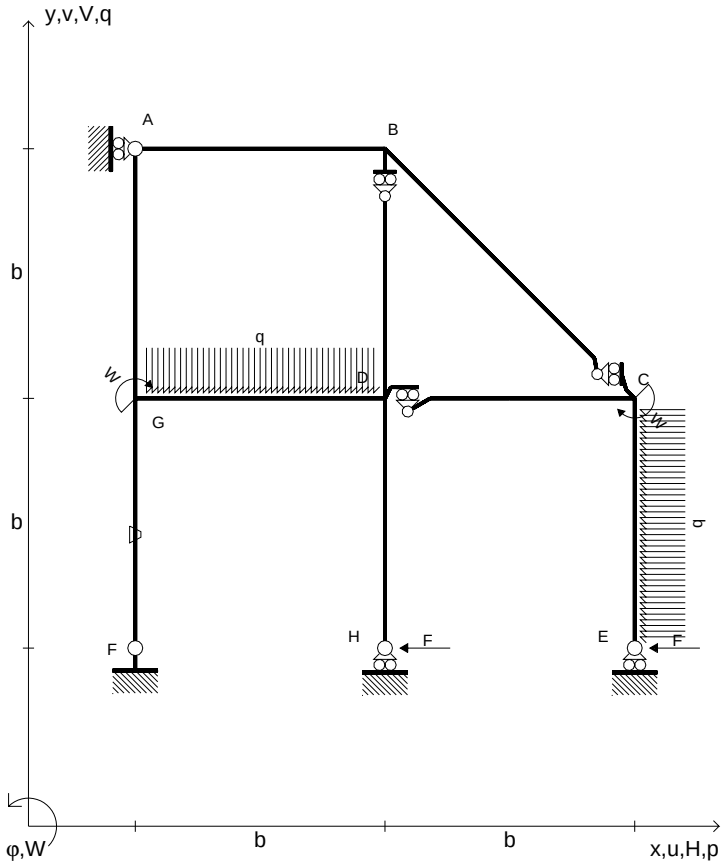
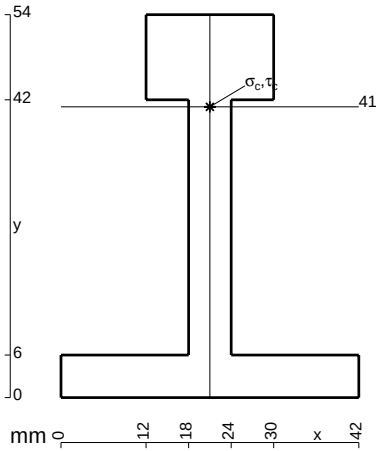
$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$p_{CE} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 510 mm, F = 1360 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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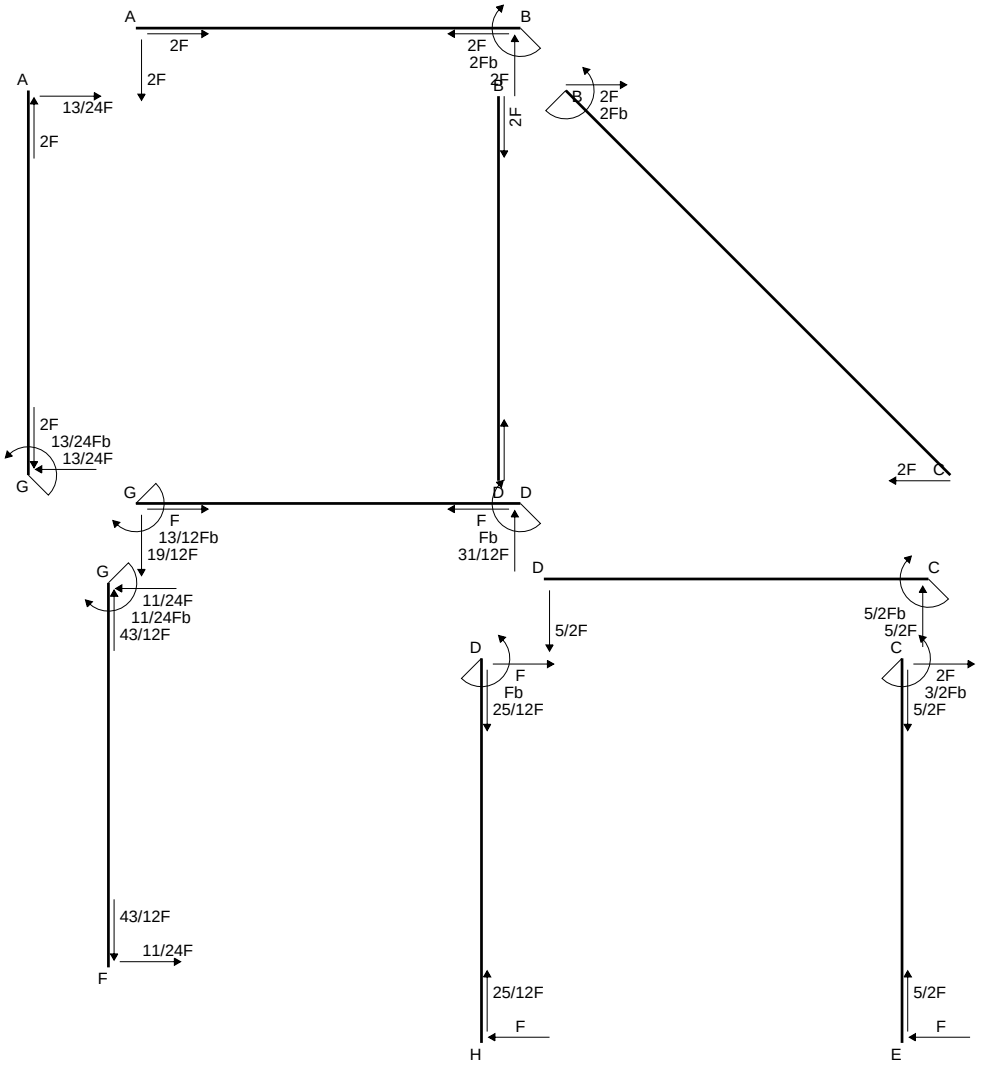

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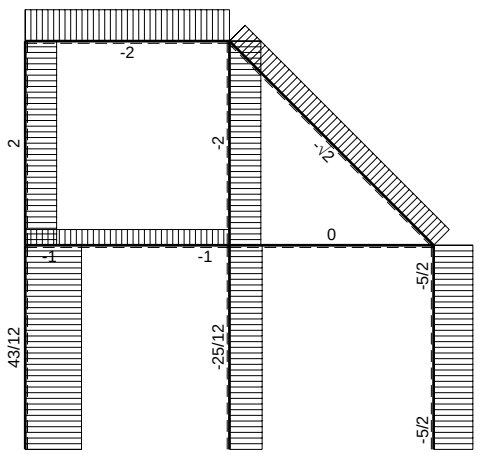
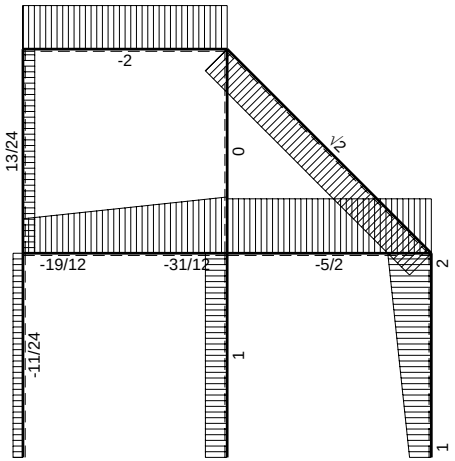
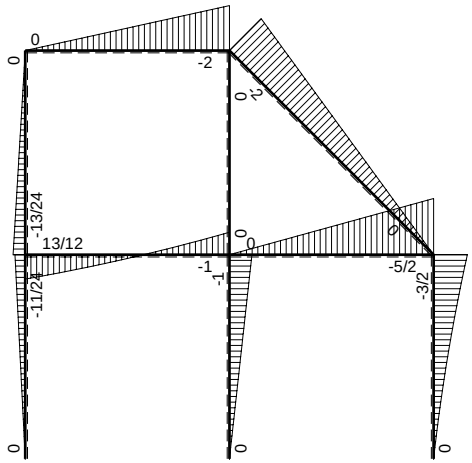


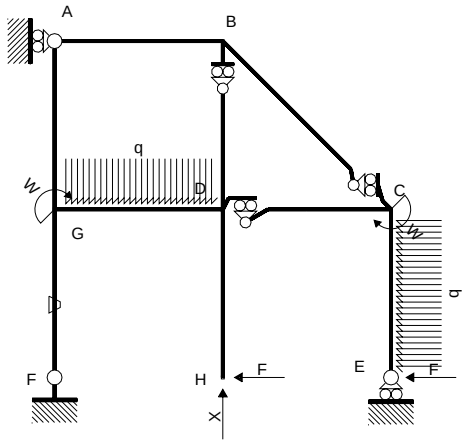
←□→ F



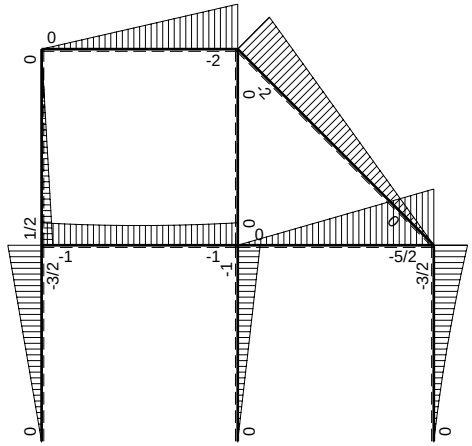
↑□↓ F



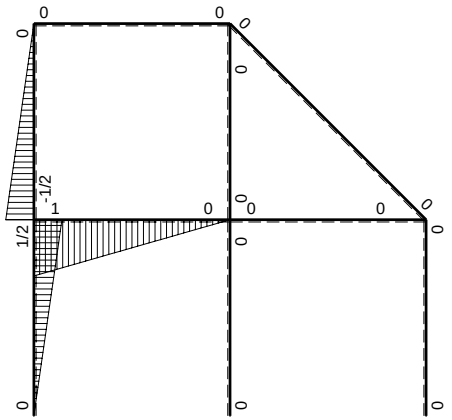
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0		
BC $\sqrt{2}$b	0	-2Fb+$\sqrt{2}$Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	-3/2Fb+2Fx-1/2qx^2	0	0	0	0	0+0	0
EC b	0	Fx+1/2qx^2	0	0	0	0		
FG b	1/2x	-3/2Fx	-Fb/EJ	-3/4Fx^2	-1/2Fxb/EJ	1/4x^2	(-1/4-1/4)Fb^3/EJ	1/12Xb^3/EJ
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	-3/4Fb^2+3/2Fbx-3/4Fx^2	-1/2Fb^2/EJ+1/2Fxb/EJ	1/4b^2-1/2bx+1/4x^2		
GD b	b-x	-Fb+1/2Fx-1/2qx^2	0	-Fb^2+3/2Fbx-Fx^2+1/2qx^3	0	b^2-2bx+x^2	(-11/24+0)Fb^3/EJ	1/3Xb^3/EJ
DG b	-x	Fb-1/2Fx+1/2qx^2	0	-Fbx+1/2Fx^2-1/2qx^3	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	1/2Fb-1/2Fx	0	-1/4Fb^2+1/2Fbx-1/4Fx^2	0	1/4b^2-1/2bx+1/4x^2	(-1/12+0)Fb^3/EJ	1/12Xb^3/EJ
AG b	1/2x	-1/2Fx	0	-1/4Fx^2	0	1/4x^2		
	totali						-25/24Fb^3/EJ	1/2Xb^3/EJ
	iperstatica $X=V_H$						25/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [-1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/4\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/4 + 3/2\ x/b - 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-3/4\ x + 3/4\ x^2/b - 1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-3/4\ b + 3/4\ b - 1/4\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-1 + 3/2\ x/b - x^2/b^2 + 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-\ x + 3/4\ x^2/b - 1/3\ x^3/b^2 + 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (-\ b + 3/4\ b - 1/3\ b + 1/8\ b)\ Fb^2\ 1/EJ = -11/24\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-\ x/b + 1/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-1/2\ x^2/b + 1/6\ x^3/b^2 - 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

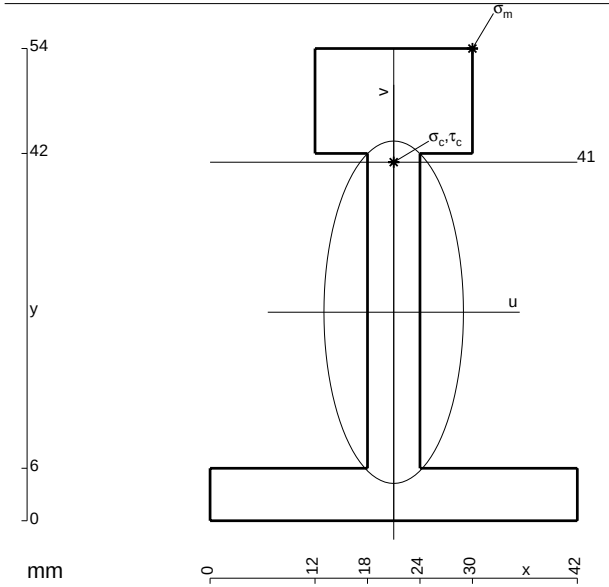
$= (-1/2\ b + 1/6\ b - 1/8\ b)\ Fb^2\ 1/EJ = -11/24\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1/4 + 1/2\ x/b - 1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-1/4\ b + 1/4\ b - 1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$

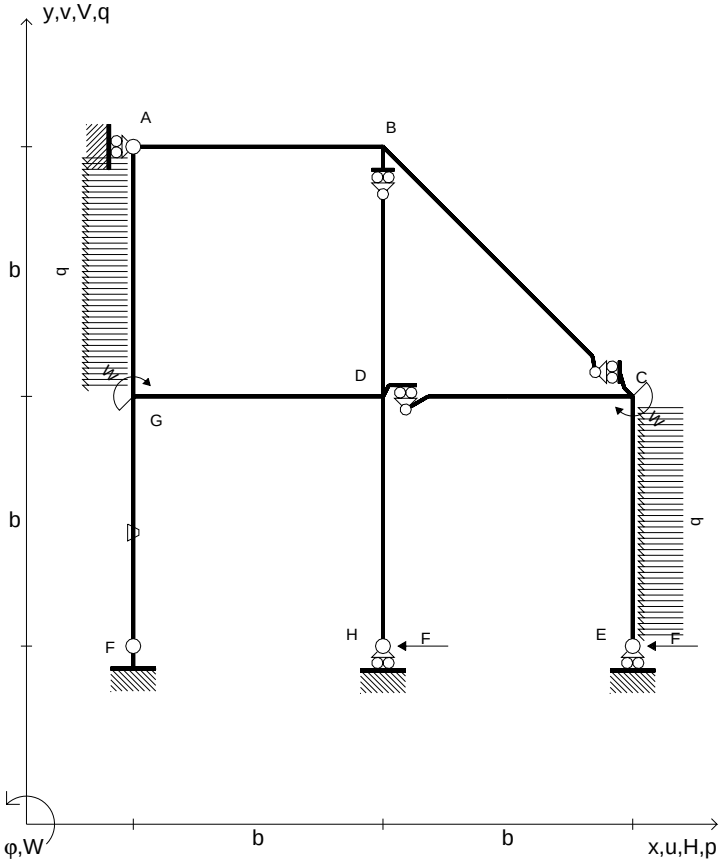
$L^{Xo}_{AG} = \int_o^b (-1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

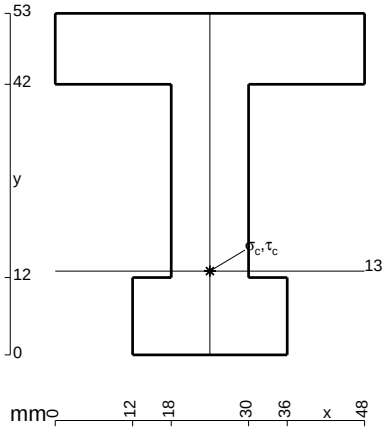
$= (-1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$



A = 684. mm^2
J_u = 262207. mm^4
J_v = 43524. mm^4
y_g = 23.84 mm
T_y = -3400. N
M_x = -1734000. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 9. mm
v_m = 30.16 mm
$\sigma_m = -Mv/J_u$ = 199.4 N/mm^2
x_c = 21. mm
y_c = 41. mm
v_c = 17.16 mm
$\sigma_c = -Mv/J_u$ = 113.5 N/mm^2
τ_c = 11.51 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 115.2 N/mm^2
S^* = 5324. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$p_{CE} = -q = -F/b$
$p_{GA} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_F$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 700 mm, F = 1610 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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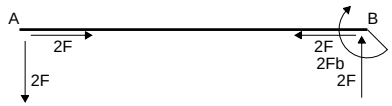

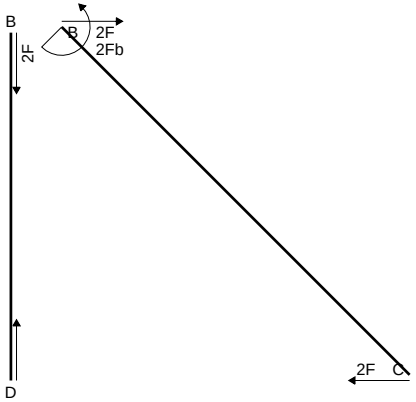

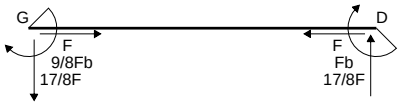

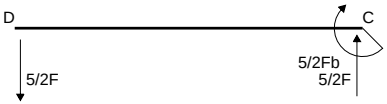

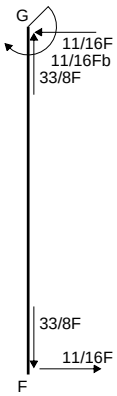

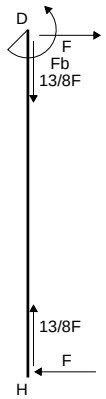

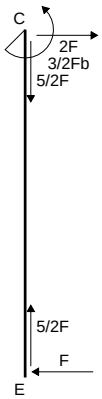

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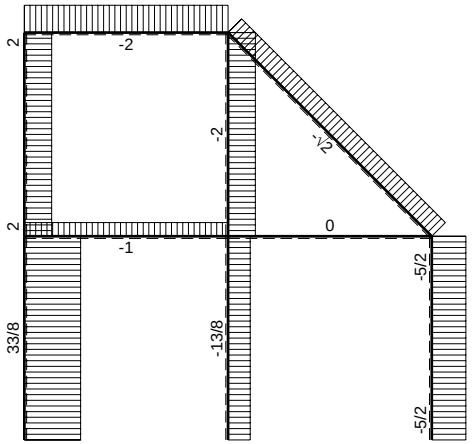
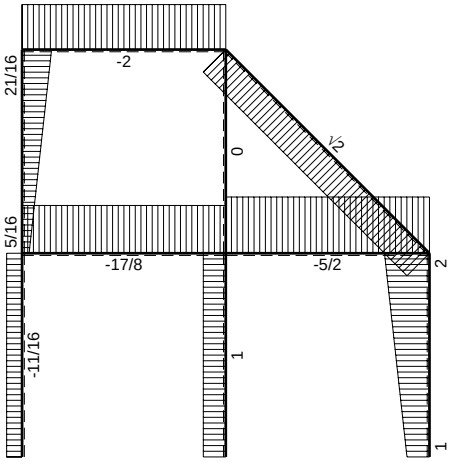
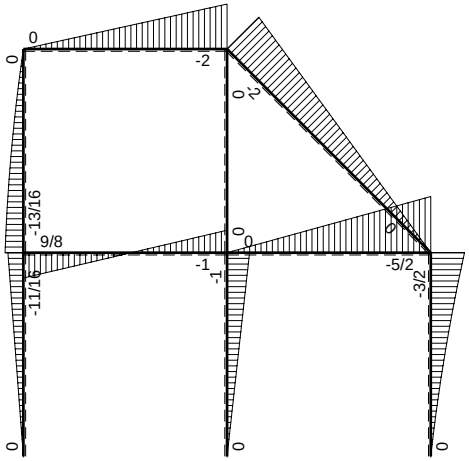


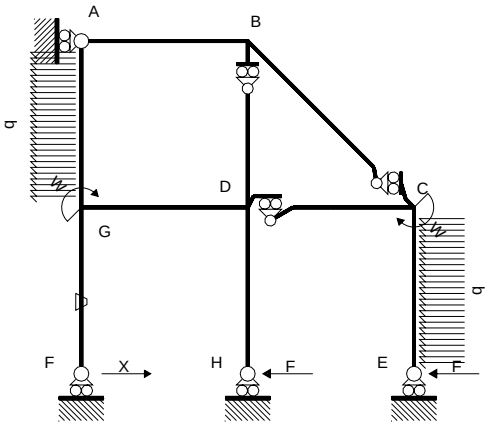
←□→ F



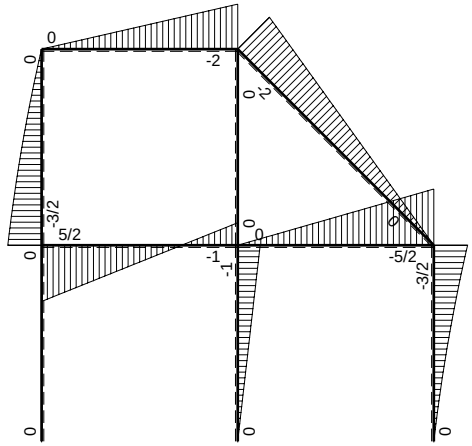
↑□↓ F



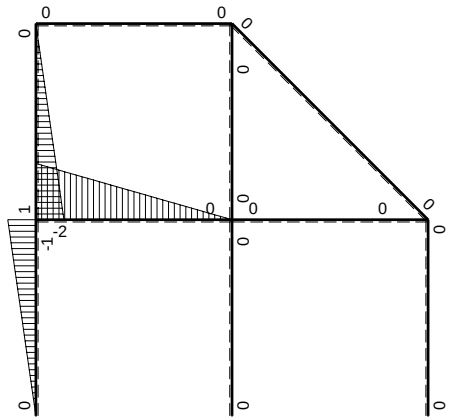
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_F$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	-2b+2x	5/2Fb-7/2Fx	0	$-5Fb^2+12Fbx-7Fx^2$	0	$4b^2-8bx+4x^2$	$(-4/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	Fb-7/2Fx	0	$2Fbx-7Fx^2$	0	$4x^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	b-x	$-3/2Fb+Fx+1/2qx^2$	0	$-3/2Fb^2+5/2Fbx-1/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$	$(-13/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	$2Fx-1/2qx^2$	0	$-2Fx^2+1/2qx^3$	0	x^2		
	totali						$-11/8Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_F$						11/16F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ b^2\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\ x/b +4\ x^2/b^2\ )\ b^2\ 1/EJ\ dx = [4\ x -4\ x^2/b +4/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (4\ b -4\ b +4/3\ b\ )\ b^2\ 1/EJ\ = 4/3\ \ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\ x^2/b^2\ )\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (4/3\ b\ )\ b^2\ 1/EJ\ = 4/3\ \ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ b^2\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2\ )\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (x/b\ )\ \theta\ \ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ \ = 1/2\ \ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1 + x/b\ )\ \theta\ \ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ \ = 1/2\ \ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-5 +12\ x/b -7\ x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [-5\ x +6\ x^2/b -7/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (-5\ b +6\ b -7/3\ b\ )\ Fb^2\ 1/EJ\ = -4/3\ \ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (2\ x/b -7\ x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [\ x^2/b -7/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (\ b -7/3\ b\ )\ Fb^2\ 1/EJ\ = -4/3\ \ Fb^3/EJ$

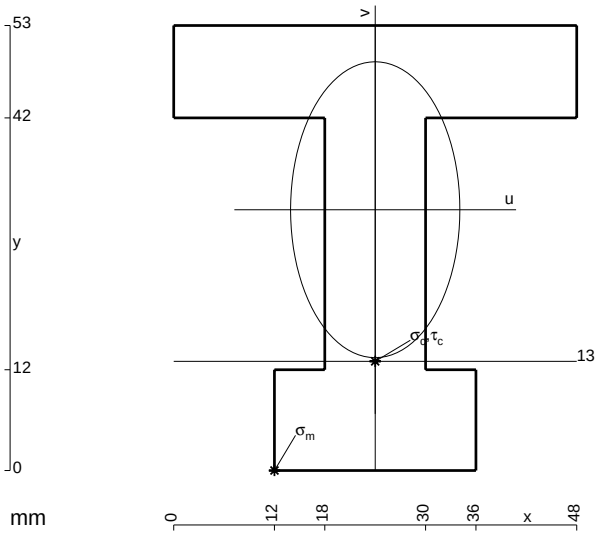
$L^{Xo}_{GA} = \int_o^b (-3/2 +5/2\ x/b -1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb^2\ 1/EJ\ dx$

$= [-3/2\ x +5/4\ x^2/b -1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ$

$= (-3/2\ b +5/4\ b -1/6\ b -1/8\ b\ )\ Fb^2\ 1/EJ\ = -13/24\ \ Fb^3/EJ$

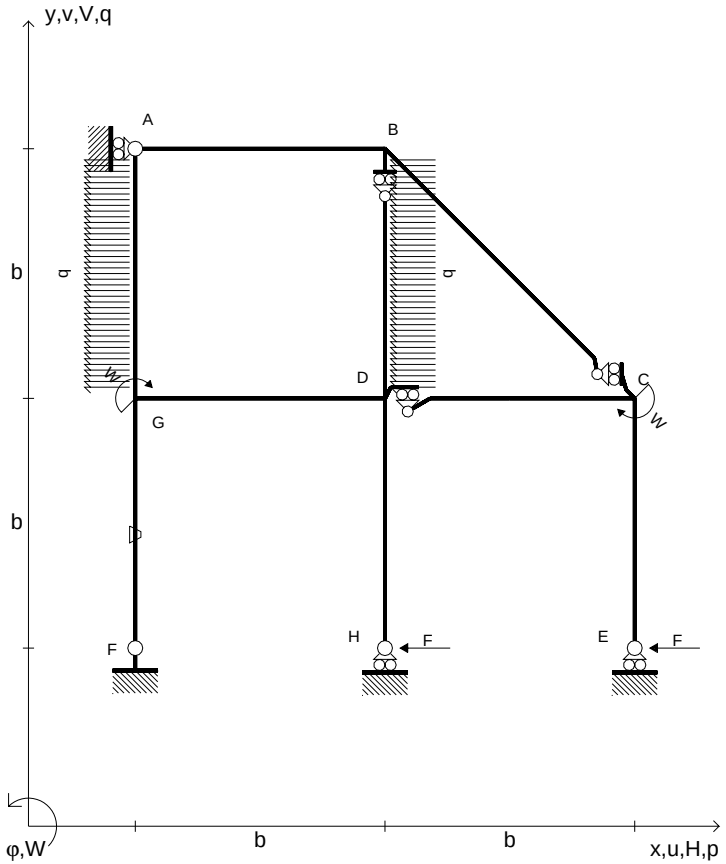
$L^{Xo}_{AG} = \int_o^b (-2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb^2\ 1/EJ\ dx = [-2/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ$

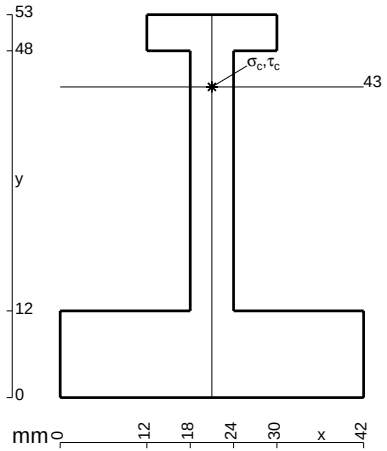
$= (-2/3\ b +1/8\ b\ )\ Fb^2\ 1/EJ\ = -13/24\ \ Fb^3/EJ$



A = 1176. mm^2
J_u = 365284. mm^4
J_v = 119520. mm^4
y_g = 31.06 mm
T_y = -4025. N
M_x = -2817500. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -31.06 mm
$\sigma_m = -Mv/J_u$ = -239.6 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -18.06 mm
$\sigma_c = -Mv/J_u$ = -139.3 N/mm^2
τ_c = 6.832 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 139.8 N/mm^2
S^* = 7440. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$p_{BD} = -q = -F/b$
$p_{GA} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_F$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 950 mm, F = 800 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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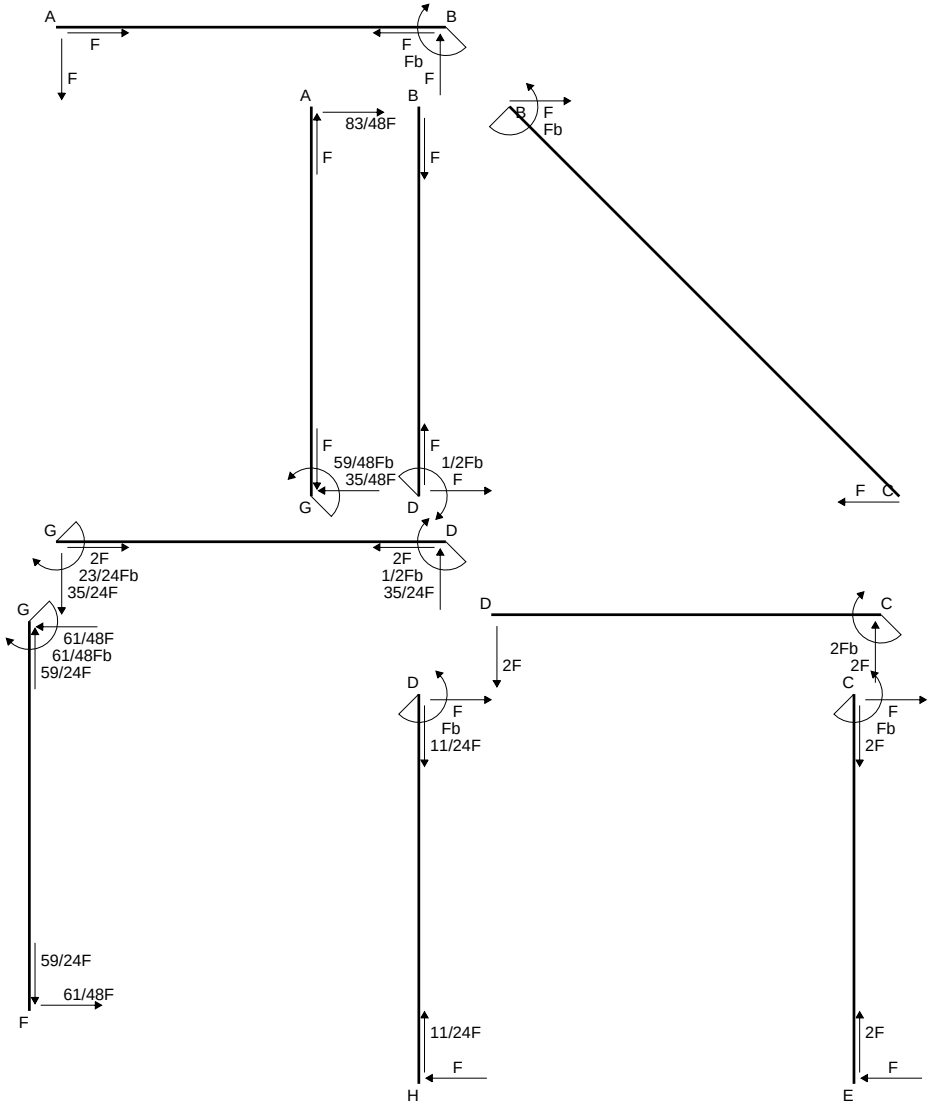

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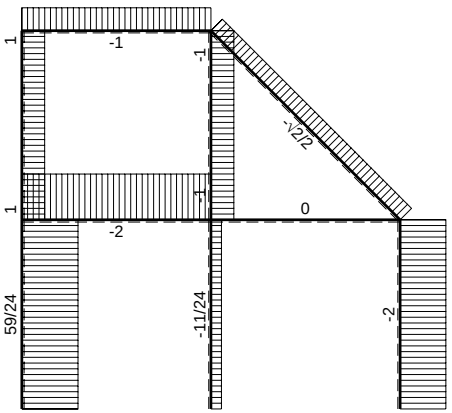
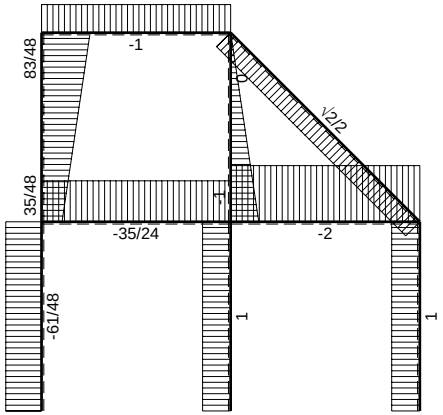
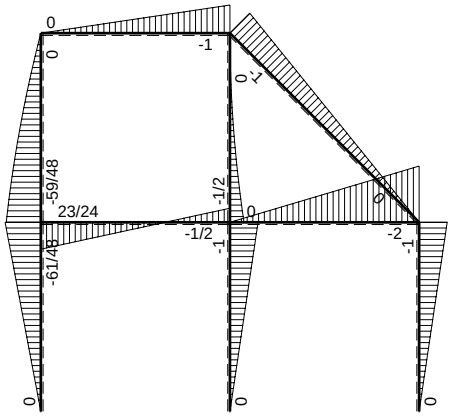


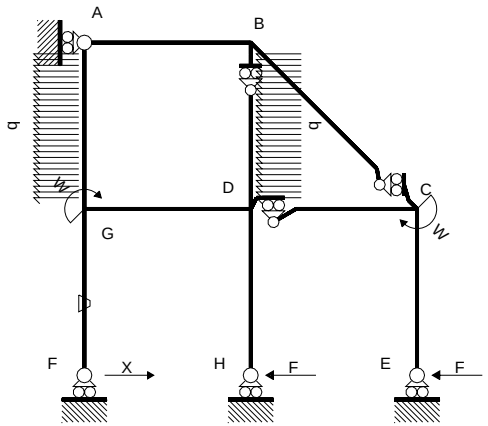
←□→ F



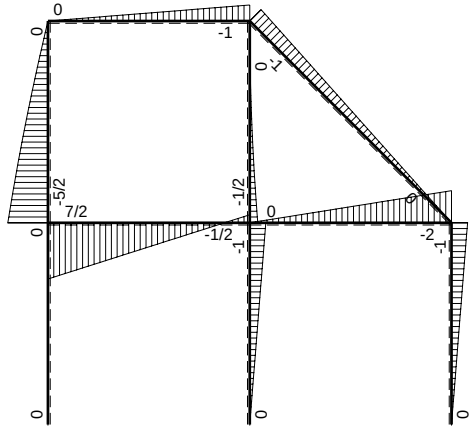
↑□↓ F



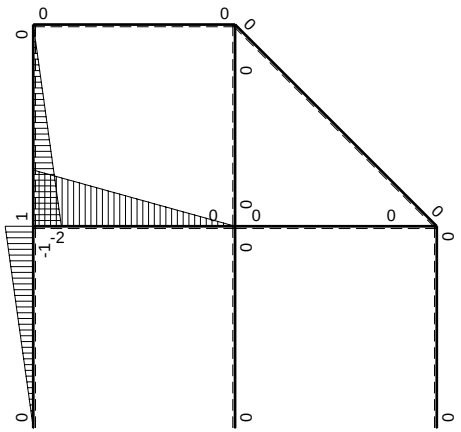
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_F$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0	0+0	0
BA b	0	Fb-Fx	0	0	0	0		
BC $\sqrt{2}$b	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	$-1/2qx^2$	0	0	0	0	0+0	0
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	-2b+2x	7/2Fb-4Fx	0	$-7Fb^2+15Fbx-8Fx^2$	0	$4b^2-8bx+4x^2$	$(-13/6+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	1/2Fb-4Fx	0	$Fbx-8Fx^2$	0	$4x^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	b-x	$-5/2Fb+2Fx+1/2qx^2$	0	$-5/2Fb^2+9/2Fbx-3/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$	$(-7/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	$3Fx-1/2qx^2$	0	$-3Fx^2+1/2qx^3$	0	x^2		
	totali						$-61/24Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_F$						61/48F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (x/b)\, \theta\ dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1 + x/b)\, \theta\ dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-7 +15\, x/b -8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-7\, x +15/2\, x^2/b -8/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-7\, b +15/2\, b -8/3\, b)\, Fb^2\, 1/EJ = -13/6\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (x/b -8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -8/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -8/3\, b)\, Fb^2\, 1/EJ = -13/6\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-5/2 +9/2\, x/b -3/2\, x^2/b^2 -1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-5/2\, x +9/4\, x^2/b -1/2\, x^3/b^2 -1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-5/2\, b +9/4\, b -1/2\, b -1/8\, b)\, Fb^2\, 1/EJ = -7/8\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-3\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [- x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (- b +1/8\, b)\, Fb^2\, 1/EJ = -7/8\ Fb^3/EJ$

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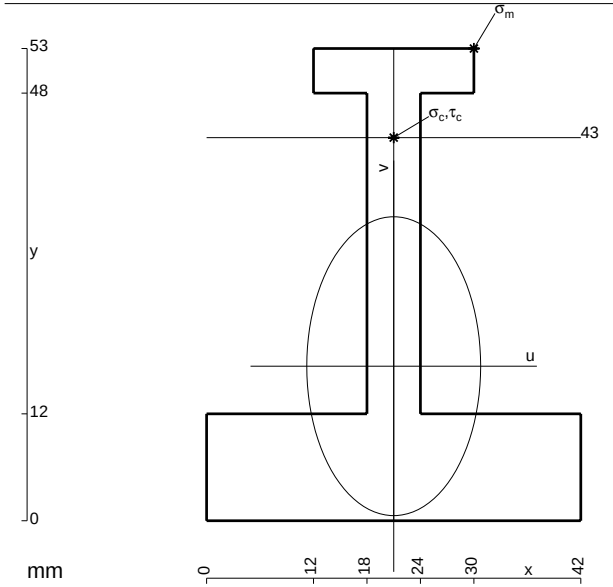

A = 810. mm^2
J_u = 227958. mm^4
J_v = 77166. mm^4
y_g = 17.34 mm
T_y = -1600. N
M_x = -1520000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 9. mm
v_m = 35.66 mm
$\sigma_m = -Mv/J_u$ = 237.7 N/mm^2
x_c = 21. mm
y_c = 43. mm
v_c = 25.66 mm
$\sigma_c = -Mv/J_u$ = 171.1 N/mm^2
τ_c = 4.479 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 171.2 N/mm^2
S^* = 3829. mm^3

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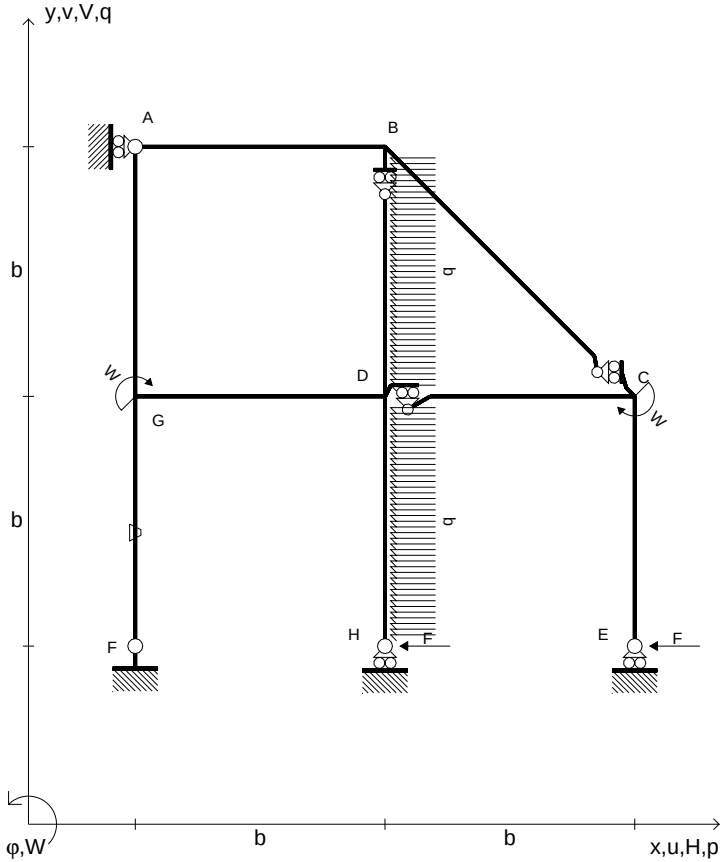
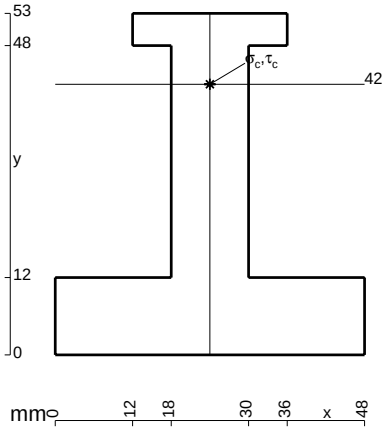
$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$p_{BD} = -q = -F/b$
$p_{DH} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_F$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 550 mm, F = 1850 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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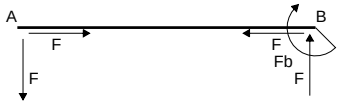

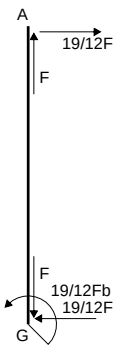

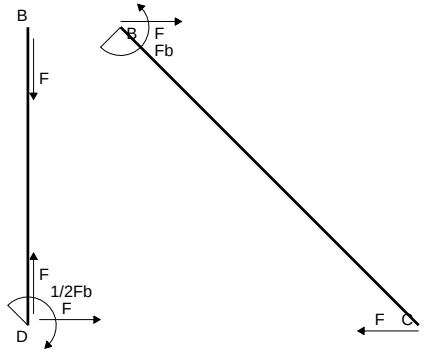

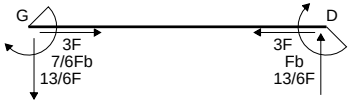

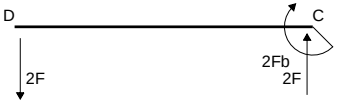

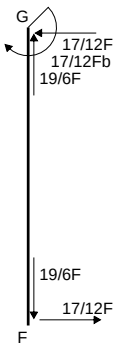

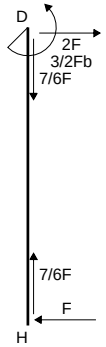

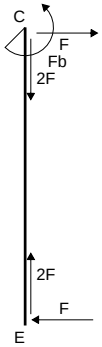

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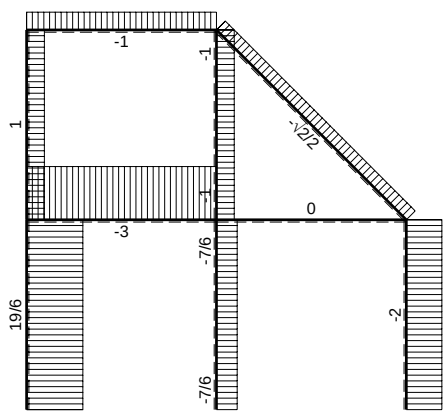
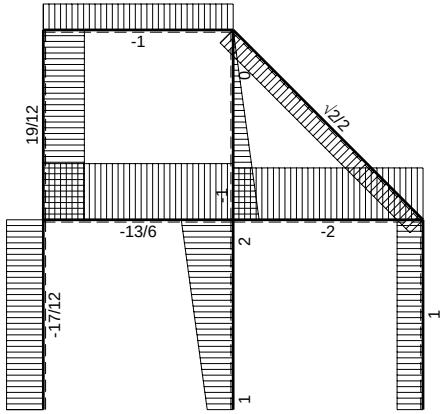
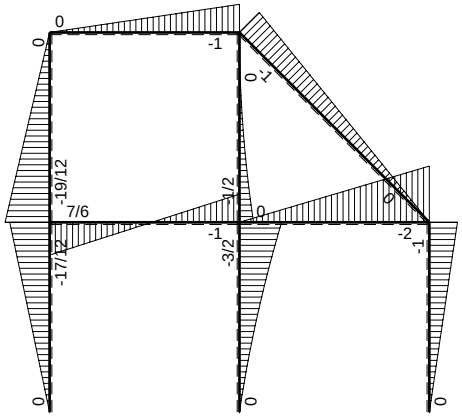


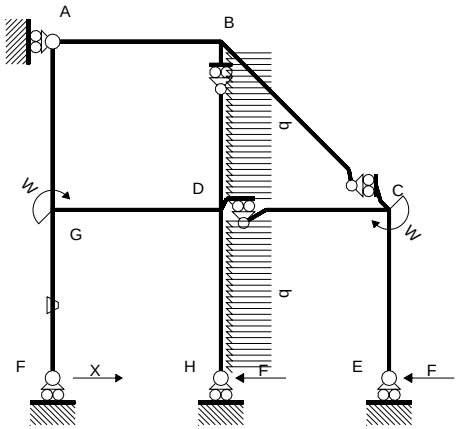
←□→ F



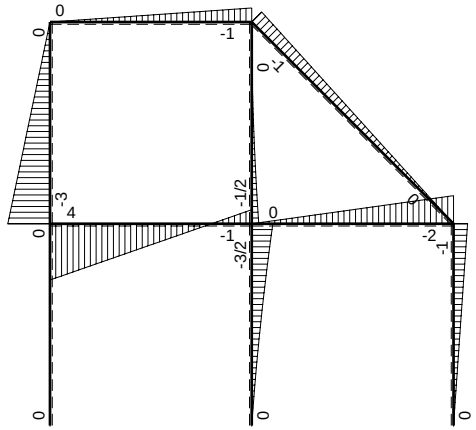
↑□↓ F



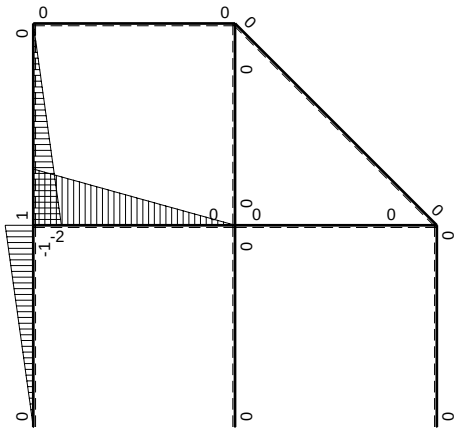
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_F$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0	0+0	0
BA b	0	Fb-Fx	0	0	0	0		
BC √2b	0	-Fb+√2/2Fx	0	0	0	0	0	0
BD b	0	$-1/2qx^2$	0	0	0	0	0+0	0
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	-2b+2x	4Fb-5Fx	0	$-8Fb^2+18Fbx-10Fx^2$	0	$4b^2-8bx+4x^2$	$(-7/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	Fb-5Fx	0	$2Fbx-10Fx^2$	0	$4x^2$		
DH b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
HD b	0	$Fx+1/2qx^2$	0	0	0	0		
GA b	b-x	-3Fb+3Fx	0	$-3Fb^2+6Fbx-3Fx^2$	0	$b^2-2bx+x^2$	$(-1+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	3Fx	0	$-3Fx^2$	0	x^2		
	totali						$-17/6Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_F$						17/12F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211730 ESΣ06.xxxx.031
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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (x/b)\, \theta\ dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1 + x/b)\, \theta\ dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-8 +18\, x/b -10\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-8\, x +9\, x^2/b -10/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-8\, b +9\, b -10/3\, b)\, Fb^2\, 1/EJ = -7/3\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (2\, x/b -10\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [\, x^2/b -10/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (\, b -10/3\, b)\, Fb^2\, 1/EJ = -7/3\ Fb^3/EJ$

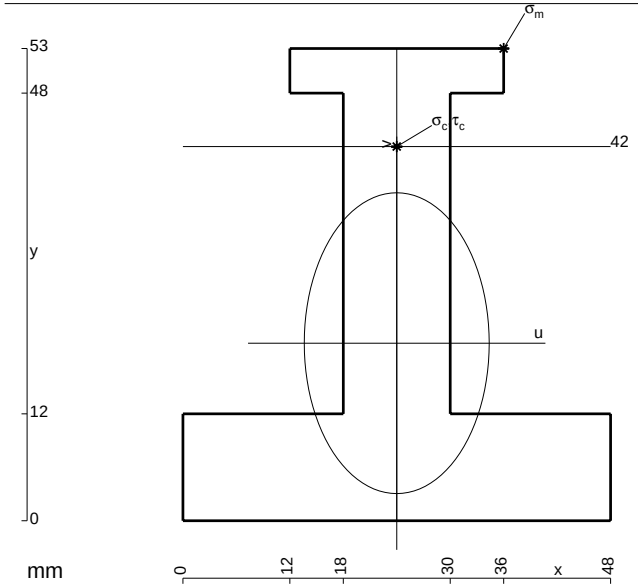
$L^{Xo}_{GA} = \int_o^b (-3 +6\, x/b -3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3\, x +3\, x^2/b - x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-3\, b +3\, b - b)\, Fb^2\, 1/EJ = - Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (- b)\, Fb^2\, 1/EJ = - Fb^3/EJ$

PROCEDIMENTO E RISULTATI 211730 ESΣ06.xxxx.031



A = 1128. mm^2
J_u = 321538. mm^4
J_v = 121536. mm^4
y_g = 19.93 mm
T_y = -3700. N
M_x = -2035000. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 12. mm
v_m = 33.07 mm
$\sigma_m = -Mv/J_u$ = 209.3 N/mm^2
x_c = 24. mm
y_c = 42. mm
v_c = 22.07 mm
$\sigma_c = -Mv/J_u$ = 139.7 N/mm^2
τ_c = 5.249 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140. N/mm^2
S^* = 5474. mm^3

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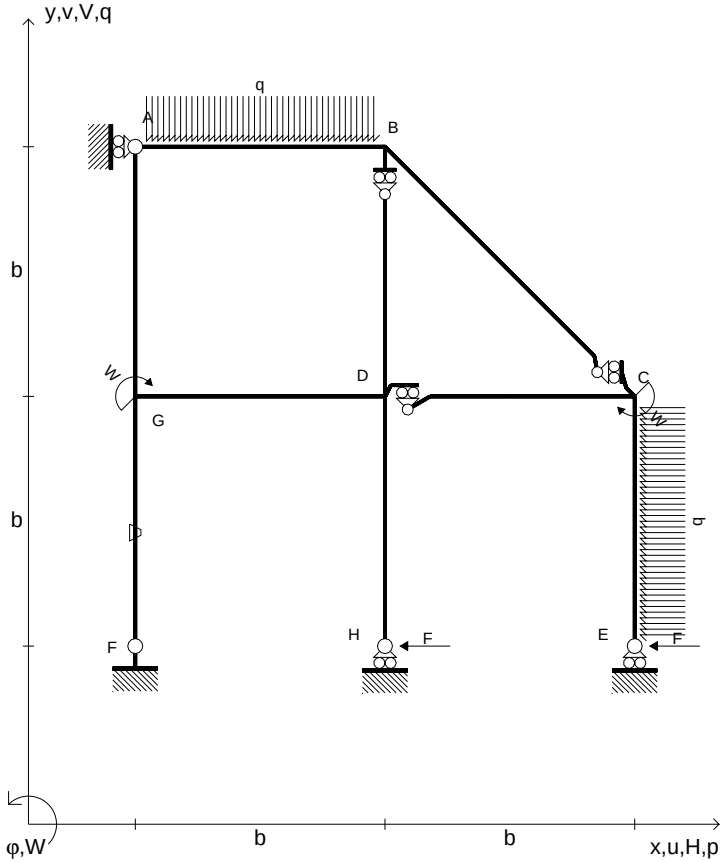
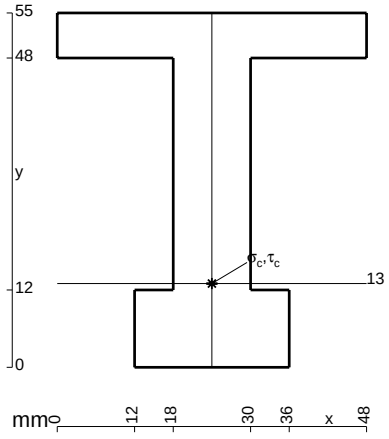
$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{CE} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_F$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 980 mm, F = 1200 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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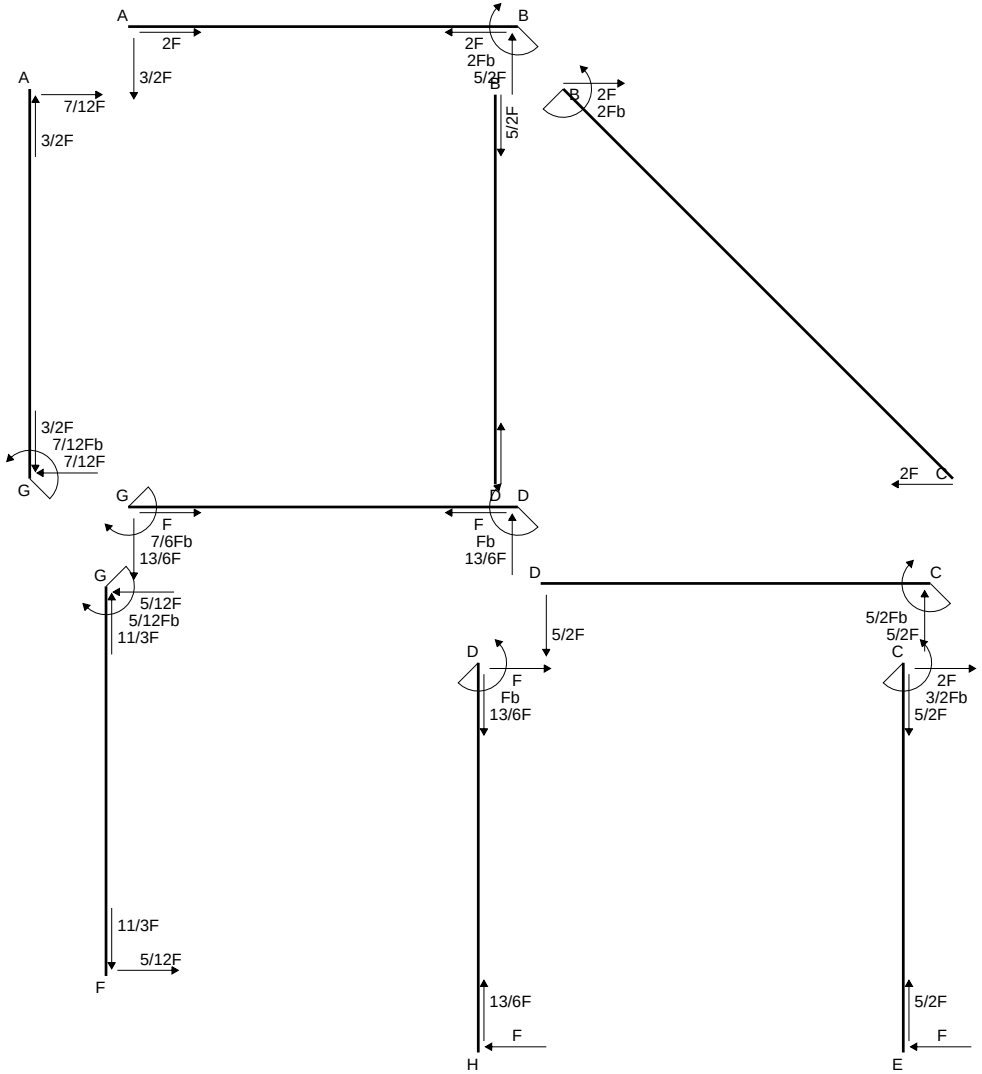

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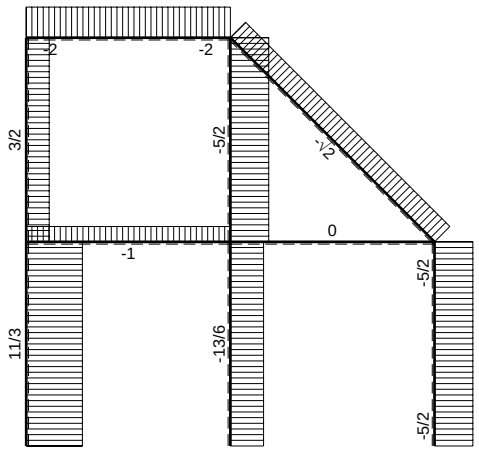
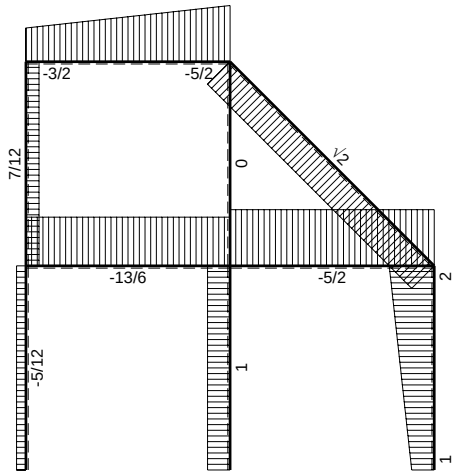
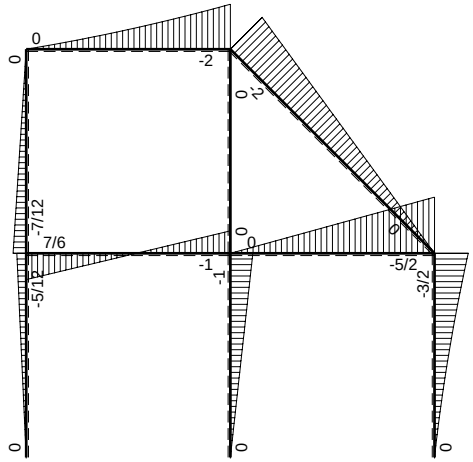


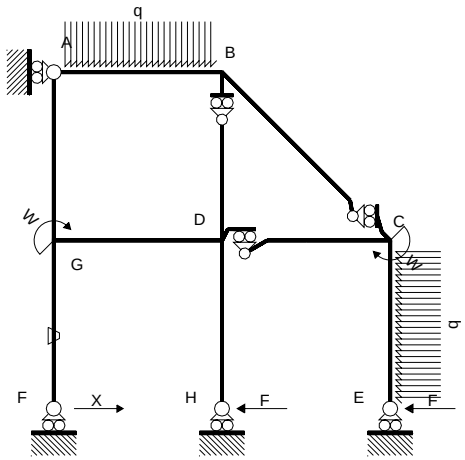
←□→ F



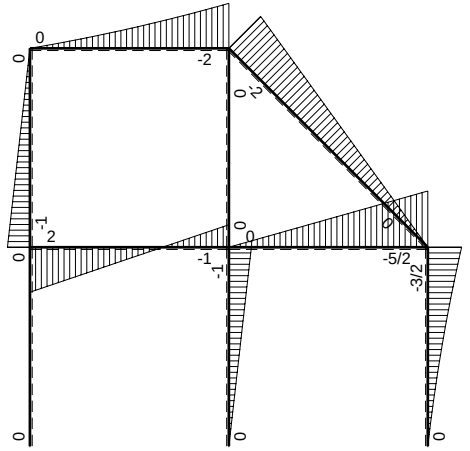
↑□↓ F



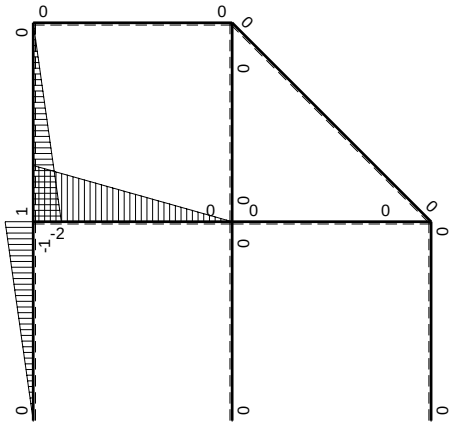
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_F$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{2}$b	0	$-2Fb+\sqrt{2}Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	$-2b+2x$	$2Fb-3Fx$	0	$-4Fb^2+10Fbx-6Fx^2$	0	$4b^2-8bx+4x^2$	$(-1+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	$Fb-3Fx$	0	$2Fbx-6Fx^2$	0	$4x^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	b-x	-Fb+Fx	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	Fx	0	$-Fx^2$	0	x^2		
	totali						$-5/6Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_F$						5/12F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213154 ESΣ06.xxxx.039
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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (x/b)\, \theta\ dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1 + x/b)\, \theta\ dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-4 +10\, x/b -6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-4\, x +5\, x^2/b -2\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-4\, b +5\, b -2\, b)\, Fb^2\, 1/EJ = - Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (2\, x/b -6\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [\, x^2/b -2\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

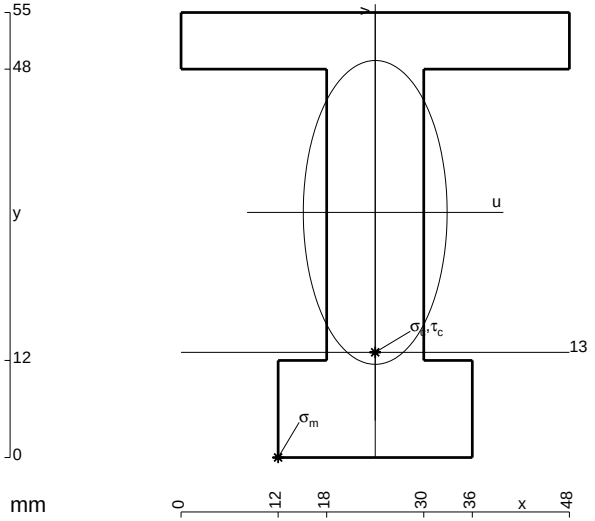
$= (\, b -2\, b)\, Fb^2\, 1/EJ = - Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1 +2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x + x^2/b -1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (- b + b -1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

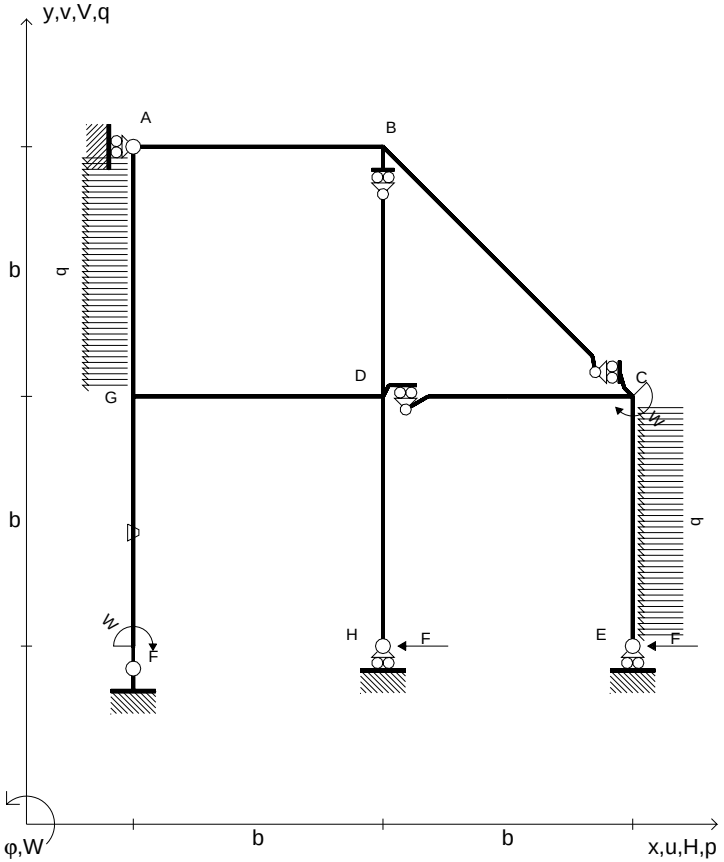
$L^{Xo}_{AG} = \int_o^b (- x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

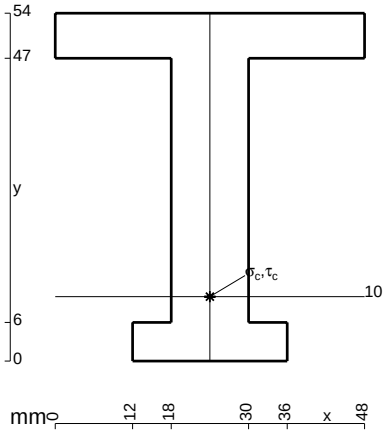
$= (-1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$



A = 1056. mm^2
J_u = 372596. mm^4
J_v = 83520. mm^4
y_g = 30.3 mm
T_y = -3000. N
M_x = -2940000. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -30.3 mm
$\sigma_m = -Mv/J_u$ = -239. N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -17.3 mm
$\sigma_c = -Mv/J_u$ = -136.5 N/mm^2
τ_c = 4.838 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 136.7 N/mm^2
S^* = 7211. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_F = -W = -Fb$
$p_{CE} = -q = -F/b$
$p_{GA} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=W_{GF}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 560 mm, F = 1530 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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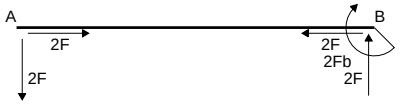

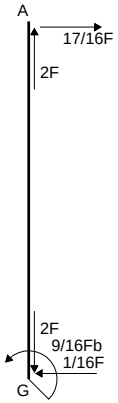

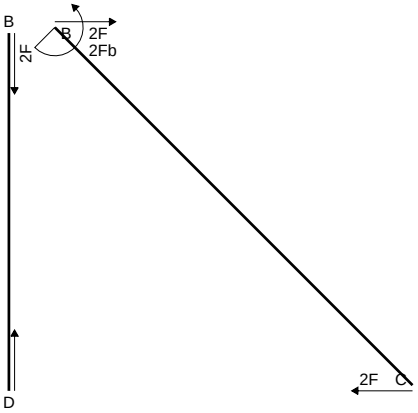

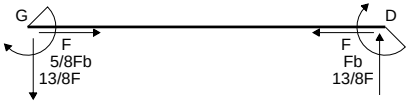

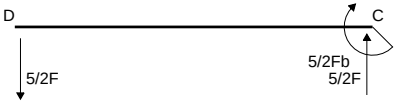

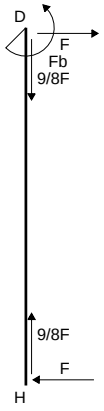

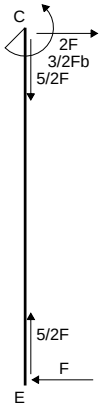

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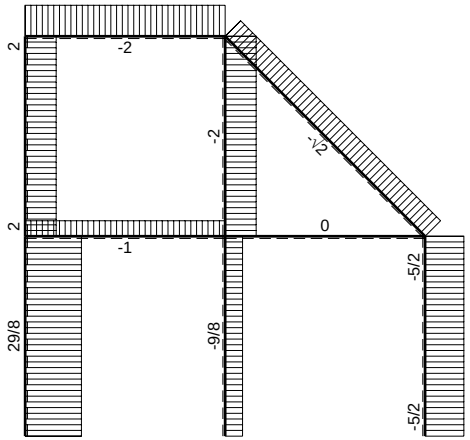
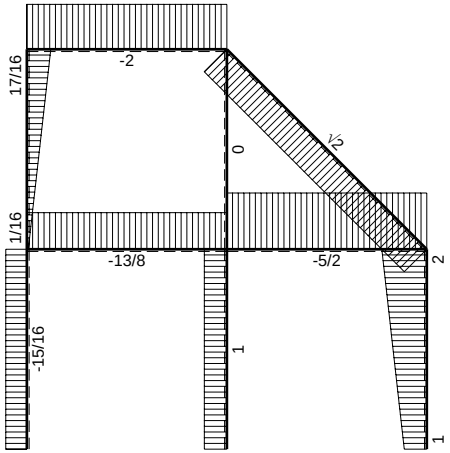
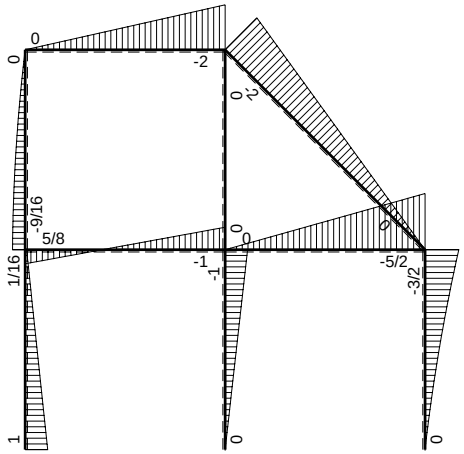


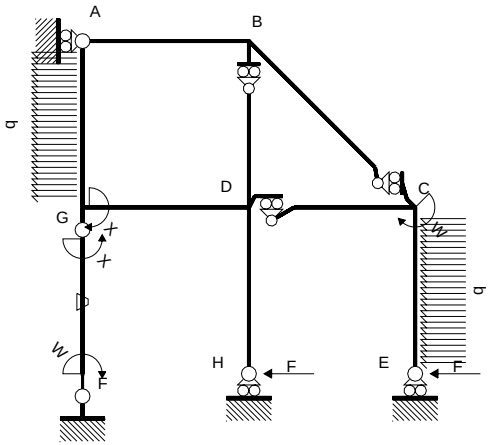
←□→ F



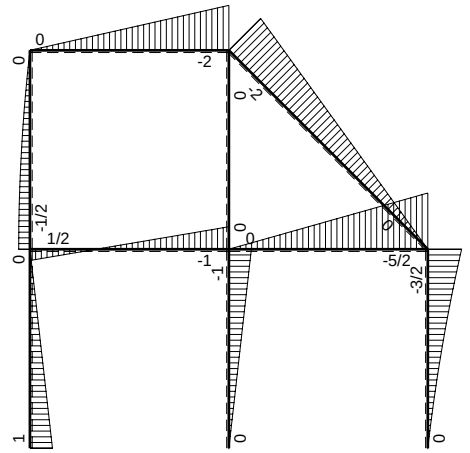
↑□↓ F



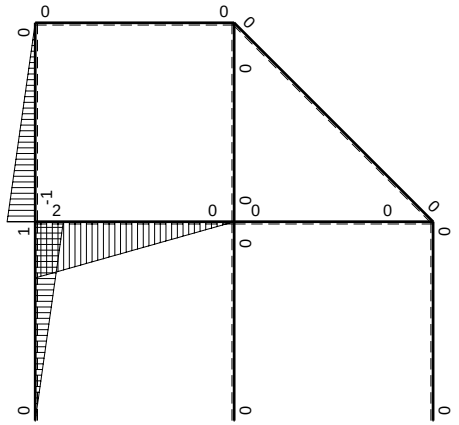
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{GF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	x/b	Fb-Fx	-Fb/EJ	$Fx-Fx^2/b$	-Fx/EJ	x^2/b^2	$(1/6-1/2)Fb^2/EJ$	1/3Xb/EJ
GF b	-1+x/b	-Fx	Fb/EJ	$Fx-Fx^2/b$	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GD b	2-2x/b	1/2Fb-3/2Fx	0	$Fb-4Fx+3Fx^2/b$	0	$4-8x/b+4x^2/b^2$	0+0	4/3Xb/EJ
DG b	-2x/b	Fb-3/2Fx	0	$-2Fx+3Fx^2/b$	0	$4x^2/b^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1+x/b	$-1/2Fb+1/2qx^2$	0	$1/2Fb-1/2Fx-1/2Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(5/24+0)Fb^2/EJ$	1/3Xb/EJ
AG b	x/b	$Fx-1/2qx^2$	0	$Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
	totali						$-1/8Fb^2/EJ$	2Xb/EJ
	iperstatica $X=W_{GF}$						1/16Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\ x/b +4\ x^2/b^2\ )\ 1/EJ\ dx = [4\ x -4\ x^2/b +4/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (4\ b -4\ b +4/3\ b\ )\ 1/EJ\ = 4/3\ b/EJ$

$L^{XX}_{DG} = \int_o^b (4\ x^2/b^2\ )\ 1/EJ\ dx = [4/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (4/3\ b\ )\ 1/EJ\ = 4/3\ b/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{AG} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{FG} = \int_o^b (\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/3\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -1/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^b (\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/3\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -1/3\ Fb^2/EJ$

$L^{Xo}_{GD} = \int_o^b (1 -4\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x -2\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b -2\ b + b\ )\ Fb\ 1/EJ\ = 0$

$L^{Xo}_{DG} = \int_o^b (-2\ x/b +3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x^2/b + x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b + b\ )\ Fb\ 1/EJ\ = 0$

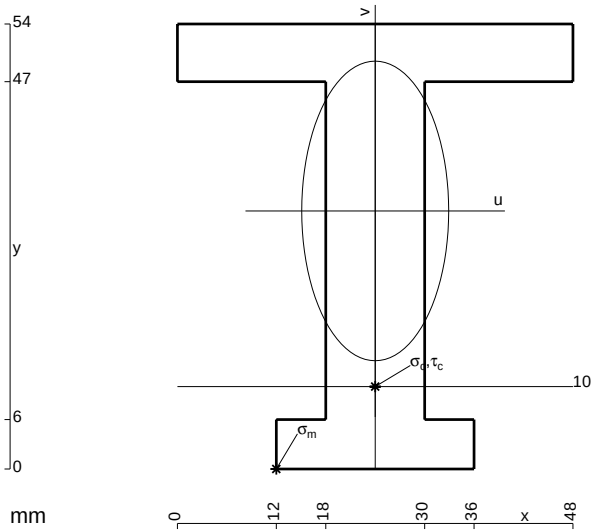
$L^{Xo}_{GA} = \int_o^b (1/2 -1/2\ x/b -1/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [1/2\ x -1/4\ x^2/b -1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/4\ b -1/6\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$

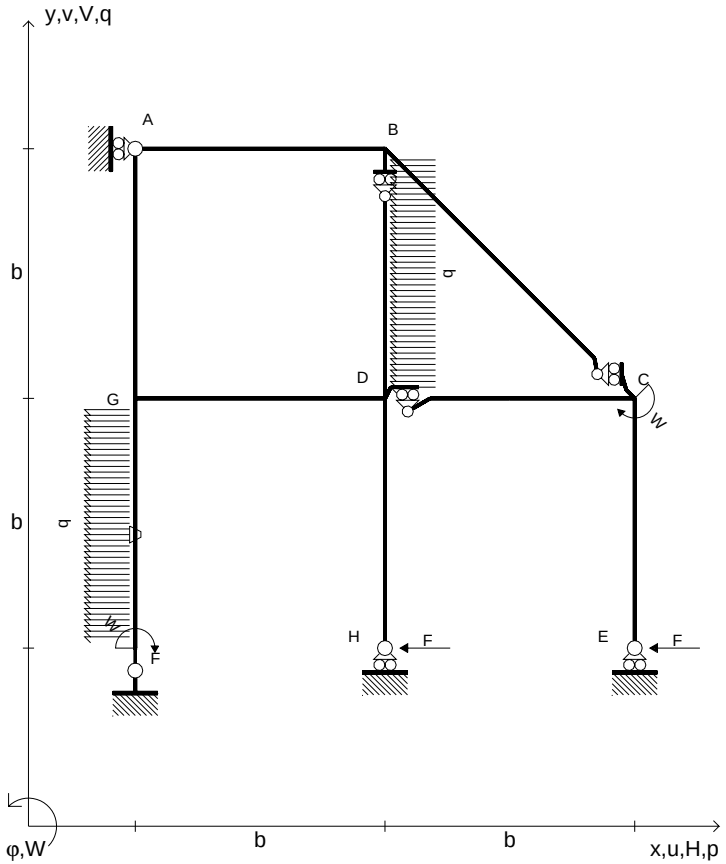
$L^{Xo}_{AG} = \int_o^b (\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

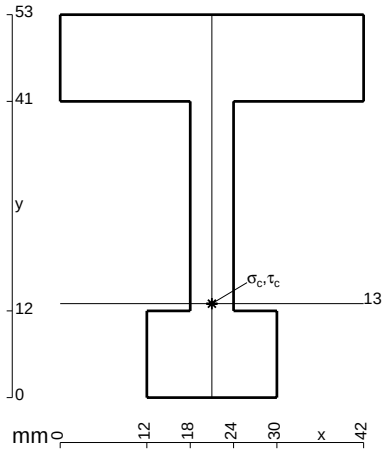
$= (1/3\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 5/24\ Fb^2/EJ$



A = 972. mm^2
J_u = 321252. mm^4
J_v = 77328. mm^4
y_g = 31.31 mm
T_y = -3825. N
M_x = -2142000. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -31.31 mm
$\sigma_m = -Mv/J_u$ = -208.8 N/mm^2
x_c = 24. mm
y_c = 10. mm
v_c = -21.31 mm
$\sigma_c = -Mv/J_u$ = -142.1 N/mm^2
τ_c = 5.156 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 142.4 N/mm^2
S^* = 5196. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_F = -W = -Fb$
$p_{BD} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 500 mm, F = 1710 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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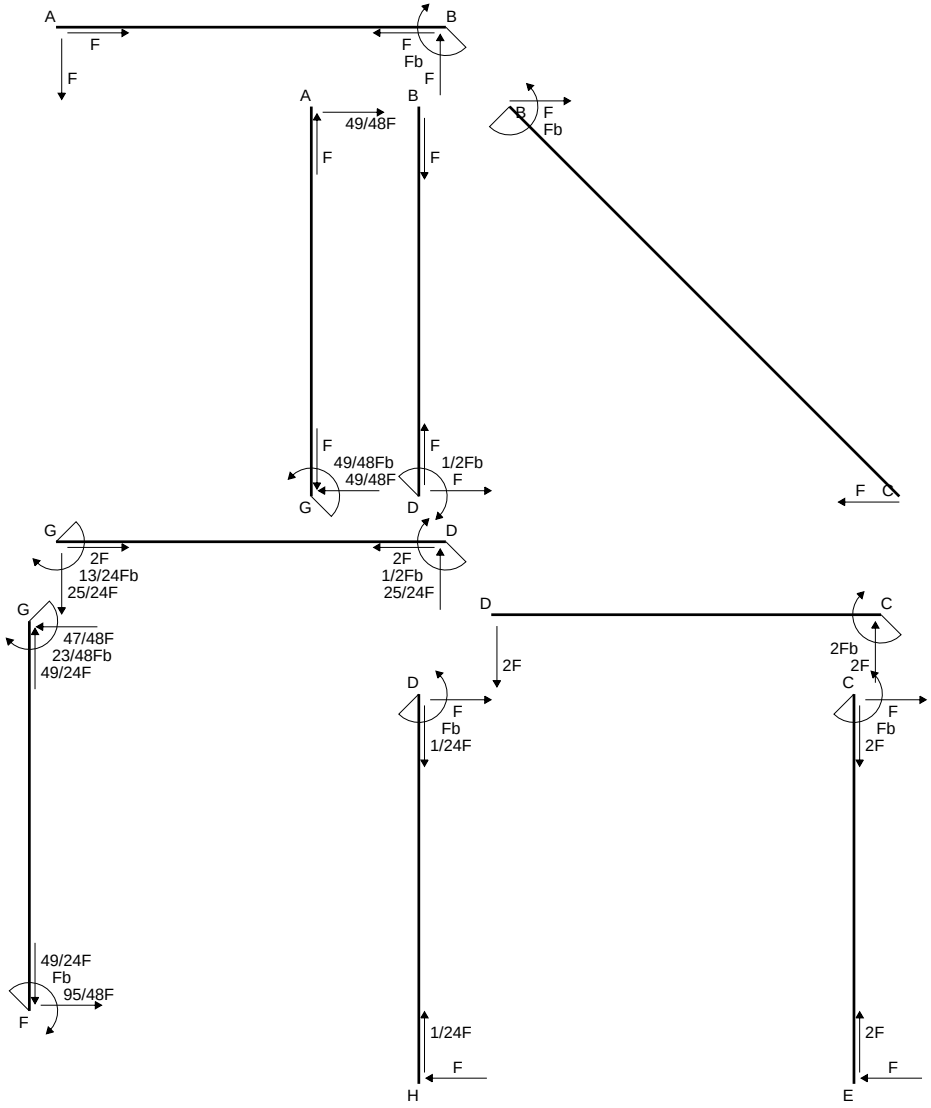

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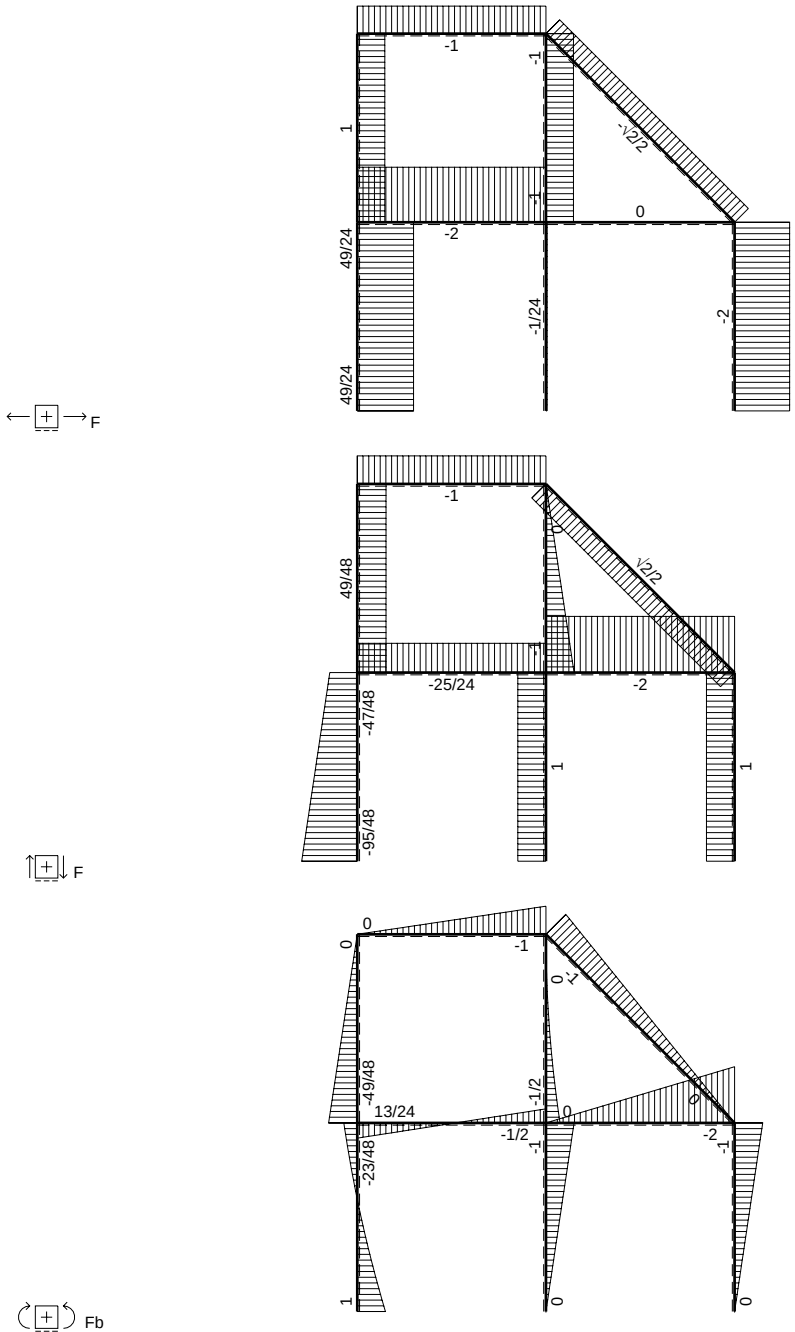
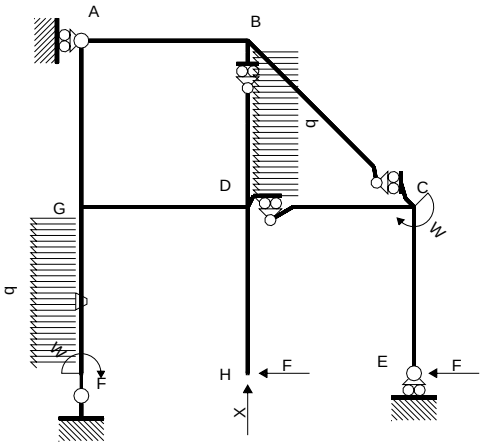
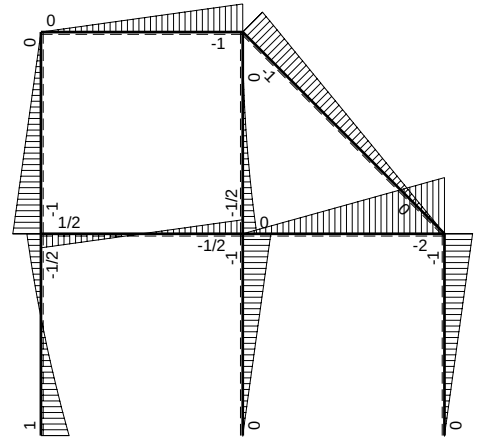
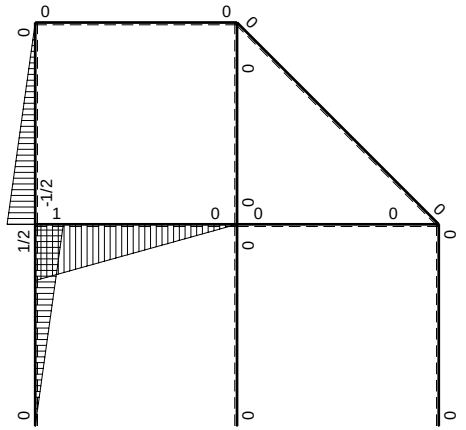




Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0	0+0	0
BA b	0	Fb-Fx	0	0	0	0		
BC √2b	0	-Fb+√2/2Fx	0	0	0	0	0	0
BD b	0	$-1/2qx^2$	0	0	0	0	0+0	0
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	1/2x	$Fb-2Fx+1/2qx^2$	-Fb/EJ	$1/2Fbx-Fx^2+1/4qx^3$	-1/2Fxb/EJ	$1/4x^2$	$(-1/48-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	$1/2Fb-Fx-1/2qx^2$	Fb/EJ	$-1/4Fb^2+3/4Fbx-1/4Fx^2-1/4qx^3$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	1/2Fb-Fx	0	$1/2Fb^2-3/2Fbx+Fx^2$	0	$b^2-2bx+x^2$	$(1/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	1/2Fb-Fx	0	$-1/2Fbx+Fx^2$	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	-Fb+Fx	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	1/2x	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
	totali						$-1/48Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						1/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (1/2\ x/b - x^2/b^2 + 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx$

$= [1/4\ x^2/b - 1/3\ x^3/b^2 + 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (1/4\ b - 1/3\ b + 1/16\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -13/48\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1/4 + 3/4\ x/b - 1/4\ x^2/b^2 - 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-1/4\ x + 3/8\ x^2/b - 1/12\ x^3/b^2 - 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-1/4\ b + 3/8\ b - 1/12\ b - 1/16\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -13/48\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (1/2 - 3/2\ x/b + x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/2\ x - 3/4\ x^2/b + 1/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (1/2\ b - 3/4\ b + 1/3\ b)\ Fb^2\ 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-1/2\ x/b + x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

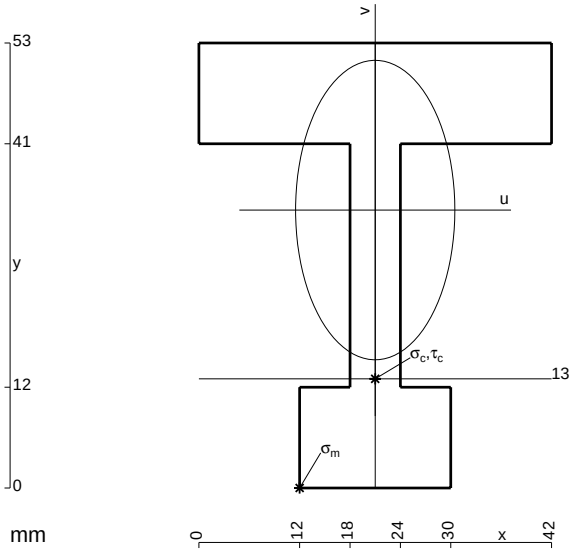
$= (-1/4\ b + 1/3\ b)\ Fb^2\ 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb^2\ 1/EJ = 1/6\ Fb^3/EJ$

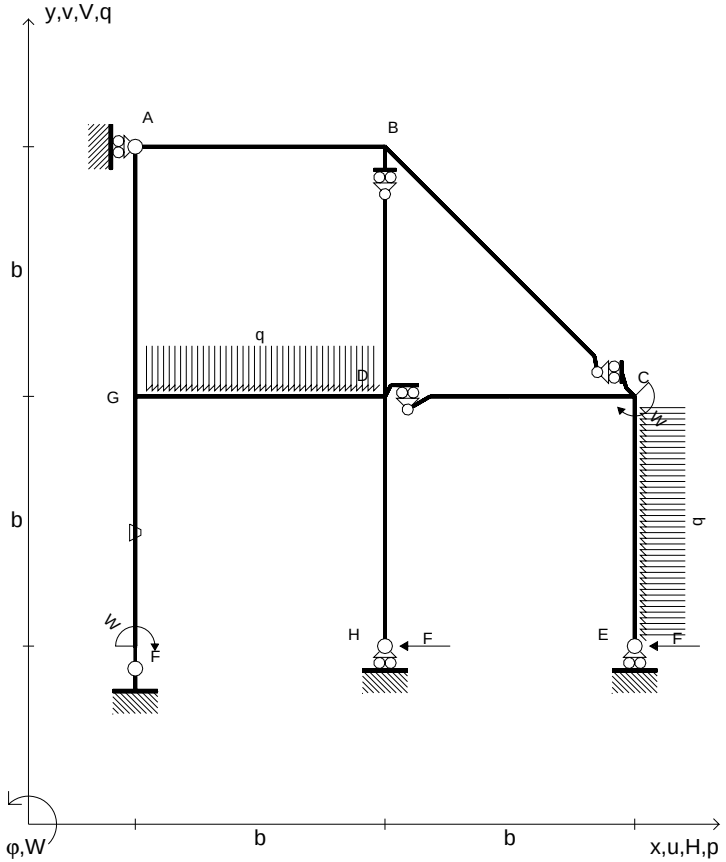
$L^{Xo}_{AG} = \int_o^b (1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

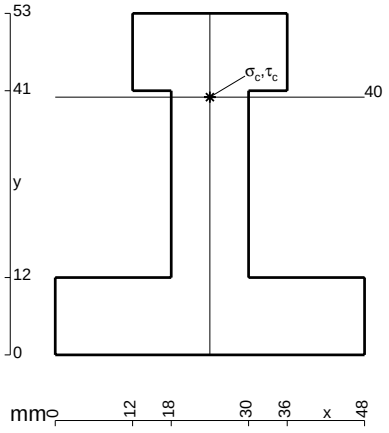
$= (1/6\ b)\ Fb^2\ 1/EJ = 1/6\ Fb^3/EJ$



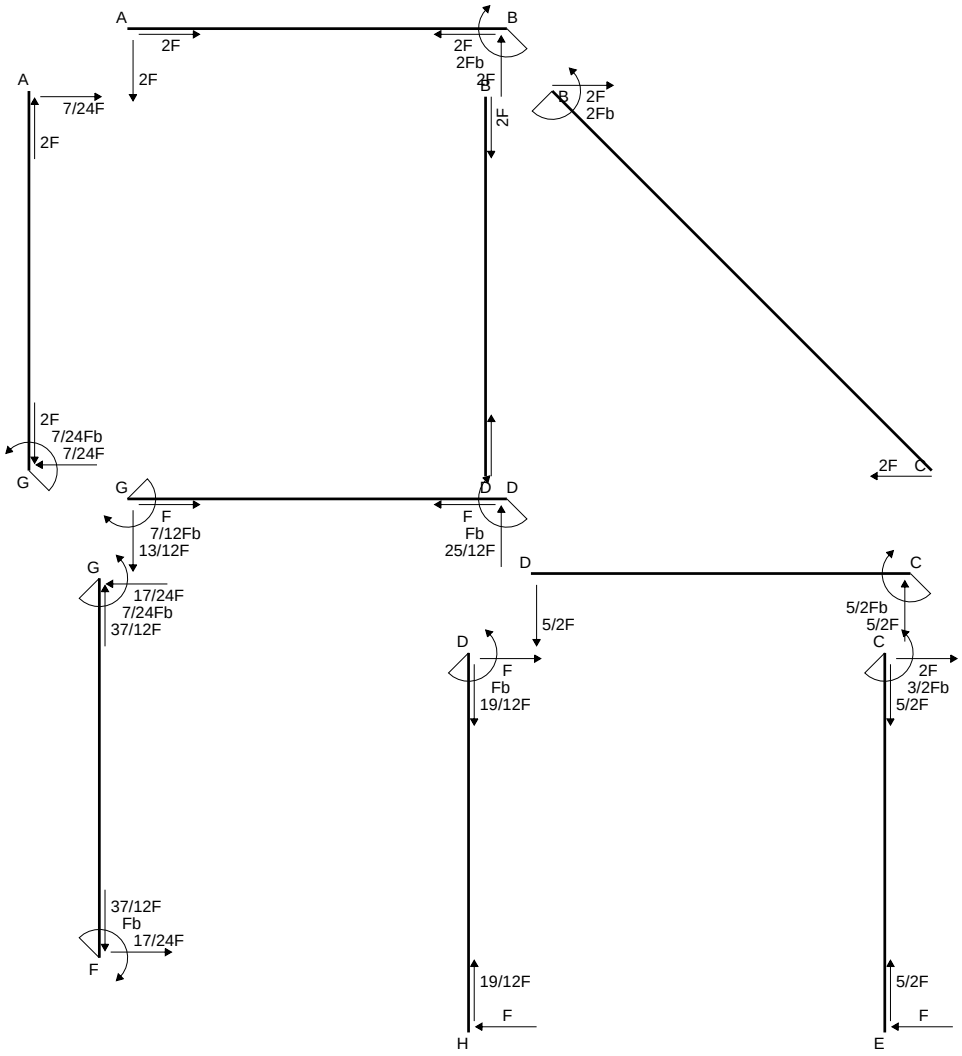
A = 894. mm^2
J_u = 284424. mm^4
J_v = 80442. mm^4
y_g = 33.1 mm
T_y = -3420. N
M_x = -1710000. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -33.1 mm
$\sigma_m = -Mv/J_u$ = -199. N/mm^2
x_c = 21. mm
y_c = 13. mm
v_c = -20.1 mm
$\sigma_c = -Mv/J_u$ = -120.9 N/mm^2
τ_c = 11.98 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 122.6 N/mm^2
S^* = 5978. mm^3

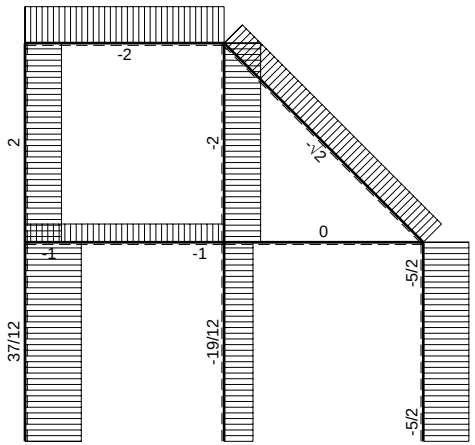
$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_F = -W = -Fb$
$p_{CE} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$



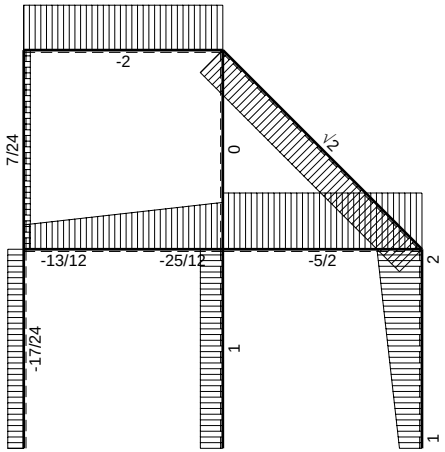


Reazioni iperstatiche in soluzione: $X=W_{GA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 650 mm, F = 1660 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

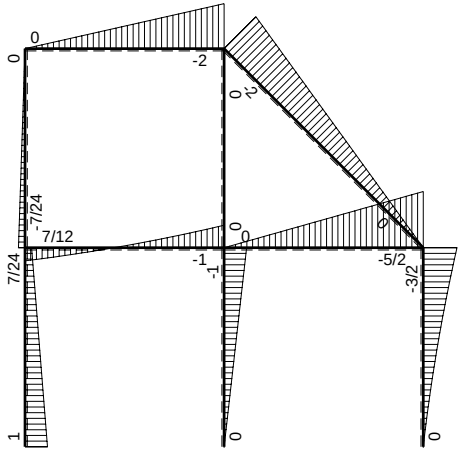




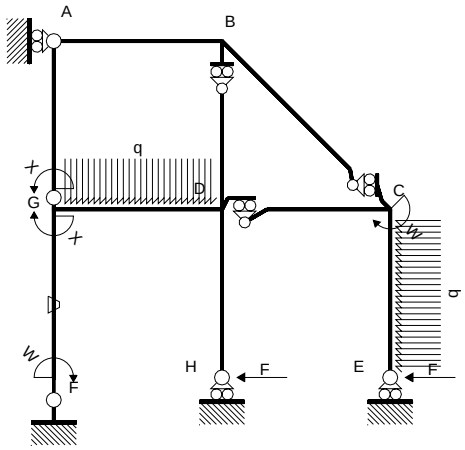
←[+]→ F



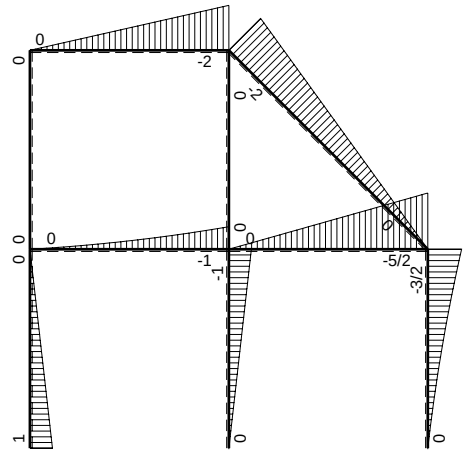
↑[+]↓ F



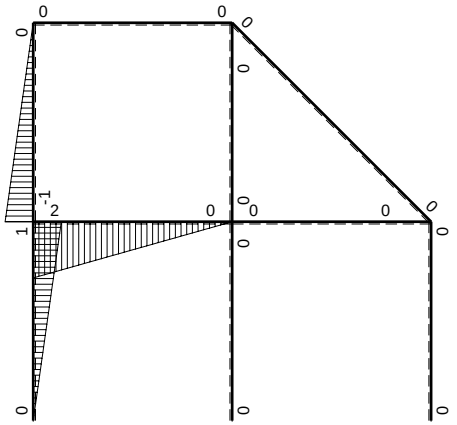
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	x/b	Fb-Fx	-Fb/EJ	$Fx-Fx^2/b$	-Fx/EJ	x^2/b^2	$(1/6-1/2)Fb^2/EJ$	1/3Xb/EJ
GF b	-1+x/b	-Fx	Fb/EJ	$Fx-Fx^2/b$	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$		
GD b	2-2x/b	$-1/2Fx-1/2qx^2$	0	$-Fx+qx^3/b$	0	$4-8x/b+4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	4/3Xb/EJ
DG b	-2x/b	$Fb-3/2Fx+1/2qx^2$	0	$-2Fx+3Fx^2/b-qx^3/b$	0	$4x^2/b^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
AG b	x/b	0	0	0	0	x^2/b^2		
	totali						$-7/12Fb^2/EJ$	2Xb/EJ
	iperstatica $X=W_{GA}$						7/24Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\ x/b +4\ x^2/b^2)\ 1/EJ\ dx = [4\ x -4\ x^2/b +4/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (4\ b -4\ b +4/3\ b\ )\ 1/EJ\ = 4/3\ b/EJ$

$L^{XX}_{DG} = \int_o^b (4\ x^2/b^2)\ 1/EJ\ dx = [4/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (4/3\ b)\ 1/EJ\ = 4/3\ b/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{FG} = \int_o^b (x/b - x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (- x/b)\ \theta\ dx$

$= [1/2\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/3\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b\ )\ \theta\ = -1/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^b (x/b - x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 - x/b)\ \theta\ dx$

$= [1/2\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

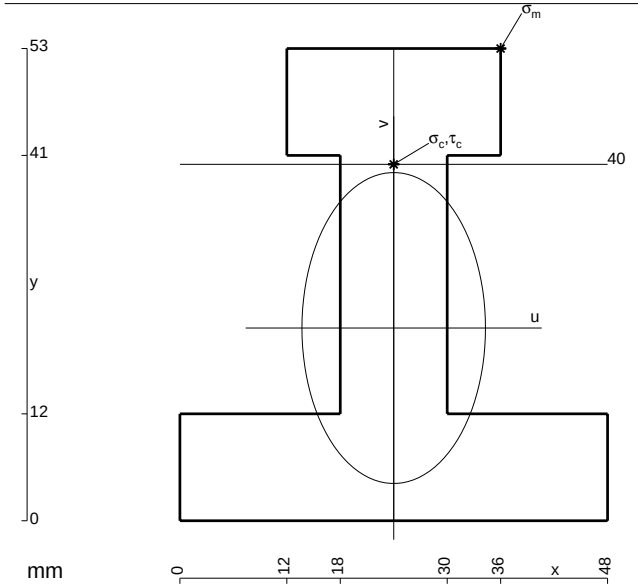
$= (1/2\ b -1/3\ b\ )\ Fb\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ = -1/3\ Fb^2/EJ$

$L^{Xo}_{GD} = \int_o^b (- x/b + x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +1/4\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$

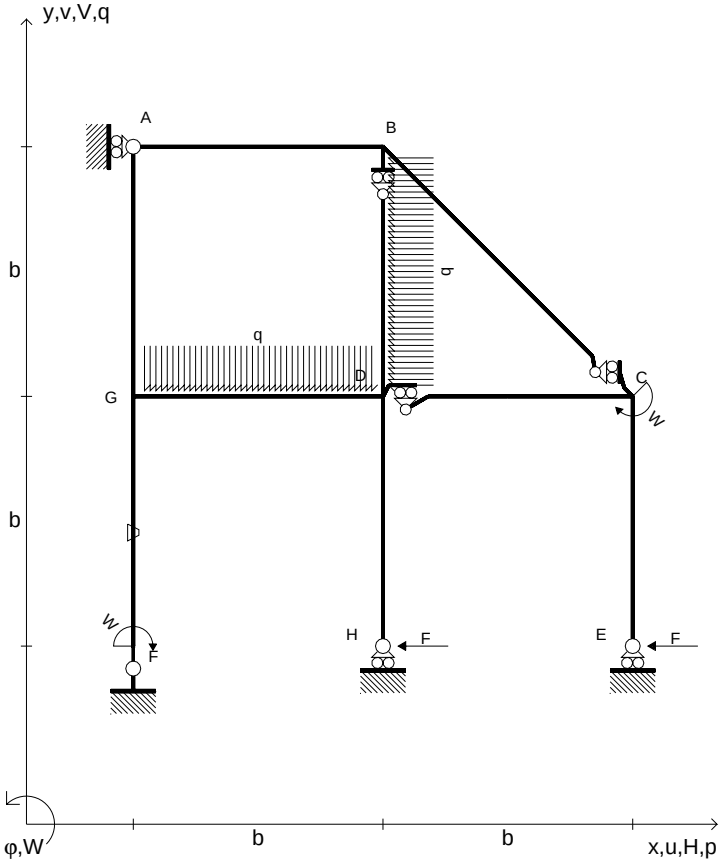
$L^{Xo}_{DG} = \int_o^b (-2\ x/b +3\ x^2/b^2 - x^3/b^3)\ Fb\ 1/EJ\ dx = [- x^2/b + x^3/b^2 -1/4\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

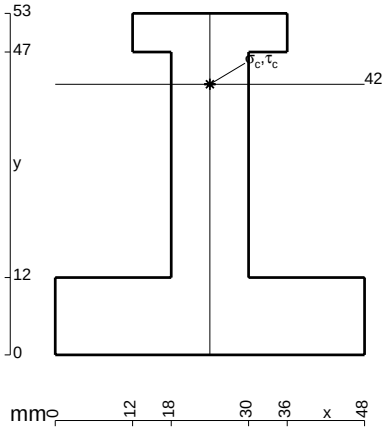
$= (-\ b + b -1/4\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$



A = 1212. mm^2
J_u = 369093. mm^4
J_v = 128592. mm^4
y_g = 21.63 mm
T_y = -4150. N
M_x = -2697500. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 12. mm
v_m = 31.37 mm
$\sigma_m = -Mv/J_u$ = 229.3 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 18.37 mm
$\sigma_c = -Mv/J_u$ = 134.3 N/mm^2
τ_c = 7.059 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.8 N/mm^2
S^* = 7533. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_F = -W = -Fb$
$p_{BD} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 950 mm, F = 1270 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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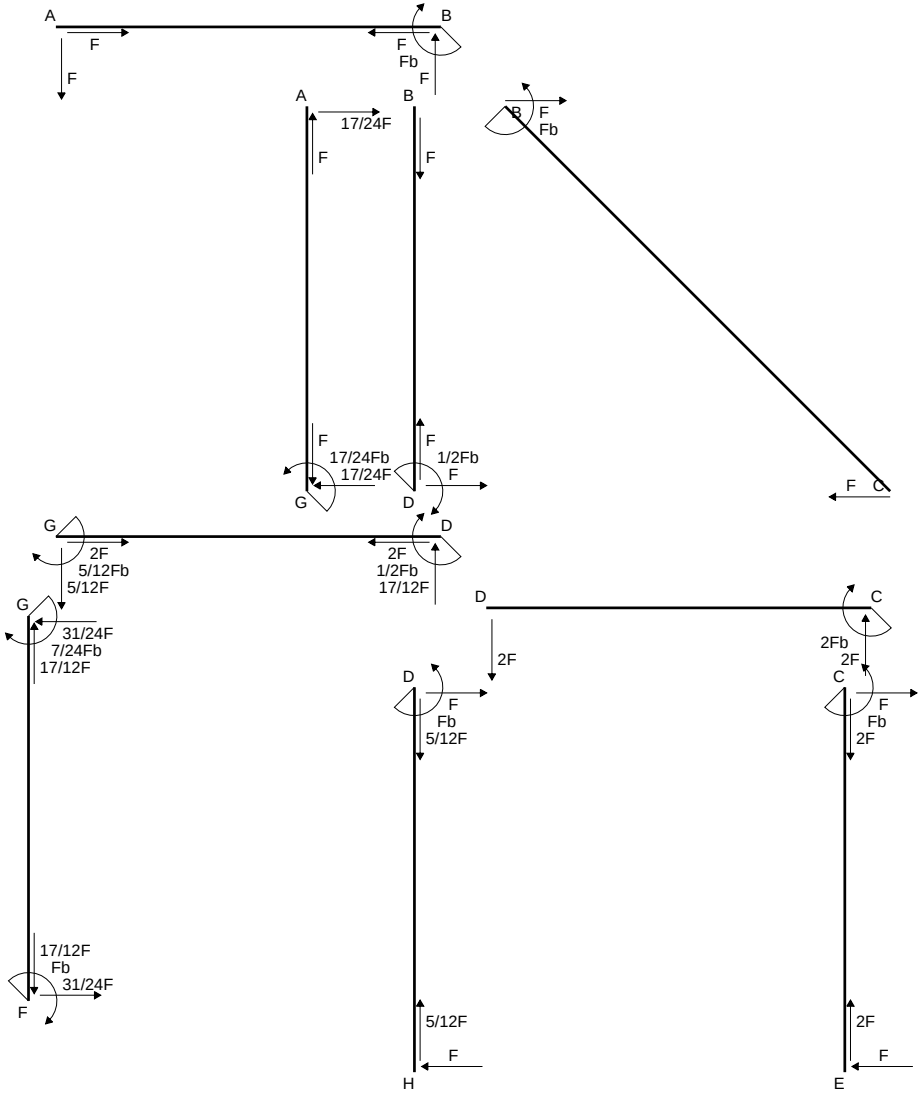

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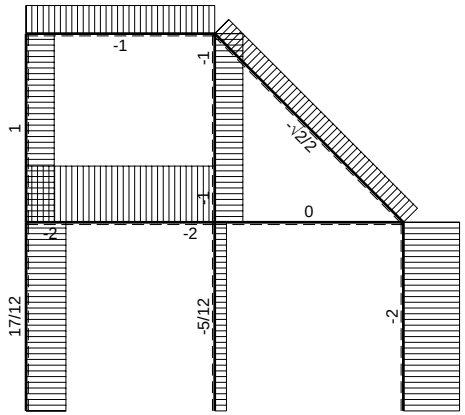
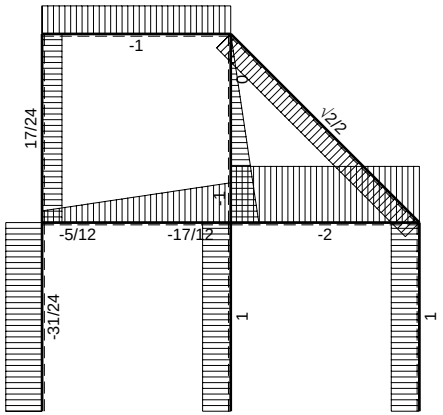
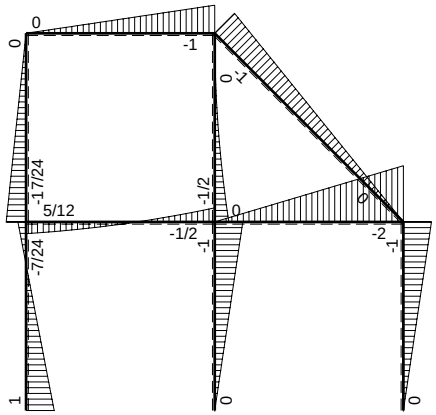


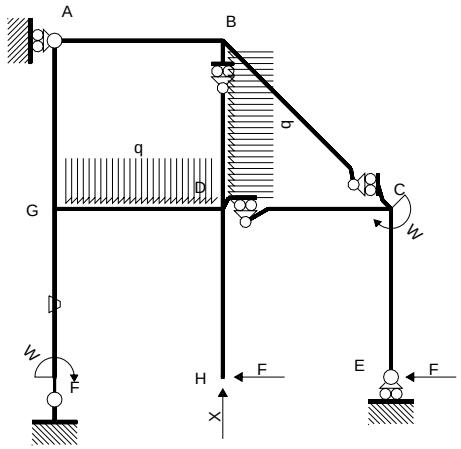
F



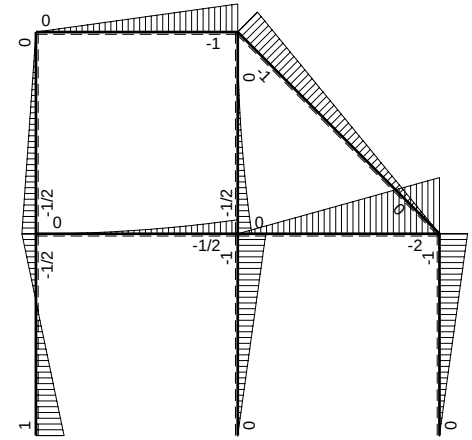
F



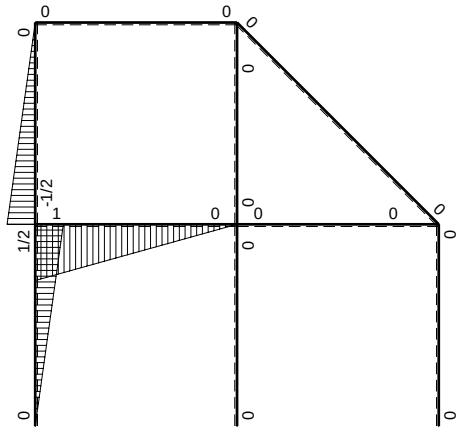
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0	0+0	0
BA b	0	Fb-Fx	0	0	0	0		
BC $\sqrt{2}$b	0	-Fb+$\sqrt{2}$/2Fx	0	0	0	0	0	0
BD b	0	-1/2qx^2	0	0	0	0	0+0	0
DB b	0	1/2Fb-Fx+1/2qx^2	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	1/2x	Fb-3/2Fx	-Fb/EJ	1/2Fbx-3/4Fx^2	-1/2Fxb/EJ	1/4x^2	(0-1/4)Fb^3/EJ	1/12Xb^3/EJ
GF b	-1/2b+1/2x	1/2Fb-3/2Fx	Fb/EJ	-1/4Fb^2+Fbx-3/4Fx^2	-1/2Fb^2/EJ+1/2Fxb/EJ	1/4b^2-1/2bx+1/4x^2		
GD b	b-x	-1/2qx^2	0	-1/2Fx^2+1/2qx^3	0	b^2-2bx+x^2	(-1/24+0)Fb^3/EJ	1/3Xb^3/EJ
DG b	-x	1/2Fb-Fx+1/2qx^2	0	-1/2Fbx+Fx^2-1/2qx^3	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb^2-1/2Fbx+1/4Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/12+0)Fb^3/EJ	1/12Xb^3/EJ
AG b	1/2x	1/2Fx	0	1/4Fx^2	0	1/4x^2		
	totali						-5/24Fb^3/EJ	1/2Xb^3/EJ
	iperstatica $X=V_H$						5/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (1/2\ x/b - 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx$

$= [1/4\ x^2/b - 1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (1/4\ b - 1/4\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -1/4\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1/4 + x/b - 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-1/4\ x + 1/2\ x^2/b - 1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-1/4\ b + 1/2\ b - 1/4\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -1/4\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-1/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-1/6\ x^3/b^2 + 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (-1/6\ b + 1/8\ b)\ Fb^2\ 1/EJ = -1/24\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-1/2\ x/b + x^2/b^2 - 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2 - 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

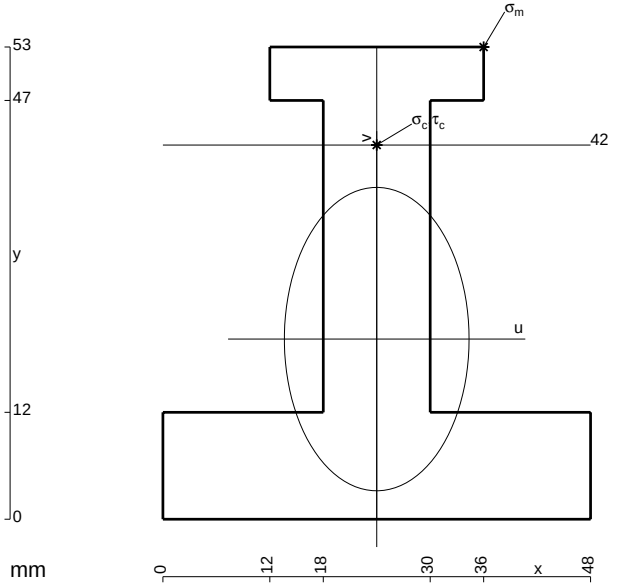
$= (-1/4\ b + 1/3\ b - 1/8\ b)\ Fb^2\ 1/EJ = -1/24\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ Fb^2\ 1/EJ = 1/12\ Fb^3/EJ$

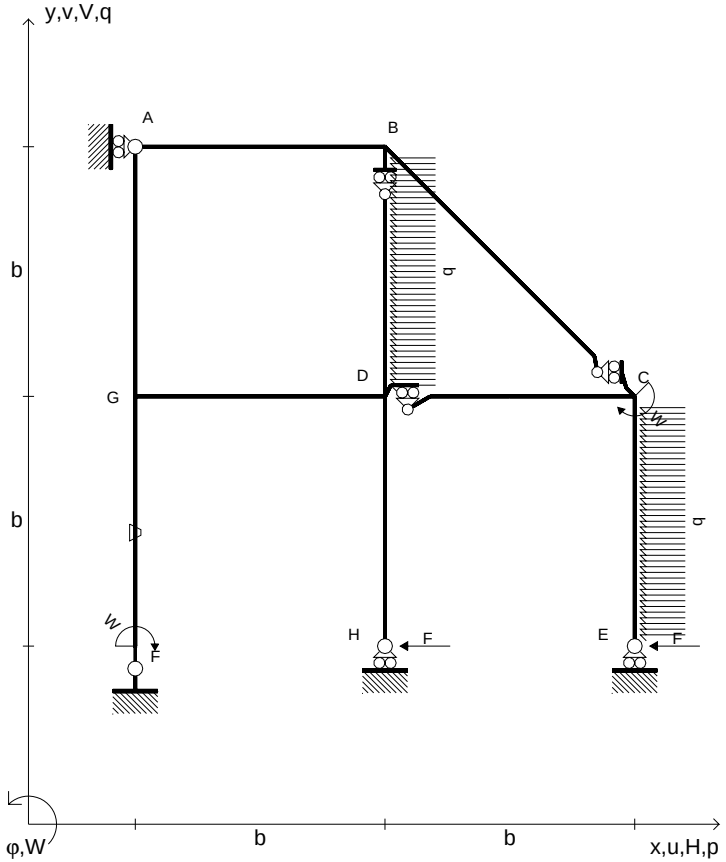
$L^{Xo}_{AG} = \int_o^b (1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

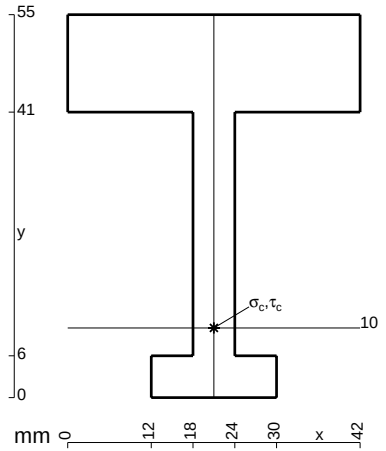
$= (1/12\ b)\ Fb^2\ 1/EJ = 1/12\ Fb^3/EJ$



A = 1140. mm^2
J_u = 330567. mm^4
J_v = 122544. mm^4
y_g = 20.22 mm
T_y = -2540. N
M_x = -2413000. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 12. mm
v_m = 32.78 mm
$\sigma_m = -Mv/J_u$ = 239.3 N/mm^2
x_c = 24. mm
y_c = 42. mm
v_c = 21.78 mm
$\sigma_c = -Mv/J_u$ = 159. N/mm^2
τ_c = 3.679 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 159.1 N/mm^2
S^* = 5746. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_F = -W = -Fb$
$p_{BD} = -q = -F/b$
$p_{CE} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 620 mm, F = 1020 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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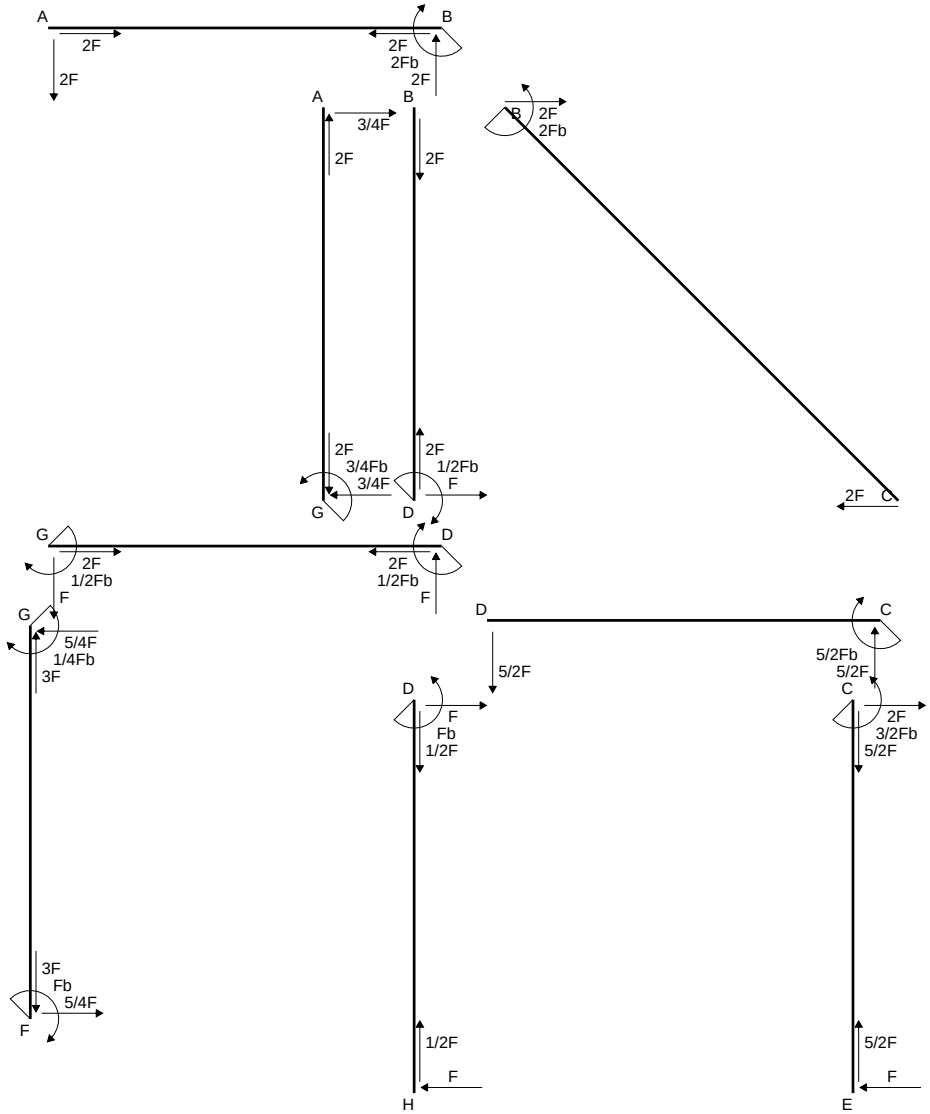

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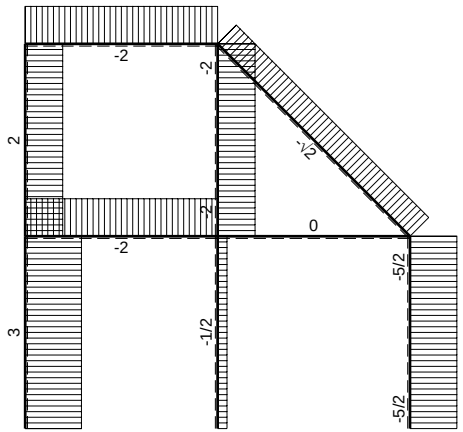
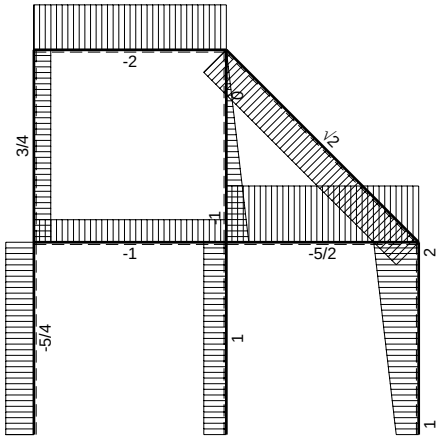
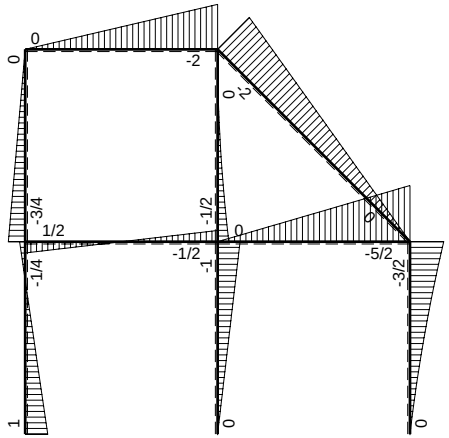


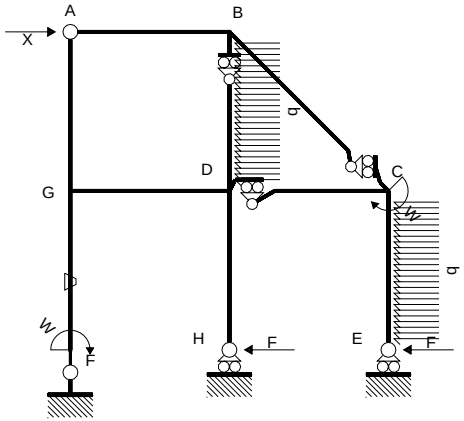
←□→ F



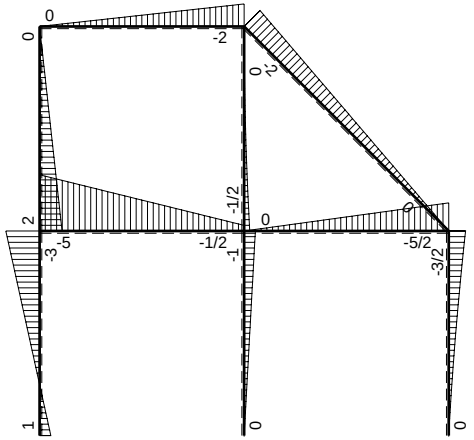
↑□↓ F



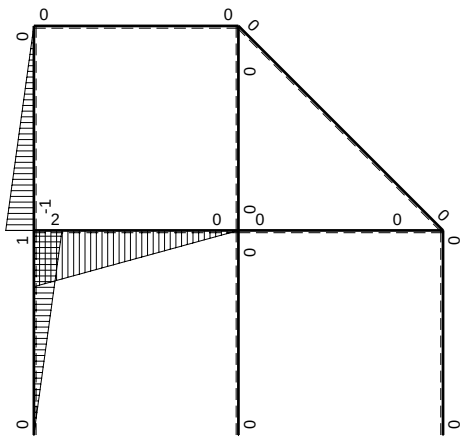
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	$-1/2qx^2$	0	0	0	0	0+0	0
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	x	Fb-4Fx	-Fb/EJ	$Fbx-4Fx^2$	-Fxb/EJ	x^2	$(-5/6-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	-b+x	3Fb-4Fx	Fb/EJ	$-3Fb^2+7Fbx-4Fx^2$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$		
GD b	2b-2x	-5Fb+9/2Fx	0	$-10Fb^2+19Fbx-9Fx^2$	0	$4b^2-8bx+4x^2$	$(-7/2+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	-2x	1/2Fb+9/2Fx	0	$-Fbx-9Fx^2$	0	$4x^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-b+x	2Fb-2Fx	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	-2Fx	0	$-2Fx^2$	0	x^2		
	totali						$-11/2Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_A$						11/4F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211841 ESΣ06.xxxx.082
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$$L^{XX}_{FG} = \int_o^b \left(x^2/b^2 \right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$
$$= \left(1/3\, b \right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{GF} = \int_o^b \left(1 -2\, x/b + x^2/b^2 \right) b^2\, 1/EJ\, dx = \left[x - x^2/b + 1/3\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$
$$= \left(b - b + 1/3\, b \right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{GD} = \int_o^b \left(4 -8\, x/b + 4\, x^2/b^2 \right) b^2\, 1/EJ\, dx = \left[4\, x -4\, x^2/b + 4/3\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$
$$= \left(4\, b -4\, b + 4/3\, b \right) b^2\, 1/EJ = 4/3\ b^3/EJ$$

$$L^{XX}_{DG} = \int_o^b \left(4\, x^2/b^2 \right) b^2\, 1/EJ\, dx = \left[4/3\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$
$$= \left(4/3\, b \right) b^2\, 1/EJ = 4/3\ b^3/EJ$$

$$L^{XX}_{GA} = \int_o^b \left(1 -2\, x/b + x^2/b^2 \right) b^2\, 1/EJ\, dx = \left[x - x^2/b + 1/3\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$
$$= \left(b - b + 1/3\, b \right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{AG} = \int_o^b \left(x^2/b^2 \right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$
$$= \left(1/3\, b \right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{Xo}_{FG} = \int_o^b \left(x/b -4\, x^2/b^2 \right) Fb^2\, 1/EJ\, dx + \int_o^b \left(- x/b \right) \theta\ dx$$
$$= \left[1/2\, x^2/b -4/3\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ + \left[-1/2\, x^2/b \right]_o^b \theta$$
$$= \left(1/2\, b -4/3\, b \right) Fb^2\, 1/EJ + \left(-1/2\, b \right) \theta = -4/3\ Fb^3/EJ$$

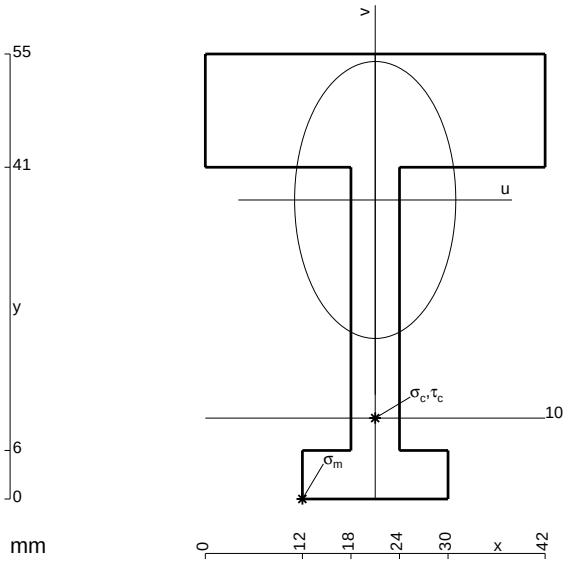
$$L^{Xo}_{GF} = \int_o^b \left(-3 +7\, x/b -4\, x^2/b^2 \right) Fb^2\, 1/EJ\, dx + \int_o^b \left(1 - x/b \right) \theta\ dx$$
$$= \left[-3\, x +7/2\, x^2/b -4/3\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ + \left[x -1/2\, x^2/b \right]_o^b \theta$$
$$= \left(-3\, b +7/2\, b -4/3\, b \right) Fb^2\, 1/EJ + \left(b -1/2\, b \right) \theta = -4/3\ Fb^3/EJ$$

$$L^{Xo}_{GD} = \int_o^b \left(-10 +19\, x/b -9\, x^2/b^2 \right) Fb^2\, 1/EJ\, dx = \left[-10\, x +19/2\, x^2/b -3\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ$$
$$= \left(-10\, b +19/2\, b -3\, b \right) Fb^2\, 1/EJ = -7/2\ Fb^3/EJ$$

$$L^{Xo}_{DG} = \int_o^b \left(- x/b -9\, x^2/b^2 \right) Fb^2\, 1/EJ\, dx = \left[-1/2\, x^2/b -3\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ$$
$$= \left(-1/2\, b -3\, b \right) Fb^2\, 1/EJ = -7/2\ Fb^3/EJ$$

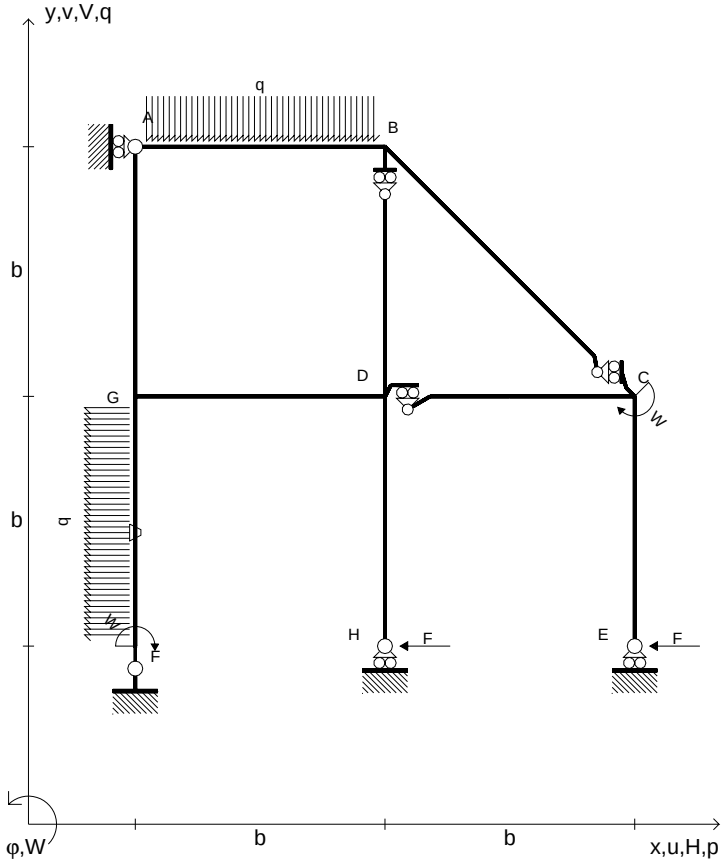
$$L^{Xo}_{GA} = \int_o^b \left(-2 +4\, x/b -2\, x^2/b^2 \right) Fb^2\, 1/EJ\, dx = \left[-2\, x +2\, x^2/b -2/3\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ$$
$$= \left(-2\, b +2\, b -2/3\, b \right) Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

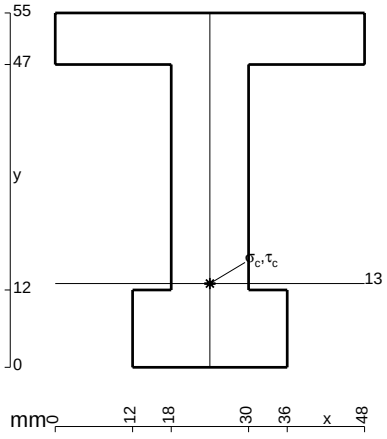
$$L^{Xo}_{AG} = \int_o^b \left(-2\, x^2/b^2 \right) Fb^2\, 1/EJ\, dx = \left[-2/3\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ$$
$$= \left(-2/3\, b \right) Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$



A = 906. mm^2
J_u = 265632. mm^4
J_v = 89982. mm^4
y_g = 36.96 mm
T_y = -2550. N
M_x = -1581000. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -36.96 mm
$\sigma_m = -Mv/J_u$ = -220. N/mm^2
x_c = 21. mm
y_c = 10. mm
v_c = -26.96 mm
$\sigma_c = -Mv/J_u$ = -160.4 N/mm^2
τ_c = 6.98 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 160.9 N/mm^2
S^* = 4362. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_F = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_F$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 880 mm, F = 1550 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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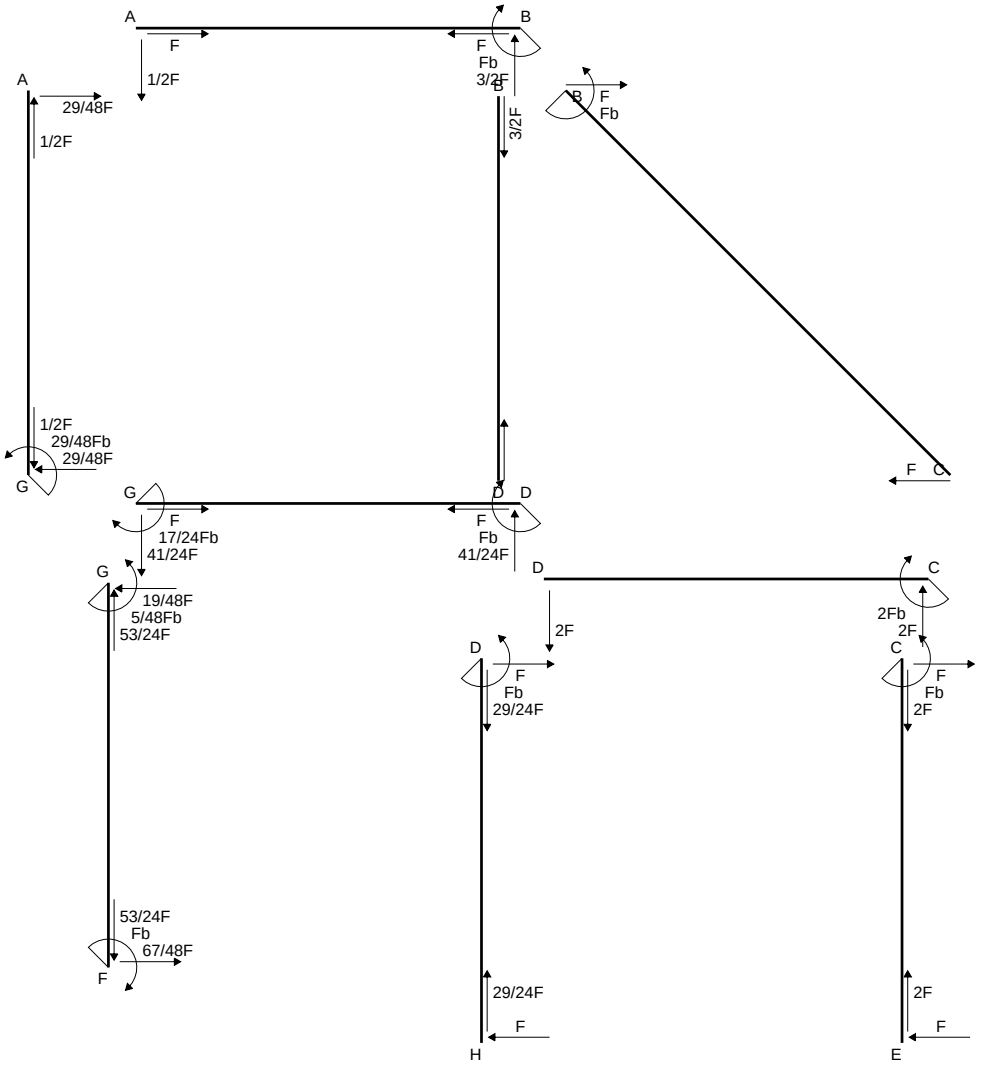

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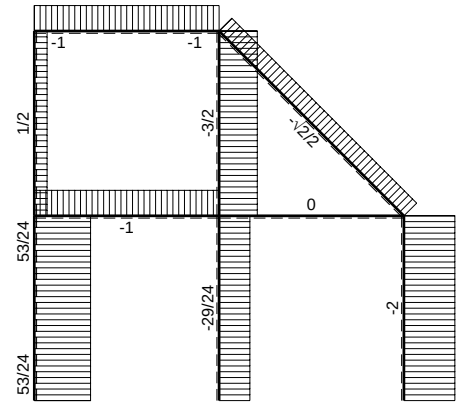
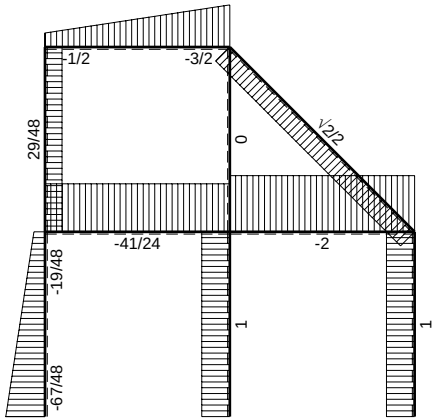
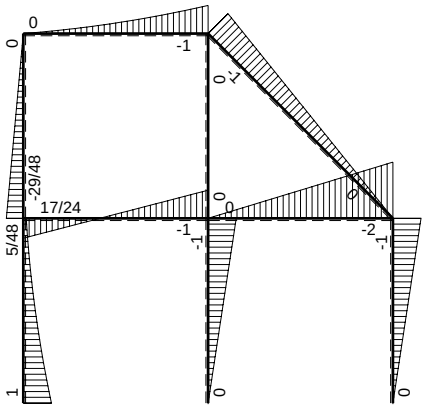


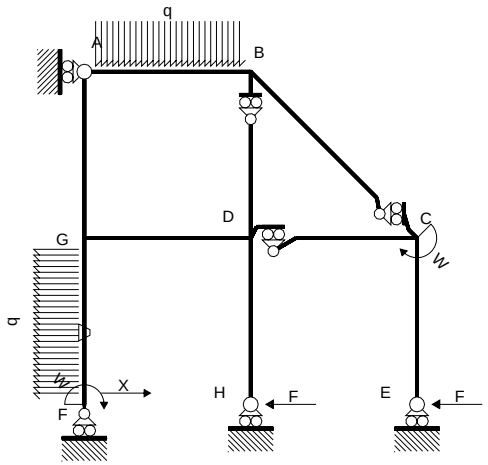


←□→ F

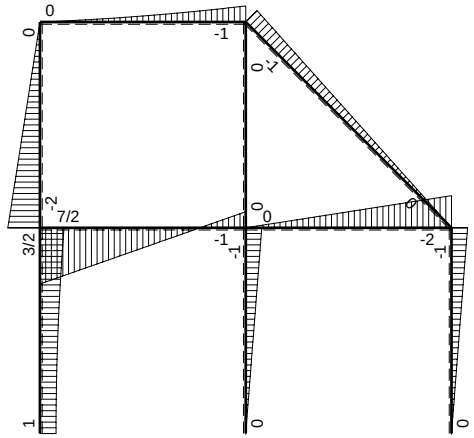
↑□↓ F



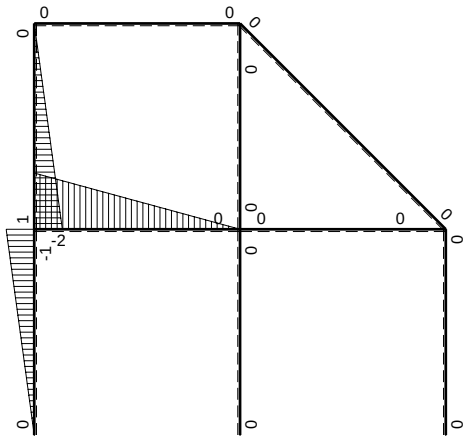
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_F$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{2}$b	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	-x	$Fb+1/2qx^2$	-Fb/EJ	$-Fbx-1/2qx^3$	Fxb/EJ	x^2	$(-5/8+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	$-3/2Fb+Fx-1/2qx^2$	Fb/EJ	$-3/2Fb^2+5/2Fbx-3/2Fx^2+1/2qx^3$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	-2b+2x	$7/2Fb-9/2Fx$	0	$-7Fb^2+16Fbx-9Fx^2$	0	$4b^2-8bx+4x^2$	$(-2+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	$Fb-9/2Fx$	0	$2Fbx-9Fx^2$	0	$4x^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	b-x	-2Fb+2Fx	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	2Fx	0	$-2Fx^2$	0	x^2		
	totali						$-67/24Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_F$						67/48F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (- x/b -1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (x/b)\, \theta\, dx$

$= [-1/2\, x^2/b -1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [1/2\, x^2/b]_o^b\, \theta$

$= (-1/2\, b -1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b)\, \theta = -1/8\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/2 +5/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 + x/b)\, \theta\, dx$

$= [-3/2\, x +5/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [- x +1/2\, x^2/b]_o^b\, \theta$

$= (-3/2\, b +5/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ + (- b +1/2\, b)\, \theta = -1/8\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-7 +16\, x/b -9\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-7\, x +8\, x^2/b -3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-7\, b +8\, b -3\, b)\, Fb^2\, 1/EJ = -2\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (2\, x/b -9\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [x^2/b -3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

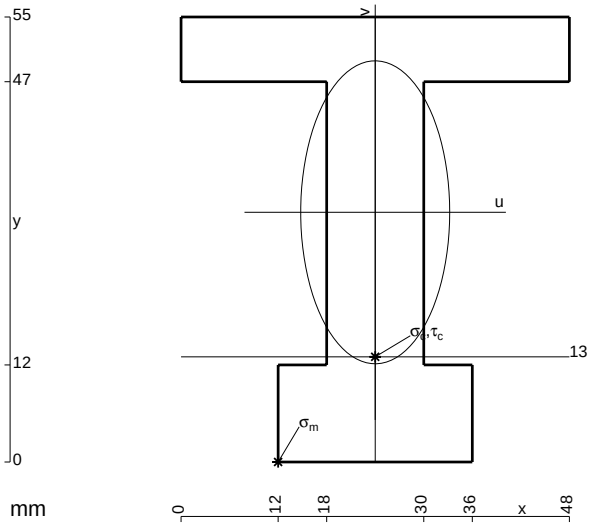
$= (b -3\, b)\, Fb^2\, 1/EJ = -2\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-2 +4\, x/b -2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\, x +2\, x^2/b -2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2\, b +2\, b -2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

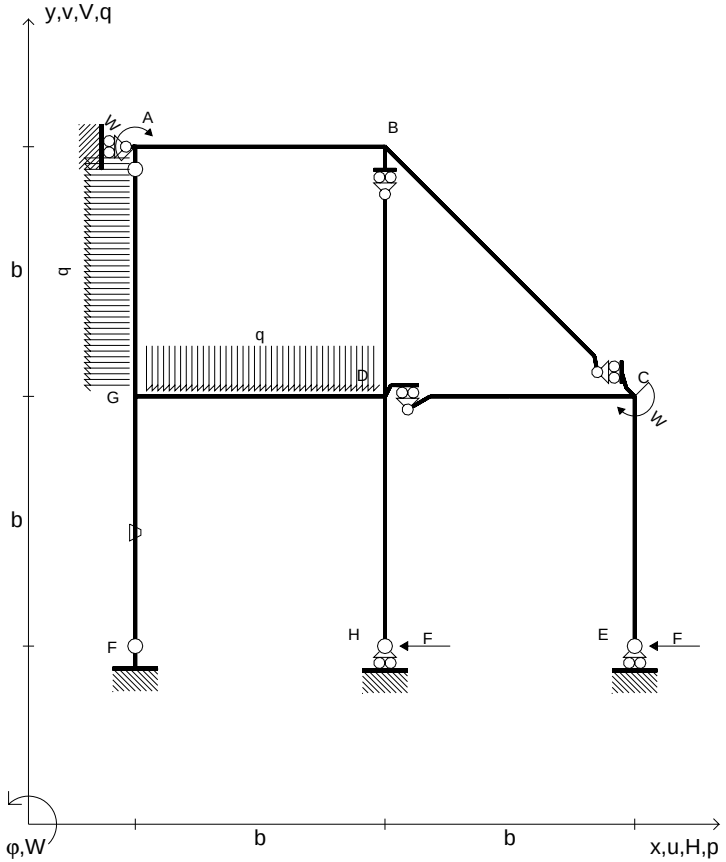
$L^{Xo}_{AG} = \int_o^b (-2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

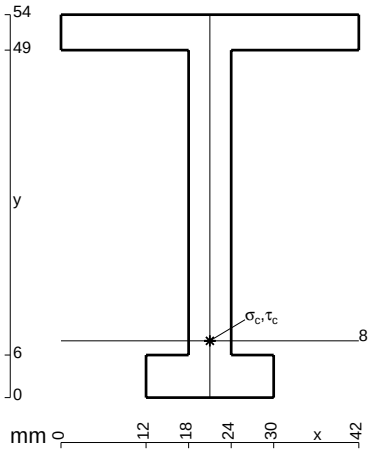
$= (-2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$



A = 1092. mm^2
J_u = 382903. mm^4
J_v = 92592. mm^4
y_g = 30.86 mm
T_y = -3100. N
M_x = -2728000. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -30.86 mm
$\sigma_m = -Mv/J_u$ = -219.9 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -17.86 mm
$\sigma_c = -Mv/J_u$ = -127.3 N/mm^2
τ_c = 4.98 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.6 N/mm^2
S^* = 7381. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_A = -W = -Fb$
$q_{GD} = -q = -F/b$
$p_{GA} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 860 mm, F = 870 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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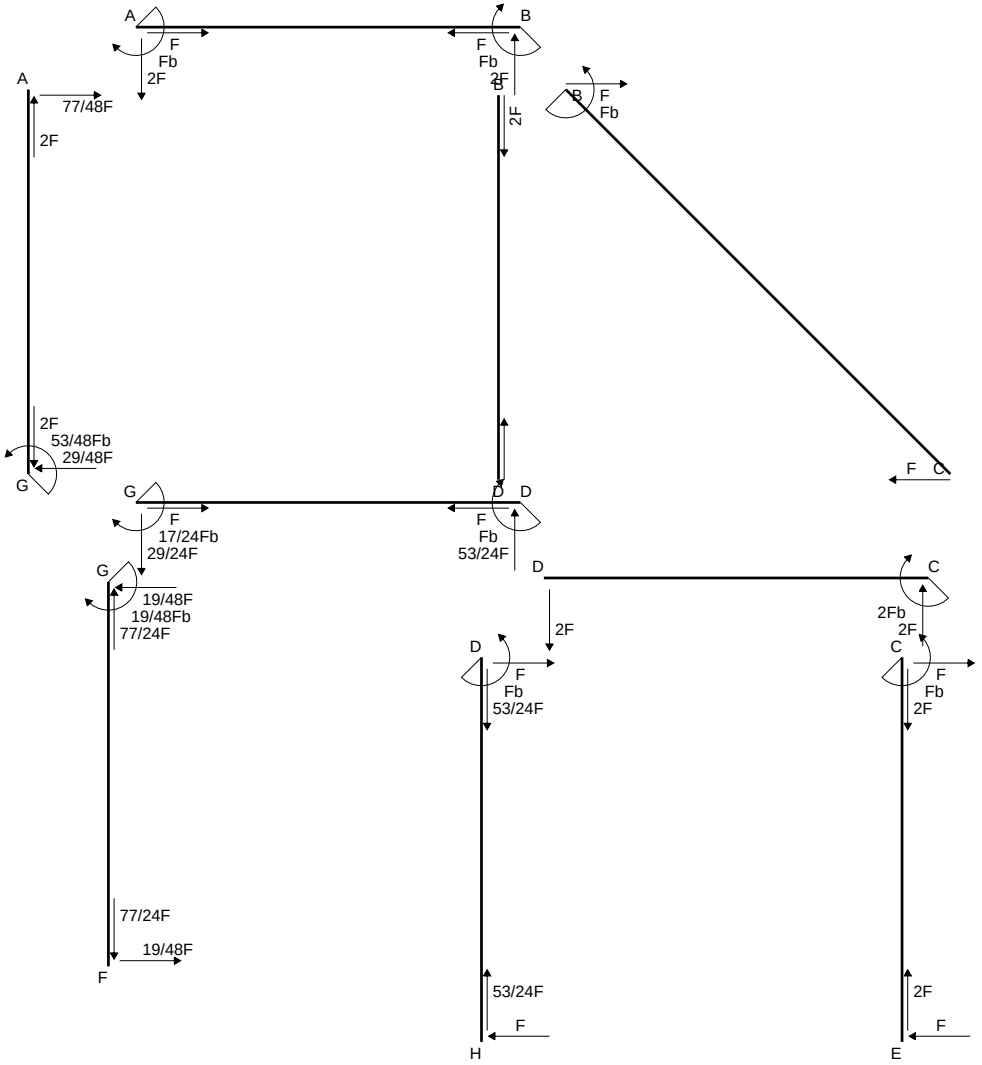

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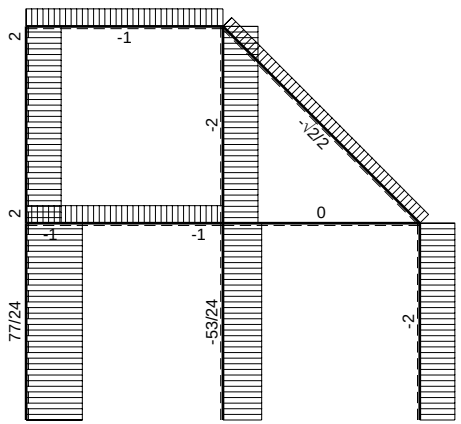
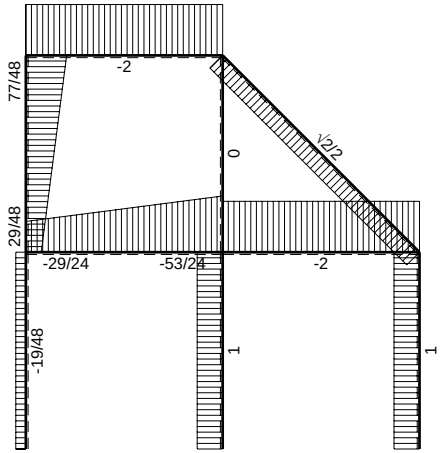
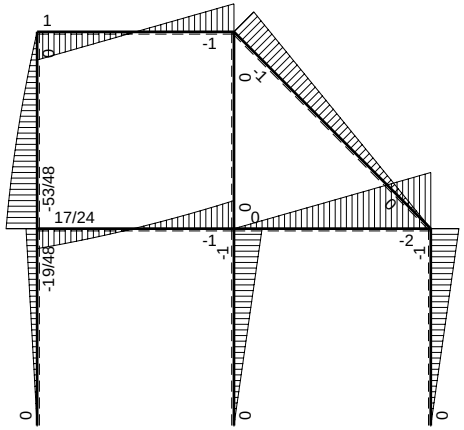


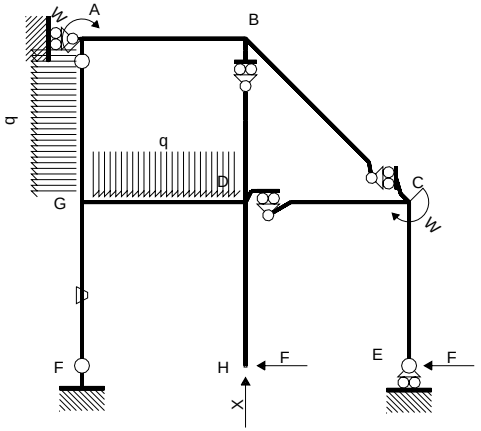
←□→ F



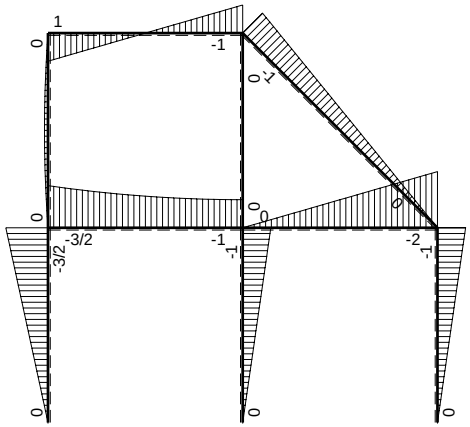
↑□↓ F



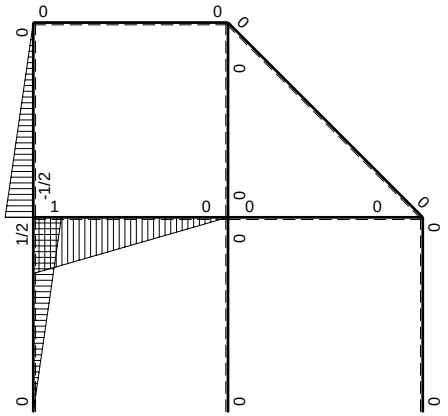
(□) Fb



Schema di calcolo iperstatico



(□) M_o flessione da carichi assegnati



(□) M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	Fb-2Fx	0	0	0	0	0+0	0
BA b	0	Fb-2Fx	0	0	0	0		
BC √2b	0	-Fb+√2/2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	1/2x	-3/2Fx	-Fb/EJ	$-3/4Fx^2$	-1/2Fxb/EJ	$1/4x^2$	$(-1/4-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	$-3/4Fb^2+3/2Fbx-3/4Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	$-3/2Fb+Fx-1/2qx^2$	0	$-3/2Fb^2+5/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(-5/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	$Fb+1/2qx^2$	0	$-Fbx-1/2qx^3$	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	$-1/2Fx+1/2qx^2$	0	$1/4Fbx-1/2Fx^2+1/4qx^3$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/48+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	1/2x	$1/2Fx-1/2qx^2$	0	$1/4Fx^2-1/4qx^3$	0	$1/4x^2$		
	totali						$-53/48Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						53/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [-1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/4\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/4 + 3/2\ x/b - 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-3/4\ x + 3/4\ x^2/b - 1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-3/4\ b + 3/4\ b - 1/4\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-3/2 + 5/2\ x/b - 3/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx$

$= [-3/2\ x + 5/4\ x^2/b - 1/2\ x^3/b^2 + 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (-3/2\ b + 5/4\ b - 1/2\ b + 1/8\ b)\ Fb^2\ 1/EJ = -5/8\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-\ x/b - 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-1/2\ x^2/b - 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

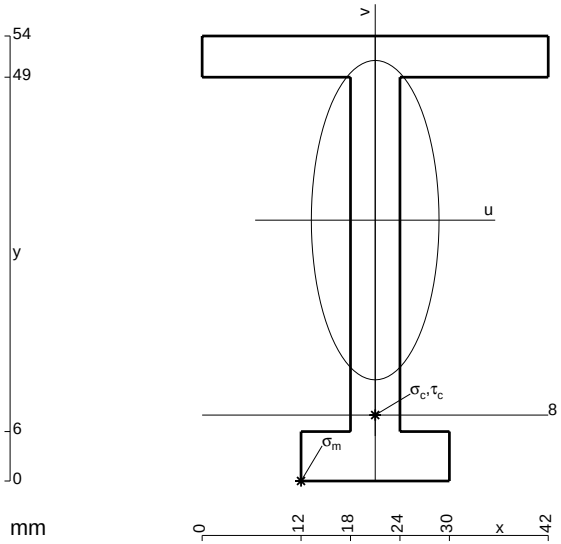
$= (-1/2\ b - 1/8\ b)\ Fb^2\ 1/EJ = -5/8\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (1/4\ x/b - 1/2\ x^2/b^2 + 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [1/8\ x^2/b - 1/6\ x^3/b^2 + 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (1/8\ b - 1/6\ b + 1/16\ b)\ Fb^2\ 1/EJ = 1/48\ Fb^3/EJ$

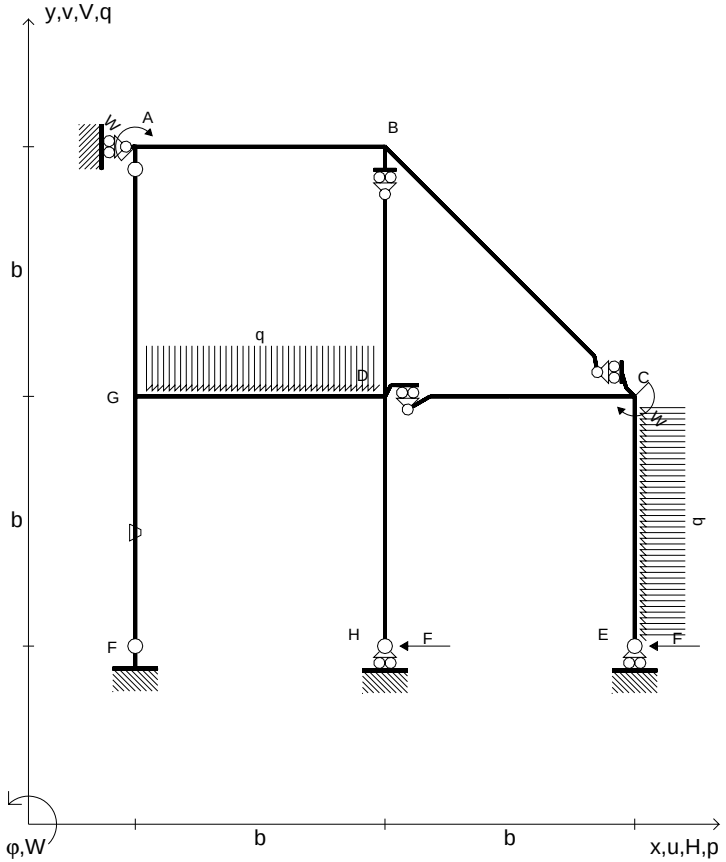
$L^{Xo}_{AG} = \int_o^b (1/4\ x^2/b^2 - 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [1/12\ x^3/b^2 - 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

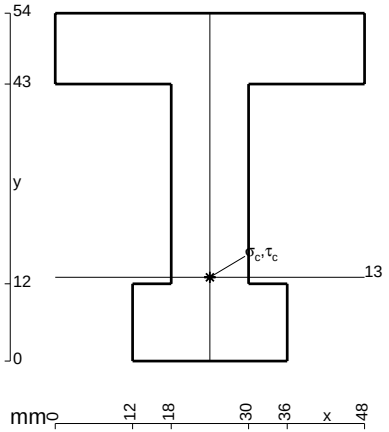
$= (1/12\ b - 1/16\ b)\ Fb^2\ 1/EJ = 1/48\ Fb^3/EJ$



A = 576. mm^2
J_u = 216352. mm^4
J_v = 34560. mm^4
y_g = 31.66 mm
T_y = -1740. N
M_x = -1496400. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -31.66 mm
$\sigma_m = -Mv/J_u$ = -219. N/mm^2
x_c = 21. mm
y_c = 8. mm
v_c = -23.66 mm
$\sigma_c = -Mv/J_u$ = -163.6 N/mm^2
τ_c = 4.545 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 163.8 N/mm^2
S^* = 3391. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_A = -W = -Fb$
$p_{CE} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=W_{GD}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 710 mm, F = 1640 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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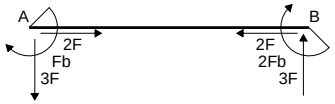

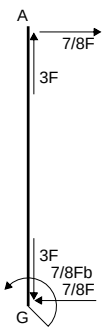

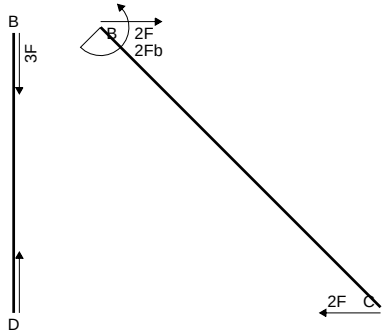

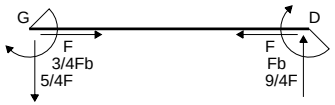

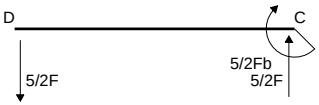

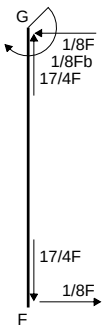

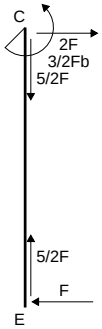

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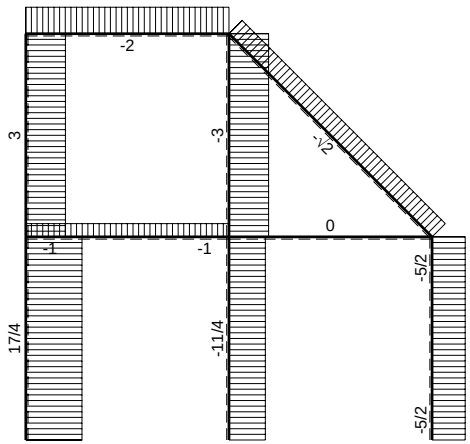
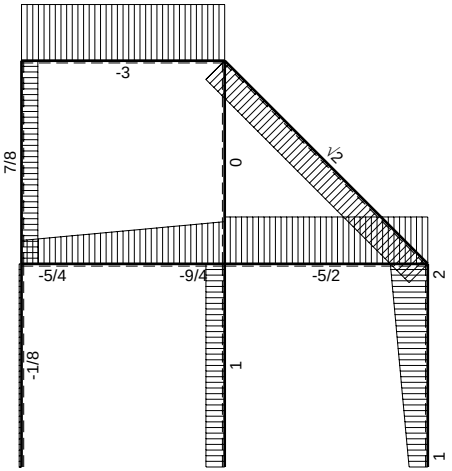
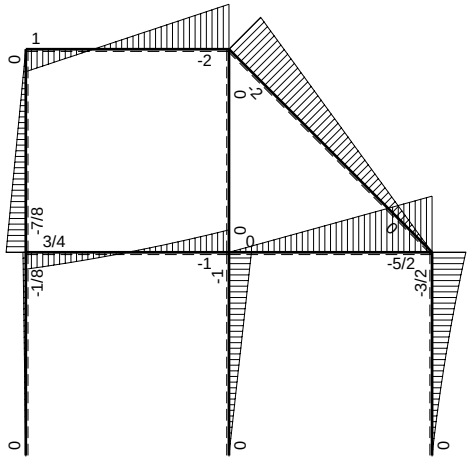


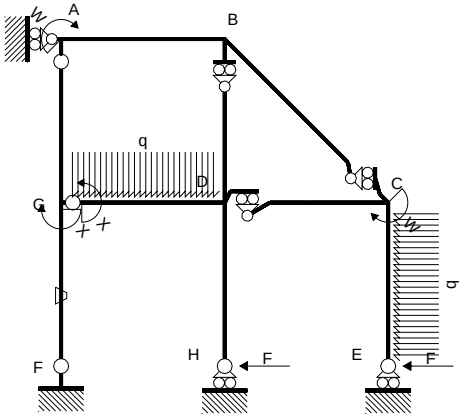
←□→ F



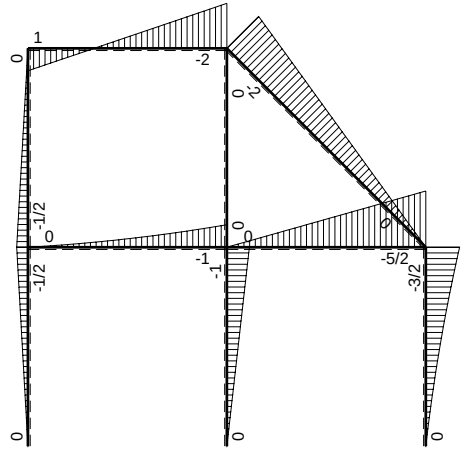
↑□↓ F



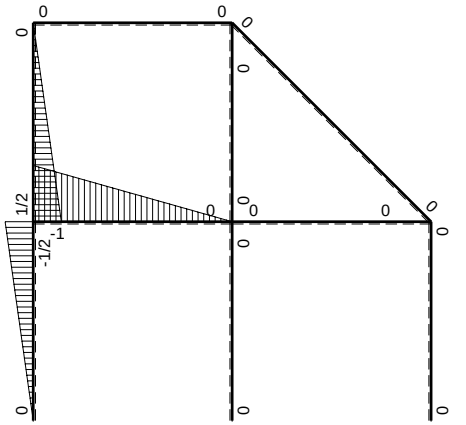
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{GD}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	Fb-3Fx	0	0	0	0	0+0	0
BA b	0	2Fb-3Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	-1/2x/b	-1/2Fx	-Fb/EJ	$1/4Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$	$(1/12+1/4)Fb^2/EJ$	1/12Xb/EJ
GF b	1/2-1/2x/b	1/2Fb-1/2Fx	Fb/EJ	$1/4Fb-1/2Fx+1/4Fx^2/b$	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$		
GD b	-1+x/b	$-1/2Fx-1/2qx^2$	0	$1/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/8+0)Fb^2/EJ$	1/3Xb/EJ
DG b	x/b	$Fb-3/2Fx+1/2qx^2$	0	$Fx-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	1/2-1/2x/b	-1/2Fb+1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
AG b	-1/2x/b	1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
	totali						$3/8Fb^2/EJ$	1/2Xb/EJ
	iperstatica $X=W_{GD}$						-3/4Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (b - b + 1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{DG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{Xo}_{FG} = \int_o^b (1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2\ x/b)\ \theta\ dx = [1/12\ x^3/b^2]_o^b\ Fb\ 1/EJ + [1/4\ x^2/b]_o^b\ \theta$

$= (1/12\ b)\ Fb\ 1/EJ + (1/4\ b)\ \theta = 1/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 + 1/2\ x/b)\ \theta\ dx$

$= [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ Fb\ 1/EJ + [-1/2\ x + 1/4\ x^2/b]_o^b\ \theta$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ Fb\ 1/EJ + (-1/2\ b + 1/4\ b)\ \theta = 1/3\ Fb^2/EJ$

$L^{Xo}_{GD} = \int_o^b (1/2\ x/b - 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b - 1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b - 1/8\ b)\ Fb\ 1/EJ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DG} = \int_o^b (x/b - 3/2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/2\ x^3/b^2 + 1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

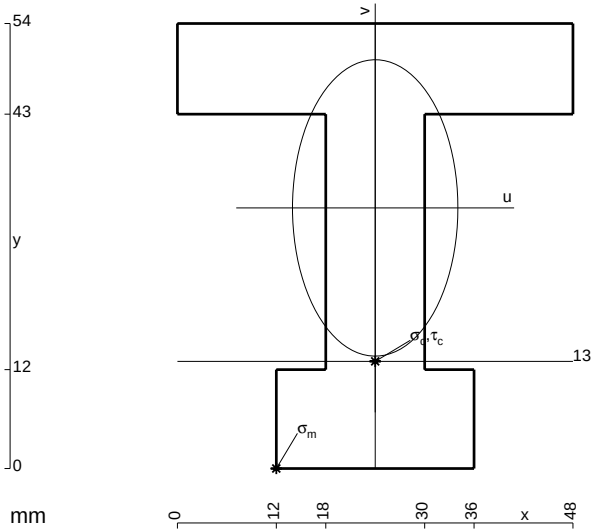
$= (1/2\ b - 1/2\ b + 1/8\ b)\ Fb\ 1/EJ = 1/8\ Fb^2/EJ$

$L^{Xo}_{GA} = \int_o^b (-1/4 + 1/2\ x/b - 1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b + 1/4\ b - 1/12\ b)\ Fb\ 1/EJ = -1/12\ Fb^2/EJ$

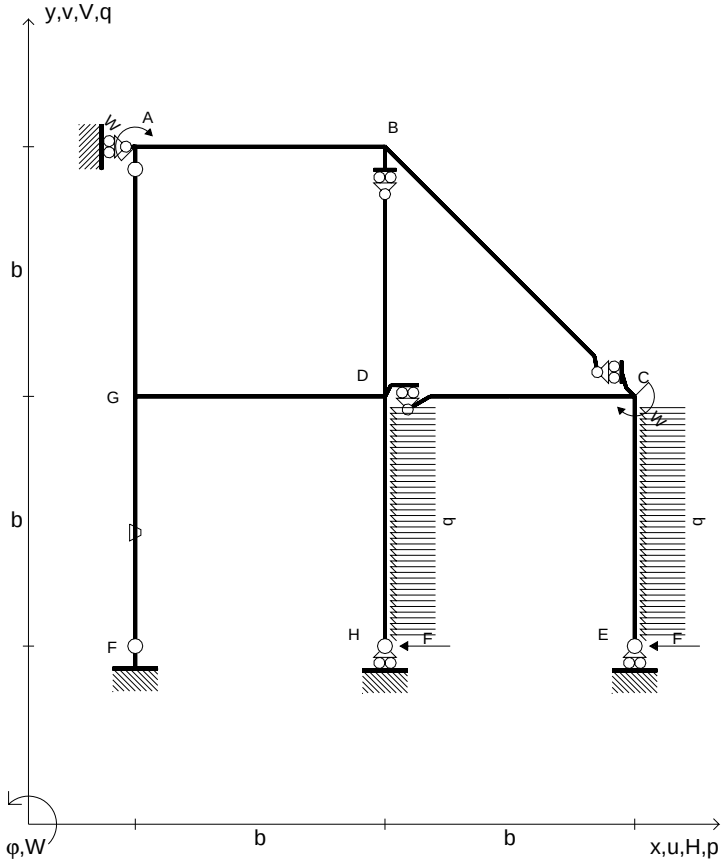
$L^{Xo}_{AG} = \int_o^b (-1/4\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/12\ x^3/b^2]_o^b\ Fb\ 1/EJ$

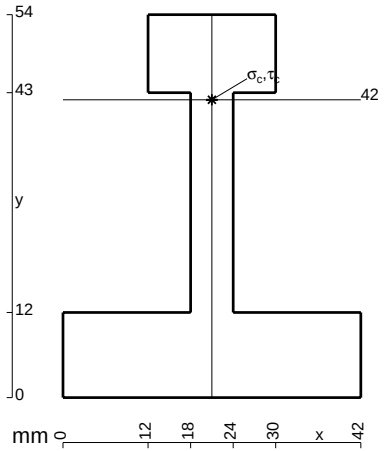
$= (-1/12\ b)\ Fb\ 1/EJ = -1/12\ Fb^2/EJ$



A = 1188. mm^2
J_u = 384370. mm^4
J_v = 119664. mm^4
y_g = 31.62 mm
T_y = -4100. N
M_x = -2911000. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -31.62 mm
$\sigma_m = -Mv/J_u$ = -239.5 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -18.62 mm
$\sigma_c = -Mv/J_u$ = -141. N/mm^2
τ_c = 6.763 N/mm^2
$\sigma_0 = \sqrt{\sigma^2 + 3\tau^2}$ = 141.5 N/mm^2
S^* = 7608. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_A = -W = -Fb$
$p_{CE} = -q = -F/b$
$p_{DH} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 910 mm, F = 870 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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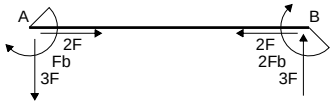

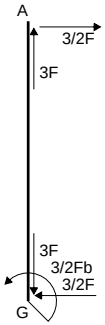

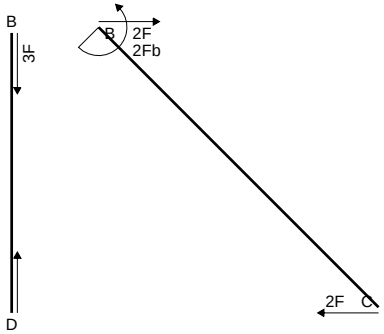

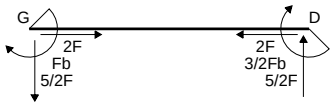

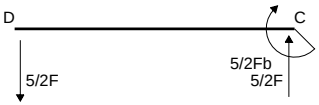

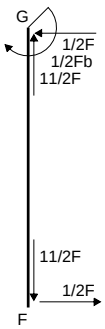

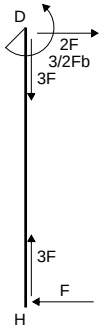

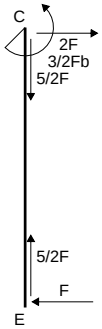

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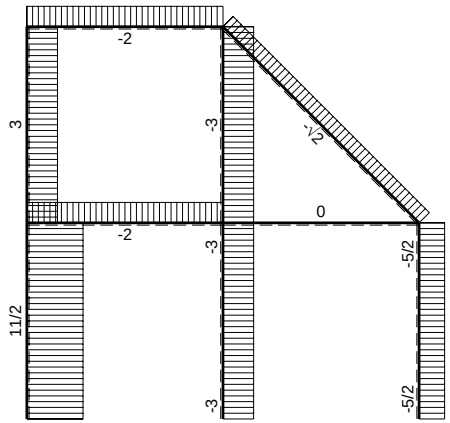
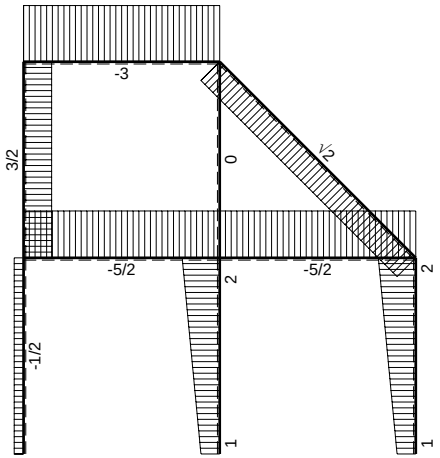
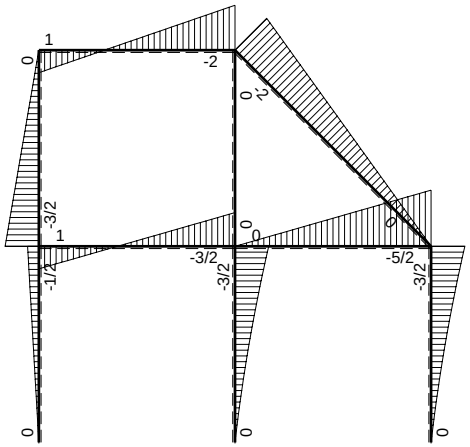


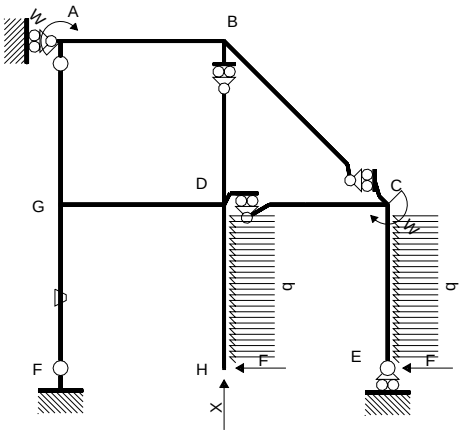
←F→



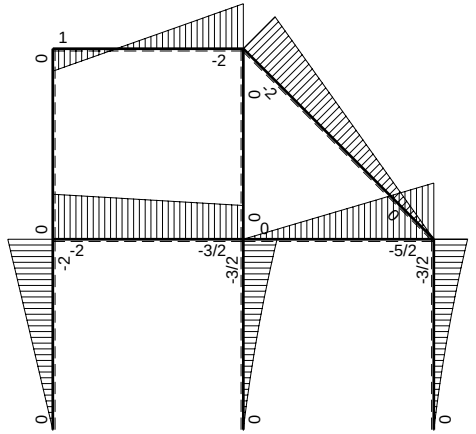
↑F↓



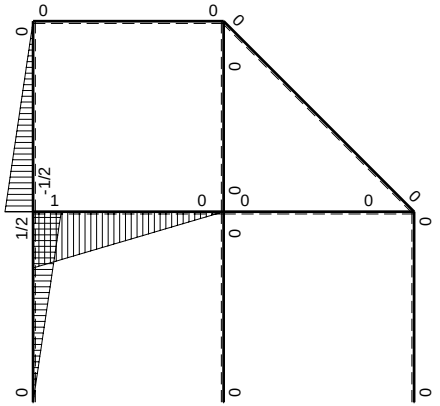
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	Fb-3Fx	0	0	0	0	0+0	0
BA b	0	2Fb-3Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	1/2x	-2Fx	-Fb/EJ	$-Fx^2$	-1/2Fxb/EJ	$1/4x^2$	$(-1/3-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	2Fb-2Fx	Fb/EJ	$-Fb^2+2Fbx-Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	-2Fb+1/2Fx	0	$-2Fb^2+5/2Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-11/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	3/2Fb+1/2Fx	0	$-3/2Fbx-1/2Fx^2$	0	x^2		
DH b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
HD b	0	$Fx+1/2qx^2$	0	0	0	0		
GA b	-1/2b+1/2x	0	0	0	0	$1/4b^2-1/2bx+1/4x^2$	0+0	$1/12Xb^3/EJ$
AG b	1/2x	0	0	0	0	$1/4x^2$		
	totali						$-3/2Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						3F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [-1/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/3\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -7/12\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1 + 2\ x/b - x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-\ x + x^2/b - 1/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

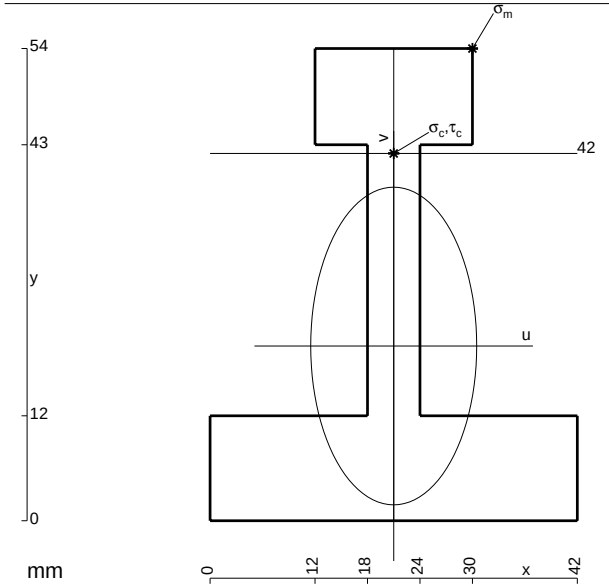
$= (-\ b + b - 1/3\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -7/12\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-2 + 5/2\ x/b - 1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-2\ x + 5/4\ x^2/b - 1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-2\ b + 5/4\ b - 1/6\ b)\ Fb^2\ 1/EJ = -11/12\ Fb^3/EJ$

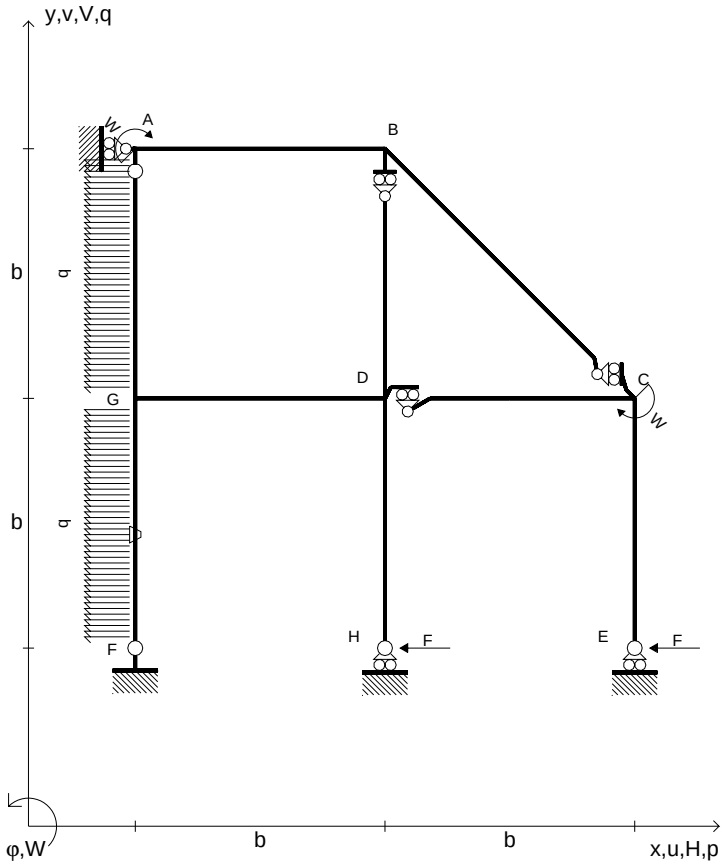
$L^{Xo}_{DG} = \int_o^b (-3/2\ x/b - 1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-3/4\ x^2/b - 1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

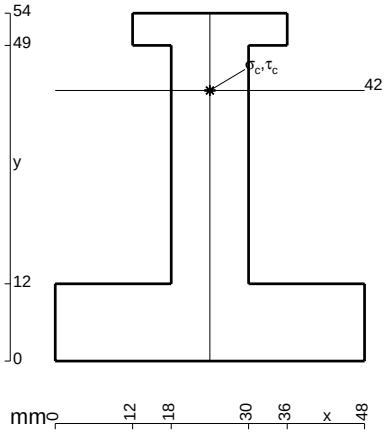
$= (-3/4\ b - 1/6\ b)\ Fb^2\ 1/EJ = -11/12\ Fb^3/EJ$



A = 888. mm^2
J_u = 293012. mm^4
J_v = 79992. mm^4
y_g = 19.98 mm
T_y = -2175. N
M_x = -1979250. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 9. mm
v_m = 34.02 mm
$\sigma_m = -Mv/J_u$ = 229.8 N/mm^2
x_c = 21. mm
y_c = 42. mm
v_c = 22.02 mm
$\sigma_c = -Mv/J_u$ = 148.7 N/mm^2
τ_c = 7.153 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 149.3 N/mm^2
S^* = 5782. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_A = -W = -Fb$
$p_{GA} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_F$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 560 mm, F = 1880 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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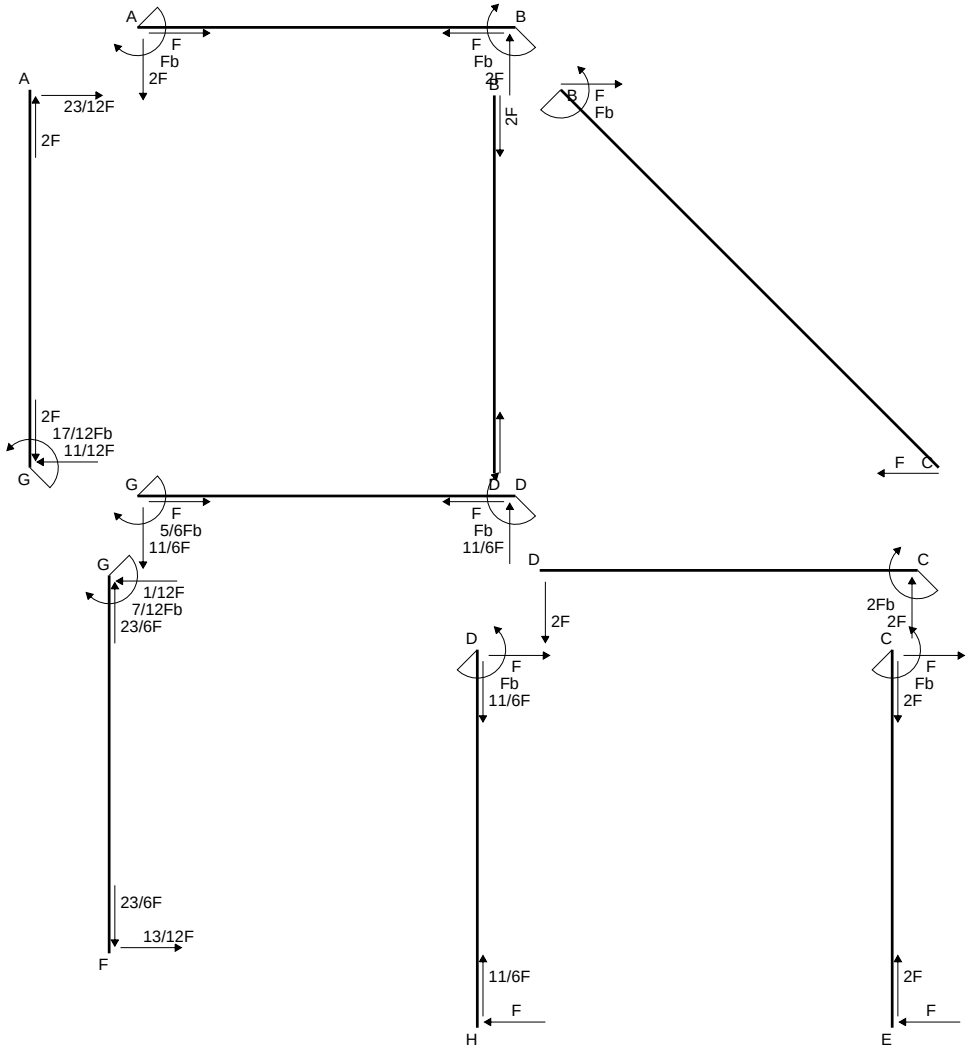

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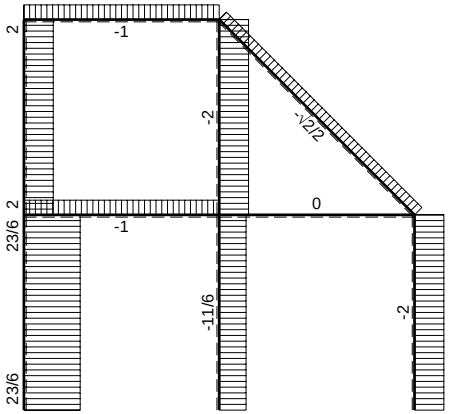
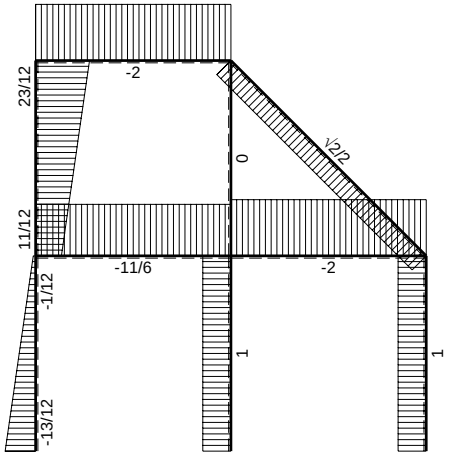
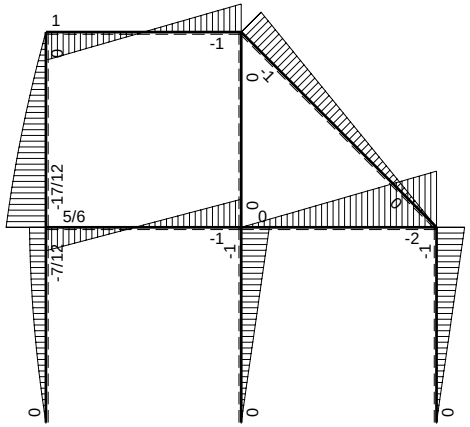


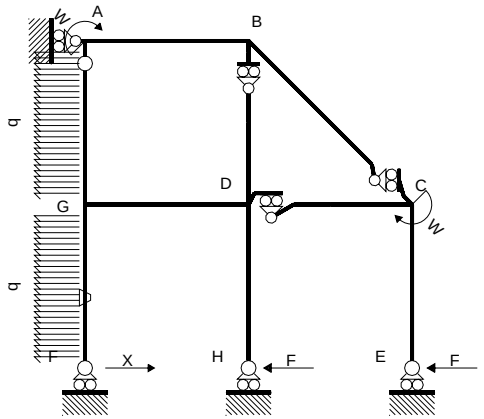
←F→



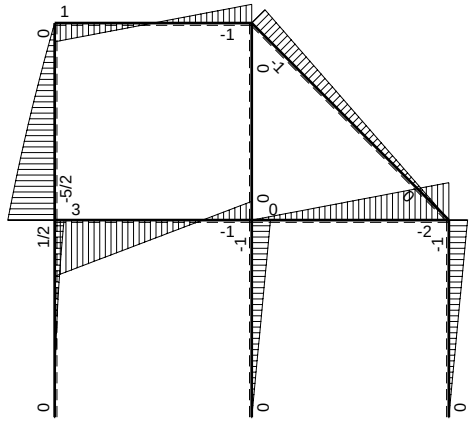
↑F↓



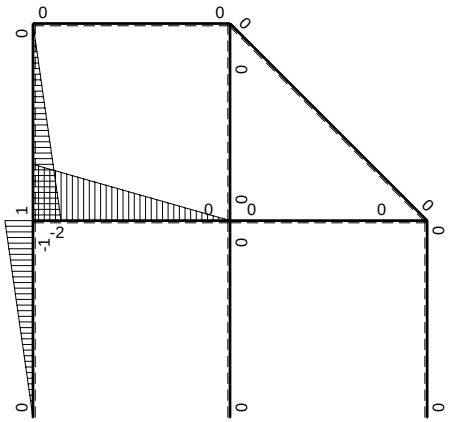
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_F$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	Fb-2Fx	0	0	0	0	0+0	0
BA b	0	Fb-2Fx	0	0	0	0		
BC √2b	0	-Fb+√2/2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	-x	$1/2qx^2$	-Fb/EJ	$-1/2qx^3$	Fxb/EJ	x^2	$(-1/8+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	-2b+2x	3Fb-4Fx	0	$-6Fb^2+14Fbx-8Fx^2$	0	$4b^2-8bx+4x^2$	$(-5/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	Fb-4Fx	0	$2Fbx-8Fx^2$	0	$4x^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	b-x	$-5/2Fb+2Fx+1/2qx^2$	0	$-5/2Fb^2+9/2Fbx-3/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$	$(-7/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	$3Fx-1/2qx^2$	0	$-3Fx^2+1/2qx^3$	0	x^2		
	totali						$-13/6Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_F$						13/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (x/b)\, \theta\ dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [1/2\, x^2/b]_o^b\, \theta$

$= (-1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 + x/b)\, \theta\ dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [- x +1/2\, x^2/b]_o^b\, \theta$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ + (- b +1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-6 +14\, x/b -8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-6\, x +7\, x^2/b -8/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-6\, b +7\, b -8/3\, b)\, Fb^2\, 1/EJ = -5/3\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (2\, x/b -8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [x^2/b -8/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (b -8/3\, b)\, Fb^2\, 1/EJ = -5/3\ Fb^3/EJ$

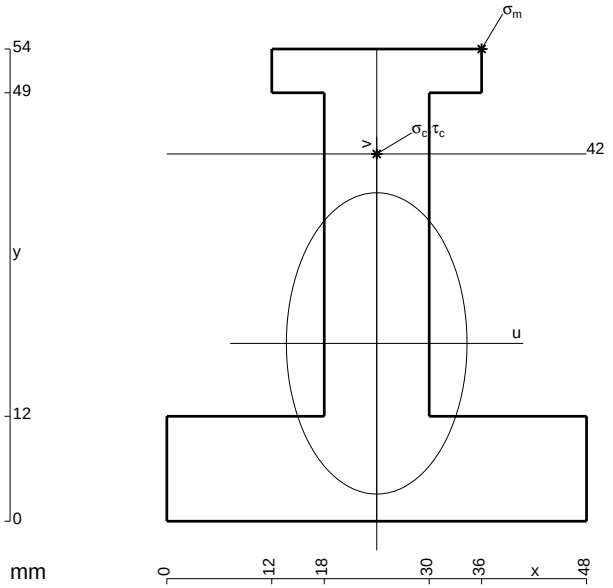
$L^{Xo}_{GA} = \int_o^b (-5/2 +9/2\, x/b -3/2\, x^2/b^2 -1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-5/2\, x +9/4\, x^2/b -1/2\, x^3/b^2 -1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-5/2\, b +9/4\, b -1/2\, b -1/8\, b)\, Fb^2\, 1/EJ = -7/8\ Fb^3/EJ$

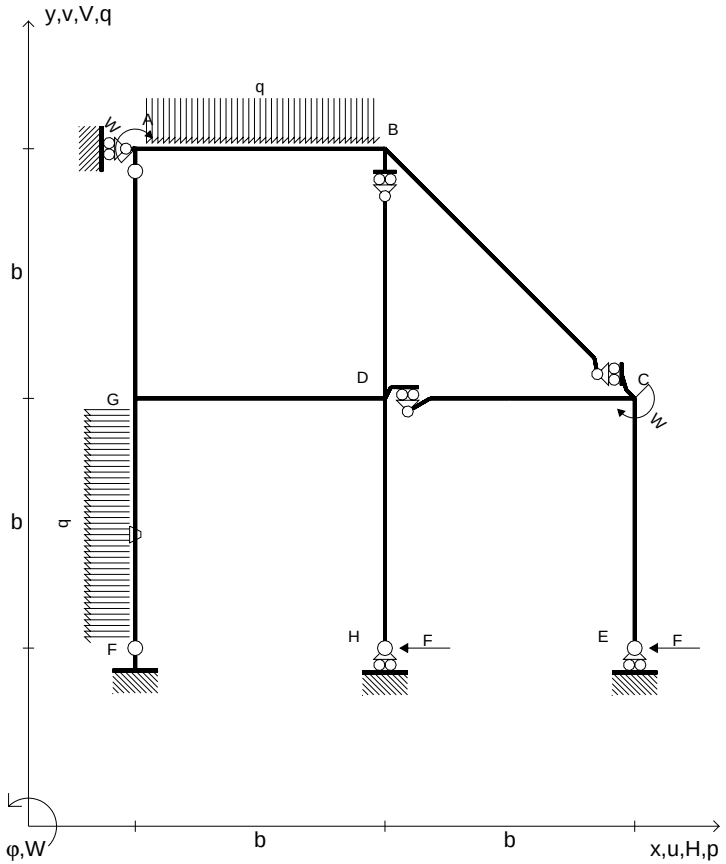
$L^{Xo}_{AG} = \int_o^b (-3\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [- x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

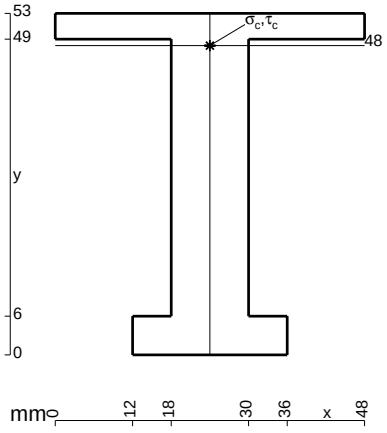
$= (- b +1/8\, b)\, Fb^2\, 1/EJ = -7/8\ Fb^3/EJ$



A = 1140. mm^2
J_u = 338607. mm^4
J_v = 121680. mm^4
y_g = 20.33 mm
T_y = -3760. N
M_x = -2105600. Nmm
x_m = 36. mm
y_m = 54. mm
u_m = 12. mm
v_m = 33.67 mm
$\sigma_m = -Mv/J_u$ = 209.4 N/mm^2
x_c = 24. mm
y_c = 42. mm
v_c = 21.67 mm
$\sigma_c = -Mv/J_u$ = 134.7 N/mm^2
τ_c = 5.417 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.1 N/mm^2
S^* = 5854. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_A = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 750 mm, F = 1260 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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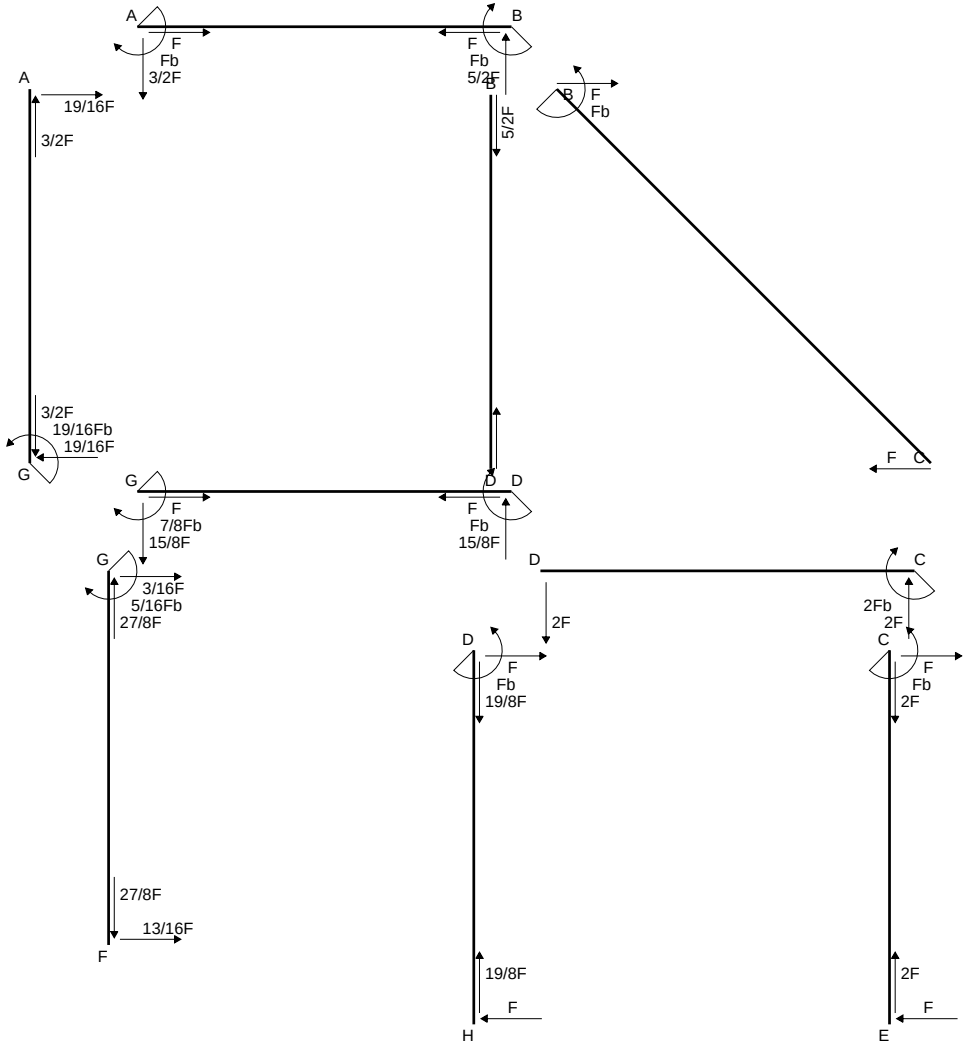

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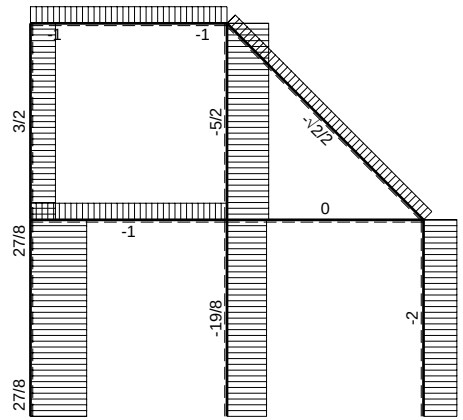
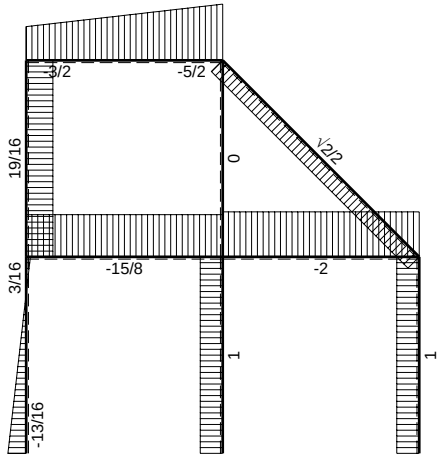
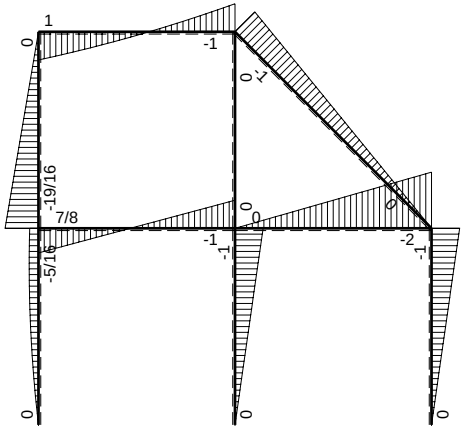


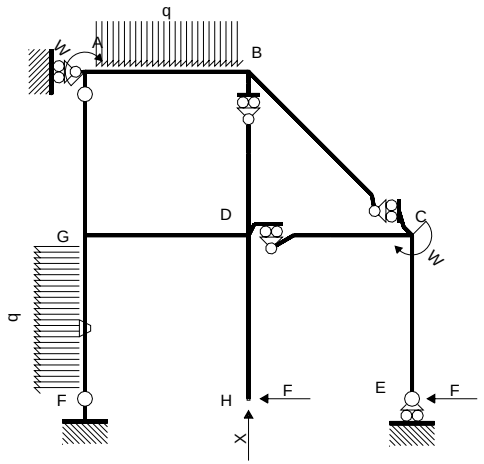
←⊞→ F



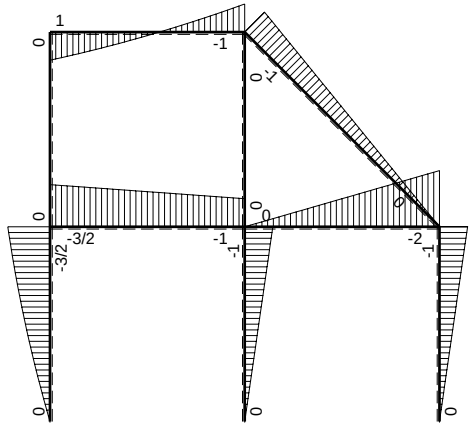
↑⊞↓ F



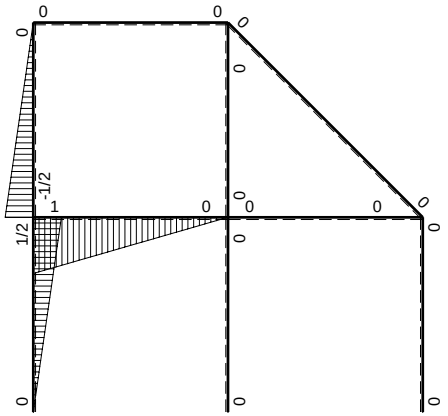
⟲⊞⟳ Fb



Schema di calcolo iperstatico



⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$Fb-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$Fb-5/2Fx+1/2qx^2$	0	0	0	0		
BC √2b	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	1/2x	$-2Fx+1/2qx^2$	$-Fb/EJ$	$-Fx^2+1/4qx^3$	$-1/2Fxb/EJ$	$1/4x^2$	$(-13/48-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	$3/2Fb-Fx-1/2qx^2$	Fb/EJ	$-3/4Fb^2+5/4Fbx-1/4Fx^2-1/4qx^3$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	$-3/2Fb+1/2Fx$	0	$-3/2Fb^2+2Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	Fb+1/2Fx	0	$-Fbx-1/2Fx^2$	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	0	0	0	0	$1/4b^2-1/2bx+1/4x^2$	0+0	$1/12Xb^3/EJ$
AG b	1/2x	0	0	0	0	$1/4x^2$		
	totali						$-19/16Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						19/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-\ x^2/b^2 + 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx$

$= [-1/3\ x^3/b^2 + 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/3\ b + 1/16\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -25/48\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/4 + 5/4\ x/b - 1/4\ x^2/b^2 - 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-3/4\ x + 5/8\ x^2/b - 1/12\ x^3/b^2 - 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

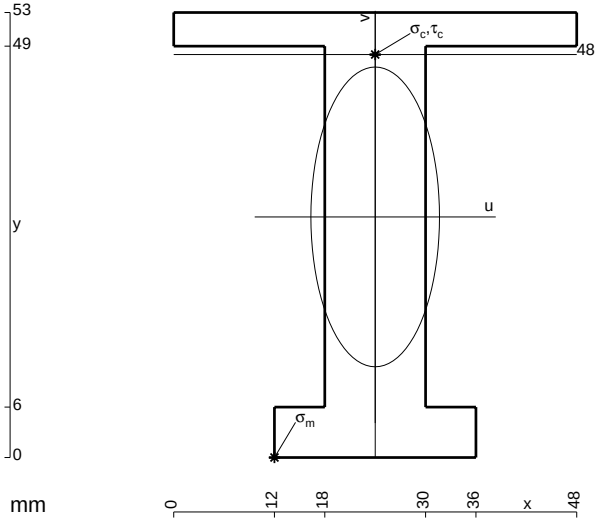
$= (-3/4\ b + 5/8\ b - 1/12\ b - 1/16\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -25/48\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-3/2 + 2\ x/b - 1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-3/2\ x + x^2/b - 1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-3/2\ b + b - 1/6\ b)\ Fb^2\ 1/EJ = -2/3\ Fb^3/EJ$

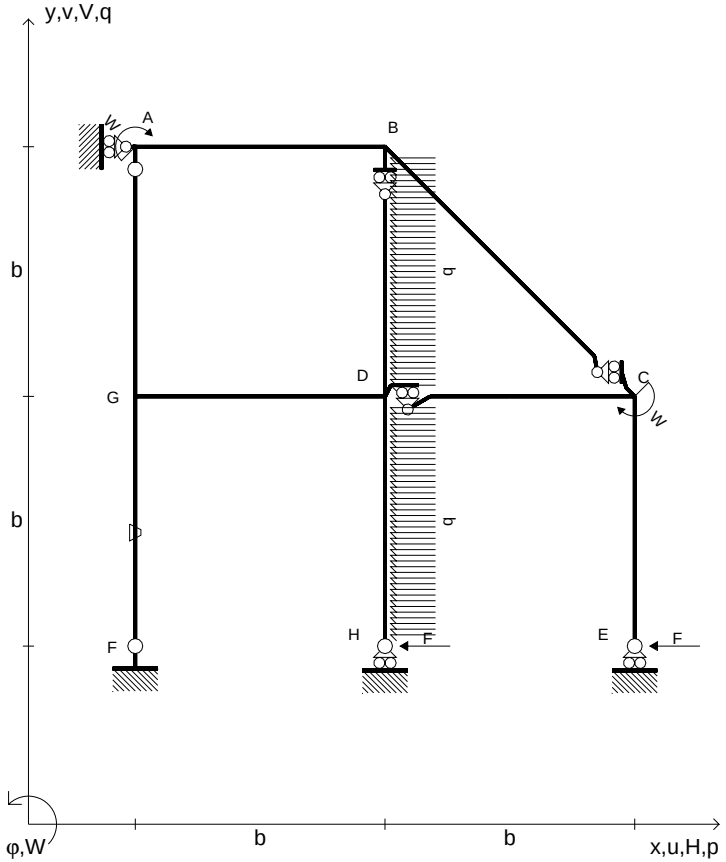
$L^{Xo}_{DG} = \int_o^b (-\ x/b - 1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/2\ x^2/b - 1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

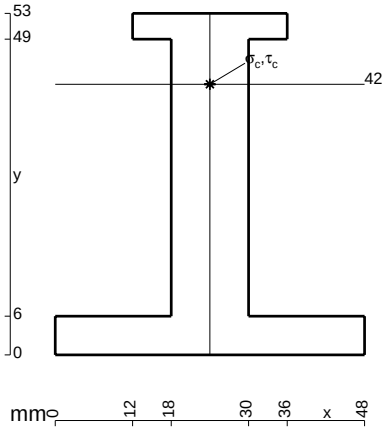
$= (-1/2\ b - 1/6\ b)\ Fb^2\ 1/EJ = -2/3\ Fb^3/EJ$



A = 852. mm^2
J_u = 271527. mm^4
J_v = 49968. mm^4
y_g = 28.65 mm
T_y = -2520. N
M_x = -1890000. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -28.65 mm
$\sigma_m = -Mv/J_u$ = -199.5 N/mm^2
x_c = 24. mm
y_c = 48. mm
v_c = 19.35 mm
$\sigma_c = -Mv/J_u$ = 134.7 N/mm^2
τ_c = 3.502 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 134.8 N/mm^2
S^* = 4528. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_A = -W = -Fb$
$p_{BD} = -q = -F/b$
$p_{DH} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 650 mm, F = 1610 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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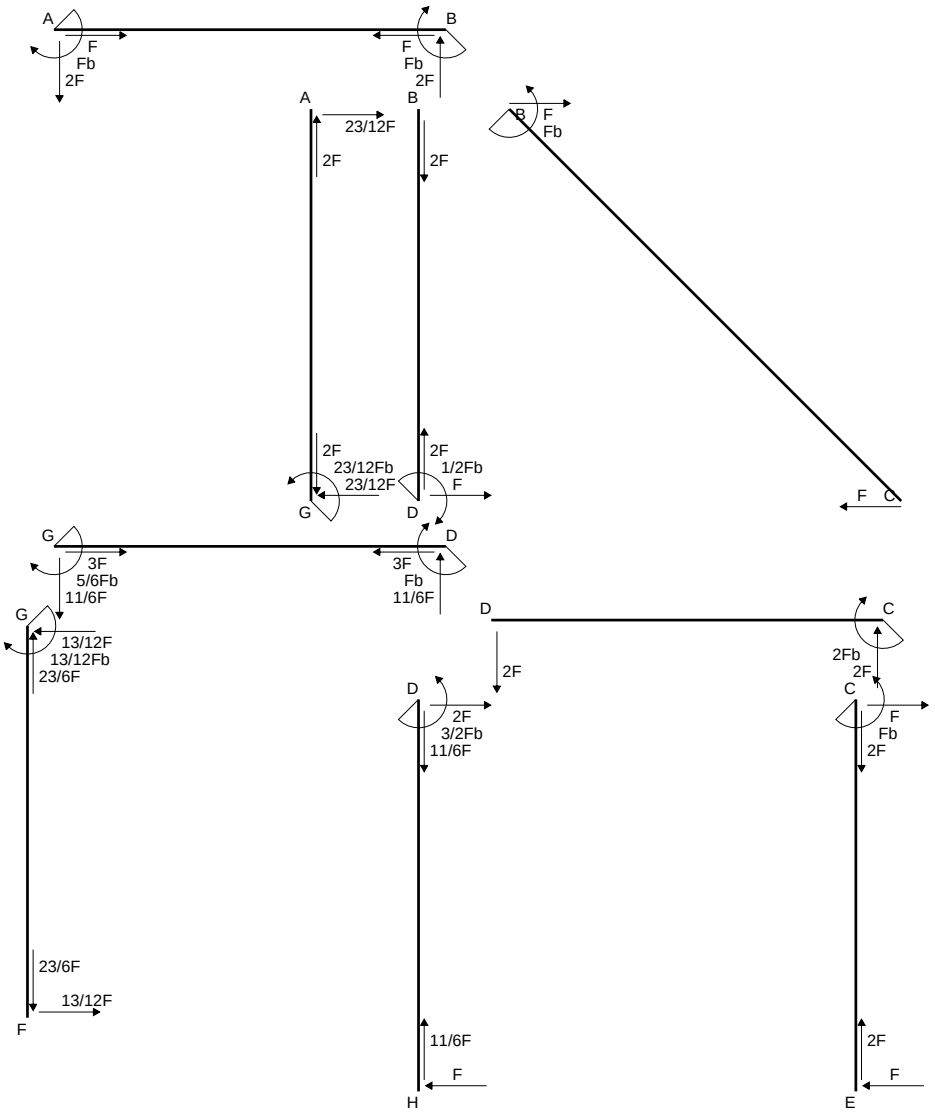

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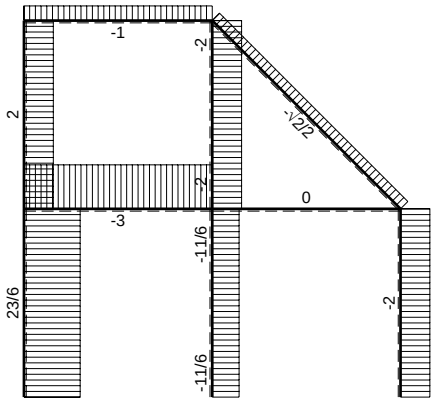
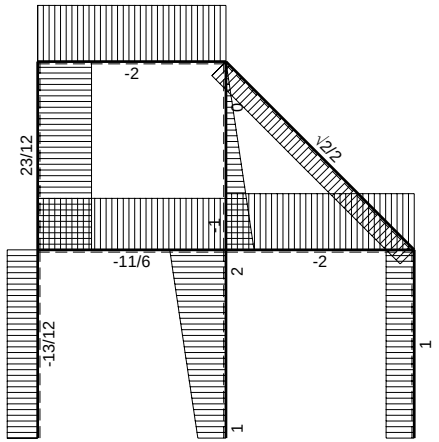
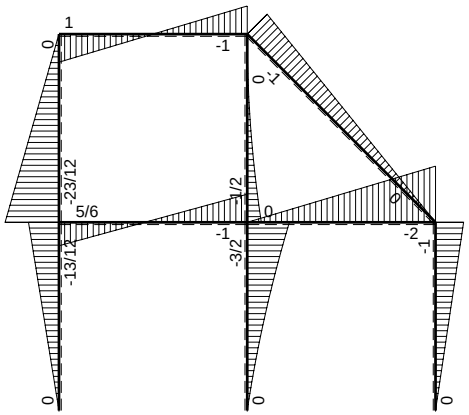


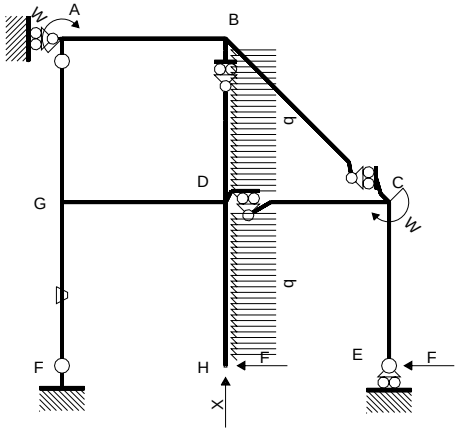
←⊞→ F



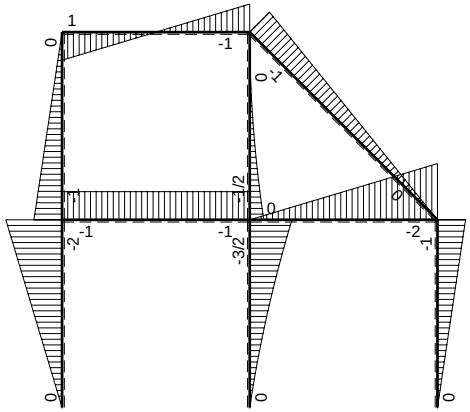
↑⊞↓ F



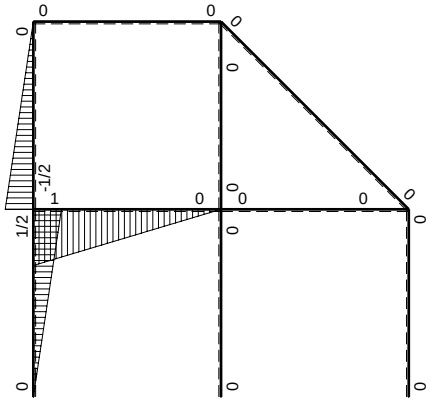
⟲⊞⟳ Fb



Schema di calcolo iperstatico



⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	Fb-2Fx	0	0	0	0	0+0	0
BA b	0	Fb-2Fx	0	0	0	0		
BC √2b	0	-Fb+√2/2Fx	0	0	0	0	0	0
BD b	0	$-1/2qx^2$	0	0	0	0	0+0	0
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	1/2x	-2Fx	-Fb/EJ	$-Fx^2$	-1/2Fxb/EJ	$1/4x^2$	$(-1/3-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	2Fb-2Fx	Fb/EJ	$-Fb^2+2Fbx-Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	-Fb	0	$-Fb^2+Fbx$	0	$b^2-2bx+x^2$	$(-1/2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	Fb	0	-Fbx	0	x^2		
DH b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
HD b	0	$Fx+1/2qx^2$	0	0	0	0		
GA b	-1/2b+1/2x	-Fb+Fx	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	1/2x	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
	totali						$-11/12Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						11/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [-1/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/3\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -7/12\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1 + 2\ x/b - x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-\ x + x^2/b - 1/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-\ b + b - 1/3\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -7/12\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-1 + x/b)\ Fb^2\ 1/EJ\ dx = [-\ x + 1/2\ x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (-\ b + 1/2\ b)\ Fb^2\ 1/EJ = -1/2\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-\ x/b)\ Fb^2\ 1/EJ\ dx = [-1/2\ x^2/b]_o^b\ Fb^2\ 1/EJ$

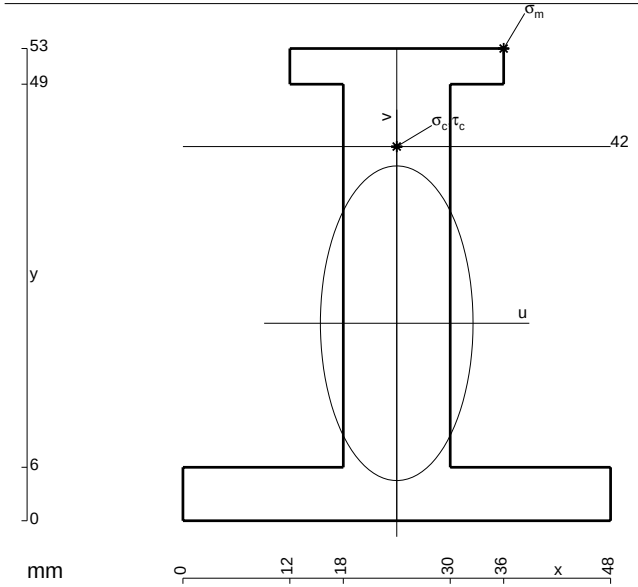
$= (-1/2\ b)\ Fb^2\ 1/EJ = -1/2\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb^2\ 1/EJ = 1/6\ Fb^3/EJ$

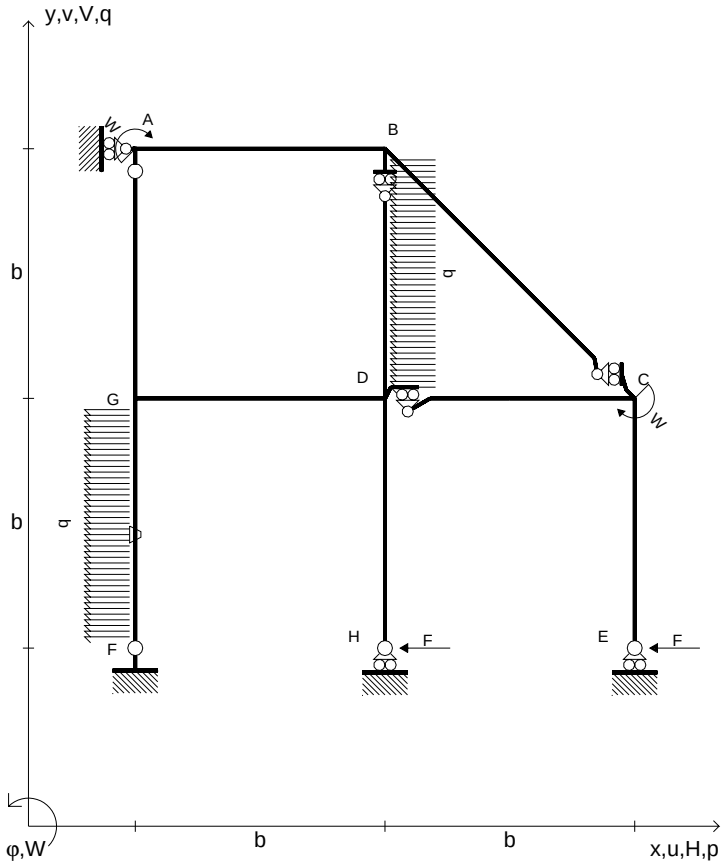
$L^{Xo}_{AG} = \int_o^b (1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

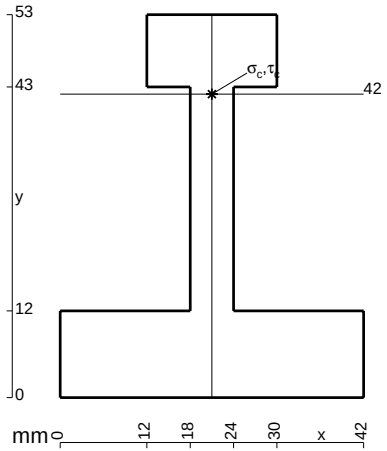
$= (1/6\ b)\ Fb^2\ 1/EJ = 1/6\ Fb^3/EJ$



A = 900. mm^2
J_u = 280787. mm^4
J_v = 66096. mm^4
y_g = 22.17 mm
T_y = -3220. N
M_x = -2093000. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 12. mm
v_m = 30.83 mm
$\sigma_m = -Mv/J_u$ = 229.8 N/mm^2
x_c = 24. mm
y_c = 42. mm
v_c = 19.83 mm
$\sigma_c = -Mv/J_u$ = 147.8 N/mm^2
τ_c = 4.518 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 148. N/mm^2
S^* = 4728. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_A = -W = -Fb$
$p_{BD} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 700 mm, F = 1380 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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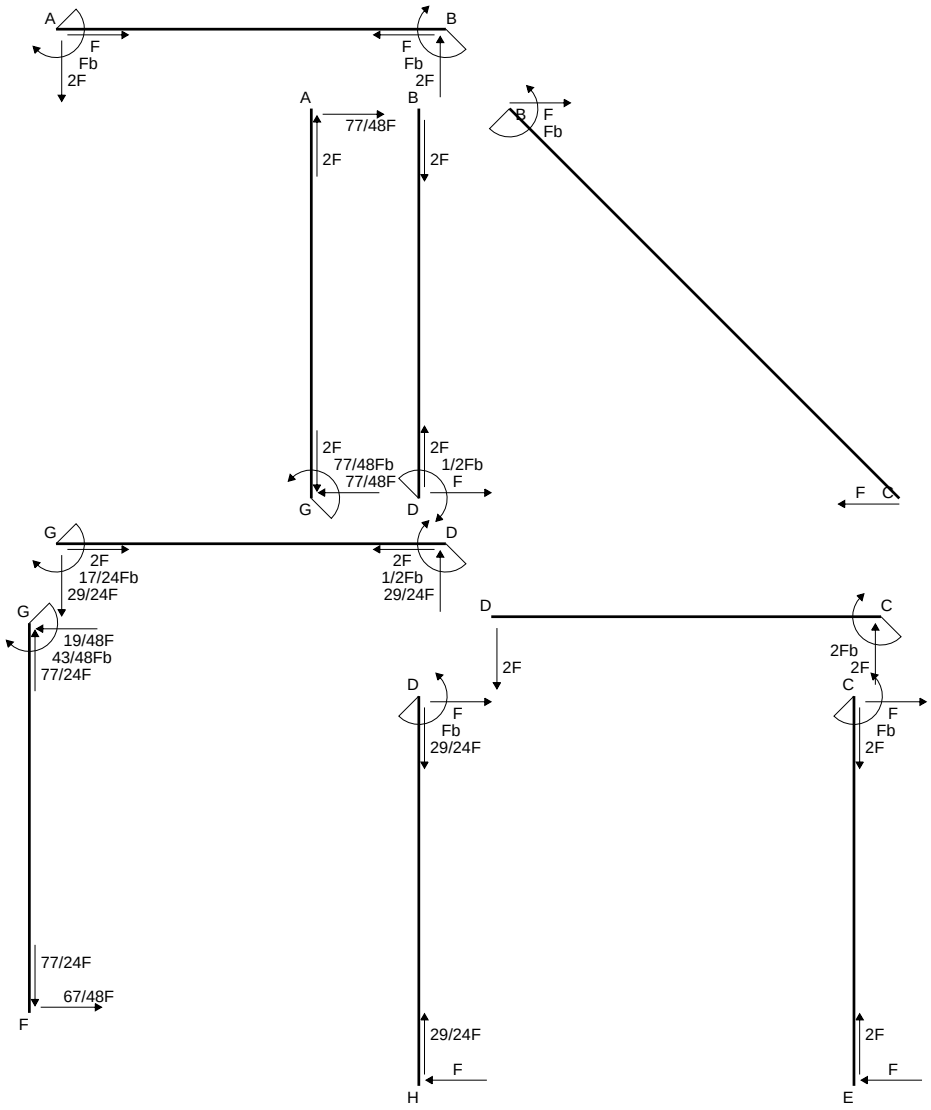

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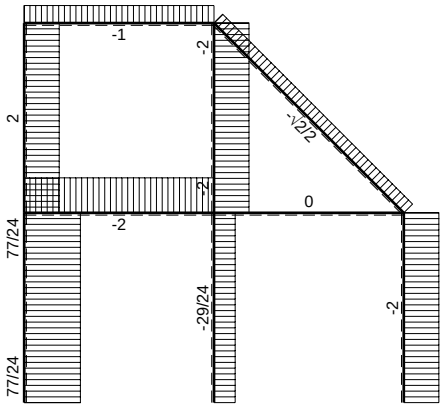
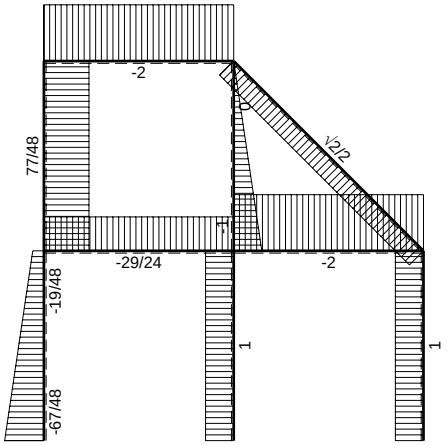
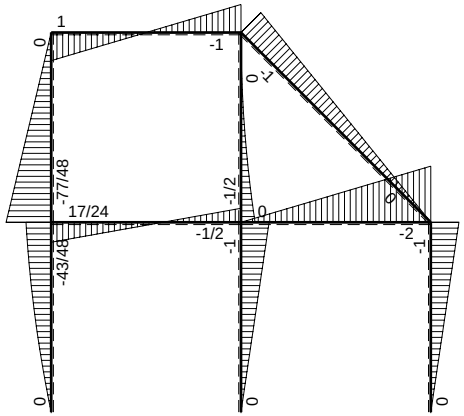


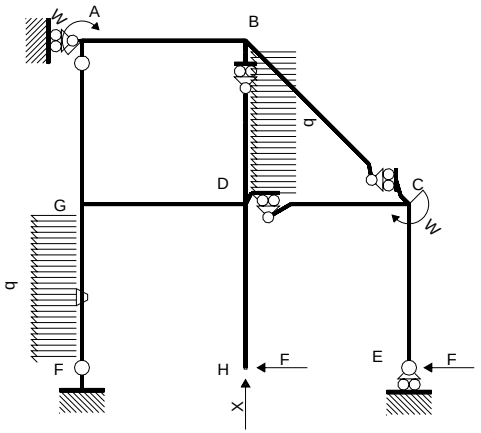
←□→ F



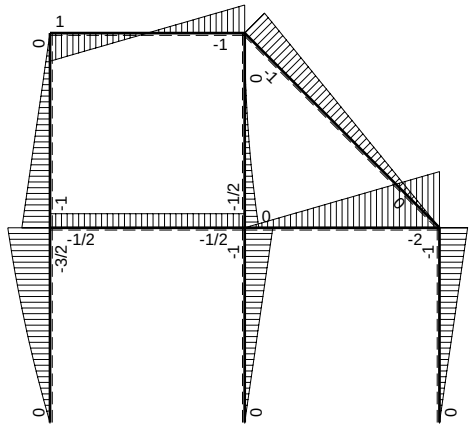
↑□↓ F



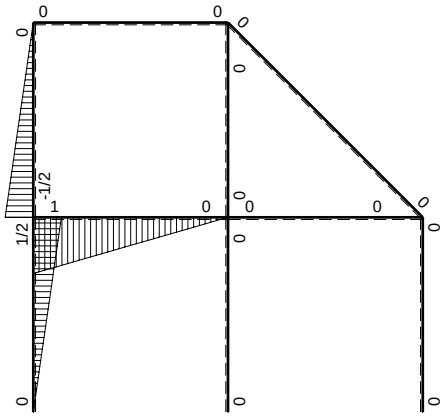
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	Fb-2Fx	0	0	0	0	0+0	0
BA b	0	Fb-2Fx	0	0	0	0		
BC √2b	0	-Fb+√2/2Fx	0	0	0	0	0	0
BD b	0	$-1/2qx^2$	0	0	0	0	0+0	0
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	1/2x	$-2Fx+1/2qx^2$	-Fb/EJ	$-Fx^2+1/4qx^3$	-1/2Fxb/EJ	$1/4x^2$	$(-13/48-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	$3/2Fb-Fx-1/2qx^2$	Fb/EJ	$-3/4Fb^2+5/4Fbx-1/4Fx^2-1/4qx^3$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	-1/2Fb	0	$-1/2Fb^2+1/2Fbx$	0	$b^2-2bx+x^2$	$(-1/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	1/2Fb	0	-1/2Fbx	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	-Fb+Fx	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	1/2x	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
	totali						$-29/48Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						29/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-\ x^2/b^2 + 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx$

$= [-1/3\ x^3/b^2 + 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/3\ b + 1/16\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -25/48\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/4 + 5/4\ x/b - 1/4\ x^2/b^2 - 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-3/4\ x + 5/8\ x^2/b - 1/12\ x^3/b^2 - 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-3/4\ b + 5/8\ b - 1/12\ b - 1/16\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -25/48\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-1/2 + 1/2\ x/b)\ Fb^2\ 1/EJ\ dx = [-1/2\ x + 1/4\ x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (-1/2\ b + 1/4\ b)\ Fb^2\ 1/EJ = -1/4\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-1/2\ x/b)\ Fb^2\ 1/EJ\ dx = [-1/4\ x^2/b]_o^b\ Fb^2\ 1/EJ$

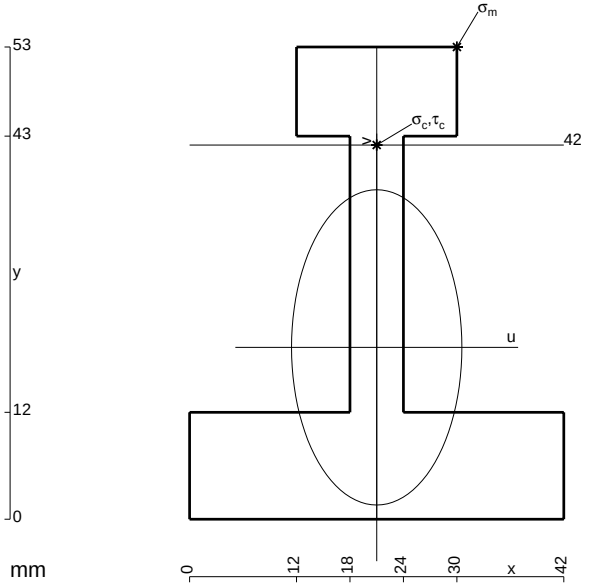
$= (-1/4\ b)\ Fb^2\ 1/EJ = -1/4\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb^2\ 1/EJ = 1/6\ Fb^3/EJ$

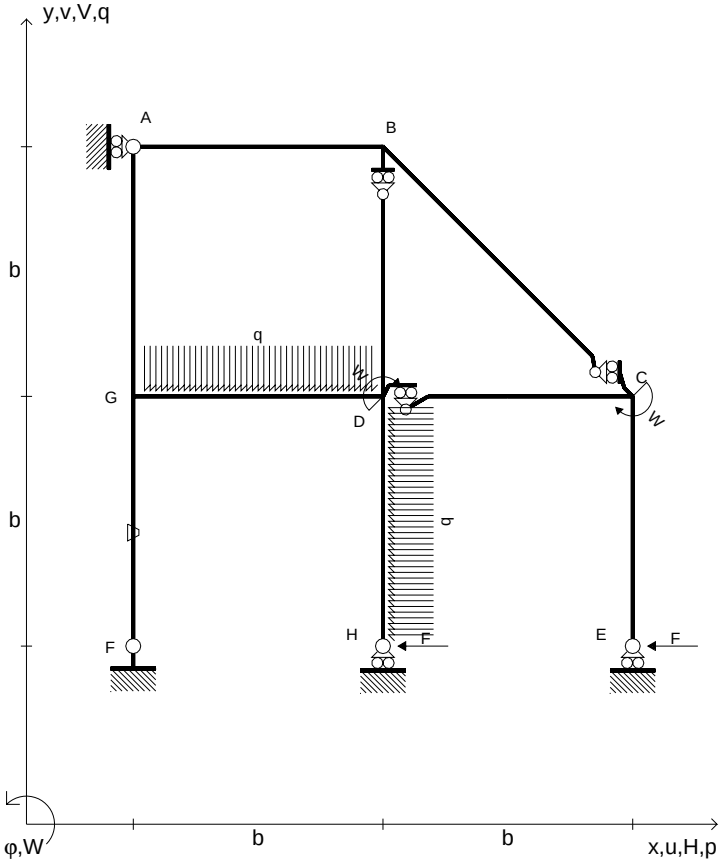
$L^{Xo}_{AG} = \int_o^b (1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

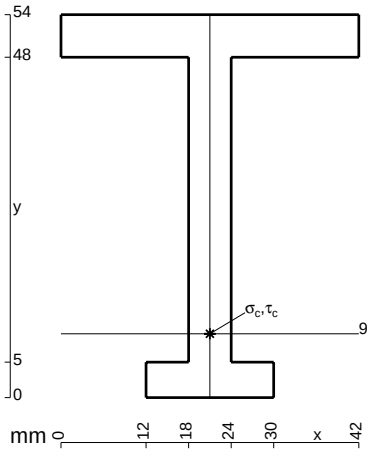
$= (1/6\ b)\ Fb^2\ 1/EJ = 1/6\ Fb^3/EJ$



A = 870. mm^2
J_u = 272367. mm^4
J_v = 79506. mm^4
y_g = 19.29 mm
T_y = -2760. N
M_x = -1932000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 9. mm
v_m = 33.71 mm
$\sigma_m = -Mv/J_u$ = 239.1 N/mm^2
x_c = 21. mm
y_c = 42. mm
v_c = 22.71 mm
$\sigma_c = -Mv/J_u$ = 161.1 N/mm^2
τ_c = 8.964 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 161.9 N/mm^2
S^* = 5308. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_D = -W = -Fb$
$p_{DH} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_F$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 760 mm, F = 860 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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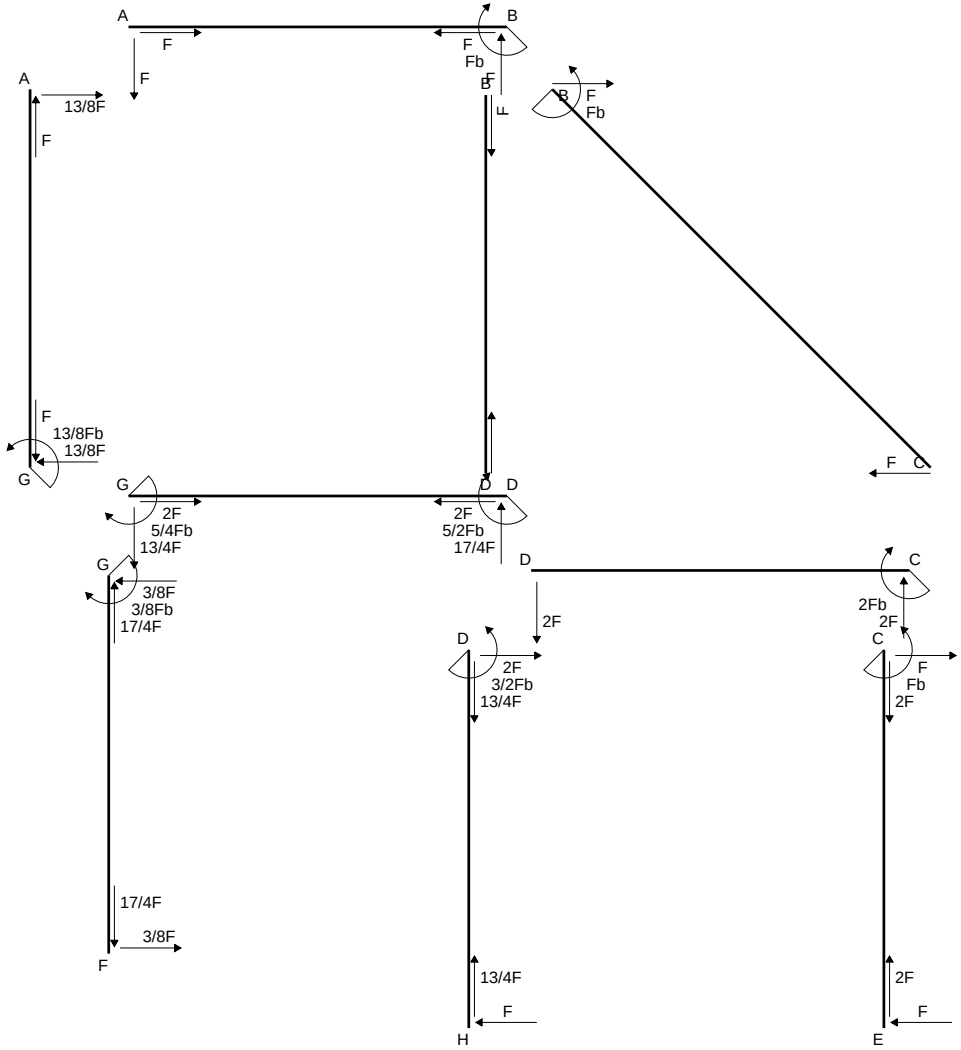

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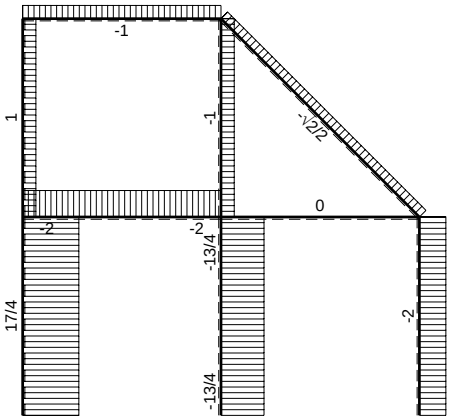
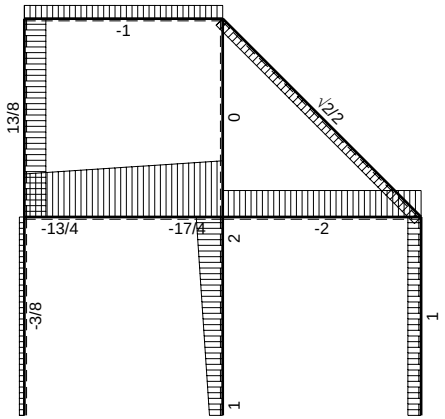
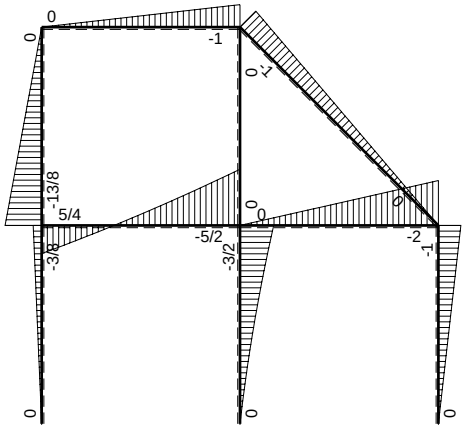


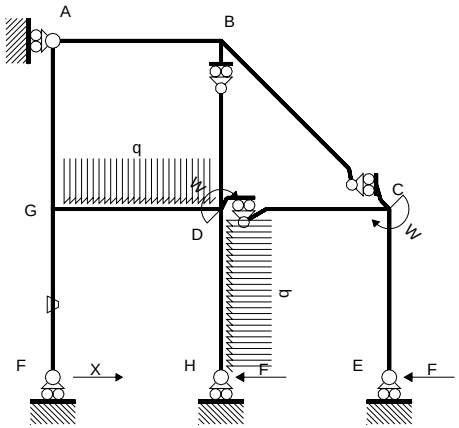
←⊞→ F



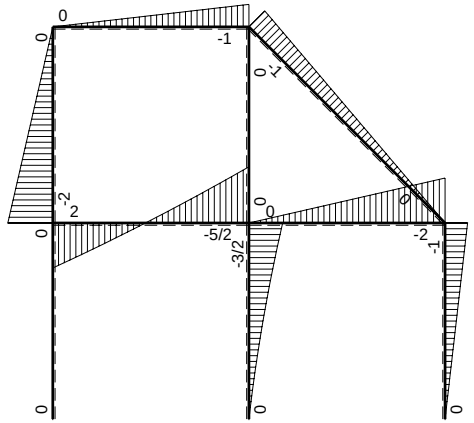
↑⊞↓ F



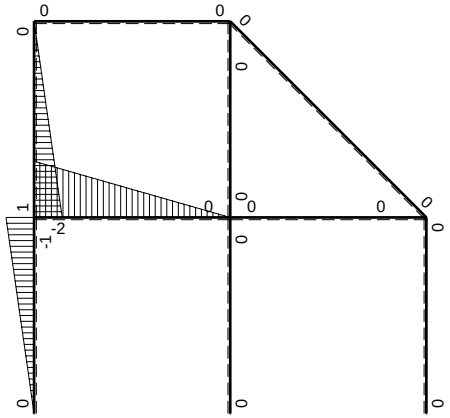
⟲⊞⟳ Fb



Schema di calcolo iperstatico



⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_F$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0	0+0	0
BA b	0	Fb-Fx	0	0	0	0		
BC √2b	0	-Fb+√2/2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	-2b+2x	$2Fb-4Fx-1/2qx^2$	0	$-4Fb^2+12Fbx-7Fx^2-qx^3$	0	$4b^2-8bx+4x^2$	$(-7/12+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	$5/2Fb-5Fx+1/2qx^2$	0	$5Fbx-10Fx^2+qx^3$	0	$4x^2$		
DH b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
HD b	0	$Fx+1/2qx^2$	0	0	0	0		
GA b	b-x	-2Fb+2Fx	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	2Fx	0	$-2Fx^2$	0	x^2		
	totali						$-3/4Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_F$						3/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (x/b)\, \theta\ dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1 + x/b)\, \theta\ dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-4 +12\, x/b -7\, x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-4\, x +6\, x^2/b -7/3\, x^3/b^2 -1/4\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-4\, b +6\, b -7/3\, b -1/4\, b)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (5\, x/b -10\, x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [5/2\, x^2/b -10/3\, x^3/b^2 +1/4\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

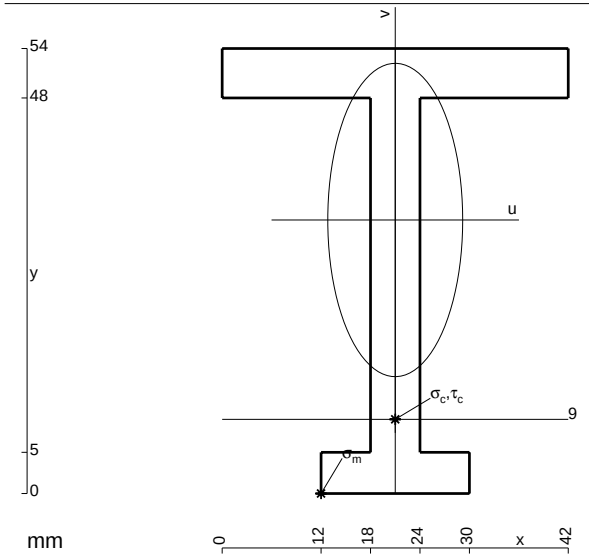
$= (5/2\, b -10/3\, b +1/4\, b)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-2 +4\, x/b -2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\, x +2\, x^2/b -2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2\, b +2\, b -2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

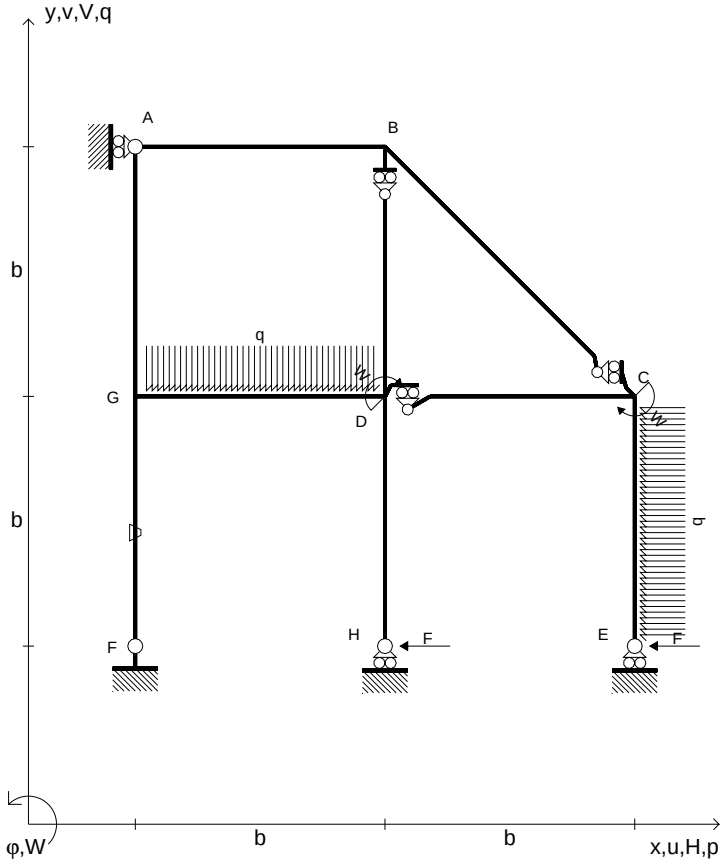
$L^{Xo}_{AG} = \int_o^b (-2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

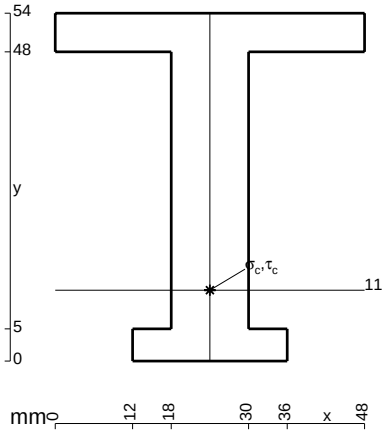
$= (-2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$



A = 600. mm^2
J_u = 216946. mm^4
J_v = 40248. mm^4
y_g = 33.19 mm
T_y = -1720. N
M_x = -1307200. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -33.19 mm
$\sigma_m = -Mv/J_u$ = -200. N/mm^2
x_c = 21. mm
y_c = 9. mm
v_c = -24.19 mm
$\sigma_c = -Mv/J_u$ = -145.8 N/mm^2
τ_c = 4.48 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 146. N/mm^2
S^* = 3391. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_D = -W = -Fb$
$p_{CE} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 860 mm, F = 1000 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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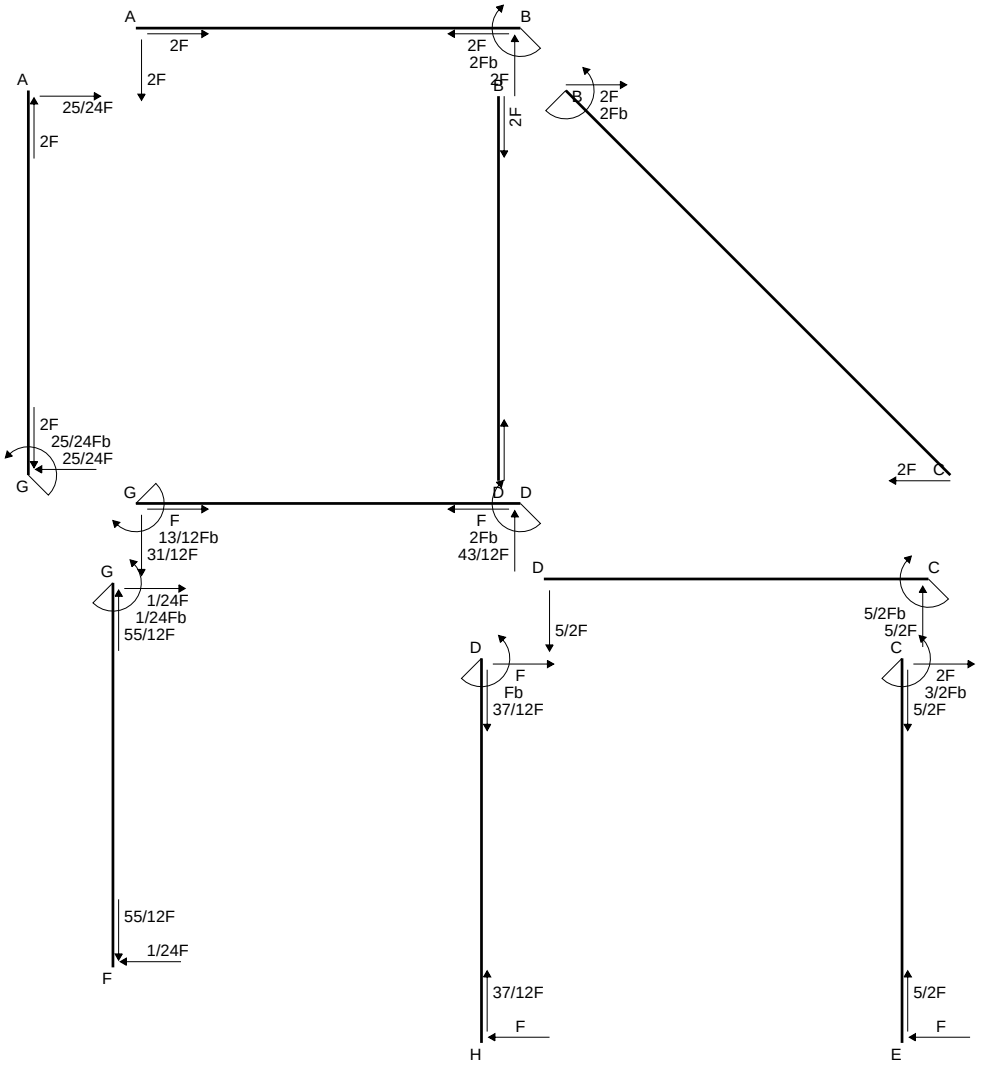

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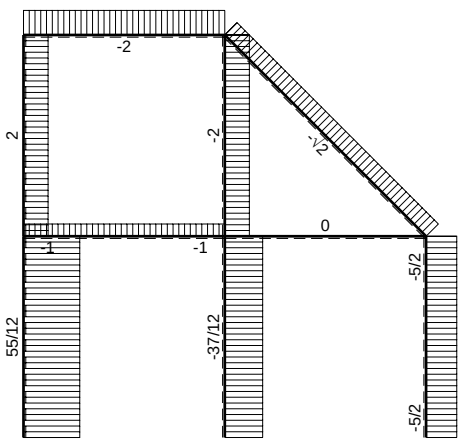
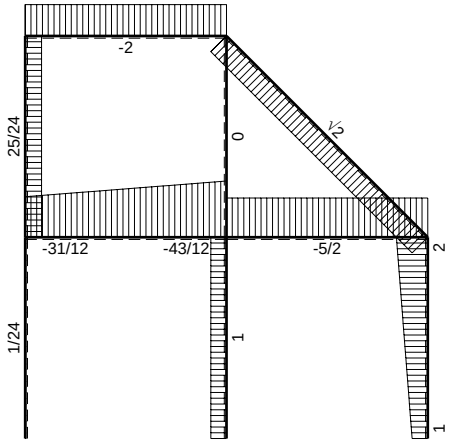
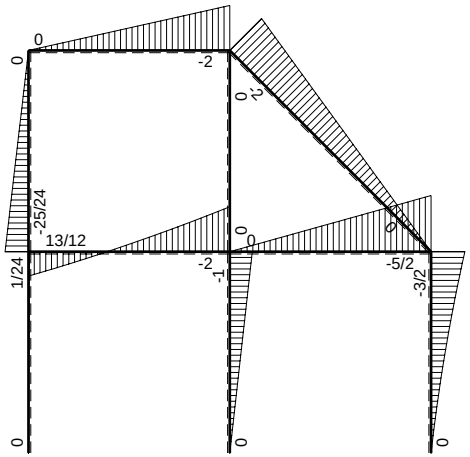


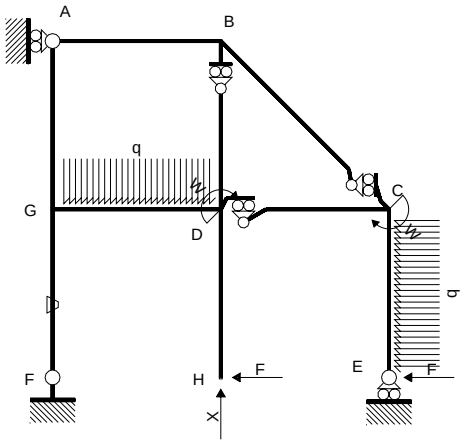
←⊞→ F



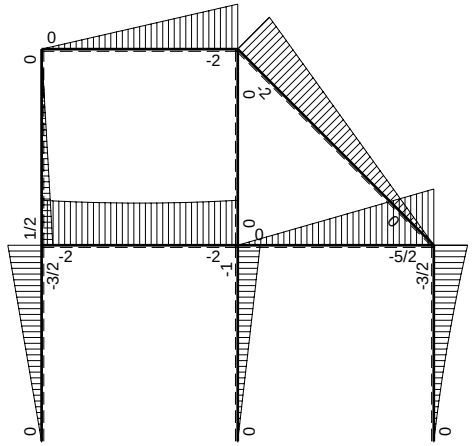
↑⊞↓ F



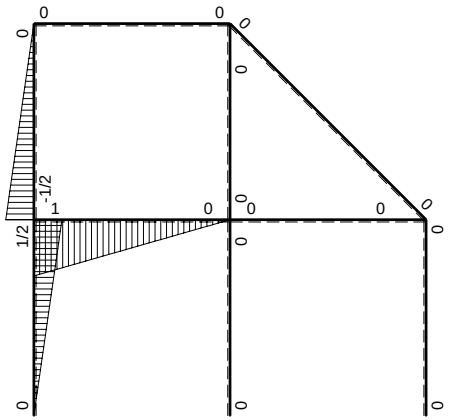
⟲⊞⟳ Fb



Schema di calcolo iperstatico



⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	1/2x	-3/2Fx	-Fb/EJ	$-3/4Fx^2$	-1/2Fxb/EJ	$1/4x^2$	$(-1/4-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	$-3/4Fb^2+3/2Fbx-3/4Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	$-2Fb+1/2Fx-1/2qx^2$	0	$-2Fb^2+5/2Fbx-Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(-23/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	$2Fb-1/2Fx+1/2qx^2$	0	$-2Fbx+1/2Fx^2-1/2qx^3$	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	1/2Fb-1/2Fx	0	$-1/4Fb^2+1/2Fbx-1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	1/2x	-1/2Fx	0	$-1/4Fx^2$	0	$1/4x^2$		
	totali						$-37/24Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						37/12F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [-1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/4\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/4 + 3/2\ x/b - 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-3/4\ x + 3/4\ x^2/b - 1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-3/4\ b + 3/4\ b - 1/4\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-2 + 5/2\ x/b - x^2/b^2 + 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-2\ x + 5/4\ x^2/b - 1/3\ x^3/b^2 + 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (-2\ b + 5/4\ b - 1/3\ b + 1/8\ b)\ Fb^2\ 1/EJ = -23/24\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-2\ x/b + 1/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-\ x^2/b + 1/6\ x^3/b^2 - 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

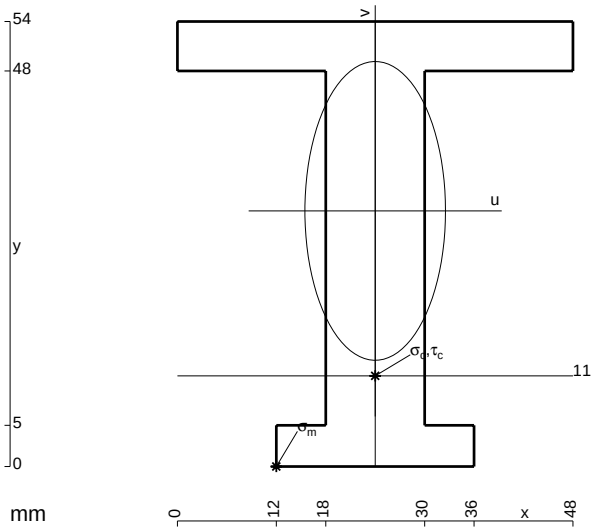
$= (-\ b + 1/6\ b - 1/8\ b)\ Fb^2\ 1/EJ = -23/24\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1/4 + 1/2\ x/b - 1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-1/4\ b + 1/4\ b - 1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$



$A = 924.\ mm^2$

$J_u = 303740.\ mm^4$

$J_v = 67248.\ mm^4$

$y_g = 31.02\ mm$

$T_y = -2500.\ N$

$M_x = -2150000.\ Nmm$

$x_m = 12.\ mm$

$u_m = -12.\ mm$

$v_m = -31.02\ mm$

$\sigma_m = -Mv/J_u = -219.6\ N/mm^2$

$x_c = 24.\ mm$

$y_c = 11.\ mm$

$v_c = -20.02\ mm$

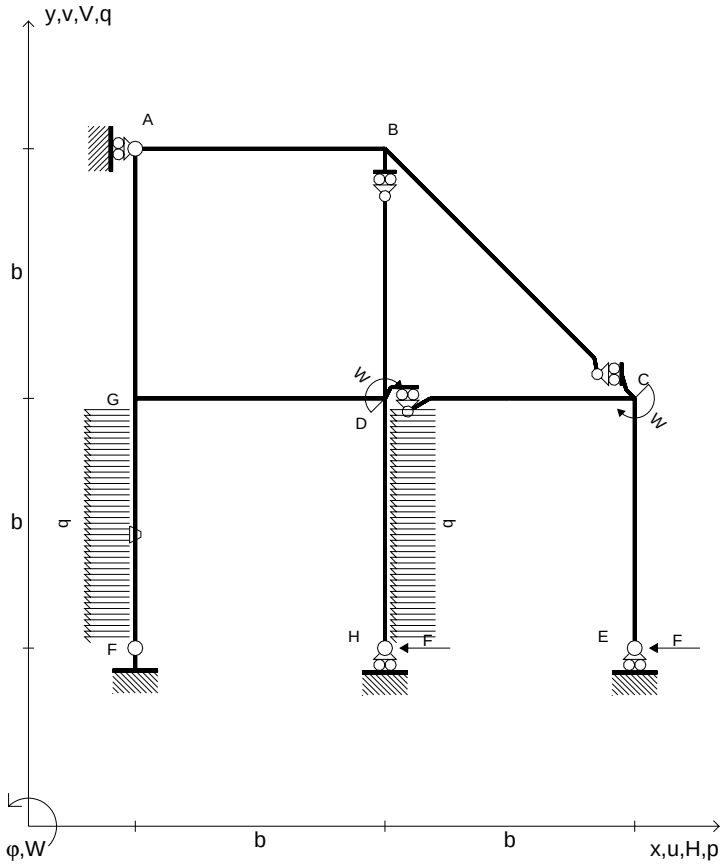
$\sigma_c = -Mv/J_u = -141.7\ N/mm^2$

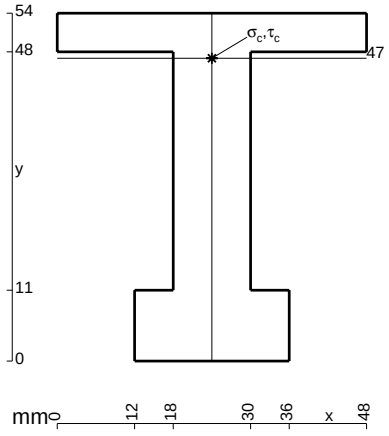
$\tau_c = 3.484\ N/mm^2$

$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 141.8\ N/mm^2$

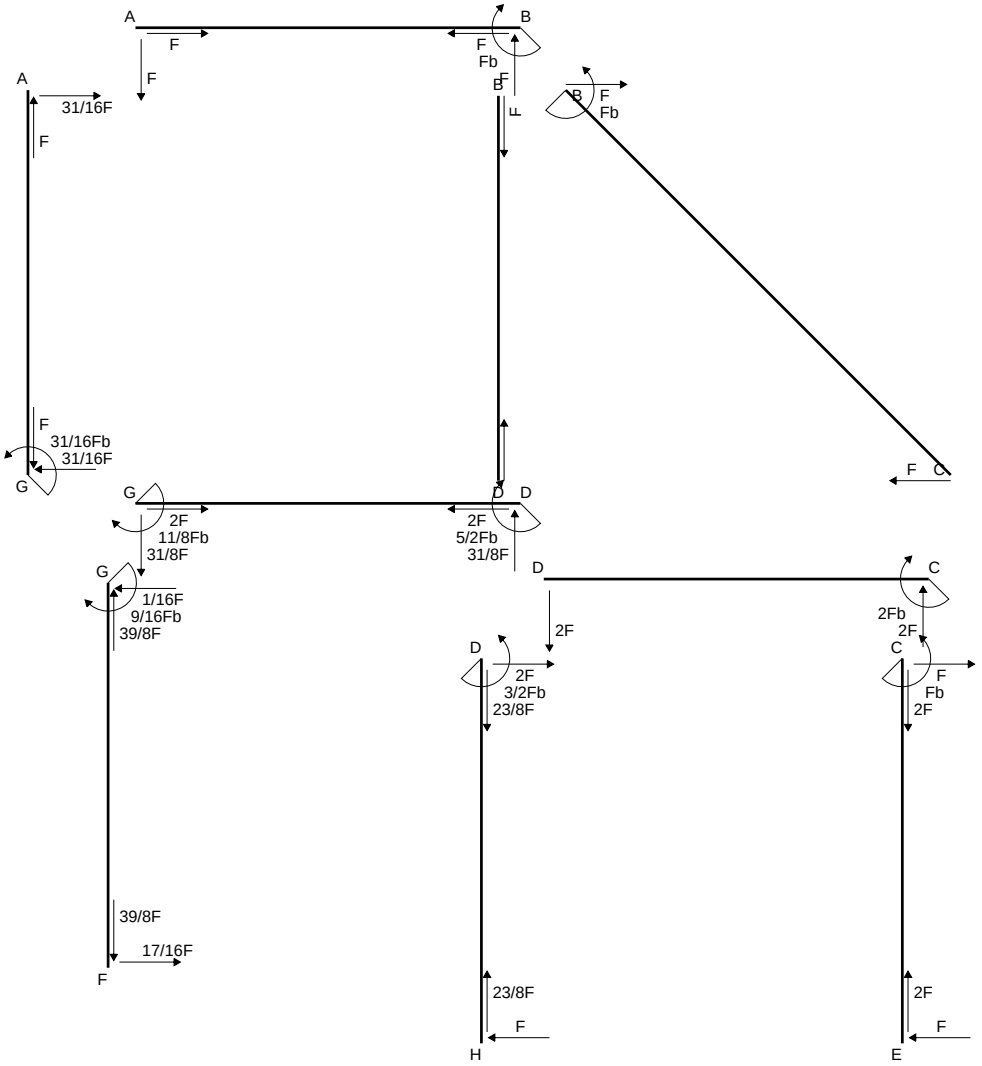
$S^* = 5080.\ mm^3$

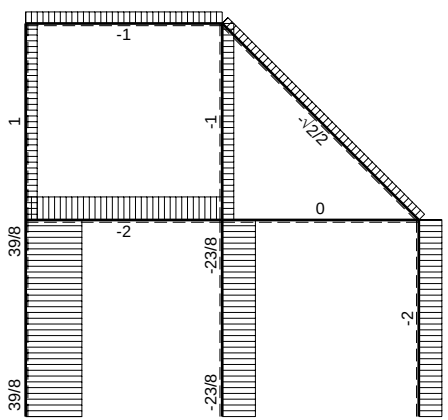
$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_D = -W = -Fb$
$p_{FG} = -q = -F/b$
$p_{DH} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$



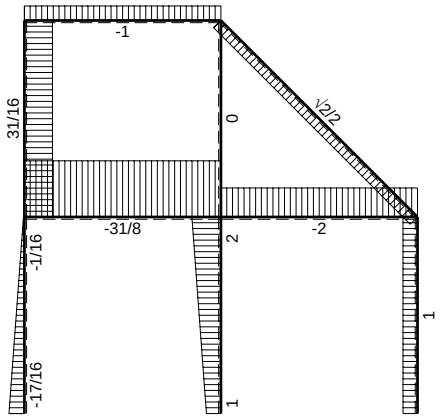


Reazioni iperstatiche in soluzione: $X=W_{GD}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 560 mm, F = 2160 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

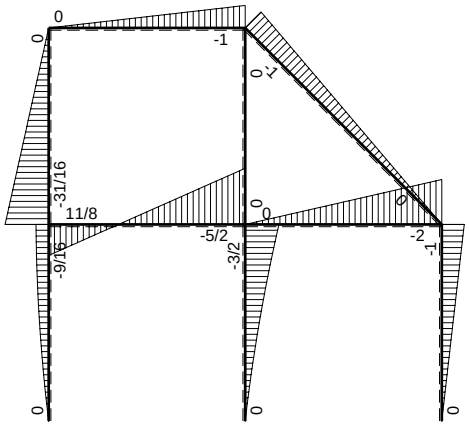




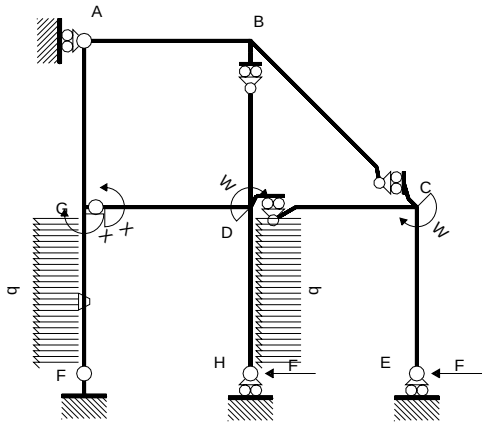
←□→ F



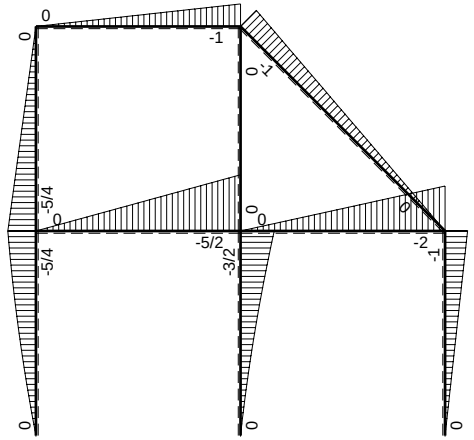
↑□↓ F



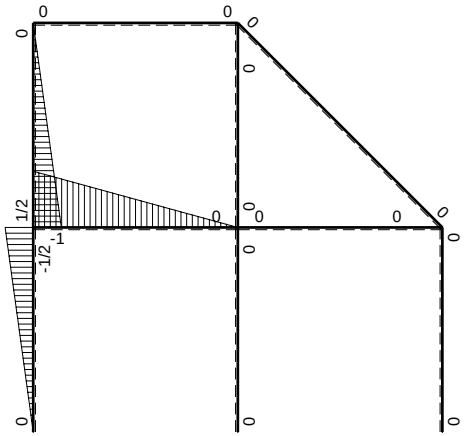
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{GD}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0	0+0	0
BA b	0	Fb-Fx	0	0	0	0		
BC $\sqrt{2}$b	0	-Fb+$\sqrt{2}$/2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	-1/2x/b	$-7/4Fx+1/2qx^2$	-Fb/EJ	$7/8Fx^2/b-1/4qx^3/b$	1/2Fx/EJ	$1/4x^2/b^2$	$(11/48+1/4)Fb^2/EJ$	1/12Xb/EJ
GF b	1/2-1/2x/b	$5/4Fb-3/4Fx-1/2qx^2$	Fb/EJ	$5/8Fb-Fx+1/8Fx^2/b+1/4qx^3/b$	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$		
GD b	-1+x/b	-5/2Fx	0	$5/2Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(5/12+0)Fb^2/EJ$	1/3Xb/EJ
DG b	x/b	5/2Fb-5/2Fx	0	$5/2Fx-5/2Fx^2/b$	0	x^2/b^2		
DH b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
HD b	0	$Fx+1/2qx^2$	0	0	0	0		
GA b	1/2-1/2x/b	-5/4Fb+5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
AG b	-1/2x/b	5/4Fx	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
	totali						$11/16Fb^2/EJ$	1/2Xb/EJ
	iperstatica $X=W_{GD}$						-11/8Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (b - b + 1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{DG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{Xo}_{FG} = \int_o^b (7/8\ x^2/b^2 - 1/4\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (1/2\ x/b)\ \theta\ dx$

$= [7/24\ x^3/b^2 - 1/16\ x^4/b^3]_o^b\ Fb\ 1/EJ + [1/4\ x^2/b]_o^b\ \theta$

$= (7/24\ b - 1/16\ b)\ Fb\ 1/EJ + (1/4\ b)\ \theta = 23/48\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^b (5/8 - x/b + 1/8\ x^2/b^2 + 1/4\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 + 1/2\ x/b)\ \theta\ dx$

$= [5/8\ x - 1/2\ x^2/b + 1/24\ x^3/b^2 + 1/16\ x^4/b^3]_o^b\ Fb\ 1/EJ + [-1/2\ x + 1/4\ x^2/b]_o^b\ \theta$

$= (5/8\ b - 1/2\ b + 1/24\ b + 1/16\ b)\ Fb\ 1/EJ + (-1/2\ b + 1/4\ b)\ \theta = 23/48\ Fb^2/EJ$

$L^{Xo}_{GD} = \int_o^b (5/2\ x/b - 5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/4\ x^2/b - 5/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (5/4\ b - 5/6\ b)\ Fb\ 1/EJ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DG} = \int_o^b (5/2\ x/b - 5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/4\ x^2/b - 5/6\ x^3/b^2]_o^b\ Fb\ 1/EJ$

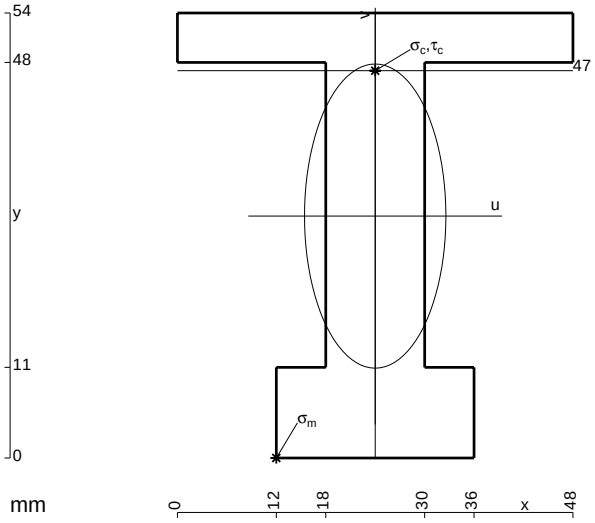
$= (5/4\ b - 5/6\ b)\ Fb\ 1/EJ = 5/12\ Fb^2/EJ$

$L^{Xo}_{GA} = \int_o^b (-5/8 + 5/4\ x/b - 5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-5/8\ b + 5/8\ b - 5/24\ b)\ Fb\ 1/EJ = -5/24\ Fb^2/EJ$

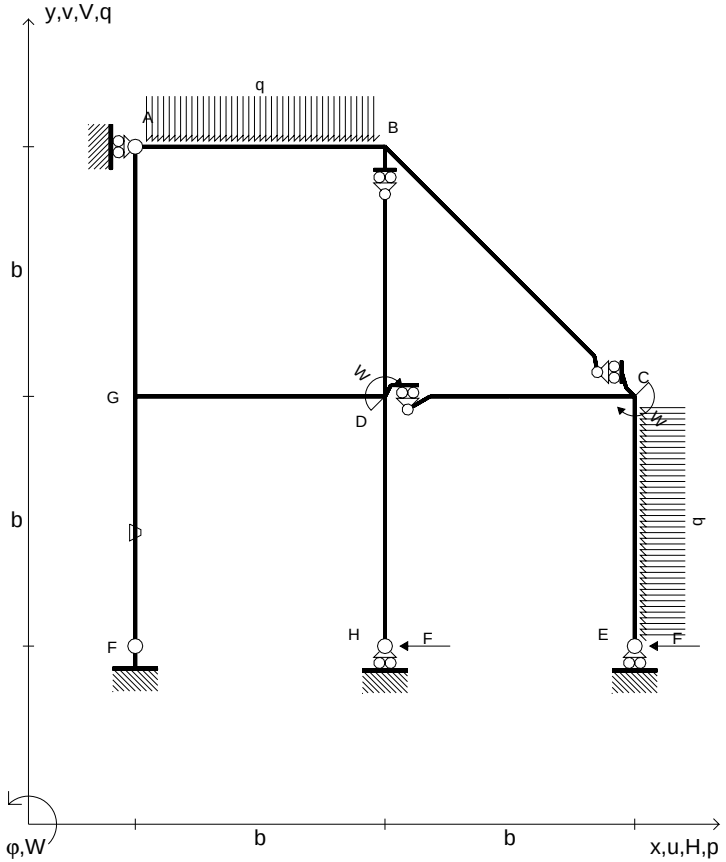
$L^{Xo}_{AG} = \int_o^b (-5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/24\ x^3/b^2]_o^b\ Fb\ 1/EJ$

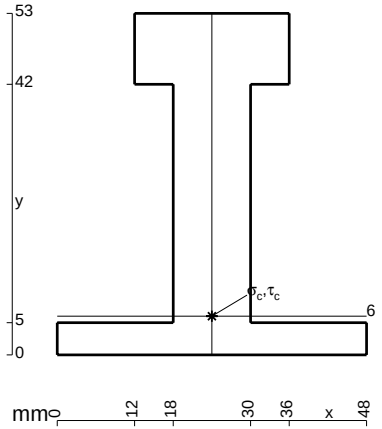
$= (-5/24\ b)\ Fb\ 1/EJ = -5/24\ Fb^2/EJ$



$A = 996.\ mm^2$
$J_u = 339350.\ mm^4$
$J_v = 73296.\ mm^4$
$y_g = 29.36\ mm$
$T_y = -4320.\ N$
$M_x = -2419200.\ Nmm$
$x_m = 12.\ mm$
$u_m = -12.\ mm$
$v_m = -29.36\ mm$
$\sigma_m = -Mv/J_u = -209.3\ N/mm^2$
$x_c = 24.\ mm$
$y_c = 47.\ mm$
$v_c = 17.64\ mm$
$\sigma_c = -Mv/J_u = 125.8\ N/mm^2$
$\tau_c = 6.844\ N/mm^2$
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 126.3\ N/mm^2$
$S^* = 6451.\ mm^3$

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_D = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{CE} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave DC ha la sezione riportata e dimensioni in mm, con:
b = 700 mm, F = 1510 N
Calcolare sulla sezione C la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da D a C
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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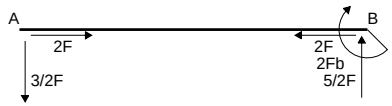

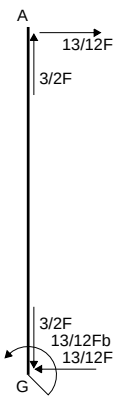

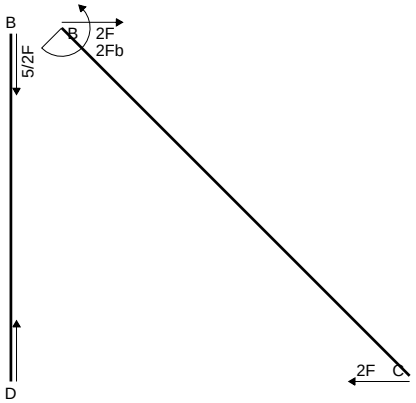

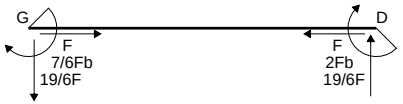

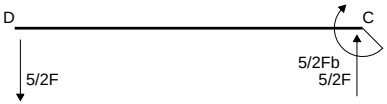

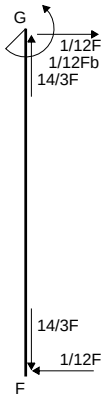

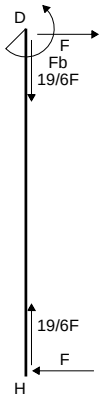

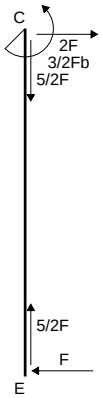

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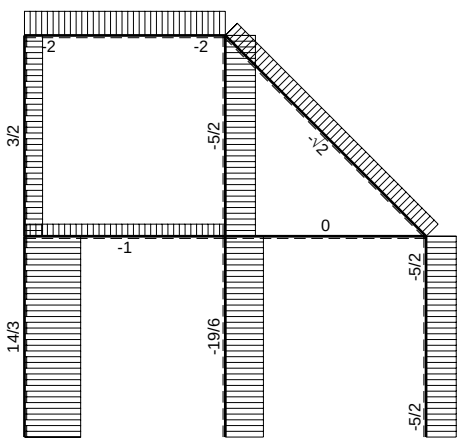
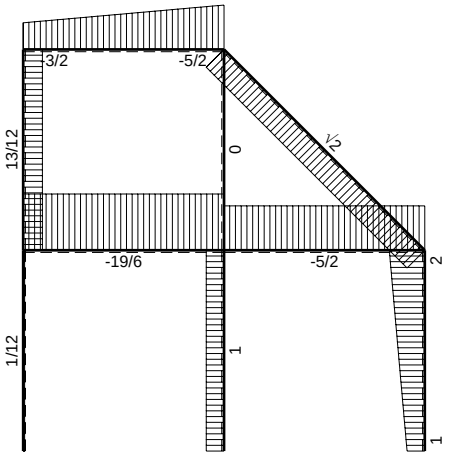
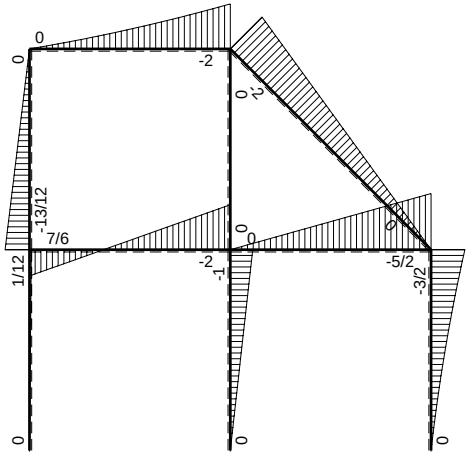


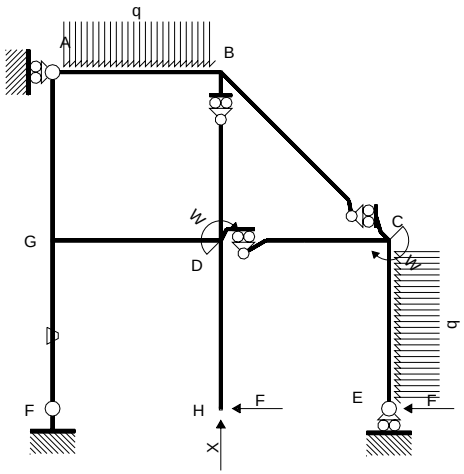
←□→ F



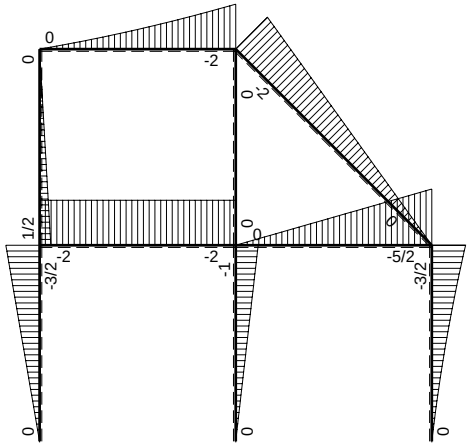
↑□↓ F



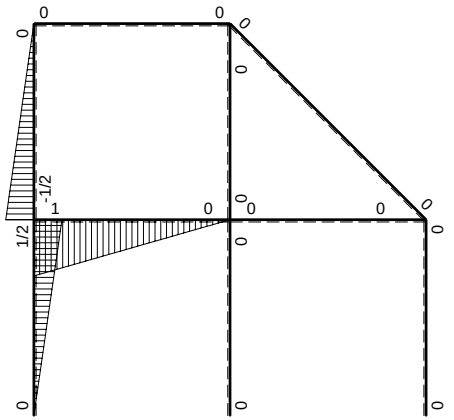
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0		
BC √2b	0	$-2Fb+\sqrt{2}Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	1/2x	-3/2Fx	-Fb/EJ	$-3/4Fx^2$	-1/2Fxb/EJ	$1/4x^2$	$(-1/4-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	$-3/4Fb^2+3/2Fbx-3/4Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	-2Fb	0	$-2Fb^2+2Fbx$	0	$b^2-2bx+x^2$	$(-1+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	2Fb	0	-2Fbx	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	1/2Fb-1/2Fx	0	$-1/4Fb^2+1/2Fbx-1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	1/2x	-1/2Fx	0	$-1/4Fx^2$	0	$1/4x^2$		
	totali						$-19/12Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						19/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [-1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/4\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/4 + 3/2\ x/b - 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-3/4\ x + 3/4\ x^2/b - 1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-3/4\ b + 3/4\ b - 1/4\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-2 + 2\ x/b)\ Fb^2\ 1/EJ\ dx = [-2\ x + x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (-2\ b + b)\ Fb^2\ 1/EJ = -\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-2\ x/b)\ Fb^2\ 1/EJ\ dx = [-\ x^2/b]_o^b\ Fb^2\ 1/EJ$

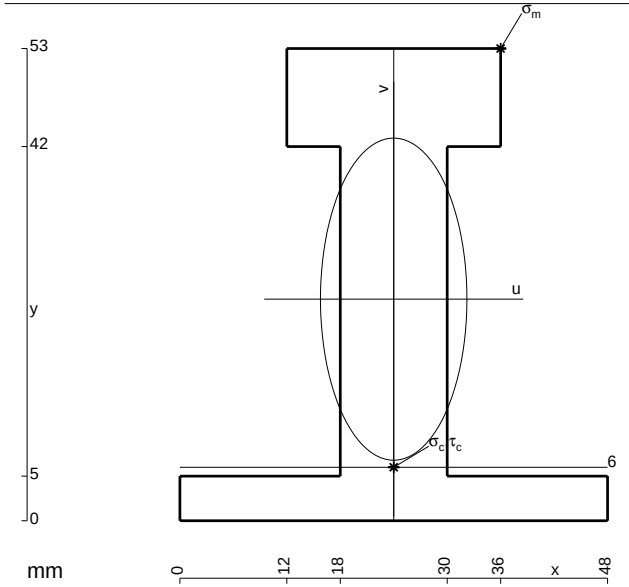
$= (-\ b)\ Fb^2\ 1/EJ = -\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1/4 + 1/2\ x/b - 1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-1/4\ b + 1/4\ b - 1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$

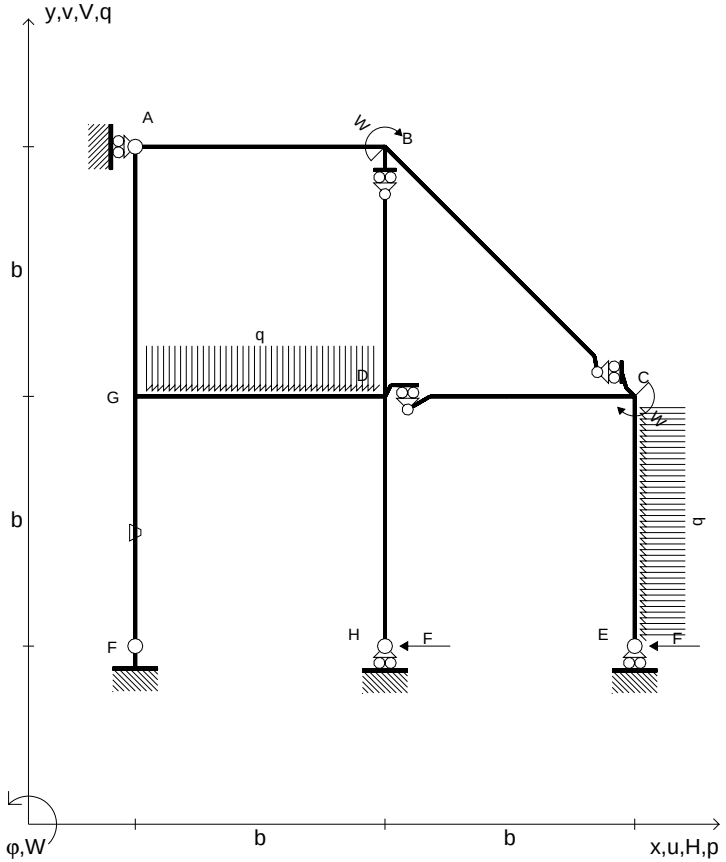
$L^{Xo}_{AG} = \int_o^b (-1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

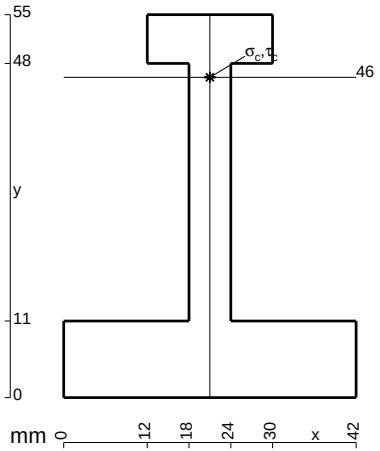
$= (-1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$



A = 948. mm^2
J_u = 309947. mm^4
J_v = 64080. mm^4
y_g = 24.87 mm
T_y = -3775. N
M_x = -2642500. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 12. mm
v_m = 28.13 mm
$\sigma_m = -Mv/J_u$ = 239.9 N/mm^2
x_c = 24. mm
y_c = 6. mm
v_c = -18.87 mm
$\sigma_c = -Mv/J_u$ = -160.9 N/mm^2
τ_c = 5.684 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 161.2 N/mm^2
S^* = 5601. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_B = -W = -Fb$
$p_{CE} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





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Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 980 mm, F = 620 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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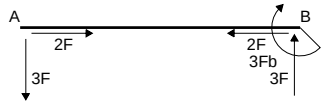

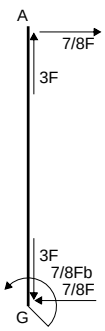

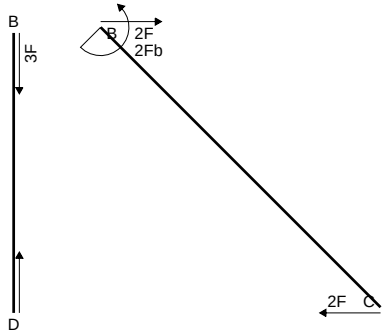

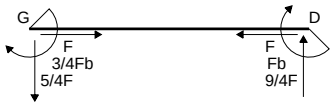

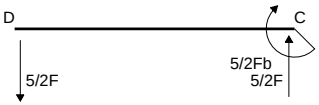

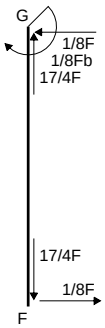

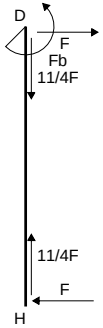

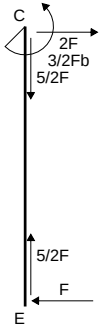

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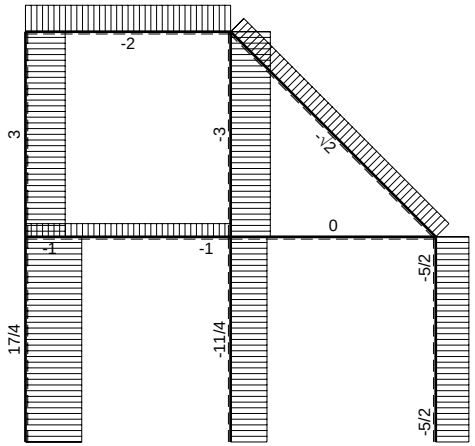
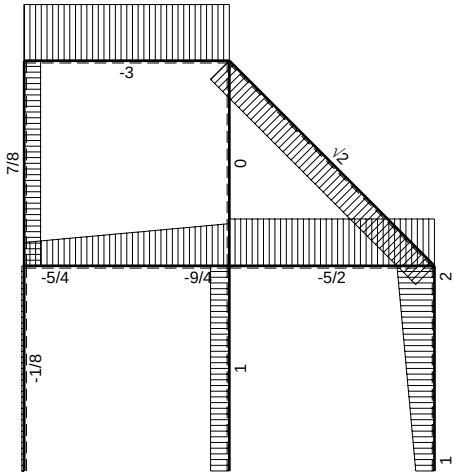
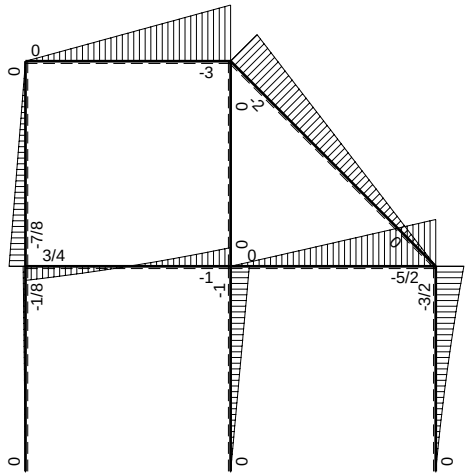


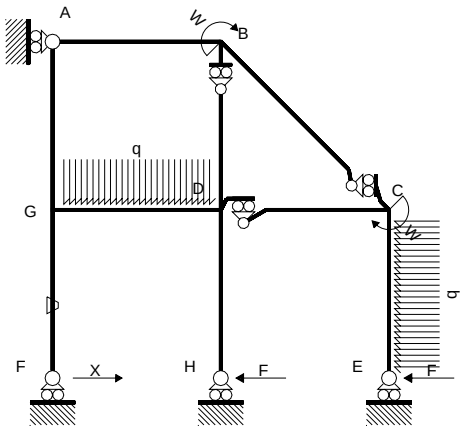
←⊞→ F



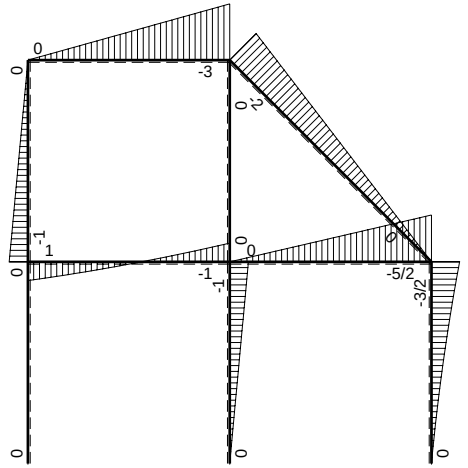
↑⊞↓ F



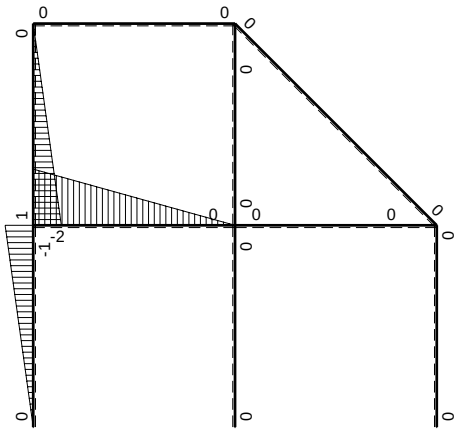
⟲⊞⟳ Fb



Schema di calcolo iperstatico



⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_F$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-3Fx	0	0	0	0	0+0	0
BA b	0	3Fb-3Fx	0	0	0	0		
BC $\sqrt{2}$b	0	$-2Fb+\sqrt{2}Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	-2b+2x	$Fb-3/2Fx-1/2qx^2$	0	$-2Fb^2+5Fbx-2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$	$(-5/12+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	$Fb-5/2Fx+1/2qx^2$	0	$2Fbx-5Fx^2+qx^3$	0	$4x^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	b-x	-Fb+Fx	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	Fx	0	$-Fx^2$	0	x^2		
	totali						$-1/4Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_F$						1/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ b^2\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\ x/b +4\ x^2/b^2\ )\ b^2\ 1/EJ\ dx = [4\ x -4\ x^2/b +4/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (4\ b -4\ b +4/3\ b\ )\ b^2\ 1/EJ\ = 4/3\ \ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\ x^2/b^2\ )\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (4/3\ b\ )\ b^2\ 1/EJ\ = 4/3\ \ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ b^2\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2\ )\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (x/b\ )\ \theta\ \ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ \theta\ \ = 1/2\ \ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1 + x/b\ )\ \theta\ \ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ \theta\ \ = 1/2\ \ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-2 +5\ x/b -2\ x^2/b^2 - x^3/b^3\ )\ Fb^2\ 1/EJ\ dx = [-2\ x +5/2\ x^2/b -2/3\ x^3/b^2 -1/4\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ$

$= (-2\ b +5/2\ b -2/3\ b -1/4\ b\ )\ Fb^2\ 1/EJ\ = -5/12\ \ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (2\ x/b -5\ x^2/b^2 + x^3/b^3\ )\ Fb^2\ 1/EJ\ dx = [\ x^2/b -5/3\ x^3/b^2 +1/4\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ$

$= (\ b -5/3\ b +1/4\ b\ )\ Fb^2\ 1/EJ\ = -5/12\ \ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1 +2\ x/b - x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [-\ x + x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (-\ b + b -1/3\ b\ )\ Fb^2\ 1/EJ\ = -1/3\ \ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-\ x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [-1/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (-1/3\ b\ )\ Fb^2\ 1/EJ\ = -1/3\ \ Fb^3/EJ$

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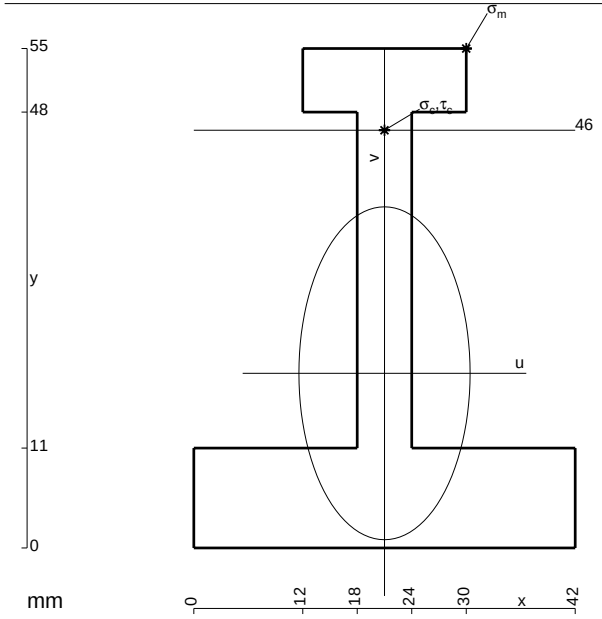

A = 810. mm^2
J_u = 272218. mm^4
J_v = 71982. mm^4
y_g = 19.23 mm
N = -1240. N
T_y = -1860. N
M_x = -1822800. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 9. mm
v_m = 35.77 mm
$\sigma_m = N/A - Mv/J_u$ = 238. N/mm^2
x_c = 21. mm
y_c = 46. mm
v_c = 26.77 mm
$\sigma_c = N/A - Mv/J_u$ = 177.7 N/mm^2
τ_c = 5.009 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 177.9 N/mm^2
S^* = 4399. mm^3

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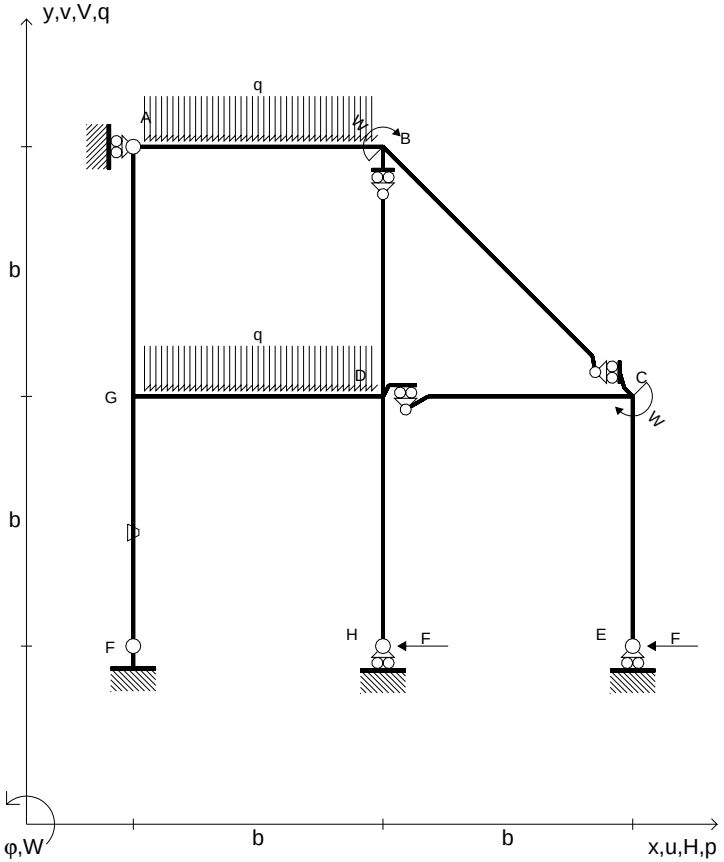
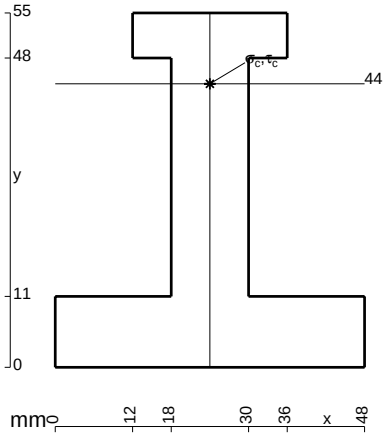
$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_B = -W = -Fb$
$q_{AB} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 570 mm, F = 2060 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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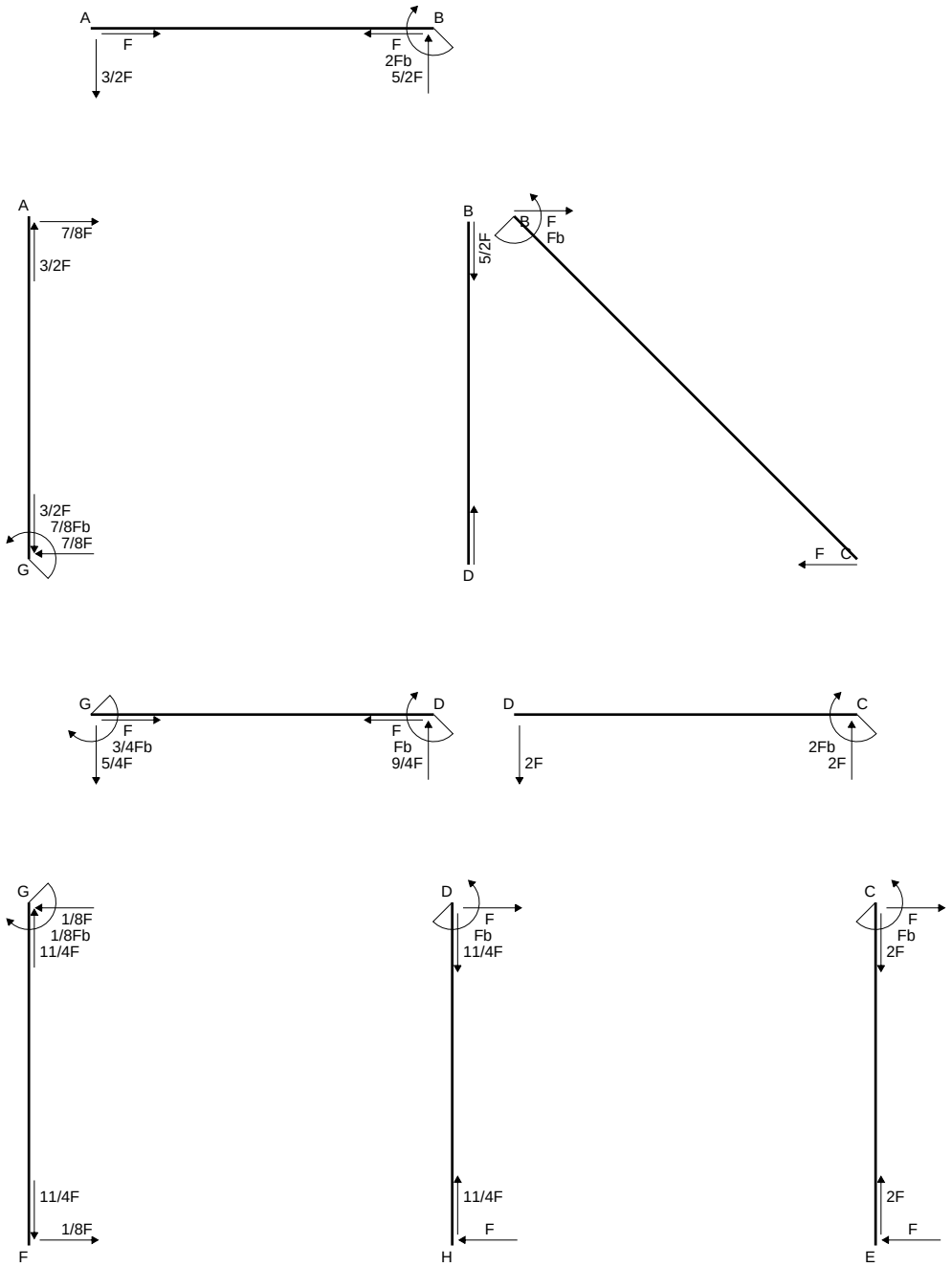

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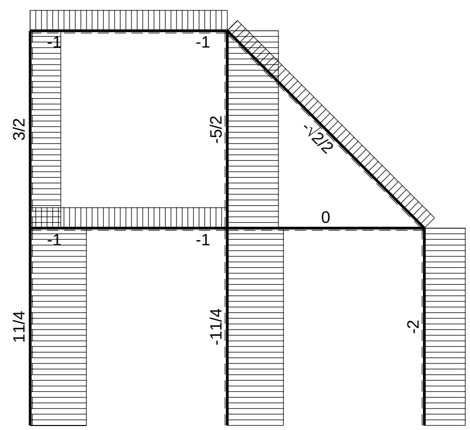
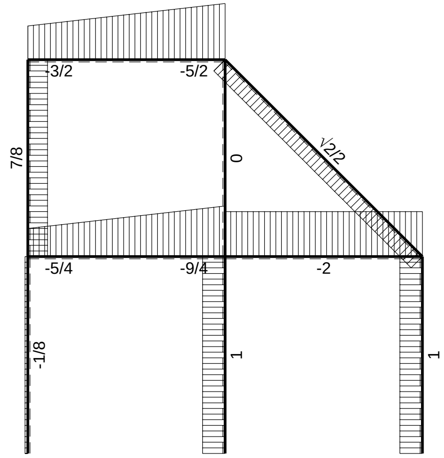
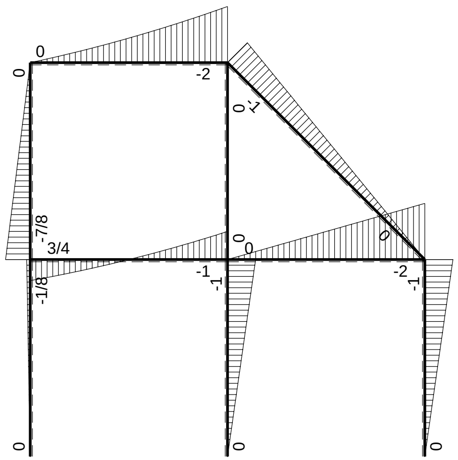


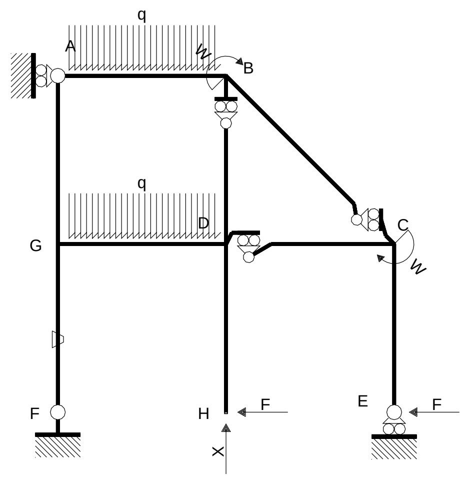
F



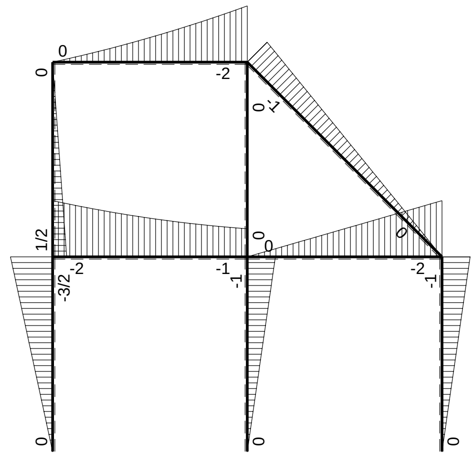
F



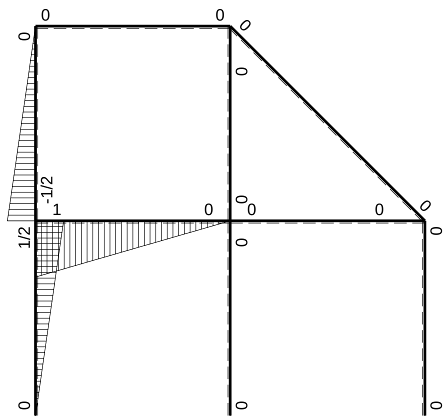
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0		
BC √2b	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	1/2x	-3/2Fx	-Fb/EJ	$-3/4Fx^2$	$-1/2Fxb/EJ$	$1/4x^2$	$(-1/4-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	$-3/4Fb^2+3/2Fbx-3/4Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	$-2Fb+3/2Fx-1/2qx^2$	0	$-2Fb^2+7/2Fbx-2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(-19/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	$Fb+1/2Fx+1/2qx^2$	0	$-Fbx-1/2Fx^2-1/2qx^3$	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	1/2Fb-1/2Fx	0	$-1/4Fb^2+1/2Fbx-1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	1/2x	-1/2Fx	0	$-1/4Fx^2$	0	$1/4x^2$		
	totali						$-11/8Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						11/4F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [-1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/4\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/4 + 3/2\ x/b - 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-3/4\ x + 3/4\ x^2/b - 1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-3/4\ b + 3/4\ b - 1/4\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-2 + 7/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-2\ x + 7/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (-2\ b + 7/4\ b - 2/3\ b + 1/8\ b)\ Fb^2\ 1/EJ = -19/24\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

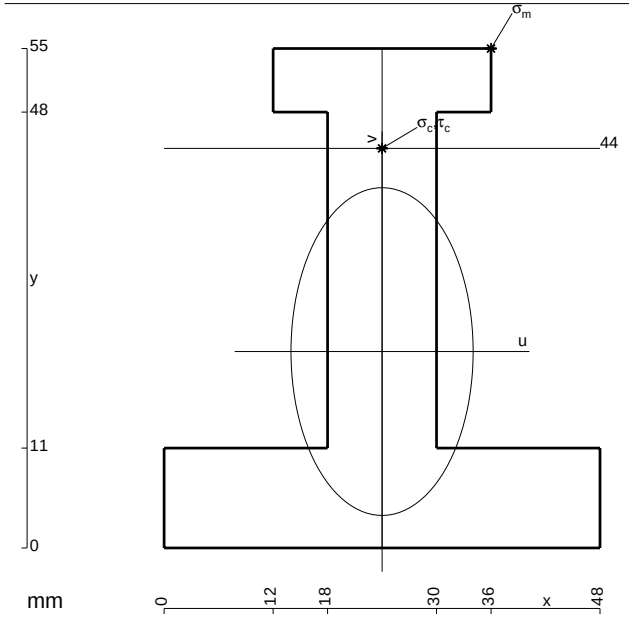
$= (-1/2\ b - 1/6\ b - 1/8\ b)\ Fb^2\ 1/EJ = -19/24\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1/4 + 1/2\ x/b - 1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-1/4\ b + 1/4\ b - 1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$

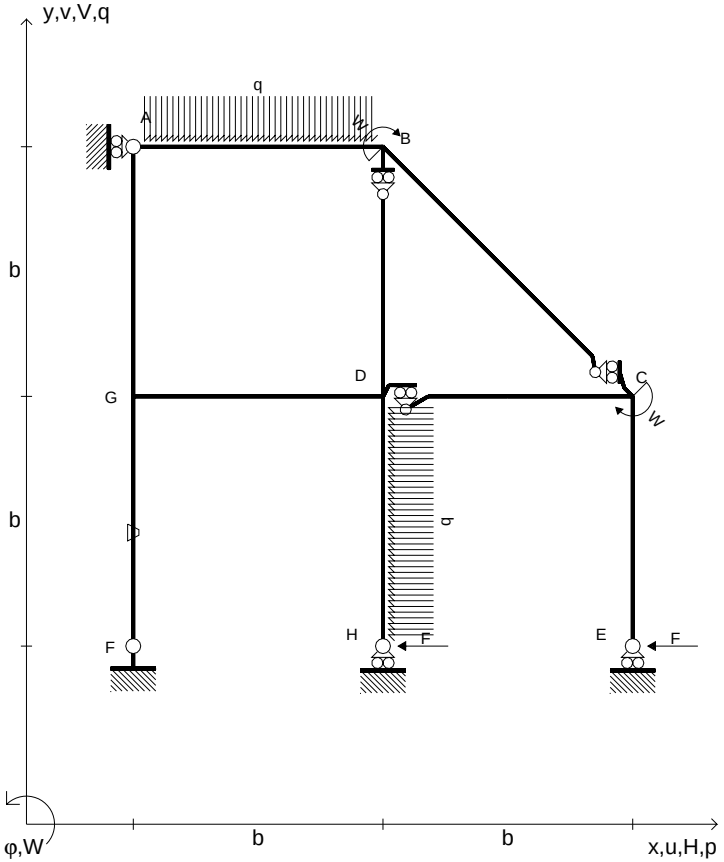
$L^{Xo}_{AG} = \int_o^b (-1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

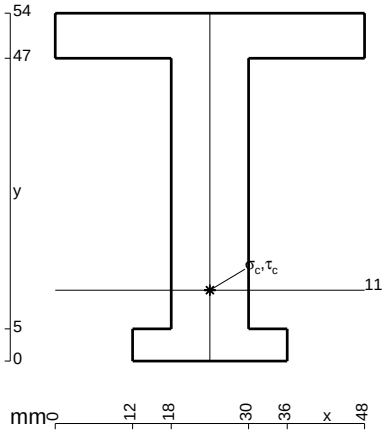
$= (-1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$



A = 1140. mm^2
J_u = 371429. mm^4
J_v = 114768. mm^4
y_g = 21.63 mm
N = -2060. N
T_y = -5150. N
M_x = -2348400. Nmm
x_m = 36. mm
y_m = 55. mm
u_m = 12. mm
v_m = 33.37 mm
$\sigma_m = N/A - Mv/J_u$ = 209.2 N/mm^2
x_c = 24. mm
y_c = 44. mm
v_c = 22.37 mm
$\sigma_c = N/A - Mv/J_u$ = 139.7 N/mm^2
τ_c = 7.151 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 140.2 N/mm^2
S^* = 6189. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_B = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{DH} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 760 mm, F = 1290 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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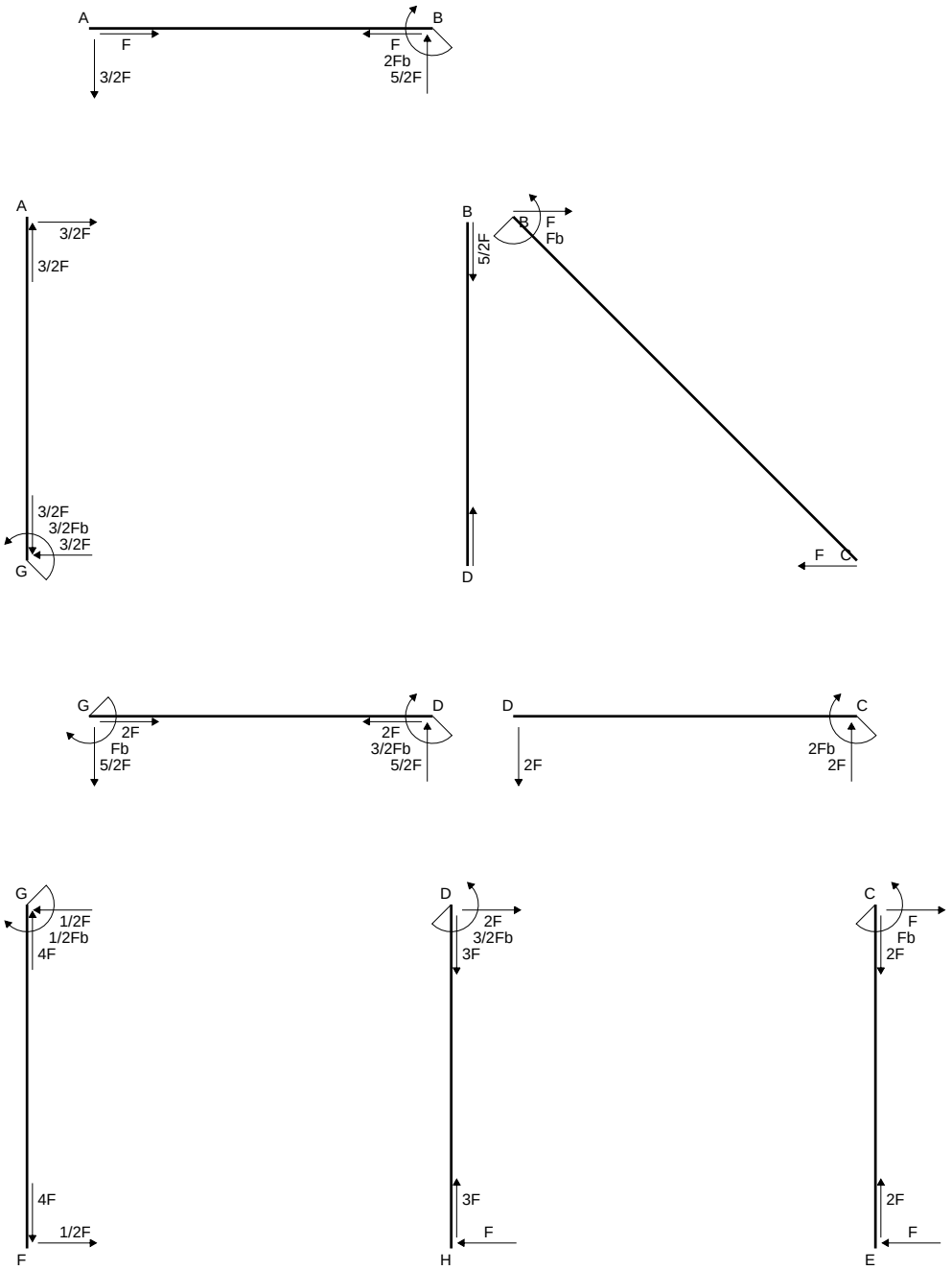

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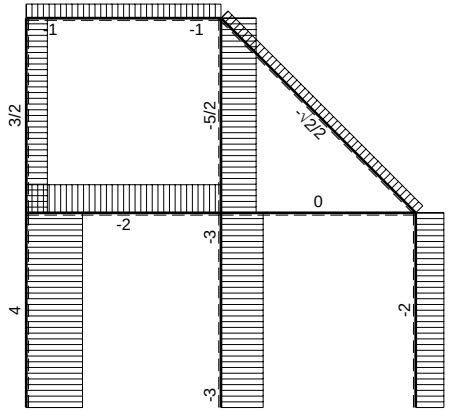
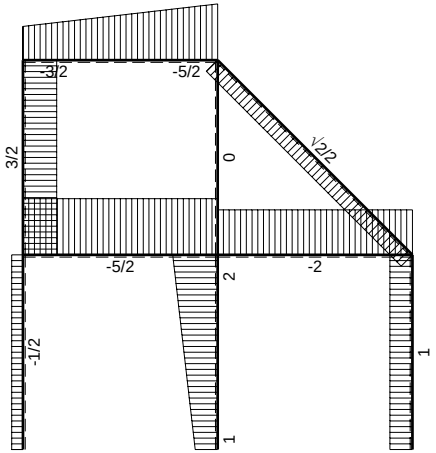
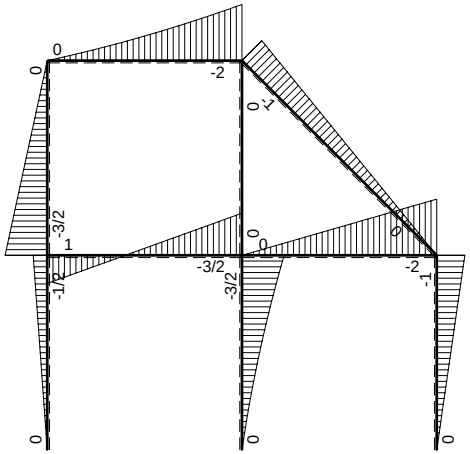


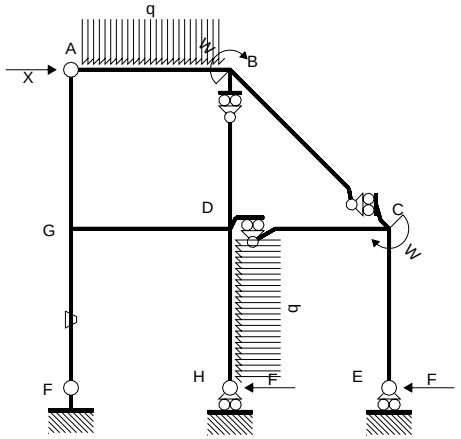
←+→ F



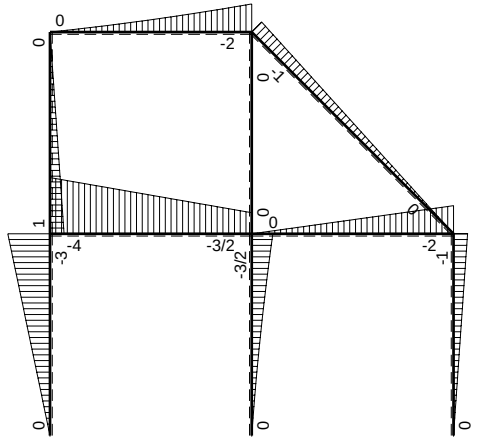
↑+↓ F



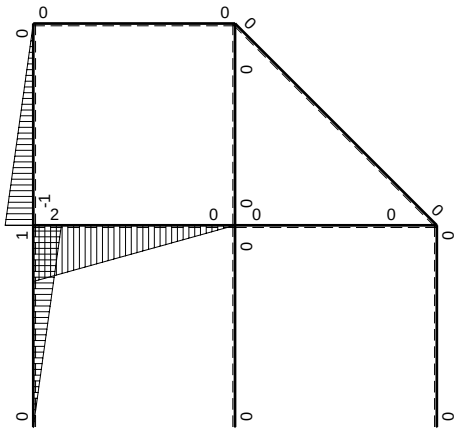
+ Fb



Schema di calcolo iperstatico



+ M_o flessione da carichi assegnati



+ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0		
BC √2b	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	$-2Fx$	0	0	0	0	0+0	0
CD b	0	$2Fb-2Fx$	0	0	0	0		
CE b	0	$-Fb+Fx$	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	x	$-3Fx$	$-Fb/EJ$	$-3Fx^2$	$-Fxb/EJ$	x^2	$(-1-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	$-b+x$	$3Fb-3Fx$	Fb/EJ	$-3Fb^2+6Fbx-3Fx^2$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$		
GD b	$2b-2x$	$-4Fb+5/2Fx$	0	$-8Fb^2+13Fbx-5Fx^2$	0	$4b^2-8bx+4x^2$	$(-19/6+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	$-2x$	$3/2Fb+5/2Fx$	0	$-3Fbx-5Fx^2$	0	$4x^2$		
DH b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
HD b	0	$Fx+1/2qx^2$	0	0	0	0		
GA b	$-b+x$	$Fb-Fx$	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	$-Fx$	0	$-Fx^2$	0	x^2		
	totali						$-5Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_A$						5/2F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (- x/b)\, \theta\ dx = [- x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x^2/b]_o^b\, \theta$

$= (- b)\, Fb^2\, 1/EJ + (-1/2\, b)\, \theta = -3/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3 +6\, x/b -3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 - x/b)\, \theta\ dx$

$= [-3\, x +3\, x^2/b - x^3/b^2]_o^b\, Fb^2\, 1/EJ + [\, x -1/2\, x^2/b]_o^b\, \theta$

$= (-3\, b +3\, b - b)\, Fb^2\, 1/EJ + (\, b -1/2\, b)\, \theta = -3/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-8 +13\, x/b -5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-8\, x +13/2\, x^2/b -5/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-8\, b +13/2\, b -5/3\, b)\, Fb^2\, 1/EJ = -19/6\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-3\, x/b -5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/2\, x^2/b -5/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

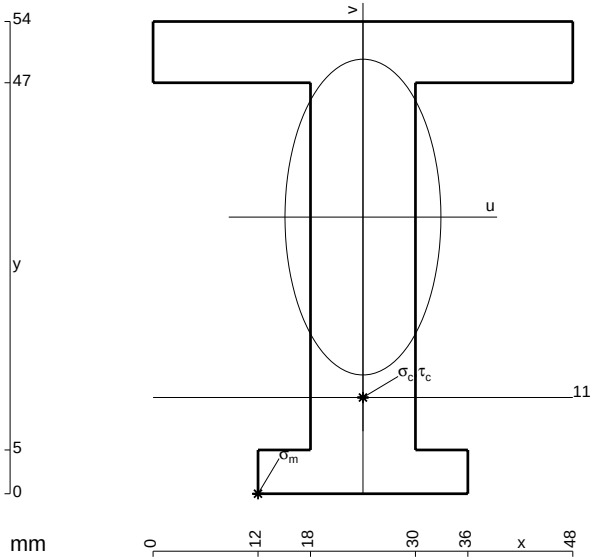
$= (-3/2\, b -5/3\, b)\, Fb^2\, 1/EJ = -19/6\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1 +2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x + x^2/b -1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (- b + b -1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

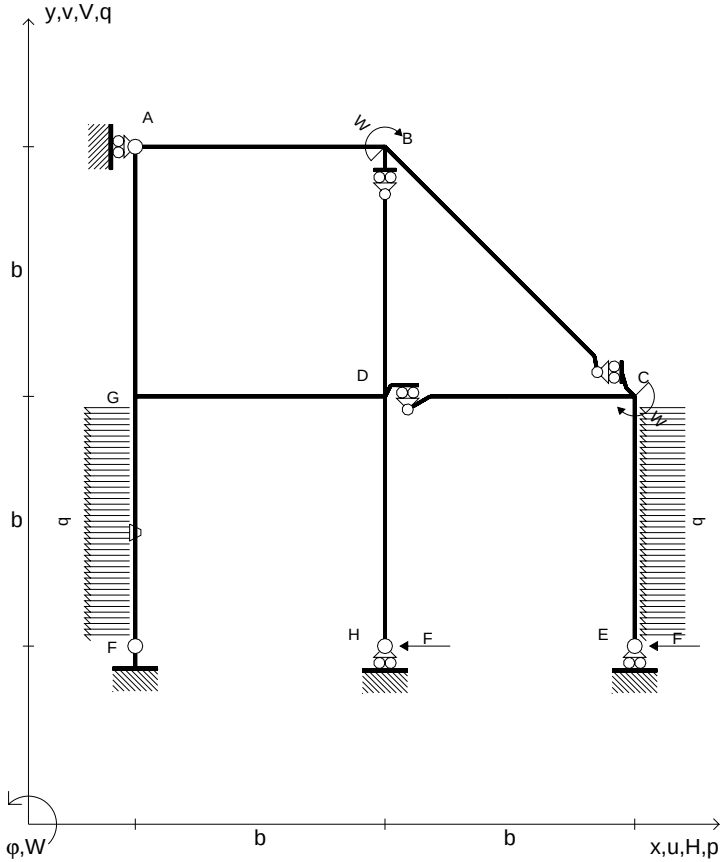
$L^{Xo}_{AG} = \int_o^b (- x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

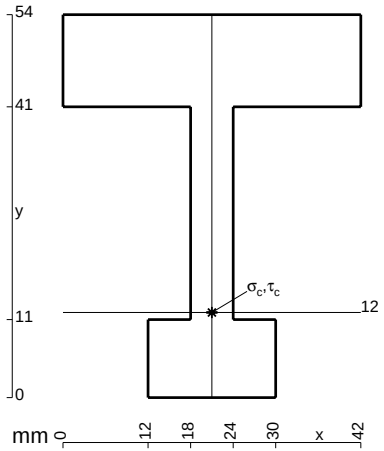
$= (-1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$



A = 960. mm^2
J_u = 313154. mm^4
J_v = 76320. mm^4
y_g = 31.64 mm
N = -1290. N
T_y = -3225. N
M_x = -1960800. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -31.64 mm
$\sigma_m = N/A - Mv/J_u$ = -199.4 N/mm^2
x_c = 24. mm
y_c = 11. mm
v_c = -20.64 mm
$\sigma_c = N/A - Mv/J_u$ = -130.6 N/mm^2
τ_c = 4.461 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 130.8 N/mm^2
S^* = 5198. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_B = -W = -Fb$
$p_{CE} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=W_{GD}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 910 mm, F = 710 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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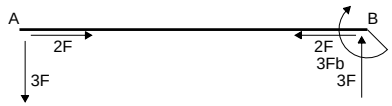

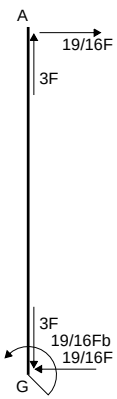

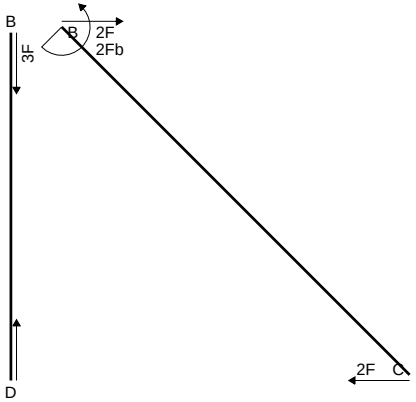

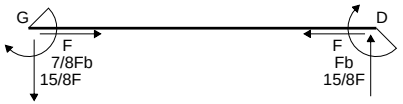

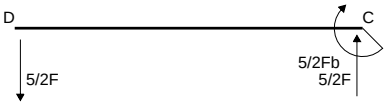

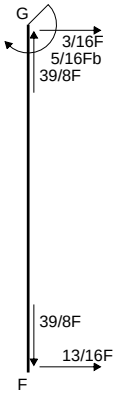

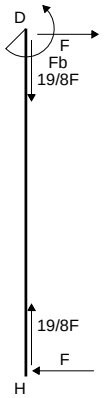

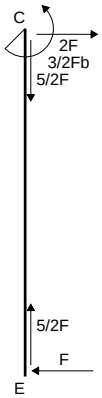

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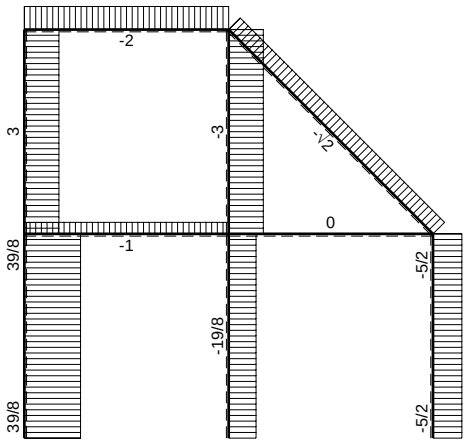
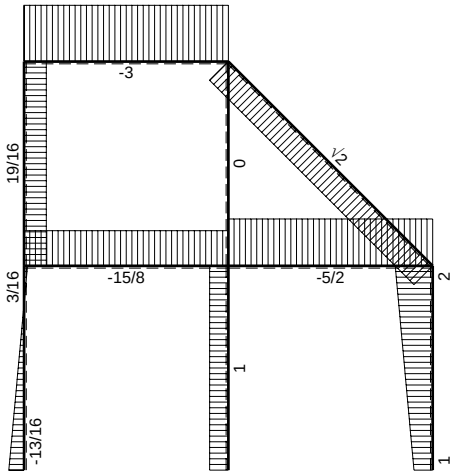
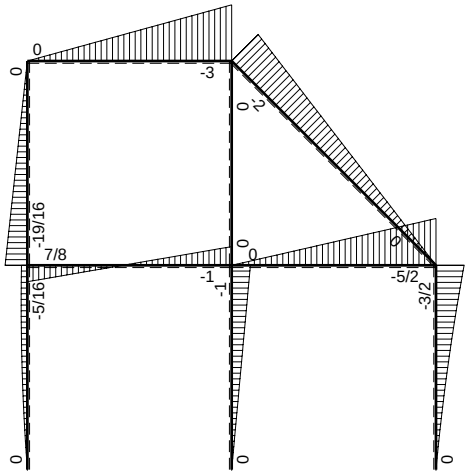


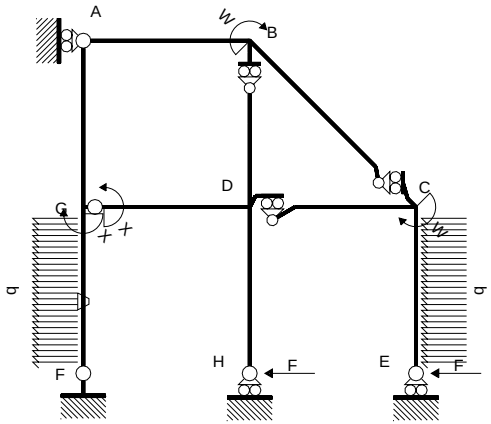
←□→ F



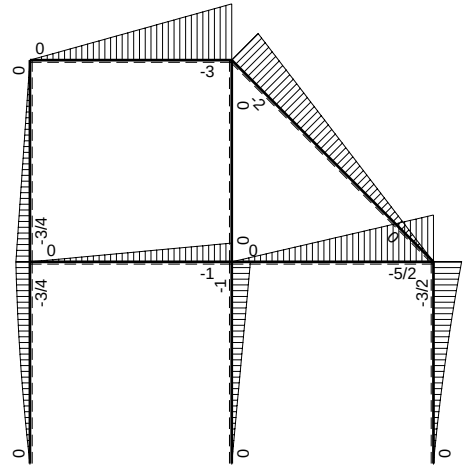
↑□↓ F



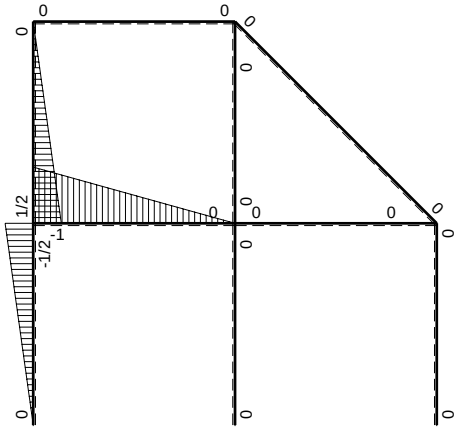
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{GD}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-3Fx	0	0	0	0	0+0	0
BA b	0	3Fb-3Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	-1/2x/b	$-5/4Fx+1/2qx^2$	-Fb/EJ	$5/8Fx^2/b-1/4qx^3/b$	1/2Fx/EJ	$1/4x^2/b^2$	$(7/48+1/4)Fb^2/EJ$	1/12Xb/EJ
GF b	1/2-1/2x/b	$3/4Fb-1/4Fx-1/2qx^2$	Fb/EJ	$3/8Fb-1/2Fx-1/8Fx^2/b+1/4qx^3/b$	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$		
GD b	-1+x/b	-Fx	0	$Fx-Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/3Xb/EJ
DG b	x/b	Fb-Fx	0	$Fx-Fx^2/b$	0	x^2/b^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	1/2-1/2x/b	-3/4Fb+3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
AG b	-1/2x/b	3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
	totali						$7/16Fb^2/EJ$	1/2Xb/EJ
	iperstatica $X=W_{GD}$						-7/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (b - b + 1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{DG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ = 1/3\ b/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ = 1/12\ b/EJ$

$L^{Xo}_{FG} = \int_o^b (5/8\ x^2/b^2 - 1/4\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (1/2\ x/b)\ \theta\ dx$

$= [5/24\ x^3/b^2 - 1/16\ x^4/b^3]_o^b\ Fb\ 1/EJ + [1/4\ x^2/b]_o^b\ \theta$

$= (5/24\ b - 1/16\ b)\ Fb\ 1/EJ + (1/4\ b)\ \theta = 19/48\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^b (3/8 - 1/2\ x/b - 1/8\ x^2/b^2 + 1/4\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 + 1/2\ x/b)\ \theta\ dx$

$= [3/8\ x - 1/4\ x^2/b - 1/24\ x^3/b^2 + 1/16\ x^4/b^3]_o^b\ Fb\ 1/EJ + [-1/2\ x + 1/4\ x^2/b]_o^b\ \theta$

$= (3/8\ b - 1/4\ b - 1/24\ b + 1/16\ b)\ Fb\ 1/EJ + (-1/2\ b + 1/4\ b)\ \theta = 19/48\ Fb^2/EJ$

$L^{Xo}_{GD} = \int_o^b (x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/3\ b)\ Fb\ 1/EJ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DG} = \int_o^b (x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/3\ x^3/b^2]_o^b\ Fb\ 1/EJ$

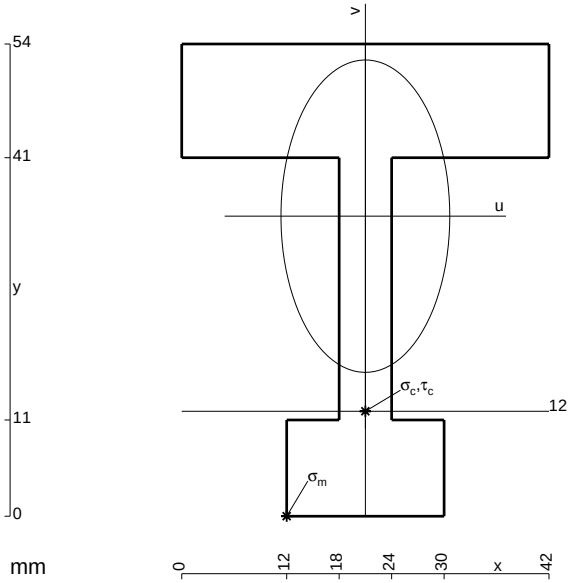
$= (1/2\ b - 1/3\ b)\ Fb\ 1/EJ = 1/6\ Fb^2/EJ$

$L^{Xo}_{GA} = \int_o^b (-3/8 + 3/4\ x/b - 3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-3/8\ b + 3/8\ b - 1/8\ b)\ Fb\ 1/EJ = -1/8\ Fb^2/EJ$

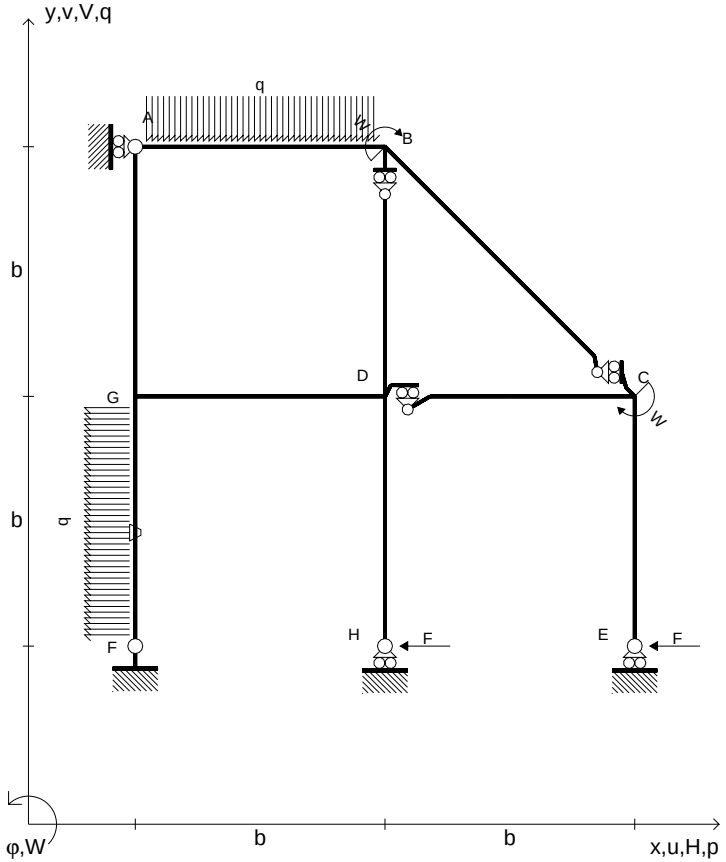
$L^{Xo}_{AG} = \int_o^b (-3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/8\ x^3/b^2]_o^b\ Fb\ 1/EJ$

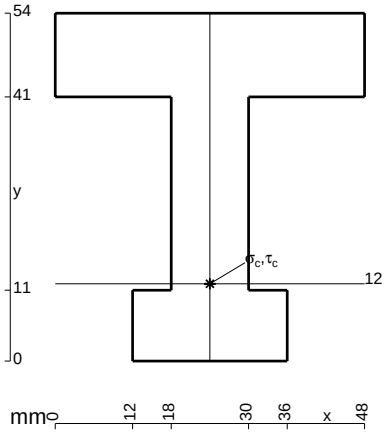
$= (-1/8\ b)\ Fb\ 1/EJ = -1/8\ Fb^2/EJ$



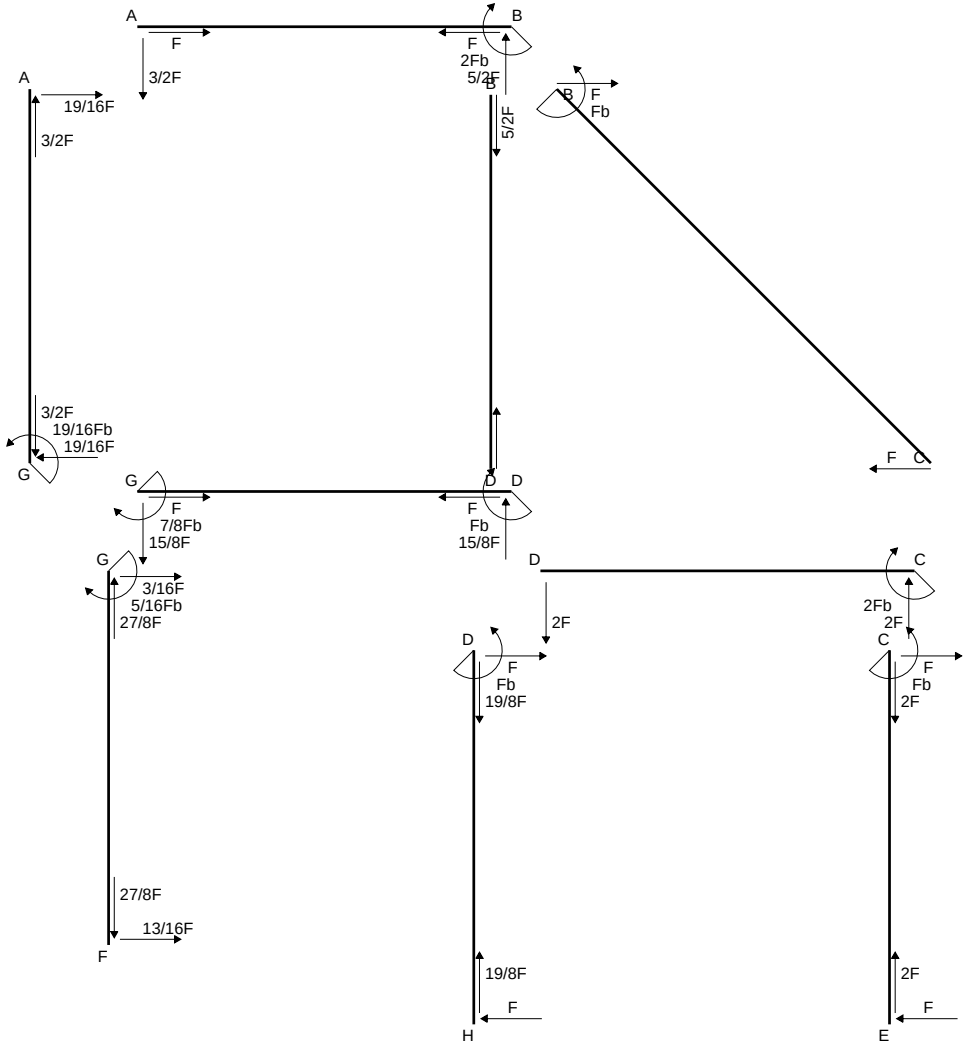
A = 924. mm^2
J_u = 294950. mm^4
J_v = 86148. mm^4
y_g = 34.31 mm
N = -1420. N
T_y = -2130. N
M_x = -1938300. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -34.31 mm
$\sigma_m = N/A - Mv/J_u$ = -227. N/mm^2
x_c = 21. mm
y_c = 12. mm
v_c = -22.31 mm
$\sigma_c = N/A - Mv/J_u$ = -148.2 N/mm^2
τ_c = 7.031 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 148.7 N/mm^2
S^* = 5842. mm^3

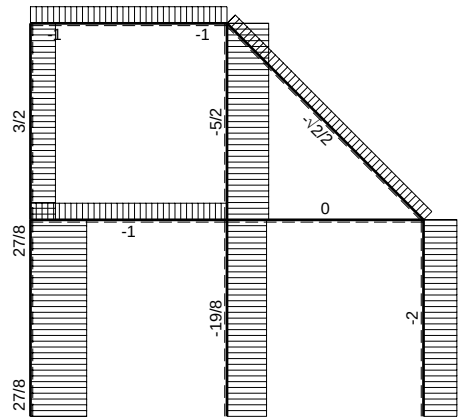
H_E = -F
H_H = -F
W_C = -W = -Fb
W_B = -W = -Fb
q_{AB} = -q = -F/b
p_{FG} = -q = -F/b
θ_{FG} = -θ = -αT/b = -bF/EJ
EJ_{AB} = EJ
EJ_{BC} = EJ
EJ_{BD} = EJ
EJ_{DC} = EJ
EJ_{CE} = EJ
EJ_{FG} = EJ
EJ_{GD} = EJ
EJ_{DH} = EJ
EJ_{GA} = EJ



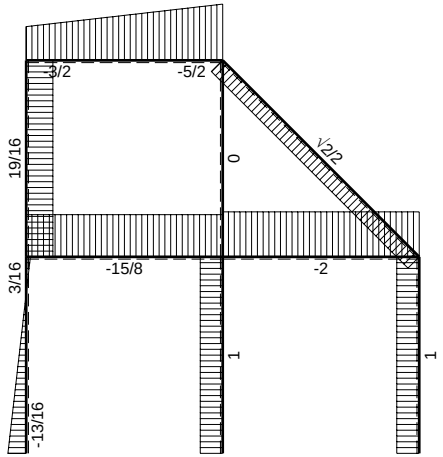


Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 510 mm, F = 2310 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.

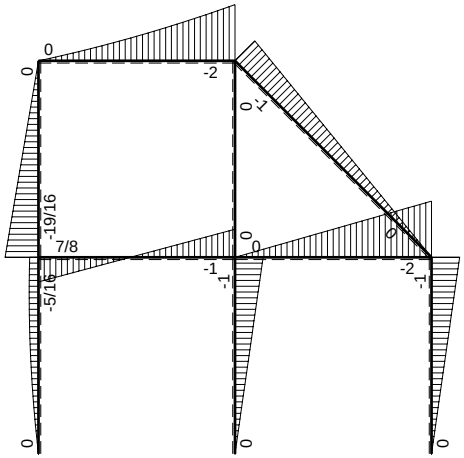




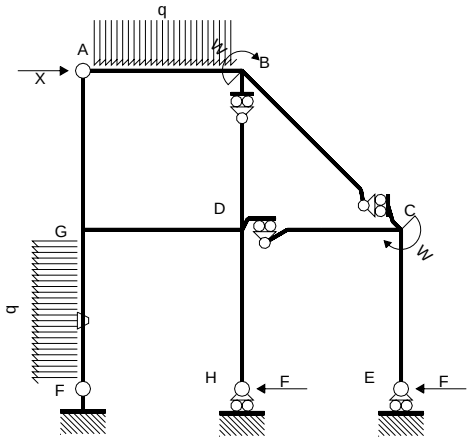
←□→ F



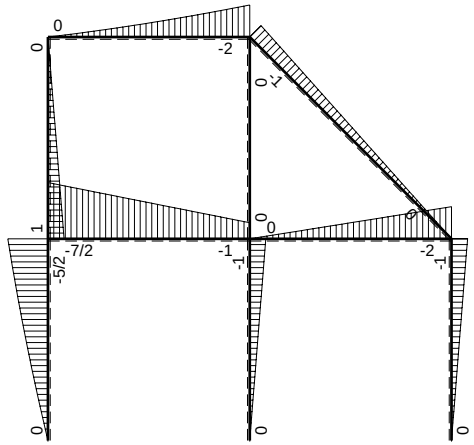
↑□↓ F



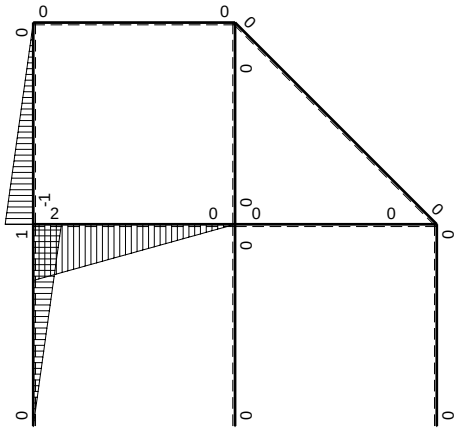
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0		
BC √2b	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	$-2Fx$	0	0	0	0	0+0	0
CD b	0	$2Fb-2Fx$	0	0	0	0		
CE b	0	$-Fb+Fx$	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	x	$-3Fx+1/2qx^2$	$-Fb/EJ$	$-3Fx^2+1/2qx^3$	$-Fxb/EJ$	x^2	$(-7/8-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	-b+x	$5/2Fb-2Fx-1/2qx^2$	Fb/EJ	$-5/2Fb^2+9/2Fbx-3/2Fx^2-1/2qx^3$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$		
GD b	2b-2x	$-7/2Fb+5/2Fx$	0	$-7Fb^2+12Fbx-5Fx^2$	0	$4b^2-8bx+4x^2$	$(-8/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	-2x	$Fb+5/2Fx$	0	$-2Fbx-5Fx^2$	0	$4x^2$		
DH b	0	$-Fb+Fx$	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-b+x	$Fb-Fx$	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	$-Fx$	0	$-Fx^2$	0	x^2		
	totali						$-35/8Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_A$						35/16F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (- x/b)\, \theta\, dx$

$= [- x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [-1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/8\, b)\, Fb^2\, 1/EJ + (-1/2\, b)\, \theta = -11/8\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-5/2 +9/2\, x/b -3/2\, x^2/b^2 -1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 - x/b)\, \theta\, dx$

$= [-5/2\, x +9/4\, x^2/b -1/2\, x^3/b^2 -1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [x -1/2\, x^2/b]_o^b\, \theta$

$= (-5/2\, b +9/4\, b -1/2\, b -1/8\, b)\, Fb^2\, 1/EJ + (b -1/2\, b)\, \theta = -11/8\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-7 +12\, x/b -5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-7\, x +6\, x^2/b -5/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-7\, b +6\, b -5/3\, b)\, Fb^2\, 1/EJ = -8/3\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-2\, x/b -5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x^2/b -5/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

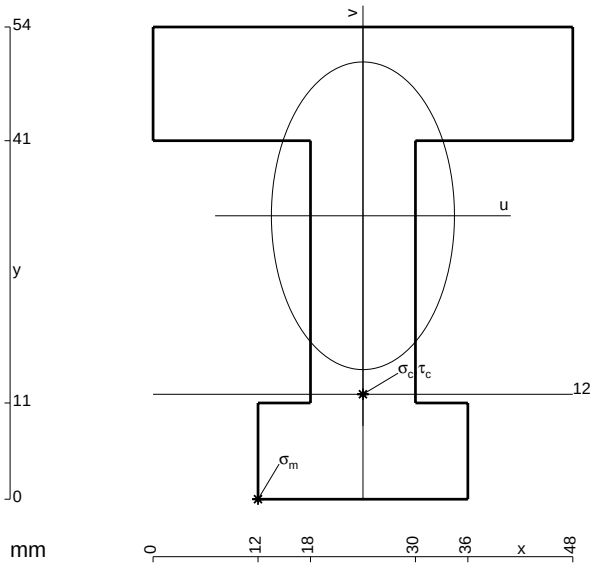
$= (- b -5/3\, b)\, Fb^2\, 1/EJ = -8/3\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1 +2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x + x^2/b -1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (- b + b -1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

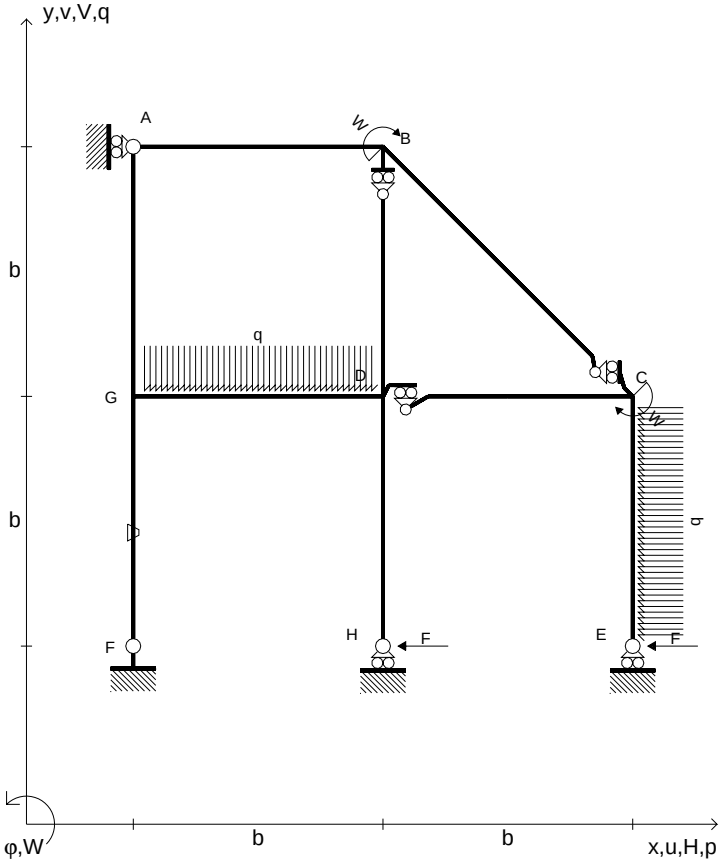
$L^{Xo}_{AG} = \int_o^b (- x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

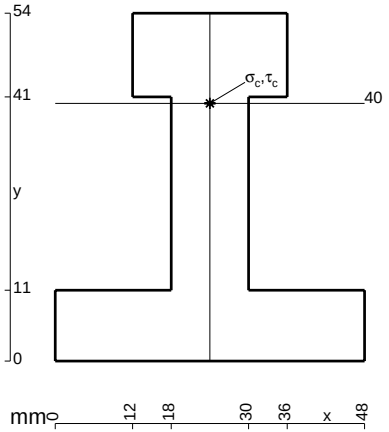


A = 1248. mm^2
J_u = 386507. mm^4
J_v = 136800. mm^4
y_g = 32.41 mm
N = -2310. N
T_y = -5775. N
M_x = -2356200. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -32.41 mm
$\sigma_m = N/A - Mv/J_u$ = -199.4 N/mm^2
x_c = 24. mm
y_c = 12. mm
v_c = -20.41 mm
$\sigma_c = N/A - Mv/J_u$ = -126.3 N/mm^2
τ_c = 9.159 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 127.3 N/mm^2
S^* = 7356. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_B = -W = -Fb$
$p_{CE} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 560 mm, F = 1560 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.



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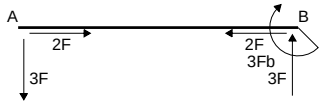

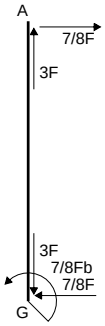

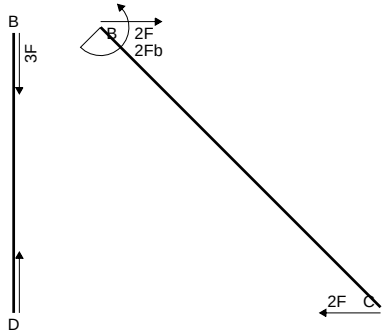

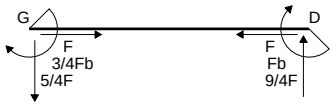

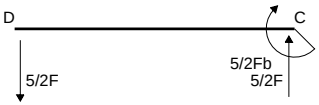

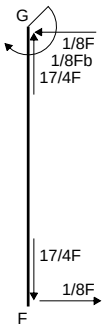

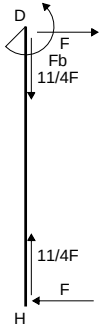

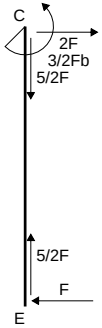

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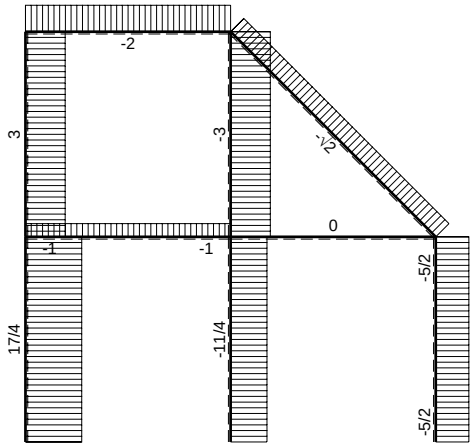
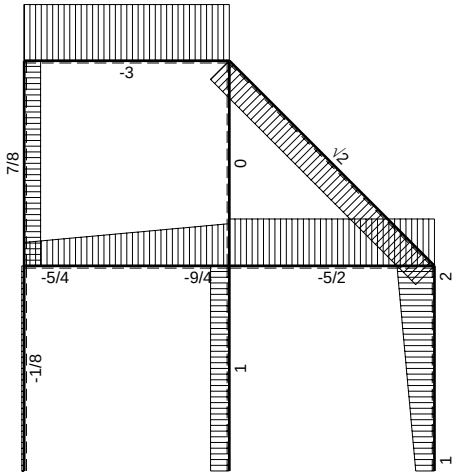
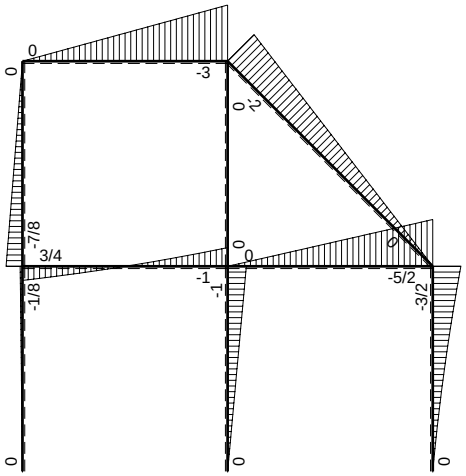


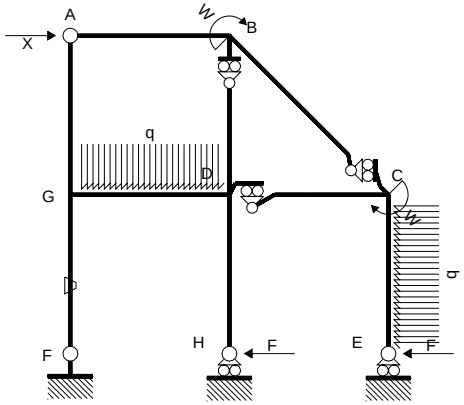
←□→ F



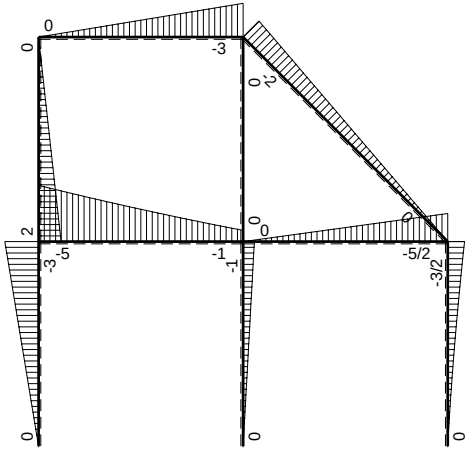
↑□↓ F



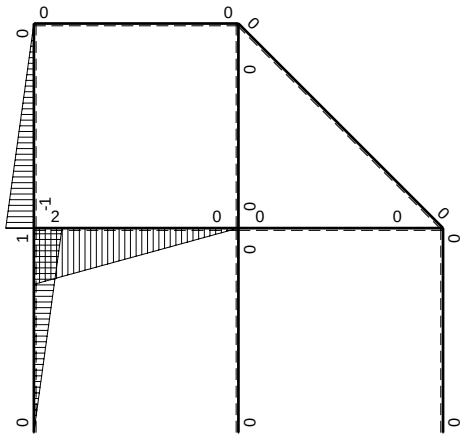
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-3Fx	0	0	0	0	0+0	0
BA b	0	3Fb-3Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0
EC b	0	$Fx+1/2qx^2$	0	0	0	0		
FG b	x	-3Fx	-Fb/EJ	$-3Fx^2$	-Fxb/EJ	x^2	$(-1-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	-b+x	3Fb-3Fx	Fb/EJ	$-3Fb^2+6Fbx-3Fx^2$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$		
GD b	2b-2x	$-5Fb+9/2Fx-1/2qx^2$	0	$-10Fb^2+19Fbx-10Fx^2+qx^3$	0	$4b^2-8bx+4x^2$	$(-43/12+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	-2x	$Fb+7/2Fx+1/2qx^2$	0	$-2Fbx-7Fx^2-qx^3$	0	$4x^2$		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-b+x	2Fb-2Fx	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	-2Fx	0	$-2Fx^2$	0	x^2		
	totali						$-23/4Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_A$						23/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (- x/b)\, \theta\ dx = [- x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x^2/b]_o^b\, \theta$

$= (- b)\, Fb^2\, 1/EJ + (-1/2\, b)\, \theta = -3/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3 +6\, x/b -3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 - x/b)\, \theta\ dx$

$= [-3\, x +3\, x^2/b - x^3/b^2]_o^b\, Fb^2\, 1/EJ + [\, x -1/2\, x^2/b]_o^b\, \theta$

$= (-3\, b +3\, b - b)\, Fb^2\, 1/EJ + (\, b -1/2\, b)\, \theta = -3/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-10 +19\, x/b -10\, x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-10\, x +19/2\, x^2/b -10/3\, x^3/b^2 +1/4\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-10\, b +19/2\, b -10/3\, b +1/4\, b)\, Fb^2\, 1/EJ = -43/12\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-2\, x/b -7\, x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [- x^2/b -7/3\, x^3/b^2 -1/4\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

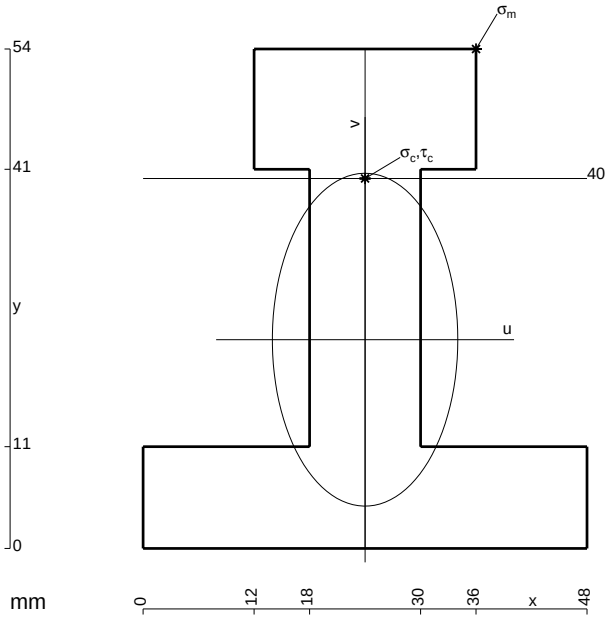
$= (- b -7/3\, b -1/4\, b)\, Fb^2\, 1/EJ = -43/12\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-2 +4\, x/b -2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\, x +2\, x^2/b -2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2\, b +2\, b -2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

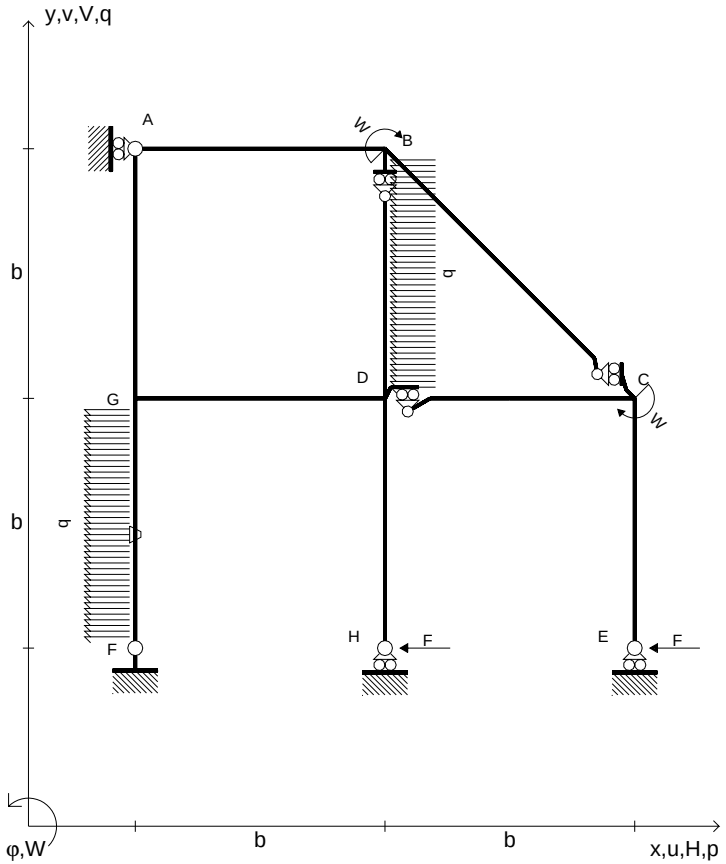
$L^{Xo}_{AG} = \int_o^b (-2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

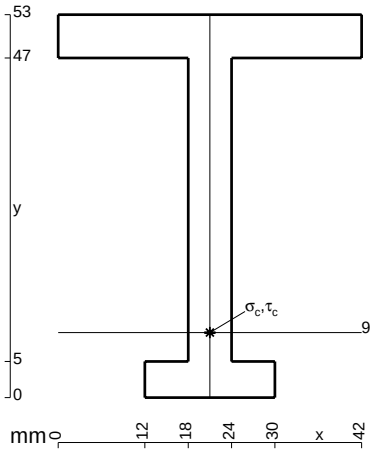
$= (-2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$



A = 1200. mm^2
J_u = 388714. mm^4
J_v = 120672. mm^4
y_g = 22.57 mm
N = -3120. N
T_y = -4680. N
M_x = -2620800. Nmm
x_m = 36. mm
y_m = 54. mm
u_m = 12. mm
v_m = 31.43 mm
$\sigma_m = N/A - Mv/J_u$ = 209.3 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 17.43 mm
$\sigma_c = N/A - Mv/J_u$ = 114.9 N/mm^2
τ_c = 8.02 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 115.8 N/mm^2
S^* = 7993. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_B = -W = -Fb$
$p_{BD} = -q = -F/b$
$p_{FG} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 950 mm, F = 790 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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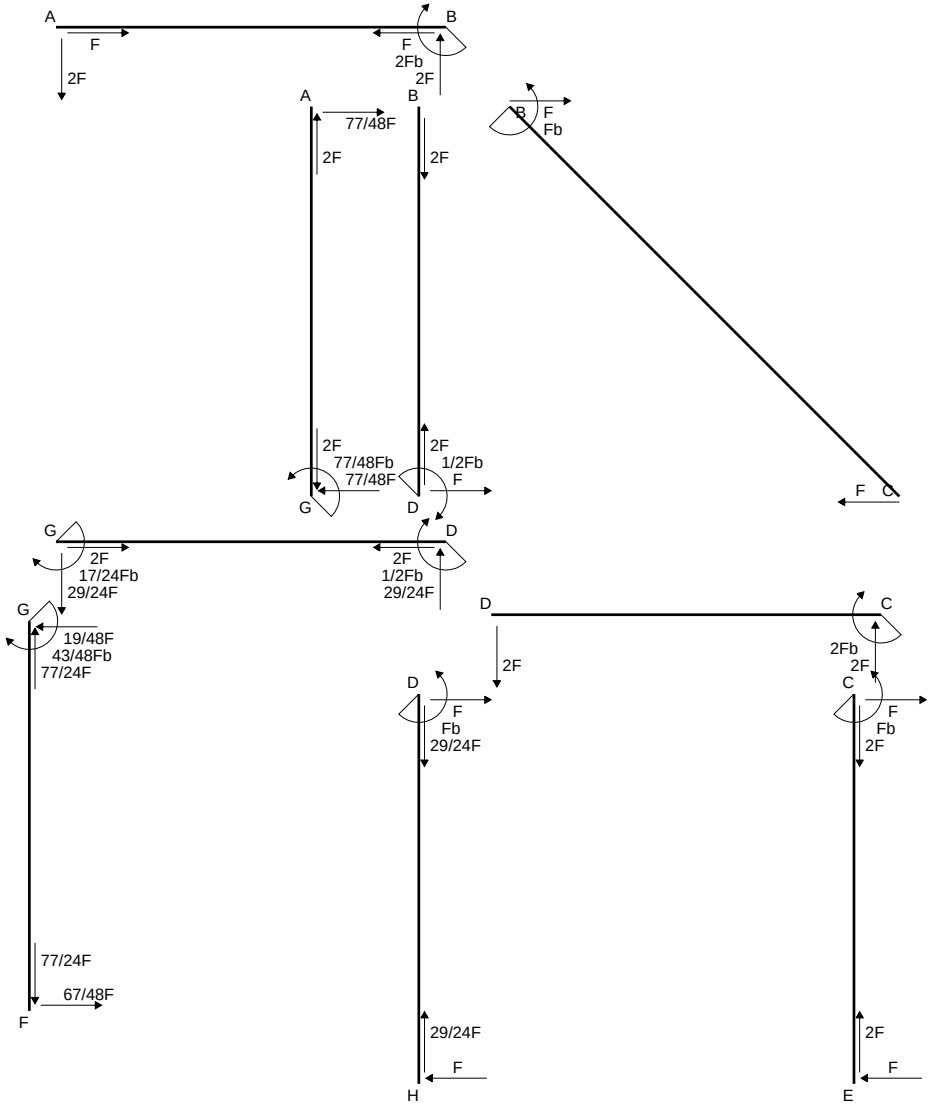

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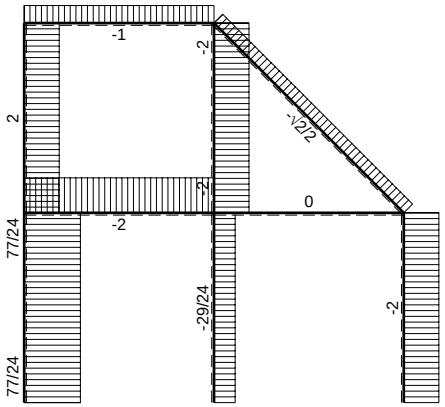
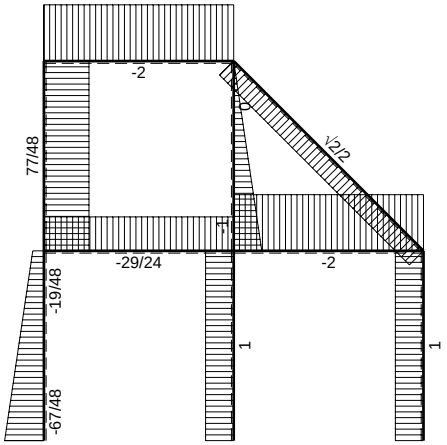
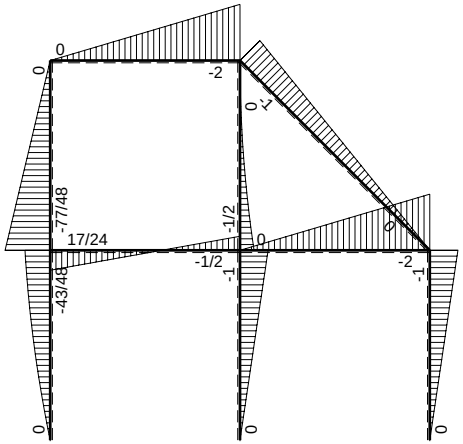


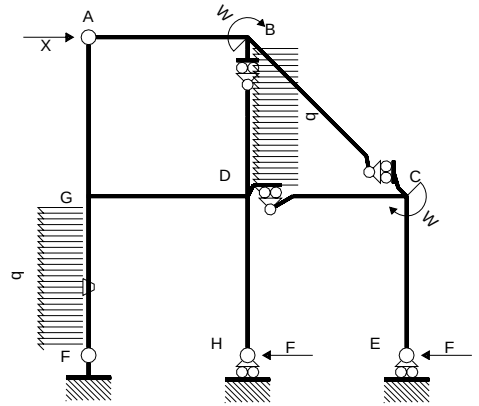
←□→ F



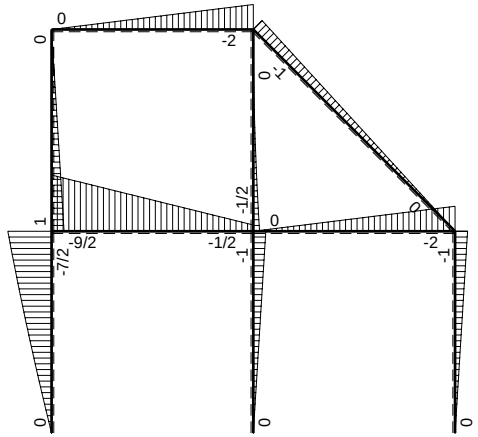
↑□↓ F



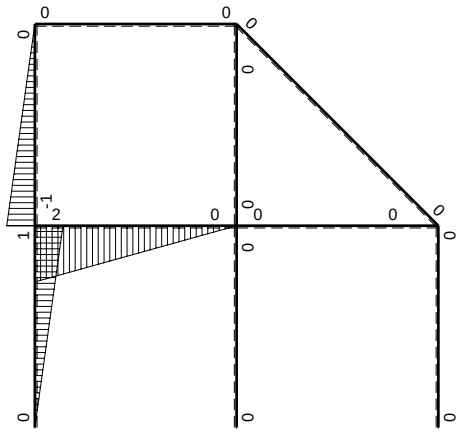
(+) Fb



Schema di calcolo iperstatico



(+) M_o flessione da carichi assegnati



(+) M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-2Fx$	0	0	0	0	0+0	0
BA b	0	$2Fb-2Fx$	0	0	0	0		
BC $\sqrt{2}b$	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	$-1/2qx^2$	0	0	0	0	0+0	0
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
DC b	0	$-2Fx$	0	0	0	0	0+0	0
CD b	0	$2Fb-2Fx$	0	0	0	0		
CE b	0	$-Fb+Fx$	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	x	$-4Fx+1/2qx^2$	$-Fb/EJ$	$-4Fx^2+1/2qx^3$	$-Fxb/EJ$	x^2	$(-29/24-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	$-b+x$	$7/2Fb-3Fx-1/2qx^2$	Fb/EJ	$-7/2Fb^2+13/2Fbx-5/2Fx^2-1/2qx^3$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$		
GD b	$2b-2x$	$-9/2Fb+4Fx$	0	$-9Fb^2+17Fbx-8Fx^2$	0	$4b^2-8bx+4x^2$	$(-19/6+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	$-2x$	$1/2Fb+4Fx$	0	$-Fbx-8Fx^2$	0	$4x^2$		
DH b	0	$-Fb+Fx$	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	$-b+x$	$Fb-Fx$	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	x	$-Fx$	0	$-Fx^2$	0	x^2		
	totali						$-125/24Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_A$						125/48F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ b^2\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\ x/b +4\ x^2/b^2\ )\ b^2\ 1/EJ\ dx = [4\ x -4\ x^2/b +4/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (4\ b -4\ b +4/3\ b\ )\ b^2\ 1/EJ\ = 4/3\ \ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\ x^2/b^2\ )\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (4/3\ b\ )\ b^2\ 1/EJ\ = 4/3\ \ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ b^2\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2\ )\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-4\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb^2\ 1/EJ\ dx + \int_o^b (-\ x/b\ )\ \theta\ dx$

$= [-4/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ\ + [-1/2\ x^2/b\ ]_o^b\ \theta$

$= (-4/3\ b +1/8\ b\ )\ Fb^2\ 1/EJ\ + (-1/2\ b\ )\ \theta\ \ = -41/24\ \ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-7/2 +13/2\ x/b -5/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb^2\ 1/EJ\ dx + \int_o^b (1 - x/b\ )\ \theta\ dx$

$= [-7/2\ x +13/4\ x^2/b -5/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ\ + [\ x -1/2\ x^2/b\ ]_o^b\ \theta$

$= (-7/2\ b +13/4\ b -5/6\ b -1/8\ b\ )\ Fb^2\ 1/EJ\ + (\ b -1/2\ b\ )\ \theta\ \ = -41/24\ \ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-9 +17\ x/b -8\ x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [-9\ x +17/2\ x^2/b -8/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (-9\ b +17/2\ b -8/3\ b\ )\ Fb^2\ 1/EJ\ = -19/6\ \ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-\ x/b -8\ x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [-1/2\ x^2/b -8/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

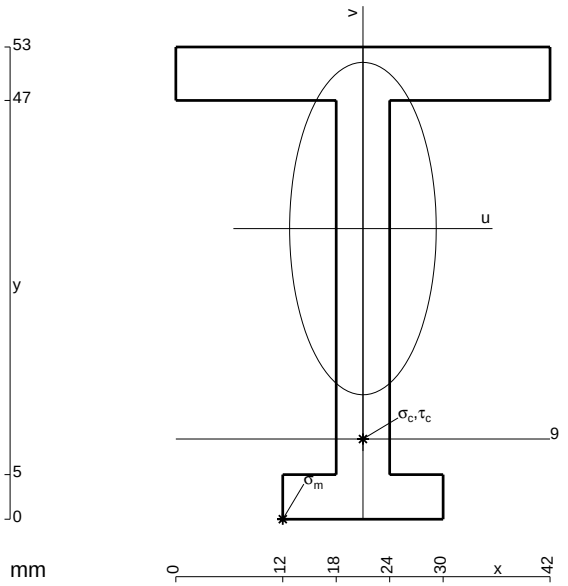
$= (-1/2\ b -8/3\ b\ )\ Fb^2\ 1/EJ\ = -19/6\ \ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1 +2\ x/b - x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [-\ x + x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (-\ b + b -1/3\ b\ )\ Fb^2\ 1/EJ\ = -1/3\ \ Fb^3/EJ$

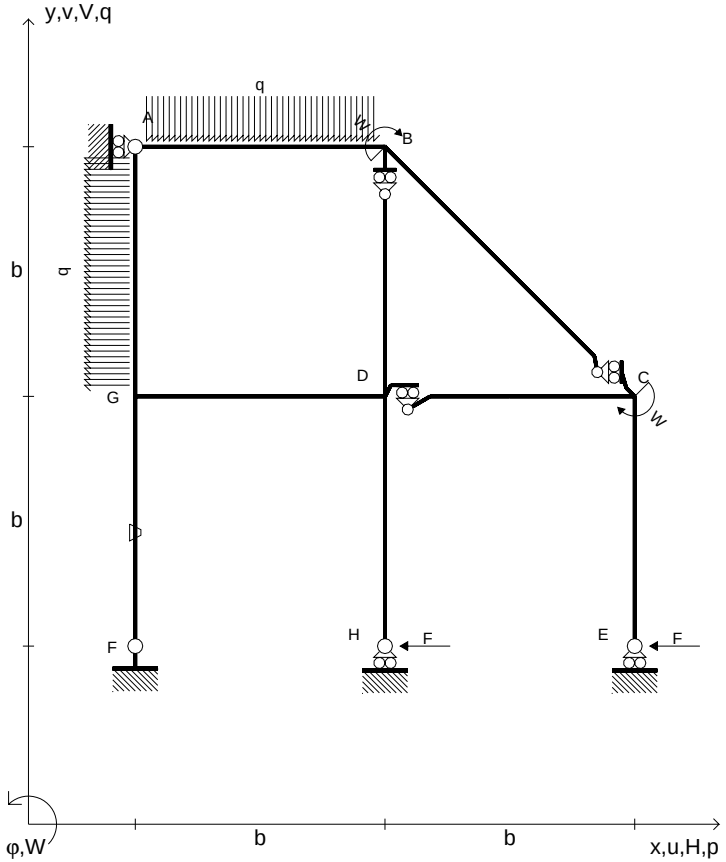
$L^{Xo}_{AG} = \int_o^b (-\ x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [-1/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

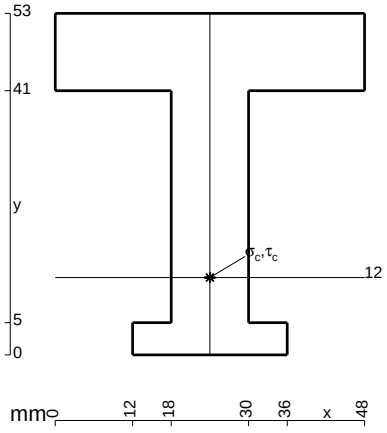
$= (-1/3\ b\ )\ Fb^2\ 1/EJ\ = -1/3\ \ Fb^3/EJ$



A = 594. mm^2
J_u = 206801. mm^4
J_v = 40230. mm^4
y_g = 32.62 mm
N = -790. N
T_y = -1580. N
M_x = -1501000. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -32.62 mm
$\sigma_m = N/A - Mv/J_u$ = -238.1 N/mm^2
x_c = 21. mm
y_c = 9. mm
v_c = -23.62 mm
$\sigma_c = N/A - Mv/J_u$ = -172.8 N/mm^2
τ_c = 4.235 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 172.9 N/mm^2
S^* = 3326. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_B = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{GA} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 600 mm, F = 1760 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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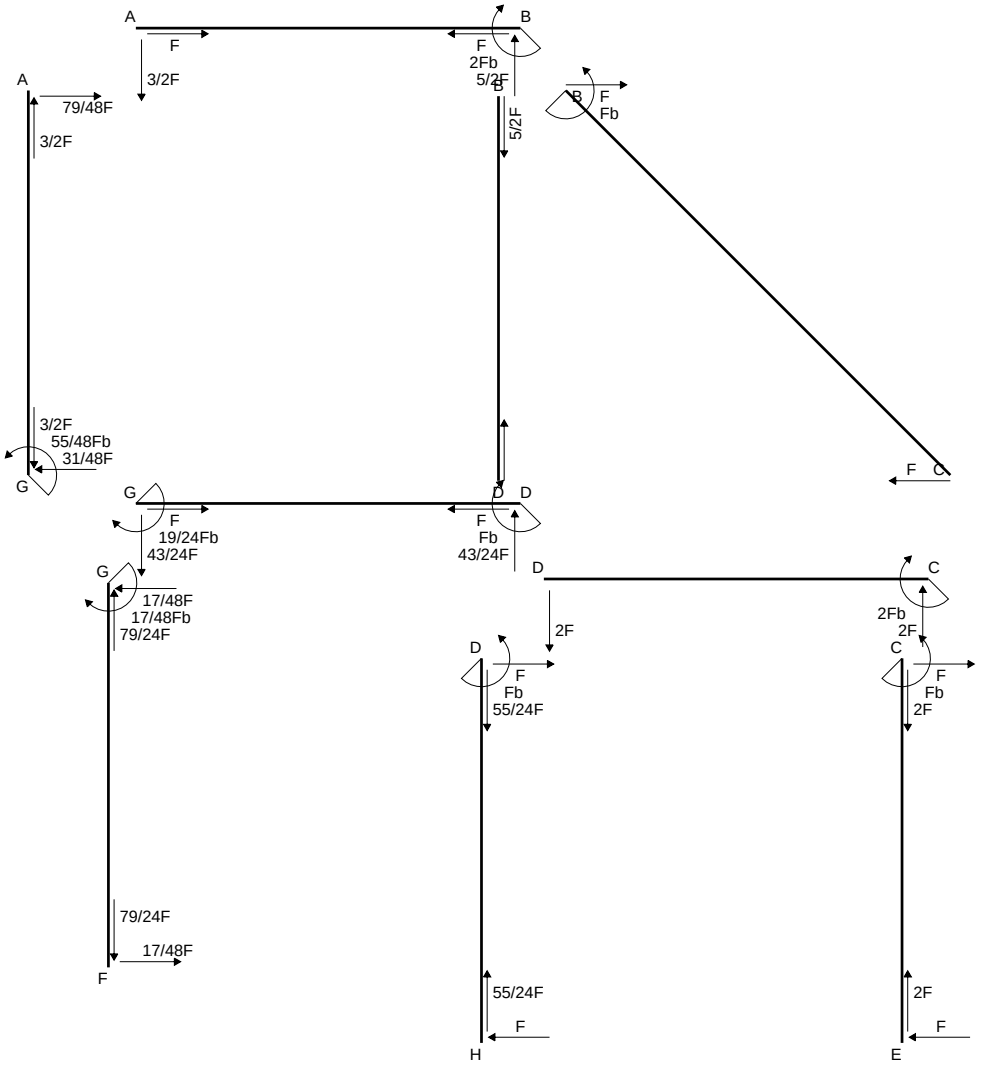

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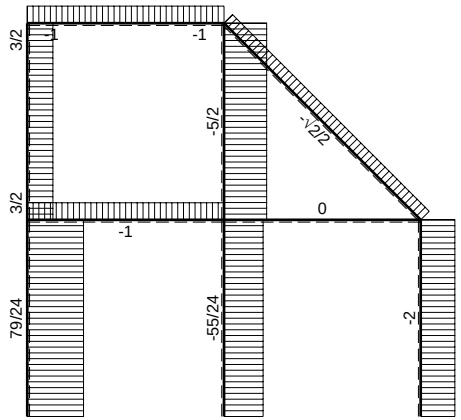
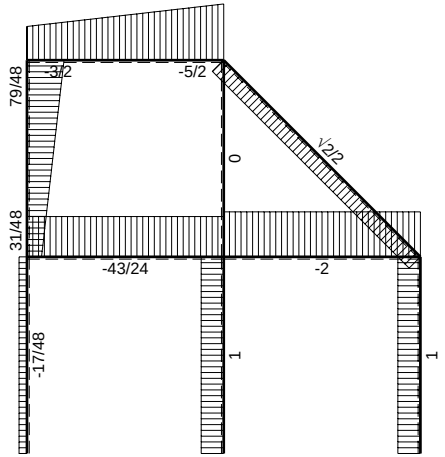
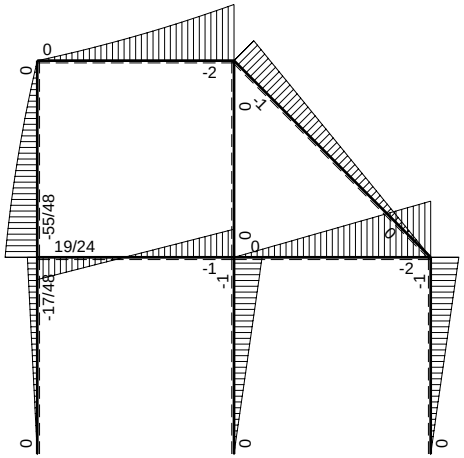


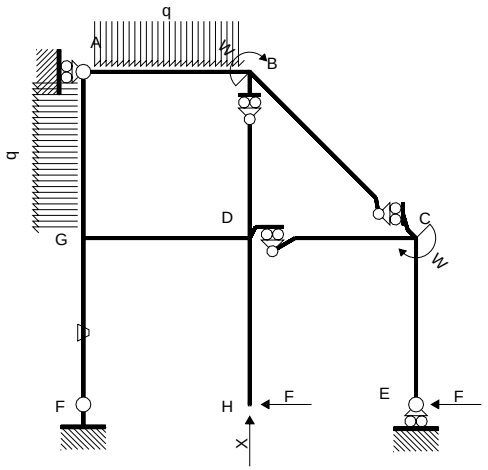
←□→ F



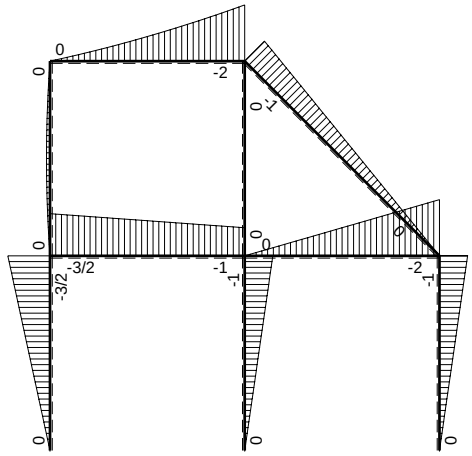
↑□↓ F



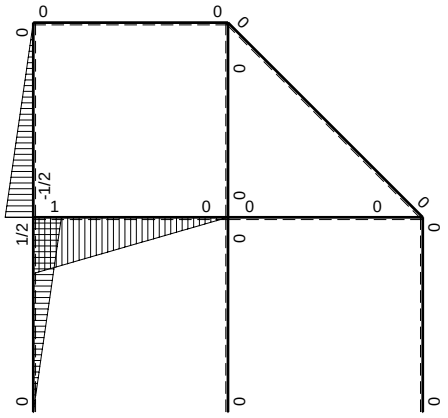
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{2}$b	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	1/2x	-3/2Fx	-Fb/EJ	$-3/4Fx^2$	$-1/2Fxb/EJ$	$1/4x^2$	$(-1/4-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	$-3/4Fb^2+3/2Fbx-3/4Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	-3/2Fb+1/2Fx	0	$-3/2Fb^2+2Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	Fb+1/2Fx	0	$-Fbx-1/2Fx^2$	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	$-1/2Fx+1/2qx^2$	0	$1/4Fbx-1/2Fx^2+1/4qx^3$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/48+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	1/2x	$1/2Fx-1/2qx^2$	0	$1/4Fx^2-1/4qx^3$	0	$1/4x^2$		
	totali						$-55/48Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						55/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [-1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/4\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/4 + 3/2\ x/b - 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-3/4\ x + 3/4\ x^2/b - 1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-3/4\ b + 3/4\ b - 1/4\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-3/2 + 2\ x/b - 1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-3/2\ x + x^2/b - 1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-3/2\ b + b - 1/6\ b)\ Fb^2\ 1/EJ = -2/3\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-\ x/b - 1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/2\ x^2/b - 1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

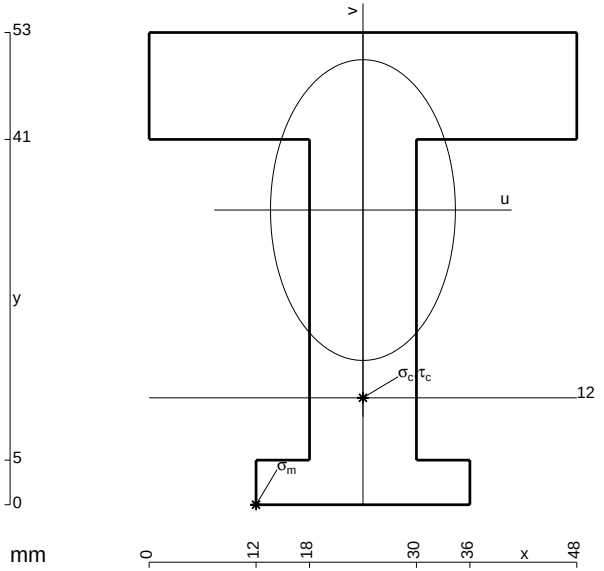
$= (-1/2\ b - 1/6\ b)\ Fb^2\ 1/EJ = -2/3\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (1/4\ x/b - 1/2\ x^2/b^2 + 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [1/8\ x^2/b - 1/6\ x^3/b^2 + 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (1/8\ b - 1/6\ b + 1/16\ b)\ Fb^2\ 1/EJ = 1/48\ Fb^3/EJ$

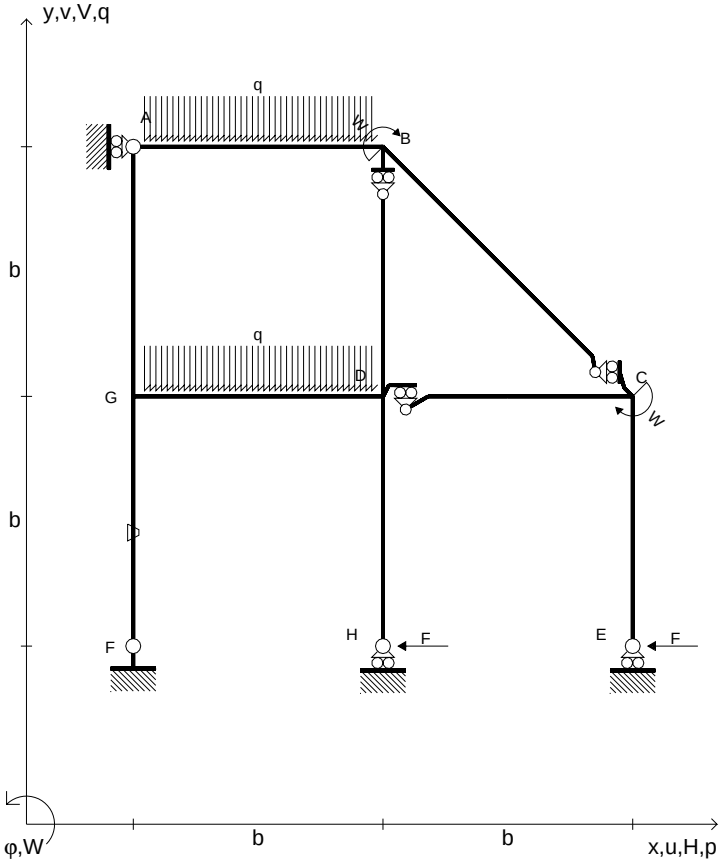
$L^{Xo}_{AG} = \int_o^b (1/4\ x^2/b^2 - 1/4\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [1/12\ x^3/b^2 - 1/16\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

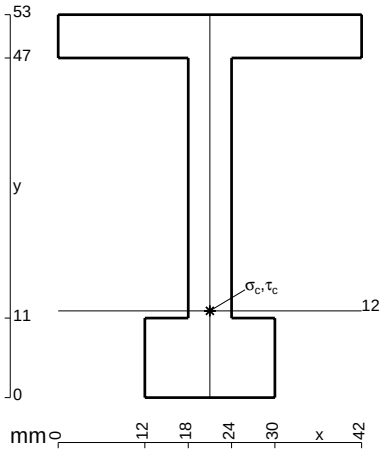
$= (1/12\ b - 1/16\ b)\ Fb^2\ 1/EJ = 1/48\ Fb^3/EJ$



A = 1128. mm^2
J_u = 321538. mm^4
J_v = 121536. mm^4
y_g = 33.07 mm
N = -1760. N
T_y = -4400. N
M_x = -2112000. Nmm
x_m = 12. mm
u_m = -12. mm
v_m = -33.07 mm
$\sigma_m = N/A - Mv/J_u$ = -218.8 N/mm^2
x_c = 24. mm
y_c = 12. mm
v_c = -21.07 mm
$\sigma_c = N/A - Mv/J_u$ = -140. N/mm^2
τ_c = 6.538 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 140.4 N/mm^2
S^* = 5733. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_B = -W = -Fb$
$q_{AB} = -q = -F/b$
$q_{GD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=V_H$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 650 mm, F = 1430 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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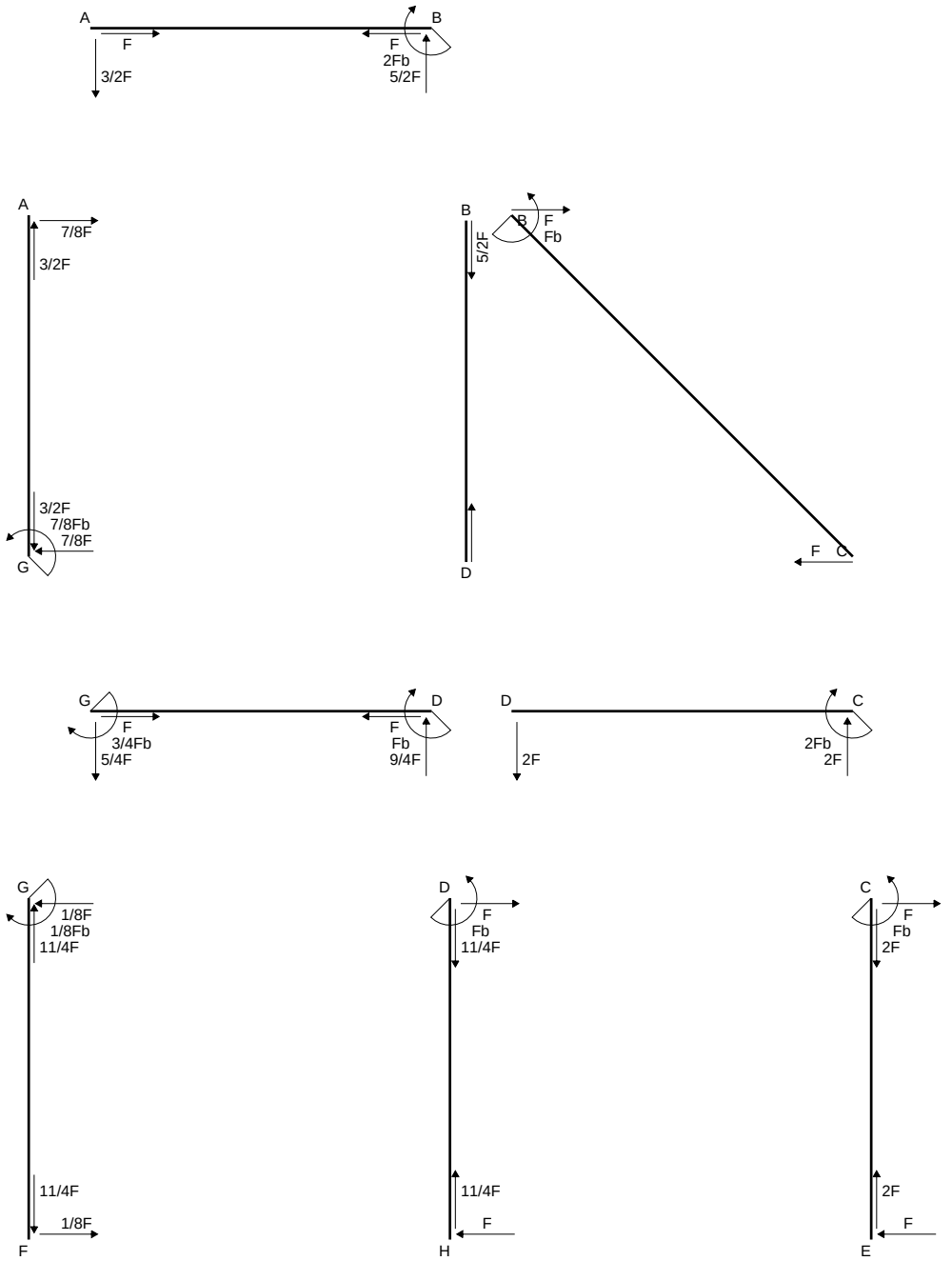

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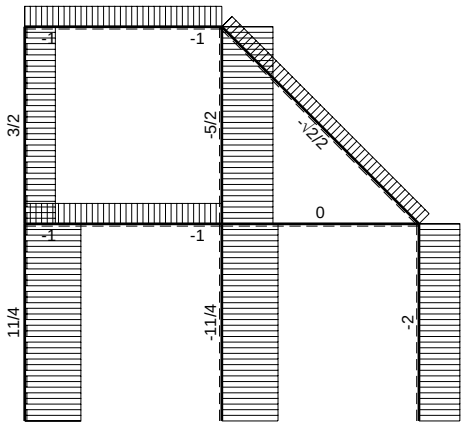
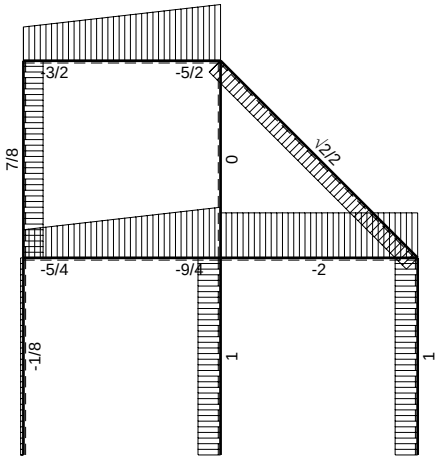
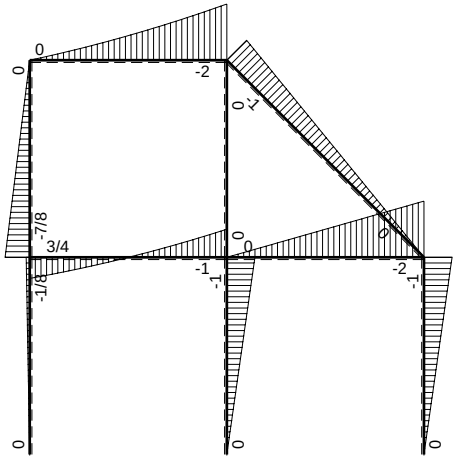


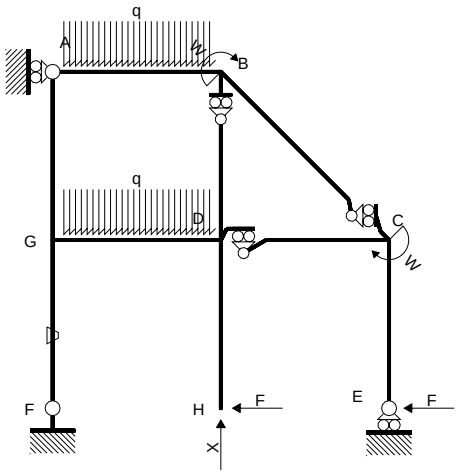
←⊞→ F



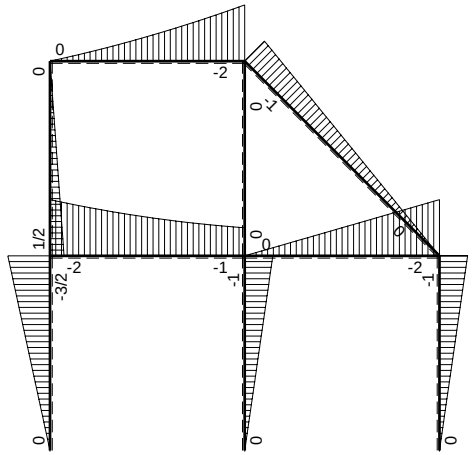
↑⊞↓ F



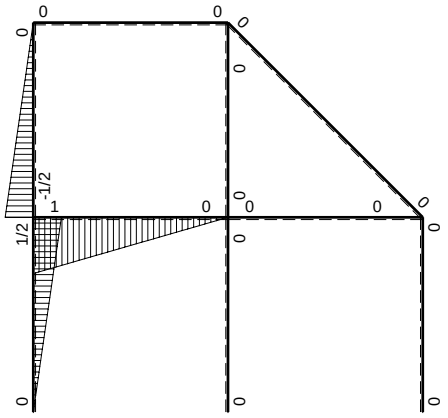
↻⊞↺ Fb



Schema di calcolo iperstatico



↻⊞↺ M_o flessione da carichi assegnati



↻⊞↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0		
BC √2b	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0		
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	1/2x	-3/2Fx	-Fb/EJ	$-3/4Fx^2$	$-1/2Fxb/EJ$	$1/4x^2$	$(-1/4-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	$-3/4Fb^2+3/2Fbx-3/4Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	b-x	$-2Fb+3/2Fx-1/2qx^2$	0	$-2Fb^2+7/2Fbx-2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$	$(-19/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	-x	$Fb+1/2Fx+1/2qx^2$	0	$-Fbx-1/2Fx^2-1/2qx^3$	0	x^2		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1/2b+1/2x	1/2Fb-1/2Fx	0	$-1/4Fb^2+1/2Fbx-1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	1/2x	-1/2Fx	0	$-1/4Fx^2$	0	$1/4x^2$		
	totali						$-11/8Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						11/4F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (-3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [-1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/4\ x^2/b]_o^b\ \theta$

$= (-1/4\ b)\ Fb^2\ 1/EJ + (-1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-3/4 + 3/2\ x/b - 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1/2 - 1/2\ x/b)\ \theta\ dx$

$= [-3/4\ x + 3/4\ x^2/b - 1/4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x - 1/4\ x^2/b]_o^b\ \theta$

$= (-3/4\ b + 3/4\ b - 1/4\ b)\ Fb^2\ 1/EJ + (1/2\ b - 1/4\ b)\ \theta = -1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-2 + 7/2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-2\ x + 7/4\ x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (-2\ b + 7/4\ b - 2/3\ b + 1/8\ b)\ Fb^2\ 1/EJ = -19/24\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (-\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [-1/2\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

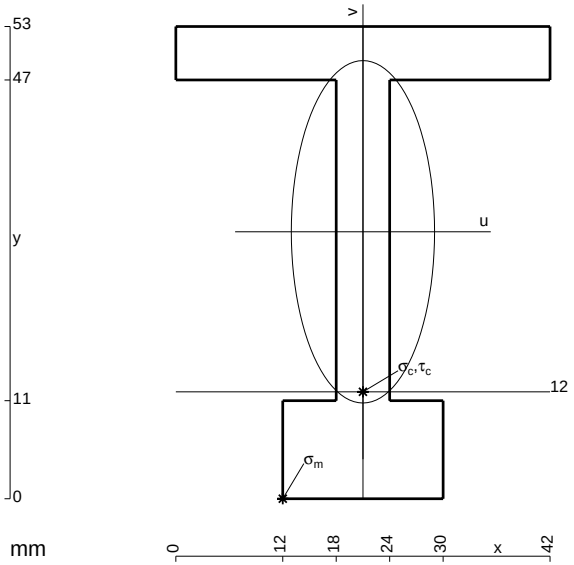
$= (-1/2\ b - 1/6\ b - 1/8\ b)\ Fb^2\ 1/EJ = -19/24\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1/4 + 1/2\ x/b - 1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-1/4\ b + 1/4\ b - 1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$

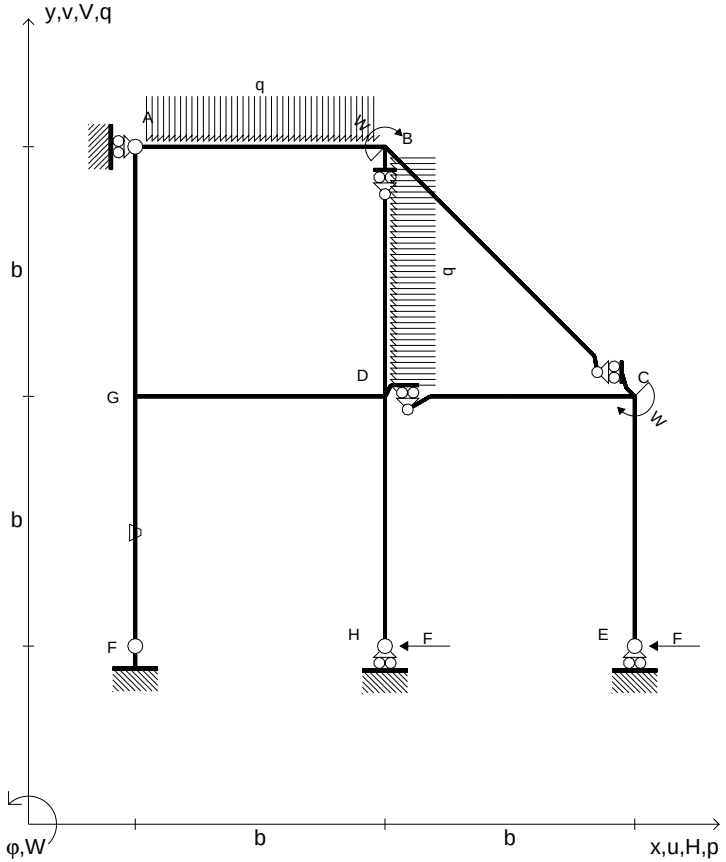
$L^{Xo}_{AG} = \int_o^b (-1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

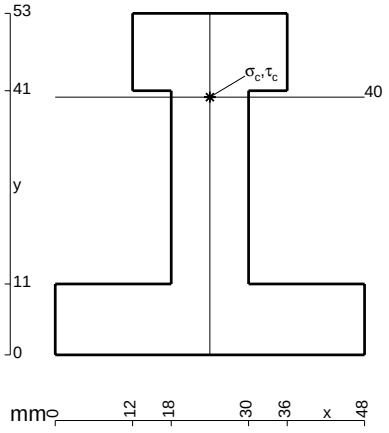
$= (-1/12\ b)\ Fb^2\ 1/EJ = -1/12\ Fb^3/EJ$



A = 666. mm^2
J_u = 245945. mm^4
J_v = 43038. mm^4
y_g = 29.96 mm
N = -1430. N
T_y = -3575. N
M_x = -1859000. Nmm
x_m = 12. mm
u_m = -9. mm
v_m = -29.96 mm
$\sigma_m = N/A - Mv/J_u = -228.6\ N/mm^2$
x_c = 21. mm
y_c = 12. mm
v_c = -17.96 mm
$\sigma_c = N/A - Mv/J_u = -137.9\ N/mm^2$
τ_c = 12. N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 139.5\ N/mm^2$
S^* = 4954. mm^3

$H_E = -F$
$H_H = -F$
$W_C = -W = -Fb$
$W_B = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{BD} = -q = -F/b$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{BD} = EJ$
$EJ_{DC} = EJ$
$EJ_{CE} = EJ$
$EJ_{FG} = EJ$
$EJ_{GD} = EJ$
$EJ_{DH} = EJ$
$EJ_{GA} = EJ$





Reazioni iperstatiche in soluzione: $X=H_F$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 850 mm, F = 1530 N
Calcolare sulla sezione B la massima tensione normale σ_m.
Calcolare in * le tensioni σ_c,τ_c e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Curvatura θ asta FG positiva se convessa a destra con inizio F.

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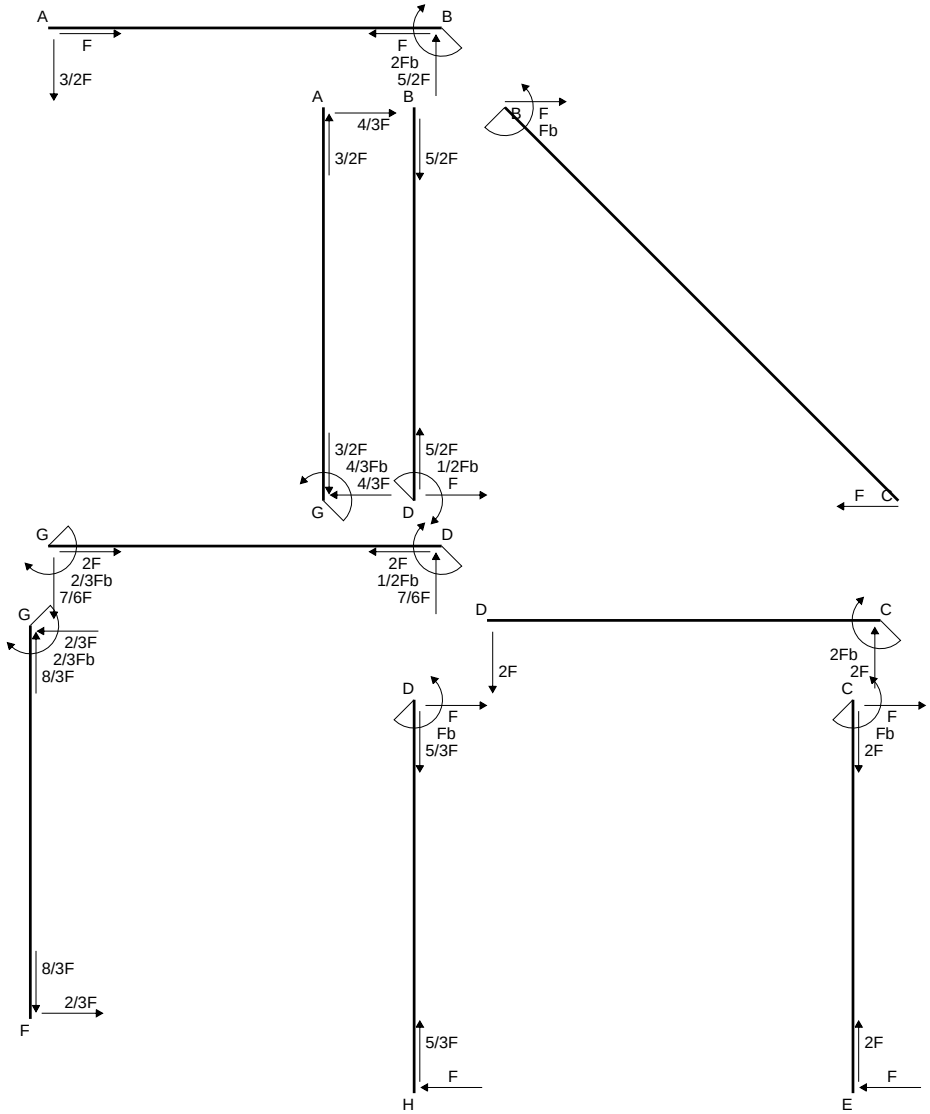

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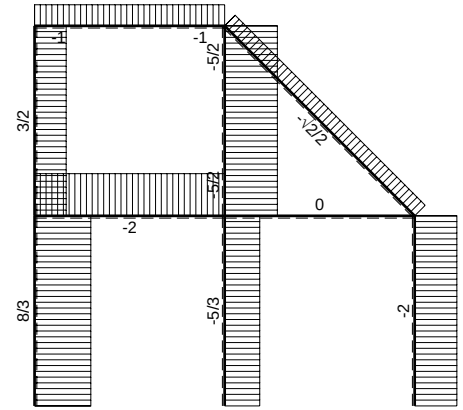
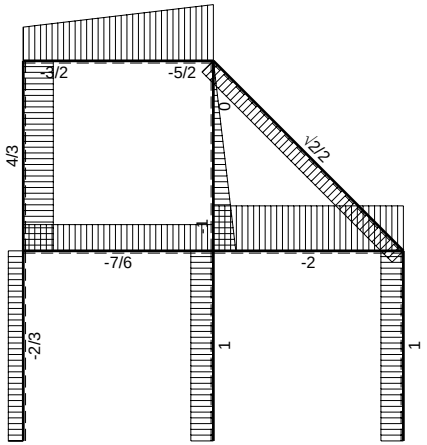
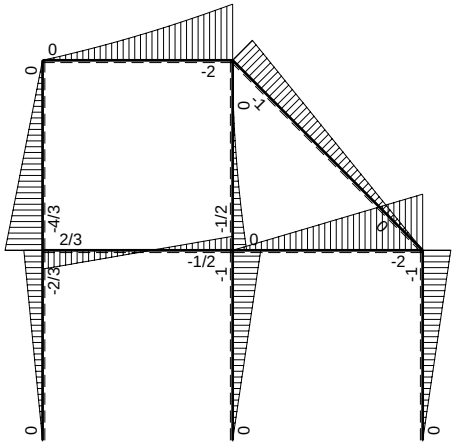


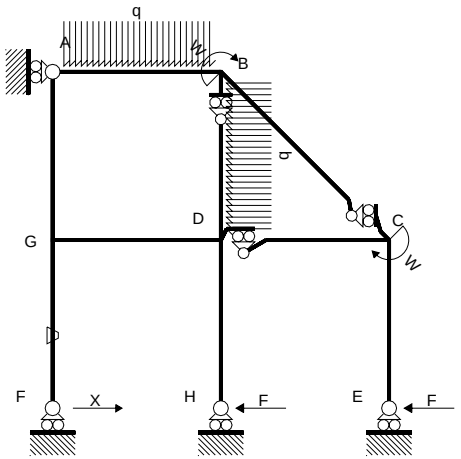
←□→ F



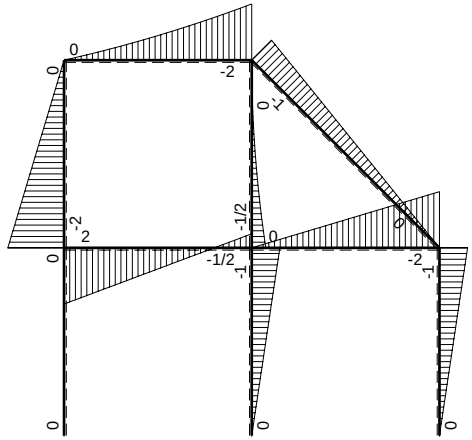
↑□↓ F



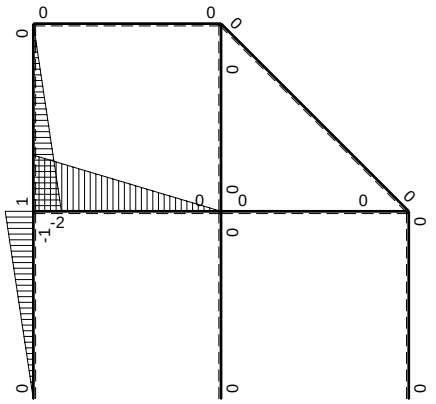
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_F$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{2}$b	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	$-1/2qx^2$	0	0	0	0	0+0	0
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
DC b	0	$-2Fx$	0	0	0	0	0+0	0
CD b	0	$2Fb-2Fx$	0	0	0	0		
CE b	0	$-Fb+Fx$	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	-x	0	$-Fb/EJ$	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	$-2b+2x$	$2Fb-5/2Fx$	0	$-4Fb^2+9Fbx-5Fx^2$	0	$4b^2-8bx+4x^2$	$(-7/6+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	$1/2Fb-5/2Fx$	0	$Fbx-5Fx^2$	0	$4x^2$		
DH b	0	$-Fb+Fx$	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	b-x	$-2Fb+2Fx$	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	$2Fx$	0	$-2Fx^2$	0	x^2		
	totali						$-4/3Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_F$						2/3F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{FG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GF} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{GD} = \int_o^b (4 -8\, x/b +4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4\, x -4\, x^2/b +4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\, b -4\, b +4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{DG} = \int_o^b (4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\, b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{GA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{FG} = \int_o^b (x/b)\, \theta\ dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GF} = \int_o^b (-1 + x/b)\, \theta\ dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{GD} = \int_o^b (-4 +9\, x/b -5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-4\, x +9/2\, x^2/b -5/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-4\, b +9/2\, b -5/3\, b)\, Fb^2\, 1/EJ = -7/6\ Fb^3/EJ$

$L^{Xo}_{DG} = \int_o^b (x/b -5\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -5/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -5/3\, b)\, Fb^2\, 1/EJ = -7/6\ Fb^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-2 +4\, x/b -2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\, x +2\, x^2/b -2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2\, b +2\, b -2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

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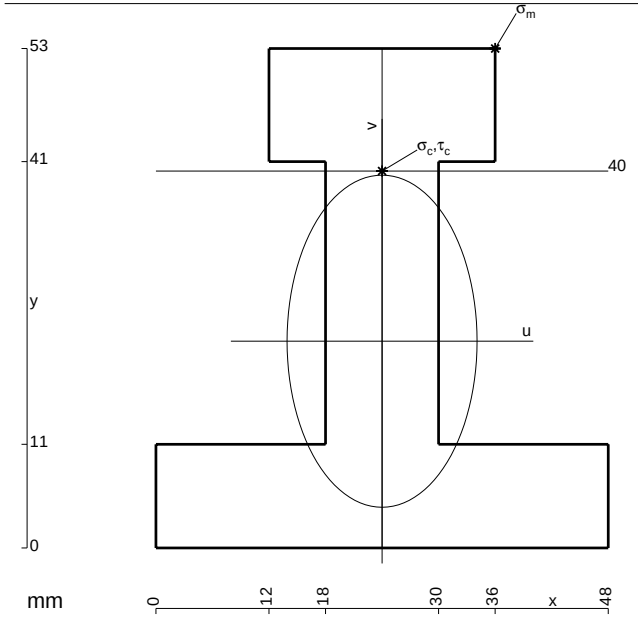

A = 1176. mm^2
J_u = 365284. mm^4
J_v = 119520. mm^4
y_g = 21.94 mm
N = -1530. N
T_y = -3825. N
M_x = -2601000. Nmm
x_m = 36. mm
y_m = 53. mm
u_m = 12. mm
v_m = 31.06 mm
$\sigma_m = N/A - Mv/J_u$ = 219.9 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 18.06 mm
$\sigma_c = N/A - Mv/J_u$ = 127.3 N/mm^2
τ_c = 6.493 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 127.8 N/mm^2
S^* = 7440. mm^3

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