

Quadro contributi PLV per iperstatica X=H_C

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	-b+1/2x	-2Fb+5/4Fx	0	2Fb ² -9/4Fbx+5/8Fx ²	0	b ² -bx+1/4x ²	(13/12+0)Fb ³ /EJ	7/12Xb ³ /EJ	
BA b	1/2b+1/2x	3/4Fb+5/4Fx	0	3/8Fb ² +Fbx+5/8Fx ²	0	1/4b ² +1/2bx+1/4x ²			
BC b	-1/2b+1/2x	-1/4Fb+1/4Fx	0	1/8Fb ² -1/4Fbx+1/8Fx ²	0	1/4b ² -1/2bx+1/4x ²	(1/24+0)Fb ³ /EJ	1/12Xb ³ /EJ	
CB b	1/2x	1/4Fx	0	1/8Fx ²	0	1/4x ²			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	-Fb+Fx	0	0	0	0			
EF b	0	Fb-Fx	-Fb/EJ	0	0	0	0+0	0	
FE b	0	-Fx	Fb/EJ	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	-2Fx-1/2qx ²	0	0	0	0	0+0	0	
HG b	0	5/2Fb-3Fx+1/2qx ²	0	0	0	0			
GI √2b	0	0	0	0	0	0	0	0	
IB b	0	Fx-1/2qx ²	0	0	0	0	0+0	0	
BI b	0	-1/2Fb+1/2qx ²	0	0	0	0			
IE √2b	0	0	0	0	0	0	0	0	
HA b	-x	-5/2Fb+Fx-1/2qx ²	0	5/2Fbx-Fx ² +1/2qx ³	0	x ²	(25/24+0)Fb ³ /EJ	1/3Xb ³ /EJ	
AH b	b-x	2Fb+1/2qx ²	0	2Fb ² -2Fbx+1/2Fx ² -1/2qx ³	0	b ² -2bx+x ²			
H	cedimento nodo -H _{1H} u _H							-Fb ³ /EJ	
	totali							7/6Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _C							-7/6F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (2 - 9/4 x/b + 5/8 x^2/b^2) Fb^2 1/EJ dx = [2x - 9/8 x^2/b + 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (2b - 9/8 b + 5/24 b) Fb^2 1/EJ = 13/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (3/8 + x/b + 5/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x + 1/2 x^2/b + 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b + 1/2 b + 5/24 b) Fb^2 1/EJ = 13/12 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/8 - 1/4 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x - 1/8 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b - 1/8 b + 1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/8 x^2/b^2) Fb^2 1/EJ dx = [1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

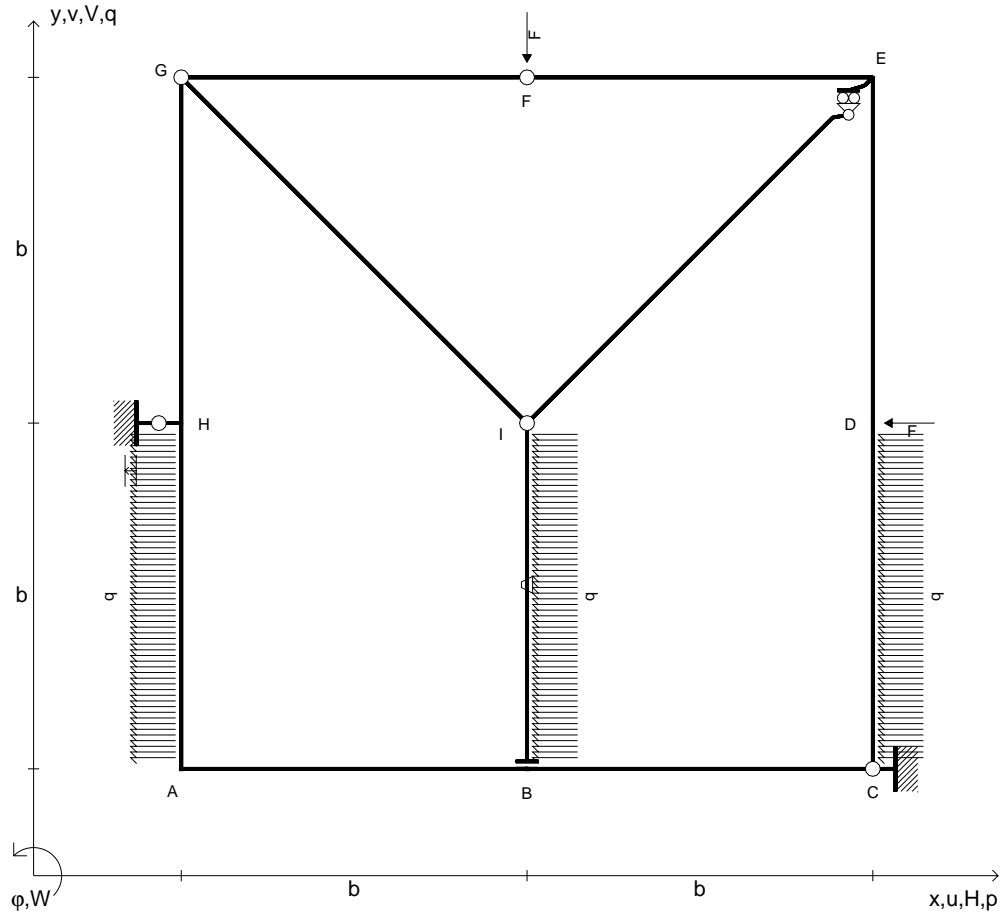
$$= (1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (5/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [5/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (5/4 b - 1/3 b + 1/8 b) Fb^2 1/EJ = 25/24 Fb^3/EJ$$

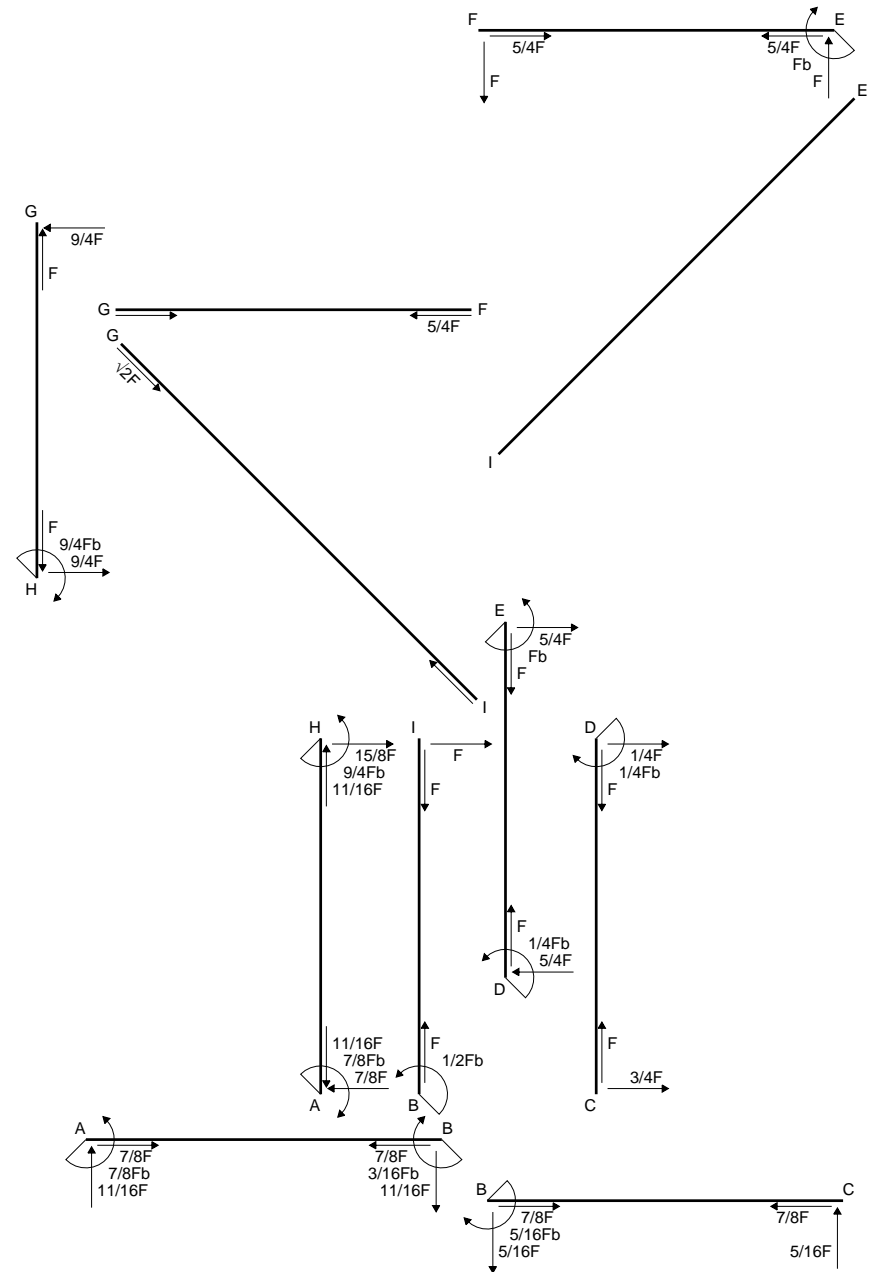
$$L_{AH}^{xo} = \int_0^b (2 - 2x/b + 1/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [2x - x^2/b + 1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

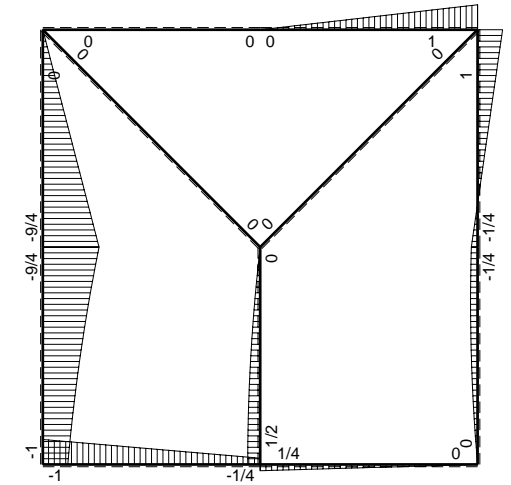
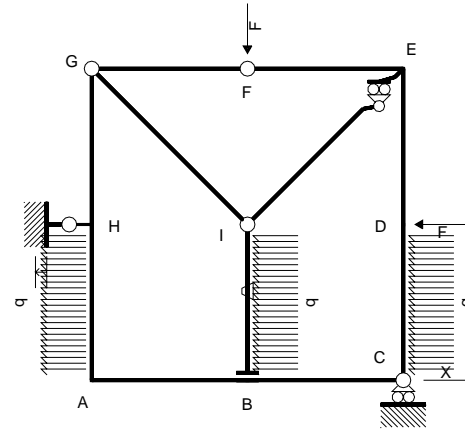
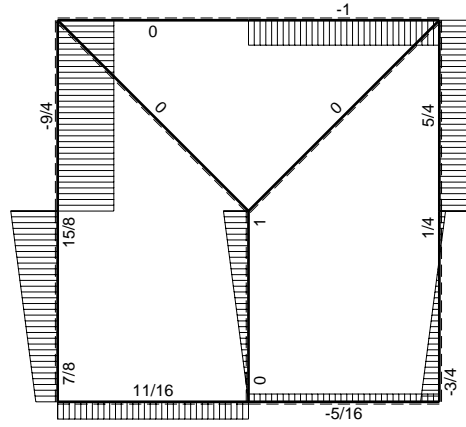
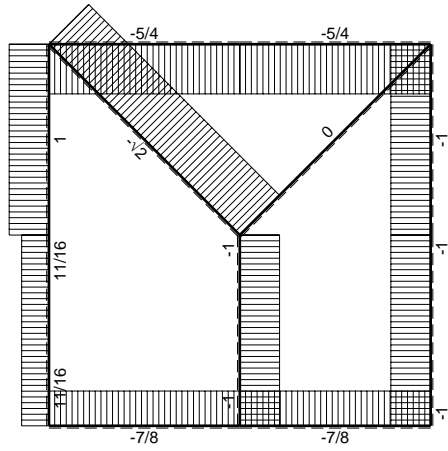
$$= (2b - b + 1/6 b - 1/8 b) Fb^2 1/EJ = 25/24 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$p_{IB} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$p_{CD} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{IB} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta IB positiva se convessa a destra con inizio I.
 Spostamento orizzontale assoluto u imposto al nodo H.
 © Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

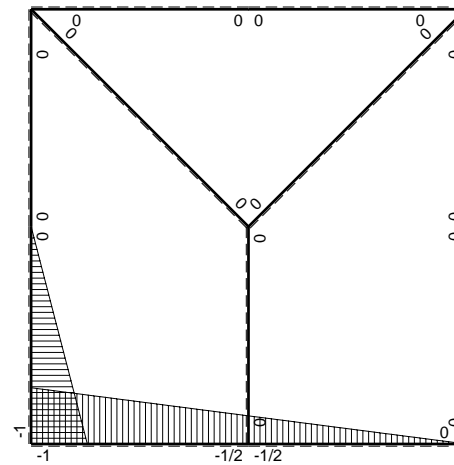
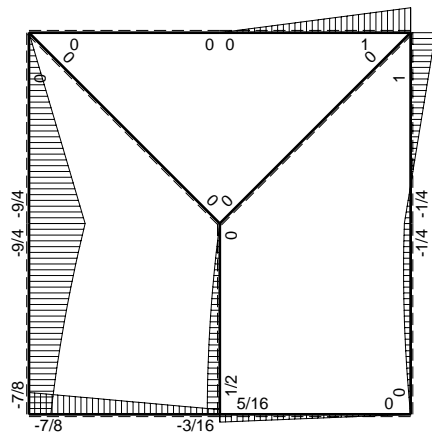




← ⊕ → F

↑ ⊕ ↓ F

⊕ M₀ flessione da carichi assegnati



⊕ F_b

⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	$-b+1/2x$	$-Fb+3/4Fx$	0	$Fb^2-5/4Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/2+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
BA b	$1/2b+1/2x$	$1/4Fb+3/4Fx$	0	$1/8Fb^2+1/2Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
BC b	$-1/2b+1/2x$	$1/4Fb-1/4Fx$	0	$-1/8Fb^2+1/4Fbx-1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
CB b	$1/2x$	$-1/4Fx$	0	$-1/8Fx^2$	0	$1/4x^2$			
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0	
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0			
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-Fb+5/4Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0	
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x$	$-9/4Fb+7/4Fx-1/2qx^2$	0	$9/4Fbx-7/4Fx^2+1/2qx^3$	0	x^2	$(2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
AH b	$b-x$	$Fb+3/4Fx+1/2qx^2$	0	$Fb^2-1/4Fbx-1/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^3/EJ$	
	totali							$1/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$							$-1/8F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (1 - 5/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [x - 5/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (b - 5/8 b + 1/8 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/8 + 1/2 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x + 1/4 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b + 1/4 b + 1/8 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/8 + 1/4 x/b - 1/8 x^2/b^2) Fb^2 1/EJ dx = [-1/8 x + 1/8 x^2/b - 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/8 b + 1/8 b - 1/24 b) Fb^2 1/EJ = -1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1/8 x^2/b^2) Fb^2 1/EJ dx = [-1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

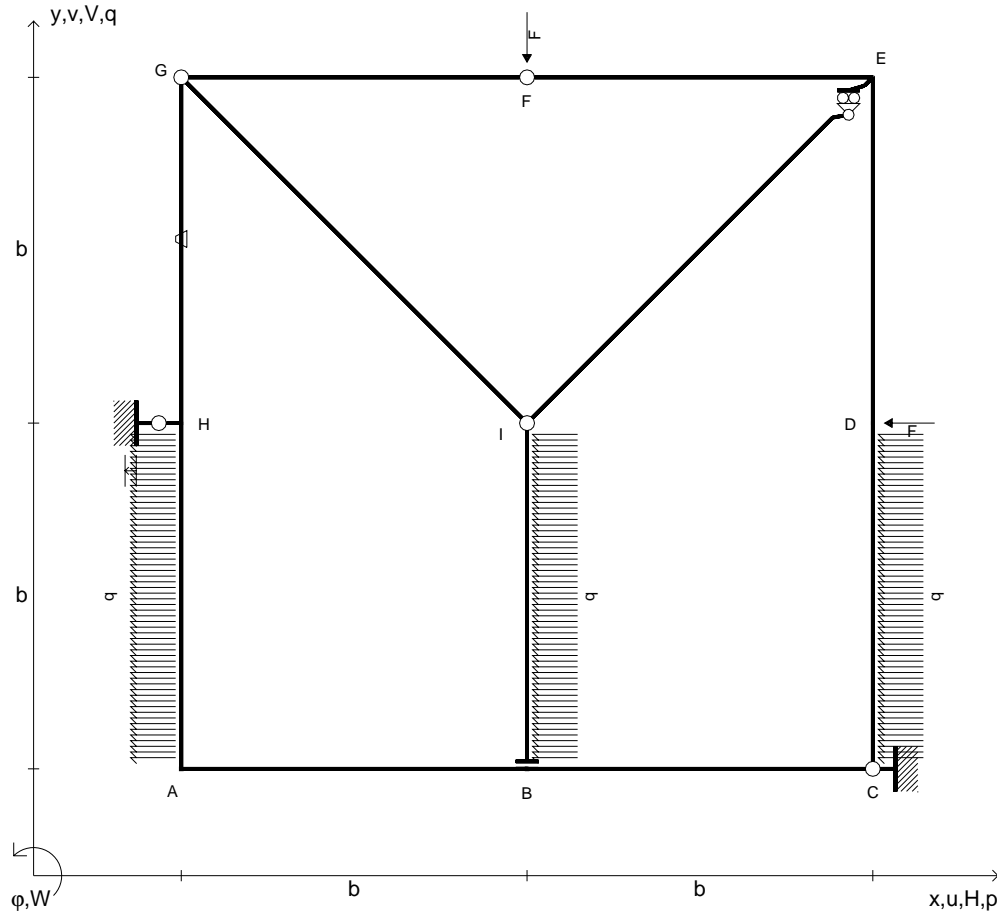
$$= (-1/24 b) Fb^2 1/EJ = -1/24 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (9/4 x/b - 7/4 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [9/8 x^2/b - 7/12 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (9/8 b - 7/12 b + 1/8 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (1 - 1/4 x/b - 1/4 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [x - 1/8 x^2/b - 1/12 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (b - 1/8 b - 1/12 b - 1/8 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$\rho_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$\rho_{IB} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$\rho_{CD} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{GH} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

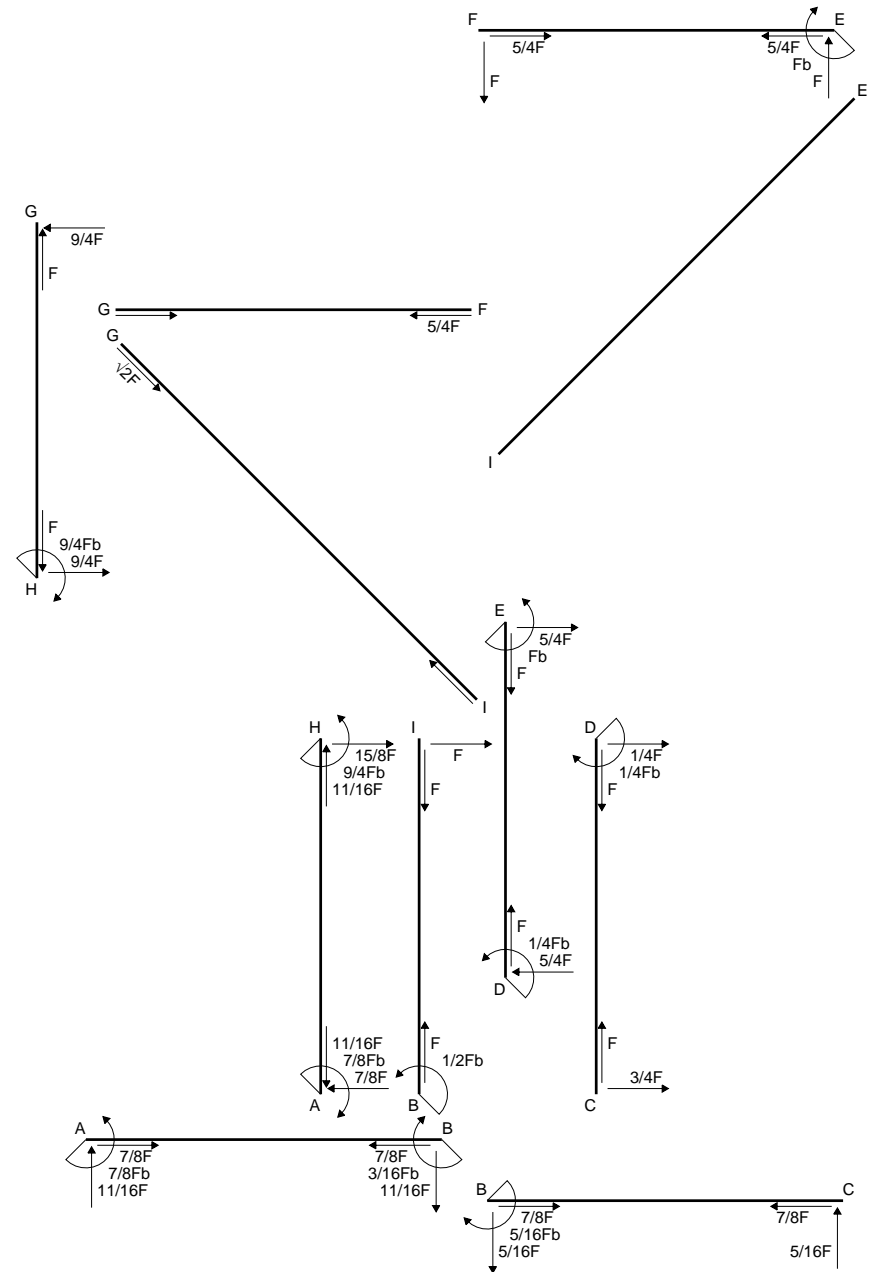
Carichi di aste curve misurati in proiezione sugli assi x,y.

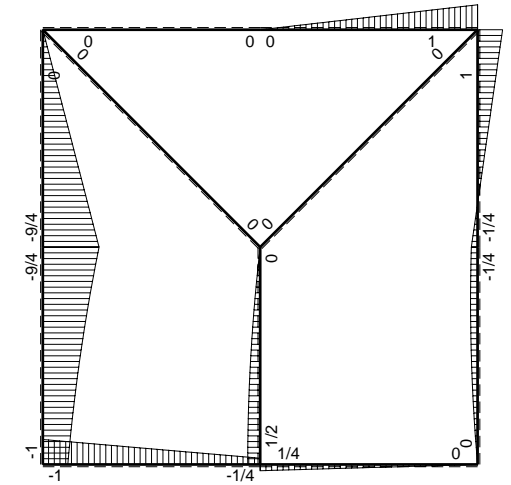
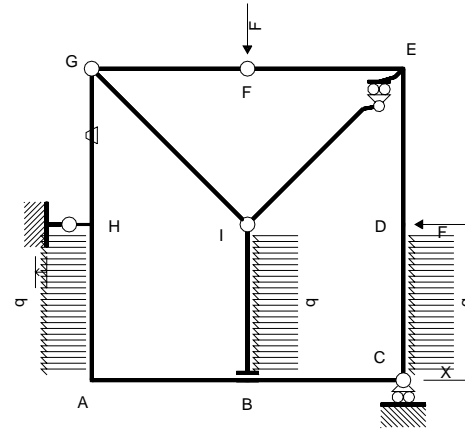
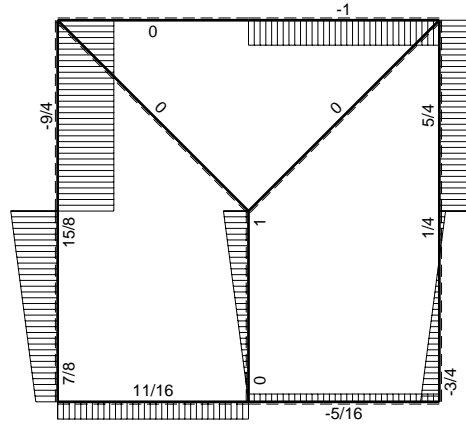
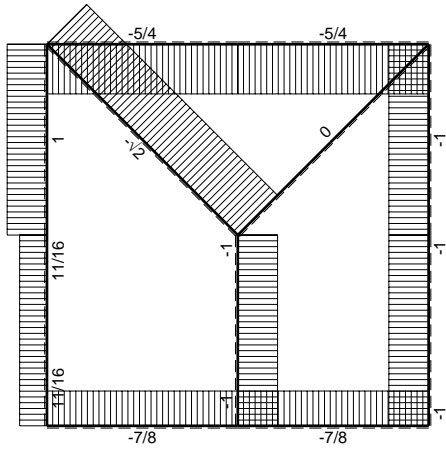
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta GH positiva se convessa a destra con inizio G.

Spostamento orizzontale assoluto u imposto al nodo H.

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

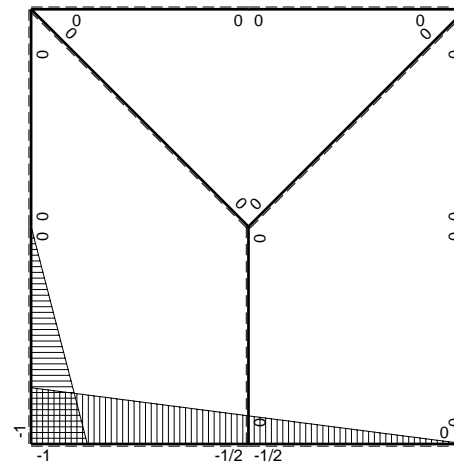
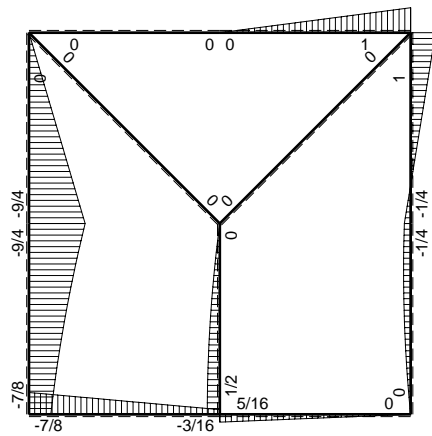




← ⊕ → F

↑ ⊕ ↓ F

⊕ M₀ flessione da carichi assegnati



⊕ F_b

⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_C

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	∫M _x (M _o /EJ+θ)dx	∫XM _x M _x /EJdx	
AB b	-b+1/2x	-Fb+3/4Fx	0	Fb ² -5/4Fbx+3/8Fx ²	0	b ² -bx+1/4x ²	(1/2+0)Fb ³ /EJ	7/12Xb ³ /EJ	
BA b	1/2b+1/2x	1/4Fb+3/4Fx	0	1/8Fb ² +1/2Fbx+3/8Fx ²	0	1/4b ² +1/2bx+1/4x ²			
BC b	-1/2b+1/2x	1/4Fb-1/4Fx	0	-1/8Fb ² +1/4Fbx-1/8Fx ²	0	1/4b ² -1/2bx+1/4x ²	(-1/24+0)Fb ³ /EJ	1/12Xb ³ /EJ	
CB b	1/2x	-1/4Fx	0	-1/8Fx ²	0	1/4x ²			
CD b	0	-3/4Fx+1/2qx ²	0	0	0	0	0+0	0	
DC b	0	1/4Fb+1/4Fx-1/2qx ²	0	0	0	0			
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0	
ED b	0	-Fb+5/4Fx	0	0	0	0			
EF b	0	Fb-Fx	0	0	0	0	0+0	0	
FE b	0	-Fx	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	-9/4Fx	-Fb/EJ	0	0	0	0+0	0	
HG b	0	9/4Fb-9/4Fx	Fb/EJ	0	0	0			
GI √2b	0	0	0	0	0	0	0	0	
IB b	0	Fx-1/2qx ²	0	0	0	0	0+0	0	
BI b	0	-1/2Fb+1/2qx ²	0	0	0	0			
IE √2b	0	0	0	0	0	0	0	0	
HA b	-x	-9/4Fb+7/4Fx-1/2qx ²	0	9/4Fbx-7/4Fx ² +1/2qx ³	0	x ²	(2/3+0)Fb ³ /EJ	1/3Xb ³ /EJ	
AH b	b-x	Fb+3/4Fx+1/2qx ²	0	Fb ² -1/4Fbx-1/4Fx ² -1/2qx ³	0	b ² -2bx+x ²			
H	cedimento nodo -H _{1H} u _H							-Fb ³ /EJ	
	totali							1/8Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _C							-1/8F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (1 - 5/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [x - 5/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (b - 5/8 b + 1/8 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/8 + 1/2 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x + 1/4 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b + 1/4 b + 1/8 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/8 + 1/4 x/b - 1/8 x^2/b^2) Fb^2 1/EJ dx = [-1/8 x + 1/8 x^2/b - 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/8 b + 1/8 b - 1/24 b) Fb^2 1/EJ = -1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1/8 x^2/b^2) Fb^2 1/EJ dx = [-1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

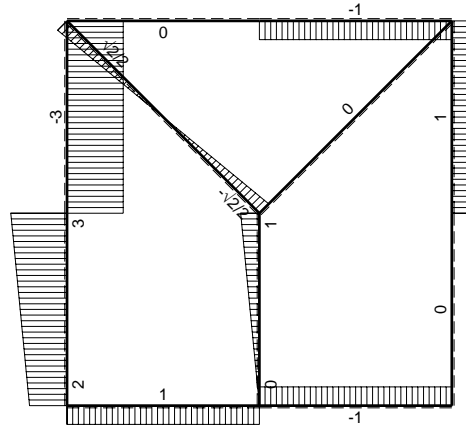
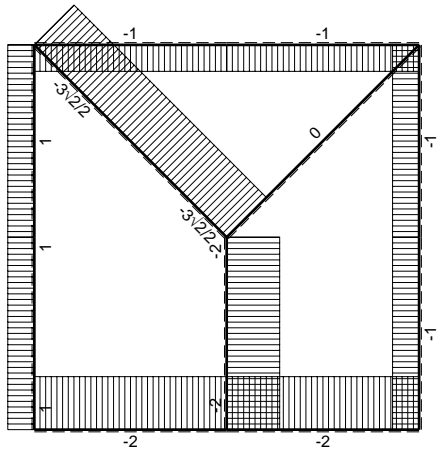
$$= (-1/24 b) Fb^2 1/EJ = -1/24 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (9/4 x/b - 7/4 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [9/8 x^2/b - 7/12 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (9/8 b - 7/12 b + 1/8 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$

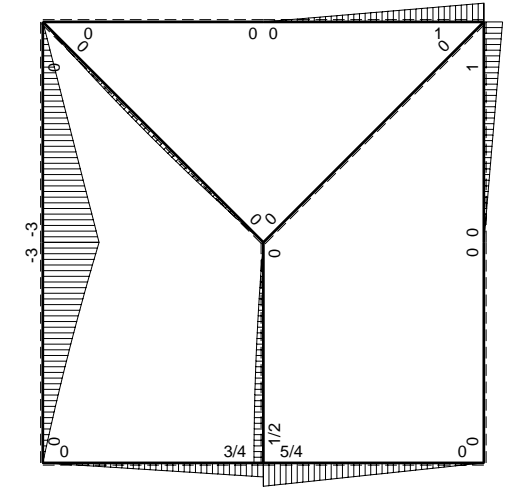
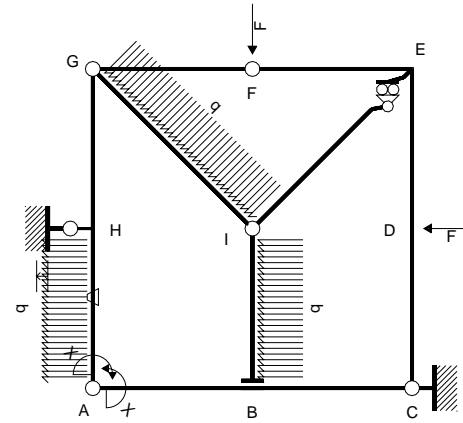
$$L_{AH}^{xo} = \int_0^b (1 - 1/4 x/b - 1/4 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [x - 1/8 x^2/b - 1/12 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (b - 1/8 b - 1/12 b - 1/8 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$

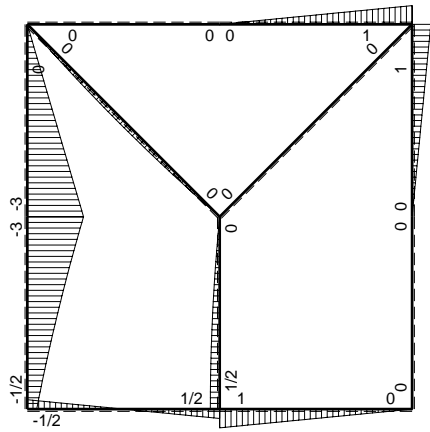


← ⊕ → F

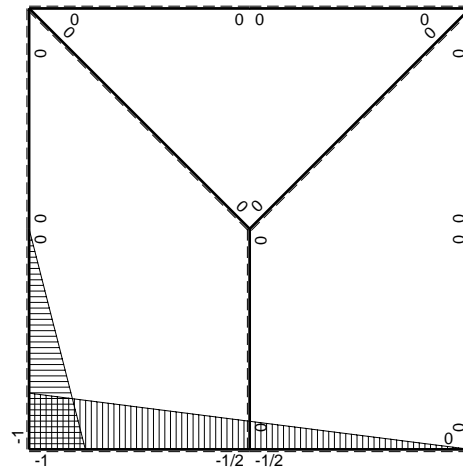
↑ ⊕ ↓ F



⊕ M₀ flessione da carichi assegnati



⊕ F_b



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	$-1+1/2x/b$	$3/4Fx$	0	$-3/4Fx+3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/4+0)Fb^2/EJ$	$7/12Xb/EJ$	
BA b	$1/2+1/2x/b$	$-3/4Fb+3/4Fx$	0	$-3/8Fb+3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	$1/12Xb/EJ$	
BC b	$-1/2+1/2x/b$	$5/4Fb-5/4Fx$	0	$-5/8Fb+5/4Fx-5/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-5/24+0)Fb^2/EJ$	$1/12Xb/EJ$	
CB b	$1/2x/b$	$-5/4Fx$	0	$-5/8Fx^2/b$	0	$1/4x^2/b^2$			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	-Fb+Fx	0	0	0	0			
EF b	0	Fb-Fx	0	0	0	0	0+0	0	
FE b	0	-Fx	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	-3Fx	0	0	0	0	0+0	0	
HG b	0	3Fb-3Fx	0	0	0	0			
GI $\sqrt{2}b$	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x/b$	$-3Fb+7/2Fx-1/2qx^2$	-Fb/EJ	$3Fx-7/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(11/24+1/2)Fb^2/EJ$	$1/3Xb/EJ$	
AH b	$1-x/b$	$5/2Fx+1/2qx^2$	Fb/EJ	$5/2Fx-2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^2/EJ$	
	totali							$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$							$1/2Fb$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = \left[x - 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = \left[1/4 x + 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = \left[1/4 x - 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = \left[1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = \left[1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (-3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = \left[-3/8 x^2/b + 1/8 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-3/8 b + 1/8 b) Fb 1/EJ = -1/4 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-3/8 + 3/8 x^2/b^2) Fb 1/EJ dx = \left[-3/8 x + 1/8 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-3/8 b + 1/8 b) Fb 1/EJ = -1/4 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (-5/8 + 5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = \left[-5/8 x + 5/8 x^2/b - 5/24 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-5/8 b + 5/8 b - 5/24 b) Fb 1/EJ = -5/24 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (-5/8 x^2/b^2) Fb 1/EJ dx = \left[-5/24 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-5/24 b) Fb 1/EJ = -5/24 Fb^2/EJ$$

$$L_{HA}^{xo} = \int_0^b (3x/b - 7/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

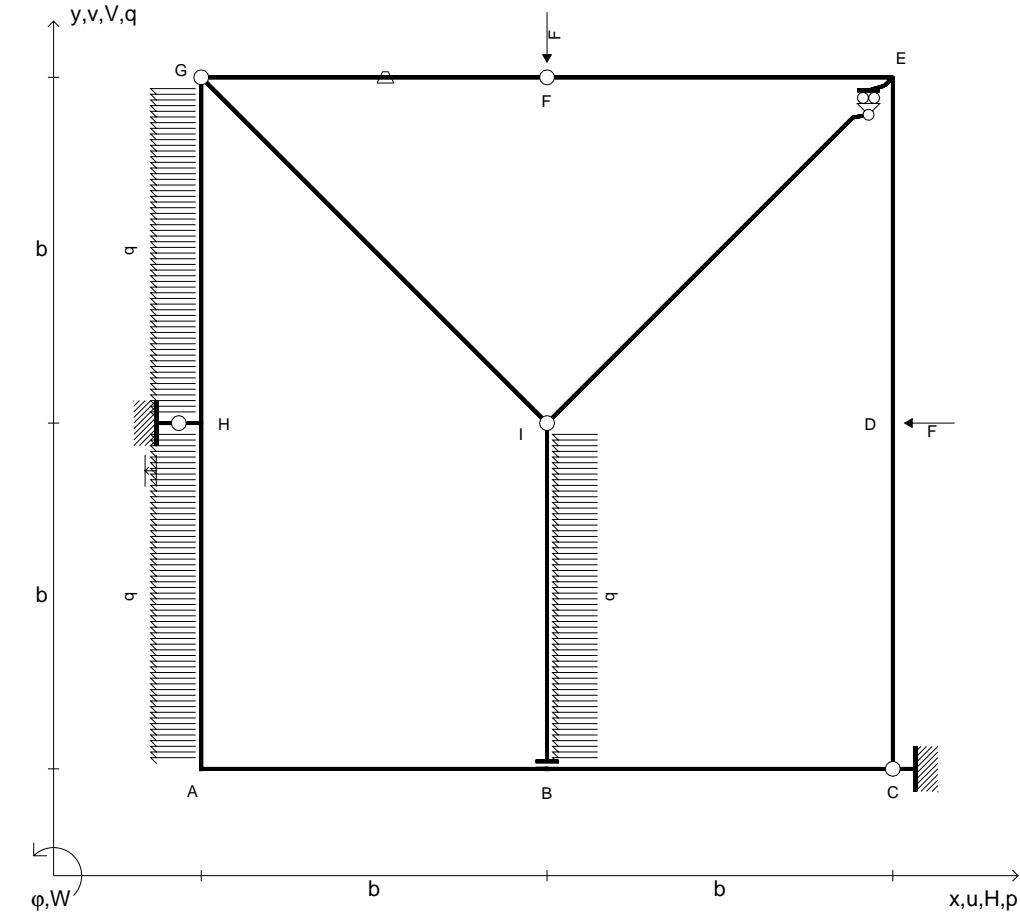
$$= \left[3/2 x^2/b - 7/6 x^3/b^2 + 1/8 x^4/b^3 \right]_0^b Fb 1/EJ + \left[1/2 x^2/b \right]_0^b \theta$$

$$= (3/2 b - 7/6 b + 1/8 b) Fb 1/EJ + (1/2 b) \theta = 23/24 Fb^2/EJ$$

$$L_{AH}^{xo} = \int_0^b (5/2 x/b - 2x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= \left[5/4 x^2/b - 2/3 x^3/b^2 - 1/8 x^4/b^3 \right]_0^b Fb 1/EJ + \left[-x + 1/2 x^2/b \right]_0^b \theta$$

$$= (5/4 b - 2/3 b - 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 23/24 Fb^2/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$\rho_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$\rho_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$\rho_{IB} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

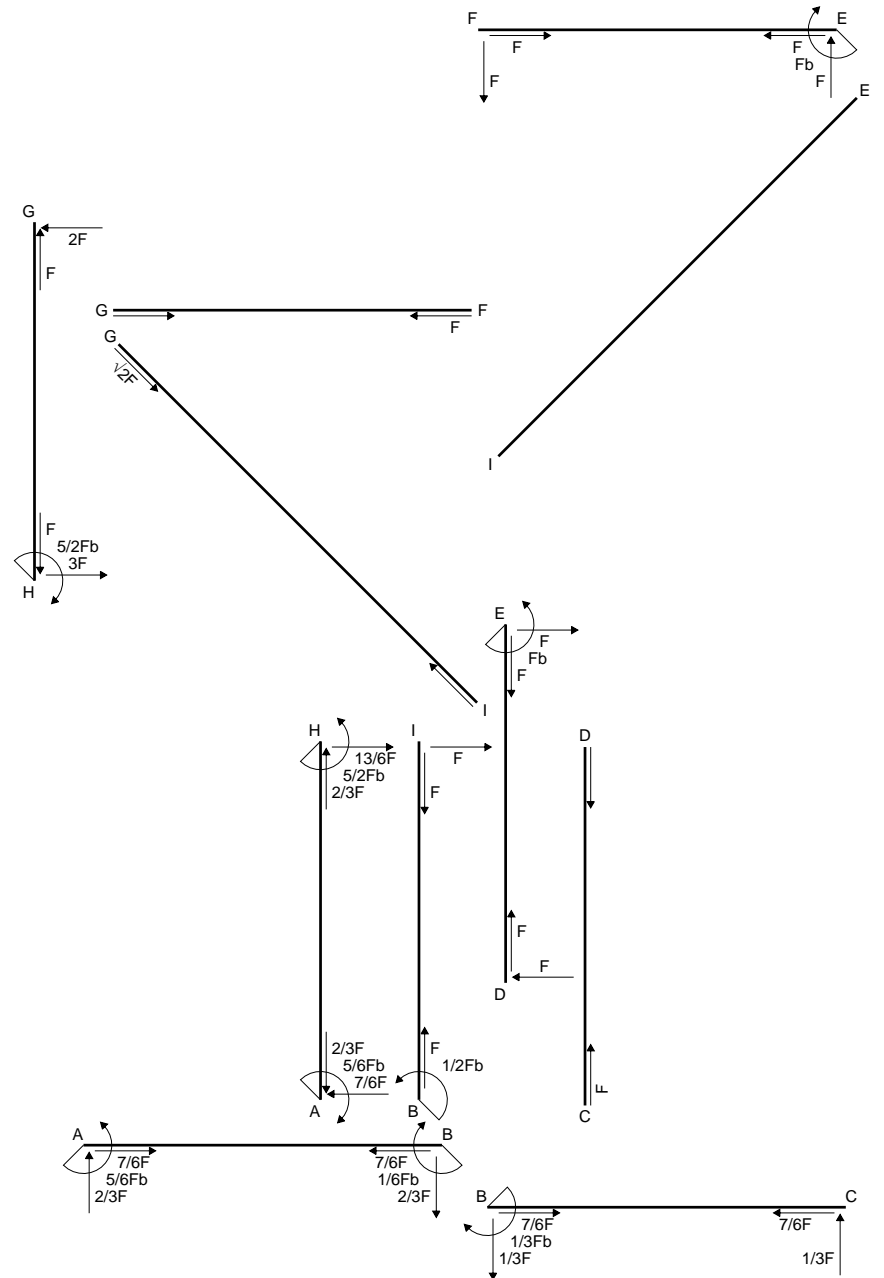
Carichi di aste curve misurati in proiezione sugli assi x,y.

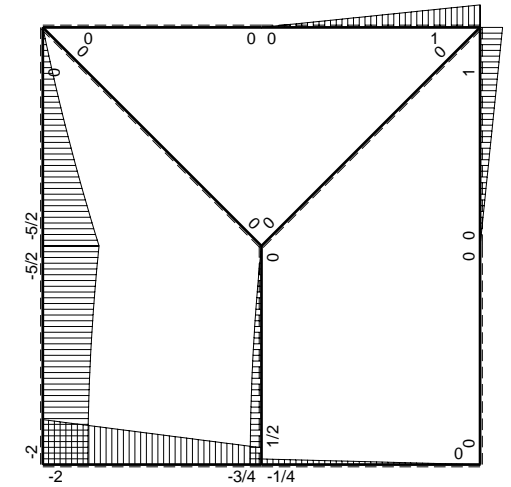
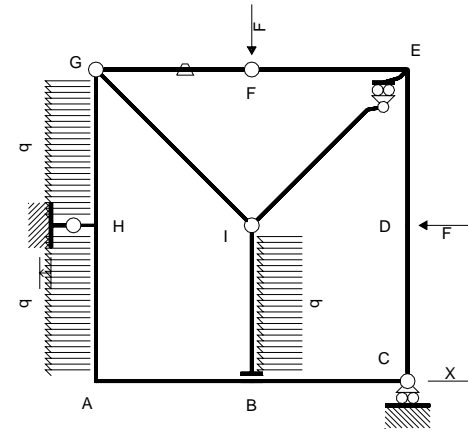
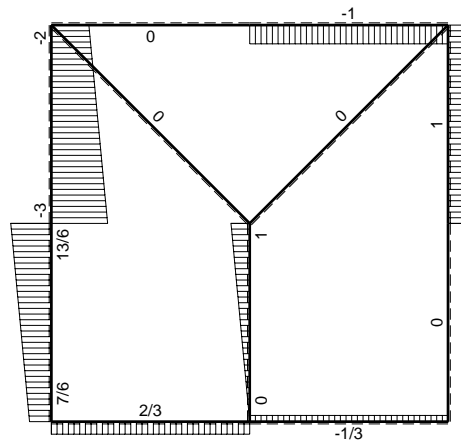
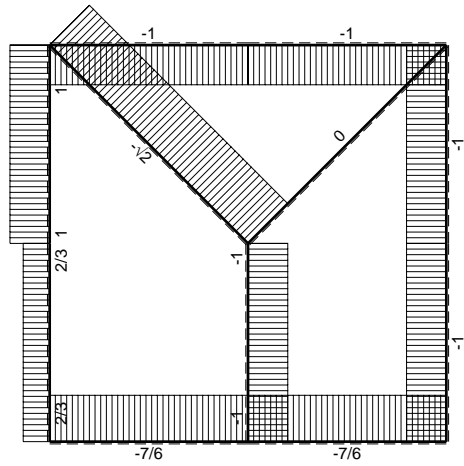
$J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta FG positiva se convessa a destra con inizio F.

Spostamento orizzontale assoluto u imposto al nodo H.

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

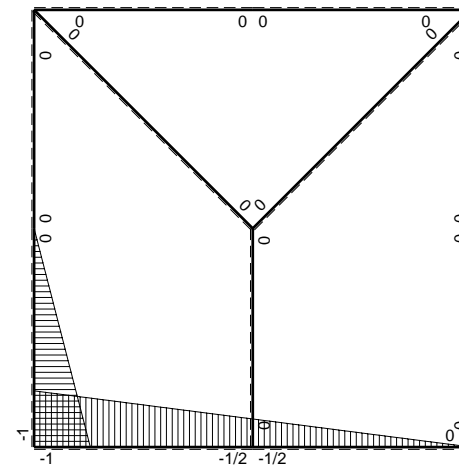
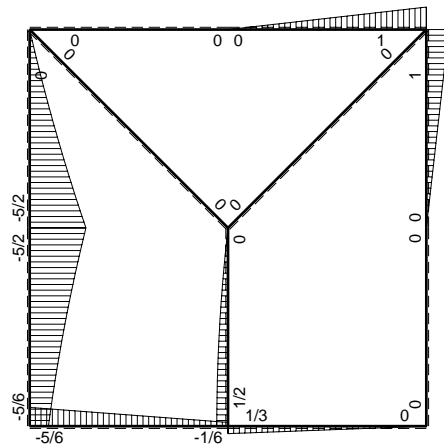




← ⊕ → F

↑ ⊕ ↓ F

⊕ M₀ flessione da carichi assegnati



⊕ F_b

⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-b+1/2x$	$-2Fb+5/4Fx$	0	$2Fb^2-9/4Fbx+5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(13/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
BA b	$1/2b+1/2x$	$3/4Fb+5/4Fx$	0	$3/8Fb^2+Fbx+5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
BC b	$-1/2b+1/2x$	$-1/4Fb+1/4Fx$	0	$1/8Fb^2-1/4Fbx+1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
CB b	$1/2x$	$1/4Fx$	0	$1/8Fx^2$	0	$1/4x^2$			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	$-Fb+Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0	
GF b	0	0	Fb/EJ	0	0	0			
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x$	$-5/2Fb+Fx-1/2qx^2$	0	$5/2Fbx-Fx^2+1/2qx^3$	0	x^2	$(25/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
AH b	$b-x$	$2Fb+1/2qx^2$	0	$2Fb^2-2Fbx+1/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^3/EJ$	
	totali							$7/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$							$-7/6F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (2 - 9/4 x/b + 5/8 x^2/b^2) Fb^2 1/EJ dx = [2x - 9/8 x^2/b + 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (2b - 9/8 b + 5/24 b) Fb^2 1/EJ = 13/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (3/8 + x/b + 5/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x + 1/2 x^2/b + 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b + 1/2 b + 5/24 b) Fb^2 1/EJ = 13/12 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/8 - 1/4 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x - 1/8 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b - 1/8 b + 1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/8 x^2/b^2) Fb^2 1/EJ dx = [1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

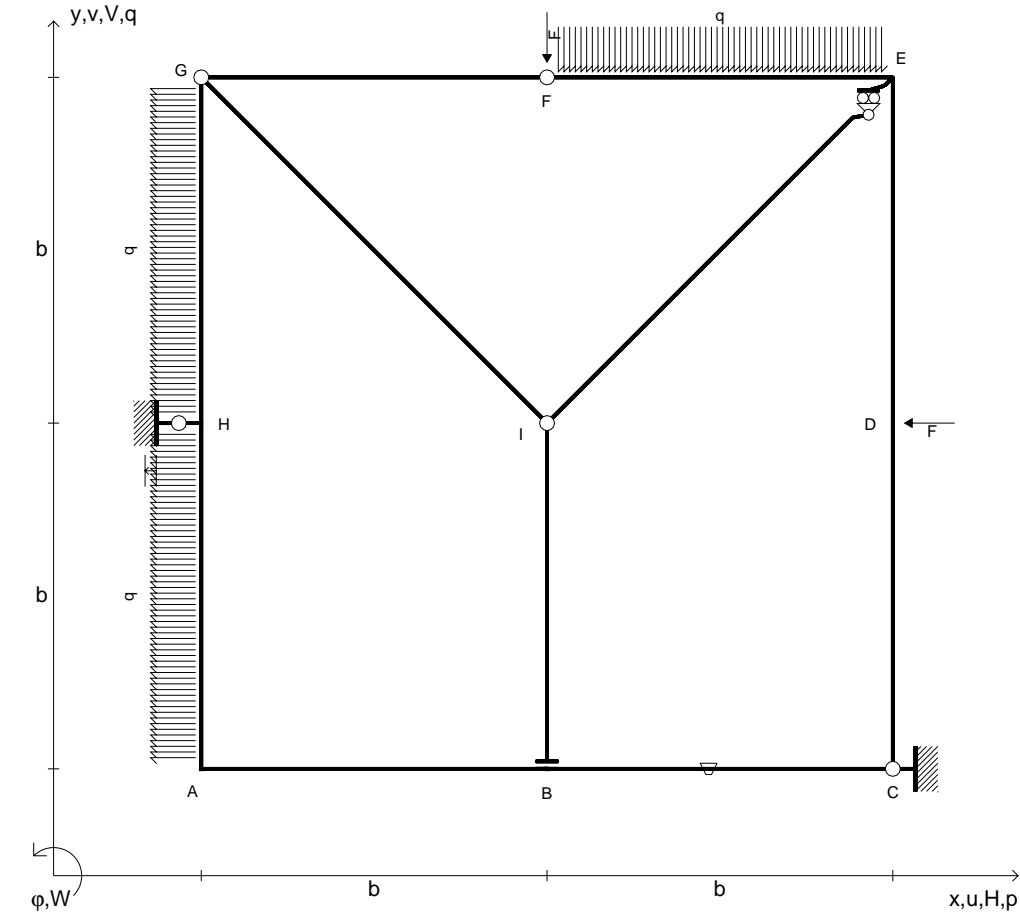
$$= (1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (5/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [5/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (5/4 b - 1/3 b + 1/8 b) Fb^2 1/EJ = 25/24 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (2 - 2x/b + 1/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [2x - x^2/b + 1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (2b - b + 1/6 b - 1/8 b) Fb^2 1/EJ = 25/24 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$\rho_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$\rho_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

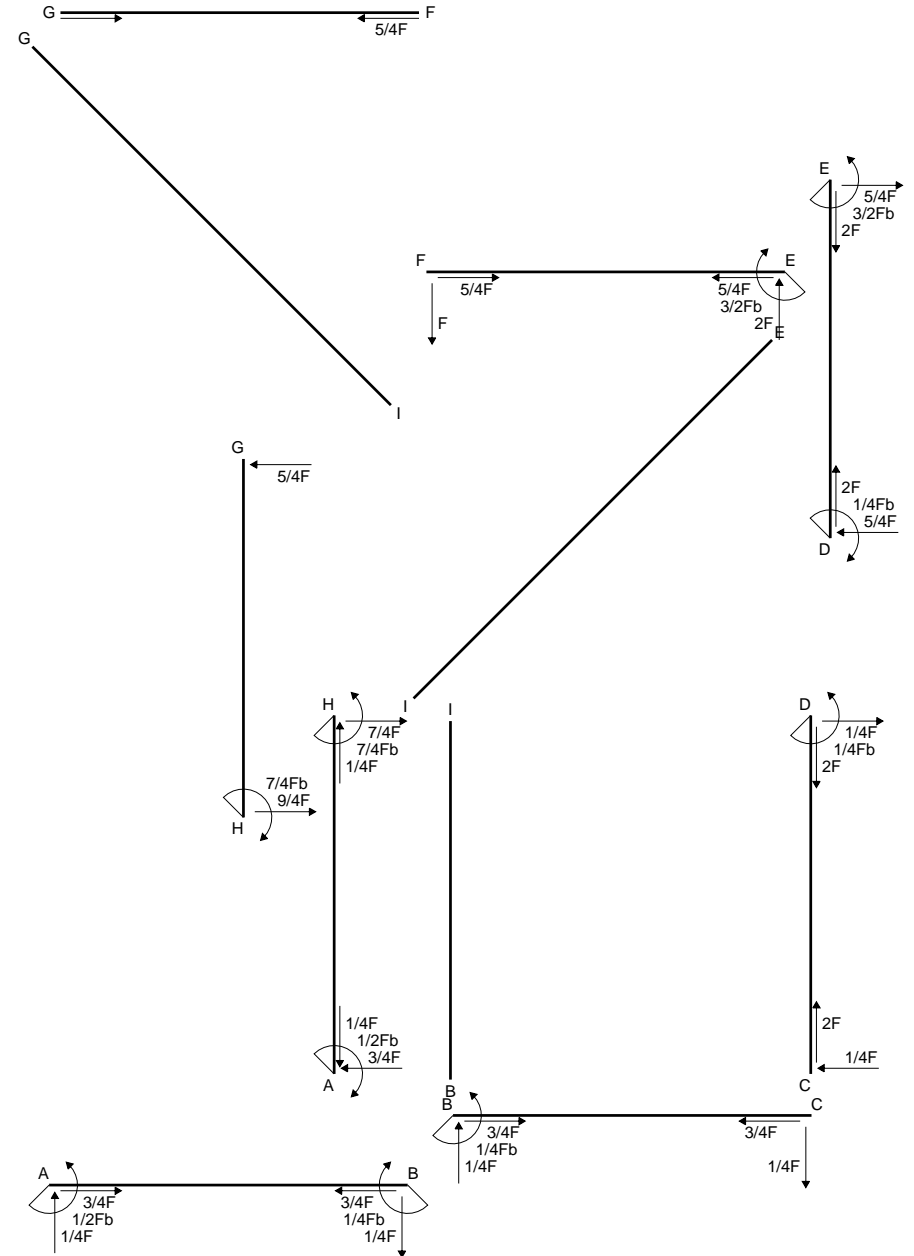
Carichi di aste curve misurati in proiezione sugli assi x,y.

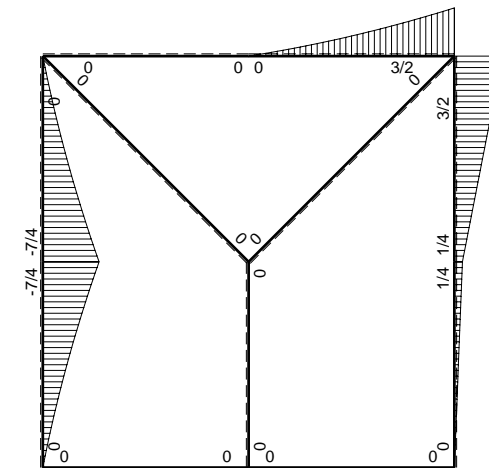
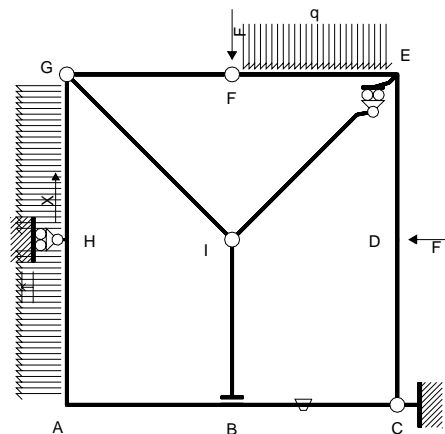
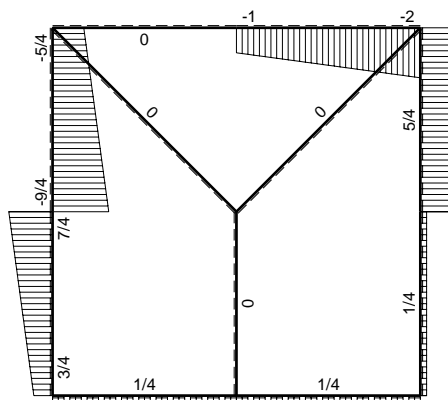
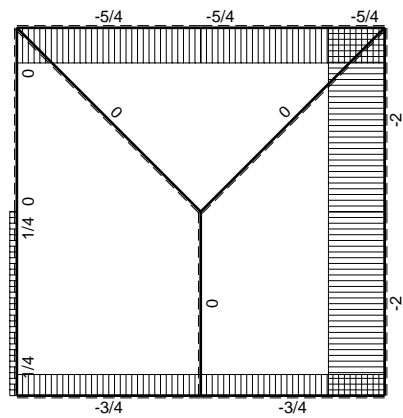
$J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta BC positiva se convessa a destra con inizio B.

Spostamento orizzontale assoluto u imposto al nodo H.

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

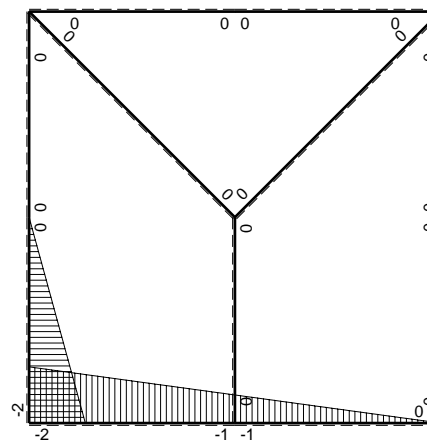
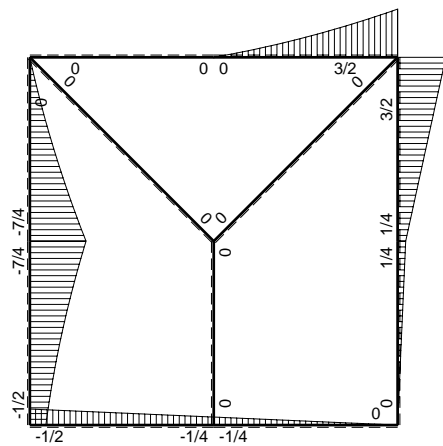




← ⊕ → F

⊕ ↓ F

⊕ ↻ M₀ flessione da carichi assegnati



⊕ ↻ F_b

⊕ ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	0	0	0	0	$4b^2-4bx+x^2$	0+0	$7/3Xb^3/EJ$	
BA b	$b+x$	0	0	0	0	$b^2+2bx+x^2$			
BC b	$-b+x$	0	$-Fb/EJ$	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$	
CB b	x	0	Fb/EJ	0	Fxb/EJ	x^2			
CD b	0	$1/4Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0			
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-Fx-1/2qx^2$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-2x$	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/2Fbx-9/2Fx^2+qx^3$	0	$4x^2$	$(1/2+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$5/4Fx+1/2qx^2$	0	$5/2Fbx-3/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$-Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							$1/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1 - x/b) \theta dx = [x - 1/2 x^2/b]_0^b \theta$$

$$= (b - 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-x/b) \theta dx = [-1/2 x^2/b]_0^b \theta$$

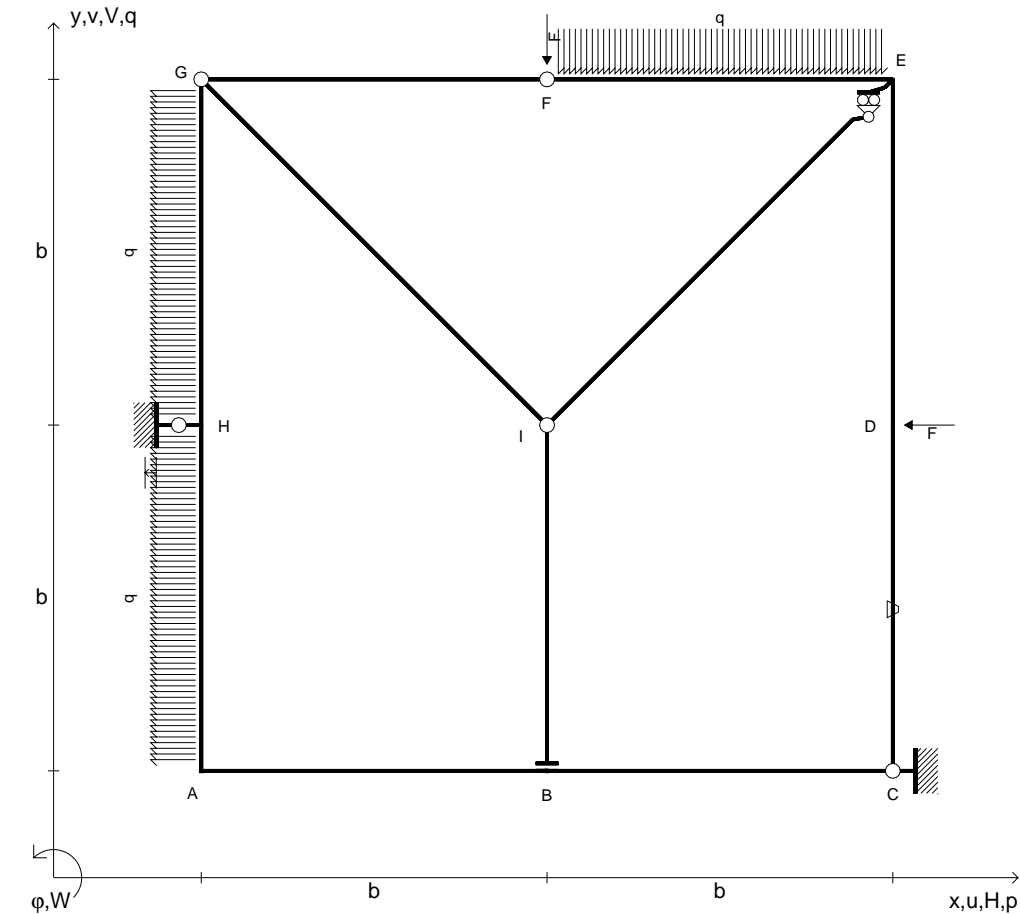
$$= (-1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (7/2 x/b - 9/2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [7/4 x^2/b - 3/2 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (7/4 b - 3/2 b + 1/4 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (5/2 x/b - 3/2 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [5/4 x^2/b - 1/2 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (5/4 b - 1/2 b - 1/4 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$\rho_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$\rho_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

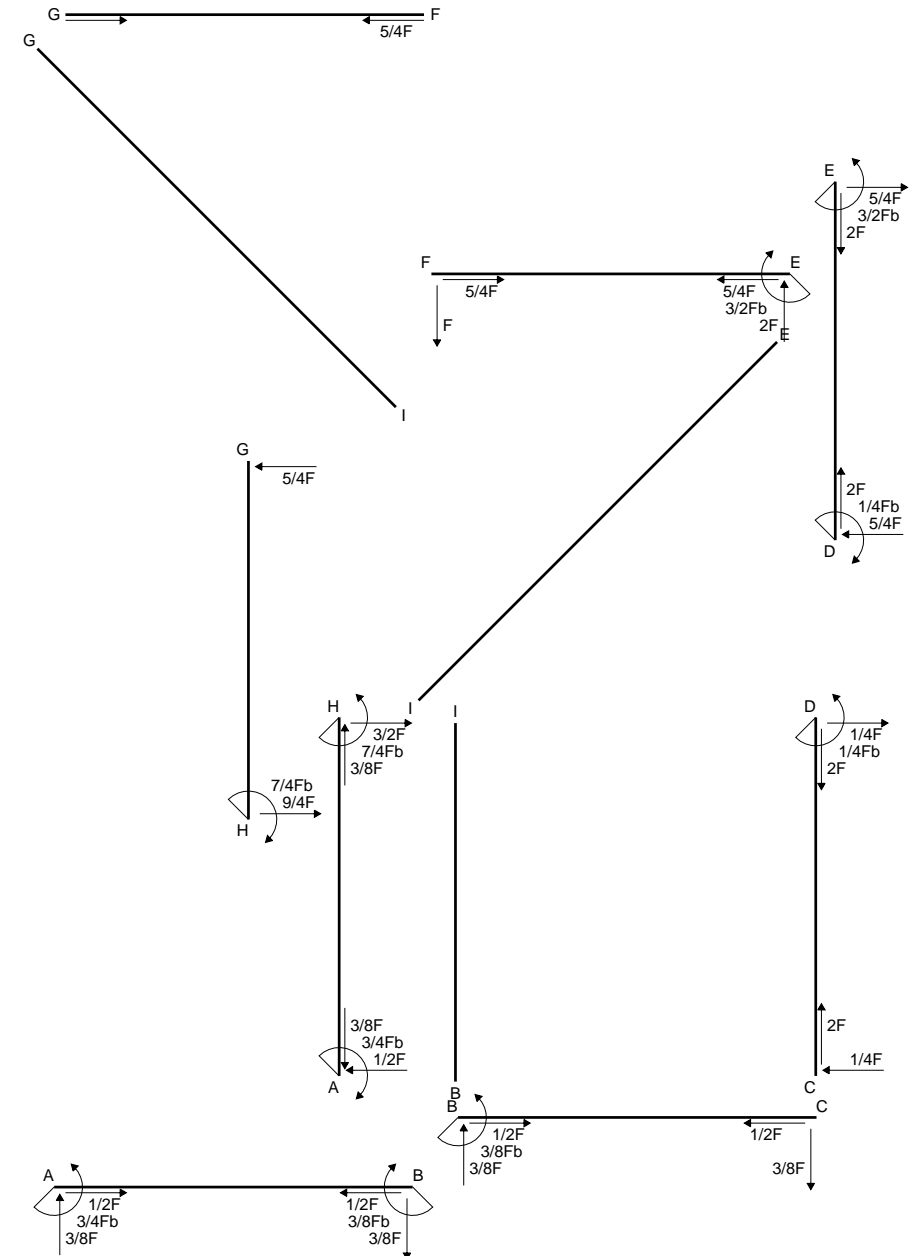
Carichi di aste curve misurati in proiezione sugli assi x,y.

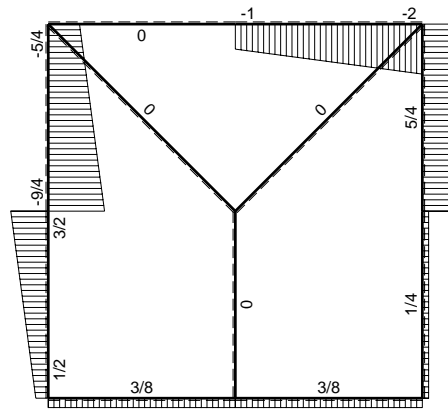
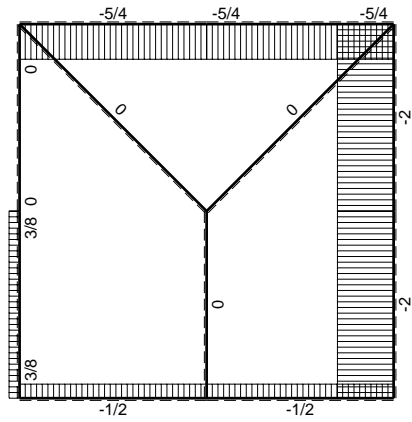
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta CD positiva se convessa a destra con inizio C.

Spostamento orizzontale assoluto u imposto al nodo H.

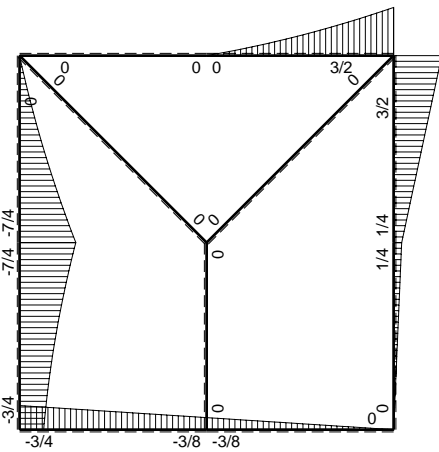
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



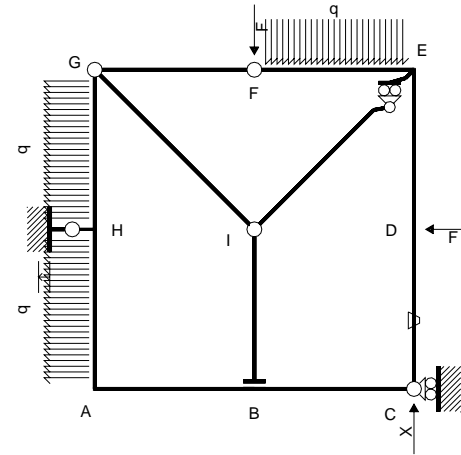


← (+) → F

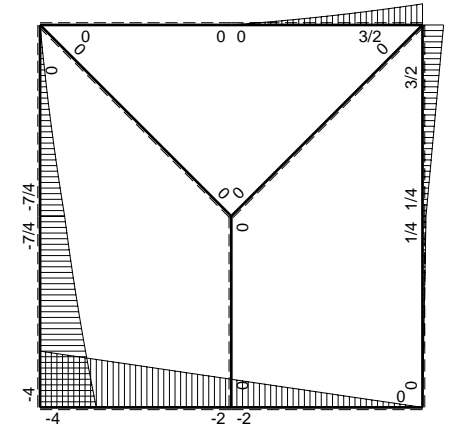
↑ (+) ↓ F



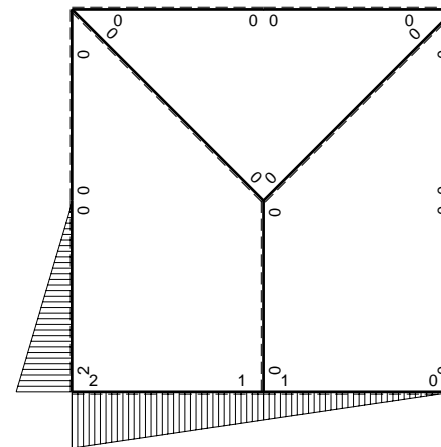
↺ (+) ↻ F_b



Schema di calcolo iperstatico



↺ (+) ↻ M₀ flessione da carichi assegnati



↺ (+) ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_c$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$2b-x$	$-4Fb+2Fx$	0	$-8Fb^2+8Fbx-2Fx^2$	0	$4b^2-4bx+x^2$	$(-14/3+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$-b-x$	$2Fb+2Fx$	0	$-2Fb^2-4Fbx-2Fx^2$	0	$b^2+2bx+x^2$			
BC b	$b-x$	$-2Fb+2Fx$	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
CB b	$-x$	$2Fx$	0	$-2Fx^2$	0	x^2			
CD b	0	$1/4Fx$	$-Fb/EJ$	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	Fb/EJ	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0			
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-Fx-1/2qx^2$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$2x$	$-7/4Fb-7/4Fx-1/2qx^2$	0	$-7/2Fbx-7/2Fx^2-qx^3$	0	$4x^2$	$(-19/6+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$-2b+2x$	$4Fb-11/4Fx+1/2qx^2$	0	$-8Fb^2+27/2Fbx-13/2Fx^2+qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$2Fb^3/EJ$	
	totali							$-13/2Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_c$							$13/8F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-8 + 8x/b - 2x^2/b^2) Fb^2 1/EJ dx = [-8x + 4x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-8b + 4b - 2/3 b) Fb^2 1/EJ = -14/3 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-2 - 4x/b - 2x^2/b^2) Fb^2 1/EJ dx = [-2x - 2x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2b - 2b - 2/3 b) Fb^2 1/EJ = -14/3 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-2 + 4x/b - 2x^2/b^2) Fb^2 1/EJ dx = [-2x + 2x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2b + 2b - 2/3 b) Fb^2 1/EJ = -2/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-2x^2/b^2) Fb^2 1/EJ dx = [-2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2/3 b) Fb^2 1/EJ = -2/3 Fb^3/EJ$$

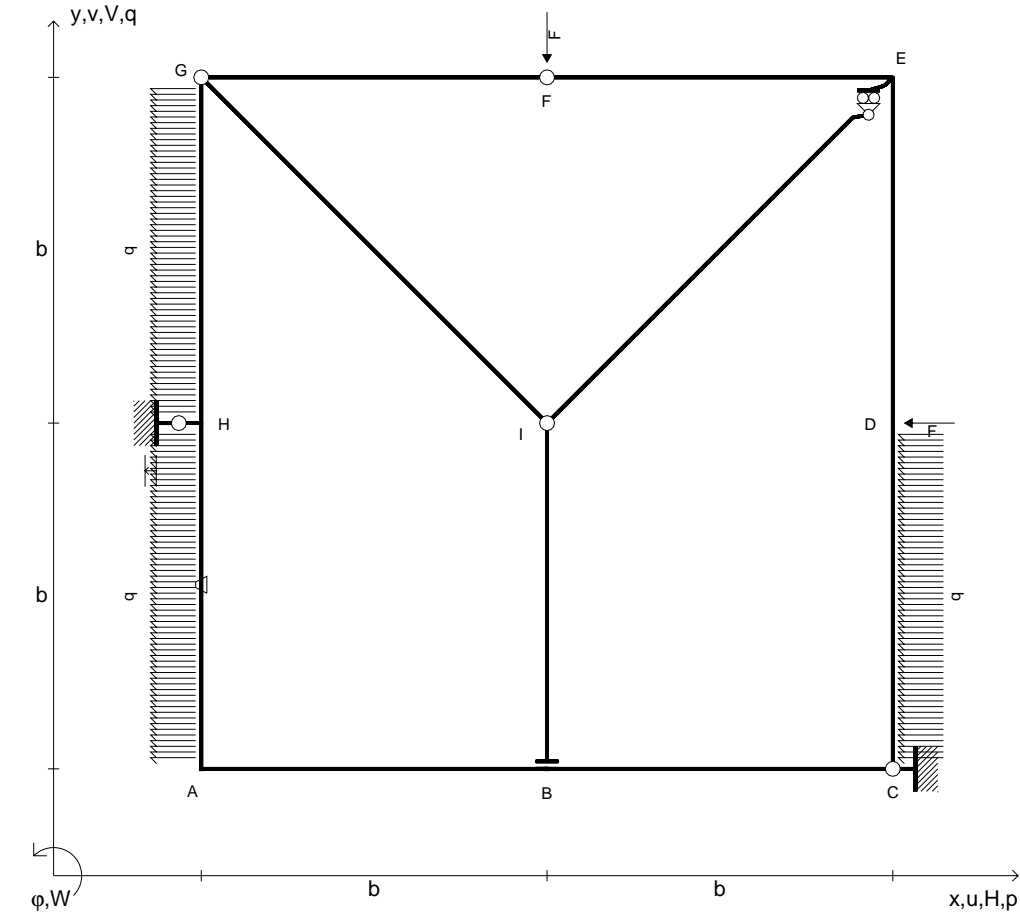
$$L_{HA}^{xo} = \int_0^b (-7/2 x/b - 7/2 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [-7/4 x^2/b - 7/6 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-7/4 b - 7/6 b - 1/4 b) Fb^2 1/EJ = -19/6 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (-8 + 27/2 x/b - 13/2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx$$

$$= [-8x + 27/4 x^2/b - 13/6 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-8b + 27/4 b - 13/6 b + 1/4 b) Fb^2 1/EJ = -19/6 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$p_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$p_{CD} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{HA} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

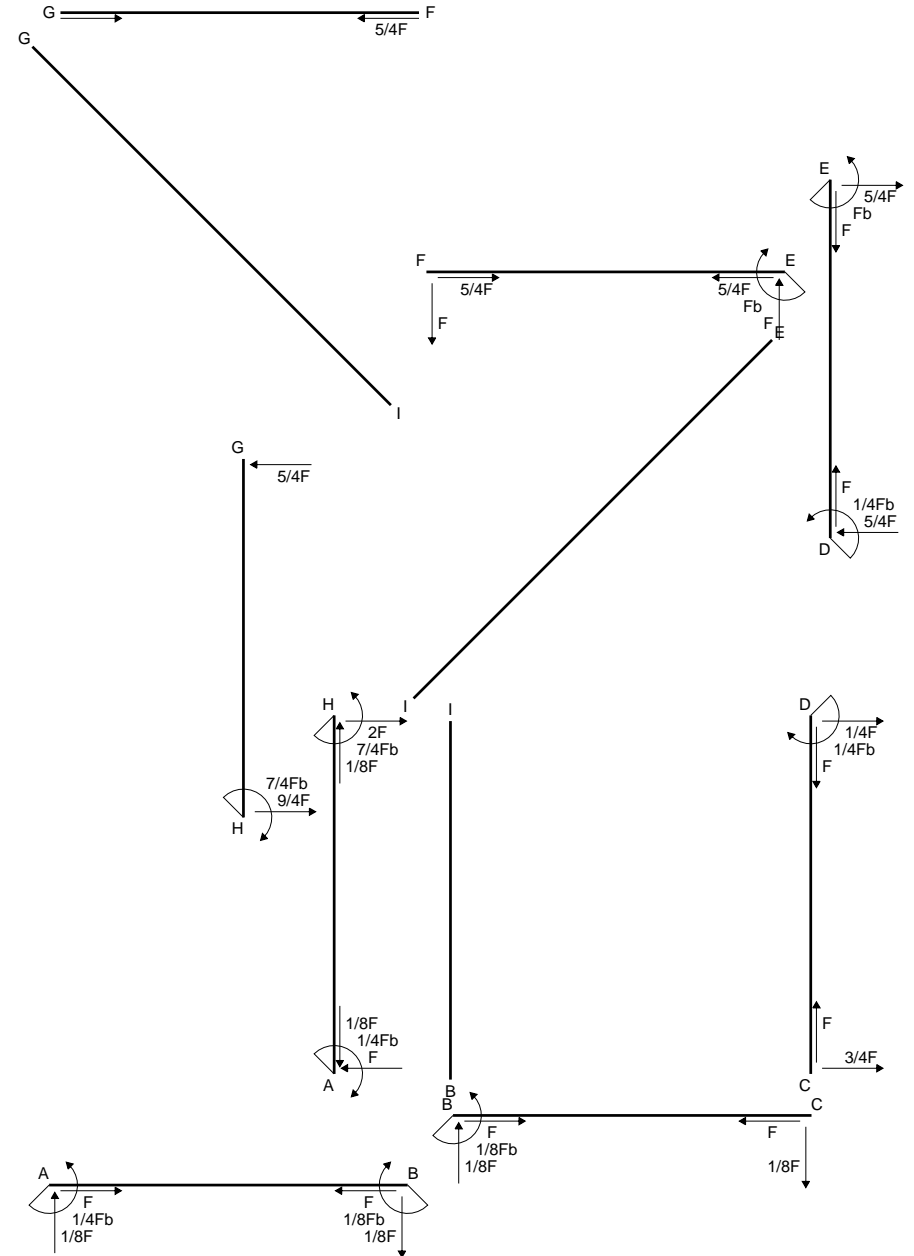
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta HA positiva se convessa a destra con inizio H.

Spostamento orizzontale assoluto u imposto al nodo H.

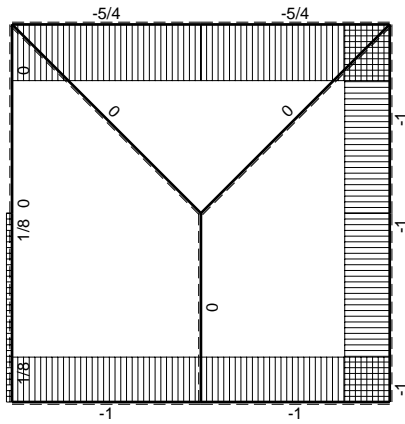
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

08.12.24

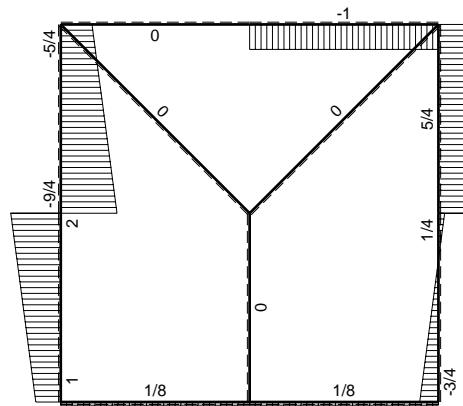


@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

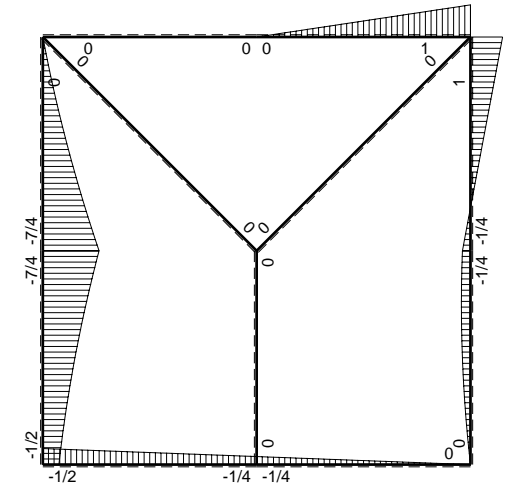
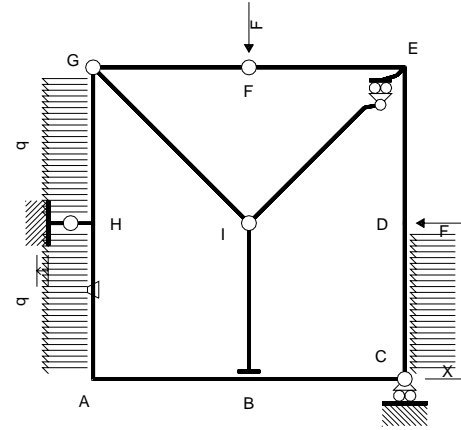
08.12.24



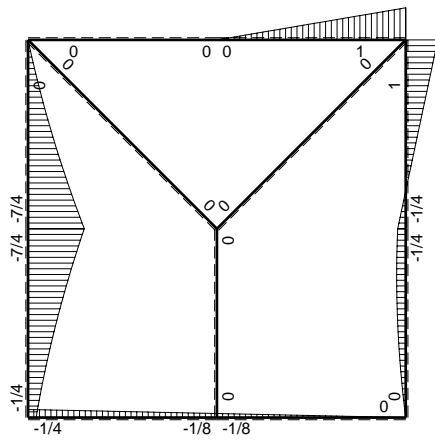
← (+) → F



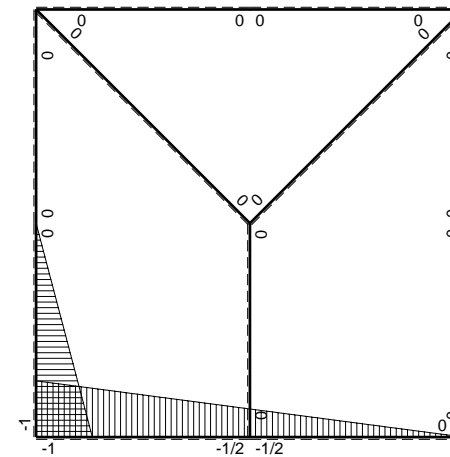
↑ (+) ↓ F



⊕ M₀ flessione da carichi assegnati



⊕ F_b



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H_C

→	M _x (x)	M _o (x)	θ	M _x M _o	M _x θ	M _x M _x	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	-b+1/2x	-1/2Fb+1/4Fx	0	1/2Fb ² -1/2Fbx+1/8Fx ²	0	b ² -bx+1/4x ²	(7/24+0)Fb ³ /EJ	7/12Xb ³ /EJ	
BA b	1/2b+1/2x	1/4Fb+1/4Fx	0	1/8Fb ² +1/4Fbx+1/8Fx ²	0	1/4b ² +1/2bx+1/4x ²			
BC b	-1/2b+1/2x	-1/4Fb+1/4Fx	0	1/8Fb ² -1/4Fbx+1/8Fx ²	0	1/4b ² -1/2bx+1/4x ²	(1/24+0)Fb ³ /EJ	1/12Xb ³ /EJ	
CB b	1/2x	1/4Fx	0	1/8Fx ²	0	1/4x ²			
CD b	0	-3/4Fx+1/2qx ²	0	0	0	0	0+0	0	
DC b	0	1/4Fb+1/4Fx-1/2qx ²	0	0	0	0			
DE b	0	-1/4Fb+5/4Fx	0	0	0	0	0+0	0	
ED b	0	-Fb+5/4Fx	0	0	0	0			
EF b	0	Fb-Fx	0	0	0	0	0+0	0	
FE b	0	-Fx	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	-5/4Fx-1/2qx ²	0	0	0	0	0+0	0	
HG b	0	7/4Fb-9/4Fx+1/2qx ²	0	0	0	0			
GI √2b	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE √2b	0	0	0	0	0	0	0	0	
HA b	-x	-7/4Fb+7/4Fx-1/2qx ²	-Fb/EJ	7/4Fbx-7/4Fx ² +1/2qx ³	Fxb/EJ	x ²	(5/12+1/2)Fb ³ /EJ	1/3Xb ³ /EJ	
AH b	b-x	1/2Fb+3/4Fx+1/2qx ²	Fb/EJ	1/2Fb ² +1/4Fbx-1/4Fx ² -1/2qx ³	Fb ² /EJ-Fxb/EJ	b ² -2bx+x ²			
H	cedimento nodo -H _{1H} u _H							-Fb ³ /EJ	
	totali							1/4Fb ³ /EJ	Xb ³ /EJ
	iperstatica X=H _C							-1/4F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 - 1/2 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/4 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/4 b + 1/24 b) Fb^2 1/EJ = 7/24 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/8 + 1/4 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x + 1/8 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b + 1/8 b + 1/24 b) Fb^2 1/EJ = 7/24 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/8 - 1/4 x/b + 1/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x - 1/8 x^2/b + 1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b - 1/8 b + 1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/8 x^2/b^2) Fb^2 1/EJ dx = [1/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (7/4 x/b - 7/4 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (x/b) \theta dx$$

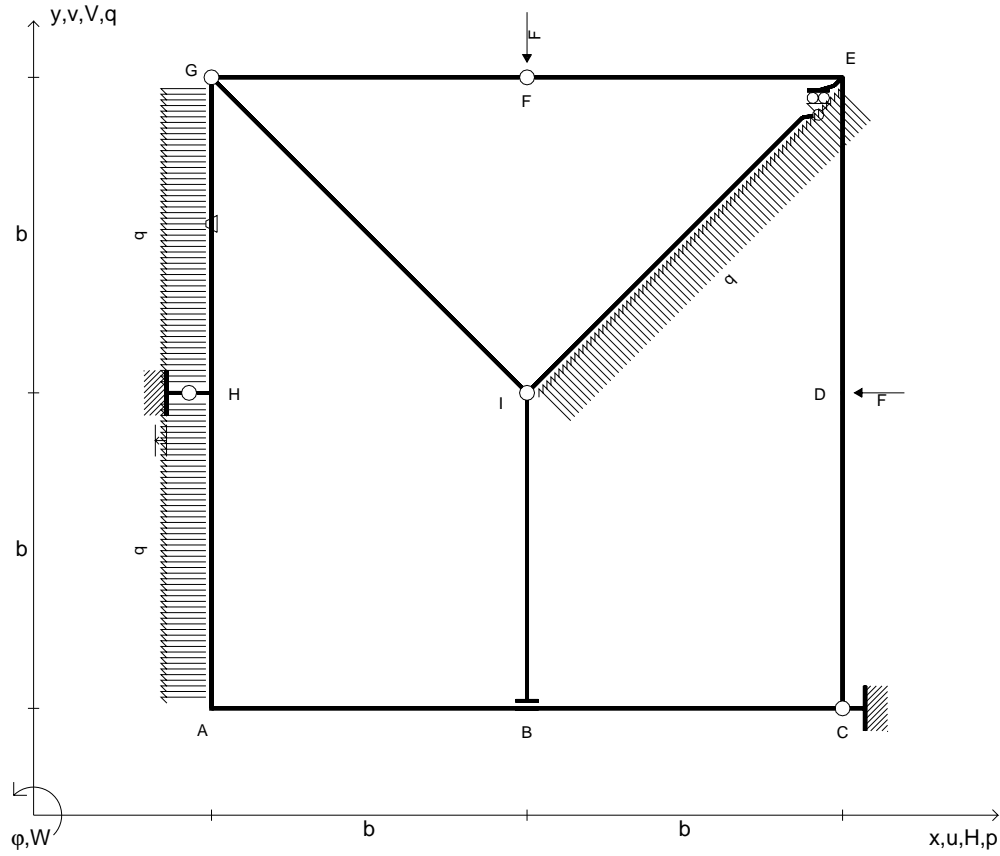
$$= [7/8 x^2/b - 7/12 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (7/8 b - 7/12 b + 1/8 b) Fb^2 1/EJ + (1/2 b) \theta = 11/12 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (1/2 + 1/4 x/b - 1/4 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/2 x + 1/8 x^2/b - 1/12 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/2 b + 1/8 b - 1/12 b - 1/8 b) Fb^2 1/EJ + (-b + 1/2 b) \theta = 11/12 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{GH} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GI} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{IB} = EJ$
$p_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IE} = EJ$
$p_{IE} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{HA} = EJ$
$q_{IE} = q = F/b$	$EJ_{EF} = EJ$	
$\theta_{GH} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{FG} = EJ$	

Reazioni iperstatiche in soluzione: $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Diagrammi di carico con valori riferiti ad asse della trave.

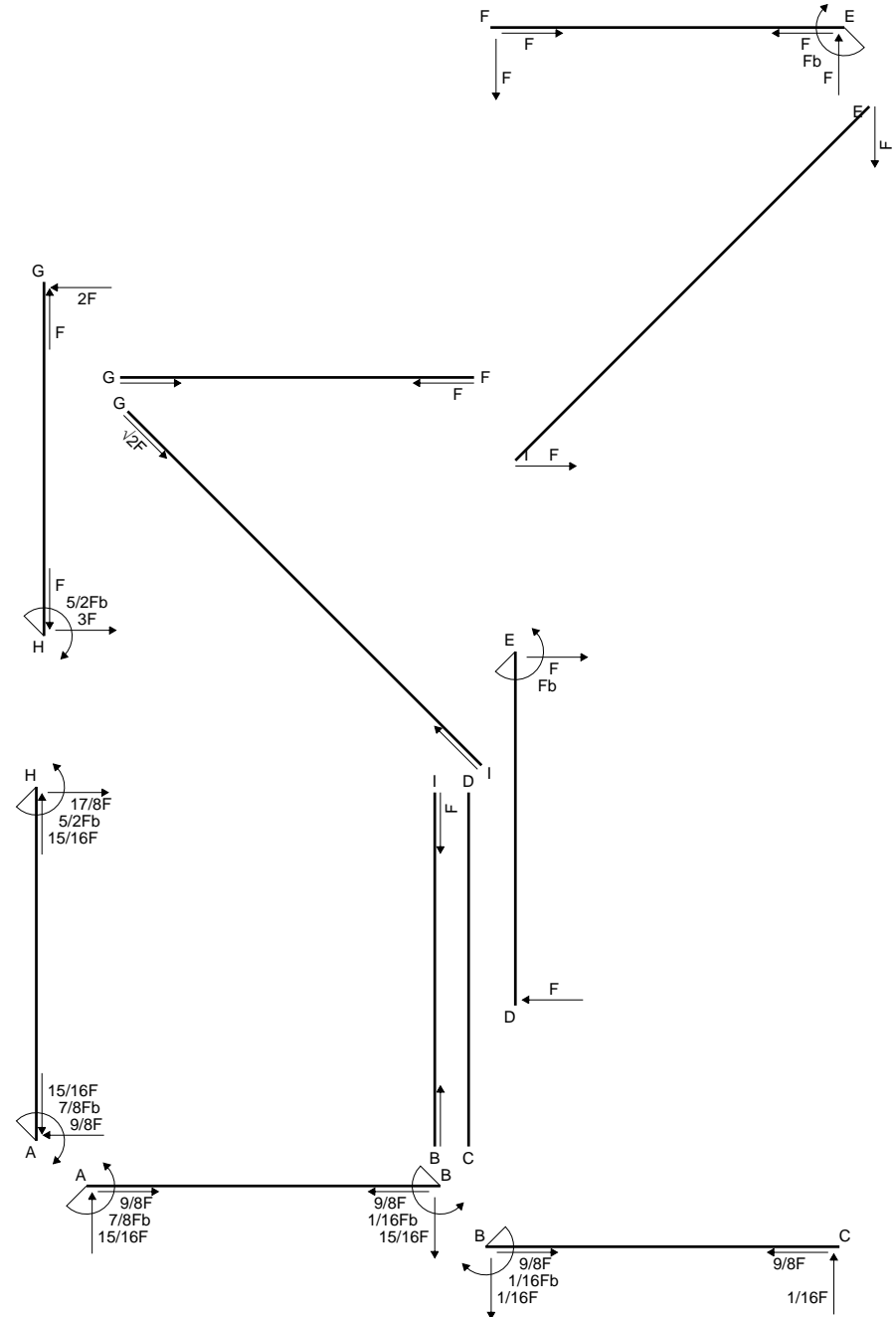
Componenti di carico distribuito riferiti ad assi ortogonali.

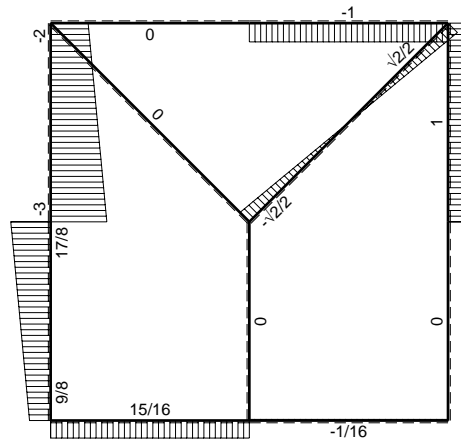
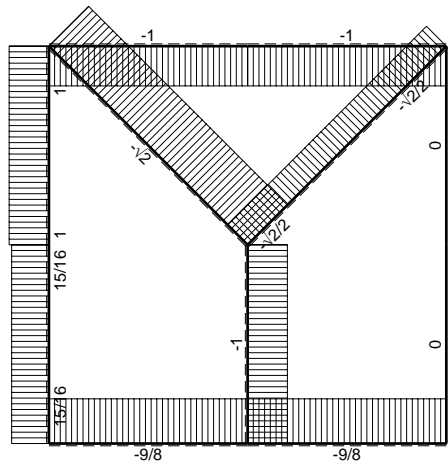
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta GH positiva se convessa a destra con inizio G.

Spostamento orizzontale assoluto u imposto al nodo H.

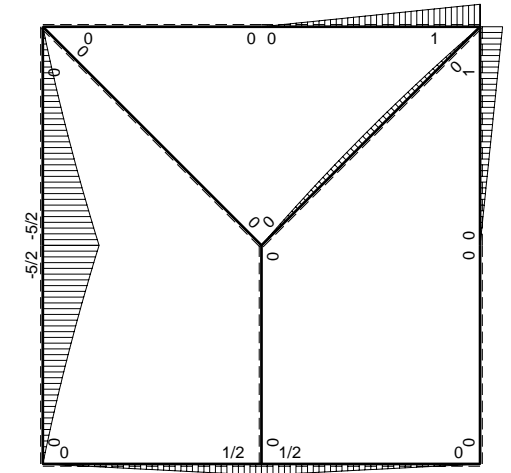
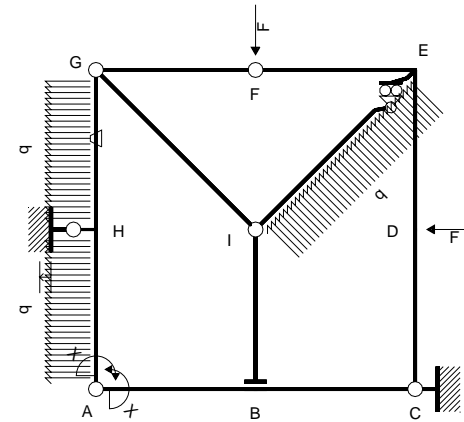
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



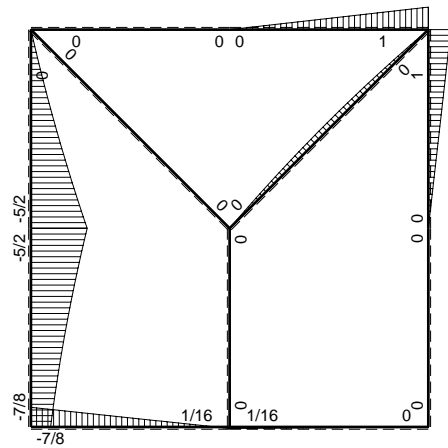


← (+) → F

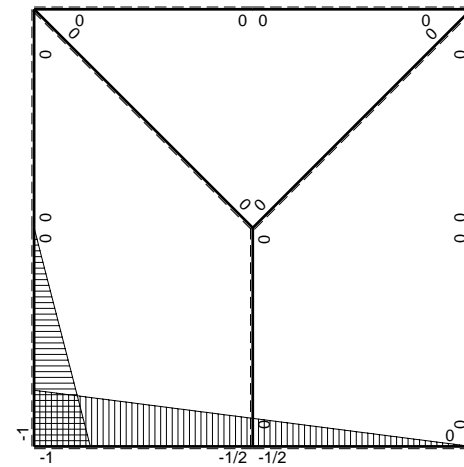
↑ (+) ↓ F



⊕ M₀ flessione da carichi assegnati



⊕ F_b



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-1+1/2x/b$	$1/2Fx$	0	$-1/2Fx+1/4Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$7/12Xb/EJ$	
BA b	$1/2+1/2x/b$	$-1/2Fb+1/2Fx$	0	$-1/4Fb+1/4Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$			
BC b	$-1/2+1/2x/b$	$1/2Fb-1/2Fx$	0	$-1/4Fb+1/2Fx-1/4Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/12+0)Fb^2/EJ$	$1/12Xb/EJ$	
CB b	$1/2x/b$	$-1/2Fx$	0	$-1/4Fx^2/b$	0	$1/4x^2/b^2$			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE b	0	Fx	0	0	0	0	0+0	0	
ED b	0	-Fb+Fx	0	0	0	0			
EF b	0	Fb-Fx	0	0	0	0	0+0	0	
FE b	0	-Fx	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-2Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0	
HG b	0	$5/2Fb-3Fx+1/2qx^2$	Fb/EJ	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0	
HA b	$-x/b$	$-5/2Fb+3Fx-1/2qx^2$	0	$5/2Fx-3Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(3/8+0)Fb^2/EJ$	$1/3Xb/EJ$	
AH b	$1-x/b$	$2Fx+1/2qx^2$	0	$2Fx-3/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^2/EJ$	
	totali							$-7/8Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$							$7/8Fb$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = \left[x - 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = \left[1/4 x + 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = \left[1/4 x - 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = \left[1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = \left[1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (-1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = \left[-1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-1/4 b + 1/12 b) Fb 1/EJ = -1/6 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/4 + 1/4 x^2/b^2) Fb 1/EJ dx = \left[-1/4 x + 1/12 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-1/4 b + 1/12 b) Fb 1/EJ = -1/6 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/4 + 1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx = \left[-1/4 x + 1/4 x^2/b - 1/12 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-1/4 b + 1/4 b - 1/12 b) Fb 1/EJ = -1/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1/4 x^2/b^2) Fb 1/EJ dx = \left[-1/12 x^3/b^2 \right]_0^b Fb 1/EJ$$

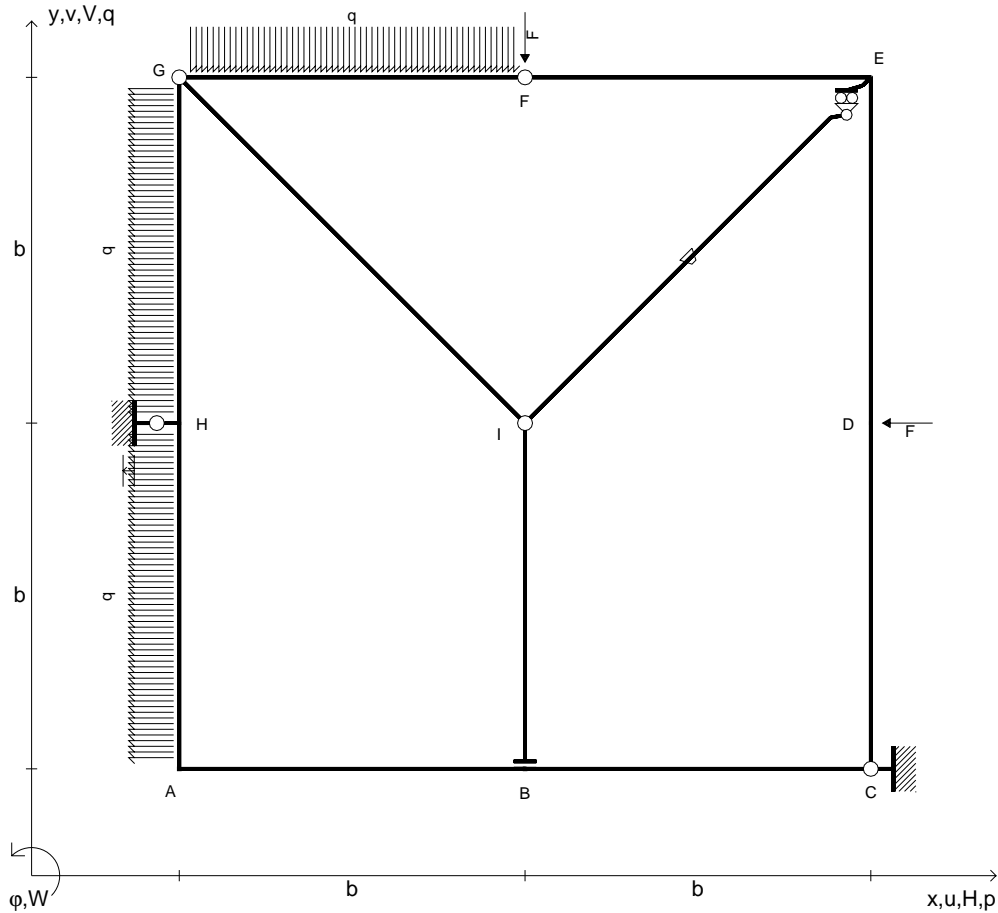
$$= (-1/12 b) Fb 1/EJ = -1/12 Fb^2/EJ$$

$$L_{HA}^{xo} = \int_0^b (5/2 x/b - 3 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = \left[5/4 x^2/b - x^3/b^2 + 1/8 x^4/b^3 \right]_0^b Fb 1/EJ$$

$$= (5/4 b - b + 1/8 b) Fb 1/EJ = 3/8 Fb^2/EJ$$

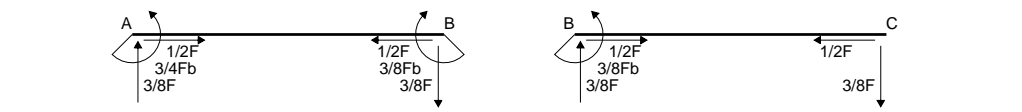
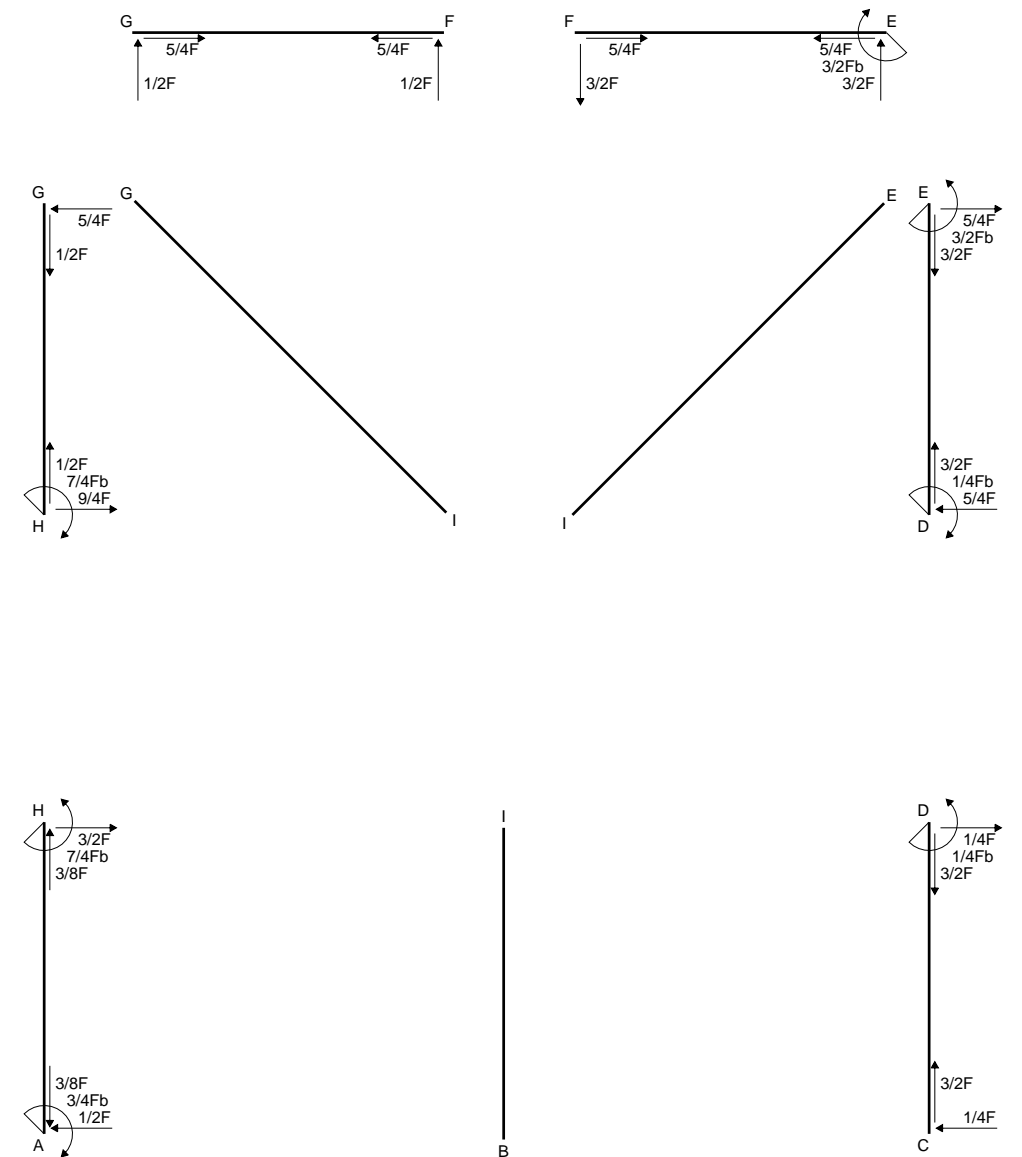
$$L_{AH}^{xo} = \int_0^b (2 x/b - 3/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = \left[x^2/b - 1/2 x^3/b^2 - 1/8 x^4/b^3 \right]_0^b Fb 1/EJ$$

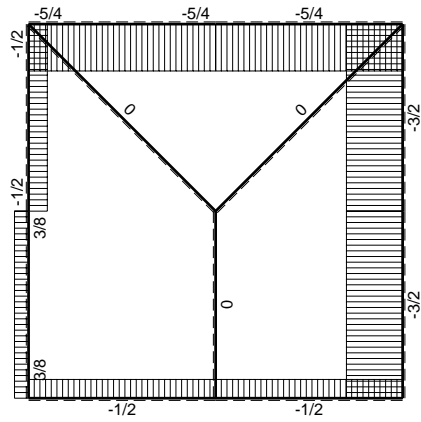
$$= (b - 1/2 b - 1/8 b) Fb 1/EJ = 3/8 Fb^2/EJ$$



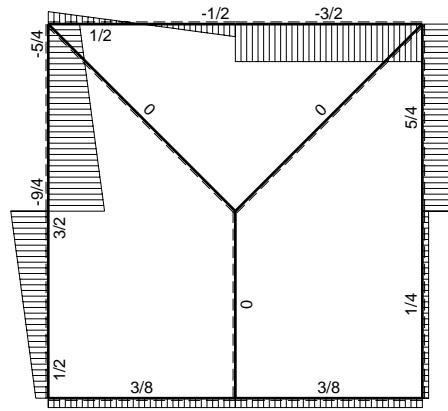
$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$p_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{FG} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta IE positiva se convessa a destra con inizio I.
 Spostamento orizzontale assoluto u imposto al nodo H.

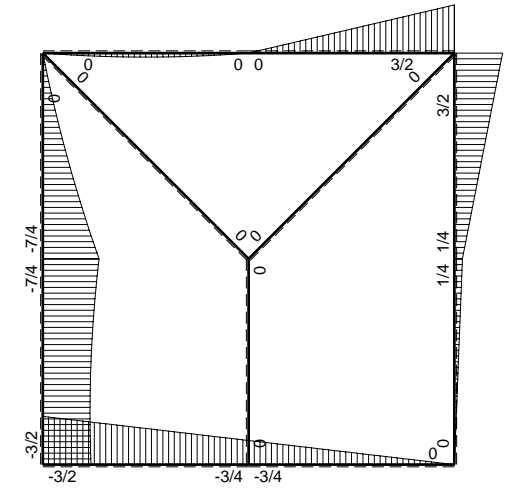
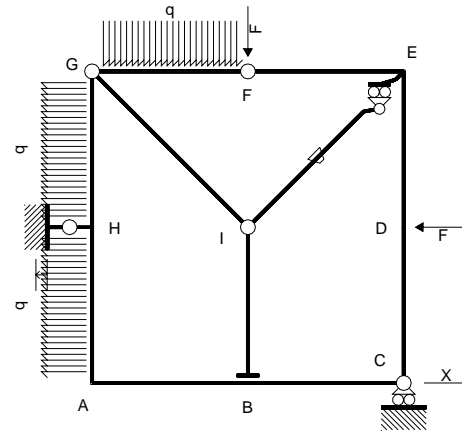




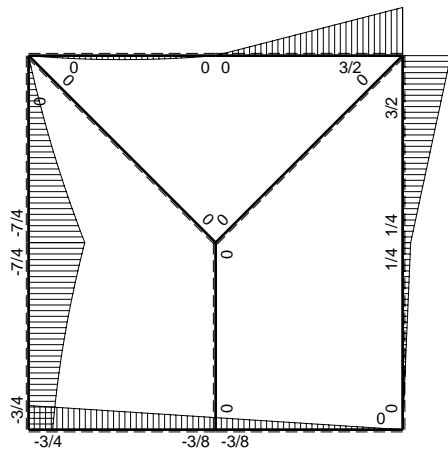
← (+) → F



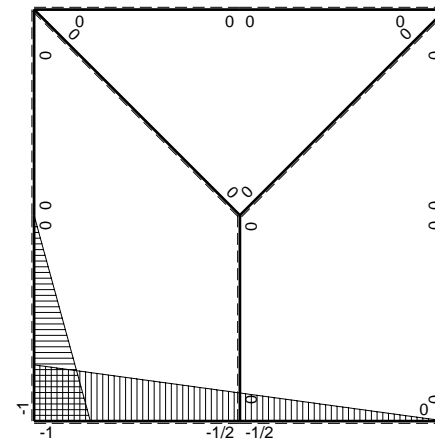
↑ (+) ↓ F



⌚ (+) ⌚ M_o flessione da carichi assegnati



⌚ (+) ⌚ F_b



⌚ (+) ⌚ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-b+1/2x$	$-3/2Fb+3/4Fx$	0	$3/2Fb^2-3/2Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
BA b	$1/2b+1/2x$	$3/4Fb+3/4Fx$	0	$3/8Fb^2+3/4Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
BC b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
CB b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$			
CD b	0	$1/4Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0			
EF b	0	$3/2Fb-3/2Fx$	0	0	0	0	0+0	0	
FE b	0	$-3/2Fx$	0	0	0	0			
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	$-Fb/EJ$	0	0	0	0	0	
HA b	$-x$	$-7/4Fb+3/4Fx-1/2qx^2$	0	$7/4Fbx-3/4Fx^2+1/2qx^3$	0	x^2	$(3/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
AH b	$b-x$	$3/2Fb-1/4Fx+1/2qx^2$	0	$3/2Fb^2-7/4Fbx+3/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^3/EJ$	
	totali							$3/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$							$-3/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (3/2 - 3/2 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/2 x - 3/4 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/2 b - 3/4 b + 1/8 b) Fb^2 1/EJ = 7/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (3/8 + 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x + 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b + 3/8 b + 1/8 b) Fb^2 1/EJ = 7/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

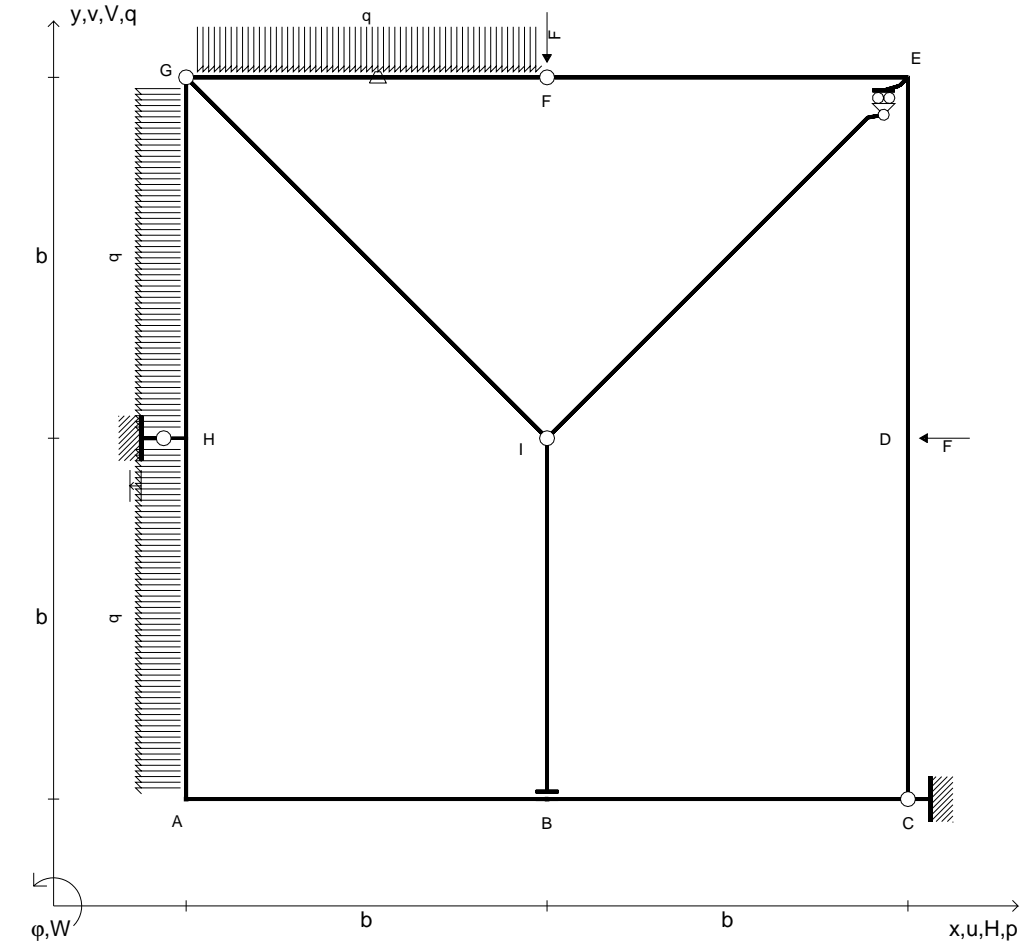
$$L_{HA}^{xo} = \int_0^b (7/4 x/b - 3/4 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [7/8 x^2/b - 1/4 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (7/8 b - 1/4 b + 1/8 b) Fb^2 1/EJ = 3/4 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (3/2 - 7/4 x/b + 3/4 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx$$

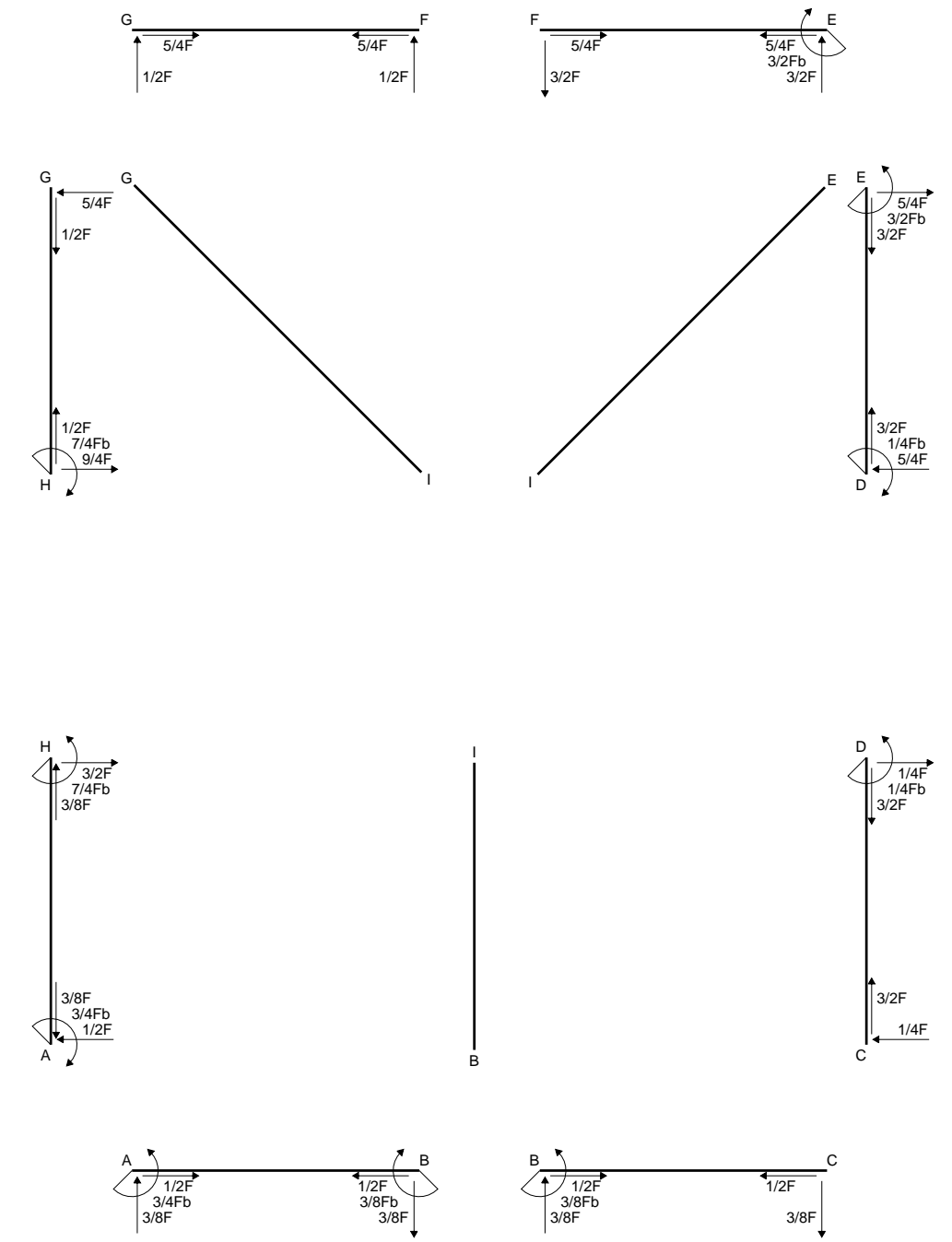
$$= [3/2 x - 7/8 x^2/b + 1/4 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

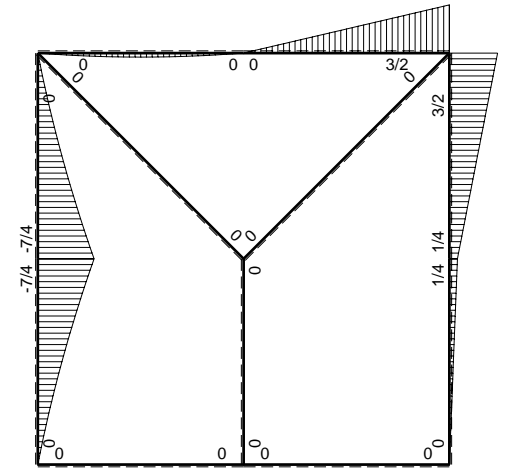
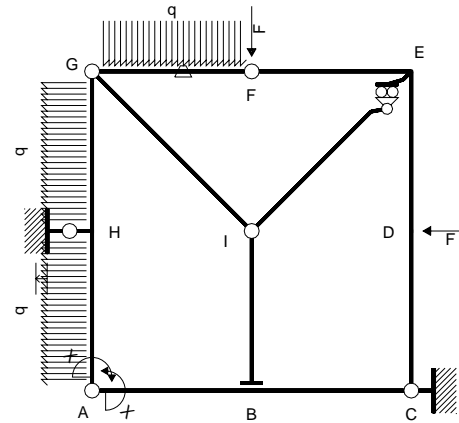
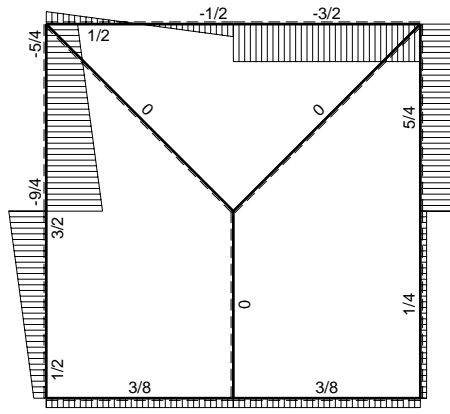
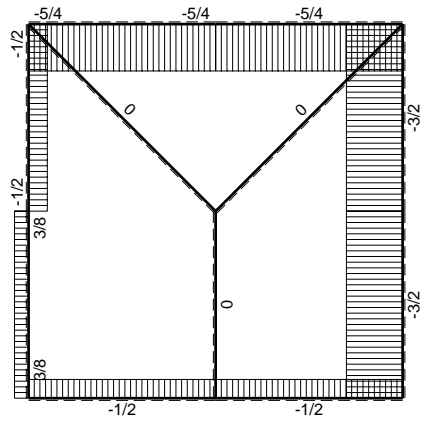
$$= (3/2 b - 7/8 b + 1/4 b - 1/8 b) Fb^2 1/EJ = 3/4 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$p_{GH} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{FG} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{FG} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta FG positiva se convessa a destra con inizio F.
 Spostamento orizzontale assoluto u imposto al nodo H.
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

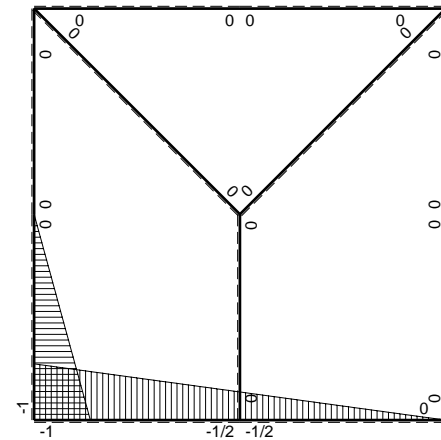
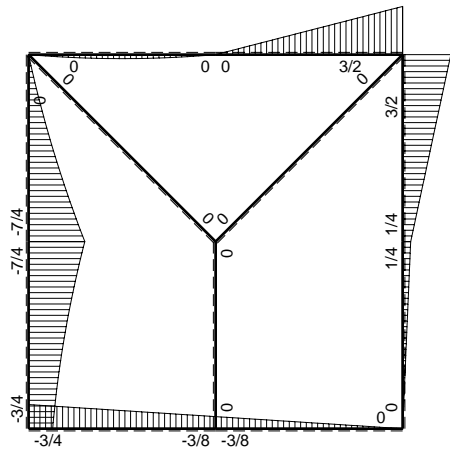




← ⊕ → F

↑ ⊕ ↓ F

⊕ M_o flessione da carichi assegnati



⊕ F_b

⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	$-1+1/2x/b$	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	$7/12Xb/EJ$	
BA b	$1/2+1/2x/b$	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$			
BC b	$-1/2+1/2x/b$	0	0	0	0	$1/4-1/2x/b+1/4x^2/b^2$	0+0	$1/12Xb/EJ$	
CB b	$1/2x/b$	0	0	0	0	$1/4x^2/b^2$			
CD b	0	$1/4Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0			
EF b	0	$3/2Fb-3/2Fx$	0	0	0	0	0+0	0	
FE b	0	$-3/2Fx$	0	0	0	0			
FG b	0	$-1/2Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0	
GF b	0	$1/2Fx-1/2qx^2$	Fb/EJ	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x/b$	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	$1/3Xb/EJ$	
AH b	$1-x/b$	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^2/EJ$	
	totali							$-3/4Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$							$3/4Fb$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = \left[x - 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = \left[1/4 x + 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = \left[1/4 x - 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = \left[1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = \left[1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = \left[x - x^2/b + 1/3 x^3/b^2 \right]_0^b 1/EJ$$

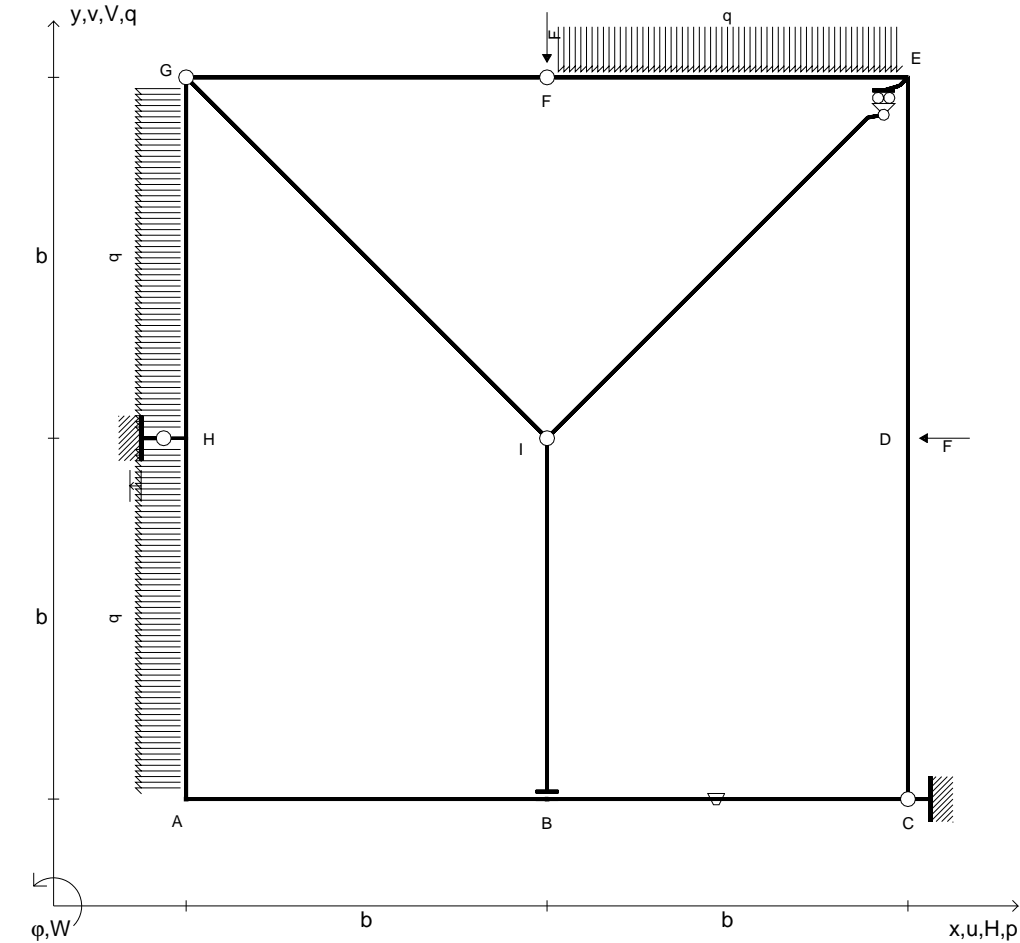
$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HA}^{xo} = \int_0^b (7/4 x/b - 9/4 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = \left[7/8 x^2/b - 3/4 x^3/b^2 + 1/8 x^4/b^3 \right]_0^b Fb 1/EJ$$

$$= (7/8 b - 3/4 b + 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

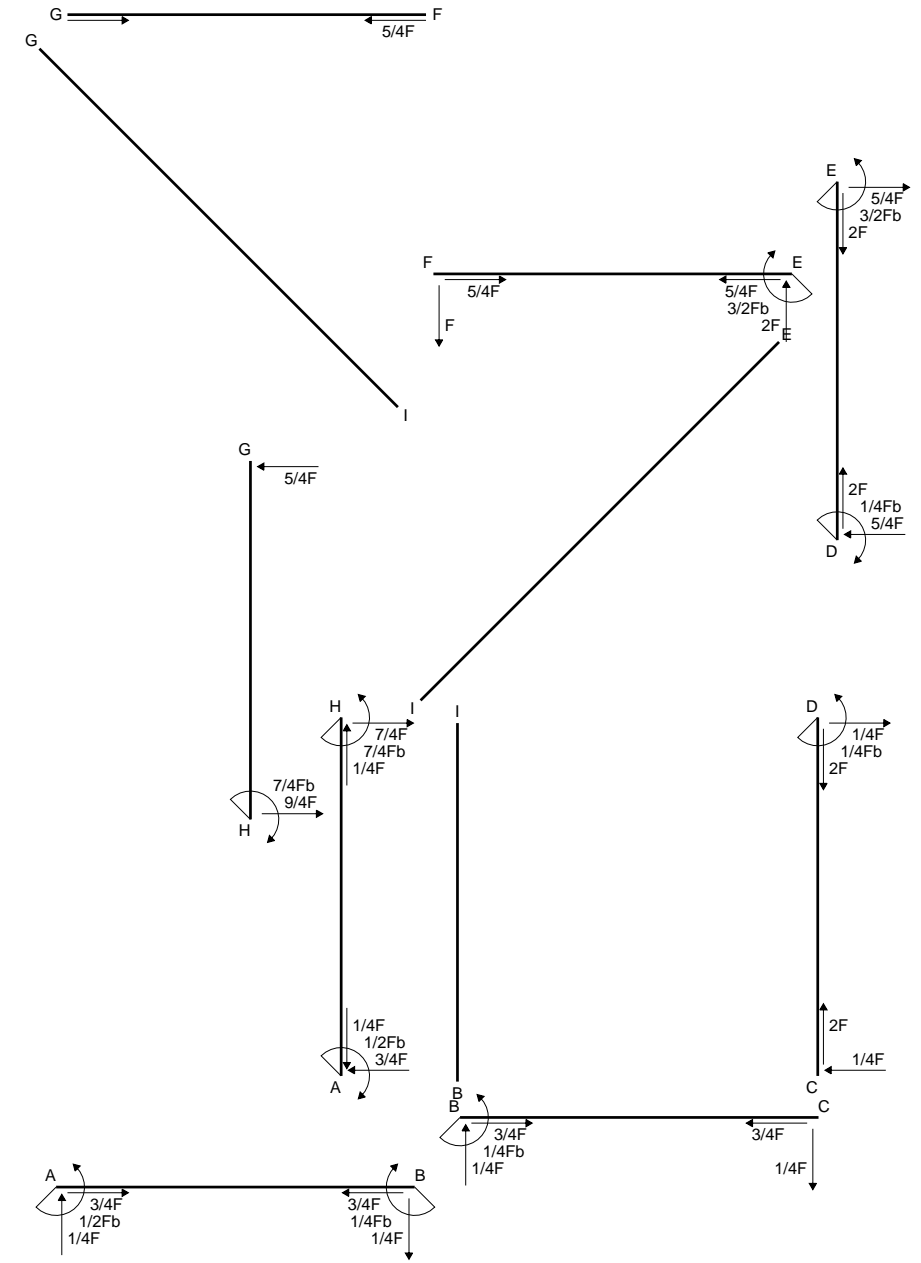
$$L_{AH}^{xo} = \int_0^b (5/4 x/b - 3/4 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = \left[5/8 x^2/b - 1/4 x^3/b^2 - 1/8 x^4/b^3 \right]_0^b Fb 1/EJ$$

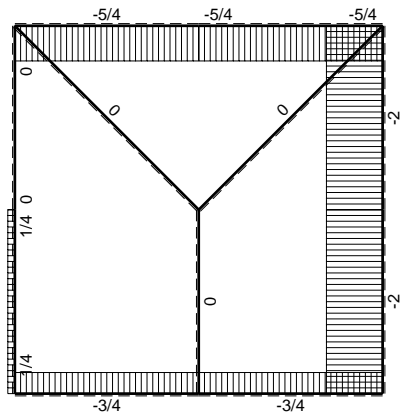
$$= (5/8 b - 1/4 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$



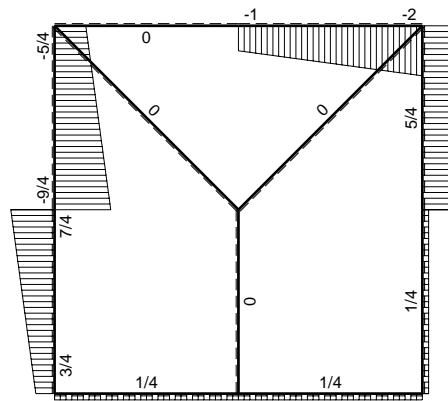
$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$p_{GH} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=W_{AB}$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta BC positiva se convessa a destra con inizio B.
 Spostamento orizzontale assoluto u imposto al nodo H.
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

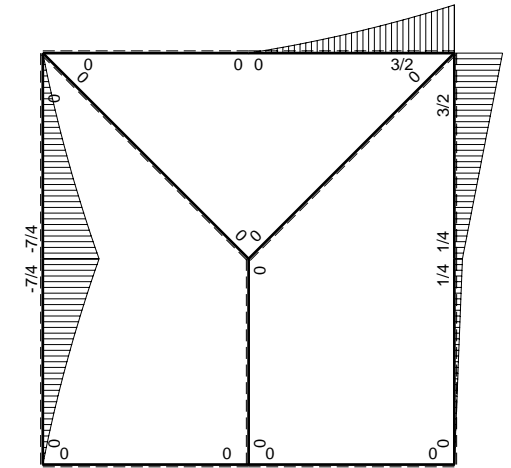
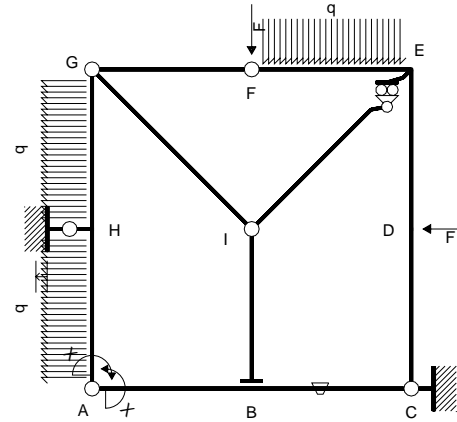




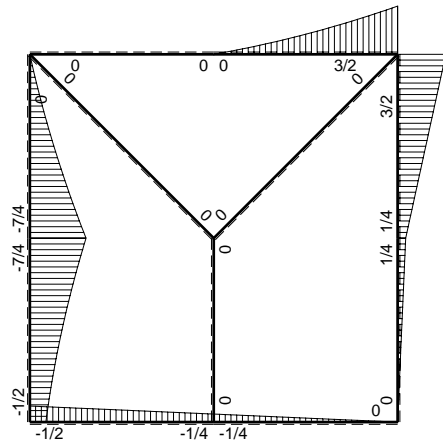
← ⊕ → F



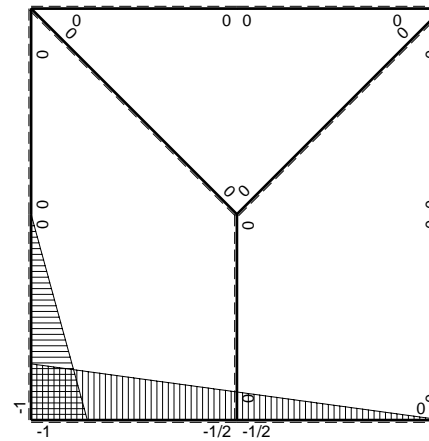
↑ ⊕ ↓ F



⊕ M₀ flessione da carichi assegnati



⊕ F_b



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$	
AB b	$-1+1/2x/b$	0	0	0	0	$1-x/b+1/4x^2/b^2$	0+0	$7/12Xb/EJ$	
BA b	$1/2+1/2x/b$	0	0	0	0	$1/4+1/2x/b+1/4x^2/b^2$			
BC b	$-1/2+1/2x/b$	0	$-Fb/EJ$	0	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(0+1/4)Fb^2/EJ$	$1/12Xb/EJ$	
CB b	$1/2x/b$	0	Fb/EJ	0	$1/2Fx/EJ$	$1/4x^2/b^2$			
CD b	0	$1/4Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0			
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-Fx-1/2qx^2$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x/b$	$-7/4Fb+9/4Fx-1/2qx^2$	0	$7/4Fx-9/4Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(1/4+0)Fb^2/EJ$	$1/3Xb/EJ$	
AH b	$1-x/b$	$5/4Fx+1/2qx^2$	0	$5/4Fx-3/4Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^2/EJ$	
	totali							$-1/2Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{AB}$							$1/2Fb$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 - 1/2 x/b) \theta dx = [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/4 b) \theta = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1/2 x/b) \theta dx = [-1/4 x^2/b]_0^b \theta$$

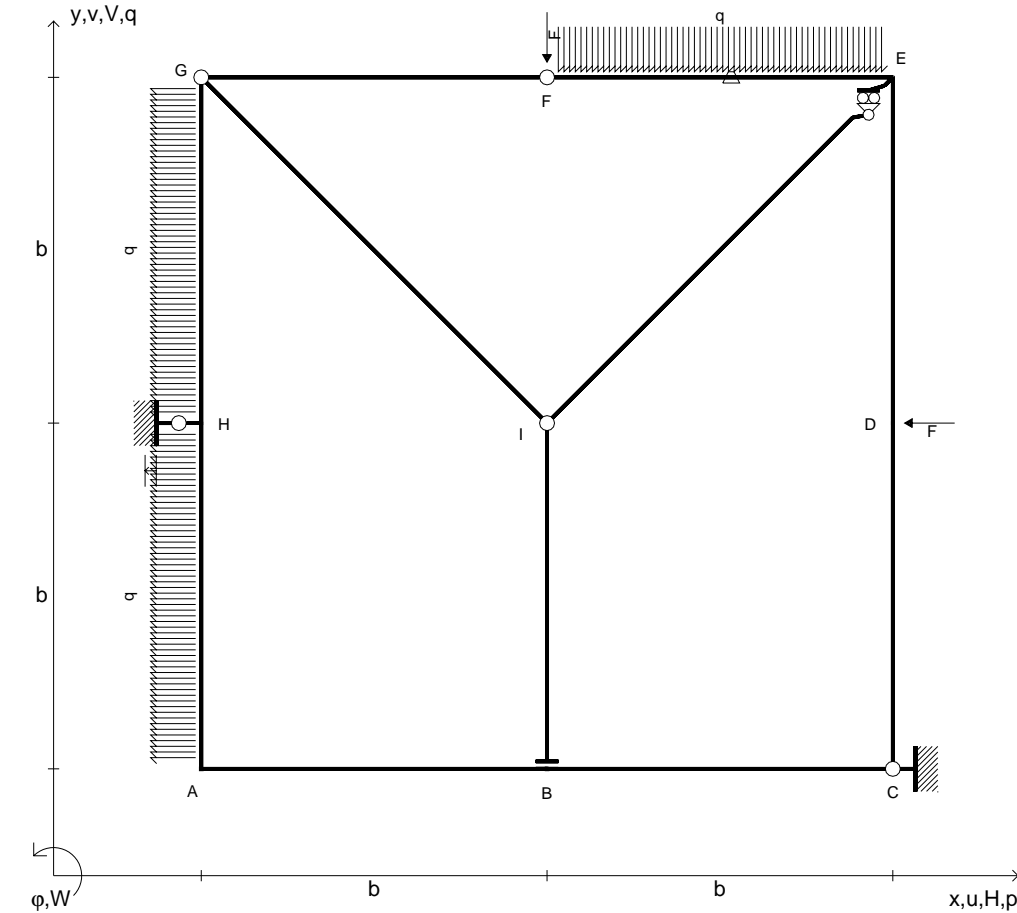
$$= (-1/4 b) \theta = 1/4 Fb^2/EJ$$

$$L_{HA}^{xo} = \int_0^b (7/4 x/b - 9/4 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [7/8 x^2/b - 3/4 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (7/8 b - 3/4 b + 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{AH}^{xo} = \int_0^b (5/4 x/b - 3/4 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [5/8 x^2/b - 1/4 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (5/8 b - 1/4 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$p_{GH} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{EF} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

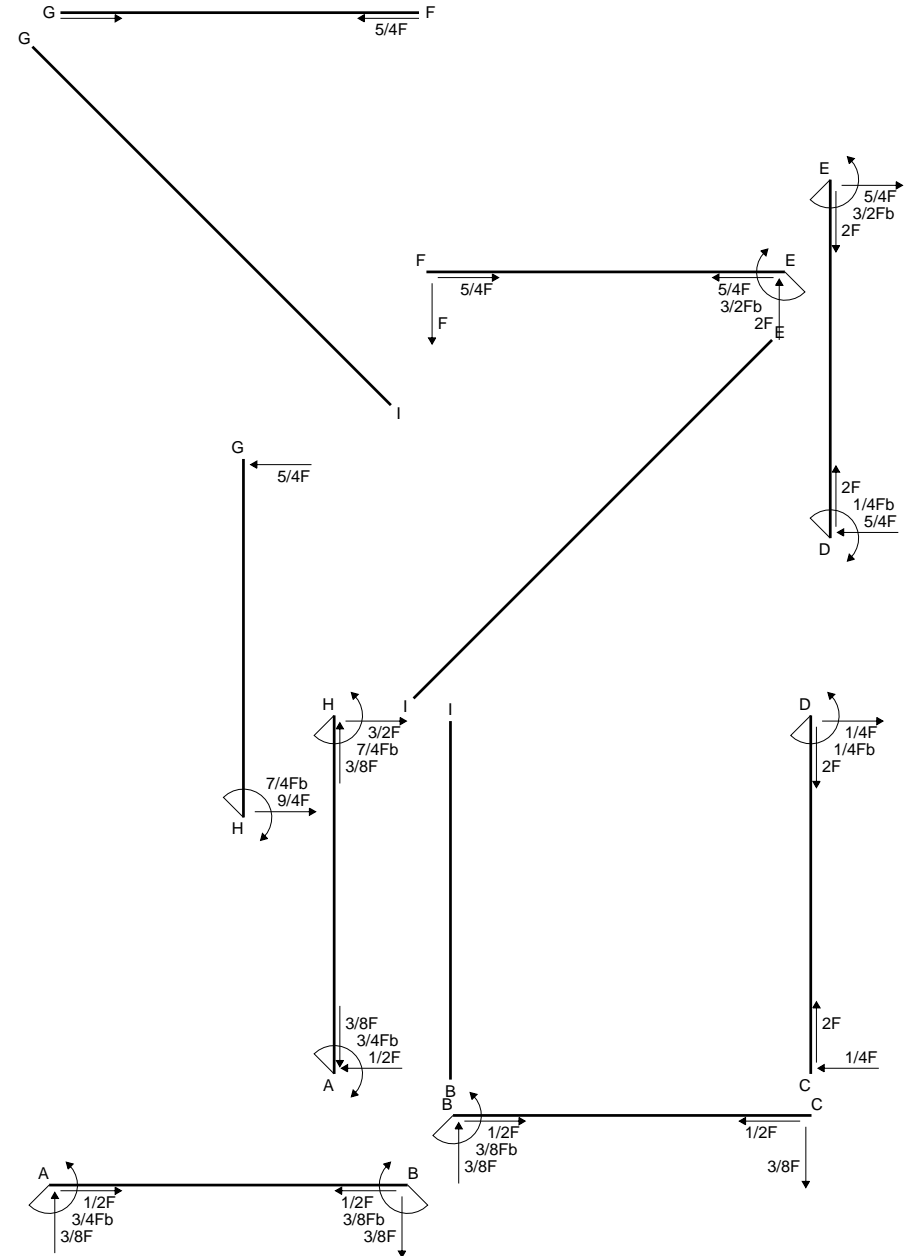
Carichi di aste curve misurati in proiezione sugli assi x,y.

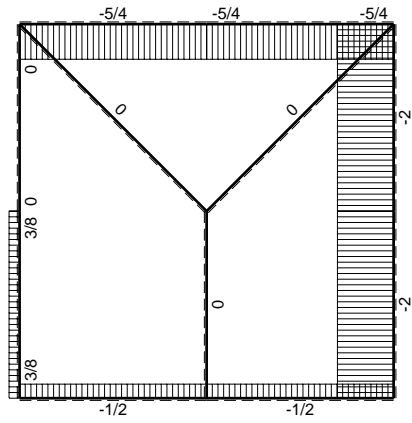
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta EF positiva se convessa a destra con inizio E.

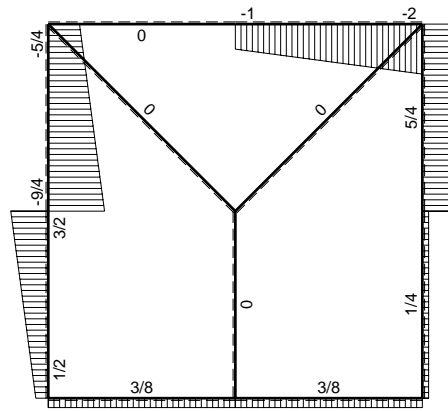
Spostamento orizzontale assoluto u imposto al nodo H.

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

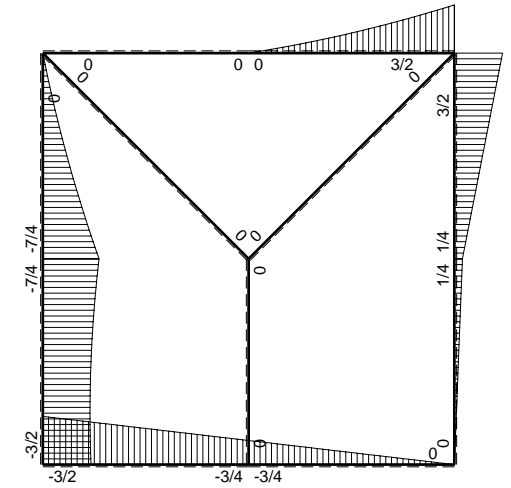
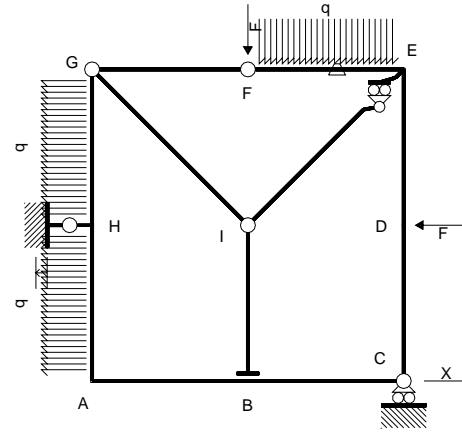




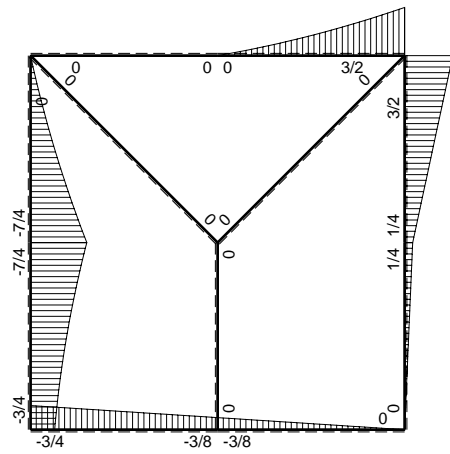
← ⊕ → F



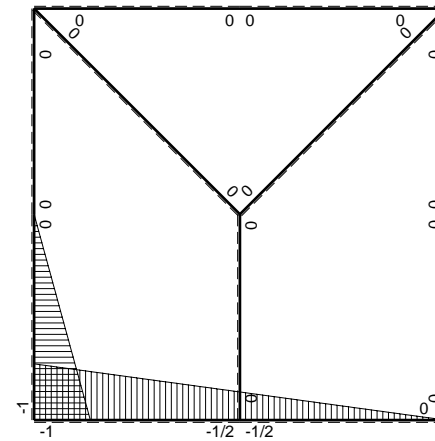
↑ ⊕ ↓ F



⊕ M₀ flessione da carichi assegnati



⊕ F_b



⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x / EJ dx$	
AB b	$-b+1/2x$	$-3/2Fb+3/4Fx$	0	$3/2Fb^2-3/2Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
BA b	$1/2b+1/2x$	$3/4Fb+3/4Fx$	0	$3/8Fb^2+3/4Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
BC b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
CB b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$			
CD b	0	$1/4Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0			
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0			
EF b	0	$3/2Fb-2Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0	
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-5/4Fx-1/2qx^2$	0	0	0	0	0+0	0	
HG b	0	$7/4Fb-9/4Fx+1/2qx^2$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x$	$-7/4Fb+3/4Fx-1/2qx^2$	0	$7/4Fbx-3/4Fx^2+1/2qx^3$	0	x^2	$(3/4+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
AH b	$b-x$	$3/2Fb-1/4Fx+1/2qx^2$	0	$3/2Fb^2-7/4Fbx+3/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^3/EJ$	
	totali							$3/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$							$-3/4F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (3/2 - 3/2 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/2 x - 3/4 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/2 b - 3/4 b + 1/8 b) Fb^2 1/EJ = 7/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (3/8 + 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x + 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b + 3/8 b + 1/8 b) Fb^2 1/EJ = 7/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

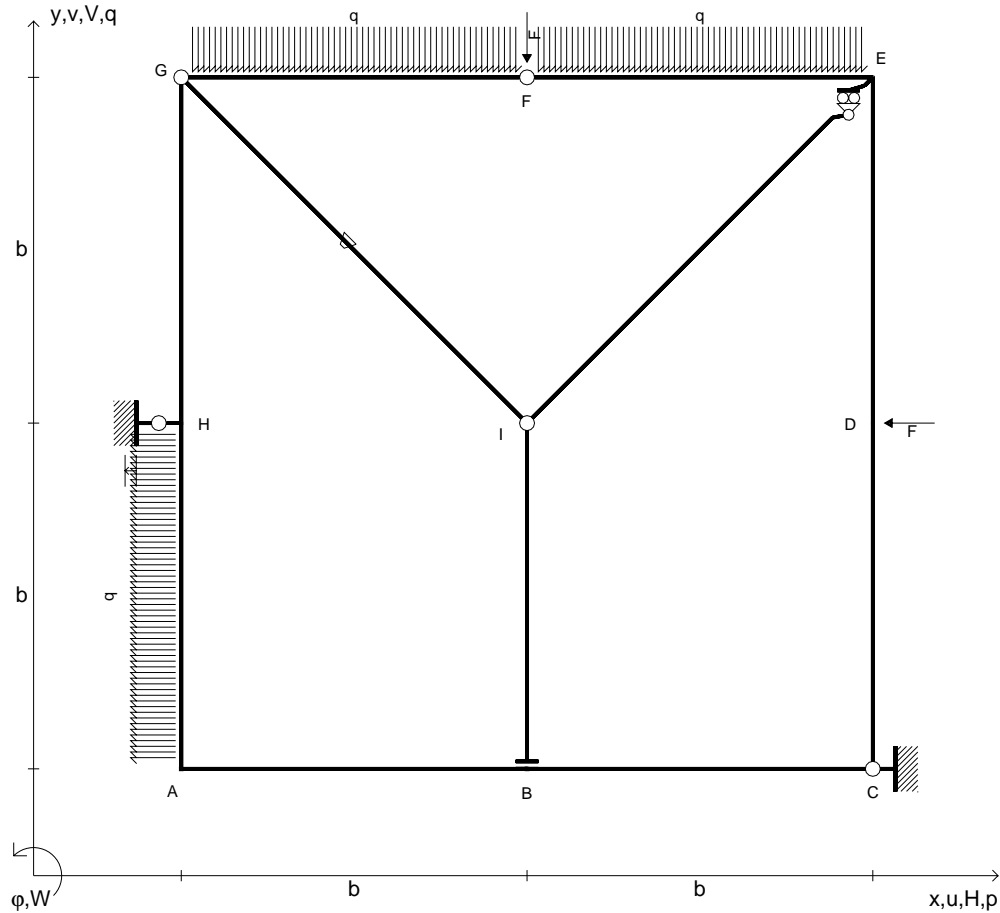
$$L_{HA}^{xo} = \int_0^b (7/4 x/b - 3/4 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [7/8 x^2/b - 1/4 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (7/8 b - 1/4 b + 1/8 b) Fb^2 1/EJ = 3/4 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (3/2 - 7/4 x/b + 3/4 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [3/2 x - 7/8 x^2/b + 1/4 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3/2 b - 7/8 b + 1/4 b - 1/8 b) Fb^2 1/EJ = 3/4 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{FG} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{GI} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=H_C$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

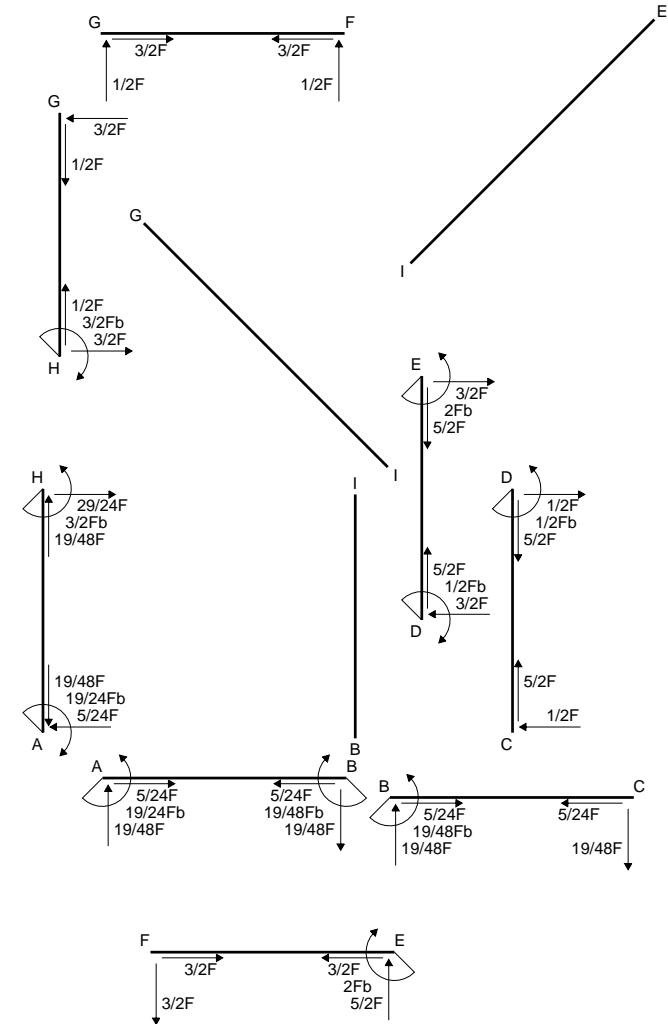
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta GI positiva se convessa a destra con inizio G.

Spostamento orizzontale assoluto u imposto al nodo H.

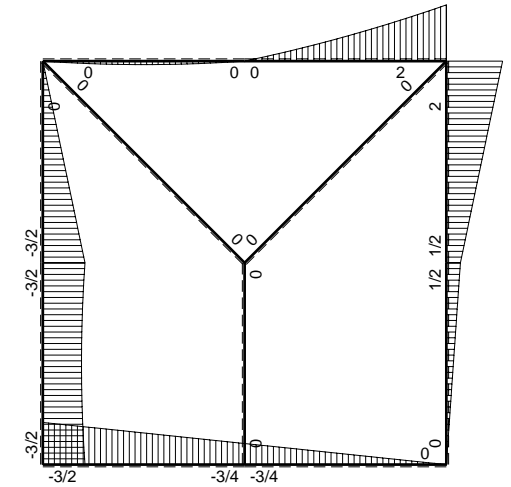
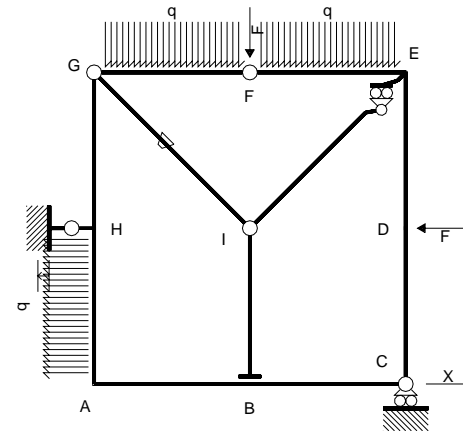
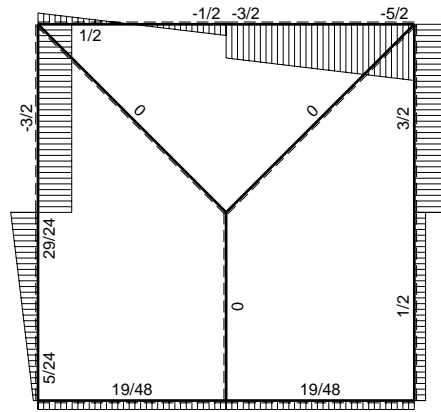
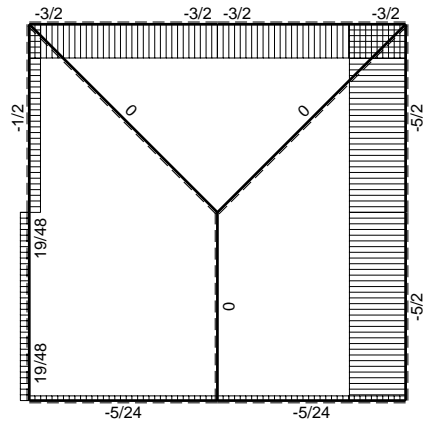
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

08.12.24



@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

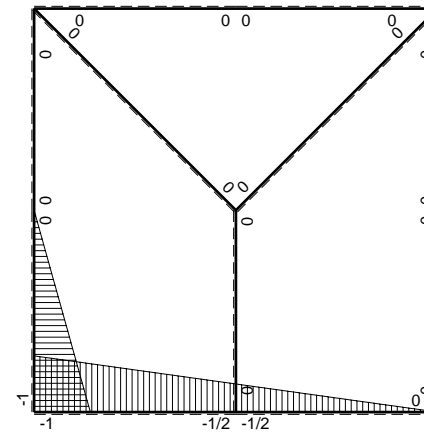
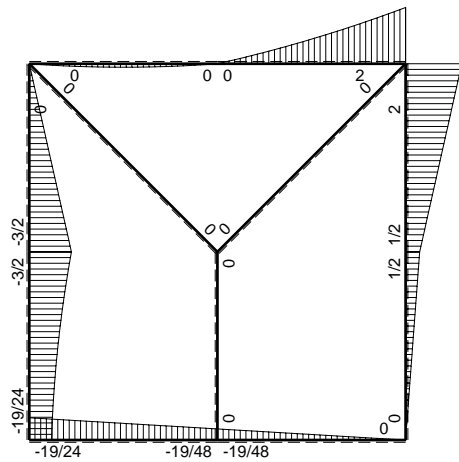
08.12.24



← ⊕ → F

⊕ ↓ F

⊕ ↻ M₀ flessione da carichi assegnati



⊕ ↻ F_b

⊕ ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-b+1/2x$	$-3/2Fb+3/4Fx$	0	$3/2Fb^2-3/2Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$	
BA b	$1/2b+1/2x$	$3/4Fb+3/4Fx$	0	$3/8Fb^2+3/4Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$			
BC b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$	
CB b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$			
CD b	0	$1/2Fx$	0	0	0	0	0+0	0	
DC b	0	$-1/2Fb+1/2Fx$	0	0	0	0			
DE b	0	$1/2Fb+3/2Fx$	0	0	0	0	0+0	0	
ED b	0	$-2Fb+3/2Fx$	0	0	0	0			
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0			
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0	
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0			
GH b	0	$-3/2Fx$	0	0	0	0	0+0	0	
HG b	0	$3/2Fb-3/2Fx$	0	0	0	0			
GI $\sqrt{2}b$	0	0	$-Fb/EJ$	0	0	0	0	0	
IB b	0	0	0	0	0	0	0+0	0	
BI b	0	0	0	0	0	0			
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0	
HA b	$-x$	$-3/2Fb+1/2Fx-1/2qx^2$	0	$3/2Fbx-1/2Fx^2+1/2qx^3$	0	x^2	$(17/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
AH b	$b-x$	$3/2Fb-1/2Fx+1/2qx^2$	0	$3/2Fb^2-2Fbx+Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-Fb^3/EJ$	
	totali							$17/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$							$-17/24F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (3/2 - 3/2 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/2 x - 3/4 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/2 b - 3/4 b + 1/8 b) Fb^2 1/EJ = 7/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (3/8 + 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x + 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b + 3/8 b + 1/8 b) Fb^2 1/EJ = 7/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

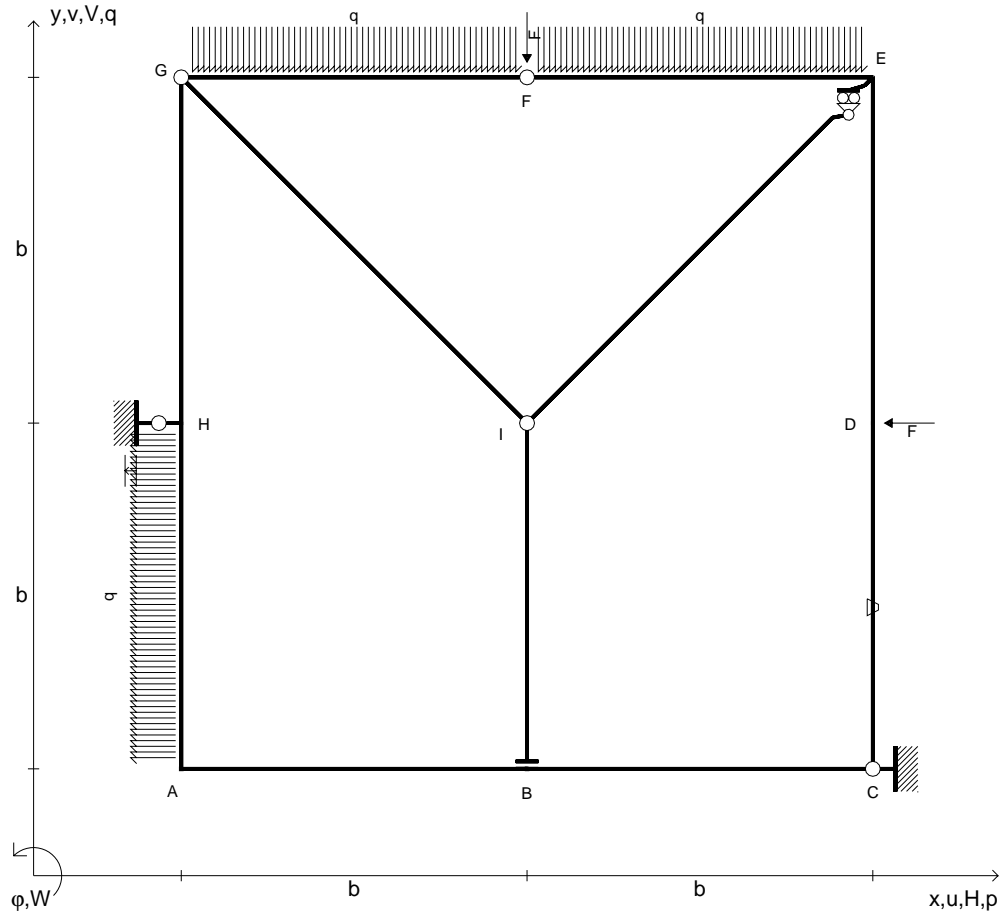
$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (3/2 x/b - 1/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [3/4 x^2/b - 1/6 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3/4 b - 1/6 b + 1/8 b) Fb^2 1/EJ = 17/24 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (3/2 - 2x/b + x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx = [3/2 x - x^2/b + 1/3 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3/2 b - b + 1/3 b - 1/8 b) Fb^2 1/EJ = 17/24 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3 F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$q_{FG} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

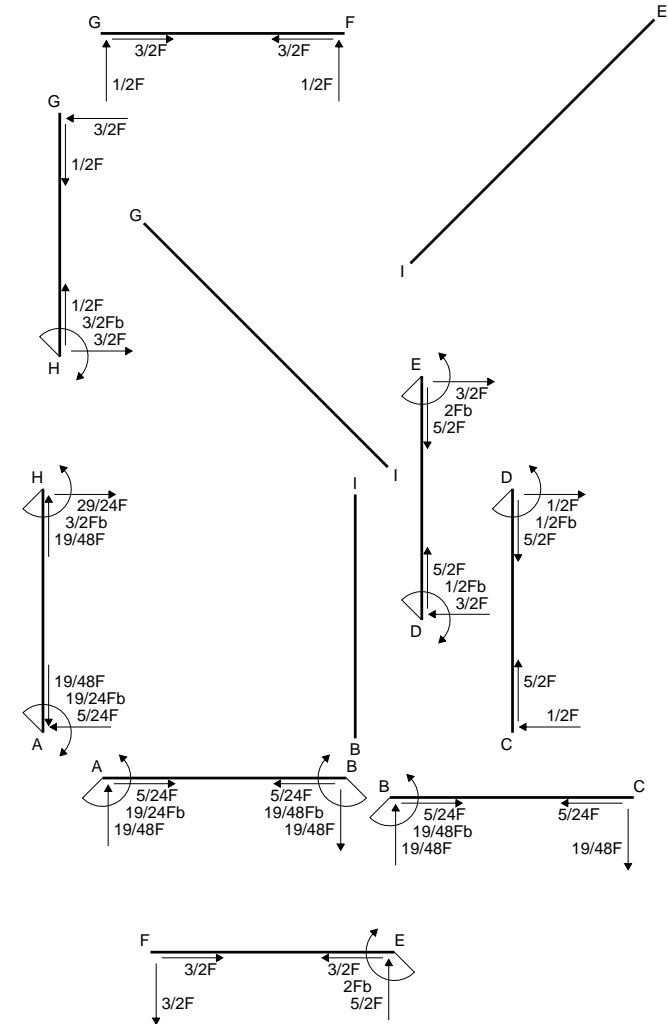
Carichi di aste curve misurati in proiezione sugli assi x,y.

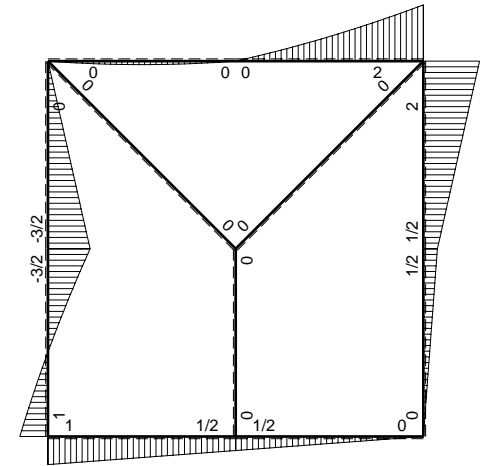
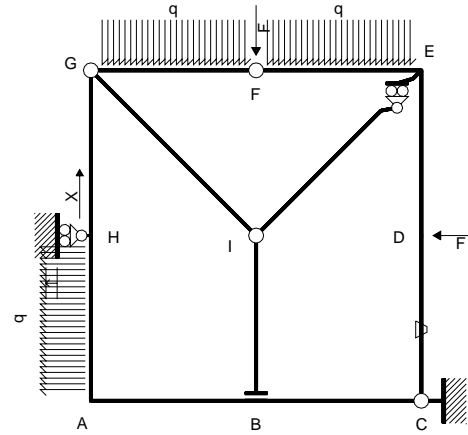
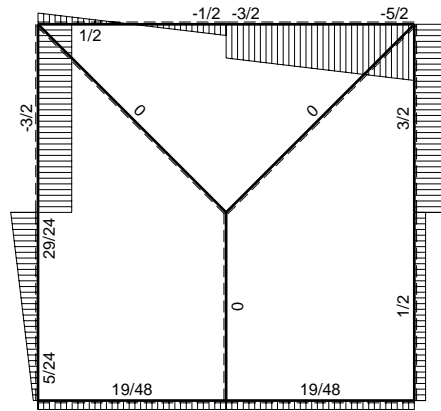
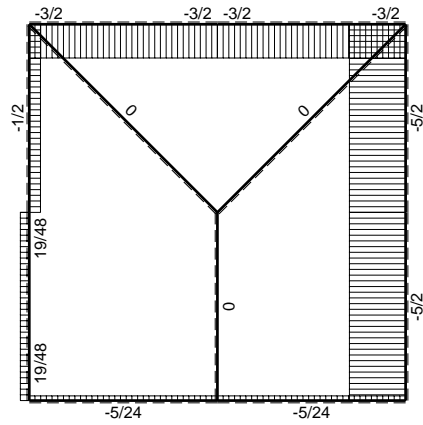
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta CD positiva se convessa a destra con inizio C.

Spostamento orizzontale assoluto u imposto al nodo H.

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

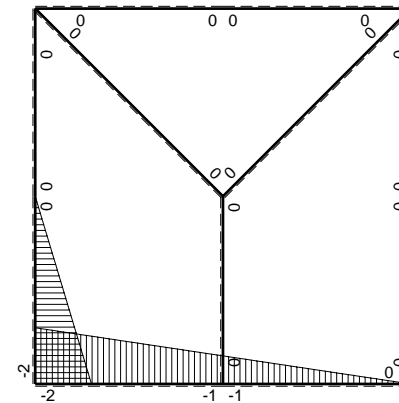
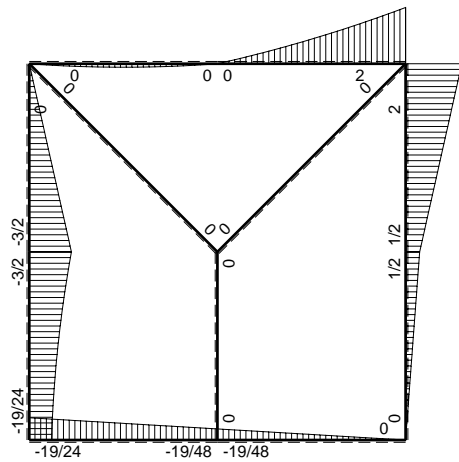




← ⊕ → F

⊕ ↓ F

⊕ ↻ M_o flessione da carichi assegnati



⊕ ↻ F_b

⊕ ↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$
AB b	$-2b+x$	$Fb-1/2Fx$	0	$-2Fb^2+2Fbx-1/2Fx^2$	0	$4b^2-4bx+x^2$	$(-7/6+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	$b+x$	$-1/2Fb-1/2Fx$	0	$-1/2Fb^2-Fbx-1/2Fx^2$	0	$b^2+2bx+x^2$		
BC b	$-b+x$	$1/2Fb-1/2Fx$	0	$-1/2Fb^2+Fbx-1/2Fx^2$	0	$b^2-2bx+x^2$	$(-1/6+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	x	$-1/2Fx$	0	$-1/2Fx^2$	0	x^2		
CD b	0	$1/2Fx$	$-Fb/EJ$	0	0	0	0+0	0
DC b	0	$-1/2Fb+1/2Fx$	Fb/EJ	0	0	0		
DE b	0	$1/2Fb+3/2Fx$	0	0	0	0	0+0	0
ED b	0	$-2Fb+3/2Fx$	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-3/2Fx$	0	0	0	0	0+0	0
HG b	0	$3/2Fb-3/2Fx$	0	0	0	0		
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0
HA b	$-2x$	$-3/2Fb+3Fx-1/2qx^2$	0	$3Fbx-6Fx^2+qx^3$	0	$4x^2$	$(-1/4+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	$2b-2x$	$-Fb+2Fx+1/2qx^2$	0	$-2Fb^2+6Fbx-3Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$-43/12Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						$43/48F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-2 + 2x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-2x + x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2b + b - 1/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 - x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x - 1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b - 1/2 b - 1/6 b) Fb^2 1/EJ = -7/6 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 1/2 b - 1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

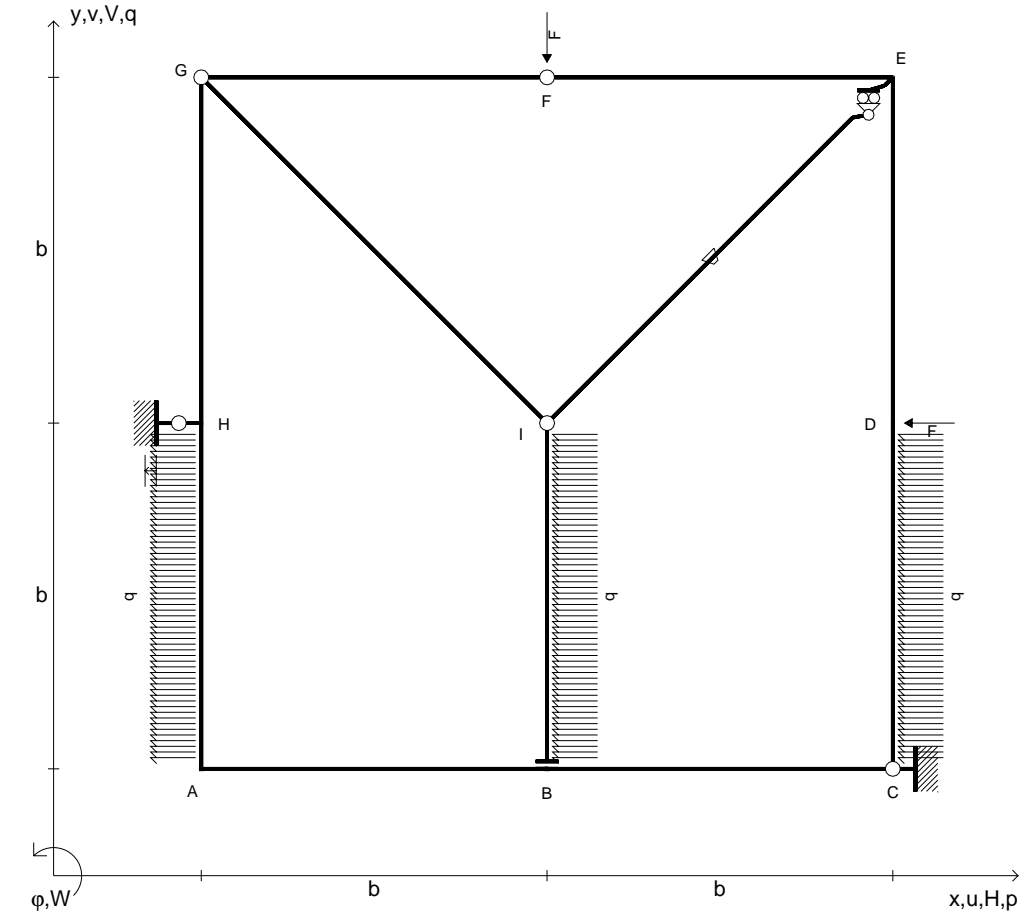
$$= (-1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (3x/b - 6x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [3/2 x^2/b - 2x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3/2 b - 2b + 1/4 b) Fb^2 1/EJ = -1/4 Fb^3/EJ$$

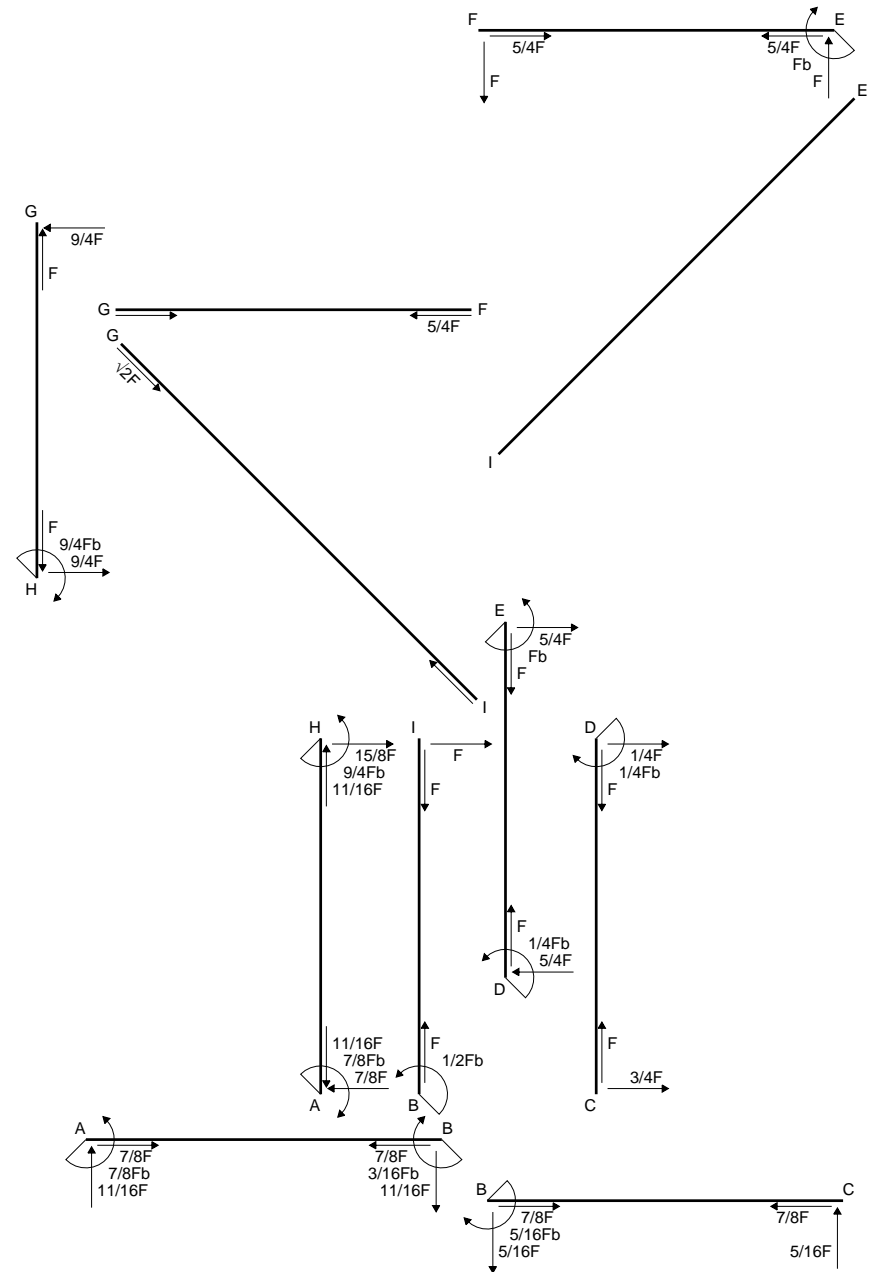
$$L_{AH}^{xo} = \int_0^b (-2 + 6x/b - 3x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [-2x + 3x^2/b - x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

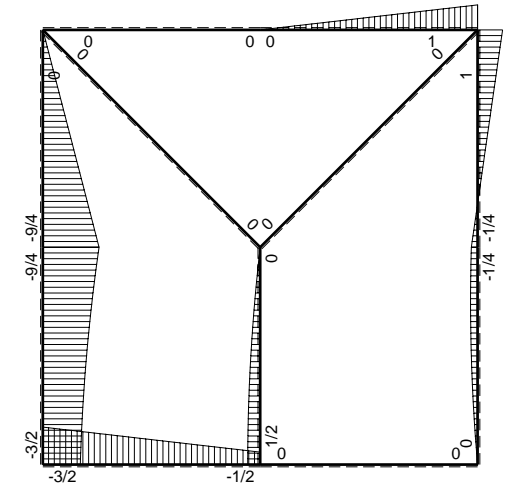
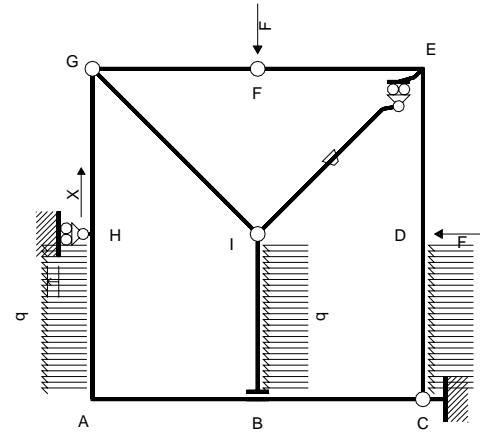
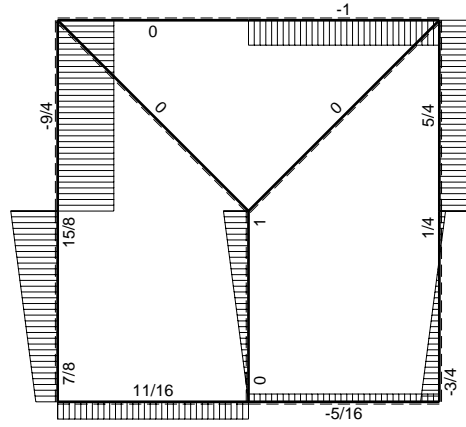
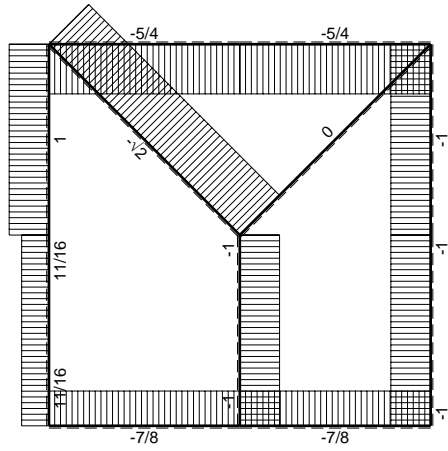
$$= (-2b + 3b - b - 1/4 b) Fb^2 1/EJ = -1/4 Fb^3/EJ$$



$V_F = -F$	$u_H = -\delta = -b^3F/EJ$	$EJ_{FG} = EJ$
$H_D = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$p_{HA} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{GI} = EJ$
$p_{CD} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IB} = EJ$
$p_{IB} = -q = -F/b$	$EJ_{DE} = EJ$	$EJ_{IE} = EJ$
$\theta_{IE} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$	$EJ_{HA} = EJ$

Reazioni iperstatiche in soluzione: $X=V_H$
 Carichi e deformazioni date hanno verso efficace in disegno.
 Calcolare reazioni vincolari della struttura e delle aste.
 Tracciare i diagrammi quotati delle azioni interne nelle aste.
 Carichi di aste curve misurati in proiezione sugli assi x,y.
 $J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.
 Curvatura θ asta IE positiva se convessa a destra con inizio I.
 Spostamento orizzontale assoluto u imposto al nodo H.
 © Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

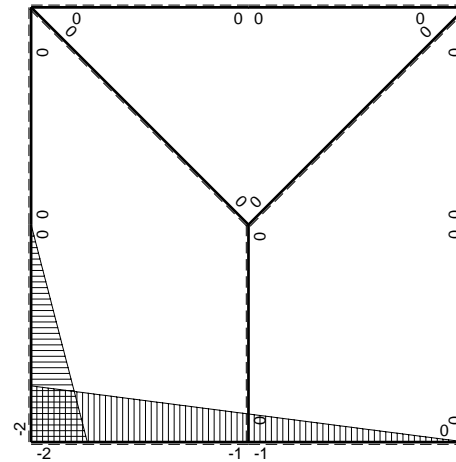
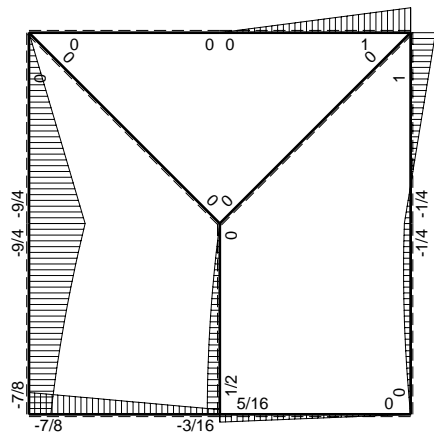




← ⊕ → F

↑ ⊕ ↓ F

⊕ M₀ flessione da carichi assegnati



⊕ F_b

⊕ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-2b+x$	$-3/2Fb+Fx$	0	$3Fb^2-7/2Fbx+Fx^2$	0	$4b^2-4bx+x^2$	$(19/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$	
BA b	$b+x$	$1/2Fb+Fx$	0	$1/2Fb^2+3/2Fbx+Fx^2$	0	$b^2+2bx+x^2$			
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$	
CB b	x	0	0	0	0	x^2			
CD b	0	$-3/4Fx+1/2qx^2$	0	0	0	0	0+0	0	
DC b	0	$1/4Fb+1/4Fx-1/2qx^2$	0	0	0	0			
DE b	0	$-1/4Fb+5/4Fx$	0	0	0	0	0+0	0	
ED b	0	$-Fb+5/4Fx$	0	0	0	0			
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0	
FE b	0	$-Fx$	0	0	0	0			
FG b	0	0	0	0	0	0	0+0	0	
GF b	0	0	0	0	0	0			
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0	
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0			
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0	
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
IE $\sqrt{2}b$	0	0	$-Fb/EJ$	0	0	0	0	0	
HA b	$-2x$	$-9/4Fb+5/4Fx-1/2qx^2$	0	$9/2Fbx-5/2Fx^2+qx^3$	0	$4x^2$	$(5/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$	
AH b	$2b-2x$	$3/2Fb+1/4Fx+1/2qx^2$	0	$3Fb^2-5/2Fbx+1/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$			
H	cedimento nodo $-H_{1H}u_H$							$-2Fb^3/EJ$	
	totali							$5/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$							$-5/16F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{HA}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AH}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (3 - 7/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [3x - 7/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3b - 7/4 b + 1/3 b) Fb^2 1/EJ = 19/12 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 + 3/2 x/b + x^2/b^2) Fb^2 1/EJ dx = [1/2 x + 3/4 x^2/b + 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b + 3/4 b + 1/3 b) Fb^2 1/EJ = 19/12 Fb^3/EJ$$

$$L_{HA}^{xo} = \int_0^b (9/2 x/b - 5/2 x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [9/4 x^2/b - 5/6 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (9/4 b - 5/6 b + 1/4 b) Fb^2 1/EJ = 5/3 Fb^3/EJ$$

$$L_{AH}^{xo} = \int_0^b (3 - 5/2 x/b + 1/2 x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [3x - 5/4 x^2/b + 1/6 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (3b - 5/4 b + 1/6 b - 1/4 b) Fb^2 1/EJ = 5/3 Fb^3/EJ$$