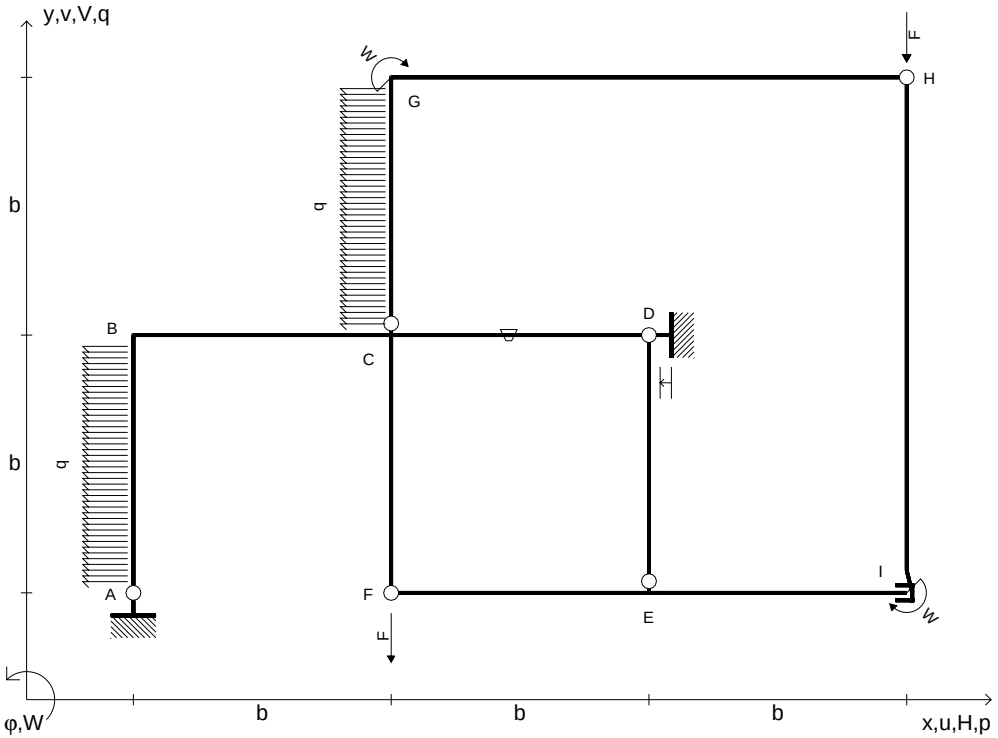
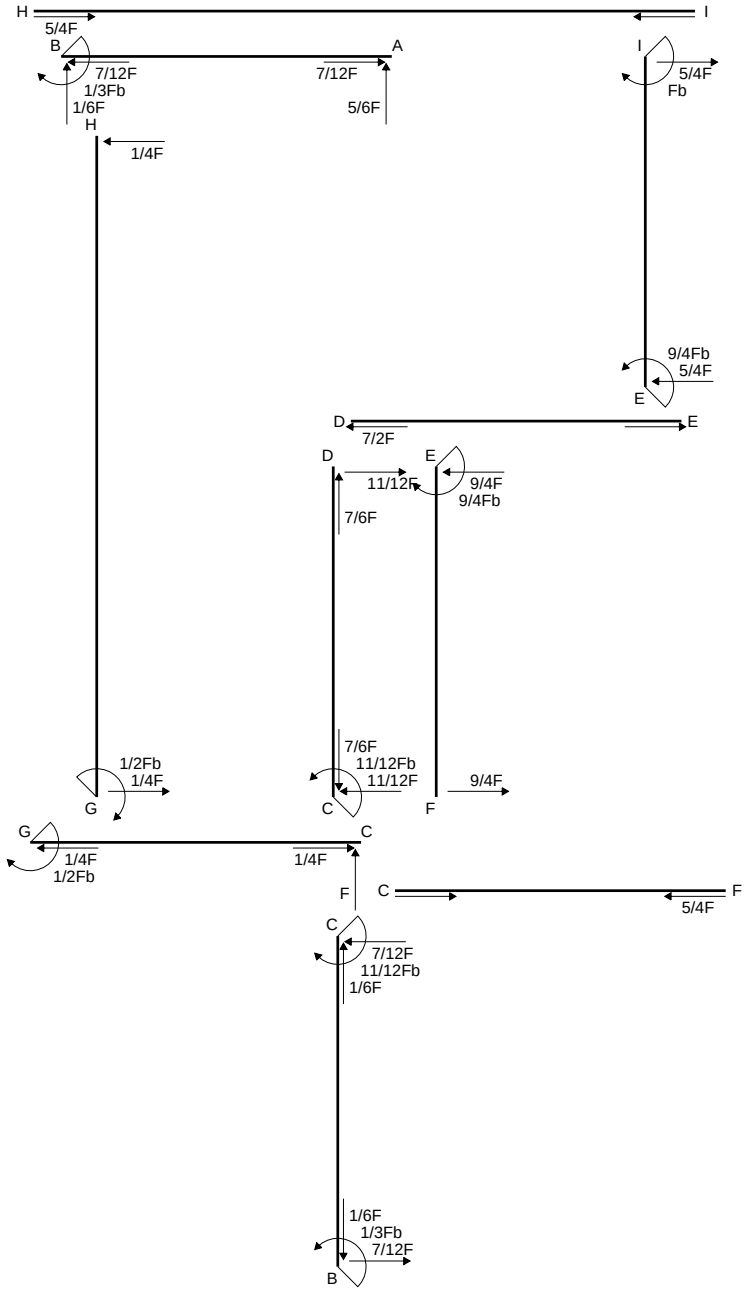


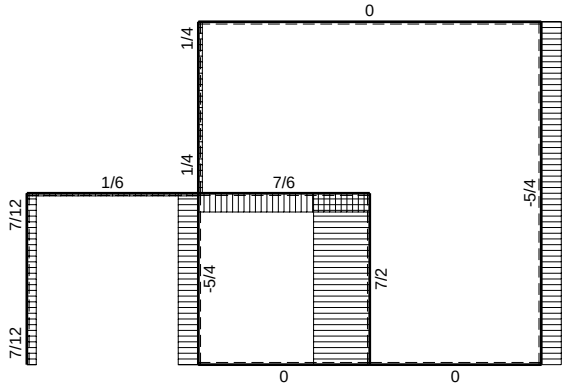
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$p_{CG} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

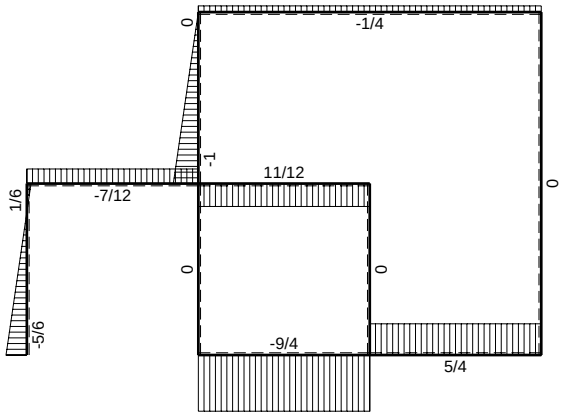
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{BC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo D.

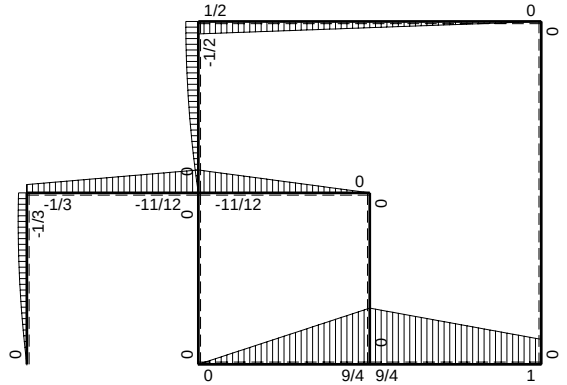




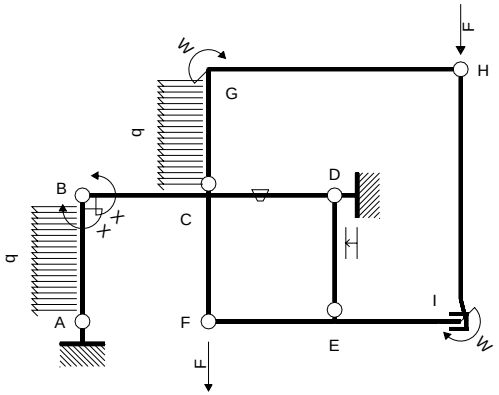
←□→ F



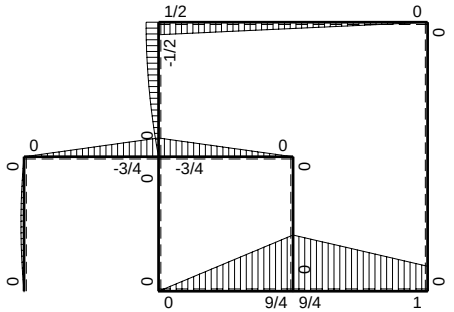
□↑↓ F



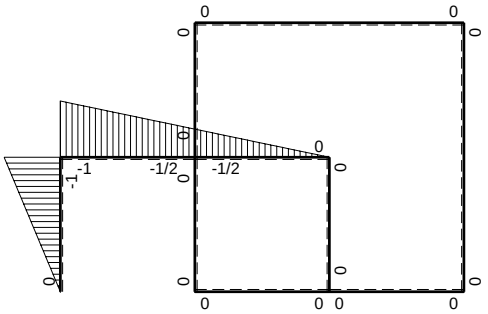
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
BC b	$-1+1/2x/b$	-3/4Fx	0	$3/4Fx-3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(1/4+0)Fb^2/EJ$	$7/12Xb/EJ$
CB b	$1/2+1/2x/b$	3/4Fb-3/4Fx	0	$3/8Fb-3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	$-1/2+1/2x/b$	-3/4Fb+3/4Fx	-Fb/EJ	$3/8Fb-3/4Fx+3/8Fx^2/b$	1/2Fb/EJ-1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(1/8+1/4)Fb^2/EJ$	$1/12Xb/EJ$
DC b	$1/2x/b$	3/4Fx	Fb/EJ	$3/8Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0
FE b	0	-9/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0
EI b	0	-9/4Fb+5/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^2/EJ$	
	totali						$-1/3Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						1/3Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BA}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{XX} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L_{AB}^{Xo} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b]_o^b\ \theta$

$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/8\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b]_o^b\ \theta$

$= (1/8\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 3/8\ Fb^2/EJ$

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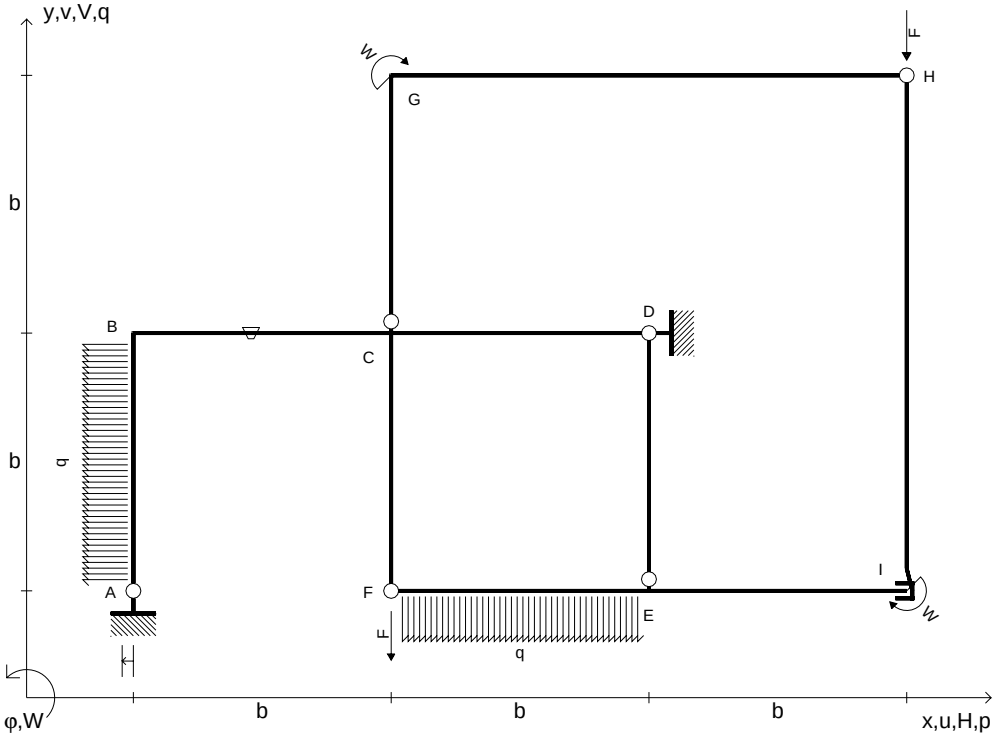


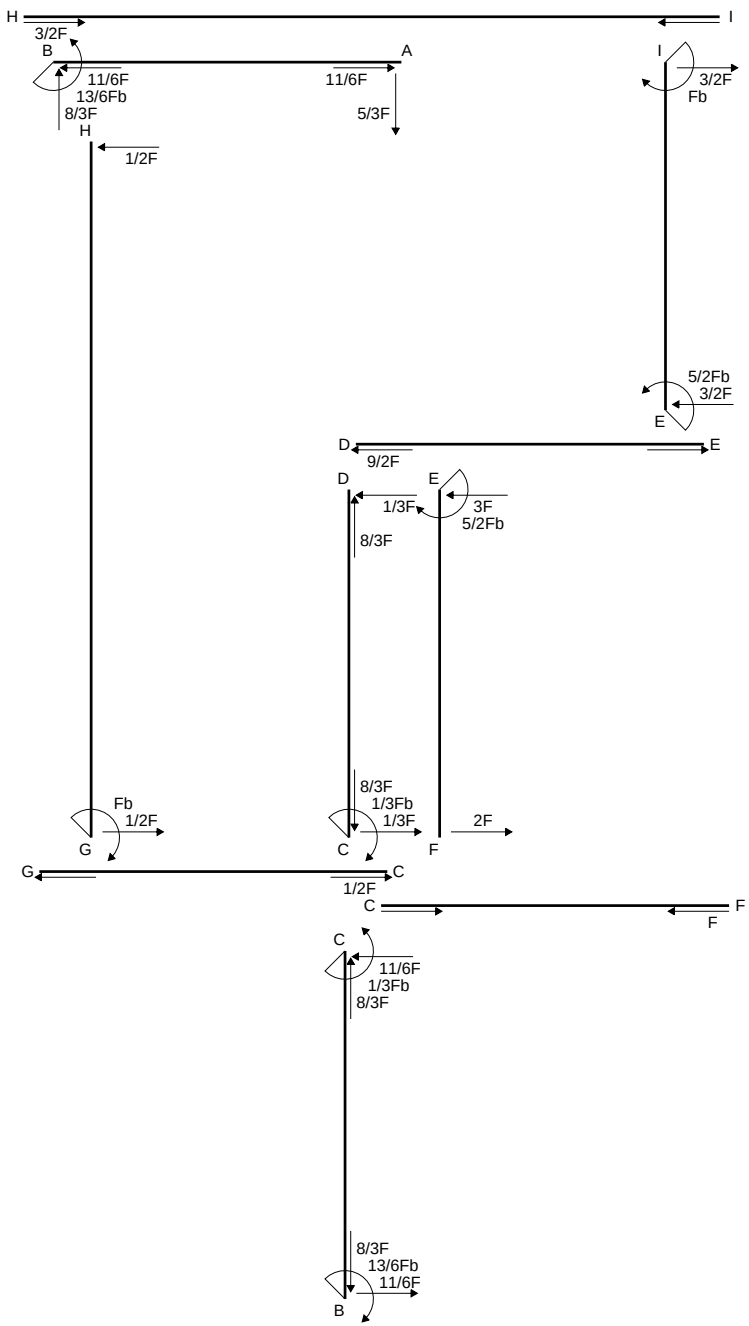
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$q_{EF} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

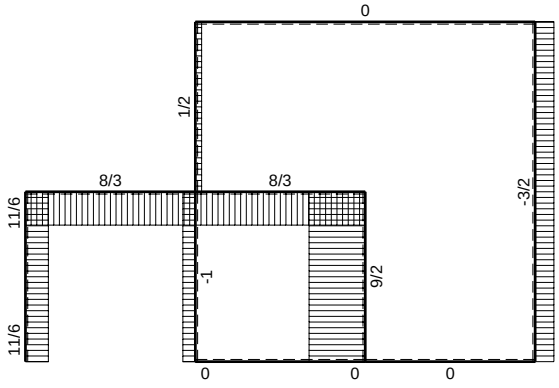
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo A.

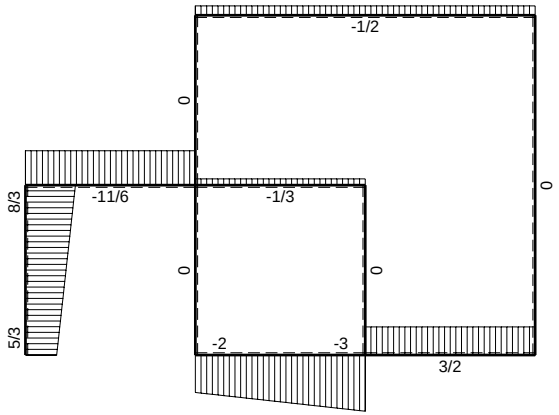
 08.01.25



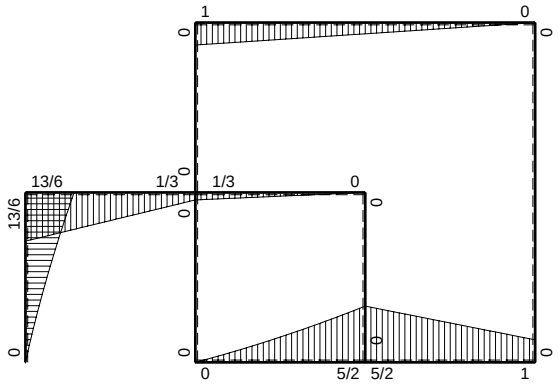
 08.01.25



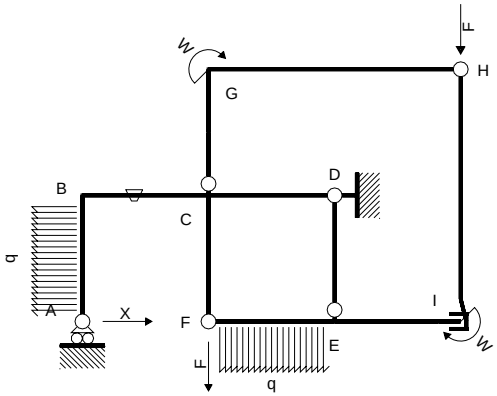
←□→ F



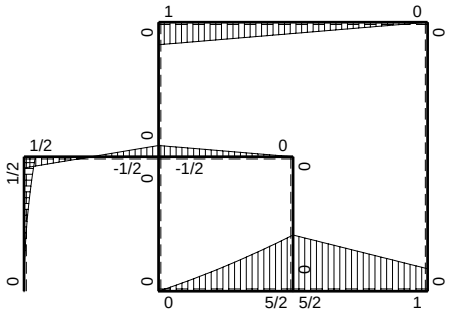
↑□↓ F



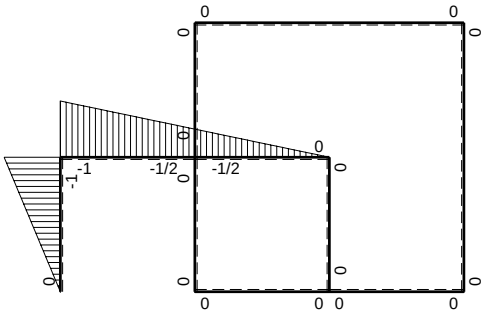
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-Fx$	$-Fb/EJ$	$-1/2Fb^2+5/4Fbx-1/2Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(-1/24+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$1/2Fb-Fx$	Fb/EJ	$1/4Fb^2-1/4Fbx-1/2Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-5/2Fb+3/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$5/3Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-5/3F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +5/4\, x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\, dx$

$= [-1/2\, x +5/8\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [\, x -1/4\, x^2/b]_o^b\, \theta$

$= (-1/2\, b +5/8\, b -1/6\, b)\, Fb^2\, 1/EJ + (\, b -1/4\, b)\, \theta = 17/24\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\, x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/4\, x -1/8\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/4\, b -1/8\, b -1/6\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 17/24\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 957901 ESΣ01.xxxx.010

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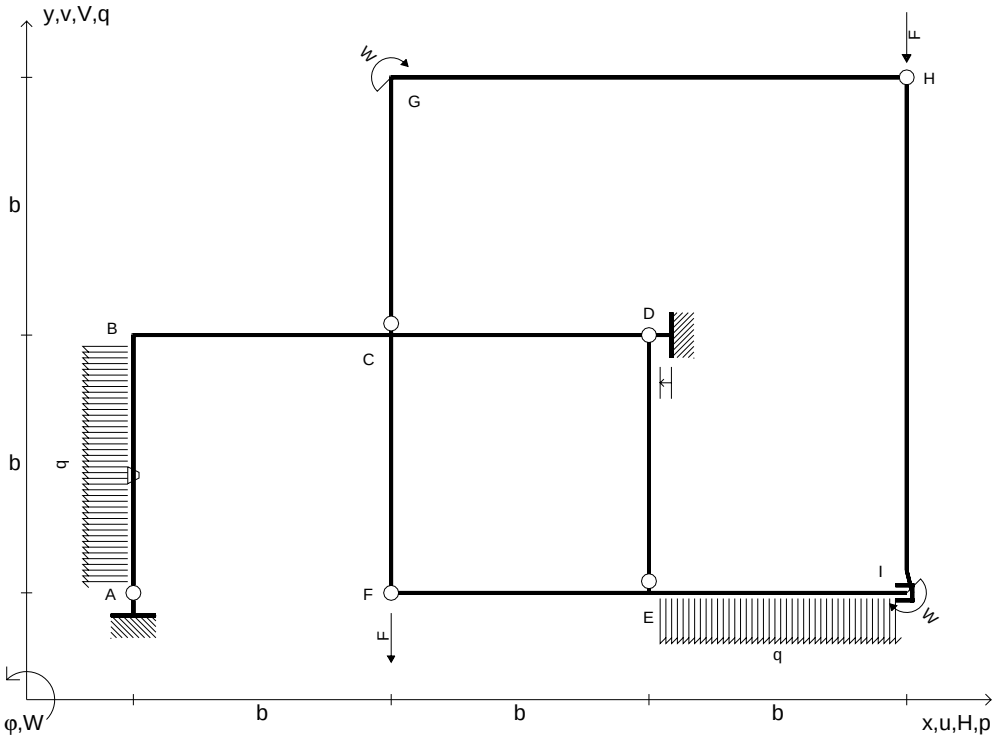


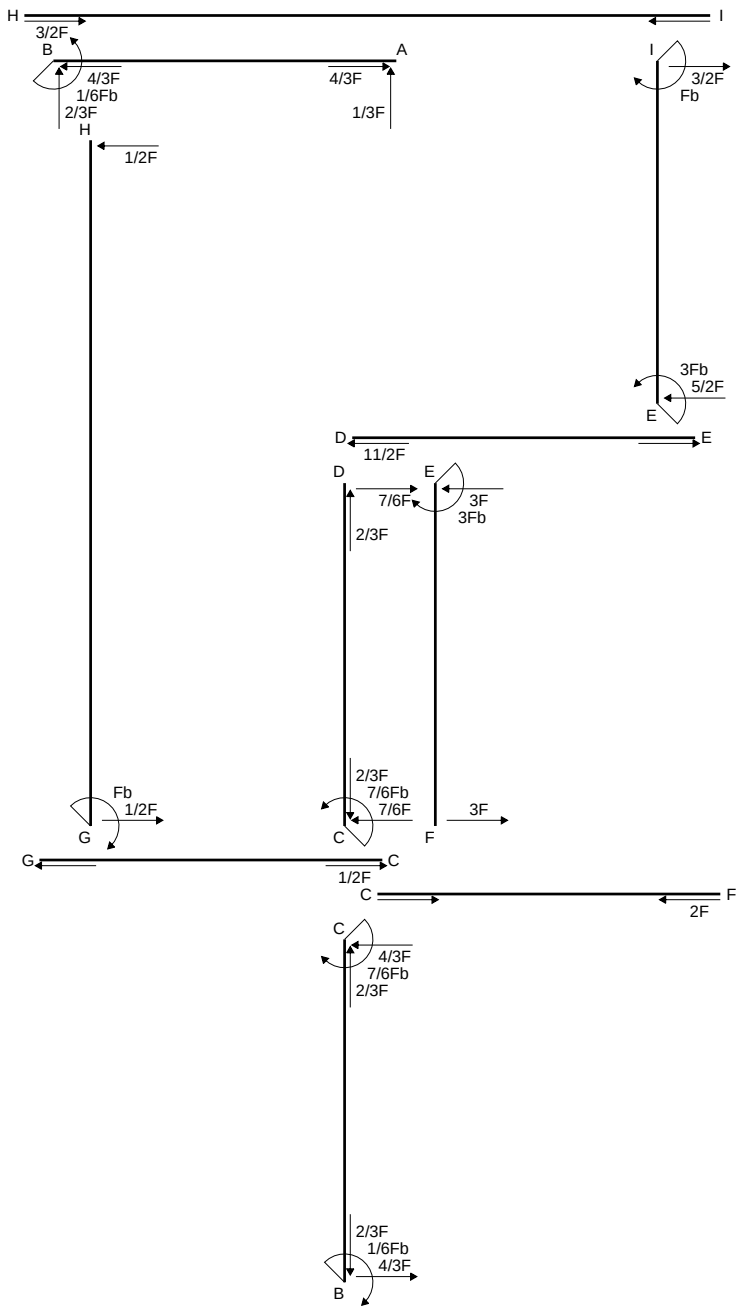
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$q_{IE} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

 08.01.25



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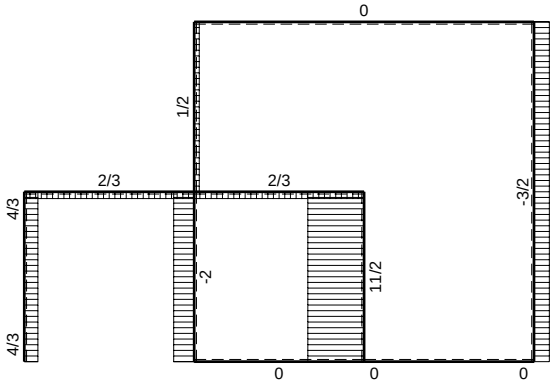
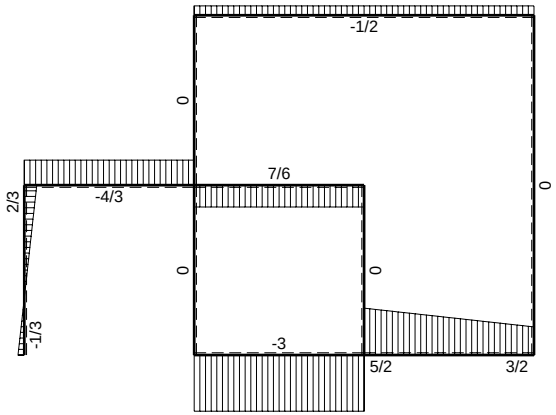
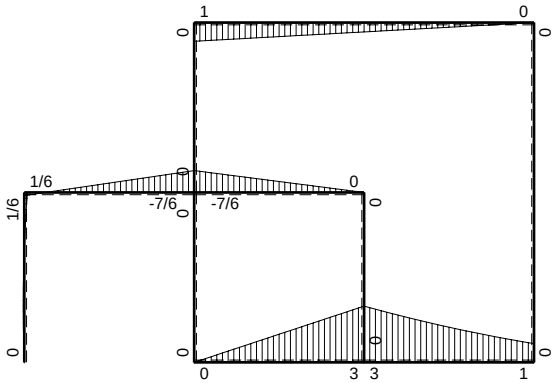


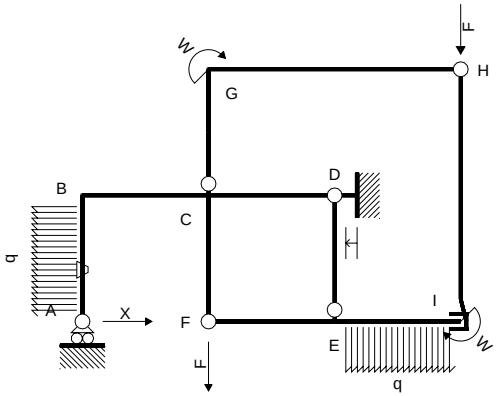
←□→ F



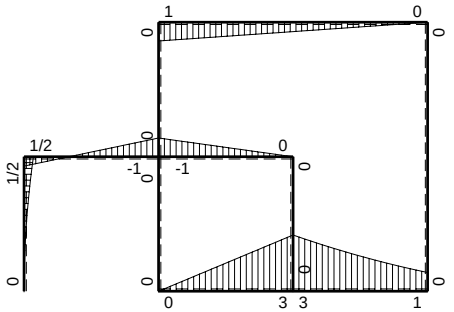
↑□↓ F



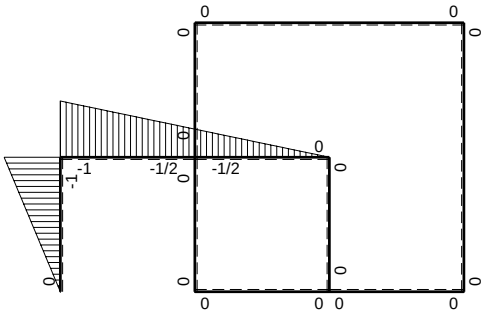
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211691 ESΣ01.xxxx.019

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	-Fb/EJ	$-1/2qx^3$	Fxb/EJ	x^2	$(-1/8+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-3/2Fx$	0	$-1/2Fb^2+7/4Fbx-3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$Fb-3/2Fx$	0	$1/2Fb^2-1/4Fbx-3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-Fb+Fx$	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	3Fb-3Fx	0	0	0	0	0+0	0
FE b	0	-3Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-1/3Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						1/3F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (x/b)\, \theta\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [1/2\, x^2/b]_o^b\, \theta$

$= (-1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 + x/b)\, \theta\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [- x +1/2\, x^2/b]_o^b\, \theta$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ + (- b +1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +7/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +7/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +7/8\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/8\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

ESΣ01.xxxx.019

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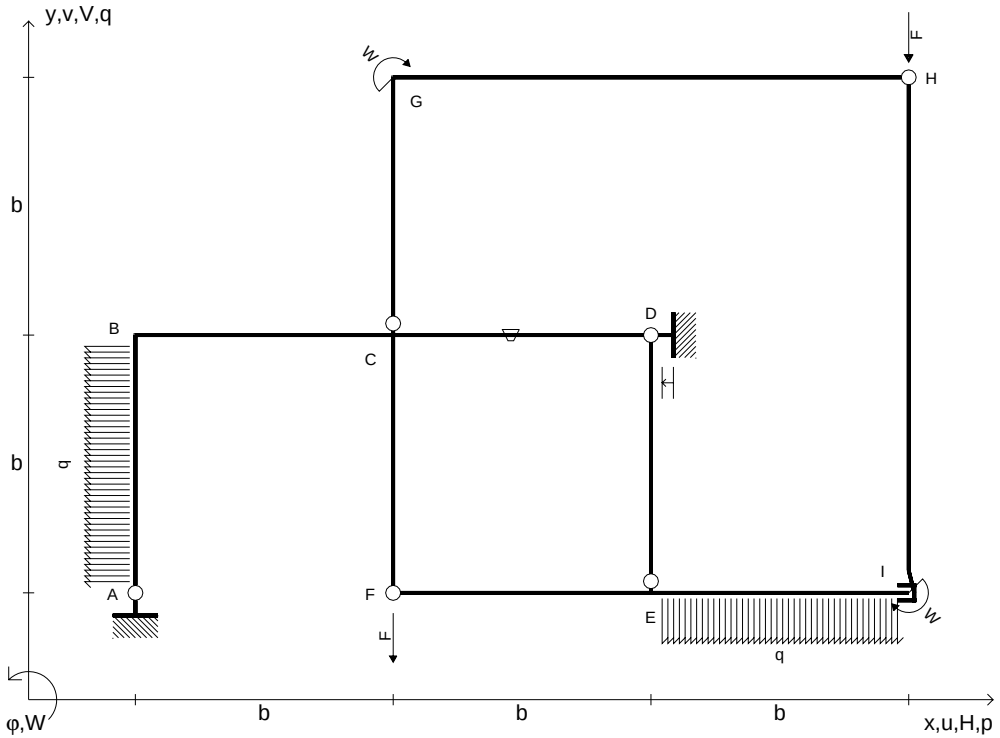

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$q_{IE} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
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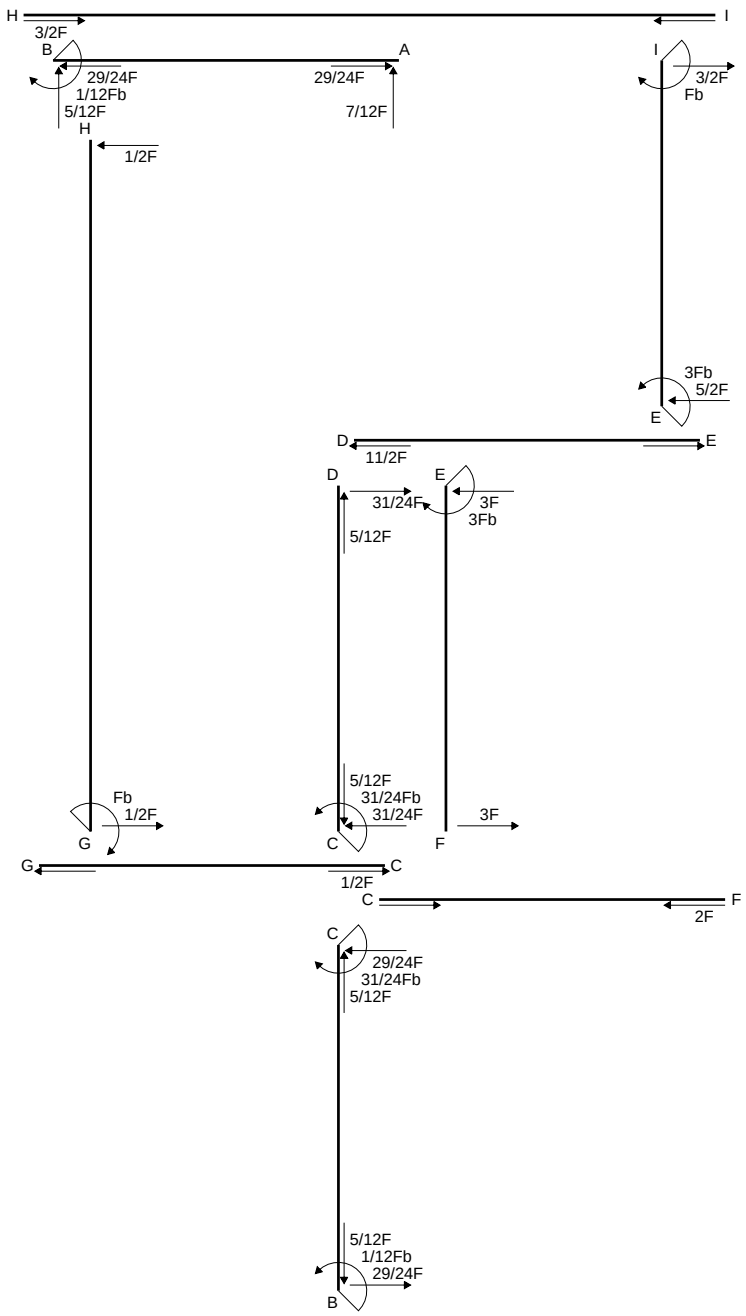
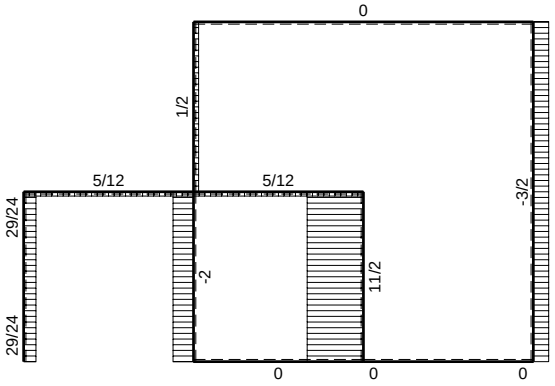
Reazioni iperstatiche in soluzione: $X=W_{BC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo D.

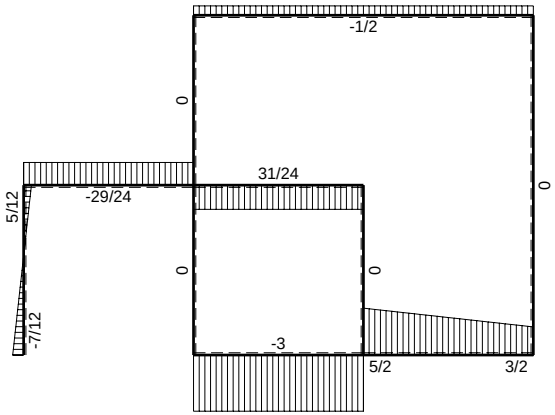
 08.01.25



 08.01.25



←□→ F



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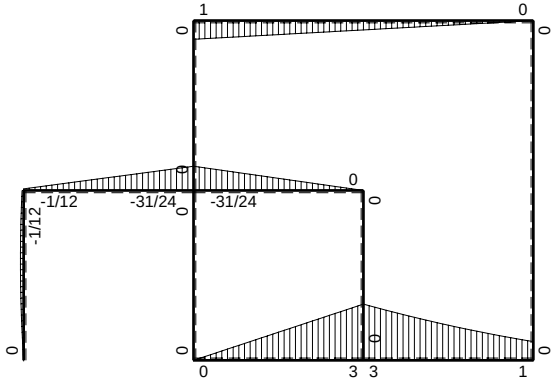
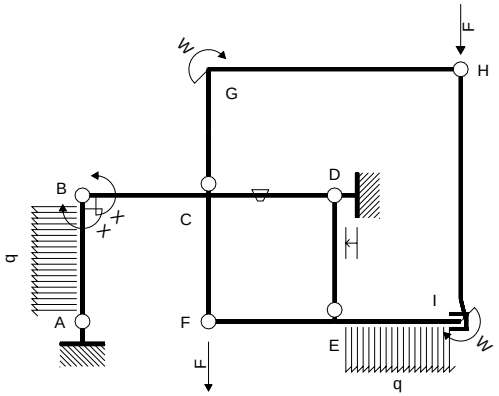
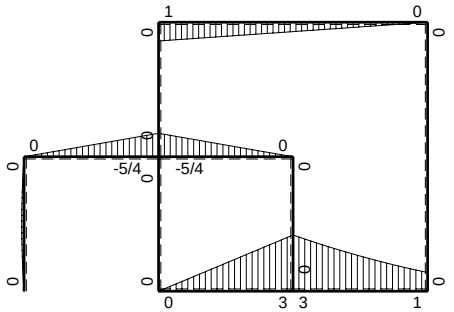


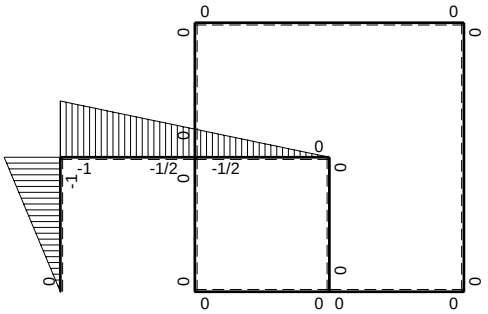
↺□↻ Fb



Schema di calcolo iperstatico



↺□↻ M_o flessione da carichi assegnati



↺□↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
BC b	$-1+1/2x/b$	-5/4Fx	0	$5/4Fx-5/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(5/12+0)Fb^2/EJ$	7/12Xb/EJ
CB b	$1/2+1/2x/b$	$5/4Fb-5/4Fx$	0	$5/8Fb-5/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	$-1/2+1/2x/b$	$-5/4Fb+5/4Fx$	-Fb/EJ	$5/8Fb-5/4Fx+5/8Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(5/24+1/4)Fb^2/EJ$	1/12Xb/EJ
DC b	$1/2x/b$	5/4Fx	Fb/EJ	$5/8Fx^2/b$	$1/2Fx/EJ$	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	3Fb-3Fx	0	0	0	0	0+0	0
FE b	0	-3Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^2/EJ$	
	totali						$-1/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						1/12Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212811 ESΣ01.xxxx.021

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$L_{AB}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BA}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{XX} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L_{AB}^{Xo} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L_{BA}^{Xo} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L_{BC}^{Xo} = \int_o^b (5/4\ x/b -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x^2/b -5/24\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (5/8 -5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [5/8\ x -5/24\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (5/8\ b -5/24\ b)\ Fb\ 1/EJ\ = 5/12\ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/8 -5/4\ x/b +5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [5/8\ x -5/8\ x^2/b +5/24\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b]_o^b\ \theta$

$= (5/8\ b -5/8\ b +5/24\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (5/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [5/24\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b]_o^b\ \theta$

$= (5/24\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 11/24\ Fb^2/EJ$

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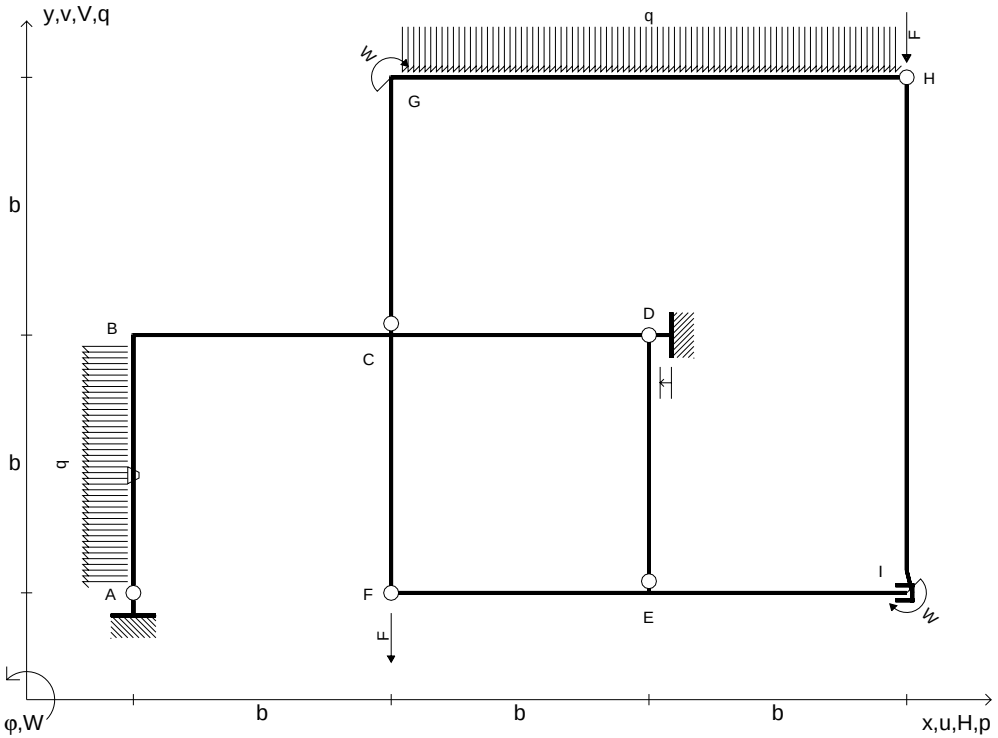
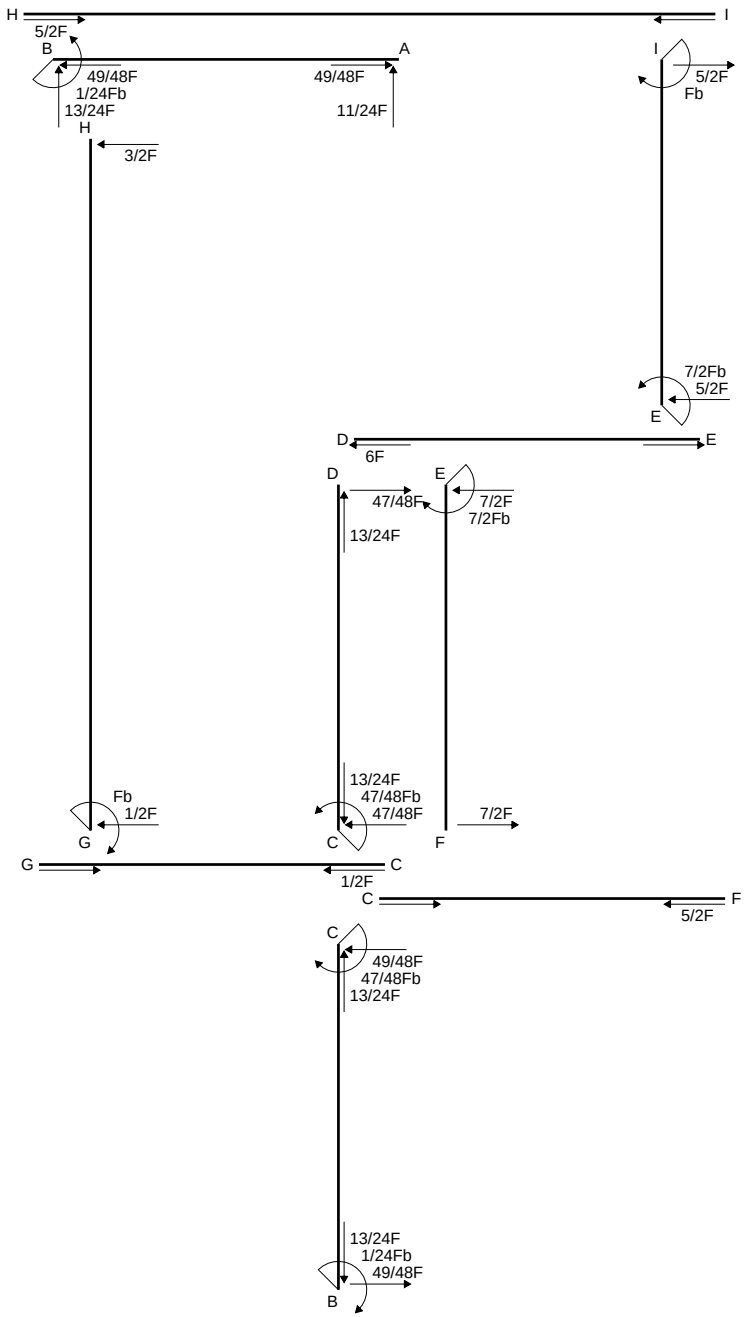


$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$q_{GH} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{BC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.



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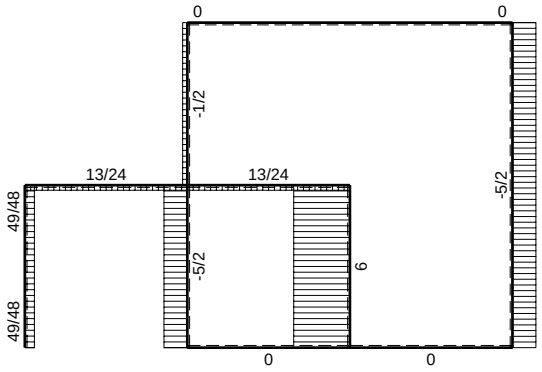
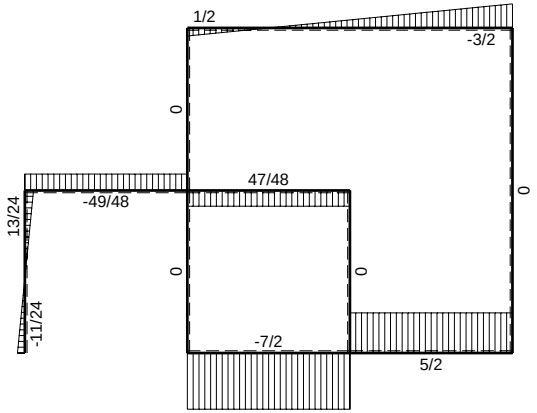
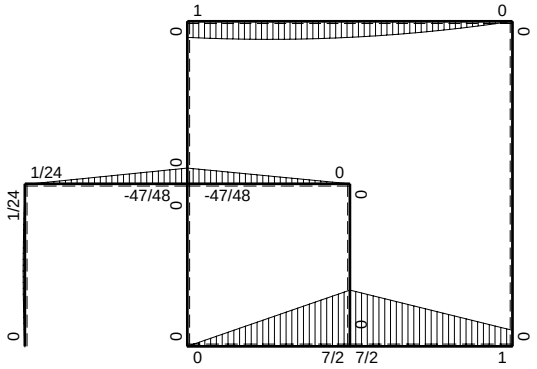


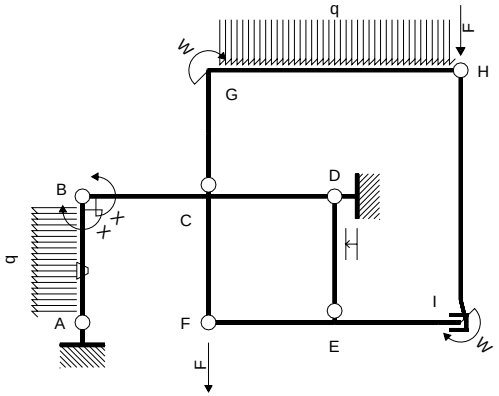
←□→ F



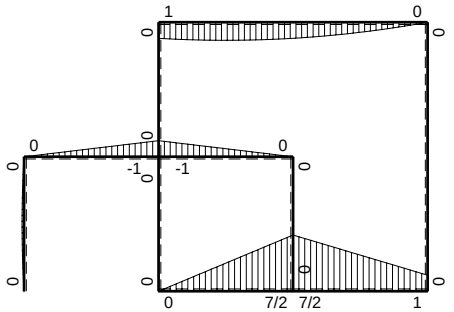
↑□↓ F



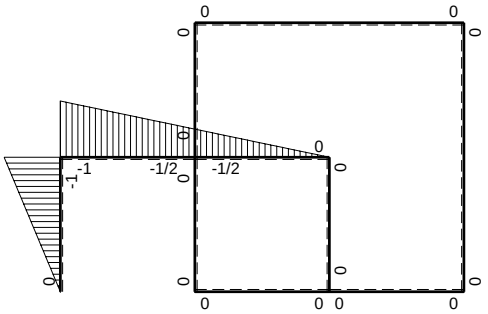
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx^2/b-1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/24+1/2)Fb^2/EJ$	1/3Xb/EJ
BA b	1-x/b	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-Fx^2/b+1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
BC b	-1+1/2x/b	-Fx	0	$Fx-1/2Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(1/3+0)Fb^2/EJ$	7/12Xb/EJ
CB b	1/2+1/2x/b	Fb-Fx	0	$1/2Fb-1/2Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	-1/2+1/2x/b	-Fb+Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/12Xb/EJ
DC b	1/2x/b	Fx	0	$1/2Fx^2/b$	0	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-7/2Fx	0	0	0	0	0+0	0
FE b	0	-7/2Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+5/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^2/EJ$	
	totali						$1/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						-1/24Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

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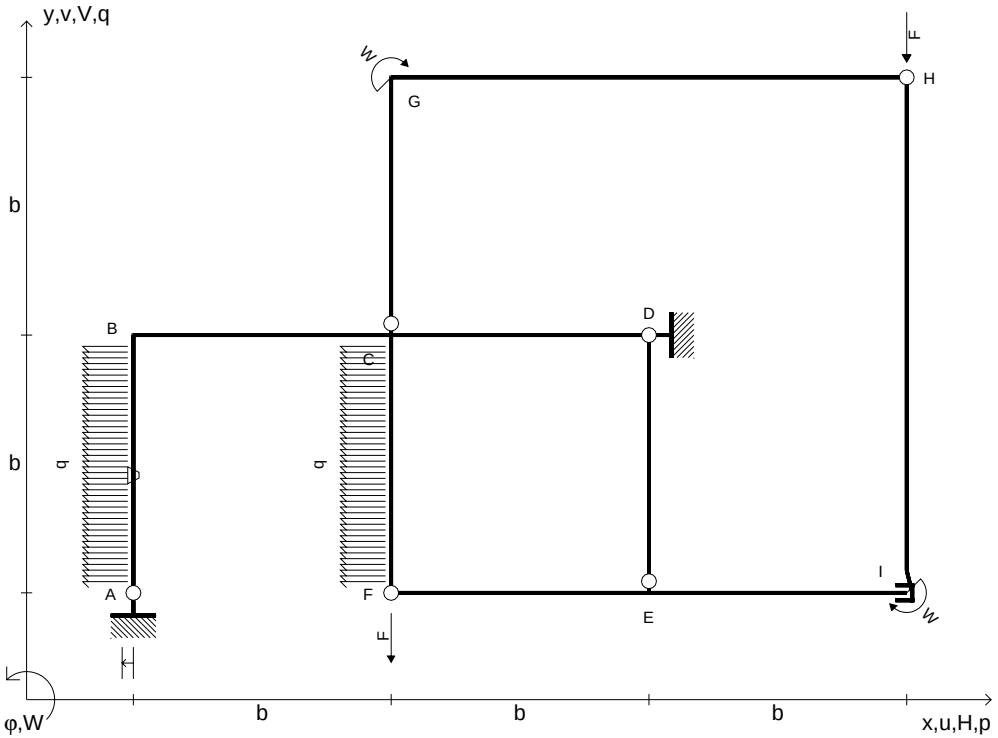


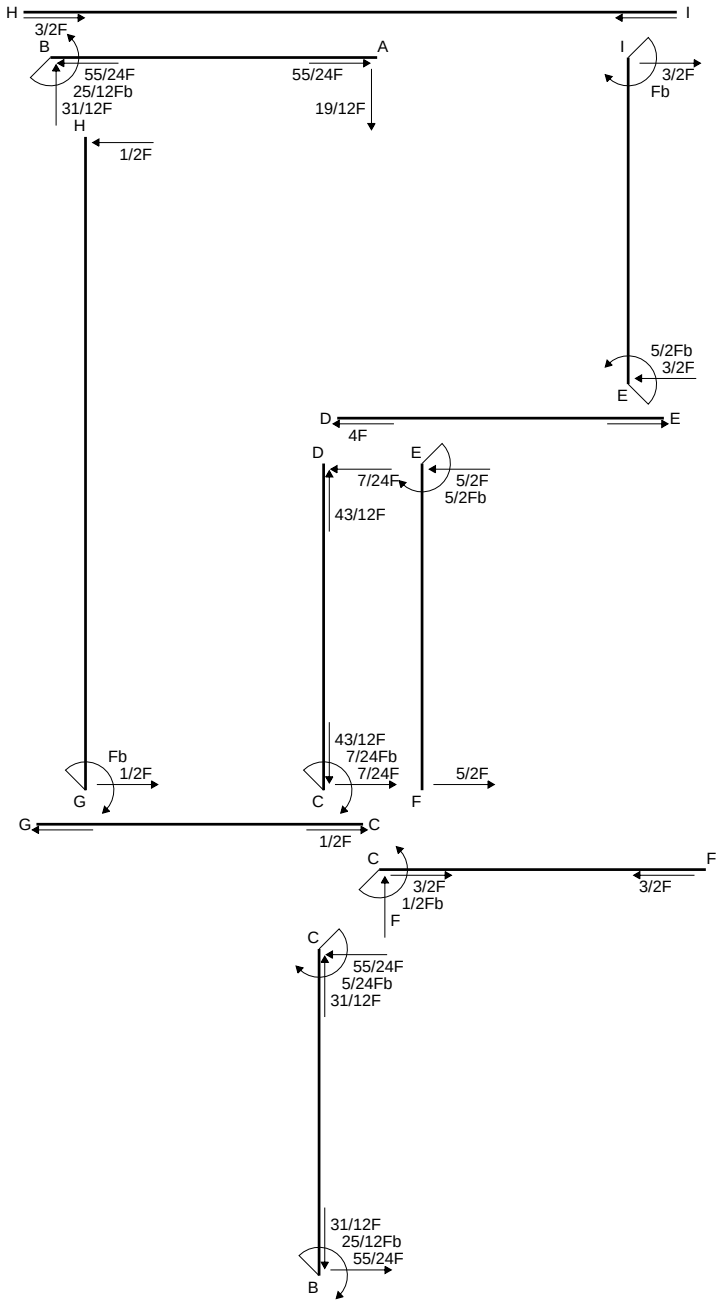
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$p_{FC} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

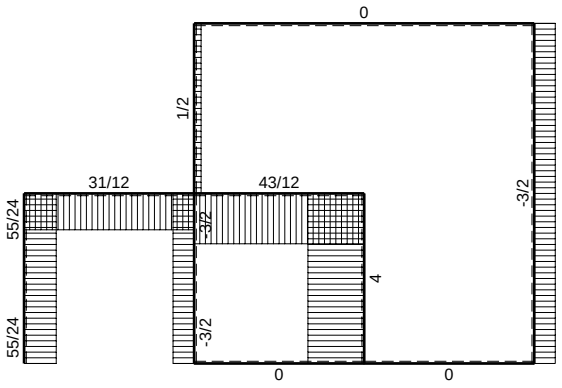
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.

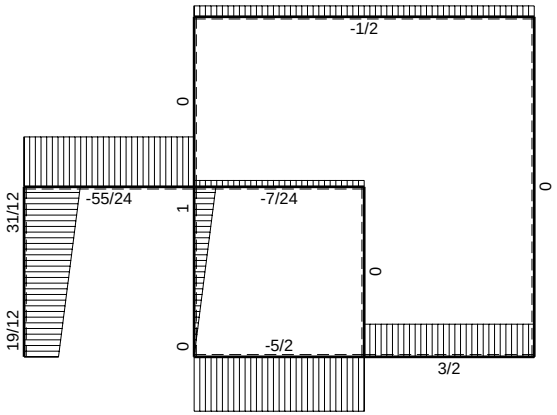
 08.01.25



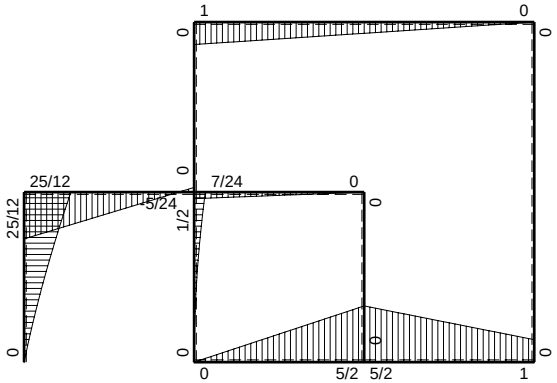
 08.01.25



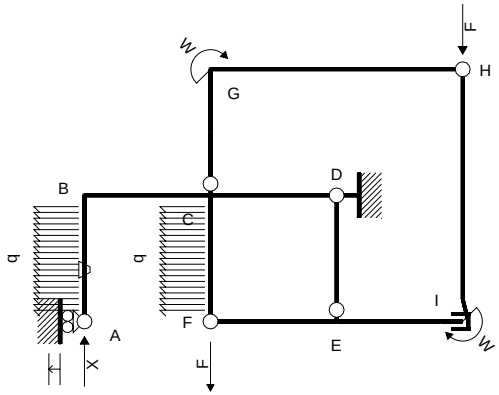
←□→ F



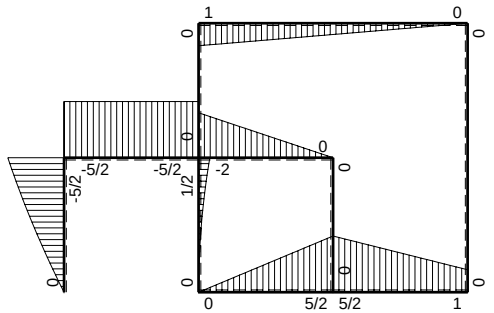
↑□↓ F



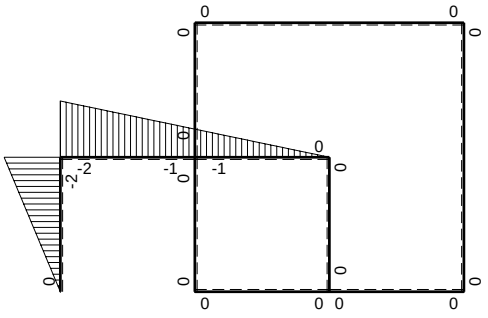
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-2x	$-3Fx+1/2qx^2$	-Fb/EJ	$6Fx^2-qx^3$	2Fxb/EJ	$4x^2$	$(7/4+1)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	2b-2x	$5/2Fb-2Fx-1/2qx^2$	Fb/EJ	$5Fb^2-9Fbx+3Fx^2+qx^3$	$2Fb^2/EJ-2Fxb/EJ$	$4b^2-8bx+4x^2$		
BC b	-2b+x	-5/2Fb	0	$5Fb^2-5/2Fbx$	0	$4b^2-4bx+x^2$	$(15/4+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	b+x	5/2Fb	0	$5/2Fb^2+5/2Fbx$	0	$b^2+2bx+x^2$		
CD b	-b+x	-2Fb+2Fx	0	$2Fb^2-4Fbx+2Fx^2$	0	$b^2-2bx+x^2$	$(2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	x	2Fx	0	$2Fx^2$	0	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	5/2Fb-5/2Fx	0	0	0	0	0+0	0
FE b	0	-5/2Fx	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-5/2Fb+3/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						$2Fb^3/EJ$	
	totali						$55/6Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_A$						-55/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 4\ b + 4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (4 - 4\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 2\ x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 2\ b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1 + 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x + x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b + b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (6\ x^2/b^2 - x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (2\ x/b)\ \theta\ dx$

$= [2\ x^3/b^2 - 1/4\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [x^2/b]_o^b\ \theta$

$= (2\ b - 1/4\ b)\ Fb^2\ 1/EJ + (b)\ \theta = 11/4\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (5 - 9\ x/b + 3\ x^2/b^2 + x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (-2 + 2\ x/b)\ \theta\ dx$

$= [5\ x - 9/2\ x^2/b + x^3/b^2 + 1/4\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [-2\ x + x^2/b]_o^b\ \theta$

$= (5\ b - 9/2\ b + b + 1/4\ b)\ Fb^2\ 1/EJ + (-2\ b + b)\ \theta = 11/4\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (5 - 5/2\ x/b)\ Fb^2\ 1/EJ\ dx = [5\ x - 5/4\ x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (5\ b - 5/4\ b)\ Fb^2\ 1/EJ = 15/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (5/2 + 5/2\ x/b)\ Fb^2\ 1/EJ\ dx = [5/2\ x + 5/4\ x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (5/2\ b + 5/4\ b)\ Fb^2\ 1/EJ = 15/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (2 - 4\ x/b + 2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [2\ x - 2\ x^2/b + 2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (2\ b - 2\ b + 2/3\ b)\ Fb^2\ 1/EJ = 2/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (2/3\ b)\ Fb^2\ 1/EJ = 2/3\ Fb^3/EJ$

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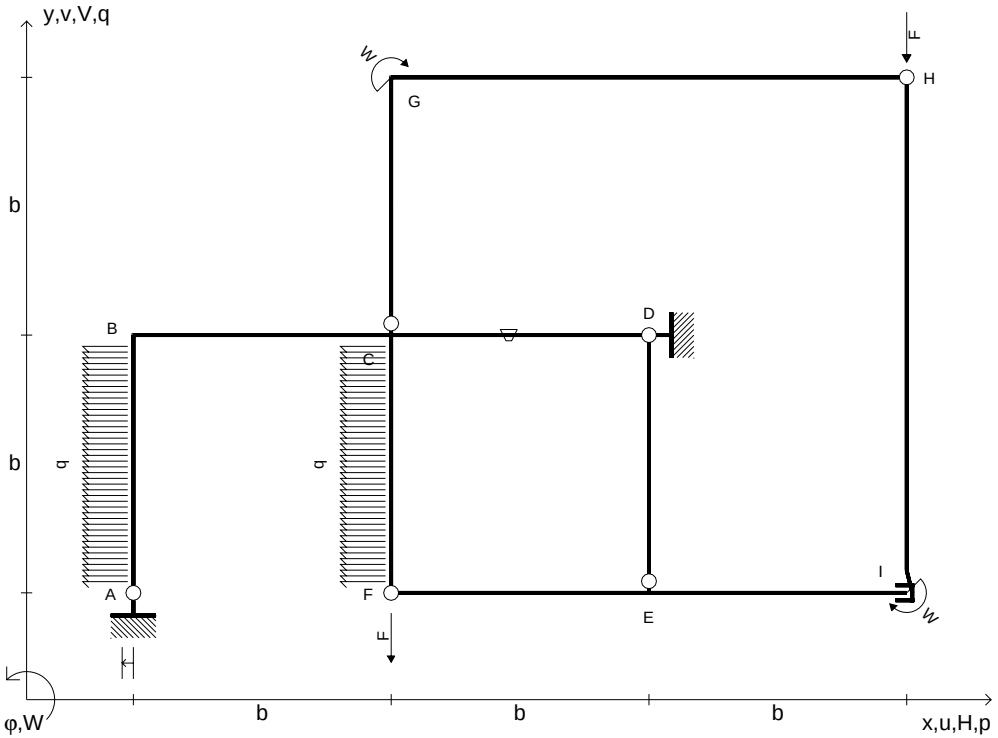
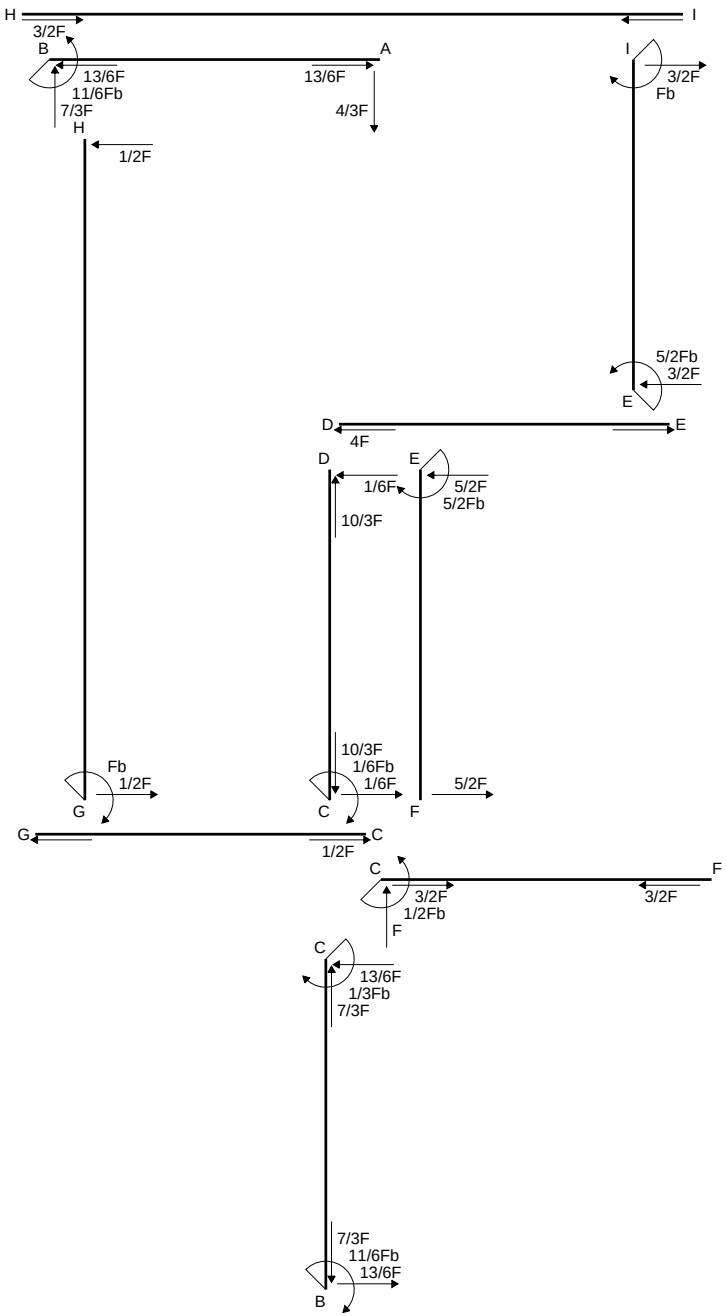


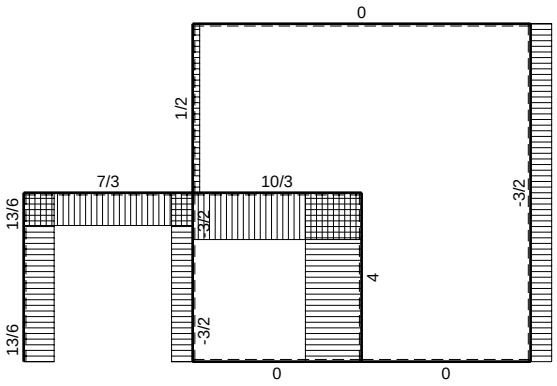
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$p_{FC} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

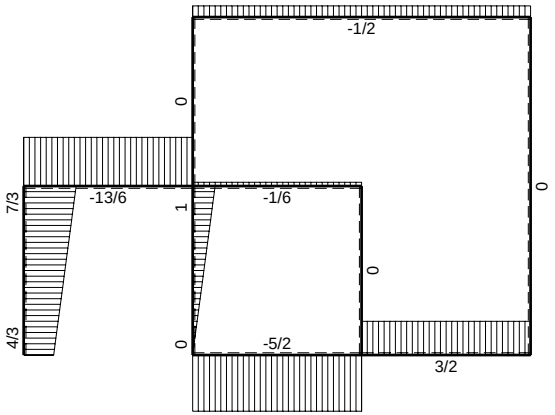
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo A.

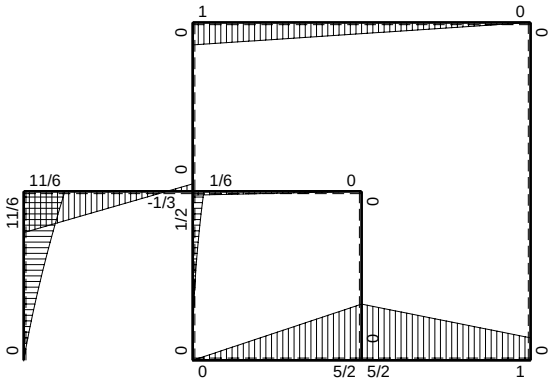




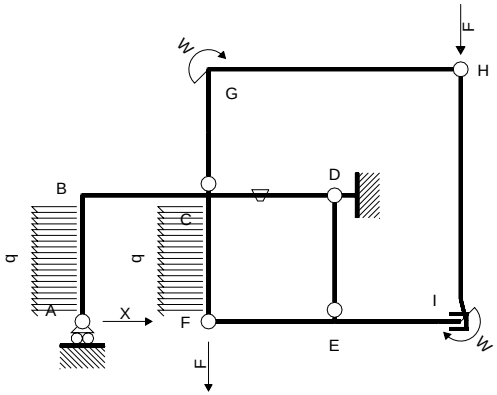
←□→ F



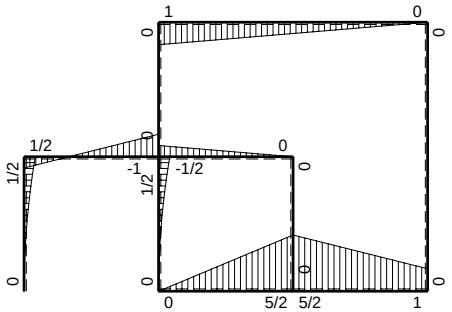
↑□↓ F



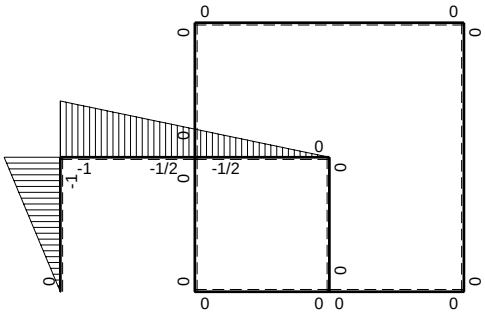
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211730 ESΣ01.xxxx.031

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-3/2Fx$	0	$-1/2Fb^2+7/4Fbx-3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$Fb-3/2Fx$	0	$1/2Fb^2-1/4Fbx-3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	-Fb/EJ	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$5/2Fb-5/2Fx$	0	0	0	0	0+0	0
FE b	0	$-5/2Fx$	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$4/3Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-4/3F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +7/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +7/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +7/8\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/8\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/12\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

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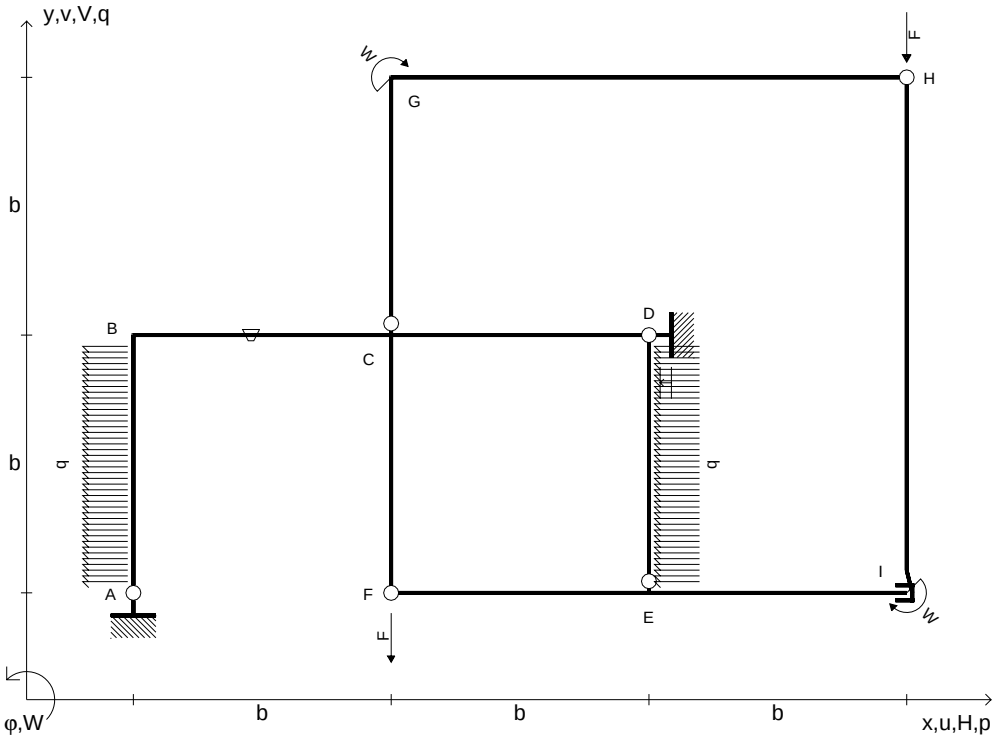

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$p_{DE} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
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Reazioni iperstatiche in soluzione: $X=H_A$
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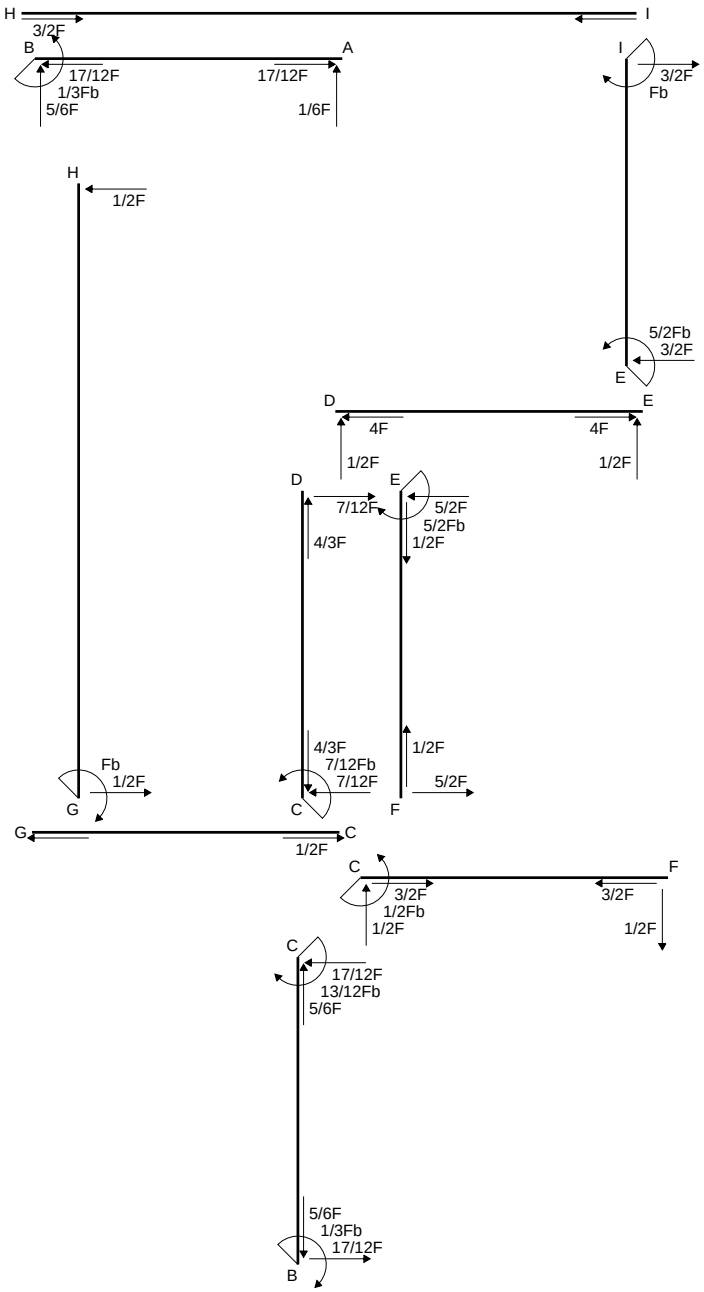
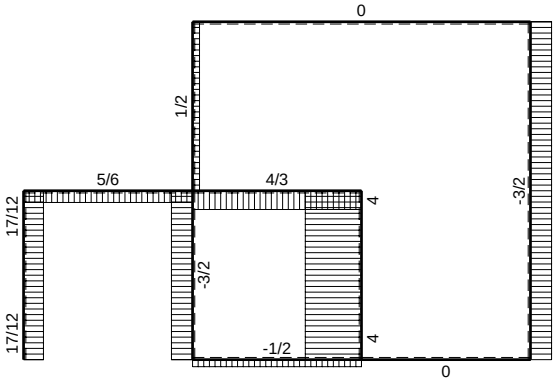
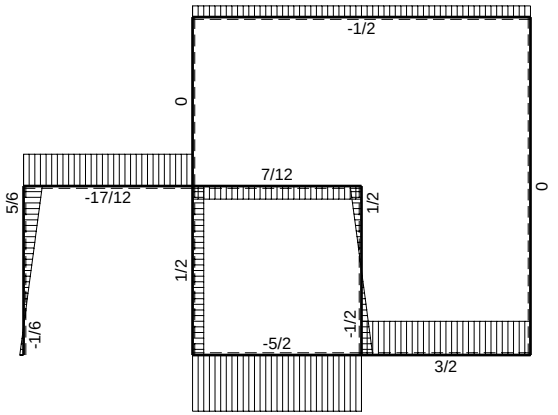
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.





F



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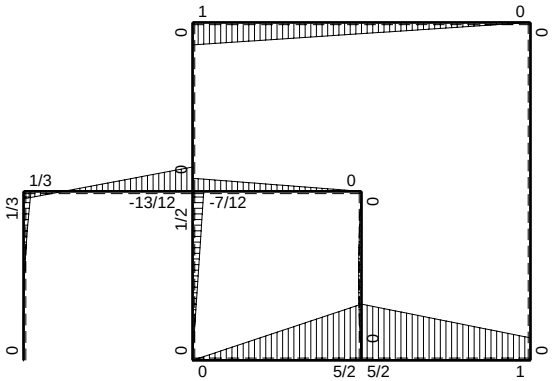
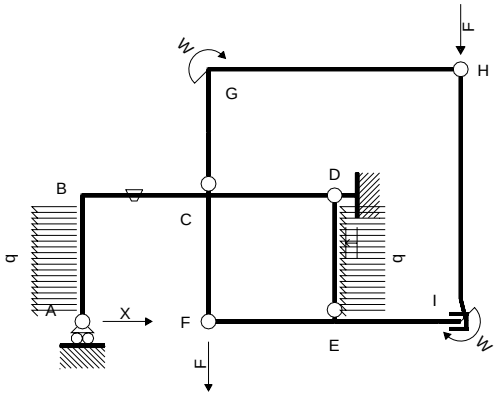
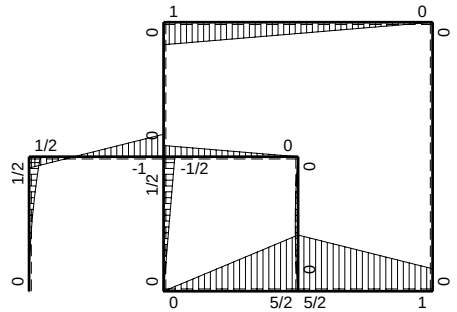


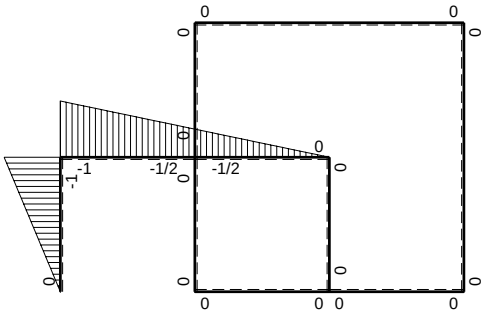
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212694 ESΣ01.xxxx.034

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-3/2Fx$	$-Fb/EJ$	$-1/2Fb^2+7/4Fbx-3/4Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/8+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$Fb-3/2Fx$	Fb/EJ	$1/2Fb^2-1/4Fbx-3/4Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	$5/2Fb-5/2Fx$	0	0	0	0	0+0	0
FE b	0	$-5/2Fx$	0	0	0	0		
FC b	0	$1/2Fx$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-1/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						1/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +7/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\, dx$

$= [-1/2\, x +7/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [\, x -1/4\, x^2/b]_o^b\, \theta$

$= (-1/2\, b +7/8\, b -1/4\, b)\, Fb^2\, 1/EJ + (\, b -1/4\, b)\, \theta = 7/8\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x -1/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/8\, b -1/4\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 7/8\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 212694 ESΣ01.xxxx.034

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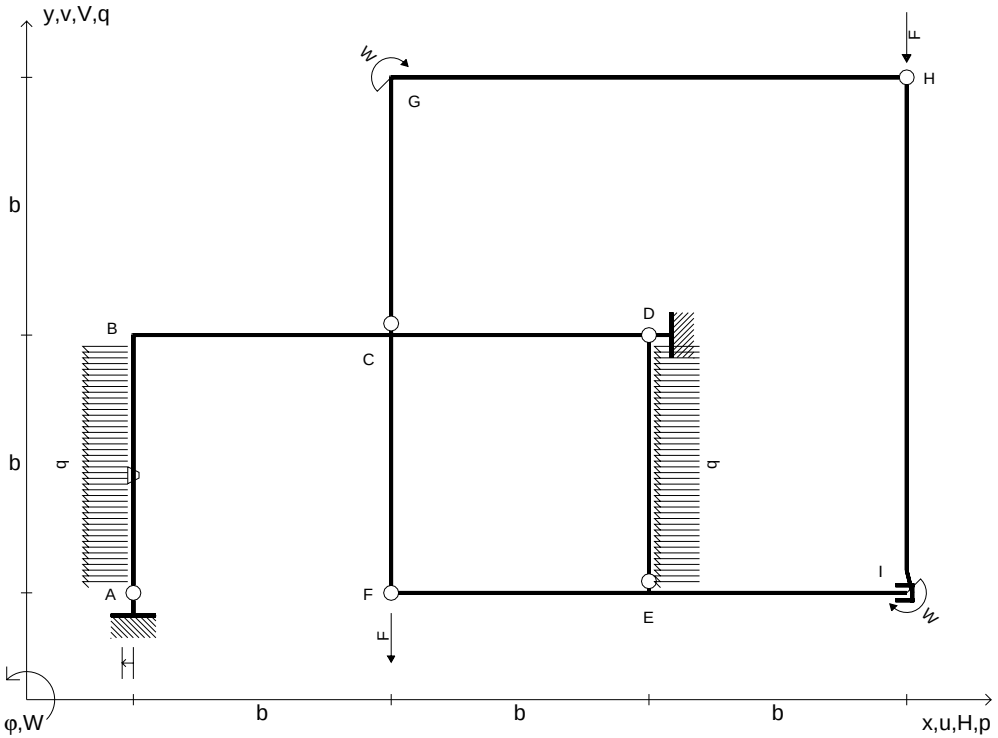


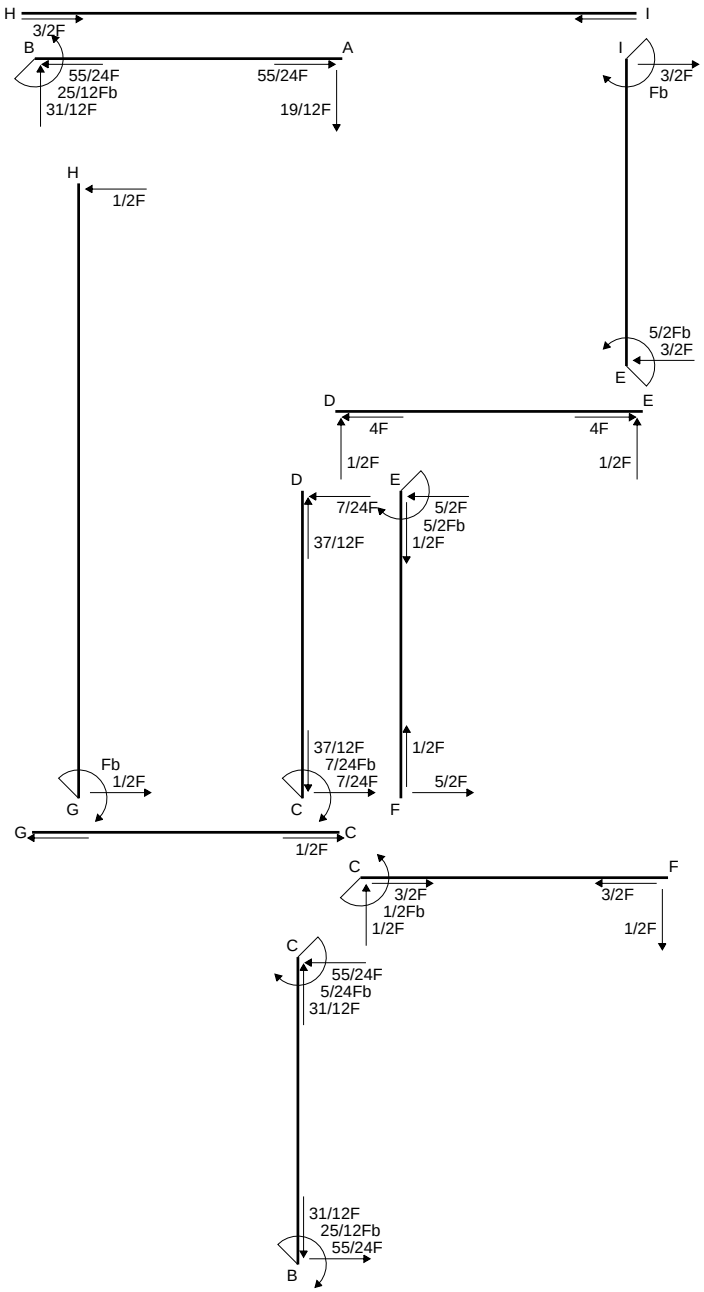
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$p_{DE} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

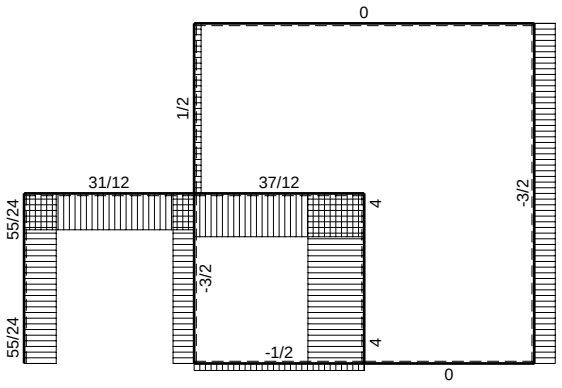
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_D$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.

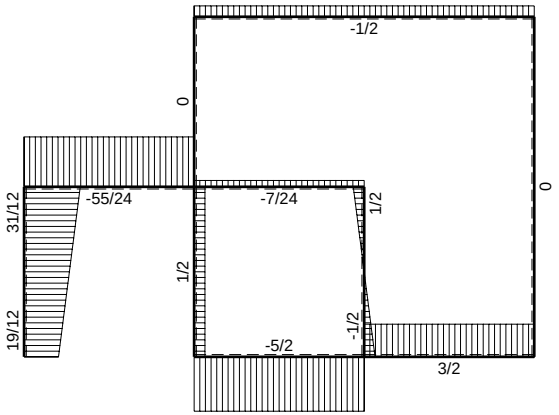
 08.01.25



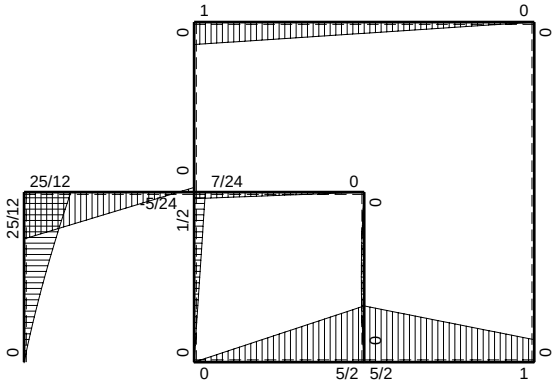
 08.01.25



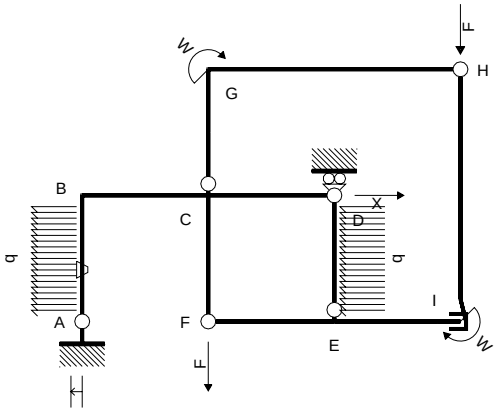
F



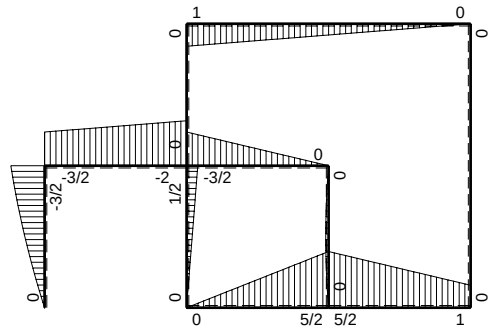
F



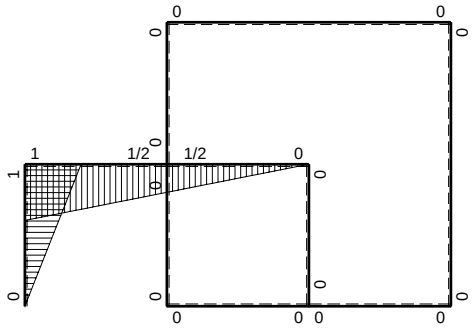
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_D$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	x	$-2Fx+1/2qx^2$	$-Fb/EJ$	$-2Fx^2+1/2qx^3$	$-Fxb/EJ$	x^2	$(-13/24-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	$-b+x$	$3/2Fb-Fx-1/2qx^2$	Fb/EJ	$-3/2Fb^2+5/2Fbx-1/2Fx^2-1/2qx^3$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$		
BC b	$b-1/2x$	$-3/2Fb-1/2Fx$	0	$-3/2Fb^2+1/4Fbx+1/4Fx^2$	0	$b^2-bx+1/4x^2$	$(-31/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$-1/2b-1/2x$	$2Fb-1/2Fx$	0	$-Fb^2-3/4Fbx+1/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$1/2b-1/2x$	$-3/2Fb+3/2Fx$	0	$-3/4Fb^2+3/2Fbx-3/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/4+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$-1/2x$	$3/2Fx$	0	$-3/4Fx^2$	0	$1/4x^2$		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	$5/2Fb-5/2Fx$	0	0	0	0	0+0	0
FE b	0	$-5/2Fx$	0	0	0	0		
FC b	0	$1/2Fx$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						$-Fb^3/EJ$	
	totali						$-43/12Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_D$						43/12F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212877 ESΣ01.xxxx.035

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (- x/b)\, \theta\, dx$

$= [-2/3\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [-1/2\, x^2/b]_o^b\, \theta$

$= (-2/3\, b +1/8\, b)\, Fb^2\, 1/EJ + (-1/2\, b)\, \theta = -25/24\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-3/2 +5/2\, x/b -1/2\, x^2/b^2 -1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 - x/b)\, \theta\, dx$

$= [-3/2\, x +5/4\, x^2/b -1/6\, x^3/b^2 -1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [\, x -1/2\, x^2/b]_o^b\, \theta$

$= (-3/2\, b +5/4\, b -1/6\, b -1/8\, b)\, Fb^2\, 1/EJ + (\, b -1/2\, b)\, \theta = -25/24\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-3/2 +1/4\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/2\, x +1/8\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b +1/8\, b +1/12\, b)\, Fb^2\, 1/EJ = -31/24\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-1 -3/4\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x -3/8\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (- b -3/8\, b +1/12\, b)\, Fb^2\, 1/EJ = -31/24\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (-3/4 +3/2\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/4\, x +3/4\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-3/4\, b +3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = -1/4\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (-3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/4\, b)\, Fb^2\, 1/EJ = -1/4\ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 212877 ESΣ01.xxxx.035

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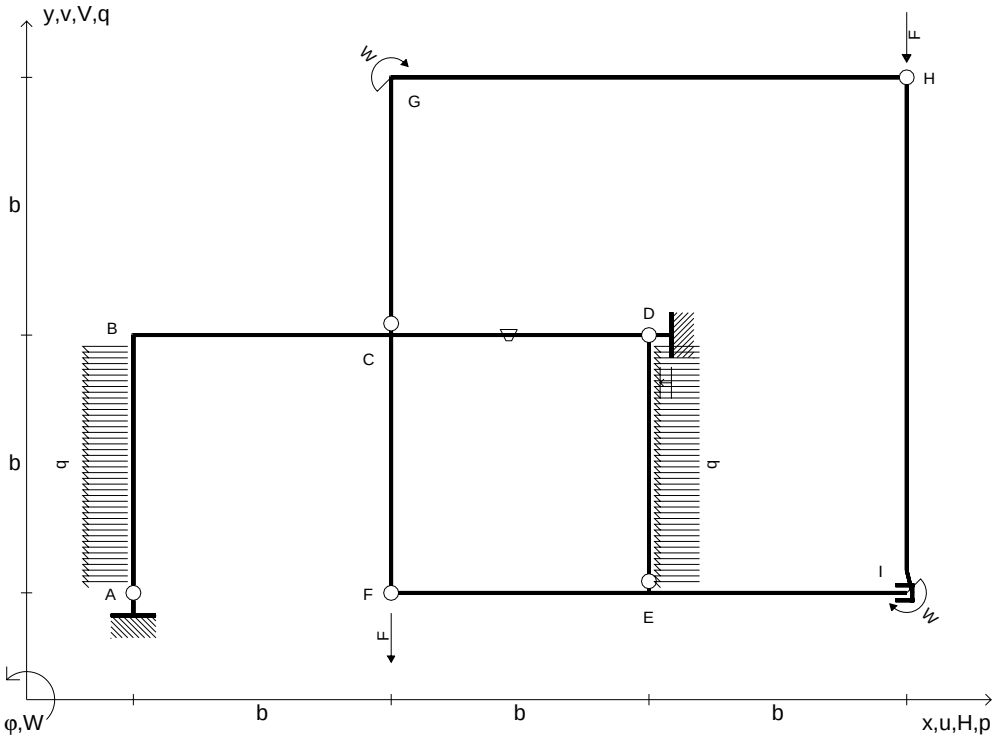


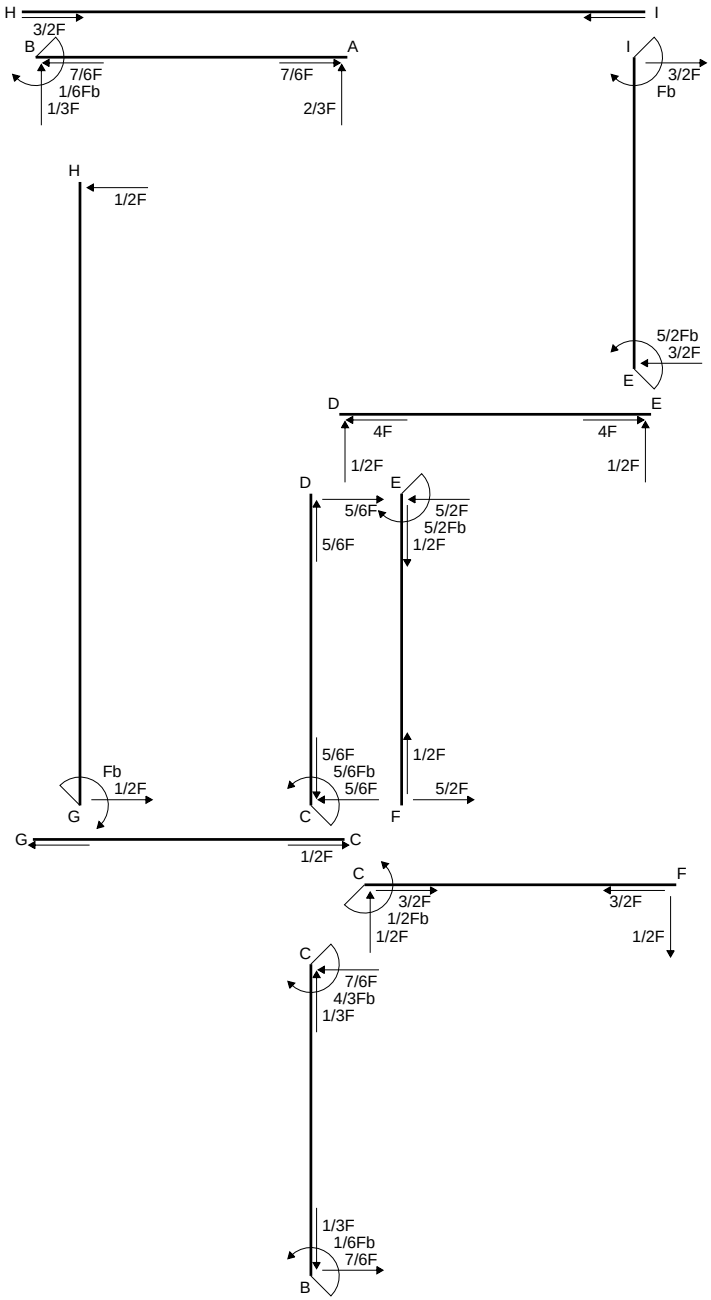
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{AB} = -q = -F/b$
$p_{DE} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

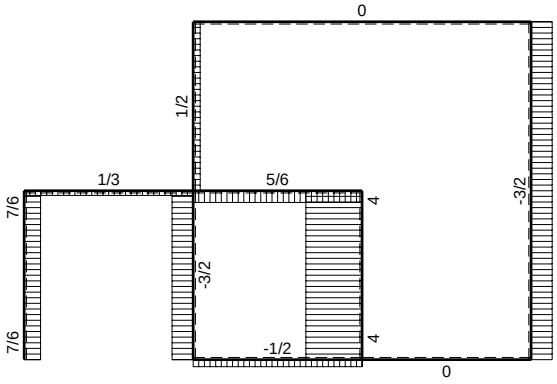
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo D.

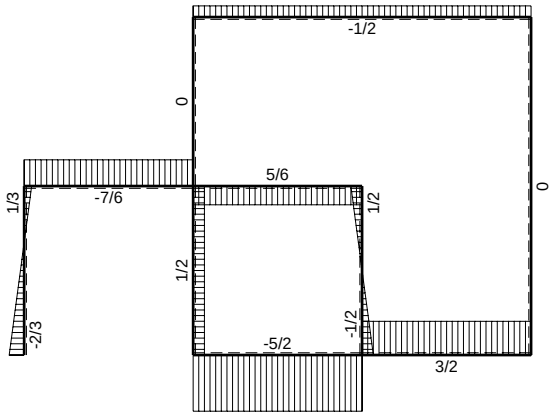
 08.01.25



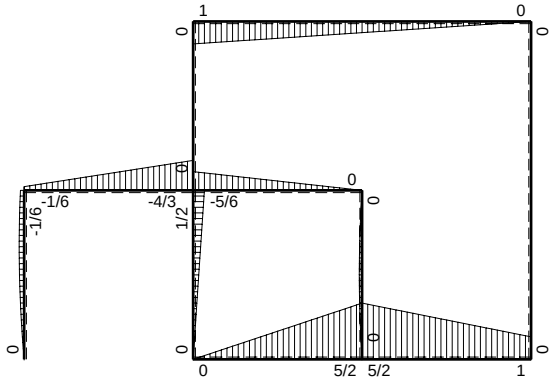
 08.01.25



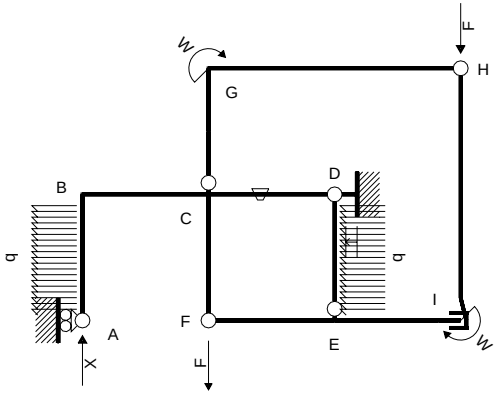
←□→ F



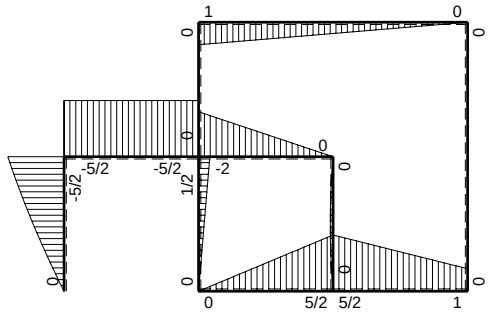
↑□↓ F



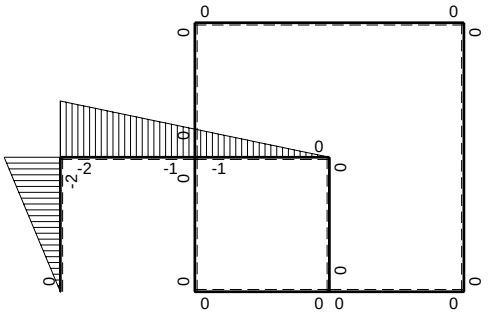
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211605 ESΣ01.xxxx.038

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-2x$	$-3Fx+1/2qx^2$	0	$6Fx^2-qx^3$	0	$4x^2$	$(7/4+0)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	$2b-2x$	$5/2Fb-2Fx-1/2qx^2$	0	$5Fb^2-9Fbx+3Fx^2+qx^3$	0	$4b^2-8bx+4x^2$		
BC b	$-2b+x$	$-5/2Fb$	0	$5Fb^2-5/2Fbx$	0	$4b^2-4bx+x^2$	$(15/4+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	$b+x$	$5/2Fb$	0	$5/2Fb^2+5/2Fbx$	0	$b^2+2bx+x^2$		
CD b	$-b+x$	$-2Fb+2Fx$	$-Fb/EJ$	$2Fb^2-4Fbx+2Fx^2$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$	$(2/3+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	x	$2Fx$	Fb/EJ	$2Fx^2$	Fxb/EJ	x^2		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	$5/2Fb-5/2Fx$	0	0	0	0	0+0	0
FE b	0	$-5/2Fx$	0	0	0	0		
FC b	0	$1/2Fx$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-2Fb^3/EJ$	
	totali						$14/3Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_A$						$-7/6F$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 4\ b + 4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (4 - 4\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 2\ x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 2\ b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1 + 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x + x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b + b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (6\ x^2/b^2 - x^3/b^3)\ Fb^2\ 1/EJ\ dx = [2\ x^3/b^2 - 1/4\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (2\ b - 1/4\ b)\ Fb^2\ 1/EJ = 7/4\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (5 - 9\ x/b + 3\ x^2/b^2 + x^3/b^3)\ Fb^2\ 1/EJ\ dx = [5\ x - 9/2\ x^2/b + x^3/b^2 + 1/4\ x^4/b^3]_o^b\ Fb^2\ 1/EJ$

$= (5\ b - 9/2\ b + b + 1/4\ b)\ Fb^2\ 1/EJ = 7/4\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (5 - 5/2\ x/b)\ Fb^2\ 1/EJ\ dx = [5\ x - 5/4\ x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (5\ b - 5/4\ b)\ Fb^2\ 1/EJ = 15/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (5/2 + 5/2\ x/b)\ Fb^2\ 1/EJ\ dx = [5/2\ x + 5/4\ x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (5/2\ b + 5/4\ b)\ Fb^2\ 1/EJ = 15/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (2 - 4\ x/b + 2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1 - x/b)\ \theta\ dx$

$= [2\ x - 2\ x^2/b + 2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [x - 1/2\ x^2/b]_o^b\ \theta$

$= (2\ b - 2\ b + 2/3\ b)\ Fb^2\ 1/EJ + (b - 1/2\ b)\ \theta = 7/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-x/b)\ \theta\ dx = [2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/2\ x^2/b]_o^b\ \theta$

$= (2/3\ b)\ Fb^2\ 1/EJ + (-1/2\ b)\ \theta = 7/6\ Fb^3/EJ$

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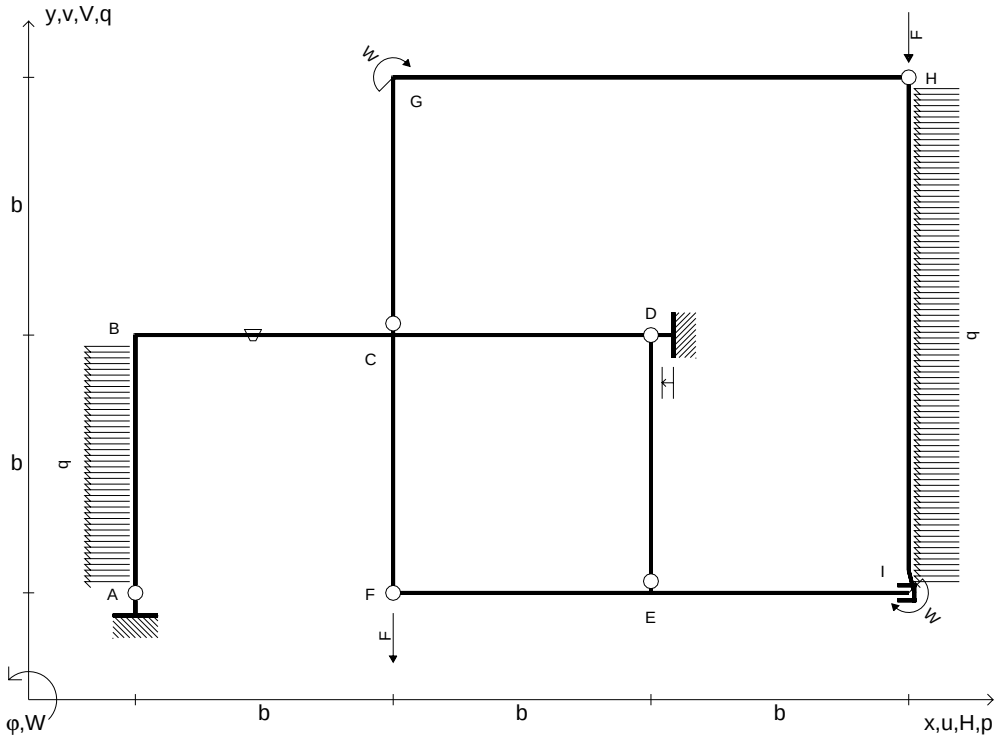
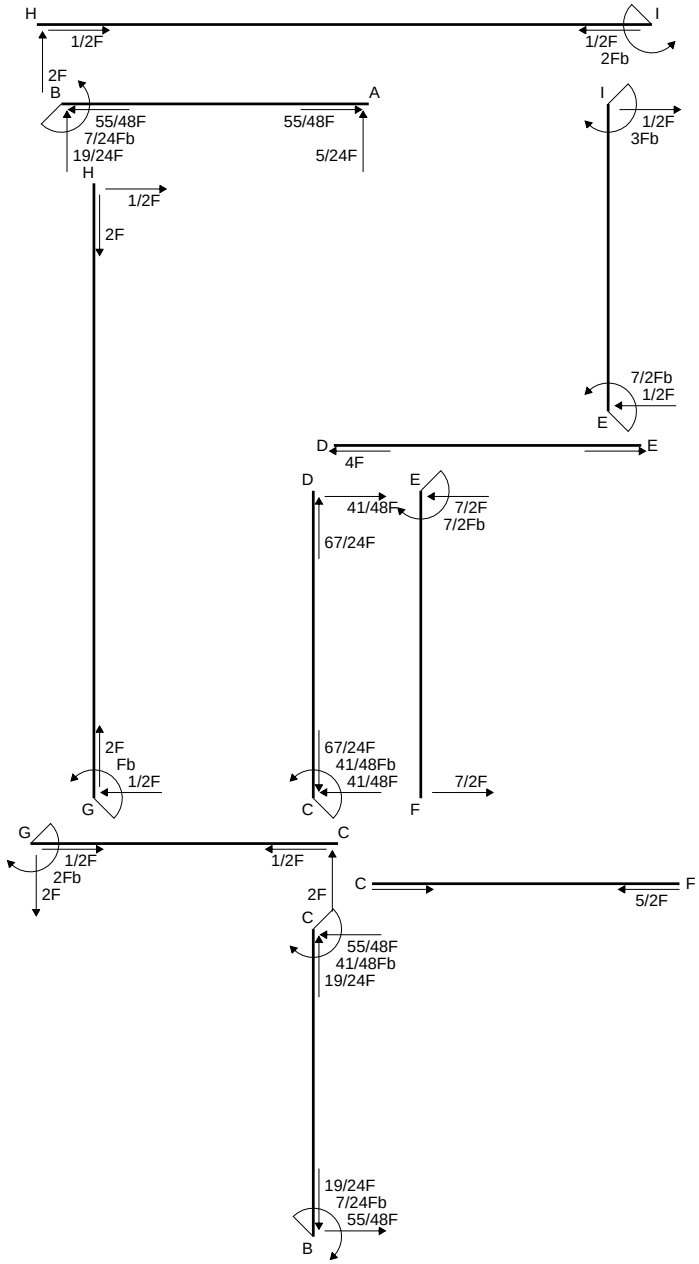


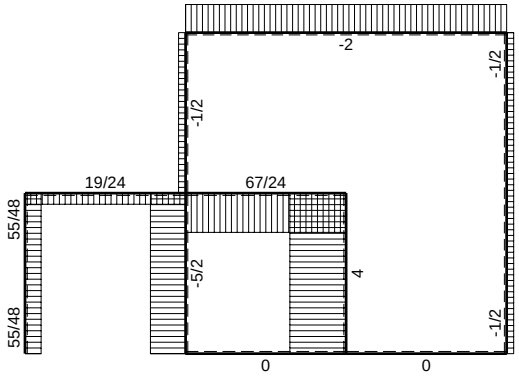
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$p_{AB} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

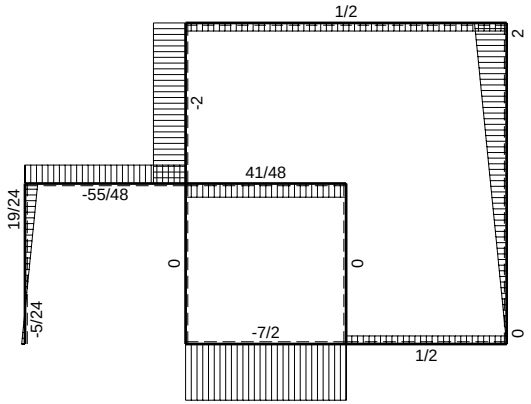
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.





F



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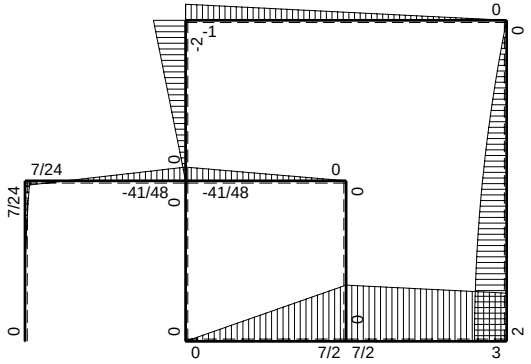
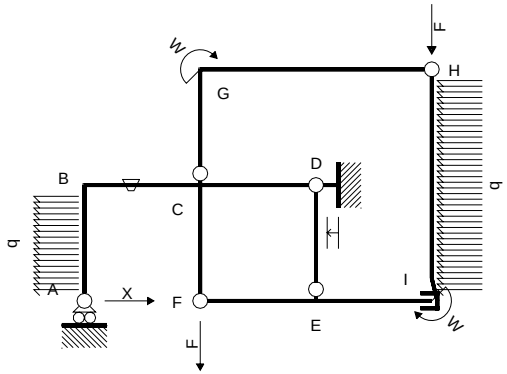
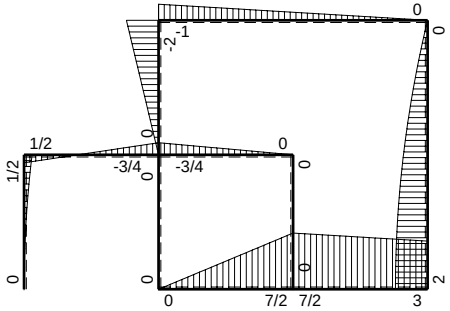


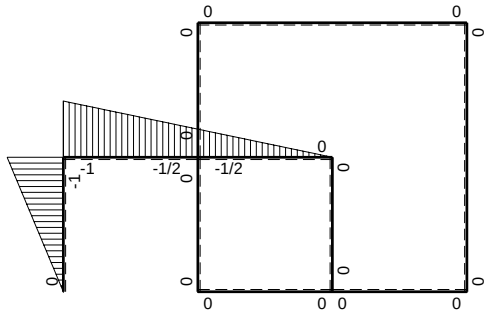
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-5/4Fx$	$-Fb/EJ$	$-1/2Fb^2+3/2Fbx-5/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/24+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-5/4Fx$	Fb/EJ	$3/8Fb^2-1/4Fbx-5/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-7/2Fx$	0	0	0	0	0+0	0
FE b	0	$-7/2Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-2Fx$	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-5/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						5/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +3/2\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\, dx$

$= [-1/2\, x +3/4\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [\, x -1/4\, x^2/b]_o^b\, \theta$

$= (-1/2\, b +3/4\, b -5/24\, b)\, Fb^2\, 1/EJ + (\, b -1/4\, b)\, \theta = 19/24\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -1/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\, dx$

$= [3/8\, x -1/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b -5/24\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 19/24\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

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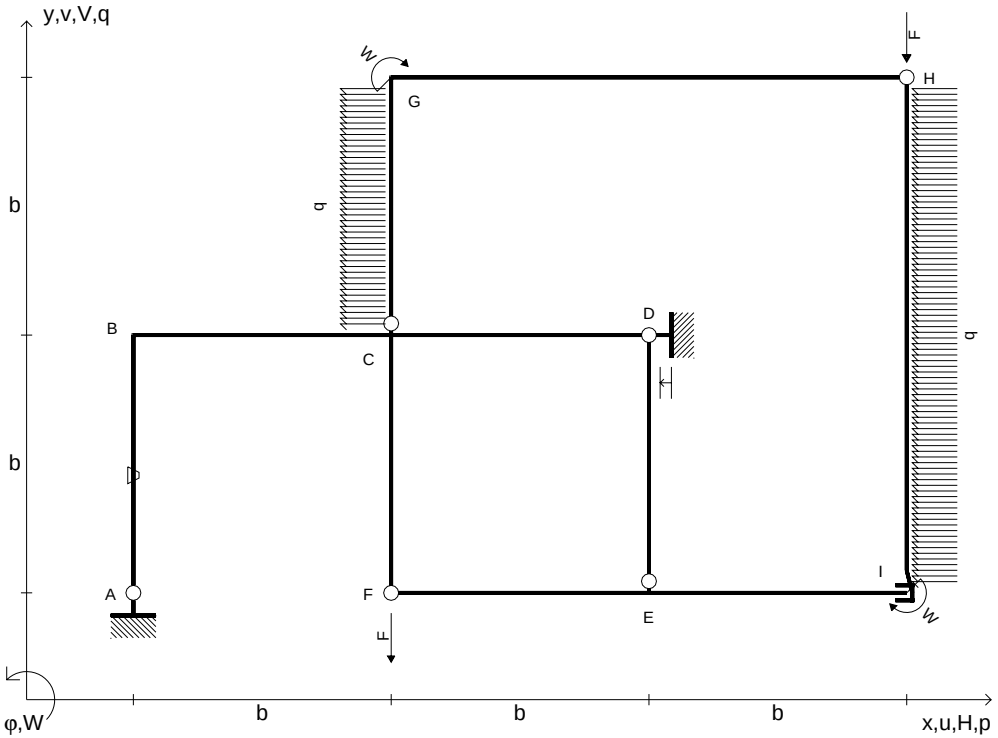

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$p_{CG} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
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Reazioni iperstatiche in soluzione: $X=H_D$
Carichi e deformazioni date hanno verso efficace in disegno.
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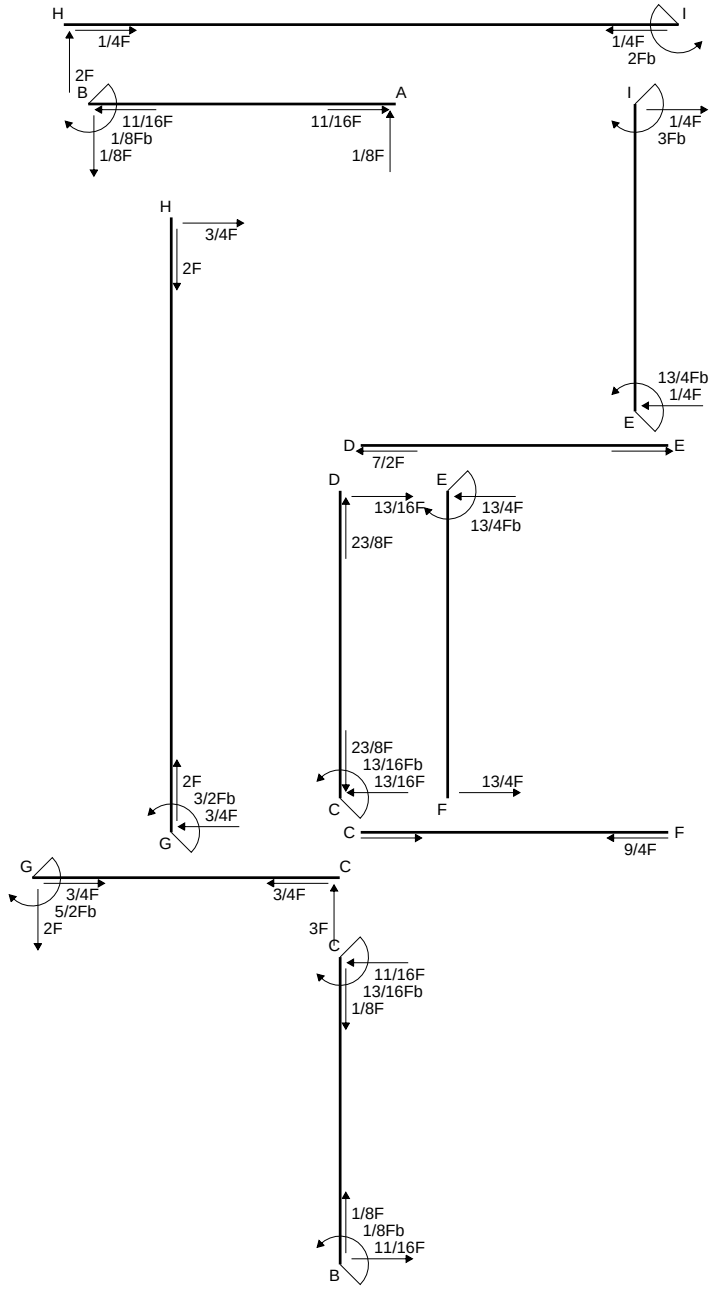
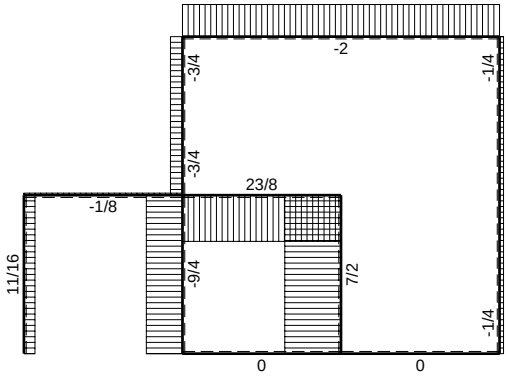
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.





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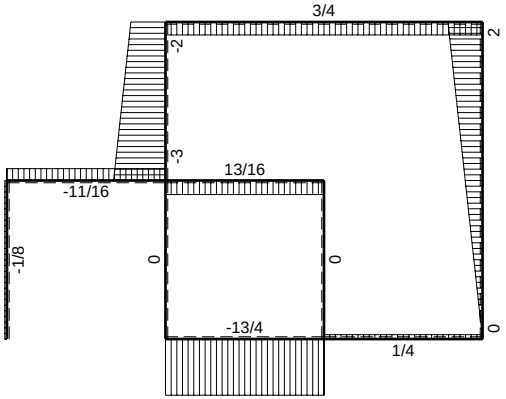

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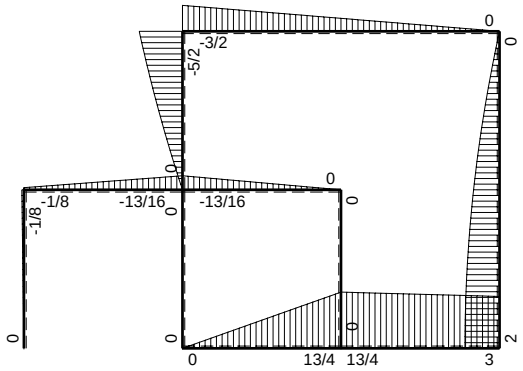
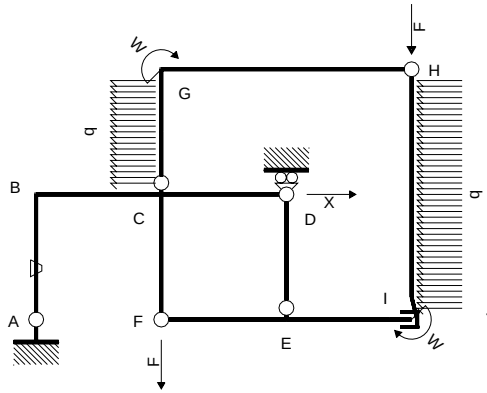
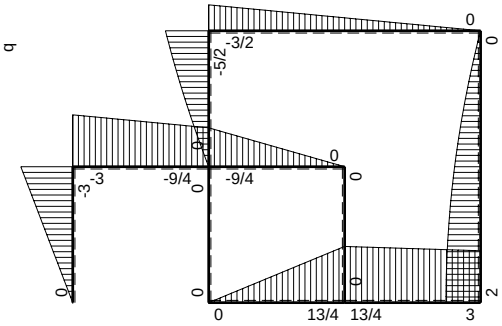


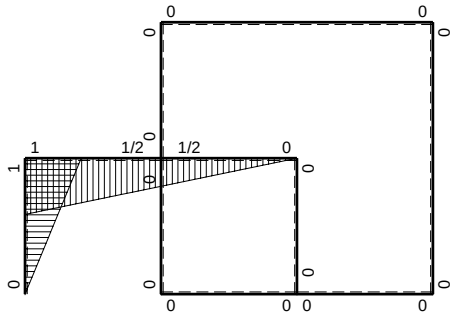
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_D$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	x	$-3Fx$	$-Fb/EJ$	$-3Fx^2$	$-Fxb/EJ$	x^2	$(-1-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	$-b+x$	$3Fb-3Fx$	Fb/EJ	$-3Fb^2+6Fbx-3Fx^2$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$		
BC b	$b-1/2x$	$-3Fb+3/4Fx$	0	$-3Fb^2+9/4Fbx-3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(-2+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$-1/2b-1/2x$	$9/4Fb+3/4Fx$	0	$-9/8Fb^2-3/2Fbx-3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$1/2b-1/2x$	$-9/4Fb+9/4Fx$	0	$-9/8Fb^2+9/4Fbx-9/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-3/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$-1/2x$	$9/4Fx$	0	$-9/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$13/4Fb-13/4Fx$	0	0	0	0	0+0	0
FE b	0	$-13/4Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-3Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$5/2Fb-2Fx-1/2qx^2$	0	0	0	0		
GH 2b	0	$-3/2Fb+3/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$3/4Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/4Fx$	0	0	0	0	0+0	0
EI b	0	$-13/4Fb+1/4Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						Fb^3/EJ	
	totali						$-23/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_D$						23/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-\, x/b)\, \theta\ dx = [-\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x^2/b]_o^b\, \theta$

$= (-\, b)\, Fb^2\, 1/EJ + (-1/2\, b)\, \theta = -3/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-3 +6\, x/b -3\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 - x/b)\, \theta\ dx$

$= [-3\, x +3\, x^2/b - x^3/b^2]_o^b\, Fb^2\, 1/EJ + [\, x -1/2\, x^2/b]_o^b\, \theta$

$= (-3\, b +3\, b - b)\, Fb^2\, 1/EJ + (\, b -1/2\, b)\, \theta = -3/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-3 +9/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3\, x +9/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-3\, b +9/8\, b -1/8\, b)\, Fb^2\, 1/EJ = -2\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-9/8 -3/2\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-9/8\, x -3/4\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-9/8\, b -3/4\, b -1/8\, b)\, Fb^2\, 1/EJ = -2\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (-9/8 +9/4\, x/b -9/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-9/8\, x +9/8\, x^2/b -3/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-9/8\, b +9/8\, b -3/8\, b)\, Fb^2\, 1/EJ = -3/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (-9/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-3/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-3/8\, b)\, Fb^2\, 1/EJ = -3/8\ Fb^3/EJ$

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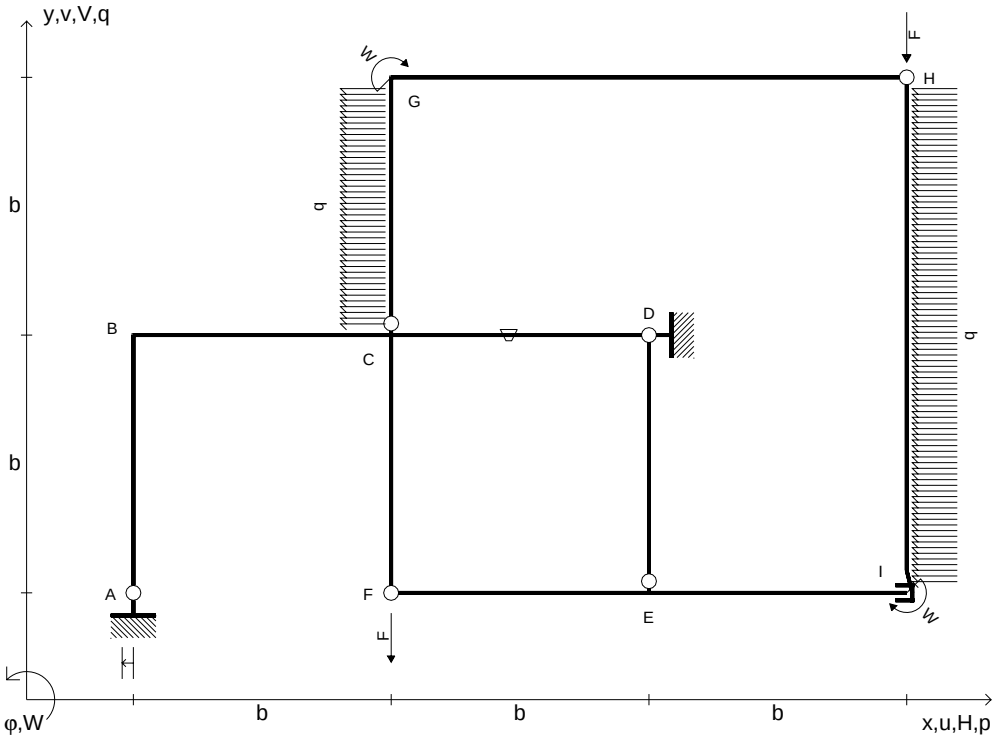
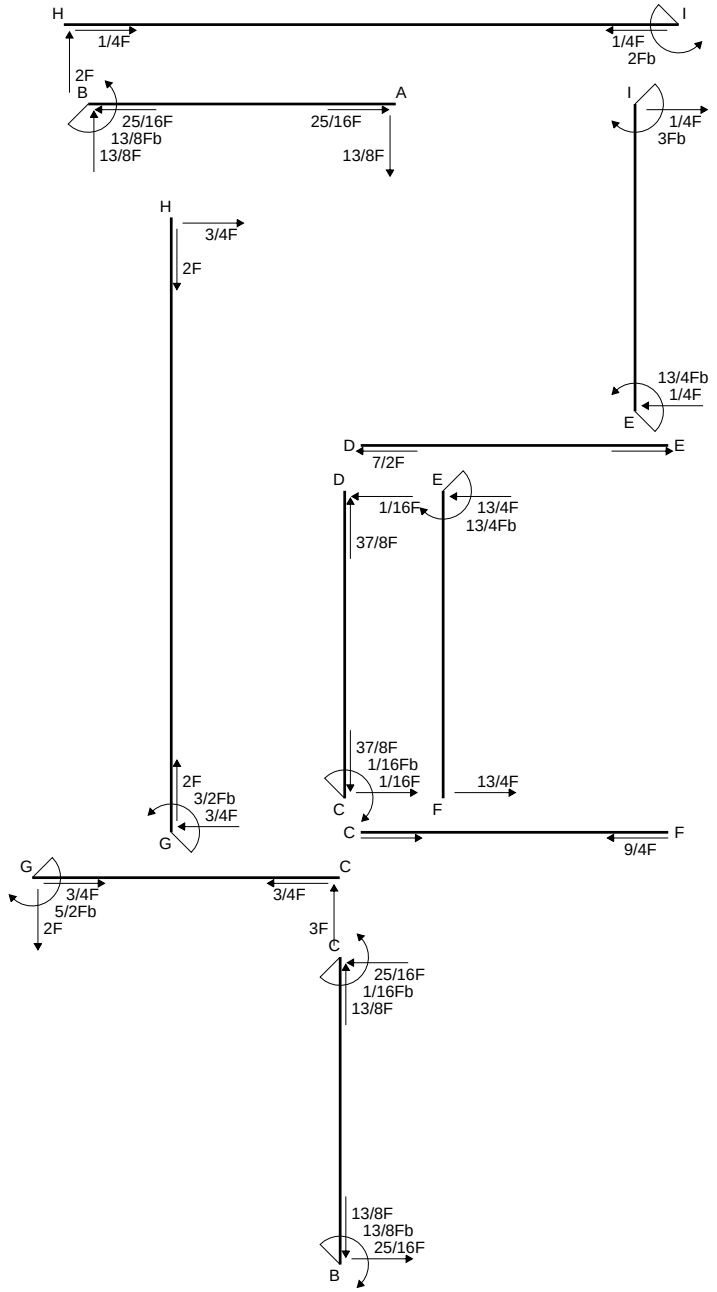


$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$p_{CG} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo A.



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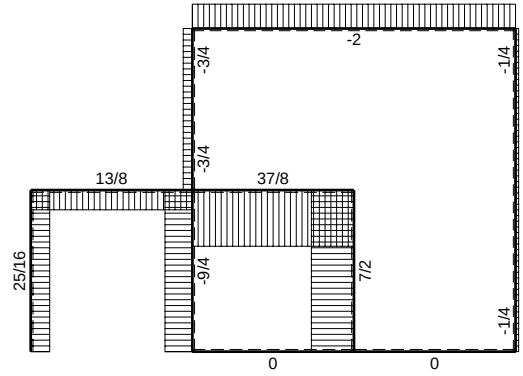

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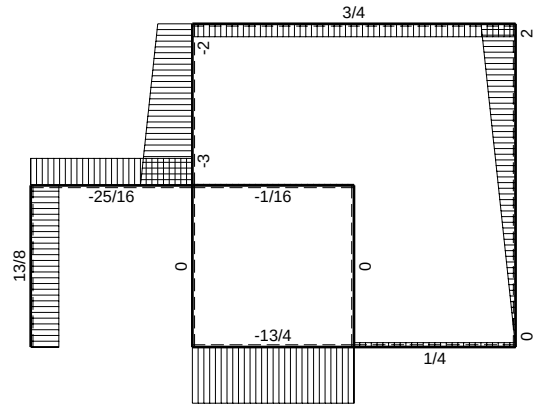

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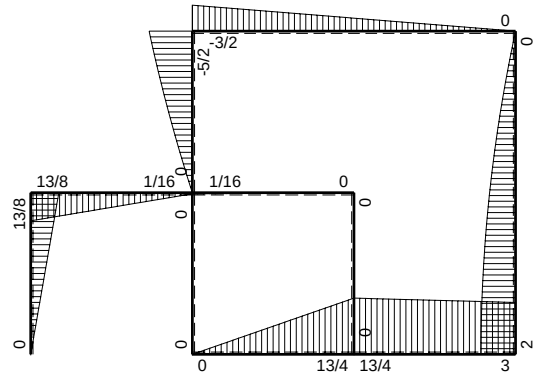
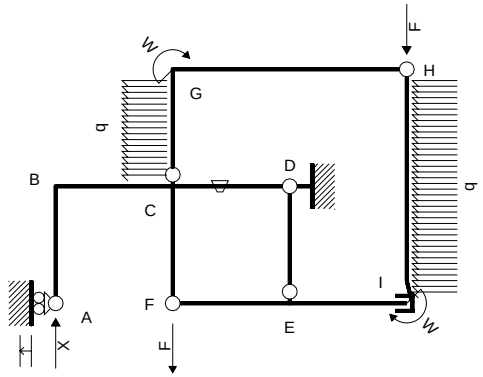
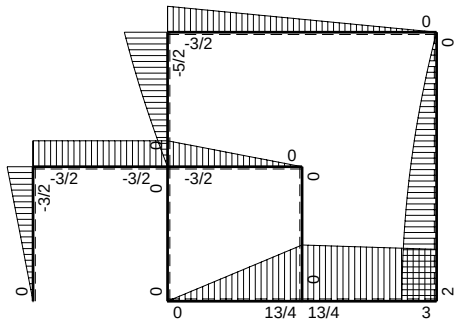


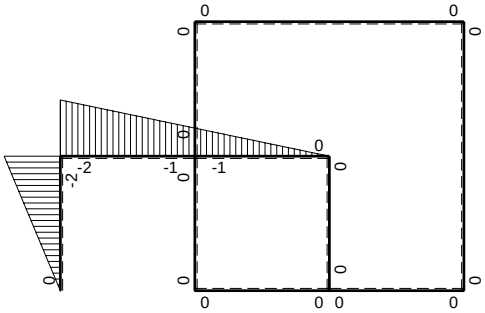
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-2x	-3/2Fx	0	$3Fx^2$	0	$4x^2$	$(1+0)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	2b-2x	3/2Fb-3/2Fx	0	$3Fb^2-6Fbx+3Fx^2$	0	$4b^2-8bx+4x^2$		
BC b	-2b+x	-3/2Fb	0	$3Fb^2-3/2Fbx$	0	$4b^2-4bx+x^2$	$(9/4+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	b+x	3/2Fb	0	$3/2Fb^2+3/2Fbx$	0	$b^2+2bx+x^2$		
CD b	-b+x	-3/2Fb+3/2Fx	-Fb/EJ	$3/2Fb^2-3Fbx+3/2Fx^2$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$	$(1/2+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	x	3/2Fx	Fb/EJ	$3/2Fx^2$	Fxb/EJ	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	13/4Fb-13/4Fx	0	0	0	0	0+0	0
FE b	0	-13/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-3Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$5/2Fb-2Fx-1/2qx^2$	0	0	0	0		
GH 2b	0	-3/2Fb+3/4Fx	0	0	0	0	0+0	0
HG 2b	0	3/4Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+1/4Fx	0	0	0	0	0+0	0
EI b	0	-13/4Fb+1/4Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						$2Fb^3/EJ$	
	totali						$25/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_A$						-25/16F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 4\ b + 4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (4 - 4\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 2\ x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 2\ b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1 + 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x + x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b + b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (3\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (b)\ Fb^2\ 1/EJ = Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (3 - 6\ x/b + 3\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [3\ x - 3\ x^2/b + x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (3\ b - 3\ b + b)\ Fb^2\ 1/EJ = Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3 - 3/2\ x/b)\ Fb^2\ 1/EJ\ dx = [3\ x - 3/4\ x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (3\ b - 3/4\ b)\ Fb^2\ 1/EJ = 9/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/2 + 3/2\ x/b)\ Fb^2\ 1/EJ\ dx = [3/2\ x + 3/4\ x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (3/2\ b + 3/4\ b)\ Fb^2\ 1/EJ = 9/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/2 - 3\ x/b + 3/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1 - x/b)\ \theta\ dx$

$= [3/2\ x - 3/2\ x^2/b + 1/2\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [x - 1/2\ x^2/b]_o^b\ \theta$

$= (3/2\ b - 3/2\ b + 1/2\ b)\ Fb^2\ 1/EJ + (b - 1/2\ b)\ \theta = Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-x/b)\ \theta\ dx = [1/2\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-1/2\ x^2/b]_o^b\ \theta$

$= (1/2\ b)\ Fb^2\ 1/EJ + (-1/2\ b)\ \theta = Fb^3/EJ$

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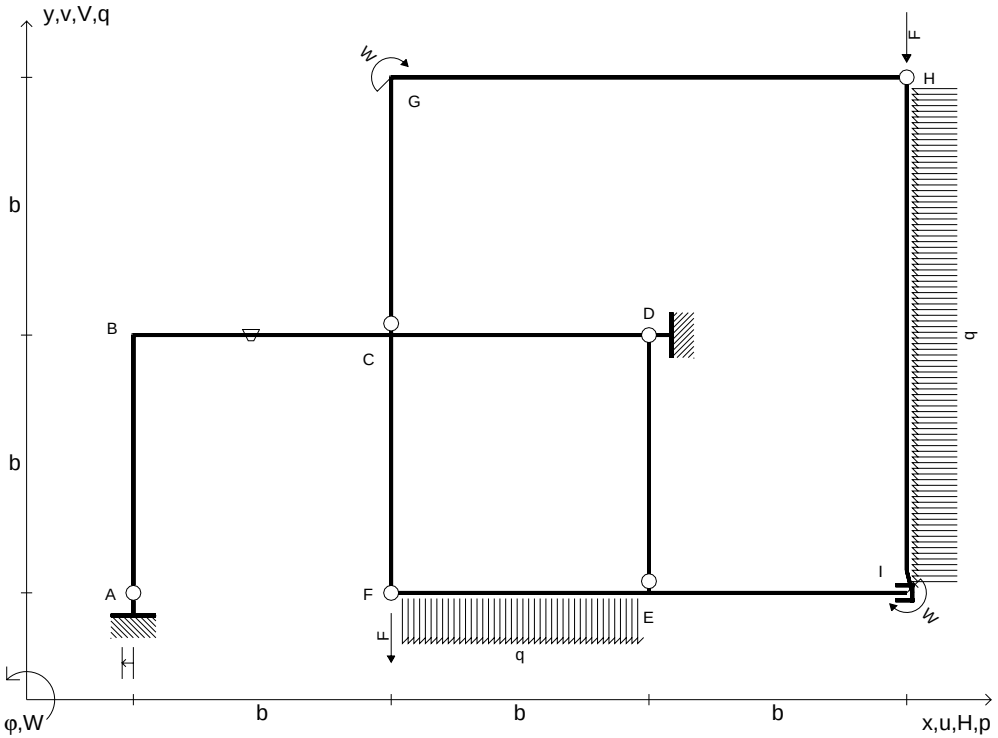


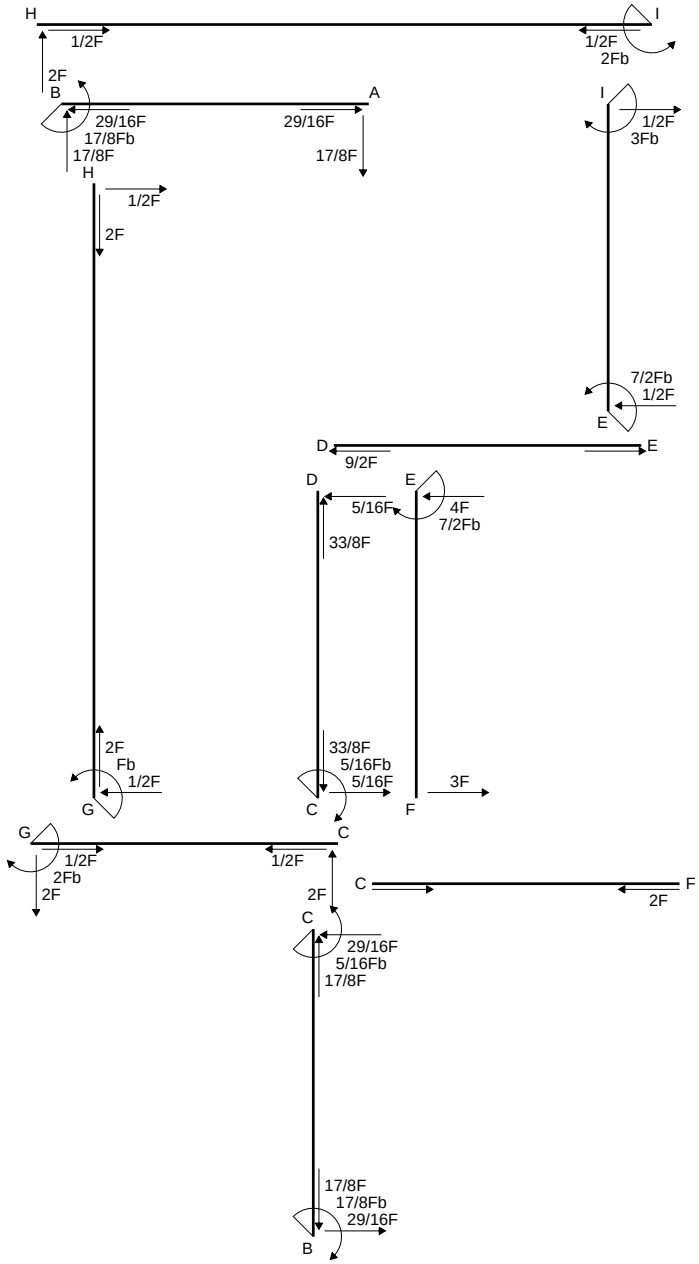
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$q_{EF} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo A.

 08.01.25



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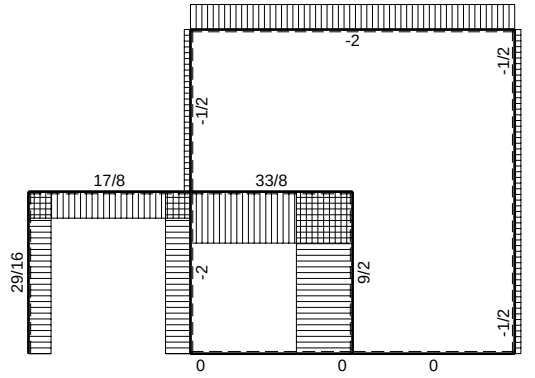

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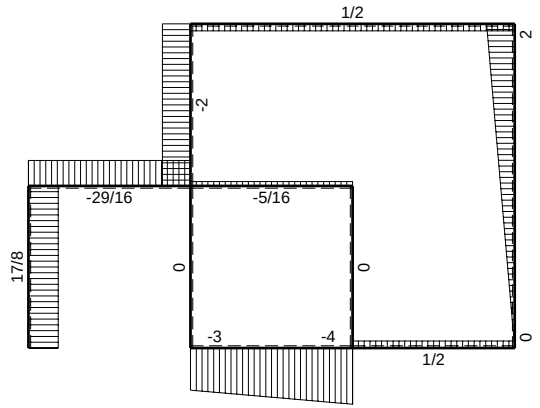

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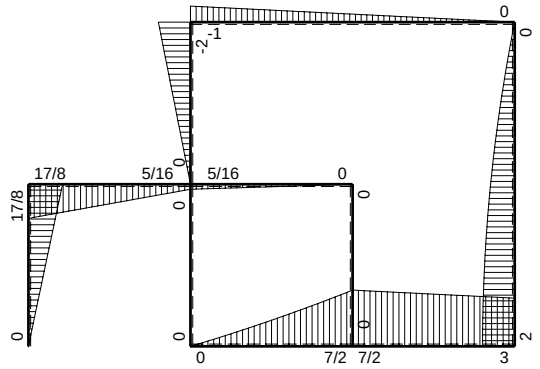
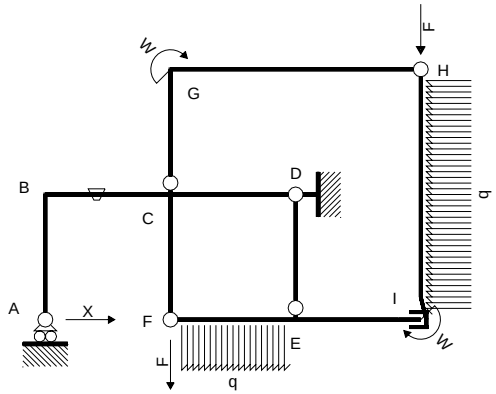
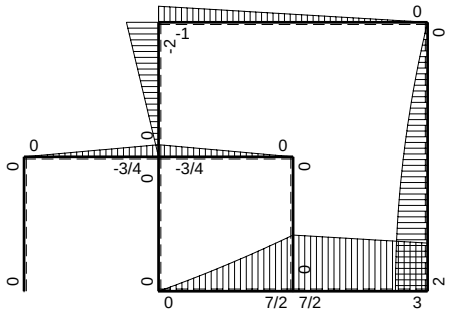


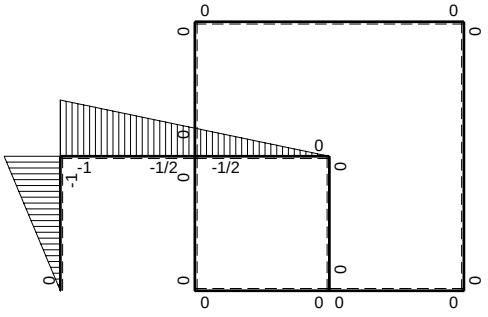
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/4Fx$	$-Fb/EJ$	$3/4Fbx-3/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/4+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	Fb/EJ	$3/8Fb^2-3/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-2Fx$	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$17/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-17/8F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210943 ESΣ01.xxxx.048

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

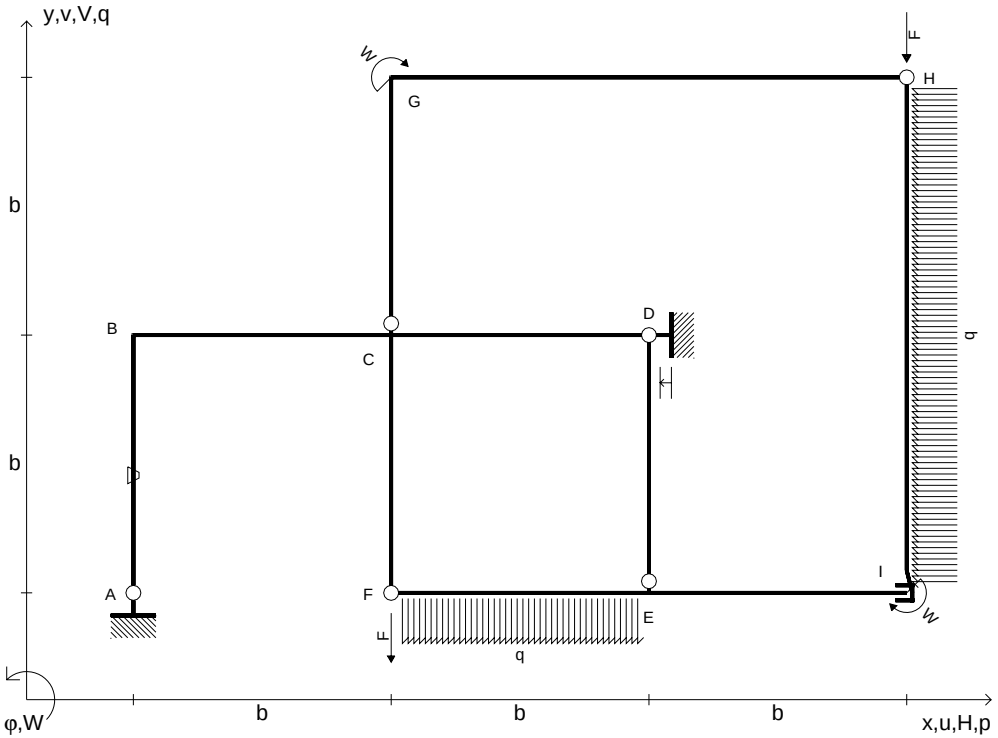
$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ01.xxxx.048

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

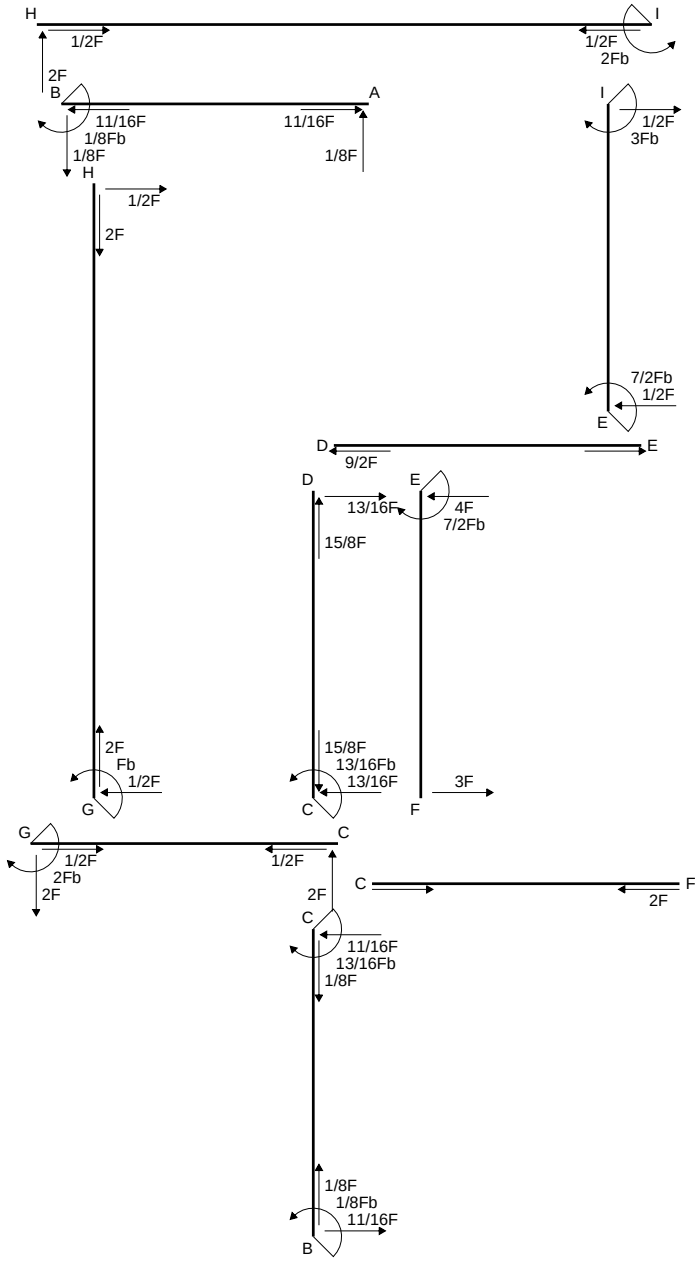


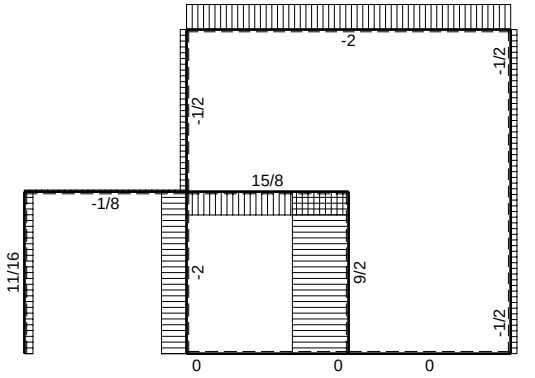
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$q_{EF} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.





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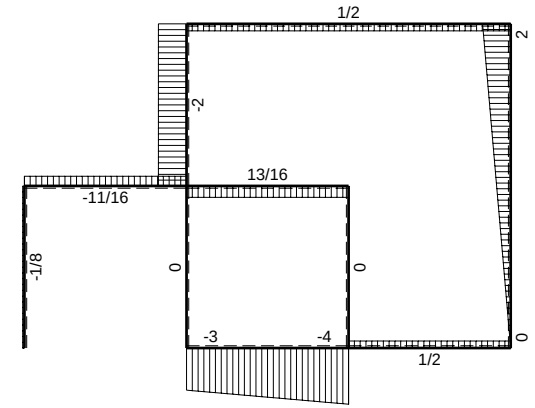

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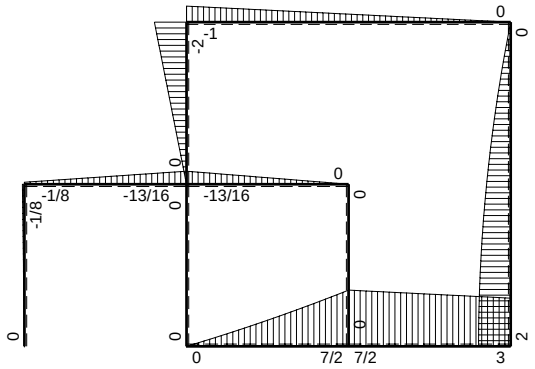
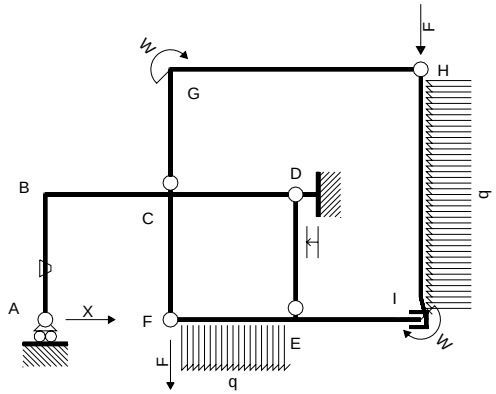
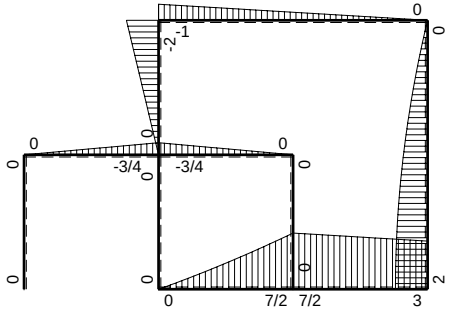


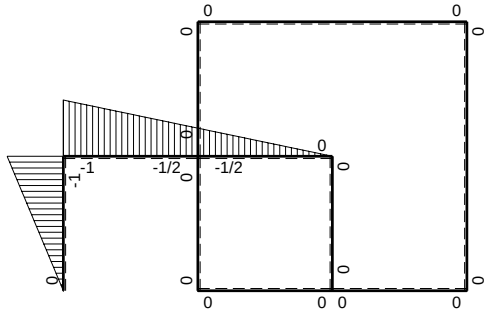
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 218425 ESΣ01.xxxx.051

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-3/4Fx	0	3/4Fbx-3/8Fx^2	0	b^2-bx+1/4x^2	(1/4+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	3/4Fb-3/4Fx	0	3/8Fb^2-3/8Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	3/8Fb^2-3/4Fbx+3/8Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/8+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	3/4Fx	0	3/8Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-4Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-3Fx-1/2qx^2	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
IH 2b	0	-2Fb+1/2qx^2	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						-Fb^3/EJ	
	totali						-1/8Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						1/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

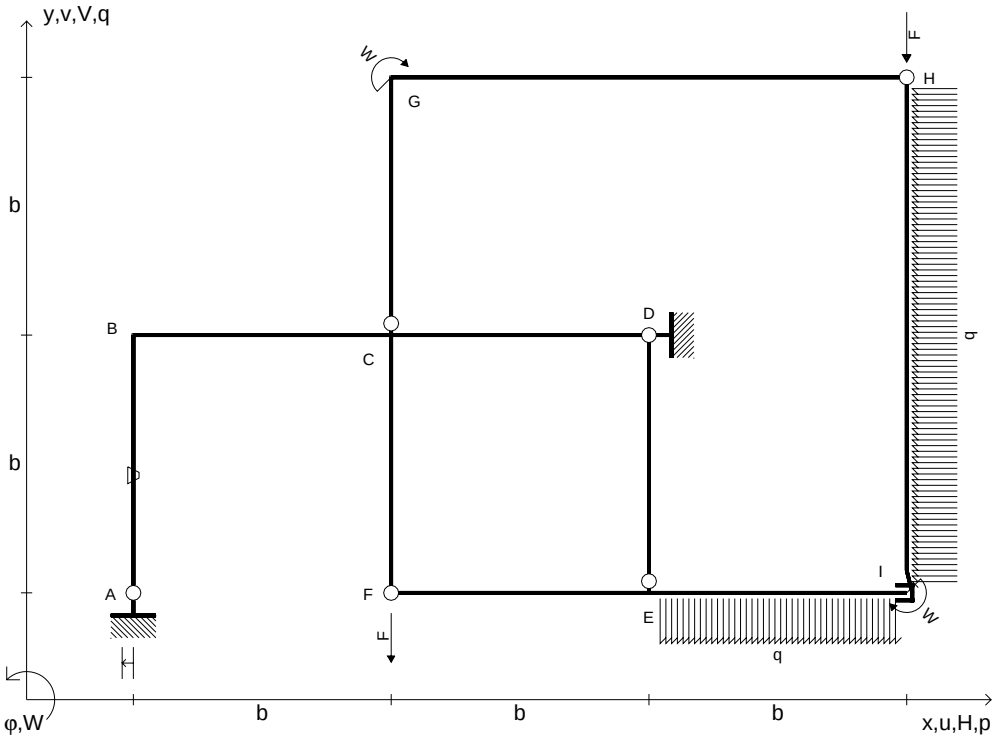
$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ01.xxxx.051

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



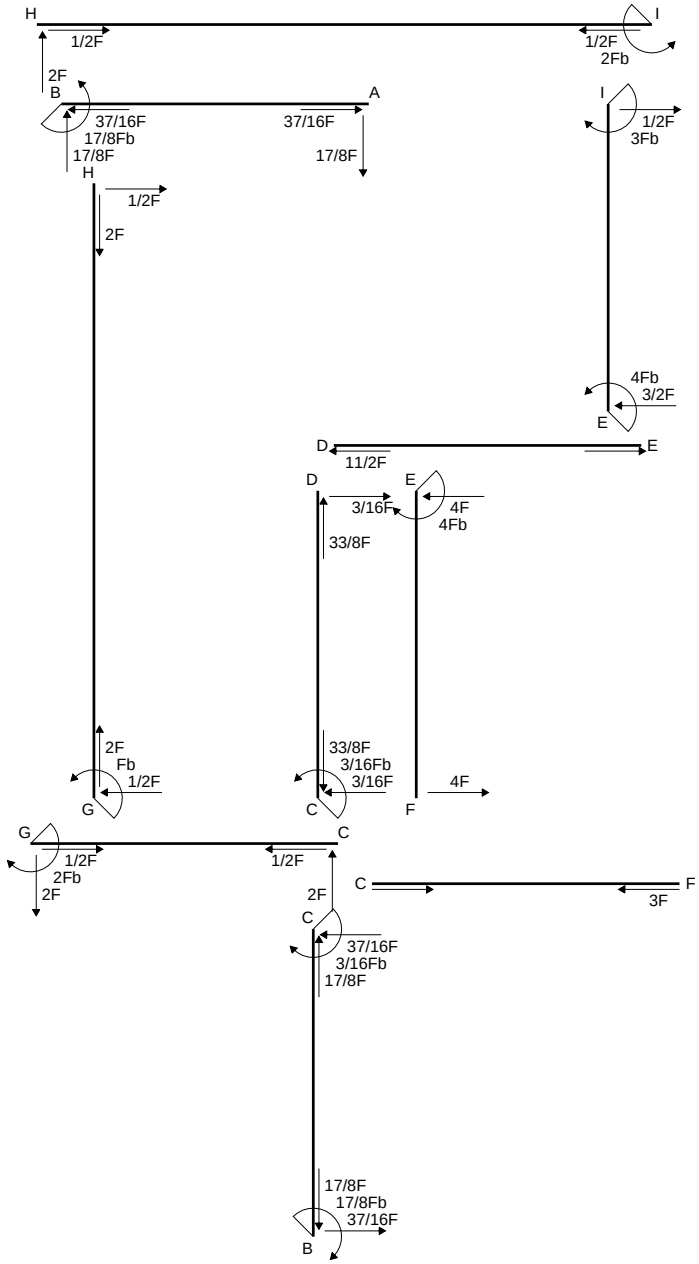
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$q_{IE} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

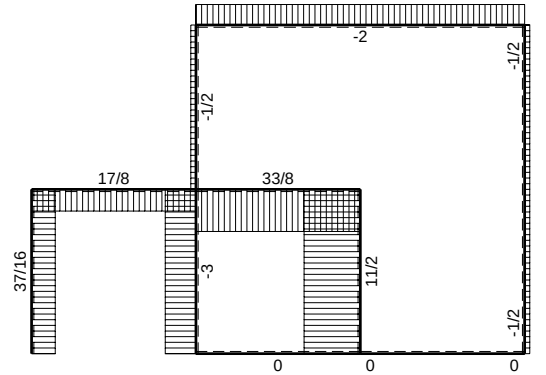
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.

 08.01.25



 08.01.25



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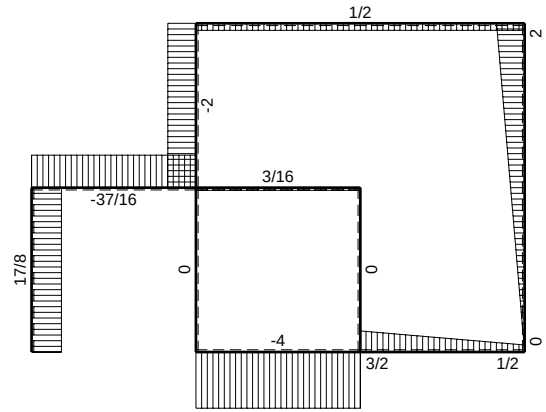

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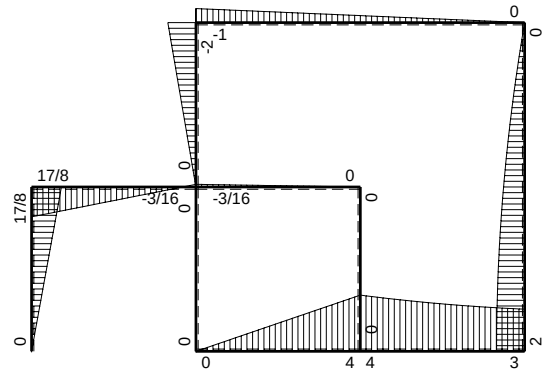

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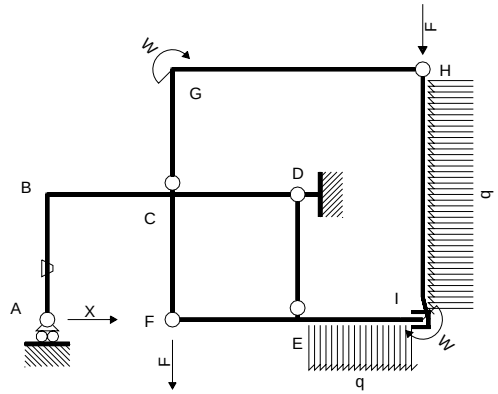
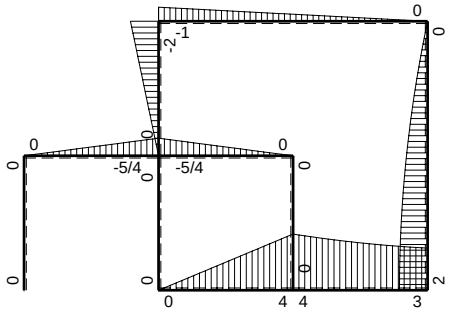
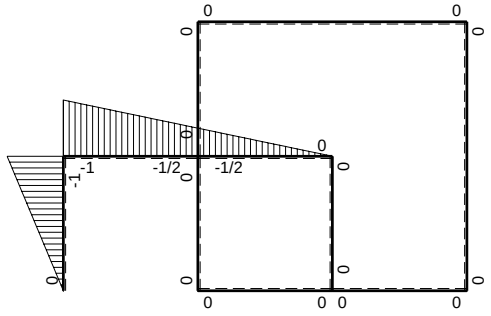


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212776 ESΣ01.xxxx.056

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	-b+1/2x	-5/4Fx	0	$5/4Fbx-5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(5/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	5/4Fb-5/4Fx	0	$5/8Fb^2-5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-5/4Fb+5/4Fx	0	$5/8Fb^2-5/4Fbx+5/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(5/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	5/4Fx	0	$5/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	4Fb-4Fx	0	0	0	0	0+0	0
FE b	0	-4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-4Fb+3/2Fx-1/2qx^2$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$17/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-17/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\, x/b +5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/8\, x^2/b +5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/8\, b +5/24\, b)\, Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/24\, b)\, Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$

ESΣ01.xxxx.056

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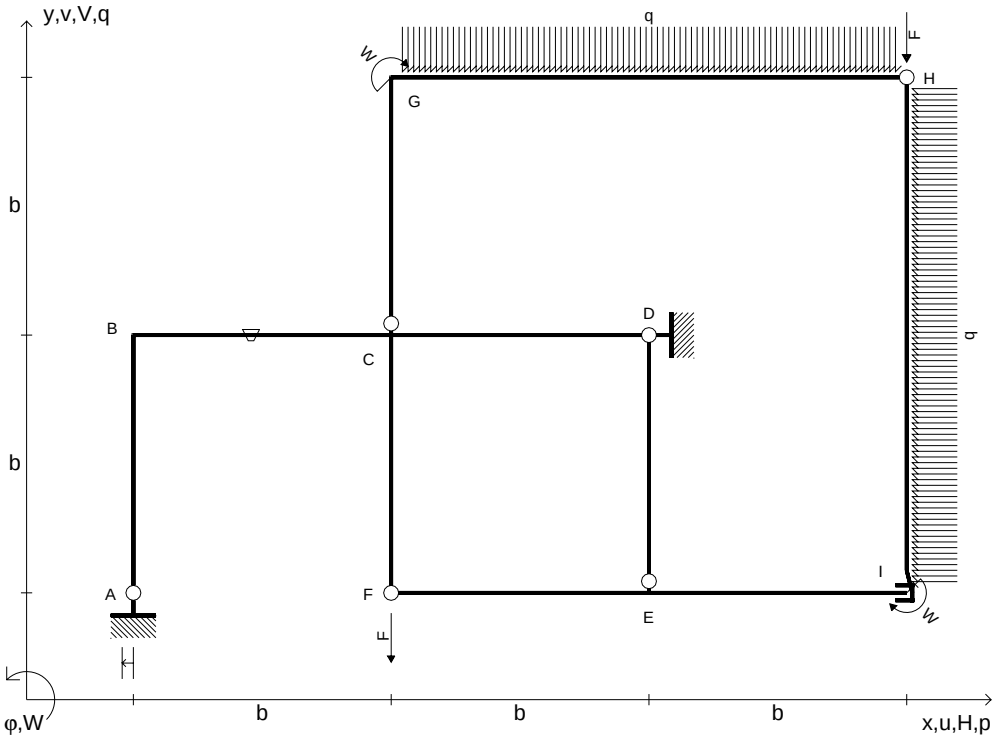


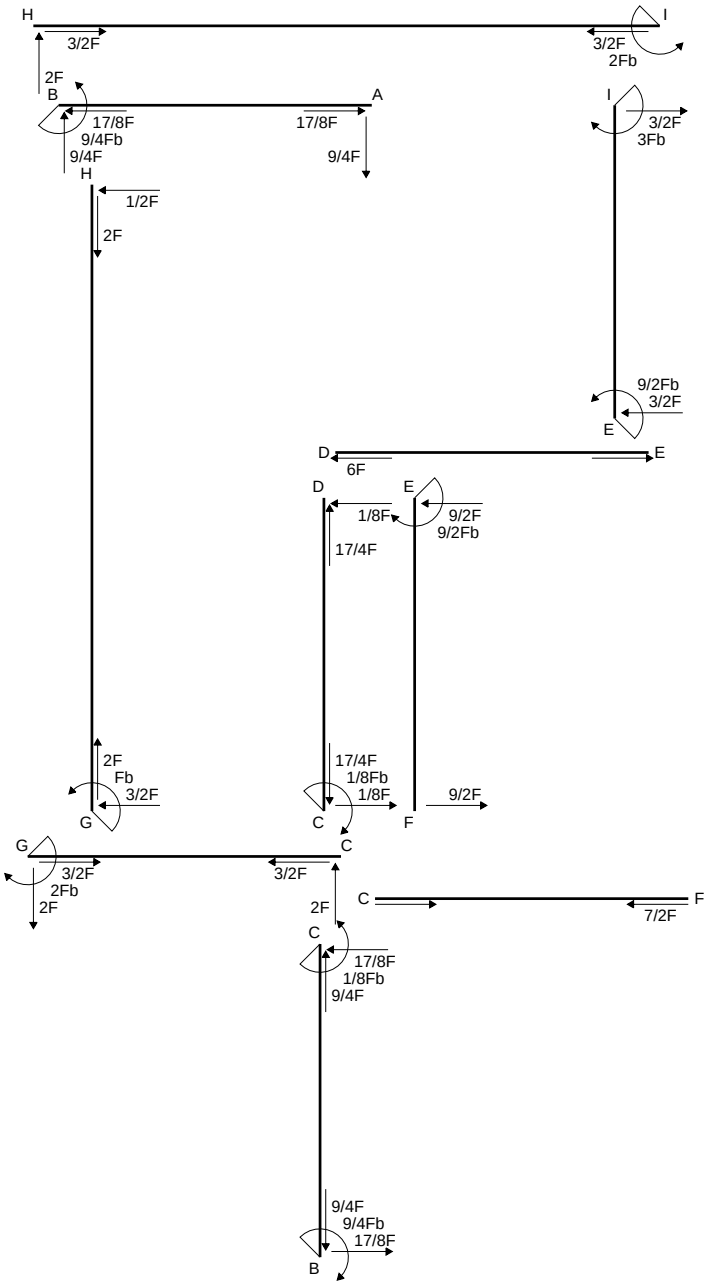
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$q_{GH} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

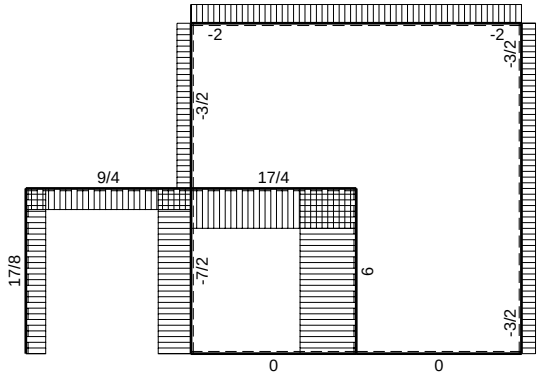
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{BC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo A.

 08.01.25



 08.01.25



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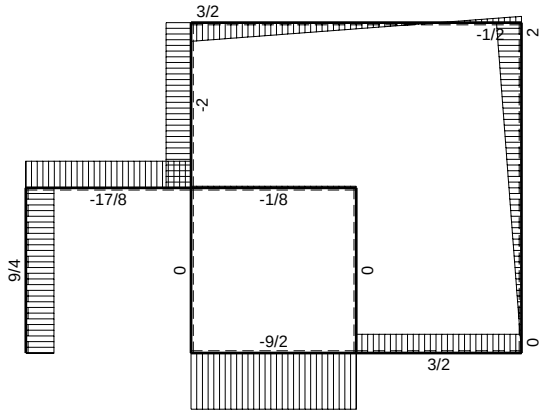

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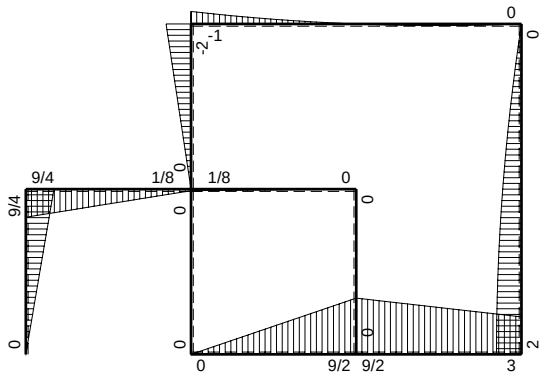
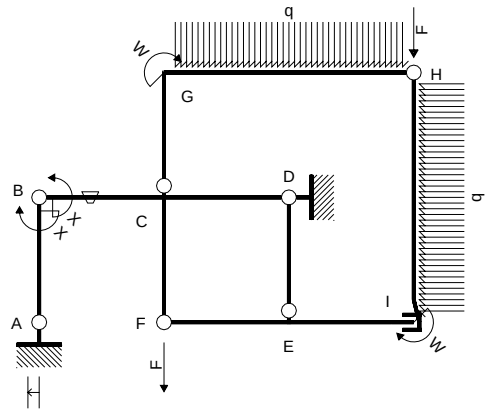
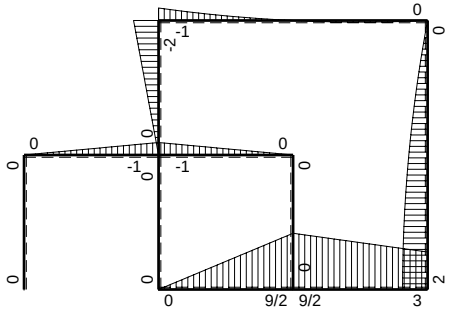


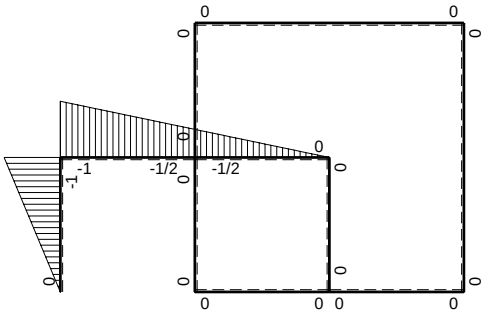
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
BA b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
BC b	$-1+1/2x/b$	-Fx	-Fb/EJ	$Fx-1/2Fx^2/b$	$Fb/EJ-1/2Fx/EJ$	$1-x/b+1/4x^2/b^2$	$(1/3+3/4)Fb^2/EJ$	7/12Xb/EJ
CB b	$1/2+1/2x/b$	Fb-Fx	Fb/EJ	$1/2Fb-1/2Fx^2/b$	$1/2Fb/EJ+1/2Fx/EJ$	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	$-1/2+1/2x/b$	-Fb+Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(1/6+0)Fb^2/EJ$	1/12Xb/EJ
DC b	1/2x/b	Fx	0	$1/2Fx^2/b$	0	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/2Fb-9/2Fx	0	0	0	0	0+0	0
FE b	0	-9/2Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	$-Fb+3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-9/2Fb+3/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^2/EJ	
	totali						$9/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						-9/4Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210884 ESΣ01.xxxx.060

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b\ )\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

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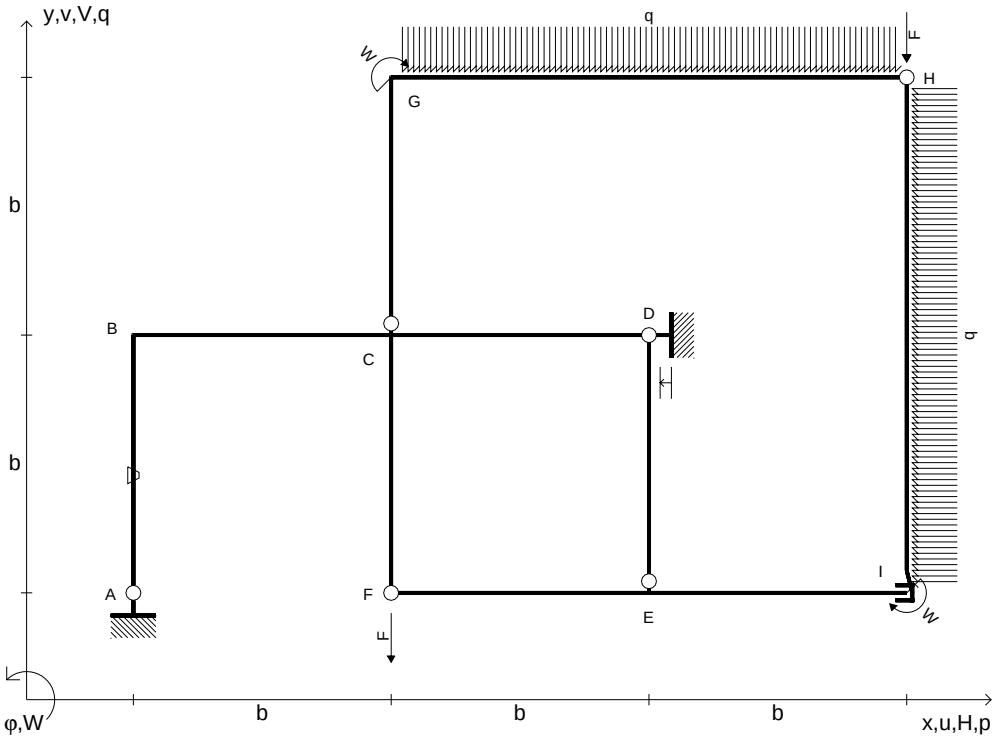
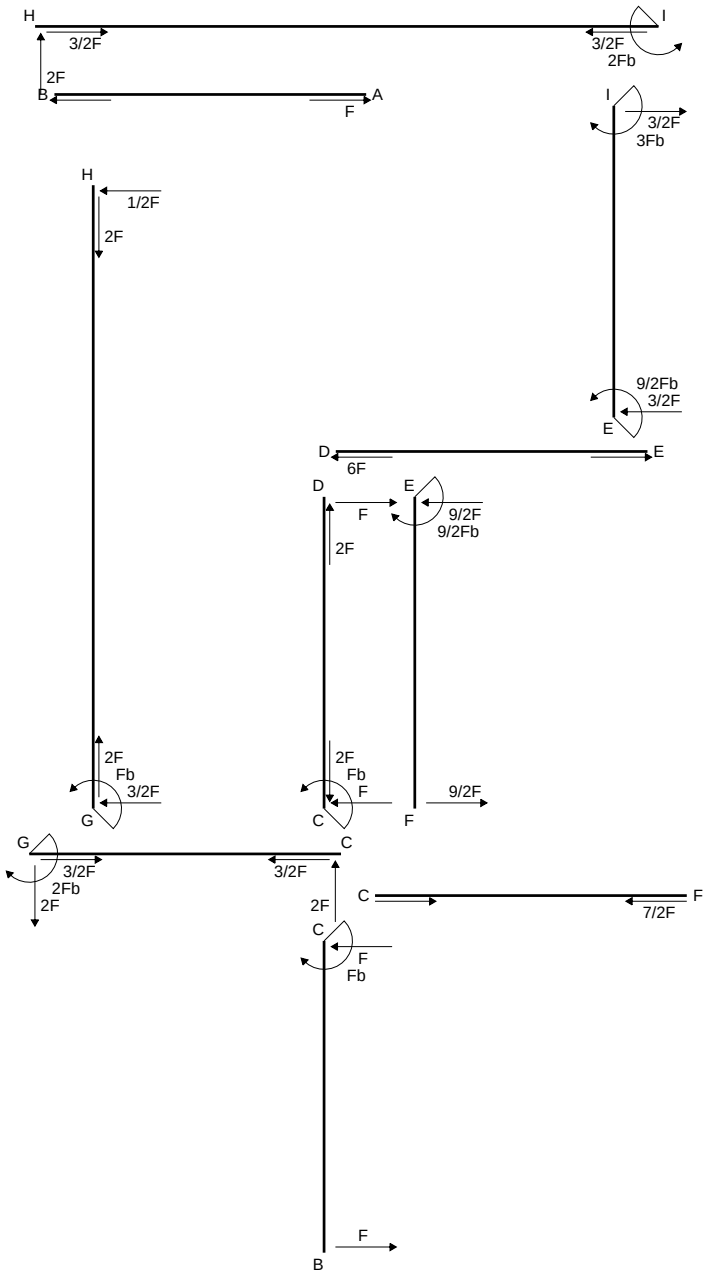


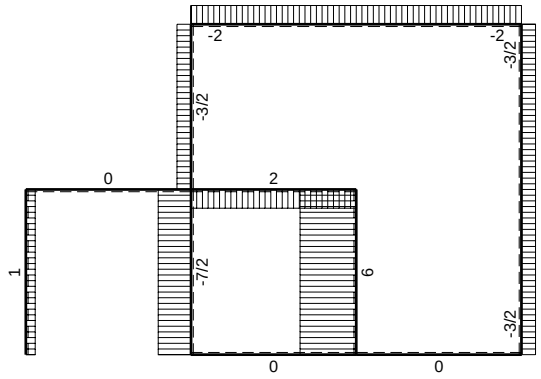
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$q_{GH} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.





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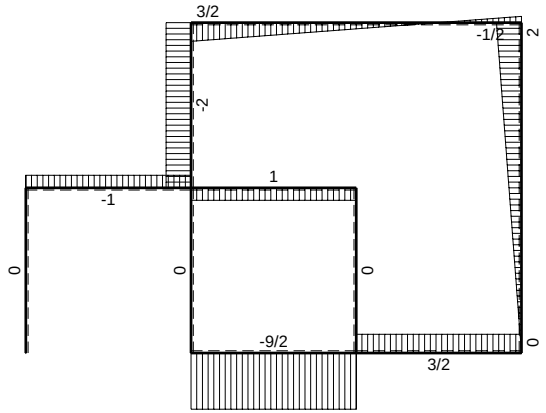

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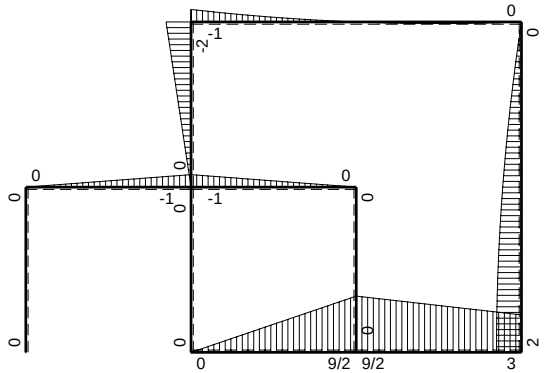

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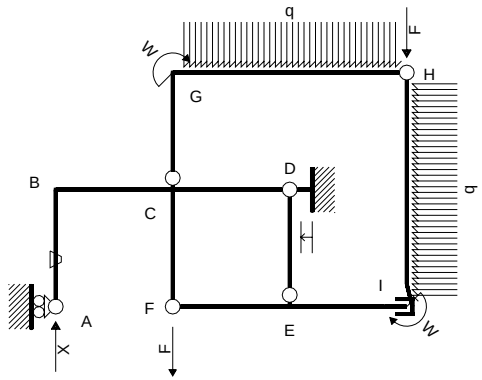
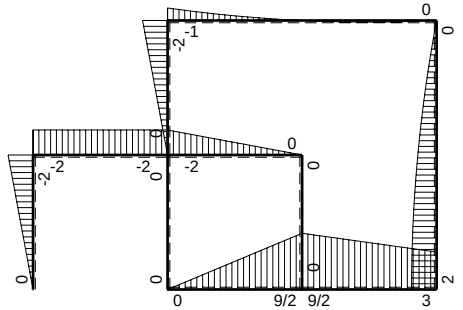
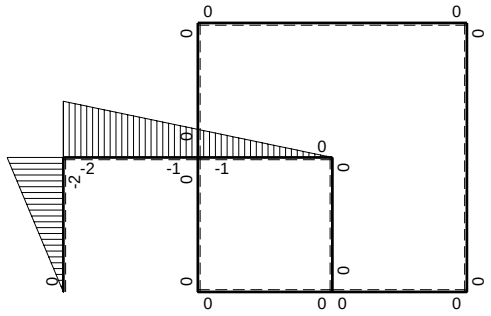


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210286 ESΣ01.xxxx.063

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-2x	-2Fx	-Fb/EJ	$4Fx^2$	2Fxb/EJ	$4x^2$	$(4/3+1)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	2b-2x	2Fb-2Fx	Fb/EJ	$4Fb^2-8Fbx+4Fx^2$	$2Fb^2/EJ-2Fxb/EJ$	$4b^2-8bx+4x^2$		
BC b	-2b+x	-2Fb	0	$4Fb^2-2Fbx$	0	$4b^2-4bx+x^2$	$(3+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	b+x	2Fb	0	$2Fb^2+2Fbx$	0	$b^2+2bx+x^2$		
CD b	-b+x	-2Fb+2Fx	0	$2Fb^2-4Fbx+2Fx^2$	0	$b^2-2bx+x^2$	$(2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	x	2Fx	0	$2Fx^2$	0	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/2Fb-9/2Fx	0	0	0	0	0+0	0
FE b	0	-9/2Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	$-Fb+3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-9/2Fb+3/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-2Fb^3/EJ$	
	totali						$4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_A$						-F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 4\ b + 4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (4 - 4\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 2\ x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 2\ b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1 + 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x + x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b + b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (2\ x/b)\ \theta\ dx = [4/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [x^2/b]_o^b\ \theta$

$= (4/3\ b)\ Fb^2\ 1/EJ + (b)\ \theta = 7/3\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-2 + 2\ x/b)\ \theta\ dx$

$= [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-2\ x + x^2/b]_o^b\ \theta$

$= (4\ b - 4\ b + 4/3\ b)\ Fb^2\ 1/EJ + (-2\ b + b)\ \theta = 7/3\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (4 - 2\ x/b)\ Fb^2\ 1/EJ\ dx = [4\ x - x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (4\ b - b)\ Fb^2\ 1/EJ = 3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (2 + 2\ x/b)\ Fb^2\ 1/EJ\ dx = [2\ x + x^2/b]_o^b\ Fb^2\ 1/EJ$

$= (2\ b + b)\ Fb^2\ 1/EJ = 3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (2 - 4\ x/b + 2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [2\ x - 2\ x^2/b + 2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (2\ b - 2\ b + 2/3\ b)\ Fb^2\ 1/EJ = 2/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (2/3\ b)\ Fb^2\ 1/EJ = 2/3\ Fb^3/EJ$

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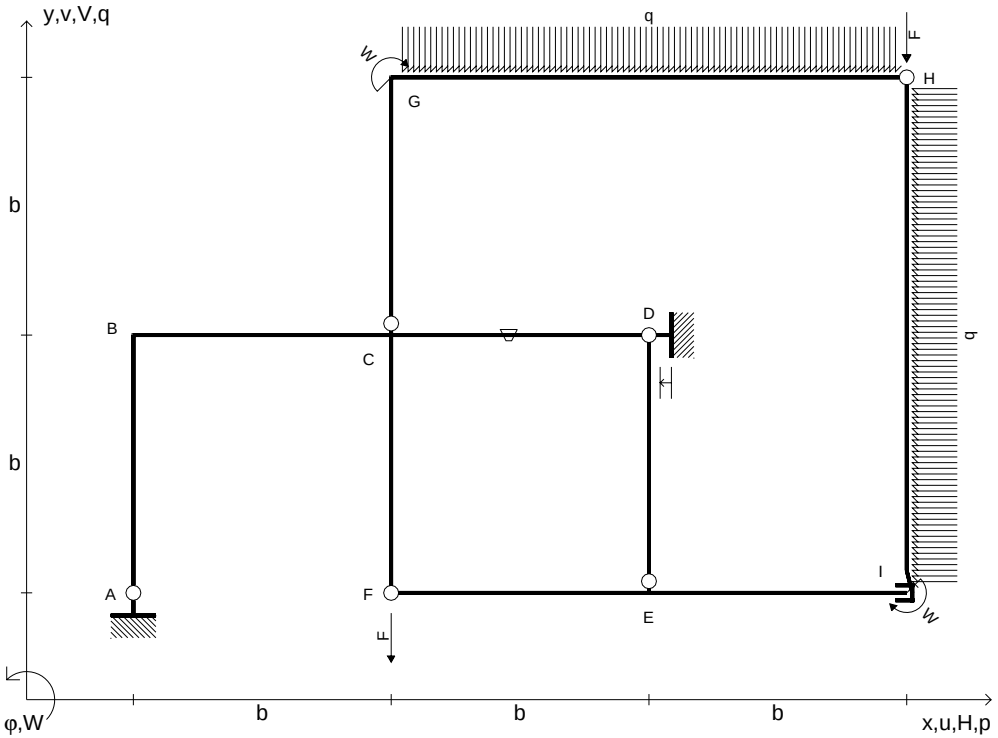


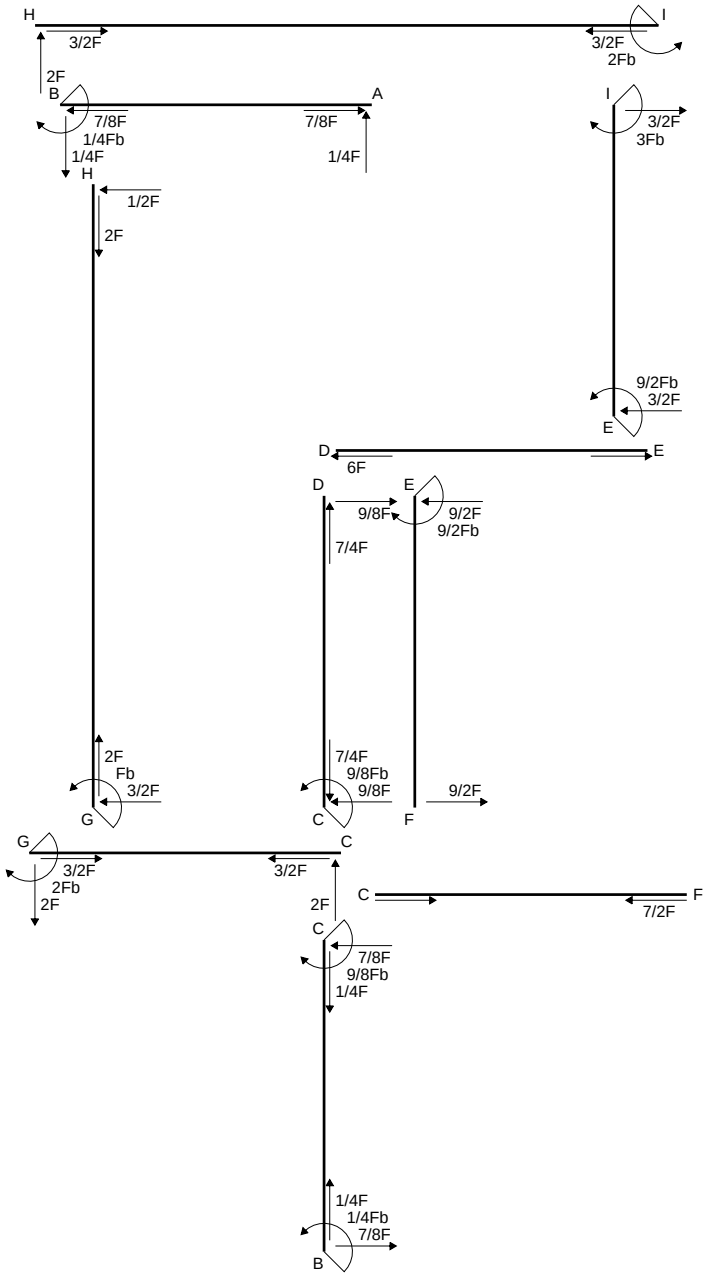
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$q_{GH} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{BC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo D.

 08.01.25



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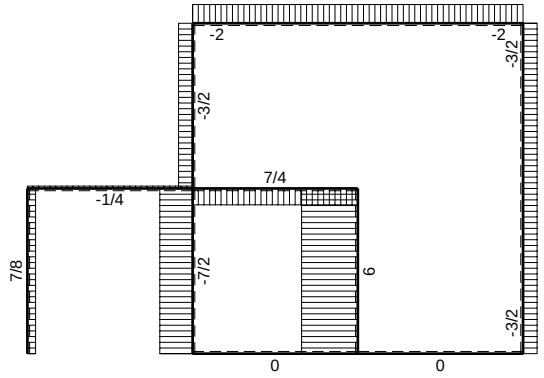
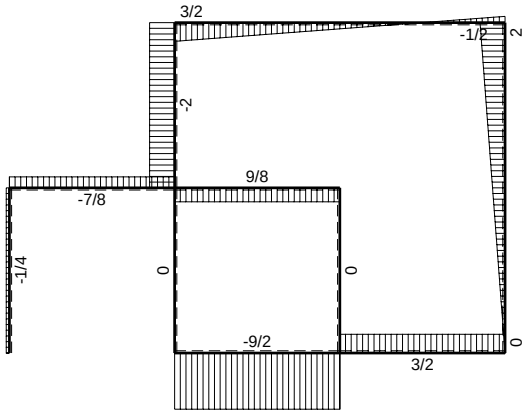
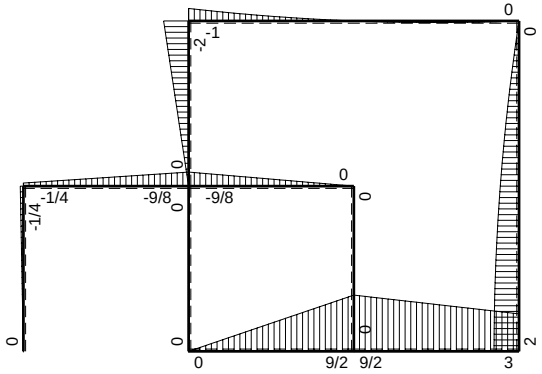


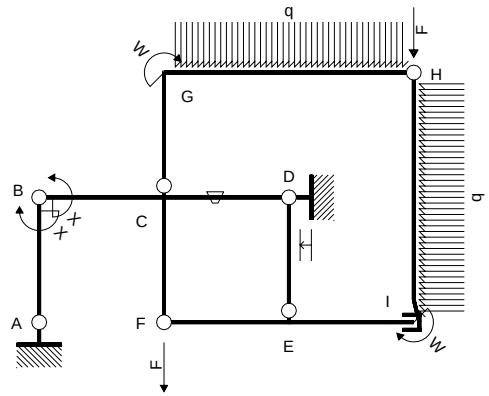
←□→ F



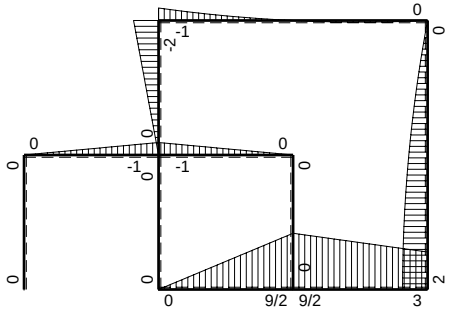
↑□↓ F



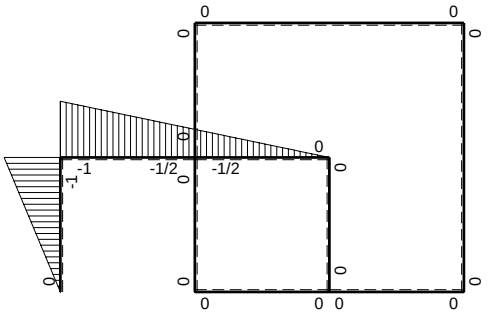
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
BA b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
BC b	$-1+1/2x/b$	-Fx	0	$Fx-1/2Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(1/3+0)Fb^2/EJ$	7/12Xb/EJ
CB b	$1/2+1/2x/b$	Fb-Fx	0	$1/2Fb-1/2Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	$-1/2+1/2x/b$	-Fb+Fx	-Fb/EJ	$1/2Fb-Fx+1/2Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(1/6+1/4)Fb^2/EJ$	1/12Xb/EJ
DC b	1/2x/b	Fx	Fb/EJ	$1/2Fx^2/b$	1/2Fx/EJ	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/2Fb-9/2Fx	0	0	0	0	0+0	0
FE b	0	-9/2Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	$-Fb+3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-9/2Fb+3/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^2/EJ$	
	totali						$-1/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						1/4Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b\ )\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b\ )\ Fb\ 1/EJ\ + (-1/4\ b\ )\ \theta\ = 5/12\ Fb^2/EJ$

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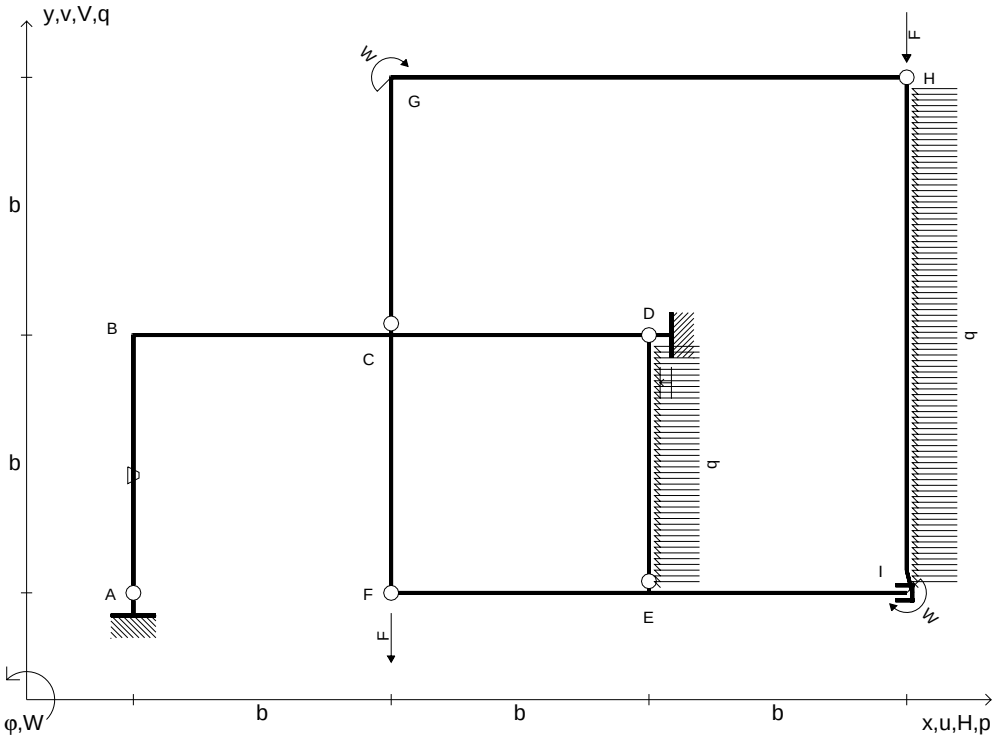
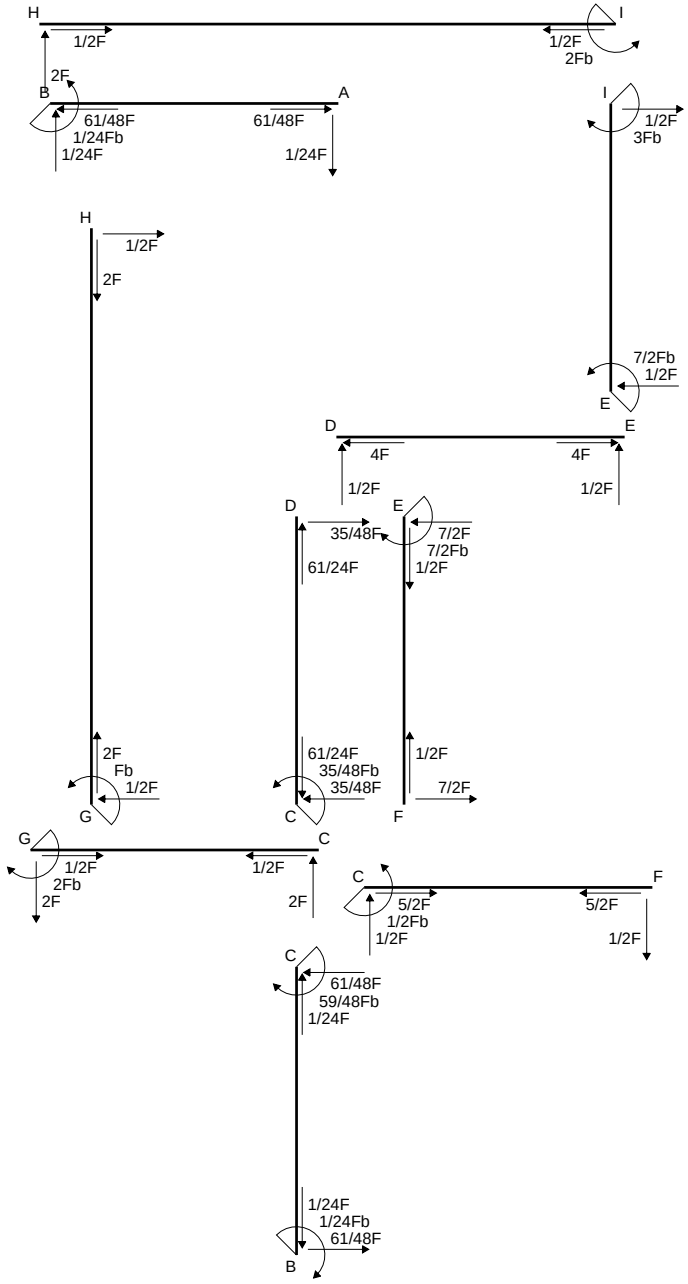


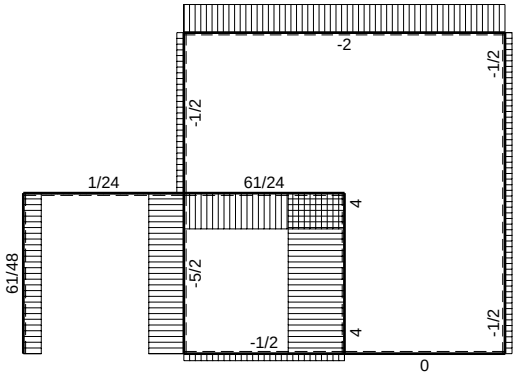
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{HI} = -q = -F/b$
$p_{DE} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.





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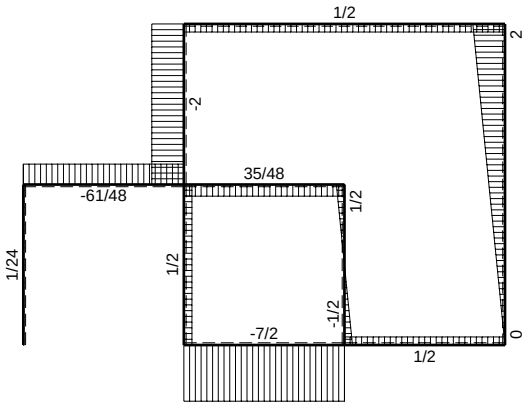

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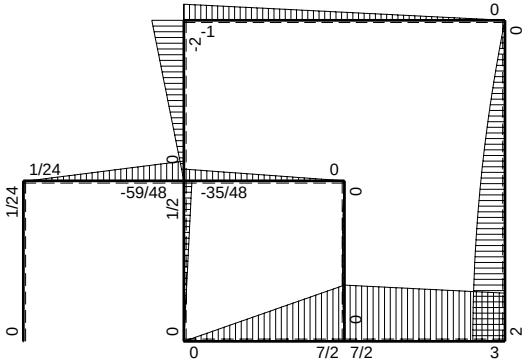
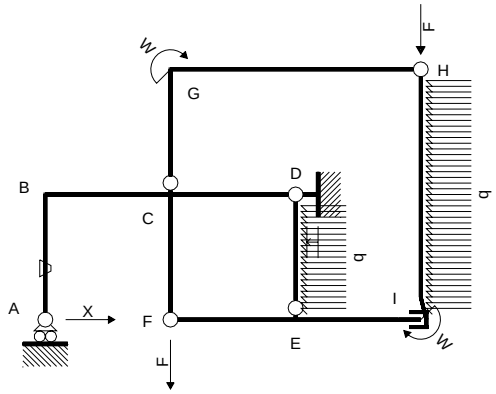
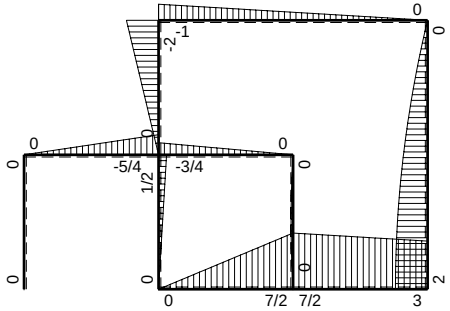


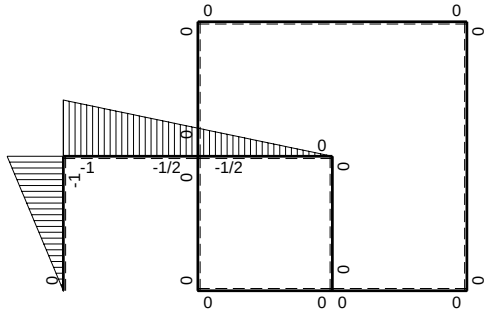
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	$b^2-2bx+x^2$		
BC b	-b+1/2x	-5/4Fx	0	5/4Fbx-5/8Fx^2	0	$b^2-bx+1/4x^2$	(5/12+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	5/4Fb-5/4Fx	0	5/8Fb^2-5/8Fx^2	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	3/8Fb^2-3/4Fbx+3/8Fx^2	0	$1/4b^2-1/2bx+1/4x^2$	(1/8+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	3/4Fx	0	3/8Fx^2	0	$1/4x^2$		
DE b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	0	7/2Fb-7/2Fx	0	0	0	0	0+0	0
FE b	0	-7/2Fx	0	0	0	0		
FC b	0	1/2Fx	0	0	0	0	0+0	0
CF b	0	-1/2Fb+1/2Fx	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
IH 2b	0	-2Fb+1/2qx^2	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						-Fb^3/EJ	
	totali						1/24Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-1/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

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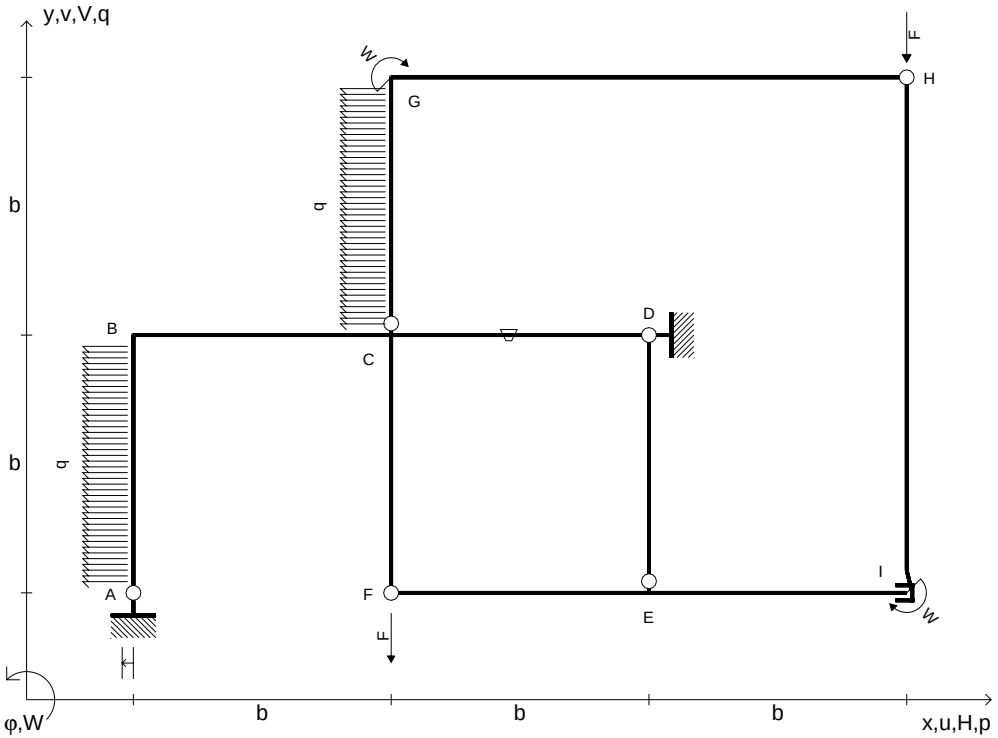
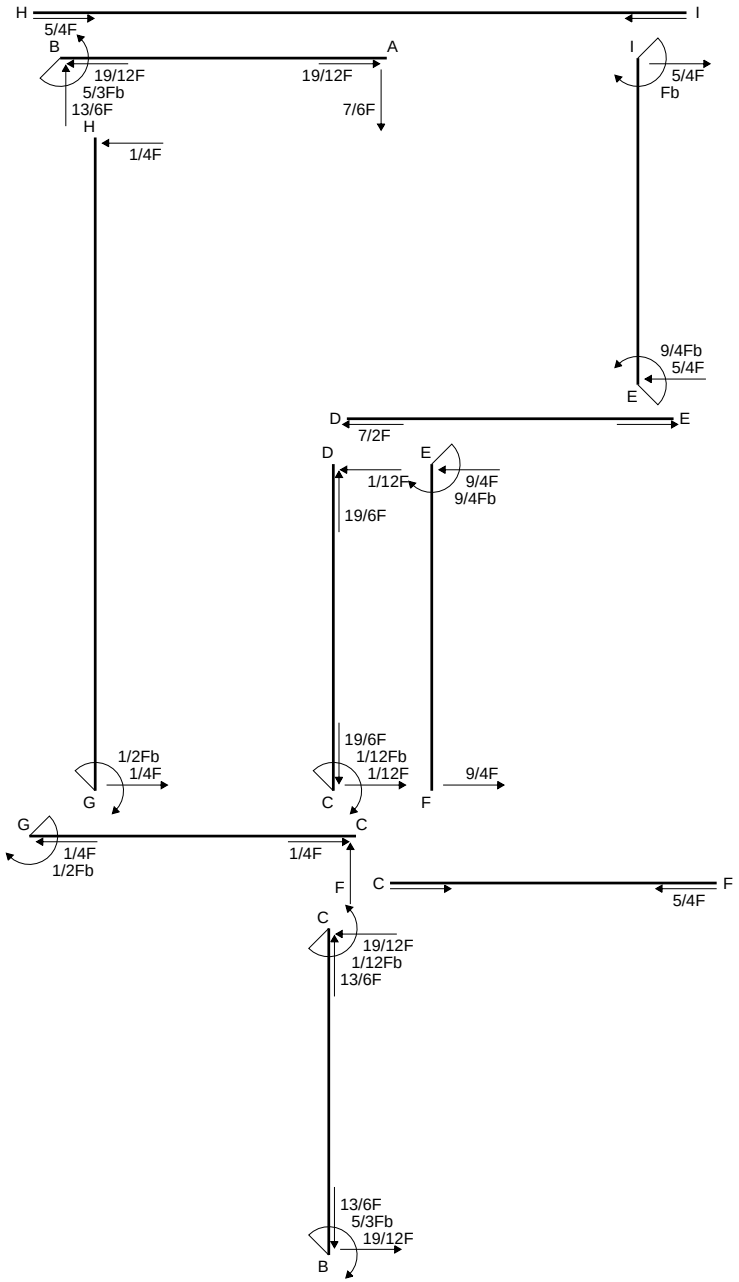


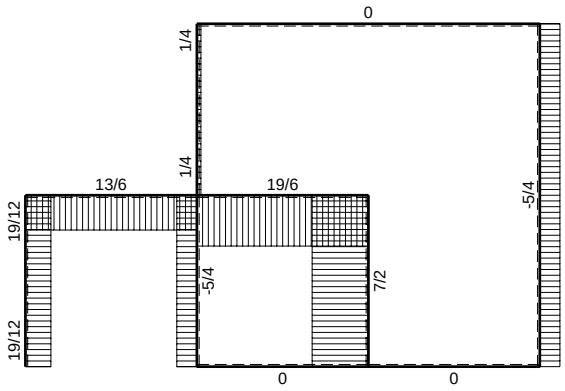
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$p_{AB} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

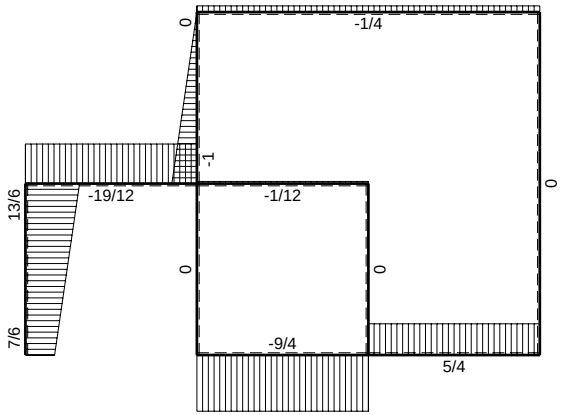
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo A.

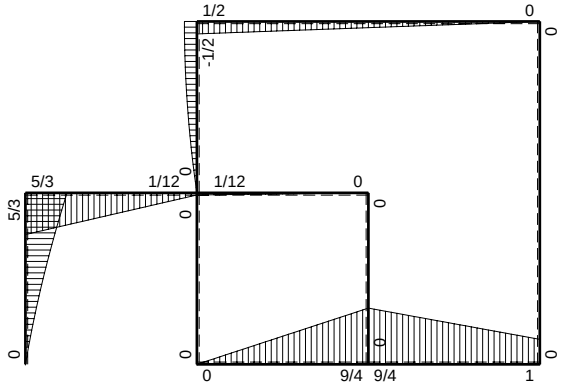




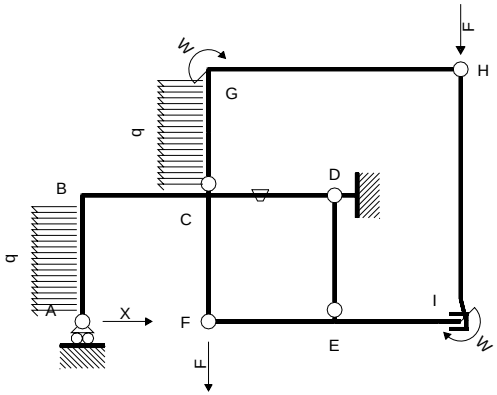
←□→ F



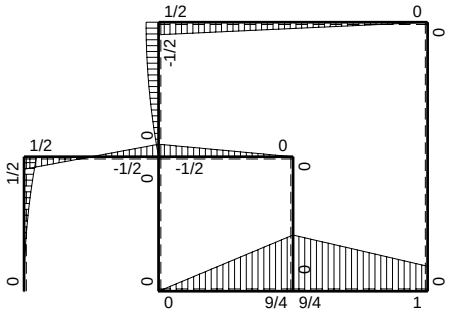
↑□↓ F



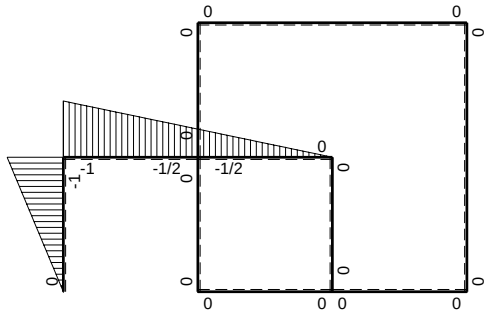
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 984883 ESΣ01.xxxx.076

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-Fx$	0	$-1/2Fb^2+5/4Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(-1/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$1/2Fb-Fx$	0	$1/4Fb^2-1/4Fbx-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	-Fb/EJ	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$9/4Fb-9/4Fx$	0	0	0	0	0+0	0
FE b	0	$-9/4Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/4Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx$	0	0	0	0	0+0	0
EI b	0	$-9/4Fb+5/4Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$7/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-7/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +5/4\, x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +5/8\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +5/8\, b -1/6\, b)\, Fb^2\, 1/EJ = -1/24\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\, x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/8\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/8\, b -1/6\, b)\, Fb^2\, 1/EJ = -1/24\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/12\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

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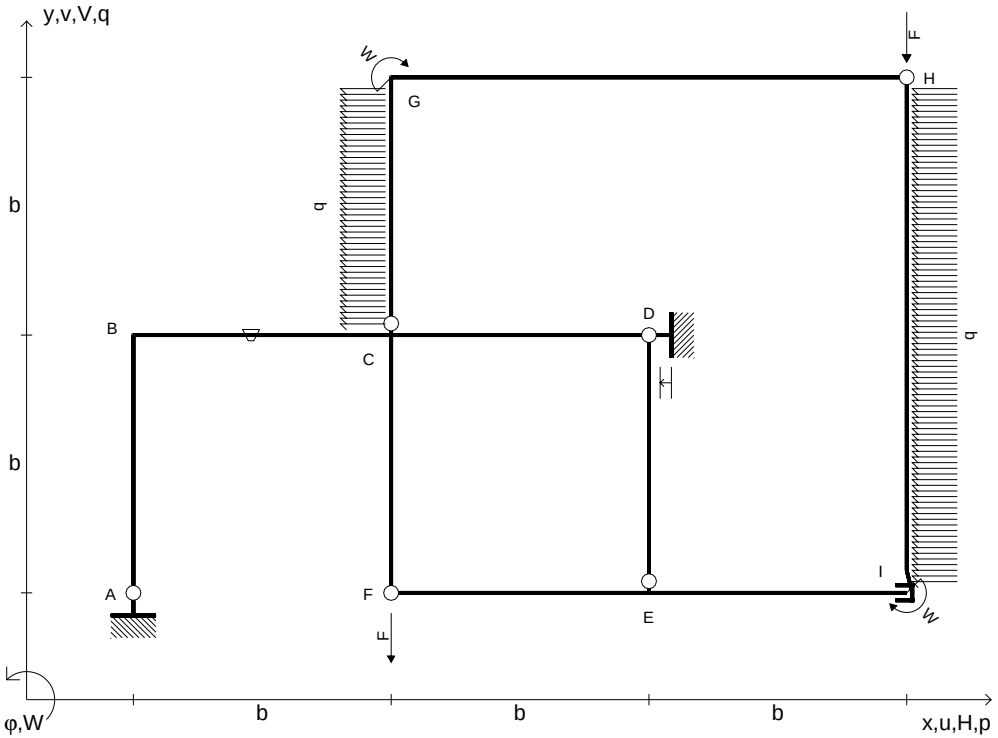

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
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Reazioni iperstatiche in soluzione: $X=H_A$
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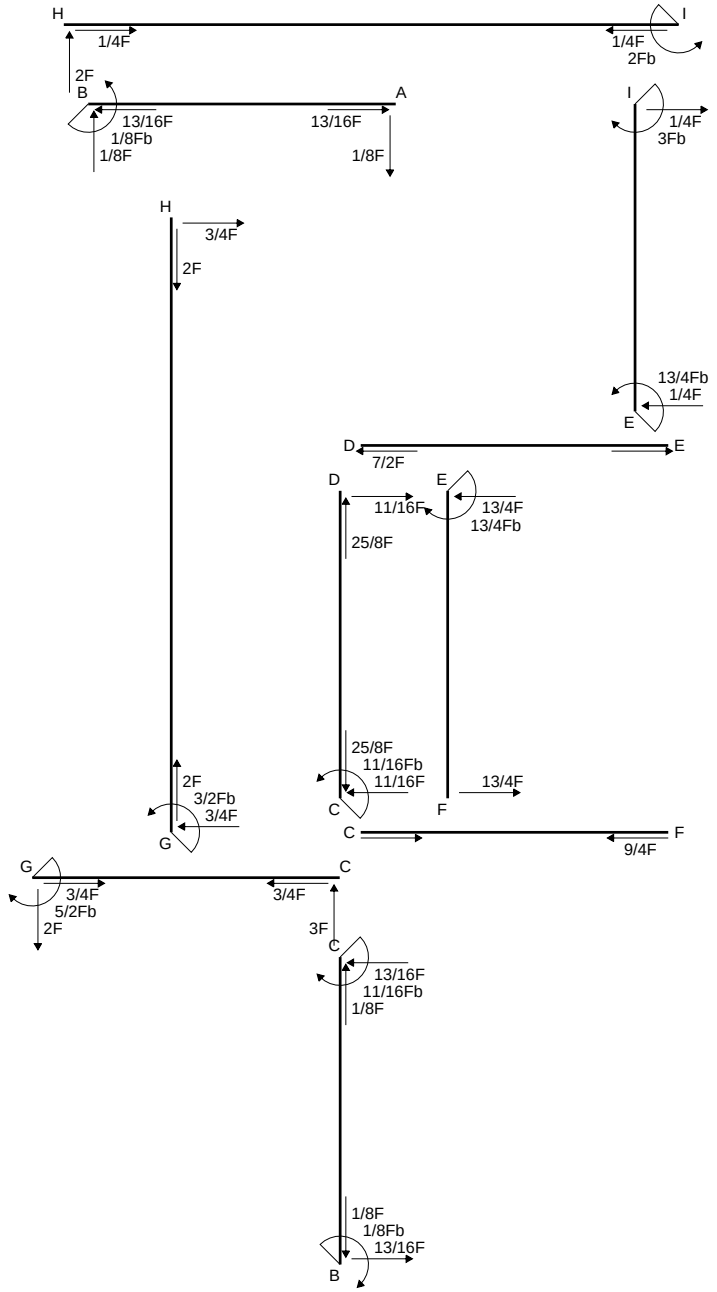
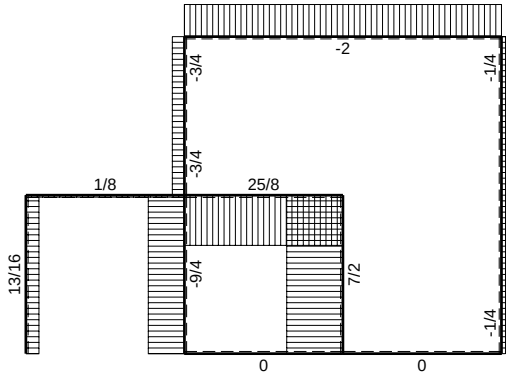
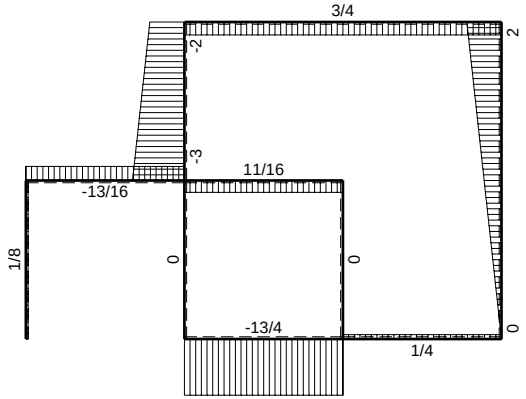
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.





F



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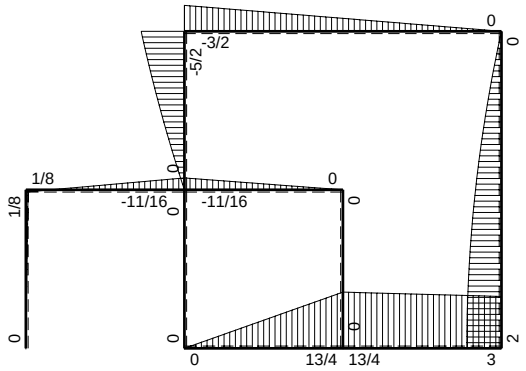
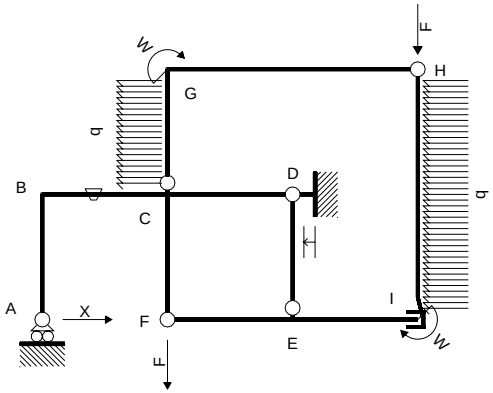
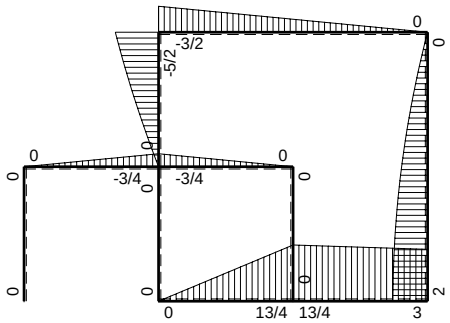


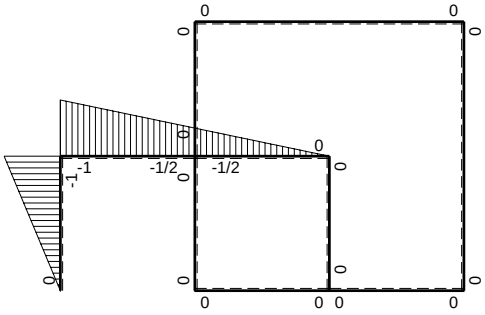
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/4Fx$	$-Fb/EJ$	$3/4Fbx-3/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/4+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	Fb/EJ	$3/8Fb^2-3/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	13/4Fb-13/4Fx	0	0	0	0	0+0	0
FE b	0	-13/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-3Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$5/2Fb-2Fx-1/2qx^2$	0	0	0	0		
GH 2b	0	$-3/2Fb+3/4Fx$	0	0	0	0	0+0	0
HG 2b	0	3/4Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+1/4Fx	0	0	0	0	0+0	0
EI b	0	-13/4Fb+1/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/8F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 983984 ESΣ01.xxxx.079

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ01.xxxx.079

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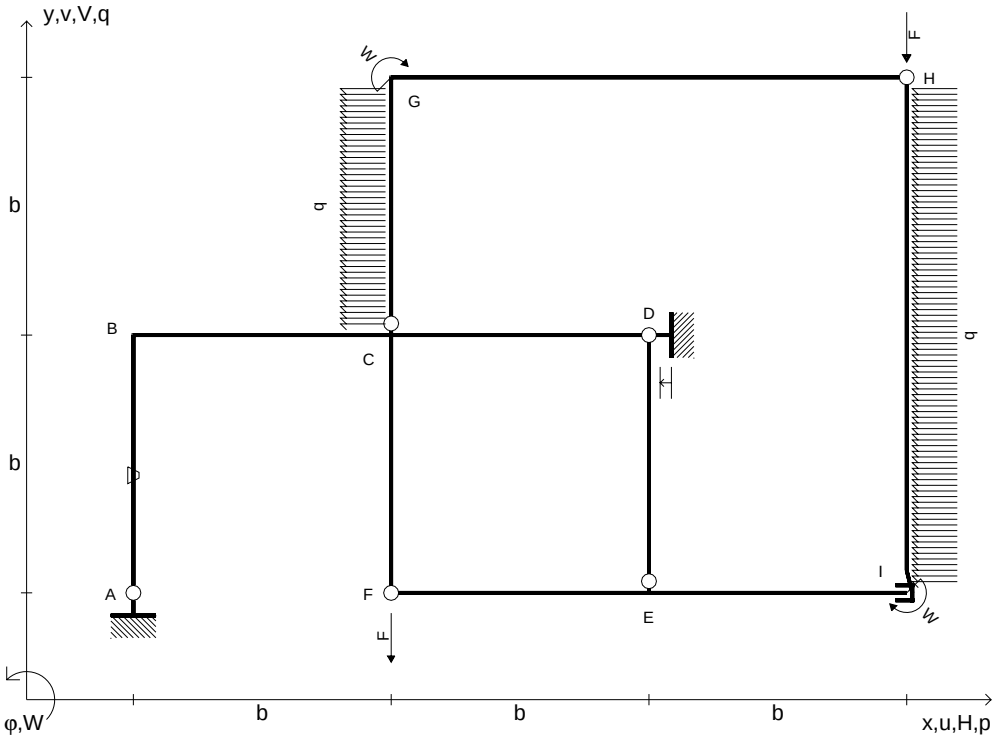


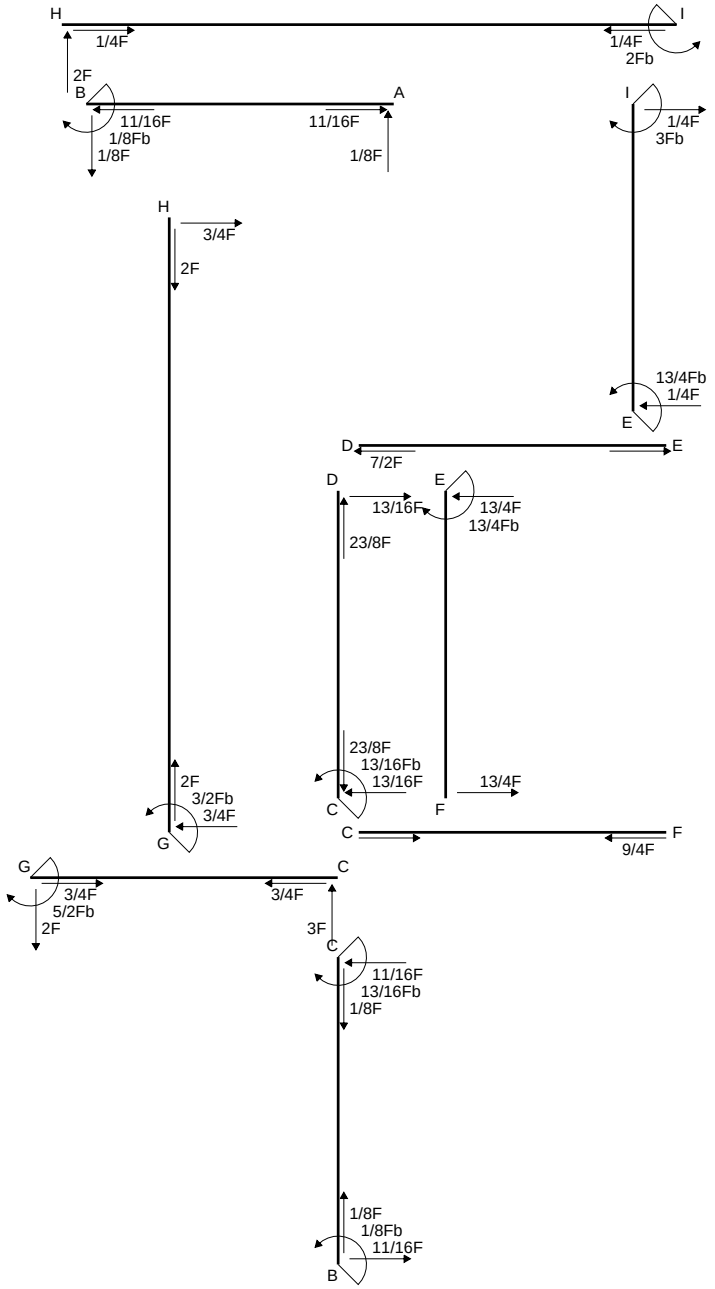
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{BC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

 08.01.25



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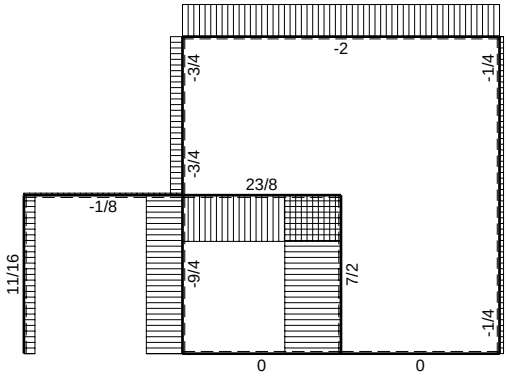
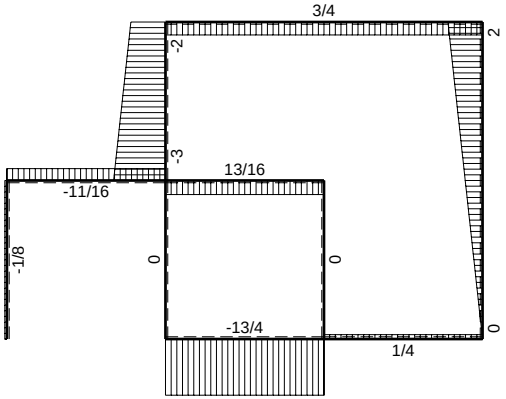
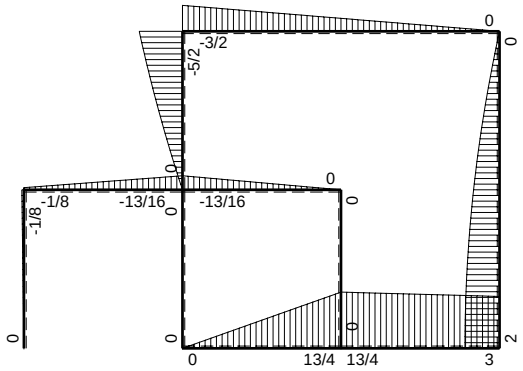


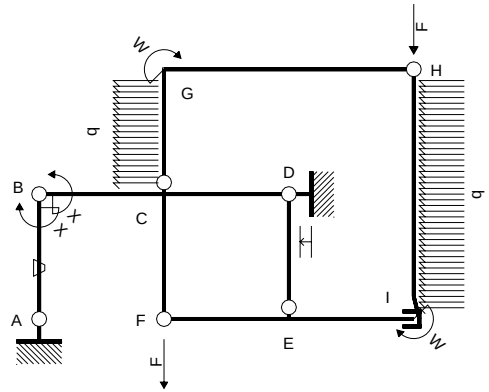
←□→ F



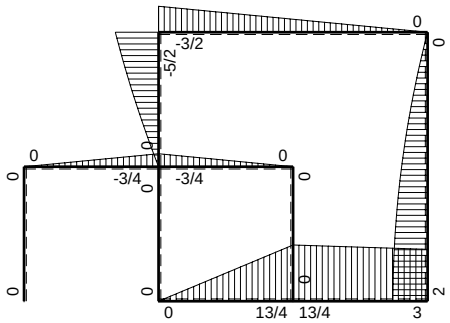
↑□↓ F



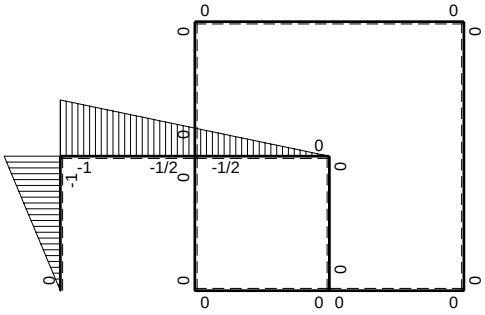
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	0	-Fb/EJ	0	Fx/EJ	x^2/b^2	$(0+1/2)Fb^2/EJ$	1/3Xb/EJ
BA b	1-x/b	0	Fb/EJ	0	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
BC b	-1+1/2x/b	-3/4Fx	0	$3/4Fx-3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(1/4+0)Fb^2/EJ$	7/12Xb/EJ
CB b	1/2+1/2x/b	3/4Fb-3/4Fx	0	$3/8Fb-3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	-1/2+1/2x/b	-3/4Fb+3/4Fx	0	$3/8Fb-3/4Fx+3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(1/8+0)Fb^2/EJ$	1/12Xb/EJ
DC b	1/2x/b	3/4Fx	0	$3/8Fx^2/b$	0	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	13/4Fb-13/4Fx	0	0	0	0	0+0	0
FE b	0	-13/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-3Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$5/2Fb-2Fx-1/2qx^2$	0	0	0	0		
GH 2b	0	-3/2Fb+3/4Fx	0	0	0	0	0+0	0
HG 2b	0	3/4Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+1/4Fx	0	0	0	0	0+0	0
EI b	0	-13/4Fb+1/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^2/EJ$	
	totali						$-1/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						1/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

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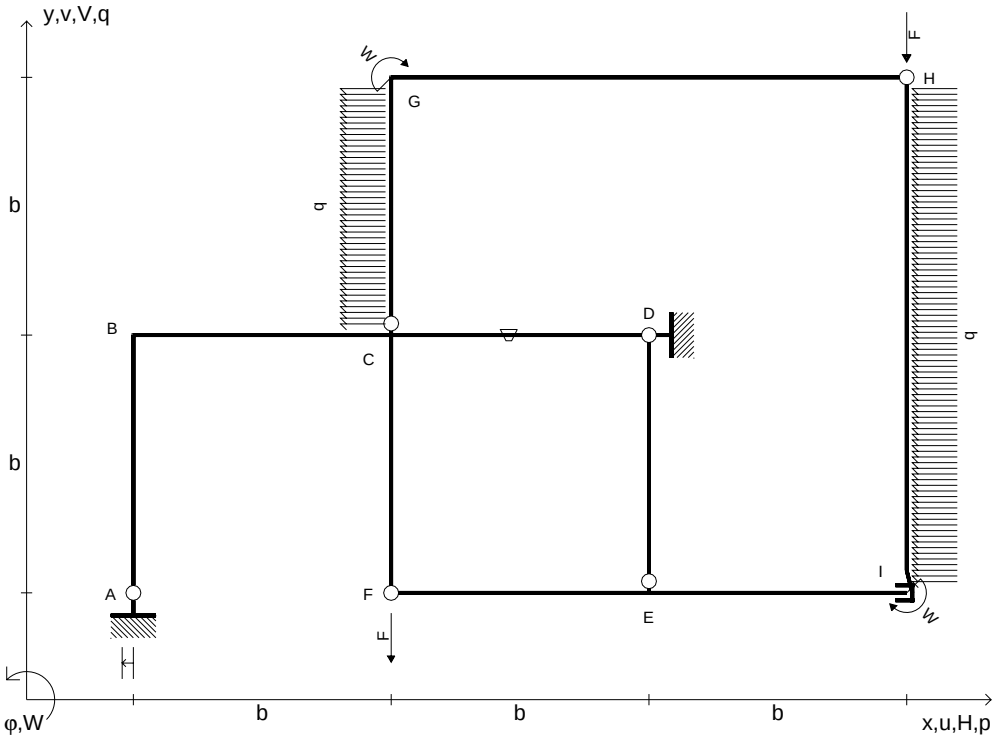


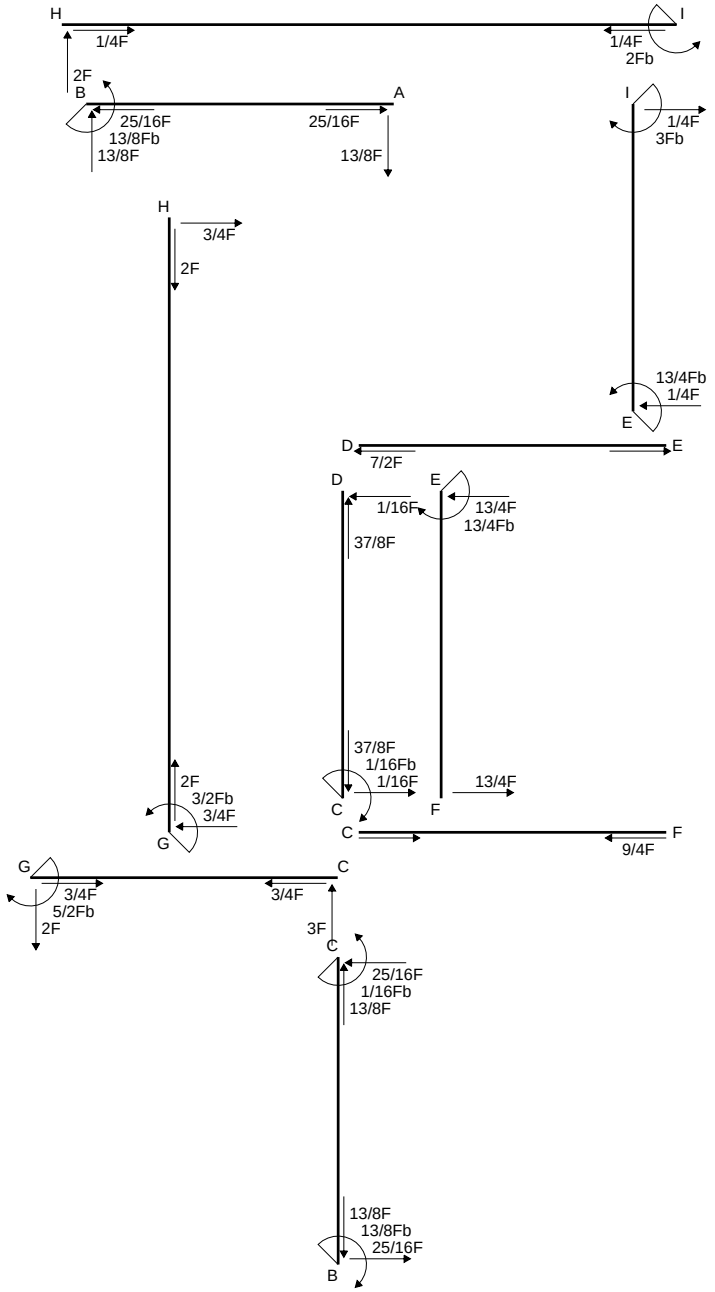
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

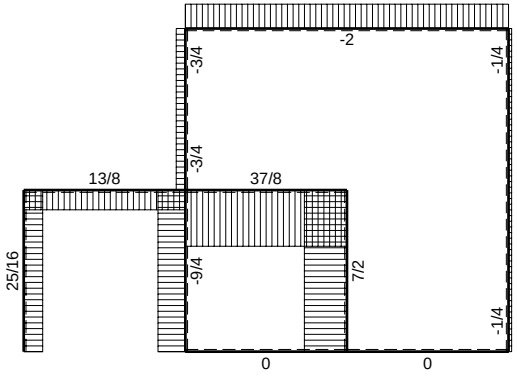
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo A.

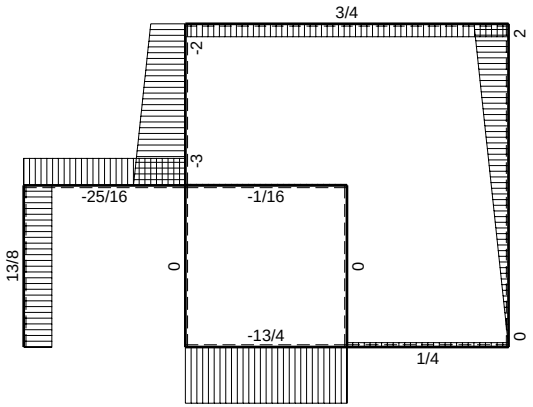
 08.01.25



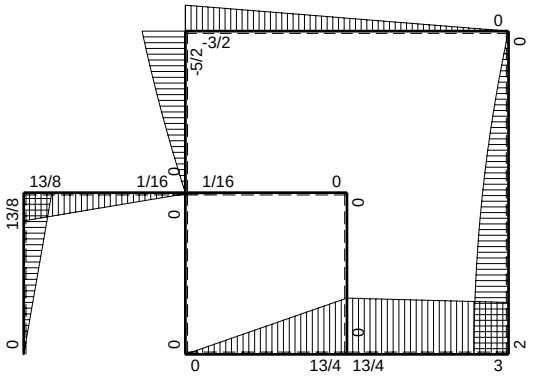
 08.01.25



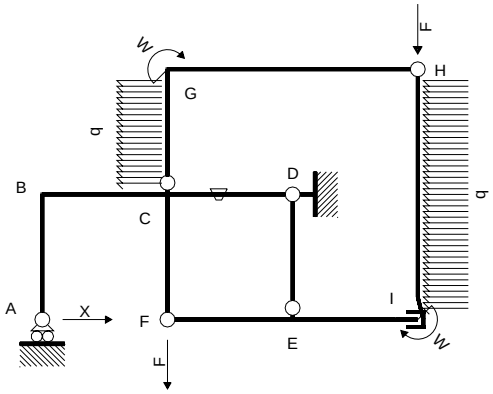
←□→ F



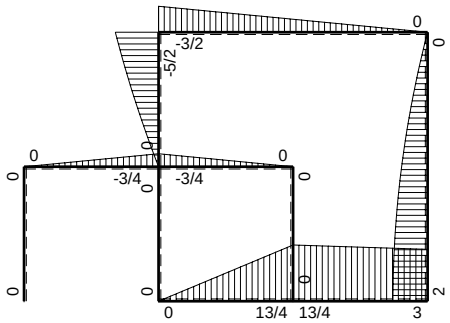
↑□↓ F



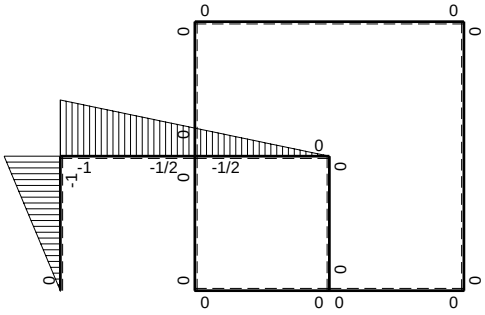
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/4Fx$	0	$3/4Fbx-3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/4+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	0	$3/8Fb^2-3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	$-Fb/EJ$	$3/8Fb^2-3/4Fbx+3/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	Fb/EJ	$3/8Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$13/4Fb-13/4Fx$	0	0	0	0	0+0	0
FE b	0	$-13/4Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-3Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$5/2Fb-2Fx-1/2qx^2$	0	0	0	0		
GH 2b	0	$-3/2Fb+3/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$3/4Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/4Fx$	0	0	0	0	0+0	0
EI b	0	$-13/4Fb+1/4Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$13/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-13/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/8\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

ESΣ01.xxxx.082

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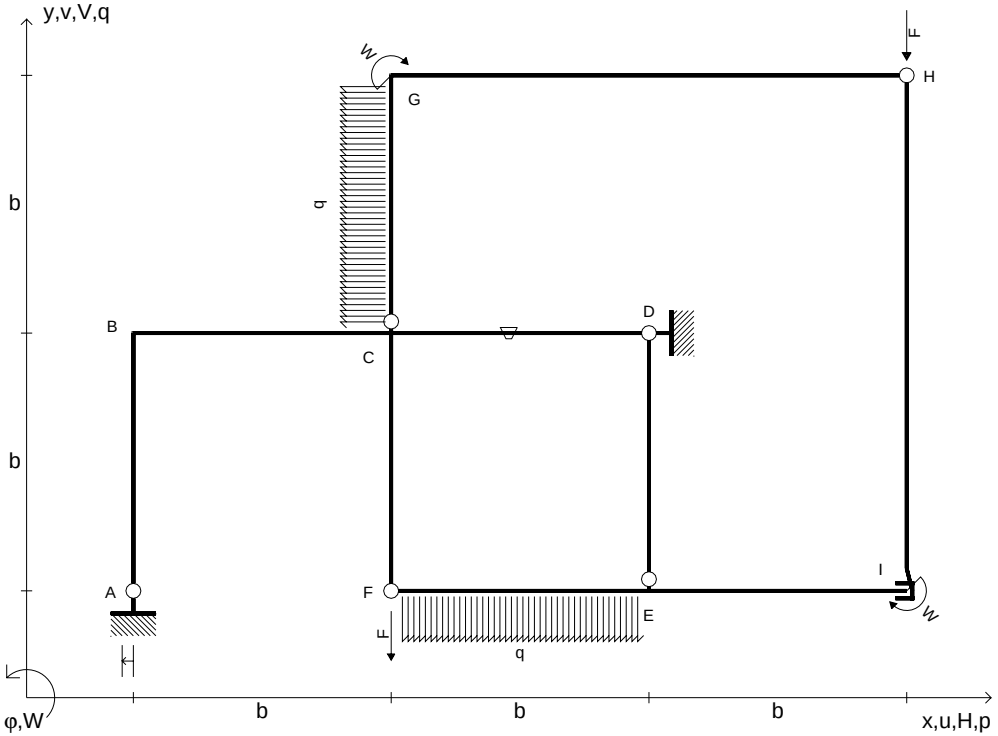

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$q_{EF} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
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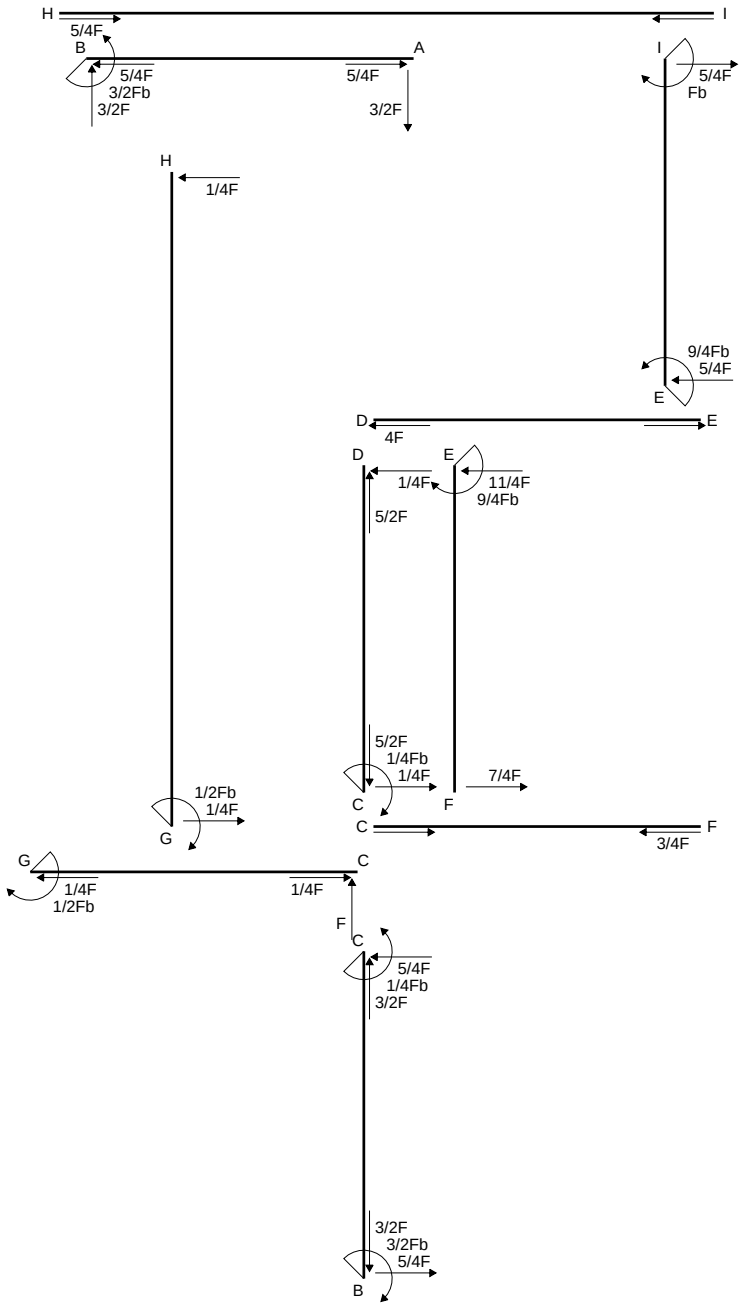
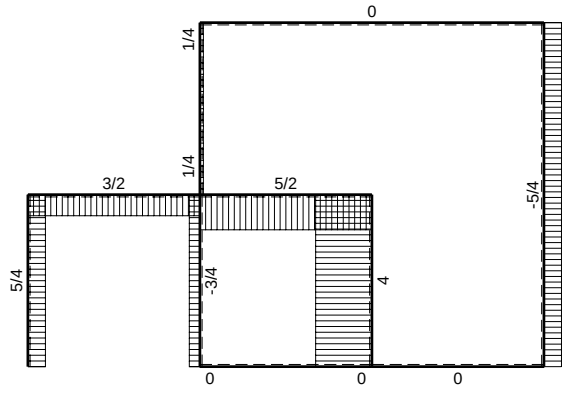
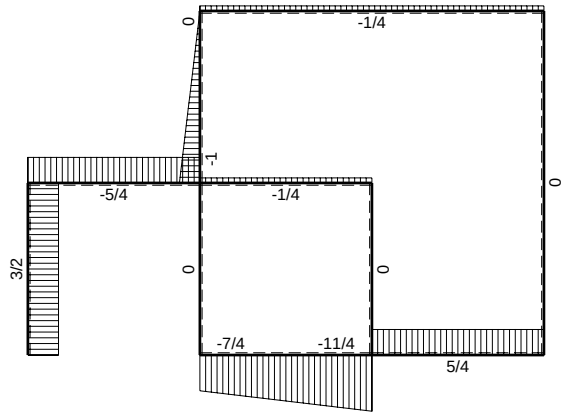
Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo A.

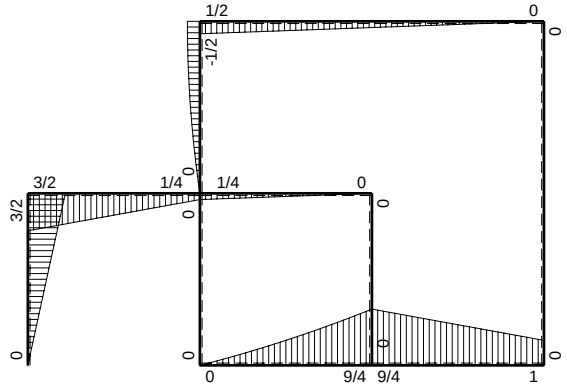




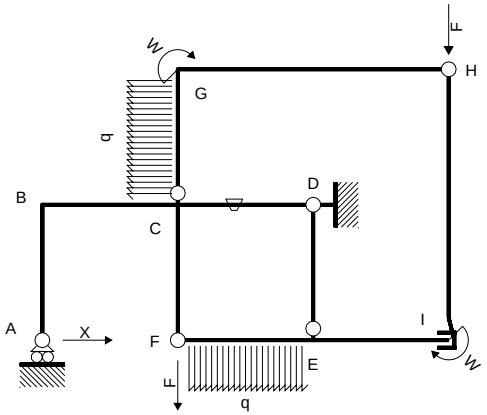
←□→ F



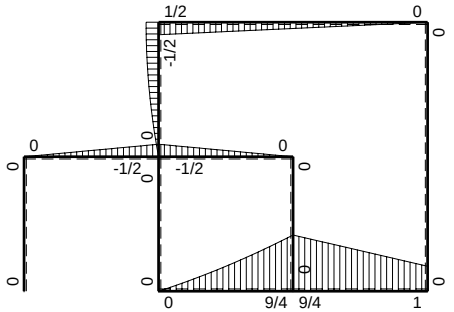
□↑↓ F



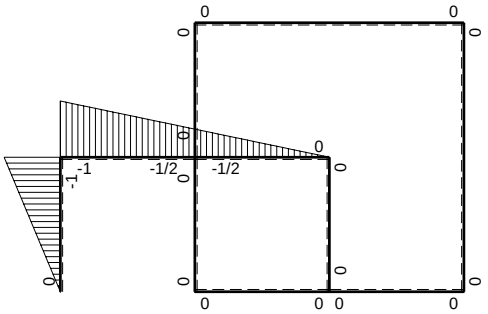
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-1/2Fx$	0	$1/2Fbx-1/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$1/2Fb-1/2Fx$	0	$1/4Fb^2-1/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	$-Fb/EJ$	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$9/4Fb-11/4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-7/4Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb-1/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/4Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx$	0	0	0	0	0+0	0
EI b	0	$-9/4Fb+5/4Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$3/2Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-3/2F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\, x/b -1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b -1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/12\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/12\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/12\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

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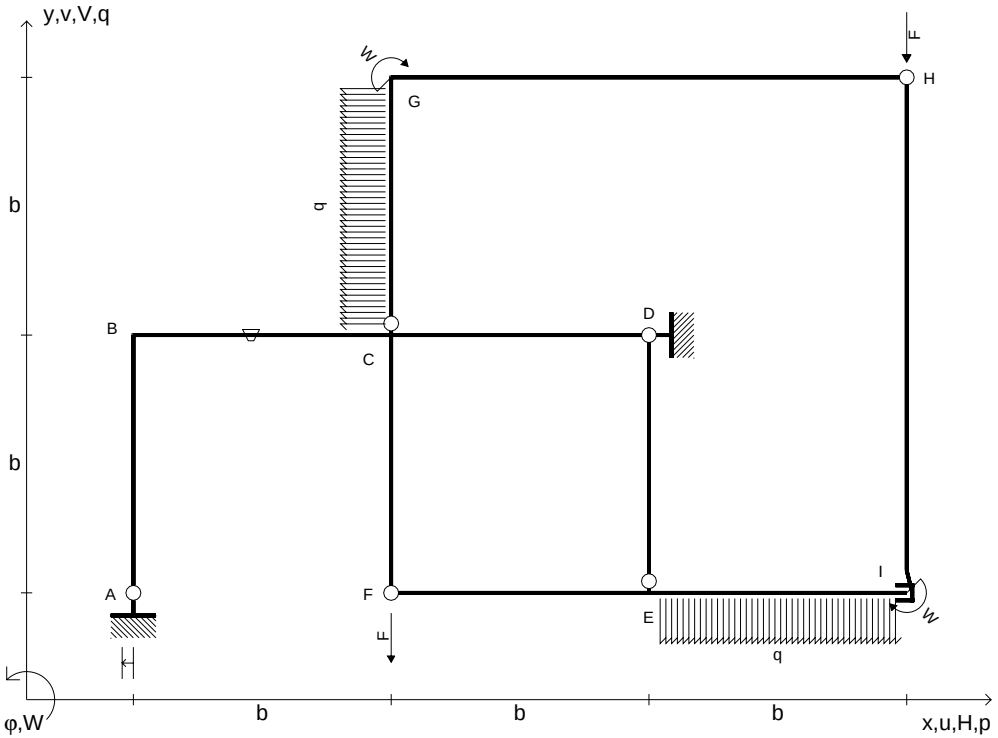
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$q_{IE} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
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$EJ_{IE} = EJ$

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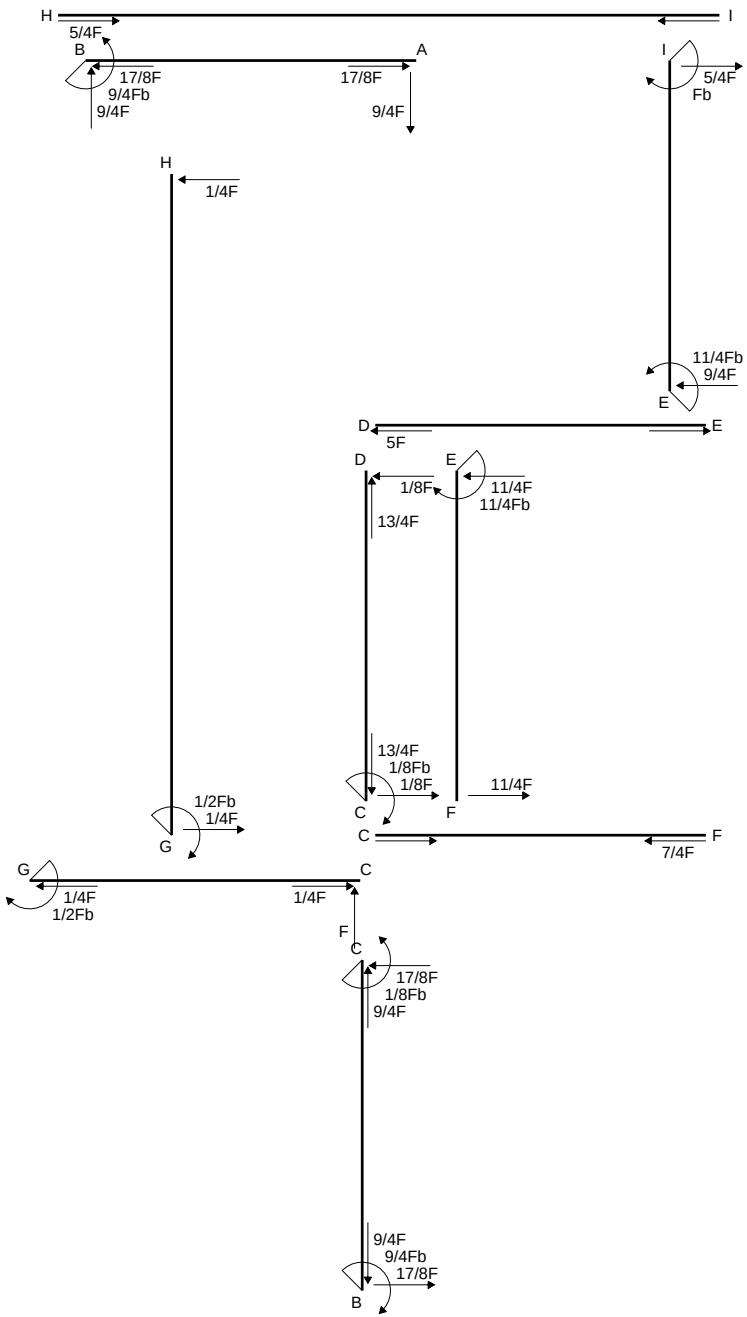
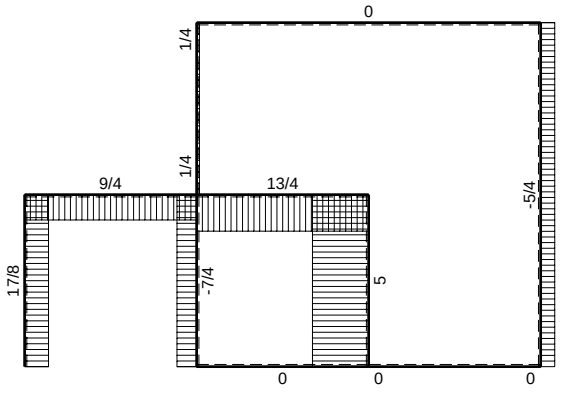
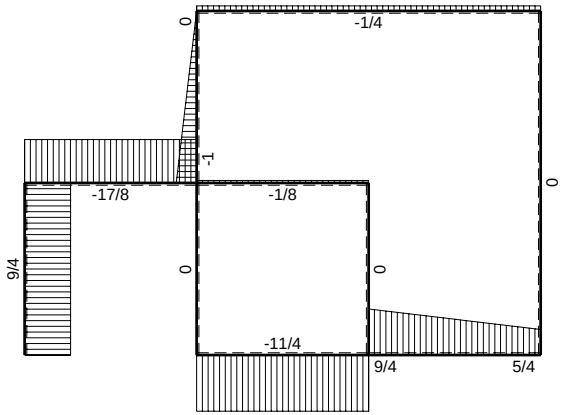
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo A.

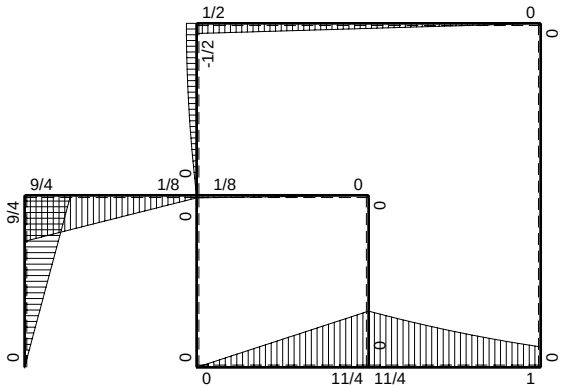




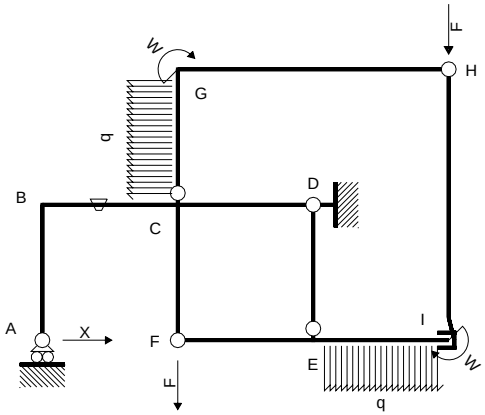
←⊞→ F



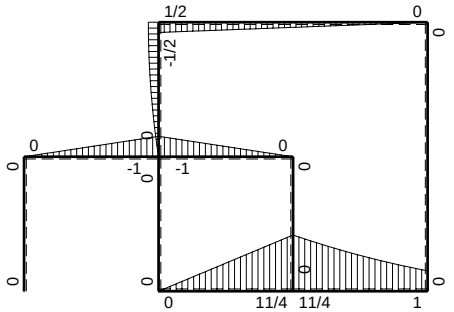
↑⊞↓ F



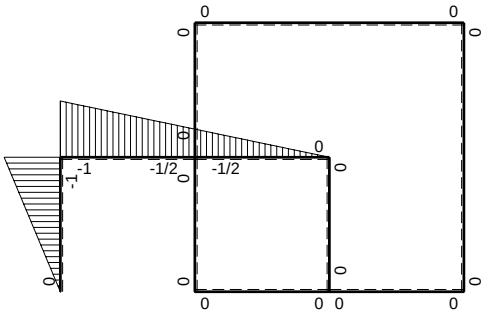
⟲⊞⟳ Fb



Schema di calcolo iperstatico



⟲⊞⟳ M_o flessione da carichi assegnati



⟲⊞⟳ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-Fx	-Fb/EJ	$Fbx-1/2Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/3+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	Fb-Fx	Fb/EJ	$1/2Fb^2-1/2Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-Fb+Fx	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	11/4Fb-11/4Fx	0	0	0	0	0+0	0
FE b	0	-11/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-11/4Fb+9/4Fx-1/2qx^2$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$9/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-9/4F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

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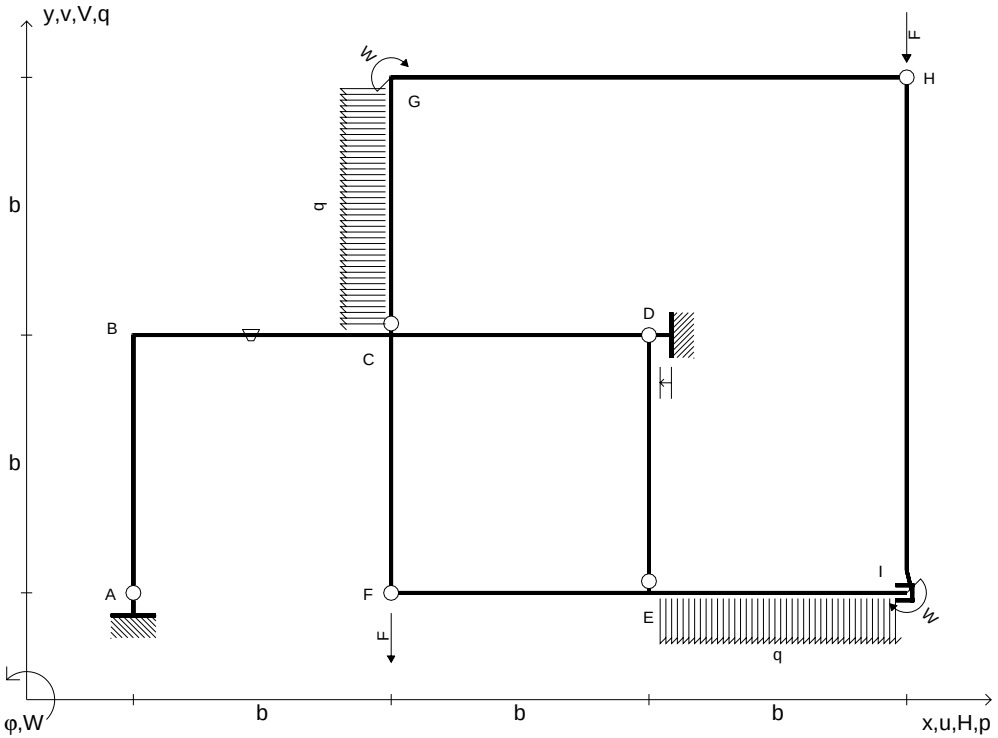
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

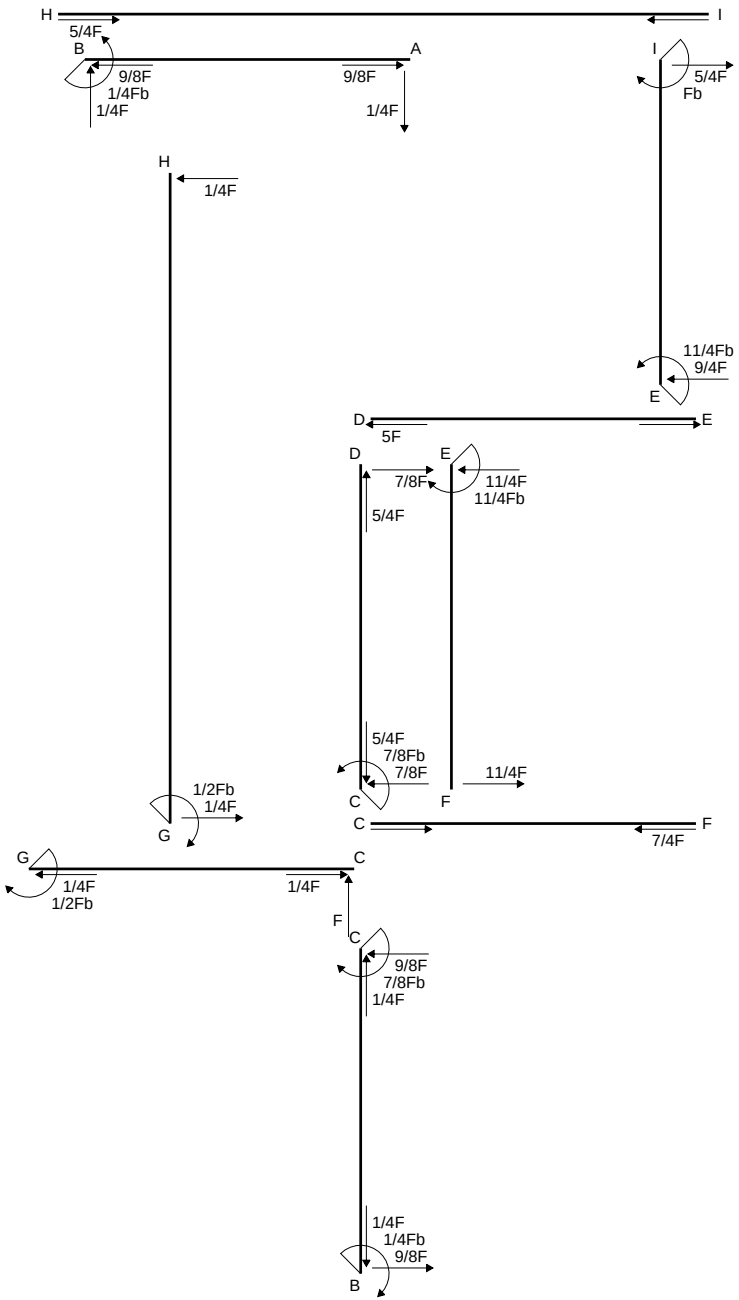


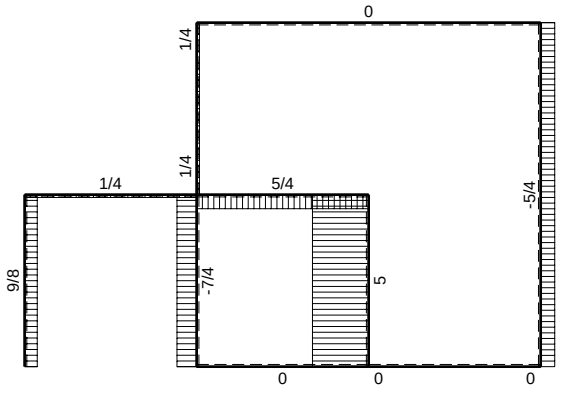
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$q_{IE} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

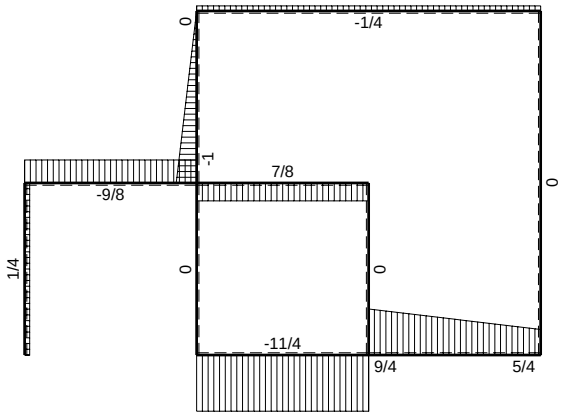
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=V_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.

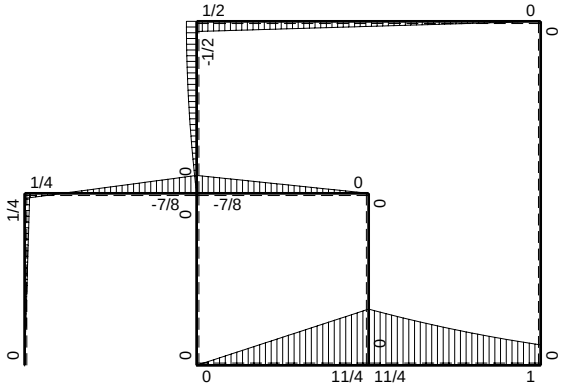




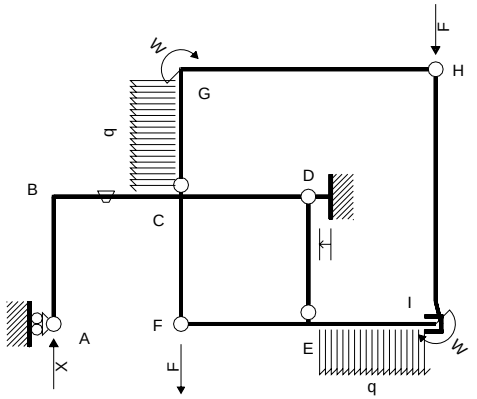
←□→ F



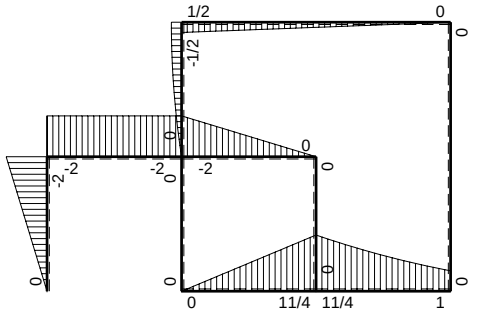
↑□↓ F



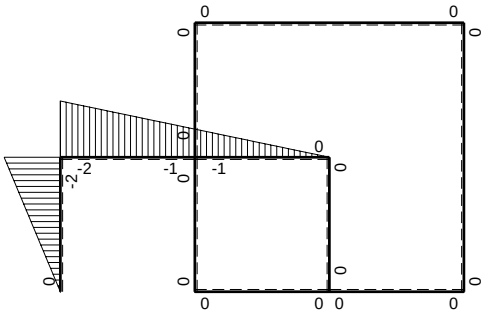
↻□ Fb



Schema di calcolo iperstatico



↻□ M_o flessione da carichi assegnati



↻□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-2x	-2Fx	0	$4Fx^2$	0	$4x^2$	$(4/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	2b-2x	2Fb-2Fx	0	$4Fb^2-8Fbx+4Fx^2$	0	$4b^2-8bx+4x^2$		
BC b	-2b+x	-2Fb	-Fb/EJ	$4Fb^2-2Fbx$	$2Fb^2/EJ-Fxb/EJ$	$4b^2-4bx+x^2$	$(3+3/2)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	b+x	2Fb	Fb/EJ	$2Fb^2+2Fbx$	$Fb^2/EJ+Fxb/EJ$	$b^2+2bx+x^2$		
CD b	-b+x	-2Fb+2Fx	0	$2Fb^2-4Fbx+2Fx^2$	0	$b^2-2bx+x^2$	$(2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	x	2Fx	0	$2Fx^2$	0	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	11/4Fb-11/4Fx	0	0	0	0	0+0	0
FE b	0	-11/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-11/4Fb+9/4Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-2Fb^3/EJ$	
	totali						$9/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_A$						-9/8F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 955375 ESΣ01.xxxx.091
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L_{AB}^{XX} = \int_o^b (4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L_{BA}^{XX} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 4\ b + 4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L_{BC}^{XX} = \int_o^b (4 - 4\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 2\ x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 2\ b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L_{CB}^{XX} = \int_o^b (1 + 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x + x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b + b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L_{CD}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L_{DC}^{XX} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L_{AB}^{Xo} = \int_o^b (4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [4/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (4/3\ b)\ Fb^2\ 1/EJ = 4/3\ Fb^3/EJ$

$L_{BA}^{Xo} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (4\ b - 4\ b + 4/3\ b)\ Fb^2\ 1/EJ = 4/3\ Fb^3/EJ$

$L_{BC}^{Xo} = \int_o^b (4 - 2\ x/b)\ Fb^2\ 1/EJ\ dx + \int_o^b (2 - x/b)\ \theta\ dx = [4\ x - x^2/b]_o^b\ Fb^2\ 1/EJ + [2\ x - 1/2\ x^2/b]_o^b\ \theta$

$= (4\ b - b)\ Fb^2\ 1/EJ + (2\ b - 1/2\ b)\ \theta = 9/2\ Fb^3/EJ$

$L_{CB}^{Xo} = \int_o^b (2 + 2\ x/b)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1 - x/b)\ \theta\ dx = [2\ x + x^2/b]_o^b\ Fb^2\ 1/EJ + [-\ x - 1/2\ x^2/b]_o^b\ \theta$

$= (2\ b + b)\ Fb^2\ 1/EJ + (-\ b - 1/2\ b)\ \theta = 9/2\ Fb^3/EJ$

$L_{CD}^{Xo} = \int_o^b (2 - 4\ x/b + 2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [2\ x - 2\ x^2/b + 2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (2\ b - 2\ b + 2/3\ b)\ Fb^2\ 1/EJ = 2/3\ Fb^3/EJ$

$L_{DC}^{Xo} = \int_o^b (2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (2/3\ b)\ Fb^2\ 1/EJ = 2/3\ Fb^3/EJ$

ESΣ01.xxxx.091

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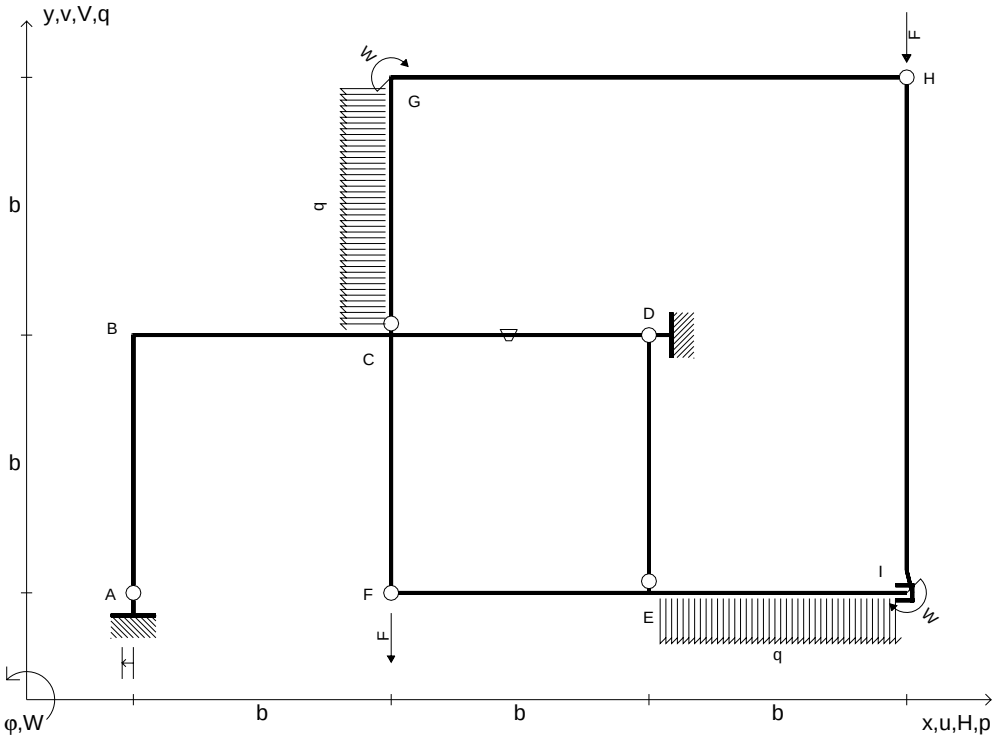

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$q_{IE} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
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Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
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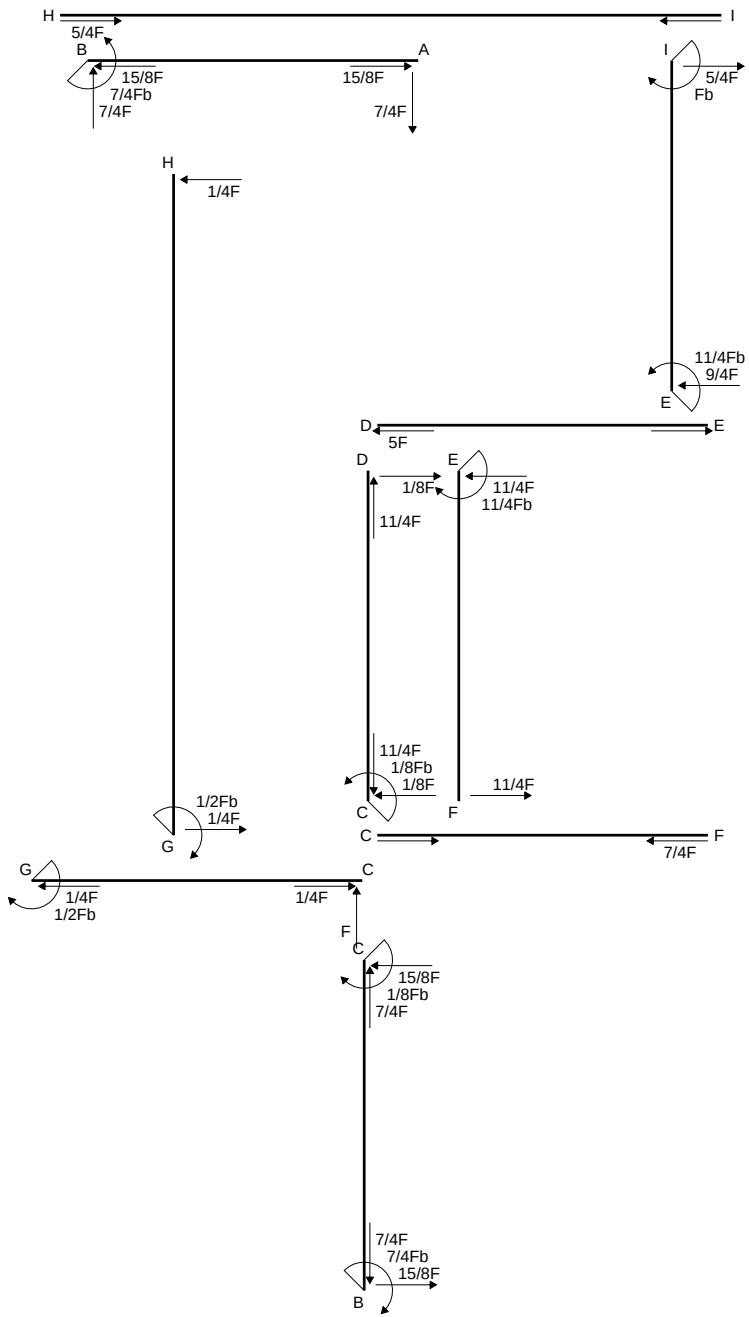
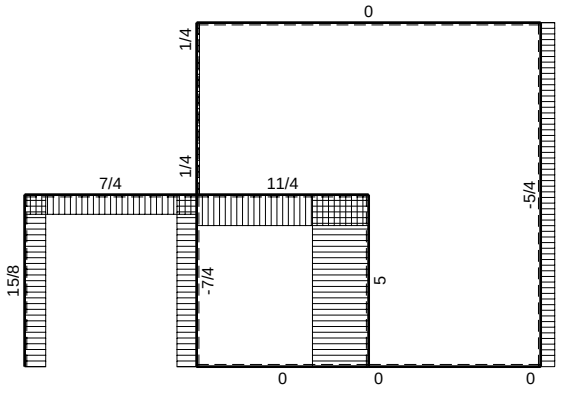
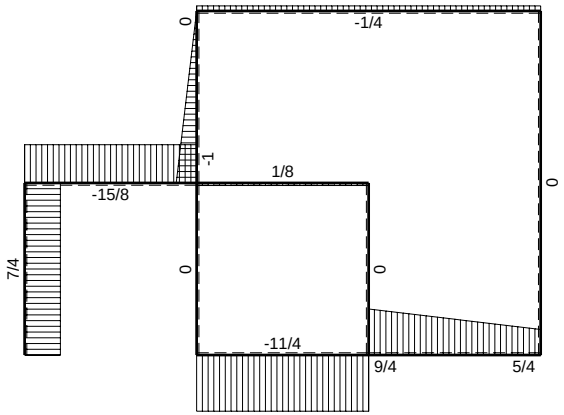
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo A.

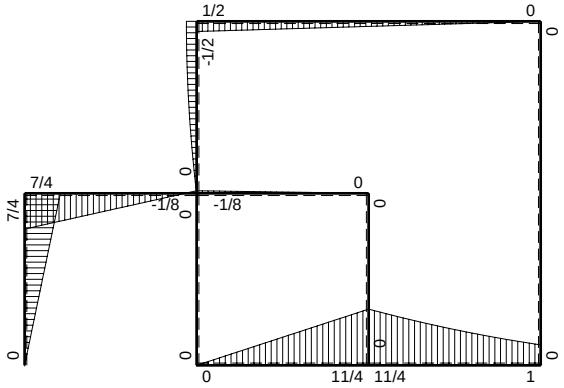




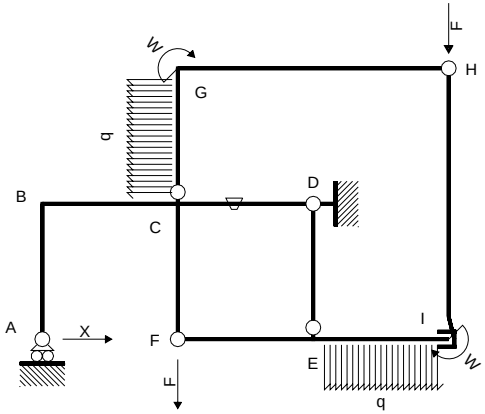
←□→ F



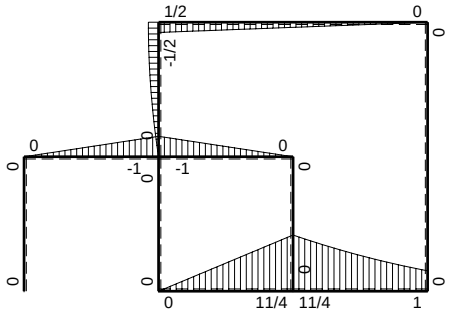
↑□↓ F



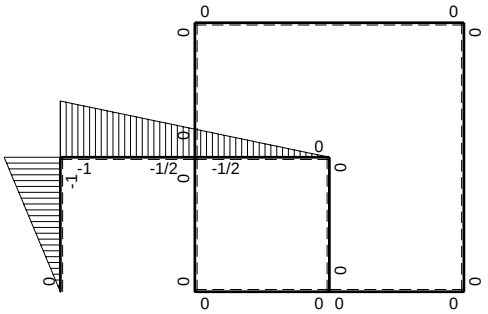
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	-Fx	0	$Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(1/3+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	Fb-Fx	0	$1/2Fb^2-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-Fb+Fx$	-Fb/EJ	$1/2Fb^2-Fbx+1/2Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/6+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	Fx	Fb/EJ	$1/2Fx^2$	1/2Fxb/EJ	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	11/4Fb-11/4Fx	0	0	0	0	0+0	0
FE b	0	-11/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-11/4Fb+9/4Fx-1/2qx^2$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$7/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-7/4F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212423 ESΣ01.xxxx.093

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 5/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/6\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 5/12\ Fb^3/EJ$

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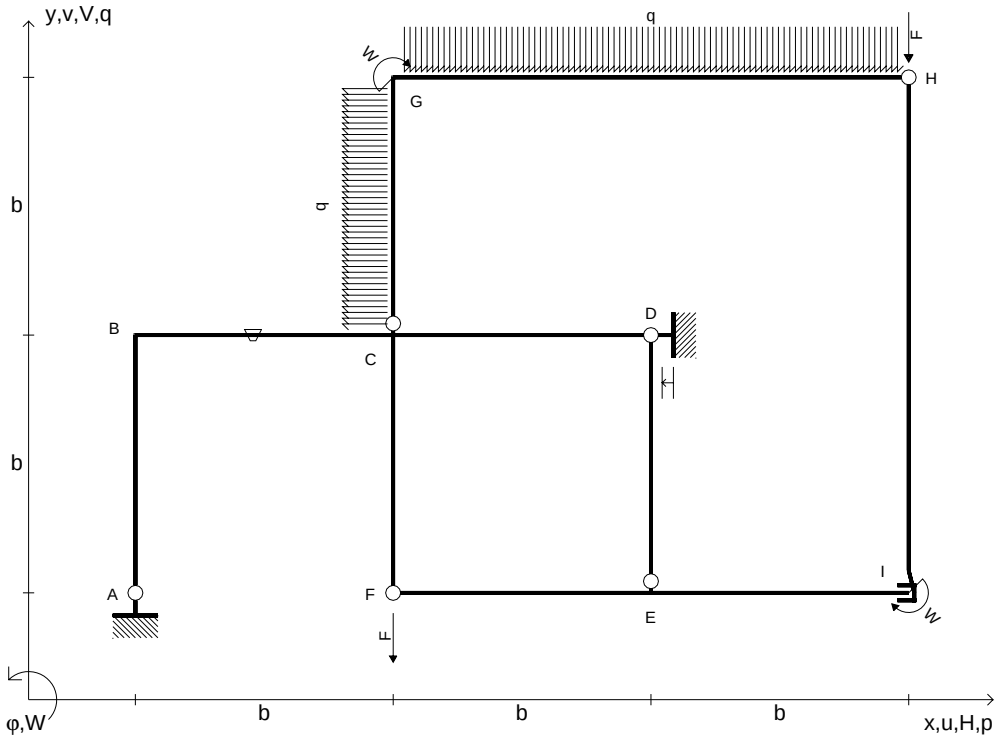
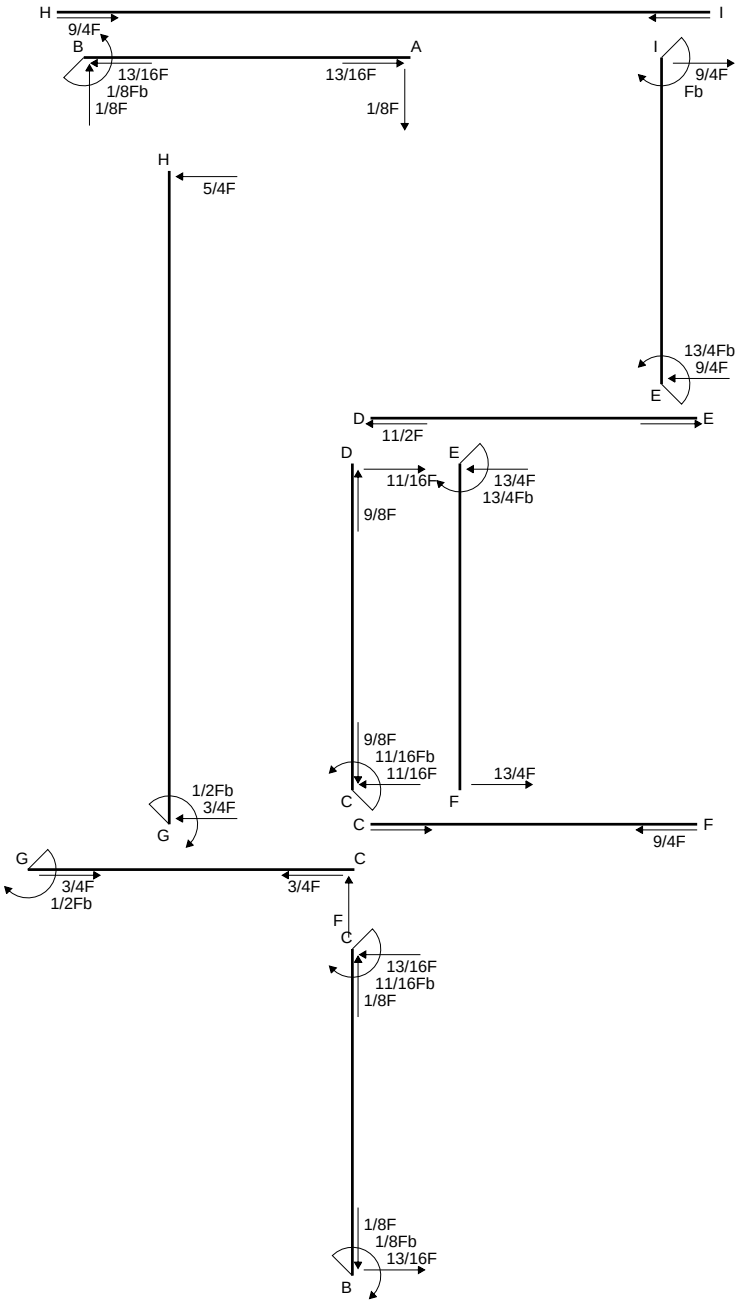


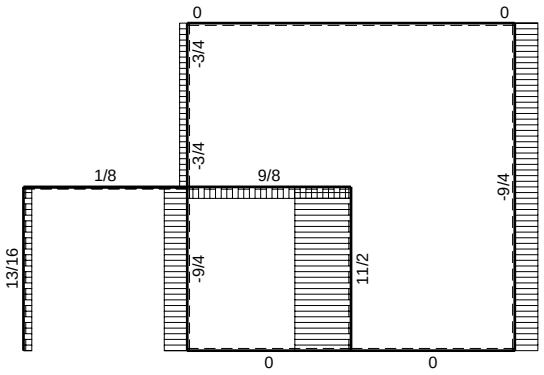
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$q_{GH} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.





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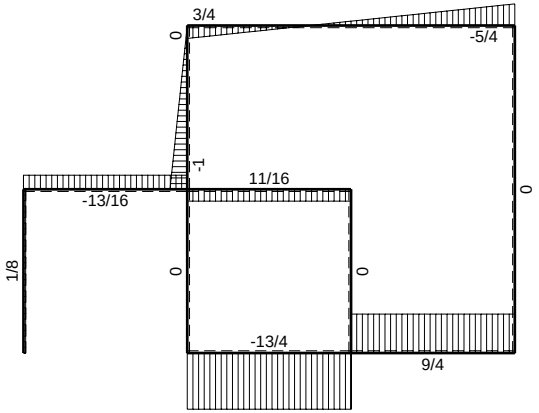

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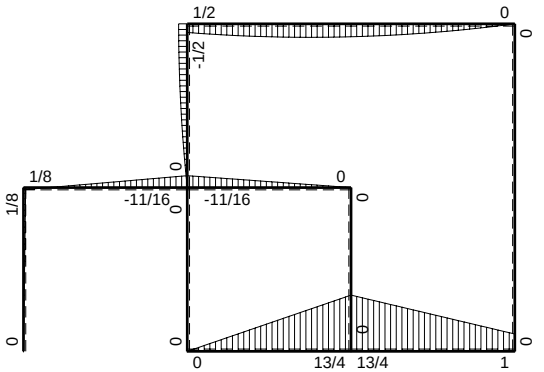
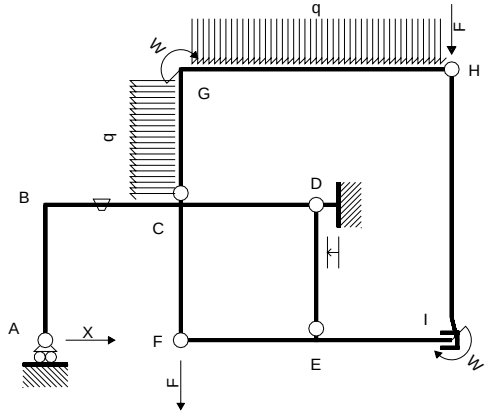
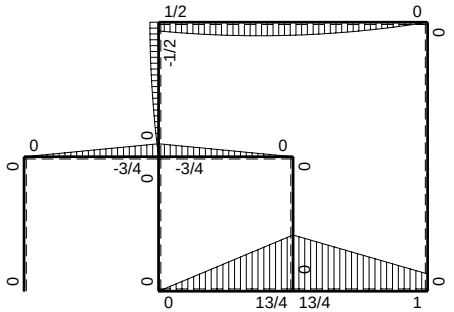


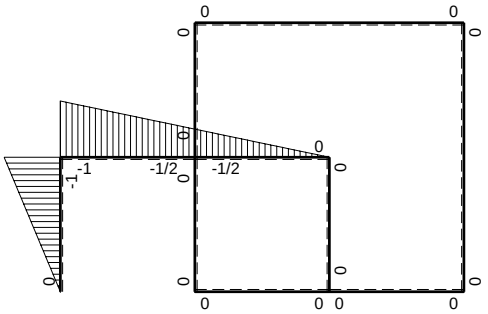
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-3/4Fx	-Fb/EJ	$3/4Fbx-3/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/4+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	3/4Fb-3/4Fx	Fb/EJ	$3/8Fb^2-3/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	3/4Fx	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	13/4Fb-13/4Fx	0	0	0	0	0+0	0
FE b	0	-13/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-5/4Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+9/4Fx	0	0	0	0	0+0	0
EI b	0	-13/4Fb+9/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

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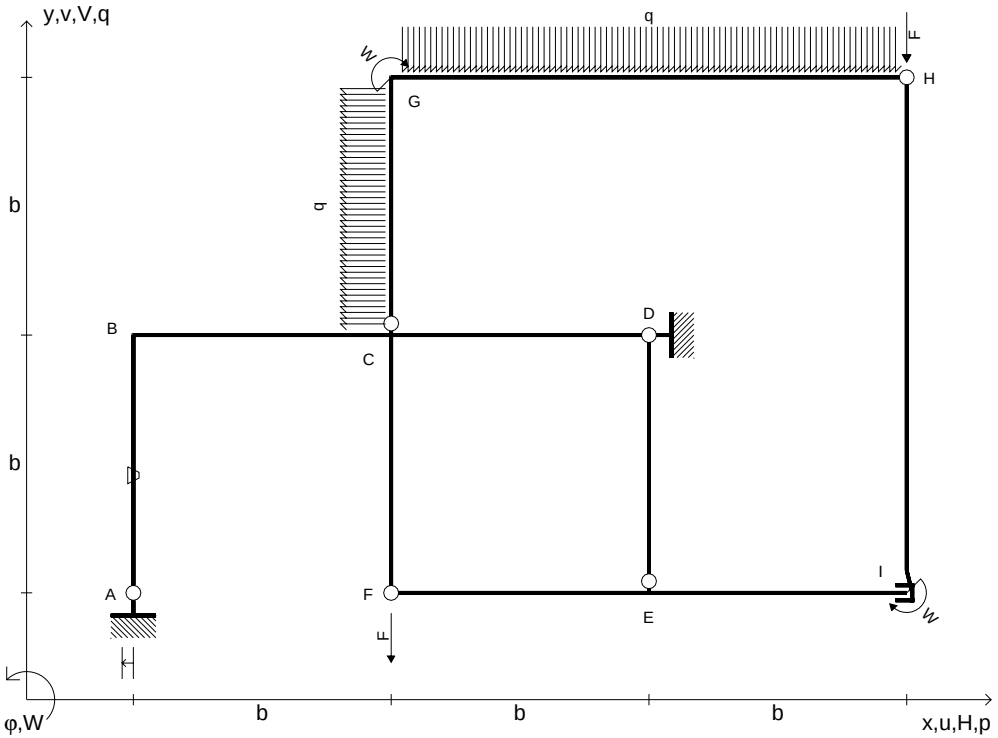

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$q_{GH} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
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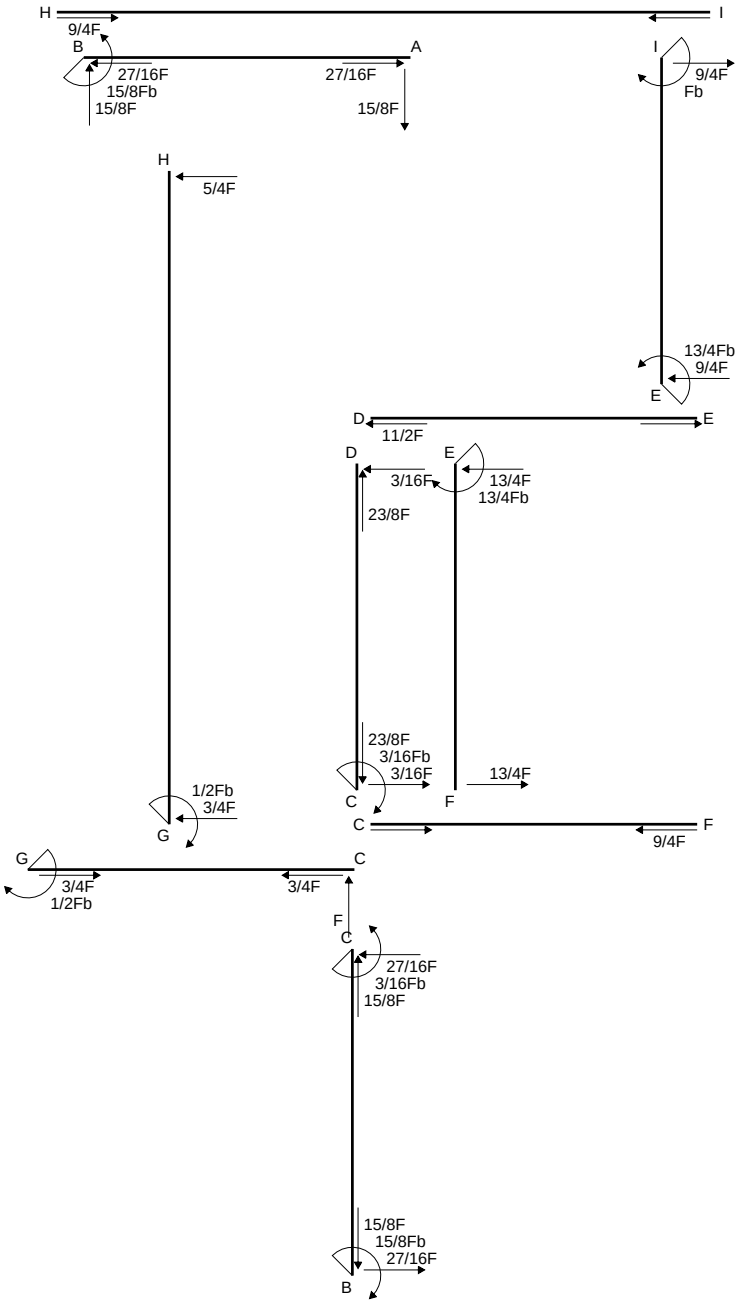
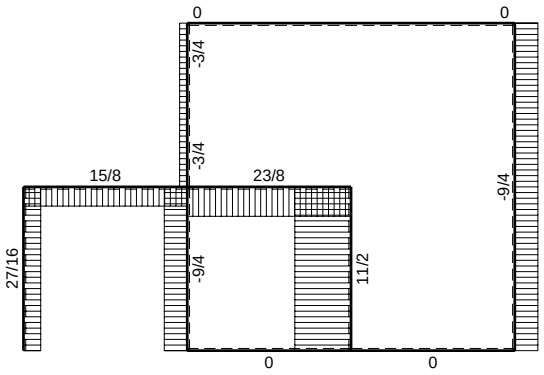
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.





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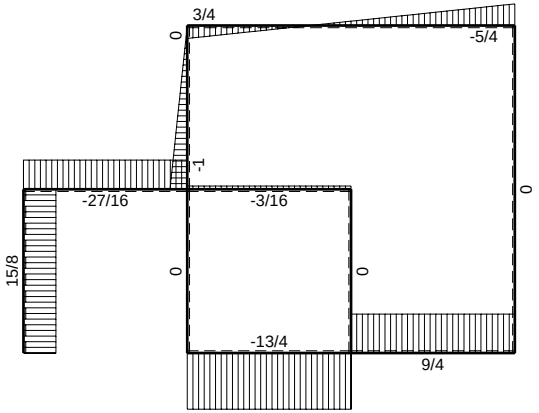

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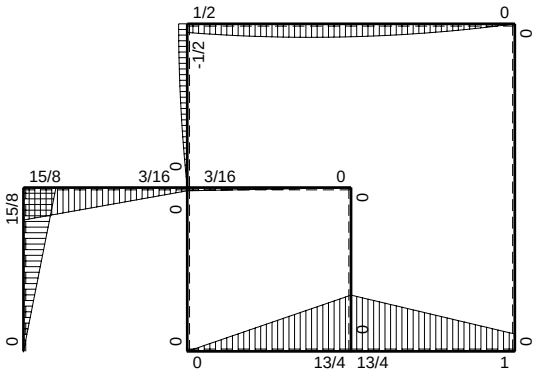

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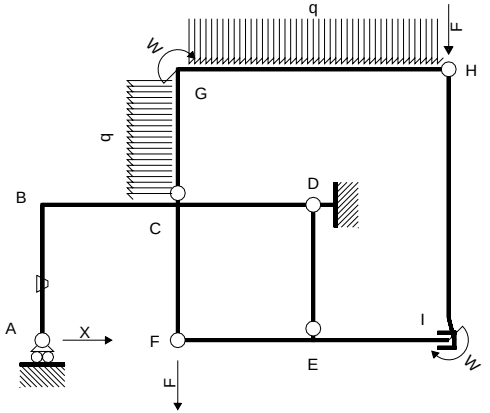
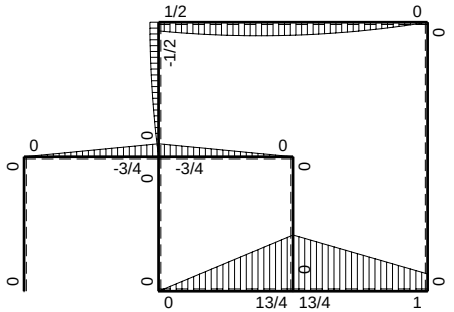
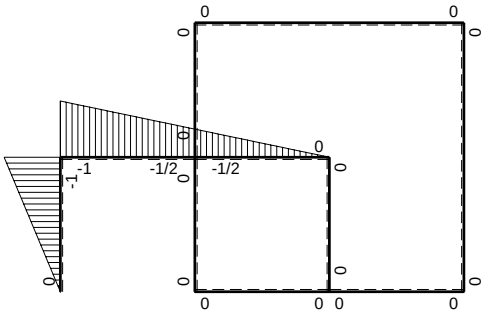


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-3/4Fx	0	3/4Fbx-3/8Fx^2	0	b^2-bx+1/4x^2	(1/4+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	3/4Fb-3/4Fx	0	3/8Fb^2-3/8Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	3/8Fb^2-3/4Fbx+3/8Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/8+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	3/4Fx	0	3/8Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	13/4Fb-13/4Fx	0	0	0	0	0+0	0
FE b	0	-13/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
GC b	0	1/2Fb-1/2qx^2	0	0	0	0		
GH 2b	0	1/2Fb+3/4Fx-1/2qx^2	0	0	0	0	0+0	0
HG 2b	0	-5/4Fx+1/2qx^2	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+9/4Fx	0	0	0	0	0+0	0
EI b	0	-13/4Fb+9/4Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						15/8Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-15/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

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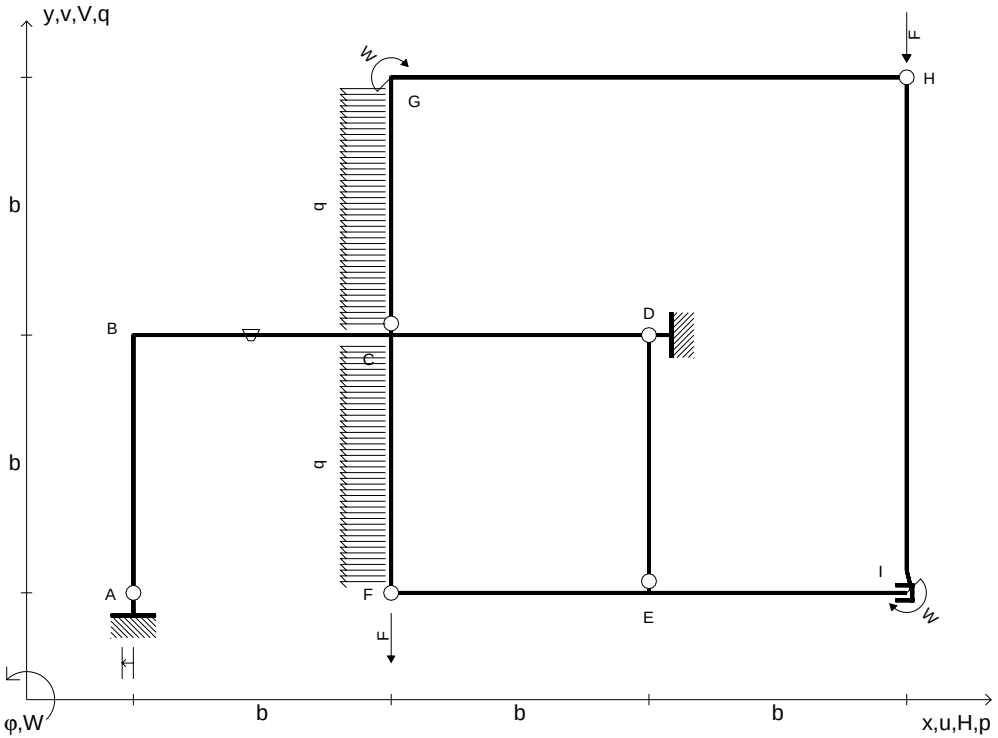

$V_H = -F$
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$W_G = -W = -Fb$
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$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
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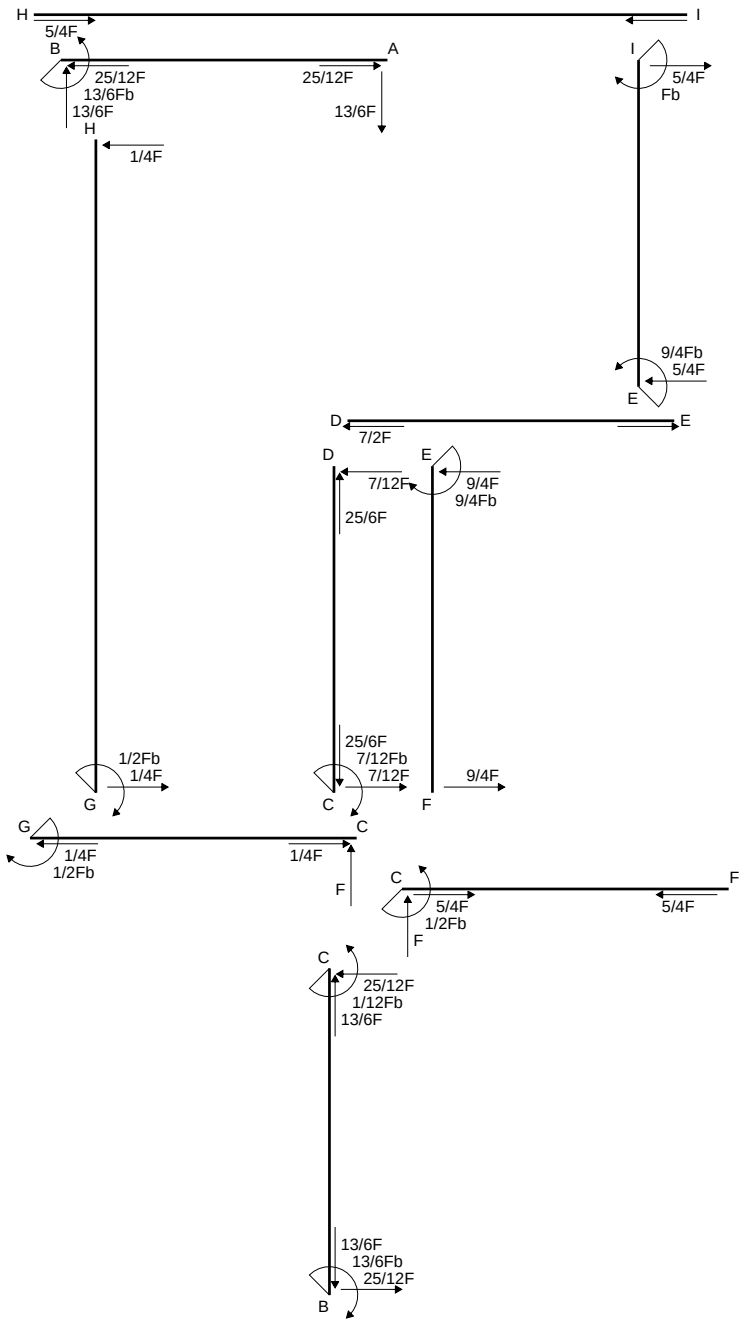
 08.01.25



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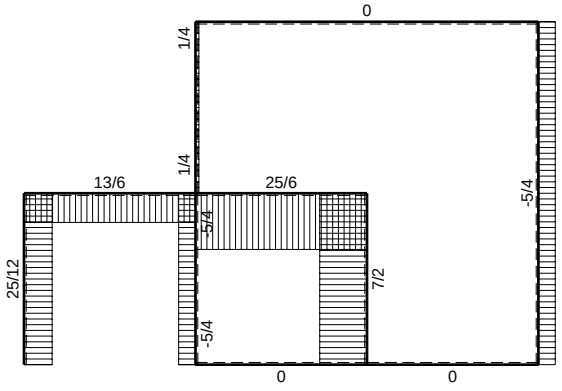
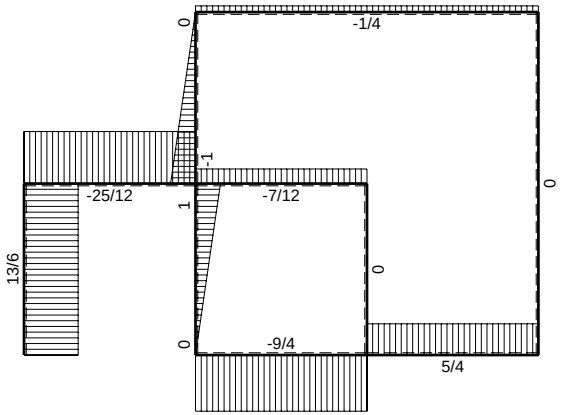
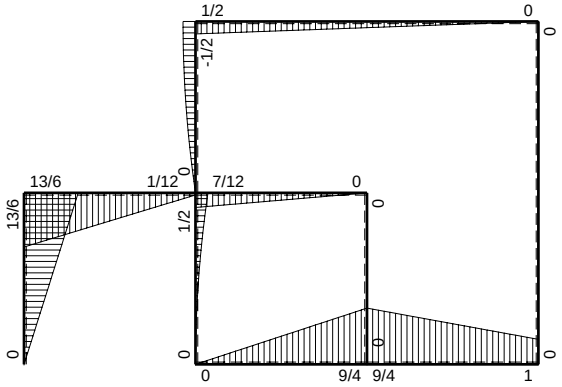


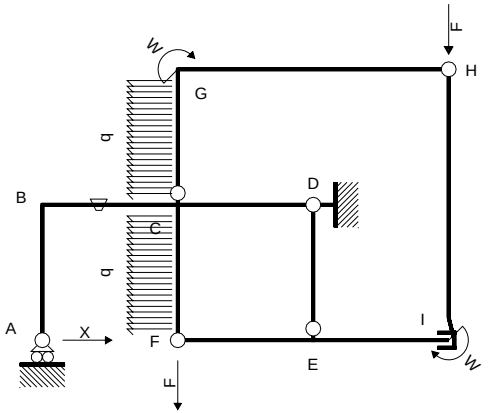
←□→ F



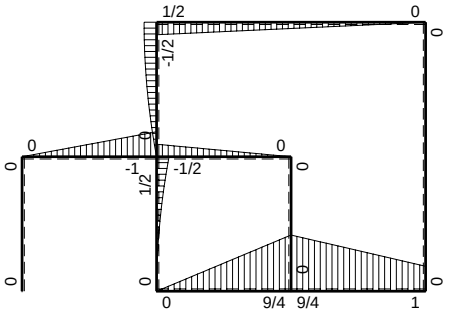
↑□↓ F



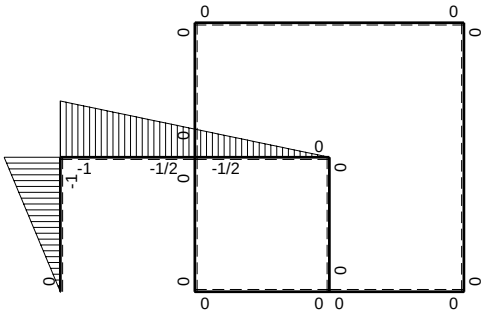
↺□↻ Fb



Schema di calcolo iperstatico



↺□↻ M_o flessione da carichi assegnati



↺□↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	-Fx	-Fb/EJ	$Fbx-1/2Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/3+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	Fb-Fx	Fb/EJ	$1/2Fb^2-1/2Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0
FE b	0	-9/4Fx	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0
EI b	0	-9/4Fb+5/4Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$13/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-13/6F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982517 ESΣ01.xxxx.101
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\ dx$

$= [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\ dx$

$= [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

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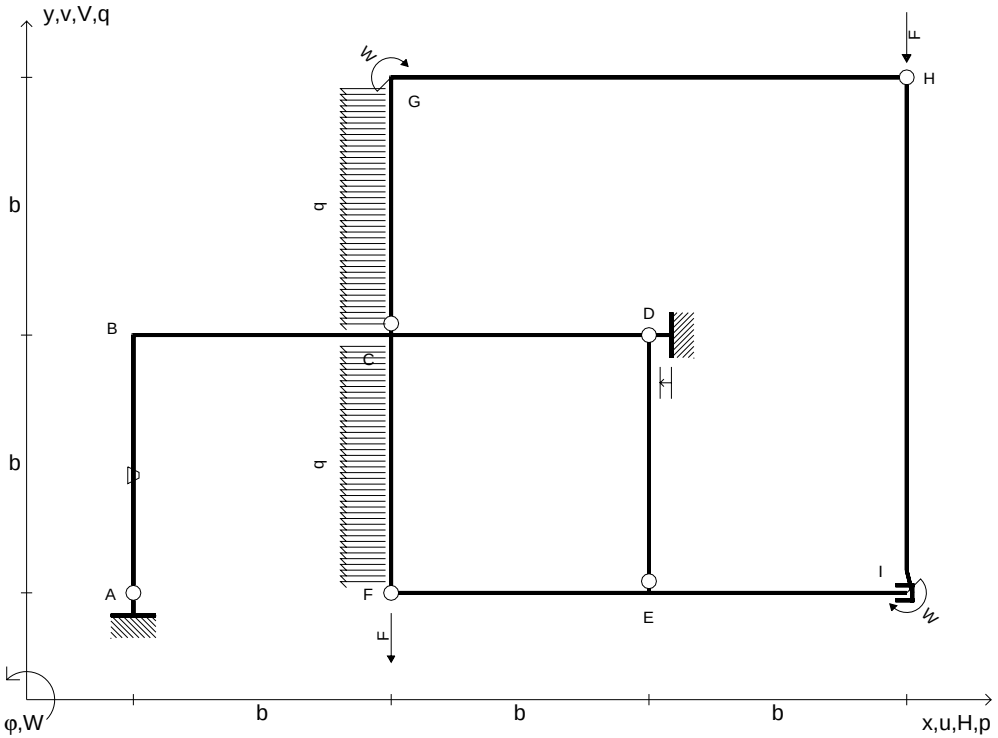
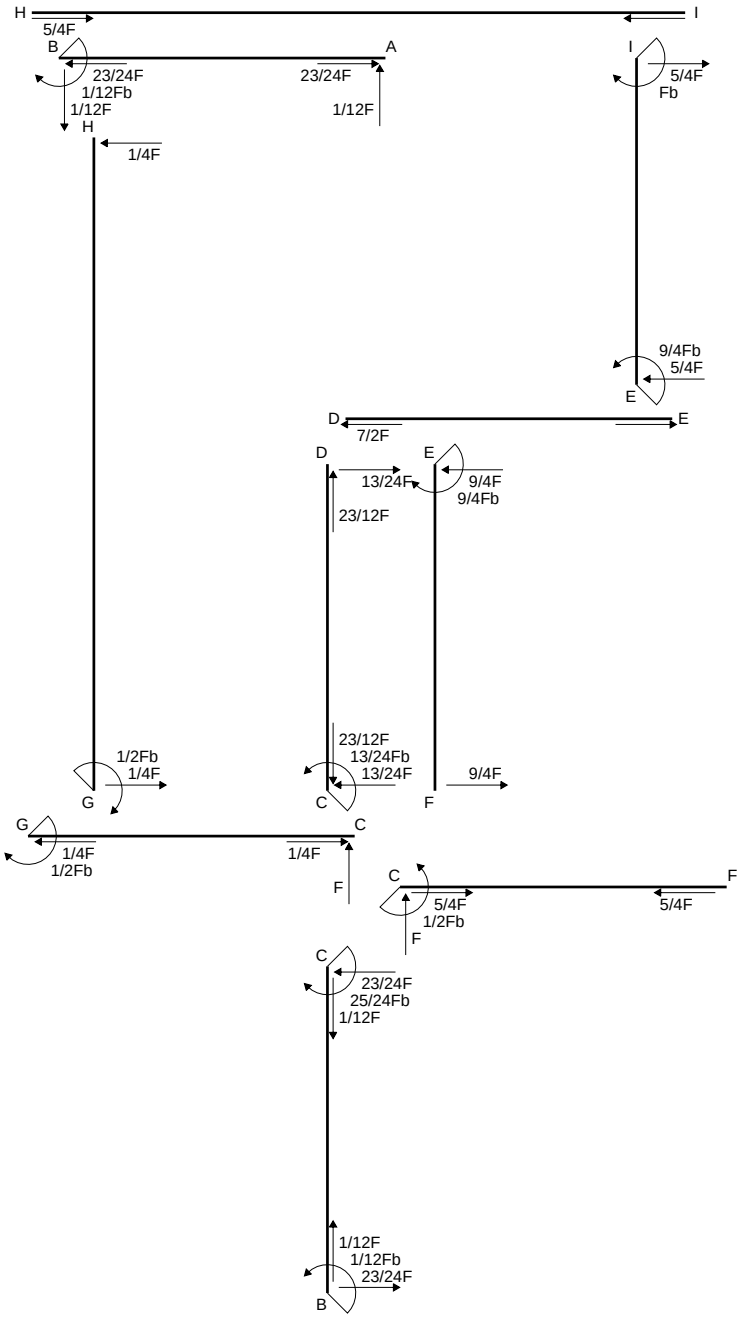


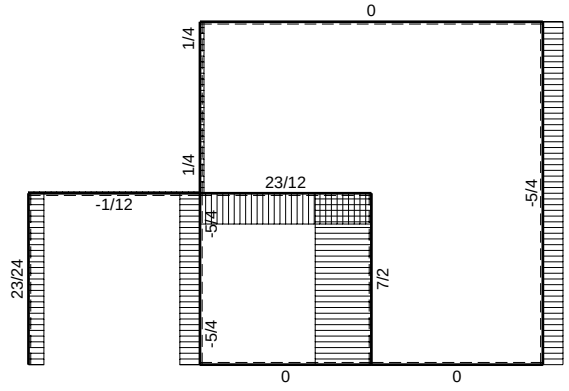
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$p_{FC} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

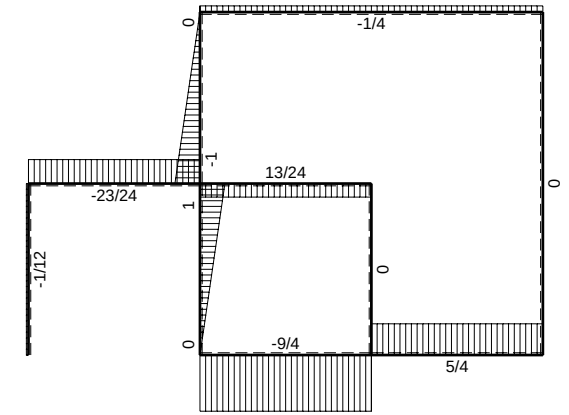
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

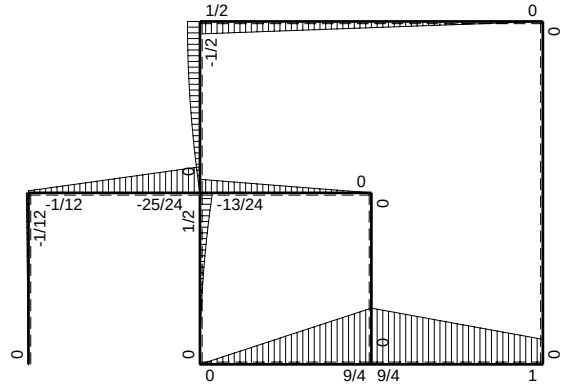




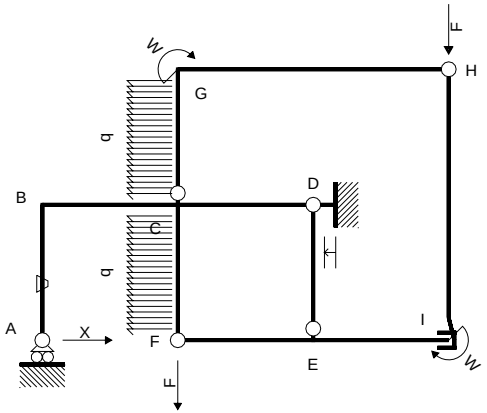
←□→ F



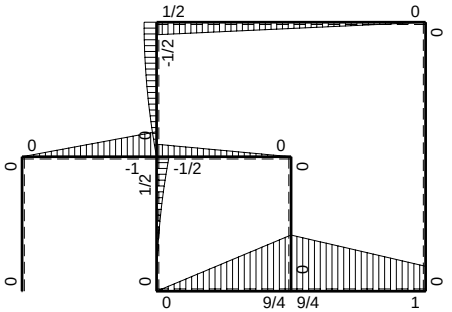
↑□↓ F



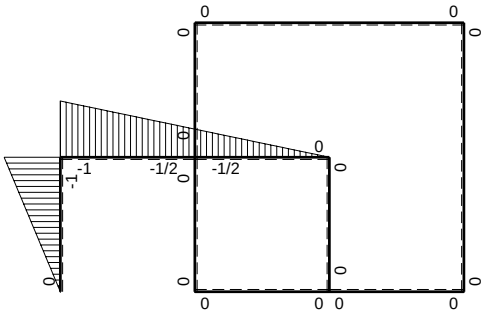
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210089 ESΣ01.xxxx.104

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-Fx	0	Fbx-1/2Fx^2	0	b^2-bx+1/4x^2	(1/3+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	Fb-Fx	0	1/2Fb^2-1/2Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb^2-1/2Fbx+1/4Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/12+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	1/2Fx	0	1/4Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0
FE b	0	-9/4Fx	0	0	0	0		
FC b	0	1/2qx^2	0	0	0	0	0+0	0
CF b	0	-1/2Fb+Fx-1/2qx^2	0	0	0	0		
CG b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
GC b	0	1/2Fb-1/2qx^2	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0
EI b	0	-9/4Fb+5/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3$/EJ	
	totali						-1/12Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						1/12F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

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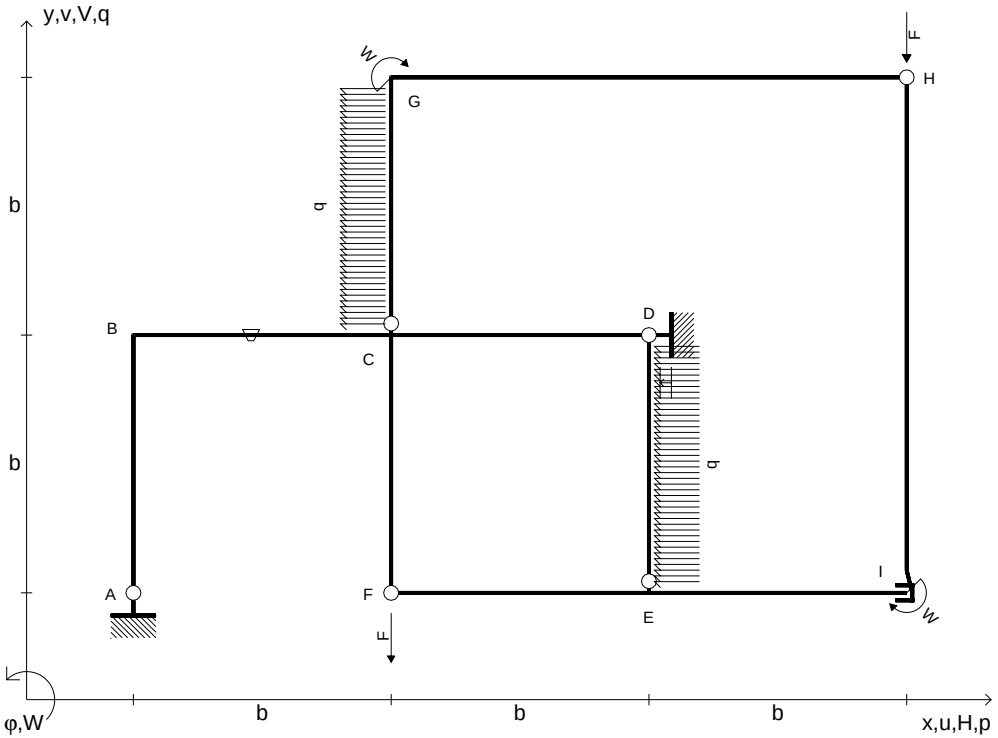

$V_H = -F$
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$W_G = -W = -Fb$
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$p_{DE} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
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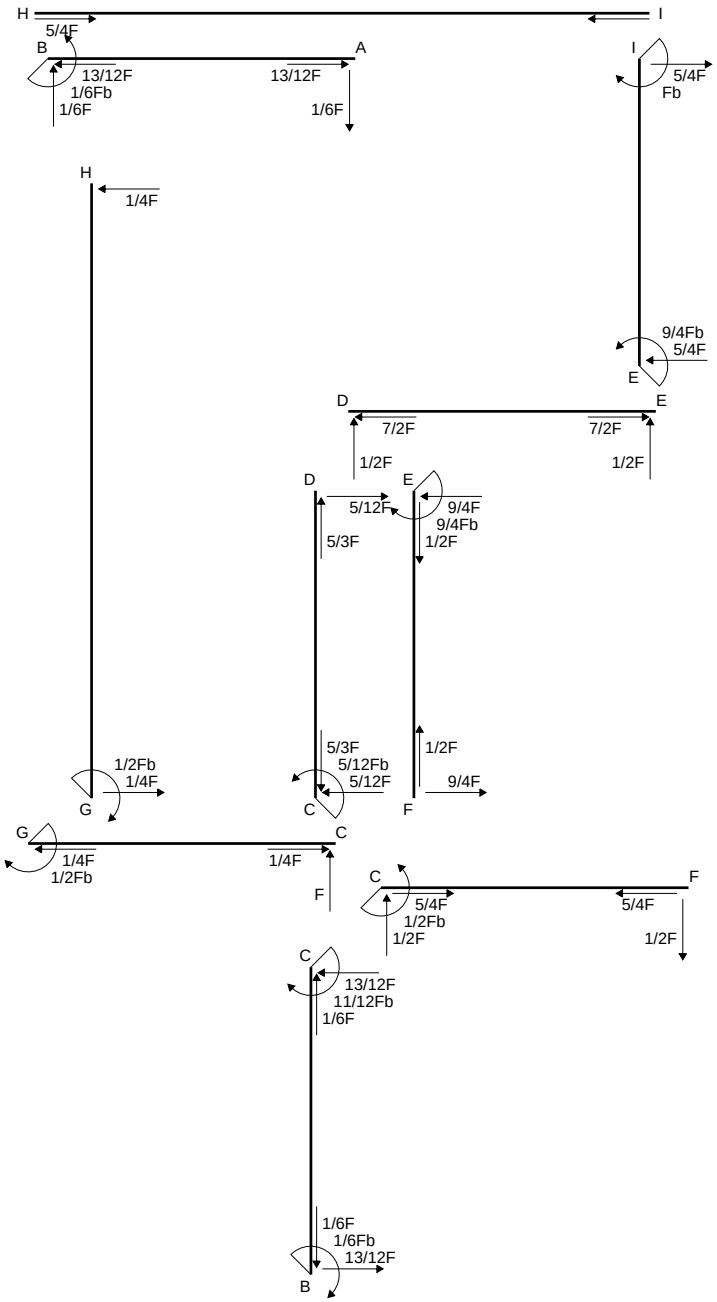
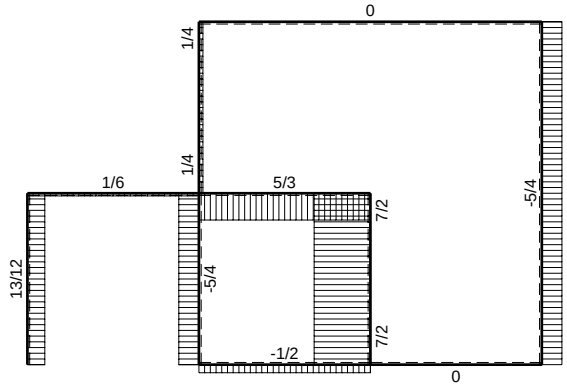
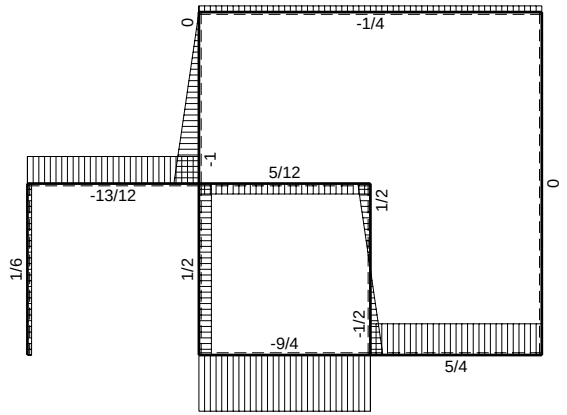
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.





F



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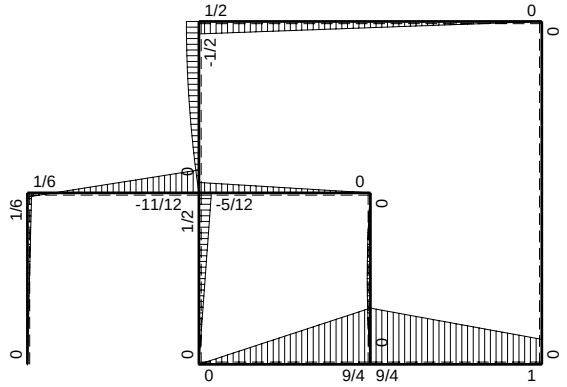
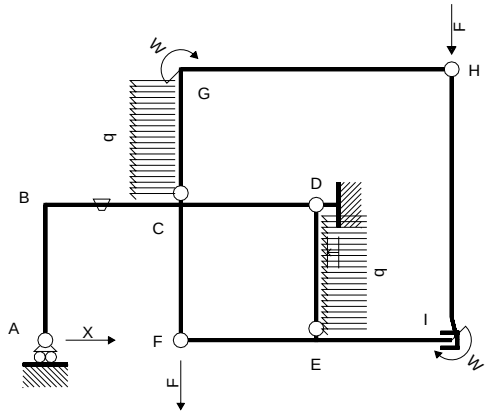
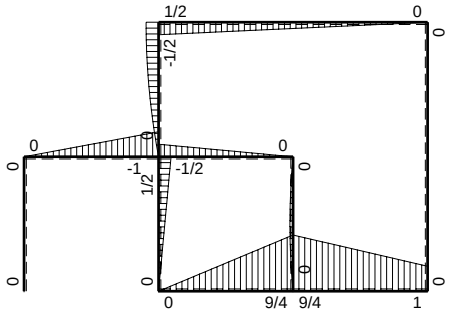


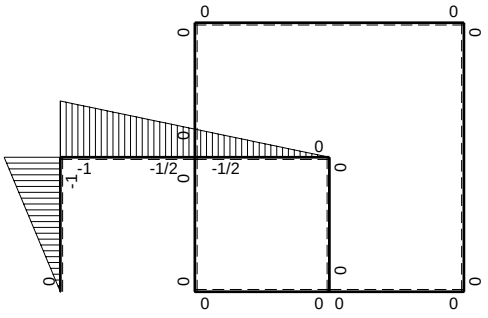
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-Fx	-Fb/EJ	$Fbx-1/2Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/3+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	Fb-Fx	Fb/EJ	$1/2Fb^2-1/2Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	1/2Fx	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0
FE b	0	-9/4Fx	0	0	0	0		
FC b	0	1/2Fx	0	0	0	0	0+0	0
CF b	0	-1/2Fb+1/2Fx	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0
EI b	0	-9/4Fb+5/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/6F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213396 ESΣ01.xxxx.108

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$L^{XX}_{AB} = \int_o^b (x^2/b^2) b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b - b +1/3\ b)\ b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/12\ b)\ b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x^2/b -1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ\ + [\ x -1/4\ x^2/b]_o^b\ \theta$

$= (1/2\ b -1/6\ b)\ Fb^2\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ = 13/12\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x -1/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b]_o^b\ \theta$

$= (1/2\ b -1/6\ b)\ Fb^2\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ = 13/12\ \ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [1/12\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (1/12\ b)\ Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

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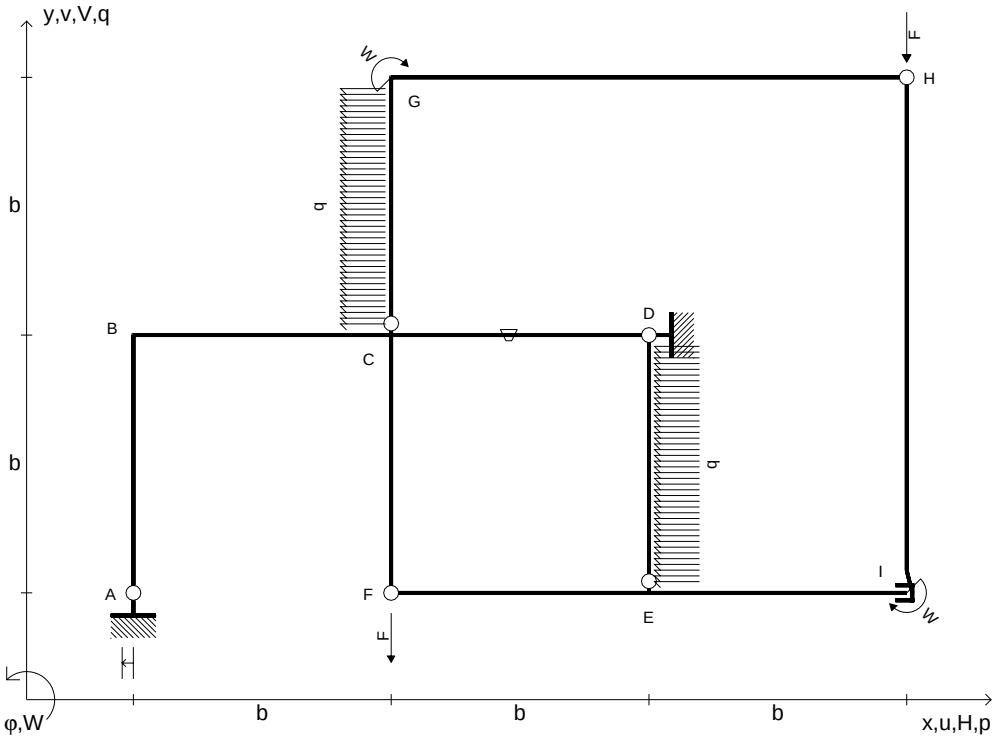
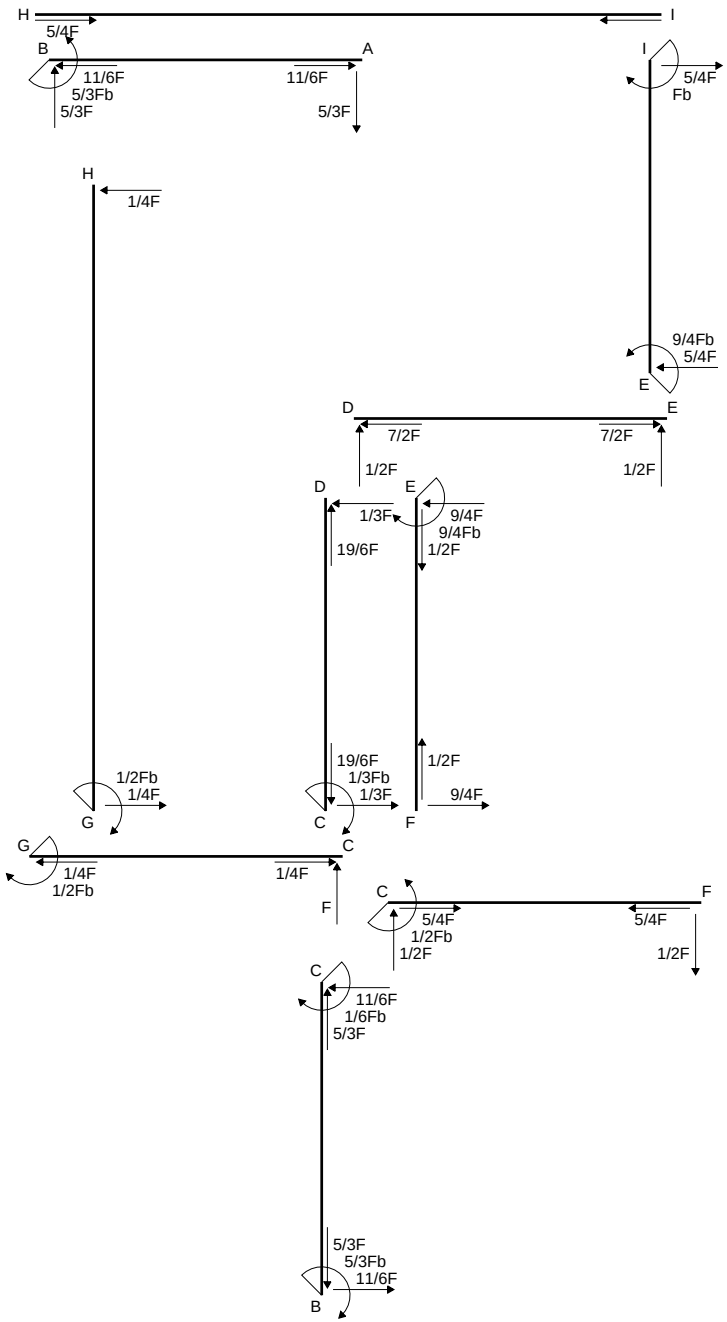


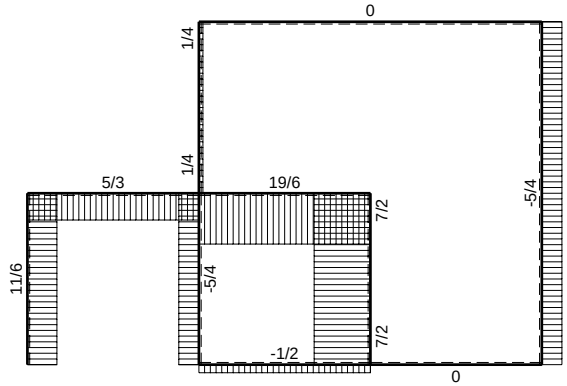
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$p_{CG} = -q = -F/b$
$p_{DE} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

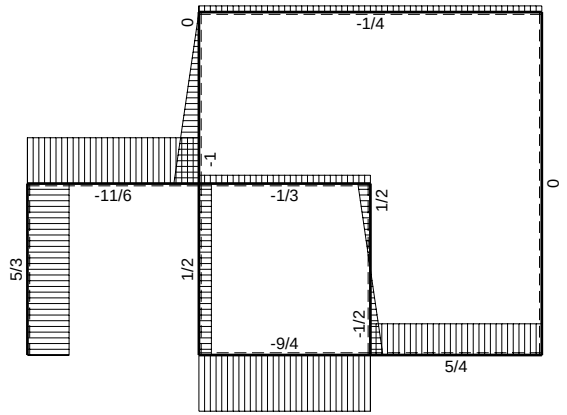
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo A.

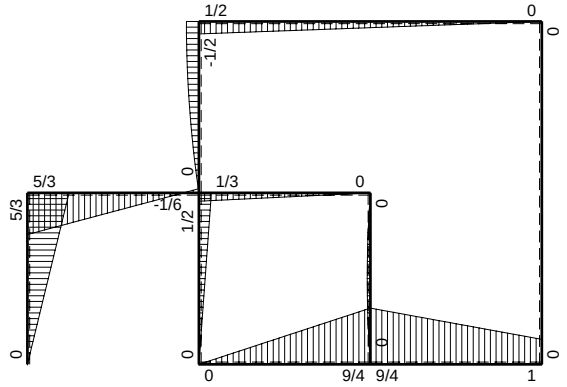




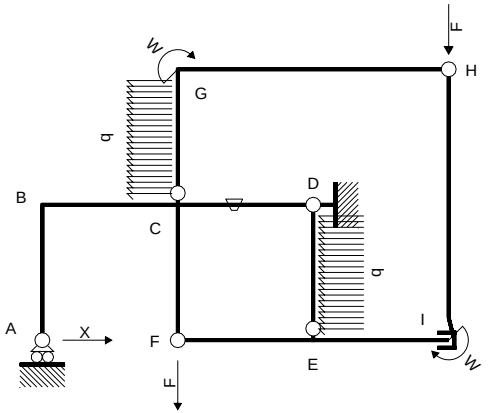
←□→ F



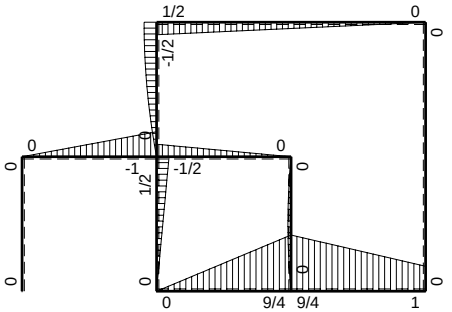
↑□↓ F



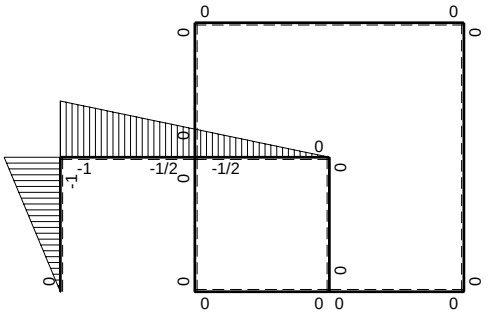
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211522 ESΣ01.xxxx.111

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	-Fx	0	$Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(1/3+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	Fb-Fx	0	$1/2Fb^2-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	-Fb/EJ	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	1/2Fx	Fb/EJ	$1/4Fx^2$	1/2Fxb/EJ	$1/4x^2$		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0
FE b	0	-9/4Fx	0	0	0	0		
FC b	0	1/2Fx	0	0	0	0	0+0	0
CF b	0	-1/2Fb+1/2Fx	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0
EI b	0	-9/4Fb+5/4Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$5/3Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-5/3F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/12\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

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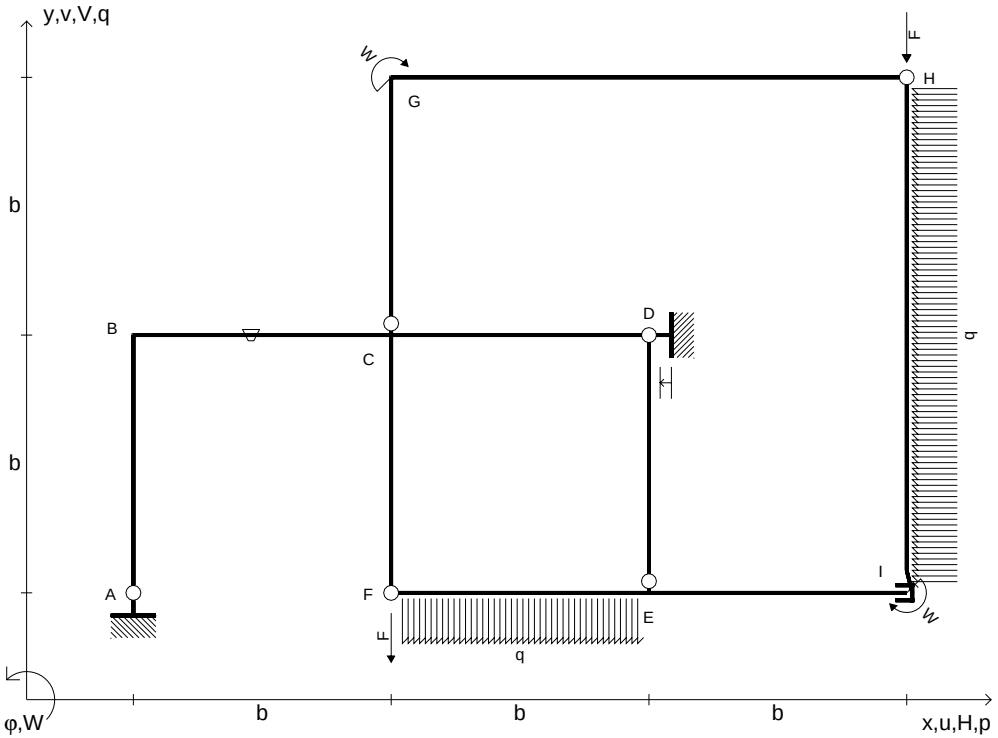
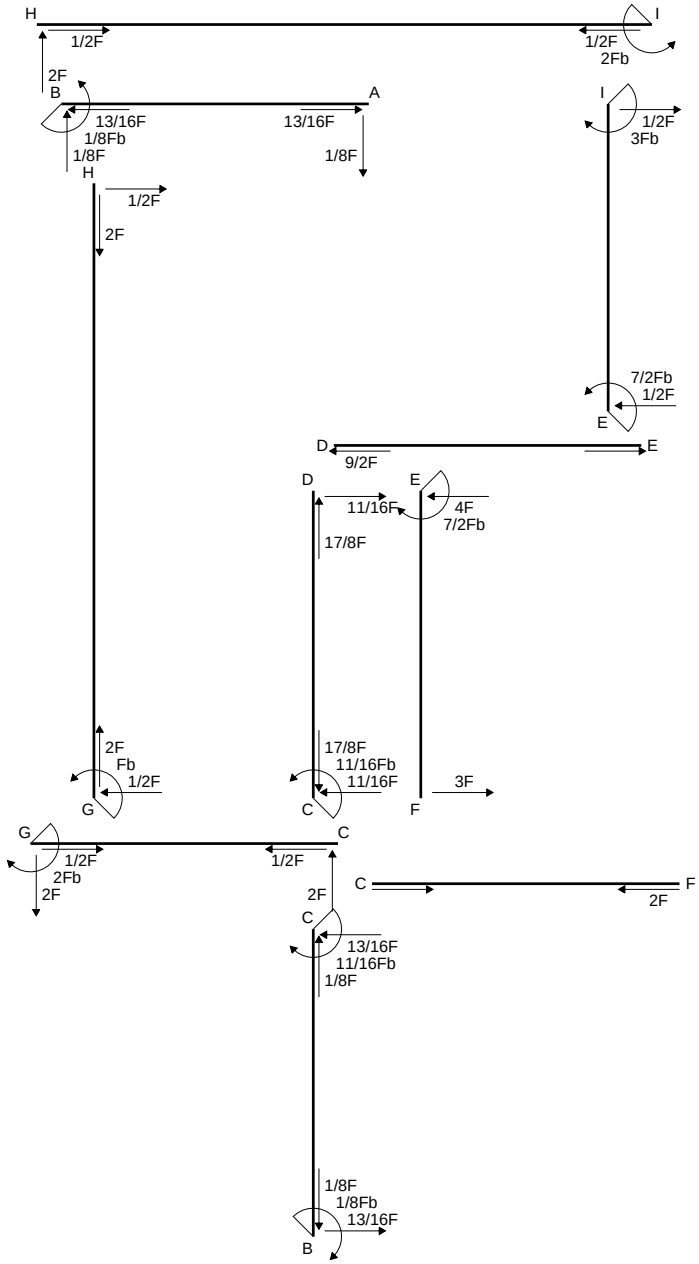


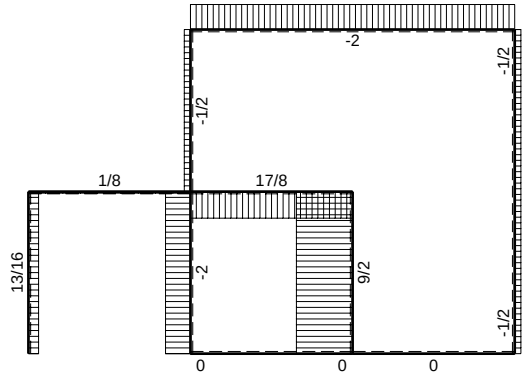
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

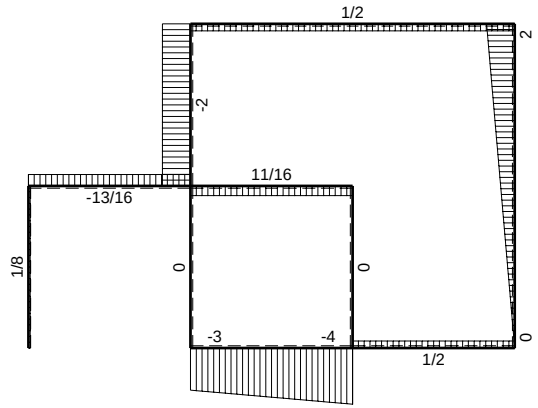
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.

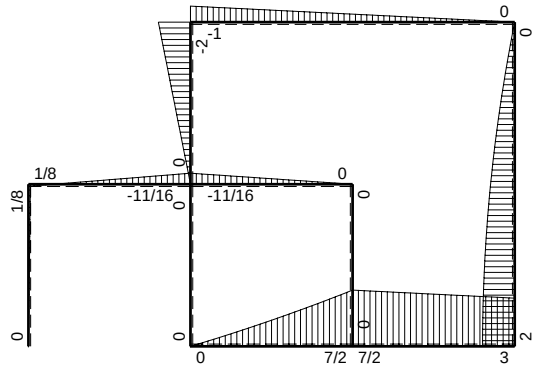




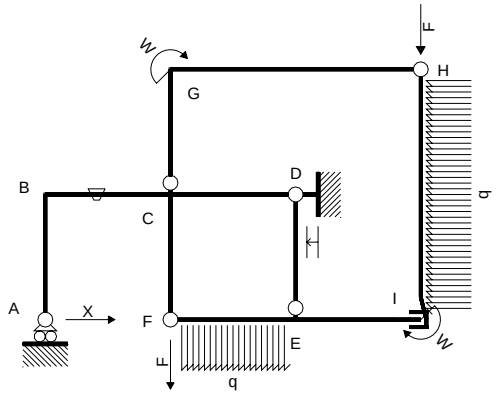
←□→ F



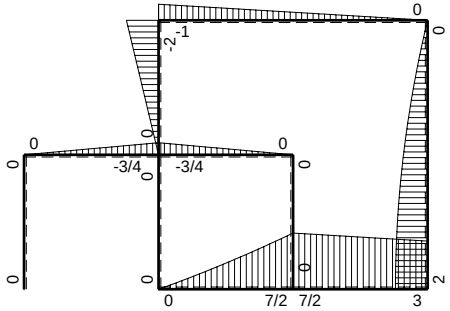
↑□↓ F



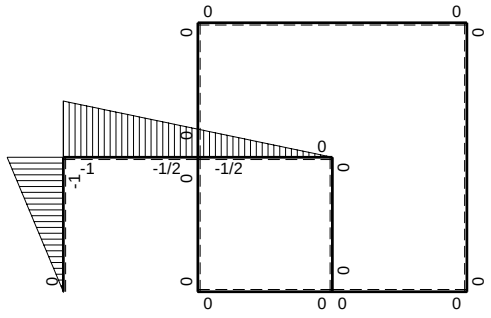
↺□↻ Fb



Schema di calcolo iperstatico



↺□↻ M_o flessione da carichi assegnati



↺□↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/4Fx$	$-Fb/EJ$	$3/4Fbx-3/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/4+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	Fb/EJ	$3/8Fb^2-3/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-2Fx$	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/8F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210011 ESΣ01.xxxx.120
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (\, b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

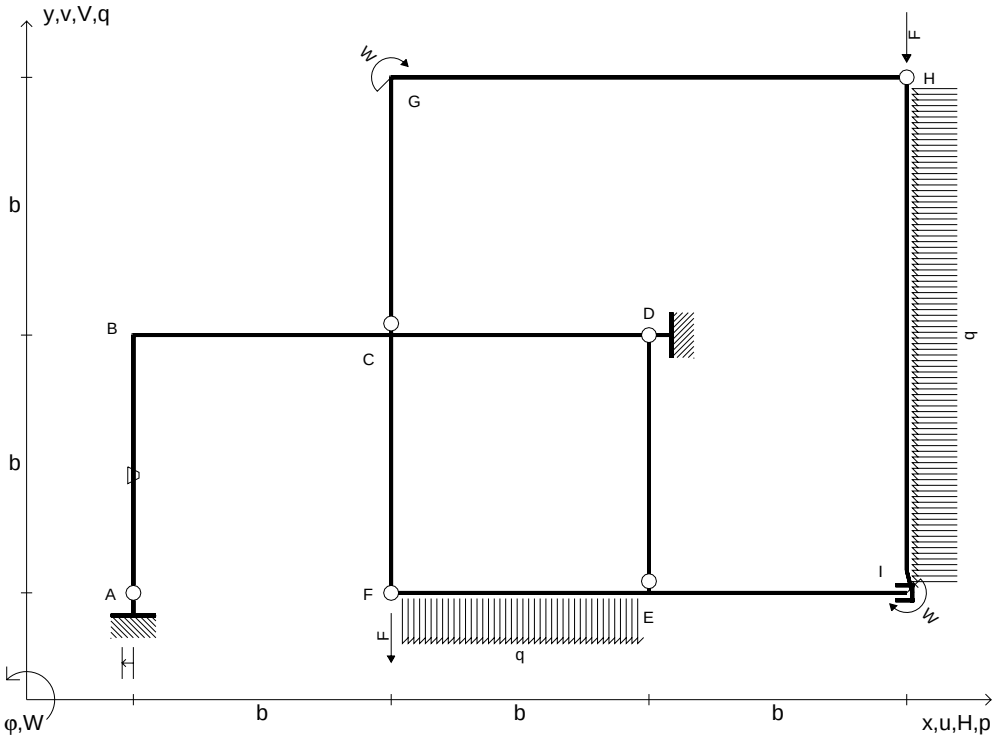
$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ01.xxxx.120

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

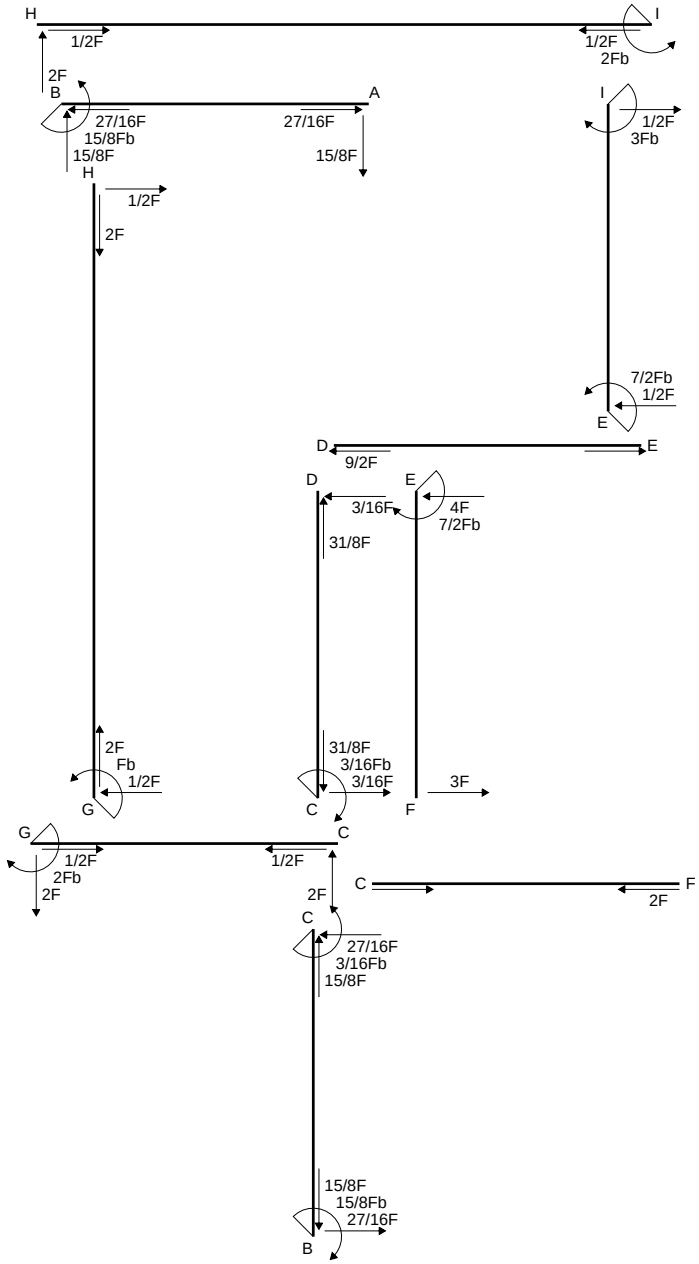


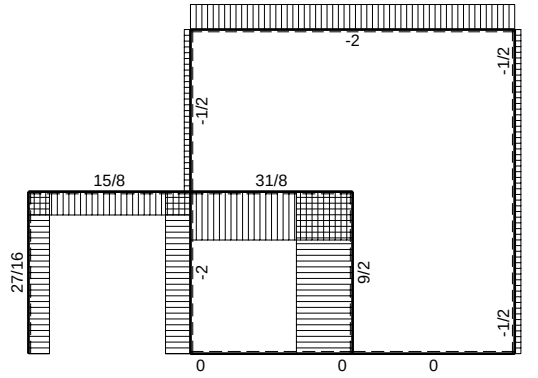
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.





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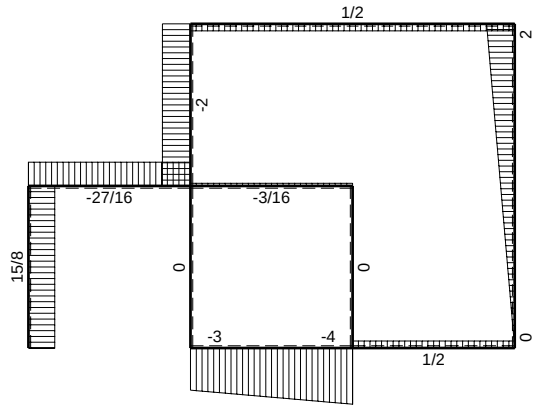

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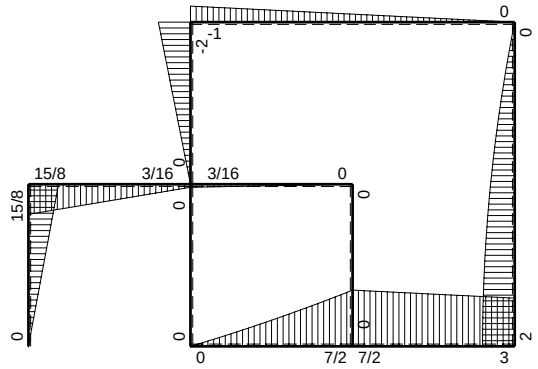
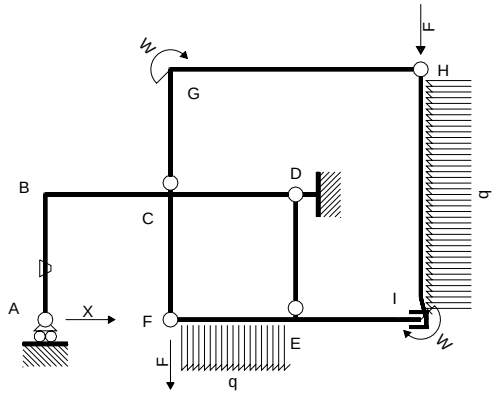
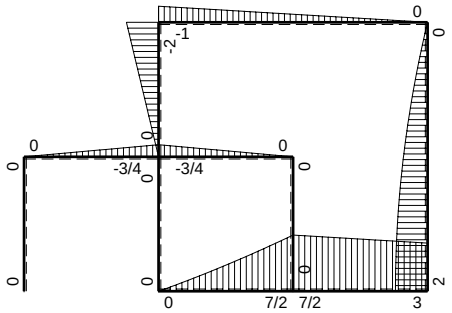


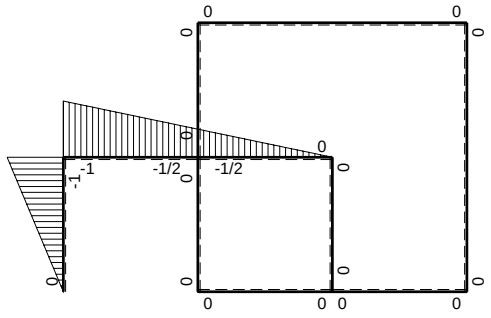
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 211210 ESΣ01.xxxx.121

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-3/4Fx	0	3/4Fbx-3/8Fx^2	0	b^2-bx+1/4x^2	(1/4+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	3/4Fb-3/4Fx	0	3/8Fb^2-3/8Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	3/8Fb^2-3/4Fbx+3/8Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/8+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	3/4Fx	0	3/8Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-4Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-3Fx-1/2qx^2	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
IH 2b	0	-2Fb+1/2qx^2	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						15/8Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-15/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

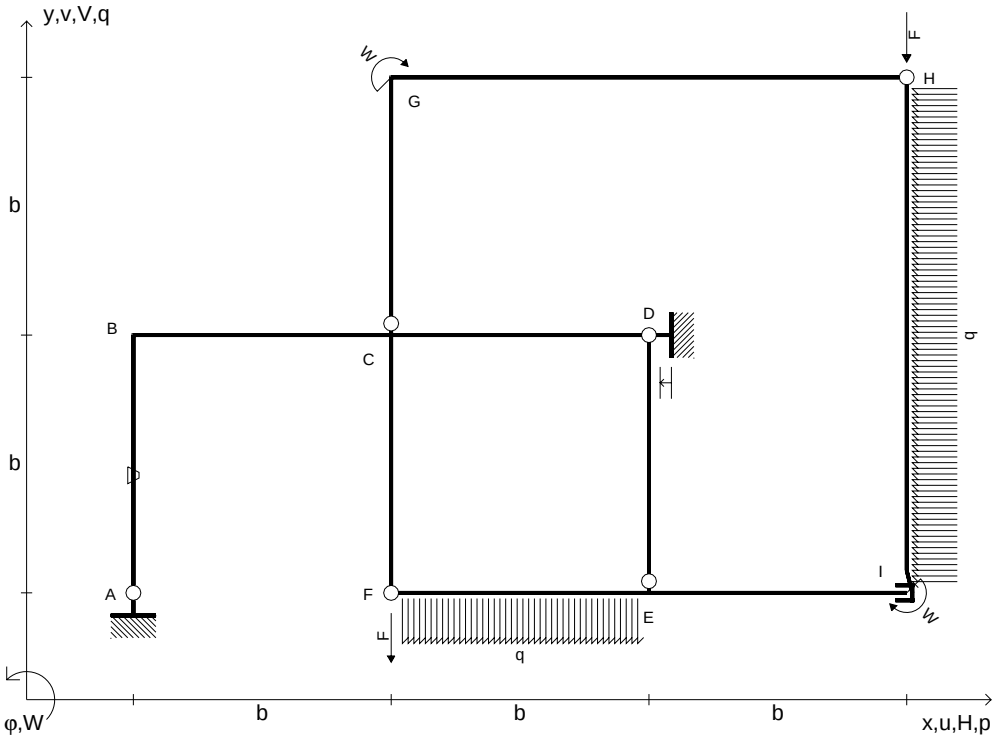
$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ01.xxxx.121

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25



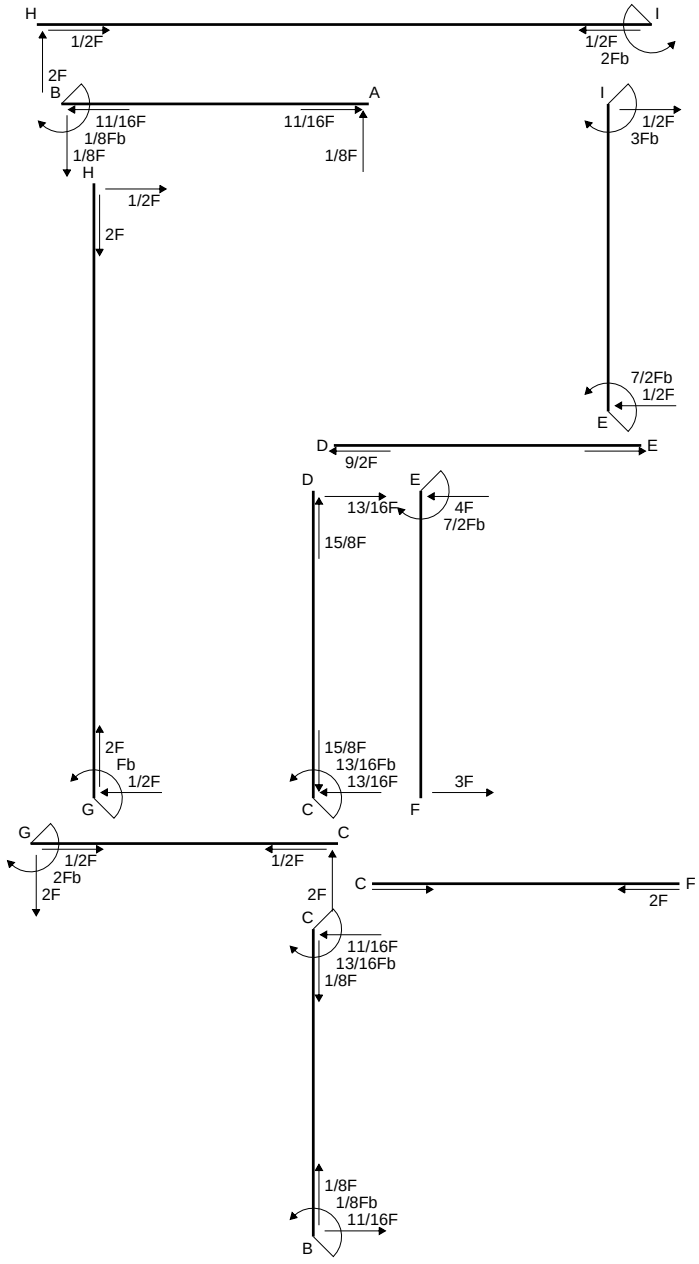
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

 08.01.25



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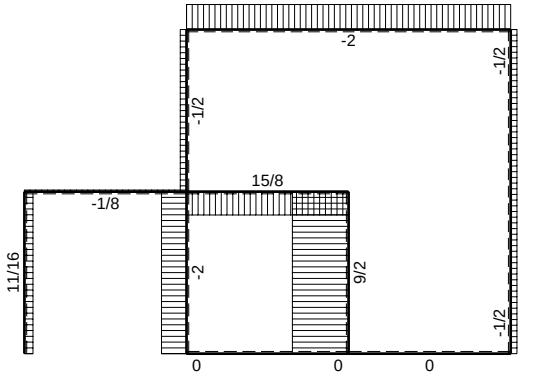
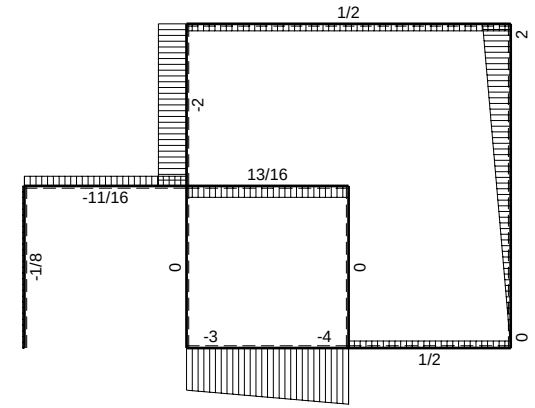


F



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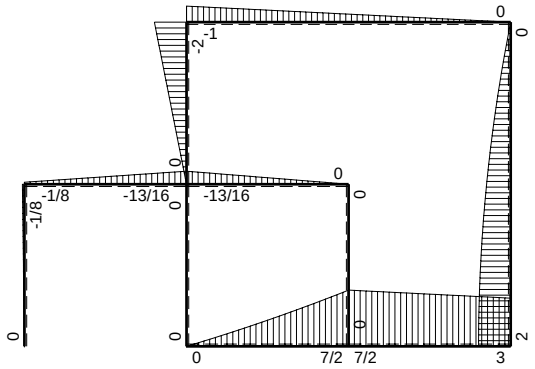
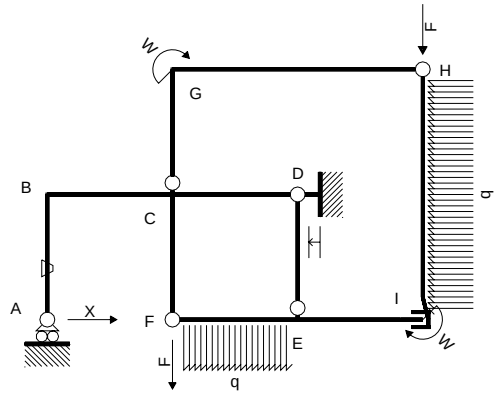
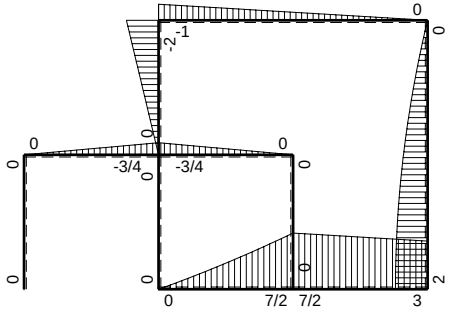


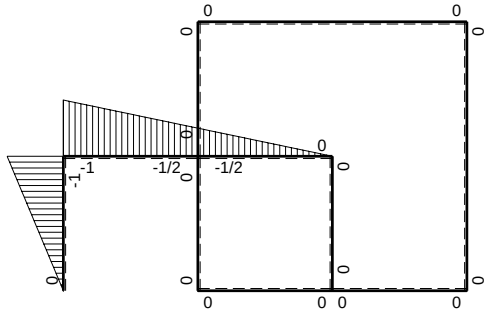
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 956427 ESΣ01.xxxx.122

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-3/4Fx	0	3/4Fbx-3/8Fx^2	0	b^2-bx+1/4x^2	(1/4+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	3/4Fb-3/4Fx	0	3/8Fb^2-3/8Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	3/8Fb^2-3/4Fbx+3/8Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/8+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	3/4Fx	0	3/8Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-4Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-3Fx-1/2qx^2	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
IH 2b	0	-2Fb+1/2qx^2	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						-Fb^3/EJ	
	totali						-1/8Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						1/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ01.xxxx.122

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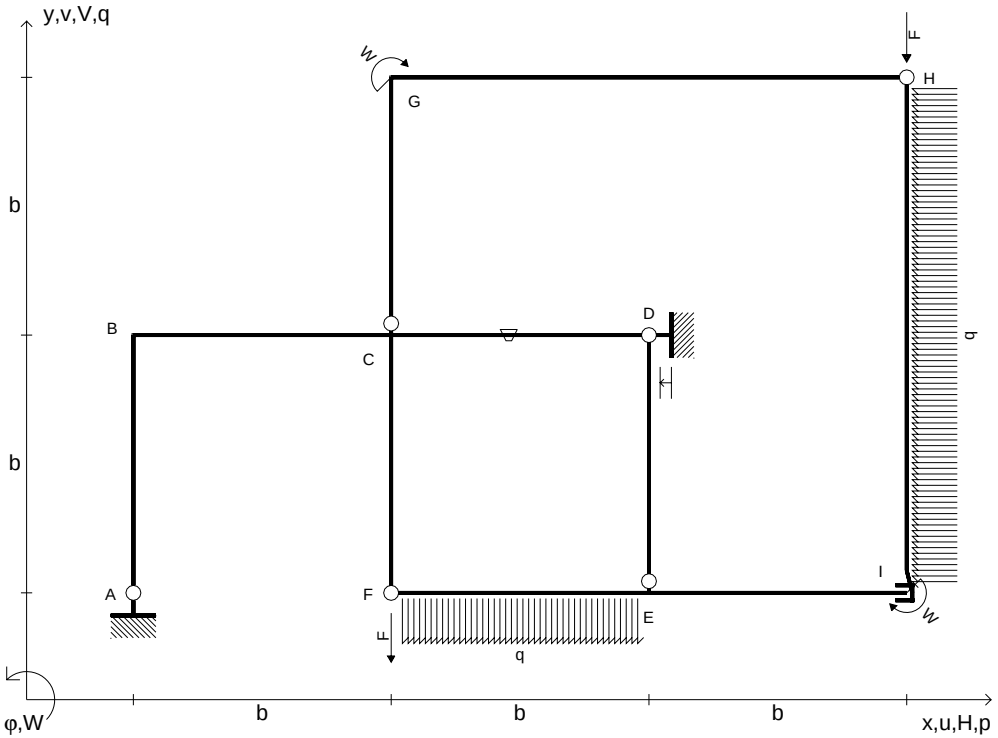


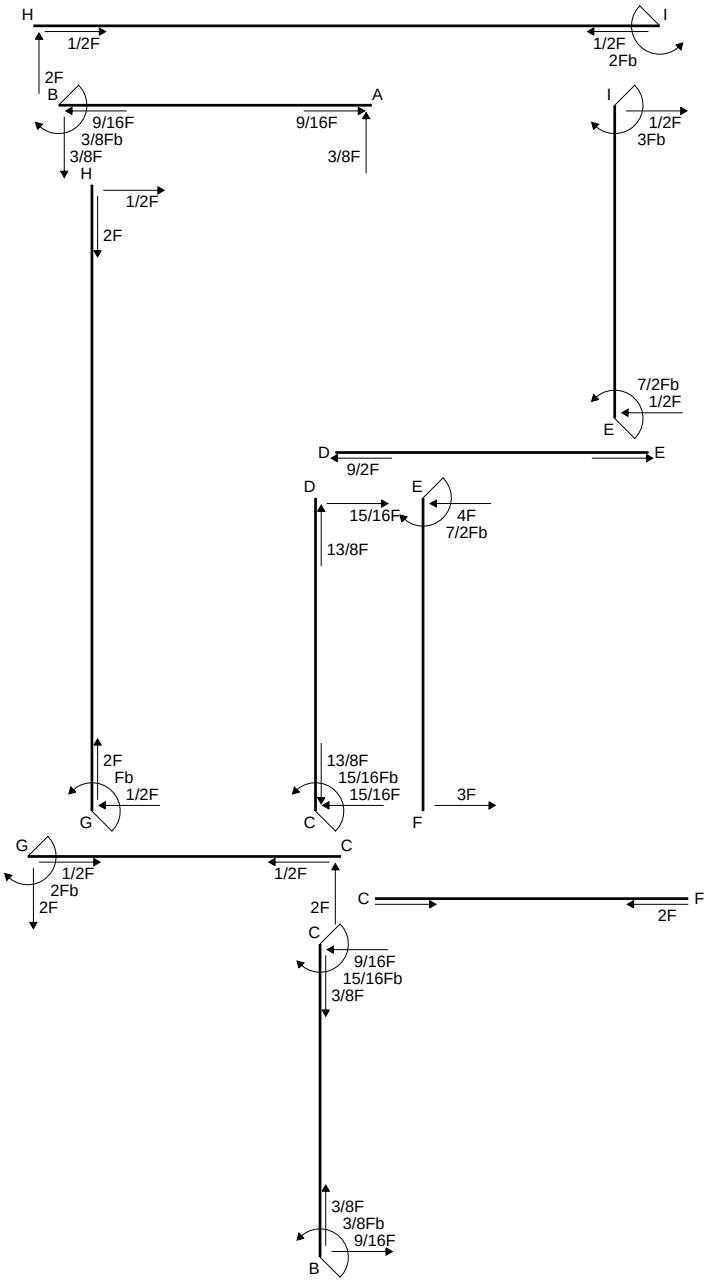
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

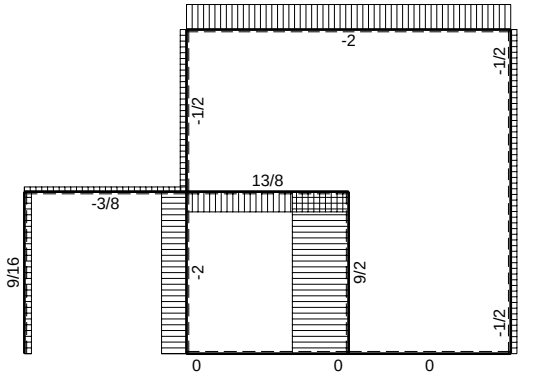
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{BA}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo D.

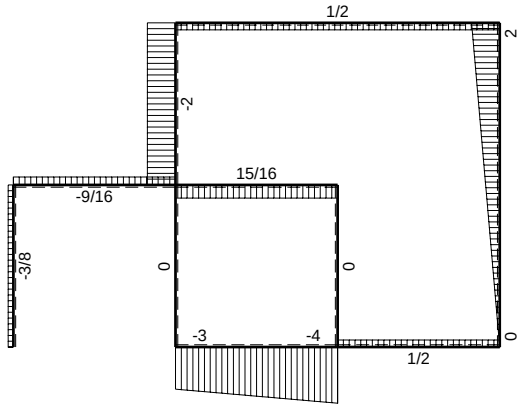
 08.01.25



 08.01.25



F



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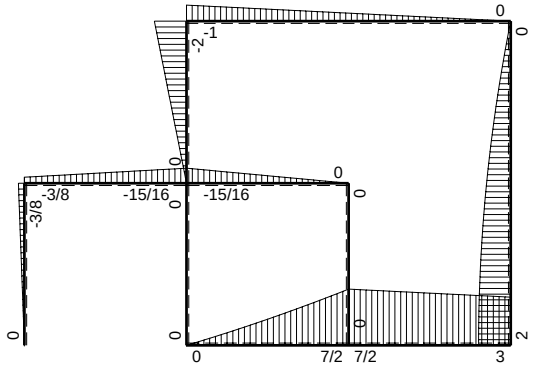
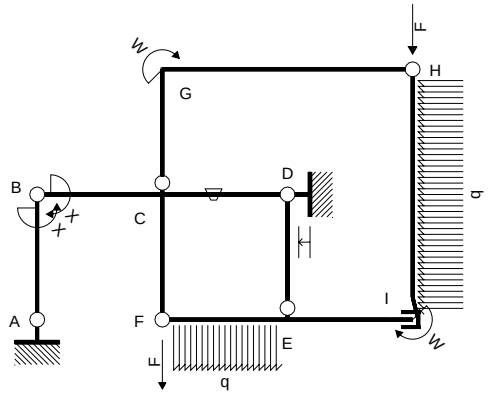
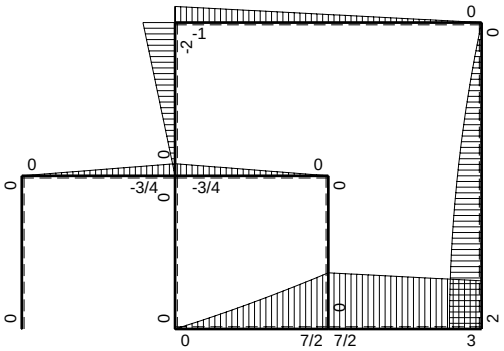


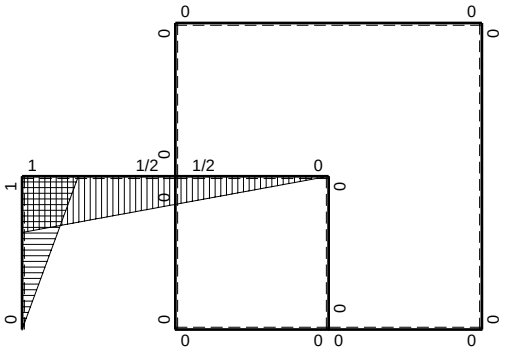
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{BA}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
BA b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
BC b	1-1/2x/b	-3/4Fx	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	7/12Xb/EJ
CB b	-1/2-1/2x/b	3/4Fb-3/4Fx	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	1/2-1/2x/b	-3/4Fb+3/4Fx	-Fb/EJ	$-3/8Fb+3/4Fx-3/8Fx^2/b$	-1/2Fb/EJ+1/2Fx/EJ	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/8-1/4)Fb^2/EJ$	1/12Xb/EJ
DC b	-1/2x/b	3/4Fx	Fb/EJ	$-3/8Fx^2/b$	-1/2Fx/EJ	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						Fb^2/EJ	
	totali						$3/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BA}$						-3/8Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (-3/8 +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/8\ x +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (-3/8 +3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 +1/2\ x/b\ )\ \theta\ dx$

$= [-3/8\ x +3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x +1/4\ x^2/b\ ]_o^b\ \theta$

$= (-3/8\ b +3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b +1/4\ b\ )\ \theta\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (-3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2\ x/b\ )\ \theta\ dx = [-1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/4\ x^2/b\ ]_o^b\ \theta$

$= (-1/8\ b\ )\ Fb\ 1/EJ\ + (1/4\ b\ )\ \theta\ = -3/8\ Fb^2/EJ$

ESΣ01.xxxx.124

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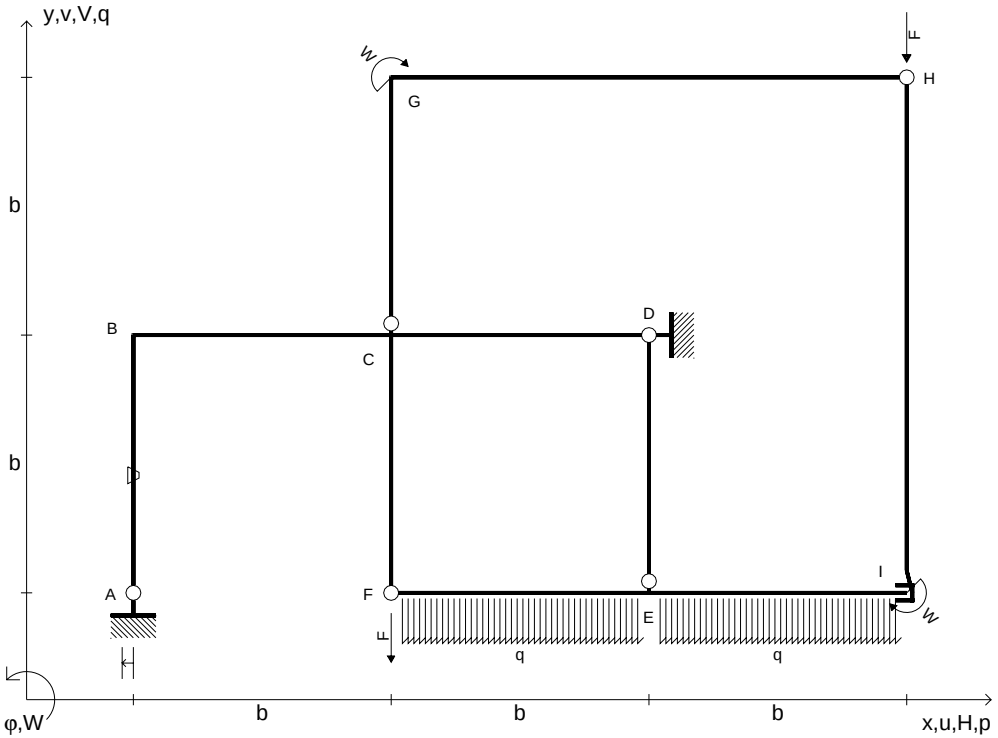


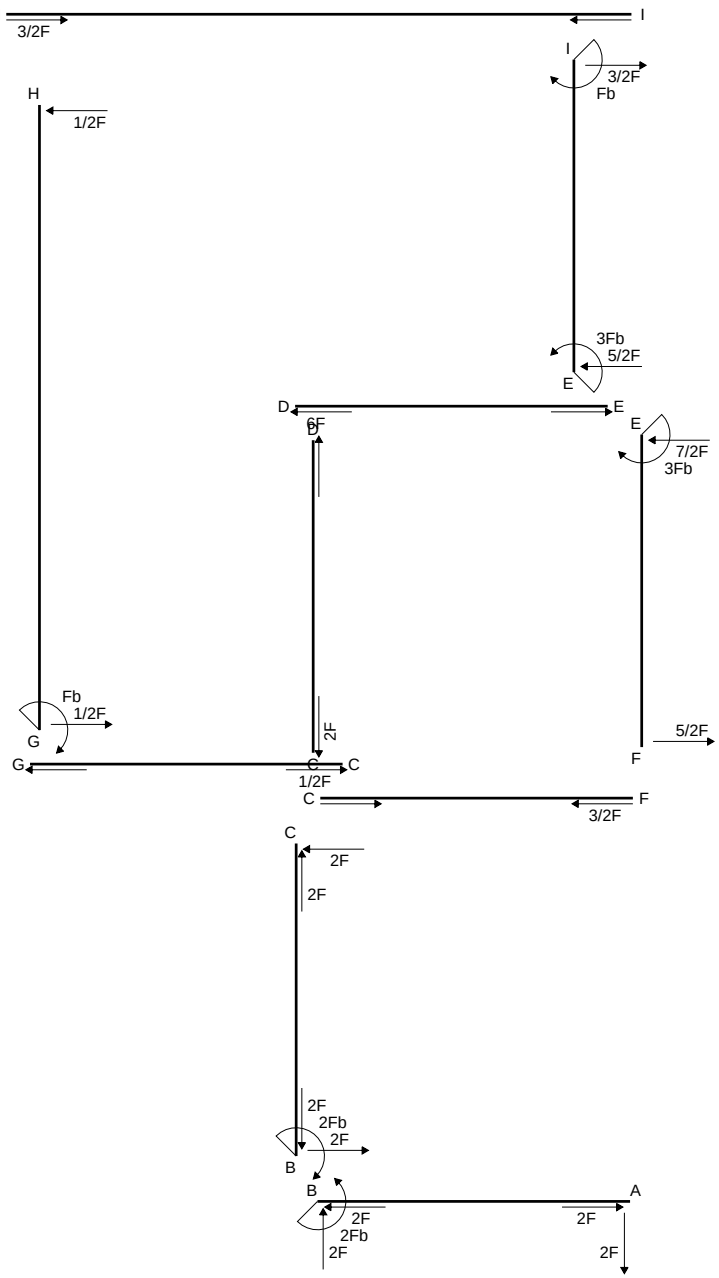
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$q_{IE} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.

 08.01.25



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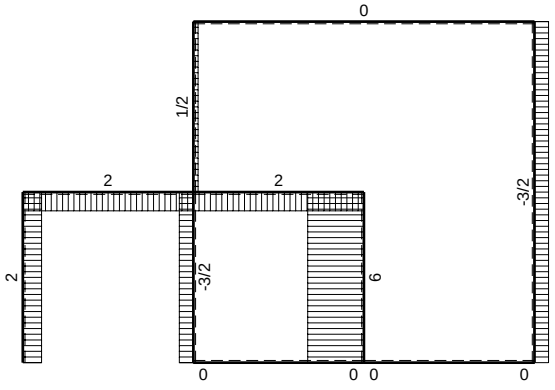

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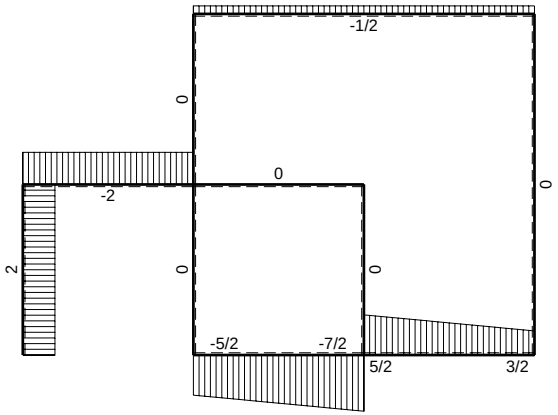

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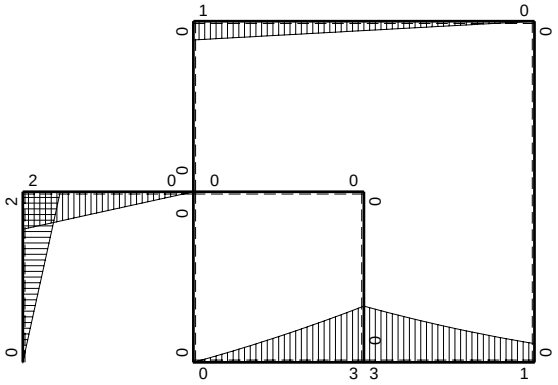

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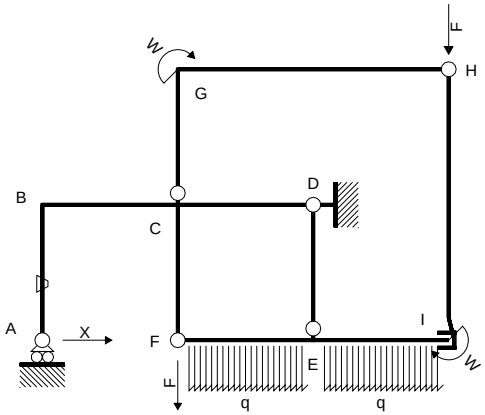
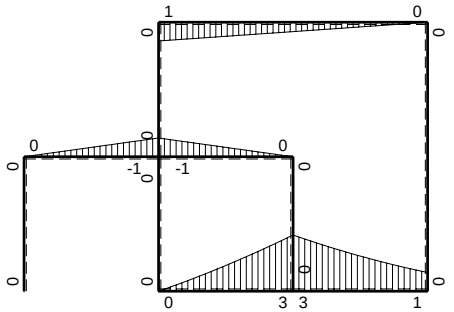
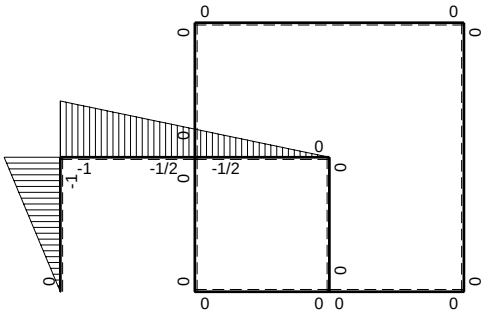


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-Fx	0	Fbx-1/2Fx^2	0	b^2-bx+1/4x^2	(1/3+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	Fb-Fx	0	1/2Fb^2-1/2Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-Fb+Fx	0	1/2Fb^2-Fbx+1/2Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/6+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	Fx	0	1/2Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	3Fb-7/2Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-5/2Fx-1/2qx^2	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx+1/2qx^2	0	0	0	0	0+0	0
EI b	0	-3Fb+5/2Fx-1/2qx^2	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$2Fb^3$/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-2F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

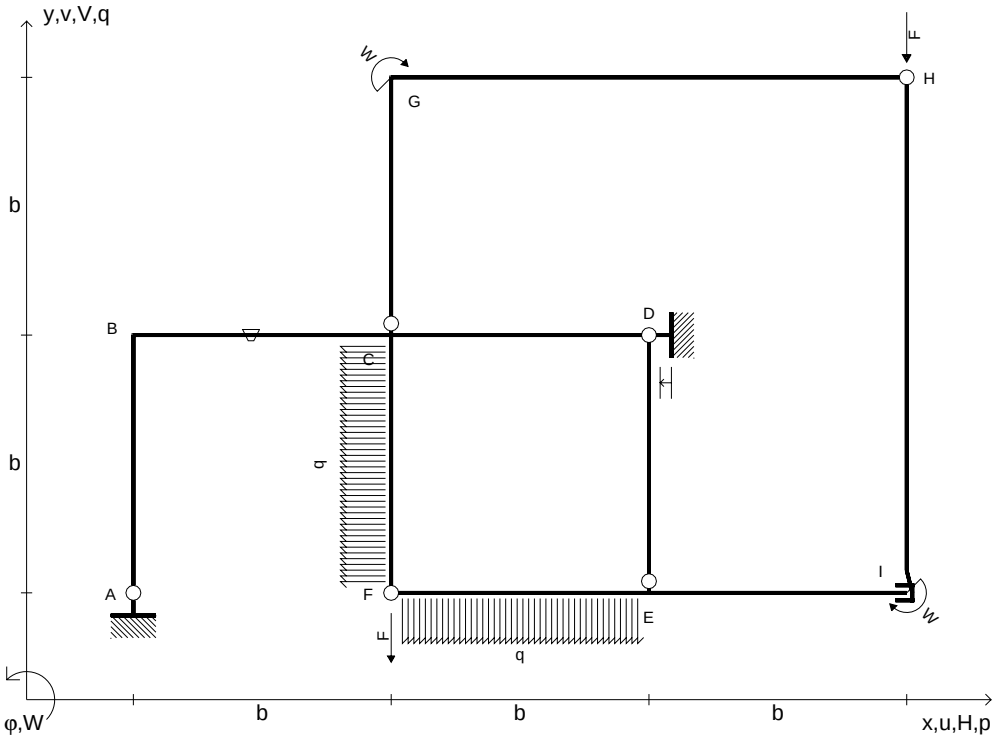
$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

ESΣ01.xxxx.133

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

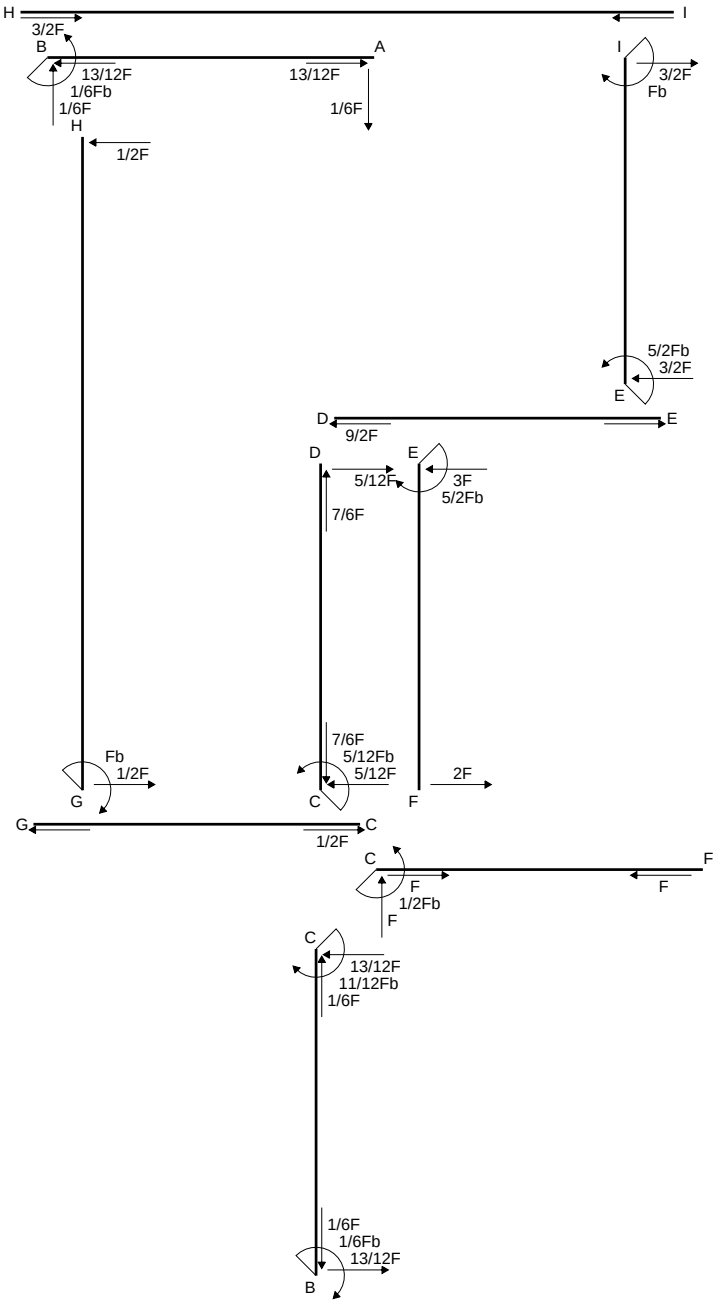


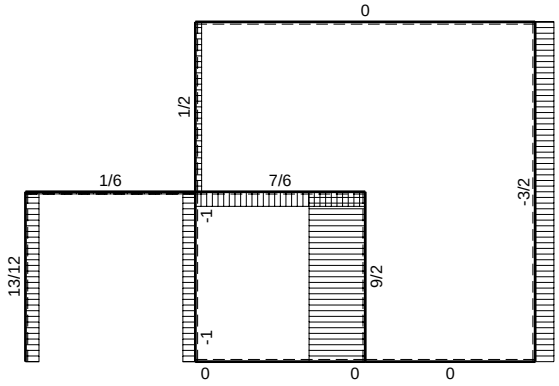
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$p_{FC} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

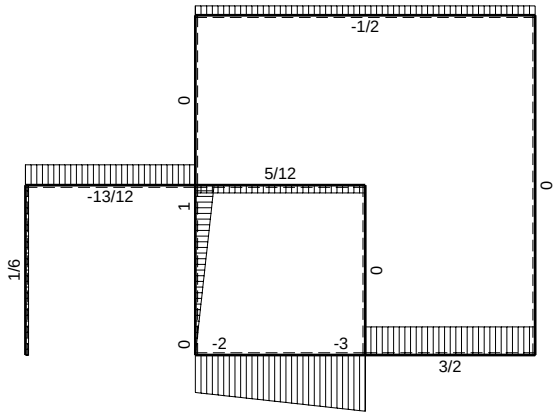
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.

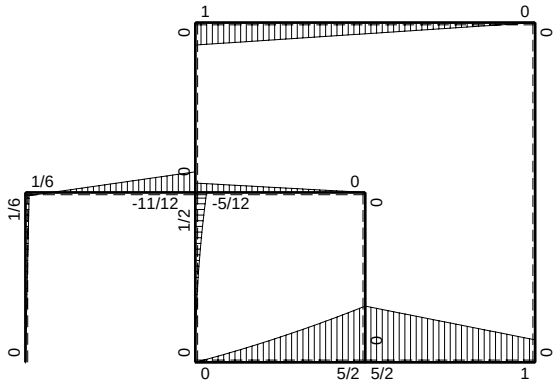




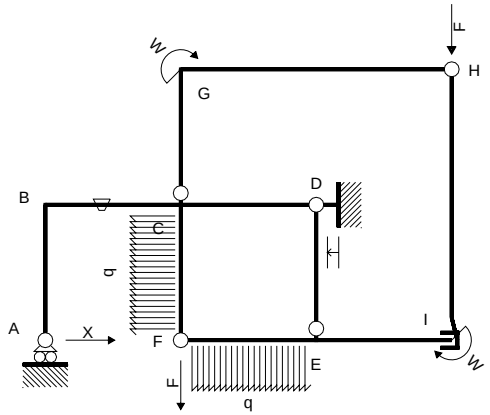
←□→ F



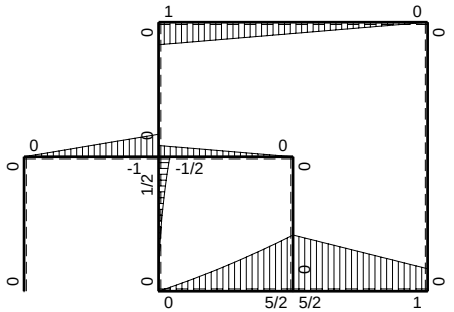
↑□↓ F



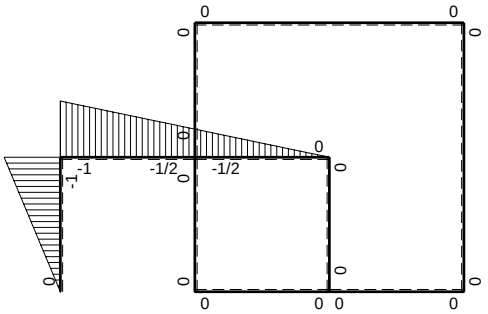
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-Fx	-Fb/EJ	$Fbx-1/2Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/3+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	Fb-Fx	Fb/EJ	$1/2Fb^2-1/2Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	1/2Fx	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-5/2Fb+3/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/6F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213310 ESΣ01.xxxx.143

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

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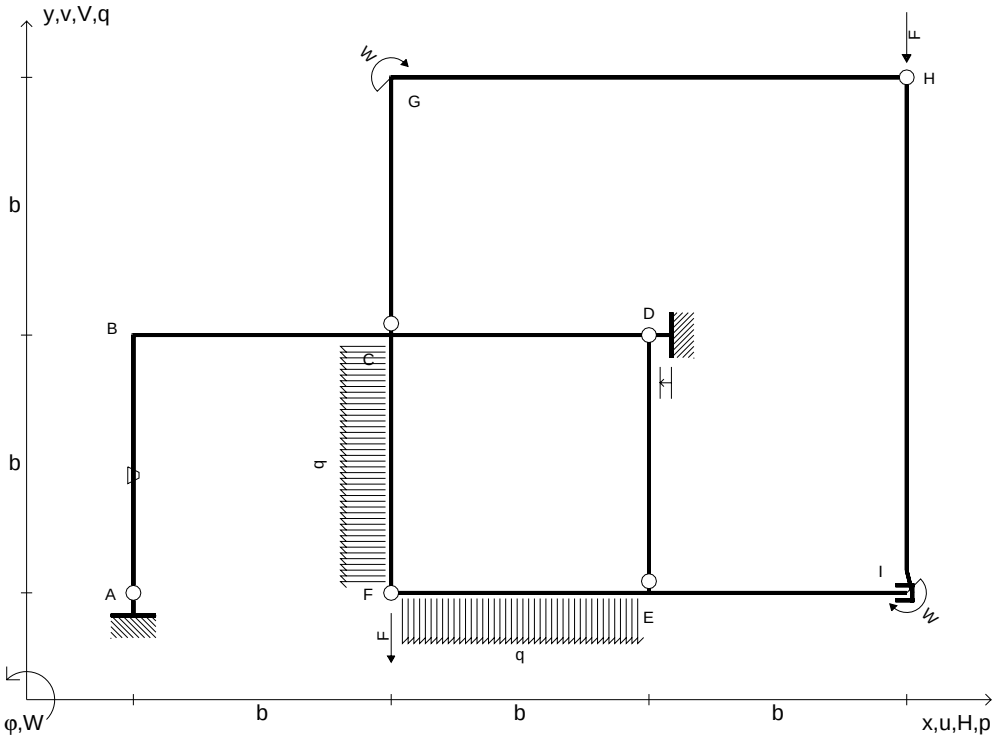
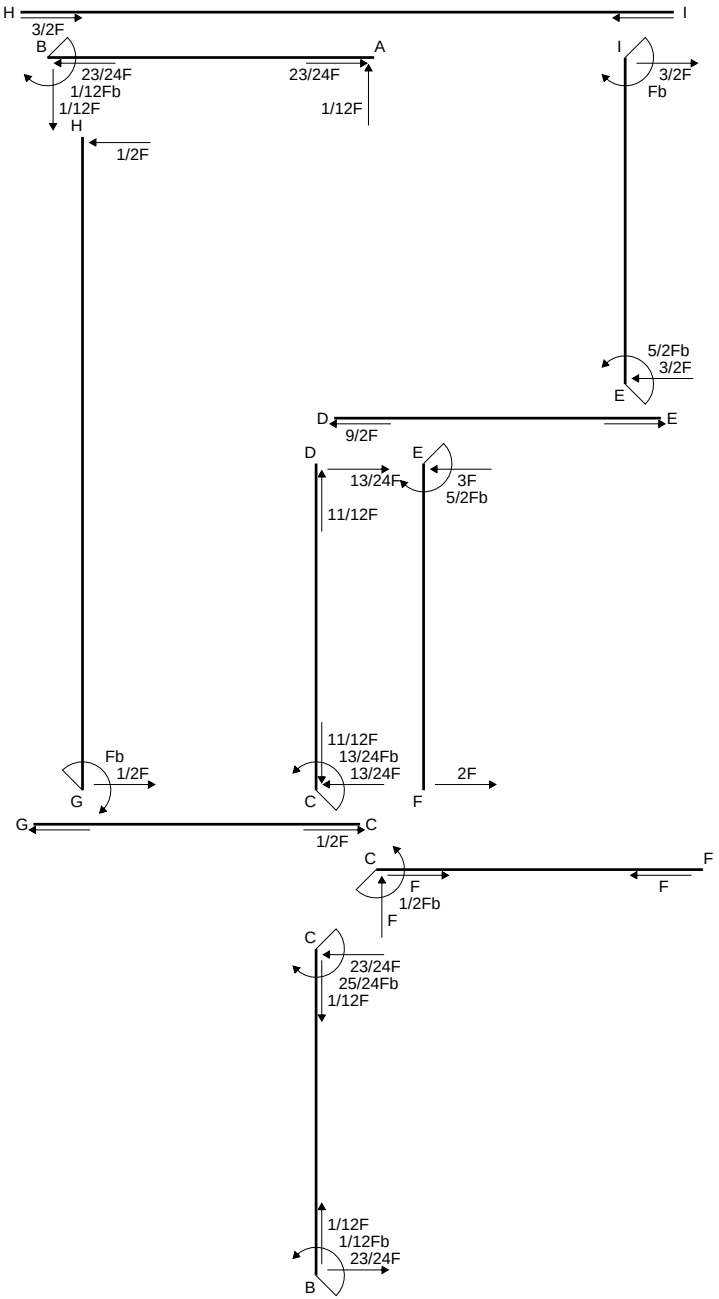


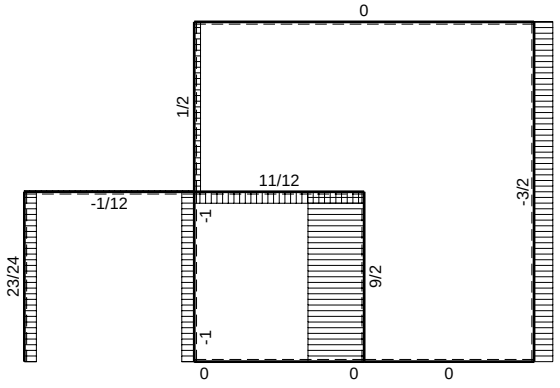
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$p_{FC} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

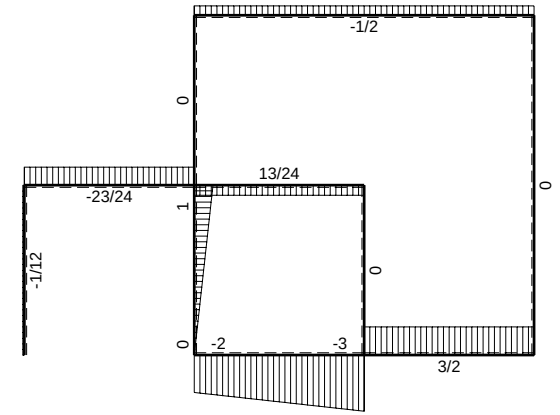
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

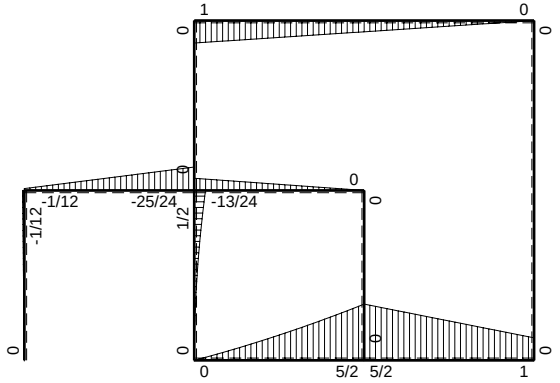




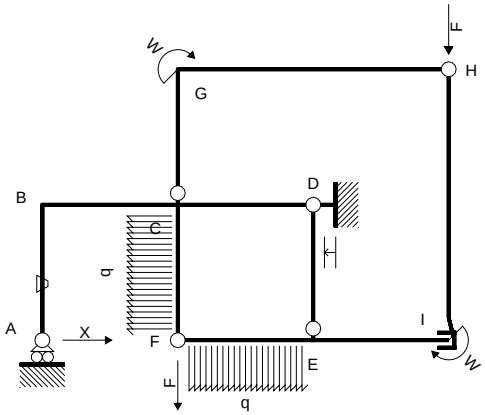
←□→ F



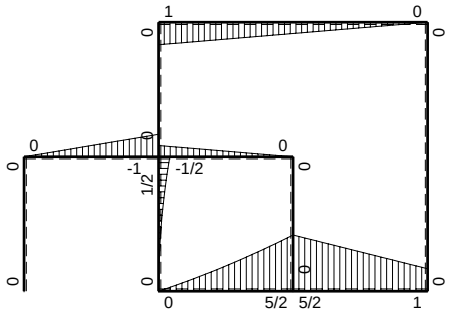
↑□↓ F



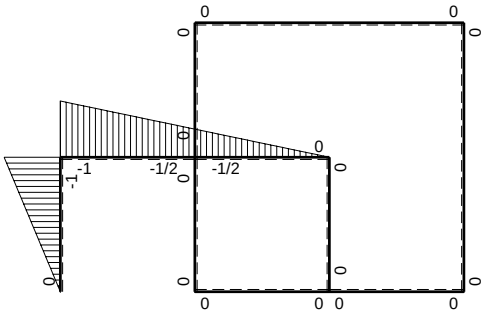
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210204 ESΣ01.xxxx.145

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	$b^2-2bx+x^2$		
BC b	-b+1/2x	-Fx	0	Fbx-1/2Fx^2	0	b^2-bx+1/4x^2	(1/3+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	Fb-Fx	0	1/2Fb^2-1/2Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb^2-1/2Fbx+1/4Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/12+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	1/2Fx	0	1/4Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	5/2Fb-3Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-2Fx-1/2qx^2	0	0	0	0		
FC b	0	1/2qx^2	0	0	0	0	0+0	0
CF b	0	-1/2Fb+Fx-1/2qx^2	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-5/2Fb+3/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						-Fb^3/EJ	
	totali						-1/12Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						1/12F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

ESΣ01.xxxx.145

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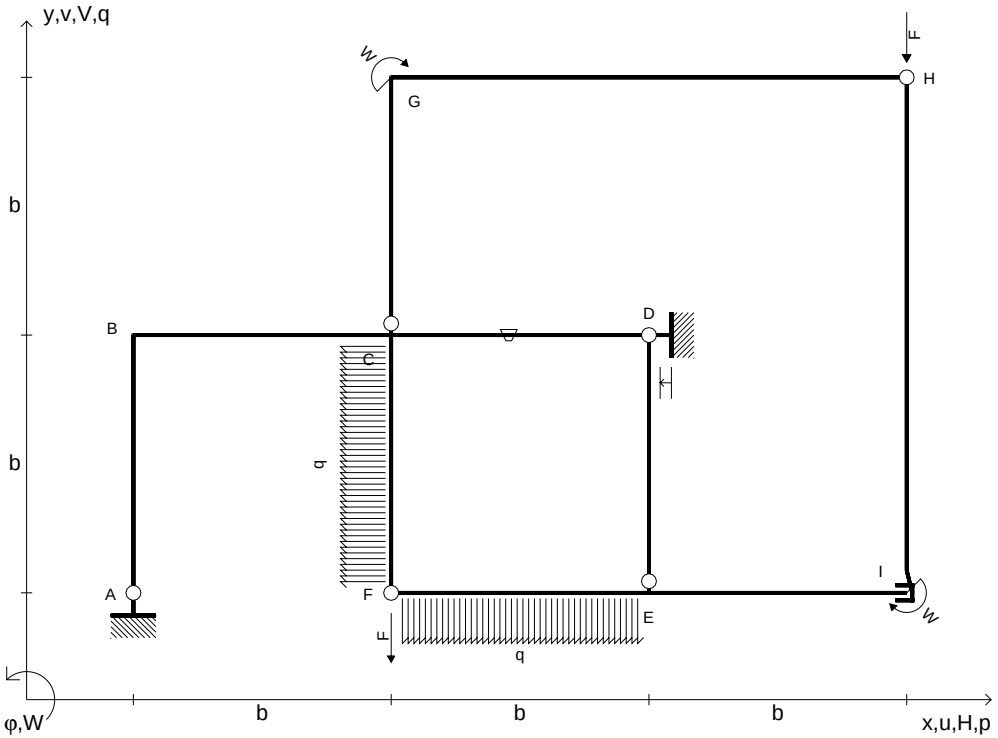

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$p_{FC} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
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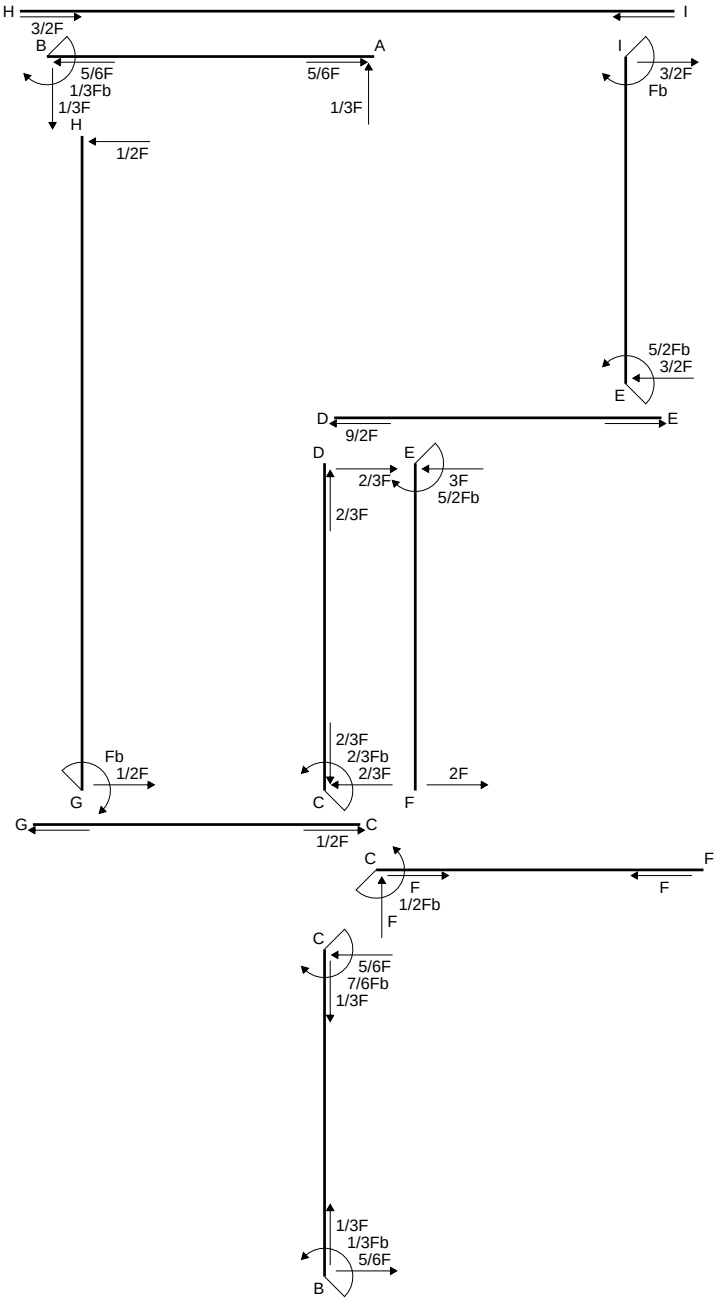
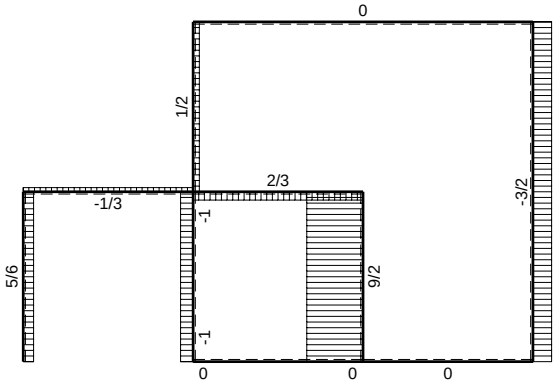
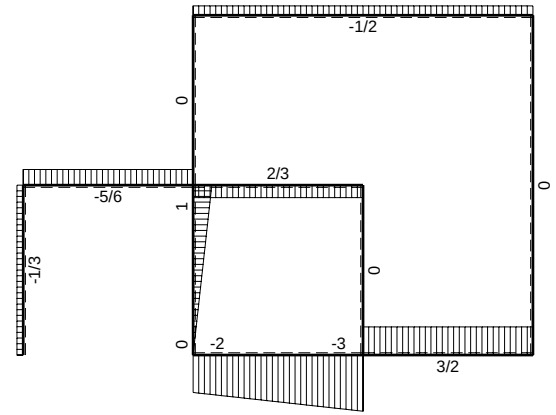
Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo D.

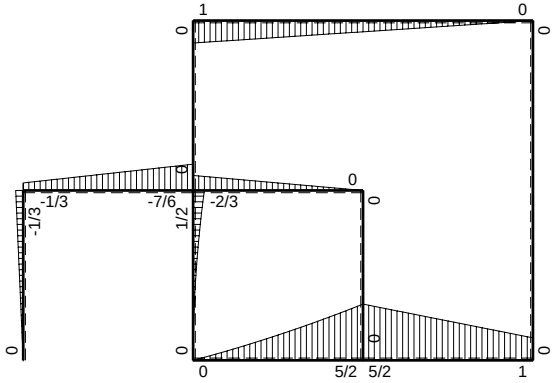




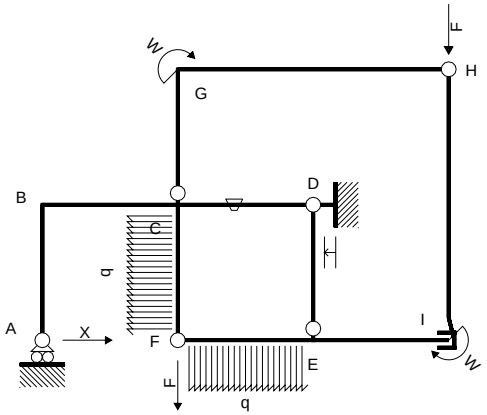
←□→ F



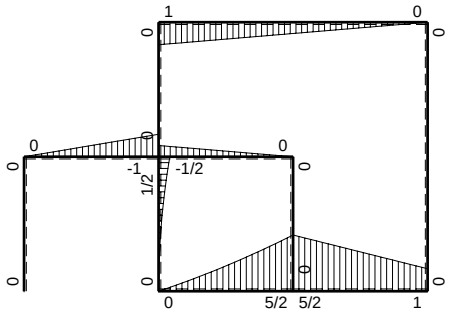
↑□↓ F



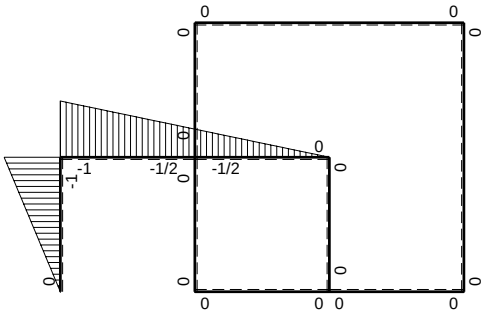
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	-Fx	0	$Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(1/3+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	Fb-Fx	0	$1/2Fb^2-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	-Fb/EJ	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-5/2Fb+3/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-1/3Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						1/3F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 981630 ESΣ01.xxxx.147

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/12\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

ESΣ01.xxxx.147

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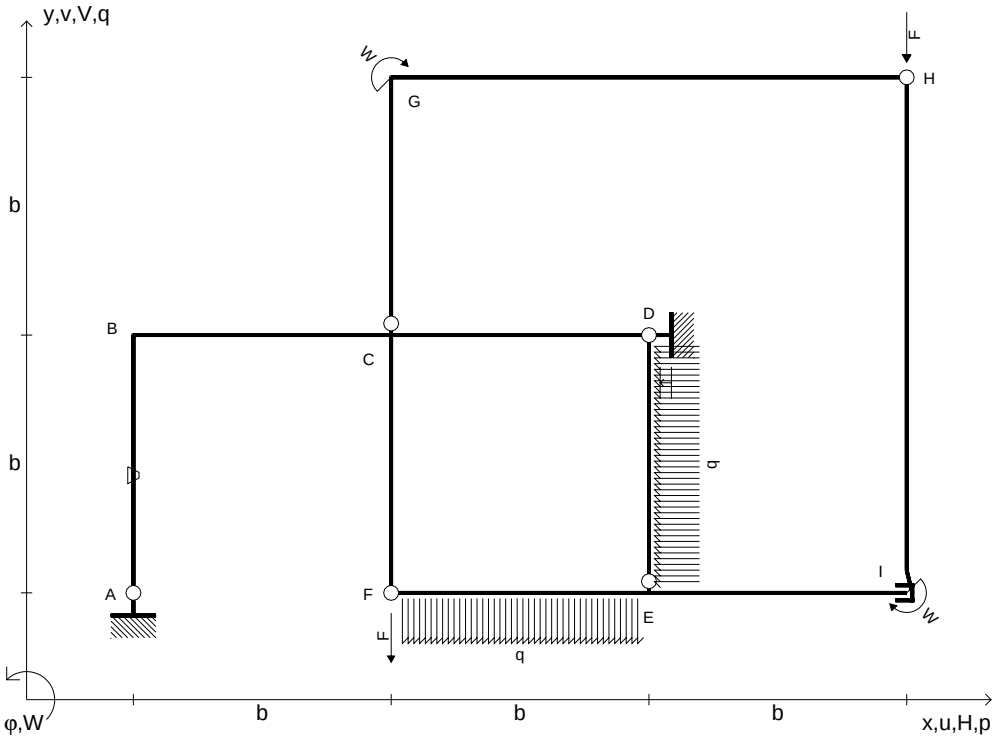

$V_H = -F$
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$W_G = -W = -Fb$
$q_{EF} = -q = -F/b$
$p_{DE} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
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Reazioni iperstatiche in soluzione: $X=H_A$
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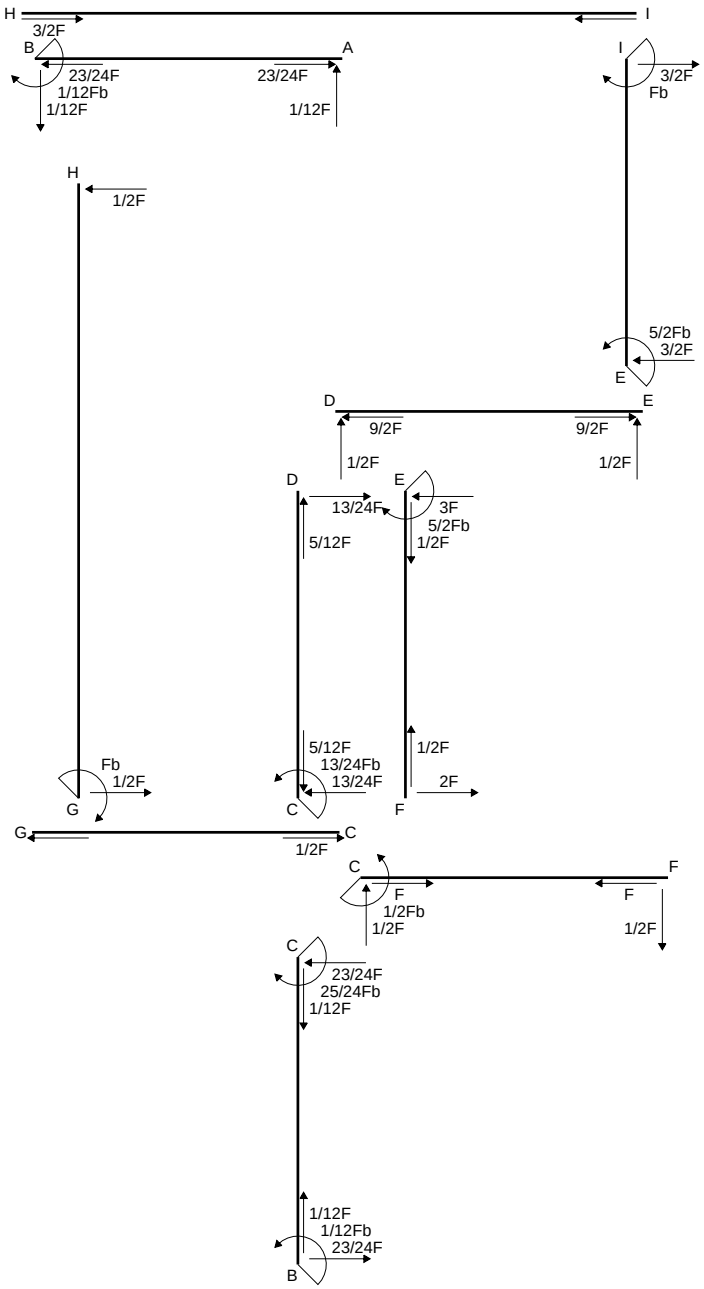
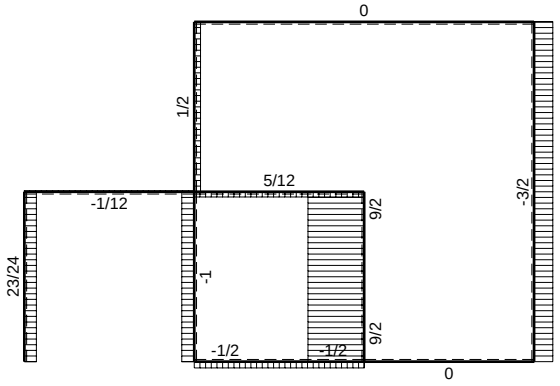
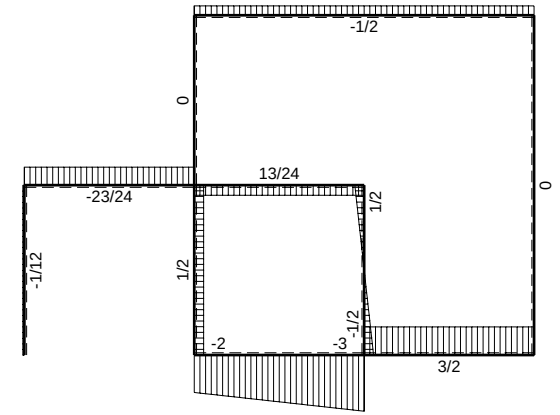
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

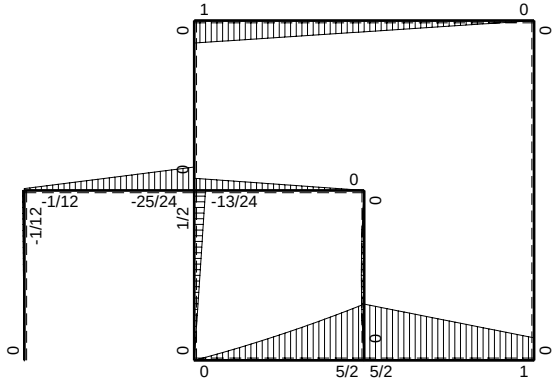




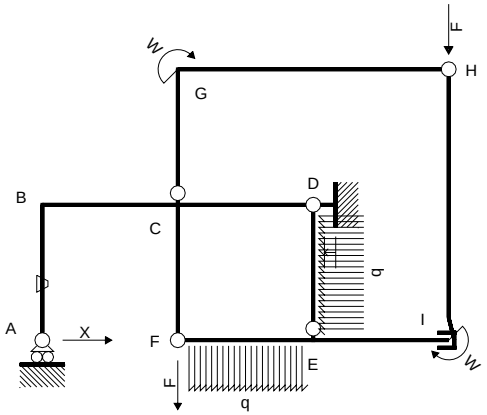
←□→ F



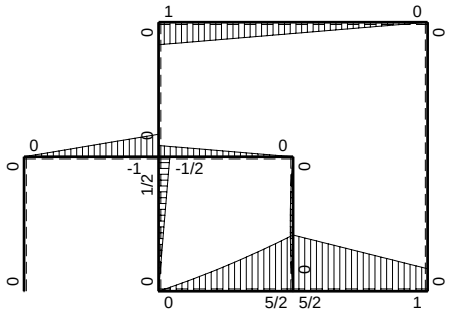
↑□↓ F



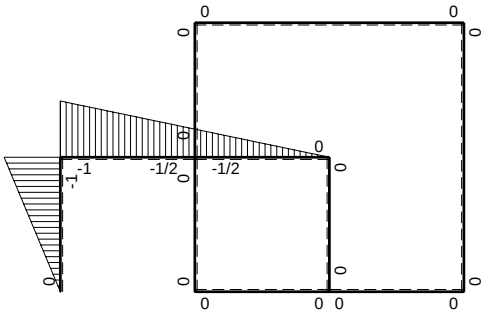
↺□↻ Fb



Schema di calcolo iperstatico



↺□↻ M_o flessione da carichi assegnati



↺□↻ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212409 ESΣ01.xxxx.151

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-Fx	0	Fbx-1/2Fx^2	0	b^2-bx+1/4x^2	(1/3+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	Fb-Fx	0	1/2Fb^2-1/2Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-1/2Fb+1/2Fx	0	1/4Fb^2-1/2Fbx+1/4Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/12+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	1/2Fx	0	1/4Fx^2	0	1/4x^2		
DE b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	0	5/2Fb-3Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-2Fx-1/2qx^2	0	0	0	0		
FC b	0	1/2Fx	0	0	0	0	0+0	0
CF b	0	-1/2Fb+1/2Fx	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-5/2Fb+3/2Fx	0	0	0	0		
D	cedimento nodo -$H_{1D}u_D$						-Fb^3/EJ	
	totali						-1/12Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						1/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

ESΣ01.xxxx.151

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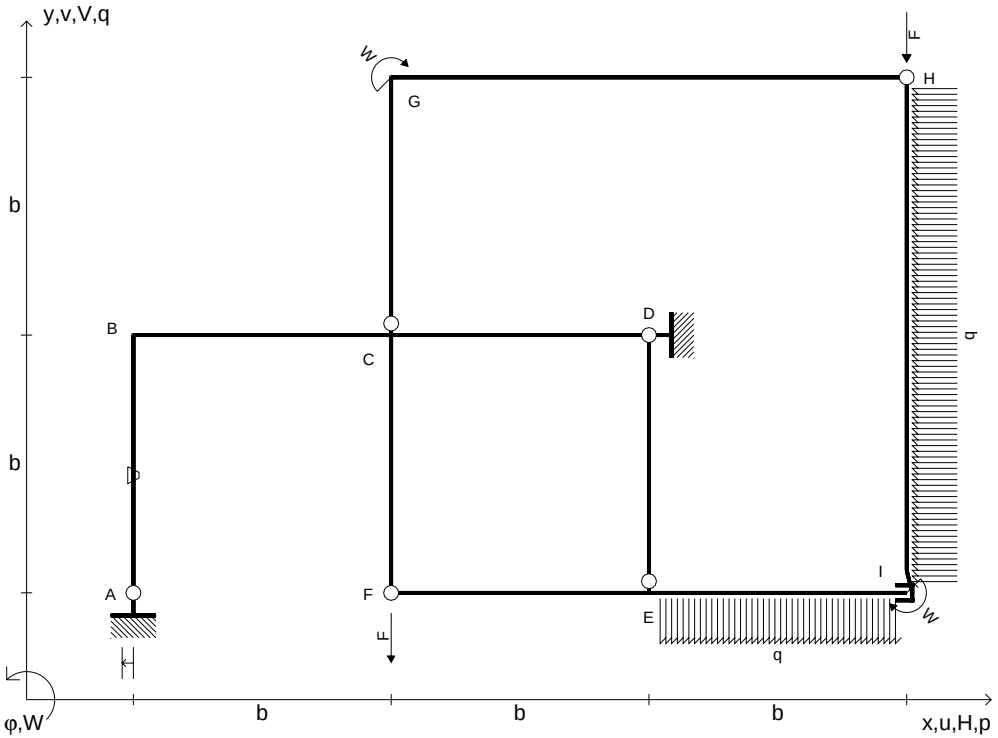


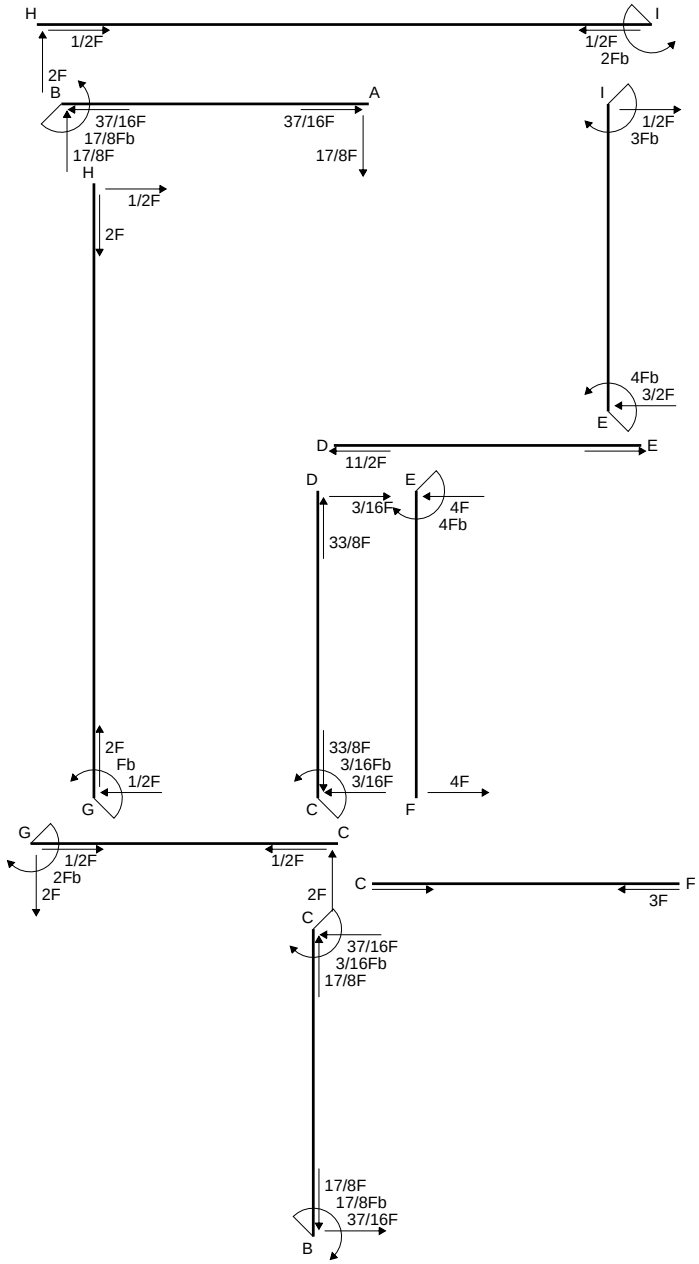
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{IE} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.

 08.01.25



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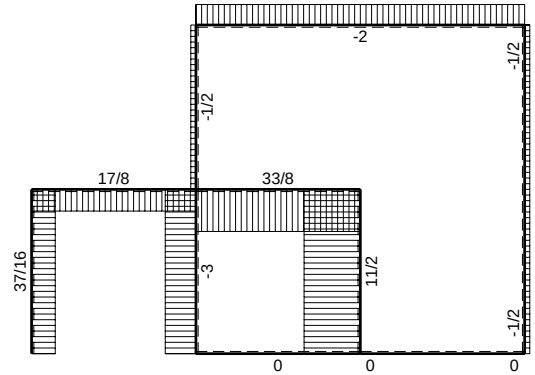

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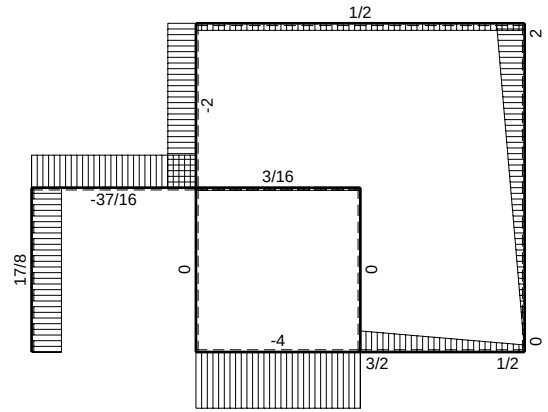

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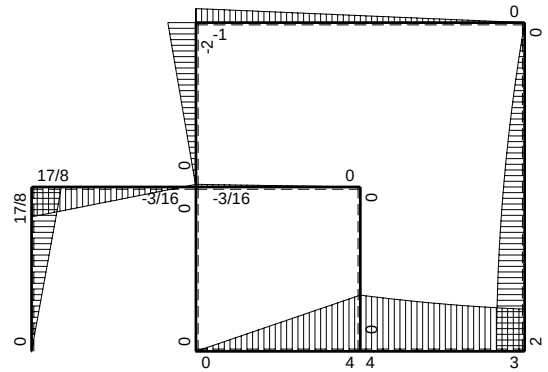
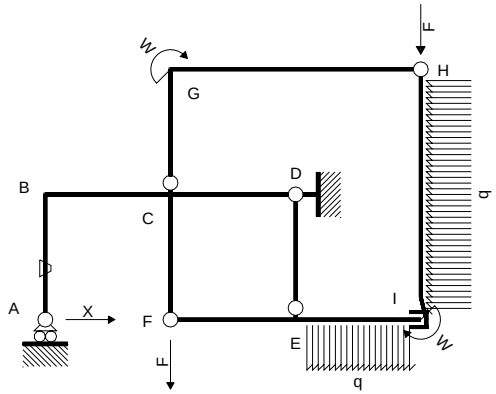
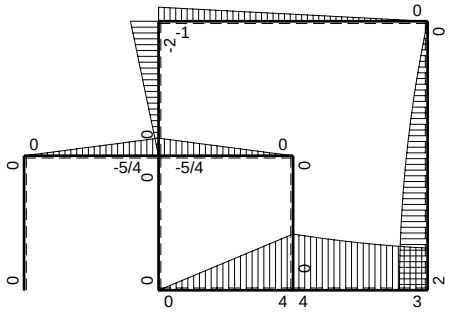


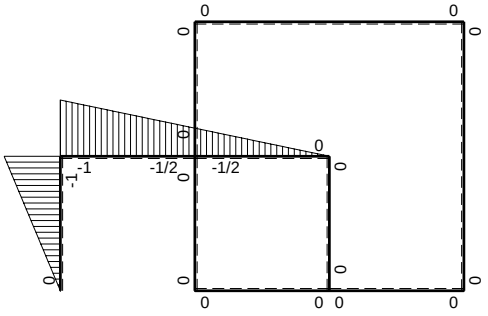
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-5/4Fx	0	5/4Fbx-5/8Fx^2	0	b^2-bx+1/4x^2	(5/12+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	5/4Fb-5/4Fx	0	5/8Fb^2-5/8Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-5/4Fb+5/4Fx	0	5/8Fb^2-5/4Fbx+5/8Fx^2	0	1/4b^2-1/2bx+1/4x^2	(5/24+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	5/4Fx	0	5/8Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	4Fb-4Fx	0	0	0	0	0+0	0
FE b	0	-4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
IH 2b	0	-2Fb+1/2qx^2	0	0	0	0		
IE b	0	3Fb+1/2Fx+1/2qx^2	0	0	0	0	0+0	0
EI b	0	-4Fb+3/2Fx-1/2qx^2	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						17/8Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-17/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\, x/b +5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/8\, x^2/b +5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/8\, b +5/24\, b)\, Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/24\, b)\, Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$

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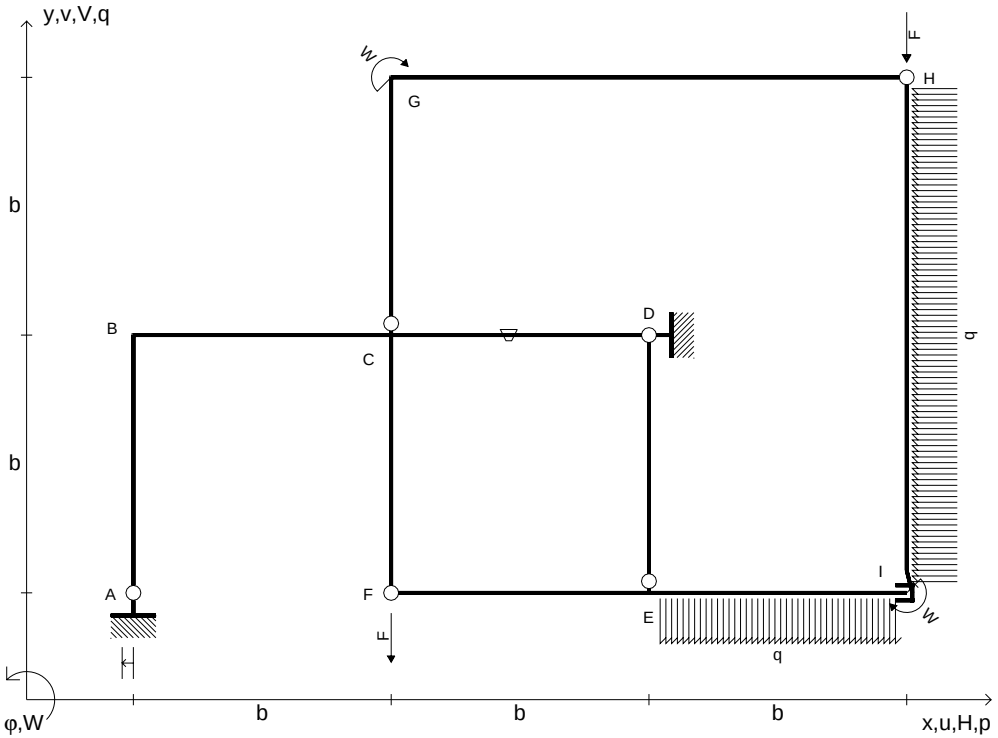

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{IE} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
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Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
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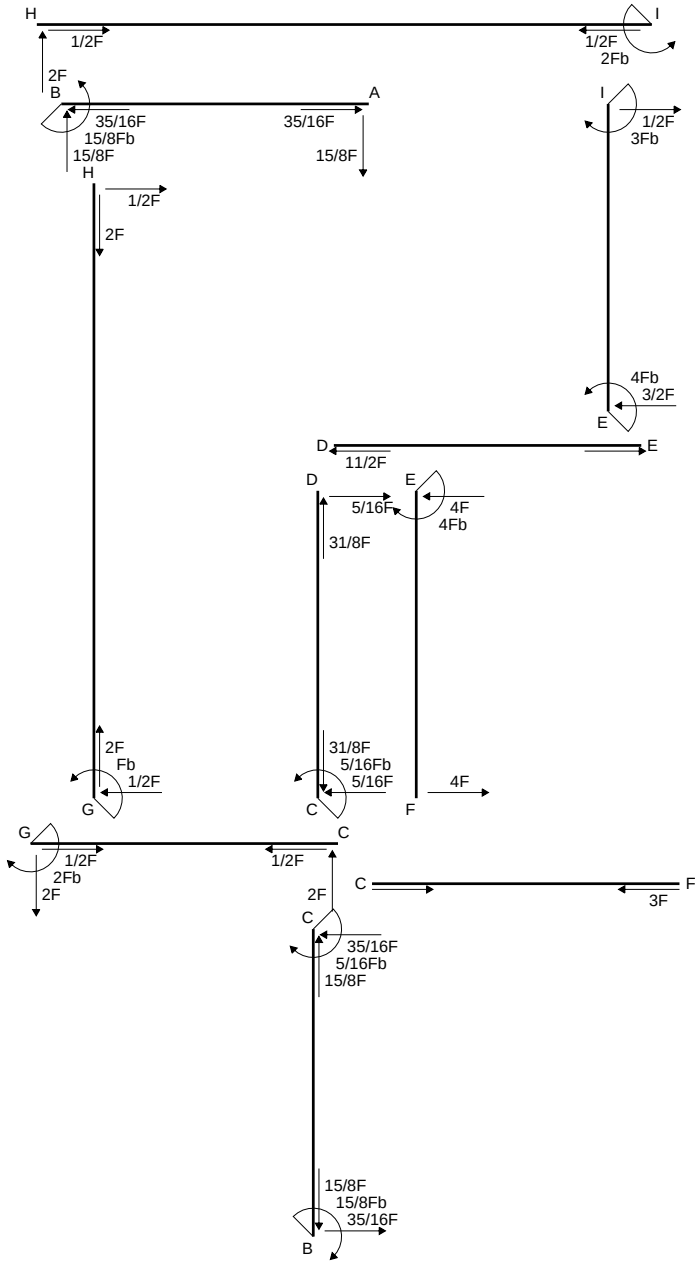
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo A.

 08.01.25



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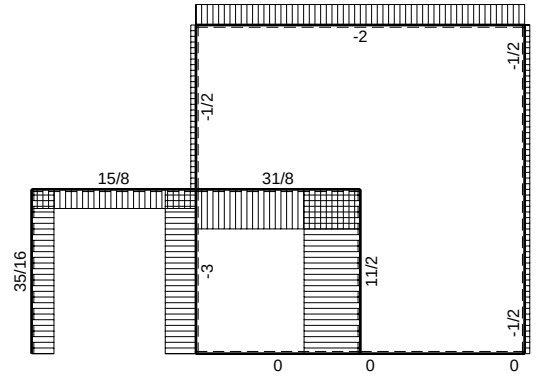

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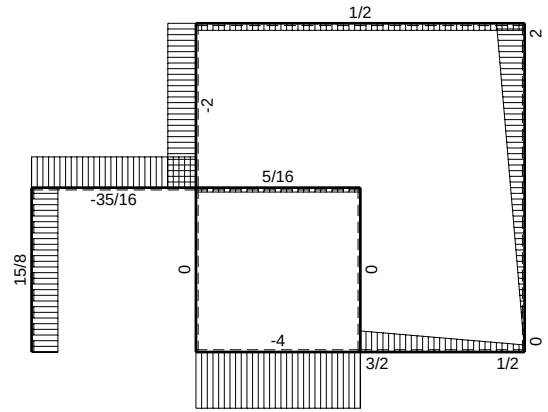

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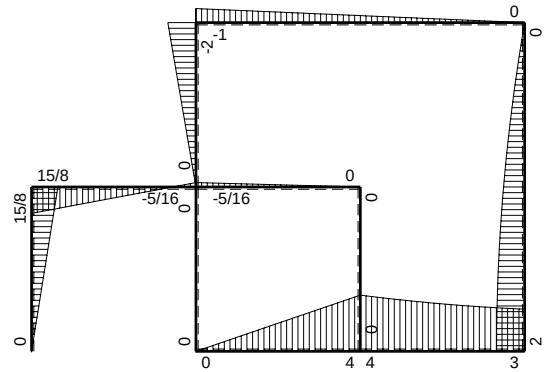

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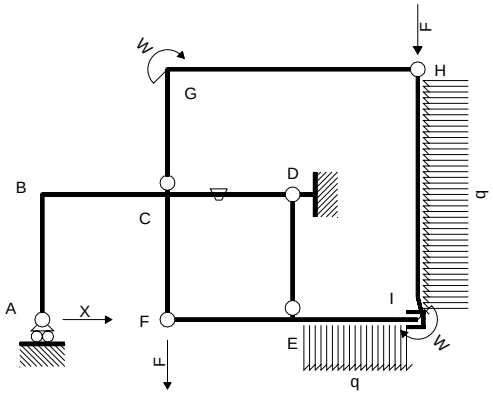
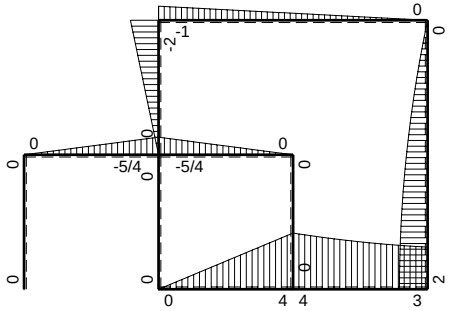
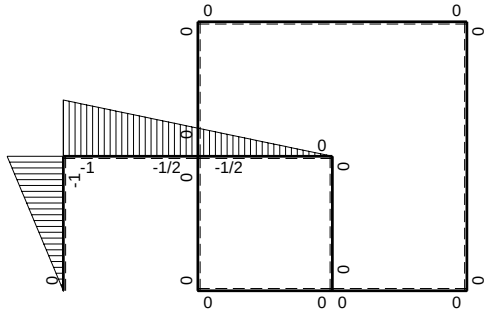


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-5/4Fx	0	$5/4Fbx-5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(5/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	5/4Fb-5/4Fx	0	$5/8Fb^2-5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-5/4Fb+5/4Fx	-Fb/EJ	$5/8Fb^2-5/4Fbx+5/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(5/24+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	5/4Fx	Fb/EJ	$5/8Fx^2$	1/2Fxb/EJ	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	4Fb-4Fx	0	0	0	0	0+0	0
FE b	0	-4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-4Fb+3/2Fx-1/2qx^2$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$15/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-15/8F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 960817 ESΣ01.xxxx.166

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\, x/b +5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [5/8\, x -5/8\, x^2/b +5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (5/8\, b -5/8\, b +5/24\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 11/24\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (5/24\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 11/24\ Fb^3/EJ$

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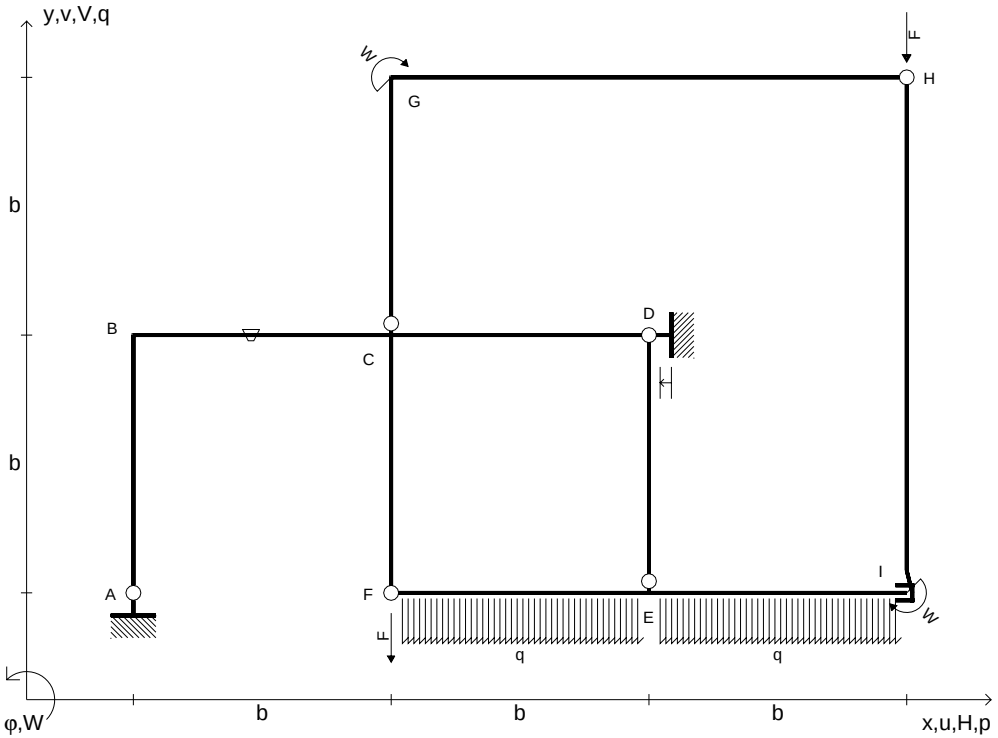

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{IE} = -q = -F/b$
$q_{EF} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
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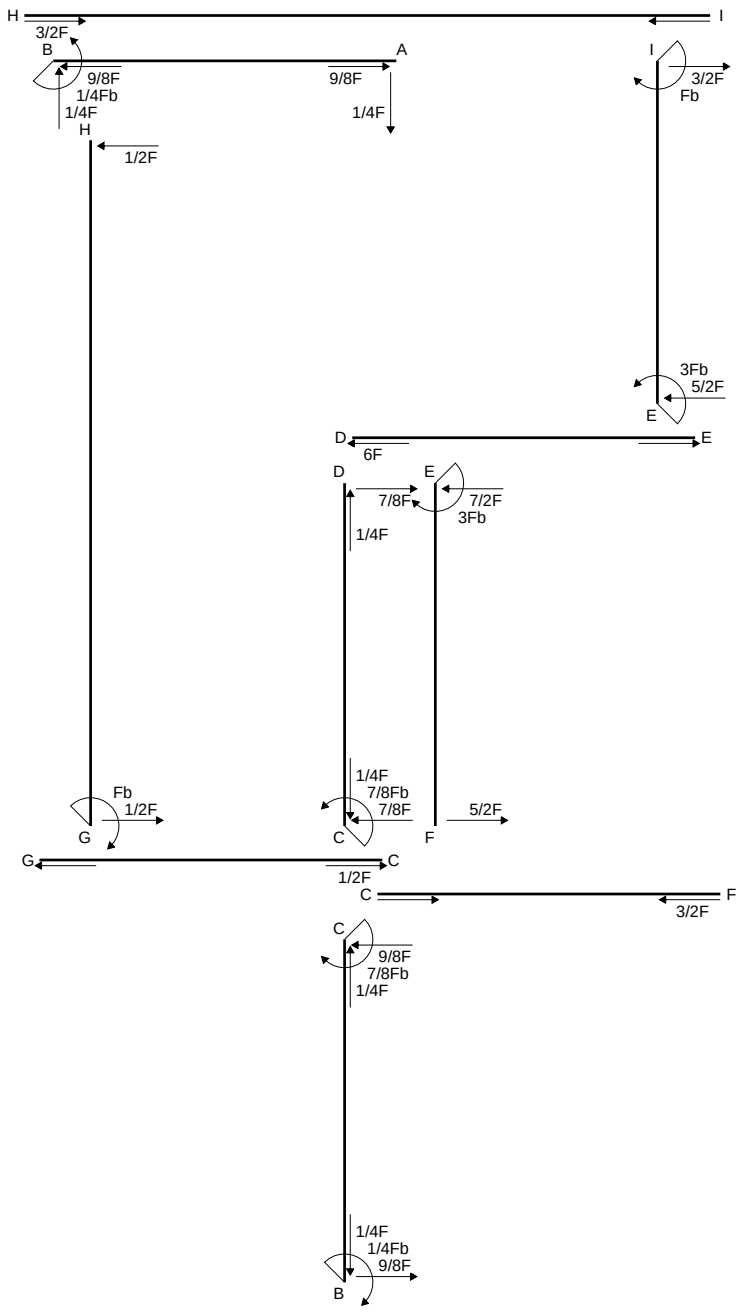
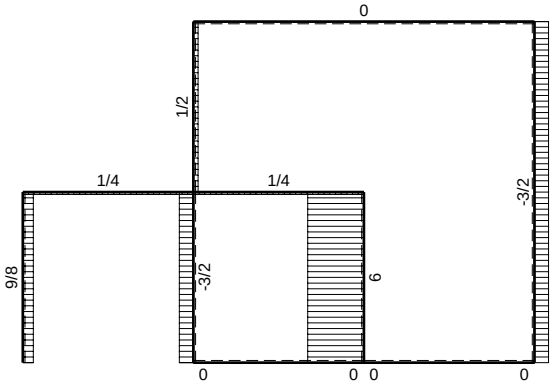
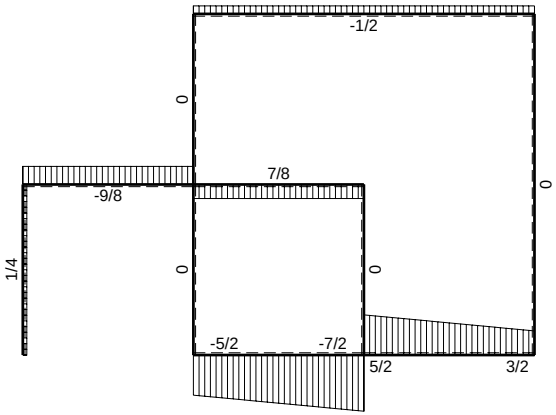
Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.

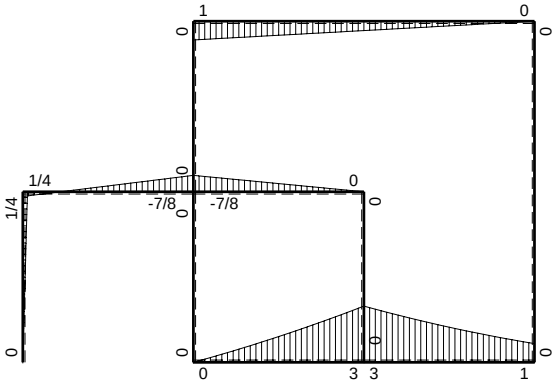




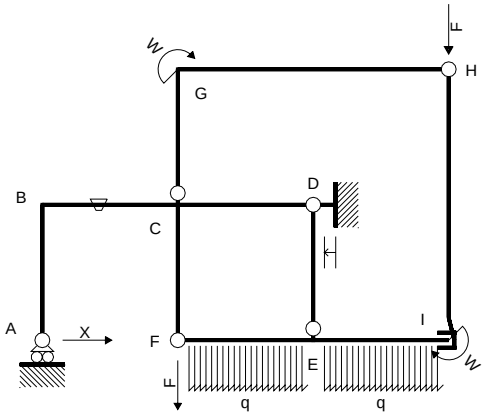
←□→ F



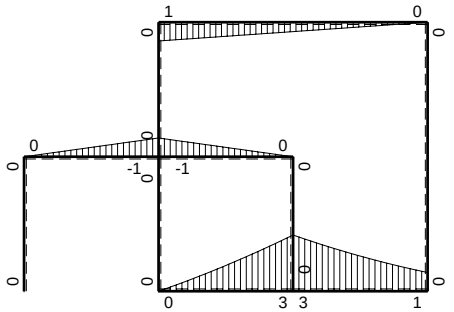
↑□↓ F



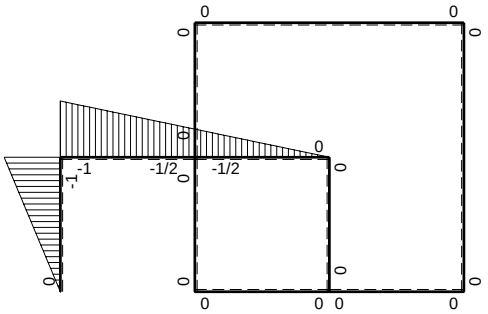
↻□ Fb



Schema di calcolo iperstatico



↻□ M_o flessione da carichi assegnati



↻□ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	-Fx	-Fb/EJ	$Fbx-1/2Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/3+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	Fb-Fx	Fb/EJ	$1/2Fb^2-1/2Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	-Fb+Fx	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$3Fb-7/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-5/2Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/4F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

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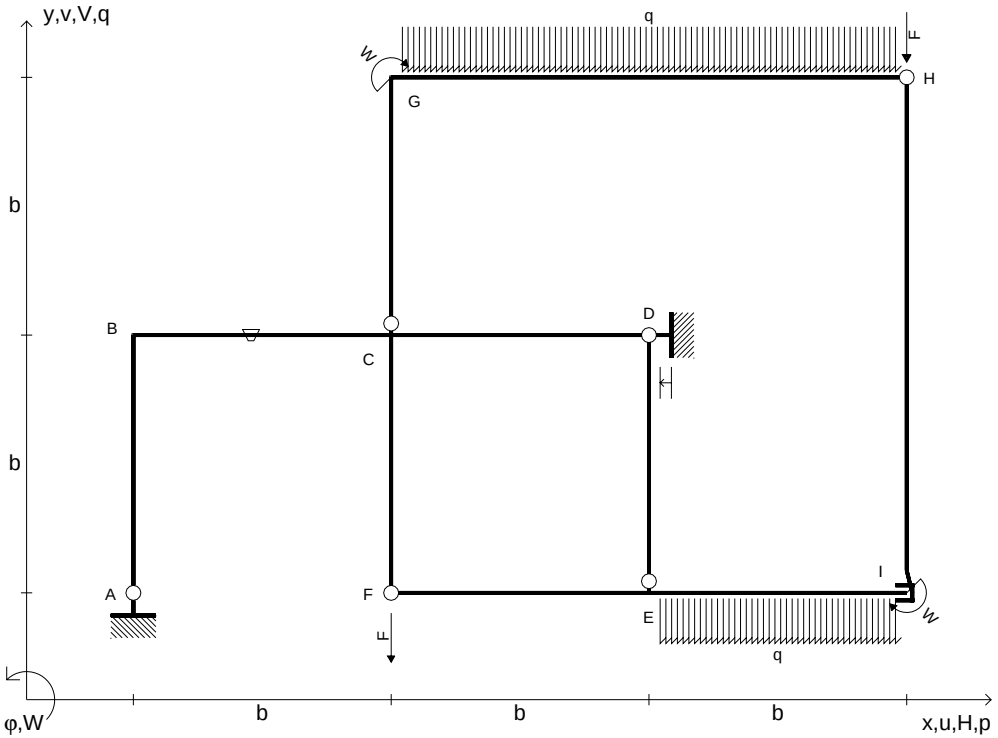

$V_H = -F$
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$W_G = -W = -Fb$
$q_{IE} = -q = -F/b$
$q_{GH} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
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$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

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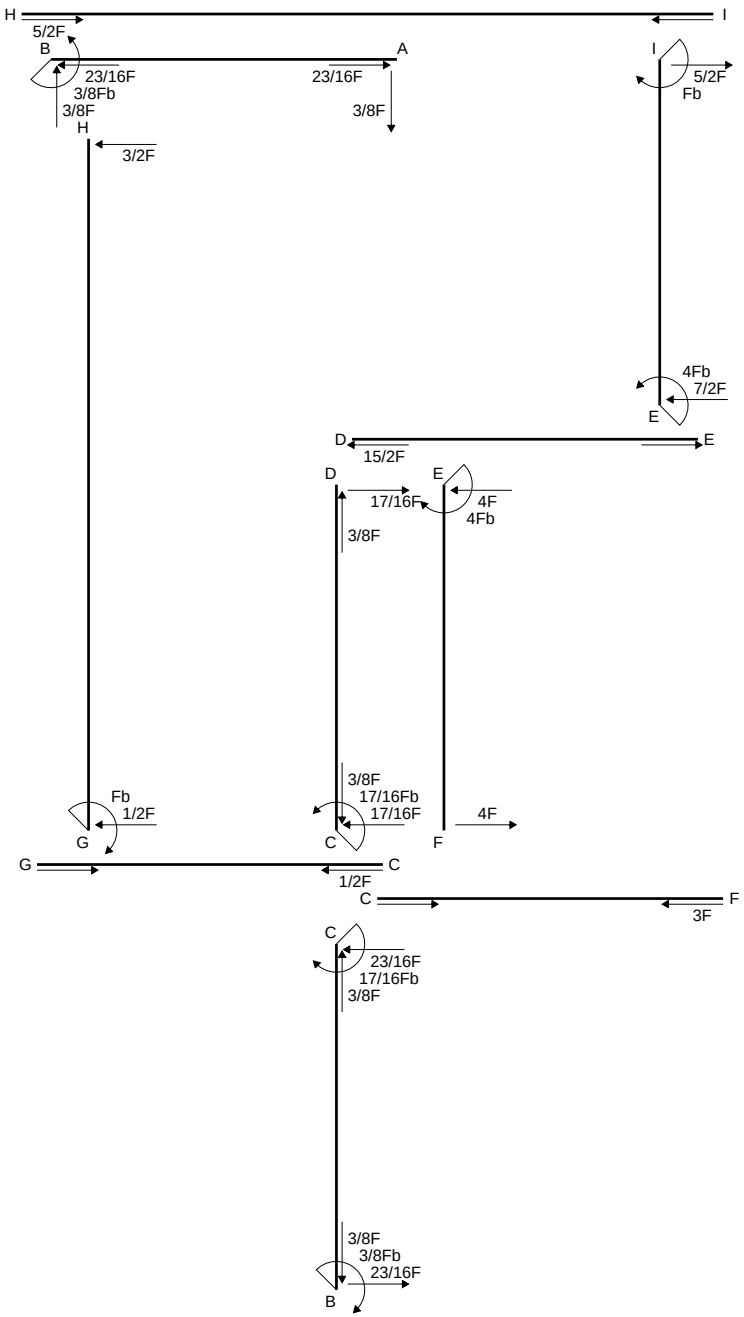
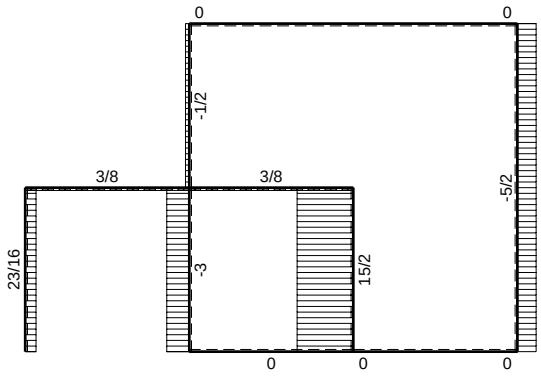
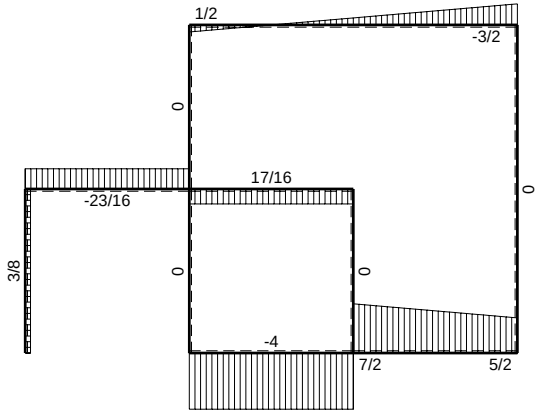
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.

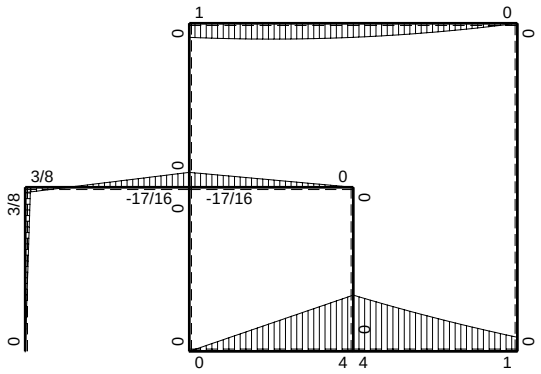




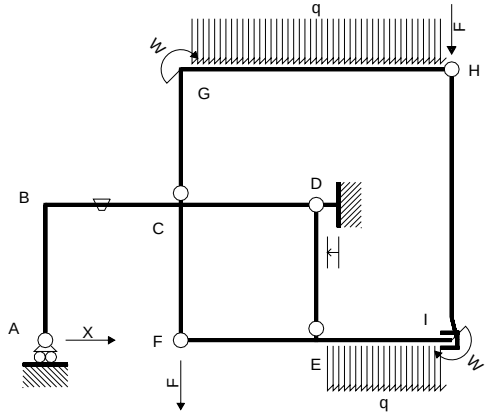
←□→ F



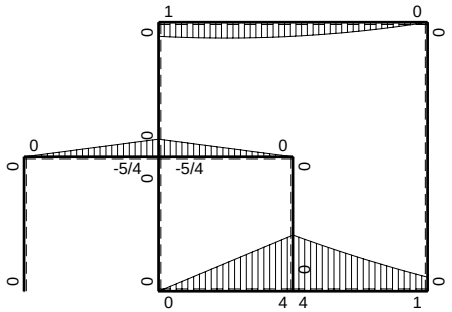
↑□↓ F



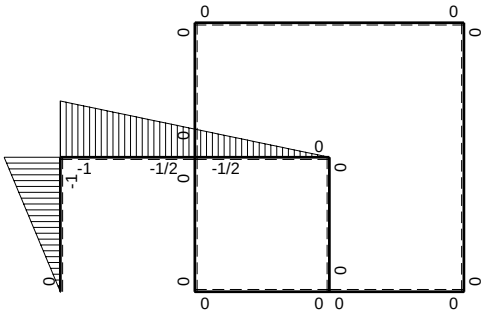
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-5/4Fx	-Fb/EJ	$5/4Fbx-5/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(5/12+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	5/4Fb-5/4Fx	Fb/EJ	$5/8Fb^2-5/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-5/4Fb+5/4Fx	0	$5/8Fb^2-5/4Fbx+5/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(5/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	5/4Fx	0	$5/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	4Fb-4Fx	0	0	0	0	0+0	0
FE b	0	-4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-4Fb+7/2Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$3/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-3/8F	

Sviluppi di calcolo iperstatica

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$$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b \right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{BA} = \int_o^b (1 - 2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = \left[x - x^2/b + 1/3\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$

$$= \left(b - b + 1/3\, b \right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{BC} = \int_o^b (1 - x/b + 1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = \left[x - 1/2\, x^2/b + 1/12\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$

$$= \left(b - 1/2\, b + 1/12\, b \right) b^2\, 1/EJ = 7/12\ b^3/EJ$$

$$L^{XX}_{CB} = \int_o^b (1/4 + 1/2\, x/b + 1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = \left[1/4\, x + 1/4\, x^2/b + 1/12\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$

$$= \left(1/4\, b + 1/4\, b + 1/12\, b \right) b^2\, 1/EJ = 7/12\ b^3/EJ$$

$$L^{XX}_{CD} = \int_o^b (1/4 - 1/2\, x/b + 1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = \left[1/4\, x - 1/4\, x^2/b + 1/12\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$

$$= \left(1/4\, b - 1/4\, b + 1/12\, b \right) b^2\, 1/EJ = 1/12\ b^3/EJ$$

$$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = \left[1/12\, x^3/b^2 \right]_o^b b^2\, 1/EJ$$

$$= \left(1/12\, b \right) b^2\, 1/EJ = 1/12\ b^3/EJ$$

$$L^{Xo}_{BC} = \int_o^b (5/4\, x/b - 5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 - 1/2\, x/b)\, \theta\ dx$$

$$= \left[5/8\, x^2/b - 5/24\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ + \left[x - 1/4\, x^2/b \right]_o^b \theta$$

$$= \left(5/8\, b - 5/24\, b \right) Fb^2\, 1/EJ + \left(b - 1/4\, b \right) \theta = 7/6\ Fb^3/EJ$$

$$L^{Xo}_{CB} = \int_o^b (5/8 - 5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 - 1/2\, x/b)\, \theta\ dx$$

$$= \left[5/8\, x - 5/24\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ + \left[-1/2\, x - 1/4\, x^2/b \right]_o^b \theta$$

$$= \left(5/8\, b - 5/24\, b \right) Fb^2\, 1/EJ + \left(-1/2\, b - 1/4\, b \right) \theta = 7/6\ Fb^3/EJ$$

$$L^{Xo}_{CD} = \int_o^b (5/8 - 5/4\, x/b + 5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = \left[5/8\, x - 5/8\, x^2/b + 5/24\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ$$

$$= \left(5/8\, b - 5/8\, b + 5/24\, b \right) Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$$

$$L^{Xo}_{DC} = \int_o^b (5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = \left[5/24\, x^3/b^2 \right]_o^b Fb^2\, 1/EJ$$

$$= \left(5/24\, b \right) Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$$

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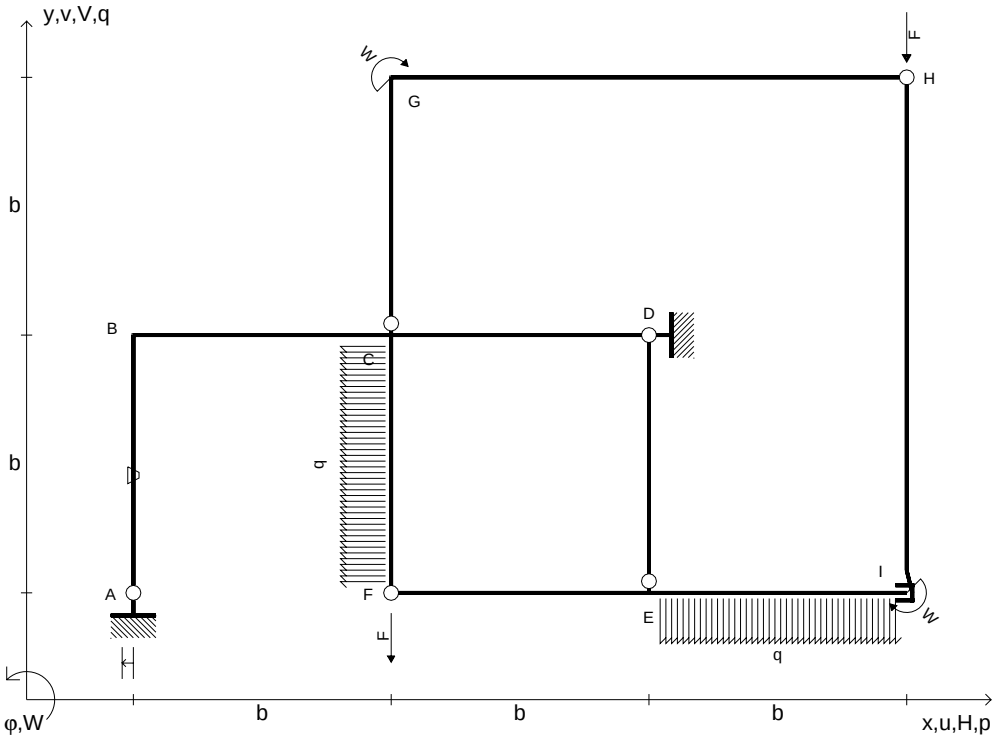
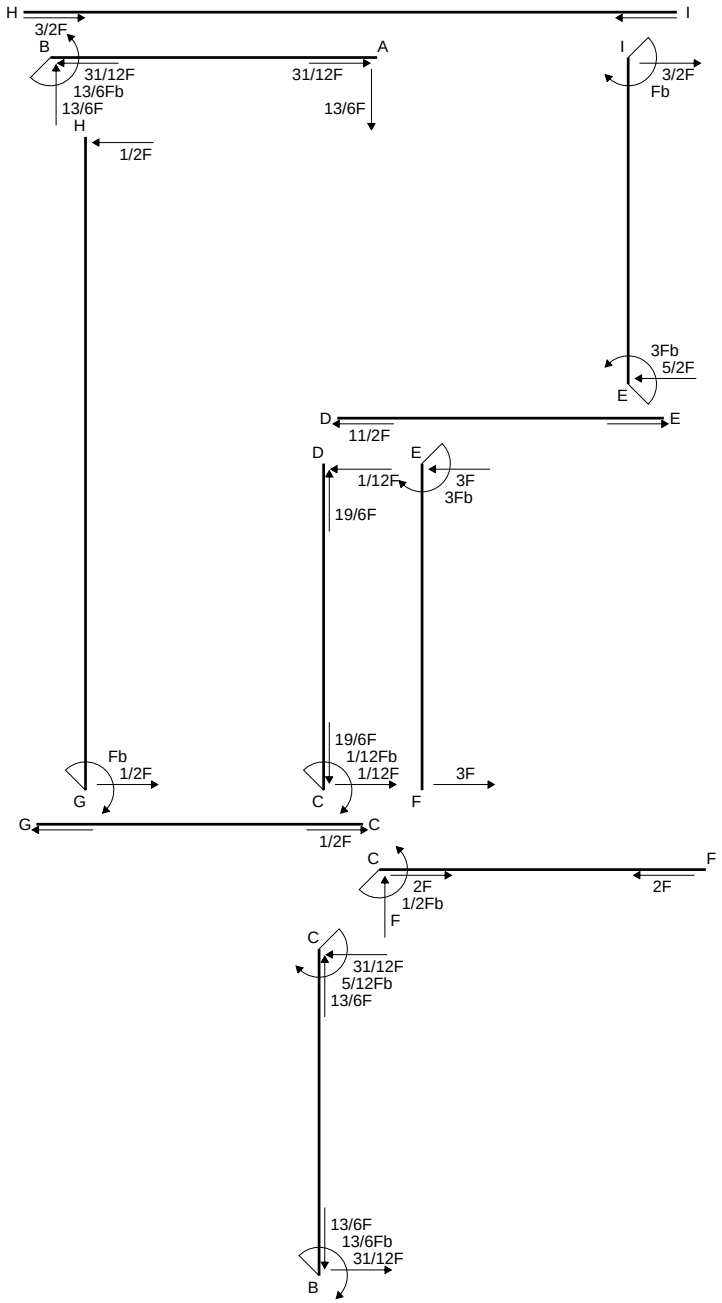


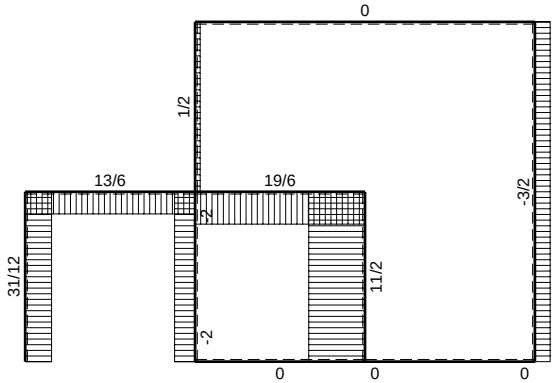
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{IE} = -q = -F/b$
$p_{FC} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

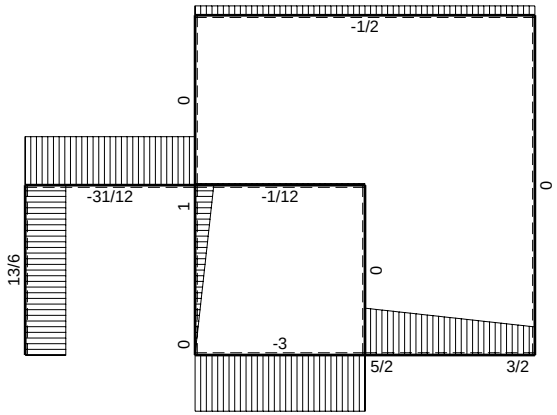
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.

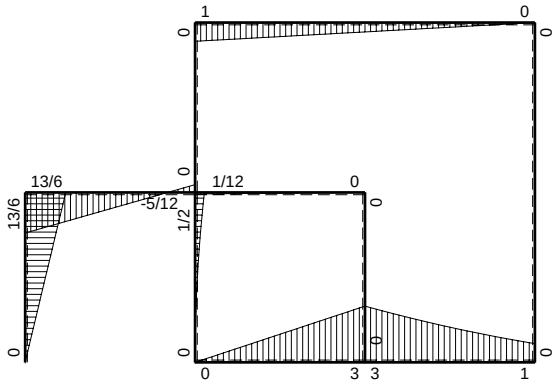




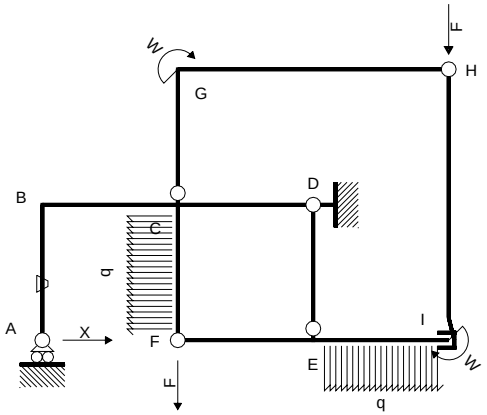
←□→ F



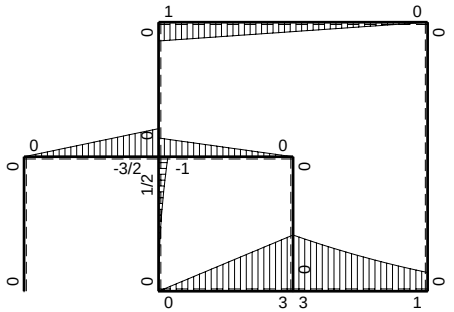
↑□↓ F



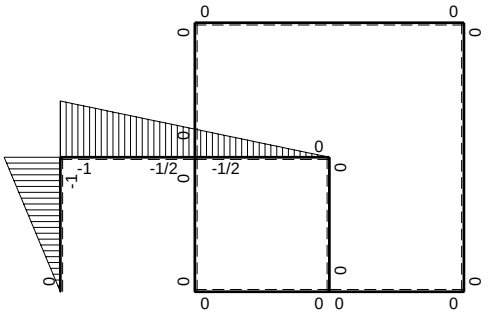
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210898 ESΣ01.xxxx.186

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-3/2Fx	0	3/2Fbx-3/4Fx^2	0	b^2-bx+1/4x^2	(1/2+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	3/2Fb-3/2Fx	0	3/4Fb^2-3/4Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-Fb+Fx	0	1/2Fb^2-Fbx+1/2Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/6+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	Fx	0	1/2Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	3Fb-3Fx	0	0	0	0	0+0	0
FE b	0	-3Fx	0	0	0	0		
FC b	0	1/2qx^2	0	0	0	0	0+0	0
CF b	0	-1/2Fb+Fx-1/2qx^2	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx+1/2qx^2	0	0	0	0	0+0	0
EI b	0	-3Fb+5/2Fx-1/2qx^2	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						13/6Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-13/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/4 -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

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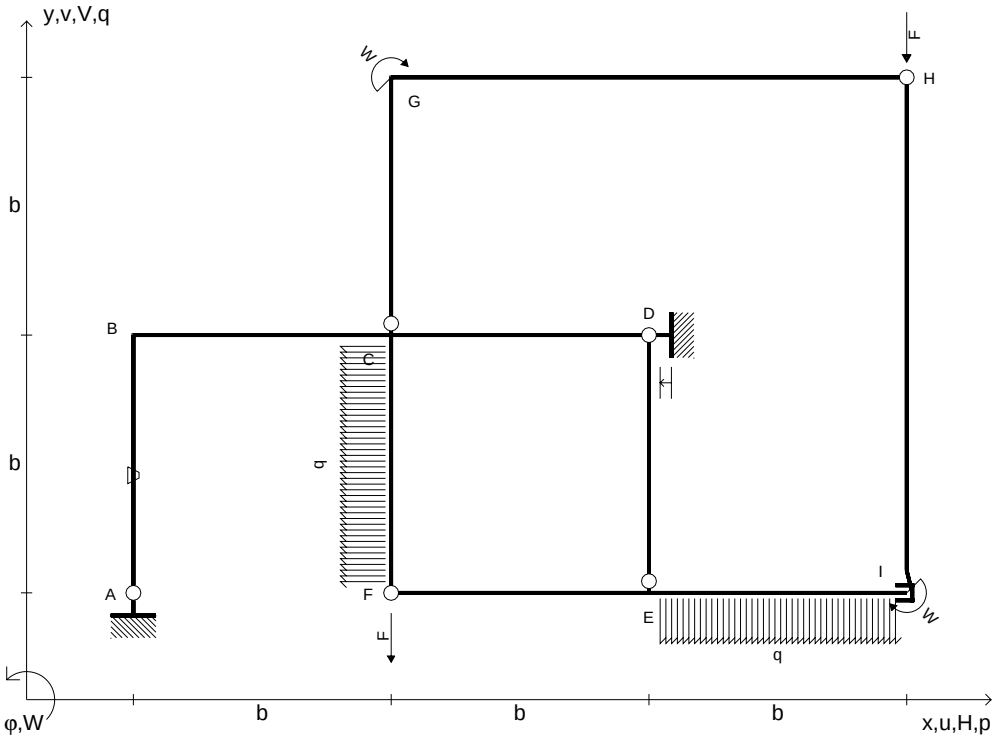

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
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$p_{FC} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

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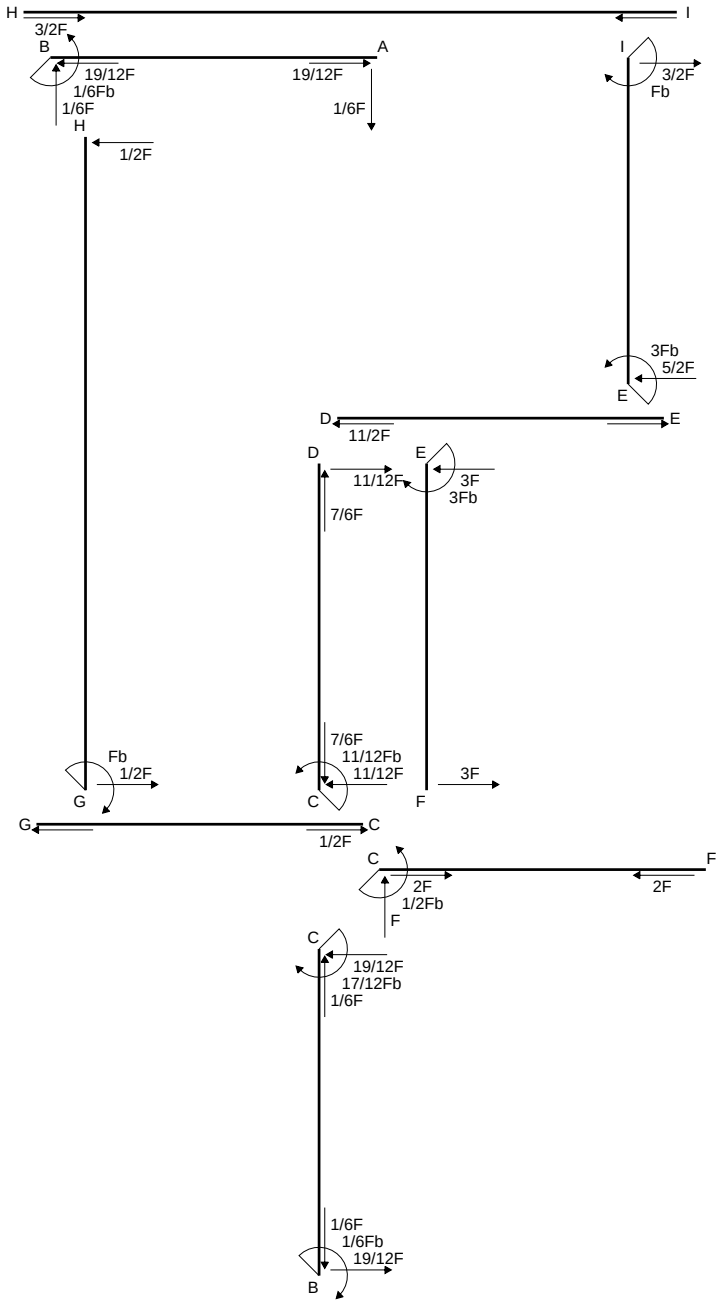
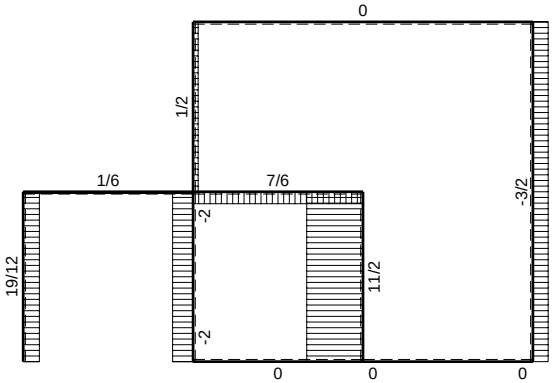
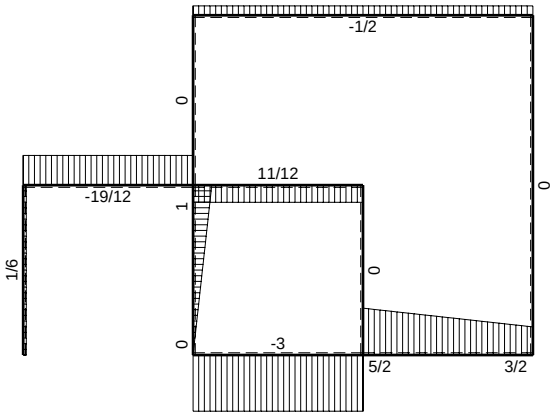
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

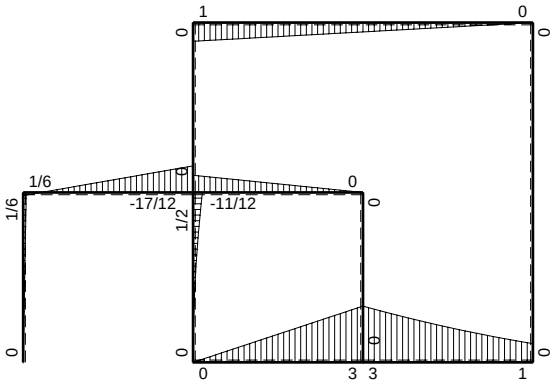




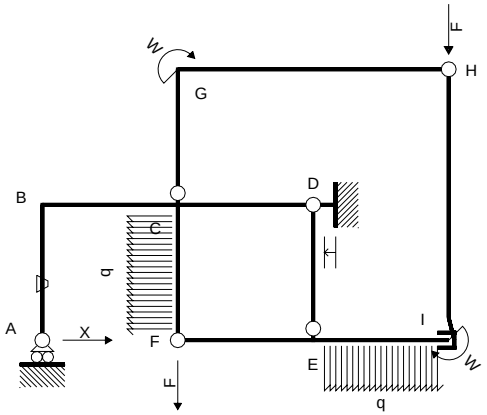
←□→ F



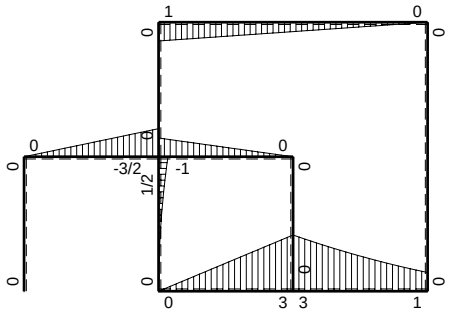
↑□↓ F



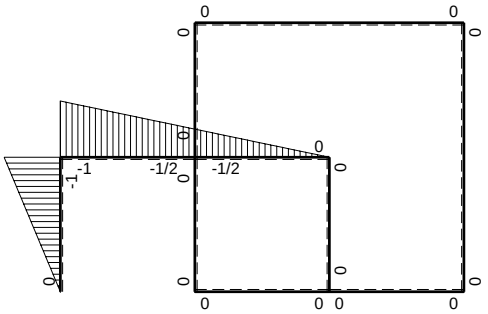
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-3/2Fx	0	3/2Fbx-3/4Fx^2	0	b^2-bx+1/4x^2	(1/2+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	3/2Fb-3/2Fx	0	3/4Fb^2-3/4Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-Fb+Fx	0	1/2Fb^2-Fbx+1/2Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/6+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	Fx	0	1/2Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	3Fb-3Fx	0	0	0	0	0+0	0
FE b	0	-3Fx	0	0	0	0		
FC b	0	1/2qx^2	0	0	0	0	0+0	0
CF b	0	-1/2Fb+Fx-1/2qx^2	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx+1/2qx^2	0	0	0	0	0+0	0
EI b	0	-3Fb+5/2Fx-1/2qx^2	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						-Fb^3/EJ	
	totali						1/6Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-1/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/4 -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

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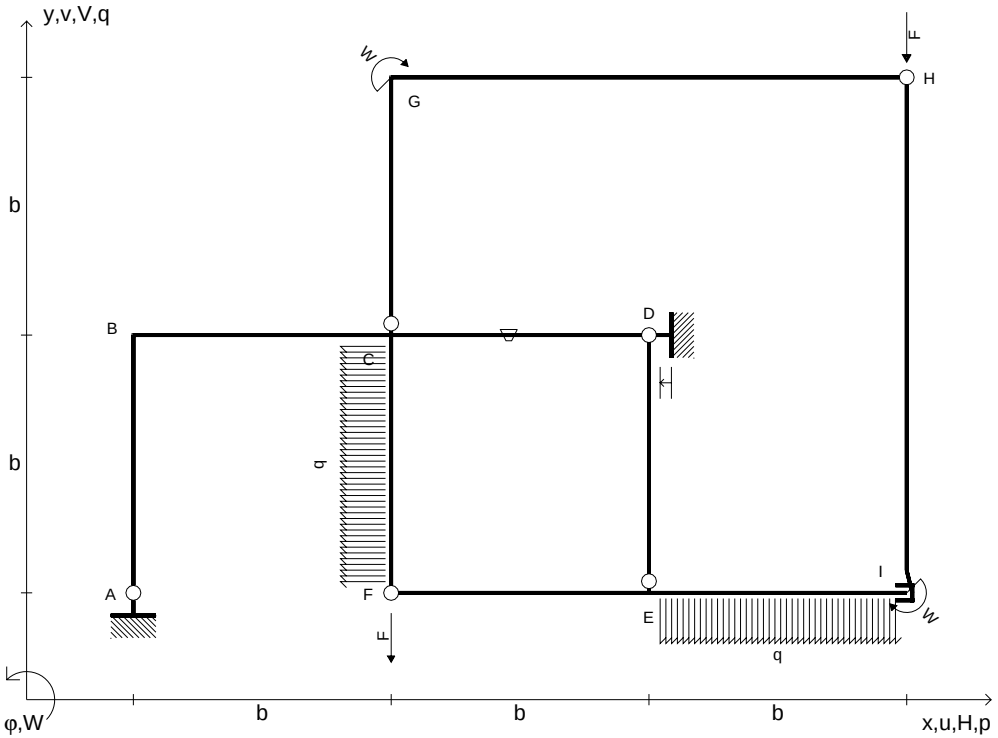
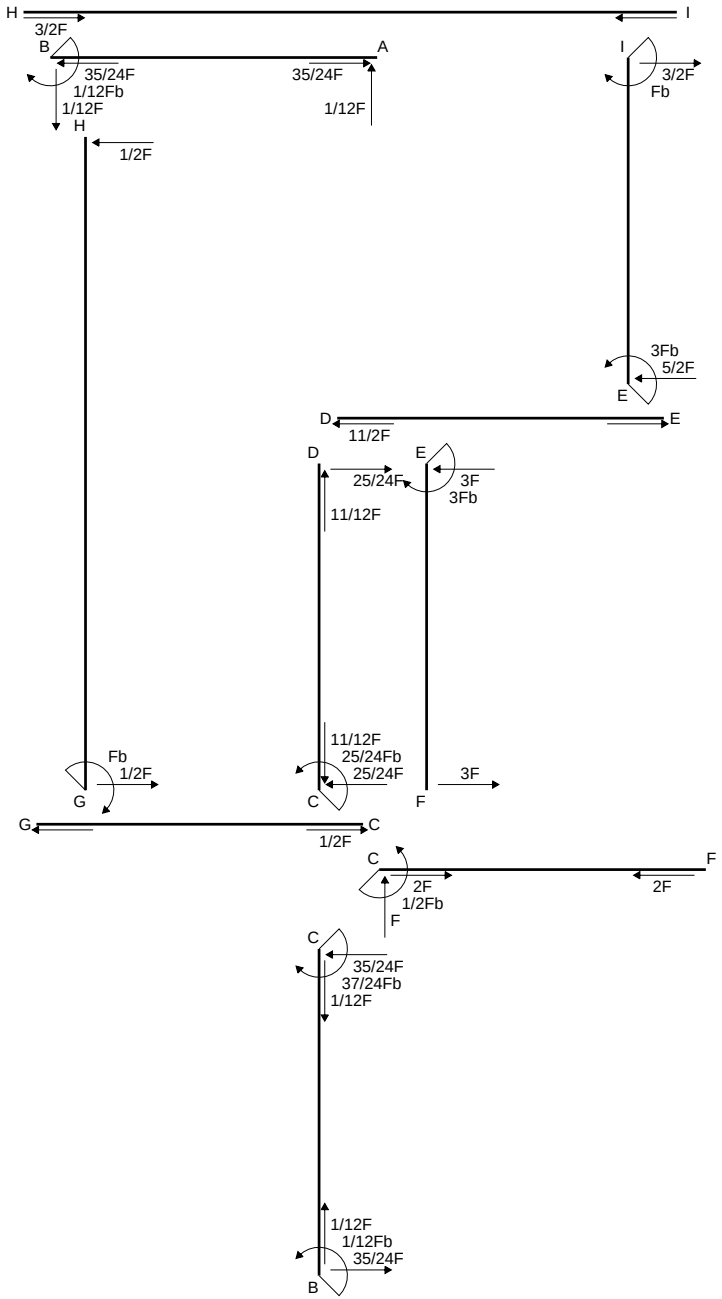


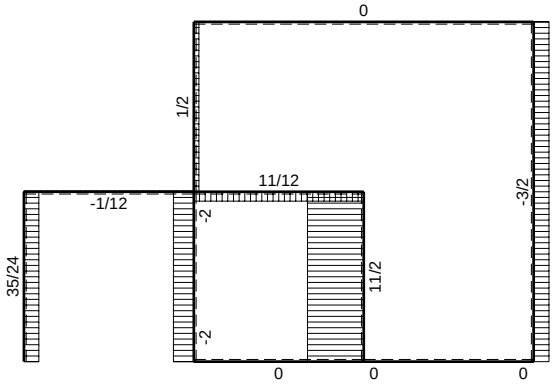
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{IE} = -q = -F/b$
$p_{FC} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

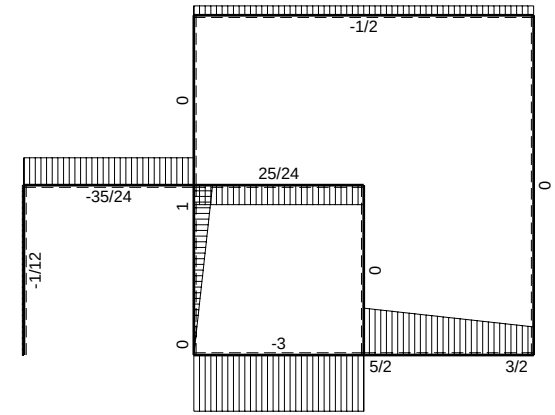
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo D.

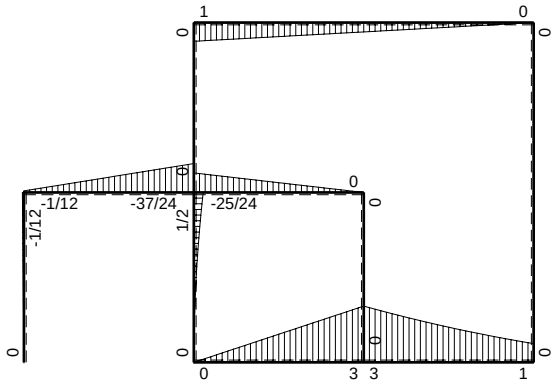




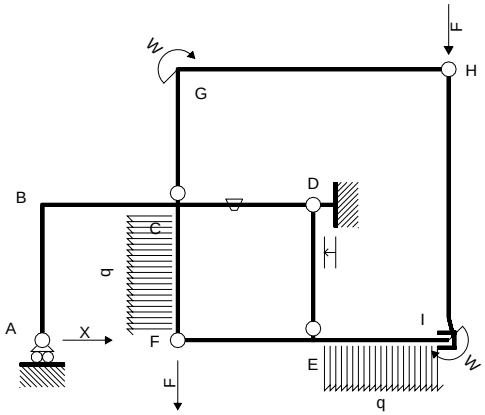
←□→ F



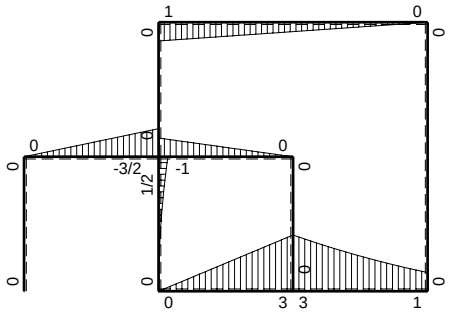
↑□↓ F



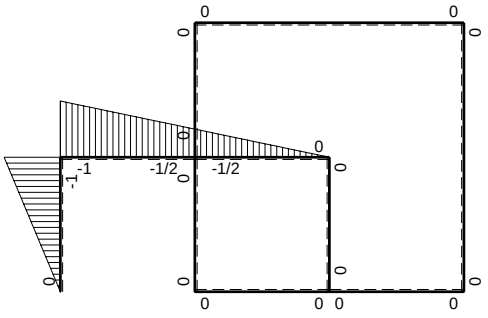
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/2Fx$	0	$3/2Fbx-3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/2+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/2Fb-3/2Fx$	0	$3/4Fb^2-3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-Fb+Fx$	$-Fb/EJ$	$1/2Fb^2-Fbx+1/2Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/6+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	Fx	Fb/EJ	$1/2Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	3Fb-3Fx	0	0	0	0	0+0	0
FE b	0	-3Fx	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-1/12Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						1/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/4 -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\ dx$

$= [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 5/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\ dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/6\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 5/12\ Fb^3/EJ$

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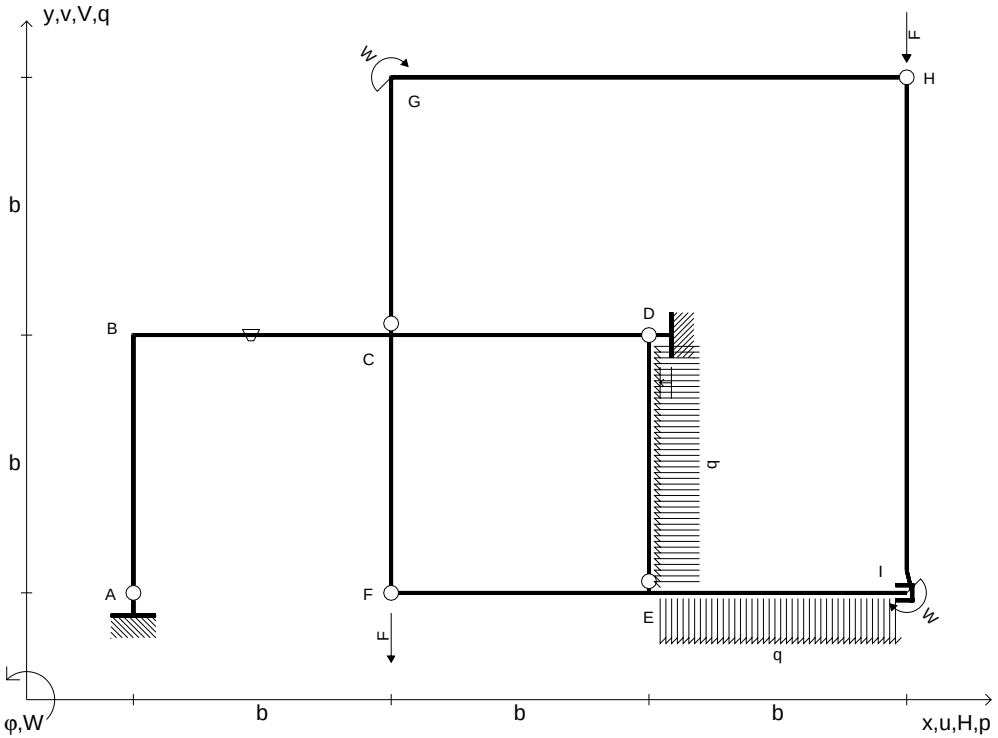

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$p_{DE} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
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Curvatura θ asta BC positiva se convessa a destra con inizio B.
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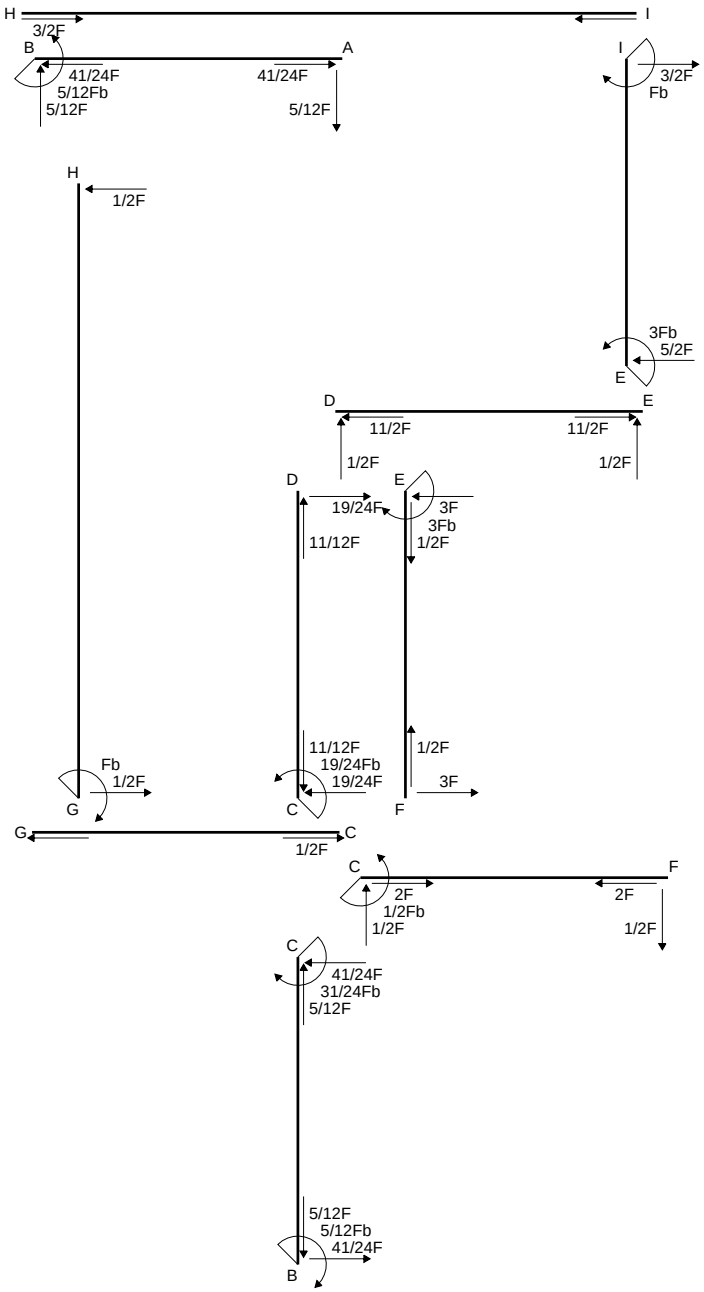

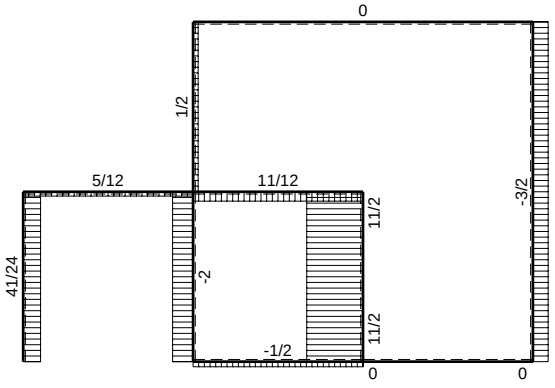

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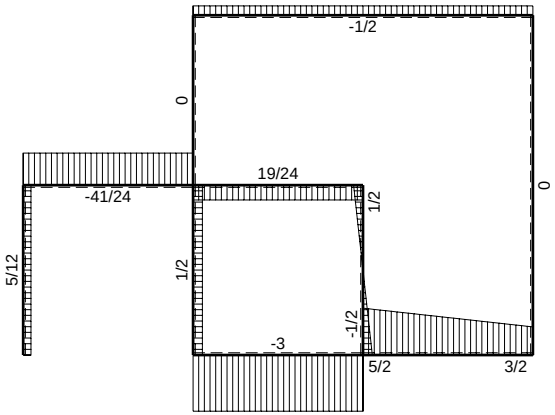

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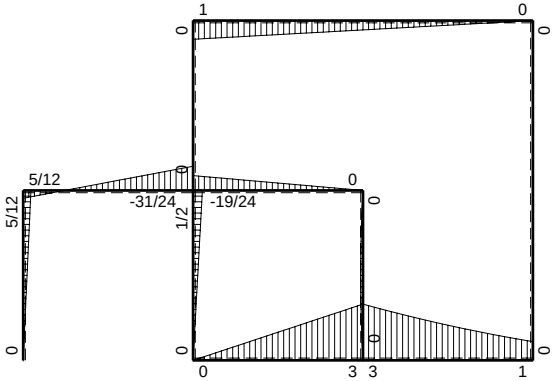
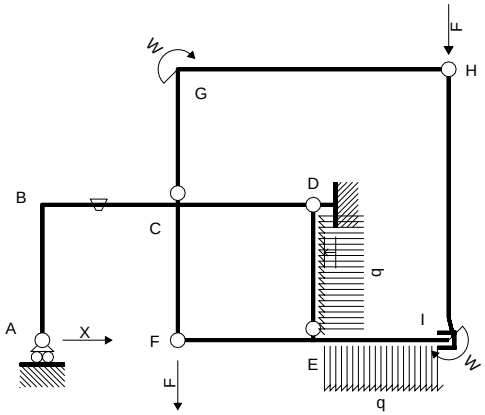
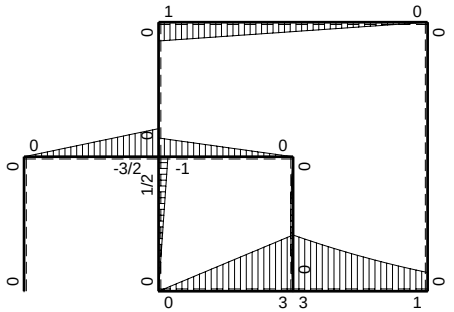


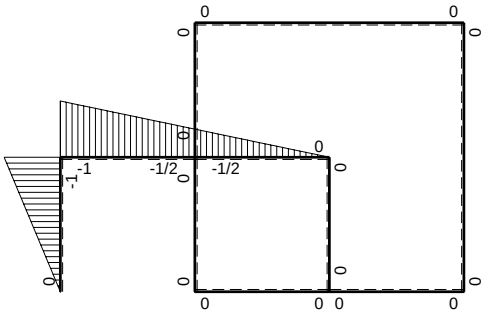
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/2Fx$	$-Fb/EJ$	$3/2Fbx-3/4Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/2+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/2Fb-3/2Fx$	Fb/EJ	$3/4Fb^2-3/4Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-Fb+Fx$	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	$3Fb-3Fx$	0	0	0	0	0+0	0
FE b	0	$-3Fx$	0	0	0	0		
FC b	0	$1/2Fx$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$5/12Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-5/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\ dx$

$= [3/4\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = 5/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/4 -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/4\, x -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 5/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

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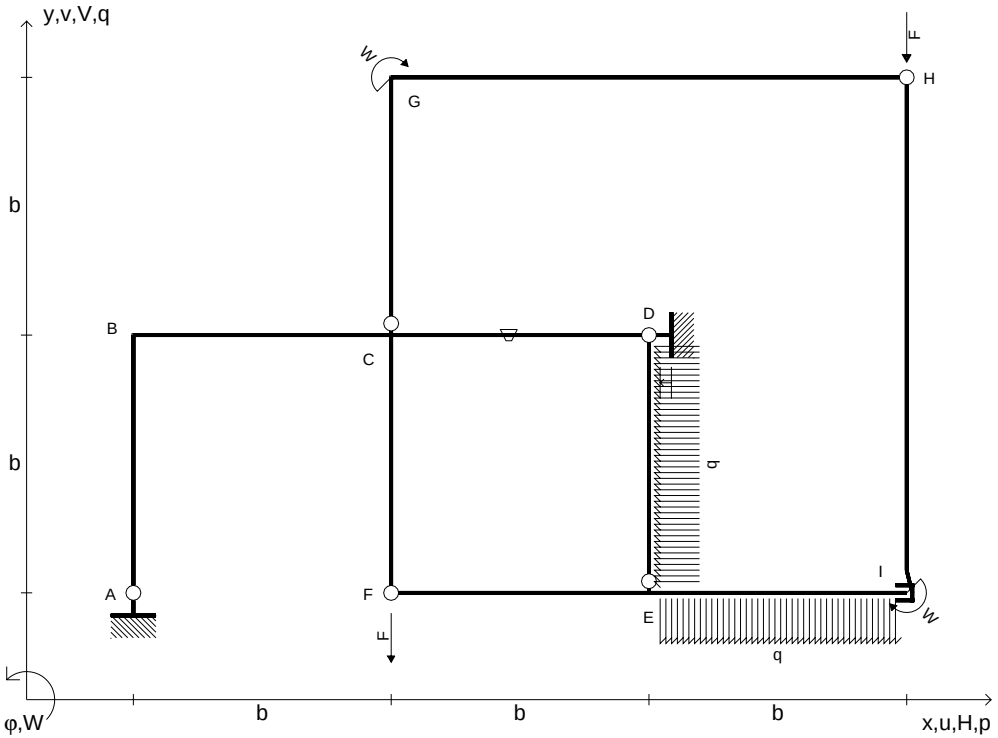


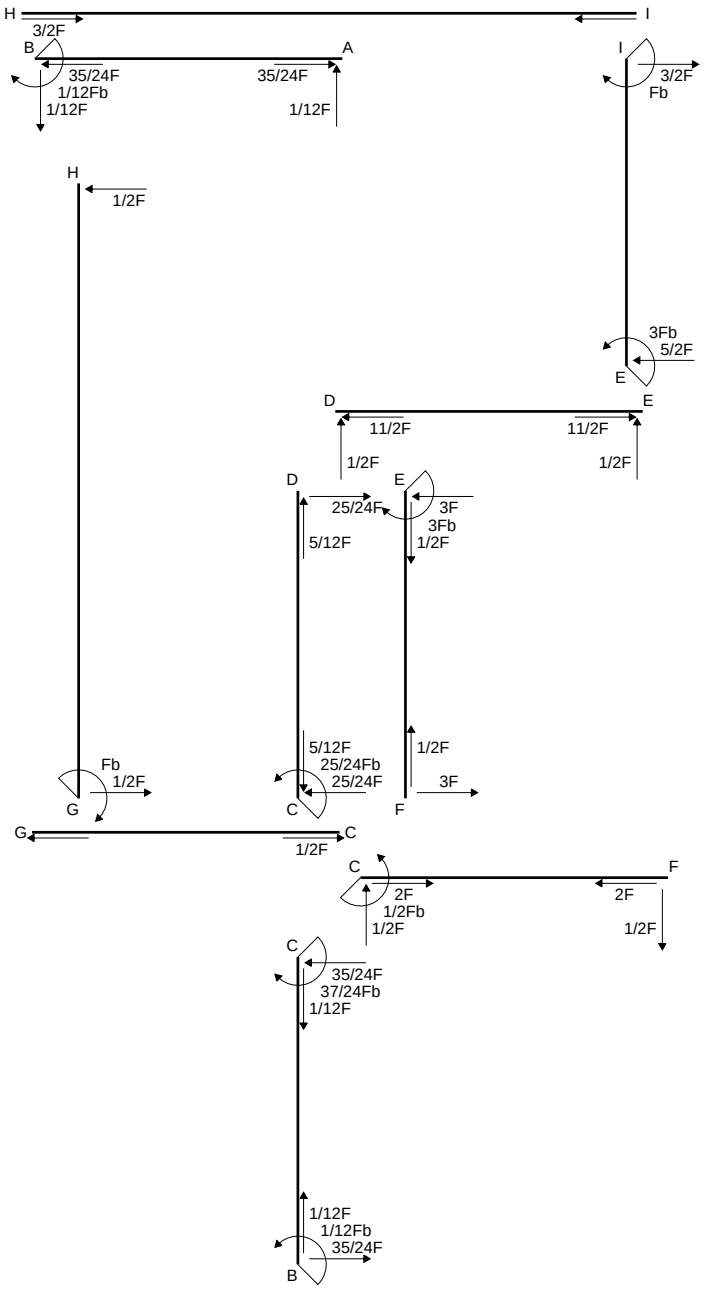
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{IE} = -q = -F/b$
$p_{DE} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

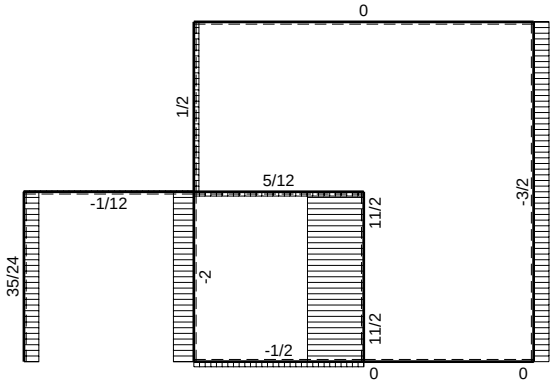
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{CB}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo D.

 08.01.25



 08.01.25



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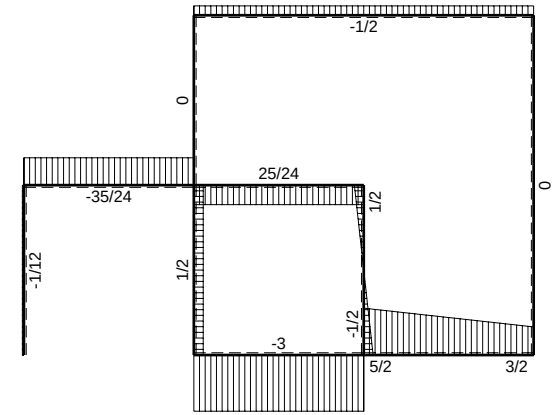

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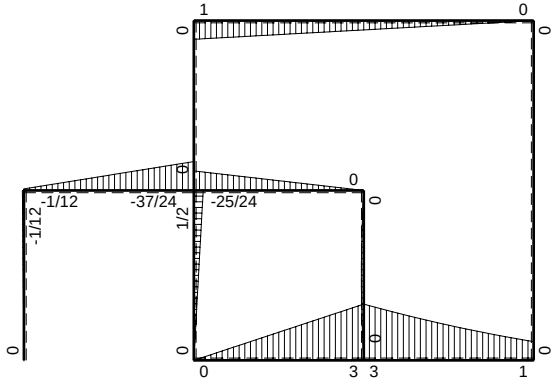
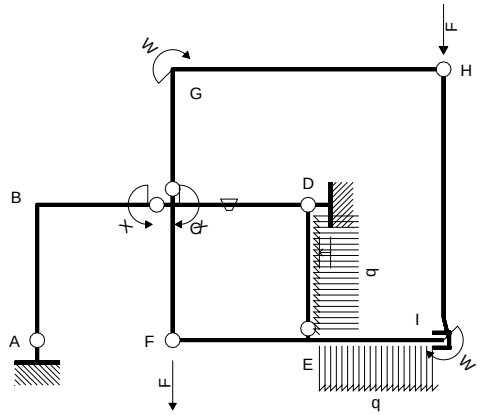
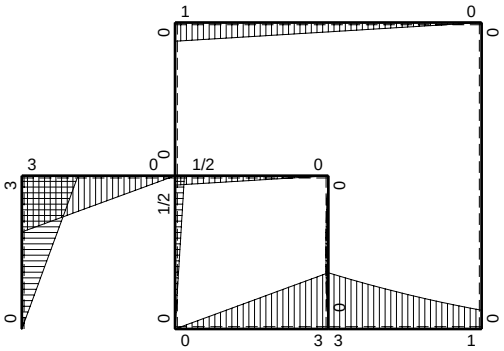


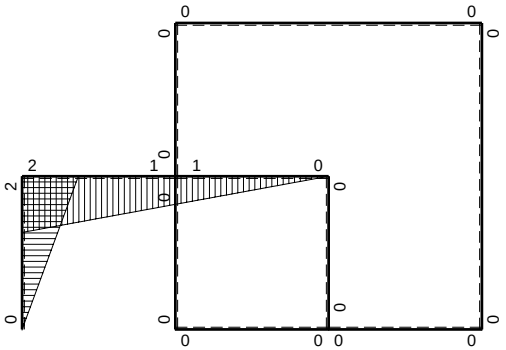
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{CB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	2x/b	3Fx	0	$6Fx^2/b$	0	$4x^2/b^2$	$(2+0)Fb^2/EJ$	4/3Xb/EJ
BA b	-2+2x/b	-3Fb+3Fx	0	$6Fb-12Fx+6Fx^2/b$	0	$4-8x/b+4x^2/b^2$		
BC b	2-x/b	3Fb-3Fx	0	$6Fb-9Fx+3Fx^2/b$	0	$4-4x/b+x^2/b^2$	$(5/2+0)Fb^2/EJ$	7/3Xb/EJ
CB b	-1-x/b	-3Fx	0	$3Fx+3Fx^2/b$	0	$1+2x/b+x^2/b^2$		
CD b	1-x/b	1/2Fb-1/2Fx	-Fb/EJ	$1/2Fb-Fx+1/2Fx^2/b$	-Fb/EJ+Fx/EJ	$1-2x/b+x^2/b^2$	$(1/6-1/2)Fb^2/EJ$	1/3Xb/EJ
DC b	-x/b	-1/2Fx	Fb/EJ	$1/2Fx^2/b$	-Fx/EJ	x^2/b^2		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	3Fb-3Fx	0	0	0	0	0+0	0
FE b	0	-3Fx	0	0	0	0		
FC b	0	1/2Fx	0	0	0	0	0+0	0
CF b	0	-1/2Fb+1/2Fx	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$2Fb^2/EJ$	
	totali						$37/6Fb^2/EJ$	4Xb/EJ
	iperstatica $X=W_{CB}$						-37/24Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4\ x^2/b^2)\ 1/EJ\ dx = [4/3\ x^3/b^2]_o^b\ 1/EJ$

$= (4/3\ b)\ 1/EJ\ = 4/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ 1/EJ\ dx = [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ 1/EJ$

$= (4\ b - 4\ b + 4/3\ b)\ 1/EJ\ = 4/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (4 - 4\ x/b + x^2/b^2)\ 1/EJ\ dx = [4\ x - 2\ x^2/b + 1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (4\ b - 2\ b + 1/3\ b)\ 1/EJ\ = 7/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 + 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x + x^2/b + 1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b + b + 1/3\ b)\ 1/EJ\ = 7/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{DC} = \int_o^b (\ x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (6\ x^2/b^2)\ Fb\ 1/EJ\ dx = [2\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (2\ b)\ Fb\ 1/EJ\ = 2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (6 - 12\ x/b + 6\ x^2/b^2)\ Fb\ 1/EJ\ dx = [6\ x - 6\ x^2/b + 2\ x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (6\ b - 6\ b + 2\ b)\ Fb\ 1/EJ\ = 2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (6 - 9\ x/b + 3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [6\ x - 9/2\ x^2/b + x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (6\ b - 9/2\ b + b)\ Fb\ 1/EJ\ = 5/2\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3\ x/b + 3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/2\ x^2/b + x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (3/2\ b + b)\ Fb\ 1/EJ\ = 5/2\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b + 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [1/2\ x - 1/2\ x^2/b + 1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b]_o^b\ \theta$

$= (1/2\ b - 1/2\ b + 1/6\ b)\ Fb\ 1/EJ\ + (-\ b + 1/2\ b)\ \theta\ = -1/3\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b)\ \theta\ dx = [1/6\ x^3/b^2]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b]_o^b\ \theta$

$= (1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = -1/3\ Fb^2/EJ$

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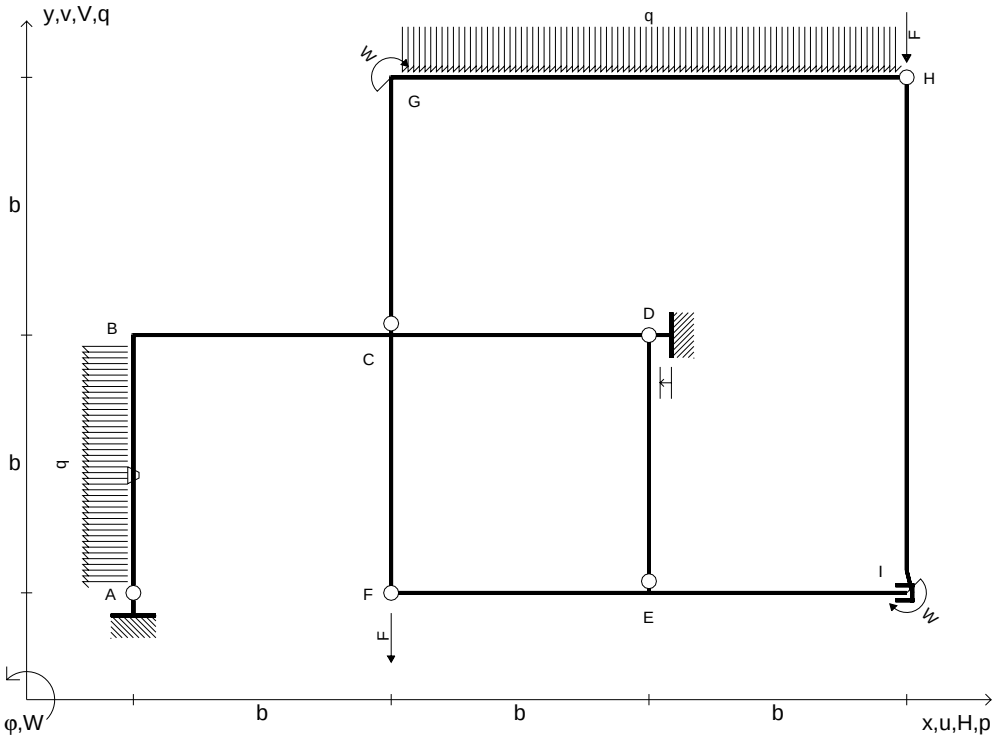
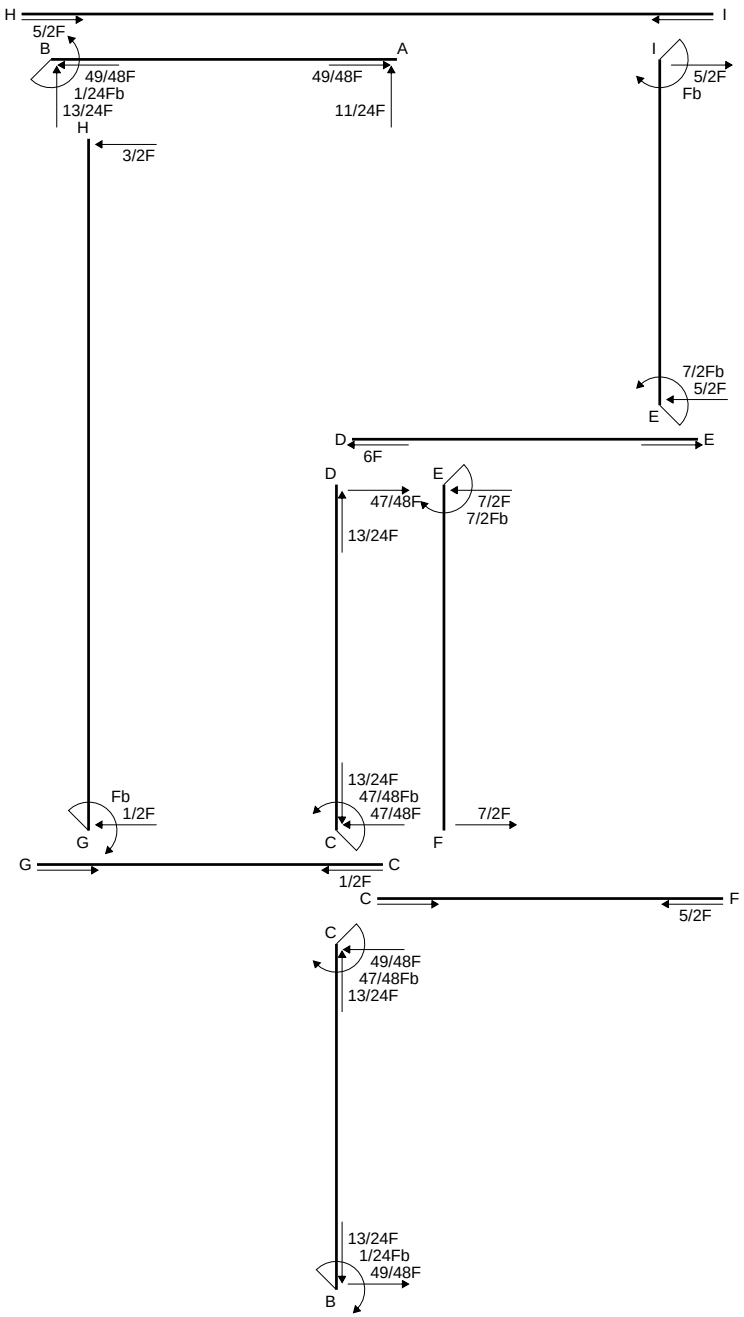


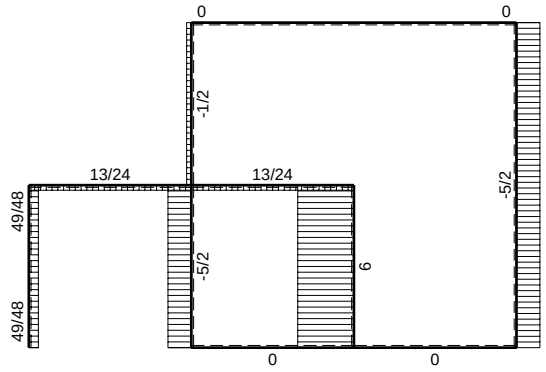
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{GH} = -q = -F/b$
$p_{AB} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

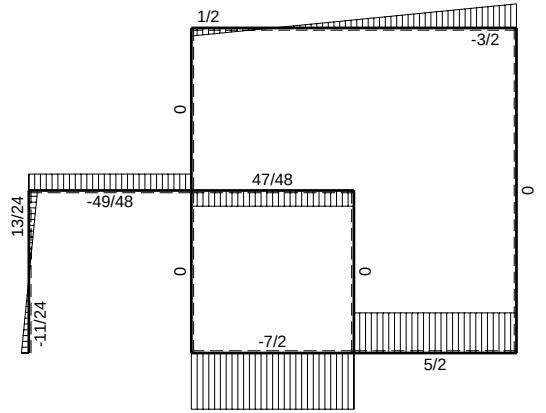
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

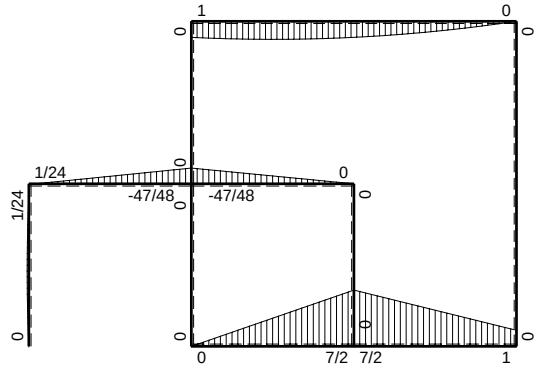




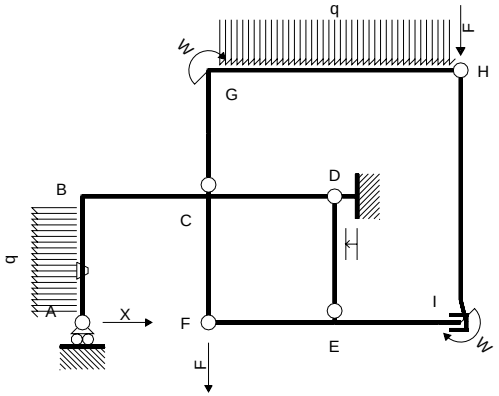
←□→ F



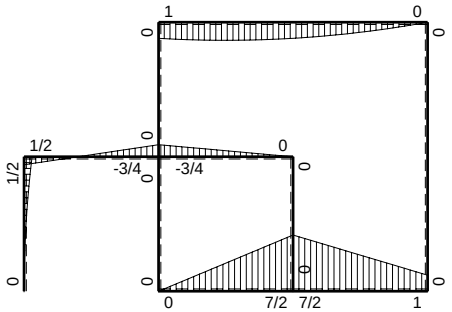
↑□↓ F



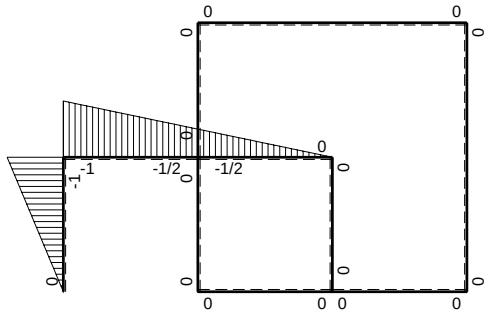
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	-Fb/EJ	$-1/2qx^3$	Fxb/EJ	x^2	$(-1/8+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-5/4Fx$	0	$-1/2Fb^2+3/2Fbx-5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-5/4Fx$	0	$3/8Fb^2-1/4Fbx-5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	3/4Fx	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-7/2Fx	0	0	0	0	0+0	0
FE b	0	-7/2Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+5/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-11/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						11/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (x/b)\, \theta\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [1/2\, x^2/b]_o^b\, \theta$

$= (-1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 + x/b)\, \theta\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [- x +1/2\, x^2/b]_o^b\, \theta$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ + (- b +1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +3/2\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +3/4\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -5/24\, b)\, Fb^2\, 1/EJ = 1/24\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -1/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 1/24\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

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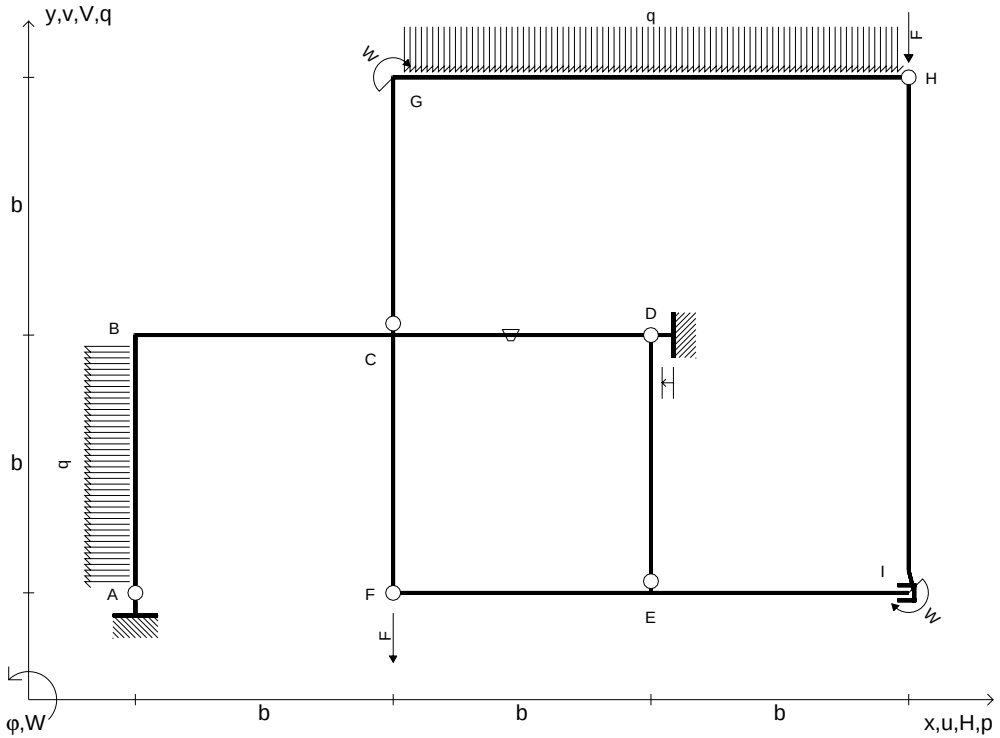

$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{GH} = -q = -F/b$
$p_{AB} = -q = -F/b$

$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
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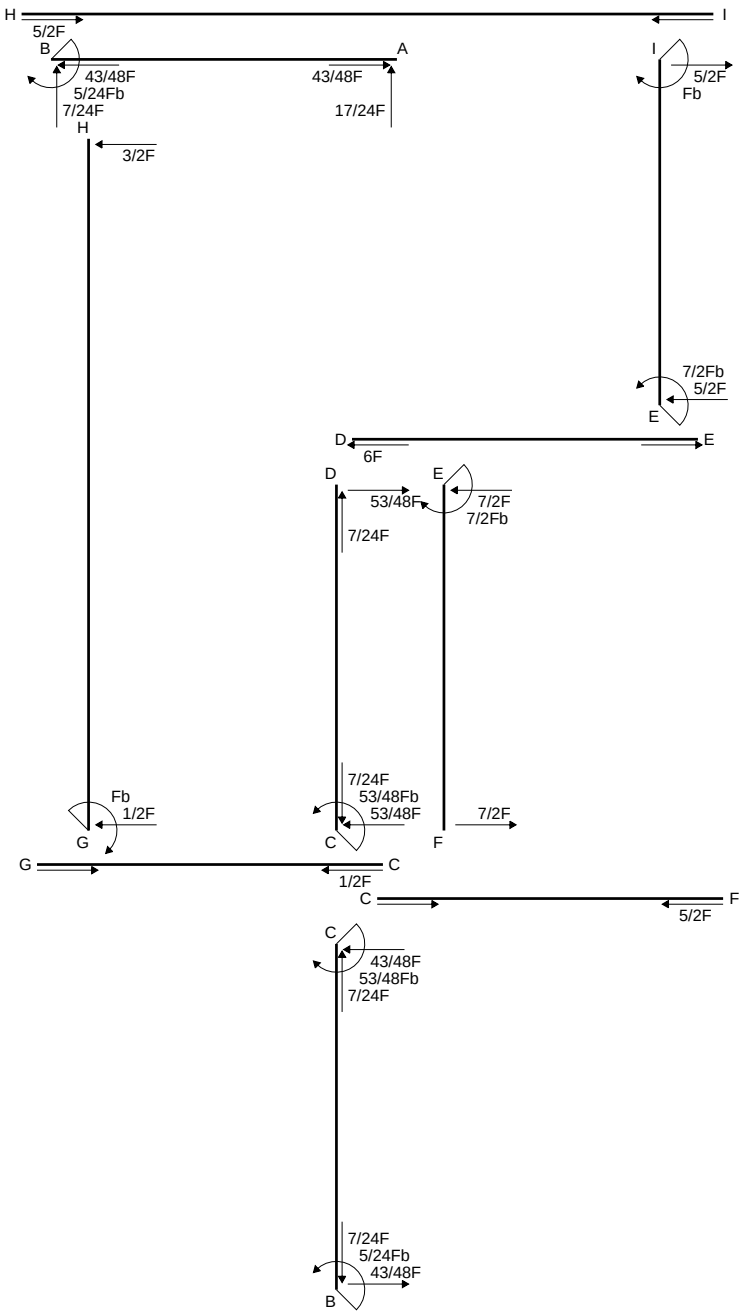
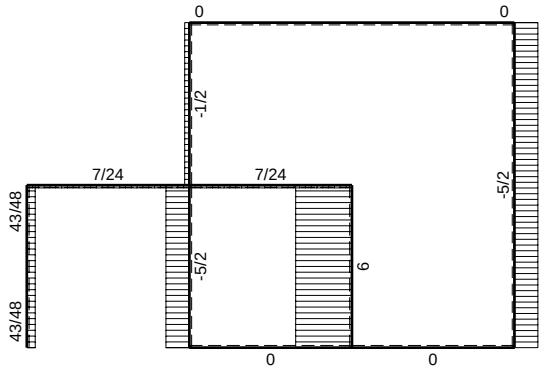
Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CD positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo D.

 08.01.25



 08.01.25



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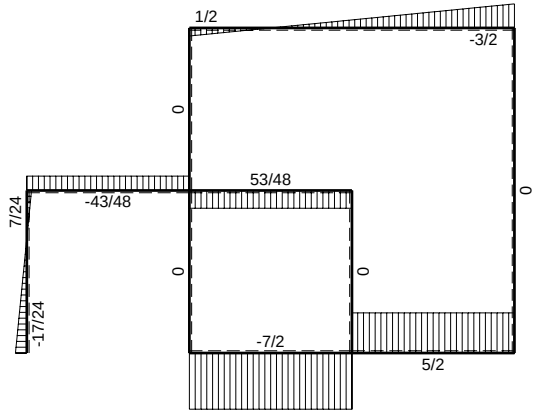

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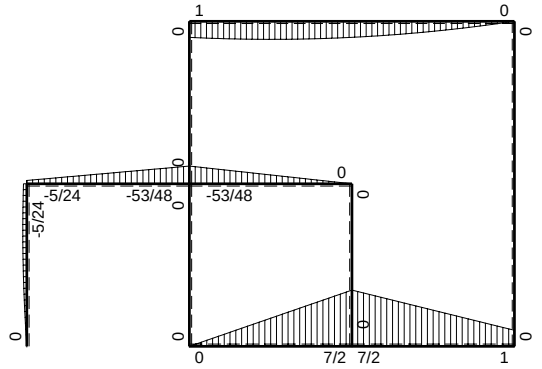

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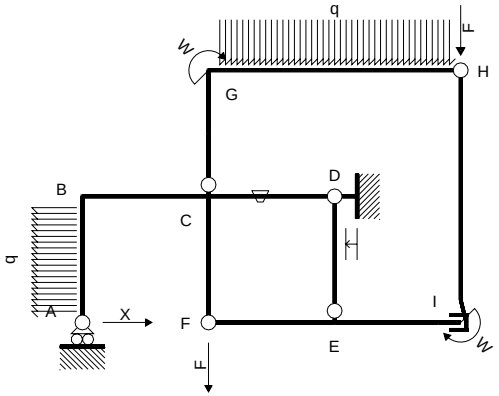
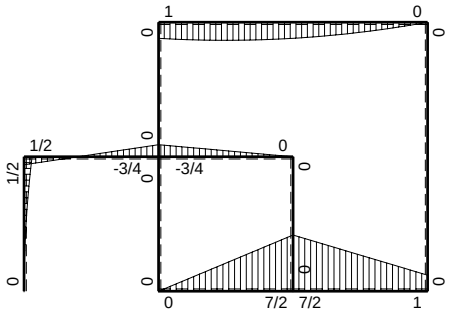
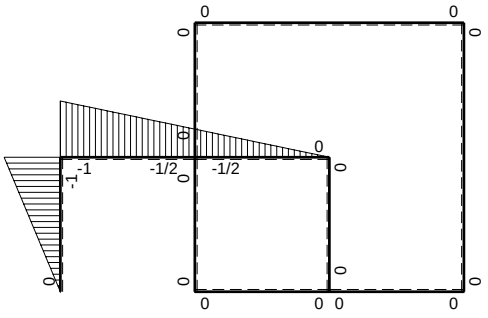


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-5/4Fx$	0	$-1/2Fb^2+3/2Fbx-5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-5/4Fx$	0	$3/8Fb^2-1/4Fbx-5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	-Fb/EJ	$3/8Fb^2-3/4Fbx+3/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	Fb/EJ	$3/8Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-7/2Fx$	0	0	0	0	0+0	0
FE b	0	$-7/2Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+5/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-17/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						17/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +3/2\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +3/4\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -5/24\, b)\, Fb^2\, 1/EJ = 1/24\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -1/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 1/24\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\ dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/8\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

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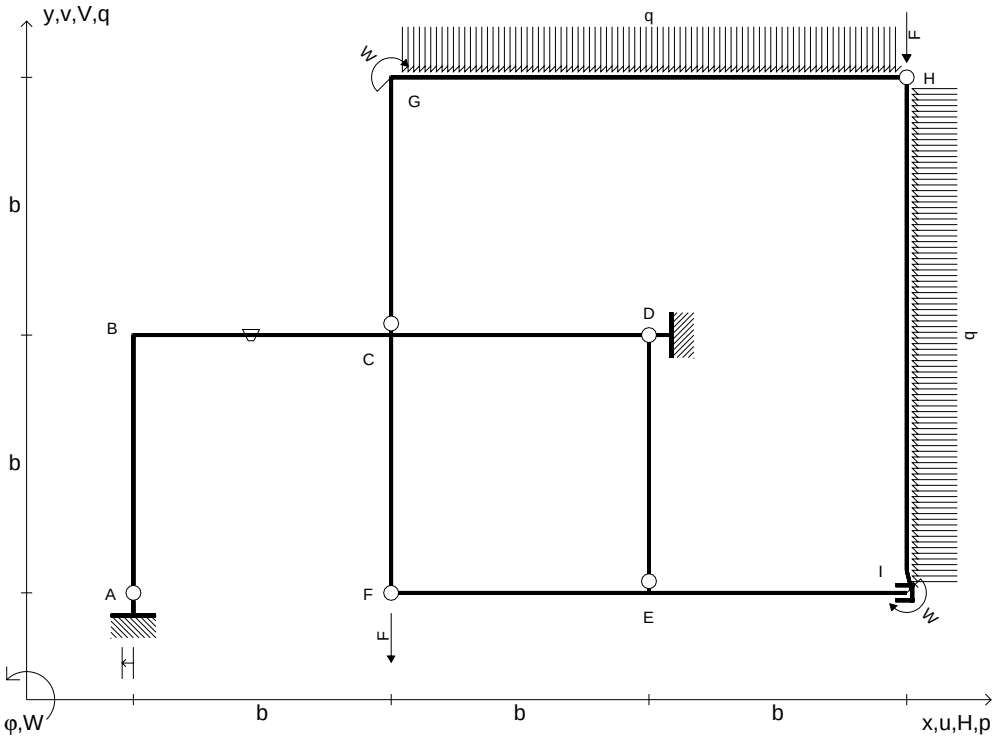
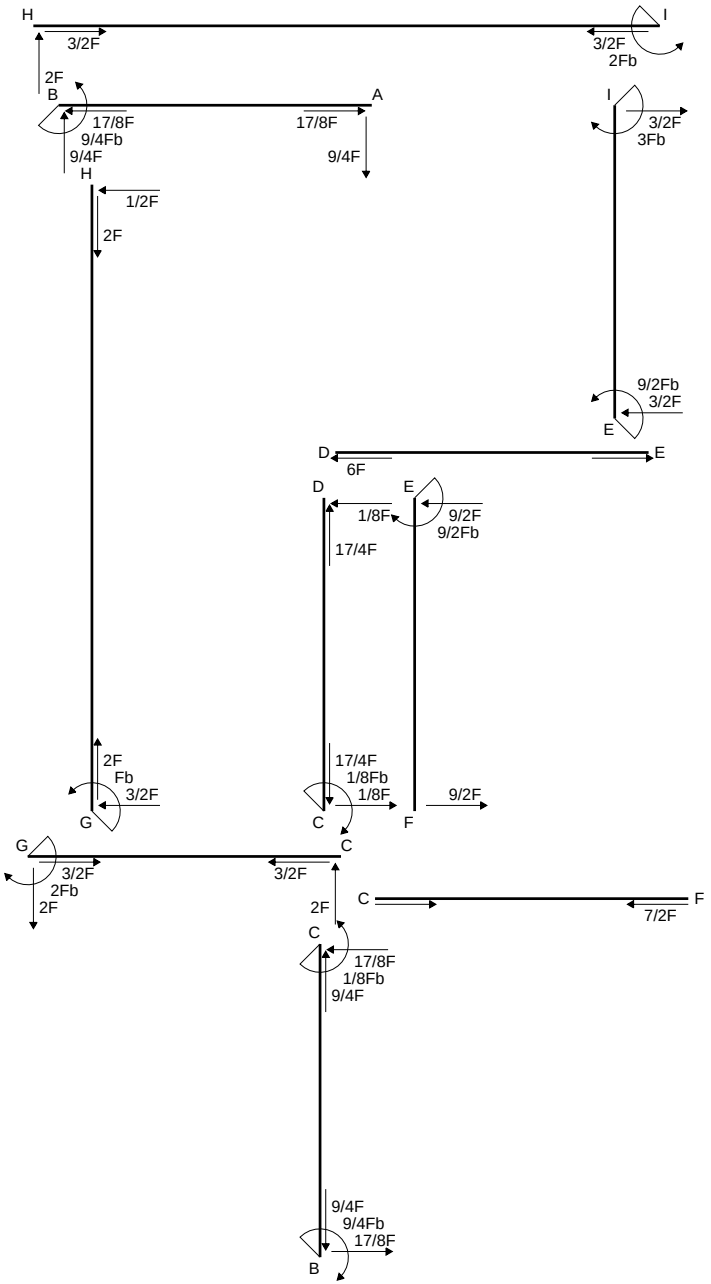


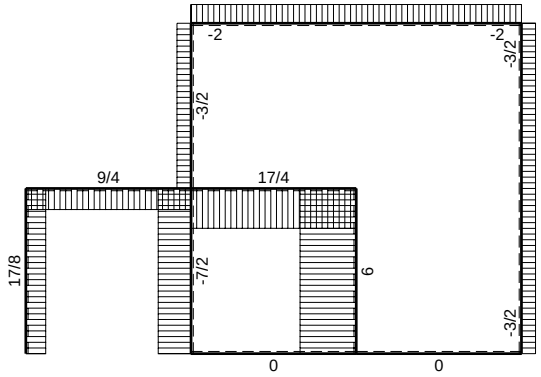
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{GH} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_D$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo A.





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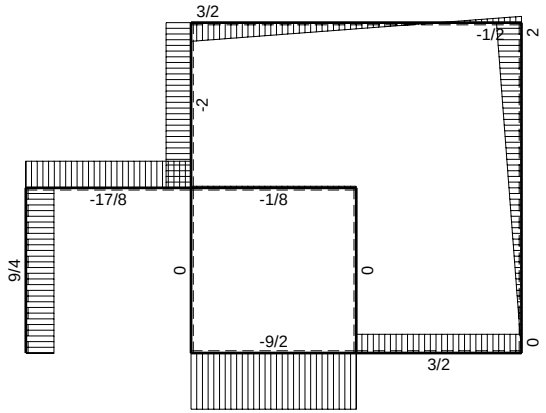

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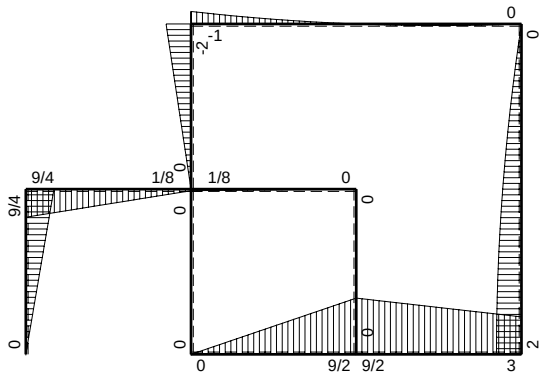

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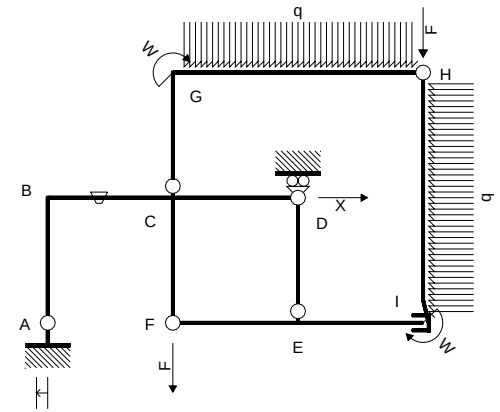
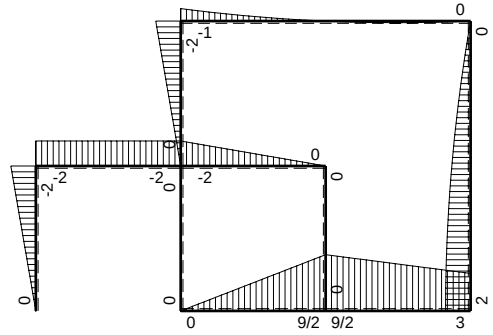
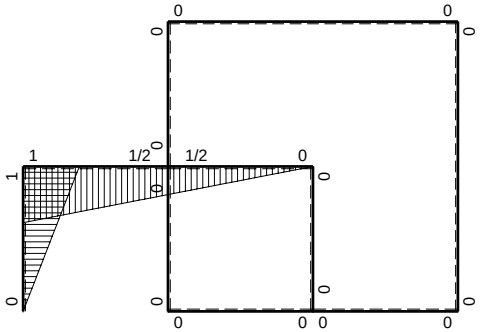


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 212600 ESΣ01.xxxx.201

Quadro contributi PLV per iperstatica $X=H_D$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	x	-2Fx	0	$-2Fx^2$	0	x^2	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	-b+x	2Fb-2Fx	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$		
BC b	b-1/2x	-2Fb	-Fb/EJ	$-2Fb^2+Fbx$	$-Fb^2/EJ+1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(-3/2-3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	-1/2b-1/2x	2Fb	Fb/EJ	$-Fb^2-Fbx$	$-1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	1/2b-1/2x	-2Fb+2Fx	0	$-Fb^2+2Fbx-Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/3+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	-1/2x	2Fx	0	$-Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/2Fb-9/2Fx	0	0	0	0	0+0	0
FE b	0	-9/2Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	$-Fb+3/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-9/2Fb+3/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						$-Fb^3/EJ$	
	totali						$-17/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_D$						17/4F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-2 +4\, x/b -2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\, x +2\, x^2/b -2/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2\, b +2\, b -2/3\, b)\, Fb^2\, 1/EJ = -2/3\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-2 + x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 +1/2\, x/b)\, \theta\, dx$

$= [-2\, x +1/2\, x^2/b]_o^b\, Fb^2\, 1/EJ + [- x +1/4\, x^2/b]_o^b\, \theta$

$= (-2\, b +1/2\, b)\, Fb^2\, 1/EJ + (- b +1/4\, b)\, \theta = -9/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-1 - x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 +1/2\, x/b)\, \theta\, dx$

$= [- x -1/2\, x^2/b]_o^b\, Fb^2\, 1/EJ + [1/2\, x +1/4\, x^2/b]_o^b\, \theta$

$= (- b -1/2\, b)\, Fb^2\, 1/EJ + (1/2\, b +1/4\, b)\, \theta = -9/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (-1 +2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x + x^2/b -1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (- b + b -1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (- x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

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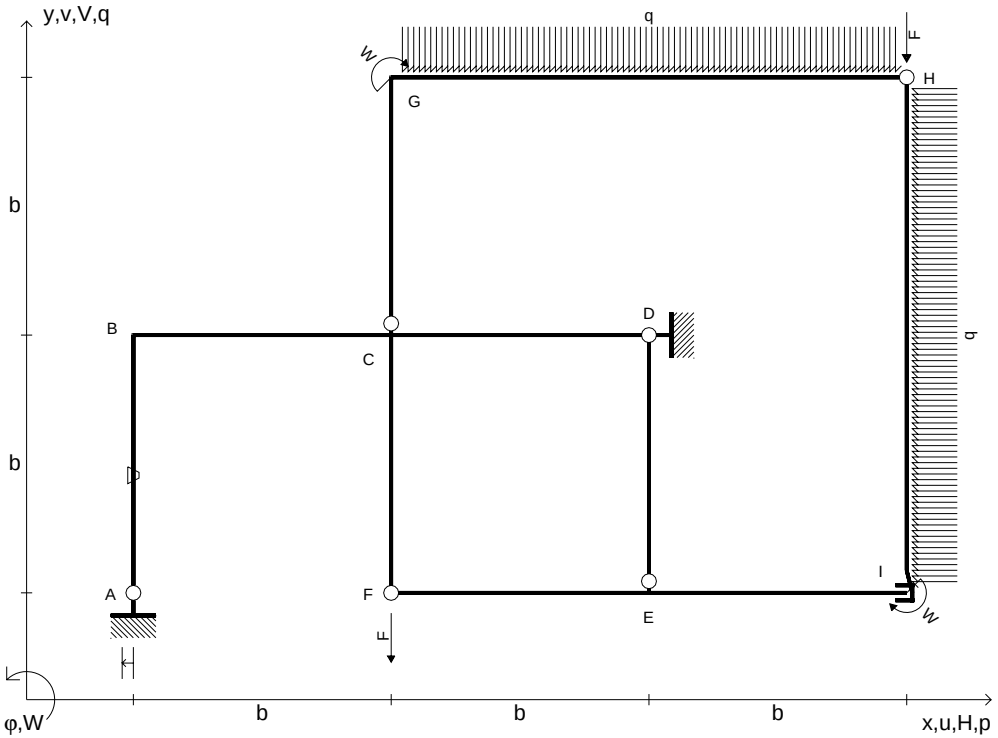


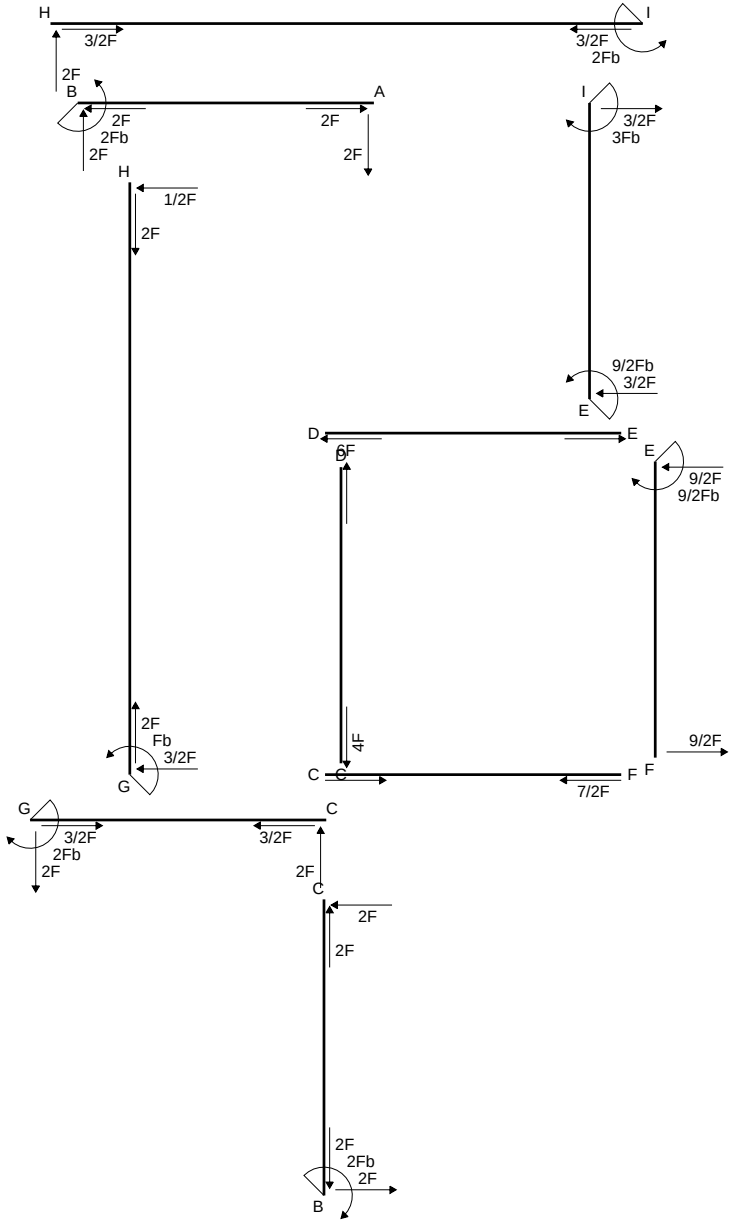
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{GH} = -q = -F/b$
$p_{HI} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

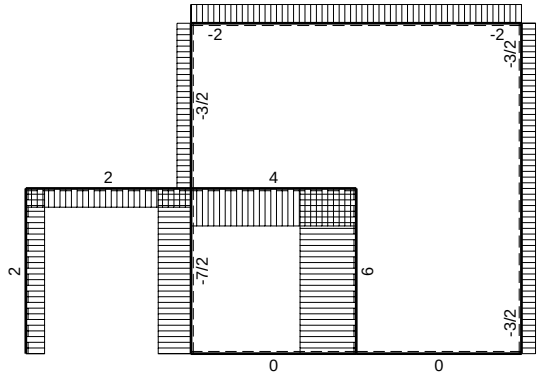
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.

 08.01.25



 08.01.25



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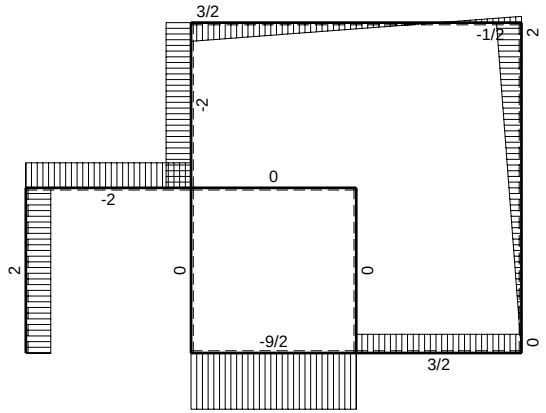

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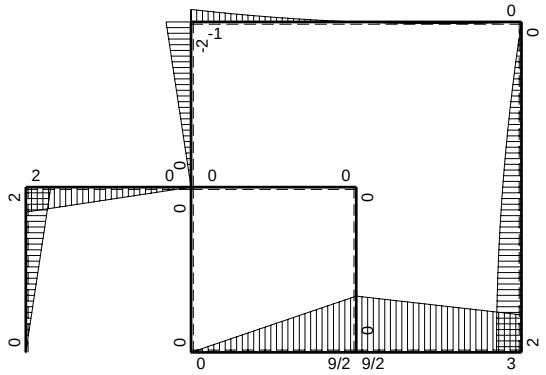
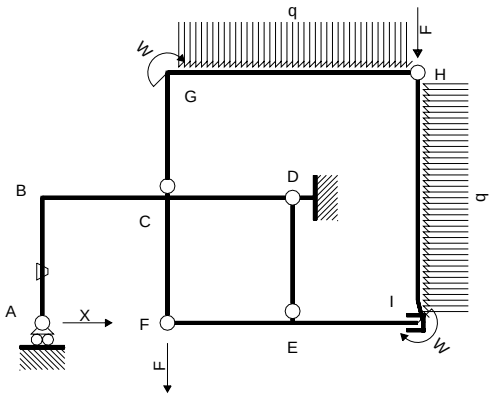
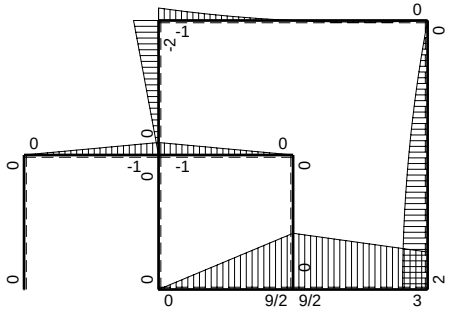


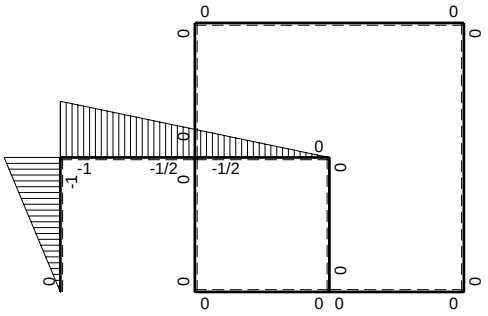
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-Fx	0	Fbx-1/2Fx^2	0	b^2-bx+1/4x^2	(1/3+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	Fb-Fx	0	1/2Fb^2-1/2Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-Fb+Fx	0	1/2Fb^2-Fbx+1/2Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/6+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	Fx	0	1/2Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/2Fb-9/2Fx	0	0	0	0	0+0	0
FE b	0	-9/2Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+3/2Fx-1/2qx^2	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx+1/2qx^2	0	0	0	0		
HI 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
IH 2b	0	-2Fb+1/2qx^2	0	0	0	0		
IE b	0	3Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-9/2Fb+3/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						2Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-2F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

ESΣ01.xxxx.203

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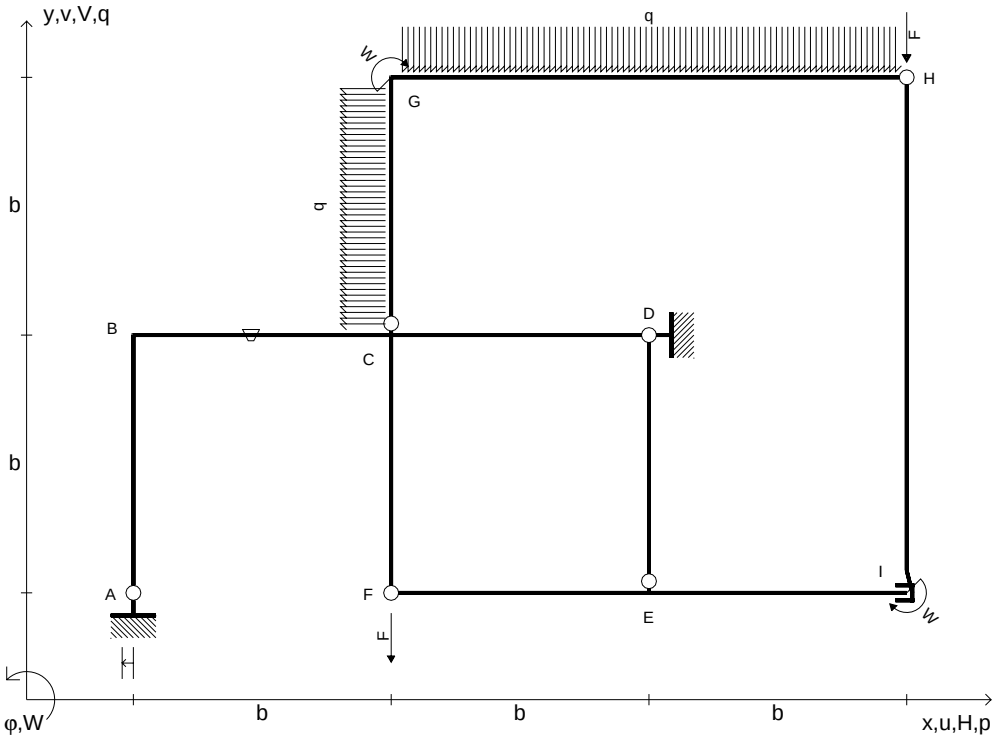


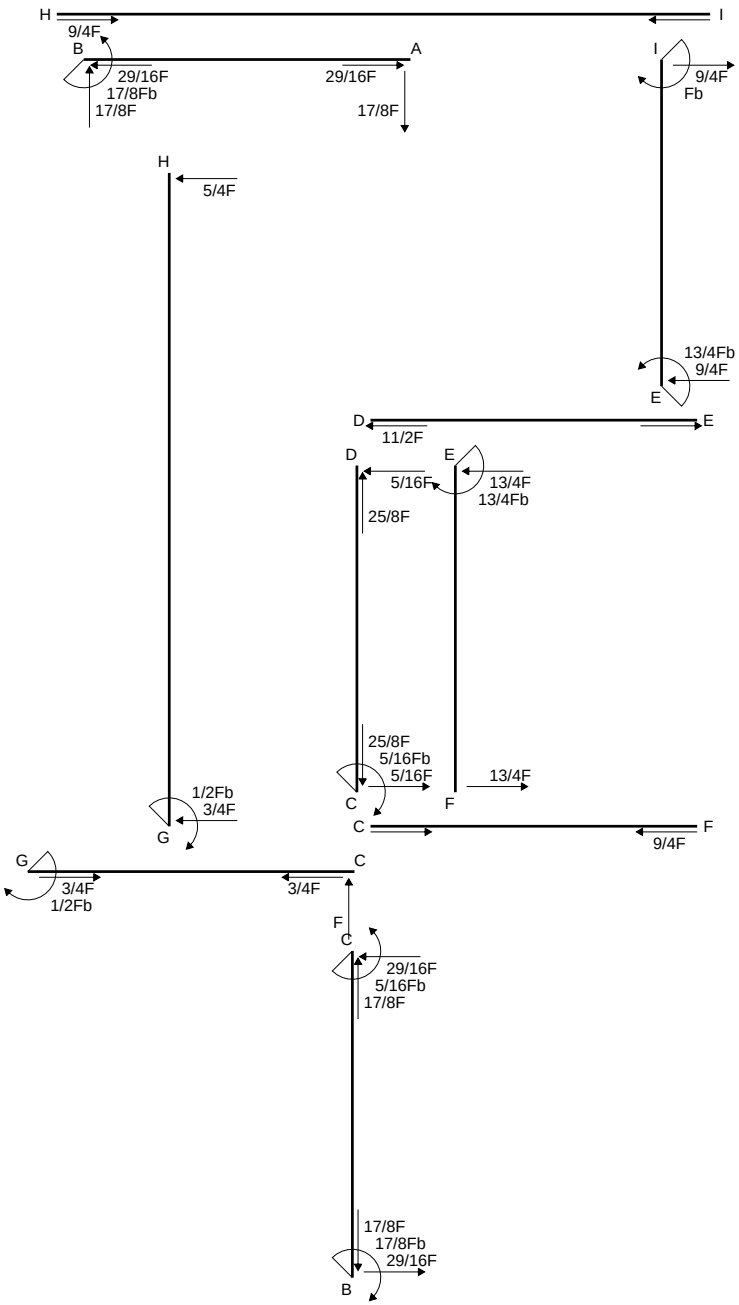
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{GH} = -q = -F/b$
$p_{CG} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{BC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo A.

 08.01.25



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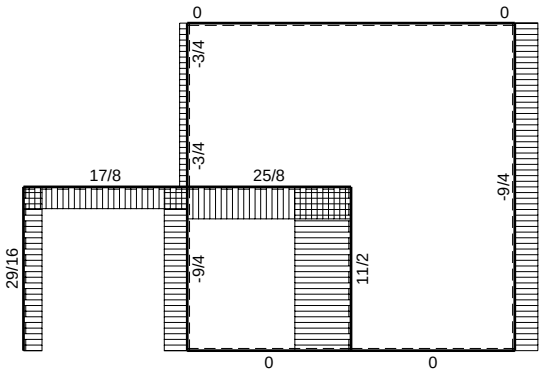

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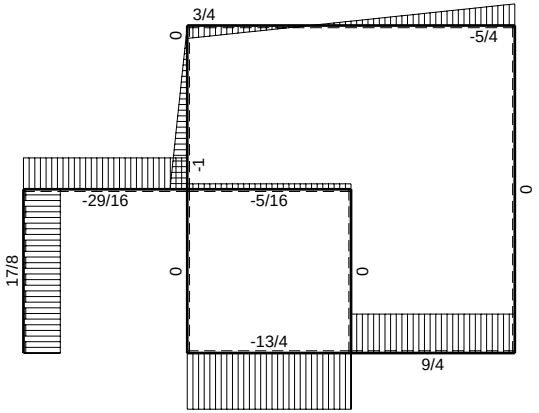

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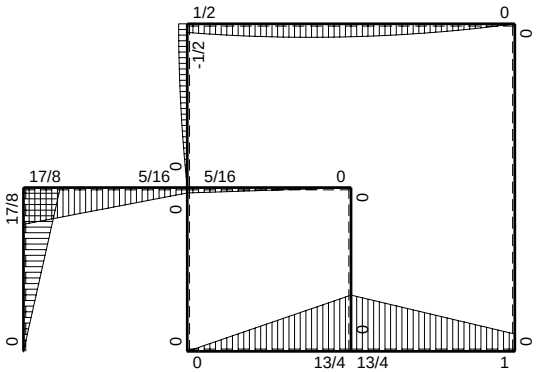

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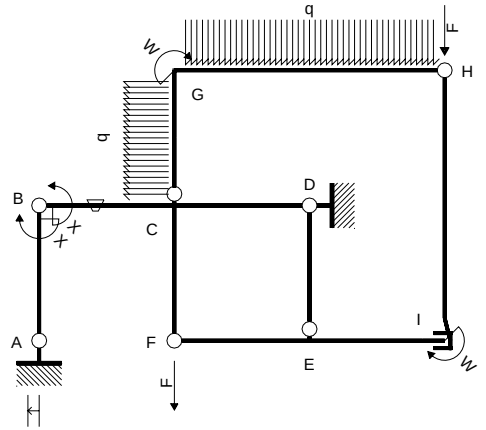
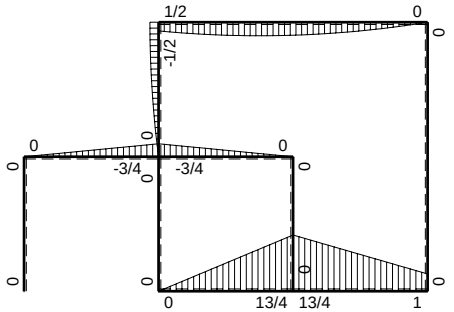
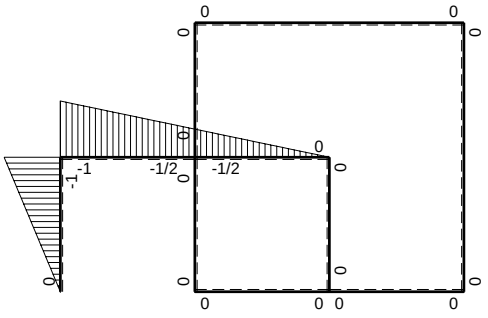


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 210484 ESΣ01.xxxx.206

Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
BA b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
BC b	-1+1/2x/b	-3/4Fx	-Fb/EJ	$3/4Fx-3/8Fx^2/b$	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(1/4+3/4)Fb^2/EJ$	7/12Xb/EJ
CB b	1/2+1/2x/b	3/4Fb-3/4Fx	Fb/EJ	$3/8Fb-3/8Fx^2/b$	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	-1/2+1/2x/b	-3/4Fb+3/4Fx	0	$3/8Fb-3/4Fx+3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(1/8+0)Fb^2/EJ$	1/12Xb/EJ
DC b	1/2x/b	3/4Fx	0	$3/8Fx^2/b$	0	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	13/4Fb-13/4Fx	0	0	0	0	0+0	0
FE b	0	-13/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	$1/2Fb+3/4Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-5/4Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+9/4Fx	0	0	0	0	0+0	0
EI b	0	-13/4Fb+9/4Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^2/EJ	
	totali						$17/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						-17/8Fb	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b\ )\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

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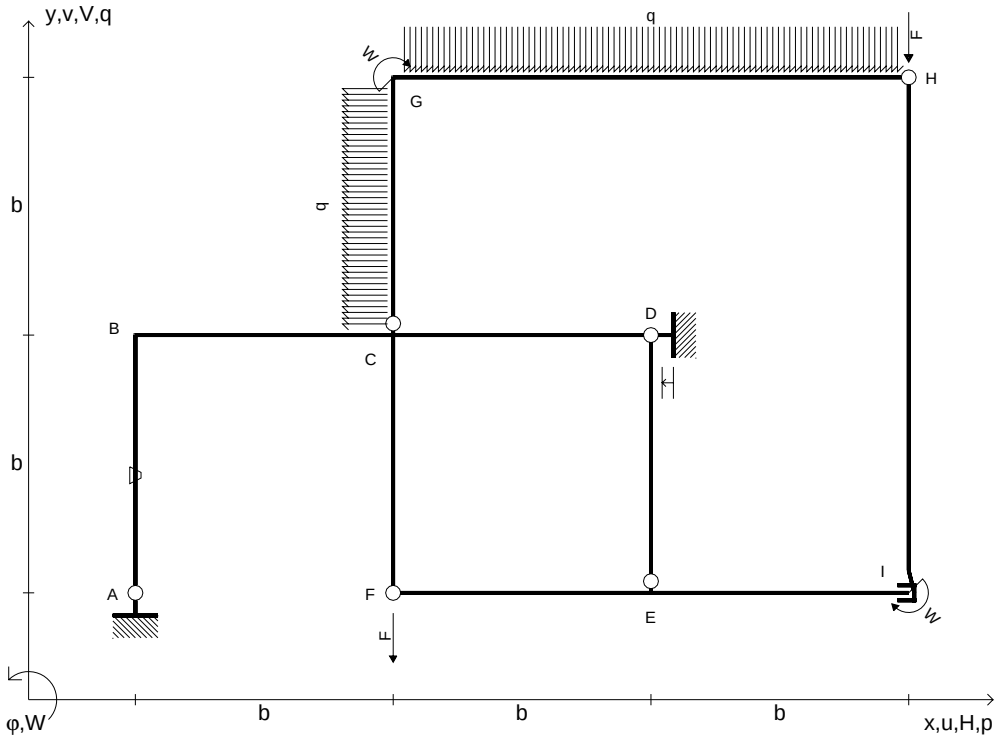
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

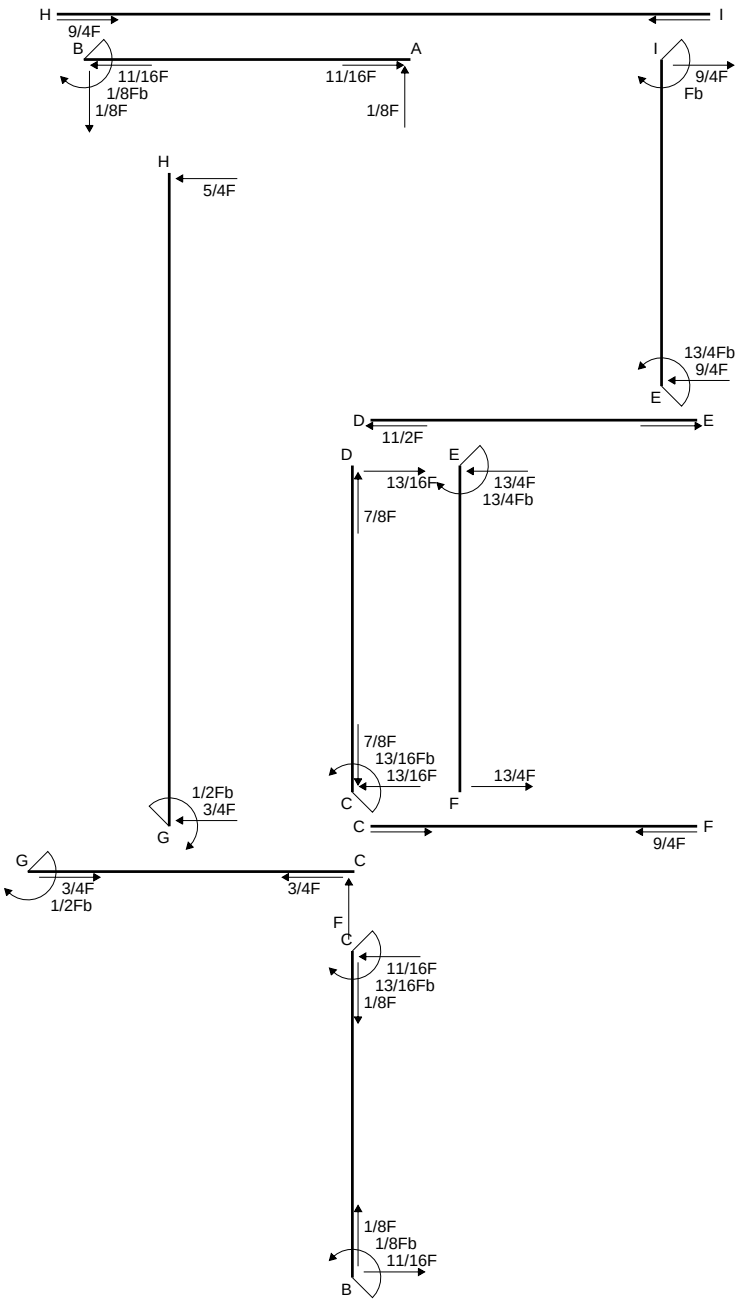


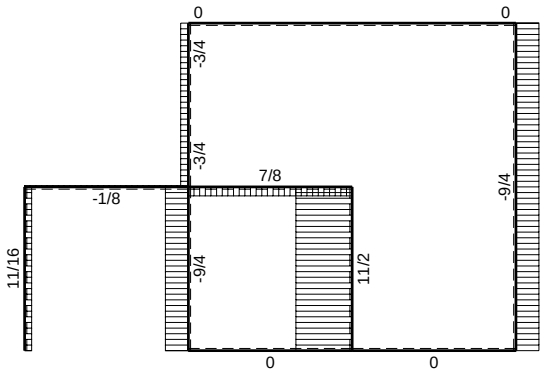
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{GH} = -q = -F/b$
$p_{CG} = -q = -F/b$

$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

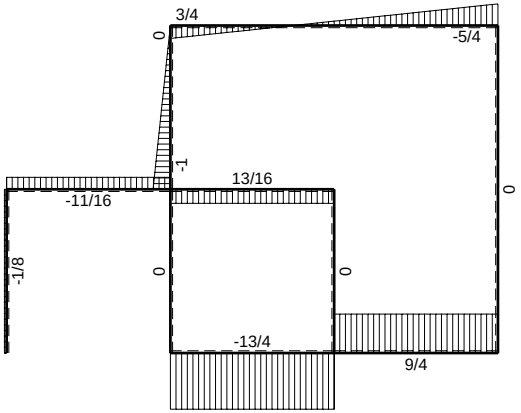
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo D.

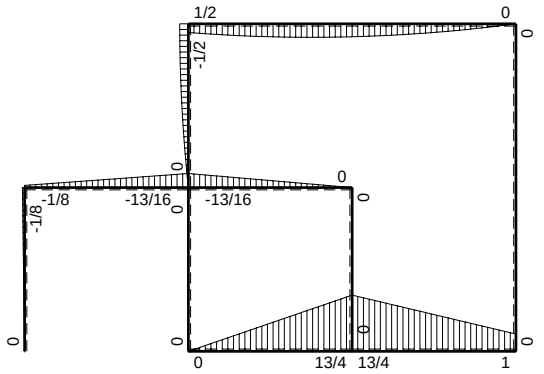




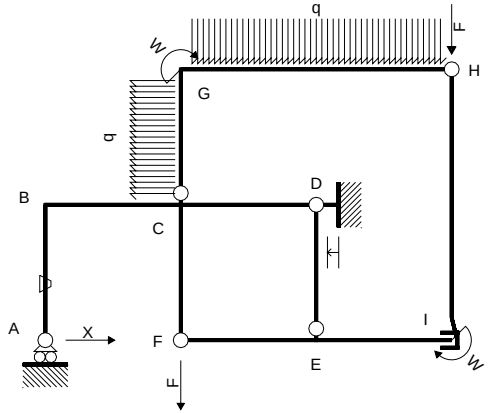
←□→ F



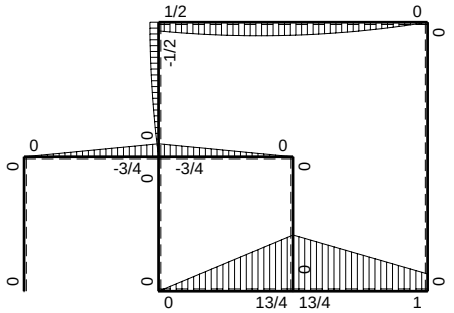
↑□↓ F



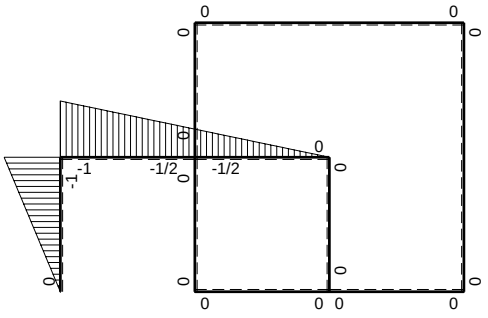
↺□↻ Fb



Schema di calcolo iperstatico



↺□↻ M_o flessione da carichi assegnati



↺□↻ M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-3/4Fx	0	3/4Fbx-3/8Fx^2	0	b^2-bx+1/4x^2	(1/4+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	3/4Fb-3/4Fx	0	3/8Fb^2-3/8Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	3/8Fb^2-3/4Fbx+3/8Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/8+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	3/4Fx	0	3/8Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	13/4Fb-13/4Fx	0	0	0	0	0+0	0
FE b	0	-13/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
GC b	0	1/2Fb-1/2qx^2	0	0	0	0		
GH 2b	0	1/2Fb+3/4Fx-1/2qx^2	0	0	0	0	0+0	0
HG 2b	0	-5/4Fx+1/2qx^2	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+9/4Fx	0	0	0	0	0+0	0
EI b	0	-13/4Fb+9/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						-Fb^3/EJ	
	totali						-1/8Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						1/8F	

Sviluppi di calcolo iperstatica

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

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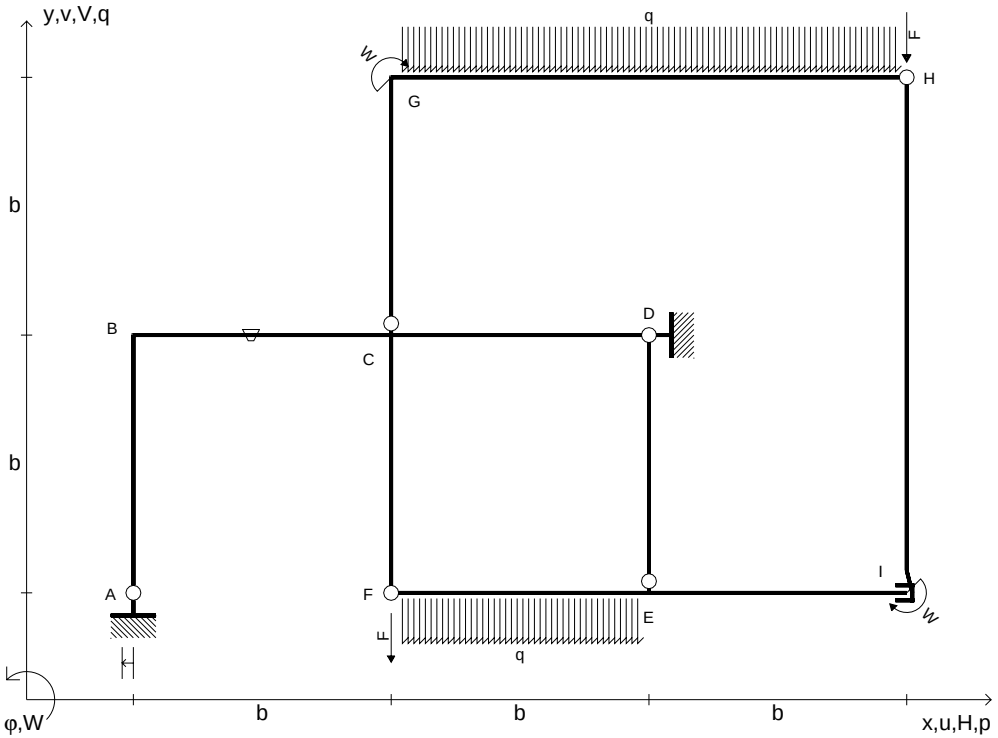
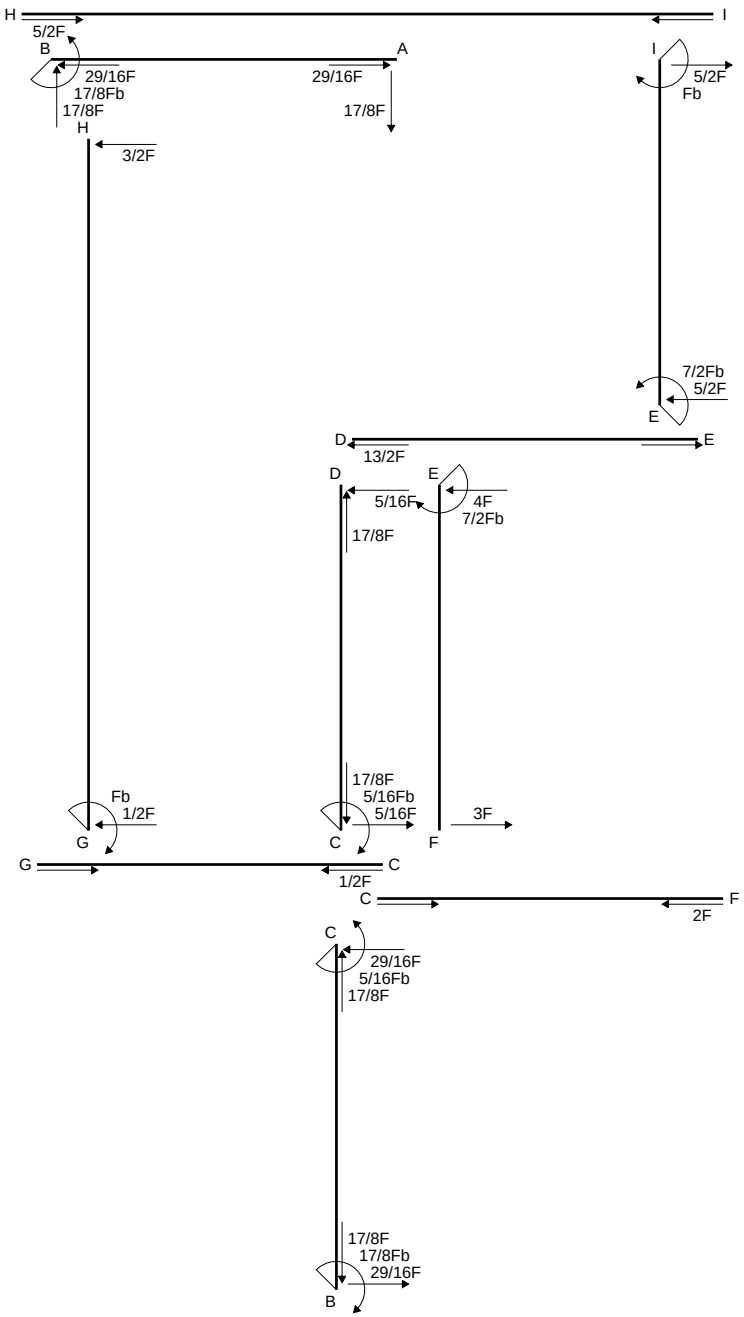


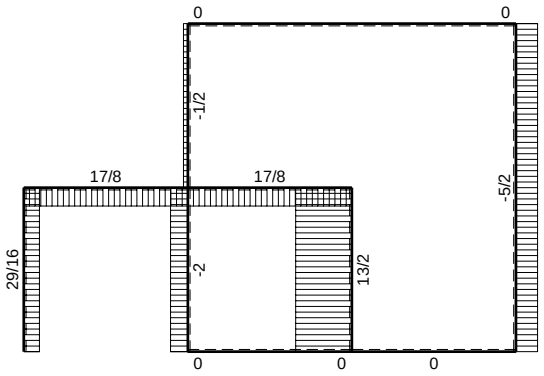
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{GH} = -q = -F/b$
$q_{EF} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

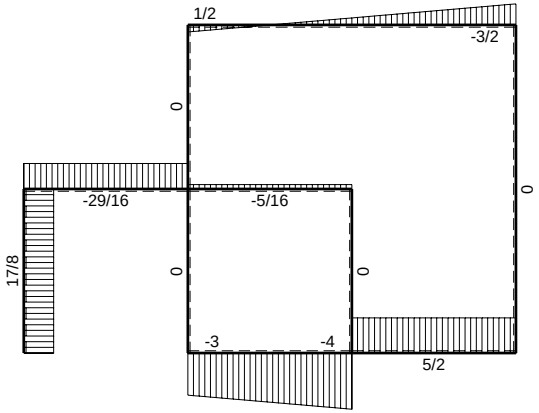
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo A.

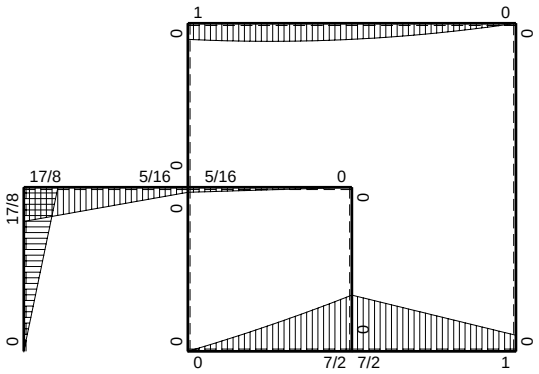




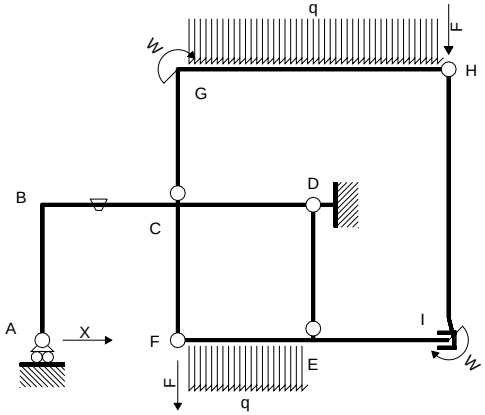
←□→ F



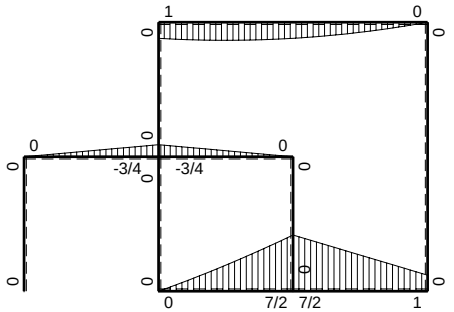
↑□↓ F



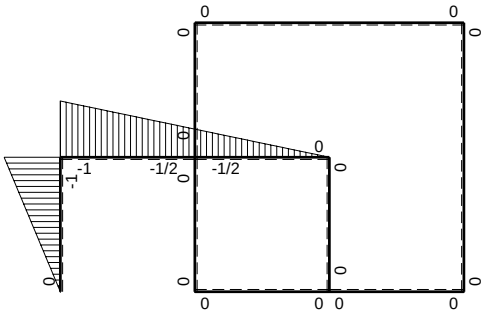
↻□ Fb



Schema di calcolo iperstatico



↻□ M_o flessione da carichi assegnati



↻□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/4Fx$	$-Fb/EJ$	$3/4Fbx-3/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/4+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	Fb/EJ	$3/8Fb^2-3/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+5/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$17/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-17/8F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984497 ESΣ01.xxxx.212
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

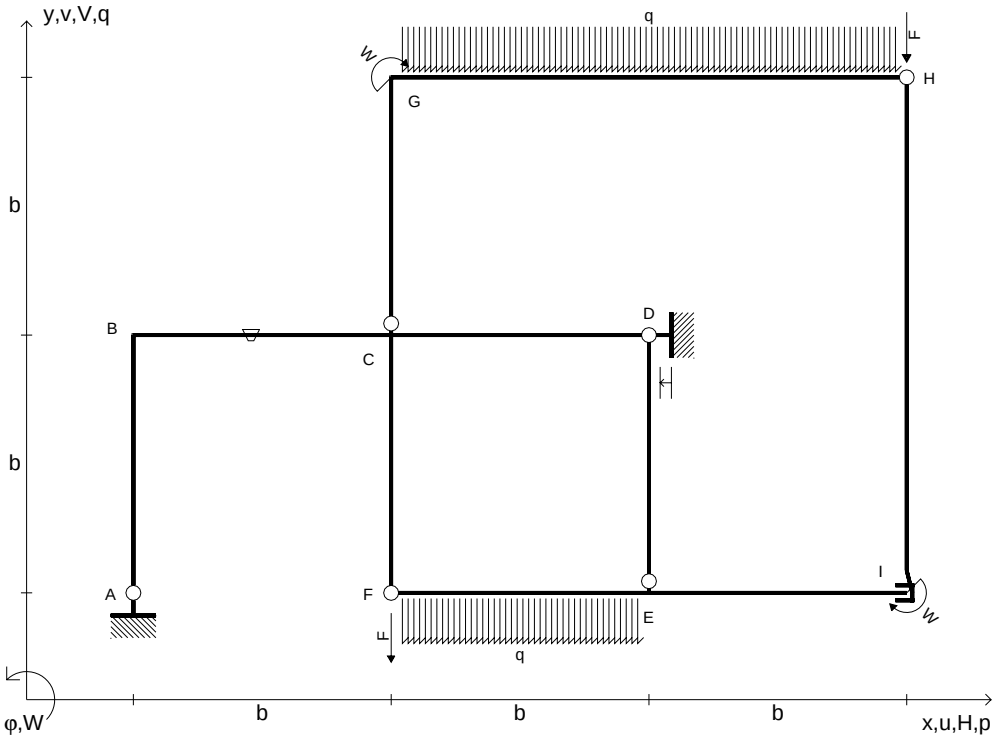
$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ01.xxxx.212

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.01.25

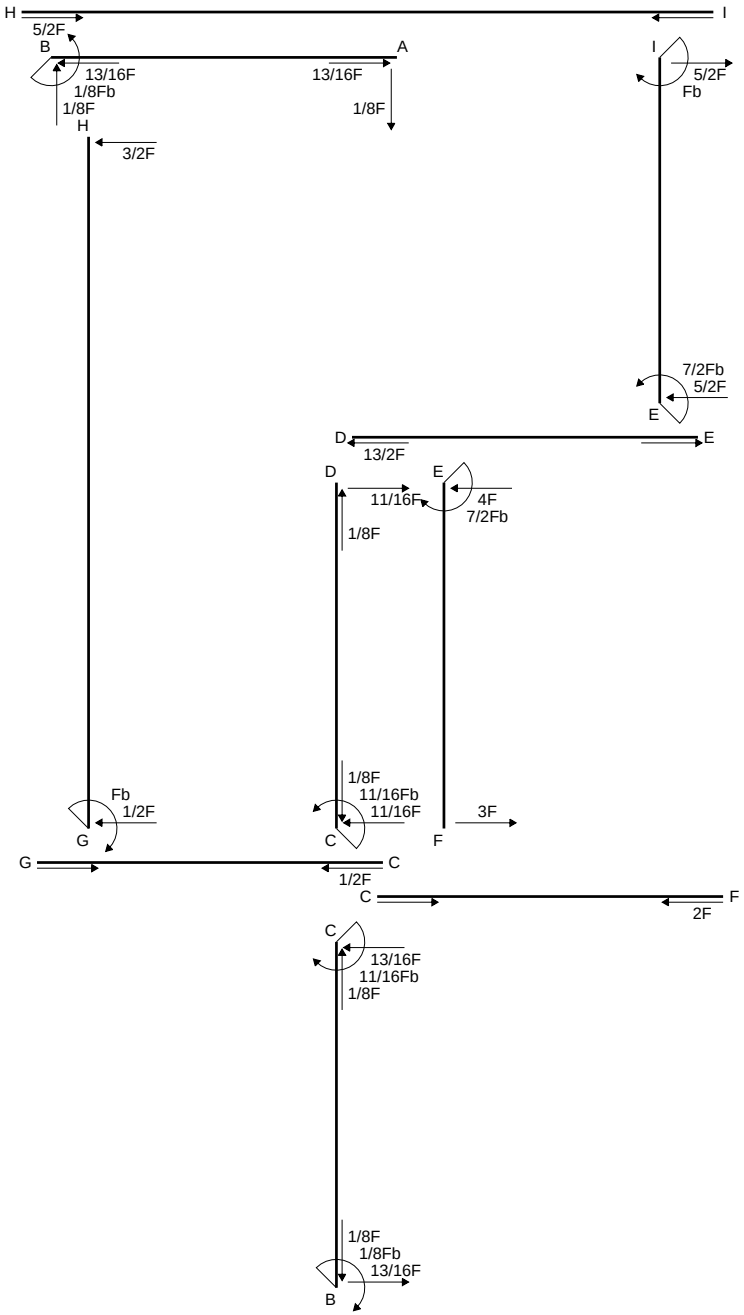


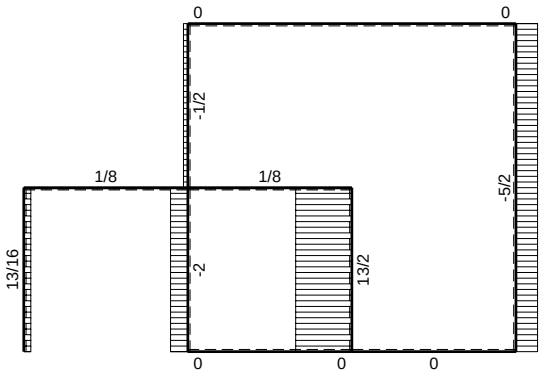
$V_H = -F$
$V_F = -F$
$W_I = -W = -Fb$
$W_G = -W = -Fb$
$q_{GH} = -q = -F/b$
$q_{EF} = -q = -F/b$

$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$
$u_D = -\delta = -b^3F/EJ$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$

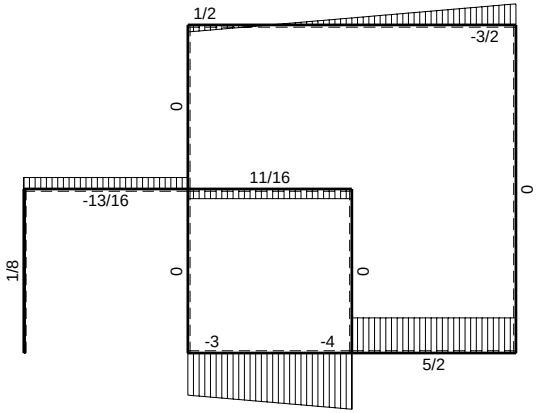
$EJ_{EF} = EJ$
$EJ_{FC} = EJ$
$EJ_{CG} = EJ$
$EJ_{GH} = EJ$
$EJ_{HI} = EJ$
$EJ_{IE} = EJ$

Reazioni iperstatiche in soluzione: $X=H_A$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Carichi di aste curve misurati in proiezione sugli assi x,y.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta BC positiva se convessa a destra con inizio B.
Spostamento orizzontale assoluto u imposto al nodo D.

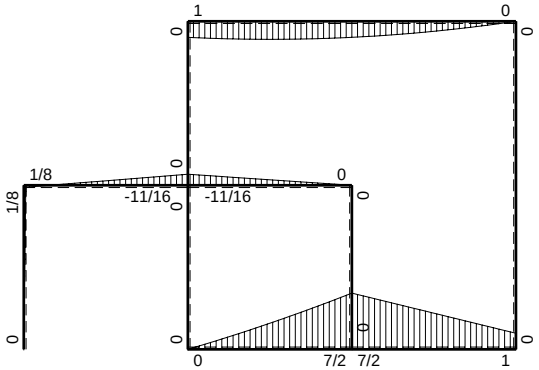




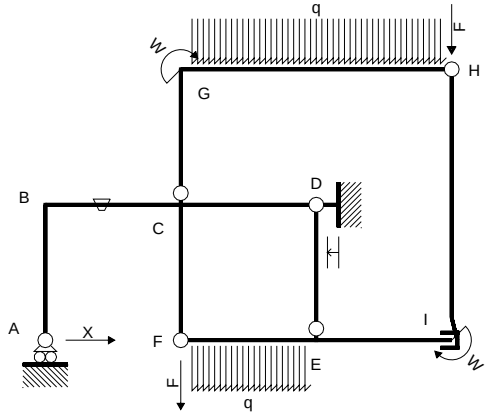
←□→ F



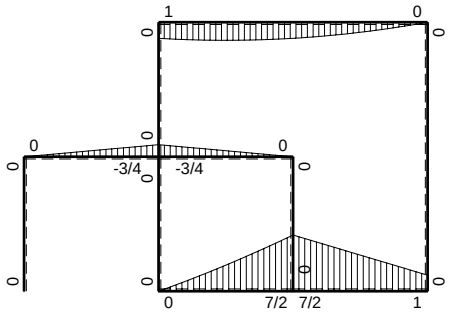
↑□↓ F



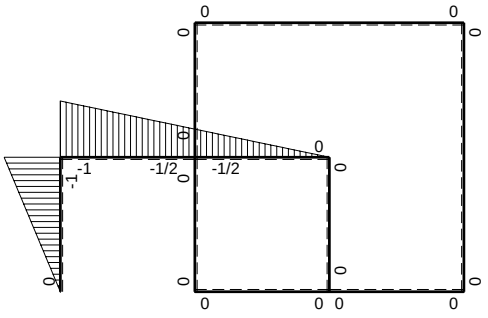
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/4Fx$	$-Fb/EJ$	$3/4Fbx-3/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/4+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	Fb/EJ	$3/8Fb^2-3/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+5/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

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