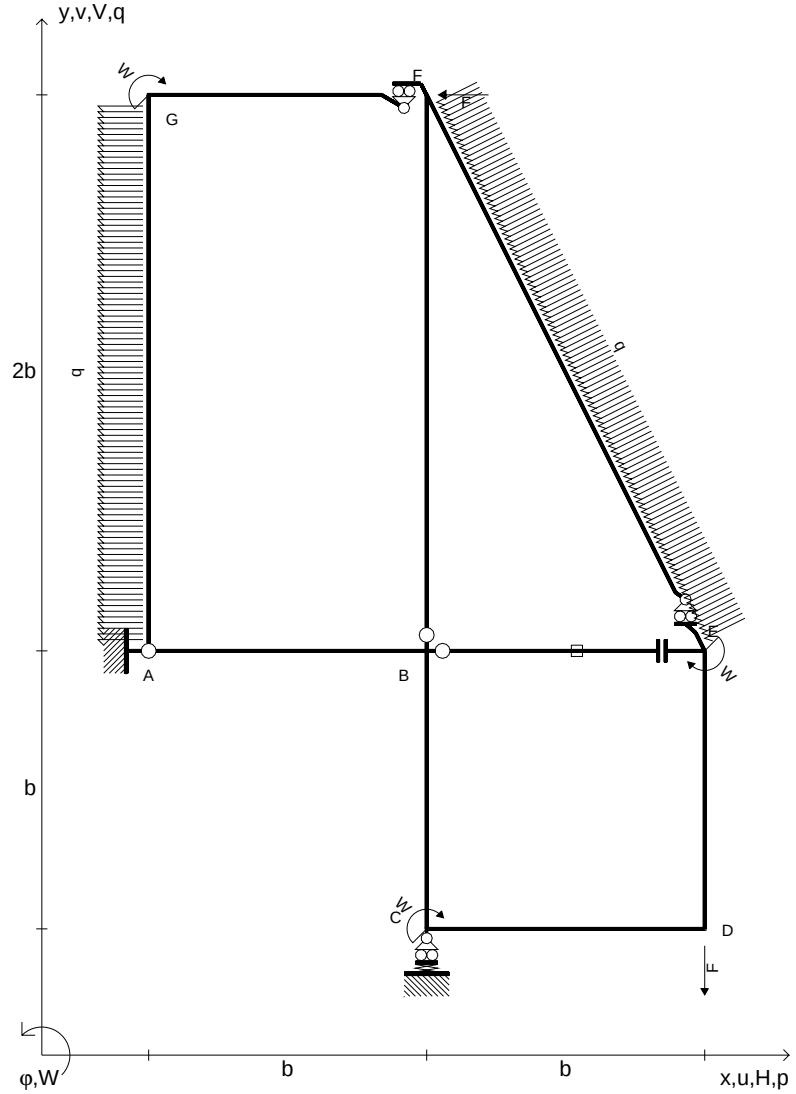
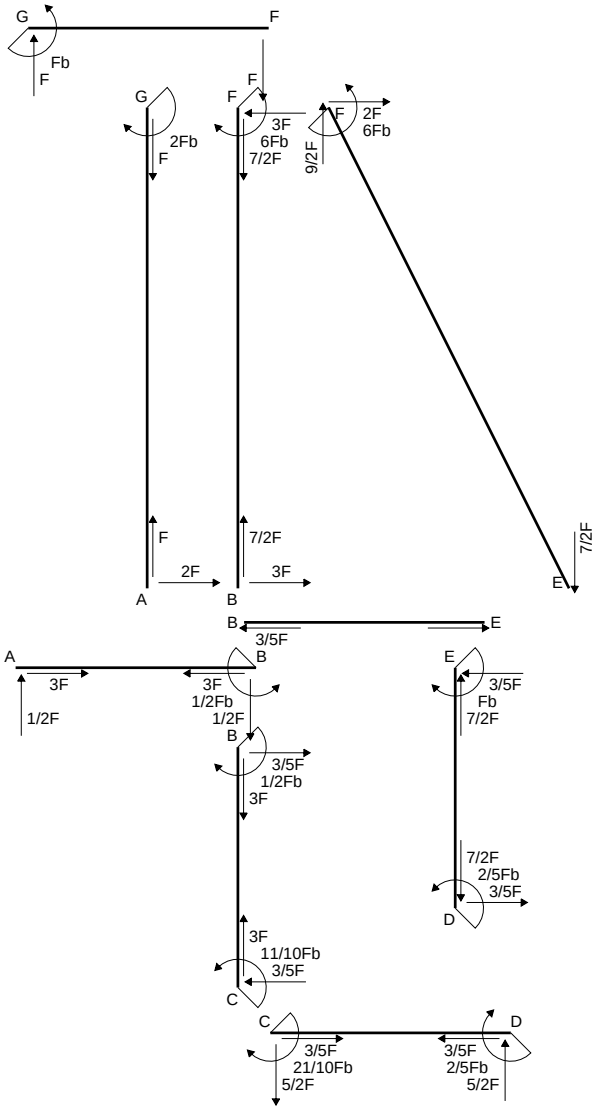
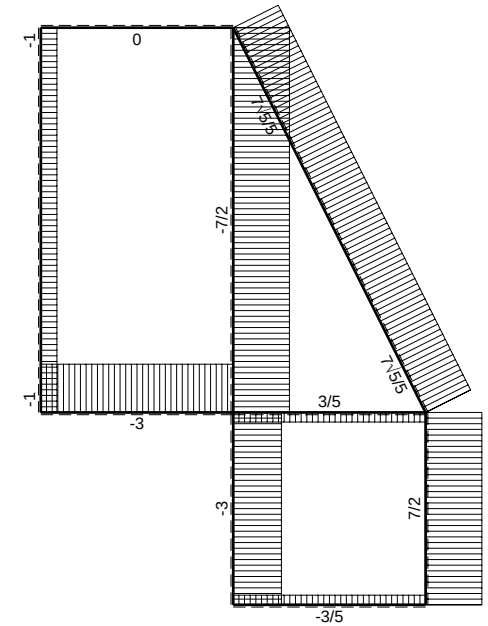
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{GA} = -q = -F/b$
$p_{EF} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.





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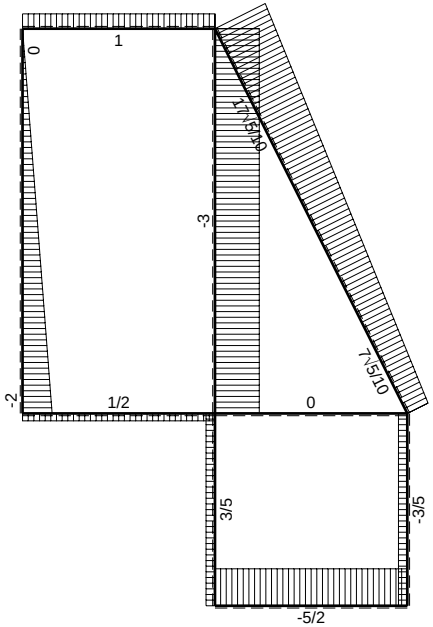

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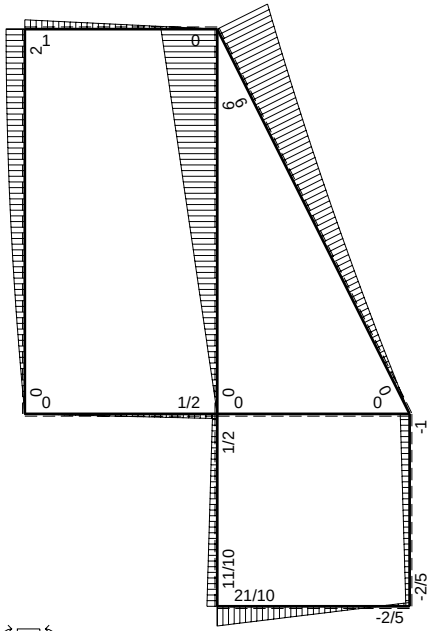
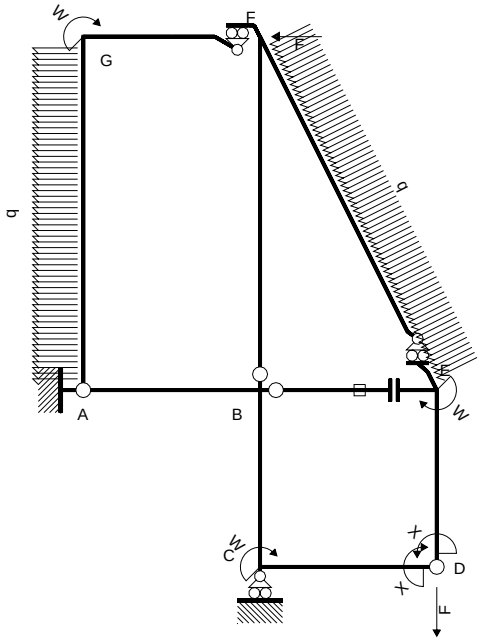
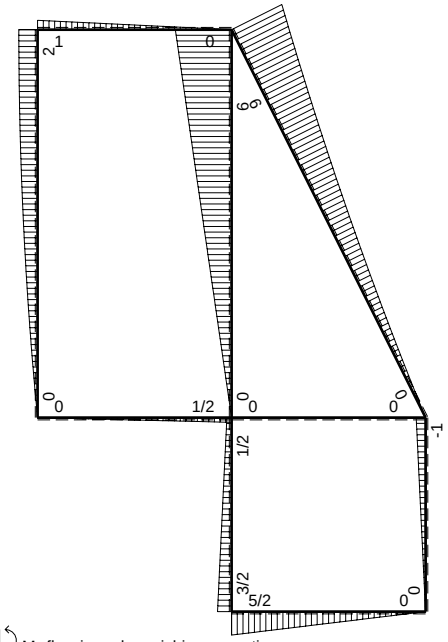


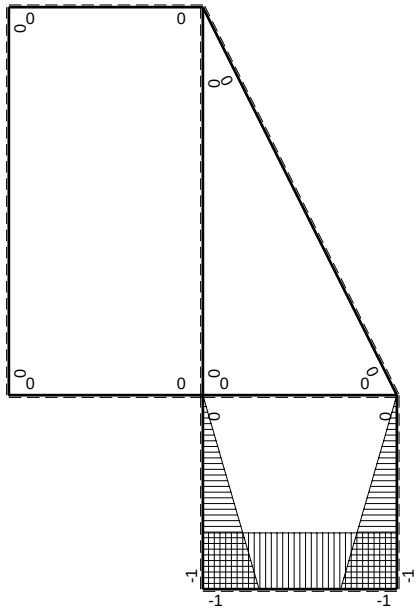
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb+Fx	$-1/2Fx-Fx^2/b$	x^2/b^2	$-7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-3/2Fb+Fx	$-3/2Fb+5/2Fx-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	5/2Fb-5/2Fx	-5/2Fb+5/2Fx	1	$-5/4Fb^2/EJ$	Xb/EJ
DC b	1	-5/2Fx	-5/2Fx	1		
DE b	-1+x/b	-Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb-Fx	$Fx-Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$-2/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				2/5Fb	

Sviluppi di calcolo iperstatica

$$L^{XX}_{BC} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CB} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CD} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DC} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{Xo}_{BC} = \int_o^b \left(-1/2 \; x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[-1/4 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-1/4 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = -7/12 \; \; Fb^2/EJ$$

$$L^{Xo}_{CB} = \int_o^b \left(-3/2 + 5/2 \; x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[-3/2 \; x + 5/4 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-3/2 \; b + 5/4 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = -7/12 \; \; Fb^2/EJ$$

$$L^{Xo}_{CD} = \int_o^b \left(-5/2 + 5/2 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[-5/2 \; x + 5/4 \; x^2/b \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-5/2 \; b + 5/4 \; b \right) \; Fb \; 1/EJ \; = -5/4 \; \; Fb^2/EJ$$

$$L^{Xo}_{DC} = \int_o^b \left(-5/2 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[-5/4 \; x^2/b \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-5/4 \; b \right) \; Fb \; 1/EJ \; = -5/4 \; \; Fb^2/EJ$$

$$L^{Xo}_{DE} = \int_o^b \left(x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(1/2 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = 1/6 \; \; Fb^2/EJ$$

$$L^{Xo}_{ED} = \int_o^b \left(x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(1/2 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = 1/6 \; \; Fb^2/EJ$$

ESΣ02.xxxx.016

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.12.24

$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{GA} = -q = -F/b$
$q_{FG} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
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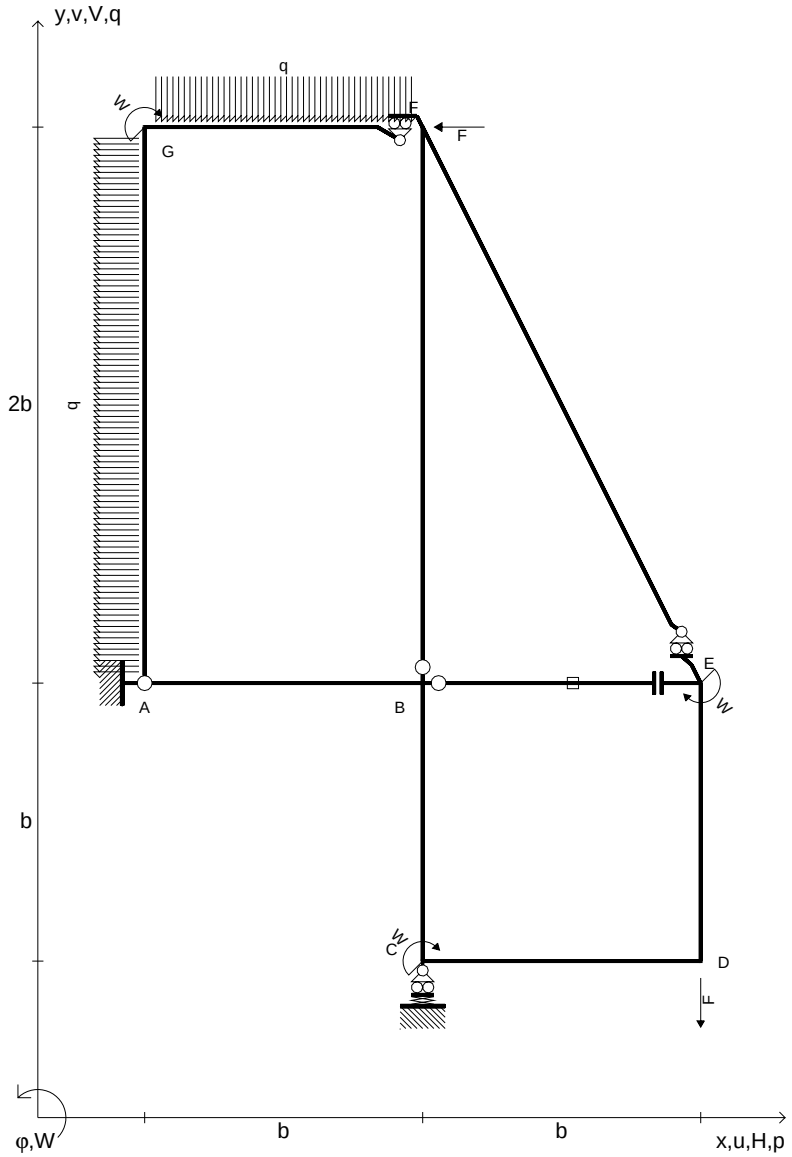

Reazioni iperstatiche in soluzione: $X=W_{DE}$
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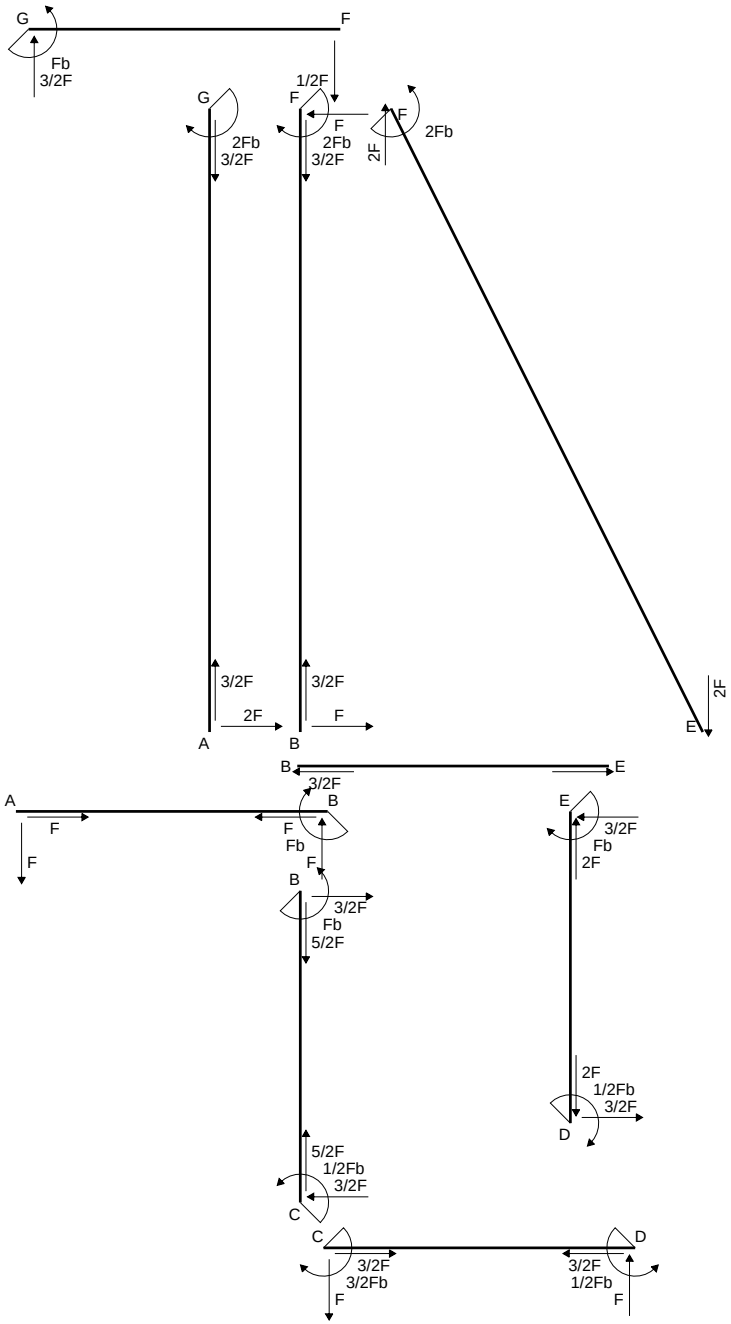
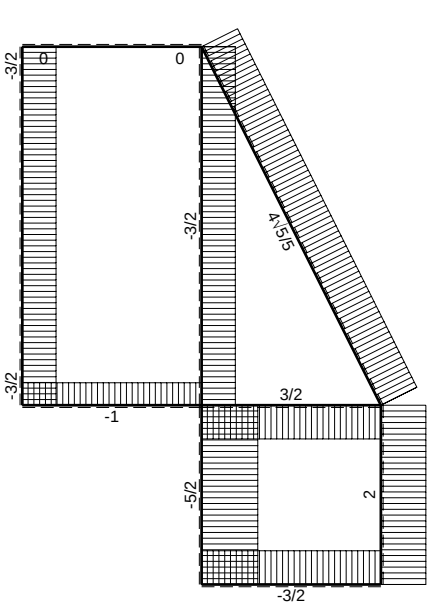
Elongazione termica specifica ε assegnata su asta BE.

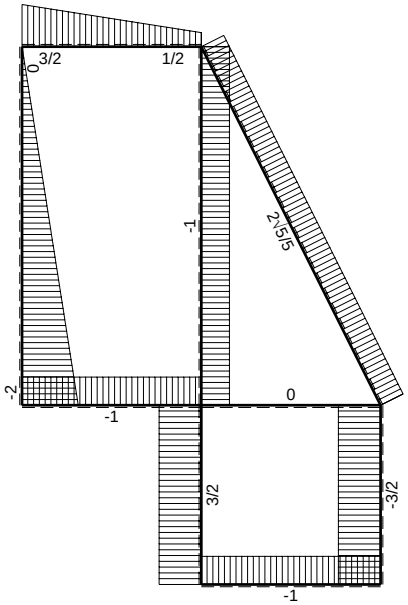
 08.12.24



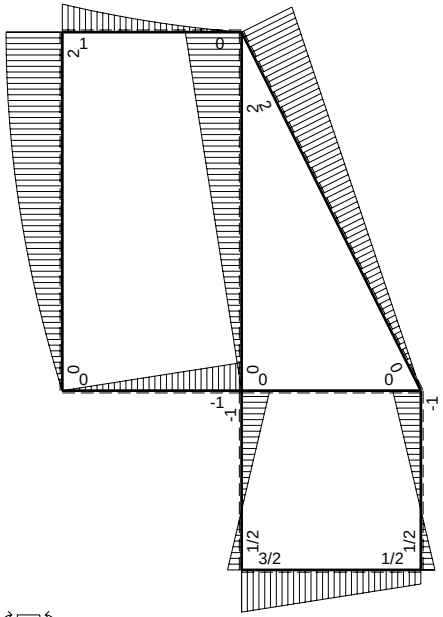
 08.12.24



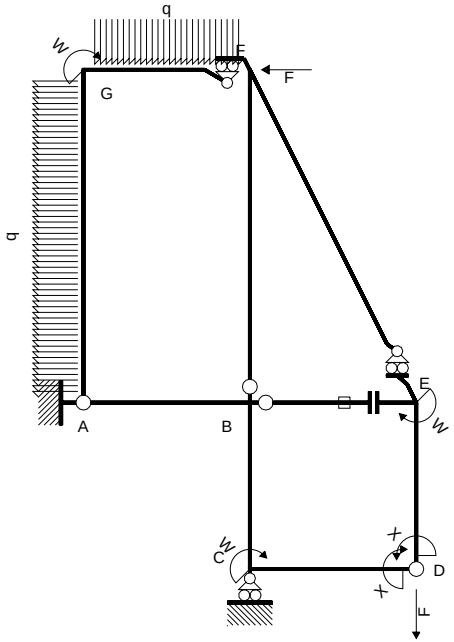
←□→ F



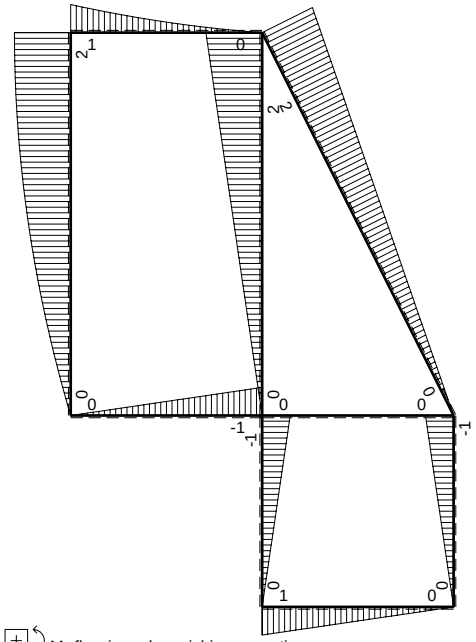
↑□↓ F



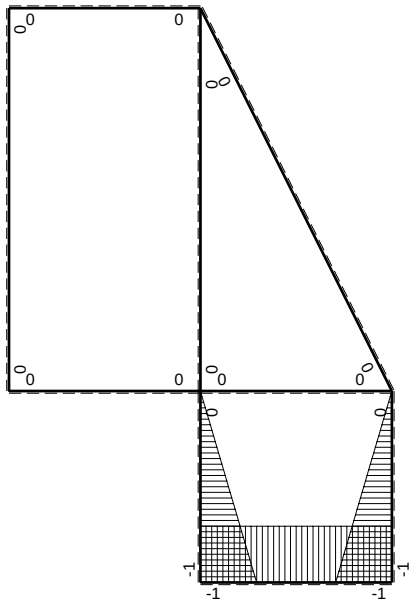
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-Fx	0	0	0	0
BA b	0	Fb-Fx	0	0		
BC b	-x/b	-Fb+Fx	$Fx-Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	Fb-Fx	-Fb+Fx	1	$-1/2Fb^2/EJ$	Xb/EJ
DC b	1	-Fx	-Fx	1		
DE b	-1+x/b	-Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	Fb-Fx	$Fx-Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$-Fb+3/2Fx-1/2qx^2$	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$5/6Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{DE}$				-1/2Fb	

Sviluppi di calcolo iperstatica

$$L^{XX}_{BC} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CB} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CD} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DC} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{Xo}_{BC} = \int_o^b \left(x/b - x^2/b^2 \right) Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b Fb \; 1/EJ$$
$$= \left(1/2 \; b - 1/3 \; b \right) Fb \; 1/EJ \; = 1/6 \; Fb^2/EJ$$

$$L^{Xo}_{CB} = \int_o^b \left(x/b - x^2/b^2 \right) Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b Fb \; 1/EJ$$
$$= \left(1/2 \; b - 1/3 \; b \right) Fb \; 1/EJ \; = 1/6 \; Fb^2/EJ$$

$$L^{Xo}_{CD} = \int_o^b \left(-1 + x/b \right) Fb \; 1/EJ \; dx = \left[- x + 1/2 \; x^2/b \right]_o^b Fb \; 1/EJ$$
$$= \left(- b + 1/2 \; b \right) Fb \; 1/EJ \; = -1/2 \; Fb^2/EJ$$

$$L^{Xo}_{DC} = \int_o^b \left(- x/b \right) Fb \; 1/EJ \; dx = \left[-1/2 \; x^2/b \right]_o^b Fb \; 1/EJ$$
$$= \left(-1/2 \; b \right) Fb \; 1/EJ \; = -1/2 \; Fb^2/EJ$$

$$L^{Xo}_{DE} = \int_o^b \left(x/b - x^2/b^2 \right) Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b Fb \; 1/EJ$$
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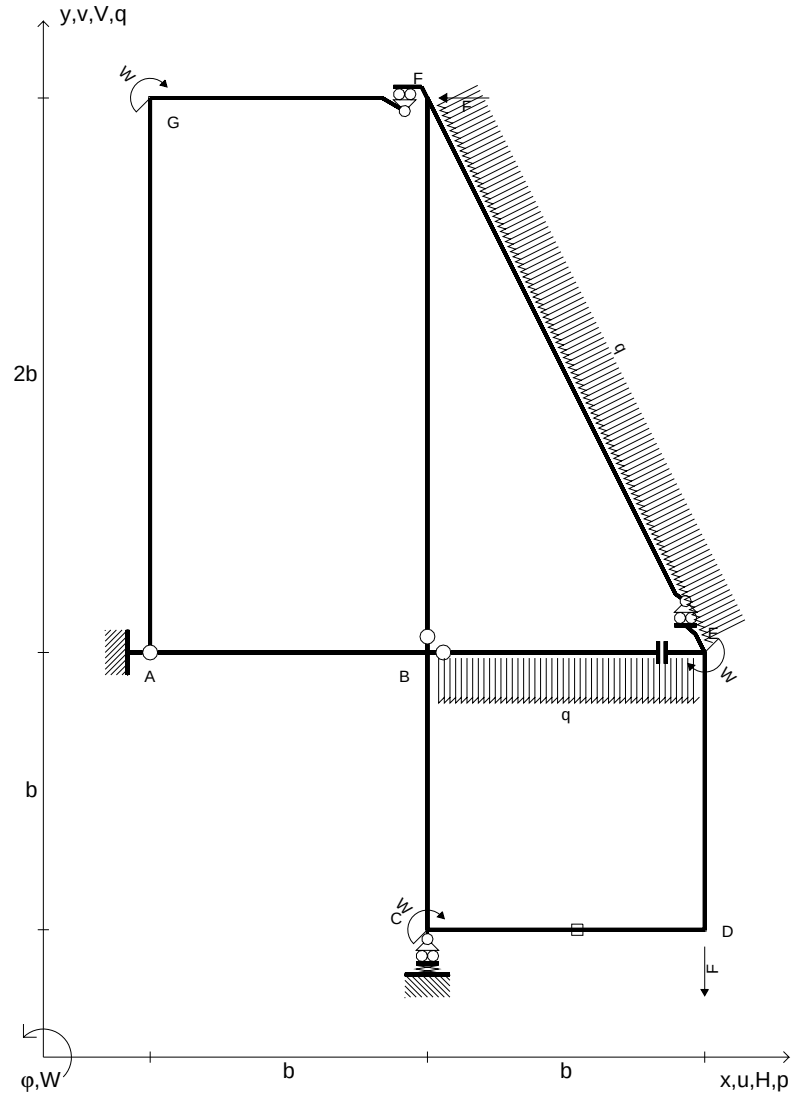
$$L^{Xo}_{ED} = \int_o^b \left(x/b - x^2/b^2 \right) Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b Fb \; 1/EJ$$
$$= \left(1/2 \; b - 1/3 \; b \right) Fb \; 1/EJ \; = 1/6 \; Fb^2/EJ$$

ESΣ02.xxxx.024

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.12.24

$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{EF} = -q = -F/b$
$q_{EF} = -q = -F/b$
$q_{BE} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



Reazioni iperstatiche in soluzione: $X=W_{DE}$
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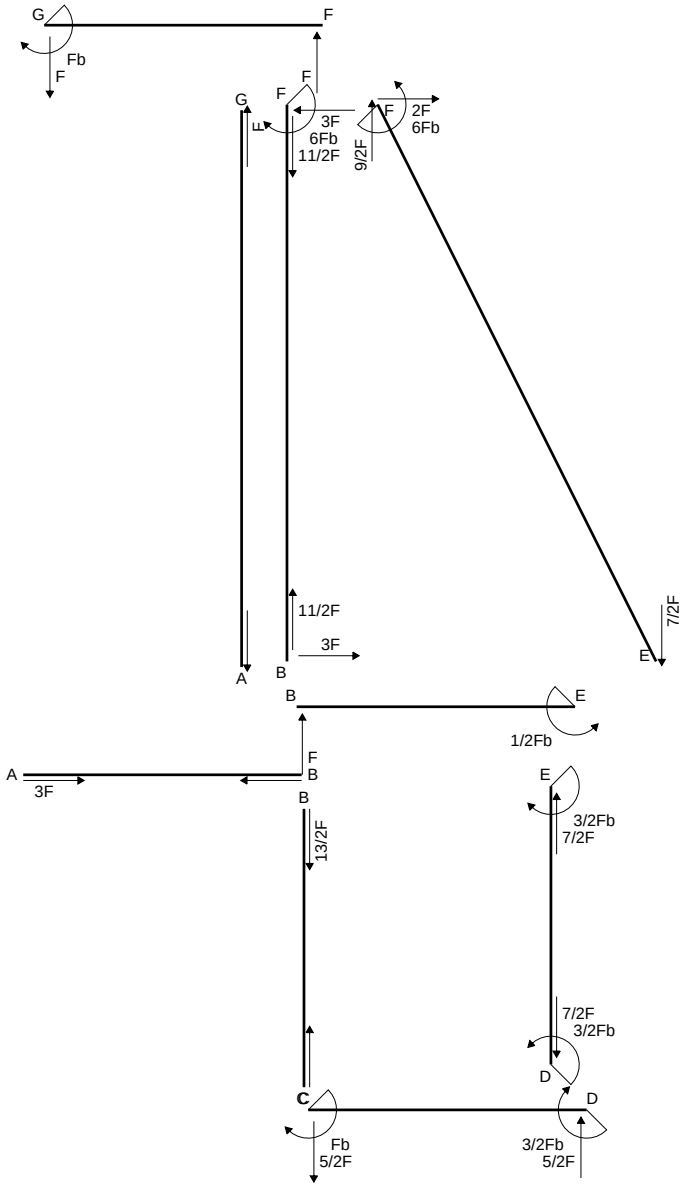
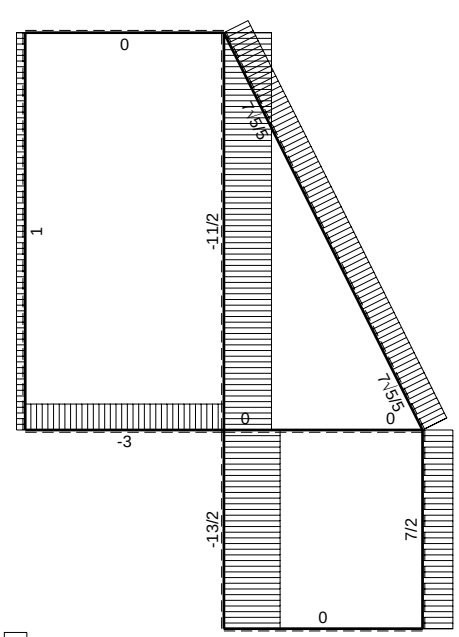
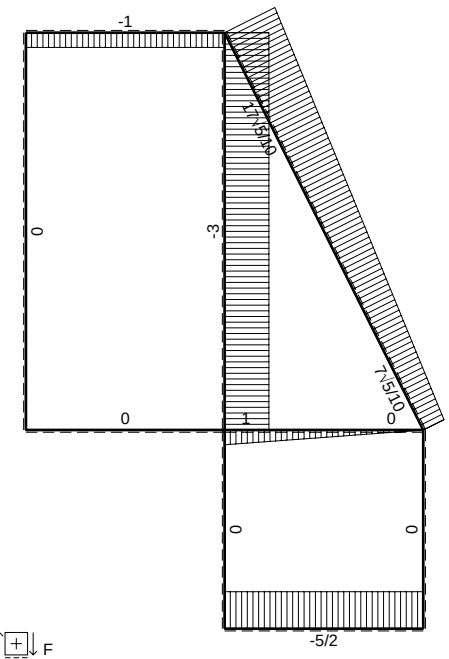
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.

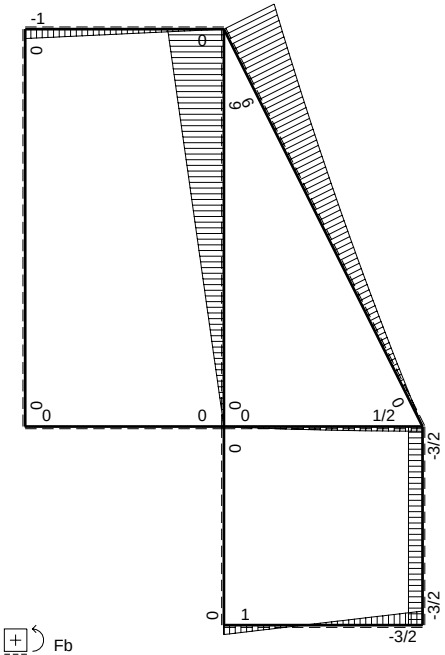




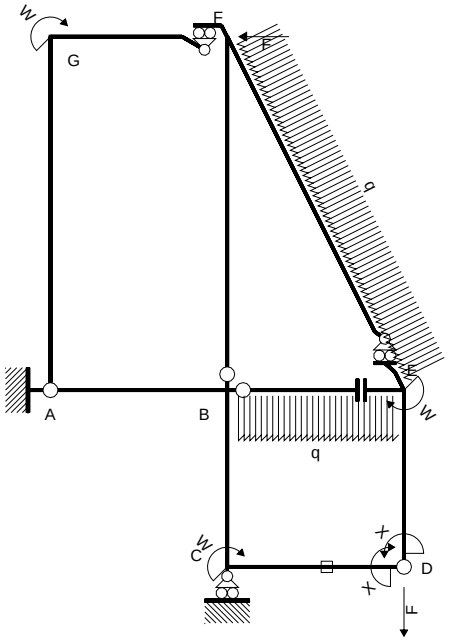
←□→ F



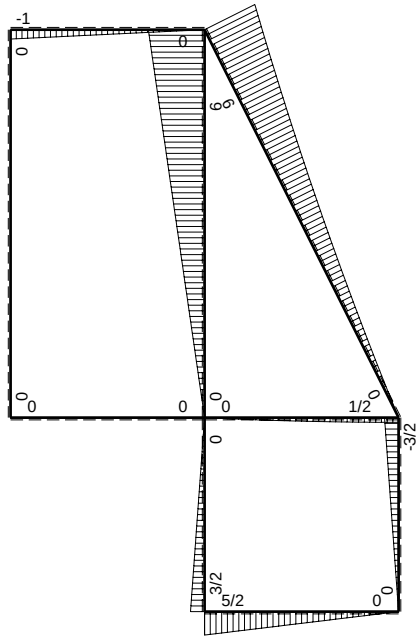
↑□↓ F



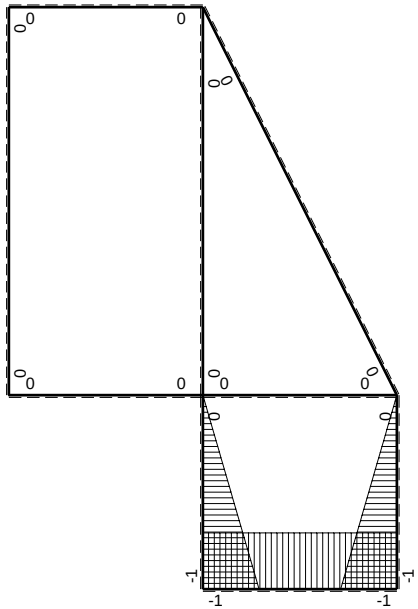
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x/b	3/2Fx	$-3/2Fx^2/b$	x^2/b^2	$-1/2Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-3/2Fb+3/2Fx	$-3/2Fb+3Fx-3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	5/2Fb-5/2Fx	-5/2Fb+5/2Fx	1	$-5/4Fb^2/EJ$	Xb/EJ
DC b	1	-5/2Fx	-5/2Fx	1		
DE b	-1+x/b	-3/2Fx	$3/2Fx-3/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/4Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	3/2Fb-3/2Fx	$3/2Fx-3/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-5/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				3/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210099 ESΣ02.xxxx.029

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (-3/2 +3\ x/b\ -3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/2\ x +3/2\ x^2/b\ -1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/2\ b +3/2\ b\ -1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -9/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -9/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3/2\ x/b\ -3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b\ -1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b\ -1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b\ -3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b\ -1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b\ -1/2\ b\ )\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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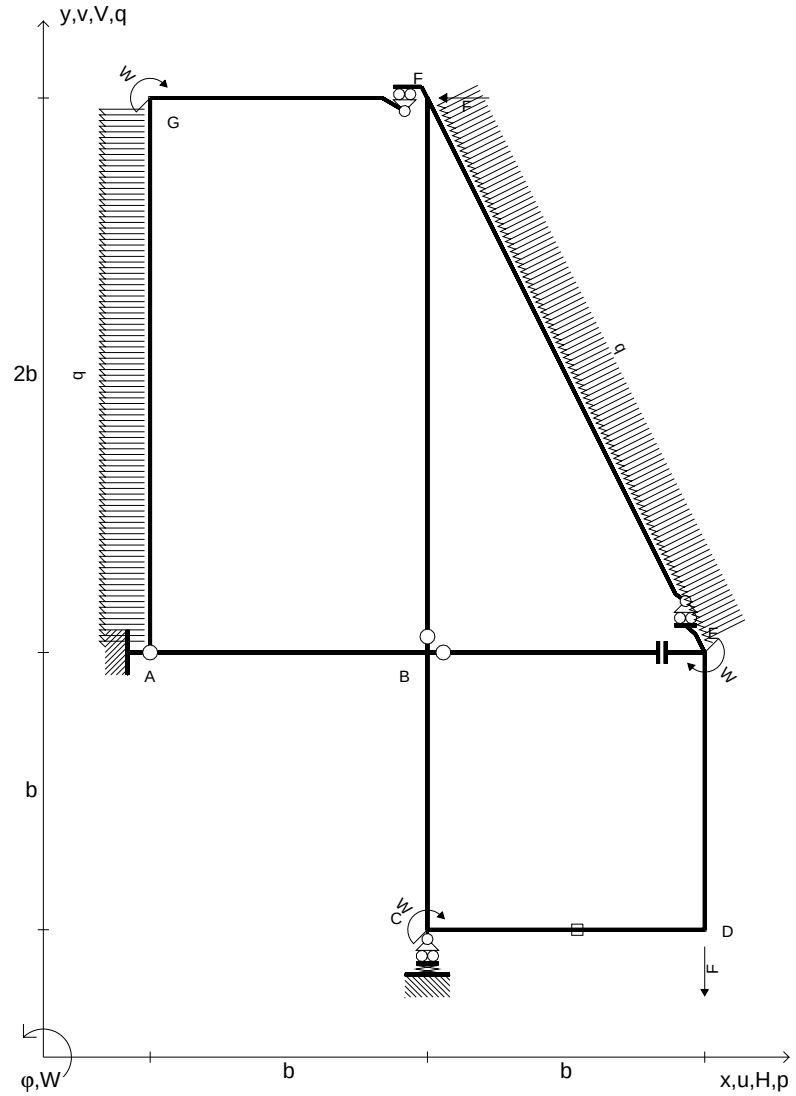
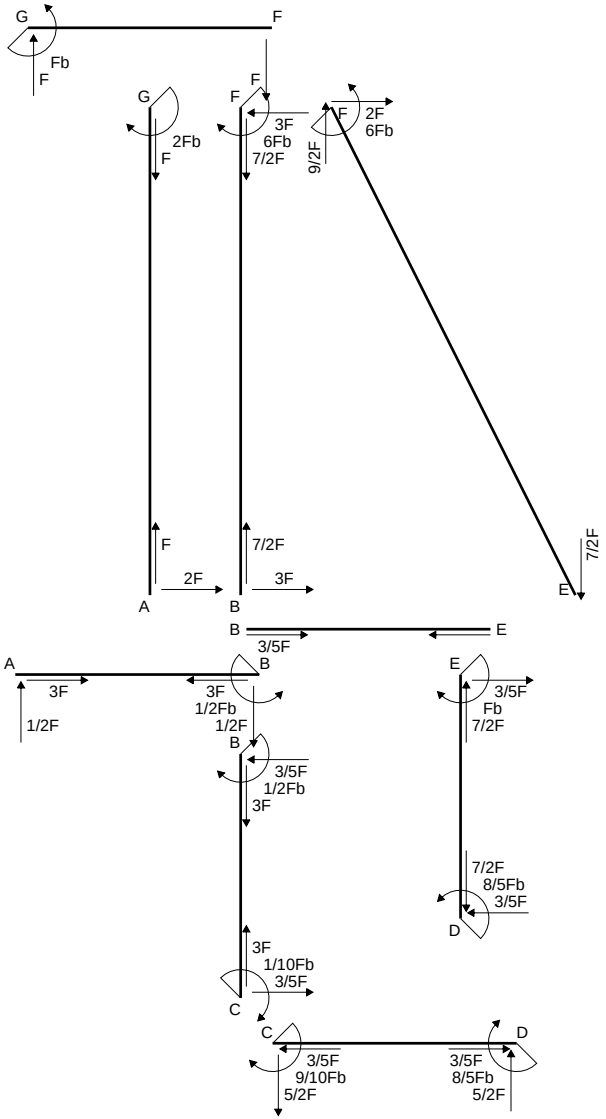
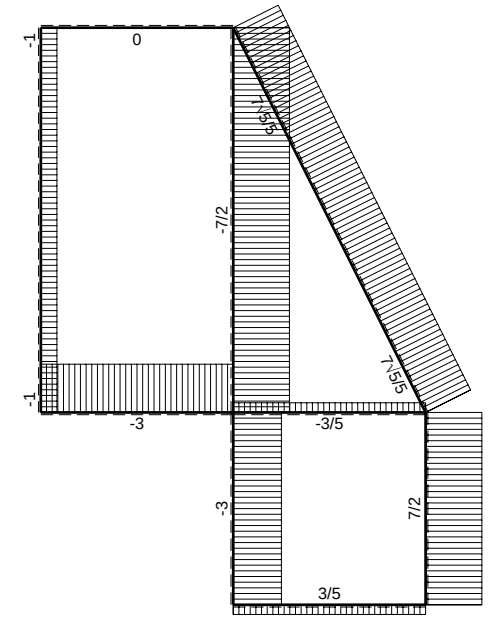
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{EF} = -q = -F/b$
$q_{EF} = -q = -F/b$
$p_{GA} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

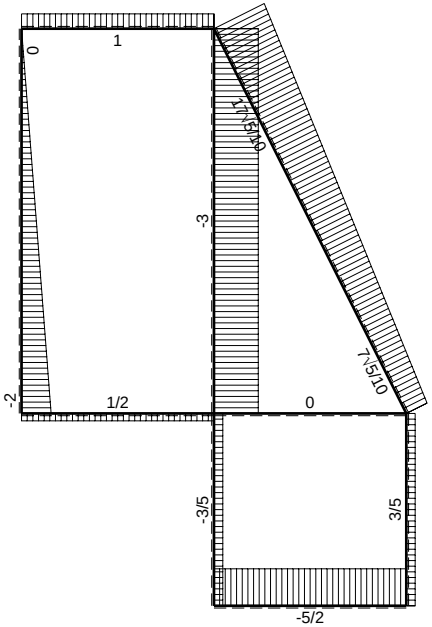


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.





F



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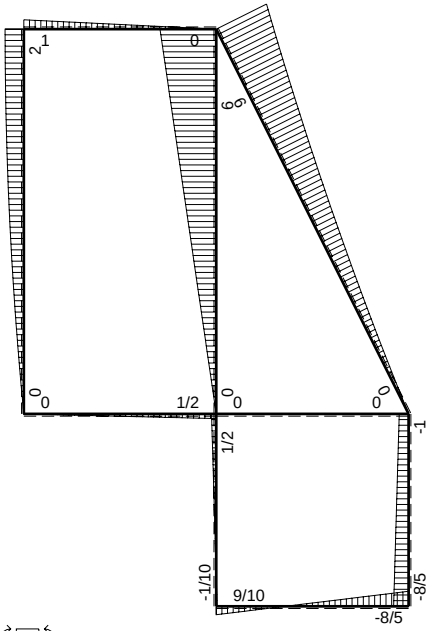
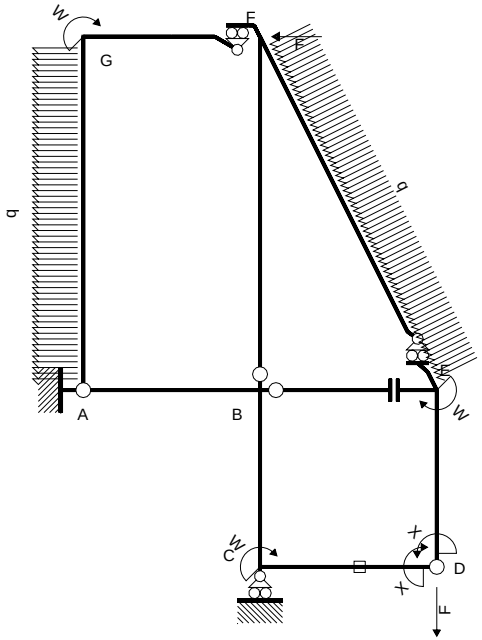
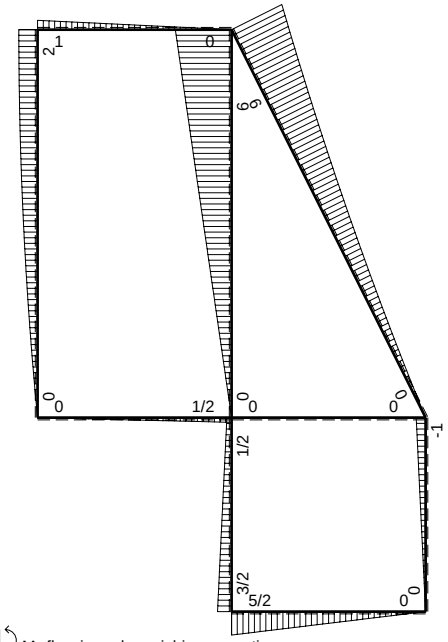


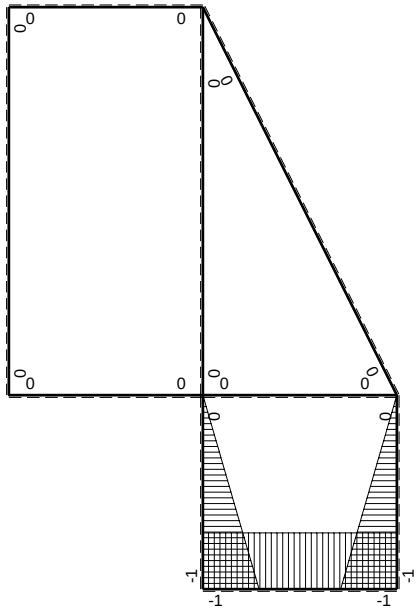
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb+Fx	$-1/2Fx-Fx^2/b$	x^2/b^2	$-7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-3/2Fb+Fx	$-3/2Fb+5/2Fx-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	5/2Fb-5/2Fx	-5/2Fb+5/2Fx	1	$-5/4Fb^2/EJ$	Xb/EJ
DC b	1	-5/2Fx	-5/2Fx	1		
DE b	-1+x/b	-Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb-Fx	$Fx-Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	Fx	0	0	0	0
GF b	0	-Fb+Fx	0	0		
GA 2b	0	$2Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$-2Fx+1/2qx^2$	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-8/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				8/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1\ -2\ x/b\ +\ x^2/b^2\ )\ 1/EJ\ dx = [\ x\ -\ x^2/b\ +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b\ -\ b\ +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1\ -2\ x/b\ +\ x^2/b^2\ )\ 1/EJ\ dx = [\ x\ -\ x^2/b\ +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b\ -\ b\ +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b\ -\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\ -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b\ -1/3\ b\ )\ Fb\ 1/EJ\ = -7/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (-3/2\ +5/2\ x/b\ -\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/2\ x\ +5/4\ x^2/b\ -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/2\ b\ +5/4\ b\ -1/3\ b\ )\ Fb\ 1/EJ\ = -7/12\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (-5/2\ +5/2\ x/b\ )\ Fb\ 1/EJ\ dx\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [-5/2\ x\ +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-5/2\ b\ +5/4\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -9/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -9/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (\ x/b\ -\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ -1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (\ x/b\ -\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ -1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

ESΣ02.xxxx.031

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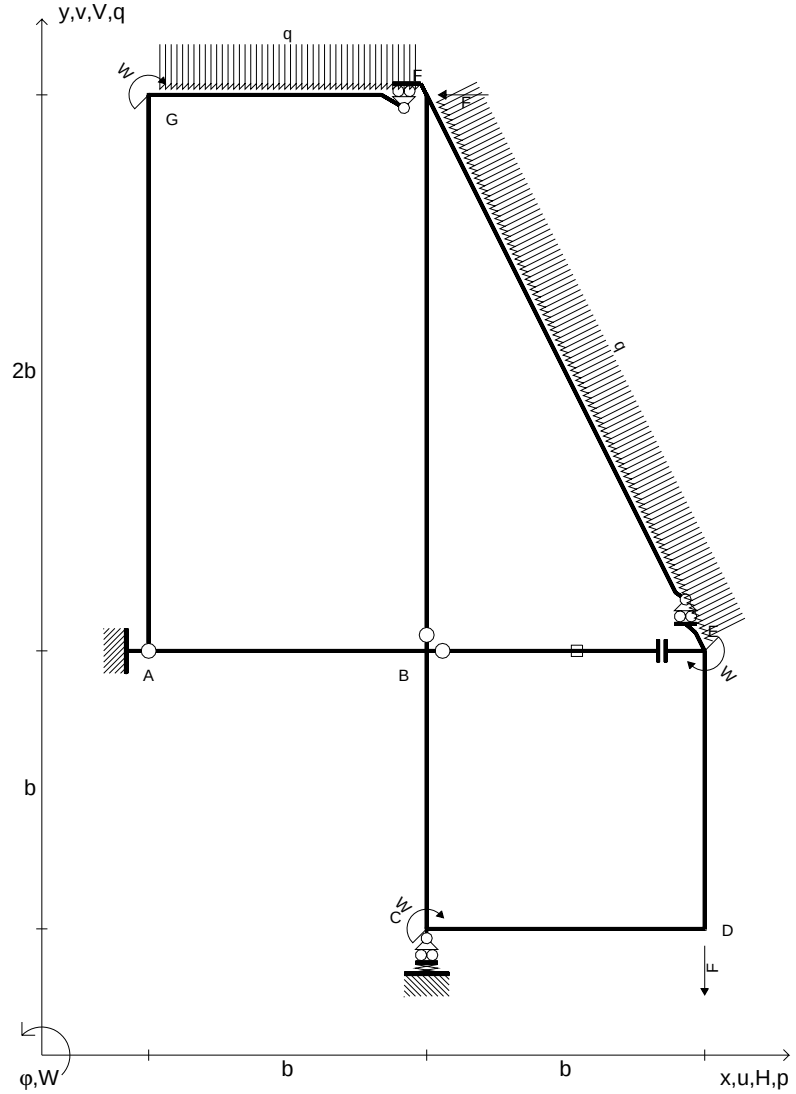
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{EF} = -q = -F/b$
$q_{EF} = -q = -F/b$
$q_{FG} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



Reazioni iperstatiche in soluzione: $X=W_{DE}$
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Calcolare reazioni vincolari della struttura e delle aste.
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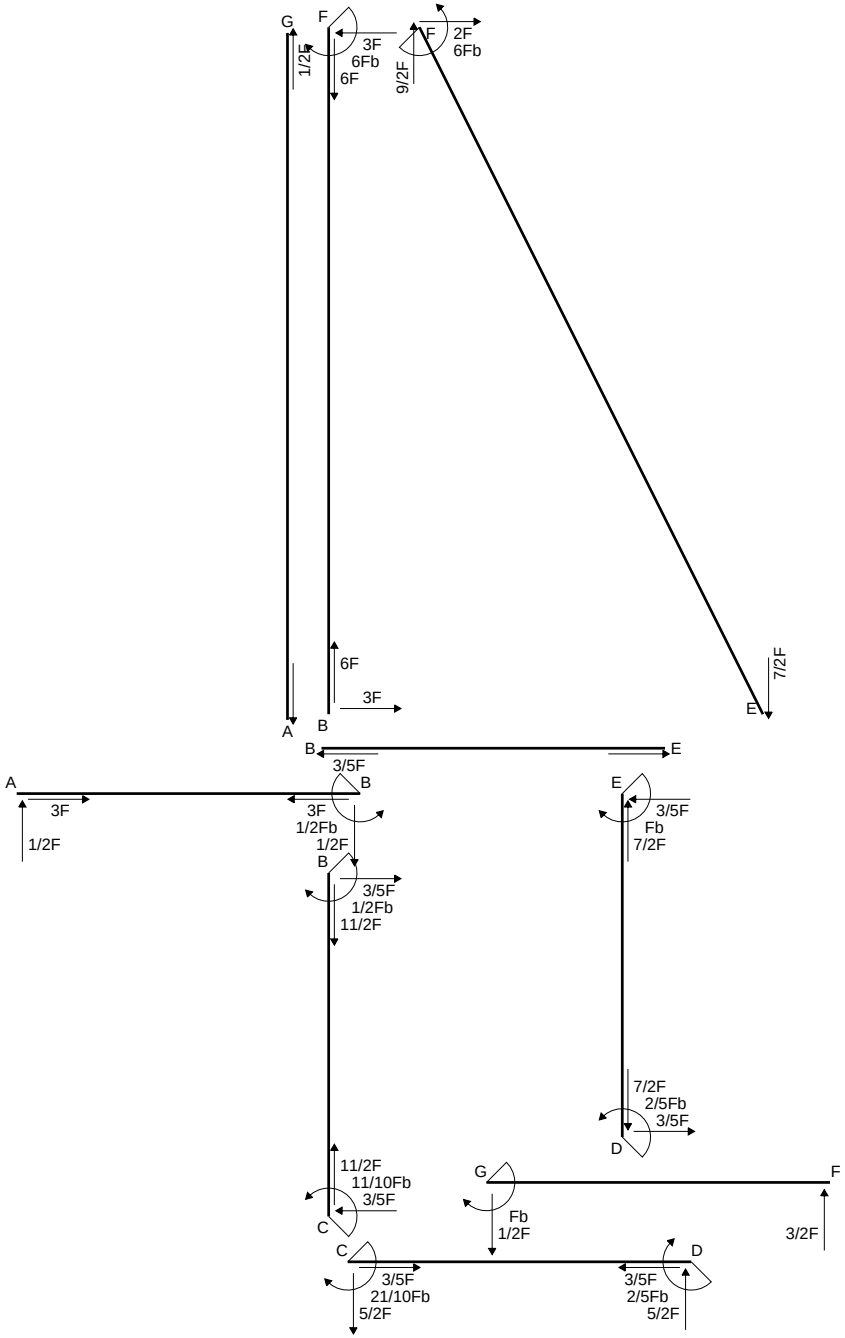
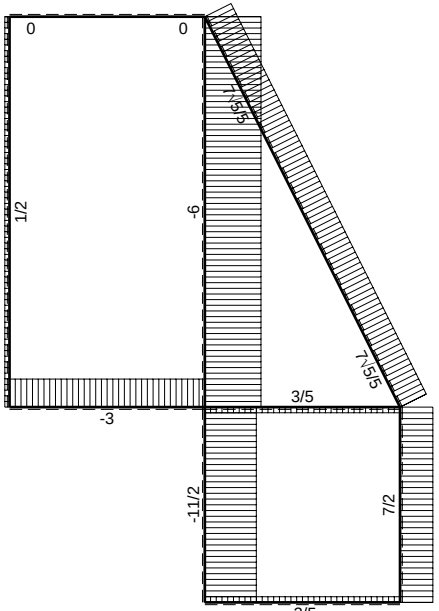
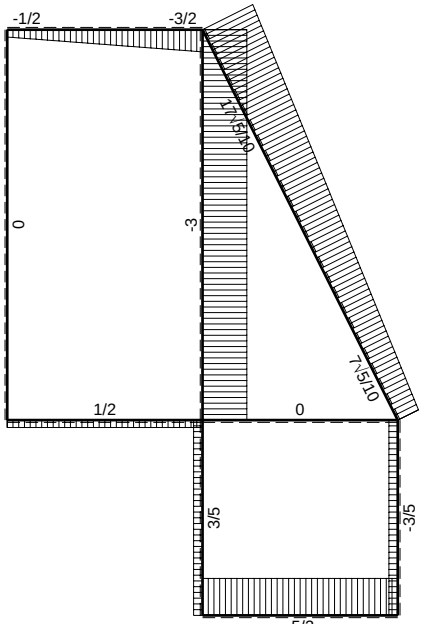
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.

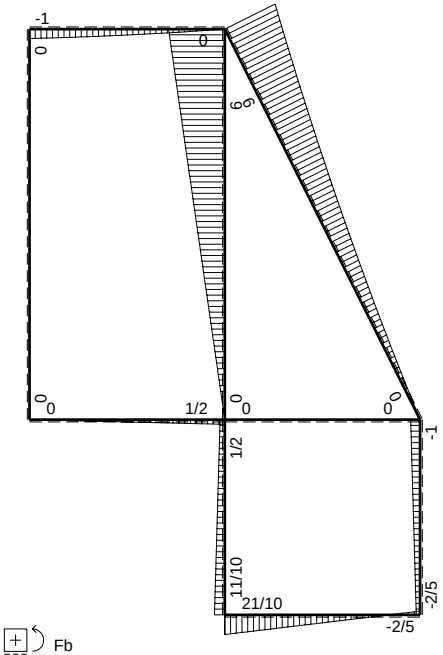




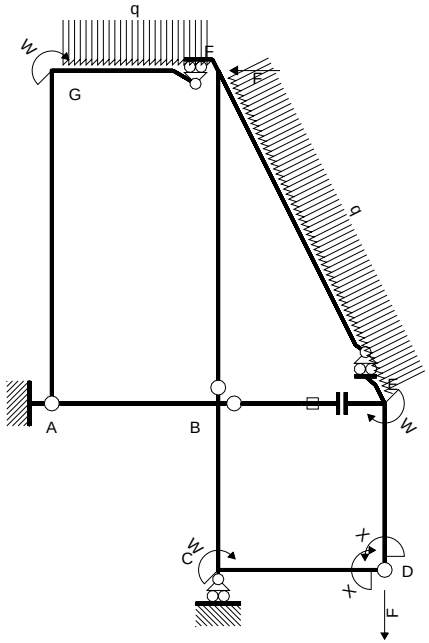
←[+]→ F



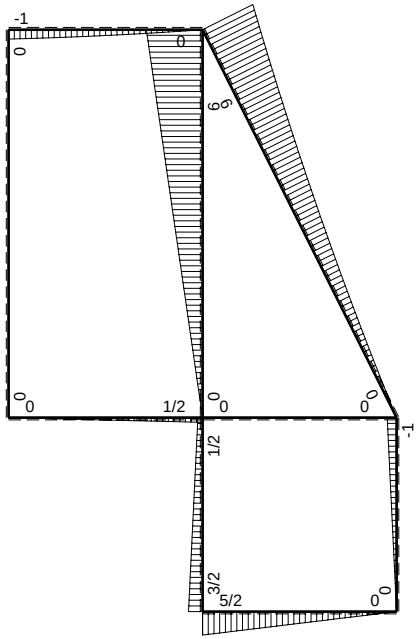
↑[+]↓ F



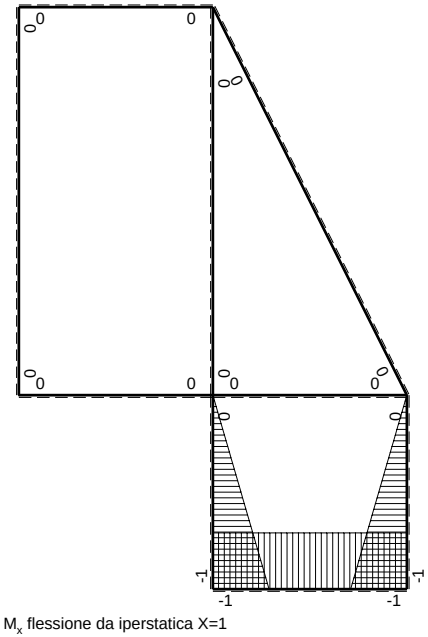
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb+Fx	$-1/2Fx-Fx^2/b$	x^2/b^2	$-7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-3/2Fb+Fx	$-3/2Fb+5/2Fx-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	5/2Fb-5/2Fx	-5/2Fb+5/2Fx	1	$-5/4Fb^2/EJ$	Xb/EJ
DC b	1	-5/2Fx	-5/2Fx	1		
DE b	-1+x/b	-Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb-Fx	$Fx-Fx^2/b$	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$-2/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				2/5Fb	

Sviluppi di calcolo iperstatica

$$L^{XX}_{BC} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CB} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CD} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DC} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{Xo}_{BC} = \int_o^b \left(-1/2 \; x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[-1/4 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-1/4 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = -7/12 \; Fb^2/EJ$$

$$L^{Xo}_{CB} = \int_o^b \left(-3/2 + 5/2 \; x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[-3/2 \; x + 5/4 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-3/2 \; b + 5/4 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = -7/12 \; Fb^2/EJ$$

$$L^{Xo}_{CD} = \int_o^b \left(-5/2 + 5/2 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[-5/2 \; x + 5/4 \; x^2/b \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-5/2 \; b + 5/4 \; b \right) \; Fb \; 1/EJ \; = -5/4 \; Fb^2/EJ$$

$$L^{Xo}_{DC} = \int_o^b \left(-5/2 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[-5/4 \; x^2/b \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-5/4 \; b \right) \; Fb \; 1/EJ \; = -5/4 \; Fb^2/EJ$$

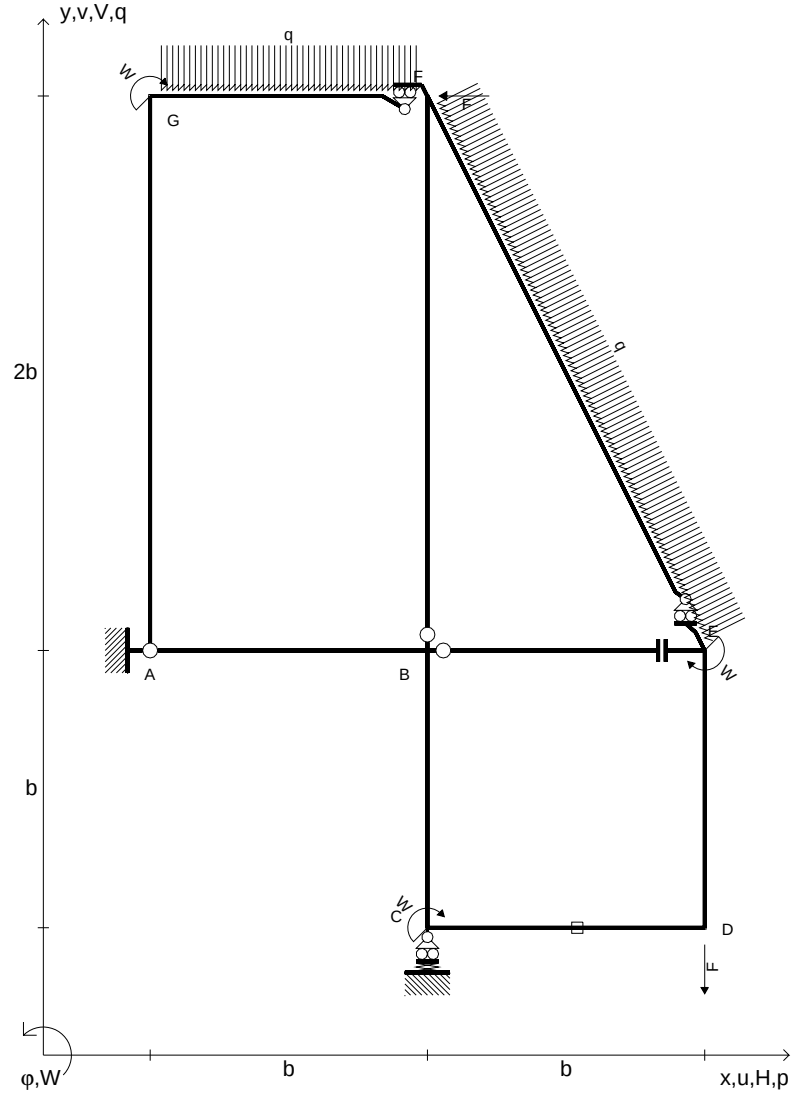
$$L^{Xo}_{DE} = \int_o^b \left(x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(1/2 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = 1/6 \; Fb^2/EJ$$

$$L^{Xo}_{ED} = \int_o^b \left(x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(1/2 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = 1/6 \; Fb^2/EJ$$

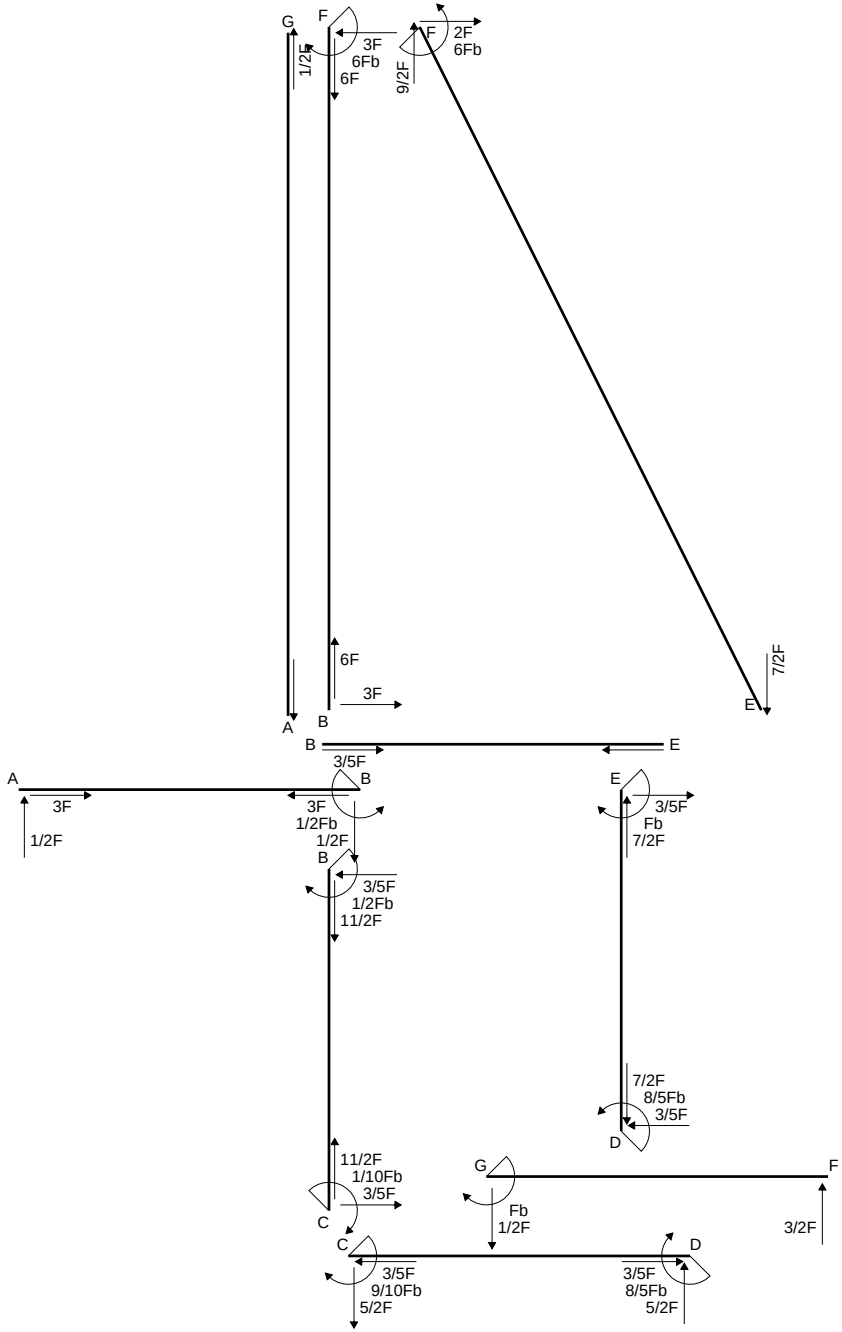
ESΣ02.xxxx.038

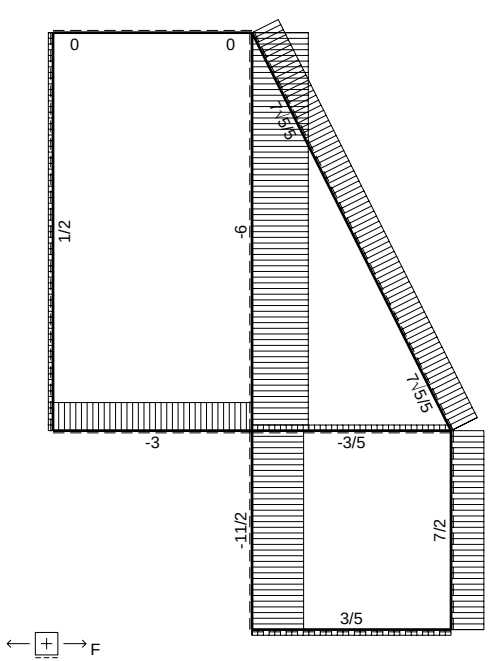
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.12.24

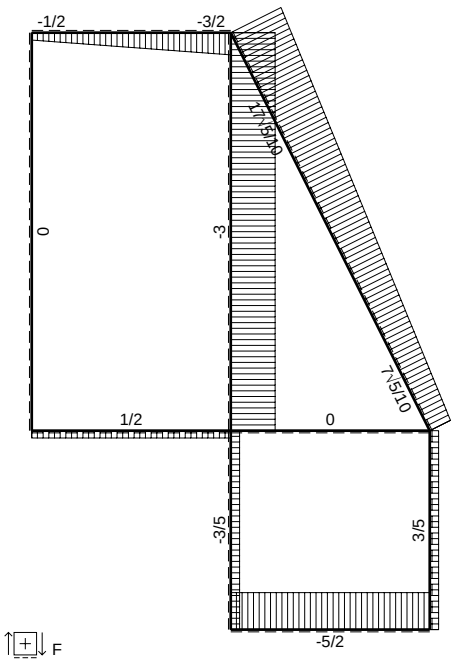
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{EF} = -q = -F/b$
$q_{EF} = -q = -F/b$
$q_{FG} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

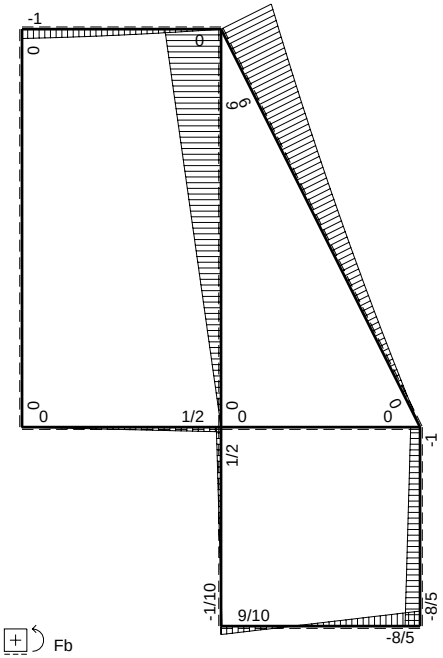


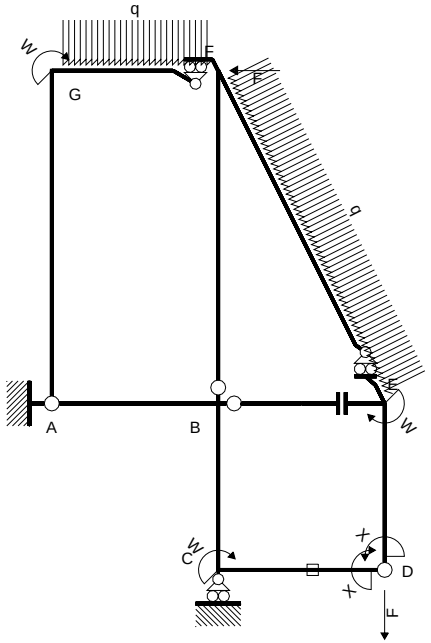
Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.



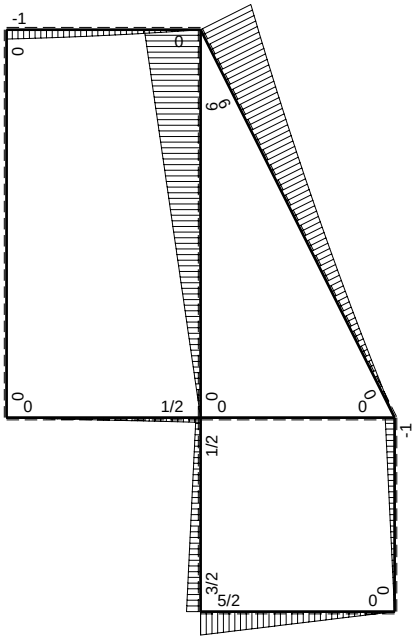




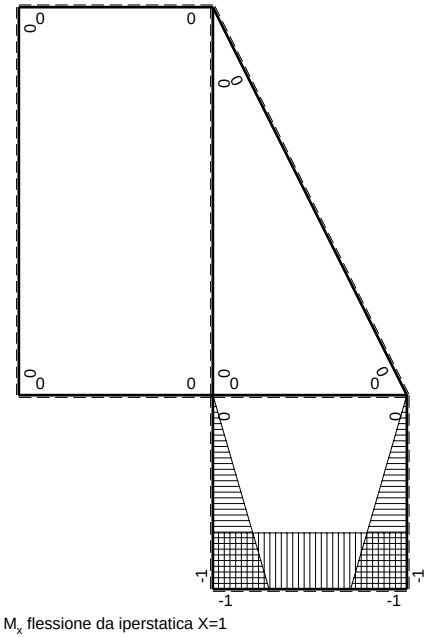




Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb+Fx	$-1/2Fx-Fx^2/b$	x^2/b^2	$-7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-3/2Fb+Fx	$-3/2Fb+5/2Fx-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	5/2Fb-5/2Fx	-5/2Fb+5/2Fx	1	$-5/4Fb^2/EJ$	Xb/EJ
DC b	1	-5/2Fx	-5/2Fx	1		
DE b	-1+x/b	-Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb-Fx	$Fx-Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-8/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				8/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b (1\ -2\ x/b\ +\ x^2/b^2\ )\ 1/EJ\ dx = [\ x\ -\ x^2/b\ +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b\ -\ b\ +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1\ -2\ x/b\ +\ x^2/b^2\ )\ 1/EJ\ dx = [\ x\ -\ x^2/b\ +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b\ -\ b\ +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b\ -\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\ -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b\ -1/3\ b\ )\ Fb\ 1/EJ\ = -7/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (-3/2\ +5/2\ x/b\ -\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/2\ x\ +5/4\ x^2/b\ -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/2\ b\ +5/4\ b\ -1/3\ b\ )\ Fb\ 1/EJ\ = -7/12\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (-5/2\ +5/2\ x/b\ )\ Fb\ 1/EJ\ dx\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [-5/2\ x\ +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-5/2\ b\ +5/4\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -9/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -9/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (\ x/b\ -\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ -1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

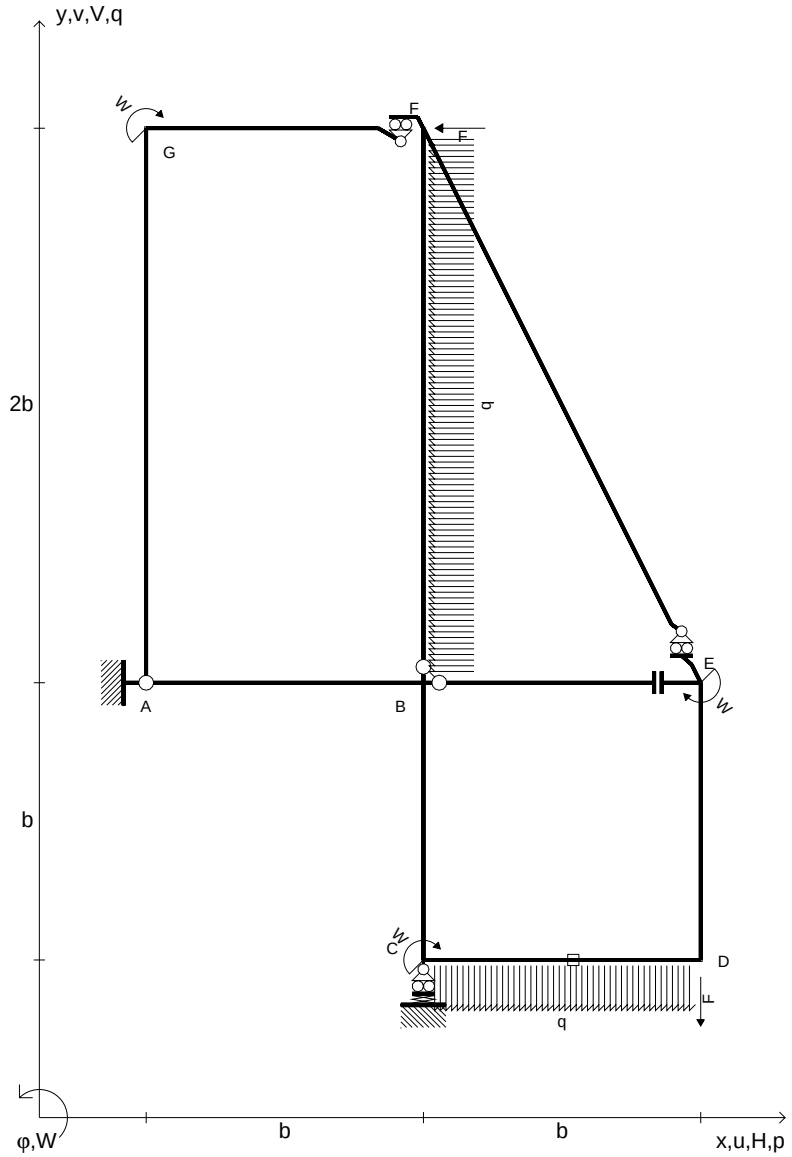
$L_{ED}^{Xo} = \int_o^b (\ x/b\ -\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ -1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

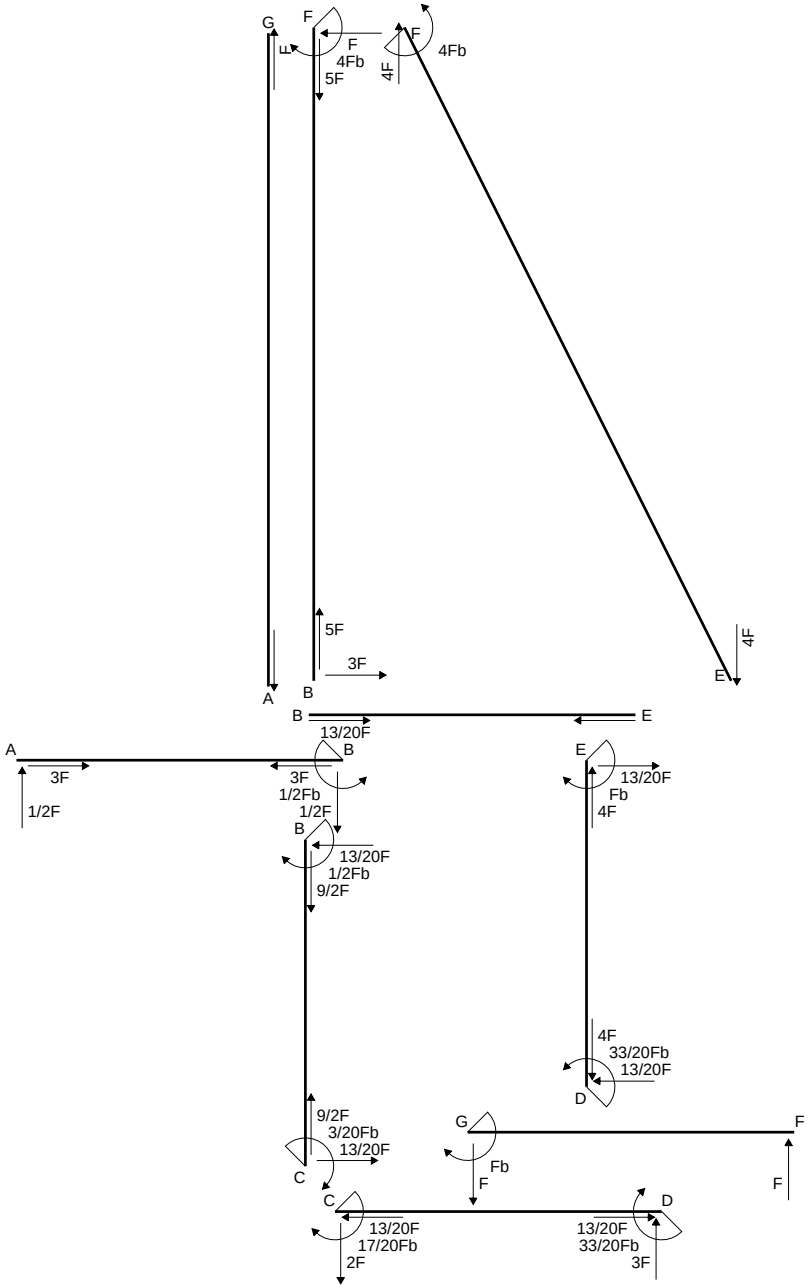
ESΣ02.xxxx.039

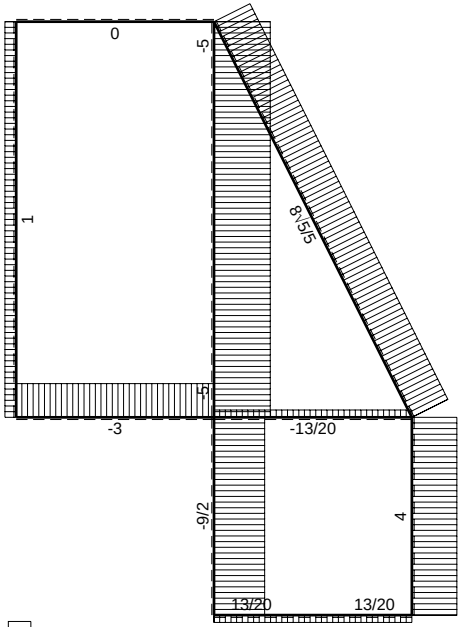
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.12.24

$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$q_{CD} = -q = -F/b$
$p_{FB} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.





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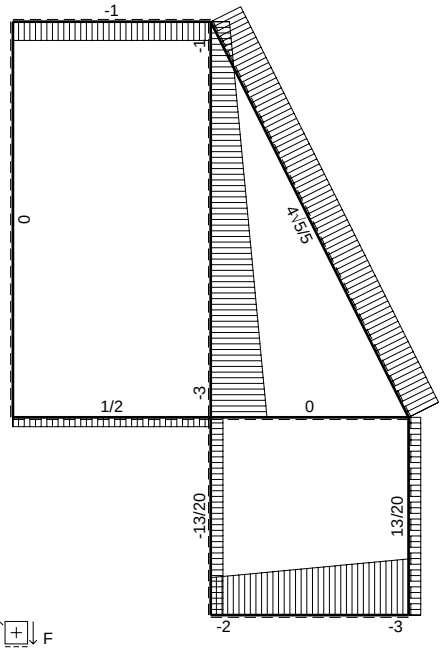
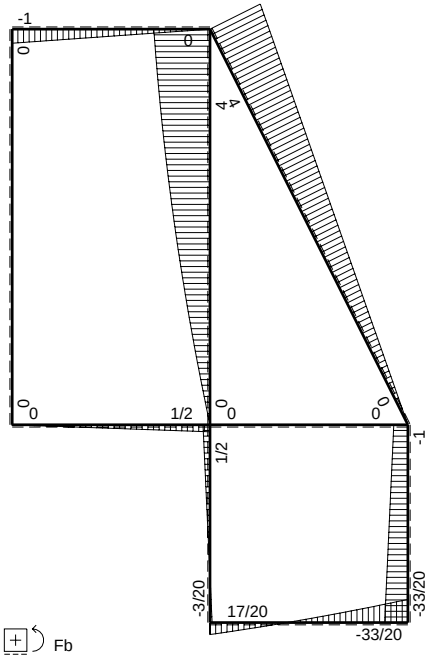
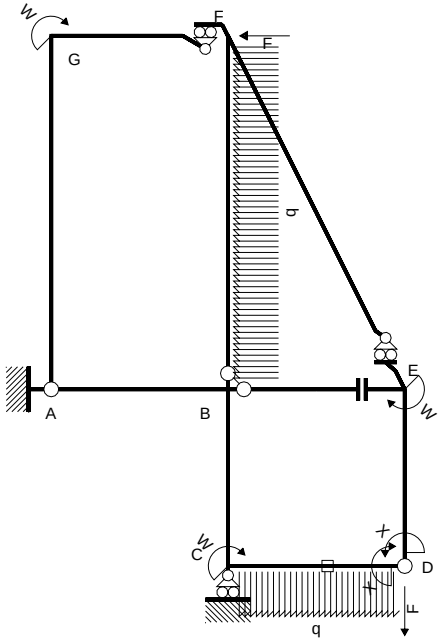


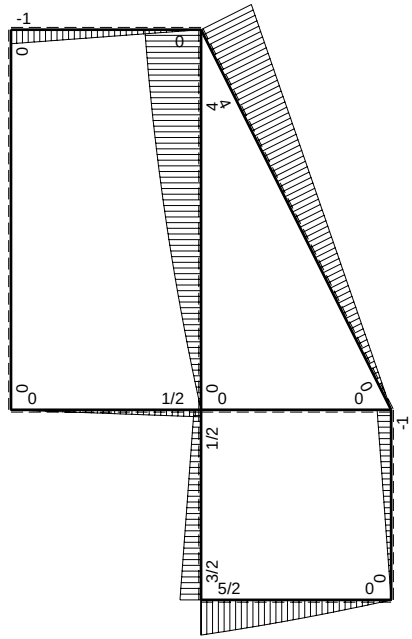
F



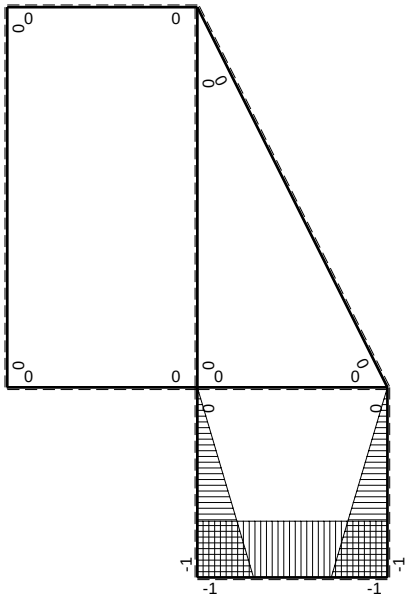
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb+Fx	$-1/2Fx-Fx^2/b$	x^2/b^2	$-7/12Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-3/2Fb+Fx	$-3/2Fb+5/2Fx-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$5/2Fb-2Fx-1/2qx^2$	$-5/2Fb+2Fx+1/2Fx^2/b$	1	$-4/3Fb^2/EJ$	Xb/EJ
DC b	1	$-3Fx+1/2qx^2$	$-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	-Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb-Fx	$Fx-Fx^2/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-11/4Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				33/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 218425 ESΣ02.xxxx.051

$$L^{XX}_{BC} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$

$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CB} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$

$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CD} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$

$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DC} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$

$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$

$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$

$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{Xo}_{BC} = \int_o^b \left(-1/2 \; x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[-1/4 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$

$$= \left(-1/4 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = -7/12 \; \; Fb^2/EJ$$

$$L^{Xo}_{CB} = \int_o^b \left(-3/2 + 5/2 \; x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[-3/2 \; x + 5/4 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$

$$= \left(-3/2 \; b + 5/4 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = -7/12 \; \; Fb^2/EJ$$

$$L^{Xo}_{CD} = \int_o^b \left(-5/2 + 2 \; x/b + 1/2 \; x^2/b^2 \right) \; Fb \; 1/EJ \; dx + 1 \quad (-1) \; \; 1 \; \; Fb^2/EJ$$

$$= \left[-5/2 \; x + x^2/b + 1/6 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ \; + 1 \quad (-1) \; \; 1 \; \; Fb^2/EJ$$

$$= \left(-5/2 \; b + b + 1/6 \; b \right) \; Fb \; 1/EJ \; + 1 \quad (-1) \; \; 1 \; \; Fb^2/EJ = -7/3 \; \; Fb^2/EJ$$

$$L^{Xo}_{DC} = \int_o^b \left(-3 \; x/b + 1/2 \; x^2/b^2 \right) \; Fb \; 1/EJ \; dx + 1 \quad (-1) \; \; 1 \; \; Fb^2/EJ$$

$$= \left[-3/2 \; x^2/b + 1/6 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ \; + 1 \quad (-1) \; \; 1 \; \; Fb^2/EJ$$

$$= \left(-3/2 \; b + 1/6 \; b \right) \; Fb \; 1/EJ \; + 1 \quad (-1) \; \; 1 \; \; Fb^2/EJ = -7/3 \; \; Fb^2/EJ$$

$$L^{Xo}_{DE} = \int_o^b \left(x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$

$$= \left(1/2 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = 1/6 \; \; Fb^2/EJ$$

$$L^{Xo}_{ED} = \int_o^b \left(x/b - x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/3 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$

$$= \left(1/2 \; b - 1/3 \; b \right) \; Fb \; 1/EJ \; = 1/6 \; \; Fb^2/EJ$$

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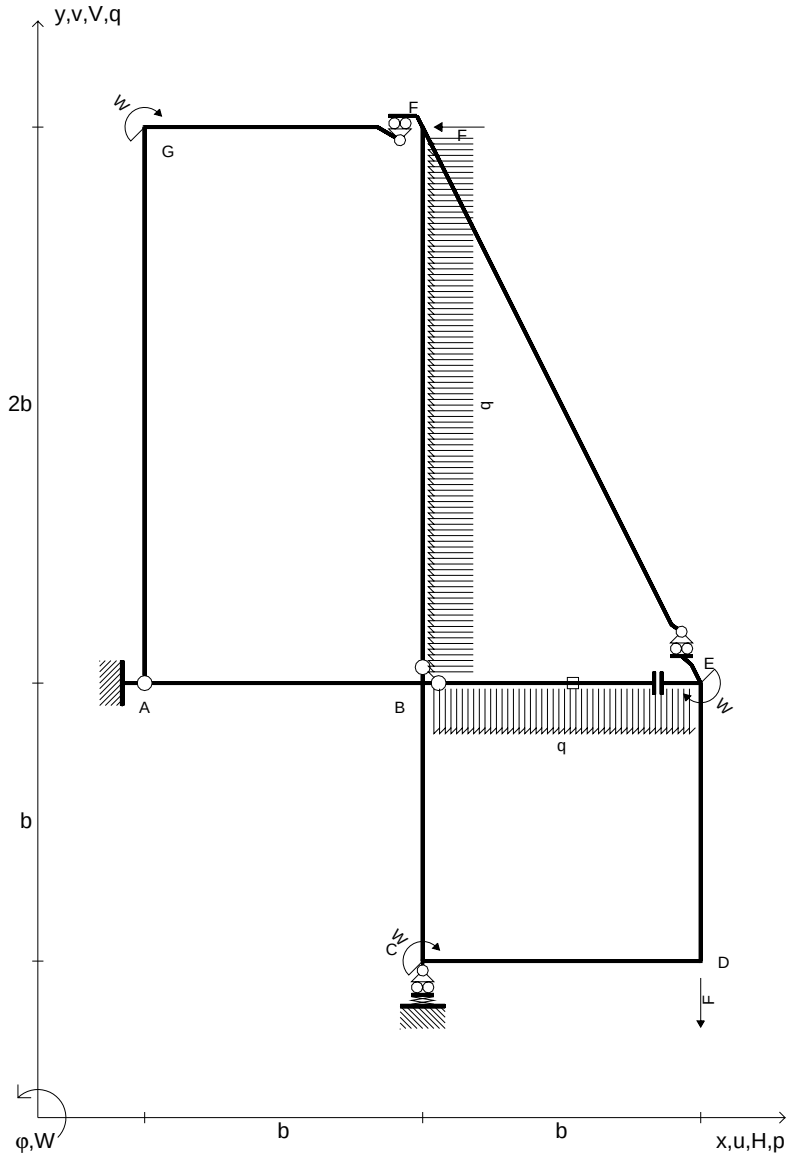
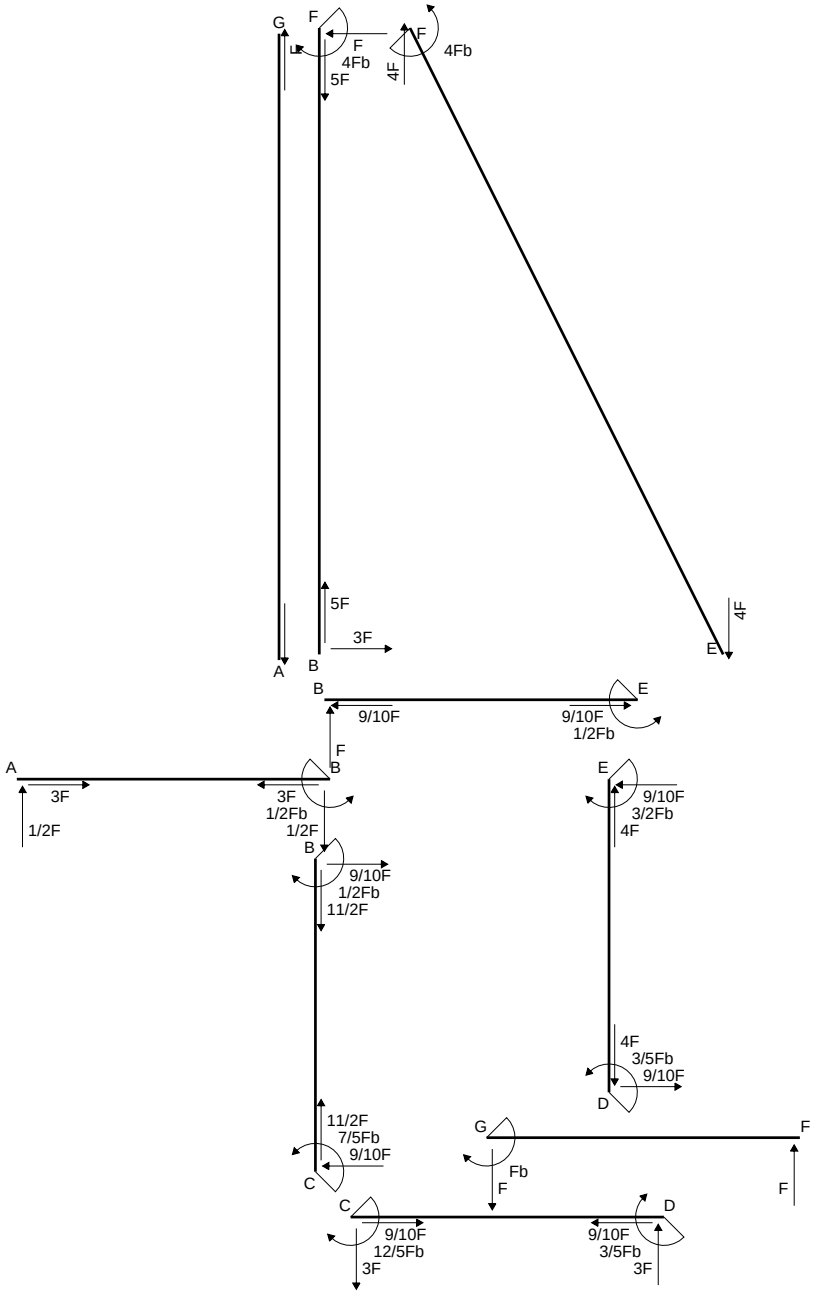
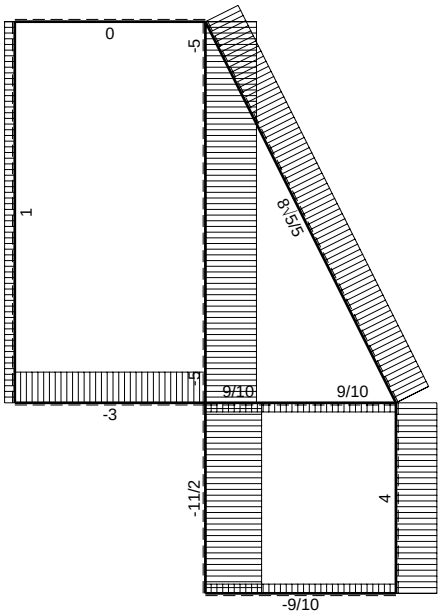
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{FB} = -q = -F/b$
$q_{BE} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.





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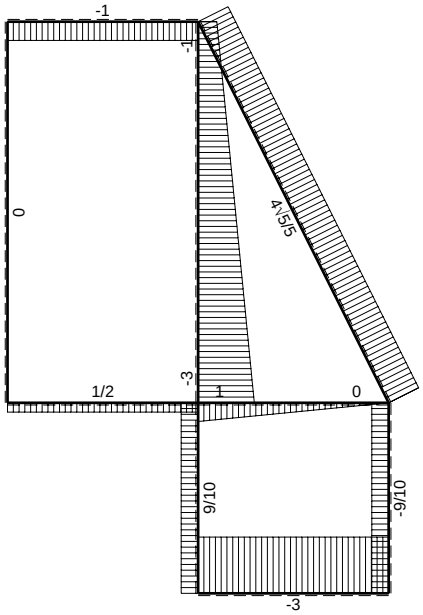
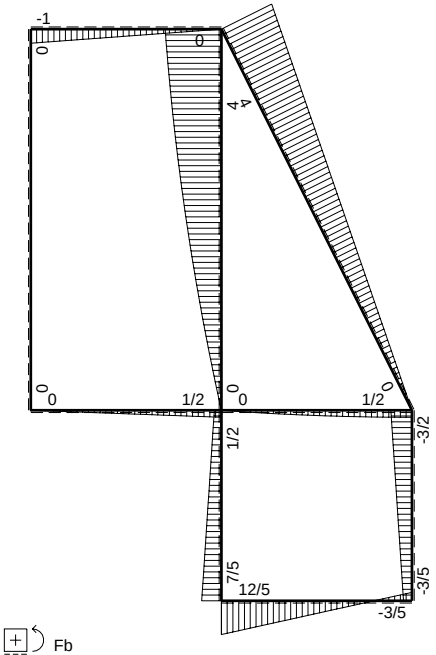
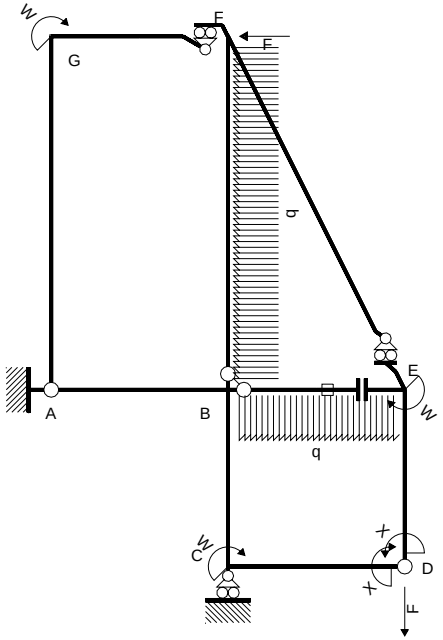


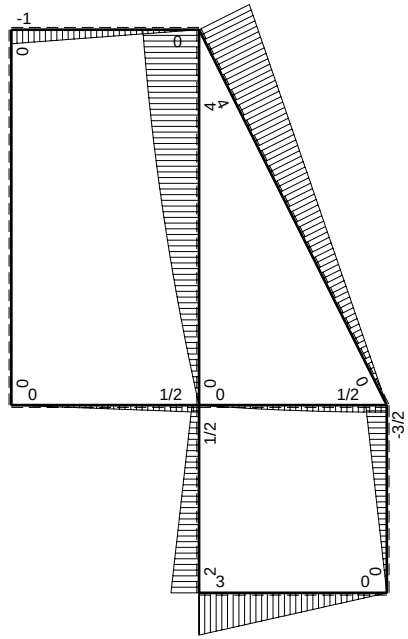
↑□↓ F



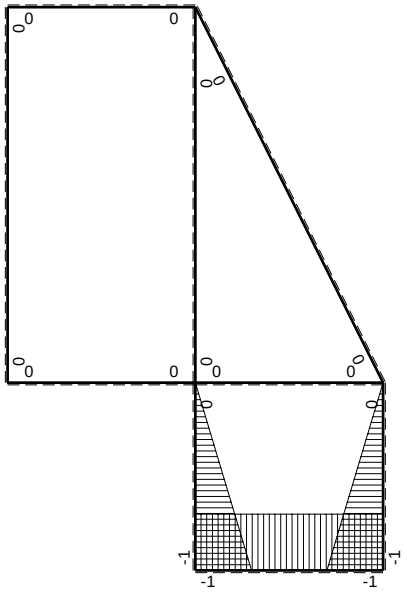
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb+3/2Fx	-1/2Fx-3/2Fx^2/b	x^2/b^2	-3/4Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	-2Fb+3/2Fx	-2Fb+7/2Fx-3/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	3Fb-3Fx	-3Fb+3Fx	1	-3/2Fb^2/EJ	Xb/EJ
DC b	1	-3Fx	-3Fx	1		
DE b	-1+x/b	-3/2Fx	3/2Fx-3/2Fx^2/b	1-2x/b+x^2/b^2	1/4Fb^2/EJ	1/3Xb/EJ
ED b	x/b	3/2Fb-3/2Fx	3/2Fx-3/2Fx^2/b	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	4Fb-Fx-1/2qx^2	0	0	0	0
BF 2b	0	-3Fx+1/2qx^2	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{DE}$				3/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b\ -3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b\ -1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b\ -1/2\ b)\ Fb\ 1/EJ\ = -3/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (-2\ +7/2\ x/b\ -3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-2\ x\ +7/4\ x^2/b\ -1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-2\ b\ +7/4\ b\ -1/2\ b)\ Fb\ 1/EJ\ = -3/4\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (-3\ +3\ x/b)\ Fb\ 1/EJ\ dx = [-3\ x\ +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b\ +3/2\ b)\ Fb\ 1/EJ\ = -3/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-3/2\ b)\ Fb\ 1/EJ\ = -3/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (3/2\ x/b\ -3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b\ -1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b\ -1/2\ b)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (3/2\ x/b\ -3/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/4\ x^2/b\ -1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b\ -1/2\ b)\ Fb\ 1/EJ\ = 1/4\ \ Fb^2/EJ$

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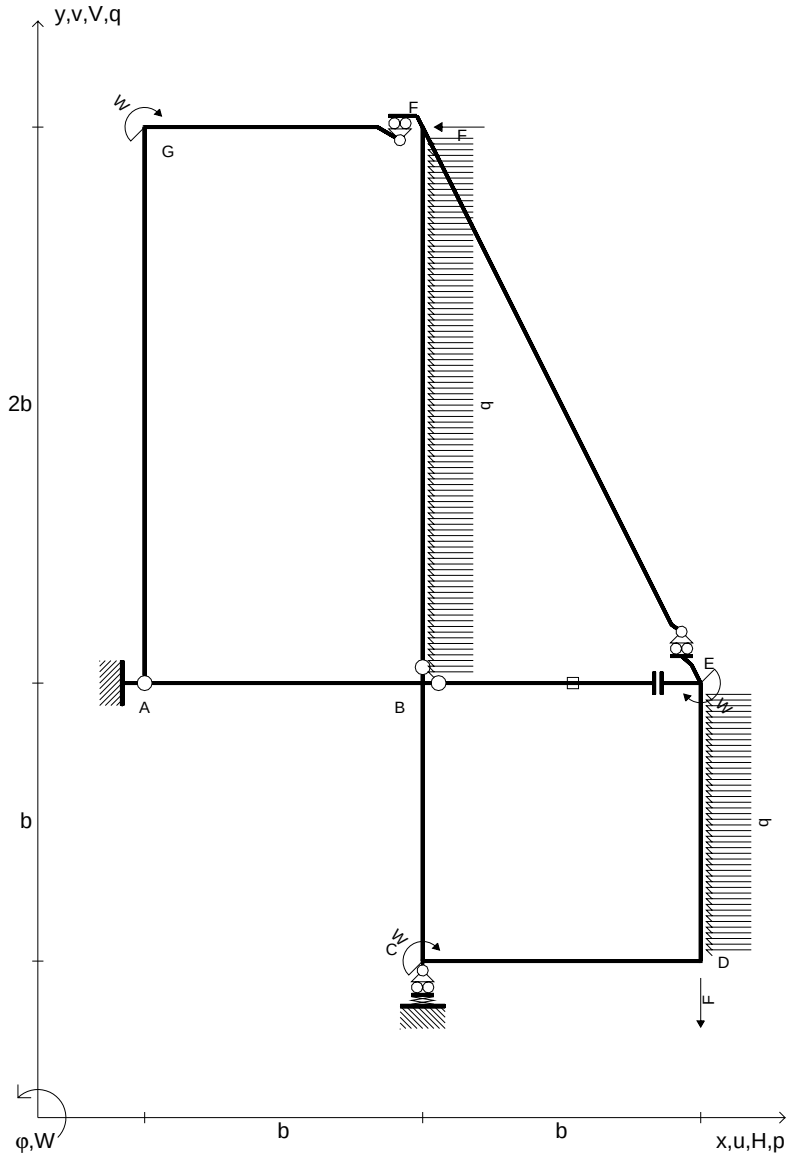
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{FB} = -q = -F/b$
$p_{DE} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



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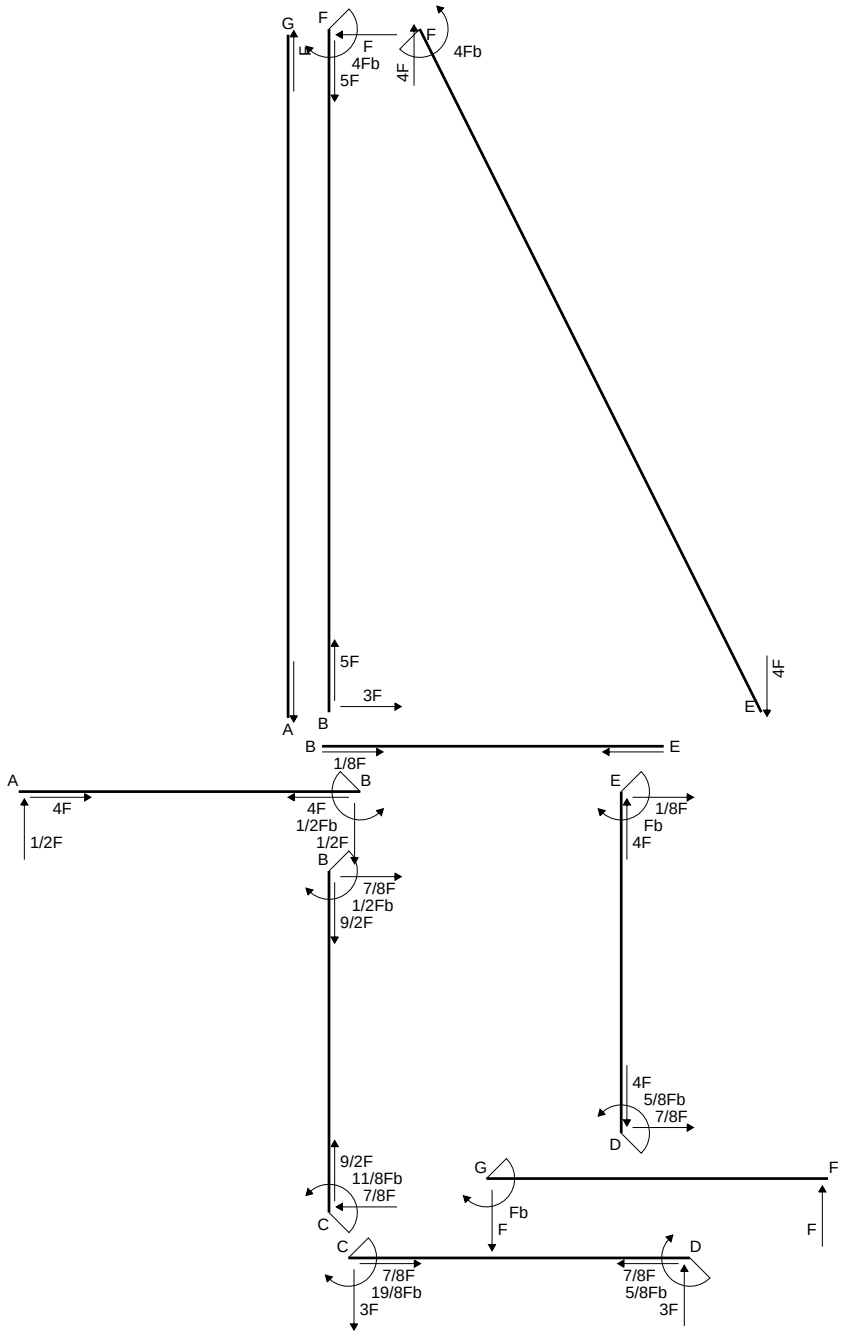
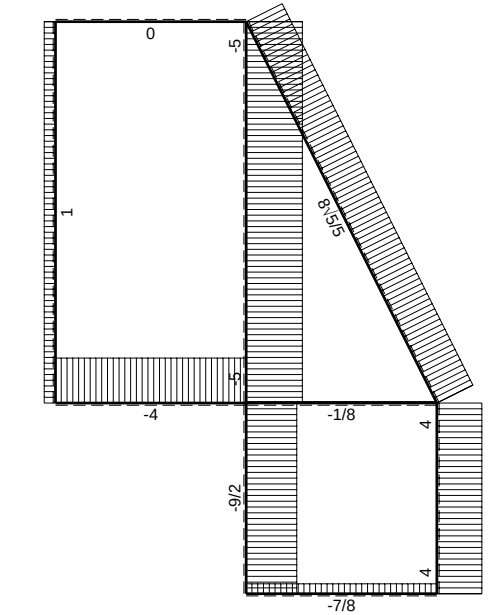
Elongazione termica specifica ε assegnata su asta BE.





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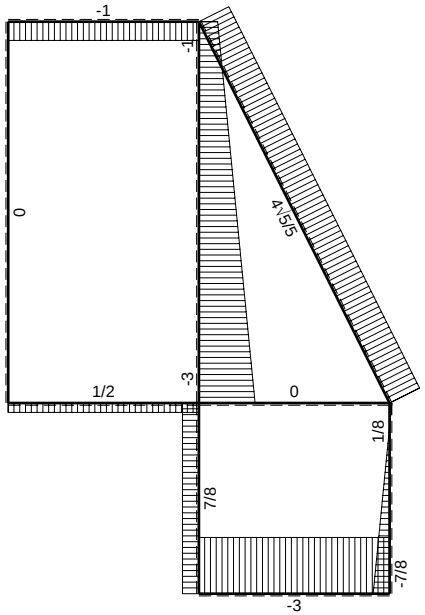
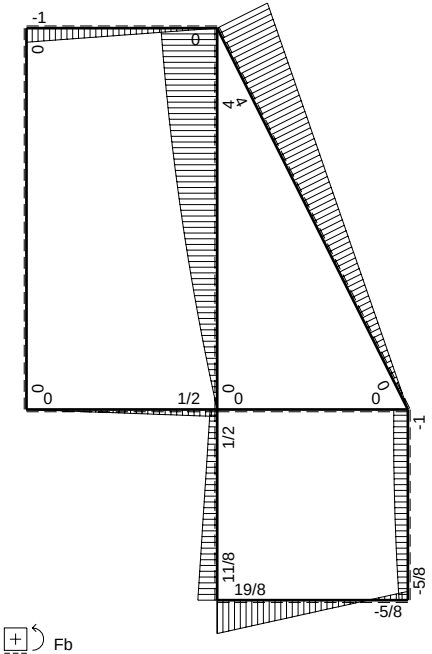
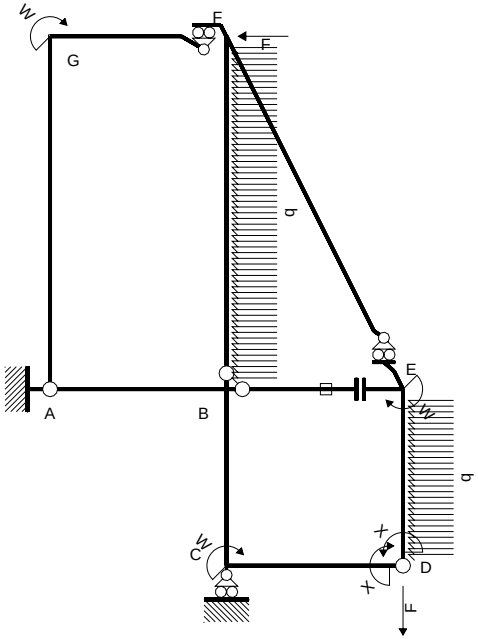


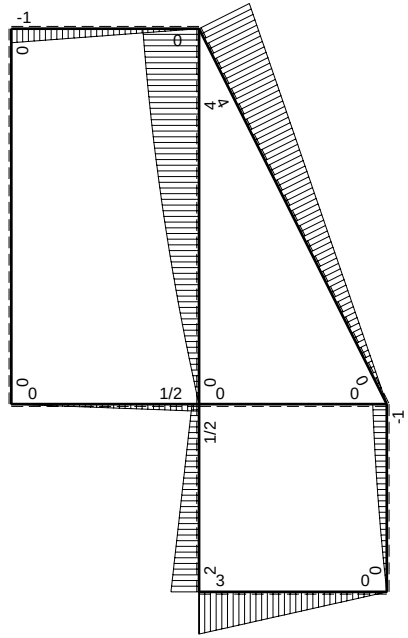
↑□↓ F



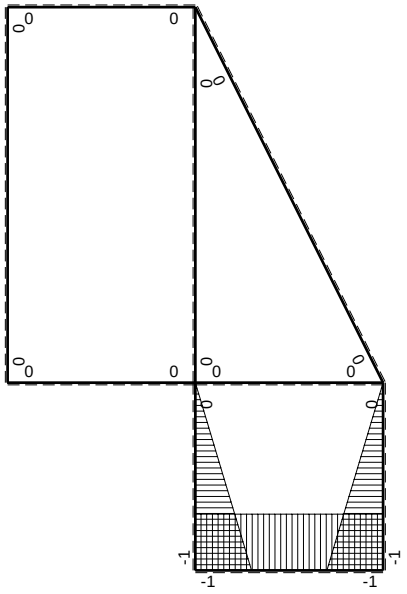
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb+3/2Fx	$-1/2Fx-3/2Fx^2/b$	x^2/b^2	$-3/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-2Fb+3/2Fx	$-2Fb+7/2Fx-3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	3Fb-3Fx	-3Fb+3Fx	1	$-3/2Fb^2/EJ$	Xb/EJ
DC b	1	-3Fx	-3Fx	1		
DE b	-1+x/b	$-3/2Fx+1/2qx^2$	$3/2Fx-2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$5/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2Fx-1/2qx^2$	$Fx-1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$-25/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				5/8Fb	

Sviluppi di calcolo iperstatica

$$L_{BC}^{XX} = \int_o^b \left(x^2/b^2 \right) \, 1/EJ \, dx = \left[1/3 \, x^3/b^2 \right]_o^b \, 1/EJ$$
$$= \left(1/3 \, b \right) \, 1/EJ = 1/3 \, b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1 - 2 \, x/b + x^2/b^2 \right) \, 1/EJ \, dx = \left[x - x^2/b + 1/3 \, x^3/b^2 \right]_o^b \, 1/EJ$$
$$= \left(b - b + 1/3 \, b \right) \, 1/EJ = 1/3 \, b/EJ$$

$$L_{CD}^{XX} = \int_o^b \left(1 \right) \, 1/EJ \, dx = \left[x \right]_o^b \, 1/EJ$$
$$= \left(b \right) \, 1/EJ = b/EJ$$

$$L_{DC}^{XX} = \int_o^b \left(1 \right) \, 1/EJ \, dx = \left[x \right]_o^b \, 1/EJ$$
$$= \left(b \right) \, 1/EJ = b/EJ$$

$$L_{DE}^{XX} = \int_o^b \left(1 - 2 \, x/b + x^2/b^2 \right) \, 1/EJ \, dx = \left[x - x^2/b + 1/3 \, x^3/b^2 \right]_o^b \, 1/EJ$$
$$= \left(b - b + 1/3 \, b \right) \, 1/EJ = 1/3 \, b/EJ$$

$$L_{ED}^{XX} = \int_o^b \left(x^2/b^2 \right) \, 1/EJ \, dx = \left[1/3 \, x^3/b^2 \right]_o^b \, 1/EJ$$
$$= \left(1/3 \, b \right) \, 1/EJ = 1/3 \, b/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-1/2 \, x/b - 3/2 \, x^2/b^2 \right) \, Fb \, 1/EJ \, dx = \left[-1/4 \, x^2/b - 1/2 \, x^3/b^2 \right]_o^b \, Fb \, 1/EJ$$
$$= \left(-1/4 \, b - 1/2 \, b \right) \, Fb \, 1/EJ = -3/4 \, Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-2 + 7/2 \, x/b - 3/2 \, x^2/b^2 \right) \, Fb \, 1/EJ \, dx = \left[-2 \, x + 7/4 \, x^2/b - 1/2 \, x^3/b^2 \right]_o^b \, Fb \, 1/EJ$$
$$= \left(-2 \, b + 7/4 \, b - 1/2 \, b \right) \, Fb \, 1/EJ = -3/4 \, Fb^2/EJ$$

$$L_{CD}^{Xo} = \int_o^b \left(-3 + 3 \, x/b \right) \, Fb \, 1/EJ \, dx = \left[-3 \, x + 3/2 \, x^2/b \right]_o^b \, Fb \, 1/EJ$$
$$= \left(-3 \, b + 3/2 \, b \right) \, Fb \, 1/EJ = -3/2 \, Fb^2/EJ$$

$$L_{DC}^{Xo} = \int_o^b \left(-3 \, x/b \right) \, Fb \, 1/EJ \, dx = \left[-3/2 \, x^2/b \right]_o^b \, Fb \, 1/EJ$$
$$= \left(-3/2 \, b \right) \, Fb \, 1/EJ = -3/2 \, Fb^2/EJ$$

$$L_{DE}^{Xo} = \int_o^b \left(3/2 \, x/b - 2 \, x^2/b^2 + 1/2 \, x^3/b^3 \right) \, Fb \, 1/EJ \, dx = \left[3/4 \, x^2/b - 2/3 \, x^3/b^2 + 1/8 \, x^4/b^3 \right]_o^b \, Fb \, 1/EJ$$
$$= \left(3/4 \, b - 2/3 \, b + 1/8 \, b \right) \, Fb \, 1/EJ = 5/24 \, Fb^2/EJ$$

$$L_{ED}^{Xo} = \int_o^b \left(x/b - 1/2 \, x^2/b^2 - 1/2 \, x^3/b^3 \right) \, Fb \, 1/EJ \, dx = \left[1/2 \, x^2/b - 1/6 \, x^3/b^2 - 1/8 \, x^4/b^3 \right]_o^b \, Fb \, 1/EJ$$
$$= \left(1/2 \, b - 1/6 \, b - 1/8 \, b \right) \, Fb \, 1/EJ = 5/24 \, Fb^2/EJ$$

ESΣ02.xxxx.082

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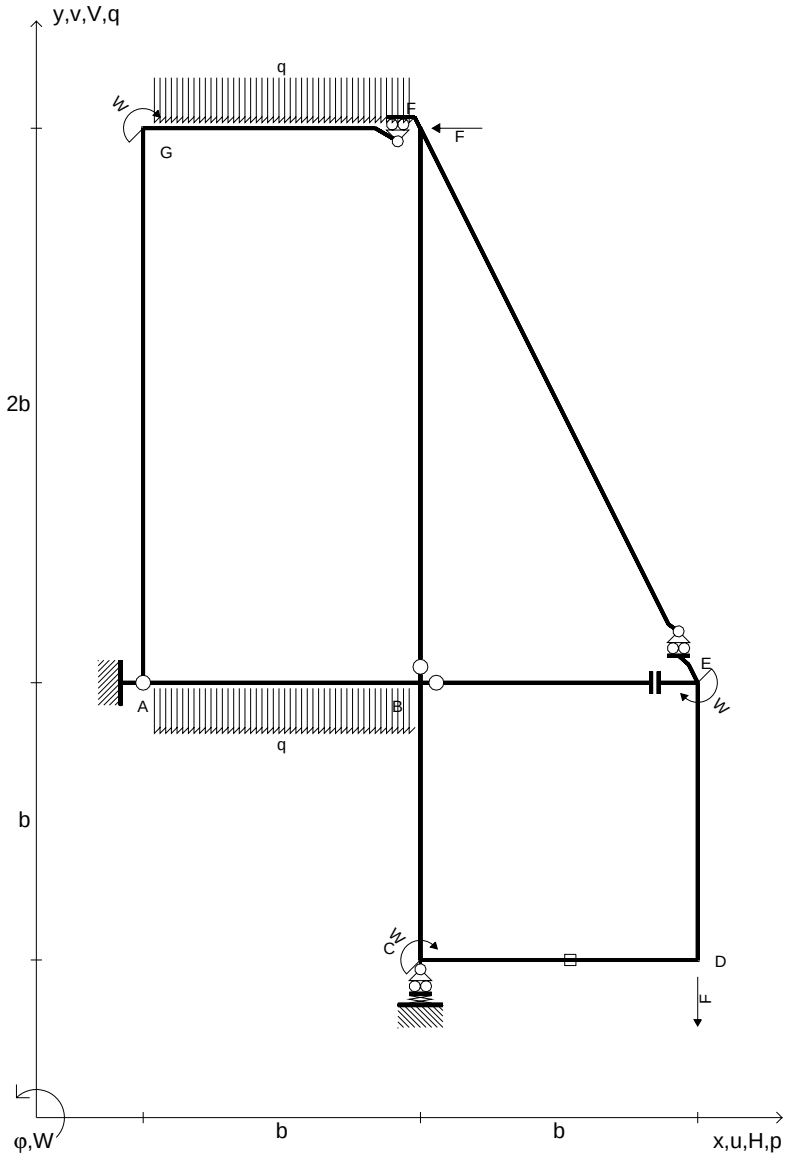
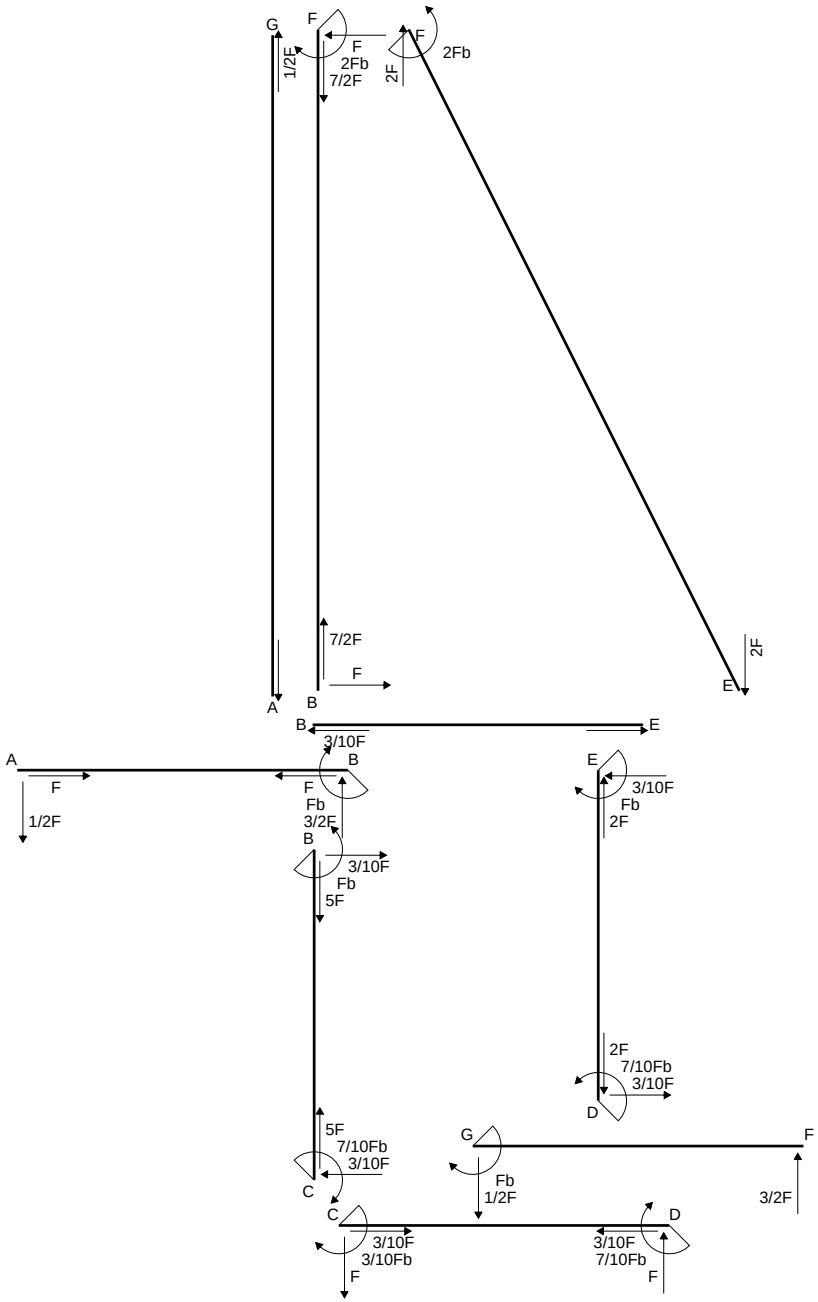
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$q_{FG} = -q = -F/b$
$q_{AB} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

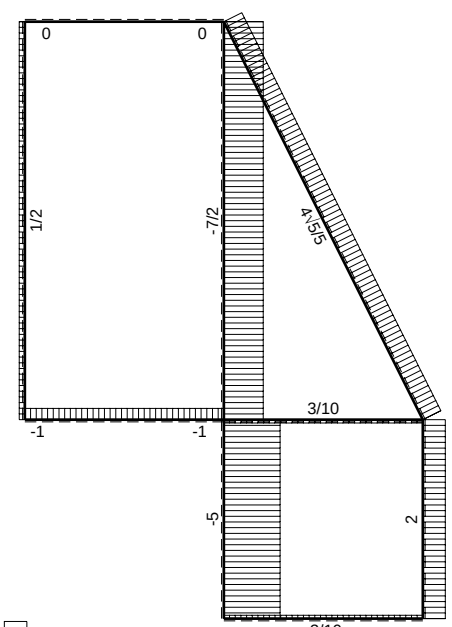


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.

 08.12.24



 08.12.24



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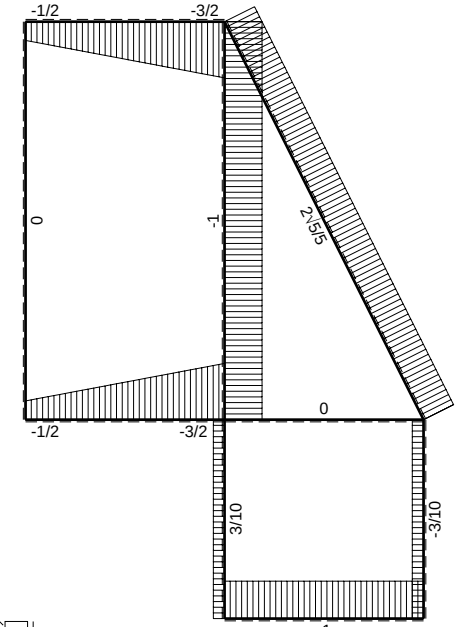

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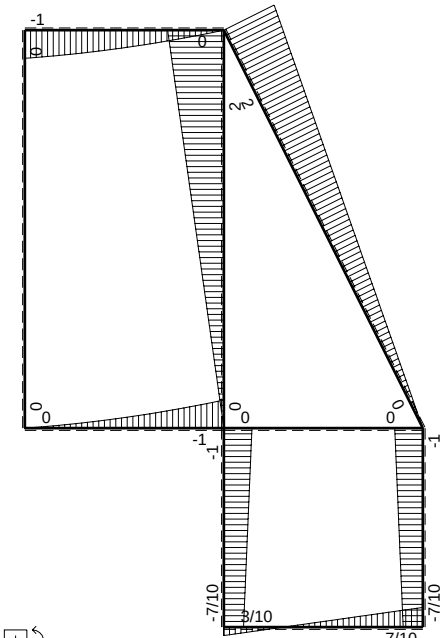
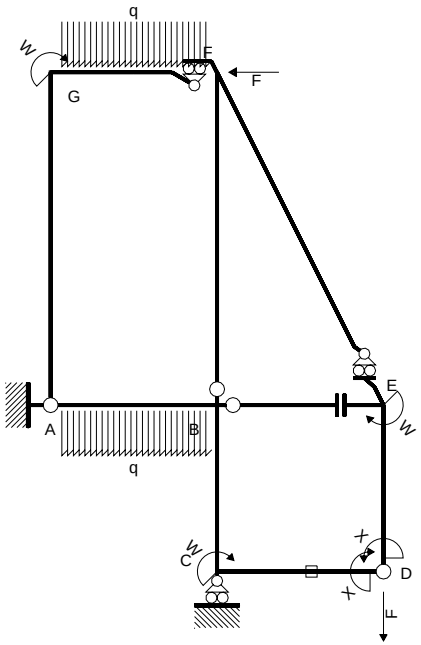
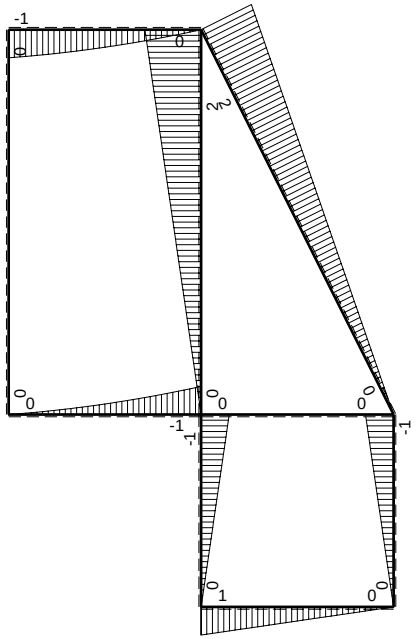


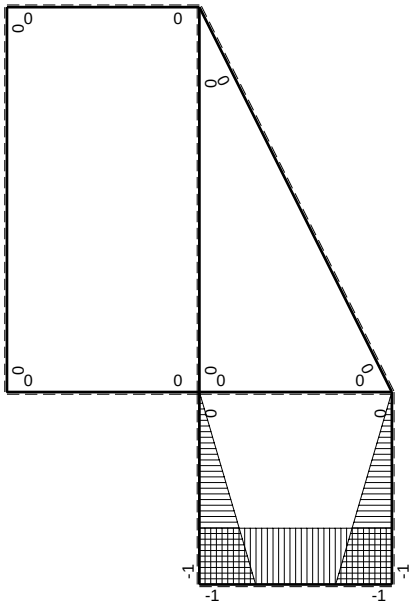
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$Fb-3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	-Fb+Fx	$Fx-Fx^2/b$	x^2/b^2	$1/6Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	Fb-Fx	-Fb+Fx	1	$-1/2Fb^2/EJ$	Xb/EJ
DC b	1	-Fx	-Fx	1		
DE b	-1+x/b	-Fx	$Fx-Fx^2/b$	$1-2x/b+x^2/b^2$	$1/6Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	Fb-Fx	$Fx-Fx^2/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-3/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$Fb-1/2Fx-1/2qx^2$	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-7/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				7/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212423 ESΣ02.xxxx.093

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b (x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [- x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -3/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ +\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -3/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (x/b - x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/3\ b\ )\ Fb\ 1/EJ\ = 1/6\ \ Fb^2/EJ$

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ESΣ02.xxxx.093

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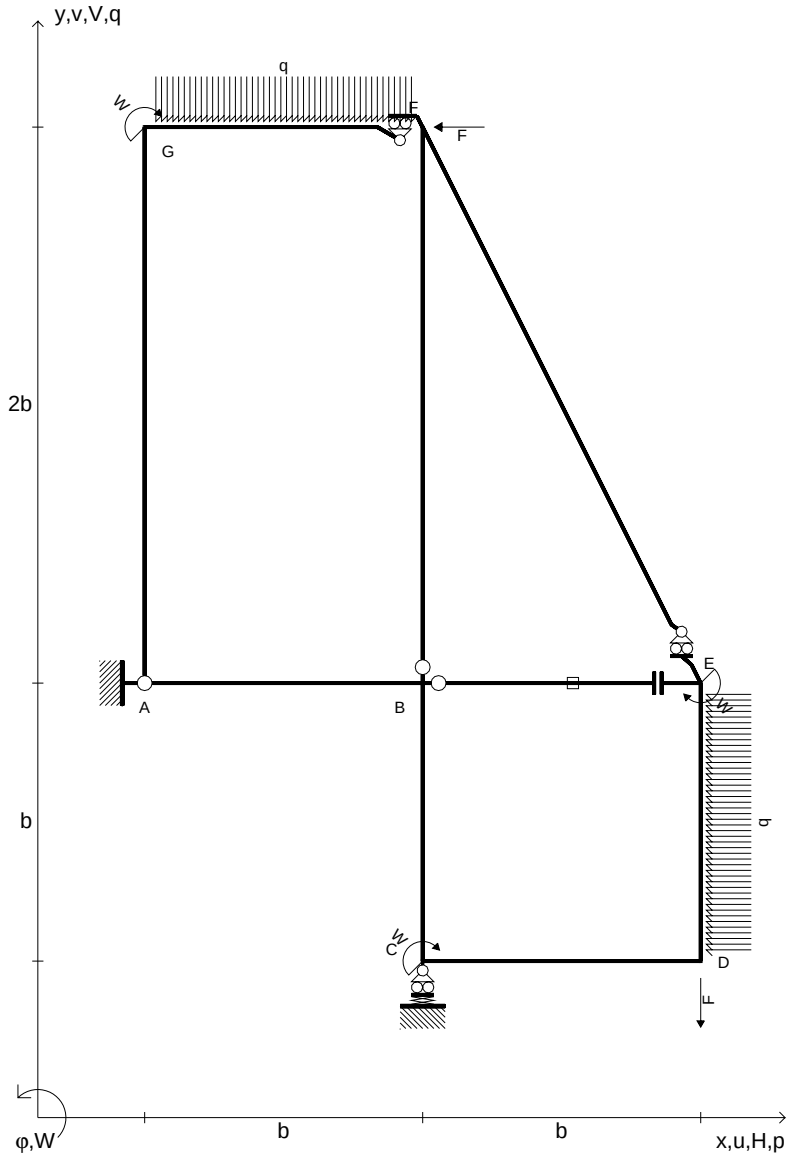
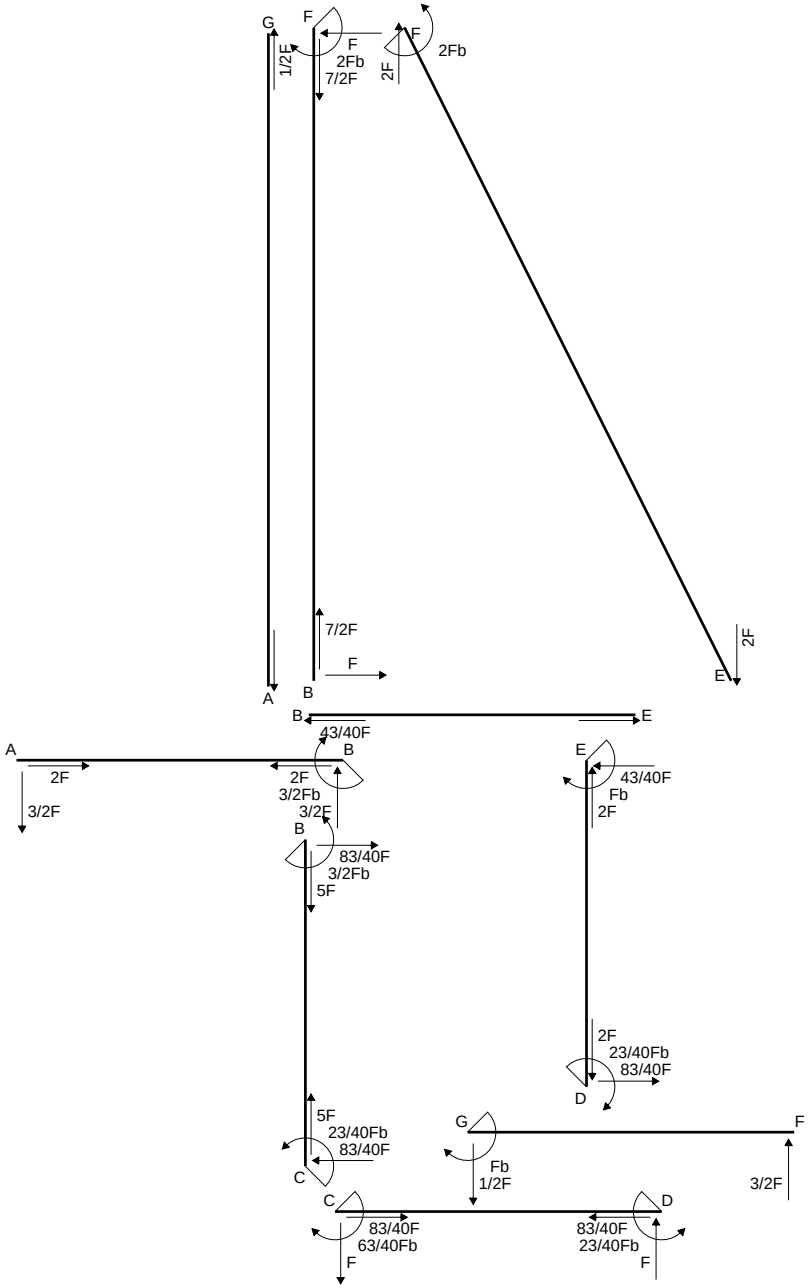
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$q_{FG} = -q = -F/b$
$p_{DE} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



Reazioni iperstatiche in soluzione: $X=W_{CD}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.

 08.12.24



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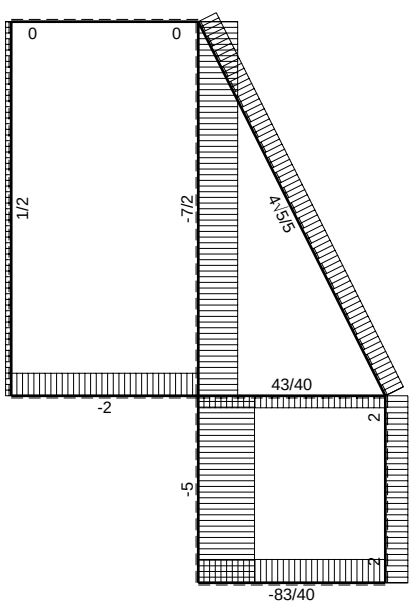
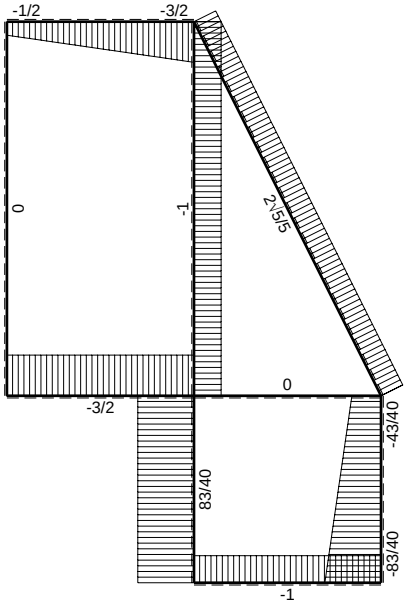
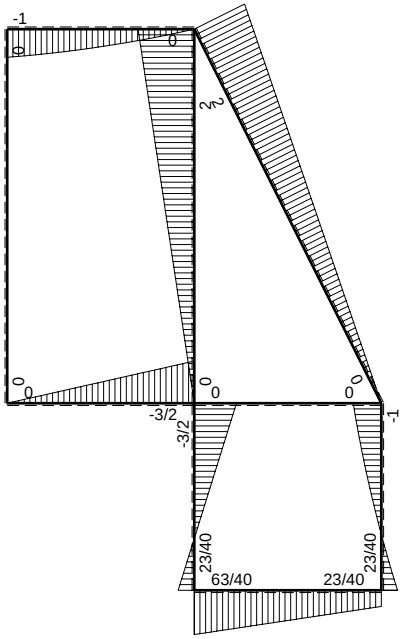


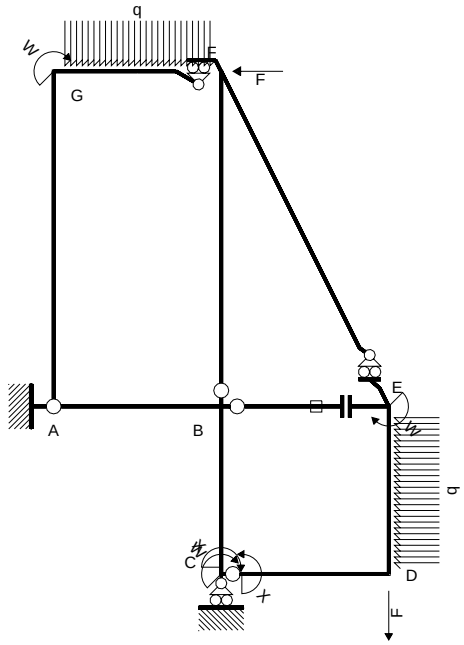
←□→ F



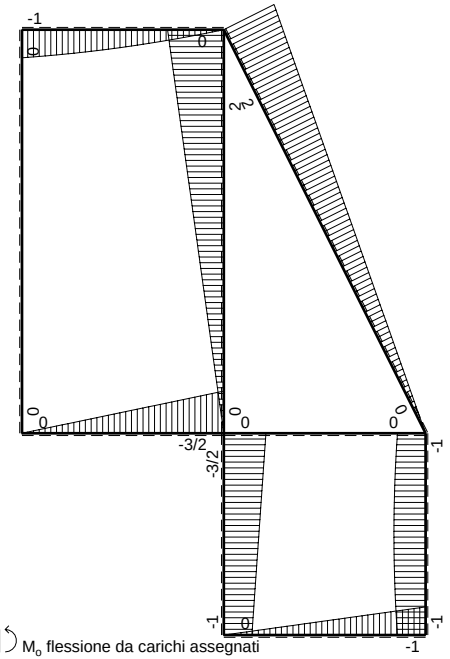
↑□↓ F



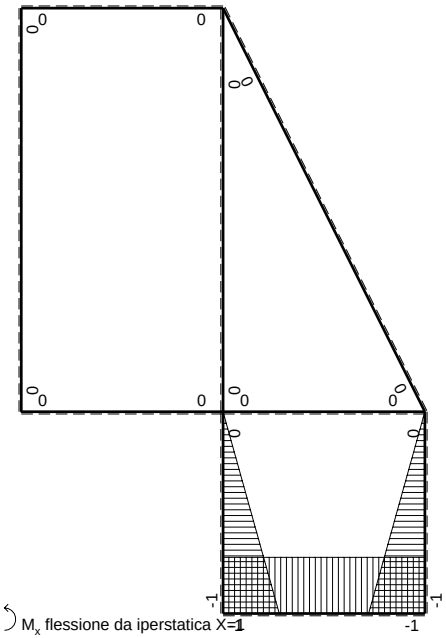
□ Fb



Schema di calcolo iperstatico



□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=-1

Quadro contributi PLV per iperstatica $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0		
BC b	-x/b	-3/2Fb+1/2Fx	3/2Fx-1/2Fx^2/b	x^2/b^2	7/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	Fb+1/2Fx	Fb-1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	-Fx	Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	1	Fb-Fx	Fb-Fx	1		
DE b	-1+x/b	-Fb-1/2Fx+1/2qx^2	Fb-1/2Fx-Fx^2/b+1/2qx^3/b	1-2x/b+x^2/b^2	13/24Fb^2/EJ	1/3Xb/EJ
ED b	x/b	Fb+1/2Fx-1/2qx^2	Fx+1/2Fx^2/b-1/2qx^3/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-3/2Fx+1/2qx^2	0	0	0	0
GF b	0	Fb-1/2Fx-1/2qx^2	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				21/8Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{CD}$				-63/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\ x/b\ -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b\ -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b\ -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1\ -1/2\ x/b\ -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [\ x\ -1/4\ x^2/b\ -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ -1/4\ b\ -1/6\ b\ )\ Fb\ 1/EJ\ = 7/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1\ -\ x/b\ )\ Fb\ 1/EJ\ dx = [\ x\ -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ -1/2\ b\ )\ Fb\ 1/EJ\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1\ -1/2\ x/b\ -\ x^2/b^2\ +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [\ x\ -1/4\ x^2/b\ -1/3\ x^3/b^2\ +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (\ b\ -1/4\ b\ -1/3\ b\ +1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (\ x/b\ +1/2\ x^2/b^2\ -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b\ +1/6\ x^3/b^2\ -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b\ +1/6\ b\ -1/8\ b\ )\ Fb\ 1/EJ\ = 13/24\ Fb^2/EJ$

ESΣ02.xxxx.096

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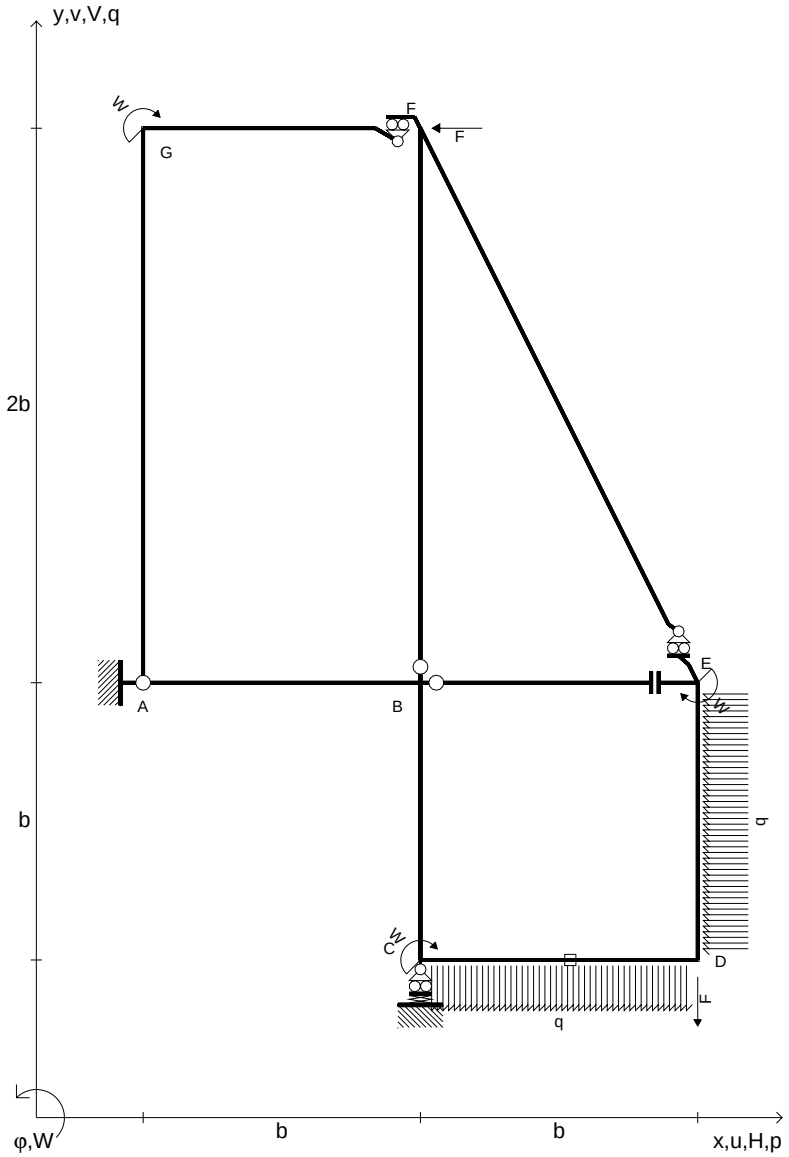
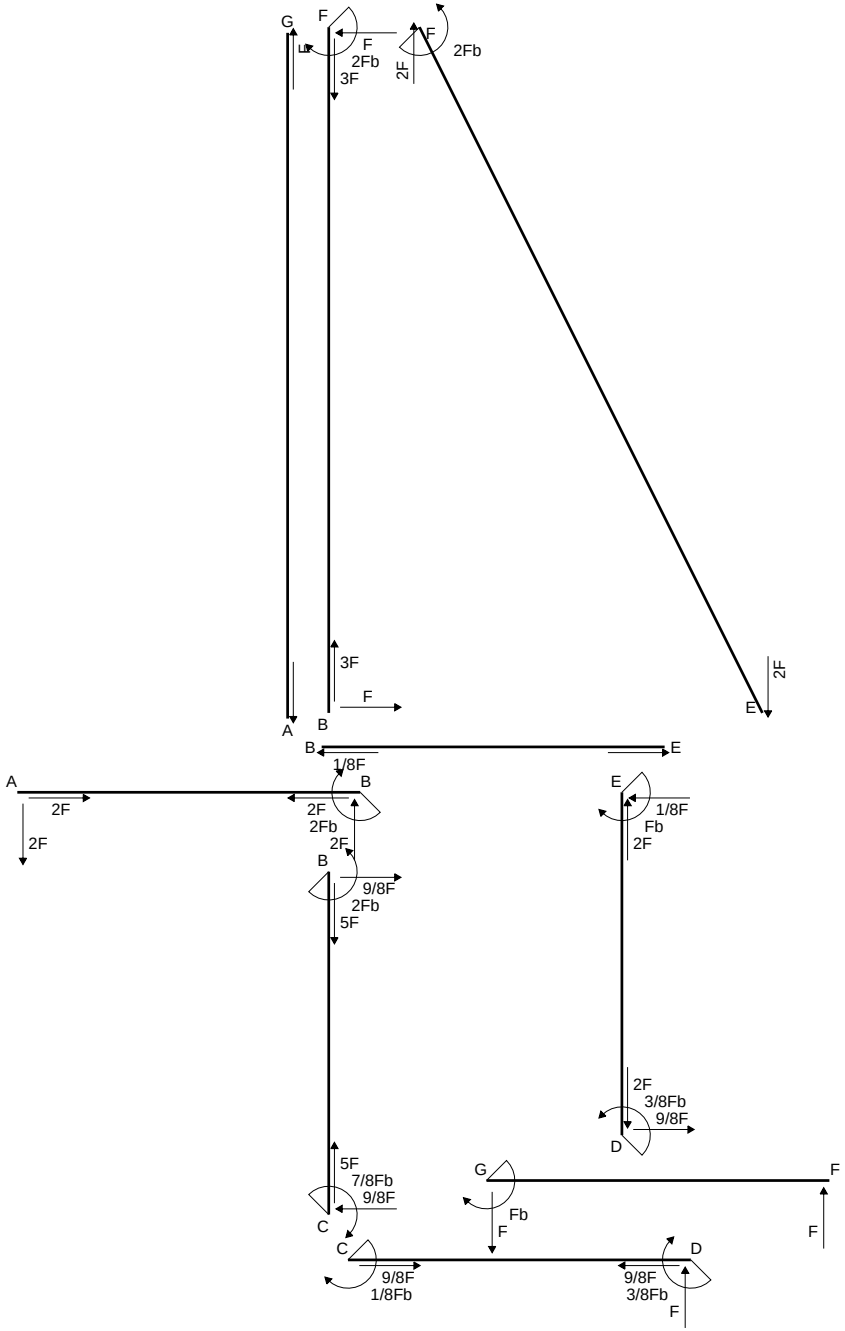
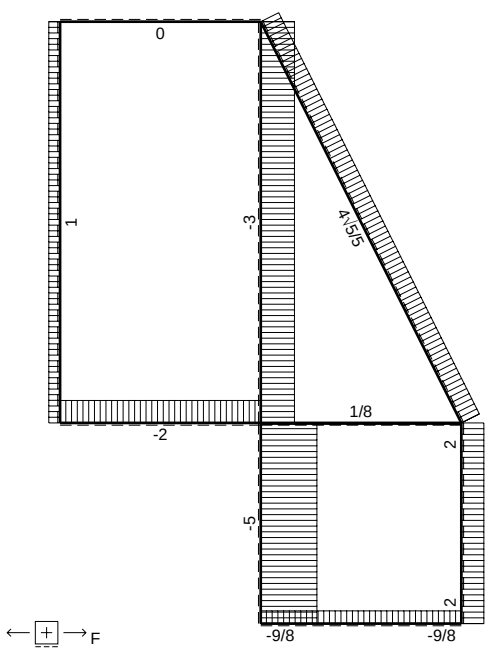
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{DE} = -q = -F/b$
$q_{CD} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



Reazioni iperstatiche in soluzione: $X=W_{DC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.

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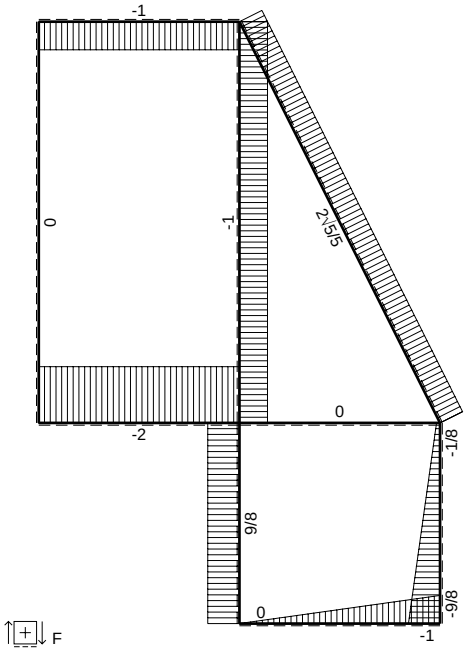
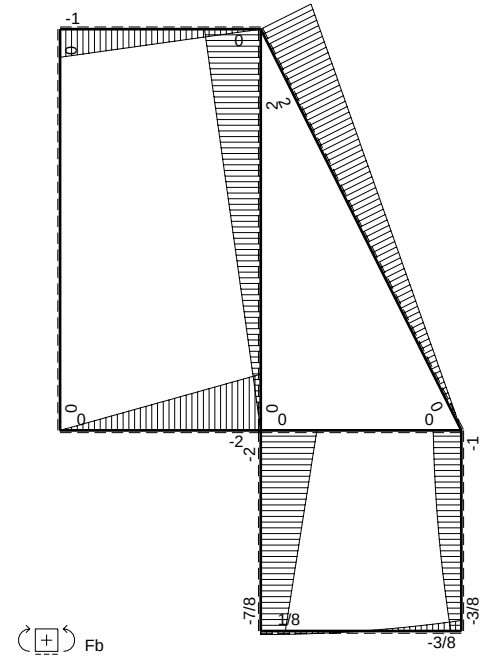
1
-3
4√5/5
1/8
-2
2
-5
2
-9/8
-9/8
F


-1
0
-1
2√5/5
0
-2
-1/8
9/8
0
-9/8
-1
F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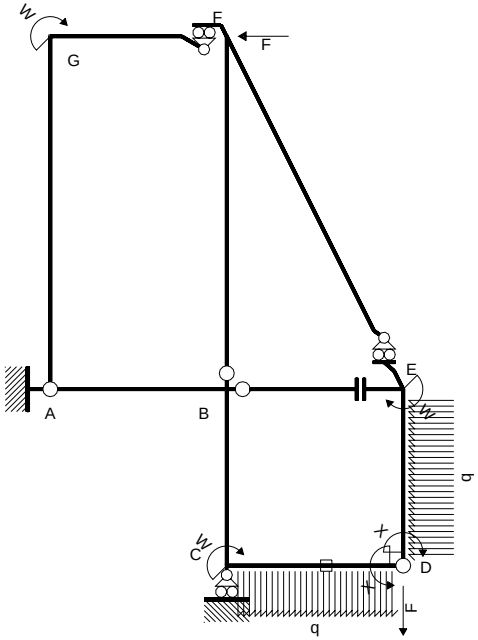
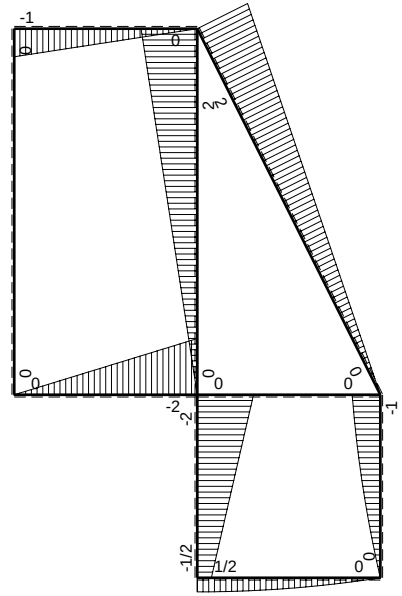
0
0
0
-2
-2
-1
-7/8
1/8
-3/8
-3/8
Fb


G
F
E
A
B
E
q
C
D
F
q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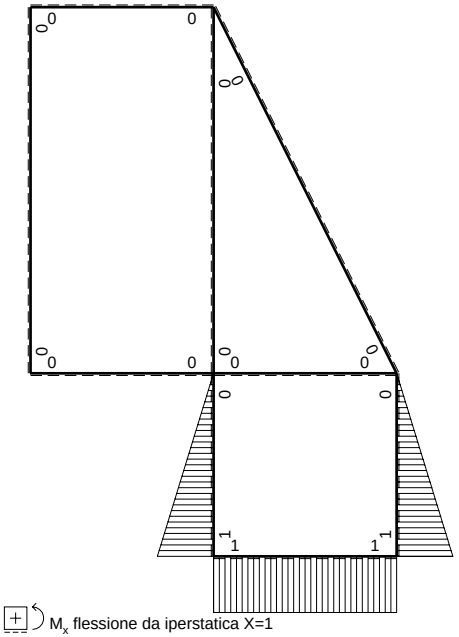
-2
-2
-1
-1/2
1/2
0
M_o flessione da carichi assegnati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0
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0
0
1
1
1
1
M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DC}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	x/b	-2Fb+3/2Fx	$-2Fx+3/2Fx^2/b$	x^2/b^2	$-1/2Fb^2/EJ$	1/3Xb/EJ
CB b	-1+x/b	1/2Fb+3/2Fx	$-1/2Fb-Fx+3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	1	$1/2Fb-1/2qx^2$	$1/2Fb-1/2Fx^2/b$	1	$1/3Fb^2/EJ$	Xb/EJ
DC b	-1	$-Fx+1/2qx^2$	$Fx-1/2Fx^2/b$	1		
DE b	1-x/b	$-3/2Fx+1/2qx^2$	$-3/2Fx+2Fx^2/b-1/2qx^3/b$	$1-2x/b+x^2/b^2$	$-5/24Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$Fb-1/2Fx-1/2qx^2$	$-Fx+1/2Fx^2/b+1/2qx^3/b$	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				Fb^2/EJ	
	totali				$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DC}$				-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-2\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (-1/2 -\ x/b +3/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x -1/2\ x^2/b +1/2\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b -1/2\ b +1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 4/3\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 4/3\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-3/2\ x/b +2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b +2/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b +2/3\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (-\ x/b +1/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/6\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ \ Fb^2/EJ$

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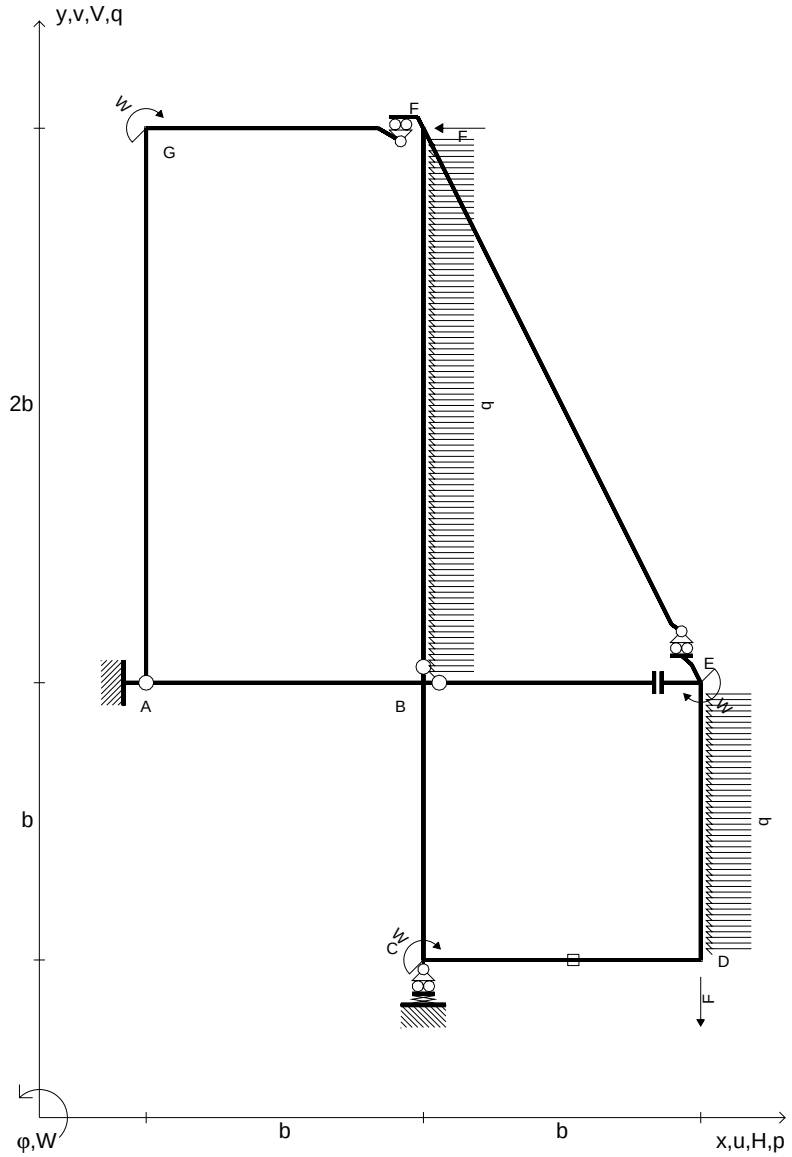
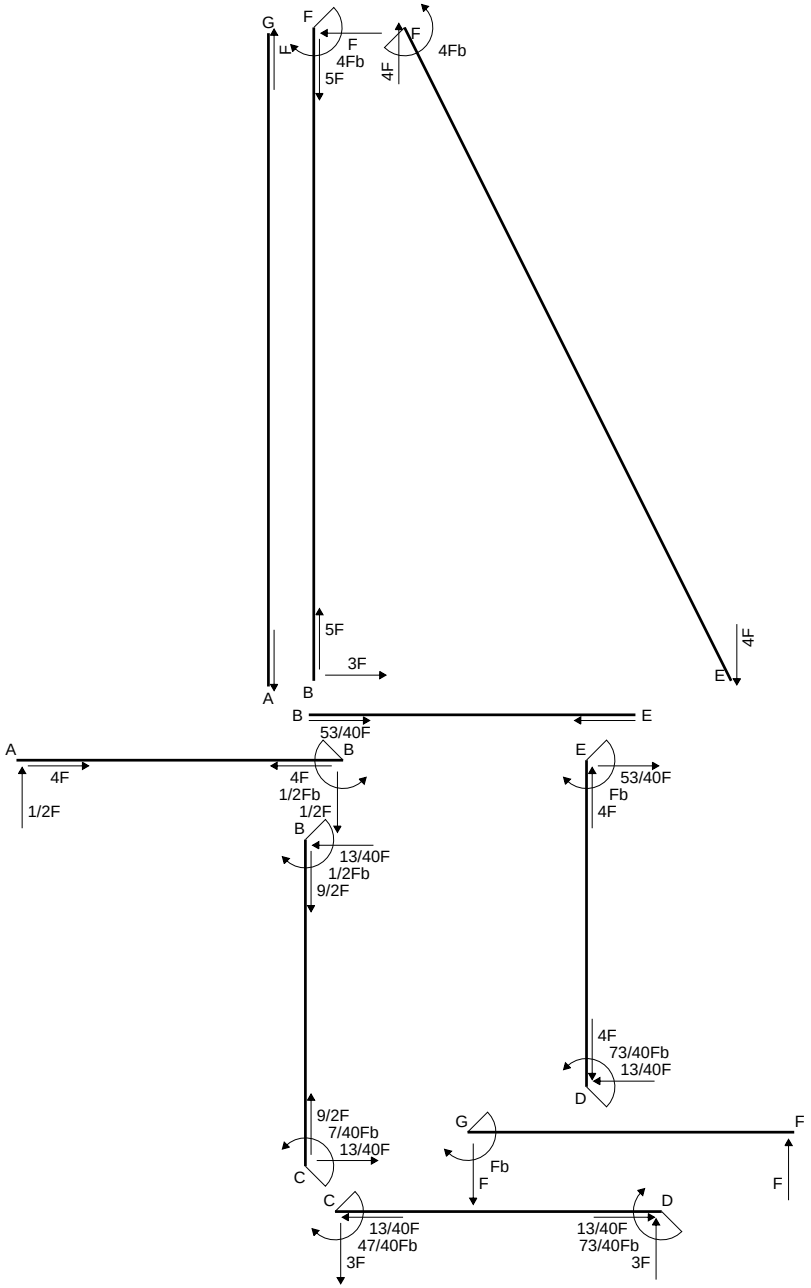
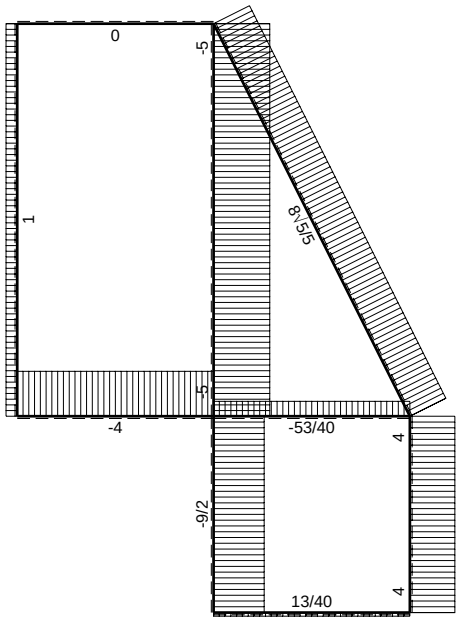
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{DE} = -q = -F/b$
$p_{FB} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

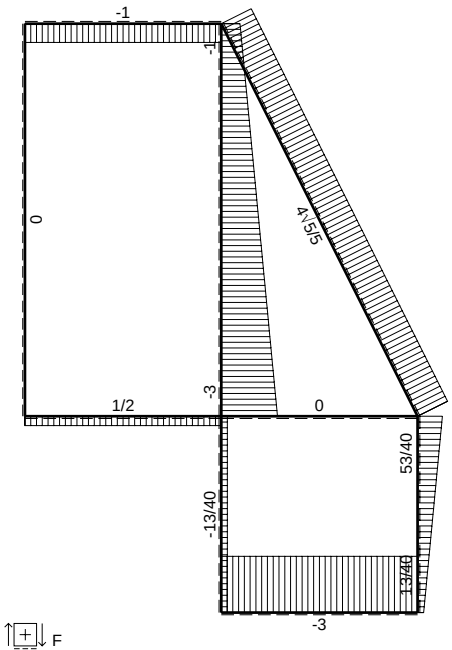


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.





F



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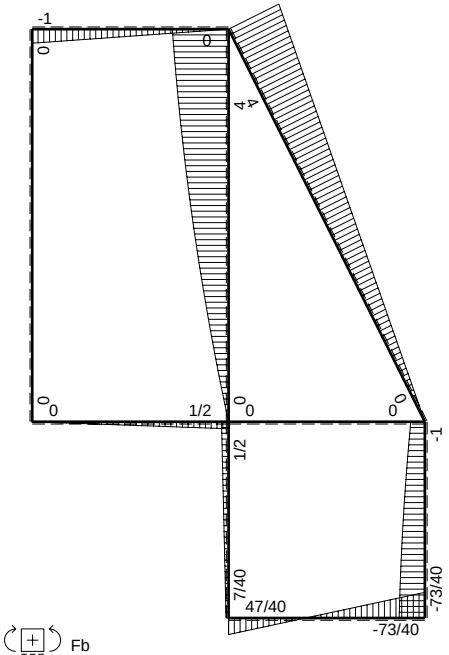
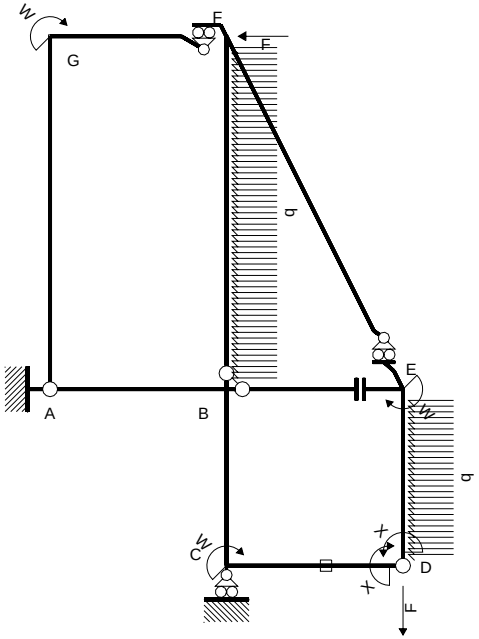
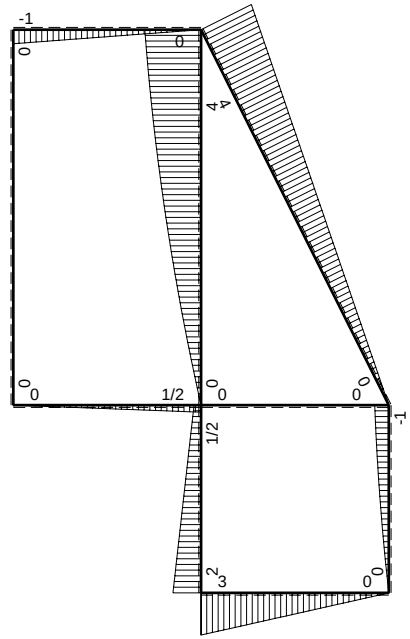


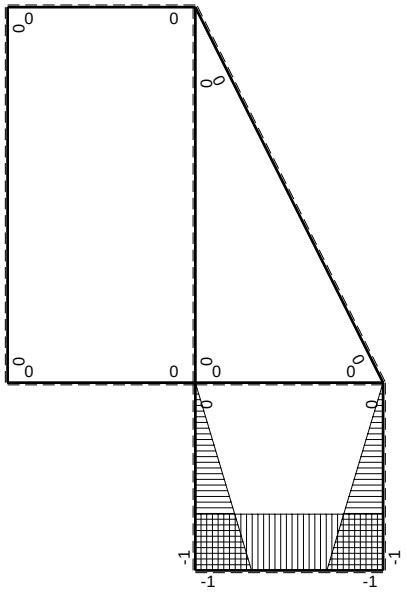
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx	0	0	0	0
BA b	0	-1/2Fb+1/2Fx	0	0		
BC b	-x/b	1/2Fb+3/2Fx	$-1/2Fx-3/2Fx^2/b$	x^2/b^2	$-3/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-2Fb+3/2Fx	$-2Fb+7/2Fx-3/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	3Fb-3Fx	-3Fb+3Fx	1	$-3/2Fb^2/EJ$	Xb/EJ
DC b	1	-3Fx	-3Fx	1		
DE b	-1+x/b	$-3/2Fx+1/2qx^2$	$3/2Fx-2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$5/24Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	$Fb-1/2Fx-1/2qx^2$	$Fx-1/2Fx^2/b-1/2qx^3/b$	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-73/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				73/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213396 ESΣ02.xxxx.108

$$L_{BC}^{XX} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L_{CD}^{XX} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L_{DC}^{XX} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L_{DE}^{XX} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L_{ED}^{XX} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-1/2 \; x/b - 3/2 \; x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[-1/4 \; x^2/b - 1/2 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-1/4 \; b - 1/2 \; b \right) \; Fb \; 1/EJ \; = -3/4 \; \; Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-2 + 7/2 \; x/b - 3/2 \; x^2/b^2 \right) \; Fb \; 1/EJ \; dx = \left[-2 \; x + 7/4 \; x^2/b - 1/2 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-2 \; b + 7/4 \; b - 1/2 \; b \right) \; Fb \; 1/EJ \; = -3/4 \; \; Fb^2/EJ$$

$$L_{CD}^{Xo} = \int_o^b \left(-3 + 3 \; x/b \right) \; Fb \; 1/EJ \; dx + 1 \;\; (-1) \;\; 1 \;\; Fb^2/EJ = \left[-3 \; x + 3/2 \; x^2/b \right]_o^b \; Fb \; 1/EJ \; + 1 \;\; (-1) \;\; 1 \;\; Fb^2/EJ$$
$$= \left(-3 \; b + 3/2 \; b \right) \; Fb \; 1/EJ \; + 1 \;\; (-1) \;\; 1 \;\; Fb^2/EJ = -5/2 \; \; Fb^2/EJ$$

$$L_{DC}^{Xo} = \int_o^b \left(-3 \; x/b \right) \; Fb \; 1/EJ \; dx + 1 \;\; (-1) \;\; 1 \;\; Fb^2/EJ = \left[-3/2 \; x^2/b \right]_o^b \; Fb \; 1/EJ \; + 1 \;\; (-1) \;\; 1 \;\; Fb^2/EJ$$
$$= \left(-3/2 \; b \right) \; Fb \; 1/EJ \; + 1 \;\; (-1) \;\; 1 \;\; Fb^2/EJ = -5/2 \; \; Fb^2/EJ$$

$$L_{DE}^{Xo} = \int_o^b \left(3/2 \; x/b - 2 \; x^2/b^2 + 1/2 \; x^3/b^3 \right) \; Fb \; 1/EJ \; dx = \left[3/4 \; x^2/b - 2/3 \; x^3/b^2 + 1/8 \; x^4/b^3 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(3/4 \; b - 2/3 \; b + 1/8 \; b \right) \; Fb \; 1/EJ \; = 5/24 \; \; Fb^2/EJ$$

$$L_{ED}^{Xo} = \int_o^b \left(x/b - 1/2 \; x^2/b^2 - 1/2 \; x^3/b^3 \right) \; Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b - 1/6 \; x^3/b^2 - 1/8 \; x^4/b^3 \right]_o^b \; Fb \; 1/EJ$$
$$= \left(1/2 \; b - 1/6 \; b - 1/8 \; b \right) \; Fb \; 1/EJ \; = 5/24 \; \; Fb^2/EJ$$

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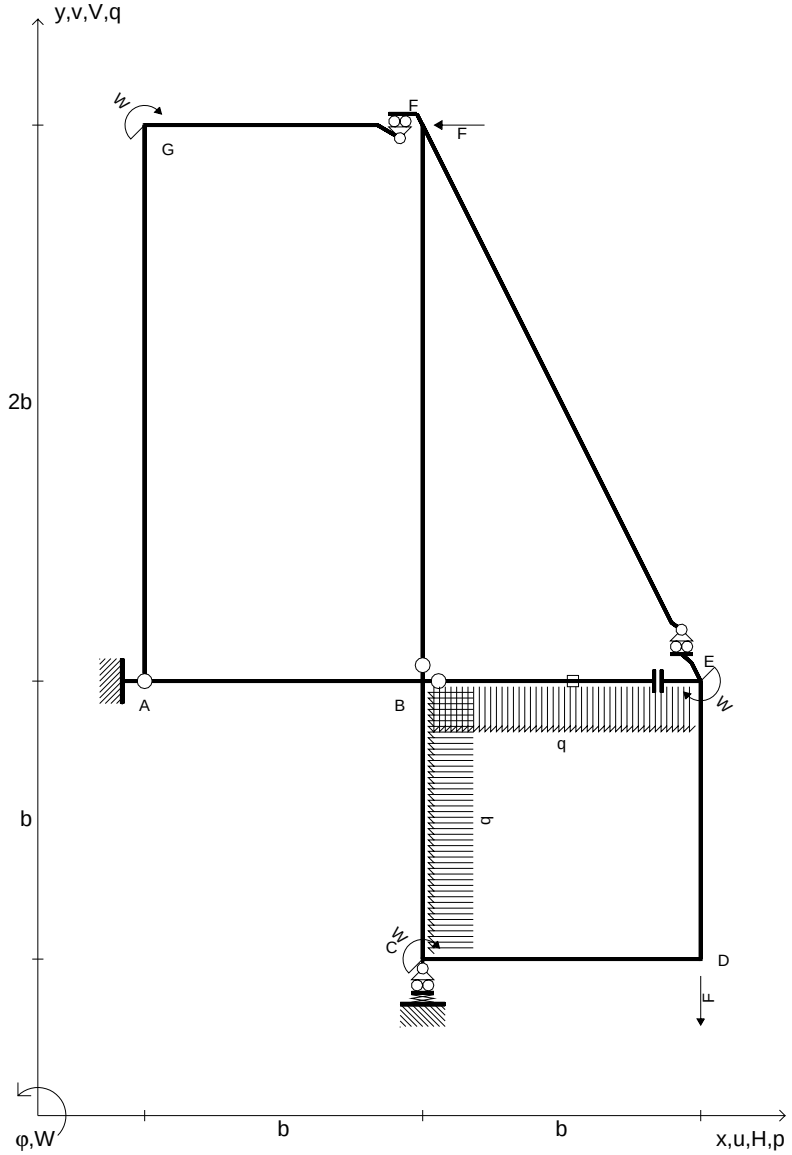
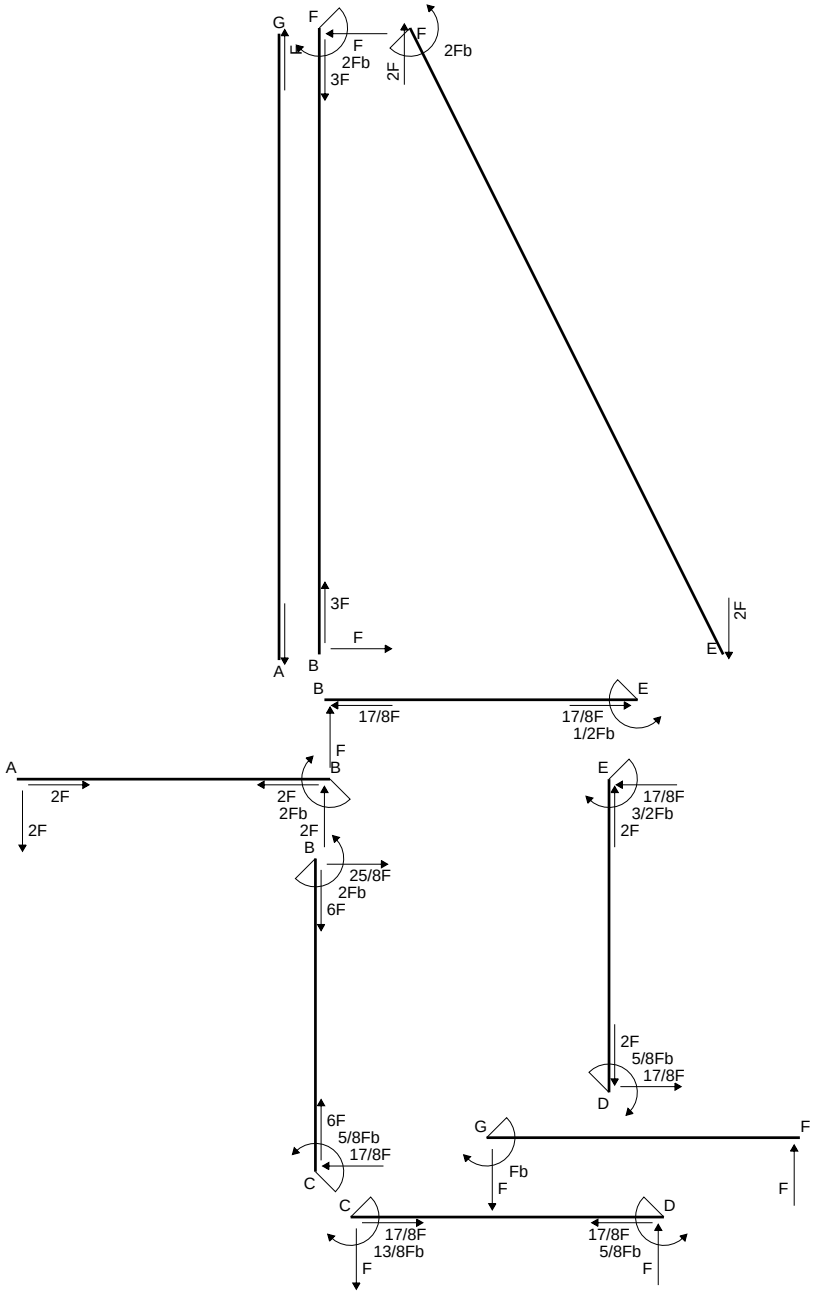
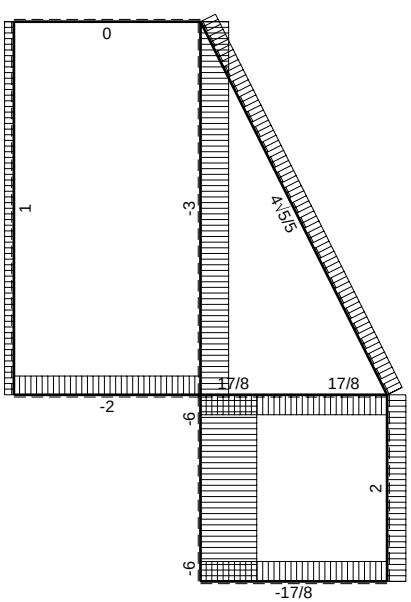
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{BC} = -q = -F/b$
$q_{BE} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

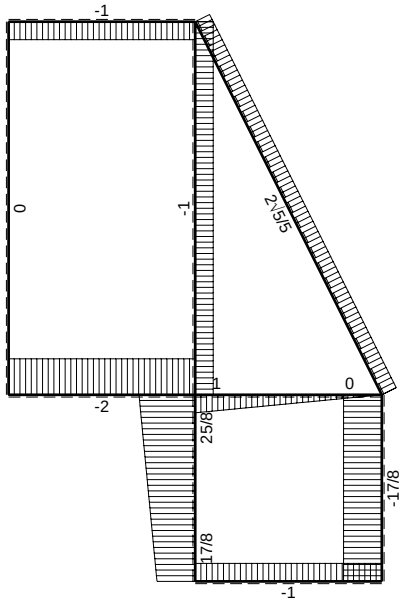


Reazioni iperstatiche in soluzione: $X=H_{BE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.

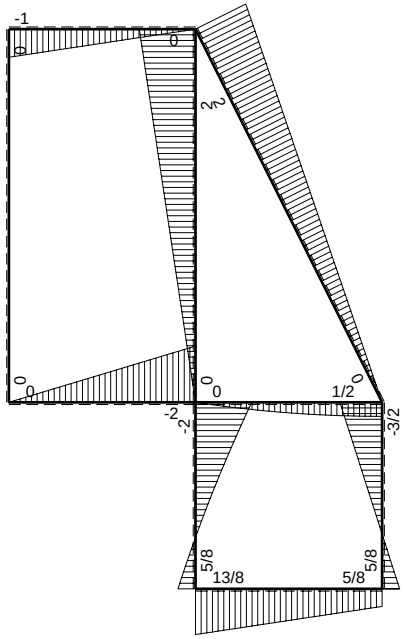




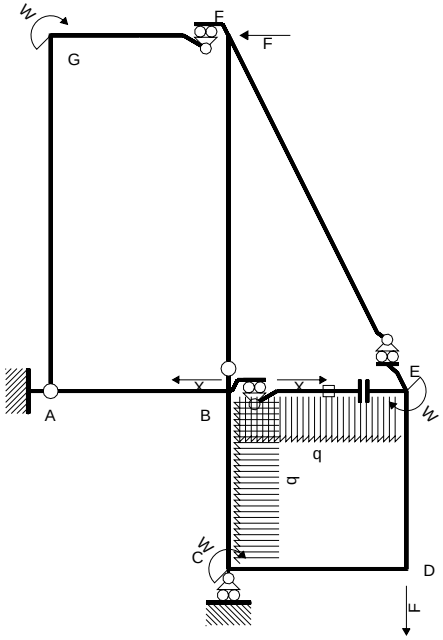
←[+]→ F



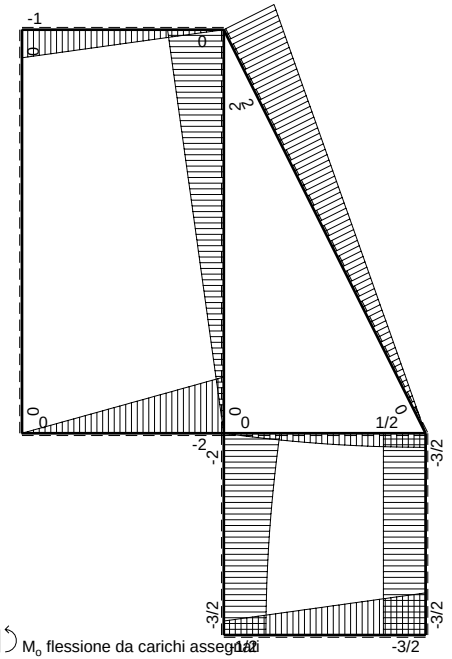
↑[+]↓ F



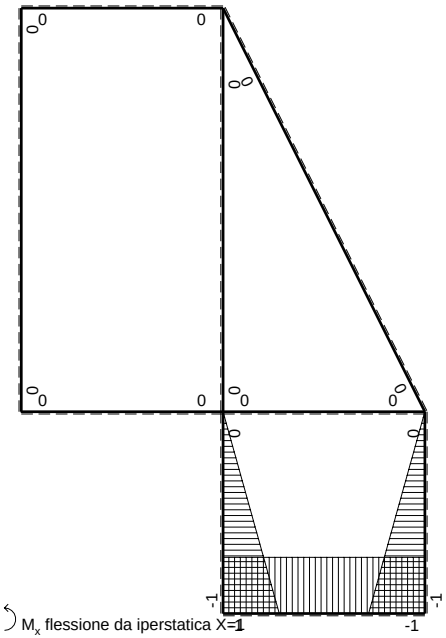
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_{BE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x	$-2Fb+Fx-1/2qx^2$	$2Fbx-Fx^2+1/2qx^3$	x^2	$19/24Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	b-x	$3/2Fb+1/2qx^2$	$3/2Fb^2-3/2Fbx+1/2Fx^2-1/2qx^3$	$b^2-2bx+x^2$		
CD b	-b	-1/2Fb-Fx	$1/2Fb^2+Fbx$	b^2	Fb^3/EJ	Xb^3/EJ
DC b	b	3/2Fb-Fx	$3/2Fb^2-Fbx$	b^2		
DE b	-b+x	-3/2Fb	$3/2Fb^2-3/2Fbx$	$b^2-2bx+x^2$	$3/4Fb^3/EJ$	$1/3Xb^3/EJ$
ED b	x	3/2Fb	3/2Fbx	x^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^3/EJ	
	totali				$85/24Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_{BE}$				-17/8F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211522 ESΣ02.xxxx.111

$$L_{BC}^{XX} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$$
$$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{CB}^{XX} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$$
$$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{CD}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x]_o^b\, b^2\, 1/EJ$$
$$= (\, b)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{DC}^{XX} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x]_o^b\, b^2\, 1/EJ$$
$$= (\, b)\, b^2\, 1/EJ = b^3/EJ$$

$$L_{DE}^{XX} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$$
$$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{ED}^{XX} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$$
$$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L_{BC}^{Xo} = \int_o^b (2\, x/b - x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [\, x^2/b -1/3\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$$
$$= (\, b -1/3\, b +1/8\, b)\, Fb^2\, 1/EJ = 19/24\ Fb^3/EJ$$

$$L_{CB}^{Xo} = \int_o^b (3/2 -3/2\, x/b +1/2\, x^2/b^2 -1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$$
$$= [3/2\, x -3/4\, x^2/b +1/6\, x^3/b^2 -1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$$
$$= (3/2\, b -3/4\, b +1/6\, b -1/8\, b)\, Fb^2\, 1/EJ = 19/24\ Fb^3/EJ$$

$$L_{CD}^{Xo} = \int_o^b (1/2 + x/b)\, Fb^2\, 1/EJ\, dx = [1/2\, x +1/2\, x^2/b]_o^b\, Fb^2\, 1/EJ$$
$$= (1/2\, b +1/2\, b)\, Fb^2\, 1/EJ = Fb^3/EJ$$

$$L_{DC}^{Xo} = \int_o^b (3/2 - x/b)\, Fb^2\, 1/EJ\, dx = [3/2\, x -1/2\, x^2/b]_o^b\, Fb^2\, 1/EJ$$
$$= (3/2\, b -1/2\, b)\, Fb^2\, 1/EJ = Fb^3/EJ$$

$$L_{DE}^{Xo} = \int_o^b (3/2 -3/2\, x/b)\, Fb^2\, 1/EJ\, dx = [3/2\, x -3/4\, x^2/b]_o^b\, Fb^2\, 1/EJ$$
$$= (3/2\, b -3/4\, b)\, Fb^2\, 1/EJ = 3/4\ Fb^3/EJ$$

$$L_{ED}^{Xo} = \int_o^b (3/2\, x/b)\, Fb^2\, 1/EJ\, dx = [3/4\, x^2/b]_o^b\, Fb^2\, 1/EJ$$
$$= (3/4\, b)\, Fb^2\, 1/EJ = 3/4\ Fb^3/EJ$$

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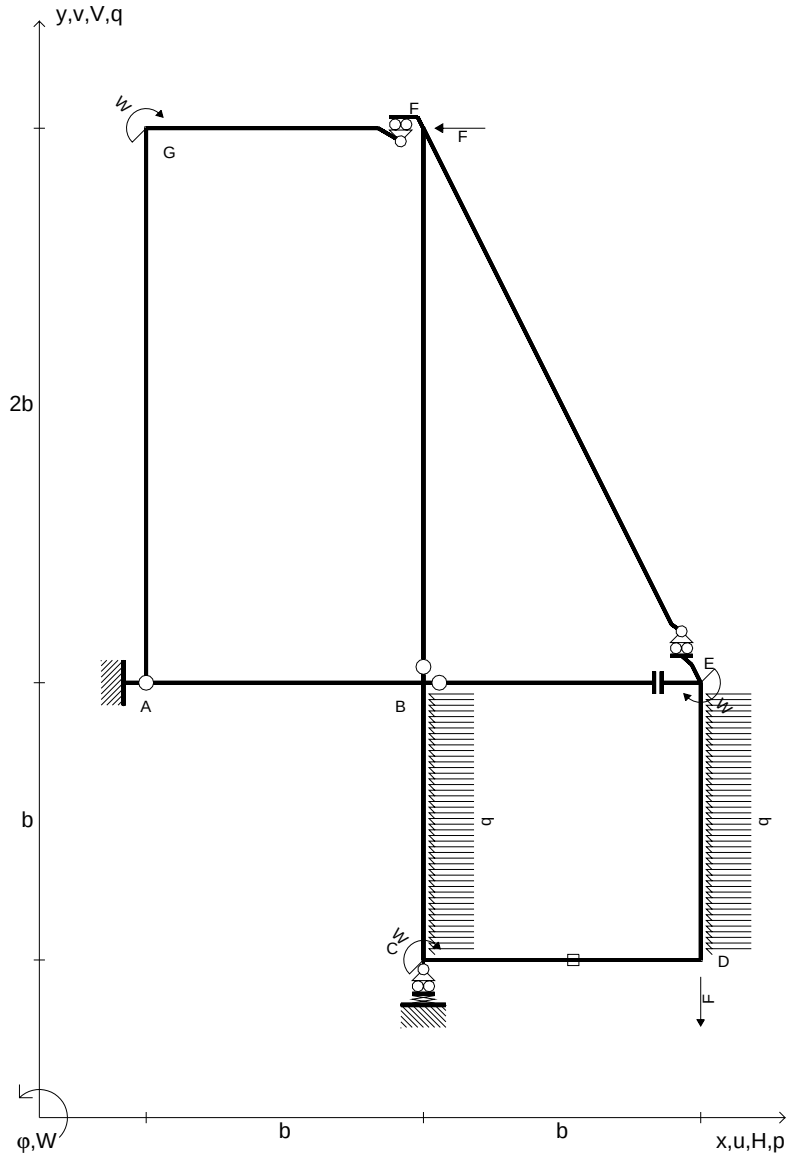
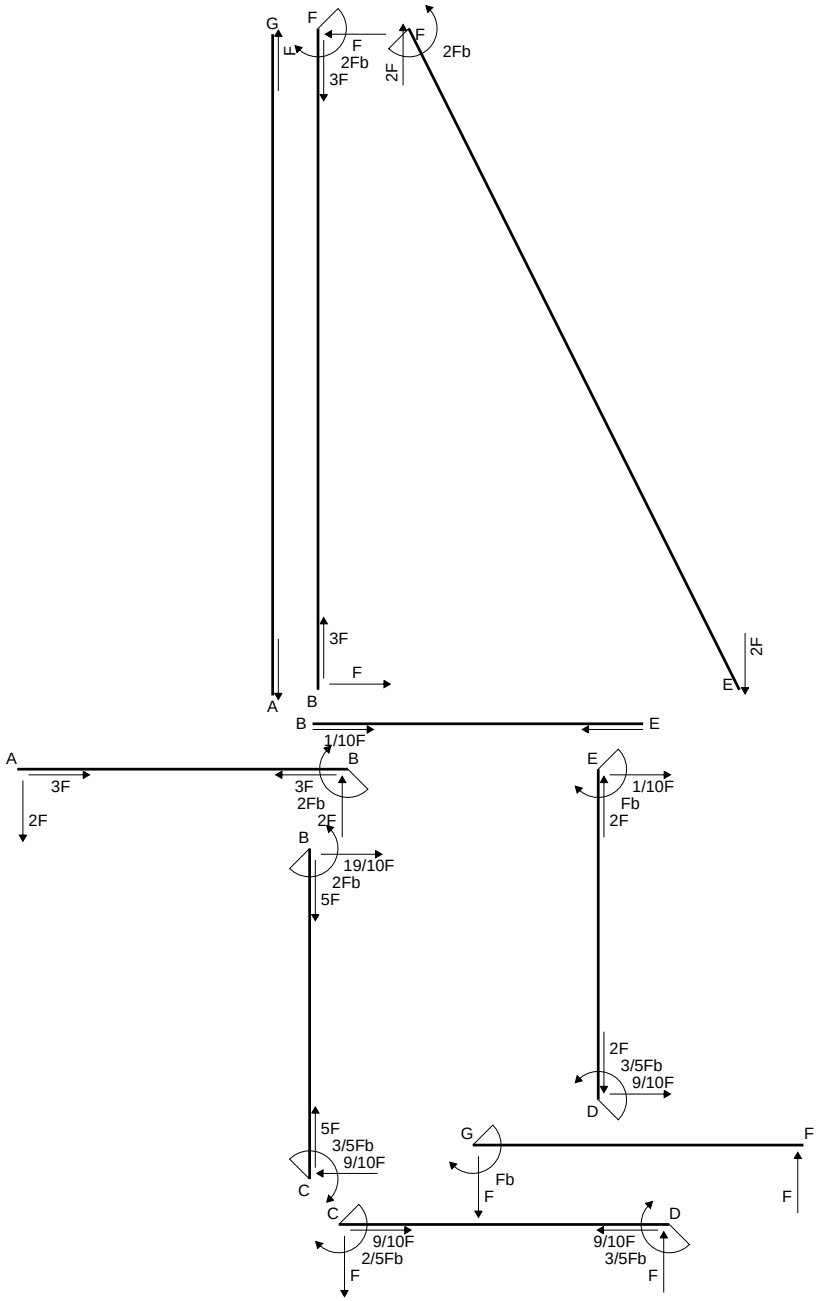
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_E = -W = -Fb$
$p_{BC} = -q = -F/b$
$p_{DE} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



Reazioni iperstatiche in soluzione: $X=H_{BE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.

 08.12.24



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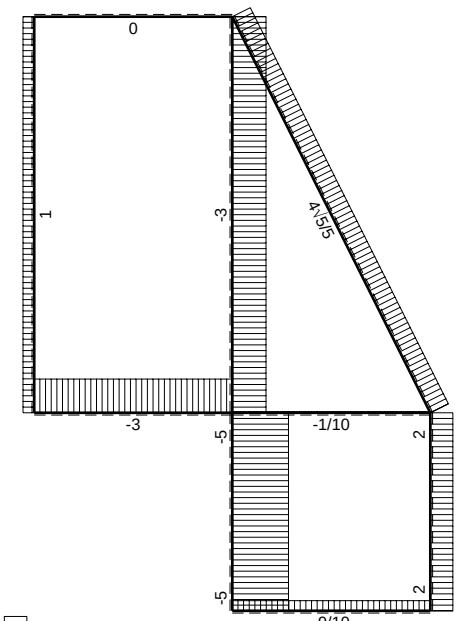

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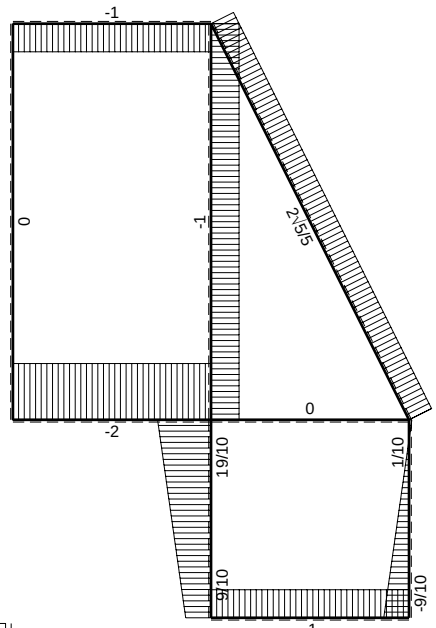

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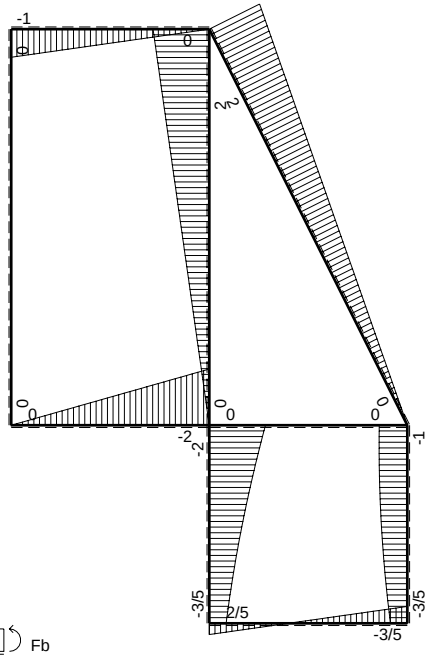
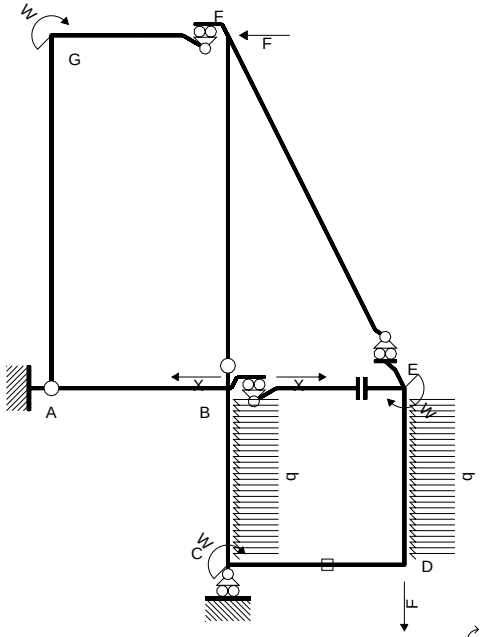
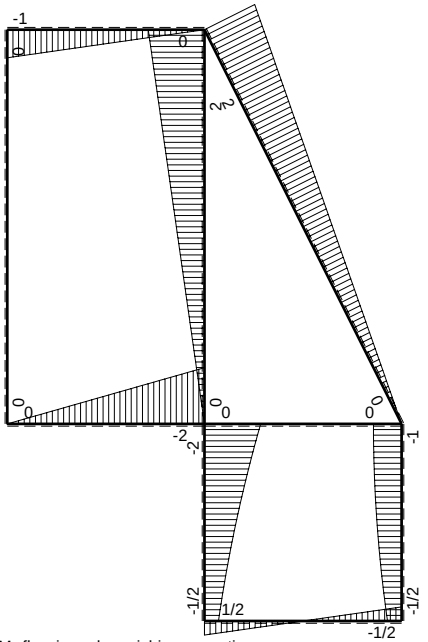


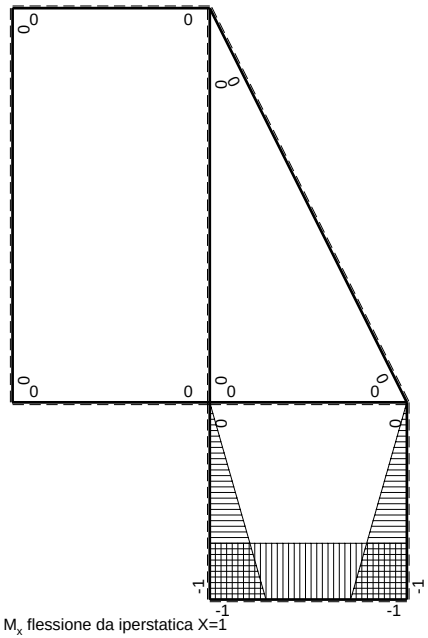
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_{BE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0		
BC b	-x	$-2Fb+2Fx-1/2qx^2$	$2Fbx-2Fx^2+1/2qx^3$	x^2	$11/24Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	b-x	$1/2Fb+Fx+1/2qx^2$	$1/2Fb^2+1/2Fbx-1/2Fx^2-1/2qx^3$	$b^2-2bx+x^2$		
CD b	-b	$1/2Fb-Fx$	$-1/2Fb^2+Fbx$	b^2	0	Xb^3/EJ
DC b	b	$1/2Fb-Fx$	$1/2Fb^2-Fbx$	b^2		
DE b	-b+x	$-1/2Fb-Fx+1/2qx^2$	$1/2Fb^2+1/2Fbx-3/2Fx^2+1/2qx^3$	$b^2-2bx+x^2$	$3/8Fb^3/EJ$	$1/3Xb^3/EJ$
ED b	x	$Fb-1/2qx^2$	$Fbx-1/2qx^3$	x^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0		
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^3/EJ$	
	totali				$-1/6Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_{BE}$				1/10F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 956427 ESΣ02.xxxx.122

$$L_{BC}^{XX} = \int_o^b \left(x^2/b^2 \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2 \right]_o^b b^2\ 1/EJ$$
$$= \left(1/3\ b \right) b^2\ 1/EJ\ = 1/3\ b^3/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2 \right) b^2\ 1/EJ\ dx = \left[x - x^2/b + 1/3\ x^3/b^2 \right]_o^b b^2\ 1/EJ$$
$$= \left(b - b + 1/3\ b \right) b^2\ 1/EJ\ = 1/3\ b^3/EJ$$

$$L_{CD}^{XX} = \int_o^b \left(1 \right) b^2\ 1/EJ\ dx = \left[x \right]_o^b b^2\ 1/EJ$$
$$= \left(b \right) b^2\ 1/EJ\ = b^3/EJ$$

$$L_{DC}^{XX} = \int_o^b \left(1 \right) b^2\ 1/EJ\ dx = \left[x \right]_o^b b^2\ 1/EJ$$
$$= \left(b \right) b^2\ 1/EJ\ = b^3/EJ$$

$$L_{DE}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2 \right) b^2\ 1/EJ\ dx = \left[x - x^2/b + 1/3\ x^3/b^2 \right]_o^b b^2\ 1/EJ$$
$$= \left(b - b + 1/3\ b \right) b^2\ 1/EJ\ = 1/3\ b^3/EJ$$

$$L_{ED}^{XX} = \int_o^b \left(x^2/b^2 \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2 \right]_o^b b^2\ 1/EJ$$
$$= \left(1/3\ b \right) b^2\ 1/EJ\ = 1/3\ b^3/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(2\ x/b - 2\ x^2/b^2 + 1/2\ x^3/b^3 \right) Fb^2\ 1/EJ\ dx = \left[x^2/b - 2/3\ x^3/b^2 + 1/8\ x^4/b^3 \right]_o^b Fb^2\ 1/EJ$$
$$= \left(b - 2/3\ b + 1/8\ b \right) Fb^2\ 1/EJ\ = 11/24\ Fb^3/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(1/2 + 1/2\ x/b - 1/2\ x^2/b^2 - 1/2\ x^3/b^3 \right) Fb^2\ 1/EJ\ dx$$
$$= \left[1/2\ x + 1/4\ x^2/b - 1/6\ x^3/b^2 - 1/8\ x^4/b^3 \right]_o^b Fb^2\ 1/EJ$$
$$= \left(1/2\ b + 1/4\ b - 1/6\ b - 1/8\ b \right) Fb^2\ 1/EJ\ = 11/24\ Fb^3/EJ$$

$$L_{CD}^{Xo} = \int_o^b \left(-1/2 + x/b \right) Fb^2\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ$$
$$= \left[-1/2\ x + 1/2\ x^2/b \right]_o^b Fb^2\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ$$
$$= \left(-1/2\ b + 1/2\ b \right) Fb^2\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ = -\ Fb^3/EJ$$

$$L_{DC}^{Xo} = \int_o^b \left(1/2 - x/b \right) Fb^2\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ = \left[1/2\ x - 1/2\ x^2/b \right]_o^b Fb^2\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ$$
$$= \left(1/2\ b - 1/2\ b \right) Fb^2\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ = -\ Fb^3/EJ$$

$$L_{DE}^{Xo} = \int_o^b \left(1/2 + 1/2\ x/b - 3/2\ x^2/b^2 + 1/2\ x^3/b^3 \right) Fb^2\ 1/EJ\ dx$$
$$= \left[1/2\ x + 1/4\ x^2/b - 1/2\ x^3/b^2 + 1/8\ x^4/b^3 \right]_o^b Fb^2\ 1/EJ$$
$$= \left(1/2\ b + 1/4\ b - 1/2\ b + 1/8\ b \right) Fb^2\ 1/EJ\ = 3/8\ Fb^3/EJ$$

$$L_{ED}^{Xo} = \int_o^b \left(x/b - 1/2\ x^3/b^3 \right) Fb^2\ 1/EJ\ dx = \left[1/2\ x^2/b - 1/8\ x^4/b^3 \right]_o^b Fb^2\ 1/EJ$$
$$= \left(1/2\ b - 1/8\ b \right) Fb^2\ 1/EJ\ = 3/8\ Fb^3/EJ$$

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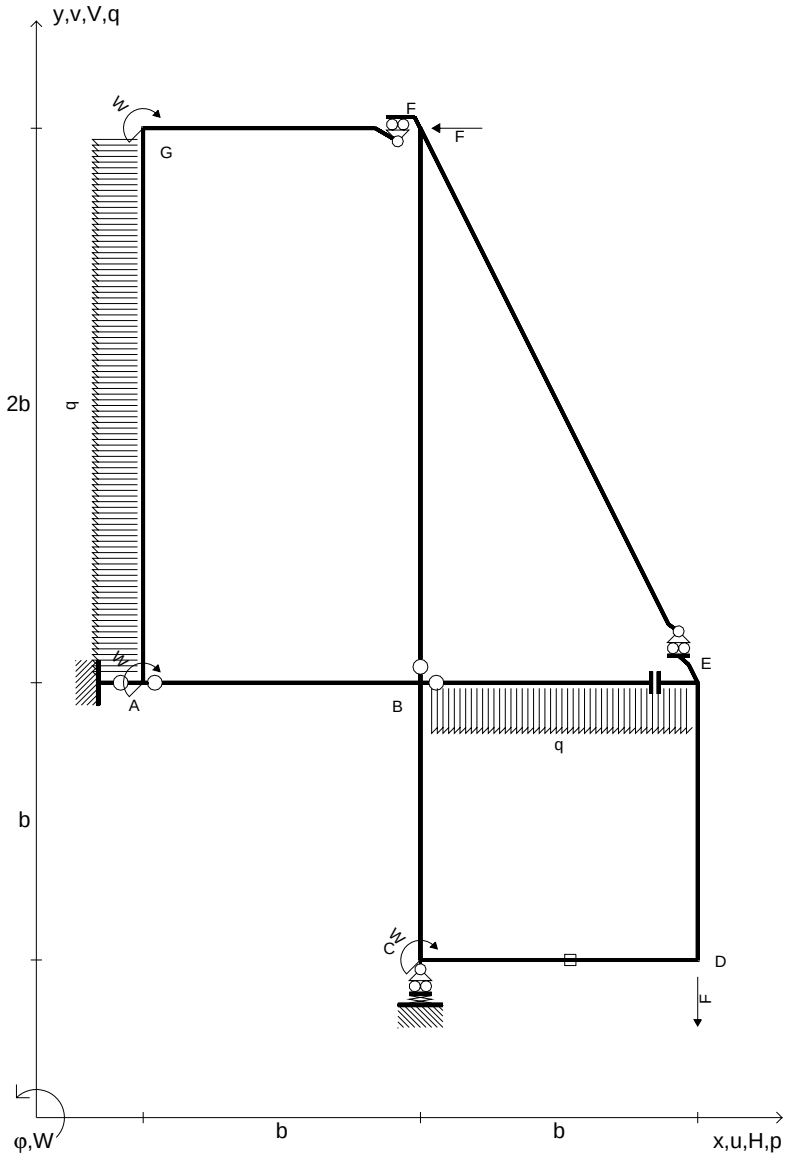
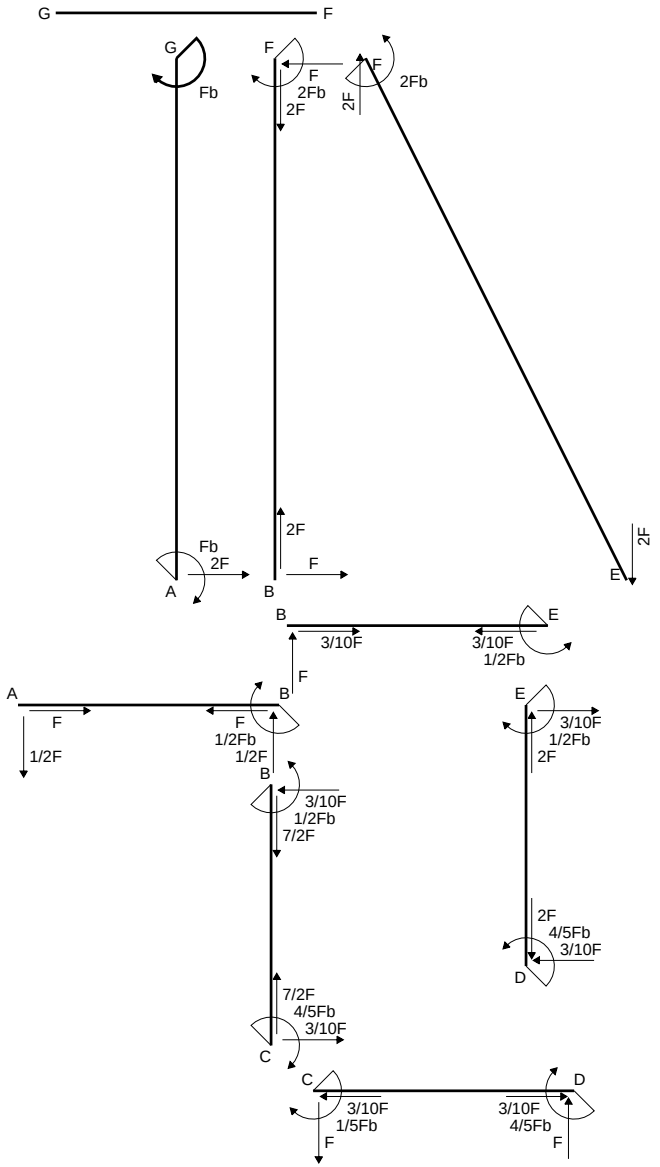
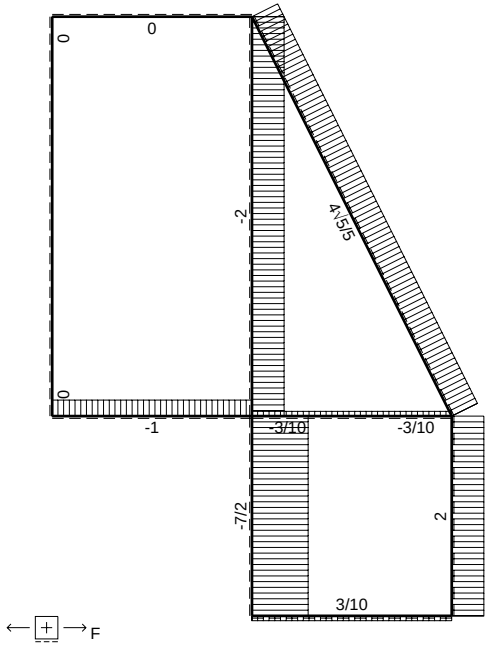
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$q_{BE} = -q = -F/b$
$p_{GA} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$



Reazioni iperstatiche in soluzione: $X=W_{DC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.





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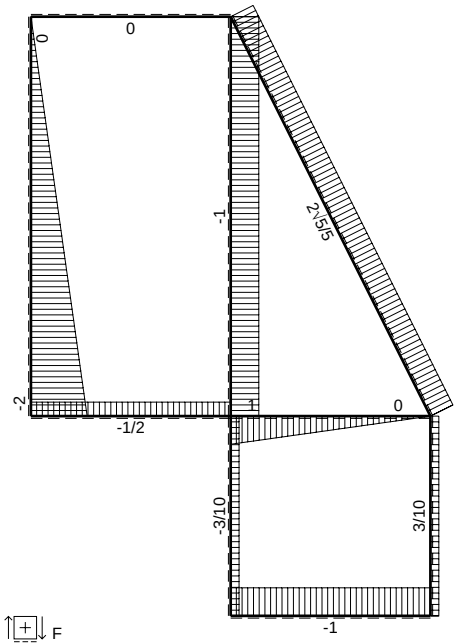

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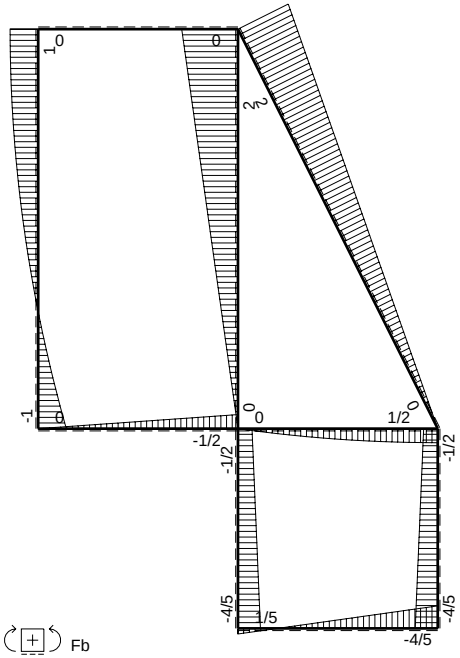

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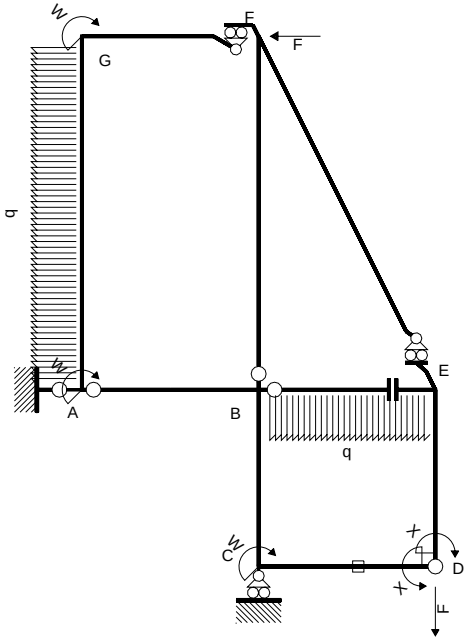
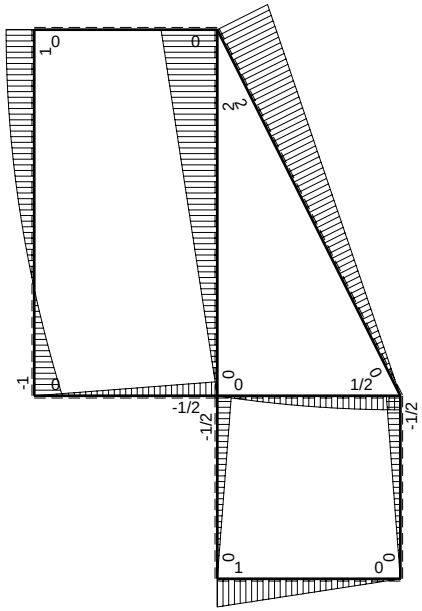
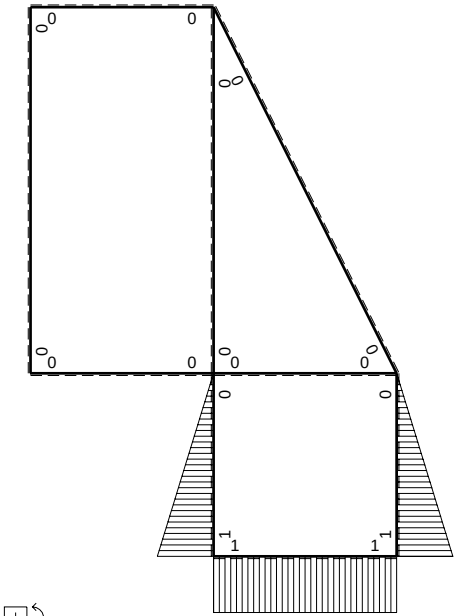




M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DC}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	x/b	-1/2Fb+1/2Fx	-1/2Fx+1/2Fx^2/b	x^2/b^2	-1/12Fb^2/EJ	1/3Xb/EJ
CB b	-1+x/b	1/2Fx	-1/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	1	Fb-Fx	Fb-Fx	1	1/2Fb^2/EJ	Xb/EJ
DC b	-1	-Fx	Fx	1		
DE b	1-x/b	-1/2Fx	-1/2Fx+1/2Fx^2/b	1-2x/b+x^2/b^2	-1/12Fb^2/EJ	1/3Xb/EJ
ED b	-x/b	1/2Fb-1/2Fx	-1/2Fx+1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	Fb-1/2qx^2	0	0	0	0
AG 2b	0	Fb-2Fx+1/2qx^2	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				Fb^2/EJ	
	totali				4/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{DC}$				-4/5Fb	

Sviluppi di calcolo iperstatica

$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (-1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 3/2\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx - 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (1/2\ b\ )\ Fb\ 1/EJ\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 3/2\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (-1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

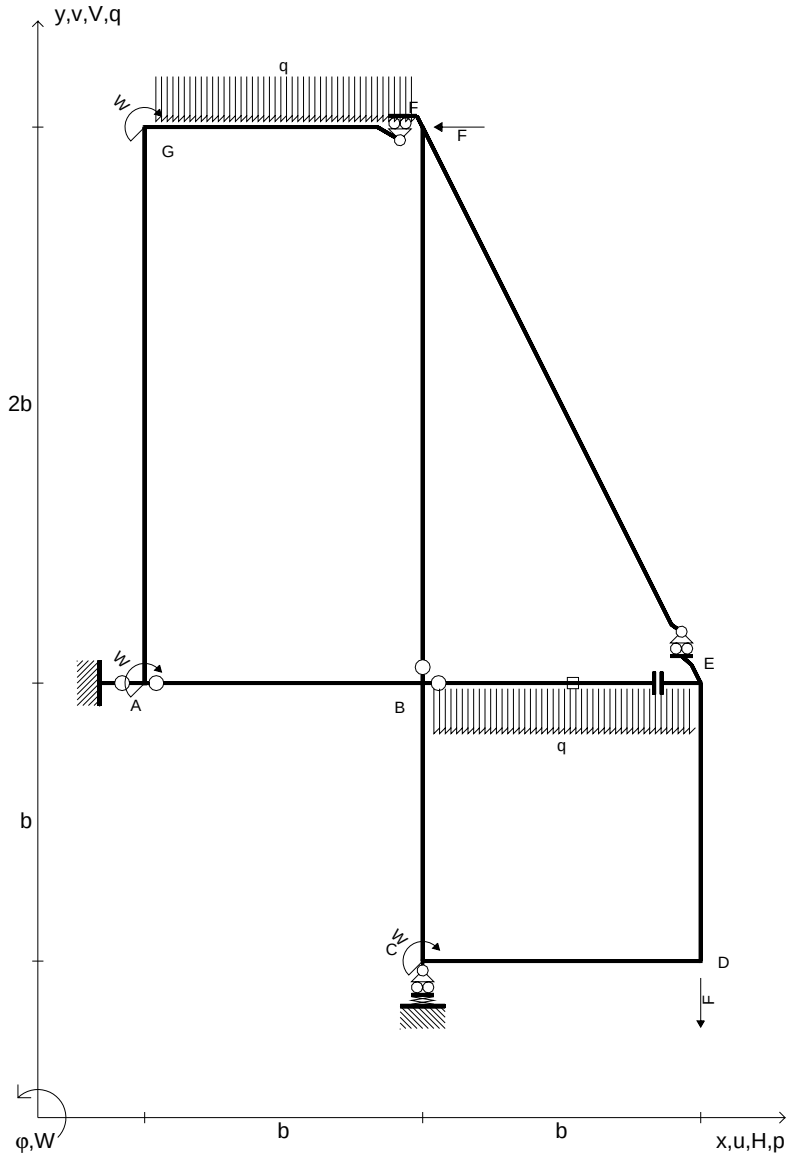
$L_{ED}^{Xo} = \int_o^b (-1/2\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/6\ b\ )\ Fb\ 1/EJ\ = -1/12\ \ Fb^2/EJ$

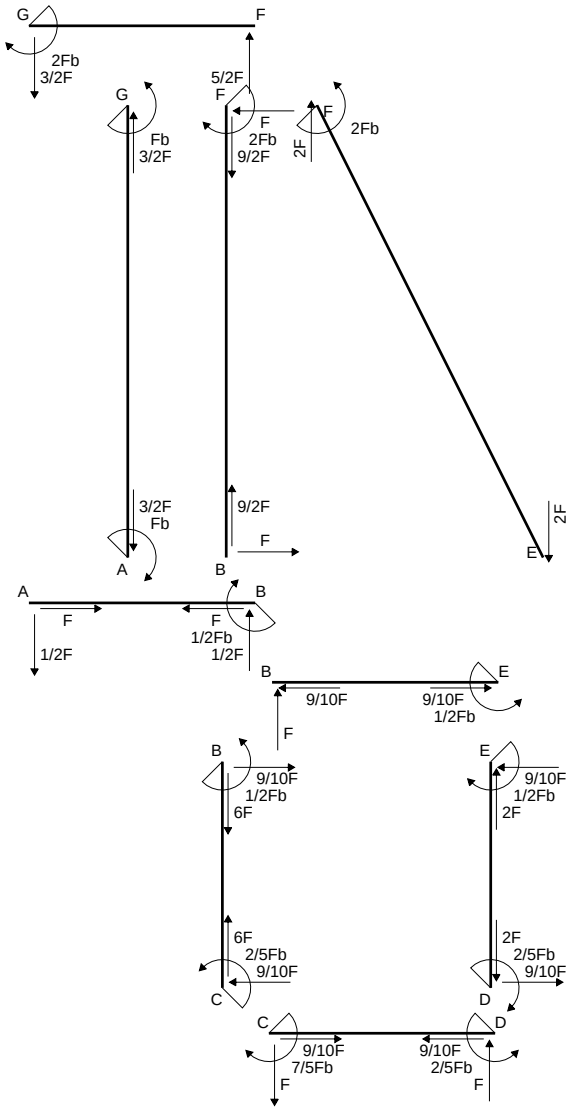
ESΣ02.xxxx.124

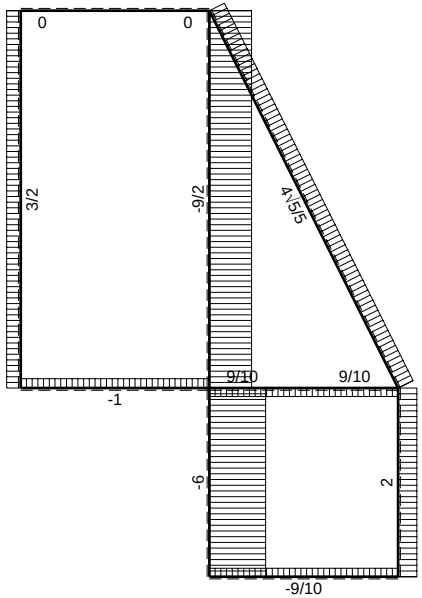
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.12.24

$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$q_{BE} = -q = -F/b$
$q_{FG} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

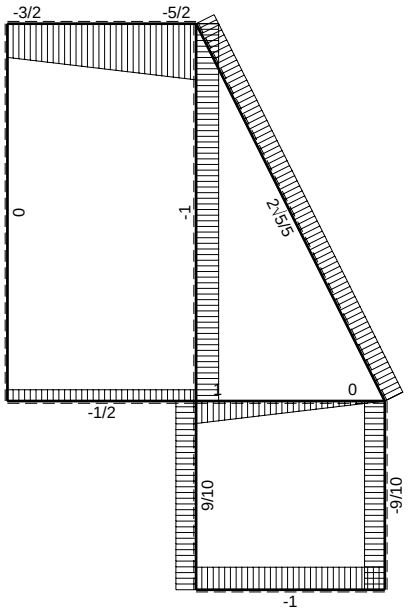


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.

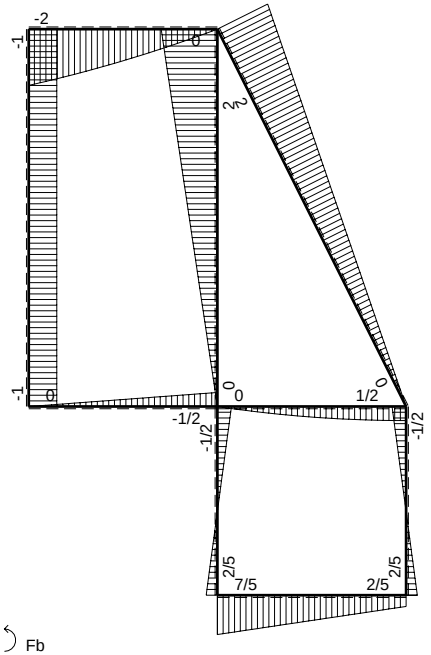




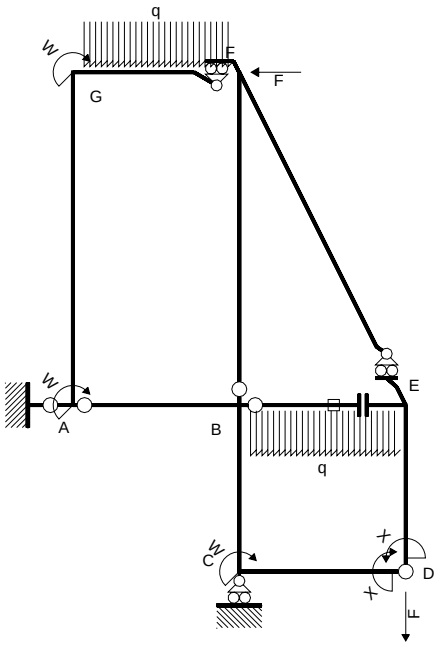
←□→ F



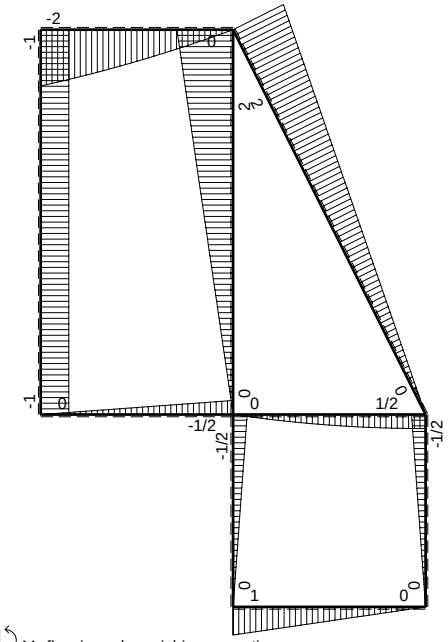
↑□↓ F



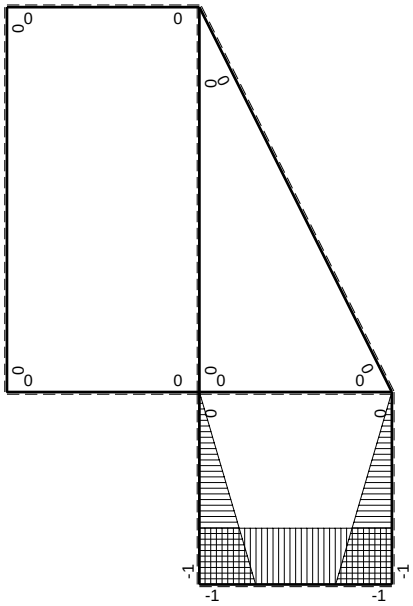
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb+1/2Fx	1/2Fx-1/2Fx^2/b	x^2/b^2	1/12Fb^2/EJ	1/3Xb/EJ
CB b	1-x/b	1/2Fx	1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2		
CD b	-1	Fb-Fx	-Fb+Fx	1	-1/2Fb^2/EJ	Xb/EJ
DC b	1	-Fx	-Fx	1		
DE b	-1+x/b	-1/2Fx	1/2Fx-1/2Fx^2/b	1-2x/b+x^2/b^2	1/12Fb^2/EJ	1/3Xb/EJ
ED b	x/b	1/2Fb-1/2Fx	1/2Fx-1/2Fx^2/b	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	-5/2Fx+1/2qx^2	0	0	0	0
GF b	0	2Fb-3/2Fx-1/2qx^2	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	Fx-1/2qx^2	0	0	0	0
EB b	0	-1/2Fb+1/2qx^2	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				2/3Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{DE}$				-2/5Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209967 ESΣ02.xxxx.133

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\ x/b\ -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b\ -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b\ -1/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2\ x/b\ -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b\ -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b\ -1/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x/b\ -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b\ -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b\ -1/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b\ -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b\ -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b\ -1/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ Fb^2/EJ$

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.12.24

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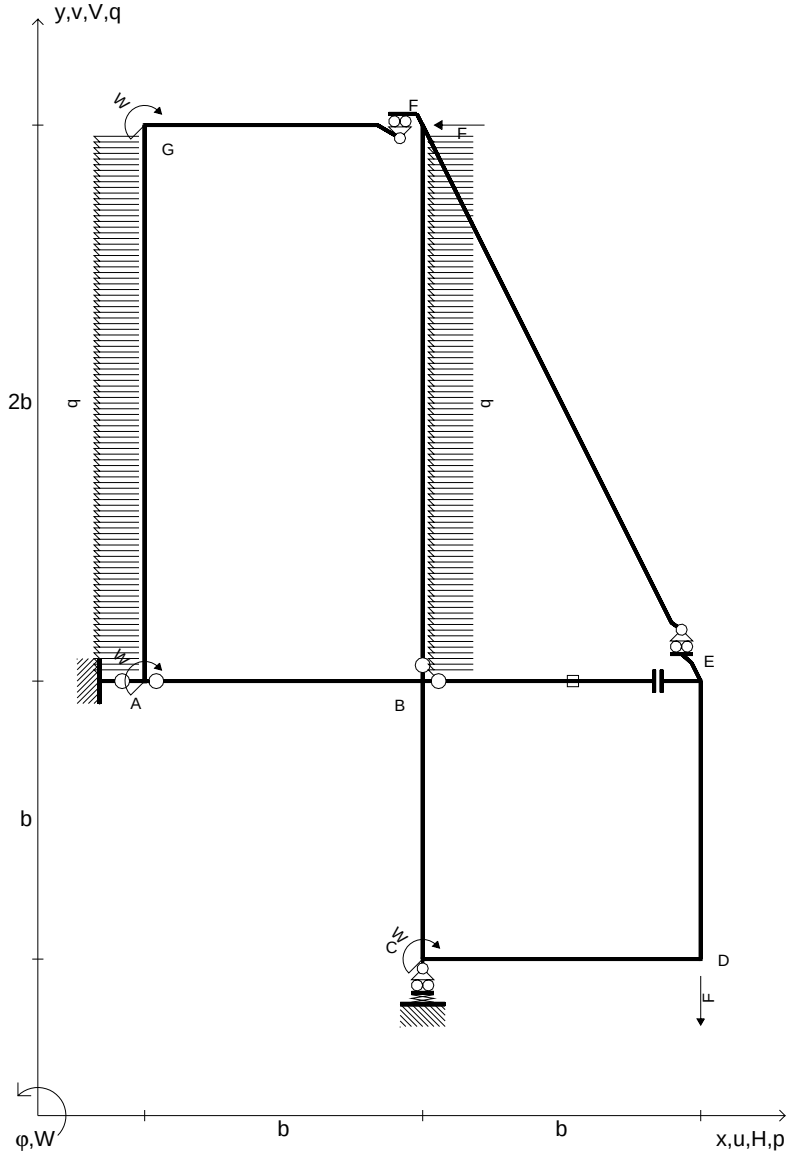
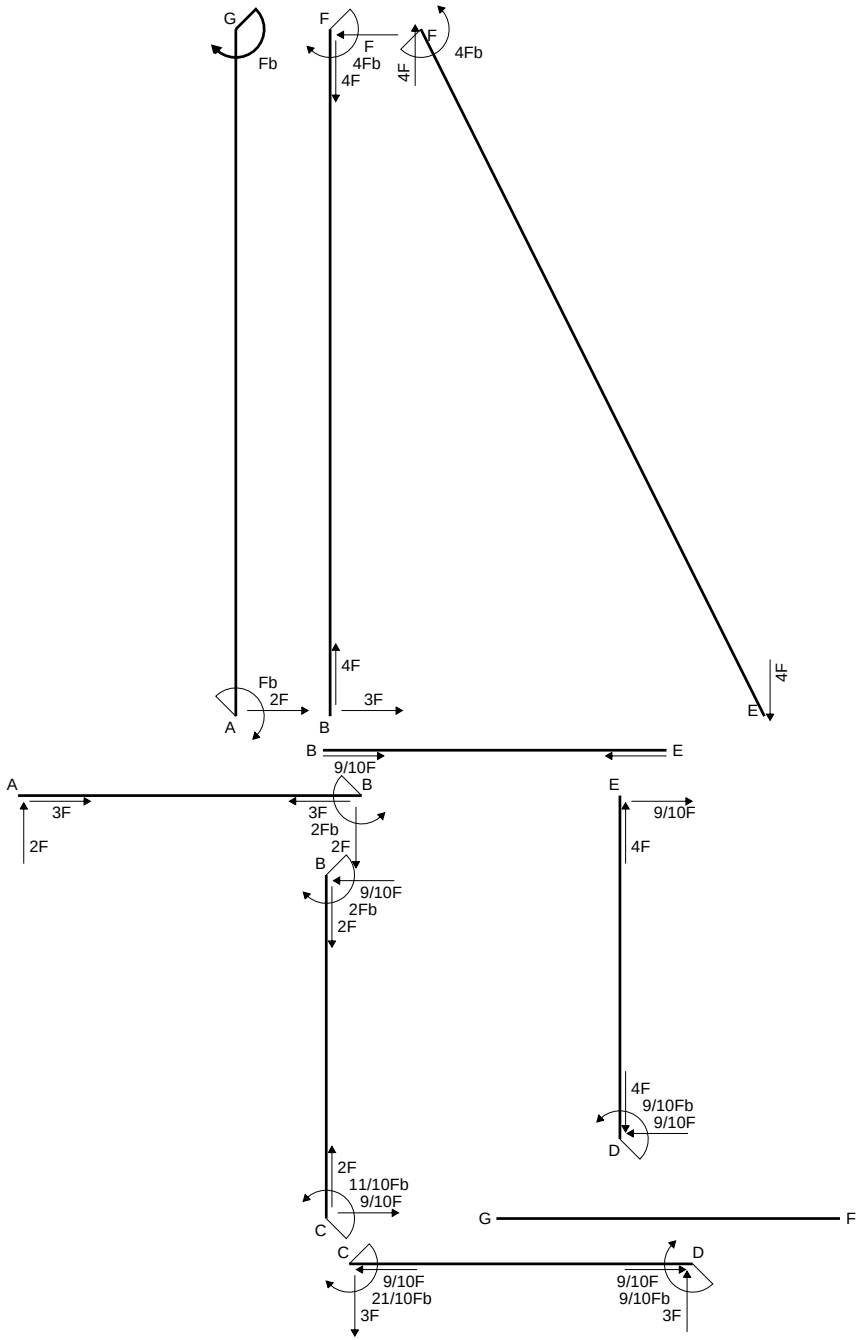
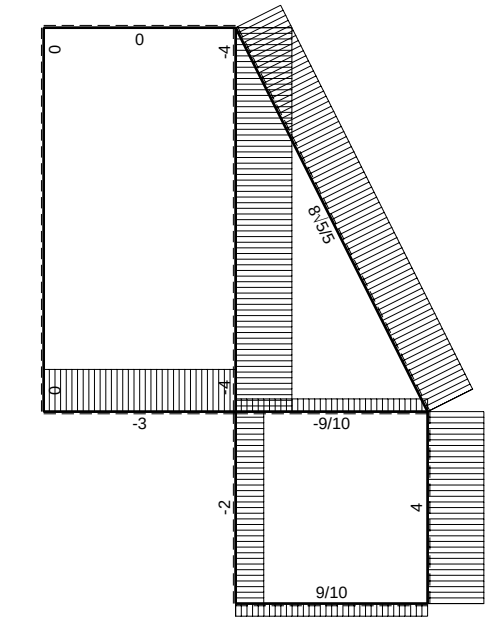
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$p_{GA} = -q = -F/b$
$p_{FB} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

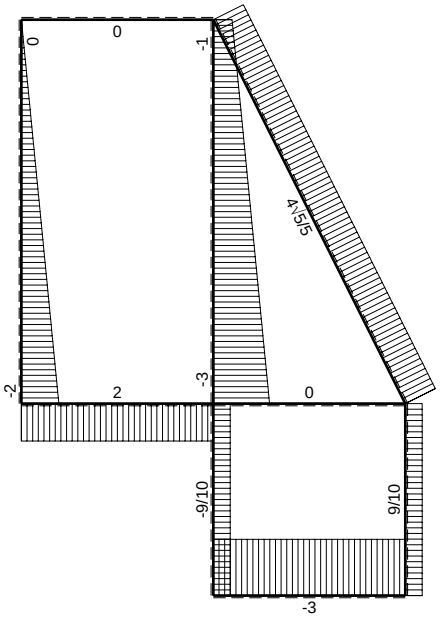


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.

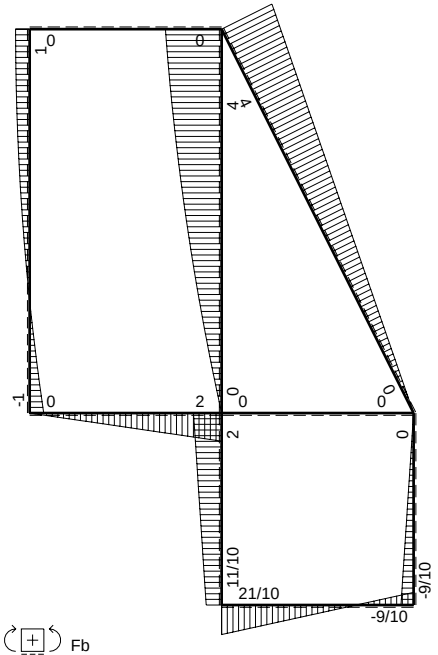




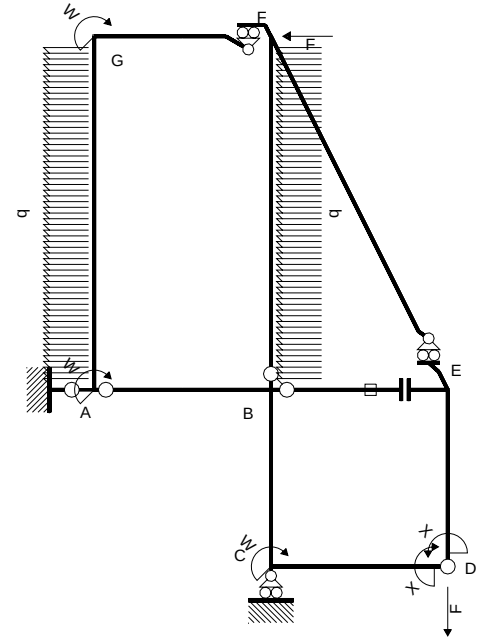
F



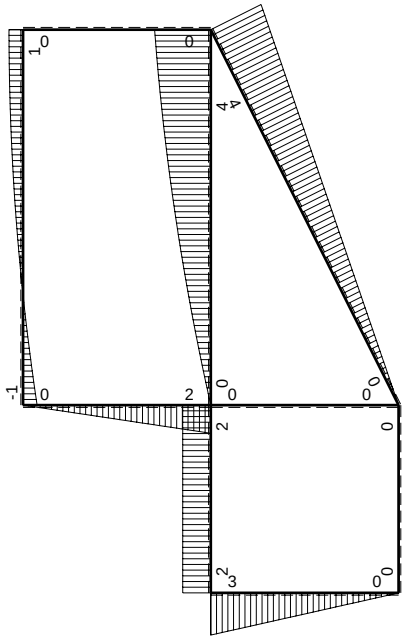
F



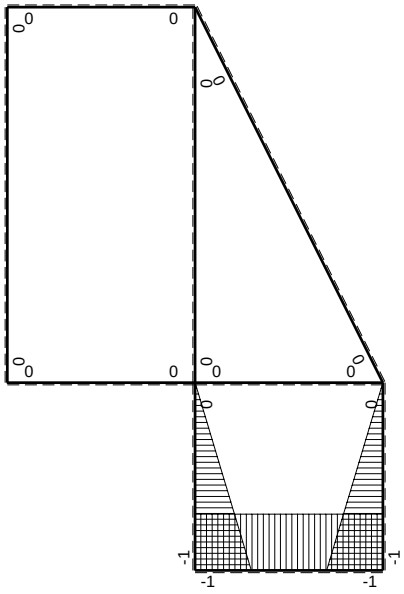
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	2Fx	0	0	0	0
BA b	0	-2Fb+2Fx	0	0		
BC b	-x/b	2Fb	-2Fx	x^2/b^2	$-Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-2Fb	-2Fb+2Fx	$1-2x/b+x^2/b^2$		
CD b	-1	3Fb-3Fx	-3Fb+3Fx	1	$-3/2Fb^2/EJ$	Xb/EJ
DC b	1	-3Fx	-3Fx	1		
DE b	-1+x/b	0	0	$1-2x/b+x^2/b^2$	0	1/3Xb/EJ
ED b	x/b	0	0	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$-3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				9/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210204 ESΣ02.xxxx.145

$$L^{XX}_{BC} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CB} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CD} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DC} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{Xo}_{BC} = \int_o^b \left(-2 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[- x^2/b \right]_o^b \; Fb \; 1/EJ$$
$$= \left(- b \right) \; Fb \; 1/EJ \; = - \; Fb^2/EJ$$

$$L^{Xo}_{CB} = \int_o^b \left(-2 + 2 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[-2 \; x + x^2/b \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-2 \; b + b \right) \; Fb \; 1/EJ \; = - \; Fb^2/EJ$$

$$L^{Xo}_{CD} = \int_o^b \left(-3 + 3 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[-3 \; x + 3/2 \; x^2/b \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-3 \; b + 3/2 \; b \right) \; Fb \; 1/EJ \; = -3/2 \; Fb^2/EJ$$

$$L^{Xo}_{DC} = \int_o^b \left(-3 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[-3/2 \; x^2/b \right]_o^b \; Fb \; 1/EJ$$
$$= \left(-3/2 \; b \right) \; Fb \; 1/EJ \; = -3/2 \; Fb^2/EJ$$

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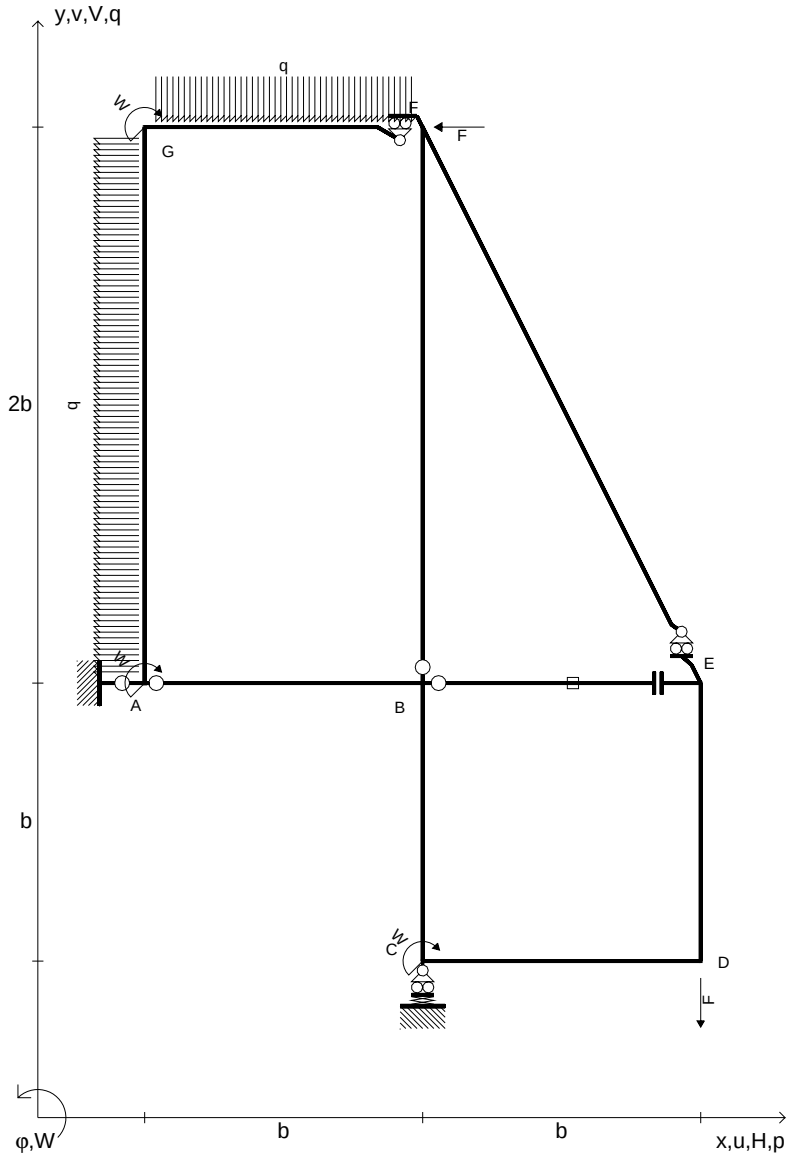
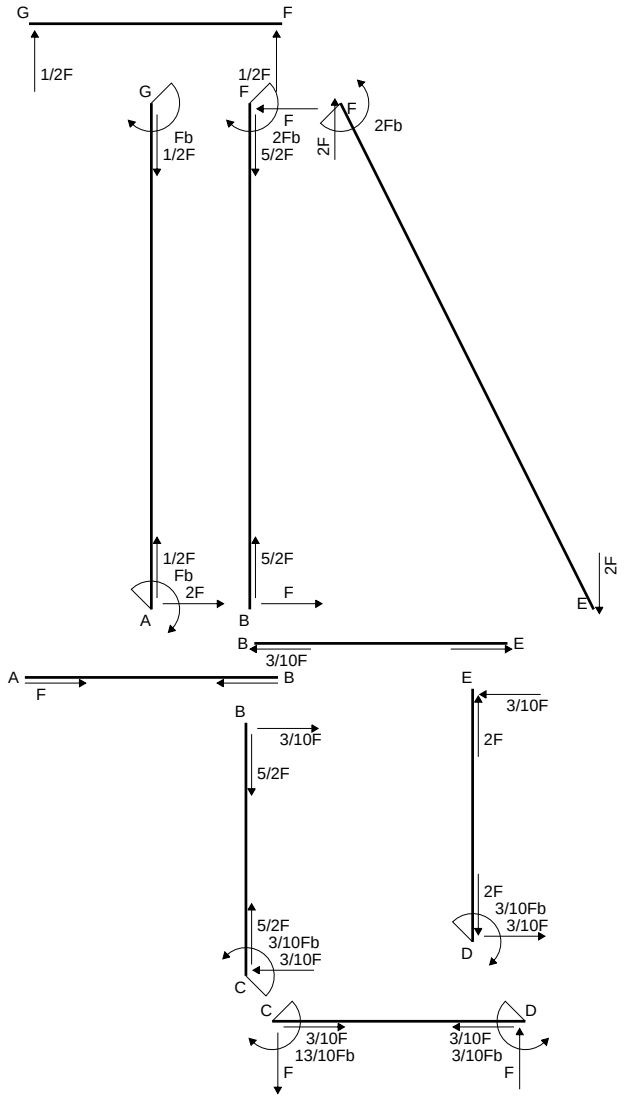
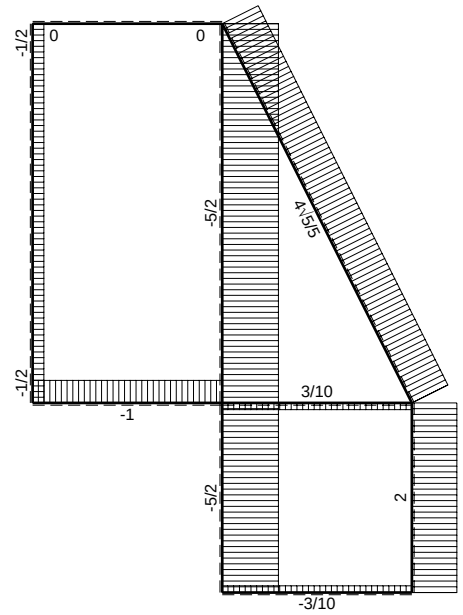
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$p_{GA} = -q = -F/b$
$q_{FG} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

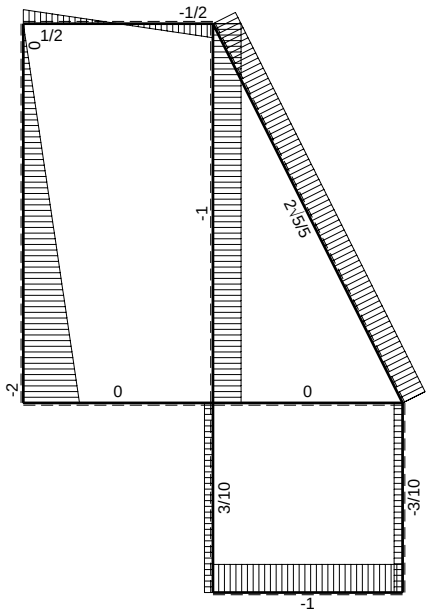


Reazioni iperstatiche in soluzione: $X=H_{BE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.

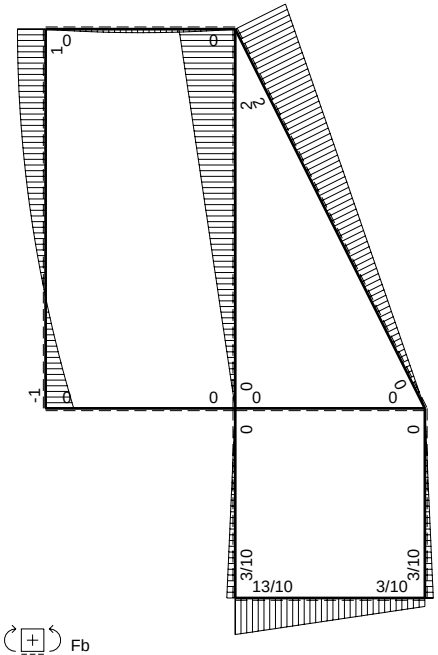




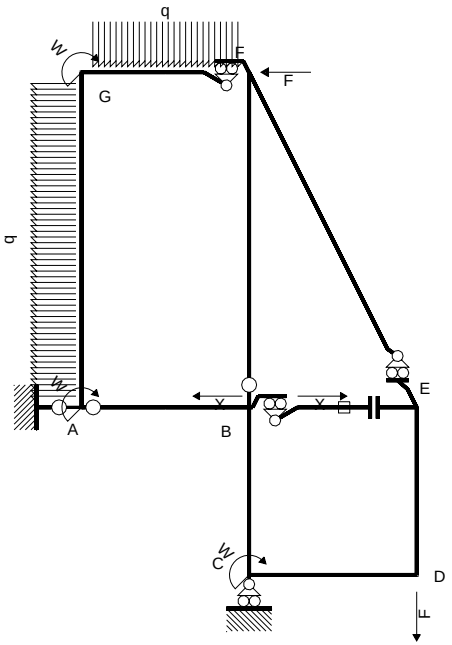
←□→ F



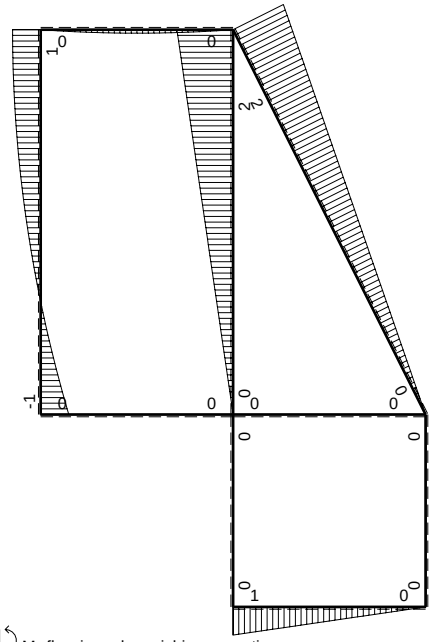
↑□↓ F



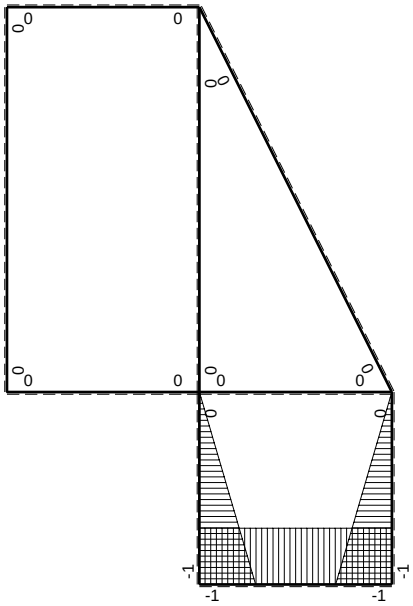
↺□↻ Fb



Schema di calcolo iperstatico



↺□↻ M_o flessione da carichi assegnati



↺□↻ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_{BE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0
BA b	0	0	0	0		
BC b	-x	0	0	x^2	0	$1/3Xb^3/EJ$
CB b	b-x	0	0	$b^2-2bx+x^2$		
CD b	-b	Fb-Fx	$-Fb^2+Fbx$	b^2	$-1/2Fb^3/EJ$	Xb^3/EJ
DC b	b	-Fx	-Fbx	b^2		
DE b	-b+x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
ED b	x	0	0	x^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0		
GA 2b	0	$Fb-1/2qx^2$	0	0	0	0
AG 2b	0	$Fb-2Fx+1/2qx^2$	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^3/EJ	
	totali				$1/2Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_{BE}$				-3/10F	

Sviluppi di calcolo iperstatica

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$$L^{XX}_{BC} = \int_o^b \left(x^2/b^2 \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2 \right]_o^b b^2\ 1/EJ$$
$$= \left(1/3\ b \right) b^2\ 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{CB} = \int_o^b \left(1 - 2\ x/b + x^2/b^2 \right) b^2\ 1/EJ\ dx = \left[x - x^2/b + 1/3\ x^3/b^2 \right]_o^b b^2\ 1/EJ$$
$$= \left(b - b + 1/3\ b \right) b^2\ 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{CD} = \int_o^b \left(1 \right) b^2\ 1/EJ\ dx = \left[x \right]_o^b b^2\ 1/EJ$$
$$= \left(b \right) b^2\ 1/EJ = b^3/EJ$$

$$L^{XX}_{DC} = \int_o^b \left(1 \right) b^2\ 1/EJ\ dx = \left[x \right]_o^b b^2\ 1/EJ$$
$$= \left(b \right) b^2\ 1/EJ = b^3/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 - 2\ x/b + x^2/b^2 \right) b^2\ 1/EJ\ dx = \left[x - x^2/b + 1/3\ x^3/b^2 \right]_o^b b^2\ 1/EJ$$
$$= \left(b - b + 1/3\ b \right) b^2\ 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(x^2/b^2 \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2 \right]_o^b b^2\ 1/EJ$$
$$= \left(1/3\ b \right) b^2\ 1/EJ = 1/3\ b^3/EJ$$

$$L^{Xo}_{CD} = \int_o^b \left(-1 + x/b \right) Fb^2\ 1/EJ\ dx = \left[-x + 1/2\ x^2/b \right]_o^b Fb^2\ 1/EJ$$
$$= \left(-b + 1/2\ b \right) Fb^2\ 1/EJ = -1/2\ Fb^3/EJ$$

$$L^{Xo}_{DC} = \int_o^b \left(-x/b \right) Fb^2\ 1/EJ\ dx = \left[-1/2\ x^2/b \right]_o^b Fb^2\ 1/EJ$$
$$= \left(-1/2\ b \right) Fb^2\ 1/EJ = -1/2\ Fb^3/EJ$$

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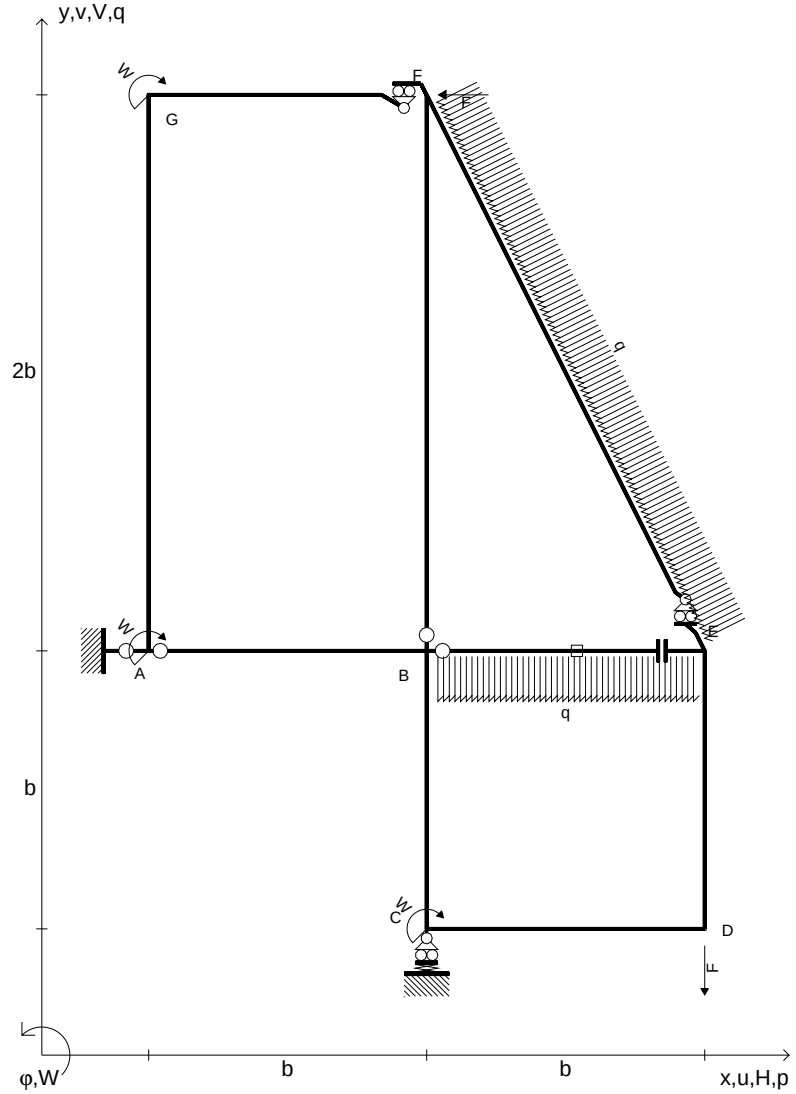
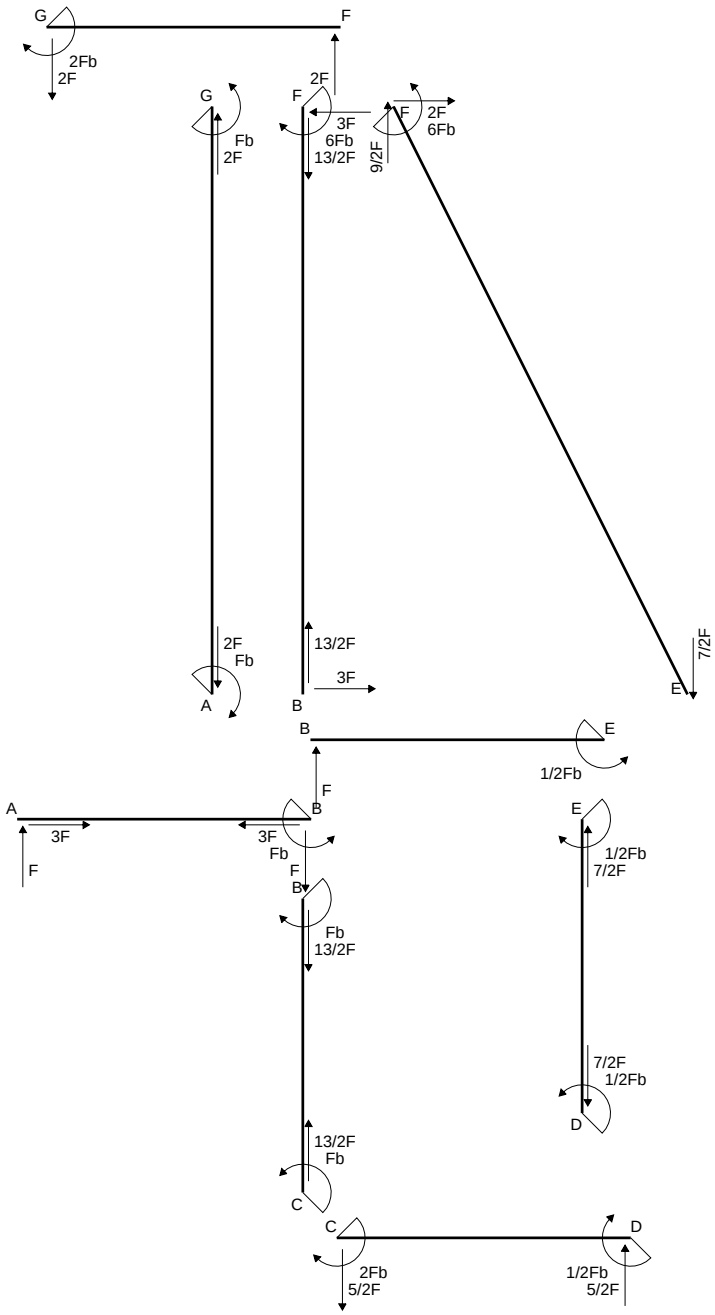
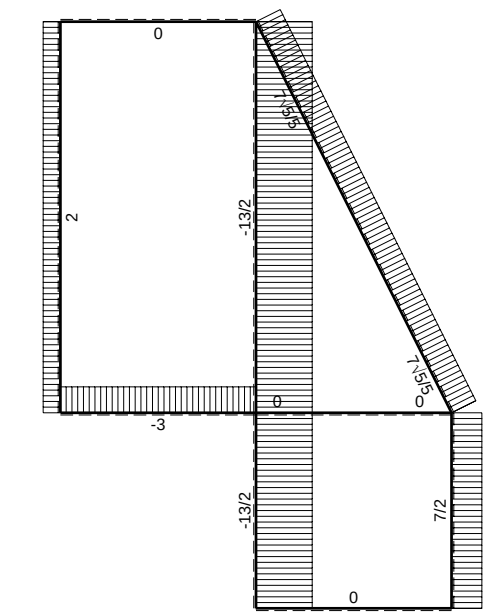
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$p_{EF} = -q = -F/b$
$q_{EF} = -q = -F/b$
$q_{BE} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

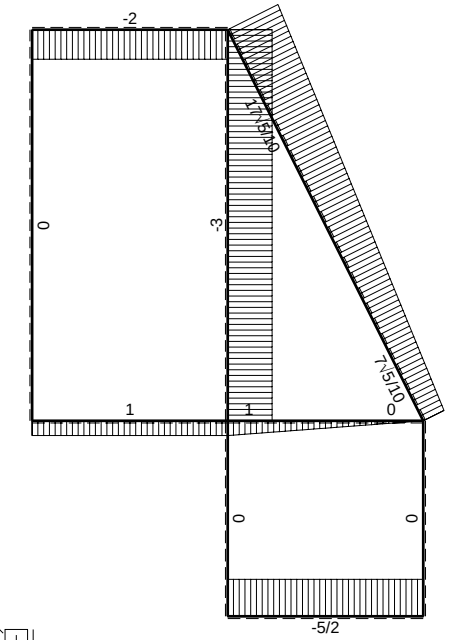


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.





F



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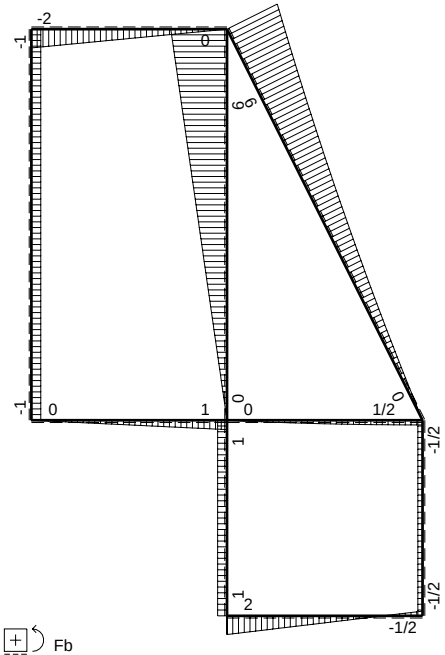
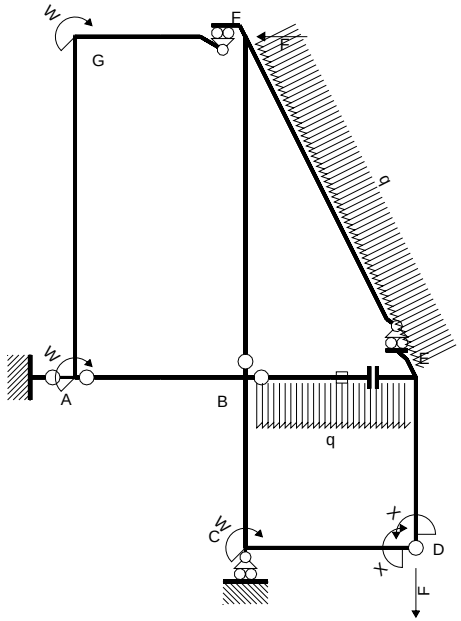
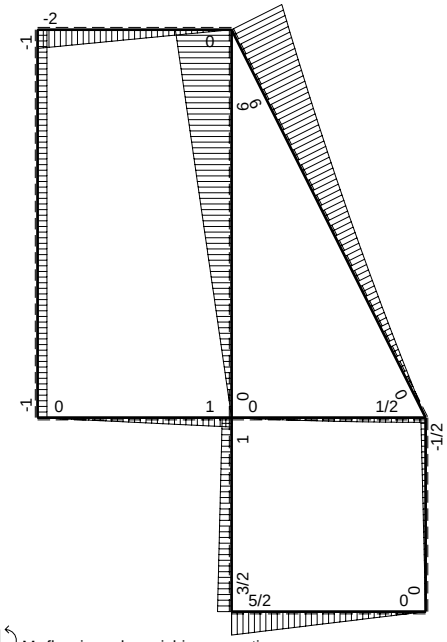


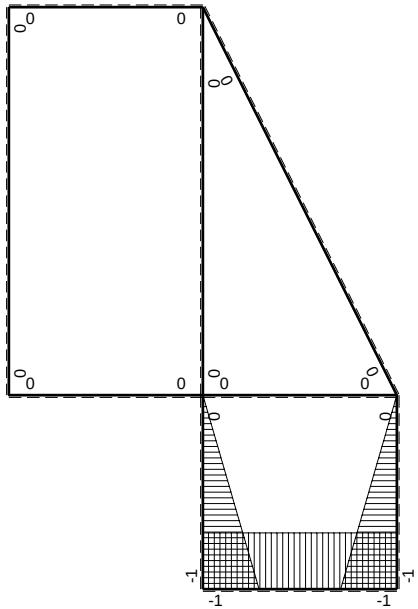
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	Fx	0	0	0	0
BA b	0	-Fb+Fx	0	0		
BC b	-x/b	Fb+1/2Fx	$-Fx-1/2Fx^2/b$	x^2/b^2	$-2/3Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-3/2Fb+1/2Fx	$-3/2Fb+2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	5/2Fb-5/2Fx	-5/2Fb+5/2Fx	1	$-5/4Fb^2/EJ$	Xb/EJ
DC b	1	-5/2Fx	-5/2Fx	1		
DE b	-1+x/b	-1/2Fx	$1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$1/12Fb^2/EJ$	1/3Xb/EJ
ED b	x/b	1/2Fb-1/2Fx	$1/2Fx-1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$-5/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				1/2Fb	

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ \ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ \ b/EJ$

$L_{BC}^{Xo} = \int_o^b (-\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -2/3\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (-3/2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3/2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3/2\ b + b -1/6\ b\ )\ Fb\ 1/EJ\ = -2/3\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ = -5/4\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ = -5/4\ \ Fb^2/EJ$

$L_{DE}^{Xo} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

$L_{ED}^{Xo} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ = 1/12\ \ Fb^2/EJ$

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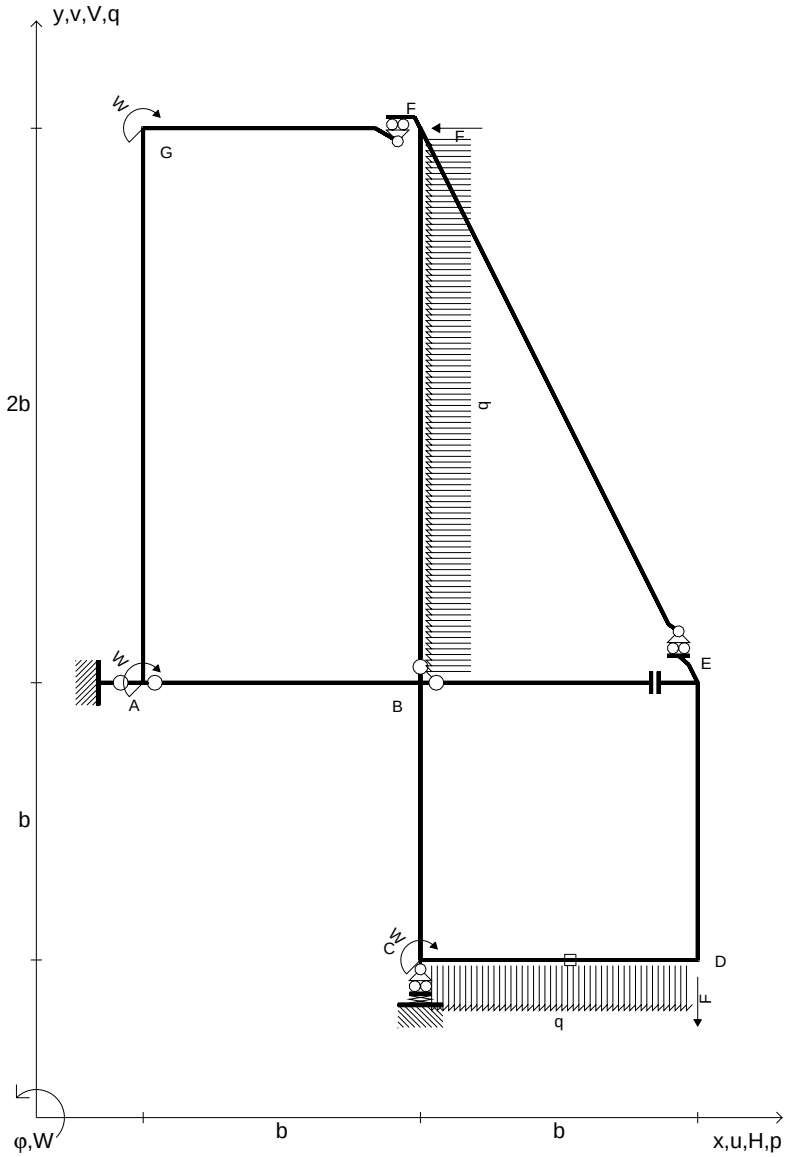
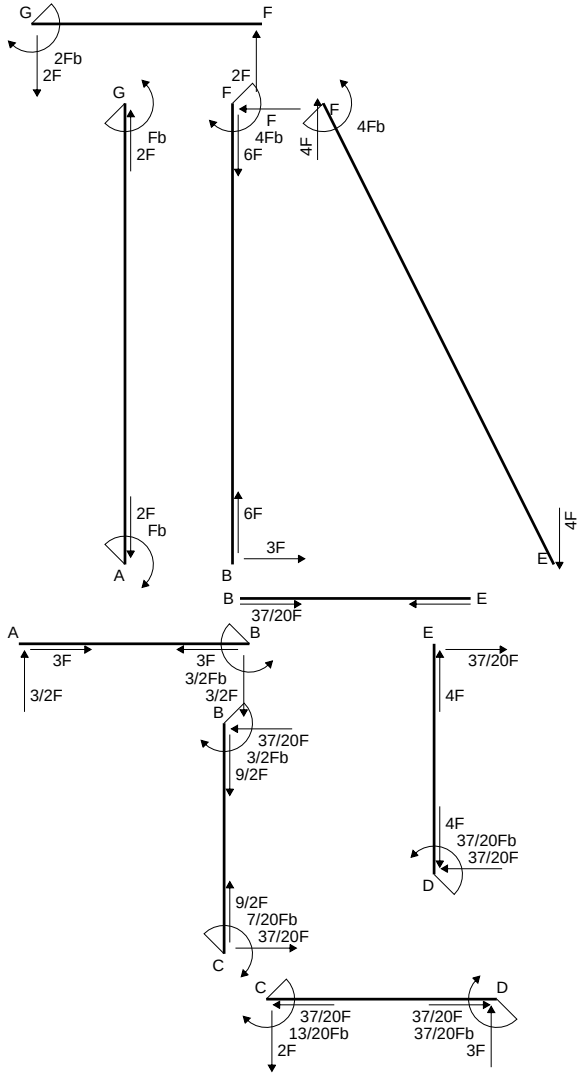
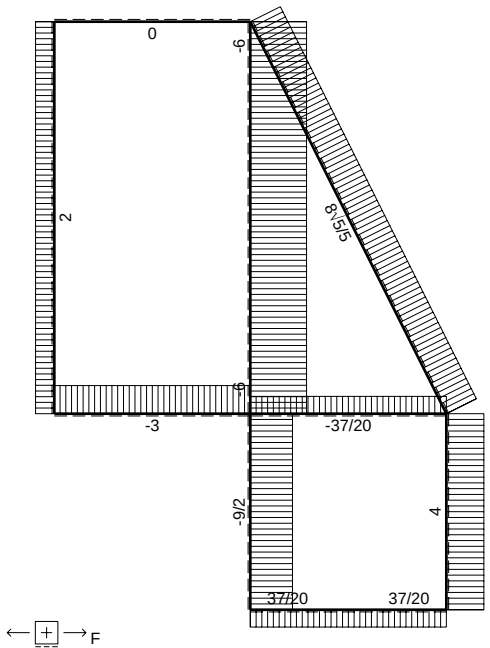
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$q_{CD} = -q = -F/b$
$p_{FB} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

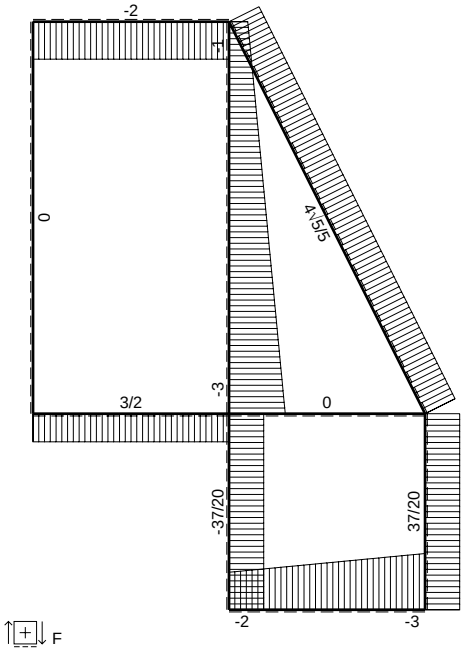


Reazioni iperstatiche in soluzione: $X=W_{DC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.





F



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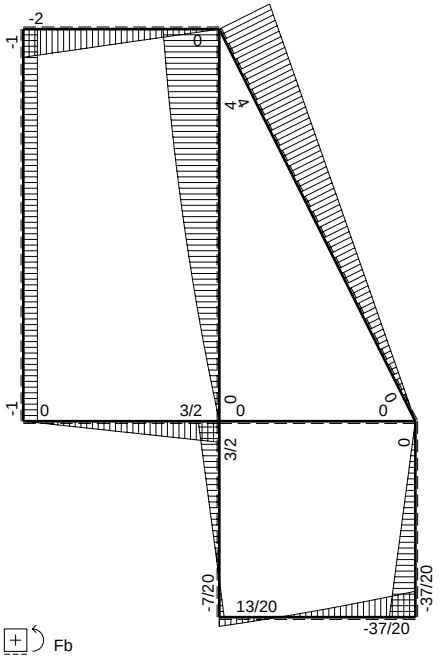
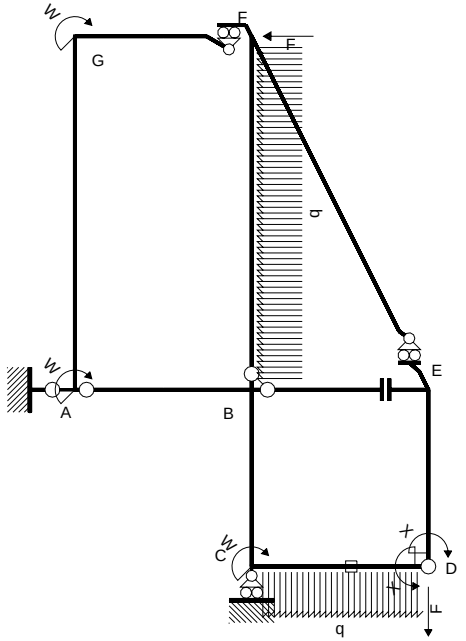
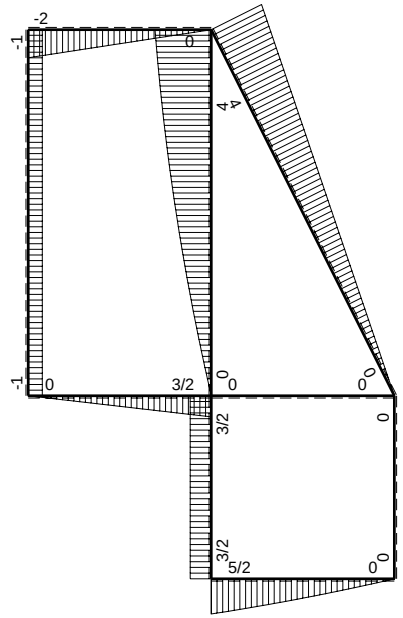
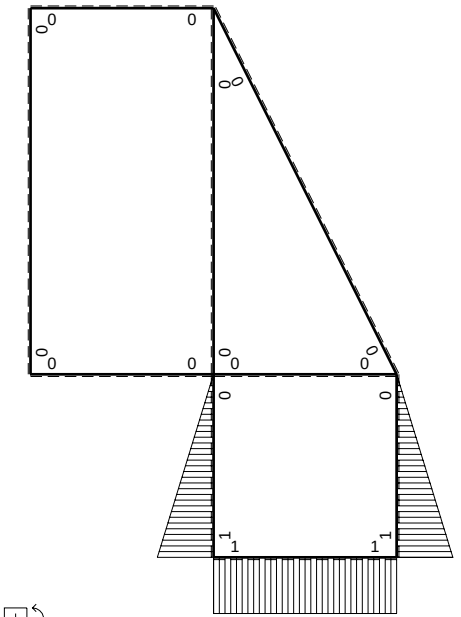


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DC}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	x/b	3/2Fb	3/2Fx	x^2/b^2	$3/4Fb^2/EJ$	1/3Xb/EJ
CB b	-1+x/b	-3/2Fb	3/2Fb-3/2Fx	$1-2x/b+x^2/b^2$		
CD b	1	$5/2Fb-2Fx-1/2qx^2$	$5/2Fb-2Fx-1/2Fx^2/b$	1	$4/3Fb^2/EJ$	Xb/EJ
DC b	-1	$-3Fx+1/2qx^2$	$3Fx-1/2Fx^2/b$	1		
DE b	1-x/b	0	0	$1-2x/b+x^2/b^2$	0	1/3Xb/EJ
ED b	-x/b	0	0	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				Fb^2/EJ	
	totali				$37/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DC}$				-37/20Fb	

Sviluppi di calcolo iperstatica

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$L_{BC}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CD}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DC}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{DE}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{BC}^{Xo} = \int_o^b (3/2\ x/b\ )\ Fb\ 1/EJ\ dx = [3/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/4\ b\ )\ Fb\ 1/EJ\ = 3/4\ \ Fb^2/EJ$

$L_{CB}^{Xo} = \int_o^b (3/2\ -3/2\ x/b\ )\ Fb\ 1/EJ\ dx = [3/2\ x\ -3/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (3/2\ b\ -3/4\ b\ )\ Fb\ 1/EJ\ = 3/4\ \ Fb^2/EJ$

$L_{CD}^{Xo} = \int_o^b (5/2\ -2\ x/b\ -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [5/2\ x\ -\ x^2/b\ -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ \ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (5/2\ b\ -\ b\ -1/6\ b\ )\ Fb\ 1/EJ\ \ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 7/3\ \ Fb^2/EJ$

$L_{DC}^{Xo} = \int_o^b (3\ x/b\ -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx\ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [3/2\ x^2/b\ -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ \ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (3/2\ b\ -1/6\ b\ )\ Fb\ 1/EJ\ \ -\ 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = 7/3\ \ Fb^2/EJ$

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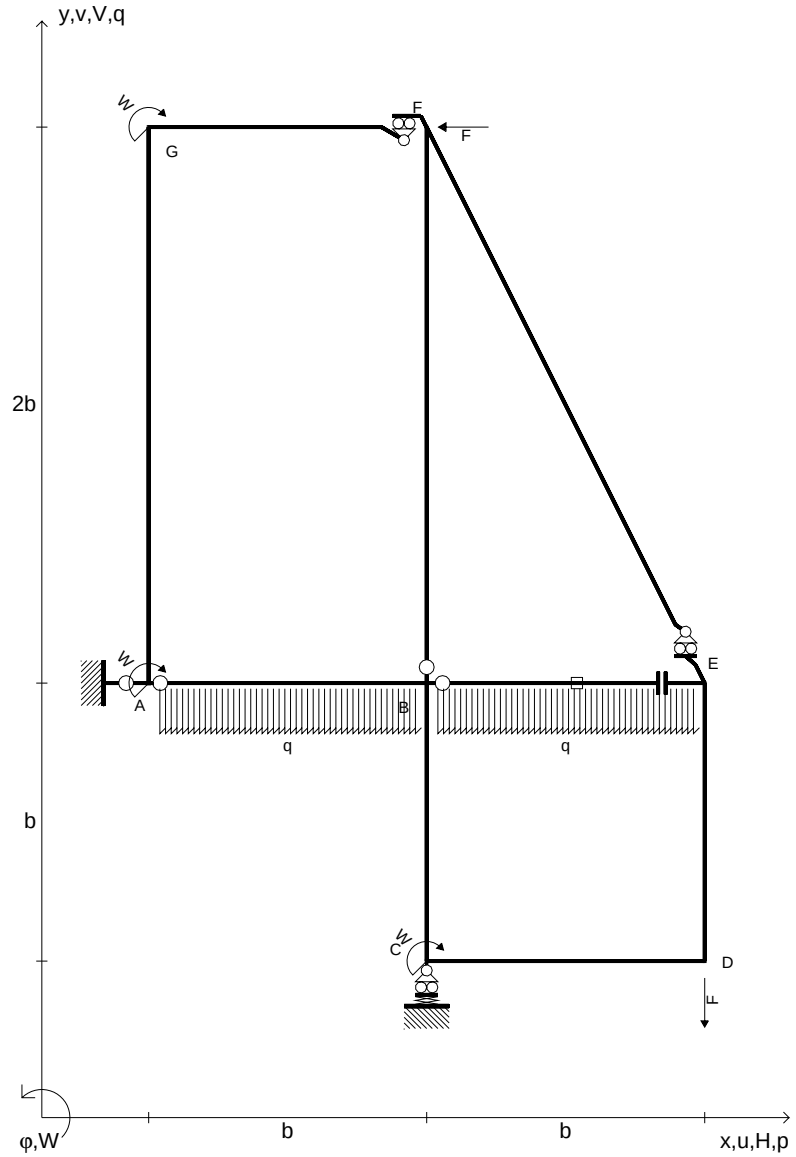
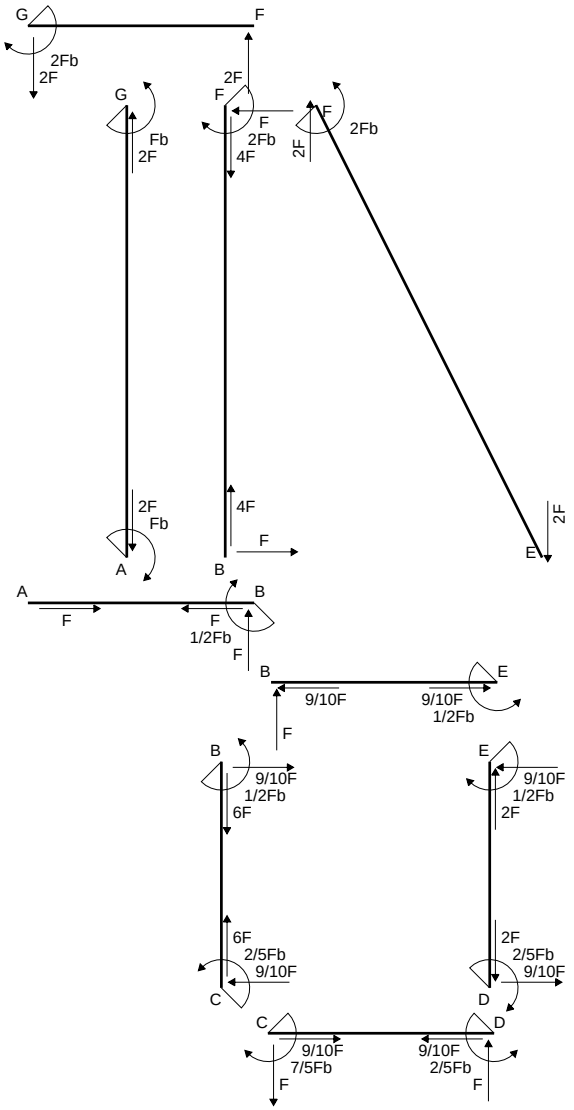
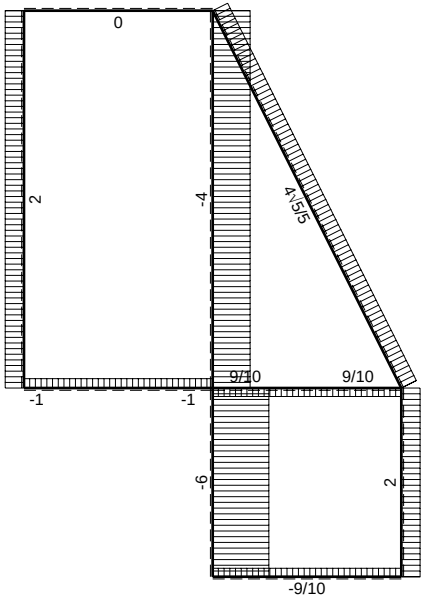
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$q_{AB} = -q = -F/b$
$q_{BE} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

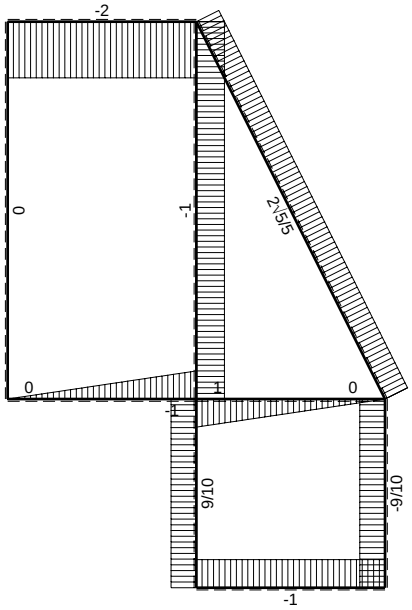


Reazioni iperstatiche in soluzione: $X=W_{DC}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.

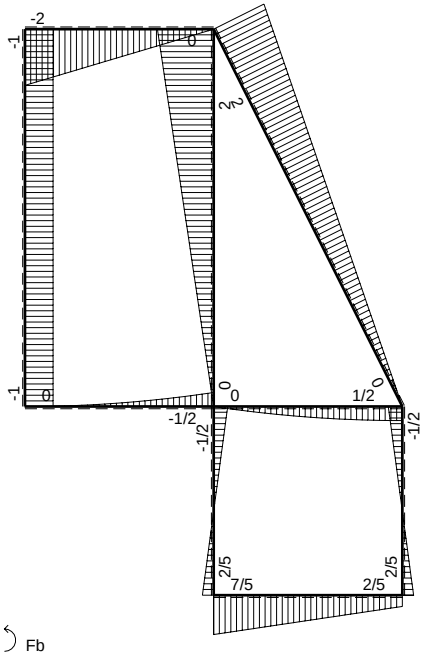




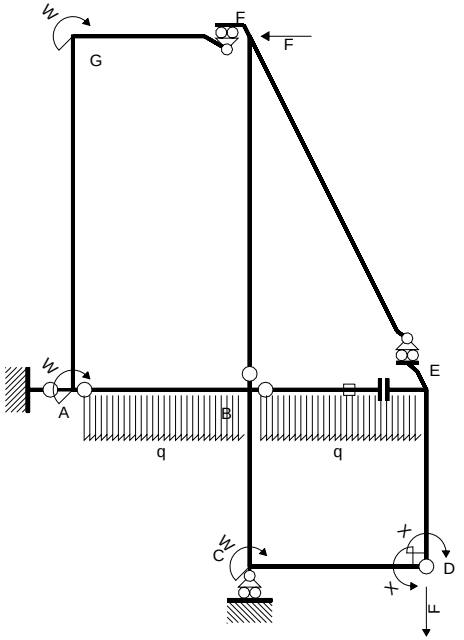
←□→ F

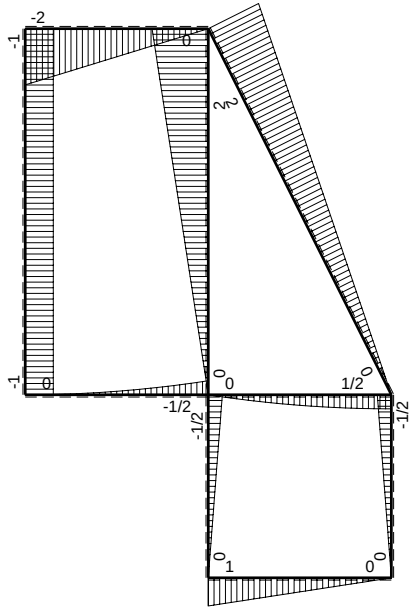


↑□↓ F

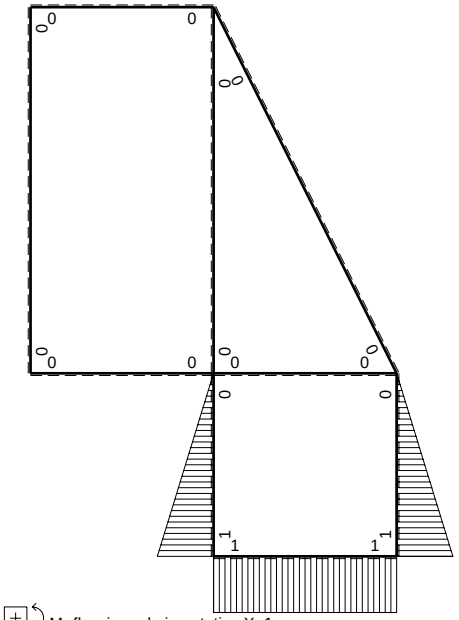


Fb





□ M_o flessione da carichi assegnati



□ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DC}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	x/b	$-1/2Fb+1/2Fx$	$-1/2Fx+1/2Fx^2/b$	x^2/b^2	$-1/12Fb^2/EJ$	1/3Xb/EJ
CB b	-1+x/b	1/2Fx	$-1/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	1	Fb-Fx	Fb-Fx	1	$1/2Fb^2/EJ$	Xb/EJ
DC b	-1	-Fx	Fx	1		
DE b	1-x/b	-1/2Fx	$-1/2Fx+1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$-1/12Fb^2/EJ$	1/3Xb/EJ
ED b	-x/b	$1/2Fb-1/2Fx$	$-1/2Fx+1/2Fx^2/b$	x^2/b^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				$-Fb^2/EJ$	
	totali				$-2/3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DC}$				2/5Fb	

Sviluppi di calcolo iperstatica

$$L^{XX}_{BC} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CB} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{CD} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DC} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$
$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L^{XX}_{DE} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{XX}_{ED} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$
$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L^{Xo}_{BC} = \int_o^b \left(-1/2 \; x/b + 1/2 \; x^2/b^2 \right) Fb \; 1/EJ \; dx = \left[-1/4 \; x^2/b + 1/6 \; x^3/b^2 \right]_o^b Fb \; 1/EJ$$
$$= \left(-1/4 \; b + 1/6 \; b \right) Fb \; 1/EJ \; = -1/12 \; Fb^2/EJ$$

$$L^{Xo}_{CB} = \int_o^b \left(-1/2 \; x/b + 1/2 \; x^2/b^2 \right) Fb \; 1/EJ \; dx = \left[-1/4 \; x^2/b + 1/6 \; x^3/b^2 \right]_o^b Fb \; 1/EJ$$
$$= \left(-1/4 \; b + 1/6 \; b \right) Fb \; 1/EJ \; = -1/12 \; Fb^2/EJ$$

$$L^{Xo}_{CD} = \int_o^b \left(1 - x/b \right) Fb \; 1/EJ \; dx = \left[x - 1/2 \; x^2/b \right]_o^b Fb \; 1/EJ$$
$$= \left(b - 1/2 \; b \right) Fb \; 1/EJ \; = 1/2 \; Fb^2/EJ$$

$$L^{Xo}_{DC} = \int_o^b \left(x/b \right) Fb \; 1/EJ \; dx = \left[1/2 \; x^2/b \right]_o^b Fb \; 1/EJ$$
$$= \left(1/2 \; b \right) Fb \; 1/EJ \; = 1/2 \; Fb^2/EJ$$

$$L^{Xo}_{DE} = \int_o^b \left(-1/2 \; x/b + 1/2 \; x^2/b^2 \right) Fb \; 1/EJ \; dx = \left[-1/4 \; x^2/b + 1/6 \; x^3/b^2 \right]_o^b Fb \; 1/EJ$$
$$= \left(-1/4 \; b + 1/6 \; b \right) Fb \; 1/EJ \; = -1/12 \; Fb^2/EJ$$

$$L^{Xo}_{ED} = \int_o^b \left(-1/2 \; x/b + 1/2 \; x^2/b^2 \right) Fb \; 1/EJ \; dx = \left[-1/4 \; x^2/b + 1/6 \; x^3/b^2 \right]_o^b Fb \; 1/EJ$$
$$= \left(-1/4 \; b + 1/6 \; b \right) Fb \; 1/EJ \; = -1/12 \; Fb^2/EJ$$

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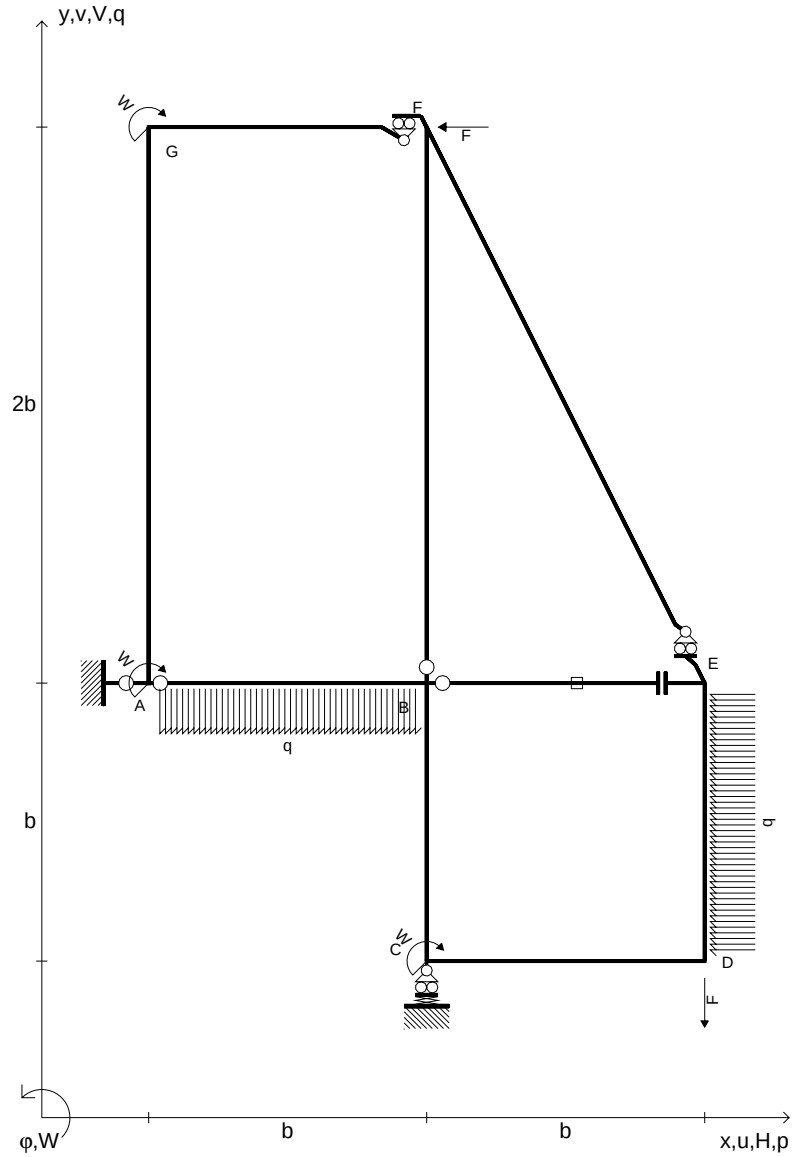
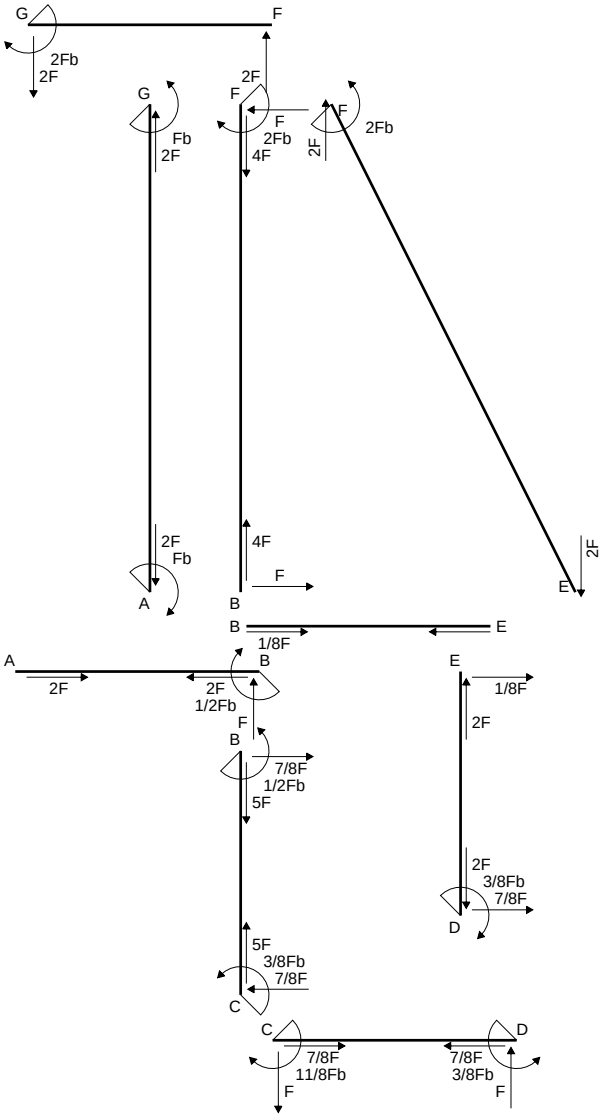
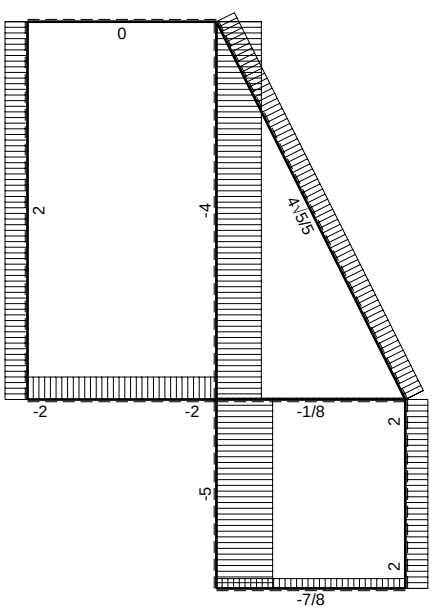
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$q_{AB} = -q = -F/b$
$p_{DE} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

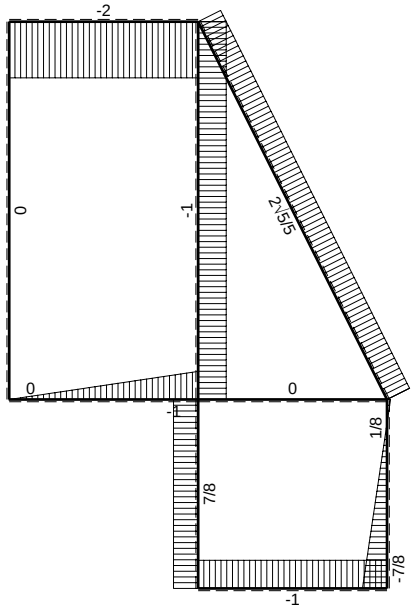


Reazioni iperstatiche in soluzione: $X=H_{BE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.





←□→ F



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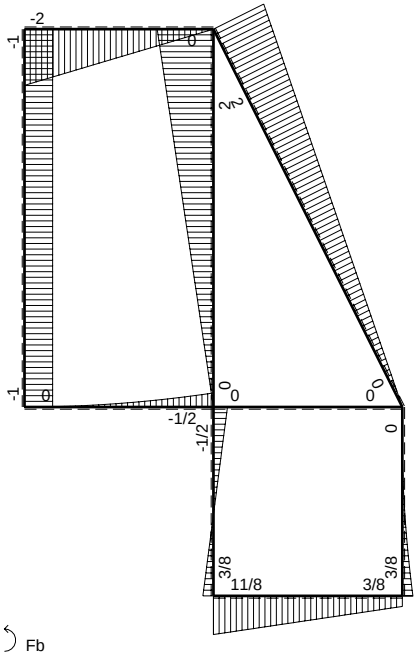
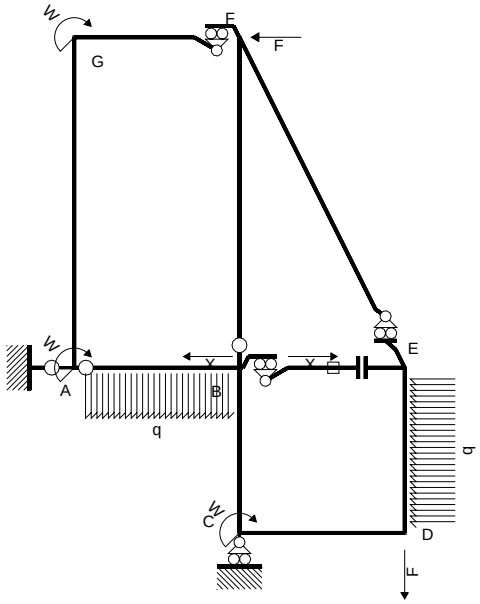
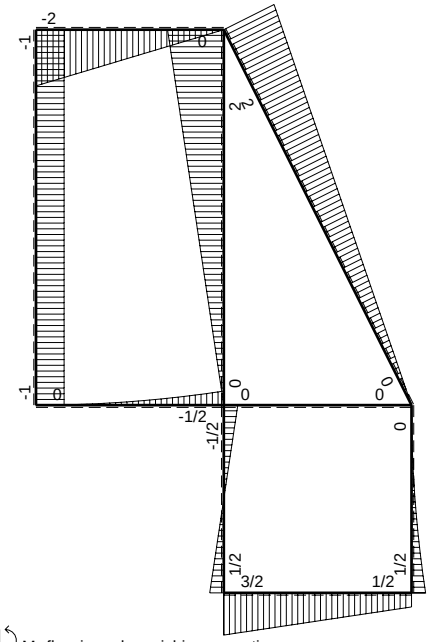


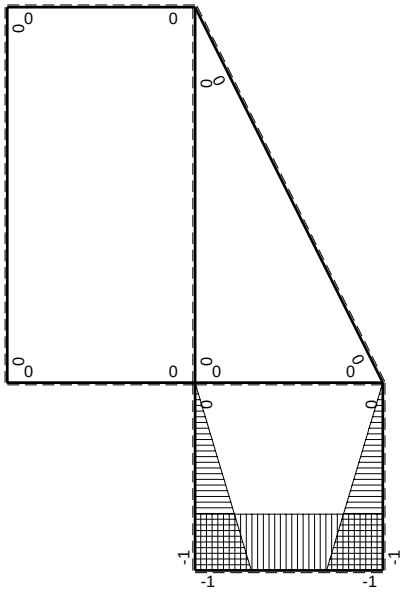
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=H_{BE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0		
BC b	-x	$-1/2Fb+Fx$	$1/2Fbx-Fx^2$	x^2	$-1/12Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	b-x	$-1/2Fb+Fx$	$-1/2Fb^2+3/2Fbx-Fx^2$	$b^2-2bx+x^2$		
CD b	-b	$3/2Fb-Fx$	$-3/2Fb^2+Fbx$	b^2	$-Fb^3/EJ$	Xb^3/EJ
DC b	b	$-1/2Fb-Fx$	$-1/2Fb^2-Fbx$	b^2		
DE b	-b+x	$1/2Fb-Fx+1/2qx^2$	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	$b^2-2bx+x^2$	$-1/8Fb^3/EJ$	$1/3Xb^3/EJ$
ED b	x	$-1/2qx^2$	$-1/2qx^3$	x^2		
EF √5b	0	$2\sqrt{5}/5Fx$	0	0	0	0
FG b	0	$-2Fx$	0	0	0	0
GF b	0	$2Fb-2Fx$	0	0		
GA 2b	0	$-Fb$	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$2Fb-Fx$	0	0	0	0
BF 2b	0	$-Fx$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^3/EJ	
	totali				$-5/24Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_{BE}$				1/8F	

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Sviluppi di calcolo iperstatica

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$L^{XX}_{BC} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x]_o^b\, b^2\, 1/EJ$

$= (\, b)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [\, x]_o^b\, b^2\, 1/EJ$

$= (\, b)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (1/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x^2/b -1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/3\, b)\, Fb^2\, 1/EJ = -1/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-1/2 +3/2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +3/4\, x^2/b -1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/3\, b)\, Fb^2\, 1/EJ = -1/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (-3/2 + x/b)\, Fb^2\, 1/EJ\, dx = [-3/2\, x +1/2\, x^2/b]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\, b +1/2\, b)\, Fb^2\, 1/EJ = - Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (-1/2 - x/b)\, Fb^2\, 1/EJ\, dx = [-1/2\, x -1/2\, x^2/b]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b -1/2\, b)\, Fb^2\, 1/EJ = - Fb^3/EJ$

$L^{Xo}_{DE} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{ED} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

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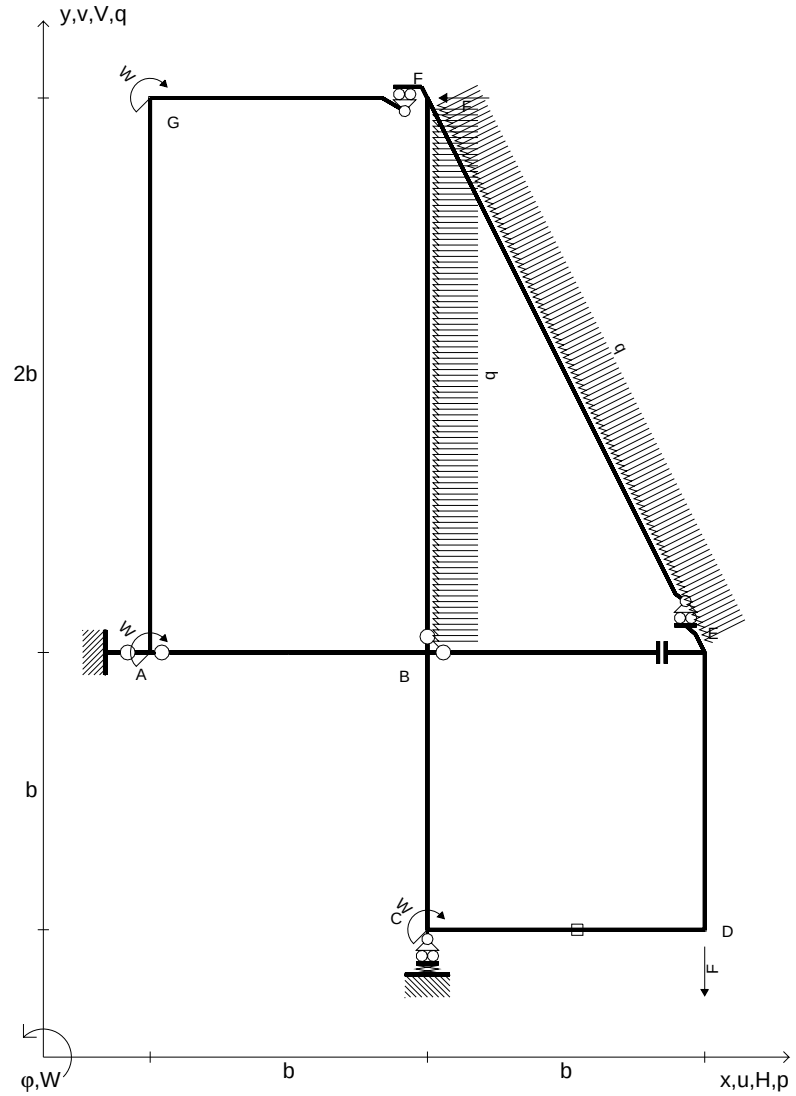
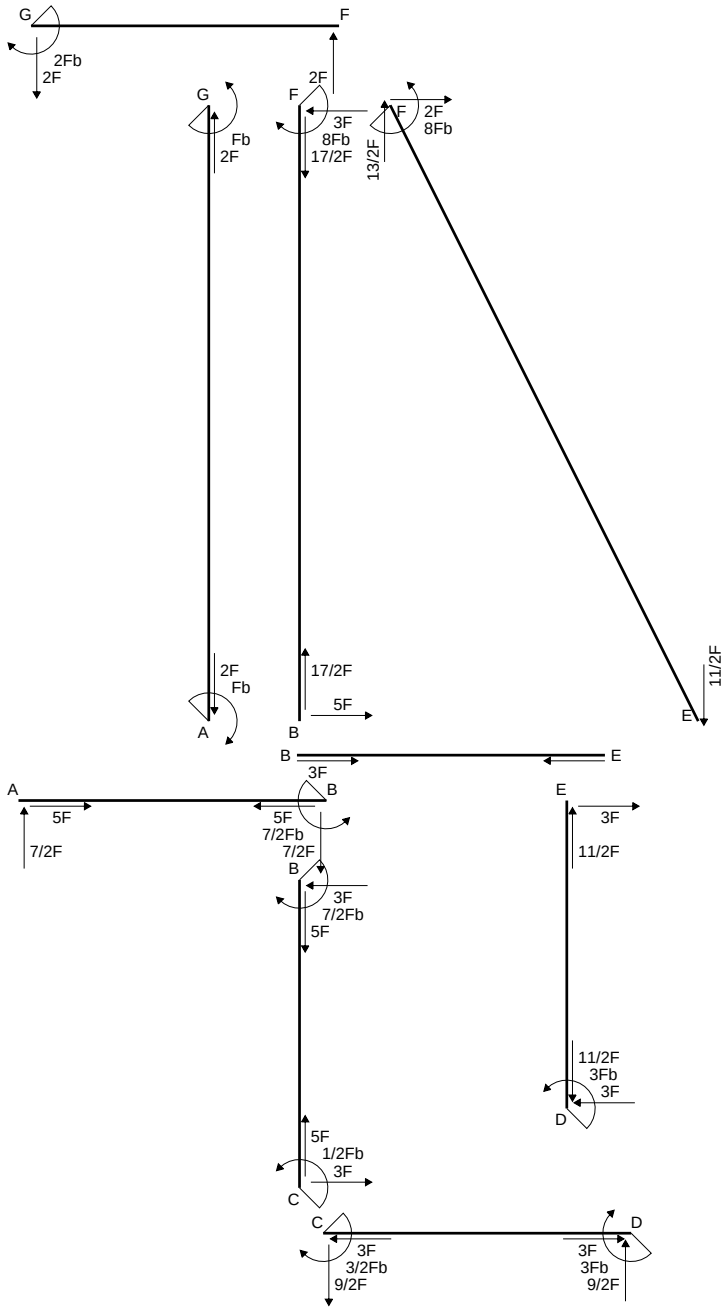
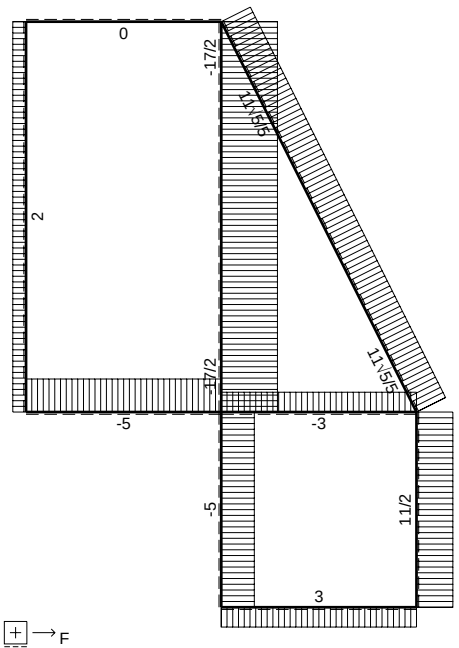
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$p_{FB} = -q = -F/b$
$p_{EF} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

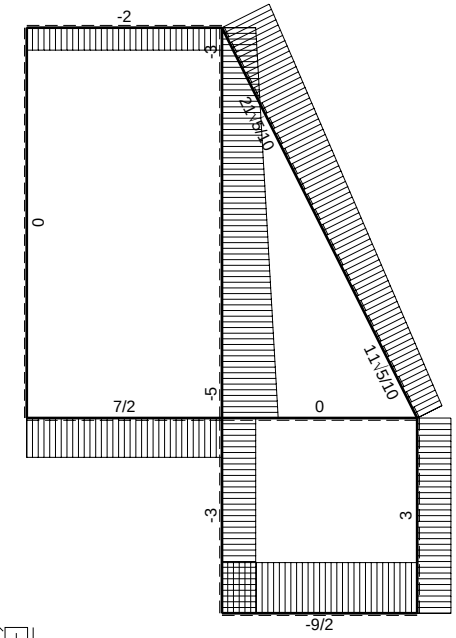


Reazioni iperstatiche in soluzione: $X=H_{BE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.

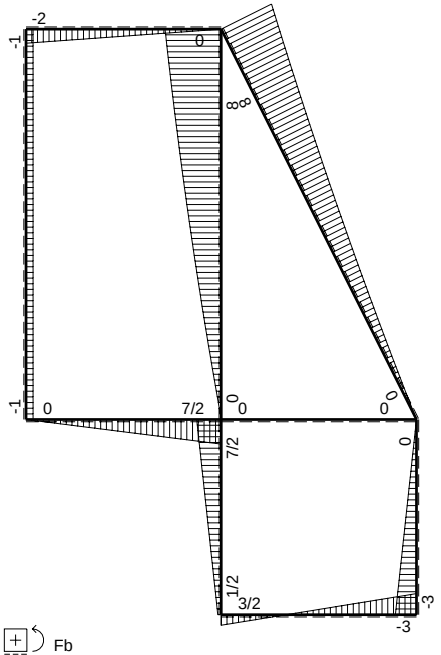




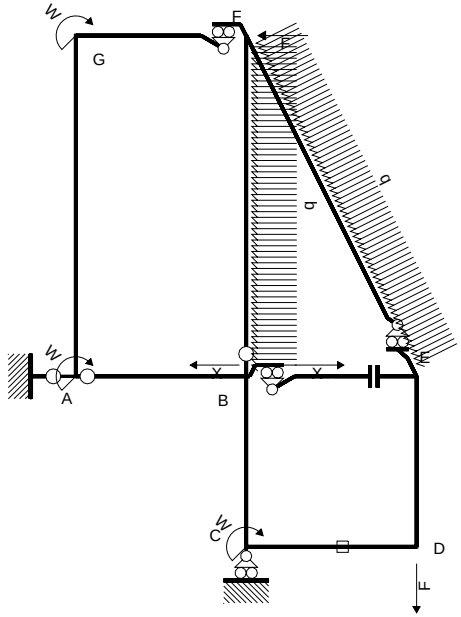
F



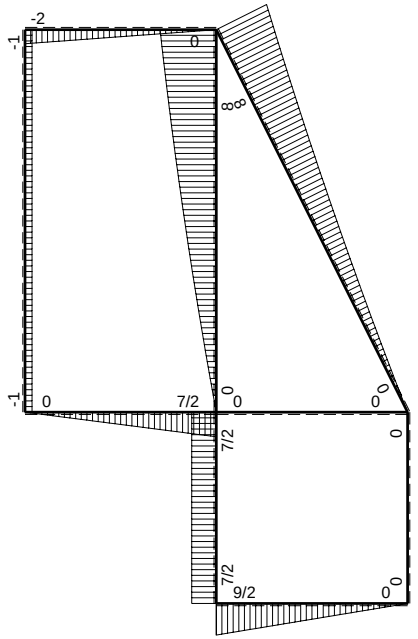
F



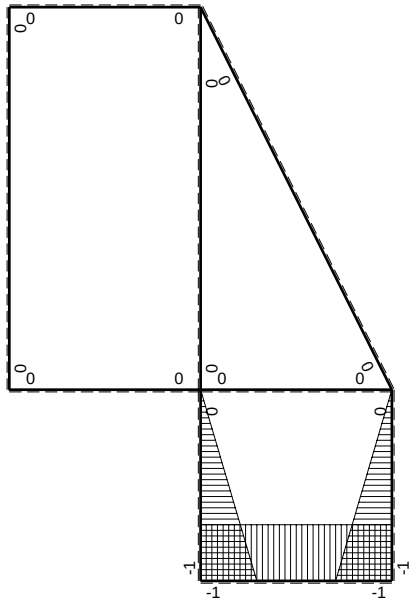
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_{BE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	7/2Fx	0	0	0	0
BA b	0	-7/2Fb+7/2Fx	0	0		
BC b	-x	7/2Fb	-7/2Fbx	x^2	$-7/4Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	b-x	-7/2Fb	$-7/2Fb^2+7/2Fbx$	$b^2-2bx+x^2$		
CD b	-b	9/2Fb-9/2Fx	$-9/2Fb^2+9/2Fbx$	b^2	$-9/4Fb^3/EJ$	Xb^3/EJ
DC b	b	-9/2Fx	-9/2Fbx	b^2		
DE b	-b+x	0	0	$b^2-2bx+x^2$	0	$1/3Xb^3/EJ$
ED b	x	0	0	x^2		
EF √5b	0	$11\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$8Fb-3Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-5Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^3/EJ$	
	totali				$-5Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_{BE}$				3F	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [x]_o^b\, b^2\, 1/EJ$

$= (b)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1)\, b^2\, 1/EJ\, dx = [x]_o^b\, b^2\, 1/EJ$

$= (b)\, b^2\, 1/EJ = b^3/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-7/2\, x/b)\, Fb^2\, 1/EJ\, dx = [-7/4\, x^2/b]_o^b\, Fb^2\, 1/EJ$

$= (-7/4\, b)\, Fb^2\, 1/EJ = -7/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-7/2 +7/2\, x/b)\, Fb^2\, 1/EJ\, dx = [-7/2\, x +7/4\, x^2/b]_o^b\, Fb^2\, 1/EJ$

$= (-7/2\, b +7/4\, b)\, Fb^2\, 1/EJ = -7/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (-9/2 +9/2\, x/b)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ$

$= [-9/2\, x +9/4\, x^2/b]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ$

$= (-9/2\, b +9/4\, b)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ = -13/4\ \ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (-9/2\, x/b)\, Fb^2\, 1/EJ\, dx + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ = [-9/4\, x^2/b]_o^b\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ$

$= (-9/4\, b)\, Fb^2\, 1/EJ + 1\ \ (-1)\ \ 1\ \ Fb^3/EJ = -13/4\ \ Fb^3/EJ$

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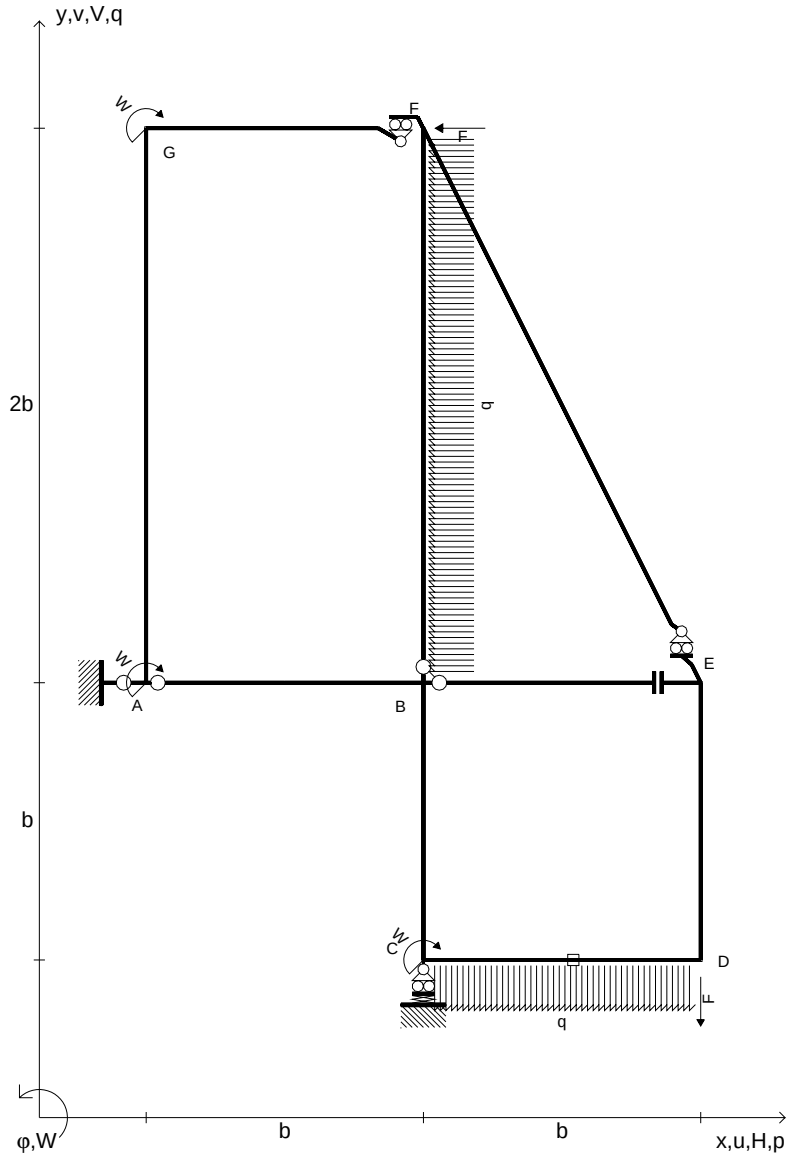
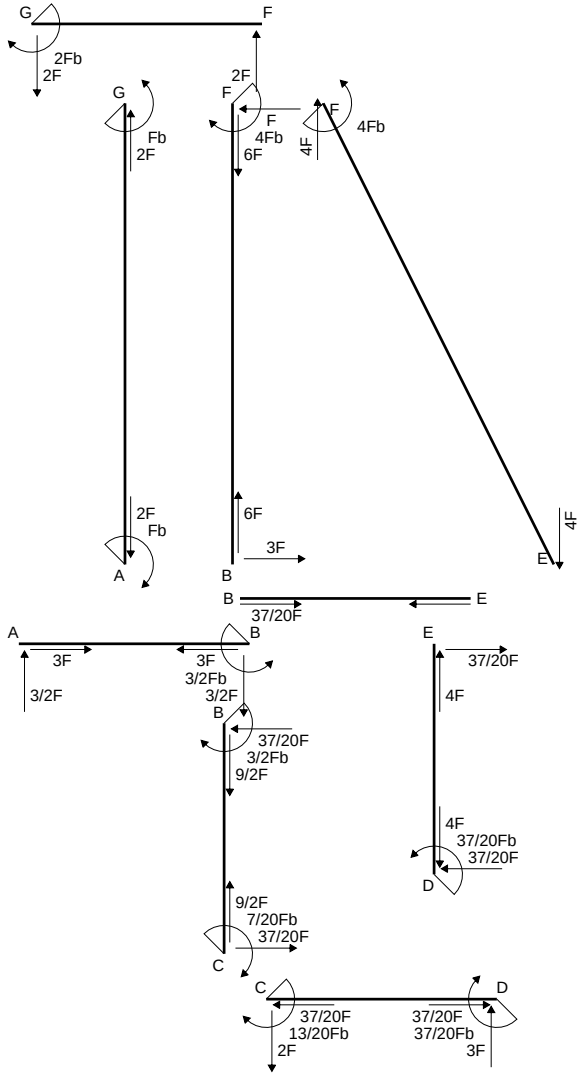
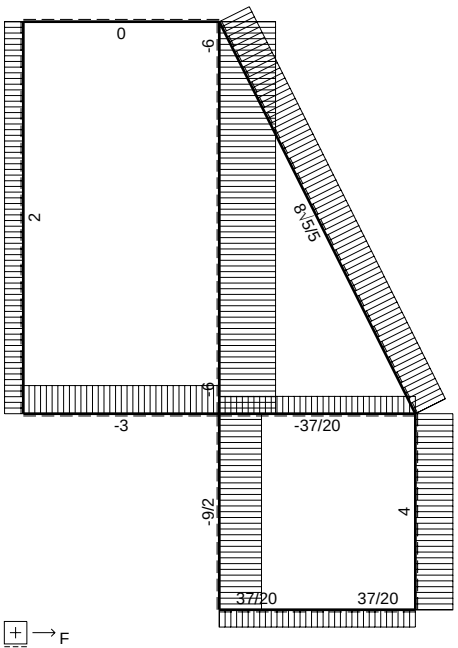
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$p_{FB} = -q = -F/b$
$q_{CD} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

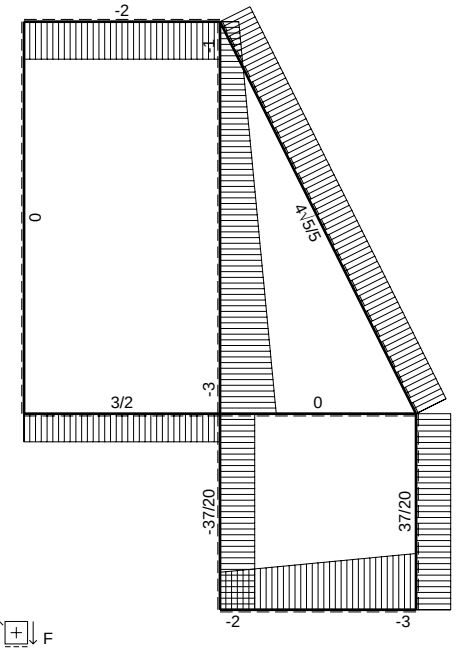


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.

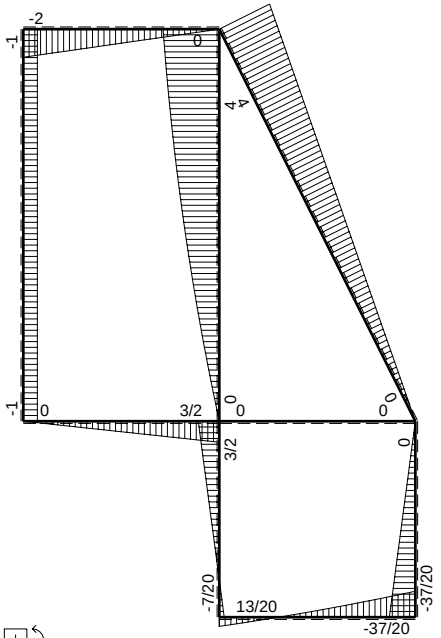




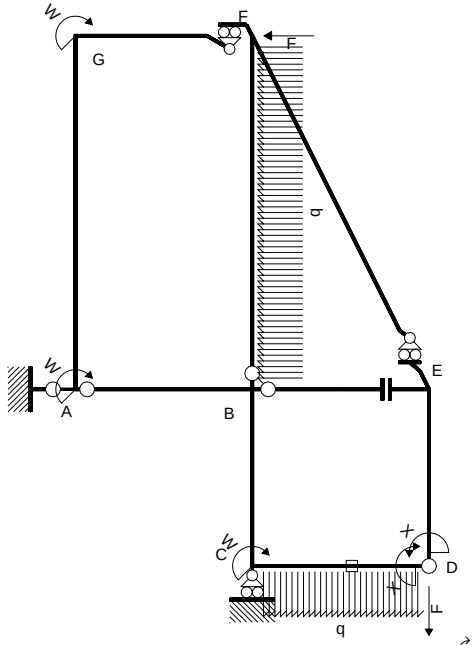
F



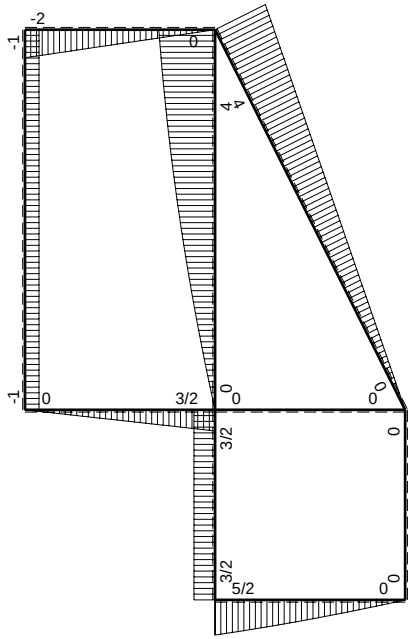
F



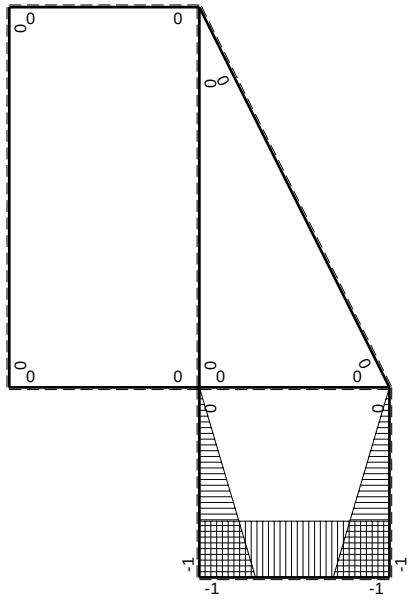
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb	-3/2Fx	x^2/b^2	$-3/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-3/2Fb	-3/2Fb+3/2Fx	$1-2x/b+x^2/b^2$		
CD b	-1	$5/2Fb-2Fx-1/2qx^2$	$-5/2Fb+2Fx+1/2Fx^2/b$	1	$-4/3Fb^2/EJ$	Xb/EJ
DC b	1	$-3Fx+1/2qx^2$	$-3Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	0	0	$1-2x/b+x^2/b^2$	0	1/3Xb/EJ
ED b	x/b	0	0	x^2/b^2		
EF $\sqrt{5}$b	0	$4\sqrt{5}/5Fx$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-37/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				37/20Fb	

Sviluppi di calcolo iperstatica

$$L_{BC}^{XX} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$

$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$

$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L_{CD}^{XX} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$

$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L_{DC}^{XX} = \int_o^b \left(1 \right) \; 1/EJ \; dx = \left[x \right]_o^b \; 1/EJ$$

$$= \left(b \right) \; 1/EJ \; = \; b/EJ$$

$$L_{DE}^{XX} = \int_o^b \left(1 - 2 \; x/b + x^2/b^2 \right) \; 1/EJ \; dx = \left[x - x^2/b + 1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$

$$= \left(b - b + 1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L_{ED}^{XX} = \int_o^b \left(x^2/b^2 \right) \; 1/EJ \; dx = \left[1/3 \; x^3/b^2 \right]_o^b \; 1/EJ$$

$$= \left(1/3 \; b \right) \; 1/EJ \; = 1/3 \; b/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(-3/2 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[-3/4 \; x^2/b \right]_o^b \; Fb \; 1/EJ$$

$$= \left(-3/4 \; b \right) \; Fb \; 1/EJ \; = -3/4 \; Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(-3/2 + 3/2 \; x/b \right) \; Fb \; 1/EJ \; dx = \left[-3/2 \; x + 3/4 \; x^2/b \right]_o^b \; Fb \; 1/EJ$$

$$= \left(-3/2 \; b + 3/4 \; b \right) \; Fb \; 1/EJ \; = -3/4 \; Fb^2/EJ$$

$$L_{CD}^{Xo} = \int_o^b \left(-5/2 + 2 \; x/b + 1/2 \; x^2/b^2 \right) \; Fb \; 1/EJ \; dx + 1 \quad (-1) \; 1 \; Fb^2/EJ$$

$$= \left[-5/2 \; x + x^2/b + 1/6 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ \; + 1 \quad (-1) \; 1 \; Fb^2/EJ$$

$$= \left(-5/2 \; b + b + 1/6 \; b \right) \; Fb \; 1/EJ \; + 1 \quad (-1) \; 1 \; Fb^2/EJ = -7/3 \; Fb^2/EJ$$

$$L_{DC}^{Xo} = \int_o^b \left(-3 \; x/b + 1/2 \; x^2/b^2 \right) \; Fb \; 1/EJ \; dx + 1 \quad (-1) \; 1 \; Fb^2/EJ$$

$$= \left[-3/2 \; x^2/b + 1/6 \; x^3/b^2 \right]_o^b \; Fb \; 1/EJ \; + 1 \quad (-1) \; 1 \; Fb^2/EJ$$

$$= \left(-3/2 \; b + 1/6 \; b \right) \; Fb \; 1/EJ \; + 1 \quad (-1) \; 1 \; Fb^2/EJ = -7/3 \; Fb^2/EJ$$

ESΣ02.xxxx.200

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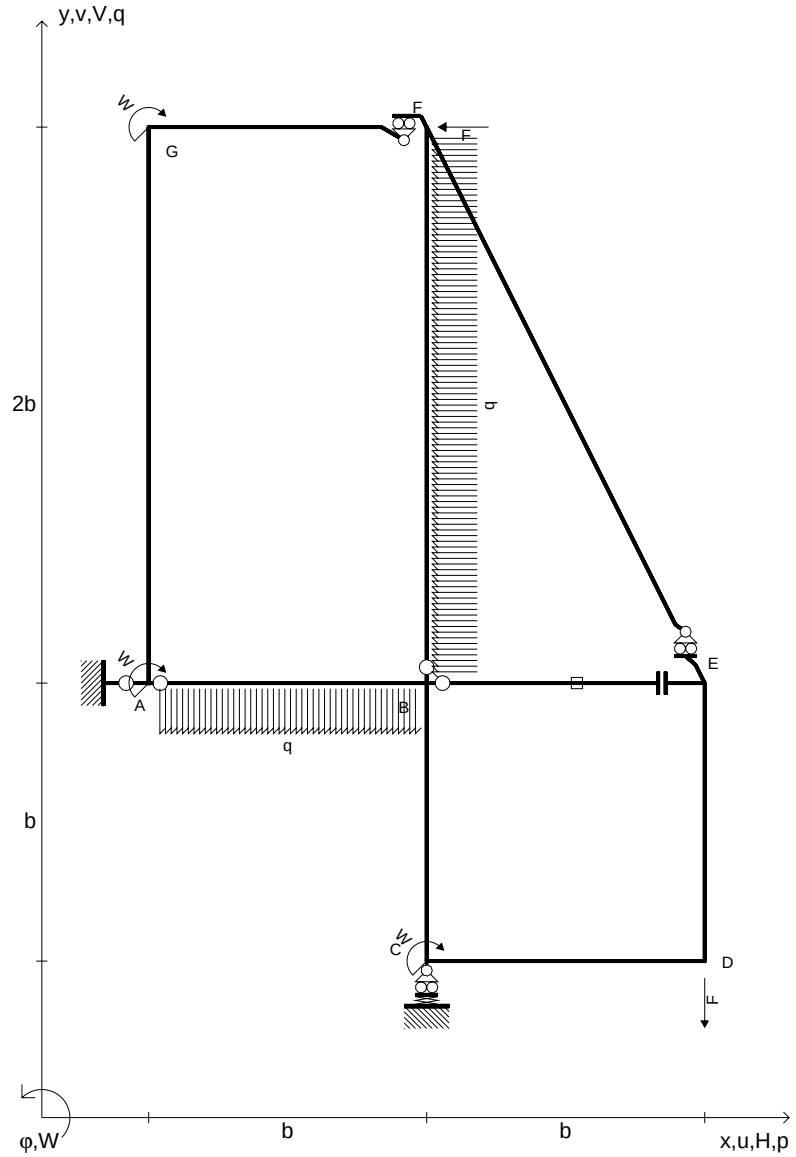
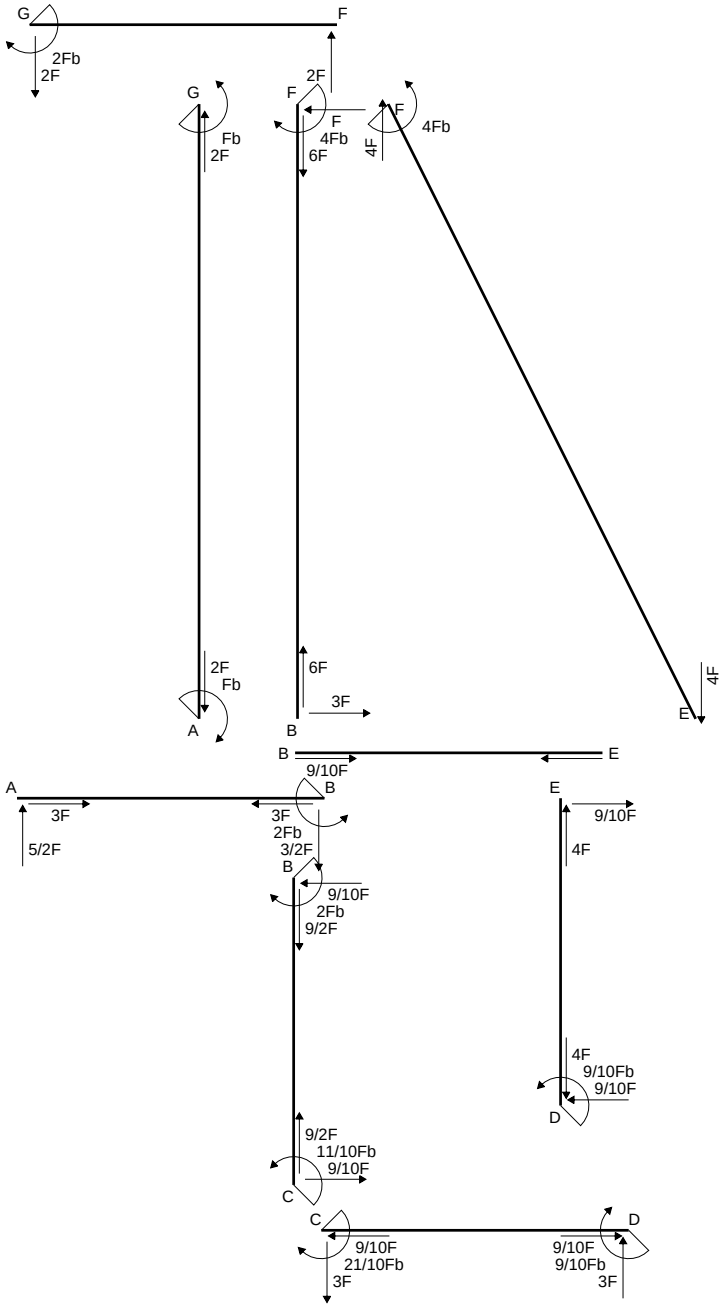
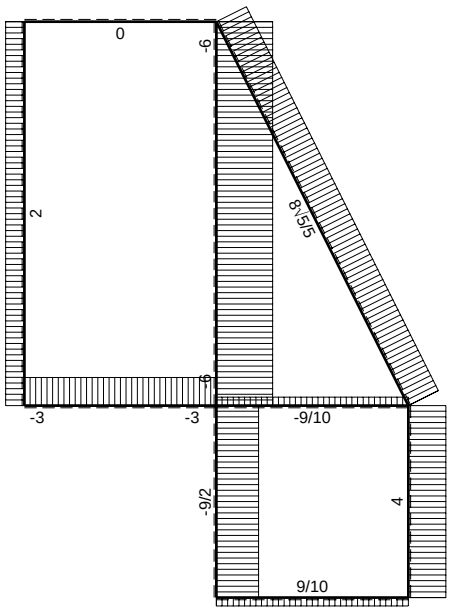
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$p_{FB} = -q = -F/b$
$q_{AB} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

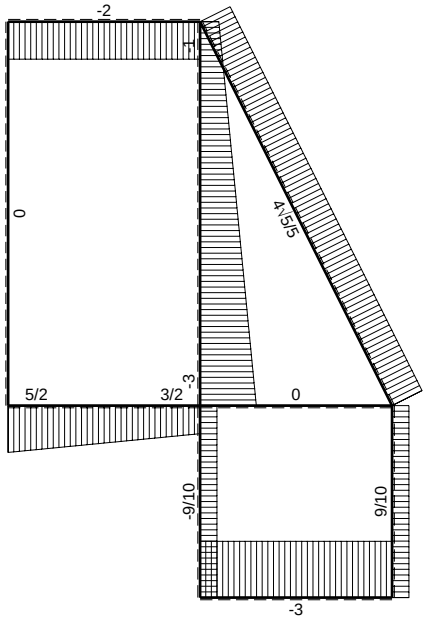


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.

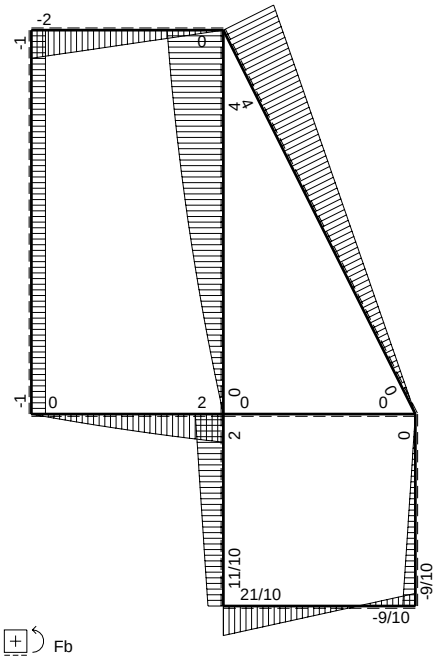




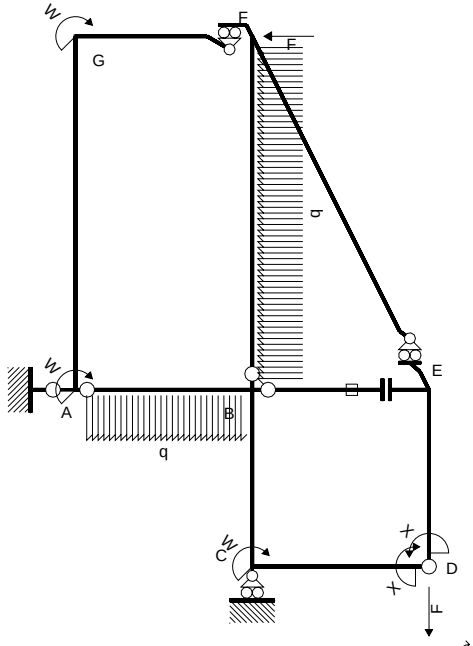
←[+]→ F



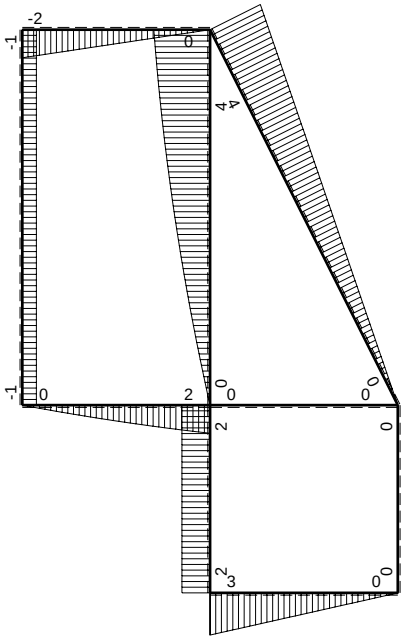
↑[+]↓ F



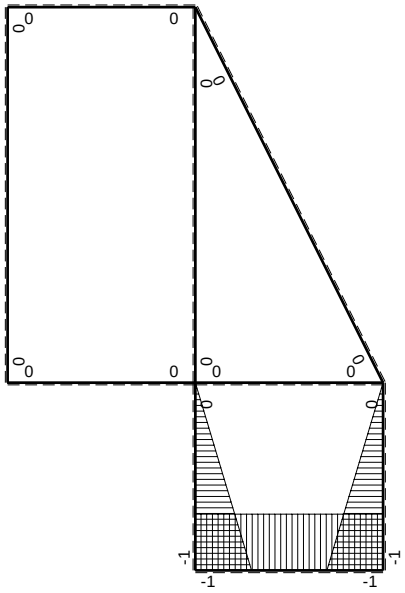
[+] Fb



Schema di calcolo iperstatico



[+] M_o flessione da carichi assegnati



[+] M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	$5/2Fx-1/2qx^2$	0	0	0	0
BA b	0	$-2Fb+3/2Fx+1/2qx^2$	0	0		
BC b	-x/b	2Fb	-2Fx	x^2/b^2	$-Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-2Fb	-2Fb+2Fx	$1-2x/b+x^2/b^2$		
CD b	-1	3Fb-3Fx	-3Fb+3Fx	1	$-3/2Fb^2/EJ$	Xb/EJ
DC b	1	-3Fx	-3Fx	1		
DE b	-1+x/b	0	0	$1-2x/b+x^2/b^2$	0	1/3Xb/EJ
ED b	x/b	0	0	x^2/b^2		
EF √5b	0	4√5/5Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BF 2b	0	$-3Fx+1/2qx^2$	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$-3/2Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				9/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-2\ x/b\ )\ Fb\ 1/EJ\ dx = [-\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (-2 +2\ x/b\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-2\ b + b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (-3 +3\ x/b\ )\ Fb\ 1/EJ\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3/2\ b\ )\ Fb\ 1/EJ\ = -3/2\ \ Fb^2/EJ$

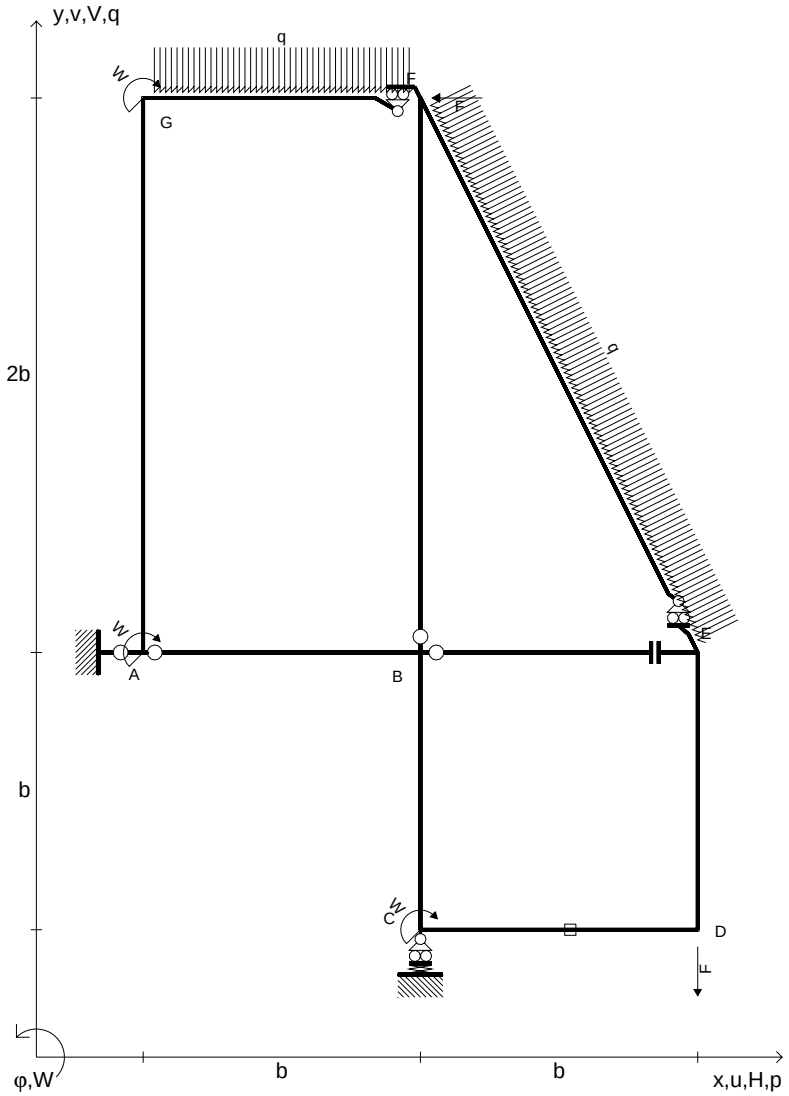
$L^{Xo}_{DC} = \int_o^b (-3\ x/b\ )\ Fb\ 1/EJ\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-3/2\ b\ )\ Fb\ 1/EJ\ = -3/2\ \ Fb^2/EJ$

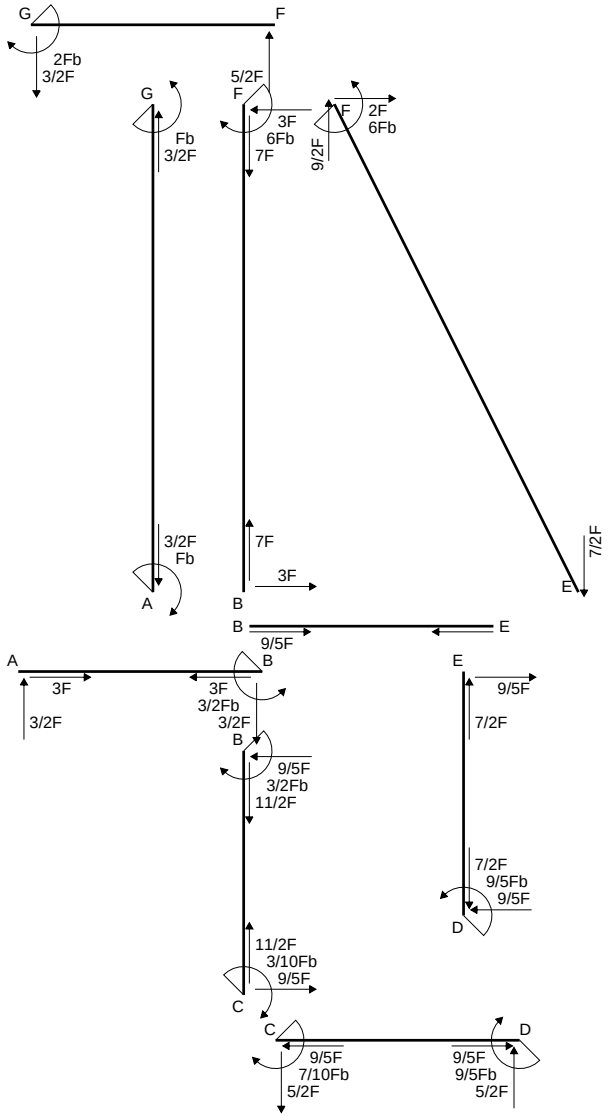
ESΣ02.xxxx.201

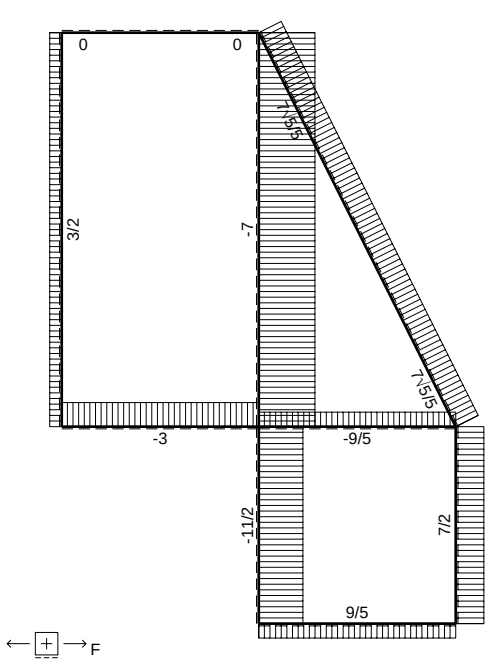
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 08.12.24

$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$q_{FG} = -q = -F/b$
$p_{EF} = -q = -F/b$
$q_{EF} = -q = -F/b$
$\varepsilon_{CD} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

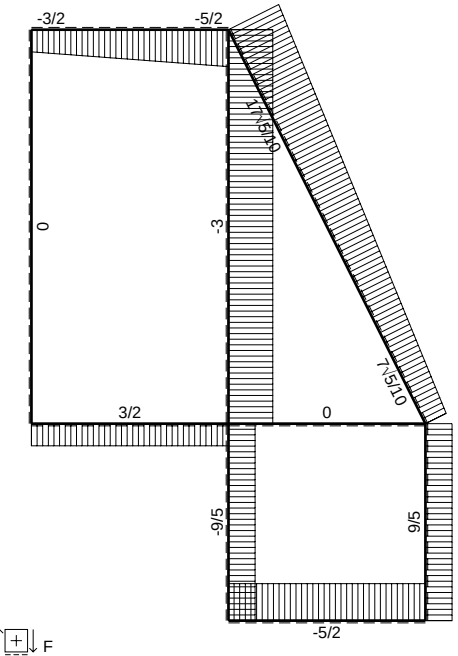


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
Diagrammi di carico con valori riferiti ad asse della trave.
Componenti di carico distribuito riferiti ad assi ortogonali.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.

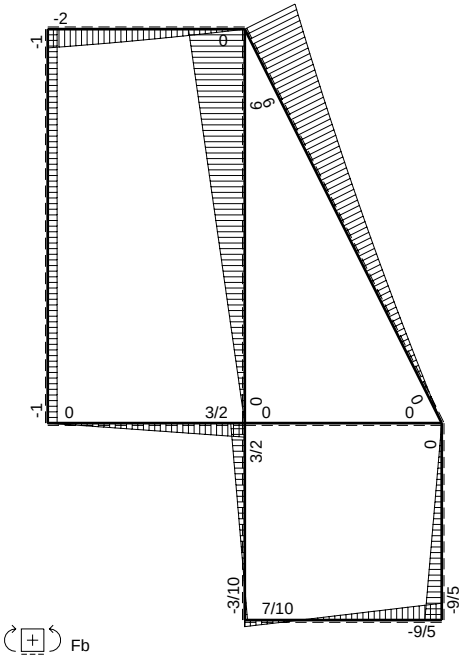




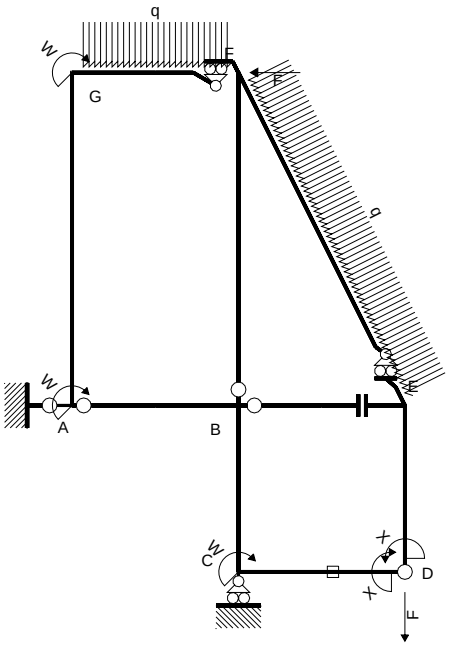
F



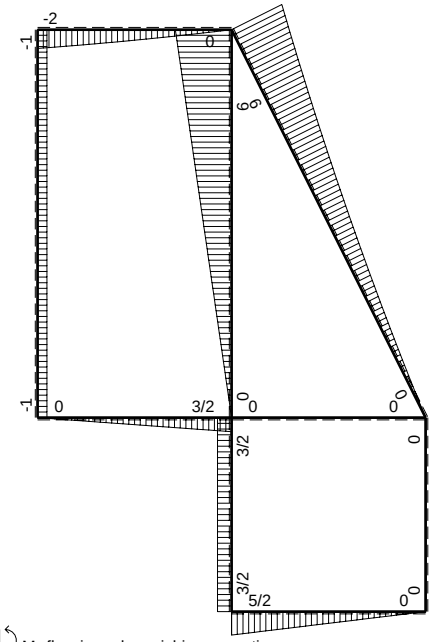
F



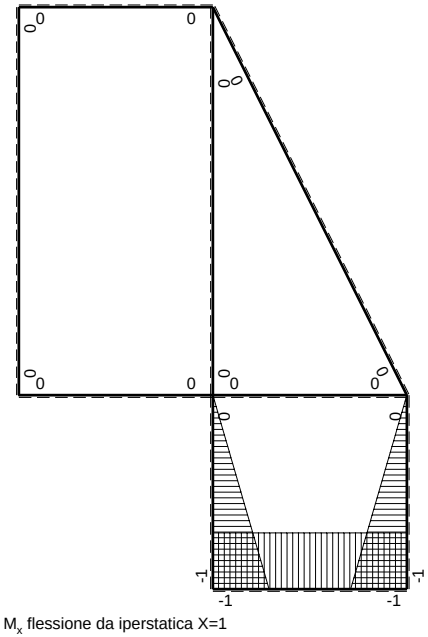
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	3/2Fx	0	0	0	0
BA b	0	-3/2Fb+3/2Fx	0	0		
BC b	-x/b	3/2Fb	-3/2Fx	x^2/b^2	$-3/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	-3/2Fb	-3/2Fb+3/2Fx	$1-2x/b+x^2/b^2$		
CD b	-1	5/2Fb-5/2Fx	-5/2Fb+5/2Fx	1	$-5/4Fb^2/EJ$	Xb/EJ
DC b	1	-5/2Fx	-5/2Fx	1		
DE b	-1+x/b	0	0	$1-2x/b+x^2/b^2$	0	1/3Xb/EJ
ED b	x/b	0	0	x^2/b^2		
EF $\sqrt{5}$b	0	$7\sqrt{5}/10Fx+1/2qx^2$	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	6Fb-3Fx	0	0	0	0
BF 2b	0	-3Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
CD	elongazione asta $N_{1CD}\varepsilon_{CD}L_{CD}$				$-Fb^2/EJ$	
	totali				$-3Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{DE}$				9/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BC} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{DE} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{BC} = \int_o^b (-3/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-3/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-3/4\ b\ )\ Fb\ 1/EJ\ = -3/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (-3/2 +3/2\ x/b\ )\ Fb\ 1/EJ\ dx = [-3/2\ x +3/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ$

$= (-3/2\ b +3/4\ b\ )\ Fb\ 1/EJ\ = -3/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -9/4\ \ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + 1\ \ (-1)\ \ 1\ \ Fb^2/EJ = -9/4\ \ Fb^2/EJ$

ESΣ02.xxxx.211

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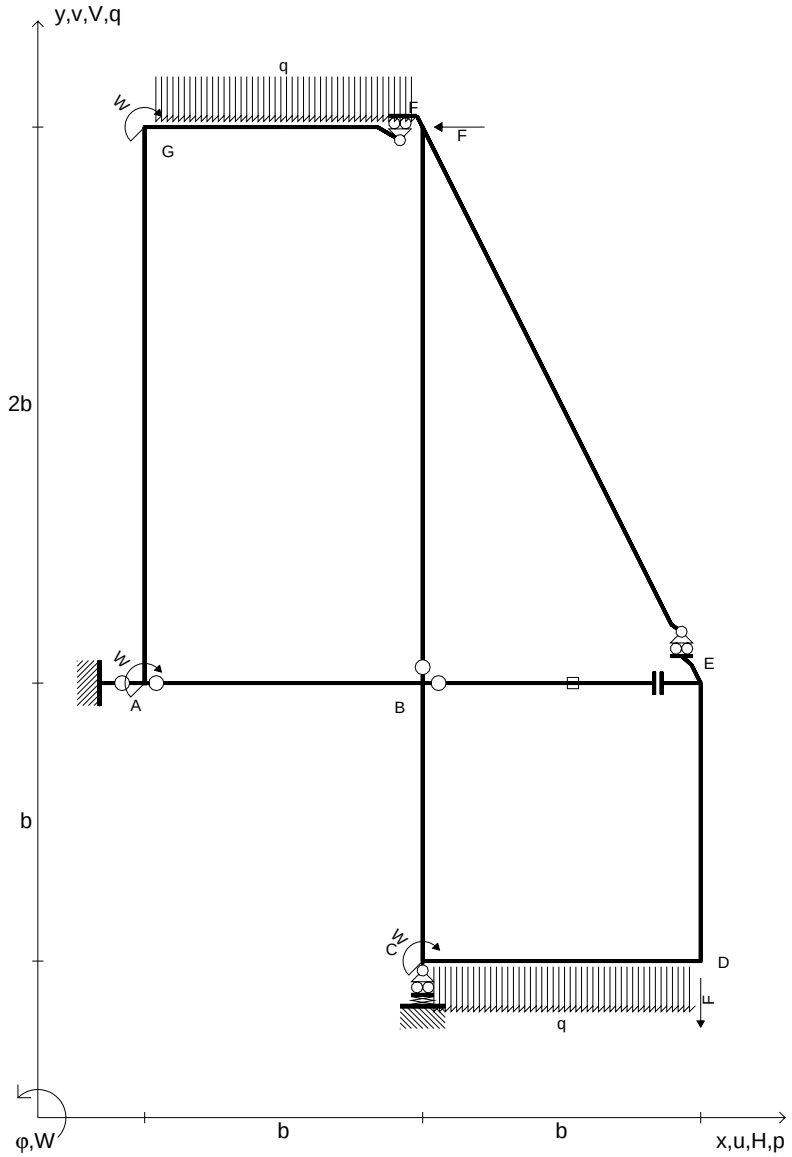
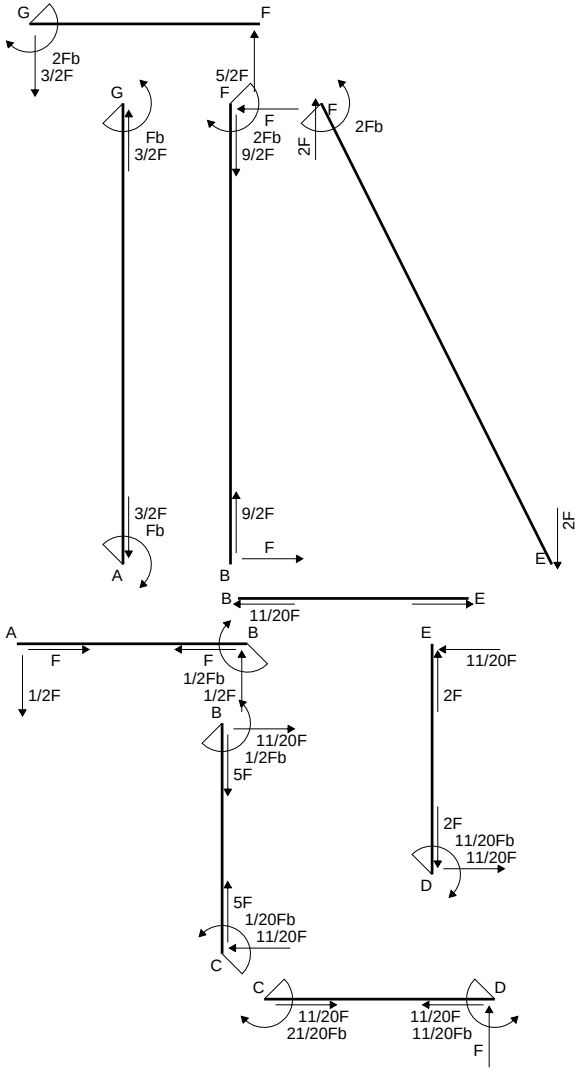
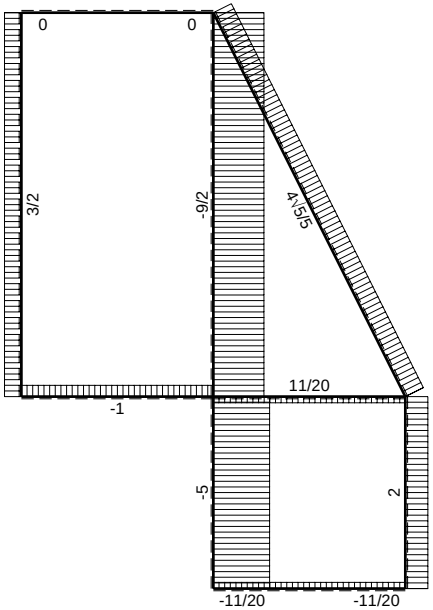
$H_{FB} = -F$
$V_D = -F$
$W_C = -W = -Fb$
$W_G = -W = -Fb$
$W_A = -W = -Fb$
$q_{FG} = -q = -F/b$
$q_{CD} = -q = -F/b$
$\varepsilon_{BE} = -\alpha T = -b^2F/EJ$
$k_C = 4EJ/b^3$
$EJ_{AB} = EJ$
$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{DE} = EJ$
$EJ_{EF} = EJ$
$EJ_{FG} = EJ$
$EJ_{GA} = EJ$
$EJ_{FB} = EJ$
$EJ_{BE} = EJ$

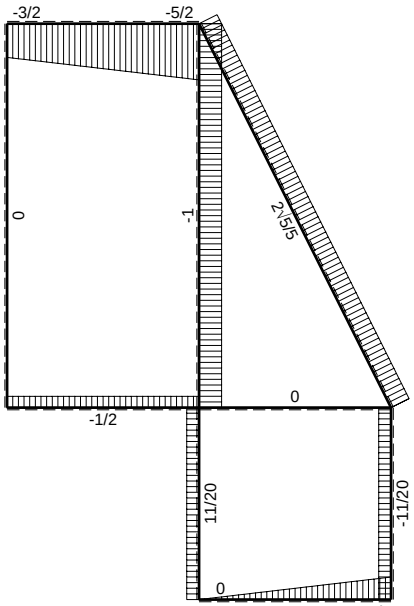


Reazioni iperstatiche in soluzione: $X=W_{DE}$
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi quotati delle azioni interne nelle aste.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.

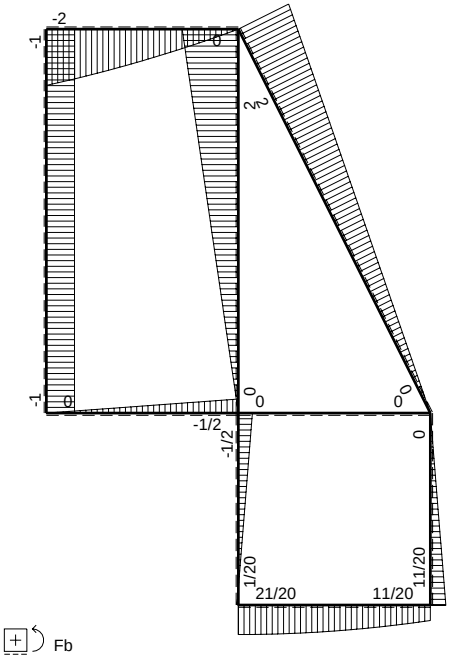




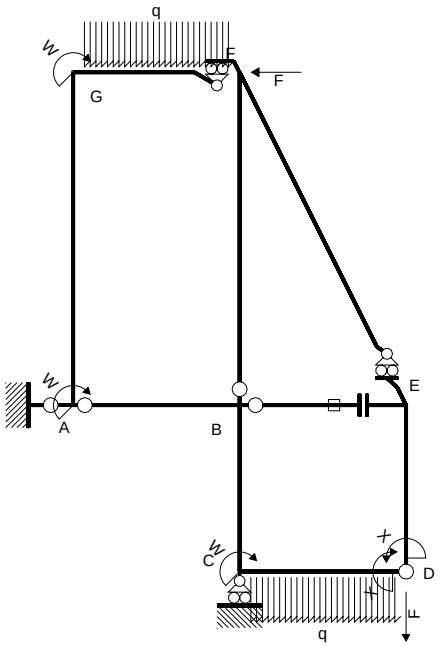
←□→ F



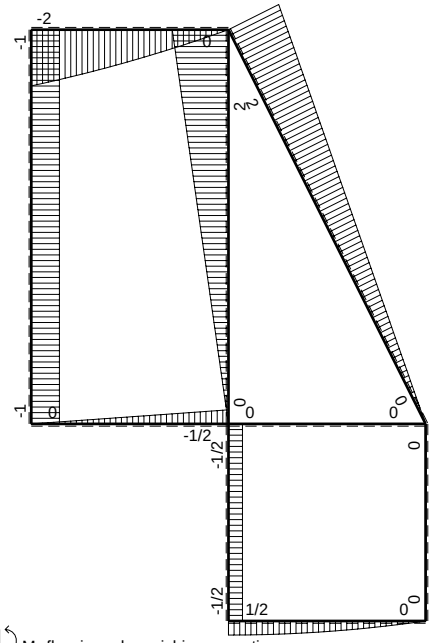
↑□↓ F



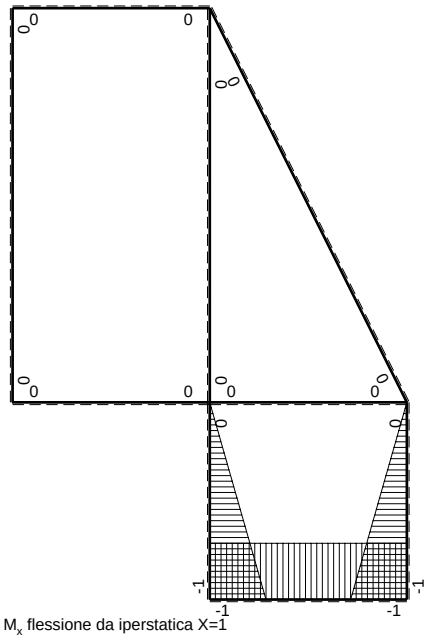
↻□↺ Fb



Schema di calcolo iperstatico



↻□↺ M_o flessione da carichi assegnati



↻□↺ M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{DE}$

→	$M_x(x)$	$M_o(x)$	M_xM_o	M_xM_x	$\int M_xM_o/EJdx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fx	0	0	0	0
BA b	0	1/2Fb-1/2Fx	0	0		
BC b	-x/b	-1/2Fb	1/2Fx	x^2/b^2	$1/4Fb^2/EJ$	1/3Xb/EJ
CB b	1-x/b	1/2Fb	1/2Fb-1/2Fx	$1-2x/b+x^2/b^2$		
CD b	-1	$1/2Fb-1/2qx^2$	$-1/2Fb+1/2Fx^2/b$	1	$-1/3Fb^2/EJ$	Xb/EJ
DC b	1	$-Fx+1/2qx^2$	$-Fx+1/2Fx^2/b$	1		
DE b	-1+x/b	0	0	$1-2x/b+x^2/b^2$	0	1/3Xb/EJ
ED b	x/b	0	0	x^2/b^2		
EF √5b	0	2√5/5Fx	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0		
GA 2b	0	-Fb	0	0	0	0
AG 2b	0	Fb	0	0		
FB 2b	0	2Fb-Fx	0	0	0	0
BF 2b	0	-Fx	0	0		
BE b	0	0	0	0	0	0
EB b	0	0	0	0		
BE	elongazione asta $N_{1BE}\varepsilon_{BE}L_{BE}$				Fb^2/EJ	
	totali				$11/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{DE}$				-11/20Fb	

Sviluppi di calcolo iperstatica

$$L_{BC}^{XX} = \int_o^b \left(x^2/b^2 \right) \, 1/EJ \, dx = \left[1/3 \, x^3/b^2 \right]_o^b \, 1/EJ$$
$$= \left(1/3 \, b \right) \, 1/EJ = 1/3 \, b/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1 - 2 \, x/b + x^2/b^2 \right) \, 1/EJ \, dx = \left[x - x^2/b + 1/3 \, x^3/b^2 \right]_o^b \, 1/EJ$$
$$= \left(b - b + 1/3 \, b \right) \, 1/EJ = 1/3 \, b/EJ$$

$$L_{CD}^{XX} = \int_o^b \left(1 \right) \, 1/EJ \, dx = \left[x \right]_o^b \, 1/EJ$$
$$= \left(b \right) \, 1/EJ = b/EJ$$

$$L_{DC}^{XX} = \int_o^b \left(1 \right) \, 1/EJ \, dx = \left[x \right]_o^b \, 1/EJ$$
$$= \left(b \right) \, 1/EJ = b/EJ$$

$$L_{DE}^{XX} = \int_o^b \left(1 - 2 \, x/b + x^2/b^2 \right) \, 1/EJ \, dx = \left[x - x^2/b + 1/3 \, x^3/b^2 \right]_o^b \, 1/EJ$$
$$= \left(b - b + 1/3 \, b \right) \, 1/EJ = 1/3 \, b/EJ$$

$$L_{ED}^{XX} = \int_o^b \left(x^2/b^2 \right) \, 1/EJ \, dx = \left[1/3 \, x^3/b^2 \right]_o^b \, 1/EJ$$
$$= \left(1/3 \, b \right) \, 1/EJ = 1/3 \, b/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(1/2 \, x/b \right) \, Fb \, 1/EJ \, dx = \left[1/4 \, x^2/b \right]_o^b \, Fb \, 1/EJ$$
$$= \left(1/4 \, b \right) \, Fb \, 1/EJ = 1/4 \, Fb^2/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(1/2 - 1/2 \, x/b \right) \, Fb \, 1/EJ \, dx = \left[1/2 \, x - 1/4 \, x^2/b \right]_o^b \, Fb \, 1/EJ$$
$$= \left(1/2 \, b - 1/4 \, b \right) \, Fb \, 1/EJ = 1/4 \, Fb^2/EJ$$

$$L_{CD}^{Xo} = \int_o^b \left(-1/2 + 1/2 \, x^2/b^2 \right) \, Fb \, 1/EJ \, dx = \left[-1/2 \, x + 1/6 \, x^3/b^2 \right]_o^b \, Fb \, 1/EJ$$
$$= \left(-1/2 \, b + 1/6 \, b \right) \, Fb \, 1/EJ = -1/3 \, Fb^2/EJ$$

$$L_{DC}^{Xo} = \int_o^b \left(- \, x/b + 1/2 \, x^2/b^2 \right) \, Fb \, 1/EJ \, dx = \left[-1/2 \, x^2/b + 1/6 \, x^3/b^2 \right]_o^b \, Fb \, 1/EJ$$
$$= \left(-1/2 \, b + 1/6 \, b \right) \, Fb \, 1/EJ = -1/3 \, Fb^2/EJ$$

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