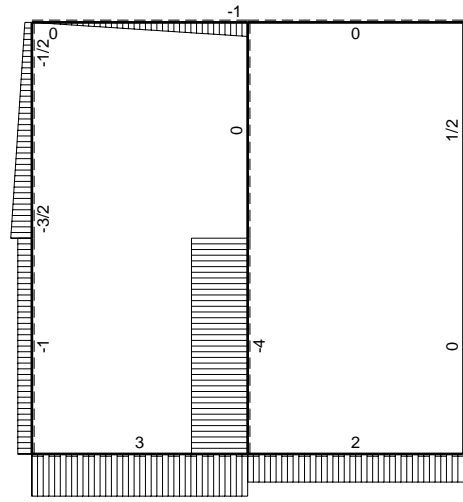
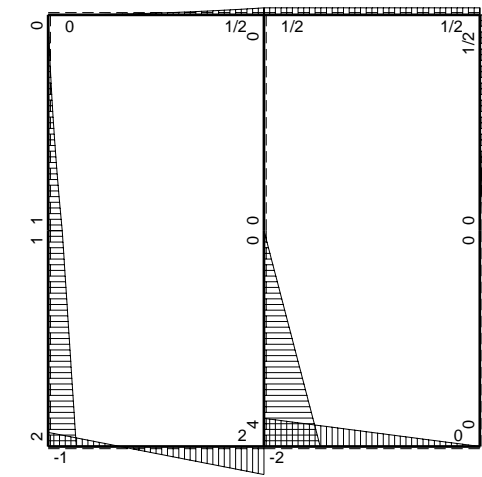


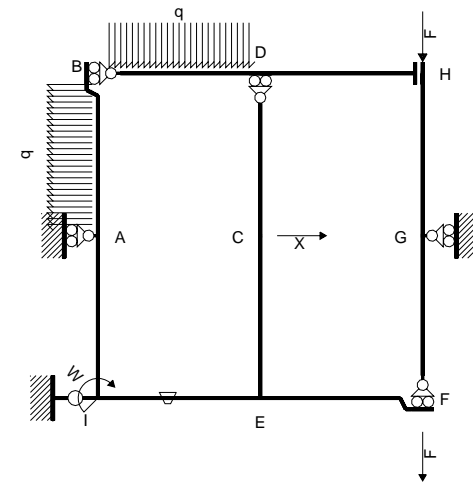
← ⊕ → F



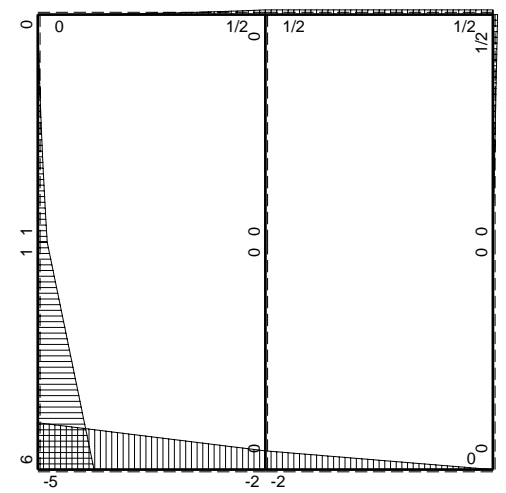
↑ ⊕ ↓ F



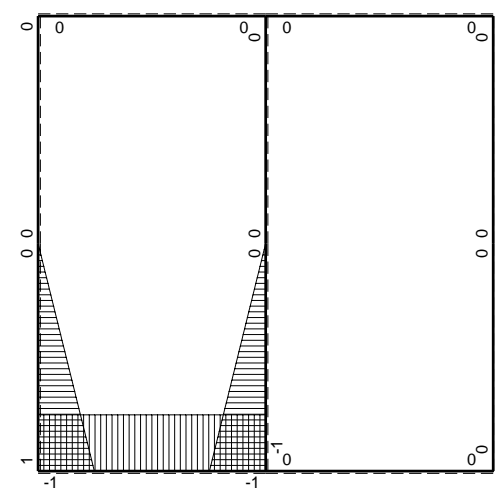
⊕ F_b



Schema di calcolo iperstatico



⊕ M₀ flessione da carichi assegnati



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_c$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int x M_x M_x/EJ dx$
AB b	0	$Fb-3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx-1/2qx^2$	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	$-2Fb+2Fx$	0	0	0	0	0+0	0
FE b	0	$2Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$1/2Fx$	0	0	0	0	0+0	0
HG b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
HD b	0	$1/2Fb$	0	0	0	0	0+0	0
DH b	0	$-1/2Fb$	0	0	0	0		
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0	0+0	0
BD b	0	$-1/2qx^2$	0	0	0	0		
IE b	-b	$-5Fb+3Fx$	$-Fb/EJ$	$5Fb^2-3Fbx$	Fb^2/EJ	b^2	$(7/2+1)Fb^3/EJ$	Xb^3/EJ
EI b	b	$2Fb+3Fx$	Fb/EJ	$2Fb^2+3Fbx$	Fb^2/EJ	b^2		
EC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CE b	x	0	0	0	0	x^2		
IA b	$b-x$	$6Fb-5Fx$	0	$6Fb^2-11Fbx+5Fx^2$	0	$b^2-2bx+x^2$	$(13/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	-x	$-Fb-5Fx$	0	$Fbx+5Fx^2$	0	x^2		
	totali						$20/3Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_c$						-4F	

Sviluppi di calcolo iperstatica

$$L_{IE}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{EI}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{EC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CE}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{IA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AI}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{IE}^{x\theta} = \int_0^b (5 - 3x/b) Fb^2 1/EJ dx + \int_0^b (1) \theta dx = [5x - 3/2 x^2/b]_0^b Fb^2 1/EJ + [x]_0^b \theta$$

$$= (5b - 3/2 b) Fb^2 1/EJ + (b) \theta = 9/2 Fb^3/EJ$$

$$L_{EI}^{x\theta} = \int_0^b (2 + 3x/b) Fb^2 1/EJ dx + \int_0^b (-1) \theta dx = [2x + 3/2 x^2/b]_0^b Fb^2 1/EJ + [-x]_0^b \theta$$

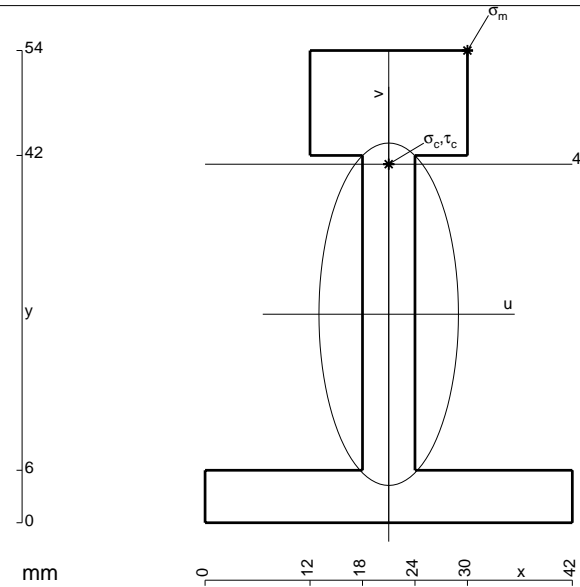
$$= (2b + 3/2 b) Fb^2 1/EJ + (-b) \theta = 9/2 Fb^3/EJ$$

$$L_{IA}^{x\theta} = \int_0^b (6 - 11x/b + 5x^2/b^2) Fb^2 1/EJ dx = [6x - 11/2 x^2/b + 5/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (6b - 11/2 b + 5/3 b) Fb^2 1/EJ = 13/6 Fb^3/EJ$$

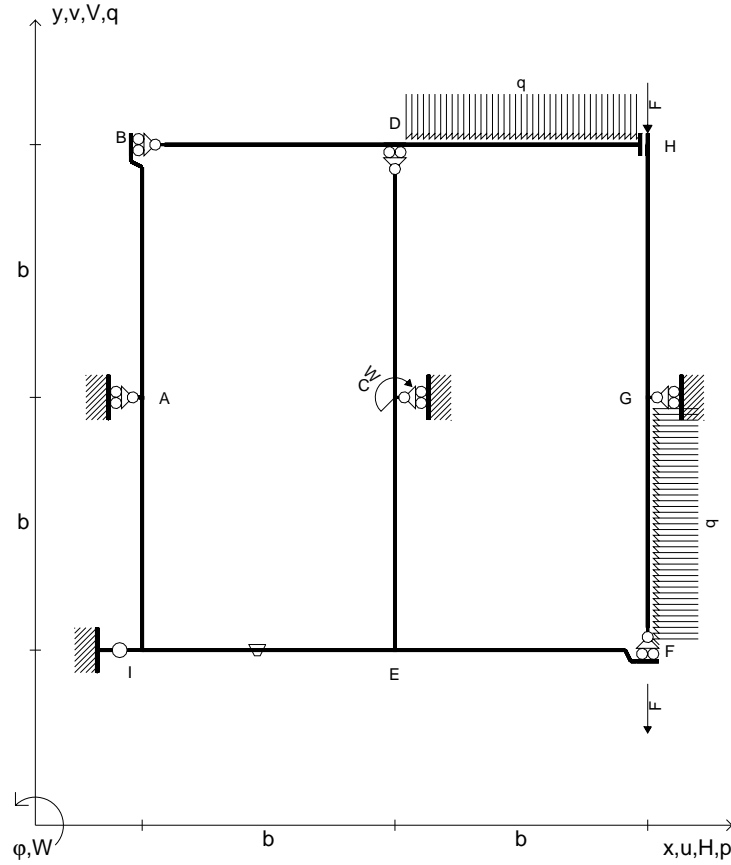
$$L_{AI}^{x\theta} = \int_0^b (x/b + 5x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b + 5/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b + 5/3 b) Fb^2 1/EJ = 13/6 Fb^3/EJ$$

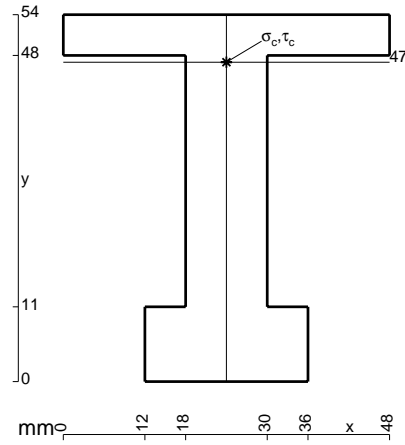


- A = 684. mm²
- J_u = 262207. mm⁴
- J_v = 43524. mm⁴
- y_g = 23.84 mm
- T_y = 3220. N
- M_x = -1738800. Nmm
- x_m = 30. mm
- y_m = 54. mm
- u_m = 9. mm
- v_m = 30.16 mm
- σ_m = -Mv/J_u = 200. N/mm²
- x_c = 21. mm
- y_c = 41. mm
- v_c = 17.16 mm
- σ_c = -Mv/J_u = 113.8 N/mm²
- τ_c = 10.9 N/mm²
- σ_q = √σ²+3τ² = 115.3 N/mm²
- S = 5324. mm³

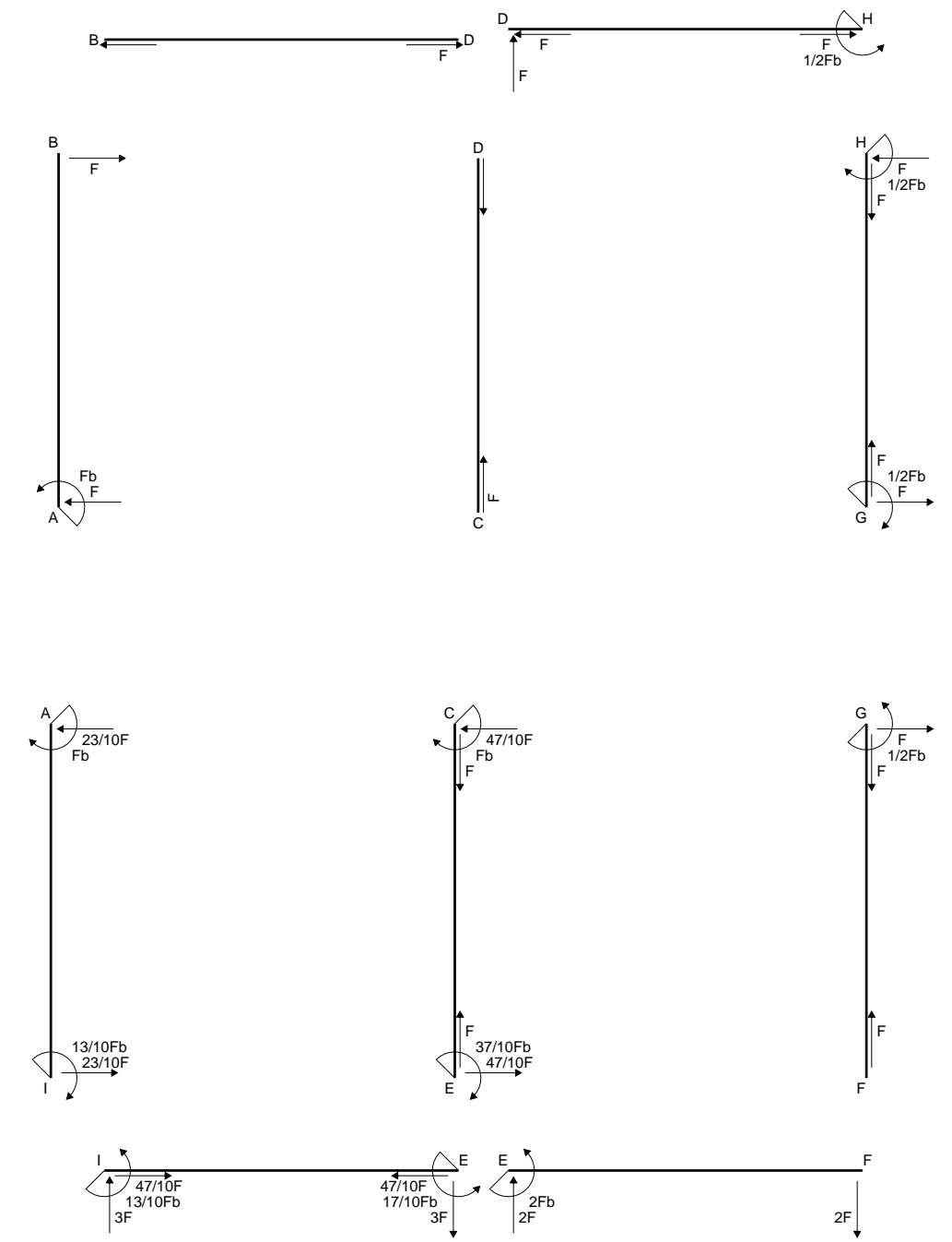
- $V_{HG} = -F$
- $V_{FE} = -F$
- $W_C = -W = -Fb$
- $p_{FG} = -q = -F/b$
- $q_{HD} = -q = -F/b$
- $\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{CD} = EJ$
- $EJ_{EF} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GH} = EJ$
- $EJ_{HD} = EJ$
- $EJ_{DB} = EJ$
- $EJ_{IE} = EJ$
- $EJ_{EC} = EJ$
- $EJ_{IA} = EJ$



Reazioni iperstatiche in soluzione: $X=H_A$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave EF ha la sezione riportata e dimensioni in mm, con:
 $b = 590 \text{ mm}$, $F = 2050 \text{ N}$
 Calcolare sulla sezione E la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da E a F
 Curvatura θ asta IE positiva se convessa a destra con inizio I.
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



08.12.24



@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

08.12.24

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	0	-Fb+Fx	0	0	0	0	0+0	0
BA b	0	Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	$1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
GH b	0	$1/2Fb-Fx$	0	0	0	0	0+0	0
HG b	0	$1/2Fb-Fx$	0	0	0	0		
HD b	0	$-1/2Fb+1/2qx^2$	0	0	0	0	0+0	0
DH b	0	$Fx-1/2qx^2$	0	0	0	0		
DB b	0	0	0	0	0	0	0+0	0
BD b	0	0	0	0	0	0		
IE b	b	2Fb+3Fx	-Fb/EJ	$2Fb^2+3Fbx$	-Fb ² /EJ	b ²	$(7/2-1)Fb^3/EJ$	Xb^3/EJ
EI b	-b	-5Fb+3Fx	Fb/EJ	$5Fb^2-3Fbx$	-Fb ² /EJ	b ²		
EC b	b-x	7Fb-8Fx	0	$7Fb^2-15Fbx+8Fx^2$	0	$b^2-2bx+x^2$	$(13/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CE b	-x	Fb-8Fx	0	$-Fbx+8Fx^2$	0	x ²		
IA b	-b+x	-2Fb+Fx	0	$2Fb^2-3Fbx+Fx^2$	0	$b^2-2bx+x^2$	$(5/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AI b	x	Fb+Fx	0	$Fbx+Fx^2$	0	x ²		
	totali						$11/2Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_A$						-33/10F	

Sviluppi di calcolo iperstatica

$$L_{IE}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{EI}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{EC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CE}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{IA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AI}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{IE}^{xo} = \int_0^b (2 + 3x/b) Fb^2 1/EJ dx + \int_0^b (-1) \theta dx = [2x + 3/2 x^2/b]_0^b Fb^2 1/EJ + [-x]_0^b \theta$$

$$= (2b + 3/2 b) Fb^2 1/EJ + (-b) \theta = 5/2 Fb^3/EJ$$

$$L_{EI}^{xo} = \int_0^b (5 - 3x/b) Fb^2 1/EJ dx + \int_0^b (1) \theta dx = [5x - 3/2 x^2/b]_0^b Fb^2 1/EJ + [x]_0^b \theta$$

$$= (5b - 3/2 b) Fb^2 1/EJ + (b) \theta = 5/2 Fb^3/EJ$$

$$L_{EC}^{xo} = \int_0^b (7 - 15x/b + 8x^2/b^2) Fb^2 1/EJ dx = [7x - 15/2 x^2/b + 8/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (7b - 15/2 b + 8/3 b) Fb^2 1/EJ = 13/6 Fb^3/EJ$$

$$L_{CE}^{xo} = \int_0^b (-x/b + 8x^2/b^2) Fb^2 1/EJ dx = [-1/2 x^2/b + 8/3 x^3/b^2]_0^b Fb^2 1/EJ$$

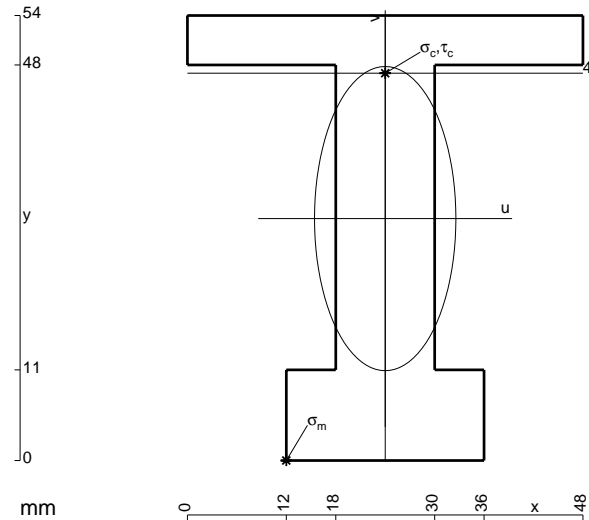
$$= (-1/2 b + 8/3 b) Fb^2 1/EJ = 13/6 Fb^3/EJ$$

$$L_{IA}^{xo} = \int_0^b (2 - 3x/b + x^2/b^2) Fb^2 1/EJ dx = [2x - 3/2 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (2b - 3/2 b + 1/3 b) Fb^2 1/EJ = 5/6 Fb^3/EJ$$

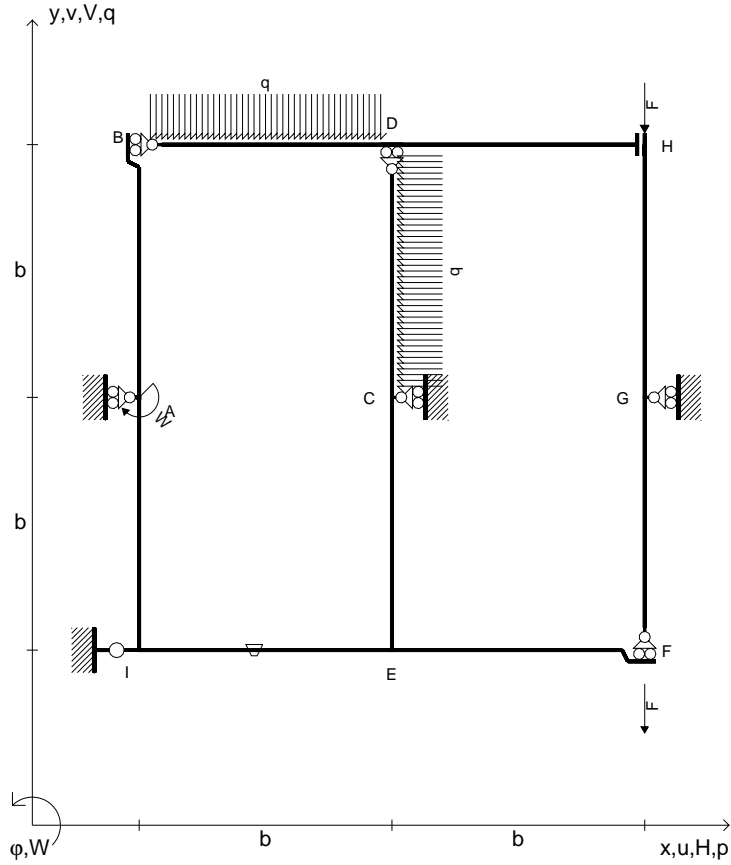
$$L_{AI}^{xo} = \int_0^b (x/b + x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b + 1/3 b) Fb^2 1/EJ = 5/6 Fb^3/EJ$$

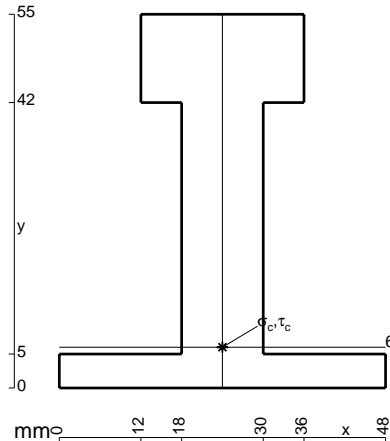


- A = 996. mm²
- J_u = 339350. mm⁴
- J_v = 73296. mm⁴
- y_g = 29.36 mm
- T_y = 4100. N
- M_x = -2419000. Nmm
- x_m = 12. mm
- u_m = -12. mm
- v_m = -29.36 mm
- σ_m = -Mv/J_u = -209.3 N/mm²
- x_c = 24. mm
- y_c = 47. mm
- v_c = 17.64 mm
- σ_c = -Mv/J_u = 125.8 N/mm²
- τ_c = 6.495 N/mm²
- σ_o = √σ²+3τ² = 126.3 N/mm²
- S = 6451. mm³

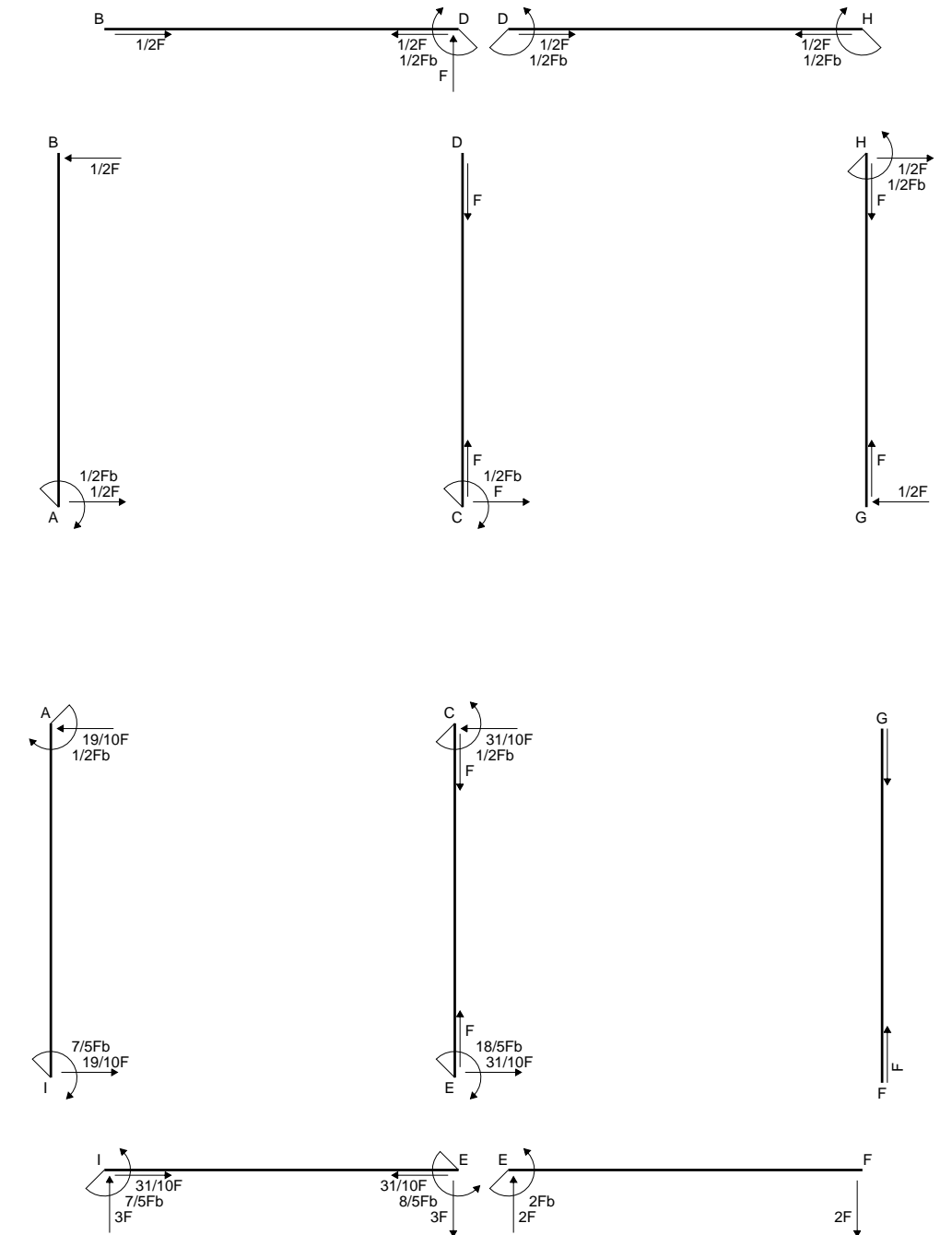
- $V_{HG} = -F$
- $V_{FE} = -F$
- $W_A = -W = -Fb$
- $p_{CD} = -q = -F/b$
- $q_{DB} = -q = -F/b$
- $\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{CD} = EJ$
- $EJ_{EF} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GH} = EJ$
- $EJ_{HD} = EJ$
- $EJ_{DB} = EJ$
- $EJ_{IE} = EJ$
- $EJ_{EC} = EJ$
- $EJ_{IA} = EJ$



Reazioni iperstatiche in soluzione: $X=H_c$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave EF ha la sezione riportata e dimensioni in mm, con:
 $b = 550 \text{ mm}$, $F = 2200 \text{ N}$
 Calcolare sulla sezione E la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da E a F
 Curvatura θ asta IE positiva se convessa a destra con inizio I.
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

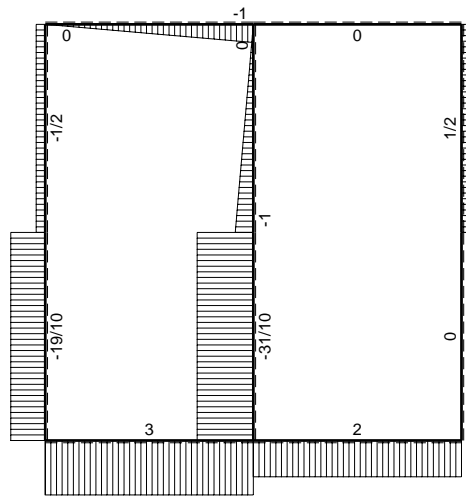
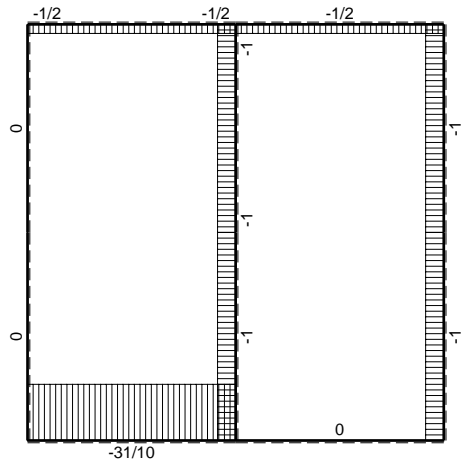


08.12.24



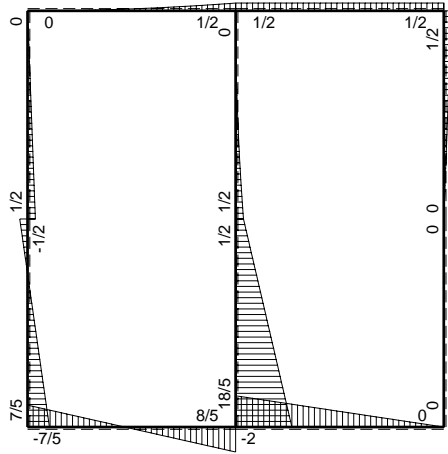
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

08.12.24

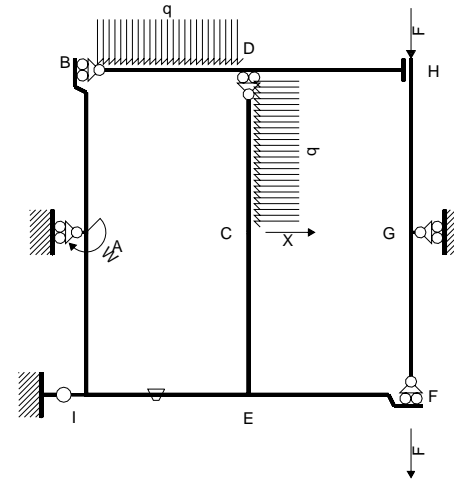


← (+) → F

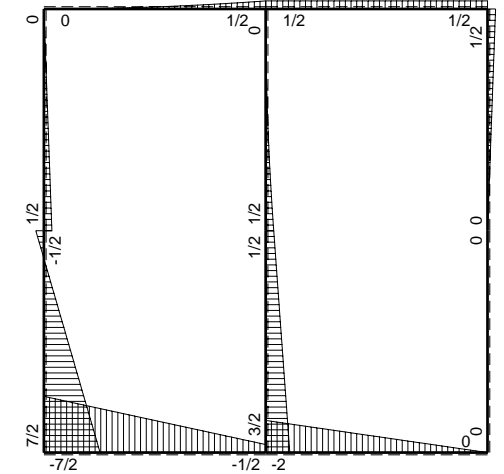
↑ (+) ↓ F



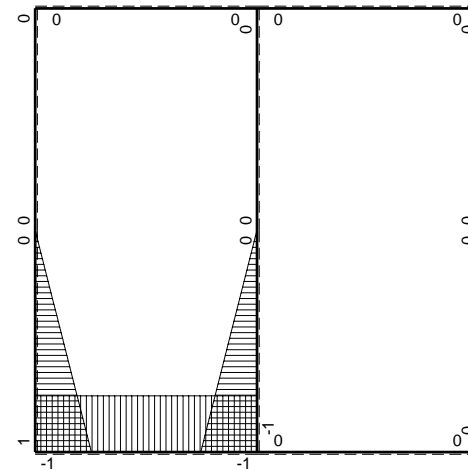
⊙ (+) ⊙ F_b



Schema di calcolo iperstatico



⊙ (+) ⊙ M_o flessione da carichi assegnati



⊙ (+) ⊙ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_C

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫XM _x M _x /EJdx
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	1/2Fb-Fx+1/2qx ²	0	0	0	0	0+0	0
DC b	0	-1/2qx ²	0	0	0	0		
EF b	0	-2Fb+2Fx	0	0	0	0	0+0	0
FE b	0	2Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	1/2Fx	0	0	0	0	0+0	0
HG b	0	-1/2Fb+1/2Fx	0	0	0	0		
HD b	0	1/2Fb	0	0	0	0	0+0	0
DH b	0	-1/2Fb	0	0	0	0		
DB b	0	1/2Fb-Fx+1/2qx ²	0	0	0	0	0+0	0
BD b	0	-1/2qx ²	0	0	0	0		
IE b	-b	-7/2Fb+3Fx	-Fb/EJ	7/2Fb ² -3Fbx	Fb ² /EJ	b ²	(2+1)Fb ³ /EJ	Xb ³ /EJ
EI b	b	1/2Fb+3Fx	Fb/EJ	1/2Fb ² +3Fbx	Fb ² /EJ	b ²		
EC b	-b+x	3/2Fb-Fx	0	-3/2Fb ² +5/2Fbx-Fx ²	0	b ² -2bx+x ²	(-7/12+0)Fb ³ /EJ	1/3Xb ³ /EJ
CE b	x	-1/2Fb-Fx	0	-1/2Fbx-Fx ²	0	x ²		
IA b	b-x	7/2Fb-4Fx	0	7/2Fb ² -15/2Fbx+4Fx ²	0	b ² -2bx+x ²	(13/12+0)Fb ³ /EJ	1/3Xb ³ /EJ
AI b	-x	1/2Fb-4Fx	0	-1/2Fbx+4Fx ²	0	x ²		
	totali						7/2Fb ³ /EJ	5/3Xb ³ /EJ
	iperstatica X=H _C						-21/10F	

Sviluppi di calcolo iperstatica

$$L_{IE}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{EI}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{EC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CE}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{IA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AI}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{IE}^{xo} = \int_0^b (7/2 - 3x/b) Fb^2 1/EJ dx + \int_0^b (1) \theta dx = [7/2 x - 3/2 x^2/b]_0^b Fb^2 1/EJ + [x]_0^b \theta$$

$$= (7/2 b - 3/2 b) Fb^2 1/EJ + (b) \theta = 3 Fb^3/EJ$$

$$L_{EI}^{xo} = \int_0^b (1/2 + 3x/b) Fb^2 1/EJ dx + \int_0^b (-1) \theta dx = [1/2 x + 3/2 x^2/b]_0^b Fb^2 1/EJ + [-x]_0^b \theta$$

$$= (1/2 b + 3/2 b) Fb^2 1/EJ + (-b) \theta = 3 Fb^3/EJ$$

$$L_{EC}^{xo} = \int_0^b (-3/2 + 5/2 x/b - x^2/b^2) Fb^2 1/EJ dx = [-3/2 x + 5/4 x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-3/2 b + 5/4 b - 1/3 b) Fb^2 1/EJ = -7/12 Fb^3/EJ$$

$$L_{CE}^{xo} = \int_0^b (-1/2 x/b - x^2/b^2) Fb^2 1/EJ dx = [-1/4 x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

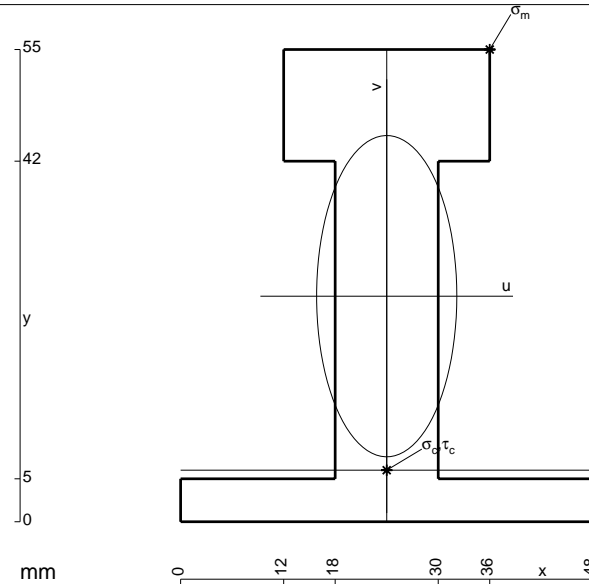
$$= (-1/4 b - 1/3 b) Fb^2 1/EJ = -7/12 Fb^3/EJ$$

$$L_{IA}^{xo} = \int_0^b (7/2 - 15/2 x/b + 4 x^2/b^2) Fb^2 1/EJ dx = [7/2 x - 15/4 x^2/b + 4/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (7/2 b - 15/4 b + 4/3 b) Fb^2 1/EJ = 13/12 Fb^3/EJ$$

$$L_{AI}^{xo} = \int_0^b (-1/2 x/b + 4 x^2/b^2) Fb^2 1/EJ dx = [-1/4 x^2/b + 4/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/4 b + 4/3 b) Fb^2 1/EJ = 13/12 Fb^3/EJ$$



- A = 996. mm²
- J_u = 348739. mm⁴
- J_v = 66384. mm⁴
- y_g = 26.27 mm
- T_y = 4400. N
- M_x = -2420000. Nmm
- x_m = 36. mm
- y_m = 55. mm
- u_m = 12. mm
- v_m = 28.73 mm
- σ_m = -Mv/J_u = 199.4 N/mm²
- x_c = 24. mm
- y_c = 6. mm
- v_c = -20.27 mm
- σ_c = -Mv/J_u = -140.7 N/mm²
- τ_c = 6.26 N/mm²
- σ_q = √(σ² + 3τ²) = 141.1 N/mm²
- S = 5954. mm³