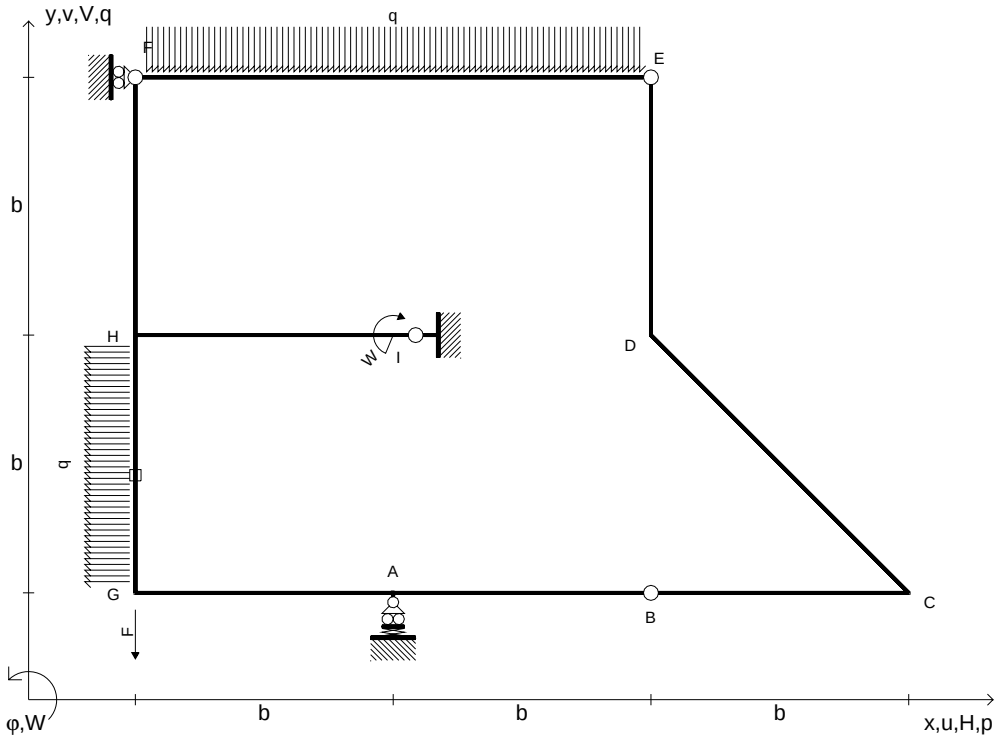
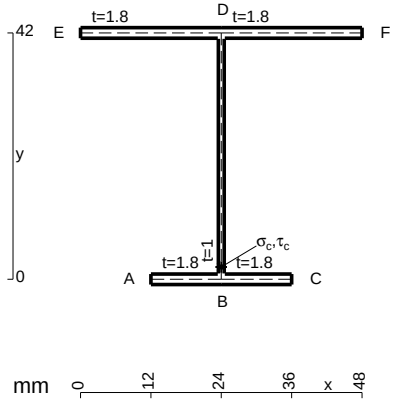



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_I = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{HG} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$




Reazioni iperstatiche in soluzione: $X=V_A$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave AB ha la sezione riportata e dimensioni in mm, con:

b = 680 mm, F = 780 N

Calcolare sulla sezione A la massima tensione normale $\sigma_m$.

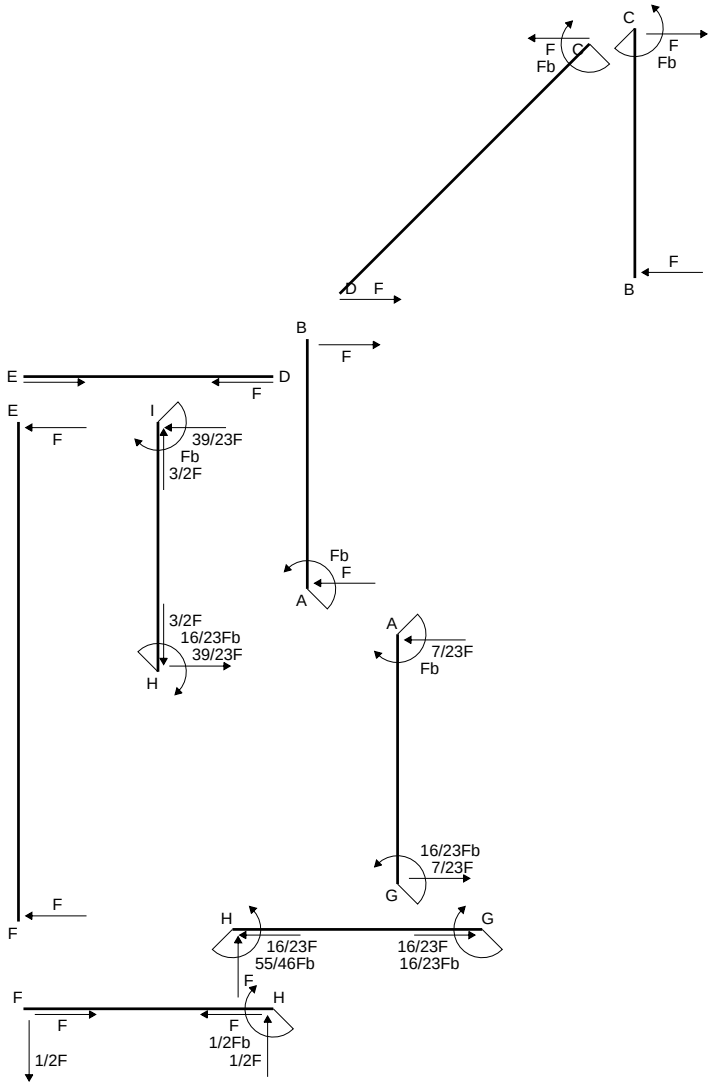
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.

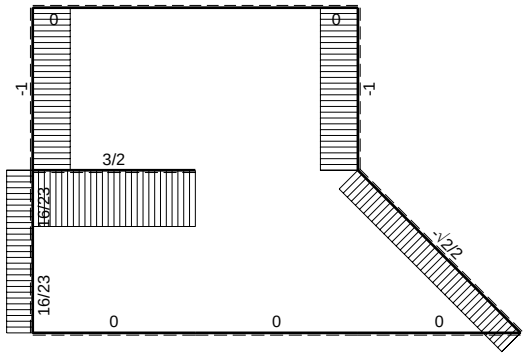
Lembo inferiore sezione su tratteggio trave, a destra da A a B

Elongazione termica specifica ε assegnata su asta HG.

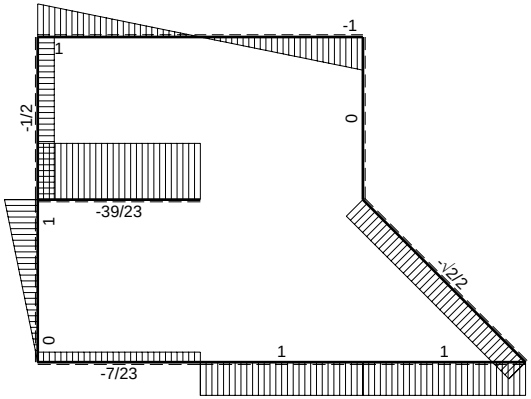




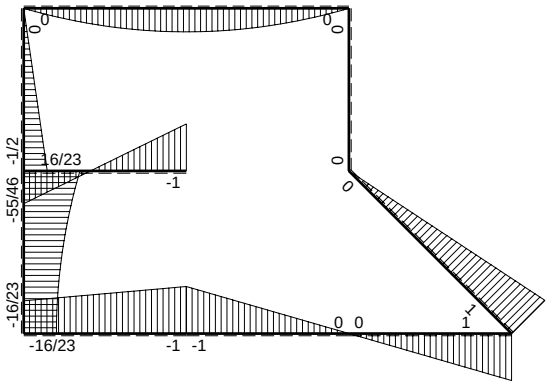





←□→ F



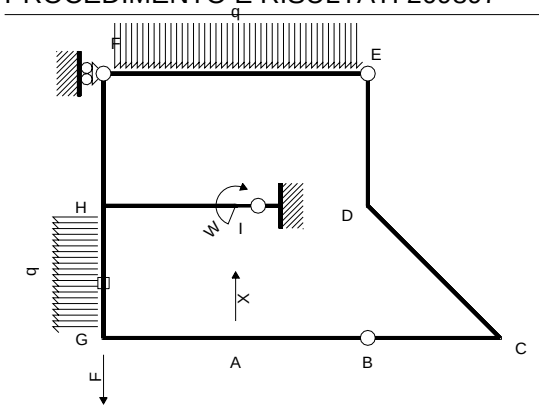

↑□↓ F



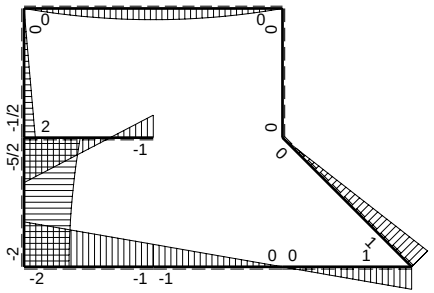

↻□↺ Fb



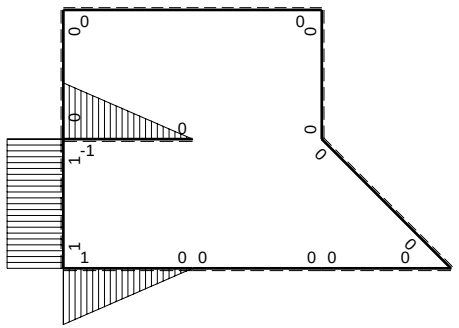





Schema di calcolo iperstatico




↻□↺ $M_o$ flessione da carichi assegnati




↻□↺ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb+Fx | 0 | 0 | | |
| CD √2b | 0 | Fb-√2/2Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | b-x | -2Fb+Fx | $-2Fb^2+3Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-5/6Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | Fb+Fx | $-Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | -1/2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 1/2Fb-1/2Fx | 0 | 0 | | |
| HI b | -b+x | 2Fb-3Fx | $-2Fb^2+5Fbx-3Fx^2$ | $b^2-2bx+x^2$ | $-1/2Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | Fb-3Fx | $Fbx-3Fx^2$ | $x^2$ | | |
| HG b | b | $-5/2Fb+Fx-1/2qx^2$ | $-5/2Fb^2+Fbx-1/2Fx^2$ | $b^2$ | $-13/6Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $2Fb+1/2qx^2$ | $-2Fb^2-1/2Fx^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-5/2Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 30/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-2 +3\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2\, x + 3/2\, x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2\, b + 3/2\, b - 1/3\, b \,\right) Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(-\, x/b - x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[-1/2\, x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/2\, b - 1/3\, b \,\right) Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-2 +5\, x/b - 3\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2\, x + 5/2\, x^2/b - x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2\, b + 5/2\, b - b \,\right) Fb^2\, 1/EJ = -1/2\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(\, x/b - 3\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[1/2\, x^2/b - x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(1/2\, b - b \,\right) Fb^2\, 1/EJ = -1/2\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-5/2 + x/b - 1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

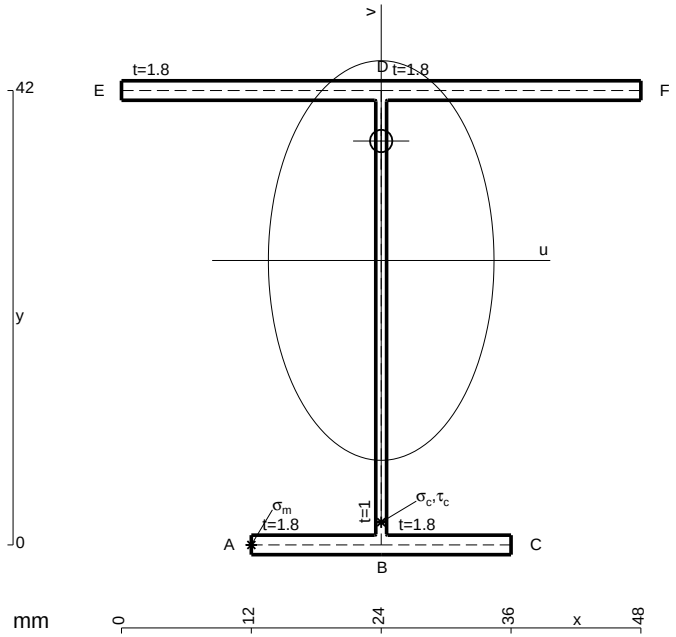
$$= \left[-5/2\, x + 1/2\, x^2/b - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-5/2\, b + 1/2\, b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -7/6\ \ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-2 -1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

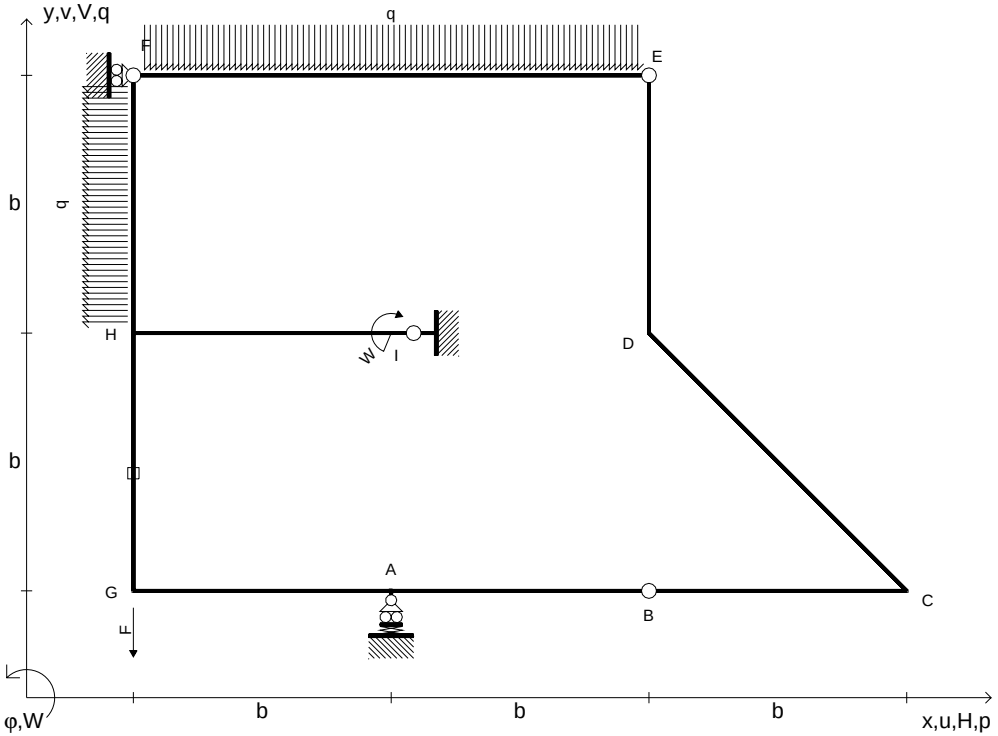
$$= \left[-2\, x - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-2\, b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -7/6\ \ Fb^3/EJ$$




A = 171.6 $mm^2$
$J_u$ = 58532. $mm^4$
$J_v$ = 18662. $mm^4$
$J_t$ = 154. $mm^4$
$y_o$ = 11.05 mm
$y_g$ = 26.29 mm
$T_y$ = 780. N
$M_x$ = -530400. Nmm
$x_m$ = 12. mm
$u_m$ = -12. mm
$v_m$ = -26.29 mm
$\sigma_m = -Mv/J_u$ = -238.2 $N/mm^2$
$x_c$ = 24. mm
$v_c$ = -26.29 mm
$\sigma_c = -Mv/J_u$ = -238.2 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 15.13 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 15.13 $N/mm^2$
$t_c$ = 780. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 239.6 $N/mm^2$


ESΣ05.xxxx.009
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

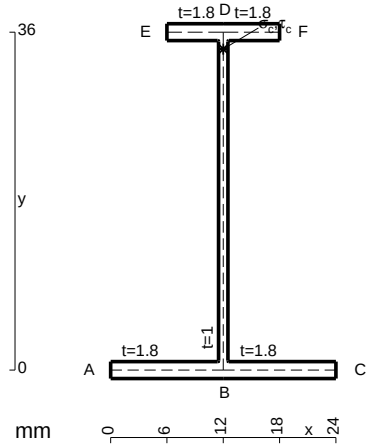



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_I = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$




Reazioni iperstatiche in soluzione: $X=V_A$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave AB ha la sezione riportata e dimensioni in mm, con:

b = 310 mm, F = 690 N

Calcolare sulla sezione A la massima tensione normale $\sigma_m$.

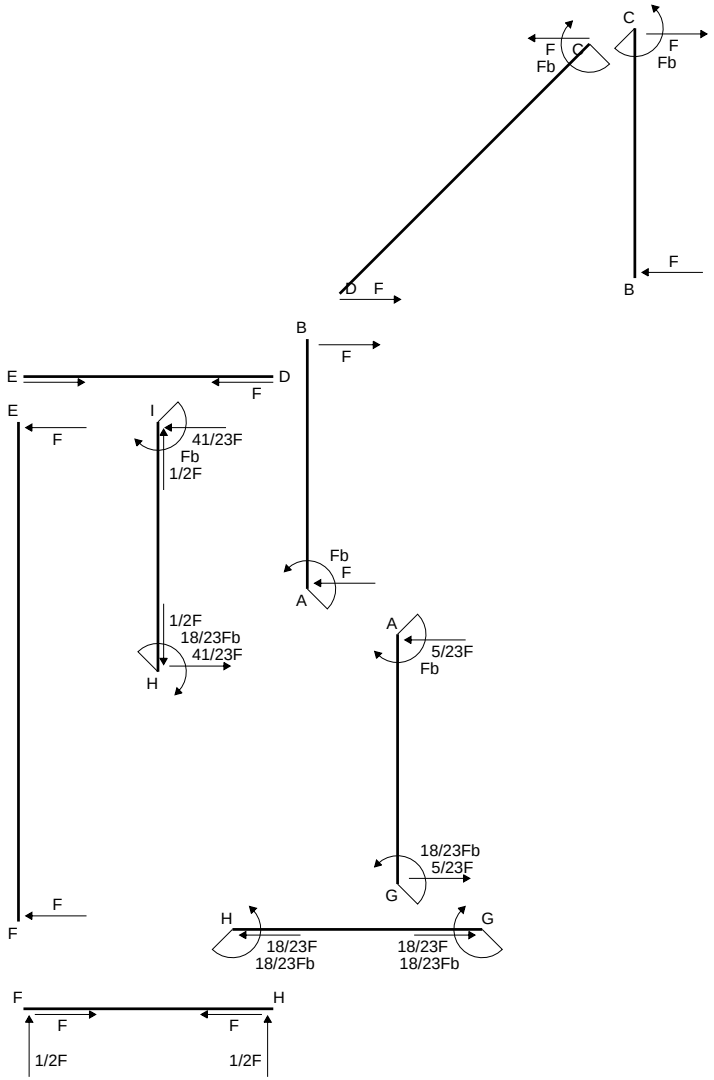
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.

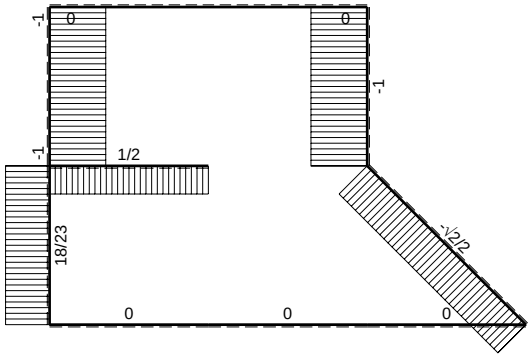
Lembo inferiore sezione su tratteggio trave, a destra da A a B

Elongazione termica specifica ε assegnata su asta HG.

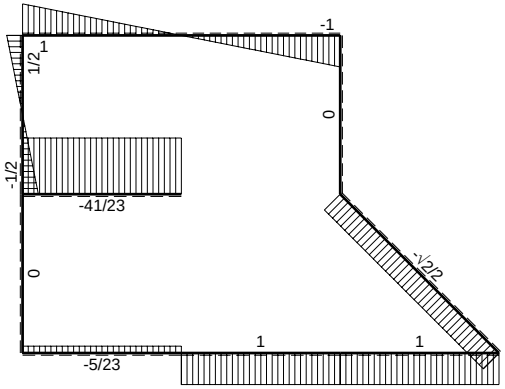




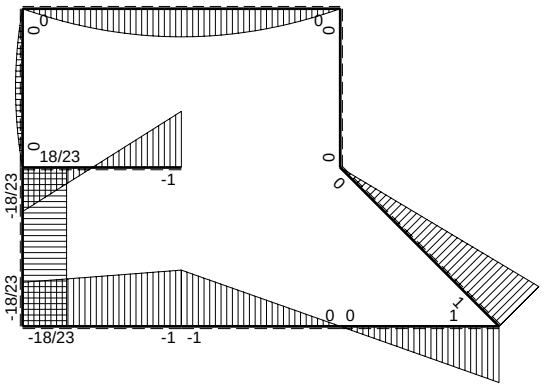





←□→ F



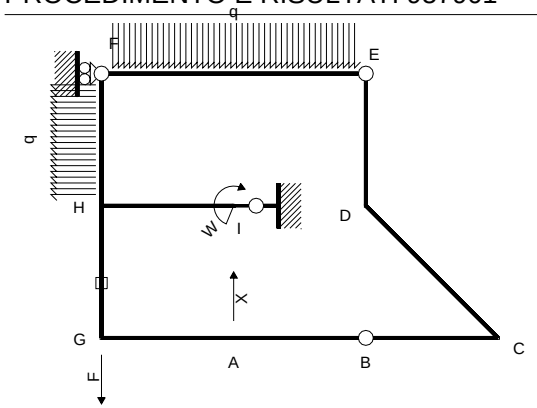

↑□↓ F



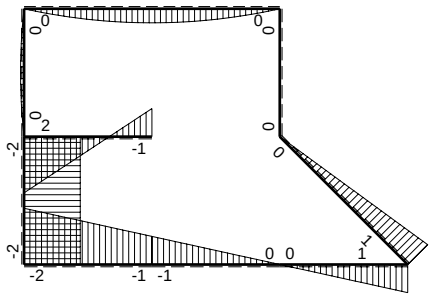

↻□↺ Fb



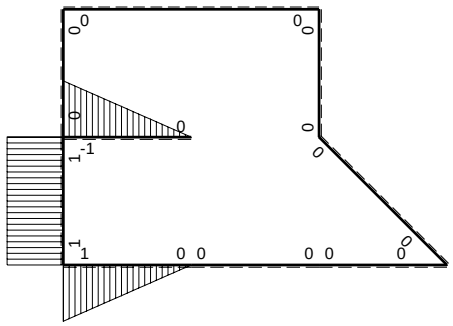





Schema di calcolo iperstatico




↻□↺ $M_o$ flessione da carichi assegnati




↻□↺ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb+Fx | 0 | 0 | | |
| CD √2b | 0 | Fb-√2/2Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | b-x | $-2Fb+Fx$ | $-2Fb^2+3Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-5/6Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | $Fb+Fx$ | $-Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | -b+x | 2Fb-3Fx | $-2Fb^2+5Fbx-3Fx^2$ | $b^2-2bx+x^2$ | $-1/2Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | Fb-3Fx | $Fbx-3Fx^2$ | $x^2$ | | |
| HG b | b | -2Fb | $-2Fb^2$ | $b^2$ | $-2Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | 2Fb | $-2Fb^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-7/3Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 28/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-2 +3\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2\, x +3/2\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2\, b +3/2\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(-\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/2\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/2\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-2 +5\, x/b -3\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2\, x +5/2\, x^2/b - x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2\, b +5/2\, b - b \,\right) Fb^2\, 1/EJ = -1/2\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(\, x/b -3\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[1/2\, x^2/b - x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

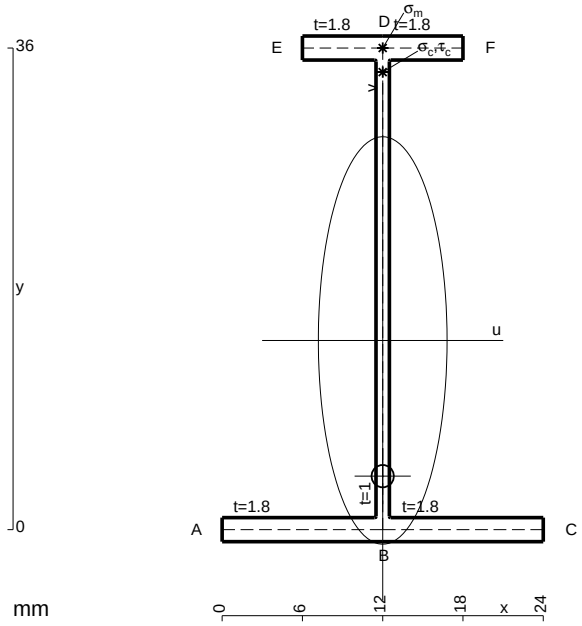
$$= \left(1/2\, b - b \,\right) Fb^2\, 1/EJ = -1/2\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-2\,\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = \left[-2\, x \,\right]_o^b Fb^2\, 1/EJ\ +1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-2\, b \,\right) Fb^2\, 1/EJ\ +1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -\ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-2\,\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = \left[-2\, x \,\right]_o^b Fb^2\, 1/EJ\ +1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-2\, b \,\right) Fb^2\, 1/EJ\ +1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -\ Fb^3/EJ$$




A = 100.8 $mm^2$
$J_u$ = 23384. $mm^4$
$J_v$ = 2333. $mm^4$
$J_t$ = 81.98 $mm^4$
$y_o$ = -10.14 mm
$y_g$ = 14.14 mm
$T_y$ = 690. N
$M_x$ = -213900. Nmm
$x_m$ = 12. mm
$y_m$ = 36. mm
$v_m$ = 21.86 mm
$\sigma_m = -Mv/J_u$ = 199.9 $N/mm^2$
$y_c$ = 2. mm
$u_c$ = -12. mm
$v_c$ = -12.14 mm
$\sigma_c = -Mv/J_u$ = 199.9 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 13.93 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 13.93 $N/mm^2$
$t_c$ = 690. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 201.4 $N/mm^2$



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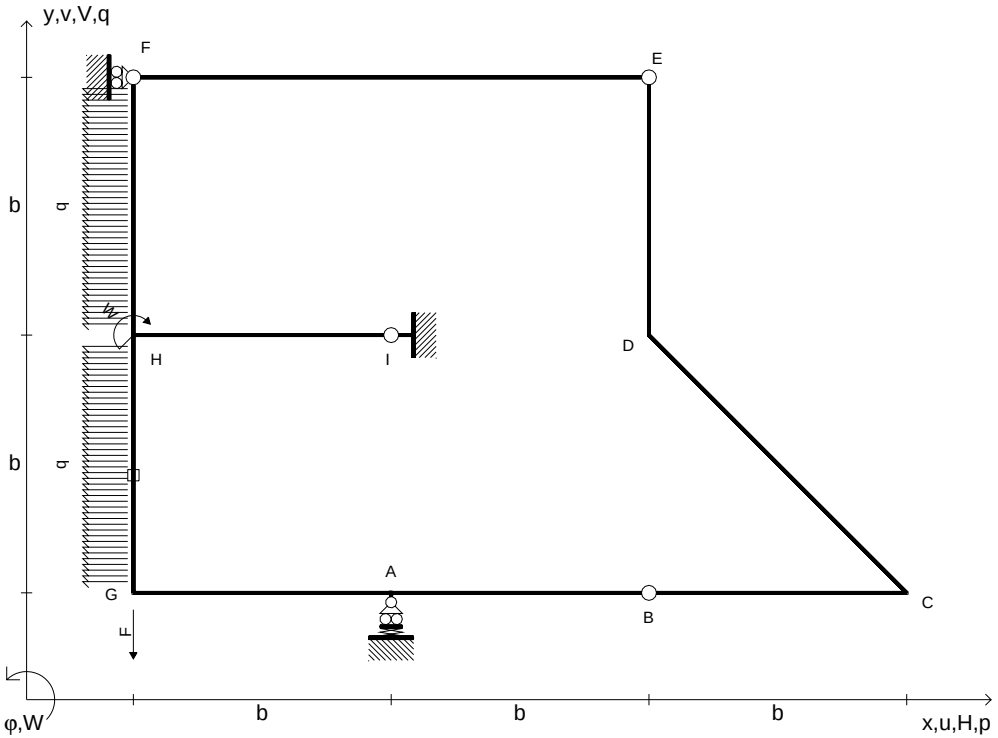
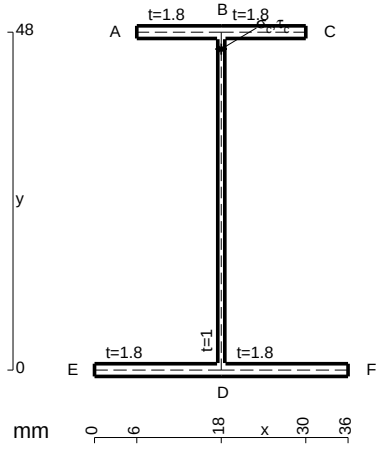



$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_H = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$p_{FH} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

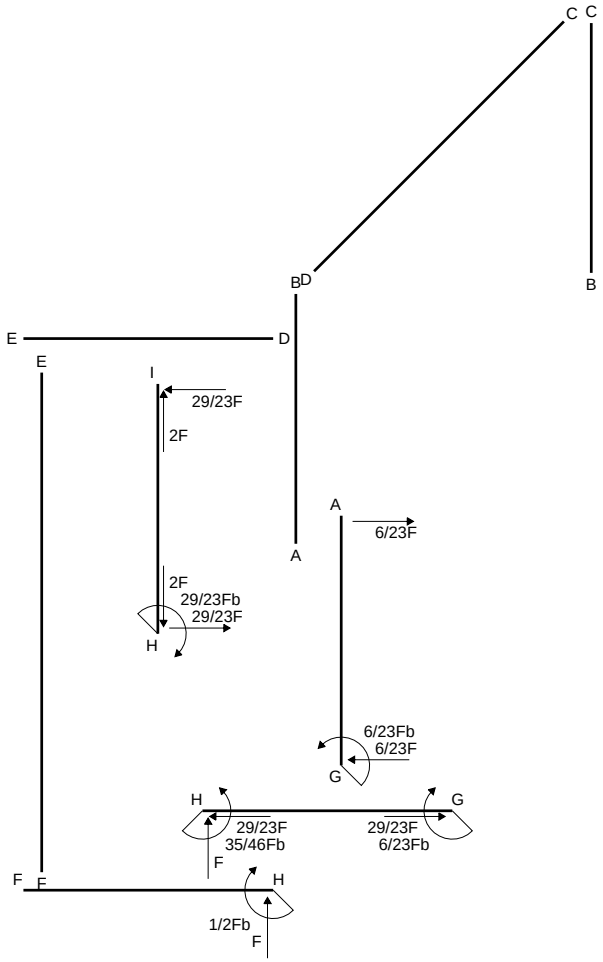
$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

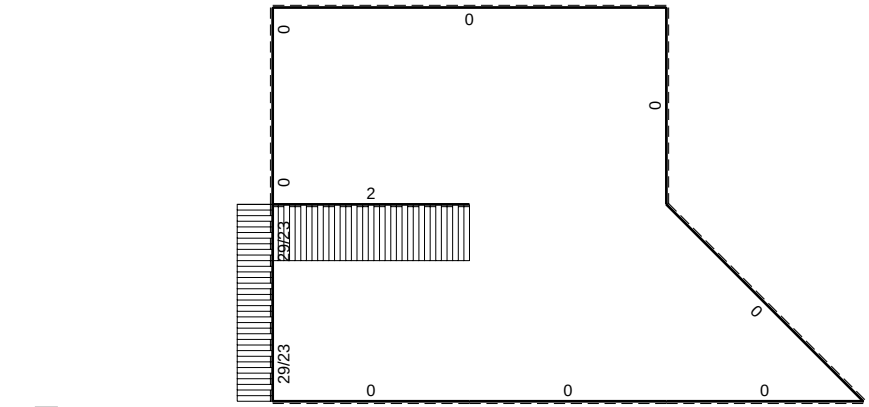



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 770 mm, F = 1590 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

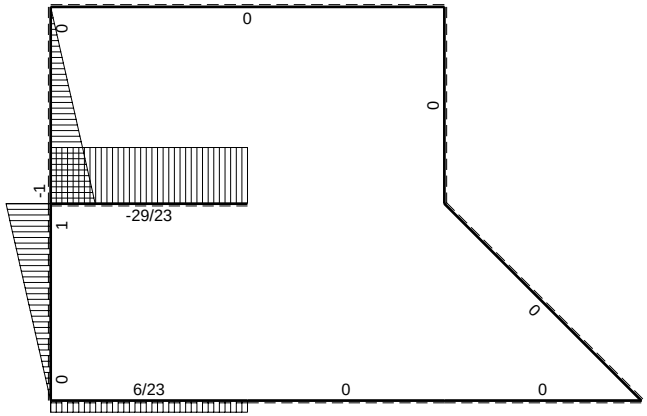




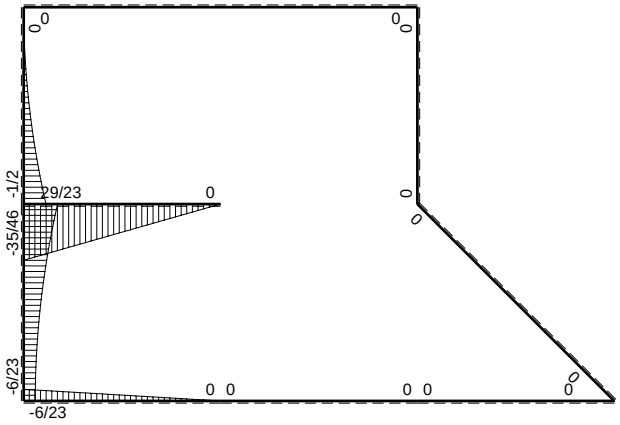






F



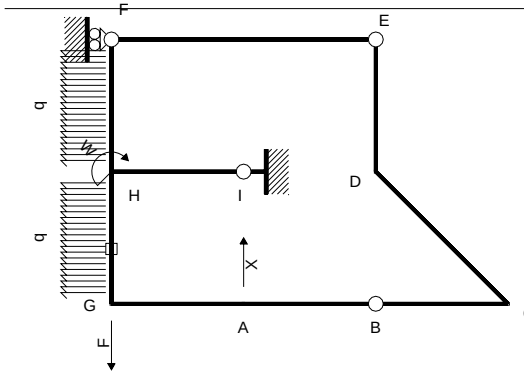

F



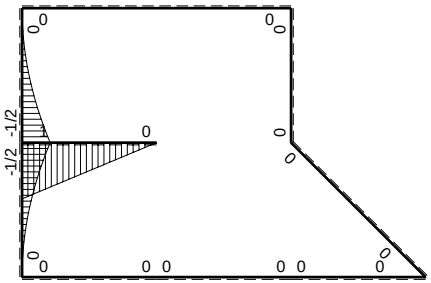

Fb



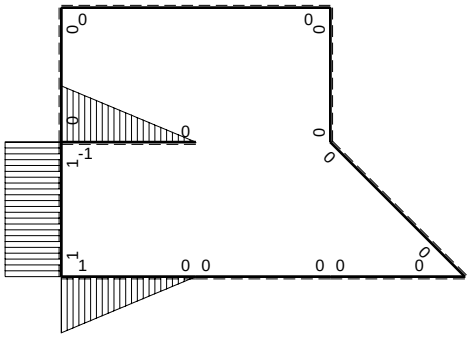





Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | 0 | 0 | 0 | 0 | 0 |
| BA b | 0 | 0 | 0 | 0 | | |
| BC b | 0 | 0 | 0 | 0 | 0 | 0 |
| CB b | 0 | 0 | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | 0 | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | b-x | 0 | 0 | $b^2-2bx+x^2$ | 0 | $1/3Xb^3/EJ$ |
| AG b | -x | 0 | 0 | $x^2$ | | |
| FH b | 0 | $-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $1/2Fb-Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | -b+x | Fb-Fx | $-Fb^2+2Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-1/3Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | -Fx | $-Fx^2$ | $x^2$ | | |
| HG b | b | $-1/2Fb+Fx-1/2qx^2$ | $-1/2Fb^2+Fbx-1/2Fx^2$ | $b^2$ | $-1/6Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $1/2qx^2$ | $-1/2Fx^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $1/2Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | -6/23F | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 211691 ESΣ05.xxxx.019

$L^{XX}_{GA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\ x^2/b^2\ )\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 -2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\ x^2/b^2\ )\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1\ )\ b^2\ 1/EJ\ dx = [\ x\ ]_o^b\ b^2\ 1/EJ$

$= (\ b\ )\ b^2\ 1/EJ\ =\ b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1\ )\ b^2\ 1/EJ\ dx = [\ x\ ]_o^b\ b^2\ 1/EJ$

$= (\ b\ )\ b^2\ 1/EJ\ =\ b^3/EJ$

$L^{Xo}_{HI} = \int_o^b (-1 +2\ x/b - x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [-\ x + x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (-\ b + b - 1/3\ b\ )\ Fb^2\ 1/EJ\ = -1/3\ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b (-\ x^2/b^2\ )\ Fb^2\ 1/EJ\ dx = [-1/3\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (-1/3\ b\ )\ Fb^2\ 1/EJ\ = -1/3\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (-1/2 + x/b - 1/2\ x^2/b^2\ )\ Fb^2\ 1/EJ\ dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [-1/2\ x + 1/2\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ\ +\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-1/2\ b + 1/2\ b - 1/6\ b\ )\ Fb^2\ 1/EJ\ +\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = 5/6\ \ Fb^3/EJ$

$L^{Xo}_{GH} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb^2\ 1/EJ\ dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = [-1/6\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ\ +\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-1/6\ b\ )\ Fb^2\ 1/EJ\ +\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = 5/6\ \ Fb^3/EJ$

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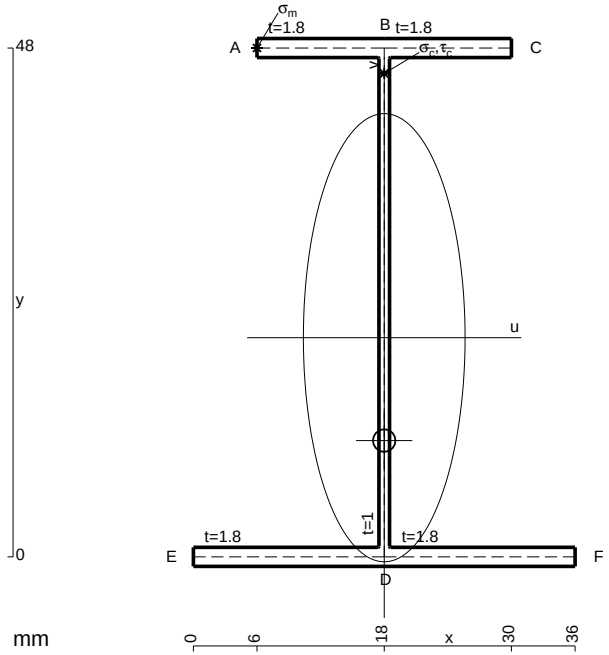


A = 156. $mm^2$
$J_u$ = 69701. $mm^4$
$J_v$ = 9072. $mm^4$
$J_t$ = 132.6 $mm^4$
$y_o$ = -9.705 mm
$y_g$ = 20.68 mm
$T_y$ = -1590. N
$M_x$ = -612150. Nmm
$x_m$ = 6. mm
$y_m$ = 48. mm
$u_m$ = -12. mm
$v_m$ = 27.32 mm
$\sigma_m = -Mv/J_u$ = 240. $N/mm^2$
$x_c$ = 18. mm
$y_c$ = 48. mm
$v_c$ = 27.32 mm
$\sigma_c = -Mv/J_u$ = 240. $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 26.93 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 26.93 $N/mm^2$
$t_c$ = 1590. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 244.5 $N/mm^2$


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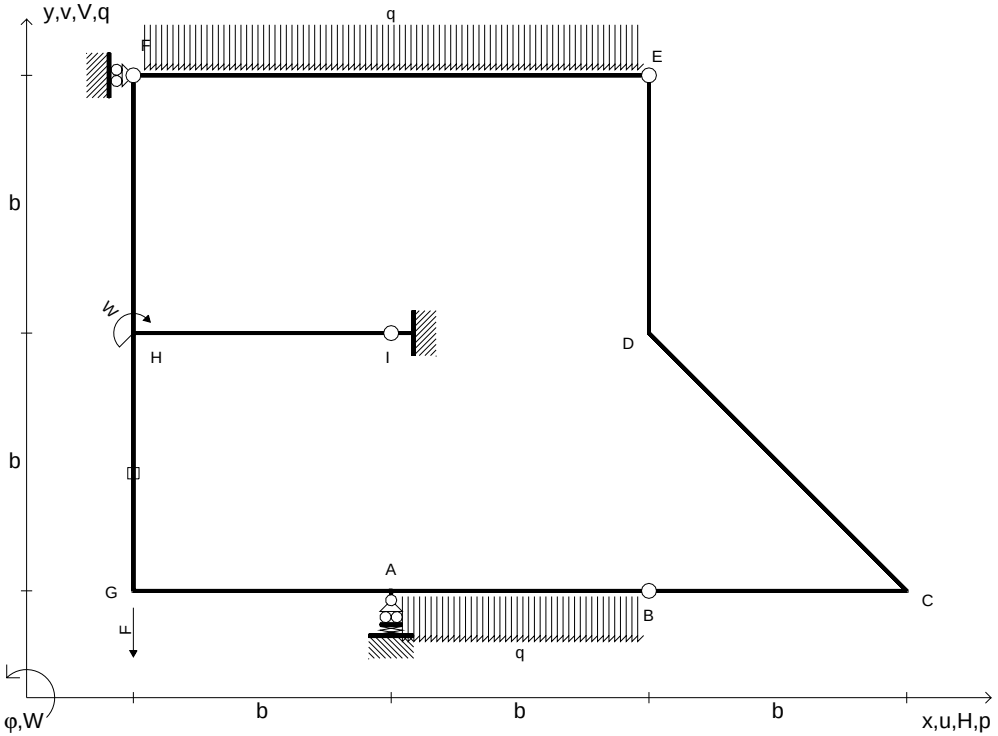
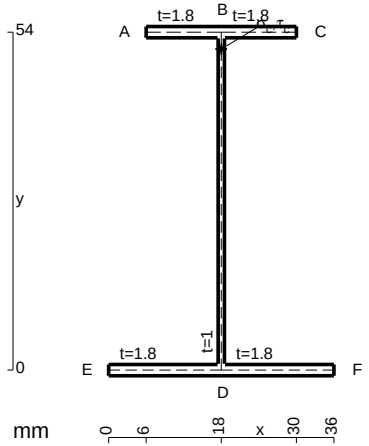



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_H = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

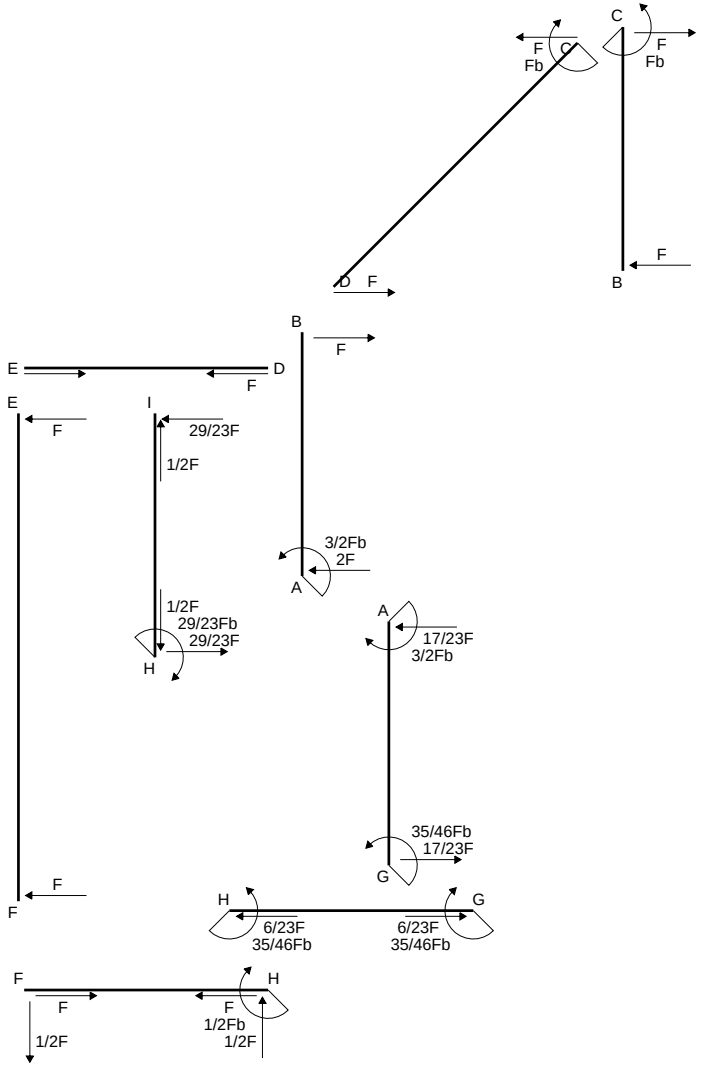
$q_{AB} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

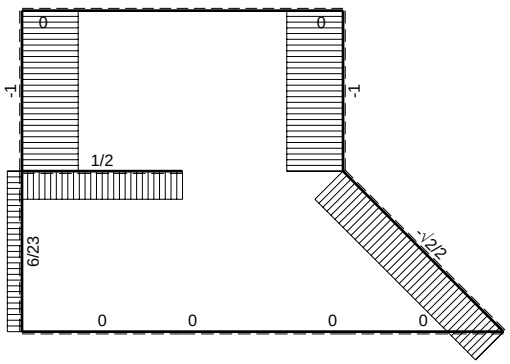



Reazioni iperstatiche in soluzione: $X=W_{GA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 640 mm, F = 730 N
Calcolare sulla sezione A la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.

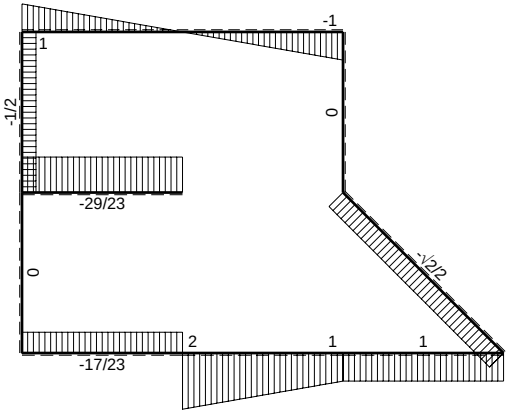




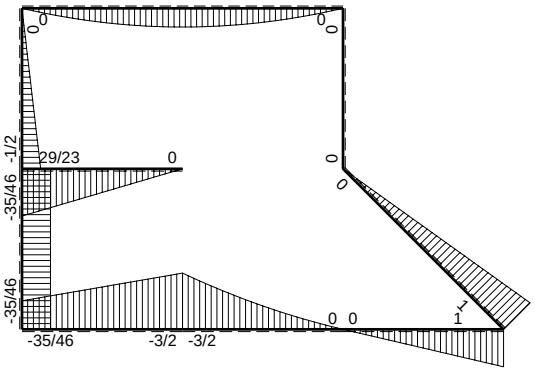





←□→ F



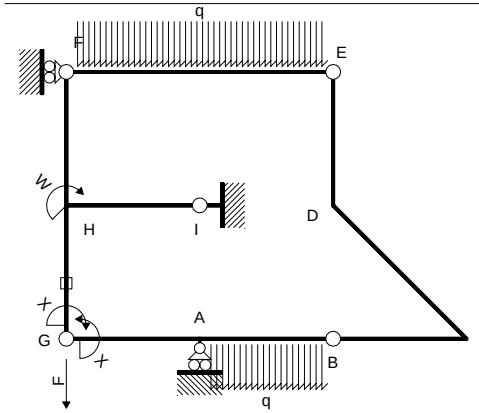

↑□↓ F

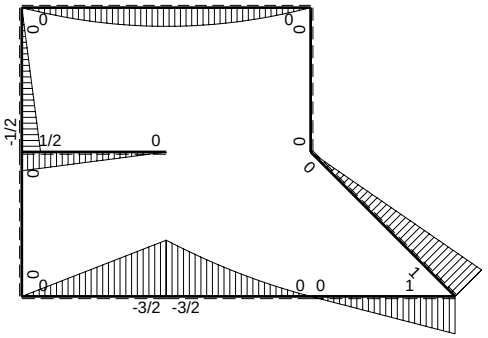



(□) Fb

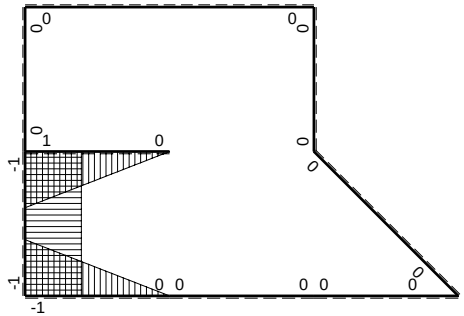







(□) $M_o$ flessione da carichi assegnati




(□) $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-3/2Fb+2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $Fx+1/2qx^2$ | 0 | 0 | | |
| BC b | 0 | Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb+Fx | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $Fb-\sqrt{2}/2Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | -1+x/b | -3/2Fx | $3/2Fx-3/2Fx^2/b$ | $1-2x/b+x^2/b^2$ | $1/4Fb^2/EJ$ | 1/3Xb/EJ |
| AG b | x/b | 3/2Fb-3/2Fx | $3/2Fx-3/2Fx^2/b$ | $x^2/b^2$ | | |
| FH b | 0 | -1/2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 1/2Fb-1/2Fx | 0 | 0 | | |
| HI b | 1-x/b | 1/2Fb-1/2Fx | $1/2Fb-Fx+1/2Fx^2/b$ | $1-2x/b+x^2/b^2$ | $1/6Fb^2/EJ$ | 1/3Xb/EJ |
| IH b | -x/b | -1/2Fx | $1/2Fx^2/b$ | $x^2/b^2$ | | |
| HG b | -1 | 0 | 0 | 1 | 0 | Xb/EJ |
| GH b | 1 | 0 | 0 | 1 | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^2/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-7/8Fb^2/EJ$ | 1/4Xb/EJ |
| | totali | | | | $-35/24Fb^2/EJ$ | 23/12Xb/EJ |
| | iperstatica $X=W_{GA}$ | | | | 35/46Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 211309 ESΣ05.xxxx.024

$L_{GA}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{AG}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{HI}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\,b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{IH}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\,x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\,b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{HG}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\,x\,]_o^b\ 1/EJ$

$= (\,b\,)\ 1/EJ = b/EJ$

$L_{GH}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\,x\,]_o^b\ 1/EJ$

$= (\,b\,)\ 1/EJ = b/EJ$

$L_{GA}^{Xo} = \int_o^b (3/2\,x/b - 3/2\,x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\,x^2/b - 1/2\,x^3/b^2\,]_o^b\ Fb\ 1/EJ$

$= (3/4\,b - 1/2\,b\,)\ Fb\ 1/EJ = 1/4\ Fb^2/EJ$

$L_{AG}^{Xo} = \int_o^b (3/2\,x/b - 3/2\,x^2/b^2\,)\ Fb\ 1/EJ\ dx = [3/4\,x^2/b - 1/2\,x^3/b^2\,]_o^b\ Fb\ 1/EJ$

$= (3/4\,b - 1/2\,b\,)\ Fb\ 1/EJ = 1/4\ Fb^2/EJ$

$L_{HI}^{Xo} = \int_o^b (1/2 - x/b + 1/2\,x^2/b^2\,)\ Fb\ 1/EJ\ dx = [1/2\,x - 1/2\,x^2/b + 1/6\,x^3/b^2\,]_o^b\ Fb\ 1/EJ$

$= (1/2\,b - 1/2\,b + 1/6\,b\,)\ Fb\ 1/EJ = 1/6\ Fb^2/EJ$

$L_{IH}^{Xo} = \int_o^b (1/2\,x^2/b^2\,)\ Fb\ 1/EJ\ dx = [1/6\,x^3/b^2\,]_o^b\ Fb\ 1/EJ$

$= (1/6\,b\,)\ Fb\ 1/EJ = 1/6\ Fb^2/EJ$

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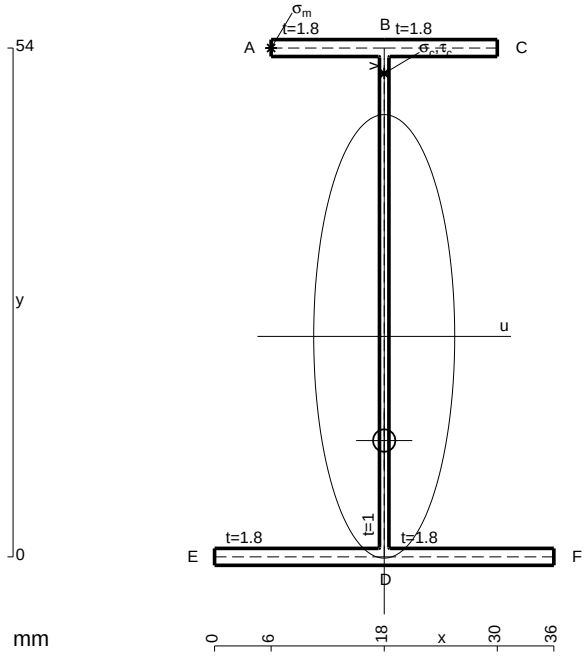


$A = 162.\ mm^2$
$J_u = 89755.\ mm^4$
$J_v = 9072.\ mm^4$
$J_t = 134.6\ mm^4$
$y_o = -11.06\ mm$
$y_g = 23.4\ mm$
$T_y = 1460.\ N$
$M_x = -700800.\ Nmm$
$x_m = 6.\ mm$
$y_m = 54.\ mm$
$u_m = -12.\ mm$
$v_m = 30.6\ mm$
$\sigma_m = -Mv/J_u = 238.9\ N/mm^2$
$x_c = 18.\ mm$
$y_c = 54.\ mm$
$v_c = 30.6\ mm$
$\sigma_c = -Mv/J_u = 238.9\ N/mm^2$
$\tau_c = TS^*/tJ_u = 21.5\ N/mm^2$
$\tau_g = TS^*/tJ_u = 21.5\ N/mm^2$
$t_c = 730.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 241.8\ N/mm^2$



ESΣ05.xxxx.024

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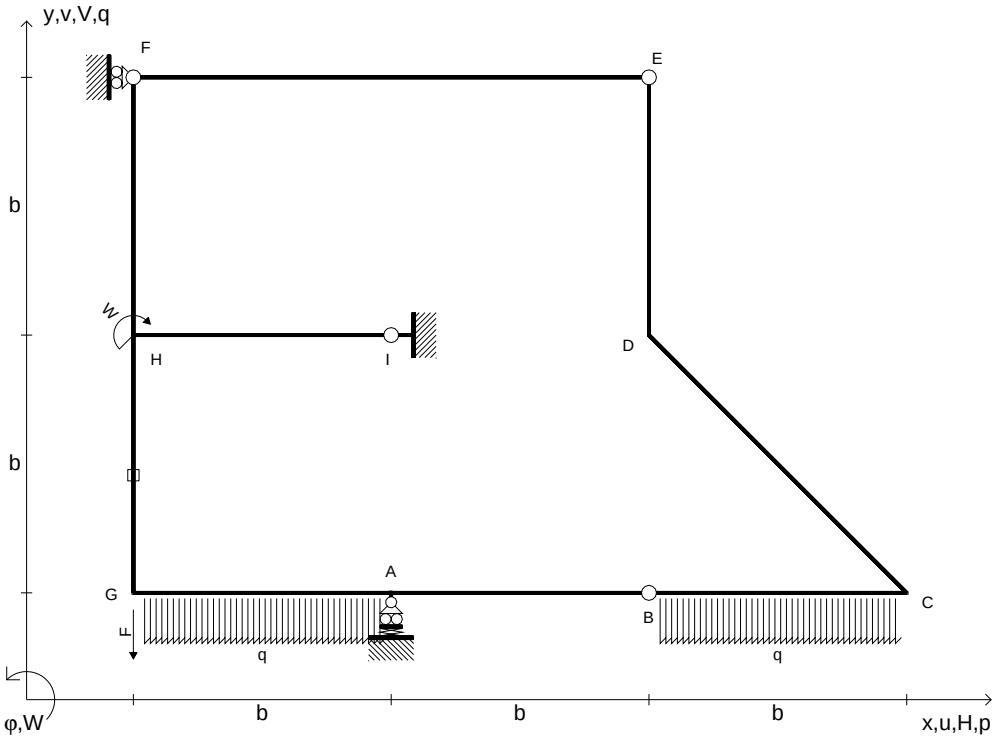
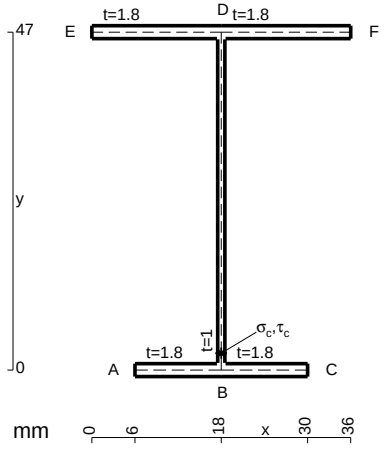



$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_H = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{BC} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

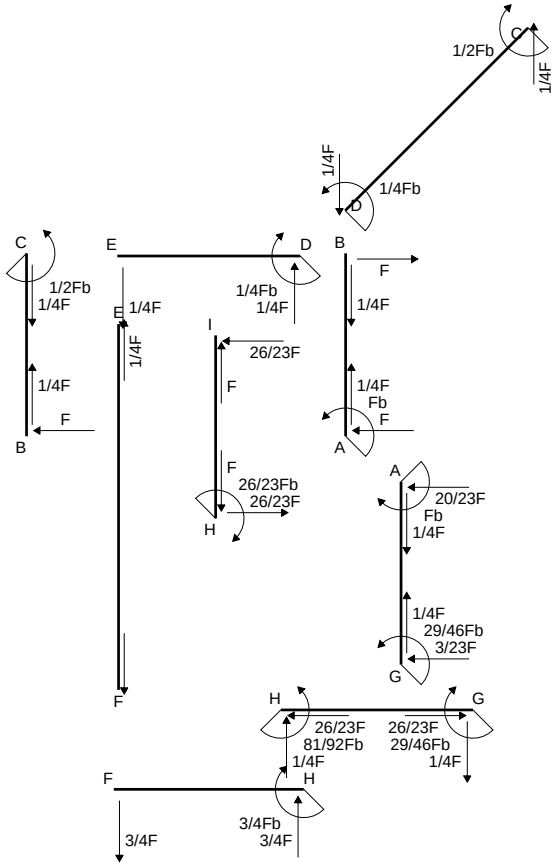
$q_{GA} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

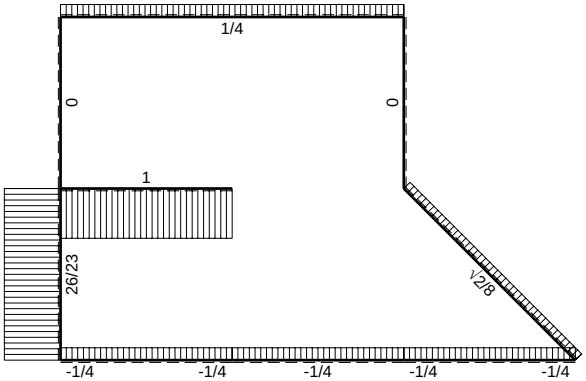



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 760 mm, F = 780 N
Calcolare sulla sezione A la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.

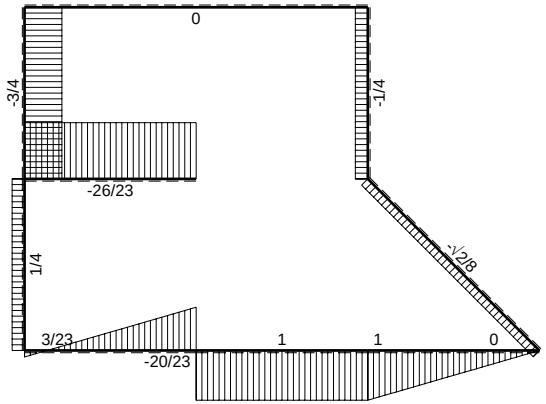




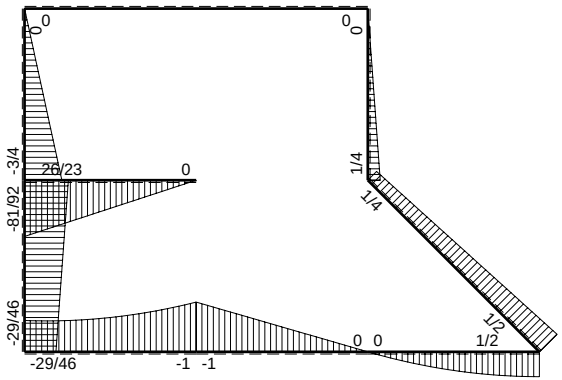





←□→ F



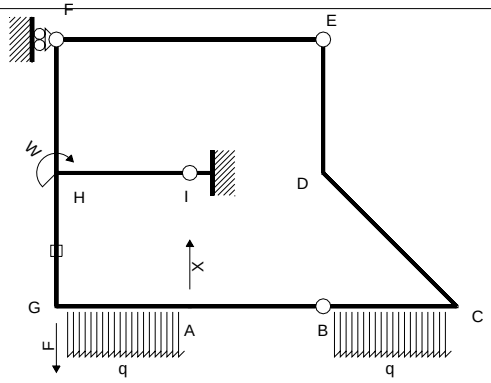

↑□↓ F



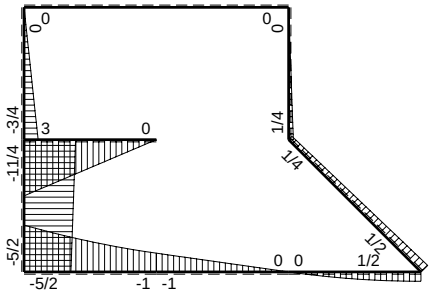

□ Fb



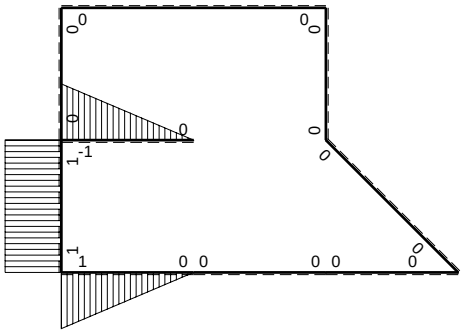





Schema di calcolo iperstatico




□ $M_o$ flessione da carichi assegnati




□ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-Fb+Fx$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $Fx$ | 0 | 0 | | |
| BC b | 0 | $Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| CB b | 0 | $-1/2Fb+1/2qx^2$ | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $1/2Fb-\sqrt{2}/8Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | $1/4Fb-1/4Fx$ | 0 | 0 | 0 | 0 |
| ED b | 0 | $-1/4Fx$ | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | b-x | $-5/2Fb+2Fx-1/2qx^2$ | $-5/2Fb^2+9/2Fbx-5/2Fx^2+1/2qx^3$ | $b^2-2bx+x^2$ | $-23/24Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | $Fb+Fx+1/2qx^2$ | $-Fbx-Fx^2-1/2qx^3$ | $x^2$ | | |
| FH b | 0 | $-3/4Fx$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $3/4Fb-3/4Fx$ | 0 | 0 | | |
| HI b | -b+x | $3Fb-3Fx$ | $-3Fb^2+6Fbx-3Fx^2$ | $b^2-2bx+x^2$ | $-Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | $-3Fx$ | $-3Fx^2$ | $x^2$ | | |
| HG b | b | $-11/4Fb+1/4Fx$ | $-11/4Fb^2+1/4Fbx$ | $b^2$ | $-21/8Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $5/2Fb+1/4Fx$ | $-5/2Fb^2-1/4Fbx$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-43/12Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 43/23F | |



PROCEDIMENTO E RISULTATI 210099 ESΣ05.xxxx.029

Sviluppi di calcolo iperstatica

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$L^{XX}_{GA} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-5/2 + 9/2\,x/b - 5/2\,x^2/b^2 + 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-5/2\,x + 9/4\,x^2/b - 5/6\,x^3/b^2 + 1/8\,x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-5/2\,b + 9/4\,b - 5/6\,b + 1/8\,b)\, Fb^2\, 1/EJ = -23/24\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-\,x/b - x^2/b^2 - 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/2\,x^2/b - 1/3\,x^3/b^2 - 1/8\,x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\,b - 1/3\,b - 1/8\,b)\, Fb^2\, 1/EJ = -23/24\ Fb^3/EJ$

$L^{Xo}_{HI} = \int_o^b (-3 + 6\,x/b - 3\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3\,x + 3\,x^2/b - x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3\,b + 3\,b - b)\, Fb^2\, 1/EJ = -\ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b (-3\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-\,x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-\,b)\, Fb^2\, 1/EJ = -\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (-11/4 + 1/4\,x/b)\, Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= [-11/4\,x + 1/8\,x^2/b \,]_o^b\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= (-11/4\,b + 1/8\,b)\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = -13/8\ Fb^3/EJ$

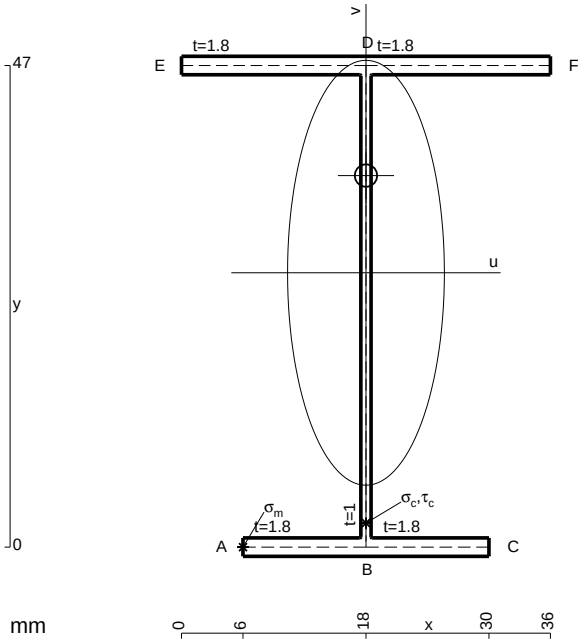
$L^{Xo}_{GH} = \int_o^b (-5/2 - 1/4\,x/b)\, Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= [-5/2\,x - 1/8\,x^2/b \,]_o^b\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

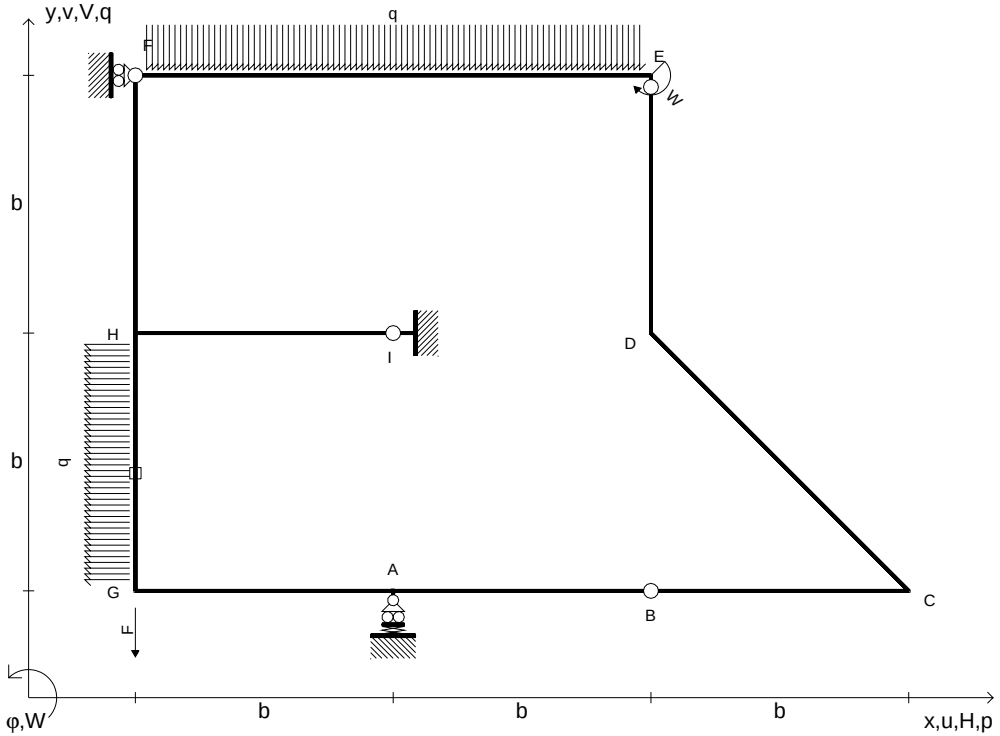
$= (-5/2\,b - 1/8\,b)\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = -13/8\ Fb^3/EJ$








A = 155. $mm^2$
$J_u$ = 66633. $mm^4$
$J_v$ = 9072. $mm^4$
$J_t$ = 132.3 $mm^4$
$y_o$ = 9.482 mm
$y_g$ = 26.77 mm
N = -195. N
$T_y$ = 780. N
$M_x$ = -592800. Nmm
$x_m$ = 6. mm
$u_m$ = -12. mm
$v_m$ = -26.77 mm
$\sigma_m = N/A - Mv/J_u$ = -239.5 $N/mm^2$
$x_c$ = 18. mm
$v_c$ = -26.77 mm
$\sigma_c = N/A - Mv/J_u$ = -239.5 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 13.54 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 13.54 $N/mm^2$
$t_c$ = 780. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 240.6 $N/mm^2$

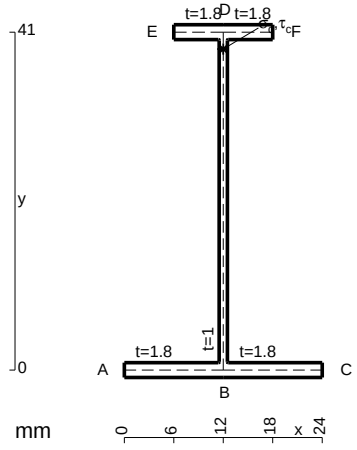



$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_E = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$




Reazioni iperstatiche in soluzione: $X=V_I$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave AB ha la sezione riportata e dimensioni in mm, con:

b = 380 mm, F = 460 N

Calcolare sulla sezione A la massima tensione normale $\sigma_m$.

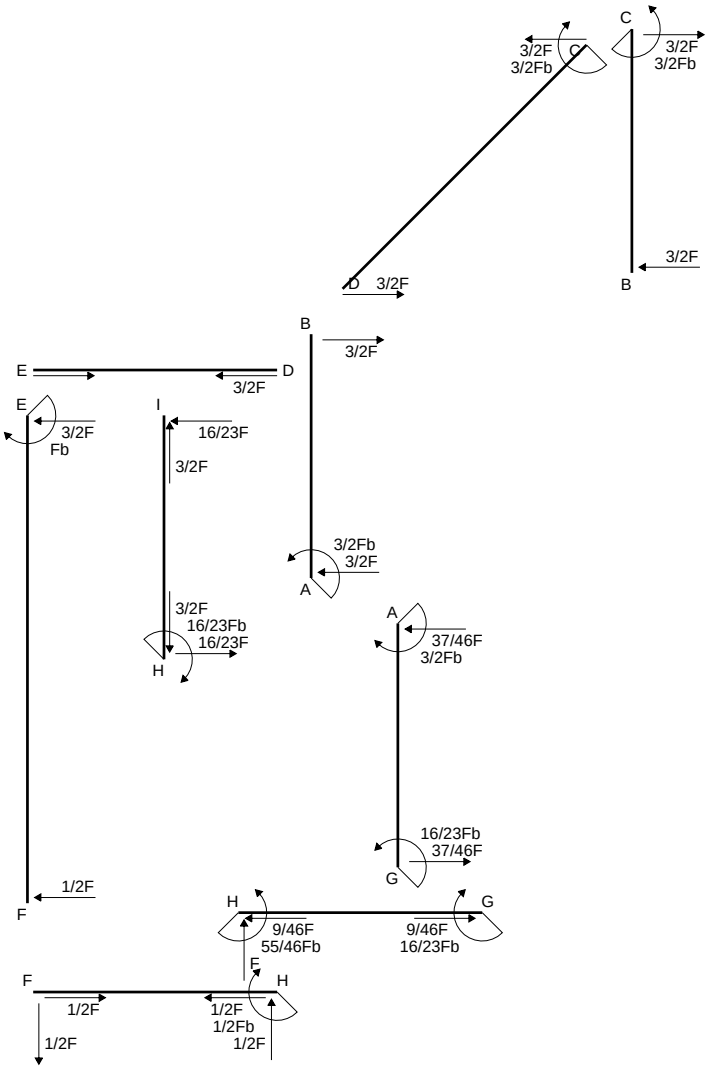
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.

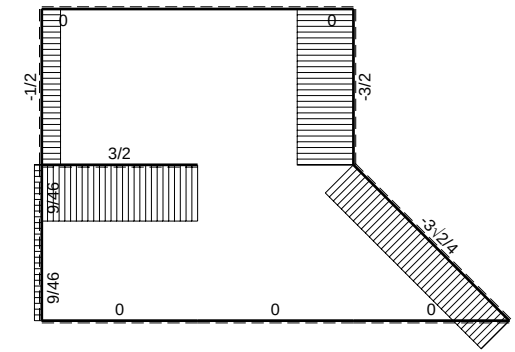
Lembo inferiore sezione su tratteggio trave, a destra da A a B

Elongazione termica specifica ε assegnata su asta HG.

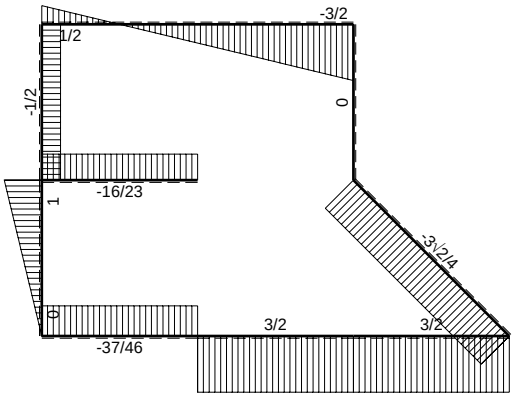




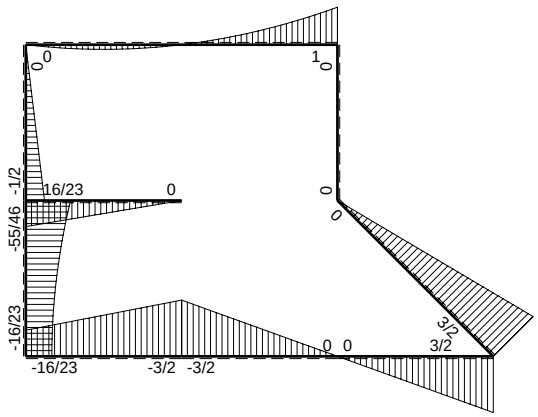





F



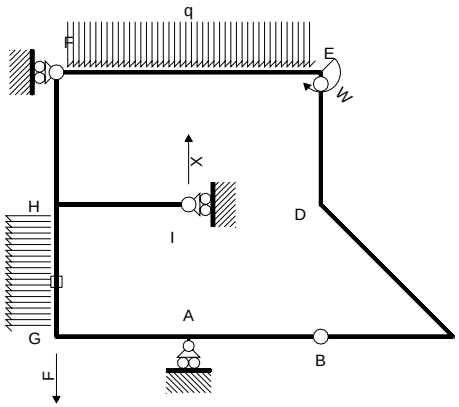

F

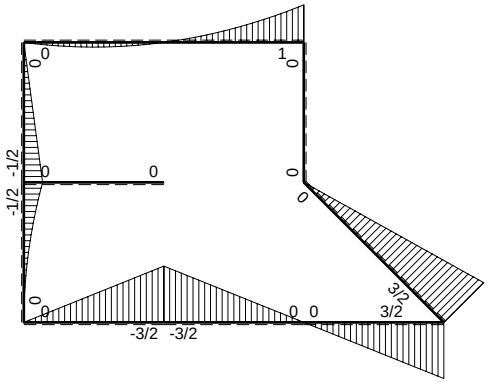



Fb

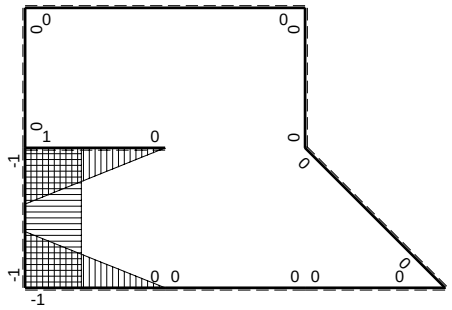







$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -3/2Fb+3/2Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | 3/2Fx | 0 | 0 | | |
| BC b | 0 | 3/2Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -3/2Fb+3/2Fx | 0 | 0 | | |
| CD √2b | 0 | 3/2Fb-3√2/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | Fb-3/2Fx+1/2qx$^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 1/2Fx-1/2qx$^2$ | 0 | 0 | | |
| GA b | -b+x | -3/2Fx | 3/2Fbx-3/2Fx$^2$ | b$^2$-2bx+x$^2$ | 1/4Fb$^3$/EJ | 1/3Xb$^3$/EJ |
| AG b | x | 3/2Fb-3/2Fx | 3/2Fbx-3/2Fx$^2$ | x$^2$ | | |
| FH b | 0 | -1/2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 1/2Fb-1/2Fx | 0 | 0 | | |
| HI b | b-x | 0 | 0 | b$^2$-2bx+x$^2$ | 0 | 1/3Xb$^3$/EJ |
| IH b | -x | 0 | 0 | x$^2$ | | |
| HG b | -b | -1/2Fb+Fx-1/2qx$^2$ | 1/2Fb$^2$-Fbx+1/2Fx$^2$ | b$^2$ | 1/6Fb$^3$/EJ | Xb$^3$/EJ |
| GH b | b | 1/2qx$^2$ | 1/2Fx$^2$ | b$^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | -Fb$^3$/EJ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | -3/4Fb$^3$/EJ | 1/4Xb$^3$/EJ |
| | totali | | | | -4/3Fb$^3$/EJ | 23/12Xb$^3$/EJ |
| | iperstatica $X=V_I$ | | | | 16/23F | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 211730 ESΣ05.xxxx.031

$L^{XX}_{GA} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [\, 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [\, 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{Xo}_{GA} = \int_o^b (3/2\, x/b - 3/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x^2/b - 1/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b - 1/2\, b \,)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (3/2\, x/b - 3/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x^2/b - 1/2\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b - 1/2\, b \,)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (1/2 - x/b + 1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [1/2\, x - 1/2\, x^2/b + 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ\ -1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (1/2\, b - 1/2\, b + 1/6\, b \,)\, Fb^2\, 1/EJ\ -1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -5/6\ \ Fb^3/EJ$

$L^{Xo}_{GH} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = [1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ\ -1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (1/6\, b \,)\, Fb^2\, 1/EJ\ -1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -5/6\ \ Fb^3/EJ$

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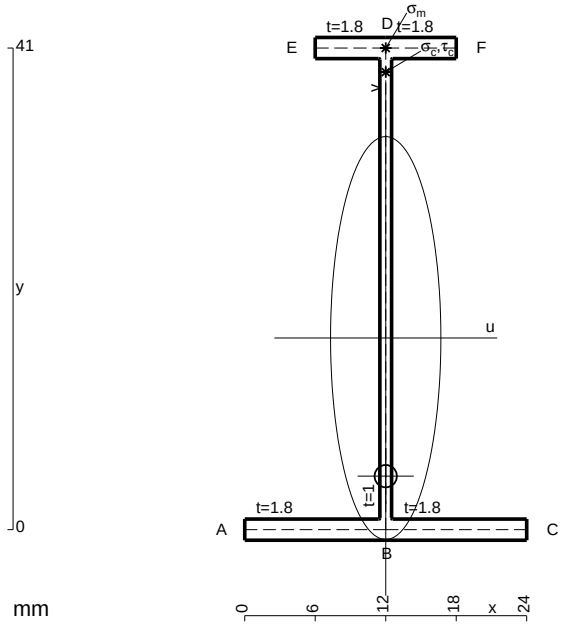


$A = 105.8\ mm^2$
$J_u = 31122.\ mm^4$
$J_v = 2333.\ mm^4$
$J_t = 83.65\ mm^4$
$y_o = -11.76\ mm$
$y_g = 16.31\ mm$
$T_y = 690.\ N$
$M_x = -262200.\ Nmm$
$x_m = 12.\ mm$
$y_m = 41.\ mm$
$v_m = 24.69\ mm$
$\sigma_m = -Mv/J_u = 208.\ N/mm^2$
$y_c = 2.\ mm$
$u_c = -12.\ mm$
$v_c = -14.31\ mm$
$\sigma_c = -Mv/J_u = 208.\ N/mm^2$
$\tau_c = TS^*/tJ_u = 11.82\ N/mm^2$
$\tau_g = TS^*/tJ_u = 11.82\ N/mm^2$
$t_c = 460.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 209.\ N/mm^2$



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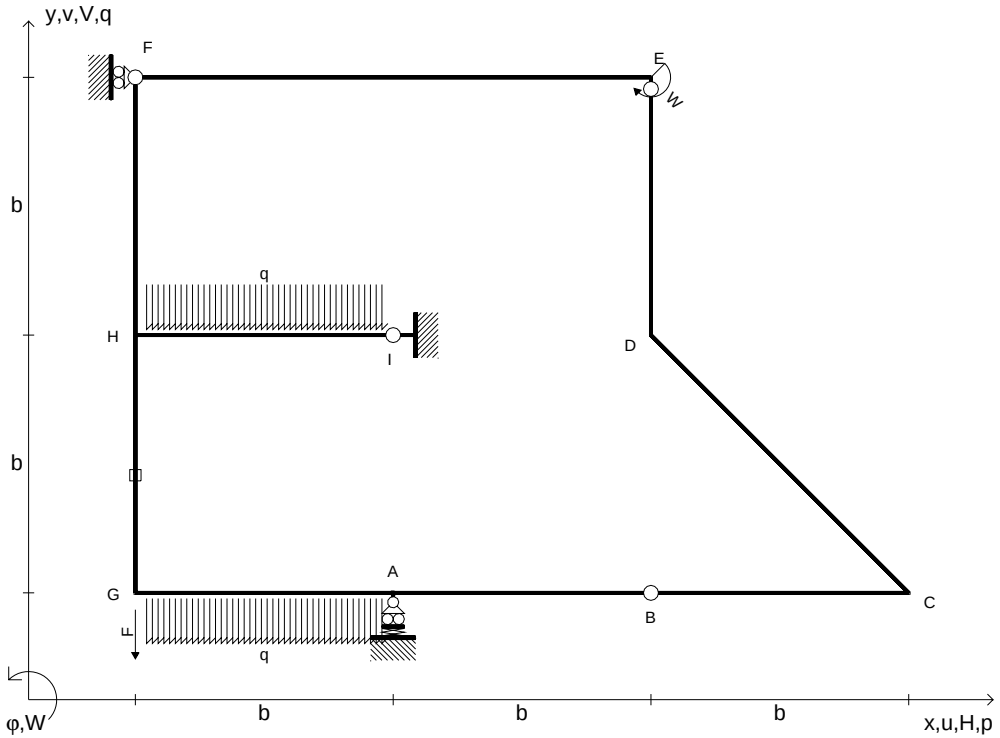
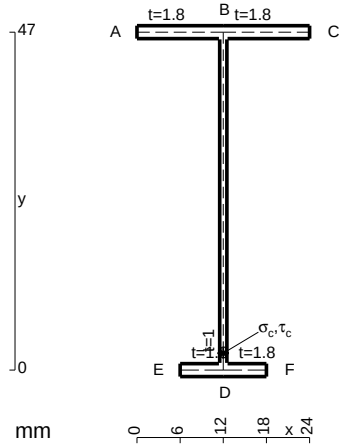



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_E = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{GA} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

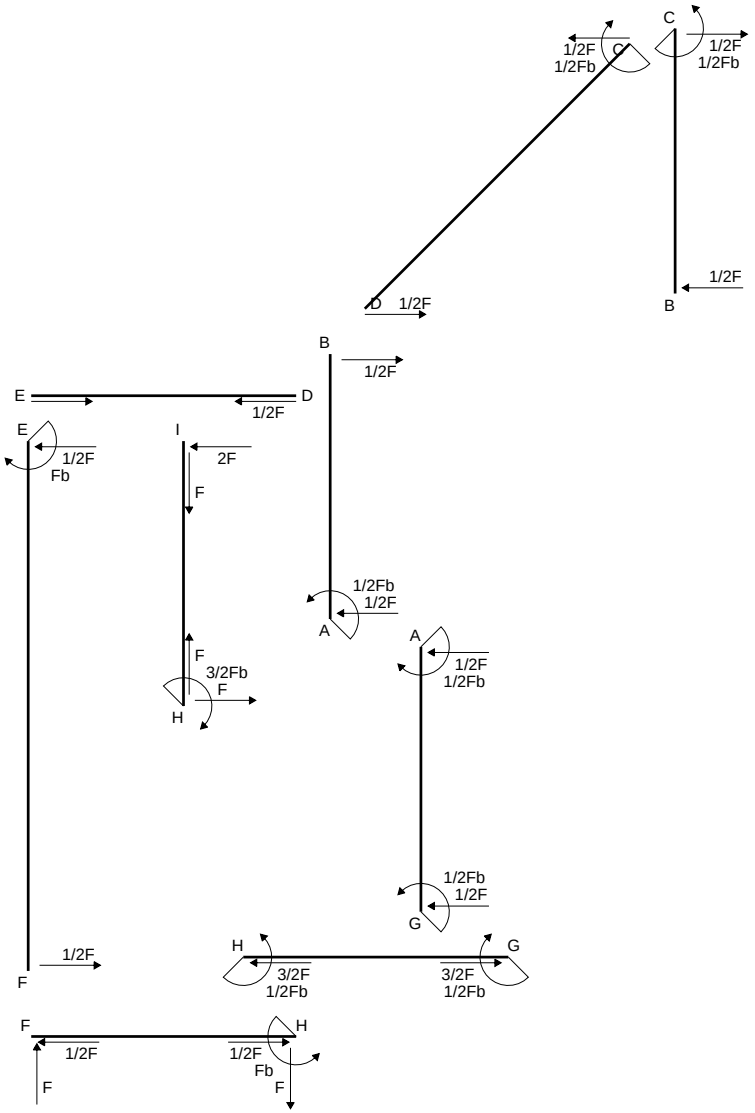
$q_{HI} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

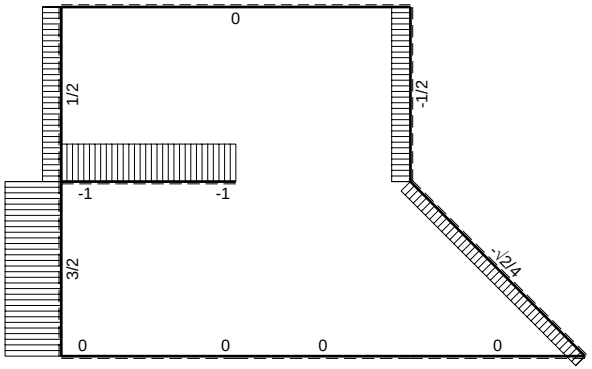



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 760 mm, F = 470 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

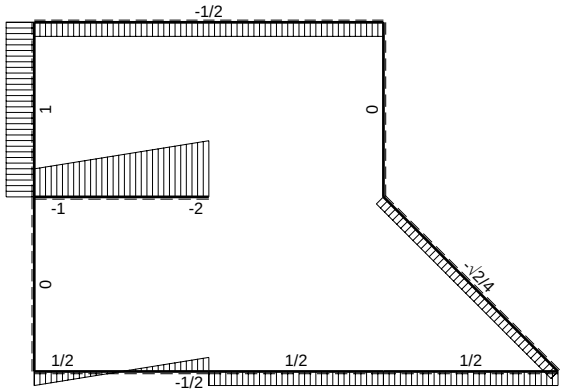




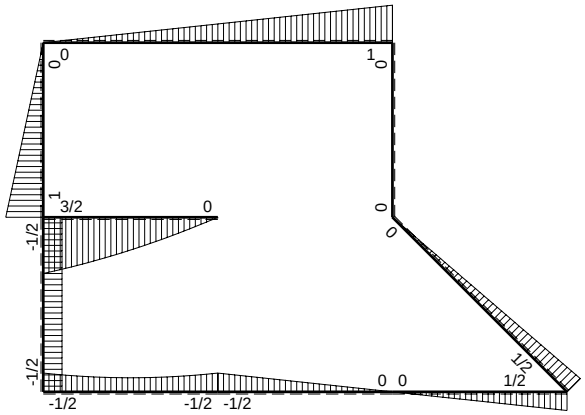






←□→ F



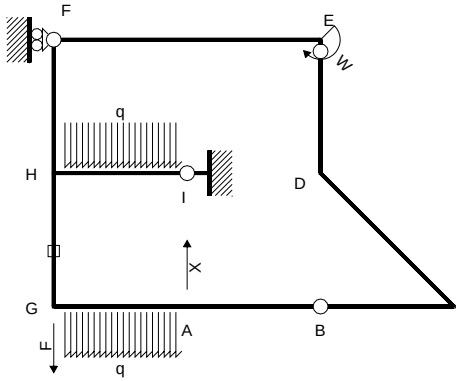

↑□↓ F



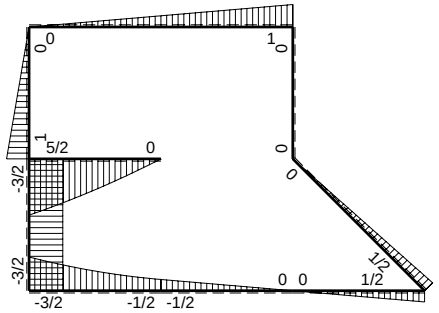

□ Fb



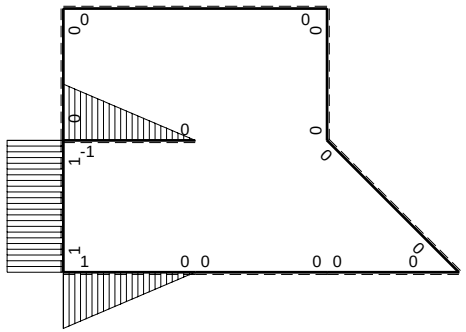





Schema di calcolo iperstatico




□ $M_o$ flessione da carichi assegnati




□ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -1/2Fb+1/2Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | 1/2Fx | 0 | 0 | | |
| BC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -1/2Fb+1/2Fx | 0 | 0 | | |
| CD √2b | 0 | 1/2Fb-√2/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 |
| FE 2b | 0 | -1/2Fx | 0 | 0 | | |
| GA b | b-x | $-3/2Fb+3/2Fx-1/2qx^2$ | $-3/2Fb^2+3Fbx-2Fx^2+1/2qx^3$ | $b^2-2bx+x^2$ | $-13/24Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | $1/2Fb+1/2Fx+1/2qx^2$ | $-1/2Fbx-1/2Fx^2-1/2qx^3$ | $x^2$ | | |
| FH b | 0 | Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | -Fb+Fx | 0 | 0 | | |
| HI b | -b+x | $5/2Fb-2Fx-1/2qx^2$ | $-5/2Fb^2+9/2Fbx-3/2Fx^2-1/2qx^3$ | $b^2-2bx+x^2$ | $-7/8Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | $-3Fx+1/2qx^2$ | $-3Fx^2+1/2qx^3$ | $x^2$ | | |
| HG b | b | $-3/2Fb$ | $-3/2Fb^2$ | $b^2$ | $-3/2Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $3/2Fb$ | $-3/2Fb^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-23/12Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | F | |



PROCEDIMENTO E RISULTATI 213154 ESΣ05.xxxx.039

Sviluppi di calcolo iperstatica

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$L^{XX}_{GA} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-3/2 + 3\,x/b - 2\,x^2/b^2 + 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-3/2\, x + 3/2\, x^2/b - 2/3\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 3/2\, b - 2/3\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = -13/24\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-1/2\,x/b - 1/2\,x^2/b^2 - 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b - 1/6\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/4\, b - 1/6\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = -13/24\ Fb^3/EJ$

$L^{Xo}_{HI} = \int_o^b (-5/2 + 9/2\,x/b - 3/2\,x^2/b^2 - 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-5/2\, x + 9/4\, x^2/b - 1/2\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-5/2\, b + 9/4\, b - 1/2\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = -7/8\ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b (-3\,x^2/b^2 + 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = -7/8\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (-3/2)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = [-3/2\, x \,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

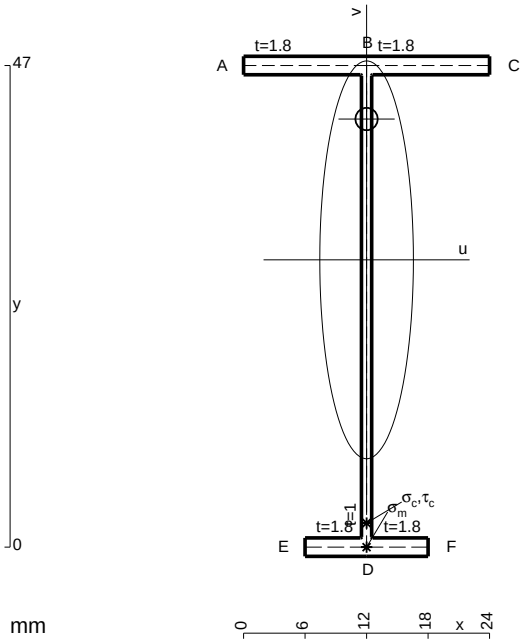
$= (-3/2\, b \,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -1/2\ Fb^3/EJ$

$L^{Xo}_{GH} = \int_o^b (-3/2)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = [-3/2\, x \,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

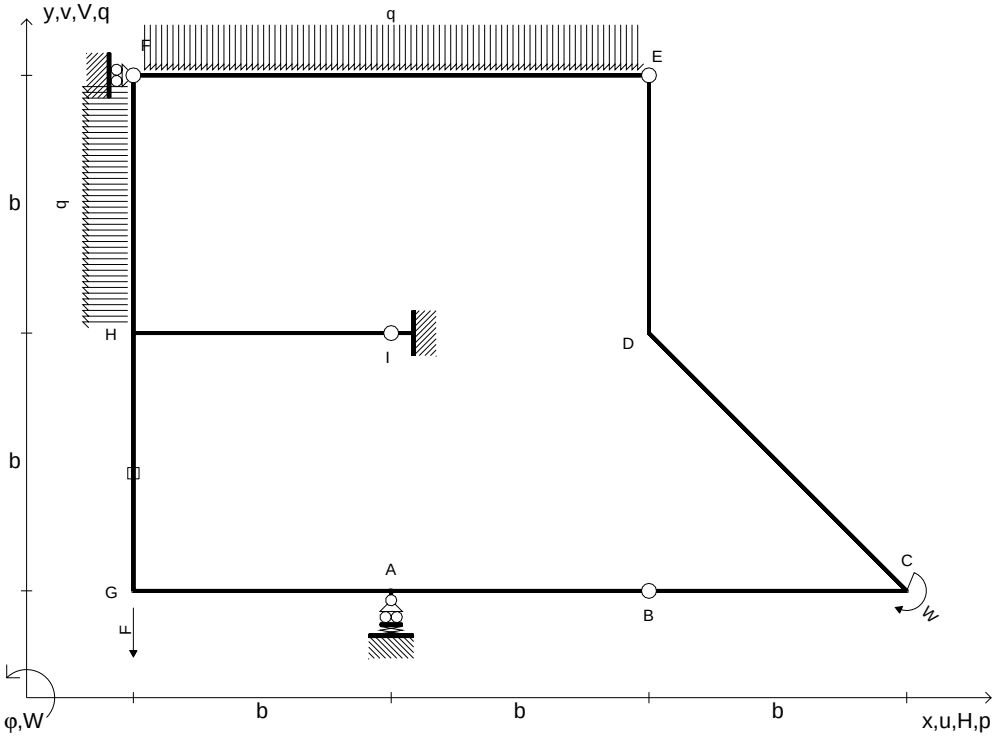
$= (-3/2\, b \,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -1/2\ Fb^3/EJ$








A = 111.8 $mm^2$
$J_u$ = 42133. $mm^4$
$J_v$ = 2333. $mm^4$
$J_t$ = 85.65 $mm^4$
$y_o$ = 13.74 mm
$y_g$ = 28.04 mm
N = 235. N
$T_y$ = 470. N
$M_x$ = 357200. Nmm
$x_m$ = 12. mm
$v_m$ = -28.04 mm
$\sigma_m = N/A - Mv/J_u$ = 239.8 $N/mm^2$
$y_c$ = 3. mm
$u_c$ = -12. mm
$v_c$ = -25.04 mm
$\sigma_c = N/A - Mv/J_u$ = 239.8 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 6.756 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 6.756 $N/mm^2$
$t_c$ = 470. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 240.1 $N/mm^2$

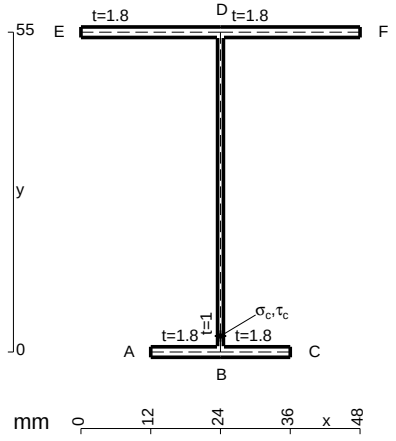



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_C = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

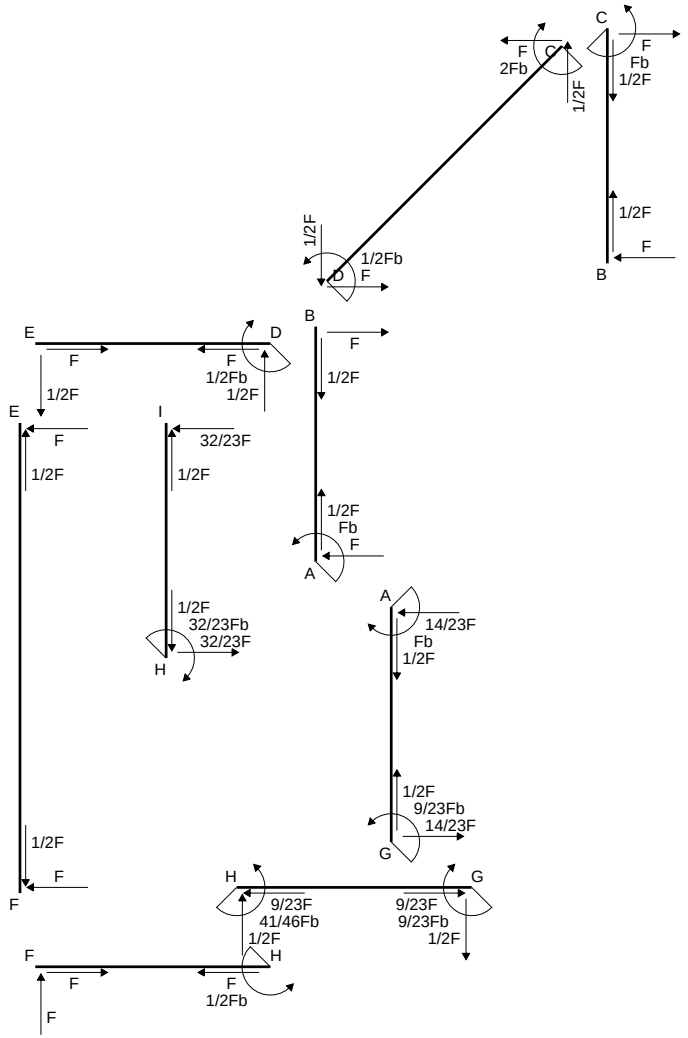
$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

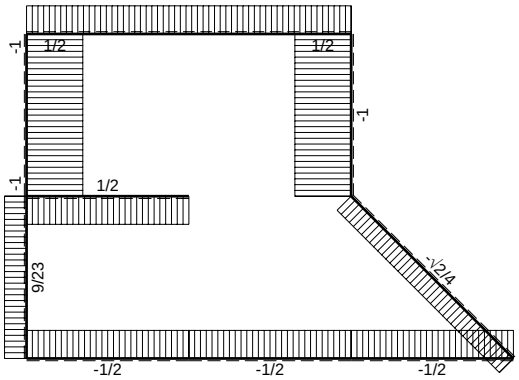



Reazioni iperstatiche in soluzione: $X=W_{GA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 510 mm, F = 630 N
Calcolare sulla sezione C la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Elongazione termica specifica ε assegnata su asta HG.

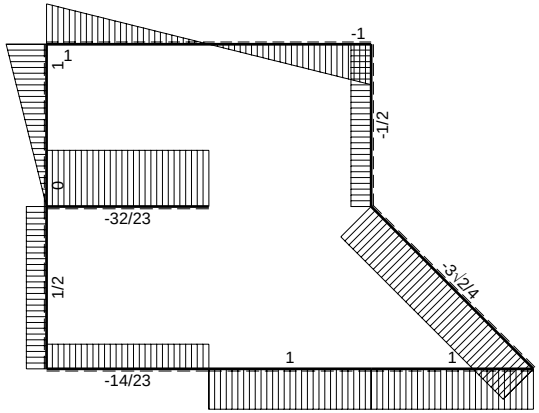




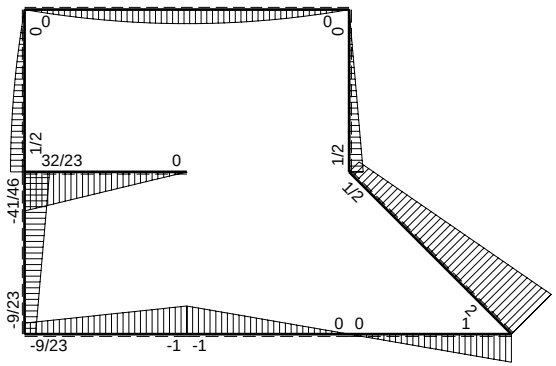






←□→ F



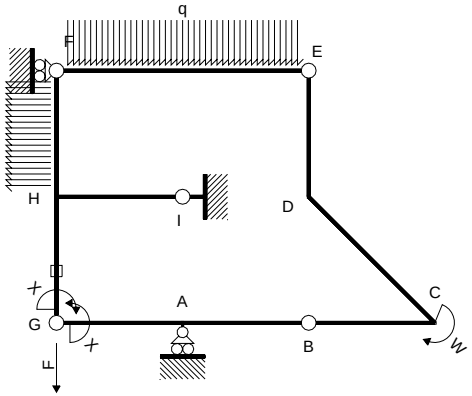

↑□↓ F

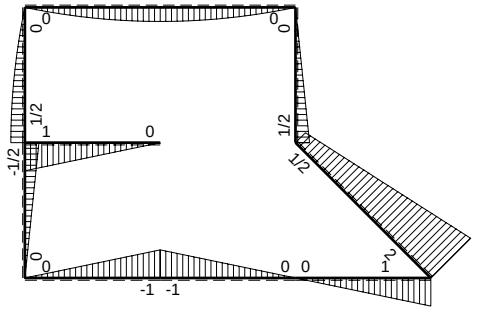



↻□↺ Fb

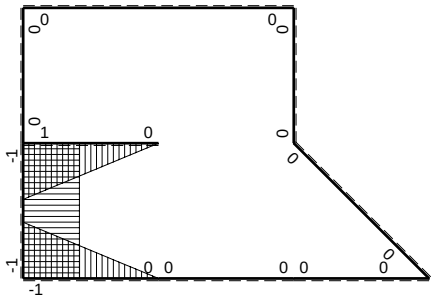







↻□↺ $M_o$ flessione da carichi assegnati




↻□↺ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb+Fx | 0 | 0 | | |
| CD √2b | 0 | 2Fb-3√2/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 1/2Fb-1/2Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -1/2Fx | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | -1+x/b | -Fx | $Fx-Fx^2/b$ | $1-2x/b+x^2/b^2$ | $1/6Fb^2/EJ$ | $1/3Xb/EJ$ |
| AG b | x/b | Fb-Fx | $Fx-Fx^2/b$ | $x^2/b^2$ | | |
| FH b | 0 | $Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $-1/2Fb+1/2qx^2$ | 0 | 0 | | |
| HI b | 1-x/b | Fb-Fx | $Fb-2Fx+Fx^2/b$ | $1-2x/b+x^2/b^2$ | $1/3Fb^2/EJ$ | $1/3Xb/EJ$ |
| IH b | -x/b | -Fx | $Fx^2/b$ | $x^2/b^2$ | | |
| HG b | -1 | -1/2Fb+1/2Fx | 1/2Fb-1/2Fx | 1 | $1/4Fb^2/EJ$ | Xb/EJ |
| GH b | 1 | 1/2Fx | 1/2Fx | 1 | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^2/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-1/2Fb^2/EJ$ | $1/4Xb/EJ$ |
| | totali | | | | $-3/4Fb^2/EJ$ | $23/12Xb/EJ$ |
| | iperstatica $X=W_{GA}$ | | | | 9/23Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 218425 ESΣ05.xxxx.051

$L_{GA}^{XX} = \int_o^b (1 - 2x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{AG}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HI}^{XX} = \int_o^b (1 - 2x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IH}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HG}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GH}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GA}^{Xo} = \int_o^b (\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{AG}^{Xo} = \int_o^b (\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{HI}^{Xo} = \int_o^b (1 - 2x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L_{IH}^{Xo} = \int_o^b (\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L_{HG}^{Xo} = \int_o^b (1/2 - 1/2\ x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= [1/2\ x - 1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/2\ b - 1/4\ b\ )\ Fb\ 1/EJ\ - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -3/4\ \ Fb^2/EJ$

$L_{GH}^{Xo} = \int_o^b (1/2\ x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = [1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/4\ b\ )\ Fb\ 1/EJ\ - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -3/4\ \ Fb^2/EJ$

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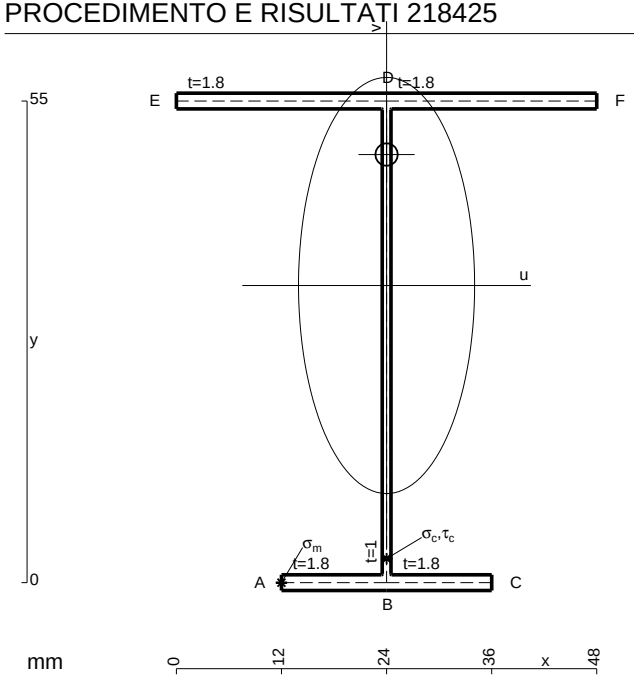


$A = 184.6\ mm^2$
$J_u = 104229.\ mm^4$
$J_v = 18662.\ mm^4$
$J_t = 158.3\ mm^4$
$y_o = 14.95\ mm$
$y_g = 33.94\ mm$
$N = -222.7\ N$
$T_y = -668.2\ N$
$M_x = 642600.\ Nmm$
$x_m = 12.\ mm$
$u_m = -12.\ mm$
$v_m = -33.94\ mm$
$\sigma_m = N/A - Mv/J_u = 208.\ N/mm^2$
$x_c = 24.\ mm$
$v_c = -33.94\ mm$
$\sigma_c = N/A - Mv/J_u = 208.\ N/mm^2$
$\tau_c = TS^*/tJ_u = 9.399\ N/mm^2$
$\tau_g = TS^*/tJ_u = 9.399\ N/mm^2$
$t_c = 630.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 208.7\ N/mm^2$



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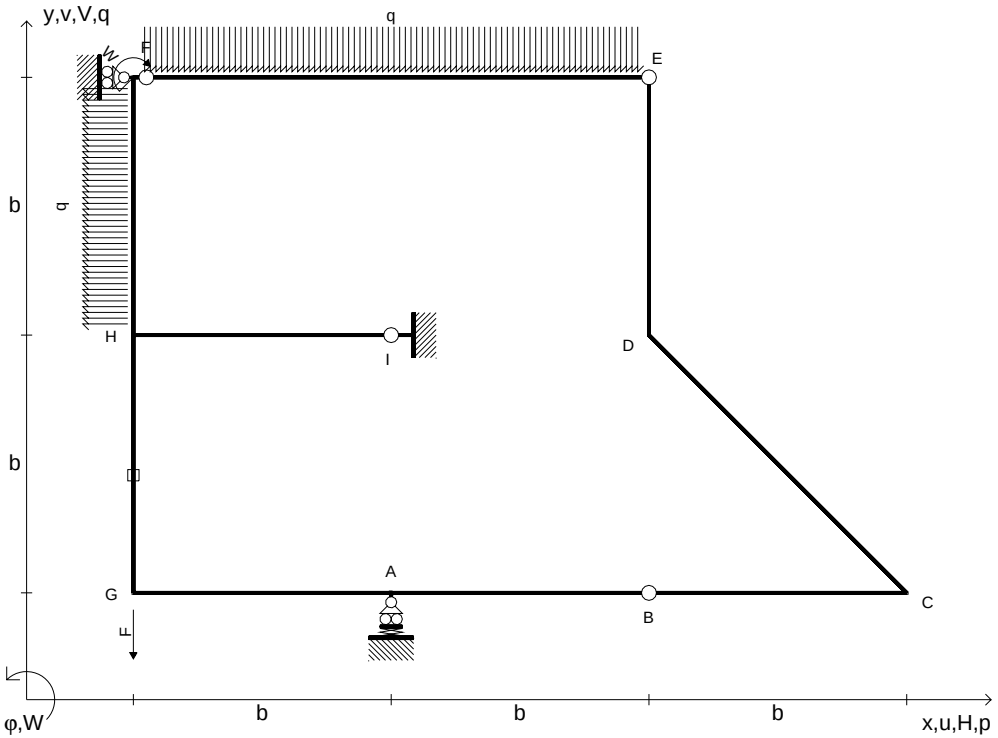
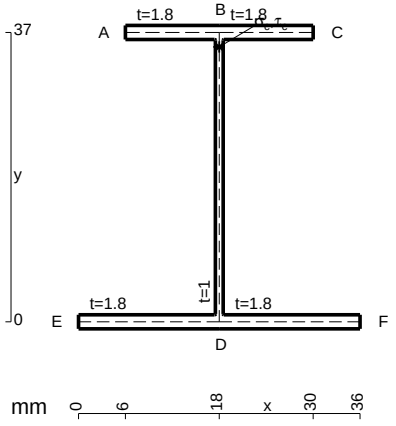



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_F = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$




Reazioni iperstatiche in soluzione: $X=V_A$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave AB ha la sezione riportata e dimensioni in mm, con:

b = 310 mm, F = 1210 N

Calcolare sulla sezione A la massima tensione normale $\sigma_m$.

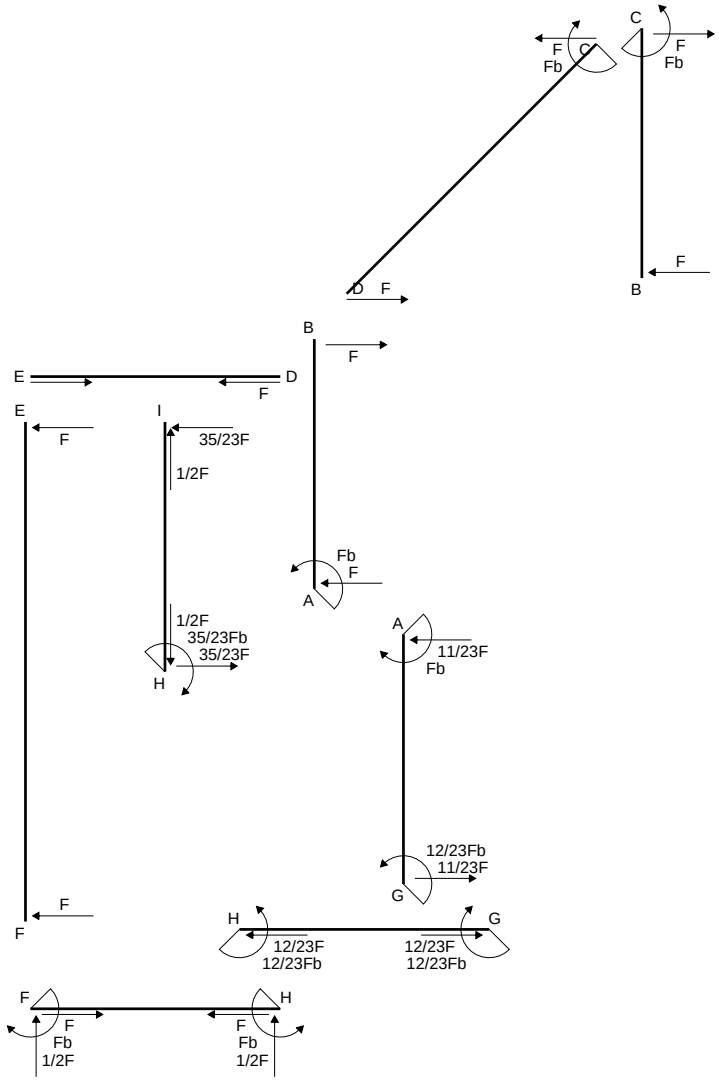
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.

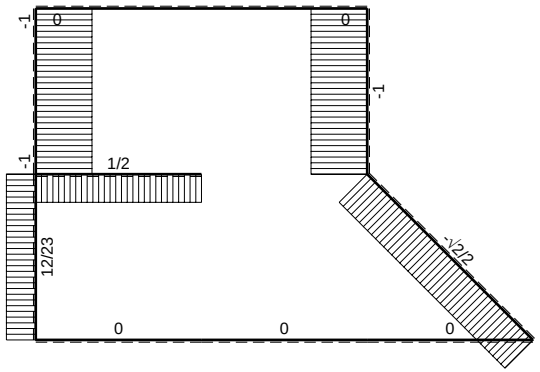
Lembo inferiore sezione su tratteggio trave, a destra da A a B

Elongazione termica specifica ε assegnata su asta HG.

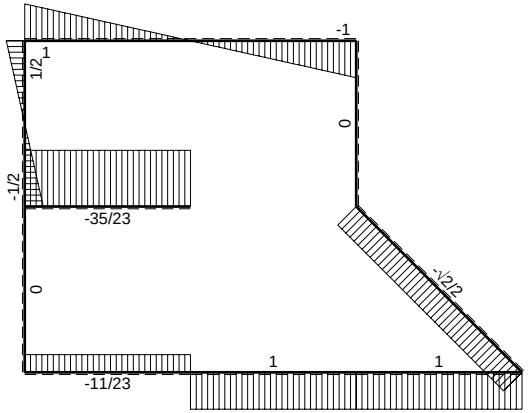




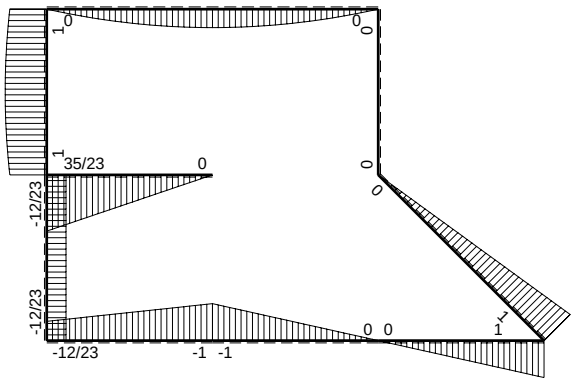





F



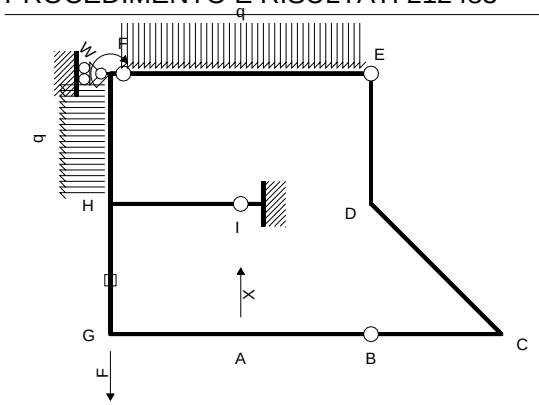

F



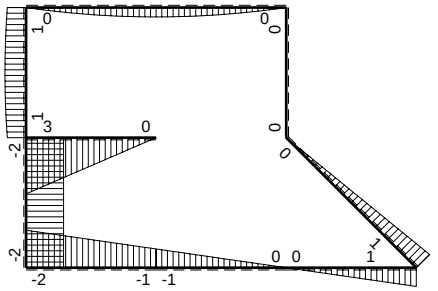

Fb



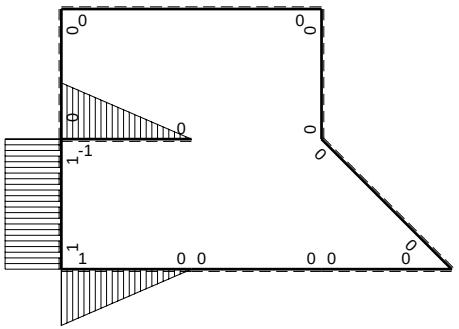





Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb+Fx | 0 | 0 | | |
| CD √2b | 0 | Fb-√2/2Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | b-x | -2Fb+Fx | $-2Fb^2+3Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-5/6Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | Fb+Fx | $-Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | $Fb+1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $-Fb-1/2Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | -b+x | 3Fb-3Fx | $-3Fb^2+6Fbx-3Fx^2$ | $b^2-2bx+x^2$ | $-Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | -3Fx | $-3Fx^2$ | $x^2$ | | |
| HG b | b | -2Fb | $-2Fb^2$ | $b^2$ | $-2Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | 2Fb | $-2Fb^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-17/6Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 34/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-2 +3\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2\, x +3/2\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(-2\, b +3/2\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

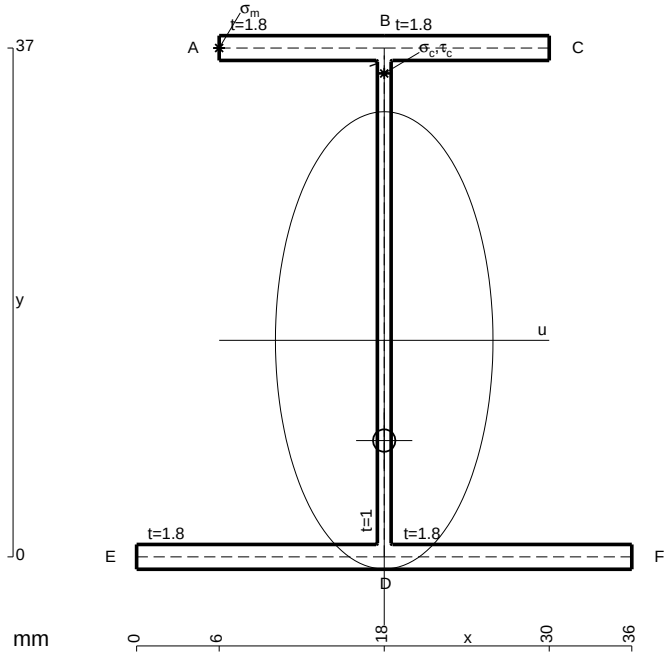
$$L_{AG}^{Xo} = \int_o^b \left(-\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/2\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(-1/2\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-3 +6\, x/b -3\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-3\, x +3\, x^2/b - x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(-3\, b +3\, b - b \,\right) Fb^2\, 1/EJ = - Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(-3\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(-\, b \,\right) Fb^2\, 1/EJ = - Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = \left[-2\, x \,\right]_o^b Fb^2\, 1/EJ\ +1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left(-2\, b \,\right) Fb^2\, 1/EJ\ +1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = - Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = \left[-2\, x \,\right]_o^b Fb^2\, 1/EJ\ +1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left(-2\, b \,\right) Fb^2\, 1/EJ\ +1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = - Fb^3/EJ$$




$A = 145.\ mm^2$
$J_u = 40083.\ mm^4$
$J_v = 9072.\ mm^4$
$J_t = 129.\ mm^4$
$y_o = -7.287$ mm
$y_g = 15.74$ mm
$T_y = 1210.$ N
$M_x = -375100.$ Nmm
$x_m = 6.$ mm
$y_m = 37.$ mm
$u_m = -12.$ mm
$v_m = 21.26$ mm
$\sigma_m = -Mv/J_u = 198.9\ N/mm^2$
$x_c = 18.$ mm
$y_c = 37.$ mm
$v_c = 21.26$ mm
$\sigma_c = -Mv/J_u = 198.9\ N/mm^2$
$\tau_c = TS^*/tJ_u = 27.72\ N/mm^2$
$\tau_g = TS^*/tJ_u = 27.72\ N/mm^2$
$t_c = 1210.$ mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 204.6\ N/mm^2$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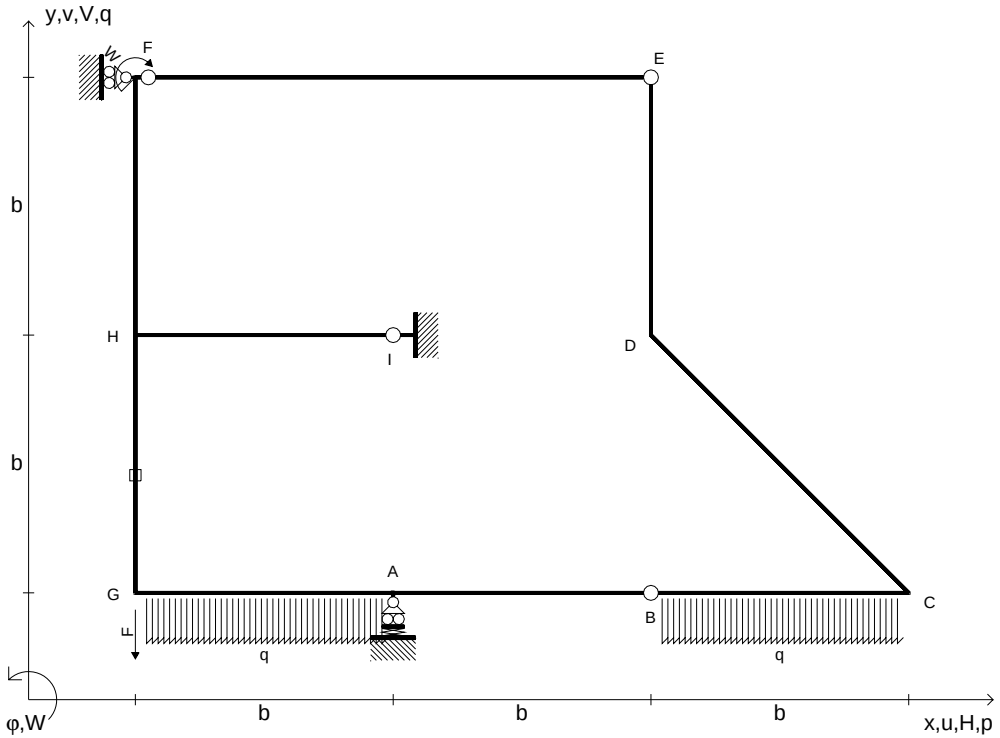
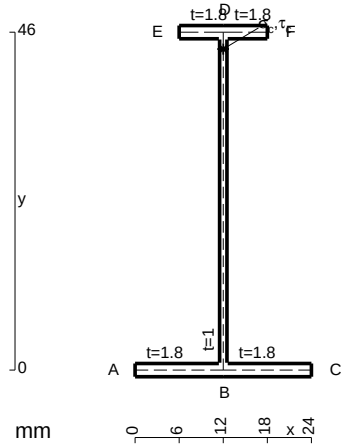



$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_F = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{BC} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

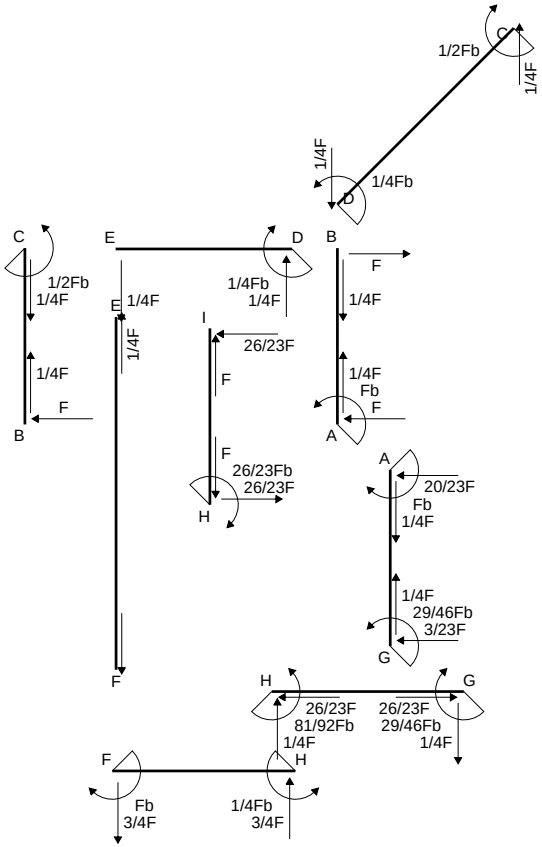
$q_{GA} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

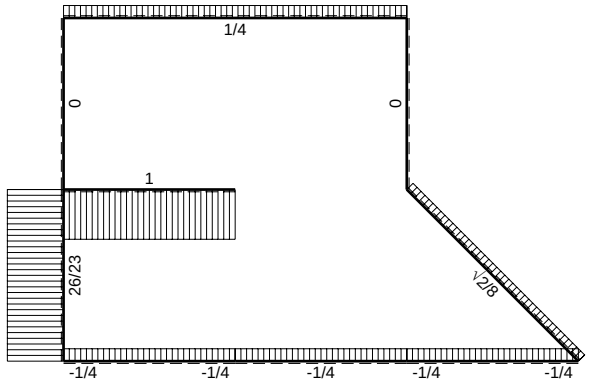



Reazioni iperstatiche in soluzione: $X=W_{GA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 620 mm, F = 490 N
Calcolare sulla sezione A la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.

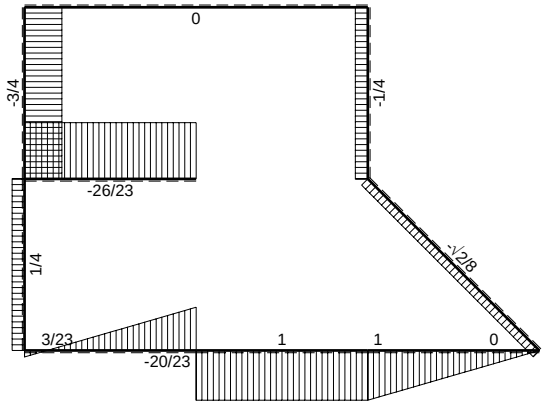




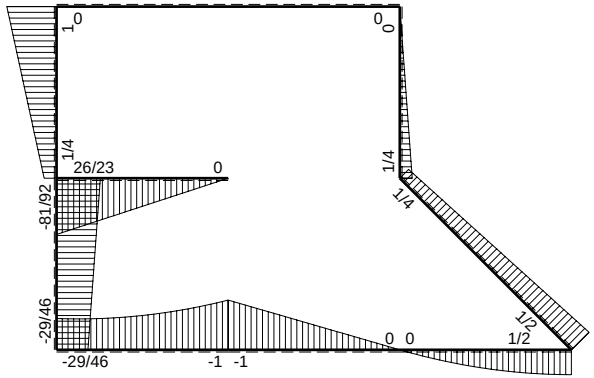





← F →



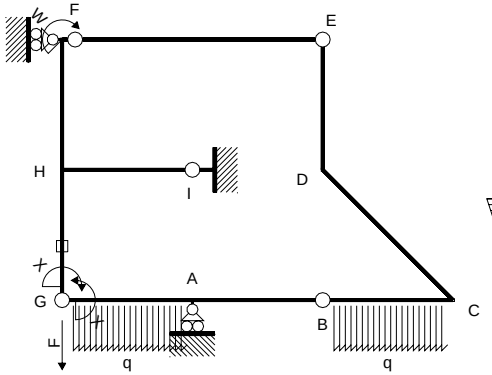

↑ F ↓



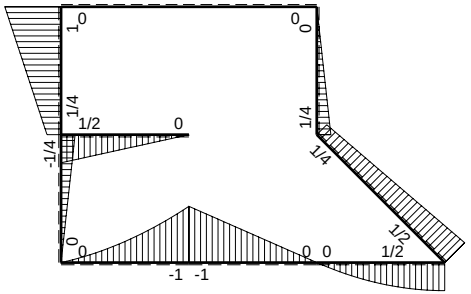

Fb



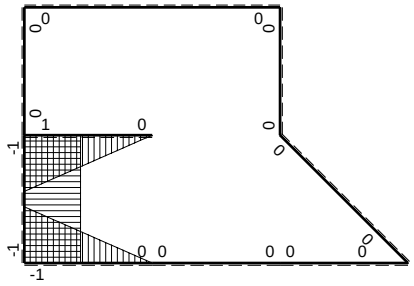





Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | $Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| CB b | 0 | $-1/2Fb+1/2qx^2$ | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $1/2Fb-\sqrt{2}/8Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | 1/4Fb-1/4Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -1/4Fx | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | -1+x/b | $-1/2Fx-1/2qx^2$ | $1/2Fx-1/2qx^3/b$ | $1-2x/b+x^2/b^2$ | $1/8Fb^2/EJ$ | 1/3Xb/EJ |
| AG b | x/b | $Fb-3/2Fx+1/2qx^2$ | $Fx-3/2Fx^2/b+1/2qx^3/b$ | $x^2/b^2$ | | |
| FH b | 0 | Fb-3/4Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | -1/4Fb-3/4Fx | 0 | 0 | | |
| HI b | 1-x/b | 1/2Fb-1/2Fx | $1/2Fb-Fx+1/2Fx^2/b$ | $1-2x/b+x^2/b^2$ | $1/6Fb^2/EJ$ | 1/3Xb/EJ |
| IH b | -x/b | -1/2Fx | $1/2Fx^2/b$ | $x^2/b^2$ | | |
| HG b | -1 | -1/4Fb+1/4Fx | 1/4Fb-1/4Fx | 1 | $1/8Fb^2/EJ$ | Xb/EJ |
| GH b | 1 | 1/4Fx | 1/4Fx | 1 | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^2/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-5/8Fb^2/EJ$ | 1/4Xb/EJ |
| | totali | | | | $-29/24Fb^2/EJ$ | 23/12Xb/EJ |
| | iperstatica $X=W_{GA}$ | | | | 29/46Fb | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 984883 ESΣ05.xxxx.076

$L_{GA}^{XX} = \int_o^b (1 - 2x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{AG}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HI}^{XX} = \int_o^b (1 - 2x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IH}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HG}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GH}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GA}^{Xo} = \int_o^b (1/2\ x/b - 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b - 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b - 1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L_{AG}^{Xo} = \int_o^b (\ x/b - 3/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/2\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/2\ b + 1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L_{HI}^{Xo} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{IH}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L_{HG}^{Xo} = \int_o^b (1/4 - 1/4\ x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

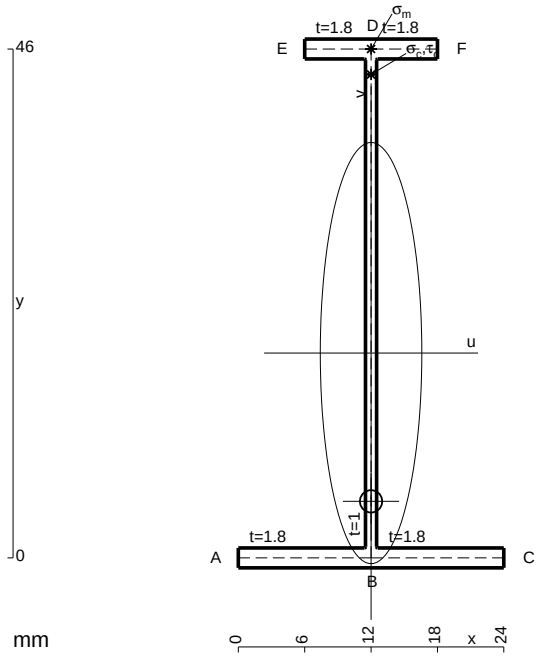
$= [1/4\ x - 1/8\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/4\ b - 1/8\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -7/8\ \ Fb^2/EJ$

$L_{GH}^{Xo} = \int_o^b (1/4\ x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = [1/8\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/8\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -7/8\ \ Fb^2/EJ$

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25




A = 110.8 $mm^2$
$J_u$ = 40163. $mm^4$
$J_v$ = 2333. $mm^4$
$J_t$ = 85.32 $mm^4$
$y_o$ = -13.41 mm
$y_g$ = 18.52 mm
N = -122.5 N
$T_y$ = 490. N
$M_x$ = -303800. Nmm
$x_m$ = 12. mm
$y_m$ = 46. mm
$v_m$ = 27.48 mm
$\sigma_m = N/A - Mv/J_u$ = 206.8 $N/mm^2$
$y_c$ = 2. mm
$u_c$ = -12. mm
$v_c$ = -16.52 mm
$\sigma_c = N/A - Mv/J_u$ = 206.8 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 7.243 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 7.243 $N/mm^2$
$t_c$ = 490. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 207.2 $N/mm^2$



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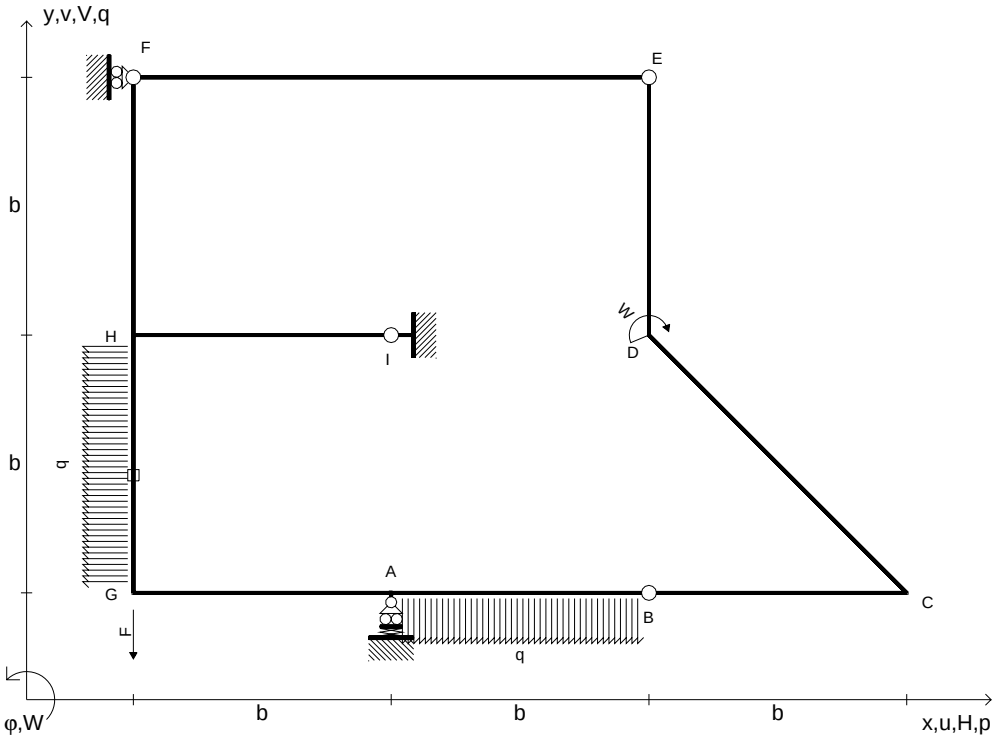
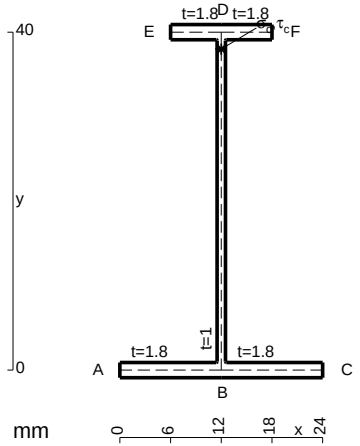



$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_D = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

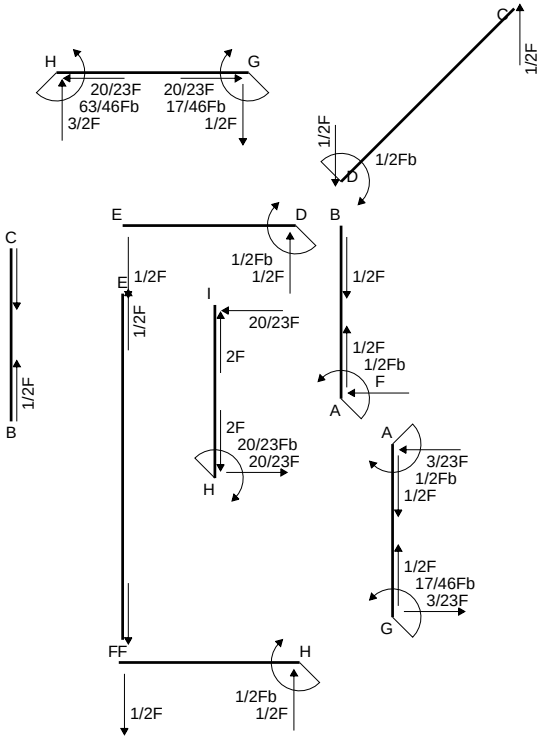
$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

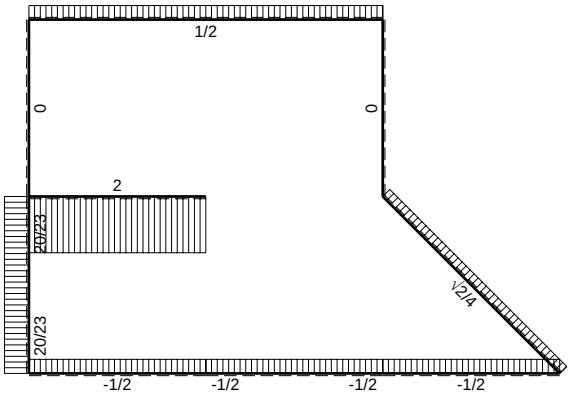



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 640 mm, F = 930 N
Calcolare sulla sezione A la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.

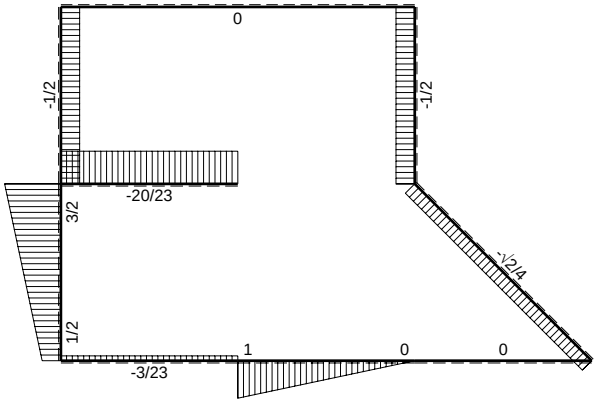




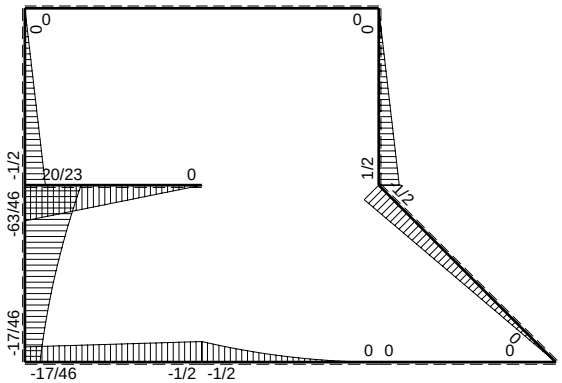





←⊞→ F



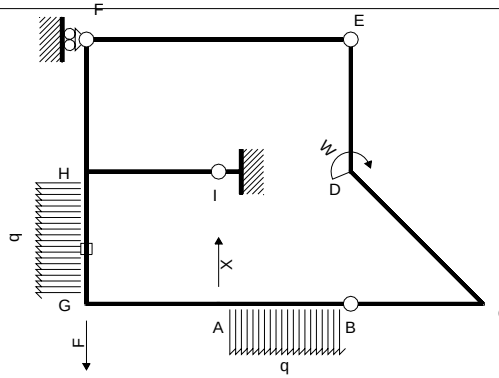

↑⊞↓ F



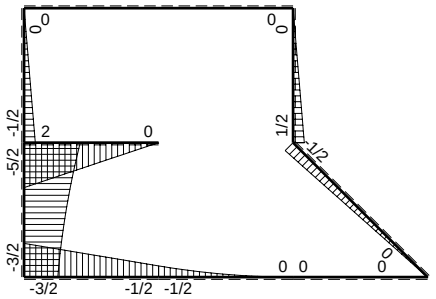

↻⊞↺ Fb



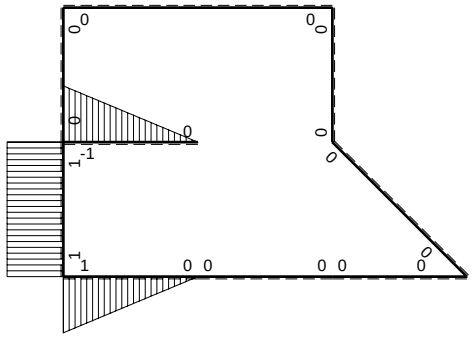





Schema di calcolo iperstatico




↻⊞↺ $M_o$ flessione da carichi assegnati




↻⊞↺ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $1/2qx^2$ | 0 | 0 | | |
| BC b | 0 | 0 | 0 | 0 | 0 | 0 |
| CB b | 0 | 0 | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $-\sqrt{2}/4Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | $1/2Fb-1/2Fx$ | 0 | 0 | 0 | 0 |
| ED b | 0 | $-1/2Fx$ | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | b-x | $-3/2Fb+Fx$ | $-3/2Fb^2+5/2Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-7/12Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | $1/2Fb+Fx$ | $-1/2Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | $-1/2Fx$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $1/2Fb-1/2Fx$ | 0 | 0 | | |
| HI b | -b+x | $2Fb-2Fx$ | $-2Fb^2+4Fbx-2Fx^2$ | $b^2-2bx+x^2$ | $-2/3Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | $-2Fx$ | $-2Fx^2$ | $x^2$ | | |
| HG b | b | $-5/2Fb+3/2Fx-1/2qx^2$ | $-5/2Fb^2+3/2Fbx-1/2Fx^2$ | $b^2$ | $-23/12Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $3/2Fb+1/2Fx+1/2qx^2$ | $-3/2Fb^2-1/2Fbx-1/2Fx^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-13/6Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 26/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-3/2 + 5/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-3/2\, x + 5/4\, x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-3/2\, b + 5/4\, b - 1/3\, b \,\right) Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(-1/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/4\, x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/4\, b - 1/3\, b \,\right) Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-2 + 4\, x/b - 2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2\, x + 2\, x^2/b - 2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2\, b + 2\, b - 2/3\, b \,\right) Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(-2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2/3\, b \,\right) Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-5/2 + 3/2\, x/b - 1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

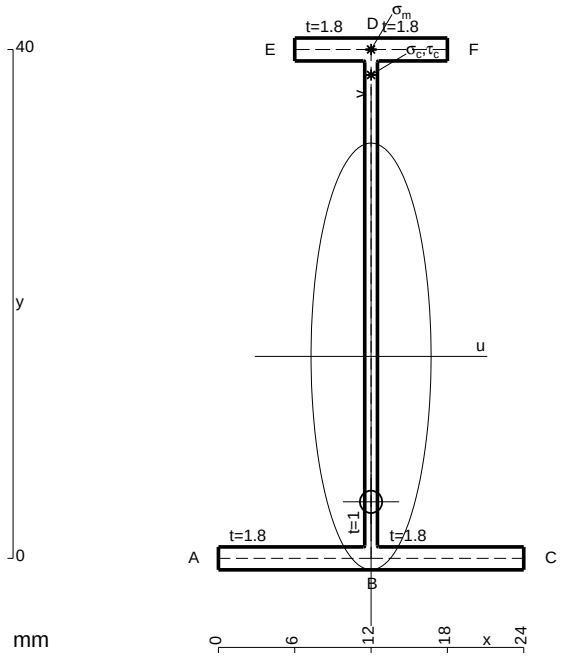
$$= \left[-5/2\, x + 3/4\, x^2/b - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-5/2\, b + 3/4\, b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -11/12\ \ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-3/2 - 1/2\, x/b - 1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left[-3/2\, x - 1/4\, x^2/b - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-3/2\, b - 1/4\, b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -11/12\ \ Fb^3/EJ$$




$A = 104.8\ mm^2$
$J_u = 29473.\ mm^4$
$J_v = 2333.\ mm^4$
$J_t = 83.32\ mm^4$
$y_o = -11.43\ mm$
$y_g = 15.88\ mm$
$N = -465.\ N$
$T_y = 930.\ N$
$M_x = -297600.\ Nmm$
$x_m = 12.\ mm$
$y_m = 40.\ mm$
$v_m = 24.12\ mm$
$\sigma_m = N/A - Mv/J_u = 239.1\ N/mm^2$
$y_c = 2.\ mm$
$u_c = -12.\ mm$
$v_c = -13.88\ mm$
$\sigma_c = N/A - Mv/J_u = 239.1\ N/mm^2$
$\tau_c = TS^*/tJ_u = 16.44\ N/mm^2$
$\tau_g = TS^*/tJ_u = 16.44\ N/mm^2$
$t_c = 930.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 240.8\ N/mm^2$



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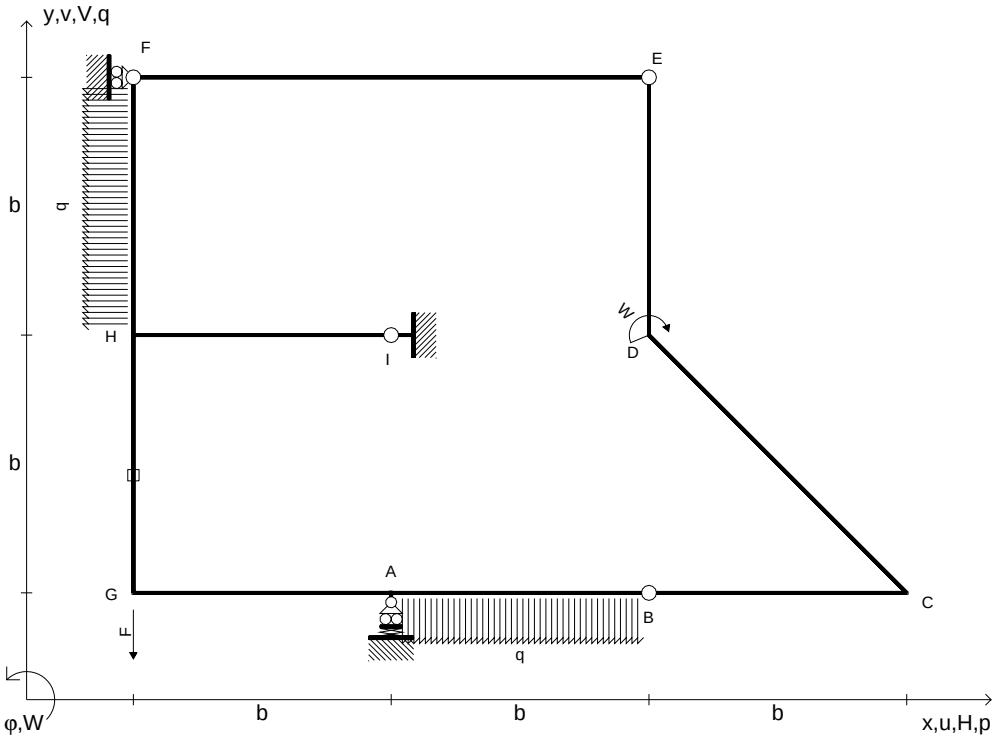
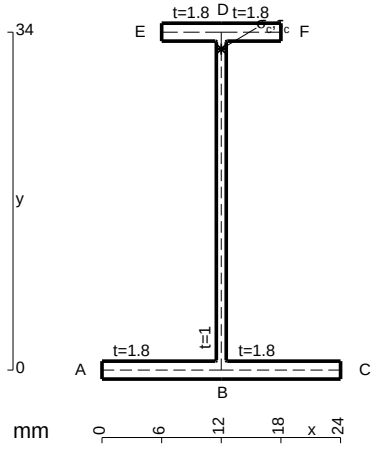



$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_D = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

$p_{FH} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$




Reazioni iperstatiche in soluzione: $X=V_I$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave AB ha la sezione riportata e dimensioni in mm, con:

b = 340 mm, F = 1320 N

Calcolare sulla sezione A la massima tensione normale $\sigma_m$.

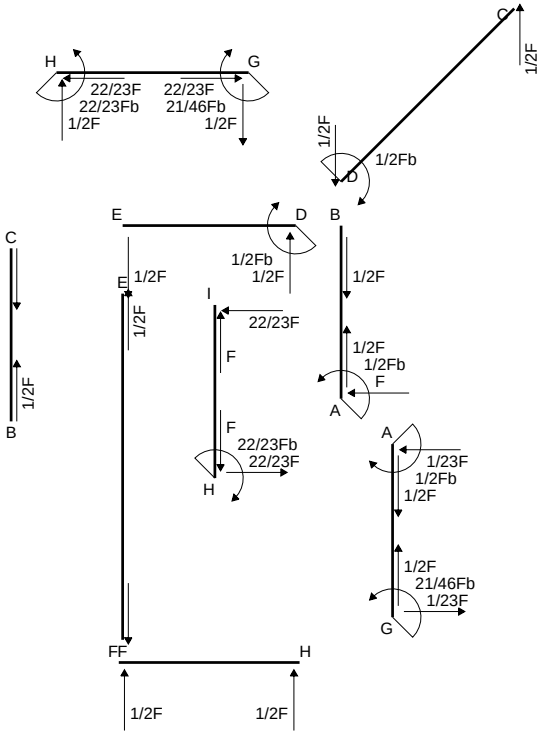
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.

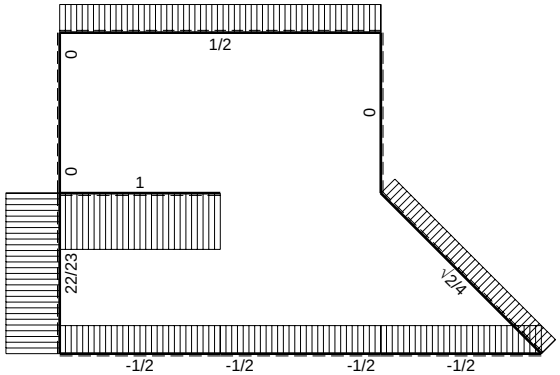
Lembo inferiore sezione su tratteggio trave, a destra da A a B

Elongazione termica specifica ε assegnata su asta HG.

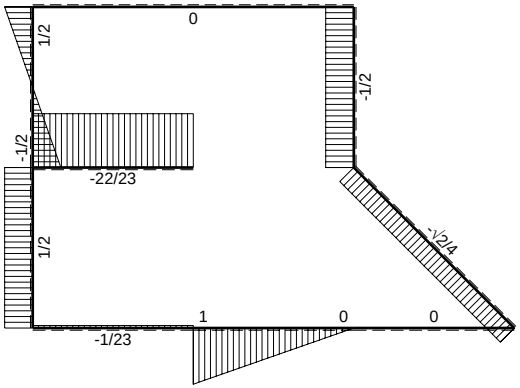




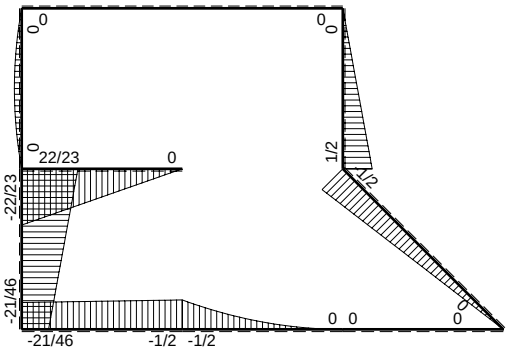





←□→ F



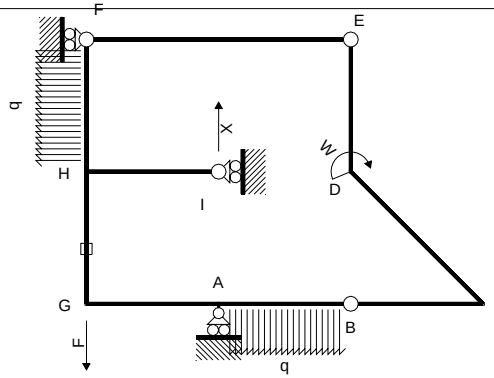

↑□↓ F



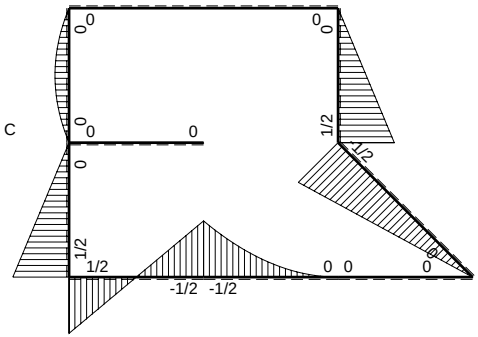

↺□↻ Fb



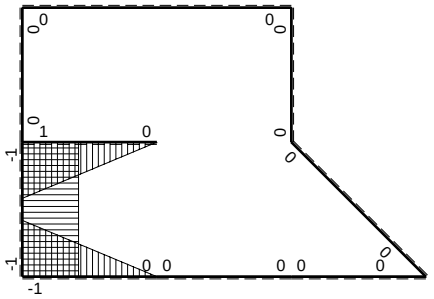





Schema di calcolo iperstatico




↺□↻ $M_o$ flessione da carichi assegnati




↺□↻ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $1/2qx^2$ | 0 | 0 | | |
| BC b | 0 | 0 | 0 | 0 | 0 | 0 |
| CB b | 0 | 0 | 0 | 0 | | |
| CD √2b | 0 | $-\sqrt{2}/4Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | $1/2Fb-1/2Fx$ | 0 | 0 | 0 | 0 |
| ED b | 0 | $-1/2Fx$ | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | $-b+x$ | $1/2Fb-Fx$ | $-1/2Fb^2+3/2Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-1/12Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | x | $1/2Fb-Fx$ | $1/2Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | b-x | 0 | 0 | $b^2-2bx+x^2$ | 0 | $1/3Xb^3/EJ$ |
| IH b | -x | 0 | 0 | $x^2$ | | |
| HG b | -b | $1/2Fx$ | $-1/2Fbx$ | $b^2$ | $-1/4Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | b | $-1/2Fb+1/2Fx$ | $-1/2Fb^2+1/2Fbx$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-1/2Fb^3/EJ$ | $1/4Xb^3/EJ$ |
| | totali | | | | $-11/6Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_I$ | | | | 22/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-1/2 + 3/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/2\, x + 3/4\, x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/2\, b + 3/4\, b - 1/3\, b \,\right) Fb^2\, 1/EJ = -1/12\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(1/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[1/4\, x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(1/4\, b - 1/3\, b \,\right) Fb^2\, 1/EJ = -1/12\ Fb^3/EJ$$

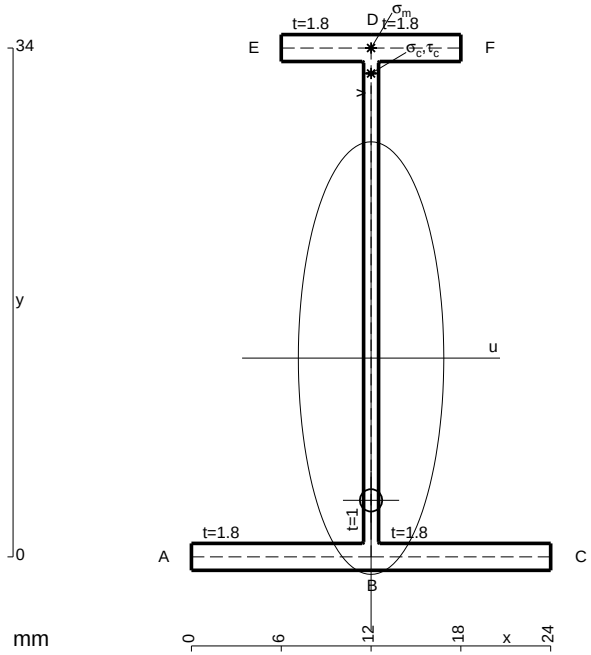
$$L_{HG}^{Xo} = \int_o^b \left(-1/2\, x/b \,\right) Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ = \left[-1/4\, x^2/b \,\right]_o^b Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(-1/4\, b \,\right) Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -5/4\ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-1/2 + 1/2\, x/b \,\right) Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[-1/2\, x + 1/4\, x^2/b \,\right]_o^b Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(-1/2\, b + 1/4\, b \,\right) Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -5/4\ Fb^3/EJ$$




$A = 98.8\ mm^2$
$J_u = 20638.\ mm^4$
$J_v = 2333.\ mm^4$
$J_t = 81.32\ mm^4$
$y_o = -9.506\ mm$
$y_g = 13.28\ mm$
$N = -660.\ N$
$T_y = 1320.\ N$
$M_x = -224400.\ Nmm$
$x_m = 12.\ mm$
$y_m = 34.\ mm$
$v_m = 20.72\ mm$
$\sigma_m = N/A - Mv/J_u = 218.6\ N/mm^2$
$y_c = 2.\ mm$
$u_c = -12.\ mm$
$v_c = -11.28\ mm$
$\sigma_c = N/A - Mv/J_u = 218.6\ N/mm^2$
$\tau_c = TS^*/tJ_u = 28.62\ N/mm^2$
$\tau_g = TS^*/tJ_u = 28.62\ N/mm^2$
$t_c = 1320.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 224.1\ N/mm^2$



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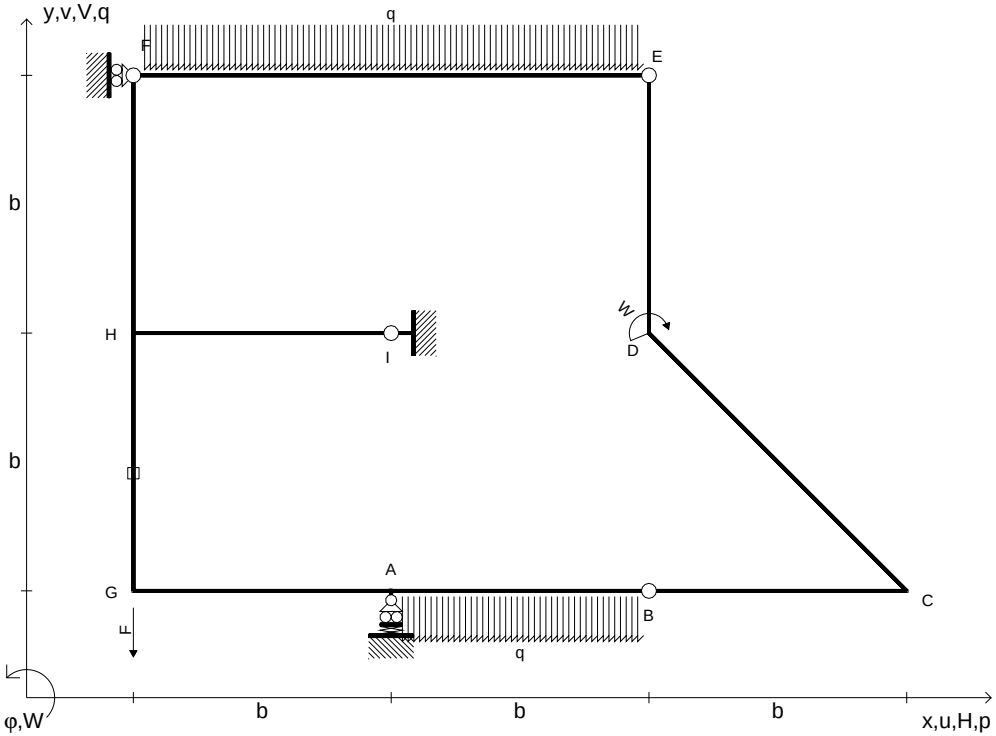
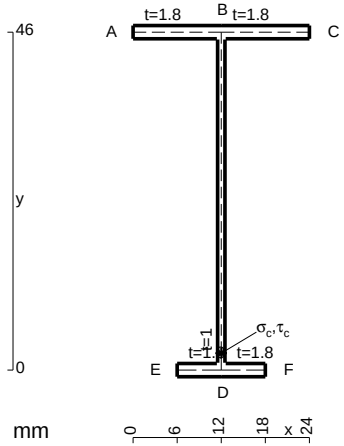



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_D = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

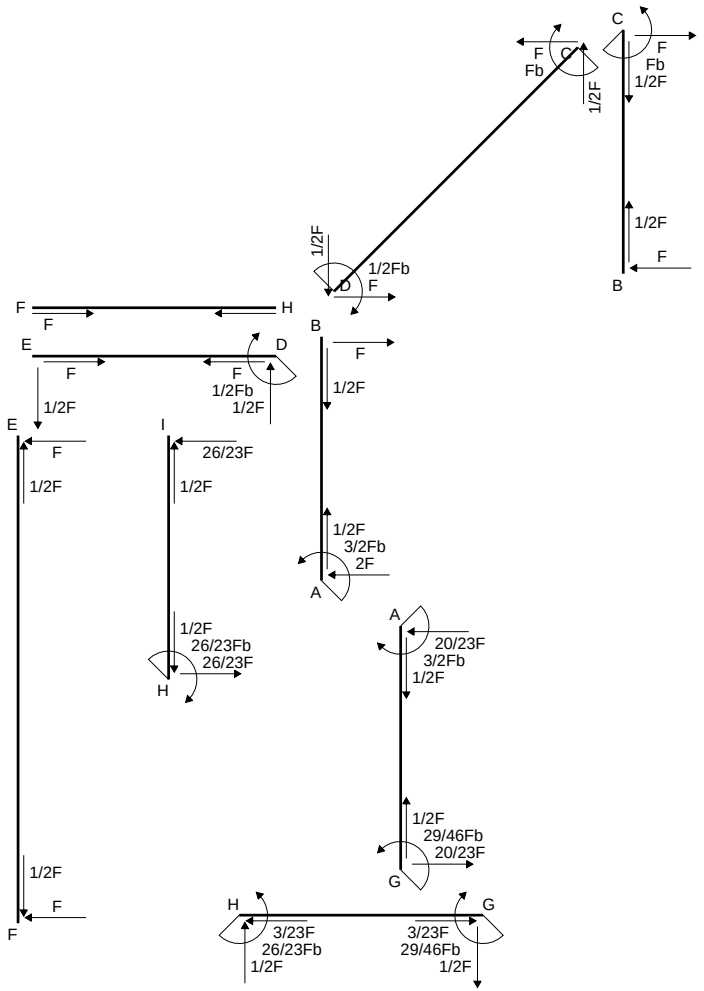
$q_{AB} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

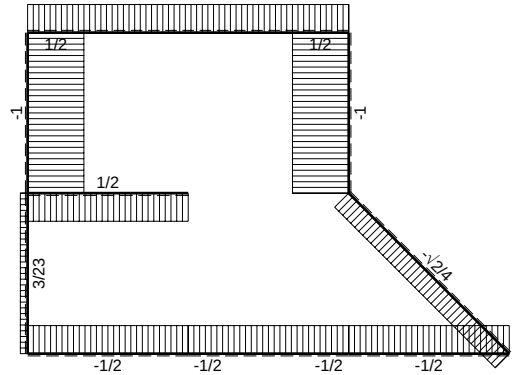



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 660 mm, F = 320 N
Calcolare sulla sezione A la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.

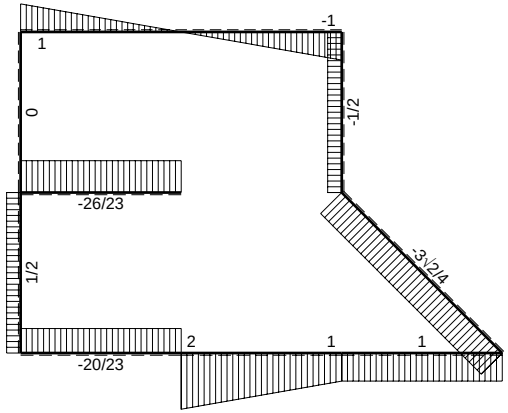




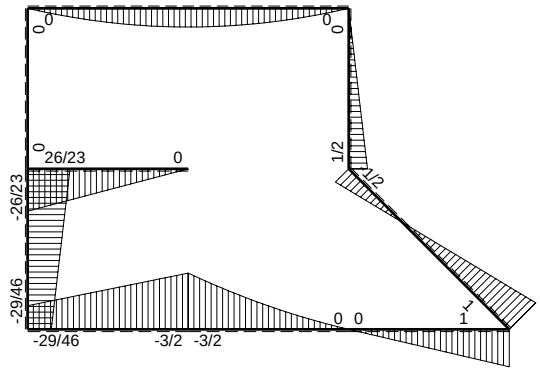






←□→ F



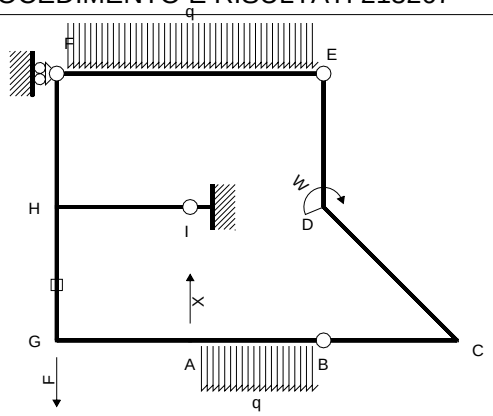

↑□↓ F



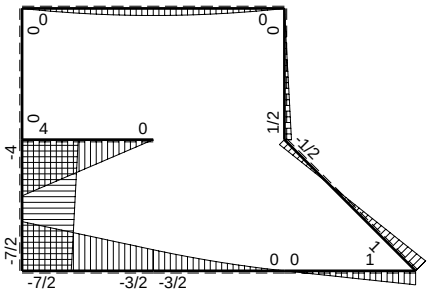

□ Fb



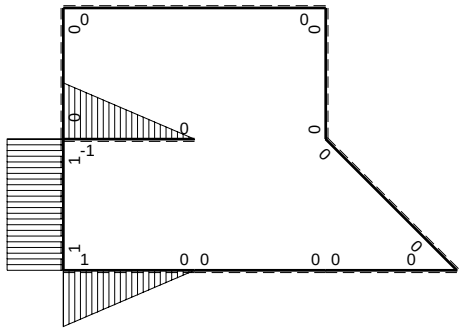





Schema di calcolo iperstatico




□ $M_o$ flessione da carichi assegnati




□ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-3/2Fb+2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $Fx+1/2qx^2$ | 0 | 0 | | |
| BC b | 0 | Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb+Fx | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $Fb-3\sqrt{2}/4Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | 1/2Fb-1/2Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -1/2Fx | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | b-x | -7/2Fb+2Fx | $-7/2Fb^2+11/2Fbx-2Fx^2$ | $b^2-2bx+x^2$ | $-17/12Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | 3/2Fb+2Fx | $-3/2Fbx-2Fx^2$ | $x^2$ | | |
| FH b | 0 | 0 | 0 | 0 | 0 | 0 |
| HF b | 0 | 0 | 0 | 0 | | |
| HI b | -b+x | 4Fb-4Fx | $-4Fb^2+8Fbx-4Fx^2$ | $b^2-2bx+x^2$ | $-4/3Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | -4Fx | $-4Fx^2$ | $x^2$ | | |
| HG b | b | -4Fb+1/2Fx | $-4Fb^2+1/2Fbx$ | $b^2$ | $-15/4Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | 7/2Fb+1/2Fx | $-7/2Fb^2-1/2Fbx$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-11/2Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 66/23F | |

Sviluppi di calcolo iperstatica

$L_{GA}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b\,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AG}^{XX} = \int_o^b (\, x^2/b^2\,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b\,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HI}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b\,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IH}^{XX} = \int_o^b (\, x^2/b^2\,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2\,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b\,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HG}^{XX} = \int_o^b (1\,)\, b^2\, 1/EJ\, dx = [\, x\,]_o^b\, b^2\, 1/EJ$

$= (\, b\,)\, b^2\, 1/EJ = b^3/EJ$

$L_{GH}^{XX} = \int_o^b (1\,)\, b^2\, 1/EJ\, dx = [\, x\,]_o^b\, b^2\, 1/EJ$

$= (\, b\,)\, b^2\, 1/EJ = b^3/EJ$

$L_{GA}^{Xo} = \int_o^b (-7/2 + 11/2\,x/b - 2\,x^2/b^2\,)\, Fb^2\, 1/EJ\, dx = [-7/2\,x + 11/4\,x^2/b - 2/3\,x^3/b^2\,]_o^b\, Fb^2\, 1/EJ$

$= (-7/2\,b + 11/4\,b - 2/3\,b\,)\, Fb^2\, 1/EJ = -17/12\ Fb^3/EJ$

$L_{AG}^{Xo} = \int_o^b (-3/2\,x/b - 2\,x^2/b^2\,)\, Fb^2\, 1/EJ\, dx = [-3/4\,x^2/b - 2/3\,x^3/b^2\,]_o^b\, Fb^2\, 1/EJ$

$= (-3/4\,b - 2/3\,b\,)\, Fb^2\, 1/EJ = -17/12\ Fb^3/EJ$

$L_{HI}^{Xo} = \int_o^b (-4 + 8\,x/b - 4\,x^2/b^2\,)\, Fb^2\, 1/EJ\, dx = [-4\,x + 4\,x^2/b - 4/3\,x^3/b^2\,]_o^b\, Fb^2\, 1/EJ$

$= (-4\,b + 4\,b - 4/3\,b\,)\, Fb^2\, 1/EJ = -4/3\ Fb^3/EJ$

$L_{IH}^{Xo} = \int_o^b (-4\,x^2/b^2\,)\, Fb^2\, 1/EJ\, dx = [-4/3\,x^3/b^2\,]_o^b\, Fb^2\, 1/EJ$

$= (-4/3\,b\,)\, Fb^2\, 1/EJ = -4/3\ Fb^3/EJ$

$L_{HG}^{Xo} = \int_o^b (-4 + 1/2\,x/b\,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [-4\,x + 1/4\,x^2/b\,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-4\,b + 1/4\,b\,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -11/4\ \ Fb^3/EJ$

$L_{GH}^{Xo} = \int_o^b (-7/2 - 1/2\,x/b\,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [-7/2\,x - 1/4\,x^2/b\,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-7/2\,b - 1/4\,b\,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -11/4\ \ Fb^3/EJ$



PROCEDIMENTO E RISULTATI 213267 ESΣ05.xxxx.087

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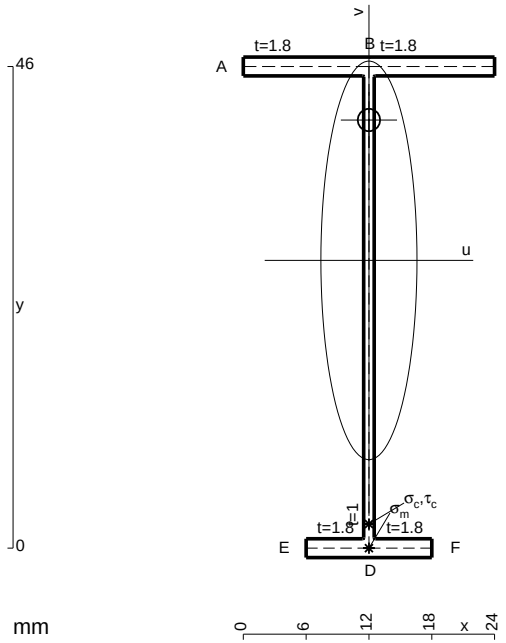


$A = 110.8\ mm^2$
$J_u = 40163.\ mm^4$
$J_v = 2333.\ mm^4$
$J_t = 85.32\ mm^4$
$y_o = 13.41\ mm$
$y_g = 27.48\ mm$
$N = -160.\ N$
$T_y = 640.\ N$
$M_x = -316800.\ Nmm$
$x_m = 12.\ mm$
$v_m = -27.48\ mm$
$\sigma_m = N/A - Mv/J_u = -218.2\ N/mm^2$
$y_c = 3.\ mm$
$u_c = -12.\ mm$
$v_c = -24.48\ mm$
$\sigma_c = N/A - Mv/J_u = -218.2\ N/mm^2$
$\tau_c = TS^*/tJ_u = 9.46\ N/mm^2$
$\tau_g = TS^*/tJ_u = 9.46\ N/mm^2$
$t_c = 320.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 218.8\ N/mm^2$



ESΣ05.xxxx.087

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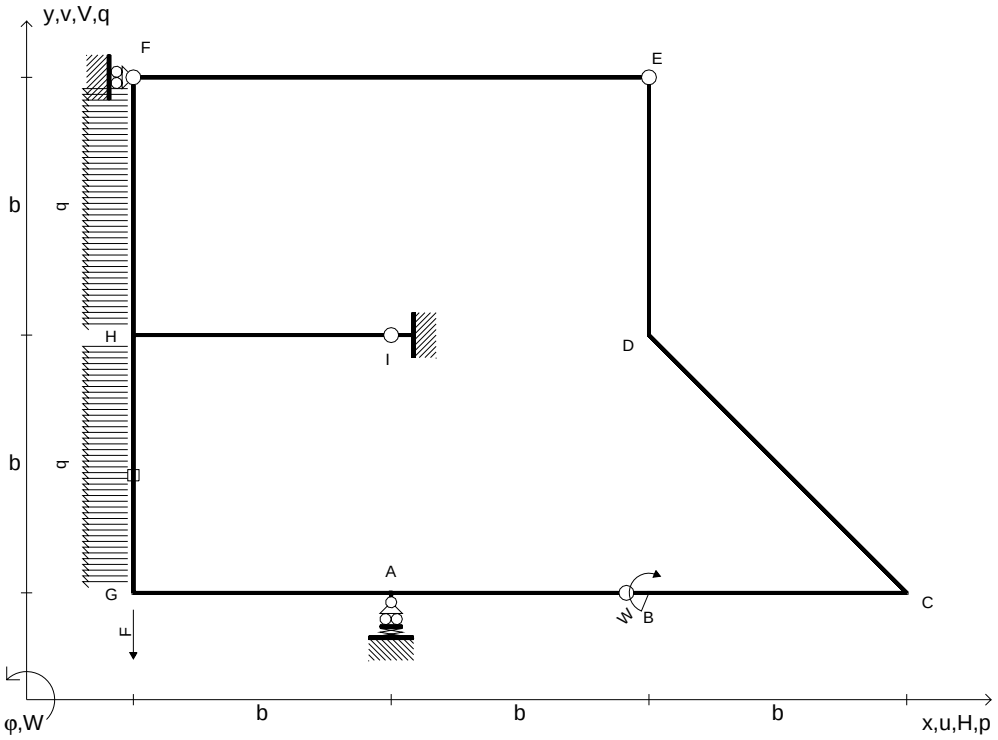
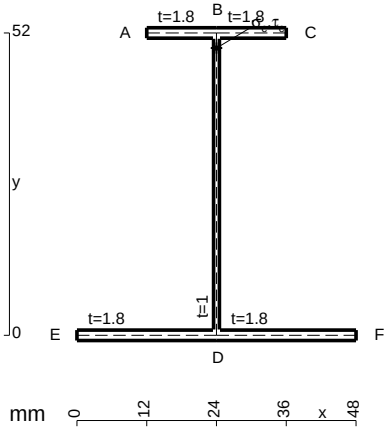



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_B = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$p_{HG} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

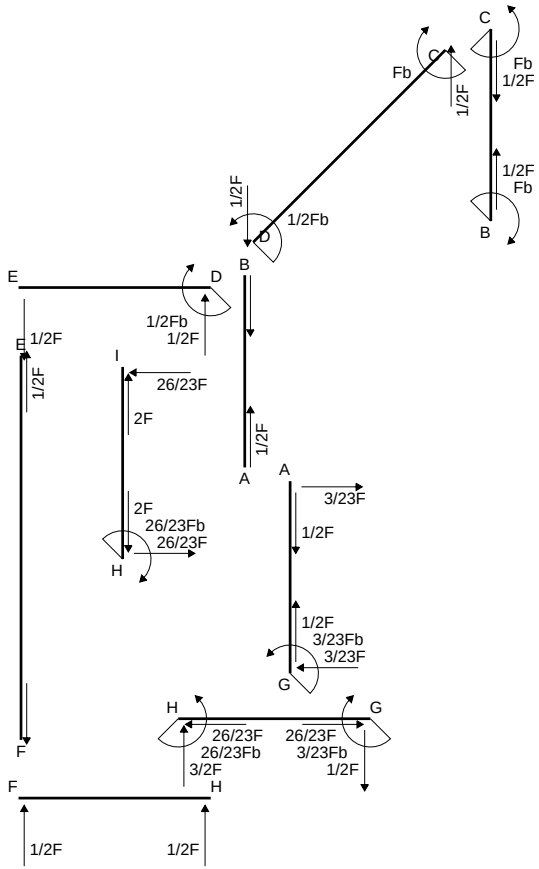
$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

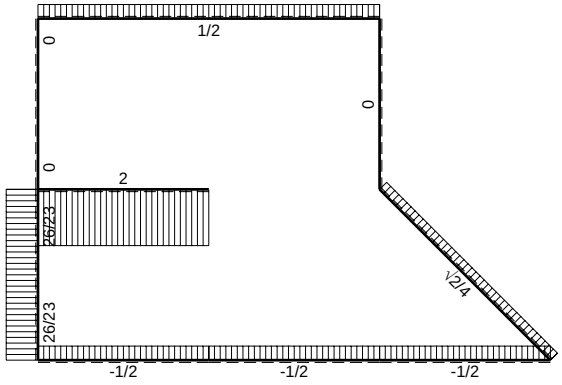



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 570 mm, F = 1160 N
Calcolare sulla sezione C la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Elongazione termica specifica ε assegnata su asta HG.

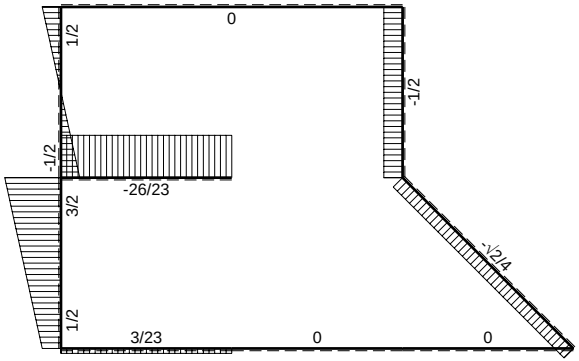




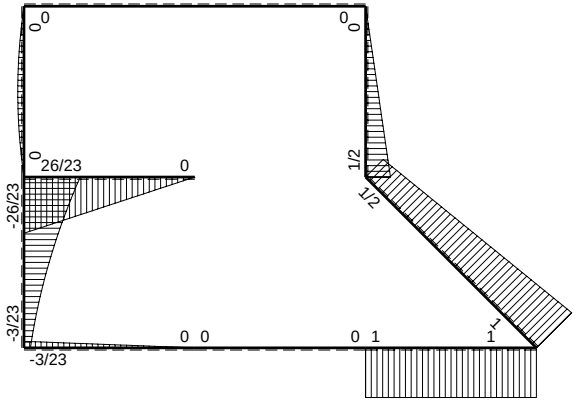





←□→ F



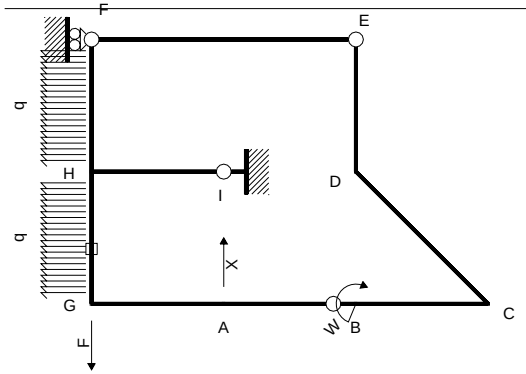

↑□↓ F



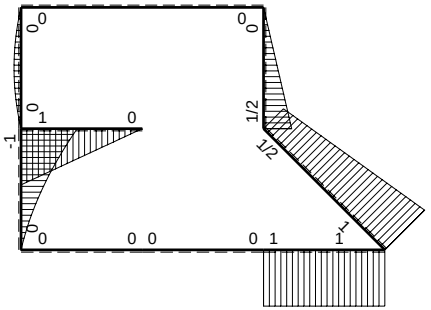

(□) Fb



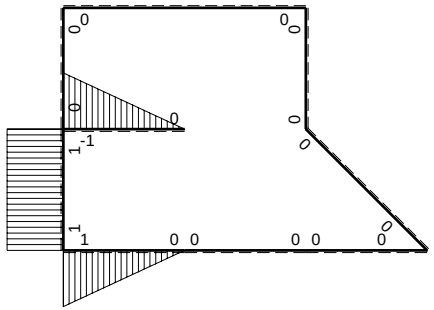





Schema di calcolo iperstatico




(□) $M_o$ flessione da carichi assegnati




(□) $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | 0 | 0 | 0 | 0 | 0 |
| BA b | 0 | 0 | 0 | 0 | | |
| BC b | 0 | Fb | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb | 0 | 0 | | |
| CD √2b | 0 | Fb-√2/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 1/2Fb-1/2Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -1/2Fx | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | b-x | 0 | 0 | $b^2-2bx+x^2$ | 0 | $1/3Xb^3/EJ$ |
| AG b | -x | 0 | 0 | $x^2$ | | |
| FH b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | -b+x | Fb-Fx | $-Fb^2+2Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-1/3Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | -Fx | $-Fx^2$ | $x^2$ | | |
| HG b | b | $-Fb+3/2Fx-1/2qx^2$ | $-Fb^2+3/2Fbx-1/2Fx^2$ | $b^2$ | $-5/12Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $1/2Fx+1/2qx^2$ | $-1/2Fbx-1/2Fx^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $1/4Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | -3/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-1 +2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[- x + x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(- b + b - 1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(- x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[-1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-1 +3/2\, x/b - 1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

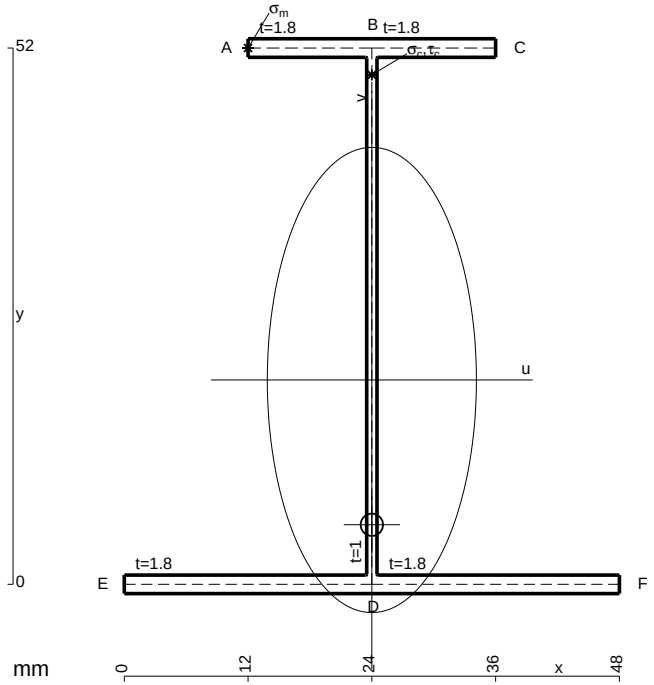
$$= \left[- x + 3/4\, x^2/b - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(- b + 3/4\, b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = 7/12\ \ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-1/2\, x/b - 1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left[-1/4\, x^2/b - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-1/4\, b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = 7/12\ \ Fb^3/EJ$$




A = 181.6 $mm^2$
$J_u$ = 92380. $mm^4$
$J_v$ = 18662. $mm^4$
$J_t$ = 157.3 $mm^4$
$y_o$ = -14.04 mm
$y_g$ = 19.82 mm
N = 410.1 N
$T_y$ = -410.1 N
$M_x$ = 661200. Nmm
$x_m$ = 12. mm
$y_m$ = 52. mm
$u_m$ = -12. mm
$v_m$ = 32.19 mm
$\sigma_m = N/A - Mv/J_u$ = -228.1 $N/mm^2$
$x_c$ = 24. mm
$y_c$ = 52. mm
$v_c$ = 32.19 mm
$\sigma_c = N/A - Mv/J_u$ = -228.1 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 6.173 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 6.173 $N/mm^2$
$t_c$ = 1160. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 228.4 $N/mm^2$



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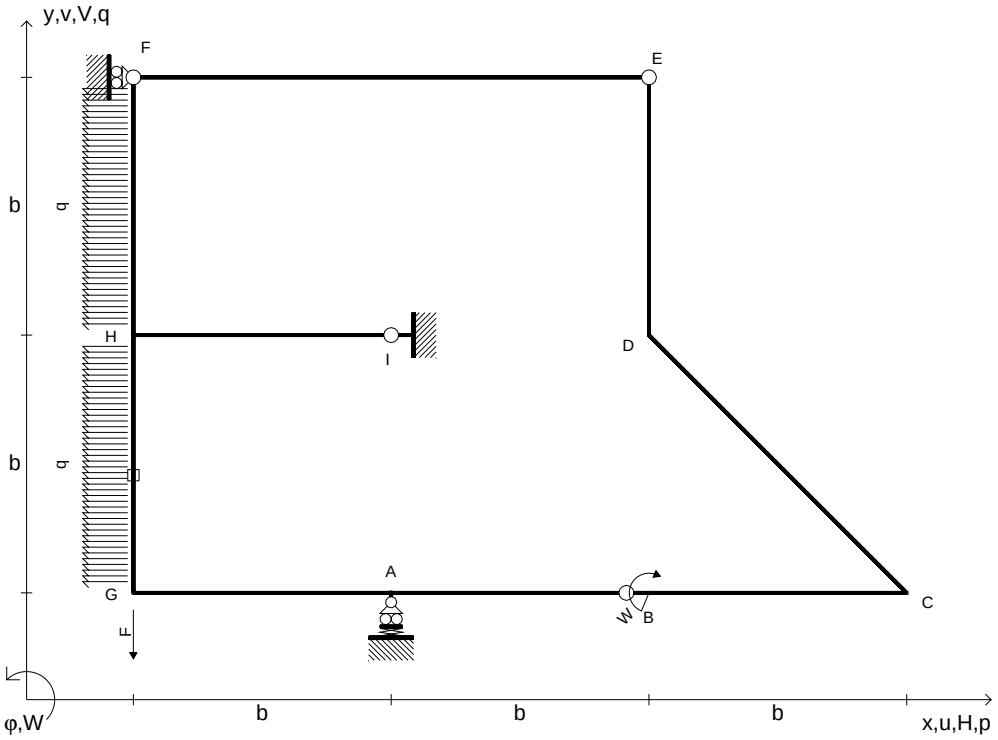
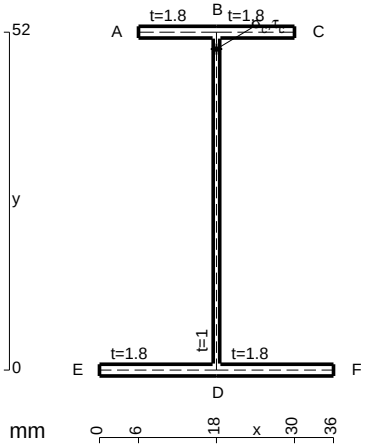



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_B = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$p_{FH} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{HG} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$




Reazioni iperstatiche in soluzione: $X=V_I$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave CD ha la sezione riportata e dimensioni in mm, con:

b = 700 mm, F = 840 N

Calcolare sulla sezione C la massima tensione normale $\sigma_m$.

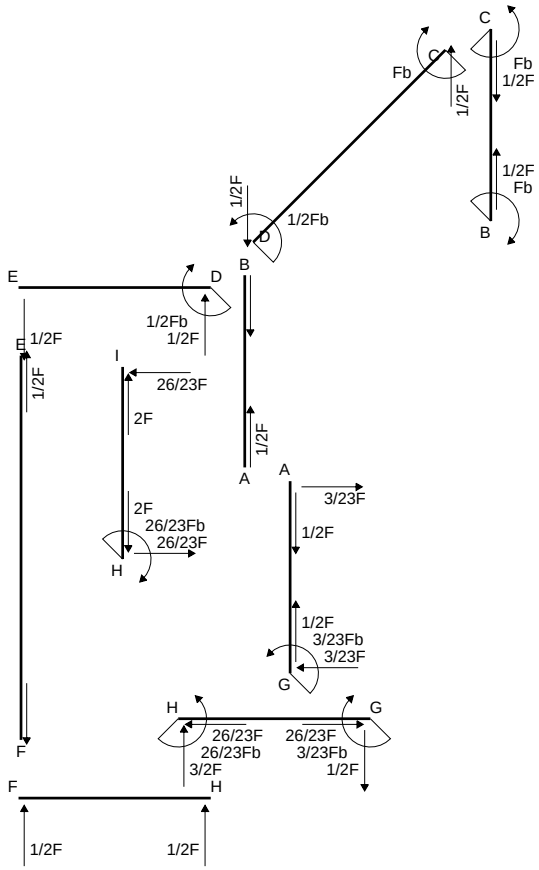
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.

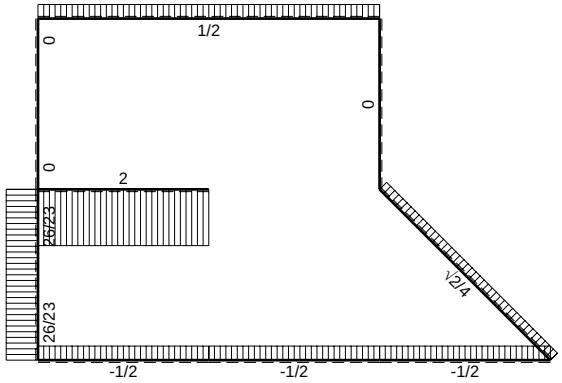
Lembo inferiore sezione su tratteggio trave, a destra da C a D

Elongazione termica specifica ε assegnata su asta HG.

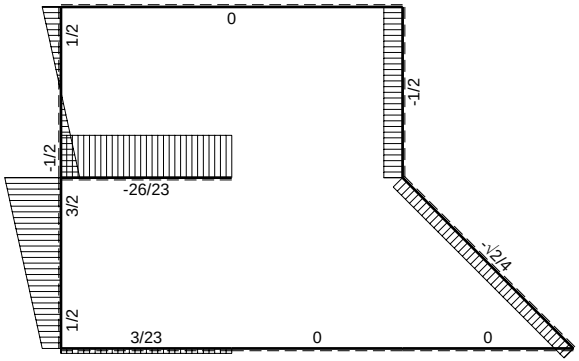




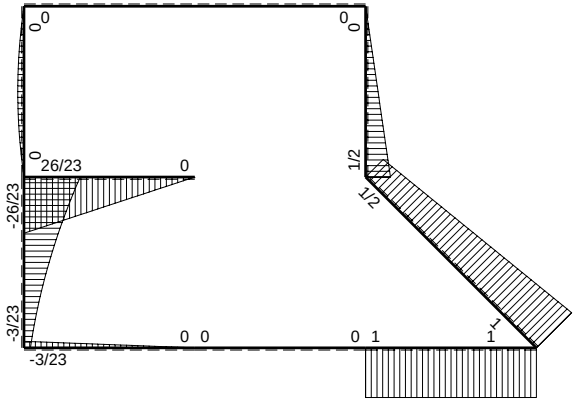





←□→ F



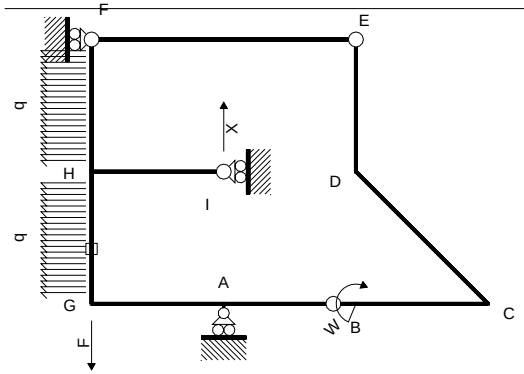

↑□↓ F



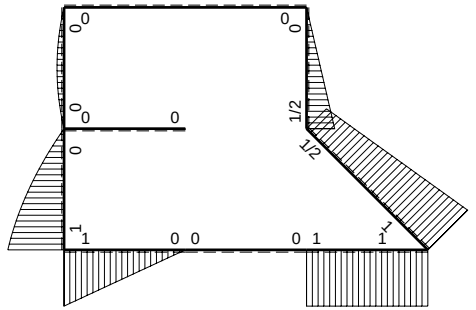

Fb



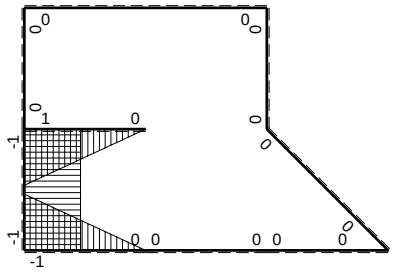





Schema di calcolo iperstatico




□ $M_o$ flessione da carichi assegnati




□ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | 0 | 0 | 0 | 0 | 0 |
| BA b | 0 | 0 | 0 | 0 | | |
| BC b | 0 | Fb | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb | 0 | 0 | | |
| CD $\sqrt{2}$b | 0 | Fb-$\sqrt{2}$/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 1/2Fb-1/2Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -1/2Fx | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | -b+x | Fb-Fx | $-Fb^2+2Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-1/3Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | x | -Fx | $-Fx^2$ | $x^2$ | | |
| FH b | 0 | $1/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $-1/2Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | b-x | 0 | 0 | $b^2-2bx+x^2$ | 0 | $1/3Xb^3/EJ$ |
| IH b | -x | 0 | 0 | $x^2$ | | |
| HG b | -b | $3/2Fx-1/2qx^2$ | $-3/2Fbx+1/2Fx^2$ | $b^2$ | $-7/12Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | b | $-Fb+1/2Fx+1/2qx^2$ | $-Fb^2+1/2Fbx+1/2Fx^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-1/4Fb^3/EJ$ | $1/4Xb^3/EJ$ |
| | totali | | | | $-13/6Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_I$ | | | | 26/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-1 +2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[- x + x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(- b + b - 1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(- x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[-1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-3/2\, x/b + 1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

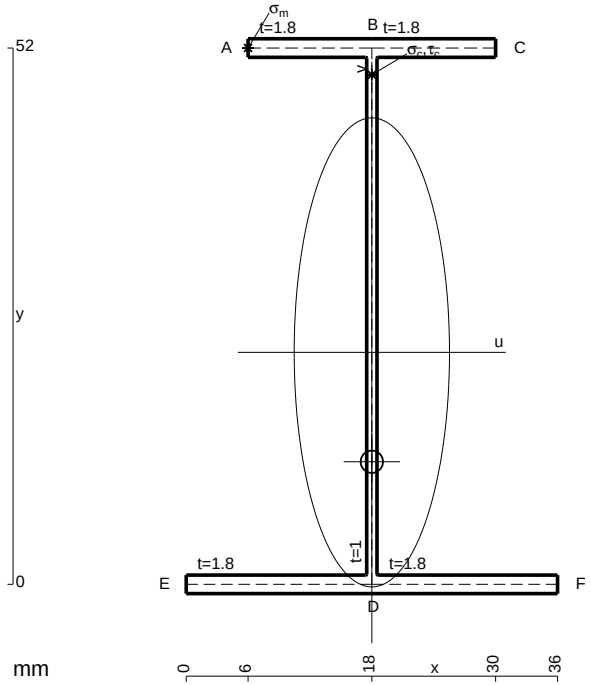
$$= \left[-3/4\, x^2/b + 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(-3/4\, b + 1/6\, b \,\right) Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -19/12\ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-1 + 1/2\, x/b + 1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[- x + 1/4\, x^2/b + 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(- b + 1/4\, b + 1/6\, b \,\right) Fb^2\, 1/EJ - 1 \quad (-1)\ (-1)\ Fb^3/EJ = -19/12\ Fb^3/EJ$$




$A = 160.\ mm^2$
$J_u = 82754.\ mm^4$
$J_v = 9072.\ mm^4$
$J_t = 134.\ mm^4$
$y_o = -10.6\ mm$
$y_g = 22.49\ mm$
$N = 297.\ N$
$T_y = -297.\ N$
$M_x = 588000.\ Nmm$
$x_m = 6.\ mm$
$y_m = 52.\ mm$
$u_m = -12.\ mm$
$v_m = 29.51\ mm$
$\sigma_m = N/A - Mv/J_u = -207.8\ N/mm^2$
$x_c = 18.\ mm$
$y_c = 52.\ mm$
$v_c = 29.51\ mm$
$\sigma_c = N/A - Mv/J_u = -207.8\ N/mm^2$
$\tau_c = TS^*/tJ_u = 4.575\ N/mm^2$
$\tau_g = TS^*/tJ_u = 4.575\ N/mm^2$
$t_c = 840.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 208.\ N/mm^2$



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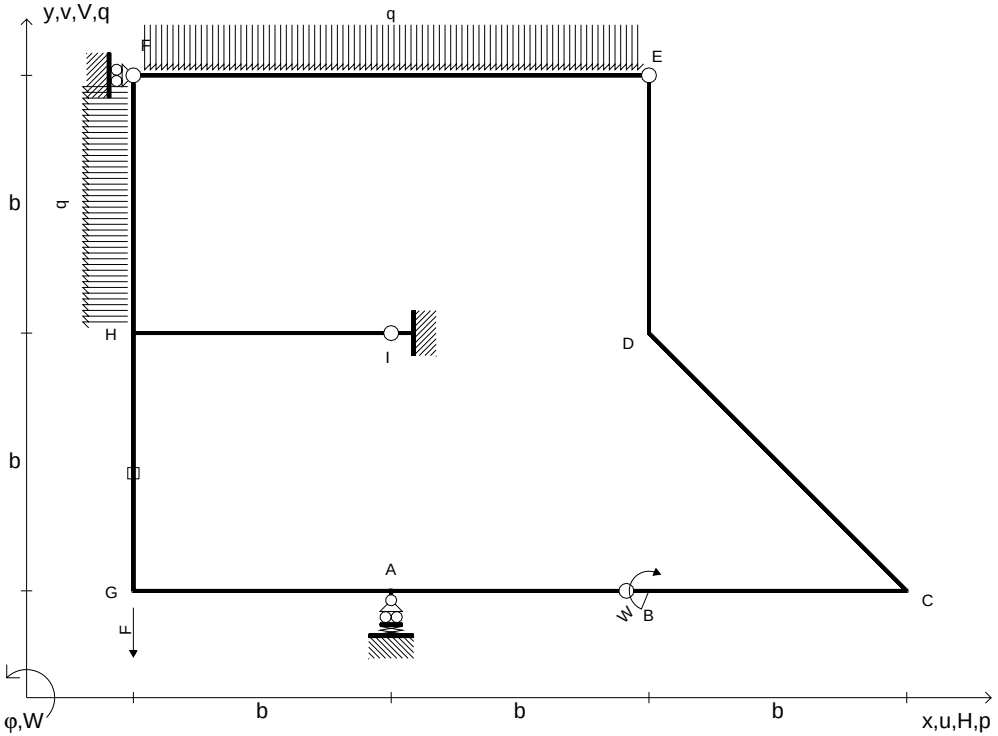
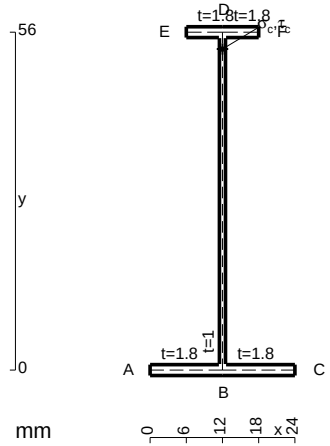



$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_B = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

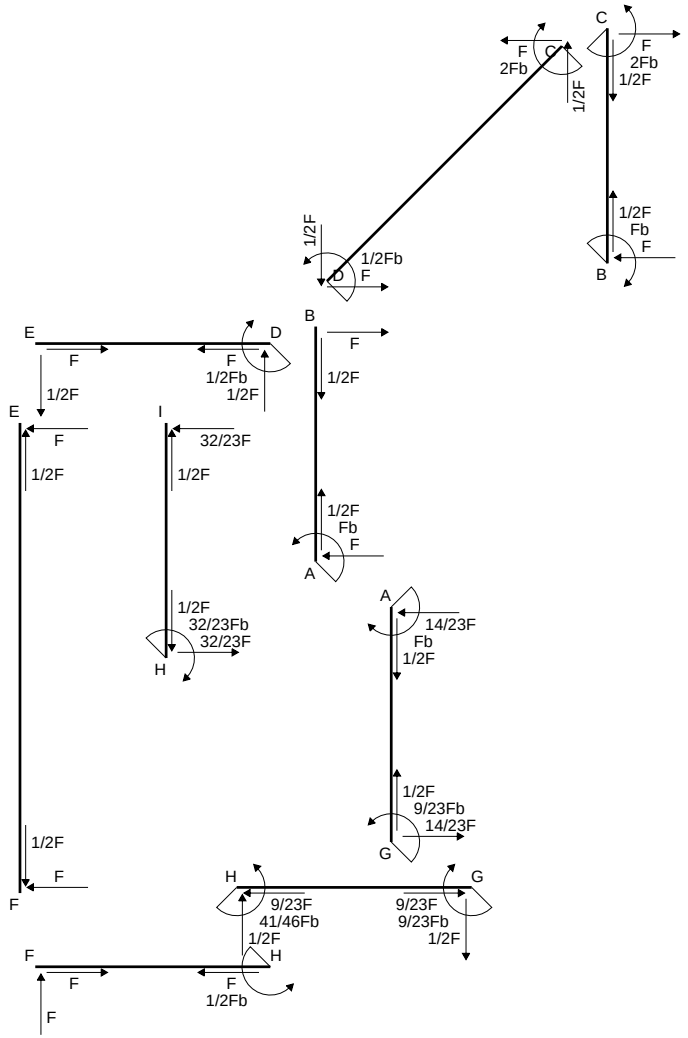
$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

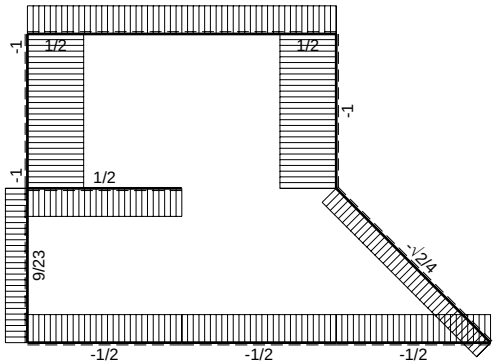



Reazioni iperstatiche in soluzione: $X=W_{GA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 810 mm, F = 250 N
Calcolare sulla sezione C la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Elongazione termica specifica ε assegnata su asta HG.

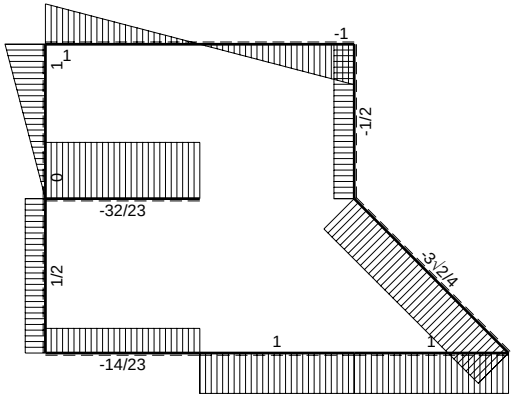




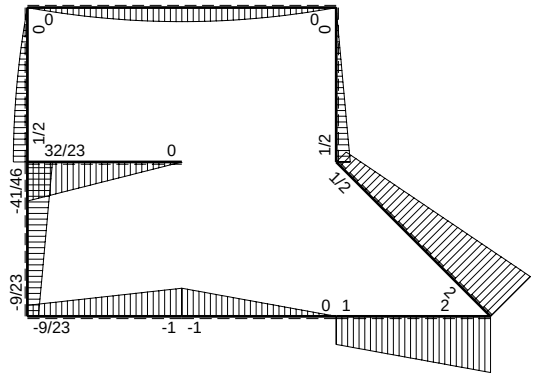





F



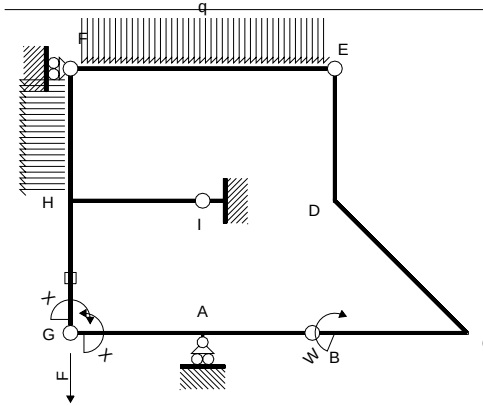

F

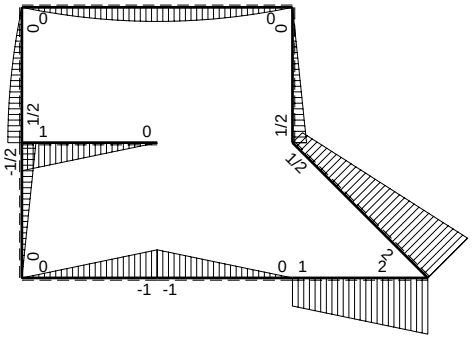



Fb

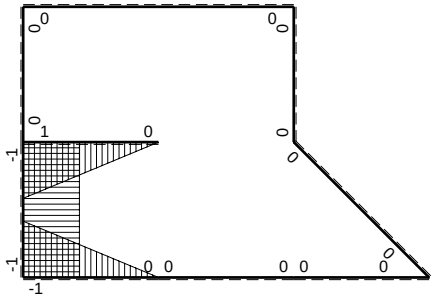







$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fb+Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -2Fb+Fx | 0 | 0 | | |
| CD √2b | 0 | 2Fb-3√2/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 1/2Fb-1/2Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -1/2Fx | 0 | 0 | | |
| EF 2b | 0 | -Fx+1/2qx$^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | Fx-1/2qx$^2$ | 0 | 0 | | |
| GA b | -1+x/b | -Fx | Fx-Fx$^2$/b | 1-2x/b+x$^2$/b$^2$ | 1/6Fb$^2$/EJ | 1/3Xb/EJ |
| AG b | x/b | Fb-Fx | Fx-Fx$^2$/b | x$^2$/b$^2$ | | |
| FH b | 0 | Fx-1/2qx$^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | -1/2Fb+1/2qx$^2$ | 0 | 0 | | |
| HI b | 1-x/b | Fb-Fx | Fb-2Fx+Fx$^2$/b | 1-2x/b+x$^2$/b$^2$ | 1/3Fb$^2$/EJ | 1/3Xb/EJ |
| IH b | -x/b | -Fx | Fx$^2$/b | x$^2$/b$^2$ | | |
| HG b | -1 | -1/2Fb+1/2Fx | 1/2Fb-1/2Fx | 1 | 1/4Fb$^2$/EJ | Xb/EJ |
| GH b | 1 | 1/2Fx | 1/2Fx | 1 | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | -Fb$^2$/EJ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | -1/2Fb$^2$/EJ | 1/4Xb/EJ |
| | totali | | | | -3/4Fb$^2$/EJ | 23/12Xb/EJ |
| | iperstatica $X=W_{GA}$ | | | | 9/23Fb | |

Sviluppi di calcolo iperstatica





$L_{GA}^{XX} = \int_o^b (1 - 2x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{AG}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HI}^{XX} = \int_o^b (1 - 2x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IH}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HG}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GH}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GA}^{Xo} = \int_o^b (\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{AG}^{Xo} = \int_o^b (\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{HI}^{Xo} = \int_o^b (1 - 2x/b + x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L_{IH}^{Xo} = \int_o^b (\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

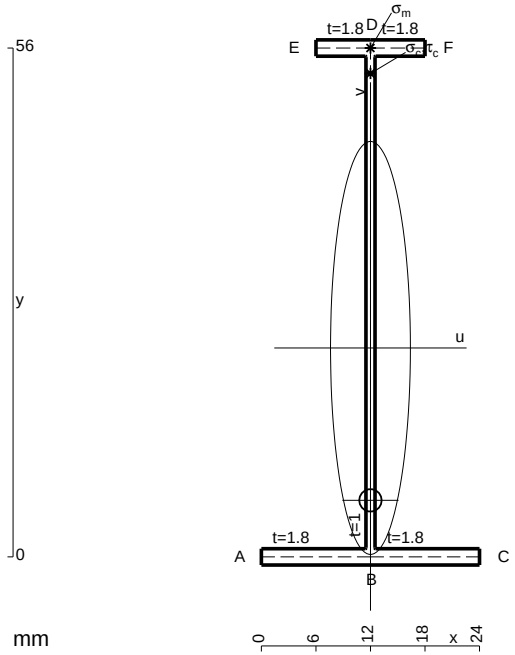
$L_{HG}^{Xo} = \int_o^b (1/2 - 1/2\ x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= [1/2\ x - 1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/2\ b - 1/4\ b\ )\ Fb\ 1/EJ\ - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -3/4\ \ Fb^2/EJ$

$L_{GH}^{Xo} = \int_o^b (1/2\ x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = [1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/4\ b\ )\ Fb\ 1/EJ\ - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -3/4\ \ Fb^2/EJ$




$A = 120.8\ mm^2$
$J_u = 62410.\ mm^4$
$J_v = 2333.\ mm^4$
$J_t = 88.65\ mm^4$
$y_o = -16.77$ mm
$y_g = 22.99$ mm
$N = -88.39$ N
$T_y = -265.2$ N
$M_x = 405000.$ Nmm
$x_m = 12.$ mm
$y_m = 56.$ mm
$v_m = 33.01$ mm
$\sigma_m = N/A - Mv/J_u = -214.9\ N/mm^2$
$y_c = 2.$ mm
$u_c = -12.$ mm
$v_c = -20.99$ mm
$\sigma_c = N/A - Mv/J_u = -214.9\ N/mm^2$
$\tau_c = TS^*/tJ_u = 3.029\ N/mm^2$
$\tau_g = TS^*/tJ_u = 3.029\ N/mm^2$
$t_c = 250.$ mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 215.\ N/mm^2$



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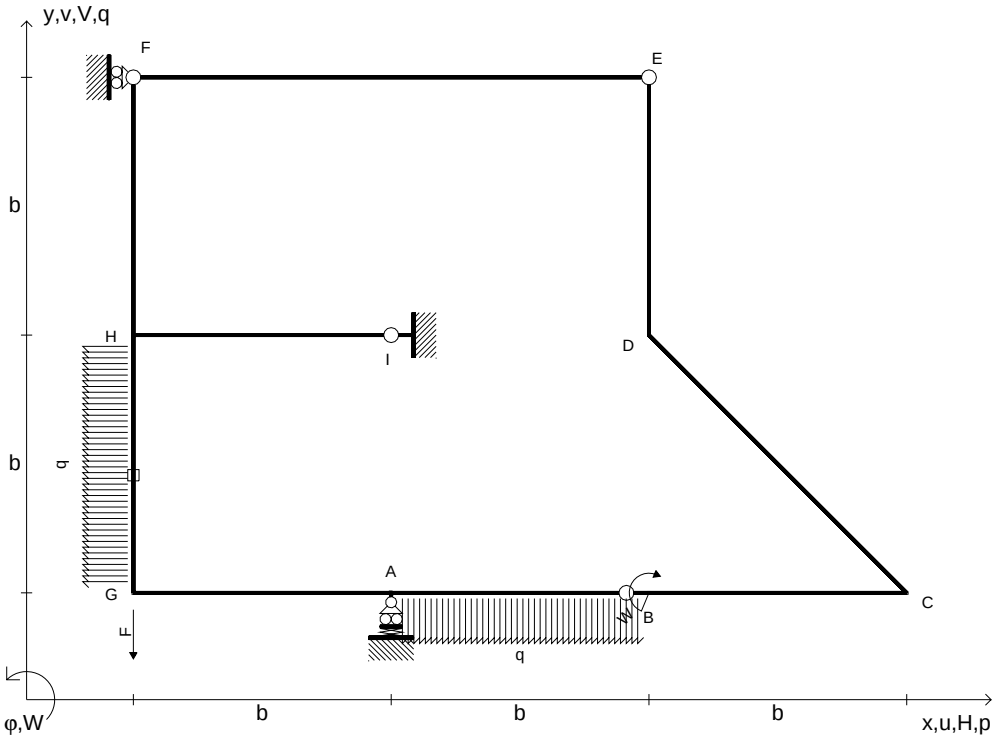
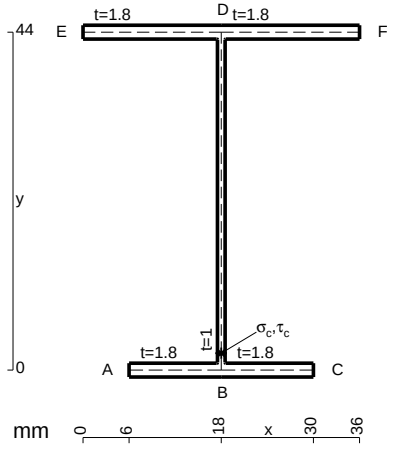



$V_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_B = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

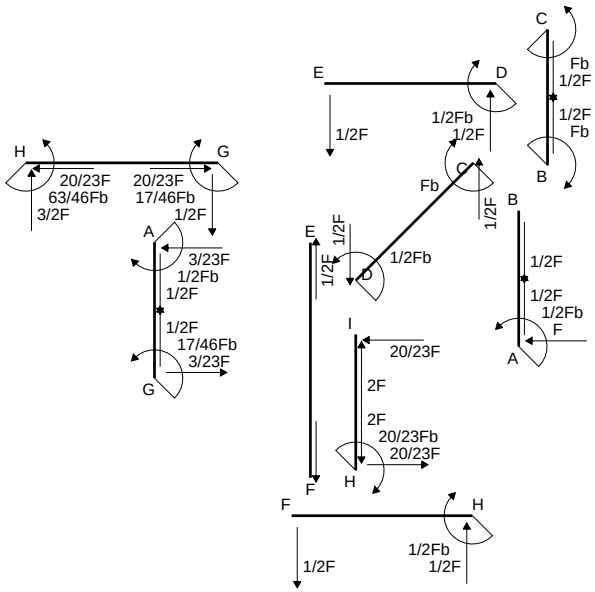
$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

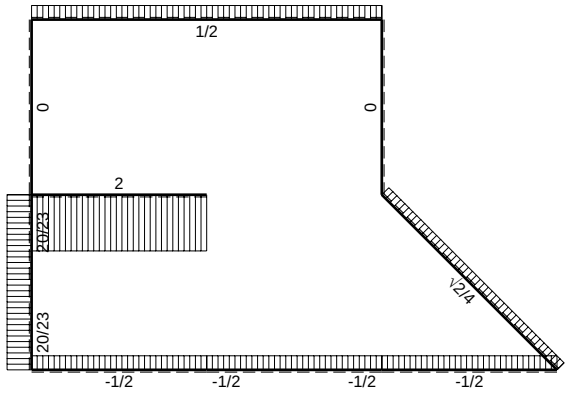



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 530 mm, F = 2020 N
Calcolare sulla sezione A la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.

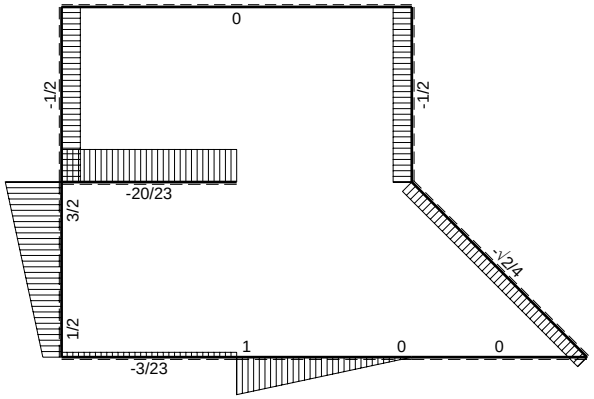




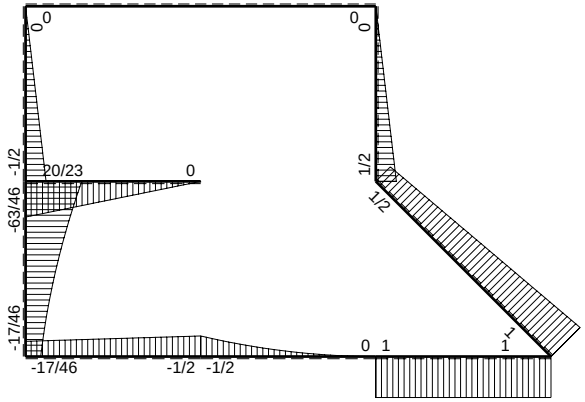





←□→ F



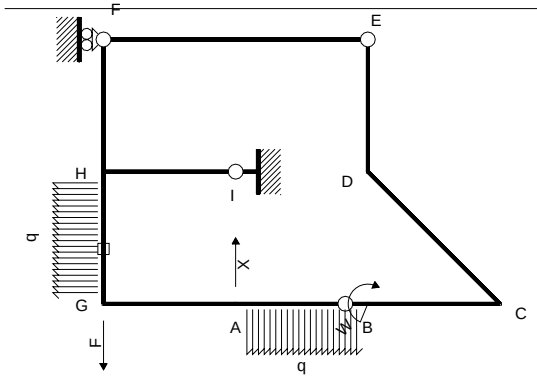

↑□↓ F



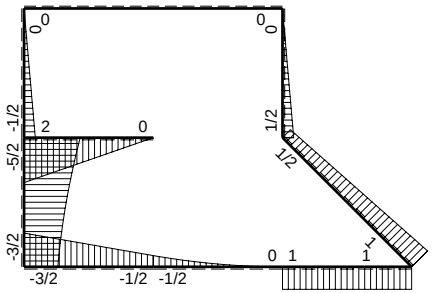

(□) Fb



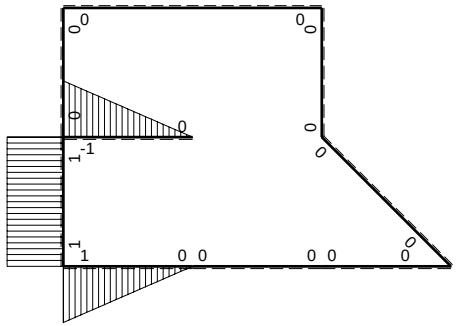





Schema di calcolo iperstatico




(□) $M_o$ flessione da carichi assegnati




(□) $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $1/2qx^2$ | 0 | 0 | | |
| BC b | 0 | Fb | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $Fb-\sqrt{2}/4Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | $1/2Fb-1/2Fx$ | 0 | 0 | 0 | 0 |
| ED b | 0 | $-1/2Fx$ | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | b-x | $-3/2Fb+Fx$ | $-3/2Fb^2+5/2Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-7/12Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | $1/2Fb+Fx$ | $-1/2Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | $-1/2Fx$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $1/2Fb-1/2Fx$ | 0 | 0 | | |
| HI b | -b+x | 2Fb-2Fx | $-2Fb^2+4Fbx-2Fx^2$ | $b^2-2bx+x^2$ | $-2/3Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | $-2Fx$ | $-2Fx^2$ | $x^2$ | | |
| HG b | b | $-5/2Fb+3/2Fx-1/2qx^2$ | $-5/2Fb^2+3/2Fbx-1/2Fx^2$ | $b^2$ | $-23/12Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $3/2Fb+1/2Fx+1/2qx^2$ | $-3/2Fb^2-1/2Fbx-1/2Fx^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-13/6Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 26/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-3/2 +5/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-3/2\, x +5/4\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-3/2\, b +5/4\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(-1/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/4\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/4\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-2 +4\, x/b -2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2\, x +2\, x^2/b -2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2\, b +2\, b -2/3\, b \,\right) Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(-2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2/3\, b \,\right) Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-5/2 +3/2\, x/b -1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

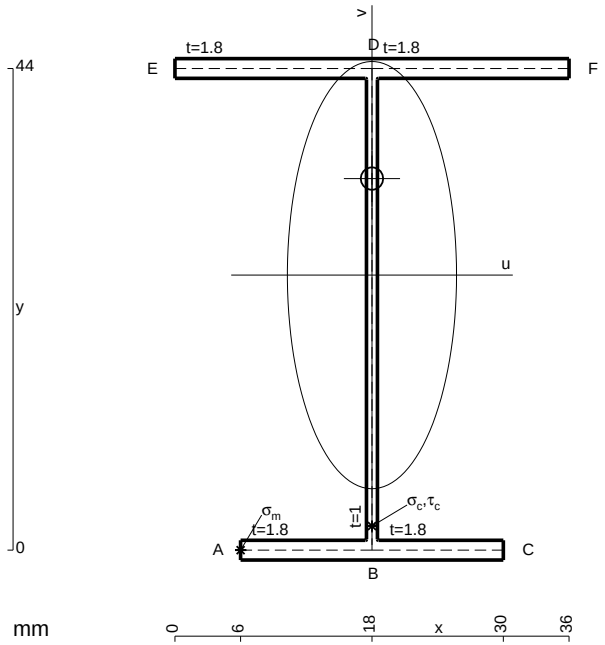
$$= \left[-5/2\, x +3/4\, x^2/b -1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ\ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-5/2\, b +3/4\, b -1/6\, b \,\right) Fb^2\, 1/EJ\ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -11/12\ \ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-3/2 -1/2\, x/b -1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left[-3/2\, x -1/4\, x^2/b -1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ\ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-3/2\, b -1/4\, b -1/6\, b \,\right) Fb^2\, 1/EJ\ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -11/12\ \ Fb^3/EJ$$




A = 152. $mm^2$
$J_u$ = 57885. $mm^4$
$J_v$ = 9072. $mm^4$
$J_t$ = 131.3 $mm^4$
$y_o$ = 8.817 mm
$y_g$ = 25.13 mm
N = -1010. N
$T_y$ = 2020. N
$M_x$ = -535300. Nmm
$x_m$ = 6. mm
$u_m$ = -12. mm
$v_m$ = -25.13 mm
$\sigma_m = N/A - Mv/J_u$ = -239. $N/mm^2$
$x_c$ = 18. mm
$v_c$ = -25.13 mm
$\sigma_c = N/A - Mv/J_u$ = -239. $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 37.88 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 37.88 $N/mm^2$
$t_c$ = 2020. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 247.8 $N/mm^2$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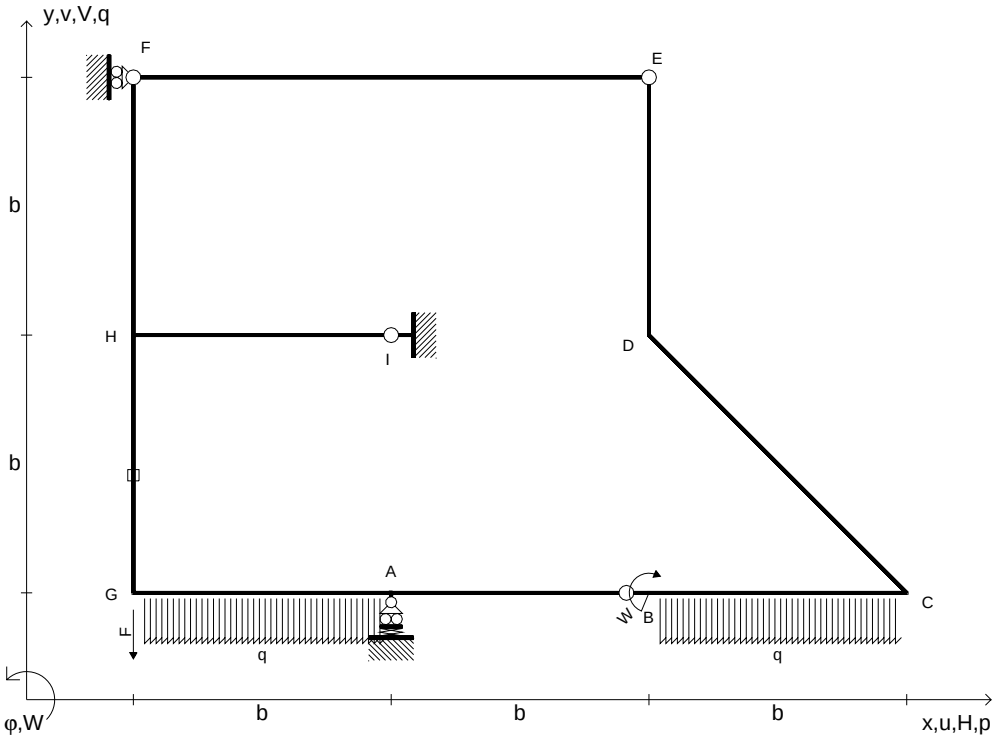


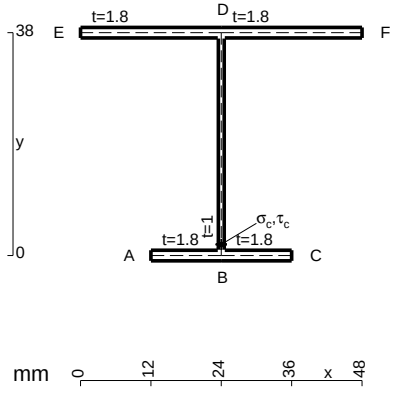

$V_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_B = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{BC} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

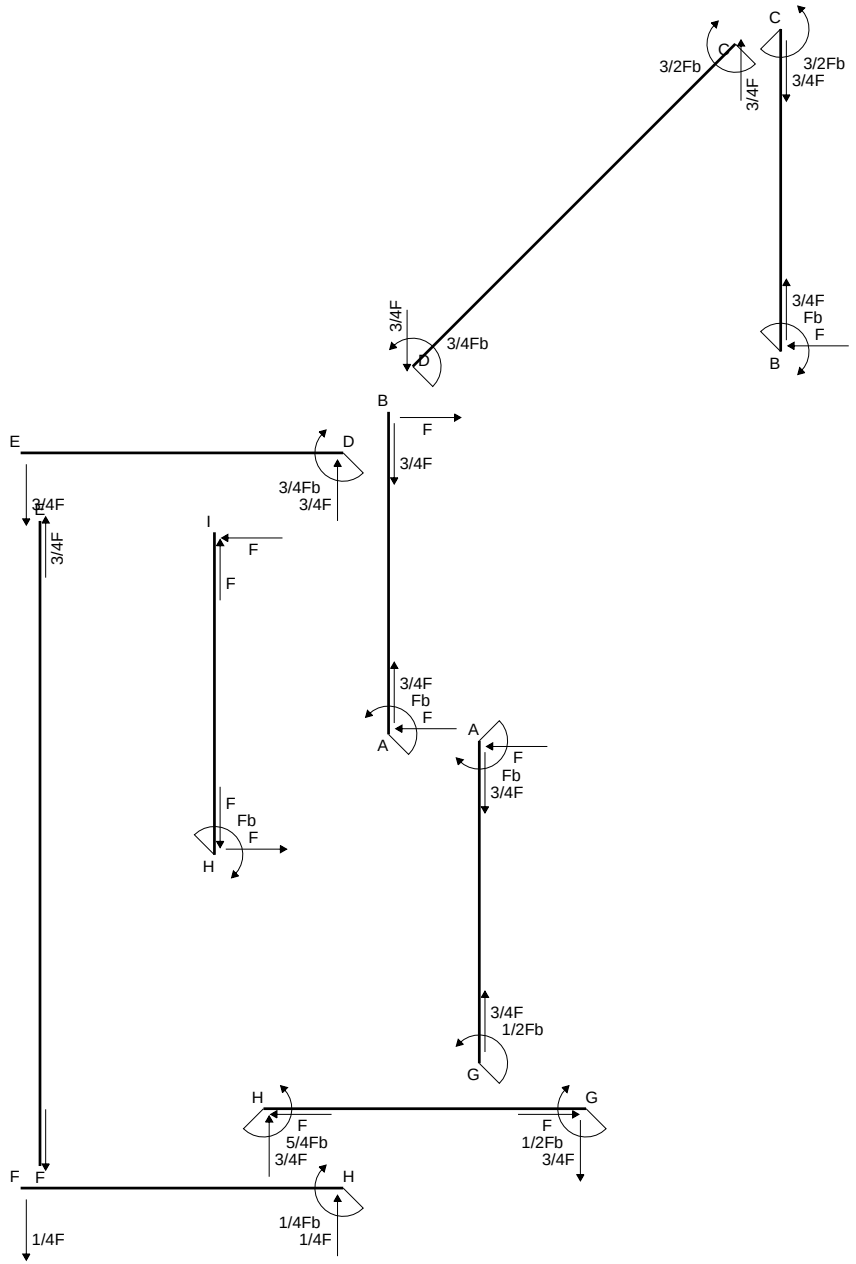
$q_{GA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

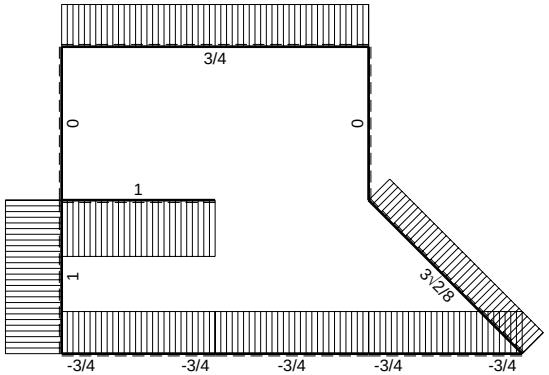
Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave AB ha la sezione riportata e dimensioni in mm, con:
b = 570 mm, F = 780 N
Calcolare sulla sezione A la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da A a B
Elongazione termica specifica ε assegnata su asta HG.



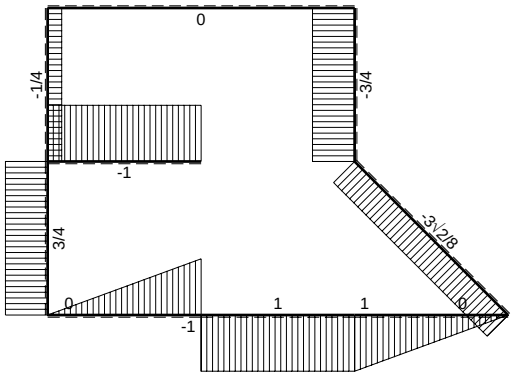




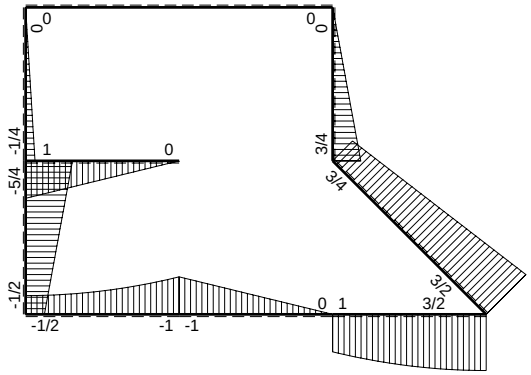






←□→ F



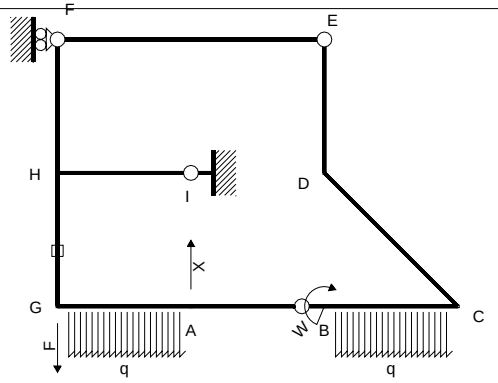

↑□↓ F



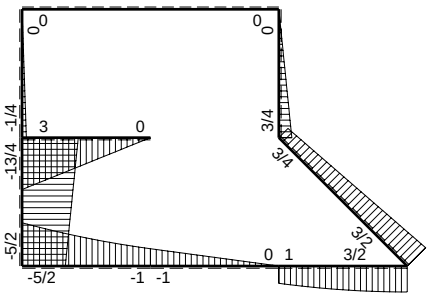

↻□↺ Fb



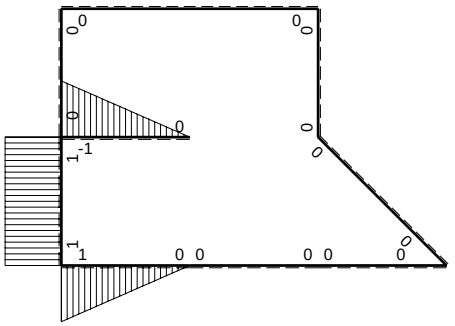





Schema di calcolo iperstatico




↻□↺ $M_o$ flessione da carichi assegnati




↻□↺ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | $Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| CB b | 0 | $-3/2Fb+1/2qx^2$ | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $3/2Fb-3\sqrt{2}/8Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | 3/4Fb-3/4Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -3/4Fx | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | b-x | $-5/2Fb+2Fx-1/2qx^2$ | $-5/2Fb^2+9/2Fbx-5/2Fx^2+1/2qx^3$ | $b^2-2bx+x^2$ | $-23/24Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | $Fb+Fx+1/2qx^2$ | $-Fbx-Fx^2-1/2qx^3$ | $x^2$ | | |
| FH b | 0 | -1/4Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 1/4Fb-1/4Fx | 0 | 0 | | |
| HI b | -b+x | 3Fb-3Fx | $-3Fb^2+6Fbx-3Fx^2$ | $b^2-2bx+x^2$ | $-Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | -3Fx | $-3Fx^2$ | $x^2$ | | |
| HG b | b | -13/4Fb+3/4Fx | $-13/4Fb^2+3/4Fbx$ | $b^2$ | $-23/8Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | 5/2Fb+3/4Fx | $-5/2Fb^2-3/4Fbx$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-23/6Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 2F | |



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Sviluppi di calcolo iperstatica

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$L^{XX}_{GA} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-5/2 + 9/2\, x/b - 5/2\, x^2/b^2 + 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-5/2\, x + 9/4\, x^2/b - 5/6\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-5/2\, b + 9/4\, b - 5/6\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = -23/24\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-\, x/b - x^2/b^2 - 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b - 1/3\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b - 1/3\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = -23/24\ Fb^3/EJ$

$L^{Xo}_{HI} = \int_o^b (-3 + 6\, x/b - 3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3\, x + 3\, x^2/b - x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3\, b + 3\, b - b \,)\, Fb^2\, 1/EJ = -\ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b (-3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-\, b \,)\, Fb^2\, 1/EJ = -\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (-13/4 + 3/4\, x/b)\, Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= [-13/4\, x + 3/8\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= (-13/4\, b + 3/8\, b \,)\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = -15/8\ Fb^3/EJ$

$L^{Xo}_{GH} = \int_o^b (-5/2 - 3/4\, x/b)\, Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= [-5/2\, x - 3/8\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= (-5/2\, b - 3/8\, b \,)\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = -15/8\ Fb^3/EJ$



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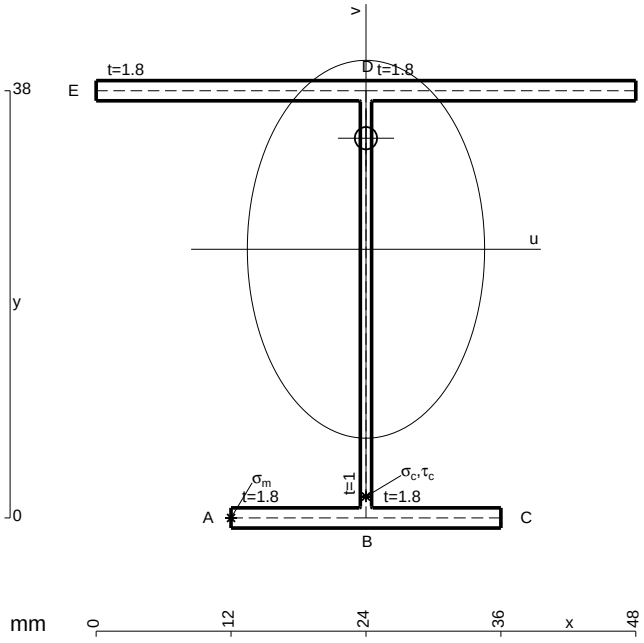


A = 167.6 $mm^2$
$J_u$ = 47339. $mm^4$
$J_v$ = 18662. $mm^4$
$J_t$ = 152.6 $mm^4$
$y_o$ = 9.88 mm
$y_g$ = 23.9 mm
N = -585. N
$T_y$ = 780. N
$M_x$ = -444600. Nmm
$x_m$ = 12. mm
$u_m$ = -12. mm
$v_m$ = -23.9 mm
$\sigma_m = N/A - Mv/J_u$ = -227.9 $N/mm^2$
$x_c$ = 24. mm
$v_c$ = -23.9 mm
$\sigma_c = N/A - Mv/J_u$ = -227.9 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 17.01 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 17.01 $N/mm^2$
$t_c$ = 780. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 229.8 $N/mm^2$

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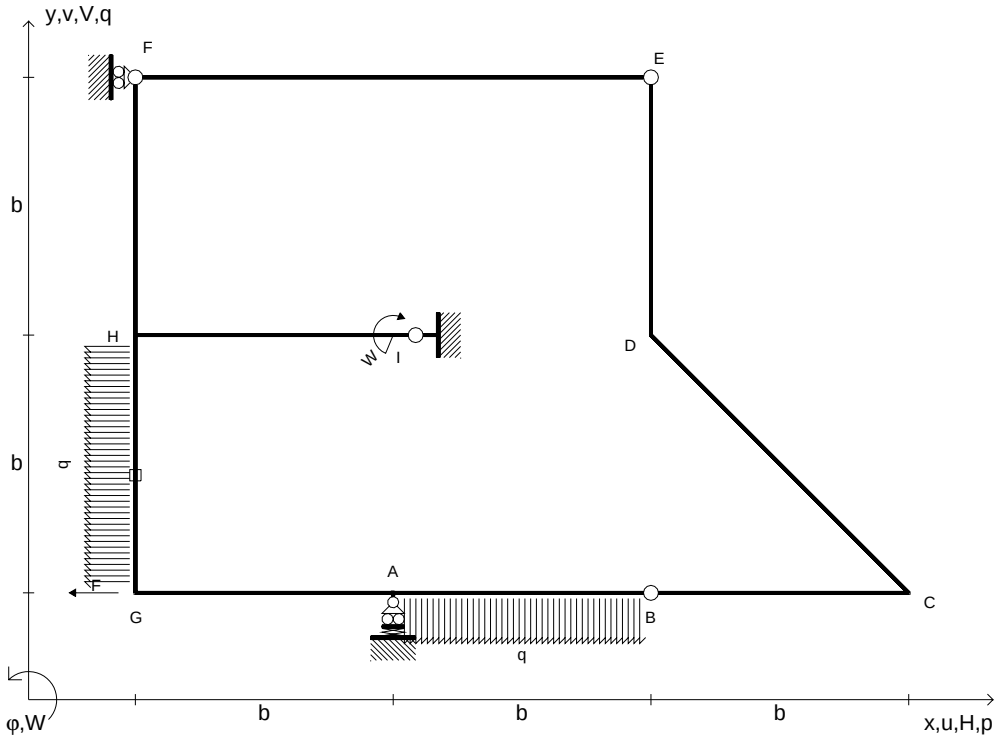
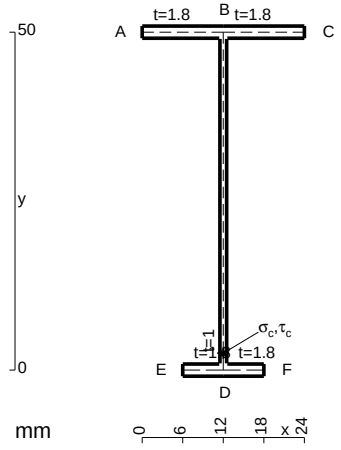



$H_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_I = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

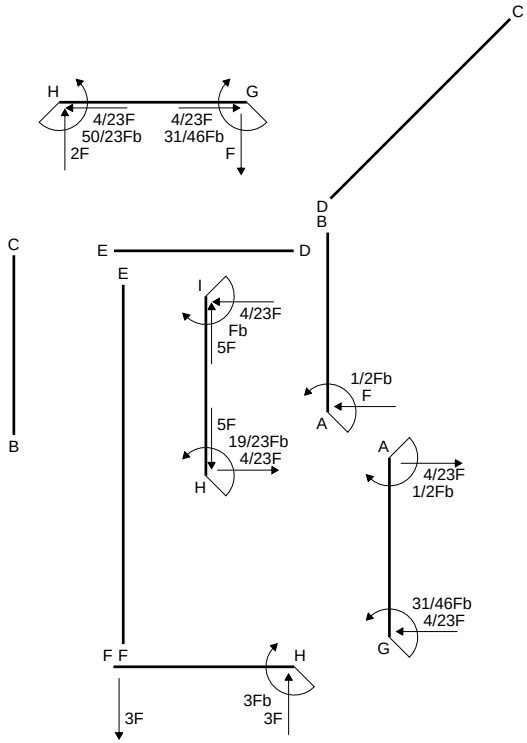
$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

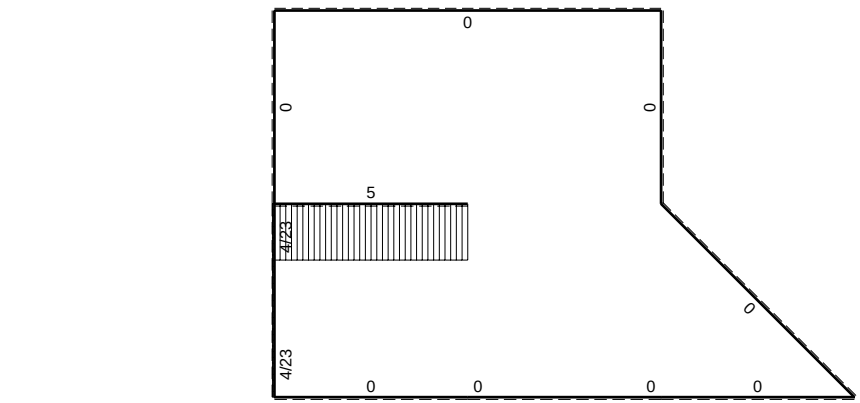



Reazioni iperstatiche in soluzione: $X=V_I$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 470 mm, F = 240 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

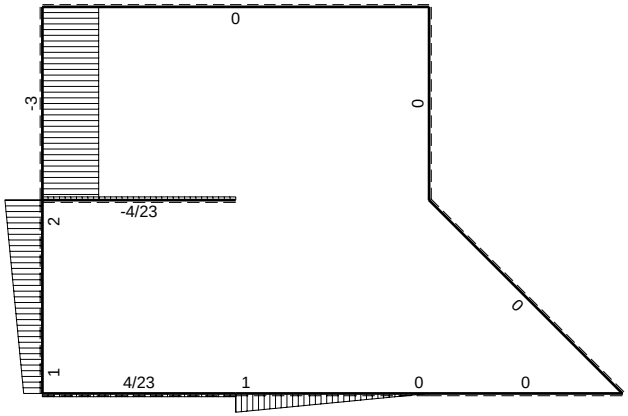




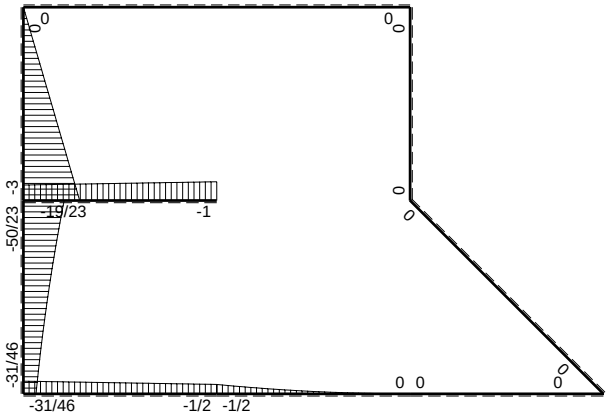





F



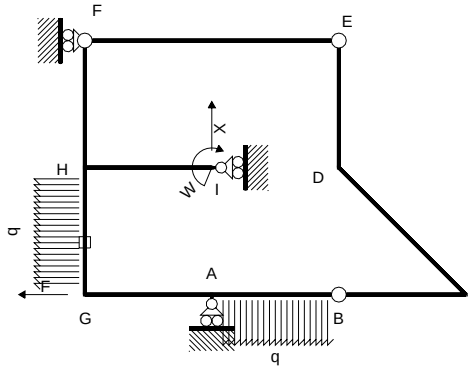

F



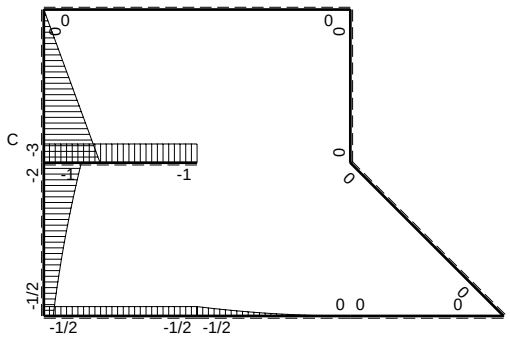

Fb



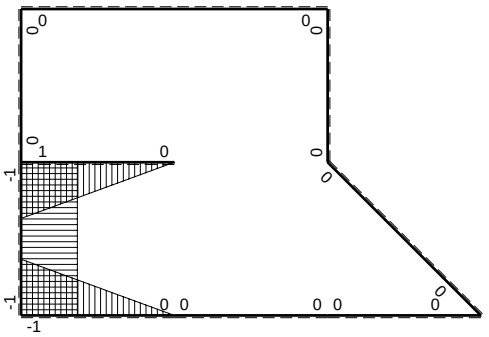





Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $1/2qx^2$ | 0 | 0 | | |
| BC b | 0 | 0 | 0 | 0 | 0 | 0 |
| CB b | 0 | 0 | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | 0 | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | $-b+x$ | $-1/2Fb$ | $1/2Fb^2-1/2Fbx$ | $b^2-2bx+x^2$ | $1/4Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | x | $1/2Fb$ | $1/2Fbx$ | $x^2$ | | |
| FH b | 0 | $-3Fx$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $3Fb-3Fx$ | 0 | 0 | | |
| HI b | b-x | $-Fb$ | $-Fb^2+Fbx$ | $b^2-2bx+x^2$ | $-1/2Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | -x | $Fb$ | $-Fbx$ | $x^2$ | | |
| HG b | -b | $-2Fb+2Fx-1/2qx^2$ | $2Fb^2-2Fbx+1/2Fx^2$ | $b^2$ | $7/6Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | b | $1/2Fb+Fx+1/2qx^2$ | $1/2Fb^2+Fbx+1/2Fx^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-1/4Fb^3/EJ$ | $1/4Xb^3/EJ$ |
| | totali | | | | $-1/3Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_I$ | | | | 4/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(1/2 -1/2\, x/b\,\right) Fb^2\, 1/EJ\, dx = \left[1/2\, x -1/4\, x^2/b\,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(1/2\, b -1/4\, b\,\right) Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(1/2\, x/b\,\right) Fb^2\, 1/EJ\, dx = \left[1/4\, x^2/b\,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(1/4\, b\,\right) Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-1 + x/b\,\right) Fb^2\, 1/EJ\, dx = \left[- x + 1/2\, x^2/b\,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(- b + 1/2\, b\,\right) Fb^2\, 1/EJ = -1/2\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(- x/b\,\right) Fb^2\, 1/EJ\, dx = \left[-1/2\, x^2/b\,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(-1/2\, b\,\right) Fb^2\, 1/EJ = -1/2\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(2 -2\, x/b + 1/2\, x^2/b^2\,\right) Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left[2\, x - x^2/b + 1/6\, x^3/b^2\,\right]_o^b Fb^2\, 1/EJ\ \ - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left(2\, b - b + 1/6\, b\,\right) Fb^2\, 1/EJ\ \ - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = 1/6\ \ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(1/2 + x/b + 1/2\, x^2/b^2\,\right) Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left[1/2\, x + 1/2\, x^2/b + 1/6\, x^3/b^2\,\right]_o^b Fb^2\, 1/EJ\ \ - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left(1/2\, b + 1/2\, b + 1/6\, b\,\right) Fb^2\, 1/EJ\ \ - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = 1/6\ \ Fb^3/EJ$$

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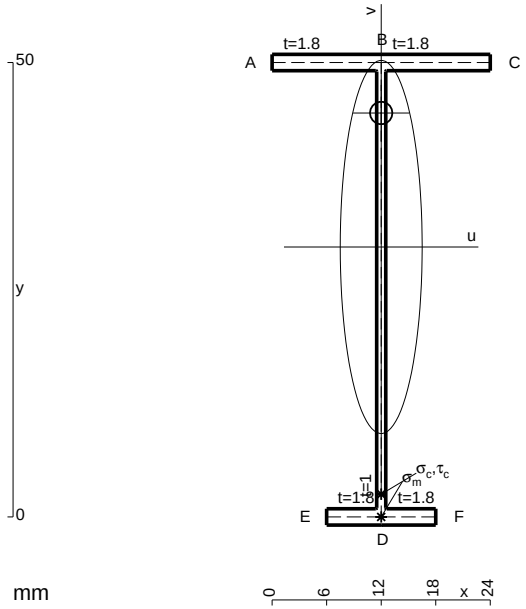
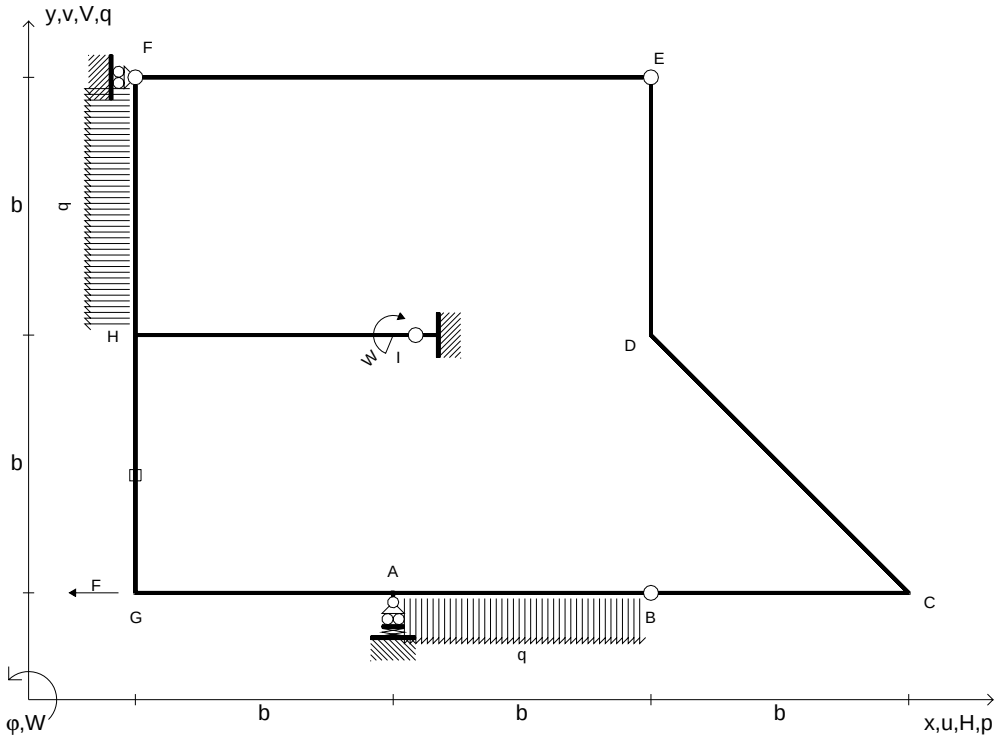



A = 114.8 $mm^2$
$J_u$ = 48377. $mm^4$
$J_v$ = 2333. $mm^4$
$J_t$ = 86.65 $mm^4$
$y_o$ = 14.74 mm
$y_g$ = 29.7 mm
$T_y$ = -720. N
$M_x$ = -338400. Nmm
$x_m$ = 12. mm
$v_m$ = -29.7 mm
$\sigma_m = -Mv/J_u$ = -207.8 $N/mm^2$
$y_c$ = 3. mm
$u_c$ = -12. mm
$v_c$ = -26.7 mm
$\sigma_c = -Mv/J_u$ = -207.8 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 9.549 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 9.549 $N/mm^2$
$t_c$ = 240. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 208.4 $N/mm^2$



@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

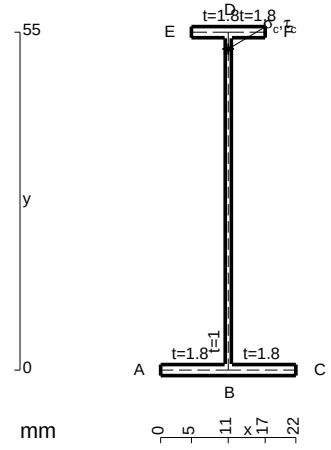



$H_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_I = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

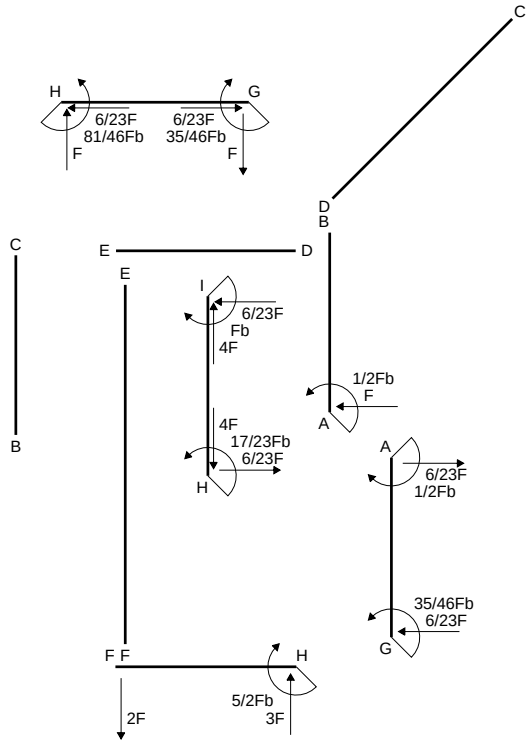
$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

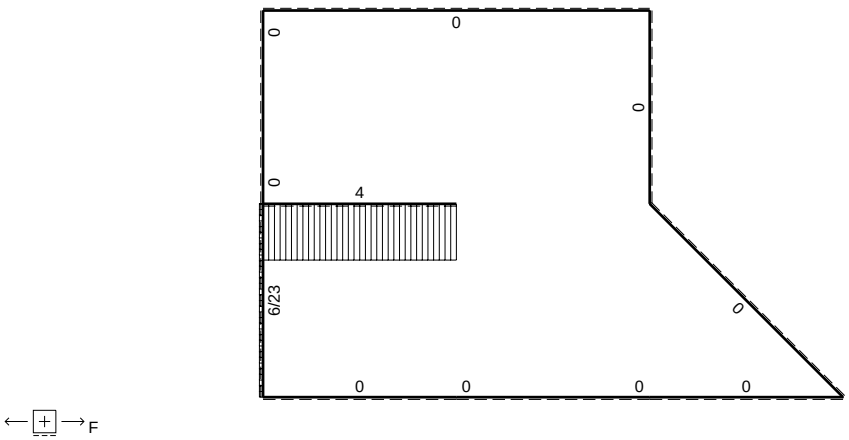


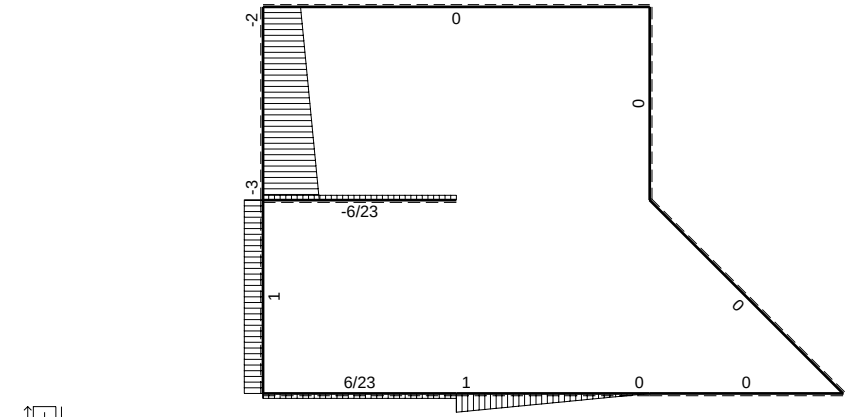

Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 510 mm, F = 300 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

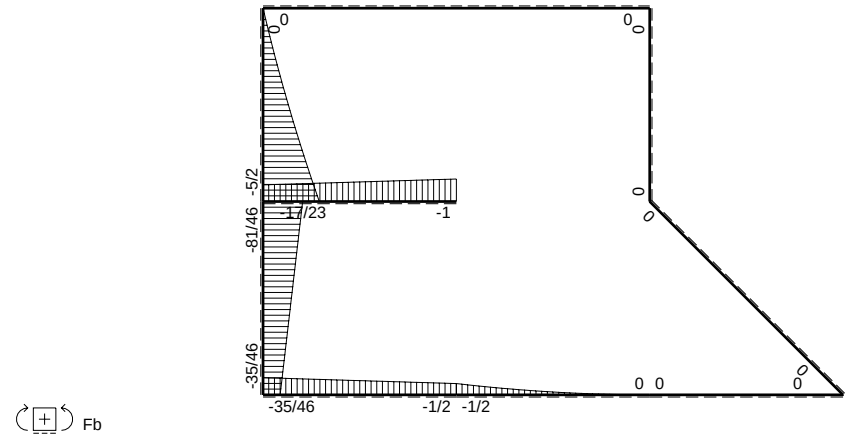






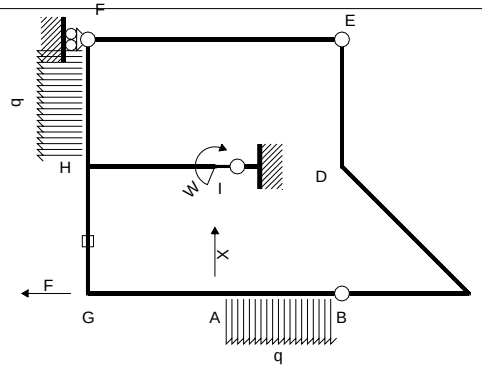


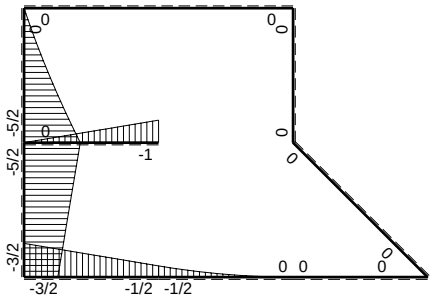





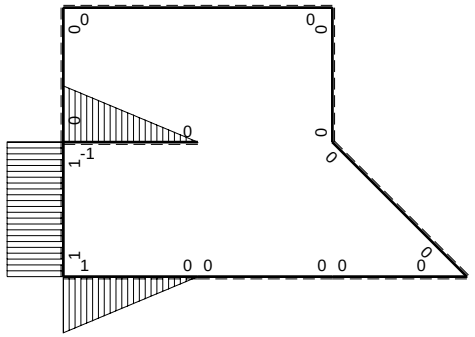



Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $1/2qx^2$ | 0 | 0 | | |
| BC b | 0 | 0 | 0 | 0 | 0 | 0 |
| CB b | 0 | 0 | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | 0 | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | b-x | $-3/2Fb+Fx$ | $-3/2Fb^2+5/2Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-7/12Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | $1/2Fb+Fx$ | $-1/2Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | -b+x | $-Fx$ | $Fbx-Fx^2$ | $b^2-2bx+x^2$ | $1/6Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | $Fb-Fx$ | $Fbx-Fx^2$ | $x^2$ | | |
| HG b | b | $-5/2Fb+Fx$ | $-5/2Fb^2+Fbx$ | $b^2$ | $-2Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $3/2Fb+Fx$ | $-3/2Fb^2-Fbx$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-17/12Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 17/23F | |

Sviluppi di calcolo iperstatica





$L_{GA}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AG}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HI}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IH}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HG}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{GH}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{GA}^{Xo} = \int_o^b (-3/2 + 5/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/2\, x + 5/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 5/4\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L_{AG}^{Xo} = \int_o^b (-1/2\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/4\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$

$L_{HI}^{Xo} = \int_o^b (\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L_{IH}^{Xo} = \int_o^b (\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L_{HG}^{Xo} = \int_o^b (-5/2 + x/b \,)\, Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

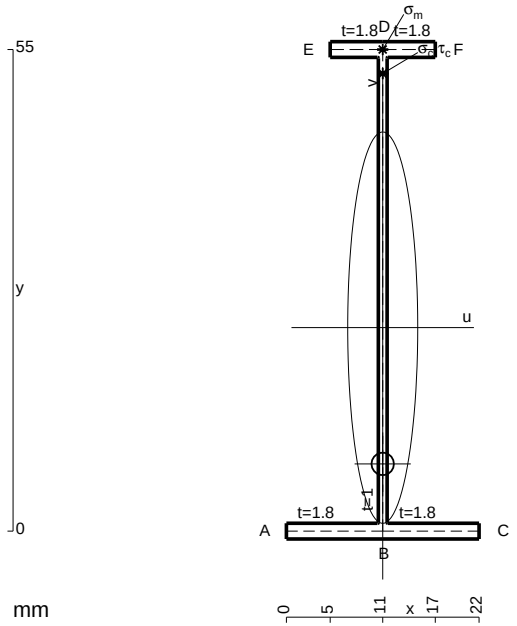
$= [-5/2\, x + 1/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= (-5/2\, b + 1/2\, b \,)\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = -\ Fb^3/EJ$

$L_{GH}^{Xo} = \int_o^b (-3/2 - x/b \,)\, Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= [-3/2\, x - 1/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$

$= (-3/2\, b - 1/2\, b \,)\, Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = -\ Fb^3/EJ$




A = 116.2 $mm^2$
$J_u$ = 58038. $mm^4$
$J_v$ = 1856. $mm^4$
$J_t$ = 84.43 $mm^4$
$y_o$ = -15.56 mm
$y_g$ = 23.24 mm
$T_y$ = -900. N
$M_x$ = -382500. Nmm
$x_m$ = 11. mm
$y_m$ = 55. mm
$v_m$ = 31.76 mm
$\sigma_m = -Mv/J_u$ = 209.3 $N/mm^2$
$y_c$ = 2. mm
$u_c$ = -11. mm
$v_c$ = -21.24 mm
$\sigma_c = -Mv/J_u$ = 209.3 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 10.64 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 10.64 $N/mm^2$
$t_c$ = 300. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 210.1 $N/mm^2$



ESΣ05.xxxx.121

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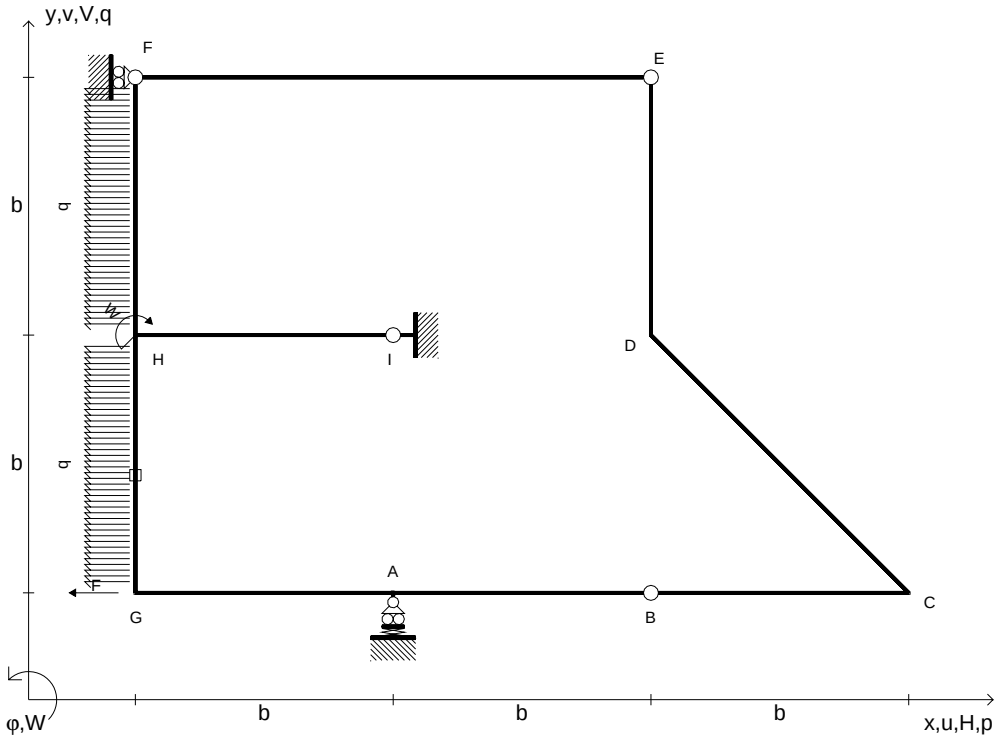
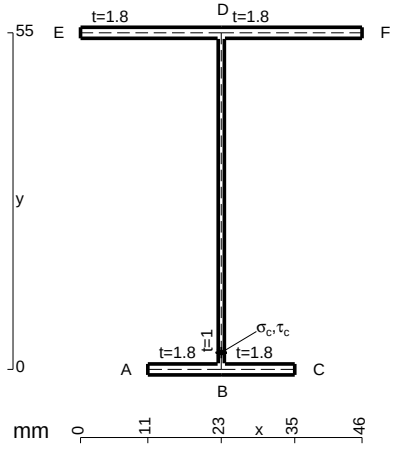



$H_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_H = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$p_{HG} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

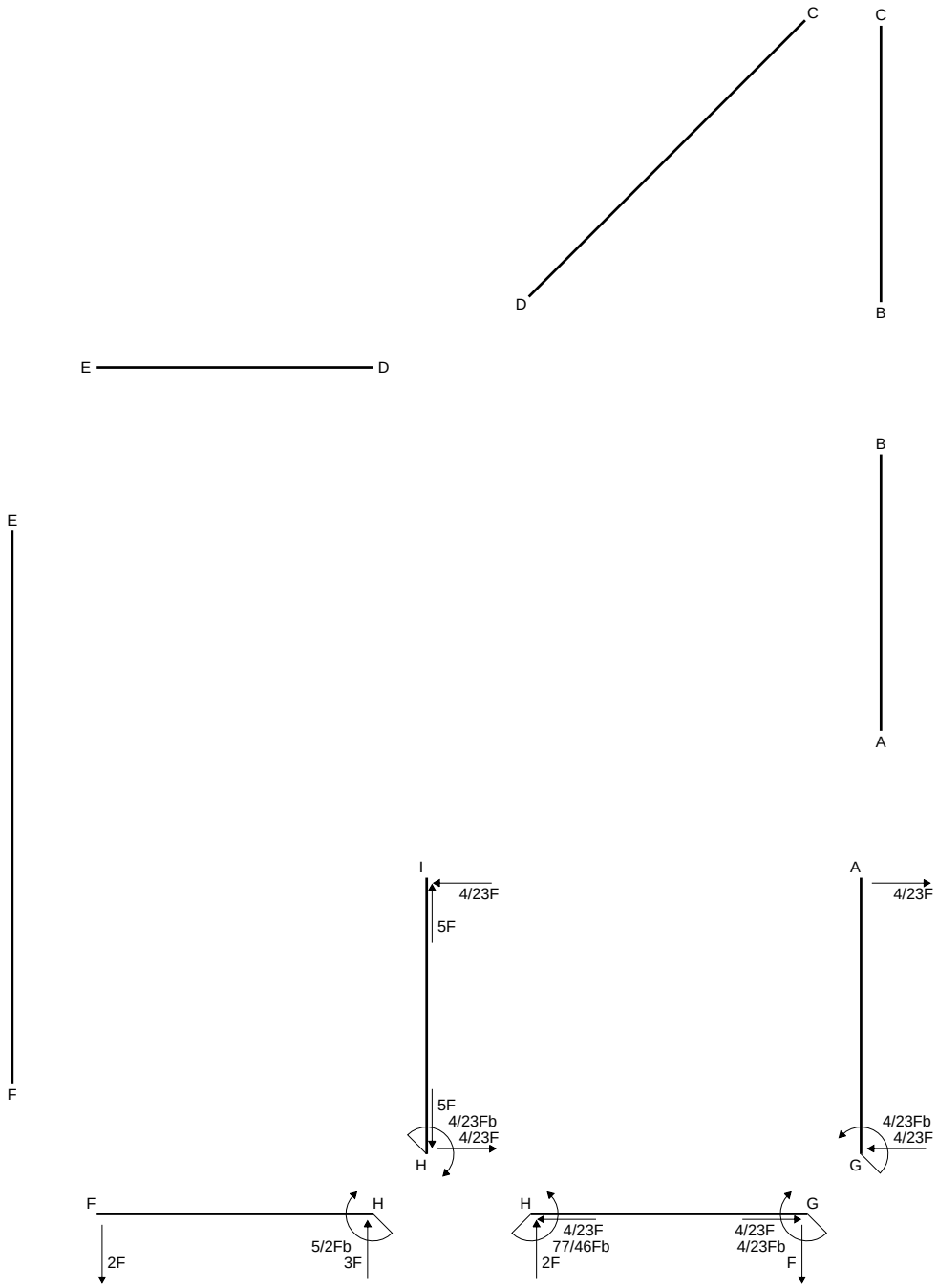
$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

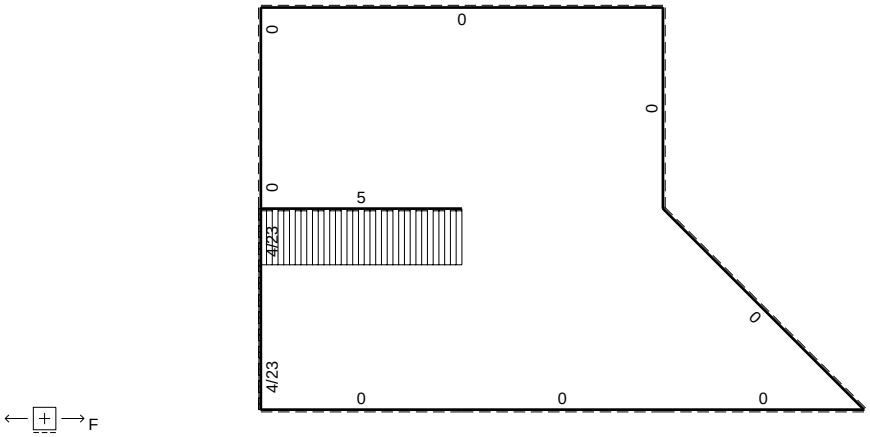



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 610 mm, F = 460 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c,\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

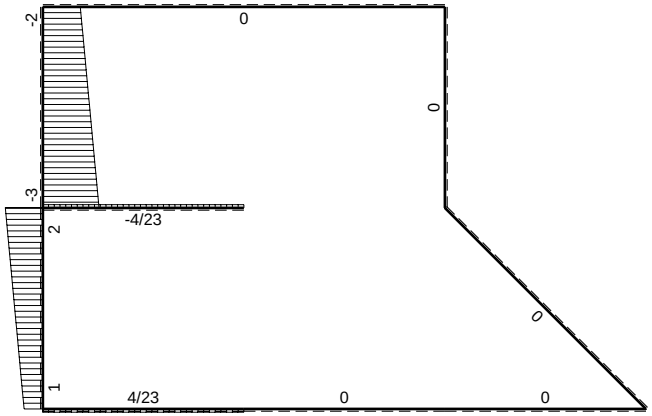




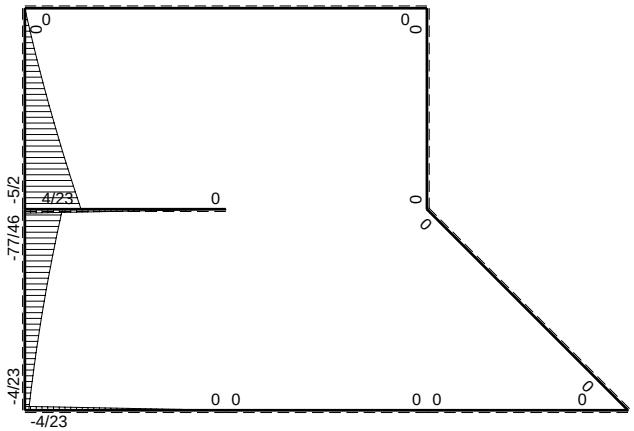






F



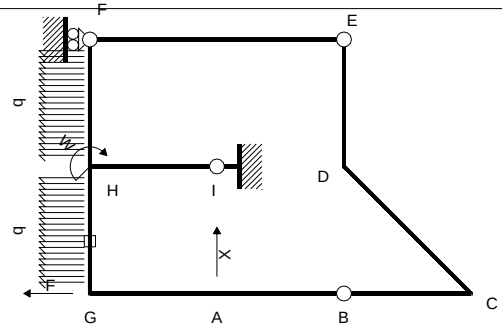

F



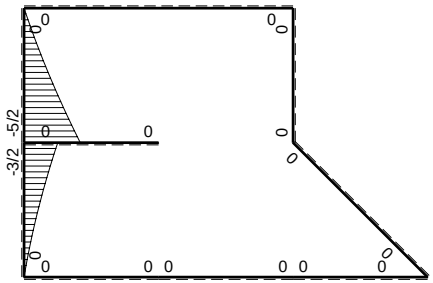

Fb



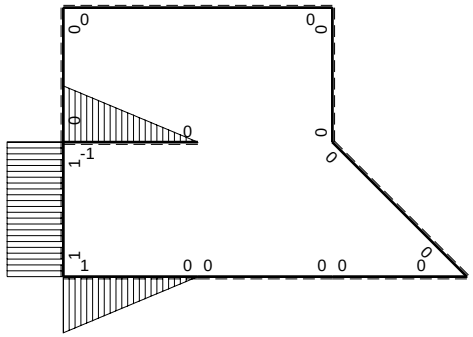





Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | 0 | 0 | 0 | 0 | 0 |
| BA b | 0 | 0 | 0 | 0 | | |
| BC b | 0 | 0 | 0 | 0 | 0 | 0 |
| CB b | 0 | 0 | 0 | 0 | | |
| CD √2b | 0 | 0 | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | b-x | 0 | 0 | $b^2-2bx+x^2$ | 0 | $1/3Xb^3/EJ$ |
| AG b | -x | 0 | 0 | $x^2$ | | |
| FH b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | -b+x | 0 | 0 | $b^2-2bx+x^2$ | 0 | $1/3Xb^3/EJ$ |
| IH b | x | 0 | 0 | $x^2$ | | |
| HG b | b | $-3/2Fb+2Fx-1/2qx^2$ | $-3/2Fb^2+2Fbx-1/2Fx^2$ | $b^2$ | $-2/3Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $Fx+1/2qx^2$ | $-Fbx-1/2Fx^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $1/3Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | -4/23F | |

Sviluppi di calcolo iperstatica



PROCEDIMENTO E RISULTATI 209852 ESΣ05.xxxx.123

$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\,\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\,\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-3/2 +2\, x/b -1/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[-3/2\, x + x^2/b -1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(-3/2\, b + b -1/6\, b \,\right) Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = 1/3\ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-\, x/b -1/2\, x^2/b^2\,\right) Fb^2\, 1/EJ\, dx + 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left[-1/2\, x^2/b -1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ$$

$$= \left(-1/2\, b -1/6\, b \,\right) Fb^2\, 1/EJ + 1 \quad (-1)\ (-1)\ Fb^3/EJ = 1/3\ Fb^3/EJ$$

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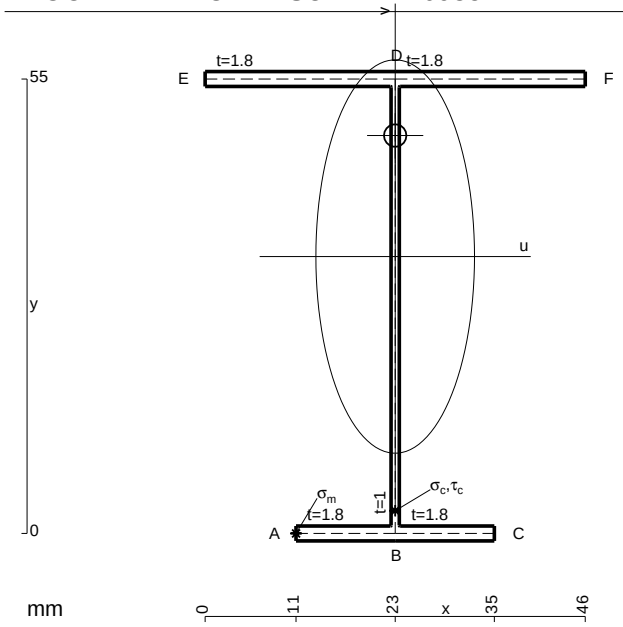


A = 181. $mm^2$
$J_u$ = 102600. $mm^4$
$J_v$ = 16674. $mm^4$
$J_t$ = 154.4 $mm^4$
$y_o$ = 14.64 mm
$y_g$ = 33.52 mm
$T_y$ = -1380. N
$M_x$ = -701500. Nmm
$x_m$ = 11. mm
$u_m$ = -12. mm
$v_m$ = -33.52 mm
$\sigma_m = -Mv/J_u$ = -229.2 $N/mm^2$
$x_c$ = 23. mm
$v_c$ = -33.52 mm
$\sigma_c = -Mv/J_u$ = -229.2 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 19.47 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 19.47 $N/mm^2$
$t_c$ = 460. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 231.6 $N/mm^2$


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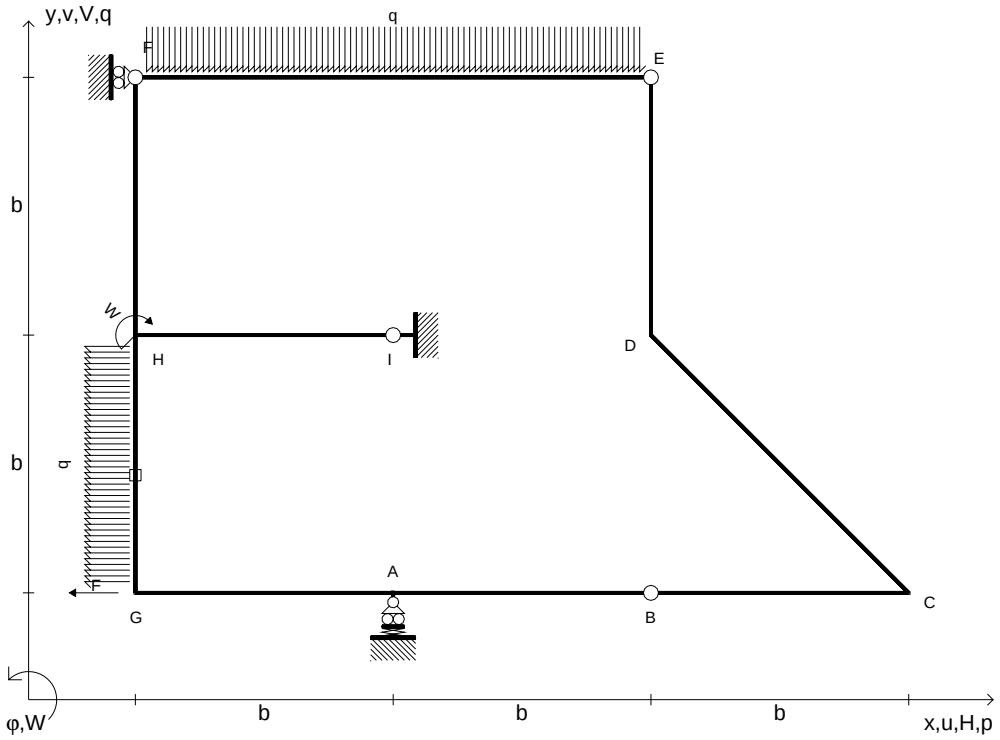
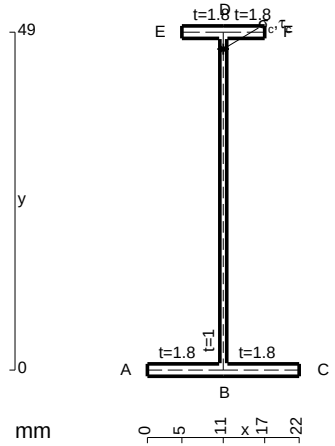



$H_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_H = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

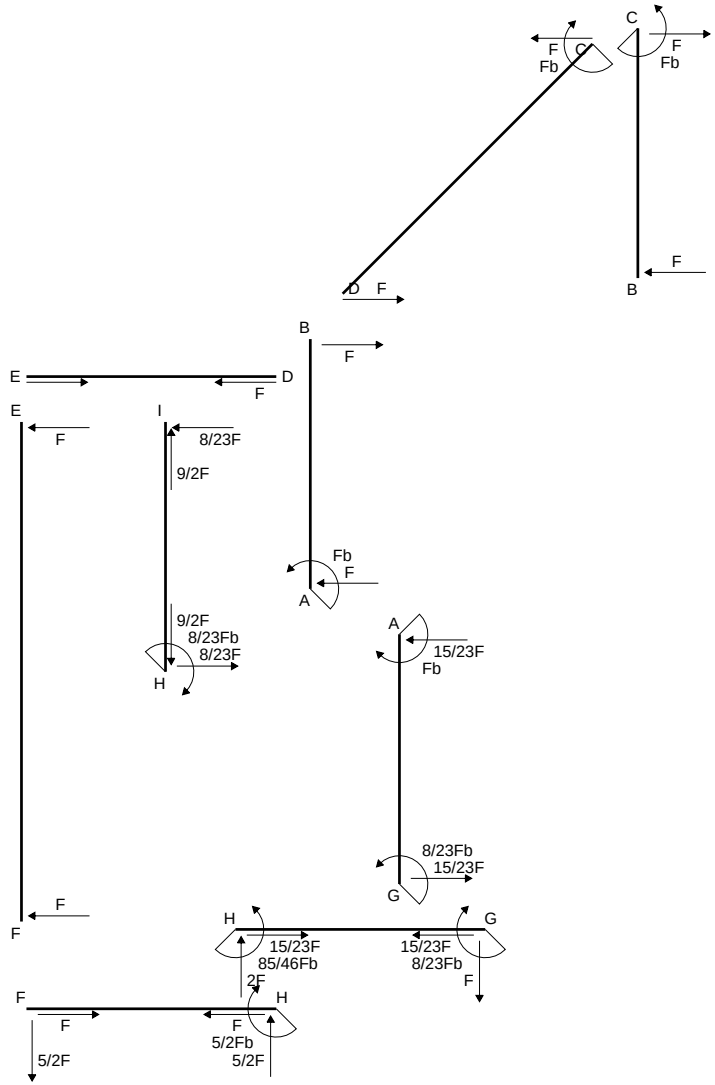
$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

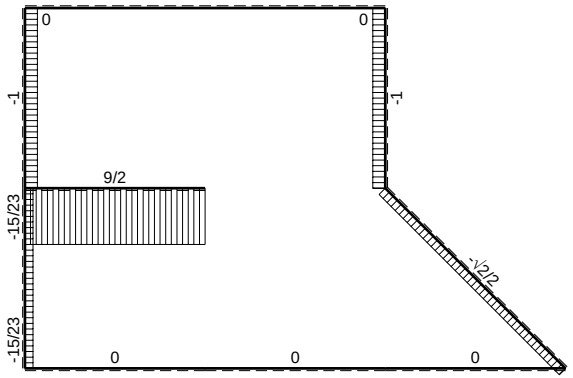



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 580 mm, F = 260 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c,\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

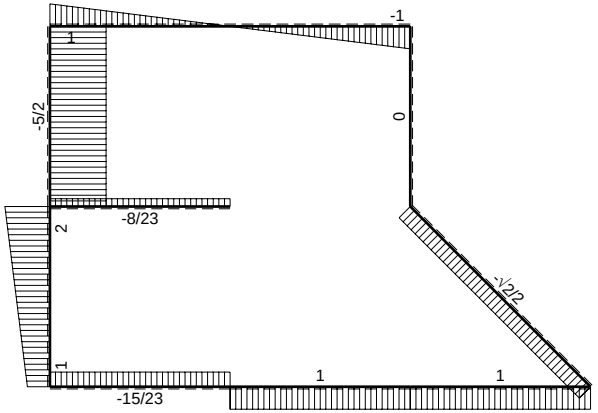




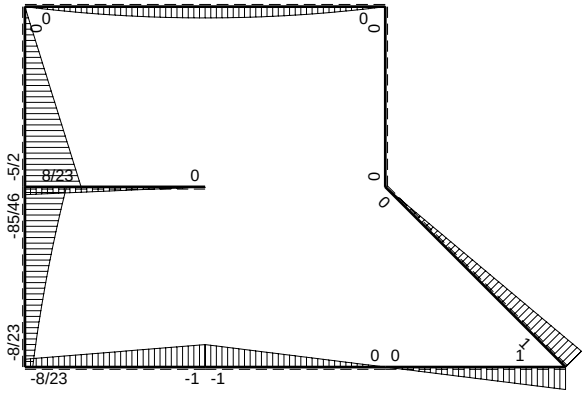





F



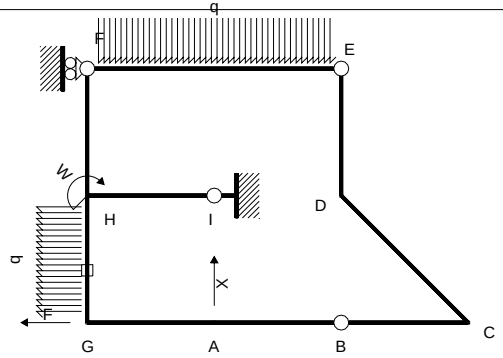

F



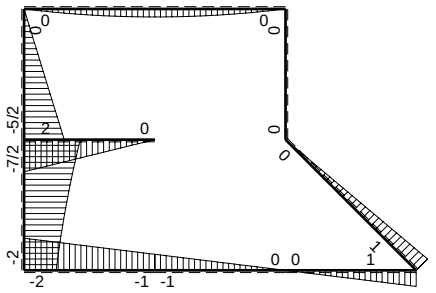

Fb



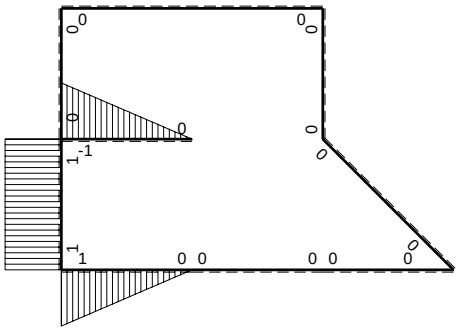





Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb+Fx | 0 | 0 | | |
| CD $\sqrt{2}$b | 0 | Fb-$\sqrt{2}$/2Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | -Fx+1/2q$x^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | Fx-1/2q$x^2$ | 0 | 0 | | |
| GA b | b-x | -2Fb+Fx | -2F$b^2$+3Fbx-F$x^2$ | $b^2$-2bx+$x^2$ | -5/6F$b^3$/EJ | 1/3X$b^3$/EJ |
| AG b | -x | Fb+Fx | -Fbx-F$x^2$ | $x^2$ | | |
| FH b | 0 | -5/2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | | |
| HI b | -b+x | 2Fb-2Fx | -2F$b^2$+4Fbx-2F$x^2$ | $b^2$-2bx+$x^2$ | -2/3F$b^3$/EJ | 1/3X$b^3$/EJ |
| IH b | x | -2Fx | -2F$x^2$ | $x^2$ | | |
| HG b | b | -7/2Fb+2Fx-1/2q$x^2$ | -7/2F$b^2$+2Fbx-1/2F$x^2$ | $b^2$ | -8/3F$b^3$/EJ | X$b^3$/EJ |
| GH b | -b | 2Fb+Fx+1/2q$x^2$ | -2F$b^2$-Fbx-1/2F$x^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | F$b^3$/EJ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | 1/4X$b^3$/EJ |
| | totali | | | | -19/6F$b^3$/EJ | 23/12X$b^3$/EJ |
| | iperstatica $X=V_A$ | | | | 38/23F | |

Sviluppi di calcolo iperstatica





$L_{GA}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{AG}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HI}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{IH}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HG}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{GH}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L_{GA}^{Xo} = \int_o^b (-2 + 3\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\,x + 3/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-2\,b + 3/2\,b - 1/3\,b \,)\, Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$

$L_{AG}^{Xo} = \int_o^b (-\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\,b - 1/3\,b \,)\, Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$

$L_{HI}^{Xo} = \int_o^b (-2 + 4\, x/b - 2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-2\,x + 2\, x^2/b - 2/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-2\,b + 2\,b - 2/3\,b \,)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

$L_{IH}^{Xo} = \int_o^b (-2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$

$= (-2/3\,b \,)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

$L_{HG}^{Xo} = \int_o^b (-7/2 + 2\, x/b - 1/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

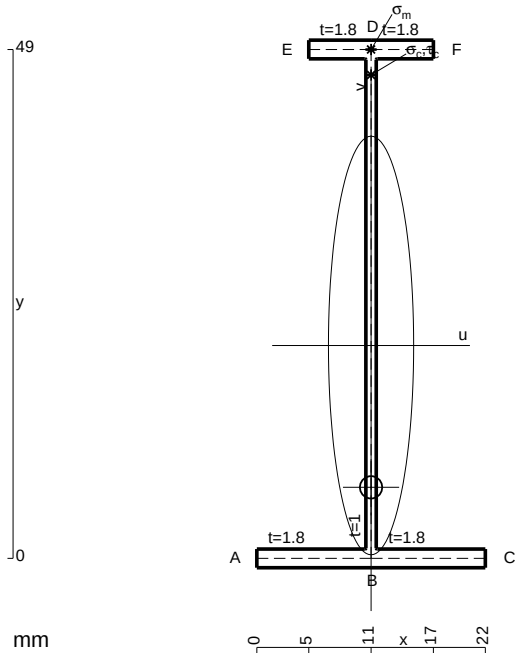
$= [-7/2\, x + x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-7/2\,b + b - 1/6\,b \,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -5/3\ \ Fb^3/EJ$

$L_{GH}^{Xo} = \int_o^b (-2 - x/b - 1/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [-2\,x - 1/2\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-2\,b - 1/2\,b - 1/6\,b \,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -5/3\ \ Fb^3/EJ$




A = 110.2 $mm^2$
$J_u$ = 44775. $mm^4$
$J_v$ = 1856. $mm^4$
$J_t$ = 82.43 $mm^4$
$y_o$ = -13.66 mm
$y_g$ = 20.5 mm
N = -260. N
$T_y$ = -650. N
$M_x$ = -377000. Nmm
$x_m$ = 11. mm
$y_m$ = 49. mm
$v_m$ = 28.5 mm
$\sigma_m = N/A - Mv/J_u$ = 237.6 $N/mm^2$
$y_c$ = 2. mm
$u_c$ = -11. mm
$v_c$ = -18.5 mm
$\sigma_c = N/A - Mv/J_u$ = 237.6 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 8.937 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 8.937 $N/mm^2$
$t_c$ = 260. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 238.1 $N/mm^2$



ESΣ05.xxxx.124

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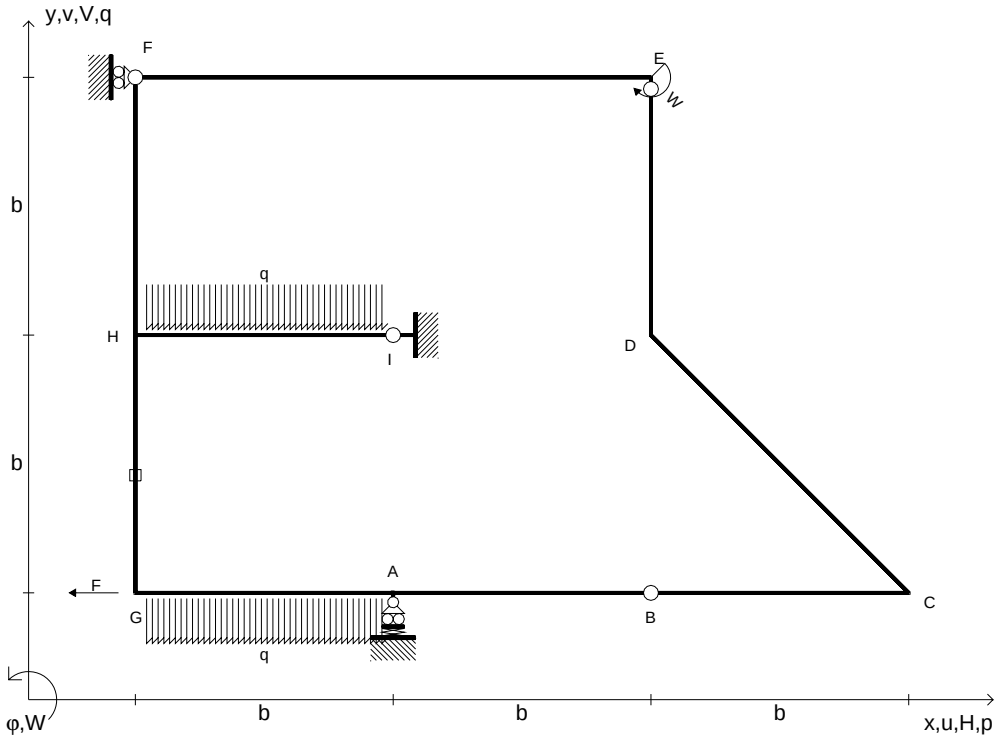
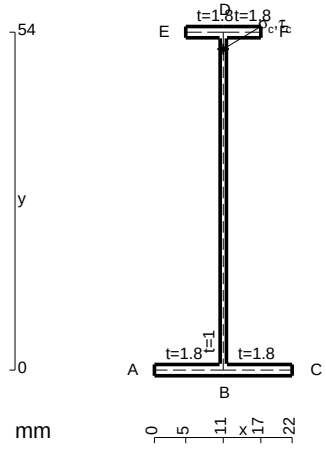



$H_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_E = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{HI} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

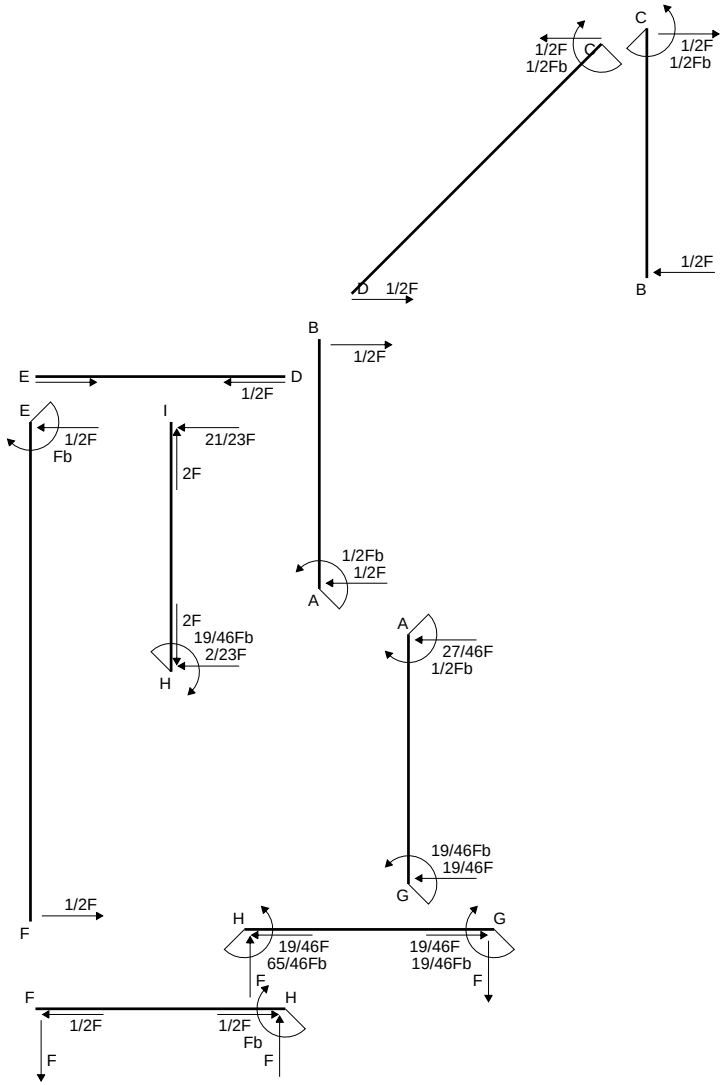
$q_{GA} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

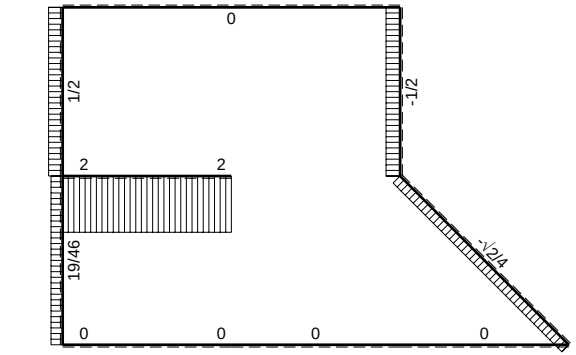



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 690 mm, F = 510 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

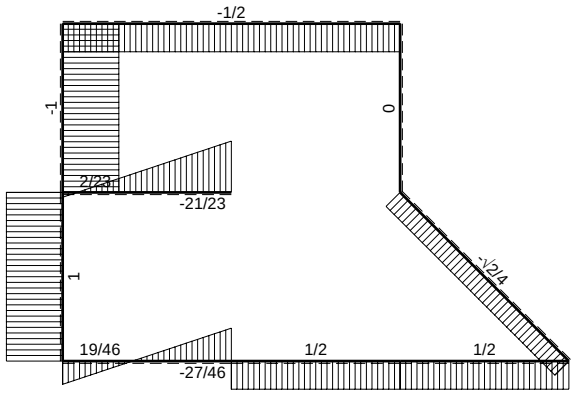




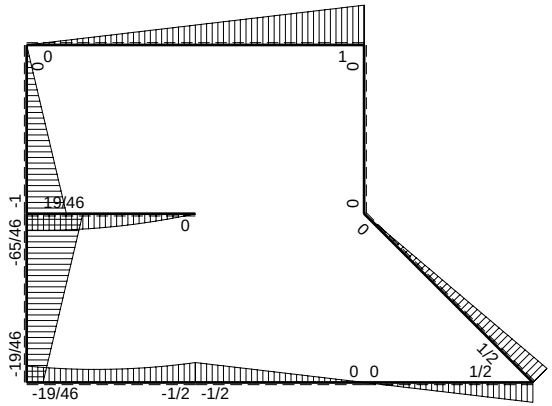





F



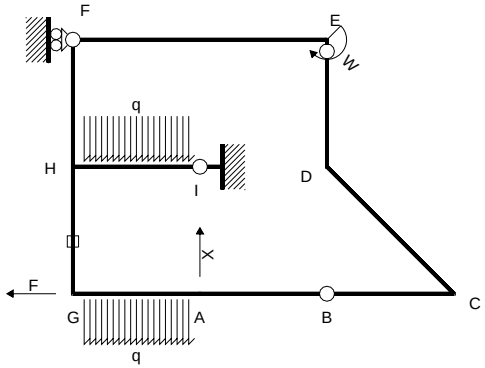

F



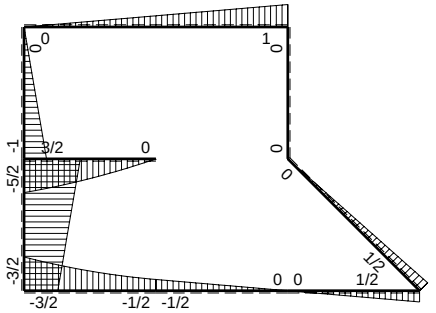

Fb



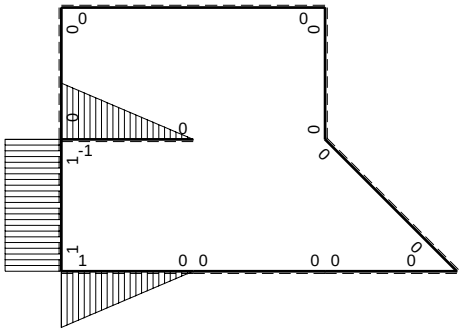





Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-1/2Fb+1/2Fx$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $1/2Fx$ | 0 | 0 | | |
| BC b | 0 | $1/2Fx$ | 0 | 0 | 0 | 0 |
| CB b | 0 | $-1/2Fb+1/2Fx$ | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $1/2Fb-\sqrt{2}/4Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | $Fb-1/2Fx$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $-1/2Fx$ | 0 | 0 | | |
| GA b | b-x | $-3/2Fb+3/2Fx-1/2qx^2$ | $-3/2Fb^2+3Fbx-2Fx^2+1/2qx^3$ | $b^2-2bx+x^2$ | $-13/24Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | $1/2Fb+1/2Fx+1/2qx^2$ | $-1/2Fbx-1/2Fx^2-1/2qx^3$ | $x^2$ | | |
| FH b | 0 | $-Fx$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $Fb-Fx$ | 0 | 0 | | |
| HI b | -b+x | $3/2Fb-Fx-1/2qx^2$ | $-3/2Fb^2+5/2Fbx-1/2Fx^2-1/2qx^3$ | $b^2-2bx+x^2$ | $-13/24Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | $-2Fx+1/2qx^2$ | $-2Fx^2+1/2qx^3$ | $x^2$ | | |
| HG b | b | $-5/2Fb+Fx$ | $-5/2Fb^2+Fbx$ | $b^2$ | $-2Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $3/2Fb+Fx$ | $-3/2Fb^2-Fbx$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-25/12Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 25/23F | |



PROCEDIMENTO E RISULTATI 210204 ESΣ05.xxxx.145

Sviluppi di calcolo iperstatica

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$L^{XX}_{GA} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-3/2 + 3\,x/b - 2\,x^2/b^2 + 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-3/2\, x + 3/2\, x^2/b - 2/3\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 3/2\, b - 2/3\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = -13/24\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (-1/2\,x/b - 1/2\,x^2/b^2 - 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^2/b - 1/6\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/4\, b - 1/6\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = -13/24\ Fb^3/EJ$

$L^{Xo}_{HI} = \int_o^b (-3/2 + 5/2\,x/b - 1/2\,x^2/b^2 - 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-3/2\, x + 5/4\, x^2/b - 1/6\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 5/4\, b - 1/6\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = -13/24\ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b (-2\,x^2/b^2 + 1/2\,x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-2/3\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = -13/24\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (-5/2 + x/b \,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [-5/2\, x + 1/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= (-5/2\, b + 1/2\, b \,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -\ Fb^3/EJ$

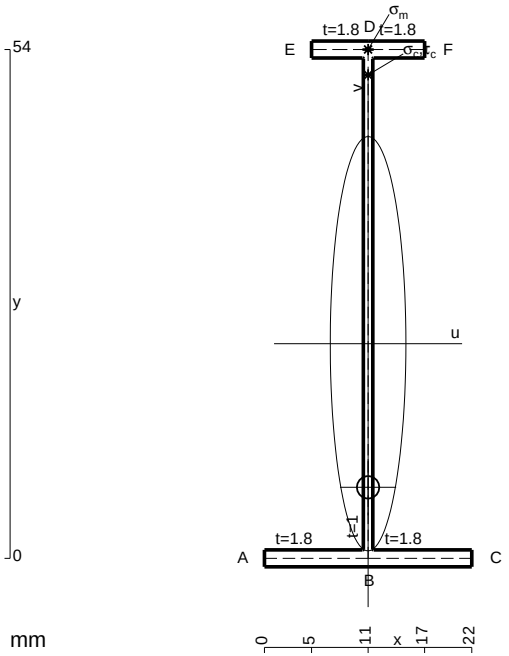
$L^{Xo}_{GH} = \int_o^b (-3/2 - x/b \,)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

$= [-3/2\, x - 1/2\, x^2/b \,]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

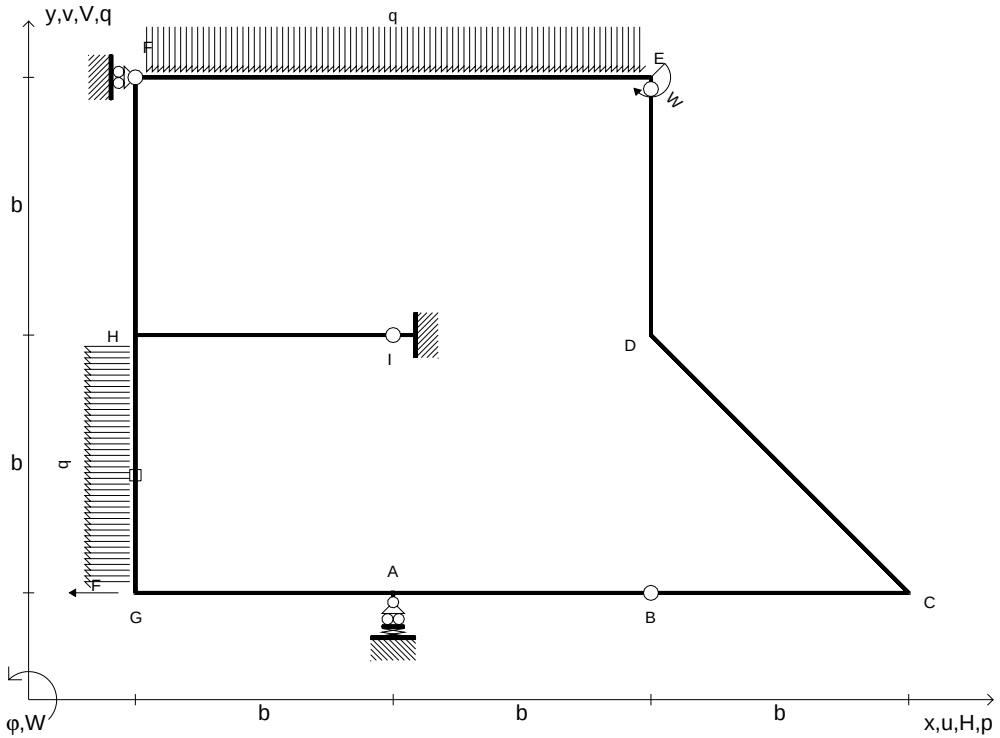
$= (-3/2\, b - 1/2\, b \,)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -\ Fb^3/EJ$








A = 115.2 $mm^2$
$J_u$ = 55687. $mm^4$
$J_v$ = 1856. $mm^4$
$J_t$ = 84.1 $mm^4$
$y_o$ = -15.24 mm
$y_g$ = 22.78 mm
N = 255. N
$T_y$ = -510. N
$M_x$ = -351900. Nmm
$x_m$ = 11. mm
$y_m$ = 54. mm
$v_m$ = 31.22 mm
$\sigma_m = N/A - Mv/J_u$ = 199.5 $N/mm^2$
$y_c$ = 2. mm
$u_c$ = -11. mm
$v_c$ = -20.78 mm
$\sigma_c = N/A - Mv/J_u$ = 199.5 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 6.176 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 6.176 $N/mm^2$
$t_c$ = 510. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 199.8 $N/mm^2$

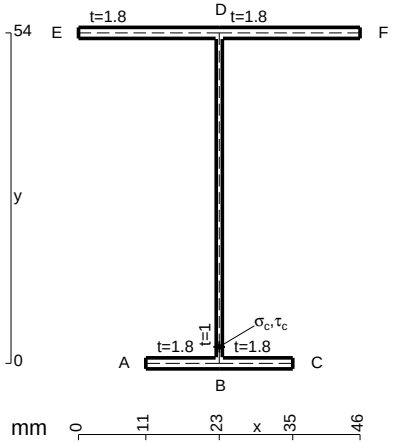



$H_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_E = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

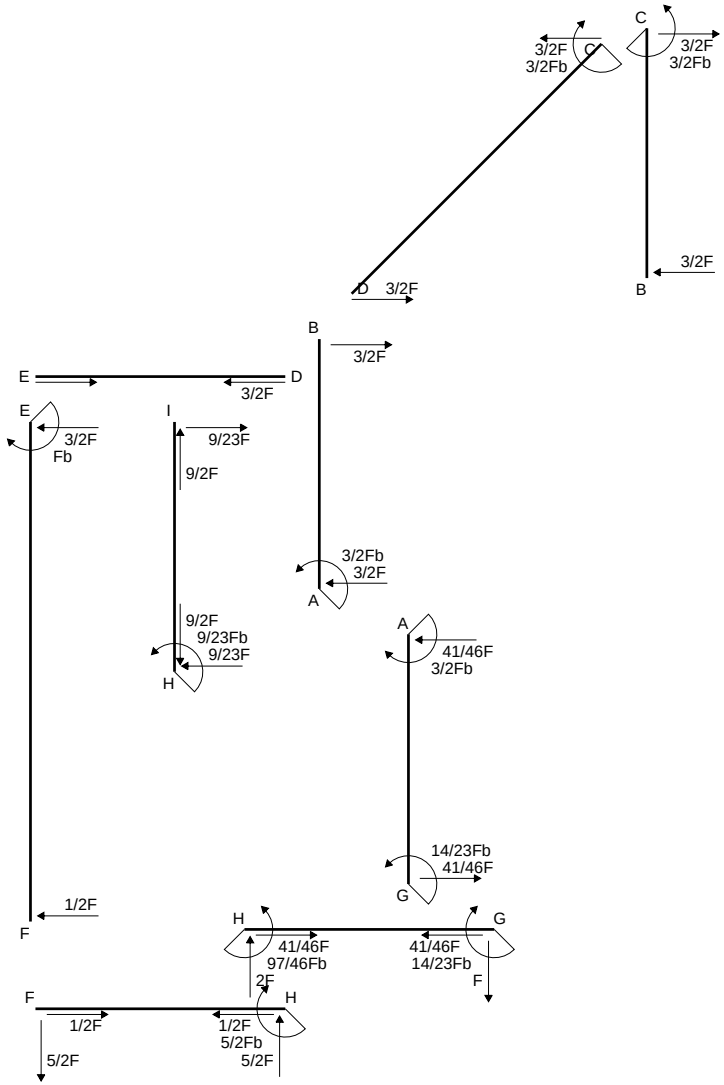
$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

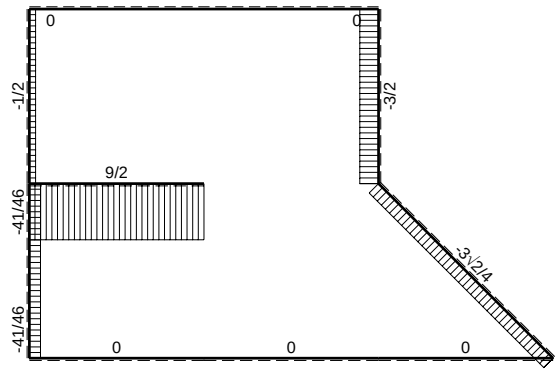



Reazioni iperstatiche in soluzione: $X=W_{GA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 780 mm, F = 330 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

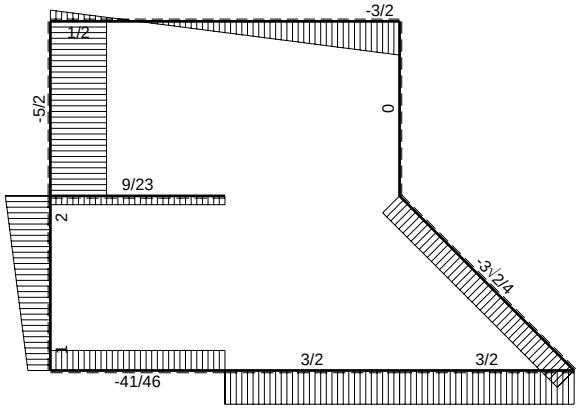




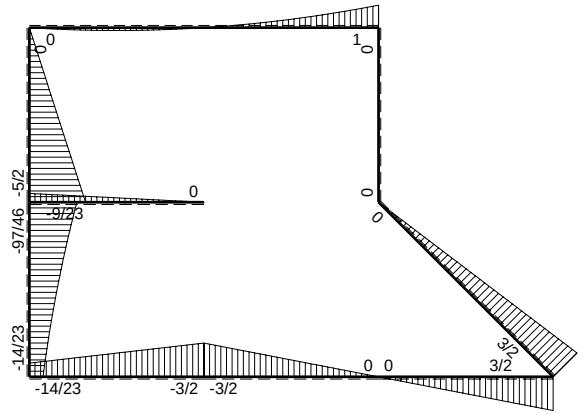





F



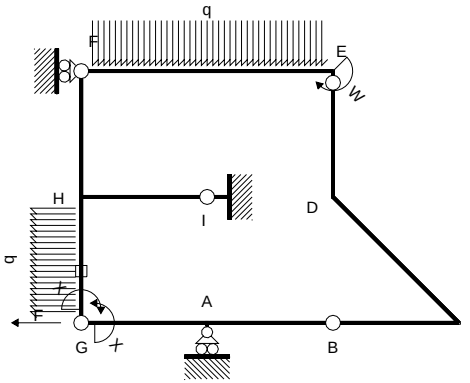

F

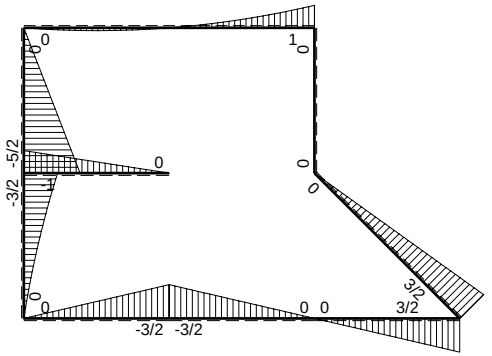



Fb

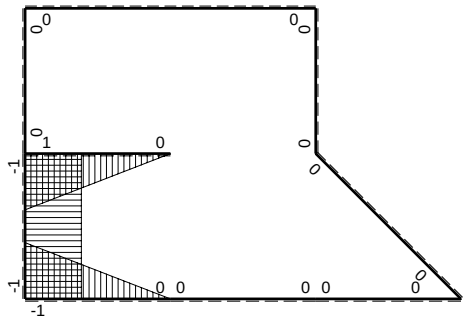







$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -3/2Fb+3/2Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | 3/2Fx | 0 | 0 | | |
| BC b | 0 | 3/2Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -3/2Fb+3/2Fx | 0 | 0 | | |
| CD √2b | 0 | 3/2Fb-3√2/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | Fb-3/2Fx+1/2qx$^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 1/2Fx-1/2qx$^2$ | 0 | 0 | | |
| GA b | -1+x/b | -3/2Fx | 3/2Fx-3/2Fx$^2$/b | 1-2x/b+x$^2$/b$^2$ | 1/4Fb$^2$/EJ | 1/3Xb/EJ |
| AG b | x/b | 3/2Fb-3/2Fx | 3/2Fx-3/2Fx$^2$/b | x$^2$/b$^2$ | | |
| FH b | 0 | -5/2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | | |
| HI b | 1-x/b | -Fb+Fx | -Fb+2Fx-Fx$^2$/b | 1-2x/b+x$^2$/b$^2$ | -1/3Fb$^2$/EJ | 1/3Xb/EJ |
| IH b | -x/b | Fx | -Fx$^2$/b | x$^2$/b$^2$ | | |
| HG b | -1 | -3/2Fb+2Fx-1/2qx$^2$ | 3/2Fb-2Fx+1/2Fx$^2$/b | 1 | 2/3Fb$^2$/EJ | Xb/EJ |
| GH b | 1 | Fx+1/2qx$^2$ | Fx+1/2Fx$^2$/b | 1 | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | -Fb$^2$/EJ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | -3/4Fb$^2$/EJ | 1/4Xb/EJ |
| | totali | | | | -7/6Fb$^2$/EJ | 23/12Xb/EJ |
| | iperstatica $X=W_{GA}$ | | | | 14/23Fb | |

Sviluppi di calcolo iperstatica





$L_{GA}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{AG}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\ b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{HI}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\,x - x^2/b + 1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (\,b - b + 1/3\ b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{IH}^{XX} = \int_o^b (\,x^2/b^2\,)\ 1/EJ\ dx = [1/3\ x^3/b^2\,]_o^b\ 1/EJ$

$= (1/3\ b\,)\ 1/EJ = 1/3\ b/EJ$

$L_{HG}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\,x\,]_o^b\ 1/EJ$

$= (\,b\,)\ 1/EJ = b/EJ$

$L_{GH}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\,x\,]_o^b\ 1/EJ$

$= (\,b\,)\ 1/EJ = b/EJ$

$L_{GA}^{Xo} = \int_o^b (3/2\ x/b - 3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b - 1/2\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

$= (3/4\ b - 1/2\ b\,)\ Fb\ 1/EJ = 1/4\ Fb^2/EJ$

$L_{AG}^{Xo} = \int_o^b (3/2\ x/b - 3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b - 1/2\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

$= (3/4\ b - 1/2\ b\,)\ Fb\ 1/EJ = 1/4\ Fb^2/EJ$

$L_{HI}^{Xo} = \int_o^b (-1 + 2\,x/b - x^2/b^2)\ Fb\ 1/EJ\ dx = [-\,x + x^2/b - 1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

$= (-\,b + b - 1/3\ b\,)\ Fb\ 1/EJ = -1/3\ Fb^2/EJ$

$L_{IH}^{Xo} = \int_o^b (-\,x^2/b^2\,)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2\,]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b\,)\ Fb\ 1/EJ = -1/3\ Fb^2/EJ$

$L_{HG}^{Xo} = \int_o^b (3/2 - 2\,x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

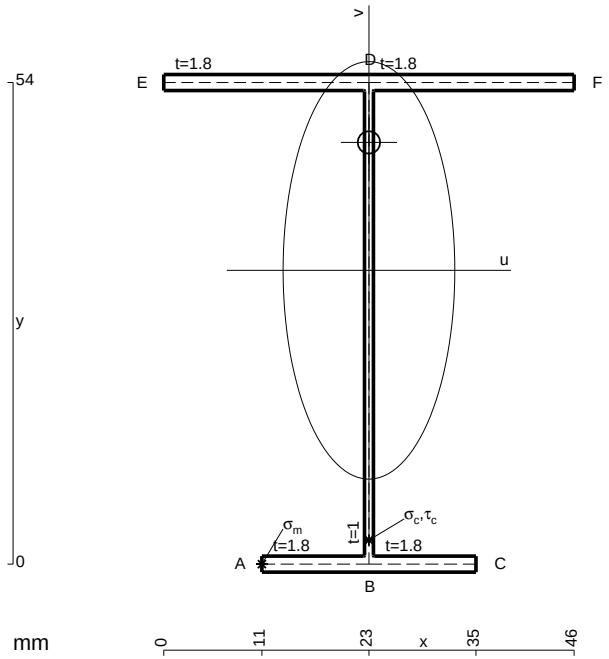
$= [3/2\ x - x^2/b + 1/6\ x^3/b^2\,]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (3/2\ b - b + 1/6\ b\,)\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -1/3\ \ Fb^2/EJ$

$L_{GH}^{Xo} = \int_o^b (\,x/b + 1/2\ x^2/b^2\,)\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= [1/2\ x^2/b + 1/6\ x^3/b^2\,]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/2\ b + 1/6\ b\,)\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -1/3\ \ Fb^2/EJ$




A = 180. $mm^2$
$J_u$ = 98625. $mm^4$
$J_v$ = 16674. $mm^4$
$J_t$ = 154.1 $mm^4$
$y_o$ = 14.34 mm
$y_g$ = 32.94 mm
N = -165. N
$T_y$ = -825. N
$M_x$ = -643500. Nmm
$x_m$ = 11. mm
$u_m$ = -12. mm
$v_m$ = -32.94 mm
$\sigma_m = N/A - Mv/J_u$ = -215.8 $N/mm^2$
$x_c$ = 23. mm
$v_c$ = -32.94 mm
$\sigma_c = N/A - Mv/J_u$ = -215.8 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 11.9 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 11.9 $N/mm^2$
$t_c$ = 330. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 216.8 $N/mm^2$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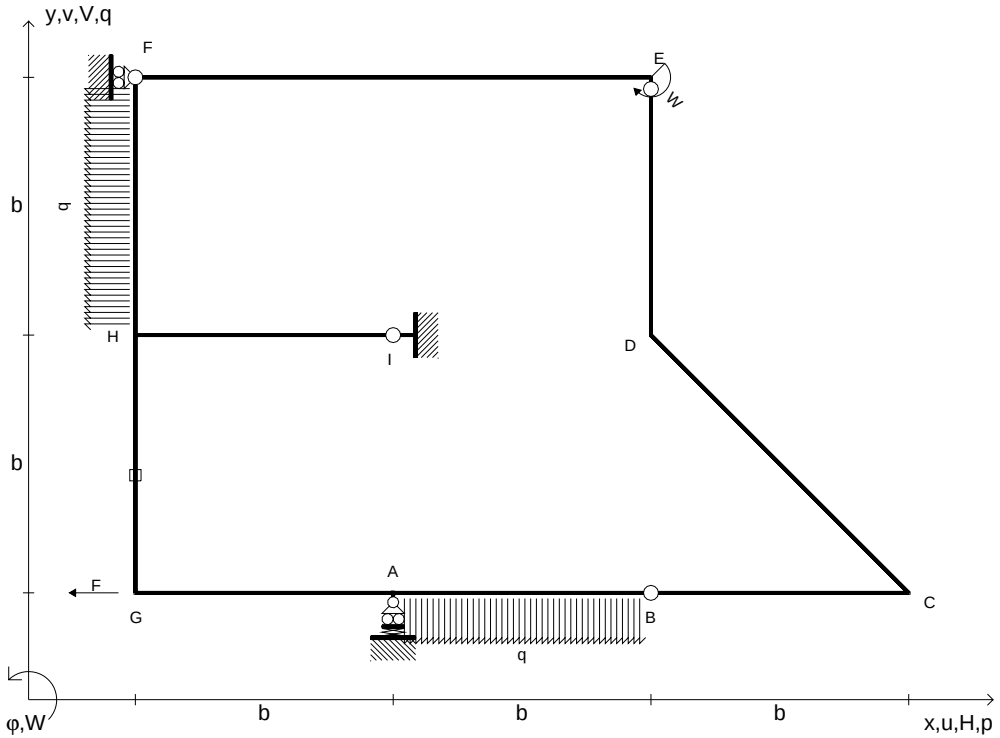
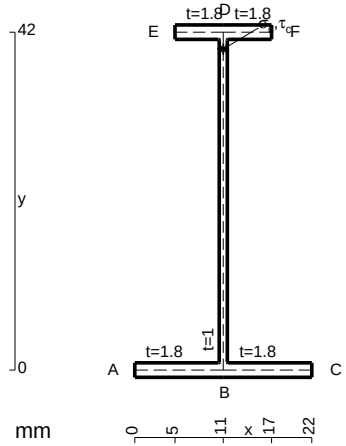



$H_G = -F$ $\quad \varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $\quad EJ_{CD} = EJ$ $\quad EJ_{FH} = EJ$

$W_E = -W = -Fb$ $\quad k_A = 4EJ/b^3$ $\quad EJ_{DE} = EJ$ $\quad EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ $\quad EJ_{AB} = EJ$ $\quad EJ_{EF} = EJ$ $\quad EJ_{HG} = EJ$

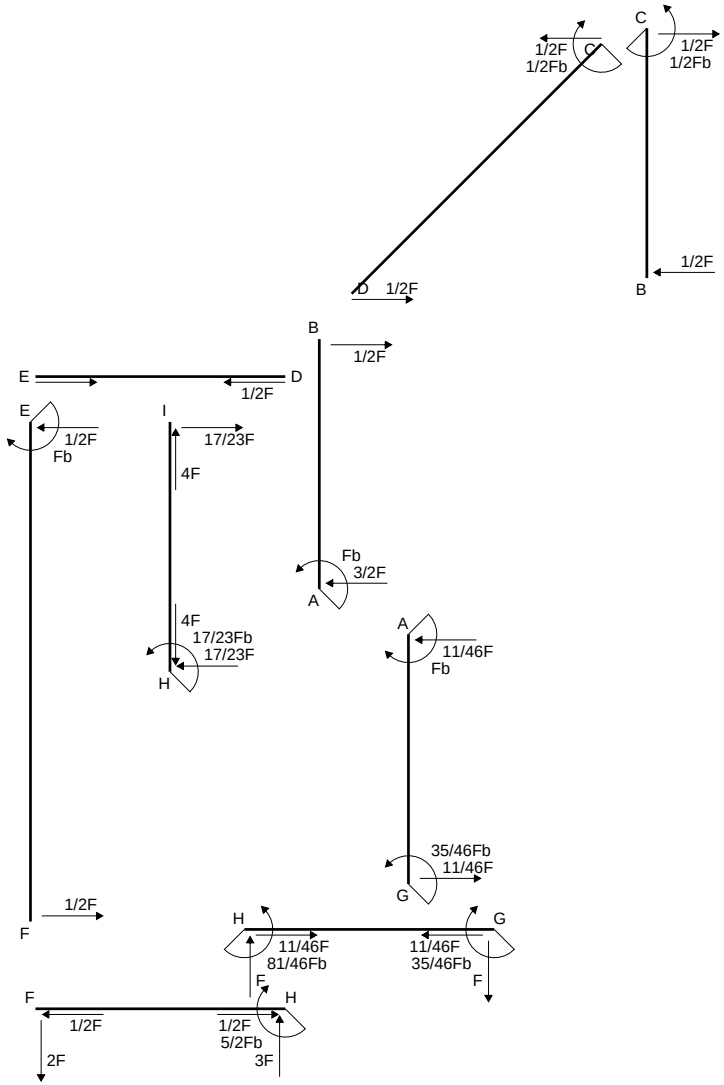
$p_{FH} = -q = -F/b$ $\quad EJ_{BC} = EJ$ $\quad EJ_{GA} = EJ$

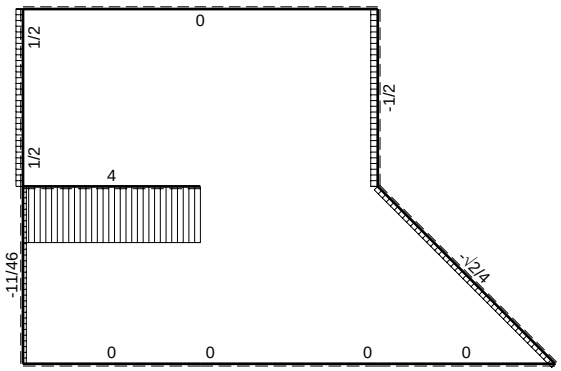



Reazioni iperstatiche in soluzione: $X=W_{GA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 390 mm, F = 270 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

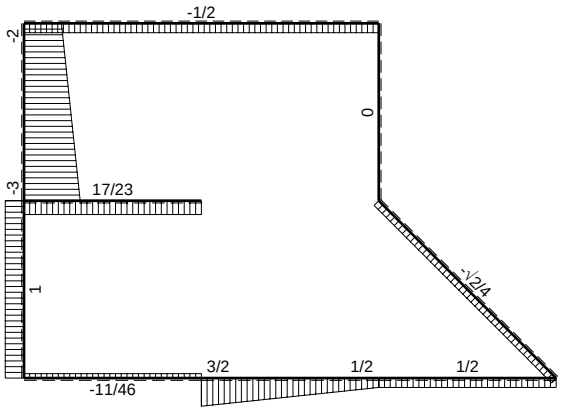




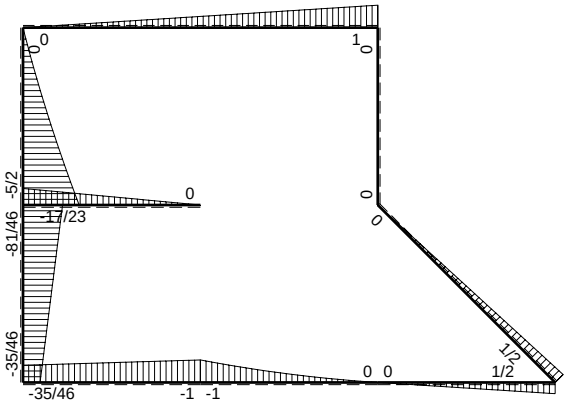





F



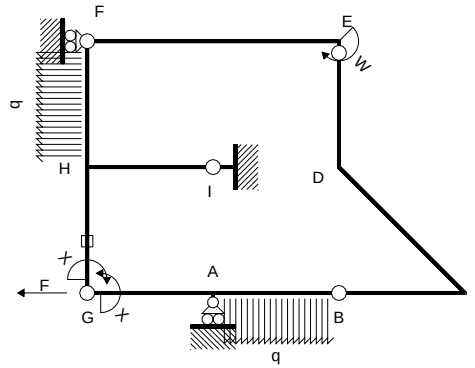

F



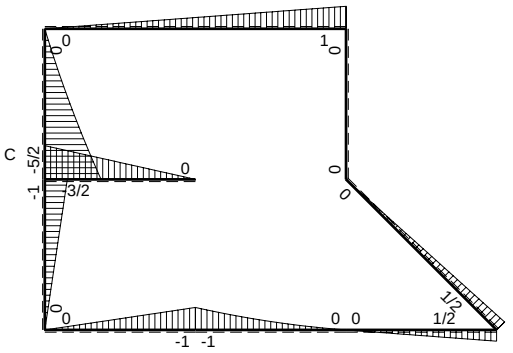

Fb



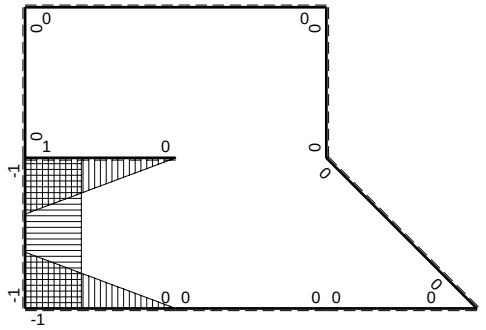





Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-Fb+3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $1/2Fx+1/2qx^2$ | 0 | 0 | | |
| BC b | 0 | 1/2Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -1/2Fb+1/2Fx | 0 | 0 | | |
| CD √2b | 0 | 1/2Fb-√2/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | Fb-1/2Fx | 0 | 0 | 0 | 0 |
| FE 2b | 0 | -1/2Fx | 0 | 0 | | |
| GA b | -1+x/b | -Fx | $Fx-Fx^2/b$ | $1-2x/b+x^2/b^2$ | $1/6Fb^2/EJ$ | 1/3Xb/EJ |
| AG b | x/b | Fb-Fx | $Fx-Fx^2/b$ | $x^2/b^2$ | | |
| FH b | 0 | $-2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $5/2Fb-3Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | 1-x/b | -3/2Fb+3/2Fx | $-3/2Fb+3Fx-3/2Fx^2/b$ | $1-2x/b+x^2/b^2$ | $-1/2Fb^2/EJ$ | 1/3Xb/EJ |
| IH b | -x/b | 3/2Fx | $-3/2Fx^2/b$ | $x^2/b^2$ | | |
| HG b | -1 | -Fb+Fx | Fb-Fx | 1 | $1/2Fb^2/EJ$ | Xb/EJ |
| GH b | 1 | Fx | Fx | 1 | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^2/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-5/8Fb^2/EJ$ | 1/4Xb/EJ |
| | totali | | | | $-35/24Fb^2/EJ$ | 23/12Xb/EJ |
| | iperstatica $X=W_{GA}$ | | | | 35/46Fb | |

Sviluppi di calcolo iperstatica



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$L^{XX}_{GA} = \int_o^b (1 -2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{AG} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{HI} = \int_o^b (1 -2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{IH} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{HG} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{GH} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{Xo}_{GA} = \int_o^b (\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{AG} = \int_o^b (\ x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L^{Xo}_{HI} = \int_o^b (-3/2 + 3\ x/b - 3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/2\ x + 3/2\ x^2/b - 1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/2\ b + 3/2\ b - 1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{IH} = \int_o^b (-3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{HG} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{GH} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -1/2\ \ Fb^2/EJ$

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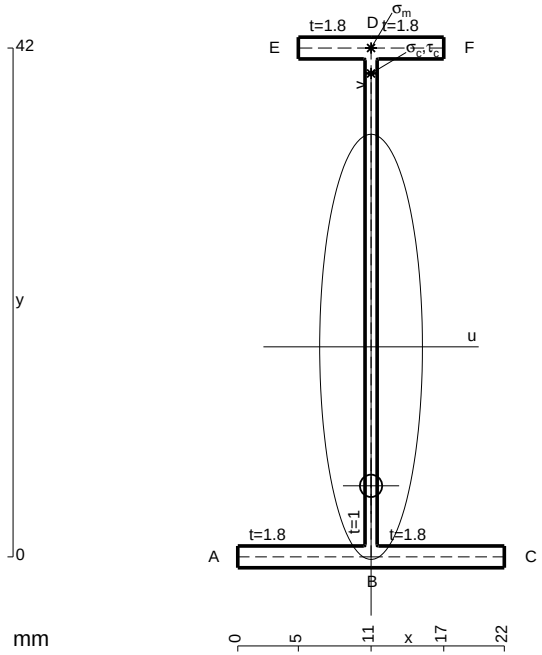


A = 103.2 mm$^2$
$J_u$ = 31779. mm$^4$
$J_v$ = 1856. mm$^4$
$J_t$ = 80.1 mm$^4$
$y_o$ = -11.47 mm
$y_g$ = 17.34 mm
N = 135. N
$T_y$ = -810. N
$M_x$ = -263250. Nmm
$x_m$ = 11. mm
$y_m$ = 42. mm
$v_m$ = 24.66 mm
$\sigma_m = N/A - Mv/J_u$ = 205.6 N/mm$^2$
$y_c$ = 2. mm
$u_c$ = -11. mm
$v_c$ = -15.34 mm
$\sigma_c = N/A - Mv/J_u$ = 205.6 N/mm$^2$
$\tau_c = TS^*/tJ_u$ = 13.58 N/mm$^2$
$\tau_g = TS^*/tJ_u$ = 13.58 N/mm$^2$
$t_c$ = 270. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 207. N/mm$^2$



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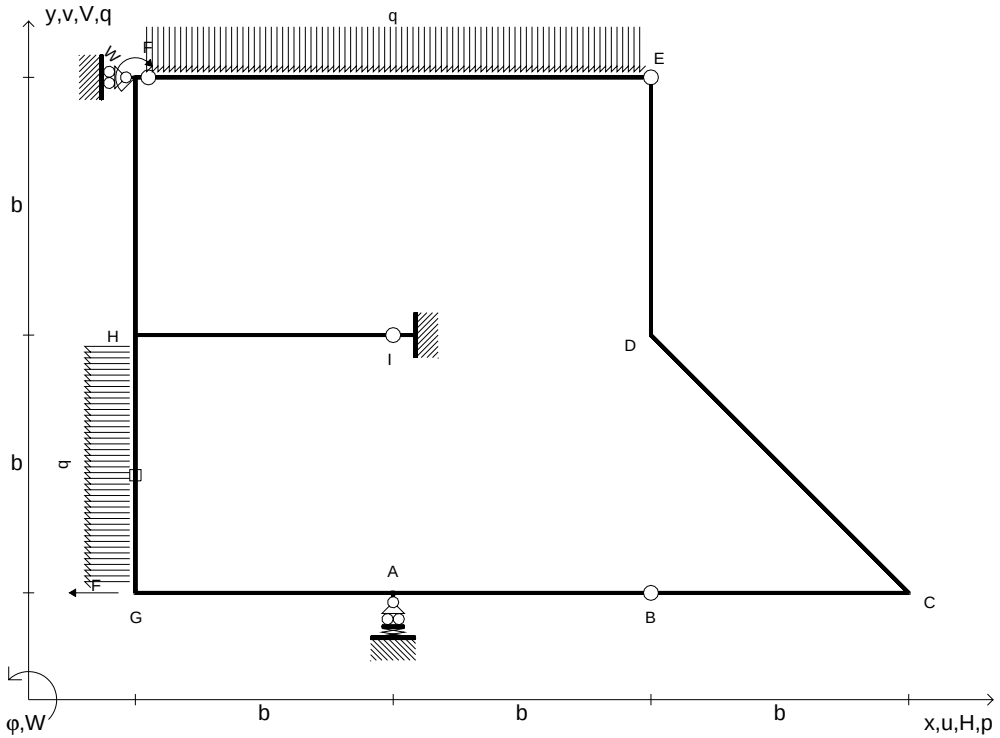
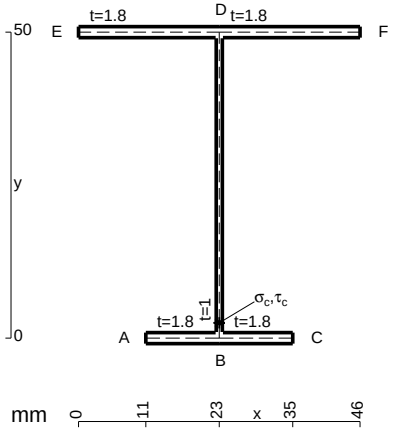



$H_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_F = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

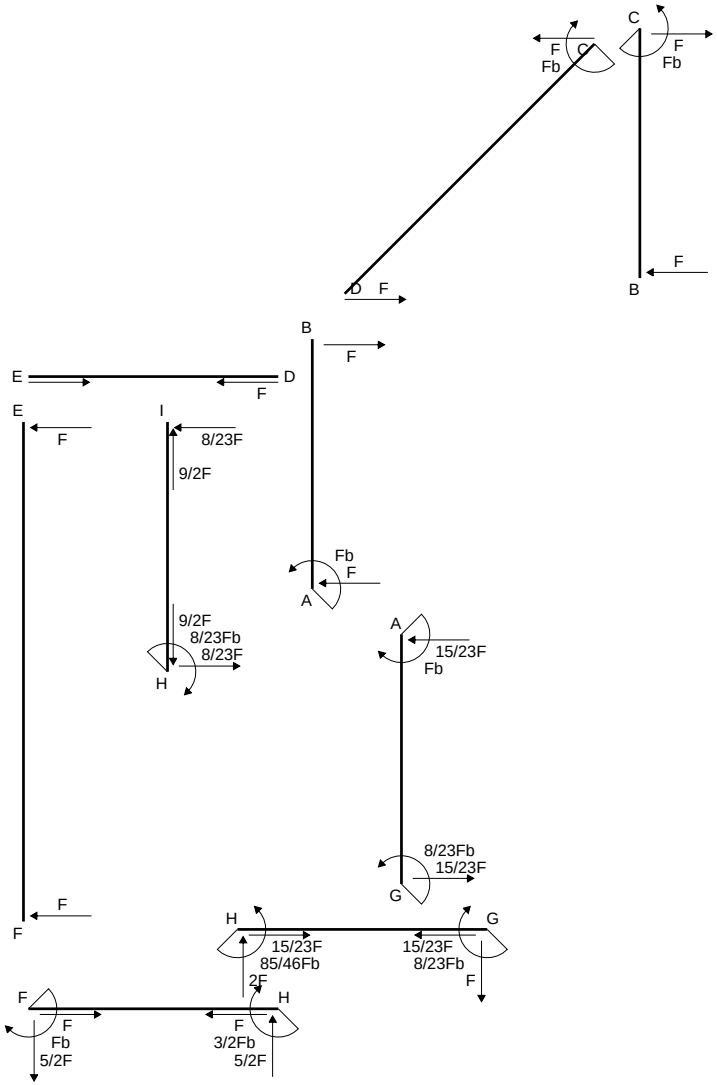
$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

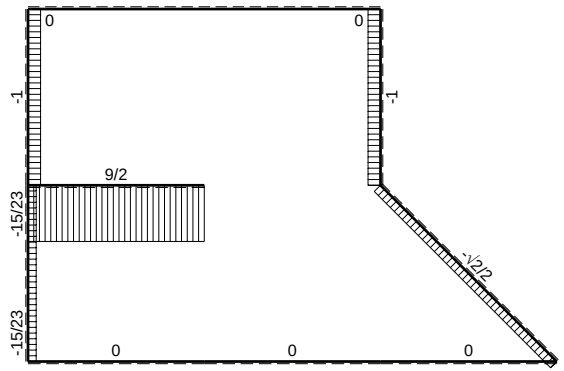



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 600 mm, F = 710 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

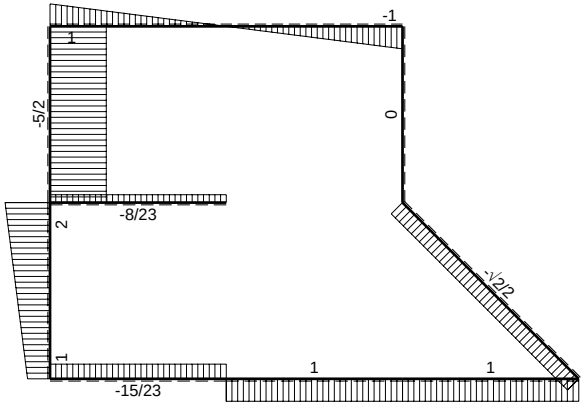







F

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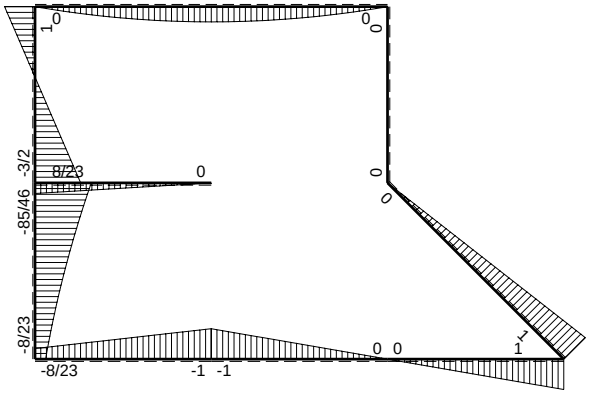
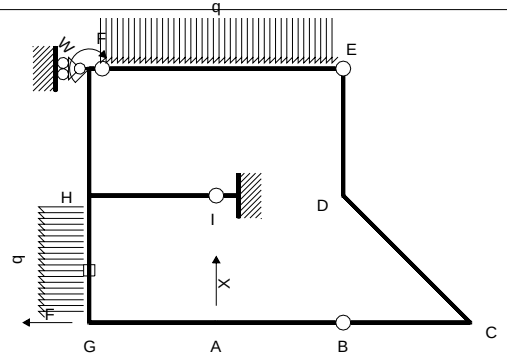


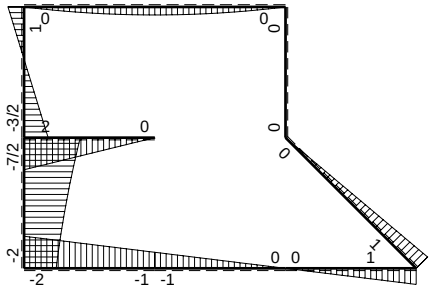

Fb



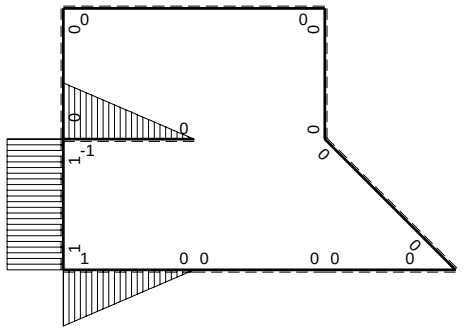





Schema di calcolo iperstatico




$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb+Fx | 0 | 0 | | |
| CD $\sqrt{2}$b | 0 | Fb-$\sqrt{2}$/2Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | -Fx+1/2q$x^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | Fx-1/2q$x^2$ | 0 | 0 | | |
| GA b | b-x | -2Fb+Fx | -2F$b^2$+3Fbx-F$x^2$ | $b^2$-2bx+$x^2$ | -5/6F$b^3$/EJ | 1/3X$b^3$/EJ |
| AG b | -x | Fb+Fx | -Fbx-F$x^2$ | $x^2$ | | |
| FH b | 0 | Fb-5/2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 3/2Fb-5/2Fx | 0 | 0 | | |
| HI b | -b+x | 2Fb-2Fx | -2F$b^2$+4Fbx-2F$x^2$ | $b^2$-2bx+$x^2$ | -2/3F$b^3$/EJ | 1/3X$b^3$/EJ |
| IH b | x | -2Fx | -2F$x^2$ | $x^2$ | | |
| HG b | b | -7/2Fb+2Fx-1/2q$x^2$ | -7/2F$b^2$+2Fbx-1/2F$x^2$ | $b^2$ | -8/3F$b^3$/EJ | X$b^3$/EJ |
| GH b | -b | 2Fb+Fx+1/2q$x^2$ | -2F$b^2$-Fbx-1/2F$x^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | F$b^3$/EJ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | 1/4X$b^3$/EJ |
| | totali | | | | -19/6F$b^3$/EJ | 23/12X$b^3$/EJ |
| | iperstatica $X=V_A$ | | | | 38/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$
$$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$
$$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$
$$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$
$$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$$
$$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$$
$$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b (-2 + 3\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\,x + 3/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$
$$= (-2\,b + 3/2\,b - 1/3\,b \,)\, Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

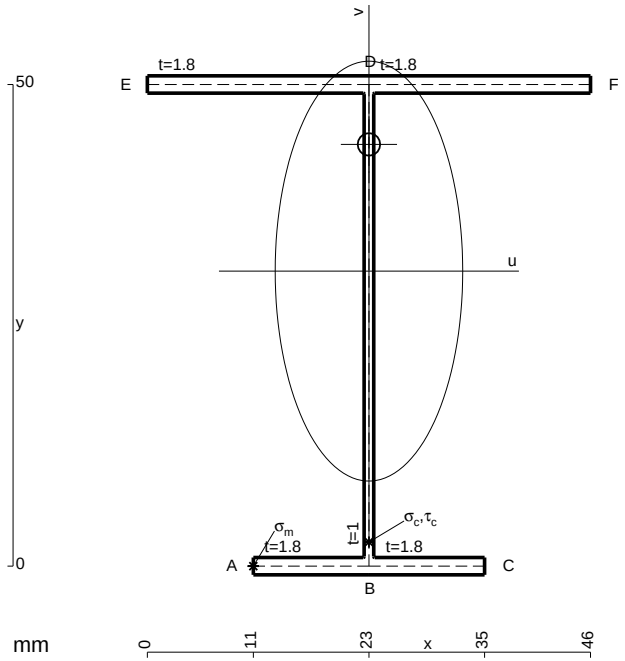
$$L_{AG}^{Xo} = \int_o^b (-\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$
$$= (-1/2\,b - 1/3\,b \,)\, Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b (-2 + 4\, x/b - 2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-2\,x + 2\, x^2/b - 2/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$
$$= (-2\,b + 2\,b - 2/3\,b \,)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b (-2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$
$$= (-2/3\,b \,)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b (-7/2 + 2\, x/b - 1/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx + 1\ (-1)\ (-1)\ Fb^3/EJ$$
$$= [-7/2\, x + x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + 1\ (-1)\ (-1)\ Fb^3/EJ$$
$$= (-7/2\,b + b - 1/6\,b \,)\, Fb^2\, 1/EJ + 1\ (-1)\ (-1)\ Fb^3/EJ = -5/3\ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b (-2 - x/b - 1/2\, x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx + 1\ (-1)\ (-1)\ Fb^3/EJ$$
$$= [-2\, x - 1/2\, x^2/b - 1/6\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ + 1\ (-1)\ (-1)\ Fb^3/EJ$$
$$= (-2\,b - 1/2\,b - 1/6\,b \,)\, Fb^2\, 1/EJ + 1\ (-1)\ (-1)\ Fb^3/EJ = -5/3\ Fb^3/EJ$$




A = 176. $mm^2$
$J_u$ = 83598. $mm^4$
$J_v$ = 16674. $mm^4$
$J_t$ = 152.7 $mm^4$
$y_o$ = 13.16 mm
$y_g$ = 30.63 mm
N = -710. N
$T_y$ = -1775. N
$M_x$ = -639000. Nmm
$x_m$ = 11. mm
$u_m$ = -12. mm
$v_m$ = -30.63 mm
$\sigma_m = N/A - Mv/J_u$ = -238.1 $N/mm^2$
$x_c$ = 23. mm
$v_c$ = -30.63 mm
$\sigma_c = N/A - Mv/J_u$ = -238.1 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 28.09 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 28.09 $N/mm^2$
$t_c$ = 710. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 243. $N/mm^2$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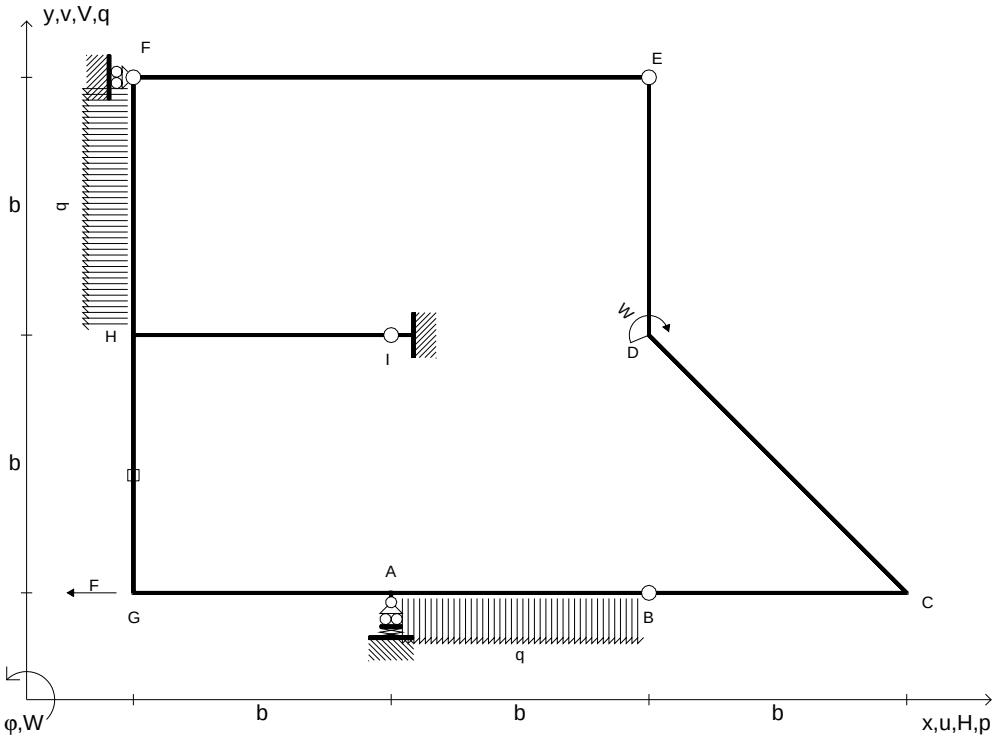
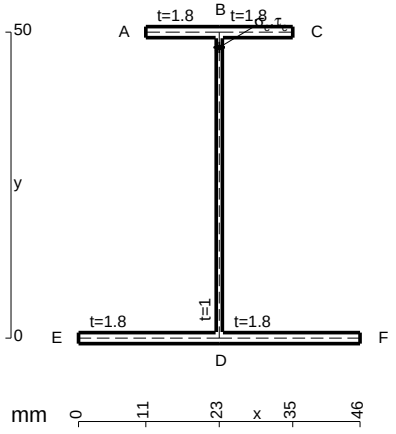



$H_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_D = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{AB} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

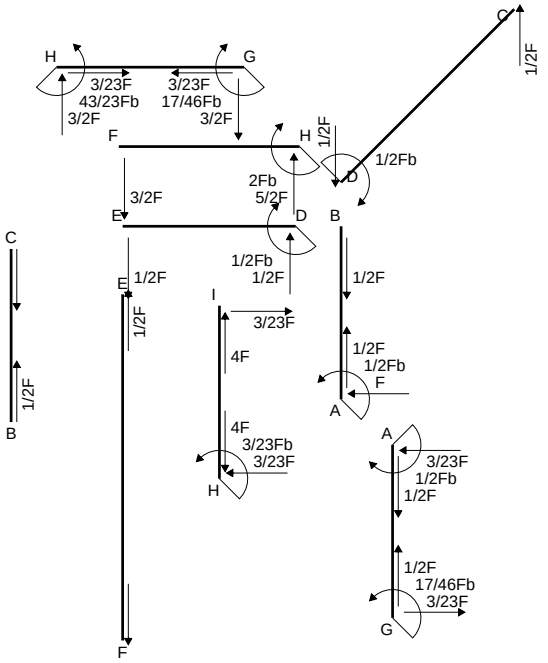
$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

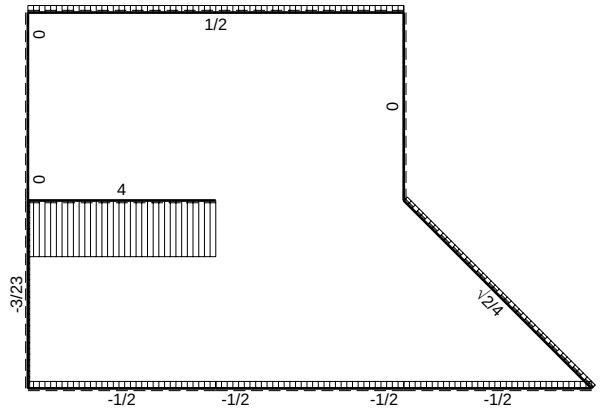



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 470 mm, F = 600 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

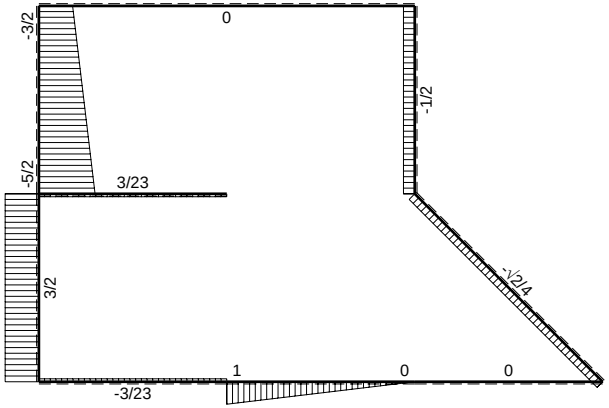




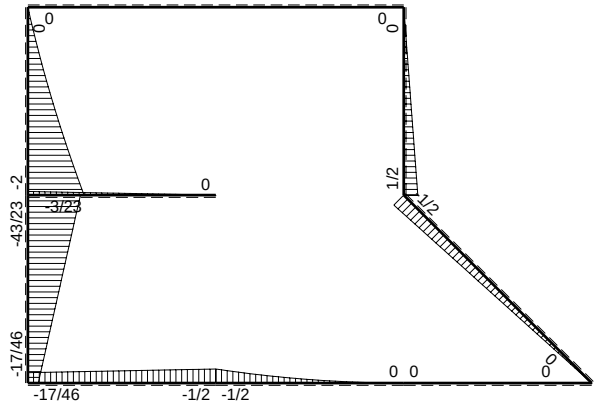





←⊞→ F



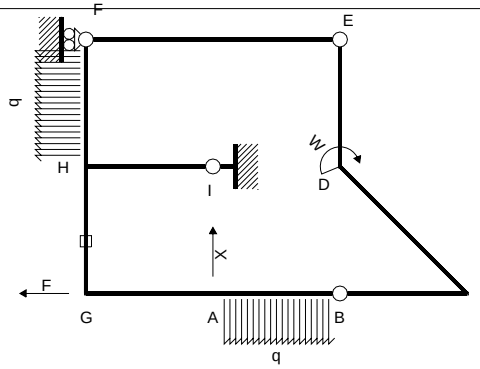

↑⊞↓ F



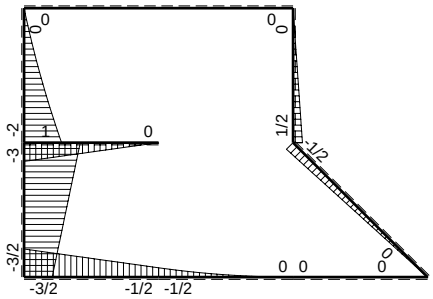

↻⊞↺ Fb



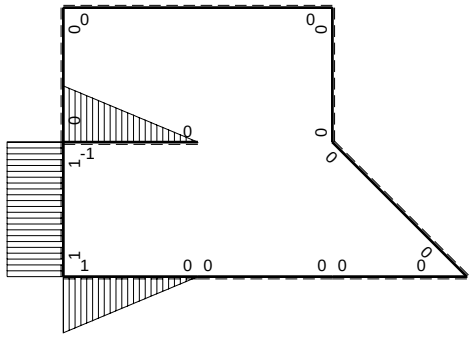





Schema di calcolo iperstatico




↻⊞↺ $M_o$ flessione da carichi assegnati




↻⊞↺ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | $-1/2Fb+Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| BA b | 0 | $1/2qx^2$ | 0 | 0 | | |
| BC b | 0 | 0 | 0 | 0 | 0 | 0 |
| CB b | 0 | 0 | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $-\sqrt{2}/4Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | $1/2Fb-1/2Fx$ | 0 | 0 | 0 | 0 |
| ED b | 0 | $-1/2Fx$ | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | b-x | $-3/2Fb+Fx$ | $-3/2Fb^2+5/2Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-7/12Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | $1/2Fb+Fx$ | $-1/2Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | $-3/2Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $2Fb-5/2Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | -b+x | Fb-Fx | $-Fb^2+2Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-1/3Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | -Fx | $-Fx^2$ | $x^2$ | | |
| HG b | b | $-3Fb+3/2Fx$ | $-3Fb^2+3/2Fbx$ | $b^2$ | $-9/4Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $3/2Fb+3/2Fx$ | $-3/2Fb^2-3/2Fbx$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-13/6Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 26/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$
$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$
$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-3/2 +5/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-3/2\, x +5/4\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(-3/2\, b +5/4\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$$

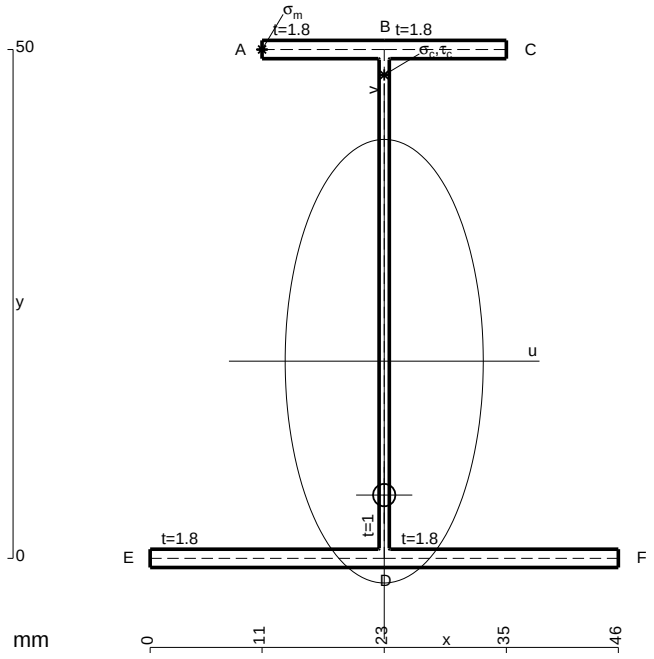
$$L_{AG}^{Xo} = \int_o^b \left(-1/2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/4\, x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(-1/4\, b -1/3\, b \,\right) Fb^2\, 1/EJ = -7/12\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-1 +2\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[- x + x^2/b -1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(- b + b -1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(- x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$
$$= \left(-1/3\, b \,\right) Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-3 +3/2\, x/b\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left[-3\, x +3/4\, x^2/b \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left(-3\, b +3/4\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -5/4\ \ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-3/2 -3/2\, x/b\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left[-3/2\, x -3/4\, x^2/b \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$
$$= \left(-3/2\, b -3/4\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -5/4\ \ Fb^3/EJ$$




A = 176. $mm^2$
$J_u$ = 83598. $mm^4$
$J_v$ = 16674. $mm^4$
$J_t$ = 152.7 $mm^4$
$y_o$ = -13.16 mm
$y_g$ = 19.38 mm
$T_y$ = -1500. N
$M_x$ = -564000. Nmm
$x_m$ = 11. mm
$y_m$ = 50. mm
$u_m$ = -12. mm
$v_m$ = 30.63 mm
$\sigma_m = -Mv/J_u$ = 206.6 $N/mm^2$
$x_c$ = 23. mm
$y_c$ = 50. mm
$v_c$ = 30.63 mm
$\sigma_c = -Mv/J_u$ = 206.6 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 23.74 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 23.74 $N/mm^2$
$t_c$ = 600. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 210.7 $N/mm^2$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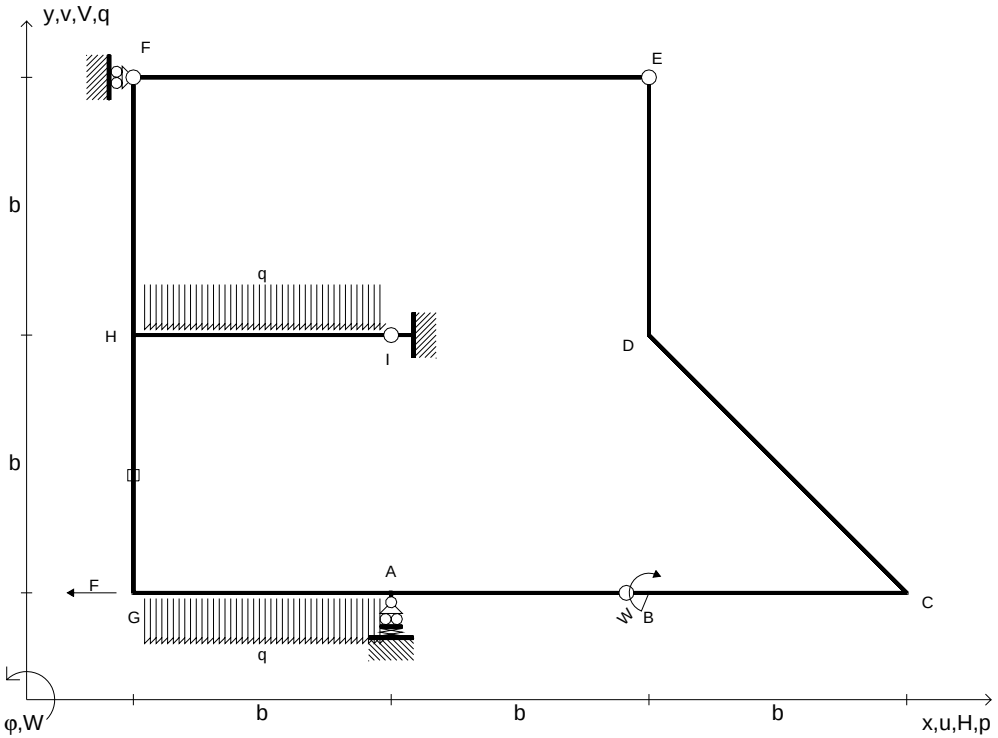
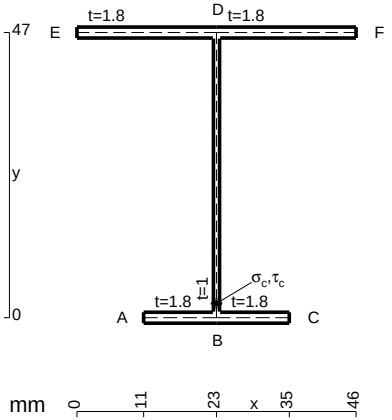



$H_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_B = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{HI} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

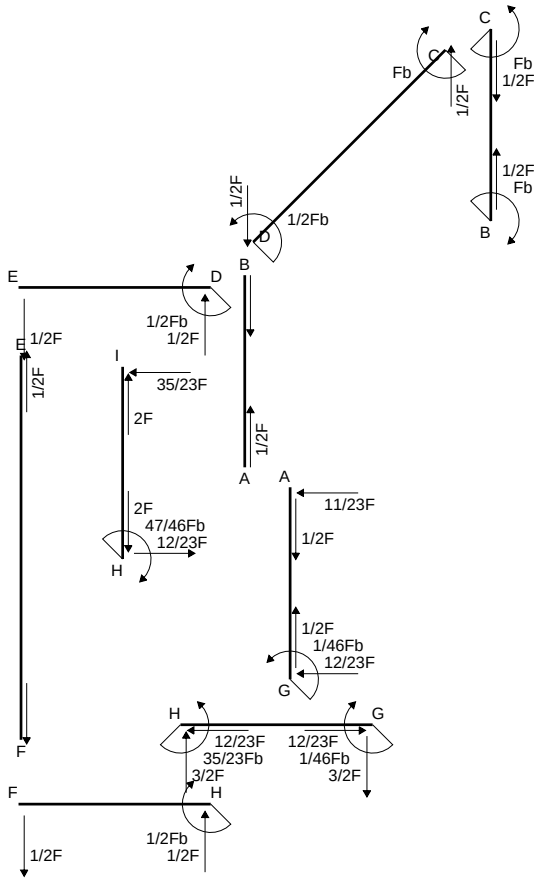
$q_{GA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

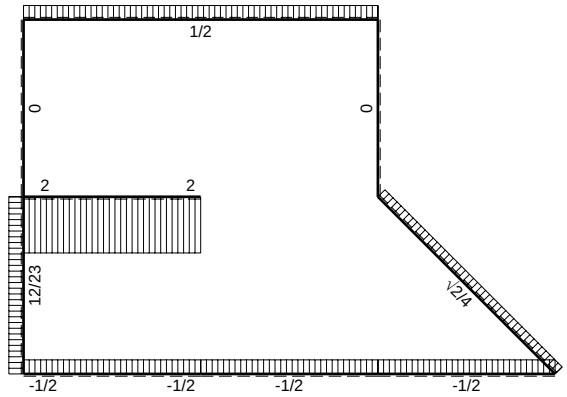



Reazioni iperstatiche in soluzione: $X=W_{GA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave CD ha la sezione riportata e dimensioni in mm, con:
b = 720 mm, F = 800 N
Calcolare sulla sezione C la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da C a D
Elongazione termica specifica ε assegnata su asta HG.

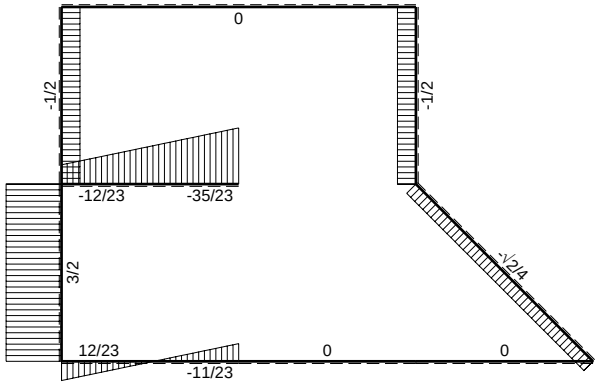




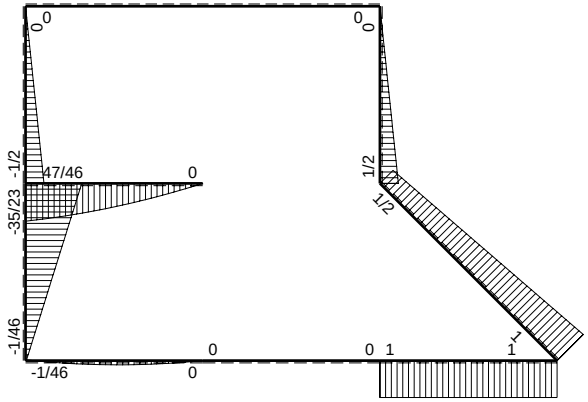





←⊞→ F



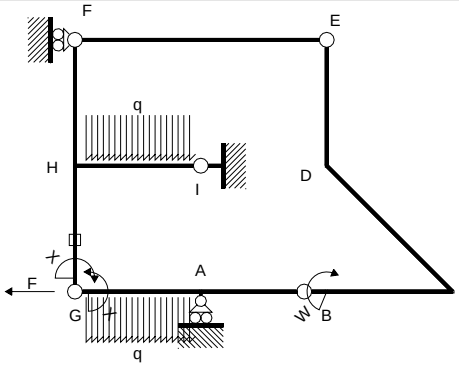

↑⊞↓ F

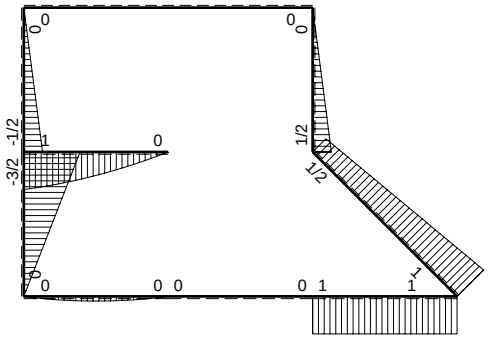



↻⊞↺ Fb

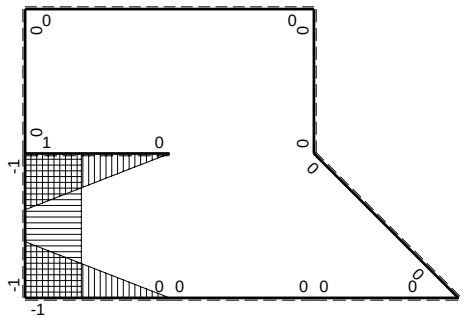







↻⊞↺ $M_o$ flessione da carichi assegnati




↻⊞↺ $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | 0 | 0 | 0 | 0 | 0 |
| BA b | 0 | 0 | 0 | 0 | | |
| BC b | 0 | Fb | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb | 0 | 0 | | |
| CD $\sqrt{2}b$ | 0 | $Fb-\sqrt{2}/4Fx$ | 0 | 0 | 0 | 0 |
| DE b | 0 | 1/2Fb-1/2Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -1/2Fx | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | -1+x/b | $1/2Fx-1/2qx^2$ | $-1/2Fx+Fx^2/b-1/2qx^3/b$ | $1-2x/b+x^2/b^2$ | $-1/24Fb^2/EJ$ | 1/3Xb/EJ |
| AG b | x/b | $-1/2Fx+1/2qx^2$ | $-1/2Fx^2/b+1/2qx^3/b$ | $x^2/b^2$ | | |
| FH b | 0 | -1/2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 1/2Fb-1/2Fx | 0 | 0 | | |
| HI b | 1-x/b | $Fb-1/2Fx-1/2qx^2$ | $Fb-3/2Fx+1/2qx^3/b$ | $1-2x/b+x^2/b^2$ | $3/8Fb^2/EJ$ | 1/3Xb/EJ |
| IH b | -x/b | $-3/2Fx+1/2qx^2$ | $3/2Fx^2/b-1/2qx^3/b$ | $x^2/b^2$ | | |
| HG b | -1 | -3/2Fb+3/2Fx | 3/2Fb-3/2Fx | 1 | $3/4Fb^2/EJ$ | Xb/EJ |
| GH b | 1 | 3/2Fx | 3/2Fx | 1 | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^2/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-1/8Fb^2/EJ$ | 1/4Xb/EJ |
| | totali | | | | $-1/24Fb^2/EJ$ | 23/12Xb/EJ |
| | iperstatica $X=W_{GA}$ | | | | 1/46Fb | |

Sviluppi di calcolo iperstatica



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$L_{GA}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{AG}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HI}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IH}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HG}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GH}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GA}^{Xo} = \int_o^b (-1/2\ x/b + x^2/b^2\ -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b + 1/3\ x^3/b^2\ -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b + 1/3\ b\ -1/8\ b\ )\ Fb\ 1/EJ\ = -1/24\ Fb^2/EJ$

$L_{AG}^{Xo} = \int_o^b (-1/2\ x^2/b^2 + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b + 1/8\ b\ )\ Fb\ 1/EJ\ = -1/24\ Fb^2/EJ$

$L_{HI}^{Xo} = \int_o^b (1\ -3/2\ x/b + 1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x\ -3/4\ x^2/b + 1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ -3/4\ b + 1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

$L_{IH}^{Xo} = \int_o^b (3/2\ x^2/b^2\ -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^3/b^2\ -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ -1/8\ b\ )\ Fb\ 1/EJ\ = 3/8\ Fb^2/EJ$

$L_{HG}^{Xo} = \int_o^b (3/2\ -3/2\ x/b\ )\ Fb\ 1/EJ\ dx\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

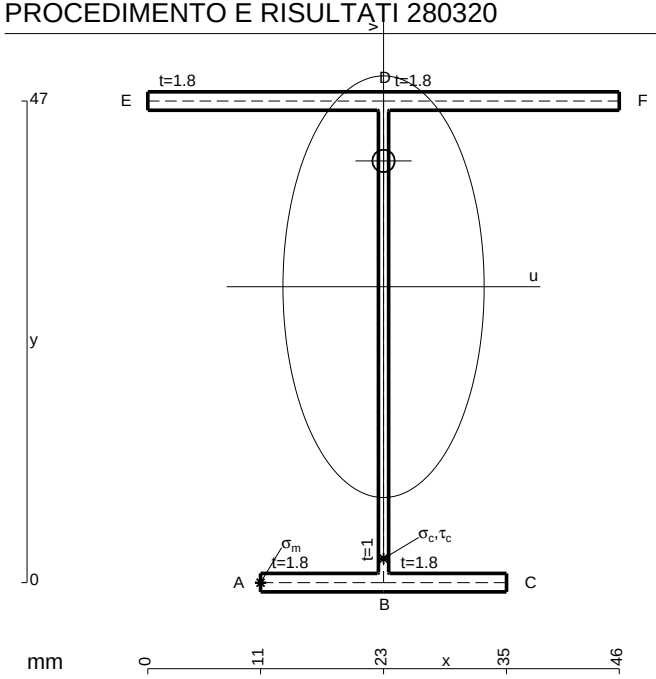
$= [3/2\ x\ -3/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ \ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (3/2\ b\ -3/4\ b\ )\ Fb\ 1/EJ\ \ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -1/4\ \ Fb^2/EJ$

$L_{GH}^{Xo} = \int_o^b (3/2\ x/b\ )\ Fb\ 1/EJ\ dx\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = [3/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ \ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (3/4\ b\ )\ Fb\ 1/EJ\ \ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -1/4\ \ Fb^2/EJ$

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25




$A = 173.\ mm^2$
$J_u = 73230.\ mm^4$
$J_v = 16674.\ mm^4$
$J_t = 151.7\ mm^4$
$y_o = 12.28\ mm$
$y_g = 28.88\ mm$
$N = 282.8\ N$
$T_y = -282.8\ N$
$M_x = 576000.\ Nmm$
$x_m = 11.\ mm$
$u_m = -12.\ mm$
$v_m = -28.88\ mm$
$\sigma_m = N/A - Mv/J_u = 228.8\ N/mm^2$
$x_c = 23.\ mm$
$v_c = -28.88\ mm$
$\sigma_c = N/A - Mv/J_u = 228.8\ N/mm^2$
$\tau_c = TS^*/tJ_u = 4.819\ N/mm^2$
$\tau_g = TS^*/tJ_u = 4.819\ N/mm^2$
$t_c = 800.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 228.9\ N/mm^2$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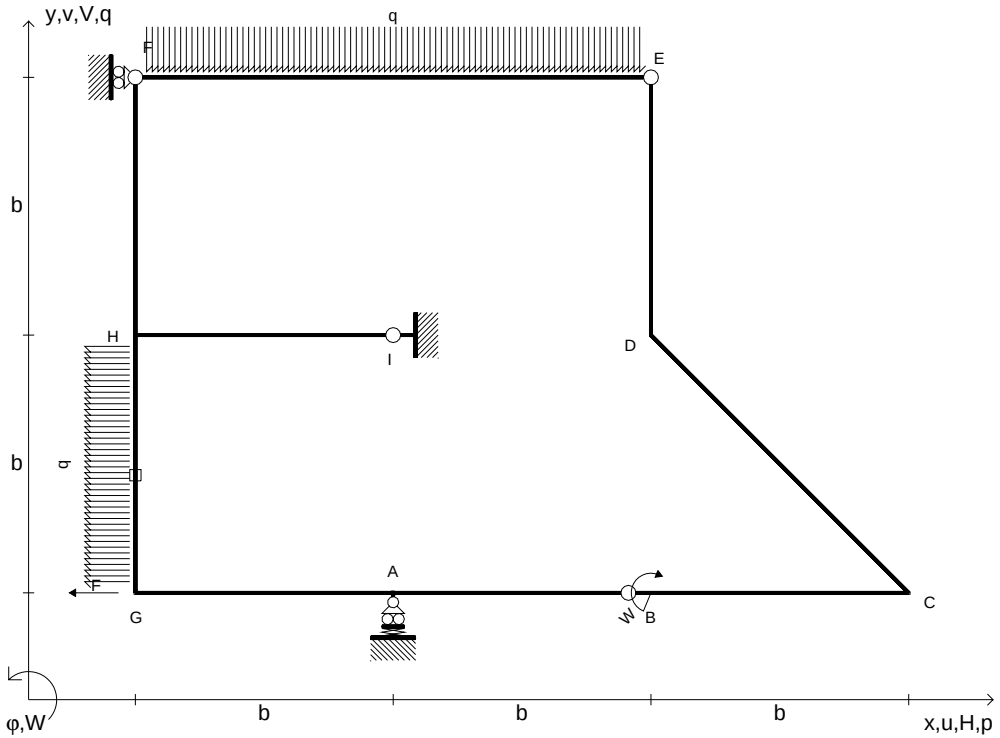
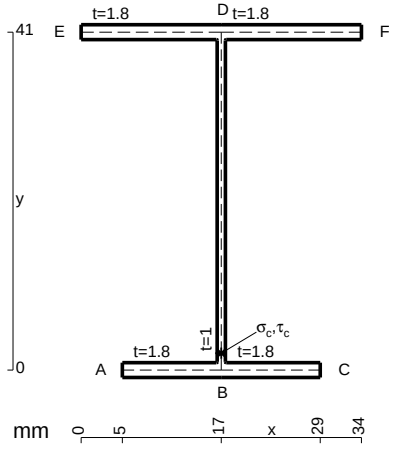



$H_G = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_B = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

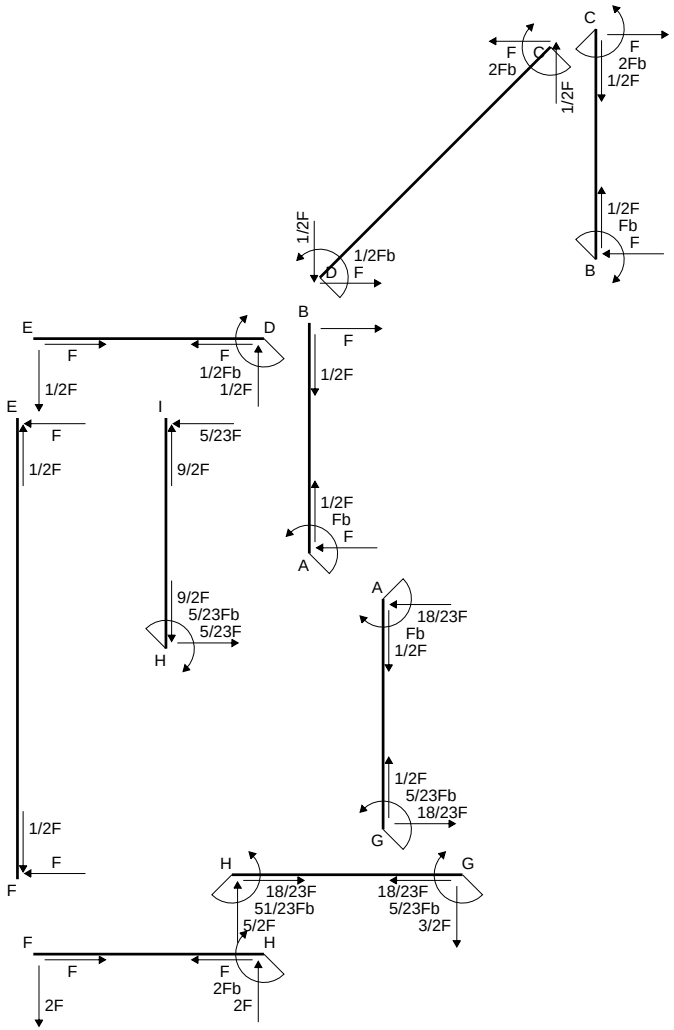
$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

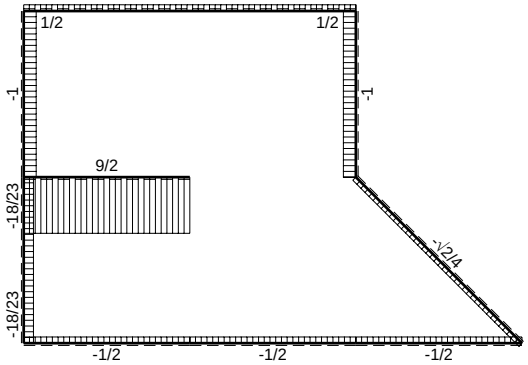



Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 350 mm, F = 590 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

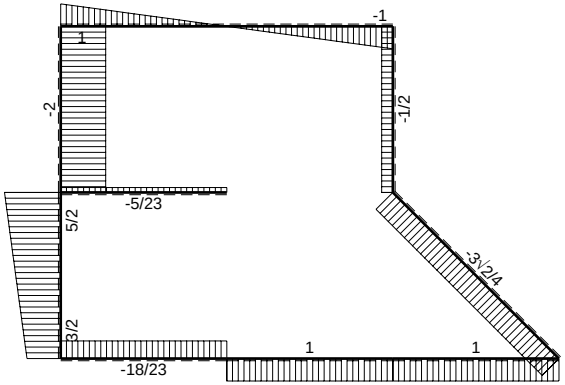




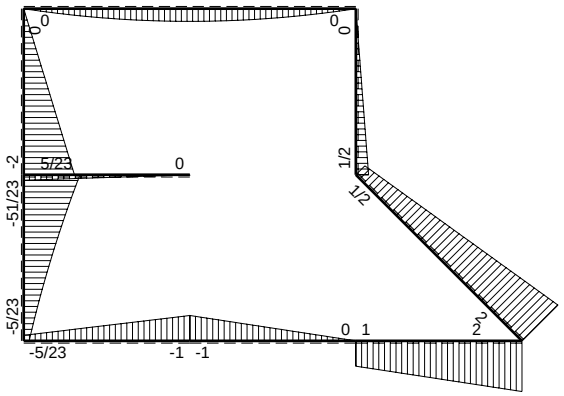





←□→ F



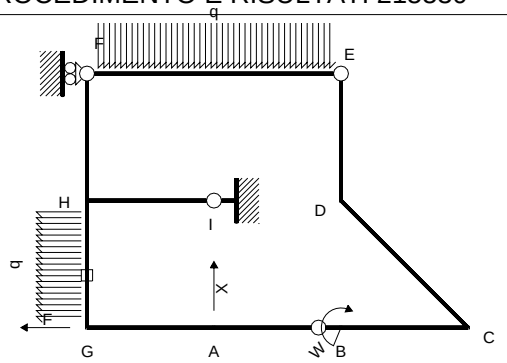

↑□↓ F



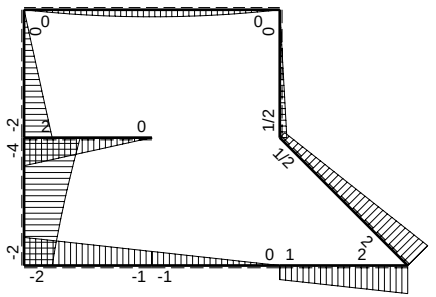

(+) Fb



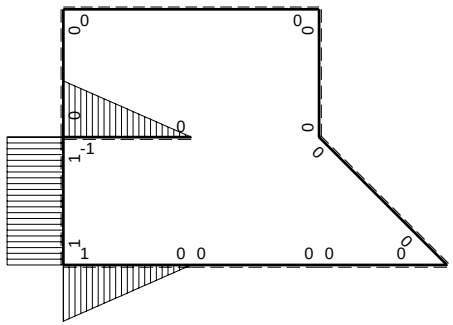





Schema di calcolo iperstatico




(+) $M_o$ flessione da carichi assegnati




(+) $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fb+Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -2Fb+Fx | 0 | 0 | | |
| CD $\sqrt{2}$b | 0 | 2Fb-3$\sqrt{2}$/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 1/2Fb-1/2Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -1/2Fx | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | b-x | -2Fb+Fx | $-2Fb^2+3Fbx-Fx^2$ | $b^2-2bx+x^2$ | $-5/6Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | -x | Fb+Fx | $-Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | -2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 2Fb-2Fx | 0 | 0 | | |
| HI b | -b+x | 2Fb-2Fx | $-2Fb^2+4Fbx-2Fx^2$ | $b^2-2bx+x^2$ | $-2/3Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | x | -2Fx | $-2Fx^2$ | $x^2$ | | |
| HG b | b | $-4Fb+5/2Fx-1/2qx^2$ | $-4Fb^2+5/2Fbx-1/2Fx^2$ | $b^2$ | $-35/12Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | -b | $2Fb+3/2Fx+1/2qx^2$ | $-2Fb^2-3/2Fbx-1/2Fx^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | | $1/4Xb^3/EJ$ |
| | totali | | | | $-41/12Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_A$ | | | | 41/23F | |

Sviluppi di calcolo iperstatica





$$L_{GA}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b \left(1 -2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[\, x - x^2/b + 1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b - b + 1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b \left(\, x^2/b^2 \,\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \,\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \,\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b \left(1\right) b^2\, 1/EJ\, dx = \left[\, x \,\right]_o^b b^2\, 1/EJ$$

$$= \left(\, b \,\right) b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b \left(-2 +3\, x/b - x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-2\, x + 3/2\, x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2\, b + 3/2\, b - 1/3\, b \,\right) Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b \left(-\, x/b - x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[-1/2\, x^2/b - 1/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-1/2\, b - 1/3\, b \,\right) Fb^2\, 1/EJ = -5/6\ Fb^3/EJ$$

$$L_{HI}^{Xo} = \int_o^b \left(-2 +4\, x/b - 2\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[-2\, x + 2\, x^2/b - 2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2\, b + 2\, b - 2/3\, b \,\right) Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{IH}^{Xo} = \int_o^b \left(-2\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx = \left[-2/3\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ$$

$$= \left(-2/3\, b \,\right) Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b \left(-4 +5/2\, x/b - 1/2\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

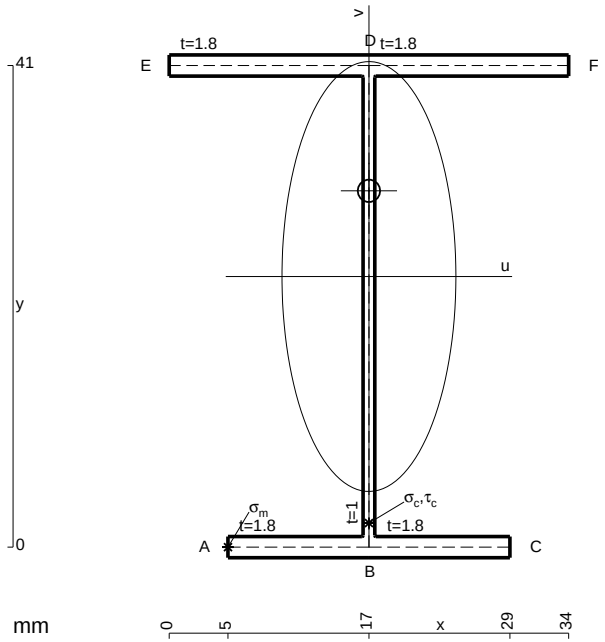
$$= \left[-4\, x + 5/4\, x^2/b - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-4\, b + 5/4\, b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -23/12\ \ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b \left(-2 -3/2\, x/b - 1/2\, x^2/b^2 \,\right) Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left[-2\, x - 3/4\, x^2/b - 1/6\, x^3/b^2 \,\right]_o^b Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$$

$$= \left(-2\, b - 3/4\, b - 1/6\, b \,\right) Fb^2\, 1/EJ + 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -23/12\ \ Fb^3/EJ$$




A = 145.4 $mm^2$
$J_u$ = 48681. $mm^4$
$J_v$ = 7969. $mm^4$
$J_t$ = 126.4 $mm^4$
$y_o$ = 7.294 mm
$y_g$ = 23.04 mm
N = -590. N
$T_y$ = -1180. N
$M_x$ = -413000. Nmm
$x_m$ = 5. mm
$u_m$ = -12. mm
$v_m$ = -23.04 mm
$\sigma_m = N/A - Mv/J_u$ = -199.5 $N/mm^2$
$x_c$ = 17. mm
$v_c$ = -23.04 mm
$\sigma_c = N/A - Mv/J_u$ = -199.5 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 24.12 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 24.12 $N/mm^2$
$t_c$ = 590. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 203.8 $N/mm^2$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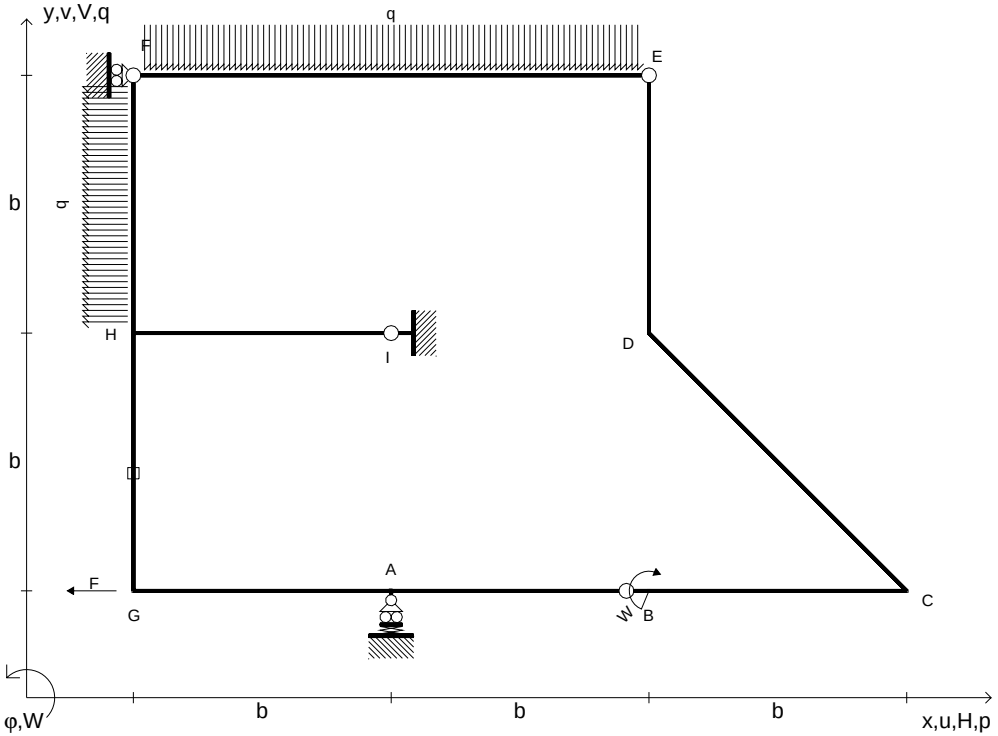



$H_G = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_B = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$p_{FH} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_I$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave FH ha la sezione riportata e dimensioni in mm, con:

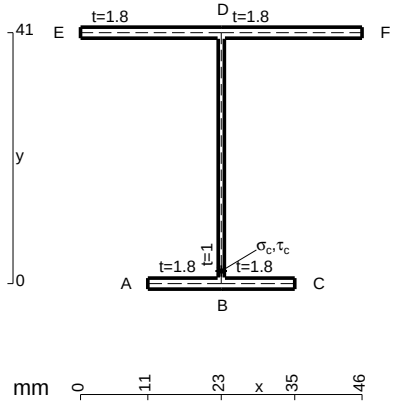
b = 380 mm, F = 770 N

Calcolare sulla sezione H la massima tensione normale $\sigma_m$.

Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.

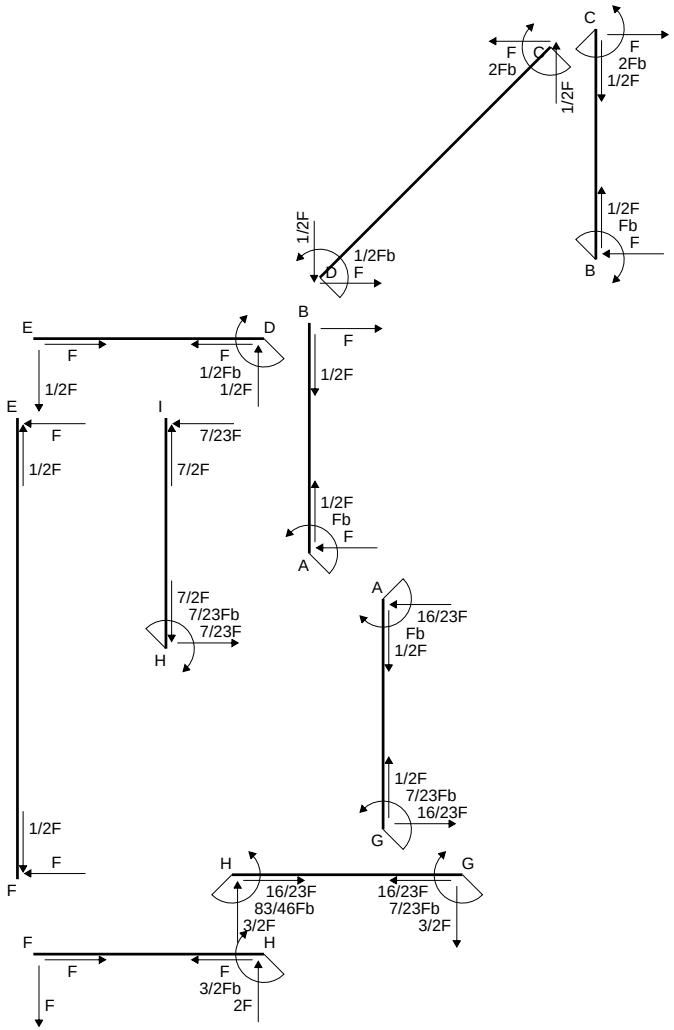
Lembo inferiore sezione su tratteggio trave, a destra da F a H

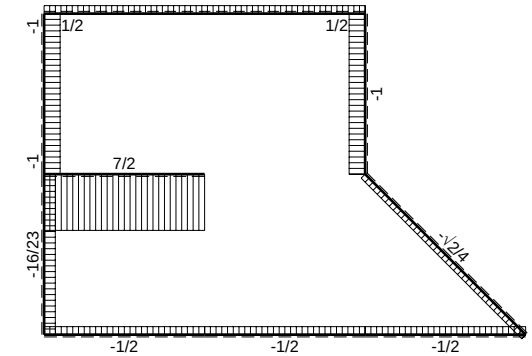
Elongazione termica specifica ε assegnata su asta HG.



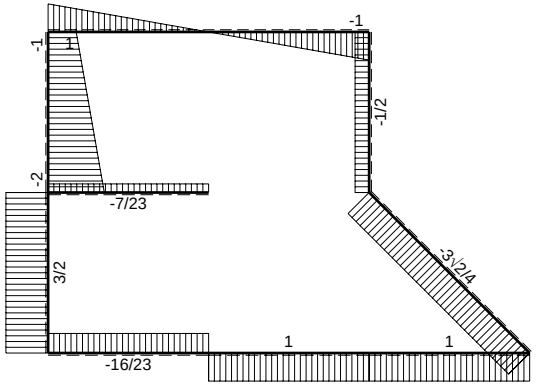




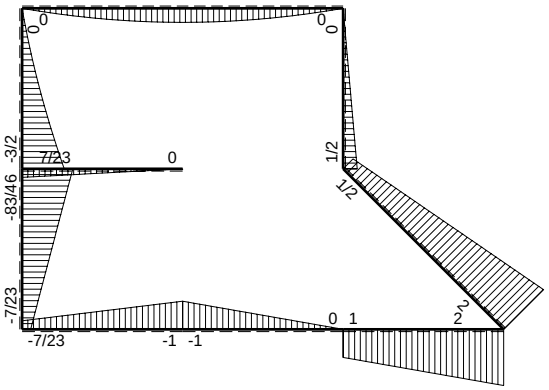






F



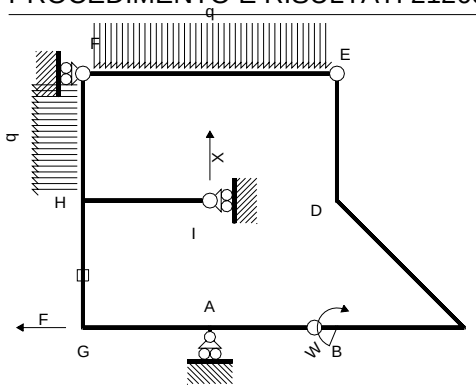

F

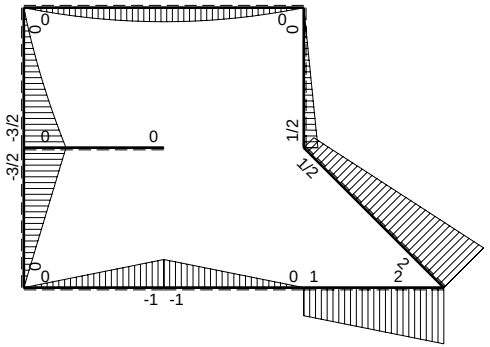



Fb

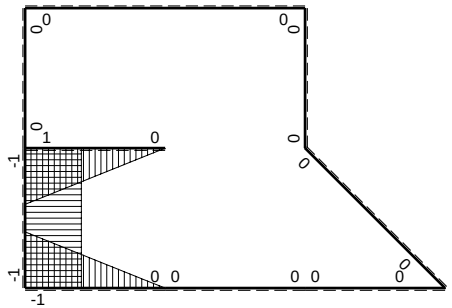







$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fb+Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -2Fb+Fx | 0 | 0 | | |
| CD √2b | 0 | 2Fb-3√2/4Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 1/2Fb-1/2Fx | 0 | 0 | 0 | 0 |
| ED b | 0 | -1/2Fx | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | -b+x | -Fx | $Fbx-Fx^2$ | $b^2-2bx+x^2$ | $1/6Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | x | Fb-Fx | $Fbx-Fx^2$ | $x^2$ | | |
| FH b | 0 | $-Fx-1/2qx^2$ | 0 | 0 | 0 | 0 |
| HF b | 0 | $3/2Fb-2Fx+1/2qx^2$ | 0 | 0 | | |
| HI b | b-x | 0 | 0 | $b^2-2bx+x^2$ | 0 | $1/3Xb^3/EJ$ |
| IH b | -x | 0 | 0 | $x^2$ | | |
| HG b | -b | -3/2Fb+3/2Fx | $3/2Fb^2-3/2Fbx$ | $b^2$ | $3/4Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | b | 3/2Fx | 3/2Fbx | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-1/2Fb^3/EJ$ | $1/4Xb^3/EJ$ |
| | totali | | | | $-7/12Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_I$ | | | | 7/23F | |

Sviluppi di calcolo iperstatica



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$$L_{GA}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$

$$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{AG}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$

$$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HI}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$

$$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{IH}^{XX} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$$

$$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{HG}^{XX} = \int_o^b (1 \,)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$$

$$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{GH}^{XX} = \int_o^b (1 \,)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$$

$$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{GA}^{Xo} = \int_o^b (\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$

$$= (1/2\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$$

$$L_{AG}^{Xo} = \int_o^b (\, x/b - x^2/b^2 \,)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b - 1/3\, x^3/b^2 \,]_o^b\, Fb^2\, 1/EJ$$

$$= (1/2\, b - 1/3\, b \,)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$$

$$L_{HG}^{Xo} = \int_o^b (3/2 - 3/2\, x/b \,)\, Fb^2\, 1/EJ\, dx - 1\ (-1)\ (-1)\ Fb^3/EJ$$

$$= [3/2\, x - 3/4\, x^2/b \,]_o^b\, Fb^2\, 1/EJ\ -1\ (-1)\ (-1)\ Fb^3/EJ$$

$$= (3/2\, b - 3/4\, b \,)\, Fb^2\, 1/EJ\ -1\ (-1)\ (-1)\ Fb^3/EJ = -1/4\ Fb^3/EJ$$

$$L_{GH}^{Xo} = \int_o^b (3/2\, x/b \,)\, Fb^2\, 1/EJ\, dx - 1\ (-1)\ (-1)\ Fb^3/EJ = [3/4\, x^2/b \,]_o^b\, Fb^2\, 1/EJ\ -1\ (-1)\ (-1)\ Fb^3/EJ$$

$$= (3/4\, b \,)\, Fb^2\, 1/EJ\ -1\ (-1)\ (-1)\ Fb^3/EJ = -1/4\ Fb^3/EJ$$

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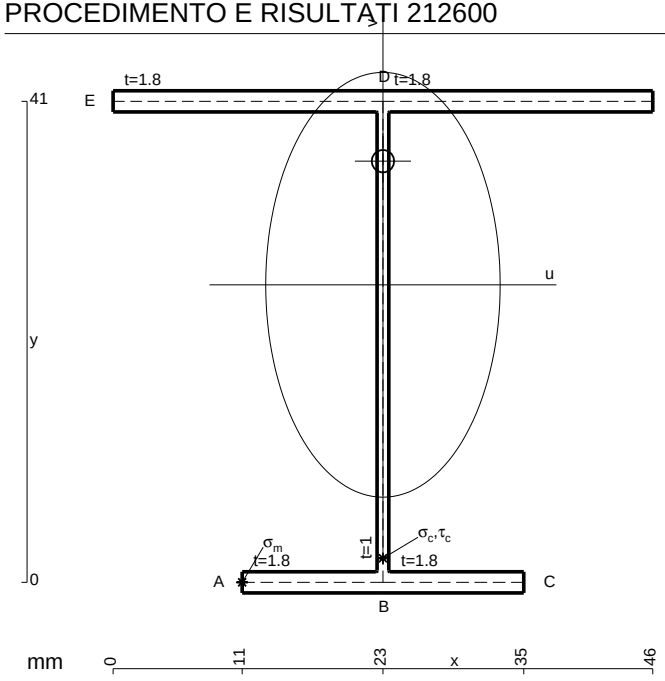


A = 167. $mm^2$
$J_u$ = 54749. $mm^4$
$J_v$ = 16674. $mm^4$
$J_t$ = 149.7 $mm^4$
$y_o$ = 10.54 mm
$y_g$ = 25.36 mm
N = -770. N
$T_y$ = -1540. N
$M_x$ = -438900. Nmm
$x_m$ = 11. mm
$u_m$ = -12. mm
$v_m$ = -25.36 mm
$\sigma_m = N/A - Mv/J_u$ = -207.9 $N/mm^2$
$x_c$ = 23. mm
$v_c$ = -25.36 mm
$\sigma_c = N/A - Mv/J_u$ = -207.9 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 30.82 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 30.82 $N/mm^2$
$t_c$ = 770. mm
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 214.7 $N/mm^2$


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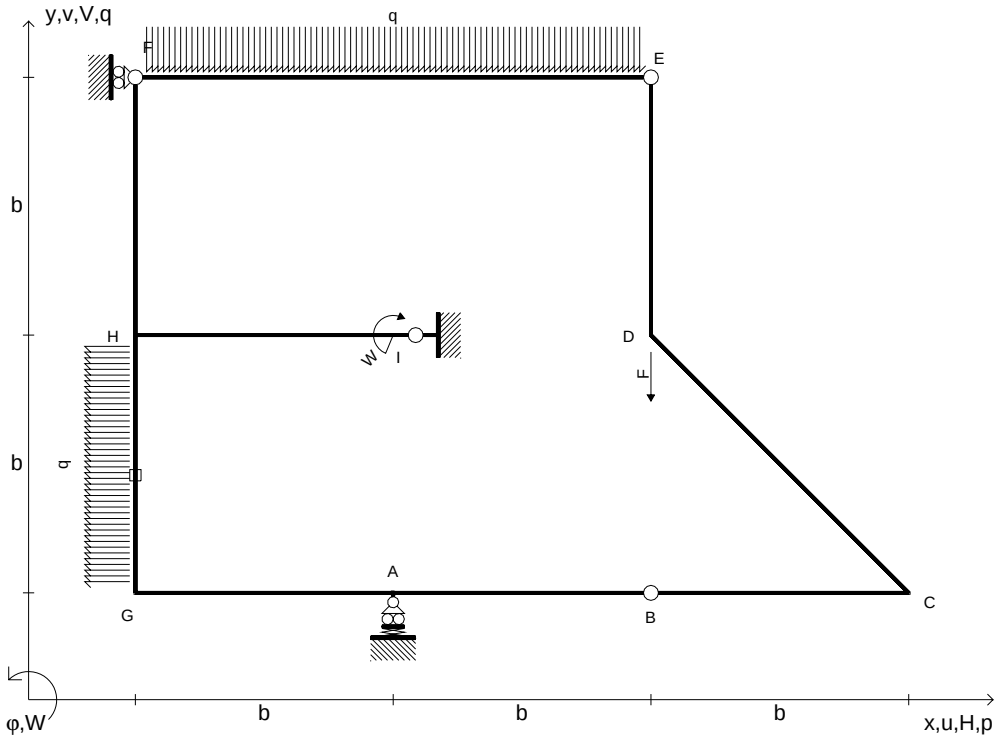
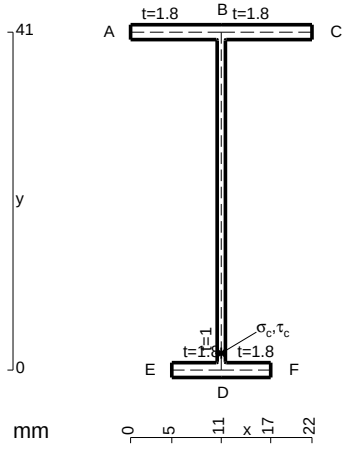



$V_D = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_I = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

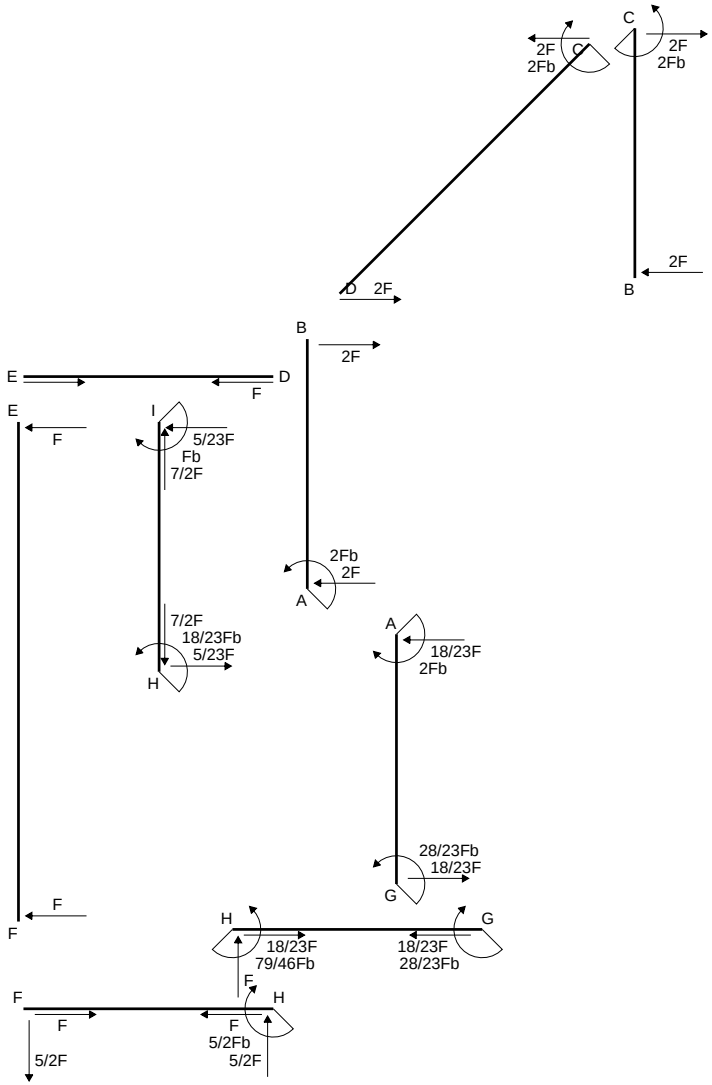
$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

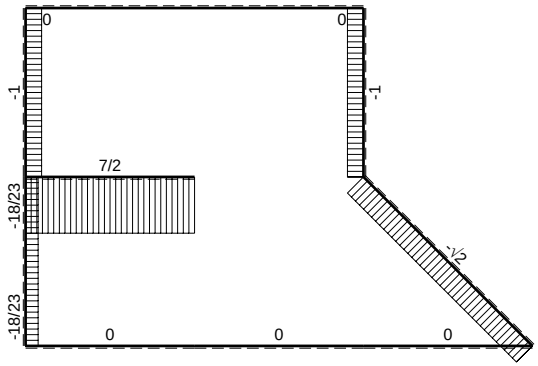



Reazioni iperstatiche in soluzione: $X=W_{GA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.
La trave FH ha la sezione riportata e dimensioni in mm, con:
b = 560 mm, F = 180 N
Calcolare sulla sezione H la massima tensione normale $\sigma_m$.
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.
Lembo inferiore sezione su tratteggio trave, a destra da F a H
Elongazione termica specifica ε assegnata su asta HG.

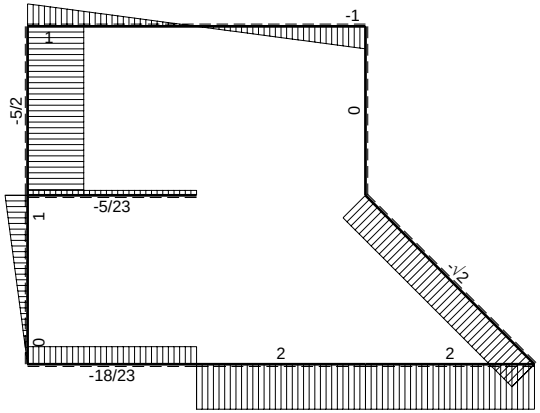









F

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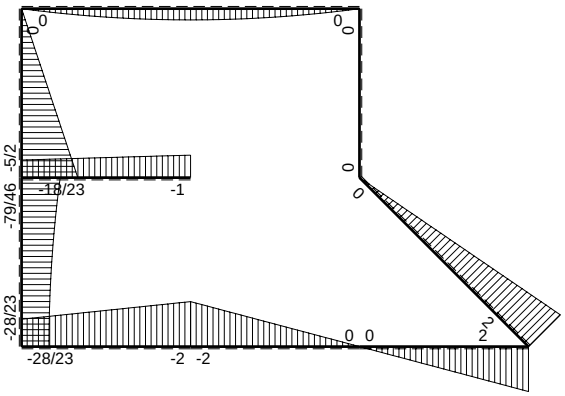
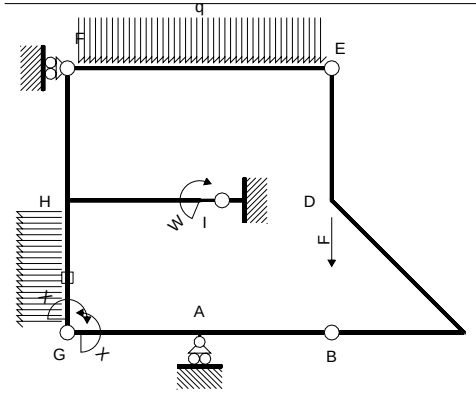
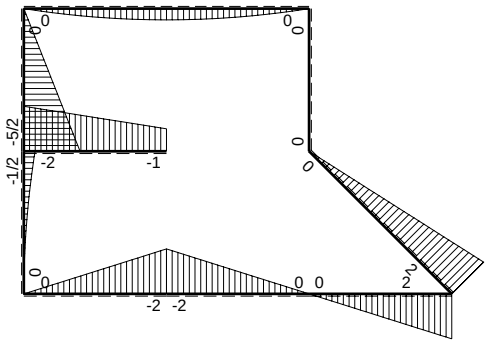



Fb

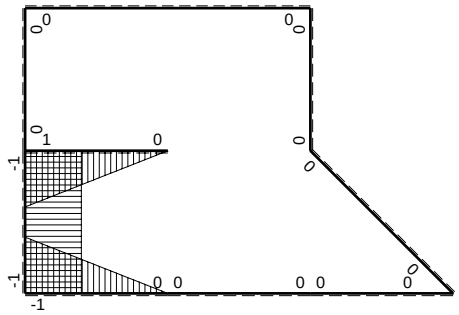







$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -2Fb+2Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | 2Fx | 0 | 0 | | |
| BC b | 0 | 2Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -2Fb+2Fx | 0 | 0 | | |
| CD √2b | 0 | 2Fb-√2Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | -1+x/b | -2Fx | $2Fx-2Fx^2/b$ | $1-2x/b+x^2/b^2$ | $1/3Fb^2/EJ$ | 1/3Xb/EJ |
| AG b | x/b | 2Fb-2Fx | $2Fx-2Fx^2/b$ | $x^2/b^2$ | | |
| FH b | 0 | -5/2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | | |
| HI b | 1-x/b | -2Fb+Fx | $-2Fb+3Fx-Fx^2/b$ | $1-2x/b+x^2/b^2$ | $-5/6Fb^2/EJ$ | 1/3Xb/EJ |
| IH b | -x/b | Fb+Fx | $-Fx-Fx^2/b$ | $x^2/b^2$ | | |
| HG b | -1 | $-1/2Fb+Fx-1/2qx^2$ | $1/2Fb-Fx+1/2Fx^2/b$ | 1 | $1/6Fb^2/EJ$ | Xb/EJ |
| GH b | 1 | $1/2qx^2$ | $1/2Fx^2/b$ | 1 | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^2/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-Fb^2/EJ$ | 1/4Xb/EJ |
| | totali | | | | $-7/3Fb^2/EJ$ | 23/12Xb/EJ |
| | iperstatica $X=W_{GA}$ | | | | 28/23Fb | |

Sviluppi di calcolo iperstatica





$L_{GA}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{AG}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HI}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IH}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HG}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GH}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GA}^{Xo} = \int_o^b (2\,x/b - 2\,x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b - 2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - 2/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{AG}^{Xo} = \int_o^b (2\,x/b - 2\,x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b - 2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - 2/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{HI}^{Xo} = \int_o^b (-2 + 3\,x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\,x + 3/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-2\ b + 3/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L_{IH}^{Xo} = \int_o^b (-\,x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

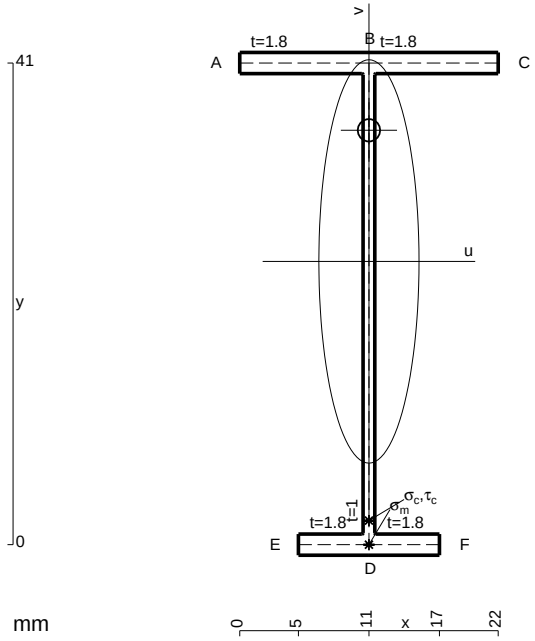
$L_{HG}^{Xo} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{GH}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$




$A = 102.2\ mm^2$
$J_u = 30130.\ mm^4$
$J_v = 1856.\ mm^4$
$J_t = 79.76\ mm^4$
$y_o = 11.16\ mm$
$y_g = 24.11\ mm$
$N = -180.\ N$
$T_y = -450.\ N$
$M_x = -252000.\ Nmm$
$x_m = 11.\ mm$
$v_m = -24.11\ mm$
$\sigma_m = N/A - Mv/J_u = -203.4\ N/mm^2$
$y_c = 3.\ mm$
$u_c = -11.\ mm$
$v_c = -21.11\ mm$
$\sigma_c = N/A - Mv/J_u = -203.4\ N/mm^2$
$\tau_c = TS^*/tJ_u = 7.778\ N/mm^2$
$\tau_g = TS^*/tJ_u = 7.778\ N/mm^2$
$t_c = 180.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 203.9\ N/mm^2$



ESΣ05.xxxx.206

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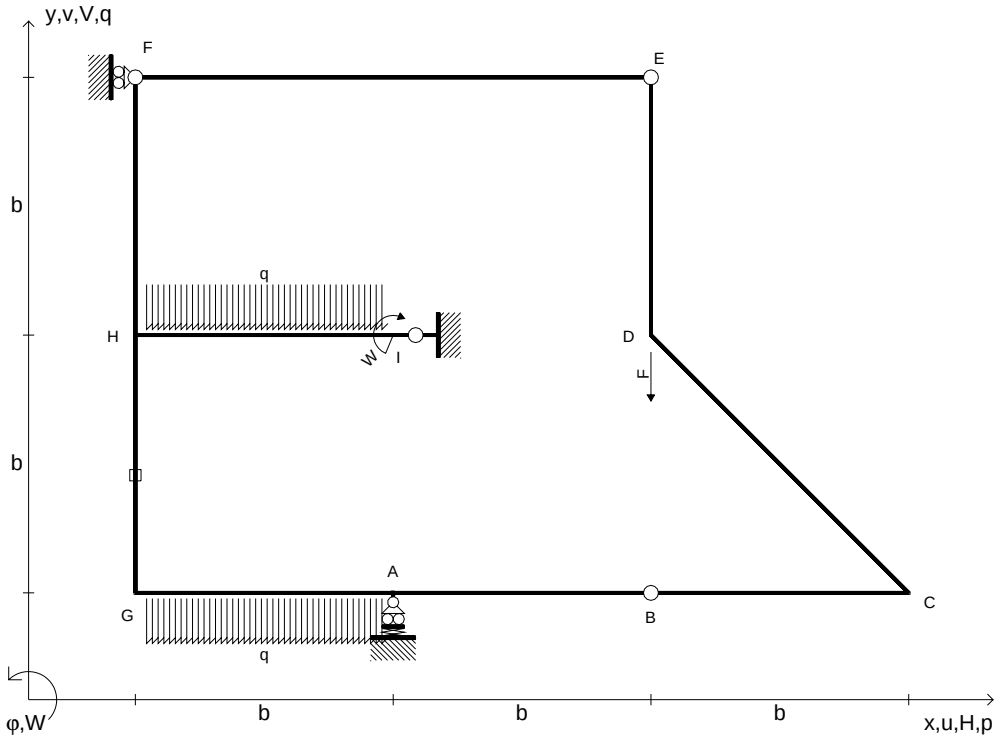
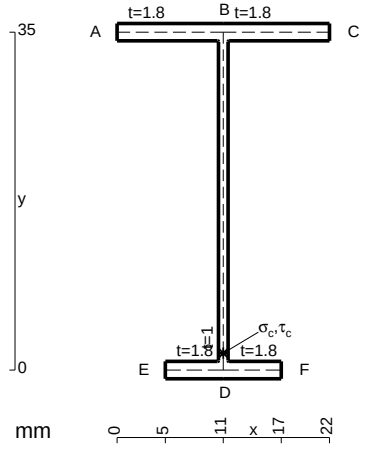



$V_D = -F$ | $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ | $EJ_{CD} = EJ$ | $EJ_{FH} = EJ$

$W_I = -W = -Fb$ | $k_A = 4EJ/b^3$ | $EJ_{DE} = EJ$ | $EJ_{HI} = EJ$

$q_{GA} = -q = -F/b$ | $EJ_{AB} = EJ$ | $EJ_{EF} = EJ$ | $EJ_{HG} = EJ$

$q_{HI} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GA} = EJ$




Reazioni iperstatiche in soluzione: $X=V_I$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave AB ha la sezione riportata e dimensioni in mm, con:

b = 360 mm, F = 620 N

Calcolare sulla sezione A la massima tensione normale $\sigma_m$.

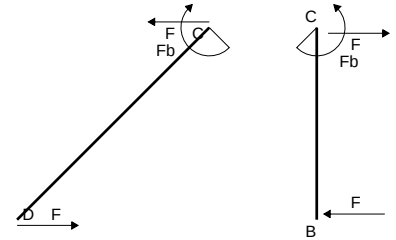
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.

Lembo inferiore sezione su tratteggio trave, a destra da A a B

Elongazione termica specifica ε assegnata su asta HG.





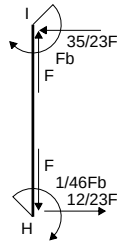


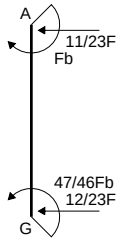


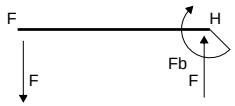


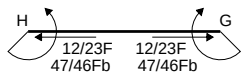


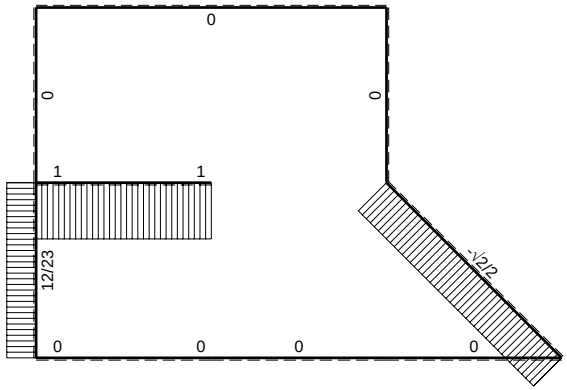




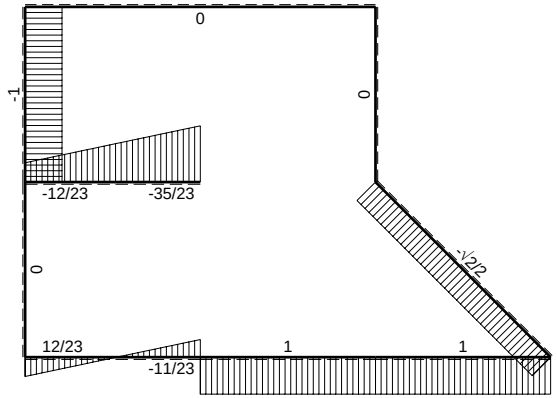



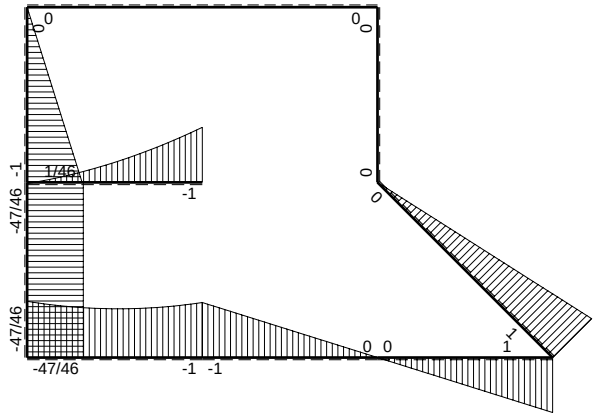






←□→ F



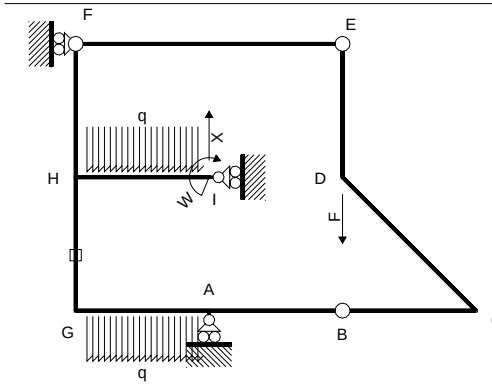

↑□↓ F



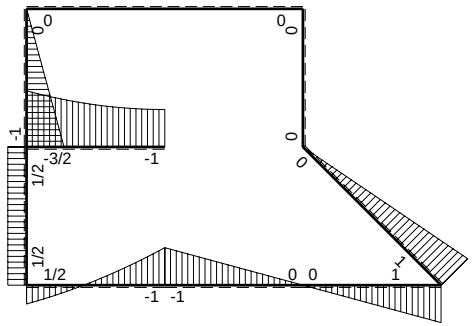

(□) Fb



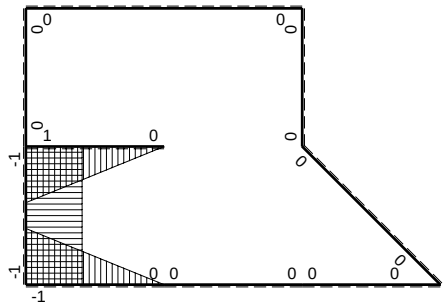





Schema di calcolo iperstatico




(□) $M_o$ flessione da carichi assegnati




(□) $M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_I$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -Fb+Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | Fx | 0 | 0 | | |
| BC b | 0 | Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -Fb+Fx | 0 | 0 | | |
| CD √2b | 0 | Fb-√2/2Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | 0 | 0 | 0 | 0 | 0 |
| FE 2b | 0 | 0 | 0 | 0 | | |
| GA b | -b+x | $1/2Fb-Fx-1/2qx^2$ | $-1/2Fb^2+3/2Fbx-1/2Fx^2-1/2qx^3$ | $b^2-2bx+x^2$ | $-1/24Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| AG b | x | $Fb-2Fx+1/2qx^2$ | $Fbx-2Fx^2+1/2qx^3$ | $x^2$ | | |
| FH b | 0 | -Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | Fb-Fx | 0 | 0 | | |
| HI b | b-x | $-3/2Fb+Fx-1/2qx^2$ | $-3/2Fb^2+5/2Fbx-3/2Fx^2+1/2qx^3$ | $b^2-2bx+x^2$ | $-5/8Fb^3/EJ$ | $1/3Xb^3/EJ$ |
| IH b | -x | $Fb+1/2qx^2$ | $-Fbx-1/2qx^3$ | $x^2$ | | |
| HG b | -b | 1/2Fb | $-1/2Fb^2$ | $b^2$ | $-1/2Fb^3/EJ$ | $Xb^3/EJ$ |
| GH b | b | -1/2Fb | $-1/2Fb^2$ | $b^2$ | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^3/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-3/4Fb^3/EJ$ | $1/4Xb^3/EJ$ |
| | totali | | | | $-35/12Fb^3/EJ$ | $23/12Xb^3/EJ$ |
| | iperstatica $X=V_I$ | | | | 35/23F | |



PROCEDIMENTO E RISULTATI 984497 ESΣ05.xxxx.212

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{GA} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{AG} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HI} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b + 1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (\, b - b + 1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{IH} = \int_o^b (\, x^2/b^2 \,)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2 \,]_o^b\, b^2\, 1/EJ$

$= (1/3\, b \,)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HG} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{GH} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x \,]_o^b\, b^2\, 1/EJ$

$= (\, b \,)\, b^2\, 1/EJ = b^3/EJ$

$L^{Xo}_{GA} = \int_o^b (-1/2 + 3/2\, x/b - 1/2\, x^2/b^2 - 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x + 3/4\, x^2/b - 1/6\, x^3/b^2 - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b + 3/4\, b - 1/6\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = -1/24\ Fb^3/EJ$

$L^{Xo}_{AG} = \int_o^b (\, x/b - 2\, x^2/b^2 + 1/2\, x^3/b^3 \,)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b - 2/3\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b - 2/3\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = -1/24\ Fb^3/EJ$

$L^{Xo}_{HI} = \int_o^b (-3/2 + 5/2\, x/b - 3/2\, x^2/b^2 + 1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-3/2\, x + 5/4\, x^2/b - 1/2\, x^3/b^2 + 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b + 5/4\, b - 1/2\, b + 1/8\, b \,)\, Fb^2\, 1/EJ = -5/8\ Fb^3/EJ$

$L^{Xo}_{IH} = \int_o^b (-\, x/b - 1/2\, x^3/b^3 \,)\, Fb^2\, 1/EJ\, dx = [-1/2\, x^2/b - 1/8\, x^4/b^3 \,]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b - 1/8\, b \,)\, Fb^2\, 1/EJ = -5/8\ Fb^3/EJ$

$L^{Xo}_{HG} = \int_o^b (-1/2 \,)\, Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = [-1/2\, x \,]_o^b\, Fb^2\, 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

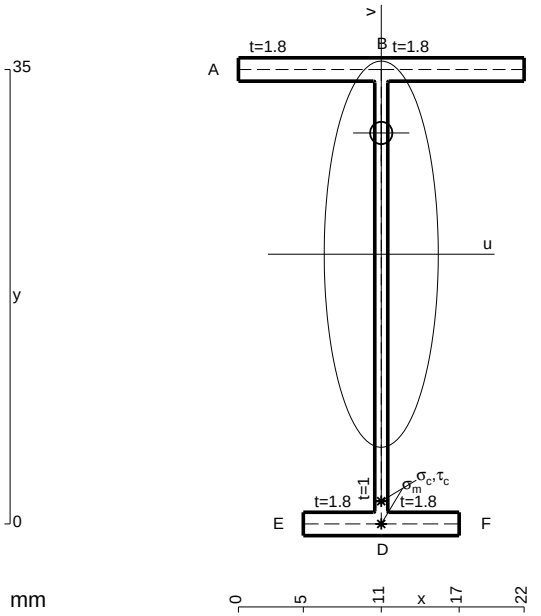
$= (-1/2\, b \,)\, Fb^2\, 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -3/2\ \ Fb^3/EJ$

$L^{Xo}_{GH} = \int_o^b (-1/2 \,)\, Fb^2\, 1/EJ\, dx - 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = [-1/2\, x \,]_o^b\, Fb^2\, 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ$

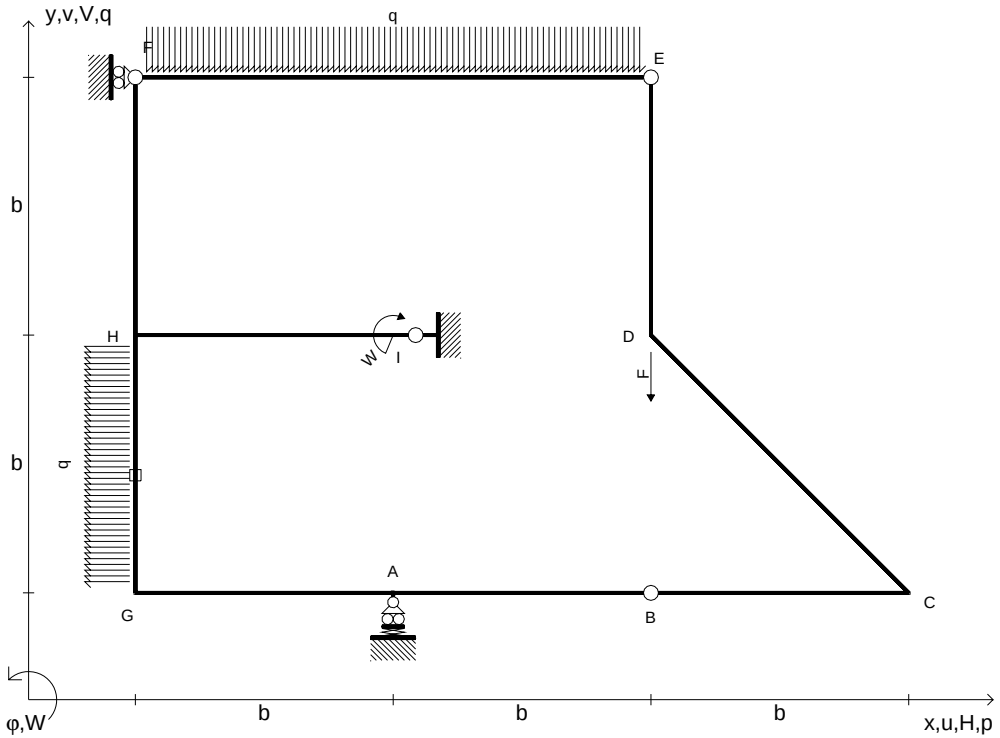
$= (-1/2\, b \,)\, Fb^2\, 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^3/EJ = -3/2\ \ Fb^3/EJ$








A = 96.2 $mm^2$
$J_u$ = 21284. $mm^4$
$J_v$ = 1856. $mm^4$
$J_t$ = 77.76 $mm^4$
$y_o$ = 9.339 mm
$y_g$ = 20.77 mm
$T_y$ = 620. N
$M_x$ = -223200. Nmm
$x_m$ = 11. mm
$v_m$ = -20.77 mm
$\sigma_m = -Mv/J_u$ = -217.9 $N/mm^2$
$y_c$ = 3. mm
$u_c$ = -11. mm
$v_c$ = -17.77 mm
$\sigma_c = -Mv/J_u$ = -217.9 $N/mm^2$
$\tau_c = TS^*/tJ_u$ = 13.07 $N/mm^2$
$\tau_g = TS^*/tJ_u$ = 13.07 $N/mm^2$
$t_c$ = 620. mm
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 219. $N/mm^2$

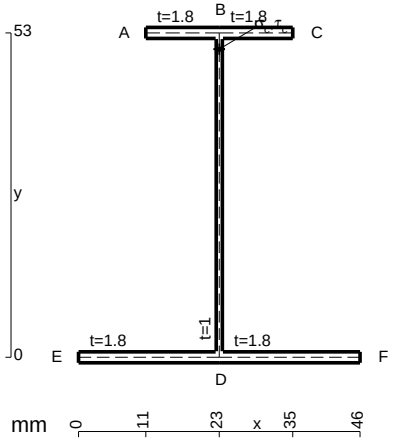



$V_D = -F$ $\varepsilon_{HG} = -\alpha T = -b^2F/EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$

$W_I = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$

$q_{EF} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$

$p_{HG} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$




Reazioni iperstatiche in soluzione: $X=W_{GA}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

$J_{YZ}$ - $x_{YZ}$ - $\theta_{YZ}$ riferimento locale asta YZ con origine in Y.

La trave FH ha la sezione riportata e dimensioni in mm, con:

b = 580 mm, F = 460 N

Calcolare sulla sezione H la massima tensione normale $\sigma_m$.

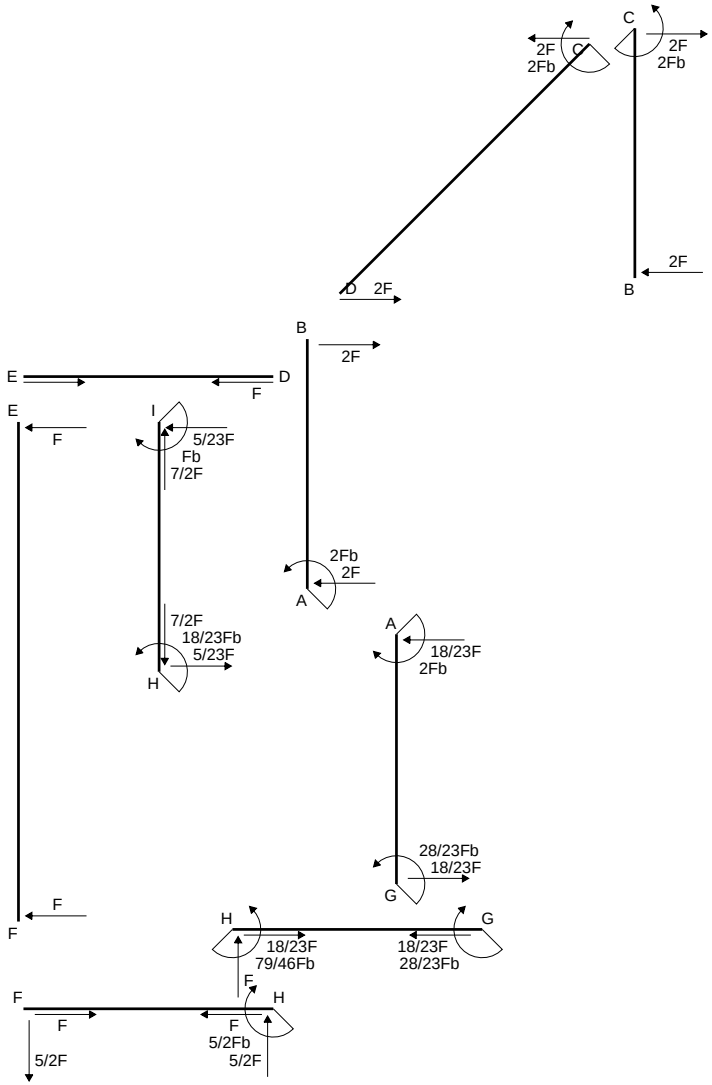
Calcolare in * le tensioni $\sigma_c$,$\tau_c$ e la tensione di von Mises.

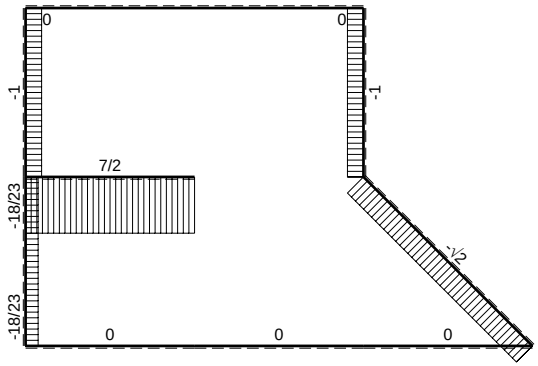
Lembo inferiore sezione su tratteggio trave, a destra da F a H

Elongazione termica specifica ε assegnata su asta HG.

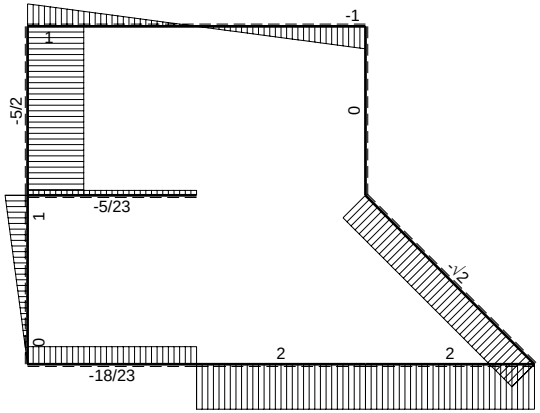




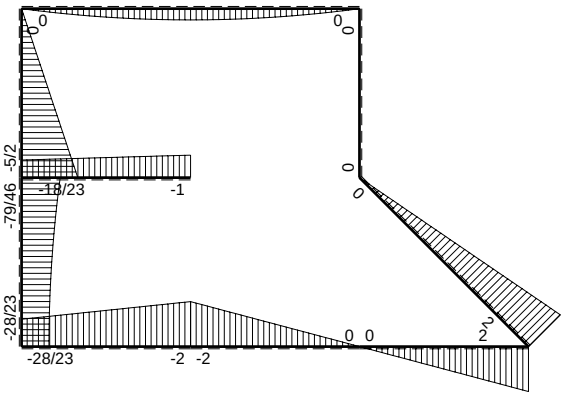





F



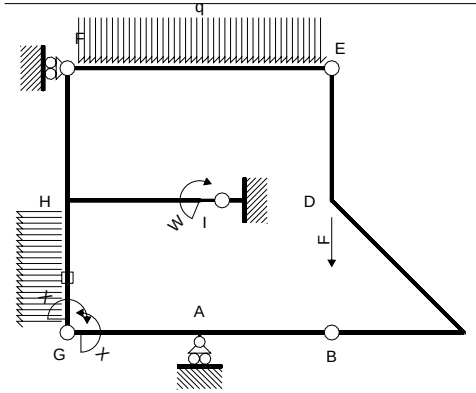

F

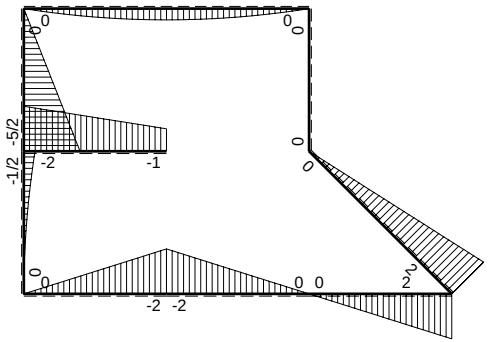



Fb

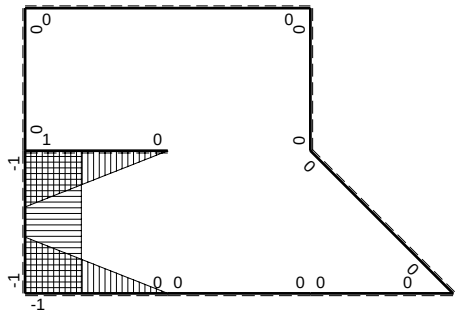







$M_o$ flessione da carichi assegnati




$M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

| → | $M_x(x)$ | $M_o(x)$ | $M_xM_o$ | $M_xM_x$ | $\int M_xM_o/EJdx$ | $\int XM_xM_x/EJdx$ |
|---|---|---|---|---|---|---|
| AB b | 0 | -2Fb+2Fx | 0 | 0 | 0 | 0 |
| BA b | 0 | 2Fx | 0 | 0 | | |
| BC b | 0 | 2Fx | 0 | 0 | 0 | 0 |
| CB b | 0 | -2Fb+2Fx | 0 | 0 | | |
| CD √2b | 0 | 2Fb-√2Fx | 0 | 0 | 0 | 0 |
| DE b | 0 | 0 | 0 | 0 | 0 | 0 |
| ED b | 0 | 0 | 0 | 0 | | |
| EF 2b | 0 | $-Fx+1/2qx^2$ | 0 | 0 | 0 | 0 |
| FE 2b | 0 | $Fx-1/2qx^2$ | 0 | 0 | | |
| GA b | -1+x/b | -2Fx | $2Fx-2Fx^2/b$ | $1-2x/b+x^2/b^2$ | $1/3Fb^2/EJ$ | 1/3Xb/EJ |
| AG b | x/b | 2Fb-2Fx | $2Fx-2Fx^2/b$ | $x^2/b^2$ | | |
| FH b | 0 | -5/2Fx | 0 | 0 | 0 | 0 |
| HF b | 0 | 5/2Fb-5/2Fx | 0 | 0 | | |
| HI b | 1-x/b | -2Fb+Fx | $-2Fb+3Fx-Fx^2/b$ | $1-2x/b+x^2/b^2$ | $-5/6Fb^2/EJ$ | 1/3Xb/EJ |
| IH b | -x/b | Fb+Fx | $-Fx-Fx^2/b$ | $x^2/b^2$ | | |
| HG b | -1 | $-1/2Fb+Fx-1/2qx^2$ | $1/2Fb-Fx+1/2Fx^2/b$ | 1 | $1/6Fb^2/EJ$ | Xb/EJ |
| GH b | 1 | $1/2qx^2$ | $1/2Fx^2/b$ | 1 | | |
| HG | elongazione asta $N_{1HG}\varepsilon_{HG}L_{HG}$ | | | | $-Fb^2/EJ$ | |
| A | molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$ | | | | $-Fb^2/EJ$ | 1/4Xb/EJ |
| | totali | | | | $-7/3Fb^2/EJ$ | 23/12Xb/EJ |
| | iperstatica $X=W_{GA}$ | | | | 28/23Fb | |

Sviluppi di calcolo iperstatica





$L_{GA}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{AG}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HI}^{XX} = \int_o^b (1 -2\,x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{IH}^{XX} = \int_o^b (\ x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{HG}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GH}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{GA}^{Xo} = \int_o^b (2\,x/b - 2\,x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b - 2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - 2/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{AG}^{Xo} = \int_o^b (2\,x/b - 2\,x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x^2/b - 2/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b - 2/3\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L_{HI}^{Xo} = \int_o^b (-2 + 3\,x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\,x + 3/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-2\ b + 3/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L_{IH}^{Xo} = \int_o^b (-\,x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b - 1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b - 1/3\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

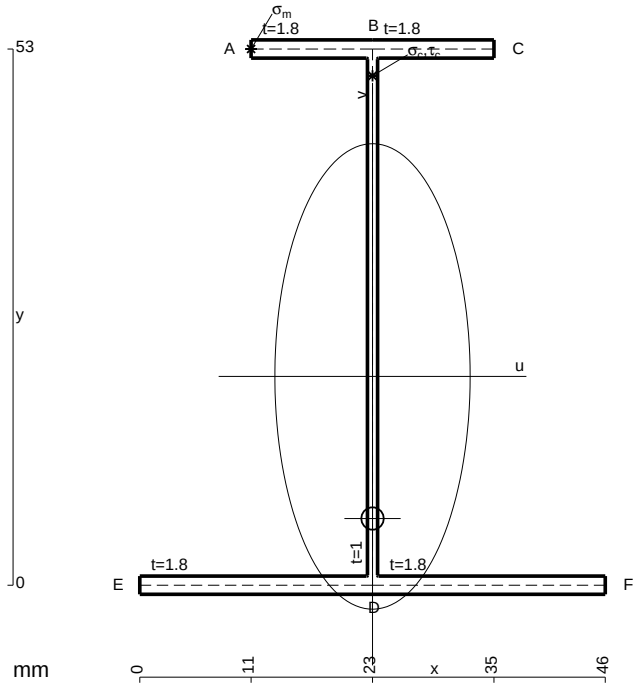
$L_{HG}^{Xo} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/2\ b - 1/2\ b + 1/6\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$

$L_{GH}^{Xo} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ (-1)\ \ Fb^2/EJ = -5/6\ \ Fb^2/EJ$




$A = 179.\ mm^2$
$J_u = 94738.\ mm^4$
$J_v = 16674.\ mm^4$
$J_t = 153.7\ mm^4$
$y_o = -14.05\ mm$
$y_g = 20.64\ mm$
$N = -460.\ N$
$T_y = -1150.\ N$
$M_x = -667000.\ Nmm$
$x_m = 11.\ mm$
$y_m = 53.\ mm$
$u_m = -12.\ mm$
$v_m = 32.36\ mm$
$\sigma_m = N/A - Mv/J_u = 225.3\ N/mm^2$
$x_c = 23.\ mm$
$y_c = 53.\ mm$
$v_c = 32.36\ mm$
$\sigma_c = N/A - Mv/J_u = 225.3\ N/mm^2$
$\tau_c = TS^*/tJ_u = 16.97\ N/mm^2$
$\tau_g = TS^*/tJ_u = 16.97\ N/mm^2$
$t_c = 460.\ mm$
$\sigma_o = \sqrt{\sigma^2+3\tau^2} = 227.2\ N/mm^2$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