, b - 3/2\, b + 1/6\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 25/6\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (2 + 2\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx$

$= [2\, x + x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (2\, b + b + 1/6\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = 25/6\ Fb^3/EJ$

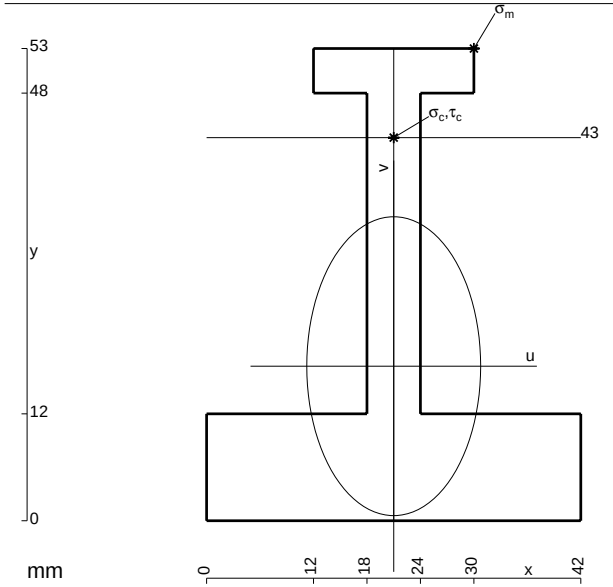
$L^{Xo}_{IA} = \int_o^b (11/2 - 23/2\, x/b + 6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [11/2\, x - 23/4\, x^2/b + 2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (11/2\, b - 23/4\, b + 2\, b)\, Fb^2\, 1/EJ = 7/4\ Fb^3/EJ$

$L^{Xo}_{AI} = \int_o^b (-1/2\, x/b + 6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b + 2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/4\, b + 2\, b)\, Fb^2\, 1/EJ = 7/4\ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 210099 ESΣ07.xxxx.029



A = 810. mm^2
J_u = 227958. mm^4
J_v = 77166. mm^4
y_g = 17.34 mm
T_y = 1520. N
M_x = -1520000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 9. mm
v_m = 35.66 mm
$\sigma_m = -Mv/J_u$ = 237.7 N/mm^2
x_c = 21. mm
y_c = 43. mm
v_c = 25.66 mm
$\sigma_c = -Mv/J_u$ = 171.1 N/mm^2
τ_c = 4.255 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 171.2 N/mm^2
S^* = 3829. mm^3

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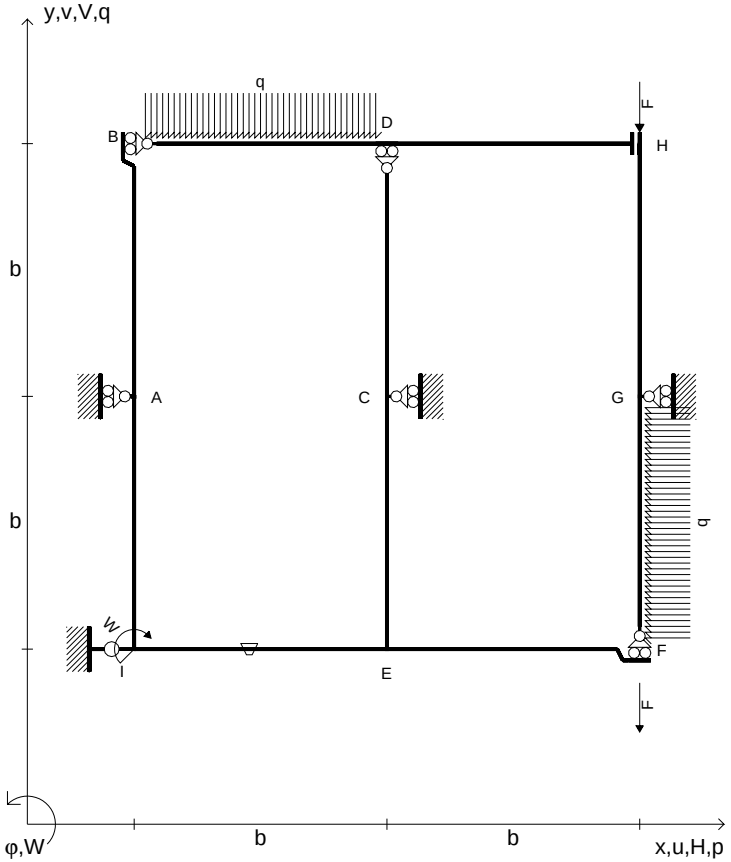
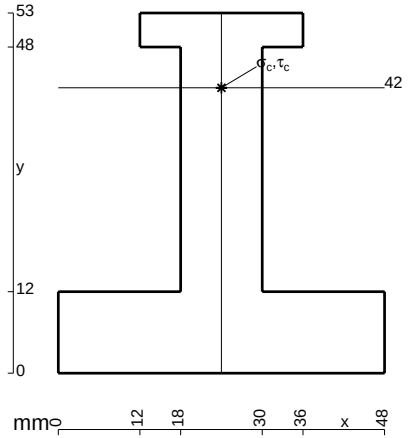
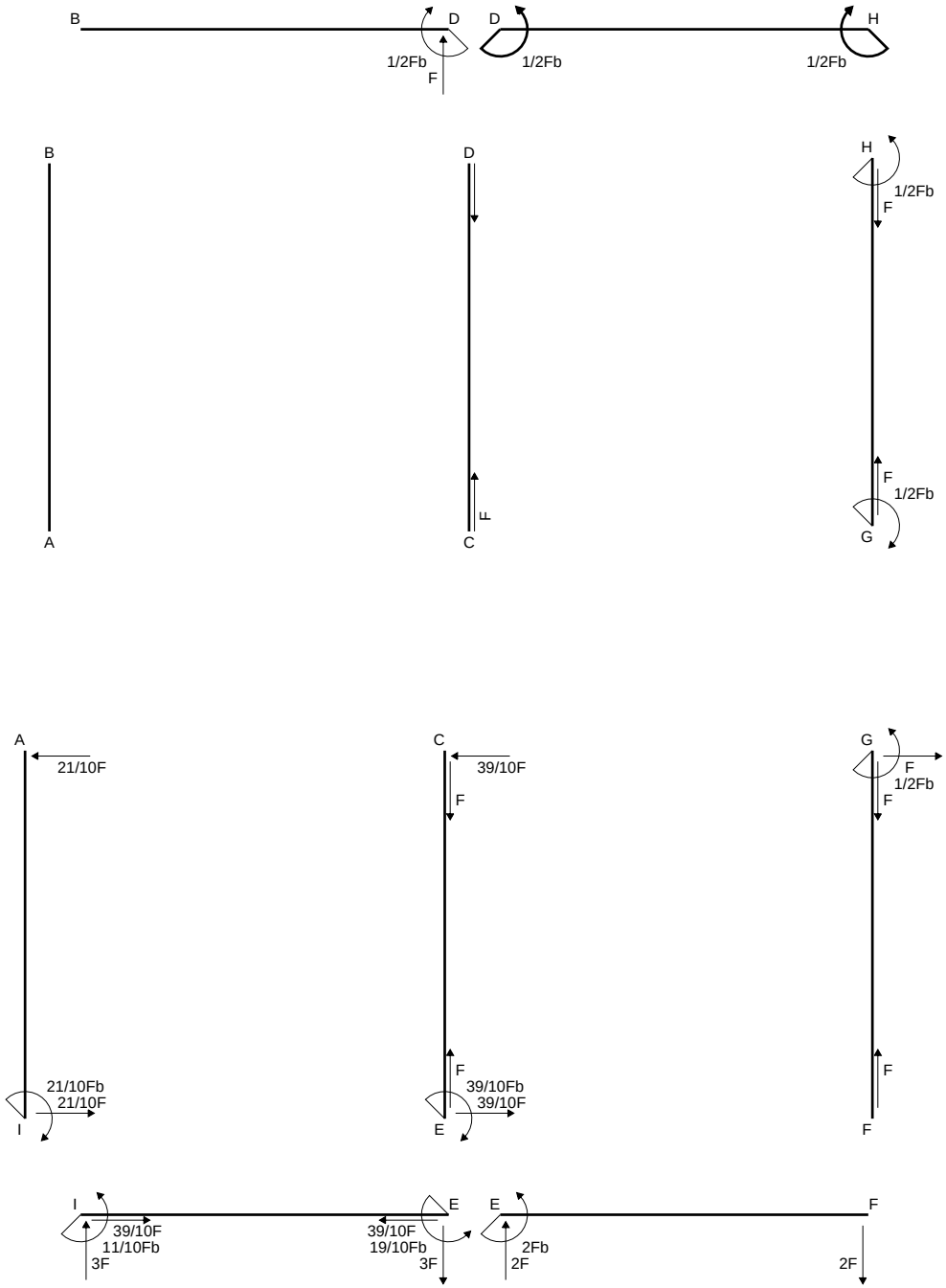
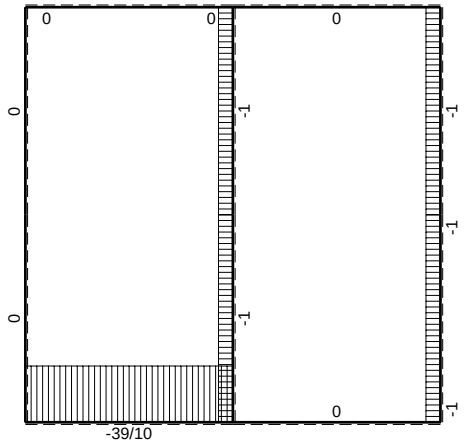
$V_{HG} = -F$
$V_{FE} = -F$
$W_I = -W = -Fb$
$p_{FG} = -q = -F/b$
$q_{DB} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



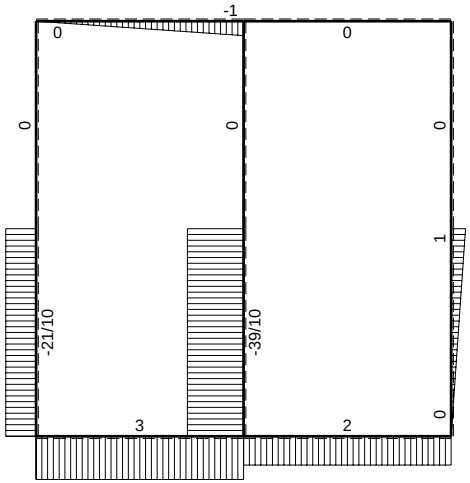


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 580 mm, F = 1750 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

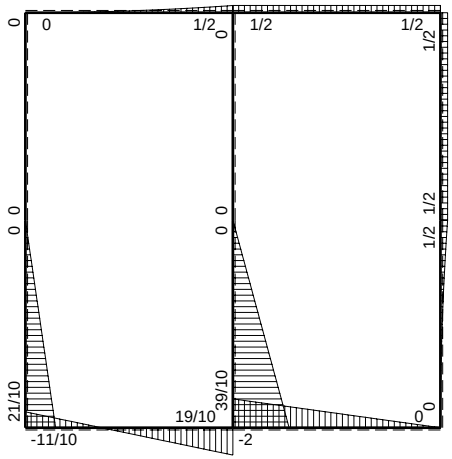




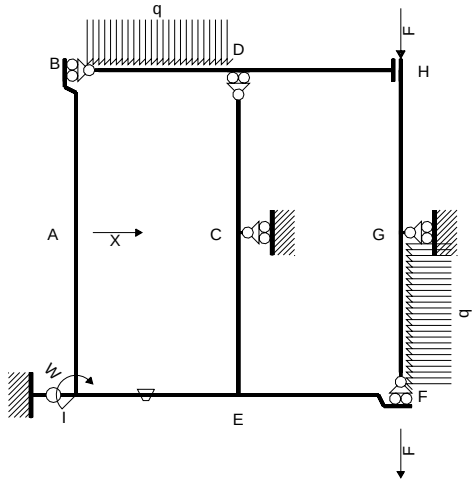
←□→ F



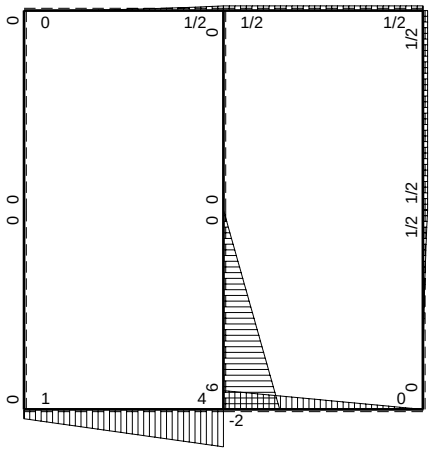
↑□↓ F



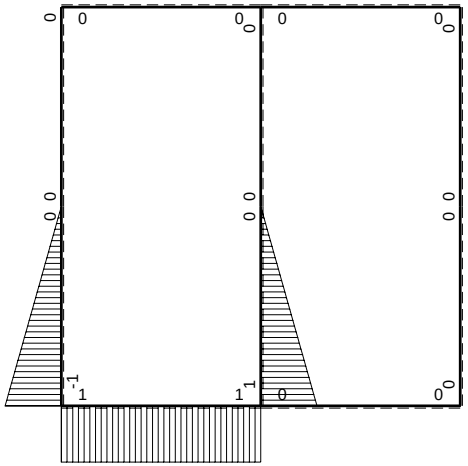
(□) Fb



Schema di calcolo iperstatico



(□) M_o flessione da carichi assegnati



(□) M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	1/2Fb	0	0	0	0	0+0	0
HG b	0	-1/2Fb	0	0	0	0		
HD b	0	1/2Fb	0	0	0	0	0+0	0
DH b	0	-1/2Fb	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	b	Fb+3Fx	-Fb/EJ	Fb^2+3Fbx	$-Fb^2/EJ$	b^2	$(5/2-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	-4Fb+3Fx	Fb/EJ	$4Fb^2-3Fbx$	$-Fb^2/EJ$	b^2		
EC b	b-x	6Fb-6Fx	0	$6Fb^2-12Fbx+6Fx^2$	0	$b^2-2bx+x^2$	$(2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	-6Fx	0	$6Fx^2$	0	x^2		
IA b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
AI b	x	0	0	0	0	x^2		
	totali						$7/2Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-21/10F	

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Sviluppi di calcolo iperstatica

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$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (1 + 3\, x/b \,)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx = [\, x + 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (\, b + 3/2\, b \,)\, Fb^2\, 1/EJ + (- b \,)\, \theta = 3/2\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (4 - 3\, x/b \,)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx = [4\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

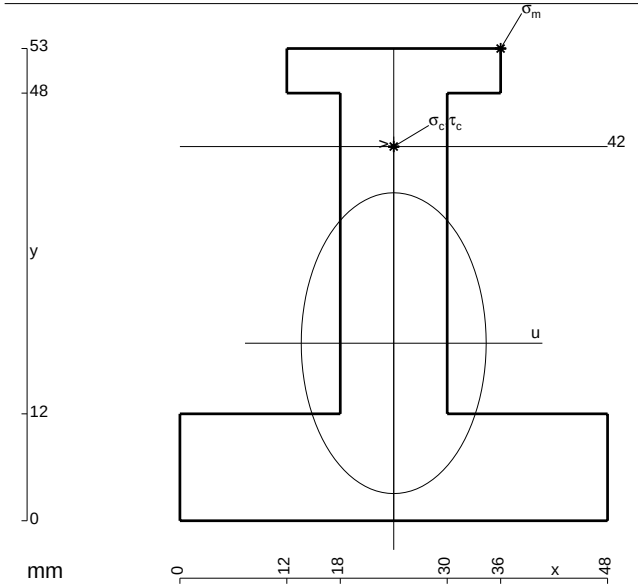
$= (4\, b - 3/2\, b \,)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 3/2\ Fb^3/EJ$

$L_{EC}^{Xo} = \int_o^b (6 - 12\, x/b + 6\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [6\, x - 6\, x^2/b + 2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (6\, b - 6\, b + 2\, b \,)\, Fb^2\, 1/EJ = 2\ Fb^3/EJ$

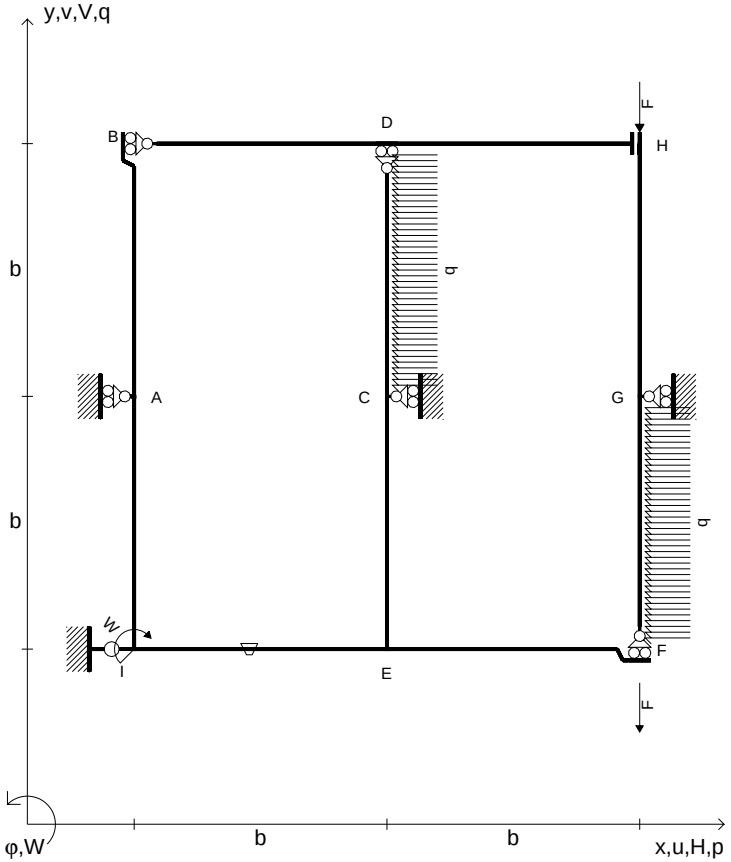
$L_{CE}^{Xo} = \int_o^b (6\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

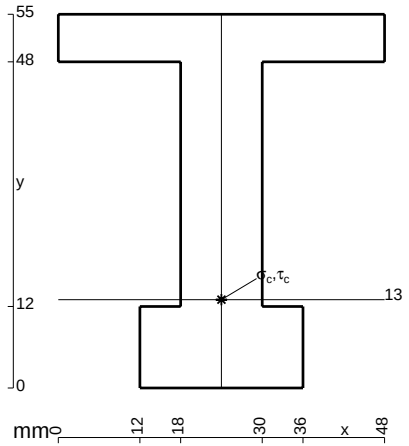
$= (2\, b \,)\, Fb^2\, 1/EJ = 2\ Fb^3/EJ$



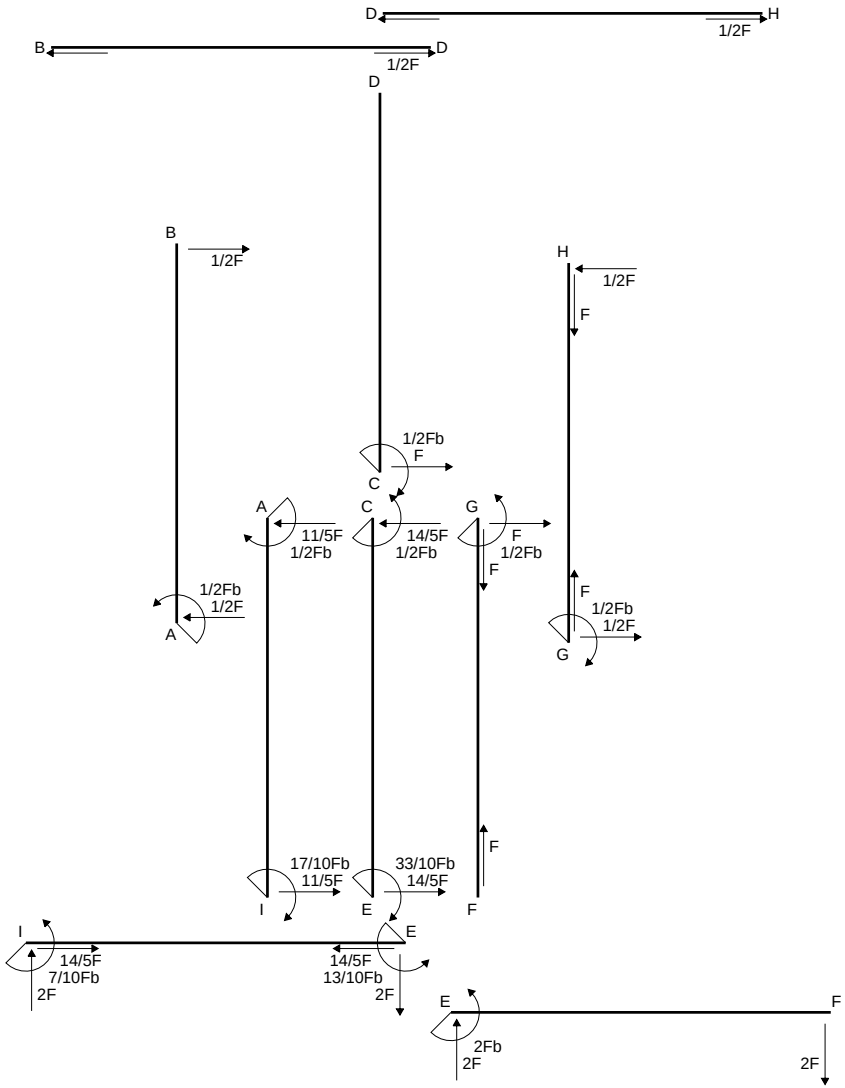
A = 1128. mm^2
J_u = 321538. mm^4
J_v = 121536. mm^4
y_g = 19.93 mm
T_y = 3500. N
M_x = -2030000. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 12. mm
v_m = 33.07 mm
$\sigma_m = -Mv/J_u$ = 208.8 N/mm^2
x_c = 24. mm
y_c = 42. mm
v_c = 22.07 mm
$\sigma_c = -Mv/J_u$ = 139.4 N/mm^2
τ_c = 4.966 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 139.6 N/mm^2
S^* = 5474. mm^3

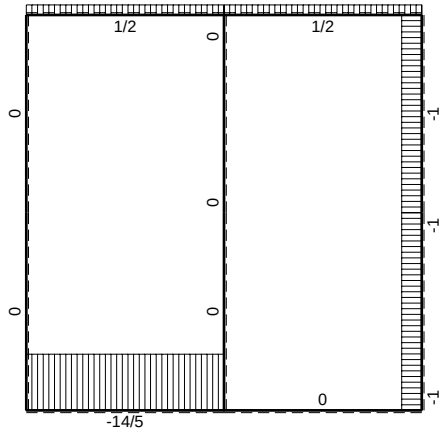
$V_{HG} = -F$
$V_{FE} = -F$
$W_I = -W = -Fb$
$p_{CD} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



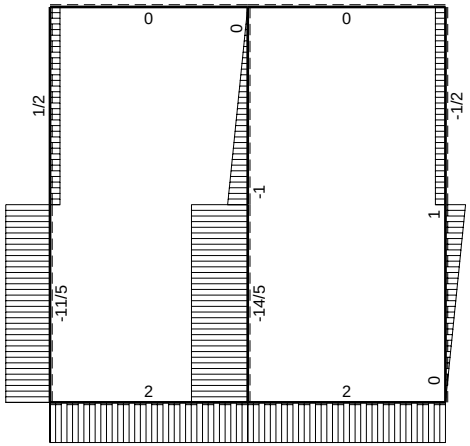


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 1040 mm, F = 1410 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

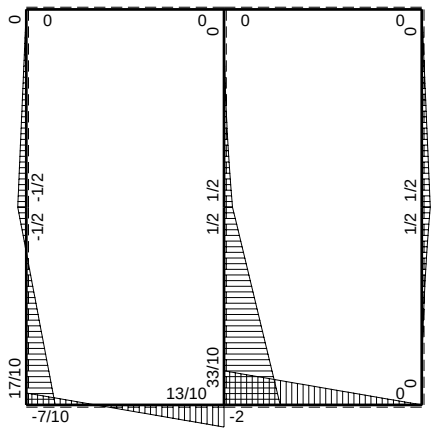




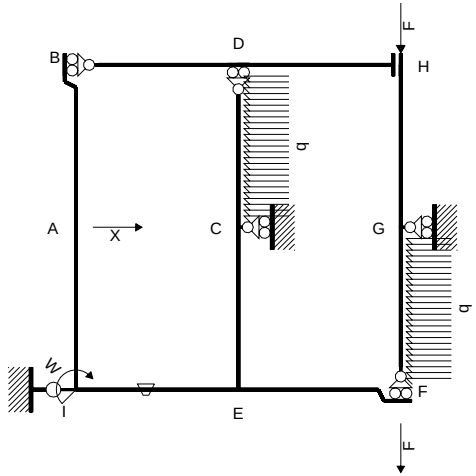
←F→



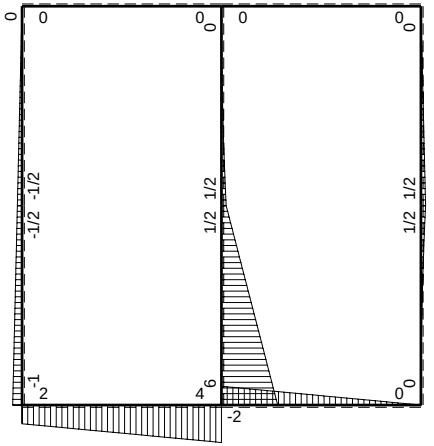
↑F↓



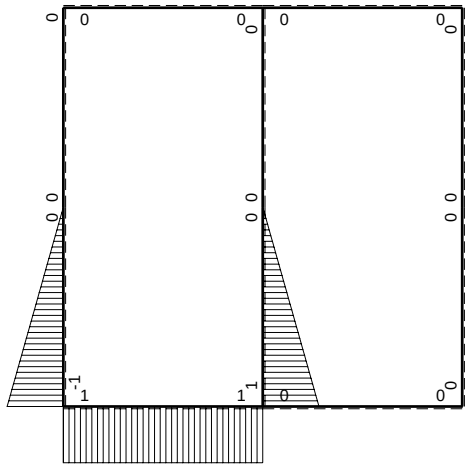
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2qx^2$	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fx	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	2Fb+2Fx	-Fb/EJ	$2Fb^2+2Fbx$	$-Fb^2/EJ$	b^2	$(3-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	-4Fb+2Fx	Fb/EJ	$4Fb^2-2Fbx$	$-Fb^2/EJ$	b^2		
EC b	b-x	6Fb-11/2Fx	0	$6Fb^2-23/2Fbx+11/2Fx^2$	0	$b^2-2bx+x^2$	$(25/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	-1/2Fb-11/2Fx	0	$1/2Fbx+11/2Fx^2$	0	x^2		
IA b	-b+x	-Fb+1/2Fx	0	$Fb^2-3/2Fbx+1/2Fx^2$	0	$b^2-2bx+x^2$	$(5/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	1/2Fb+1/2Fx	0	$1/2Fbx+1/2Fx^2$	0	x^2		
	totali						$9/2Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-27/10F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (2 + 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [2\, x + x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (2\, b + b)\, Fb^2\, 1/EJ + (- b)\, \theta = 2\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (4 - 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [4\, x - x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (4\, b - b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 2\ Fb^3/EJ$

$L^{Xo}_{EC} = \int_o^b (6 - 23/2\, x/b + 11/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [6\, x - 23/4\, x^2/b + 11/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (6\, b - 23/4\, b + 11/6\, b)\, Fb^2\, 1/EJ = 25/12\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (1/2\, x/b + 11/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b + 11/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

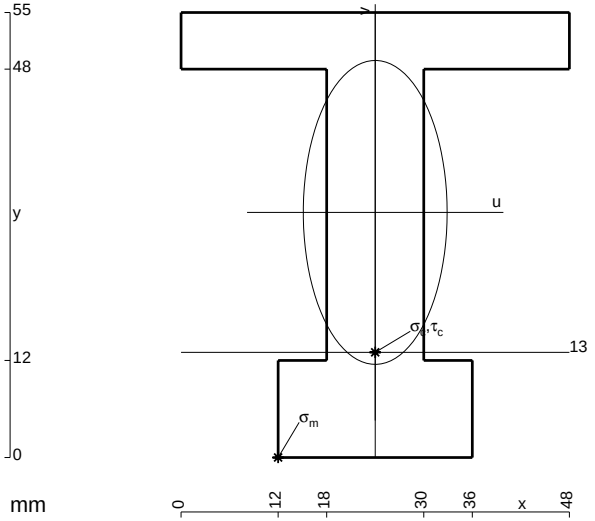
$= (1/4\, b + 11/6\, b)\, Fb^2\, 1/EJ = 25/12\ Fb^3/EJ$

$L^{Xo}_{IA} = \int_o^b (1 - 3/2\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [\, x - 3/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (\, b - 3/4\, b + 1/6\, b \,)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

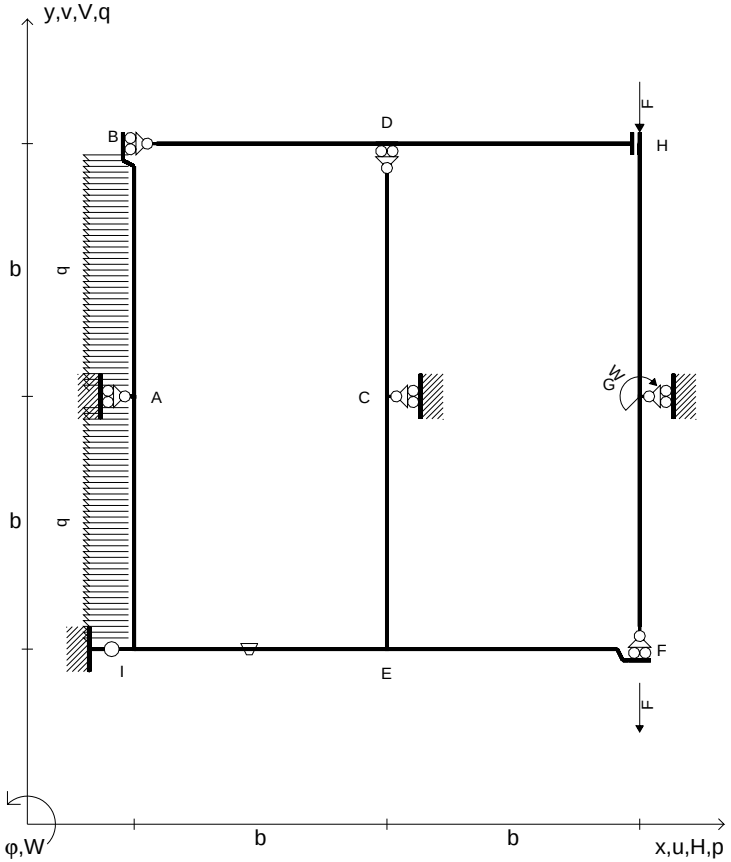
$L^{Xo}_{AI} = \int_o^b (1/2\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

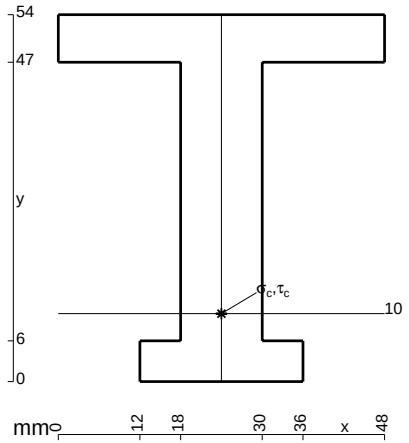
$= (1/4\, b + 1/6\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$



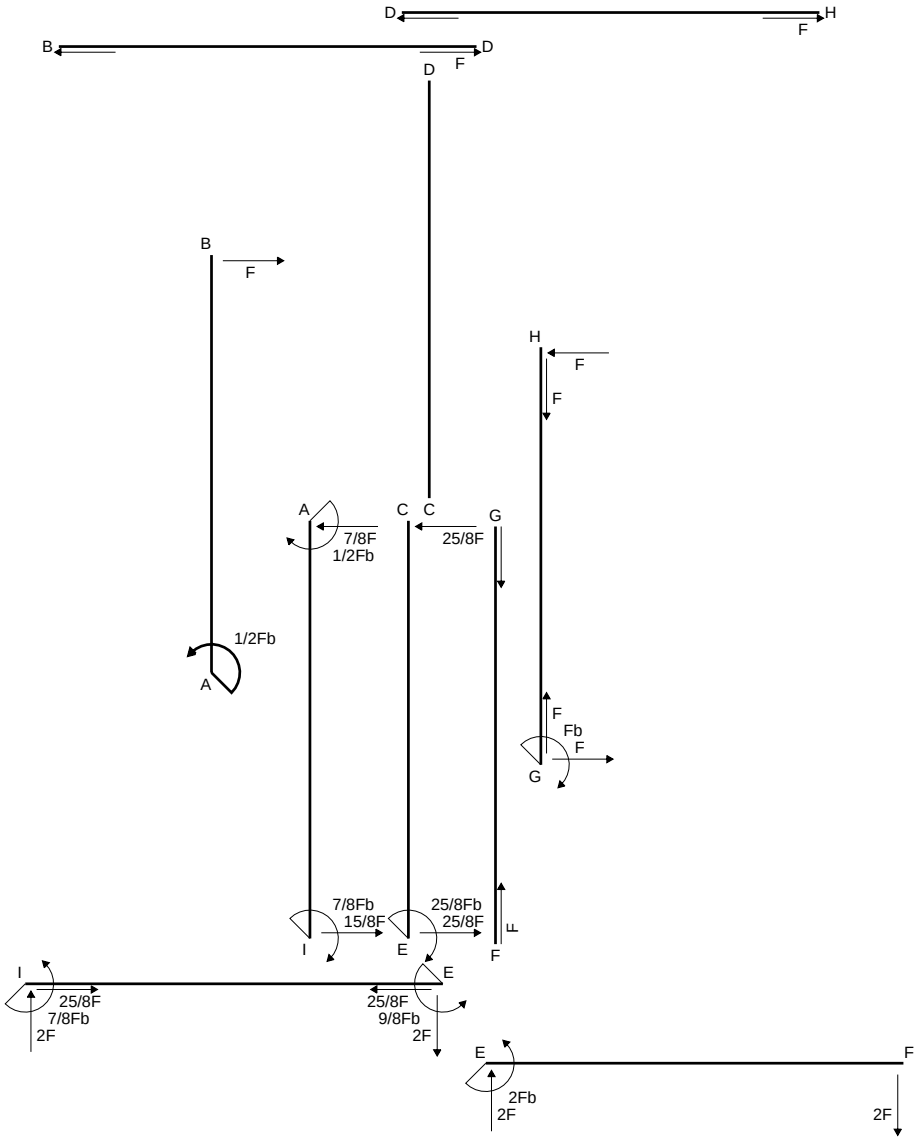
A = 1056. mm^2
J_u = 372596. mm^4
J_v = 83520. mm^4
y_g = 30.3 mm
T_y = 2820. N
M_x = -2932800. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -30.3 mm
$\sigma_m = -Mv/J_u$ = -238.5 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -17.3 mm
$\sigma_c = -Mv/J_u$ = -136.1 N/mm^2
τ_c = 4.548 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 136.4 N/mm^2
S^* = 7211. mm^3

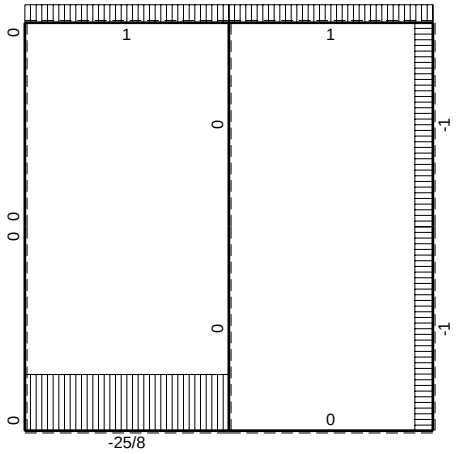
$V_{HG} = -F$
$V_{FE} = -F$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$p_{IA} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



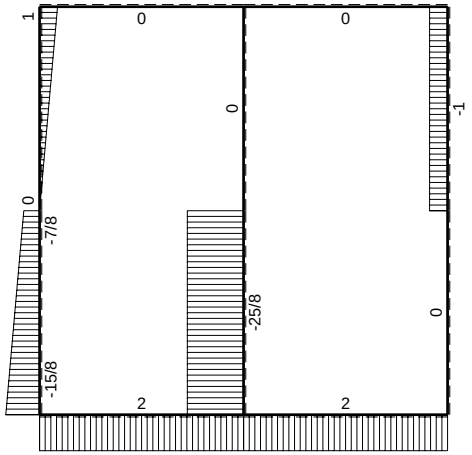


Reazioni iperstatiche in soluzione: $X=W_{EI}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 590 mm, F = 1820 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

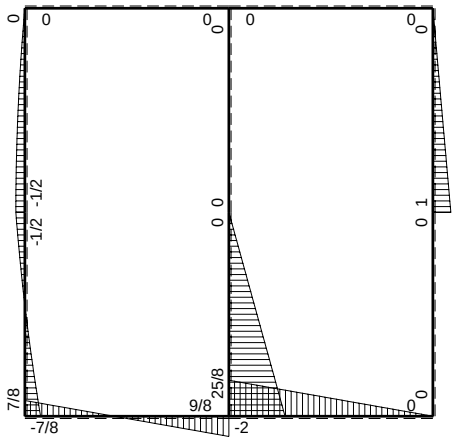




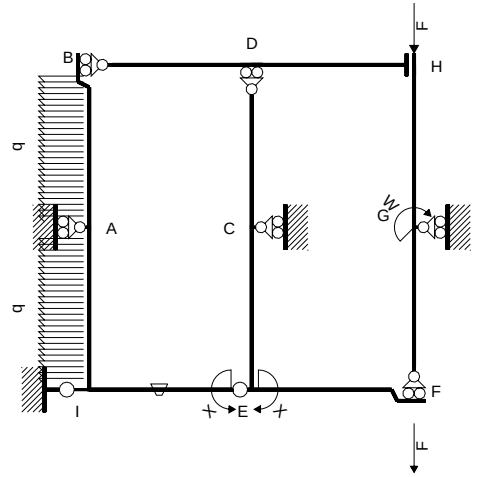
←F→



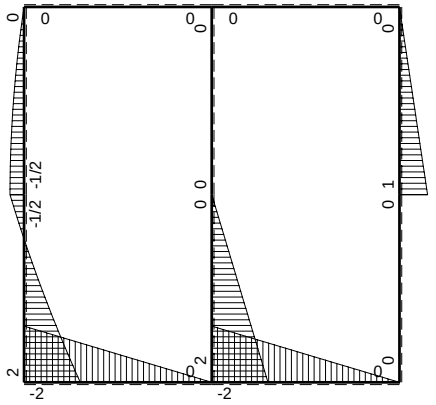
↑F↓



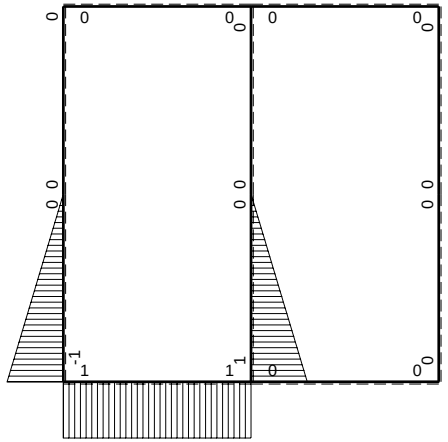
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EI}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$Fx-1/2qx^2$	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	Fb-Fx	0	0	0	0	0+0	0
HG b	0	-Fx	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	1	-2Fb+2Fx	-Fb/EJ	-2Fb+2Fx	-Fb/EJ	1	$(-1-1)Fb^2/EJ$	Xb/EJ
EI b	-1	2Fx	Fb/EJ	-2Fx	-Fb/EJ	1		
EC b	1-x/b	2Fb-2Fx	0	$2Fb-4Fx+2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(2/3+0)Fb^2/EJ$	1/3Xb/EJ
CE b	-x/b	-2Fx	0	$2Fx^2/b$	0	x^2/b^2		
IA b	-1+x/b	$2Fb-3Fx+1/2qx^2$	0	$-2Fb+5Fx-7/2Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-13/24+0)Fb^2/EJ$	1/3Xb/EJ
AI b	x/b	$1/2Fb-2Fx-1/2qx^2$	0	$1/2Fx-2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
	totali						$-15/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EI}$						9/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 218425 ESΣ07.xxxx.051
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{EI} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CE} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{AI} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{IE} = \int_o^b (-2 + 2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-2\ x + x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-2\ b + b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -2\ Fb^2/EJ$

$L^{Xo}_{EI} = \int_o^b (-2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -2\ Fb^2/EJ$

$L^{Xo}_{EC} = \int_o^b (2 - 4\ x/b + 2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [2\ x - 2\ x^2/b + 2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2\ b - 2\ b + 2/3\ b)\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

$L^{Xo}_{CE} = \int_o^b (2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (2/3\ b)\ Fb\ 1/EJ\ = 2/3\ Fb^2/EJ$

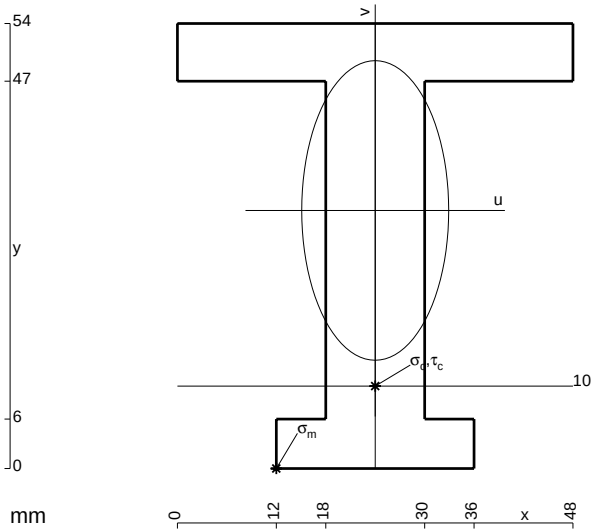
$L^{Xo}_{IA} = \int_o^b (-2 + 5\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-2\ x + 5/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-2\ b + 5/2\ b - 7/6\ b + 1/8\ b)\ Fb\ 1/EJ\ = -13/24\ Fb^2/EJ$

$L^{Xo}_{AI} = \int_o^b (1/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b - 2/3\ b - 1/8\ b)\ Fb\ 1/EJ\ = -13/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 218425 ESΣ07.xxxx.051



$A = 972.\ mm^2$

$J_u = 321252.\ mm^4$

$J_v = 77328.\ mm^4$

$y_g = 31.31\ mm$

$T_y = 3640.\ N$

$M_x = -2147600.\ Nmm$

$x_m = 12.\ mm$

$u_m = -12.\ mm$

$v_m = -31.31\ mm$

$\sigma_m = -Mv/J_u = -209.3\ N/mm^2$

$x_c = 24.\ mm$

$y_c = 10.\ mm$

$v_c = -21.31\ mm$

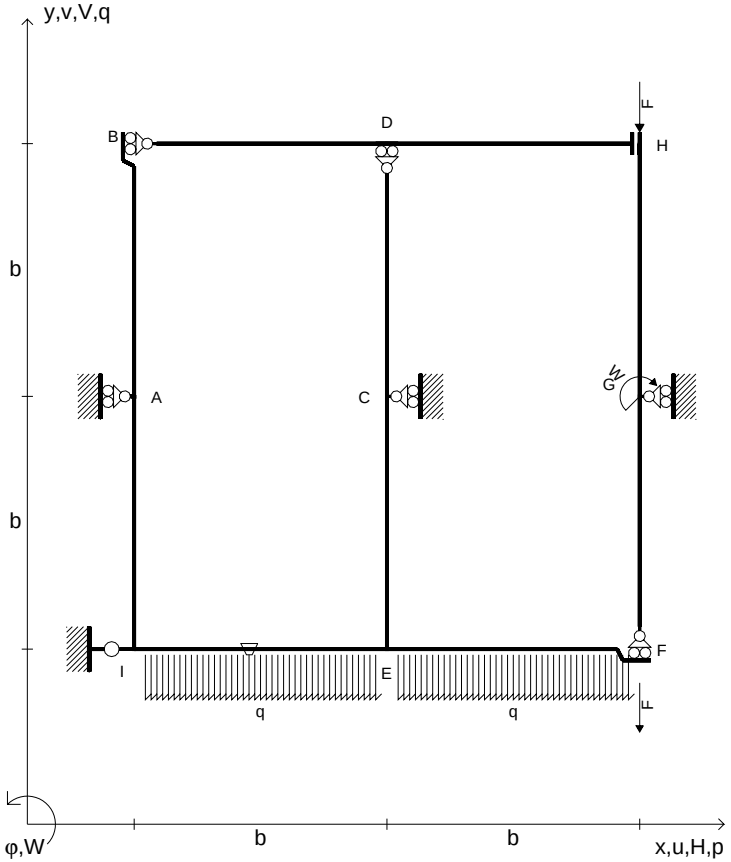
$\sigma_c = -Mv/J_u = -142.5\ N/mm^2$

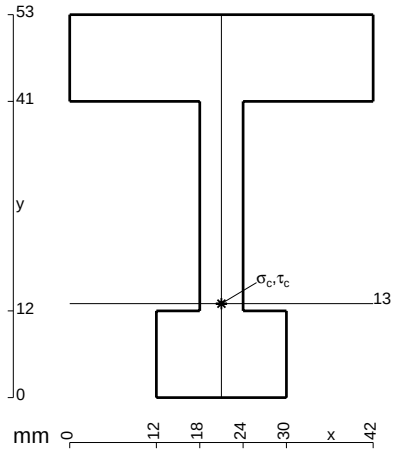
$\tau_c = 4.907\ N/mm^2$

$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 142.7\ N/mm^2$

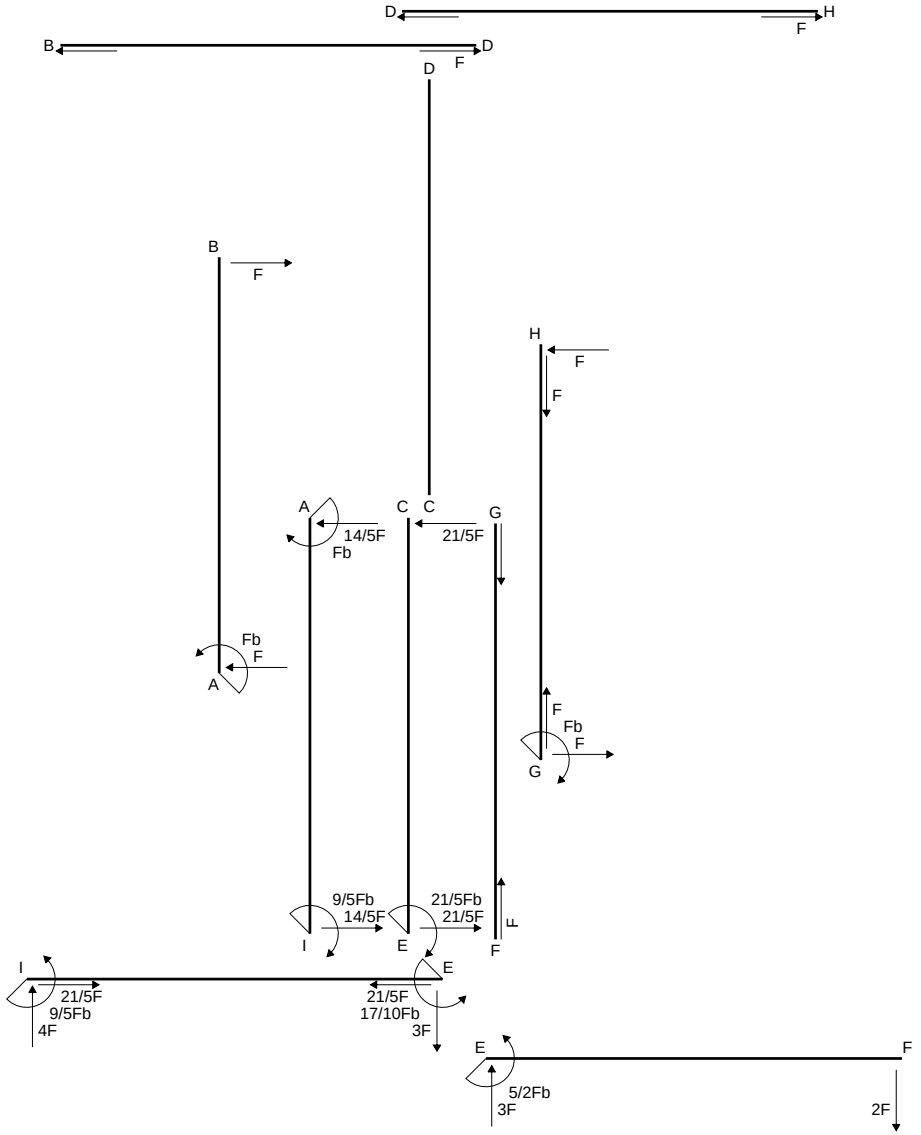
$S^* = 5196.\ mm^3$

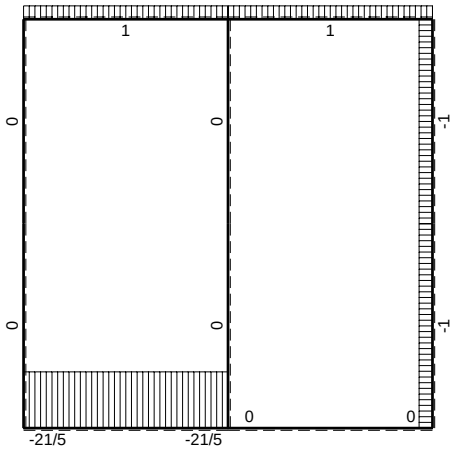
$V_{HG} = -F$
$V_{FE} = -F$
$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$q_{IE} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



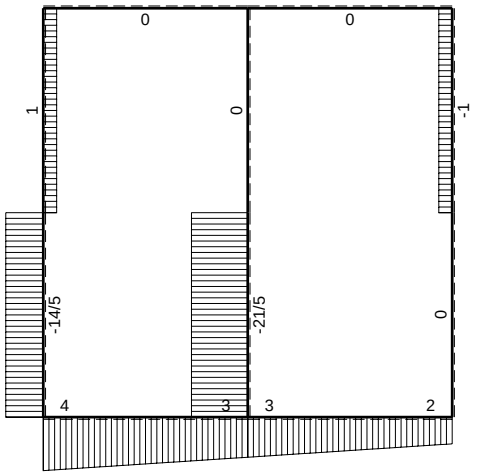


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 530 mm, F = 1290 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

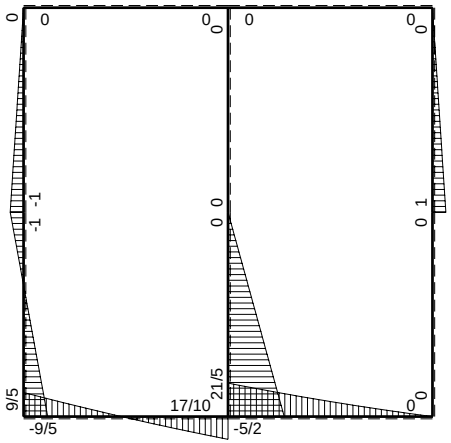




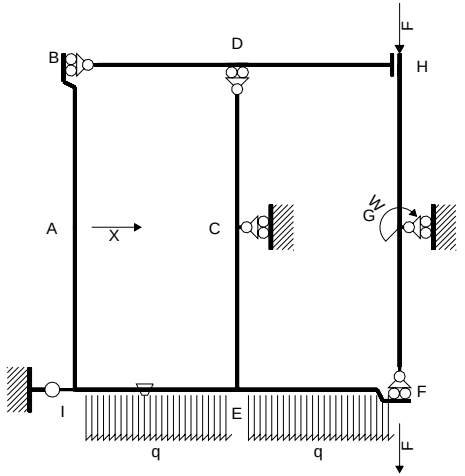
←□→ F



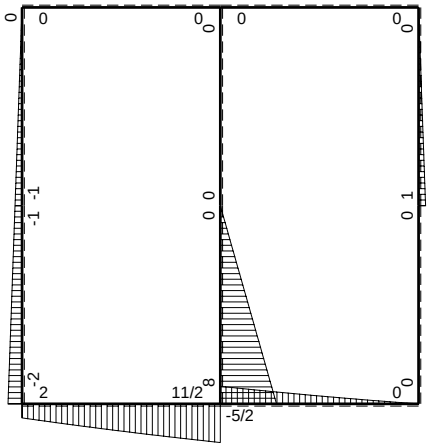
↑□↓ F



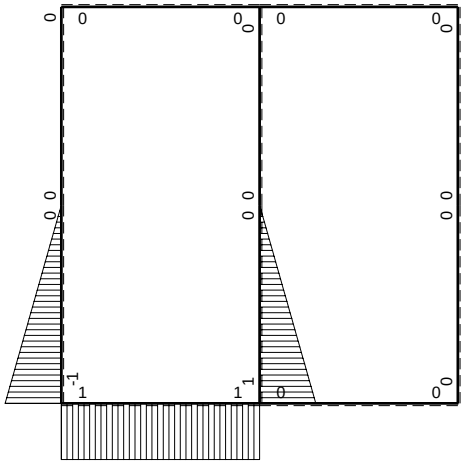
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212483 ESΣ07.xxxx.070

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fb+Fx	0	0	0	0	0+0	0
BA b	0	Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-5/2Fb+3Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$2Fx+1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	Fb-Fx	0	0	0	0	0+0	0
HG b	0	-Fx	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	$2Fb+4Fx-1/2qx^2$	-Fb/EJ	$2Fb^2+4Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2	$(23/6-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	$-11/2Fb+3Fx+1/2qx^2$	Fb/EJ	$11/2Fb^2-3Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2		
EC b	b-x	8Fb-8Fx	0	$8Fb^2-16Fbx+8Fx^2$	0	$b^2-2bx+x^2$	$(8/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	-8Fx	0	$8Fx^2$	0	x^2		
IA b	-b+x	-2Fb+Fx	0	$2Fb^2-3Fbx+Fx^2$	0	$b^2-2bx+x^2$	$(5/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	Fb+Fx	0	$Fbx+Fx^2$	0	x^2		
	totali						$19/3Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-19/5F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (2 + 4\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx$

$= [2\, x + 2\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (2\, b + 2\, b - 1/6\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = 17/6\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (11/2 - 3\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx$

$= [11/2\, x - 3/2\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (11/2\, b - 3/2\, b - 1/6\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 17/6\ Fb^3/EJ$

$L^{Xo}_{EC} = \int_o^b (8 - 16\, x/b + 8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [8\, x - 8\, x^2/b + 8/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (8\, b - 8\, b + 8/3\, b)\, Fb^2\, 1/EJ = 8/3\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [8/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

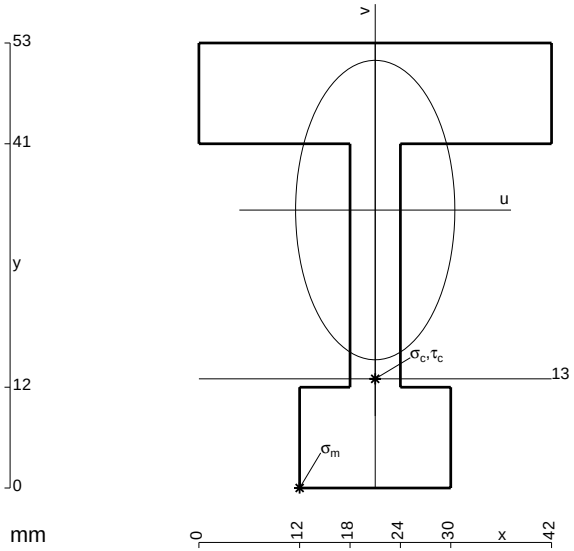
$= (8/3\, b)\, Fb^2\, 1/EJ = 8/3\ Fb^3/EJ$

$L^{Xo}_{IA} = \int_o^b (2 - 3\, x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx = [2\, x - 3/2\, x^2/b + 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (2\, b - 3/2\, b + 1/3\, b)\, Fb^2\, 1/EJ = 5/6\ Fb^3/EJ$

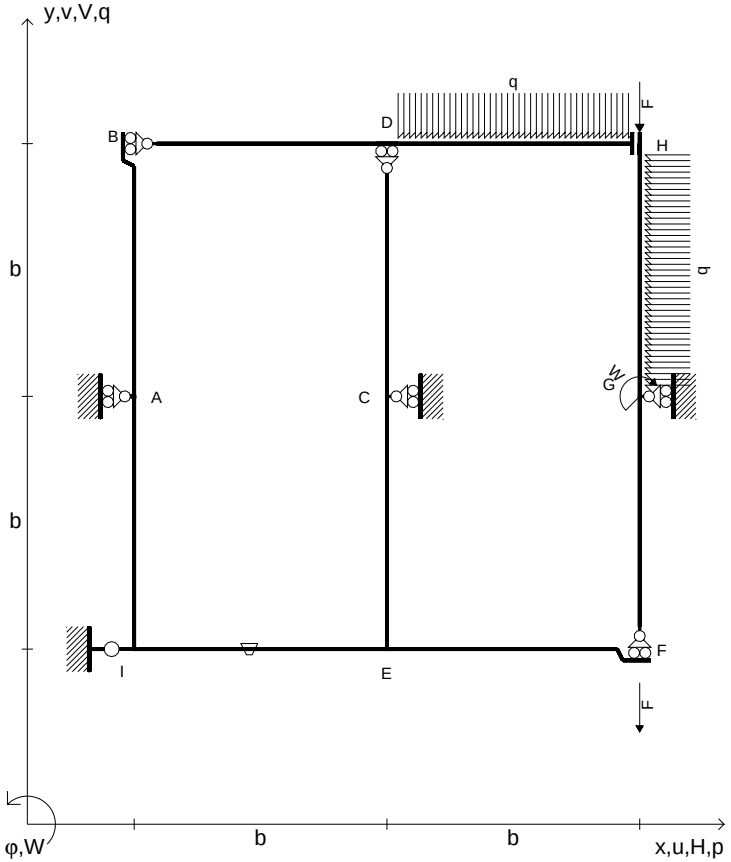
$L^{Xo}_{AI} = \int_o^b (\, x/b + x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b + 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

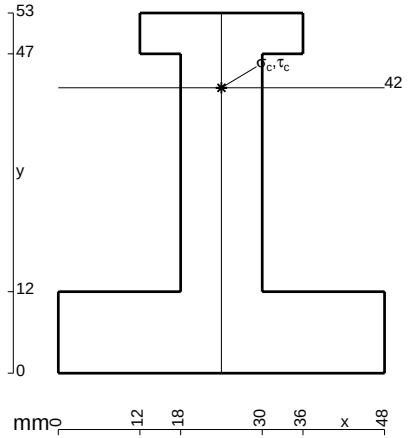
$= (1/2\, b + 1/3\, b)\, Fb^2\, 1/EJ = 5/6\ Fb^3/EJ$



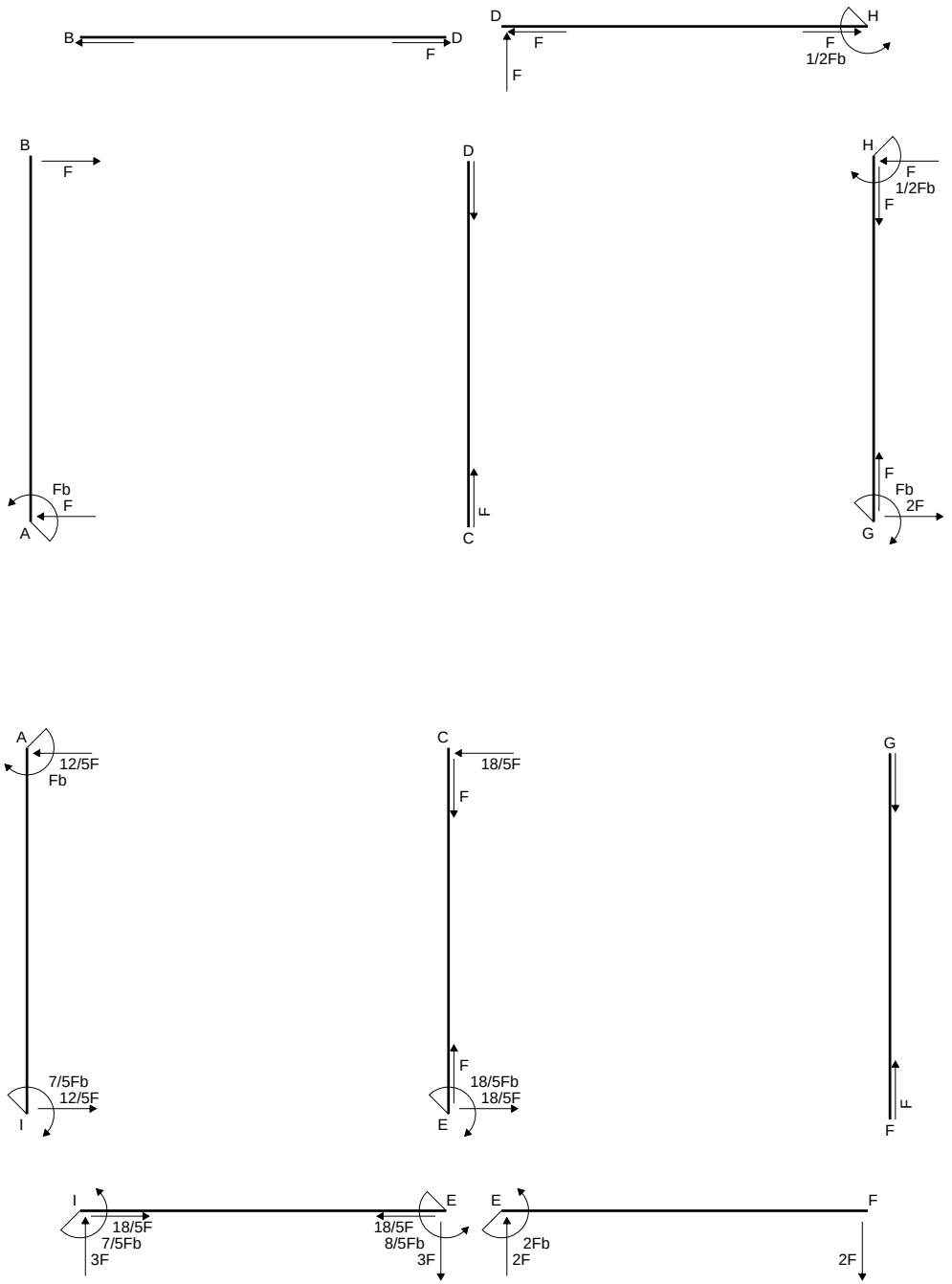
A = 894. mm^2
J_u = 284424. mm^4
J_v = 80442. mm^4
y_g = 33.1 mm
T_y = 3870. N
M_x = -1709250. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -33.1 mm
$\sigma_m = -Mv/J_u$ = -198.9 N/mm^2
x_c = 21. mm
y_c = 13. mm
v_c = -20.1 mm
$\sigma_c = -Mv/J_u$ = -120.8 N/mm^2
τ_c = 13.56 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 123.1 N/mm^2
S^* = 5978. mm^3

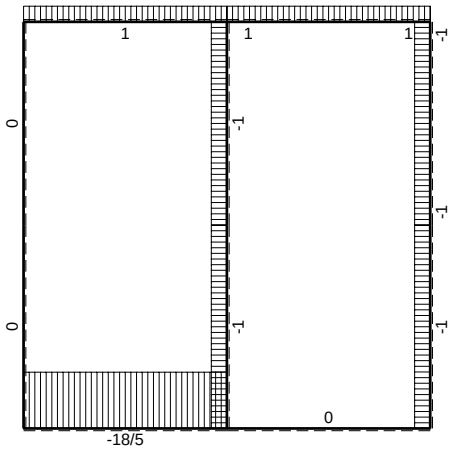
$V_{HG} = -F$
$V_{FE} = -F$
$W_G = -W = -Fb$
$q_{HD} = -q = -F/b$
$p_{GH} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



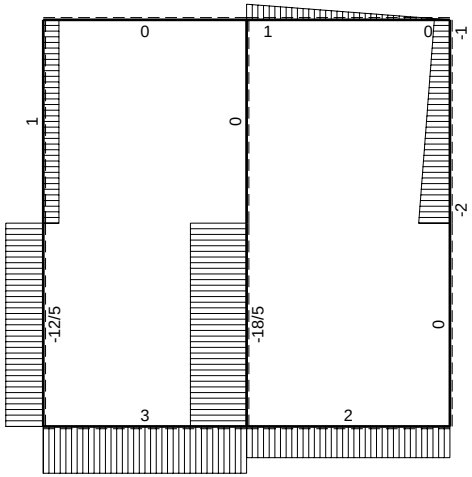


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 1000 mm, F = 1200 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

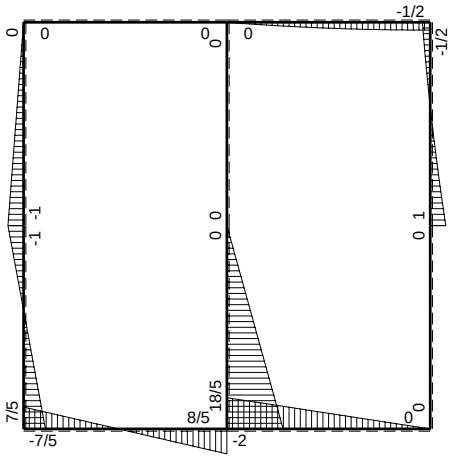




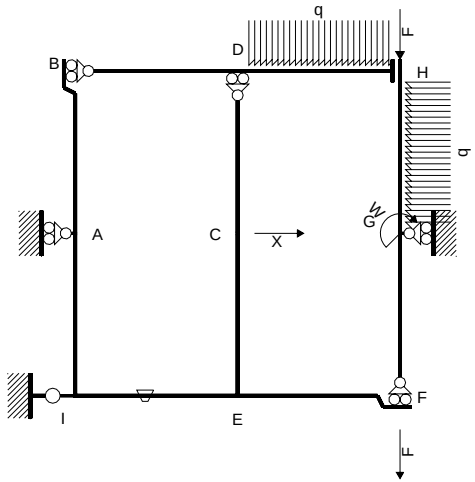
←F→



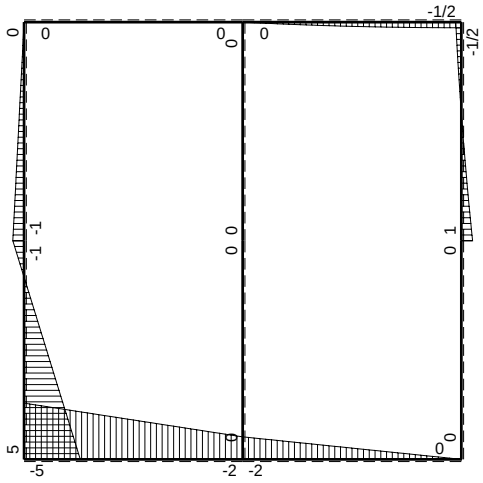
↑F↓



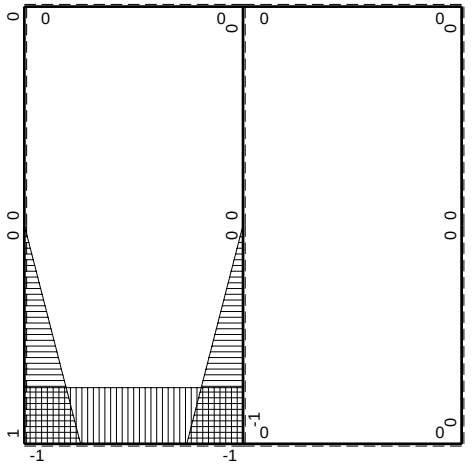
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fb+Fx	0	0	0	0	0+0	0
BA b	0	Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$1/2Fb-Fx-1/2qx^2$	0	0	0	0		
HD b	0	$-1/2Fb+1/2qx^2$	0	0	0	0	0+0	0
DH b	0	$Fx-1/2qx^2$	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-b	$-5Fb+3Fx$	$-Fb/EJ$	$5Fb^2-3Fbx$	Fb^2/EJ	b^2	$(7/2+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$2Fb+3Fx$	Fb/EJ	$2Fb^2+3Fbx$	Fb^2/EJ	b^2		
EC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CE b	x	0	0	0	0	x^2		
IA b	b-x	5Fb-6Fx	0	$5Fb^2-11Fbx+6Fx^2$	0	$b^2-2bx+x^2$	$(3/2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	Fb-6Fx	0	$-Fbx+6Fx^2$	0	x^2		
	totali						$6Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-18/5F	

PROCEDIMENTO E RISULTATI 983984 ESΣ07.xxxx.079

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (5 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [5\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (5\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 9/2\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (2 + 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [2\, x + 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

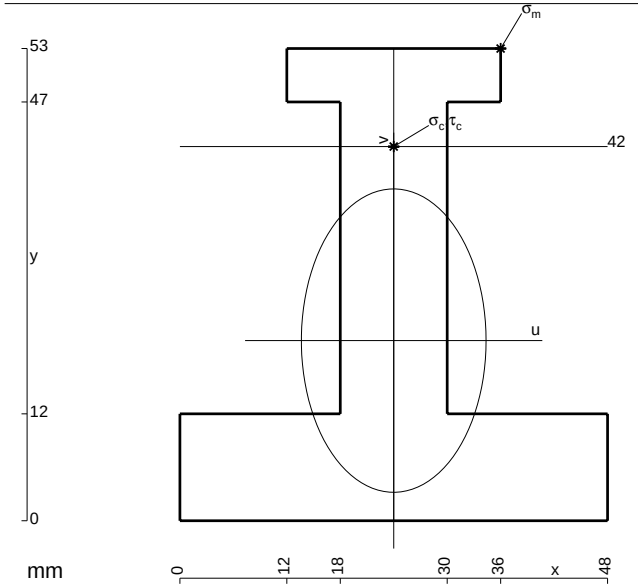
$= (2\, b + 3/2\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = 9/2\ Fb^3/EJ$

$L_{IA}^{Xo} = \int_o^b (5 - 11\, x/b + 6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5\, x - 11/2\, x^2/b + 2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (5\, b - 11/2\, b + 2\, b)\, Fb^2\, 1/EJ = 3/2\ Fb^3/EJ$

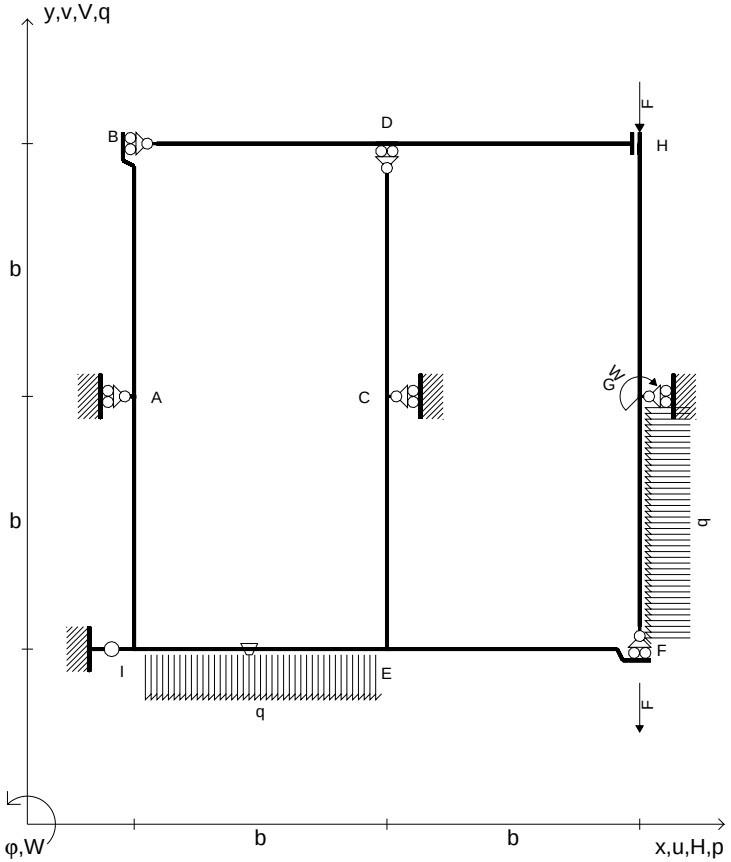
$L_{AI}^{Xo} = \int_o^b (- x/b + 6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b + 2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

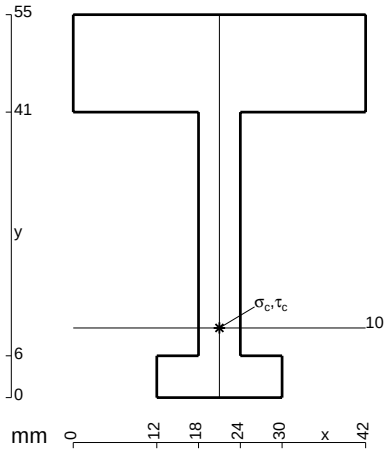
$= (-1/2\, b + 2\, b)\, Fb^2\, 1/EJ = 3/2\ Fb^3/EJ$



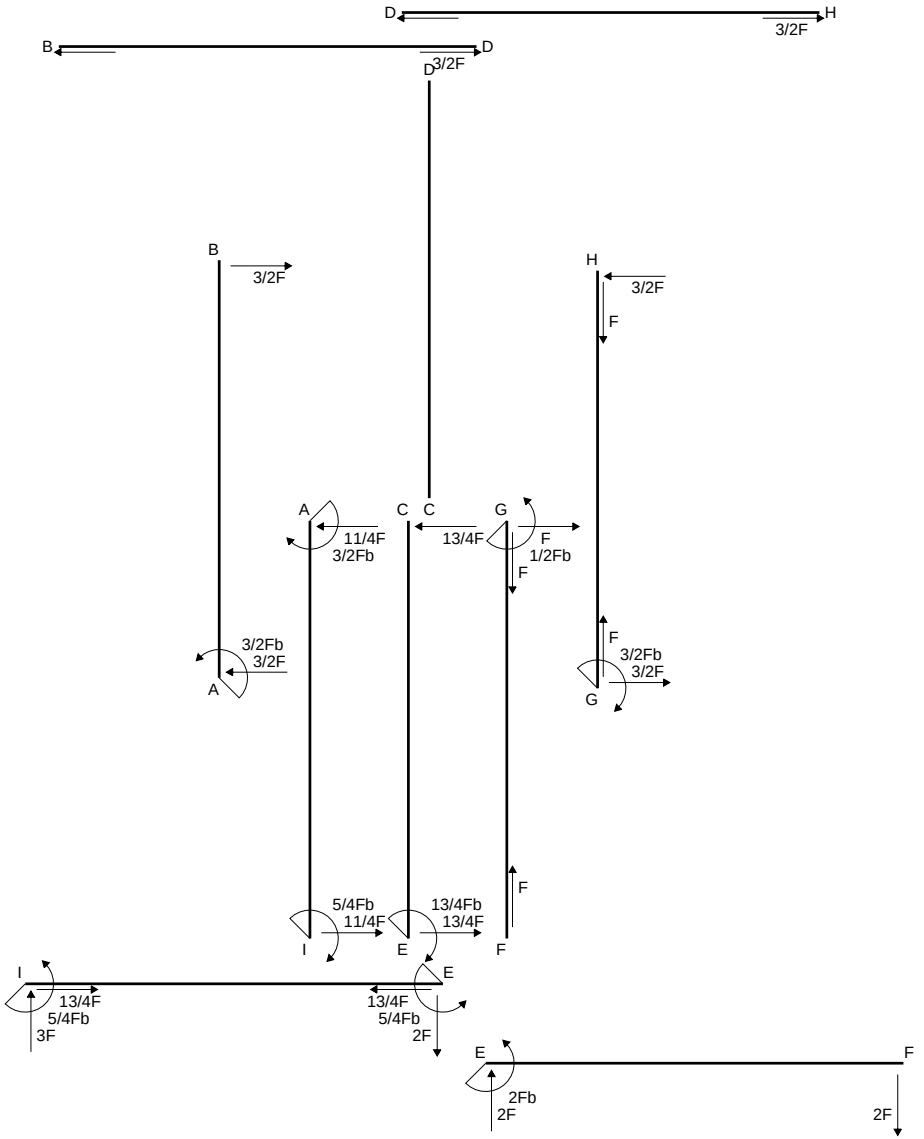
A = 1140. mm^2
J_u = 330567. mm^4
J_v = 122544. mm^4
y_g = 20.22 mm
T_y = 2400. N
M_x = -2400000. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 12. mm
v_m = 32.78 mm
$\sigma_m = -Mv/J_u$ = 238. N/mm^2
x_c = 24. mm
y_c = 42. mm
v_c = 21.78 mm
$\sigma_c = -Mv/J_u$ = 158.2 N/mm^2
τ_c = 3.476 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 158.3 N/mm^2
S^* = 5746. mm^3

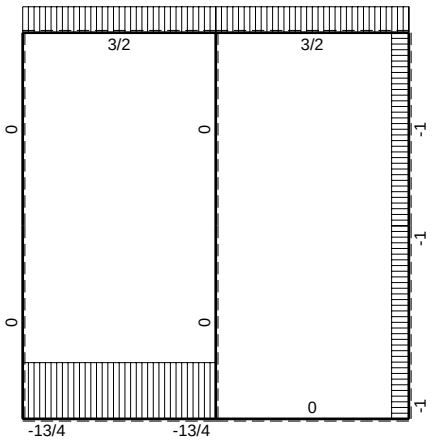
$V_{HG} = -F$
$V_{FE} = -F$
$W_G = -W = -Fb$
$p_{FG} = -q = -F/b$
$q_{IE} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



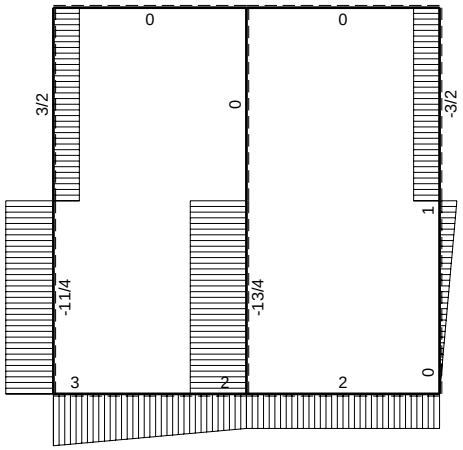


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 660 mm, F = 1190 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

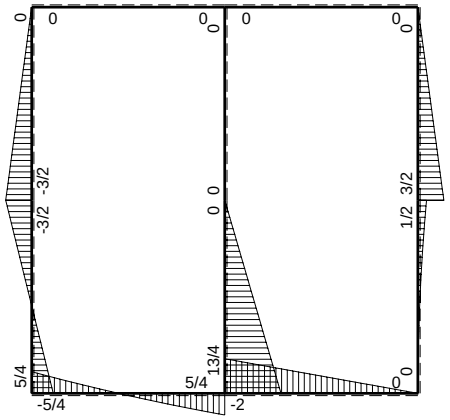




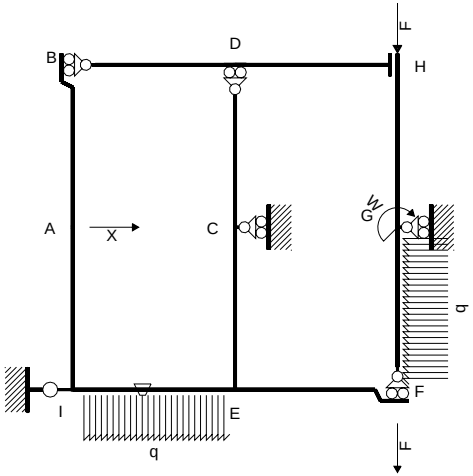
←⊞→ F



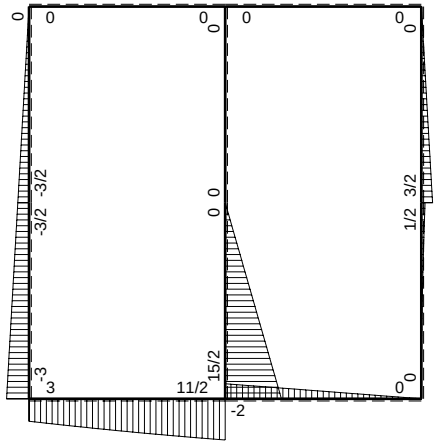
↑⊞↓ F



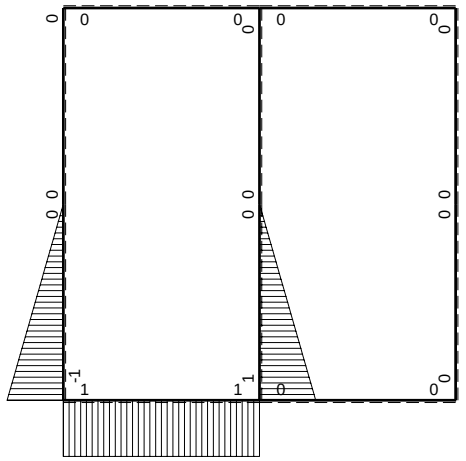
⟲⊞⟳ Fb



Schema di calcolo iperstatico



⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211841 ESΣ07.xxxx.082

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fb+3/2Fx$	0	0	0	0	0+0	0
BA b	0	$3/2Fx$	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
FE b	0	$2Fx$	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	$3/2Fb-3/2Fx$	0	0	0	0	0+0	0
HG b	0	$-3/2Fx$	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	$3Fb+3Fx-1/2qx^2$	$-Fb/EJ$	$3Fb^2+3Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2	$(13/3-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	$-11/2Fb+2Fx+1/2qx^2$	Fb/EJ	$11/2Fb^2-2Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2		
EC b	b-x	$15/2Fb-15/2Fx$	0	$15/2Fb^2-15Fbx+15/2Fx^2$	0	$b^2-2bx+x^2$	$(5/2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	$-15/2Fx$	0	$15/2Fx^2$	0	x^2		
IA b	-b+x	$-3Fb+3/2Fx$	0	$3Fb^2-9/2Fbx+3/2Fx^2$	0	$b^2-2bx+x^2$	$(5/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	$3/2Fb+3/2Fx$	0	$3/2Fbx+3/2Fx^2$	0	x^2		
	totali						$85/12Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						$-17/4F$	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (3 + 3\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx$

$= [\, 3\, x + 3/2\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, - x \,]_o^b\, \theta$

$= (3\, b + 3/2\, b - 1/6\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = 10/3\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (11/2 - 2\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx$

$= [\, 11/2\, x - x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (11/2\, b - b - 1/6\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 10/3\ Fb^3/EJ$

$L^{Xo}_{EC} = \int_o^b (15/2 - 15\, x/b + 15/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [15/2\, x - 15/2\, x^2/b + 5/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (15/2\, b - 15/2\, b + 5/2\, b)\, Fb^2\, 1/EJ = 5/2\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (15/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

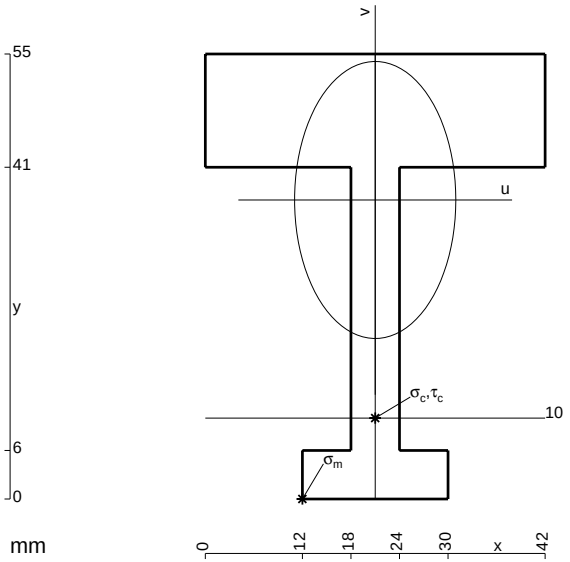
$= (5/2\, b)\, Fb^2\, 1/EJ = 5/2\ Fb^3/EJ$

$L^{Xo}_{IA} = \int_o^b (3 - 9/2\, x/b + 3/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [\, 3\, x - 9/4\, x^2/b + 1/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (3\, b - 9/4\, b + 1/2\, b)\, Fb^2\, 1/EJ = 5/4\ Fb^3/EJ$

$L^{Xo}_{AI} = \int_o^b (3/2\, x/b + 3/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x^2/b + 1/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b + 1/2\, b)\, Fb^2\, 1/EJ = 5/4\ Fb^3/EJ$



$A = 906.\ mm^2$

$J_u = 265632.\ mm^4$

$J_v = 89982.\ mm^4$

$y_g = 36.96\ mm$

$T_y = 2380.\ N$

$M_x = -1570800.\ Nmm$

$x_m = 12.\ mm$

$u_m = -9.\ mm$

$v_m = -36.96\ mm$

$\sigma_m = -Mv/J_u = -218.5\ N/mm^2$

$x_c = 21.\ mm$

$y_c = 10.\ mm$

$v_c = -26.96\ mm$

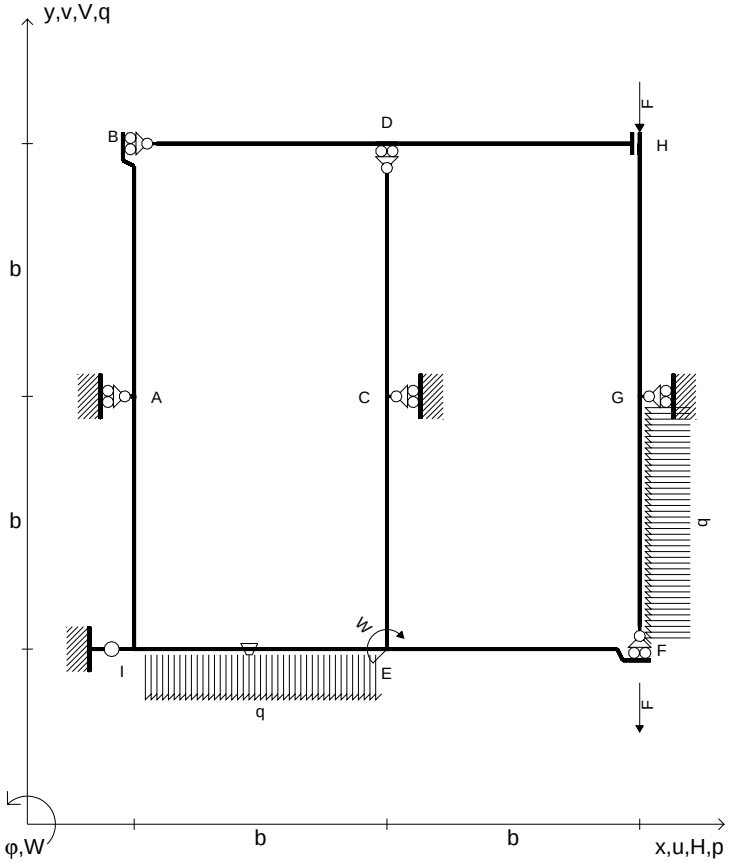
$\sigma_c = -Mv/J_u = -159.4\ N/mm^2$

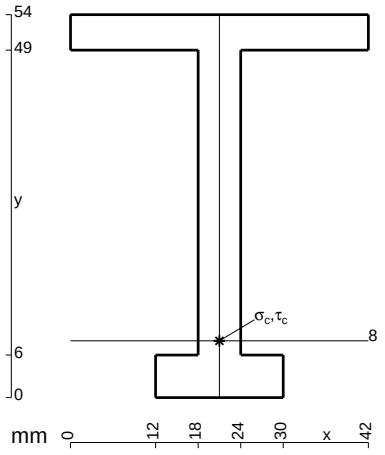
$\tau_c = 6.514\ N/mm^2$

$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 159.8\ N/mm^2$

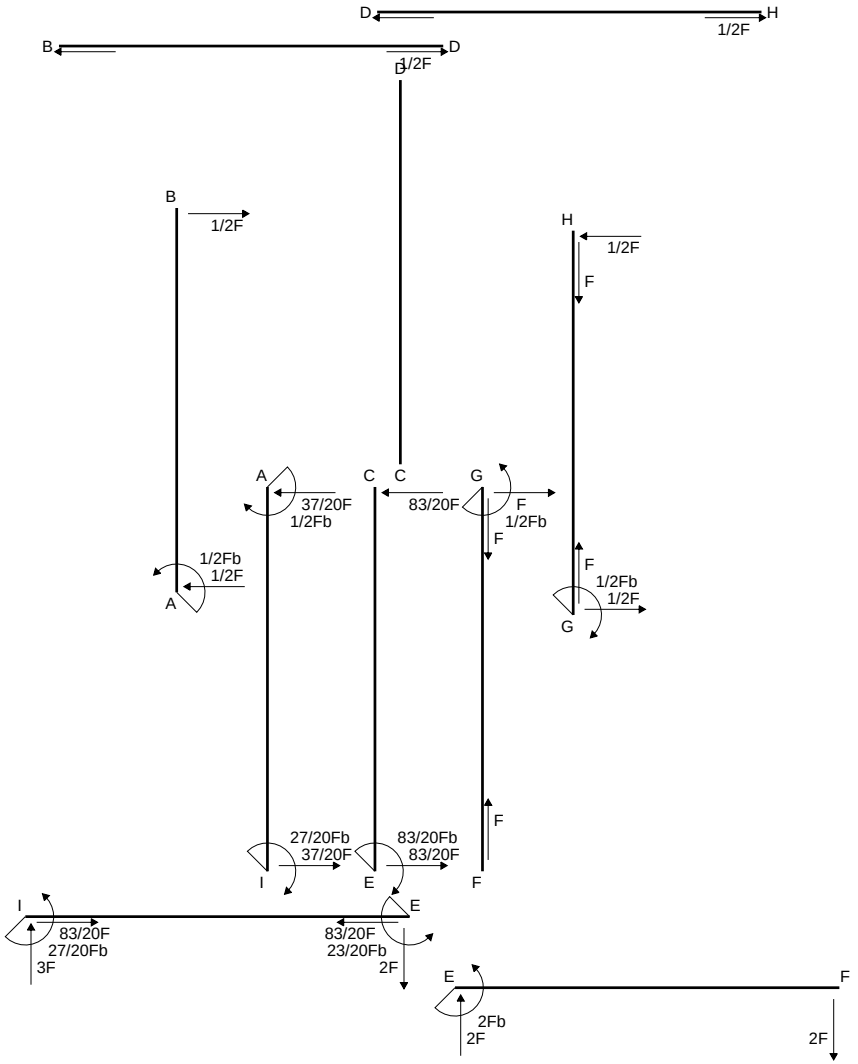
$S^* = 4362.\ mm^3$

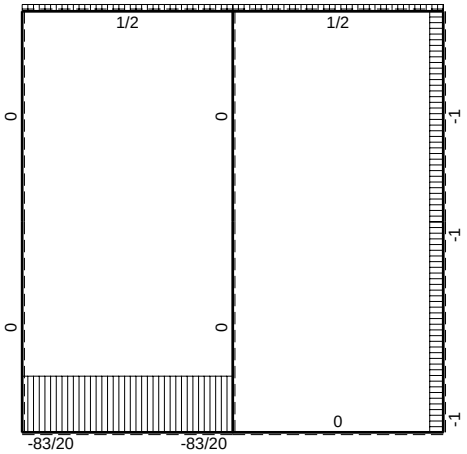
$V_{HG} = -F$
$V_{FE} = -F$
$W_E = -W = -Fb$
$p_{FG} = -q = -F/b$
$q_{IE} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



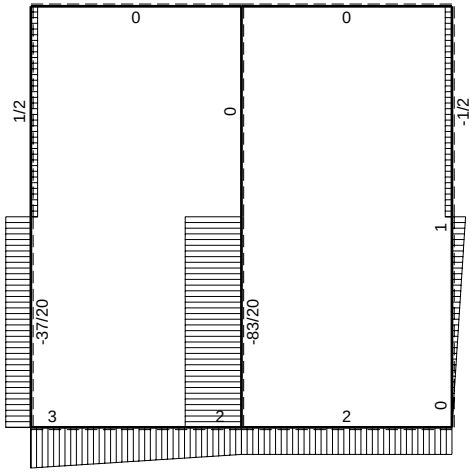


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 910 mm, F = 820 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

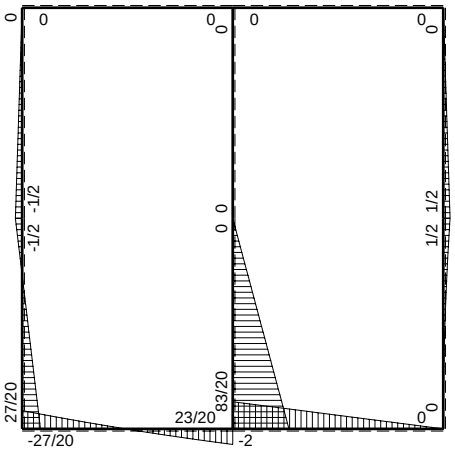




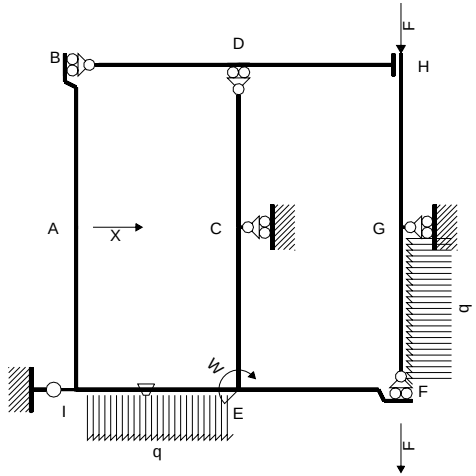
←□→ F



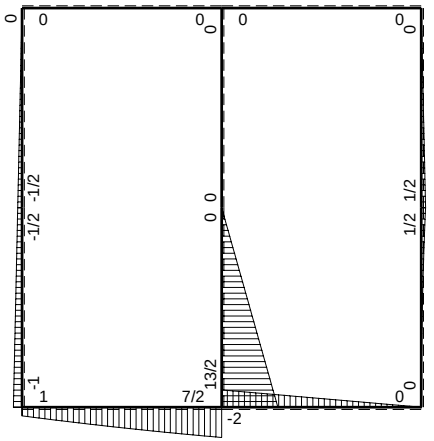
↑□↓ F



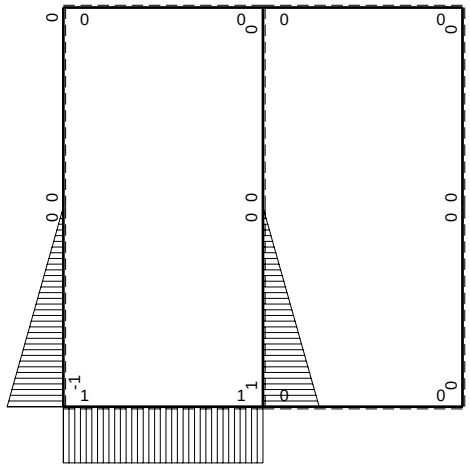
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
FE b	0	$2Fx$	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	$1/2Fb-1/2Fx$	0	0	0	0	0+0	0
HG b	0	$-1/2Fx$	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	$Fb+3Fx-1/2qx^2$	$-Fb/EJ$	$Fb^2+3Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2	$(7/3-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	$-7/2Fb+2Fx+1/2qx^2$	Fb/EJ	$7/2Fb^2-2Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2		
EC b	b-x	$13/2Fb-13/2Fx$	0	$13/2Fb^2-13Fbx+13/2Fx^2$	0	$b^2-2bx+x^2$	$(13/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	$-13/2Fx$	0	$13/2Fx^2$	0	x^2		
IA b	-b+x	$-Fb+1/2Fx$	0	$Fb^2-3/2Fbx+1/2Fx^2$	0	$b^2-2bx+x^2$	$(5/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	$1/2Fb+1/2Fx$	0	$1/2Fbx+1/2Fx^2$	0	x^2		
	totali						$47/12Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						$-47/20F$	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (1 + 3\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx$

$= [\, x + 3/2\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, - x \,]_o^b\, \theta$

$= (\, b + 3/2\, b - 1/6\, b \,)\, Fb^2\, 1/EJ + (- b)\, \theta = 4/3\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (7/2 - 2\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx$

$= [7/2\, x - x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (7/2\, b - b - 1/6\, b \,)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 4/3\ Fb^3/EJ$

$L^{Xo}_{EC} = \int_o^b (13/2 - 13\, x/b + 13/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [13/2\, x - 13/2\, x^2/b + 13/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (13/2\, b - 13/2\, b + 13/6\, b \,)\, Fb^2\, 1/EJ = 13/6\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (13/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [13/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

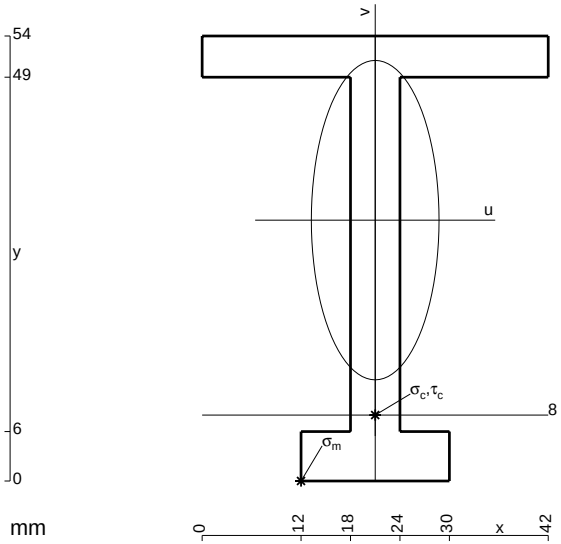
$= (13/6\, b \,)\, Fb^2\, 1/EJ = 13/6\ Fb^3/EJ$

$L^{Xo}_{IA} = \int_o^b (1 - 3/2\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [\, x - 3/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (\, b - 3/4\, b + 1/6\, b \,)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

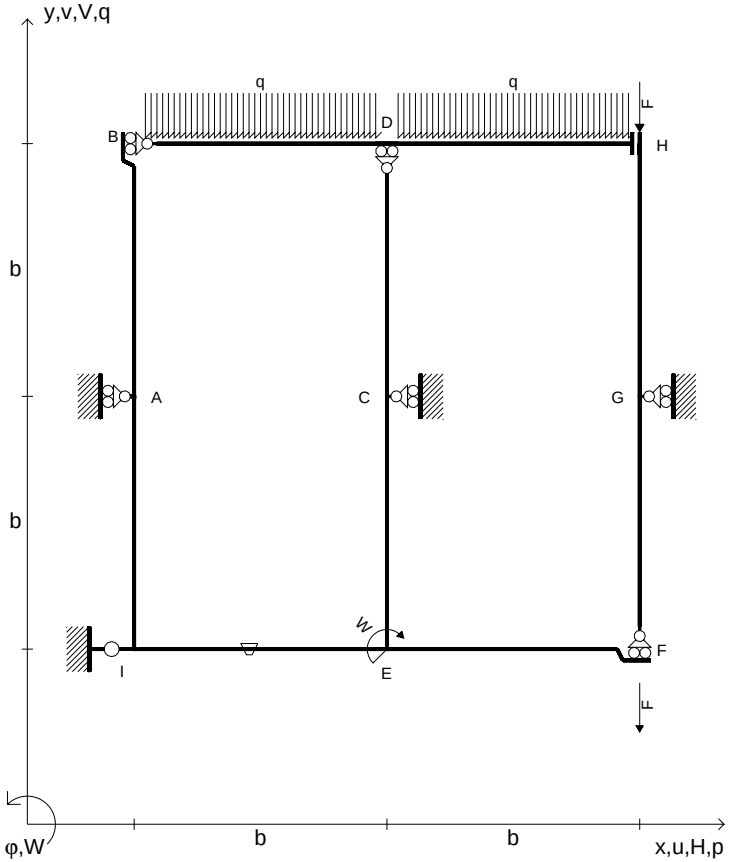
$L^{Xo}_{AI} = \int_o^b (1/2\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

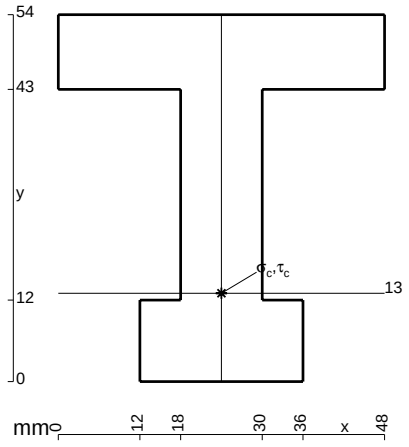
$= (1/4\, b + 1/6\, b \,)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$



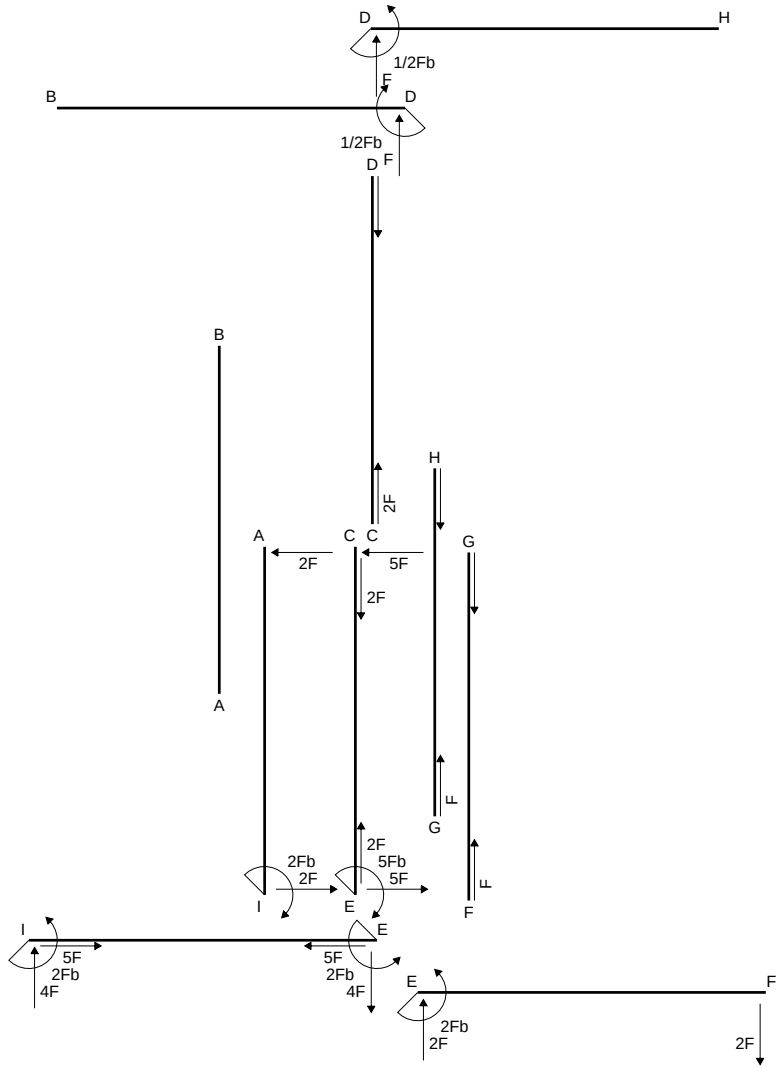
A = 576. mm^2
J_u = 216352. mm^4
J_v = 34560. mm^4
y_g = 31.66 mm
T_y = 1640. N
M_x = -1492400. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -31.66 mm
$\sigma_m = -Mv/J_u$ = -218.4 N/mm^2
x_c = 21. mm
y_c = 8. mm
v_c = -23.66 mm
$\sigma_c = -Mv/J_u$ = -163.2 N/mm^2
τ_c = 4.284 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 163.4 N/mm^2
S^* = 3391. mm^3

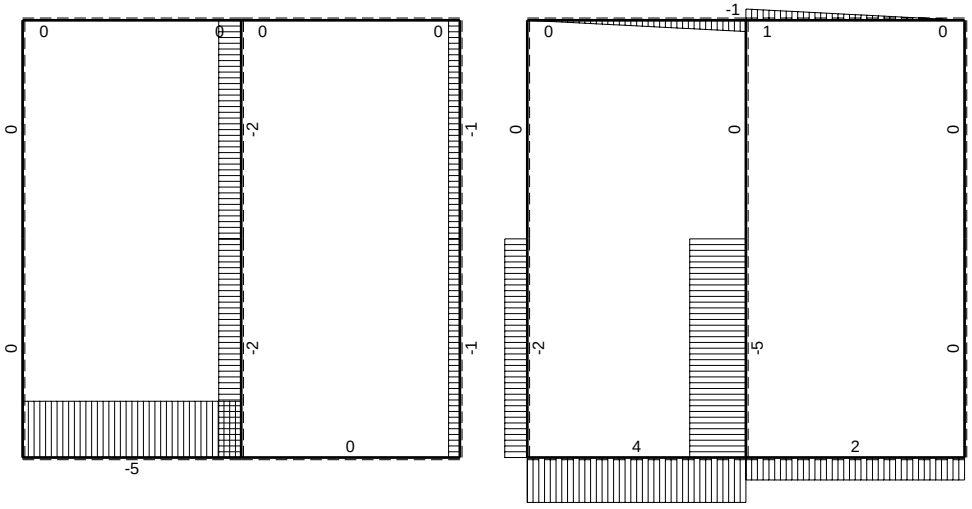
$V_{HG} = -F$
$V_{FE} = -F$
$W_E = -W = -Fb$
$q_{DB} = -q = -F/b$
$q_{HD} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$





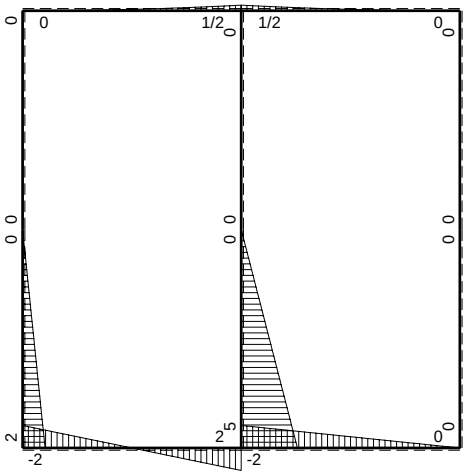
Reazioni iperstatiche in soluzione: $X=W_{IE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 750 mm, F = 1940 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.



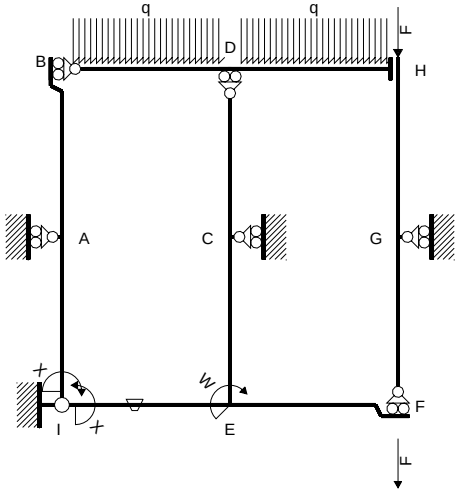


←□→ F

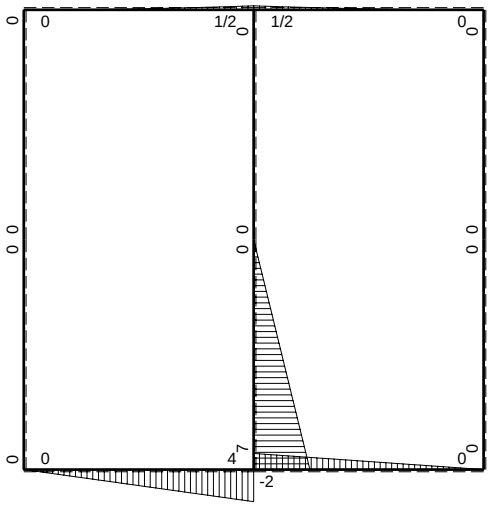
↑□↓ F



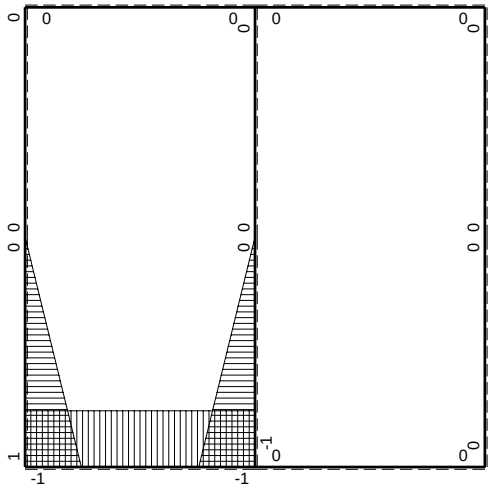
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{IE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	0	0	0	0	0	0+0	0
HG b	0	0	0	0	0	0		
HD b	0	$1/2qx^2$	0	0	0	0	0+0	0
DH b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	-1	4Fx	-Fb/EJ	-4Fx	Fb/EJ	1	$(-2+1)Fb^2/EJ$	Xb/EJ
EI b	1	-4Fb+4Fx	Fb/EJ	-4Fb+4Fx	Fb/EJ	1		
EC b	$-1+x/b$	7Fb-7Fx	0	$-7Fb+14Fx-7Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-7/3+0)Fb^2/EJ$	1/3Xb/EJ
CE b	x/b	-7Fx	0	$-7Fx^2/b$	0	x^2/b^2		
IA b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
AI b	-x/b	0	0	0	0	x^2/b^2		
	totali						$-10/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{IE}$						2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210089 ESΣ07.xxxx.104
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$L_{IE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{EI}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CE}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{AI}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IE}^{Xo} = \int_o^b (-4\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L_{EI}^{Xo} = \int_o^b (-4 + 4\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-4\ x + 2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

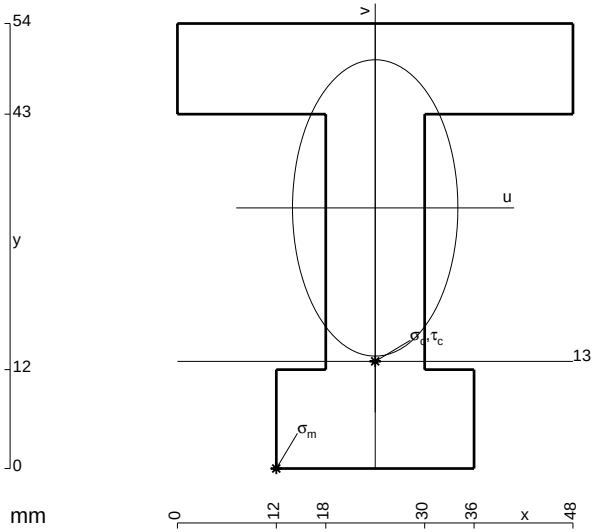
$= (-4\ b + 2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L_{EC}^{Xo} = \int_o^b (-7 + 14\ x/b - 7\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-7\ x + 7\ x^2/b - 7/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-7\ b + 7\ b - 7/3\ b\ )\ Fb\ 1/EJ\ = -7/3\ Fb^2/EJ$

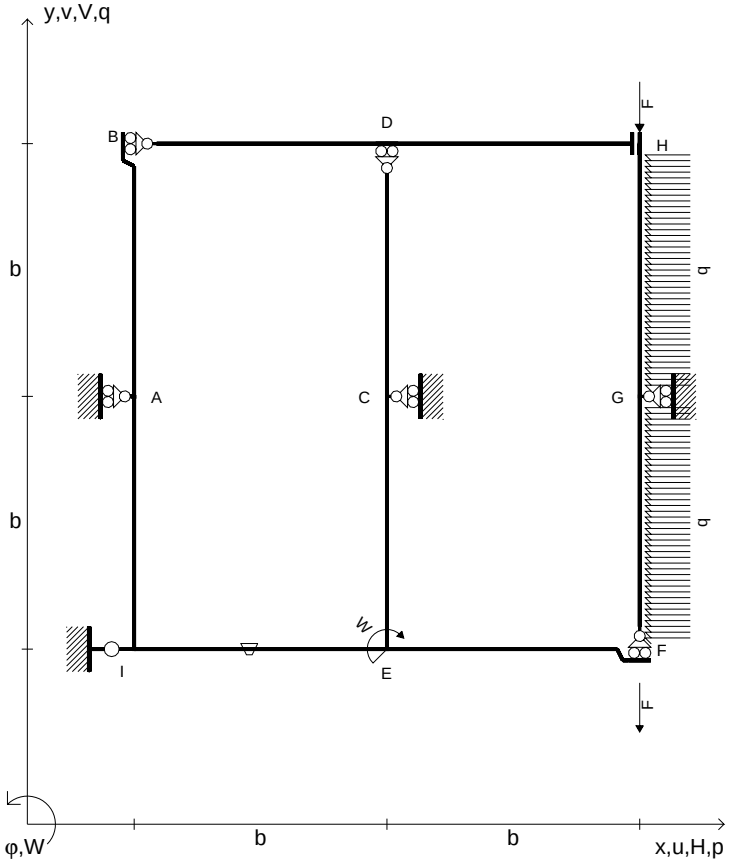
$L_{CE}^{Xo} = \int_o^b (-7\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-7/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

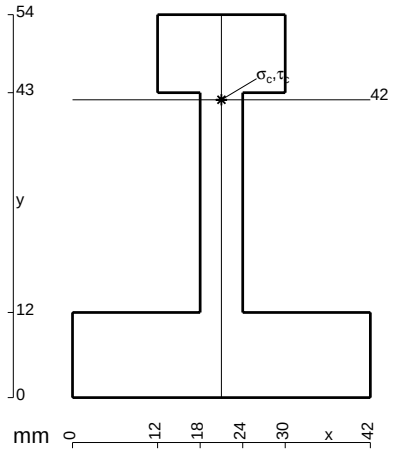
$= (-7/3\ b\ )\ Fb\ 1/EJ\ = -7/3\ Fb^2/EJ$



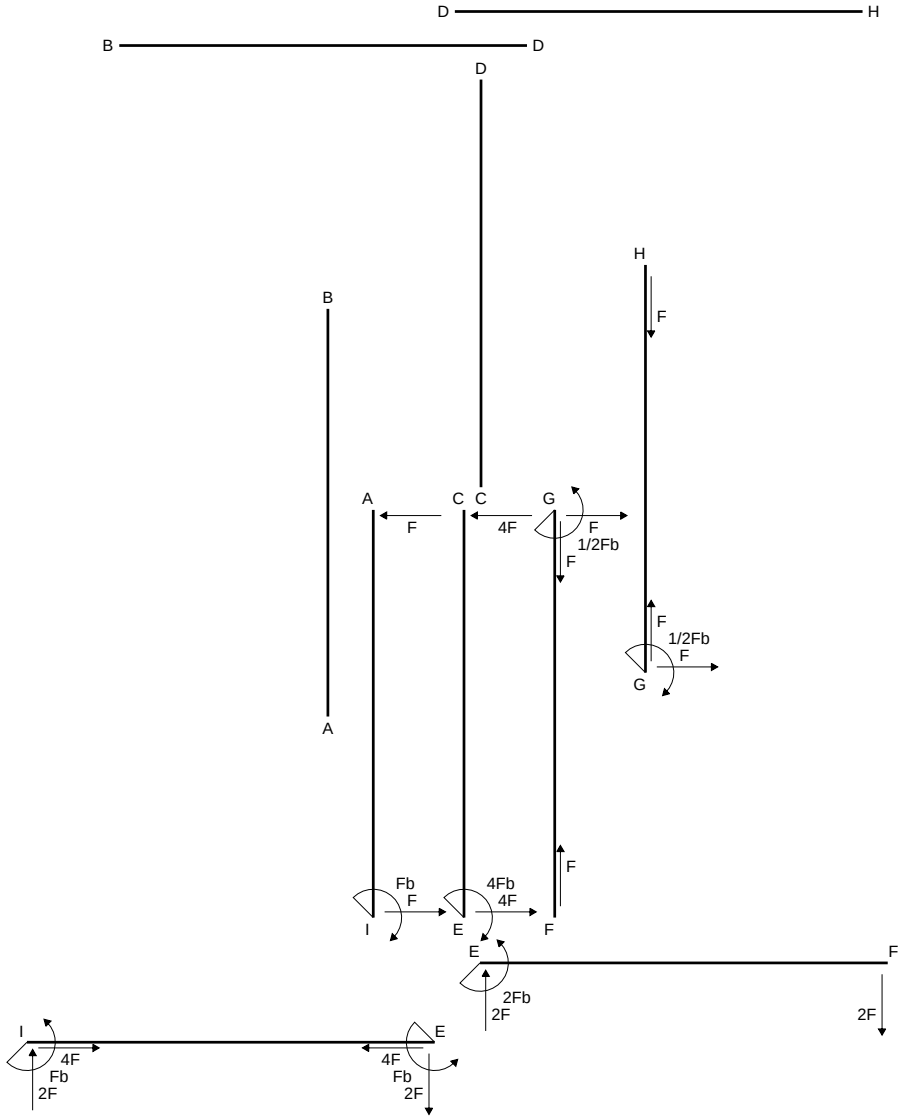
A = 1188. mm^2
J_u = 384370. mm^4
J_v = 119664. mm^4
y_g = 31.62 mm
T_y = 3880. N
M_x = -2910000. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -31.62 mm
$\sigma_m = -Mv/J_u$ = -239.4 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -18.62 mm
$\sigma_c = -Mv/J_u$ = -141. N/mm^2
τ_c = 6.4 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 141.4 N/mm^2
S^* = 7608. mm^3

$V_{HG} = -F$
$V_{FE} = -F$
$W_E = -W = -Fb$
$p_{GH} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 970 mm, F = 1020 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.



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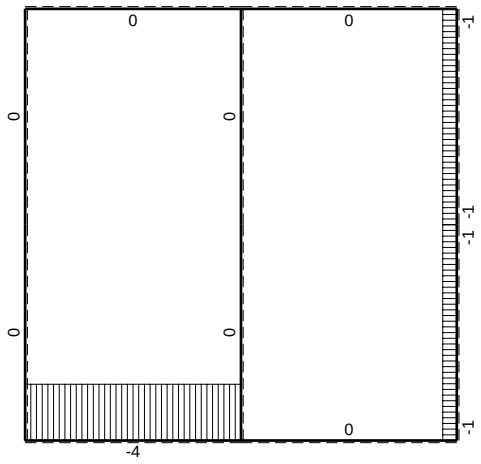
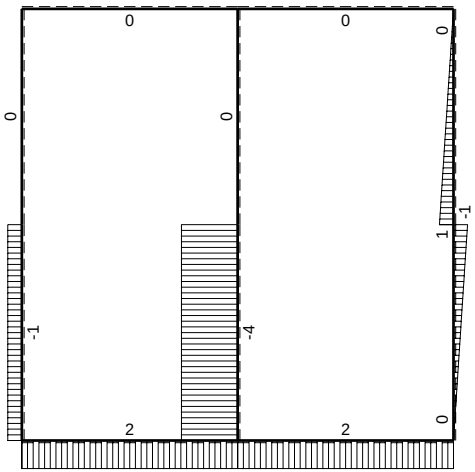
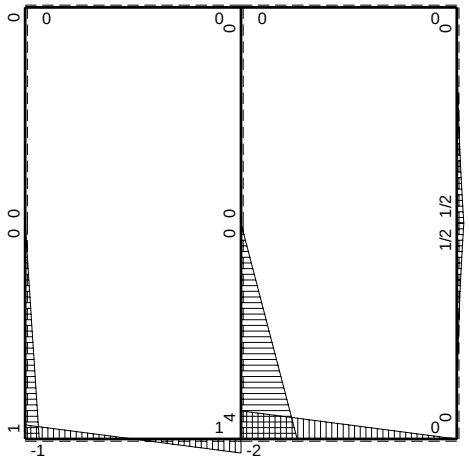


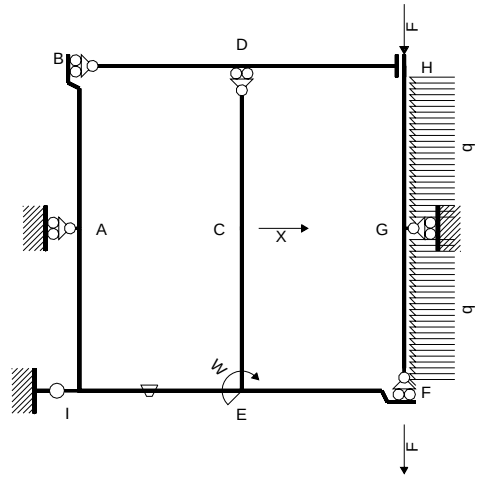
←□→ F



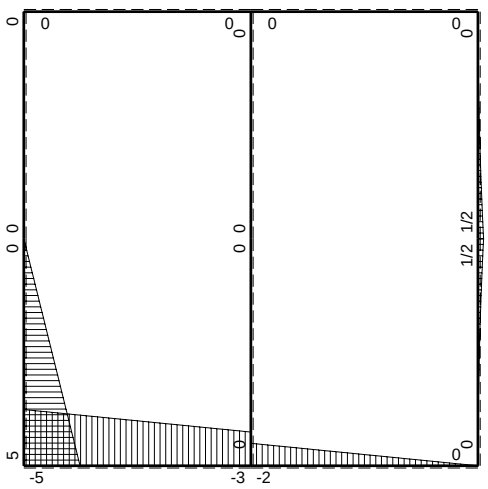
↑□↓ F



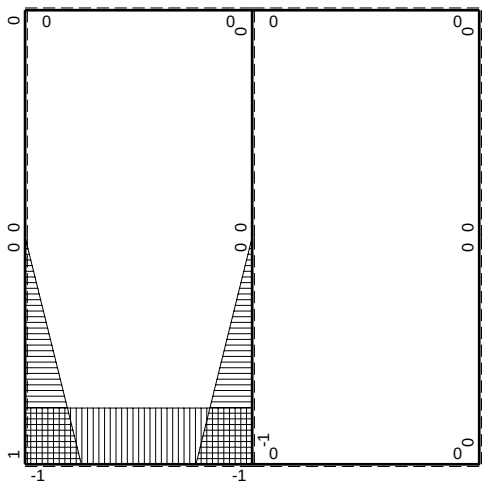
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$-1/2qx^2$	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-b	-5Fb+2Fx	-Fb/EJ	$5Fb^2-2Fbx$	Fb^2/EJ	b^2	$(4+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	3Fb+2Fx	Fb/EJ	$3Fb^2+2Fbx$	Fb^2/EJ	b^2		
EC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CE b	x	0	0	0	0	x^2		
IA b	b-x	5Fb-5Fx	0	$5Fb^2-10Fbx+5Fx^2$	0	$b^2-2bx+x^2$	$(5/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	-5Fx	0	$5Fx^2$	0	x^2		
	totali						$20/3Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-4F	

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Sviluppi di calcolo iperstatica

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$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (5 - 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx = [5\, x - x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (5\, b - b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 5\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (3 + 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx = [3\, x + x^2/b \,]_o^b\, Fb^2\, 1/EJ + [-\, x \,]_o^b\, \theta$

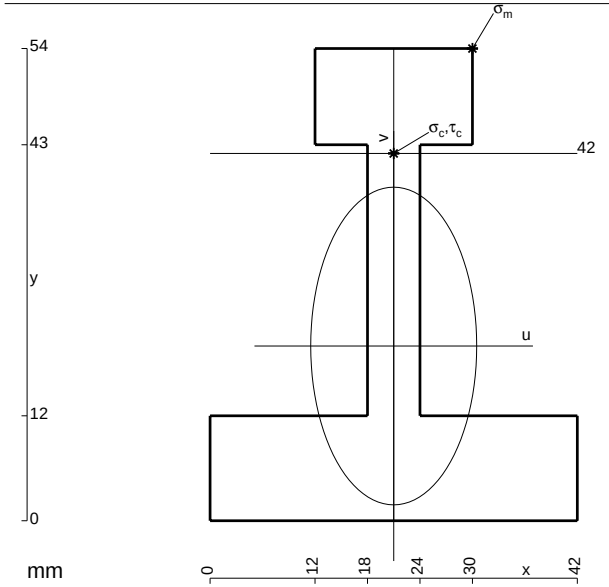
$= (3\, b + b)\, Fb^2\, 1/EJ + (-\, b)\, \theta = 5\ Fb^3/EJ$

$L_{IA}^{Xo} = \int_o^b (5 - 10\, x/b + 5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5\, x - 5\, x^2/b + 5/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (5\, b - 5\, b + 5/3\, b)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

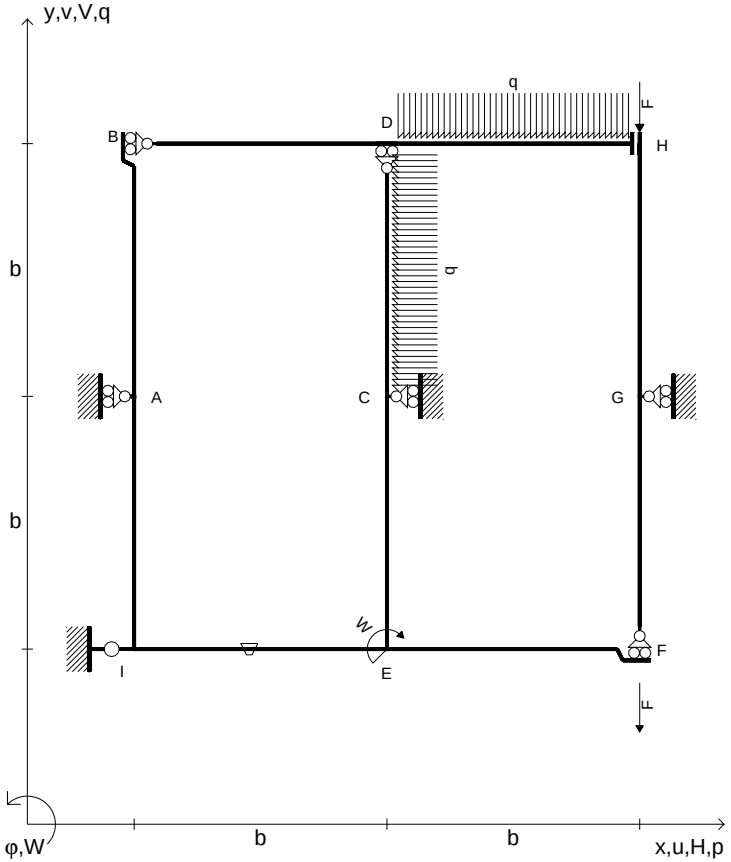
$L_{AI}^{Xo} = \int_o^b (5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

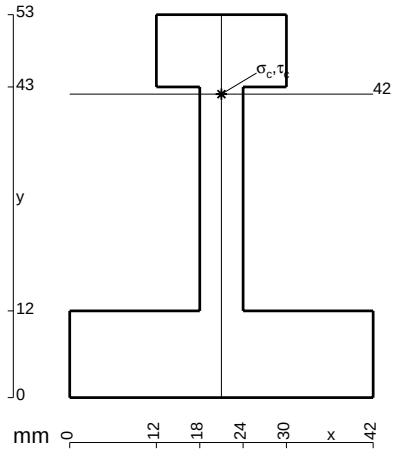
$= (5/3\, b)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$



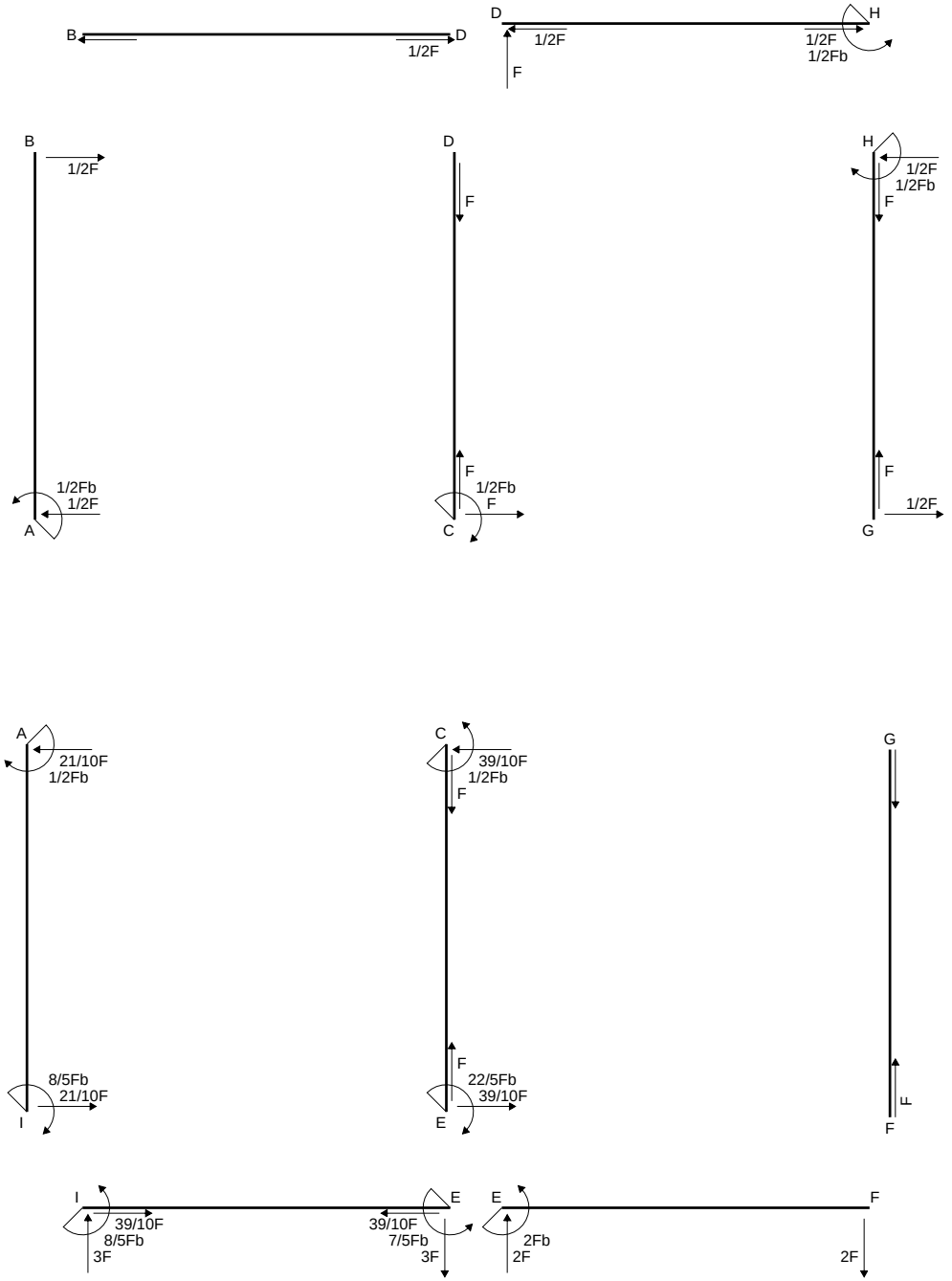
A = 888. mm^2
J_u = 293012. mm^4
J_v = 79992. mm^4
y_g = 19.98 mm
T_y = 2040. N
M_x = -1978800. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 9. mm
v_m = 34.02 mm
$\sigma_m = -Mv/J_u$ = 229.8 N/mm^2
x_c = 21. mm
y_c = 42. mm
v_c = 22.02 mm
$\sigma_c = -Mv/J_u$ = 148.7 N/mm^2
τ_c = 6.709 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 149.2 N/mm^2
S^* = 5782. mm^3

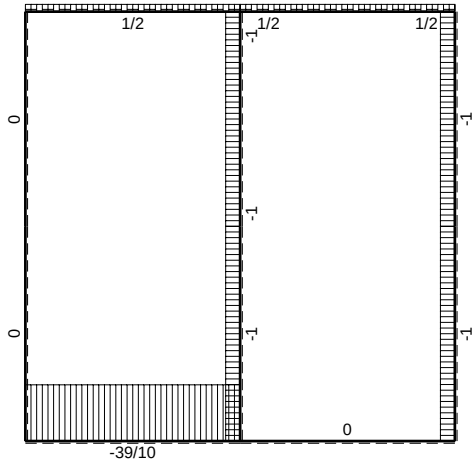
$V_{HG} = -F$
$V_{FE} = -F$
$W_E = -W = -Fb$
$p_{CD} = -q = -F/b$
$q_{HD} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



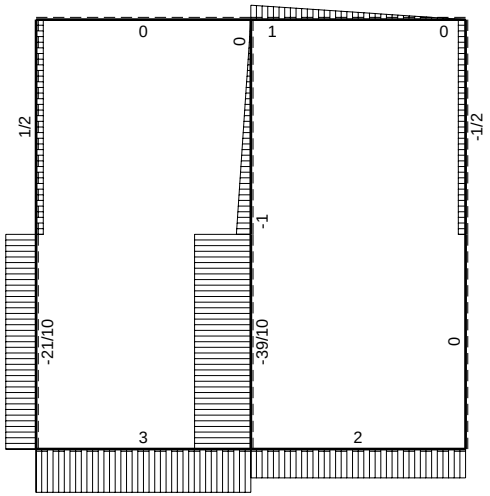


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 740 mm, F = 1310 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

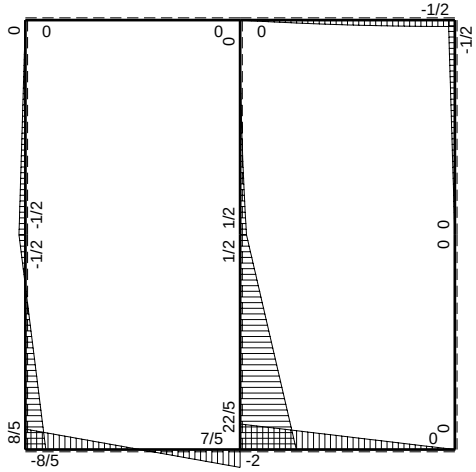




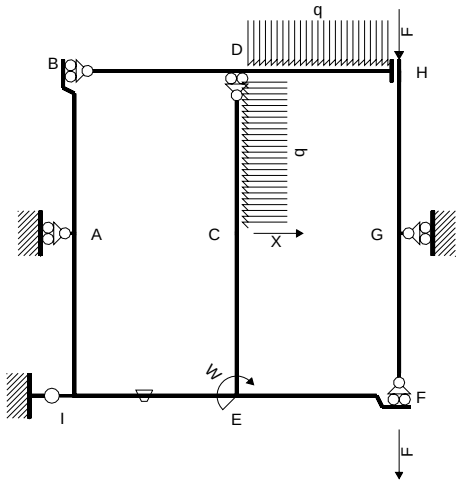
←⊞→ F



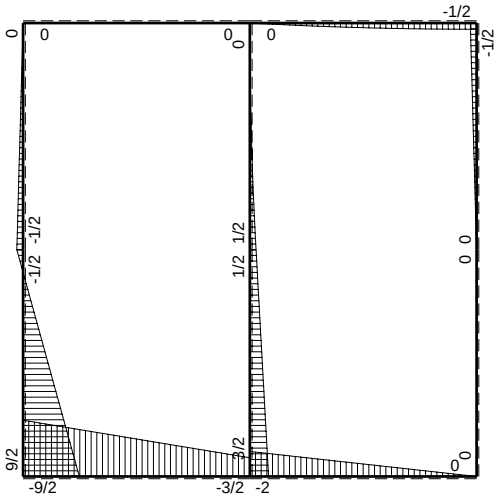
↑⊞↓ F



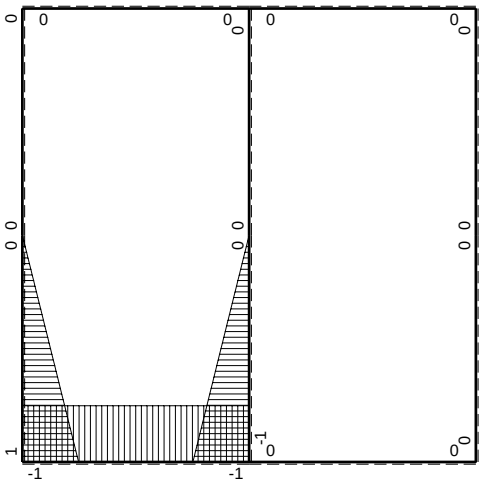
↻⊞↺ Fb



Schema di calcolo iperstatico



↻⊞↺ M_o flessione da carichi assegnati



↻⊞↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2qx^2$	0	0	0	0		
EF b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
FE b	0	$2Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-1/2Fx$	0	0	0	0	0+0	0
HG b	0	$1/2Fb-1/2Fx$	0	0	0	0		
HD b	0	$-1/2Fb+1/2qx^2$	0	0	0	0	0+0	0
DH b	0	$Fx-1/2qx^2$	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-b	$-9/2Fb+3Fx$	$-Fb/EJ$	$9/2Fb^2-3Fbx$	Fb^2/EJ	b^2	$(3+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$3/2Fb+3Fx$	Fb/EJ	$3/2Fb^2+3Fbx$	Fb^2/EJ	b^2		
EC b	-b+x	$3/2Fb-Fx$	0	$-3/2Fb^2+5/2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-7/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	x	$-1/2Fb-Fx$	0	$-1/2Fbx-Fx^2$	0	x^2		
IA b	b-x	$9/2Fb-5Fx$	0	$9/2Fb^2-19/2Fbx+5Fx^2$	0	$b^2-2bx+x^2$	$(17/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	$1/2Fb-5Fx$	0	$-1/2Fbx+5Fx^2$	0	x^2		
	totali						$29/6Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-29/10F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213131 ESΣ07.xxxx.124
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (9/2 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx = [9/2\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (9/2\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 4\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (3/2 + 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx = [3/2\, x + 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [-\, x \,]_o^b\, \theta$

$= (3/2\, b + 3/2\, b)\, Fb^2\, 1/EJ + (-\, b)\, \theta = 4\ Fb^3/EJ$

$L_{EC}^{Xo} = \int_o^b (-3/2 + 5/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/2\, x + 5/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 5/4\, b - 1/3\, b)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L_{CE}^{Xo} = \int_o^b (-1/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

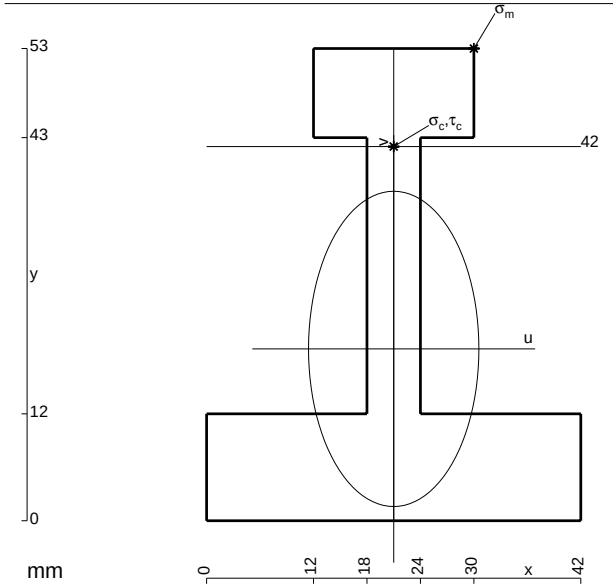
$= (-1/4\, b - 1/3\, b)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L_{IA}^{Xo} = \int_o^b (9/2 - 19/2\, x/b + 5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [9/2\, x - 19/4\, x^2/b + 5/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (9/2\, b - 19/4\, b + 5/3\, b)\, Fb^2\, 1/EJ = 17/12\ Fb^3/EJ$

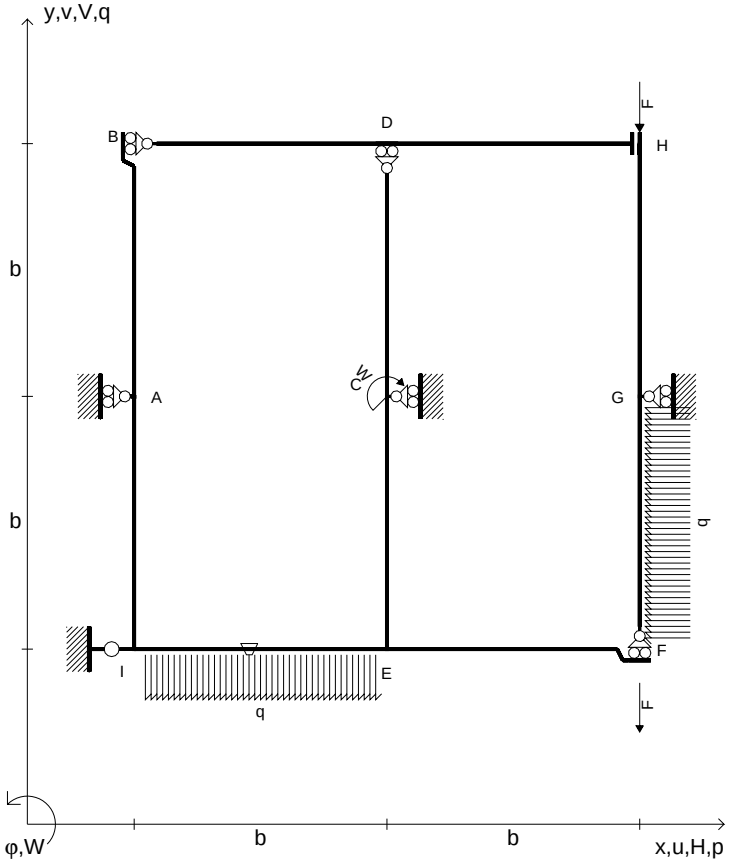
$L_{AI}^{Xo} = \int_o^b (-1/2\, x/b + 5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b + 5/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

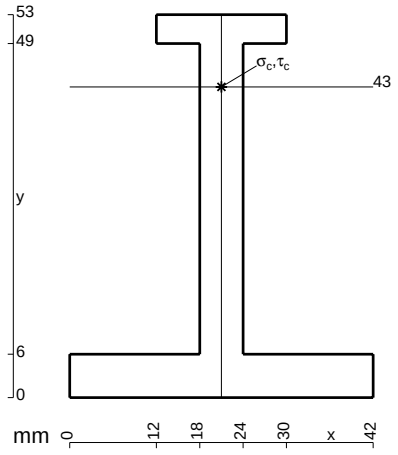
$= (-1/4\, b + 5/3\, b)\, Fb^2\, 1/EJ = 17/12\ Fb^3/EJ$



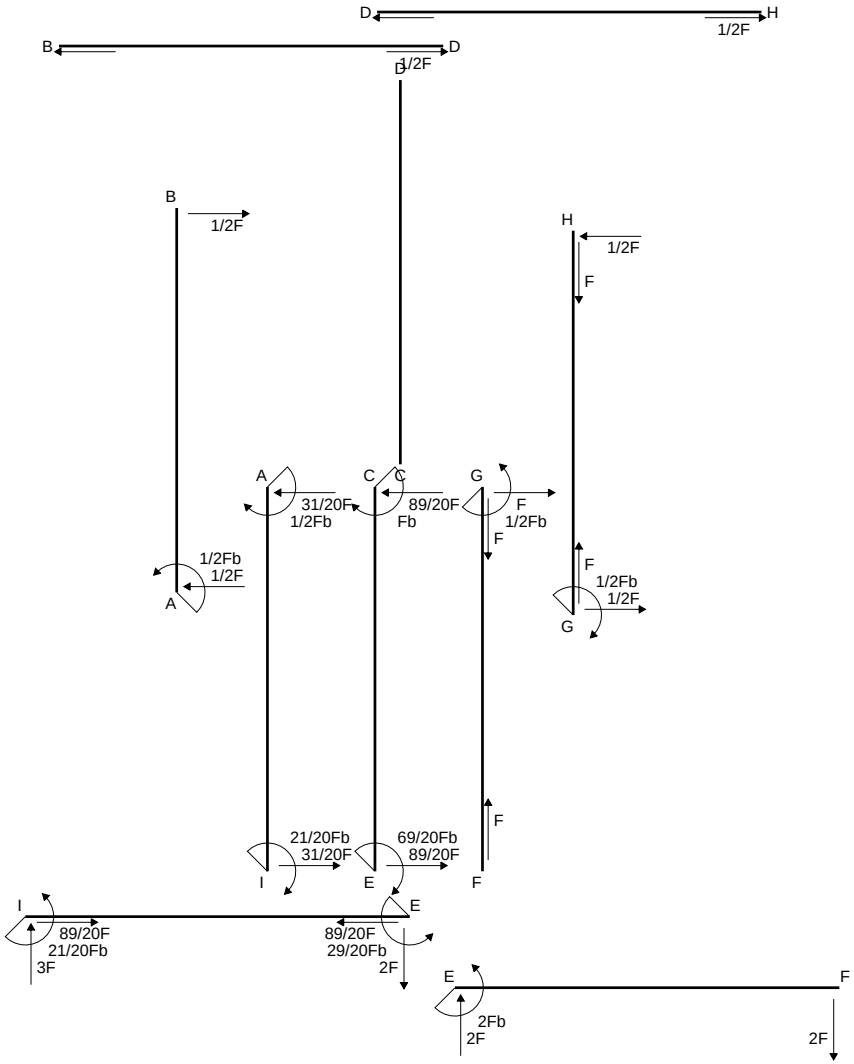
A = 870. mm^2
J_u = 272367. mm^4
J_v = 79506. mm^4
y_g = 19.29 mm
T_y = 2620. N
M_x = -1938800. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 9. mm
v_m = 33.71 mm
$\sigma_m = -Mv/J_u$ = 240. N/mm^2
x_c = 21. mm
y_c = 42. mm
v_c = 22.71 mm
$\sigma_c = -Mv/J_u$ = 161.7 N/mm^2
τ_c = 8.51 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 162.4 N/mm^2
S^* = 5308. mm^3

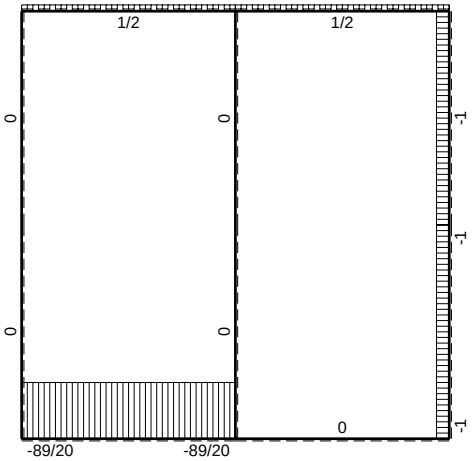
$V_{HG} = -F$
$V_{FE} = -F$
$W_C = -W = -Fb$
$p_{FG} = -q = -F/b$
$q_{IE} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



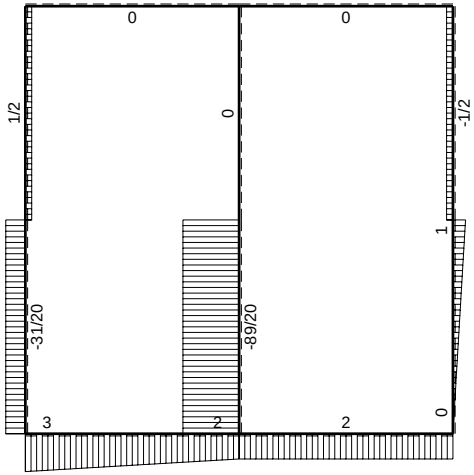


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 950 mm, F = 710 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

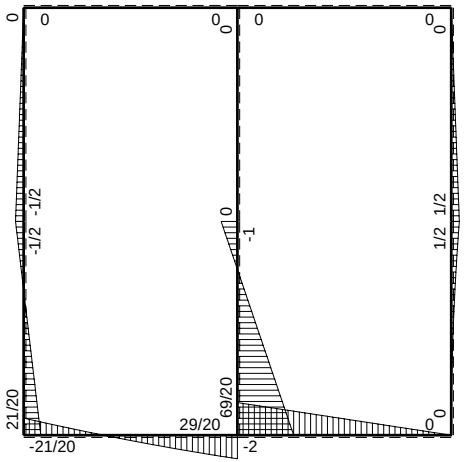




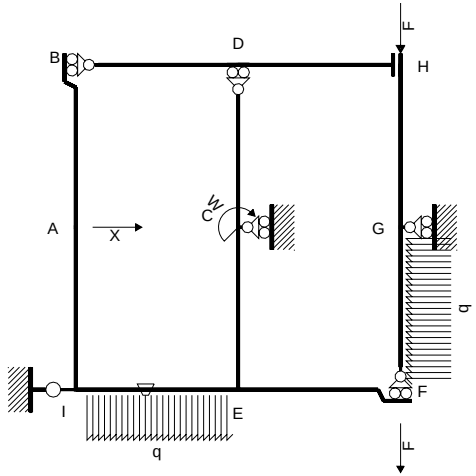
←□→ F



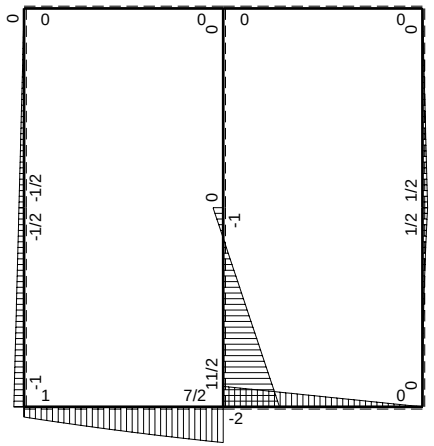
↑□↓ F



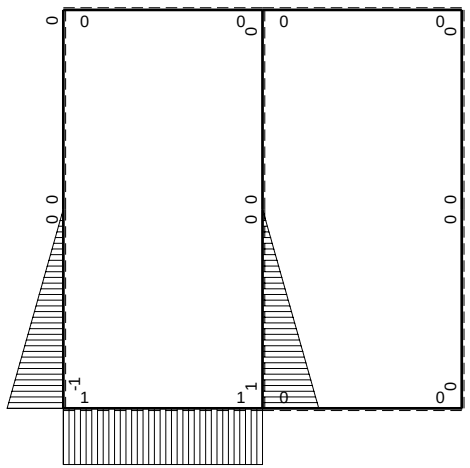
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fx	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	$Fb+3Fx-1/2qx^2$	$-Fb/EJ$	$Fb^2+3Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2	$(7/3-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	$-7/2Fb+2Fx+1/2qx^2$	Fb/EJ	$7/2Fb^2-2Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2		
EC b	b-x	11/2Fb-13/2Fx	0	$11/2Fb^2-12Fbx+13/2Fx^2$	0	$b^2-2bx+x^2$	$(5/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	Fb-13/2Fx	0	$-Fbx+13/2Fx^2$	0	x^2		
IA b	-b+x	-Fb+1/2Fx	0	$Fb^2-3/2Fbx+1/2Fx^2$	0	$b^2-2bx+x^2$	$(5/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	1/2Fb+1/2Fx	0	$1/2Fbx+1/2Fx^2$	0	x^2		
	totali						$41/12Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-41/20F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (1 + 3\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx$

$= [\, x + 3/2\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, - x \,]_o^b\, \theta$

$= (\, b + 3/2\, b - 1/6\, b \,)\, Fb^2\, 1/EJ + (\, - b \,)\, \theta = 4/3\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (7/2 - 2\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx$

$= [7/2\, x - x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (7/2\, b - b - 1/6\, b \,)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 4/3\ Fb^3/EJ$

$L^{Xo}_{EC} = \int_o^b (11/2 - 12\, x/b + 13/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [11/2\, x - 6\, x^2/b + 13/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (11/2\, b - 6\, b + 13/6\, b \,)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (- x/b + 13/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b + 13/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b + 13/6\, b \,)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

$L^{Xo}_{IA} = \int_o^b (1 - 3/2\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [\, x - 3/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (\, b - 3/4\, b + 1/6\, b \,)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{AI} = \int_o^b (1/2\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b + 1/6\, b \,)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

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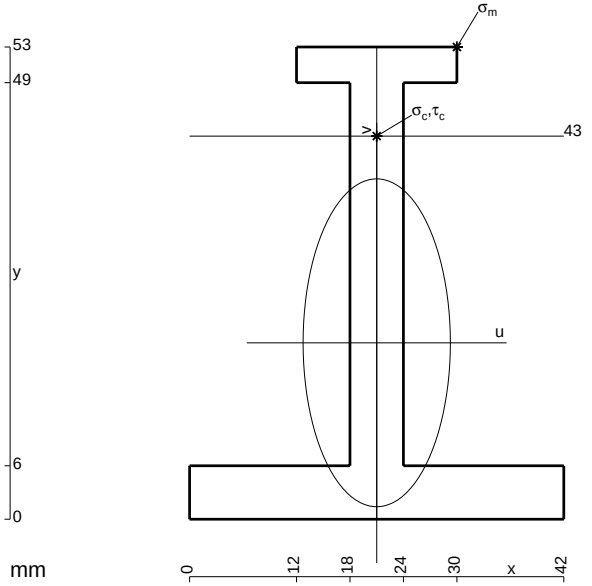

A = 582. mm^2
J_u = 197114. mm^4
J_v = 39762. mm^4
y_g = 19.8 mm
T_y = 1420. N
M_x = -1349000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 9. mm
v_m = 33.2 mm
$\sigma_m = -Mv/J_u$ = 227.2 N/mm^2
x_c = 21. mm
y_c = 43. mm
v_c = 23.2 mm
$\sigma_c = -Mv/J_u$ = 158.8 N/mm^2
τ_c = 3.83 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 158.9 N/mm^2
S^* = 3190. mm^3

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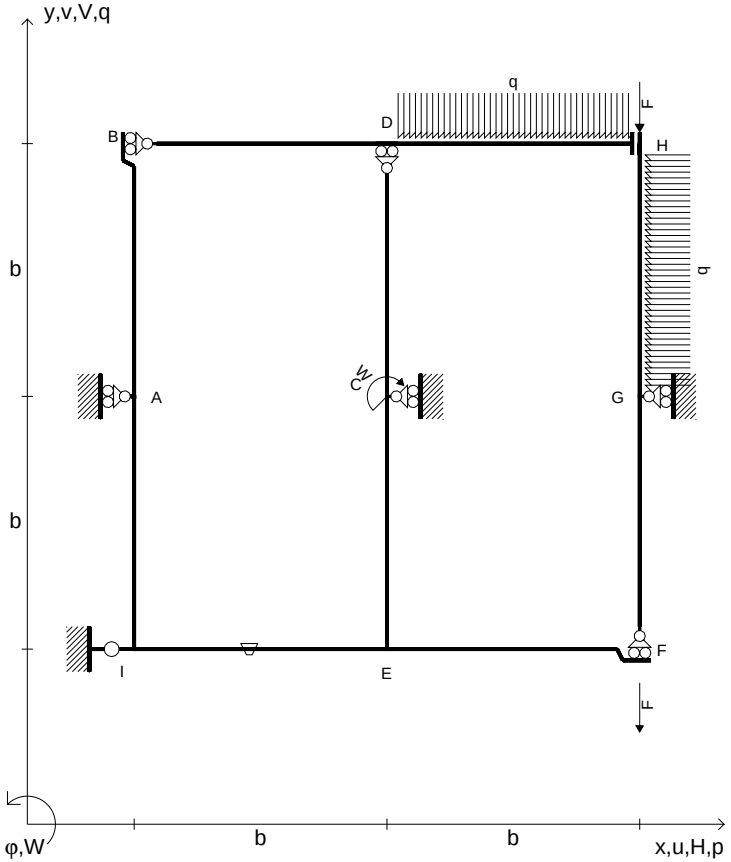
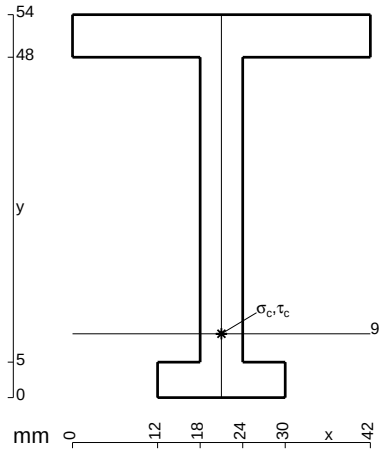
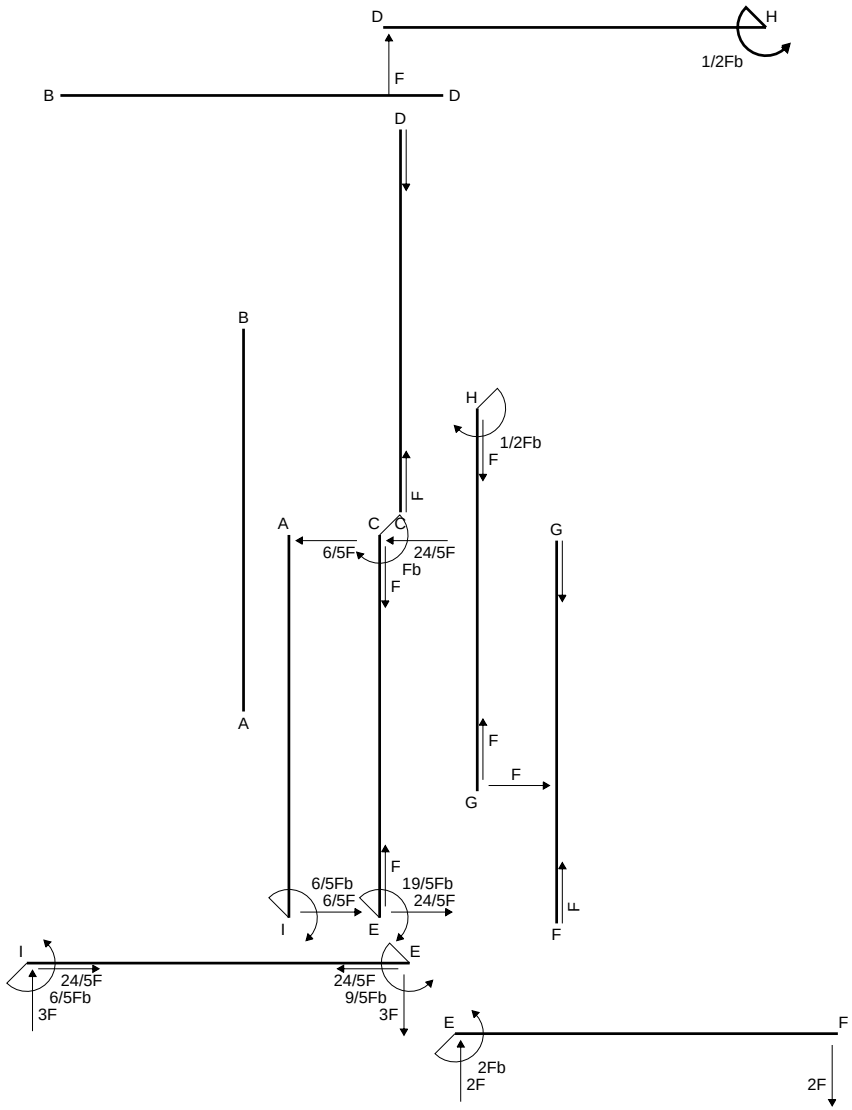
$V_{HG} = -F$
$V_{FE} = -F$
$W_C = -W = -Fb$
$q_{HD} = -q = -F/b$
$p_{GH} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 810 mm, F = 800 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.



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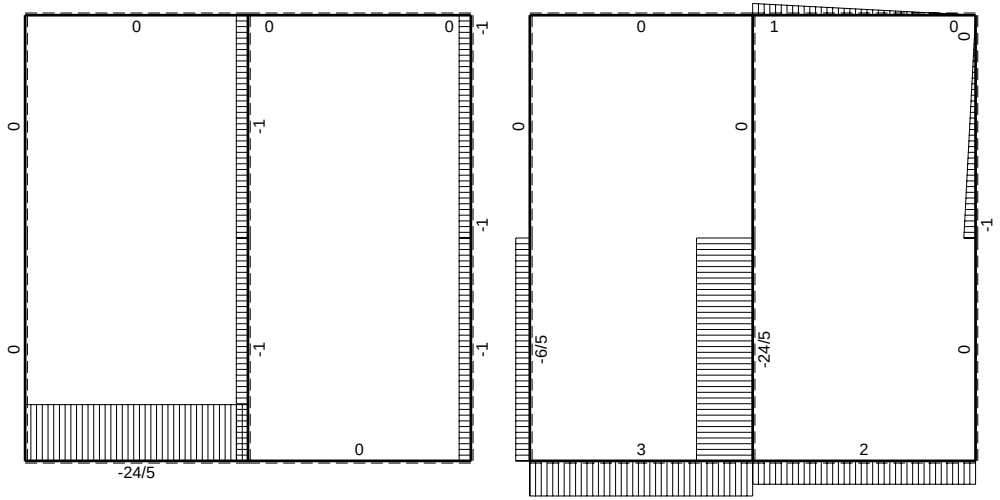
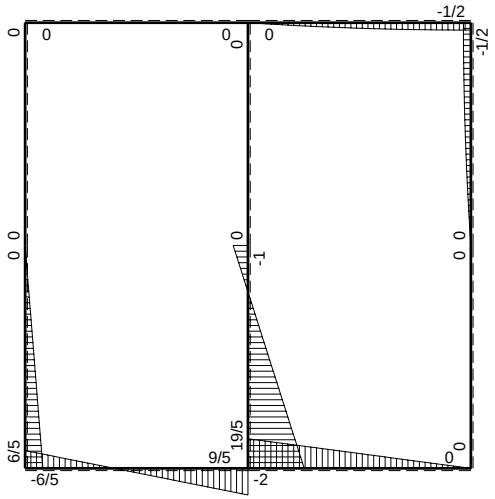
08.01.25

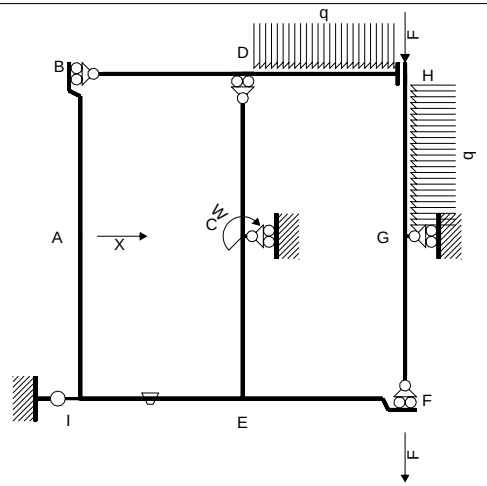


←□→ F

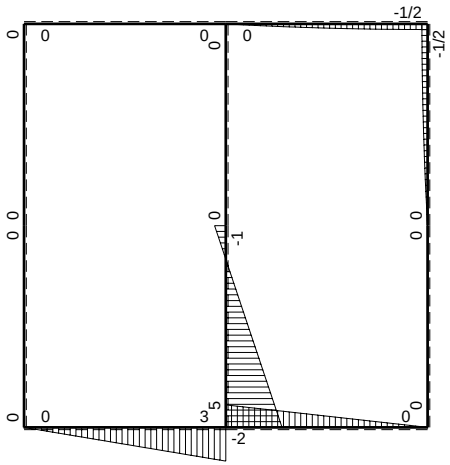
↑□↓ F



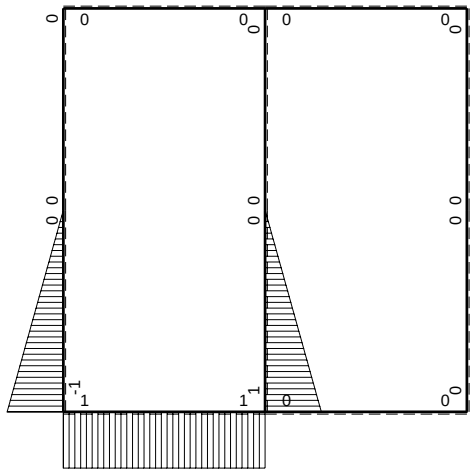
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
HD b	0	$-1/2Fb+1/2qx^2$	0	0	0	0	0+0	0
DH b	0	$Fx-1/2qx^2$	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	3Fx	-Fb/EJ	3Fbx	$-Fb^2/EJ$	b^2	$(3/2-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	$-3Fb+3Fx$	Fb/EJ	$3Fb^2-3Fbx$	$-Fb^2/EJ$	b^2		
EC b	b-x	5Fb-6Fx	0	$5Fb^2-11Fbx+6Fx^2$	0	$b^2-2bx+x^2$	$(3/2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	Fb-6Fx	0	$-Fbx+6Fx^2$	0	x^2		
IA b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
AI b	x	0	0	0	0	x^2		
	totali						$2Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-6/5F	

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Sviluppi di calcolo iperstatica

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$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (3/2\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (3 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [3\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (3\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 1/2\ Fb^3/EJ$

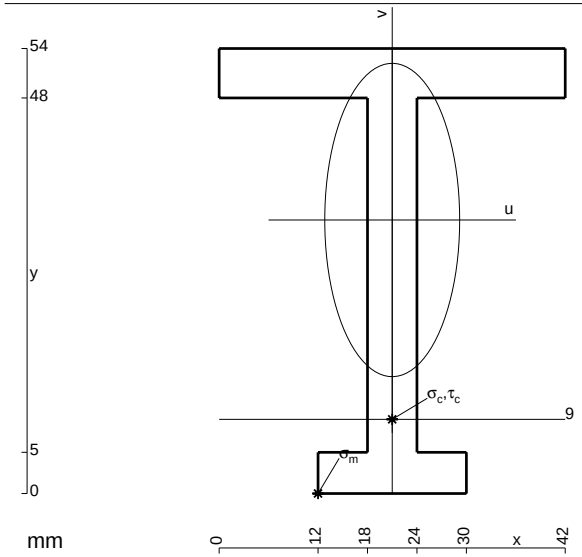
$L^{Xo}_{EC} = \int_o^b (5 - 11\, x/b + 6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5\, x - 11/2\, x^2/b + 2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (5\, b - 11/2\, b + 2\, b)\, Fb^2\, 1/EJ = 3/2\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (- x/b + 6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b + 2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b + 2\, b)\, Fb^2\, 1/EJ = 3/2\ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 210204 ESΣ07.xxxx.145



A = 600. mm^2
J_u = 216946. mm^4
J_v = 40248. mm^4
y_g = 33.19 mm
T_y = 1600. N
M_x = -1296000. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -33.19 mm
$\sigma_m = -Mv/J_u$ = -198.3 N/mm^2
x_c = 21. mm
y_c = 9. mm
v_c = -24.19 mm
$\sigma_c = -Mv/J_u$ = -144.5 N/mm^2
τ_c = 4.168 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 144.7 N/mm^2
S^* = 3391. mm^3

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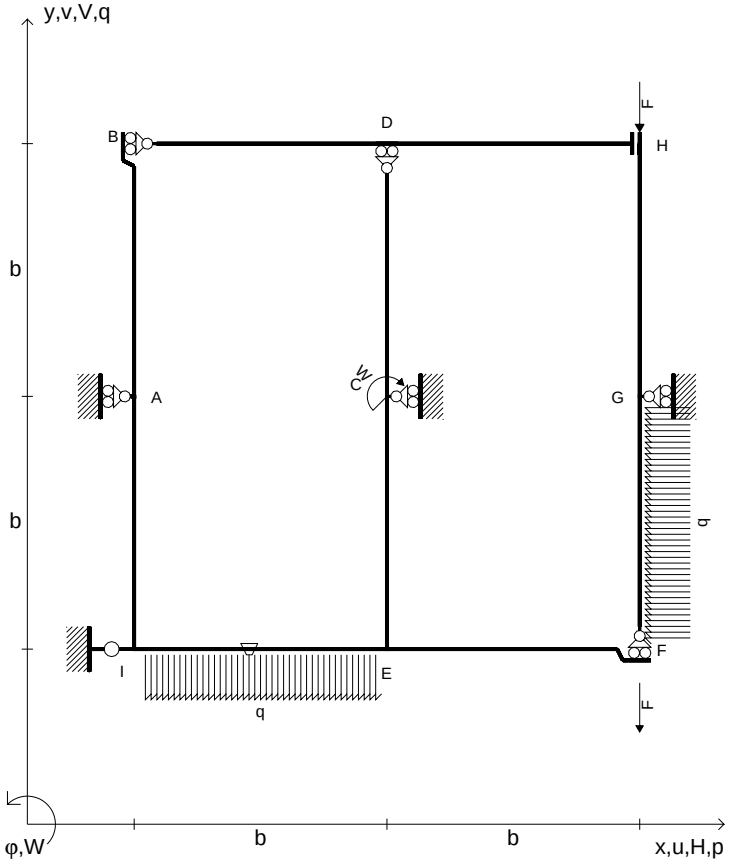
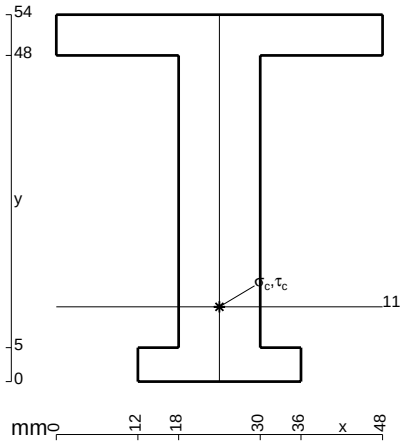
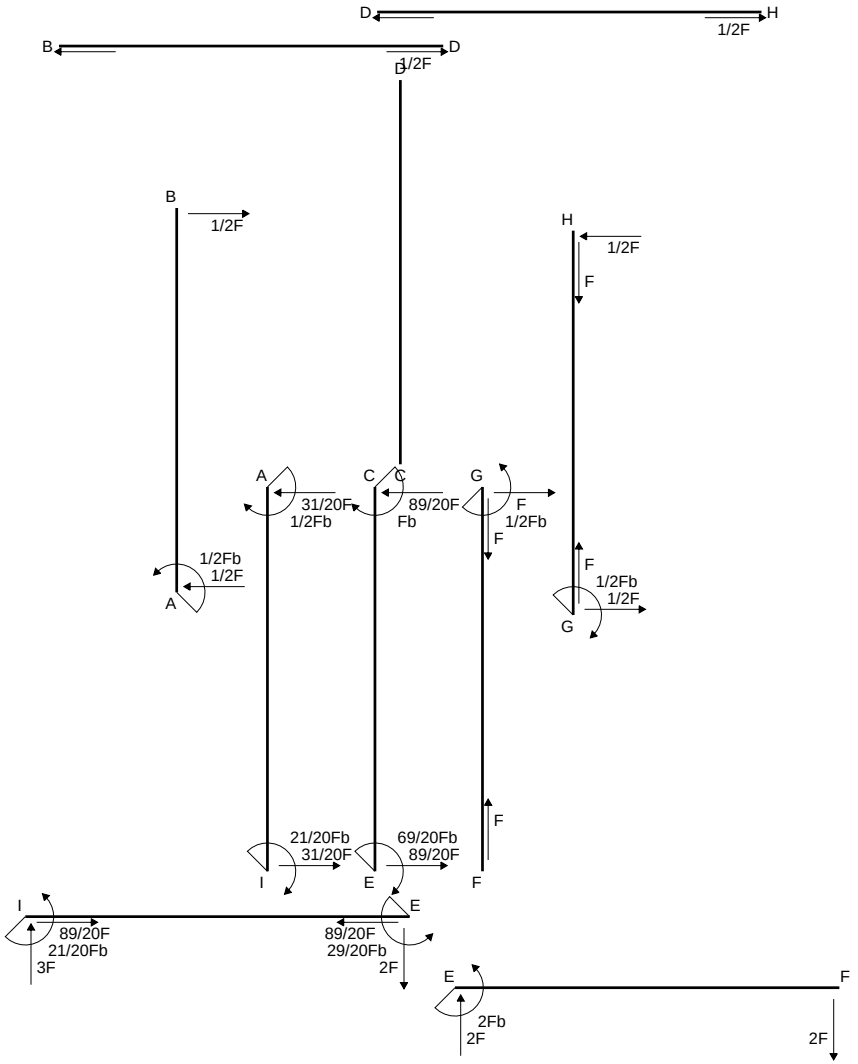
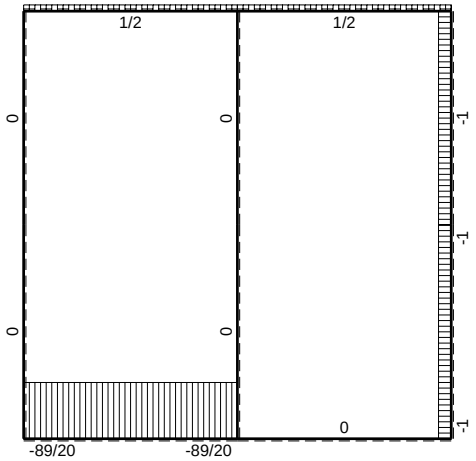
$V_{HG} = -F$
$V_{FE} = -F$
$W_C = -W = -Fb$
$p_{FG} = -q = -F/b$
$q_{IE} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



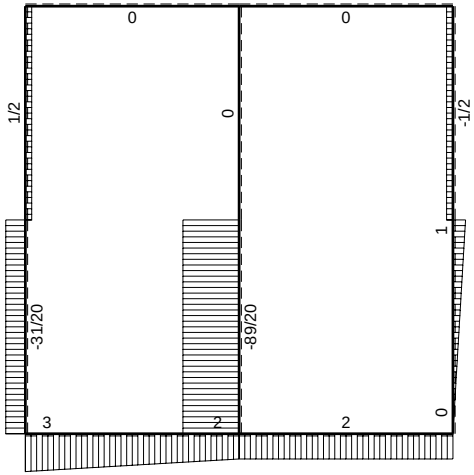


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 910 mm, F = 1180 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

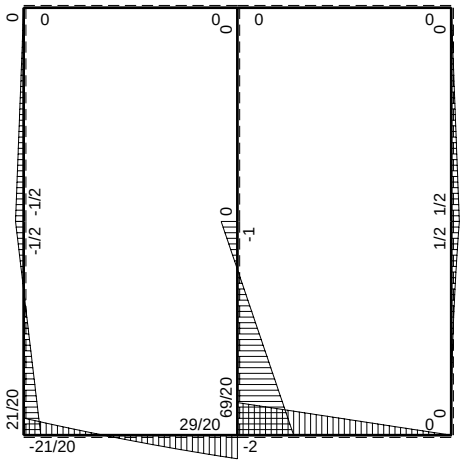




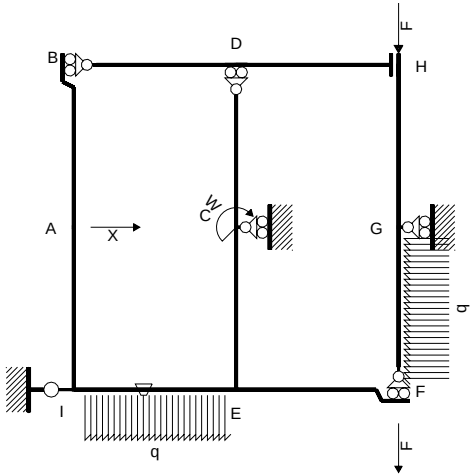
←[+]→ F



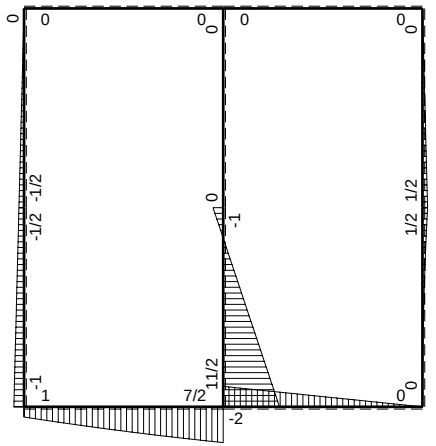
↑[+]↓ F



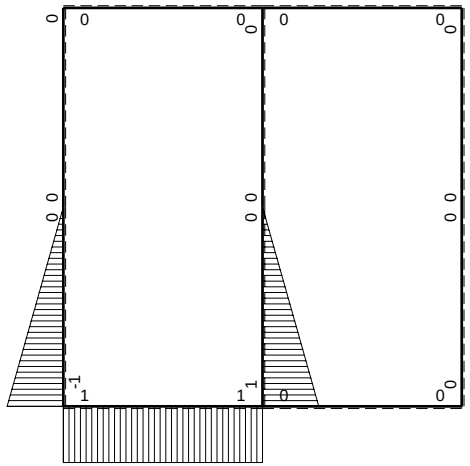
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981630 ESΣ07.xxxx.147

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fx	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	$Fb+3Fx-1/2qx^2$	$-Fb/EJ$	$Fb^2+3Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2	$(7/3-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	$-7/2Fb+2Fx+1/2qx^2$	Fb/EJ	$7/2Fb^2-2Fbx-1/2Fx^2$	$-Fb^2/EJ$	b^2		
EC b	b-x	11/2Fb-13/2Fx	0	$11/2Fb^2-12Fbx+13/2Fx^2$	0	$b^2-2bx+x^2$	$(5/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	Fb-13/2Fx	0	$-Fbx+13/2Fx^2$	0	x^2		
IA b	-b+x	-Fb+1/2Fx	0	$Fb^2-3/2Fbx+1/2Fx^2$	0	$b^2-2bx+x^2$	$(5/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	1/2Fb+1/2Fx	0	$1/2Fbx+1/2Fx^2$	0	x^2		
	totali						$41/12Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-41/20F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (1 + 3\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx$

$= [\, x + 3/2\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, - x \,]_o^b\, \theta$

$= (\, b + 3/2\, b - 1/6\, b \,)\, Fb^2\, 1/EJ + (\, - b \,)\, \theta = 4/3\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (7/2 - 2\, x/b - 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx$

$= [7/2\, x - x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (7/2\, b - b - 1/6\, b \,)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 4/3\ Fb^3/EJ$

$L^{Xo}_{EC} = \int_o^b (11/2 - 12\, x/b + 13/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [11/2\, x - 6\, x^2/b + 13/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (11/2\, b - 6\, b + 13/6\, b \,)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (- x/b + 13/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b + 13/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

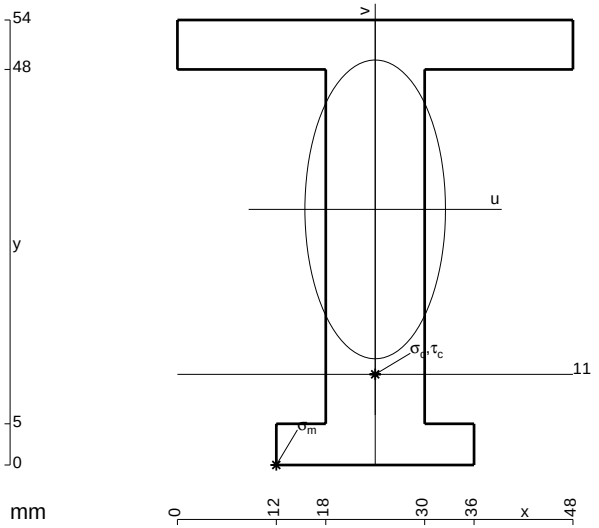
$= (-1/2\, b + 13/6\, b \,)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

$L^{Xo}_{IA} = \int_o^b (1 - 3/2\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [\, x - 3/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (\, b - 3/4\, b + 1/6\, b \,)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

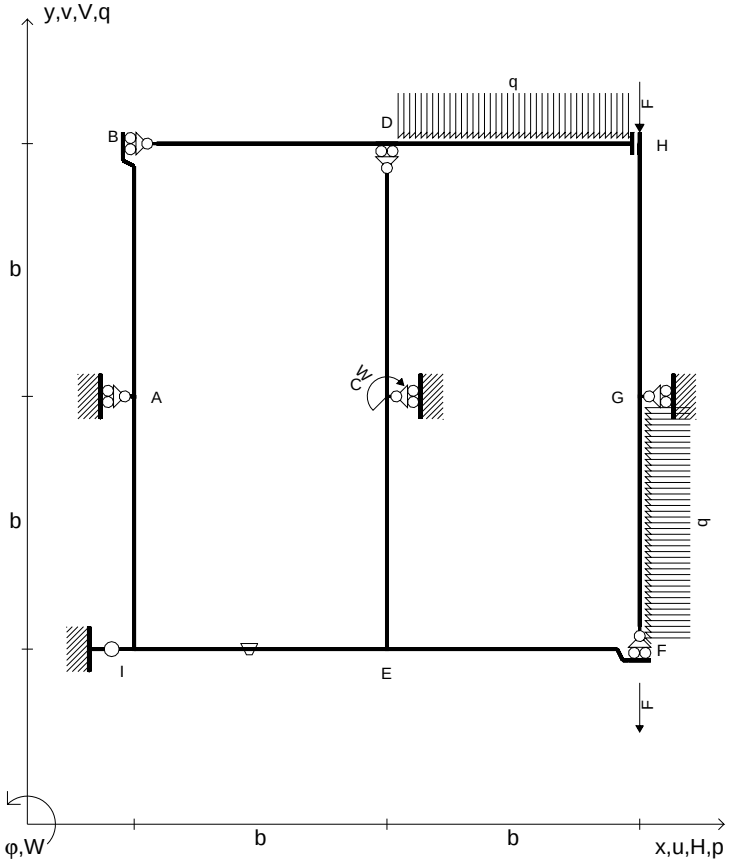
$L^{Xo}_{AI} = \int_o^b (1/2\, x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

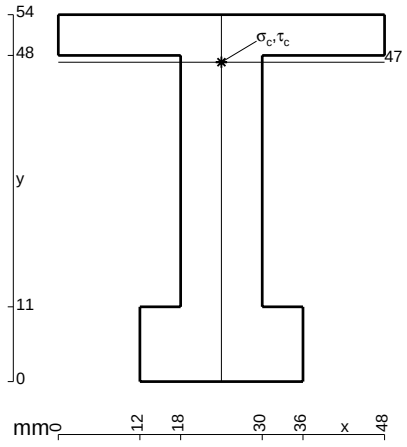
$= (1/4\, b + 1/6\, b \,)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$



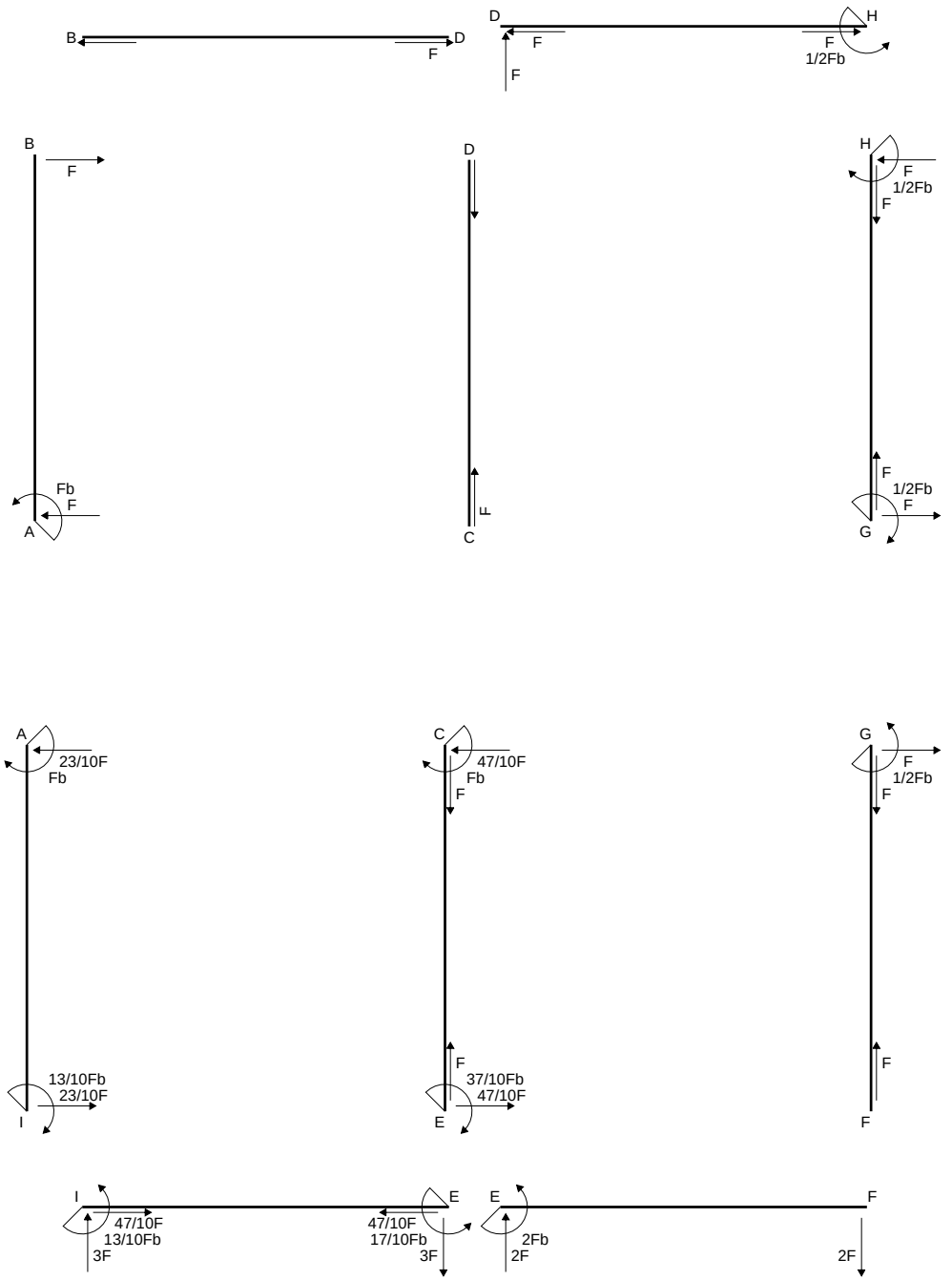
A = 924. mm^2
J_u = 303740. mm^4
J_v = 67248. mm^4
y_g = 31.02 mm
T_y = 2360. N
M_x = -2147600. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -31.02 mm
$\sigma_m = -Mv/J_u$ = -219.3 N/mm^2
x_c = 24. mm
y_c = 11. mm
v_c = -20.02 mm
$\sigma_c = -Mv/J_u$ = -141.5 N/mm^2
τ_c = 3.289 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 141.7 N/mm^2
S^* = 5080. mm^3

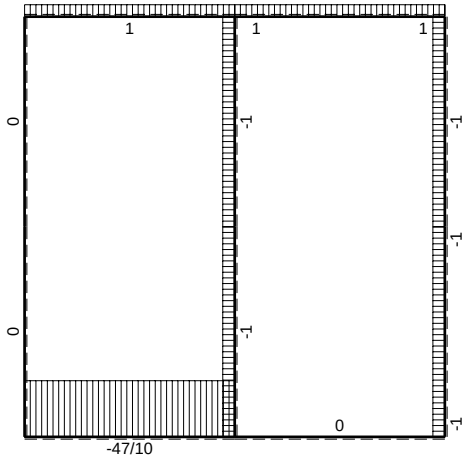
$V_{HG} = -F$
$V_{FE} = -F$
$W_C = -W = -Fb$
$p_{FG} = -q = -F/b$
$q_{HD} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



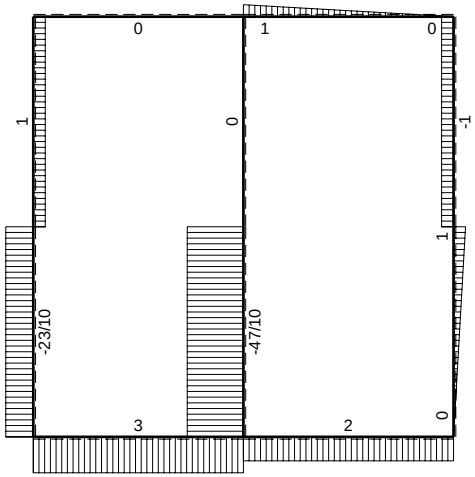


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 590 mm, F = 2050 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

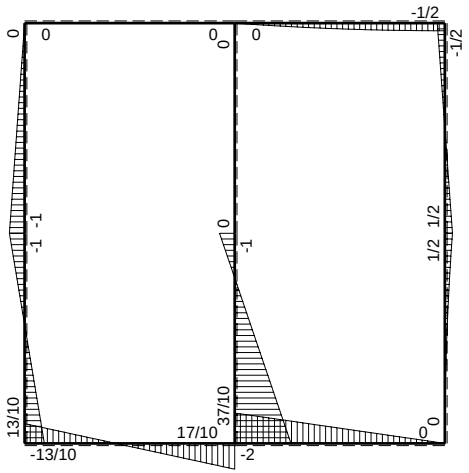




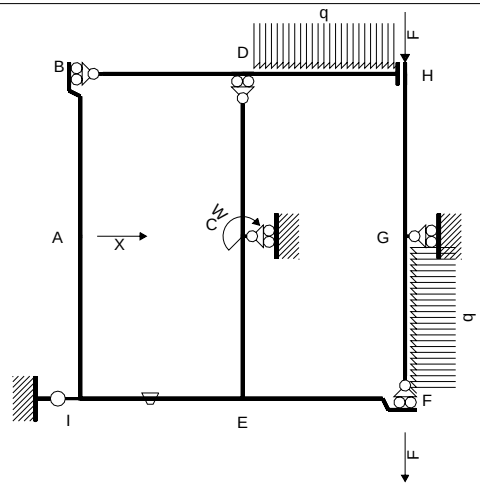
←□→ F



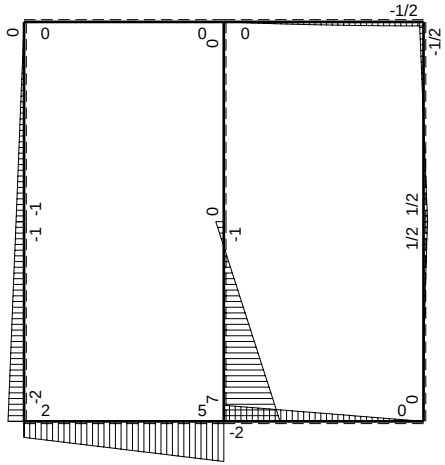
↑□↓ F



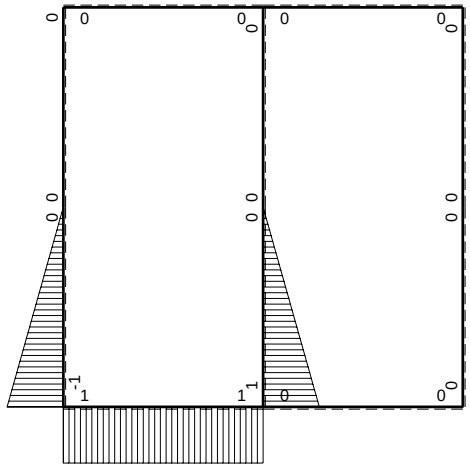
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fb+Fx	0	0	0	0	0+0	0
BA b	0	Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	1/2Fb-Fx	0	0	0	0	0+0	0
HG b	0	1/2Fb-Fx	0	0	0	0		
HD b	0	$-1/2Fb+1/2qx^2$	0	0	0	0	0+0	0
DH b	0	$Fx-1/2qx^2$	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	2Fb+3Fx	-Fb/EJ	$2Fb^2+3Fbx$	$-Fb^2/EJ$	b^2	$(7/2-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	-5Fb+3Fx	Fb/EJ	$5Fb^2-3Fbx$	$-Fb^2/EJ$	b^2		
EC b	b-x	7Fb-8Fx	0	$7Fb^2-15Fbx+8Fx^2$	0	$b^2-2bx+x^2$	$(13/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	Fb-8Fx	0	$-Fbx+8Fx^2$	0	x^2		
IA b	-b+x	-2Fb+Fx	0	$2Fb^2-3Fbx+Fx^2$	0	$b^2-2bx+x^2$	$(5/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	Fb+Fx	0	$Fbx+Fx^2$	0	x^2		
	totali						$11/2Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-33/10F	

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Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (2 + 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx = [2\, x + 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (2\, b + 3/2\, b)\, Fb^2\, 1/EJ + (- b)\, \theta = 5/2\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (5 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx = [5\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (5\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 5/2\ Fb^3/EJ$

$L^{Xo}_{EC} = \int_o^b (7 - 15\, x/b + 8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [7\, x - 15/2\, x^2/b + 8/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (7\, b - 15/2\, b + 8/3\, b)\, Fb^2\, 1/EJ = 13/6\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (- x/b + 8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b + 8/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

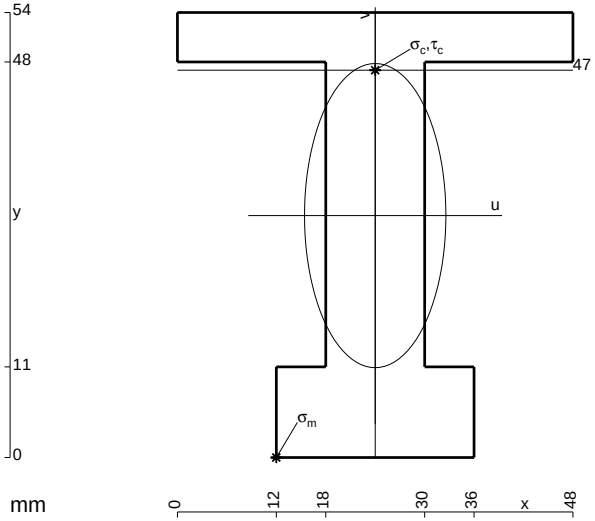
$= (-1/2\, b + 8/3\, b)\, Fb^2\, 1/EJ = 13/6\ Fb^3/EJ$

$L^{Xo}_{IA} = \int_o^b (2 - 3\, x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx = [2\, x - 3/2\, x^2/b + 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (2\, b - 3/2\, b + 1/3\, b)\, Fb^2\, 1/EJ = 5/6\ Fb^3/EJ$

$L^{Xo}_{AI} = \int_o^b (\, x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b + 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b + 1/3\, b)\, Fb^2\, 1/EJ = 5/6\ Fb^3/EJ$



$A = 996.\ mm^2$

$J_u = 339350.\ mm^4$

$J_v = 73296.\ mm^4$

$y_g = 29.36\ mm$

$T_y = 4100.\ N$

$M_x = -2419000.\ Nmm$

$x_m = 12.\ mm$

$u_m = -12.\ mm$

$v_m = -29.36\ mm$

$\sigma_m = -Mv/J_u = -209.3\ N/mm^2$

$x_c = 24.\ mm$

$y_c = 47.\ mm$

$v_c = 17.64\ mm$

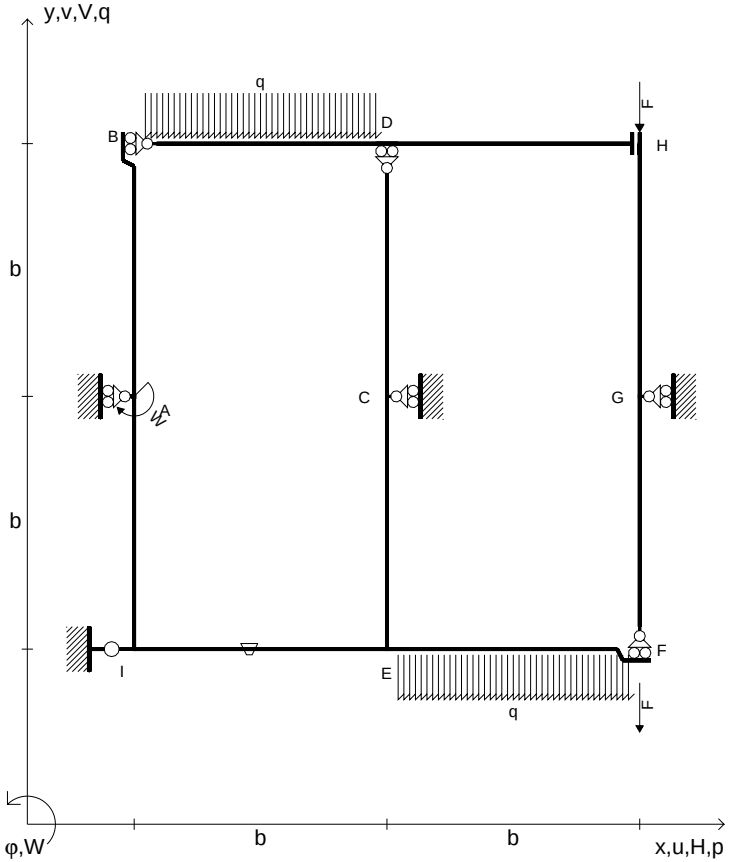
$\sigma_c = -Mv/J_u = 125.8\ N/mm^2$

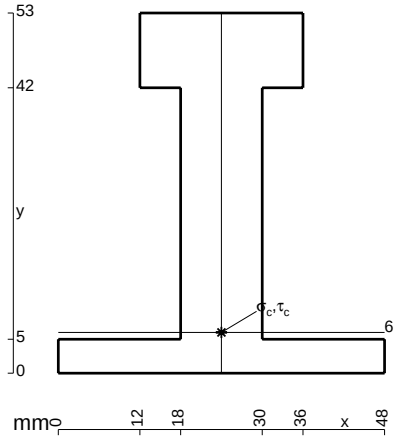
$\tau_c = 6.495\ N/mm^2$

$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 126.3\ N/mm^2$

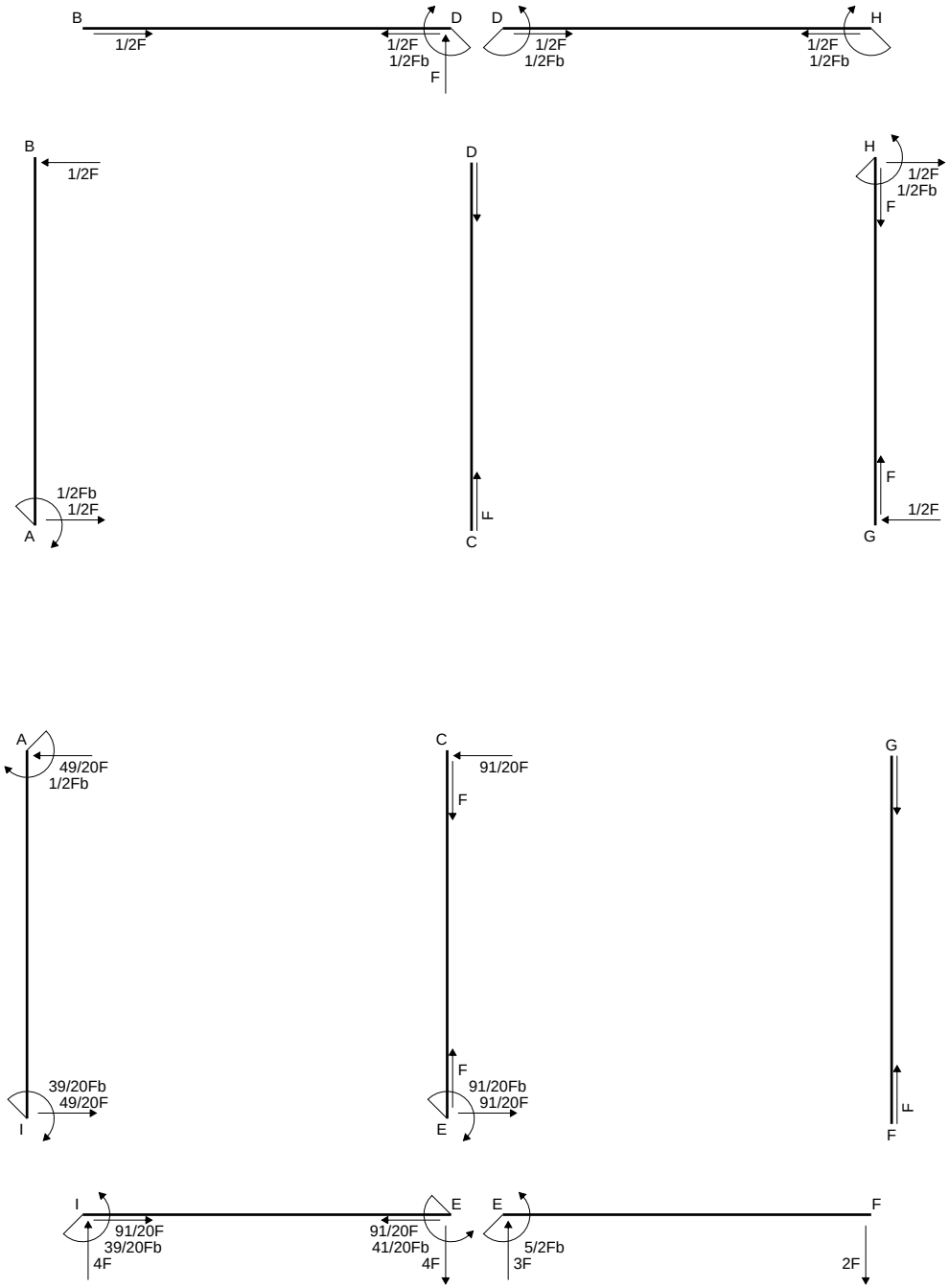
$S^* = 6451.\ mm^3$

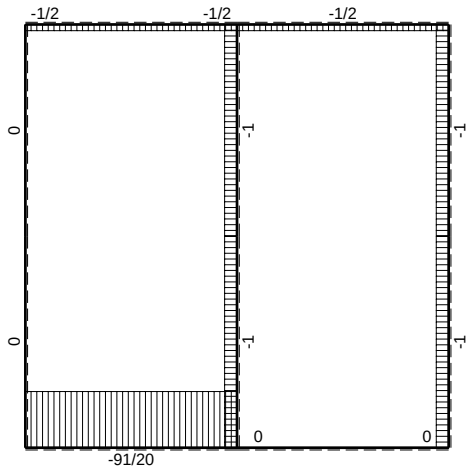
$V_{HG} = -F$
$V_{FE} = -F$
$W_A = -W = -Fb$
$q_{EF} = -q = -F/b$
$q_{DB} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$

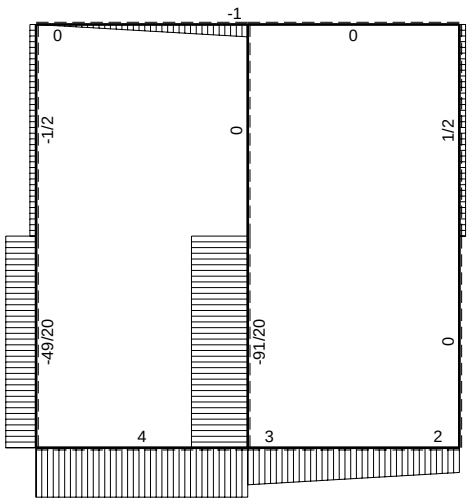




Reazioni iperstatiche in soluzione: $X=W_{IE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 740 mm, F = 1420 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

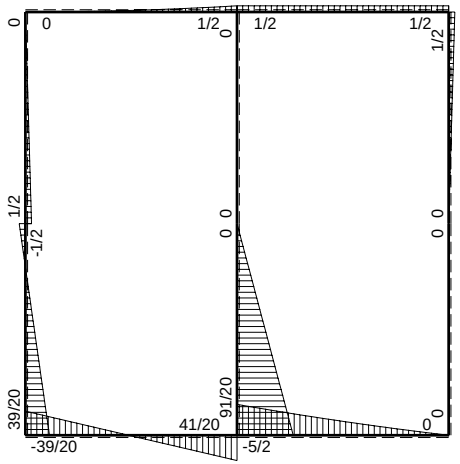




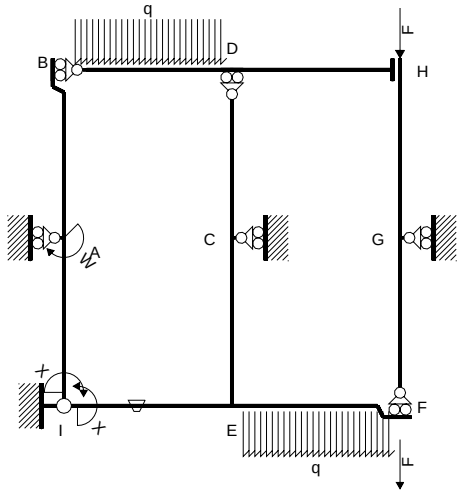


←□→ F

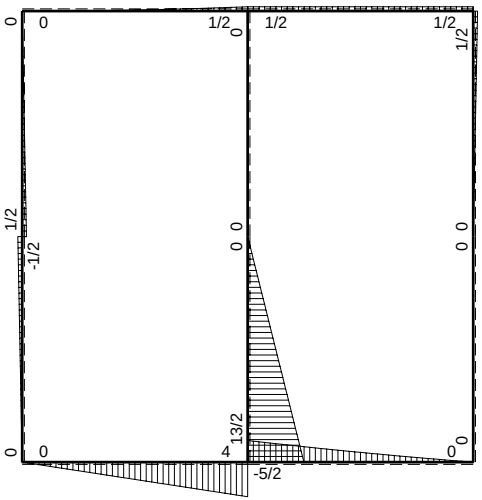
↑□↓ F



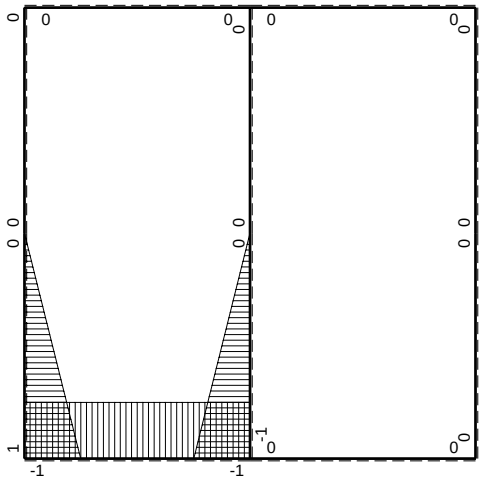
(□) Fb



Schema di calcolo iperstatico



(□) M_o flessione da carichi assegnati



(□) M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{IE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-5/2Fb+3Fx-1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$2Fx+1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fb+1/2Fx	0	0	0	0		
HD b	0	1/2Fb	0	0	0	0	0+0	0
DH b	0	-1/2Fb	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	-1	4Fx	-Fb/EJ	-4Fx	Fb/EJ	1	$(-2+1)Fb^2/EJ$	Xb/EJ
EI b	1	-4Fb+4Fx	Fb/EJ	-4Fb+4Fx	Fb/EJ	1		
EC b	-1+x/b	13/2Fb-13/2Fx	0	$-13/2Fb+13Fx-13/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-13/6+0)Fb^2/EJ$	1/3Xb/EJ
CE b	x/b	-13/2Fx	0	$-13/2Fx^2/b$	0	x^2/b^2		
IA b	1-x/b	-1/2Fx	0	$-1/2Fx+1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/3Xb/EJ
AI b	-x/b	1/2Fb-1/2Fx	0	$-1/2Fx+1/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{IE}$						39/20Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{IE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{EI}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CE}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{AI}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IE}^{Xo} = \int_o^b (-4\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L_{EI}^{Xo} = \int_o^b (-4 + 4\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-4\ x + 2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-4\ b + 2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -\ Fb^2/EJ$

$L_{EC}^{Xo} = \int_o^b (-13/2 + 13\ x/b - 13/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-13/2\ x + 13/2\ x^2/b - 13/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-13/2\ b + 13/2\ b - 13/6\ b\ )\ Fb\ 1/EJ\ = -13/6\ Fb^2/EJ$

$L_{CE}^{Xo} = \int_o^b (-13/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-13/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-13/6\ b\ )\ Fb\ 1/EJ\ = -13/6\ Fb^2/EJ$

$L_{IA}^{Xo} = \int_o^b (-1/2\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

$L_{AI}^{Xo} = \int_o^b (-1/2\ x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ Fb^2/EJ$

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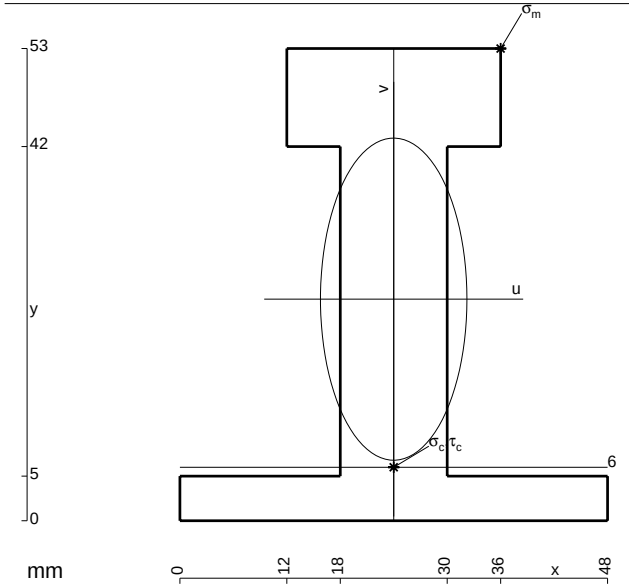
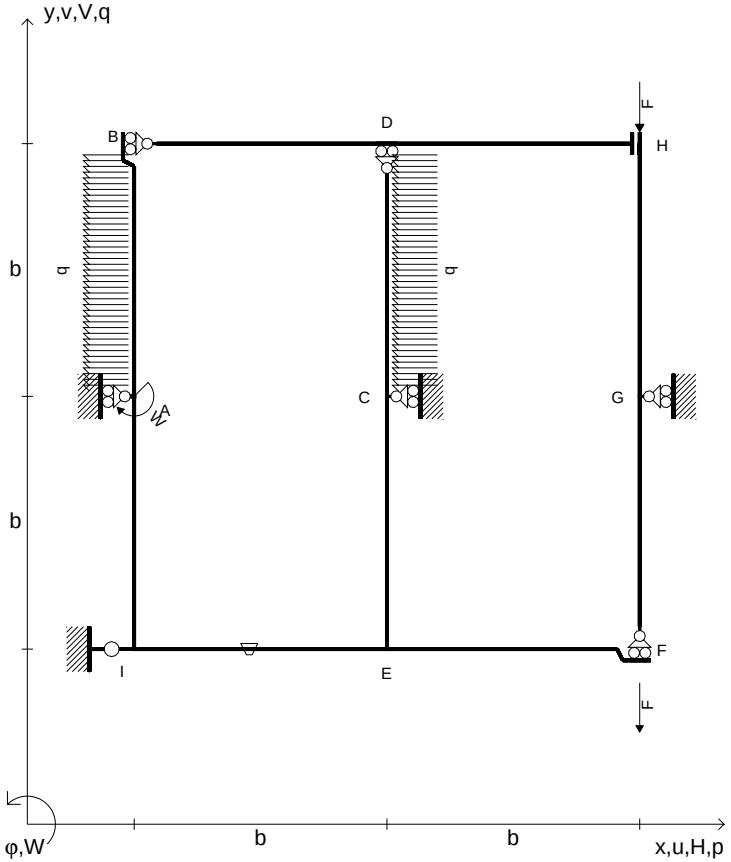
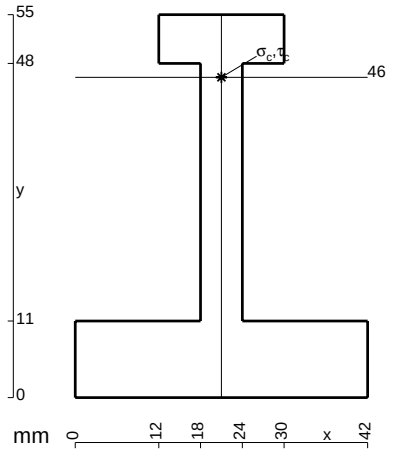


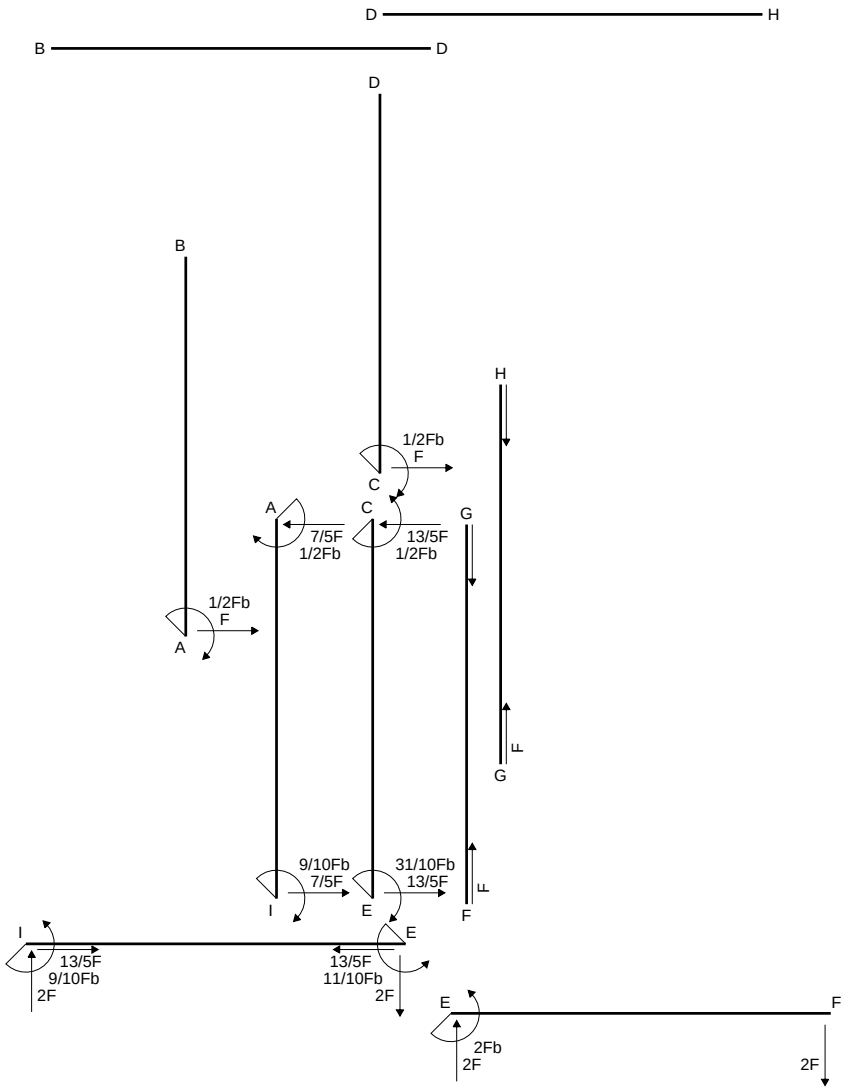
A = 948. mm^2
J_u = 309947. mm^4
J_v = 64080. mm^4
y_g = 24.87 mm
T_y = 4260. N
M_x = -2627000. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 12. mm
v_m = 28.13 mm
$\sigma_m = -Mv/J_u$ = 238.4 N/mm^2
x_c = 24. mm
y_c = 6. mm
v_c = -18.87 mm
$\sigma_c = -Mv/J_u$ = -159.9 N/mm^2
τ_c = 6.415 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 160.3 N/mm^2
S^* = 5601. mm^3

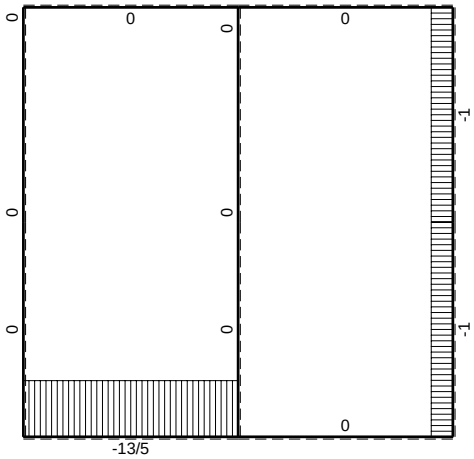
$V_{HG} = -F$
$V_{FE} = -F$
$W_A = -W = -Fb$
$p_{CD} = -q = -F/b$
$p_{AB} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



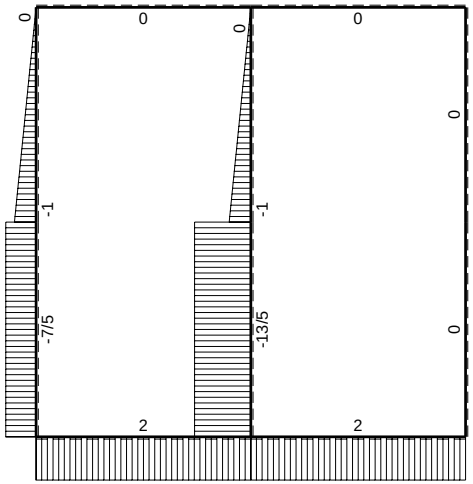


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 1040 mm, F = 870 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

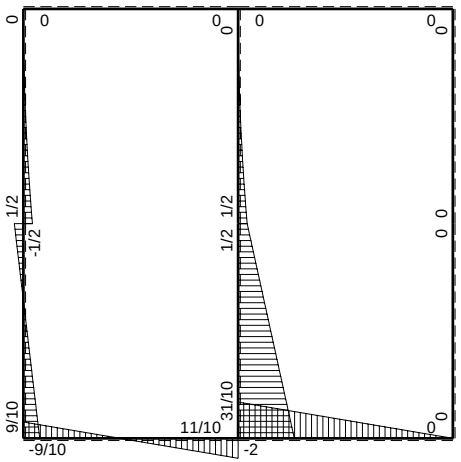




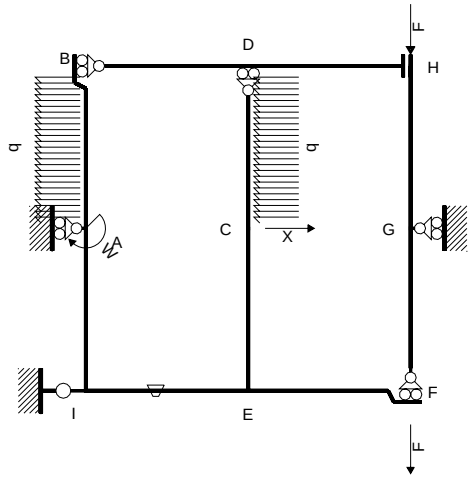
←□→ F



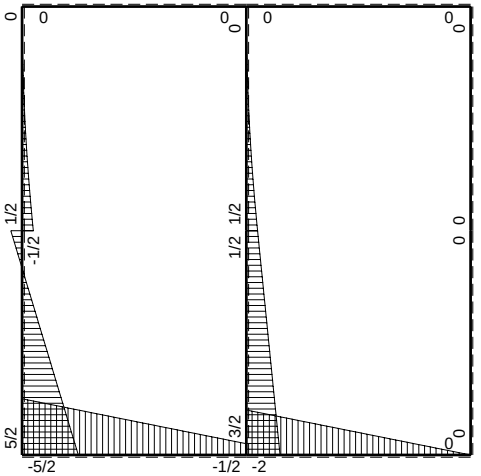
↑□↓ F



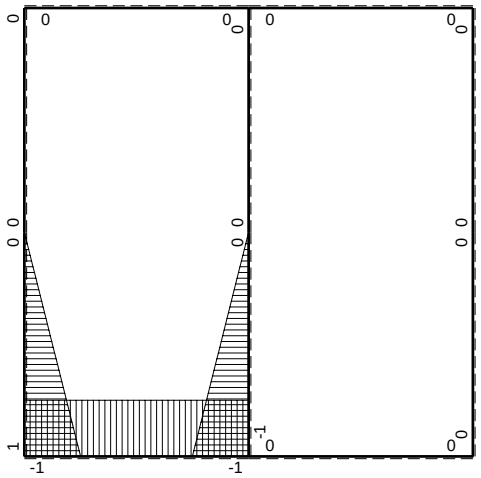
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2qx^2$	0	0	0	0		
CD b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2qx^2$	0	0	0	0		
EF b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
FE b	0	$2Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	0	0	0	0	0	0+0	0
HG b	0	0	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-b	$-5/2Fb+2Fx$	$-Fb/EJ$	$5/2Fb^2-2Fbx$	Fb^2/EJ	b^2	$(3/2+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$1/2Fb+2Fx$	Fb/EJ	$1/2Fb^2+2Fbx$	Fb^2/EJ	b^2		
EC b	-b+x	$3/2Fb-Fx$	0	$-3/2Fb^2+5/2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-7/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	x	$-1/2Fb-Fx$	0	$-1/2Fbx-Fx^2$	0	x^2		
IA b	b-x	$5/2Fb-3Fx$	0	$5/2Fb^2-11/2Fbx+3Fx^2$	0	$b^2-2bx+x^2$	$(3/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	$1/2Fb-3Fx$	0	$-1/2Fbx+3Fx^2$	0	x^2		
	totali						$8/3Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						$-8/5F$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212317 ESΣ07.xxxx.189
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (5/2 - 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx = [5/2\, x - x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (5/2\, b - b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 5/2\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (1/2 + 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx = [1/2\, x + x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (1/2\, b + b)\, Fb^2\, 1/EJ + (- b)\, \theta = 5/2\ Fb^3/EJ$

$L_{EC}^{Xo} = \int_o^b (-3/2 + 5/2\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-3/2\, x + 5/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 5/4\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L_{CE}^{Xo} = \int_o^b (-1/2\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

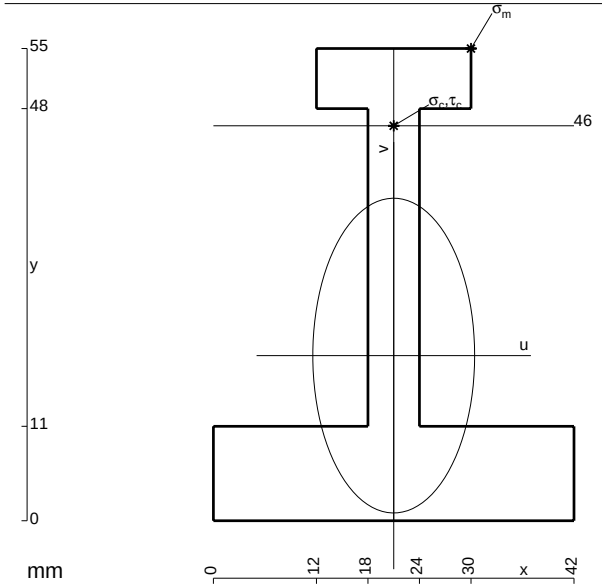
$= (-1/4\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L_{IA}^{Xo} = \int_o^b (5/2 - 11/2\, x/b + 3\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [5/2\, x - 11/4\, x^2/b + x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (5/2\, b - 11/4\, b + b \,)\, Fb^2\, 1/EJ = 3/4\ Fb^3/EJ$

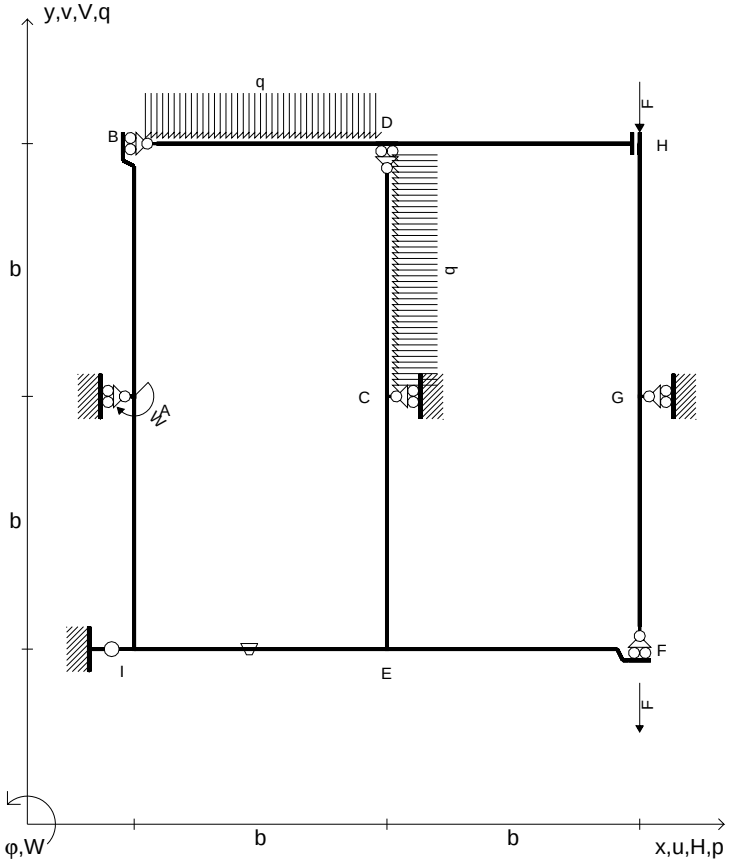
$L_{AI}^{Xo} = \int_o^b (-1/2\, x/b + 3\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b + x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

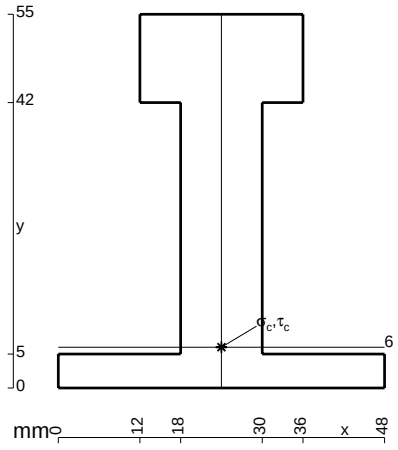
$= (-1/4\, b + b \,)\, Fb^2\, 1/EJ = 3/4\ Fb^3/EJ$



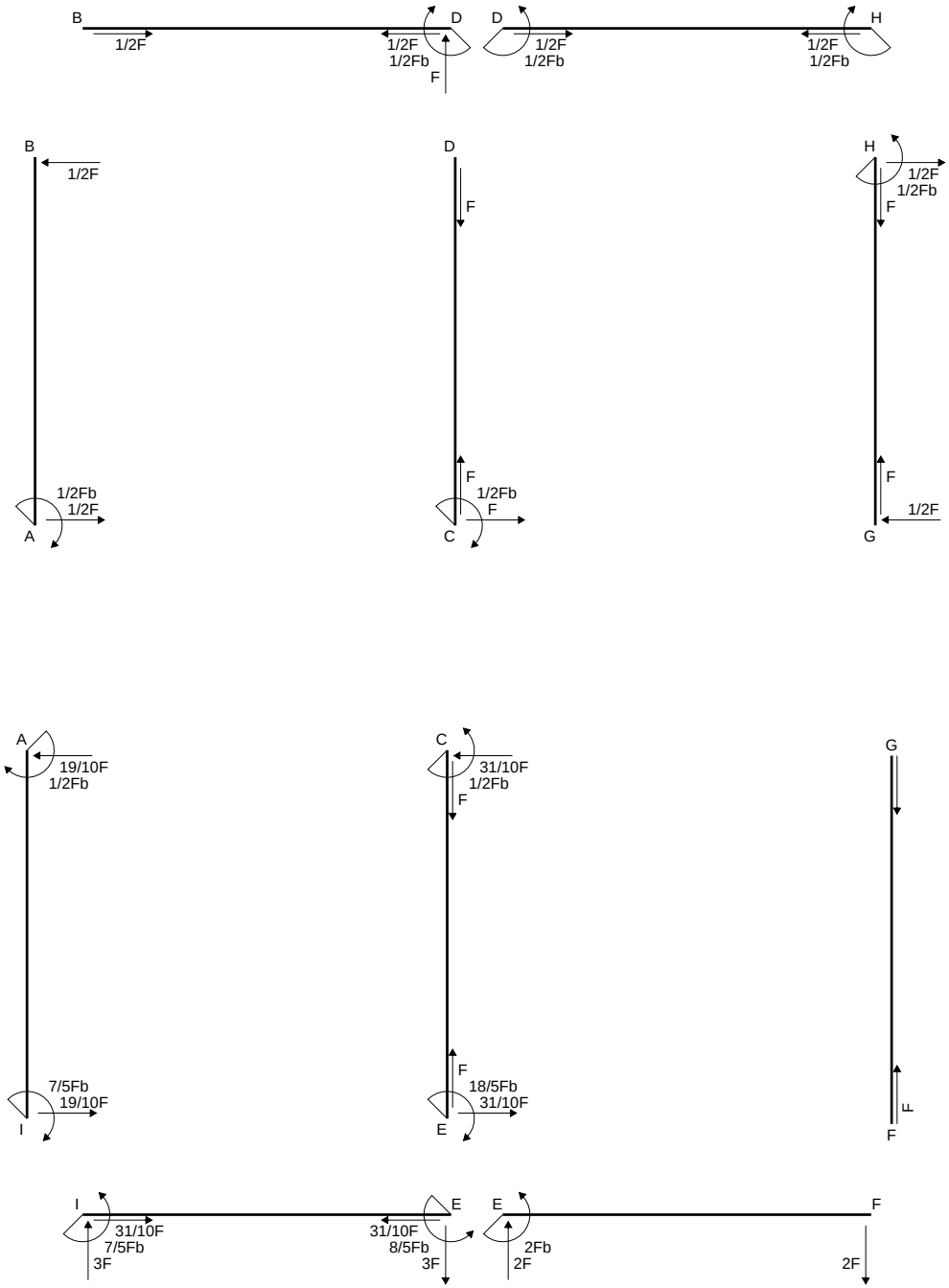
A = 810. mm^2
J_u = 272218. mm^4
J_v = 71982. mm^4
y_g = 19.23 mm
T_y = 1740. N
M_x = -1809600. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 9. mm
v_m = 35.77 mm
$\sigma_m = -Mv/J_u$ = 237.8 N/mm^2
x_c = 21. mm
y_c = 46. mm
v_c = 26.77 mm
$\sigma_c = -Mv/J_u$ = 177.9 N/mm^2
τ_c = 4.686 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 178.1 N/mm^2
S^* = 4399. mm^3

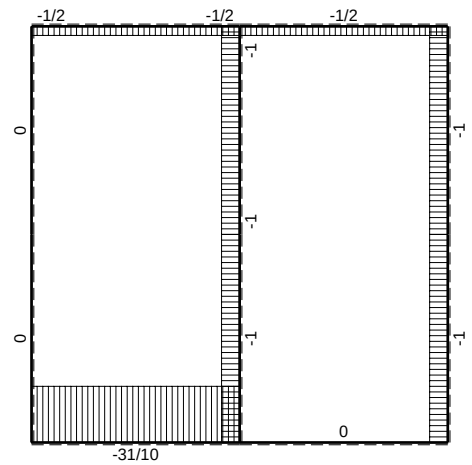
$V_{HG} = -F$
$V_{FE} = -F$
$W_A = -W = -Fb$
$p_{CD} = -q = -F/b$
$q_{DB} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



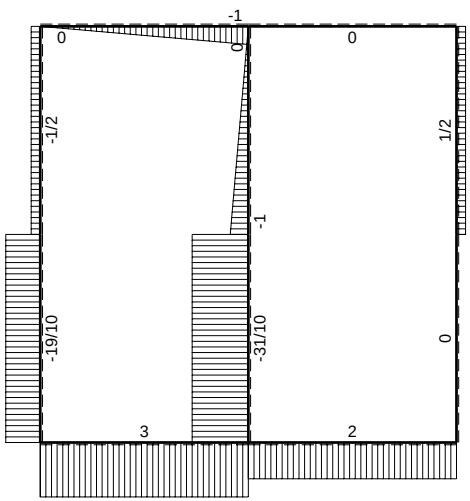


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 550 mm, F = 2200 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

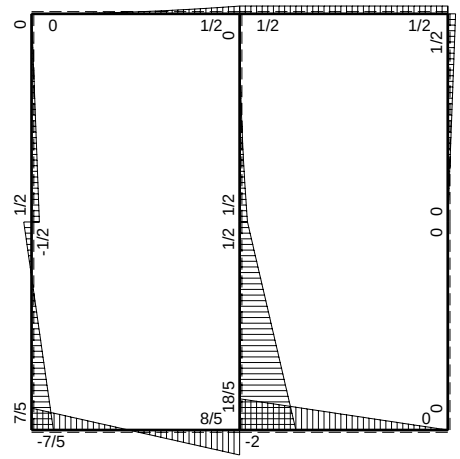




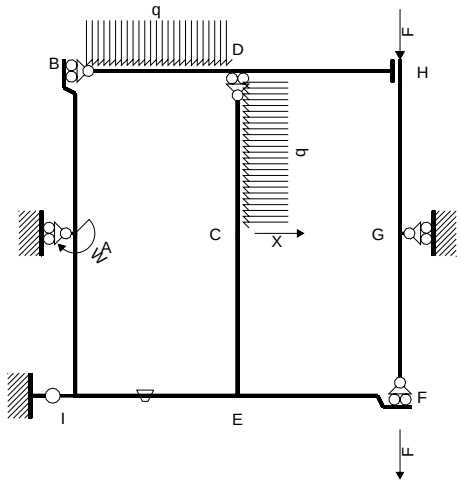
←[+]→ F



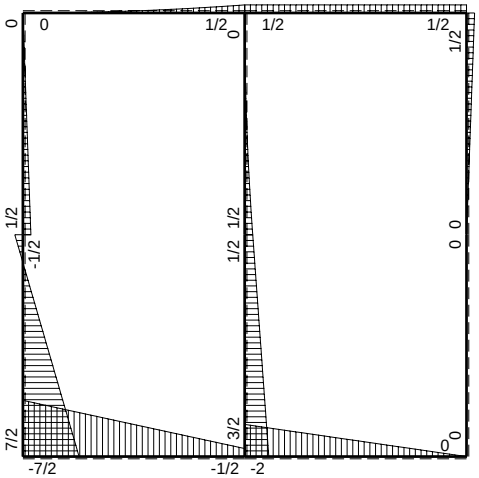
↑[+]↓ F



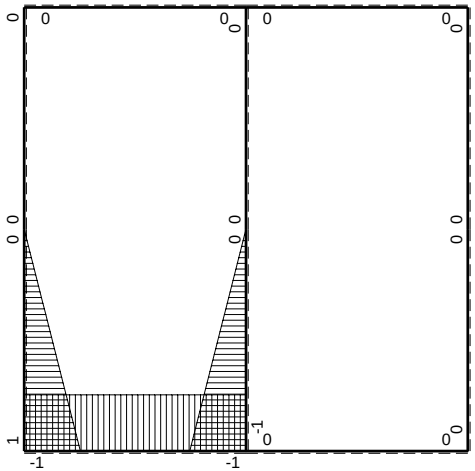
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2qx^2$	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fb+1/2Fx	0	0	0	0		
HD b	0	1/2Fb	0	0	0	0	0+0	0
DH b	0	-1/2Fb	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	-b	$-7/2Fb+3Fx$	-Fb/EJ	$7/2Fb^2-3Fbx$	Fb^2/EJ	b^2	$(2+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$1/2Fb+3Fx$	Fb/EJ	$1/2Fb^2+3Fbx$	Fb^2/EJ	b^2		
EC b	-b+x	$3/2Fb-Fx$	0	$-3/2Fb^2+5/2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-7/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	x	$-1/2Fb-Fx$	0	$-1/2Fbx-Fx^2$	0	x^2		
IA b	b-x	7/2Fb-4Fx	0	$7/2Fb^2-15/2Fbx+4Fx^2$	0	$b^2-2bx+x^2$	$(13/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	1/2Fb-4Fx	0	$-1/2Fbx+4Fx^2$	0	x^2		
	totali						$7/2Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-21/10F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211486 ESΣ07.xxxx.190
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (7/2 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [7/2\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (7/2\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 3\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (1/2 + 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [1/2\, x + 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [-\, x \,]_o^b\, \theta$

$= (1/2\, b + 3/2\, b)\, Fb^2\, 1/EJ + (-\, b)\, \theta = 3\ Fb^3/EJ$

$L_{EC}^{Xo} = \int_o^b (-3/2 + 5/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/2\, x + 5/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 5/4\, b - 1/3\, b)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L_{CE}^{Xo} = \int_o^b (-1/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/4\, b - 1/3\, b)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

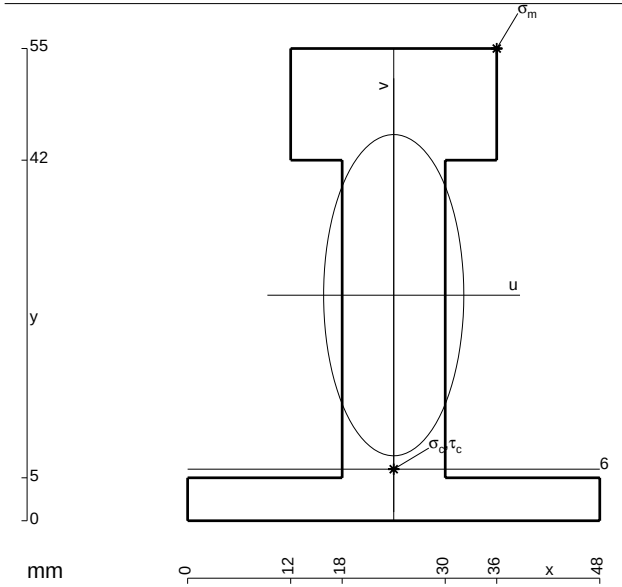
$L_{IA}^{Xo} = \int_o^b (7/2 - 15/2\, x/b + 4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [7/2\, x - 15/4\, x^2/b + 4/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (7/2\, b - 15/4\, b + 4/3\, b)\, Fb^2\, 1/EJ = 13/12\ Fb^3/EJ$

$L_{AI}^{Xo} = \int_o^b (-1/2\, x/b + 4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b + 4/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/4\, b + 4/3\, b)\, Fb^2\, 1/EJ = 13/12\ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 211486 ESΣ07.xxxx.190



A = 996. mm^2
J_u = 348739. mm^4
J_v = 66384. mm^4
y_g = 26.27 mm
T_y = 4400. N
M_x = -2420000. Nmm
x_m = 36. mm
y_m = 55. mm
u_m = 12. mm
v_m = 28.73 mm
$\sigma_m = -Mv/J_u$ = 199.4 N/mm^2
x_c = 24. mm
y_c = 6. mm
v_c = -20.27 mm
$\sigma_c = -Mv/J_u$ = -140.7 N/mm^2
τ_c = 6.26 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 141.1 N/mm^2
S^* = 5954. mm^3

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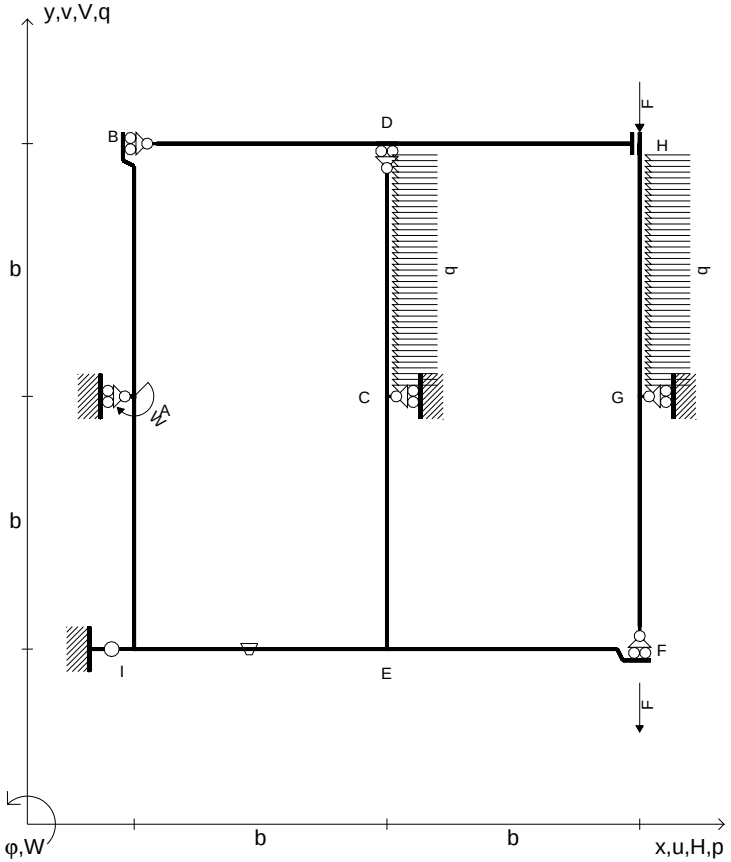
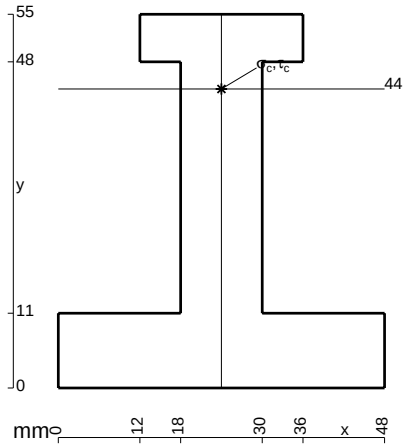
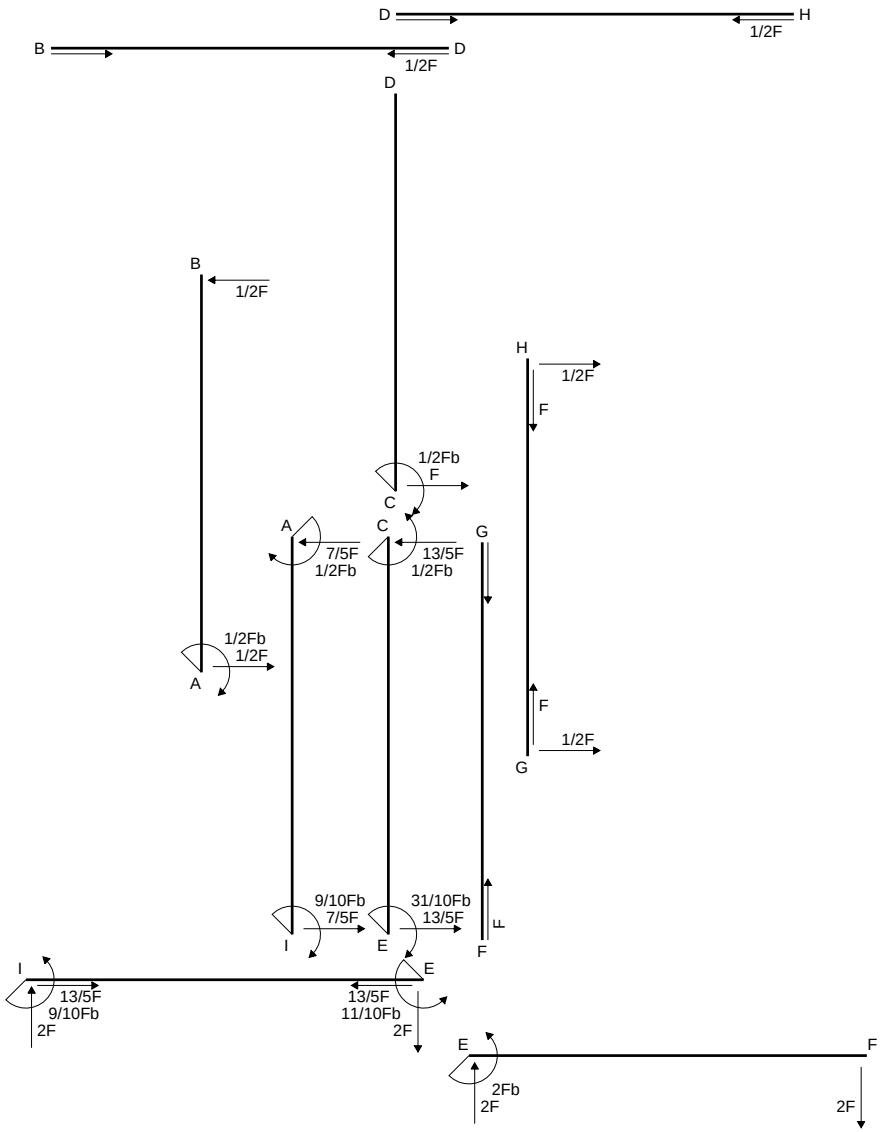
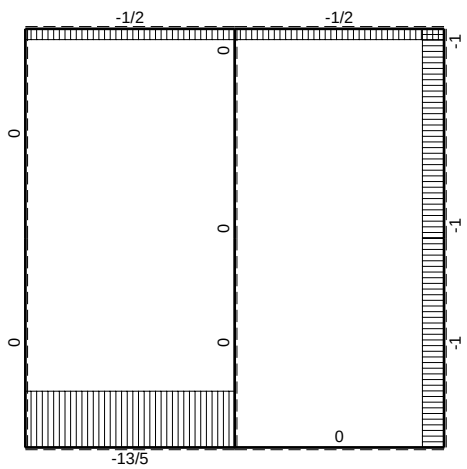
$V_{HG} = -F$
$V_{FE} = -F$
$W_A = -W = -Fb$
$p_{CD} = -q = -F/b$
$p_{GH} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



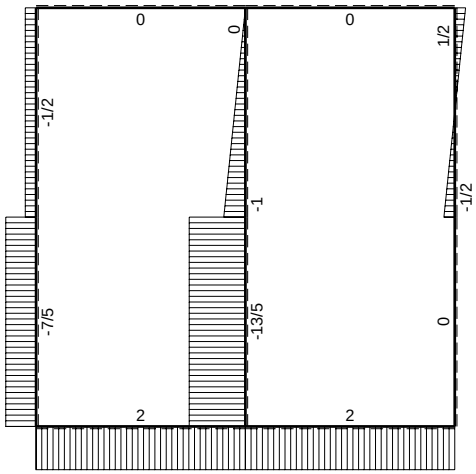


Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 600 mm, F = 1940 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

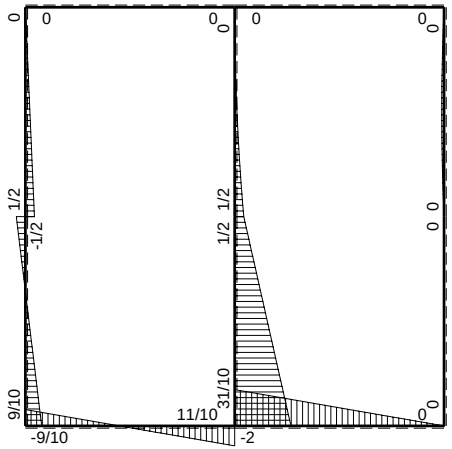




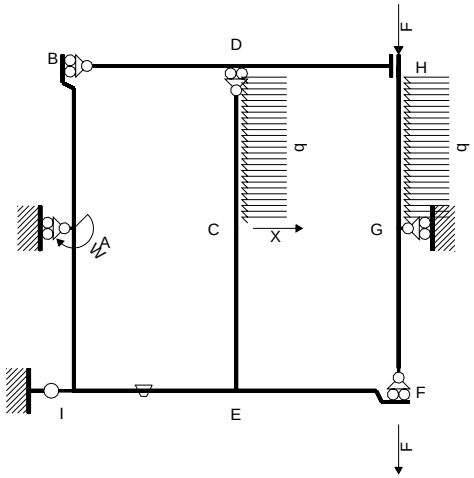
←F→



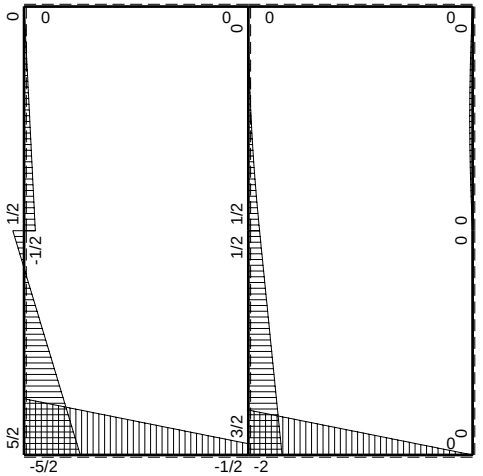
↑F↓



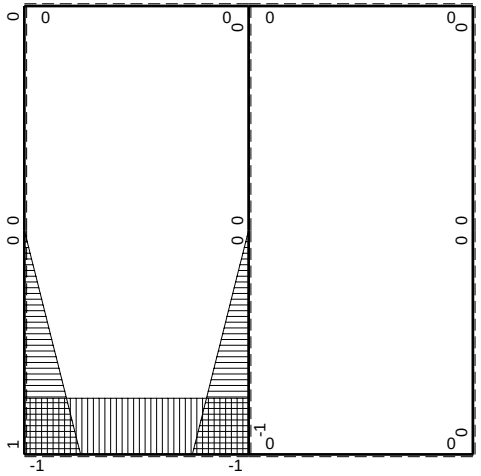
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2qx^2$	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
HD b	0	0	0	0	0	0	0+0	0
DH b	0	0	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-b	$-5/2Fb+2Fx$	-Fb/EJ	$5/2Fb^2-2Fbx$	Fb^2/EJ	b^2	$(3/2+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$1/2Fb+2Fx$	Fb/EJ	$1/2Fb^2+2Fbx$	Fb^2/EJ	b^2		
EC b	-b+x	3/2Fb-Fx	0	$-3/2Fb^2+5/2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-7/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	x	-1/2Fb-Fx	0	$-1/2Fbx-Fx^2$	0	x^2		
IA b	b-x	5/2Fb-3Fx	0	$5/2Fb^2-11/2Fbx+3Fx^2$	0	$b^2-2bx+x^2$	$(3/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	1/2Fb-3Fx	0	$-1/2Fbx+3Fx^2$	0	x^2		
	totali						$8/3Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_C$						-8/5F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 981317 ESΣ07.xxxx.191
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$L_{IE}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EI}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{EC}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CE}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IA}^{XX} = \int_o^b (1 - 2\, x/b + x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AI}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IE}^{Xo} = \int_o^b (5/2 - 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\, dx = [5/2\, x - x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (5/2\, b - b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 5/2\ Fb^3/EJ$

$L_{EI}^{Xo} = \int_o^b (1/2 + 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\, dx = [1/2\, x + x^2/b \,]_o^b\, Fb^2\, 1/EJ + [-\, x \,]_o^b\, \theta$

$= (1/2\, b + b)\, Fb^2\, 1/EJ + (-\, b \,)\, \theta = 5/2\ Fb^3/EJ$

$L_{EC}^{Xo} = \int_o^b (-3/2 + 5/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/2\, x + 5/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 5/4\, b - 1/3\, b)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L_{CE}^{Xo} = \int_o^b (-1/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

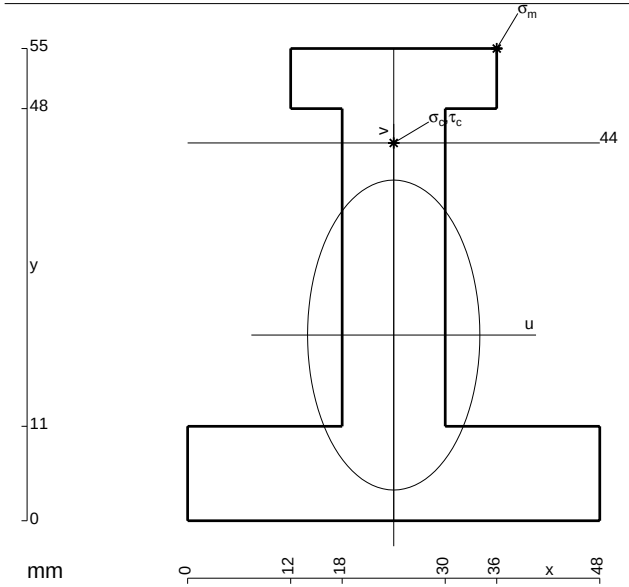
$= (-1/4\, b - 1/3\, b)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L_{IA}^{Xo} = \int_o^b (5/2 - 11/2\, x/b + 3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/2\, x - 11/4\, x^2/b + x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (5/2\, b - 11/4\, b + b)\, Fb^2\, 1/EJ = 3/4\ Fb^3/EJ$

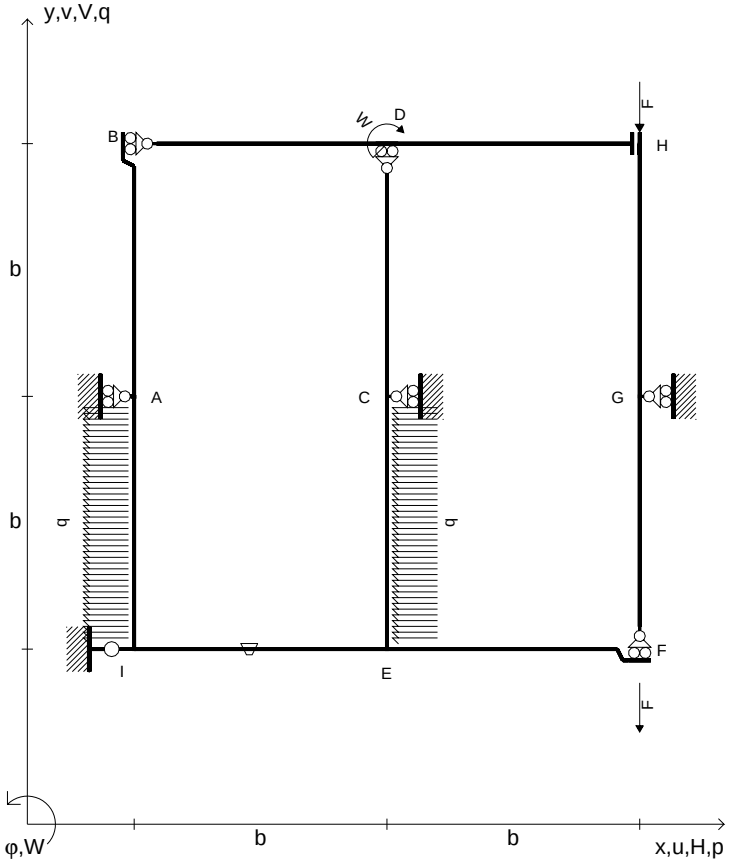
$L_{AI}^{Xo} = \int_o^b (-1/2\, x/b + 3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b + x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

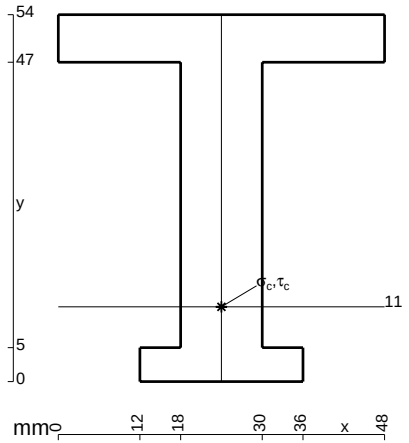
$= (-1/4\, b + b)\, Fb^2\, 1/EJ = 3/4\ Fb^3/EJ$



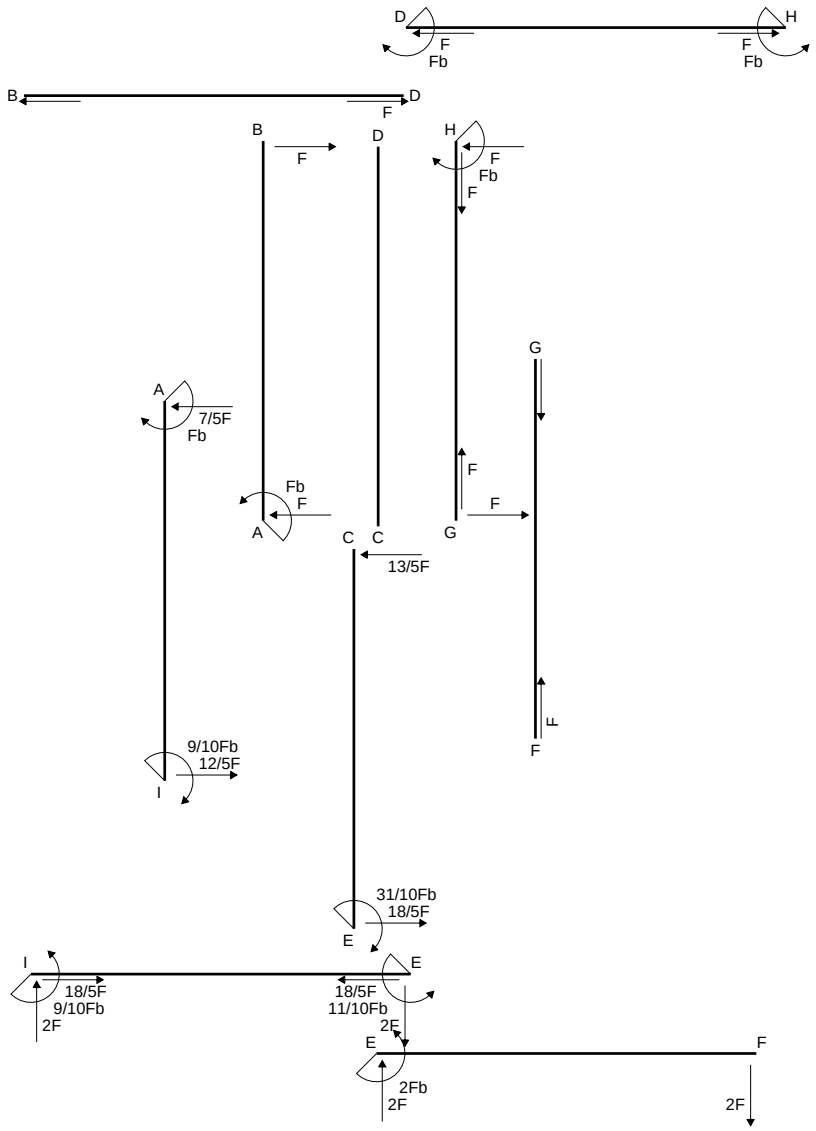
A = 1140. mm^2
J_u = 371429. mm^4
J_v = 114768. mm^4
y_g = 21.63 mm
T_y = 3880. N
M_x = -2328000. Nmm
x_m = 36. mm
y_m = 55. mm
u_m = 12. mm
v_m = 33.37 mm
$\sigma_m = -Mv/J_u$ = 209.2 N/mm^2
x_c = 24. mm
y_c = 44. mm
v_c = 22.37 mm
$\sigma_c = -Mv/J_u$ = 140.2 N/mm^2
τ_c = 5.387 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 140.5 N/mm^2
S^* = 6189. mm^3

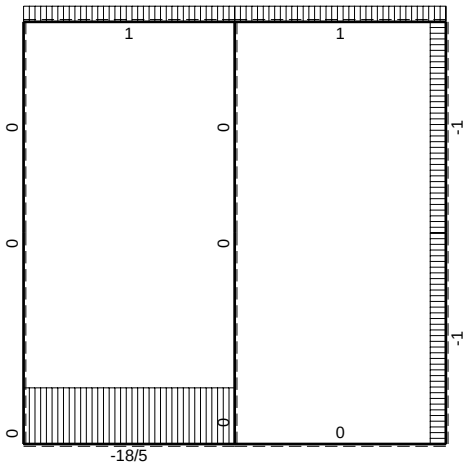
$V_{HG} = -F$
$V_{FE} = -F$
$W_D = -W = -Fb$
$p_{IA} = -q = -F/b$
$p_{EC} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$



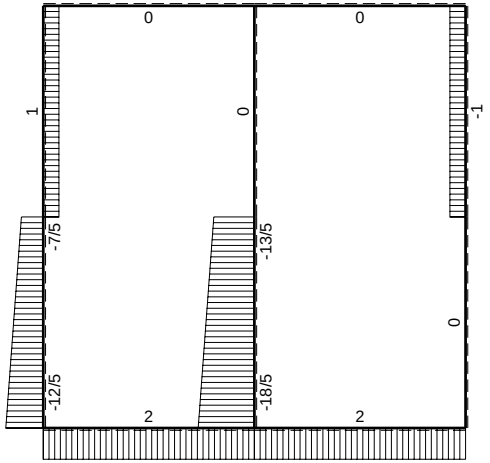


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 810 mm, F = 1220 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

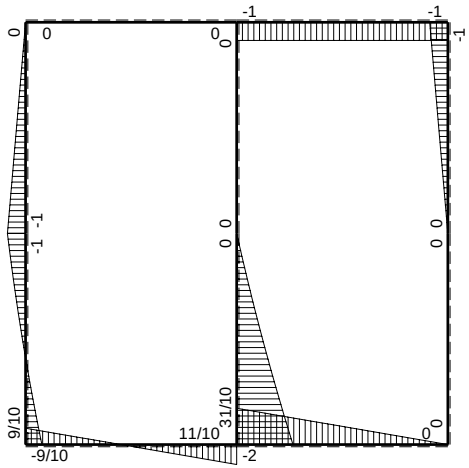




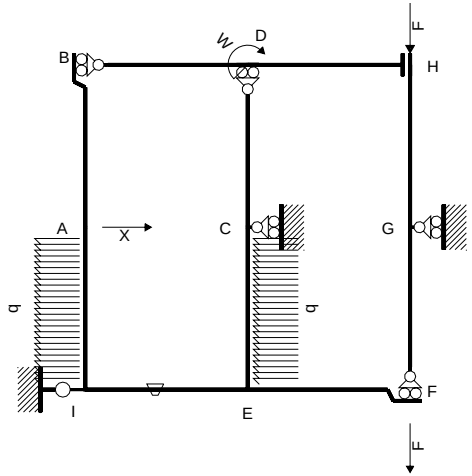
←⊞→ F



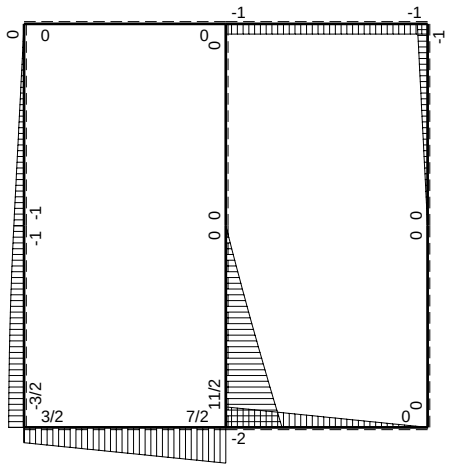
↑⊞↓ F



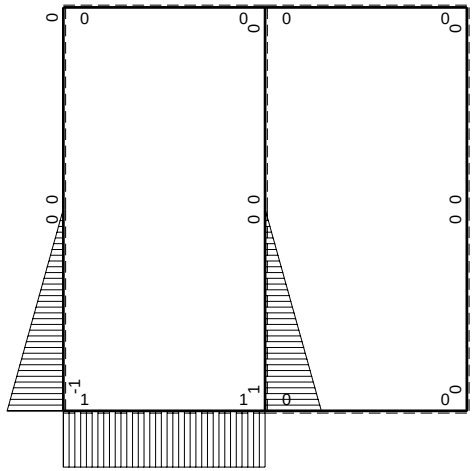
⟲⊞⟳ Fb



Schema di calcolo iperstatico



⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fb+Fx	0	0	0	0	0+0	0
BA b	0	Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-Fx	0	0	0	0	0+0	0
HG b	0	Fb-Fx	0	0	0	0		
HD b	0	-Fb	0	0	0	0	0+0	0
DH b	0	Fb	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	3/2Fb+2Fx	-Fb/EJ	$3/2Fb^2+2Fbx$	$-Fb^2/EJ$	b^2	$(5/2-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	-7/2Fb+2Fx	Fb/EJ	$7/2Fb^2-2Fbx$	$-Fb^2/EJ$	b^2		
EC b	b-x	$11/2Fb-6Fx+1/2qx^2$	0	$11/2Fb^2-23/2Fbx+13/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$	$(43/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	$-5Fx-1/2qx^2$	0	$5Fx^2+1/2qx^3$	0	x^2		
IA b	-b+x	$-3/2Fb+1/2qx^2$	0	$3/2Fb^2-3/2Fbx-1/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(17/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	$Fb+Fx-1/2qx^2$	0	$Fbx+Fx^2-1/2qx^3$	0	x^2		
	totali						$4Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-12/5F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211434 ESΣ07.xxxx.195
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (3/2 + 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx = [3/2\, x + x^2/b \,]_o^b\, Fb^2\, 1/EJ + [- x \,]_o^b\, \theta$

$= (3/2\, b + b \,)\, Fb^2\, 1/EJ + (- b \,)\, \theta = 3/2\ Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (7/2 - 2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx = [7/2\, x - x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (7/2\, b - b \,)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = 3/2\ Fb^3/EJ$

$L^{Xo}_{EC} = \int_o^b (11/2 - 23/2\, x/b + 13/2\, x^2/b^2 - 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [11/2\, x - 23/4\, x^2/b + 13/6\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (11/2\, b - 23/4\, b + 13/6\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = 43/24\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (5\, x^2/b^2 + 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [5/3\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (5/3\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = 43/24\ Fb^3/EJ$

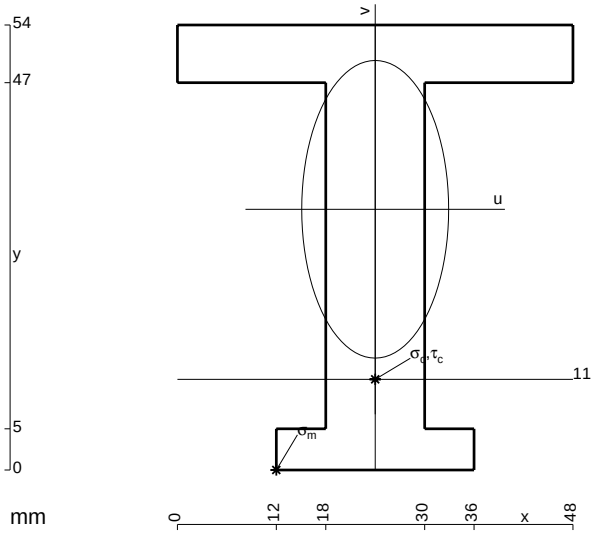
$L^{Xo}_{IA} = \int_o^b (3/2 - 3/2\, x/b - 1/2\, x^2/b^2 + 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [3/2\, x - 3/4\, x^2/b - 1/6\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (3/2\, b - 3/4\, b - 1/6\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = 17/24\ Fb^3/EJ$

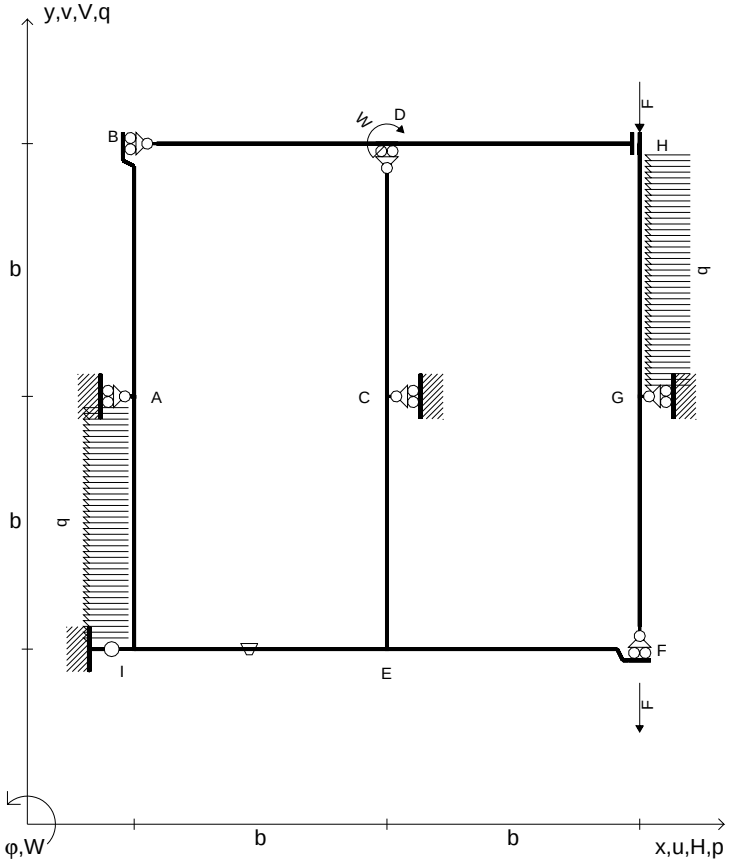
$L^{Xo}_{AI} = \int_o^b (\, x/b + x^2/b^2 - 1/2\, x^3/b^3 \,)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b + 1/3\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

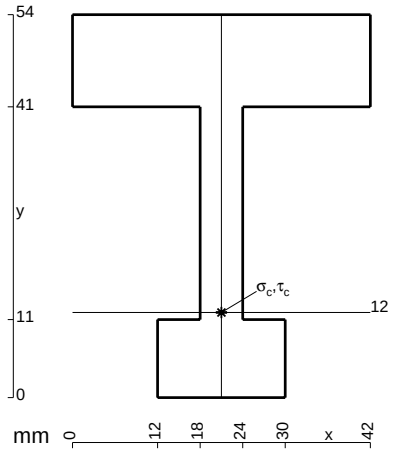
$= (1/2\, b + 1/3\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = 17/24\ Fb^3/EJ$



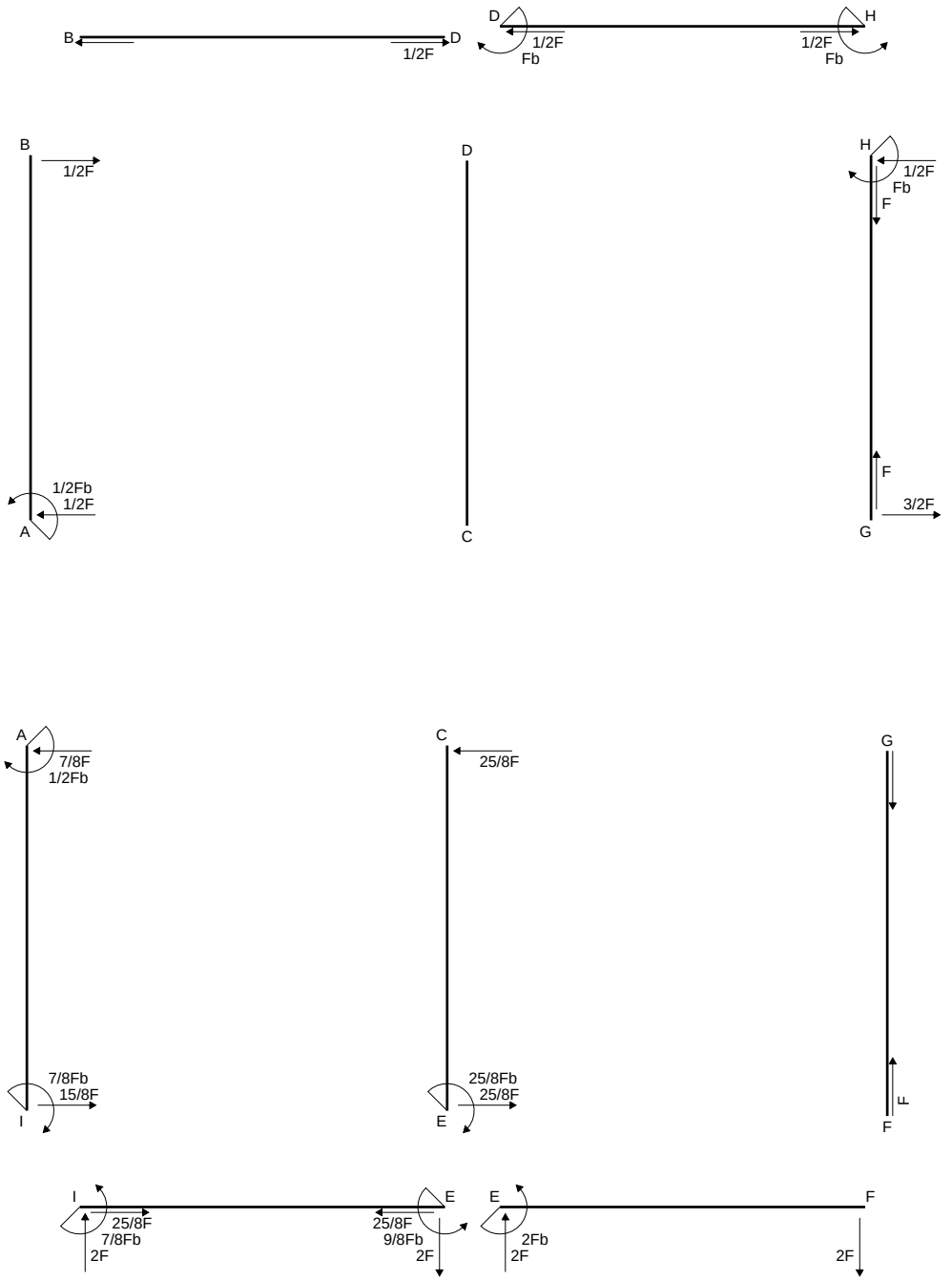
A = 960. mm^2
J_u = 313154. mm^4
J_v = 76320. mm^4
y_g = 31.64 mm
T_y = 2440. N
M_x = -1976400. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -31.64 mm
$\sigma_m = -Mv/J_u$ = -199.7 N/mm^2
x_c = 24. mm
y_c = 11. mm
v_c = -20.64 mm
$\sigma_c = -Mv/J_u$ = -130.2 N/mm^2
τ_c = 3.375 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 130.4 N/mm^2
S^* = 5198. mm^3

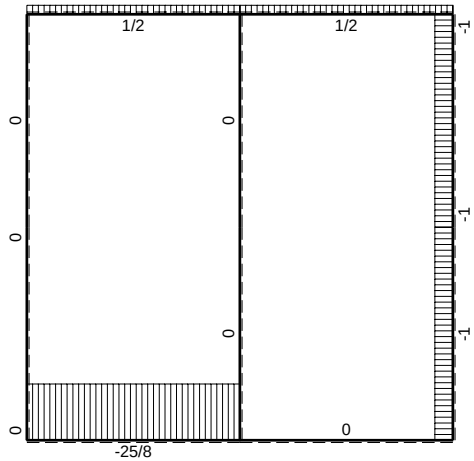
$V_{HG} = -F$
$V_{FE} = -F$
$W_D = -W = -Fb$
$p_{GH} = -q = -F/b$
$p_{IA} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$

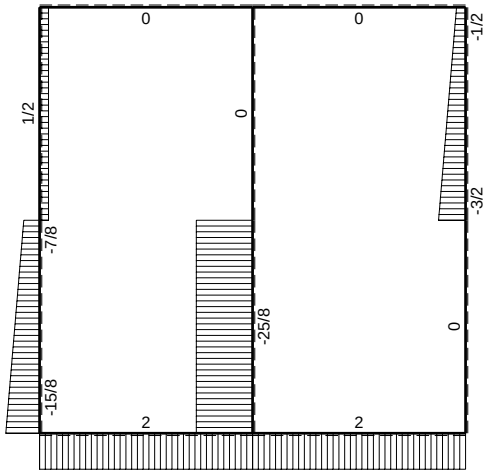




Reazioni iperstatiche in soluzione: $X=W_{IE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 970 mm, F = 1010 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

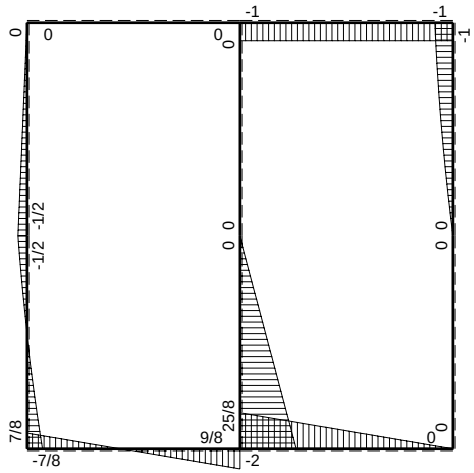




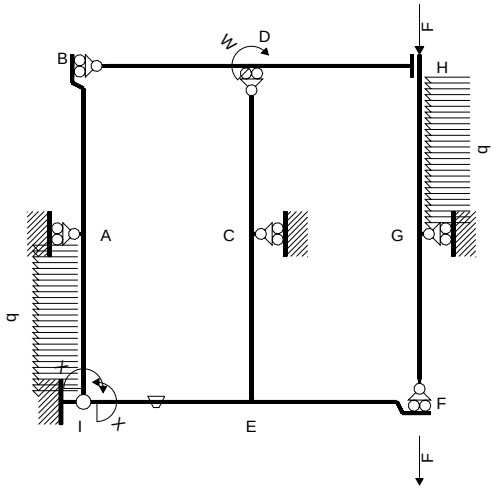


←[+]→ F

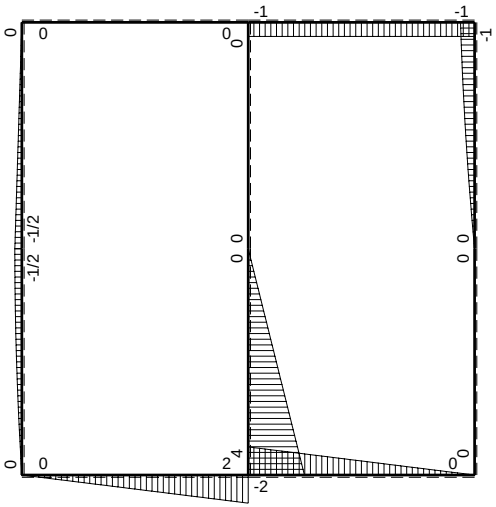
↑[+]↓ F



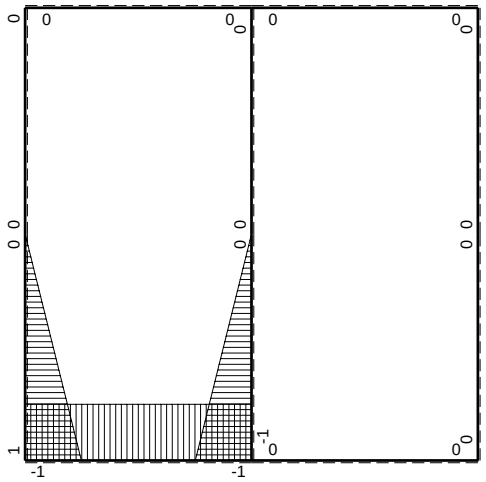
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{IE}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
FE b	0	$2Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$Fb-1/2Fx-1/2qx^2$	0	0	0	0		
HD b	0	$-Fb$	0	0	0	0	0+0	0
DH b	0	Fb	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	-1	$2Fx$	$-Fb/EJ$	$-2Fx$	Fb/EJ	1	$(-1+1)Fb^2/EJ$	Xb/EJ
EI b	1	$-2Fb+2Fx$	Fb/EJ	$-2Fb+2Fx$	Fb/EJ	1		
EC b	$-1+x/b$	$4Fb-4Fx$	0	$-4Fb+8Fx-4Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-4/3+0)Fb^2/EJ$	$1/3Xb/EJ$
CE b	x/b	$-4Fx$	0	$-4Fx^2/b$	0	x^2/b^2		
IA b	$1-x/b$	$-Fx+1/2qx^2$	0	$-Fx+3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-1/8+0)Fb^2/EJ$	$1/3Xb/EJ$
AI b	$-x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fx+1/2qx^3/b$	0	x^2/b^2		
	totali						$-35/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{IE}$						$7/8Fb$	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{EI} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CE} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{AI} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{IE} = \int_o^b (-2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 0$

$L^{Xo}_{EI} = \int_o^b (-2 + 2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-2\ x + x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-2\ b + b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 0$

$L^{Xo}_{EC} = \int_o^b (-4 + 8\ x/b - 4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-4\ x + 4\ x^2/b - 4/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-4\ b + 4\ b - 4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{CE} = \int_o^b (-4\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-4/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

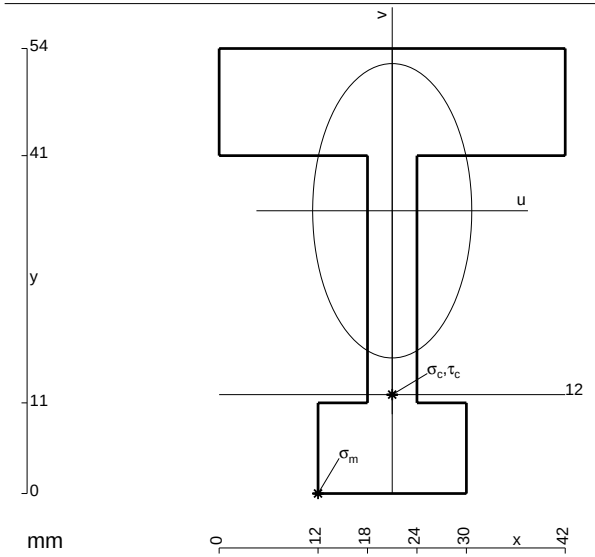
$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{IA} = \int_o^b (-\ x/b + 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b + 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b + 1/2\ b - 1/8\ b\ )\ Fb\ 1/EJ\ = -1/8\ Fb^2/EJ$

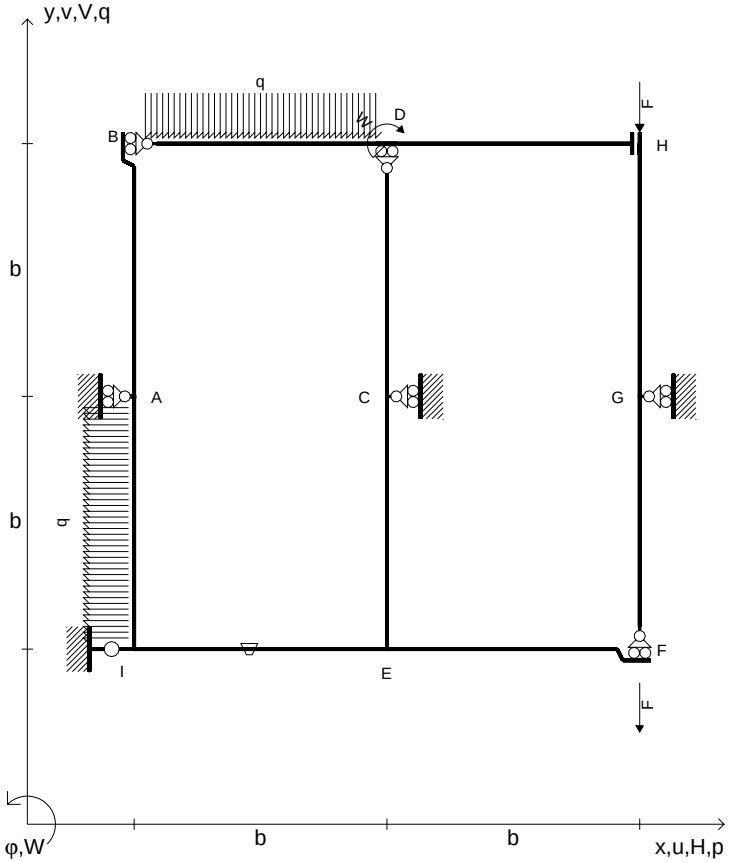
$L^{Xo}_{AI} = \int_o^b (-1/2\ x/b + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

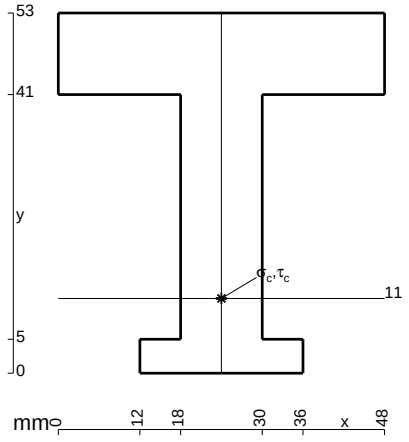
$= (-1/4\ b + 1/8\ b\ )\ Fb\ 1/EJ\ = -1/8\ Fb^2/EJ$



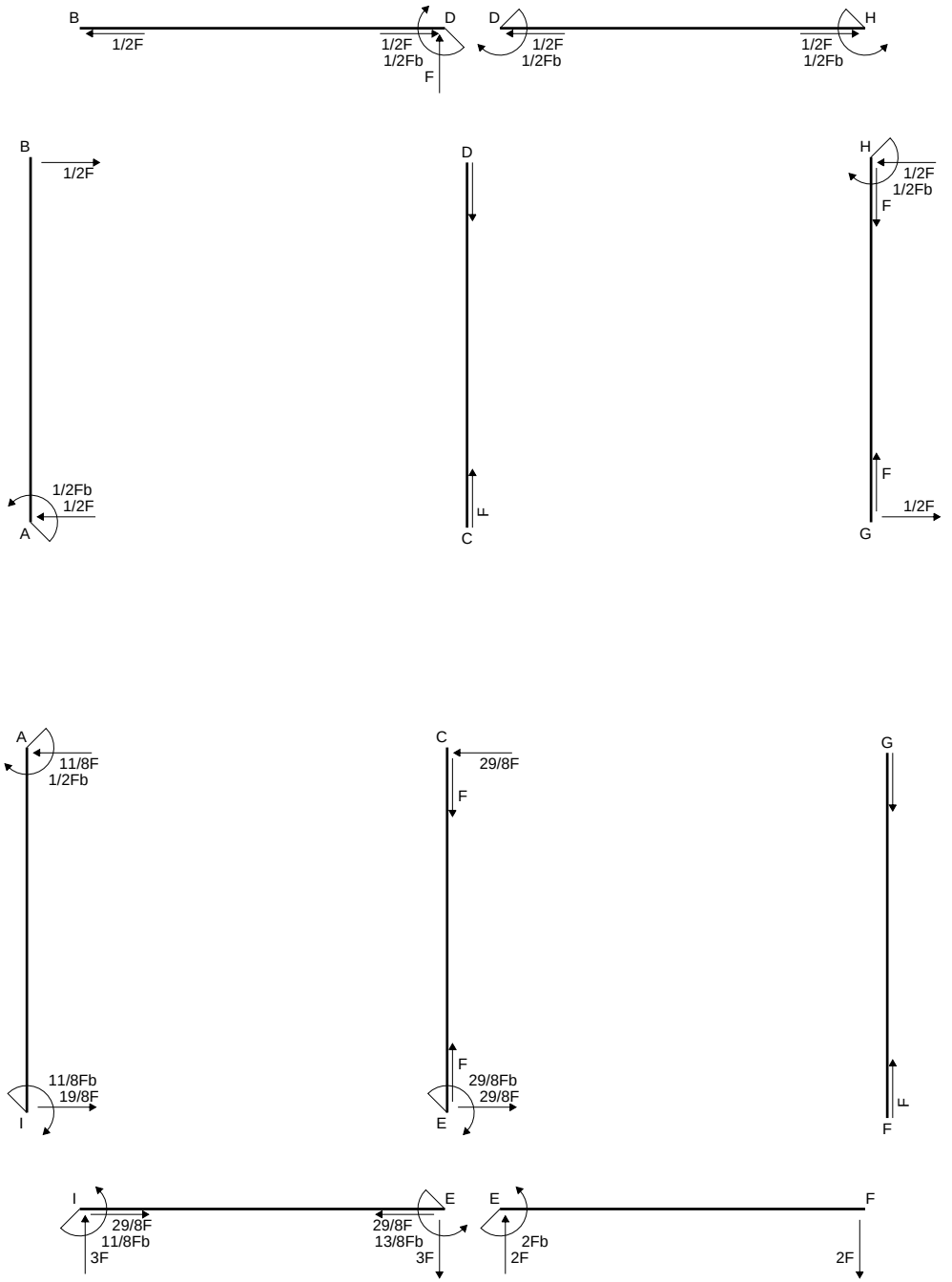
A = 924. mm^2
J_u = 294950. mm^4
J_v = 86148. mm^4
y_g = 34.31 mm
T_y = 2020. N
M_x = -1959400. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -34.31 mm
$\sigma_m = -Mv/J_u$ = -227.9 N/mm^2
x_c = 21. mm
y_c = 12. mm
v_c = -22.31 mm
$\sigma_c = -Mv/J_u$ = -148.2 N/mm^2
τ_c = 6.668 N/mm^2
$\sigma_0 = \sqrt{\sigma^2 + 3\tau^2}$ = 148.7 N/mm^2
S^* = 5842. mm^3

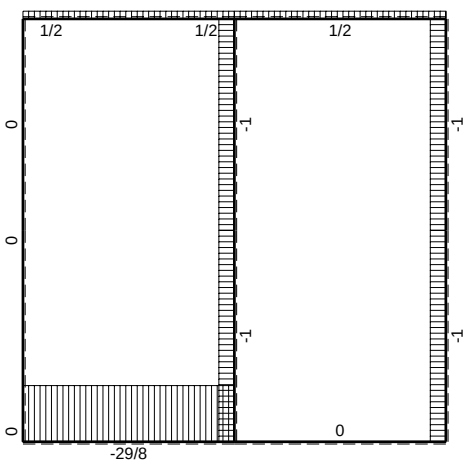
$V_{HG} = -F$
$V_{FE} = -F$
$W_D = -W = -Fb$
$q_{DB} = -q = -F/b$
$p_{IA} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{CD} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HD} = EJ$
$EJ_{DB} = EJ$
$EJ_{IE} = EJ$
$EJ_{EC} = EJ$
$EJ_{IA} = EJ$

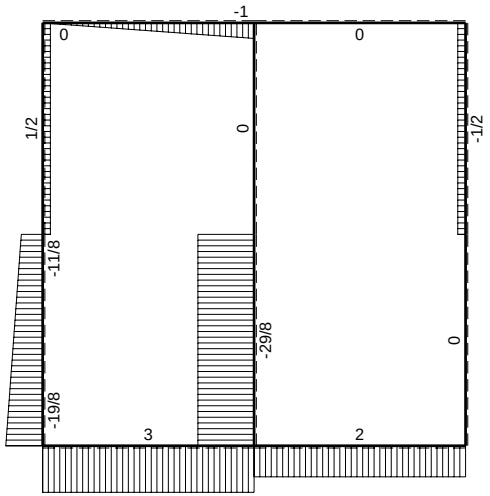




Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave EF ha la sezione riportata e dimensioni in mm, con:
b = 630 mm, F = 1690 N
Calcolare sulla sezione E la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da E a F
Curvatura θ asta IE positiva se convessa a destra con inizio I.

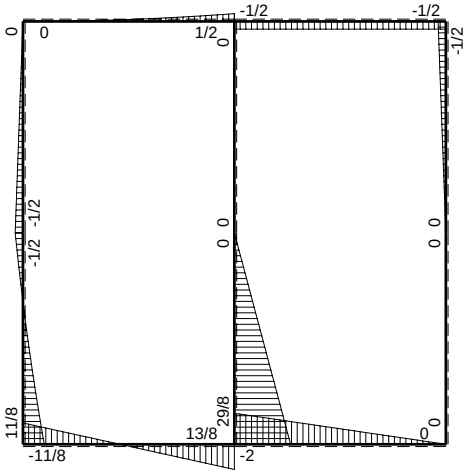




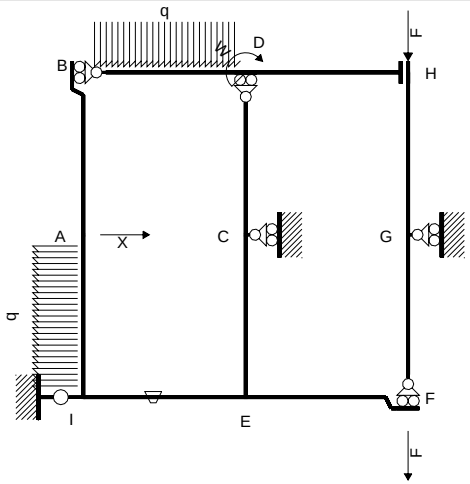


←□→ F

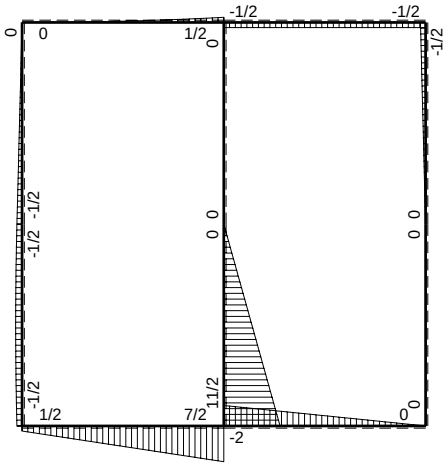
↑□↓ F



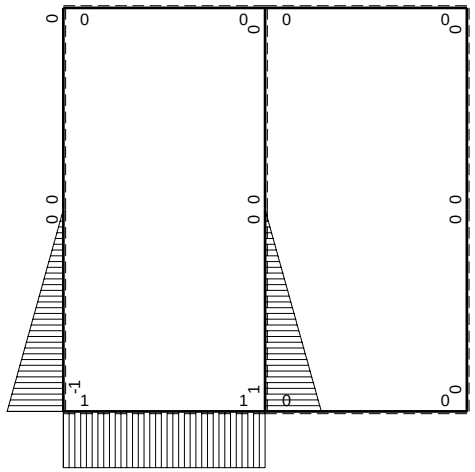
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-1/2Fx	0	0	0	0	0+0	0
HG b	0	1/2Fb-1/2Fx	0	0	0	0		
HD b	0	-1/2Fb	0	0	0	0	0+0	0
DH b	0	1/2Fb	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	b	1/2Fb+3Fx	-Fb/EJ	$1/2Fb^2+3Fbx$	$-Fb^2/EJ$	b^2	$(2-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	-7/2Fb+3Fx	Fb/EJ	$7/2Fb^2-3Fbx$	$-Fb^2/EJ$	b^2		
EC b	b-x	11/2Fb-11/2Fx	0	$11/2Fb^2-11Fbx+11/2Fx^2$	0	$b^2-2bx+x^2$	$(11/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	-11/2Fx	0	$11/2Fx^2$	0	x^2		
IA b	-b+x	$-1/2Fb-1/2Fx+1/2qx^2$	0	$1/2Fb^2-Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(7/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	$1/2Fb+1/2Fx-1/2qx^2$	0	$1/2Fbx+1/2Fx^2-1/2qx^3$	0	x^2		
	totali						$25/8Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-15/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{IE} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EI} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{EC} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CE} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AI} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{IE} = \int_o^b (1/2 + 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1)\, \theta\ dx = [1/2\, x + 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [-x]_o^b\, \theta$

$= (1/2\, b + 3/2\, b)\, Fb^2\, 1/EJ + (-b)\, \theta = Fb^3/EJ$

$L^{Xo}_{EI} = \int_o^b (7/2 - 3\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1)\, \theta\ dx = [7/2\, x - 3/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + [\, x \,]_o^b\, \theta$

$= (7/2\, b - 3/2\, b)\, Fb^2\, 1/EJ + (\, b \,)\, \theta = Fb^3/EJ$

$L^{Xo}_{EC} = \int_o^b (11/2 - 11\, x/b + 11/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [11/2\, x - 11/2\, x^2/b + 11/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (11/2\, b - 11/2\, b + 11/6\, b)\, Fb^2\, 1/EJ = 11/6\ Fb^3/EJ$

$L^{Xo}_{CE} = \int_o^b (11/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [11/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (11/6\, b)\, Fb^2\, 1/EJ = 11/6\ Fb^3/EJ$

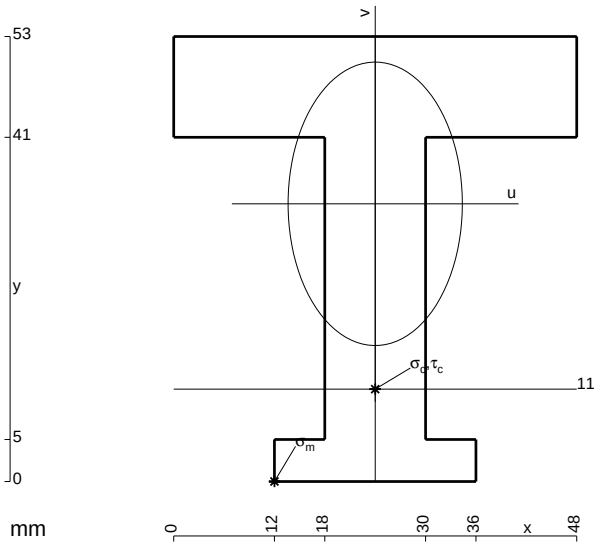
$L^{Xo}_{IA} = \int_o^b (1/2 - x^2/b^2 + 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [1/2\, x - 1/3\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b - 1/3\, b + 1/8\, b)\, Fb^2\, 1/EJ = 7/24\ Fb^3/EJ$

$L^{Xo}_{AI} = \int_o^b (1/2\, x/b + 1/2\, x^2/b^2 - 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b + 1/6\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b + 1/6\, b - 1/8\, b)\, Fb^2\, 1/EJ = 7/24\ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 984497 ESΣ07.xxxx.212



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J_u = 321538. mm^4
J_v = 121536. mm^4
y_g = 33.07 mm
T_y = 3380. N
M_x = -2129400. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -33.07 mm
$\sigma_m = -Mv/J_u$ = -219. N/mm^2
x_c = 24. mm
y_c = 11. mm
v_c = -22.07 mm
$\sigma_c = -Mv/J_u$ = -146.2 N/mm^2
τ_c = 4.795 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 146.4 N/mm^2
S^* = 5474. mm^3