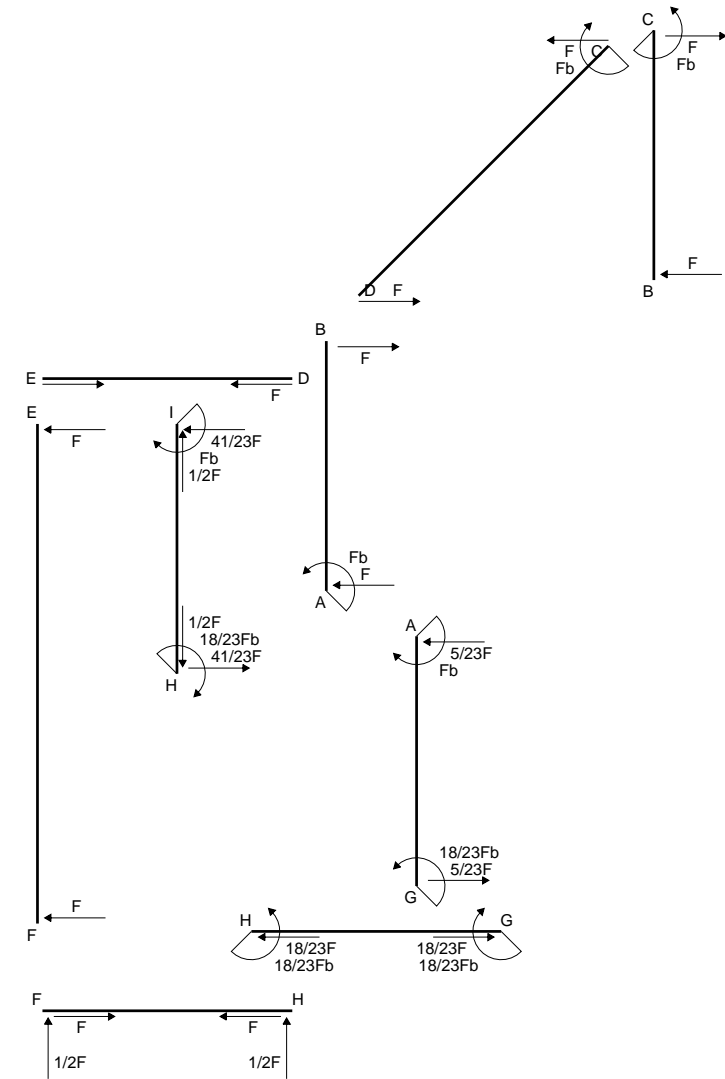
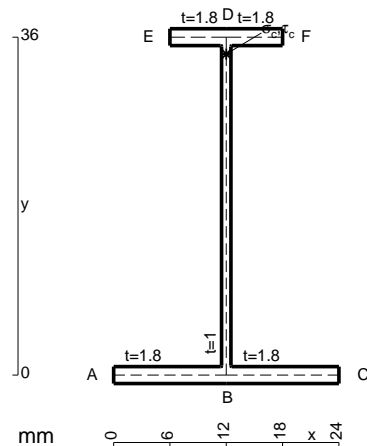
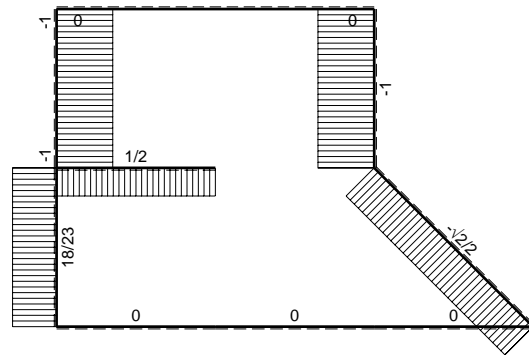


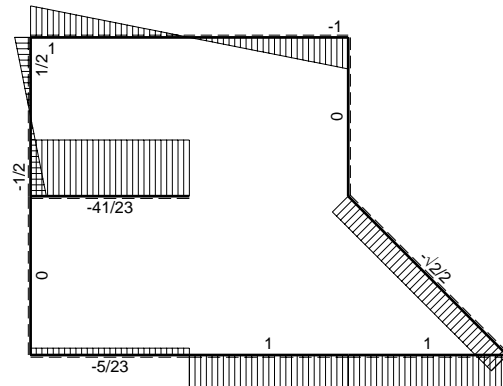
$V_G = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_I = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$p_{FH} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave AB ha la sezione riportata e dimensioni in mm, con:
 $b = 310 \text{ mm}$, $F = 690 \text{ N}$
 Calcolare sulla sezione A la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da A a B
 Elongazione termica specifica ϵ assegnata su asta HG.

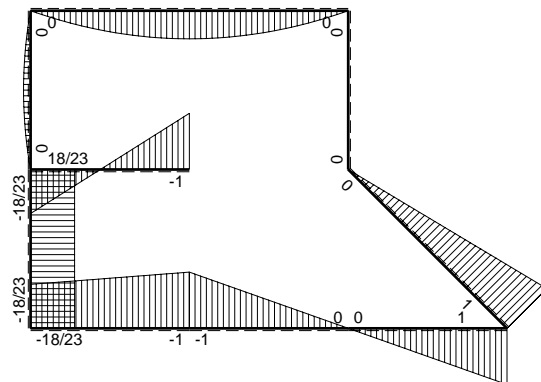




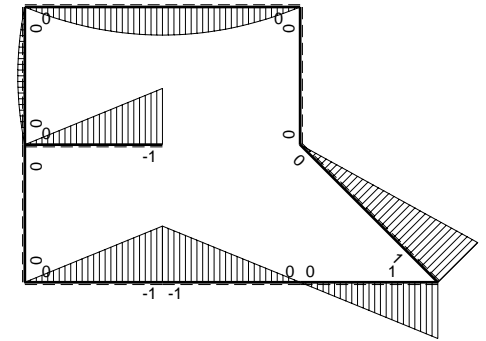
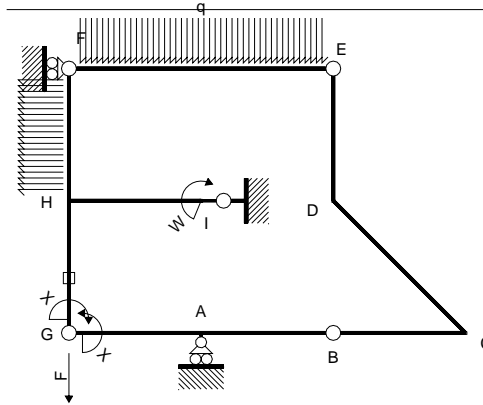
← (+) → F



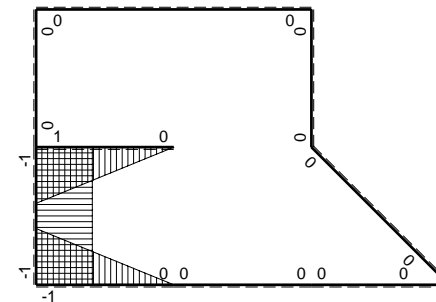
↑ (+) ↓ F



⊕ F_b



⊕ M₀ flessione da carichi assegnati



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-Fb+Fx	0	0	0	0
BA b	0	Fx	0	0		
BC b	0	Fx	0	0	0	0
CB b	0	-Fb+Fx	0	0		
CD $\sqrt{2}b$	0	Fb- $\sqrt{2}/2Fx$	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0		
EF 2b	0	-Fx+1/2qx ²	0	0	0	0
FE 2b	0	Fx-1/2qx ²	0	0		
GA b	-1+x/b	-Fx	Fx-Fx ² /b	1-2x/b+x ² /b ²	1/6Fb ² /EJ	1/3Xb/EJ
AG b	x/b	Fb-Fx	Fx-Fx ² /b	x ² /b ²		
FH b	0	1/2Fx-1/2qx ²	0	0	0	0
HF b	0	-1/2Fx+1/2qx ²	0	0		
HI b	1-x/b	-Fx	-Fx+Fx ² /b	1-2x/b+x ² /b ²	-1/6Fb ² /EJ	1/3Xb/EJ
IH b	-x/b	Fb-Fx	-Fx+Fx ² /b	x ² /b ²		
HG b	-1	0	0	1	0	Xb/EJ
GH b	1	0	0	1		
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				-Fb ² /EJ	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				-1/2Fb ² /EJ	1/4Xb/EJ
	totali				-3/2Fb ² /EJ	23/12Xb/EJ
	iperstatica $X=W_{GA}$				18/23Fb	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GA}^{Xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{AG}^{Xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

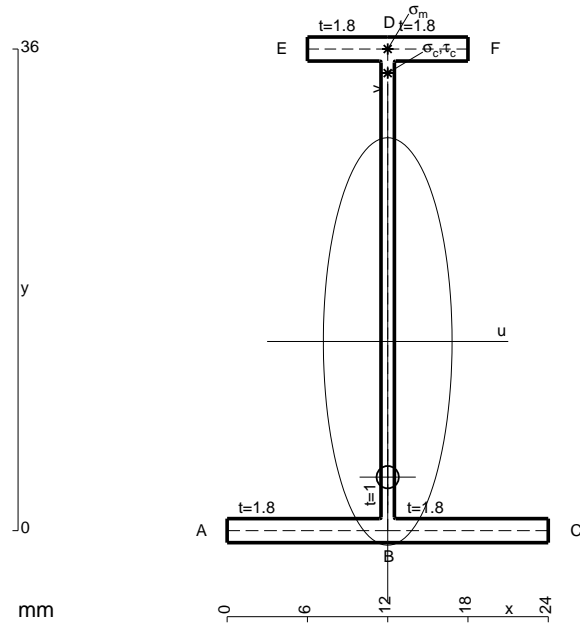
$$= (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{HI}^{Xo} = \int_0^b (-x/b + x^2/b^2) Fb 1/EJ dx = [-1/2 x^2/b + 1/3 x^3/b^2]_0^b Fb 1/EJ$$

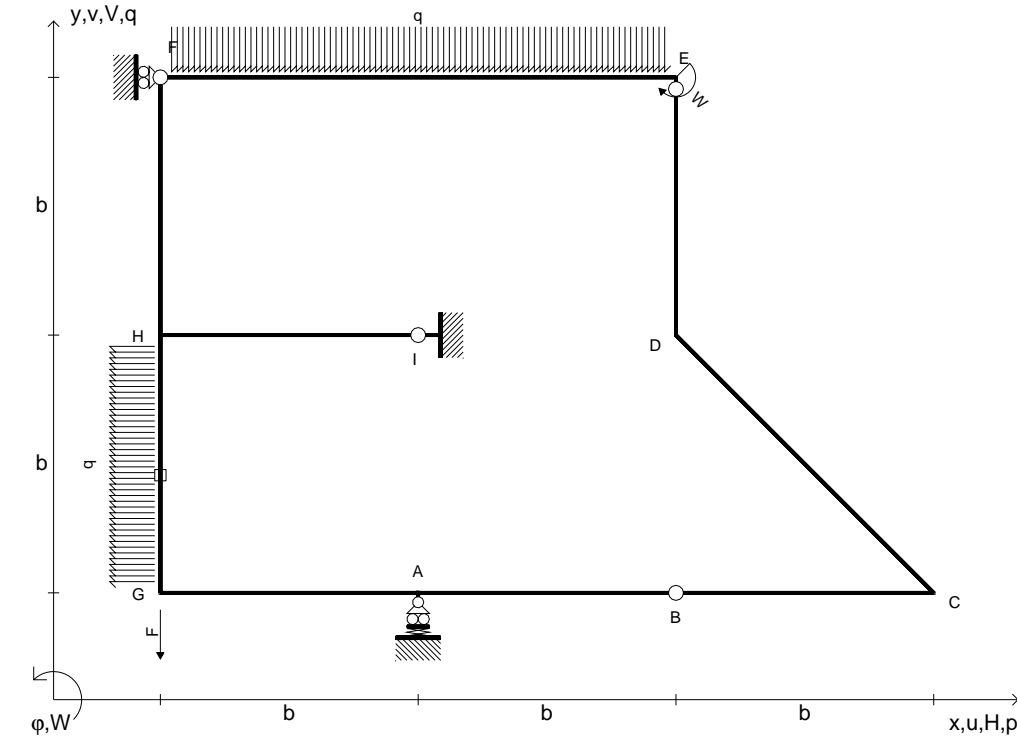
$$= (-1/2 b + 1/3 b) Fb 1/EJ = -1/6 Fb^2/EJ$$

$$L_{IH}^{Xo} = \int_0^b (-x/b + x^2/b^2) Fb 1/EJ dx = [-1/2 x^2/b + 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-1/2 b + 1/3 b) Fb 1/EJ = -1/6 Fb^2/EJ$$

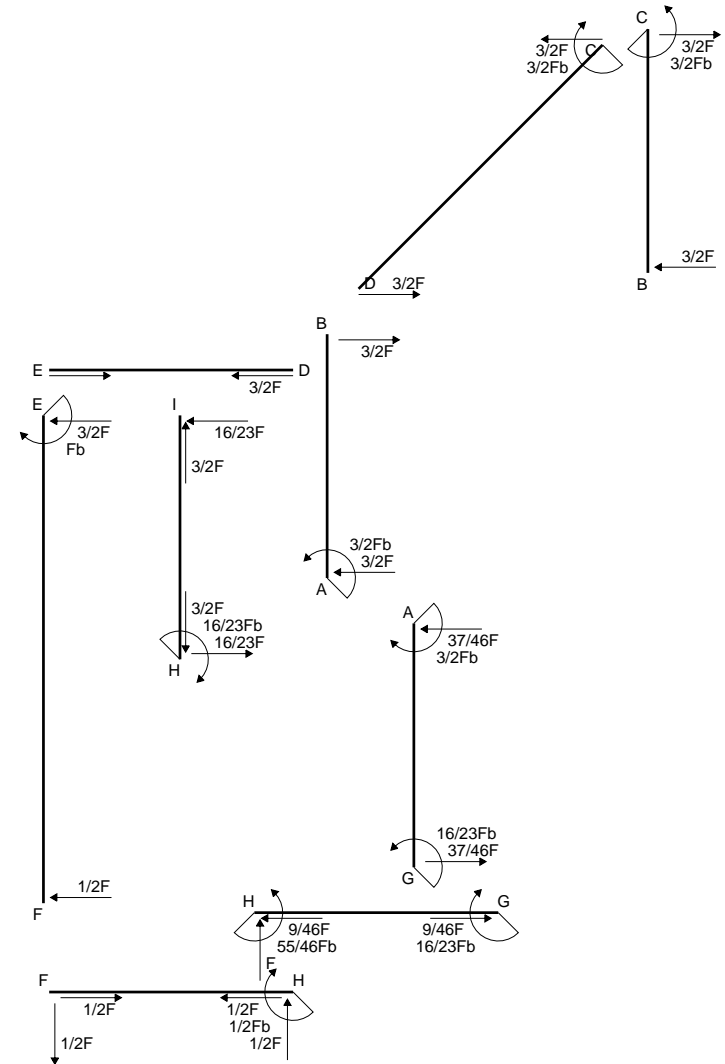
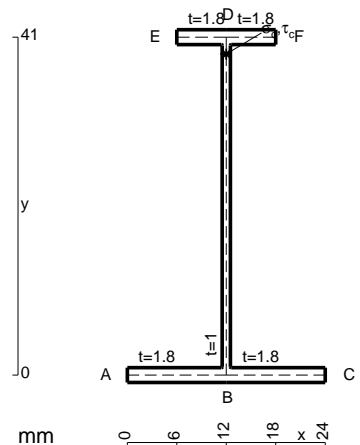


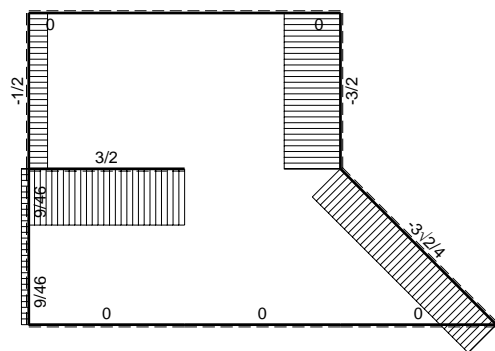
- $A = 100.8 \text{ mm}^2$
- $J_u = 23384. \text{ mm}^4$
- $J_v = 2333. \text{ mm}^4$
- $J_t = 81.98 \text{ mm}^4$
- $y_o = -10.14 \text{ mm}$
- $y_g = 14.14 \text{ mm}$
- $T_y = 690. \text{ N}$
- $M_x = -213900. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $y_m = 36. \text{ mm}$
- $v_m = 21.86 \text{ mm}$
- $\sigma_m = -Mv/J_u = 199.9 \text{ N/mm}^2$
- $y_c = 2. \text{ mm}$
- $u_c = -12. \text{ mm}$
- $v_c = -12.14 \text{ mm}$
- $\sigma_c = -Mv/J_u = 199.9 \text{ N/mm}^2$
- $\tau_c = TS/tJ_u = 13.93 \text{ N/mm}^2$
- $\tau_g = TS/tJ_u = 13.93 \text{ N/mm}^2$
- $t_c = 690. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2+3\tau^2} = 201.4 \text{ N/mm}^2$



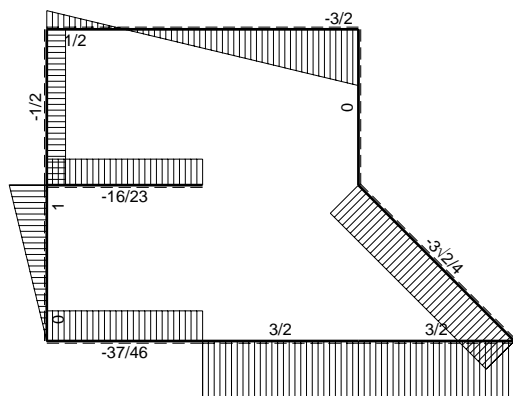
$V_G = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_E = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$p_{HG} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave AB ha la sezione riportata e dimensioni in mm, con:
 $b = 380 \text{ mm}$, $F = 460 \text{ N}$
 Calcolare sulla sezione A la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da A a B
 Elongazione termica specifica ϵ assegnata su asta HG.

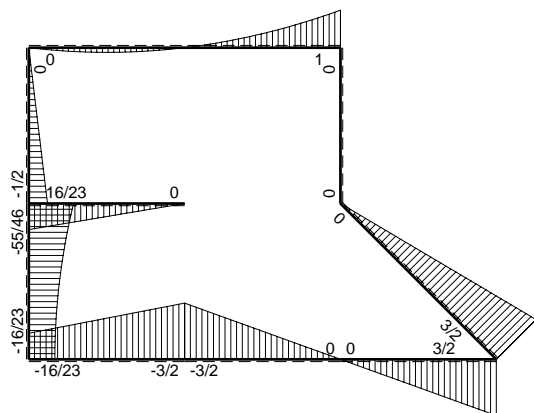




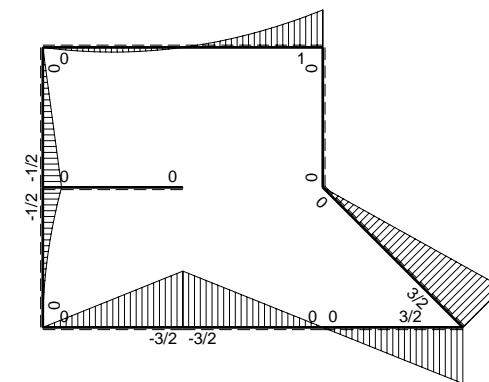
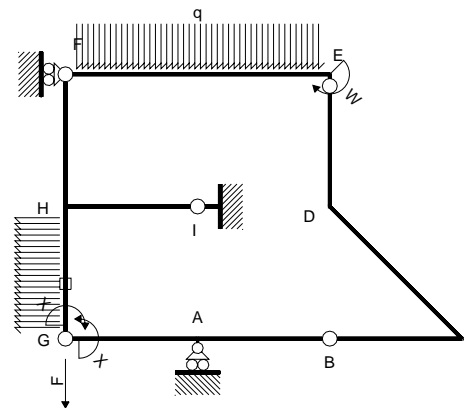
← (+) → F



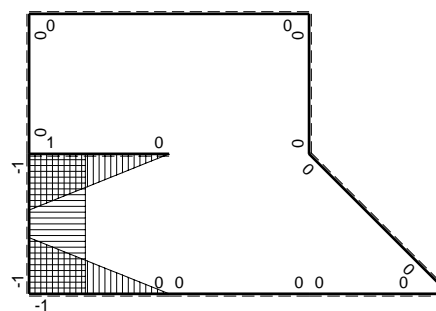
↑ (+) ↓ F



⊕ (+) ⊖ F_b



⊕ (+) ⊖ M₀ flessione da carichi assegnati



⊕ (+) ⊖ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	$-3/2Fb+3/2Fx$	0	0	0	0
BA b	0	$3/2Fx$	0	0	0	0
BC b	0	$3/2Fx$	0	0	0	0
CB b	0	$-3/2Fb+3/2Fx$	0	0	0	0
CD $\sqrt{2}b$	0	$3/2Fb-3\sqrt{2}/4Fx$	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0	0	0
EF 2b	0	$Fb-3/2Fx+1/2qx^2$	0	0	0	0
FE 2b	0	$1/2Fx-1/2qx^2$	0	0	0	0
GA b	$-1+x/b$	$-3/2Fx$	$3/2Fx-3/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/4Fb^2/EJ$	$1/3Xb/EJ$
AG b	x/b	$3/2Fb-3/2Fx$	$3/2Fx-3/2Fx^2/b$	x^2/b^2		
FH b	0	$-1/2Fx$	0	0	0	0
HF b	0	$1/2Fb-1/2Fx$	0	0	0	0
HI b	$1-x/b$	0	0	$1-2x/b+x^2/b^2$	0	$1/3Xb/EJ$
IH b	$-x/b$	0	0	x^2/b^2		
HG b	-1	$-1/2Fb+Fx-1/2qx^2$	$1/2Fb-Fx+1/2Fx^2/b$	1	$1/6Fb^2/EJ$	Xb/EJ
GH b	1	$1/2qx^2$	$1/2Fx^2/b$	1		
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				$-Fb^2/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-3/4Fb^2/EJ$	$1/4Xb/EJ$
	totali				$-4/3Fb^2/EJ$	$23/12Xb/EJ$
	iperstatica $X=W_{GA}$				$16/23Fb$	

Sviluppi di calcolo iperstatica

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GA}^{xo} = \int_0^b (3/2 x/b - 3/2 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/2 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/2 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{AG}^{xo} = \int_0^b (3/2 x/b - 3/2 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/2 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/2 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

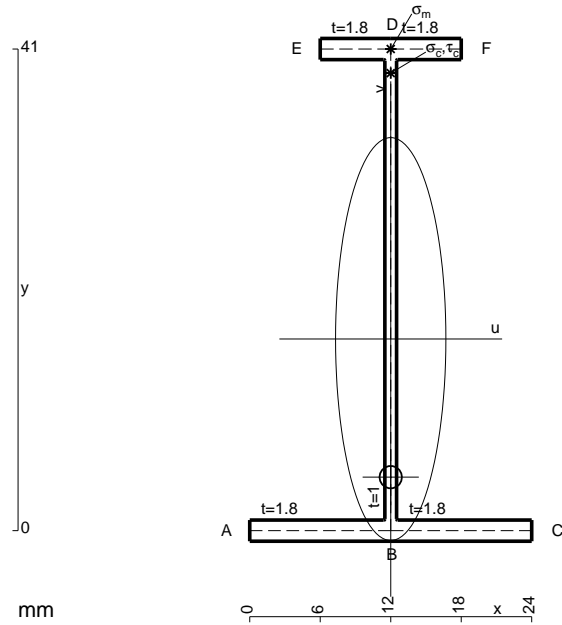
$$L_{HG}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ$$

$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

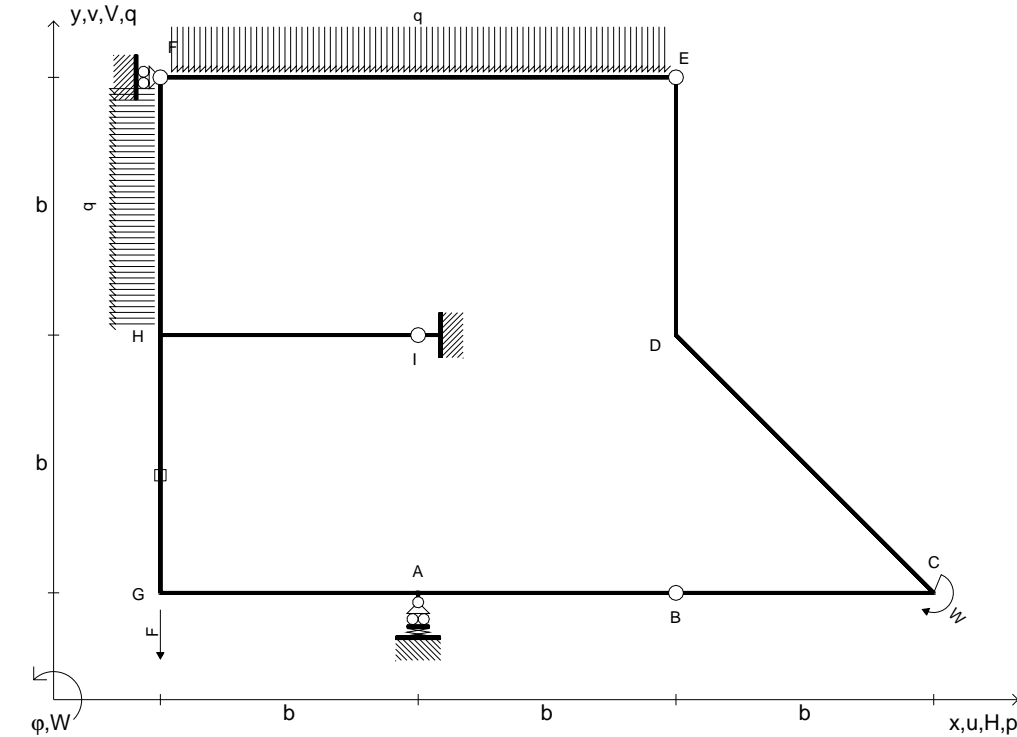
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -5/6 Fb^2/EJ$$

$$L_{GH}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ = [1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

$$= (1/6 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -5/6 Fb^2/EJ$$

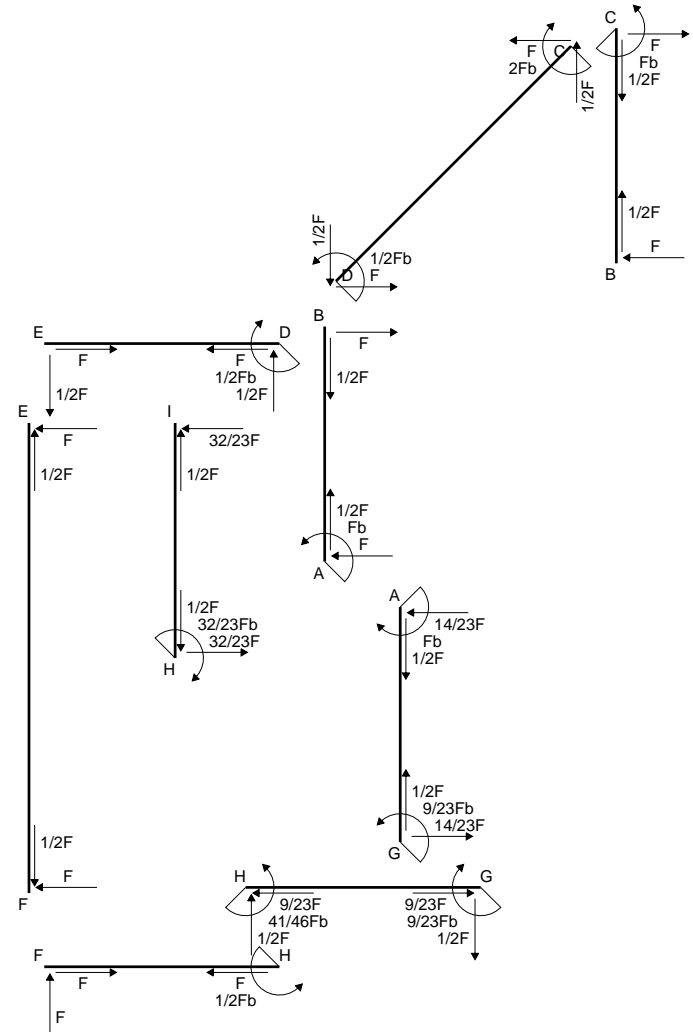
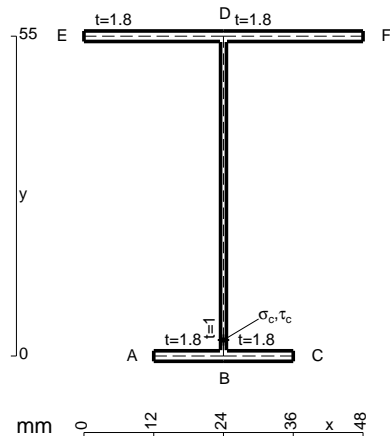


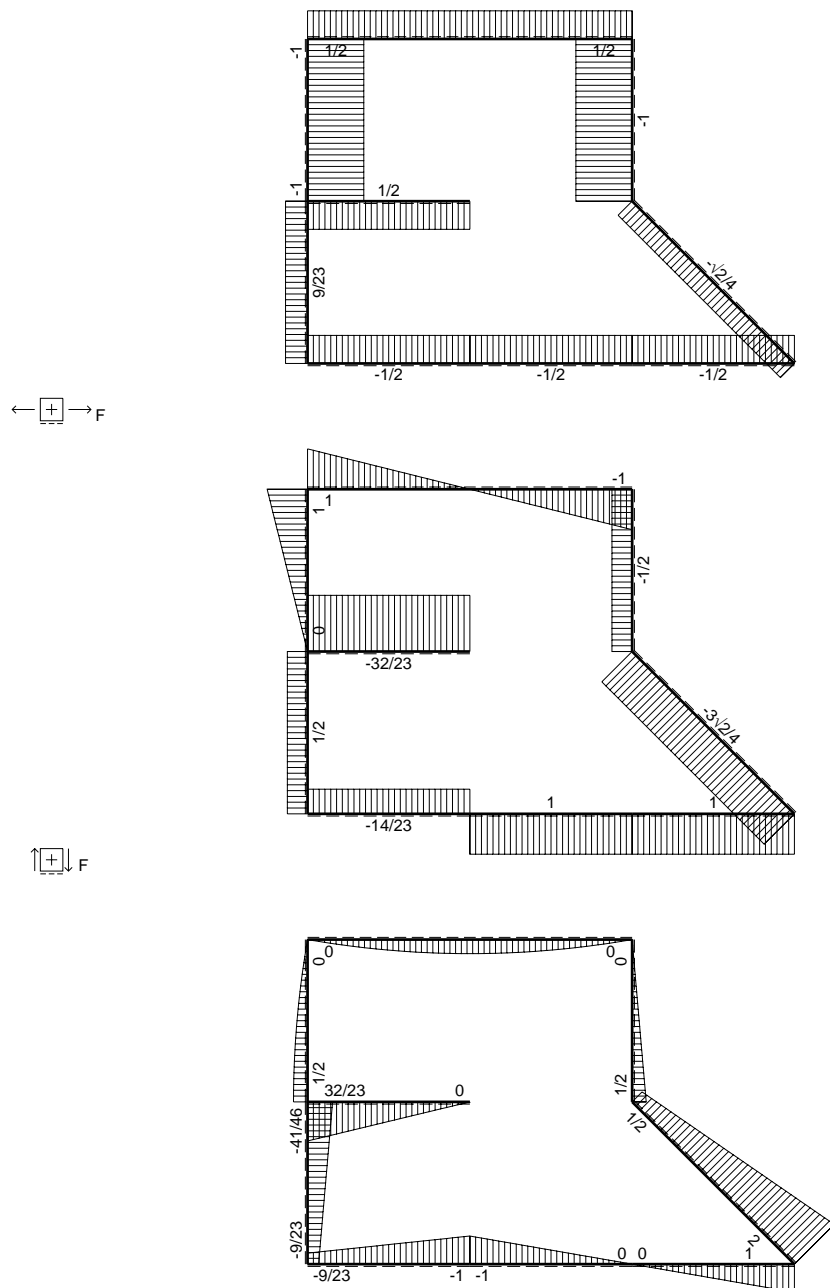
- $A = 105.8 \text{ mm}^2$
- $J_u = 31122. \text{ mm}^4$
- $J_v = 2333. \text{ mm}^4$
- $J_x = 83.65 \text{ mm}^4$
- $y_o = -11.76 \text{ mm}$
- $y_g = 16.31 \text{ mm}$
- $T_y = 690. \text{ N}$
- $M_x = -262200. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $y_m = 41. \text{ mm}$
- $v_m = 24.69 \text{ mm}$
- $\sigma_m = -Mv/J_u = 208. \text{ N/mm}^2$
- $y_c = 2. \text{ mm}$
- $u_c = -12. \text{ mm}$
- $v_c = -14.31 \text{ mm}$
- $\sigma_c = -Mv/J_u = 208. \text{ N/mm}^2$
- $\tau_c = TS^*/tJ_u = 11.82 \text{ N/mm}^2$
- $\tau_g = TS^*/tJ_u = 11.82 \text{ N/mm}^2$
- $t_c = 460. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 209. \text{ N/mm}^2$



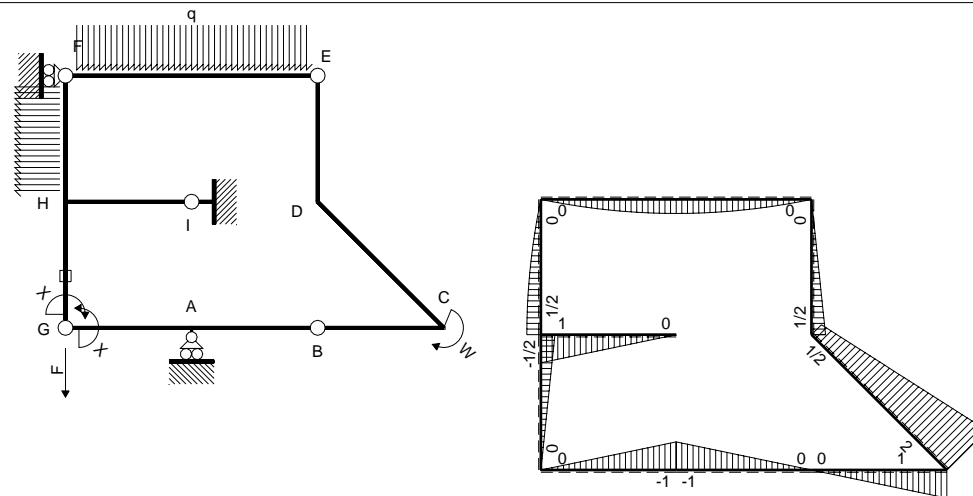
$V_G = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_C = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$p_{FH} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave CD ha la sezione riportata e dimensioni in mm, con:
 $b = 510 \text{ mm}$, $F = 630 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da C a D
 Elongazione termica specifica ϵ assegnata su asta HG.

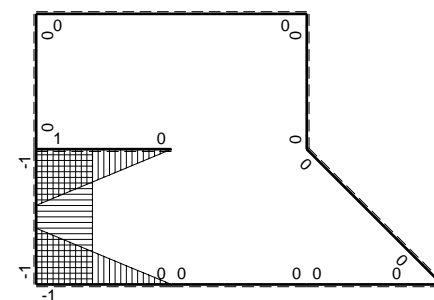




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M_0 flessione da carichi assegnati



M_x flessione da iperstatica $X=1$

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Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-Fb+Fx	0	0	0	0
BA b	0	Fx	0	0	0	0
BC b	0	Fx	0	0	0	0
CB b	0	-Fb+Fx	0	0	0	0
CD $\sqrt{2}b$	0	$2Fb-3\sqrt{2}/4Fx$	0	0	0	0
DE b	0	$1/2Fb-1/2Fx$	0	0	0	0
ED b	0	-1/2Fx	0	0	0	0
EF 2b	0	$-Fx+1/2qx^2$	0	0	0	0
FE 2b	0	$Fx-1/2qx^2$	0	0	0	0
GA b	-1+x/b	-Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
AG b	x/b	Fb-Fx	$Fx-Fx^2/b$	x^2/b^2		
FH b	0	$Fx-1/2qx^2$	0	0	0	0
HF b	0	$-1/2Fb+1/2qx^2$	0	0	0	0
HI b	1-x/b	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
IH b	-x/b	-Fx	Fx^2/b	x^2/b^2		
HG b	-1	$-1/2Fb+1/2Fx$	$1/2Fb-1/2Fx$	1	$1/4Fb^2/EJ$	Xb/EJ
GH b	1	$1/2Fx$	$1/2Fx$	1		
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				$-Fb^2/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-1/2Fb^2/EJ$	$1/4Xb/EJ$
	totali				$-3/4Fb^2/EJ$	$23/12Xb/EJ$
	iperstatica $X=W_{GA}$				$9/23Fb$	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GA}^{Xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{AG}^{Xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{HI}^{Xo} = \int_0^b (1 - 2x/b + x^2/b^2) Fb 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (b - b + 1/3 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{IH}^{Xo} = \int_0^b (x^2/b^2) Fb 1/EJ dx = [1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/3 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

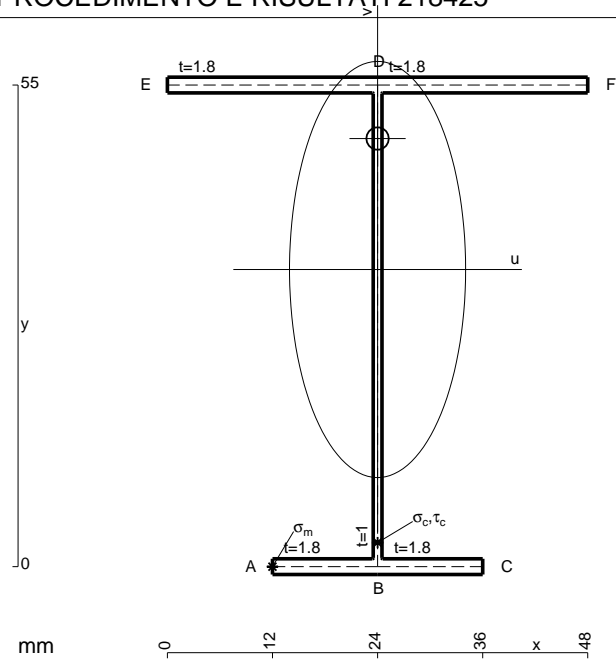
$$L_{HG}^{Xo} = \int_0^b (1/2 - 1/2 x/b) Fb 1/EJ dx - 1 \cdot (-1) \cdot (-1) Fb^2/EJ$$

$$= [1/2 x - 1/4 x^2/b]_0^b Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ$$

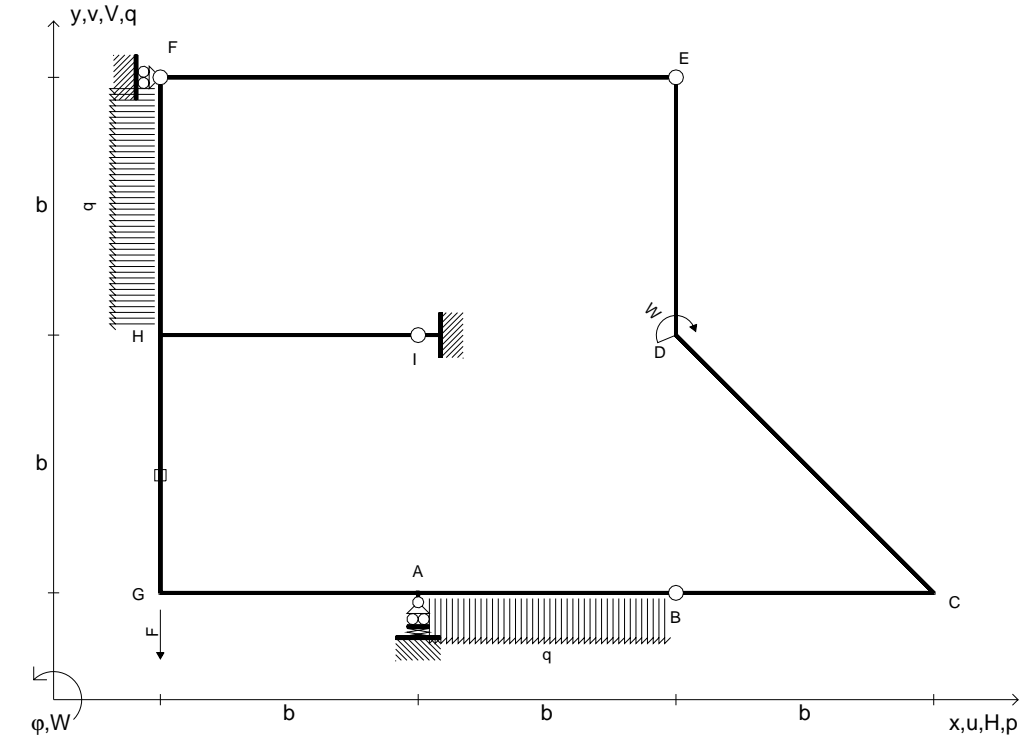
$$= (1/2 b - 1/4 b) Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ = -3/4 Fb^2/EJ$$

$$L_{GH}^{Xo} = \int_0^b (1/2 x/b) Fb 1/EJ dx - 1 \cdot (-1) \cdot (-1) Fb^2/EJ = [1/4 x^2/b]_0^b Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ$$

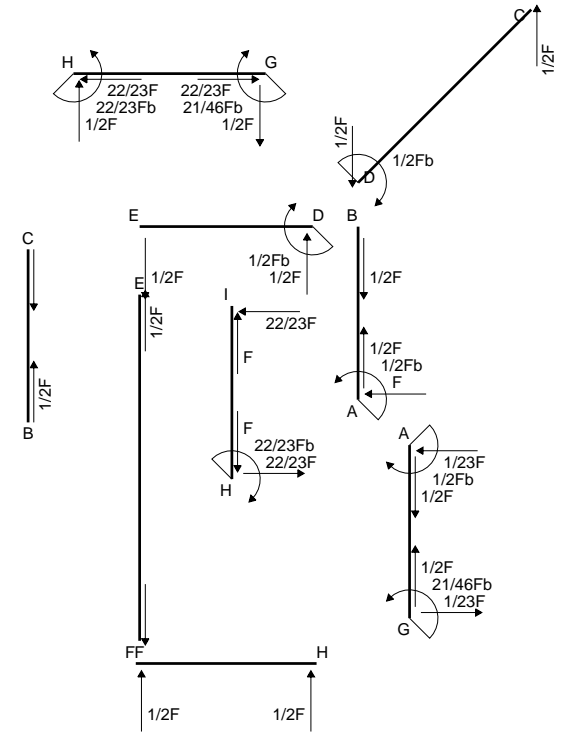
$$= (1/4 b) Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ = -3/4 Fb^2/EJ$$



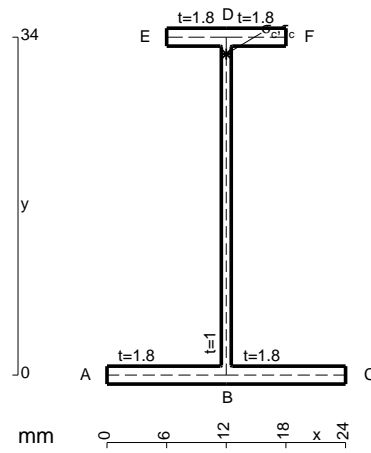
- $A = 184.6 \text{ mm}^2$
- $J_u = 104229. \text{ mm}^4$
- $J_v = 18662. \text{ mm}^4$
- $J_t = 158.3 \text{ mm}^4$
- $y_o = 14.95 \text{ mm}$
- $y_g = 33.94 \text{ mm}$
- $N = -222.7 \text{ N}$
- $T_y = -668.2 \text{ N}$
- $M_x = 642600. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $u_m = -12. \text{ mm}$
- $v_m = -33.94 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = 208. \text{ N/mm}^2$
- $x_c = 24. \text{ mm}$
- $v_c = -33.94 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = 208. \text{ N/mm}^2$
- $\tau_c = TS'/tJ_u = 9.399 \text{ N/mm}^2$
- $\tau_g = TS'/tJ_u = 9.399 \text{ N/mm}^2$
- $t_c = 630. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 208.7 \text{ N/mm}^2$

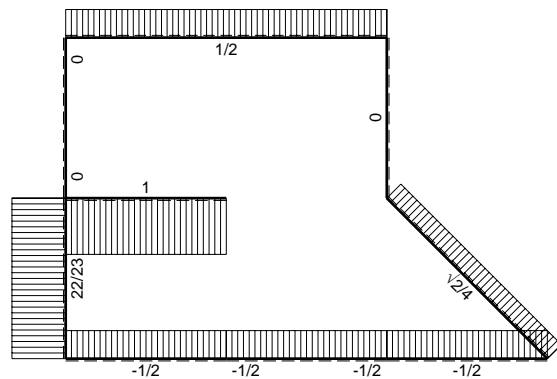


$V_G = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_D = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$q_{AB} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$p_{FH} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

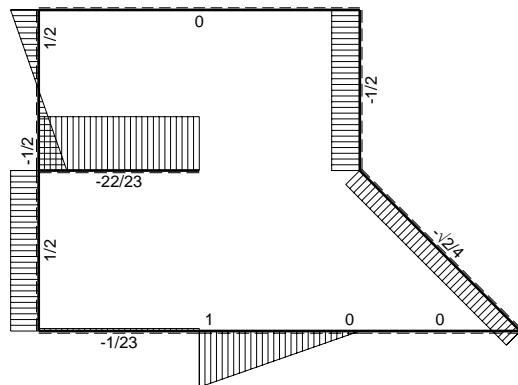


Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave AB ha la sezione riportata e dimensioni in mm, con:
 $b = 340$ mm, $F = 1320$ N
 Calcolare sulla sezione A la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da A a B
 Elongazione termica specifica ϵ assegnata su asta HG.
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

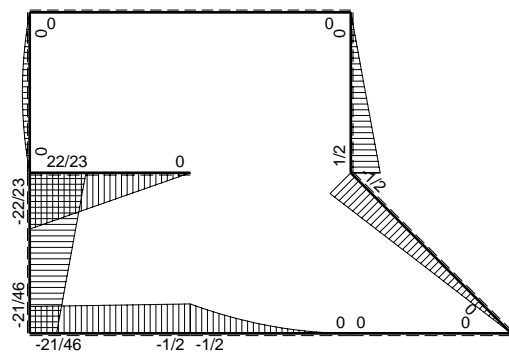




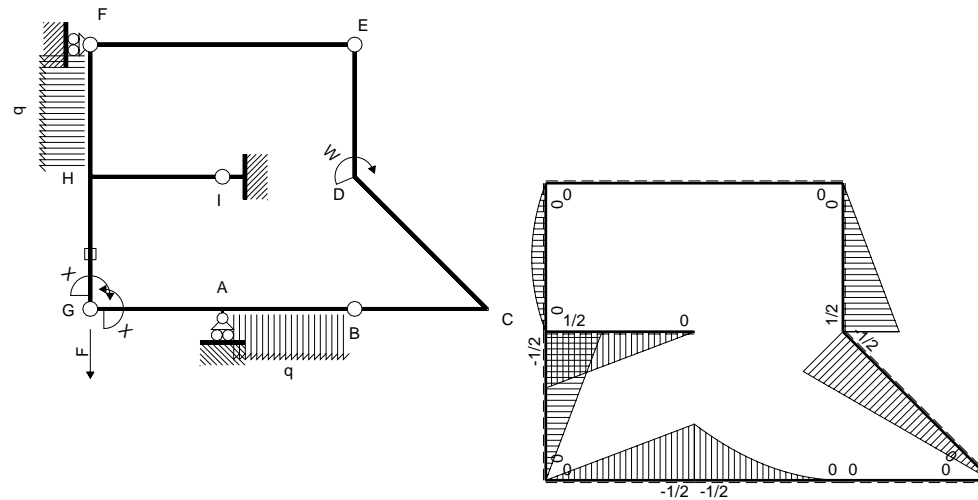
← ⊕ → F



↑ ⊕ ↓ F

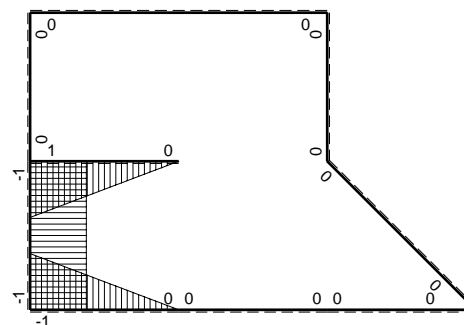


⊕ ↺ F_b



Schema di calcolo iperstatico

⊕ ↺ M₀ flessione da carichi assegnati



⊕ ↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	$-1/2Fb + Fx - 1/2qx^2$	0	0	0	0
BA b	0	$1/2qx^2$	0	0	0	0
BC b	0	0	0	0	0	0
CB b	0	0	0	0	0	0
CD $\sqrt{2}b$	0	$-\sqrt{2}/4Fx$	0	0	0	0
DE b	0	$1/2Fb - 1/2Fx$	0	0	0	0
ED b	0	$-1/2Fx$	0	0	0	0
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0	0	0
GA b	$-1+x/b$	$-1/2Fx$	$1/2Fx - 1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/12Fb^2/EJ$	$1/3Xb/EJ$
AG b	x/b	$1/2Fb - 1/2Fx$	$1/2Fx - 1/2Fx^2/b$	x^2/b^2		
FH b	0	$1/2Fx - 1/2qx^2$	0	0	0	0
HF b	0	$-1/2Fx + 1/2qx^2$	0	0	0	0
HI b	$1-x/b$	$1/2Fb - 1/2Fx$	$1/2Fb - Fx + 1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
IH b	$-x/b$	$-1/2Fx$	$1/2Fx^2/b$	x^2/b^2		
HG b	-1	$-1/2Fb + 1/2Fx$	$1/2Fb - 1/2Fx$	1	$1/4Fb^2/EJ$	Xb/EJ
GH b	1	$1/2Fx$	$1/2Fx$	1		
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				$-Fb^2/EJ$	
A	molla nodo $-V_{1A}(V_{oA} + XV_{1A})/k_A$				$-3/8Fb^2/EJ$	$1/4Xb/EJ$
	totali				$-7/8Fb^2/EJ$	$23/12Xb/EJ$
	iperstatica $X=W_{GA}$				$21/46Fb$	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GA}^{Xo} = \int_0^b (1/2 x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/4 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/6 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{AG}^{Xo} = \int_0^b (1/2 x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/4 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/6 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{HI}^{Xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{IH}^{Xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

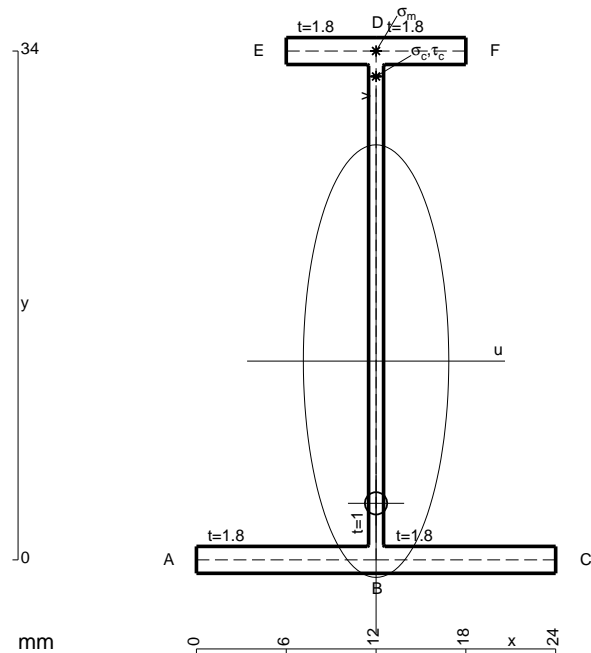
$$L_{HG}^{Xo} = \int_0^b (1/2 - 1/2 x/b) Fb 1/EJ dx - 1 \cdot (-1) \cdot (-1) Fb^2/EJ$$

$$= [1/2 x - 1/4 x^2/b]_0^b Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ$$

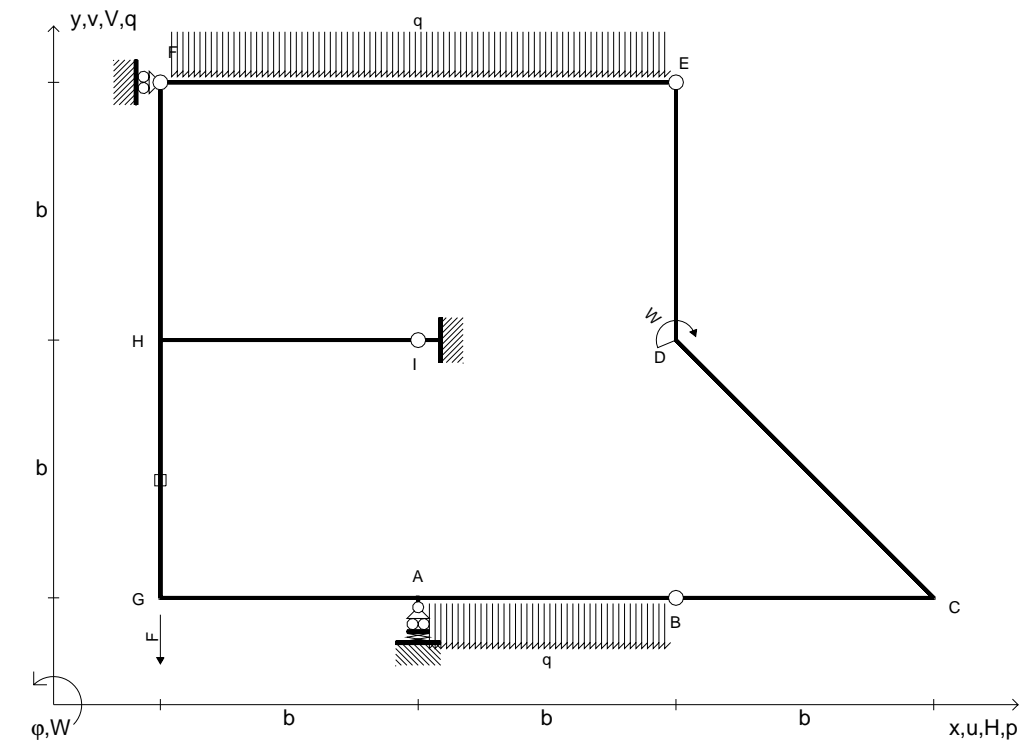
$$= (1/2 b - 1/4 b) Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ = -3/4 Fb^2/EJ$$

$$L_{GH}^{Xo} = \int_0^b (1/2 x/b) Fb 1/EJ dx - 1 \cdot (-1) \cdot (-1) Fb^2/EJ = [1/4 x^2/b]_0^b Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ$$

$$= (1/4 b) Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ = -3/4 Fb^2/EJ$$

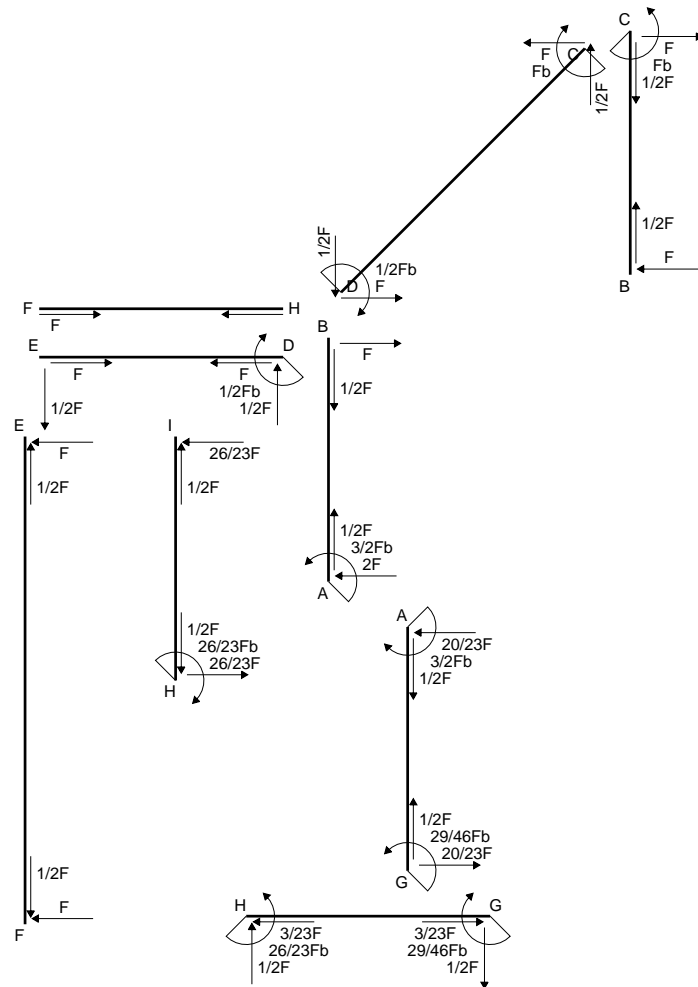
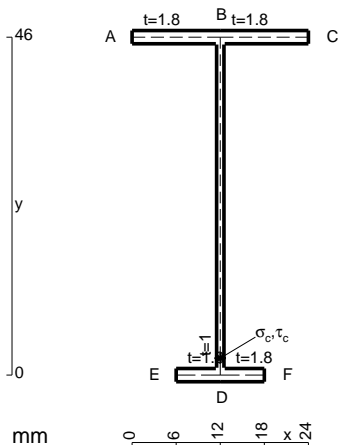


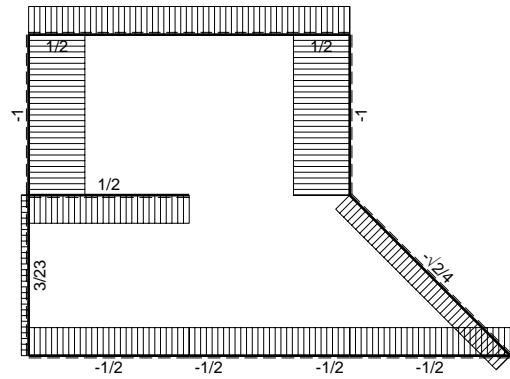
- $A = 98.8 \text{ mm}^2$
- $J_u = 20638. \text{ mm}^4$
- $J_v = 2333. \text{ mm}^4$
- $J_t = 81.32 \text{ mm}^4$
- $y_o = -9.506 \text{ mm}$
- $y_g = 13.28 \text{ mm}$
- $N = -660. \text{ N}$
- $T_y = 1320. \text{ N}$
- $M_x = -224400. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $y_m = 34. \text{ mm}$
- $v_m = 20.72 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = 218.6 \text{ N/mm}^2$
- $y_c = 2. \text{ mm}$
- $u_c = -12. \text{ mm}$
- $v_c = -11.28 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = 218.6 \text{ N/mm}^2$
- $\tau_c = TS'/tJ_u = 28.62 \text{ N/mm}^2$
- $\tau_g = TS'/tJ_u = 28.62 \text{ N/mm}^2$
- $t_c = 1320. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 224.1 \text{ N/mm}^2$



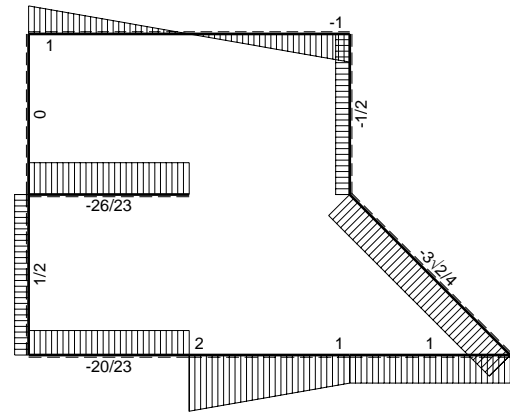
$V_G = -F$ $\epsilon_{HG} = -\alpha T = -b^2 F / EJ$ $EJ_{CD} = EJ$ $EJ_{FH} = EJ$
 $W_D = -W = -Fb$ $k_A = 4EJ/b^3$ $EJ_{DE} = EJ$ $EJ_{HI} = EJ$
 $q_{EF} = -q = -F/b$ $EJ_{AB} = EJ$ $EJ_{EF} = EJ$ $EJ_{HG} = EJ$
 $q_{AB} = -q = -F/b$ $EJ_{BC} = EJ$ $EJ_{GA} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave AB ha la sezione riportata e dimensioni in mm, con:
 $b = 660$ mm, $F = 320$ N
 Calcolare sulla sezione A la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da A a B
 Elongazione termica specifica ϵ assegnata su asta HG.

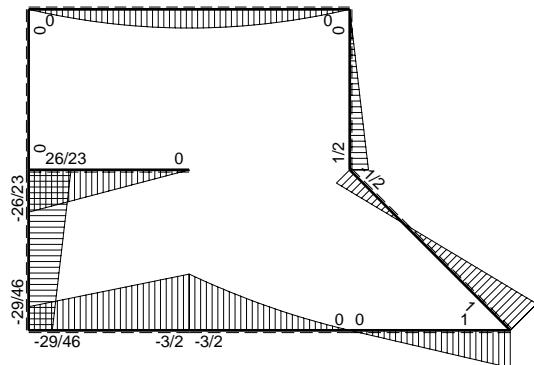




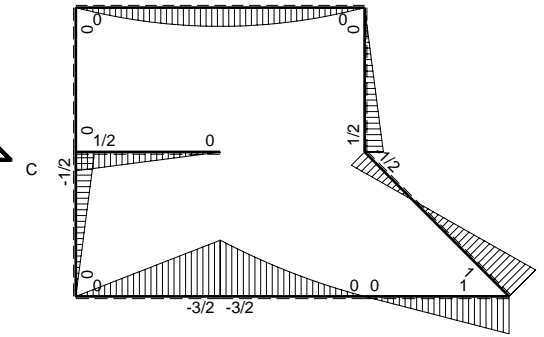
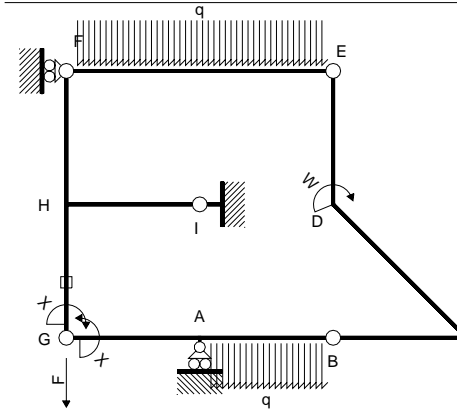
$\left[\begin{matrix} + \\ - \end{matrix} \right] \rightarrow F$



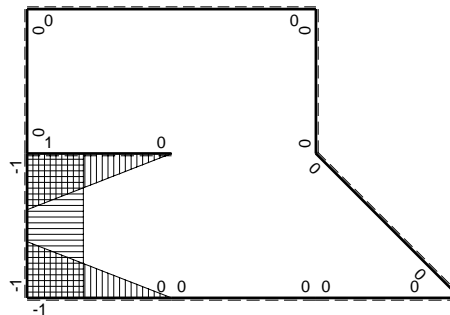
$\left[\begin{matrix} + \\ - \end{matrix} \right] \uparrow F$



$\left[\begin{matrix} + \\ - \end{matrix} \right] \curvearrowright F_b$



$\left[\begin{matrix} + \\ - \end{matrix} \right] \curvearrowright M_0$ flessione da carichi assegnati



$\left[\begin{matrix} + \\ - \end{matrix} \right] \curvearrowright M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	$-3/2Fb+2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fx+1/2qx^2$	0	0	0	0
BC b	0	Fx	0	0	0	0
CB b	0	$-Fb+Fx$	0	0	0	0
CD $\sqrt{2}b$	0	$Fb-3\sqrt{2}/4Fx$	0	0	0	0
DE b	0	$1/2Fb-1/2Fx$	0	0	0	0
ED b	0	$-1/2Fx$	0	0	0	0
EF 2b	0	$-Fx+1/2qx^2$	0	0	0	0
FE 2b	0	$Fx-1/2qx^2$	0	0	0	0
GA b	$-1+x/b$	$-3/2Fx$	$3/2Fx-3/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/4Fb^2/EJ$	$1/3Xb/EJ$
AG b	x/b	$3/2Fb-3/2Fx$	$3/2Fx-3/2Fx^2/b$	x^2/b^2		
FH b	0	0	0	0	0	0
HF b	0	0	0	0	0	0
HI b	$1-x/b$	$1/2Fb-1/2Fx$	$1/2Fb-Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
IH b	$-x/b$	$-1/2Fx$	$1/2Fx^2/b$	x^2/b^2		
HG b	-1	$-1/2Fb+1/2Fx$	$1/2Fb-1/2Fx$	1	$1/4Fb^2/EJ$	Xb/EJ
GH b	1	$1/2Fx$	$1/2Fx$	1		
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				$-Fb^2/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-7/8Fb^2/EJ$	$1/4Xb/EJ$
	totali				$-29/24Fb^2/EJ$	$23/12Xb/EJ$
	iperstatica $X=W_{GA}$				$29/46Fb$	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GA}^{Xo} = \int_0^b (3/2 x/b - 3/2 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/2 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/2 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{AG}^{Xo} = \int_0^b (3/2 x/b - 3/2 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/2 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/2 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{HI}^{Xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{IH}^{Xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

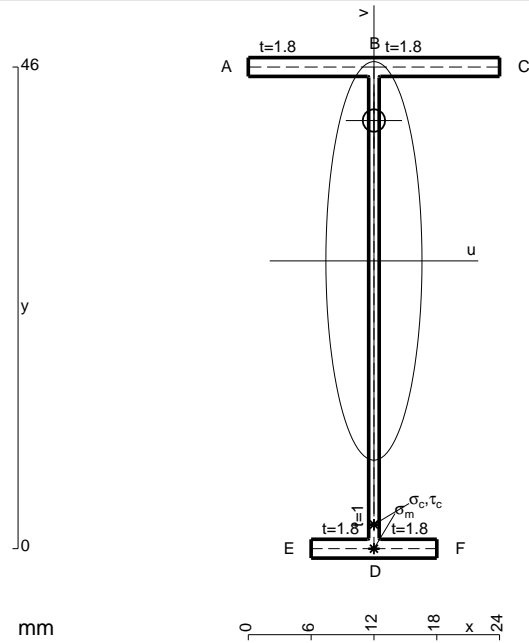
$$L_{HG}^{Xo} = \int_0^b (1/2 - 1/2 x/b) Fb 1/EJ dx - 1 \cdot (-1) \cdot (-1) Fb^2/EJ$$

$$= [1/2 x - 1/4 x^2/b]_0^b Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ$$

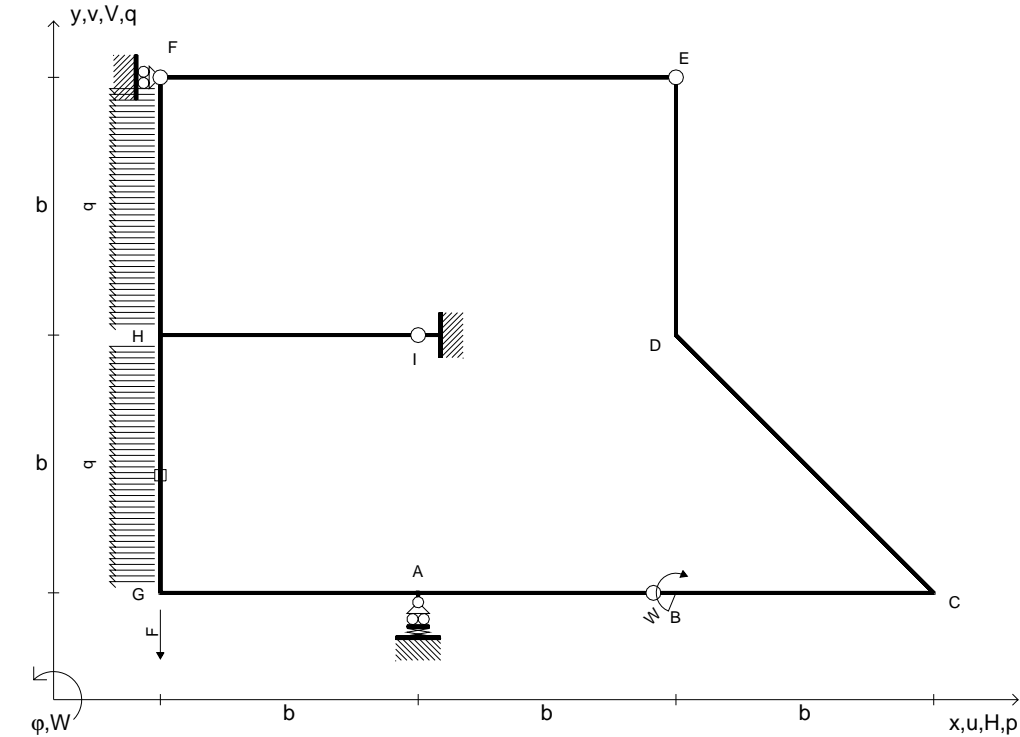
$$= (1/2 b - 1/4 b) Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ = -3/4 Fb^2/EJ$$

$$L_{GH}^{Xo} = \int_0^b (1/2 x/b) Fb 1/EJ dx - 1 \cdot (-1) \cdot (-1) Fb^2/EJ = [1/4 x^2/b]_0^b Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ$$

$$= (1/4 b) Fb 1/EJ - 1 \cdot (-1) \cdot (-1) Fb^2/EJ = -3/4 Fb^2/EJ$$

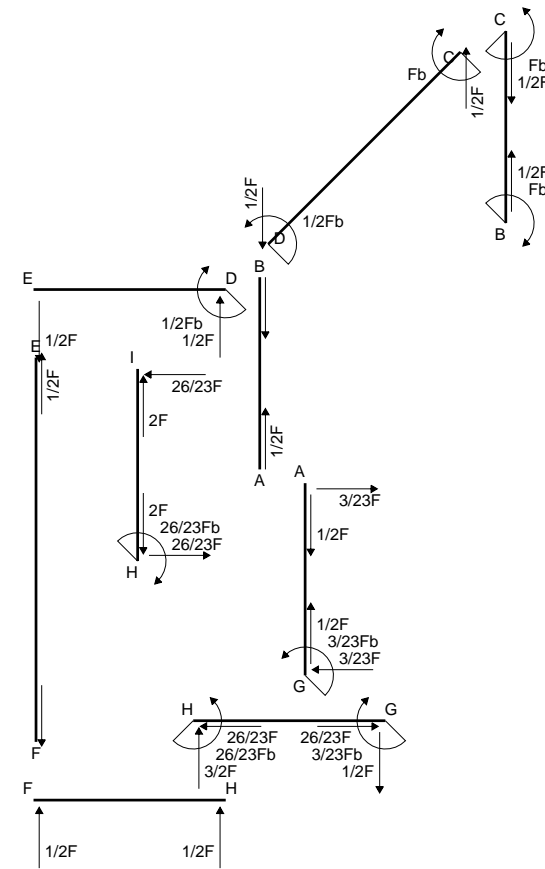
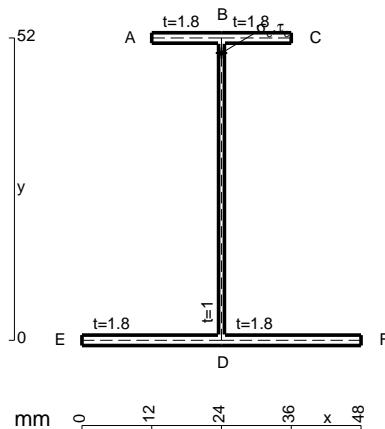


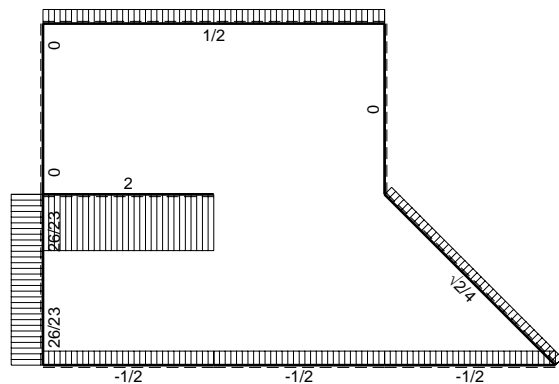
- $A = 110.8 \text{ mm}^2$
- $J_u = 40163. \text{ mm}^4$
- $J_v = 2333. \text{ mm}^4$
- $J_t = 85.32 \text{ mm}^4$
- $y_o = 13.41 \text{ mm}$
- $y_g = 27.48 \text{ mm}$
- $N = -160. \text{ N}$
- $T_y = 640. \text{ N}$
- $M_x = -316800. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $v_m = -27.48 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = -218.2 \text{ N/mm}^2$
- $y_c = 3. \text{ mm}$
- $u_c = -12. \text{ mm}$
- $v_c = -24.48 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = -218.2 \text{ N/mm}^2$
- $\tau_c = TS'/tJ_u = 9.46 \text{ N/mm}^2$
- $\tau_g = TS'/tJ_u = 9.46 \text{ N/mm}^2$
- $t_c = 320. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 218.8 \text{ N/mm}^2$



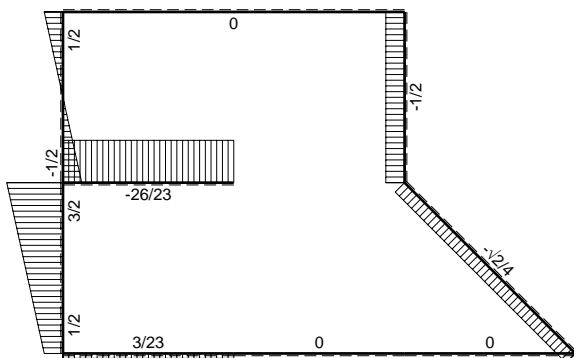
$V_G = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_B = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$p_{HG} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$p_{FH} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave CD ha la sezione riportata e dimensioni in mm, con:
 $b = 570 \text{ mm}$, $F = 1160 \text{ N}$
 Calcolare sulla sezione C la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da C a D
 Elongazione termica specifica ϵ assegnata su asta HG.

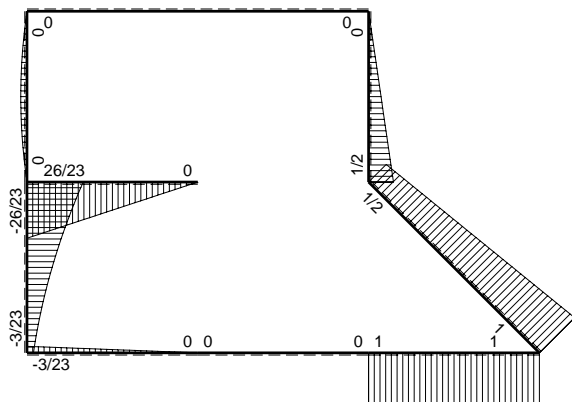




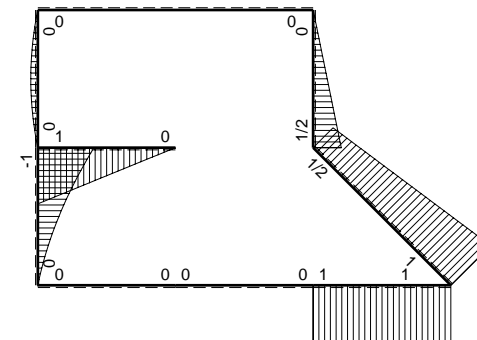
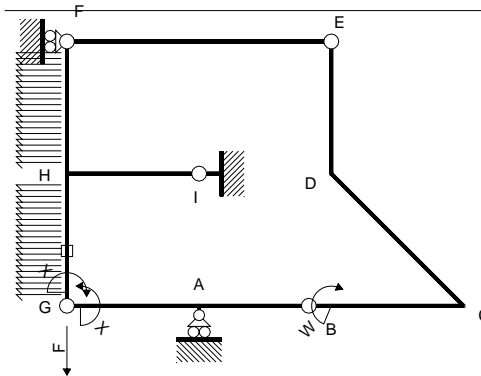
← (+) → F



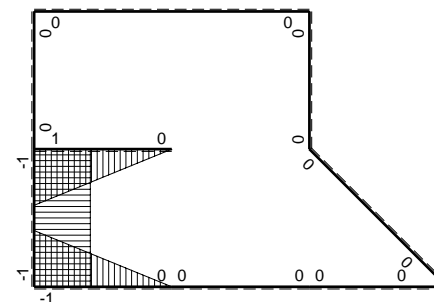
↑ (+) ↓ F



⌚ (+) ⌚ F_b



⌚ (+) ⌚ M_o flessione da carichi assegnati



⌚ (+) ⌚ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0	0	0
BC b	0	Fb	0	0	0	0
CB b	0	-Fb	0	0	0	0
CD $\sqrt{2}b$	0	$Fb - \sqrt{2}/4Fx$	0	0	0	0
DE b	0	$1/2Fb - 1/2Fx$	0	0	0	0
ED b	0	$-1/2Fx$	0	0	0	0
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0	0	0
GA b	$-1+x/b$	0	0	$1-2x/b+x^2/b^2$	0	$1/3Xb/EJ$
AG b	x/b	0	0	x^2/b^2	0	$1/3Xb/EJ$
FH b	0	$1/2Fx - 1/2qx^2$	0	0	0	0
HF b	0	$-1/2Fx + 1/2qx^2$	0	0	0	0
HI b	$1-x/b$	Fb-Fx	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
IH b	$-x/b$	-Fx	Fx^2/b	x^2/b^2	$1/3Fb^2/EJ$	$1/3Xb/EJ$
HG b	-1	$-Fb+3/2Fx-1/2qx^2$	$Fb-3/2Fx+1/2Fx^2/b$	1	$5/12Fb^2/EJ$	Xb/EJ
GH b	1	$1/2Fx+1/2qx^2$	$1/2Fx+1/2Fx^2/b$	1	$5/12Fb^2/EJ$	Xb/EJ
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				$-Fb^2/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$					$1/4Xb/EJ$
	totali				$-1/4Fb^2/EJ$	$23/12Xb/EJ$
	iperstatica $X=W_{GA}$				$3/23Fb$	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{HI}^{Xo} = \int_0^b (1 - 2x/b + x^2/b^2) Fb 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (b - b + 1/3 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{IH}^{Xo} = \int_0^b (x^2/b^2) Fb 1/EJ dx = [1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/3 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{HG}^{Xo} = \int_0^b (1 - 3/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ$$

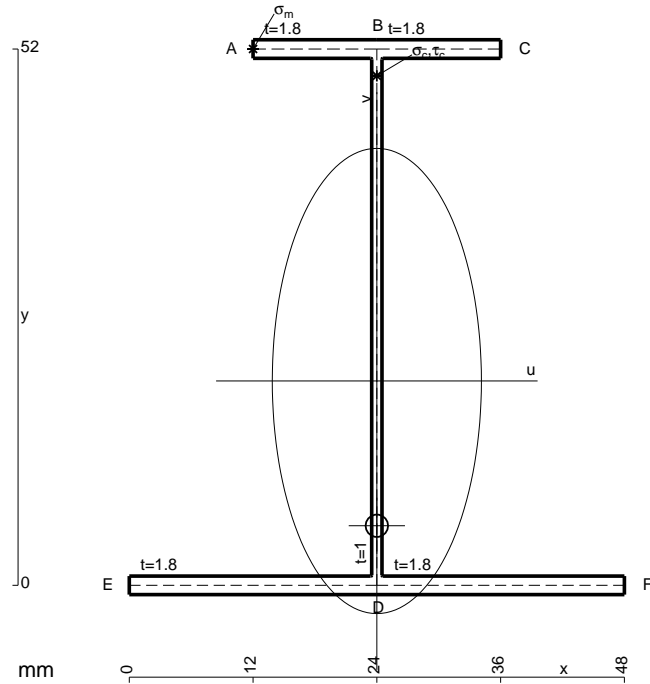
$$= [x - 3/4 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

$$= (b - 3/4 b + 1/6 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -7/12 Fb^2/EJ$$

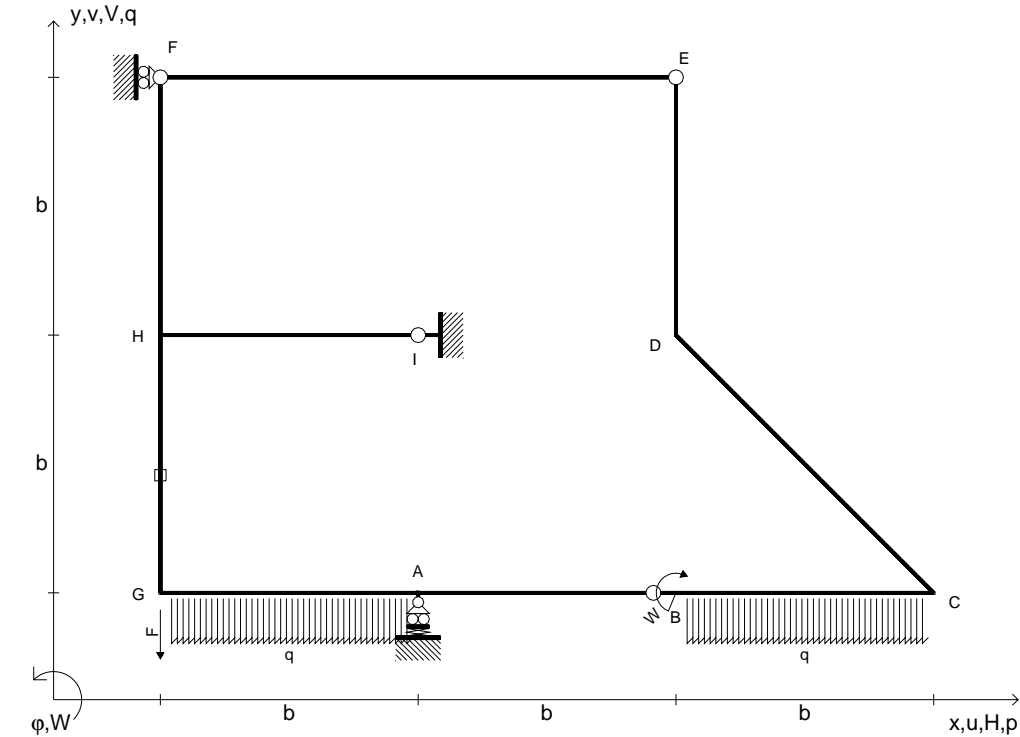
$$L_{GH}^{Xo} = \int_0^b (1/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ$$

$$= [1/4 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

$$= (1/4 b + 1/6 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -7/12 Fb^2/EJ$$



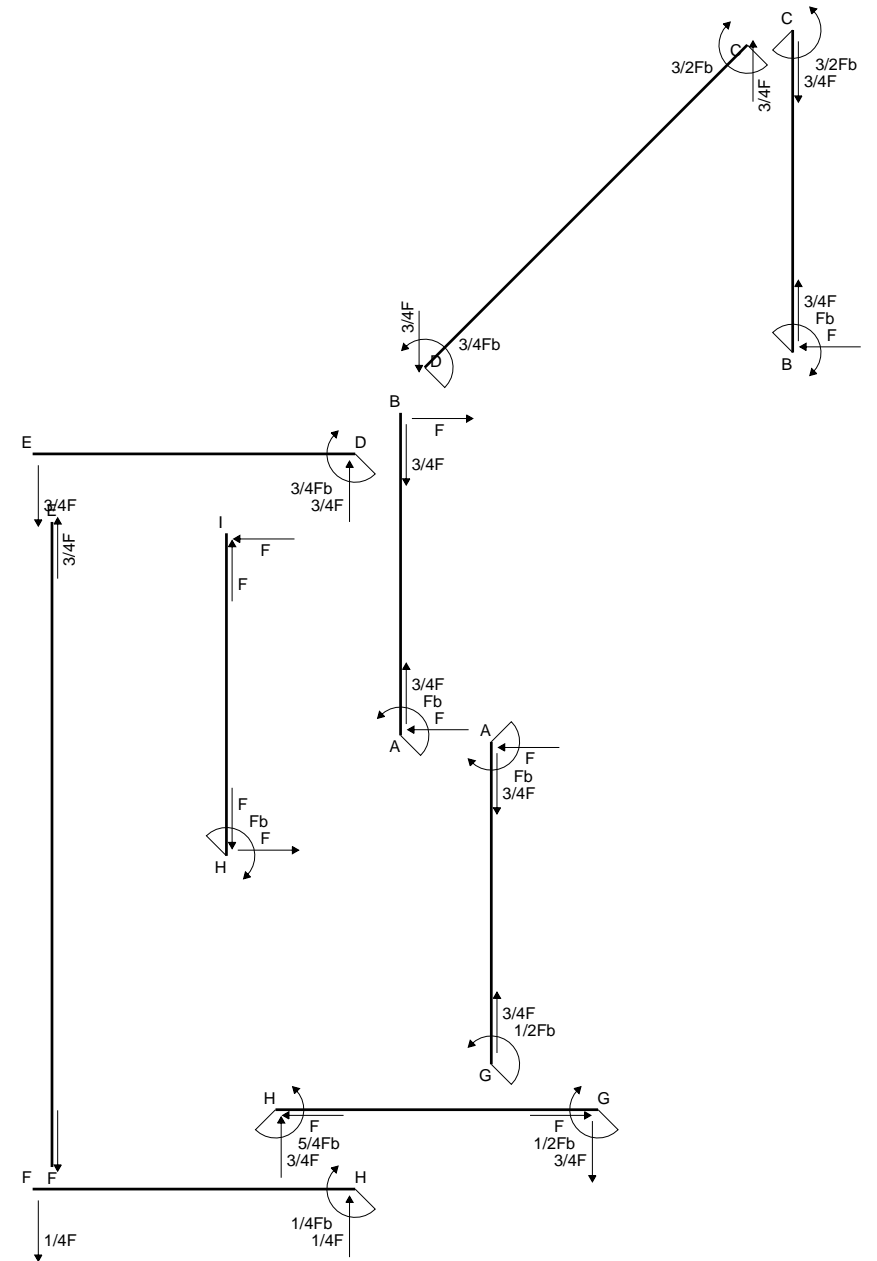
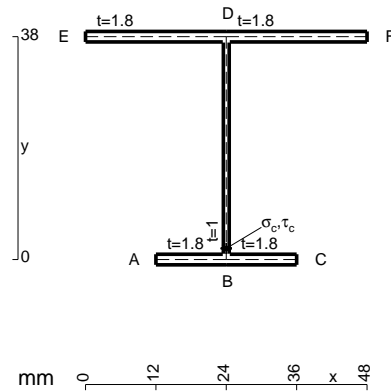
- $A = 181.6 \text{ mm}^2$
- $J_u = 92380. \text{ mm}^4$
- $J_v = 18662. \text{ mm}^4$
- $J_t = 157.3 \text{ mm}^4$
- $y_o = -14.04 \text{ mm}$
- $y_g = 19.82 \text{ mm}$
- $N = 410.1 \text{ N}$
- $T_y = -410.1 \text{ N}$
- $M_x = 661200. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $y_m = 52. \text{ mm}$
- $u_m = -12. \text{ mm}$
- $v_m = 32.19 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = -228.1 \text{ N/mm}^2$
- $x_c = 24. \text{ mm}$
- $y_c = 52. \text{ mm}$
- $v_c = 32.19 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = -228.1 \text{ N/mm}^2$
- $\tau_c = TS/tJ_u = 6.173 \text{ N/mm}^2$
- $\tau_g = TS/tJ_u = 6.173 \text{ N/mm}^2$
- $t_c = 1160. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 228.4 \text{ N/mm}^2$

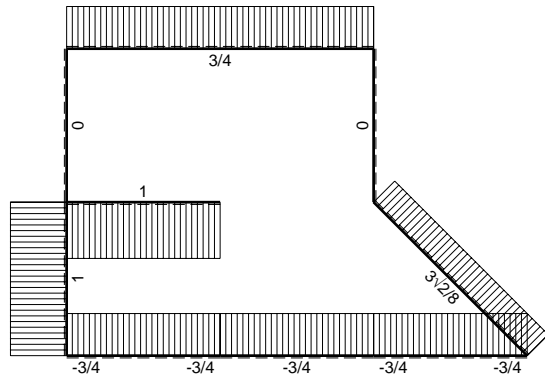


$V_G = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_B = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$q_{BC} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$q_{GA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

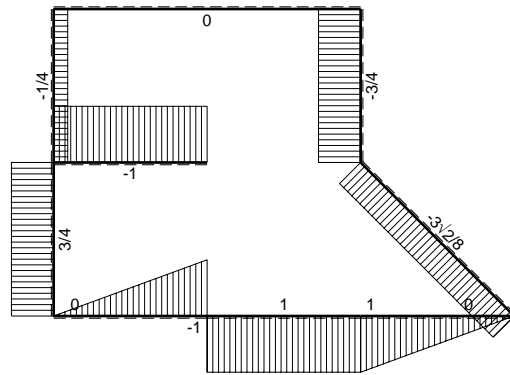
Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave AB ha la sezione riportata e dimensioni in mm, con:
 $b = 570 \text{ mm}$, $F = 780 \text{ N}$

Calcolare sulla sezione A la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da A a B
 Elongazione termica specifica ϵ assegnata su asta HG.

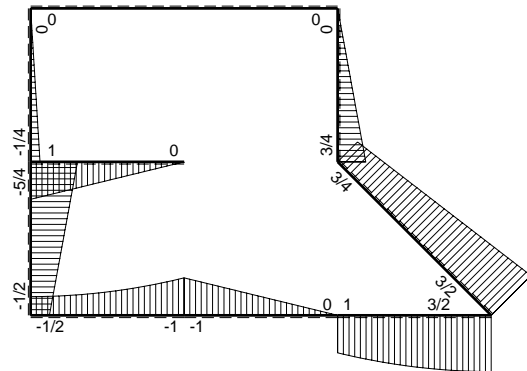




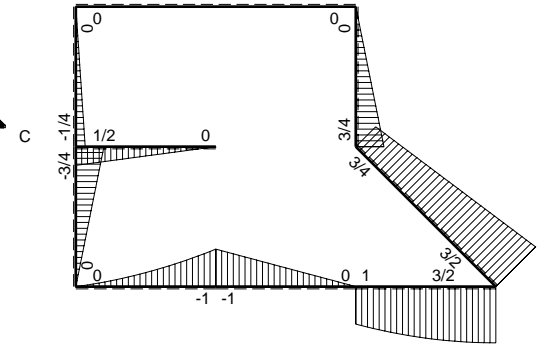
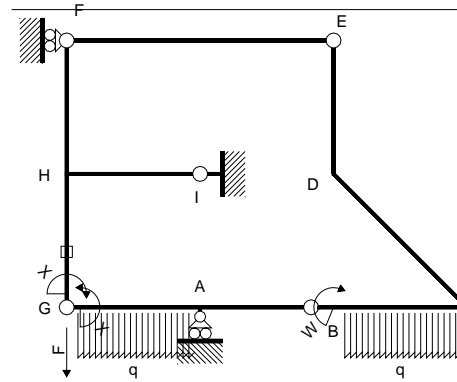
← (+) → F



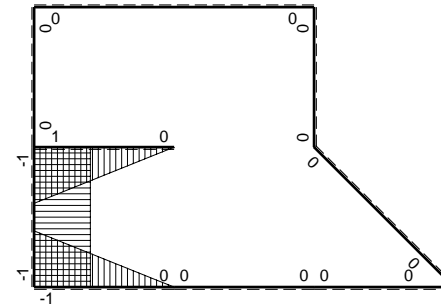
↑ (+) ↓ F



⊕ (+) ⊖ (-) F_b



⊕ (+) ⊖ (-) M₀ flessione da carichi assegnati



⊕ (+) ⊖ (-) M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-Fb+Fx	0	0	0	0
BA b	0	Fx	0	0	0	0
BC b	0	Fb+Fx-1/2qx ²	0	0	0	0
CB b	0	-3/2Fb+1/2qx ²	0	0	0	0
CD √2b	0	3/2Fb-3√2/8Fx	0	0	0	0
DE b	0	3/4Fb-3/4Fx	0	0	0	0
ED b	0	-3/4Fx	0	0	0	0
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0	0	0
GA b	-1+x/b	-1/2Fx-1/2qx ²	1/2Fx-1/2qx ³ /b	1-2x/b+x ² /b ²	1/8Fb ² /EJ	1/3Xb/EJ
AG b	x/b	Fb-3/2Fx+1/2qx ²	Fx-3/2Fx ² /b+1/2qx ³ /b	x ² /b ²	0	0
FH b	0	-1/4Fx	0	0	0	0
HF b	0	1/4Fb-1/4Fx	0	0	0	0
HI b	1-x/b	1/2Fb-1/2Fx	1/2Fb-Fx+1/2Fx ² /b	1-2x/b+x ² /b ²	1/6Fb ² /EJ	1/3Xb/EJ
IH b	-x/b	-1/2Fx	1/2Fx ² /b	x ² /b ²	0	0
HG b	-1	-3/4Fb+3/4Fx	3/4Fb-3/4Fx	1	3/8Fb ² /EJ	Xb/EJ
GH b	1	3/4Fx	3/4Fx	1	0	0
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				-Fb ² /EJ	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				-5/8Fb ² /EJ	1/4Xb/EJ
	totali				-23/24Fb ² /EJ	23/12Xb/EJ
	iperstatica $X=W_{GA}$				1/2Fb	

Sviluppi di calcolo iperstatica

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GA}^{xo} = \int_0^b (1/2 x/b - 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{AG}^{xo} = \int_0^b (x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/2 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{HI}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{IH}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

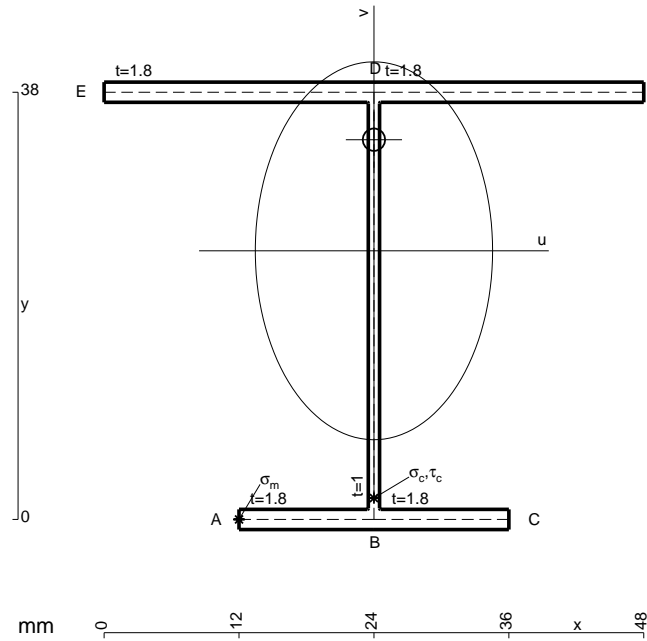
$$L_{HG}^{xo} = \int_0^b (3/4 - 3/4 x/b) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ$$

$$= [3/4 x - 3/8 x^2/b]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

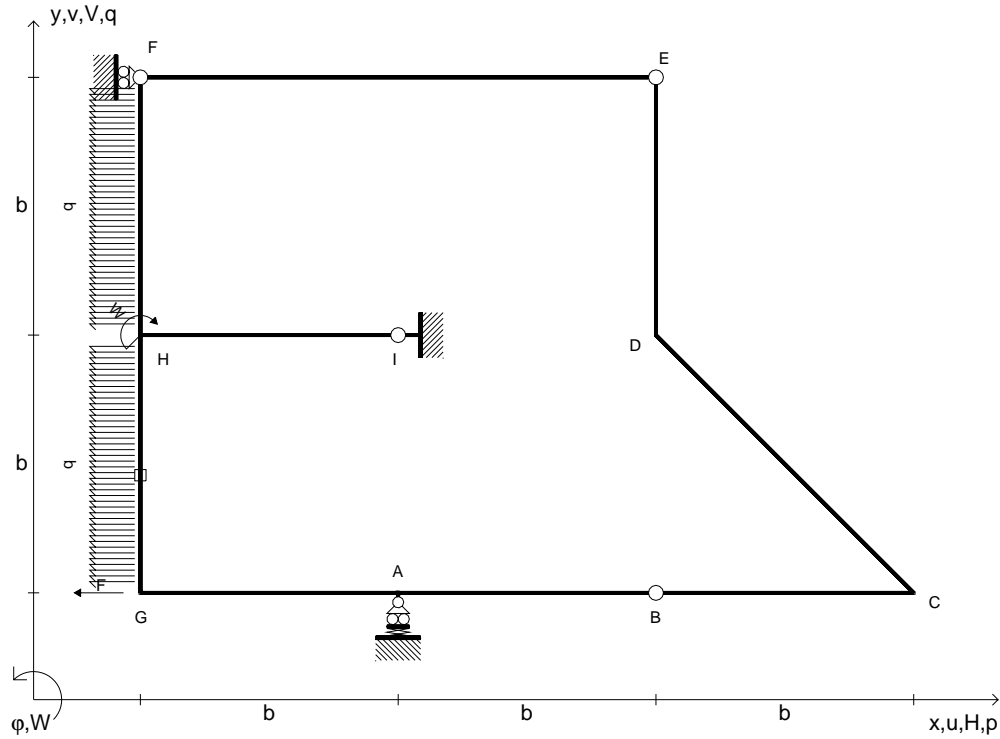
$$= (3/4 b - 3/8 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -5/8 Fb^2/EJ$$

$$L_{GH}^{xo} = \int_0^b (3/4 x/b) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ = [3/8 x^2/b]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

$$= (3/8 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -5/8 Fb^2/EJ$$

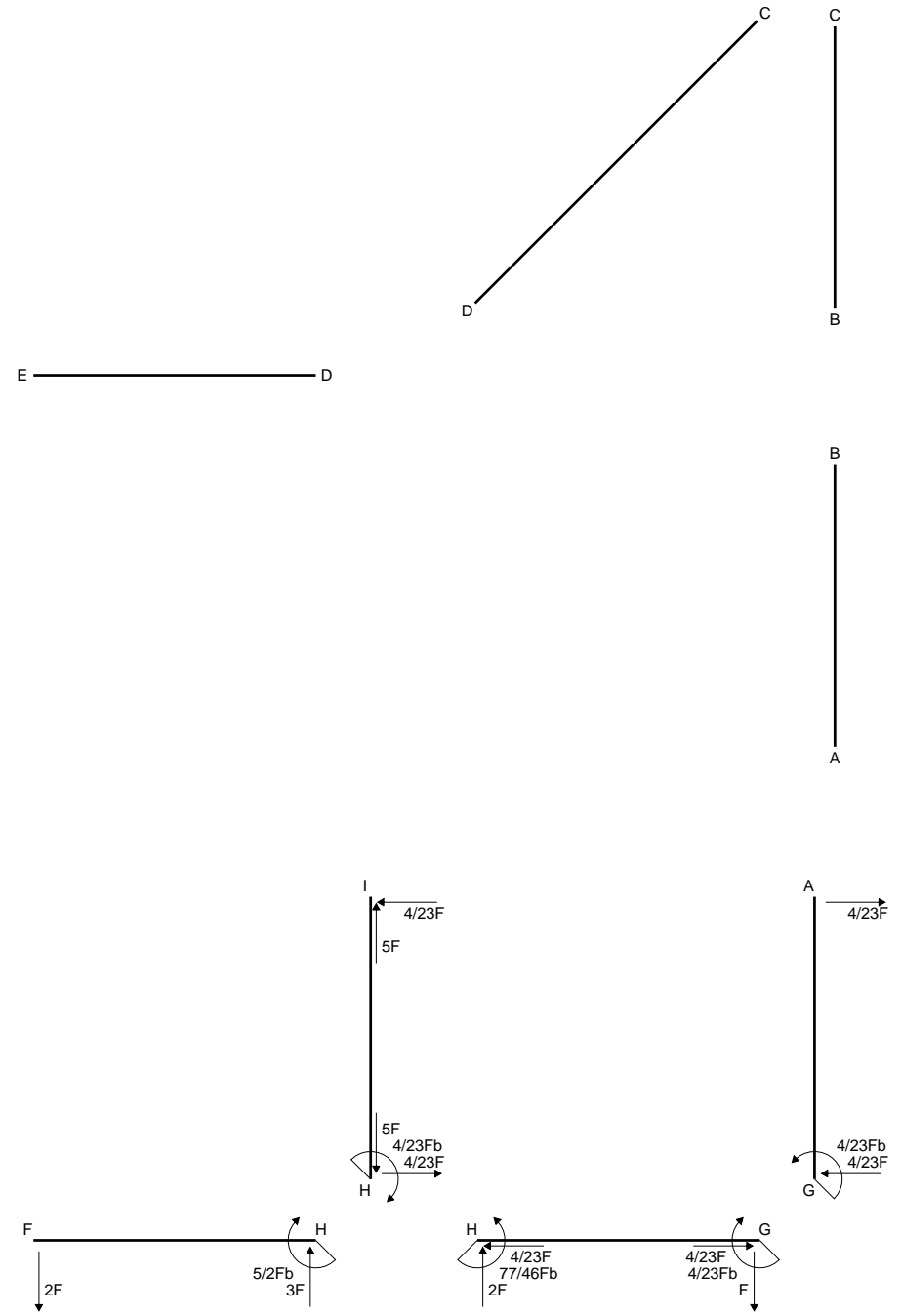
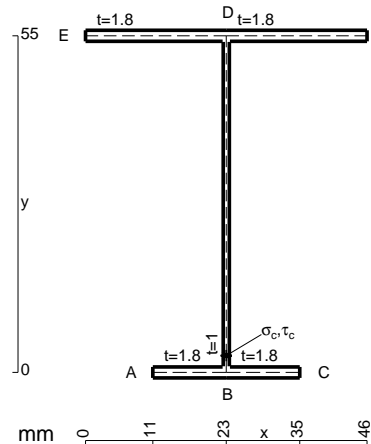


- $A = 167.6 \text{ mm}^2$
- $J_u = 47339. \text{ mm}^4$
- $J_v = 18662. \text{ mm}^4$
- $J_t = 152.6 \text{ mm}^4$
- $y_o = 9.88 \text{ mm}$
- $y_g = 23.9 \text{ mm}$
- $N = -585. \text{ N}$
- $T_y = 780. \text{ N}$
- $M_x = -444600. \text{ Nmm}$
- $x_m = 12. \text{ mm}$
- $u_m = -12. \text{ mm}$
- $v_m = -23.9 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = -227.9 \text{ N/mm}^2$
- $x_c = 24. \text{ mm}$
- $v_c = -23.9 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = -227.9 \text{ N/mm}^2$
- $\tau_c = TS/tJ_u = 17.01 \text{ N/mm}^2$
- $\tau_g = TS/tJ_u = 17.01 \text{ N/mm}^2$
- $t_c = 780. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 229.8 \text{ N/mm}^2$

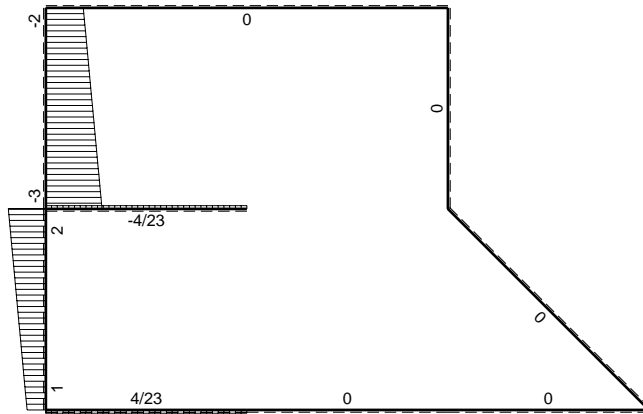
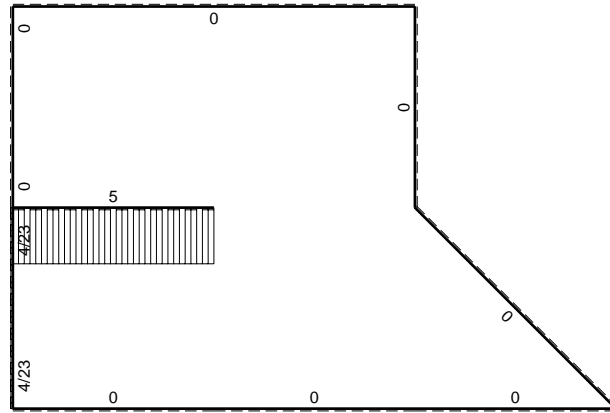


$H_G = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_H = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$\rho_{HG} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$\rho_{FH} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

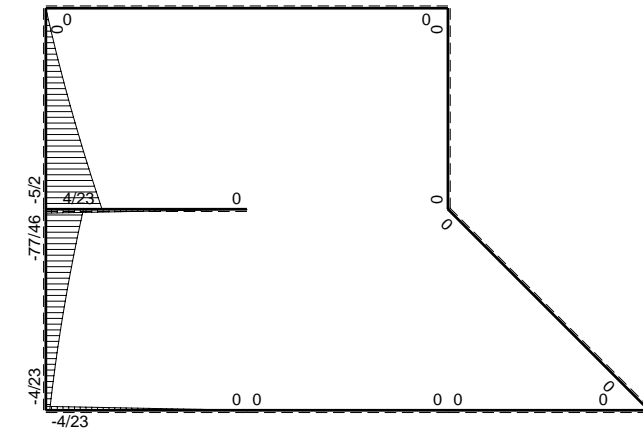
Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave FH ha la sezione riportata e dimensioni in mm, con:
 $b = 610 \text{ mm}$, $F = 460 \text{ N}$
 Calcolare sulla sezione H la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da F a H
 Elongazione termica specifica ϵ assegnata su asta HG.



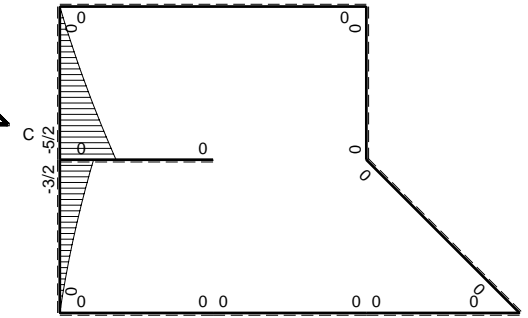
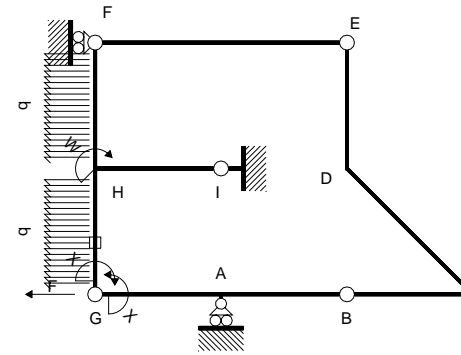
← (+) → F



↑ (+) ↓ F

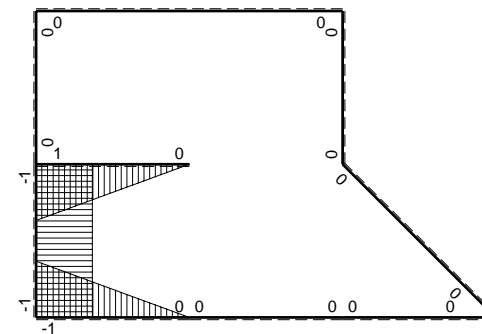


⊕ (+) ⊖ F_b



Schema di calcolo iperstatico

⊕ (+) ⊖ M₀ flessione da carichi assegnati



⊕ (+) ⊖ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0	0	0
BC b	0	0	0	0	0	0
CB b	0	0	0	0	0	0
CD $\sqrt{2}b$	0	0	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0	0	0
EF 2b	0	0	0	0	0	0
FE 2b	0	0	0	0	0	0
GA b	$-1+x/b$	0	0	$1-2x/b+x^2/b^2$	0	$1/3Xb/EJ$
AG b	x/b	0	0	x^2/b^2	0	$1/3Xb/EJ$
FH b	0	$-2Fx-1/2qx^2$	0	0	0	0
HF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0
HI b	$1-x/b$	0	0	$1-2x/b+x^2/b^2$	0	$1/3Xb/EJ$
IH b	$-x/b$	0	0	x^2/b^2	0	$1/3Xb/EJ$
HG b	-1	$-3/2Fb+2Fx-1/2qx^2$	$3/2Fb-2Fx+1/2Fx^2/b$	1	$2/3Fb^2/EJ$	Xb/EJ
GH b	1	$Fx+1/2qx^2$	$Fx+1/2Fx^2/b$	1	$2/3Fb^2/EJ$	Xb/EJ
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				$-Fb^2/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$					$1/4Xb/EJ$
	totali				$-1/3Fb^2/EJ$	$23/12Xb/EJ$
	iperstatica $X=W_{GA}$				$4/23Fb$	

Sviluppi di calcolo iperstatica

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{HG}^{xo} = \int_0^b (3/2 - 2x/b + 1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ$$

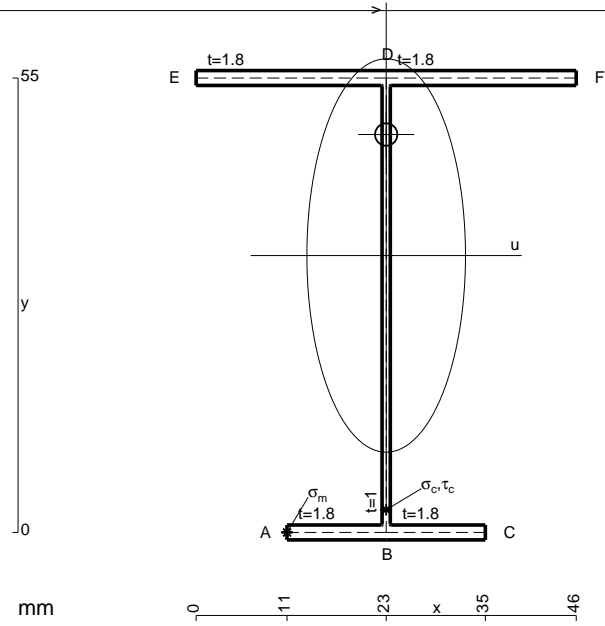
$$= [3/2 x - x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

$$= (3/2 b - b + 1/6 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -1/3 Fb^2/EJ$$

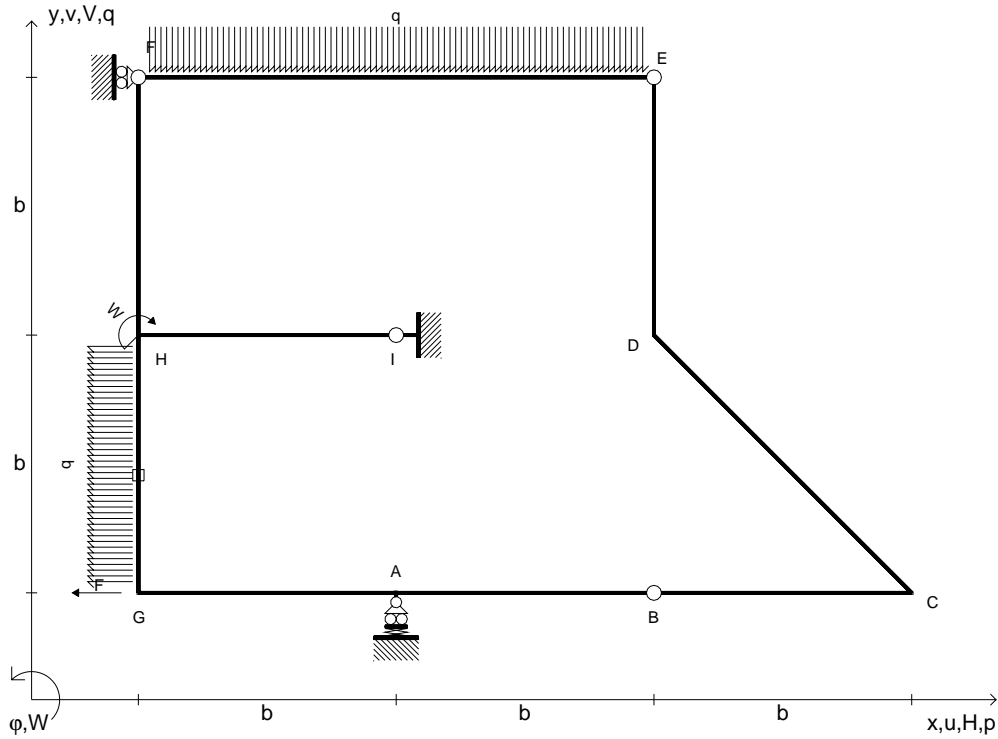
$$L_{GH}^{xo} = \int_0^b (x/b + 1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ$$

$$= [1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

$$= (1/2 b + 1/6 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -1/3 Fb^2/EJ$$

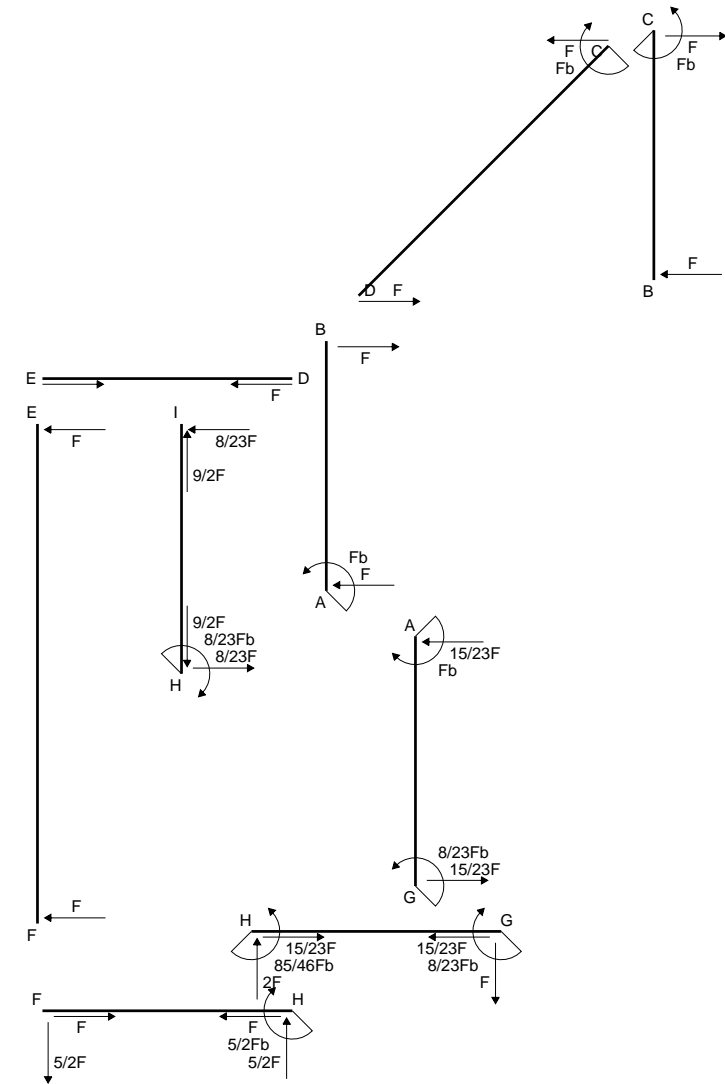
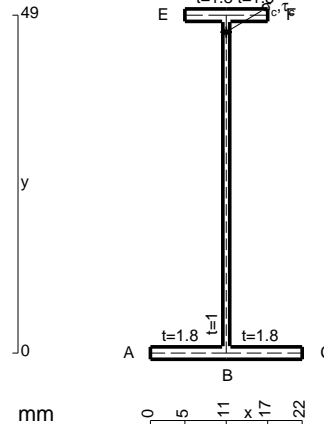


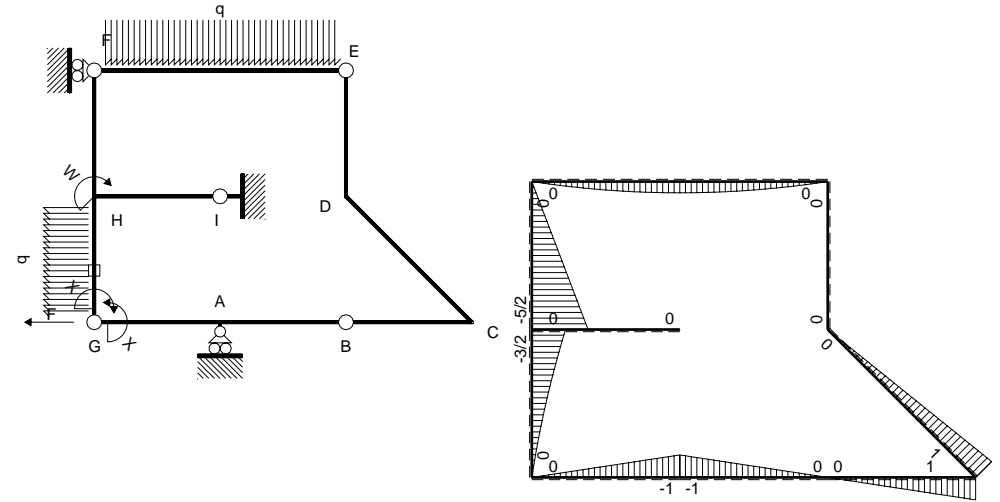
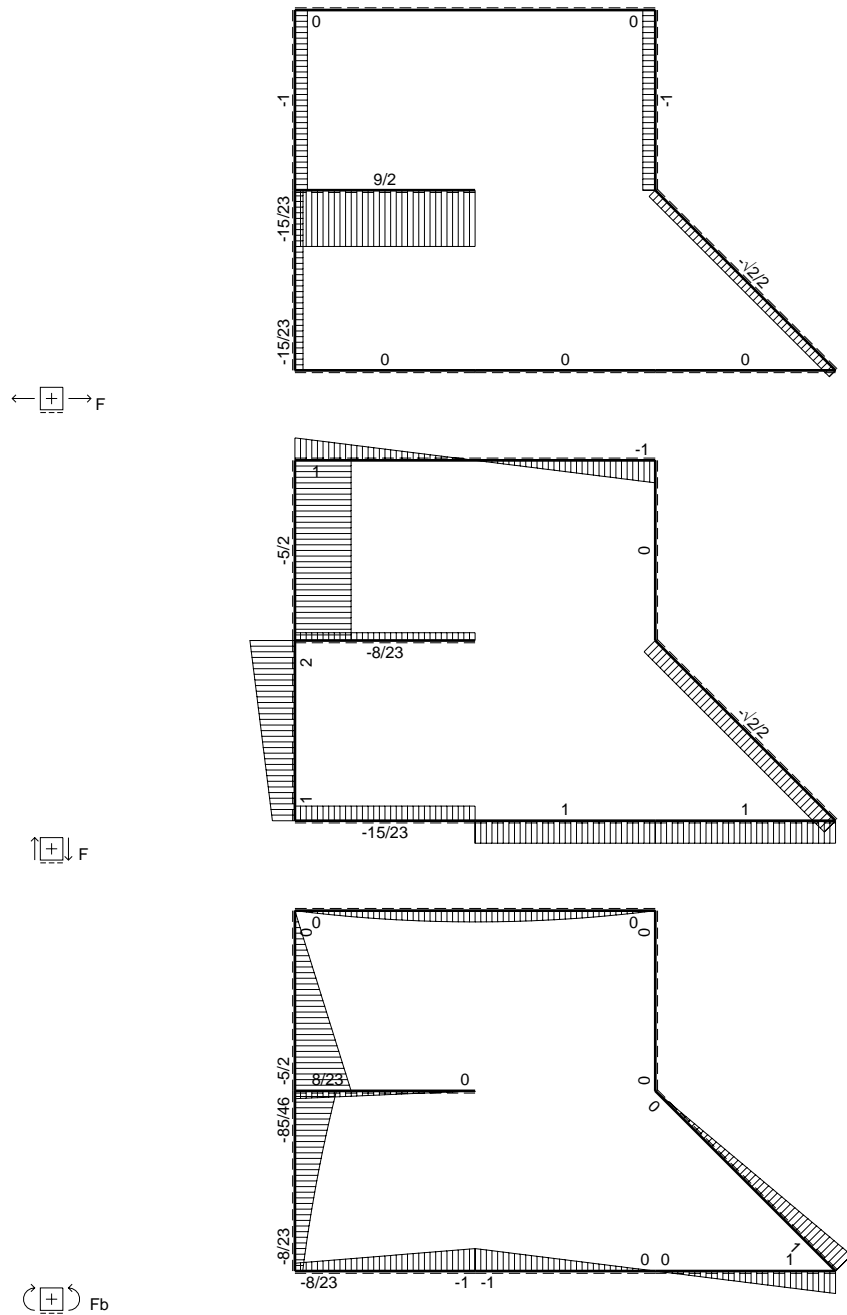
- $A = 181. \text{ mm}^2$
- $J_u = 102600. \text{ mm}^4$
- $J_v = 16674. \text{ mm}^4$
- $J_t = 154.4 \text{ mm}^4$
- $y_o = 14.64 \text{ mm}$
- $y_g = 33.52 \text{ mm}$
- $T_y = -1380. \text{ N}$
- $M_x = -701500. \text{ Nmm}$
- $x_m = 11. \text{ mm}$
- $u_m = -12. \text{ mm}$
- $v_m = -33.52 \text{ mm}$
- $\sigma_m = -Mv/J_u = -229.2 \text{ N/mm}^2$
- $x_c = 23. \text{ mm}$
- $v_c = -33.52 \text{ mm}$
- $\sigma_c = -Mv/J_u = -229.2 \text{ N/mm}^2$
- $\tau_c = TS'/tJ_u = 19.47 \text{ N/mm}^2$
- $\tau_g = TS'/tJ_u = 19.47 \text{ N/mm}^2$
- $t_c = 460. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 231.6 \text{ N/mm}^2$



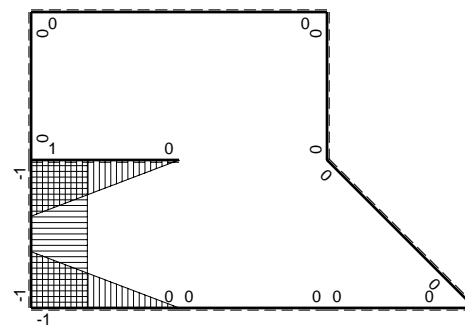
$H_G = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_H = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$p_{HG} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave FH ha la sezione riportata e dimensioni in mm, con:
 $b = 580 \text{ mm}$, $F = 260 \text{ N}$
 Calcolare sulla sezione H la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da F a H
 Elongazione termica specifica ϵ assegnata su asta HG.





$\left[\begin{array}{c} + \\ - \end{array} \right] M_o$ flessione da carichi assegnati



$\left[\begin{array}{c} + \\ - \end{array} \right] M_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-Fb+Fx	0	0	0	0
BA b	0	Fx	0	0	0	0
BC b	0	Fx	0	0	0	0
CB b	0	-Fb+Fx	0	0	0	0
CD $\sqrt{2}b$	0	Fb- $\sqrt{2}/2Fx$	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0	0	0
EF 2b	0	-Fx+1/2qx ²	0	0	0	0
FE 2b	0	Fx-1/2qx ²	0	0	0	0
GA b	-1+x/b	-Fx	Fx-Fx ² /b	1-2x/b+x ² /b ²	1/6Fb ² /EJ	1/3Xb/EJ
AG b	x/b	Fb-Fx	Fx-Fx ² /b	x ² /b ²	0	0
FH b	0	-5/2Fx	0	0	0	0
HF b	0	5/2Fb-5/2Fx	0	0	0	0
HI b	1-x/b	0	0	1-2x/b+x ² /b ²	0	1/3Xb/EJ
IH b	-x/b	0	0	x ² /b ²	0	0
HG b	-1	-3/2Fb+2Fx-1/2qx ²	3/2Fb-2Fx+1/2Fx ² /b	1	2/3Fb ² /EJ	Xb/EJ
GH b	1	Fx+1/2qx ²	Fx+1/2Fx ² /b	1	0	0
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				-Fb ² /EJ	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				-1/2Fb ² /EJ	1/4Xb/EJ
	totali				-2/3Fb ² /EJ	23/12Xb/EJ
	iperstatica $X=W_{GA}$				8/23Fb	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GA}^{Xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{AG}^{Xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{HG}^{Xo} = \int_0^b (3/2 - 2x/b + 1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ$$

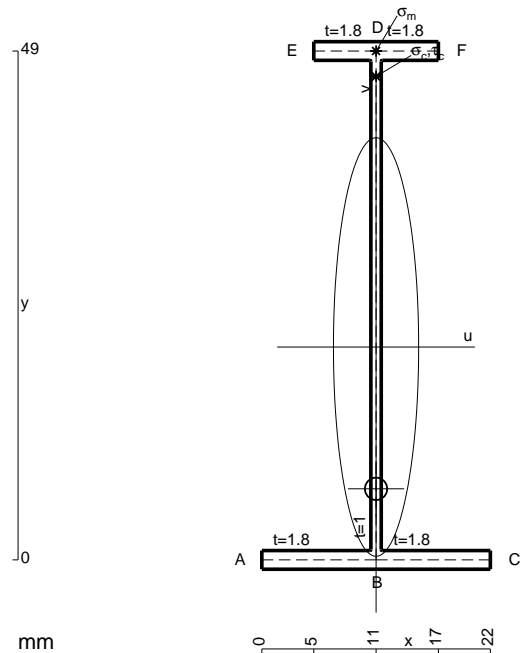
$$= [3/2 x - x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

$$= (3/2 b - b + 1/6 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -1/3 Fb^2/EJ$$

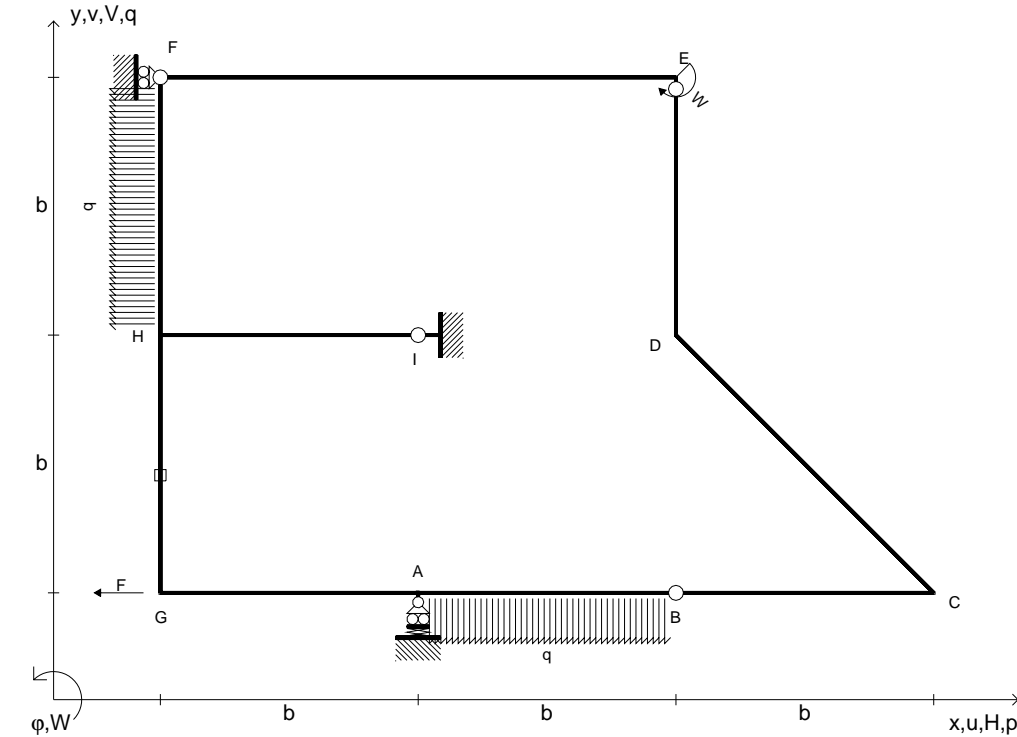
$$L_{GH}^{Xo} = \int_0^b (x/b + 1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ$$

$$= [1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ$$

$$= (1/2 b + 1/6 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -1/3 Fb^2/EJ$$

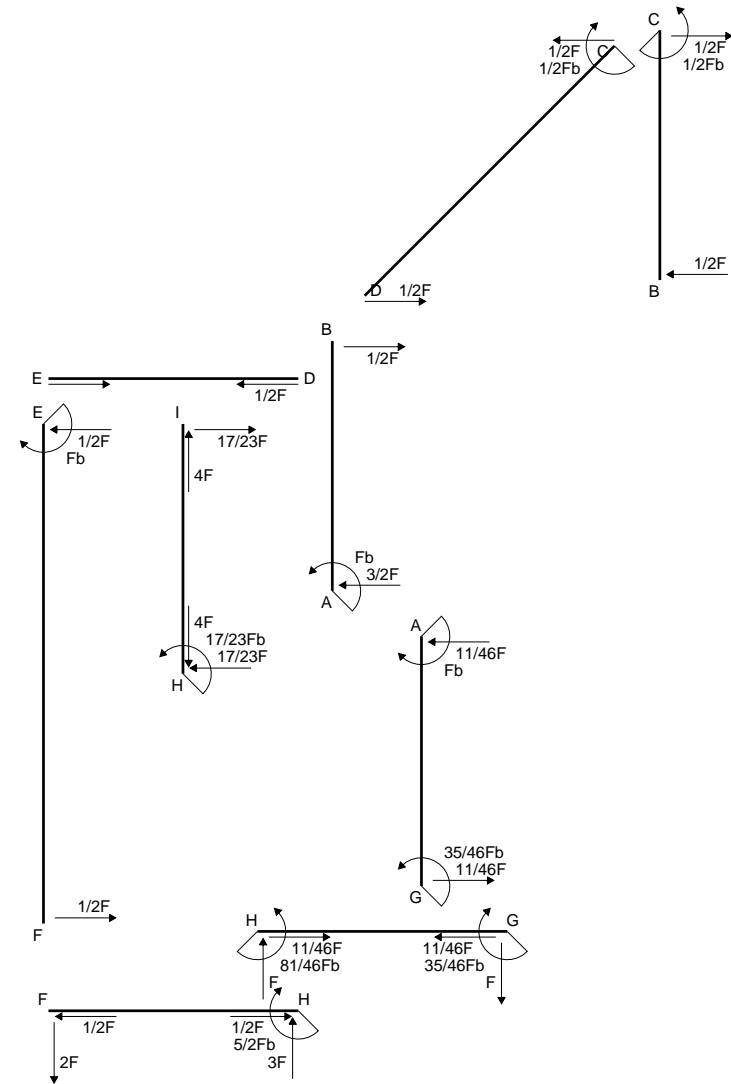
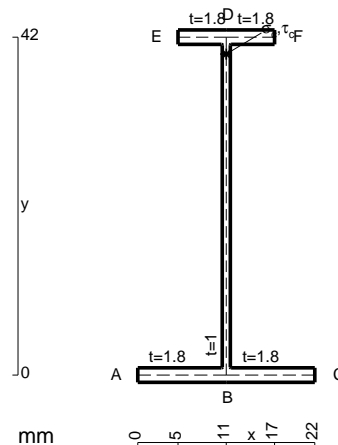


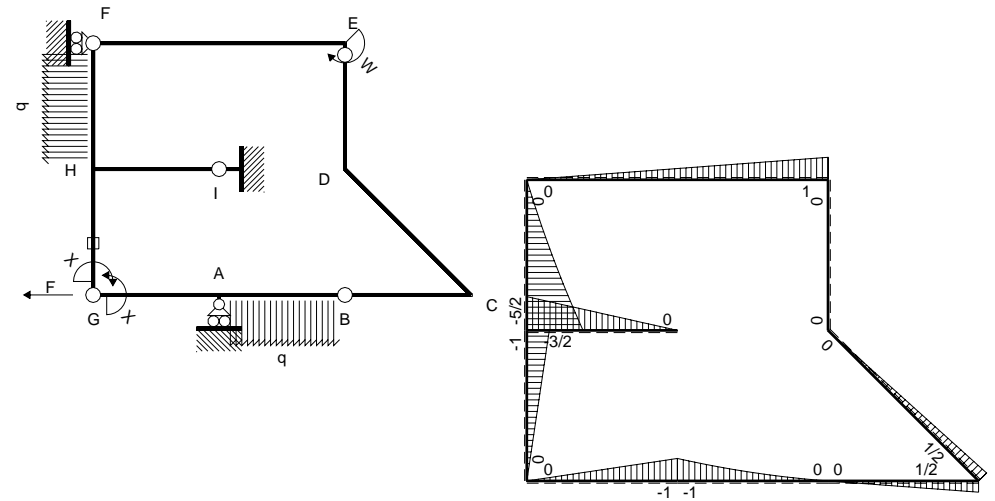
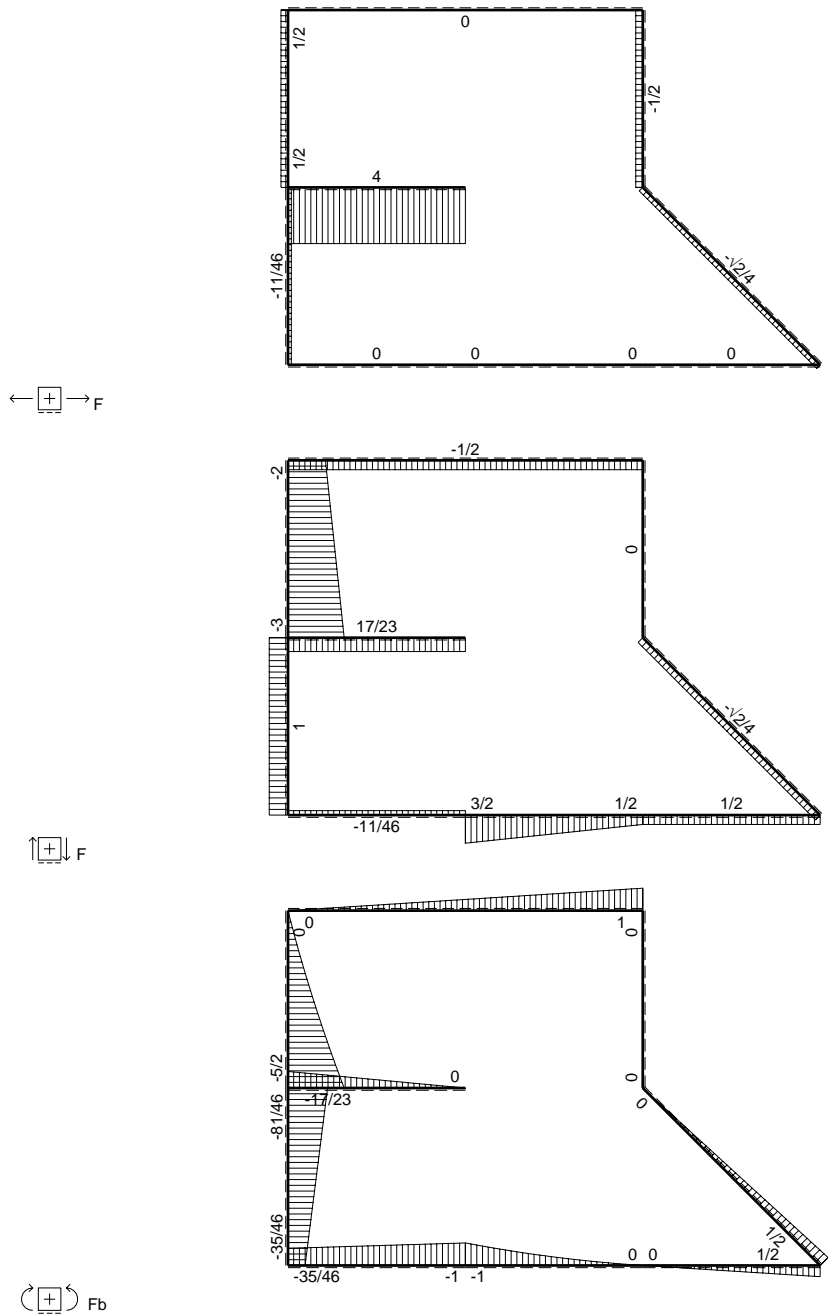
$A = 110.2 \text{ mm}^2$
 $J_u = 44775. \text{ mm}^4$
 $J_v = 1856. \text{ mm}^4$
 $J_t = 82.43 \text{ mm}^4$
 $y_o = -13.66 \text{ mm}$
 $y_g = 20.5 \text{ mm}$
 $N = -260. \text{ N}$
 $T_y = -650. \text{ N}$
 $M_x = -377000. \text{ Nmm}$
 $x_m = 11. \text{ mm}$
 $y_m = 49. \text{ mm}$
 $v_m = 28.5 \text{ mm}$
 $\sigma_m = N/A - Mv/J_u = 237.6 \text{ N/mm}^2$
 $y_c = 2. \text{ mm}$
 $u_c = -11. \text{ mm}$
 $v_c = -18.5 \text{ mm}$
 $\sigma_c = N/A - Mv/J_u = 237.6 \text{ N/mm}^2$
 $\tau_c = TS'/tJ_u = 8.937 \text{ N/mm}^2$
 $\tau_g = TS'/tJ_u = 8.937 \text{ N/mm}^2$
 $t_c = 260. \text{ mm}$
 $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 238.1 \text{ N/mm}^2$



$H_G = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_E = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$q_{AB} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$p_{FH} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

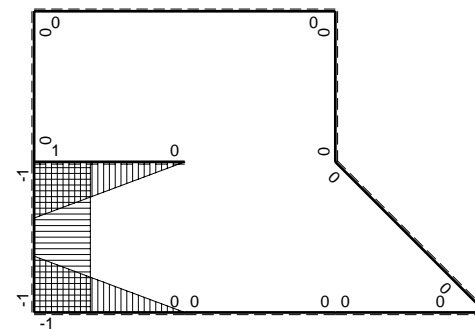
Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave FH ha la sezione riportata e dimensioni in mm, con:
 $b = 390 \text{ mm}$, $F = 270 \text{ N}$
 Calcolare sulla sezione H la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da F a H
 Elongazione termica specifica ϵ assegnata su asta HG.





Schema di calcolo iperstatico

⊕ ↻ M_o flessione da carichi assegnati



⊕ ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	$-Fb+3/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fx+1/2qx^2$	0	0	0	0
BC b	0	$1/2Fx$	0	0	0	0
CB b	0	$-1/2Fb+1/2Fx$	0	0	0	0
CD $\sqrt{2}b$	0	$1/2Fb-\sqrt{2}4Fx$	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0	0	0
EF 2b	0	$Fb-1/2Fx$	0	0	0	0
FE 2b	0	$-1/2Fx$	0	0	0	0
GA b	$-1+x/b$	$-Fx$	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
AG b	x/b	$Fb-Fx$	$Fx-Fx^2/b$	x^2/b^2		
FH b	0	$-2Fx-1/2qx^2$	0	0	0	0
HF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0
HI b	$1-x/b$	$-3/2Fb+3/2Fx$	$-3/2Fb+3Fx-3/2Fx^2/b$	$1-2x/b+x^2/b^2$	$-1/2Fb^2/EJ$	$1/3Xb/EJ$
IH b	$-x/b$	$3/2Fx$	$-3/2Fx^2/b$	x^2/b^2		
HG b	-1	$-Fb+Fx$	$Fb-Fx$	1	$1/2Fb^2/EJ$	Xb/EJ
GH b	1	Fx	Fx	1		
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				$-Fb^2/EJ$	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				$-5/8Fb^2/EJ$	$1/4Xb/EJ$
	totali				$-35/24Fb^2/EJ$	$23/12Xb/EJ$
	iperstatica $X=W_{GA}$				$35/46Fb$	

Sviluppi di calcolo iperstatica

$$L_{GA}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ = (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ = (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{GH}^{XX} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ = (b) 1/EJ = b/EJ$$

$$L_{GA}^{Xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ = (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

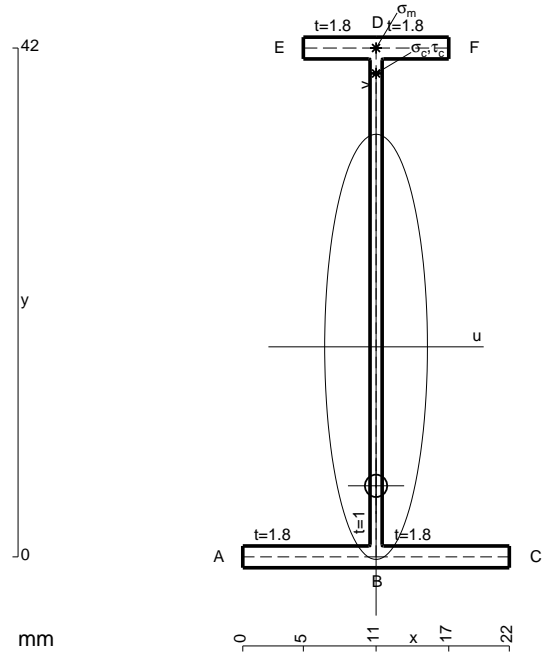
$$L_{AG}^{Xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ = (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{HI}^{Xo} = \int_0^b (-3/2 + 3x/b - 3/2 x^2/b^2) Fb 1/EJ dx = [-3/2 x + 3/2 x^2/b - 1/2 x^3/b^2]_0^b Fb 1/EJ = (-3/2 b + 3/2 b - 1/2 b) Fb 1/EJ = -1/2 Fb^2/EJ$$

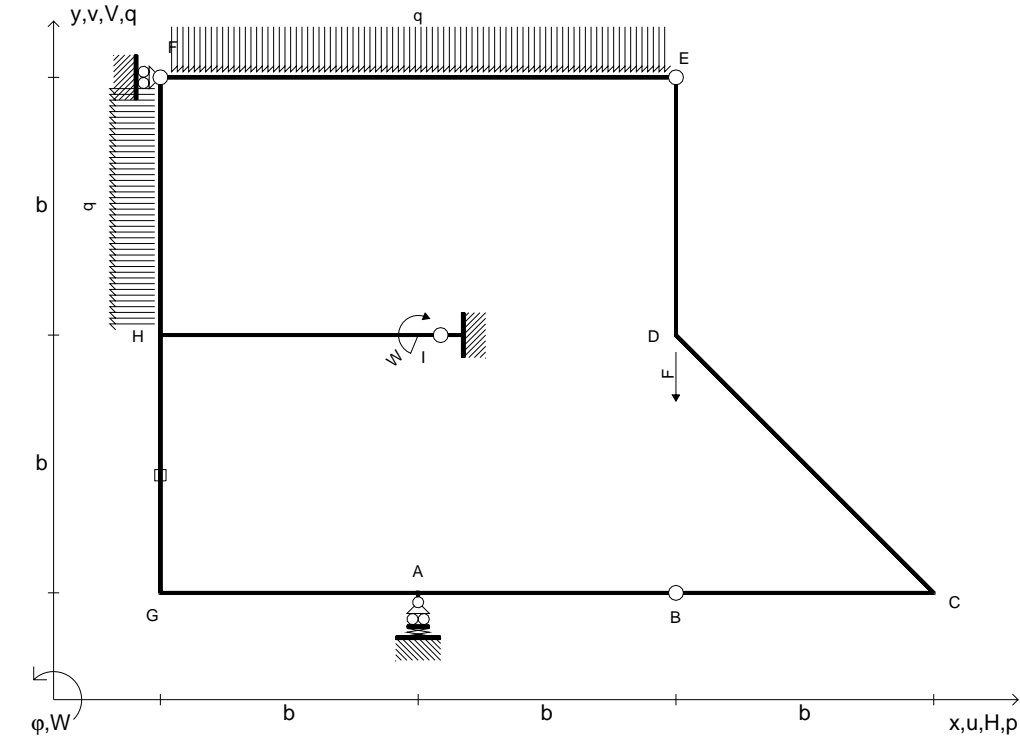
$$L_{IH}^{Xo} = \int_0^b (-3/2 x^2/b^2) Fb 1/EJ dx = [-1/2 x^3/b^2]_0^b Fb 1/EJ = (-1/2 b) Fb 1/EJ = -1/2 Fb^2/EJ$$

$$L_{HG}^{Xo} = \int_0^b (1 - x/b) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ = [x - 1/2 x^2/b]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = (b - 1/2 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -1/2 Fb^2/EJ$$

$$L_{GH}^{Xo} = \int_0^b (x/b) Fb 1/EJ dx - 1 (-1) (-1) Fb^2/EJ = [1/2 x^2/b]_0^b Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = (1/2 b) Fb 1/EJ - 1 (-1) (-1) Fb^2/EJ = -1/2 Fb^2/EJ$$

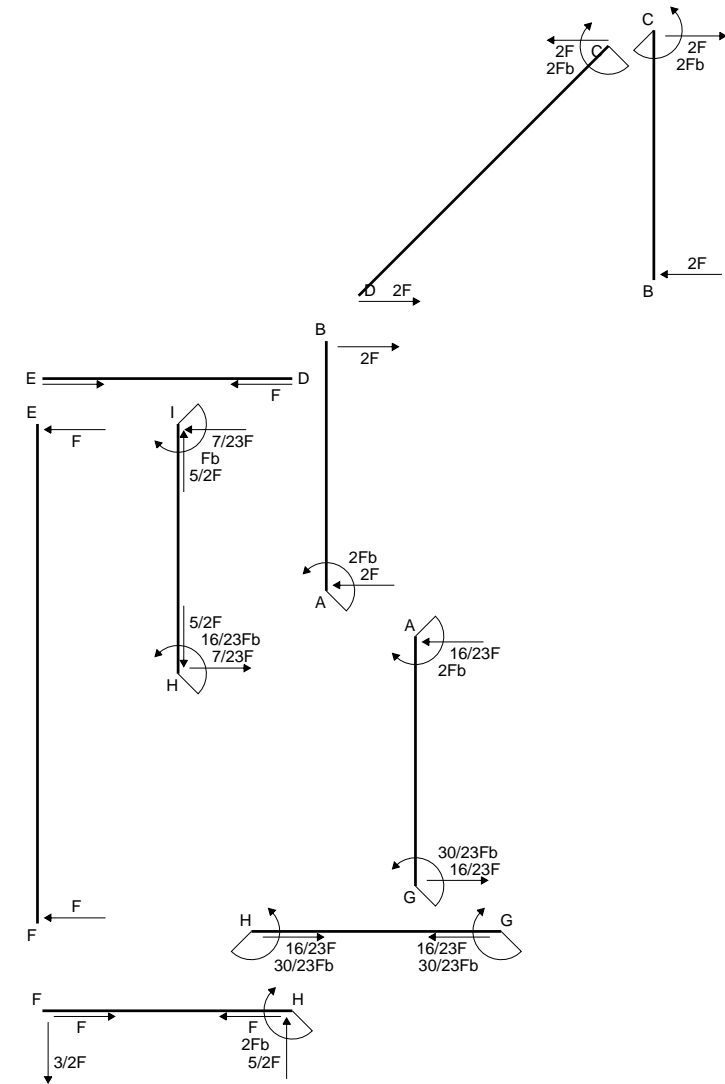
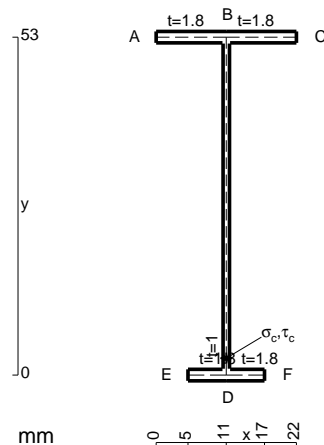


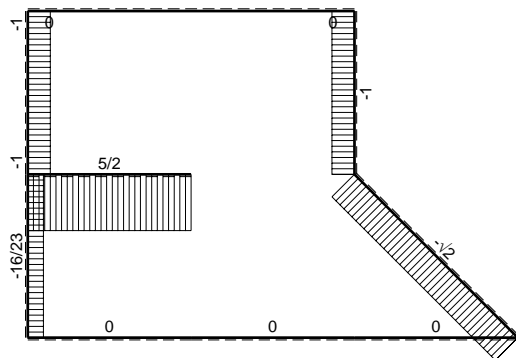
- $A = 103.2 \text{ mm}^2$
- $J_u = 31779. \text{ mm}^4$
- $J_v = 1856. \text{ mm}^4$
- $J_x = 80.1 \text{ mm}^4$
- $y_o = -11.47 \text{ mm}$
- $y_g = 17.34 \text{ mm}$
- $N = 135. \text{ N}$
- $T_y = -810. \text{ N}$
- $M_x = -263250. \text{ Nmm}$
- $x_m = 11. \text{ mm}$
- $y_m = 42. \text{ mm}$
- $v_m = 24.66 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = 205.6 \text{ N/mm}^2$
- $y_c = 2. \text{ mm}$
- $u_c = -11. \text{ mm}$
- $v_c = -15.34 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = 205.6 \text{ N/mm}^2$
- $\tau_c = TS'/tJ_u = 13.58 \text{ N/mm}^2$
- $\tau_g = TS'/tJ_u = 13.58 \text{ N/mm}^2$
- $t_c = 270. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 207. \text{ N/mm}^2$



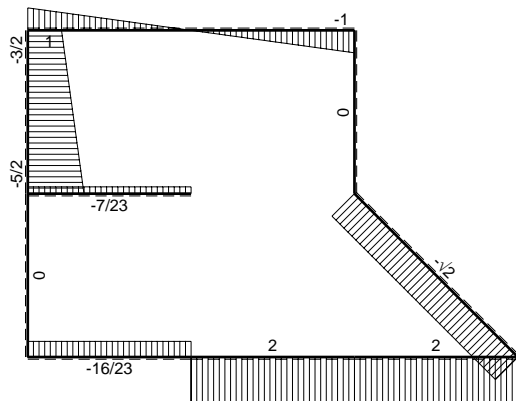
$V_D = -F$	$\epsilon_{HG} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{FH} = EJ$
$W_I = -W = -Fb$	$k_A = 4EJ/b^3$	$EJ_{DE} = EJ$	$EJ_{HI} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	$EJ_{HG} = EJ$
$p_{FH} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GA} = EJ$	

Reazioni iperstatiche in soluzione: $X=W_{GA}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 La trave FH ha la sezione riportata e dimensioni in mm, con:
 $b = 850 \text{ mm}$, $F = 240 \text{ N}$
 Calcolare sulla sezione H la massima tensione normale σ_m .
 Calcolare in * le tensioni σ_c, τ_c e la tensione di von Mises.
 Lembo inferiore sezione su tratteggio trave, a destra da F a H
 Elongazione termica specifica ϵ assegnata su asta HG.

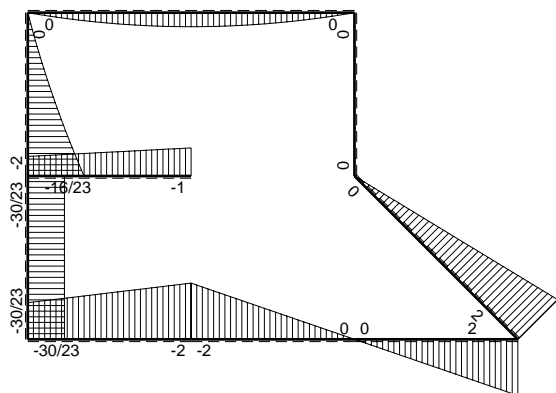




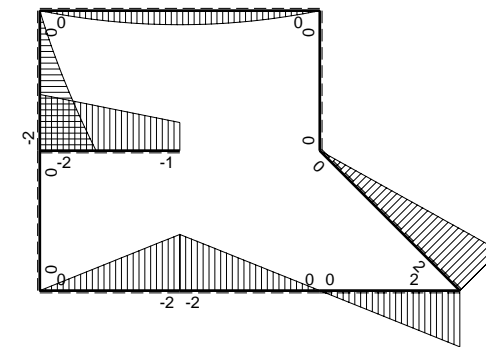
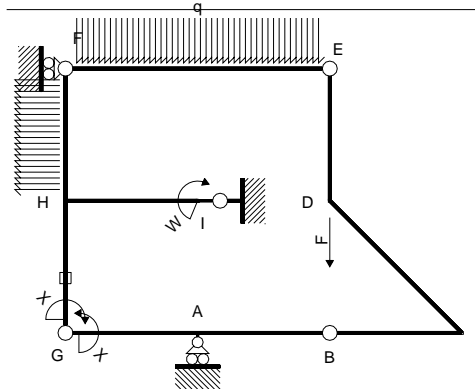
← (+) → F



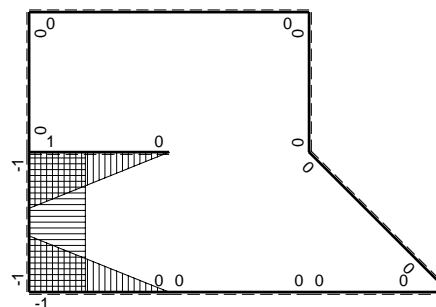
↑ (+) ↓ F



⌚ (+) ⌚ F_b



⌚ (+) ⌚ M_o flessione da carichi assegnati



⌚ (+) ⌚ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GA}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-2Fb+2Fx	0	0	0	0
BA b	0	2Fx	0	0		
BC b	0	2Fx	0	0	0	0
CB b	0	-2Fb+2Fx	0	0		
CD $\sqrt{2}b$	0	2Fb- $\sqrt{2}$ Fx	0	0	0	0
DE b	0	0	0	0	0	0
ED b	0	0	0	0		
EF 2b	0	-Fx+1/2qx ²	0	0	0	0
FE 2b	0	Fx-1/2qx ²	0	0		
GA b	-1+x/b	-2Fx	2Fx-2Fx ² /b	1-2x/b+x ² /b ²	1/3Fb ² /EJ	1/3Xb/EJ
AG b	x/b	2Fb-2Fx	2Fx-2Fx ² /b	x ² /b ²		
FH b	0	-3/2Fx-1/2qx ²	0	0	0	0
HF b	0	2Fb-5/2Fx+1/2qx ²	0	0		
HI b	1-x/b	-2Fb+Fx	-2Fb+3Fx-Fx ² /b	1-2x/b+x ² /b ²	-5/6Fb ² /EJ	1/3Xb/EJ
IH b	-x/b	Fb+Fx	-Fx-Fx ² /b	x ² /b ²		
HG b	-1	0	0	1	0	Xb/EJ
GH b	1	0	0	1		
HG	elongazione asta $N_{1HG} \epsilon_{HG} L_{HG}$				-Fb ² /EJ	
A	molla nodo $-V_{1A}(V_{oA}+XV_{1A})/k_A$				-Fb ² /EJ	1/4Xb/EJ
	totali				-5/2Fb ² /EJ	23/12Xb/EJ
	iperstatica $X=W_{GA}$				30/23Fb	

Sviluppi di calcolo iperstatica

$$L_{GA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HI}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{IH}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GA}^{xo} = \int_0^b (2x/b - 2x^2/b^2) Fb 1/EJ dx = [x^2/b - 2/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (b - 2/3 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{AG}^{xo} = \int_0^b (2x/b - 2x^2/b^2) Fb 1/EJ dx = [x^2/b - 2/3 x^3/b^2]_0^b Fb 1/EJ$$

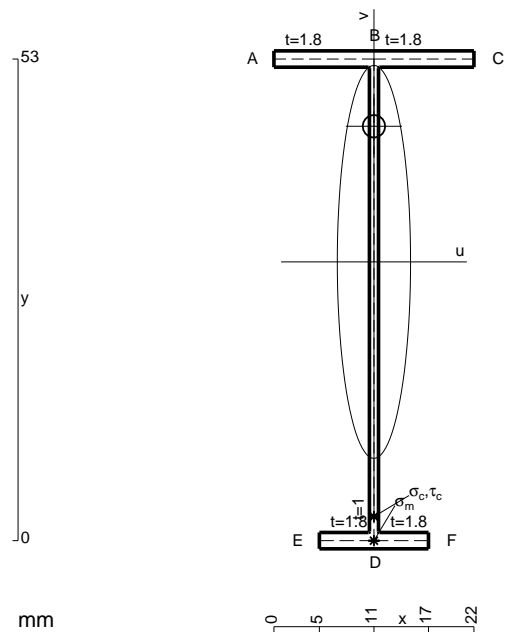
$$= (b - 2/3 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{HI}^{xo} = \int_0^b (-2 + 3x/b - x^2/b^2) Fb 1/EJ dx = [-2x + 3/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-2b + 3/2 b - 1/3 b) Fb 1/EJ = -5/6 Fb^2/EJ$$

$$L_{IH}^{xo} = \int_0^b (-x/b - x^2/b^2) Fb 1/EJ dx = [-1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-1/2 b - 1/3 b) Fb 1/EJ = -5/6 Fb^2/EJ$$



- $A = 114.2 \text{ mm}^2$
- $J_u = 53392. \text{ mm}^4$
- $J_v = 1856. \text{ mm}^4$
- $J_t = 83.76 \text{ mm}^4$
- $y_o = 14.92 \text{ mm}$
- $y_g = 30.68 \text{ mm}$
- $N = -240. \text{ N}$
- $T_y = -600. \text{ N}$
- $M_x = -408000. \text{ Nmm}$
- $x_m = 11. \text{ mm}$
- $v_m = -30.68 \text{ mm}$
- $\sigma_m = N/A - Mv/J_u = -236.5 \text{ N/mm}^2$
- $y_c = 3. \text{ mm}$
- $u_c = -11. \text{ mm}$
- $v_c = -27.68 \text{ mm}$
- $\sigma_c = N/A - Mv/J_u = -236.5 \text{ N/mm}^2$
- $\tau_c = TS'/tJ_u = 7.446 \text{ N/mm}^2$
- $\tau_g = TS'/tJ_u = 7.446 \text{ N/mm}^2$
- $t_c = 240. \text{ mm}$
- $\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 236.9 \text{ N/mm}^2$