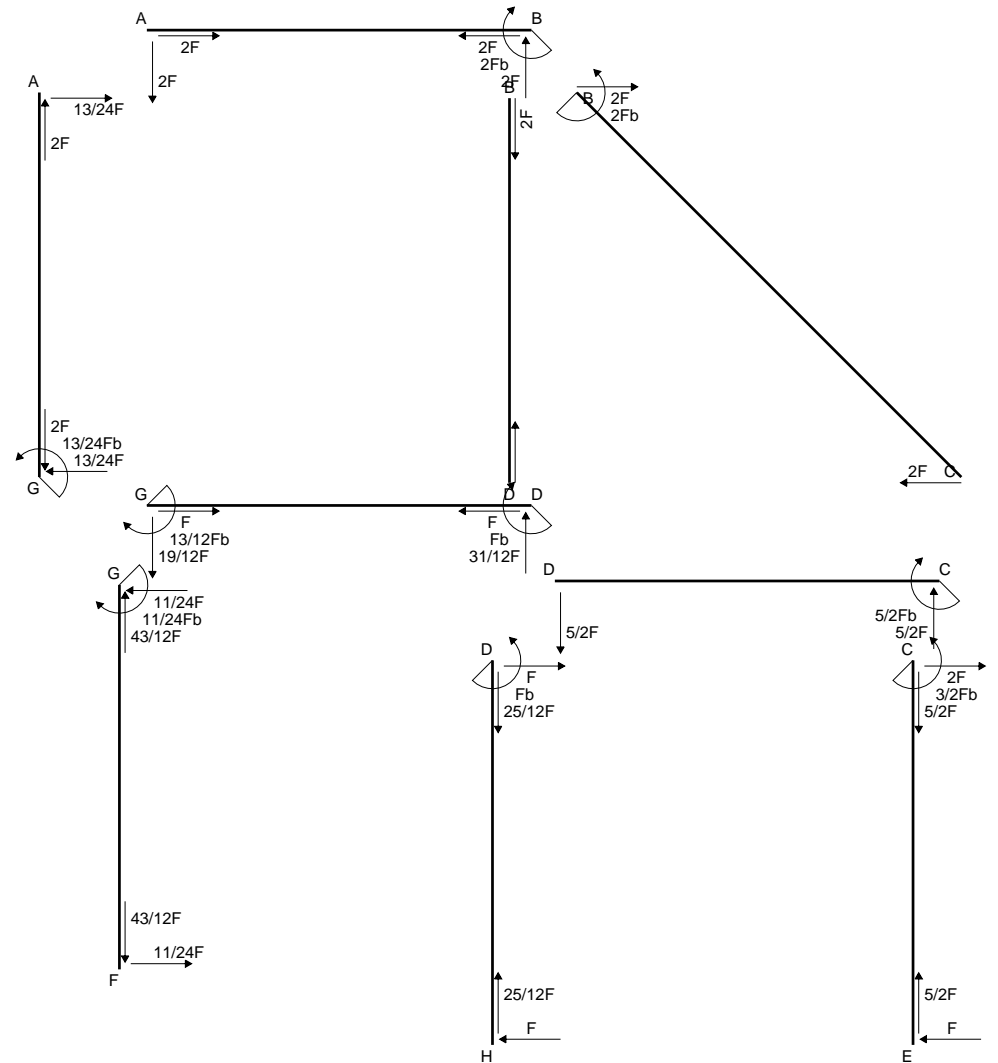
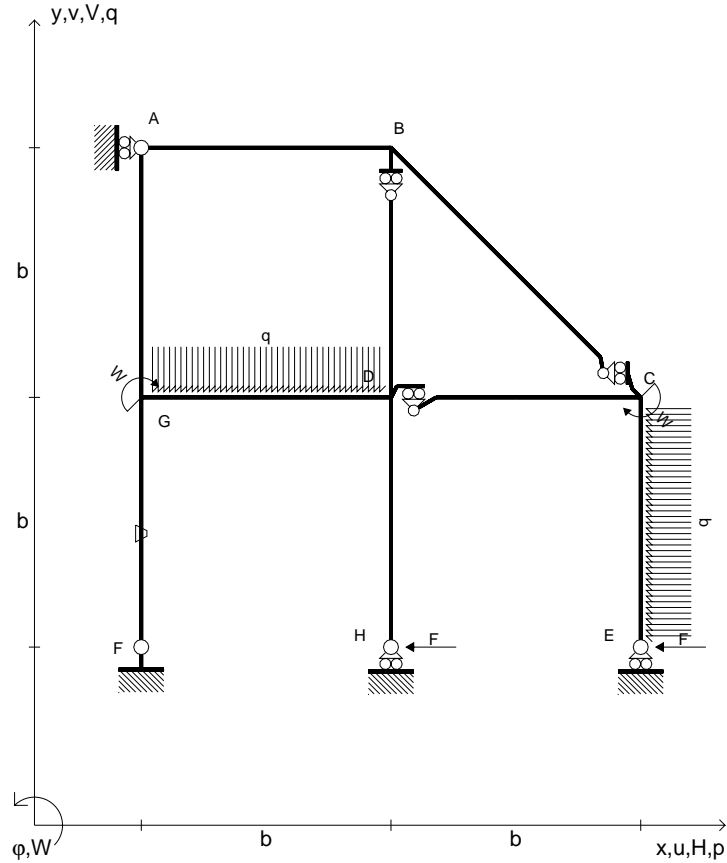
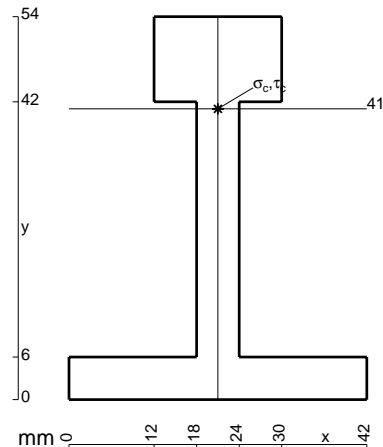
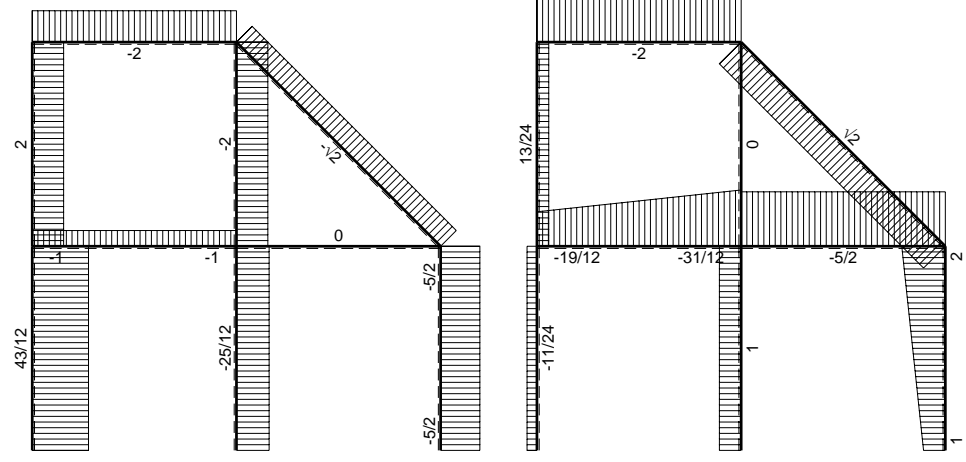


- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_G = -W = -Fb$
- $p_{CE} = -q = -F/b$
- $q_{GD} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



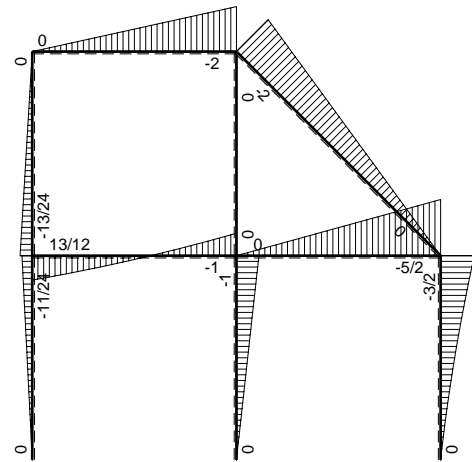
Reazioni iperstatiche in soluzione: $X=V_H$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave DC ha la sezione riportata e dimensioni in mm, con:
 $b = 510 \text{ mm}$, $F = 1360 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da D a C
 Curvatura θ asta FG positiva se convessa a destra con inizio F.
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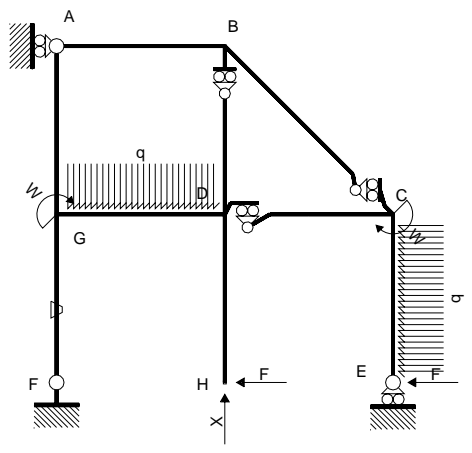


← ⊕ → F

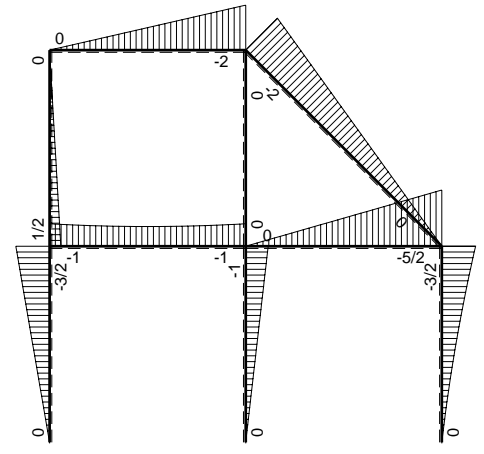
↑ ⊕ ↓ F



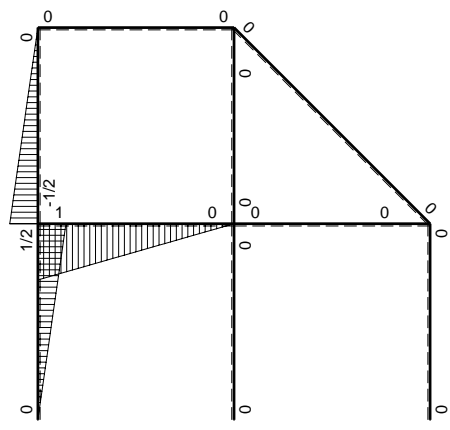
⊕ ↺ F_b



Schema di calcolo iperstatico



⊕ ↺ M_o flessione da carichi assegnati



⊕ ↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0	0	0
BC $\sqrt{2}b$	0	-2Fb+ $\sqrt{2}Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0	0	0
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0	0	0
CE b	0	-3/2Fb+2Fx-1/2qx ²	0	0	0	0	0+0	0
EC b	0	Fx+1/2qx ²	0	0	0	0	0	0
FG b	1/2x	-3/2Fx	-Fb/EJ	-3/4Fx ²	-1/2Fxb/EJ	1/4x ²	(-1/4-1/4)Fb ³ /EJ	1/12Xb ³ /EJ
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	-3/4Fb ² +3/2Fbx-3/4Fx ²	-1/2Fb ² /EJ+1/2Fxb/EJ	1/4b ² -1/2bx+1/4x ²		
GD b	b-x	-Fb+1/2Fx-1/2qx ²	0	-Fb ² +3/2Fbx-Fx ² +1/2qx ³	0	b ² -2bx+x ²	(-11/24+0)Fb ³ /EJ	1/3Xb ³ /EJ
DG b	-x	Fb-1/2Fx+1/2qx ²	0	-Fbx+1/2Fx ² -1/2qx ³	0	x ²		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0	0	0
GA b	-1/2b+1/2x	1/2Fb-1/2Fx	0	-1/4Fb ² +1/2Fbx-1/4Fx ²	0	1/4b ² -1/2bx+1/4x ²	(-1/12+0)Fb ³ /EJ	1/12Xb ³ /EJ
AG b	1/2x	-1/2Fx	0	-1/4Fx ²	0	1/4x ²		
	totali						-25/24Fb ³ /EJ	1/2Xb ³ /EJ
	iperstatica $X=V_H$						25/12F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [-1/4 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (-1/4 b) Fb^2 1/EJ + (-1/4 b) \theta = -1/2 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-3/4 + 3/2 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [-3/4 x + 3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (-3/4 b + 3/4 b - 1/4 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = -1/2 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-1 + 3/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [-x + 3/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-b + 3/4 b - 1/3 b + 1/8 b) Fb^2 1/EJ = -11/24 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-x/b + 1/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [-1/2 x^2/b + 1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

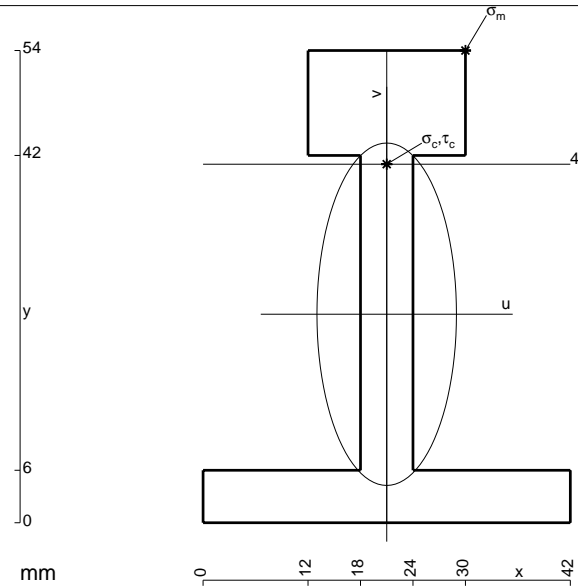
$$= (-1/2 b + 1/6 b - 1/8 b) Fb^2 1/EJ = -11/24 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (-1/4 + 1/2 x/b - 1/4 x^2/b^2) Fb^2 1/EJ dx = [-1/4 x + 1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/4 b + 1/4 b - 1/12 b) Fb^2 1/EJ = -1/12 Fb^3/EJ$$

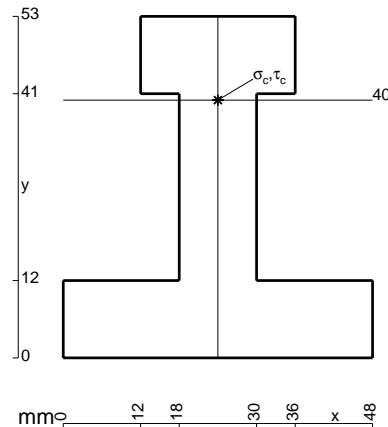
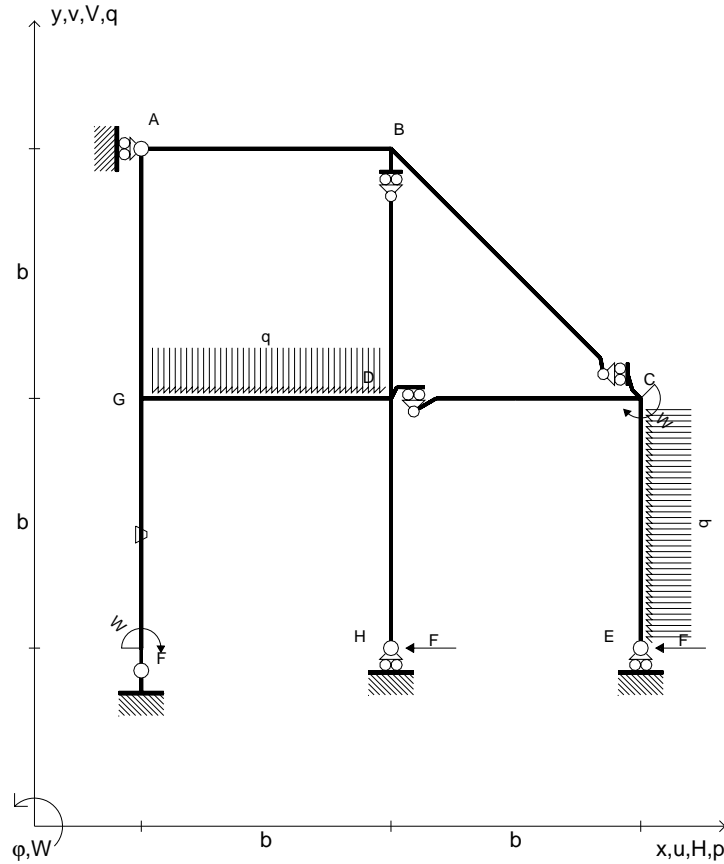
$$L_{AG}^{xo} = \int_0^b (-1/4 x^2/b^2) Fb^2 1/EJ dx = [-1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/12 b) Fb^2 1/EJ = -1/12 Fb^3/EJ$$

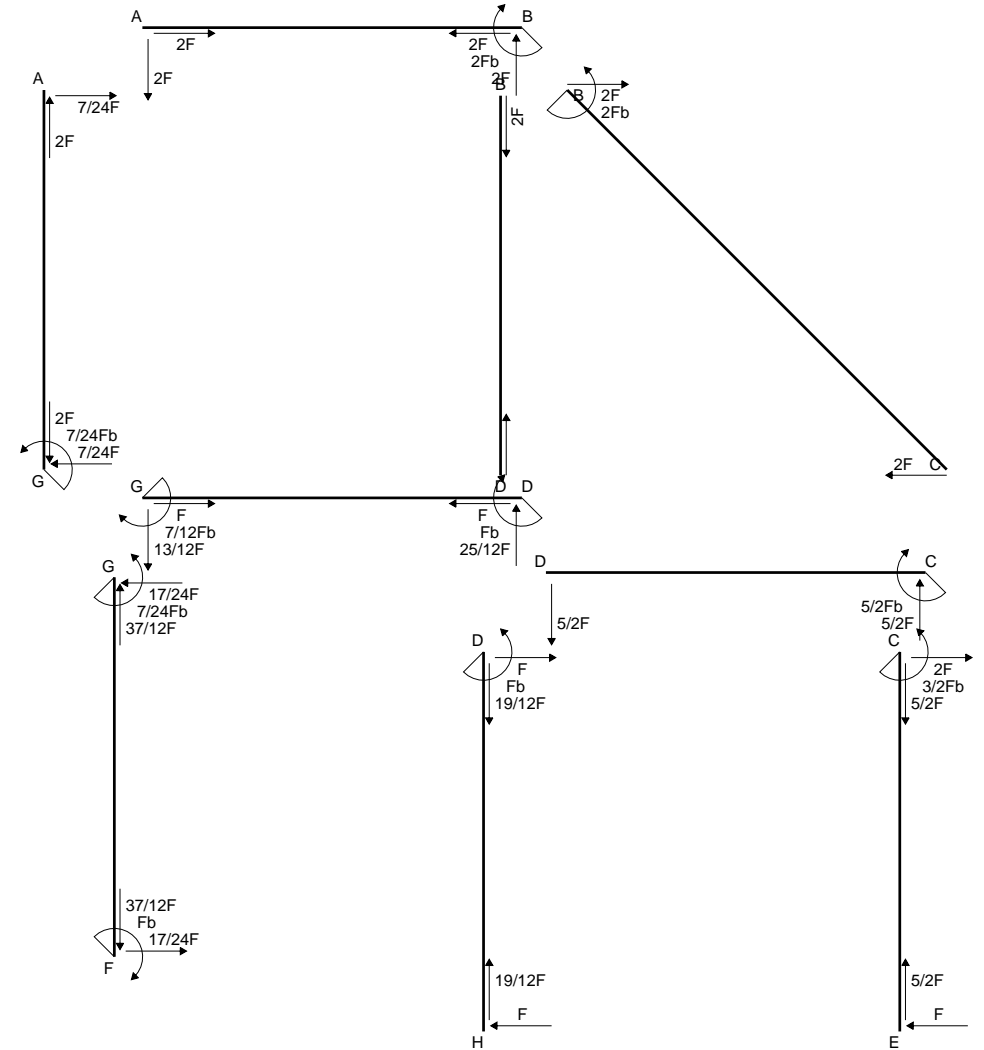


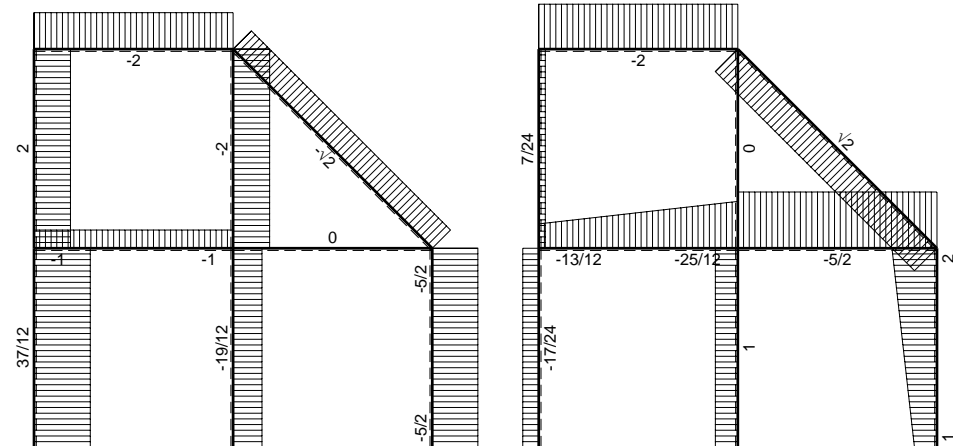
- A = 684. mm²
- J_u = 262207. mm⁴
- J_v = 43524. mm⁴
- y_g = 23.84 mm
- T_y = -3400. N
- M_x = -1734000. Nmm
- x_m = 30. mm
- y_m = 54. mm
- u_m = 9. mm
- v_m = 30.16 mm
- σ_m = -Mv/J_u = 199.4 N/mm²
- x_c = 21. mm
- y_c = 41. mm
- v_c = 17.16 mm
- σ_c = -Mv/J_u = 113.5 N/mm²
- τ_c = 11.51 N/mm²
- σ_q = √σ²+3τ² = 115.2 N/mm²
- S = 5324. mm³

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_F = -W = -Fb$
- $p_{CE} = -q = -F/b$
- $q_{GD} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



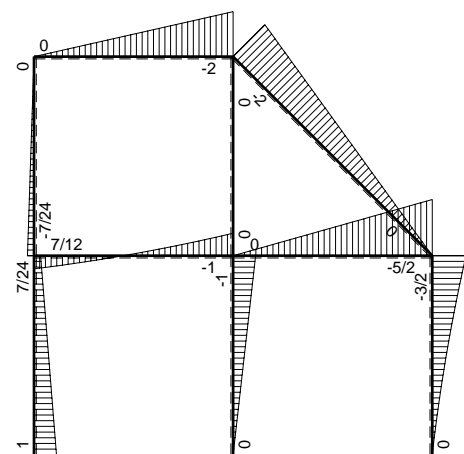
Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave DC ha la sezione riportata e dimensioni in mm, con:
 $b = 650 \text{ mm}$, $F = 1660 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da D a C
 Curvatura θ asta FG positiva se convessa a destra con inizio F.
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



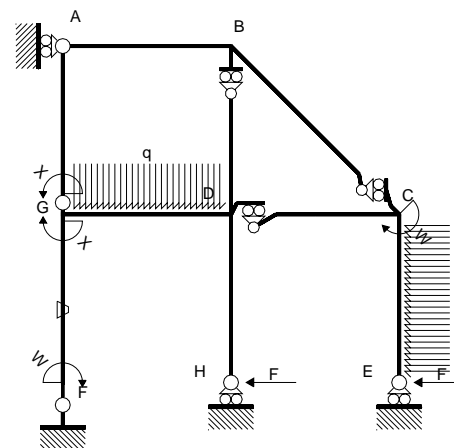


← (+) → F

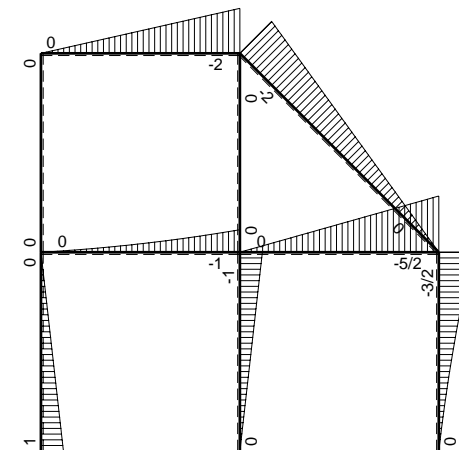
↑ (+) ↓ F



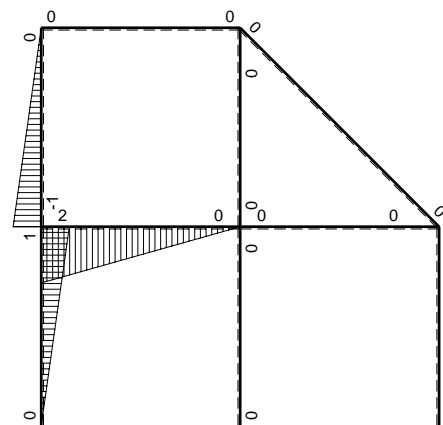
⤵ (+) ⤴ F_b



Schema di calcolo iperstatico



⤵ (+) ⤴ M₀ flessione da carichi assegnati



⤵ (+) ⤴ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0		
BC $\sqrt{2}b$	0	-2Fb+ $\sqrt{2}Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	-3/2Fb+2Fx-1/2qx ²	0	0	0	0	0+0	0
EC b	0	Fx+1/2qx ²	0	0	0	0		
FG b	x/b	Fb-Fx	-Fb/EJ	Fx-Fx ² /b	-Fx/EJ	x ² /b ²	(1/6-1/2)Fb ² /EJ	1/3Xb/EJ
GF b	-1+x/b	-Fx	Fb/EJ	Fx-Fx ² /b	-Fb/EJ+Fx/EJ	1-2x/b+x ² /b ²		
GD b	2-2x/b	-1/2Fx-1/2qx ²	0	-Fx+qx ³ /b	0	4-8x/b+4x ² /b ²	(-1/4+0)Fb ² /EJ	4/3Xb/EJ
DG b	-2x/b	Fb-3/2Fx+1/2qx ²	0	-2Fx+3Fx ² /b-qx ³ /b	0	4x ² /b ²		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-1+x/b	0	0	0	0	1-2x/b+x ² /b ²	0+0	1/3Xb/EJ
AG b	x/b	0	0	0	0	x ² /b ²		
	totali						-7/12Fb ² /EJ	2Xb/EJ
	iperstatica $X=W_{GA}$						7/24Fb	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{GF}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{GD}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b 1/EJ$$

$$= (4b - 4b + 4/3 b) 1/EJ = 4/3 b/EJ$$

$$L_{DG}^{xx} = \int_0^b (4x^2/b^2) 1/EJ dx = [4/3 x^3/b^2]_0^b 1/EJ$$

$$= (4/3 b) 1/EJ = 4/3 b/EJ$$

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FG}^{xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx + \int_0^b (-x/b) \theta dx$$

$$= [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/3 b) Fb 1/EJ + (-1/2 b) \theta = -1/3 Fb^2/EJ$$

$$L_{GF}^{xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

$$= [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ + [x - 1/2 x^2/b]_0^b \theta$$

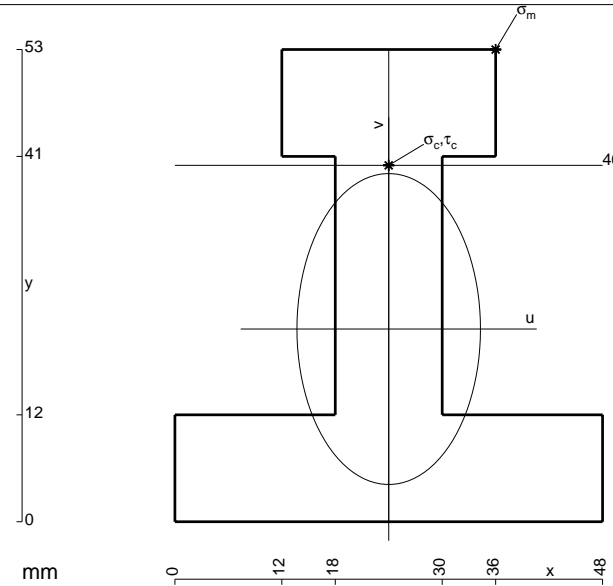
$$= (1/2 b - 1/3 b) Fb 1/EJ + (b - 1/2 b) \theta = -1/3 Fb^2/EJ$$

$$L_{GD}^{xo} = \int_0^b (-x/b + x^3/b^3) Fb 1/EJ dx = [-1/2 x^2/b + 1/4 x^4/b^3]_0^b Fb 1/EJ$$

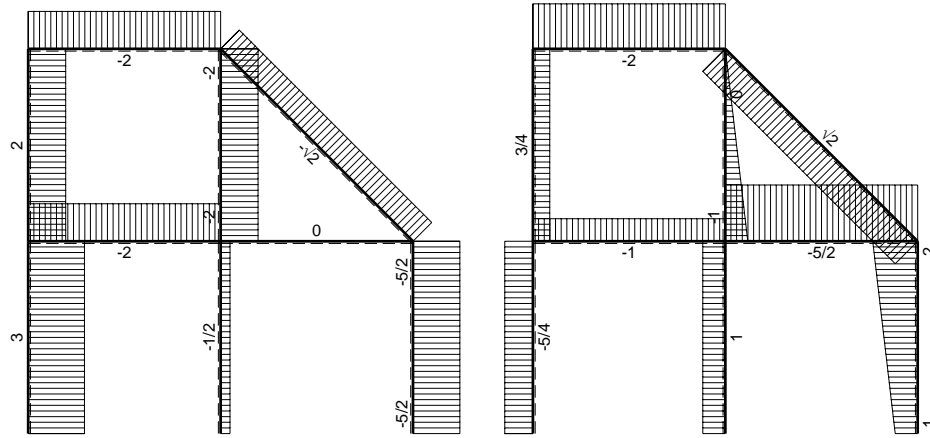
$$= (-1/2 b + 1/4 b) Fb 1/EJ = -1/4 Fb^2/EJ$$

$$L_{DG}^{xo} = \int_0^b (-2x/b + 3x^2/b^2 - x^3/b^3) Fb 1/EJ dx = [-x^2/b + x^3/b^2 - 1/4 x^4/b^3]_0^b Fb 1/EJ$$

$$= (-b + b - 1/4 b) Fb 1/EJ = -1/4 Fb^2/EJ$$

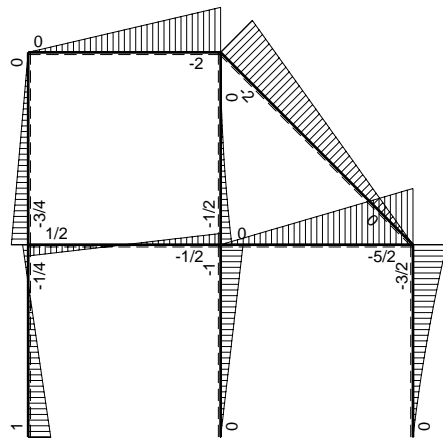


- A = 1212. mm²
- J_u = 369093. mm⁴
- J_v = 128592. mm⁴
- y_g = 21.63 mm
- T_y = -4150. N
- M_x = -2697500. Nmm
- x_m = 36. mm
- y_m = 53. mm
- u_m = 12. mm
- v_m = 31.37 mm
- σ_m = -Mv/J_u = 229.3 N/mm²
- x_c = 24. mm
- y_c = 40. mm
- v_c = 18.37 mm
- σ_c = -Mv/J_u = 134.3 N/mm²
- τ_c = 7.059 N/mm²
- σ_q = √σ²+3τ² = 134.8 N/mm²
- S = 7533. mm³

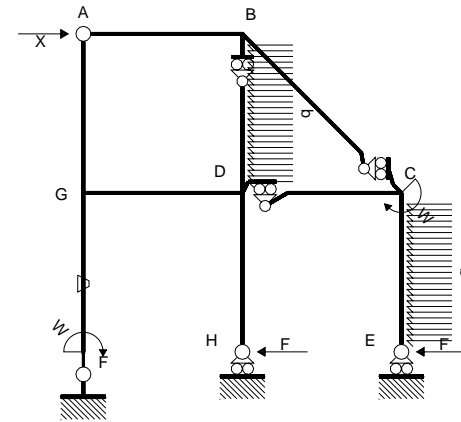


← (+) → F

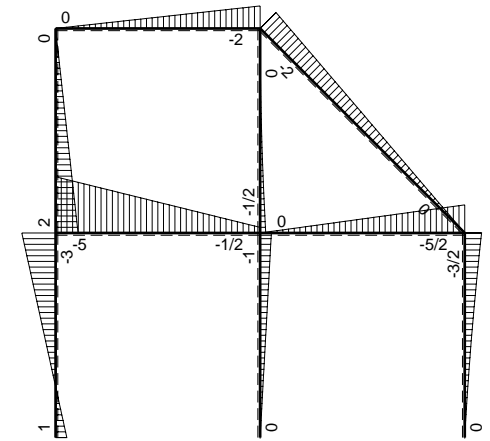
↑ (+) ↓ F



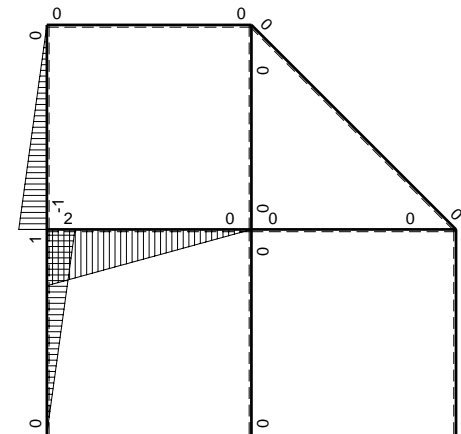
⌚ (+) ⌚ Mb



Schema di calcolo iperstatico



⌚ (+) ⌚ Mo flessione da carichi assegnati



⌚ (+) ⌚ Mx flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_A

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫xM _x M _x /EJdx
AB b	0	-2Fx	0	0	0	0	0+0	0
BA b	0	2Fb-2Fx	0	0	0	0		
BC √2b	0	-2Fb+√2Fx	0	0	0	0	0	0
BD b	0	-1/2qx ²	0	0	0	0	0+0	0
DB b	0	1/2Fb-Fx+1/2qx ²	0	0	0	0		
DC b	0	-5/2Fx	0	0	0	0	0+0	0
CD b	0	5/2Fb-5/2Fx	0	0	0	0		
CE b	0	-3/2Fb+2Fx-1/2qx ²	0	0	0	0	0+0	0
EC b	0	Fx+1/2qx ²	0	0	0	0		
FG b	x	Fb-4Fx	-Fb/EJ	Fbx-4Fx ²	-Fxb/EJ	x ²	(-5/6-1/2)Fb ³ /EJ	1/3Xb ³ /EJ
GF b	-b+x	3Fb-4Fx	Fb/EJ	-3Fb ² +7Fbx-4Fx ²	-Fb ² /EJ+Fxb/EJ	b ² -2bx+x ²		
GD b	2b-2x	-5Fb+9/2Fx	0	-10Fb ² +19Fbx-9Fx ²	0	4b ² -8bx+4x ²	(-7/2+0)Fb ³ /EJ	4/3Xb ³ /EJ
DG b	-2x	1/2Fb+9/2Fx	0	-Fbx-9Fx ²	0	4x ²		
DH b	0	-Fb+Fx	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	-b+x	2Fb-2Fx	0	-2Fb ² +4Fbx-2Fx ²	0	b ² -2bx+x ²	(-2/3+0)Fb ³ /EJ	1/3Xb ³ /EJ
AG b	x	-2Fx	0	-2Fx ²	0	x ²		
	totali						-11/2Fb ³ /EJ	2Xb ³ /EJ
	iperstatica X=H _A						11/4F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (x/b - 4x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx$$

$$= [1/2 x^2/b - 4/3 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x^2/b]_0^b \theta$$

$$= (1/2 b - 4/3 b) Fb^2 1/EJ + (-1/2 b) \theta = -4/3 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-3 + 7x/b - 4x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

$$= [-3x + 7/2 x^2/b - 4/3 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/2 x^2/b]_0^b \theta$$

$$= (-3b + 7/2 b - 4/3 b) Fb^2 1/EJ + (b - 1/2 b) \theta = -4/3 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-10 + 19x/b - 9x^2/b^2) Fb^2 1/EJ dx = [-10x + 19/2 x^2/b - 3x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-10b + 19/2 b - 3b) Fb^2 1/EJ = -7/2 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-x/b - 9x^2/b^2) Fb^2 1/EJ dx = [-1/2 x^2/b - 3x^3/b^2]_0^b Fb^2 1/EJ$$

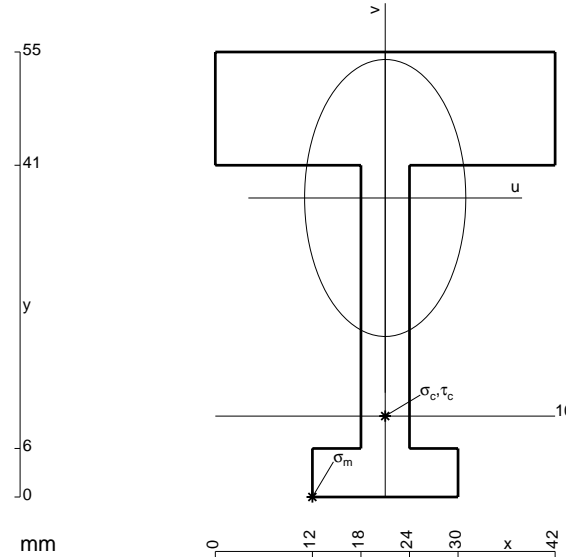
$$= (-1/2 b - 3b) Fb^2 1/EJ = -7/2 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (-2 + 4x/b - 2x^2/b^2) Fb^2 1/EJ dx = [-2x + 2x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2b + 2b - 2/3 b) Fb^2 1/EJ = -2/3 Fb^3/EJ$$

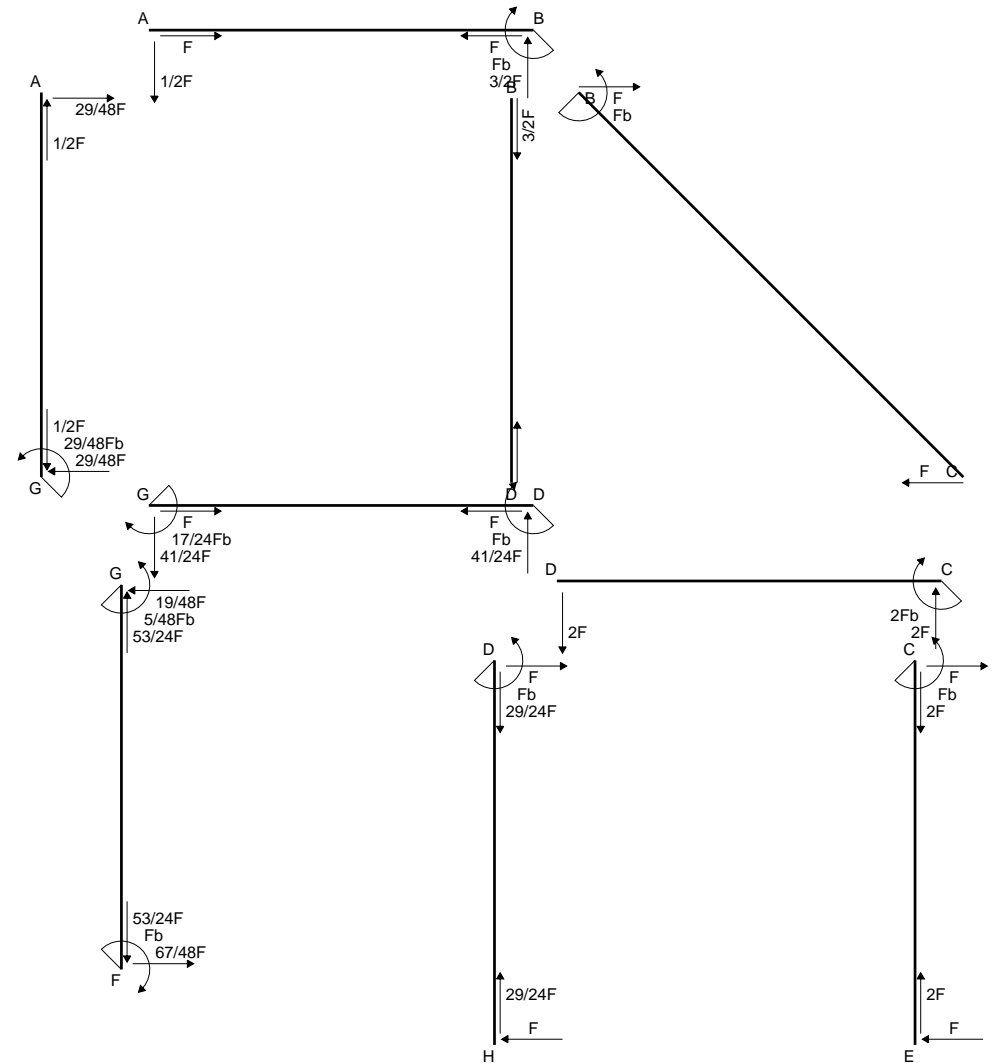
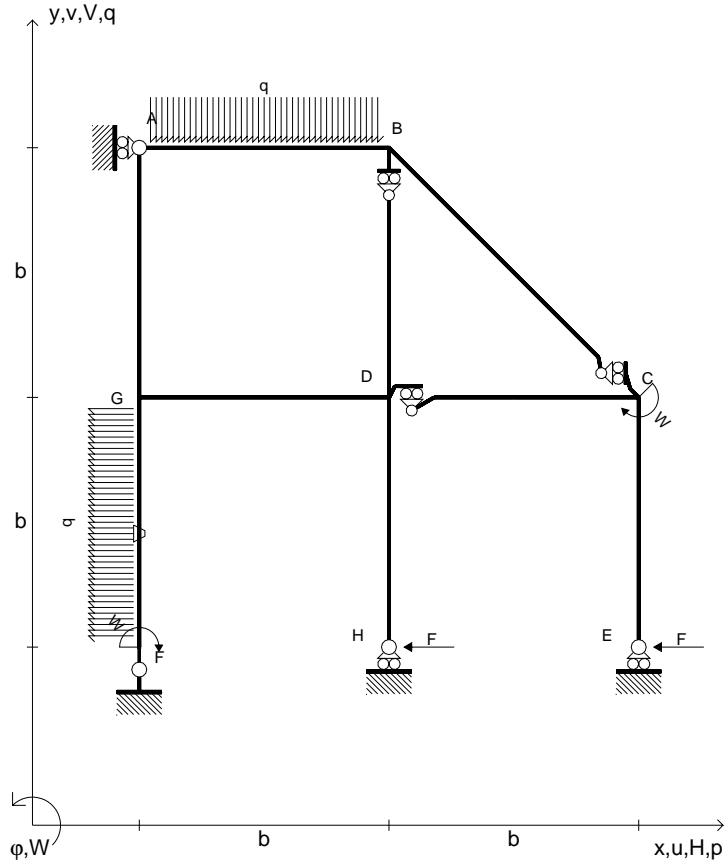
$$L_{AG}^{xo} = \int_0^b (-2x^2/b^2) Fb^2 1/EJ dx = [-2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2/3 b) Fb^2 1/EJ = -2/3 Fb^3/EJ$$

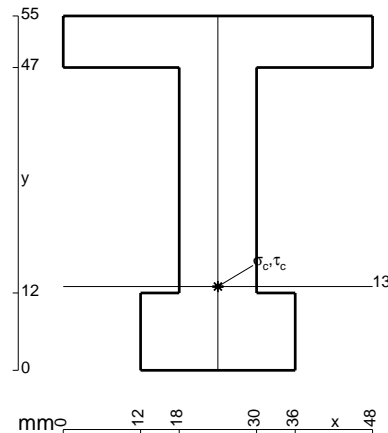


- A = 906. mm²
- J_u = 265632. mm⁴
- J_v = 89982. mm⁴
- y_g = 36.96 mm
- T_y = -2550. N
- M_x = -1581000. Nmm
- x_m = 12. mm
- u_m = -9. mm
- v_m = -36.96 mm
- σ_m = -Mv/J_u = -220. N/mm²
- x_c = 21. mm
- y_c = 10. mm
- v_c = -26.96 mm
- σ_c = -Mv/J_u = -160.4 N/mm²
- τ_c = 6.98 N/mm²
- σ_q = √σ²+3τ² = 160.9 N/mm²
- S = 4362. mm³

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_F = -W = -Fb$
- $q_{AB} = -q = -F/b$
- $p_{FG} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione: $X=H_F$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave DC ha la sezione riportata e dimensioni in mm, con:
 $b = 880 \text{ mm}$, $F = 1550 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da D a C
 Curvatura θ asta FG positiva se convessa a destra con inizio F.
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Quadro contributi PLV per iperstatica $X=H_f$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0	0	0	0	0
BC $\sqrt{2}b$	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0	0	0
DC b	0	$-2Fx$	0	0	0	0	0+0	0
CD b	0	$2Fb-2Fx$	0	0	0	0	0	0
CE b	0	$-Fb+Fx$	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0	0	0
FG b	-x	$Fb+1/2qx^2$	$-Fb/EJ$	$-Fbx-1/2qx^3$	Fxb/EJ	x^2	$(-5/8+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
GF b	b-x	$-3/2Fb+Fx-1/2qx^2$	Fb/EJ	$-3/2Fb^2+5/2Fbx-3/2Fx^2+1/2qx^3$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
GD b	$-2b+2x$	$7/2Fb-9/2Fx$	0	$-7Fb^2+16Fbx-9Fx^2$	0	$4b^2-8bx+4x^2$	$(-2+0)Fb^3/EJ$	$4/3Xb^3/EJ$
DG b	2x	$Fb-9/2Fx$	0	$2Fbx-9Fx^2$	0	$4x^2$		
DH b	0	$-Fb+Fx$	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	b-x	$-2Fb+2Fx$	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AG b	-x	$2Fx$	0	$-2Fx^2$	0	x^2		
	totali						$-67/24Fb^3/EJ$	$2Xb^3/EJ$
	iperstatica $X=H_f$						$67/48F$	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-x/b - 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [-1/2 x^2/b - 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (-1/2 b - 1/8 b) Fb^2 1/EJ + (1/2 b) \theta = -1/8 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-3/2 + 5/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [-3/2 x + 5/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-3/2 b + 5/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ + (-b + 1/2 b) \theta = -1/8 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-7 + 16x/b - 9x^2/b^2) Fb^2 1/EJ dx = [-7x + 8x^2/b - 3x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-7b + 8b - 3b) Fb^2 1/EJ = -2 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (2x/b - 9x^2/b^2) Fb^2 1/EJ dx = [x^2/b - 3x^3/b^2]_0^b Fb^2 1/EJ$$

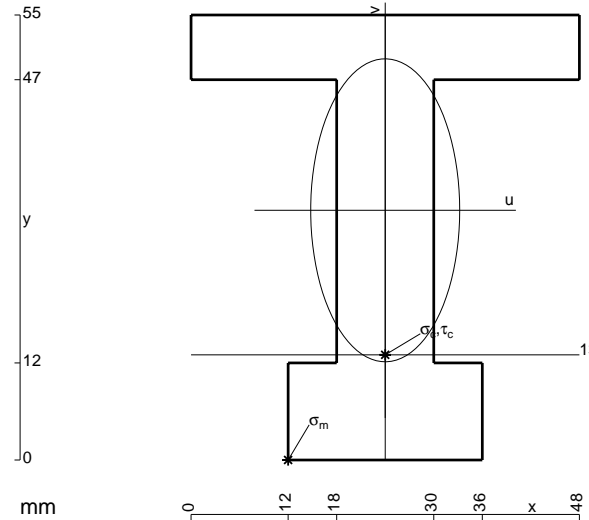
$$= (b - 3b) Fb^2 1/EJ = -2 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (-2 + 4x/b - 2x^2/b^2) Fb^2 1/EJ dx = [-2x + 2x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2b + 2b - 2/3 b) Fb^2 1/EJ = -2/3 Fb^3/EJ$$

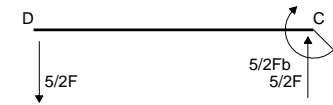
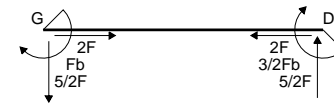
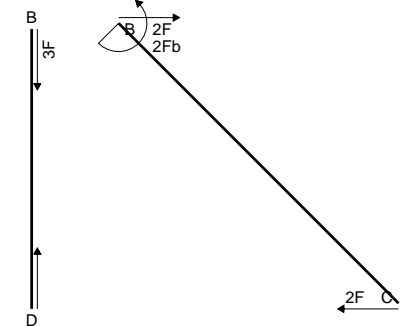
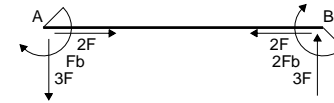
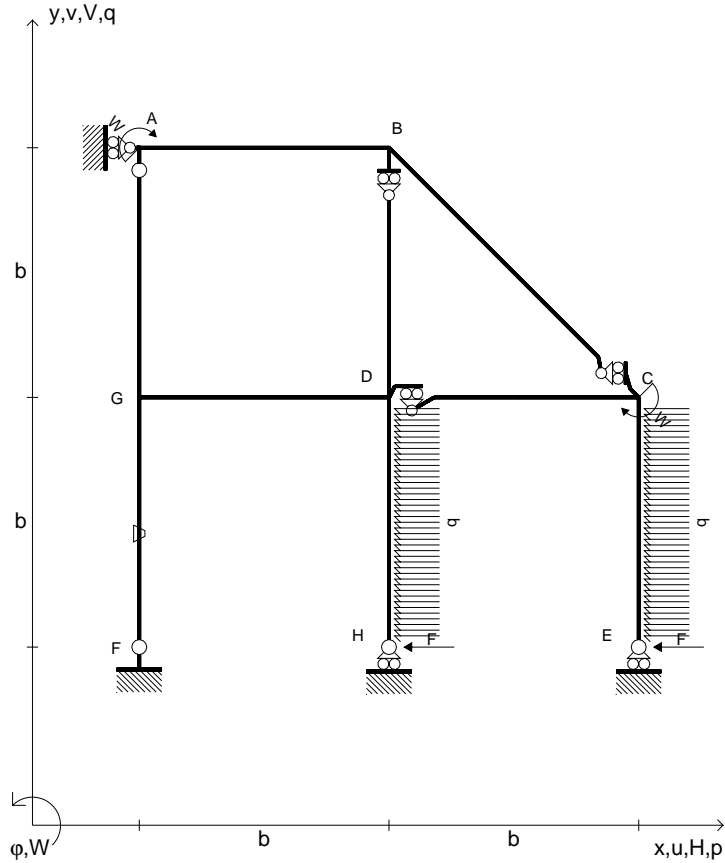
$$L_{AG}^{xo} = \int_0^b (-2x^2/b^2) Fb^2 1/EJ dx = [-2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2/3 b) Fb^2 1/EJ = -2/3 Fb^3/EJ$$

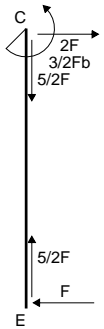
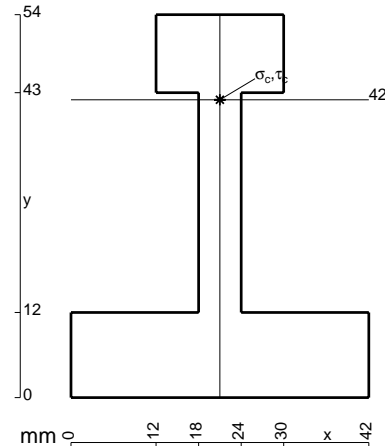


- A = 1092. mm²
- J_u = 382903. mm⁴
- J_v = 92592. mm⁴
- y_g = 30.86 mm
- T_y = -3100. N
- M_x = -2728000. Nmm
- x_m = 12. mm
- u_m = -12. mm
- v_m = -30.86 mm
- σ_m = -Mv/J_u = -219.9 N/mm²
- x_c = 24. mm
- y_c = 13. mm
- v_c = -17.86 mm
- σ_c = -Mv/J_v = -127.3 N/mm²
- τ_c = 4.98 N/mm²
- σ_q = √σ²+3τ² = 127.6 N/mm²
- S = 7381. mm³

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_A = -W = -Fb$
- $p_{CE} = -q = -F/b$
- $p_{DH} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



Reazioni iperstatiche in soluzione: $X=V_H$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave DC ha la sezione riportata e dimensioni in mm, con:
 $b = 910 \text{ mm}$, $F = 870 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da D a C
 Curvatura θ asta FG positiva se convessa a destra con inizio F.
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$	
AB b	0	$Fb-3Fx$	0	0	0	0	0+0	0	
BA b	0	$2Fb-3Fx$	0	0	0	0	0	0	
BC $\sqrt{2}b$	0	$-2Fb+\sqrt{2}Fx$	0	0	0	0	0	0	
BD b	0	0	0	0	0	0	0+0	0	
DB b	0	0	0	0	0	0	0	0	
DC b	0	$-5/2Fx$	0	0	0	0	0+0	0	
CD b	0	$5/2Fb-5/2Fx$	0	0	0	0	0	0	
CE b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0	
EC b	0	$Fx+1/2qx^2$	0	0	0	0	0	0	
FG b	$1/2x$	$-2Fx$	$-Fb/EJ$	$-Fx^2$	$-1/2Fxb/EJ$	$1/4x^2$	$(-1/3-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$	
GF b	$-1/2b+1/2x$	$2Fb-2Fx$	Fb/EJ	$-Fb^2+2Fbx-Fx^2$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$			
GD b	$b-x$	$-2Fb+1/2Fx$	0	$-2Fb^2+5/2Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-11/12+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
DG b	$-x$	$3/2Fb+1/2Fx$	0	$-3/2Fbx-1/2Fx^2$	0	x^2			
DH b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0	0+0	0	
HD b	0	$Fx+1/2qx^2$	0	0	0	0	0	0	
GA b	$-1/2b+1/2x$	0	0	0	0	$1/4b^2-1/2bx+1/4x^2$	0+0	$1/12Xb^3/EJ$	
AG b	$1/2x$	0	0	0	0	$1/4x^2$			
	totali							$-3/2Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$							3F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [-1/3 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (-1/3 b) Fb^2 1/EJ + (-1/4 b) \theta = -7/12 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-1 + 2x/b - x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [-x + x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

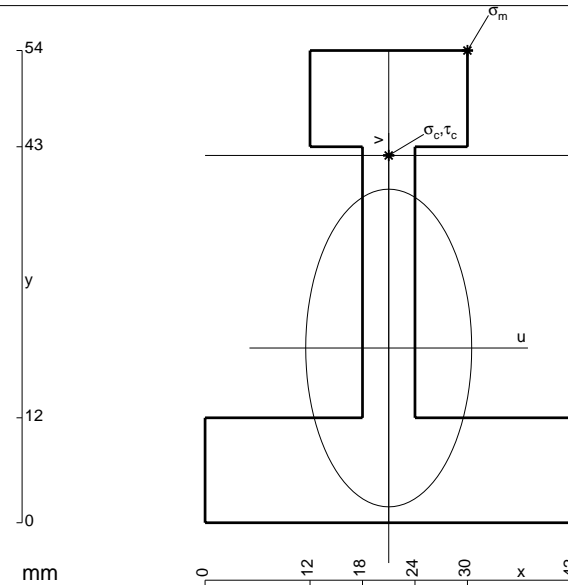
$$= (-b + b - 1/3 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = -7/12 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-2 + 5/2 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-2x + 5/4 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2b + 5/4 b - 1/6 b) Fb^2 1/EJ = -11/12 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-3/2 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-3/4 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-3/4 b - 1/6 b) Fb^2 1/EJ = -11/12 Fb^3/EJ$$



$$A = 888. \text{ mm}^2$$

$$J_u = 293012. \text{ mm}^4$$

$$J_v = 79992. \text{ mm}^4$$

$$y_g = 19.98 \text{ mm}$$

$$T_y = -2175. \text{ N}$$

$$M_x = -1979250. \text{ Nmm}$$

$$x_m = 30. \text{ mm}$$

$$y_m = 54. \text{ mm}$$

$$u_m = 9. \text{ mm}$$

$$v_m = 34.02 \text{ mm}$$

$$\sigma_m = -Mv/J_u = 229.8 \text{ N/mm}^2$$

$$x_c = 21. \text{ mm}$$

$$y_c = 42. \text{ mm}$$

$$v_c = 22.02 \text{ mm}$$

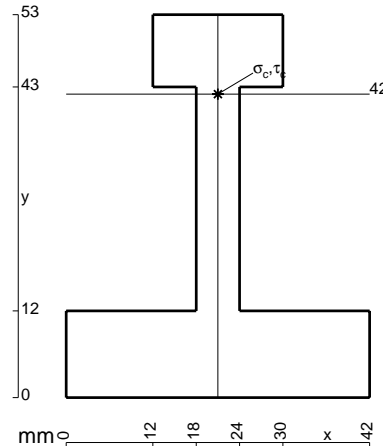
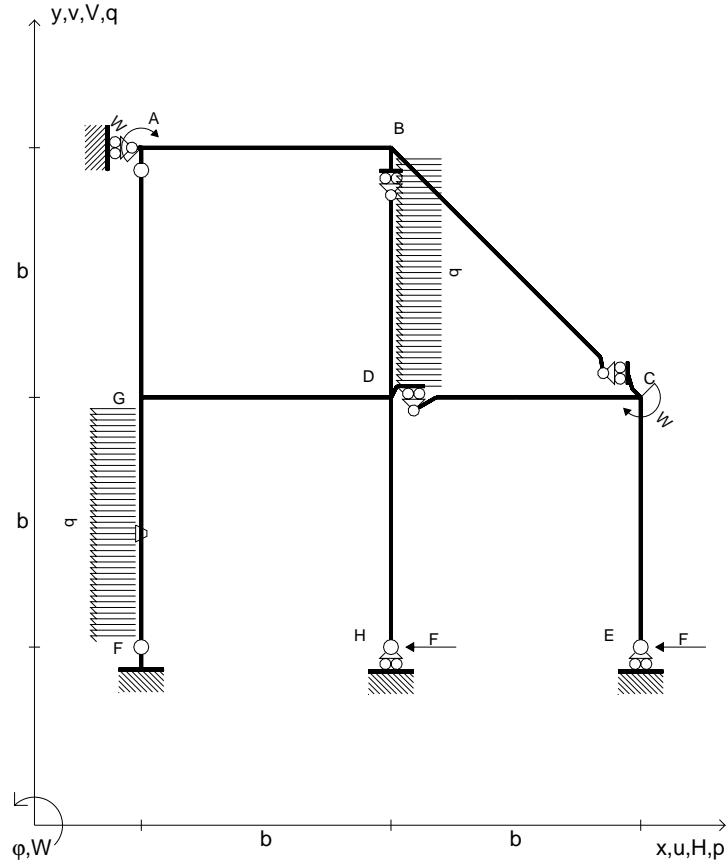
$$\sigma_c = -Mv/J_u = 148.7 \text{ N/mm}^2$$

$$\tau_c = 7.153 \text{ N/mm}^2$$

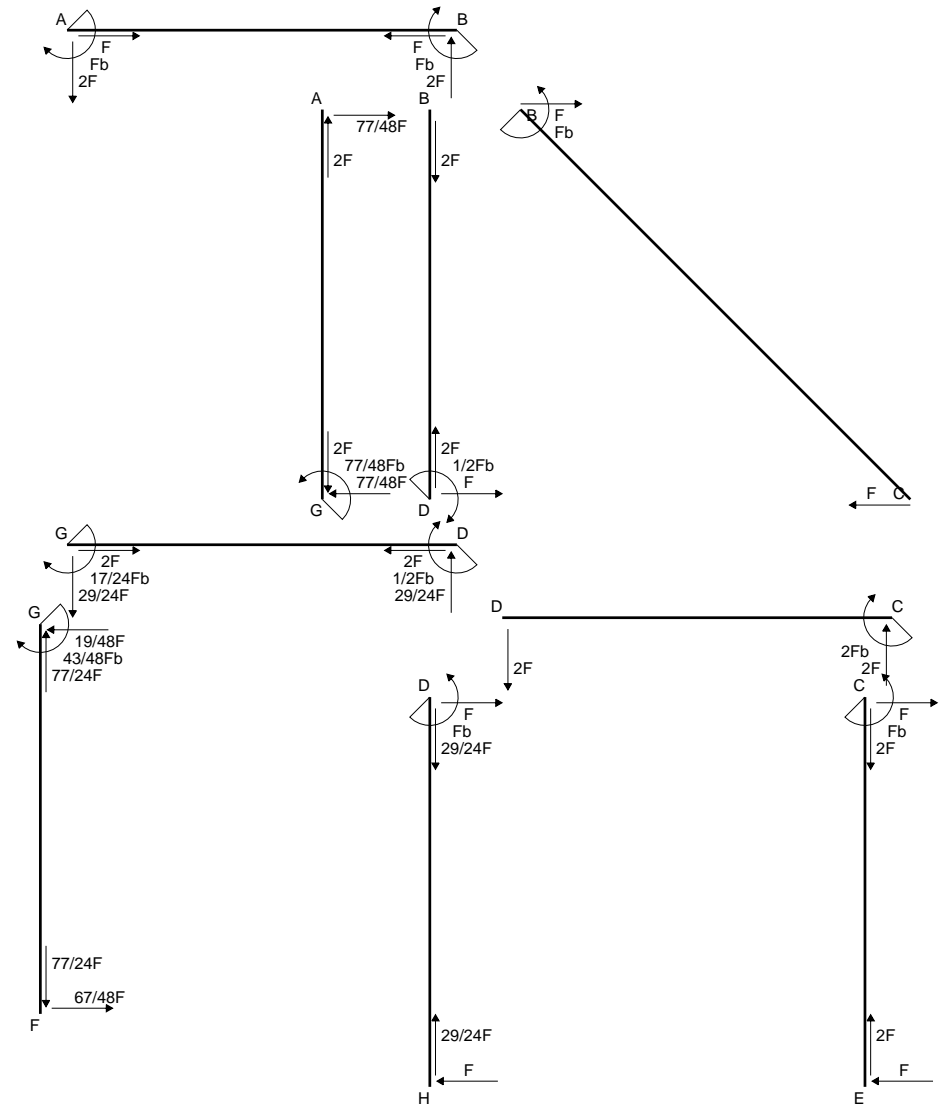
$$\sigma_\varphi = \sqrt{\sigma^2 + 3\tau^2} = 149.3 \text{ N/mm}^2$$

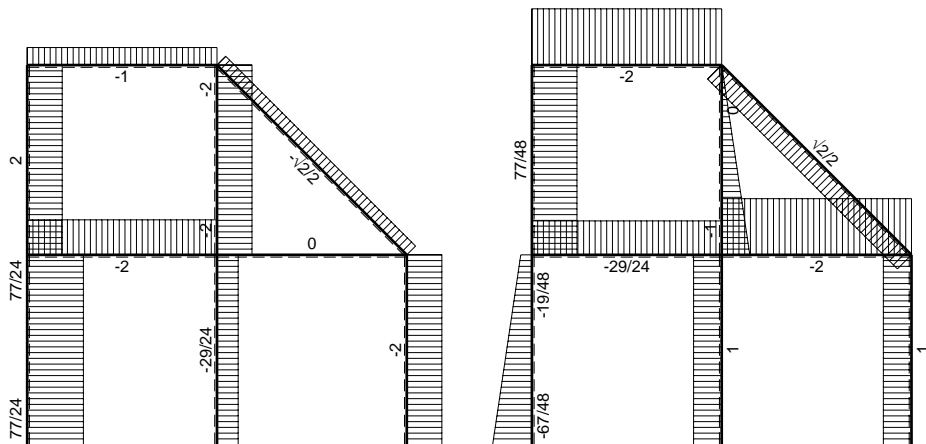
$$S = 5782. \text{ mm}^3$$

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_A = -W = -Fb$
- $p_{BD} = -q = -F/b$
- $p_{FG} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



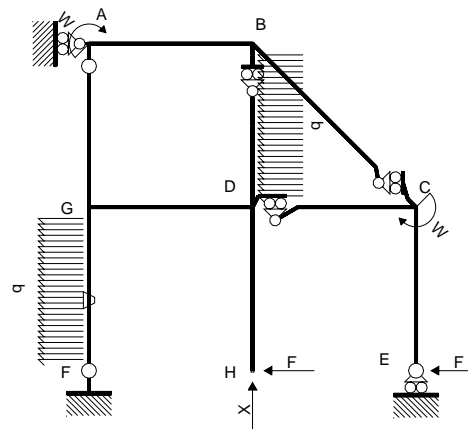
Reazioni iperstatiche in soluzione: $X=V_H$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave DC ha la sezione riportata e dimensioni in mm, con:
 $b = 700 \text{ mm}$, $F = 1380 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da D a C
 Curvatura θ asta FG positiva se convessa a destra con inizio F.
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



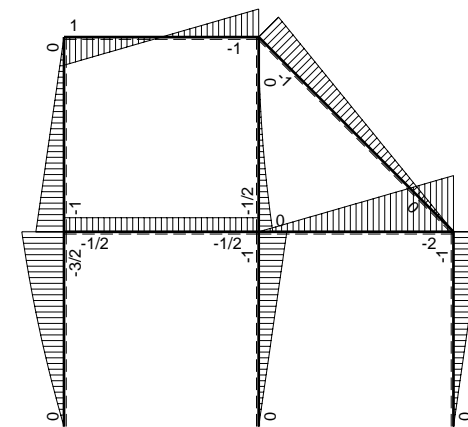


← (+) → F

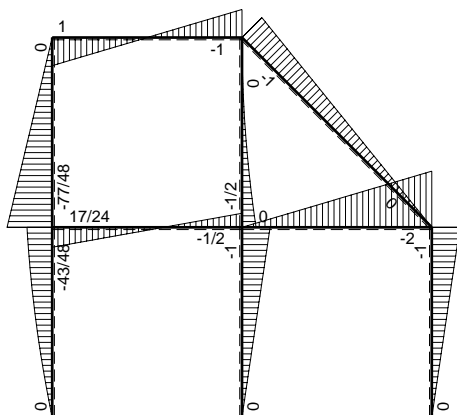
↑ (+) ↓ F



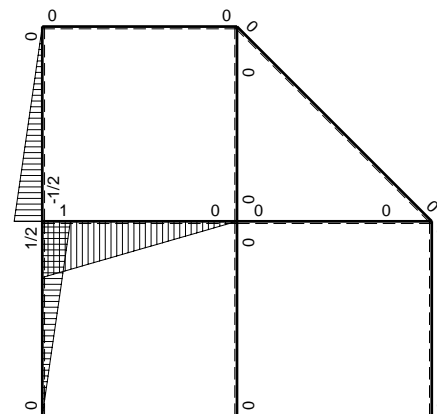
Schema di calcolo iperstatico



⌚ (+) ⌚ M₀ flessione da carichi assegnati



⌚ (+) ⌚ F_b



⌚ (+) ⌚ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	0	$Fb-2Fx$	0	0	0	0	0+0	0
BA b	0	$Fb-2Fx$	0	0	0	0		
BC $\sqrt{2}b$	0	$-Fb+\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	$-1/2qx^2$	0	0	0	0	0+0	0
DB b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
DC b	0	$-2Fx$	0	0	0	0	0+0	0
CD b	0	$2Fb-2Fx$	0	0	0	0		
CE b	0	$-Fb+Fx$	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0		
FG b	$1/2x$	$-2Fx+1/2qx^2$	$-Fb/EJ$	$-Fx^2+1/4qx^3$	$-1/2Fxb/EJ$	$1/4x^2$	$(-13/48-1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
GF b	$-1/2b+1/2x$	$3/2Fb-Fx-1/2qx^2$	Fb/EJ	$-3/4Fb^2+5/4Fbx-1/4Fx^2-1/4qx^3$	$-1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$		
GD b	$b-x$	$-1/2Fb$	0	$-1/2Fb^2+1/2Fbx$	0	$b^2-2bx+x^2$	$(-1/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DG b	$-x$	$1/2Fb$	0	$-1/2Fbx$	0	x^2		
DH b	0	$-Fb+Fx$	0	0	0	0	0+0	0
HD b	0	Fx	0	0	0	0		
GA b	$-1/2b+1/2x$	$-Fb+Fx$	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
AG b	$1/2x$	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
	totali						$-29/48Fb^3/EJ$	$1/2Xb^3/EJ$
	iperstatica $X=V_H$						$29/24F$	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-x^2/b^2 + 1/4 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx$$

$$= [-1/3 x^3/b^2 + 1/16 x^4/b^3]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (-1/3 b + 1/16 b) Fb^2 1/EJ + (-1/4 b) \theta = -25/48 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-3/4 + 5/4 x/b - 1/4 x^2/b^2 - 1/4 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [-3/4 x + 5/8 x^2/b - 1/12 x^3/b^2 - 1/16 x^4/b^3]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (-3/4 b + 5/8 b - 1/12 b - 1/16 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = -25/48 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-1/2 + 1/2 x/b) Fb^2 1/EJ dx = [-1/2 x + 1/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 1/4 b) Fb^2 1/EJ = -1/4 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-1/2 x/b) Fb^2 1/EJ dx = [-1/4 x^2/b]_0^b Fb^2 1/EJ$$

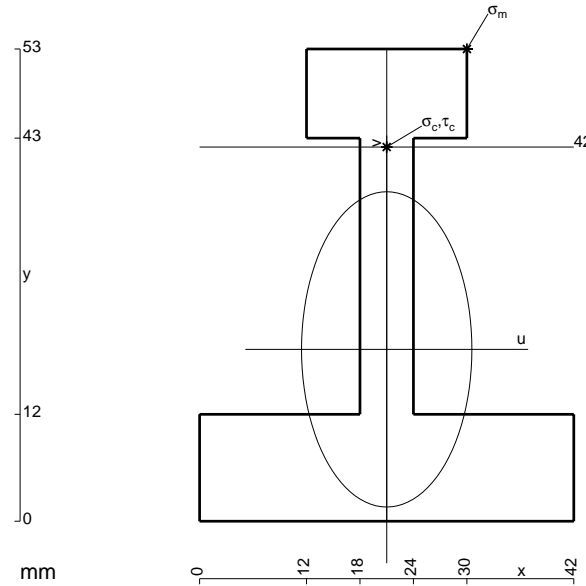
$$= (-1/4 b) Fb^2 1/EJ = -1/4 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

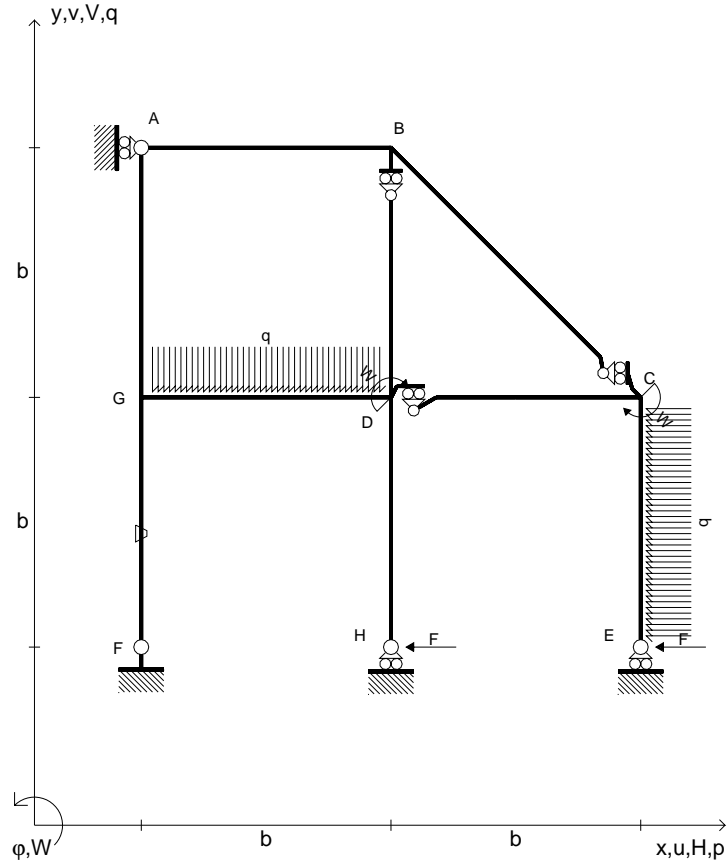
$$L_{AG}^{xo} = \int_0^b (1/2 x^2/b^2) Fb^2 1/EJ dx = [1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

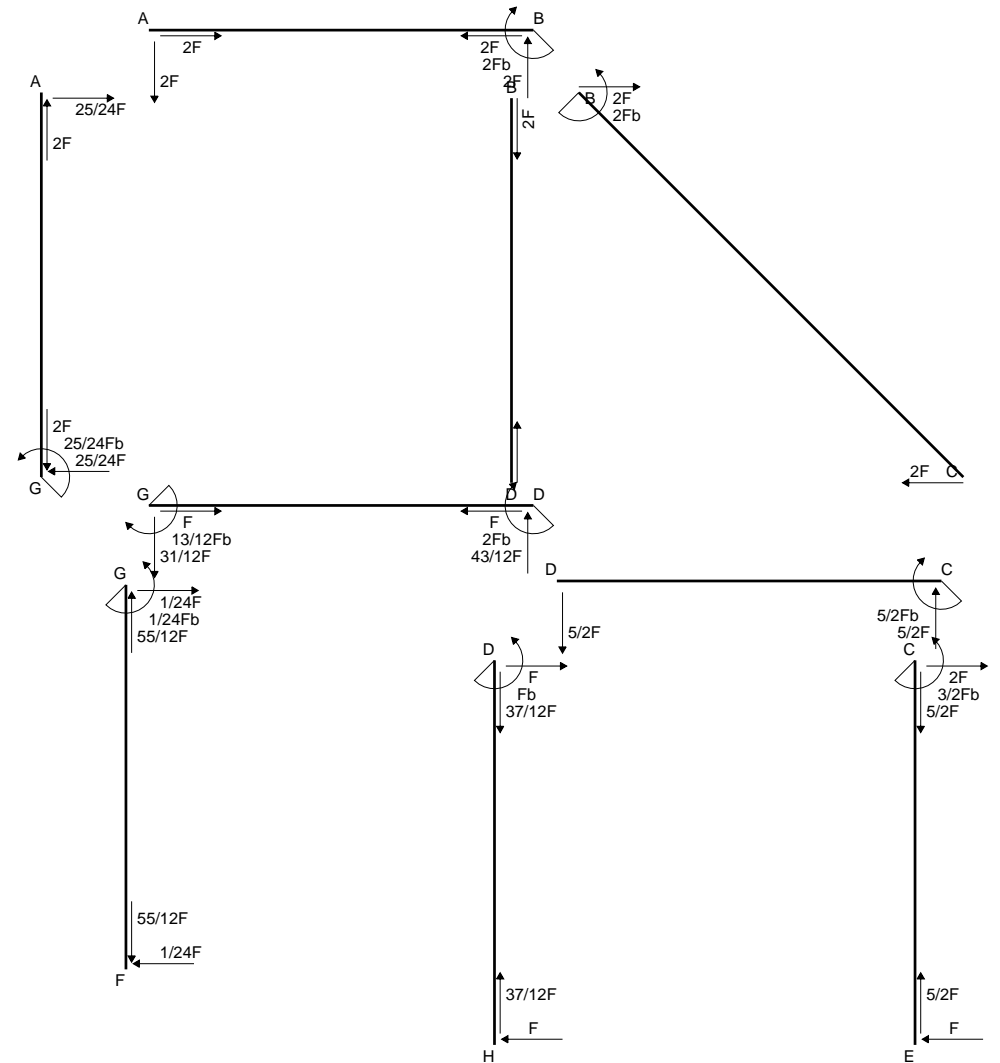
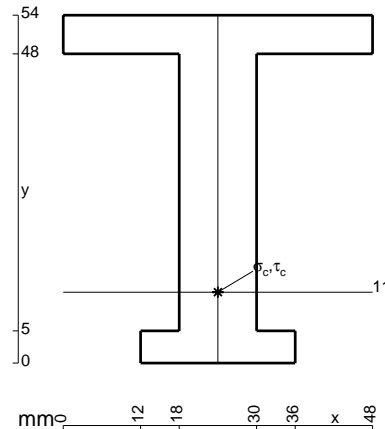


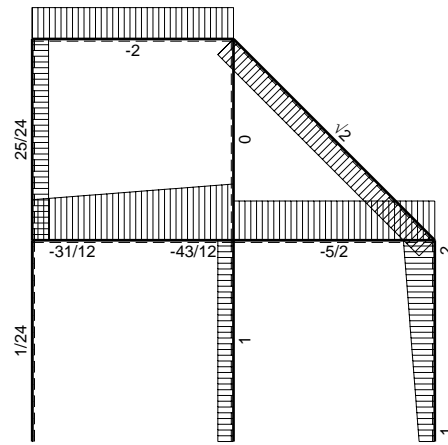
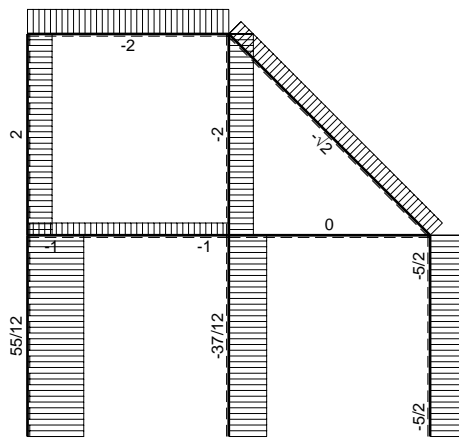
- A = 870. mm²
- J_u = 272367. mm⁴
- J_v = 79506. mm⁴
- y_g = 19.29 mm
- T_y = -2760. N
- M_x = -1932000. Nmm
- x_m = 30. mm
- y_m = 53. mm
- u_m = 9. mm
- v_m = 33.71 mm
- σ_m = -Mv/J_u = 239.1 N/mm²
- x_c = 21. mm
- y_c = 42. mm
- v_c = 22.71 mm
- σ_c = -Mv/J_u = 161.1 N/mm²
- τ_c = 8.964 N/mm²
- σ_o = √σ²+3τ² = 161.9 N/mm²
- S = 5308. mm³

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_D = -W = -Fb$
- $p_{CE} = -q = -F/b$
- $q_{GD} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$



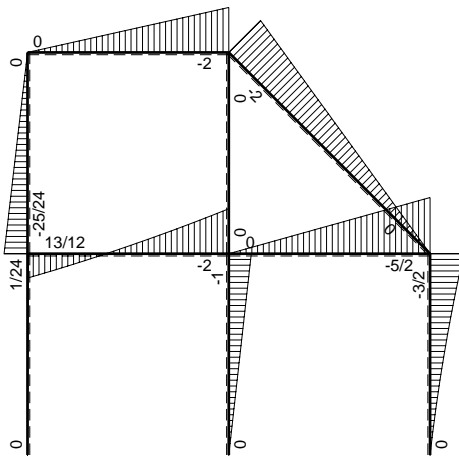
Reazioni iperstatiche in soluzione: $X=V_H$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave DC ha la sezione riportata e dimensioni in mm, con:
 $b = 860 \text{ mm}$, $F = 1000 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da D a C
 Curvatura θ asta FG positiva se convessa a destra con inizio F.
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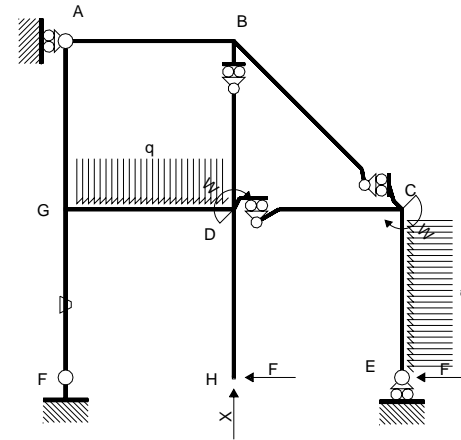


← ⊕ → F

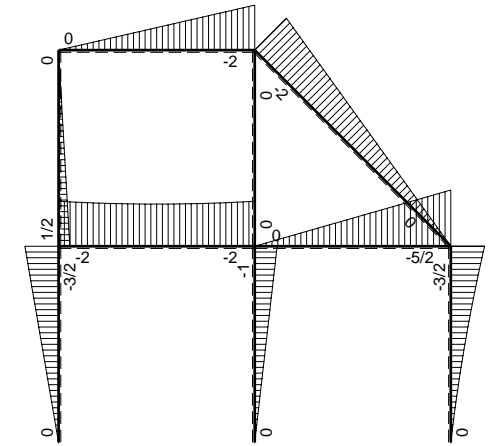
⊕ ↓ F



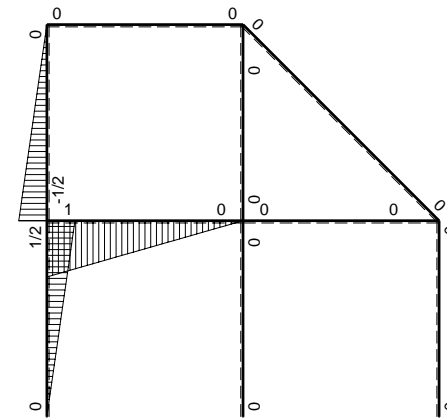
⊕ ↺ F_b



Schema di calcolo iperstatico



⊕ ↺ M₀ flessione da carichi assegnati



⊕ ↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	0	-2Fx	0	0	0	0	0+0	0	
BA b	0	2Fb-2Fx	0	0	0	0			
BC $\sqrt{2}b$	0	-2Fb+ $\sqrt{2}Fx$	0	0	0	0	0	0	
BD b	0	0	0	0	0	0	0+0	0	
DB b	0	0	0	0	0	0			
DC b	0	-5/2Fx	0	0	0	0	0+0	0	
CD b	0	5/2Fb-5/2Fx	0	0	0	0			
CE b	0	-3/2Fb+2Fx-1/2qx ²	0	0	0	0	0+0	0	
EC b	0	Fx+1/2qx ²	0	0	0	0			
FG b	1/2x	-3/2Fx	-Fb/EJ	-3/4Fx ²	-1/2Fxb/EJ	1/4x ²	(-1/4-1/4)Fb ³ /EJ	1/12Xb ³ /EJ	
GF b	-1/2b+1/2x	3/2Fb-3/2Fx	Fb/EJ	-3/4Fb ² +3/2Fbx-3/4Fx ²	-1/2Fb ² /EJ+1/2Fxb/EJ	1/4b ² -1/2bx+1/4x ²			
GD b	b-x	-2Fb+1/2Fx-1/2qx ²	0	-2Fb ² +5/2Fbx-Fx ² +1/2qx ³	0	b ² -2bx+x ²	(-23/24+0)Fb ³ /EJ	1/3Xb ³ /EJ	
DG b	-x	2Fb-1/2Fx+1/2qx ²	0	-2Fbx+1/2Fx ² -1/2qx ³	0	x ²			
DH b	0	-Fb+Fx	0	0	0	0	0+0	0	
HD b	0	Fx	0	0	0	0			
GA b	-1/2b+1/2x	1/2Fb-1/2Fx	0	-1/4Fb ² +1/2Fbx-1/4Fx ²	0	1/4b ² -1/2bx+1/4x ²	(-1/12+0)Fb ³ /EJ	1/12Xb ³ /EJ	
AG b	1/2x	-1/2Fx	0	-1/4Fx ²	0	1/4x ²			
	totali							-37/24Fb ³ /EJ	1/2Xb ³ /EJ
	iperstatica $X=V_H$							37/12F	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{FG}^{xo} = \int_0^b (-3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [-1/4 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (-1/4 b) Fb^2 1/EJ + (-1/4 b) \theta = -1/2 Fb^3/EJ$$

$$L_{GF}^{xo} = \int_0^b (-3/4 + 3/2 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [-3/4 x + 3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (-3/4 b + 3/4 b - 1/4 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = -1/2 Fb^3/EJ$$

$$L_{GD}^{xo} = \int_0^b (-2 + 5/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [-2 x + 5/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-2 b + 5/4 b - 1/3 b + 1/8 b) Fb^2 1/EJ = -23/24 Fb^3/EJ$$

$$L_{DG}^{xo} = \int_0^b (-2 x/b + 1/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [-x^2/b + 1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

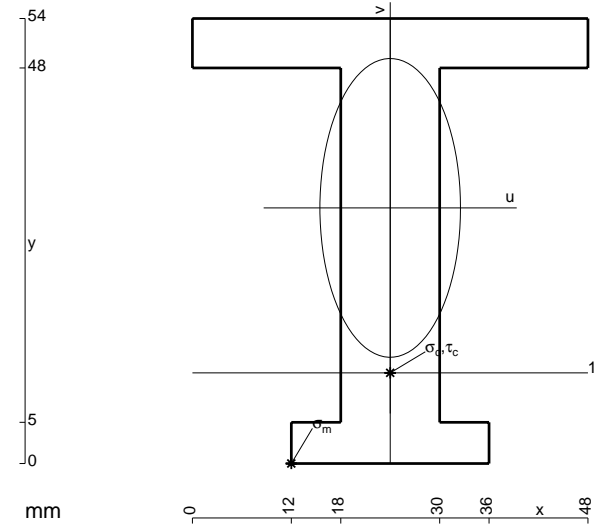
$$= (-b + 1/6 b - 1/8 b) Fb^2 1/EJ = -23/24 Fb^3/EJ$$

$$L_{GA}^{xo} = \int_0^b (-1/4 + 1/2 x/b - 1/4 x^2/b^2) Fb^2 1/EJ dx = [-1/4 x + 1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/4 b + 1/4 b - 1/12 b) Fb^2 1/EJ = -1/12 Fb^3/EJ$$

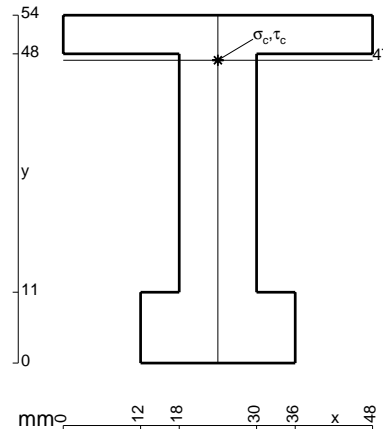
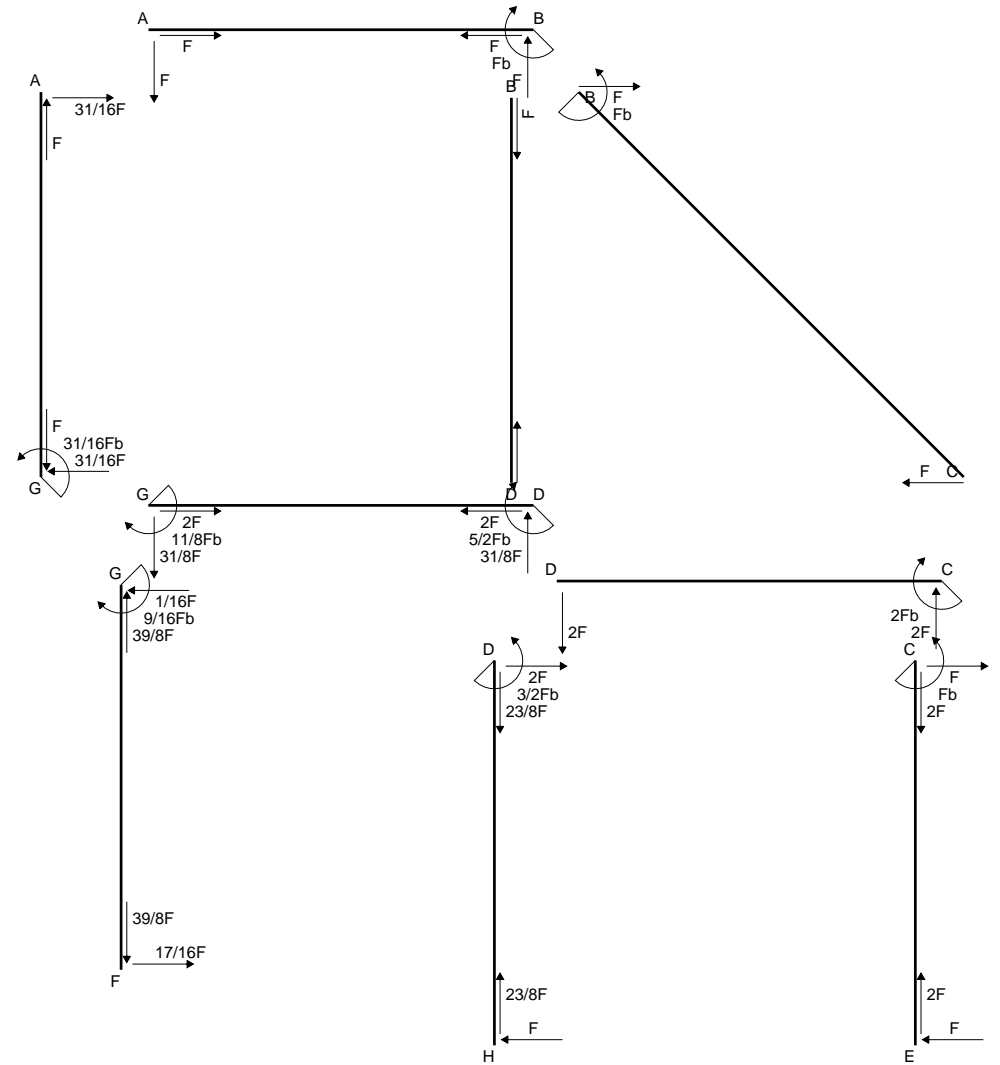
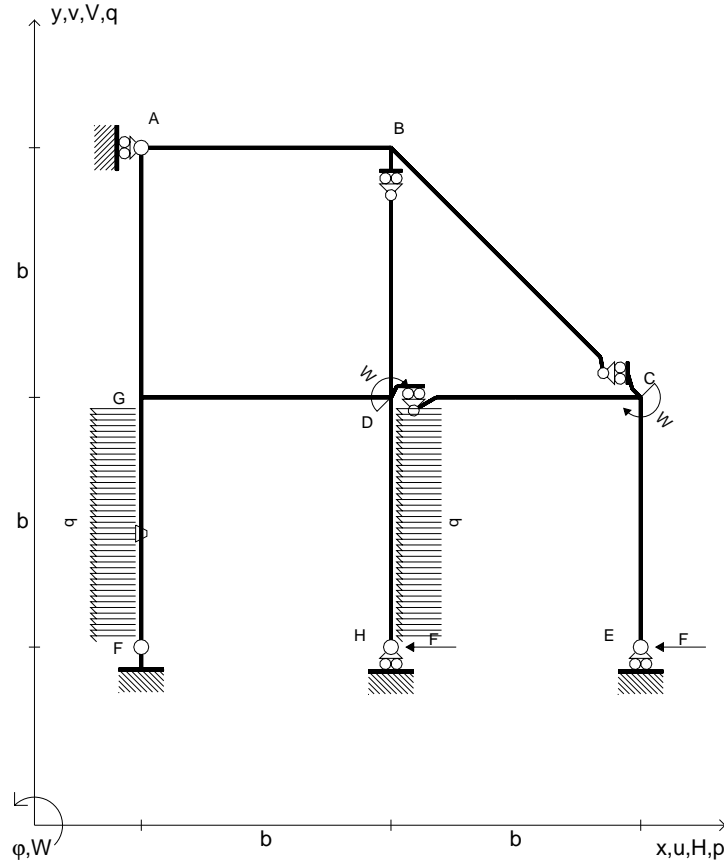
$$L_{AG}^{xo} = \int_0^b (-1/4 x^2/b^2) Fb^2 1/EJ dx = [-1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/12 b) Fb^2 1/EJ = -1/12 Fb^3/EJ$$

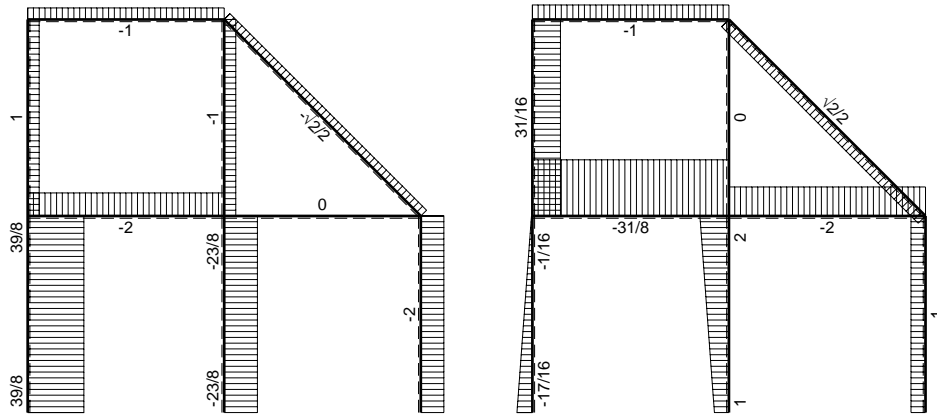


- A = 924. mm²
- J_u = 303740. mm⁴
- J_v = 67248. mm⁴
- y_g = 31.02 mm
- T_y = -2500. N
- M_x = -2150000. Nmm
- x_m = 12. mm
- u_m = -12. mm
- v_m = -31.02 mm
- σ_m = -Mv/J_u = -219.6 N/mm²
- x_c = 24. mm
- y_c = 11. mm
- v_c = -20.02 mm
- σ_c = -Mv/J_u = -141.7 N/mm²
- τ_c = 3.484 N/mm²
- σ_p = √σ²+3τ² = 141.8 N/mm²
- S = 5080. mm³

- $H_E = -F$
- $H_H = -F$
- $W_C = -W = -Fb$
- $W_D = -W = -Fb$
- $p_{FG} = -q = -F/b$
- $p_{DH} = -q = -F/b$
- $\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{BD} = EJ$
- $EJ_{DC} = EJ$
- $EJ_{CE} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GD} = EJ$
- $EJ_{DH} = EJ$
- $EJ_{GA} = EJ$

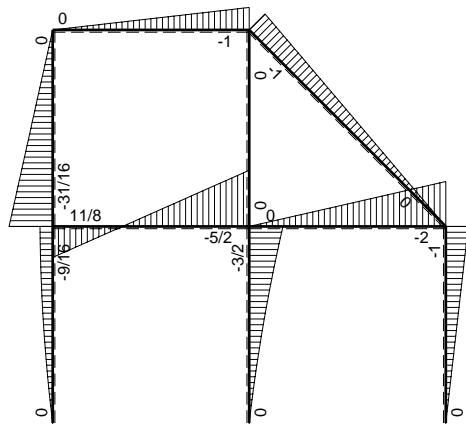


Reazioni iperstatiche in soluzione: $X=W_{GD}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave DC ha la sezione riportata e dimensioni in mm, con:
 $b = 560 \text{ mm}$, $F = 2160 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da D a C
 Curvatura θ asta FG positiva se convessa a destra con inizio F.
 © Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

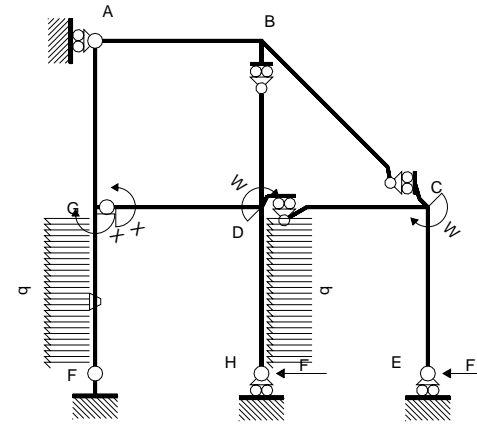


← ⊕ → F

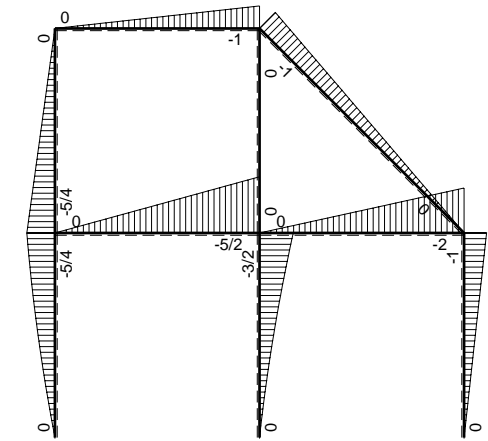
↑ ⊕ ↓ F



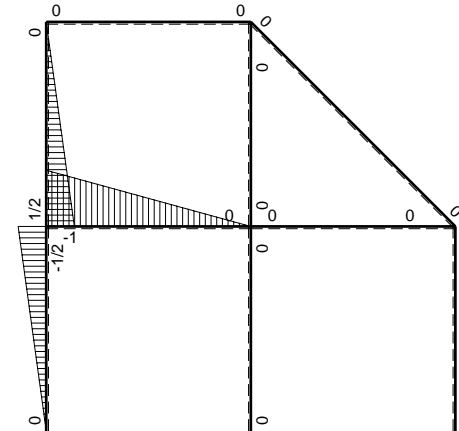
⊕ Mb



Schema di calcolo iperstatico



⊕ Mo flessione da carichi assegnati



⊕ Mx flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GD}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$
AB b	0	-Fx	0	0	0	0	0+0	0
BA b	0	Fb-Fx	0	0	0	0	0	0
BC $\sqrt{2}b$	0	-Fb+ $\sqrt{2}/2Fx$	0	0	0	0	0	0
BD b	0	0	0	0	0	0	0+0	0
DB b	0	0	0	0	0	0	0	0
DC b	0	-2Fx	0	0	0	0	0+0	0
CD b	0	2Fb-2Fx	0	0	0	0	0	0
CE b	0	-Fb+Fx	0	0	0	0	0+0	0
EC b	0	Fx	0	0	0	0	0	0
FG b	-1/2x/b	-7/4Fx+1/2qx ²	-Fb/EJ	7/8Fx ² /b-1/4qx ³ /b	1/2Fx/EJ	1/4x ² /b ²	(11/48+1/4)Fb ² /EJ	1/12Xb/EJ
GF b	1/2-1/2x/b	5/4Fb-3/4Fx-1/2qx ²	Fb/EJ	5/8Fb-Fx+1/8Fx ² /b+1/4qx ³ /b	1/2Fb/EJ-1/2Fx/EJ	1/4-1/2x/b+1/4x ² /b ²		
GD b	-1+x/b	-5/2Fx	0	5/2Fx-5/2Fx ² /b	0	1-2x/b+x ² /b ²	(5/12+0)Fb ² /EJ	1/3Xb/EJ
DG b	x/b	5/2Fb-5/2Fx	0	5/2Fx-5/2Fx ² /b	0	x ² /b ²		
DH b	0	-3/2Fb+2Fx-1/2qx ²	0	0	0	0	0+0	0
HD b	0	Fx+1/2qx ²	0	0	0	0		
GA b	1/2-1/2x/b	-5/4Fb+5/4Fx	0	-5/8Fb+5/4Fx-5/8Fx ² /b	0	1/4-1/2x/b+1/4x ² /b ²	(-5/24+0)Fb ² /EJ	1/12Xb/EJ
AG b	-1/2x/b	5/4Fx	0	-5/8Fx ² /b	0	1/4x ² /b ²		
	totali						11/16Fb ² /EJ	1/2Xb/EJ
	iperstatica $X=W_{GD}$						-11/8Fb	

Sviluppi di calcolo iperstatica

$$L_{FG}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{GF}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{GD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DG}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{GA}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AG}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{FG}^{xo} = \int_0^b (7/8 x^2/b^2 - 1/4 x^3/b^3) Fb 1/EJ dx + \int_0^b (1/2 x/b) \theta dx$$

$$= [7/24 x^3/b^2 - 1/16 x^4/b^3]_0^b Fb 1/EJ + [1/4 x^2/b]_0^b \theta$$

$$= (7/24 b - 1/16 b) Fb 1/EJ + (1/4 b) \theta = 23/48 Fb^2/EJ$$

$$L_{GF}^{xo} = \int_0^b (5/8 - x/b + 1/8 x^2/b^2 + 1/4 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1/2 + 1/2 x/b) \theta dx$$

$$= [5/8 x - 1/2 x^2/b + 1/24 x^3/b^2 + 1/16 x^4/b^3]_0^b Fb 1/EJ + [-1/2 x + 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 1/2 b + 1/24 b + 1/16 b) Fb 1/EJ + (-1/2 b + 1/4 b) \theta = 23/48 Fb^2/EJ$$

$$L_{GD}^{xo} = \int_0^b (5/2 x/b - 5/2 x^2/b^2) Fb 1/EJ dx = [5/4 x^2/b - 5/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/4 b - 5/6 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{DG}^{xo} = \int_0^b (5/2 x/b - 5/2 x^2/b^2) Fb 1/EJ dx = [5/4 x^2/b - 5/6 x^3/b^2]_0^b Fb 1/EJ$$

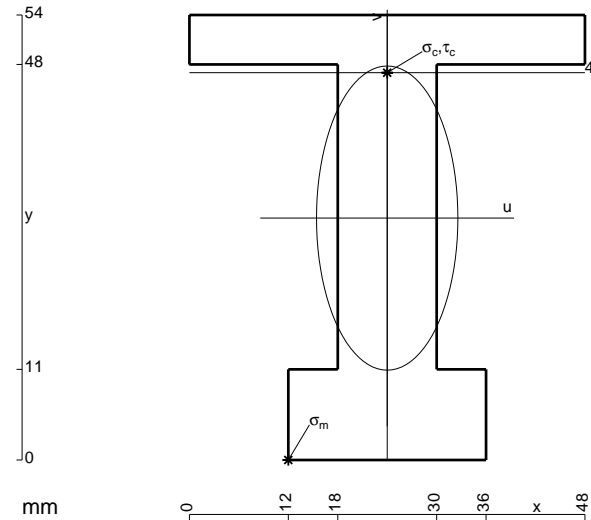
$$= (5/4 b - 5/6 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{GA}^{xo} = \int_0^b (-5/8 + 5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [-5/8 x + 5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-5/8 b + 5/8 b - 5/24 b) Fb 1/EJ = -5/24 Fb^2/EJ$$

$$L_{AG}^{xo} = \int_0^b (-5/8 x^2/b^2) Fb 1/EJ dx = [-5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-5/24 b) Fb 1/EJ = -5/24 Fb^2/EJ$$



- A = 996. mm²
- J_u = 339350. mm⁴
- J_v = 73296. mm⁴
- y_g = 29.36 mm
- T_y = -4320. N
- M_x = -2419200. Nmm
- x_m = 12. mm
- u_m = -12. mm
- v_m = -29.36 mm
- σ_m = -Mv/J_u = -209.3 N/mm²
- x_c = 24. mm
- y_c = 47. mm
- v_c = 17.64 mm
- σ_c = -Mv/J_u = 125.8 N/mm²
- τ_c = 6.844 N/mm²
- σ_q = √σ²+3τ² = 126.3 N/mm²
- S = 6451. mm³