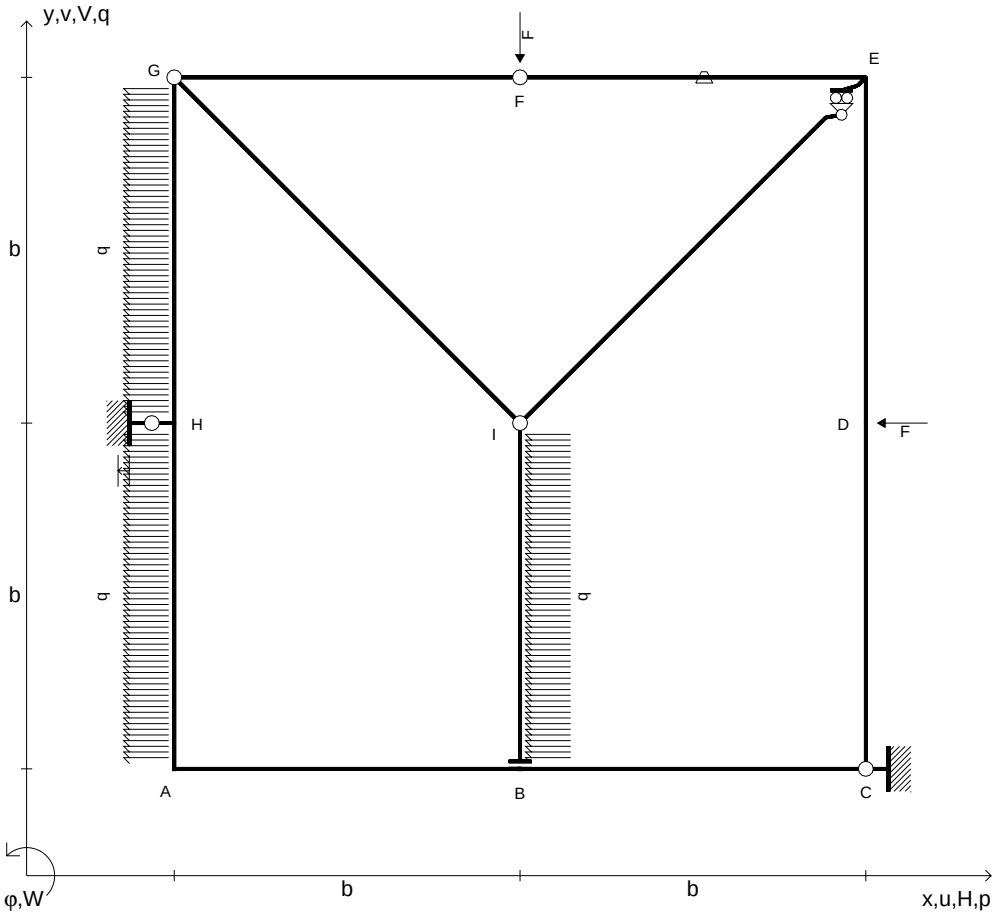


$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{IB} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

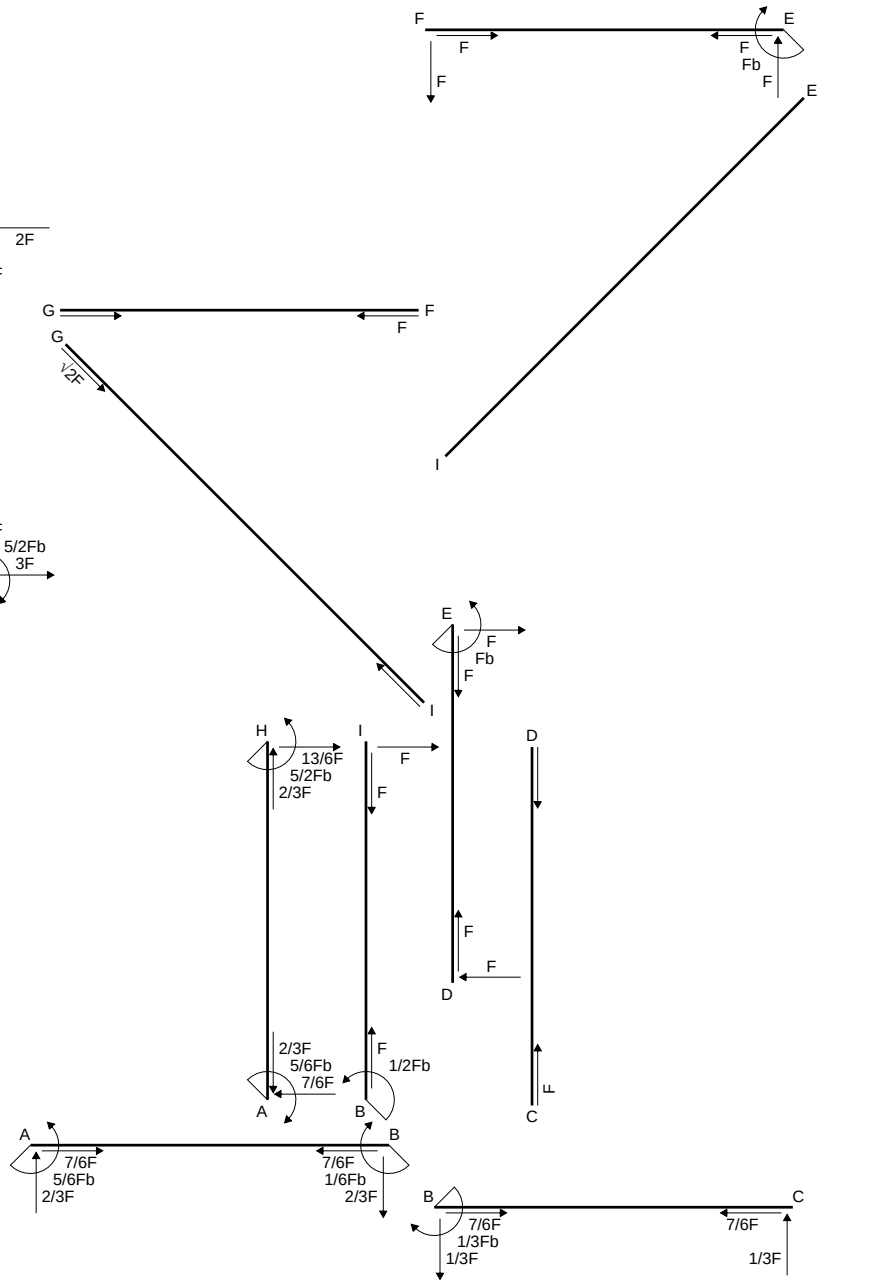
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

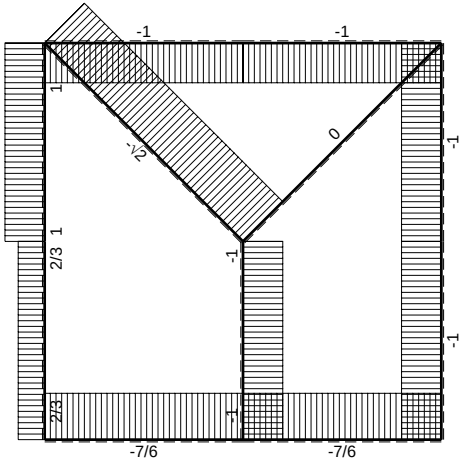
Curvatura θ asta EF positiva se convessa a destra con inizio E.

Spostamento orizzontale assoluto u imposto al nodo H.

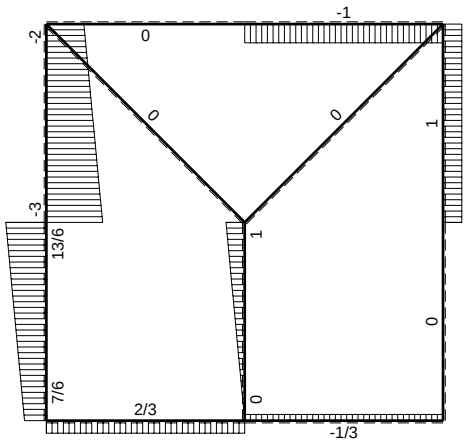
 08.01.25



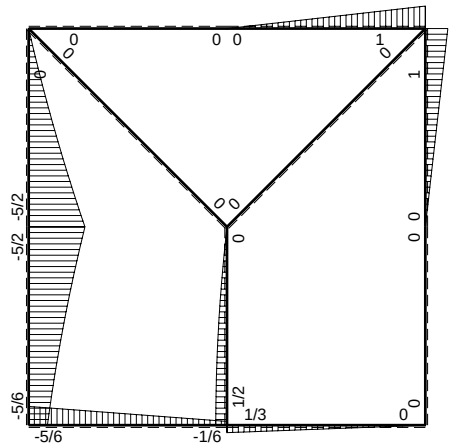
 08.01.25



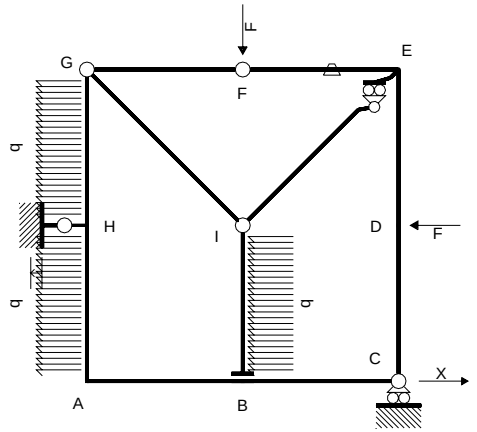
←□→ F

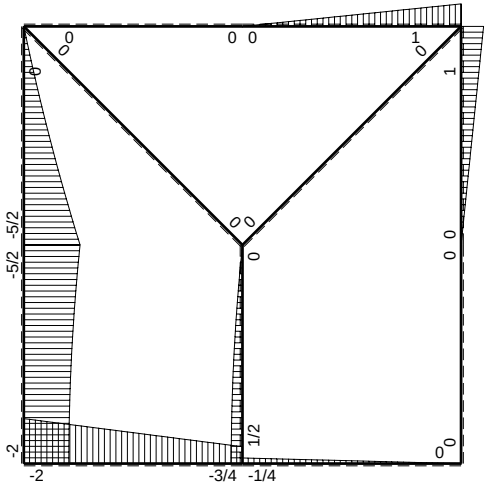


↑□↓ F

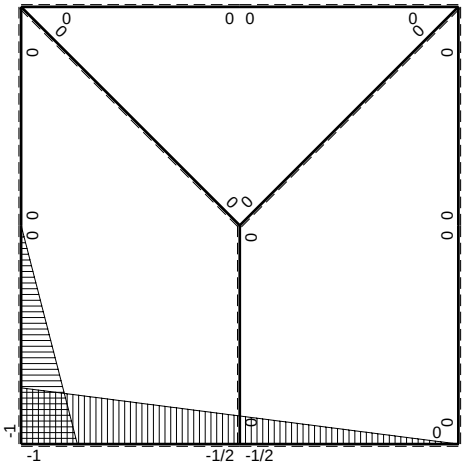


↻□↺ Fb





↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-b+1/2x	-2Fb+5/4Fx	0	$2Fb^2-9/4Fbx+5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(13/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	1/2b+1/2x	3/4Fb+5/4Fx	0	$3/8Fb^2+Fbx+5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	-1/2b+1/2x	-1/4Fb+1/4Fx	0	$1/8Fb^2-1/4Fbx+1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	1/2x	1/4Fx	0	$1/8Fx^2$	0	$1/4x^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x	$-5/2Fb+Fx-1/2qx^2$	0	$5/2Fbx-Fx^2+1/2qx^3$	0	x^2	$(25/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$2Fb+1/2qx^2$	0	$2Fb^2-2Fbx+1/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$7/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-7/6F	

PROCEDIMENTO E RISULTATI 957901 ESΣ04.xxxx.010

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$$

$$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/4\ b + 1/4\ b + 1/12\ b\right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$$

$$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/4\ b - 1/4\ b + 1/12\ b\right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$$

$$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/12\ b\right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$$

$$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/3\ b\right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$$

$$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(\ b - b + 1/3\ b\right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$$

$$L^{Xo}_{AB} = \int_o^b \left(2 - 9/4\ x/b + 5/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[2\ x - 9/8\ x^2/b + 5/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(2\ b - 9/8\ b + 5/24\ b\right) Fb^2\ 1/EJ\ = 13/12\ \ Fb^3/EJ$$

$$L^{Xo}_{BA} = \int_o^b \left(3/8 + x/b + 5/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x + 1/2\ x^2/b + 5/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(3/8\ b + 1/2\ b + 5/24\ b\right) Fb^2\ 1/EJ\ = 13/12\ \ Fb^3/EJ$$

$$L^{Xo}_{BC} = \int_o^b \left(1/8 - 1/4\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(1/8\ b - 1/8\ b + 1/24\ b\right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$$

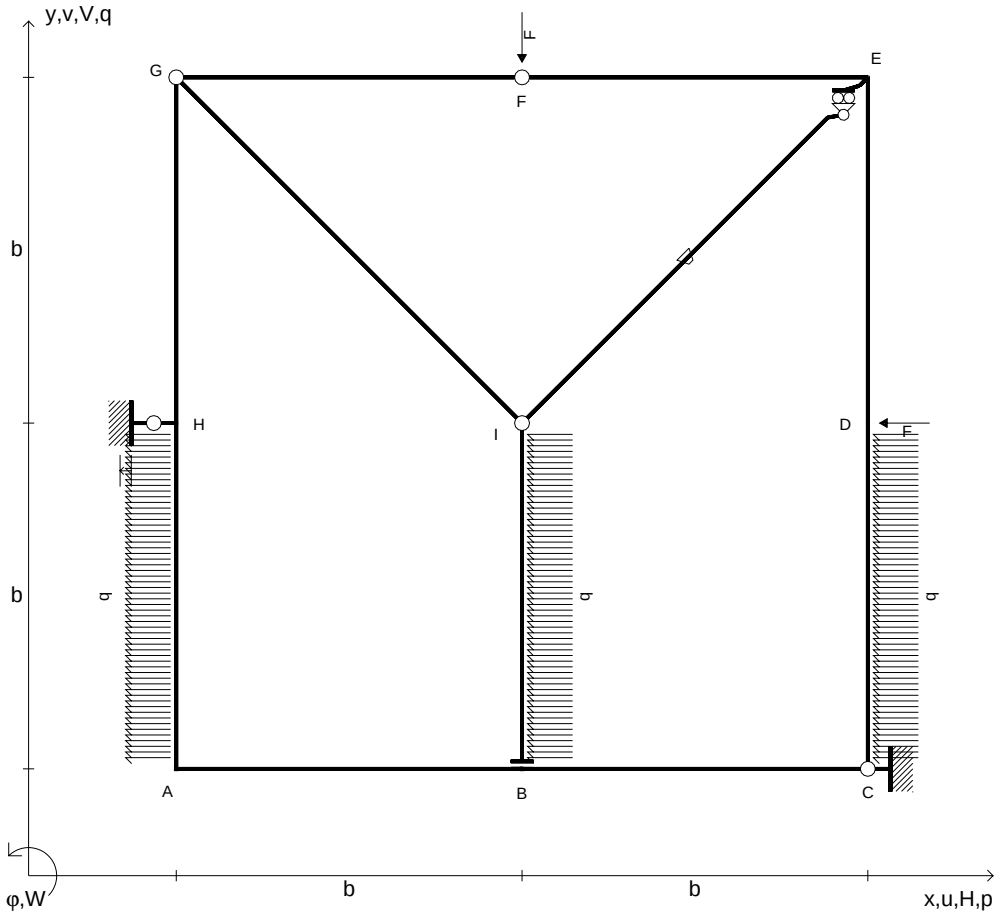
$$L^{Xo}_{CB} = \int_o^b \left(1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(1/24\ b\right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$$

$$L^{Xo}_{HA} = \int_o^b \left(5/2\ x/b - x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[5/4\ x^2/b - 1/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(5/4\ b - 1/3\ b + 1/8\ b\right) Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$$

$$L^{Xo}_{AH} = \int_o^b \left(2 - 2\ x/b + 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[2\ x - x^2/b + 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(2\ b - b + 1/6\ b - 1/8\ b\right) Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$$

ESΣ04.xxxx.010

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



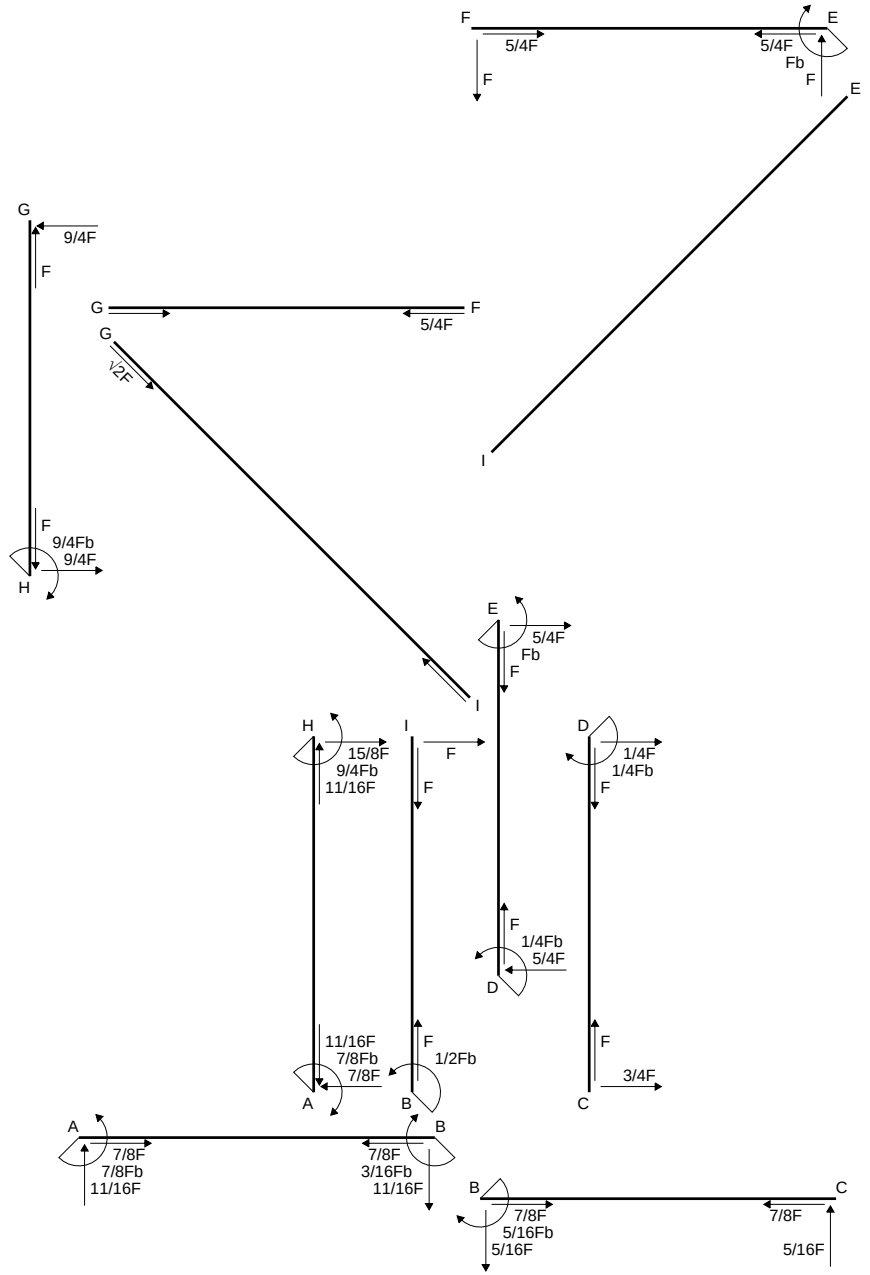
$V_F = -F$
$H_D = -F$
$p_{HA} = -q = -F/b$
$p_{IB} = -q = -F/b$
$p_{CD} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$

$u_H = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$

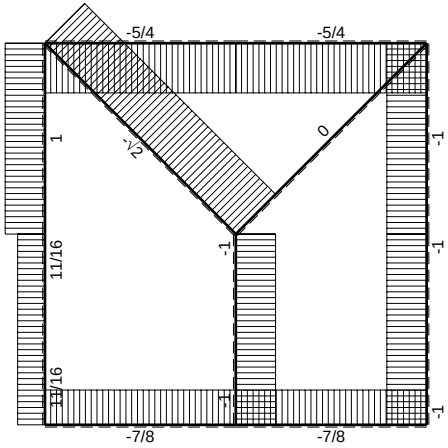
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{GI} = EJ$
$EJ_{IB} = EJ$
$EJ_{IE} = EJ$
$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta IE positiva se convessa a destra con inizio I.
Spostamento orizzontale assoluto u imposto al nodo H.

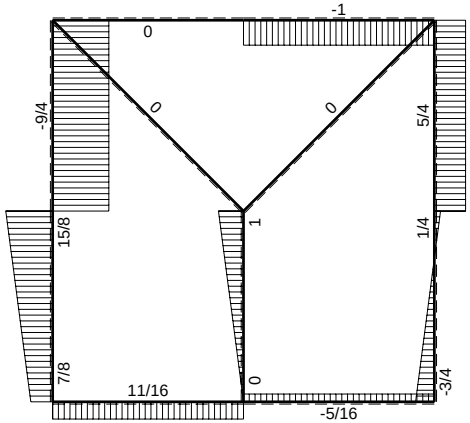
 08.01.25



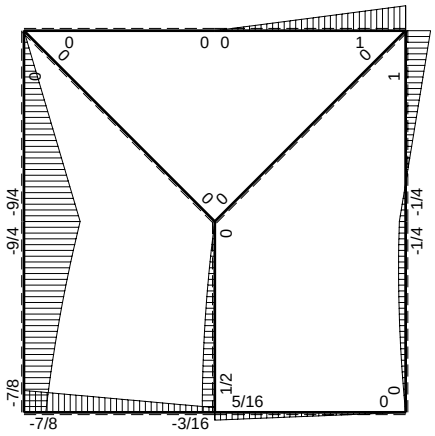
 08.01.25



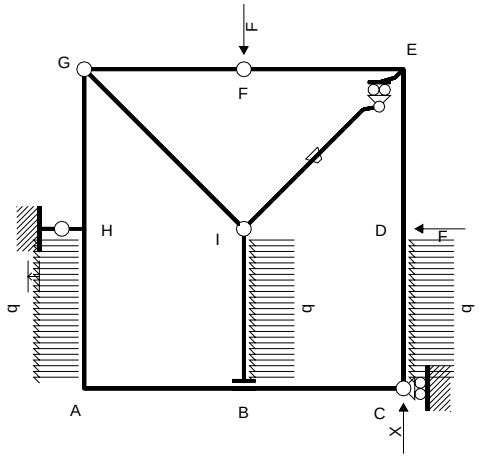
←□→ F



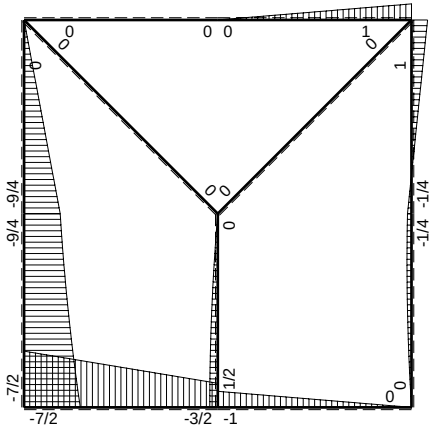
↑□↓ F



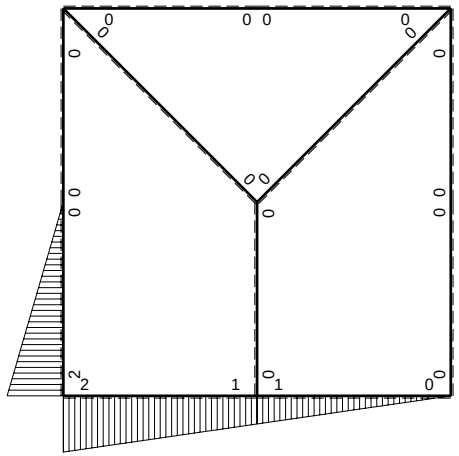
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	2b-x	-7/2Fb+2Fx	0	$-7Fb^2+15/2Fbx-2Fx^2$	0	$4b^2-4bx+x^2$	$(-47/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	-b-x	3/2Fb+2Fx	0	$-3/2Fb^2-7/2Fbx-2Fx^2$	0	$b^2+2bx+x^2$		
BC b	b-x	-Fb+Fx	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	-x	Fx	0	$-Fx^2$	0	x^2		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	2x	$-9/4Fb-3/4Fx-1/2qx^2$	0	$-9/2Fbx-3/2Fx^2-qx^3$	0	$4x^2$	$(-3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	-2b+2x	$7/2Fb-7/4Fx+1/2qx^2$	0	$-7Fb^2+21/2Fbx-9/2Fx^2+qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$2Fb^3/EJ$	
	totali						$-21/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_C$						21/16F	

PROCEDIMENTO E RISULTATI 211309 ESΣ04.xxxx.024

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-7 + 15/2\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-7\,x + 15/4\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-7\,b + 15/4\,b - 2/3\,b)\, Fb^2\, 1/EJ = -47/12\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-3/2 - 7/2\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/2\,x - 7/4\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\,b - 7/4\,b - 2/3\,b)\, Fb^2\, 1/EJ = -47/12\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1 + 2\,x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-x + x^2/b - 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-b + b - 1/3\,b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/3\,b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (-9/2\,x/b - 3/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-9/4\,x^2/b - 1/2\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-9/4\,b - 1/2\,b - 1/4\,b)\, Fb^2\, 1/EJ = -3\ Fb^3/EJ$

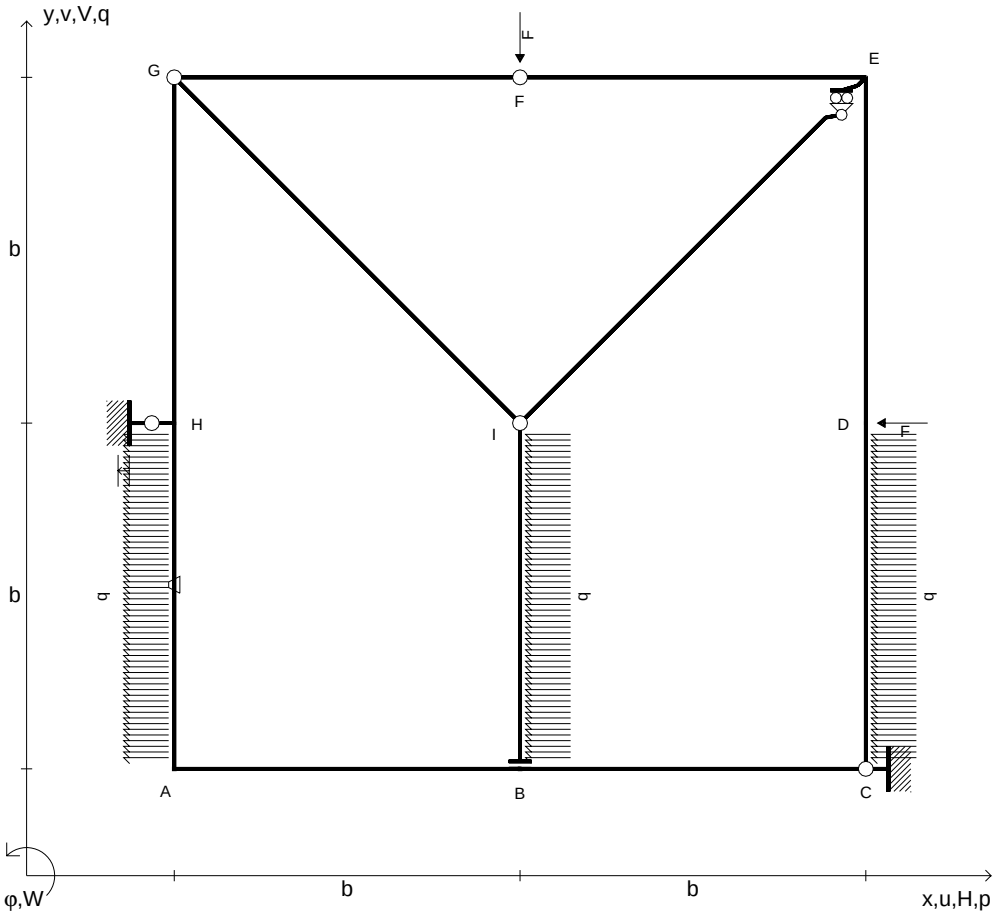
$L^{Xo}_{AH} = \int_o^b (-7 + 21/2\,x/b - 9/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-7\,x + 21/4\,x^2/b - 3/2\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-7\,b + 21/4\,b - 3/2\,b + 1/4\,b)\, Fb^2\, 1/EJ = -3\ Fb^3/EJ$

ESΣ04.xxxx.024

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{IB} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{HA} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

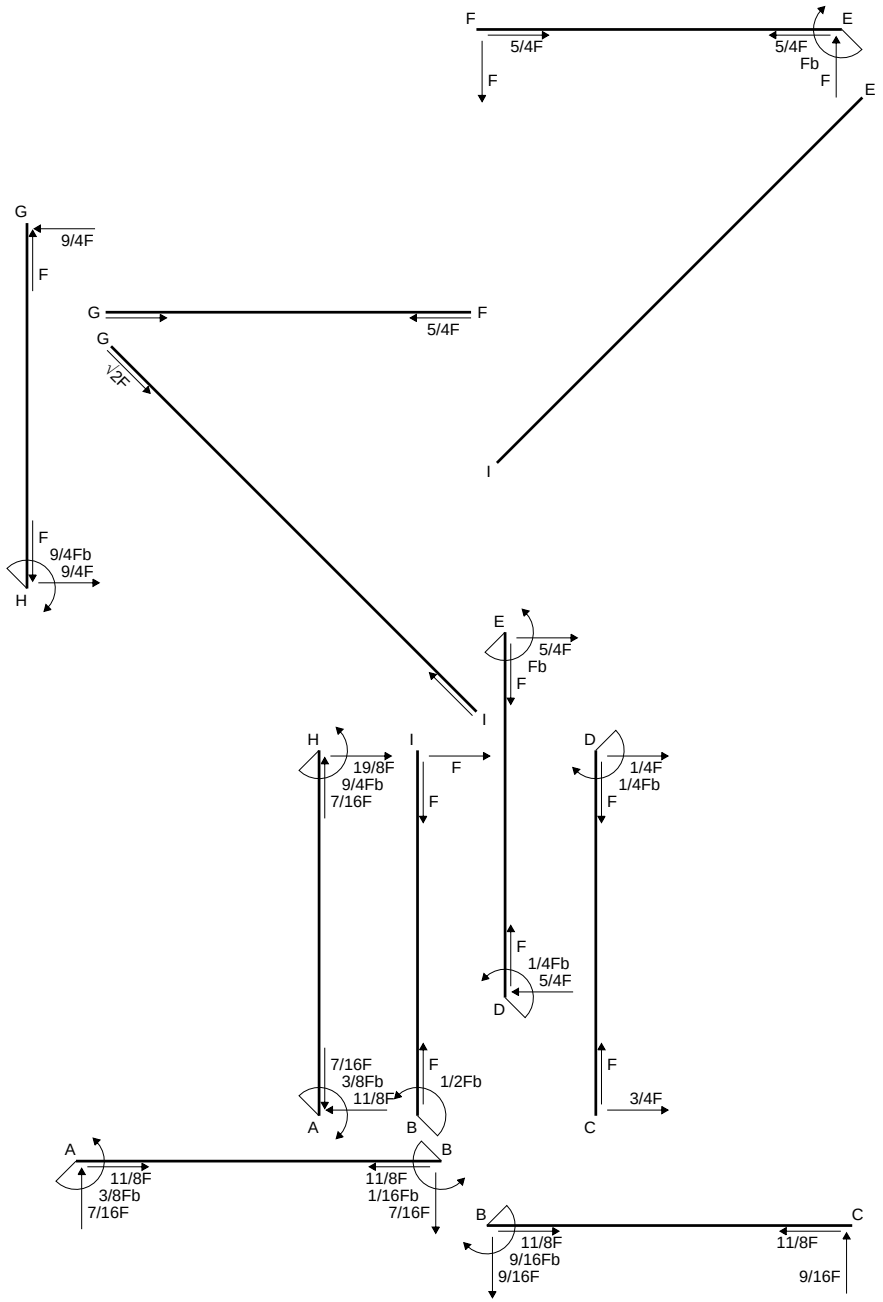
Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

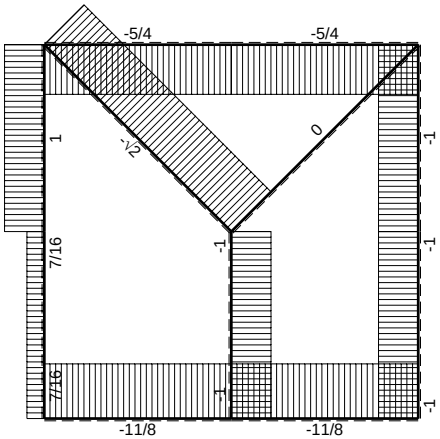
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

Curvatura θ asta HA positiva se convessa a destra con inizio H.

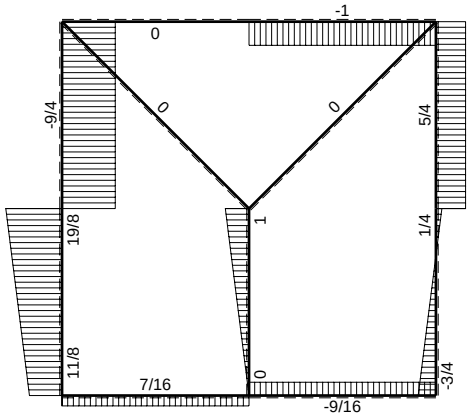
Spostamento orizzontale assoluto u imposto al nodo H.



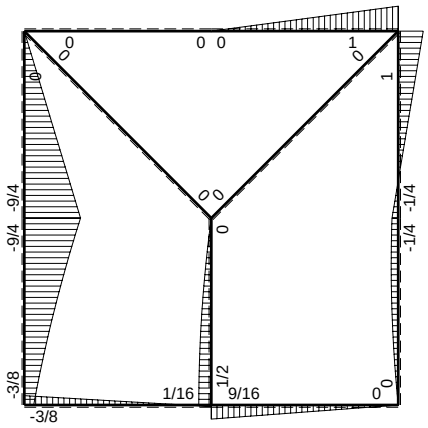
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



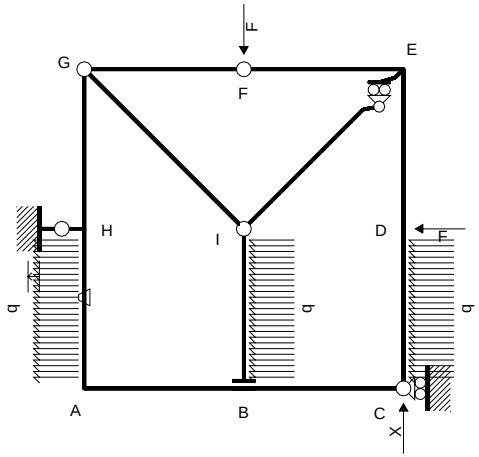
←F→



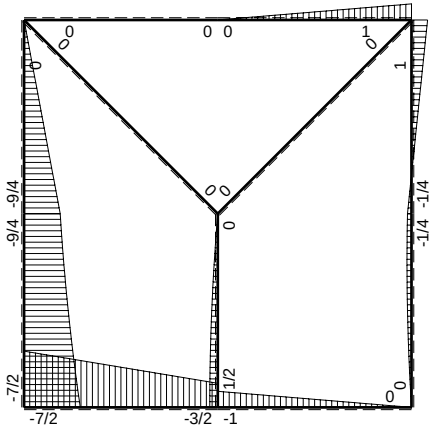
↑F↓



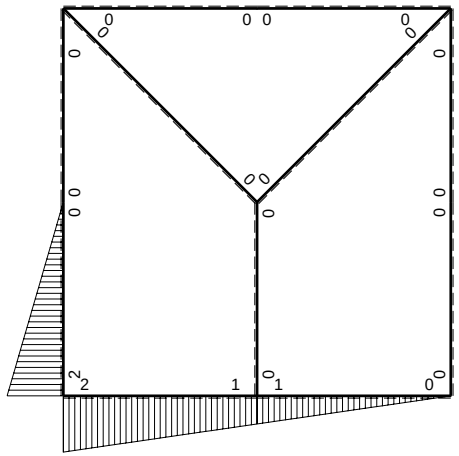
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210099 ESΣ04.xxxx.029

Quadro contributi PLV per iperstatica $X=V_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	2b-x	-7/2Fb+2Fx	0	$-7Fb^2+15/2Fbx-2Fx^2$	0	$4b^2-4bx+x^2$	$(-47/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	-b-x	3/2Fb+2Fx	0	$-3/2Fb^2-7/2Fbx-2Fx^2$	0	$b^2+2bx+x^2$		
BC b	b-x	-Fb+Fx	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	-x	Fx	0	$-Fx^2$	0	x^2		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	2x	$-9/4Fb-3/4Fx-1/2qx^2$	-Fb/EJ	$-9/2Fbx-3/2Fx^2-qx^3$	-2Fxb/EJ	$4x^2$	$(-3-1)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	-2b+2x	$7/2Fb-7/4Fx+1/2qx^2$	Fb/EJ	$-7Fb^2+21/2Fbx-9/2Fx^2+qx^3$	$-2Fb^2/EJ+2Fxb/EJ$	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$2Fb^3/EJ$	
	totali						$-25/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_C$						25/16F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-7 + 15/2\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-7\,x + 15/4\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-7\,b + 15/4\,b - 2/3\,b)\, Fb^2\, 1/EJ = -47/12\; Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-3/2 - 7/2\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/2\,x - 7/4\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\,b - 7/4\,b - 2/3\,b)\, Fb^2\, 1/EJ = -47/12\; Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1 + 2\,x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-x + x^2/b - 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-b + b - 1/3\,b)\, Fb^2\, 1/EJ = -1/3\; Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/3\,b)\, Fb^2\, 1/EJ = -1/3\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (-9/2\,x/b - 3/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (-2\,x/b)\,\theta\; dx$

$= [-9/4\,x^2/b - 1/2\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ + [-x^2/b]_o^b\,\theta$

$= (-9/4\,b - 1/2\,b - 1/4\,b)\, Fb^2\, 1/EJ + (-b)\,\theta = -4\; Fb^3/EJ$

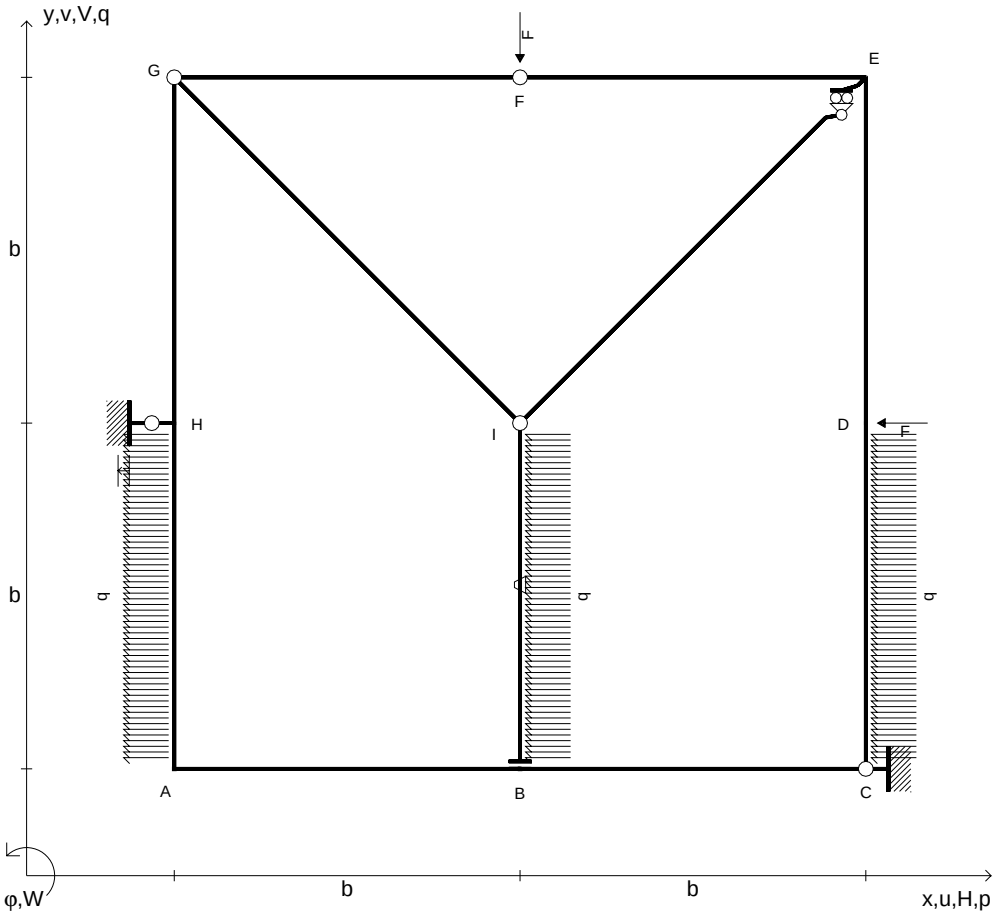
$L^{Xo}_{AH} = \int_o^b (-7 + 21/2\,x/b - 9/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (2 - 2\,x/b)\,\theta\; dx$

$= [-7\,x + 21/4\,x^2/b - 3/2\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ + [2\,x - x^2/b]_o^b\,\theta$

$= (-7\,b + 21/4\,b - 3/2\,b + 1/4\,b)\, Fb^2\, 1/EJ + (2\,b - b)\,\theta = -4\; Fb^3/EJ$

PROCEDIMENTO E RISULTATI 210099 ESΣ04.xxxx.029

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{IB} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{IB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

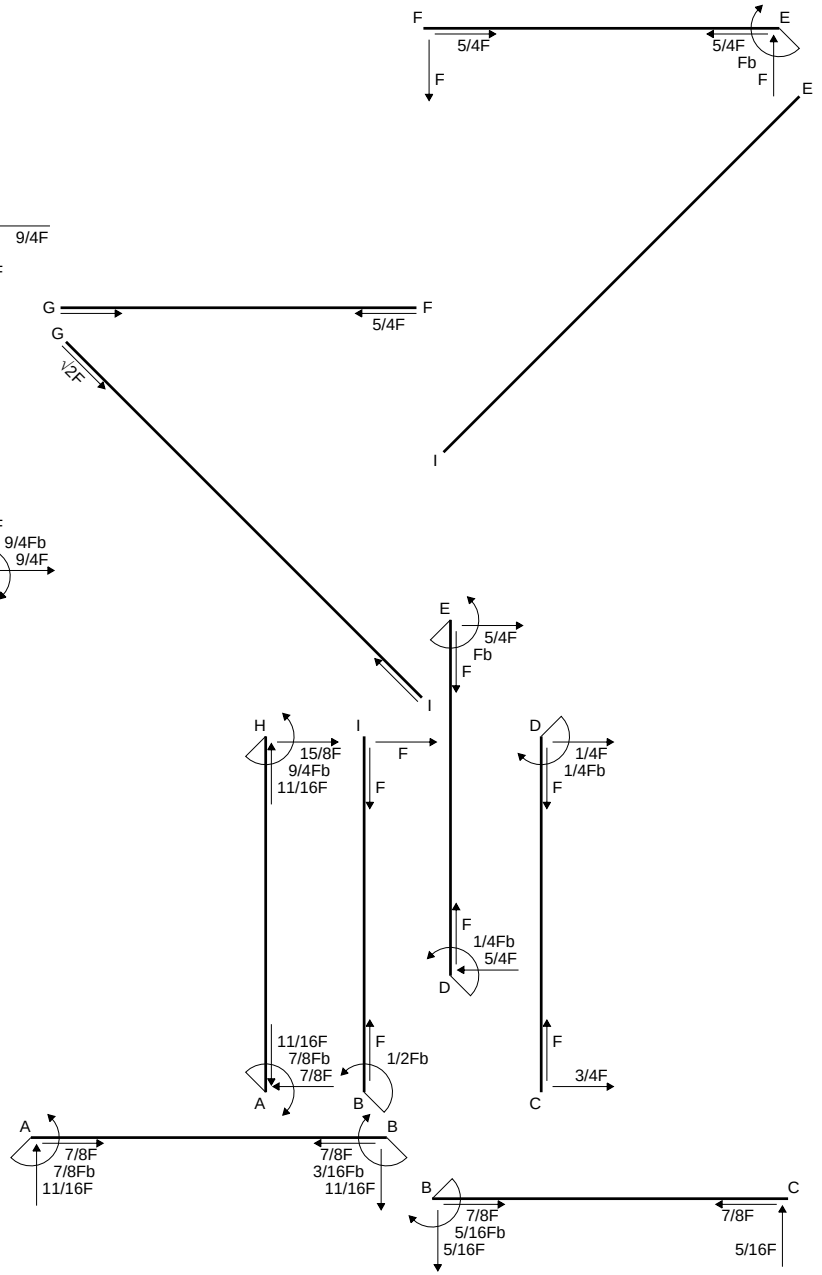
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

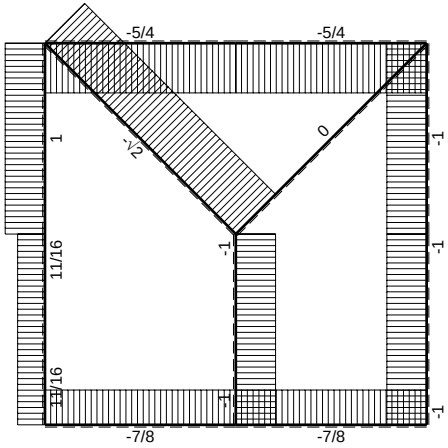
Curvatura θ asta IB positiva se convessa a destra con inizio I.

Spostamento orizzontale assoluto u imposto al nodo H.

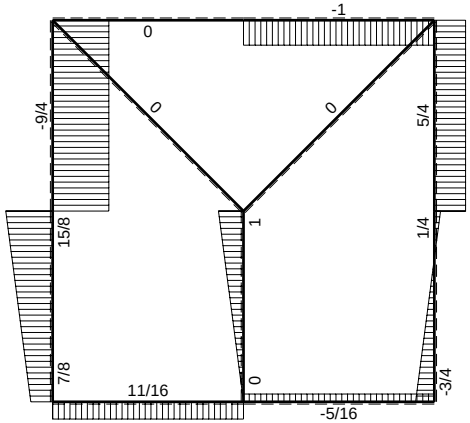
 08.01.25



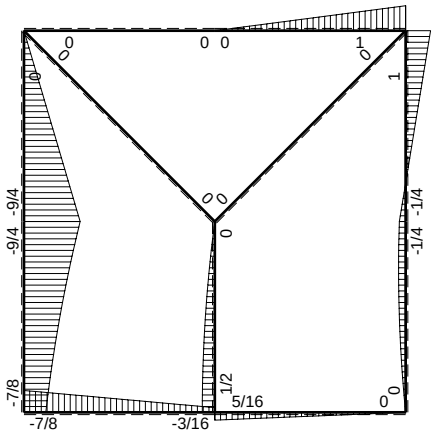
 08.01.25



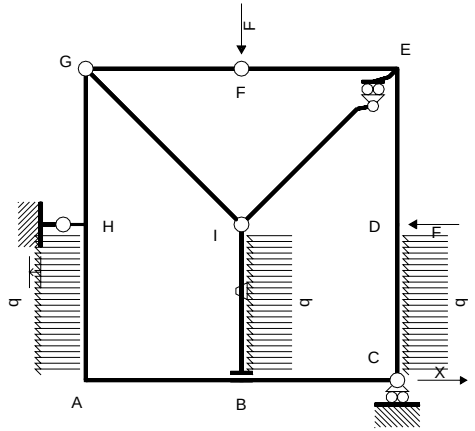
←F→

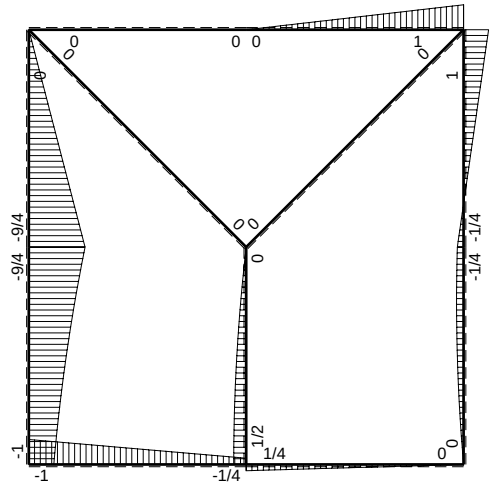


↑F↓

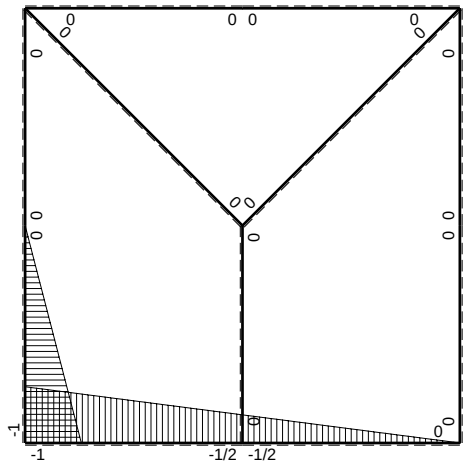


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212663 ESΣ04.xxxx.030

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-Fb+3/4Fx$	0	$Fb^2-5/4Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/2+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$1/4Fb+3/4Fx$	0	$1/8Fb^2+1/2Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$1/4Fb-1/4Fx$	0	$-1/8Fb^2+1/4Fbx-1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$-1/4Fx$	0	$-1/8Fx^2$	0	$1/4x^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-Fb+5/4Fx$	0	0	0	0		
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0
FE b	0	$-Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	$-x$	$-9/4Fb+7/4Fx-1/2qx^2$	0	$9/4Fbx-7/4Fx^2+1/2qx^3$	0	x^2	$(2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	$b-x$	$Fb+3/4Fx+1/2qx^2$	0	$Fb^2-1/4Fbx-1/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$1/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-1/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(1 - 5/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[\ x - 5/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(\ b - 5/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/2\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(1/8 + 1/2\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x + 1/4\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b + 1/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/2\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-1/8 + 1/4\ x/b - 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-1/8\ x + 1/8\ x^2/b - 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-1/8\ b + 1/8\ b - 1/24\ b\ \right) Fb^2\ 1/EJ\ = -1/24\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-1/24\ b\ \right) Fb^2\ 1/EJ\ = -1/24\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(9/4\ x/b - 7/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[9/8\ x^2/b - 7/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

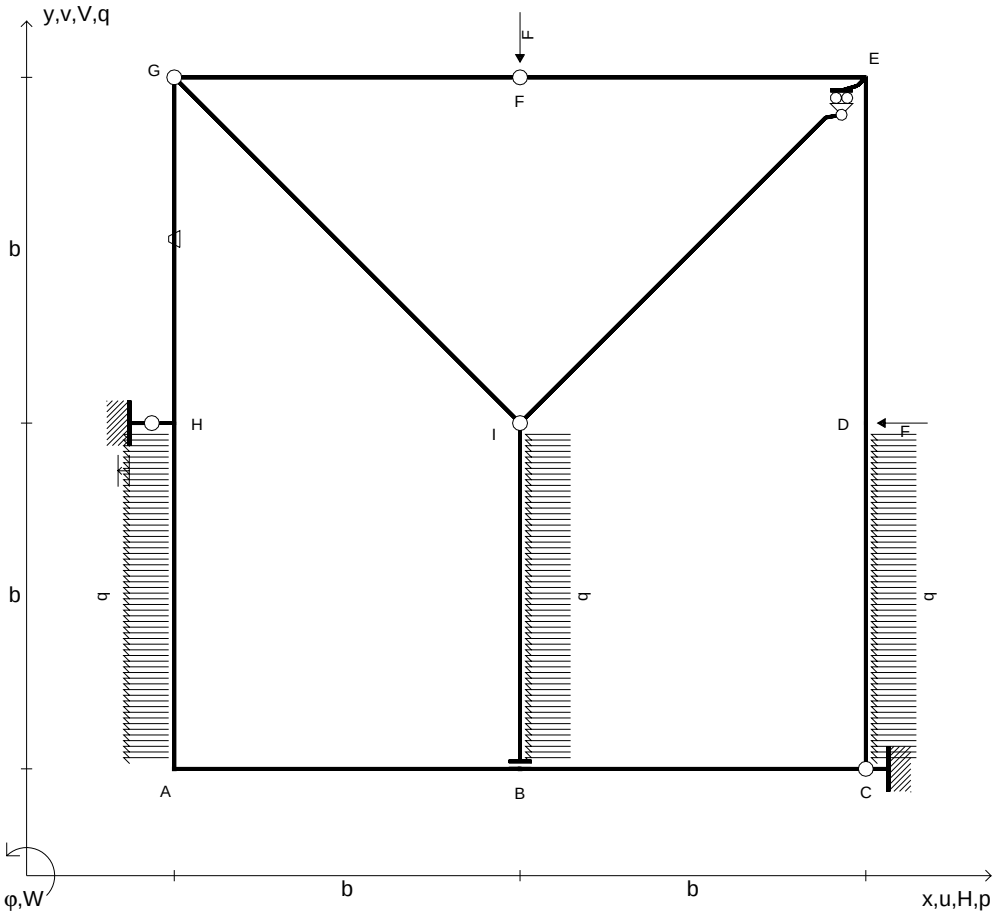
$= \left(9/8\ b - 7/12\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 2/3\ \ Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b \left(1 - 1/4\ x/b - 1/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[\ x - 1/8\ x^2/b - 1/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(\ b - 1/8\ b - 1/12\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 2/3\ \ Fb^3/EJ$

ESΣ04.xxxx.030

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{IB} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{GH} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

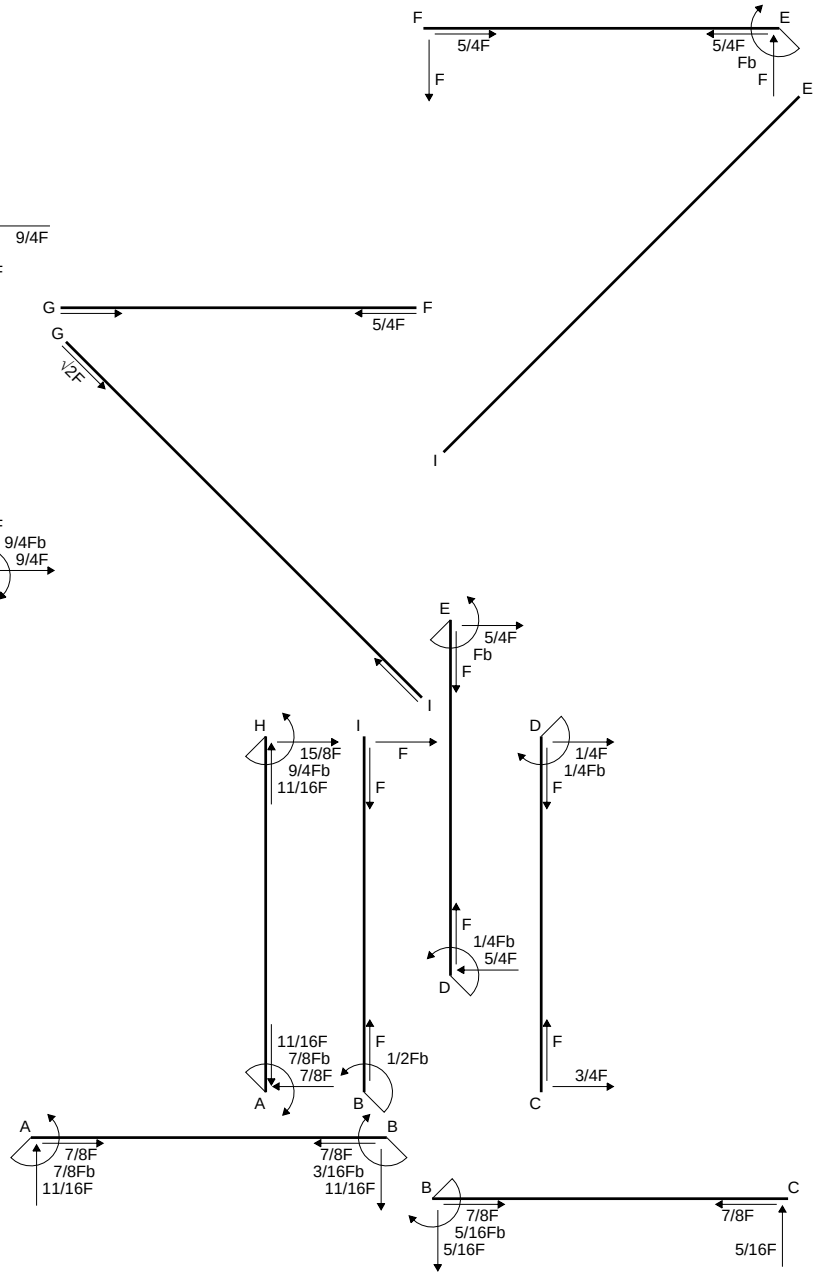
Tracciare i diagrammi quotati delle azioni interne nelle aste.

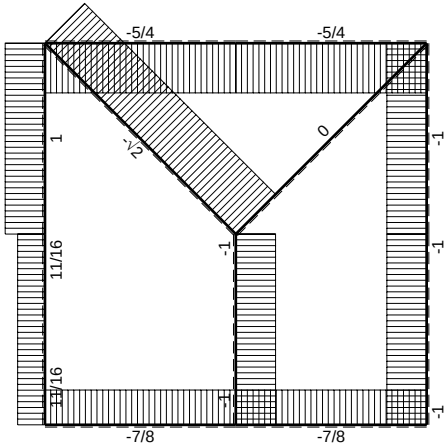
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

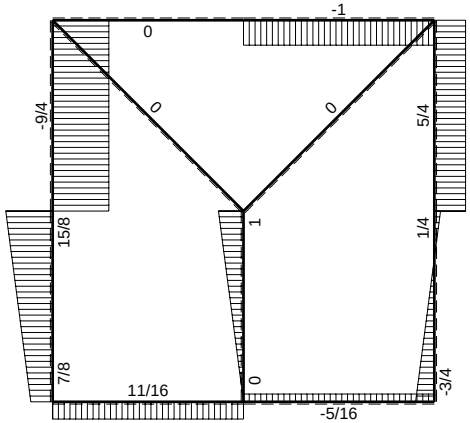
Curvatura θ asta GH positiva se convessa a destra con inizio G.

Spostamento orizzontale assoluto u imposto al nodo H.

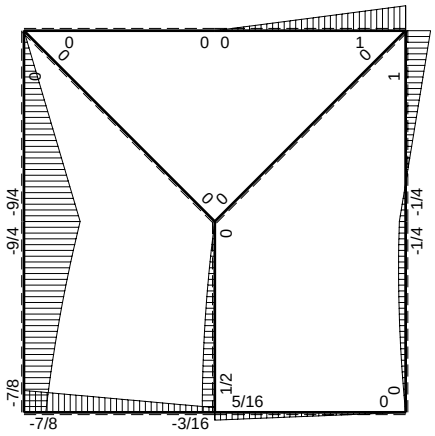




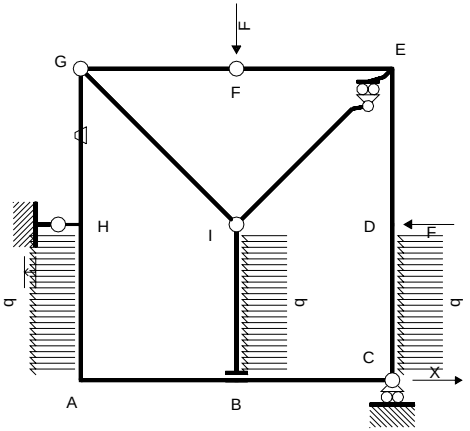
←⊞→ F

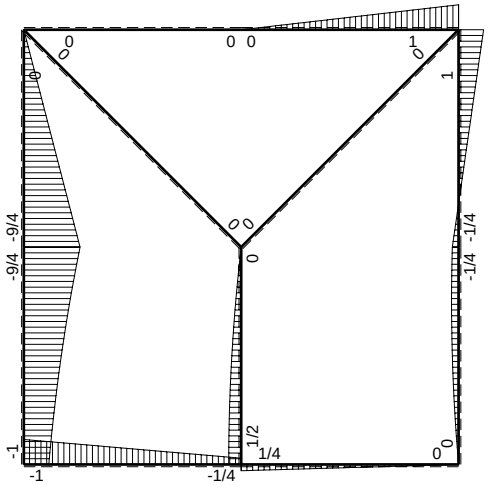


↑⊞↓ F

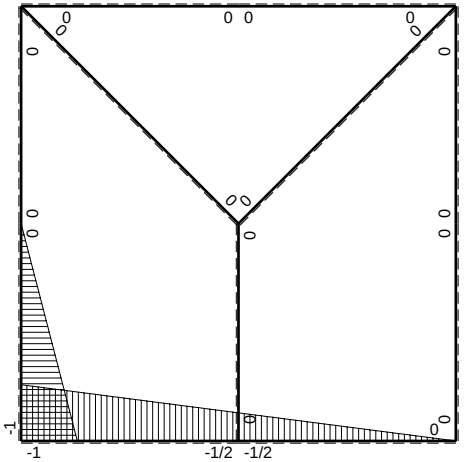


↻⊞↺ Fb





↻⊞↺ M_o flessione da carichi assegnati



↻⊞↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211730 ESΣ04.xxxx.031

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-Fb+3/4Fx$	0	$Fb^2-5/4Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/2+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$1/4Fb+3/4Fx$	0	$1/8Fb^2+1/2Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$1/4Fb-1/4Fx$	0	$-1/8Fb^2+1/4Fbx-1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$-1/4Fx$	0	$-1/8Fx^2$	0	$1/4x^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-Fb+5/4Fx$	0	0	0	0		
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0
FE b	0	$-Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-9/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
HG b	0	$9/4Fb-9/4Fx$	Fb/EJ	0	0	0		
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0
HA b	$-x$	$-9/4Fb+7/4Fx-1/2qx^2$	0	$9/4Fbx-7/4Fx^2+1/2qx^3$	0	x^2	$(2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	$b-x$	$Fb+3/4Fx+1/2qx^2$	0	$Fb^2-1/4Fbx-1/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$1/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-1/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(1 - 5/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[\ x - 5/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(\ b - 5/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/2\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(1/8 + 1/2\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x + 1/4\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b + 1/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/2\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-1/8 + 1/4\ x/b - 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-1/8\ x + 1/8\ x^2/b - 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-1/8\ b + 1/8\ b - 1/24\ b\ \right) Fb^2\ 1/EJ\ = -1/24\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-1/24\ b\ \right) Fb^2\ 1/EJ\ = -1/24\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(9/4\ x/b - 7/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[9/8\ x^2/b - 7/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

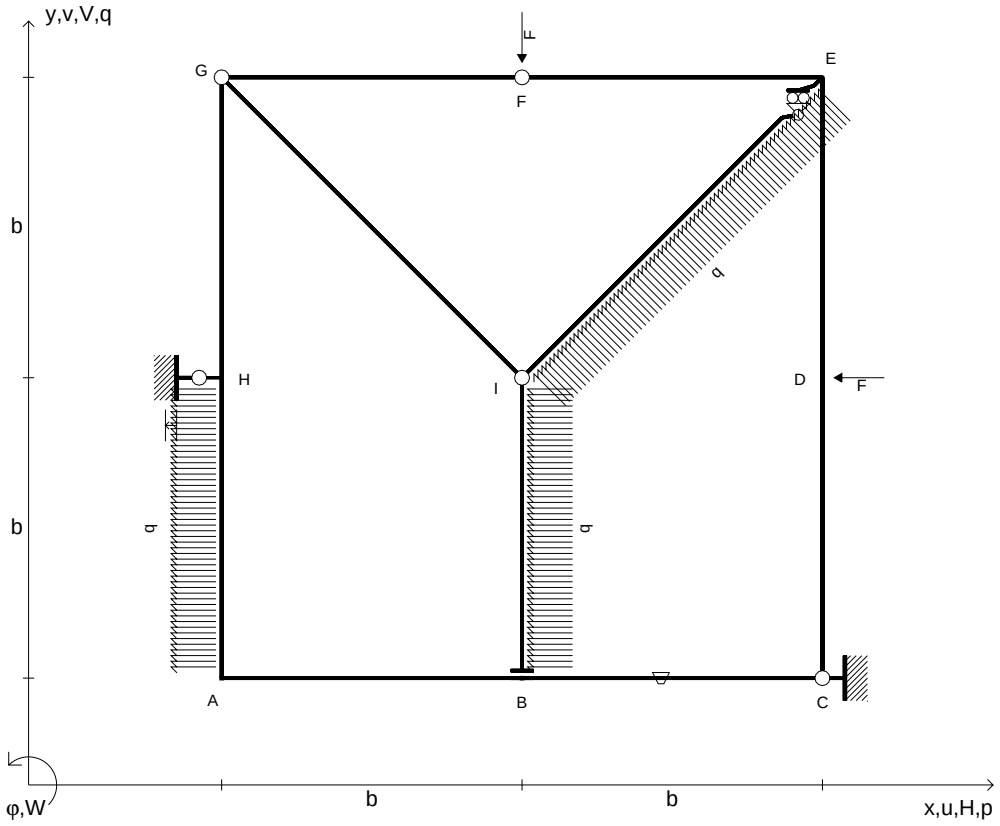
$= \left(9/8\ b - 7/12\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 2/3\ \ Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b \left(1 - 1/4\ x/b - 1/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[\ x - 1/8\ x^2/b - 1/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(\ b - 1/8\ b - 1/12\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 2/3\ \ Fb^3/EJ$

ESΣ04.xxxx.031

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



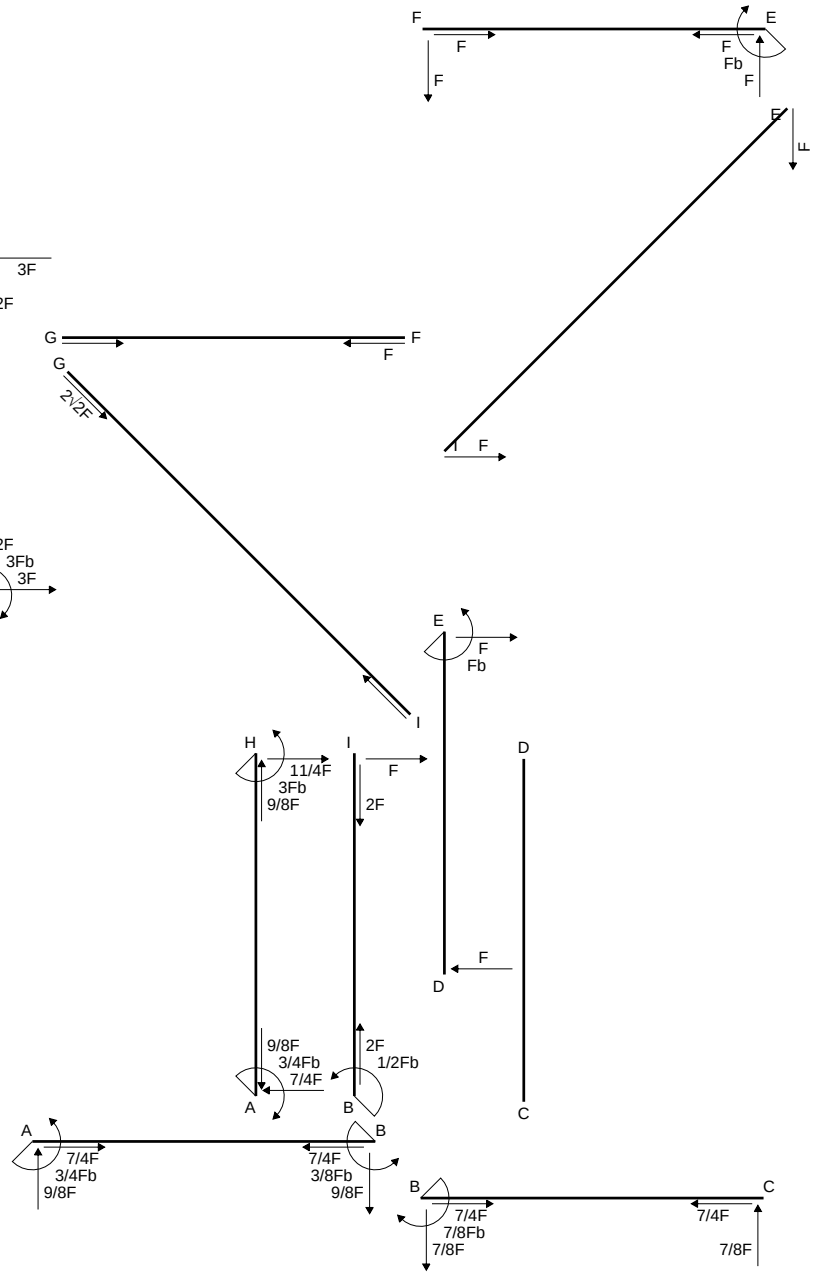
$V_F = -F$
$H_D = -F$
$p_{HA} = -q = -F/b$
$p_{IB} = -q = -F/b$
$p_{IE} = -q = -F/b$
$q_{IE} = q = F/b$
$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$

$u_H = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$

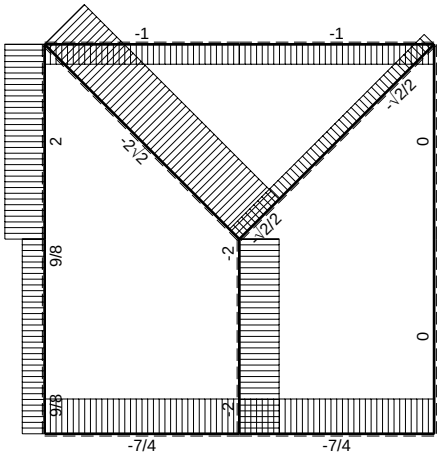
$EJ_{GH} = EJ$
$EJ_{GI} = EJ$
$EJ_{IB} = EJ$
$EJ_{IE} = EJ$
$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo H.

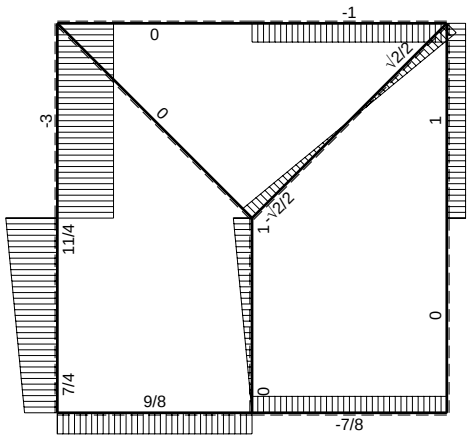
 08.01.25



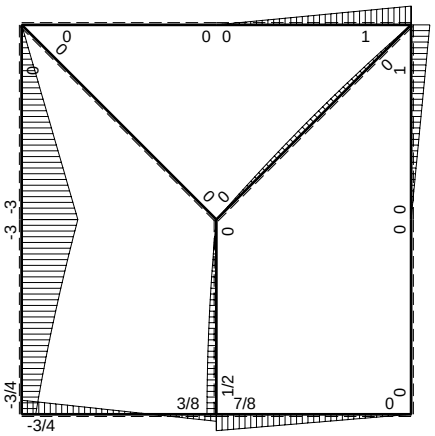
 08.01.25



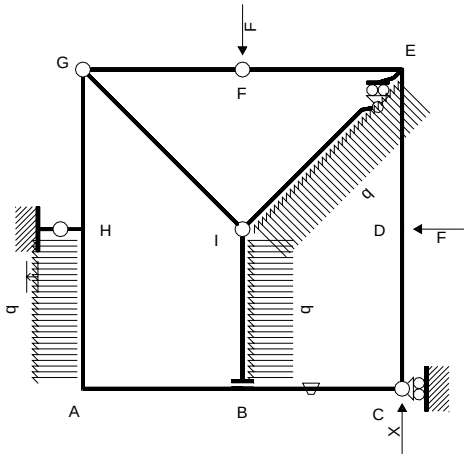
←□→ F



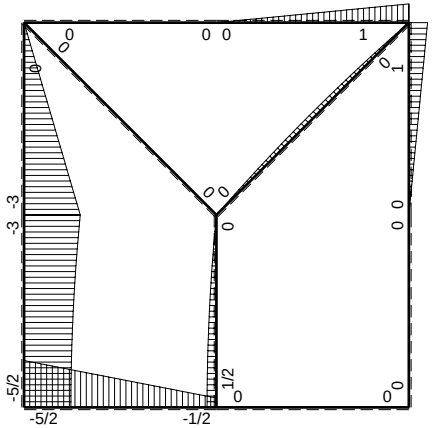
↑□↓ F



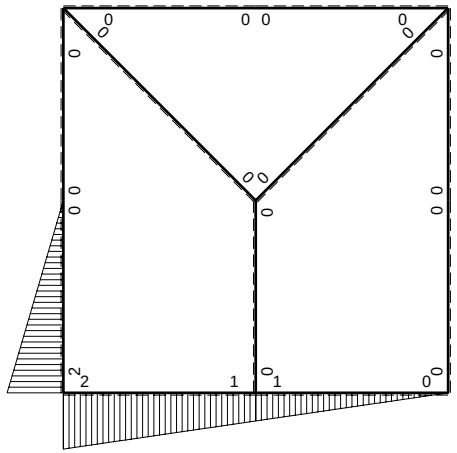
(□) Fb



Schema di calcolo iperstatico



(□) M_o flessione da carichi assegnati



(□) M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	2b-x	-5/2Fb+2Fx	0	$-5Fb^2+13/2Fbx-2Fx^2$	0	$4b^2-4bx+x^2$	$(-29/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	-b-x	1/2Fb+2Fx	0	$-1/2Fb^2-5/2Fbx-2Fx^2$	0	$b^2+2bx+x^2$		
BC b	b-x	0	-Fb/EJ	0	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$	$(0-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	-x	0	Fb/EJ	0	-Fxb/EJ	x^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	2x	$-3Fb+Fx-1/2qx^2$	0	$-6Fbx+2Fx^2-qx^3$	0	$4x^2$	$(-31/12+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	-2b+2x	$5/2Fb+1/2qx^2$	0	$-5Fb^2+5Fbx-Fx^2+qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$2Fb^3/EJ$	
	totali						$-7/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_C$						7/8F	

PROCEDIMENTO E RISULTATI 213154 ESΣ04.xxxx.039

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-5 + 13/2\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-5\,x + 13/4\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-5\,b + 13/4\,b - 2/3\,b)\, Fb^2\, 1/EJ = -29/12\; Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 - 5/2\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\,x - 5/4\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\,b - 5/4\,b - 2/3\,b)\, Fb^2\, 1/EJ = -29/12\; Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1 + x/b)\, \theta\, dx = [-x + 1/2\,x^2/b]_o^b\, \theta$

$= (-b + 1/2\,b)\, \theta = -1/2\; Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (x/b)\, \theta\, dx = [1/2\,x^2/b]_o^b\, \theta$

$= (1/2\,b)\, \theta = -1/2\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (-6\,x/b + 2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-3\,x^2/b + 2/3\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

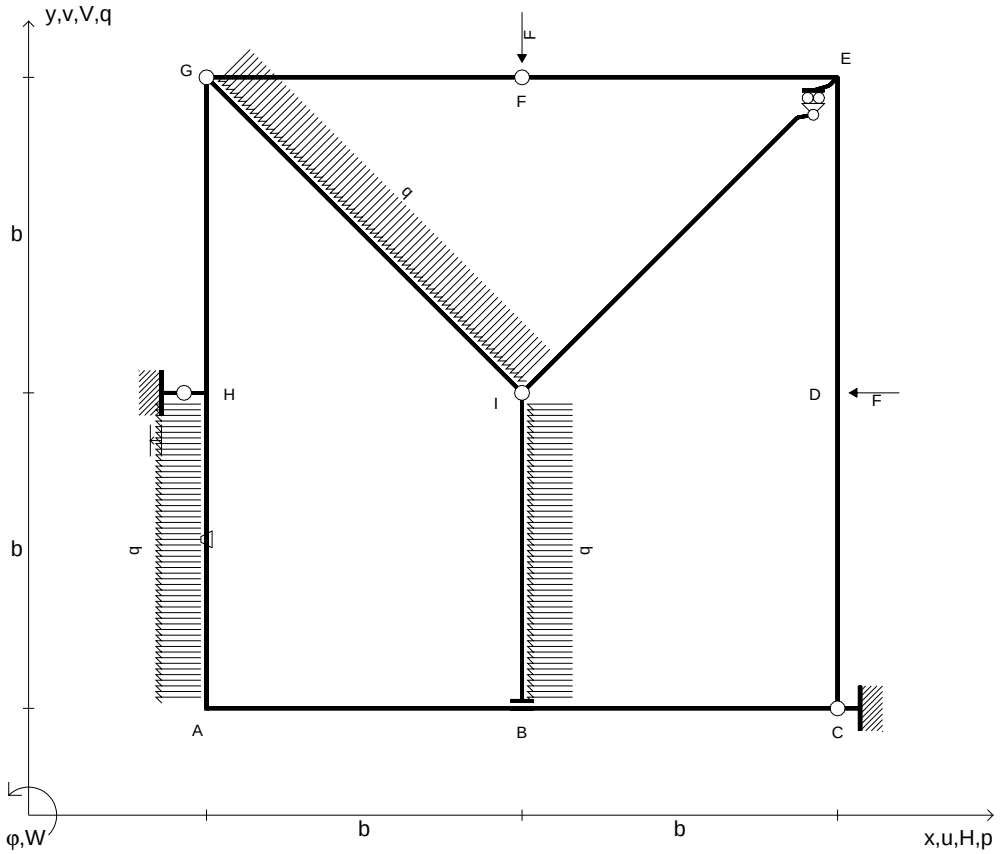
$= (-3\,b + 2/3\,b - 1/4\,b)\, Fb^2\, 1/EJ = -31/12\; Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (-5 + 5\,x/b - x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-5\,x + 5/2\,x^2/b - 1/3\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-5\,b + 5/2\,b - 1/3\,b + 1/4\,b)\, Fb^2\, 1/EJ = -31/12\; Fb^3/EJ$

ESΣ04.xxxx.039

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

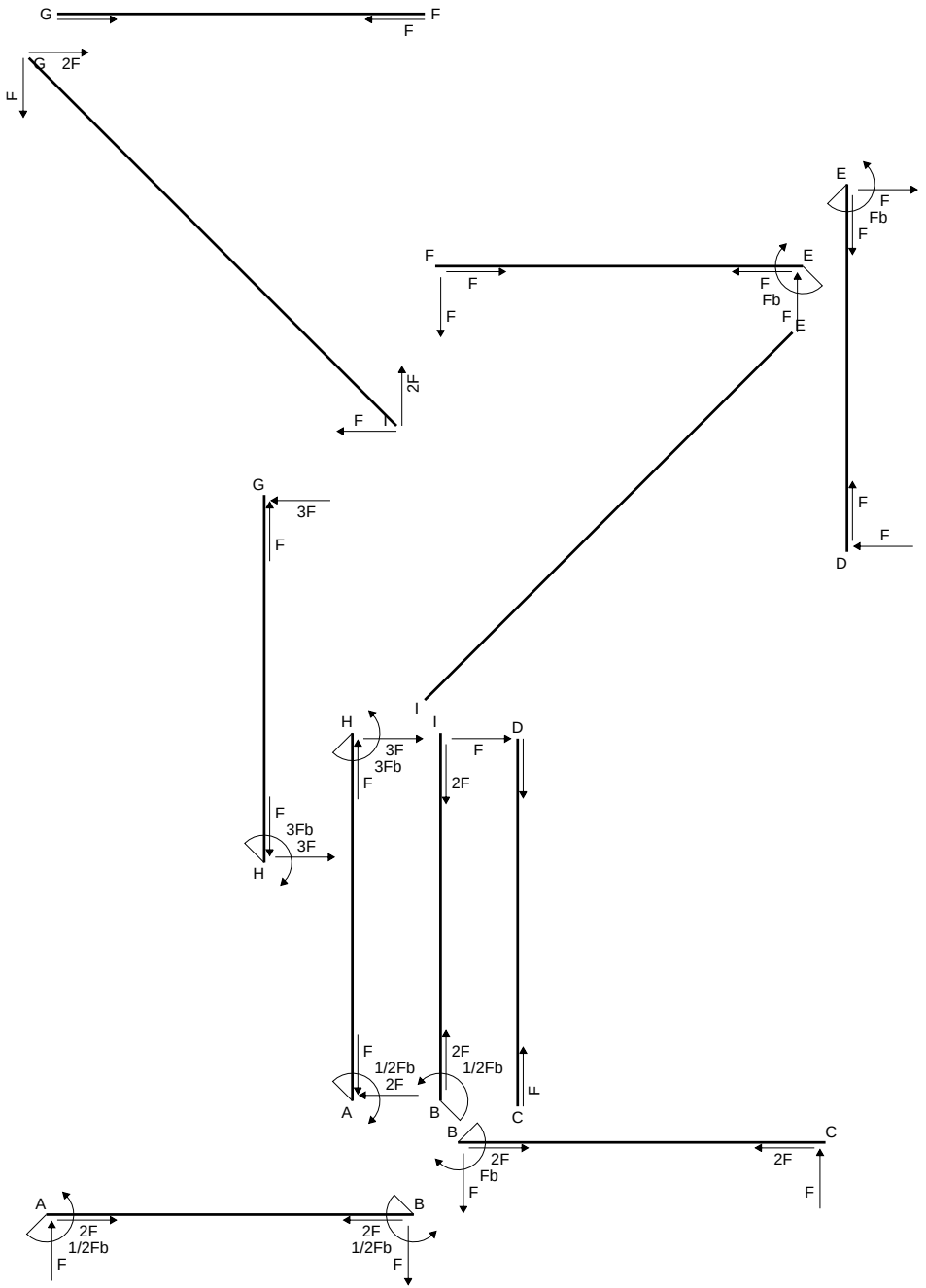


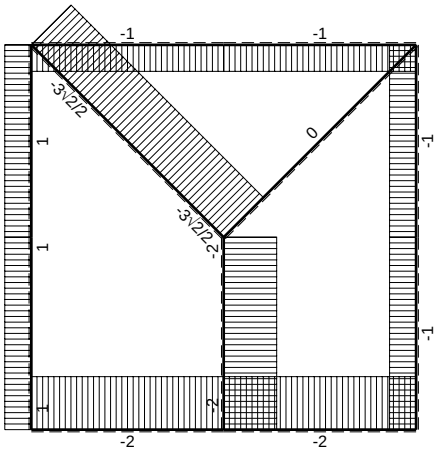
$V_F = -F$
$H_D = -F$
$p_{HA} = -q = -F/b$
$p_{IB} = -q = -F/b$
$p_{GI} = -q = -F/b$
$q_{GI} = -q = -F/b$
$\theta_{HA} = -\theta = -\alpha T/b = -bF/EJ$

$u_H = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$

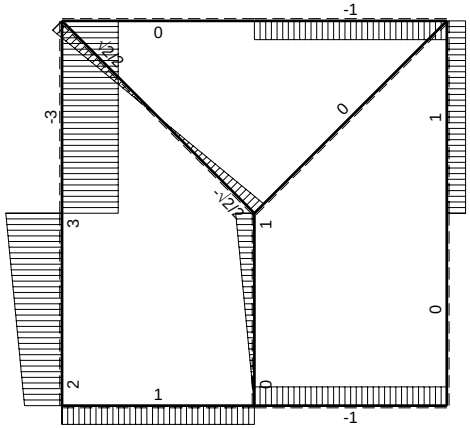
$EJ_{GH} = EJ$
$EJ_{GI} = EJ$
$EJ_{IB} = EJ$
$EJ_{IE} = EJ$
$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta HA positiva se convessa a destra con inizio H.
Spostamento orizzontale assoluto u imposto al nodo H.

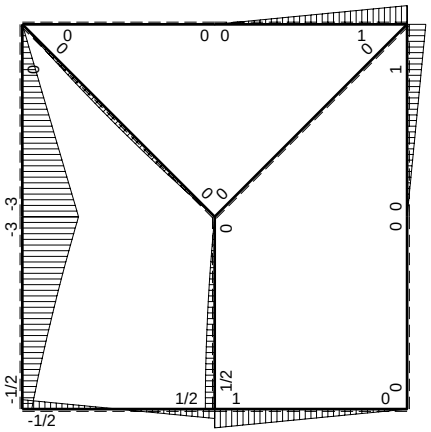




←F→

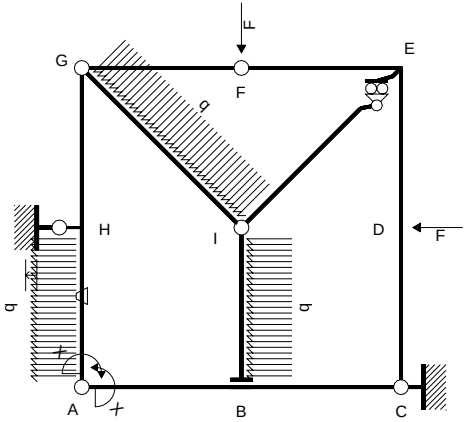


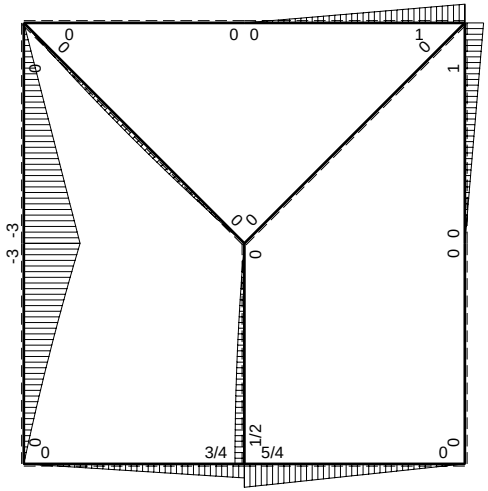
↑F↓



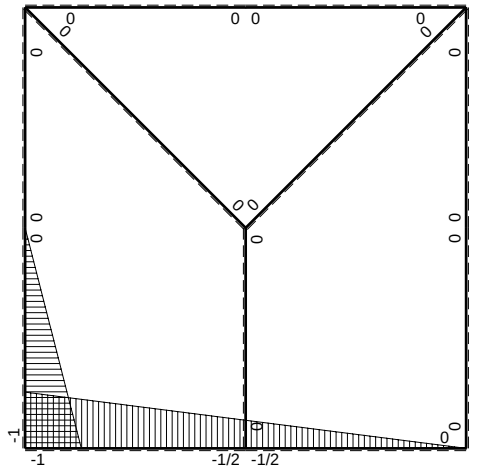
Fb

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	5/4Fb-5/4Fx	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-3Fb+7/2Fx-1/2qx^2$	-Fb/EJ	$3Fx-7/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/2Fx+1/2qx^2$	Fb/EJ	$5/2Fx-2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 218425 ESΣ04.xxxx.051
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-3/4\ x/b + 3/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right) Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-3/8 + 3/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-3/8\ x + 1/8\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-3/8\ b + 1/8\ b\right) Fb\ 1/EJ\ = -1/4\ \ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-5/8 + 5/4\ x/b - 5/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-5/8\ x + 5/8\ x^2/b - 5/24\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-5/8\ b + 5/8\ b - 5/24\ b\right) Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-5/8\ x^2/b^2\right) Fb\ 1/EJ\ dx = \left[-5/24\ x^3/b^2\ \right]_o^b Fb\ 1/EJ$

$= \left(-5/24\ b\right) Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L^{Xo}_{HA} = \int_o^b \left(3\ x/b - 7/2\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right)\theta\ dx$

$= \left[3/2\ x^2/b - 7/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b \theta$

$= \left(3/2\ b - 7/6\ b + 1/8\ b\right) Fb\ 1/EJ\ + \left(1/2\ b\right)\theta\ \ = 23/24\ \ Fb^2/EJ$

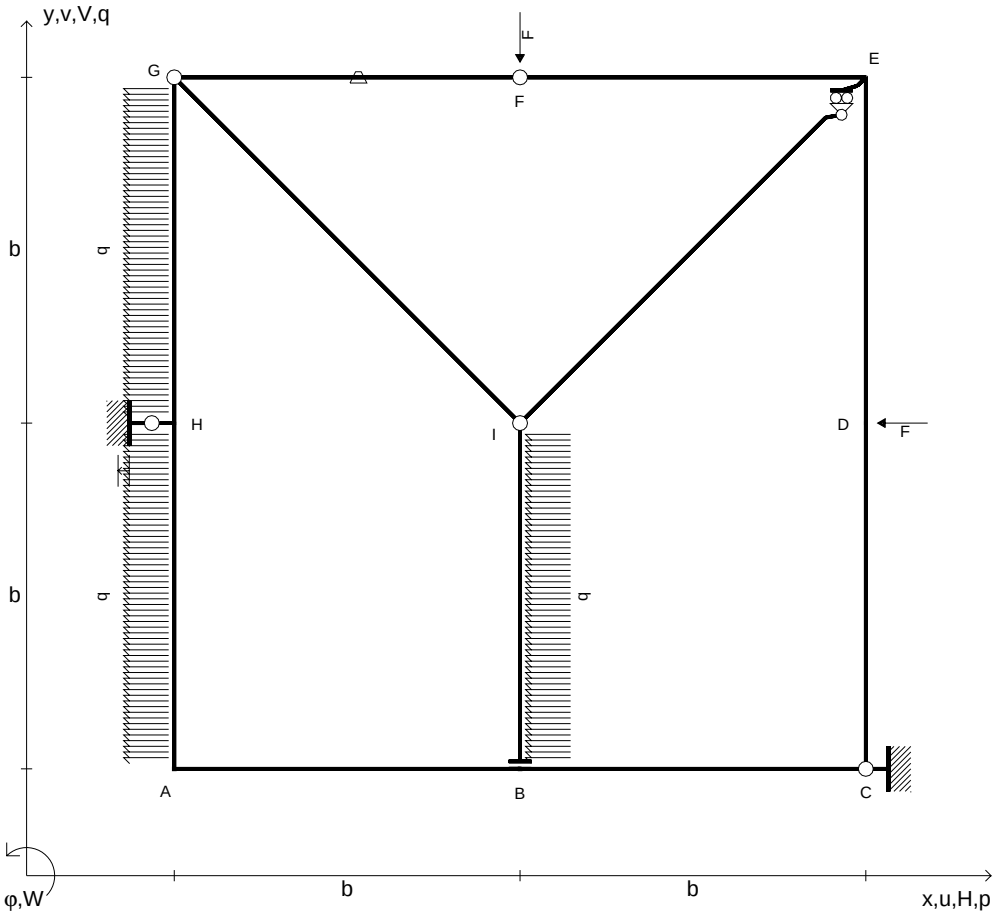
$L^{Xo}_{AH} = \int_o^b \left(5/2\ x/b - 2\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\right)\theta\ dx$

$= \left[5/4\ x^2/b - 2/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b \theta$

$= \left(5/4\ b - 2/3\ b - 1/8\ b\right) Fb\ 1/EJ\ + \left(-\ b + 1/2\ b\right)\theta\ \ = 23/24\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 218425 ESΣ04.xxxx.051

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{IB} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

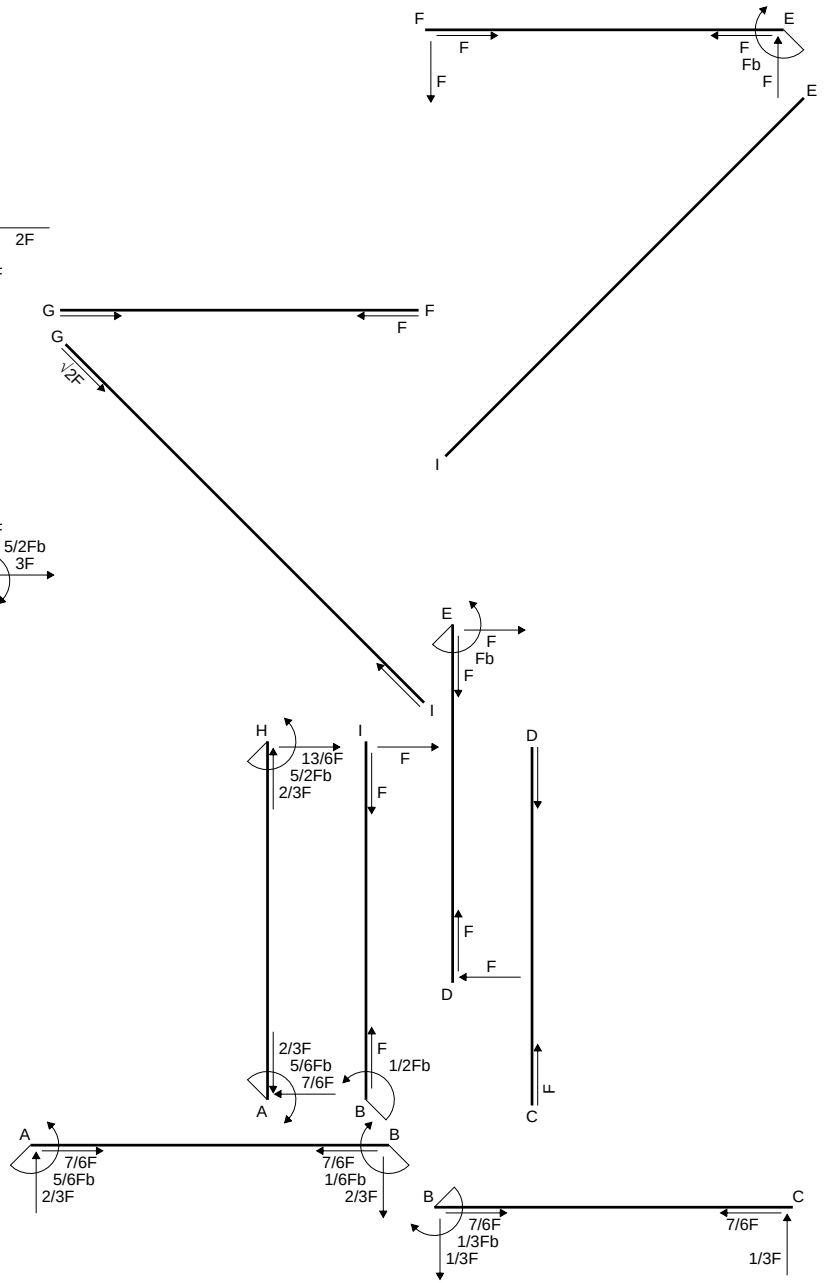
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

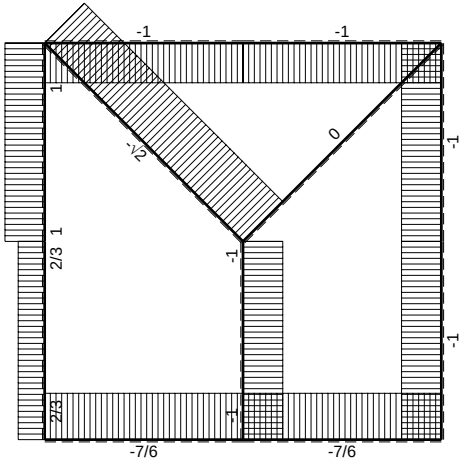
Curvatura θ asta FG positiva se convessa a destra con inizio F.

Spostamento orizzontale assoluto u imposto al nodo H.

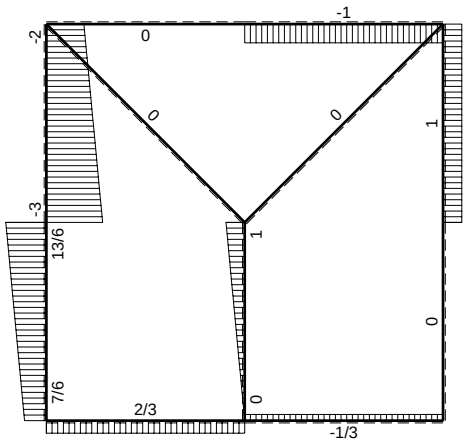
 08.01.25



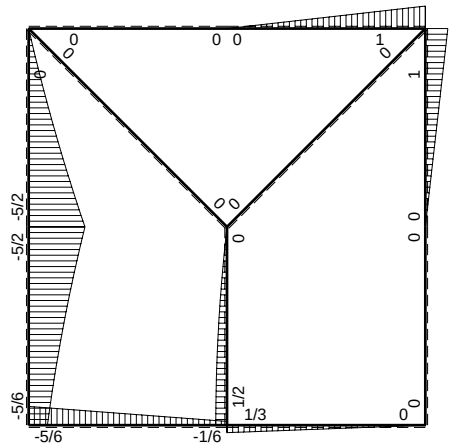
 08.01.25



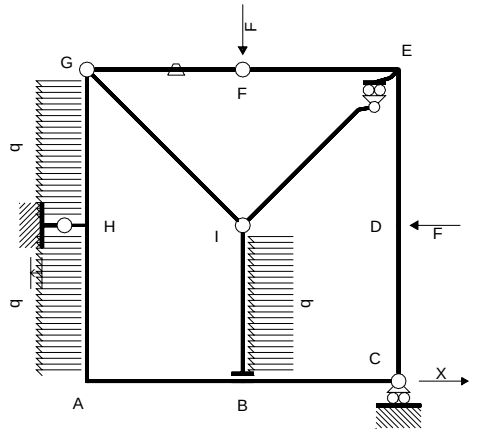
←[+]→ F

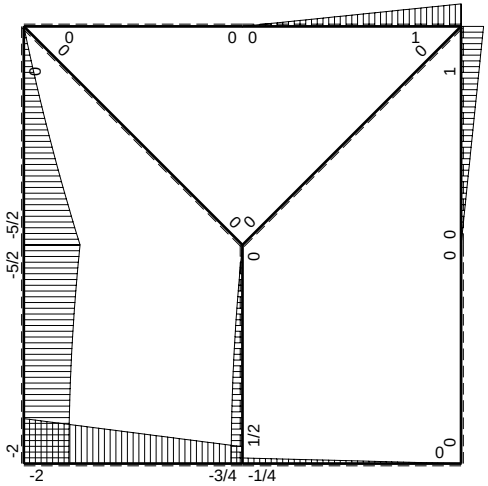


↑[+]↓ F

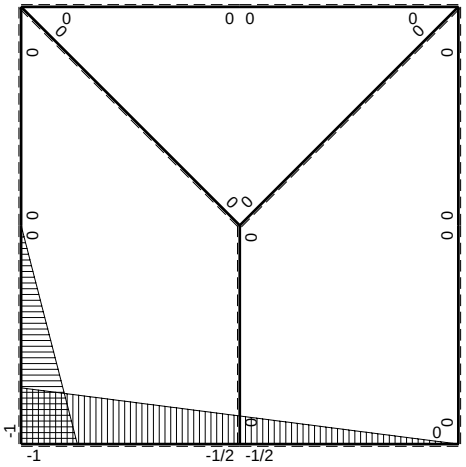


[+] Fb





[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-b+1/2x	-2Fb+5/4Fx	0	$2Fb^2-9/4Fbx+5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(13/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	1/2b+1/2x	3/4Fb+5/4Fx	0	$3/8Fb^2+Fbx+5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	-1/2b+1/2x	-1/4Fb+1/4Fx	0	$1/8Fb^2-1/4Fbx+1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	1/2x	1/4Fx	0	$1/8Fx^2$	0	$1/4x^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	-Fb/EJ	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x	$-5/2Fb+Fx-1/2qx^2$	0	$5/2Fbx-Fx^2+1/2qx^3$	0	x^2	$(25/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$2Fb+1/2qx^2$	0	$2Fb^2-2Fbx+1/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$7/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-7/6F	

PROCEDIMENTO E RISULTATI 212483 ESΣ04.xxxx.070

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(2 - 9/4\ x/b + 5/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[2\ x - 9/8\ x^2/b + 5/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(2\ b - 9/8\ b + 5/24\ b\ \right) Fb^2\ 1/EJ\ = 13/12\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(3/8 + x/b + 5/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x + 1/2\ x^2/b + 5/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/8\ b + 1/2\ b + 5/24\ b\ \right) Fb^2\ 1/EJ\ = 13/12\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(1/8 - 1/4\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b - 1/8\ b + 1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(5/2\ x/b - x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[5/4\ x^2/b - 1/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

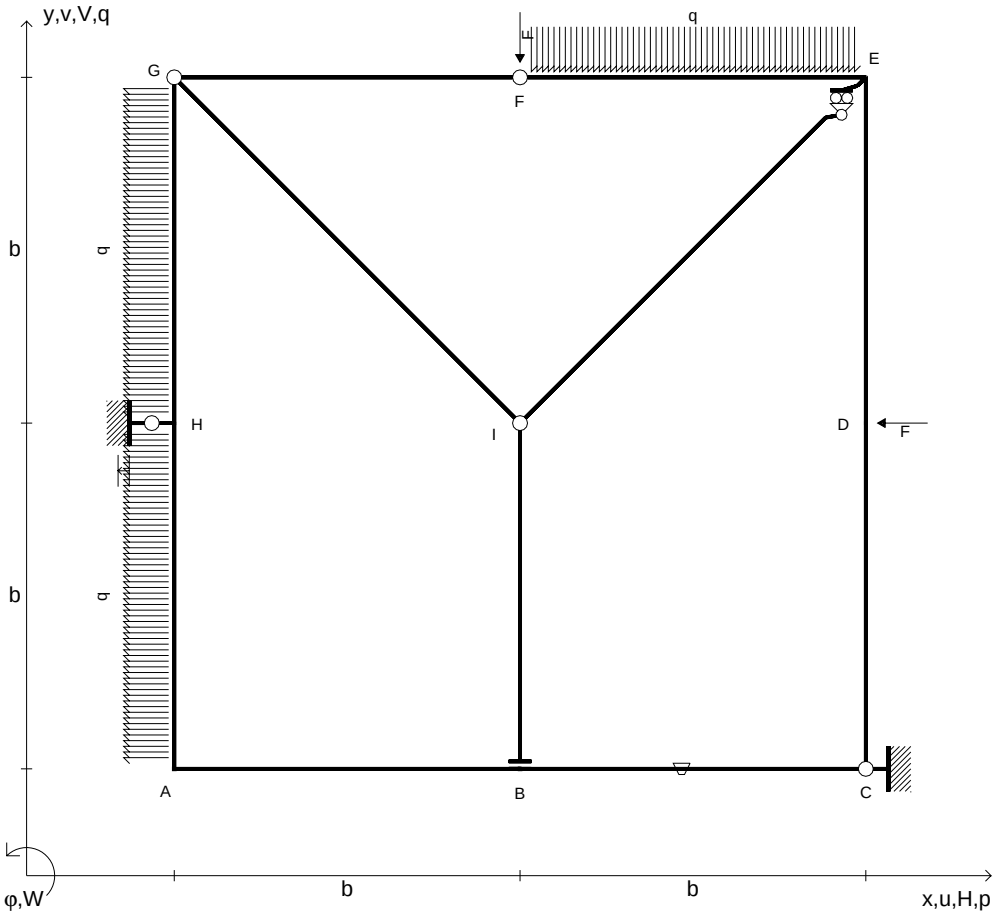
$= \left(5/4\ b - 1/3\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b \left(2 - 2\ x/b + 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[2\ x - x^2/b + 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(2\ b - b + 1/6\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$

ESΣ04.xxxx.070

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

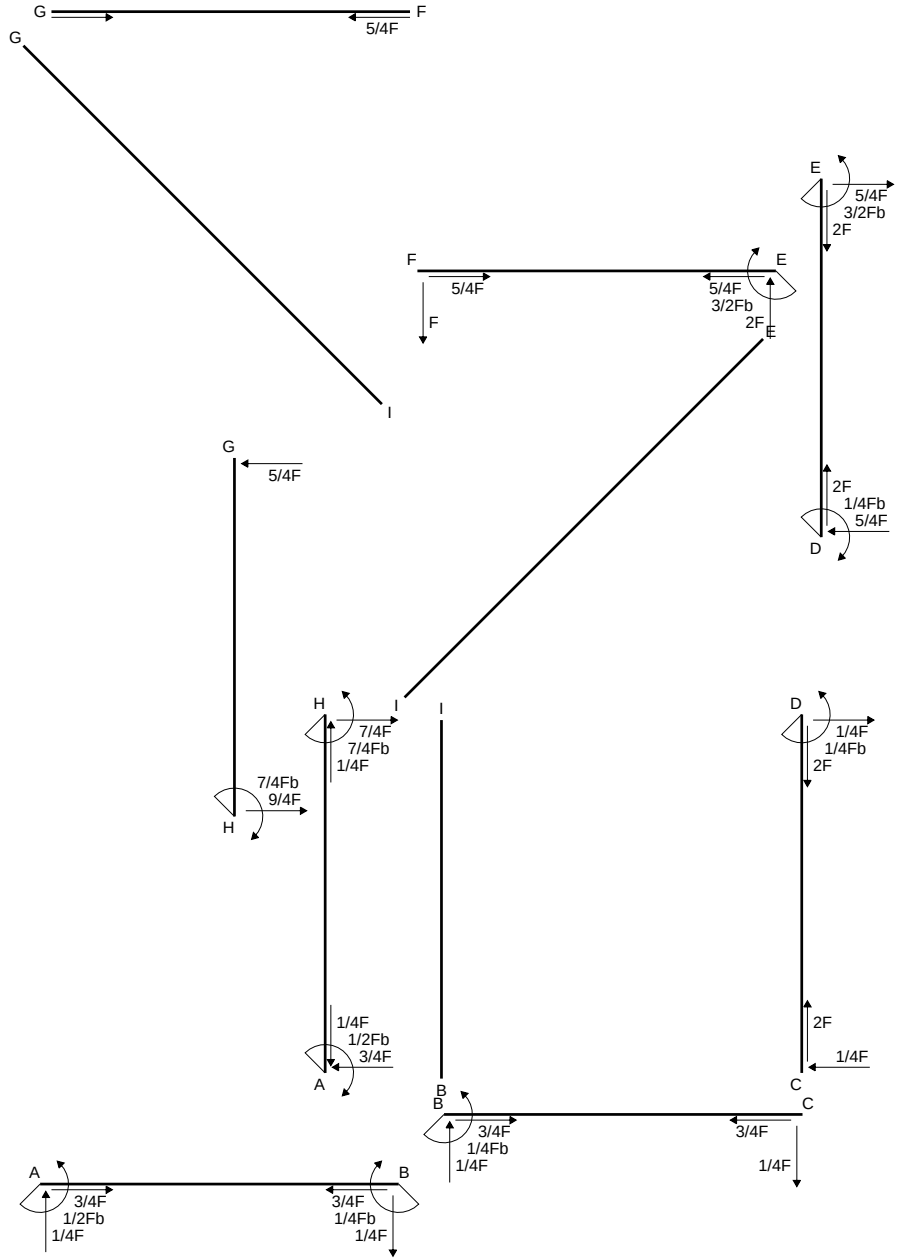
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

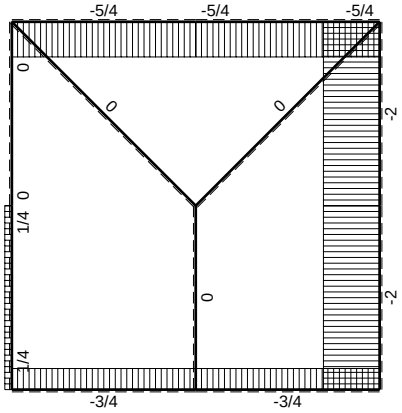
Curvatura θ asta BC positiva se convessa a destra con inizio B.

Spostamento orizzontale assoluto u imposto al nodo H.

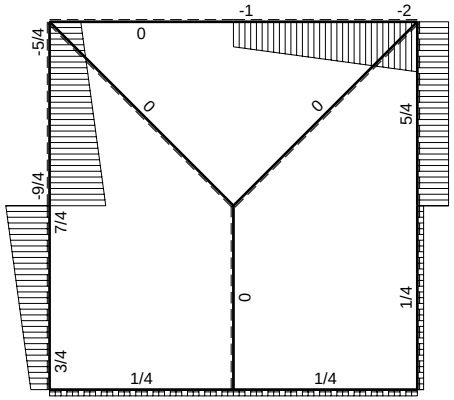
 08.01.25



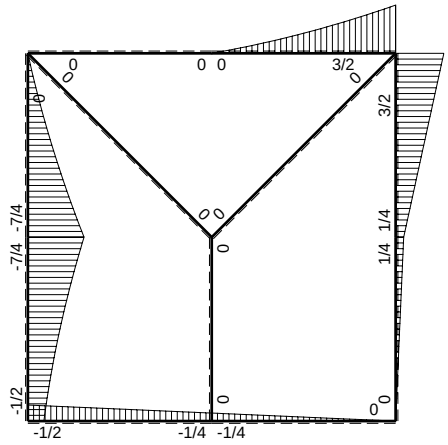
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



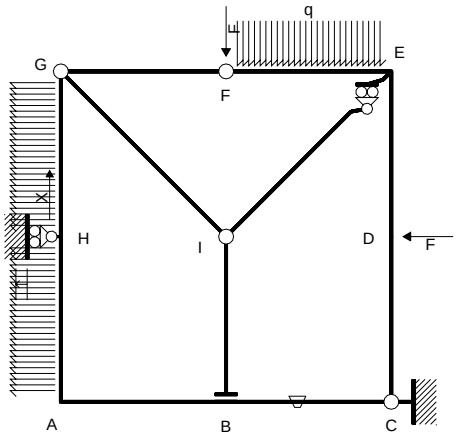
←□→ F

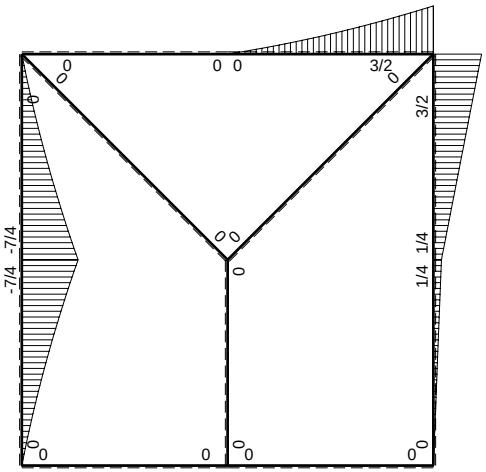


↑□↓ F

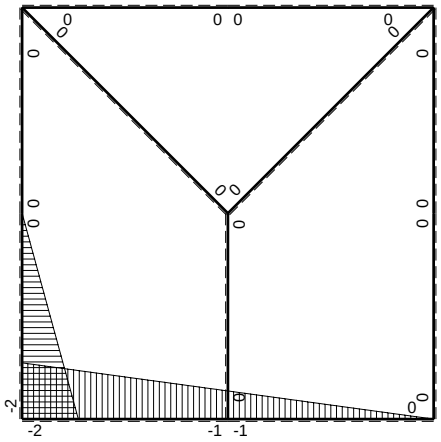


□ Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-2b+x$	0	0	0	0	$4b^2-4bx+x^2$	0+0	$7/3Xb^3/EJ$
BA b	$b+x$	0	0	0	0	$b^2+2bx+x^2$		
BC b	$-b+x$	0	$-Fb/EJ$	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	x	0	Fb/EJ	0	Fxb/EJ	x^2		
CD b	0	$1/4Fx$	0	0	0	0	0+0	0
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0		
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	$-2x$	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/2Fbx-9/2Fx^2+qx^3$	0	$4x^2$	$(1/2+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	$2b-2x$	$5/4Fx+1/2qx^2$	0	$5/2Fbx-3/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$-Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						1/4F	

PROCEDIMENTO E RISULTATI 211841 ESΣ04.xxxx.082

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (1 - x/b)\, \theta\, dx = [x - 1/2\,x^2/b]_o^b\, \theta$

$= (b - 1/2\,b)\, \theta = 1/2\; Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-x/b)\, \theta\, dx = [-1/2\,x^2/b]_o^b\, \theta$

$= (-1/2\,b)\, \theta = 1/2\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (7/2\,x/b - 9/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [7/4\,x^2/b - 3/2\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

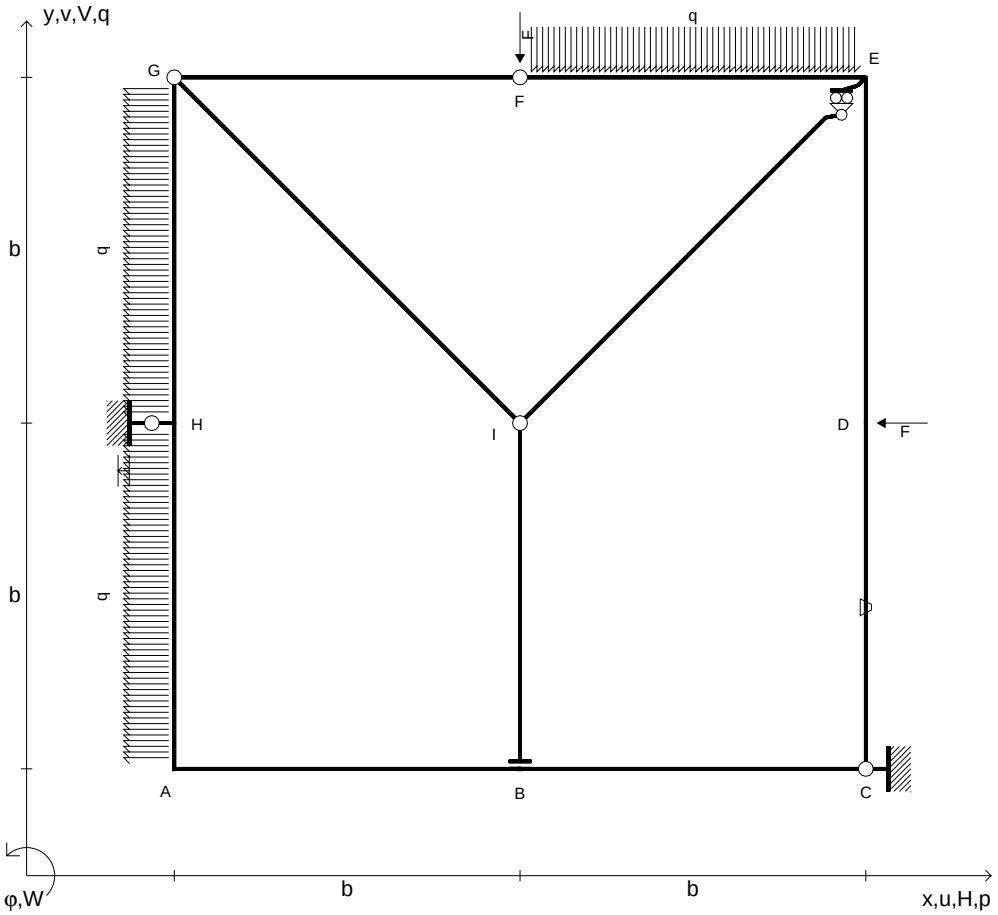
$= (7/4\,b - 3/2\,b + 1/4\,b)\, Fb^2\, 1/EJ = 1/2\; Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (5/2\,x/b - 3/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [5/4\,x^2/b - 1/2\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (5/4\,b - 1/2\,b - 1/4\,b)\, Fb^2\, 1/EJ = 1/2\; Fb^3/EJ$

ESΣ04.xxxx.082

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

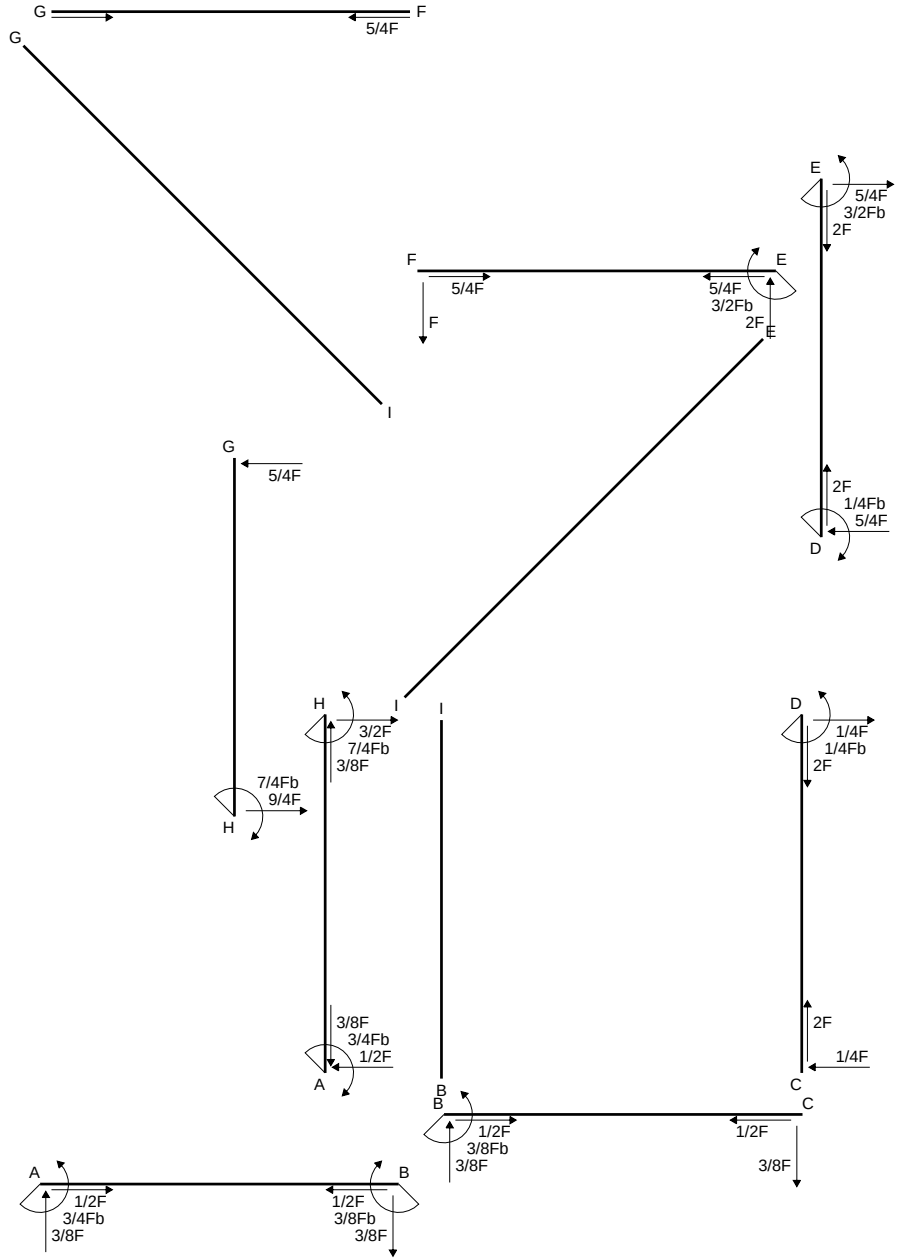
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

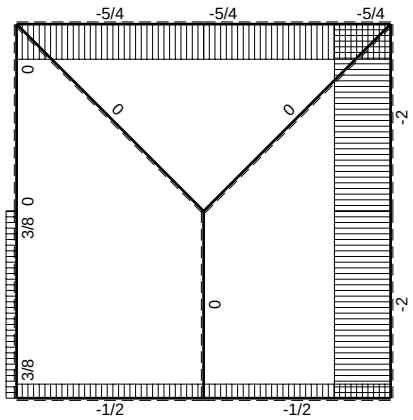
Curvatura θ asta CD positiva se convessa a destra con inizio C.

Spostamento orizzontale assoluto u imposto al nodo H.

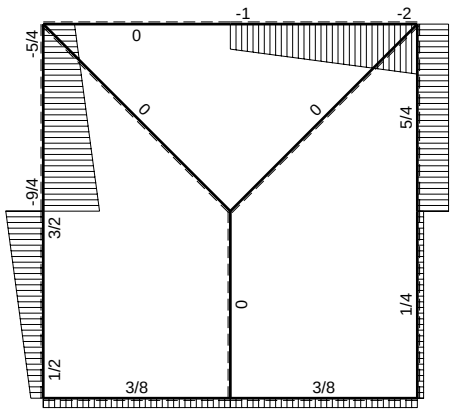
 08.01.25



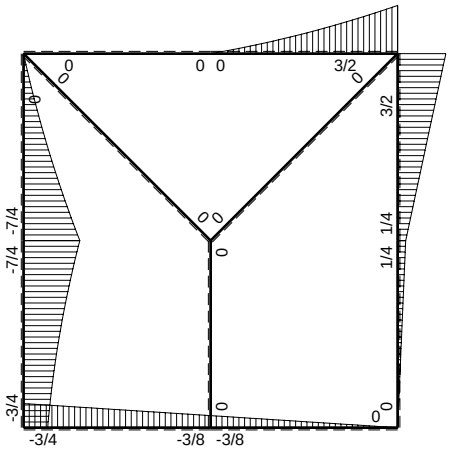
 08.01.25



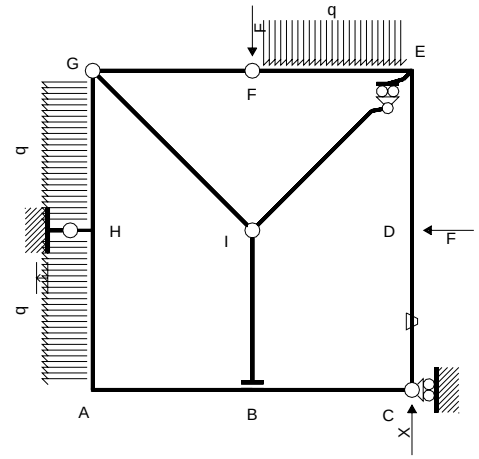
←□→ F



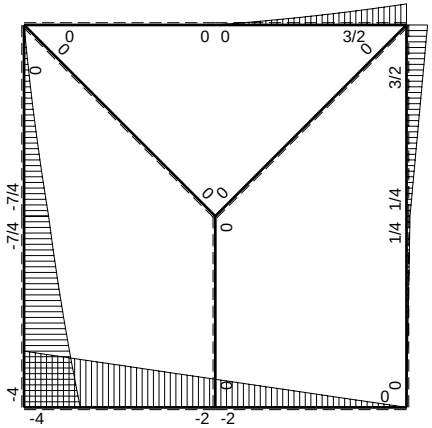
↑□↓ F



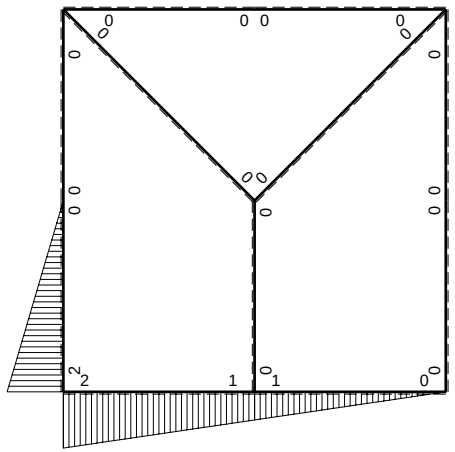
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	2b-x	-4Fb+2Fx	0	$-8Fb^2+8Fbx-2Fx^2$	0	$4b^2-4bx+x^2$	$(-14/3+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	-b-x	2Fb+2Fx	0	$-2Fb^2-4Fbx-2Fx^2$	0	$b^2+2bx+x^2$		
BC b	b-x	-2Fb+2Fx	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	-x	2Fx	0	$-2Fx^2$	0	x^2		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	2x	$-7/4Fb-7/4Fx-1/2qx^2$	0	$-7/2Fbx-7/2Fx^2-qx^3$	0	$4x^2$	$(-19/6+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	-2b+2x	$4Fb-11/4Fx+1/2qx^2$	0	$-8Fb^2+27/2Fbx-13/2Fx^2+qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$2Fb^3/EJ$	
	totali						$-13/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_C$						13/8F	

PROCEDIMENTO E RISULTATI 213267 ESΣ04.xxxx.087

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-8 + 8\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-8\,x + 4\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-8\,b + 4\,b - 2/3\,b)\, Fb^2\, 1/EJ = -14/3\; Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-2 - 4\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\,x - 2\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2\,b - 2\,b - 2/3\,b)\, Fb^2\, 1/EJ = -14/3\; Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-2 + 4\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\,x + 2\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2\,b + 2\,b - 2/3\,b)\, Fb^2\, 1/EJ = -2/3\; Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2/3\,b)\, Fb^2\, 1/EJ = -2/3\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (-7/2\,x/b - 7/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-7/4\,x^2/b - 7/6\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-7/4\,b - 7/6\,b - 1/4\,b)\, Fb^2\, 1/EJ = -19/6\; Fb^3/EJ$

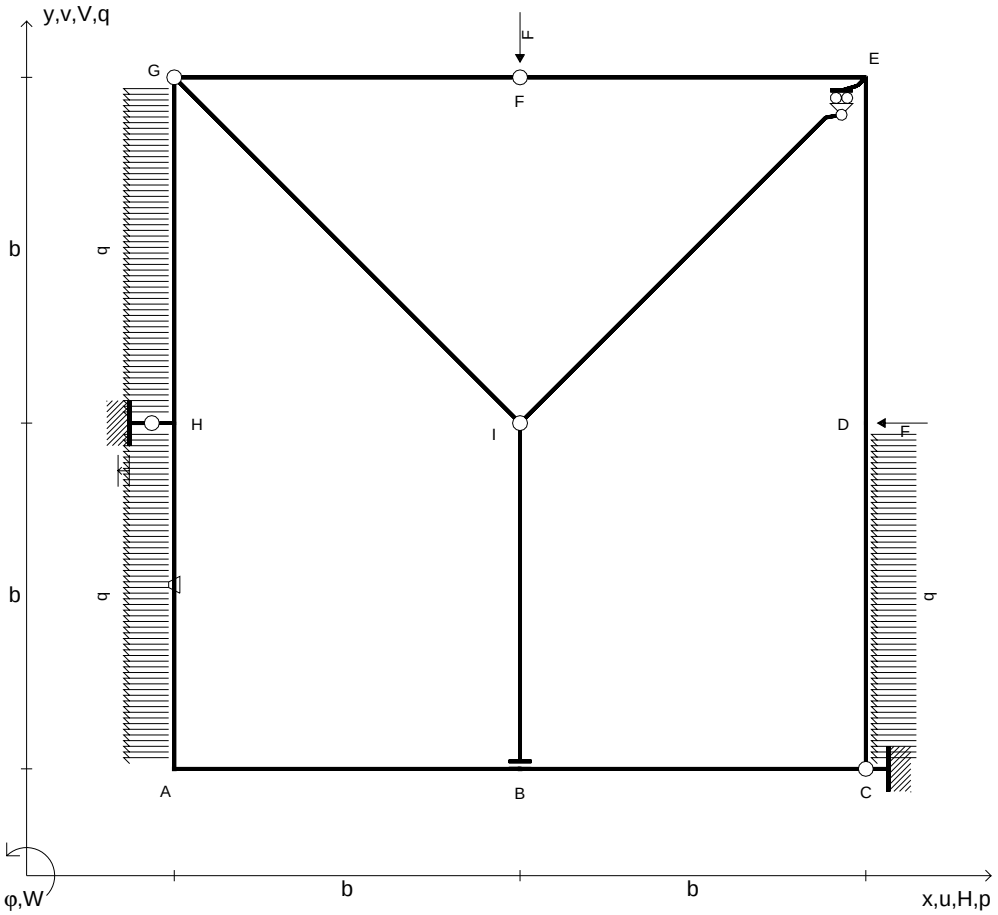
$L^{Xo}_{AH} = \int_o^b (-8 + 27/2\,x/b - 13/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-8\,x + 27/4\,x^2/b - 13/6\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-8\,b + 27/4\,b - 13/6\,b + 1/4\,b)\, Fb^2\, 1/EJ = -19/6\; Fb^3/EJ$

ESΣ04.xxxx.087

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{HA} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

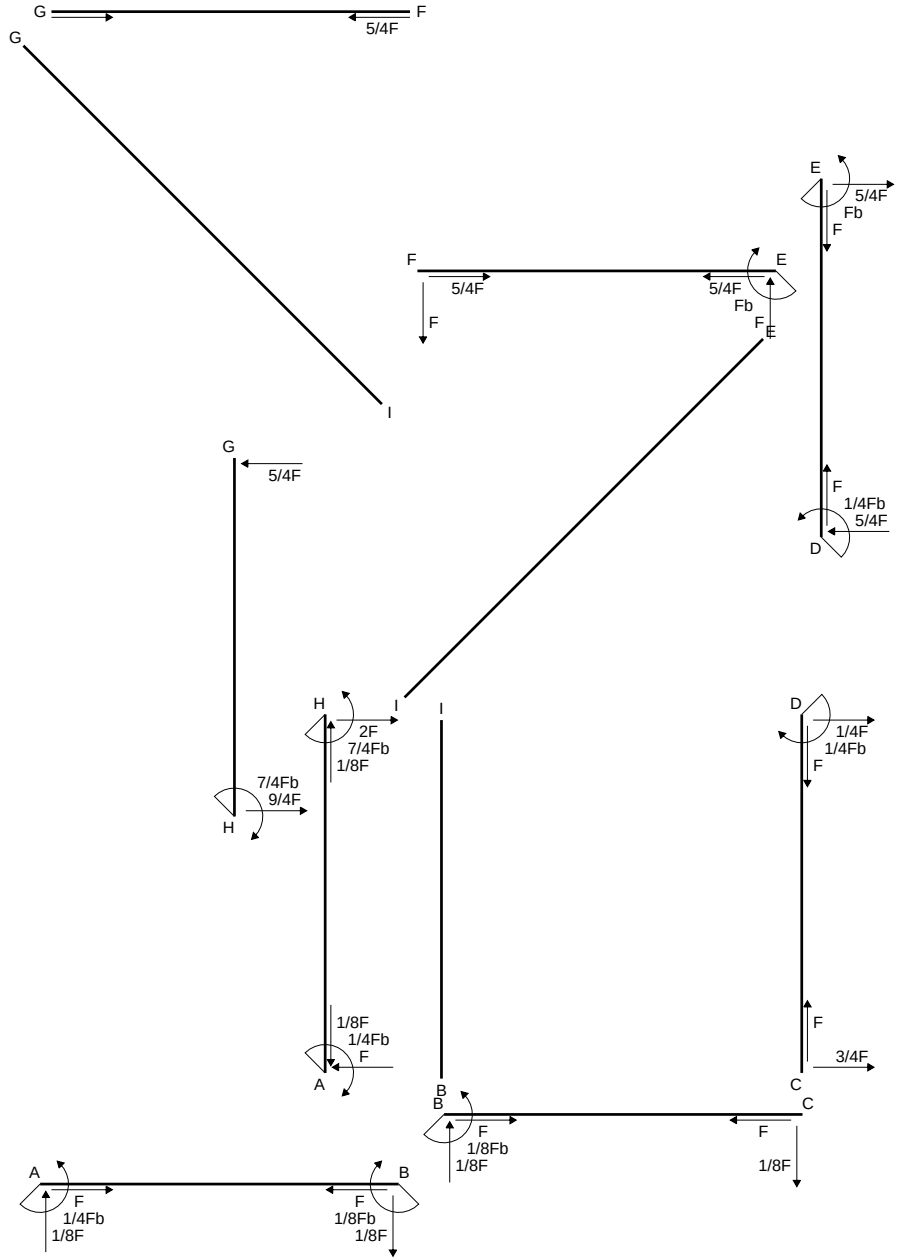
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

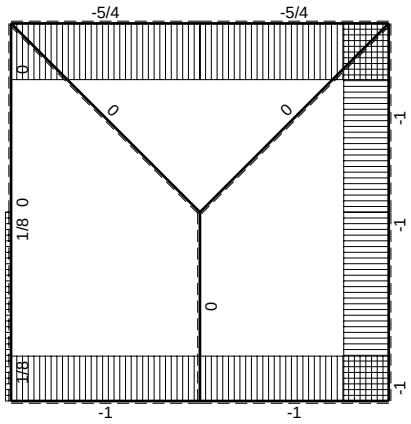
Curvatura θ asta HA positiva se convessa a destra con inizio H.

Spostamento orizzontale assoluto u imposto al nodo H.

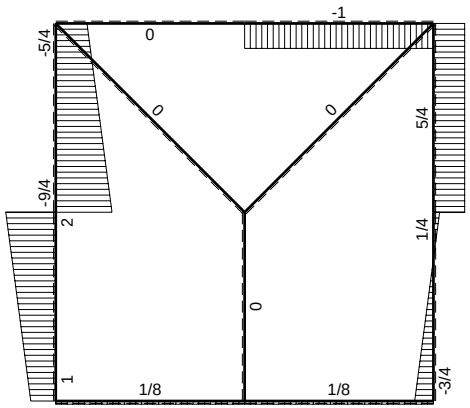
 08.01.25



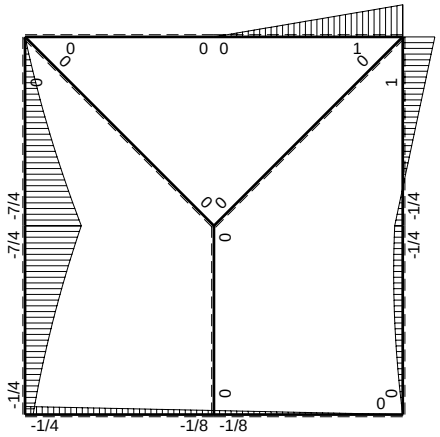
 08.01.25



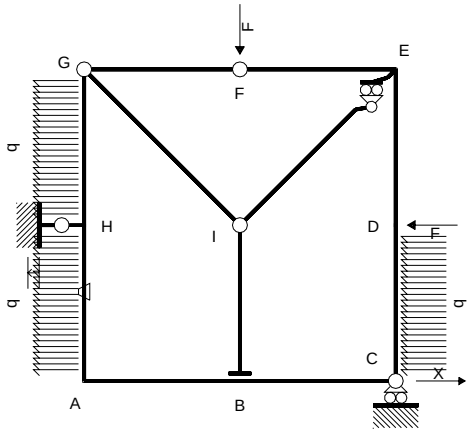
←⊞→ F

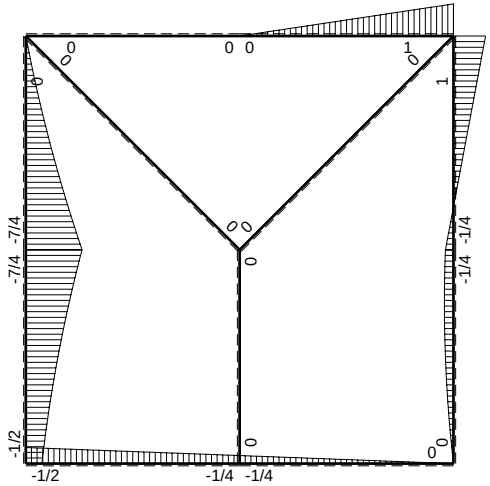


↑⊞↓ F

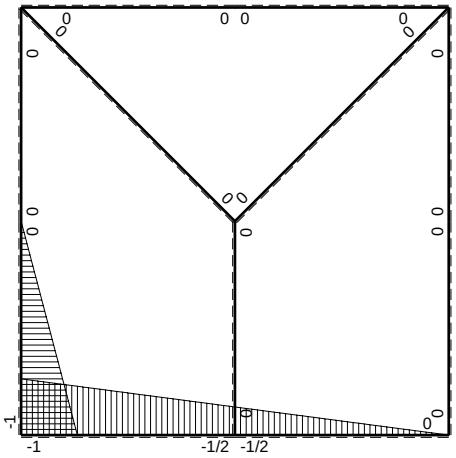


↻⊞↺ Fb





↻⊞↺ M_o flessione da carichi assegnati



↻⊞↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-1/2Fb+1/4Fx$	0	$1/2Fb^2-1/2Fbx+1/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$1/4Fb+1/4Fx$	0	$1/8Fb^2+1/4Fbx+1/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-1/4Fb+1/4Fx$	0	$1/8Fb^2-1/4Fbx+1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$1/4Fx$	0	$1/8Fx^2$	0	$1/4x^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-Fb+5/4Fx$	0	0	0	0		
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0
FE b	0	$-Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	$-x$	$-7/4Fb+7/4Fx-1/2qx^2$	$-Fb/EJ$	$7/4Fbx-7/4Fx^2+1/2qx^3$	Fxb/EJ	x^2	$(5/12+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	$b-x$	$1/2Fb+3/4Fx+1/2qx^2$	Fb/EJ	$1/2Fb^2+1/4Fbx-1/4Fx^2-1/2qx^3$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$1/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						$-1/4F$	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212423 ESΣ04.xxxx.093
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(1/2 - 1/2\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/2\ x - 1/4\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/2\ b - 1/4\ b + 1/24\ b\ \right) Fb^2\ 1/EJ\ = 7/24\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(1/8 + 1/4\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x + 1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b + 1/8\ b + 1/24\ b\ \right) Fb^2\ 1/EJ\ = 7/24\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(1/8 - 1/4\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b - 1/8\ b + 1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(7/4\ x/b - 7/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(\ x/b\ \right) \theta\ dx$

$= \left[7/8\ x^2/b - 7/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ\ + \left[1/2\ x^2/b\ \right]_o^b \theta$

$= \left(7/8\ b - 7/12\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ + \left(1/2\ b\ \right) \theta\ \ = 11/12\ \ Fb^3/EJ$

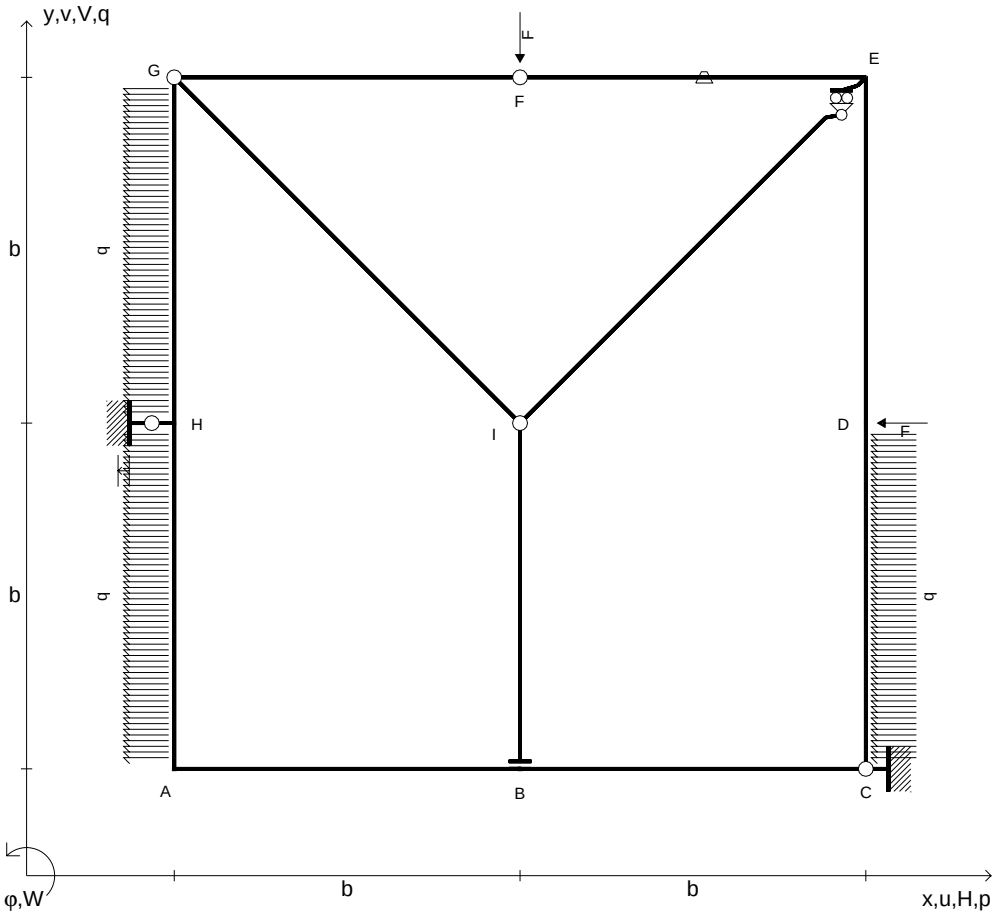
$L^{Xo}_{AH} = \int_o^b \left(1/2 + 1/4\ x/b - 1/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(-1 + x/b\ \right) \theta\ dx$

$= \left[1/2\ x + 1/8\ x^2/b - 1/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ\ + \left[-\ x + 1/2\ x^2/b\ \right]_o^b \theta$

$= \left(1/2\ b + 1/8\ b - 1/12\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ + \left(-\ b + 1/2\ b\ \right) \theta\ \ = 11/12\ \ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 212423 ESΣ04.xxxx.093

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

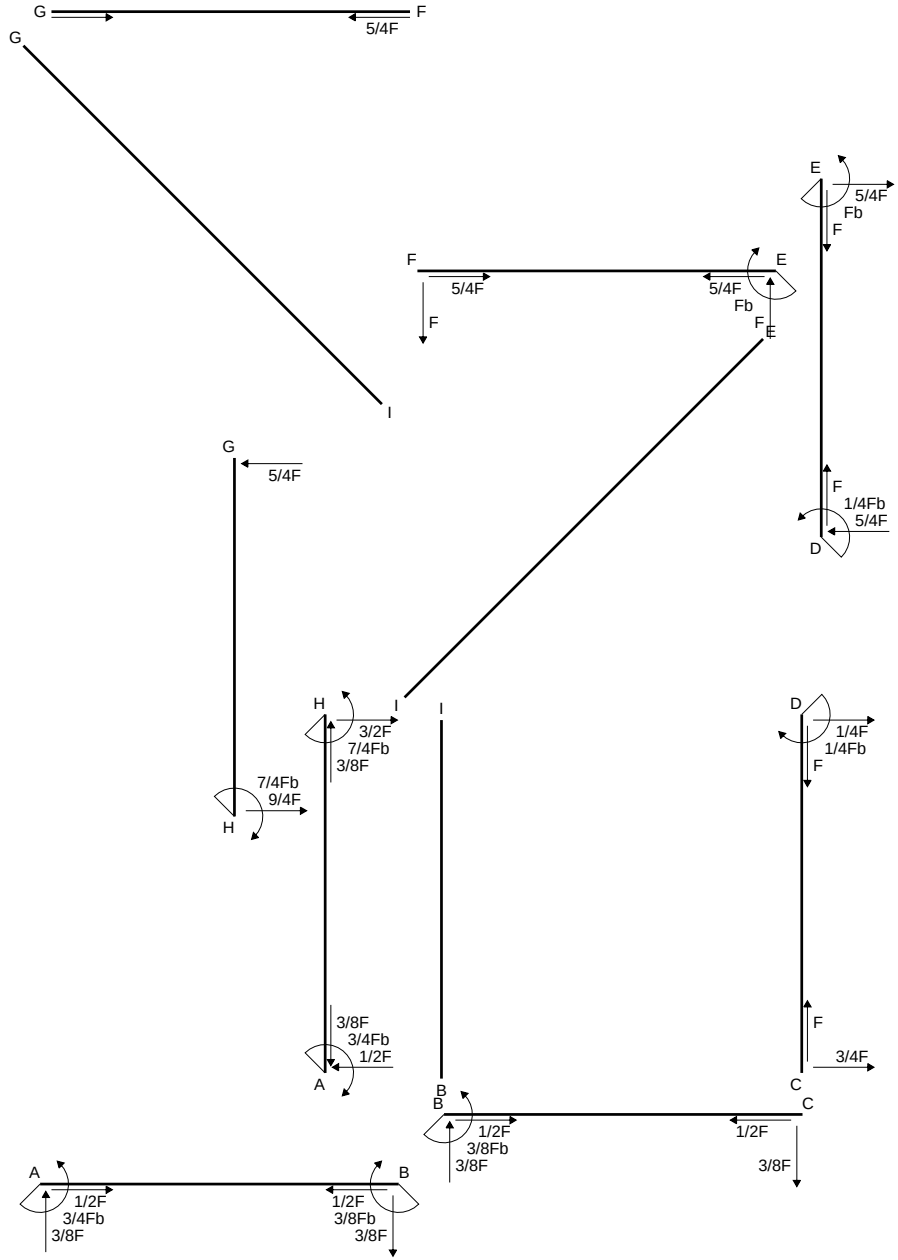
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

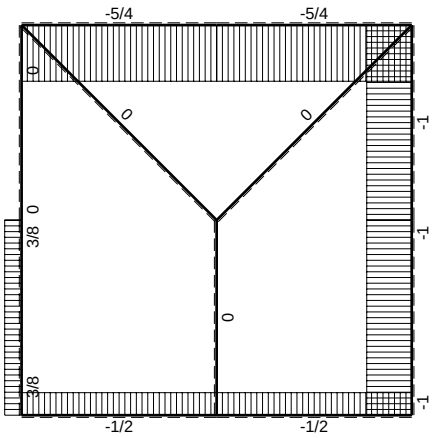
Curvatura θ asta EF positiva se convessa a destra con inizio E.

Spostamento orizzontale assoluto u imposto al nodo H.

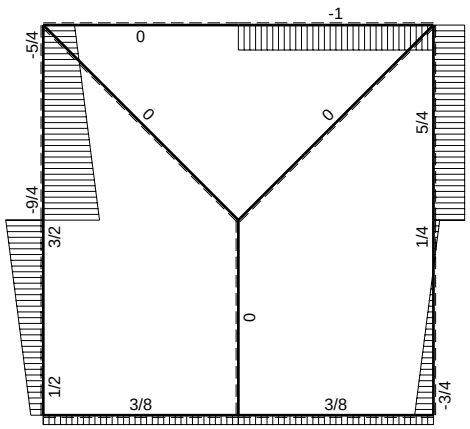
 08.01.25



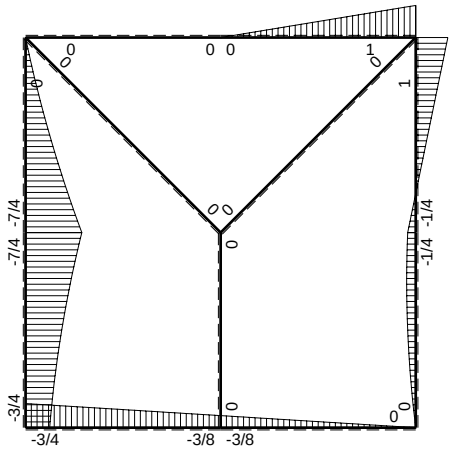
 08.01.25



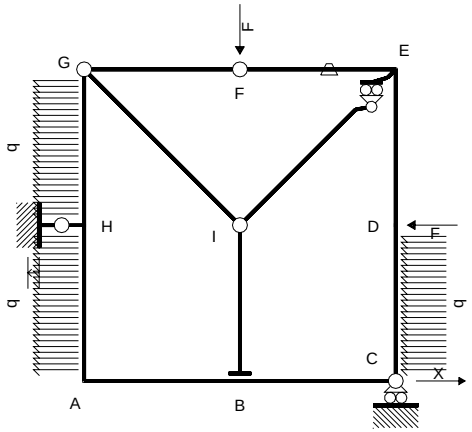
←F→

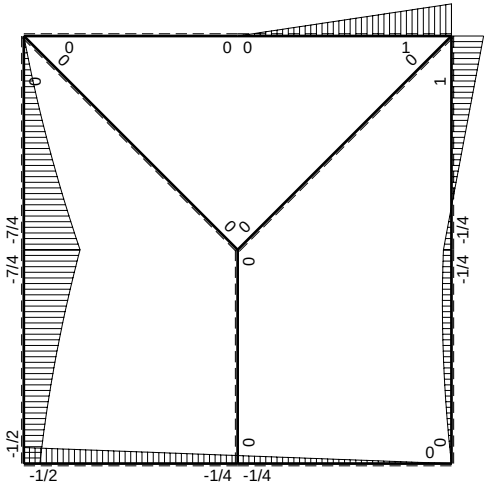


↑F↓

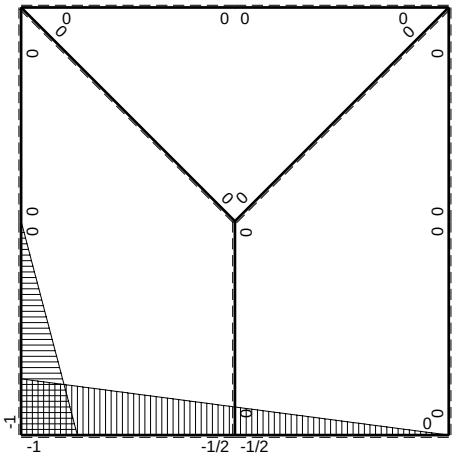


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-b+1/2x	-1/2Fb+1/4Fx	0	$1/2Fb^2-1/2Fbx+1/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	1/2b+1/2x	1/4Fb+1/4Fx	0	$1/8Fb^2+1/4Fbx+1/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	-1/2b+1/2x	-1/4Fb+1/4Fx	0	$1/8Fb^2-1/4Fbx+1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	1/2x	1/4Fx	0	$1/8Fx^2$	0	$1/4x^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0
FE b	0	-Fx	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x	$-7/4Fb+7/4Fx-1/2qx^2$	0	$7/4Fbx-7/4Fx^2+1/2qx^3$	0	x^2	$(5/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$1/2Fb+3/4Fx+1/2qx^2$	0	$1/2Fb^2+1/4Fbx-1/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$-1/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						1/4F	

PROCEDIMENTO E RISULTATI 934988 ESΣ04.xxxx.096

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(1/2 - 1/2\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/2\ x - 1/4\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/2\ b - 1/4\ b + 1/24\ b\ \right) Fb^2\ 1/EJ\ = 7/24\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(1/8 + 1/4\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x + 1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b + 1/8\ b + 1/24\ b\ \right) Fb^2\ 1/EJ\ = 7/24\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(1/8 - 1/4\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b - 1/8\ b + 1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(7/4\ x/b - 7/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[7/8\ x^2/b - 7/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(7/8\ b - 7/12\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 5/12\ \ Fb^3/EJ$

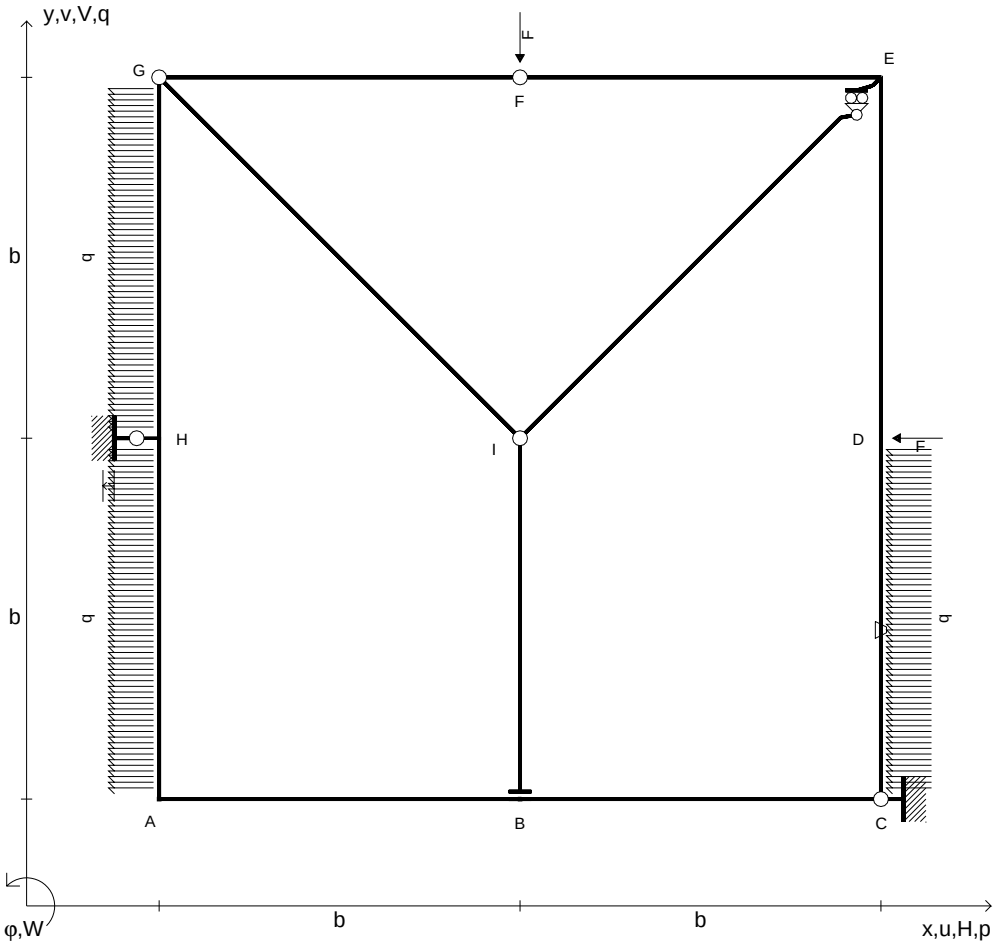
$L^{Xo}_{AH} = \int_o^b \left(1/2 + 1/4\ x/b - 1/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx$

$= \left[1/2\ x + 1/8\ x^2/b - 1/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/2\ b + 1/8\ b - 1/12\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 5/12\ \ Fb^3/EJ$

ESΣ04.xxxx.096

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

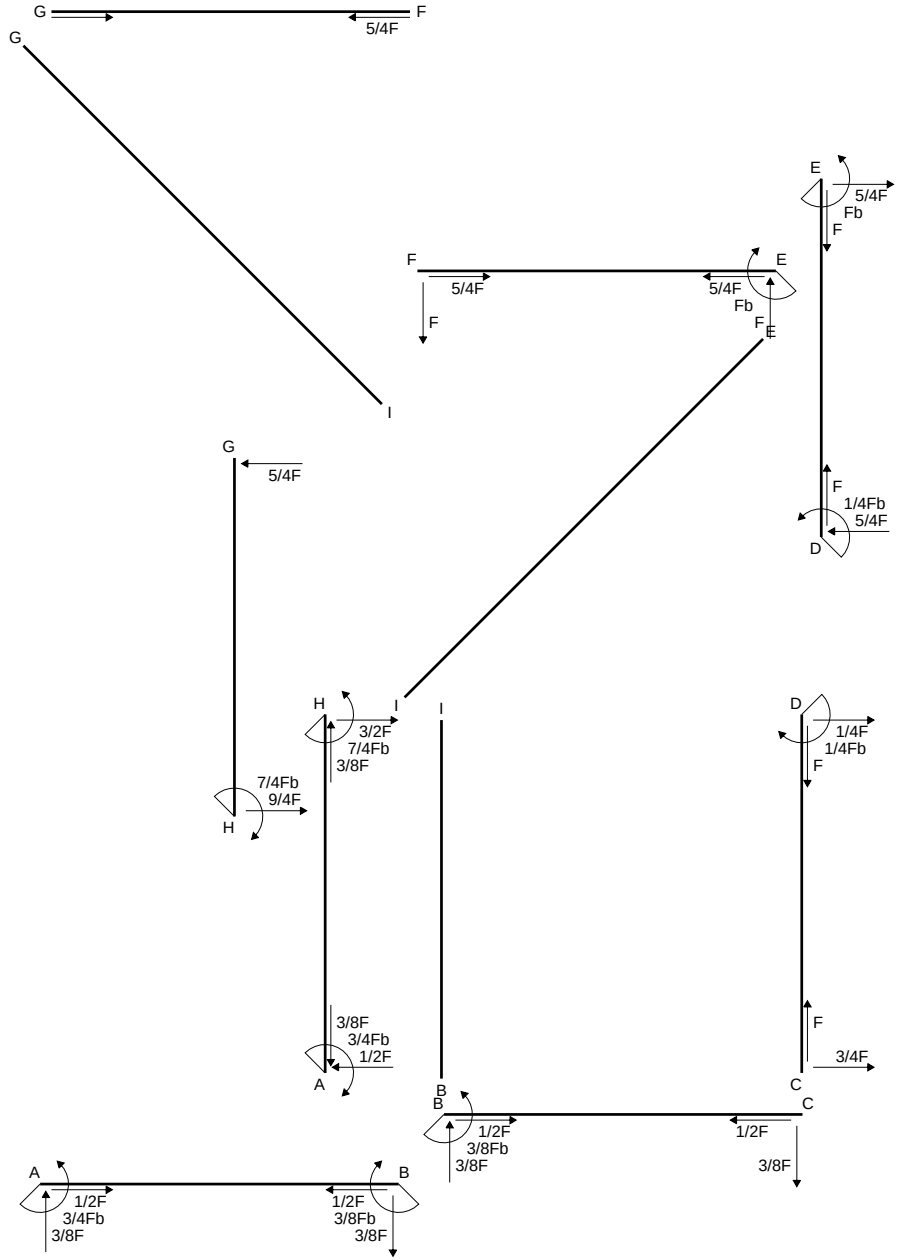
Tracciare i diagrammi quotati delle azioni interne nelle aste.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

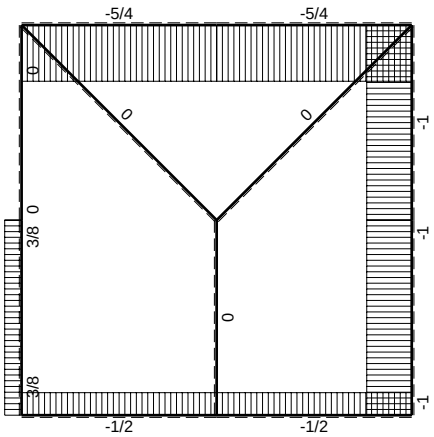
Curvatura θ asta CD positiva se convessa a destra con inizio C.

Spostamento orizzontale assoluto u imposto al nodo H.

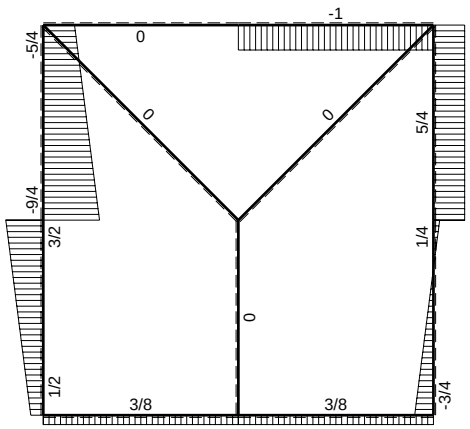
 08.01.25



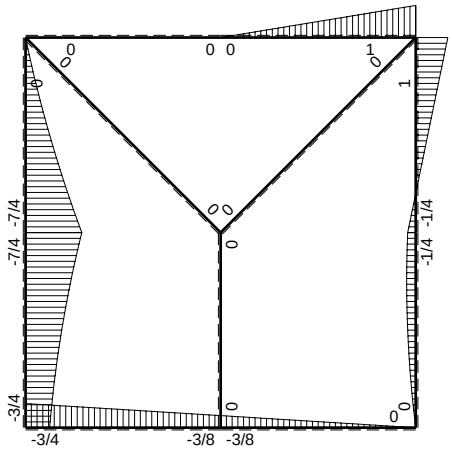
 08.01.25



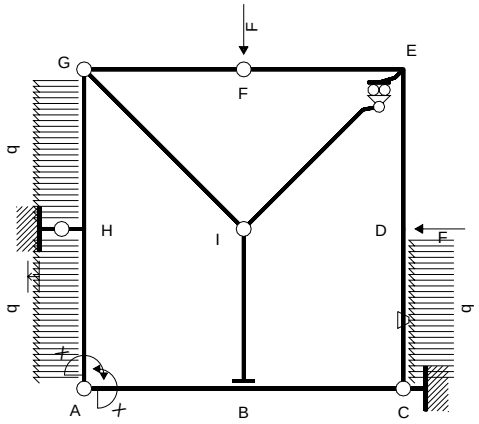
←□→ F

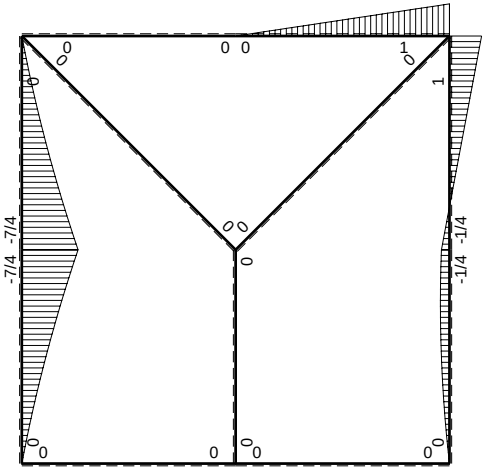


↑□↓ F

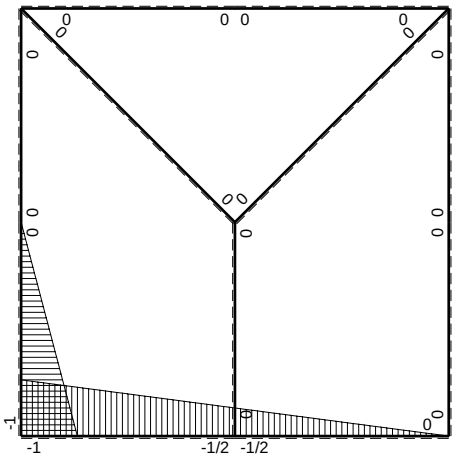


□ Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	Fb/EJ	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209775 ESΣ04.xxxx.097
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

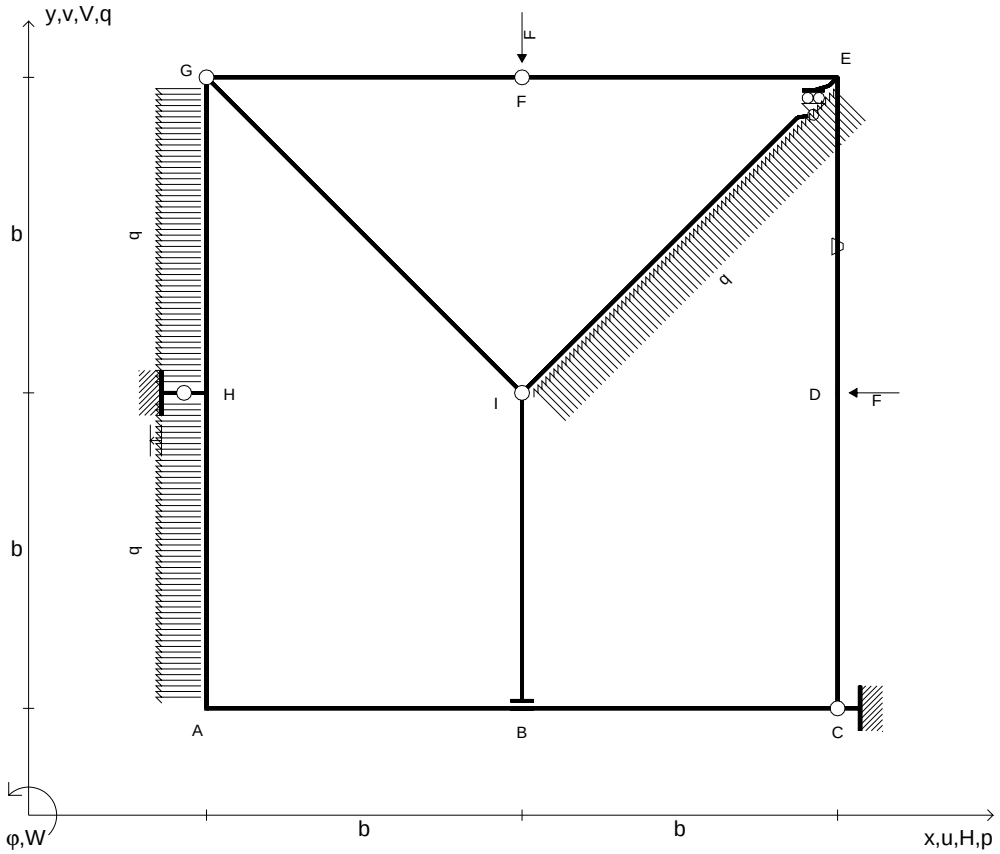
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

ESΣ04.xxxx.097

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

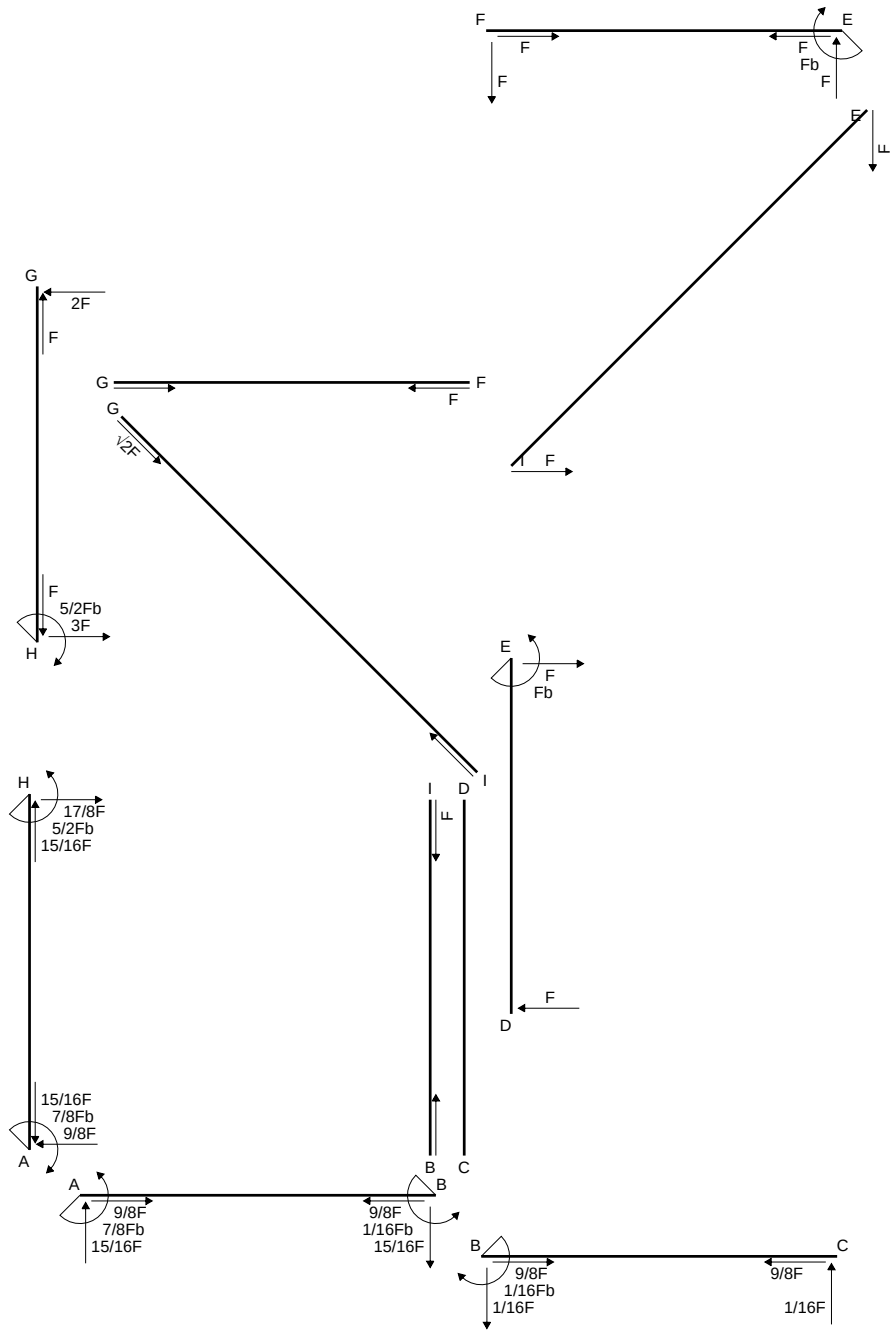


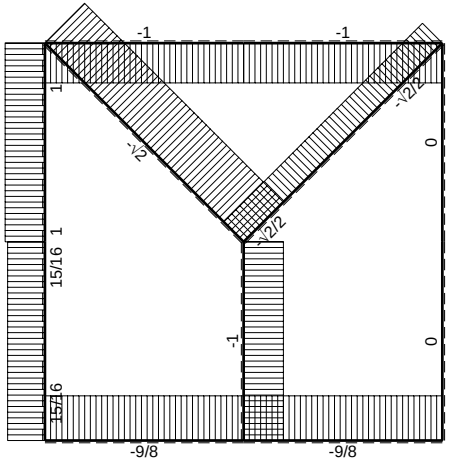
$V_F = -F$
$H_D = -F$
$p_{HA} = -q = -F/b$
$p_{GH} = -q = -F/b$
$p_{IE} = -q = -F/b$
$q_{IE} = q = F/b$
$\theta_{DE} = -\theta = -\alpha T/b = -bF/EJ$

$u_H = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$

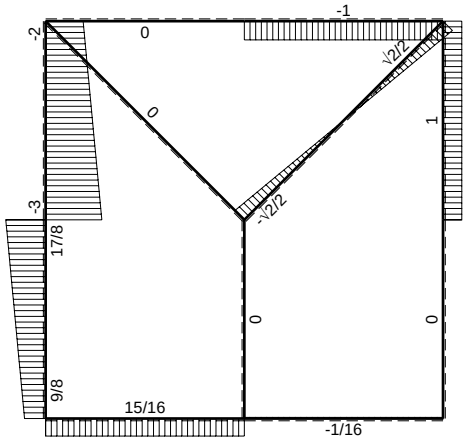
$EJ_{GH} = EJ$
$EJ_{GI} = EJ$
$EJ_{IB} = EJ$
$EJ_{IE} = EJ$
$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta DE positiva se convessa a destra con inizio D.
Spostamento orizzontale assoluto u imposto al nodo H.

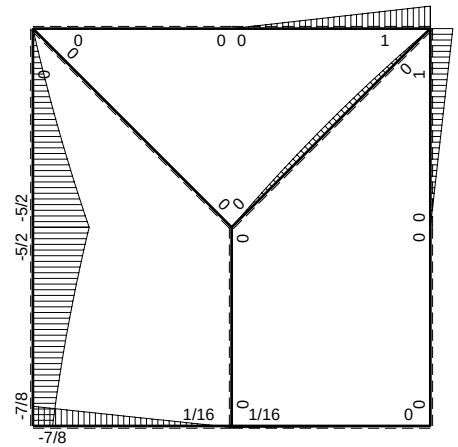




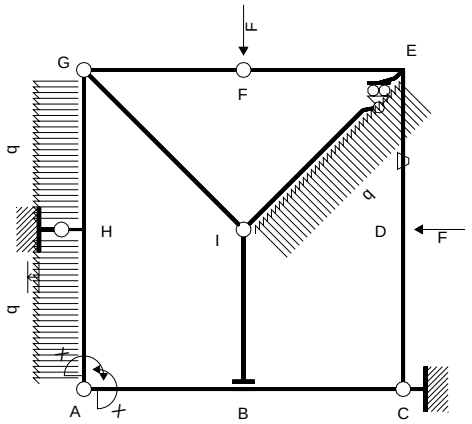
←F→

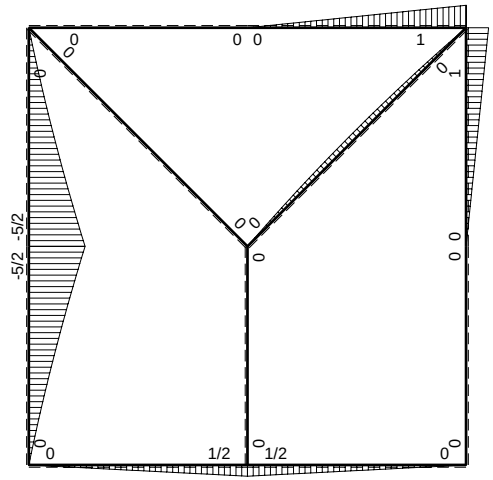


↑F↓

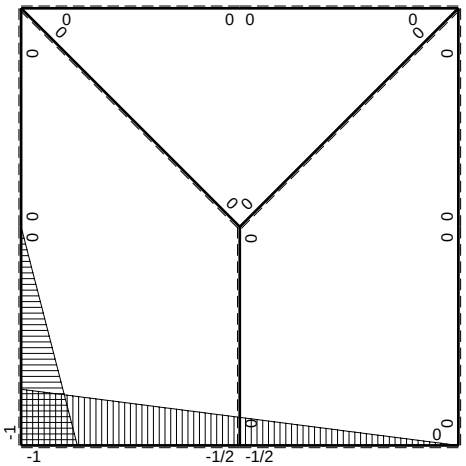


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210089 ESΣ04.xxxx.104

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-1/2Fx	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-Fb+Fx	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

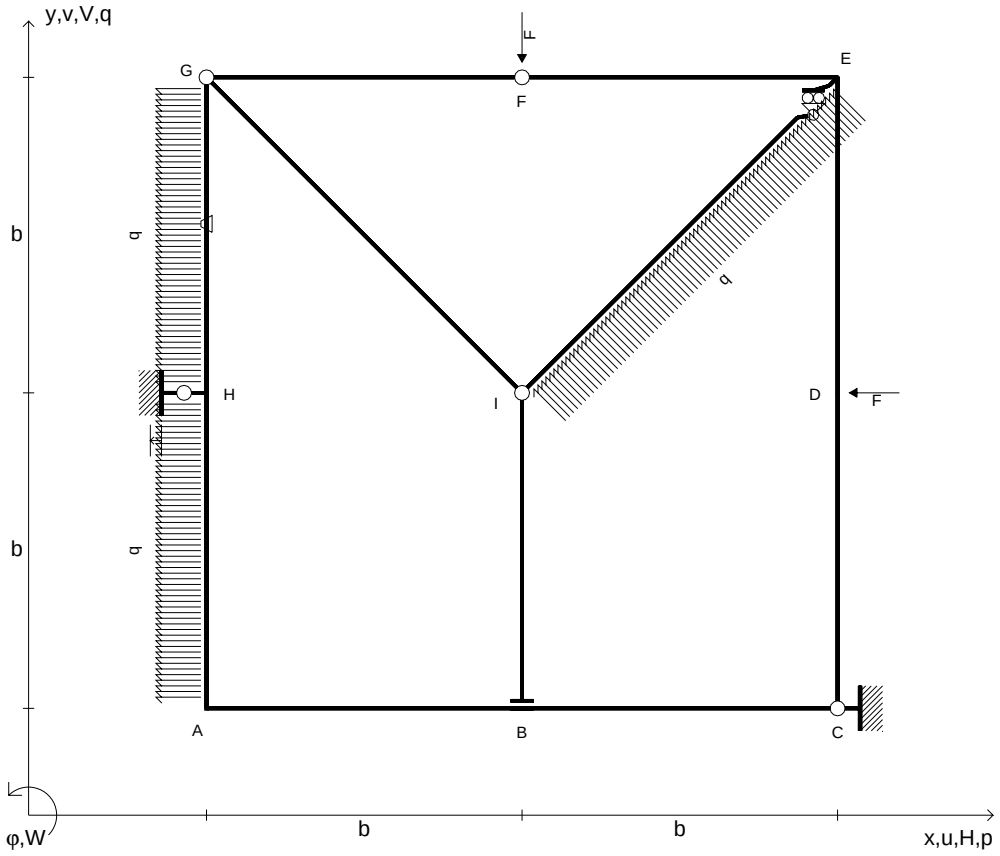
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

ESΣ04.xxxx.104

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{GH} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GI} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{IB} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IE} = EJ$

$p_{IE} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{HA} = EJ$

$q_{IE} = q = F/b$ | $EJ_{EF} = EJ$

$\theta_{GH} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{FG} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Diagrammi di carico con valori riferiti ad asse della trave.

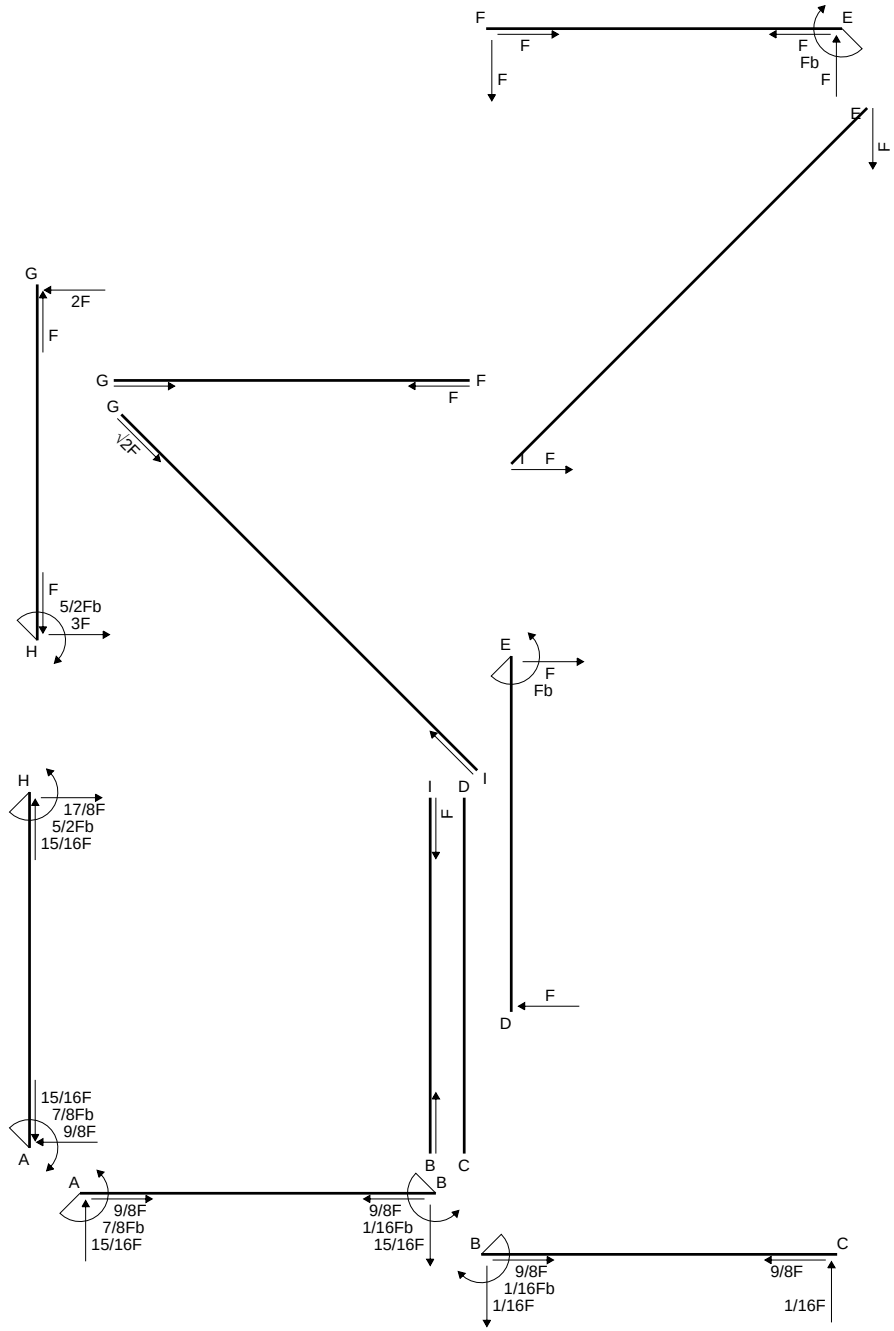
Componenti di carico distribuito riferiti ad assi ortogonali.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

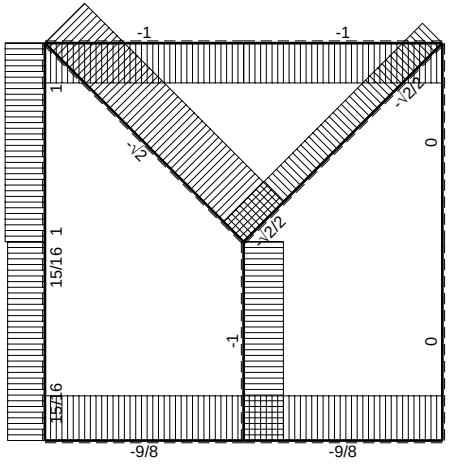
Curvatura θ asta GH positiva se convessa a destra con inizio G.

Spostamento orizzontale assoluto u imposto al nodo H.

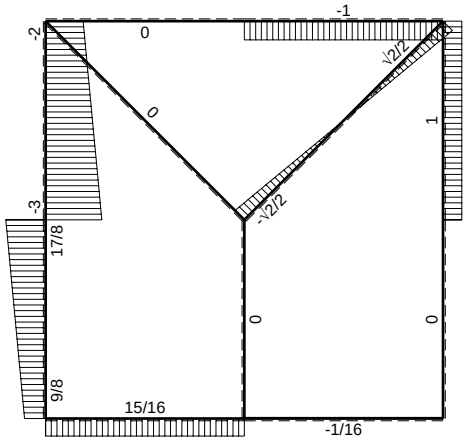
 08.01.25



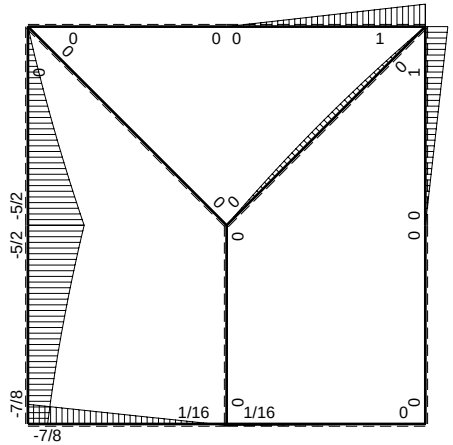
 08.01.25



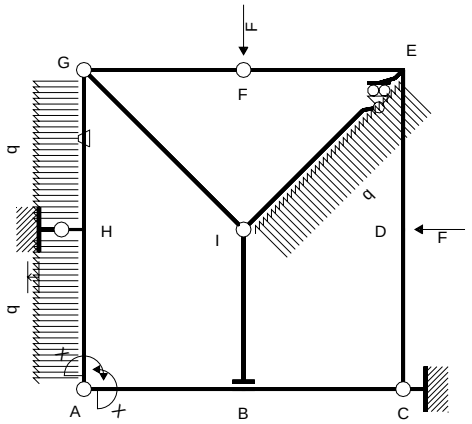
←□→ F

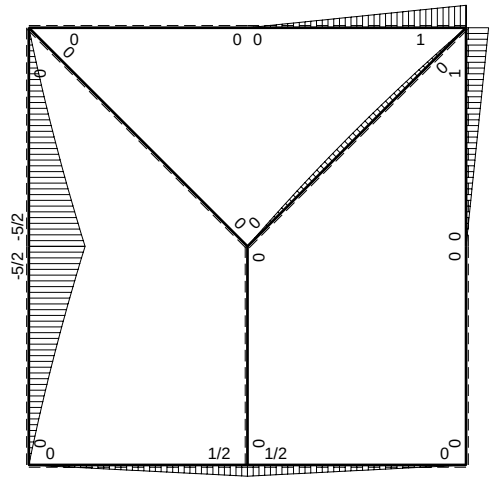


↑□↓ F

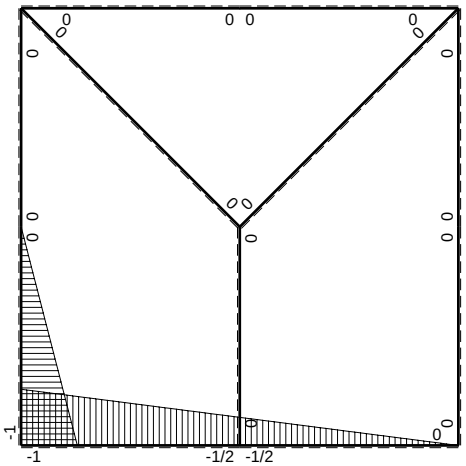


□ Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/2Fx	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	1/2Fb-1/2Fx	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	-x/b	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

PROCEDIMENTO E RISULTATI 213396 ESΣ04.xxxx.108

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/2\ x/b + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/4 + 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/12\ b\right)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1/4 + 1/2\ x/b - 1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/4\ x + 1/4\ x^2/b - 1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/4\ b + 1/4\ b - 1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/4\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/12\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/12\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/4\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

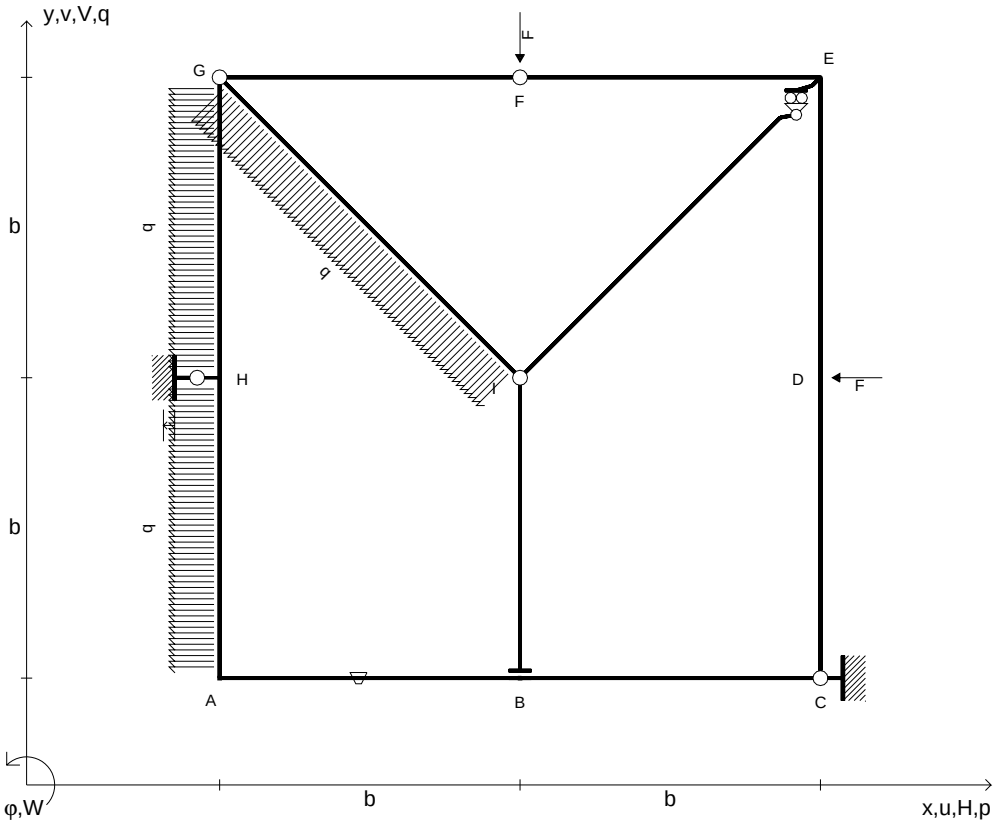
$= \left(5/4\ b - b + 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

$L_{AH}^{Xo} = \int_o^b \left(2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(\ b - 1/2\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 3/8\ \ Fb^2/EJ$

ESΣ04.xxxx.108

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

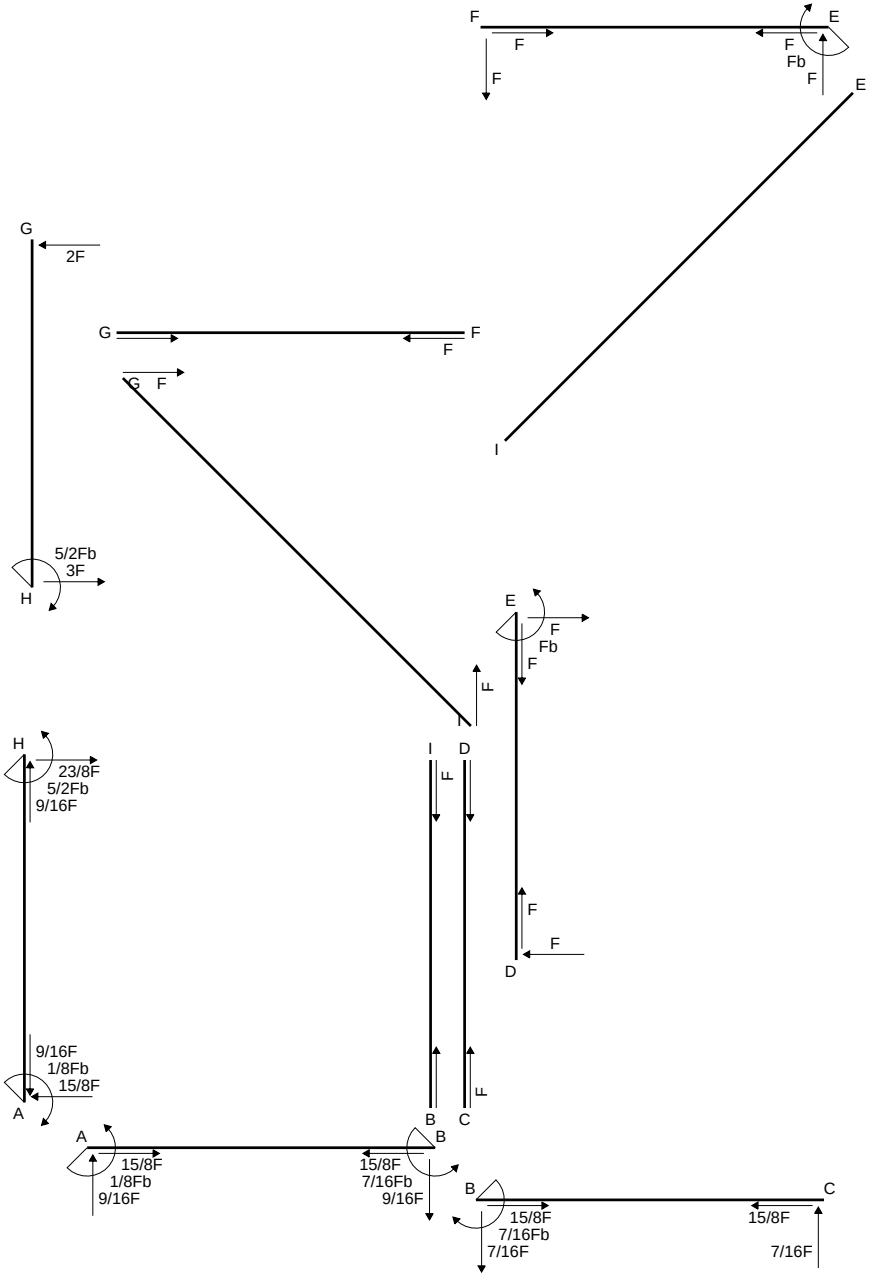


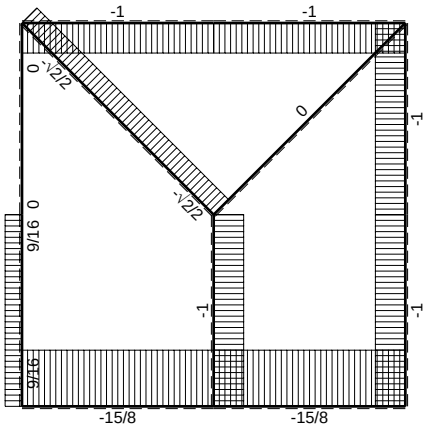
$V_F = -F$
$H_D = -F$
$p_{HA} = -q = -F/b$
$p_{GH} = -q = -F/b$
$p_{GI} = -q = -F/b$
$q_{GI} = -q = -F/b$
$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$

$u_H = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$

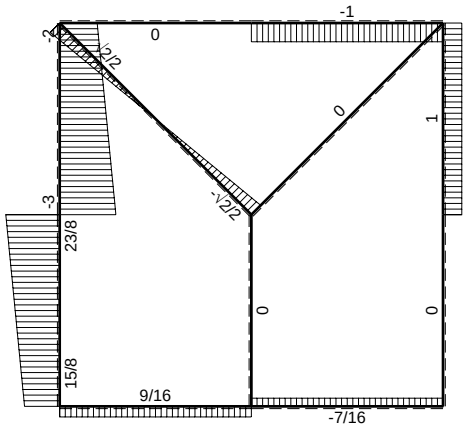
$EJ_{GH} = EJ$
$EJ_{GI} = EJ$
$EJ_{IB} = EJ$
$EJ_{IE} = EJ$
$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo H.

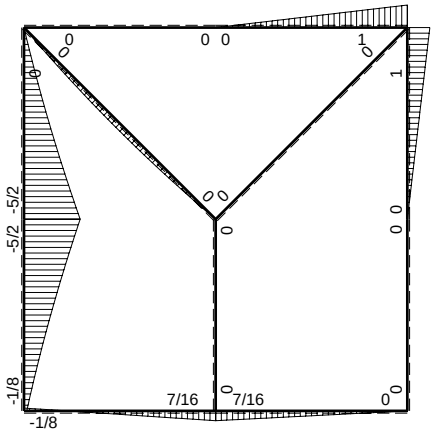




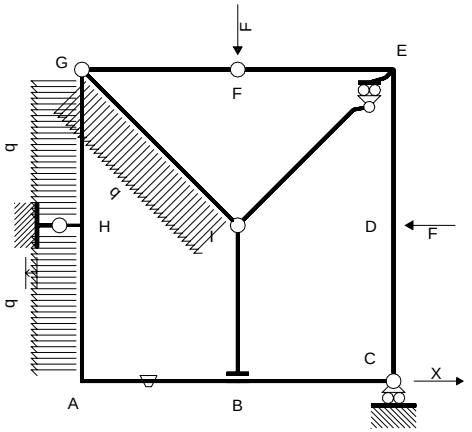
←□→ F

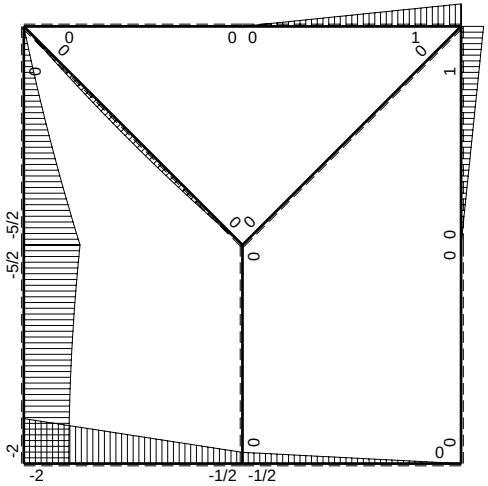


↑□↓ F

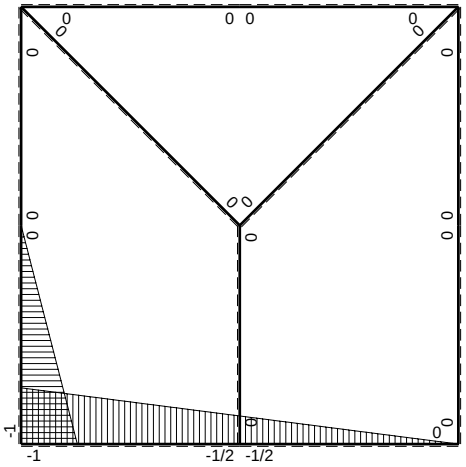


↻□↺ Fb





↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211522 ESΣ04.xxxx.111

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-b+1/2x	-2Fb+3/2Fx	-Fb/EJ	$2Fb^2-5/2Fbx+3/4Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	1/2b+1/2x	1/2Fb+3/2Fx	Fb/EJ	$1/4Fb^2+Fbx+3/4Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
BC b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	1/2x	1/2Fx	0	$1/4Fx^2$	0	$1/4x^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x	$-5/2Fb+Fx-1/2qx^2$	0	$5/2Fbx-Fx^2+1/2qx^3$	0	x^2	$(25/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$2Fb+1/2qx^2$	0	$2Fb^2-2Fbx+1/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$15/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-15/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(2 - 5/2\ x/b + 3/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(1 - 1/2\ x/b\ \right) \theta\ \ dx$

$= \left[2\ x - 5/4\ x^2/b + 1/4\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ\ + \left[\ x - 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(2\ b - 5/4\ b + 1/4\ b\ \right) Fb^2\ 1/EJ\ + \left(\ b - 1/4\ b\ \right) \theta\ \ = 7/4\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(1/4 + x/b + 3/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(-1/2 - 1/2\ x/b\ \right) \theta\ \ dx$

$= \left[1/4\ x + 1/2\ x^2/b + 1/4\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ\ + \left[-1/2\ x - 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(1/4\ b + 1/2\ b + 1/4\ b\ \right) Fb^2\ 1/EJ\ + \left(-1/2\ b - 1/4\ b\ \right) \theta\ \ = 7/4\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/12\ b\ \right) Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(5/2\ x/b - x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[5/4\ x^2/b - 1/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

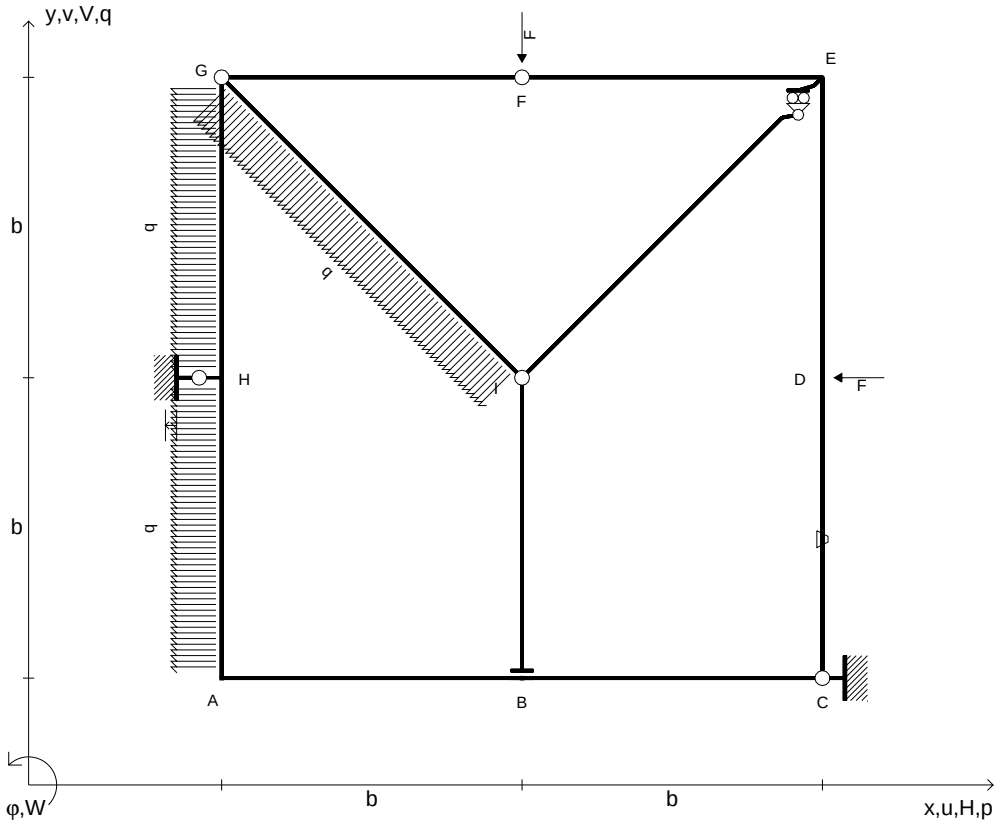
$= \left(5/4\ b - 1/3\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b \left(2 - 2\ x/b + 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[2\ x - x^2/b + 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(2\ b - b + 1/6\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 211522 ESΣ04.xxxx.111

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{GH} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GI} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{IB} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IE} = EJ$

$p_{GI} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{HA} = EJ$

$q_{GI} = -q = -F/b$ | $EJ_{EF} = EJ$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{FG} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

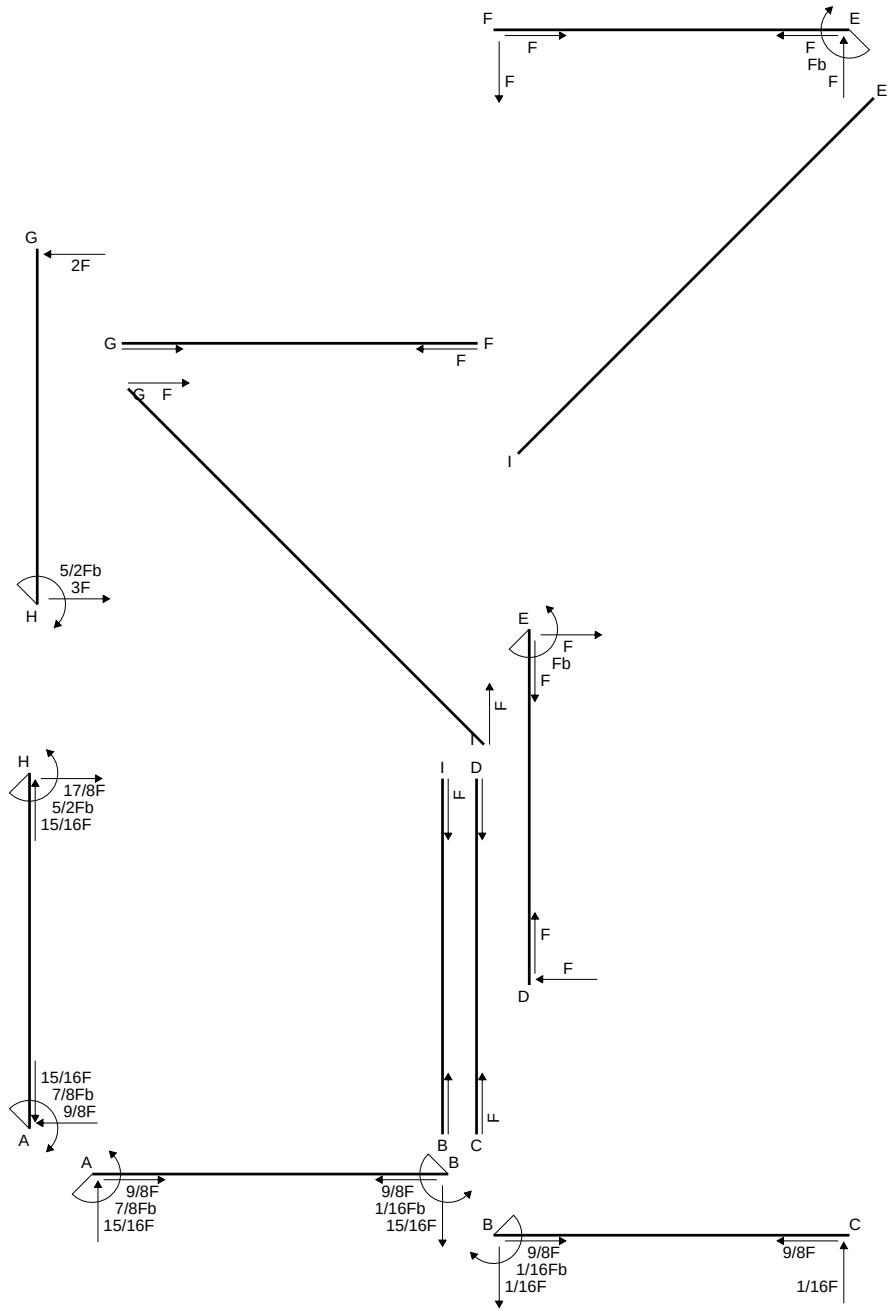
Diagrammi di carico con valori riferiti ad asse della trave.

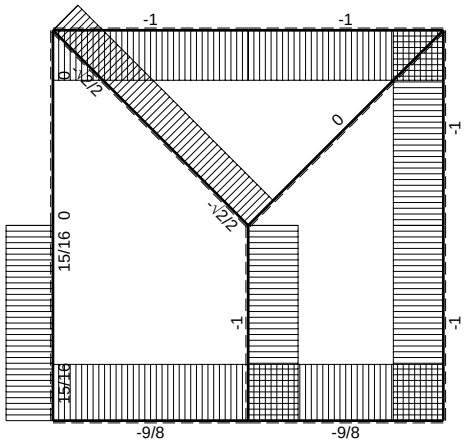
Componenti di carico distribuito riferiti ad assi ortogonali.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

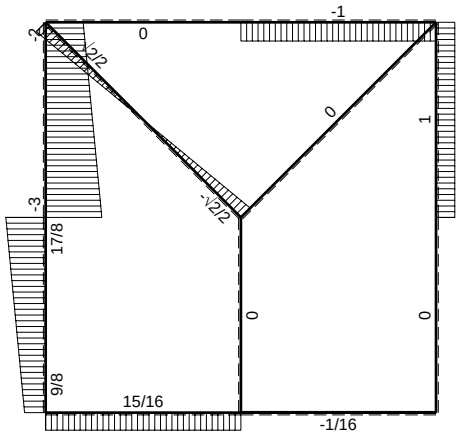
Curvatura θ asta CD positiva se convessa a destra con inizio C.

Spostamento orizzontale assoluto u imposto al nodo H.

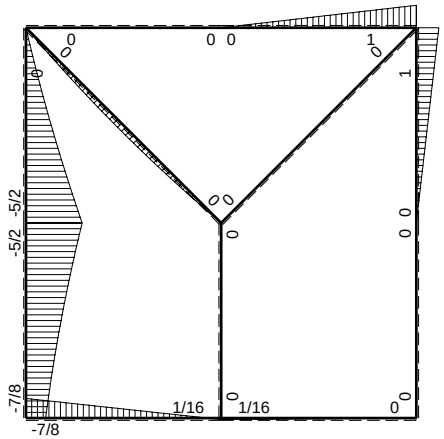




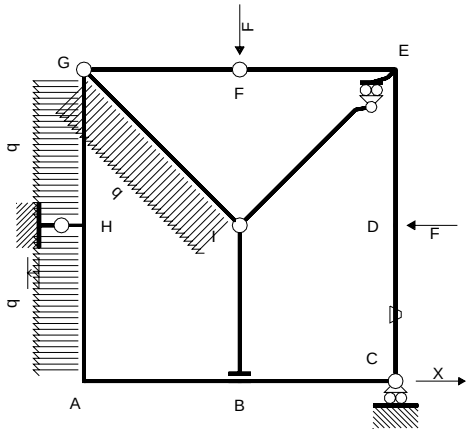
←□→ F

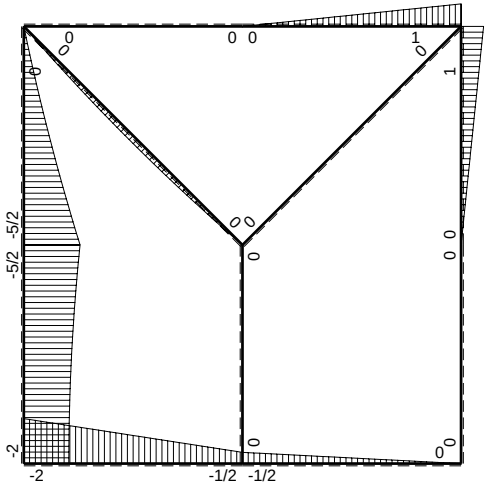


↑□↓ F

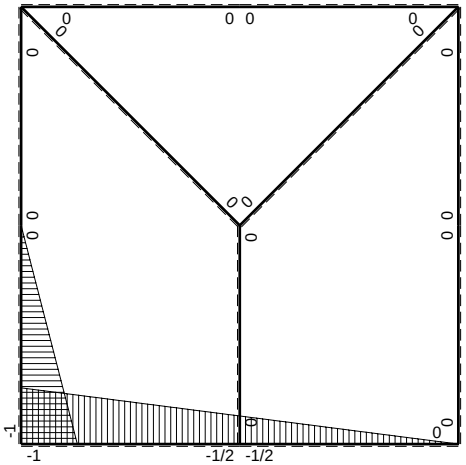


(□) Fb





(□) M_o flessione da carichi assegnati



(□) M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211210 ESΣ04.xxxx.121

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-2Fb+3/2Fx$	0	$2Fb^2-5/2Fbx+3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$1/2Fb+3/2Fx$	0	$1/4Fb^2+Fbx+3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
CD b	0	0	$-Fb/EJ$	0	0	0	0+0	0
DC b	0	0	Fb/EJ	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	$-Fb+Fx$	0	0	0	0		
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0
FE b	0	$-Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	$-x$	$-5/2Fb+Fx-1/2qx^2$	0	$5/2Fbx-Fx^2+1/2qx^3$	0	x^2	$(25/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	$b-x$	$2Fb+1/2qx^2$	0	$2Fb^2-2Fbx+1/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$9/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						$-9/8F$	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (1 - x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - 1/2\ b + 1/12\ b\ )\ b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1/4 + 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/4\ b + 1/4\ b + 1/12\ b)\ b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (2 - 5/2\ x/b + 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [2\ x - 5/4\ x^2/b + 1/4\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (2\ b - 5/4\ b + 1/4\ b)\ Fb^2\ 1/EJ\ = \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (1/4 + x/b + 3/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/4\ x + 1/2\ x^2/b + 1/4\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (1/4\ b + 1/2\ b + 1/4\ b)\ Fb^2\ 1/EJ\ = \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (1/12\ b)\ Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (5/2\ x/b - x^2/b^2 + 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [5/4\ x^2/b - 1/3\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ$

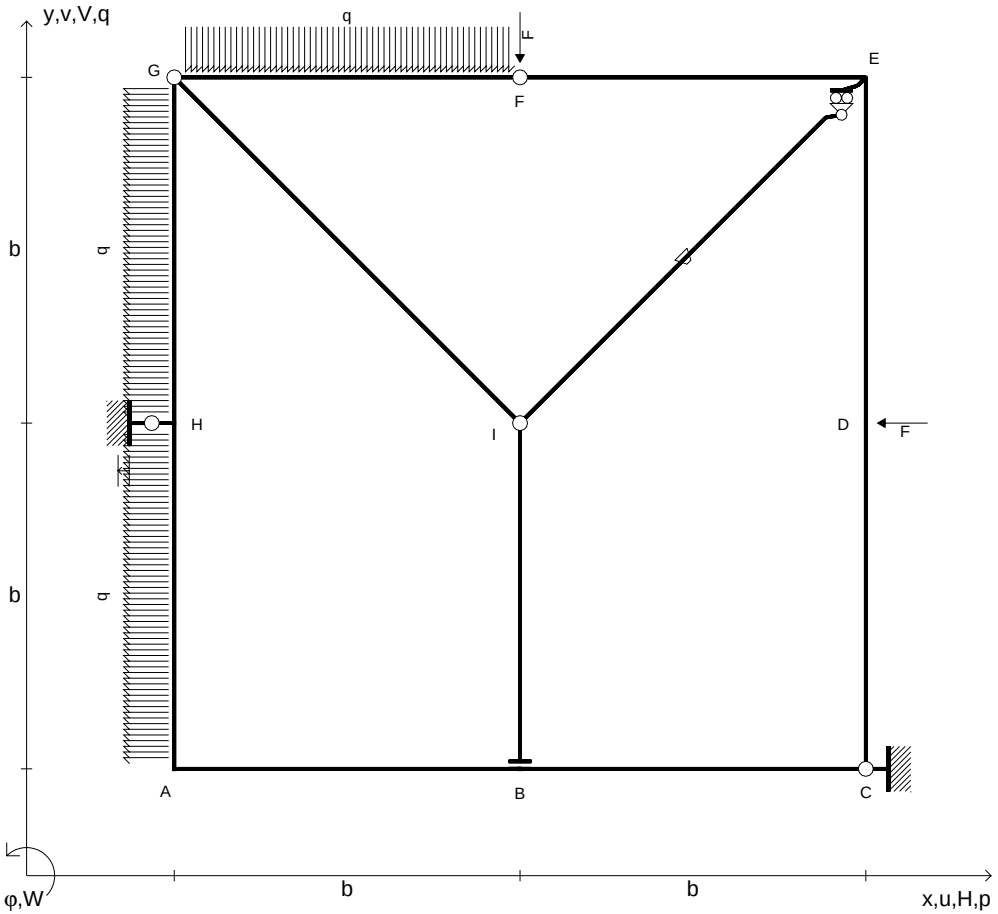
$= (5/4\ b - 1/3\ b + 1/8\ b)\ Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (2 - 2\ x/b + 1/2\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [2\ x - x^2/b + 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ$

$= (2\ b - b + 1/6\ b - 1/8\ b)\ Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$

ESΣ04.xxxx.121

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$q_{FG} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

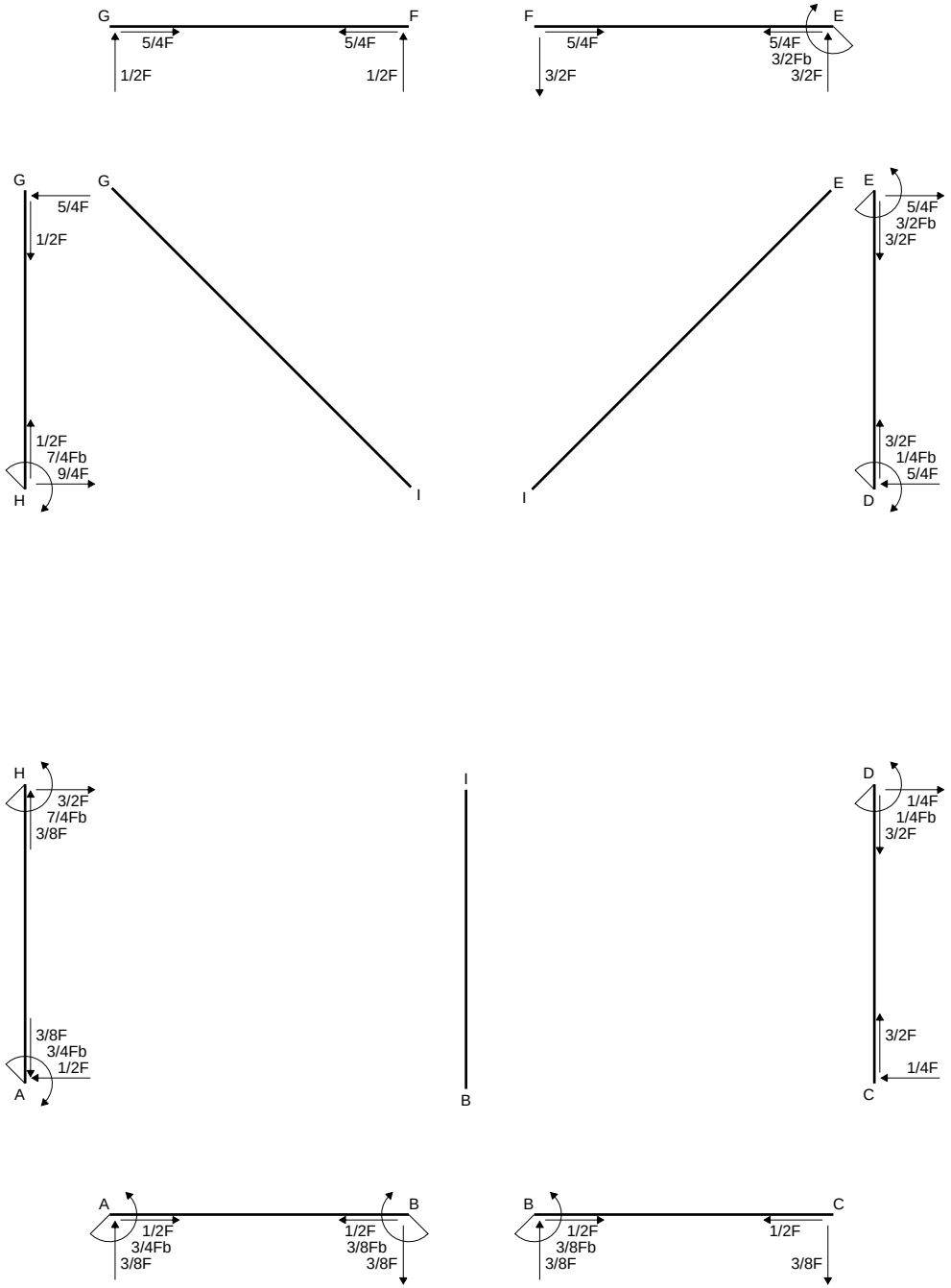
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

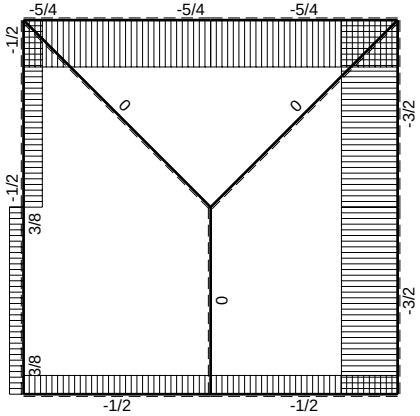
Curvatura θ asta IE positiva se convessa a destra con inizio I.

Spostamento orizzontale assoluto u imposto al nodo H.

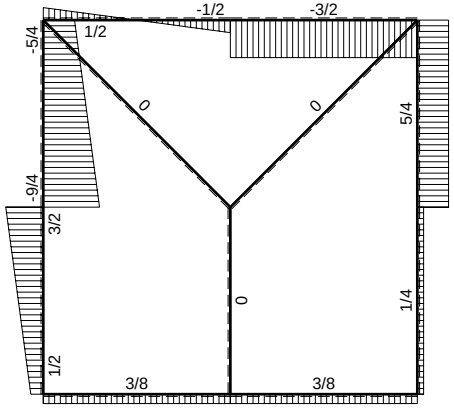
 08.01.25



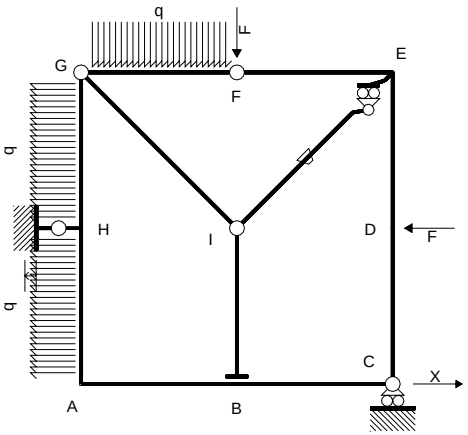
 08.01.25

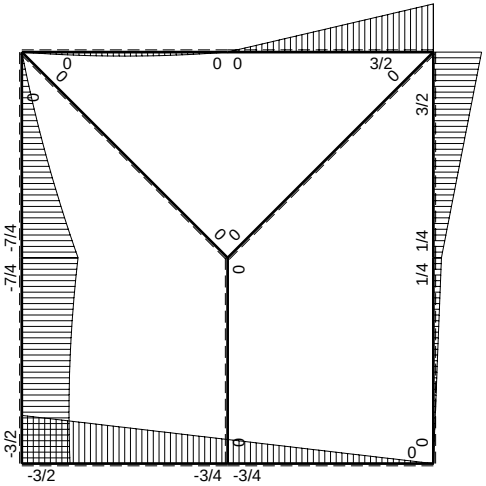


←□→ F

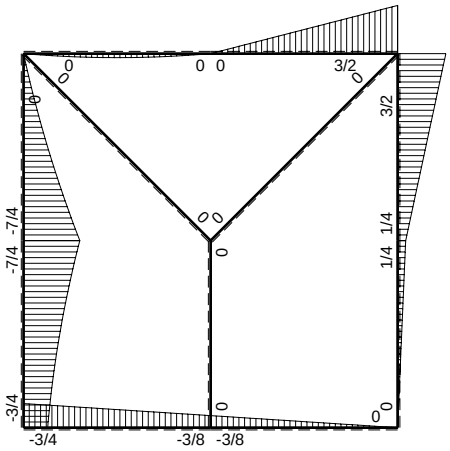


↑□↓ F

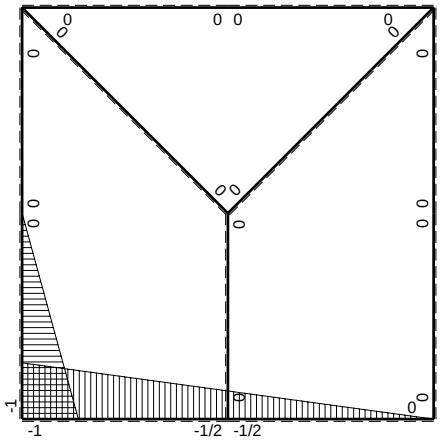




□ M_o flessione da carichi assegnati



□ Fb



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-b+1/2x	-3/2Fb+3/4Fx	0	$3/2Fb^2-3/2Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	1/2b+1/2x	3/4Fb+3/4Fx	0	$3/8Fb^2+3/4Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	1/2x	3/4Fx	0	$3/8Fx^2$	0	$1/4x^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x	$-7/4Fb+3/4Fx-1/2qx^2$	0	$7/4Fbx-3/4Fx^2+1/2qx^3$	0	x^2	$(3/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$3/2Fb-1/4Fx+1/2qx^2$	0	$3/2Fb^2-7/4Fbx+3/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$3/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-3/4F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 956427 ESΣ04.xxxx.122
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2 - 3/2\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/2\ x - 3/4\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/2\ b - 3/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(3/8 + 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x + 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/8\ b + 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(3/8 - 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x - 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/8\ b - 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(7/4\ x/b - 3/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[7/8\ x^2/b - 1/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(7/8\ b - 1/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 3/4\ \ Fb^3/EJ$

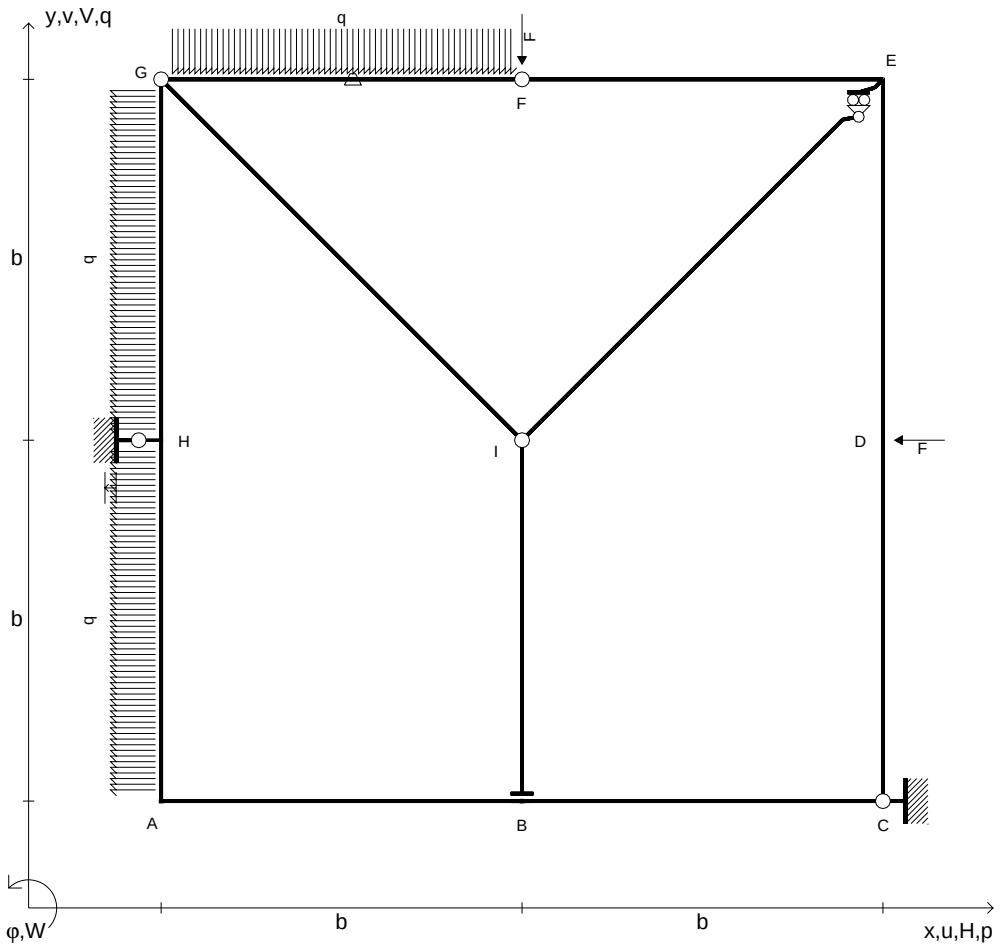
$L^{Xo}_{AH} = \int_o^b \left(3/2 - 7/4\ x/b + 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx$

$= \left[3/2\ x - 7/8\ x^2/b + 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/2\ b - 7/8\ b + 1/4\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 3/4\ \ Fb^3/EJ$

ESΣ04.xxxx.122

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$q_{FG} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

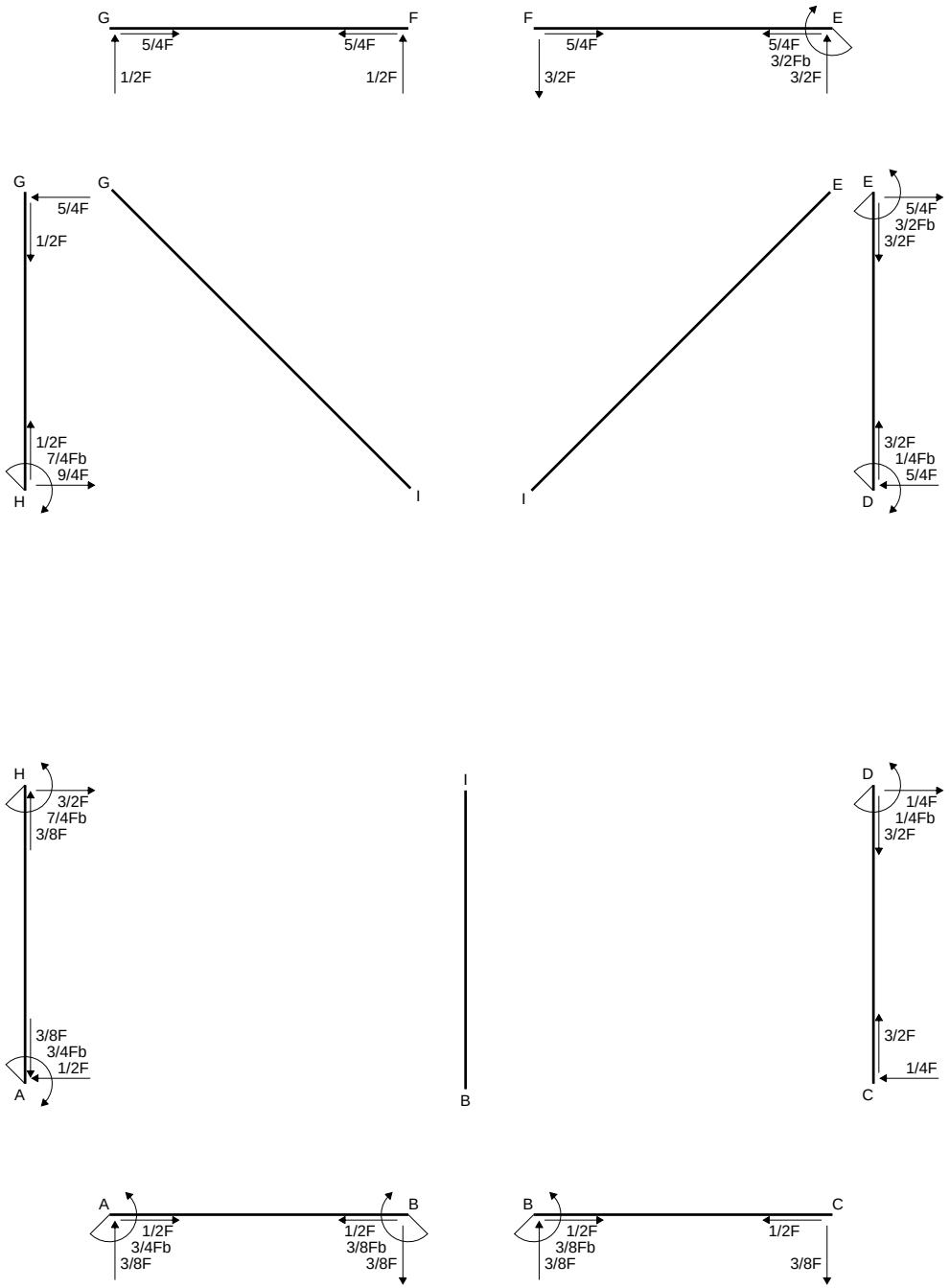
Tracciare i diagrammi quotati delle azioni interne nelle aste.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

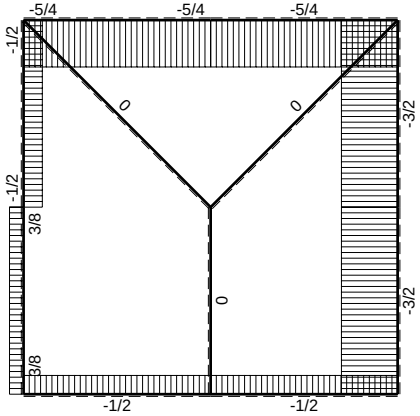
Curvatura θ asta FG positiva se convessa a destra con inizio F.

Spostamento orizzontale assoluto u imposto al nodo H.

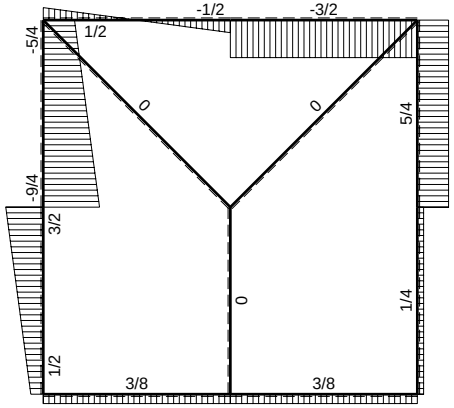
 08.01.25



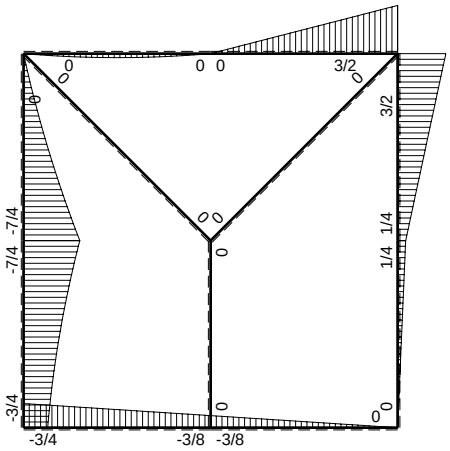
 08.01.25



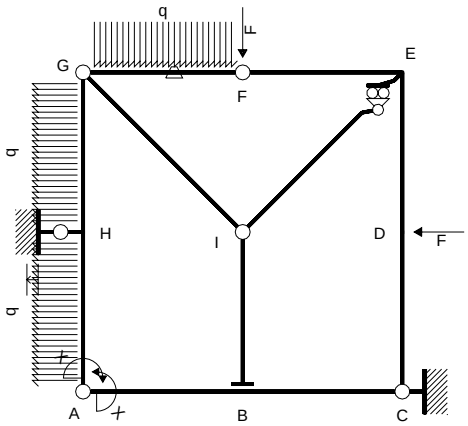
←□→ F

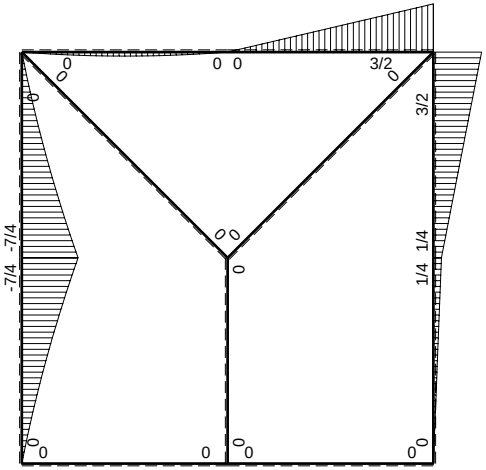


↑□↓ F

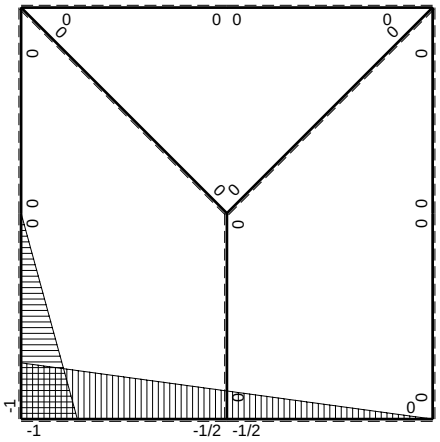


Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 213131 ESΣ04.xxxx.124

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

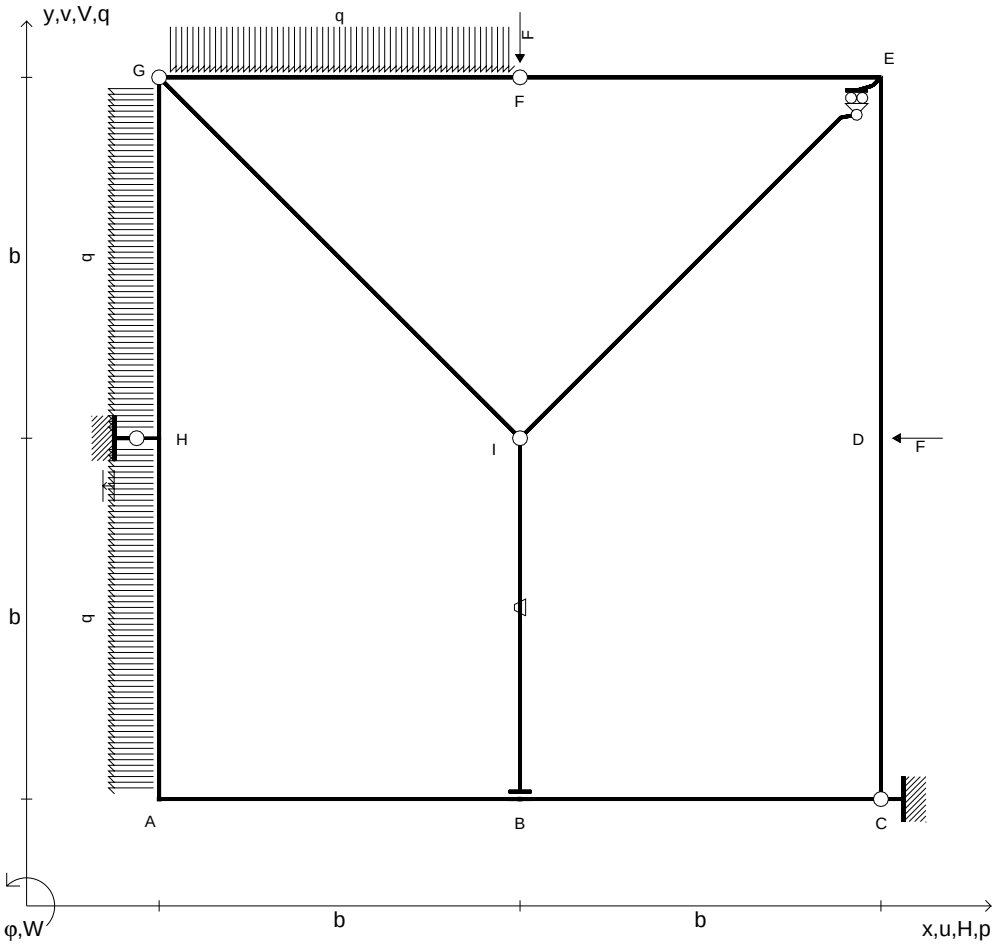
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

ESΣ04.xxxx.124

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$q_{FG} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{IB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

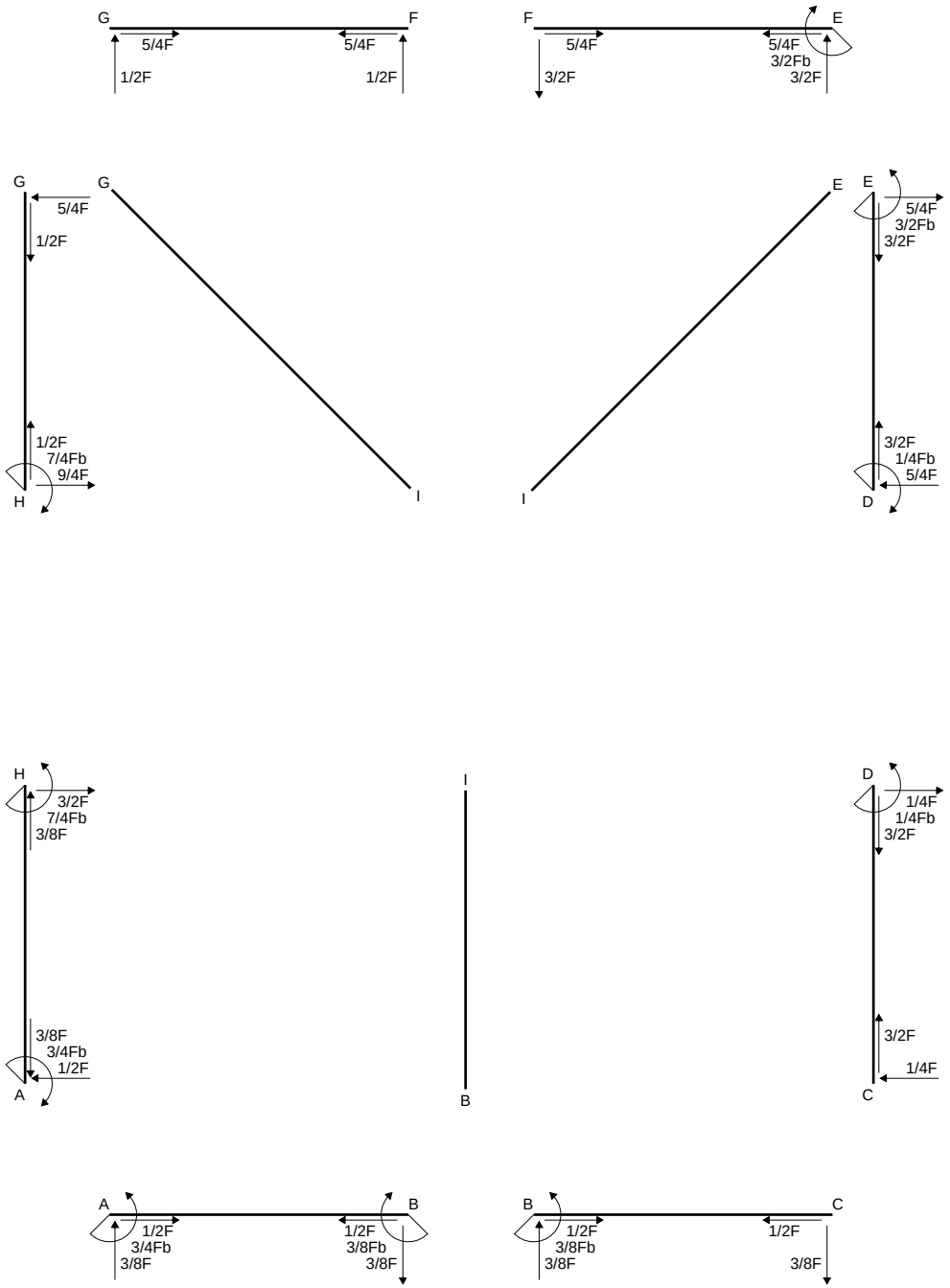
Tracciare i diagrammi quotati delle azioni interne nelle aste.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

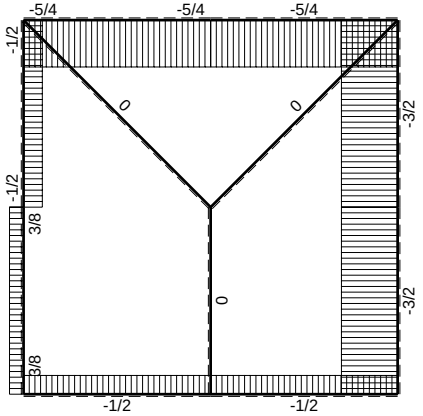
Curvatura θ asta IB positiva se convessa a destra con inizio I.

Spostamento orizzontale assoluto u imposto al nodo H.

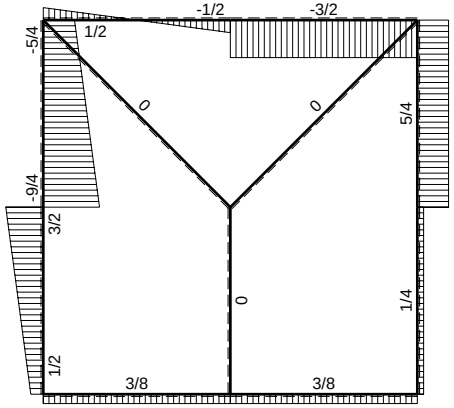
 08.01.25



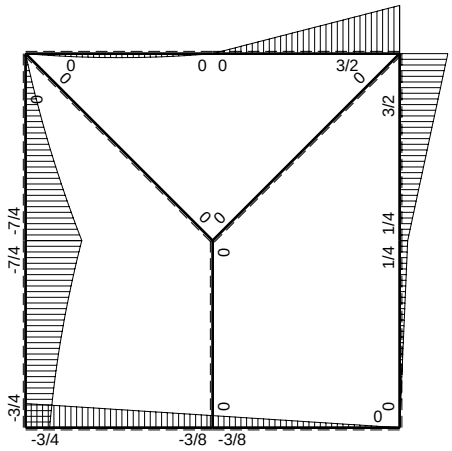
 08.01.25



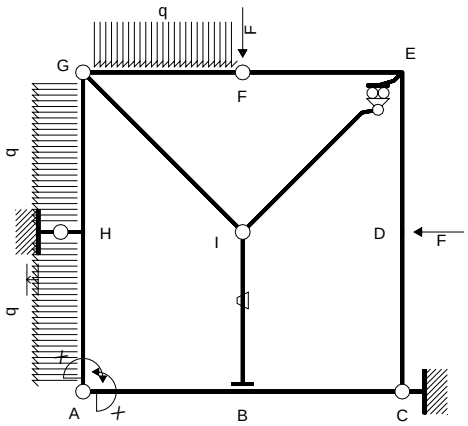
←□→ F

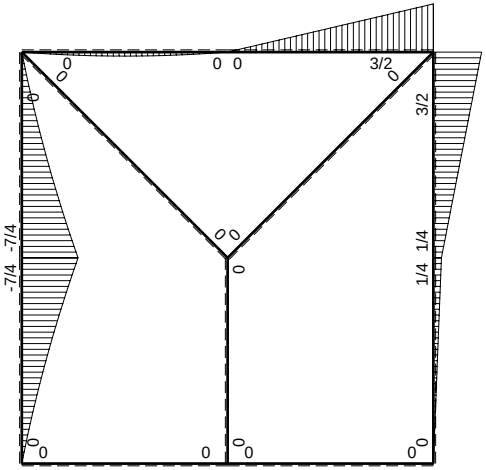


↑□↓ F

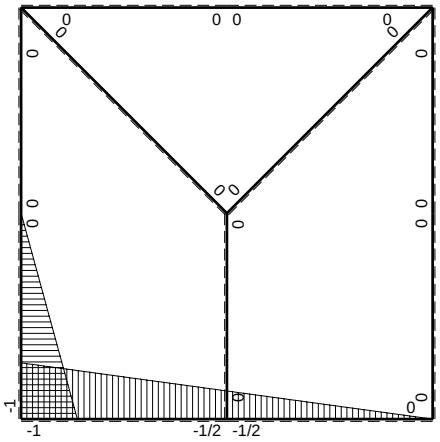


↻□↺ Fb





↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 984418 ESΣ04.xxxx.128

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	1/12Xb/EJ
CB b	1/2x/b	0	0	0	0	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	-Fb/EJ	0	0	0	0+0	0
BI b	0	0	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						3/4Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

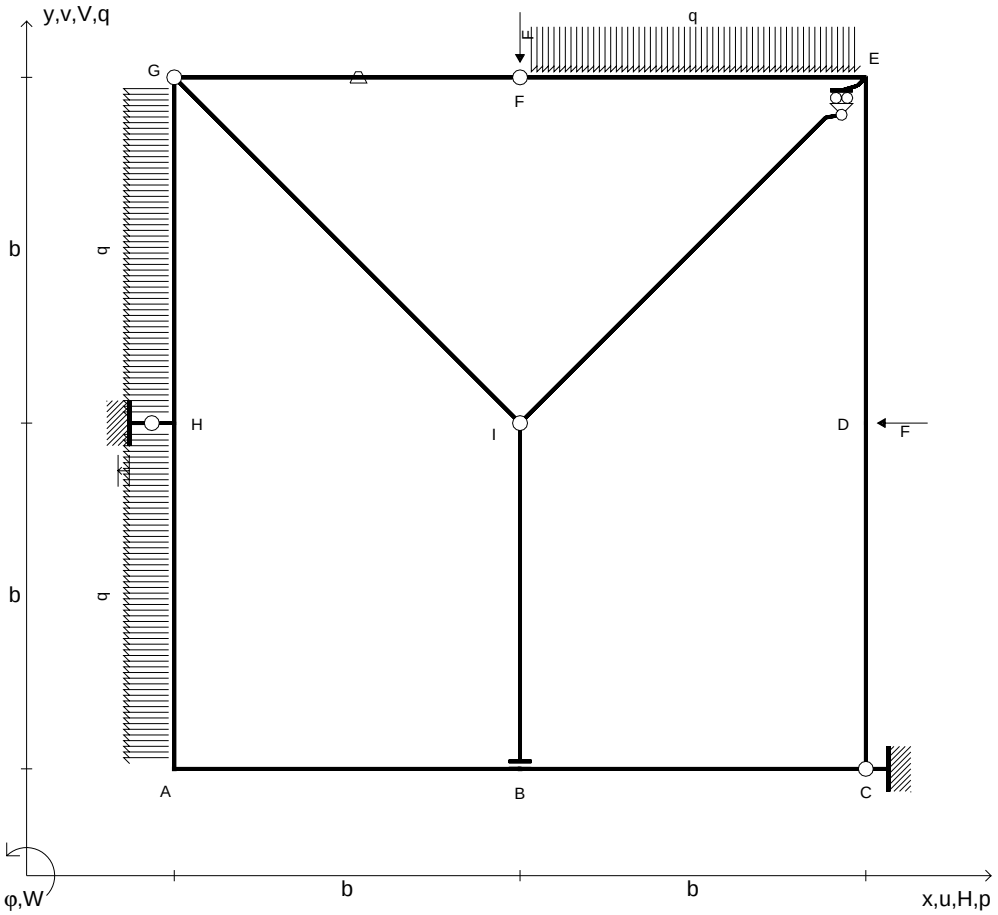
$= \left(7/8\ b - 3/4\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(5/8\ b - 1/4\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

ESΣ04.xxxx.128

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

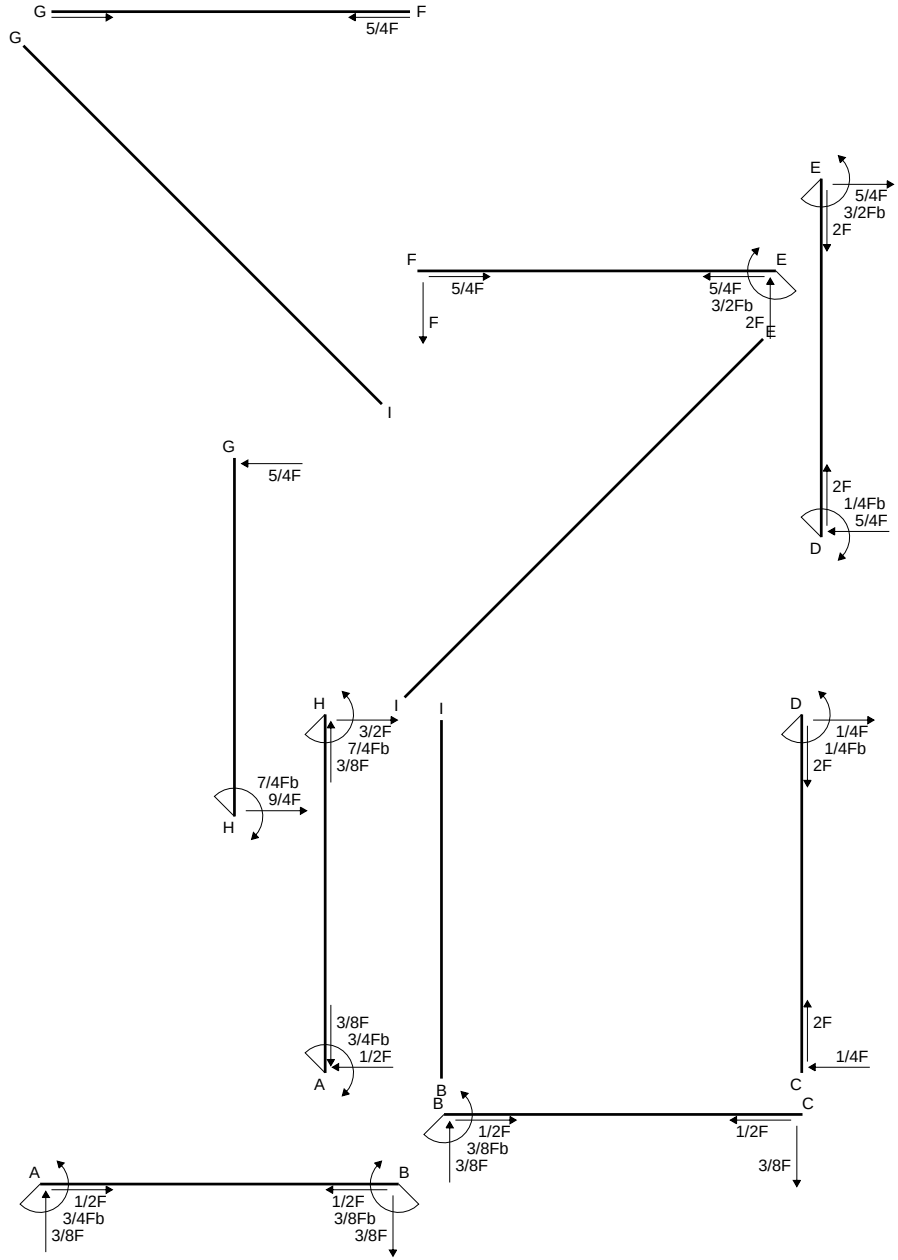
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

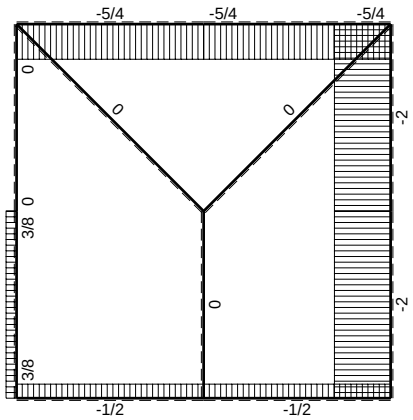
Curvatura θ asta FG positiva se convessa a destra con inizio F.

Spostamento orizzontale assoluto u imposto al nodo H.

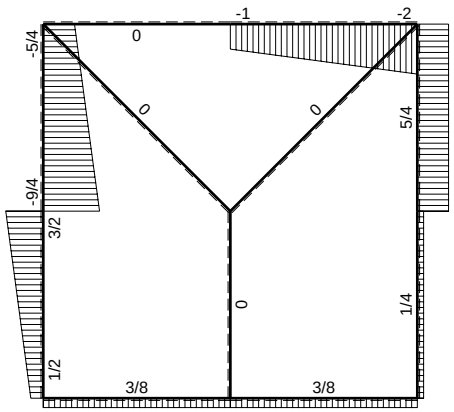
 08.01.25



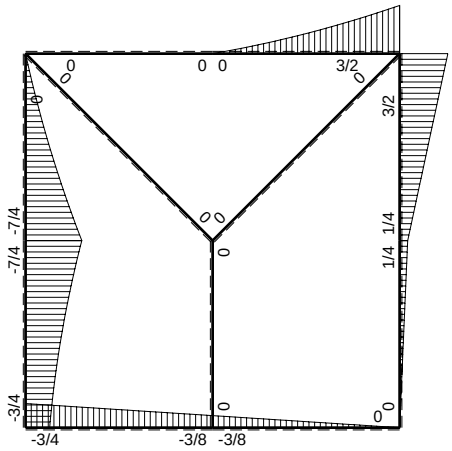
 08.01.25



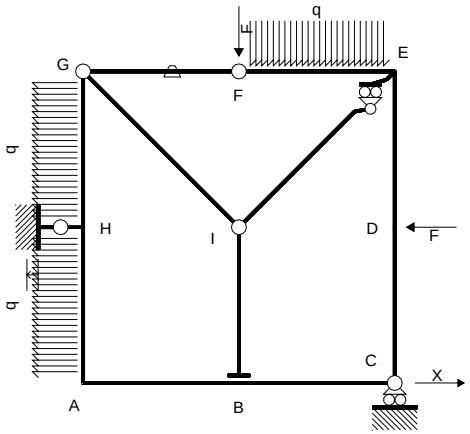
←⊞→ F

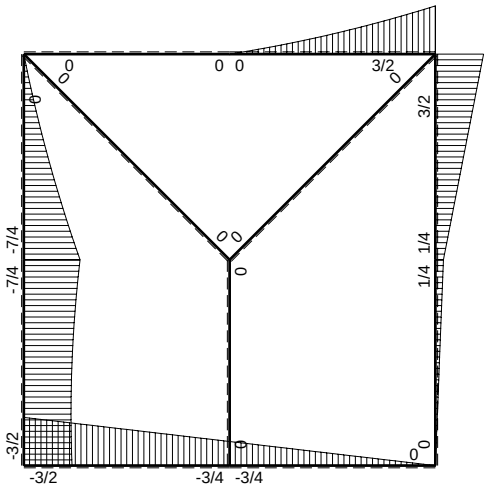


↑⊞↓ F

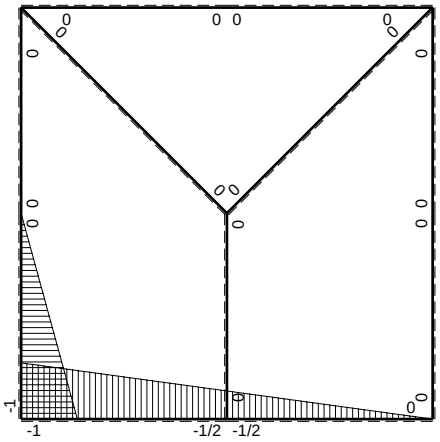


⟲⊞⟳ Fb





⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210204 ESΣ04.xxxx.145

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-3/2Fb+3/4Fx$	0	$3/2Fb^2-3/2Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$3/4Fb+3/4Fx$	0	$3/8Fb^2+3/4Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
CD b	0	$1/4Fx$	0	0	0	0	0+0	0
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0		
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	$-x$	$-7/4Fb+3/4Fx-1/2qx^2$	0	$7/4Fbx-3/4Fx^2+1/2qx^3$	0	x^2	$(3/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	$b-x$	$3/2Fb-1/4Fx+1/2qx^2$	0	$3/2Fb^2-7/4Fbx+3/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$3/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						$-3/4F$	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b (1 - x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - 1/2\ b + 1/12\ b\ )\ b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BA}^{XX} = \int_o^b (1/4 + 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/4\ b + 1/4\ b + 1/12\ b)\ b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BC}^{XX} = \int_o^b (1/4 - 1/2\ x/b + 1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/4\ b - 1/4\ b + 1/12\ b)\ b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{CB}^{XX} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{HA}^{XX} = \int_o^b (\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AH}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ b^2\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AB}^{Xo} = \int_o^b (3/2 - 3/2\ x/b + 3/8\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [3/2\ x - 3/4\ x^2/b + 1/8\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (3/2\ b - 3/4\ b + 1/8\ b)\ Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L_{BA}^{Xo} = \int_o^b (3/8 + 3/4\ x/b + 3/8\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [3/8\ x + 3/8\ x^2/b + 1/8\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (3/8\ b + 3/8\ b + 1/8\ b)\ Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L_{BC}^{Xo} = \int_o^b (3/8 - 3/4\ x/b + 3/8\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [3/8\ x - 3/8\ x^2/b + 1/8\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (3/8\ b - 3/8\ b + 1/8\ b)\ Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$

$L_{CB}^{Xo} = \int_o^b (3/8\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb^2\ 1/EJ$

$= (1/8\ b)\ Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$

$L_{HA}^{Xo} = \int_o^b (7/4\ x/b - 3/4\ x^2/b^2 + 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx = [7/8\ x^2/b - 1/4\ x^3/b^2 + 1/8\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ$

$= (7/8\ b - 1/4\ b + 1/8\ b)\ Fb^2\ 1/EJ\ = 3/4\ \ Fb^3/EJ$

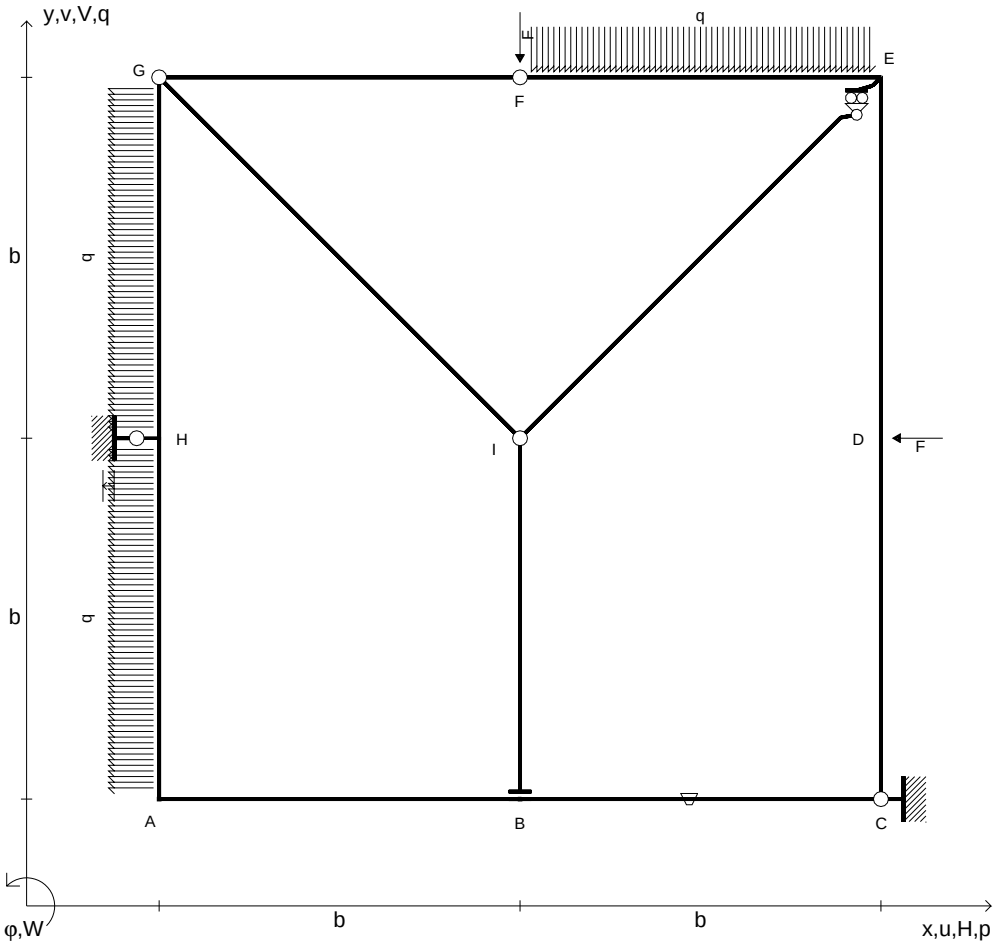
$L_{AH}^{Xo} = \int_o^b (3/2 - 7/4\ x/b + 3/4\ x^2/b^2 - 1/2\ x^3/b^3)\ Fb^2\ 1/EJ\ dx$

$= [3/2\ x - 7/8\ x^2/b + 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ ]_o^b\ Fb^2\ 1/EJ$

$= (3/2\ b - 7/8\ b + 1/4\ b - 1/8\ b)\ Fb^2\ 1/EJ\ = 3/4\ \ Fb^3/EJ$

ESΣ04.xxxx.145

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

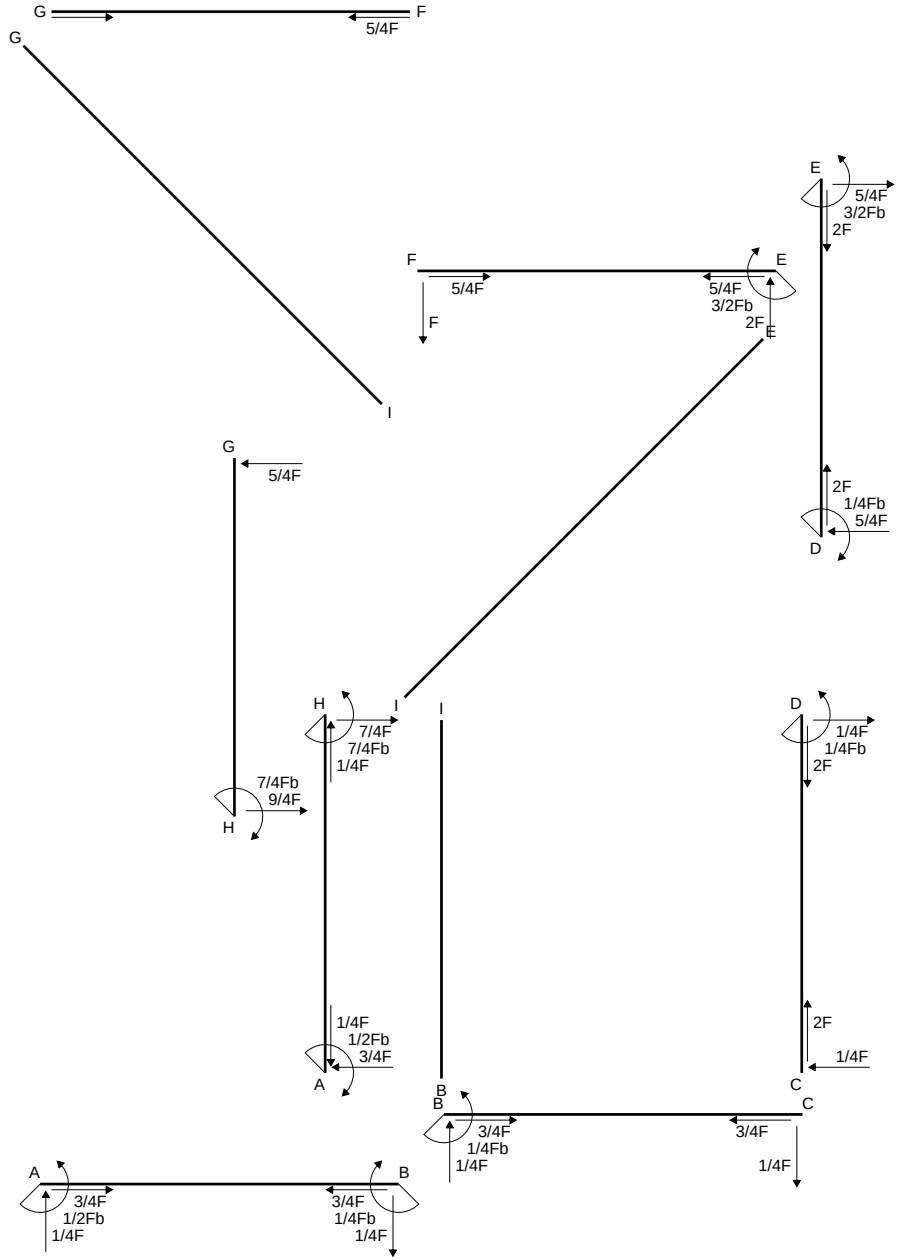


$V_F = -F$
$H_D = -F$
$p_{HA} = -q = -F/b$
$q_{EF} = -q = -F/b$
$p_{GH} = -q = -F/b$
$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$

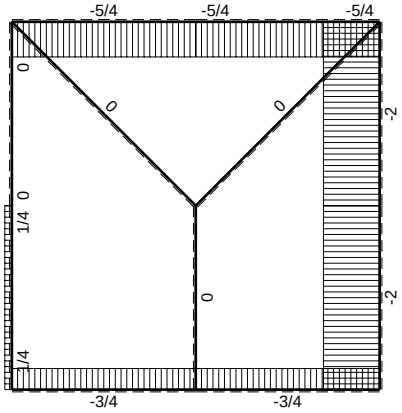
$u_H = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$

$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{GI} = EJ$
$EJ_{IB} = EJ$
$EJ_{IE} = EJ$
$EJ_{HA} = EJ$

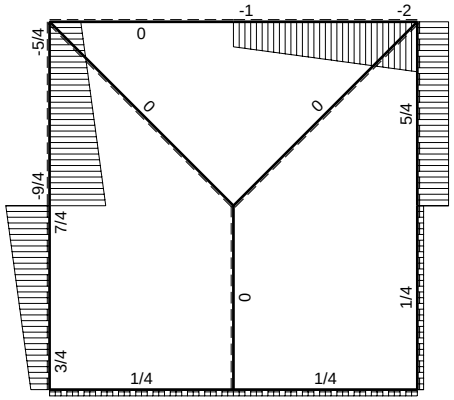
Reazioni iperstatiche in soluzione: $X=W_{AB}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo H.



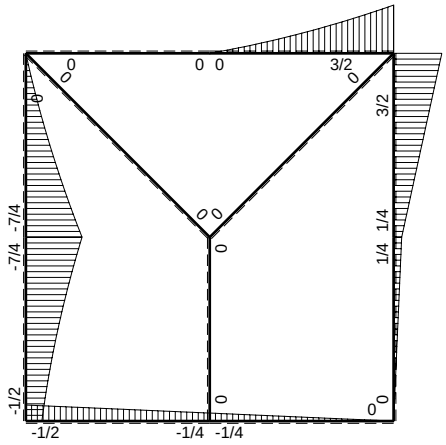
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13
08.01.25



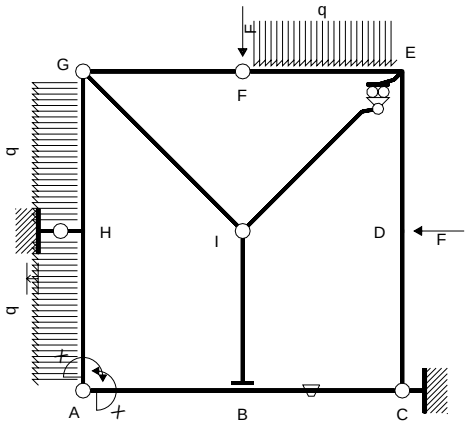
←□→ F

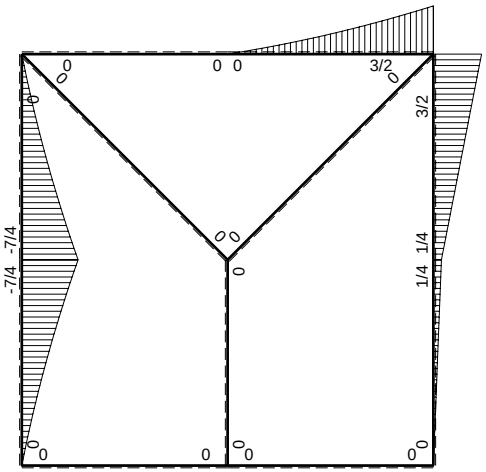


↑□↓ F

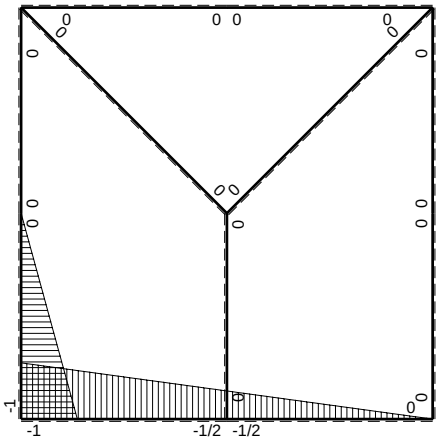


□ Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	7/12Xb/EJ
BA b	1/2+1/2x/b	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	0	-Fb/EJ	0	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(0+1/4)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	0	Fb/EJ	0	1/2Fx/EJ	$1/4x^2/b^2$		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						1/2Fb	

PROCEDIMENTO E RISULTATI 981630 ESΣ04.xxxx.147

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/2 - 1/2\ x/b\ \right)\theta\ \ dx = \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(1/2\ b - 1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-1/2\ x/b\ \right)\theta\ \ dx = \left[-1/4\ x^2/b\ \right]_o^b\ \theta$

$= \left(-1/4\ b\ \right)\theta\ \ = 1/4\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(7/4\ x/b - 9/4\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 3/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

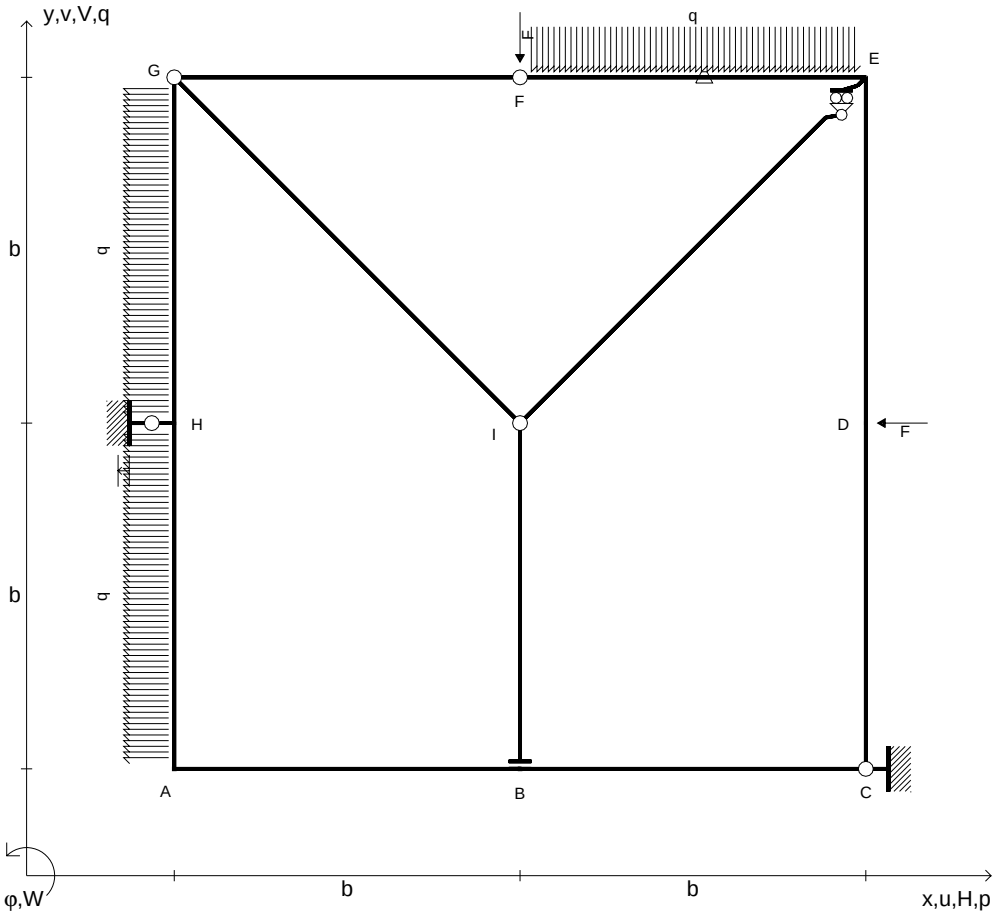
$= \left(7/8\ b - 3/4\ b + 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{AH}^{Xo} = \int_o^b \left(5/4\ x/b - 3/4\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb\ 1/EJ\ dx = \left[5/8\ x^2/b - 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(5/8\ b - 1/4\ b - 1/8\ b\ \right) Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

ESΣ04.xxxx.147

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

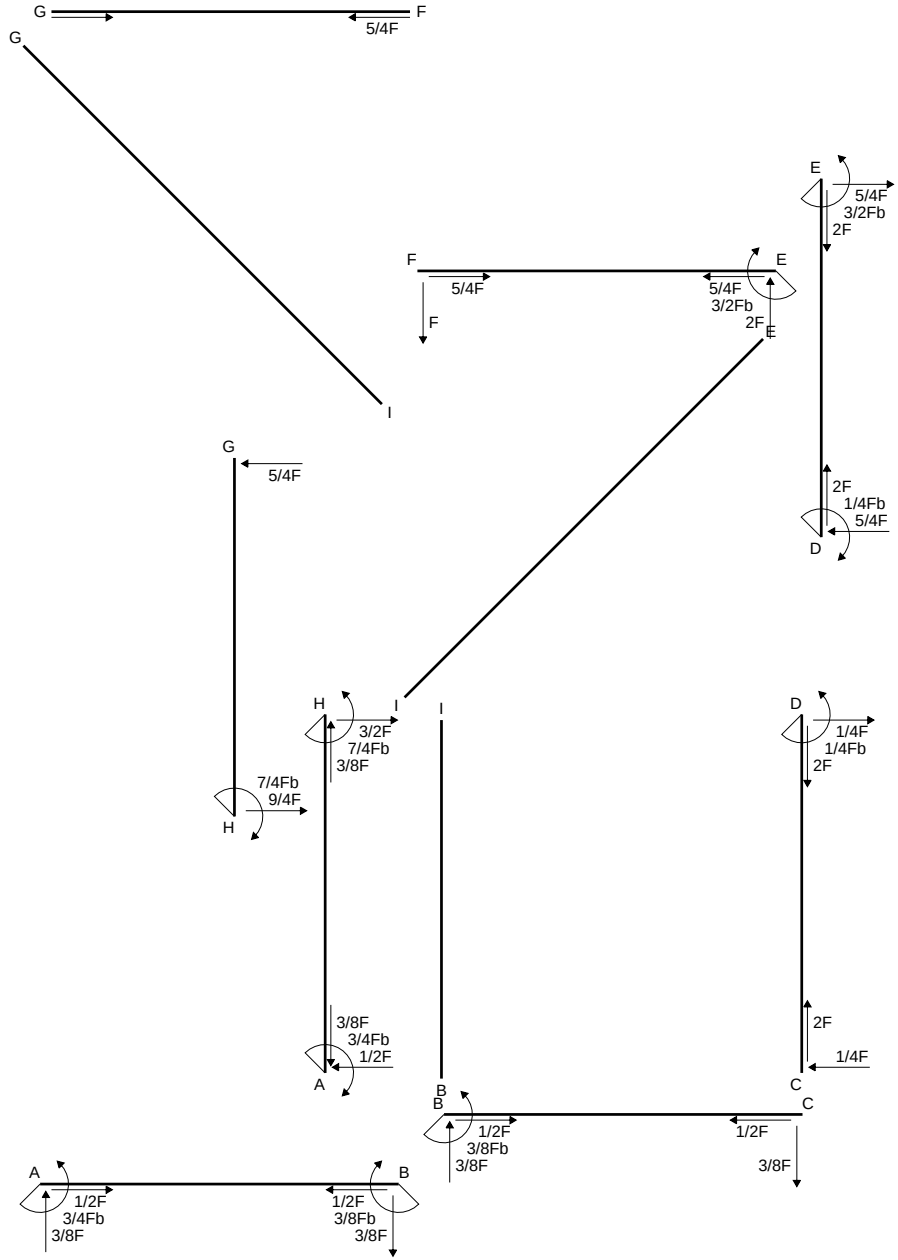
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

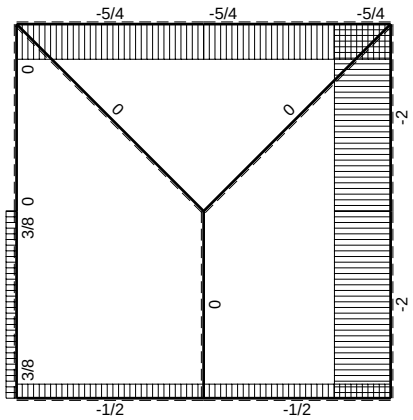
Curvatura θ asta EF positiva se convessa a destra con inizio E.

Spostamento orizzontale assoluto u imposto al nodo H.

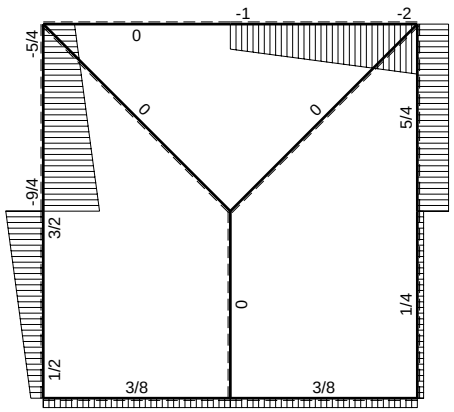
 08.01.25



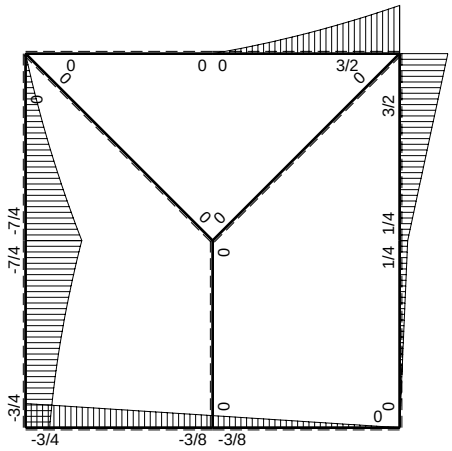
 08.01.25



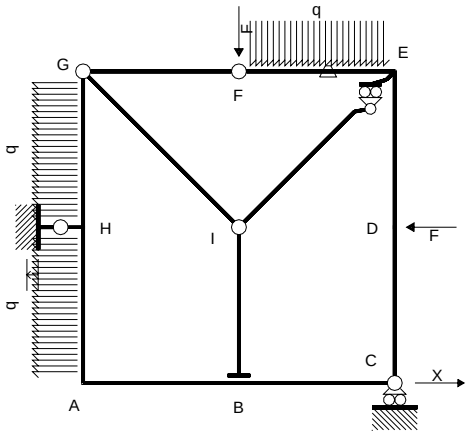
←□→ F

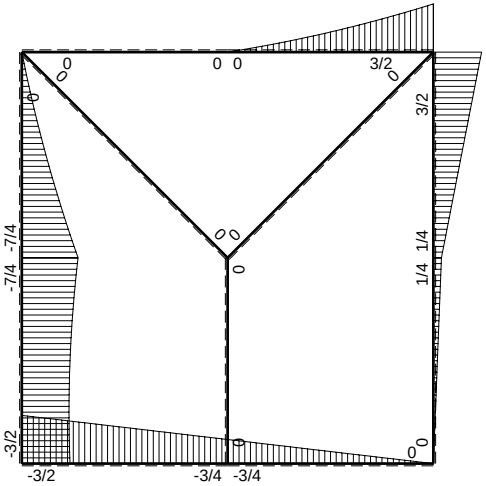


↑□↓ F

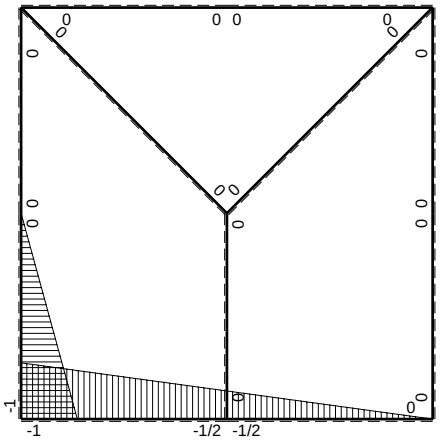


□ Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212409 ESΣ04.xxxx.151

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-3/2Fb+3/4Fx$	0	$3/2Fb^2-3/2Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$3/4Fb+3/4Fx$	0	$3/8Fb^2+3/4Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
CD b	0	$1/4Fx$	0	0	0	0	0+0	0
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0		
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x	$-7/4Fb+3/4Fx-1/2qx^2$	0	$7/4Fbx-3/4Fx^2+1/2qx^3$	0	x^2	$(3/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$3/2Fb-1/4Fx+1/2qx^2$	0	$3/2Fb^2-7/4Fbx+3/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$3/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-3/4F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2 - 3/2\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/2\ x - 3/4\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/2\ b - 3/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(3/8 + 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x + 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/8\ b + 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(3/8 - 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x - 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/8\ b - 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(7/4\ x/b - 3/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[7/8\ x^2/b - 1/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(7/8\ b - 1/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 3/4\ \ Fb^3/EJ$

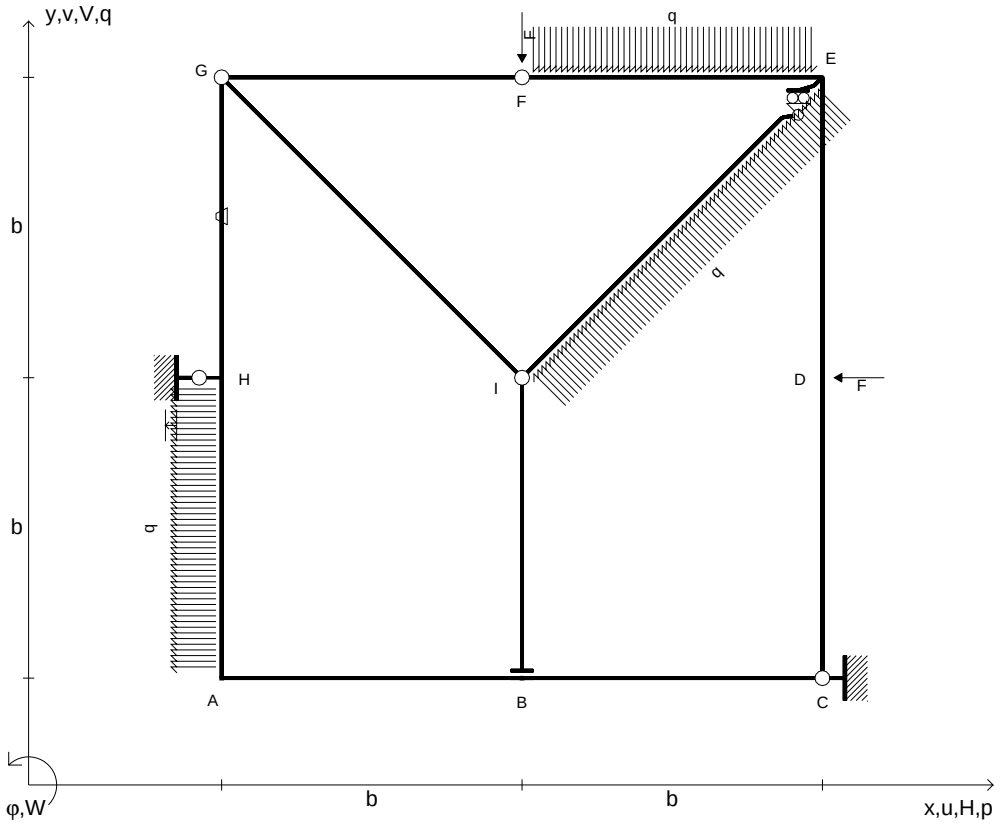
$L^{Xo}_{AH} = \int_o^b \left(3/2 - 7/4\ x/b + 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx$

$= \left[3/2\ x - 7/8\ x^2/b + 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/2\ b - 7/8\ b + 1/4\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 3/4\ \ Fb^3/EJ$

ESΣ04.xxxx.151

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



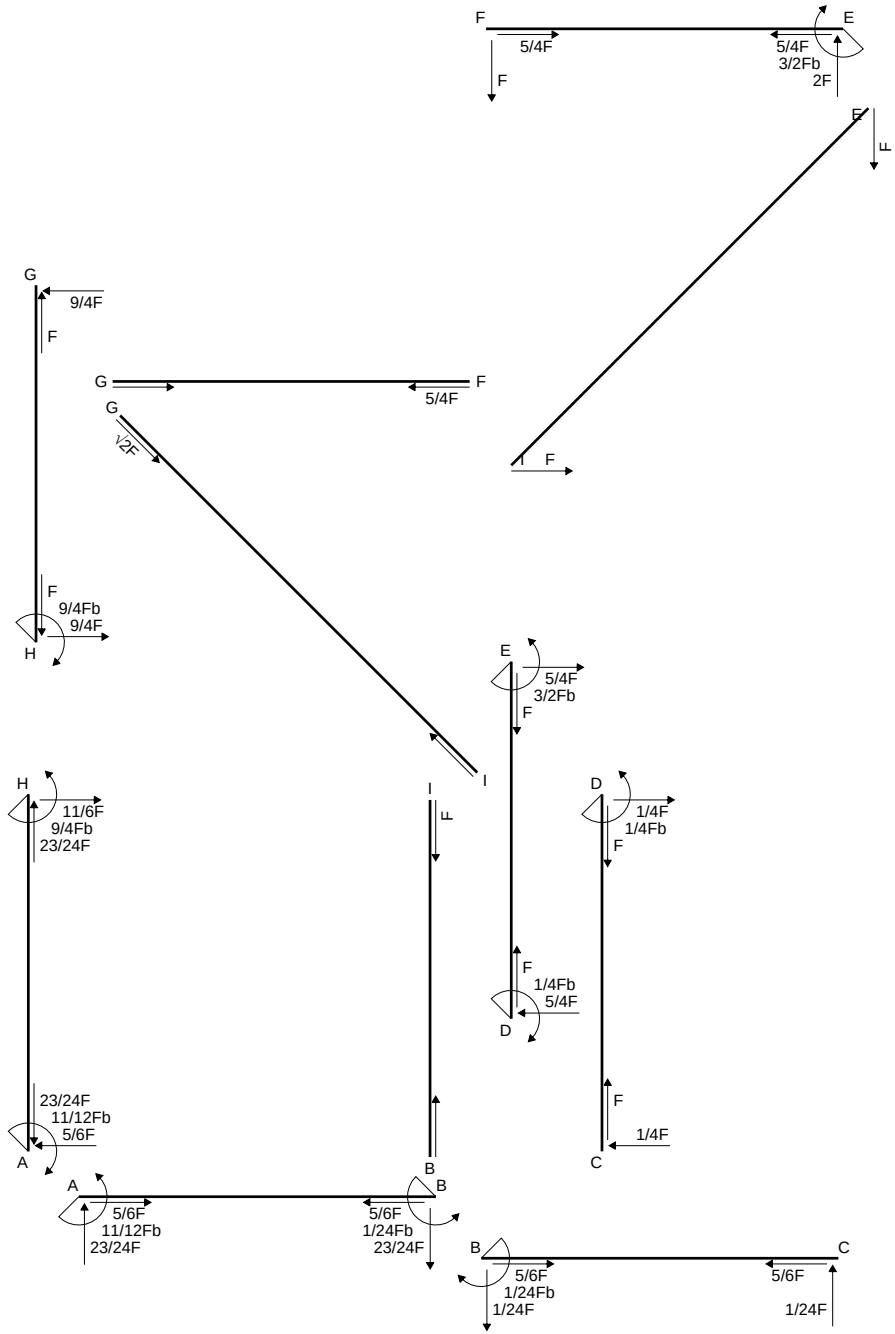
$V_F = -F$
$H_D = -F$
$p_{HA} = -q = -F/b$
$q_{EF} = -q = -F/b$
$p_{IE} = -q = -F/b$
$q_{IE} = q = F/b$
$\theta_{GH} = -\theta = -\alpha T/b = -bF/EJ$

$u_H = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$

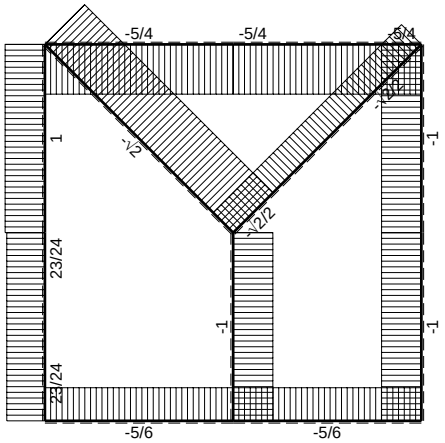
$EJ_{GH} = EJ$
$EJ_{GI} = EJ$
$EJ_{IB} = EJ$
$EJ_{IE} = EJ$
$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta GH positiva se convessa a destra con inizio G.
Spostamento orizzontale assoluto u imposto al nodo H.

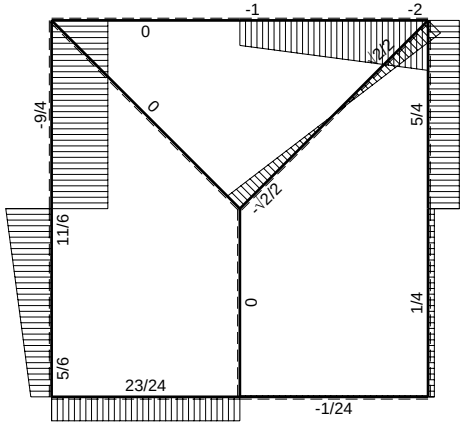
 08.01.25



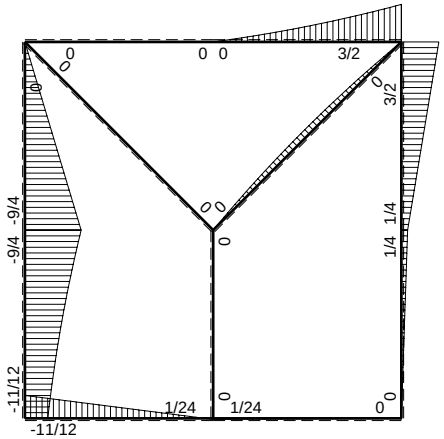
 08.01.25



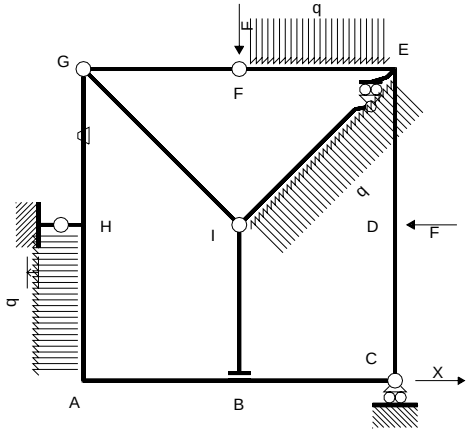
←F→

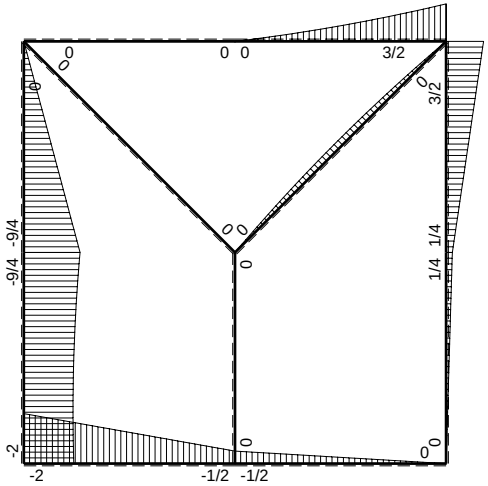


↑F↓

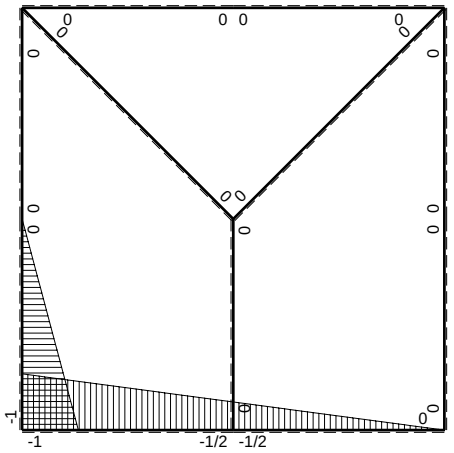


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212046 ESΣ04.xxxx.174

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-2Fb+3/2Fx$	0	$2Fb^2-5/2Fbx+3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$1/2Fb+3/2Fx$	0	$1/4Fb^2+Fbx+3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
CD b	0	$1/4Fx$	0	0	0	0	0+0	0
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0		
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-9/4Fx$	$-Fb/EJ$	0	0	0	0+0	0
HG b	0	$9/4Fb-9/4Fx$	Fb/EJ	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	$-x$	$-9/4Fb+3/4Fx-1/2qx^2$	0	$9/4Fbx-3/4Fx^2+1/2qx^3$	0	x^2	$(1+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$2Fb-1/4Fx+1/2qx^2$	0	$2Fb^2-9/4Fbx+3/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$13/12Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						$-13/12F$	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AB}^{Xo} = \int_o^b \left(2 - 5/2\ x/b + 3/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[2\ x - 5/4\ x^2/b + 1/4\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(2\ b - 5/4\ b + 1/4\ b\ \right) Fb^2\ 1/EJ\ = \ Fb^3/EJ$

$L_{BA}^{Xo} = \int_o^b \left(1/4 + x/b + 3/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/4\ x + 1/2\ x^2/b + 1/4\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/4\ b + 1/2\ b + 1/4\ b\ \right) Fb^2\ 1/EJ\ = \ Fb^3/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

$L_{CB}^{Xo} = \int_o^b \left(1/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/12\ b\ \right) Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 3/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[9/8\ x^2/b - 1/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

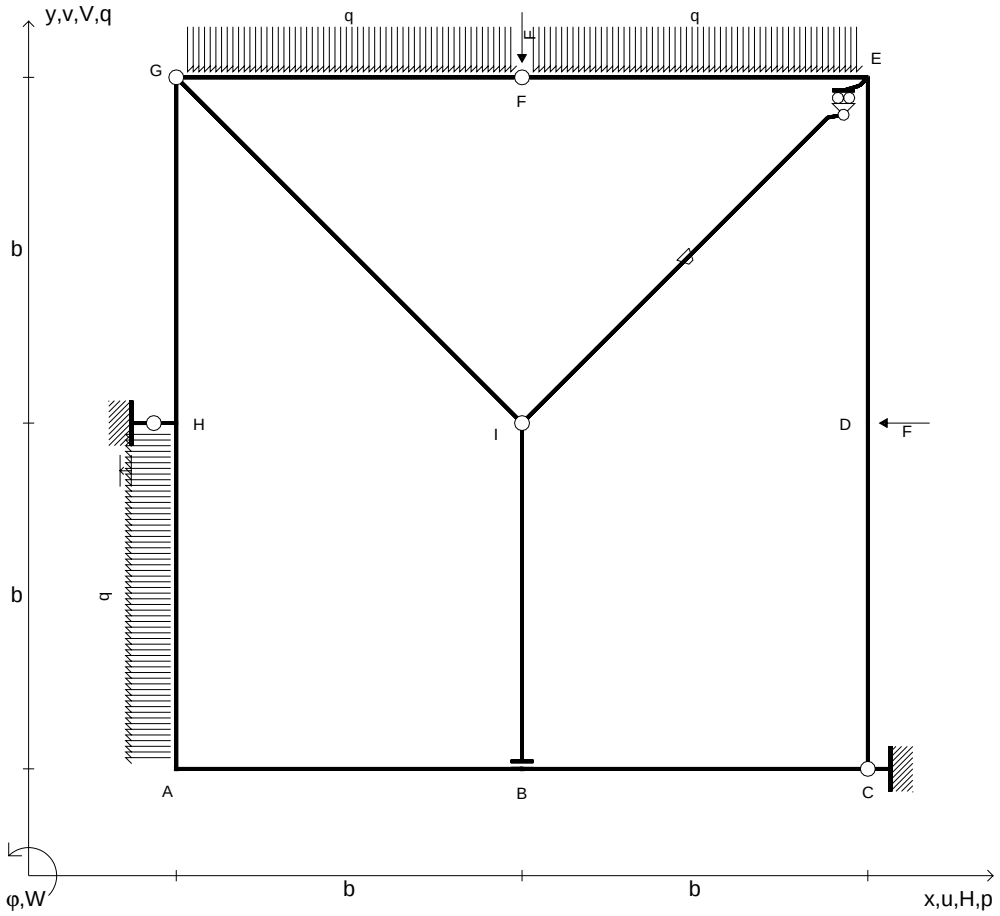
$= \left(9/8\ b - 1/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = \ Fb^3/EJ$

$L_{AH}^{Xo} = \int_o^b \left(2 - 9/4\ x/b + 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[2\ x - 9/8\ x^2/b + 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(2\ b - 9/8\ b + 1/4\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = \ Fb^3/EJ$

ESΣ04.xxxx.174

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

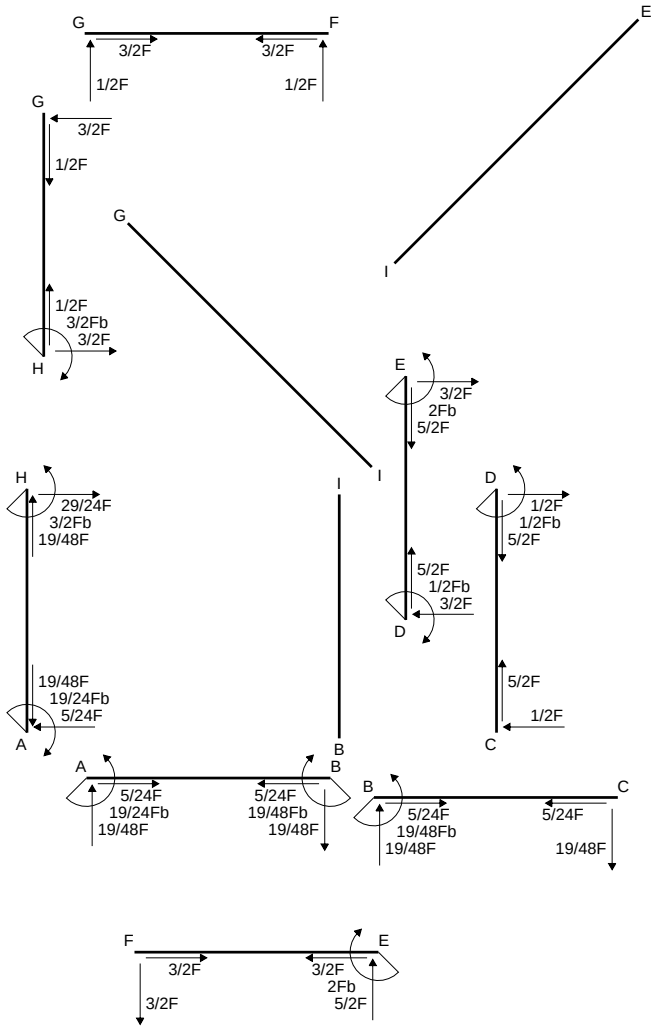


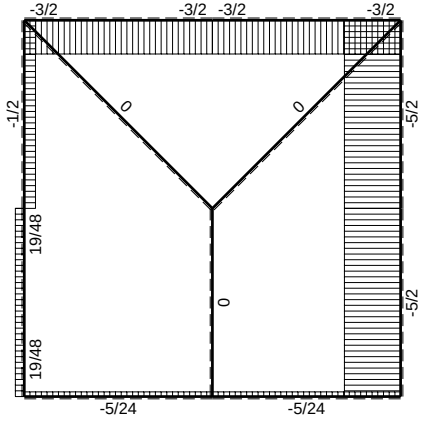
$V_F = -F$
$H_D = -F$
$p_{HA} = -q = -F/b$
$q_{EF} = -q = -F/b$
$q_{FG} = -q = -F/b$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$

$u_H = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$

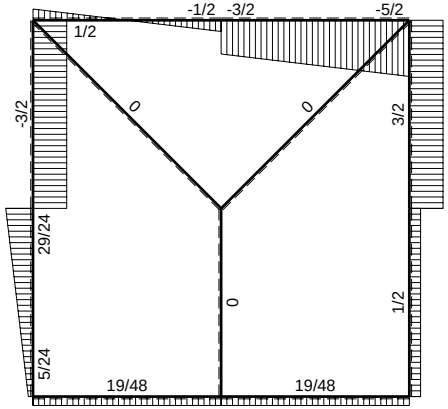
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{GI} = EJ$
$EJ_{IB} = EJ$
$EJ_{IE} = EJ$
$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta IE positiva se convessa a destra con inizio I.
Spostamento orizzontale assoluto u imposto al nodo H.

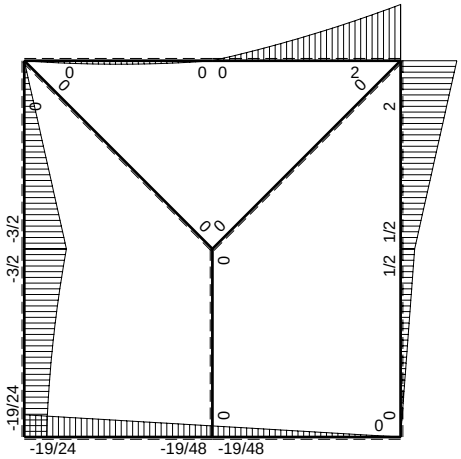




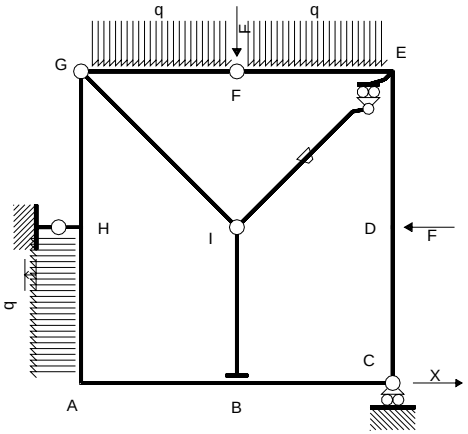
←F→

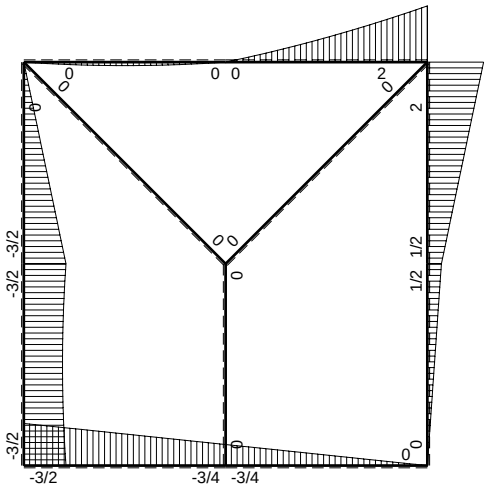


↑F↓

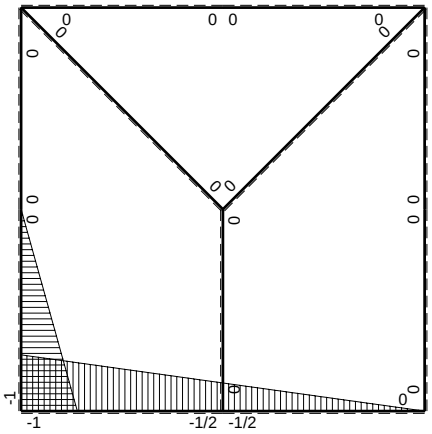


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

PROCEDIMENTO E RISULTATI 212317 ESΣ04.xxxx.189

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-3/2Fb+3/4Fx$	0	$3/2Fb^2-3/2Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$3/4Fb+3/4Fx$	0	$3/8Fb^2+3/4Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
CD b	0	$1/2Fx$	0	0	0	0	0+0	0
DC b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
DE b	0	$1/2Fb+3/2Fx$	0	0	0	0	0+0	0
ED b	0	$-2Fb+3/2Fx$	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-3/2Fx$	0	0	0	0	0+0	0
HG b	0	$3/2Fb-3/2Fx$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
HA b	-x	$-3/2Fb+1/2Fx-1/2qx^2$	0	$3/2Fbx-1/2Fx^2+1/2qx^3$	0	x^2	$(17/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$3/2Fb-1/2Fx+1/2qx^2$	0	$3/2Fb^2-2Fbx+Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$17/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-17/24F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AB}^{Xo} = \int_o^b \left(3/2 - 3/2\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/2\ x - 3/4\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/2\ b - 3/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L_{BA}^{Xo} = \int_o^b \left(3/8 + 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x + 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/8\ b + 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L_{BC}^{Xo} = \int_o^b \left(3/8 - 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x - 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/8\ b - 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$

$L_{CB}^{Xo} = \int_o^b \left(3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$

$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 1/2\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[3/4\ x^2/b - 1/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

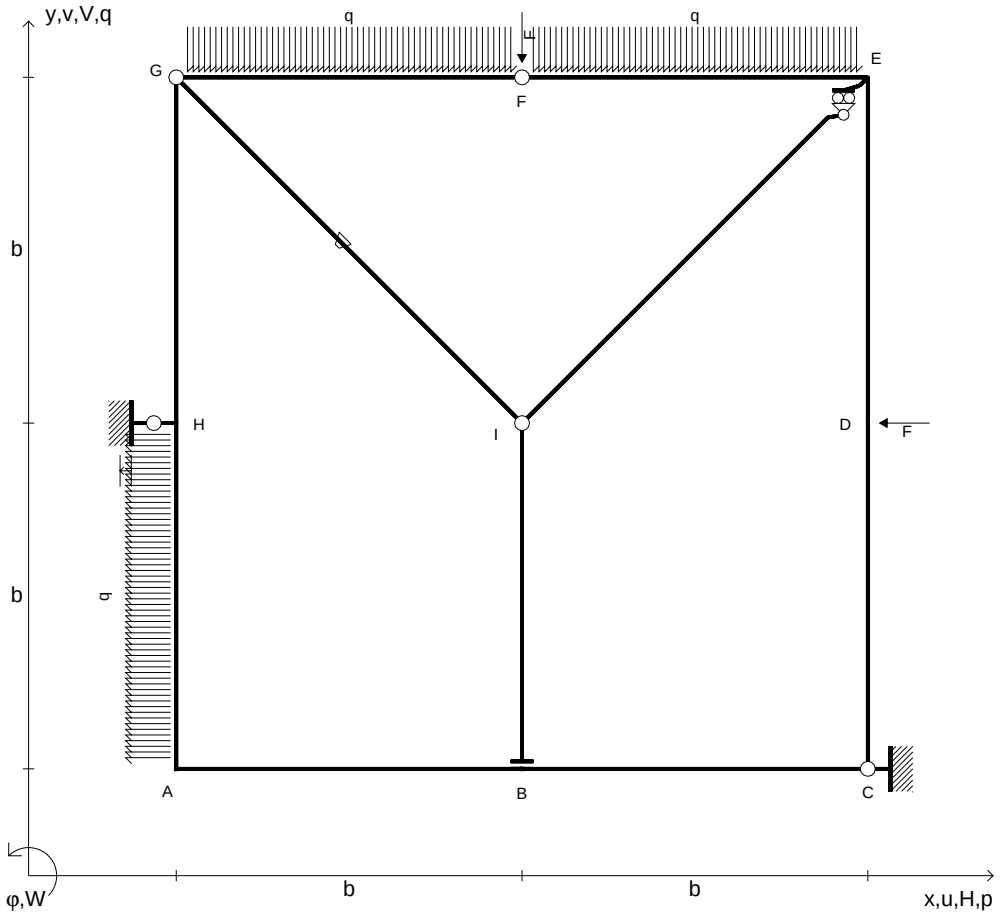
$= \left(3/4\ b - 1/6\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 17/24\ \ Fb^3/EJ$

$L_{AH}^{Xo} = \int_o^b \left(3/2 - 2\ x/b + x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[3/2\ x - x^2/b + 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/2\ b - b + 1/3\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 17/24\ \ Fb^3/EJ$

ESΣ04.xxxx.189

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

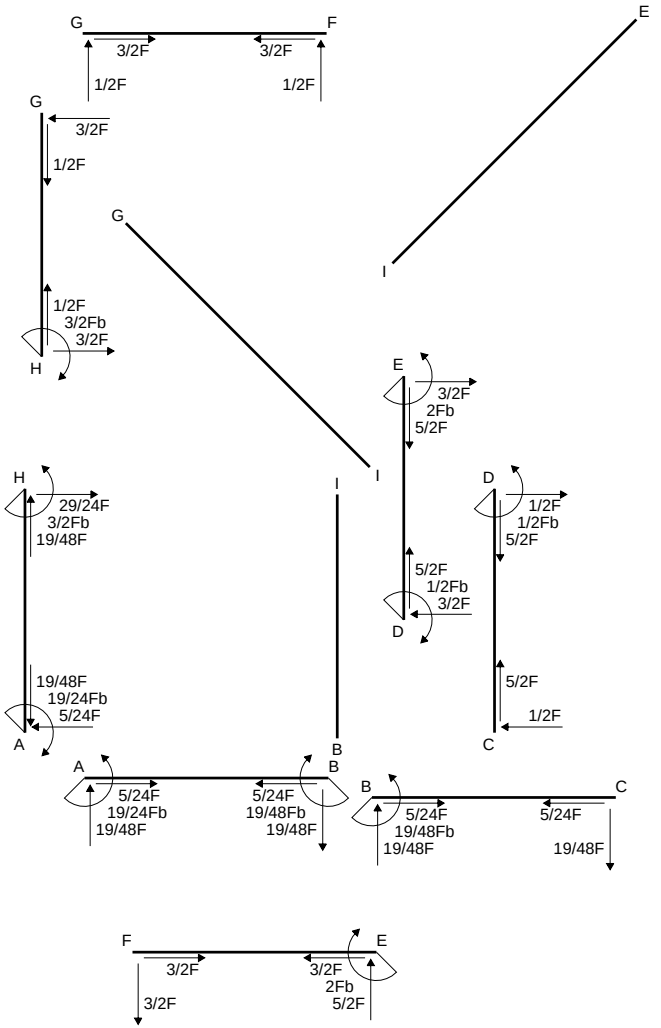


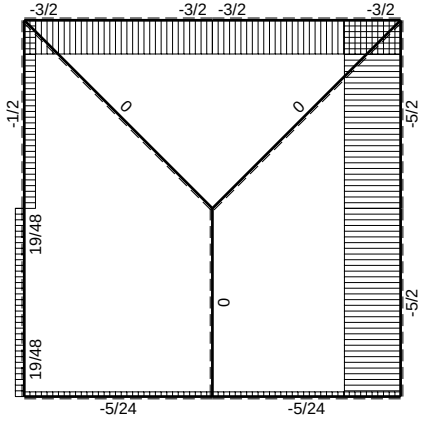
$V_F = -F$
$H_D = -F$
$p_{HA} = -q = -F/b$
$q_{EF} = -q = -F/b$
$q_{FG} = -q = -F/b$
$\theta_{GI} = -\theta = -\alpha T/b = -bF/EJ$

$u_H = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$

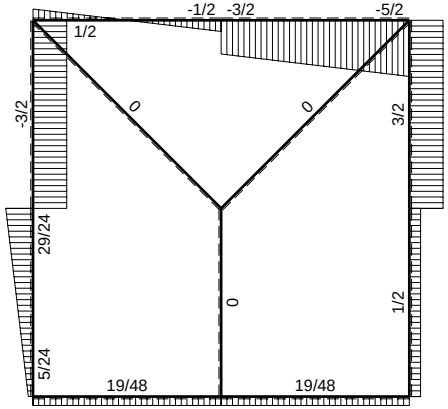
$EJ_{FG} = EJ$
$EJ_{GH} = EJ$
$EJ_{GI} = EJ$
$EJ_{IB} = EJ$
$EJ_{IE} = EJ$
$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta GI positiva se convessa a destra con inizio G.
Spostamento orizzontale assoluto u imposto al nodo H.

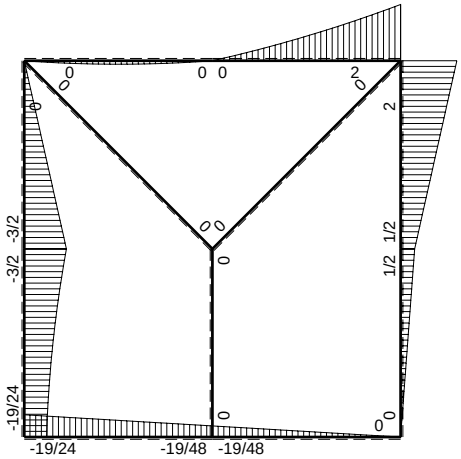




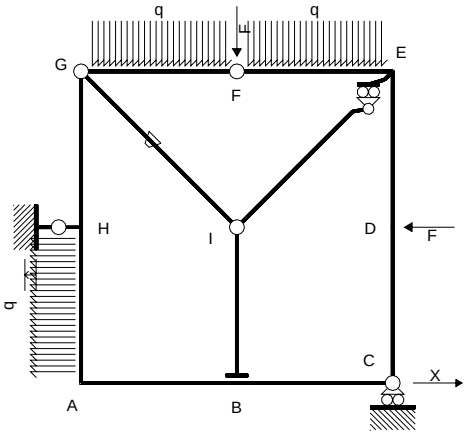
←F→

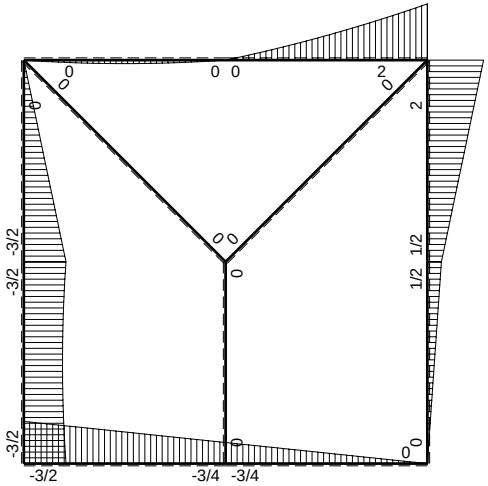


↑F↓

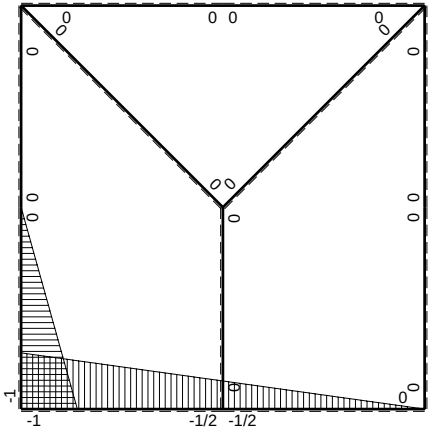


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211486 ESΣ04.xxxx.190

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-3/2Fb+3/4Fx$	0	$3/2Fb^2-3/2Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$3/4Fb+3/4Fx$	0	$3/8Fb^2+3/4Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
CD b	0	$1/2Fx$	0	0	0	0	0+0	0
DC b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
DE b	0	$1/2Fb+3/2Fx$	0	0	0	0	0+0	0
ED b	0	$-2Fb+3/2Fx$	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-3/2Fx$	0	0	0	0	0+0	0
HG b	0	$3/2Fb-3/2Fx$	0	0	0	0		
GI $\sqrt{2}$b	0	0	$-Fb/EJ$	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x	$-3/2Fb+1/2Fx-1/2qx^2$	0	$3/2Fbx-1/2Fx^2+1/2qx^3$	0	x^2	$(17/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$3/2Fb-1/2Fx+1/2qx^2$	0	$3/2Fb^2-2Fbx+Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$17/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-17/24F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/4\ b + 1/4\ b + 1/12\ b\right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/4\ b - 1/4\ b + 1/12\ b\right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/12\ b\right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/3\ b\right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(3/2 - 3/2\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/2\ x - 3/4\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(3/2\ b - 3/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(3/8 + 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x + 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(3/8\ b + 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(3/8 - 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x - 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(3/8\ b - 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$$

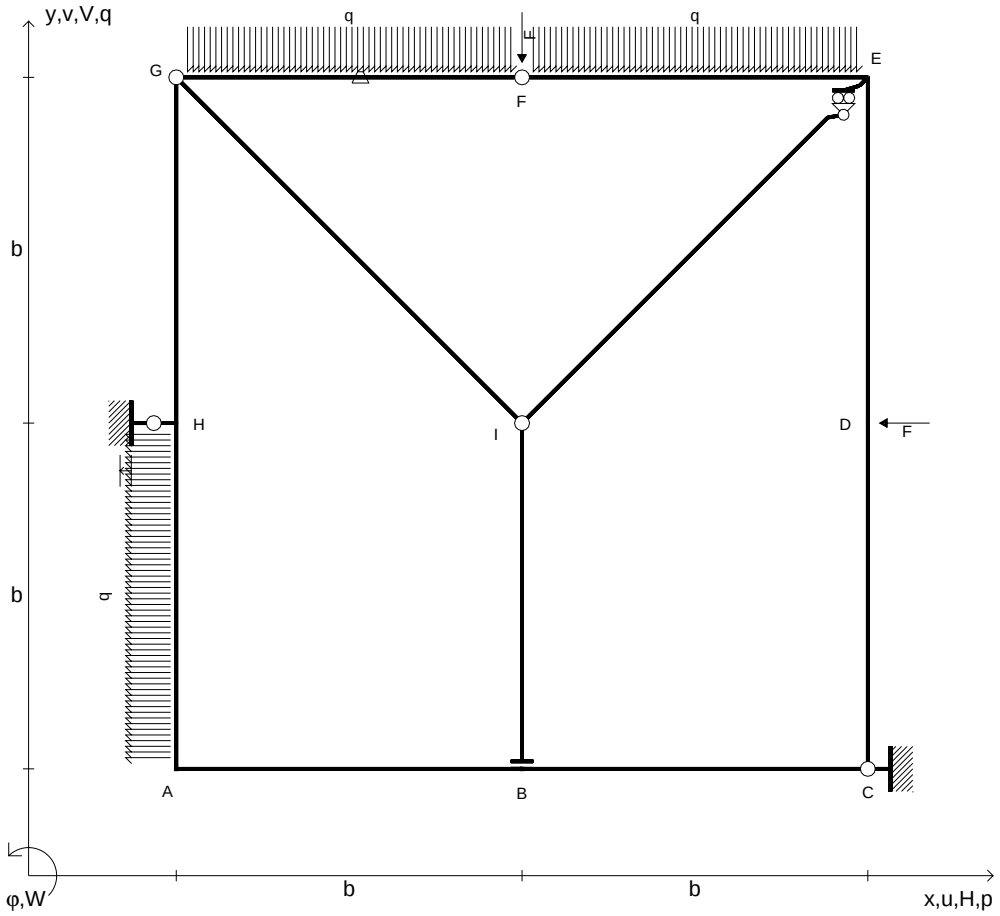
$$L_{CB}^{Xo} = \int_o^b \left(3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(1/8\ b\ \right) Fb^2\ 1/EJ\ = 1/8\ \ Fb^3/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(3/2\ x/b - 1/2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx = \left[3/4\ x^2/b - 1/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(3/4\ b - 1/6\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 17/24\ \ Fb^3/EJ$$

$$L_{AH}^{Xo} = \int_o^b \left(3/2 - 2\ x/b + x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx = \left[3/2\ x - x^2/b + 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(3/2\ b - b + 1/3\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 17/24\ \ Fb^3/EJ$$

ESΣ04.xxxx.190

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$q_{FG} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

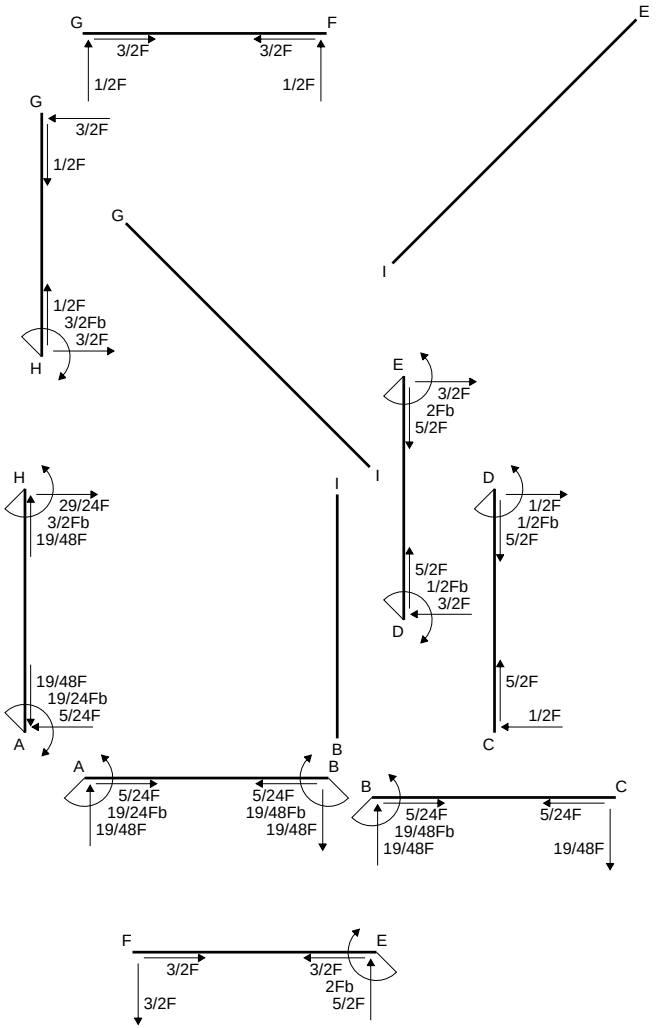
Tracciare i diagrammi quotati delle azioni interne nelle aste.

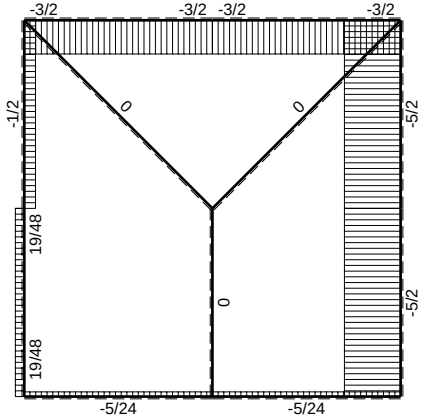
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

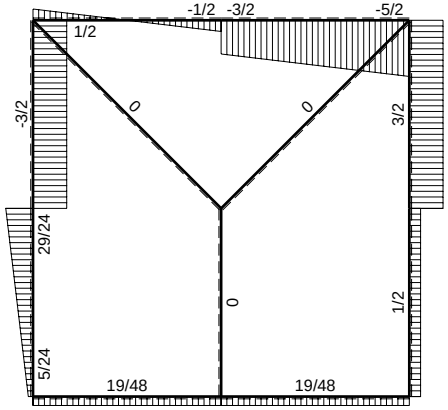
Curvatura θ asta FG positiva se convessa a destra con inizio F.

Spostamento orizzontale assoluto u imposto al nodo H.

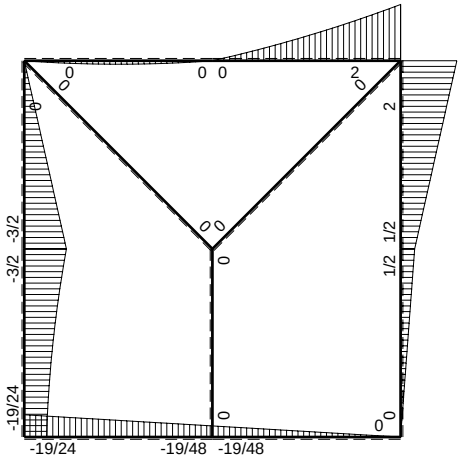




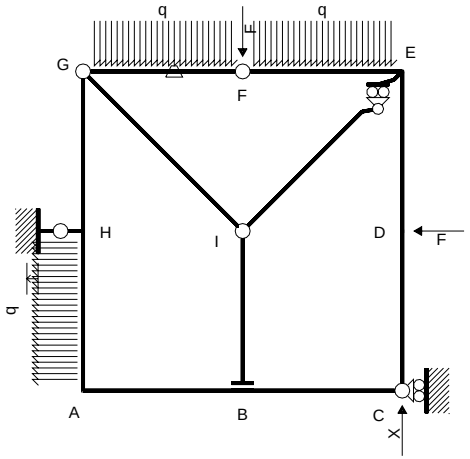
←F→



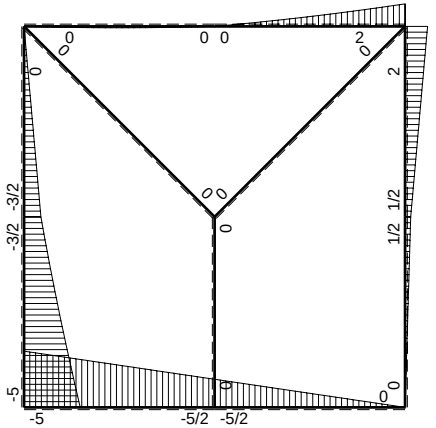
↑F↓



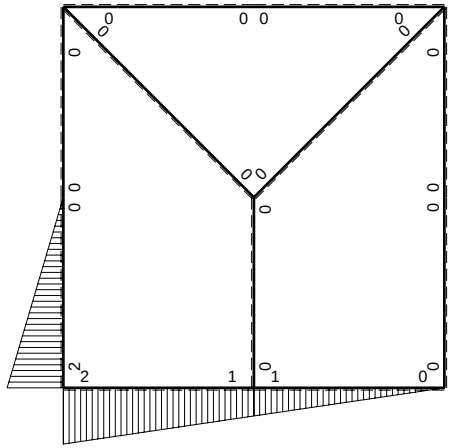
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	2b-x	-5Fb+5/2Fx	0	$-10Fb^2+10Fbx-5/2Fx^2$	0	$4b^2-4bx+x^2$	$(-35/6+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	-b-x	5/2Fb+5/2Fx	0	$-5/2Fb^2-5Fbx-5/2Fx^2$	0	$b^2+2bx+x^2$		
BC b	b-x	-5/2Fb+5/2Fx	0	$-5/2Fb^2+5Fbx-5/2Fx^2$	0	$b^2-2bx+x^2$	$(-5/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	-x	5/2Fx	0	$-5/2Fx^2$	0	x^2		
CD b	0	1/2Fx	0	0	0	0	0+0	0
DC b	0	-1/2Fb+1/2Fx	0	0	0	0		
DE b	0	1/2Fb+3/2Fx	0	0	0	0	0+0	0
ED b	0	-2Fb+3/2Fx	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	2x	$-3/2Fb-3Fx-1/2qx^2$	0	$-3Fbx-6Fx^2-qx^3$	0	$4x^2$	$(-15/4+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	-2b+2x	$5Fb-4Fx+1/2qx^2$	0	$-10Fb^2+18Fbx-9Fx^2+qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$2Fb^3/EJ$	
	totali						$-101/12Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_C$						101/48F	

PROCEDIMENTO E RISULTATI 981317 ESΣ04.xxxx.191

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-10 + 10\,x/b - 5/2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-10\,x + 5\,x^2/b - 5/6\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-10\,b + 5\,b - 5/6\,b)\, Fb^2\, 1/EJ = -35/6\; Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-5/2 - 5\,x/b - 5/2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-5/2\,x - 5/2\,x^2/b - 5/6\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-5/2\,b - 5/2\,b - 5/6\,b)\, Fb^2\, 1/EJ = -35/6\; Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-5/2 + 5\,x/b - 5/2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-5/2\,x + 5/2\,x^2/b - 5/6\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-5/2\,b + 5/2\,b - 5/6\,b)\, Fb^2\, 1/EJ = -5/6\; Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-5/2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-5/6\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-5/6\,b)\, Fb^2\, 1/EJ = -5/6\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (-3\,x/b - 6\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-3/2\,x^2/b - 2\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

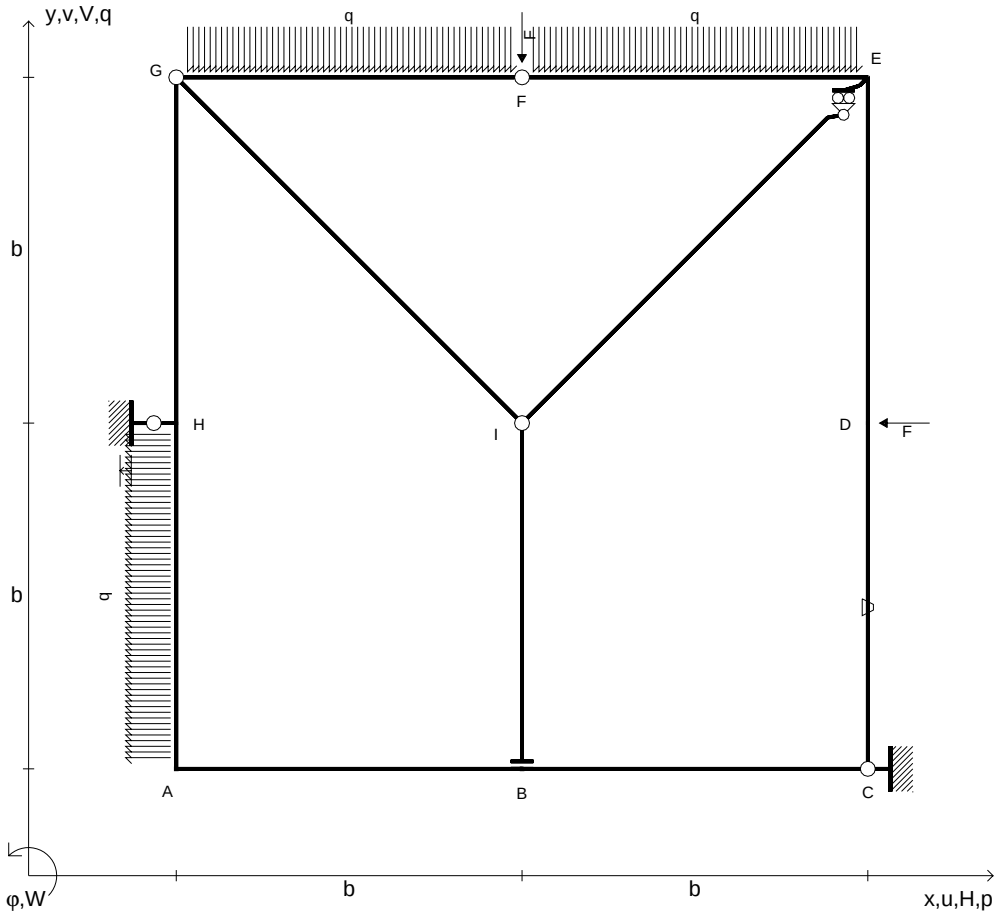
$= (-3/2\,b - 2\,b - 1/4\,b)\, Fb^2\, 1/EJ = -15/4\; Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (-10 + 18\,x/b - 9\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-10\,x + 9\,x^2/b - 3\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-10\,b + 9\,b - 3\,b + 1/4\,b)\, Fb^2\, 1/EJ = -15/4\; Fb^3/EJ$

ESΣ04.xxxx.191

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$q_{FG} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

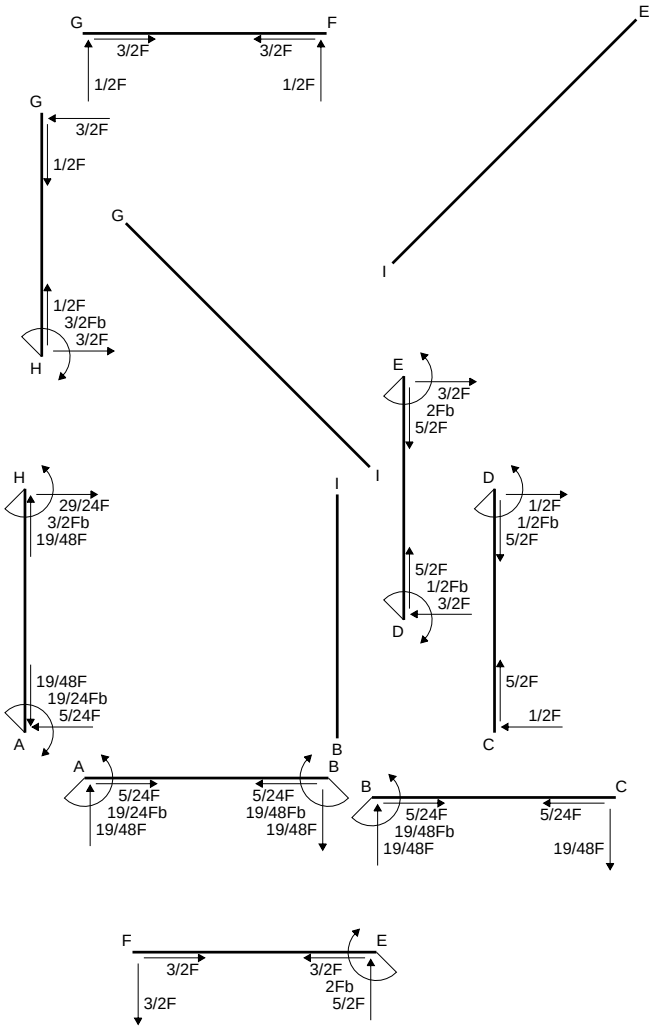
Tracciare i diagrammi quotati delle azioni interne nelle aste.

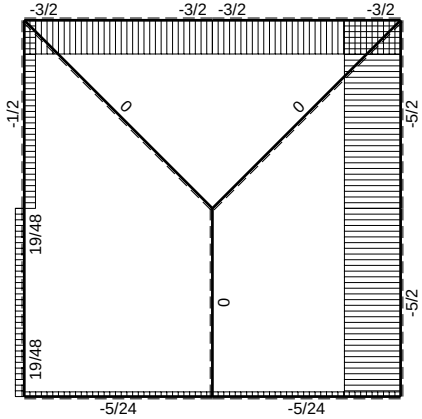
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

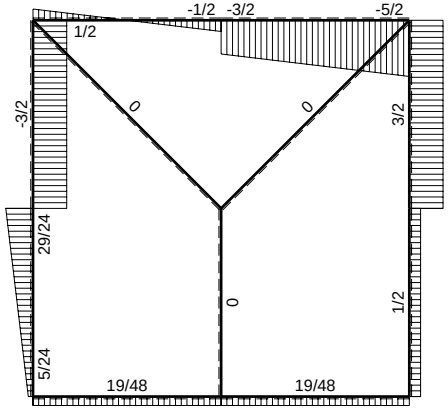
Curvatura θ asta CD positiva se convessa a destra con inizio C.

Spostamento orizzontale assoluto u imposto al nodo H.

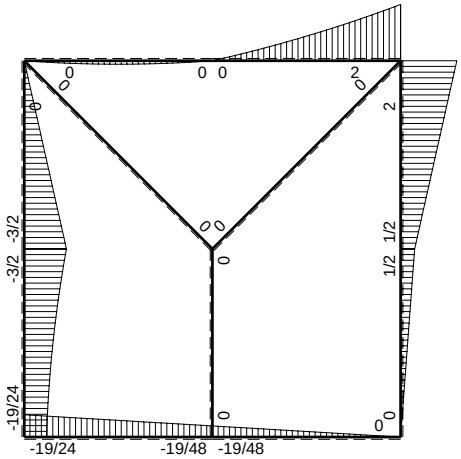




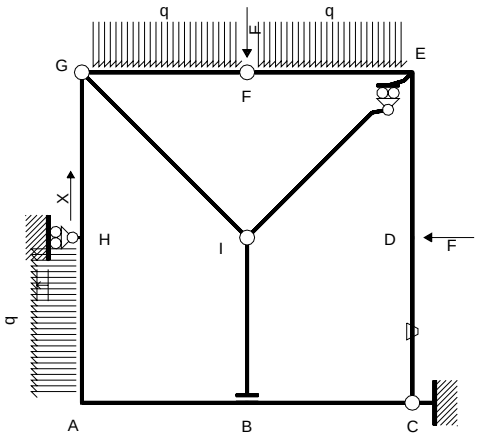
←F→

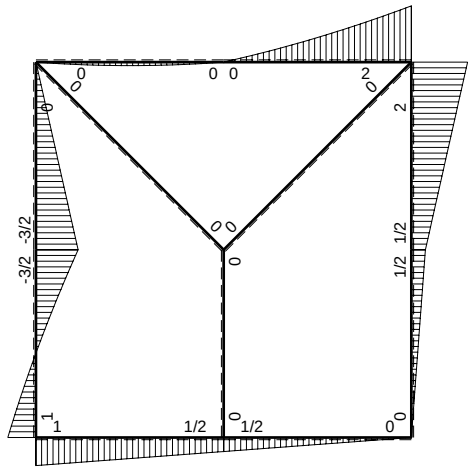


↑F↓

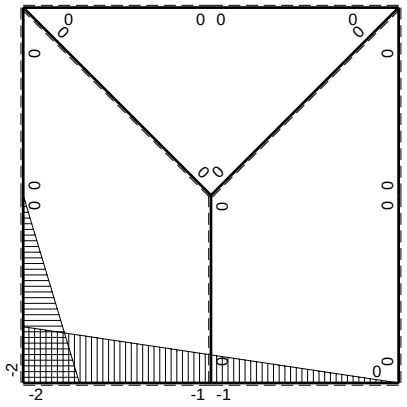


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-2b+x$	$Fb-1/2Fx$	0	$-2Fb^2+2Fbx-1/2Fx^2$	0	$4b^2-4bx+x^2$	$(-7/6+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	$b+x$	$-1/2Fb-1/2Fx$	0	$-1/2Fb^2-Fbx-1/2Fx^2$	0	$b^2+2bx+x^2$		
BC b	$-b+x$	$1/2Fb-1/2Fx$	0	$-1/2Fb^2+Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-1/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	x	$-1/2Fx$	0	$-1/2Fx^2$	0	x^2		
CD b	0	$1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
DC b	0	$-1/2Fb+1/2Fx$	Fb/EJ	0	0	0		
DE b	0	$1/2Fb+3/2Fx$	0	0	0	0	0+0	0
ED b	0	$-2Fb+3/2Fx$	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-3/2Fx$	0	0	0	0	0+0	0
HG b	0	$3/2Fb-3/2Fx$	0	0	0	0		
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0
HA b	$-2x$	$-3/2Fb+3Fx-1/2qx^2$	0	$3Fbx-6Fx^2+qx^3$	0	$4x^2$	$(-1/4+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	$2b-2x$	$-Fb+2Fx+1/2qx^2$	0	$-2Fb^2+6Fbx-3Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$-43/12Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						43/48F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 280320 ESΣ04.xxxx.198
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-2 + 2\,x/b - 1/2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\,x + x^2/b - 1/6\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2\,b + b - 1/6\,b)\, Fb^2\, 1/EJ = -7/6\; Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 - x/b - 1/2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\,x - 1/2\,x^2/b - 1/6\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\,b - 1/2\,b - 1/6\,b)\, Fb^2\, 1/EJ = -7/6\; Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 + x/b - 1/2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\,x + 1/2\,x^2/b - 1/6\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\,b + 1/2\,b - 1/6\,b)\, Fb^2\, 1/EJ = -1/6\; Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-1/2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/6\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/6\,b)\, Fb^2\, 1/EJ = -1/6\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (3\,x/b - 6\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [3/2\,x^2/b - 2\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

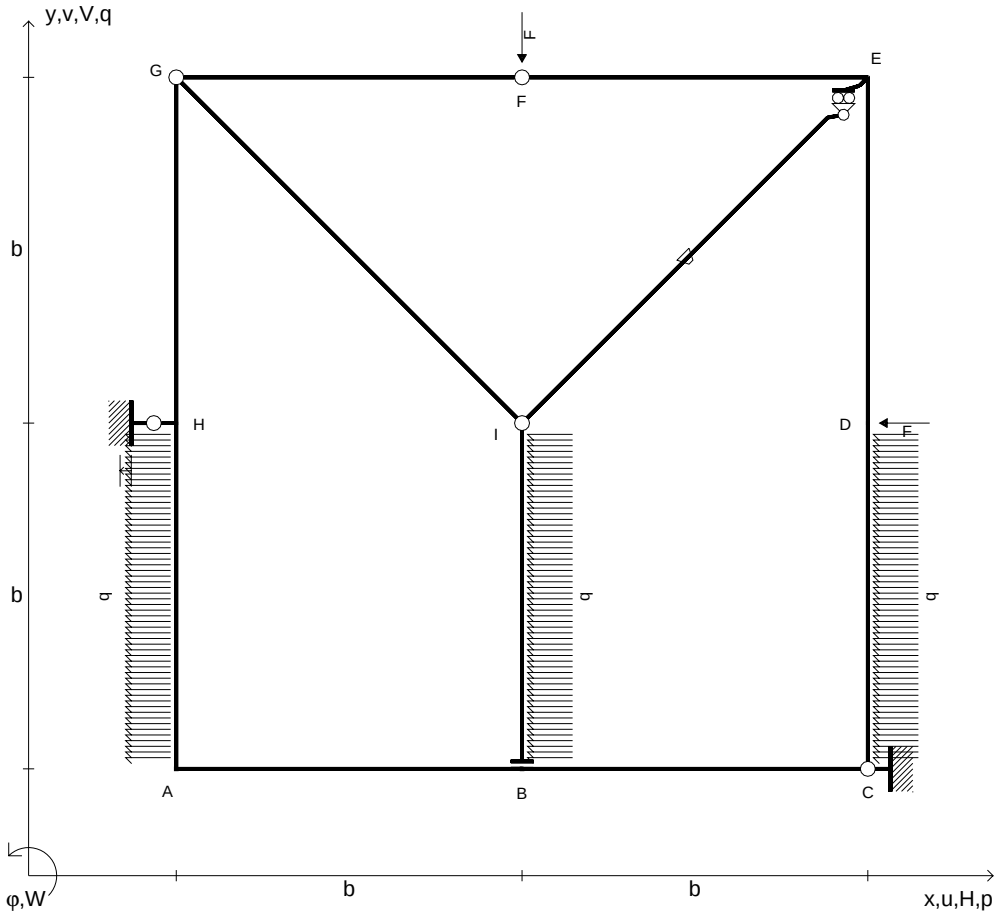
$= (3/2\,b - 2\,b + 1/4\,b)\, Fb^2\, 1/EJ = -1/4\; Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (-2 + 6\,x/b - 3\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-2\,x + 3\,x^2/b - x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-2\,b + 3\,b - b - 1/4\,b)\, Fb^2\, 1/EJ = -1/4\; Fb^3/EJ$

ESΣ04.xxxx.198

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{IB} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

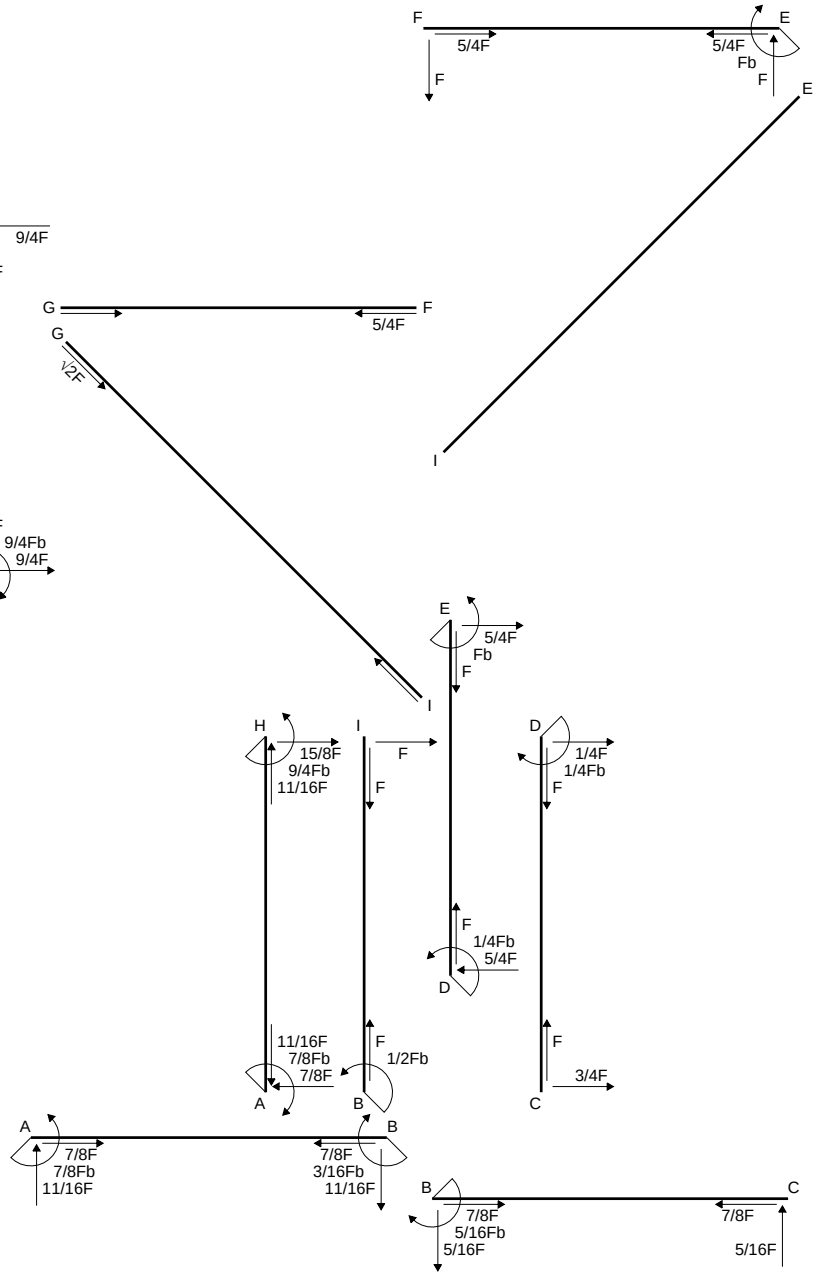
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

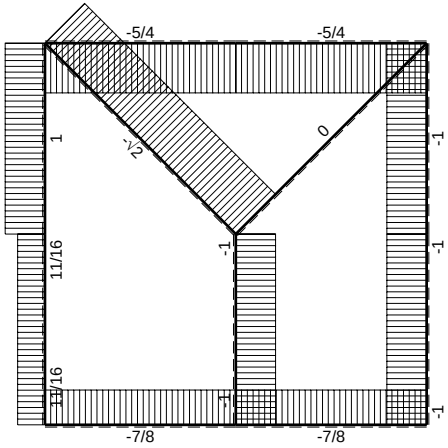
Curvatura θ asta IE positiva se convessa a destra con inizio I.

Spostamento orizzontale assoluto u imposto al nodo H.

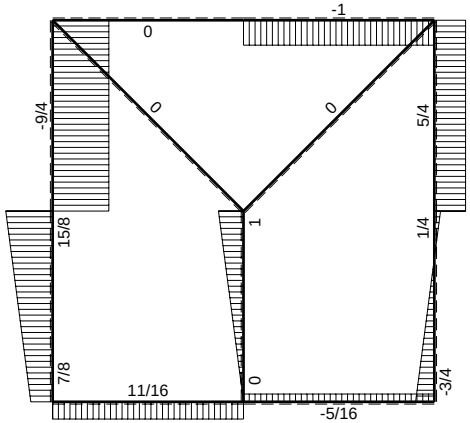
 08.01.25



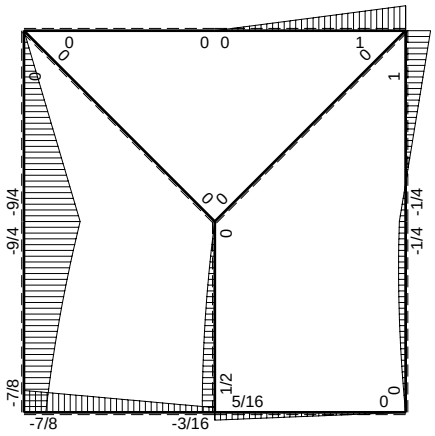
 08.01.25



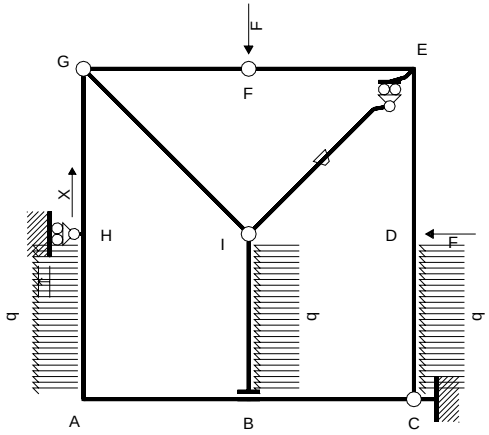
←[+]→ F

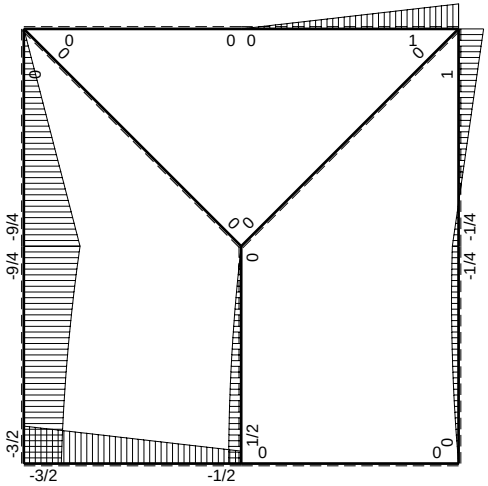


↑[+]↓ F

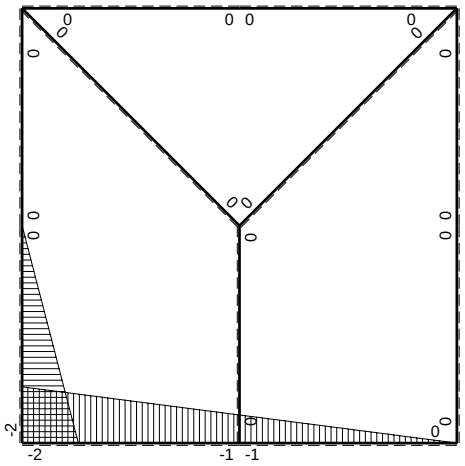


[+] Fb





[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-2b+x$	$-3/2Fb+Fx$	0	$3Fb^2-7/2Fbx+Fx^2$	0	$4b^2-4bx+x^2$	$(19/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	$b+x$	$1/2Fb+Fx$	0	$1/2Fb^2+3/2Fbx+Fx^2$	0	$b^2+2bx+x^2$		
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CB b	x	0	0	0	0	x^2		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-Fb+5/4Fx$	0	0	0	0		
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0
FE b	0	$-Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0		
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}b$	0	0	$-Fb/EJ$	0	0	0	0	0
HA b	$-2x$	$-9/4Fb+5/4Fx-1/2qx^2$	0	$9/2Fbx-5/2Fx^2+qx^3$	0	$4x^2$	$(5/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	$2b-2x$	$3/2Fb+1/4Fx+1/2qx^2$	0	$3Fb^2-5/2Fbx+1/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$5/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						$-5/16F$	

PROCEDIMENTO E RISULTATI 215530 ESΣ04.xxxx.200

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\ b^3/EJ$

$L_{BA}^{XX} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\ b^3/EJ$

$L_{BC}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CB}^{XX} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HA}^{XX} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L_{AH}^{XX} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L_{AB}^{Xo} = \int_o^b (3 - 7/2\,x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3\,x - 7/4\,x^2/b + 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3\,b - 7/4\,b + 1/3\,b)\, Fb^2\, 1/EJ = 19/12\ Fb^3/EJ$

$L_{BA}^{Xo} = \int_o^b (1/2 + 3/2\,x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\,x + 3/4\,x^2/b + 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\,b + 3/4\,b + 1/3\,b)\, Fb^2\, 1/EJ = 19/12\ Fb^3/EJ$

$L_{HA}^{Xo} = \int_o^b (9/2\,x/b - 5/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [9/4\,x^2/b - 5/6\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

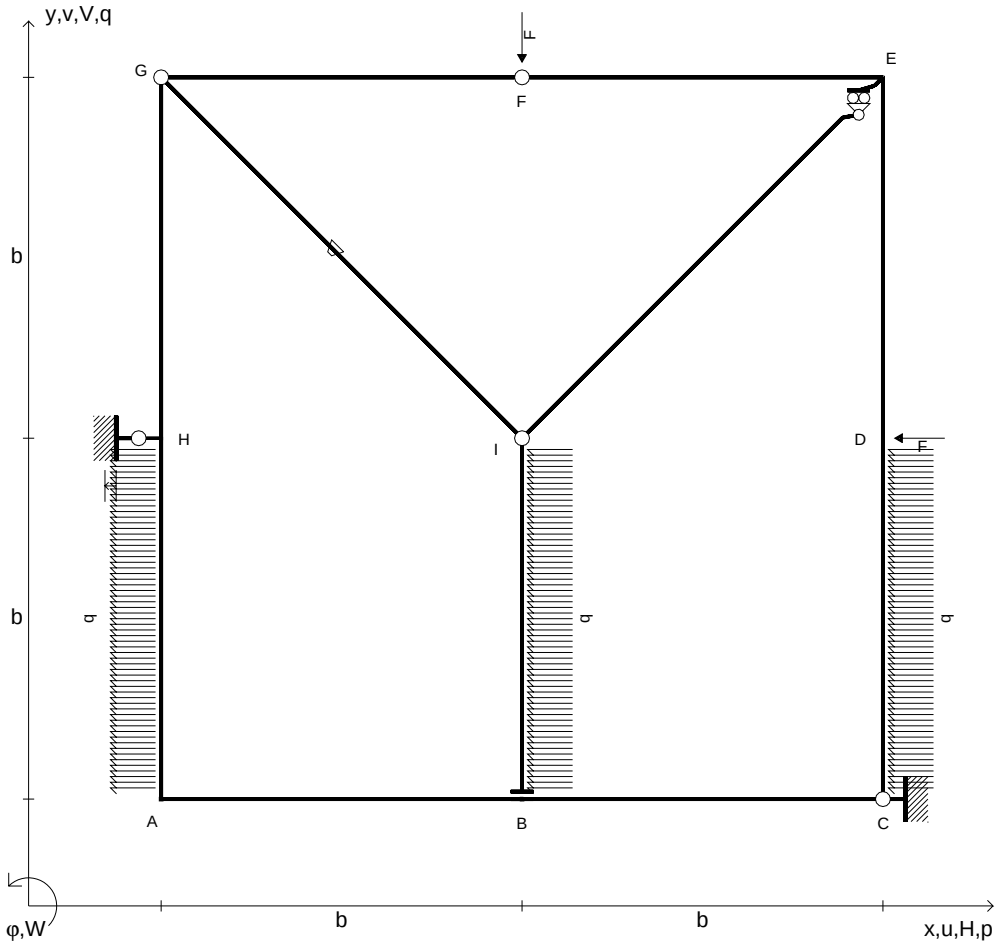
$= (9/4\,b - 5/6\,b + 1/4\,b)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

$L_{AH}^{Xo} = \int_o^b (3 - 5/2\,x/b + 1/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [3\,x - 5/4\,x^2/b + 1/6\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (3\,b - 5/4\,b + 1/6\,b - 1/4\,b)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

ESΣ04.xxxx.200

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{IB} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{GI} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

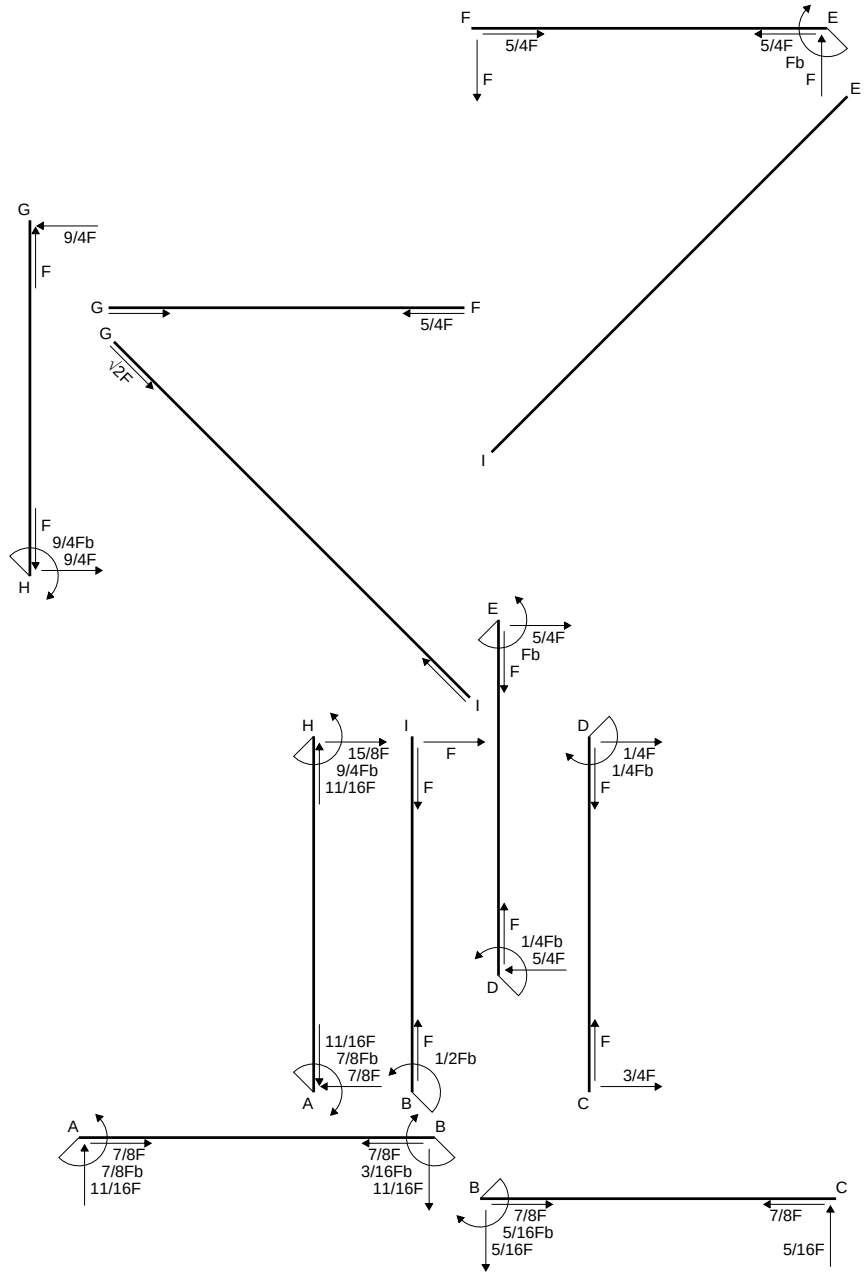
Calcolare reazioni vincolari della struttura e delle aste.

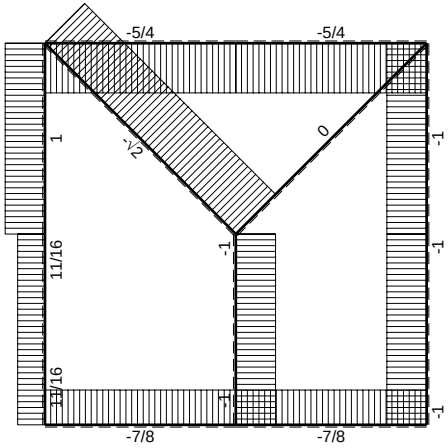
Tracciare i diagrammi quotati delle azioni interne nelle aste.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

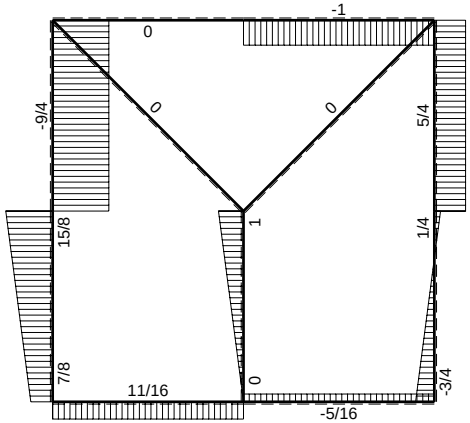
Curvatura θ asta GI positiva se convessa a destra con inizio G.

Spostamento orizzontale assoluto u imposto al nodo H.

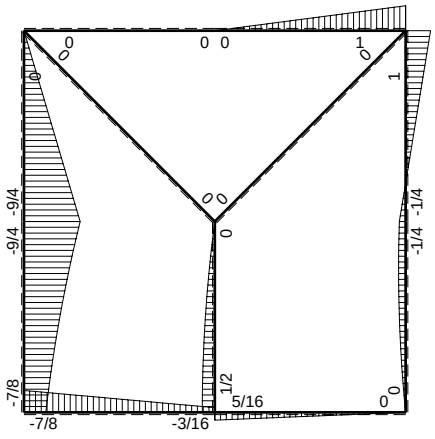




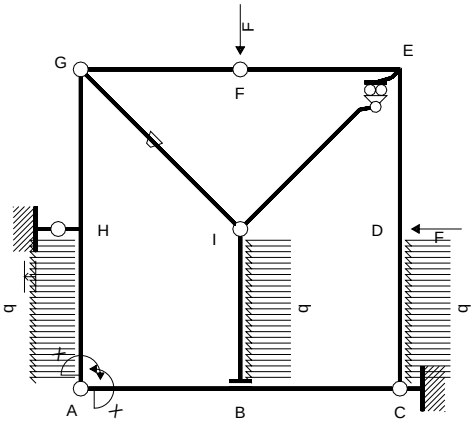
←□→ F

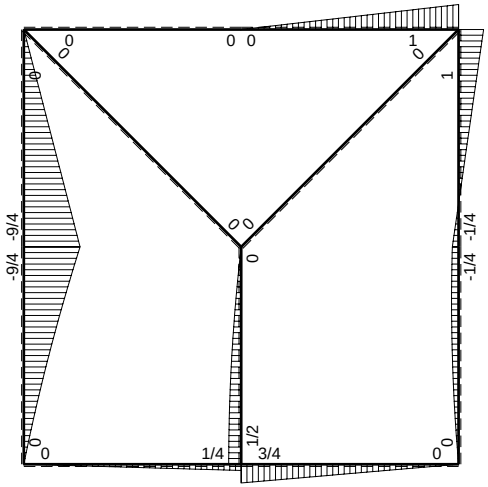


↑□↓ F

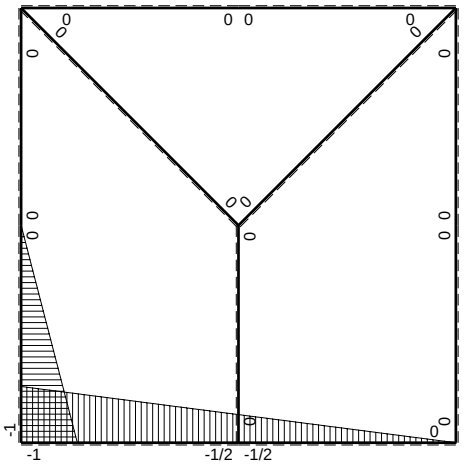


↻□↺ Fb





↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212600 ESΣ04.xxxx.201

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-1+1/2x/b	1/4Fx	0	$-1/4Fx+1/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	7/12Xb/EJ
BA b	1/2+1/2x/b	-1/4Fb+1/4Fx	0	$-1/8Fb+1/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
BC b	-1/2+1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/4Fx-3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8+0)Fb^2/EJ$	1/12Xb/EJ
CB b	1/2x/b	-3/4Fx	0	$-3/8Fx^2/b$	0	$1/4x^2/b^2$		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-Fb+5/4Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	-Fb/EJ	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x/b	$-9/4Fb+11/4Fx-1/2qx^2$	0	$9/4Fx-11/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/3+0)Fb^2/EJ$	1/3Xb/EJ
AH b	1-x/b	$7/4Fx+1/2qx^2$	0	$7/4Fx-5/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^2/EJ$	
	totali						$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$						7/8Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 7/12\ \ b/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/12\ b\right)\ 1/EJ\ = 1/12\ \ b/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\right)\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right)\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b\ 1/EJ$

$= \left(\ b - b + 1/3\ b\right)\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-1/4\ x/b + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-1/8 + 1/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x + 1/24\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b + 1/24\ b\right)\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-3/8 + 3/4\ x/b - 3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-3/8\ x + 3/8\ x^2/b - 1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-3/8\ b + 3/8\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-3/8\ x^2/b^2\right)\ Fb\ 1/EJ\ dx = \left[-1/8\ x^3/b^2\ \right]_o^b\ Fb\ 1/EJ$

$= \left(-1/8\ b\right)\ Fb\ 1/EJ\ = -1/8\ \ Fb^2/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 11/4\ x^2/b^2 + 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[9/8\ x^2/b - 11/12\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

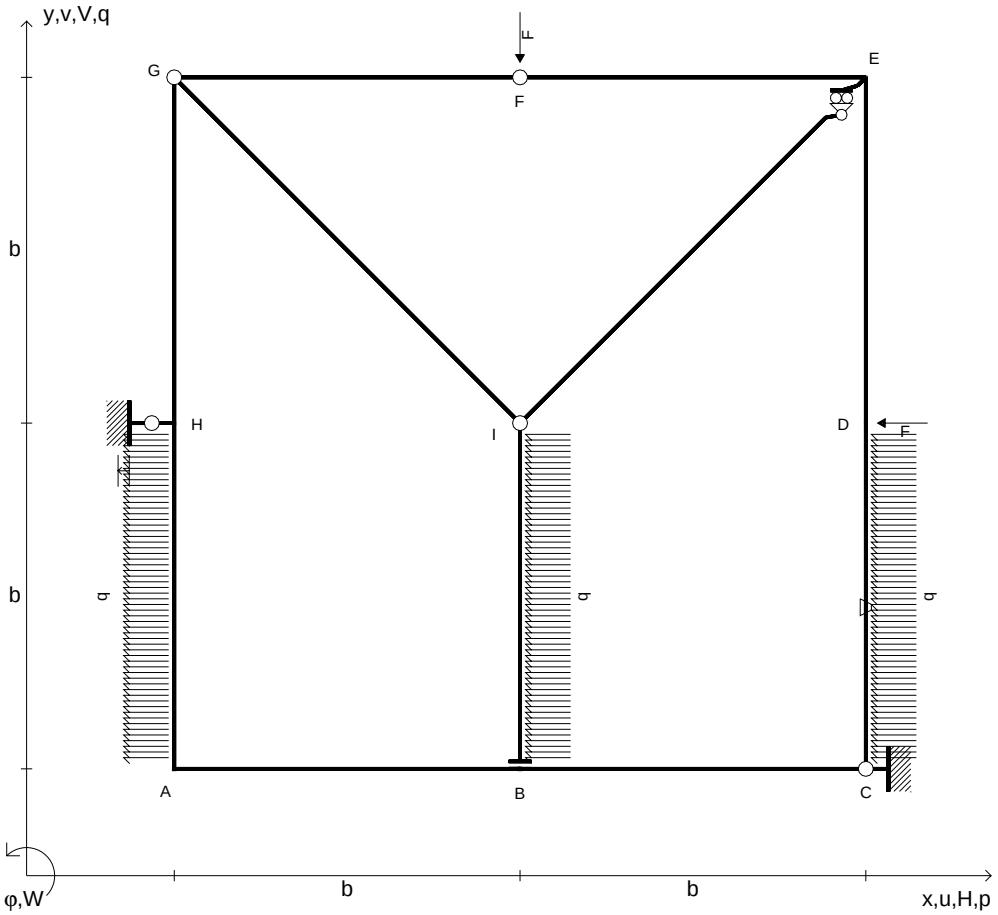
$= \left(9/8\ b - 11/12\ b + 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

$L_{AH}^{Xo} = \int_o^b \left(7/4\ x/b - 5/4\ x^2/b^2 - 1/2\ x^3/b^3\right)\ Fb\ 1/EJ\ dx = \left[7/8\ x^2/b - 5/12\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b\ Fb\ 1/EJ$

$= \left(7/8\ b - 5/12\ b - 1/8\ b\right)\ Fb\ 1/EJ\ = 1/3\ \ Fb^2/EJ$

ESΣ04.xxxx.201

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{IB} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

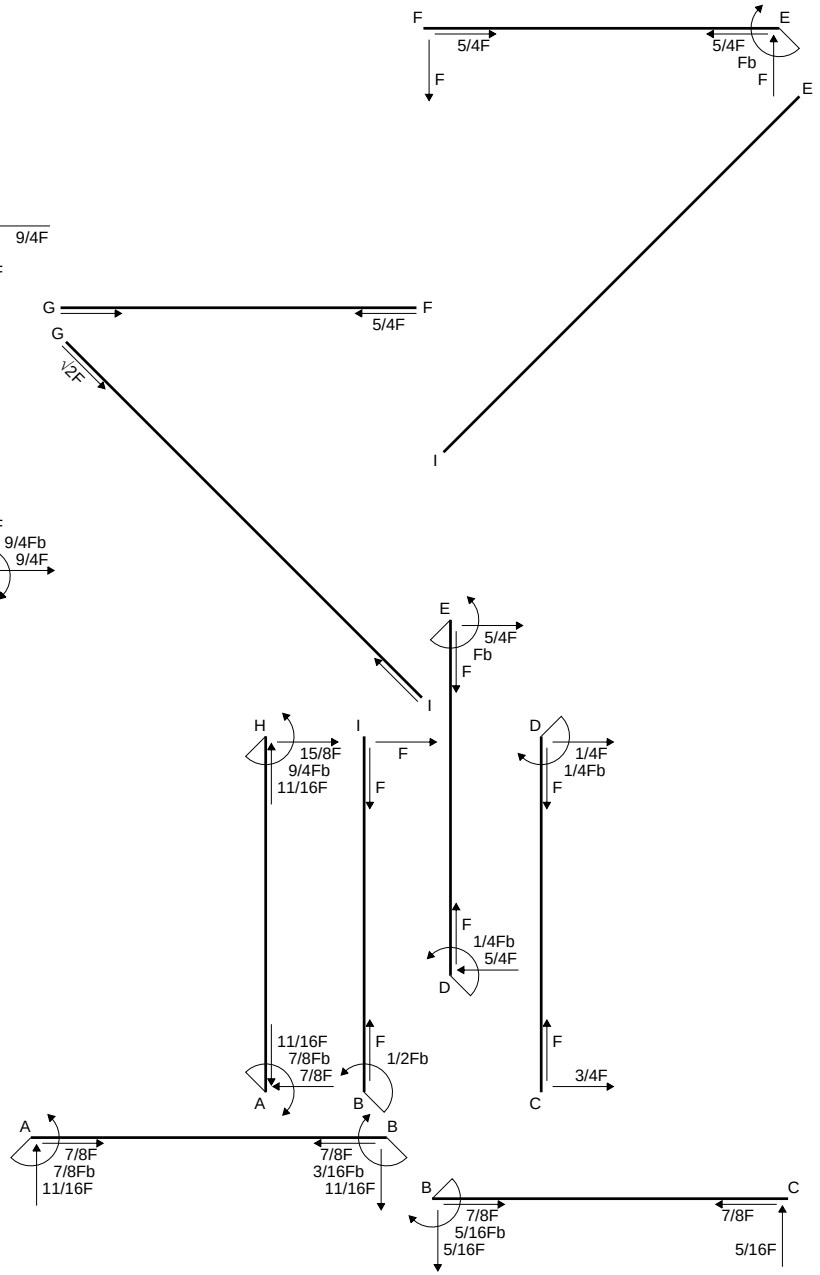
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

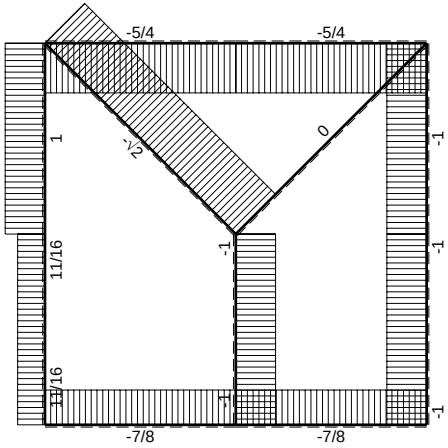
Curvatura θ asta CD positiva se convessa a destra con inizio C.

Spostamento orizzontale assoluto u imposto al nodo H.

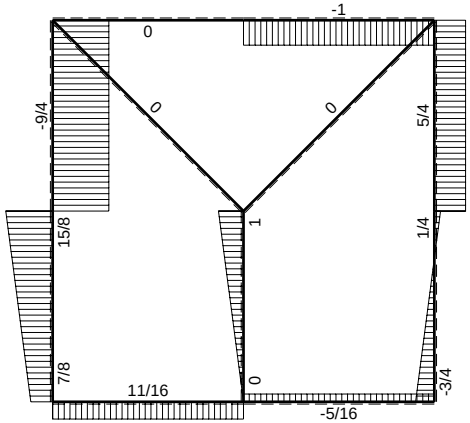
 08.01.25



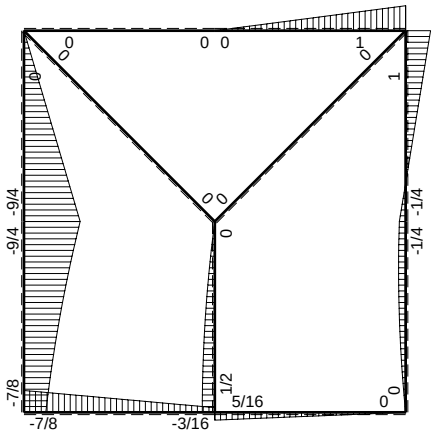
 08.01.25



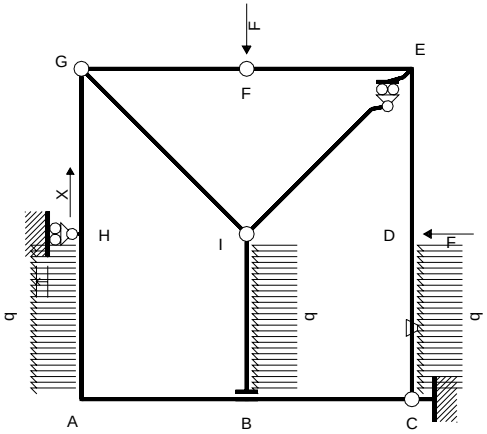
←[+]→ F

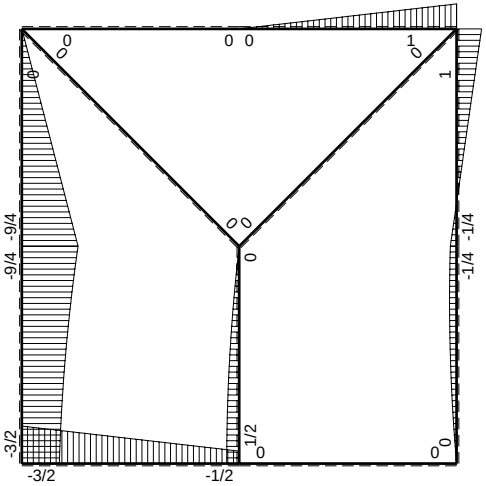


↑[+]↓ F

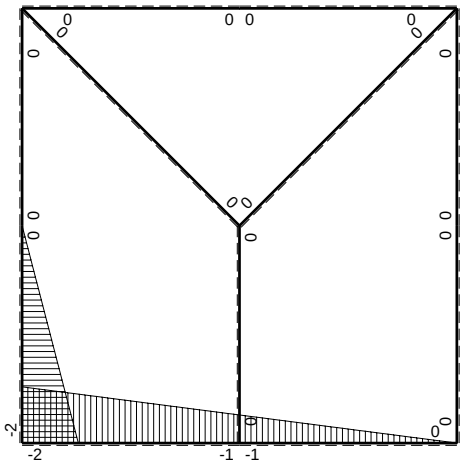


(+) Fb





(+) M_o flessione da carichi assegnati



(+) M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-2b+x$	$-3/2Fb+Fx$	0	$3Fb^2-7/2Fbx+Fx^2$	0	$4b^2-4bx+x^2$	$(19/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	$b+x$	$1/2Fb+Fx$	0	$1/2Fb^2+3/2Fbx+Fx^2$	0	$b^2+2bx+x^2$		
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CB b	x	0	0	0	0	x^2		
CD b	0	$-3/4Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	Fb/EJ	0	0	0		
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-Fb+5/4Fx$	0	0	0	0		
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0
FE b	0	$-Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0		
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0
HA b	$-2x$	$-9/4Fb+5/4Fx-1/2qx^2$	0	$9/2Fbx-5/2Fx^2+qx^3$	0	$4x^2$	$(5/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	$2b-2x$	$3/2Fb+1/4Fx+1/2qx^2$	0	$3Fb^2-5/2Fbx+1/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$5/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						$-5/16F$	

PROCEDIMENTO E RISULTATI 982827 ESΣ04.xxxx.209

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\ b^3/EJ$

$L_{BA}^{XX} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\ b^3/EJ$

$L_{BC}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{CB}^{XX} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L_{HA}^{XX} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L_{AH}^{XX} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\ b^3/EJ$

$L_{AB}^{Xo} = \int_o^b (3 - 7/2\,x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3\,x - 7/4\,x^2/b + 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3\,b - 7/4\,b + 1/3\,b)\, Fb^2\, 1/EJ = 19/12\ Fb^3/EJ$

$L_{BA}^{Xo} = \int_o^b (1/2 + 3/2\,x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\,x + 3/4\,x^2/b + 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\,b + 3/4\,b + 1/3\,b)\, Fb^2\, 1/EJ = 19/12\ Fb^3/EJ$

$L_{HA}^{Xo} = \int_o^b (9/2\,x/b - 5/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [9/4\,x^2/b - 5/6\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

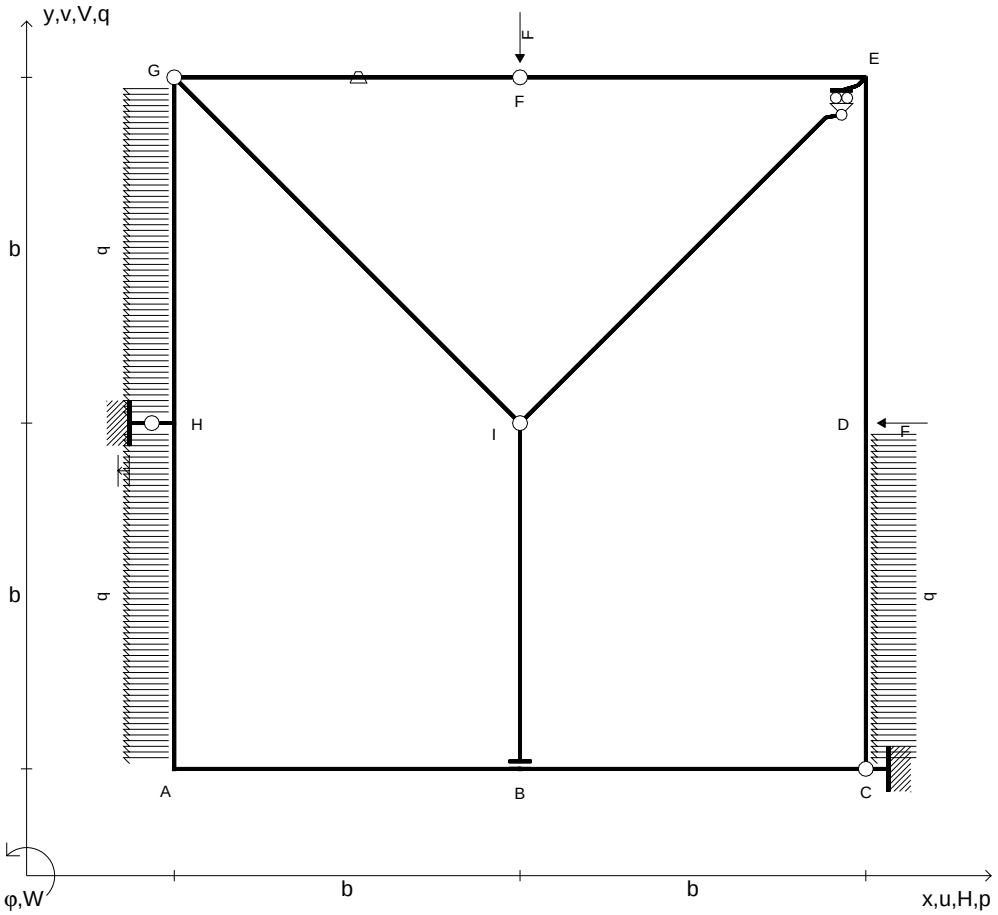
$= (9/4\,b - 5/6\,b + 1/4\,b)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

$L_{AH}^{Xo} = \int_o^b (3 - 5/2\,x/b + 1/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [3\,x - 5/4\,x^2/b + 1/6\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (3\,b - 5/4\,b + 1/6\,b - 1/4\,b)\, Fb^2\, 1/EJ = 5/3\ Fb^3/EJ$

ESΣ04.xxxx.209

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

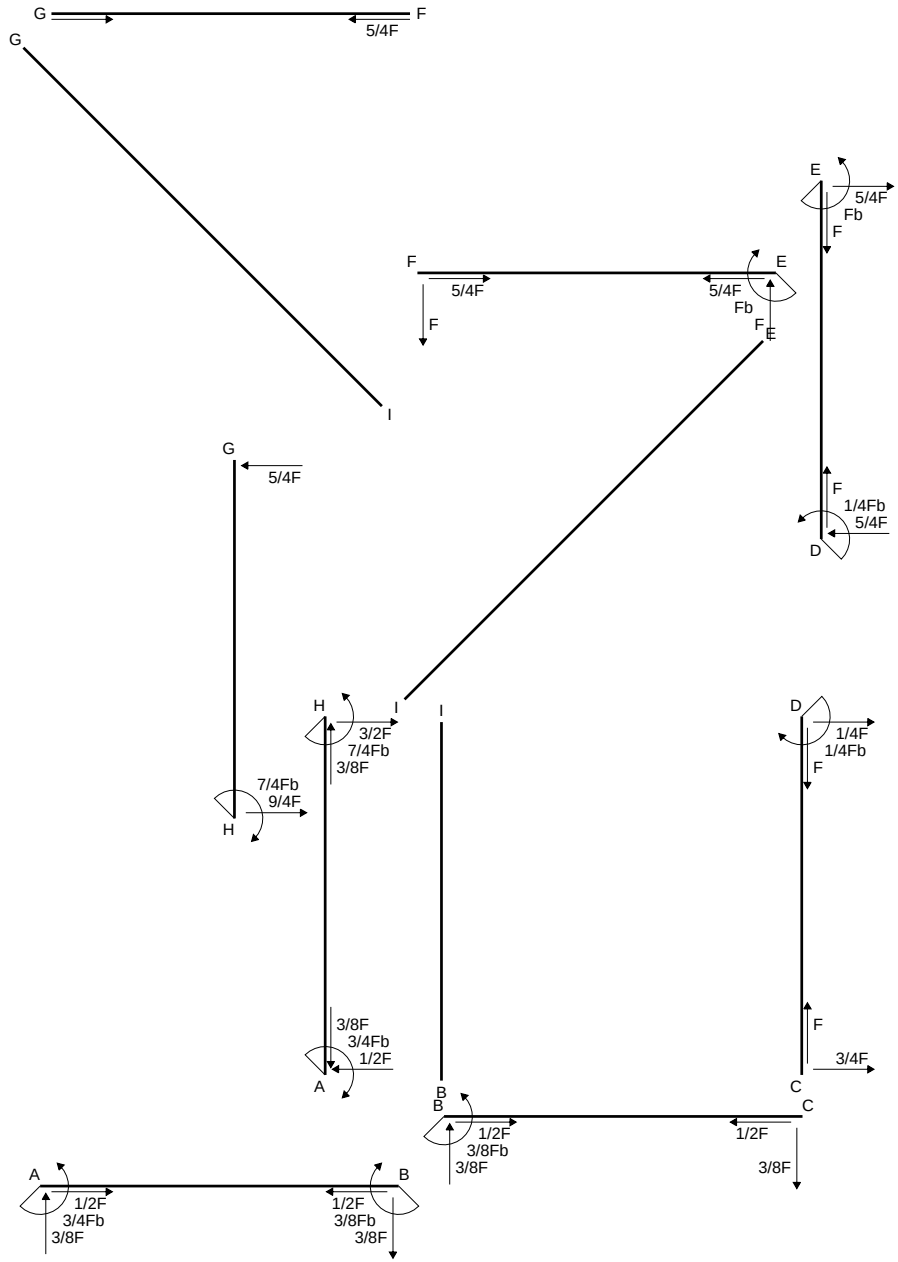
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

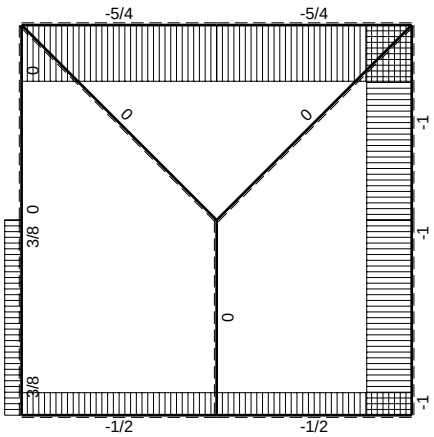
Curvatura θ asta FG positiva se convessa a destra con inizio F.

Spostamento orizzontale assoluto u imposto al nodo H.

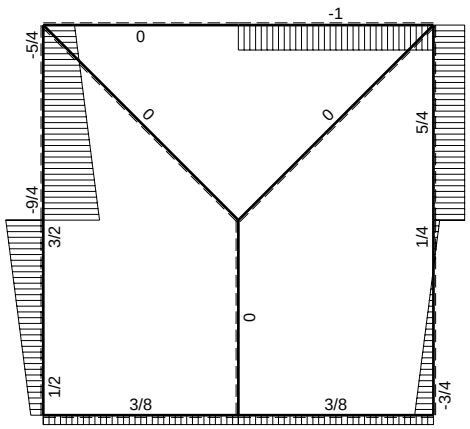
 08.01.25



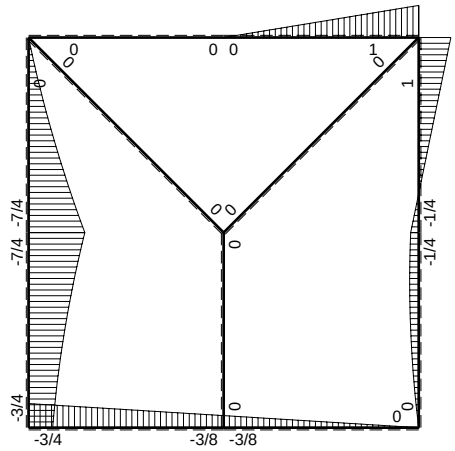
 08.01.25



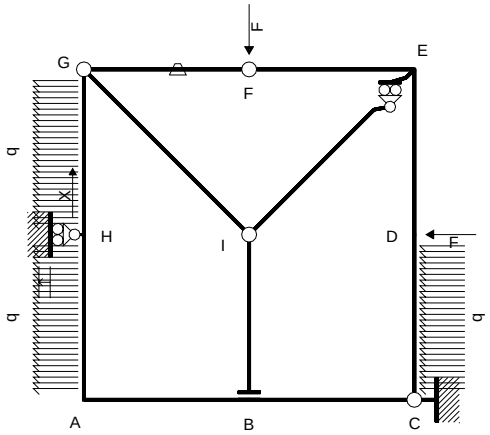
←□→ F

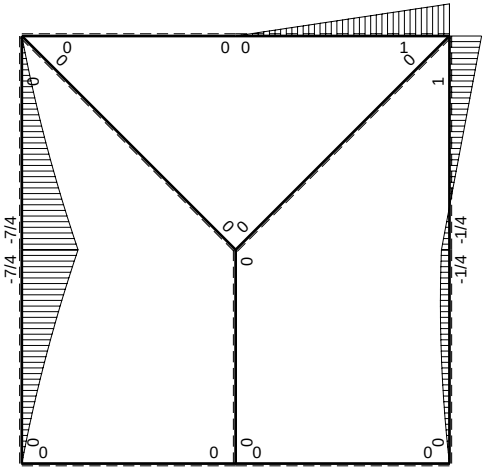


↑□↓ F

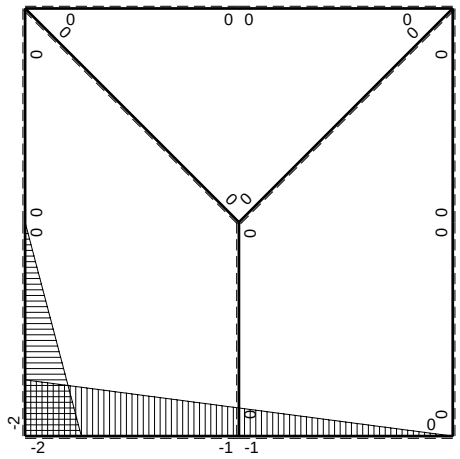


□ Fb





↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-2b+x$	0	0	0	0	$4b^2-4bx+x^2$	0+0	$7/3Xb^3/EJ$
BA b	$b+x$	0	0	0	0	$b^2+2bx+x^2$		
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CB b	x	0	0	0	0	x^2		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-Fb+5/4Fx$	0	0	0	0		
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0
FE b	0	$-Fx$	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0
HA b	$-2x$	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/2Fbx-9/2Fx^2+qx^3$	0	$4x^2$	$(1/2+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	$2b-2x$	$5/4Fx+1/2qx^2$	0	$5/2Fbx-3/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$-3/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						3/8F	

PROCEDIMENTO E RISULTATI 984497 ESΣ04.xxxx.212

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{HA} = \int_o^b (7/2\,x/b - 9/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [7/4\,x^2/b - 3/2\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

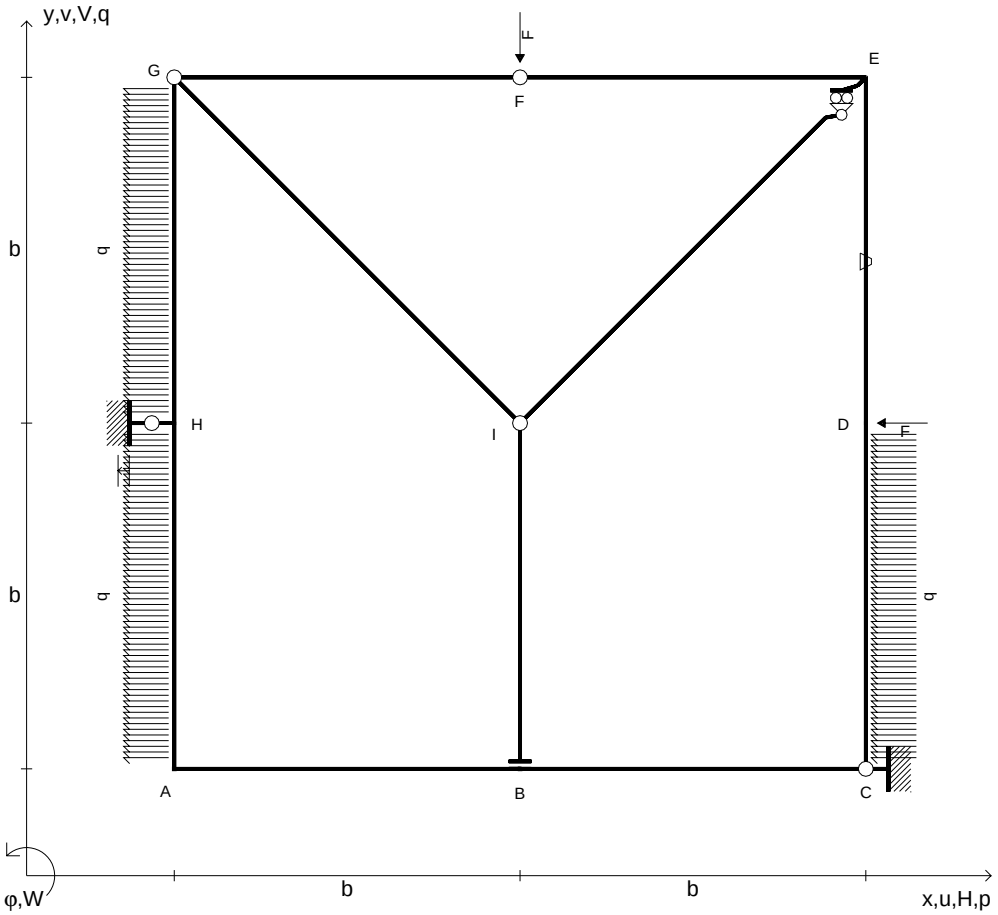
$= (7/4\,b - 3/2\,b + 1/4\,b)\, Fb^2\, 1/EJ = 1/2\; Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (5/2\,x/b - 3/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [5/4\,x^2/b - 1/2\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (5/4\,b - 1/2\,b - 1/4\,b)\, Fb^2\, 1/EJ = 1/2\; Fb^3/EJ$

ESΣ04.xxxx.212

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_F = -F$ | $u_H = -\delta = -b^3F/EJ$ | $EJ_{FG} = EJ$

$H_D = -F$ | $EJ_{AB} = EJ$ | $EJ_{GH} = EJ$

$p_{HA} = -q = -F/b$ | $EJ_{BC} = EJ$ | $EJ_{GI} = EJ$

$p_{CD} = -q = -F/b$ | $EJ_{CD} = EJ$ | $EJ_{IB} = EJ$

$p_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$ | $EJ_{IE} = EJ$

$\theta_{DE} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ | $EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

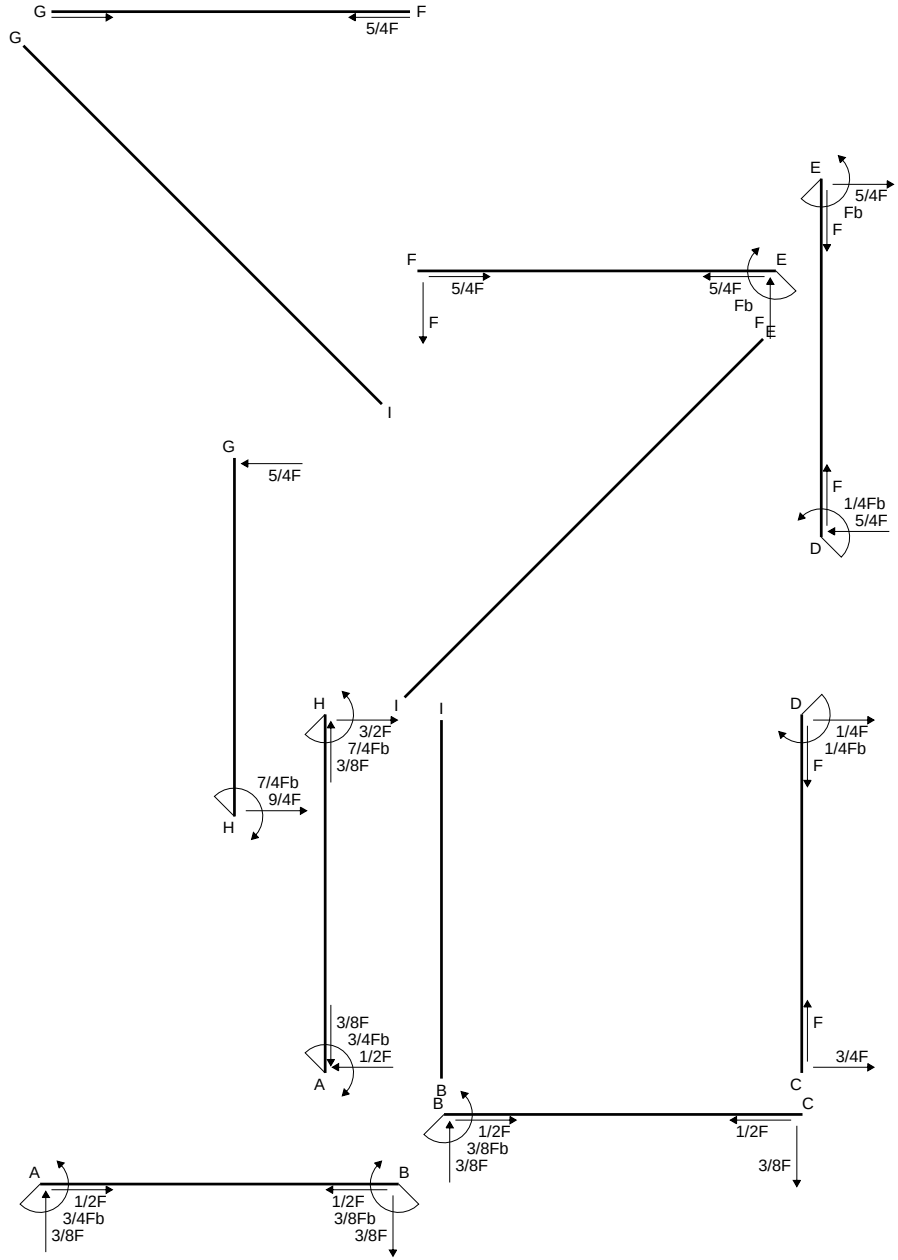
Carichi di aste curve misurati in proiezione sugli assi x,y.

J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.

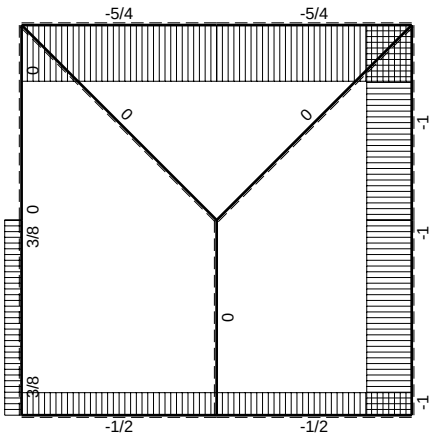
Curvatura θ asta DE positiva se convessa a destra con inizio D.

Spostamento orizzontale assoluto u imposto al nodo H.

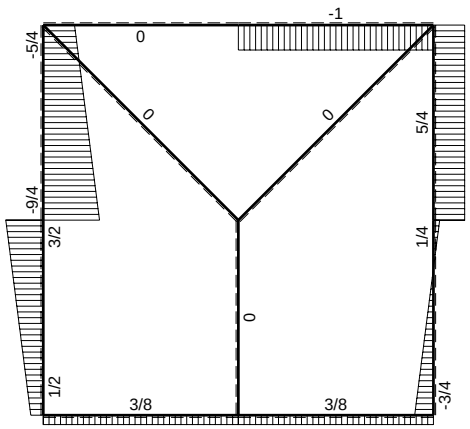
 08.01.25



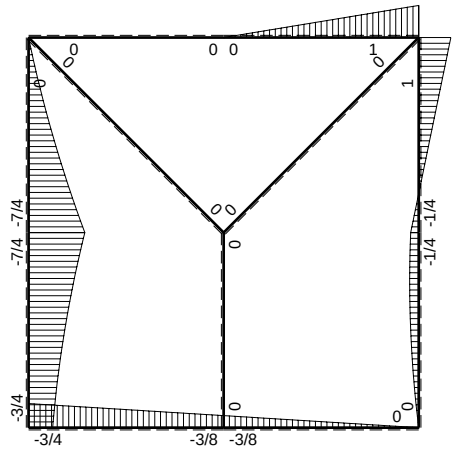
 08.01.25



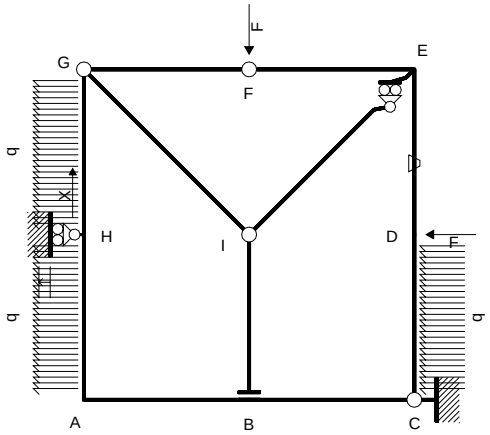
←□→ F

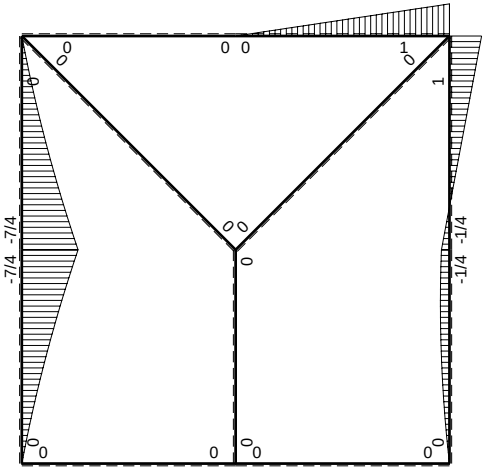


↑□↓ F

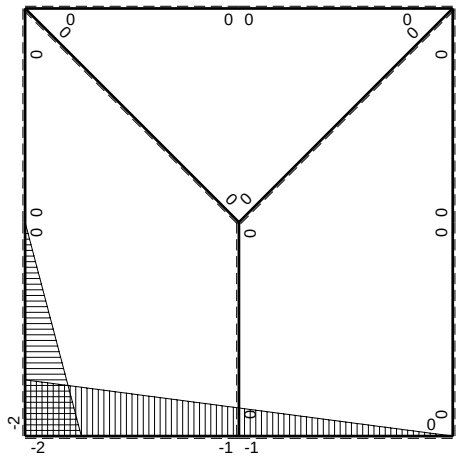


↻□↺ Fb





↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-2b+x	0	0	0	0	$4b^2-4bx+x^2$	0+0	$7/3Xb^3/EJ$
BA b	b+x	0	0	0	0	$b^2+2bx+x^2$		
BC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CB b	x	0	0	0	0	x^2		
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0		
DE b	0	$-1/4Fb+5/4Fx$	-Fb/EJ	0	0	0	0+0	0
ED b	0	$-Fb+5/4Fx$	Fb/EJ	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-2x	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/2Fbx-9/2Fx^2+qx^3$	0	$4x^2$	$(1/2+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	2b-2x	$5/4Fx+1/2qx^2$	0	$5/2Fbx-3/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$-3/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						3/8F	

PROCEDIMENTO E RISULTATI 213144 ESΣ04.xxxx.213

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L_{BA}^{XX} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L_{BC}^{XX} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L_{CB}^{XX} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L_{HA}^{XX} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L_{AH}^{XX} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L_{HA}^{Xo} = \int_o^b (7/2\,x/b - 9/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [7/4\,x^2/b - 3/2\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (7/4\,b - 3/2\,b + 1/4\,b)\, Fb^2\, 1/EJ = 1/2\; Fb^3/EJ$

$L_{AH}^{Xo} = \int_o^b (5/2\,x/b - 3/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [5/4\,x^2/b - 1/2\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (5/4\,b - 1/2\,b - 1/4\,b)\, Fb^2\, 1/EJ = 1/2\; Fb^3/EJ$

ESΣ04.xxxx.213

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25