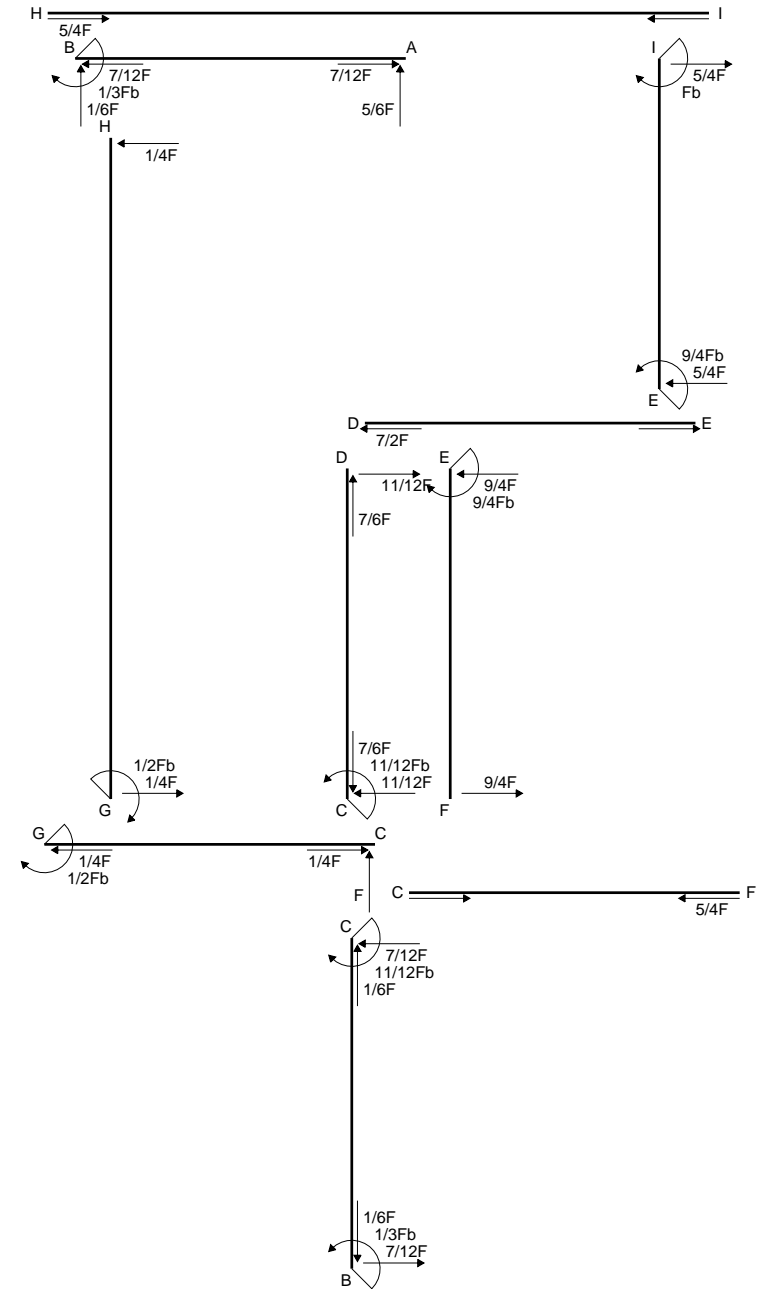
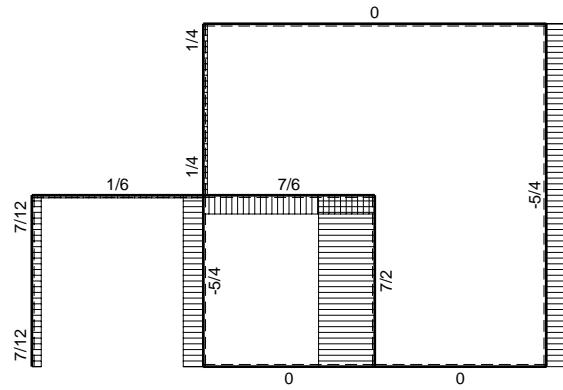


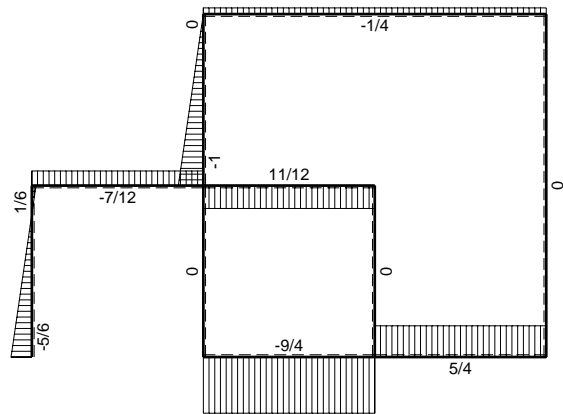
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3 F/EJ$                    | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=W_{BC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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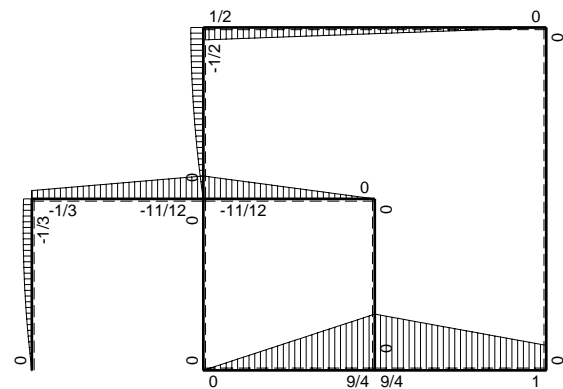




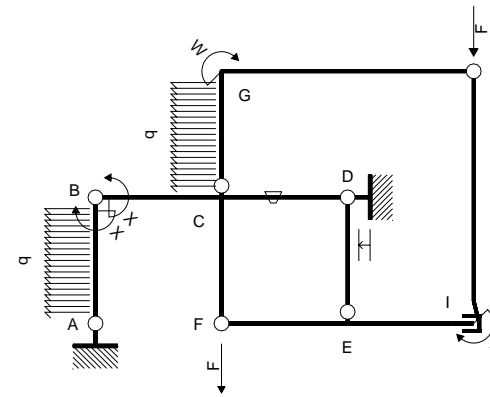
← (+) → F



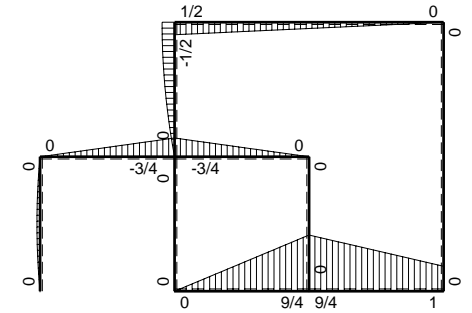
↑ (+) ↓ F



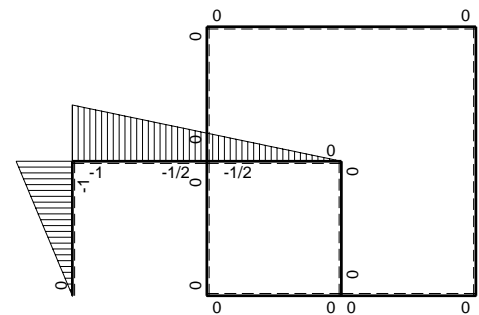
⤵ (+) ⤴ F<sub>b</sub>



Schema di calcolo iperstatico



⤵ (+) ⤴ M<sub>o</sub> flessione da carichi assegnati



⤵ (+) ⤴ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | 0        | $3/4Fx-3/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | 0        | $3/8Fb-3/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $-1/3Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $1/3Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

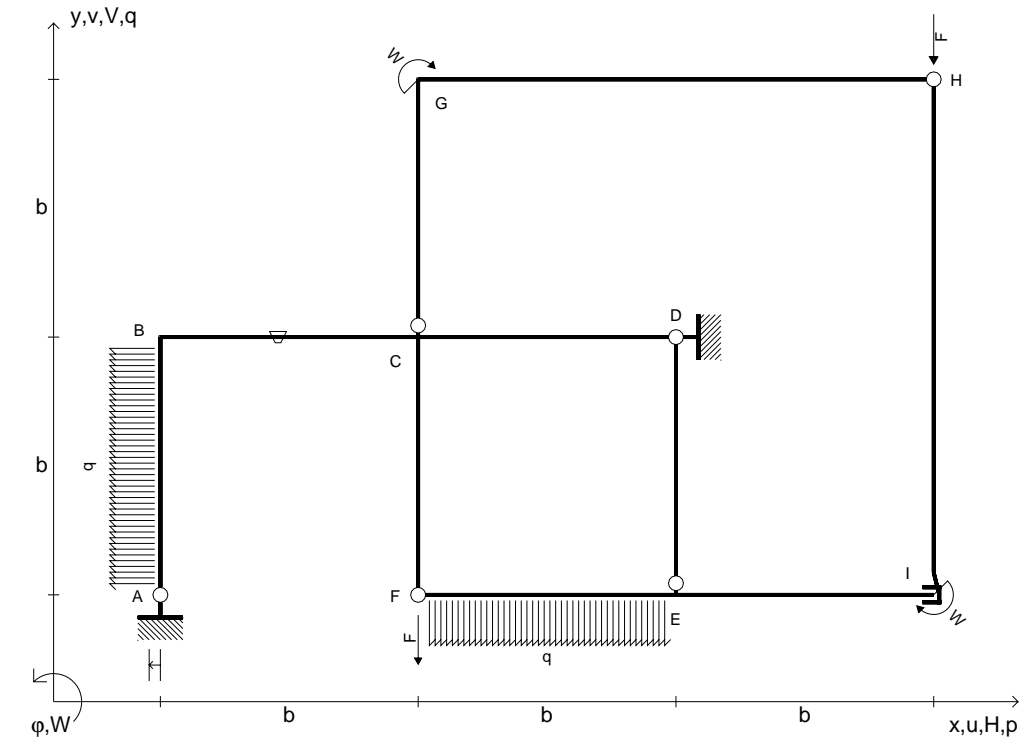
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

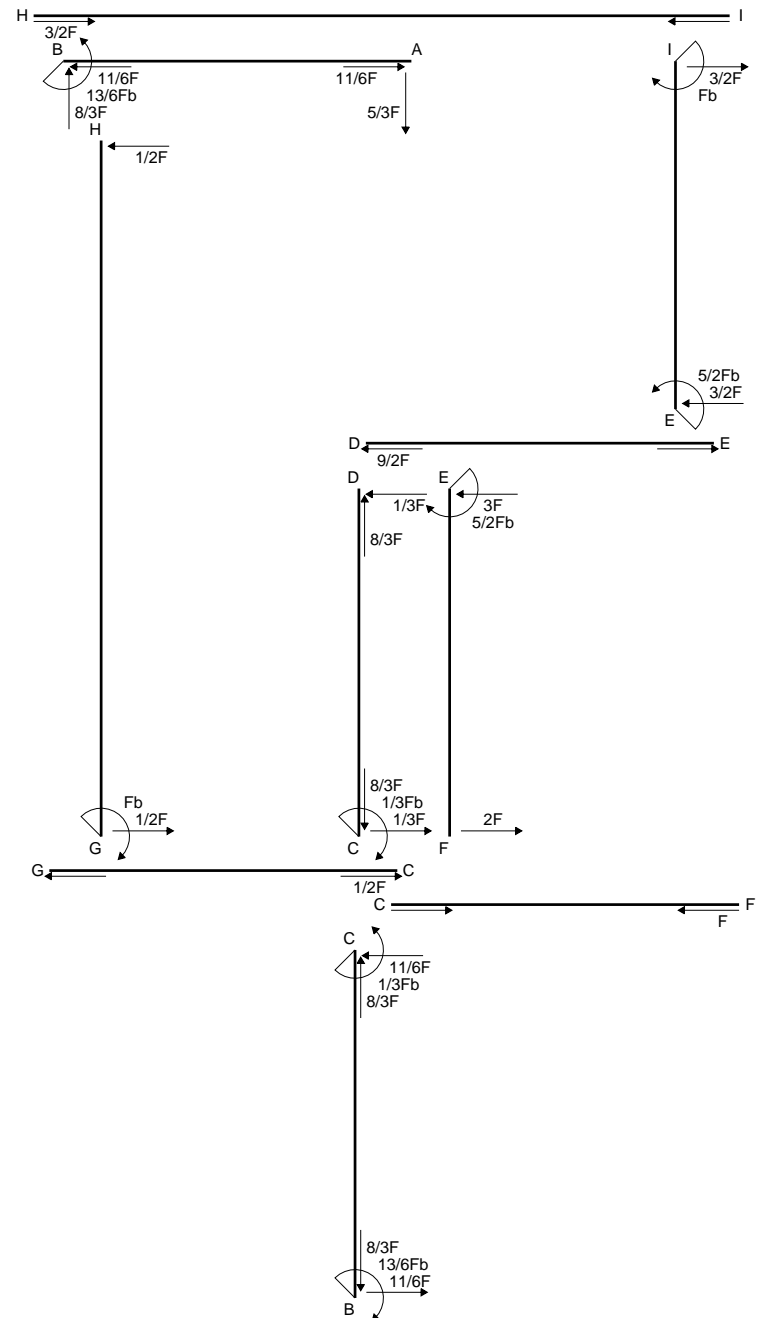
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

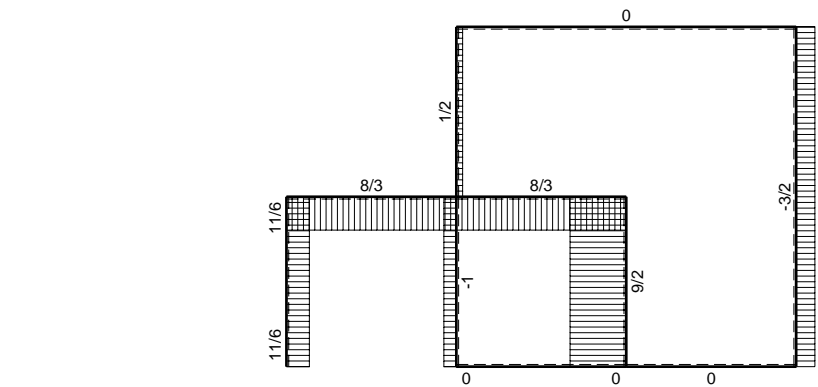
$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$



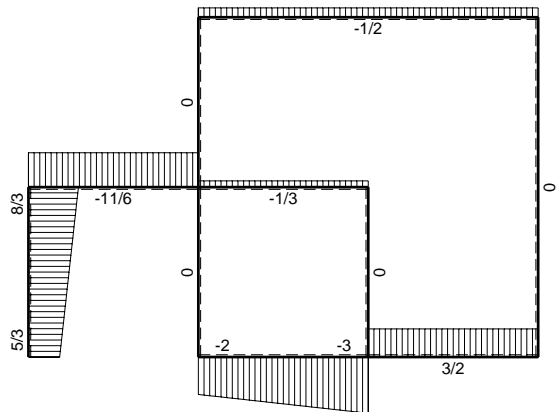
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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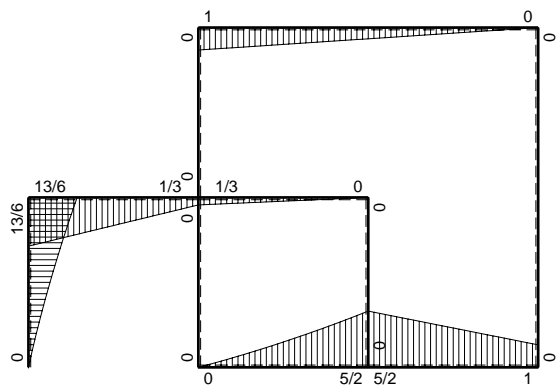




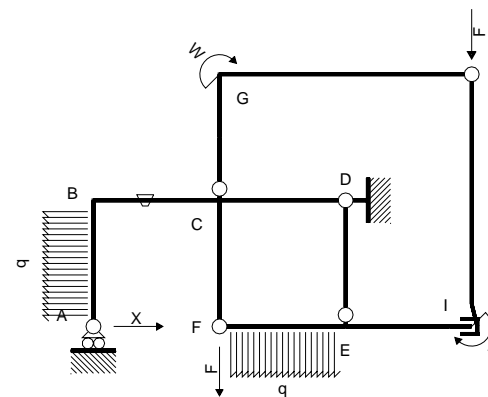
← (+) → F



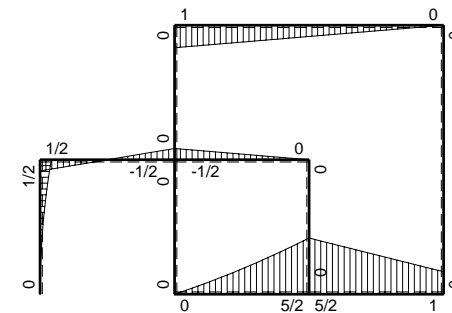
↑ (+) ↓ F



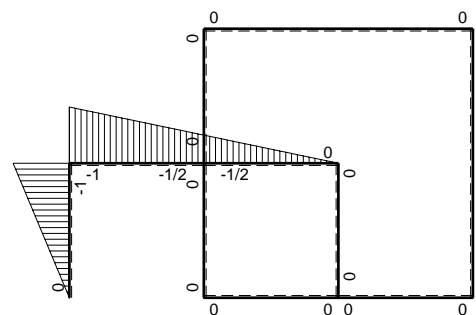
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>0</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|---------------------|----------|-----------------------------------|------------------------|-----------------------|-----------------------------|--------------------------|
| AB b  | -x                          | $1/2qx^2$           | 0        | $-1/2qx^3$                        | 0                      | $x^2$                 | $(-1/8+0)Fb^3/EJ$           | $1/3Xb^3/EJ$             |
| BA b  | b-x                         | $-1/2Fb+Fx-1/2qx^2$ | 0        | $-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$ | 0                      | $b^2-2bx+x^2$         |                             |                          |
| BC b  | -b+1/2x                     | $1/2Fb-Fx$          | $-Fb/EJ$ | $-1/2Fb^2+5/4Fbx-1/2Fx^2$         | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(-1/24+3/4)Fb^3/EJ$        | $7/12Xb^3/EJ$            |
| CB b  | $1/2b+1/2x$                 | $1/2Fb-Fx$          | $Fb/EJ$  | $1/4Fb^2-1/4Fbx-1/2Fx^2$          | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                          |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb^2-1/2Fbx+1/4Fx^2$          | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+0)Fb^3/EJ$           | $1/12Xb^3/EJ$            |
| DC b  | $1/2x$                      | $1/2Fx$             | 0        | $1/4Fx^2$                         | 0                      | $1/4x^2$              |                             |                          |
| DE b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                        |
| ED b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                          |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                        |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                                 | 0                      | 0                     |                             |                          |
| FC b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                        |
| CF b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                          |
| CG b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                        |
| GC b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                          |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                        |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                                 | 0                      | 0                     |                             |                          |
| HI 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                        |
| IH 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                          |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                        |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                                 | 0                      | 0                     |                             |                          |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                                   |                        |                       | $Fb^3/EJ$                   |                          |
|       | totali                      |                     |          |                                   |                        |                       | $5/3Fb^3/EJ$                | $Xb^3/EJ$                |
|       | iperstatica $X=H_A$         |                     |          |                                   |                        |                       | $-5/3F$                     |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-1/2 x^3/b^3) Fb^2 1/EJ dx = [-1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [-1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + 5/4 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [-1/2 x + 5/8 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (-1/2 b + 5/8 b - 1/6 b) Fb^2 1/EJ + (b - 1/4 b) \theta = 17/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/8 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/8 b - 1/6 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = 17/24 Fb^3/EJ$$

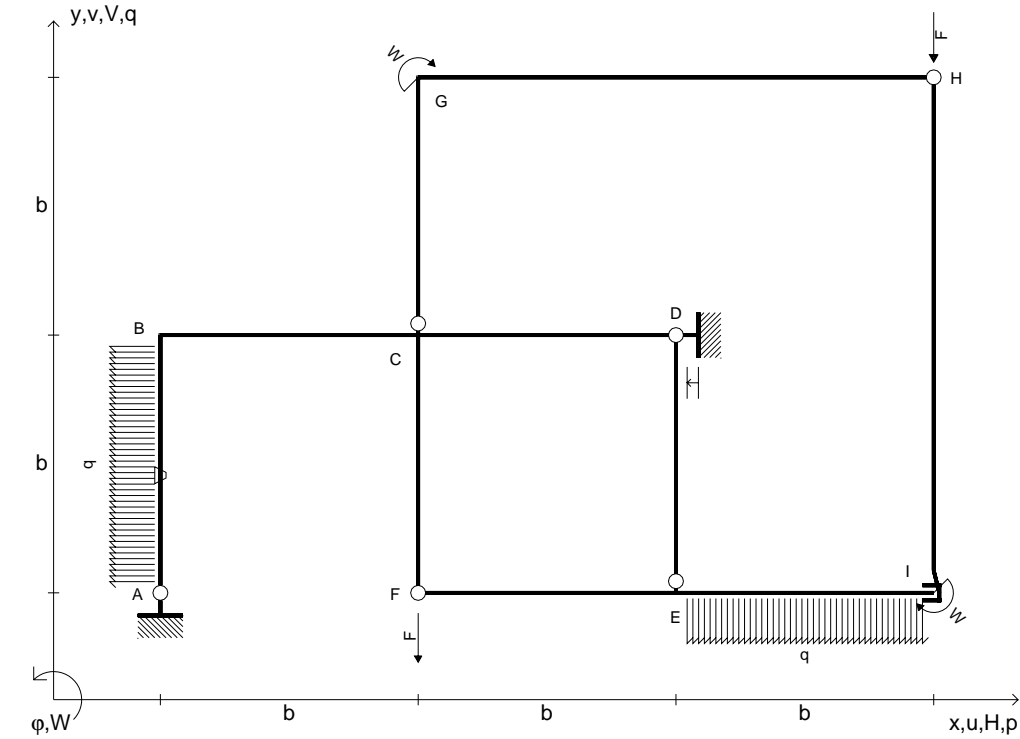
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

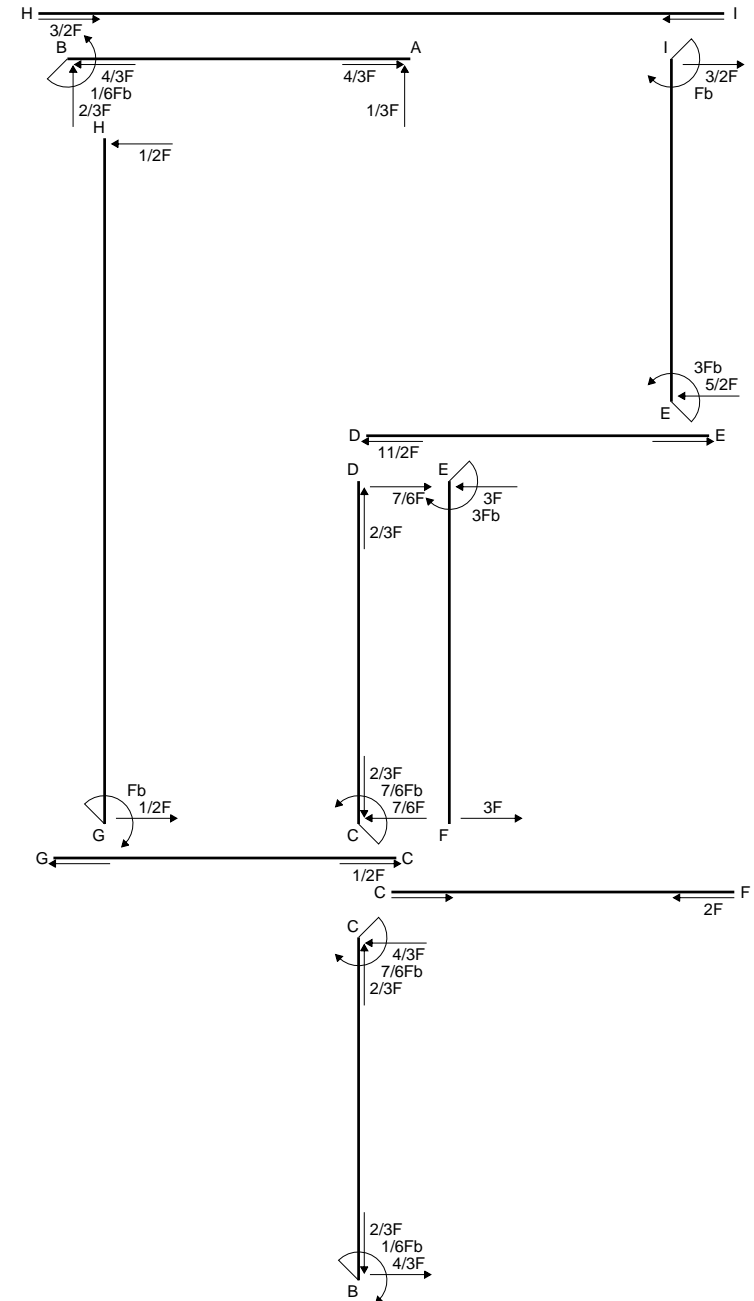
$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

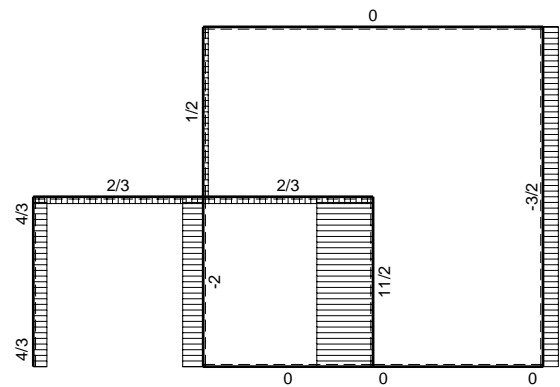




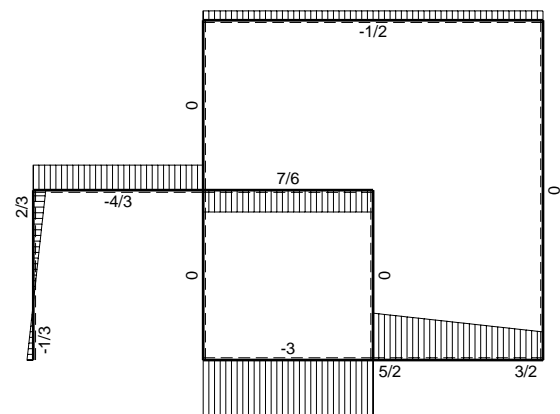
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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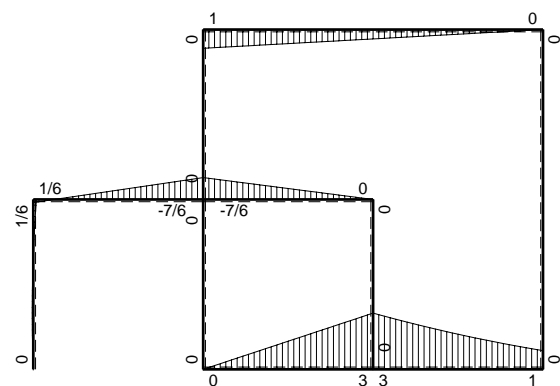




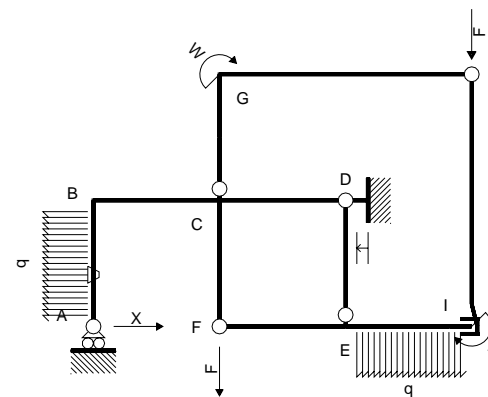
← (+) → F



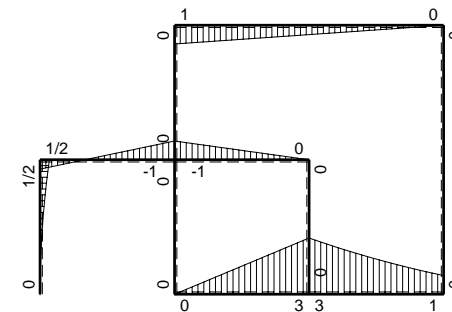
↑ (+) ↓ F



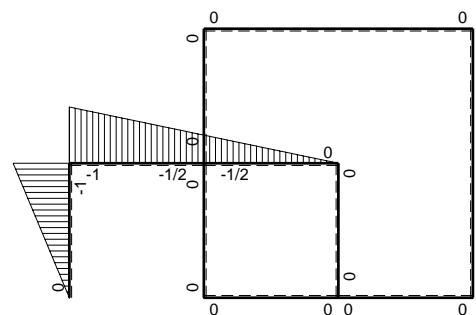
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>o</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H<sub>A</sub>

| →     | M <sub>x</sub> (x)                             | M <sub>o</sub> (x)            | θ      | M <sub>x</sub> M <sub>o</sub>                                      | M <sub>x</sub> θ           | M <sub>x</sub> M <sub>x</sub>              | ∫M <sub>x</sub> (M <sub>o</sub> /EJ+θ)dx | ∫xM <sub>x</sub> M <sub>x</sub> /EJdx |                     |
|-------|--|-------------------------------|--------|--|----------------------------|--|--|---------------------------------------|---------------------|
| AB b  | -x   | 1/2qx <sup>2</sup>            | -Fb/EJ | -1/2qx <sup>3</sup>  | Fxb/EJ                     | x <sup>2</sup>                             | (-1/8+1/2)Fb <sup>3</sup> /EJ            | 1/3Xb <sup>3</sup> /EJ                |                     |
| BA b  | b-x  | -1/2Fb+Fx-1/2qx <sup>2</sup>  | Fb/EJ  | -1/2Fb <sup>2</sup> +3/2Fbx-3/2Fx <sup>2</sup> +1/2qx <sup>3</sup> | Fb <sup>2</sup> /EJ-Fxb/EJ | b <sup>2</sup> -2bx+x <sup>2</sup>         |  |                                       |                     |
| BC b  | -b+1/2x  | 1/2Fb-3/2Fx                   | 0      | -1/2Fb <sup>2</sup> +7/4Fbx-3/4Fx <sup>2</sup>                     | 0                          | b <sup>2</sup> -bx+1/4x <sup>2</sup>       | (1/8+0)Fb <sup>3</sup> /EJ               | 7/12Xb <sup>3</sup> /EJ               |                     |
| CB b  | 1/2b+1/2x                                      | Fb-3/2Fx                      | 0      | 1/2Fb <sup>2</sup> -1/4Fbx-3/4Fx <sup>2</sup>                      | 0                          | 1/4b <sup>2</sup> +1/2bx+1/4x <sup>2</sup> |  |                                       |                     |
| CD b  | -1/2b+1/2x                                     | -Fb+Fx                        | 0      | 1/2Fb <sup>2</sup> -Fbx+1/2Fx <sup>2</sup>                         | 0                          | 1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup> | (1/6+0)Fb <sup>3</sup> /EJ               | 1/12Xb <sup>3</sup> /EJ               |                     |
| DC b  | 1/2x   | Fx                            | 0      | 1/2Fx <sup>2</sup>   | 0                          | 1/4x <sup>2</sup>                          |  |                                       |                     |
| DE b  | 0  | 0                             | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| ED b  | 0  | 0                             | 0      | 0  | 0                          | 0  |  |                                       |                     |
| EF b  | 0  | 3Fb-3Fx                       | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| FE b  | 0  | -3Fx                          | 0      | 0  | 0                          | 0  |  |                                       |                     |
| FC b  | 0  | 0                             | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| CF b  | 0  | 0                             | 0      | 0  | 0                          | 0  |  |                                       |                     |
| CG b  | 0  | 0                             | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| GC b  | 0  | 0                             | 0      | 0  | 0                          | 0  |  |                                       |                     |
| GH 2b | 0  | Fb-1/2Fx                      | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| HG 2b | 0  | -1/2Fx                        | 0      | 0  | 0                          | 0  |  |                                       |                     |
| HI 2b | 0  | 0                             | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| IH 2b | 0  | 0                             | 0      | 0  | 0                          | 0  |  |                                       |                     |
| IE b  | 0  | Fb+3/2Fx+1/2qx <sup>2</sup>   | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| EI b  | 0  | -3Fb+5/2Fx-1/2qx <sup>2</sup> | 0      | 0  | 0                          | 0  |  |                                       |                     |
| D     | cedimento nodo -H <sub>1D</sub> u <sub>D</sub> |                               |        |  |                            |  |  | -Fb <sup>3</sup> /EJ                  |                     |
|       | totali   |                               |        |  |                            |  |  | -1/3Fb <sup>3</sup> /EJ               | Xb <sup>3</sup> /EJ |
|       | iperstatica X=H <sub>A</sub>                   |                               |        |  |                            |  |  | 1/3F                                  |                     |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left( -\frac{1}{2} \frac{x^3}{b^3} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( \frac{x}{b} \right) \theta dx = \left[ -\frac{1}{8} \frac{x^4}{b^3} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( -\frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( \frac{1}{2} b \right) \theta = \frac{3}{8} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left( -\frac{1}{2} + \frac{3}{2} \frac{x}{b} - \frac{3}{2} \frac{x^2}{b^2} + \frac{1}{2} \frac{x^3}{b^3} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -1 + \frac{x}{b} \right) \theta dx$$

$$= \left[ -\frac{1}{2} x + \frac{3}{4} \frac{x^2}{b} - \frac{1}{2} \frac{x^3}{b^2} + \frac{1}{8} \frac{x^4}{b^3} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( -\frac{1}{2} b + \frac{3}{4} b - \frac{1}{2} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( -b + \frac{1}{2} b \right) \theta = \frac{3}{8} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( -\frac{1}{2} + \frac{7}{4} \frac{x}{b} - \frac{3}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ -\frac{1}{2} x + \frac{7}{8} \frac{x^2}{b} - \frac{1}{4} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( -\frac{1}{2} b + \frac{7}{8} b - \frac{1}{4} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{1}{4} \frac{x}{b} - \frac{3}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{2} x - \frac{1}{8} \frac{x^2}{b} - \frac{1}{4} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

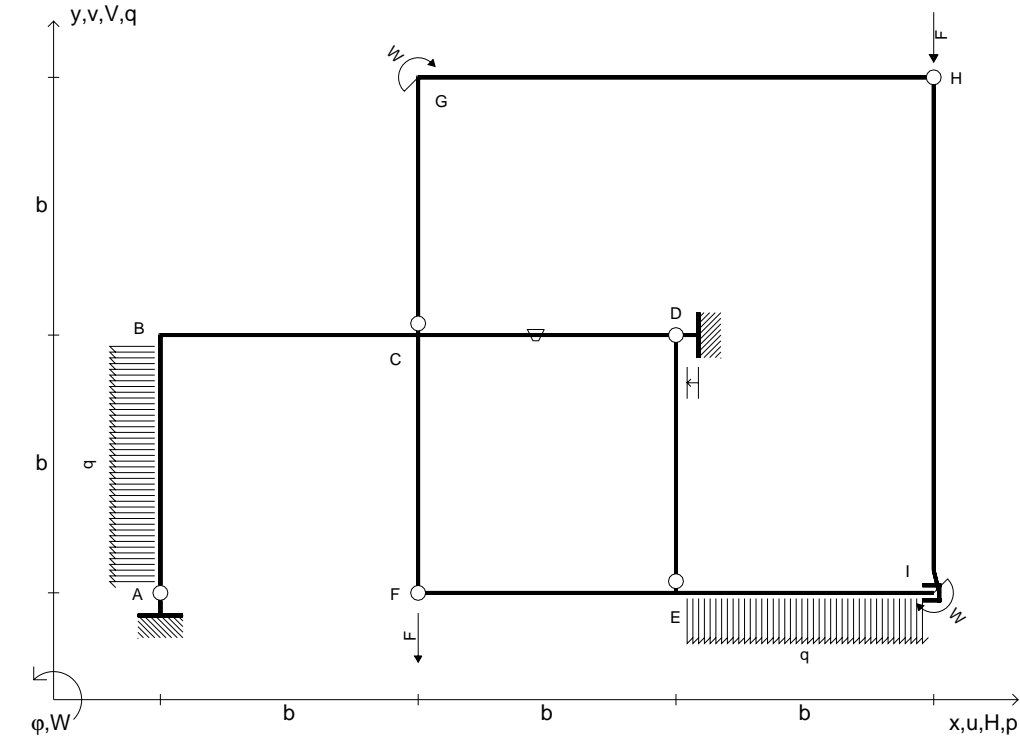
$$= \left( \frac{1}{2} b - \frac{1}{8} b - \frac{1}{4} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{x}{b} + \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{2} x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{2} b - \frac{1}{2} b + \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$

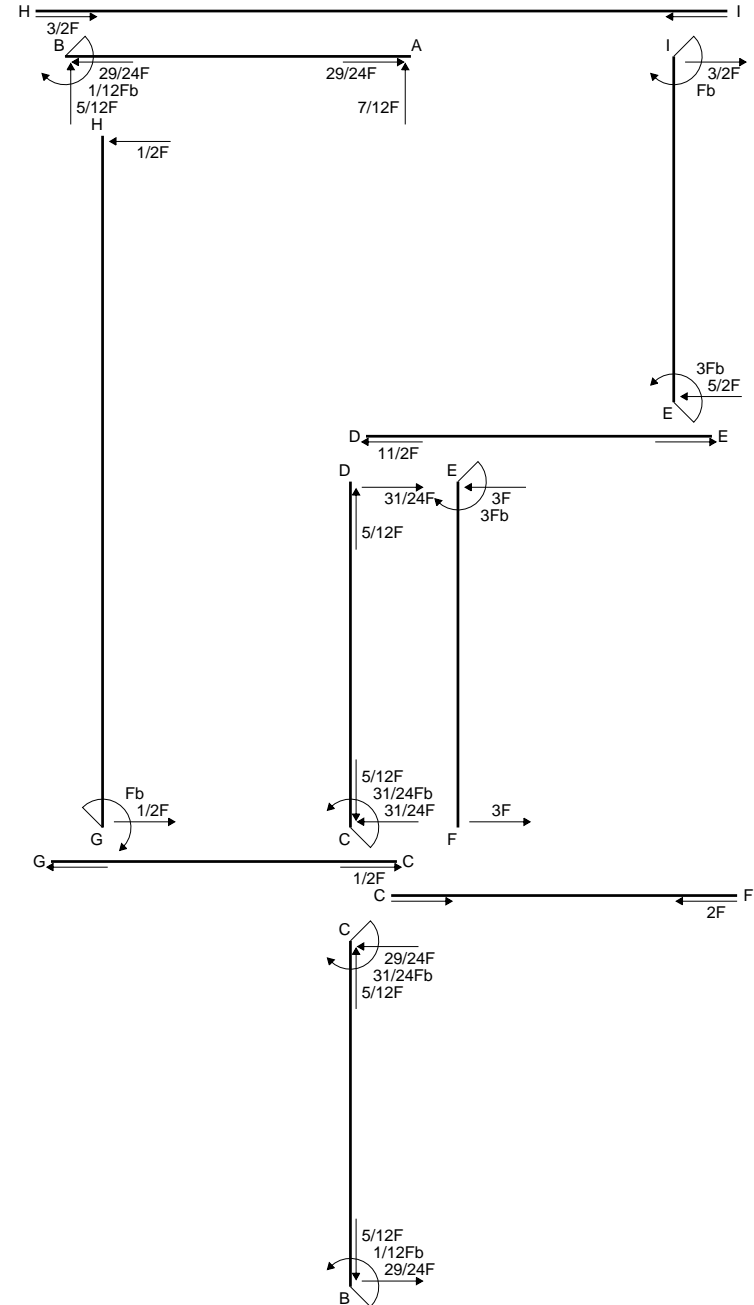
$$L_{DC}^{xo} = \int_0^b \left( \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$



|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=W_{BC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13





Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                          |                     |                         |                             | $-1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |                     |                         |                             | $1/12Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

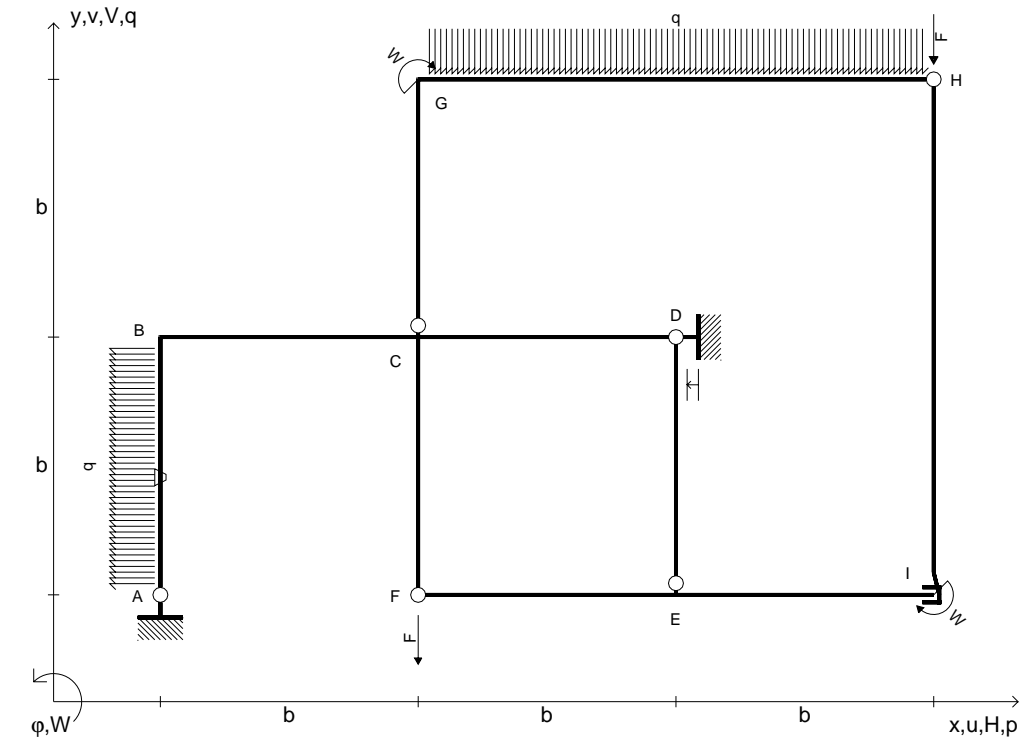
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

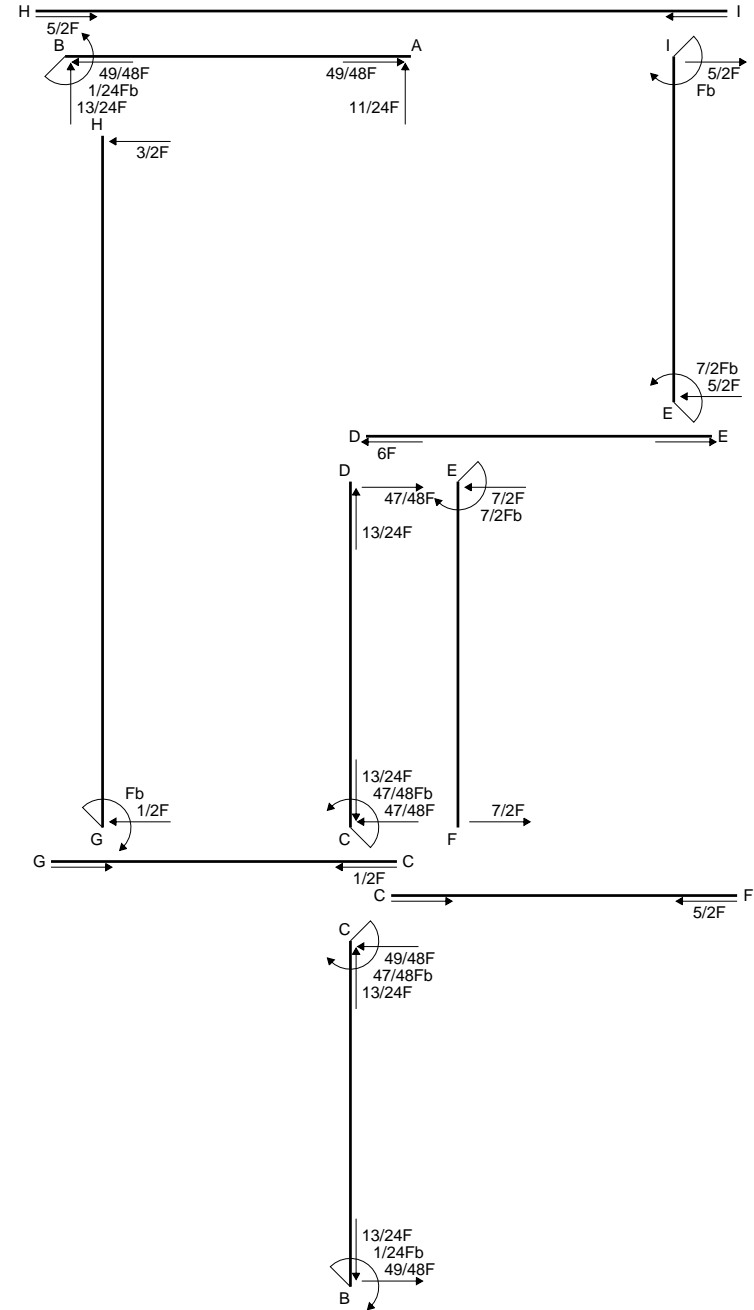
$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$

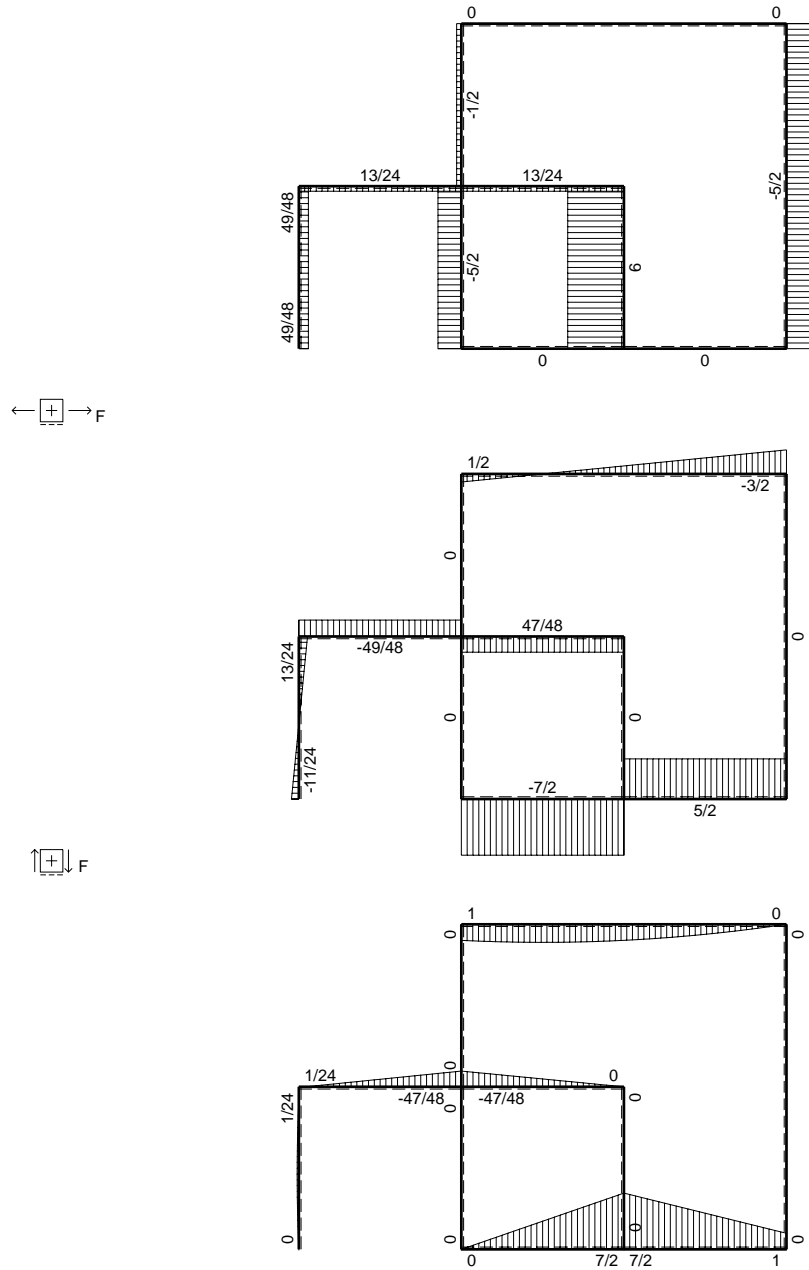




|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=W_{BC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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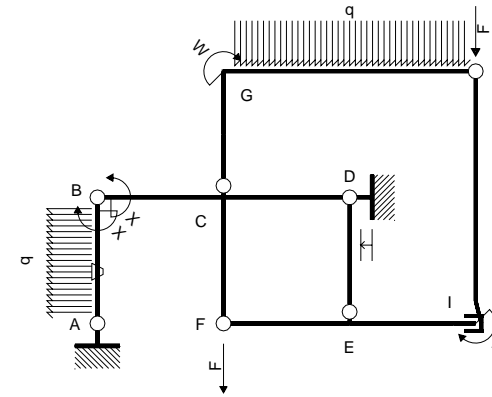




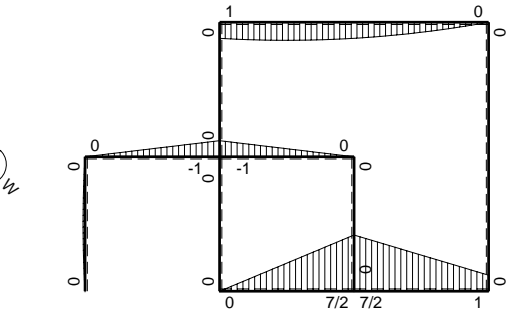
← (+) → F

↑ (+) ↓ F

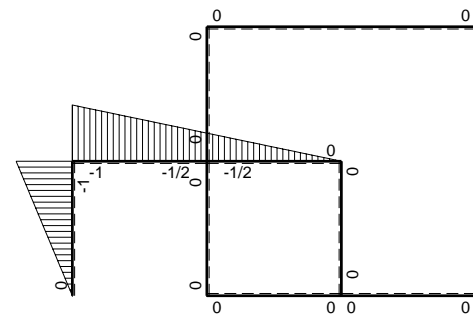
⌚ (+) ⌚ F<sub>b</sub>



Schema di calcolo iperstatico



⌚ (+) ⌚ M<sub>0</sub> flessione da carichi assegnati



⌚ (+) ⌚ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |         |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|-----------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$            |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                       |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | 0        | $Fx-1/2Fx^2/b$           | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$           |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | 0        | $1/2Fb-1/2Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$           |         |
| DC b  | $1/2x/b$                    | $Fx$               | 0        | $1/2Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                       |         |
| DE b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| ED b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |         |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0             | 0                       |                             |                       |         |
| FC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| CF b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |         |
| CG b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| GC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |         |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0             | 0                       |                             |                       |         |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |         |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0             | 0                       |                             |                       |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                    |          |                          |               |                         |                             | $-Fb^2/EJ$            |         |
|       | totali                      |                    |          |                          |               |                         |                             | $1/24Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |               |                         |                             | $-1/24Fb$             |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

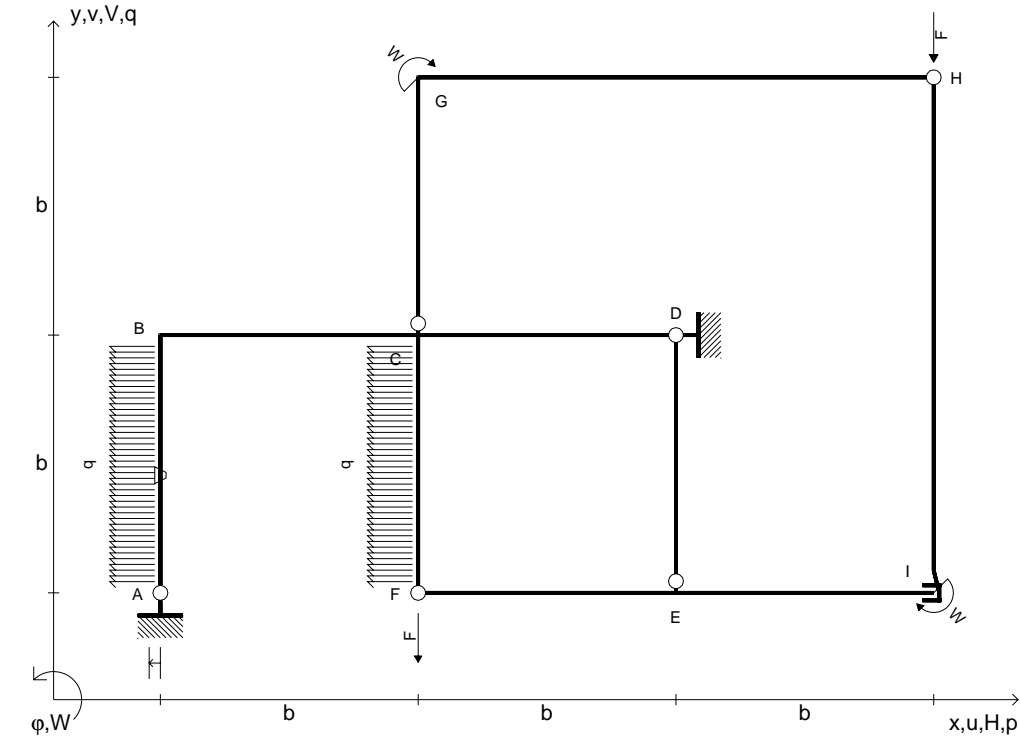
$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

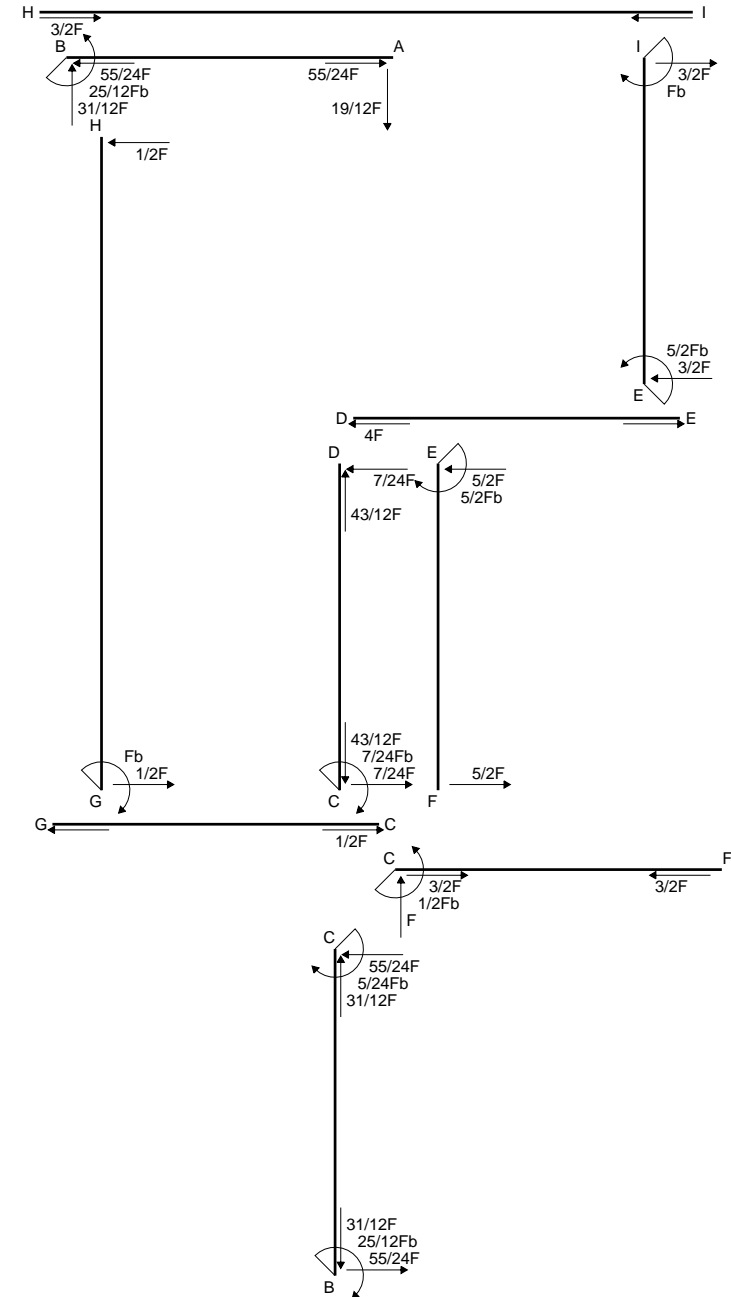
$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

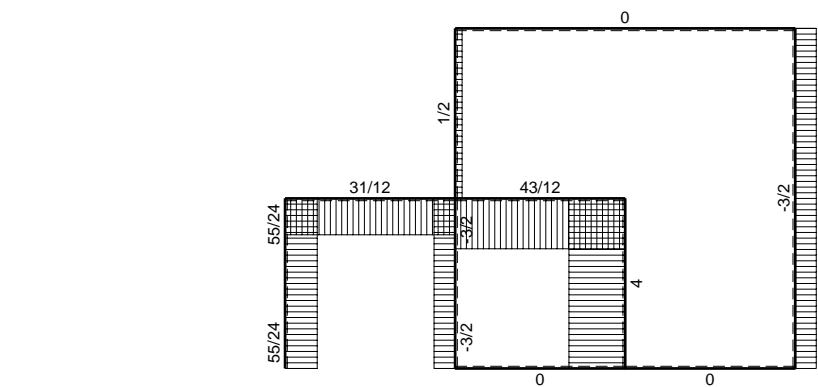
$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$



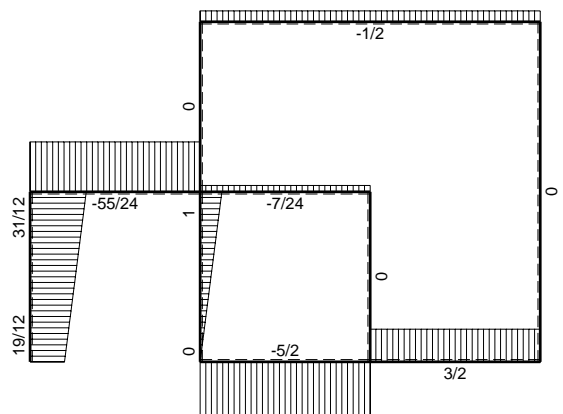
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{FC} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=V_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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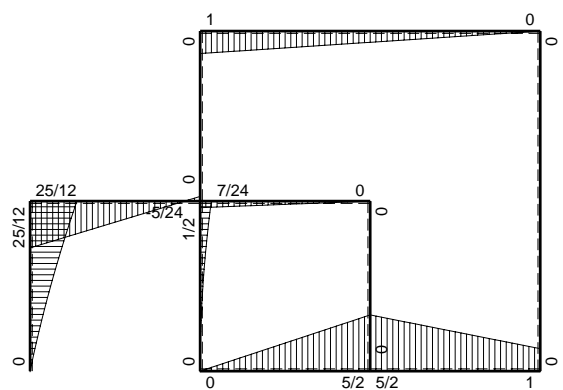




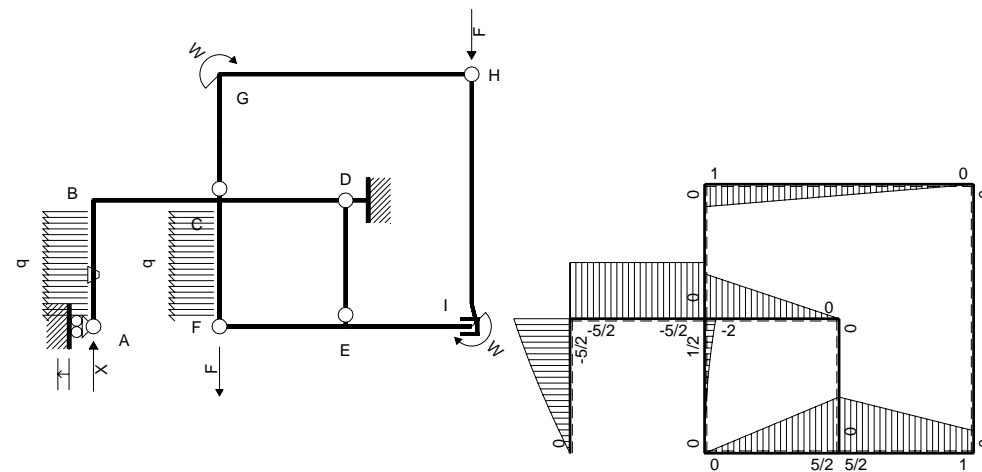
← (+) → F



↑ (+) ↓ F

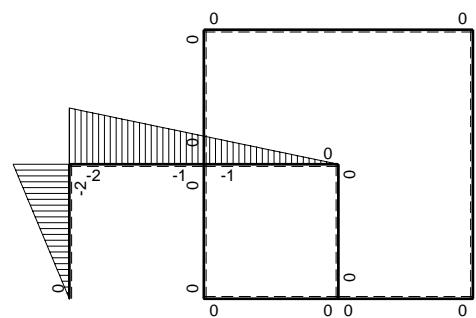


⤵ (+) ⤴ F<sub>b</sub>



Schema di calcolo iperstatico

⤵ (+) ⤴ M<sub>o</sub> flessione da carichi assegnati



⤵ (+) ⤴ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=V_A$ 

| →     | $M_x(x)$                                       | $M_o(x)$                     | $\theta$ | $M_x M_o$  | $M_x \theta$                 | $M_x M_x$                            | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$  |                      |
|-------|--|------------------------------|----------|--|------------------------------|--------------------------------------|-----------------------------|-------------------------|----------------------|
| AB b  | -2x  | -3Fx+1/2qx <sup>2</sup>      | -Fb/EJ   | 6Fx <sup>2</sup> -qx <sup>3</sup>                        | 2Fxb/EJ                      | 4x <sup>2</sup>                      | (7/4+1)Fb <sup>3</sup> /EJ  | 4/3Xb <sup>3</sup> /EJ  |                      |
| BA b  | 2b-2x  | 5/2Fb-2Fx-1/2qx <sup>2</sup> | Fb/EJ    | 5Fb <sup>2</sup> -9Fbx+3Fx <sup>2</sup> +qx <sup>3</sup> | 2Fb <sup>2</sup> /EJ-2Fxb/EJ | 4b <sup>2</sup> -8bx+4x <sup>2</sup> |                             |                         |                      |
| BC b  | -2b+x  | -5/2Fb                       | 0        | 5Fb <sup>2</sup> -5/2Fbx                                 | 0                            | 4b <sup>2</sup> -4bx+x <sup>2</sup>  | (15/4+0)Fb <sup>3</sup> /EJ | 7/3Xb <sup>3</sup> /EJ  |                      |
| CB b  | b+x  | 5/2Fb                        | 0        | 5/2Fb <sup>2</sup> +5/2Fbx                               | 0                            | b <sup>2</sup> +2bx+x <sup>2</sup>   |                             |                         |                      |
| CD b  | -b+x   | -2Fb+2Fx                     | 0        | 2Fb <sup>2</sup> -4Fbx+2Fx <sup>2</sup>                  | 0                            | b <sup>2</sup> -2bx+x <sup>2</sup>   | (2/3+0)Fb <sup>3</sup> /EJ  | 1/3Xb <sup>3</sup> /EJ  |                      |
| DC b  | x  | 2Fx                          | 0        | 2Fx <sup>2</sup>   | 0                            | x <sup>2</sup>                       |                             |                         |                      |
| DE b  | 0  | 0                            | 0        | 0  | 0                            | 0                                    | 0+0                         | 0                       |                      |
| ED b  | 0  | 0                            | 0        | 0  | 0                            | 0                                    |                             |                         |                      |
| EF b  | 0  | 5/2Fb-5/2Fx                  | 0        | 0  | 0                            | 0                                    | 0+0                         | 0                       |                      |
| FE b  | 0  | -5/2Fx                       | 0        | 0  | 0                            | 0                                    |                             |                         |                      |
| FC b  | 0  | 1/2qx <sup>2</sup>           | 0        | 0  | 0                            | 0                                    | 0+0                         | 0                       |                      |
| CF b  | 0  | -1/2Fb+Fx-1/2qx <sup>2</sup> | 0        | 0  | 0                            | 0                                    |                             |                         |                      |
| CG b  | 0  | 0                            | 0        | 0  | 0                            | 0                                    | 0+0                         | 0                       |                      |
| GC b  | 0  | 0                            | 0        | 0  | 0                            | 0                                    |                             |                         |                      |
| GH 2b | 0  | Fb-1/2Fx                     | 0        | 0  | 0                            | 0                                    | 0+0                         | 0                       |                      |
| HG 2b | 0  | -1/2Fx                       | 0        | 0  | 0                            | 0                                    |                             |                         |                      |
| HI 2b | 0  | 0                            | 0        | 0  | 0                            | 0                                    | 0+0                         | 0                       |                      |
| IH 2b | 0  | 0                            | 0        | 0  | 0                            | 0                                    |                             |                         |                      |
| IE b  | 0  | Fb+3/2Fx                     | 0        | 0  | 0                            | 0                                    | 0+0                         | 0                       |                      |
| EI b  | 0  | -5/2Fb+3/2Fx                 | 0        | 0  | 0                            | 0                                    |                             |                         |                      |
| A     | cedimento nodo -H <sub>1A</sub> u <sub>A</sub> |                              |          |  |                              |                                      |                             | 2Fb <sup>3</sup> /EJ    |                      |
|       | totali   |                              |          |  |                              |                                      |                             | 55/6Fb <sup>3</sup> /EJ | 4Xb <sup>3</sup> /EJ |
|       | iperstatica X=V <sub>A</sub>                   |                              |          |  |                              |                                      |                             | -55/24F                 |                      |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (6x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx + \int_0^b (2x/b) \theta dx$$

$$= [2x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ + [x^2/b]_0^b \theta$$

$$= (2b - 1/4 b) Fb^2 1/EJ + (b) \theta = 11/4 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (5 - 9x/b + 3x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-2 + 2x/b) \theta dx$$

$$= [5x - 9/2 x^2/b + x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ + [-2x + x^2/b]_0^b \theta$$

$$= (5b - 9/2 b + b + 1/4 b) Fb^2 1/EJ + (-2b + b) \theta = 11/4 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (5 - 5/2 x/b) Fb^2 1/EJ dx = [5x - 5/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (5b - 5/4 b) Fb^2 1/EJ = 15/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/2 + 5/2 x/b) Fb^2 1/EJ dx = [5/2 x + 5/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (5/2 b + 5/4 b) Fb^2 1/EJ = 15/4 Fb^3/EJ$$

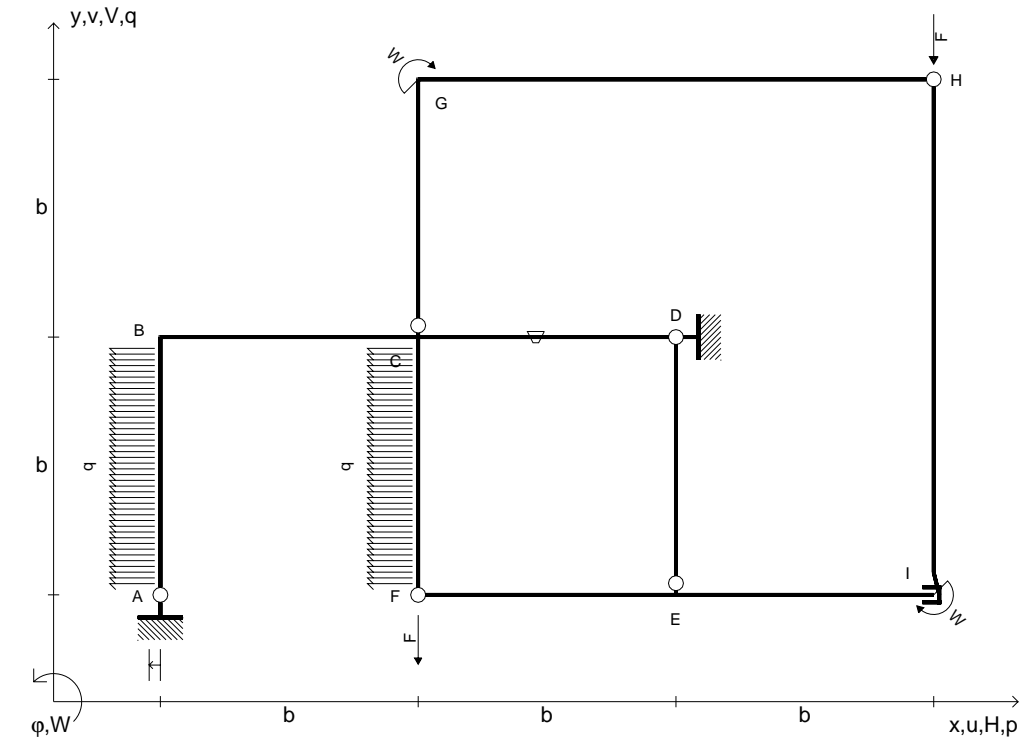
$$L_{CD}^{xo} = \int_0^b (2 - 4x/b + 2x^2/b^2) Fb^2 1/EJ dx = [2x - 2x^2/b + 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (2b - 2b + 2/3 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (2x^2/b^2) Fb^2 1/EJ dx = [2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

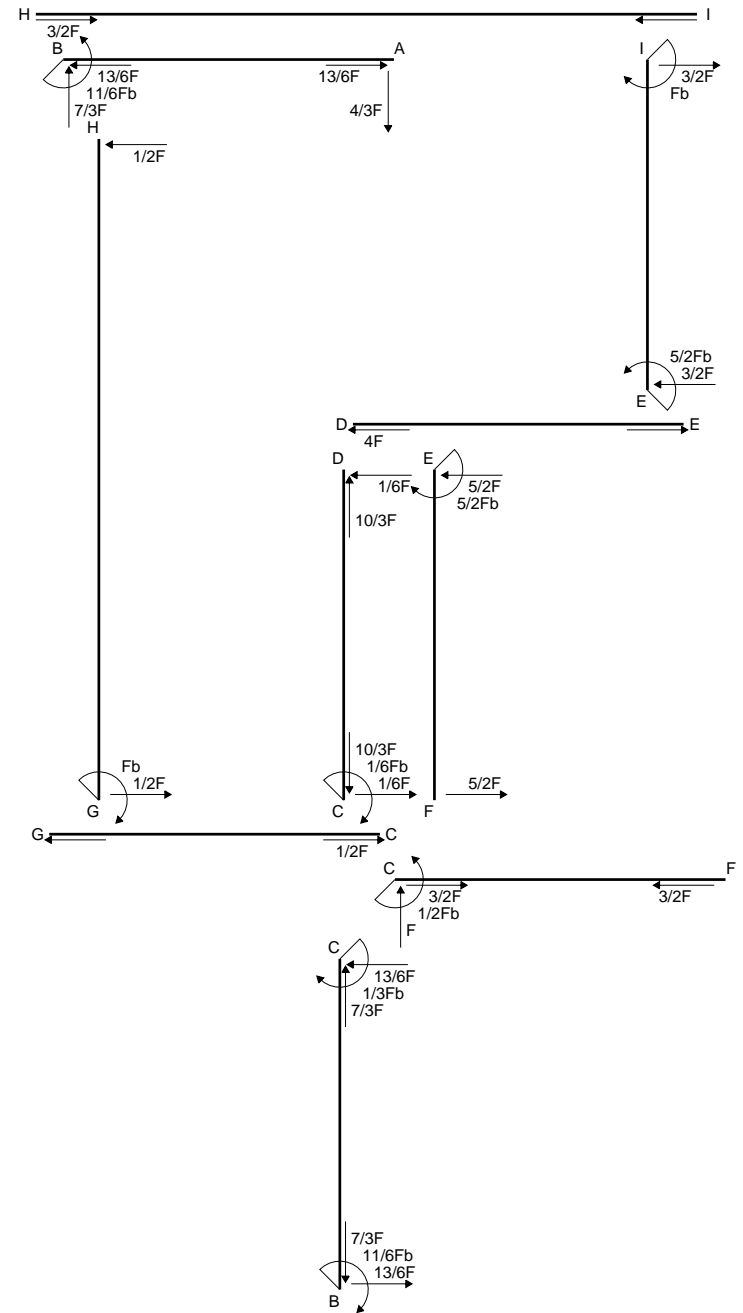
$$= (2/3 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$

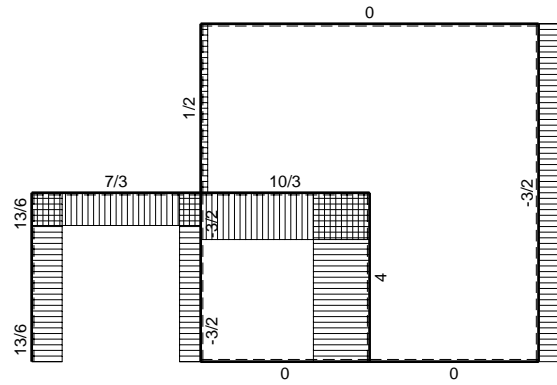




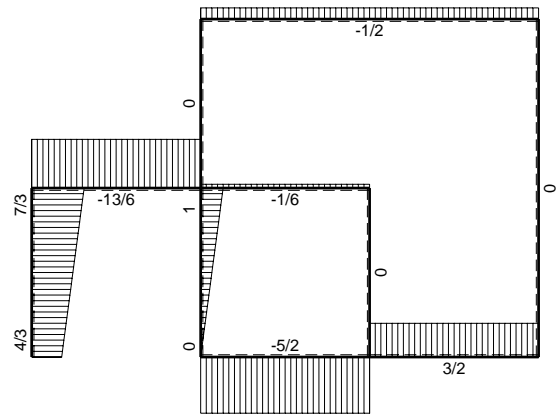
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{FC} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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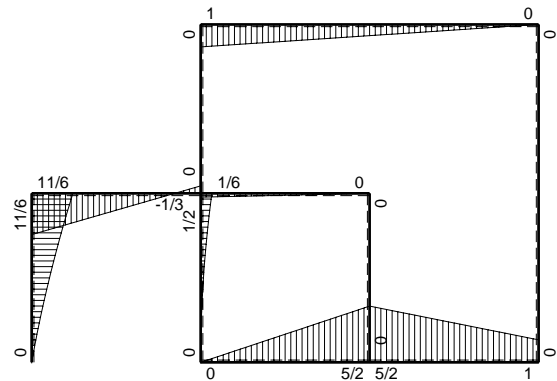




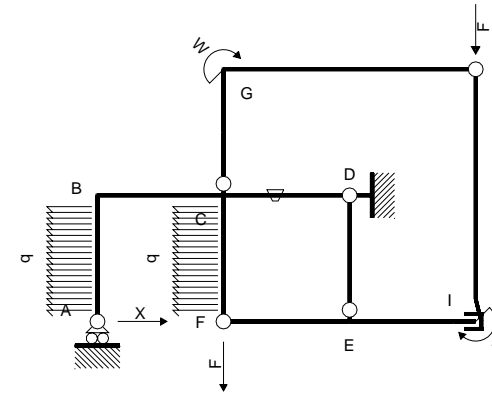
← ⊕ → F



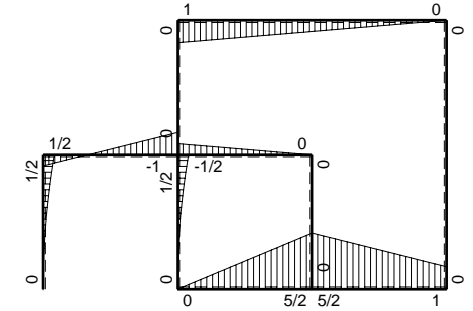
↑ ⊕ ↓ F



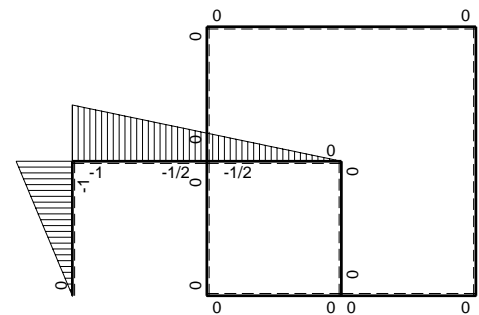
⊕ ↺ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ ↺ M<sub>o</sub> flessione da carichi assegnati



⊕ ↺ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-----------------------------------|------------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | $1/2qx^2$           | 0        | $-1/2qx^3$                        | 0                      | $x^2$                 | $(-1/8+0)Fb^3/EJ$           | $1/3Xb^3/EJ$           |
| BA b  | b-x                         | $-1/2Fb+Fx-1/2qx^2$ | 0        | $-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$ | 0                      | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | $1/2Fb-3/2Fx$       | 0        | $-1/2Fb^2+7/4Fbx-3/4Fx^2$         | 0                      | $b^2-bx+1/4x^2$       | $(1/8+0)Fb^3/EJ$            | $7/12Xb^3/EJ$          |
| CB b  | $1/2b+1/2x$                 | $Fb-3/2Fx$          | 0        | $1/2Fb^2-1/4Fbx-3/4Fx^2$          | 0                      | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$      | $-Fb/EJ$ | $1/4Fb^2-1/2Fbx+1/4Fx^2$          | $1/2Fb^2/EJ-1/2Fxb/EJ$ | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+1/4)Fb^3/EJ$         | $1/12Xb^3/EJ$          |
| DC b  | $1/2x$                      | $1/2Fx$             | $Fb/EJ$  | $1/4Fx^2$                         | $1/2Fxb/EJ$            | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| EF b  | 0                           | $5/2Fb-5/2Fx$       | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx$            | 0        | 0                                 | 0                      | 0                     |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                                 | 0                      | 0                     |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                                 | 0                      | 0                     |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                                 | 0                      | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                                   |                        |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                     |          |                                   |                        |                       | $4/3Fb^3/EJ$                | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                     |          |                                   |                        |                       | $-4/3F$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-1/2 x^3/b^3) Fb^2 1/EJ dx = [-1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [-1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + 7/4 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 7/8 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 7/8 b - 1/4 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/4 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/8 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/8 b - 1/4 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

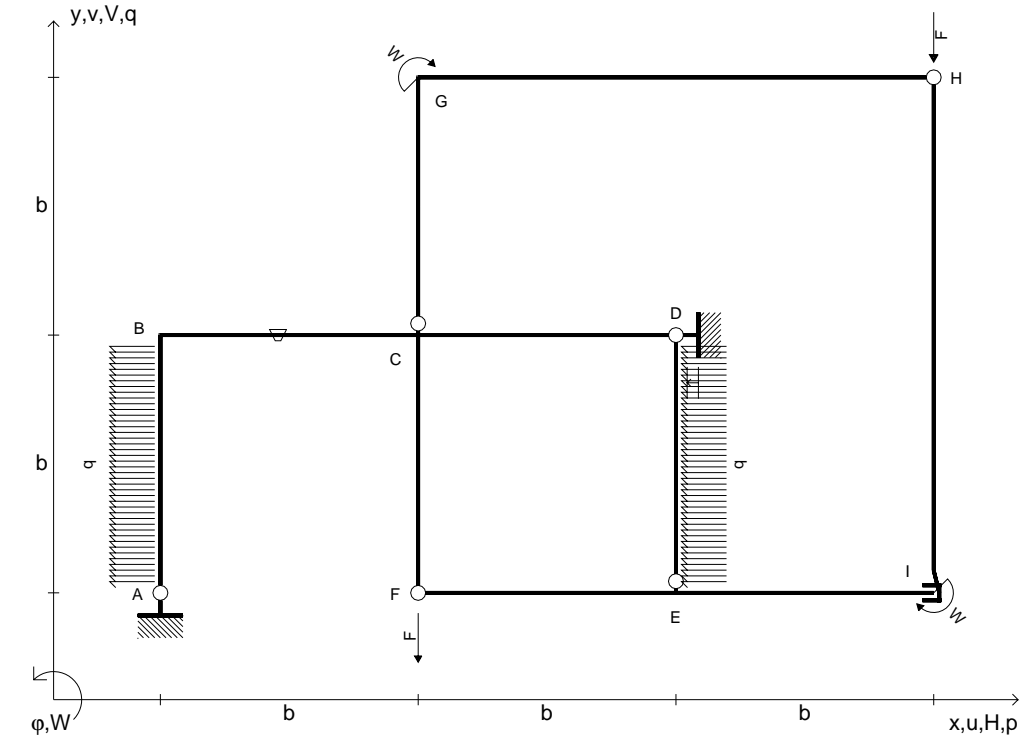
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^3/EJ$$

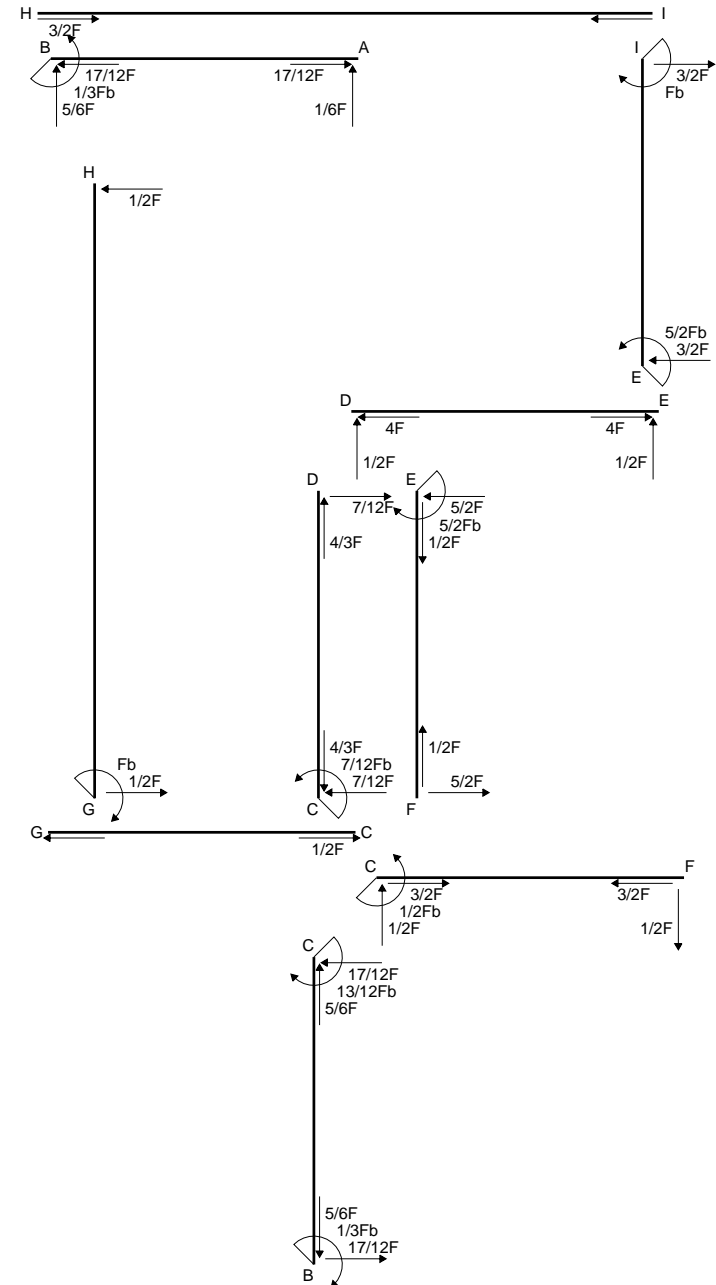
$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

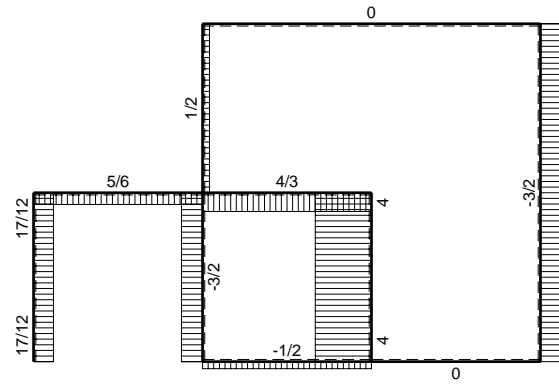
$$= (1/12 b) Fb^2 1/EJ + (-1/4 b) \theta = 1/3 Fb^3/EJ$$



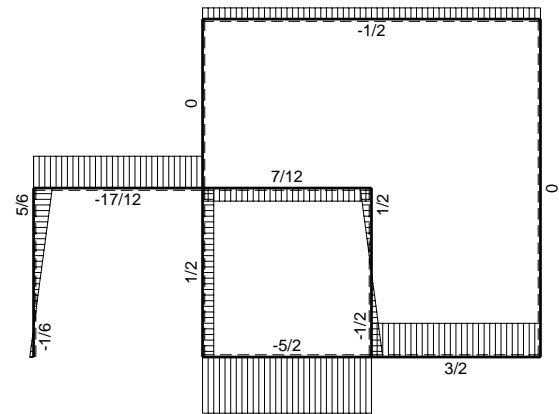
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{DE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

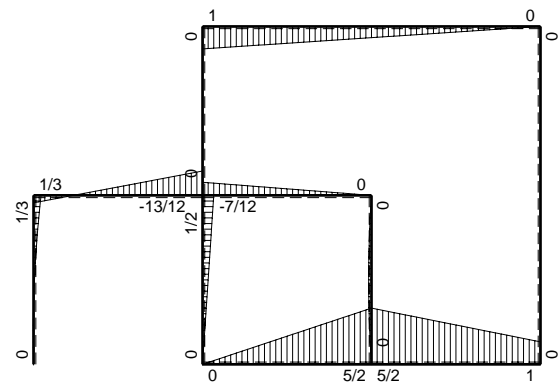




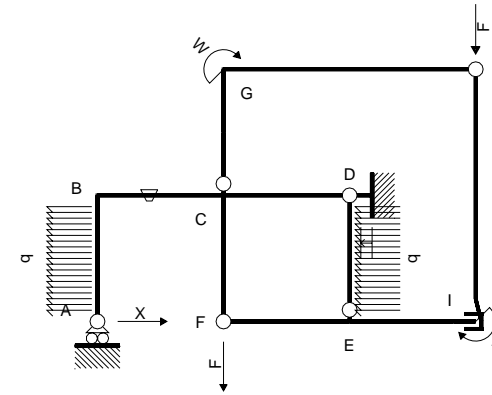
← (+) → F



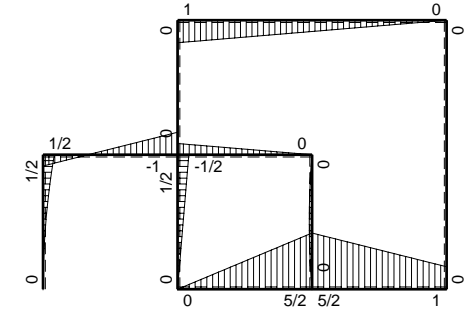
↑ (+) ↓ F



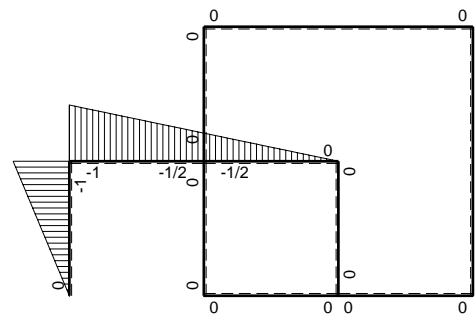
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>o</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |           |
|-------|-----------------------------|---------------------|----------|-----------------------------------|------------------------|-----------------------|-----------------------------|-----------------------|-----------|
| AB b  | -x                          | $1/2qx^2$           | 0        | $-1/2qx^3$                        | 0                      | $x^2$                 | $(-1/8+0)Fb^3/EJ$           | $1/3Xb^3/EJ$          |           |
| BA b  | b-x                         | $-1/2Fb+Fx-1/2qx^2$ | 0        | $-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$ | 0                      | $b^2-2bx+x^2$         |                             |                       |           |
| BC b  | -b+1/2x                     | $1/2Fb-3/2Fx$       | -Fb/EJ   | $-1/2Fb^2+7/4Fbx-3/4Fx^2$         | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/8+3/4)Fb^3/EJ$          | $7/12Xb^3/EJ$         |           |
| CB b  | $1/2b+1/2x$                 | $Fb-3/2Fx$          | Fb/EJ    | $1/2Fb^2-1/4Fbx-3/4Fx^2$          | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                       |           |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb^2-1/2Fbx+1/4Fx^2$          | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+0)Fb^3/EJ$           | $1/12Xb^3/EJ$         |           |
| DC b  | $1/2x$                      | $1/2Fx$             | 0        | $1/4Fx^2$                         | 0                      | $1/4x^2$              |                             |                       |           |
| DE b  | 0                           | $1/2Fx-1/2qx^2$     | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                     |           |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                                 | 0                      | 0                     |                             |                       |           |
| EF b  | 0                           | $5/2Fb-5/2Fx$       | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                     |           |
| FE b  | 0                           | $-5/2Fx$            | 0        | 0                                 | 0                      | 0                     |                             |                       |           |
| FC b  | 0                           | $1/2Fx$             | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                     |           |
| CF b  | 0                           | $-1/2Fb+1/2Fx$      | 0        | 0                                 | 0                      | 0                     |                             |                       |           |
| CG b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                     |           |
| GC b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                       |           |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                     |           |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                                 | 0                      | 0                     |                             |                       |           |
| HI 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                     |           |
| IH 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                       |           |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                     |           |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                                 | 0                      | 0                     |                             |                       |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                                   |                        |                       |                             | $-Fb^3/EJ$            |           |
|       | totali                      |                     |          |                                   |                        |                       |                             | $-1/6Fb^3/EJ$         | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                                   |                        |                       |                             | $1/6F$                |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left( -\frac{1}{2} \frac{x^3}{b^3} \right) Fb^2 \frac{1}{EJ} dx = \left[ -\frac{1}{8} \frac{x^4}{b^3} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( -\frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = -\frac{1}{8} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left( -\frac{1}{2} + \frac{3}{2} \frac{x}{b} - \frac{3}{2} \frac{x^2}{b^2} + \frac{1}{2} \frac{x^3}{b^3} \right) Fb^2 \frac{1}{EJ} dx$$

$$= \left[ -\frac{1}{2} x + \frac{3}{4} \frac{x^2}{b} - \frac{1}{2} \frac{x^3}{b^2} + \frac{1}{8} \frac{x^4}{b^3} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( -\frac{1}{2} b + \frac{3}{4} b - \frac{1}{2} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = -\frac{1}{8} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( -\frac{1}{2} + \frac{7}{4} \frac{x}{b} - \frac{3}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( 1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ -\frac{1}{2} x + \frac{7}{8} \frac{x^2}{b} - \frac{1}{4} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( -\frac{1}{2} b + \frac{7}{8} b - \frac{1}{4} b \right) Fb^2 \frac{1}{EJ} + \left( b - \frac{1}{4} b \right) \theta = \frac{7}{8} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{1}{4} \frac{x}{b} - \frac{3}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{1}{2} x - \frac{1}{8} \frac{x^2}{b} - \frac{1}{4} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b - \frac{1}{8} b - \frac{1}{4} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{7}{8} Fb^3/EJ$$

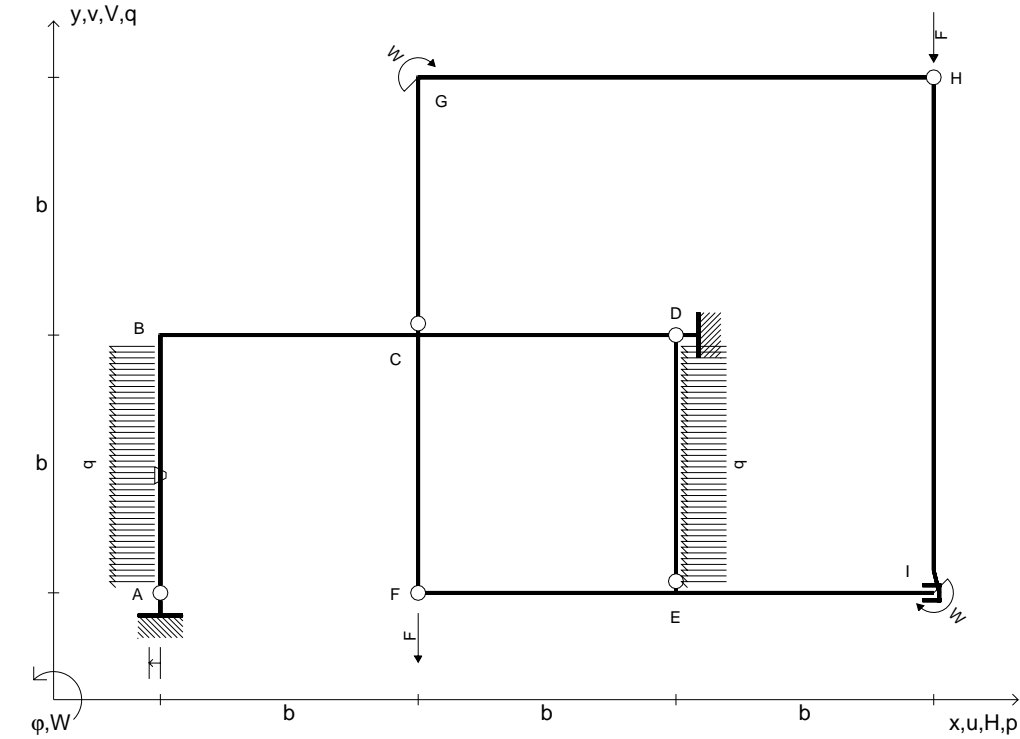
$$L_{CD}^{xo} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

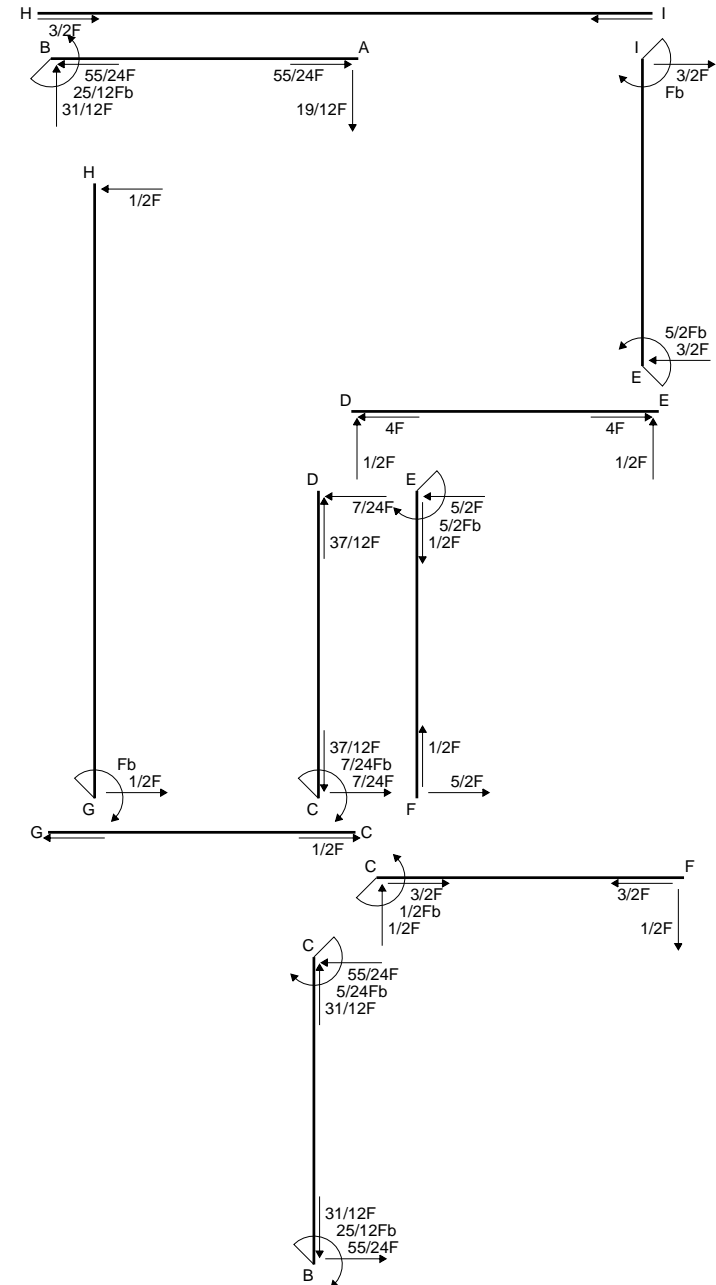
$$= \left( \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

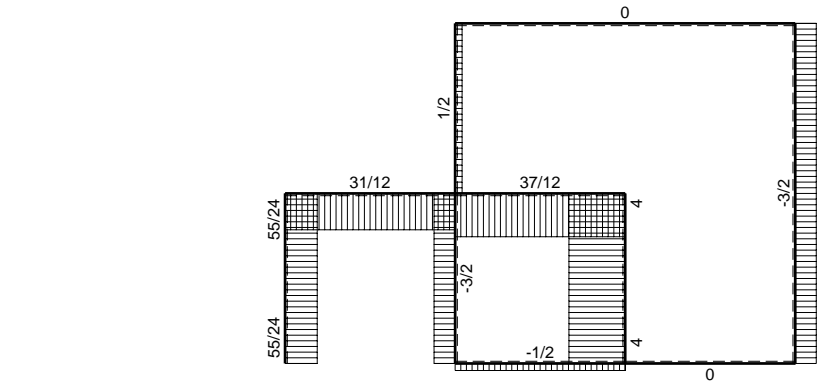




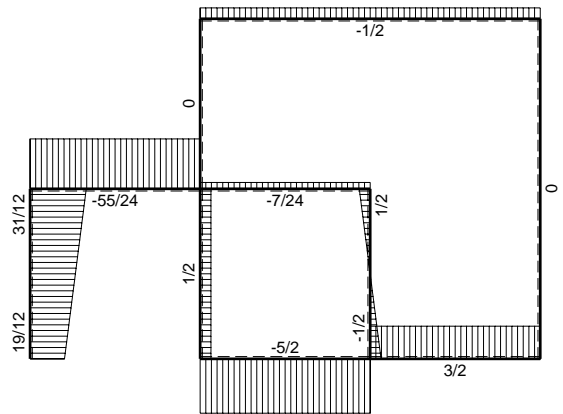
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{DE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_D$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

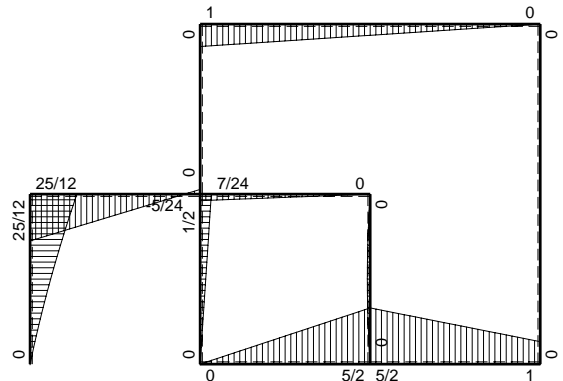




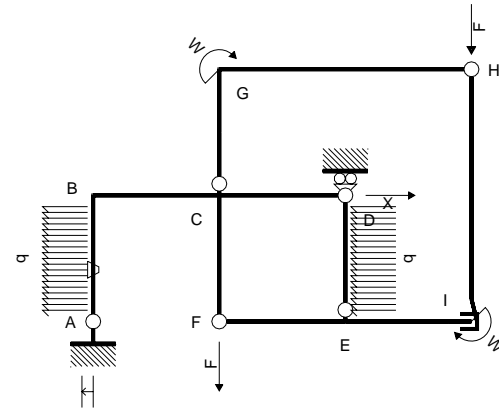
← (+) → F



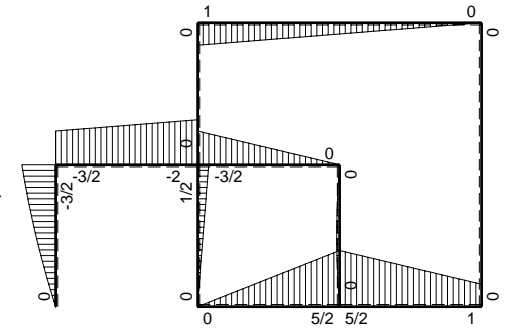
↑ (+) ↓ F



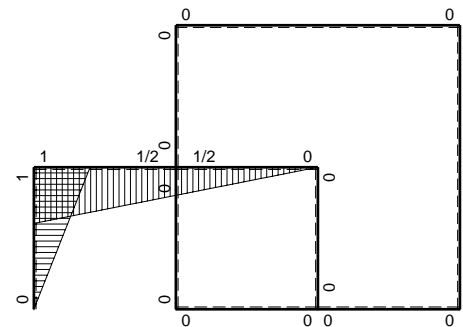
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>o</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_D$ 

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                         | $M_x \theta$      | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|--------------------|----------|-----------------------------------|-------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | x                           | $-2Fx+1/2qx^2$     | $-Fb/EJ$ | $-2Fx^2+1/2qx^3$                  | $-Fxb/EJ$         | $x^2$                 | $(-13/24-1/2)Fb^3/EJ$       | $1/3Xb^3/EJ$           |           |
| BA b  | $-b+x$                      | $3/2Fb-Fx-1/2qx^2$ | $Fb/EJ$  | $-3/2Fb^2+5/2Fbx-1/2Fx^2-1/2qx^3$ | $-Fb^2/EJ+Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | $b-1/2x$                    | $-3/2Fb-1/2Fx$     | 0        | $-3/2Fb^2+1/4Fbx+1/4Fx^2$         | 0                 | $b^2-bx+1/4x^2$       | $(-31/24+0)Fb^3/EJ$         | $7/12Xb^3/EJ$          |           |
| CB b  | $-1/2b-1/2x$                | $2Fb-1/2Fx$        | 0        | $-Fb^2-3/4Fbx+1/4Fx^2$            | 0                 | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | $1/2b-1/2x$                 | $-3/2Fb+3/2Fx$     | 0        | $-3/4Fb^2+3/2Fbx-3/4Fx^2$         | 0                 | $1/4b^2-1/2bx+1/4x^2$ | $(-1/4+0)Fb^3/EJ$           | $1/12Xb^3/EJ$          |           |
| DC b  | $-1/2x$                     | $3/2Fx$            | 0        | $-3/4Fx^2$                        | 0                 | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | $1/2Fx-1/2qx^2$    | 0        | 0                                 | 0                 | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$   | 0        | 0                                 | 0                 | 0                     |                             |                        |           |
| EF b  | 0                           | $5/2Fb-5/2Fx$      | 0        | 0                                 | 0                 | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | $-5/2Fx$           | 0        | 0                                 | 0                 | 0                     |                             |                        |           |
| FC b  | 0                           | $1/2Fx$            | 0        | 0                                 | 0                 | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | $-1/2Fb+1/2Fx$     | 0        | 0                                 | 0                 | 0                     |                             |                        |           |
| CG b  | 0                           | 0                  | 0        | 0                                 | 0                 | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | 0                  | 0        | 0                                 | 0                 | 0                     |                             |                        |           |
| GH 2b | 0                           | $Fb-1/2Fx$         | 0        | 0                                 | 0                 | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | $-1/2Fx$           | 0        | 0                                 | 0                 | 0                     |                             |                        |           |
| HI 2b | 0                           | 0                  | 0        | 0                                 | 0                 | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | 0                  | 0        | 0                                 | 0                 | 0                     |                             |                        |           |
| IE b  | 0                           | $Fb+3/2Fx$         | 0        | 0                                 | 0                 | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-5/2Fb+3/2Fx$     | 0        | 0                                 | 0                 | 0                     |                             |                        |           |
| A     | cedimento nodo $-H_{1A}u_A$ |                    |          |                                   |                   |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                    |          |                                   |                   |                       |                             | $-43/12Fb^3/EJ$        | $Xb^3/EJ$ |
|       | iperstatica $X=H_D$         |                    |          |                                   |                   |                       |                             | $43/12F$               |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-2x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx$$

$$= [-2/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [-1/2 x^2/b]_0^b \theta$$

$$= (-2/3 b + 1/8 b) Fb^2 1/EJ + (-1/2 b) \theta = -25/24 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-3/2 + 5/2 x/b - 1/2 x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

$$= [-3/2 x + 5/4 x^2/b - 1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [x - 1/2 x^2/b]_0^b \theta$$

$$= (-3/2 b + 5/4 b - 1/6 b - 1/8 b) Fb^2 1/EJ + (b - 1/2 b) \theta = -25/24 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-3/2 + 1/4 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [-3/2 x + 1/8 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-3/2 b + 1/8 b + 1/12 b) Fb^2 1/EJ = -31/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1 - 3/4 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [-x - 3/8 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

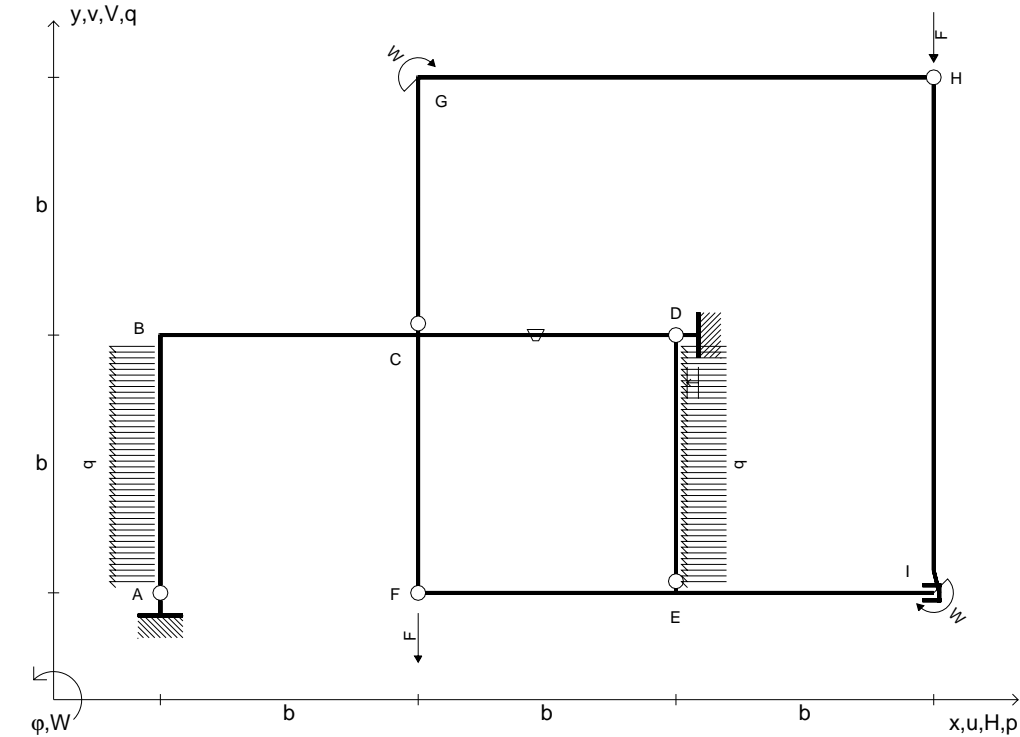
$$= (-b - 3/8 b + 1/12 b) Fb^2 1/EJ = -31/24 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (-3/4 + 3/2 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx = [-3/4 x + 3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-3/4 b + 3/4 b - 1/4 b) Fb^2 1/EJ = -1/4 Fb^3/EJ$$

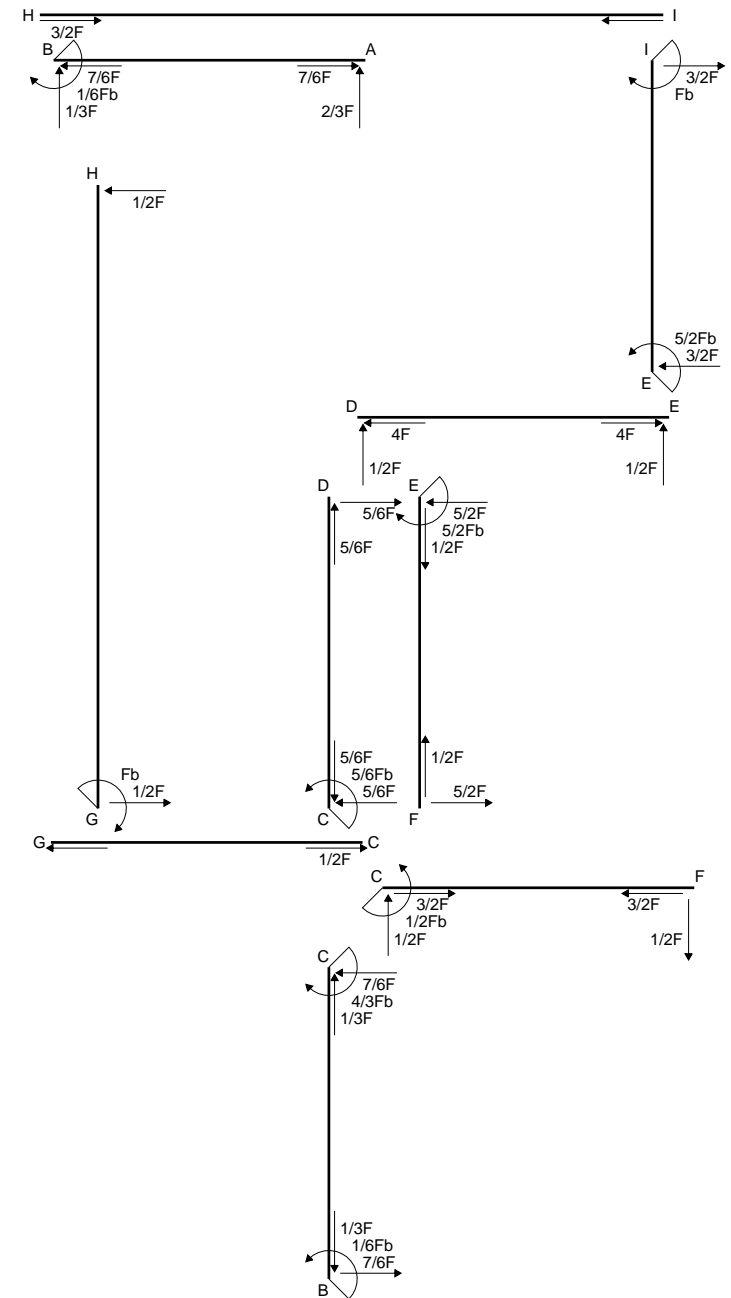
$$L_{DC}^{xo} = \int_0^b (-3/4 x^2/b^2) Fb^2 1/EJ dx = [-1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

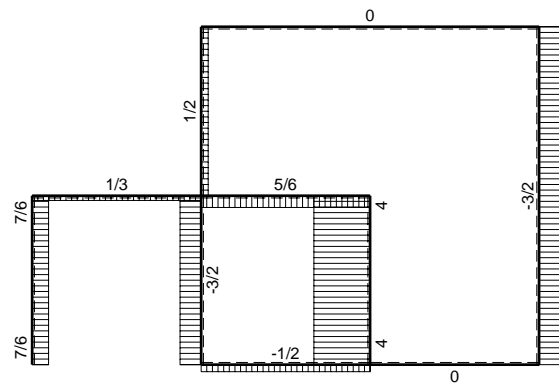
$$= (-1/4 b) Fb^2 1/EJ = -1/4 Fb^3/EJ$$



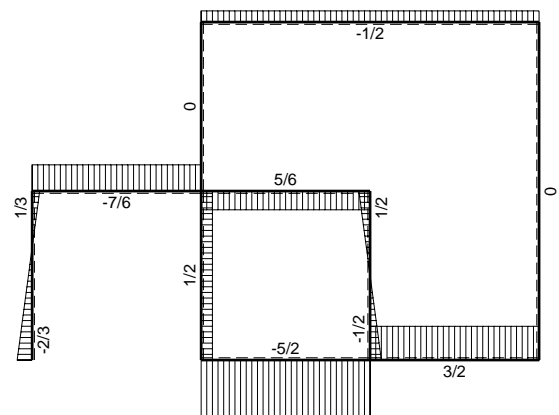
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{DE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=V_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
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 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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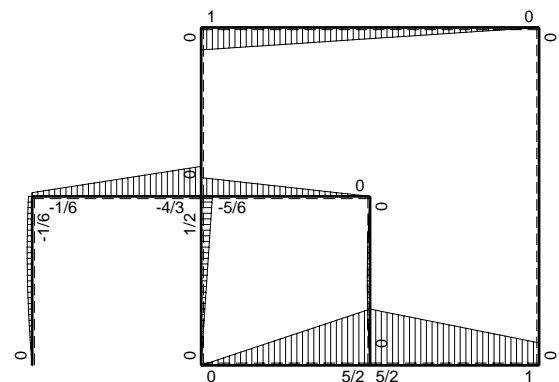




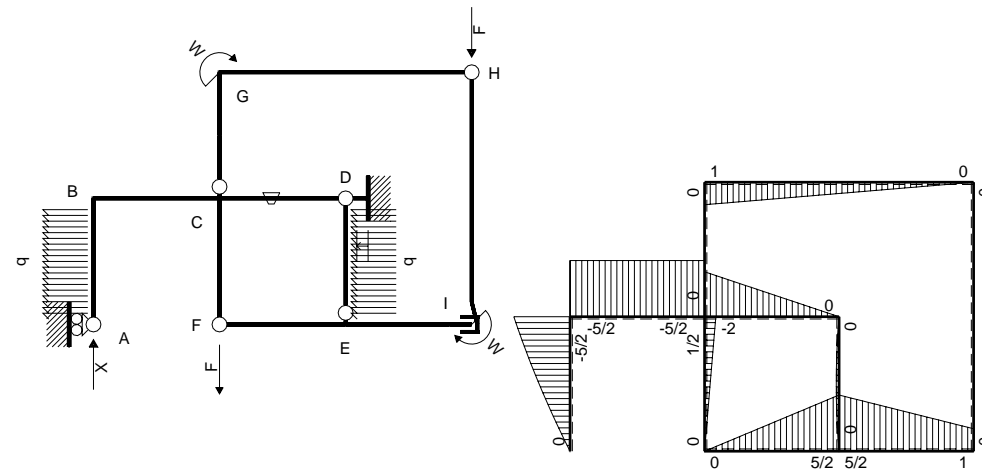
← (+) → F



↑ (+) ↓ F

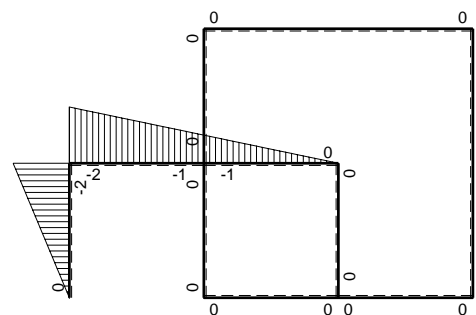


⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico

⊕ (+) ⊖ M<sub>0</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=V_A$ 

| →     | $M_x(x)$                                       | $M_o(x)$                     | $\theta$ | $M_x M_o$  | $M_x \theta$               | $M_x M_x$                            | $\int M_x(M_o/EJ+\theta)dx$  | $\int X M_x M_x/EJ dx$  |                      |
|-------|--|------------------------------|----------|--|----------------------------|--------------------------------------|------------------------------|-------------------------|----------------------|
| AB b  | -2x  | -3Fx+1/2qx <sup>2</sup>      | 0        | 6Fx <sup>2</sup> -qx <sup>3</sup>                        | 0                          | 4x <sup>2</sup>                      | (7/4+0)Fb <sup>3</sup> /EJ   | 4/3Xb <sup>3</sup> /EJ  |                      |
| BA b  | 2b-2x  | 5/2Fb-2Fx-1/2qx <sup>2</sup> | 0        | 5Fb <sup>2</sup> -9Fbx+3Fx <sup>2</sup> +qx <sup>3</sup> | 0                          | 4b <sup>2</sup> -8bx+4x <sup>2</sup> |                              |                         |                      |
| BC b  | -2b+x  | -5/2Fb                       | 0        | 5Fb <sup>2</sup> -5/2Fbx                                 | 0                          | 4b <sup>2</sup> -4bx+x <sup>2</sup>  | (15/4+0)Fb <sup>3</sup> /EJ  | 7/3Xb <sup>3</sup> /EJ  |                      |
| CB b  | b+x  | 5/2Fb                        | 0        | 5/2Fb <sup>2</sup> +5/2Fbx                               | 0                          | b <sup>2</sup> +2bx+x <sup>2</sup>   |                              |                         |                      |
| CD b  | -b+x   | -2Fb+2Fx                     | -Fb/EJ   | 2Fb <sup>2</sup> -4Fbx+2Fx <sup>2</sup>                  | Fb <sup>2</sup> /EJ-Fxb/EJ | b <sup>2</sup> -2bx+x <sup>2</sup>   | (2/3+1/2)Fb <sup>3</sup> /EJ | 1/3Xb <sup>3</sup> /EJ  |                      |
| DC b  | x  | 2Fx                          | Fb/EJ    | 2Fx <sup>2</sup>   | Fxb/EJ                     | x <sup>2</sup>                       |                              |                         |                      |
| DE b  | 0  | 1/2Fx-1/2qx <sup>2</sup>     | 0        | 0  | 0                          | 0                                    | 0+0                          | 0                       |                      |
| ED b  | 0  | -1/2Fx+1/2qx <sup>2</sup>    | 0        | 0  | 0                          | 0                                    |                              |                         |                      |
| EF b  | 0  | 5/2Fb-5/2Fx                  | 0        | 0  | 0                          | 0                                    | 0+0                          | 0                       |                      |
| FE b  | 0  | -5/2Fx                       | 0        | 0  | 0                          | 0                                    |                              |                         |                      |
| FC b  | 0  | 1/2Fx                        | 0        | 0  | 0                          | 0                                    | 0+0                          | 0                       |                      |
| CF b  | 0  | -1/2Fb+1/2Fx                 | 0        | 0  | 0                          | 0                                    |                              |                         |                      |
| CG b  | 0  | 0                            | 0        | 0  | 0                          | 0                                    | 0+0                          | 0                       |                      |
| GC b  | 0  | 0                            | 0        | 0  | 0                          | 0                                    |                              |                         |                      |
| GH 2b | 0  | Fb-1/2Fx                     | 0        | 0  | 0                          | 0                                    | 0+0                          | 0                       |                      |
| HG 2b | 0  | -1/2Fx                       | 0        | 0  | 0                          | 0                                    |                              |                         |                      |
| HI 2b | 0  | 0                            | 0        | 0  | 0                          | 0                                    | 0+0                          | 0                       |                      |
| IH 2b | 0  | 0                            | 0        | 0  | 0                          | 0                                    |                              |                         |                      |
| IE b  | 0  | Fb+3/2Fx                     | 0        | 0  | 0                          | 0                                    | 0+0                          | 0                       |                      |
| EI b  | 0  | -5/2Fb+3/2Fx                 | 0        | 0  | 0                          | 0                                    |                              |                         |                      |
| D     | cedimento nodo -H <sub>1D</sub> u <sub>D</sub> |                              |          |  |                            |                                      |                              | -2Fb <sup>3</sup> /EJ   |                      |
|       | totali   |                              |          |  |                            |                                      |                              | 14/3Fb <sup>3</sup> /EJ | 4Xb <sup>3</sup> /EJ |
|       | iperstatica X=V <sub>A</sub>                   |                              |          |  |                            |                                      |                              | -7/6F                   |                      |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (6x^2/b^2 - x^3/b^3) Fb^2 1/EJ dx = [2x^3/b^2 - 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (2b - 1/4 b) Fb^2 1/EJ = 7/4 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (5 - 9x/b + 3x^2/b^2 + x^3/b^3) Fb^2 1/EJ dx = [5x - 9/2 x^2/b + x^3/b^2 + 1/4 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (5b - 9/2 b + b + 1/4 b) Fb^2 1/EJ = 7/4 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (5 - 5/2 x/b) Fb^2 1/EJ dx = [5x - 5/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (5b - 5/4 b) Fb^2 1/EJ = 15/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/2 + 5/2 x/b) Fb^2 1/EJ dx = [5/2 x + 5/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (5/2 b + 5/4 b) Fb^2 1/EJ = 15/4 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (2 - 4x/b + 2x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

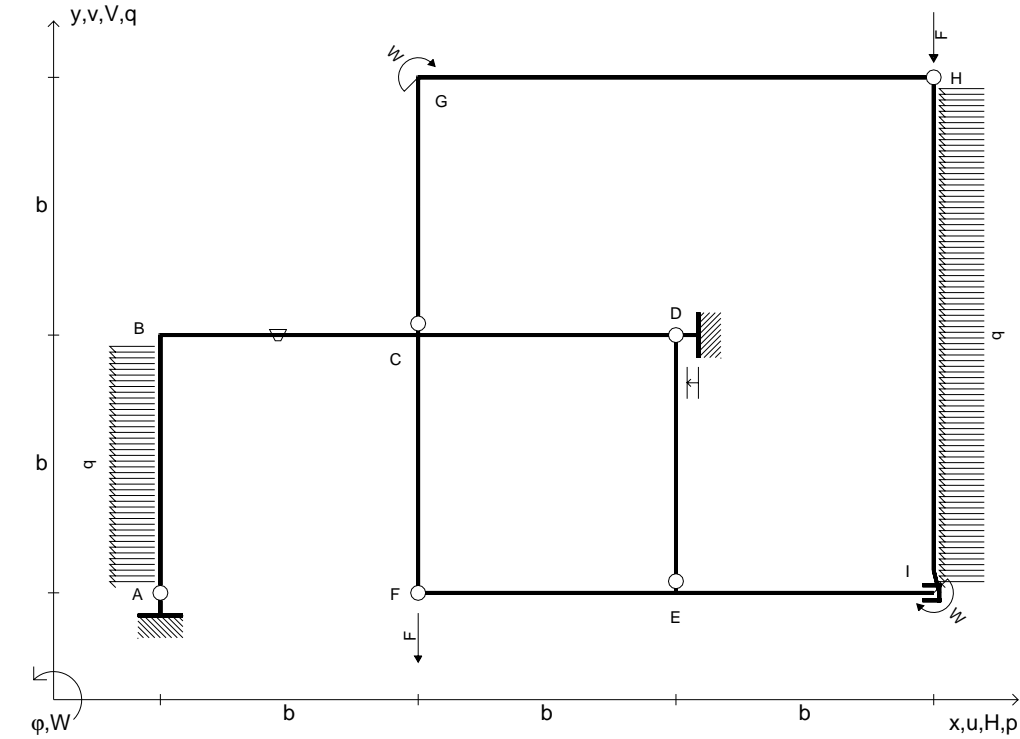
$$= [2x - 2x^2/b + 2/3 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/2 x^2/b]_0^b \theta$$

$$= (2b - 2b + 2/3 b) Fb^2 1/EJ + (b - 1/2 b) \theta = 7/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (2x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx = [2/3 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x^2/b]_0^b \theta$$

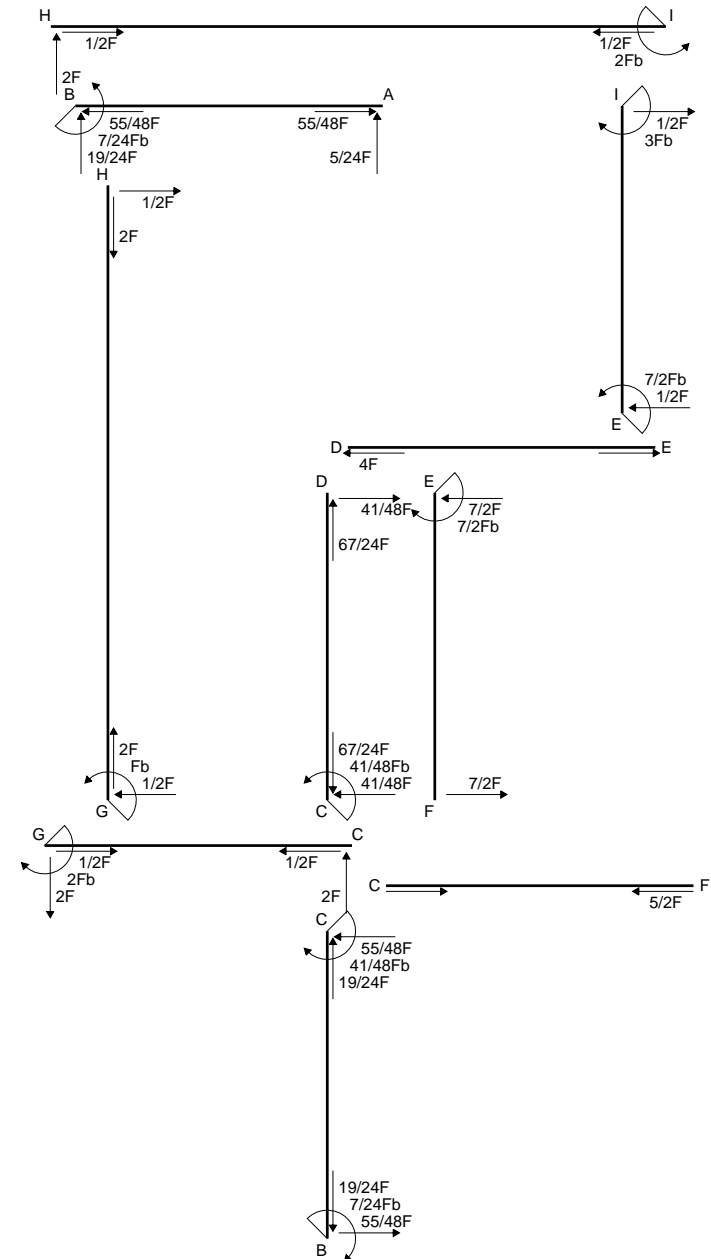
$$= (2/3 b) Fb^2 1/EJ + (-1/2 b) \theta = 7/6 Fb^3/EJ$$

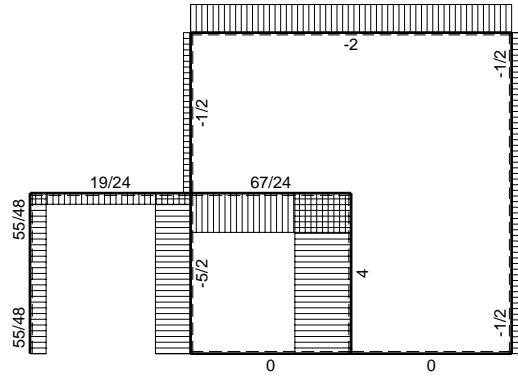




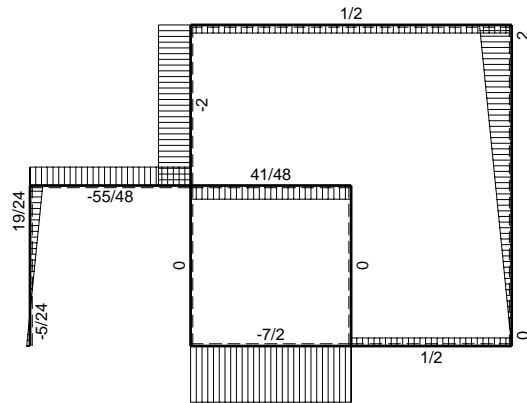
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $P_{HI} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $P_{AB} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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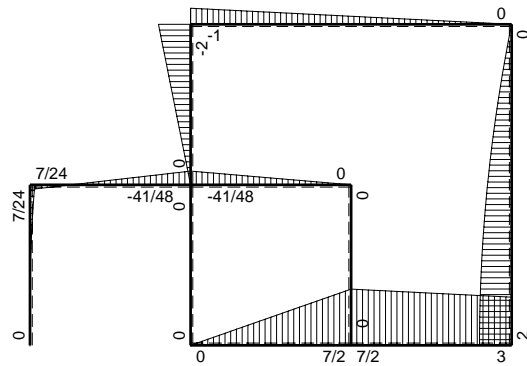




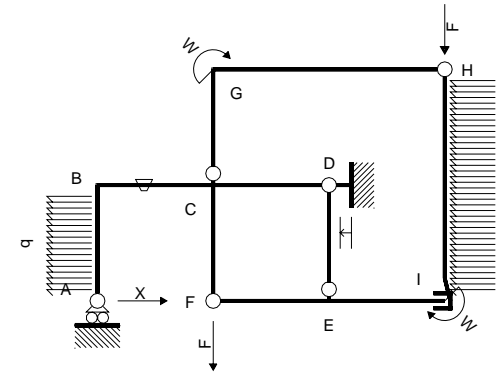
← (+) → F



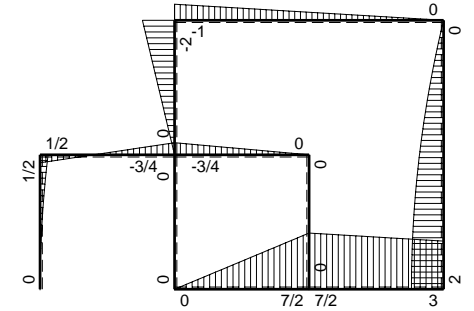
↑ (+) ↓ F



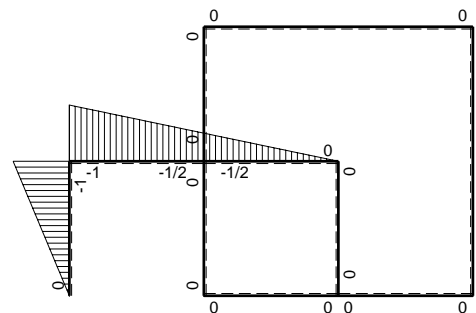
⌚ (+) ⌚ F<sub>b</sub>



Schema di calcolo iperstatico



⌚ (+) ⌚ M<sub>o</sub> flessione da carichi assegnati



⌚ (+) ⌚ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|-----------------------------------|------------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | -x                          | $1/2qx^2$           | 0        | $-1/2qx^3$                        | 0                      | $x^2$                 | $(-1/8+0)Fb^3/EJ$           | $1/3Xb^3/EJ$           |           |
| BA b  | b-x                         | $-1/2Fb+Fx-1/2qx^2$ | 0        | $-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$ | 0                      | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | $-b+1/2x$                   | $1/2Fb-5/4Fx$       | $-Fb/EJ$ | $-1/2Fb^2+3/2Fbx-5/8Fx^2$         | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/24+3/4)Fb^3/EJ$         | $7/12Xb^3/EJ$          |           |
| CB b  | $1/2b+1/2x$                 | $3/4Fb-5/4Fx$       | $Fb/EJ$  | $3/8Fb^2-1/4Fbx-5/8Fx^2$          | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$          | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12Xb^3/EJ$          |           |
| DC b  | $1/2x$                      | $3/4Fx$             | 0        | $3/8Fx^2$                         | 0                      | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |           |
| EF b  | 0                           | $7/2Fb-7/2Fx$       | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | $-7/2Fx$            | 0        | 0                                 | 0                      | 0                     |                             |                        |           |
| FC b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |           |
| CG b  | 0                           | $-2Fx$              | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                                 | 0                      | 0                     |                             |                        |           |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                                 | 0                      | 0                     |                             |                        |           |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                                 | 0                      | 0                     |                             |                        |           |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                                 | 0                      | 0                     |                             |                        |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                                   |                        |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                     |          |                                   |                        |                       |                             | $-5/24Fb^3/EJ$         | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                                   |                        |                       |                             | $5/24F$                |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-1/2 x^3/b^3) Fb^2 1/EJ dx = [-1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [-1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 5/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [-1/2 x + 3/4 x^2/b - 5/24 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (-1/2 b + 3/4 b - 5/24 b) Fb^2 1/EJ + (b - 1/4 b) \theta = 19/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 1/4 x/b - 5/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^2/b - 5/24 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

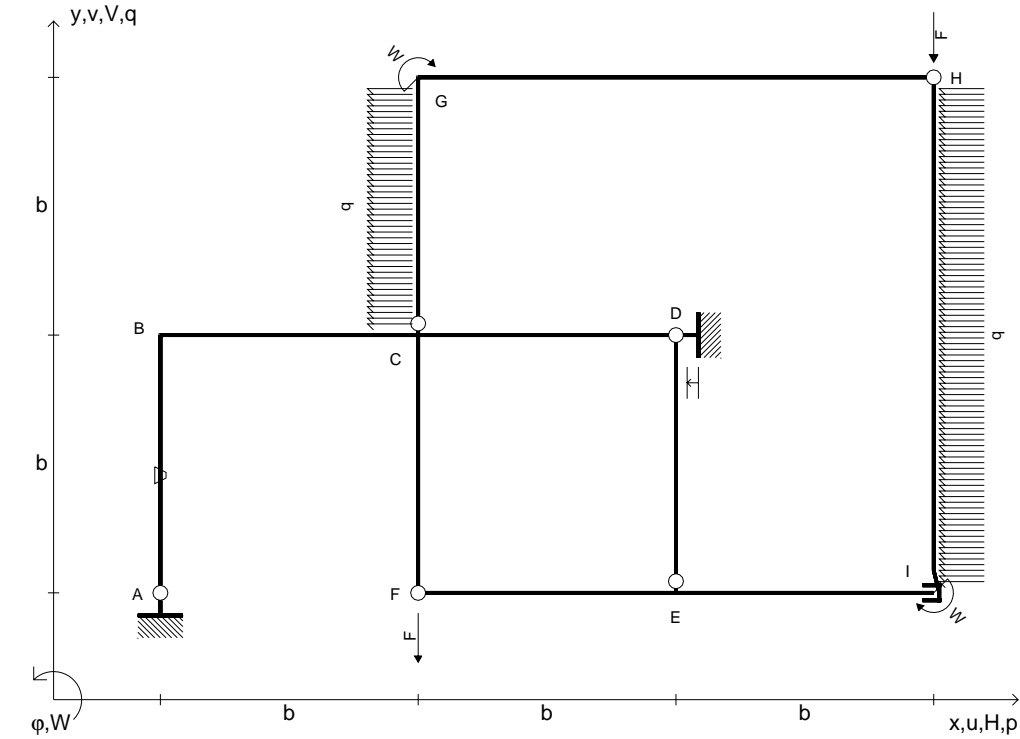
$$= (3/8 b - 1/8 b - 5/24 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = 19/24 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

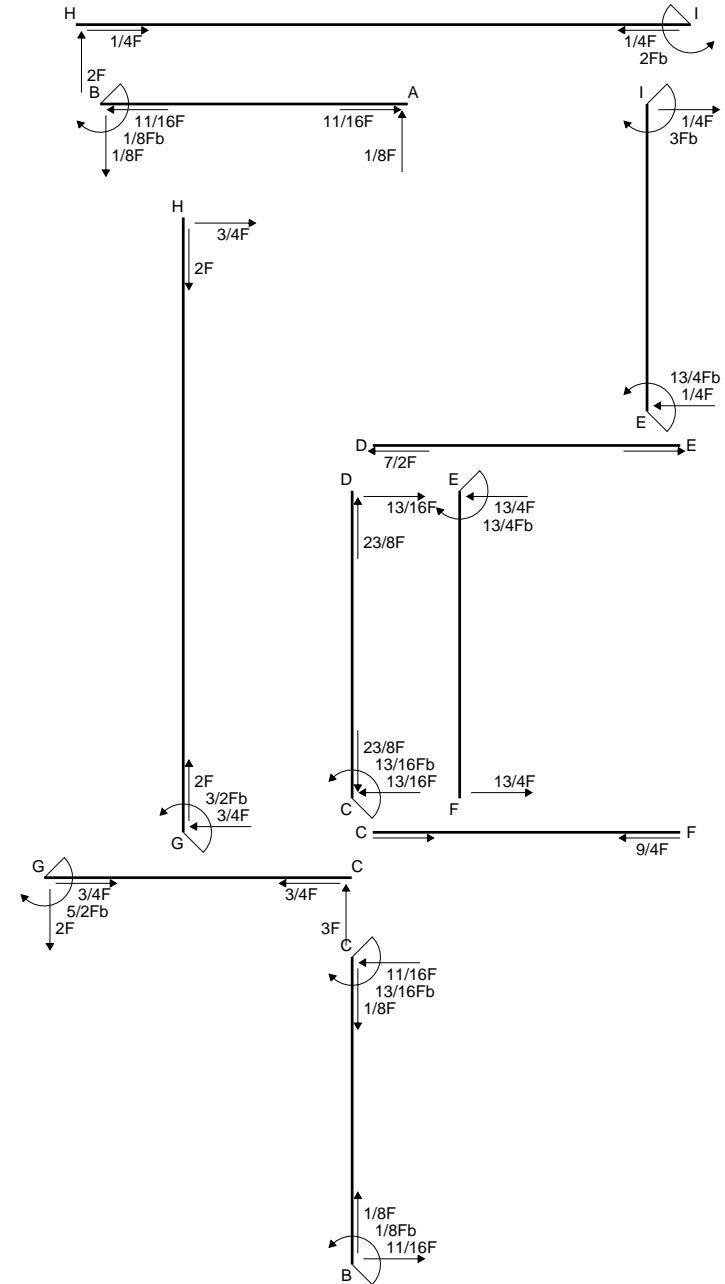
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

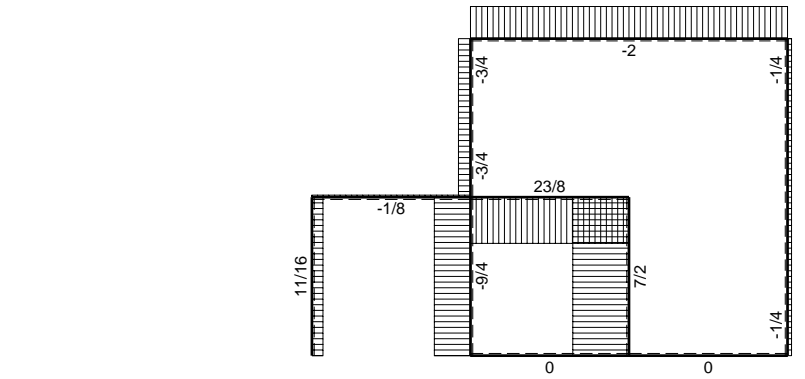
$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$



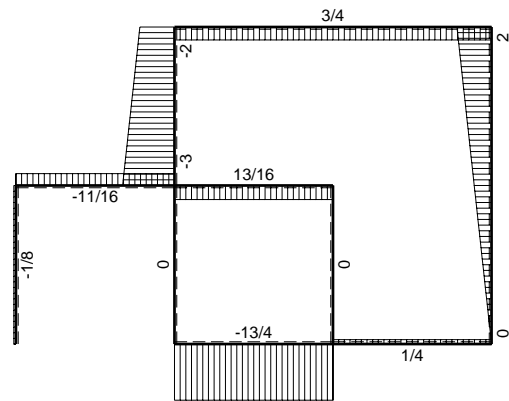
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_D$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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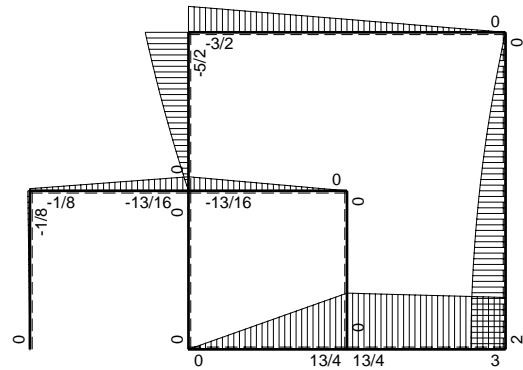




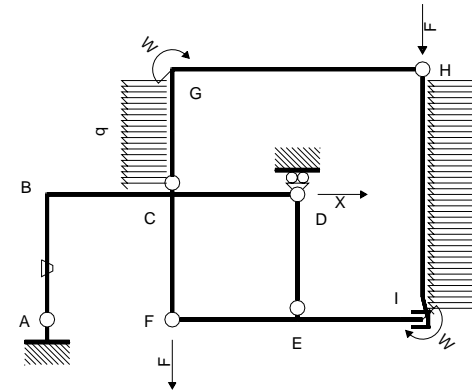
← (+) → F



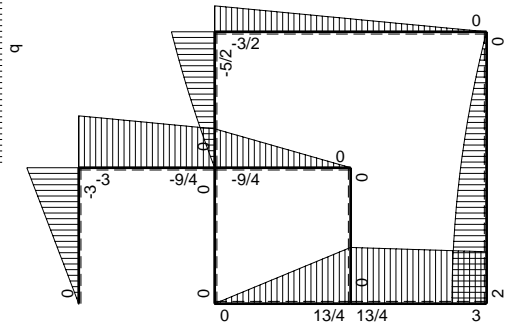
↑ (+) ↓ F



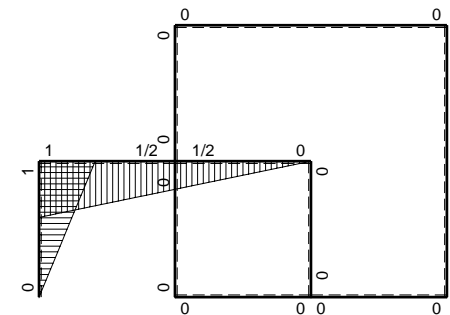
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>o</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_D$ 

| →     | $M_x(x)$                                       | $M_o(x)$                     | $\theta$ | $M_x M_o$                                      | $M_x \theta$                | $M_x M_x$                                  | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$  |
|-------|--|------------------------------|----------|--|-----------------------------|--|-----------------------------|-------------------------|
| AB b  | x  | -3Fx                         | -Fb/EJ   | -3Fx <sup>2</sup>                              | -Fxb/EJ                     | x <sup>2</sup>                             | (-1-1/2)Fb <sup>3</sup> /EJ | 1/3Xb <sup>3</sup> /EJ  |
| BA b  | -b+x   | 3Fb-3Fx                      | Fb/EJ    | -3Fb <sup>2</sup> +6Fbx-3Fx <sup>2</sup>       | -Fb <sup>2</sup> /EJ+Fxb/EJ | b <sup>2</sup> -2bx+x <sup>2</sup>         |                             |                         |
| BC b  | b-1/2x   | -3Fb+3/4Fx                   | 0        | -3Fb <sup>2</sup> +9/4Fbx-3/8Fx <sup>2</sup>   | 0                           | b <sup>2</sup> -bx+1/4x <sup>2</sup>       | (-2+0)Fb <sup>3</sup> /EJ   | 7/12Xb <sup>3</sup> /EJ |
| CB b  | -1/2b-1/2x                                     | 9/4Fb+3/4Fx                  | 0        | -9/8Fb <sup>2</sup> -3/2Fbx-3/8Fx <sup>2</sup> | 0                           | 1/4b <sup>2</sup> +1/2bx+1/4x <sup>2</sup> |                             |                         |
| CD b  | 1/2b-1/2x                                      | -9/4Fb+9/4Fx                 | 0        | -9/8Fb <sup>2</sup> +9/4Fbx-9/8Fx <sup>2</sup> | 0                           | 1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup> | (-3/8+0)Fb <sup>3</sup> /EJ | 1/12Xb <sup>3</sup> /EJ |
| DC b  | -1/2x  | 9/4Fx                        | 0        | -9/8Fx <sup>2</sup>                            | 0                           | 1/4x <sup>2</sup>                          |                             |                         |
| DE b  | 0  | 0                            | 0        | 0  | 0                           | 0  | 0+0                         | 0                       |
| ED b  | 0  | 0                            | 0        | 0  | 0                           | 0  |                             |                         |
| EF b  | 0  | 13/4Fb-13/4Fx                | 0        | 0  | 0                           | 0  | 0+0                         | 0                       |
| FE b  | 0  | -13/4Fx                      | 0        | 0  | 0                           | 0  |                             |                         |
| FC b  | 0  | 0                            | 0        | 0  | 0                           | 0  | 0+0                         | 0                       |
| CF b  | 0  | 0                            | 0        | 0  | 0                           | 0  |                             |                         |
| CG b  | 0  | -3Fx+1/2qx <sup>2</sup>      | 0        | 0  | 0                           | 0  | 0+0                         | 0                       |
| GC b  | 0  | 5/2Fb-2Fx-1/2qx <sup>2</sup> | 0        | 0  | 0                           | 0  |                             |                         |
| GH 2b | 0  | -3/2Fb+3/4Fx                 | 0        | 0  | 0                           | 0  | 0+0                         | 0                       |
| HG 2b | 0  | 3/4Fx                        | 0        | 0  | 0                           | 0  |                             |                         |
| HI 2b | 0  | 2Fx-1/2qx <sup>2</sup>       | 0        | 0  | 0                           | 0  | 0+0                         | 0                       |
| IH 2b | 0  | -2Fb+1/2qx <sup>2</sup>      | 0        | 0  | 0                           | 0  |                             |                         |
| IE b  | 0  | 3Fb+1/4Fx                    | 0        | 0  | 0                           | 0  | 0+0                         | 0                       |
| EI b  | 0  | -13/4Fb+1/4Fx                | 0        | 0  | 0                           | 0  |                             |                         |
| D     | cedimento nodo -H <sub>1D</sub> u <sub>D</sub> |                              |          |  |                             |  | Fb <sup>3</sup> /EJ         |                         |
|       | totali   |                              |          |  |                             |  | -23/8Fb <sup>3</sup> /EJ    | Xb <sup>3</sup> /EJ     |
|       | iperstatica X=H <sub>D</sub>                   |                              |          |  |                             |  | 23/8F                       |                         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-3x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx = [-x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x^2/b]_0^b \theta$$

$$= (-b) Fb^2 1/EJ + (-1/2 b) \theta = -3/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-3 + 6x/b - 3x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

$$= [-3x + 3x^2/b - x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/2 x^2/b]_0^b \theta$$

$$= (-3b + 3b - b) Fb^2 1/EJ + (b - 1/2 b) \theta = -3/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-3 + 9/4 x/b - 3/8 x^2/b^2) Fb^2 1/EJ dx = [-3x + 9/8 x^2/b - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-3b + 9/8 b - 1/8 b) Fb^2 1/EJ = -2 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-9/8 - 3/2 x/b - 3/8 x^2/b^2) Fb^2 1/EJ dx = [-9/8 x - 3/4 x^2/b - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-9/8 b - 3/4 b - 1/8 b) Fb^2 1/EJ = -2 Fb^3/EJ$$

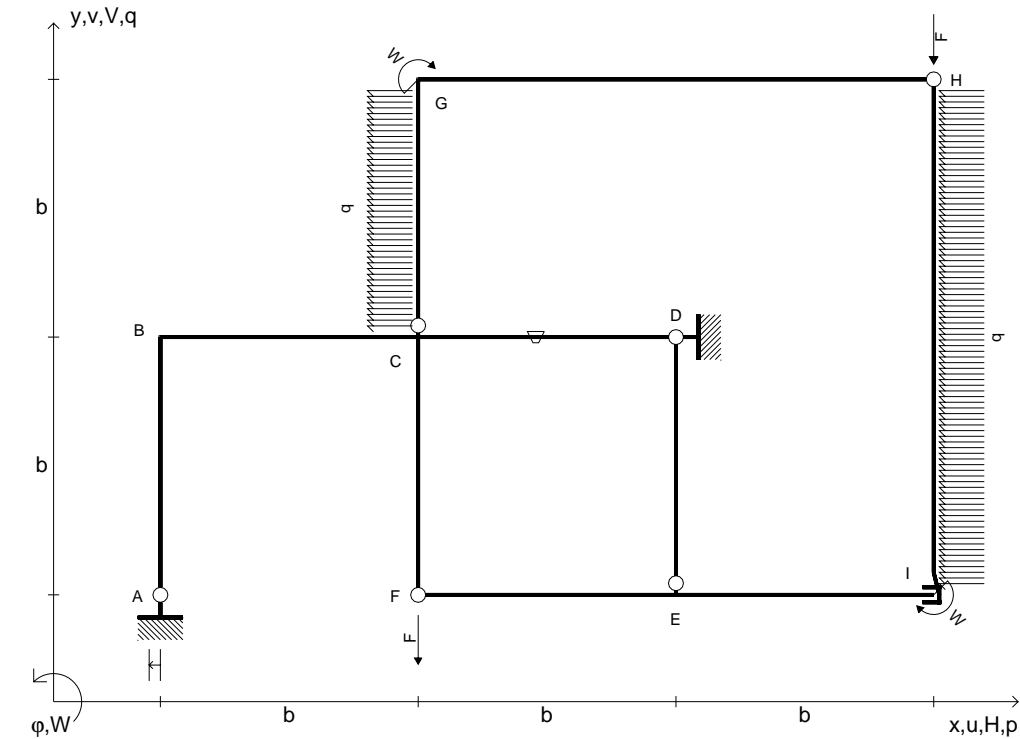
$$L_{CD}^{xo} = \int_0^b (-9/8 + 9/4 x/b - 9/8 x^2/b^2) Fb^2 1/EJ dx = [-9/8 x + 9/8 x^2/b - 3/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-9/8 b + 9/8 b - 3/8 b) Fb^2 1/EJ = -3/8 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (-9/8 x^2/b^2) Fb^2 1/EJ dx = [-3/8 x^3/b^2]_0^b Fb^2 1/EJ$$

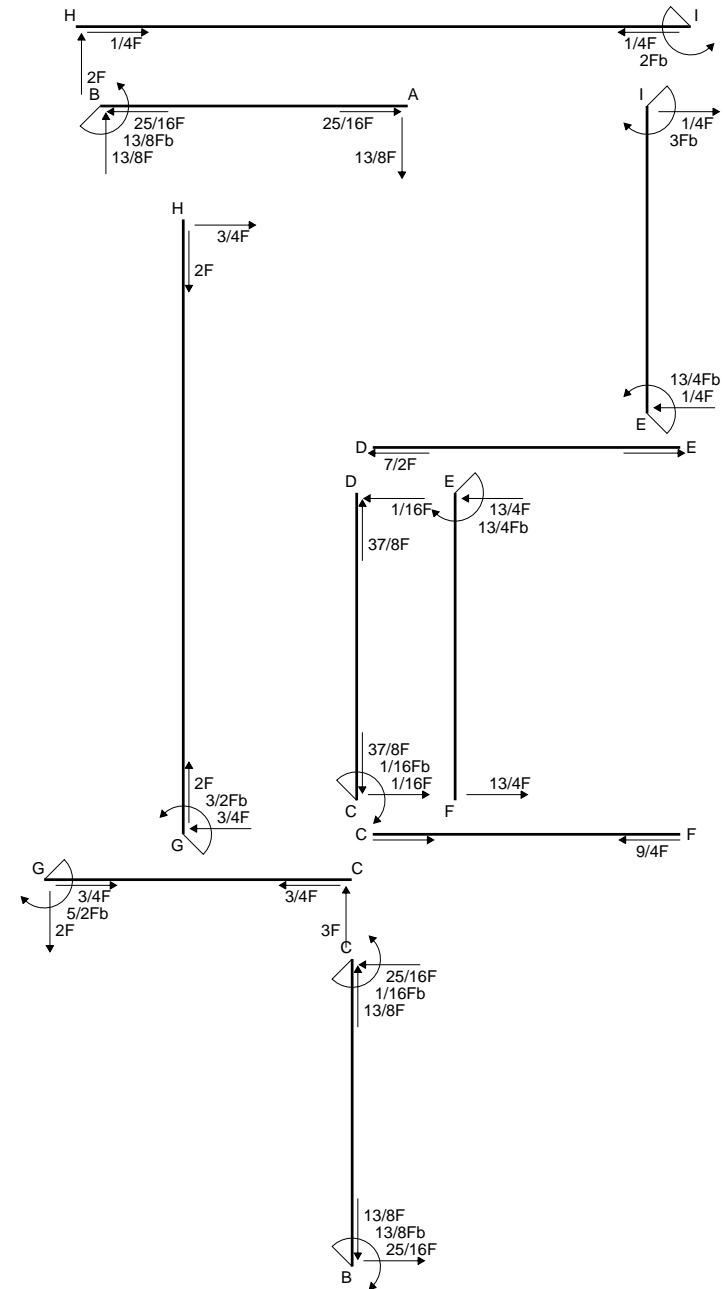
$$= (-3/8 b) Fb^2 1/EJ = -3/8 Fb^3/EJ$$



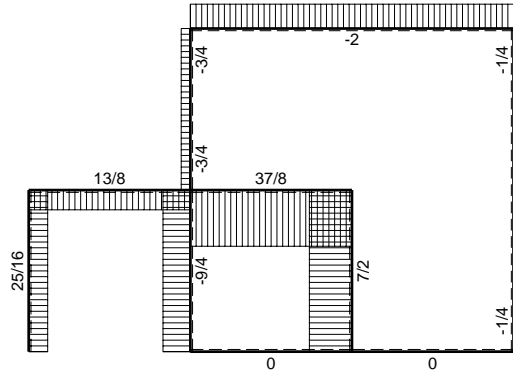


|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

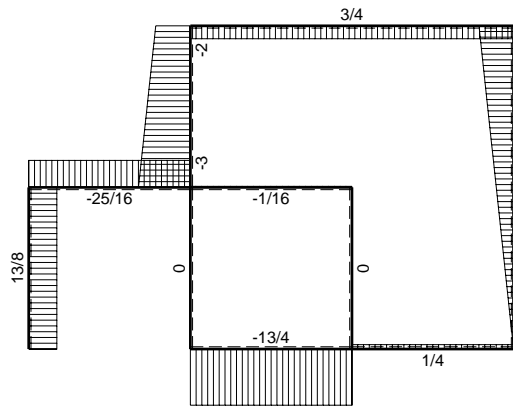
Reazioni iperstatiche in soluzione:  $X=V_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
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 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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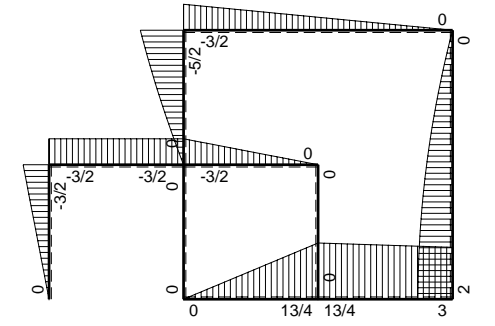
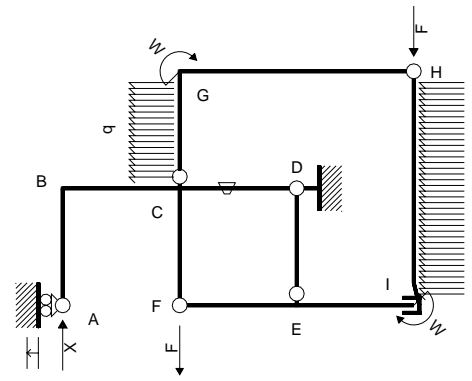
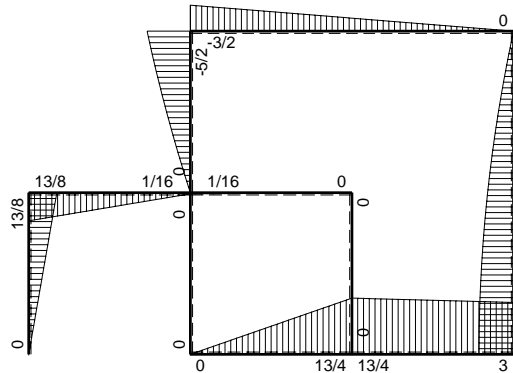
← (+) → F



↑ (+) ↓ F

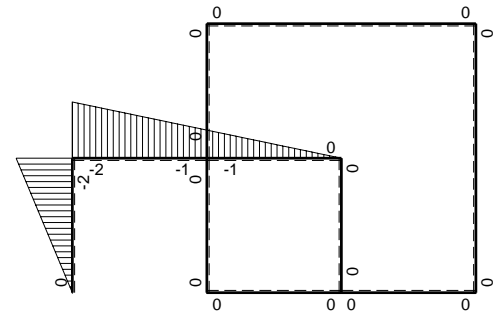


⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico

⊕ (+) ⊖ M<sub>0</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=V_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$              | $M_x \theta$     | $M_x M_x$       | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|---------------------|----------|------------------------|------------------|-----------------|-----------------------------|--------------------------|
| AB b  | -2x                         | -3/2Fx              | 0        | $3Fx^2$                | 0                | $4x^2$          | $(1+0)Fb^3/EJ$              | $4/3Xb^3/EJ$             |
| BA b  | 2b-2x                       | $3/2Fb-3/2Fx$       | 0        | $3Fb^2-6Fbx+3Fx^2$     | 0                | $4b^2-8bx+4x^2$ |                             |                          |
| BC b  | -2b+x                       | -3/2Fb              | 0        | $3Fb^2-3/2Fbx$         | 0                | $4b^2-4bx+x^2$  | $(9/4+0)Fb^3/EJ$            | $7/3Xb^3/EJ$             |
| CB b  | b+x                         | $3/2Fb$             | 0        | $3/2Fb^2+3/2Fbx$       | 0                | $b^2+2bx+x^2$   |                             |                          |
| CD b  | -b+x                        | $-3/2Fb+3/2Fx$      | -Fb/EJ   | $3/2Fb^2-3Fbx+3/2Fx^2$ | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$   | $(1/2+1/2)Fb^3/EJ$          | $1/3Xb^3/EJ$             |
| DC b  | x                           | $3/2Fx$             | Fb/EJ    | $3/2Fx^2$              | $Fxb/EJ$         | $x^2$           |                             |                          |
| DE b  | 0                           | 0                   | 0        | 0                      | 0                | 0               | 0+0                         | 0                        |
| ED b  | 0                           | 0                   | 0        | 0                      | 0                | 0               |                             |                          |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                      | 0                | 0               | 0+0                         | 0                        |
| FE b  | 0                           | -13/4Fx             | 0        | 0                      | 0                | 0               |                             |                          |
| FC b  | 0                           | 0                   | 0        | 0                      | 0                | 0               | 0+0                         | 0                        |
| CF b  | 0                           | 0                   | 0        | 0                      | 0                | 0               |                             |                          |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                      | 0                | 0               | 0+0                         | 0                        |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                      | 0                | 0               |                             |                          |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                      | 0                | 0               | 0+0                         | 0                        |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                      | 0                | 0               |                             |                          |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                      | 0                | 0               | 0+0                         | 0                        |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                      | 0                | 0               |                             |                          |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                      | 0                | 0               | 0+0                         | 0                        |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                      | 0                | 0               |                             |                          |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                        |                  |                 | $2Fb^3/EJ$                  |                          |
|       | totali                      |                     |          |                        |                  |                 | $25/4Fb^3/EJ$               | $4Xb^3/EJ$               |
|       | iperstatica $X=V_A$         |                     |          |                        |                  |                 | $-25/16F$                   |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = [4 x - 4 x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = [4 x - 2 x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (3 x^2/b^2) Fb^2 1/EJ dx = [x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (3 - 6 x/b + 3 x^2/b^2) Fb^2 1/EJ dx = [3 x - 3 x^2/b + x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3 b - 3 b + b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3 - 3/2 x/b) Fb^2 1/EJ dx = [3 x - 3/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (3 b - 3/4 b) Fb^2 1/EJ = 9/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/2 + 3/2 x/b) Fb^2 1/EJ dx = [3/2 x + 3/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (3/2 b + 3/4 b) Fb^2 1/EJ = 9/4 Fb^3/EJ$$

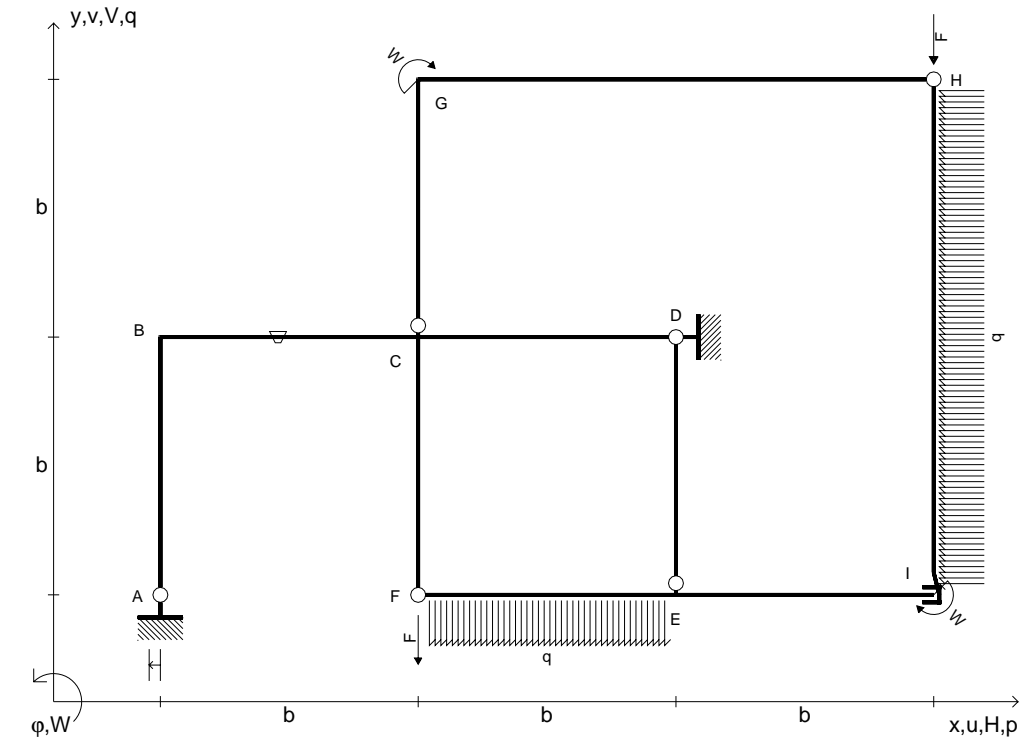
$$L_{CD}^{xo} = \int_0^b (3/2 - 3 x/b + 3/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx$$

$$= [3/2 x - 3/2 x^2/b + 1/2 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/2 x^2/b]_0^b \theta$$

$$= (3/2 b - 3/2 b + 1/2 b) Fb^2 1/EJ + (b - 1/2 b) \theta = Fb^3/EJ$$

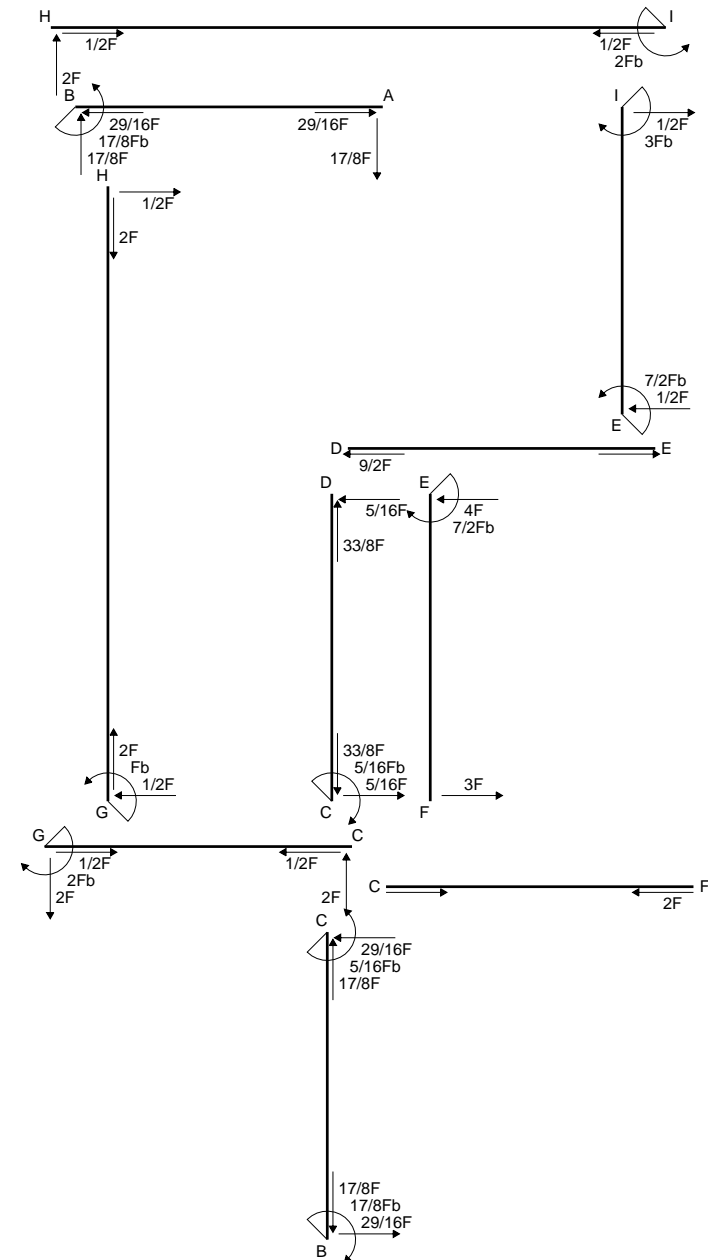
$$L_{DC}^{xo} = \int_0^b (3/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx = [1/2 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x^2/b]_0^b \theta$$

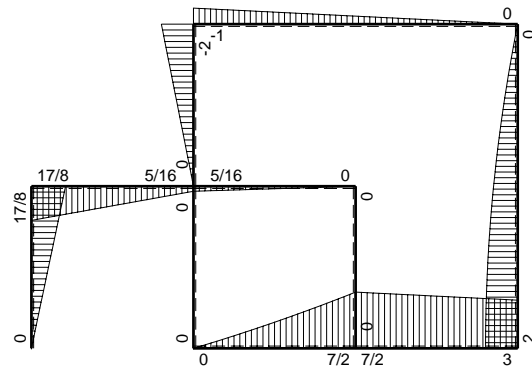
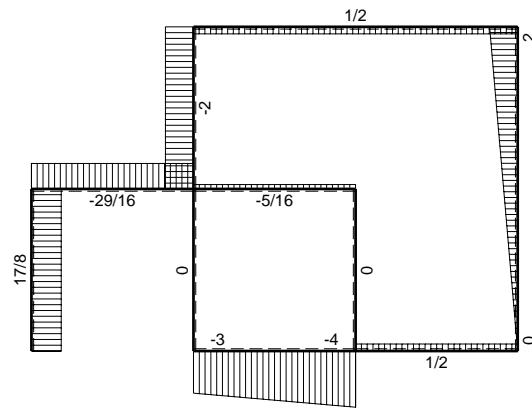
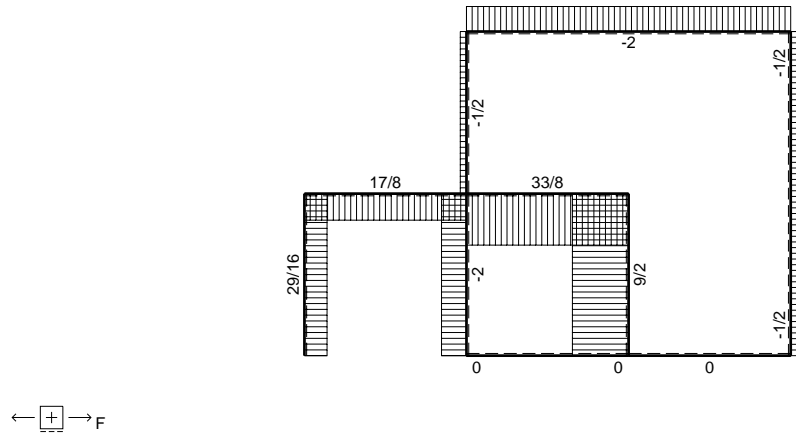
$$= (1/2 b) Fb^2 1/EJ + (-1/2 b) \theta = Fb^3/EJ$$



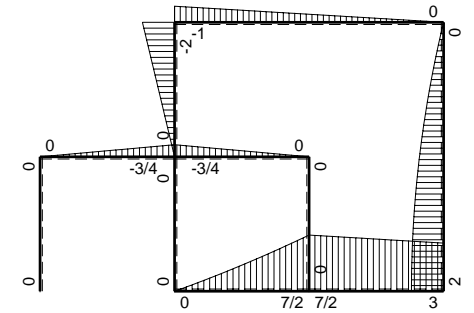
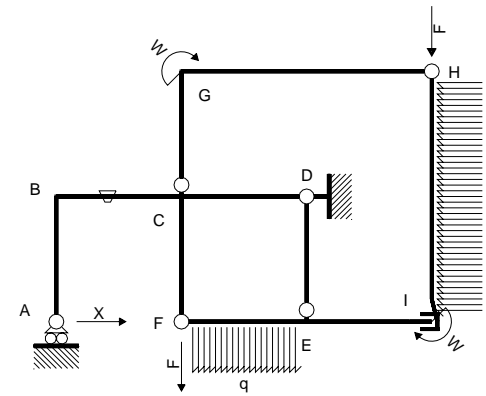
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $P_{HI} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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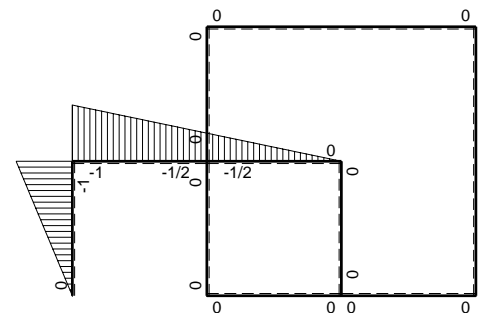


$\left[ \begin{array}{c} + \\ \curvearrowright \end{array} \right] F_b$



Schema di calcolo iperstatico

$\left[ \begin{array}{c} + \\ \curvearrowright \end{array} \right] M_o$  flessione da carichi assegnati



$\left[ \begin{array}{c} + \\ \curvearrowright \end{array} \right] M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | 0                   | 0        | 0                        | 0                      | $x^2$                 | 0+0                         | $1/3Xb^3/EJ$           |
| BA b  | b-x                         | 0                   | 0        | 0                        | 0                      | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | -3/4Fx              | -Fb/EJ   | $3/4Fbx-3/8Fx^2$         | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/4+3/4)Fb^3/EJ$          | $7/12Xb^3/EJ$          |
| CB b  | $1/2b+1/2x$                 | $3/4Fb-3/4Fx$       | Fb/EJ    | $3/8Fb^2-3/8Fx^2$        | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12Xb^3/EJ$          |
| DC b  | $1/2x$                      | $3/4Fx$             | 0        | $3/8Fx^2$                | 0                      | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                        | 0                      | 0                     |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |
| CG b  | 0                           | -2Fx                | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                        | 0                      | 0                     |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                        | 0                      | 0                     |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                        | 0                      | 0                     |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                        | 0                      | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |                        |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                     |          |                          |                        |                       | $17/8Fb^3/EJ$               | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                     |          |                          |                        |                       | -17/8F                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{3}{4} \frac{x}{b} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( 1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{3}{8} \frac{x^2}{b} - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( b - \frac{1}{4} b \right) \theta = Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{3}{8} x - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{2} b - \frac{1}{4} b \right) \theta = Fb^3/EJ$$

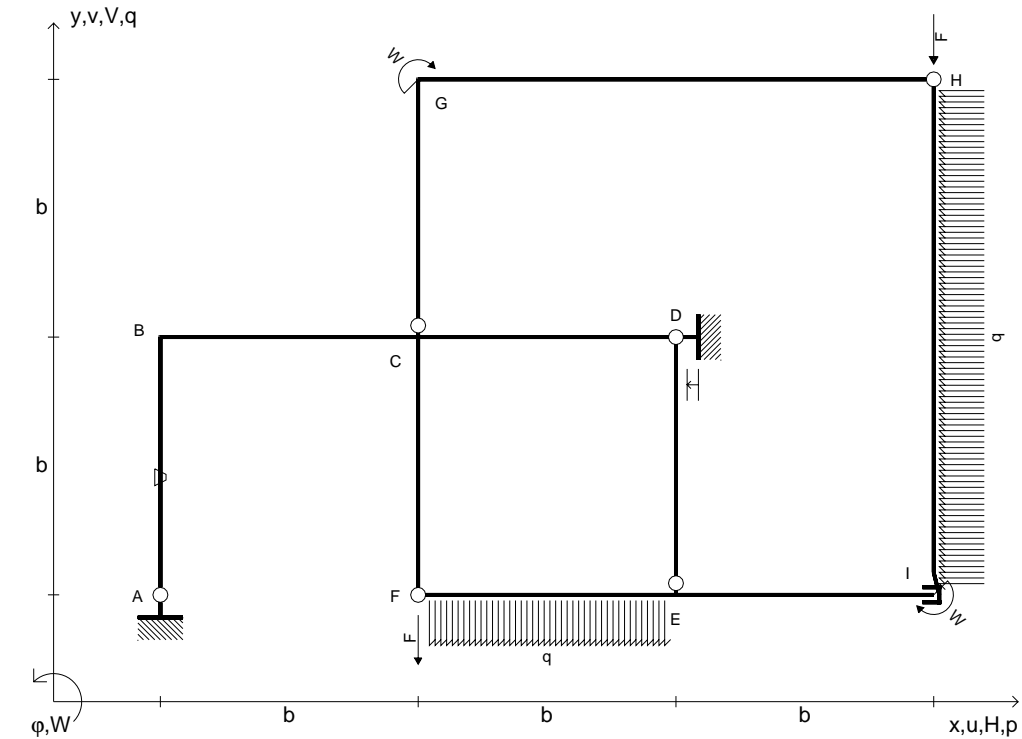
$$L_{CD}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{4} \frac{x}{b} + \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} x - \frac{3}{8} \frac{x^2}{b} + \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{3}{8} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

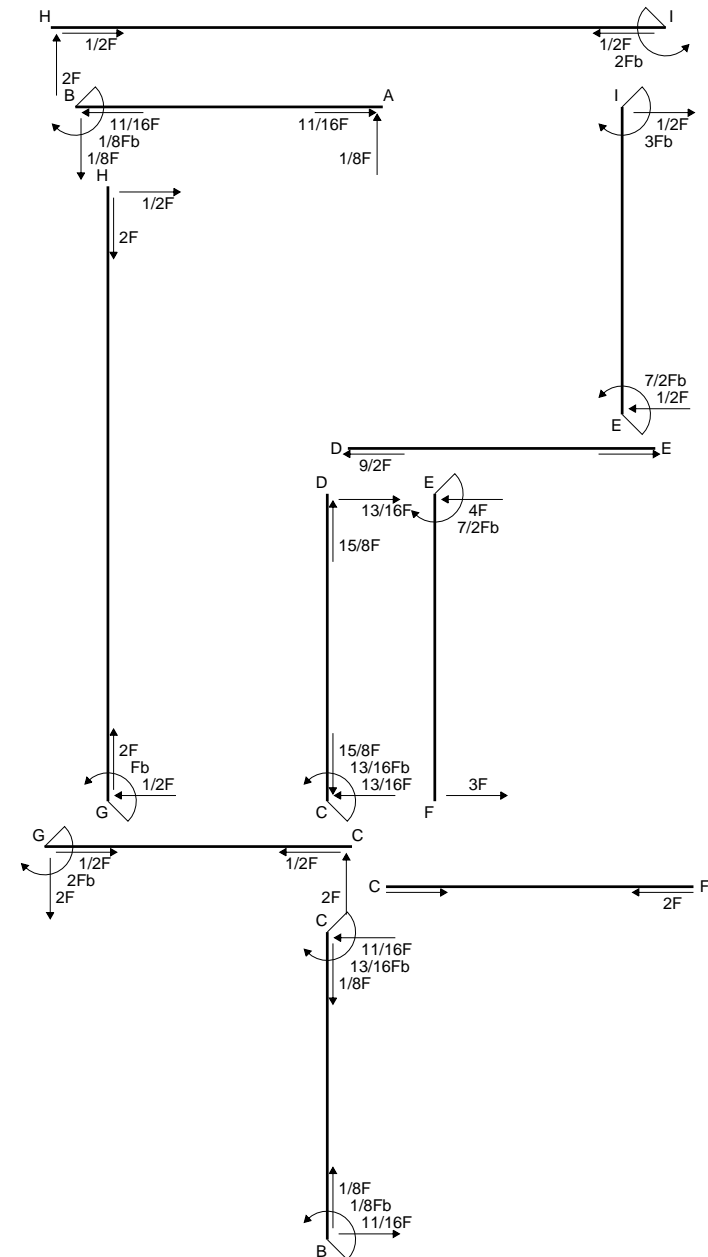
$$= \left( \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

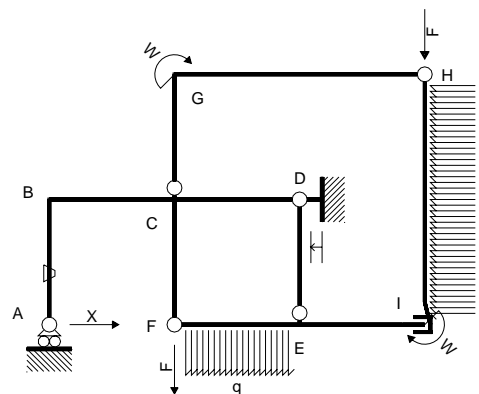
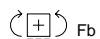
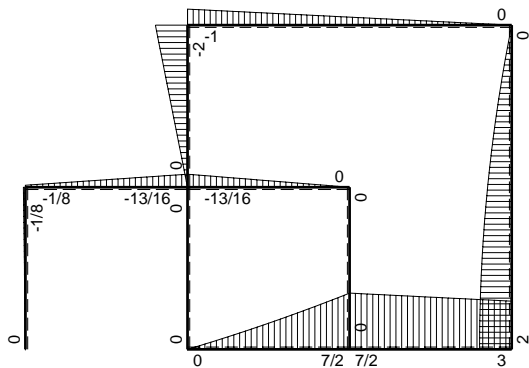
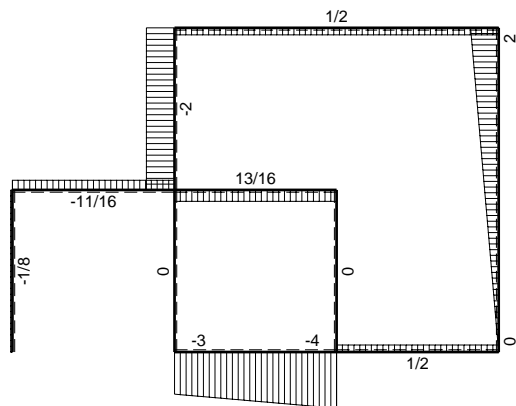
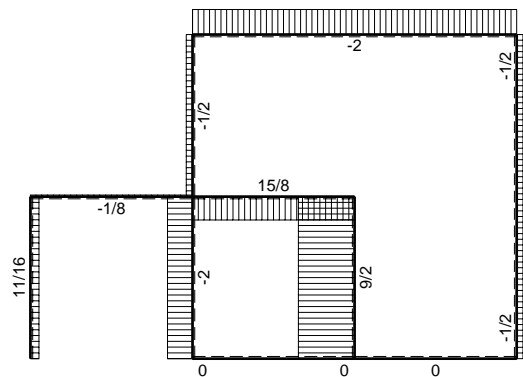
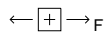




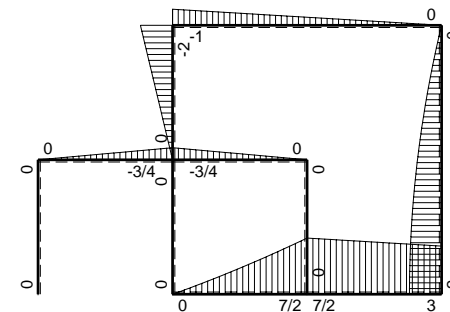
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $P_{HI} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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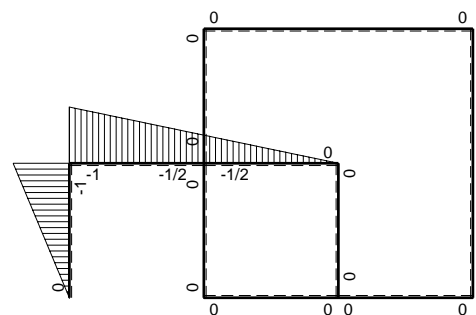




Schema di calcolo iperstatico



$M_o$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|--------------------------|-----------|
| AB b  | -x                          | 0                   | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$             |           |
| BA b  | b-x                         | 0                   | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                          |           |
| BC b  | -b+1/2x                     | -3/4Fx              | 0        | $3/4Fbx-3/8Fx^2$         | 0                | $b^2-bx+1/4x^2$       | $(1/4+0)Fb^3/EJ$            | $7/12Xb^3/EJ$            |           |
| CB b  | $1/2b+1/2x$                 | $3/4Fb-3/4Fx$       | 0        | $3/8Fb^2-3/8Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                          |           |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12Xb^3/EJ$            |           |
| DC b  | $1/2x$                      | $3/4Fx$             | 0        | $3/8Fx^2$                | 0                | $1/4x^2$              |                             |                          |           |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                          |           |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                          |           |
| CG b  | 0                           | -2Fx                | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                        | 0                | 0                     |                             |                          |           |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                        | 0                | 0                     |                             |                          |           |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                  |                       |                             | $-Fb^3/EJ$               |           |
|       | totali                      |                     |          |                          |                  |                       |                             | $-1/8Fb^3/EJ$            | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                          |                  |                       |                             | $1/8F$                   |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

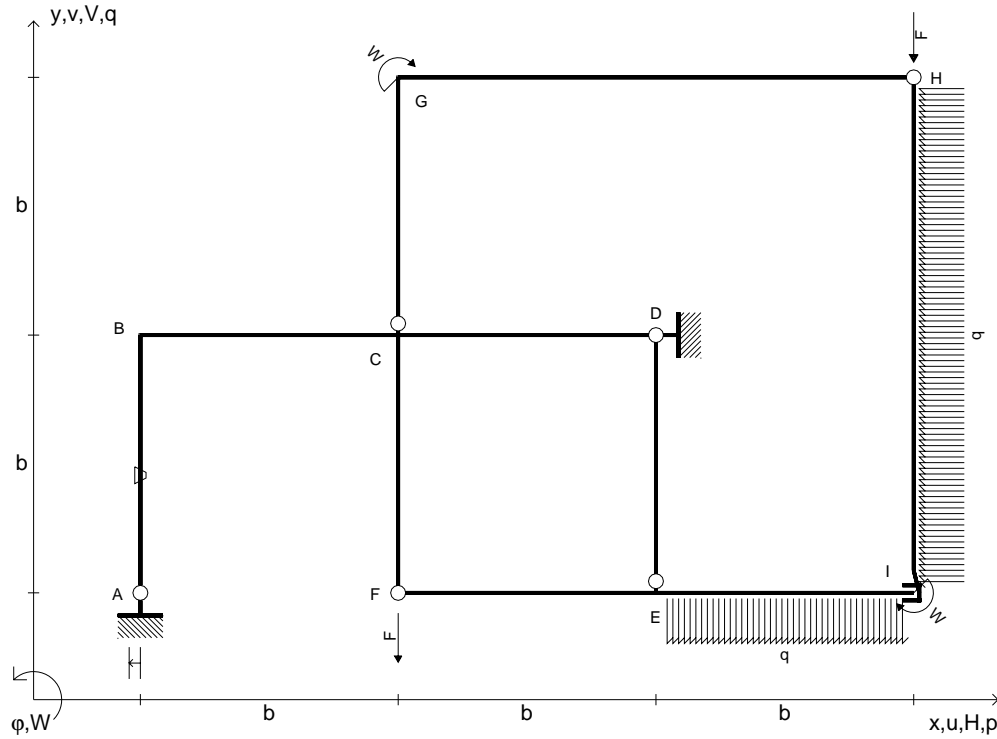
$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

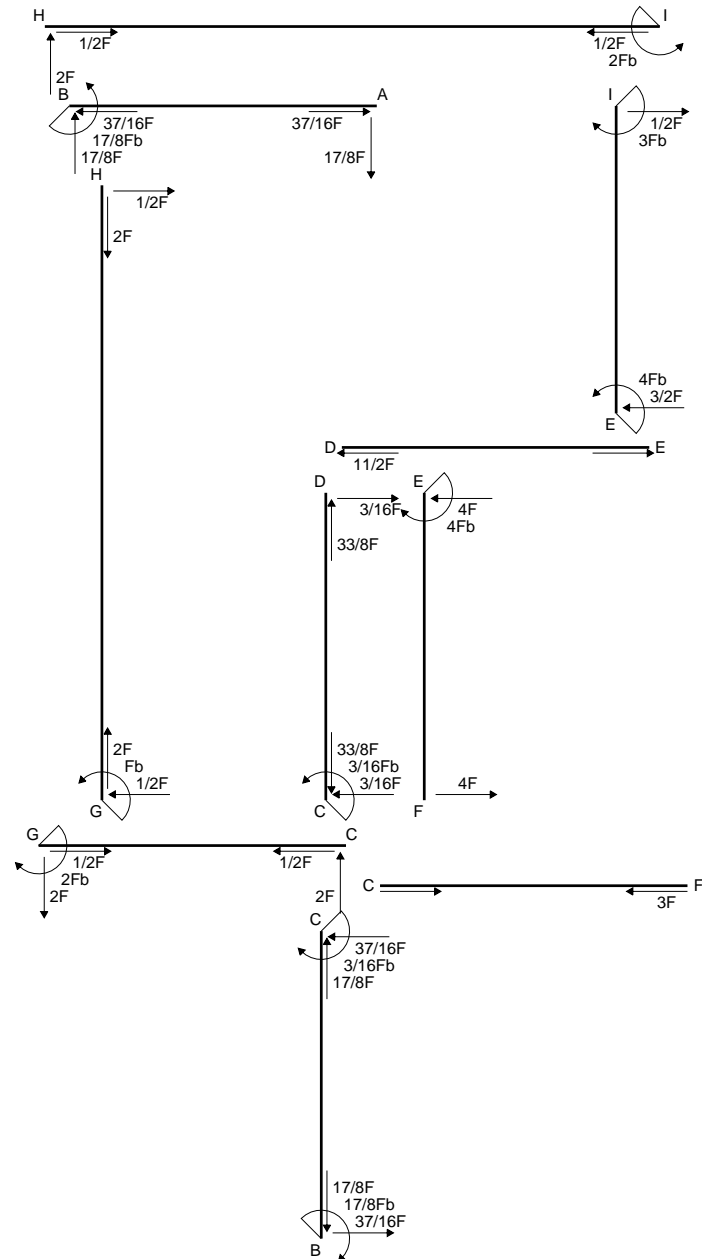
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

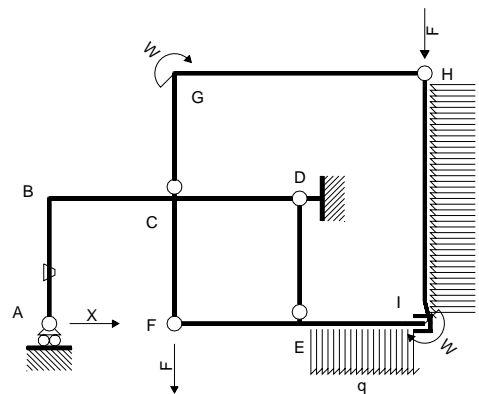
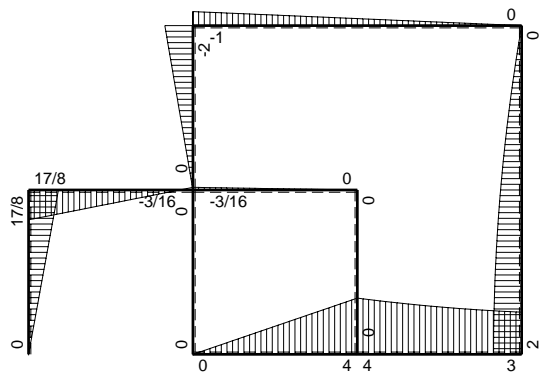
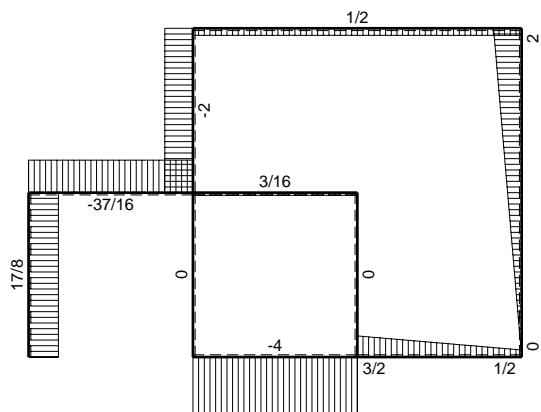
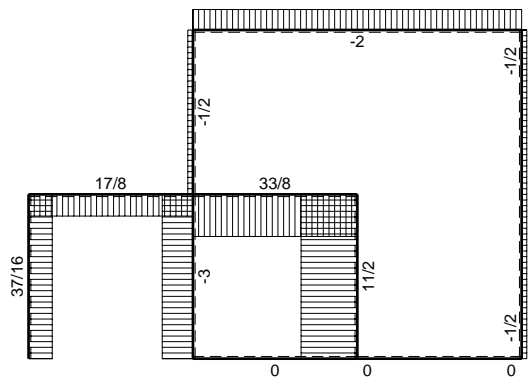
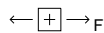
$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$



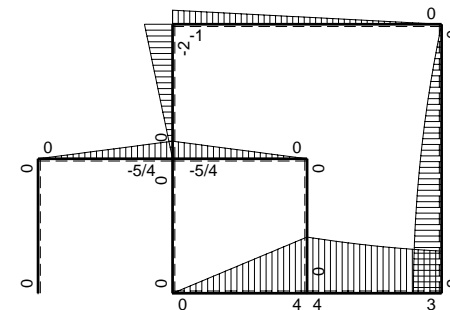
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
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 Calcolare reazioni vincolari della struttura e delle aste.  
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 Carichi di aste curve misurati in proiezione sugli assi  $x, y$ .  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta  $YZ$  con origine in  $Y$ .  
 Curvatura  $\theta$  asta  $AB$  positiva se convessa a destra con inizio  $A$ .  
 Spostamento orizzontale assoluto  $u$  imposto al nodo  $A$ .  
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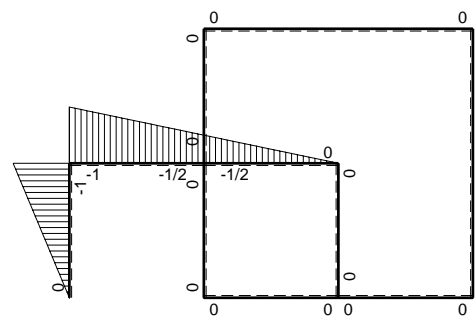




Schema di calcolo iperstatico



M<sub>0</sub> flessione da carichi assegnati



M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | 0                    | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$           |
| BA b  | b-x                         | 0                    | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | -5/4Fx               | 0        | $5/4Fbx-5/8Fx^2$         | 0                | $b^2-bx+1/4x^2$       | $(5/12+0)Fb^3/EJ$           | $7/12Xb^3/EJ$          |
| CB b  | $1/2b+1/2x$                 | $5/4Fb-5/4Fx$        | 0        | $5/8Fb^2-5/8Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb^2-5/4Fbx+5/8Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(5/24+0)Fb^3/EJ$           | $1/12Xb^3/EJ$          |
| DC b  | $1/2x$                      | $5/4Fx$              | 0        | $5/8Fx^2$                | 0                | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                | 0                     |                             |                        |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | -4Fx                 | 0        | 0                        | 0                | 0                     |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                | 0                     |                             |                        |
| CG b  | 0                           | -2Fx                 | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                        | 0                | 0                     |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                        | 0                | 0                     |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                        | 0                | 0                     |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                        | 0                | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                          |                  |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                      |          |                          |                  |                       | $17/8Fb^3/EJ$               | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                      |          |                          |                  |                       | -17/8F                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left( \frac{x}{b} \right) \theta dx = \left[ \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left( -1 + x/b \right) \theta dx = \left[ -x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( -b + \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{5}{4} \frac{x}{b} - \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{8} \frac{x^2}{b} - \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{5}{8} b - \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{12} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{5}{8} - \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{8} x - \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{5}{8} b - \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{12} Fb^3/EJ$$

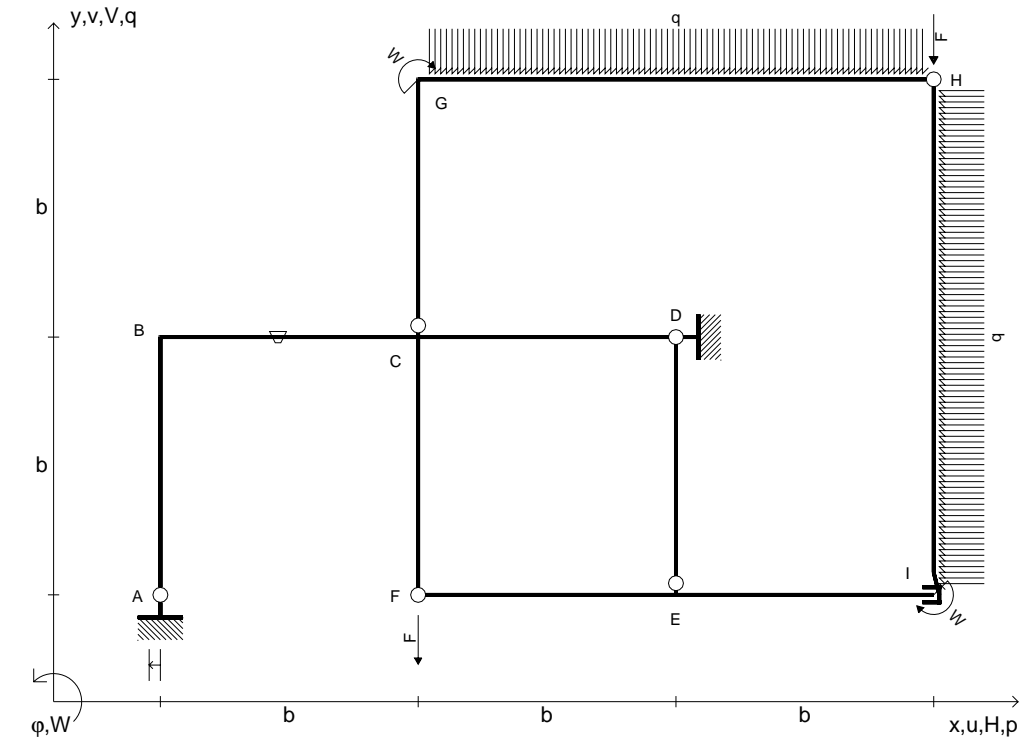
$$L_{CD}^{xo} = \int_0^b \left( \frac{5}{8} - \frac{5}{4} \frac{x}{b} + \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{8} x - \frac{5}{8} \frac{x^2}{b} + \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{5}{8} b - \frac{5}{8} b + \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{24} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

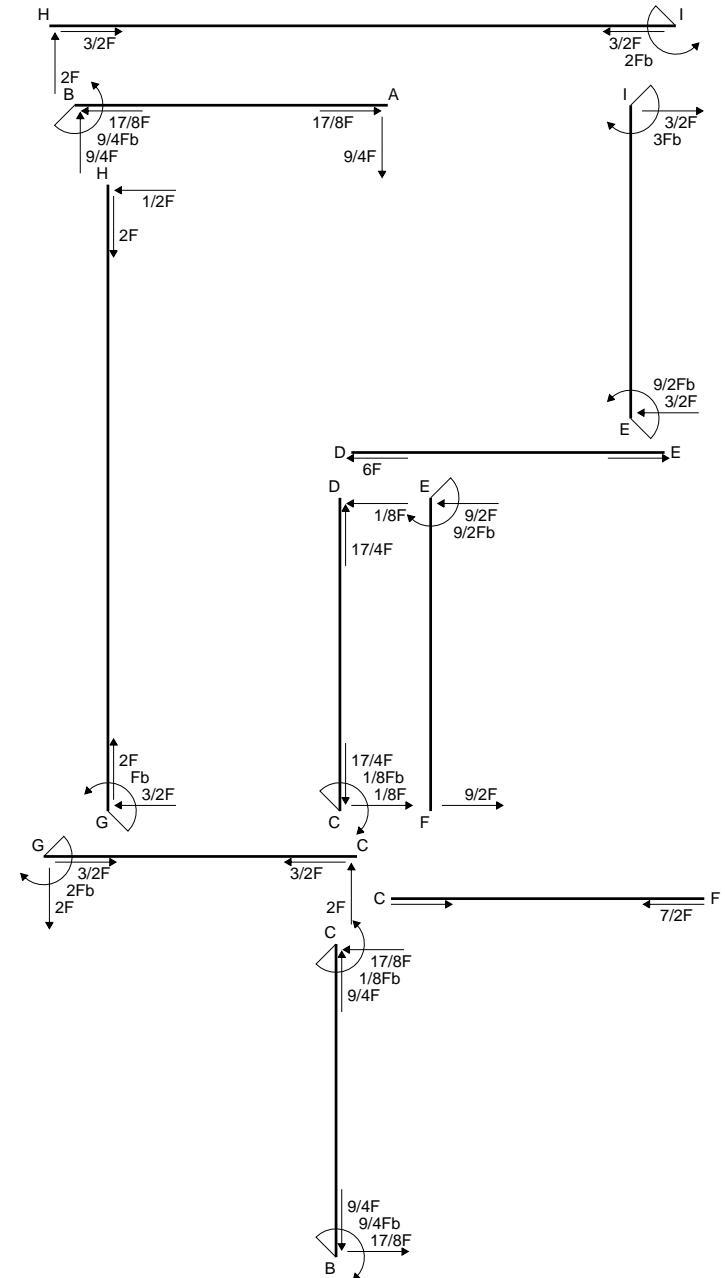
$$= \left( \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{24} Fb^3/EJ$$

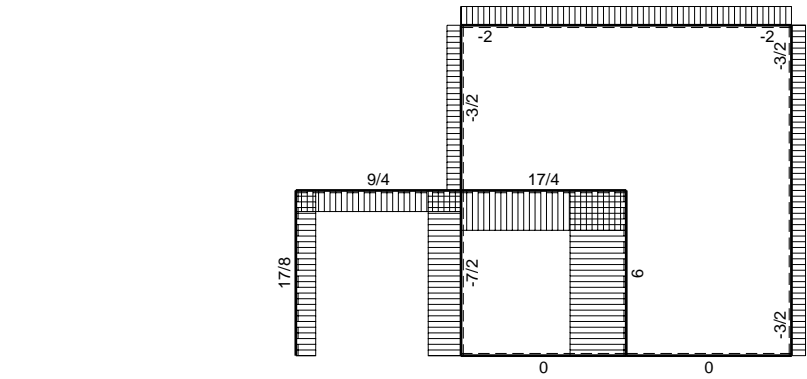




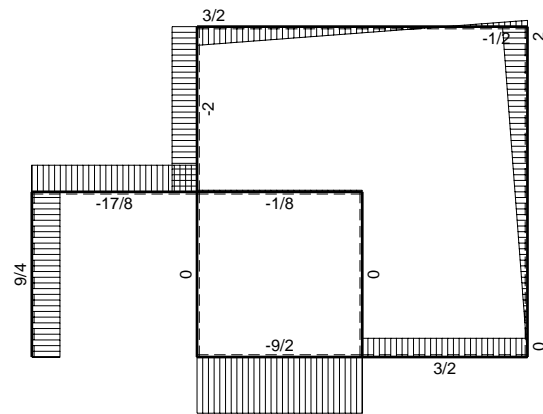
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=W_{BC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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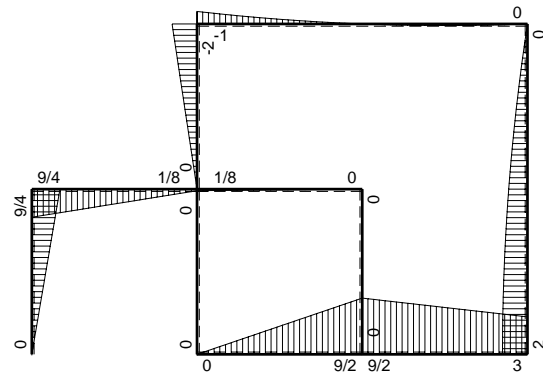




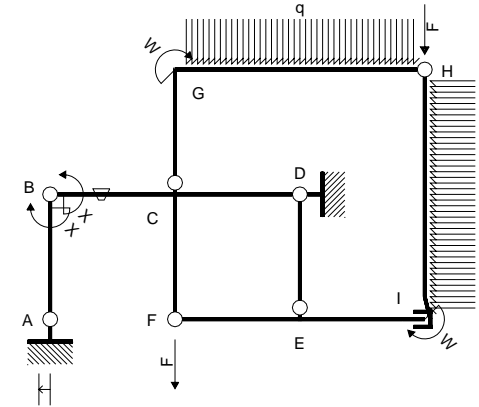
← (+) → F



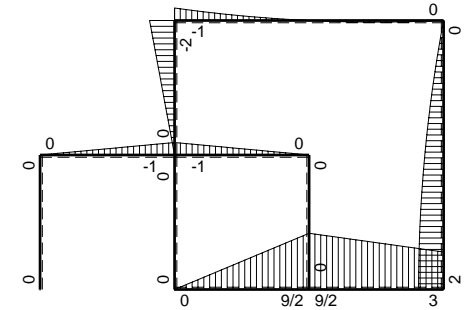
↑ (+) ↓ F



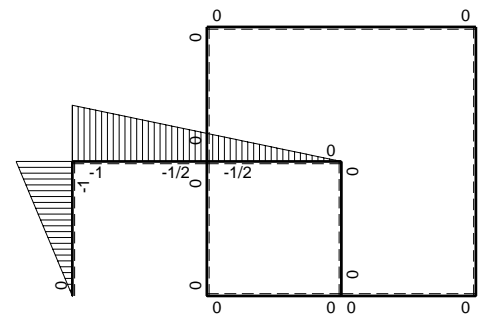
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>o</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$            | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/2Fb-9/2Fx$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/2Fx$            | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+3/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+3/2Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/2Fb+3/2Fx$      | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                      |                     |                         | $9/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                      |                     |                         | $-9/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

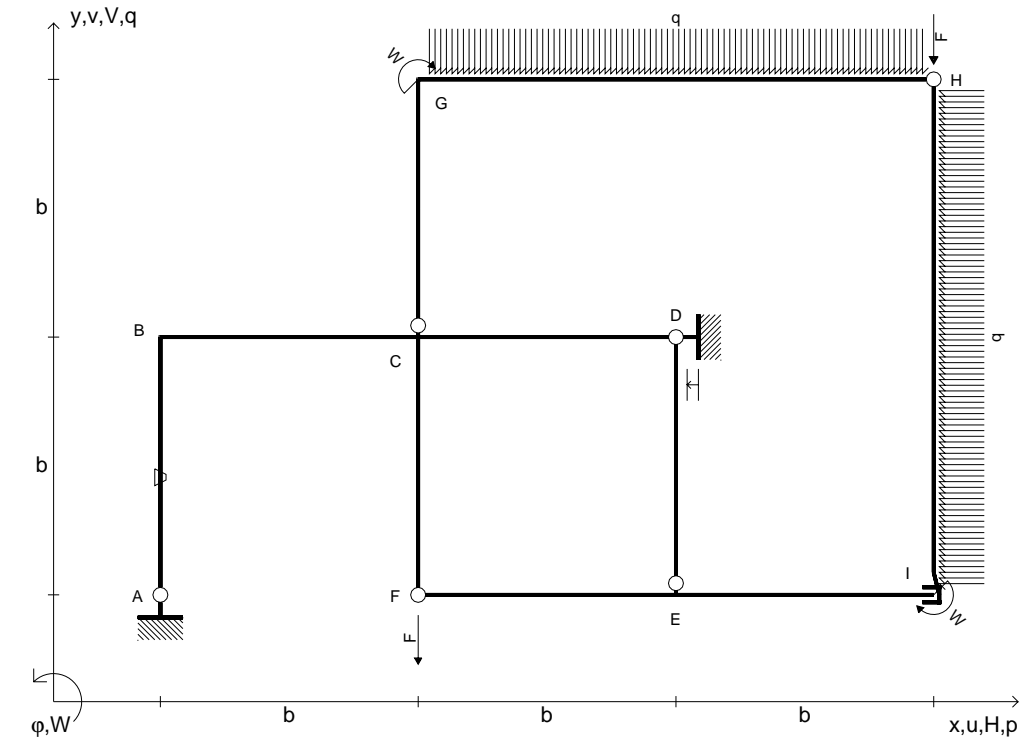
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

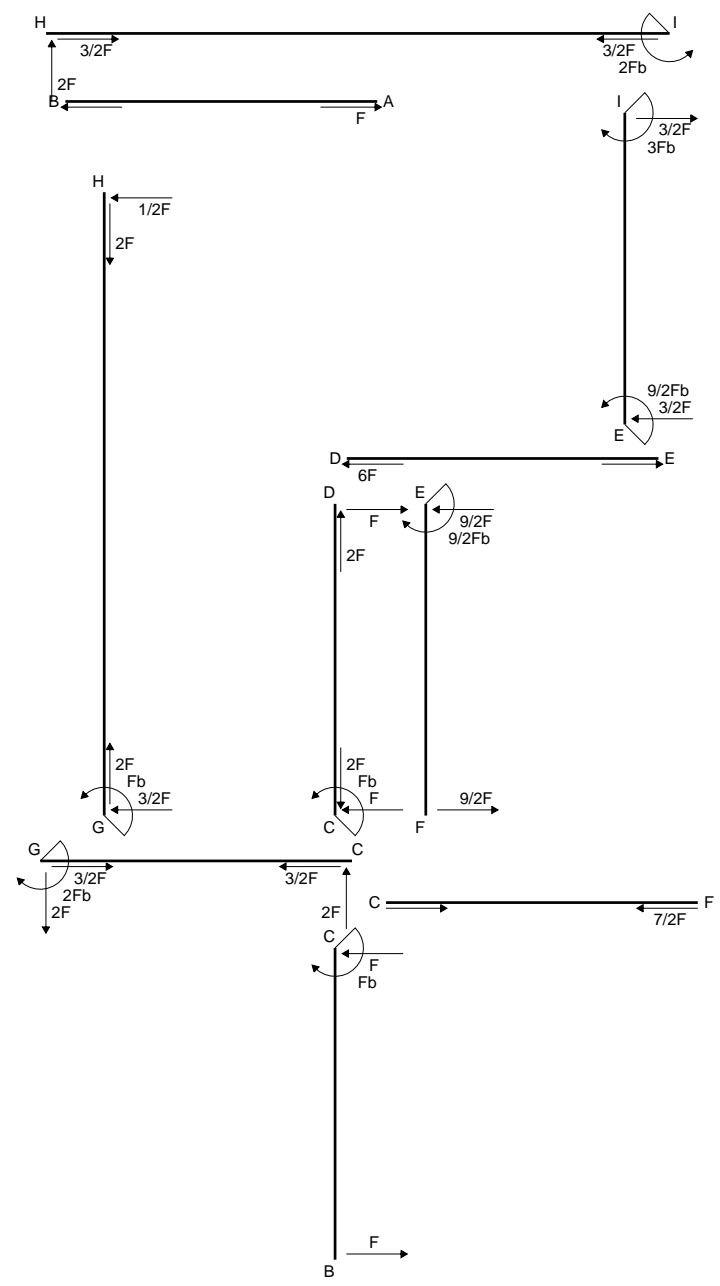
$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

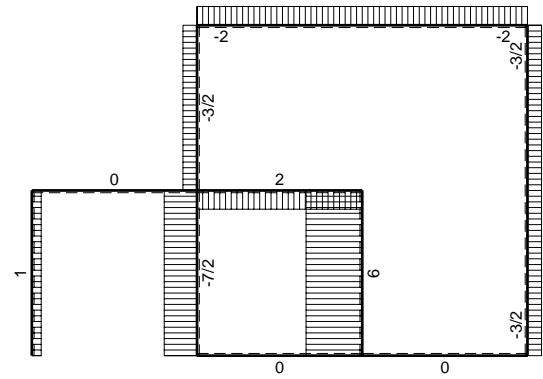
$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$



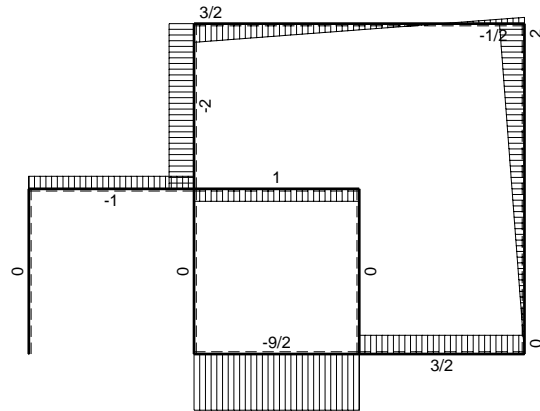
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=V_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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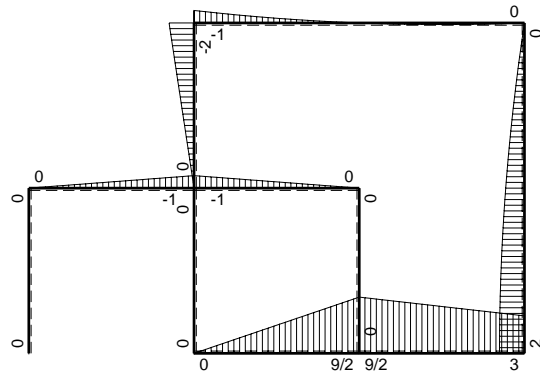




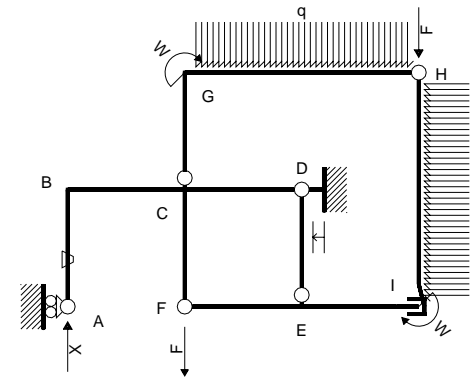
← (+) → F



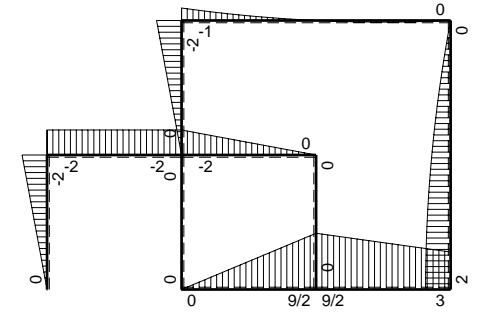
↑ (+) ↓ F



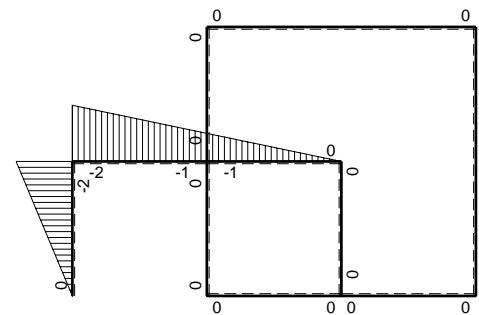
⤵ (+) ⤶ F<sub>b</sub>



Schema di calcolo iperstatico



⤵ (+) ⤶ M<sub>0</sub> flessione da carichi assegnati



⤵ (+) ⤶ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=V_A$ 

| →     | $M_x(x)$                    | $M_0(x)$            | $\theta$ | $M_x M_0$          | $M_x \theta$       | $M_x M_x$       | $\int M_x(M_0/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |            |
|-------|-----------------------------|---------------------|----------|--------------------|--------------------|-----------------|-----------------------------|--------------------------|------------|
| AB b  | -2x                         | -2Fx                | -Fb/EJ   | $4Fx^2$            | $2Fxb/EJ$          | $4x^2$          | $(4/3+1)Fb^3/EJ$            | $4/3Xb^3/EJ$             |            |
| BA b  | $2b-2x$                     | $2Fb-2Fx$           | Fb/EJ    | $4Fb^2-8Fbx+4Fx^2$ | $2Fb^2/EJ-2Fxb/EJ$ | $4b^2-8bx+4x^2$ |                             |                          |            |
| BC b  | $-2b+x$                     | -2Fb                | 0        | $4Fb^2-2Fbx$       | 0                  | $4b^2-4bx+x^2$  | $(3+0)Fb^3/EJ$              | $7/3Xb^3/EJ$             |            |
| CB b  | $b+x$                       | 2Fb                 | 0        | $2Fb^2+2Fbx$       | 0                  | $b^2+2bx+x^2$   |                             |                          |            |
| CD b  | $-b+x$                      | $-2Fb+2Fx$          | 0        | $2Fb^2-4Fbx+2Fx^2$ | 0                  | $b^2-2bx+x^2$   | $(2/3+0)Fb^3/EJ$            | $1/3Xb^3/EJ$             |            |
| DC b  | x                           | 2Fx                 | 0        | $2Fx^2$            | 0                  | $x^2$           |                             |                          |            |
| DE b  | 0                           | 0                   | 0        | 0                  | 0                  | 0               | 0+0                         | 0                        |            |
| ED b  | 0                           | 0                   | 0        | 0                  | 0                  | 0               |                             |                          |            |
| EF b  | 0                           | $9/2Fb-9/2Fx$       | 0        | 0                  | 0                  | 0               | 0+0                         | 0                        |            |
| FE b  | 0                           | $-9/2Fx$            | 0        | 0                  | 0                  | 0               |                             |                          |            |
| FC b  | 0                           | 0                   | 0        | 0                  | 0                  | 0               | 0+0                         | 0                        |            |
| CF b  | 0                           | 0                   | 0        | 0                  | 0                  | 0               |                             |                          |            |
| CG b  | 0                           | -2Fx                | 0        | 0                  | 0                  | 0               | 0+0                         | 0                        |            |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                  | 0                  | 0               |                             |                          |            |
| GH 2b | 0                           | $-Fb+3/2Fx-1/2qx^2$ | 0        | 0                  | 0                  | 0               | 0+0                         | 0                        |            |
| HG 2b | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                  | 0                  | 0               |                             |                          |            |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                  | 0                  | 0               | 0+0                         | 0                        |            |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                  | 0                  | 0               |                             |                          |            |
| IE b  | 0                           | $3Fb+3/2Fx$         | 0        | 0                  | 0                  | 0               | 0+0                         | 0                        |            |
| EI b  | 0                           | $-9/2Fb+3/2Fx$      | 0        | 0                  | 0                  | 0               |                             |                          |            |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                    |                    |                 |                             | $-2Fb^3/EJ$              |            |
|       | totali                      |                     |          |                    |                    |                 |                             | $4Fb^3/EJ$               | $4Xb^3/EJ$ |
|       | iperstatica $X=V_A$         |                     |          |                    |                    |                 |                             | -F                       |            |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8x/b + 4x^2/b^2) b^2 1/EJ dx = [4x - 4x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 4b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4b - 2b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (4x^2/b^2) Fb^2 1/EJ dx + \int_0^b (2x/b) \theta dx = [4/3 x^3/b^2]_0^b Fb^2 1/EJ + [x^2/b]_0^b \theta$$

$$= (4/3 b) Fb^2 1/EJ + (b) \theta = 7/3 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (4 - 8x/b + 4x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-2 + 2x/b) \theta dx$$

$$= [4x - 4x^2/b + 4/3 x^3/b^2]_0^b Fb^2 1/EJ + [-2x + x^2/b]_0^b \theta$$

$$= (4b - 4b + 4/3 b) Fb^2 1/EJ + (-2b + b) \theta = 7/3 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (4 - 2x/b) Fb^2 1/EJ dx = [4x - x^2/b]_0^b Fb^2 1/EJ$$

$$= (4b - b) Fb^2 1/EJ = 3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (2 + 2x/b) Fb^2 1/EJ dx = [2x + x^2/b]_0^b Fb^2 1/EJ$$

$$= (2b + b) Fb^2 1/EJ = 3 Fb^3/EJ$$

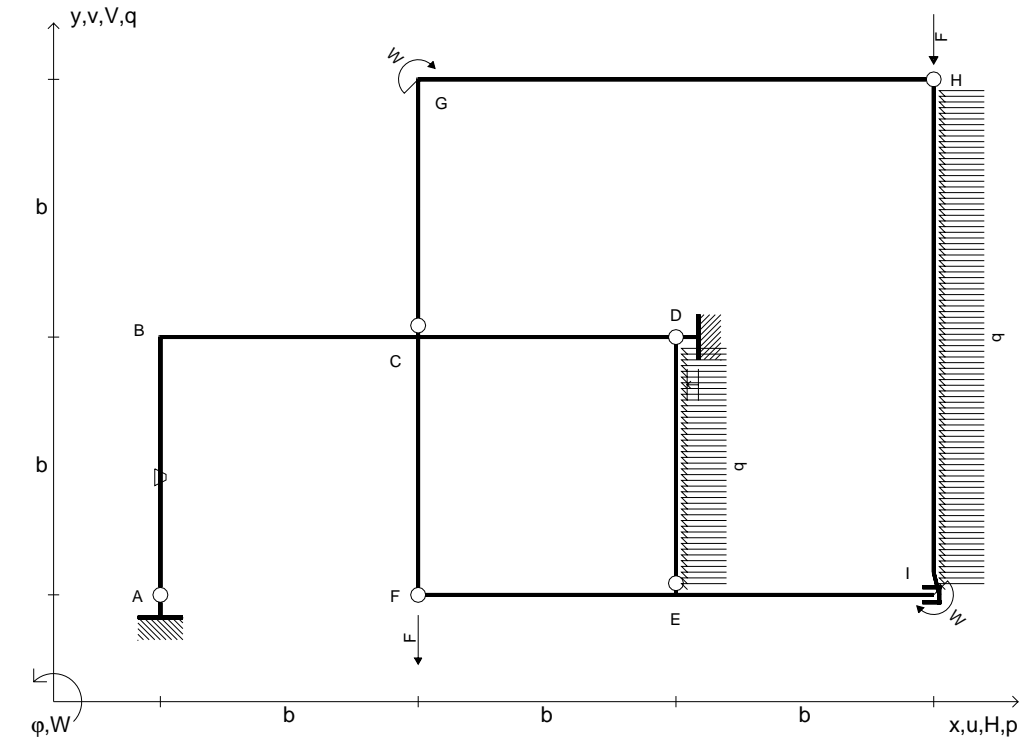
$$L_{CD}^{xo} = \int_0^b (2 - 4x/b + 2x^2/b^2) Fb^2 1/EJ dx = [2x - 2x^2/b + 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (2b - 2b + 2/3 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (2x^2/b^2) Fb^2 1/EJ dx = [2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

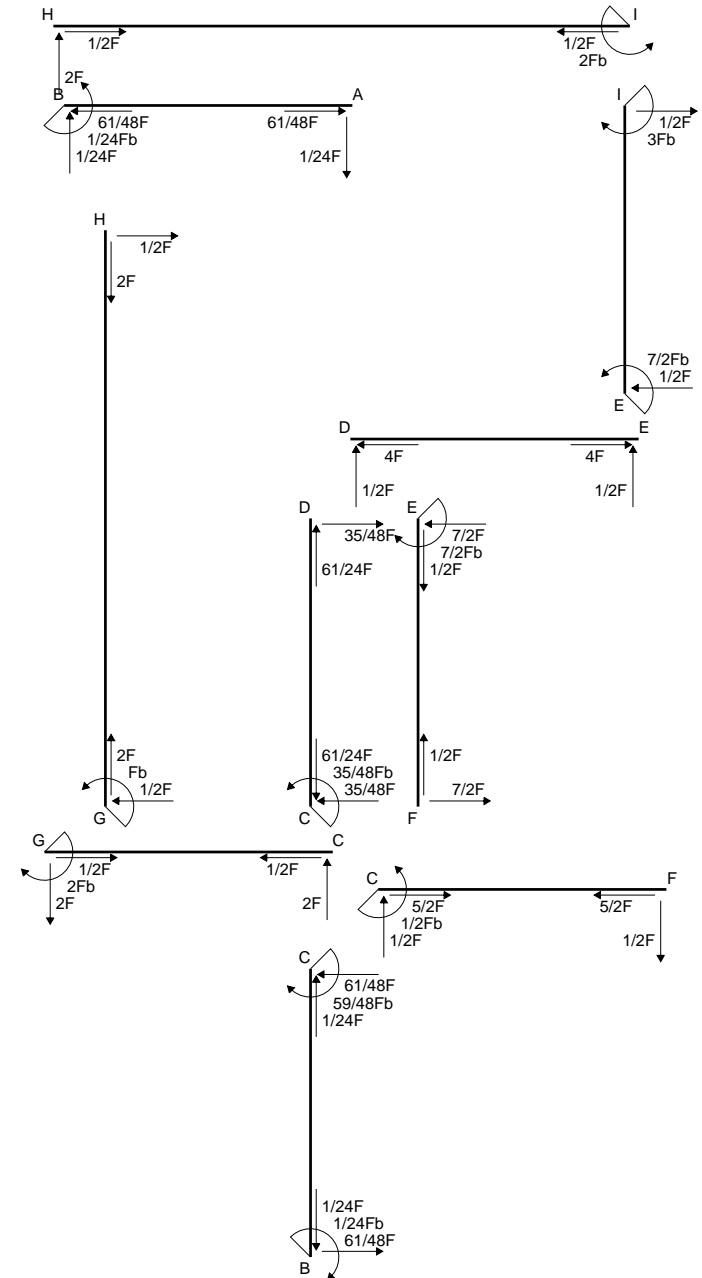
$$= (2/3 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$

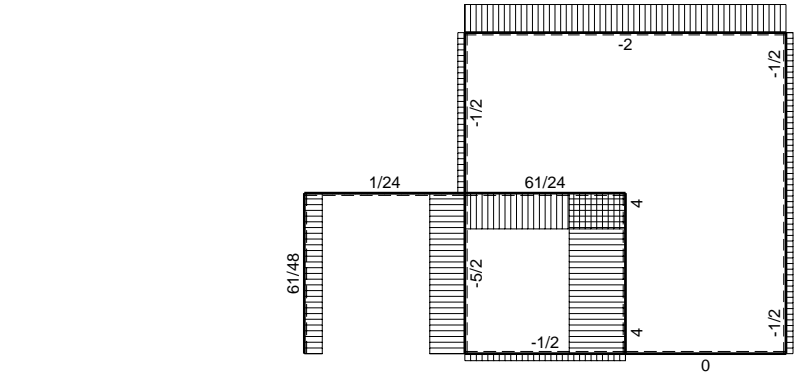




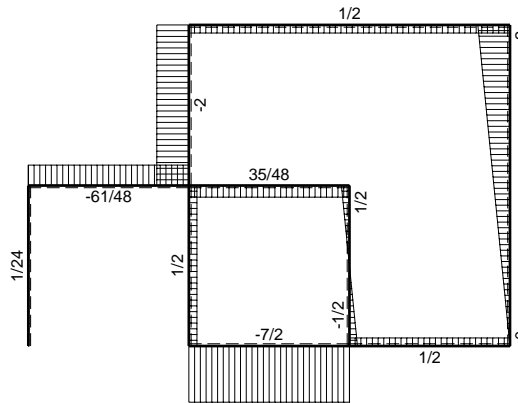
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{DE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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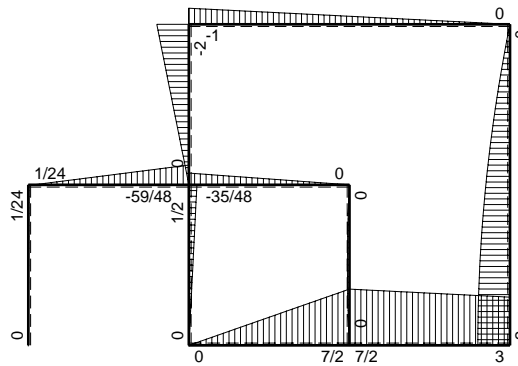




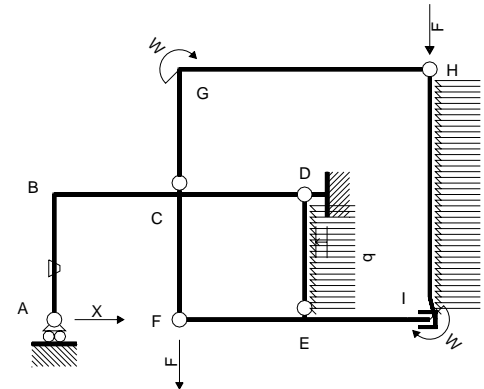
← (+) → F



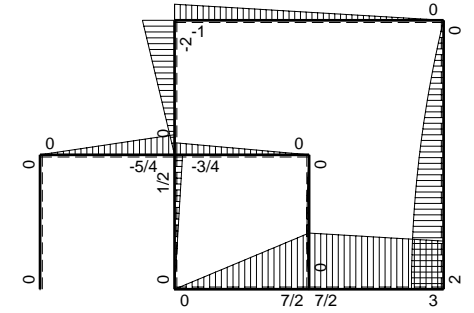
↑ (+) ↓ F



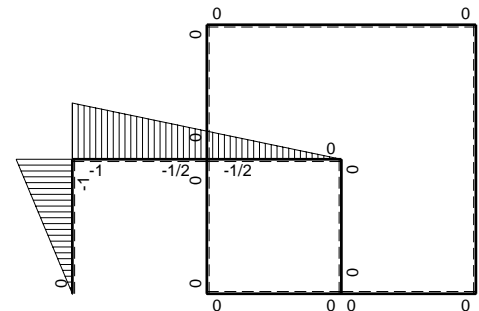
⊙ (+) ⊙ F<sub>b</sub>



Schema di calcolo iperstatico



⊙ (+) ⊙ M<sub>0</sub> flessione da carichi assegnati



⊙ (+) ⊙ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_0(x)$         | $\theta$ | $M_x M_0$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_0/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | -x                          | 0                | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$           |           |
| BA b  | b-x                         | 0                | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | -b+1/2x                     | -5/4Fx           | 0        | $5/4Fbx-5/8Fx^2$         | 0                | $b^2-bx+1/4x^2$       | $(5/12+0)Fb^3/EJ$           | $7/12Xb^3/EJ$          |           |
| CB b  | $1/2b+1/2x$                 | $5/4Fb-5/4Fx$    | 0        | $5/8Fb^2-5/8Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12Xb^3/EJ$          |           |
| DC b  | $1/2x$                      | $3/4Fx$          | 0        | $3/8Fx^2$                | 0                | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                        | 0                | 0                     |                             |                        |           |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                        | 0                | 0                     |                             |                        |           |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                        | 0                | 0                     |                             |                        |           |
| CG b  | 0                           | $-2Fx$           | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                        | 0                | 0                     |                             |                        |           |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                        | 0                | 0                     |                             |                        |           |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                        | 0                | 0                     |                             |                        |           |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                        | 0                | 0                     |                             |                        |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                  |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                  |          |                          |                  |                       |                             | $1/24Fb^3/EJ$          | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                  |          |                          |                  |                       |                             | $-1/24F$               |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb^2 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (5/8 b - 5/24 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb^2 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

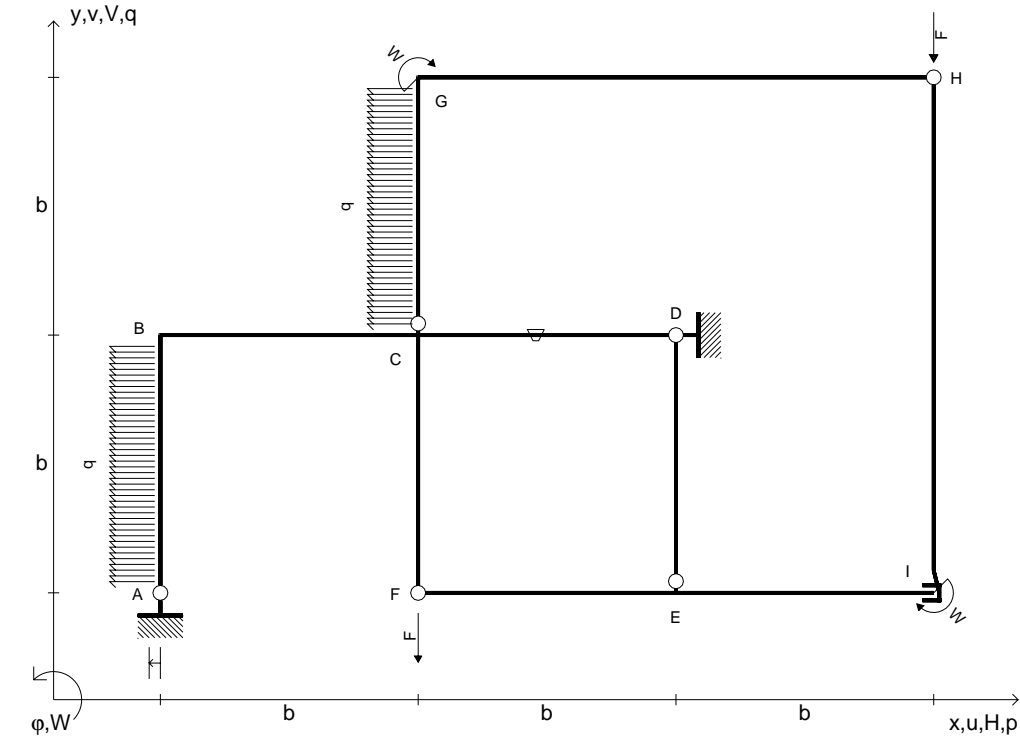
$$= (5/8 b - 5/24 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

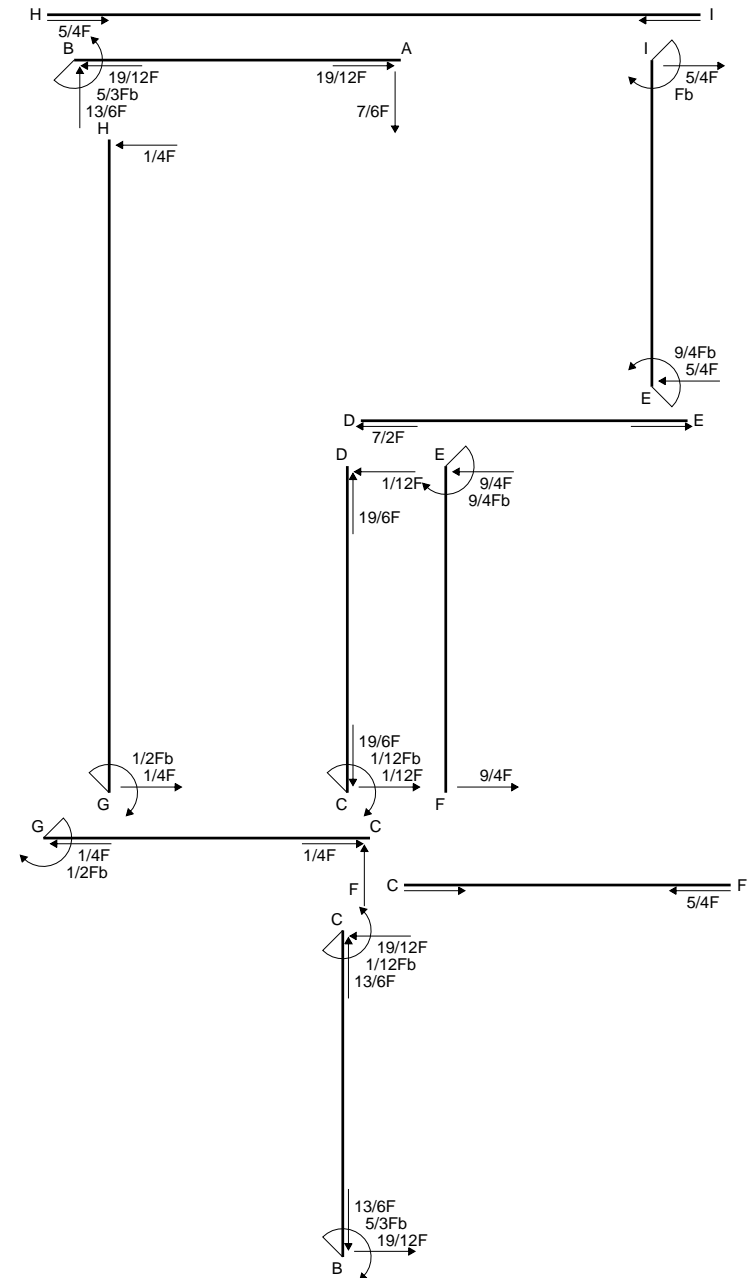
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

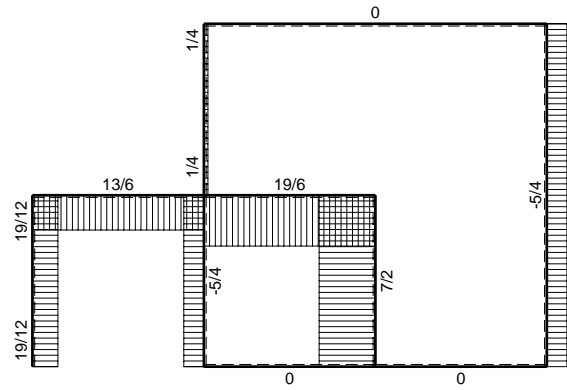
$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$



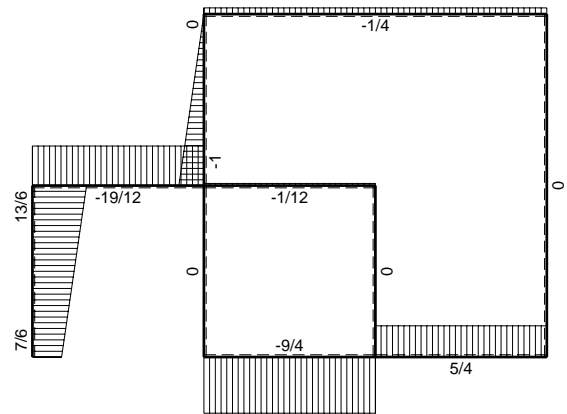
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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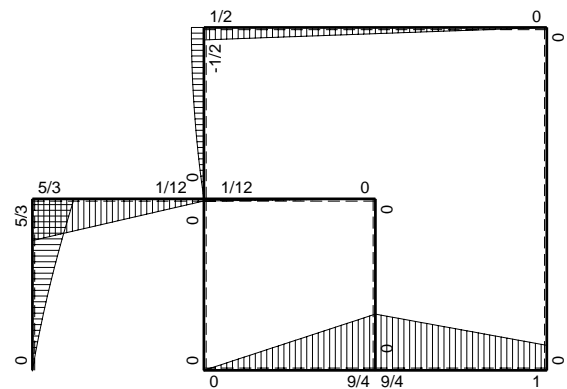




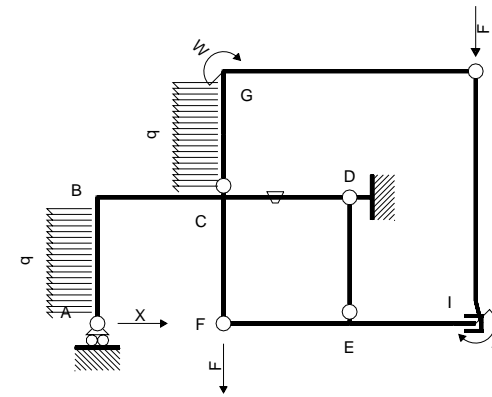
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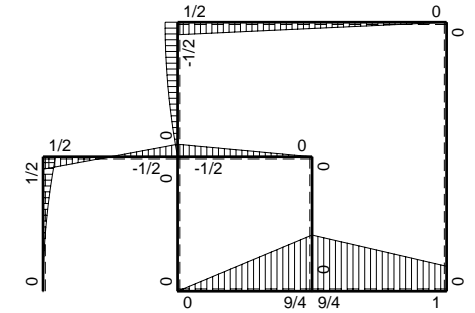
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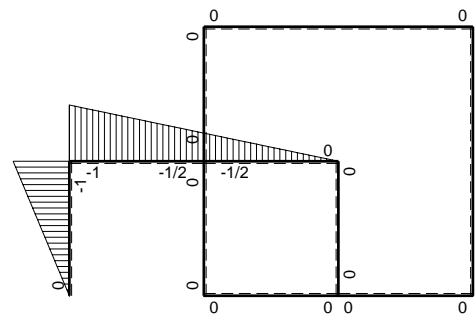
⤵ (+) ⤴ F<sub>b</sub>



Schema di calcolo iperstatico



⤵ (+) ⤴ M<sub>o</sub> flessione da carichi assegnati



⤵ (+) ⤴ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-----------------------------------|------------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | $1/2qx^2$           | 0        | $-1/2qx^3$                        | 0                      | $x^2$                 | $(-1/8+0)Fb^3/EJ$           | $1/3Xb^3/EJ$           |
| BA b  | b-x                         | $-1/2Fb+Fx-1/2qx^2$ | 0        | $-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$ | 0                      | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | $1/2Fb-Fx$          | 0        | $-1/2Fb^2+5/4Fbx-1/2Fx^2$         | 0                      | $b^2-bx+1/4x^2$       | $(-1/24+0)Fb^3/EJ$          | $7/12Xb^3/EJ$          |
| CB b  | $1/2b+1/2x$                 | $1/2Fb-Fx$          | 0        | $1/4Fb^2-1/4Fbx-1/2Fx^2$          | 0                      | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$      | $-Fb/EJ$ | $1/4Fb^2-1/2Fbx+1/4Fx^2$          | $1/2Fb^2/EJ-1/2Fxb/EJ$ | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+1/4)Fb^3/EJ$         | $1/12Xb^3/EJ$          |
| DC b  | $1/2x$                      | $1/2Fx$             | $Fb/EJ$  | $1/4Fx^2$                         | $1/2Fxb/EJ$            | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$            | 0        | 0                                 | 0                      | 0                     |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                                 | 0                      | 0                     |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$            | 0        | 0                                 | 0                      | 0                     |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                                 | 0                      | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                                   |                        |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                     |          |                                   |                        |                       | $7/6Fb^3/EJ$                | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                     |          |                                   |                        |                       | $-7/6F$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left( -\frac{1}{2} \frac{x^3}{b^3} \right) Fb^2 \frac{1}{EJ} dx = \left[ -\frac{1}{8} \frac{x^4}{b^3} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( -\frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = -\frac{1}{8} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left( -\frac{1}{2} + \frac{3}{2} \frac{x}{b} - \frac{3}{2} \frac{x^2}{b^2} + \frac{1}{2} \frac{x^3}{b^3} \right) Fb^2 \frac{1}{EJ} dx$$

$$= \left[ -\frac{1}{2} x + \frac{3}{4} \frac{x^2}{b} - \frac{1}{2} \frac{x^3}{b^2} + \frac{1}{8} \frac{x^4}{b^3} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( -\frac{1}{2} b + \frac{3}{4} b - \frac{1}{2} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = -\frac{1}{8} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( -\frac{1}{2} + \frac{5}{4} \frac{x}{b} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ -\frac{1}{2} x + \frac{5}{8} \frac{x^2}{b} - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( -\frac{1}{2} b + \frac{5}{8} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = -\frac{1}{24} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{1}{4} - \frac{1}{4} \frac{x}{b} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{8} \frac{x^2}{b} - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{8} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = -\frac{1}{24} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( \frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

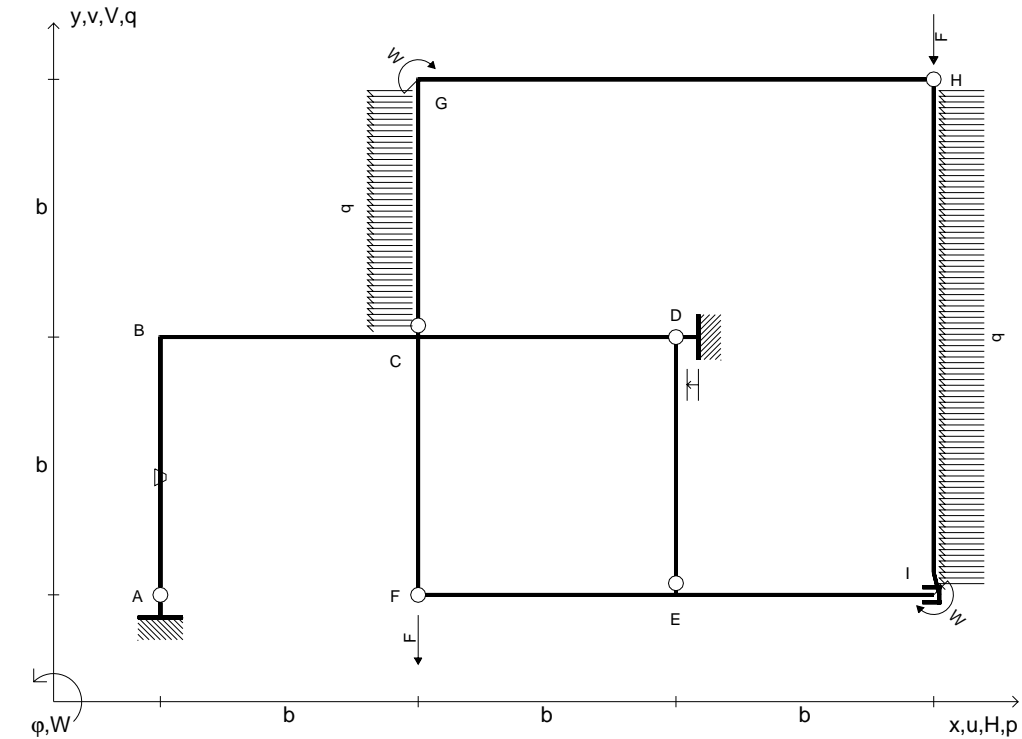
$$= \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ \frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left( \frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{1}{3} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} \frac{x}{b} \right) \theta dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

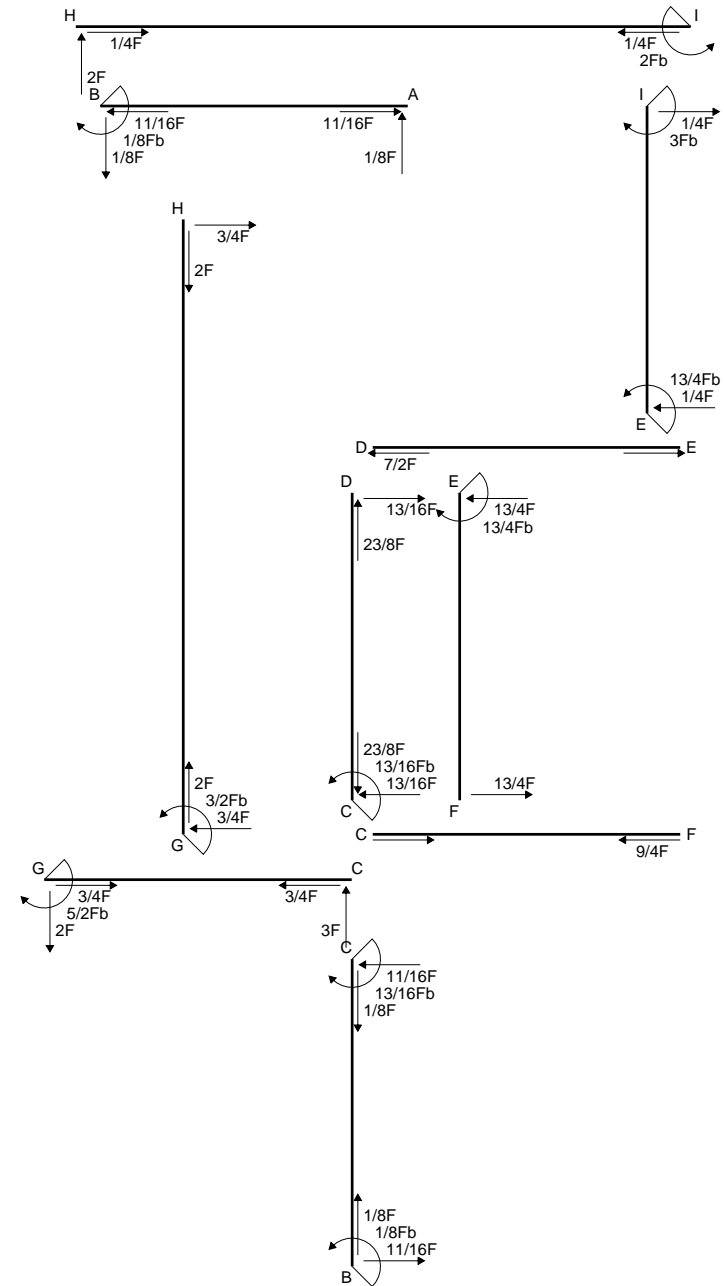
$$= \left( \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{4} b \right) \theta = \frac{1}{3} Fb^3/EJ$$

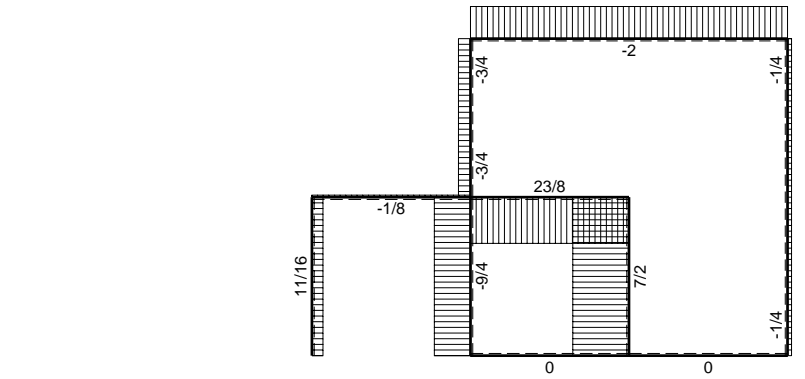




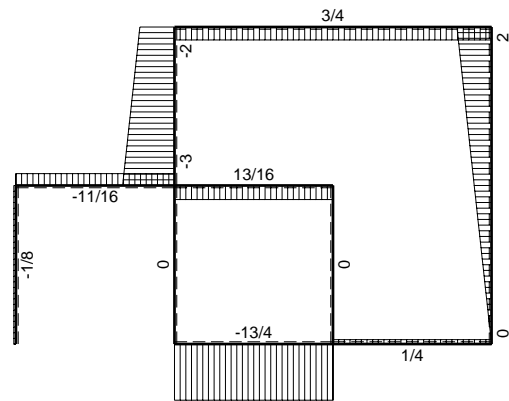
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=W_{BC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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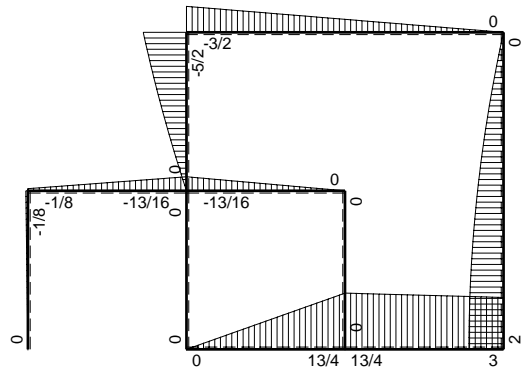




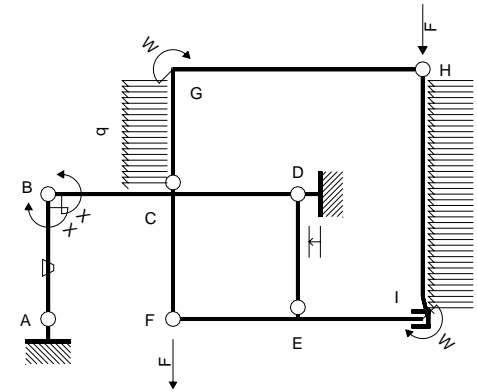
← (+) → F



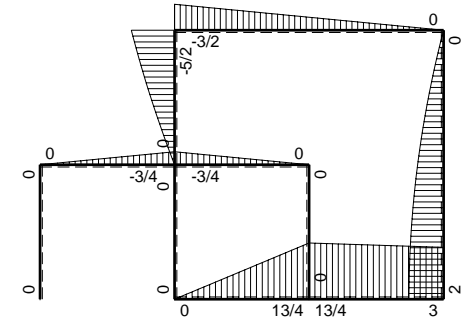
↑ (+) ↓ F



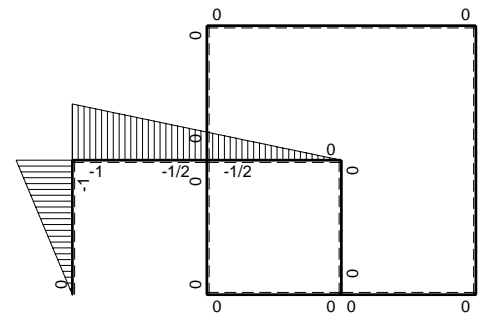
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>0</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |               |                         |                             | $-1/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         |                             | $1/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

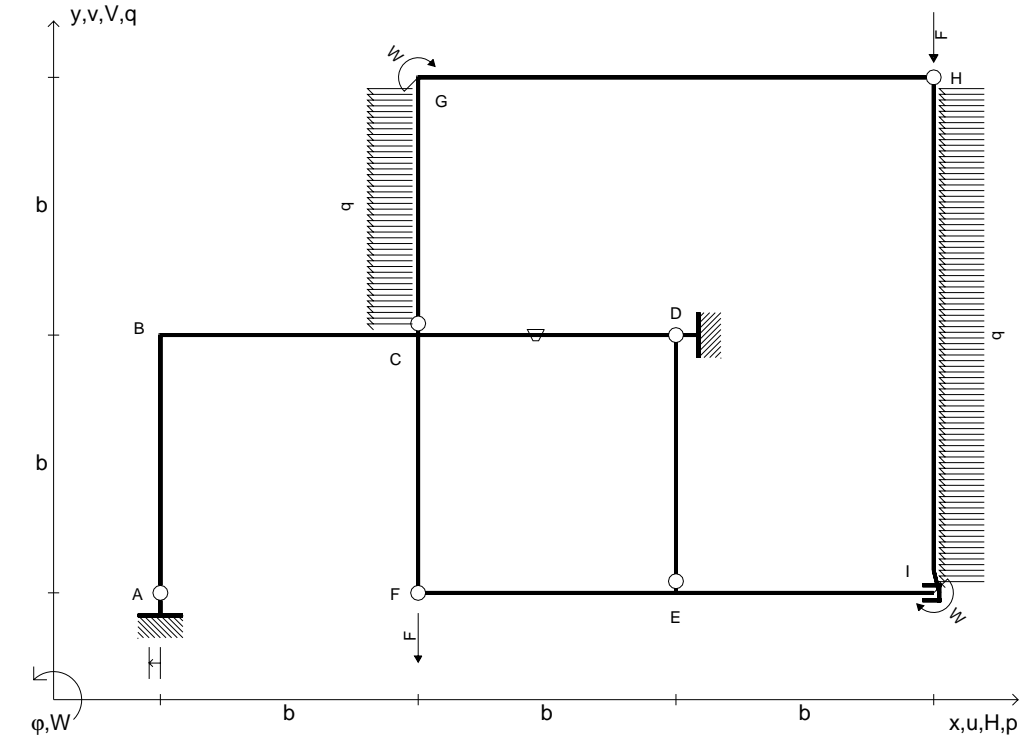
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

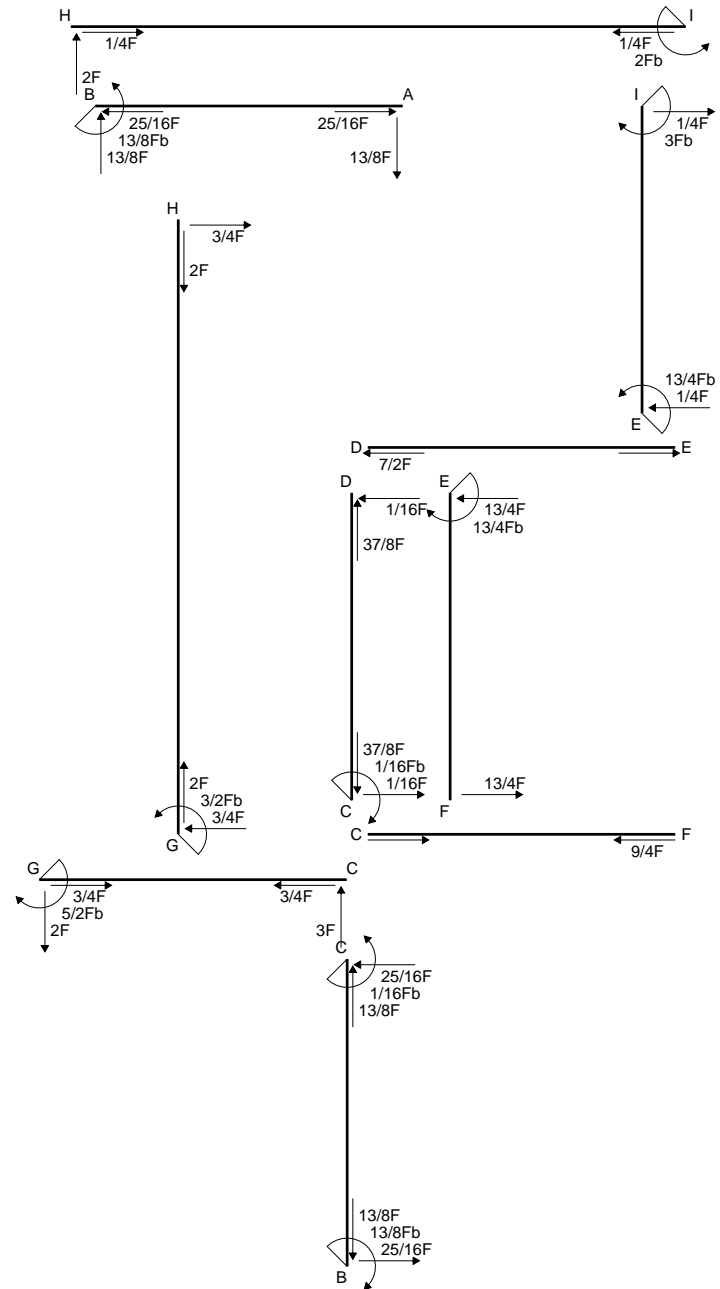
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

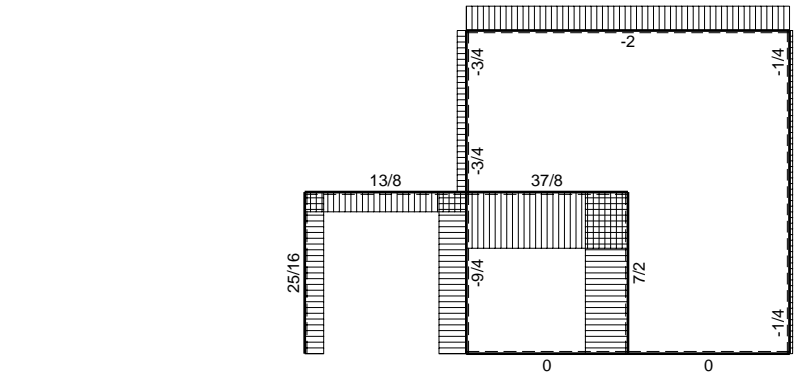
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$



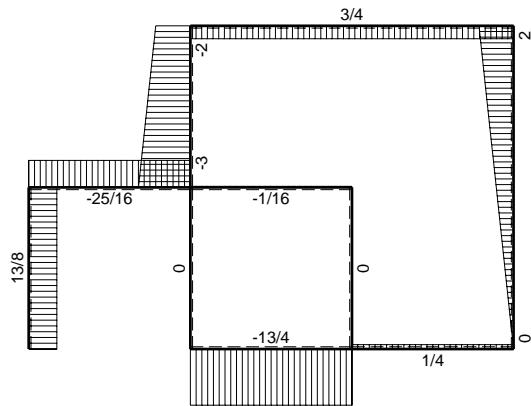
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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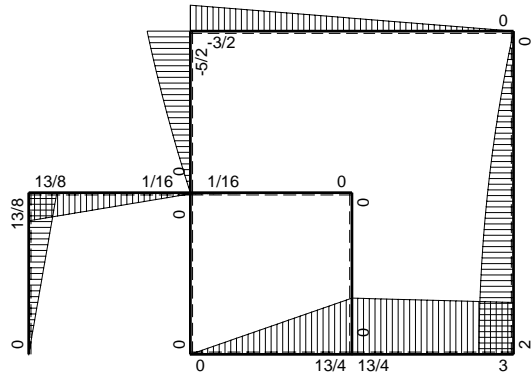




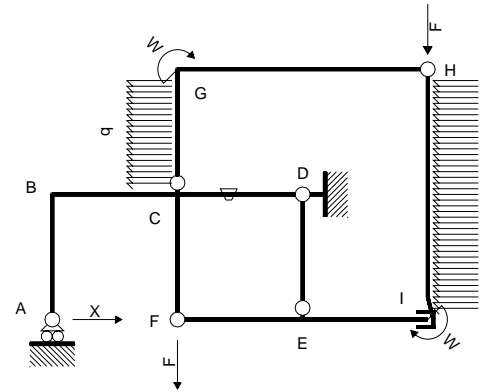
← (+) → F



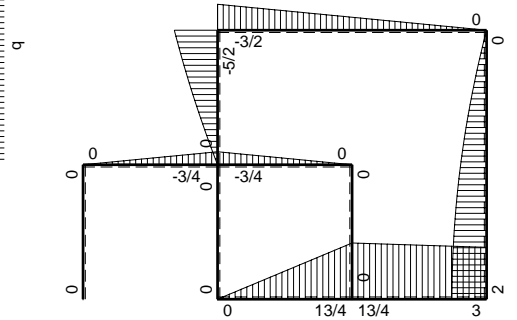
↑ (+) ↓ F



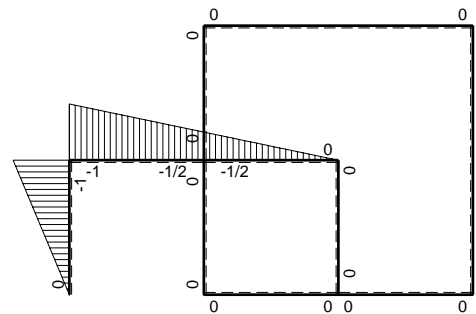
⊕ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ M<sub>o</sub> flessione da carichi assegnati



⊕ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | 0                   | 0        | 0                        | 0                      | $x^2$                 | 0+0                         | $1/3 X b^3/EJ$         |
| BA b  | b-x                         | 0                   | 0        | 0                        | 0                      | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | -3/4Fx              | 0        | $3/4Fbx-3/8Fx^2$         | 0                      | $b^2-bx+1/4x^2$       | $(1/4+0)Fb^3/EJ$            | $7/12 X b^3/EJ$        |
| CB b  | 1/2b+1/2x                   | 3/4Fb-3/4Fx         | 0        | $3/8Fb^2-3/8Fx^2$        | 0                      | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | -1/2b+1/2x                  | -3/4Fb+3/4Fx        | -Fb/EJ   | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | $1/2Fb^2/EJ-1/2Fxb/EJ$ | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+1/4)Fb^3/EJ$          | $1/12 X b^3/EJ$        |
| DC b  | 1/2x                        | 3/4Fx               | Fb/EJ    | $3/8Fx^2$                | $1/2Fxb/EJ$            | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | -13/4Fx             | 0        | 0                        | 0                      | 0                     |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                        | 0                      | 0                     |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | 3/4Fx               | 0        | 0                        | 0                      | 0                     |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                        | 0                      | 0                     |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                        | 0                      | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |                        |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                     |          |                          |                        |                       | $13/8Fb^3/EJ$               | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                     |          |                          |                        |                       | -13/8F                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

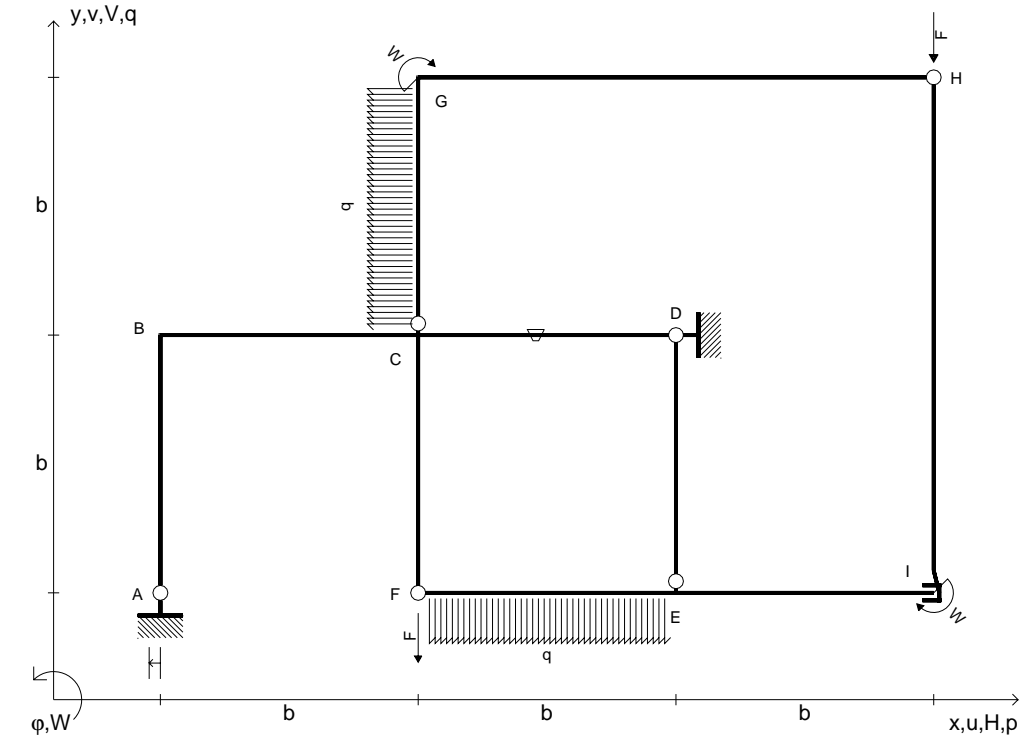
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

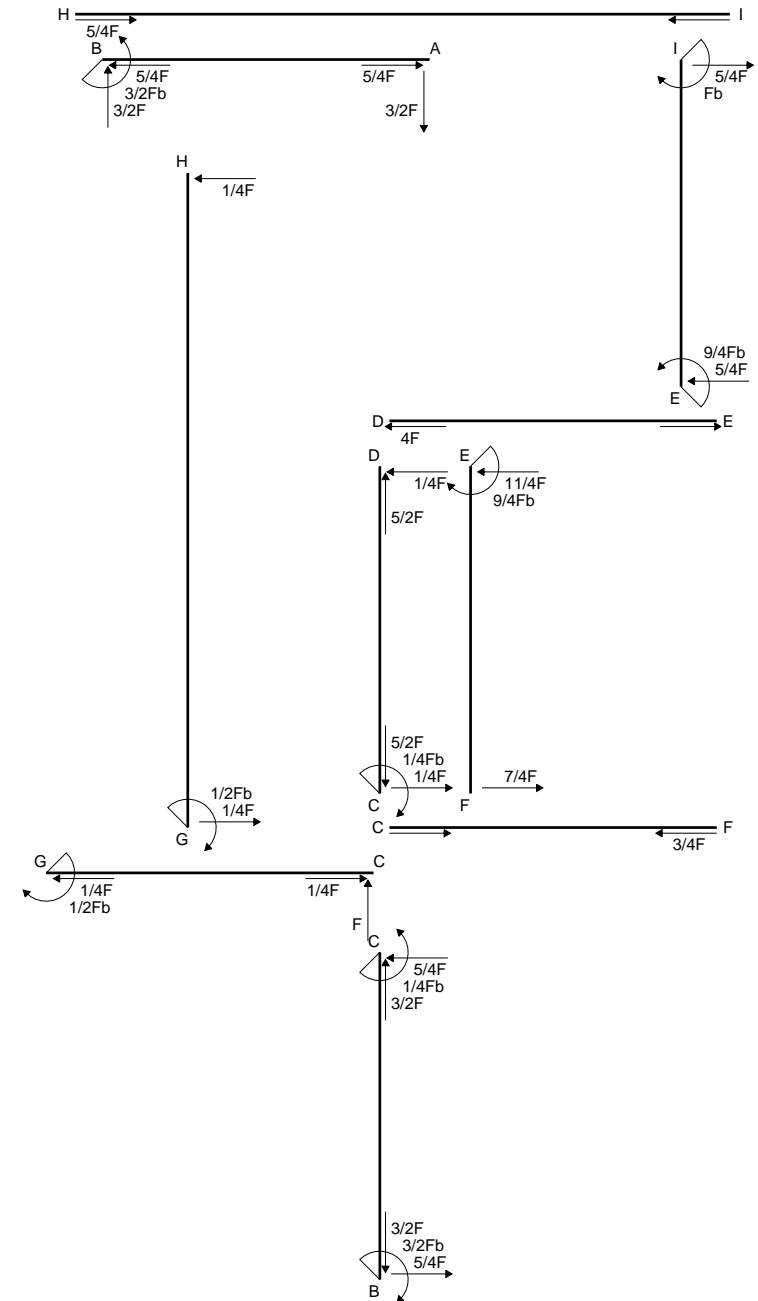
$$= (1/8 b) Fb^2 1/EJ + (-1/4 b) \theta = 3/8 Fb^3/EJ$$

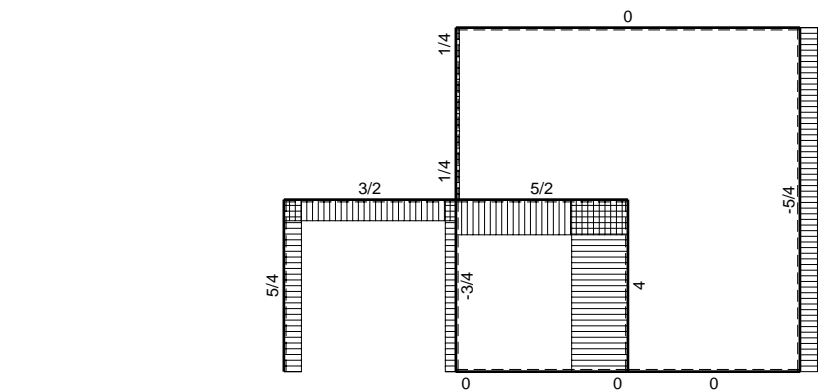




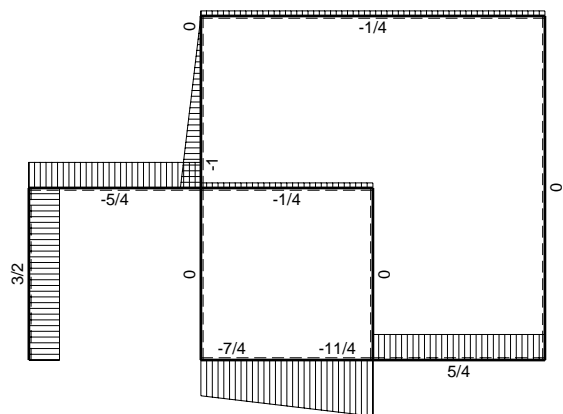
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3 F/EJ$                    | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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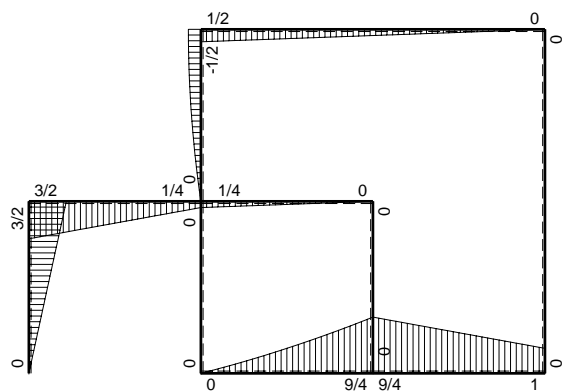




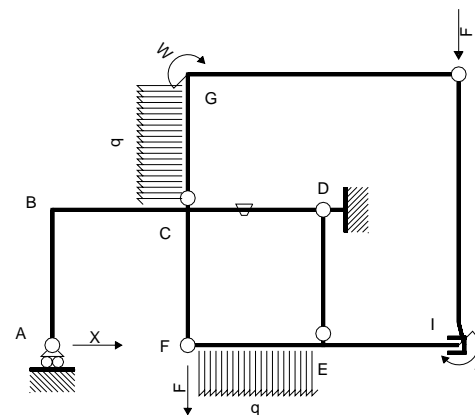
← (+) → F



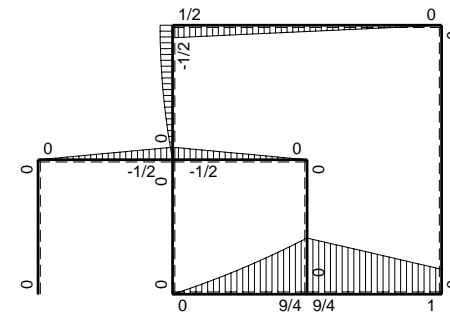
↑ (+) ↓ F



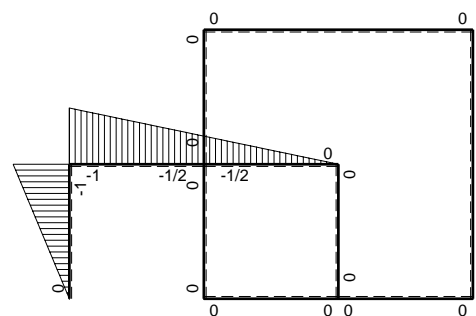
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>o</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$                | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|--------------------------|------------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | 0                      | 0        | 0                        | 0                      | $x^2$                 | 0+0                         | $1/3Xb^3/EJ$           |
| BA b  | b-x                         | 0                      | 0        | 0                        | 0                      | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | -1/2Fx                 | 0        | $1/2Fbx-1/4Fx^2$         | 0                      | $b^2-bx+1/4x^2$       | $(1/6+0)Fb^3/EJ$            | $7/12Xb^3/EJ$          |
| CB b  | $1/2b+1/2x$                 | $1/2Fb-1/2Fx$          | 0        | $1/4Fb^2-1/4Fx^2$        | 0                      | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$         | $-Fb/EJ$ | $1/4Fb^2-1/2Fbx+1/4Fx^2$ | $1/2Fb^2/EJ-1/2Fxb/EJ$ | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+1/4)Fb^3/EJ$         | $1/12Xb^3/EJ$          |
| DC b  | $1/2x$                      | $1/2Fx$                | $Fb/EJ$  | $1/4Fx^2$                | $1/2Fxb/EJ$            | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                        | 0                      | 0                     |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                        | 0                      | 0                     |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                        | 0                      | 0                     |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                        | 0                      | 0                     |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                        | 0                      | 0                     |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                        | 0                      | 0                     |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                        | 0                      | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                        |          |                          |                        |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                        |          |                          |                        |                       | $3/2Fb^3/EJ$                | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                        |          |                          |                        |                       | $-3/2F$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/12 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

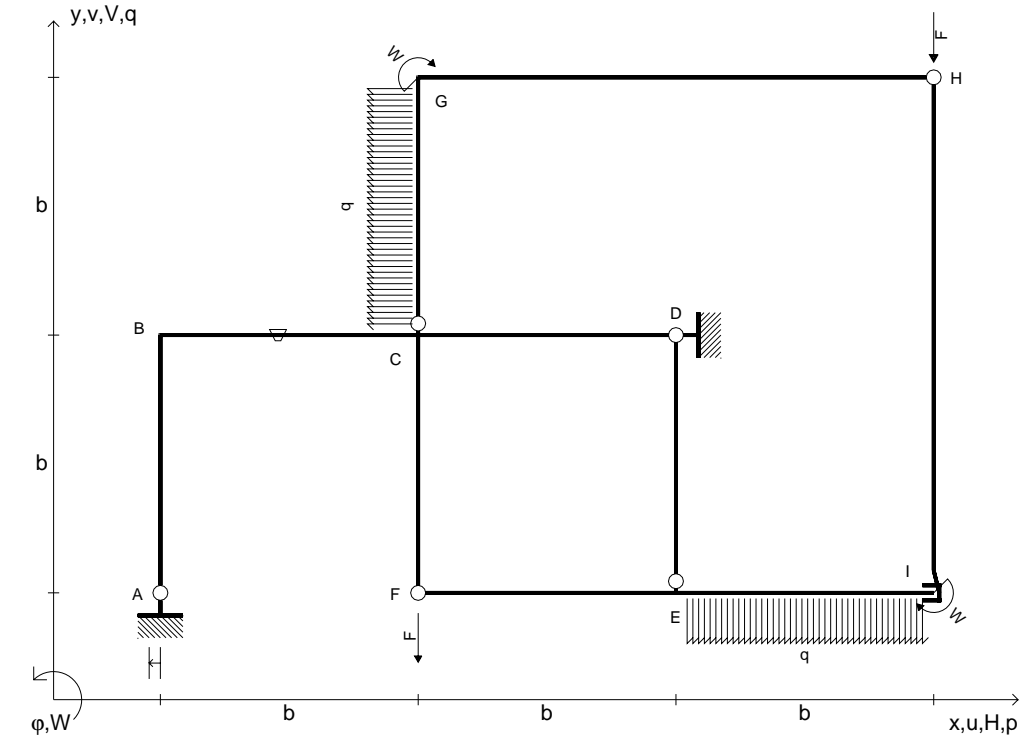
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^3/EJ$$

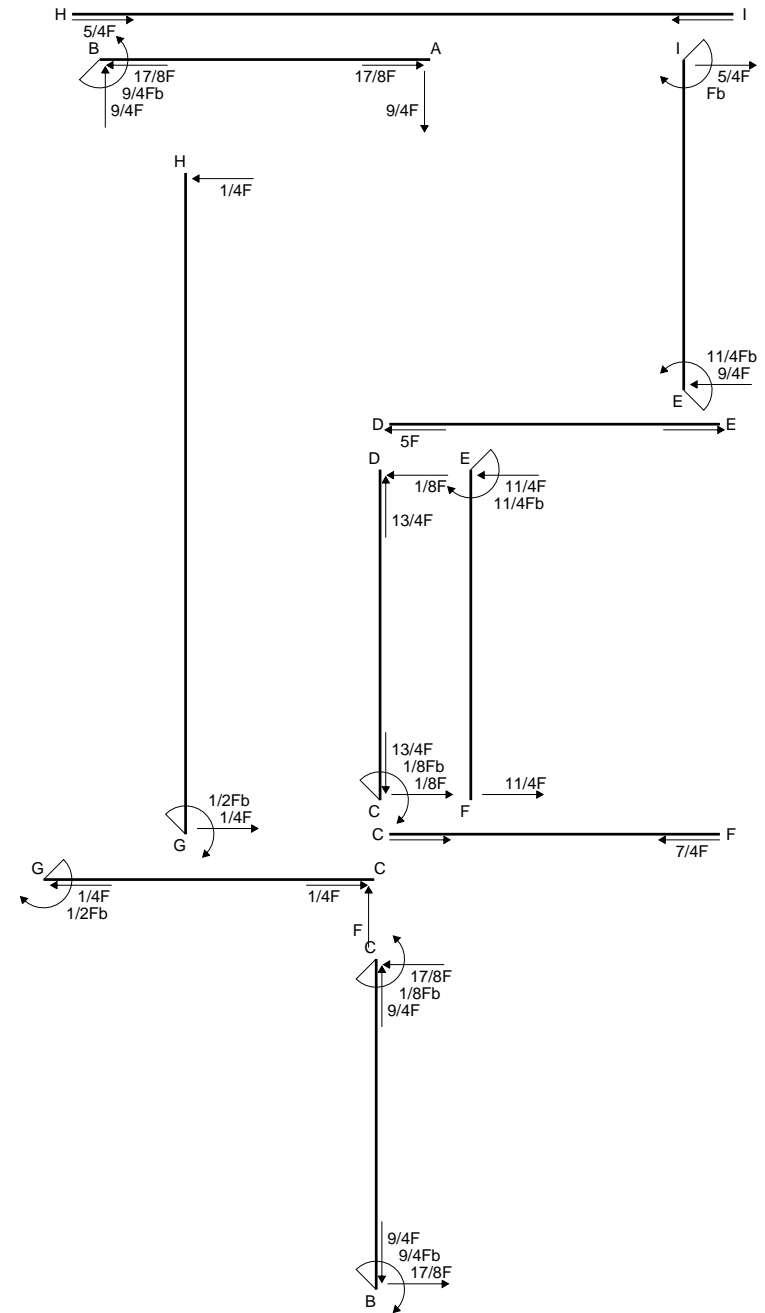
$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

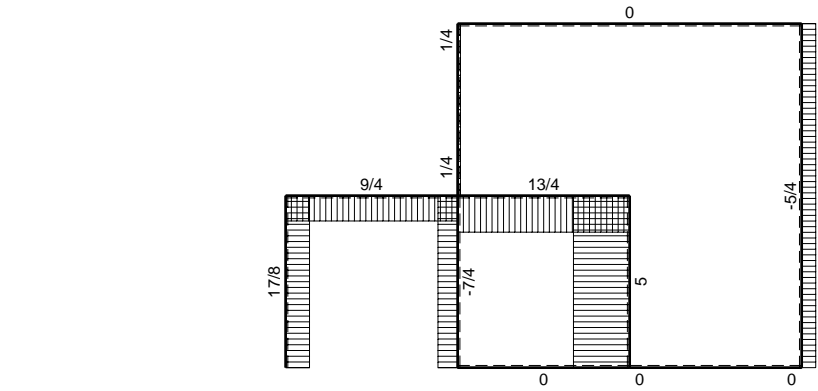
$$= (1/12 b) Fb^2 1/EJ + (-1/4 b) \theta = 1/3 Fb^3/EJ$$



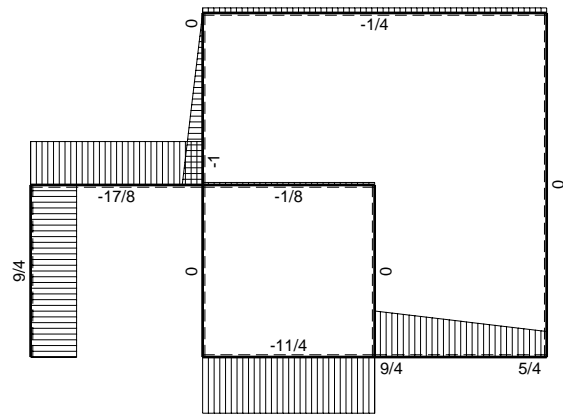
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
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 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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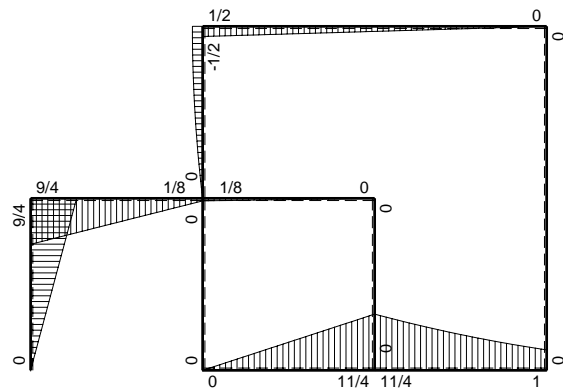




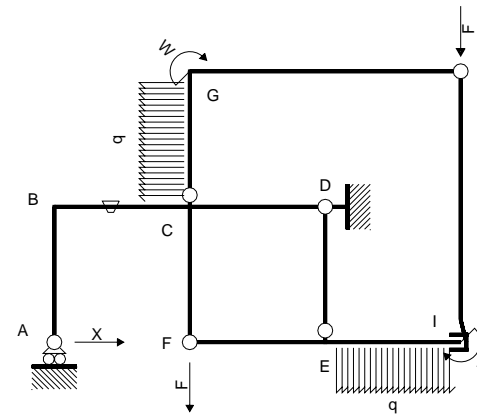
← (+) → F



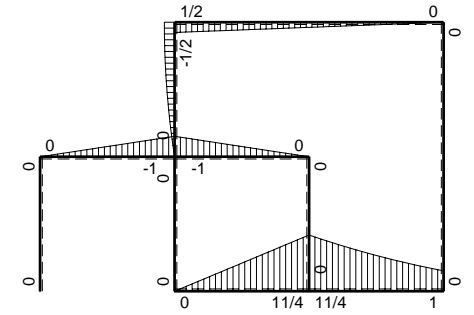
↑ (+) ↓ F



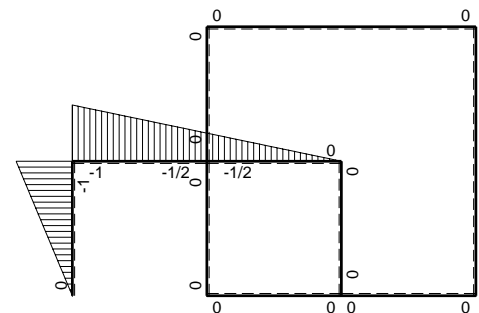
⊕ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ M<sub>o</sub> flessione da carichi assegnati



⊕ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$             | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|-----------------------|------------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | 0                       | 0        | 0                     | 0                      | $x^2$                 | 0+0                         | $1/3 X b^3/EJ$         |
| BA b  | b-x                         | 0                       | 0        | 0                     | 0                      | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | -Fx                     | -Fb/EJ   | $Fbx-1/2Fx^2$         | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/3+3/4)Fb^3/EJ$          | $7/12 X b^3/EJ$        |
| CB b  | $1/2b+1/2x$                 | Fb-Fx                   | Fb/EJ    | $1/2Fb^2-1/2Fx^2$     | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | -Fb+Fx                  | 0        | $1/2Fb^2-Fbx+1/2Fx^2$ | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/6+0)Fb^3/EJ$            | $1/12 X b^3/EJ$        |
| DC b  | $1/2x$                      | Fx                      | 0        | $1/2Fx^2$             | 0                      | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                     | 0                      | 0                     |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                     | 0                      | 0                     |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                     | 0                      | 0                     |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                     | 0                      | 0                     |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                     | 0                      | 0                     |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                     | 0                      | 0                     |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                     | 0                      | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                         |          |                       |                        |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                         |          |                       |                        |                       | $9/4Fb^3/EJ$                | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                         |          |                       |                        |                       | $-9/4F$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^3/EJ$$

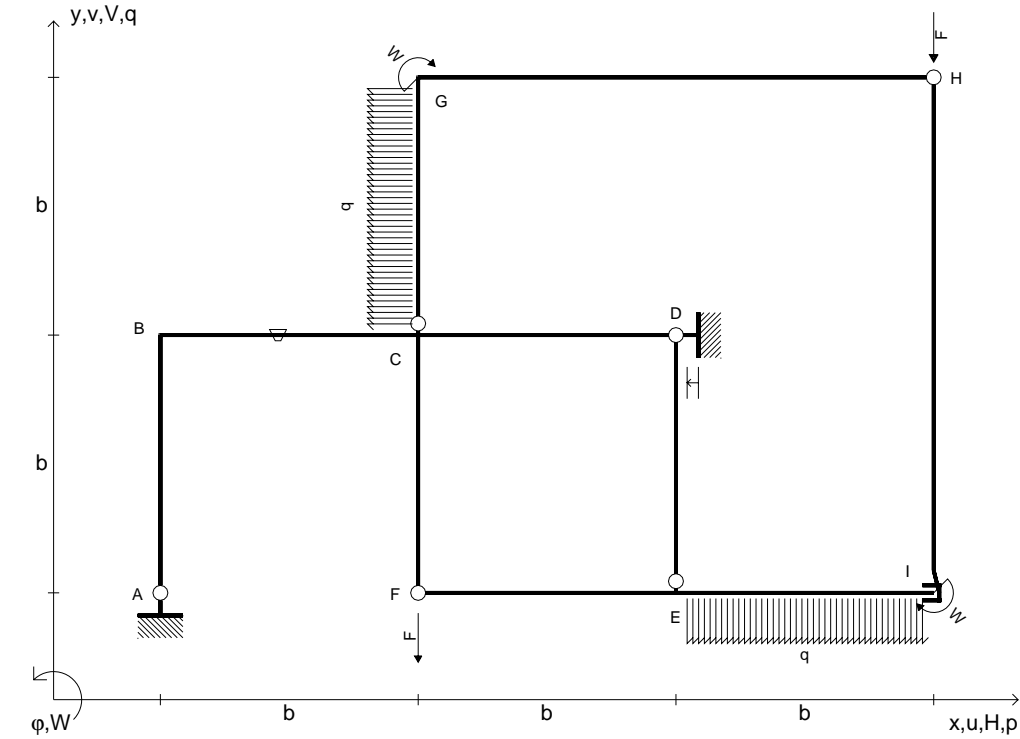
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb^2 1/EJ dx = [1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

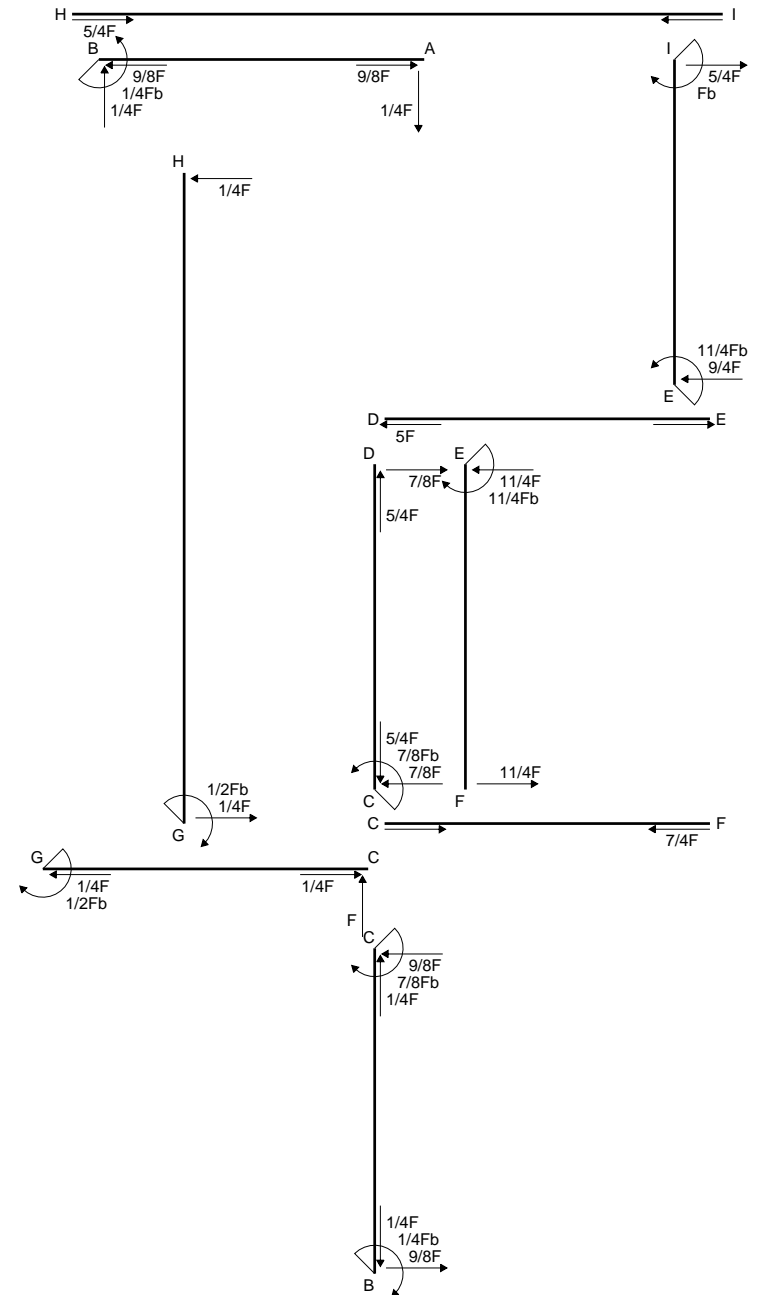
$$= (1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

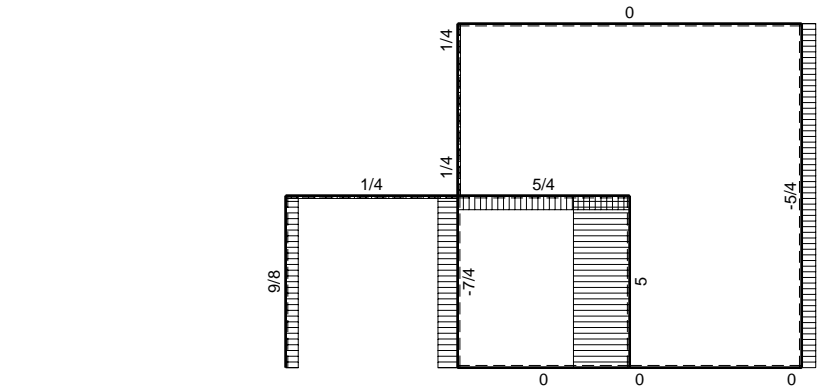




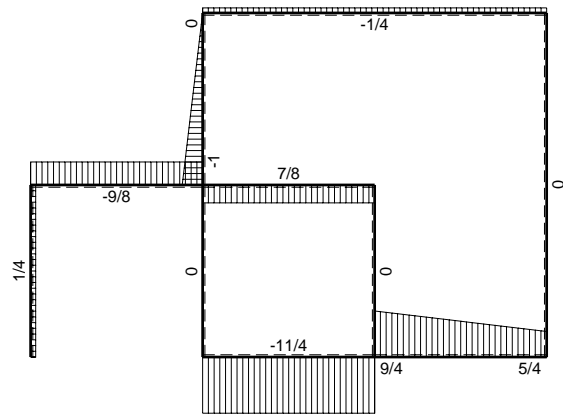
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=V_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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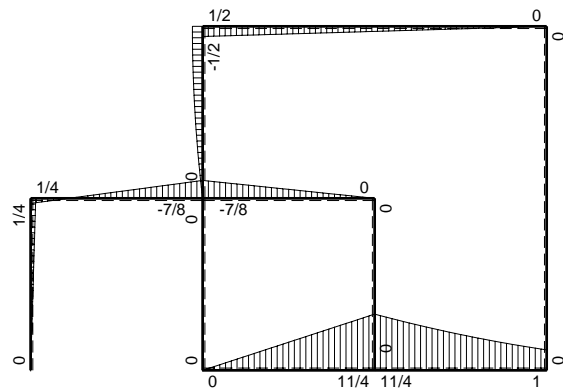




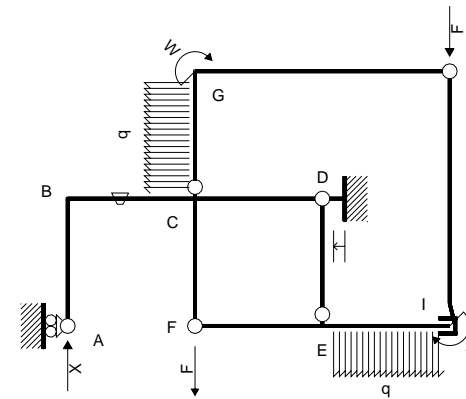
← (+) → F



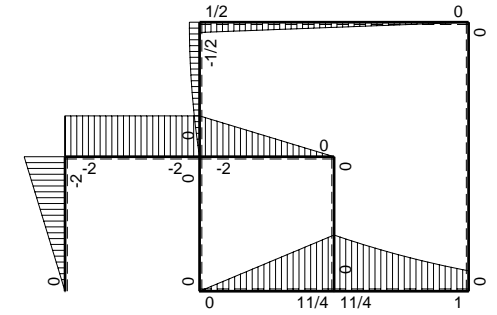
↑ (+) ↓ F



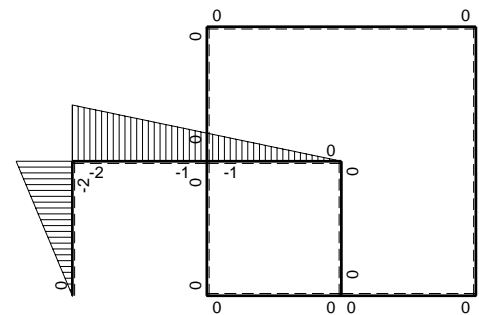
⊕ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ M<sub>o</sub> flessione da carichi assegnati



⊕ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=V_A$ 

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$          | $M_x \theta$      | $M_x M_x$       | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|--------------------|-------------------|-----------------|-----------------------------|------------------------|
| AB b  | -2x                         | -2Fx                    | 0        | $4Fx^2$            | 0                 | $4x^2$          | $(4/3+0)Fb^3/EJ$            | $4/3Xb^3/EJ$           |
| BA b  | $2b-2x$                     | $2Fb-2Fx$               | 0        | $4Fb^2-8Fbx+4Fx^2$ | 0                 | $4b^2-8bx+4x^2$ |                             |                        |
| BC b  | $-2b+x$                     | -2Fb                    | $-Fb/EJ$ | $4Fb^2-2Fbx$       | $2Fb^2/EJ-Fxb/EJ$ | $4b^2-4bx+x^2$  | $(3+3/2)Fb^3/EJ$            | $7/3Xb^3/EJ$           |
| CB b  | $b+x$                       | 2Fb                     | $Fb/EJ$  | $2Fb^2+2Fbx$       | $Fb^2/EJ+Fxb/EJ$  | $b^2+2bx+x^2$   |                             |                        |
| CD b  | $-b+x$                      | $-2Fb+2Fx$              | 0        | $2Fb^2-4Fbx+2Fx^2$ | 0                 | $b^2-2bx+x^2$   | $(2/3+0)Fb^3/EJ$            | $1/3Xb^3/EJ$           |
| DC b  | x                           | 2Fx                     | 0        | $2Fx^2$            | 0                 | $x^2$           |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                  | 0                 | 0               | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                  | 0                 | 0               |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                  | 0                 | 0               | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                  | 0                 | 0               |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                  | 0                 | 0               | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                  | 0                 | 0               |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                  | 0                 | 0               | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                  | 0                 | 0               |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                  | 0                 | 0               | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                  | 0                 | 0               |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                  | 0                 | 0               | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                  | 0                 | 0               |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                  | 0                 | 0               | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                  | 0                 | 0               |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                         |          |                    |                   |                 | $-2Fb^3/EJ$                 |                        |
|       | totali                      |                         |          |                    |                   |                 | $9/2Fb^3/EJ$                | $4Xb^3/EJ$             |
|       | iperstatica $X=V_A$         |                         |          |                    |                   |                 | $-9/8F$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (4 x^2/b^2) b^2 1/EJ dx = [4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) b^2 1/EJ dx = [4 x - 4 x^2/b + 4/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) b^2 1/EJ = 4/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (4 - 4 x/b + x^2/b^2) b^2 1/EJ dx = [4 x - 2 x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (4 b - 2 b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 + 2 x/b + x^2/b^2) b^2 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b + b + 1/3 b) b^2 1/EJ = 7/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2 x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (4 x^2/b^2) Fb^2 1/EJ dx = [4/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (4/3 b) Fb^2 1/EJ = 4/3 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (4 - 8 x/b + 4 x^2/b^2) Fb^2 1/EJ dx = [4 x - 4 x^2/b + 4/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (4 b - 4 b + 4/3 b) Fb^2 1/EJ = 4/3 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (4 - 2 x/b) Fb^2 1/EJ dx + \int_0^b (2 - x/b) \theta dx = [4 x - x^2/b]_0^b Fb^2 1/EJ + [2 x - 1/2 x^2/b]_0^b \theta$$

$$= (4 b - b) Fb^2 1/EJ + (2 b - 1/2 b) \theta = 9/2 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (2 + 2 x/b) Fb^2 1/EJ dx + \int_0^b (-1 - x/b) \theta dx = [2 x + x^2/b]_0^b Fb^2 1/EJ + [-x - 1/2 x^2/b]_0^b \theta$$

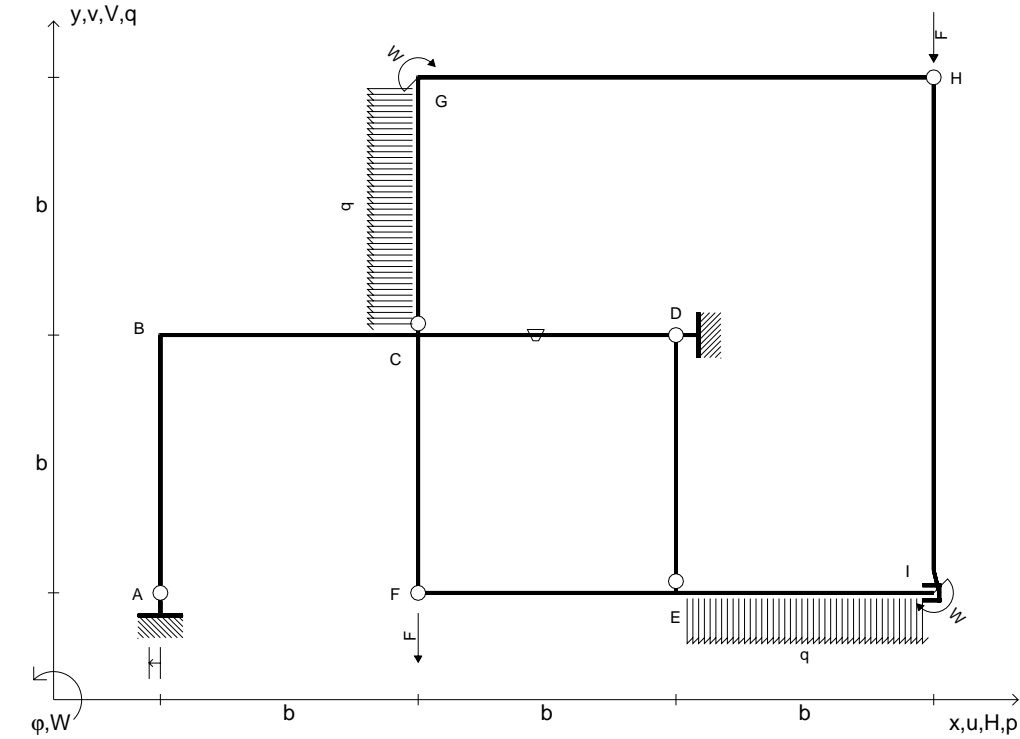
$$= (2 b + b) Fb^2 1/EJ + (-b - 1/2 b) \theta = 9/2 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (2 - 4 x/b + 2 x^2/b^2) Fb^2 1/EJ dx = [2 x - 2 x^2/b + 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (2 b - 2 b + 2/3 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$

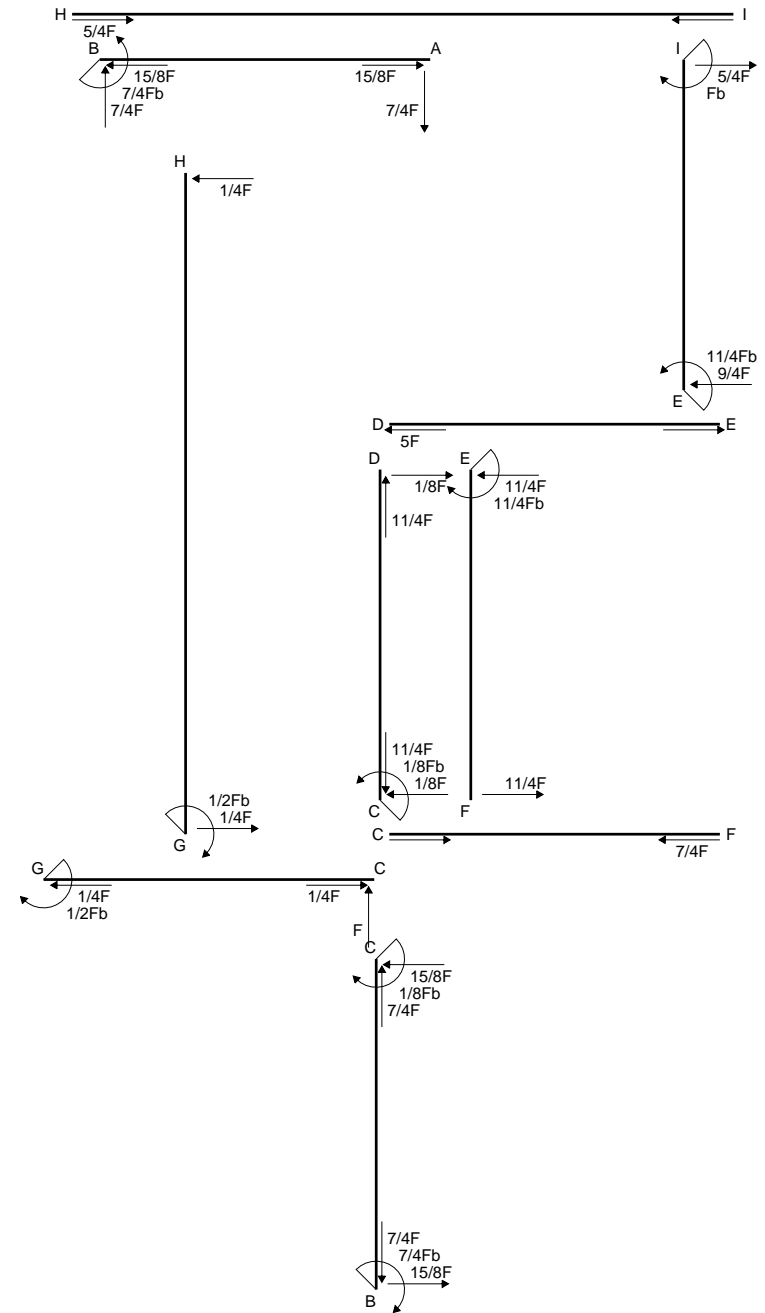
$$L_{DC}^{xo} = \int_0^b (2 x^2/b^2) Fb^2 1/EJ dx = [2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

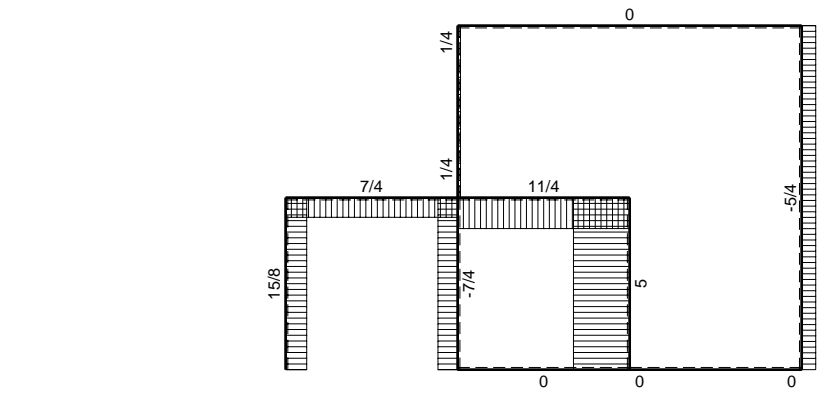
$$= (2/3 b) Fb^2 1/EJ = 2/3 Fb^3/EJ$$



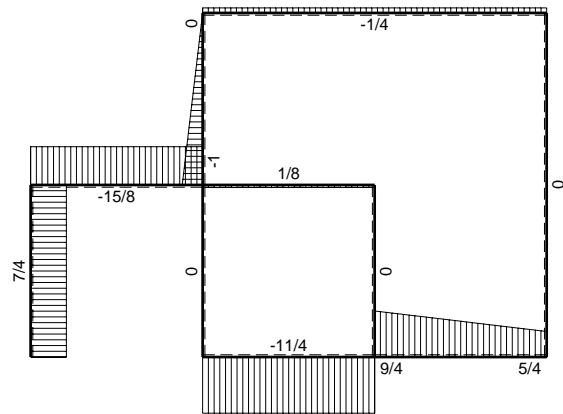
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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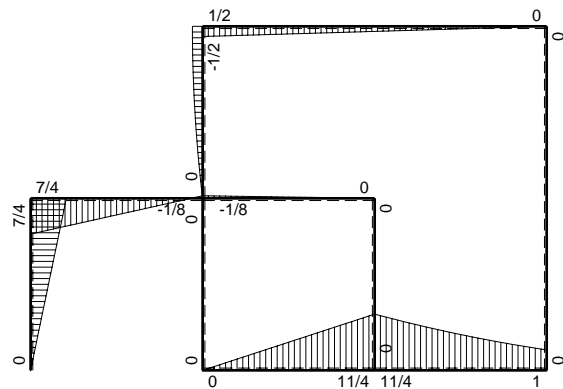




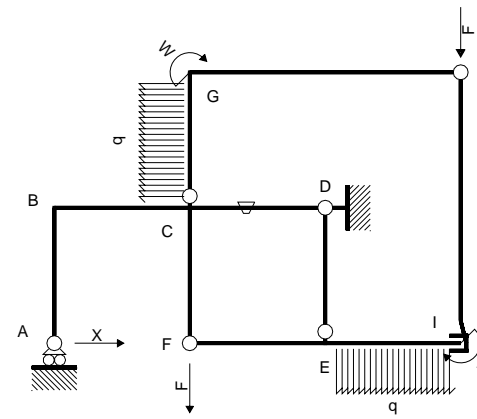
← (+) → F



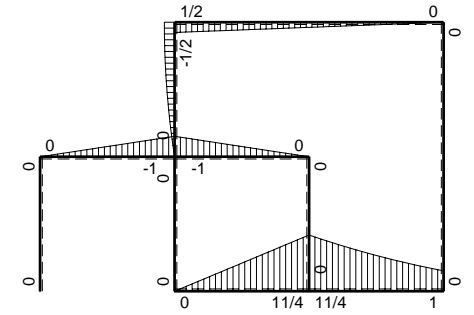
↑ (+) ↓ F



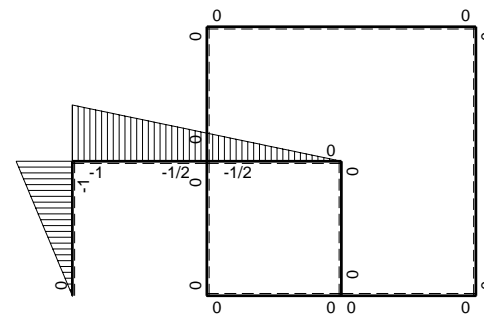
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>o</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$             | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|-----------------------|------------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | 0                       | 0        | 0                     | 0                      | $x^2$                 | 0+0                         | $1/3 X b^3/EJ$         |
| BA b  | b-x                         | 0                       | 0        | 0                     | 0                      | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | -Fx                     | 0        | $Fbx-1/2Fx^2$         | 0                      | $b^2-bx+1/4x^2$       | $(1/3+0)Fb^3/EJ$            | $7/12 X b^3/EJ$        |
| CB b  | $1/2b+1/2x$                 | $Fb-Fx$                 | 0        | $1/2Fb^2-1/2Fx^2$     | 0                      | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | $-Fb+Fx$                | $-Fb/EJ$ | $1/2Fb^2-Fbx+1/2Fx^2$ | $1/2Fb^2/EJ-1/2Fxb/EJ$ | $1/4b^2-1/2bx+1/4x^2$ | $(1/6+1/4)Fb^3/EJ$          | $1/12 X b^3/EJ$        |
| DC b  | $1/2x$                      | $Fx$                    | $Fb/EJ$  | $1/2Fx^2$             | $1/2Fxb/EJ$            | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                     | 0                      | 0                     |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                     | 0                      | 0                     |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                     | 0                      | 0                     |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                     | 0                      | 0                     |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                     | 0                      | 0                     |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                     | 0                      | 0                     |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                     | 0                      | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                         |          |                       |                        |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                         |          |                       |                        |                       | $7/4Fb^3/EJ$                | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                         |          |                       |                        |                       | $-7/4F$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

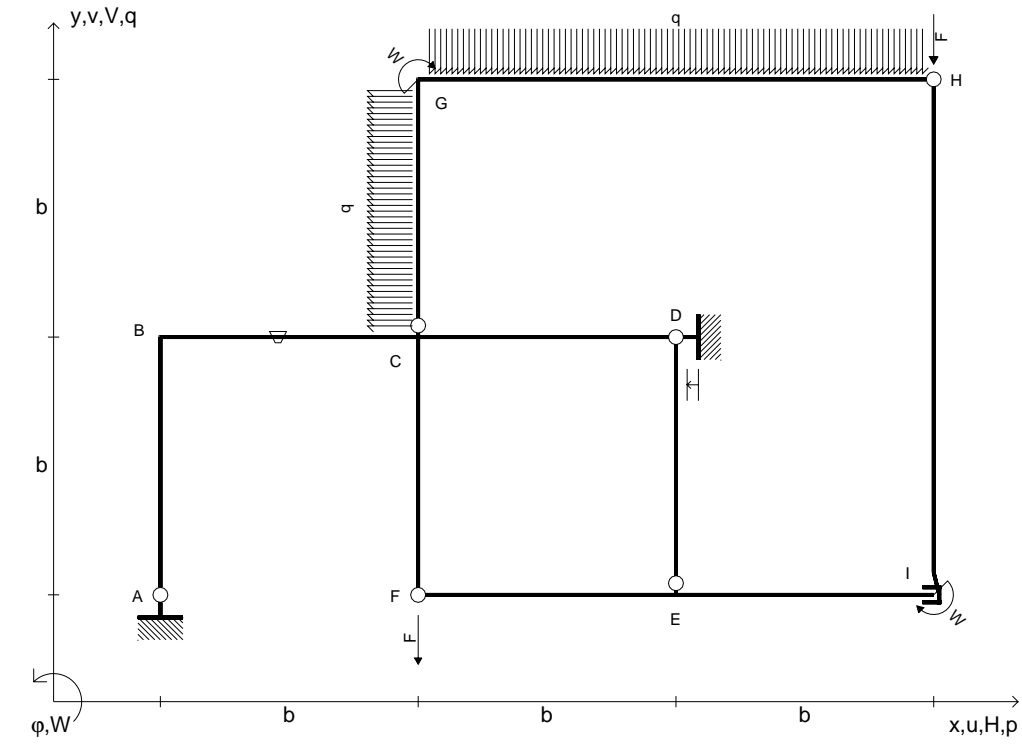
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

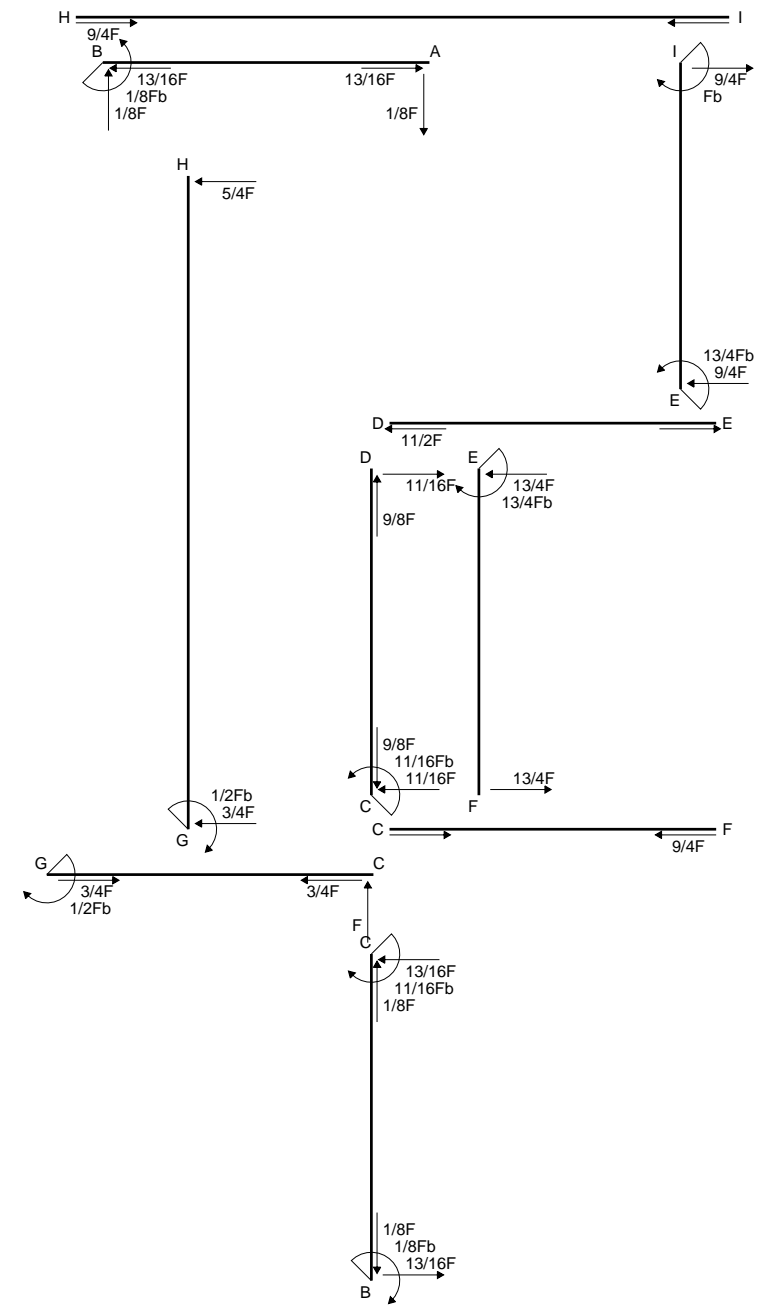
$$= (1/6 b) Fb^2 1/EJ + (-1/4 b) \theta = 5/12 Fb^3/EJ$$

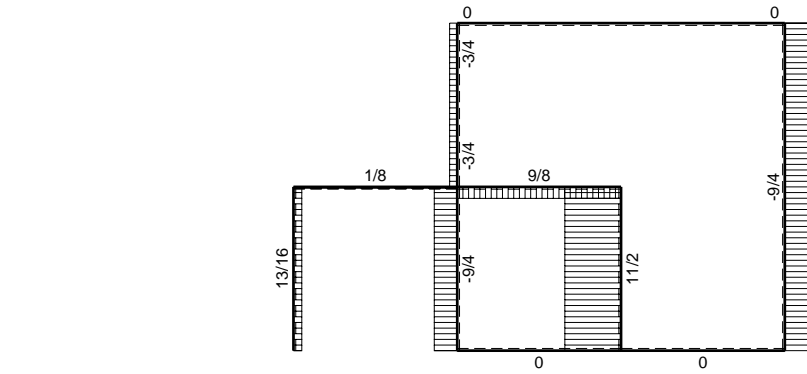




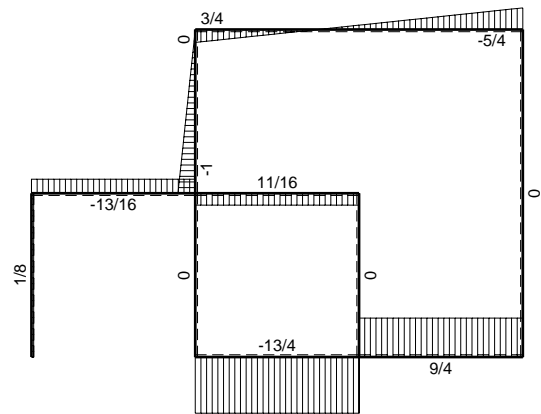
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi  $x, y$ .  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta  $YZ$  con origine in  $Y$ .  
 Curvatura  $\theta$  asta  $BC$  positiva se convessa a destra con inizio  $B$ .  
 Spostamento orizzontale assoluto  $u$  imposto al nodo  $D$ .  
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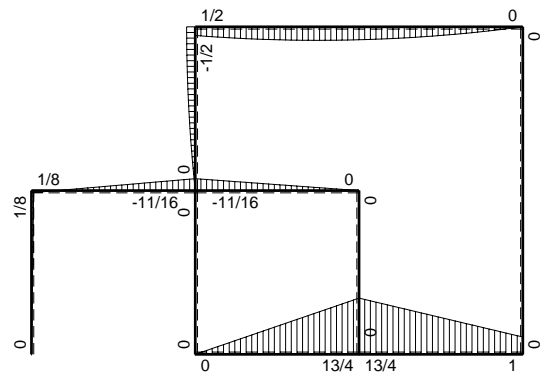




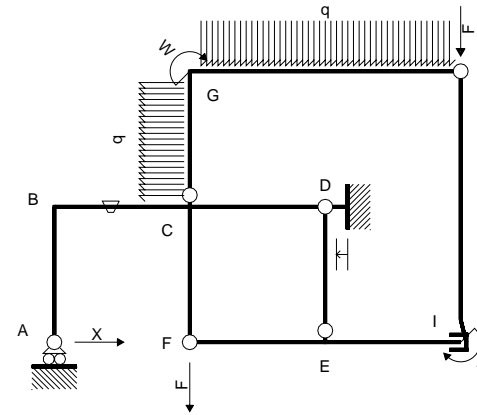
← (+) → F



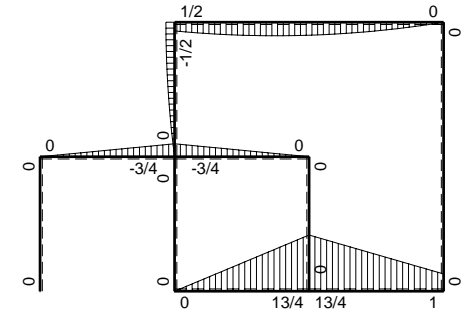
↑ (+) ↓ F



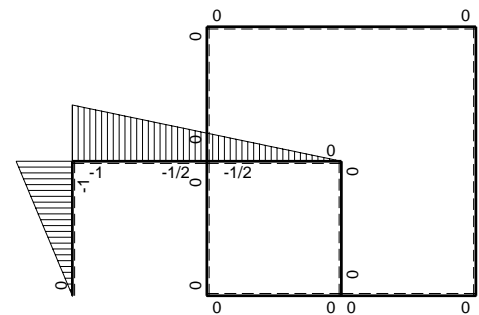
⊕ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ M<sub>o</sub> flessione da carichi assegnati



⊕ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H<sub>A</sub>

| →     | M <sub>x</sub> (x)                             | M <sub>o</sub> (x)             | θ      | M <sub>x</sub> M <sub>o</sub>                 | M <sub>x</sub> θ                 | M <sub>x</sub> M <sub>x</sub>              | ∫M <sub>x</sub> (M <sub>o</sub> /EJ+θ)dx | ∫XM <sub>x</sub> M <sub>x</sub> /EJdx |                     |
|-------|--|--------------------------------|--------|---|----------------------------------|--|--|---------------------------------------|---------------------|
| AB b  | -x   | 0                              | 0      | 0   | 0                                | x <sup>2</sup>                             | 0+0                                      | 1/3Xb <sup>3</sup> /EJ                |                     |
| BA b  | b-x  | 0                              | 0      | 0   | 0                                | b <sup>2</sup> -2bx+x <sup>2</sup>         |  |                                       |                     |
| BC b  | -b+1/2x  | -3/4Fx                         | -Fb/EJ | 3/4Fbx-3/8Fx <sup>2</sup>                     | Fb <sup>2</sup> /EJ-1/2Fxb/EJ    | b <sup>2</sup> -bx+1/4x <sup>2</sup>       | (1/4+3/4)Fb <sup>3</sup> /EJ             | 7/12Xb <sup>3</sup> /EJ               |                     |
| CB b  | 1/2b+1/2x                                      | 3/4Fb-3/4Fx                    | Fb/EJ  | 3/8Fb <sup>2</sup> -3/8Fx <sup>2</sup>        | 1/2Fb <sup>2</sup> /EJ+1/2Fxb/EJ | 1/4b <sup>2</sup> +1/2bx+1/4x <sup>2</sup> |  |                                       |                     |
| CD b  | -1/2b+1/2x                                     | -3/4Fb+3/4Fx                   | 0      | 3/8Fb <sup>2</sup> -3/4Fbx+3/8Fx <sup>2</sup> | 0                                | 1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup> | (1/8+0)Fb <sup>3</sup> /EJ               | 1/12Xb <sup>3</sup> /EJ               |                     |
| DC b  | 1/2x   | 3/4Fx                          | 0      | 3/8Fx <sup>2</sup>                            | 0                                | 1/4x <sup>2</sup>                          |  |                                       |                     |
| DE b  | 0  | 0                              | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| ED b  | 0  | 0                              | 0      | 0   | 0                                | 0  |  |                                       |                     |
| EF b  | 0  | 13/4Fb-13/4Fx                  | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| FE b  | 0  | -13/4Fx                        | 0      | 0   | 0                                | 0  |  |                                       |                     |
| FC b  | 0  | 0                              | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| CF b  | 0  | 0                              | 0      | 0   | 0                                | 0  |  |                                       |                     |
| CG b  | 0  | -Fx+1/2qx <sup>2</sup>         | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| GC b  | 0  | 1/2Fb-1/2qx <sup>2</sup>       | 0      | 0   | 0                                | 0  |  |                                       |                     |
| GH 2b | 0  | 1/2Fb+3/4Fx-1/2qx <sup>2</sup> | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| HG 2b | 0  | -5/4Fx+1/2qx <sup>2</sup>      | 0      | 0   | 0                                | 0  |  |                                       |                     |
| HI 2b | 0  | 0                              | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| IH 2b | 0  | 0                              | 0      | 0   | 0                                | 0  |  |                                       |                     |
| IE b  | 0  | Fb+9/4Fx                       | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| EI b  | 0  | -13/4Fb+9/4Fx                  | 0      | 0   | 0                                | 0  |  |                                       |                     |
| D     | cedimento nodo -H <sub>1D</sub> u <sub>D</sub> |                                |        |   |                                  |  |  | -Fb <sup>3</sup> /EJ                  |                     |
|       | totali   |                                |        |   |                                  |  |  | 1/8Fb <sup>3</sup> /EJ                | Xb <sup>3</sup> /EJ |
|       | iperstatica X=H <sub>A</sub>                   |                                |        |   |                                  |  |  | -1/8F                                 |                     |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{3}{4} \frac{x}{b} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( 1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{3}{8} \frac{x^2}{b} - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( b - \frac{1}{4} b \right) \theta = Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{3}{8} x - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

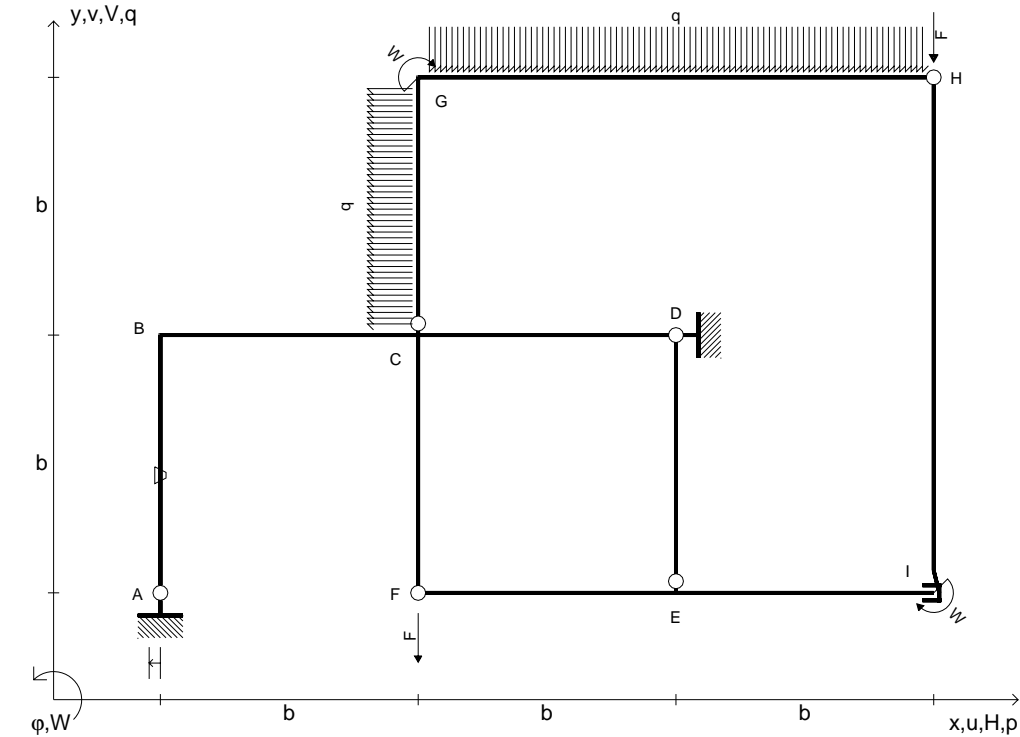
$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{2} b - \frac{1}{4} b \right) \theta = Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{4} \frac{x}{b} + \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} x - \frac{3}{8} \frac{x^2}{b} + \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{3}{8} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

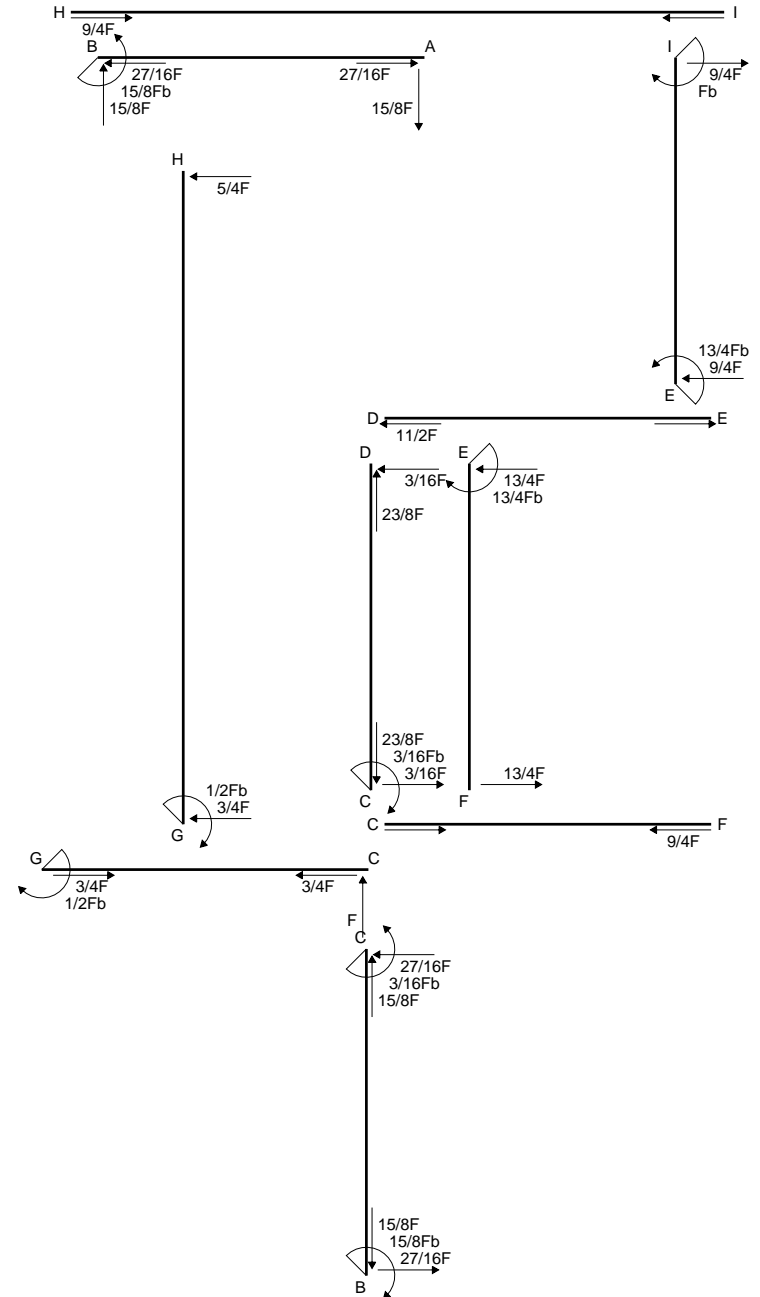
$$L_{DC}^{xo} = \int_0^b \left( \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

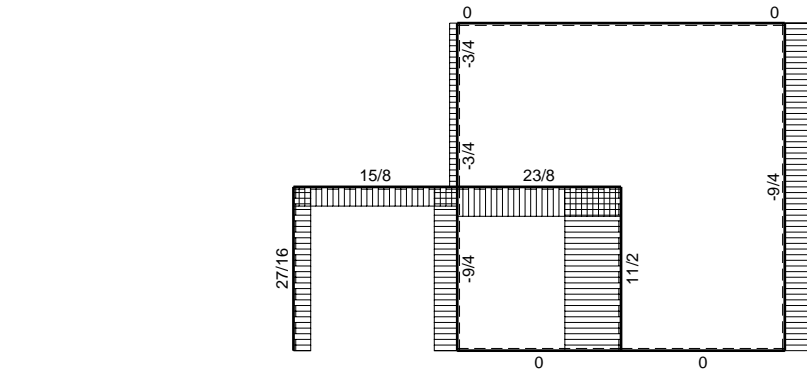
$$= \left( \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$



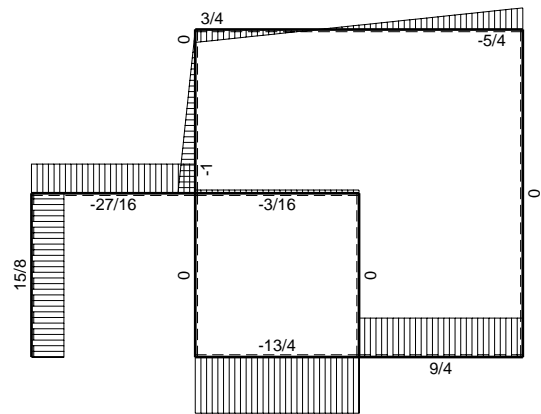
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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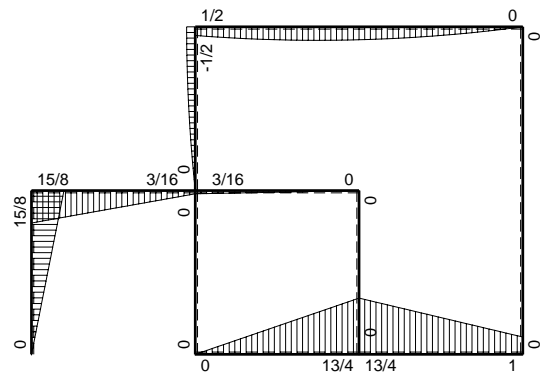




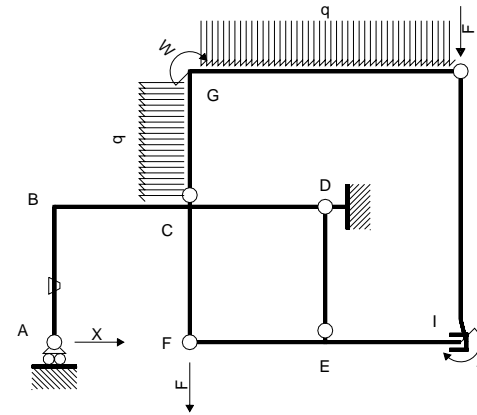
← (+) → F



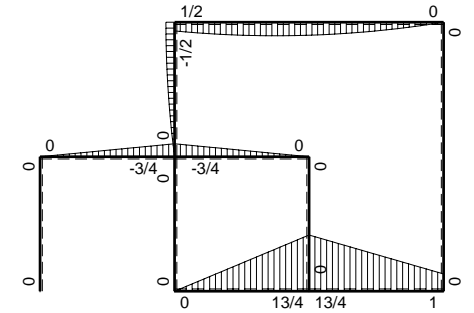
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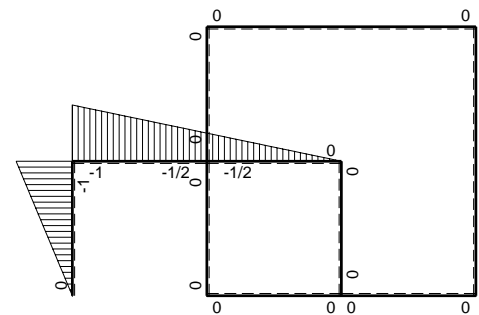
⤵ (+) ⤴ F<sub>b</sub>



Schema di calcolo iperstatico



⤵ (+) ⤴ M<sub>o</sub> flessione da carichi assegnati



⤵ (+) ⤴ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$              | $\theta$ | $M_x M_o$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|-----------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|--------------------------|
| AB b  | -x                          | 0                     | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$             |
| BA b  | b-x                         | 0                     | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                          |
| BC b  | -b+1/2x                     | -3/4Fx                | 0        | $3/4Fbx-3/8Fx^2$         | 0                | $b^2-bx+1/4x^2$       | $(1/4+0)Fb^3/EJ$            | $7/12Xb^3/EJ$            |
| CB b  | $1/2b+1/2x$                 | $3/4Fb-3/4Fx$         | 0        | $3/8Fb^2-3/8Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                          |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$        | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12Xb^3/EJ$            |
| DC b  | $1/2x$                      | $3/4Fx$               | 0        | $3/8Fx^2$                | 0                | $1/4x^2$              |                             |                          |
| DE b  | 0                           | 0                     | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |
| ED b  | 0                           | 0                     | 0        | 0                        | 0                | 0                     |                             |                          |
| EF b  | 0                           | $13/4Fb-13/4Fx$       | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |
| FE b  | 0                           | $-13/4Fx$             | 0        | 0                        | 0                | 0                     |                             |                          |
| FC b  | 0                           | 0                     | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |
| CF b  | 0                           | 0                     | 0        | 0                        | 0                | 0                     |                             |                          |
| CG b  | 0                           | $-Fx+1/2qx^2$         | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |
| GC b  | 0                           | $1/2Fb-1/2qx^2$       | 0        | 0                        | 0                | 0                     |                             |                          |
| GH 2b | 0                           | $1/2Fb+3/4Fx-1/2qx^2$ | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |
| HG 2b | 0                           | $-5/4Fx+1/2qx^2$      | 0        | 0                        | 0                | 0                     |                             |                          |
| HI 2b | 0                           | 0                     | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |
| IH 2b | 0                           | 0                     | 0        | 0                        | 0                | 0                     |                             |                          |
| IE b  | 0                           | $Fb+9/4Fx$            | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |
| EI b  | 0                           | $-13/4Fb+9/4Fx$       | 0        | 0                        | 0                | 0                     |                             |                          |
| A     | cedimento nodo $-H_{1A}u_A$ |                       |          |                          |                  |                       | $Fb^3/EJ$                   |                          |
|       | totali                      |                       |          |                          |                  |                       | $15/8Fb^3/EJ$               | $Xb^3/EJ$                |
|       | iperstatica $X=H_A$         |                       |          |                          |                  |                       | $-15/8F$                    |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left( \frac{x}{b} \right) \theta dx = \left[ \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left( -1 + x/b \right) \theta dx = \left[ -x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( -b + \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{3}{4} \frac{x}{b} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} \frac{x^2}{b} - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{4} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} x - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{4} Fb^3/EJ$$

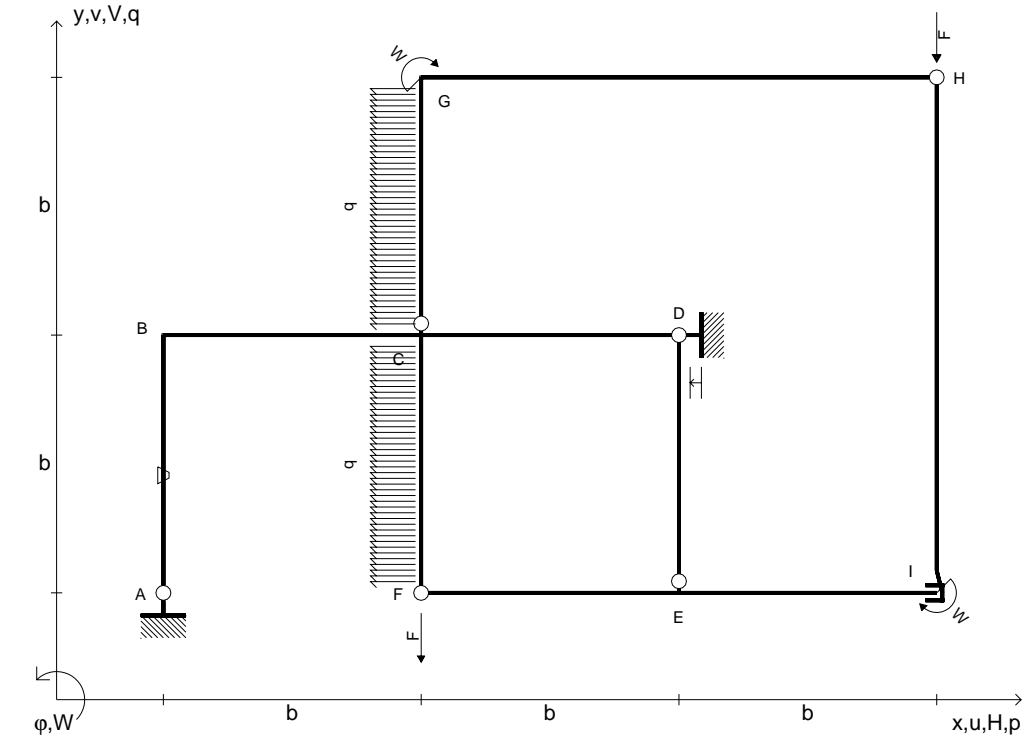
$$L_{CD}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{4} \frac{x}{b} + \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} x - \frac{3}{8} \frac{x^2}{b} + \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{3}{8} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

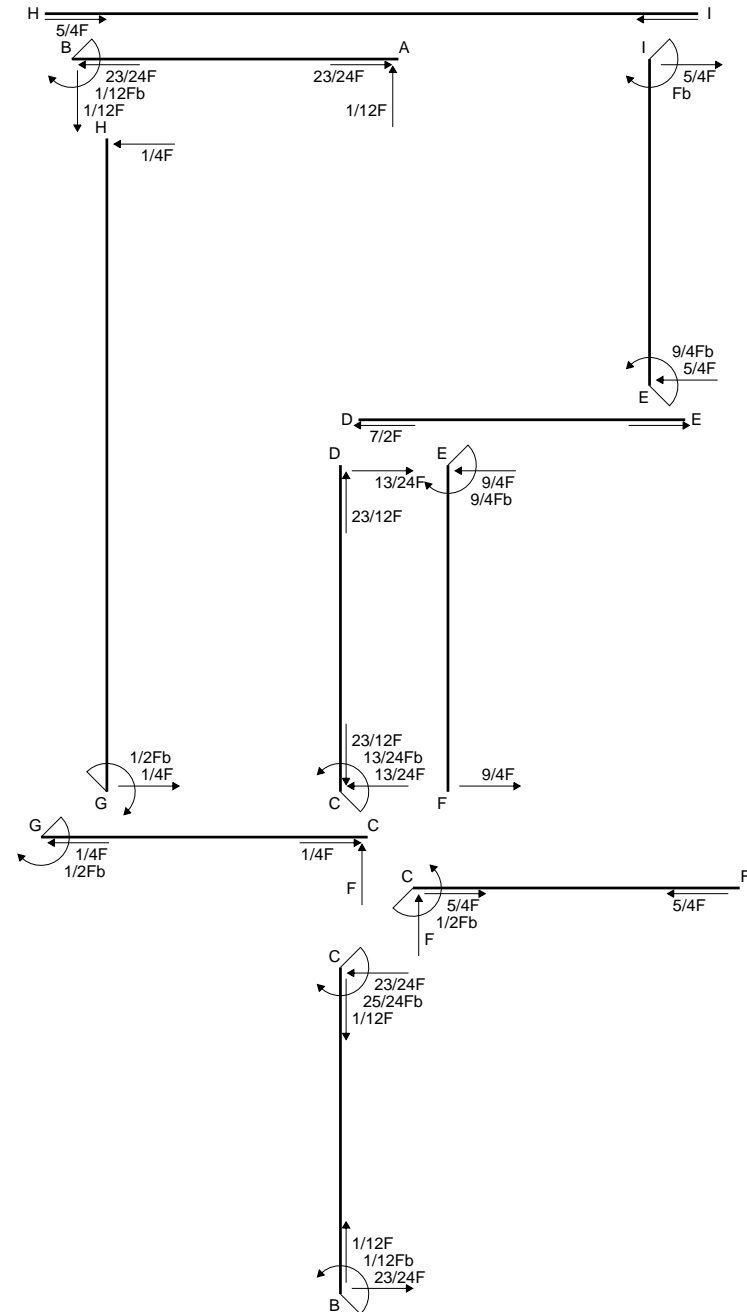
$$= \left( \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

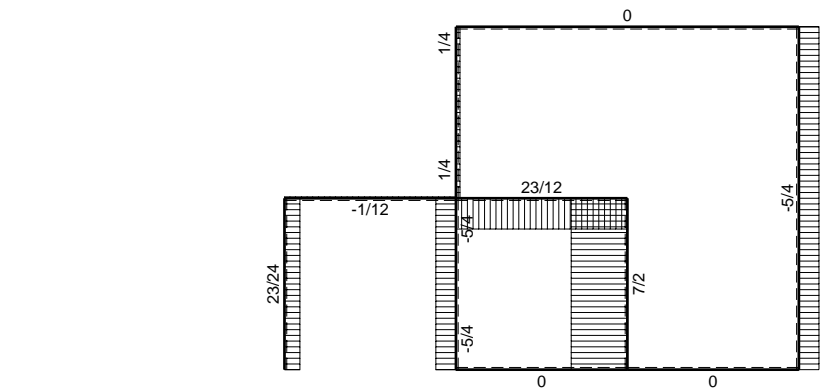




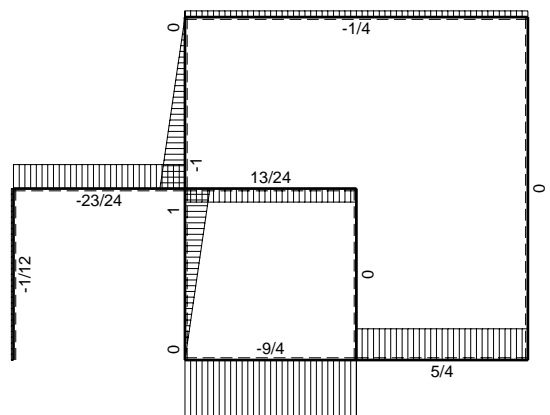
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{FC} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi  $x, y$ .  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta  $YZ$  con origine in  $Y$ .  
 Curvatura  $\theta$  asta  $AB$  positiva se convessa a destra con inizio  $A$ .  
 Spostamento orizzontale assoluto  $u$  imposto al nodo  $D$ .  
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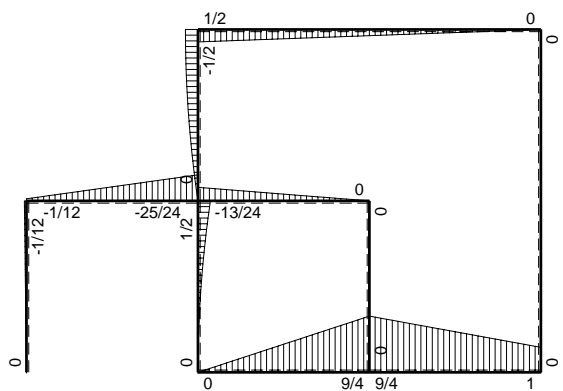




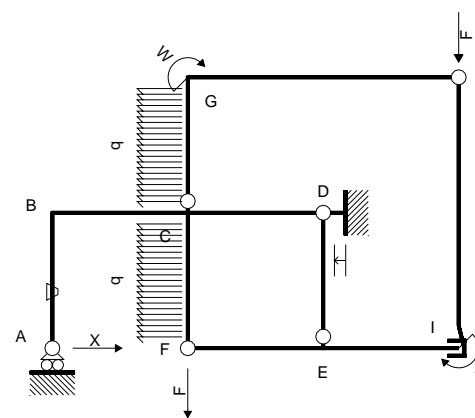
← (+) → F



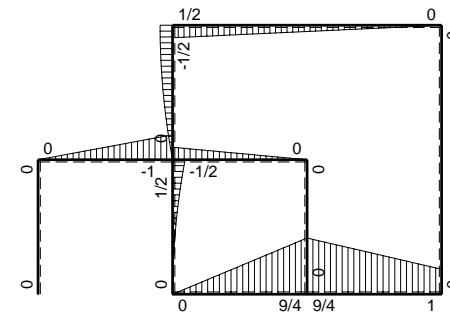
↑ (+) ↓ F



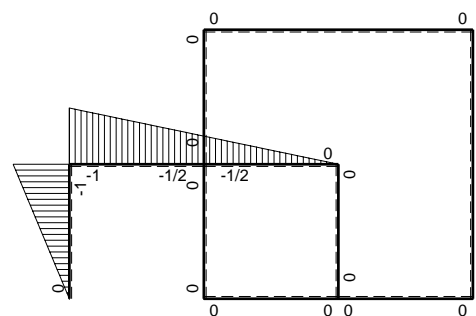
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>0</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_0(x)$            | $\theta$ | $M_x M_0$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_0/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|--------------------------|-----------|
| AB b  | -x                          | 0                   | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$             |           |
| BA b  | b-x                         | 0                   | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                          |           |
| BC b  | -b+1/2x                     | -Fx                 | 0        | $Fbx-1/2Fx^2$            | 0                | $b^2-bx+1/4x^2$       | $(1/3+0)Fb^3/EJ$            | $7/12Xb^3/EJ$            |           |
| CB b  | $1/2b+1/2x$                 | Fb-Fx               | 0        | $1/2Fb^2-1/2Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                          |           |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb^2-1/2Fbx+1/4Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+0)Fb^3/EJ$           | $1/12Xb^3/EJ$            |           |
| DC b  | $1/2x$                      | $1/2Fx$             | 0        | $1/4Fx^2$                | 0                | $1/4x^2$              |                             |                          |           |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                          |           |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| FE b  | 0                           | $-9/4Fx$            | 0        | 0                        | 0                | 0                     |                             |                          |           |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0                | 0                     |                             |                          |           |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                        | 0                | 0                     |                             |                          |           |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| HG 2b | 0                           | $-1/4Fx$            | 0        | 0                        | 0                | 0                     |                             |                          |           |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                          |           |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                  |                       |                             | $-Fb^3/EJ$               |           |
|       | totali                      |                     |          |                          |                  |                       |                             | $-1/12Fb^3/EJ$           | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                          |                  |                       |                             | $1/12F$                  |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

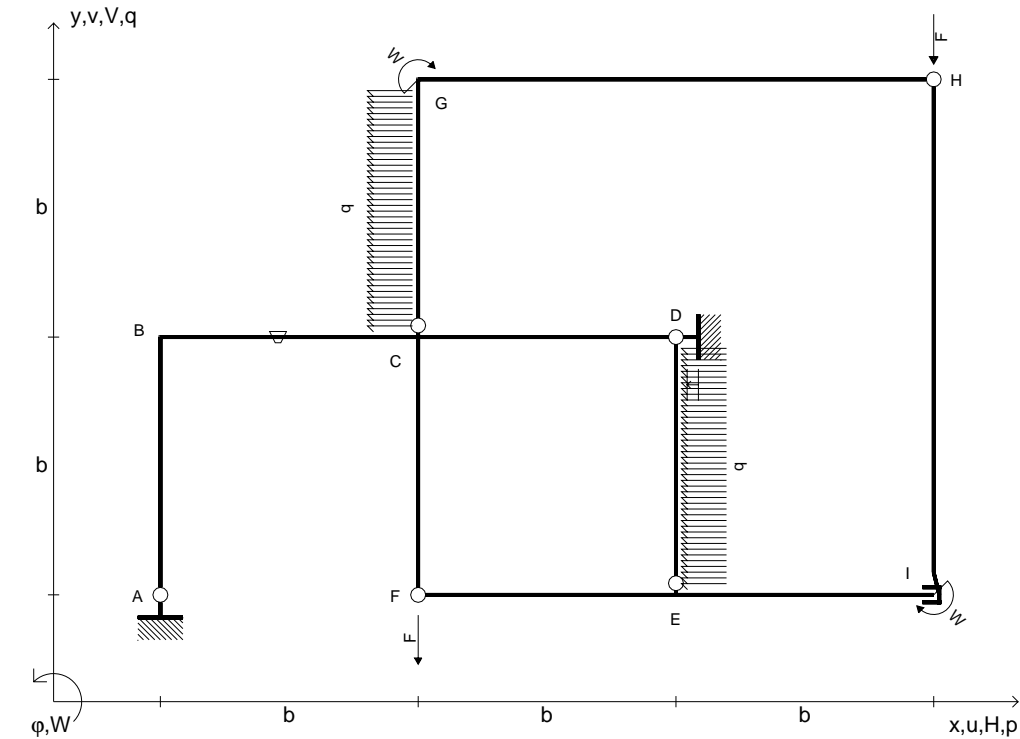
$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

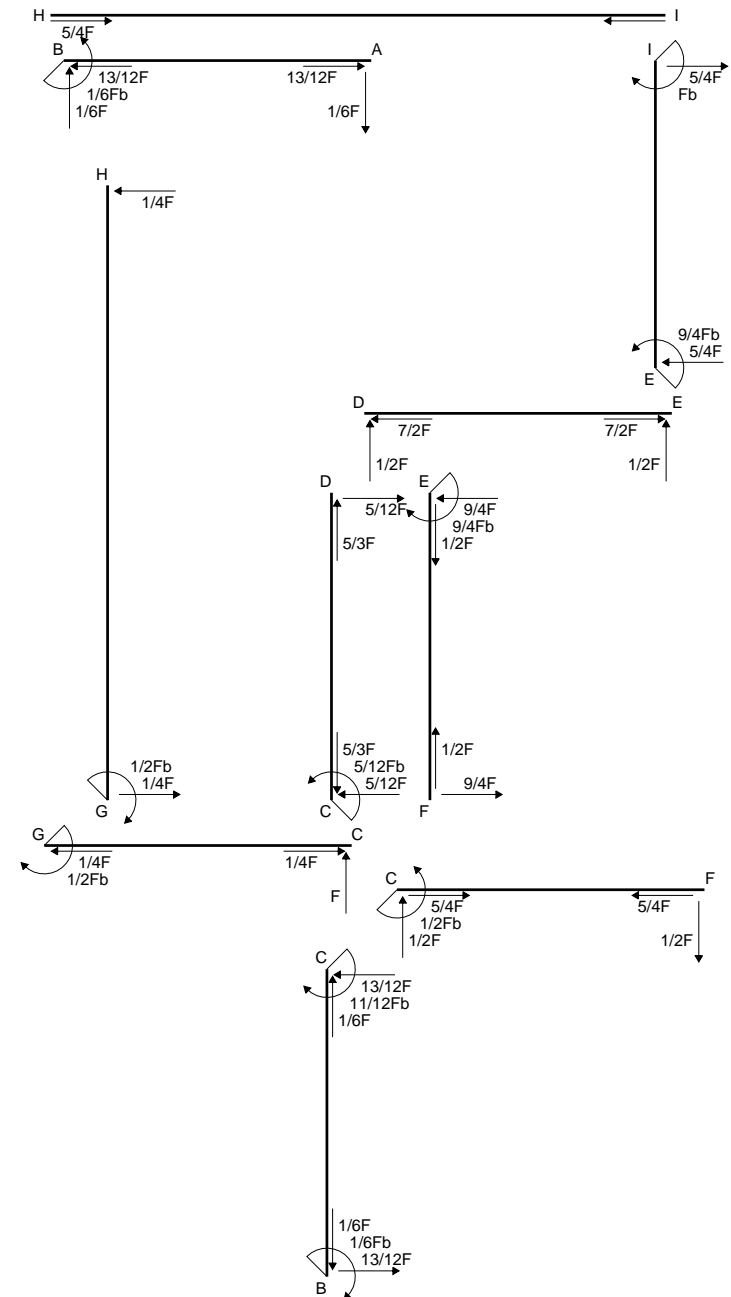
$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

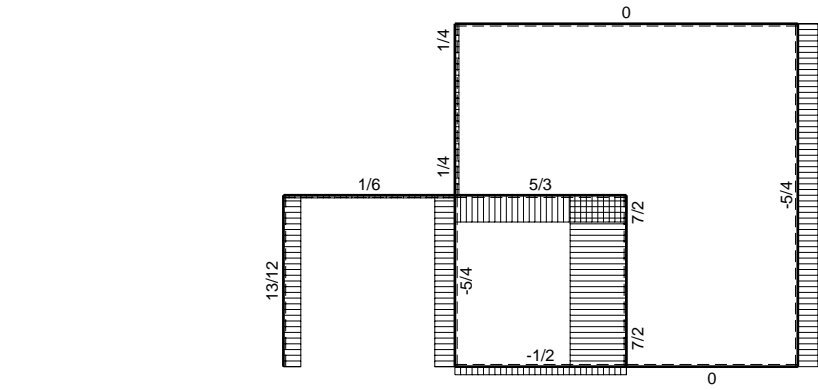
$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$



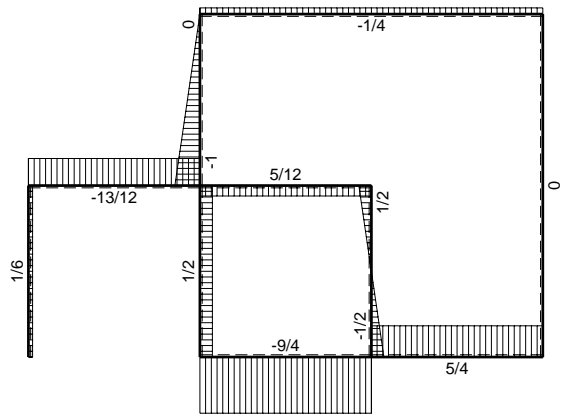
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{DE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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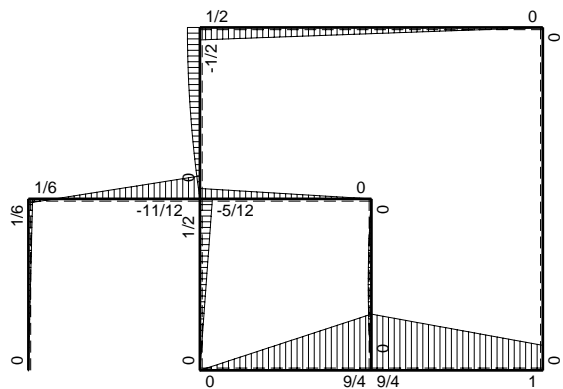




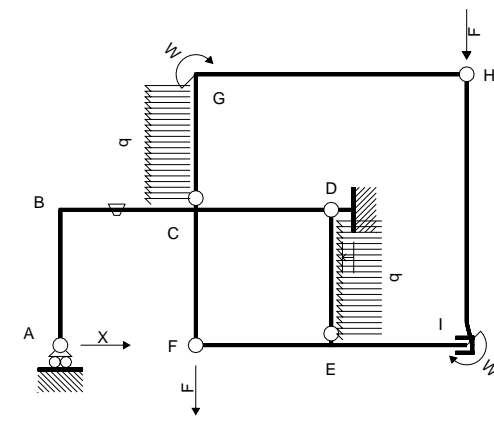
← (+) → F



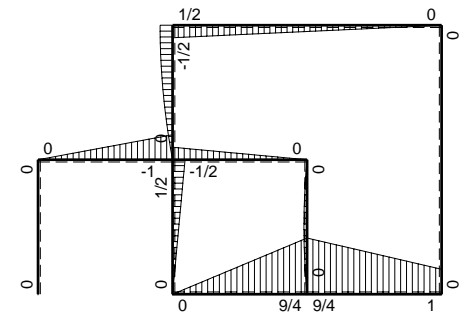
↑ (+) ↓ F



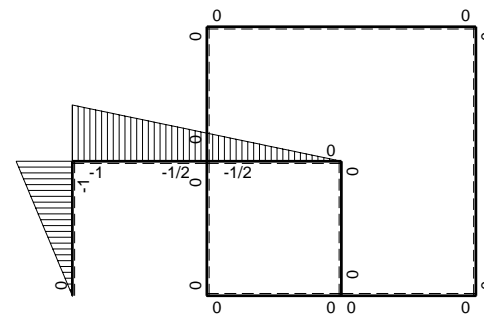
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>o</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H<sub>A</sub>

| →     | M <sub>x</sub> (x)                             | M <sub>0</sub> (x)        | θ      | M <sub>x</sub> M <sub>0</sub>                 | M <sub>x</sub> θ                 | M <sub>x</sub> M <sub>x</sub>              | ∫M <sub>x</sub> (M <sub>0</sub> /EJ+θ)dx | ∫XM <sub>x</sub> M <sub>x</sub> /EJdx |                     |
|-------|--|---------------------------|--------|---|----------------------------------|--|--|---------------------------------------|---------------------|
| AB b  | -x   | 0                         | 0      | 0   | 0                                | x <sup>2</sup>                             | 0+0                                      | 1/3Xb <sup>3</sup> /EJ                |                     |
| BA b  | b-x  | 0                         | 0      | 0   | 0                                | b <sup>2</sup> -2bx+x <sup>2</sup>         |  |                                       |                     |
| BC b  | -b+1/2x  | -Fx                       | -Fb/EJ | Fbx-1/2Fx <sup>2</sup>                        | Fb <sup>2</sup> /EJ-1/2Fxb/EJ    | b <sup>2</sup> -bx+1/4x <sup>2</sup>       | (1/3+3/4)Fb <sup>3</sup> /EJ             | 7/12Xb <sup>3</sup> /EJ               |                     |
| CB b  | 1/2b+1/2x                                      | Fb-Fx                     | Fb/EJ  | 1/2Fb <sup>2</sup> -1/2Fx <sup>2</sup>        | 1/2Fb <sup>2</sup> /EJ+1/2Fxb/EJ | 1/4b <sup>2</sup> +1/2bx+1/4x <sup>2</sup> |  |                                       |                     |
| CD b  | -1/2b+1/2x                                     | -1/2Fb+1/2Fx              | 0      | 1/4Fb <sup>2</sup> -1/2Fbx+1/4Fx <sup>2</sup> | 0                                | 1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup> | (1/12+0)Fb <sup>3</sup> /EJ              | 1/12Xb <sup>3</sup> /EJ               |                     |
| DC b  | 1/2x   | 1/2Fx                     | 0      | 1/4Fx <sup>2</sup>                            | 0                                | 1/4x <sup>2</sup>                          |  |                                       |                     |
| DE b  | 0  | 1/2Fx-1/2qx <sup>2</sup>  | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| ED b  | 0  | -1/2Fx+1/2qx <sup>2</sup> | 0      | 0   | 0                                | 0  |  |                                       |                     |
| EF b  | 0  | 9/4Fb-9/4Fx               | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| FE b  | 0  | -9/4Fx                    | 0      | 0   | 0                                | 0  |  |                                       |                     |
| FC b  | 0  | 1/2Fx                     | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| CF b  | 0  | -1/2Fb+1/2Fx              | 0      | 0   | 0                                | 0  |  |                                       |                     |
| CG b  | 0  | -Fx+1/2qx <sup>2</sup>    | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| GC b  | 0  | 1/2Fb-1/2qx <sup>2</sup>  | 0      | 0   | 0                                | 0  |  |                                       |                     |
| GH 2b | 0  | 1/2Fb-1/4Fx               | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| HG 2b | 0  | -1/4Fx                    | 0      | 0   | 0                                | 0  |  |                                       |                     |
| HI 2b | 0  | 0                         | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| IH 2b | 0  | 0                         | 0      | 0   | 0                                | 0  |  |                                       |                     |
| IE b  | 0  | Fb+5/4Fx                  | 0      | 0   | 0                                | 0  | 0+0                                      | 0                                     |                     |
| EI b  | 0  | -9/4Fb+5/4Fx              | 0      | 0   | 0                                | 0  |  |                                       |                     |
| D     | cedimento nodo -H <sub>1D</sub> u <sub>D</sub> |                           |        |   |                                  |  |  | -Fb <sup>3</sup> /EJ                  |                     |
|       | totali   |                           |        |   |                                  |  |  | 1/6Fb <sup>3</sup> /EJ                | Xb <sup>3</sup> /EJ |
|       | iperstatica X=H <sub>A</sub>                   |                           |        |   |                                  |  |  | -1/6F                                 |                     |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{x}{b} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( 1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{1}{2} \frac{x^2}{b} - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} + \left( b - \frac{1}{4} b \right) \theta = \frac{13}{12} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{1}{2} x - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{13}{12} Fb^3/EJ$$

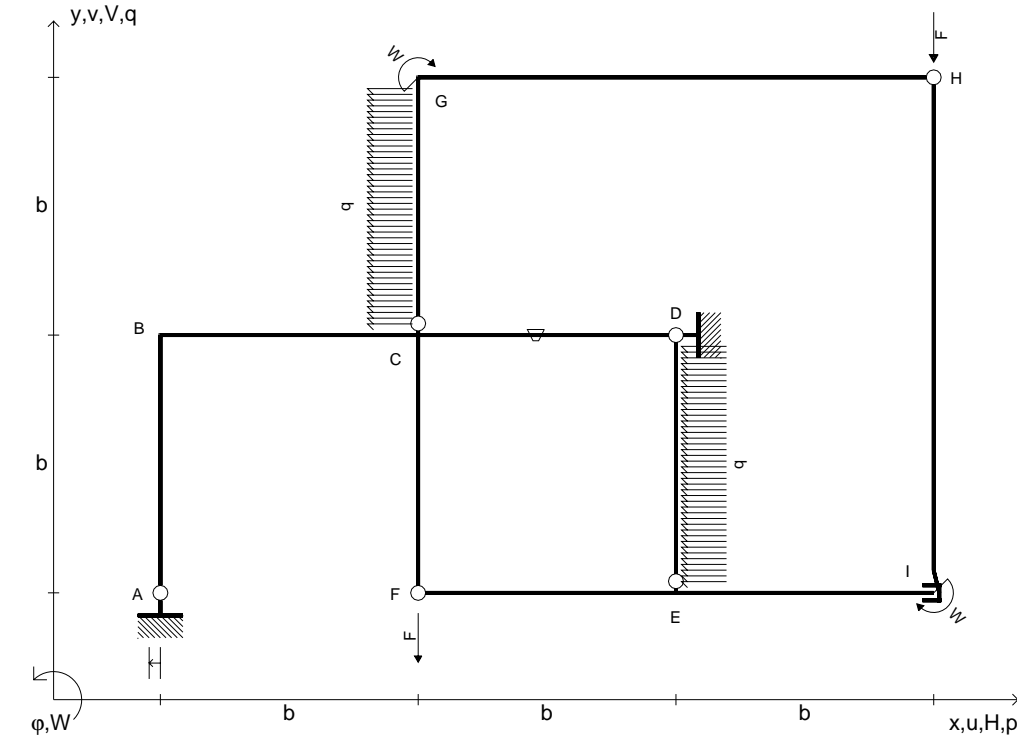
$$L_{CD}^{xo} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

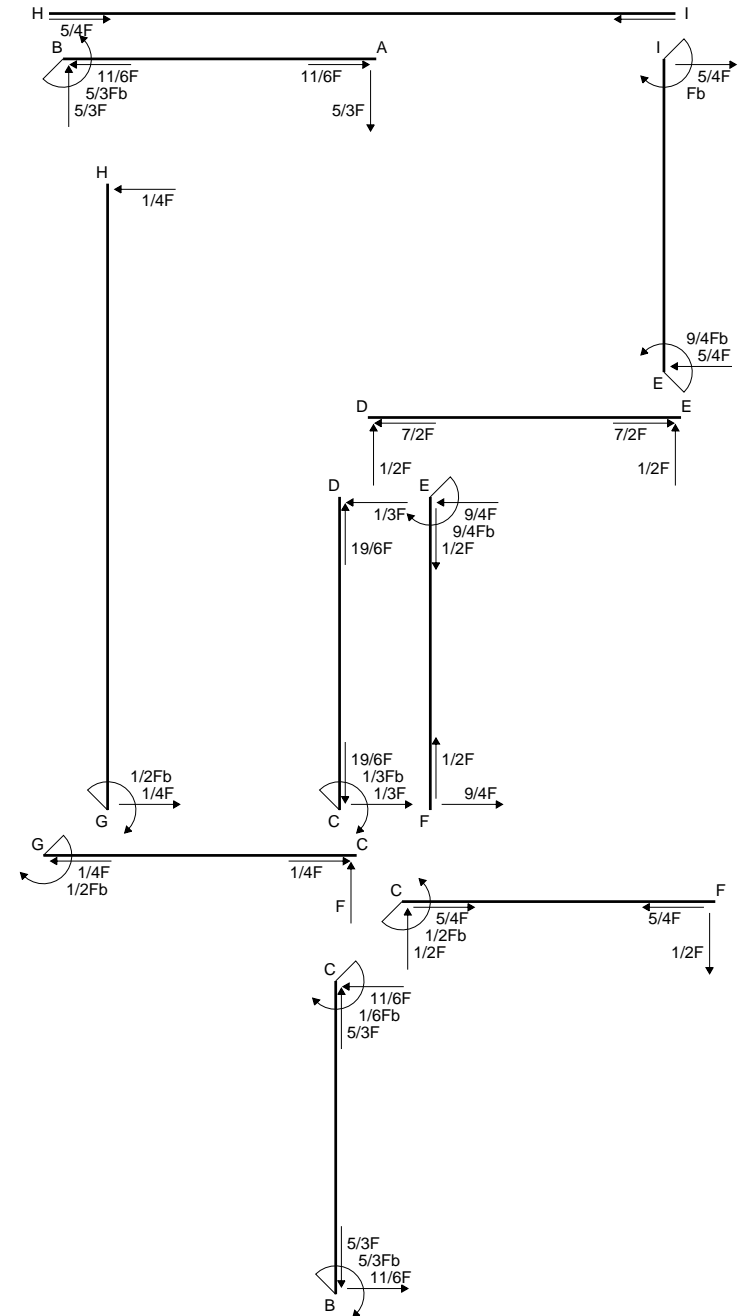
$$= \left( \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{12} Fb^3/EJ$$

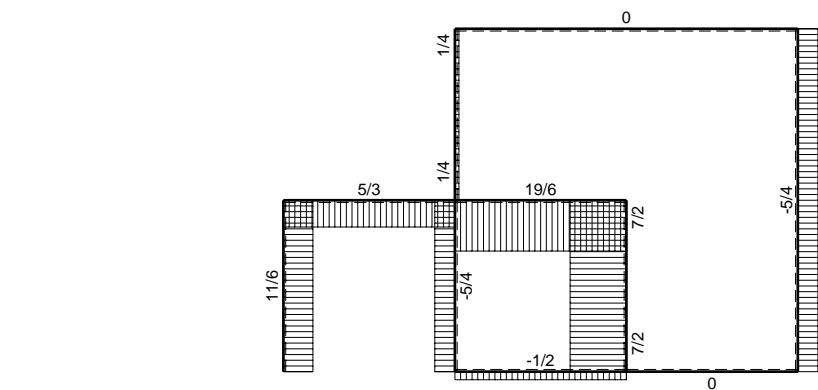




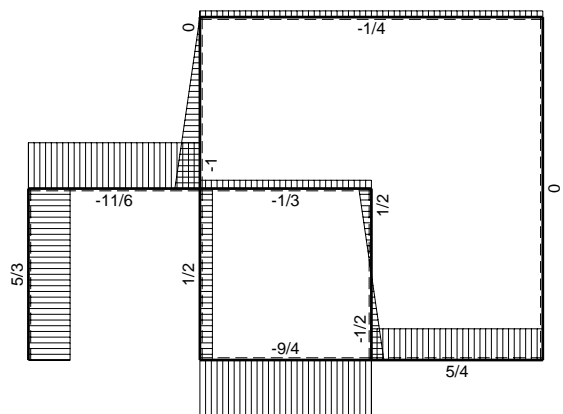
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $p_{CG} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{DE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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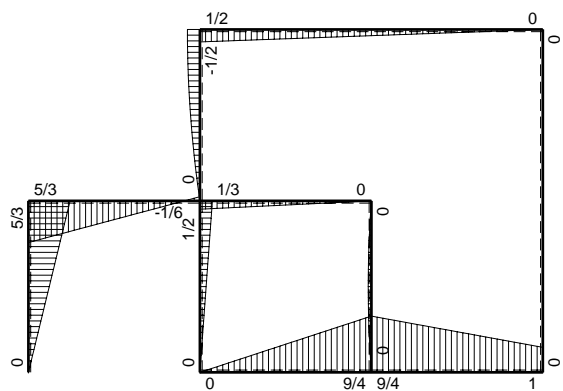




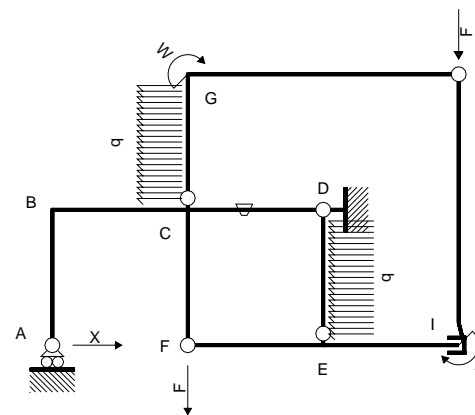
← (+) → F



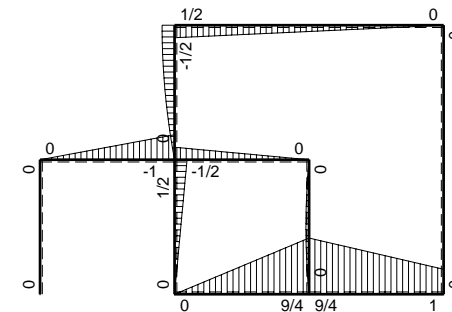
↑ (+) ↓ F



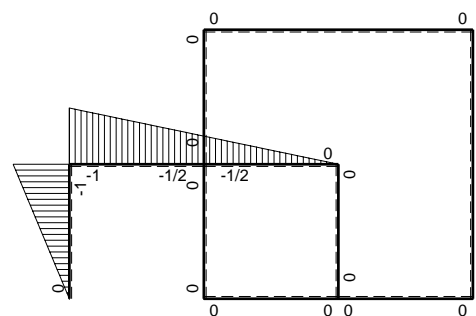
⤵ (+) ⤴ F<sub>b</sub>



Schema di calcolo iperstatico



⤵ (+) ⤴ M<sub>0</sub> flessione da carichi assegnati



⤵ (+) ⤴ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                     | $M_o(x)$           | $\theta$ | $M_x M_o$                    | $M_x \theta$             | $M_x M_x$                 | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|------------------------------|--------------------|----------|------------------------------|--------------------------|---------------------------|-----------------------------|--------------------------|
| AB b  | -x                           | 0                  | 0        | 0                            | 0                        | $x^2$                     | 0+0                         | $1/3 X b^3 / EJ$         |
| BA b  | b-x                          | 0                  | 0        | 0                            | 0                        | $b^2 - 2bx + x^2$         |                             |                          |
| BC b  | -b+1/2x                      | -Fx                | 0        | $Fbx - 1/2Fx^2$              | 0                        | $b^2 - bx + 1/4x^2$       | $(1/3+0)Fb^3/EJ$            | $7/12 X b^3 / EJ$        |
| CB b  | 1/2b+1/2x                    | Fb-Fx              | 0        | $1/2Fb^2 - 1/2Fx^2$          | 0                        | $1/4b^2 + 1/2bx + 1/4x^2$ |                             |                          |
| CD b  | -1/2b+1/2x                   | -1/2Fb+1/2Fx       | -Fb/EJ   | $1/4Fb^2 - 1/2Fbx + 1/4Fx^2$ | $1/2Fb^2/EJ - 1/2Fxb/EJ$ | $1/4b^2 - 1/2bx + 1/4x^2$ | $(1/12+1/4)Fb^3/EJ$         | $1/12 X b^3 / EJ$        |
| DC b  | 1/2x                         | 1/2Fx              | Fb/EJ    | $1/4Fx^2$                    | $1/2Fxb/EJ$              | $1/4x^2$                  |                             |                          |
| DE b  | 0                            | $1/2Fx - 1/2qx^2$  | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |
| ED b  | 0                            | $-1/2Fx + 1/2qx^2$ | 0        | 0                            | 0                        | 0                         |                             |                          |
| EF b  | 0                            | $9/4Fb - 9/4Fx$    | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |
| FE b  | 0                            | $-9/4Fx$           | 0        | 0                            | 0                        | 0                         |                             |                          |
| FC b  | 0                            | 1/2Fx              | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |
| CF b  | 0                            | $-1/2Fb + 1/2Fx$   | 0        | 0                            | 0                        | 0                         |                             |                          |
| CG b  | 0                            | $-Fx + 1/2qx^2$    | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |
| GC b  | 0                            | $1/2Fb - 1/2qx^2$  | 0        | 0                            | 0                        | 0                         |                             |                          |
| GH 2b | 0                            | $1/2Fb - 1/4Fx$    | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |
| HG 2b | 0                            | $-1/4Fx$           | 0        | 0                            | 0                        | 0                         |                             |                          |
| HI 2b | 0                            | 0                  | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |
| IH 2b | 0                            | 0                  | 0        | 0                            | 0                        | 0                         |                             |                          |
| IE b  | 0                            | $Fb + 5/4Fx$       | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |
| EI b  | 0                            | $-9/4Fb + 5/4Fx$   | 0        | 0                            | 0                        | 0                         |                             |                          |
| A     | cedimento nodo $-H_{1A} u_A$ |                    |          |                              |                          |                           | $Fb^3/EJ$                   |                          |
|       | totali                       |                    |          |                              |                          |                           | $5/3 Fb^3/EJ$               | $Xb^3/EJ$                |
|       | iperstatica $X=H_A$          |                    |          |                              |                          |                           | $-5/3F$                     |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{x}{b} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{2} \frac{x^2}{b} - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{3} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{2} x - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{3} Fb^3/EJ$$

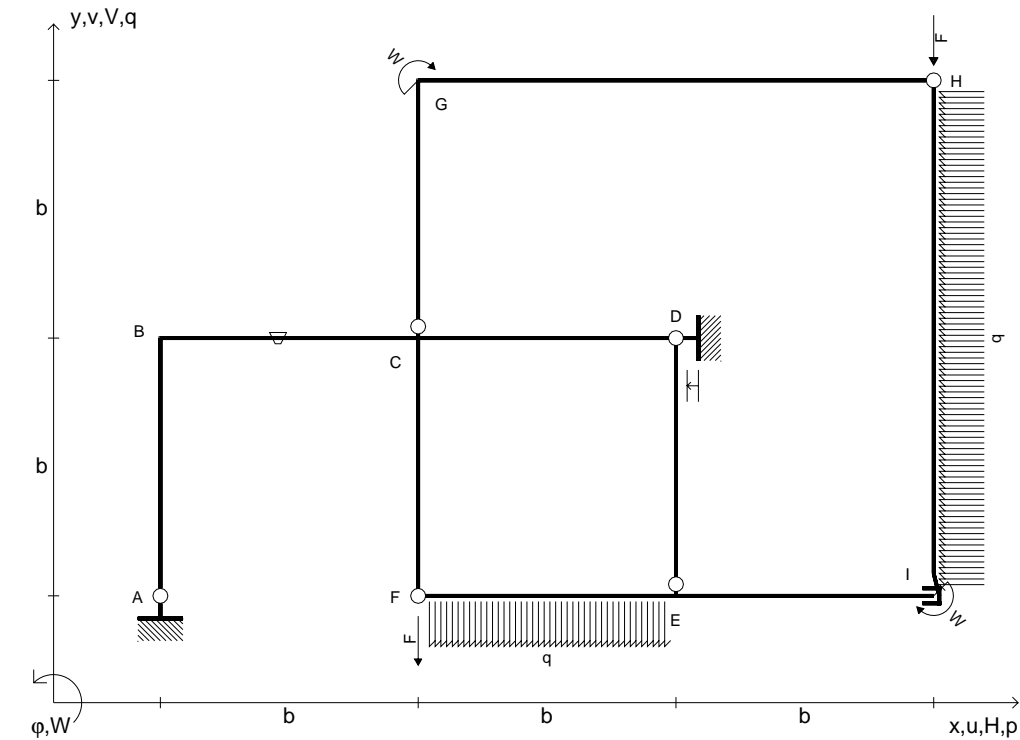
$$L_{CD}^{xo} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( \frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ \frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left( \frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{1}{3} Fb^3/EJ$$

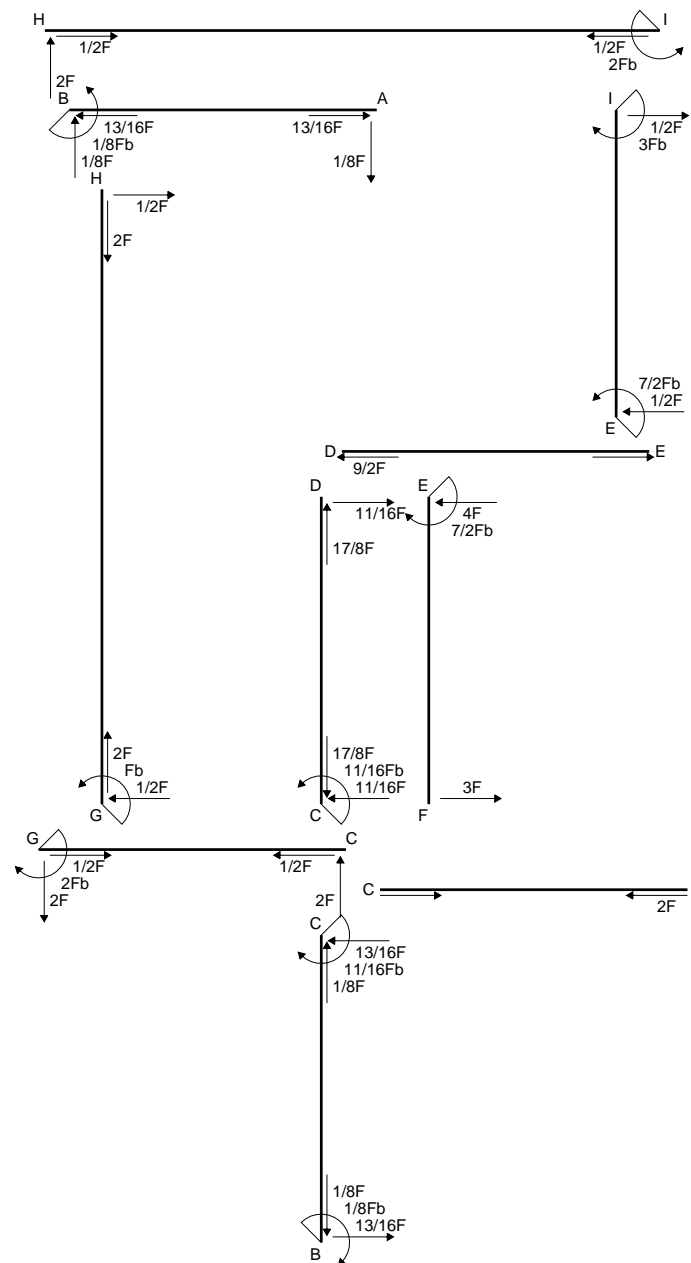
$$L_{DC}^{xo} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} \frac{x}{b} \right) \theta dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

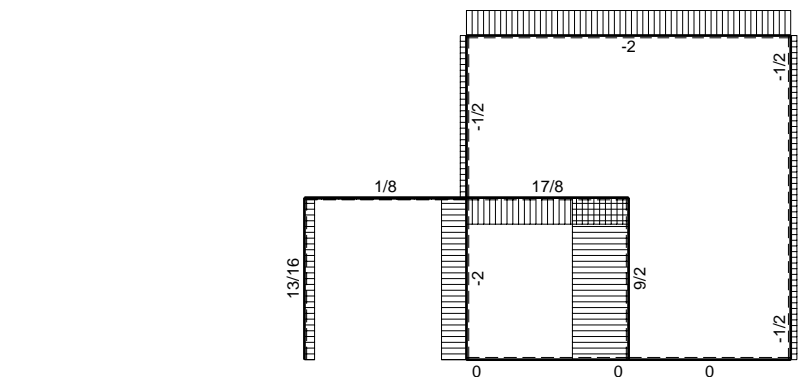
$$= \left( \frac{1}{12} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{4} b \right) \theta = \frac{1}{3} Fb^3/EJ$$



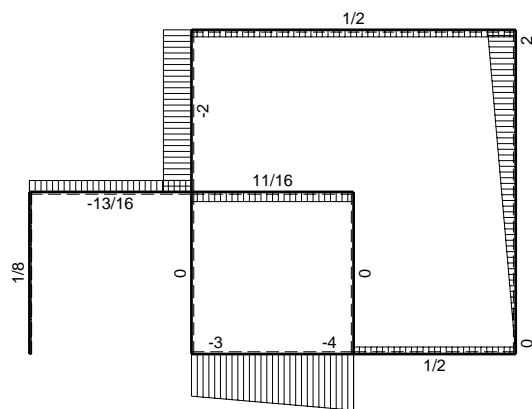
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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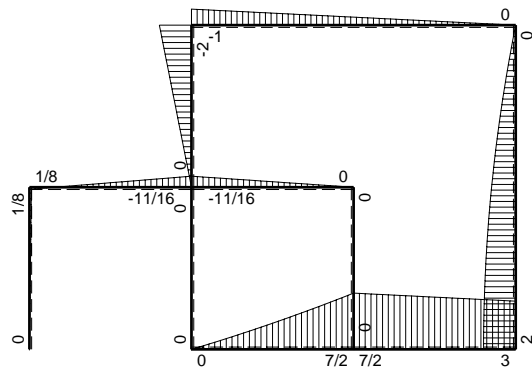




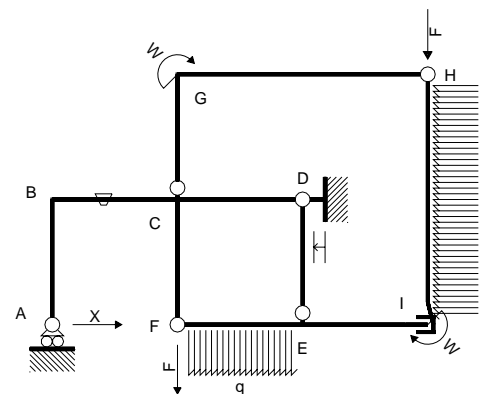
← (+) → F



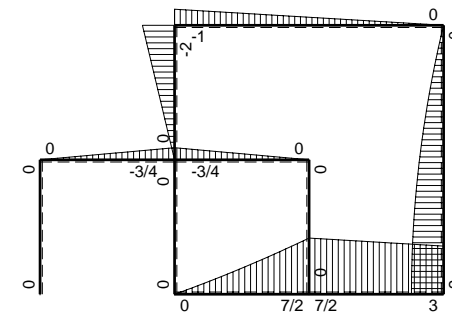
↑ (+) ↓ F



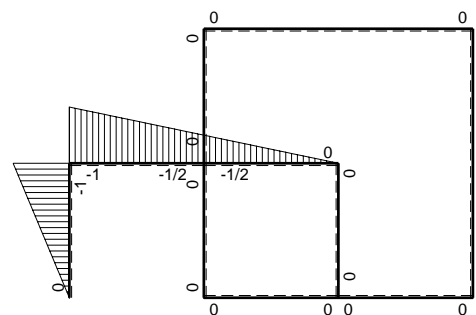
⊞ (+) ⊞ F<sub>b</sub>



Schema di calcolo iperstatico



⊞ (+) ⊞ M<sub>o</sub> flessione da carichi assegnati



⊞ (+) ⊞ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | -x                          | 0                   | 0        | 0                        | 0                      | $x^2$                 | 0+0                         | $1/3 X b^3/EJ$         |           |
| BA b  | b-x                         | 0                   | 0        | 0                        | 0                      | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | -b+1/2x                     | -3/4Fx              | -Fb/EJ   | $3/4Fbx-3/8Fx^2$         | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/4+3/4)Fb^3/EJ$          | $7/12 X b^3/EJ$        |           |
| CB b  | $1/2b+1/2x$                 | $3/4Fb-3/4Fx$       | Fb/EJ    | $3/8Fb^2-3/8Fx^2$        | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12 X b^3/EJ$        |           |
| DC b  | $1/2x$                      | $3/4Fx$             | 0        | $3/8Fx^2$                | 0                      | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| CG b  | 0                           | -2Fx                | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                        |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                     |          |                          |                        |                       |                             | $1/8Fb^3/EJ$           | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                          |                        |                       |                             | $-1/8F$                |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ + (b - 1/4 b) \theta = Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^3/EJ$$

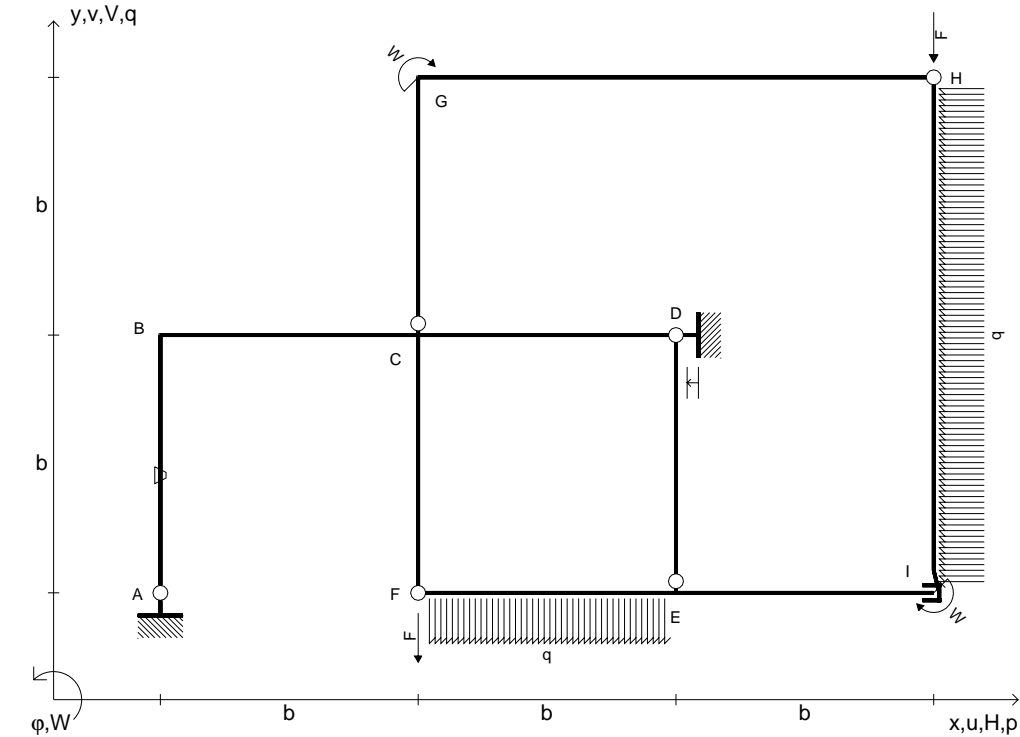
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

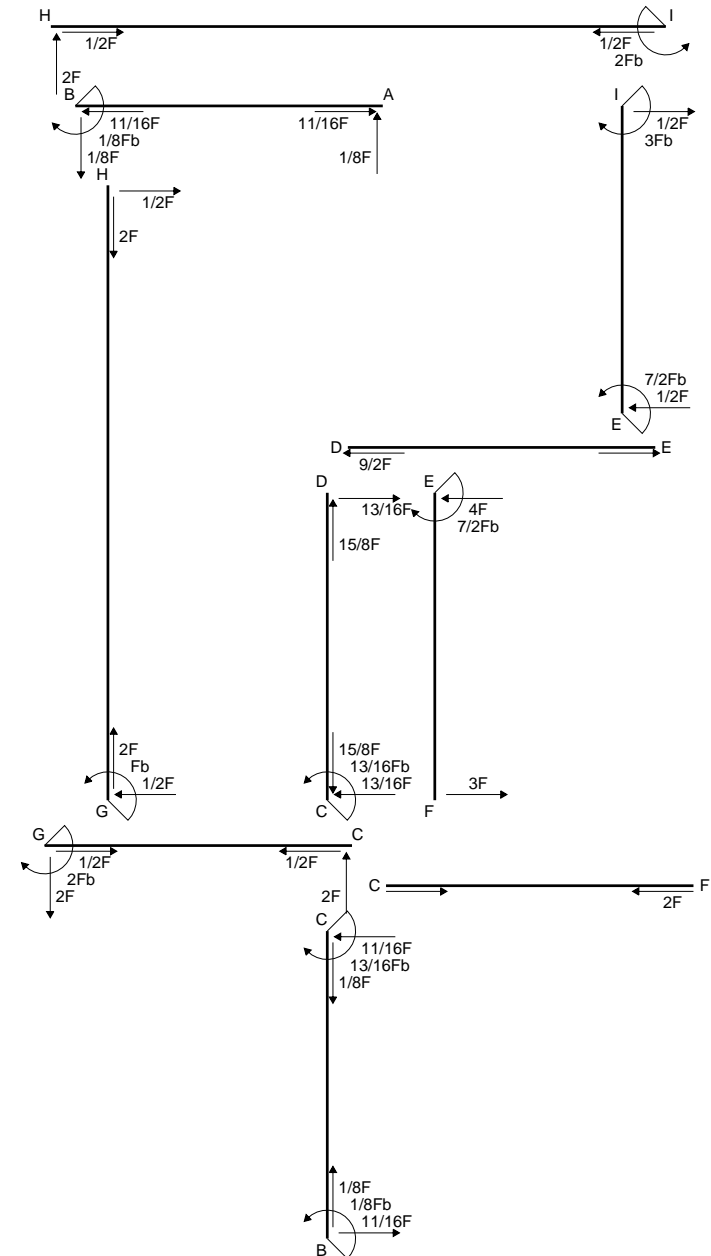
$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

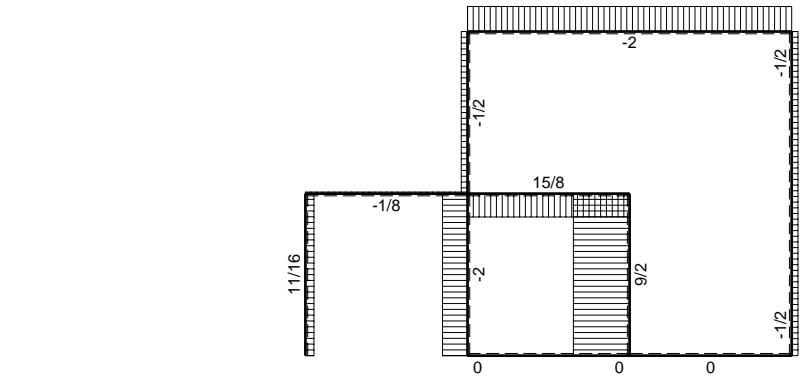




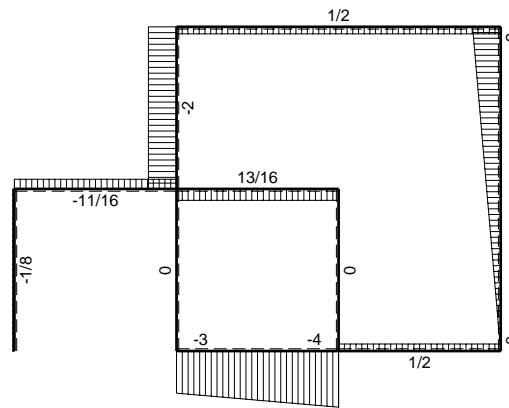
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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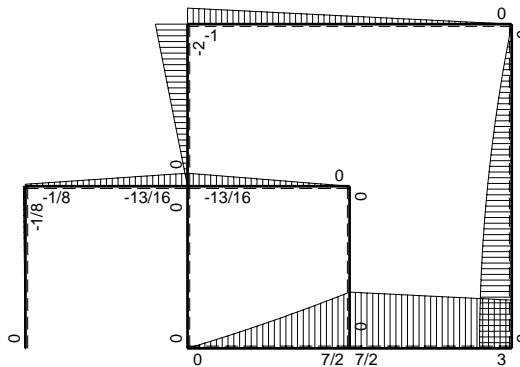




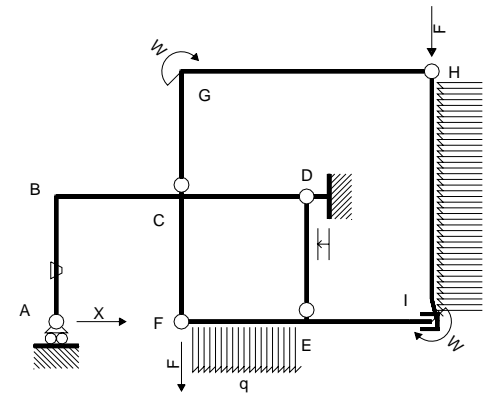
← (+) → F



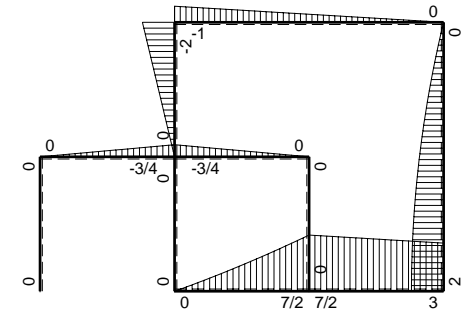
↑ (+) ↓ F



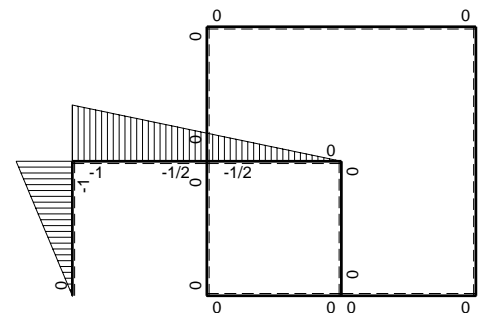
⌚ (+) ⌚ F<sub>b</sub>



Schema di calcolo iperstatico



⌚ (+) ⌚ M<sub>o</sub> flessione da carichi assegnati



⌚ (+) ⌚ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | -x                          | 0                   | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$           |           |
| BA b  | b-x                         | 0                   | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | -b+1/2x                     | -3/4Fx              | 0        | $3/4Fbx-3/8Fx^2$         | 0                | $b^2-bx+1/4x^2$       | $(1/4+0)Fb^3/EJ$            | $7/12Xb^3/EJ$          |           |
| CB b  | 1/2b+1/2x                   | 3/4Fb-3/4Fx         | 0        | $3/8Fb^2-3/8Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | -1/2b+1/2x                  | -3/4Fb+3/4Fx        | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12Xb^3/EJ$          |           |
| DC b  | 1/2x                        | 3/4Fx               | 0        | $3/8Fx^2$                | 0                | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                        |           |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                        | 0                | 0                     |                             |                        |           |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                        |           |
| CG b  | 0                           | -2Fx                | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | 2Fb-2Fx             | 0        | 0                        | 0                | 0                     |                             |                        |           |
| GH 2b | 0                           | -Fb+1/2Fx           | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | 1/2Fx               | 0        | 0                        | 0                | 0                     |                             |                        |           |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                        | 0                | 0                     |                             |                        |           |
| IE b  | 0                           | 3Fb+1/2Fx           | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                        | 0                | 0                     |                             |                        |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                  |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                     |          |                          |                  |                       |                             | $-1/8Fb^3/EJ$          | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                          |                  |                       |                             | 1/8F                   |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

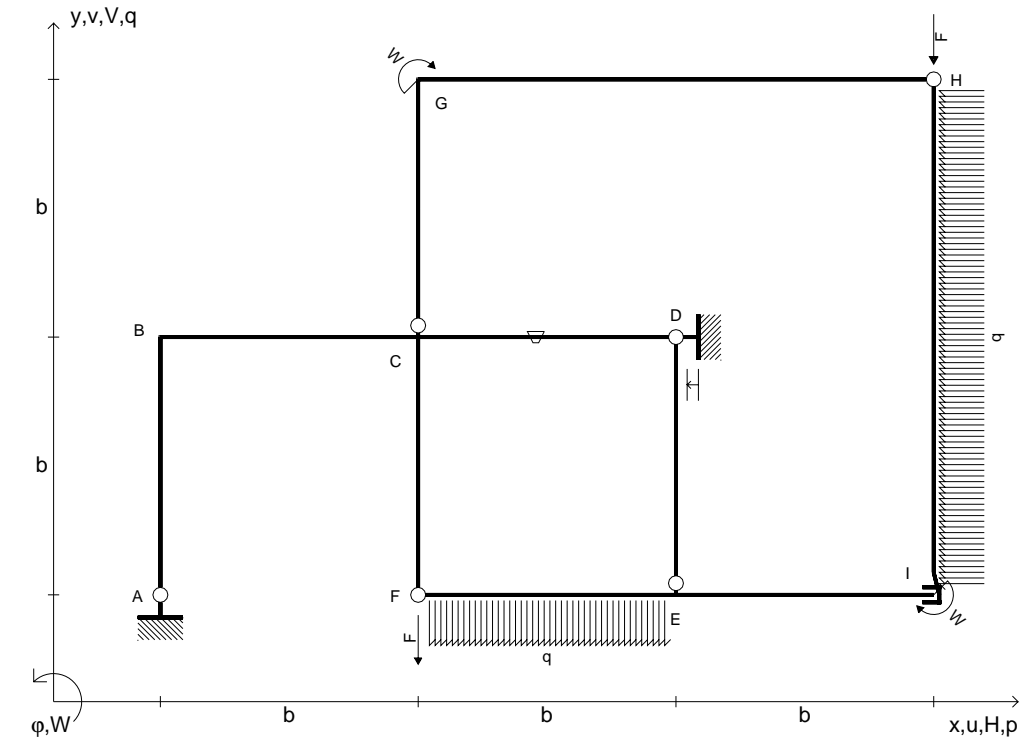
$$= (3/8 b - 1/8 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

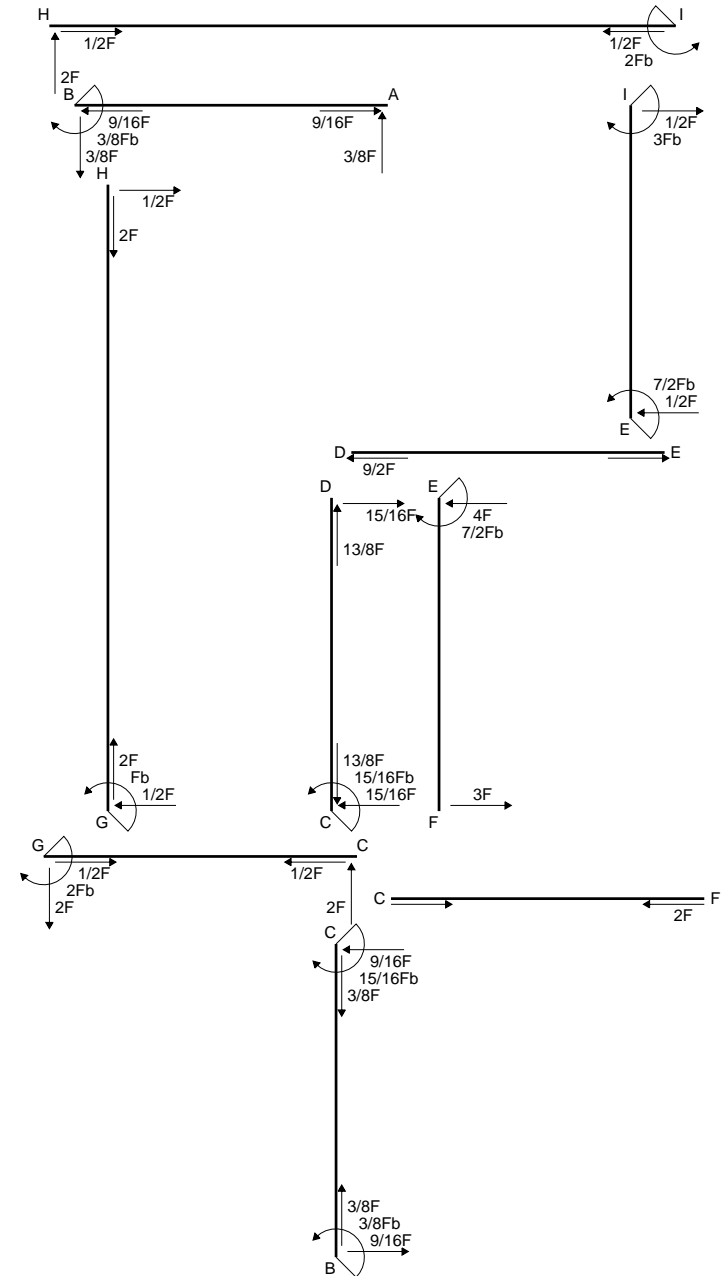
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

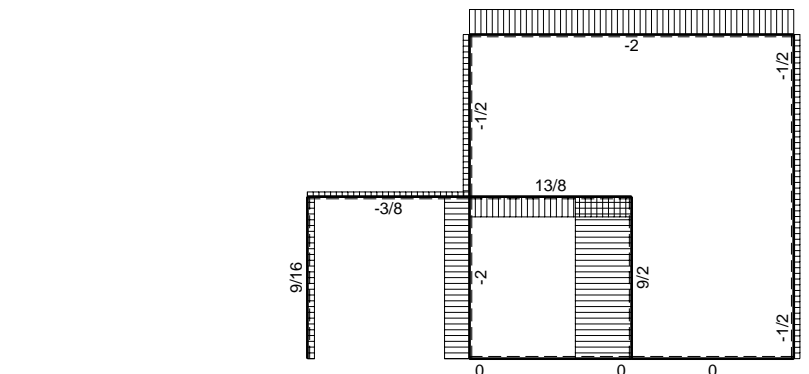
$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$



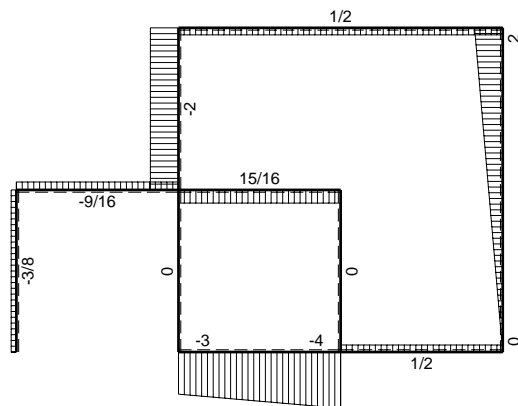
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $P_{HI} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=W_{BA}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

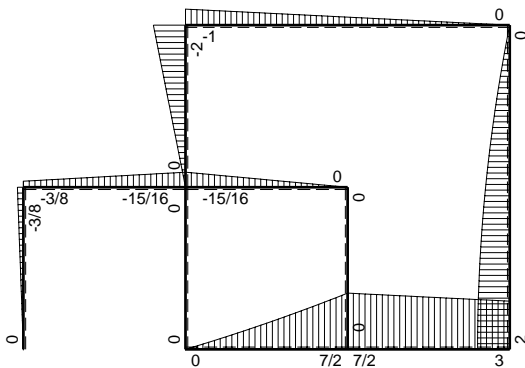




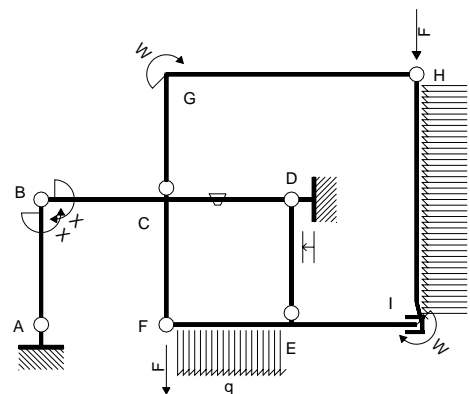
← (+) → F



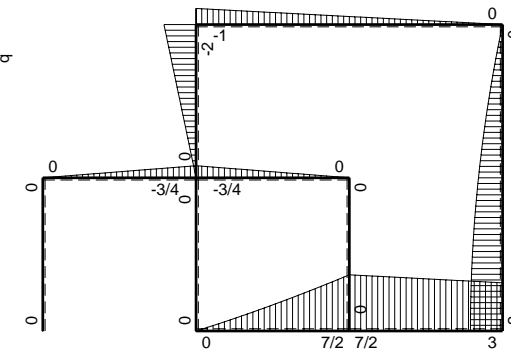
↑ (+) ↓ F



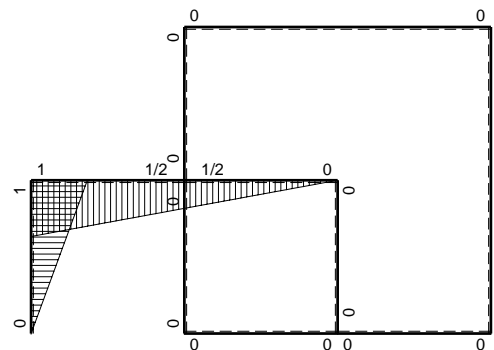
⤵ (+) ⤴ F<sub>b</sub>



Schema di calcolo iperstatico



⤵ (+) ⤴ M<sub>0</sub> flessione da carichi assegnati



⤵ (+) ⤴ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BA}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$         | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|----------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $x/b$                       | 0                   | 0        | 0                        | 0                    | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $-1+x/b$                    | 0                   | 0        | 0                        | 0                    | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $1-1/2x/b$                  | $-3/4Fx$            | 0        | $-3/4Fx+3/8Fx^2/b$       | 0                    | $1-x/b+1/4x^2/b^2$      | $(-1/4+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $-1/2-1/2x/b$               | $3/4Fb-3/4Fx$       | 0        | $-3/8Fb+3/8Fx^2/b$       | 0                    | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $1/2-1/2x/b$                | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $-3/8Fb+3/4Fx-3/8Fx^2/b$ | $-1/2Fb/EJ+1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(-1/8-1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $-1/2x/b$                   | $3/4Fx$             | $Fb/EJ$  | $-3/8Fx^2/b$             | $-1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                    | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                    | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                        | 0                    | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                        | 0                    | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                    | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                    | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$              | 0        | 0                        | 0                    | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                        | 0                    | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                        | 0                    | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                        | 0                    | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                        | 0                    | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                        | 0                    | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                        | 0                    | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                        | 0                    | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                      |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                          |                      |                         | $3/8Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BA}$      |                     |          |                          |                      |                         | $-3/8Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (-3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [-3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-3/8 b + 1/8 b) Fb 1/EJ = -1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (-3/8 + 3/8 x^2/b^2) Fb 1/EJ dx = [-3/8 x + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-3/8 b + 1/8 b) Fb 1/EJ = -1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (-3/8 + 3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 + 1/2 x/b) \theta dx$$

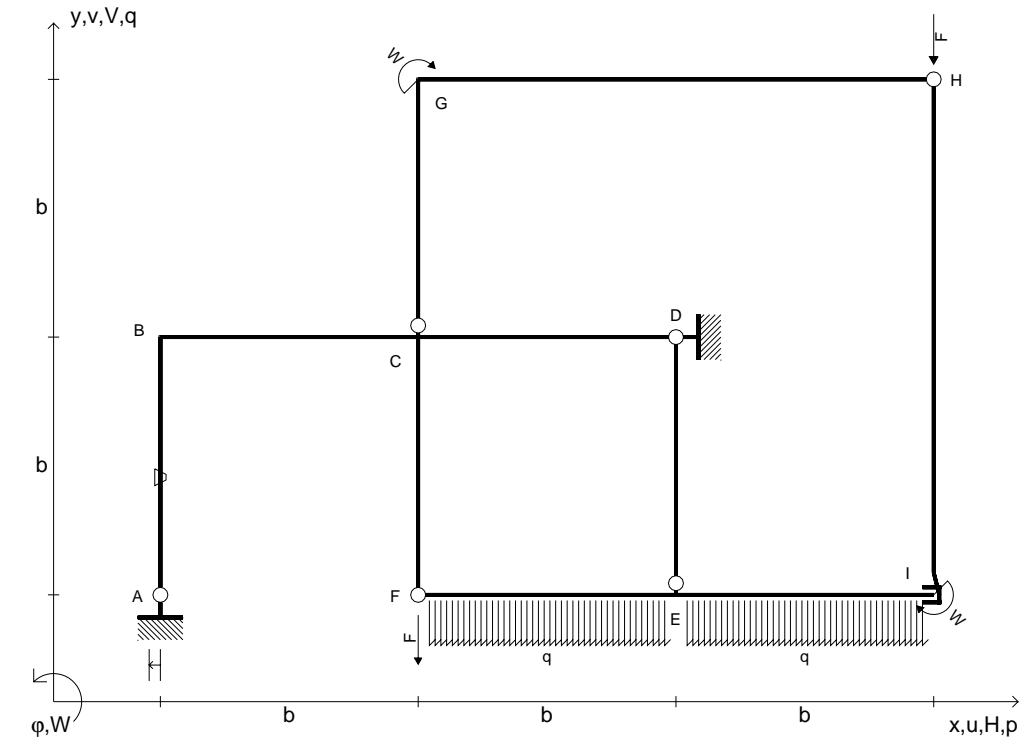
$$= [-3/8 x + 3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x + 1/4 x^2/b]_0^b \theta$$

$$= (-3/8 b + 3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b + 1/4 b) \theta = -3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (-3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 x/b) \theta dx = [-1/8 x^3/b^2]_0^b Fb 1/EJ + [1/4 x^2/b]_0^b \theta$$

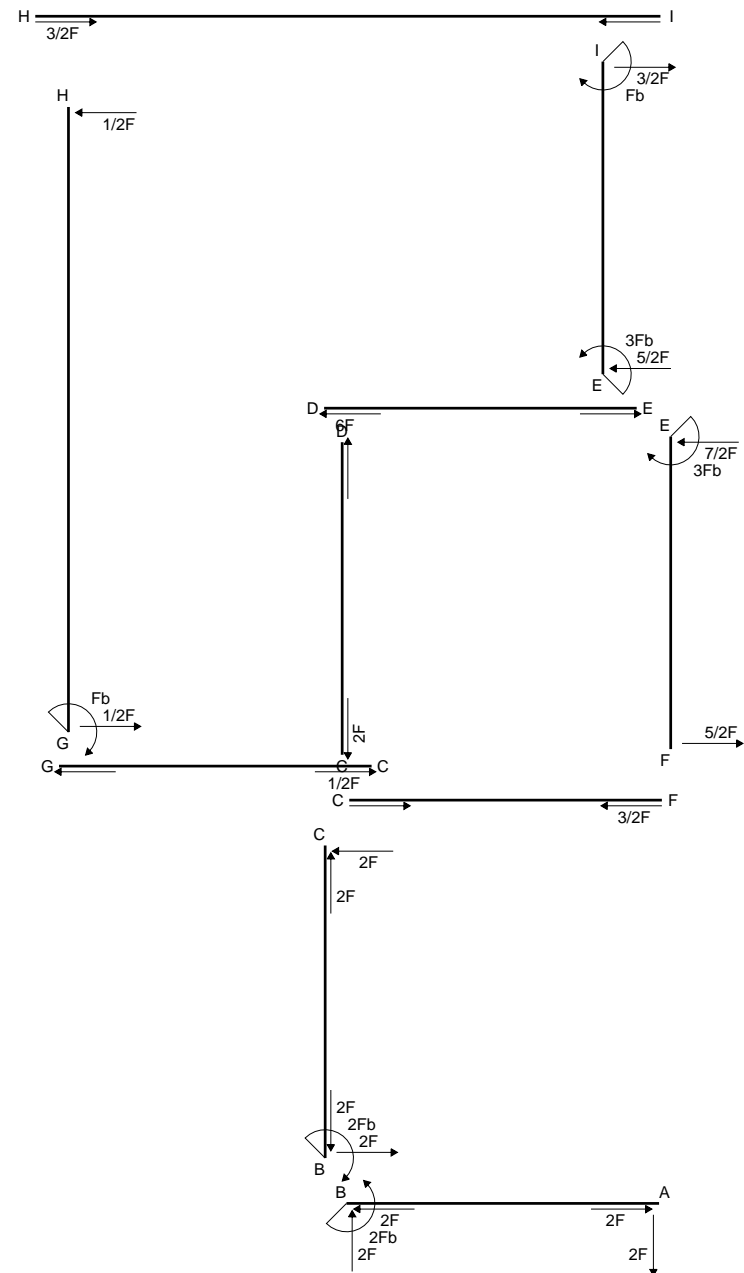
$$= (-1/8 b) Fb 1/EJ + (1/4 b) \theta = -3/8 Fb^2/EJ$$

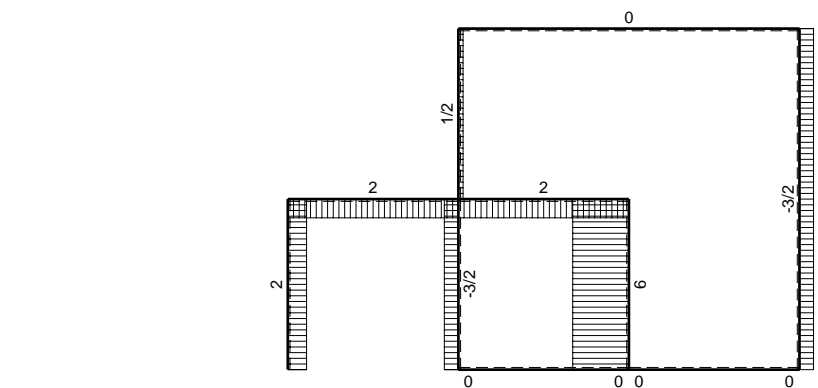




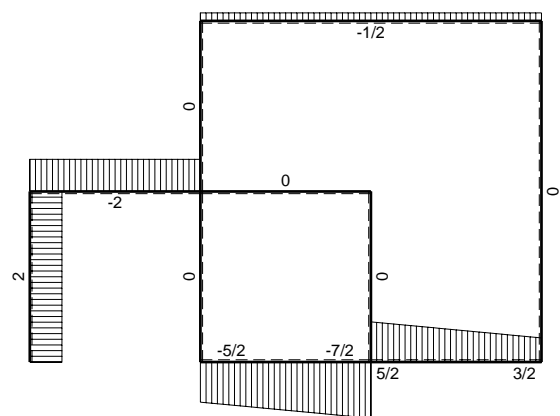
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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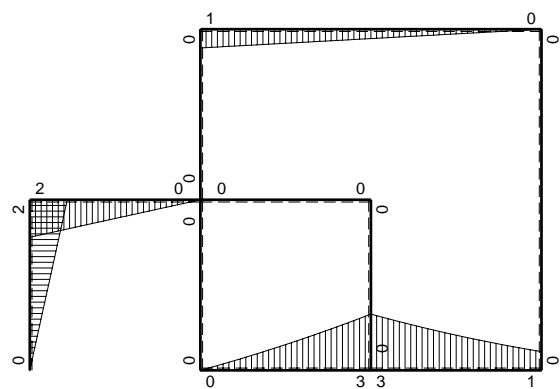




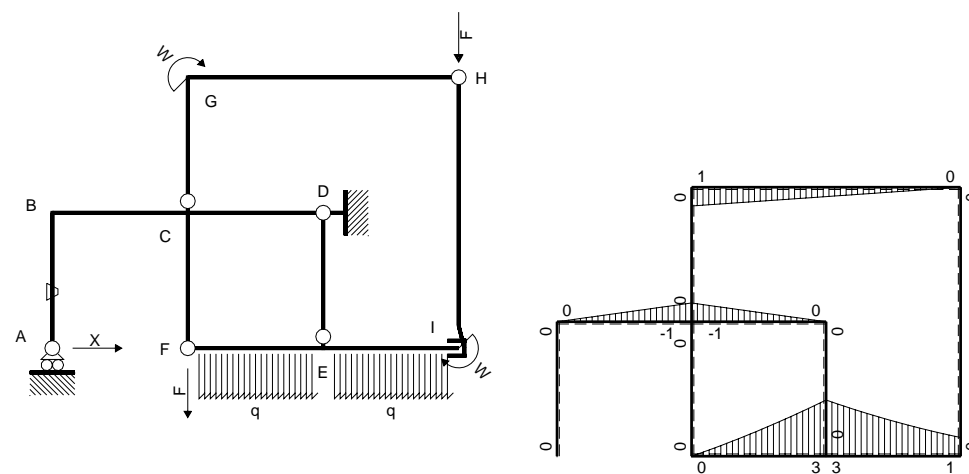
← (+) → F



↑ (+) ↓ F

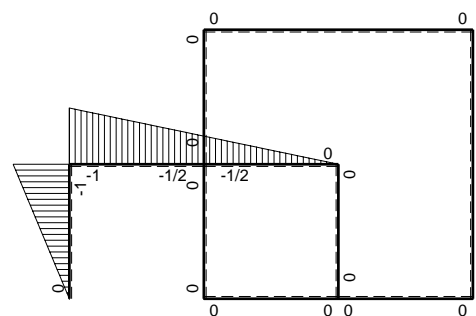


⤵ (+) ⤴ F<sub>b</sub>



Schema di calcolo iperstatico

⤵ (+) ⤴ M<sub>o</sub> flessione da carichi assegnati



⤵ (+) ⤴ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H<sub>A</sub>

| →     | M <sub>x</sub> (x)                             | M <sub>o</sub> (x)            | θ      | M <sub>x</sub> M <sub>o</sub>              | M <sub>x</sub> θ           | M <sub>x</sub> M <sub>x</sub>              | ∫M <sub>x</sub> (M <sub>o</sub> /EJ+θ)dx | ∫xM <sub>x</sub> M <sub>x</sub> /EJdx |
|-------|--|-------------------------------|--------|--|----------------------------|--|--|---------------------------------------|
| AB b  | -x   | 0                             | -Fb/EJ | 0  | Fxb/EJ                     | x <sup>2</sup>                             | (0+1/2)Fb <sup>3</sup> /EJ               | 1/3Xb <sup>3</sup> /EJ                |
| BA b  | b-x  | 0                             | Fb/EJ  | 0  | Fb <sup>2</sup> /EJ-Fxb/EJ | b <sup>2</sup> -2bx+x <sup>2</sup>         |  |                                       |
| BC b  | -b+1/2x  | -Fx                           | 0      | Fbx-1/2Fx <sup>2</sup>                     | 0                          | b <sup>2</sup> -bx+1/4x <sup>2</sup>       | (1/3+0)Fb <sup>3</sup> /EJ               | 7/12Xb <sup>3</sup> /EJ               |
| CB b  | 1/2b+1/2x                                      | Fb-Fx                         | 0      | 1/2Fb <sup>2</sup> -1/2Fx <sup>2</sup>     | 0                          | 1/4b <sup>2</sup> +1/2bx+1/4x <sup>2</sup> |  |                                       |
| CD b  | -1/2b+1/2x                                     | -Fb+Fx                        | 0      | 1/2Fb <sup>2</sup> -Fbx+1/2Fx <sup>2</sup> | 0                          | 1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup> | (1/6+0)Fb <sup>3</sup> /EJ               | 1/12Xb <sup>3</sup> /EJ               |
| DC b  | 1/2x   | Fx                            | 0      | 1/2Fx <sup>2</sup>                         | 0                          | 1/4x <sup>2</sup>                          |  |                                       |
| DE b  | 0  | 0                             | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| ED b  | 0  | 0                             | 0      | 0  | 0                          | 0  |  |                                       |
| EF b  | 0  | 3Fb-7/2Fx+1/2qx <sup>2</sup>  | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| FE b  | 0  | -5/2Fx-1/2qx <sup>2</sup>     | 0      | 0  | 0                          | 0  |  |                                       |
| FC b  | 0  | 0                             | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| CF b  | 0  | 0                             | 0      | 0  | 0                          | 0  |  |                                       |
| CG b  | 0  | 0                             | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| GC b  | 0  | 0                             | 0      | 0  | 0                          | 0  |  |                                       |
| GH 2b | 0  | Fb-1/2Fx                      | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| HG 2b | 0  | -1/2Fx                        | 0      | 0  | 0                          | 0  |  |                                       |
| HI 2b | 0  | 0                             | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| IH 2b | 0  | 0                             | 0      | 0  | 0                          | 0  |  |                                       |
| IE b  | 0  | Fb+3/2Fx+1/2qx <sup>2</sup>   | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| EI b  | 0  | -3Fb+5/2Fx-1/2qx <sup>2</sup> | 0      | 0  | 0                          | 0  |  |                                       |
| A     | cedimento nodo -H <sub>1A</sub> u <sub>A</sub> |                               |        |  |                            |  | Fb <sup>3</sup> /EJ                      |                                       |
|       | totali   |                               |        |  |                            |  | 2Fb <sup>3</sup> /EJ                     | Xb <sup>3</sup> /EJ                   |
|       | iperstatica X=H <sub>A</sub>                   |                               |        |  |                            |  | -2F                                      |                                       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

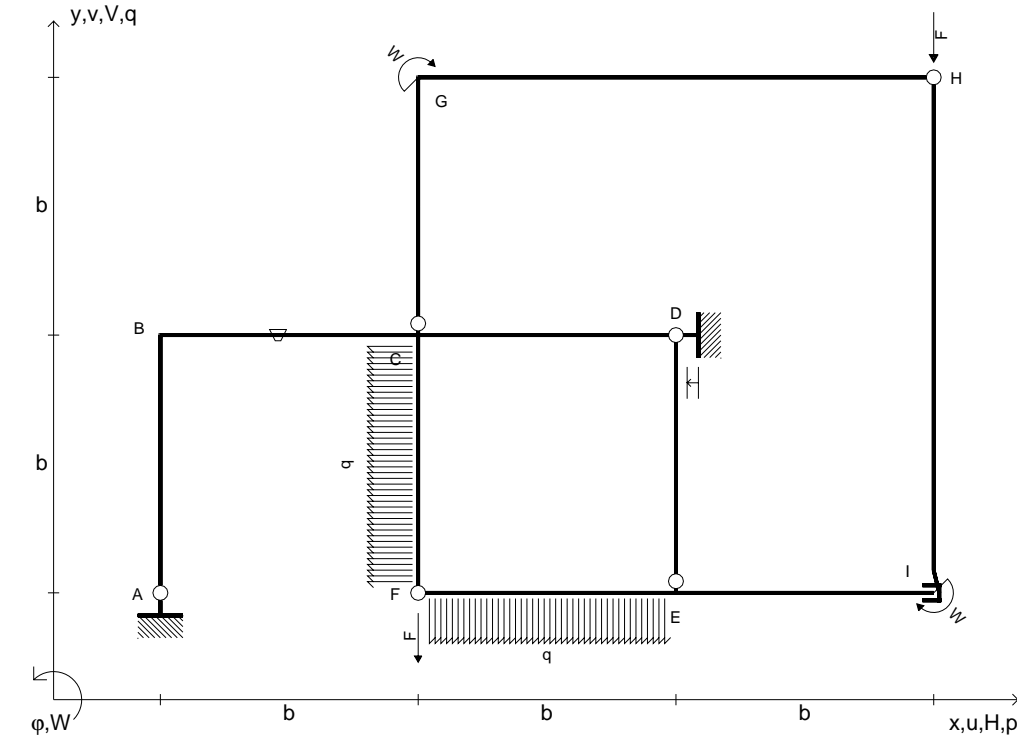
$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

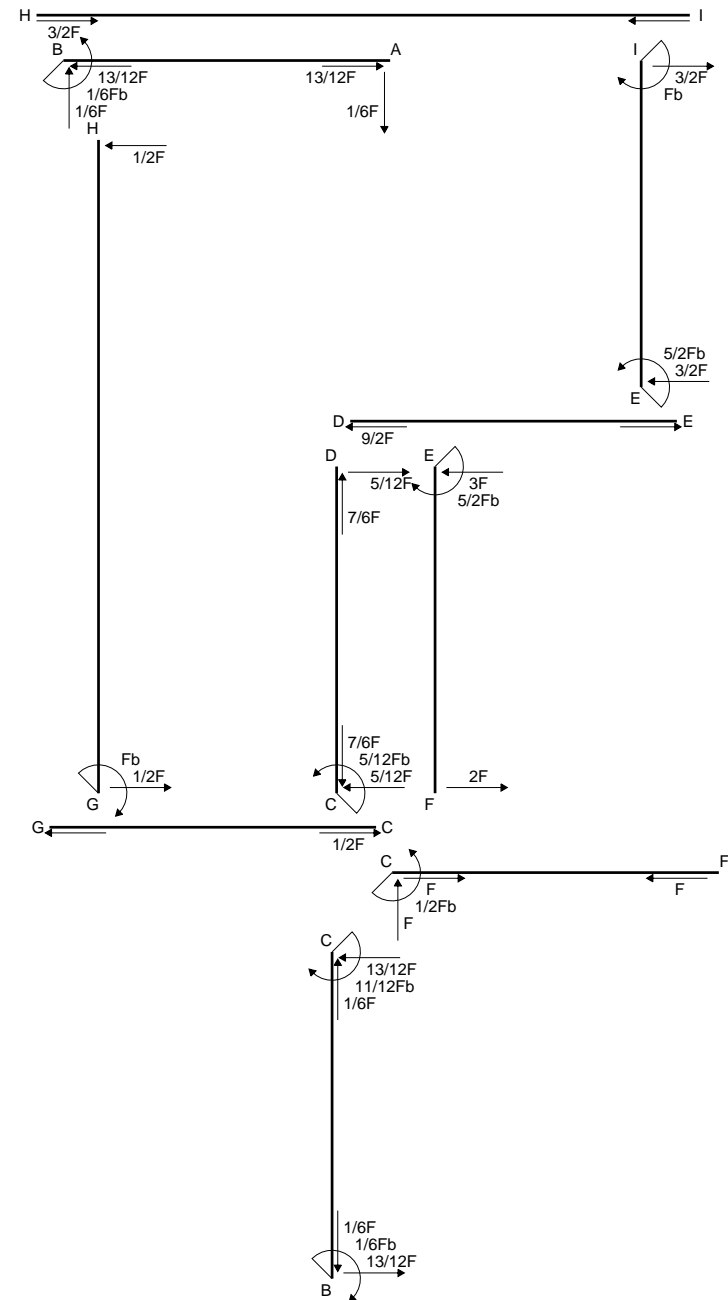
$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb^2 1/EJ dx = [1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

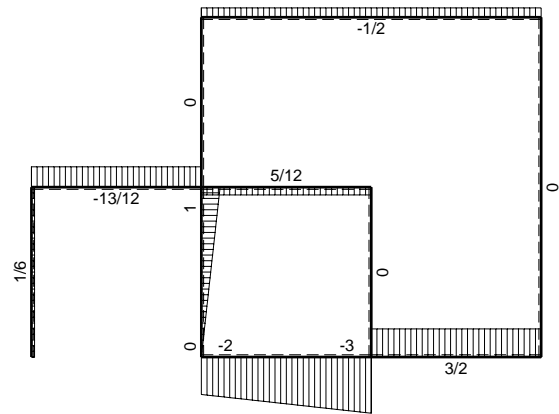
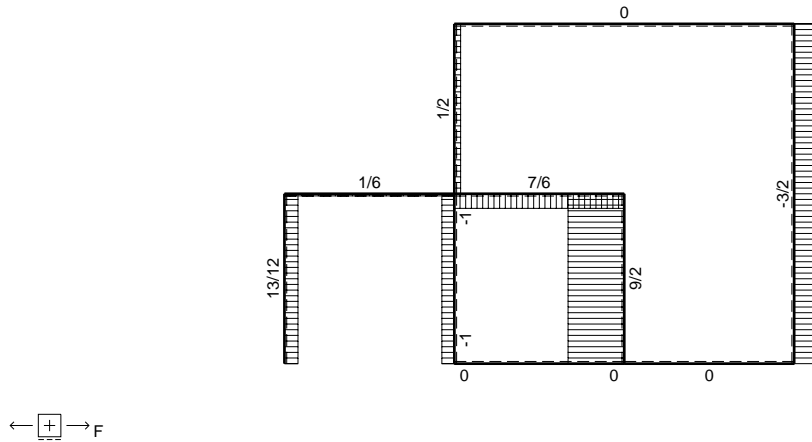
$$= (1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$



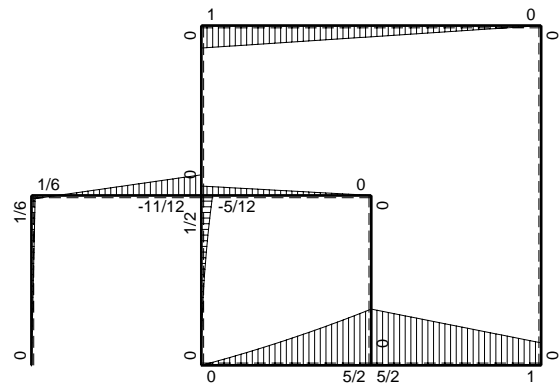
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3 F/EJ$                    | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{FC} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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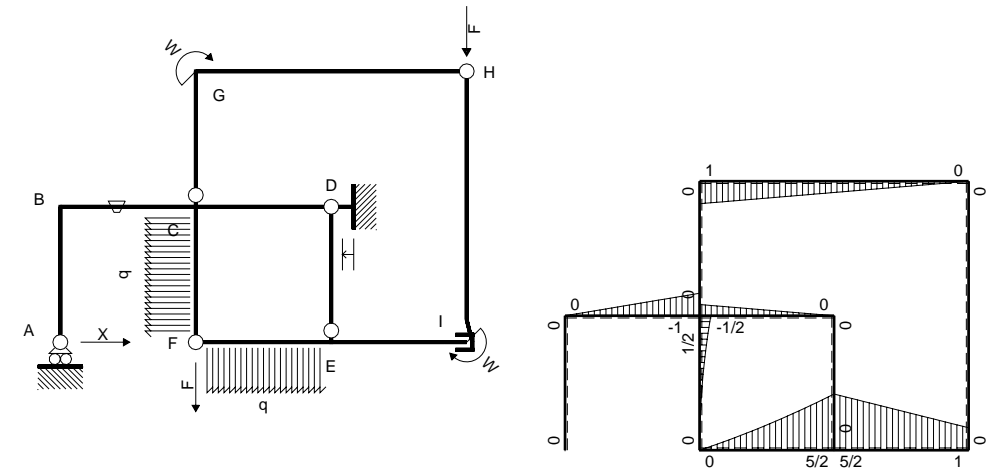




↑ (+) ↓ F

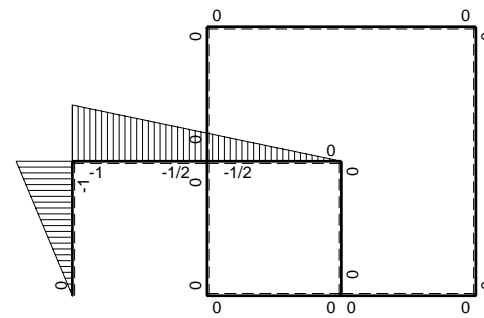


⤵ (+) ⤶ F<sub>b</sub>



Schema di calcolo iperstatico

⤵ (+) ⤶ M<sub>o</sub> flessione da carichi assegnati



⤵ (+) ⤶ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | -x                          | 0                   | 0        | 0                        | 0                      | $x^2$                 | 0+0                         | $1/3 X b^3/EJ$         |           |
| BA b  | b-x                         | 0                   | 0        | 0                        | 0                      | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | -b+1/2x                     | -Fx                 | -Fb/EJ   | $Fbx-1/2Fx^2$            | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/3+3/4)Fb^3/EJ$          | $7/12 X b^3/EJ$        |           |
| CB b  | $1/2b+1/2x$                 | Fb-Fx               | Fb/EJ    | $1/2Fb^2-1/2Fx^2$        | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb^2-1/2Fbx+1/4Fx^2$ | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+0)Fb^3/EJ$           | $1/12 X b^3/EJ$        |           |
| DC b  | $1/2x$                      | $1/2Fx$             | 0        | $1/4Fx^2$                | 0                      | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| GH 2b | 0                           | Fb-1/2Fx            | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | -1/2Fx              | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| IE b  | 0                           | Fb+3/2Fx            | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                        |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                     |          |                          |                        |                       |                             | $1/6Fb^3/EJ$           | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                          |                        |                       |                             | $-1/6F$                |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^3/EJ$$

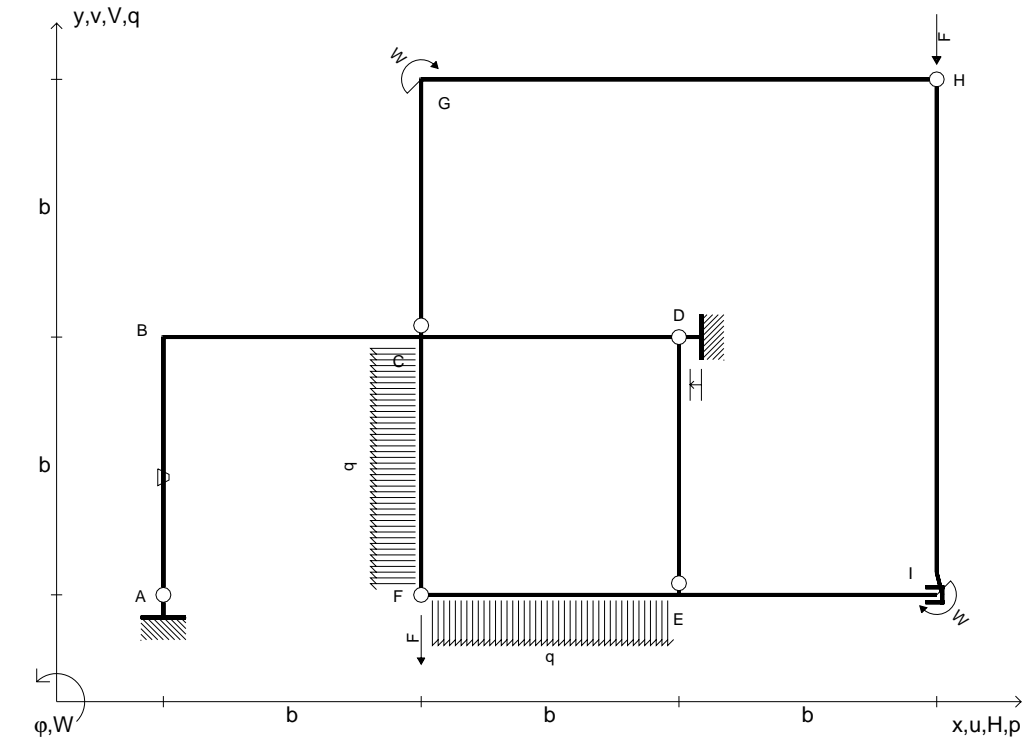
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

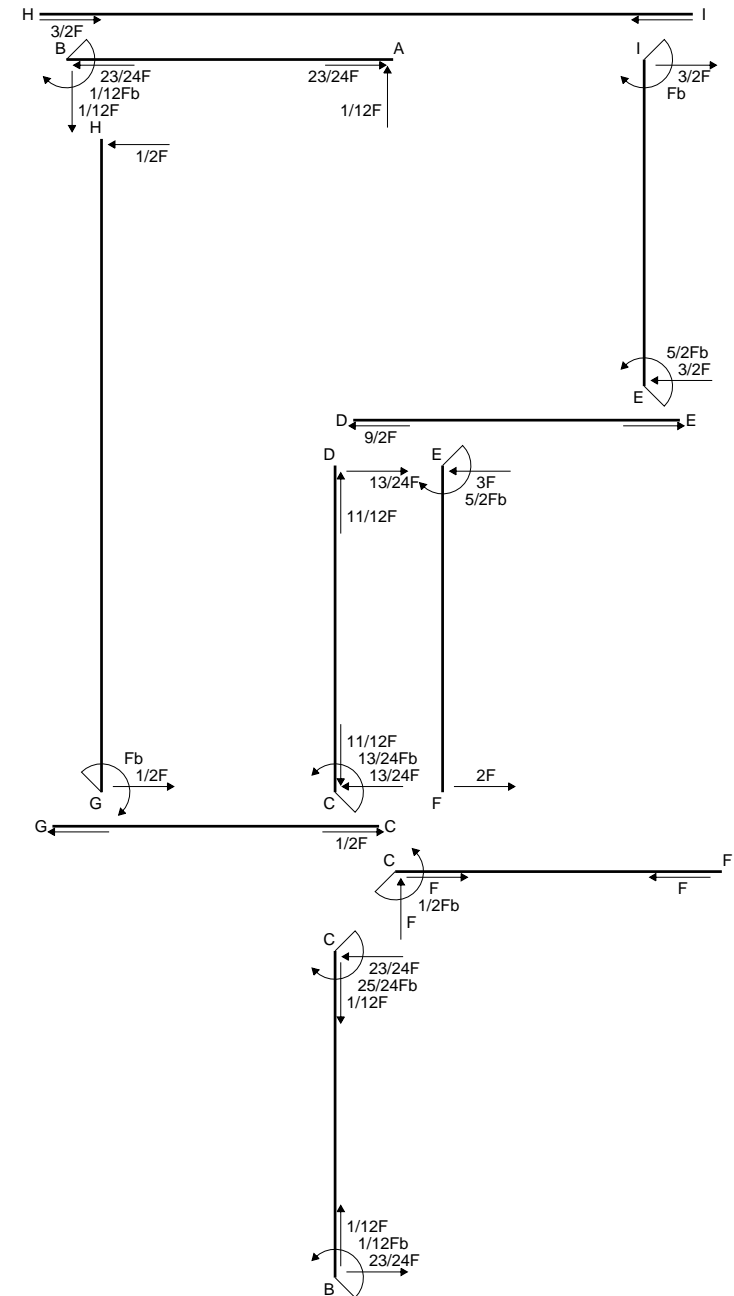
$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

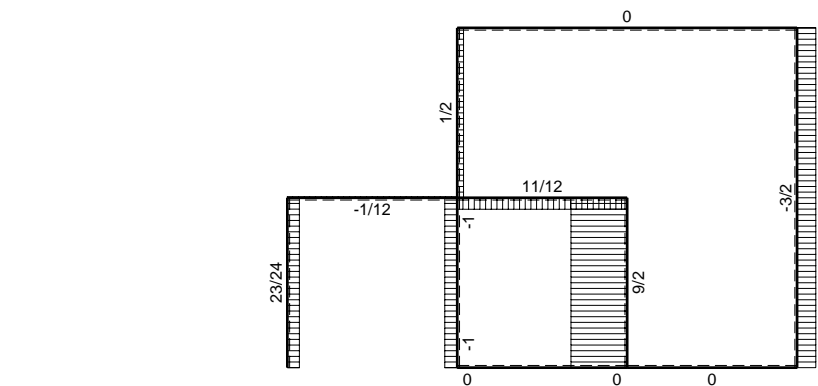




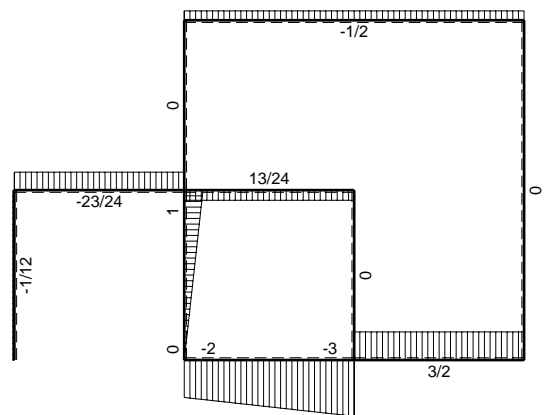
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3 F/EJ$                    | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{FC} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi  $x, y$ .  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto  $u$  imposto al nodo D.  
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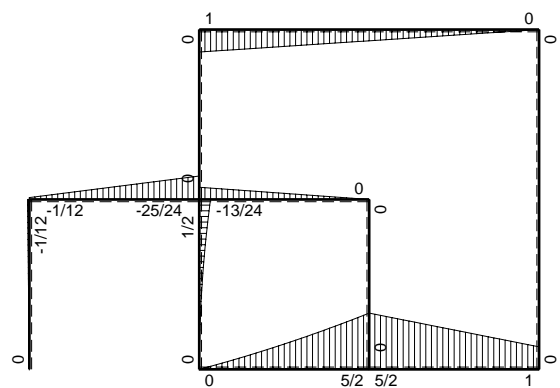




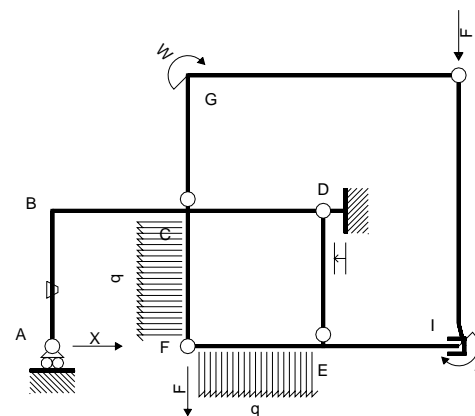
← (+) →  $F$



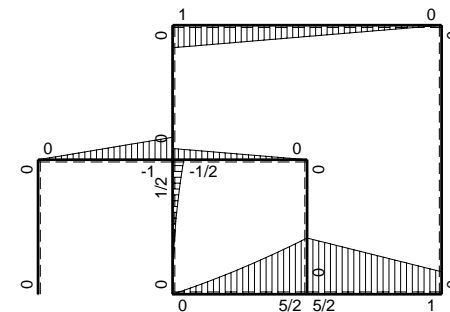
↑ (+) ↓  $F$



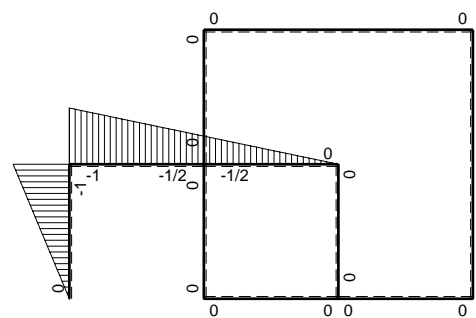
⤵ (+) ⤴  $F_b$



Schema di calcolo iperstatico



⤵ (+) ⤴  $M_0$  flessione da carichi assegnati



⤵ (+) ⤴  $M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|--------------------------|-----------|
| AB b  | -x                          | 0                   | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$             |           |
| BA b  | b-x                         | 0                   | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                          |           |
| BC b  | -b+1/2x                     | -Fx                 | 0        | $Fbx-1/2Fx^2$            | 0                | $b^2-bx+1/4x^2$       | $(1/3+0)Fb^3/EJ$            | $7/12Xb^3/EJ$            |           |
| CB b  | $1/2b+1/2x$                 | Fb-Fx               | 0        | $1/2Fb^2-1/2Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                          |           |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb^2-1/2Fbx+1/4Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+0)Fb^3/EJ$           | $1/12Xb^3/EJ$            |           |
| DC b  | $1/2x$                      | $1/2Fx$             | 0        | $1/4Fx^2$                | 0                | $1/4x^2$              |                             |                          |           |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                          |           |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0                | 0                     |                             |                          |           |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                          |           |
| GH 2b | 0                           | Fb-1/2Fx            | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| HG 2b | 0                           | -1/2Fx              | 0        | 0                        | 0                | 0                     |                             |                          |           |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                          |           |
| IE b  | 0                           | Fb+3/2Fx            | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                  |                       |                             | $-Fb^3/EJ$               |           |
|       | totali                      |                     |          |                          |                  |                       |                             | $-1/12Fb^3/EJ$           | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                          |                  |                       |                             | $1/12F$                  |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

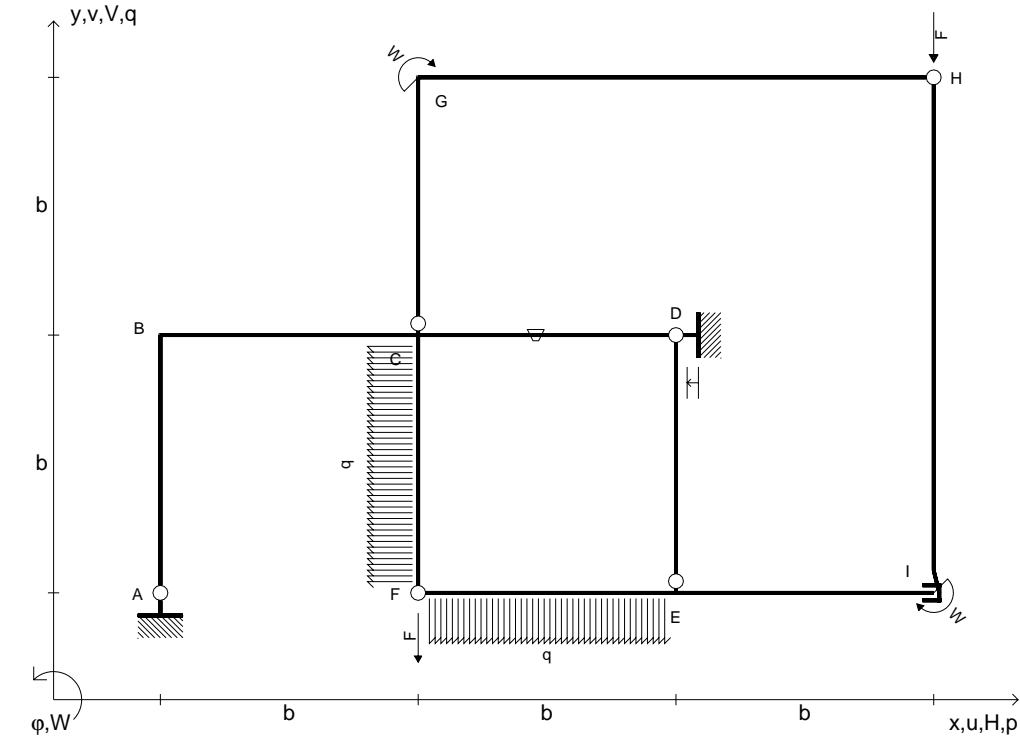
$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

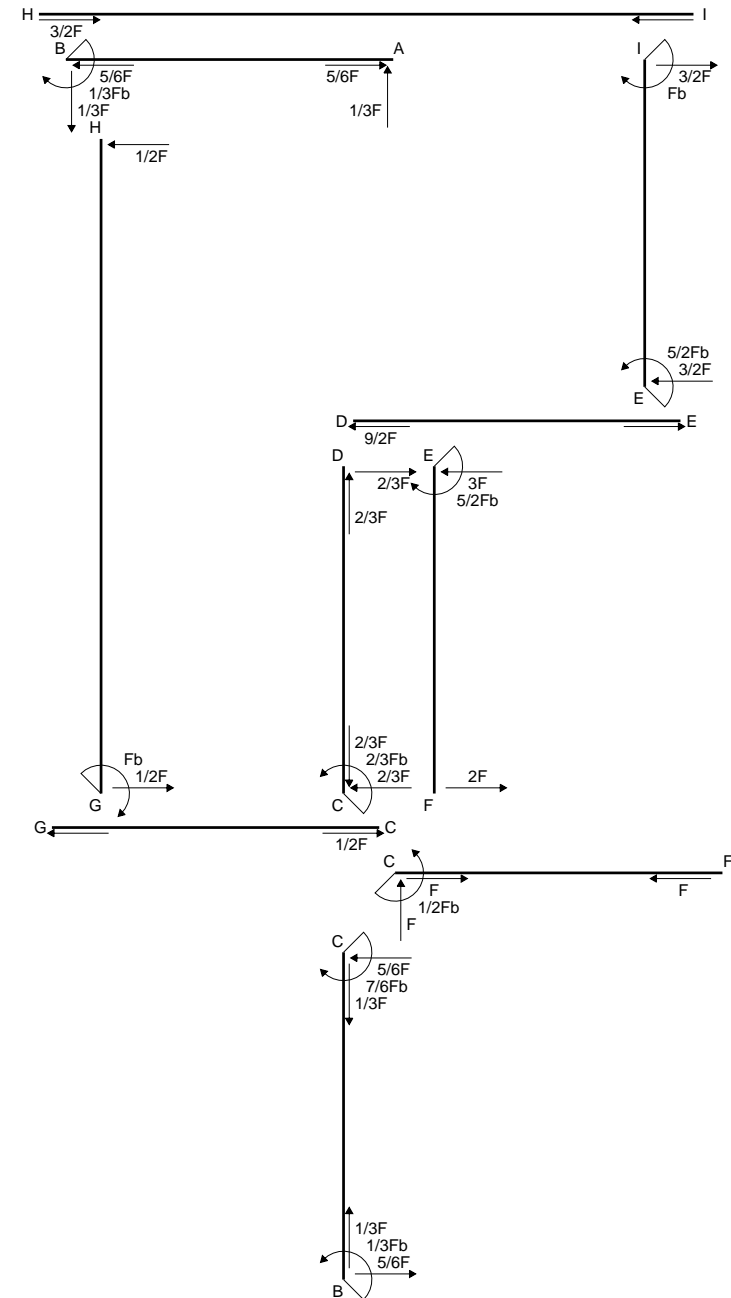
$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

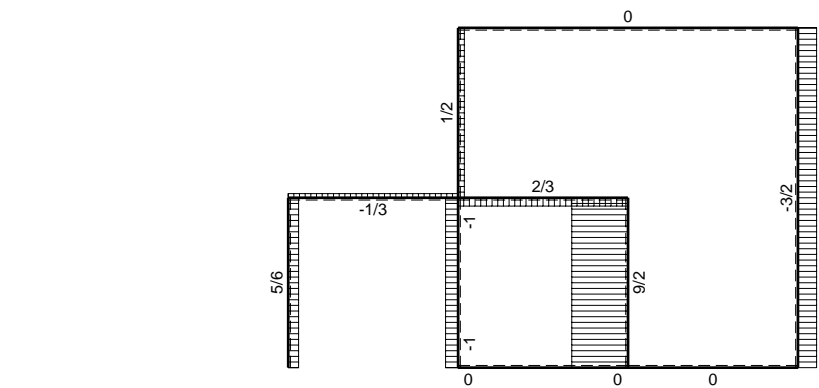
$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$



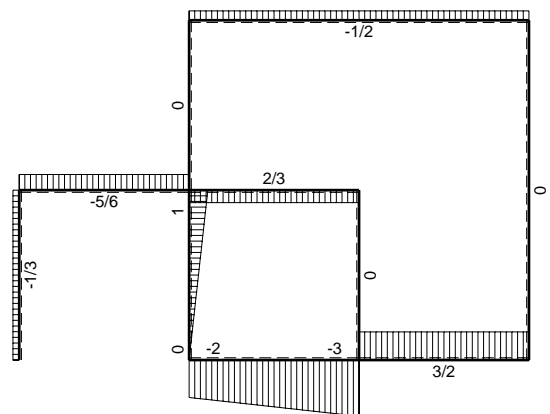
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $P_{FC} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

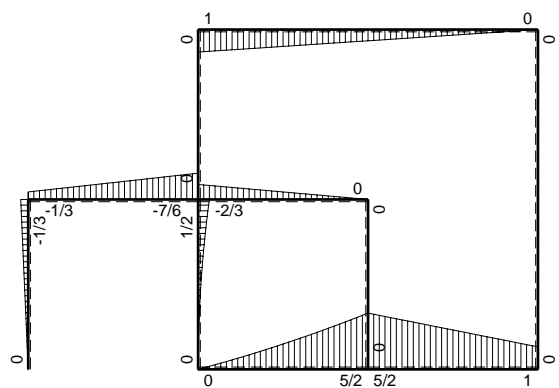




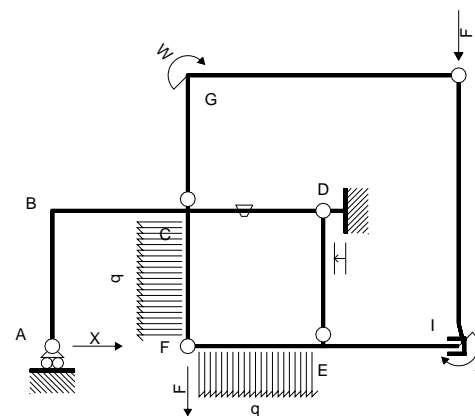
$\left[ \begin{matrix} + \\ - \end{matrix} \right] F$



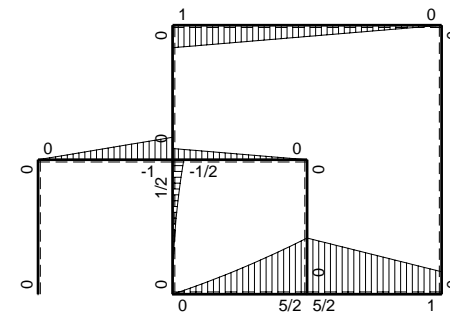
$\left[ \begin{matrix} + \\ - \end{matrix} \right] F$



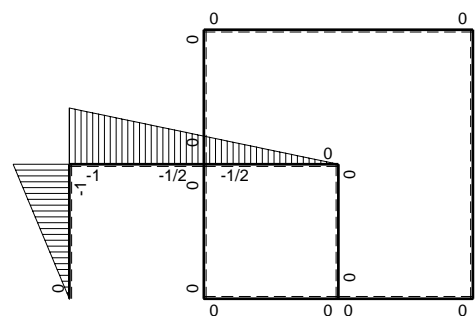
$\left[ \begin{matrix} + \\ - \end{matrix} \right] F_b$



Schema di calcolo iperstatico



$\left[ \begin{matrix} + \\ - \end{matrix} \right] M_o$  flessione da carichi assegnati



$\left[ \begin{matrix} + \\ - \end{matrix} \right] M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | -x                          | 0                   | 0        | 0                        | 0                      | $x^2$                 | 0+0                         | $1/3 X b^3/EJ$         |           |
| BA b  | b-x                         | 0                   | 0        | 0                        | 0                      | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | -b+1/2x                     | -Fx                 | 0        | $Fbx-1/2Fx^2$            | 0                      | $b^2-bx+1/4x^2$       | $(1/3+0)Fb^3/EJ$            | $7/12 X b^3/EJ$        |           |
| CB b  | $1/2b+1/2x$                 | $Fb-Fx$             | 0        | $1/2Fb^2-1/2Fx^2$        | 0                      | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$      | $-Fb/EJ$ | $1/4Fb^2-1/2Fbx+1/4Fx^2$ | $1/2Fb^2/EJ-1/2Fxb/EJ$ | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+1/4)Fb^3/EJ$         | $1/12 X b^3/EJ$        |           |
| DC b  | $1/2x$                      | $1/2Fx$             | $Fb/EJ$  | $1/4Fx^2$                | $1/2Fxb/EJ$            | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                        |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                     |          |                          |                        |                       |                             | $-1/3Fb^3/EJ$          | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                          |                        |                       |                             | $1/3F$                 |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

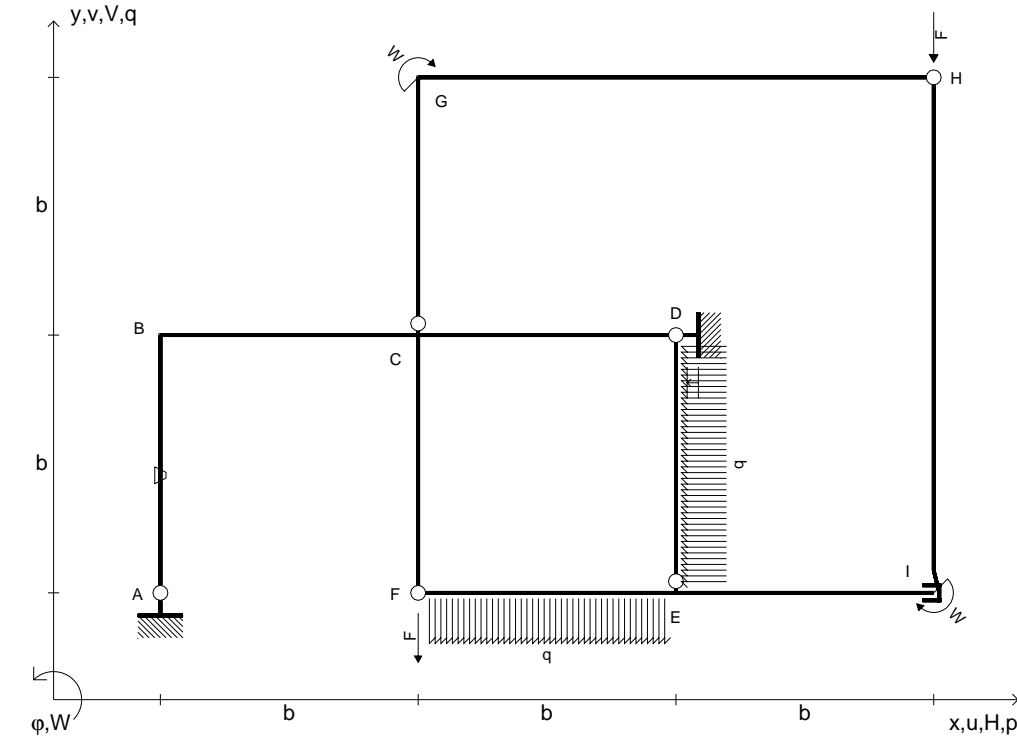
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

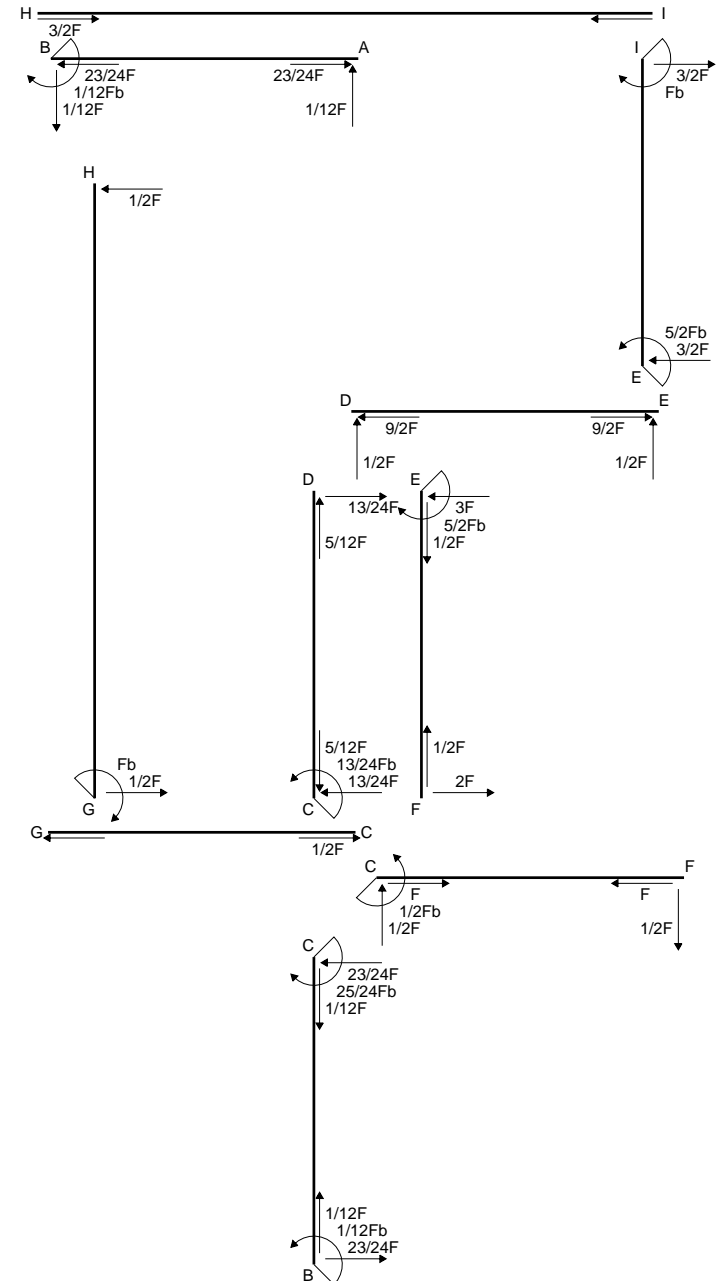
$$= (1/12 b) Fb^2 1/EJ + (-1/4 b) \theta = 1/3 Fb^3/EJ$$

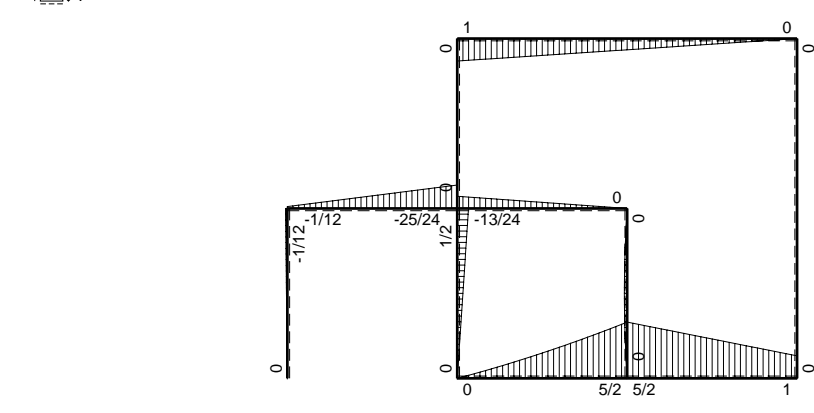
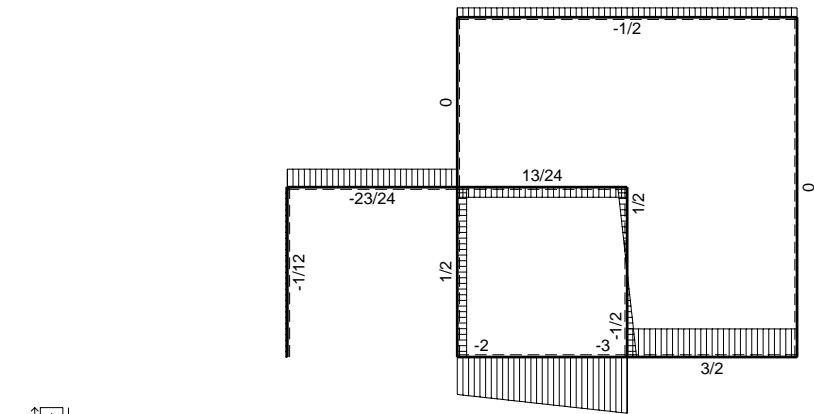
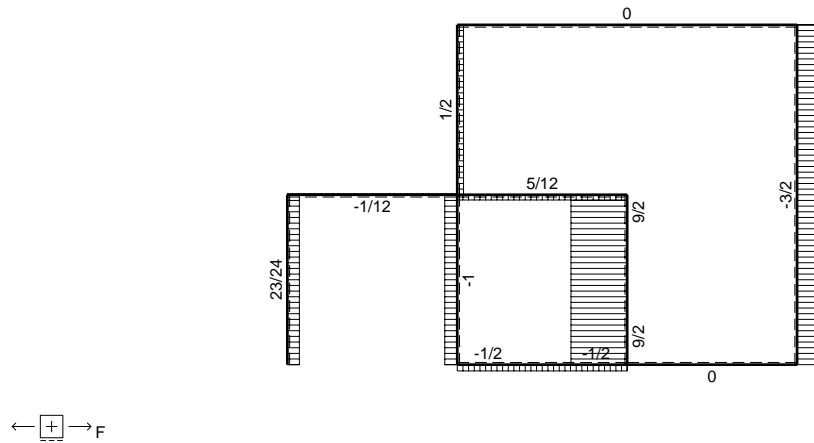




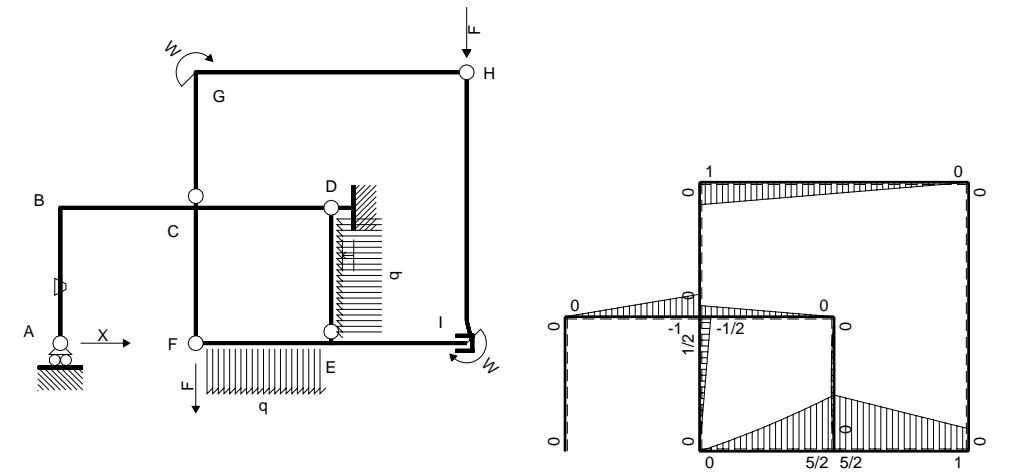
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{DE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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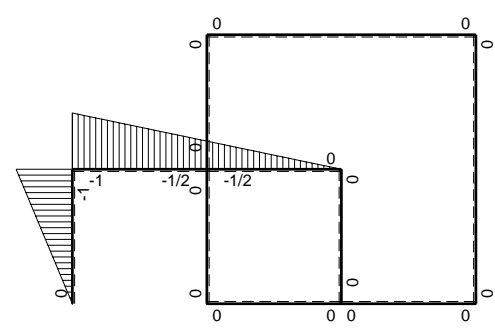


$F_b$



Schema di calcolo iperstatico

$M_o$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|--------------------------|-----------|
| AB b  | -x                          | 0                   | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$             |           |
| BA b  | b-x                         | 0                   | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                          |           |
| BC b  | -b+1/2x                     | -Fx                 | 0        | $Fbx-1/2Fx^2$            | 0                | $b^2-bx+1/4x^2$       | $(1/3+0)Fb^3/EJ$            | $7/12Xb^3/EJ$            |           |
| CB b  | $1/2b+1/2x$                 | Fb-Fx               | 0        | $1/2Fb^2-1/2Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                          |           |
| CD b  | $-1/2b+1/2x$                | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb^2-1/2Fbx+1/4Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/12+0)Fb^3/EJ$           | $1/12Xb^3/EJ$            |           |
| DC b  | $1/2x$                      | $1/2Fx$             | 0        | $1/4Fx^2$                | 0                | $1/4x^2$              |                             |                          |           |
| DE b  | 0                           | $1/2Fx-1/2qx^2$     | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                        | 0                | 0                     |                             |                          |           |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| FC b  | 0                           | $1/2Fx$             | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| CF b  | 0                           | $-1/2Fb+1/2Fx$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                          |           |
| GH 2b | 0                           | Fb-1/2Fx            | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| HG 2b | 0                           | -1/2Fx              | 0        | 0                        | 0                | 0                     |                             |                          |           |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                | 0                     |                             |                          |           |
| IE b  | 0                           | Fb+3/2Fx            | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                  |                       |                             | $-Fb^3/EJ$               |           |
|       | totali                      |                     |          |                          |                  |                       |                             | $-1/12Fb^3/EJ$           | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                          |                  |                       |                             | $1/12F$                  |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

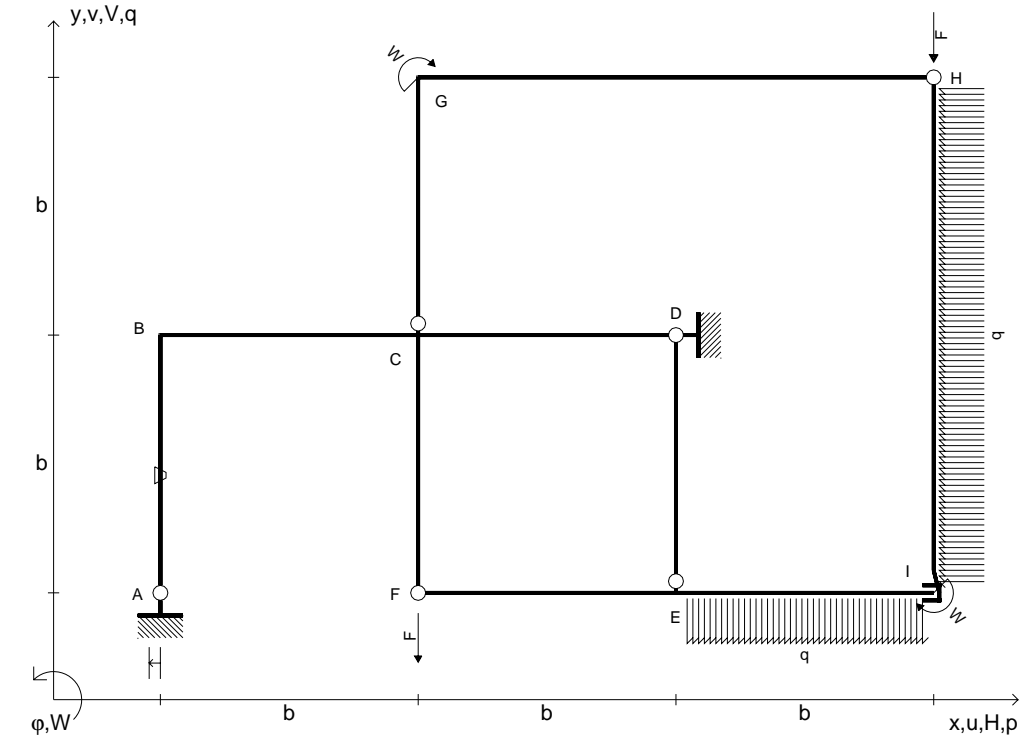
$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$

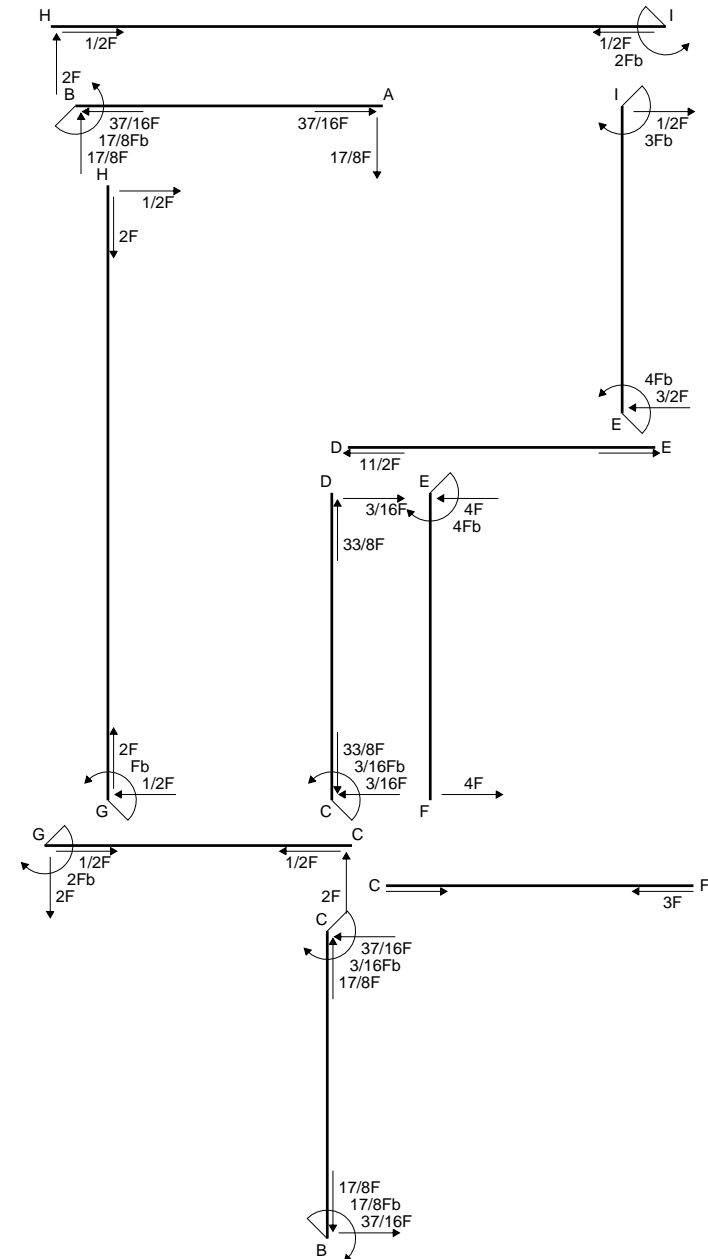
$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb^2 1/EJ dx = [1/12 x^3/b^2]_0^b Fb^2 1/EJ$$

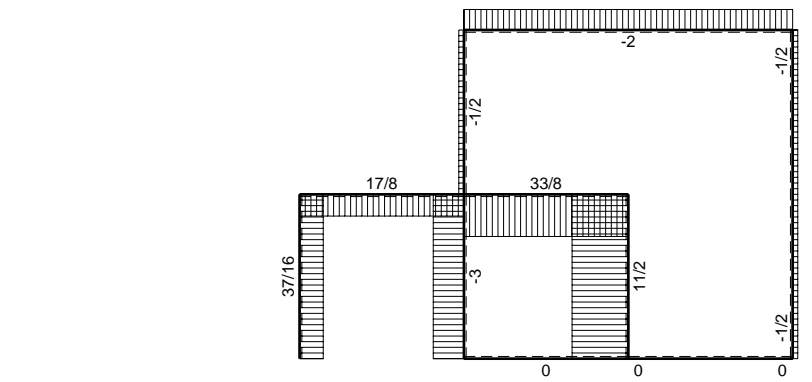
$$= (1/12 b) Fb^2 1/EJ = 1/12 Fb^3/EJ$$



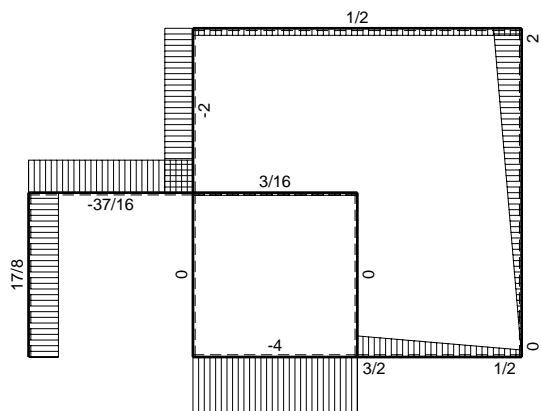
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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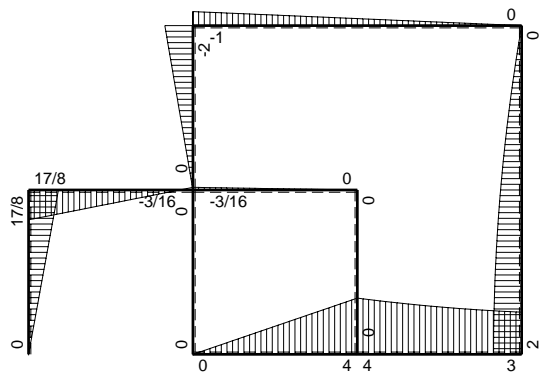




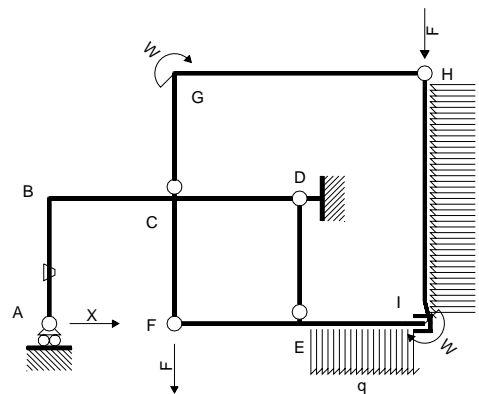
← (+) → F



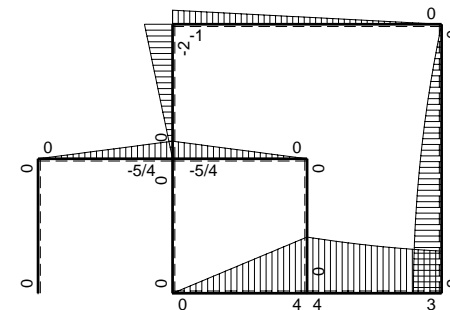
↑ (+) ↓ F



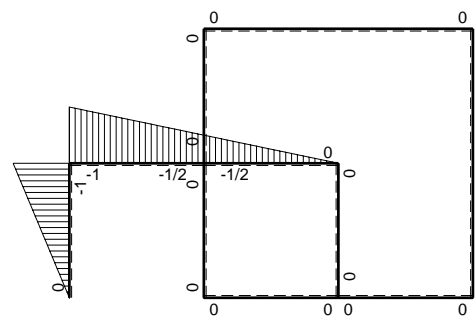
⤵ (+) ⤵ F<sub>b</sub>



Schema di calcolo iperstatico



⤵ (+) ⤵ M<sub>o</sub> flessione da carichi assegnati



⤵ (+) ⤵ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | 0                    | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$           |
| BA b  | b-x                         | 0                    | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | -5/4Fx               | 0        | $5/4Fbx-5/8Fx^2$         | 0                | $b^2-bx+1/4x^2$       | $(5/12+0)Fb^3/EJ$           | $7/12Xb^3/EJ$          |
| CB b  | 1/2b+1/2x                   | 5/4Fb-5/4Fx          | 0        | $5/8Fb^2-5/8Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | -1/2b+1/2x                  | -5/4Fb+5/4Fx         | 0        | $5/8Fb^2-5/4Fbx+5/8Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(5/24+0)Fb^3/EJ$           | $1/12Xb^3/EJ$          |
| DC b  | 1/2x                        | 5/4Fx                | 0        | $5/8Fx^2$                | 0                | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                | 0                     |                             |                        |
| EF b  | 0                           | 4Fb-4Fx              | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | -4Fx                 | 0        | 0                        | 0                | 0                     |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                | 0                     |                             |                        |
| CG b  | 0                           | -2Fx                 | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | 2Fb-2Fx              | 0        | 0                        | 0                | 0                     |                             |                        |
| GH 2b | 0                           | -Fb+1/2Fx            | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | 1/2Fx                | 0        | 0                        | 0                | 0                     |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                        | 0                | 0                     |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                        | 0                | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                        | 0                | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                          |                  |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                      |          |                          |                  |                       | $17/8Fb^3/EJ$               | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                      |          |                          |                  |                       | -17/8F                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left( \frac{x}{b} \right) \theta dx = \left[ \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left( -1 + x/b \right) \theta dx = \left[ -x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( -b + \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{5}{4} \frac{x}{b} - \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{8} \frac{x^2}{b} - \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{5}{8} b - \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{12} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{5}{8} - \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{8} x - \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{5}{8} b - \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{12} Fb^3/EJ$$

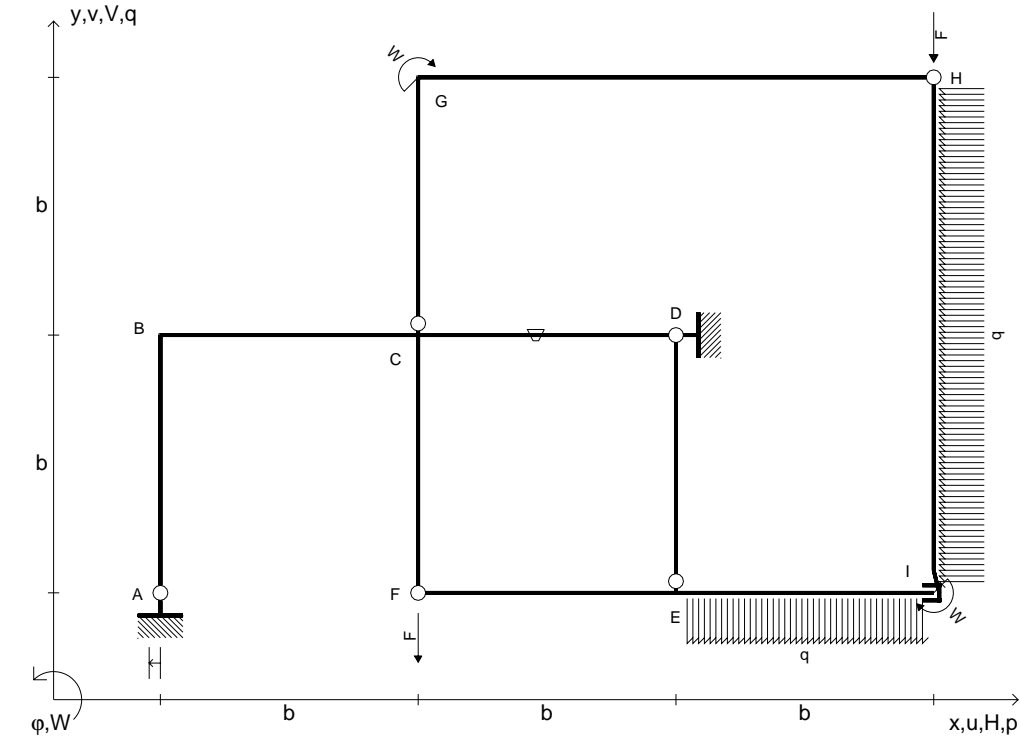
$$L_{CD}^{xo} = \int_0^b \left( \frac{5}{8} - \frac{5}{4} \frac{x}{b} + \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{8} x - \frac{5}{8} \frac{x^2}{b} + \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{5}{8} b - \frac{5}{8} b + \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{24} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

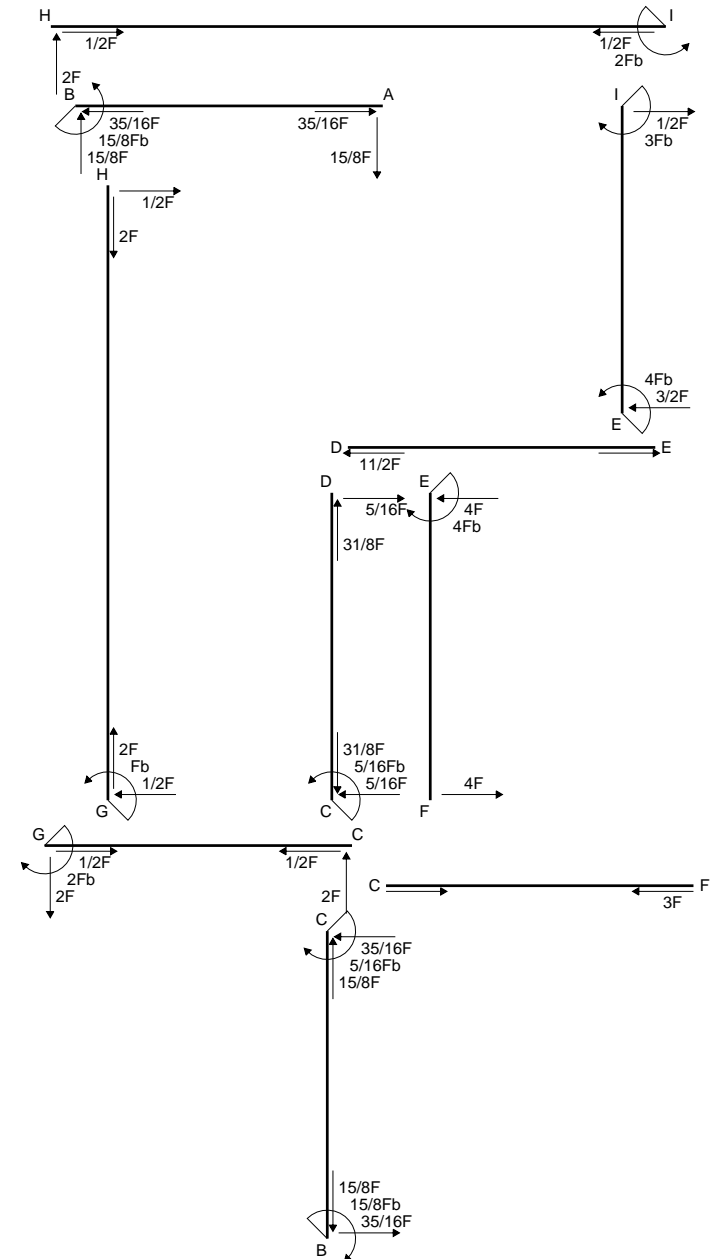
$$= \left( \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{24} Fb^3/EJ$$

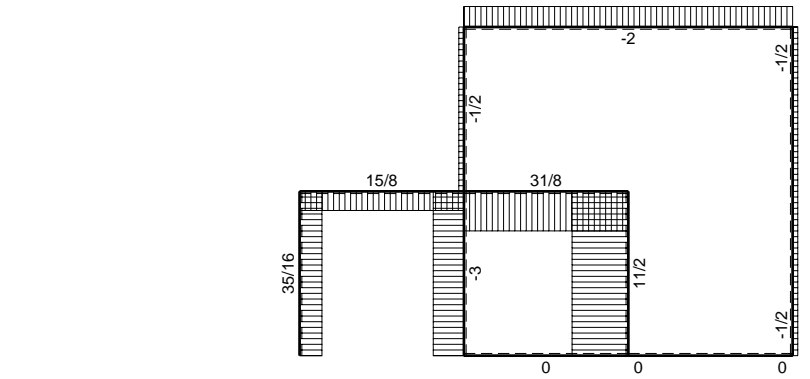




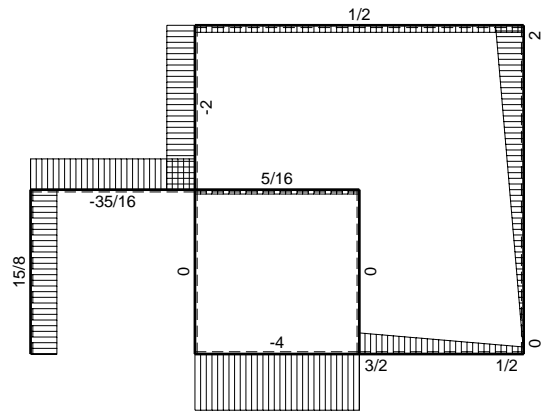
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{HI} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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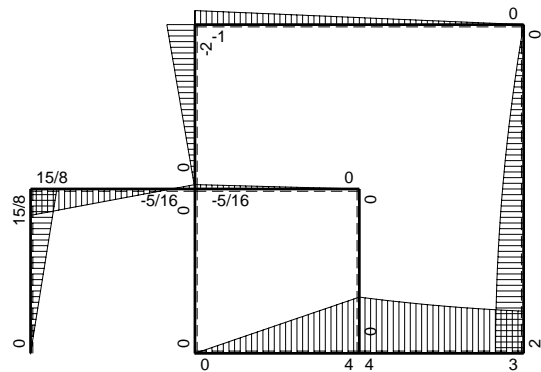




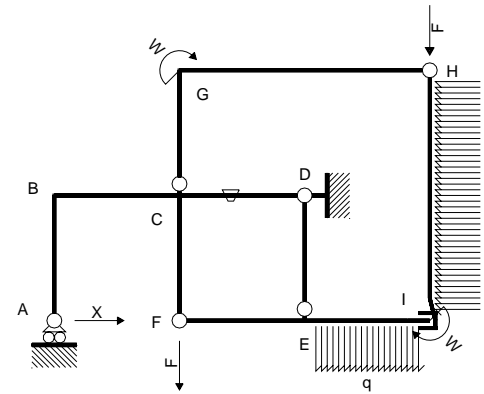
← (+) → F



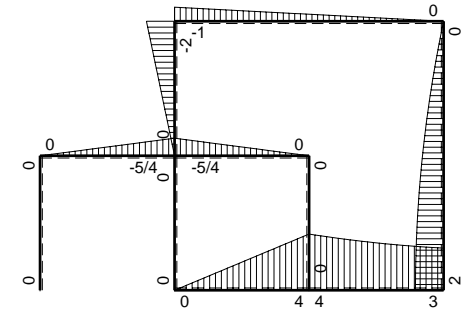
↑ (+) ↓ F



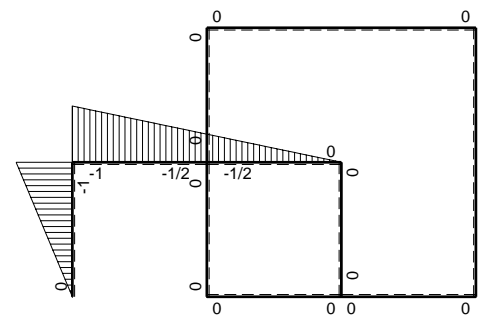
⌚ (+) ⌚ F<sub>b</sub>



Schema di calcolo iperstatico



⌚ (+) ⌚ M<sub>o</sub> flessione da carichi assegnati



⌚ (+) ⌚ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|------------------------|-----------------------|-----------------------------|-----------------------|
| AB b  | -x                          | 0                    | 0        | 0                        | 0                      | $x^2$                 | 0+0                         | $1/3 X b^3/EJ$        |
| BA b  | b-x                         | 0                    | 0        | 0                        | 0                      | $b^2-2bx+x^2$         |                             |                       |
| BC b  | -b+1/2x                     | -5/4Fx               | 0        | $5/4Fbx-5/8Fx^2$         | 0                      | $b^2-bx+1/4x^2$       | $(5/12+0)Fb^3/EJ$           | $7/12 X b^3/EJ$       |
| CB b  | $1/2b+1/2x$                 | $5/4Fb-5/4Fx$        | 0        | $5/8Fb^2-5/8Fx^2$        | 0                      | $1/4b^2+1/2bx+1/4x^2$ |                             |                       |
| CD b  | $-1/2b+1/2x$                | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb^2-5/4Fbx+5/8Fx^2$ | $1/2Fb^2/EJ-1/2Fxb/EJ$ | $1/4b^2-1/2bx+1/4x^2$ | $(5/24+1/4)Fb^3/EJ$         | $1/12 X b^3/EJ$       |
| DC b  | $1/2x$                      | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2$                | $1/2Fxb/EJ$            | $1/4x^2$              |                             |                       |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                     |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                      | 0                     |                             |                       |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                     |
| FE b  | 0                           | $-4Fx$               | 0        | 0                        | 0                      | 0                     |                             |                       |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                     |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                      | 0                     |                             |                       |
| CG b  | 0                           | $-2Fx$               | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                     |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                        | 0                      | 0                     |                             |                       |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                     |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                        | 0                      | 0                     |                             |                       |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                     |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                        | 0                      | 0                     |                             |                       |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                     |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                        | 0                      | 0                     |                             |                       |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                          |                        |                       | $Fb^3/EJ$                   |                       |
|       | totali                      |                      |          |                          |                        |                       | $15/8Fb^3/EJ$               | $Xb^3/EJ$             |
|       | iperstatica $X=H_A$         |                      |          |                          |                        |                       | $-15/8F$                    |                       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb^2 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (5/8 b - 5/24 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb^2 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (5/8 b - 5/24 b) Fb^2 1/EJ = 5/12 Fb^3/EJ$$

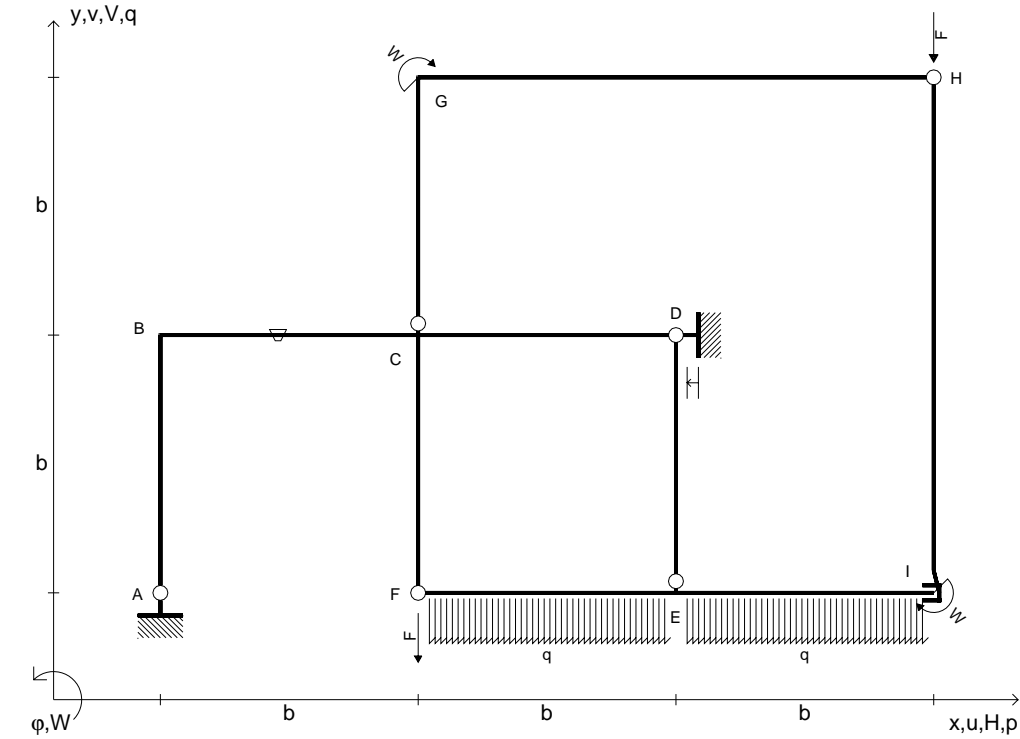
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^3/EJ$$

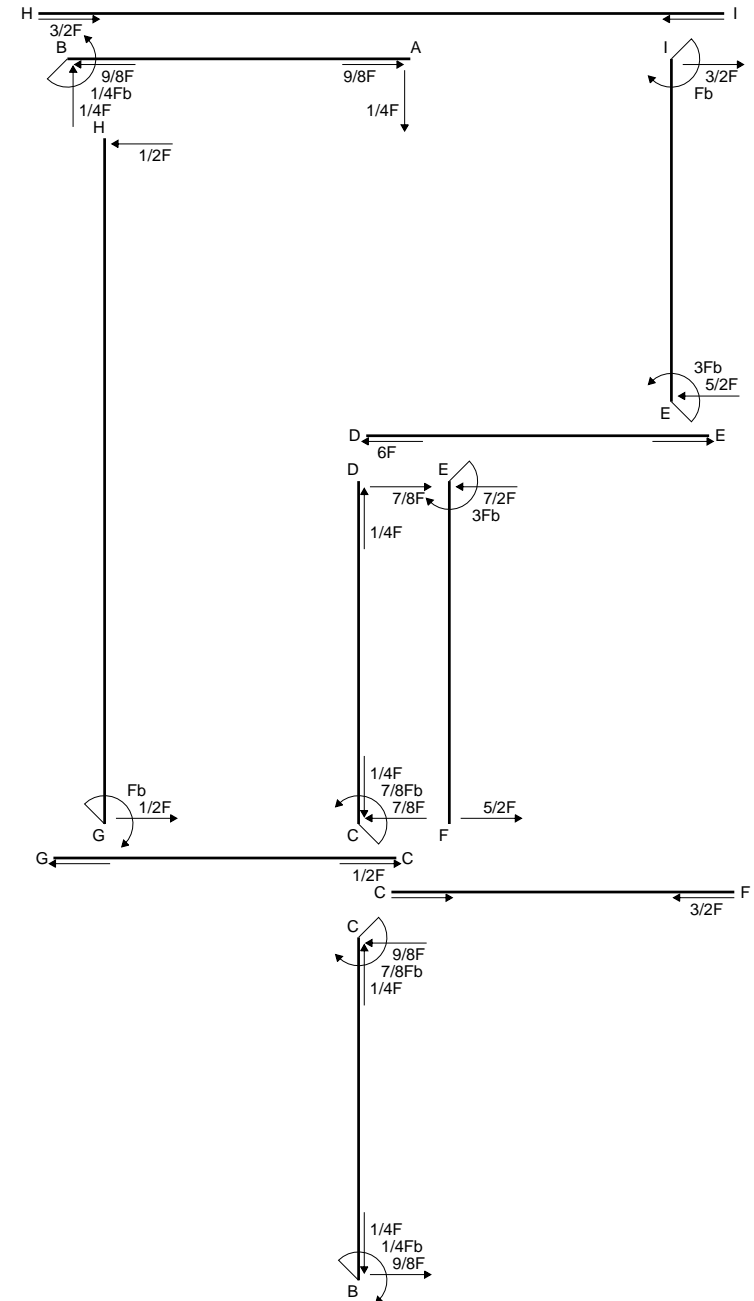
$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

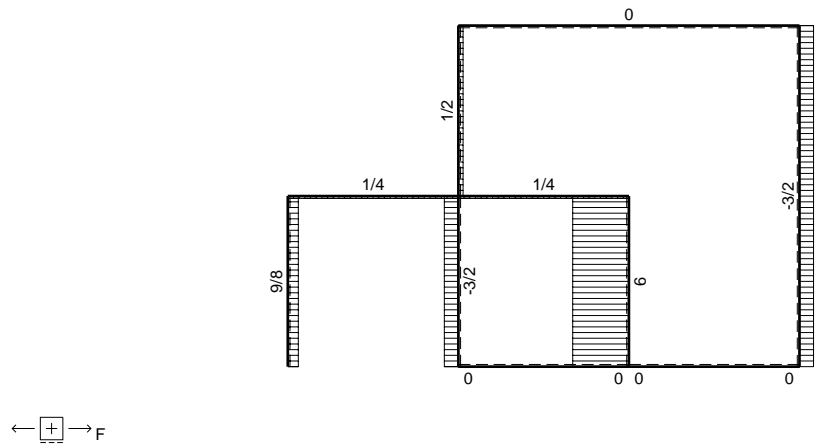
$$= (5/24 b) Fb^2 1/EJ + (-1/4 b) \theta = 11/24 Fb^3/EJ$$



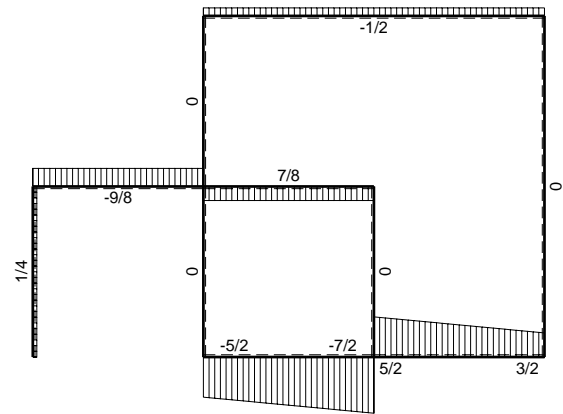
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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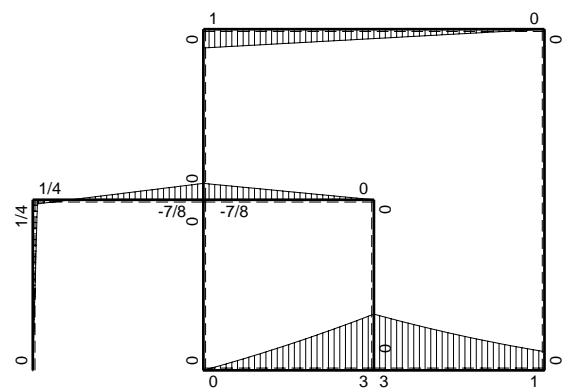




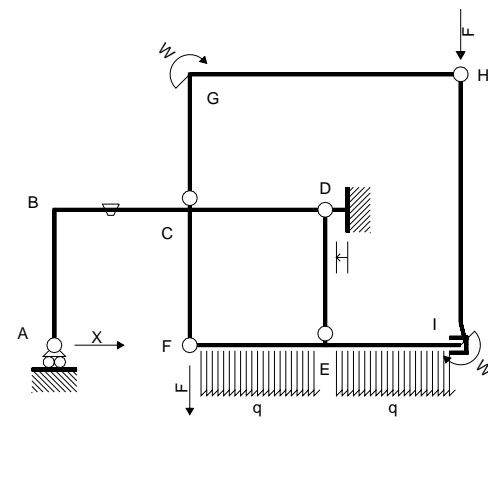
← ⊕ →  $F$



↑ ⊕ ↓  $F$

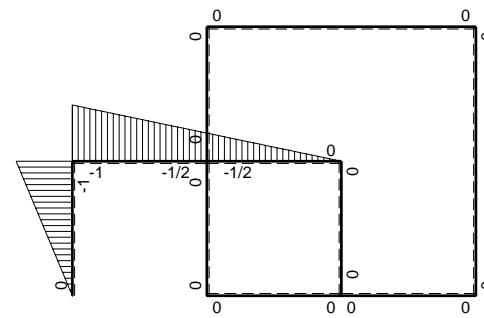


⊕ ⊖  $F_b$



Schema di calcolo iperstatico

⊕ ⊖  $M_o$  flessione da carichi assegnati



⊕ ⊖  $M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$             | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |  |
|-------|-----------------------------|----------------------|----------|-----------------------|------------------------|-----------------------|-----------------------------|------------------------|--|
| AB b  | -x                          | 0                    | 0        | 0                     | 0                      | $x^2$                 | 0+0                         | $1/3 X b^3/EJ$         |  |
| BA b  | b-x                         | 0                    | 0        | 0                     | 0                      | $b^2-2bx+x^2$         |                             |                        |  |
| BC b  | -b+1/2x                     | -Fx                  | -Fb/EJ   | $Fbx-1/2Fx^2$         | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/3+3/4)Fb^3/EJ$          | $7/12 X b^3/EJ$        |  |
| CB b  | $1/2b+1/2x$                 | $Fb-Fx$              | $Fb/EJ$  | $1/2Fb^2-1/2Fx^2$     | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |  |
| CD b  | $-1/2b+1/2x$                | $-Fb+Fx$             | 0        | $1/2Fb^2-Fbx+1/2Fx^2$ | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/6+0)Fb^3/EJ$            | $1/12 X b^3/EJ$        |  |
| DC b  | $1/2x$                      | $Fx$                 | 0        | $1/2Fx^2$             | 0                      | $1/4x^2$              |                             |                        |  |
| DE b  | 0                           | 0                    | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |  |
| ED b  | 0                           | 0                    | 0        | 0                     | 0                      | 0                     |                             |                        |  |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |  |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                     | 0                      | 0                     |                             |                        |  |
| FC b  | 0                           | 0                    | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |  |
| CF b  | 0                           | 0                    | 0        | 0                     | 0                      | 0                     |                             |                        |  |
| CG b  | 0                           | 0                    | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |  |
| GC b  | 0                           | 0                    | 0        | 0                     | 0                      | 0                     |                             |                        |  |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |  |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                     | 0                      | 0                     |                             |                        |  |
| HI 2b | 0                           | 0                    | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |  |
| IH 2b | 0                           | 0                    | 0        | 0                     | 0                      | 0                     |                             |                        |  |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |  |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                     | 0                      | 0                     |                             |                        |  |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                       |                        |                       | $-Fb^3/EJ$                  |                        |  |
|       | totali                      |                      |          |                       |                        |                       | $1/4Fb^3/EJ$                | $Xb^3/EJ$              |  |
|       | iperstatica $X=H_A$         |                      |          |                       |                        |                       | $-1/4F$                     |                        |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{x}{b} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( 1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{1}{2} \frac{x^2}{b} - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} + \left( b - \frac{1}{4} b \right) \theta = \frac{13}{12} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{1}{2} x - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b - \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{13}{12} Fb^3/EJ$$

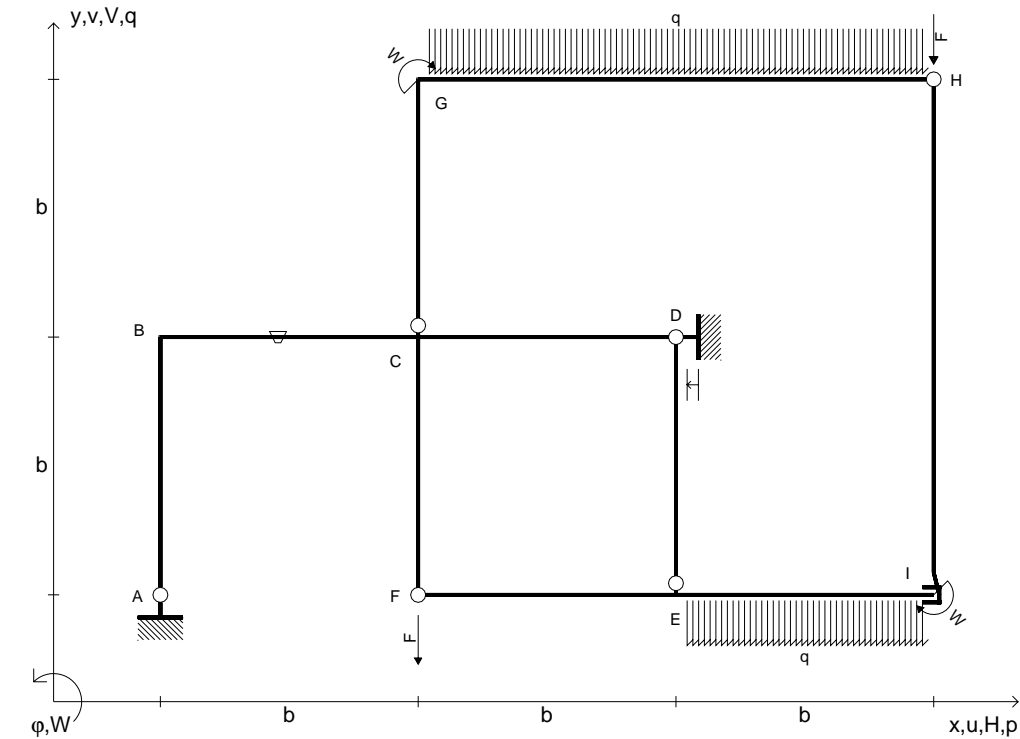
$$L_{CD}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{x}{b} + \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{2} x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{2} b - \frac{1}{2} b + \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

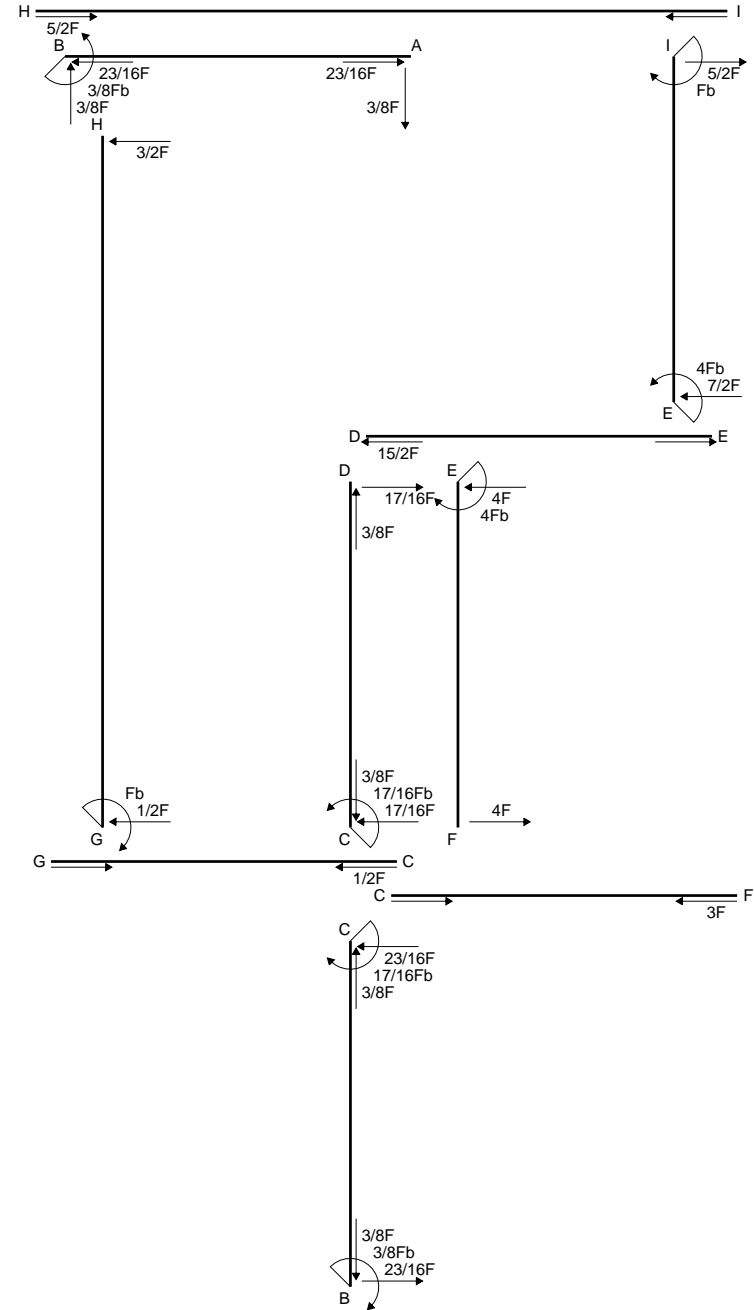
$$= \left( \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$

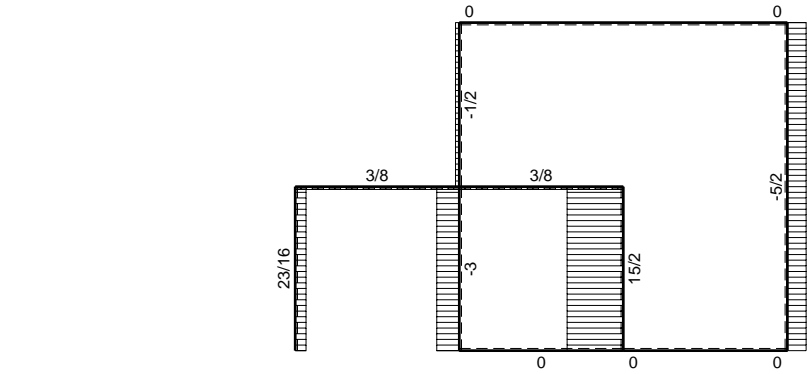




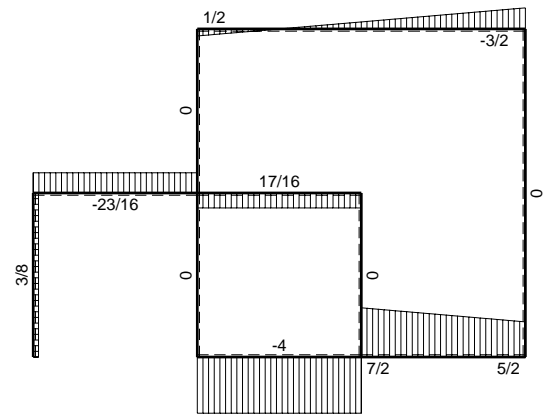
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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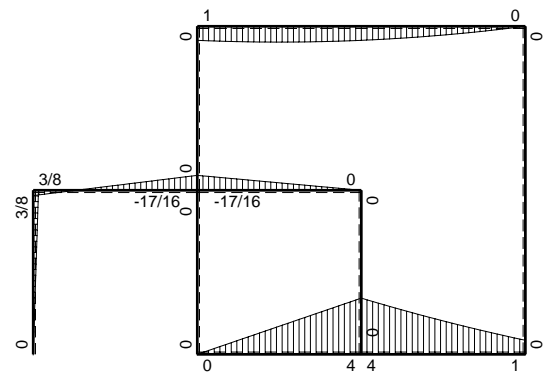




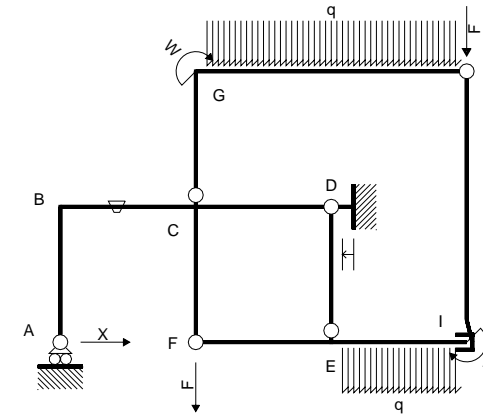
← (+) → F



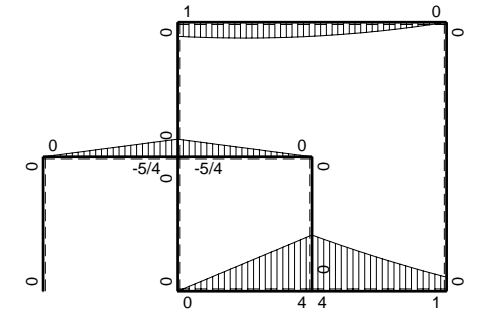
↑ (+) ↓ F



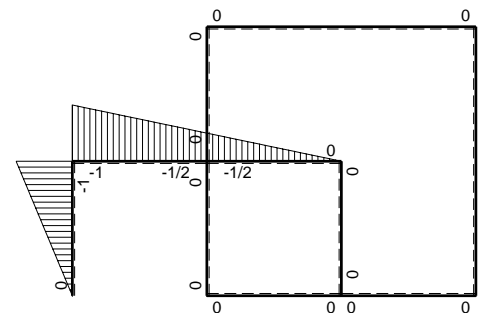
⊕ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ M<sub>o</sub> flessione da carichi assegnati



⊕ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$                 | $\theta$ | $M_x M_o$                    | $M_x \theta$             | $M_x M_x$                 | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |           |
|-------|-----------------------------|--------------------------|----------|------------------------------|--------------------------|---------------------------|-----------------------------|--------------------------|-----------|
| AB b  | -x                          | 0                        | 0        | 0                            | 0                        | $x^2$                     | 0+0                         | $1/3 X b^3 / EJ$         |           |
| BA b  | b-x                         | 0                        | 0        | 0                            | 0                        | $b^2 - 2bx + x^2$         |                             |                          |           |
| BC b  | -b+1/2x                     | -5/4Fx                   | -Fb/EJ   | $5/4Fbx - 5/8Fx^2$           | $Fb^2/EJ - 1/2Fxb/EJ$    | $b^2 - bx + 1/4x^2$       | $(5/12 + 3/4)Fb^3/EJ$       | $7/12 X b^3 / EJ$        |           |
| CB b  | $1/2b + 1/2x$               | $5/4Fb - 5/4Fx$          | Fb/EJ    | $5/8Fb^2 - 5/8Fx^2$          | $1/2Fb^2/EJ + 1/2Fxb/EJ$ | $1/4b^2 + 1/2bx + 1/4x^2$ |                             |                          |           |
| CD b  | $-1/2b + 1/2x$              | $-5/4Fb + 5/4Fx$         | 0        | $5/8Fb^2 - 5/4Fbx + 5/8Fx^2$ | 0                        | $1/4b^2 - 1/2bx + 1/4x^2$ | $(5/24 + 0)Fb^3/EJ$         | $1/12 X b^3 / EJ$        |           |
| DC b  | $1/2x$                      | $5/4Fx$                  | 0        | $5/8Fx^2$                    | 0                        | $1/4x^2$                  |                             |                          |           |
| DE b  | 0                           | 0                        | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |           |
| ED b  | 0                           | 0                        | 0        | 0                            | 0                        | 0                         |                             |                          |           |
| EF b  | 0                           | $4Fb - 4Fx$              | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |           |
| FE b  | 0                           | -4Fx                     | 0        | 0                            | 0                        | 0                         |                             |                          |           |
| FC b  | 0                           | 0                        | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |           |
| CF b  | 0                           | 0                        | 0        | 0                            | 0                        | 0                         |                             |                          |           |
| CG b  | 0                           | 0                        | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |           |
| GC b  | 0                           | 0                        | 0        | 0                            | 0                        | 0                         |                             |                          |           |
| GH 2b | 0                           | $Fb + 1/2Fx - 1/2qx^2$   | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |           |
| HG 2b | 0                           | $-3/2Fx + 1/2qx^2$       | 0        | 0                            | 0                        | 0                         |                             |                          |           |
| HI 2b | 0                           | 0                        | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |           |
| IH 2b | 0                           | 0                        | 0        | 0                            | 0                        | 0                         |                             |                          |           |
| IE b  | 0                           | $Fb + 5/2Fx + 1/2qx^2$   | 0        | 0                            | 0                        | 0                         | 0+0                         | 0                        |           |
| EI b  | 0                           | $-4Fb + 7/2Fx - 1/2qx^2$ | 0        | 0                            | 0                        | 0                         |                             |                          |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                          |          |                              |                          |                           |                             | $-Fb^3/EJ$               |           |
|       | totali                      |                          |          |                              |                          |                           |                             | $3/8Fb^3/EJ$             | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                          |          |                              |                          |                           |                             | $-3/8F$                  |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{5}{4} \frac{x}{b} - \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( 1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{5}{8} \frac{x^2}{b} - \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{5}{8} b - \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} + \left( b - \frac{1}{4} b \right) \theta = \frac{7}{6} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{5}{8} - \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{5}{8} x - \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

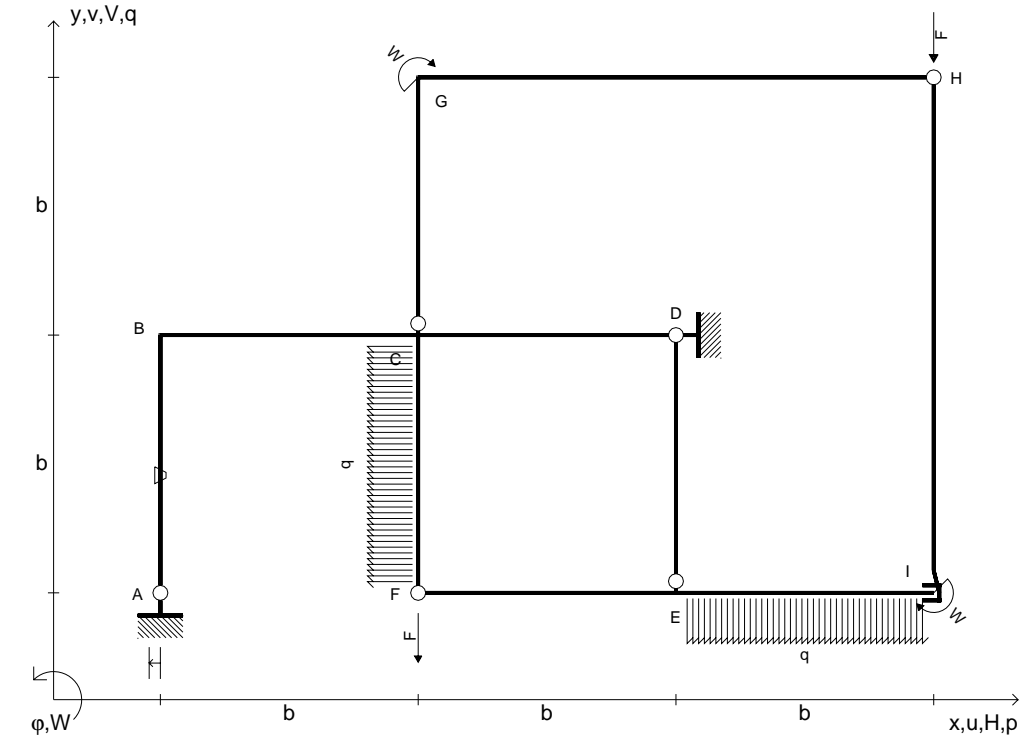
$$= \left( \frac{5}{8} b - \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{7}{6} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left( \frac{5}{8} - \frac{5}{4} \frac{x}{b} + \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{8} x - \frac{5}{8} \frac{x^2}{b} + \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{5}{8} b - \frac{5}{8} b + \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{24} Fb^3/EJ$$

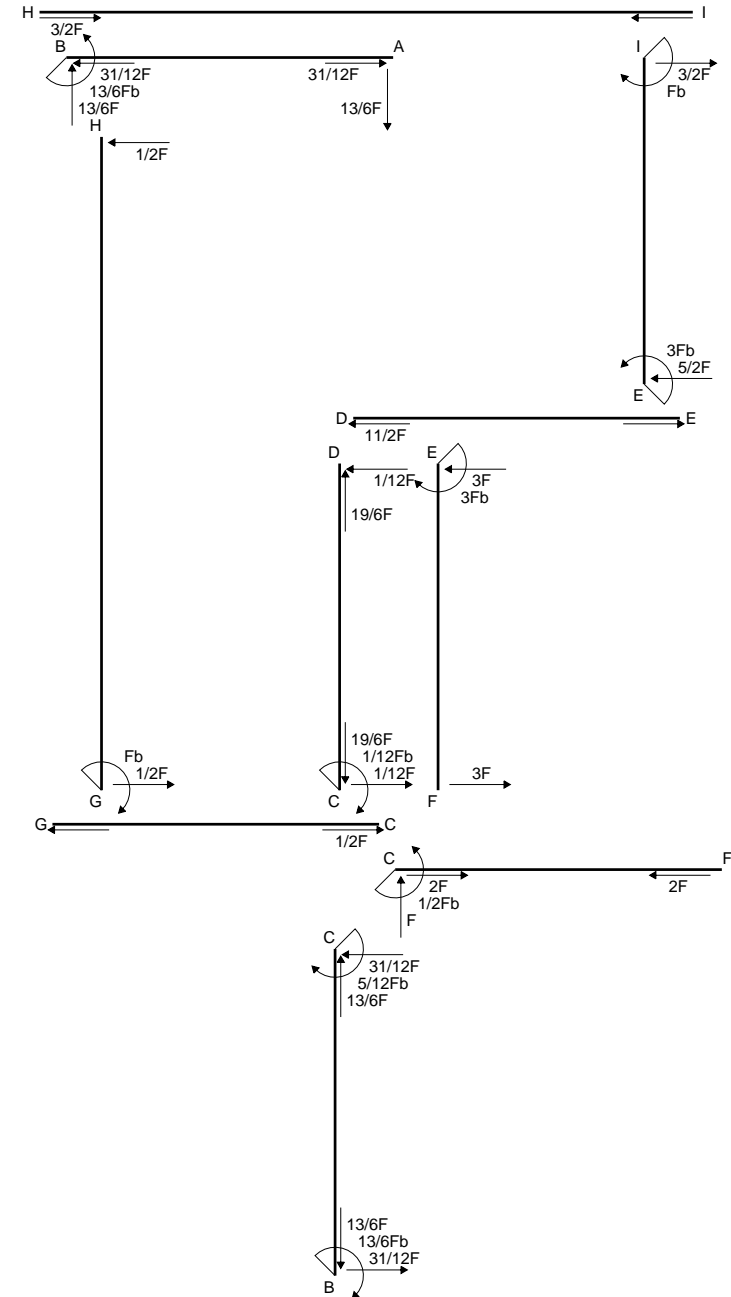
$$L_{DC}^{xo} = \int_0^b \left( \frac{5}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{5}{24} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

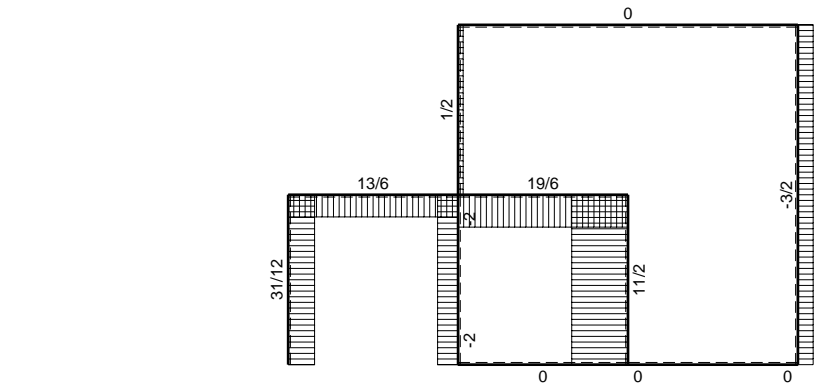
$$= \left( \frac{5}{24} b \right) Fb^2 \frac{1}{EJ} = \frac{5}{24} Fb^3/EJ$$



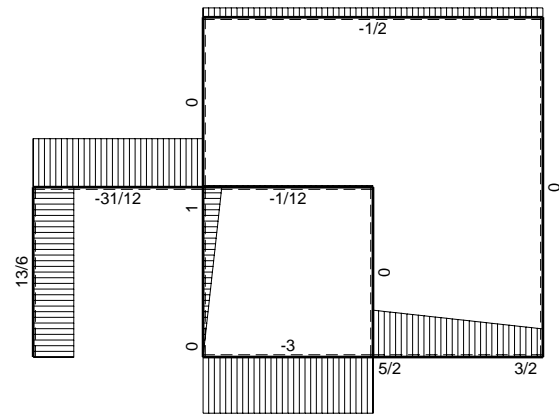
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{FC} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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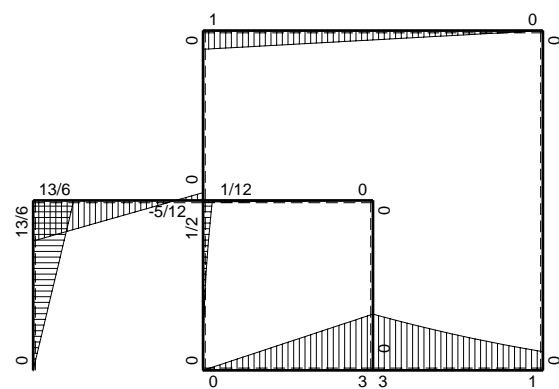




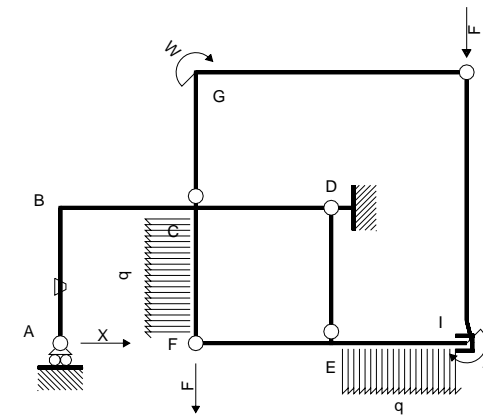
← ⊕ → F



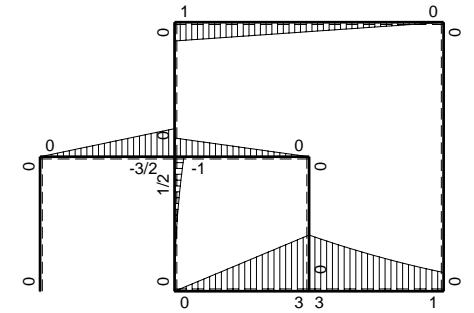
↑ ⊕ ↓ F



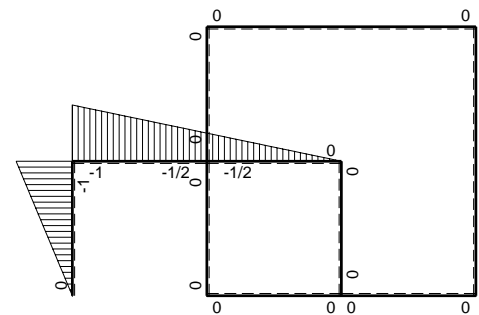
⊕ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ M<sub>o</sub> flessione da carichi assegnati



⊕ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$             | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|-----------------------|------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | 0                    | -Fb/EJ   | 0                     | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$           |
| BA b  | b-x                         | 0                    | Fb/EJ    | 0                     | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | -3/2Fx               | 0        | $3/2Fbx-3/4Fx^2$      | 0                | $b^2-bx+1/4x^2$       | $(1/2+0)Fb^3/EJ$            | $7/12Xb^3/EJ$          |
| CB b  | $1/2b+1/2x$                 | $3/2Fb-3/2Fx$        | 0        | $3/4Fb^2-3/4Fx^2$     | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | -Fb+Fx               | 0        | $1/2Fb^2-Fbx+1/2Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/6+0)Fb^3/EJ$            | $1/12Xb^3/EJ$          |
| DC b  | $1/2x$                      | Fx                   | 0        | $1/2Fx^2$             | 0                | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                     | 0                | 0                     |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | -3Fx                 | 0        | 0                     | 0                | 0                     |                             |                        |
| FC b  | 0                           | $1/2qx^2$            | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$  | 0        | 0                     | 0                | 0                     |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                     | 0                | 0                     |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                     | 0                | 0                     |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                     | 0                | 0                     |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                     | 0                | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                       |                  |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                      |          |                       |                  |                       | $13/6Fb^3/EJ$               | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                      |          |                       |                  |                       | -13/6F                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/4 b - 1/4 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb^2 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/4 b - 1/4 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

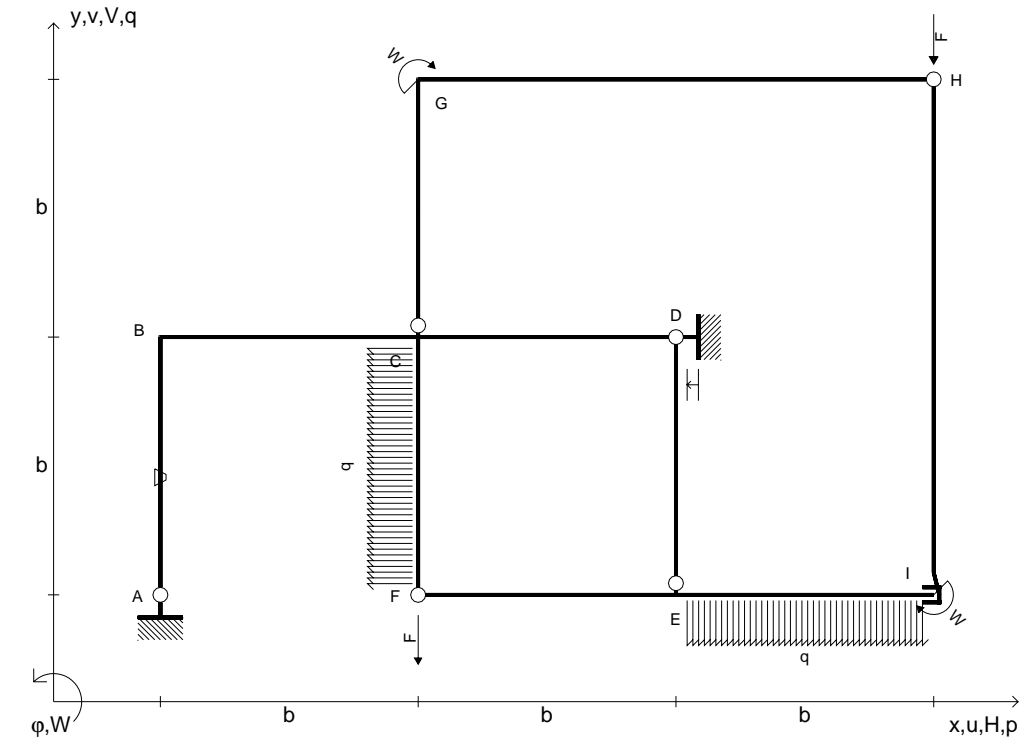
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb^2 1/EJ dx = [1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

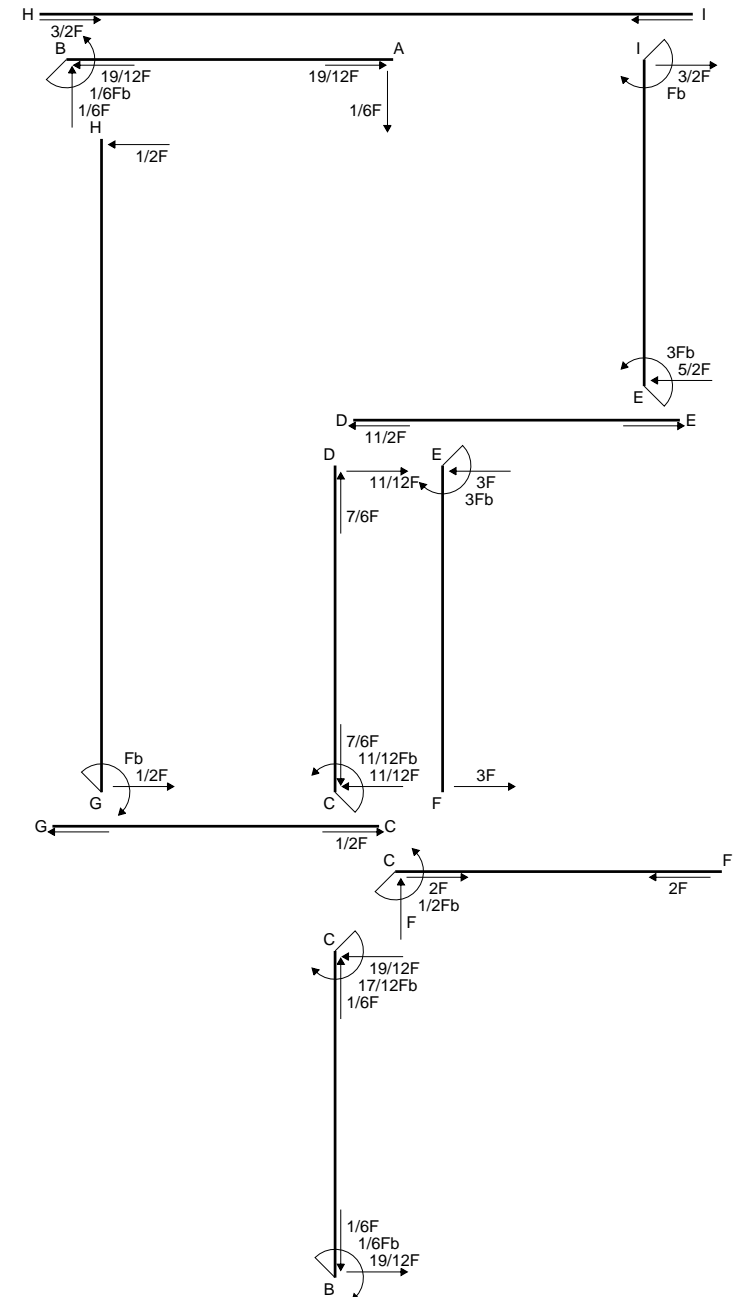
$$= (1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

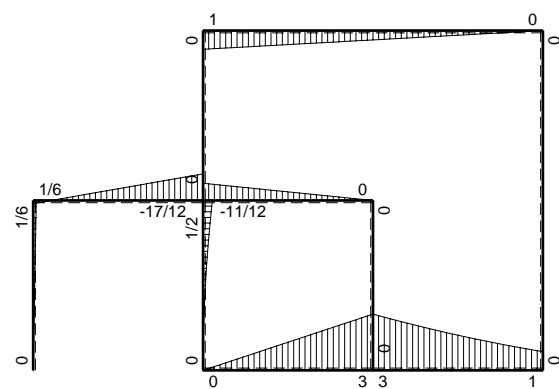
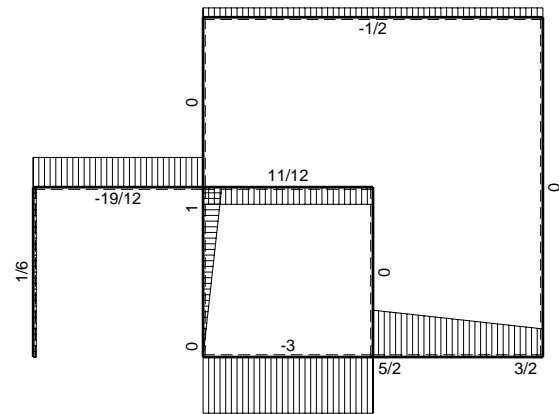
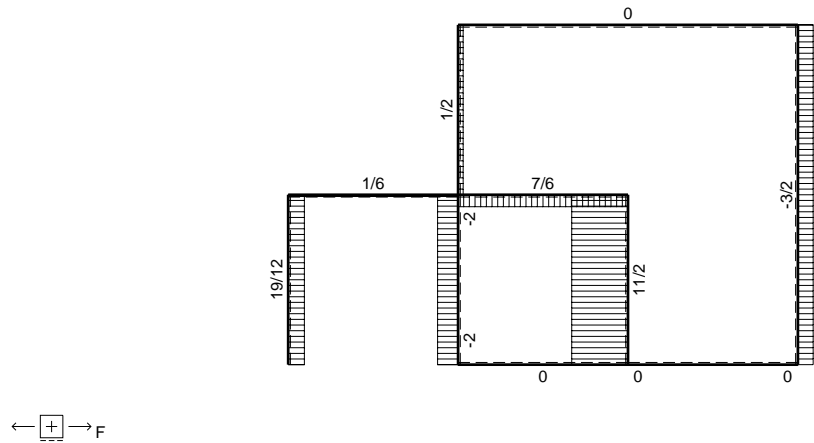




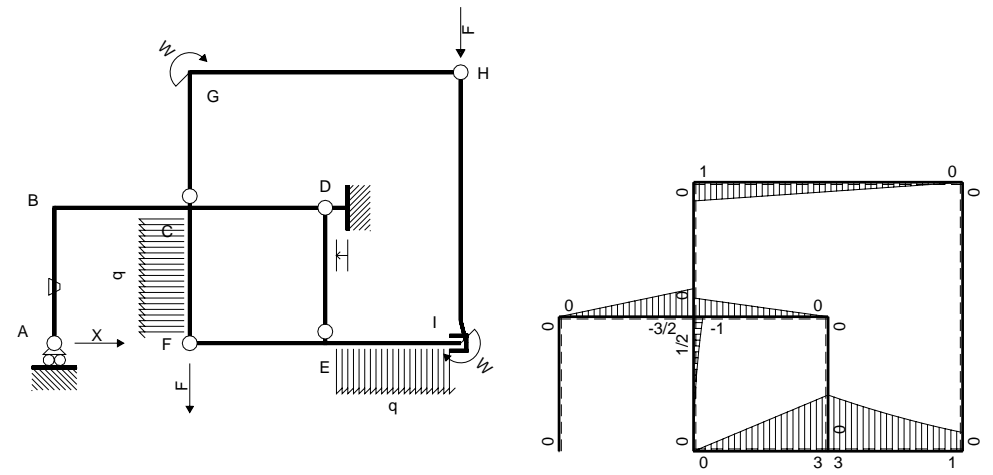
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3 F/EJ$                    | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{FC} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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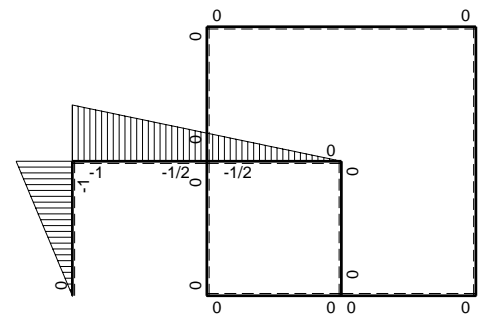


$\curvearrowright (+)$   $F_b$



Schema di calcolo iperstatico

$\curvearrowright (+)$   $M_0$  flessione da carichi assegnati



$\curvearrowright (+)$   $M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$             | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|----------------------|----------|-----------------------|------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | -x                          | 0                    | -Fb/EJ   | 0                     | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$           |           |
| BA b  | b-x                         | 0                    | Fb/EJ    | 0                     | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | -b+1/2x                     | -3/2Fx               | 0        | $3/2Fbx-3/4Fx^2$      | 0                | $b^2-bx+1/4x^2$       | $(1/2+0)Fb^3/EJ$            | $7/12Xb^3/EJ$          |           |
| CB b  | $1/2b+1/2x$                 | $3/2Fb-3/2Fx$        | 0        | $3/4Fb^2-3/4Fx^2$     | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | $-1/2b+1/2x$                | -Fb+Fx               | 0        | $1/2Fb^2-Fbx+1/2Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/6+0)Fb^3/EJ$            | $1/12Xb^3/EJ$          |           |
| DC b  | $1/2x$                      | Fx                   | 0        | $1/2Fx^2$             | 0                | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | 0                    | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | 0                    | 0        | 0                     | 0                | 0                     |                             |                        |           |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | -3Fx                 | 0        | 0                     | 0                | 0                     |                             |                        |           |
| FC b  | 0                           | $1/2qx^2$            | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$  | 0        | 0                     | 0                | 0                     |                             |                        |           |
| CG b  | 0                           | 0                    | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | 0                    | 0        | 0                     | 0                | 0                     |                             |                        |           |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                     | 0                | 0                     |                             |                        |           |
| HI 2b | 0                           | 0                    | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | 0                    | 0        | 0                     | 0                | 0                     |                             |                        |           |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                     | 0                | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                     | 0                | 0                     |                             |                        |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                       |                  |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                      |          |                       |                  |                       |                             | $1/6Fb^3/EJ$           | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                      |          |                       |                  |                       |                             | $-1/6F$                |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left( \frac{x}{b} \right) \theta dx = \left[ \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left( -1 + x/b \right) \theta dx = \left[ -x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( -b + \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{3}{2} \frac{x}{b} - \frac{3}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{4} \frac{x^2}{b} - \frac{1}{4} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{4} b - \frac{1}{4} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{2} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{3}{4} - \frac{3}{4} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{4} x - \frac{1}{4} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

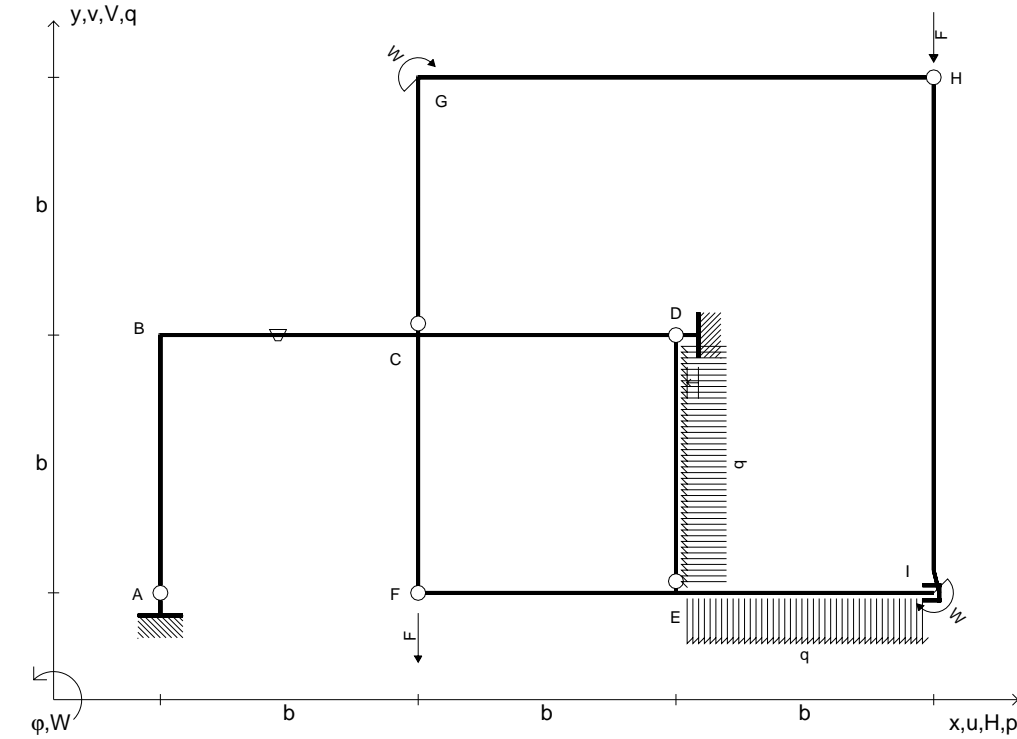
$$= \left( \frac{3}{4} b - \frac{1}{4} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{2} Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{x}{b} + \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{2} x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{2} b - \frac{1}{2} b + \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$

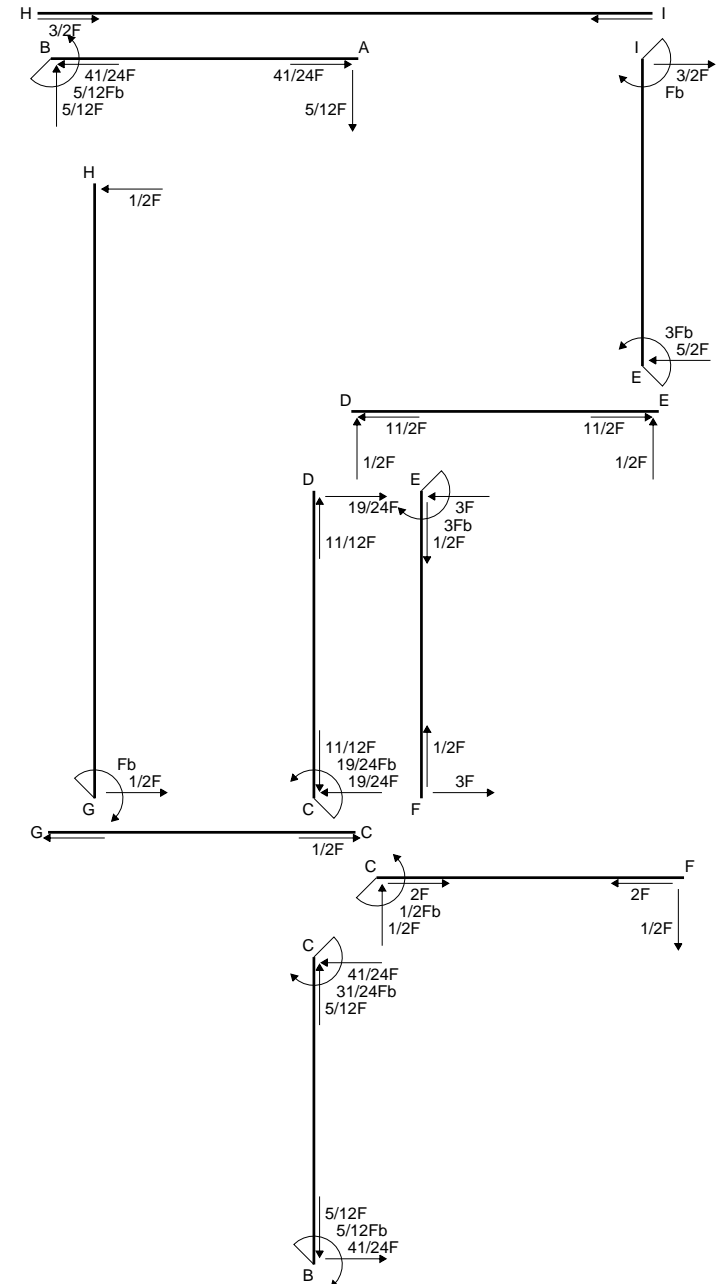
$$L_{DC}^{xo} = \int_0^b \left( \frac{1}{2} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

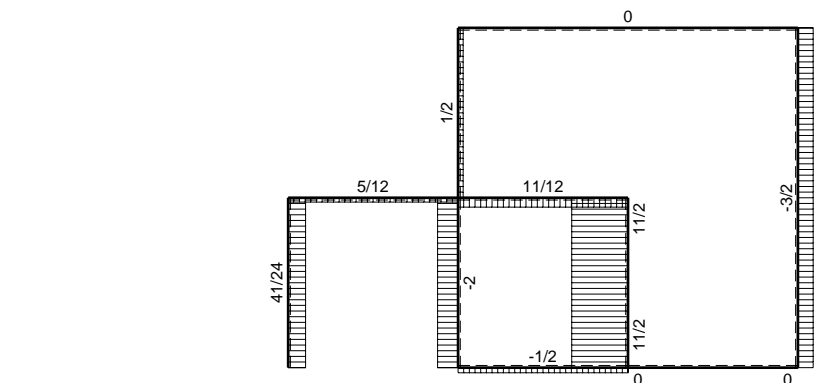
$$= \left( \frac{1}{6} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{6} Fb^3/EJ$$



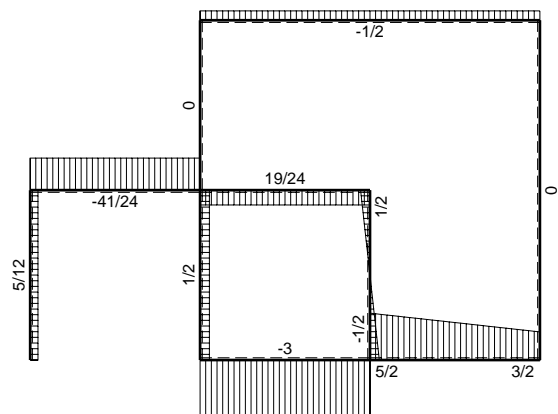
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{DE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
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 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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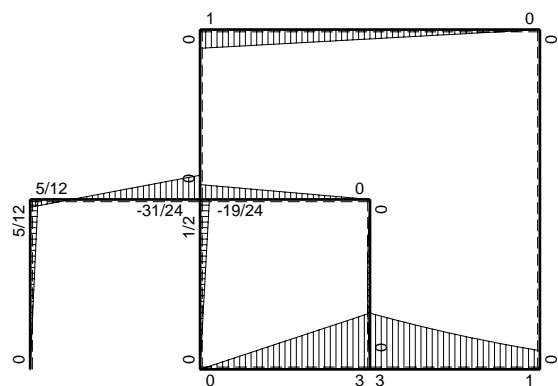




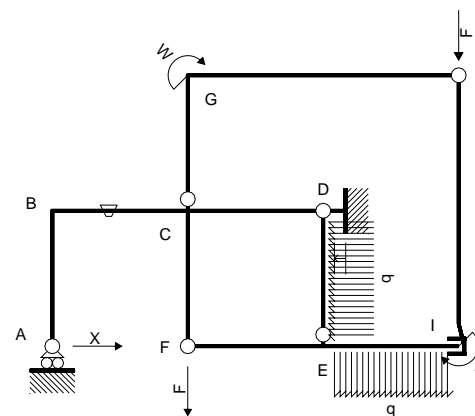
← (+) → F



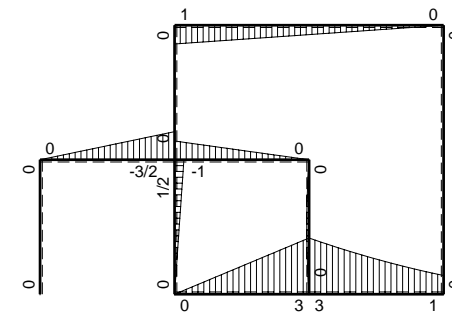
↑ (+) ↓ F



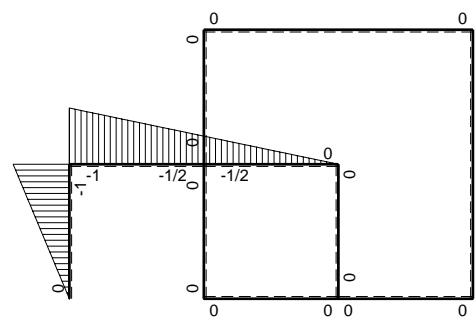
⊙ (+) ⊙ F<sub>b</sub>



Schema di calcolo iperstatico



⊙ (+) ⊙ M<sub>o</sub> flessione da carichi assegnati



⊙ (+) ⊙ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$             | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|----------------------|----------|-----------------------|------------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | -x                          | 0                    | 0        | 0                     | 0                      | $x^2$                 | 0+0                         | $1/3 X b^3/EJ$         |           |
| BA b  | b-x                         | 0                    | 0        | 0                     | 0                      | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | -b+1/2x                     | -3/2Fx               | -Fb/EJ   | $3/2Fbx-3/4Fx^2$      | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/2+3/4)Fb^3/EJ$          | $7/12 X b^3/EJ$        |           |
| CB b  | $1/2b+1/2x$                 | $3/2Fb-3/2Fx$        | Fb/EJ    | $3/4Fb^2-3/4Fx^2$     | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | $-1/2b+1/2x$                | -Fb+Fx               | 0        | $1/2Fb^2-Fbx+1/2Fx^2$ | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/6+0)Fb^3/EJ$            | $1/12 X b^3/EJ$        |           |
| DC b  | $1/2x$                      | Fx                   | 0        | $1/2Fx^2$             | 0                      | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | $1/2Fx-1/2qx^2$      | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$     | 0        | 0                     | 0                      | 0                     |                             |                        |           |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | -3Fx                 | 0        | 0                     | 0                      | 0                     |                             |                        |           |
| FC b  | 0                           | $1/2Fx$              | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | $-1/2Fb+1/2Fx$       | 0        | 0                     | 0                      | 0                     |                             |                        |           |
| CG b  | 0                           | 0                    | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | 0                    | 0        | 0                     | 0                      | 0                     |                             |                        |           |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                     | 0                      | 0                     |                             |                        |           |
| HI 2b | 0                           | 0                    | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | 0                    | 0        | 0                     | 0                      | 0                     |                             |                        |           |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                     | 0                      | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                     | 0                      | 0                     |                             |                        |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                       |                        |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                      |          |                       |                        |                       |                             | $5/12 Fb^3/EJ$         | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                      |          |                       |                        |                       |                             | $-5/12 F$              |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/4 b - 1/4 b) Fb^2 1/EJ + (b - 1/4 b) \theta = 5/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/4 x - 1/4 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/4 b - 1/4 b) Fb^2 1/EJ + (-1/2 b - 1/4 b) \theta = 5/4 Fb^3/EJ$$

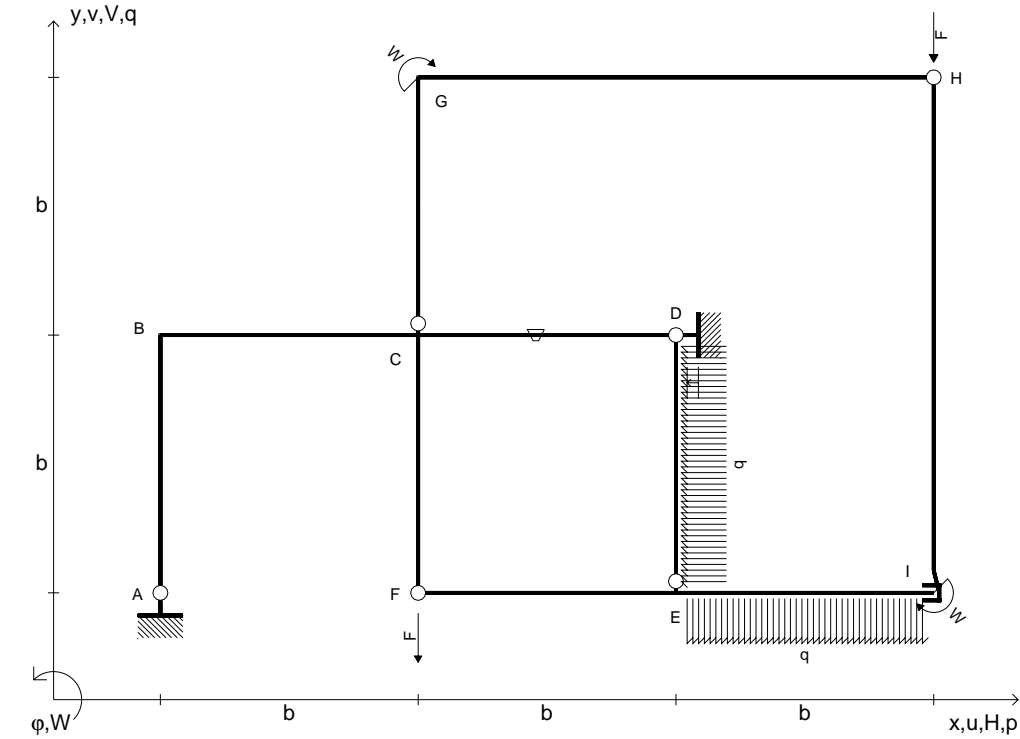
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb^2 1/EJ dx = [1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

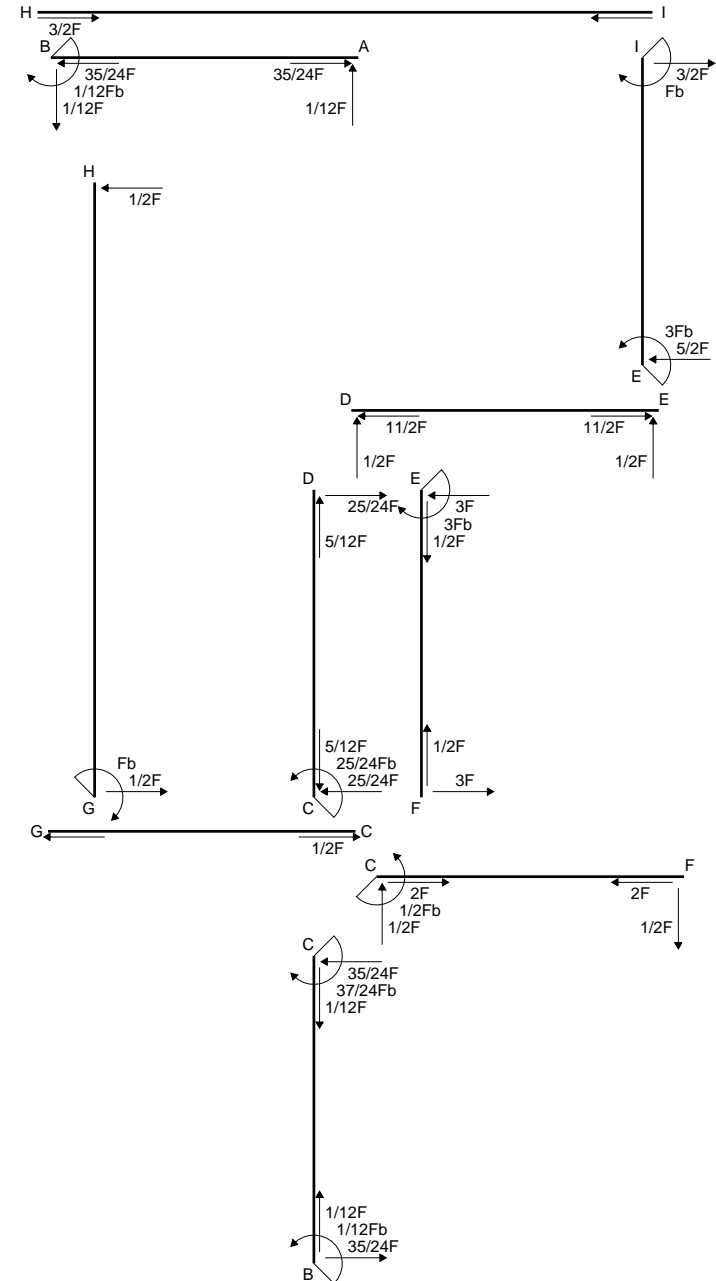
$$= (1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

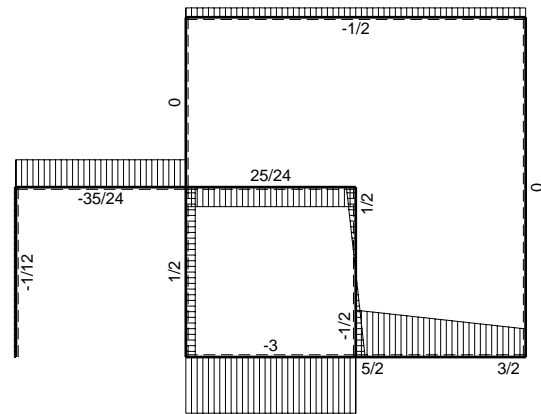
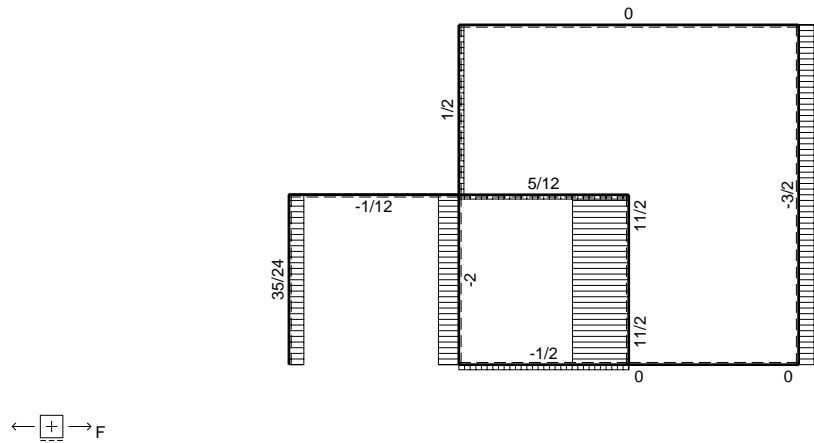




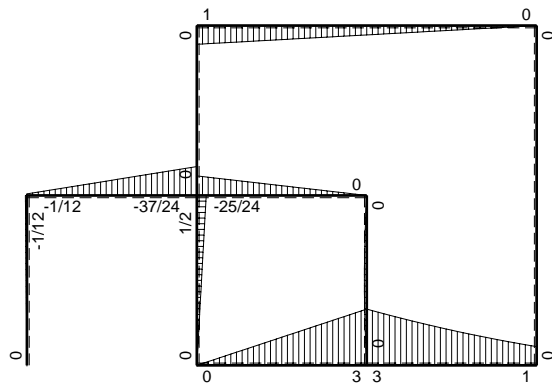
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{IE} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{DE} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=W_{BC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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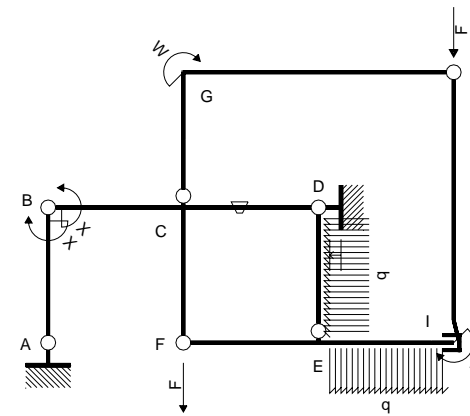




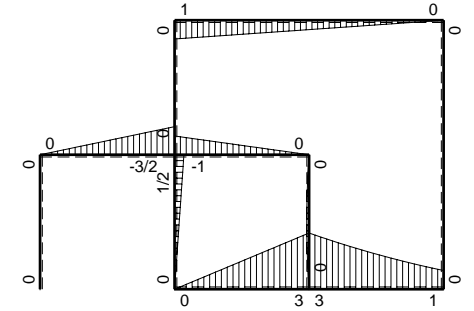
↑ (+) ↓ F



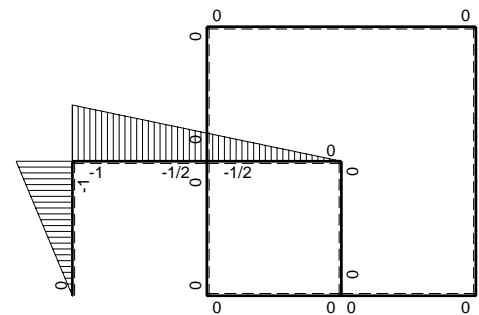
⊙ (+) ⊙ F<sub>b</sub>



Schema di calcolo iperstatico



⊙ (+) ⊙ M<sub>0</sub> flessione da carichi assegnati



⊙ (+) ⊙ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | 0        | $3/2Fx-3/4Fx^2/b$    | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/2+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | 0        | $3/4Fb-3/4Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$       | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                      |          |                      |                     |                         | $-1/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $1/12Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

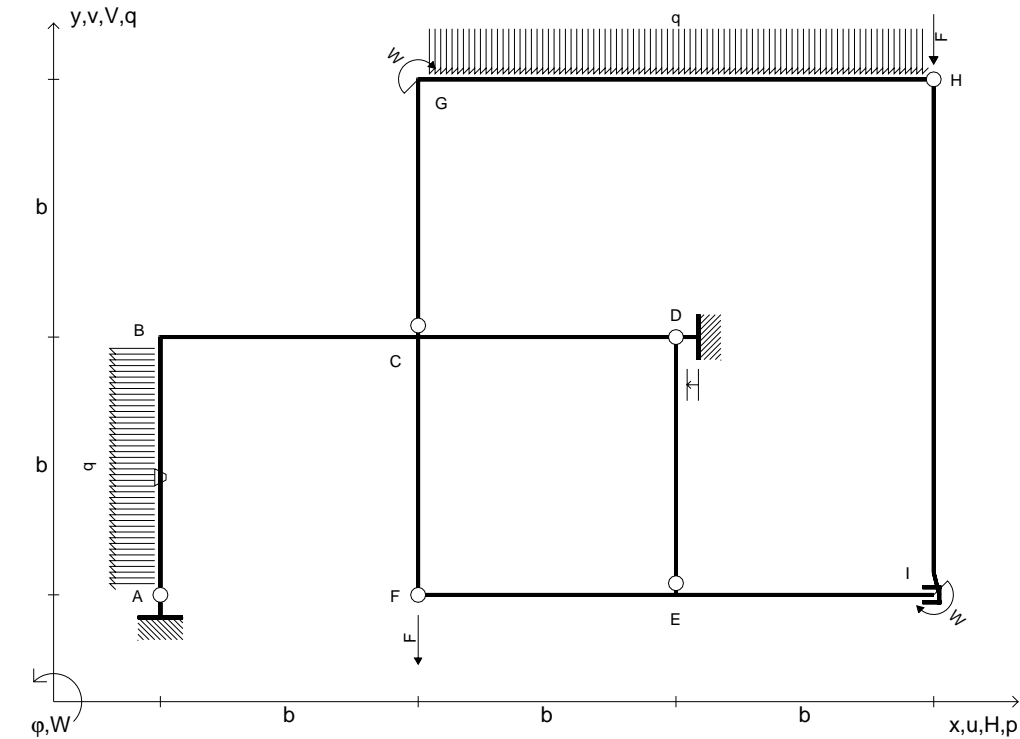
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

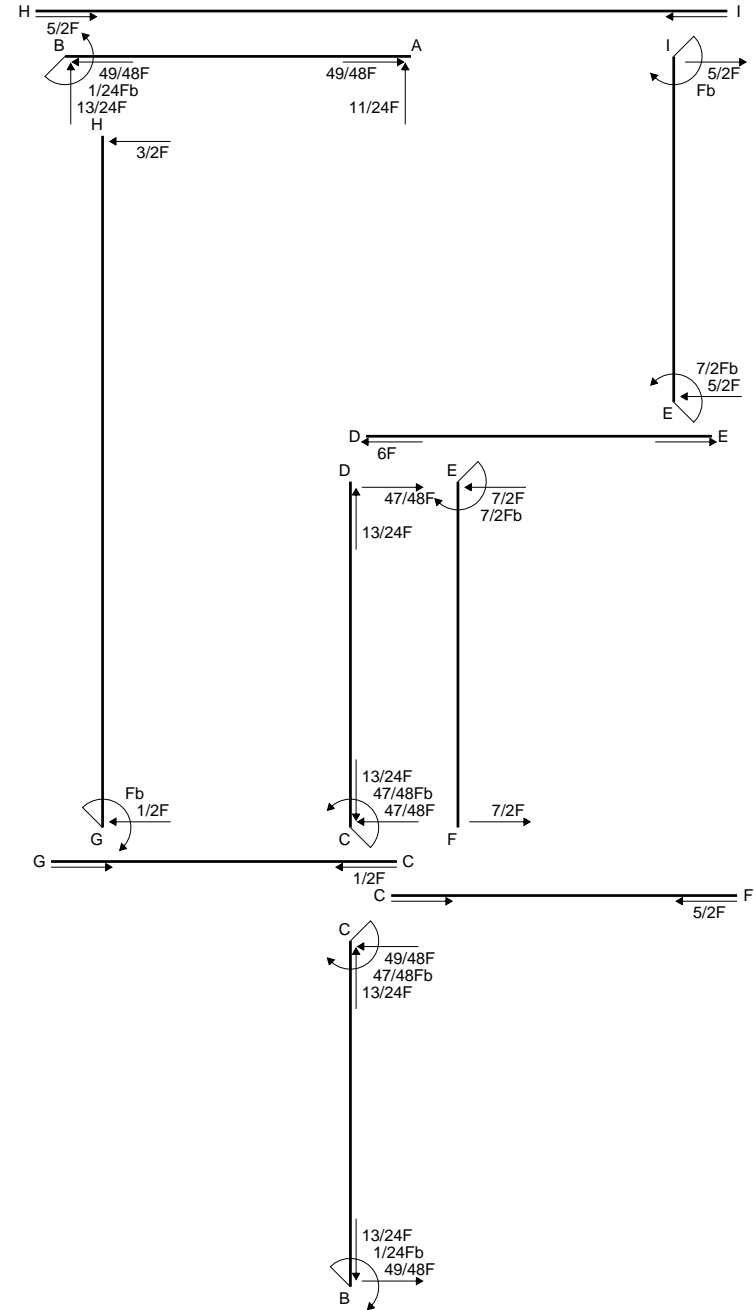
$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

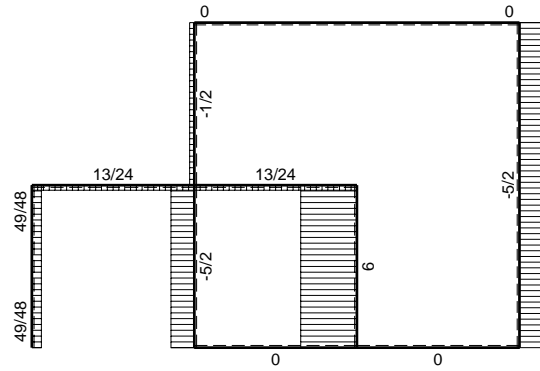
$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$



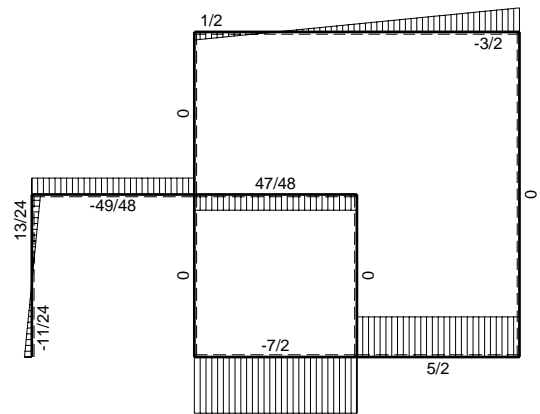
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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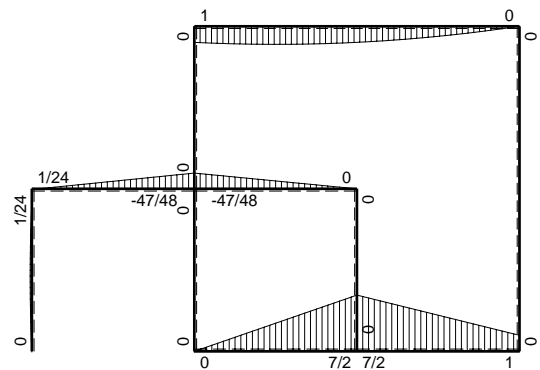




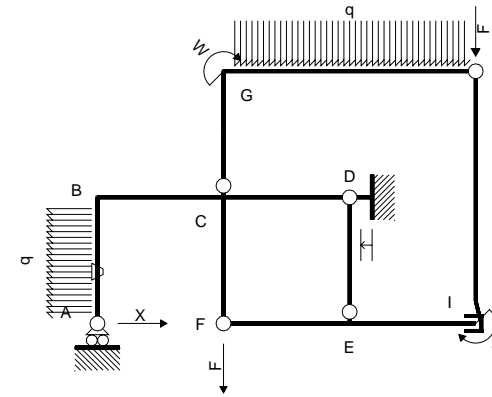
← (+) → F



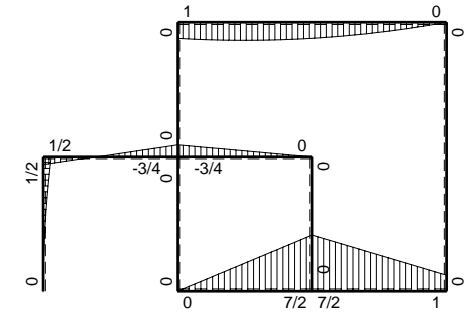
↑ (+) ↓ F



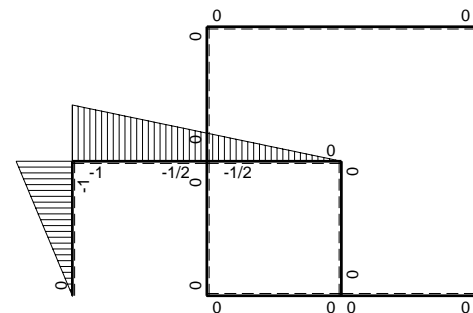
⤵ (+) ⤴ F<sub>b</sub>



Schema di calcolo iperstatico



⤵ (+) ⤴ M<sub>o</sub> flessione da carichi assegnati



⤵ (+) ⤴ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H<sub>A</sub>

| →     | M <sub>x</sub> (x)                             | M <sub>o</sub> (x)           | θ      | M <sub>x</sub> M <sub>o</sub>                                      | M <sub>x</sub> θ           | M <sub>x</sub> M <sub>x</sub>              | ∫M <sub>x</sub> (M <sub>o</sub> /EJ+θ)dx | ∫XM <sub>x</sub> M <sub>x</sub> /EJdx |                     |
|-------|--|------------------------------|--------|--|----------------------------|--|--|---------------------------------------|---------------------|
| AB b  | -x   | 1/2qx <sup>2</sup>           | -Fb/EJ | -1/2qx <sup>3</sup>  | Fxb/EJ                     | x <sup>2</sup>                             | (-1/8+1/2)Fb <sup>3</sup> /EJ            | 1/3Xb <sup>3</sup> /EJ                |                     |
| BA b  | b-x  | -1/2Fb+Fx-1/2qx <sup>2</sup> | Fb/EJ  | -1/2Fb <sup>2</sup> +3/2Fbx-3/2Fx <sup>2</sup> +1/2qx <sup>3</sup> | Fb <sup>2</sup> /EJ-Fxb/EJ | b <sup>2</sup> -2bx+x <sup>2</sup>         |  |                                       |                     |
| BC b  | -b+1/2x  | 1/2Fb-5/4Fx                  | 0      | -1/2Fb <sup>2</sup> +3/2Fbx-5/8Fx <sup>2</sup>                     | 0                          | b <sup>2</sup> -bx+1/4x <sup>2</sup>       | (1/24+0)Fb <sup>3</sup> /EJ              | 7/12Xb <sup>3</sup> /EJ               |                     |
| CB b  | 1/2b+1/2x                                      | 3/4Fb-5/4Fx                  | 0      | 3/8Fb <sup>2</sup> -1/4Fbx-5/8Fx <sup>2</sup>                      | 0                          | 1/4b <sup>2</sup> +1/2bx+1/4x <sup>2</sup> |  |                                       |                     |
| CD b  | -1/2b+1/2x                                     | -3/4Fb+3/4Fx                 | 0      | 3/8Fb <sup>2</sup> -3/4Fbx+3/8Fx <sup>2</sup>                      | 0                          | 1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup> | (1/8+0)Fb <sup>3</sup> /EJ               | 1/12Xb <sup>3</sup> /EJ               |                     |
| DC b  | 1/2x   | 3/4Fx                        | 0      | 3/8Fx <sup>2</sup>   | 0                          | 1/4x <sup>2</sup>                          |  |                                       |                     |
| DE b  | 0  | 0                            | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| ED b  | 0  | 0                            | 0      | 0  | 0                          | 0  |  |                                       |                     |
| EF b  | 0  | 7/2Fb-7/2Fx                  | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| FE b  | 0  | -7/2Fx                       | 0      | 0  | 0                          | 0  |  |                                       |                     |
| FC b  | 0  | 0                            | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| CF b  | 0  | 0                            | 0      | 0  | 0                          | 0  |  |                                       |                     |
| CG b  | 0  | 0                            | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| GC b  | 0  | 0                            | 0      | 0  | 0                          | 0  |  |                                       |                     |
| GH 2b | 0  | Fb+1/2Fx-1/2qx <sup>2</sup>  | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| HG 2b | 0  | -3/2Fx+1/2qx <sup>2</sup>    | 0      | 0  | 0                          | 0  |  |                                       |                     |
| HI 2b | 0  | 0                            | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| IH 2b | 0  | 0                            | 0      | 0  | 0                          | 0  |  |                                       |                     |
| IE b  | 0  | Fb+5/2Fx                     | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |                     |
| EI b  | 0  | -7/2Fb+5/2Fx                 | 0      | 0  | 0                          | 0  |  |                                       |                     |
| D     | cedimento nodo -H <sub>1D</sub> u <sub>D</sub> |                              |        |  |                            |  |  | -Fb <sup>3</sup> /EJ                  |                     |
|       | totali   |                              |        |  |                            |  |  | -11/24Fb <sup>3</sup> /EJ             | Xb <sup>3</sup> /EJ |
|       | iperstatica X=H <sub>A</sub>                   |                              |        |  |                            |  |  | 11/24F                                |                     |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (x/b) \theta dx = [-1/8 x^4/b^3]_0^b Fb^2 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (-1/8 b) Fb^2 1/EJ + (1/2 b) \theta = 3/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [-1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ + (-b + 1/2 b) \theta = 3/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 5/8 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 3/4 x^2/b - 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 3/4 b - 5/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 1/4 x/b - 5/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 1/8 x^2/b - 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 1/8 b - 5/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

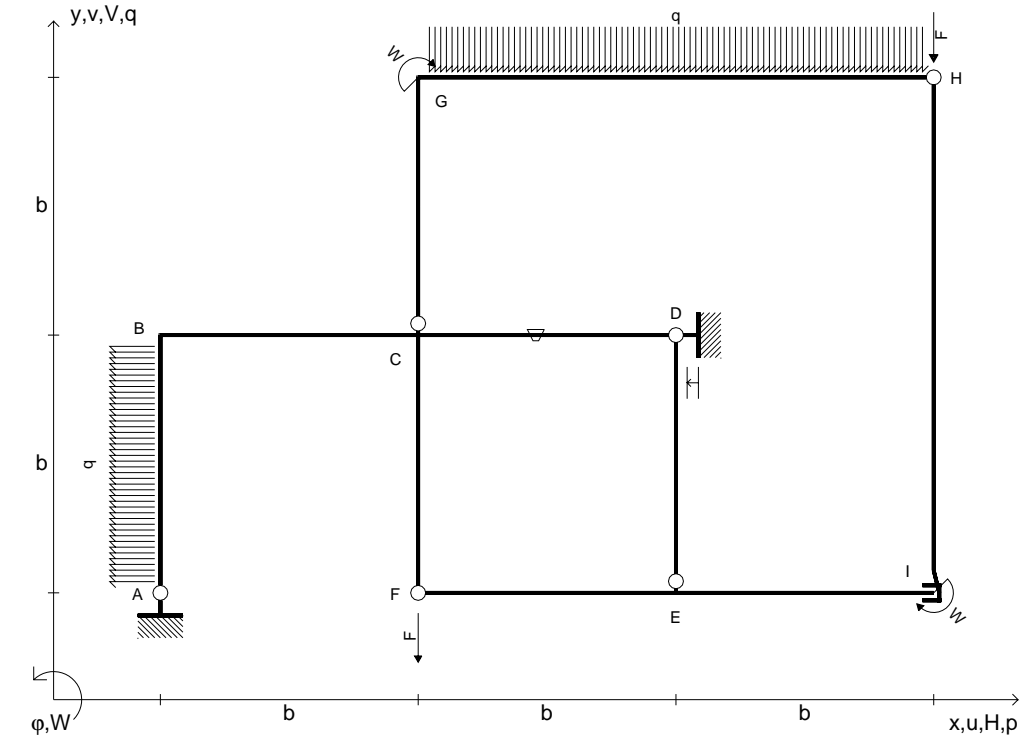
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ$$

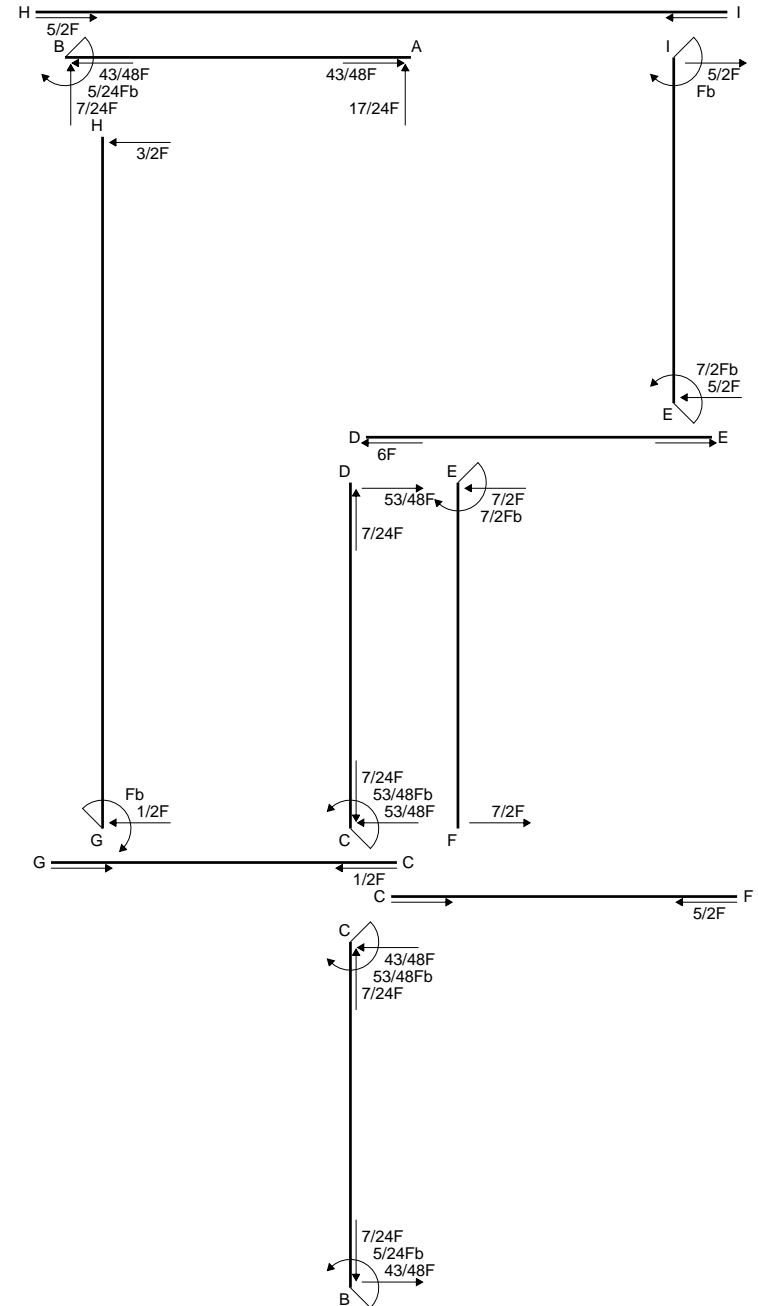
$$= (1/8 b) Fb^2 1/EJ = 1/8 Fb^3/EJ$$

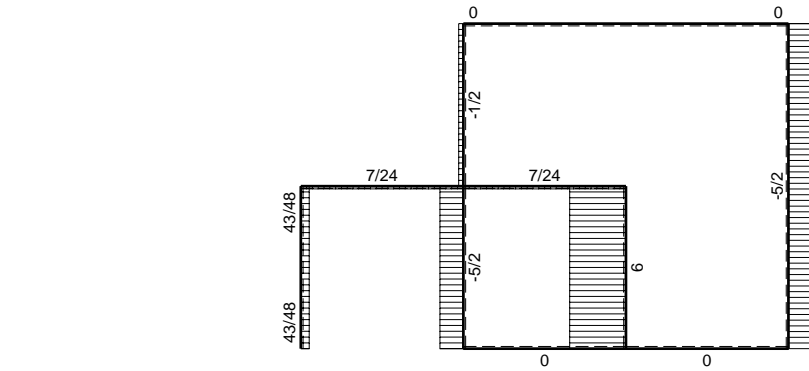




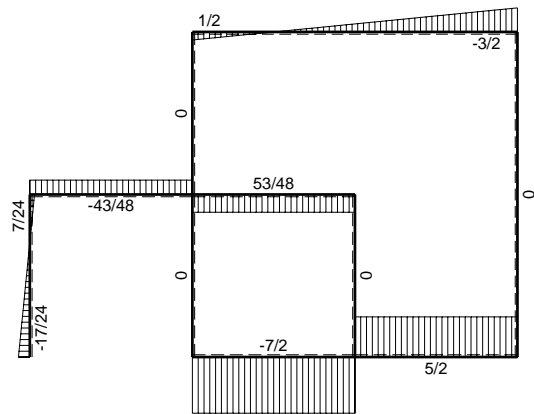
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{CD} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $p_{AB} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta CD positiva se convessa a destra con inizio C.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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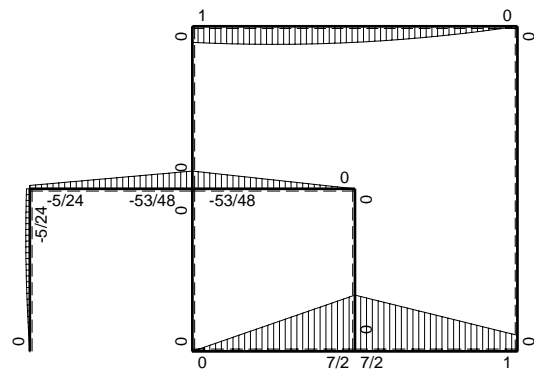




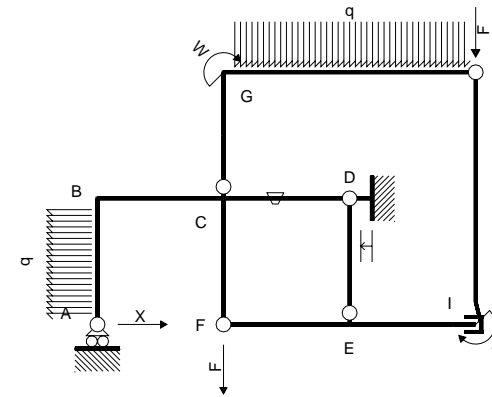
← (+) → F



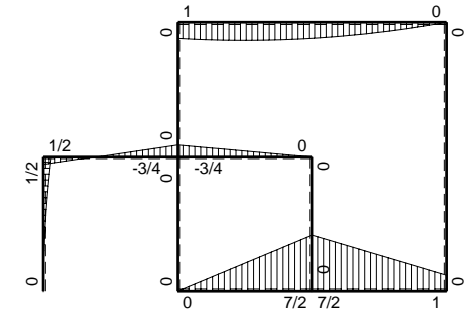
↑ (+) ↓ F



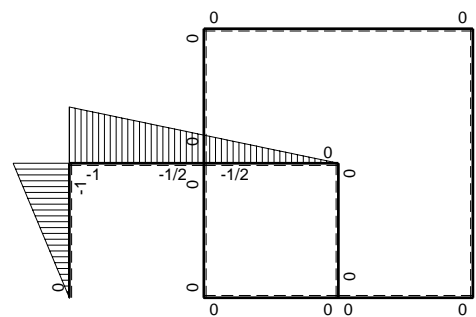
⌚ (+) ⌚ F<sub>b</sub>



Schema di calcolo iperstatico



⌚ (+) ⌚ M<sub>o</sub> flessione da carichi assegnati



⌚ (+) ⌚ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-----------------------------------|------------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | $1/2qx^2$           | 0        | $-1/2qx^3$                        | 0                      | $x^2$                 | $(-1/8+0)Fb^3/EJ$           | $1/3Xb^3/EJ$           |
| BA b  | b-x                         | $-1/2Fb+Fx-1/2qx^2$ | 0        | $-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$ | 0                      | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | $1/2Fb-5/4Fx$       | 0        | $-1/2Fb^2+3/2Fbx-5/8Fx^2$         | 0                      | $b^2-bx+1/4x^2$       | $(1/24+0)Fb^3/EJ$           | $7/12Xb^3/EJ$          |
| CB b  | $1/2b+1/2x$                 | $3/4Fb-5/4Fx$       | 0        | $3/8Fb^2-1/4Fbx-5/8Fx^2$          | 0                      | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb^2-3/4Fbx+3/8Fx^2$          | $1/2Fb^2/EJ-1/2Fxb/EJ$ | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+1/4)Fb^3/EJ$          | $1/12Xb^3/EJ$          |
| DC b  | $1/2x$                      | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2$                         | $1/2Fxb/EJ$            | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| EF b  | 0                           | $7/2Fb-7/2Fx$       | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | $-7/2Fx$            | 0        | 0                                 | 0                      | 0                     |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$  | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$    | 0        | 0                                 | 0                      | 0                     |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                                 | 0                      | 0                     |                             |                        |
| IE b  | 0                           | $Fb+5/2Fx$          | 0        | 0                                 | 0                      | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+5/2Fx$      | 0        | 0                                 | 0                      | 0                     |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                                   |                        |                       | $-Fb^3/EJ$                  |                        |
|       | totali                      |                     |          |                                   |                        |                       | $-17/24Fb^3/EJ$             | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                     |          |                                   |                        |                       | $17/24F$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-1/2 x^3/b^3) Fb^2 1/EJ dx = [-1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [-1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb^2 1/EJ = -1/8 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 5/8 x^2/b^2) Fb^2 1/EJ dx = [-1/2 x + 3/4 x^2/b - 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-1/2 b + 3/4 b - 5/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 1/4 x/b - 5/8 x^2/b^2) Fb^2 1/EJ dx = [3/8 x - 1/8 x^2/b - 5/24 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (3/8 b - 1/8 b - 5/24 b) Fb^2 1/EJ = 1/24 Fb^3/EJ$$

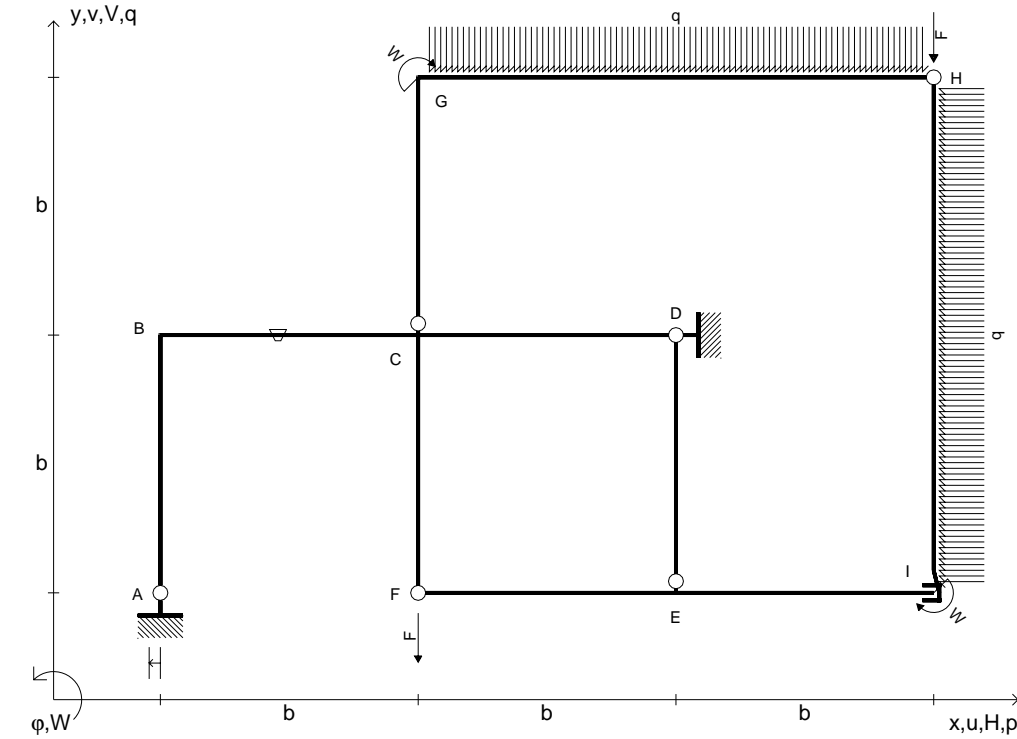
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb^2 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb^2 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^3/EJ$$

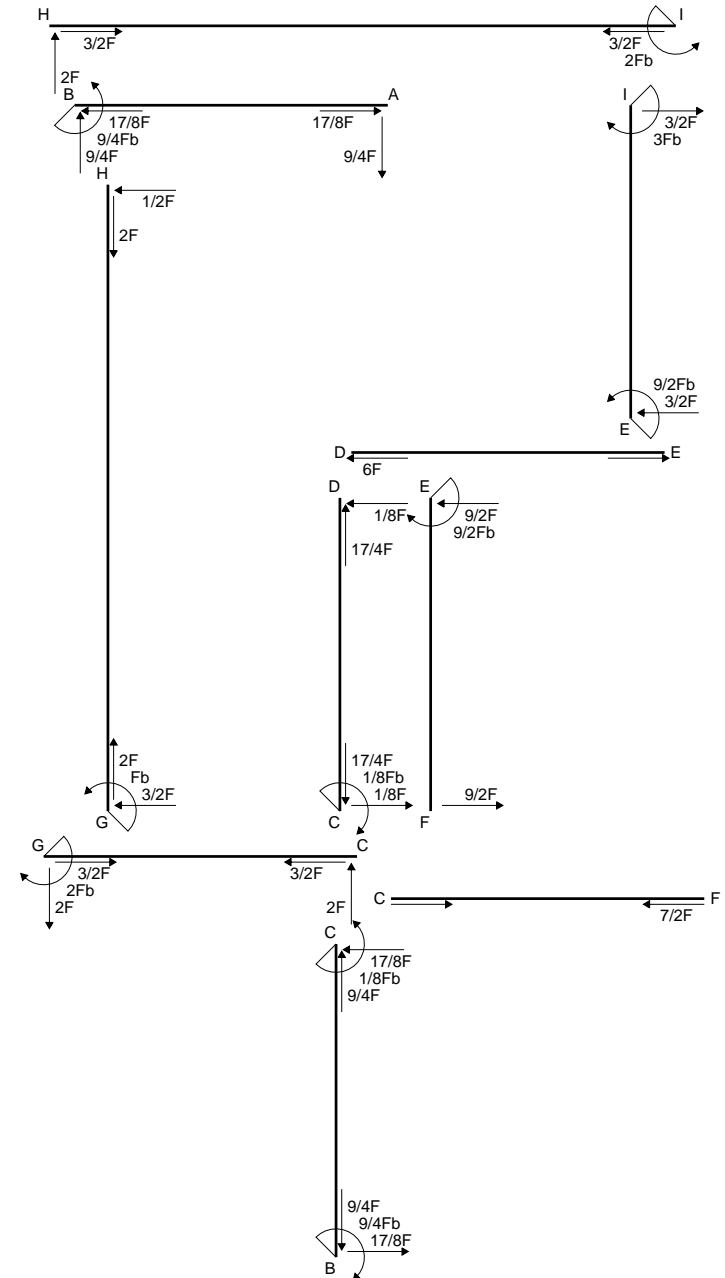
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x^2/b]_0^b \theta$$

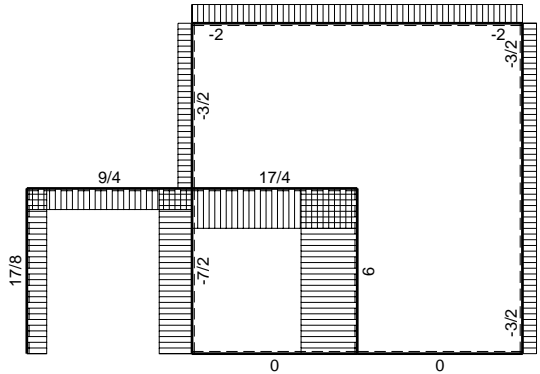
$$= (1/8 b) Fb^2 1/EJ + (-1/4 b) \theta = 3/8 Fb^3/EJ$$



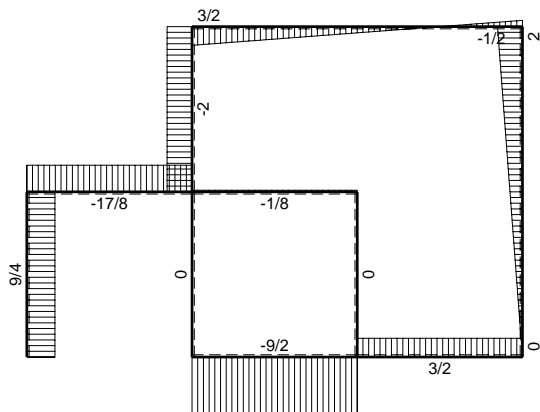
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $P_{HI} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_D$   
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 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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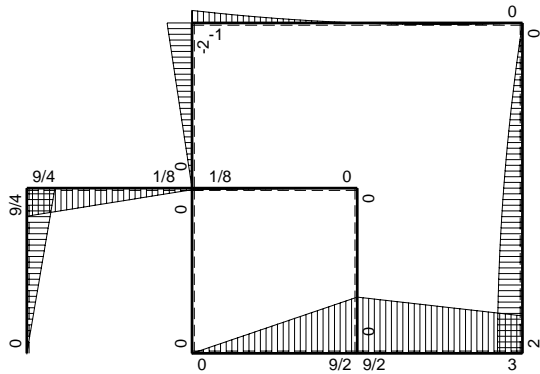




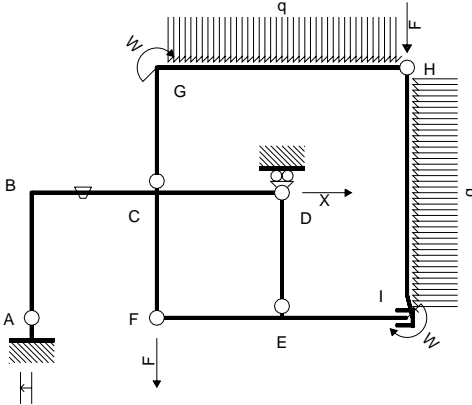
← (+) → F



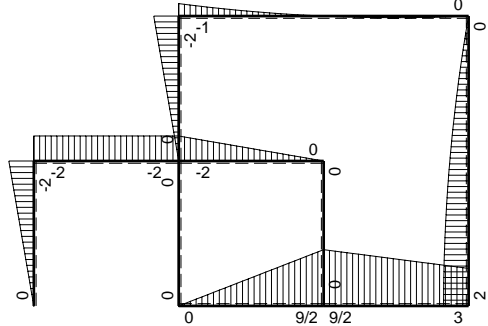
↑ (+) ↓ F



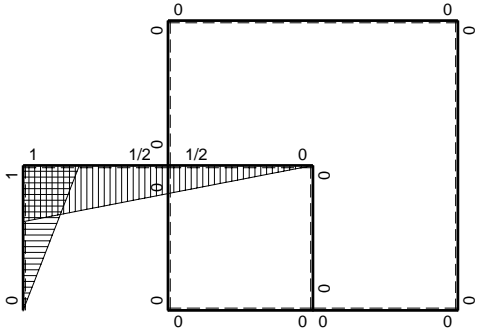
⌚ (+) ↻ F<sub>v</sub>



Schema di calcolo iperstatico



⌚ (+) ↻ M<sub>o</sub> flessione da carichi assegnati



⌚ (+) ↻ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_D$ 

| →     | $M_x(x)$                                       | $M_o(x)$                     | $\theta$ | $M_x M_o$                                | $M_x \theta$                      | $M_x M_x$                                  | $\int M_x(M_o/EJ+\theta)dx$   | $\int X M_x M_x/EJdx$   |
|-------|--|------------------------------|----------|--|-----------------------------------|--|-------------------------------|-------------------------|
| AB b  | x  | -2Fx                         | 0        | -2Fx <sup>2</sup>                        | 0                                 | x <sup>2</sup>                             | (-2/3+0)Fb <sup>3</sup> /EJ   | 1/3Xb <sup>3</sup> /EJ  |
| BA b  | -b+x   | 2Fb-2Fx                      | 0        | -2Fb <sup>2</sup> +4Fbx-2Fx <sup>2</sup> | 0                                 | b <sup>2</sup> -2bx+x <sup>2</sup>         |                               |                         |
| BC b  | b-1/2x   | -2Fb                         | -Fb/EJ   | -2Fb <sup>2</sup> +Fbx                   | -Fb <sup>2</sup> /EJ+1/2Fxb/EJ    | b <sup>2</sup> -bx+1/4x <sup>2</sup>       | (-3/2-3/4)Fb <sup>3</sup> /EJ | 7/12Xb <sup>3</sup> /EJ |
| CB b  | -1/2b-1/2x                                     | 2Fb                          | Fb/EJ    | -Fb <sup>2</sup> -Fbx                    | -1/2Fb <sup>2</sup> /EJ-1/2Fxb/EJ | 1/4b <sup>2</sup> +1/2bx+1/4x <sup>2</sup> |                               |                         |
| CD b  | 1/2b-1/2x                                      | -2Fb+2Fx                     | 0        | -Fb <sup>2</sup> +2Fbx-Fx <sup>2</sup>   | 0                                 | 1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup> | (-1/3+0)Fb <sup>3</sup> /EJ   | 1/12Xb <sup>3</sup> /EJ |
| DC b  | -1/2x  | 2Fx                          | 0        | -Fx <sup>2</sup>                         | 0                                 | 1/4x <sup>2</sup>                          |                               |                         |
| DE b  | 0  | 0                            | 0        | 0  | 0                                 | 0  | 0+0                           | 0                       |
| ED b  | 0  | 0                            | 0        | 0  | 0                                 | 0  |                               |                         |
| EF b  | 0  | 9/2Fb-9/2Fx                  | 0        | 0  | 0                                 | 0  | 0+0                           | 0                       |
| FE b  | 0  | -9/2Fx                       | 0        | 0  | 0                                 | 0  |                               |                         |
| FC b  | 0  | 0                            | 0        | 0  | 0                                 | 0  | 0+0                           | 0                       |
| CF b  | 0  | 0                            | 0        | 0  | 0                                 | 0  |                               |                         |
| CG b  | 0  | -2Fx                         | 0        | 0  | 0                                 | 0  | 0+0                           | 0                       |
| GC b  | 0  | 2Fb-2Fx                      | 0        | 0  | 0                                 | 0  |                               |                         |
| GH 2b | 0  | -Fb+3/2Fx-1/2qx <sup>2</sup> | 0        | 0  | 0                                 | 0  | 0+0                           | 0                       |
| HG 2b | 0  | -1/2Fx+1/2qx <sup>2</sup>    | 0        | 0  | 0                                 | 0  |                               |                         |
| HI 2b | 0  | 2Fx-1/2qx <sup>2</sup>       | 0        | 0  | 0                                 | 0  | 0+0                           | 0                       |
| IH 2b | 0  | -2Fb+1/2qx <sup>2</sup>      | 0        | 0  | 0                                 | 0  |                               |                         |
| IE b  | 0  | 3Fb+3/2Fx                    | 0        | 0  | 0                                 | 0  | 0+0                           | 0                       |
| EI b  | 0  | -9/2Fb+3/2Fx                 | 0        | 0  | 0                                 | 0  |                               |                         |
| A     | cedimento nodo -H <sub>1A</sub> u <sub>A</sub> |                              |          |  |                                   |  | -Fb <sup>3</sup> /EJ          |                         |
|       | totali   |                              |          |  |                                   |  | -17/4Fb <sup>3</sup> /EJ      | Xb <sup>3</sup> /EJ     |
|       | iperstatica X=H <sub>D</sub>                   |                              |          |  |                                   |  | 17/4F                         |                         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-2x^2/b^2) Fb^2 1/EJ dx = [-2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2/3 b) Fb^2 1/EJ = -2/3 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-2 + 4x/b - 2x^2/b^2) Fb^2 1/EJ dx = [-2x + 2x^2/b - 2/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-2b + 2b - 2/3 b) Fb^2 1/EJ = -2/3 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-2 + x/b) Fb^2 1/EJ dx + \int_0^b (-1 + 1/2 x/b) \theta dx$$

$$= [-2x + 1/2 x^2/b]_0^b Fb^2 1/EJ + [-x + 1/4 x^2/b]_0^b \theta$$

$$= (-2b + 1/2 b) Fb^2 1/EJ + (-b + 1/4 b) \theta = -9/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1 - x/b) Fb^2 1/EJ dx + \int_0^b (1/2 + 1/2 x/b) \theta dx$$

$$= [-x - 1/2 x^2/b]_0^b Fb^2 1/EJ + [1/2 x + 1/4 x^2/b]_0^b \theta$$

$$= (-b - 1/2 b) Fb^2 1/EJ + (1/2 b + 1/4 b) \theta = -9/4 Fb^3/EJ$$

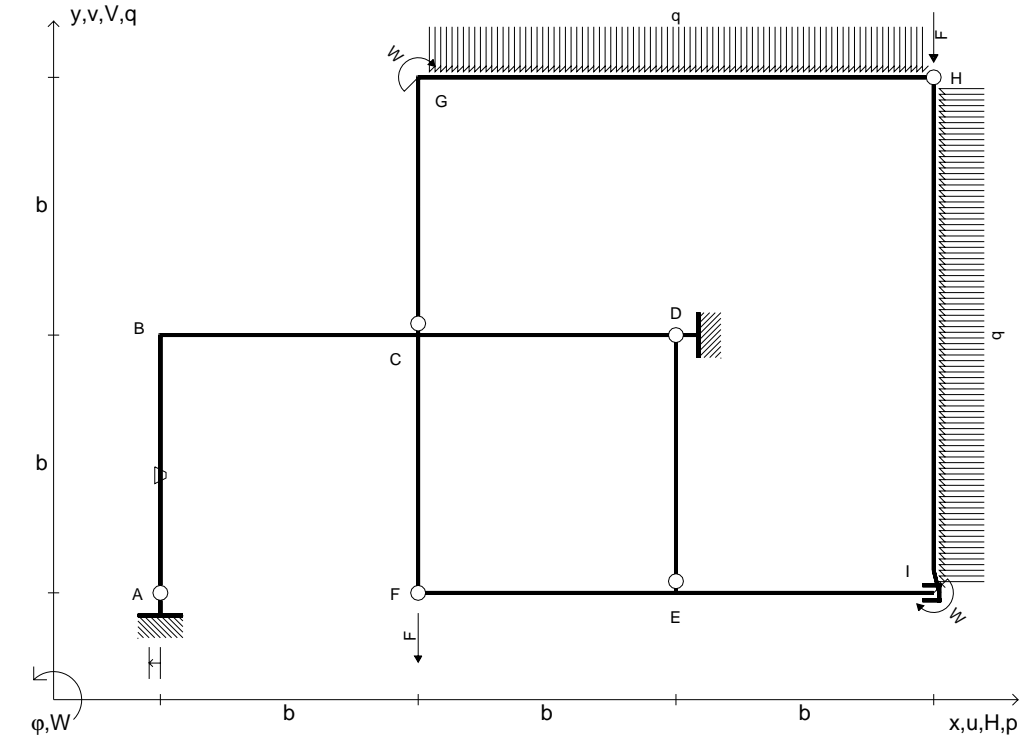
$$L_{CD}^{xo} = \int_0^b (-1 + 2x/b - x^2/b^2) Fb^2 1/EJ dx = [-x + x^2/b - 1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (-b + b - 1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (-x^2/b^2) Fb^2 1/EJ dx = [-1/3 x^3/b^2]_0^b Fb^2 1/EJ$$

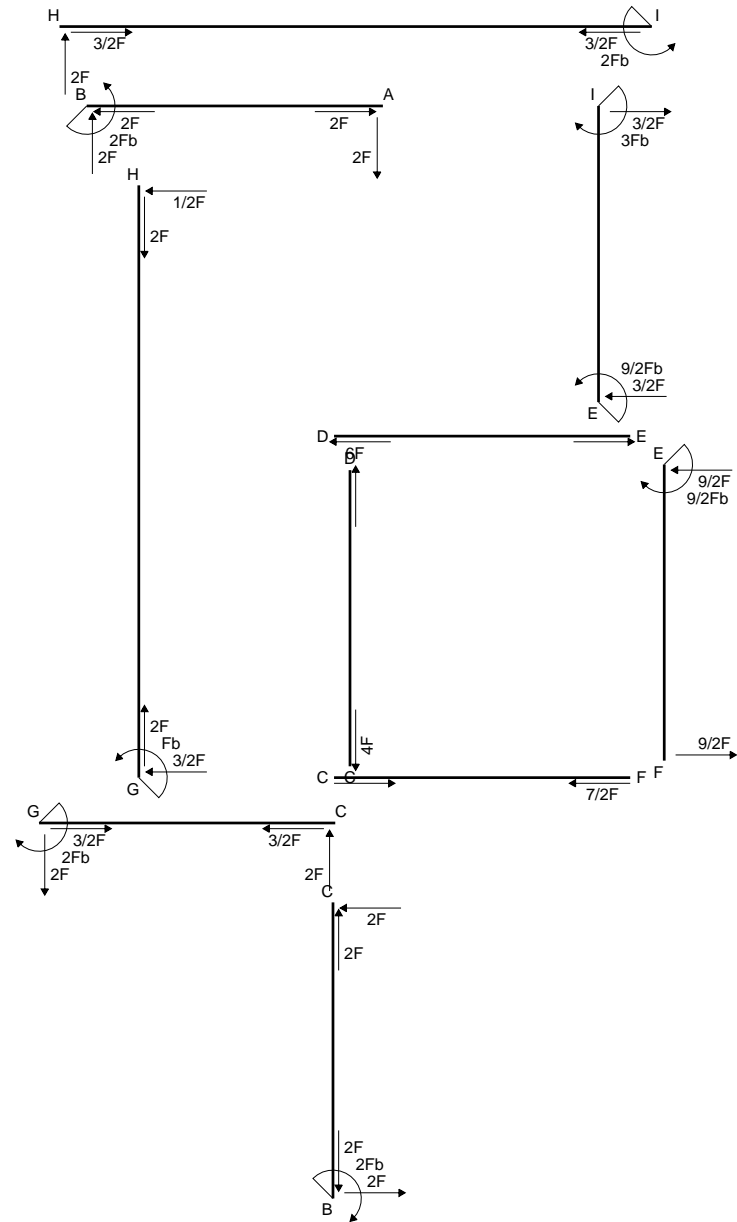
$$= (-1/3 b) Fb^2 1/EJ = -1/3 Fb^3/EJ$$

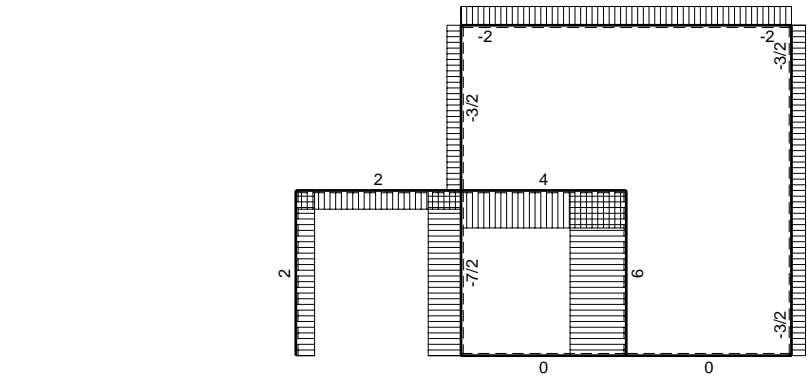




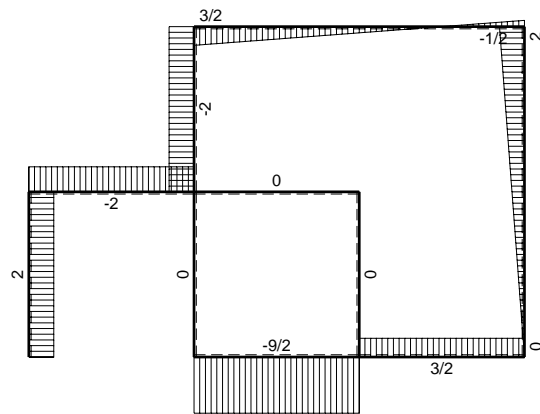
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $P_{HI} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
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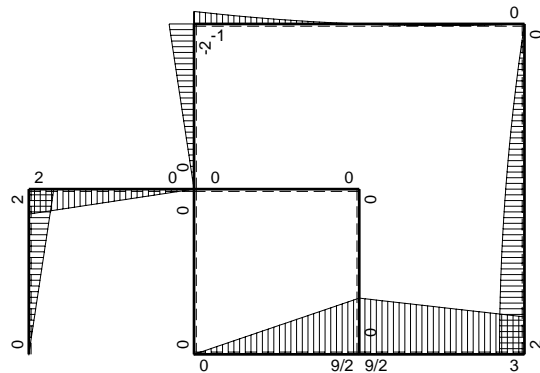




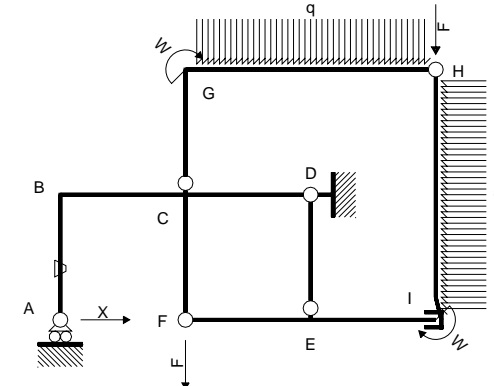
← (+) → F



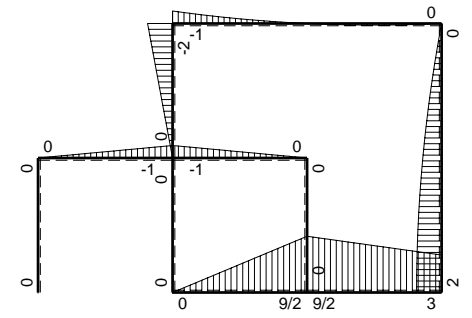
↑ (+) ↓ F



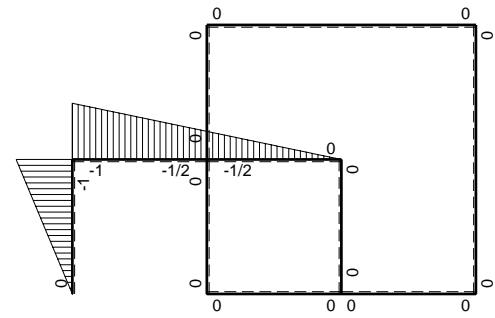
⊙ (+) ⊙ F<sub>b</sub>



Schema di calcolo iperstatico



⊙ (+) ⊙ M<sub>o</sub> flessione da carichi assegnati



⊙ (+) ⊙ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H<sub>A</sub>

| →     | M <sub>x</sub> (x)                             | M <sub>0</sub> (x)           | θ      | M <sub>x</sub> M <sub>0</sub>              | M <sub>x</sub> θ           | M <sub>x</sub> M <sub>x</sub>              | ∫M <sub>x</sub> (M <sub>0</sub> /EJ+θ)dx | ∫XM <sub>x</sub> M <sub>x</sub> /EJdx |
|-------|--|------------------------------|--------|--|----------------------------|--|--|---------------------------------------|
| AB b  | -x   | 0                            | -Fb/EJ | 0  | Fxb/EJ                     | x <sup>2</sup>                             | (0+1/2)Fb <sup>3</sup> /EJ               | 1/3Xb <sup>3</sup> /EJ                |
| BA b  | b-x  | 0                            | Fb/EJ  | 0  | Fb <sup>2</sup> /EJ-Fxb/EJ | b <sup>2</sup> -2bx+x <sup>2</sup>         |  |                                       |
| BC b  | -b+1/2x  | -Fx                          | 0      | Fbx-1/2Fx <sup>2</sup>                     | 0                          | b <sup>2</sup> -bx+1/4x <sup>2</sup>       | (1/3+0)Fb <sup>3</sup> /EJ               | 7/12Xb <sup>3</sup> /EJ               |
| CB b  | 1/2b+1/2x                                      | Fb-Fx                        | 0      | 1/2Fb <sup>2</sup> -1/2Fx <sup>2</sup>     | 0                          | 1/4b <sup>2</sup> +1/2bx+1/4x <sup>2</sup> |  |                                       |
| CD b  | -1/2b+1/2x                                     | -Fb+Fx                       | 0      | 1/2Fb <sup>2</sup> -Fbx+1/2Fx <sup>2</sup> | 0                          | 1/4b <sup>2</sup> -1/2bx+1/4x <sup>2</sup> | (1/6+0)Fb <sup>3</sup> /EJ               | 1/12Xb <sup>3</sup> /EJ               |
| DC b  | 1/2x   | Fx                           | 0      | 1/2Fx <sup>2</sup>                         | 0                          | 1/4x <sup>2</sup>                          |  |                                       |
| DE b  | 0  | 0                            | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| ED b  | 0  | 0                            | 0      | 0  | 0                          | 0  |  |                                       |
| EF b  | 0  | 9/2Fb-9/2Fx                  | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| FE b  | 0  | -9/2Fx                       | 0      | 0  | 0                          | 0  |  |                                       |
| FC b  | 0  | 0                            | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| CF b  | 0  | 0                            | 0      | 0  | 0                          | 0  |  |                                       |
| CG b  | 0  | -2Fx                         | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| GC b  | 0  | 2Fb-2Fx                      | 0      | 0  | 0                          | 0  |  |                                       |
| GH 2b | 0  | -Fb+3/2Fx-1/2qx <sup>2</sup> | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| HG 2b | 0  | -1/2Fx+1/2qx <sup>2</sup>    | 0      | 0  | 0                          | 0  |  |                                       |
| HI 2b | 0  | 2Fx-1/2qx <sup>2</sup>       | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| IH 2b | 0  | -2Fb+1/2qx <sup>2</sup>      | 0      | 0  | 0                          | 0  |  |                                       |
| IE b  | 0  | 3Fb+3/2Fx                    | 0      | 0  | 0                          | 0  | 0+0                                      | 0                                     |
| EI b  | 0  | -9/2Fb+3/2Fx                 | 0      | 0  | 0                          | 0  |  |                                       |
| A     | cedimento nodo -H <sub>1A</sub> u <sub>A</sub> |                              |        |  |                            |  | Fb <sup>3</sup> /EJ                      |                                       |
|       | totali   |                              |        |  |                            |  | 2Fb <sup>3</sup> /EJ                     | Xb <sup>3</sup> /EJ                   |
|       | iperstatica X=H <sub>A</sub>                   |                              |        |  |                            |  | -2F                                      |                                       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - 1/2 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) b^2 1/EJ = 7/12 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) b^2 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) b^2 1/EJ dx = [1/12 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/12 b) b^2 1/EJ = 1/12 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

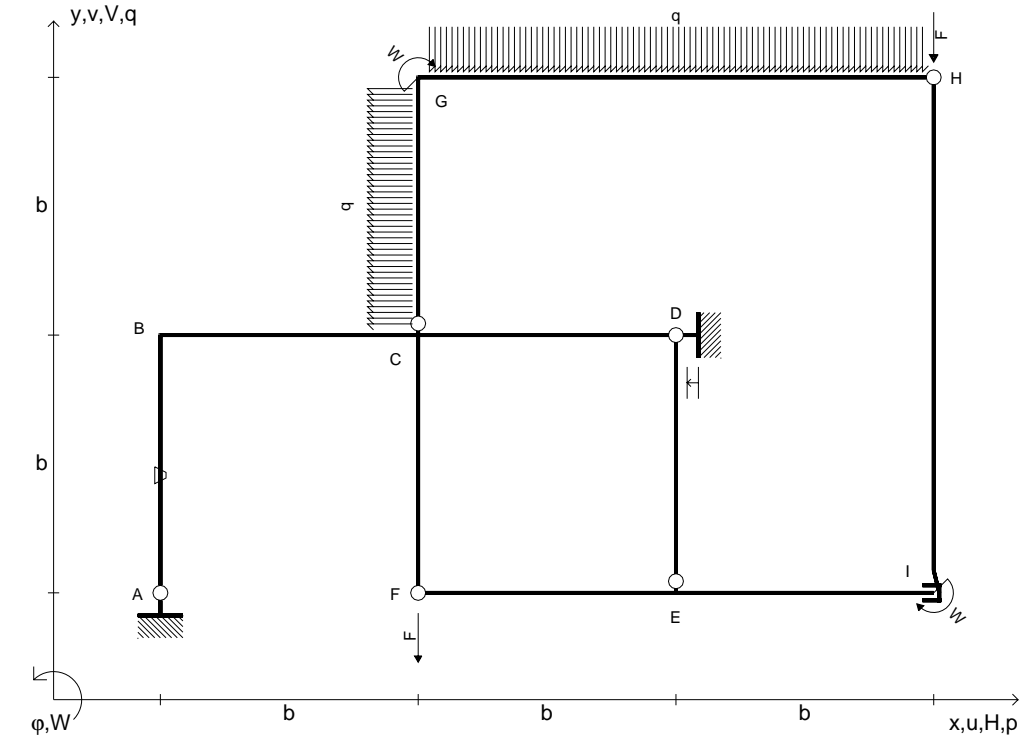
$$= (1/2 b - 1/6 b) Fb^2 1/EJ = 1/3 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb^2 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$

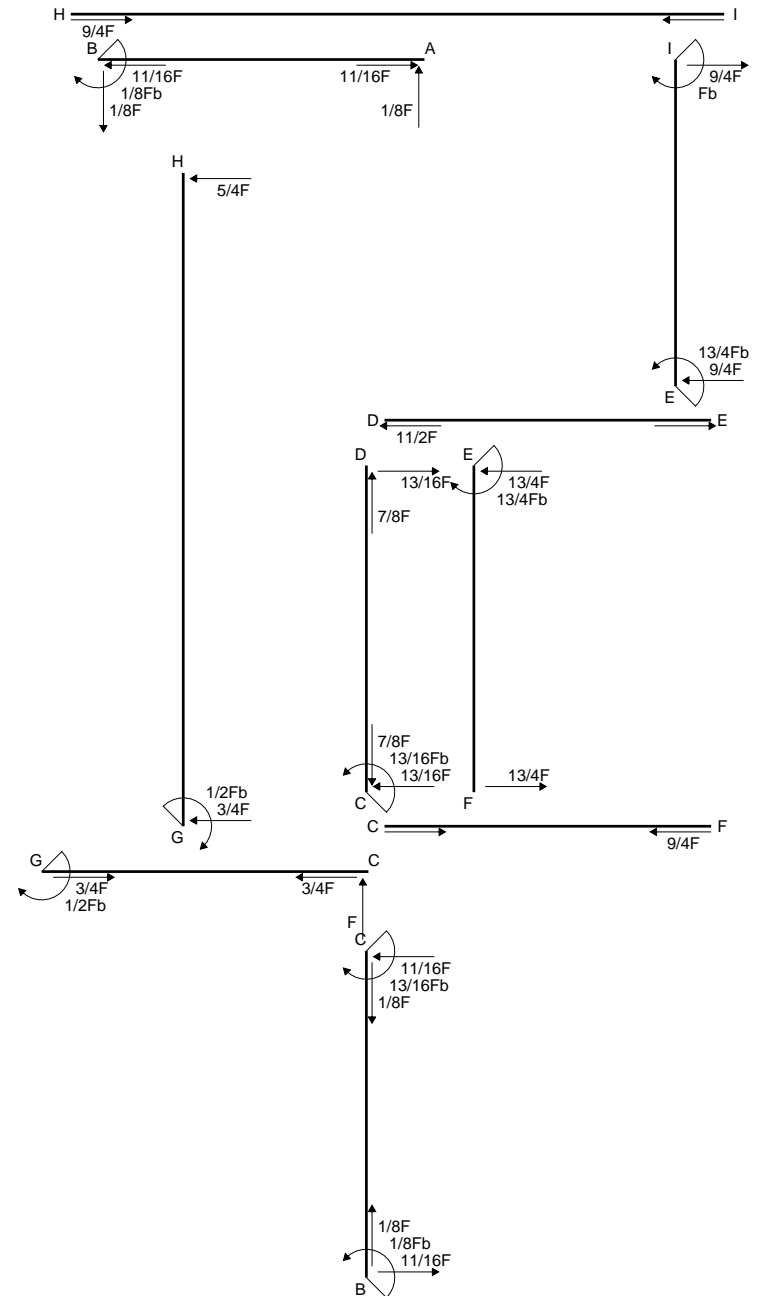
$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb^2 1/EJ dx = [1/6 x^3/b^2]_0^b Fb^2 1/EJ$$

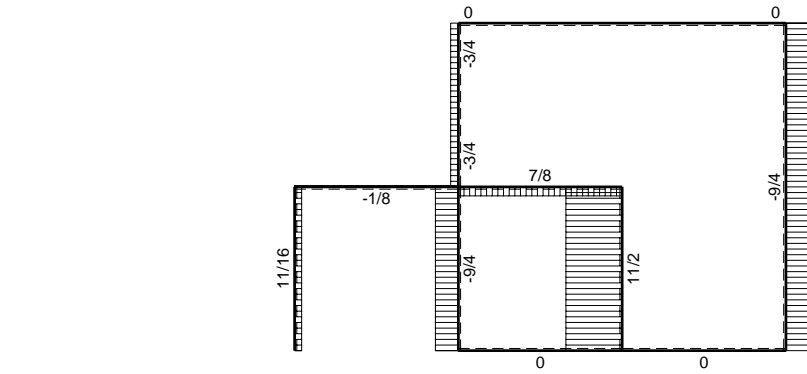
$$= (1/6 b) Fb^2 1/EJ = 1/6 Fb^3/EJ$$



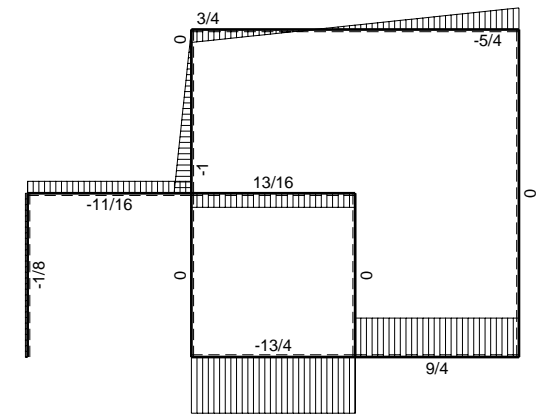
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $P_{CG} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
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 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
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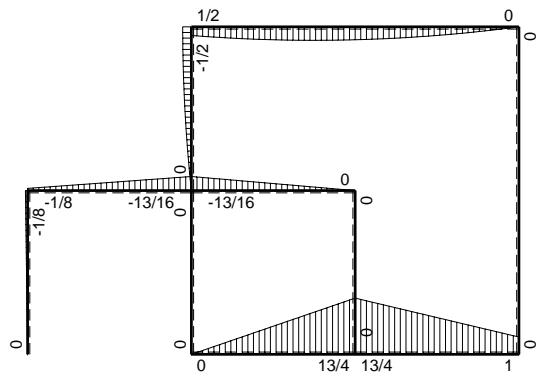




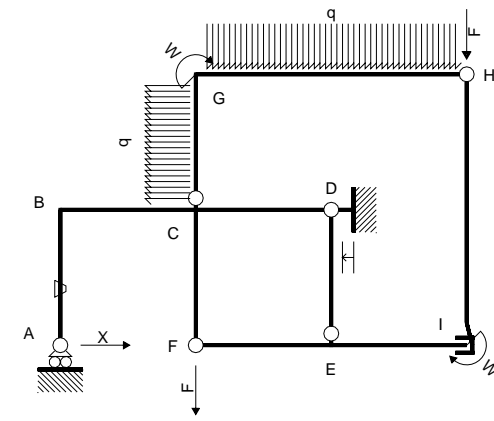
← (+) → F



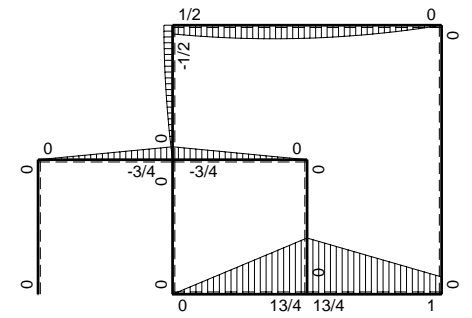
↑ (+) ↓ F



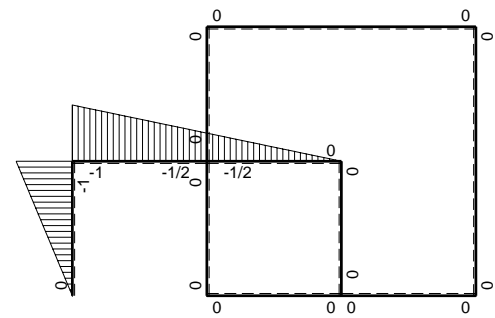
⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico



⊕ (+) ⊖ M<sub>o</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$              | $\theta$ | $M_x M_o$                | $M_x \theta$     | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |           |
|-------|-----------------------------|-----------------------|----------|--------------------------|------------------|-----------------------|-----------------------------|--------------------------|-----------|
| AB b  | -x                          | 0                     | -Fb/EJ   | 0                        | Fxb/EJ           | $x^2$                 | $(0+1/2)Fb^3/EJ$            | $1/3Xb^3/EJ$             |           |
| BA b  | b-x                         | 0                     | Fb/EJ    | 0                        | $Fb^2/EJ-Fxb/EJ$ | $b^2-2bx+x^2$         |                             |                          |           |
| BC b  | -b+1/2x                     | -3/4Fx                | 0        | $3/4Fbx-3/8Fx^2$         | 0                | $b^2-bx+1/4x^2$       | $(1/4+0)Fb^3/EJ$            | $7/12Xb^3/EJ$            |           |
| CB b  | $1/2b+1/2x$                 | $3/4Fb-3/4Fx$         | 0        | $3/8Fb^2-3/8Fx^2$        | 0                | $1/4b^2+1/2bx+1/4x^2$ |                             |                          |           |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$        | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | 0                | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12Xb^3/EJ$            |           |
| DC b  | $1/2x$                      | $3/4Fx$               | 0        | $3/8Fx^2$                | 0                | $1/4x^2$              |                             |                          |           |
| DE b  | 0                           | 0                     | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| ED b  | 0                           | 0                     | 0        | 0                        | 0                | 0                     |                             |                          |           |
| EF b  | 0                           | $13/4Fb-13/4Fx$       | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| FE b  | 0                           | $-13/4Fx$             | 0        | 0                        | 0                | 0                     |                             |                          |           |
| FC b  | 0                           | 0                     | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| CF b  | 0                           | 0                     | 0        | 0                        | 0                | 0                     |                             |                          |           |
| CG b  | 0                           | $-Fx+1/2qx^2$         | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| GC b  | 0                           | $1/2Fb-1/2qx^2$       | 0        | 0                        | 0                | 0                     |                             |                          |           |
| GH 2b | 0                           | $1/2Fb+3/4Fx-1/2qx^2$ | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| HG 2b | 0                           | $-5/4Fx+1/2qx^2$      | 0        | 0                        | 0                | 0                     |                             |                          |           |
| HI 2b | 0                           | 0                     | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| IH 2b | 0                           | 0                     | 0        | 0                        | 0                | 0                     |                             |                          |           |
| IE b  | 0                           | $Fb+9/4Fx$            | 0        | 0                        | 0                | 0                     | 0+0                         | 0                        |           |
| EI b  | 0                           | $-13/4Fb+9/4Fx$       | 0        | 0                        | 0                | 0                     |                             |                          |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                       |          |                          |                  |                       |                             | $-Fb^3/EJ$               |           |
|       | totali                      |                       |          |                          |                  |                       |                             | $-1/8Fb^3/EJ$            | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                       |          |                          |                  |                       |                             | $1/8F$                   |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b \left( \frac{x}{b} \right) \theta dx = \left[ \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b \left( -1 + x/b \right) \theta dx = \left[ -x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( -b + \frac{1}{2} b \right) \theta = \frac{1}{2} Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{3}{4} \frac{x}{b} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} \frac{x^2}{b} - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{4} Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} x - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{4} Fb^3/EJ$$

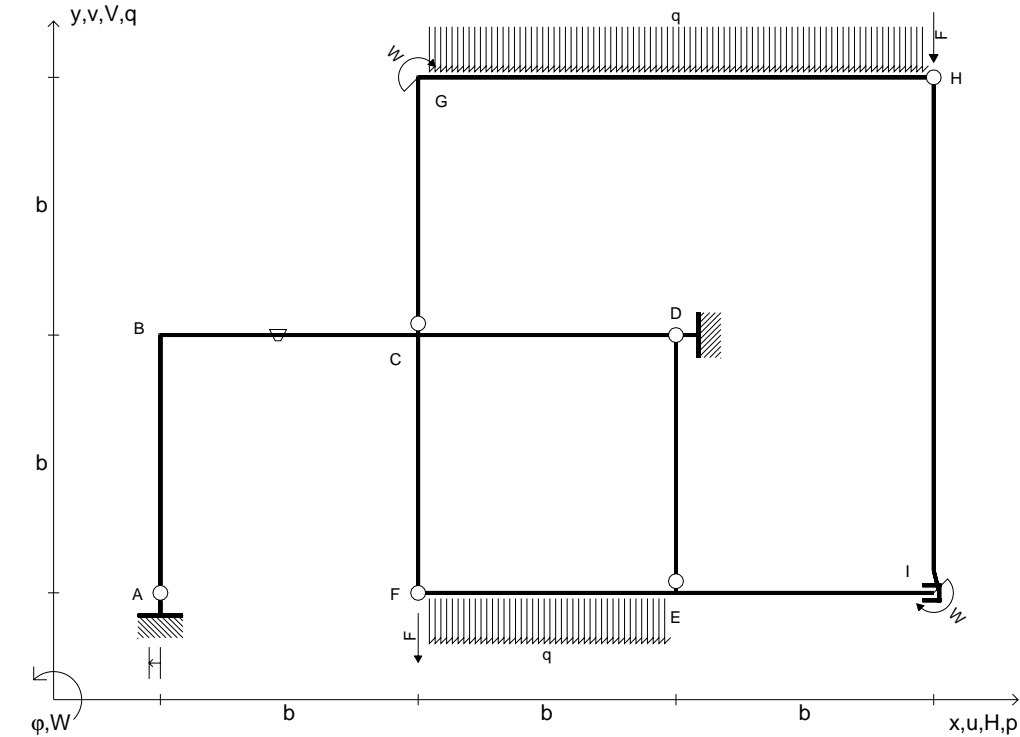
$$L_{CD}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{4} \frac{x}{b} + \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} x - \frac{3}{8} \frac{x^2}{b} + \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{3}{8} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

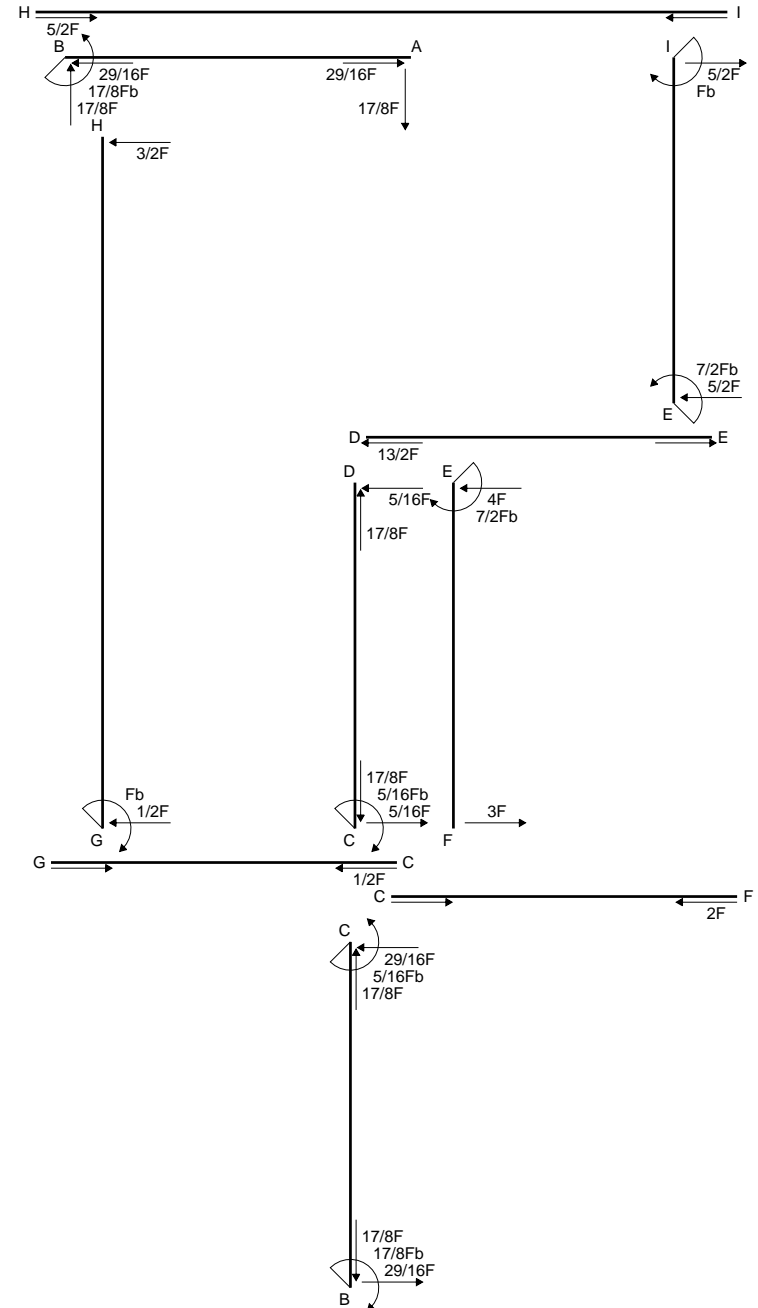
$$= \left( \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

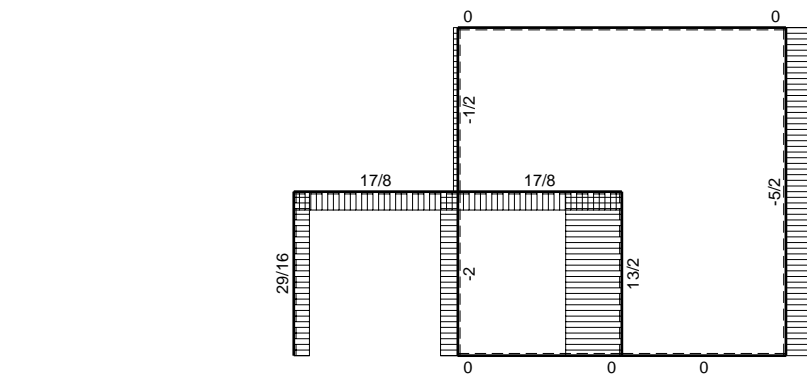




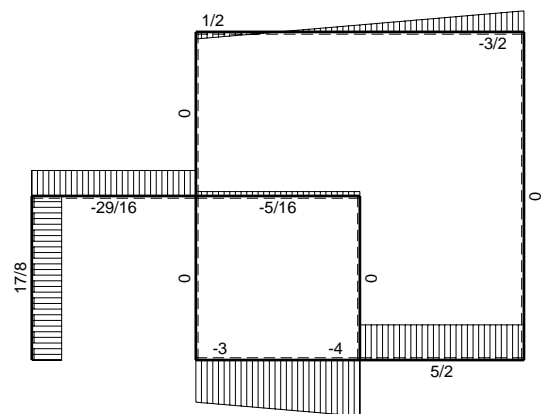
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_A = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo A.  
 © Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

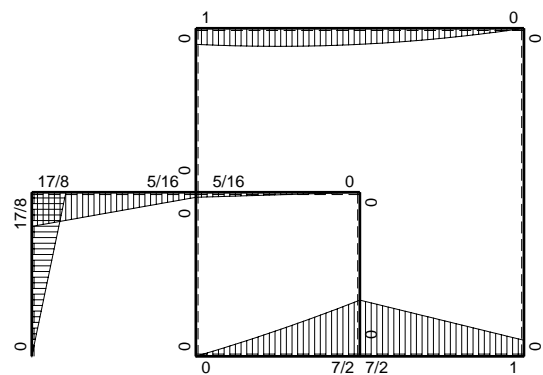




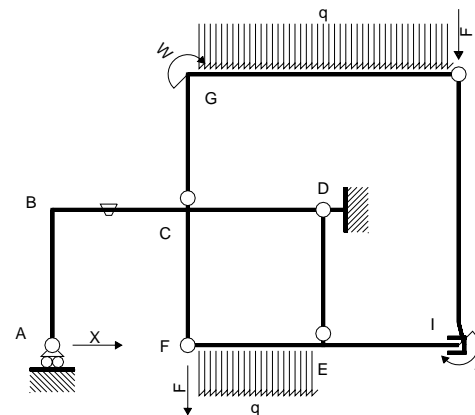
← (+) → F



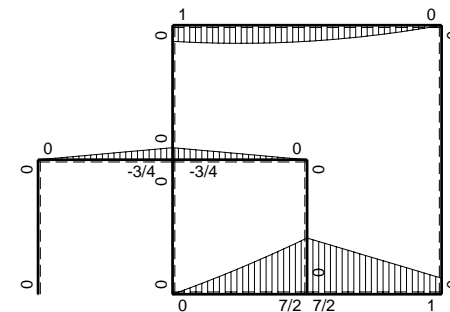
↑ (+) ↓ F



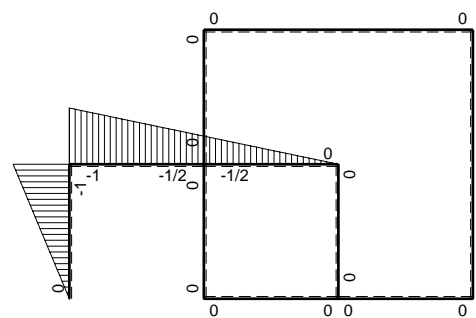
⌚ (+) ⌚ F<sub>b</sub>



Schema di calcolo iperstatico



⌚ (+) ⌚ M<sub>o</sub> flessione da carichi assegnati



⌚ (+) ⌚ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------------|-----------------------|-----------------------------|------------------------|
| AB b  | -x                          | 0                   | 0        | 0                        | 0                      | $x^2$                 | 0+0                         | $1/3Xb^3/EJ$           |
| BA b  | b-x                         | 0                   | 0        | 0                        | 0                      | $b^2-2bx+x^2$         |                             |                        |
| BC b  | -b+1/2x                     | -3/4Fx              | -Fb/EJ   | $3/4Fbx-3/8Fx^2$         | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/4+3/4)Fb^3/EJ$          | $7/12Xb^3/EJ$          |
| CB b  | $1/2b+1/2x$                 | $3/4Fb-3/4Fx$       | Fb/EJ    | $3/8Fb^2-3/8Fx^2$        | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12Xb^3/EJ$          |
| DC b  | $1/2x$                      | $3/4Fx$             | 0        | $3/8Fx^2$                | 0                      | $1/4x^2$              |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                        | 0                      | 0                     |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$  | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$    | 0        | 0                        | 0                      | 0                     |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |
| IE b  | 0                           | $Fb+5/2Fx$          | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+5/2Fx$      | 0        | 0                        | 0                      | 0                     |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |                        |                       | $Fb^3/EJ$                   |                        |
|       | totali                      |                     |          |                          |                        |                       | $17/8Fb^3/EJ$               | $Xb^3/EJ$              |
|       | iperstatica $X=H_A$         |                     |          |                          |                        |                       | -17/8F                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{3}{4} \frac{x}{b} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( 1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{3}{8} \frac{x^2}{b} - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( b - \frac{1}{4} b \right) \theta = Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{3}{8} x - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

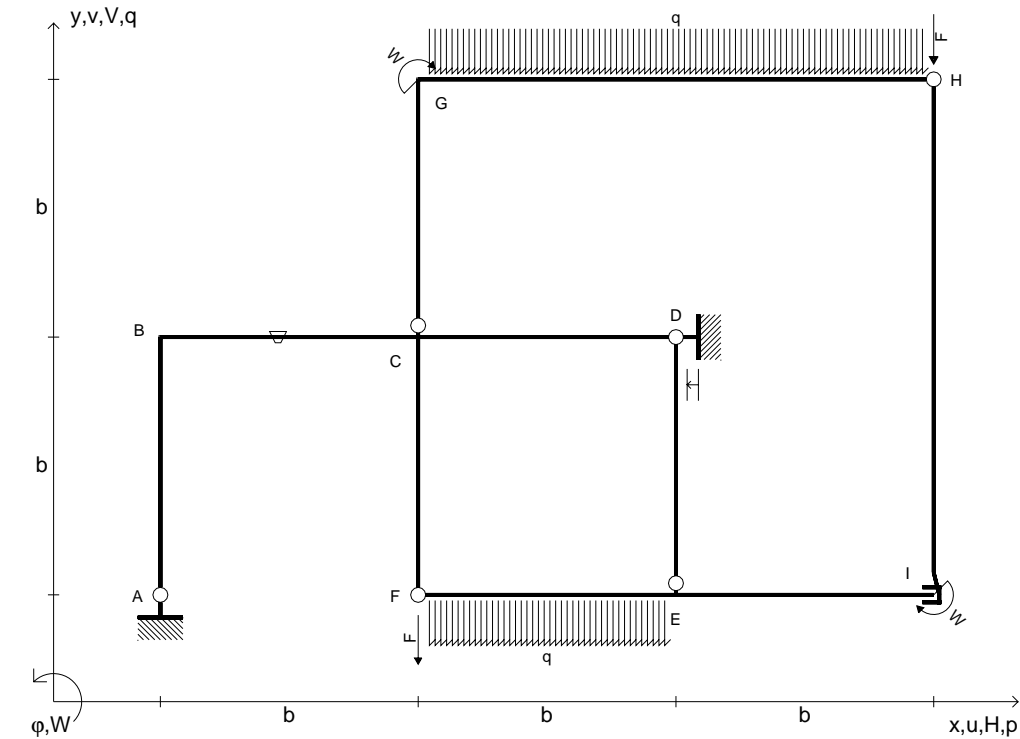
$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{2} b - \frac{1}{4} b \right) \theta = Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{4} \frac{x}{b} + \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} x - \frac{3}{8} \frac{x^2}{b} + \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{3}{8} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

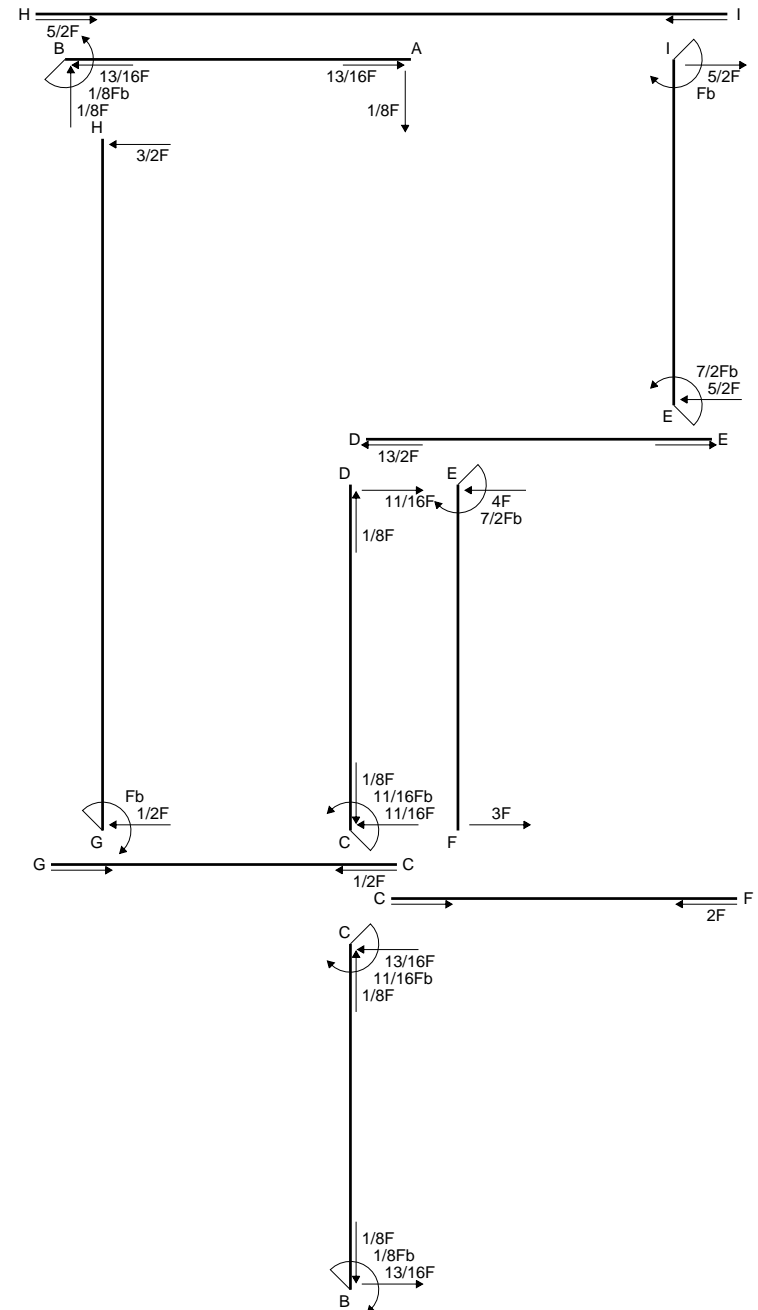
$$L_{DC}^{xo} = \int_0^b \left( \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

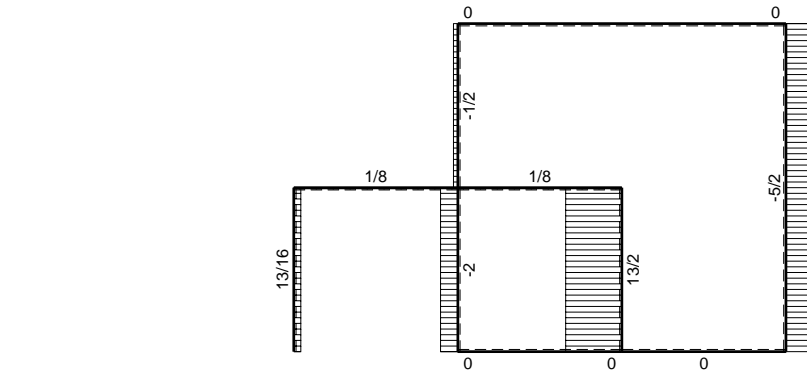
$$= \left( \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$



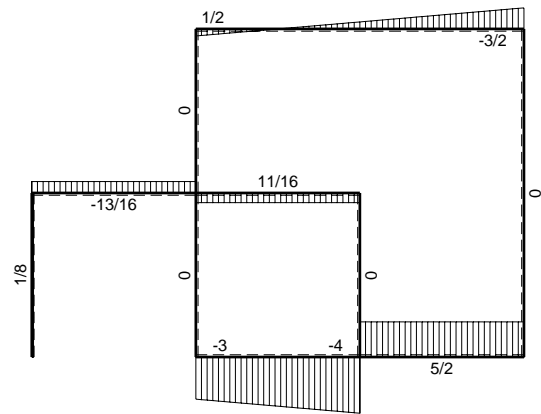
|                      |  |                |
|----------------------|--|----------------|
| $V_H = -F$           | $\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$ | $EJ_{EF} = EJ$ |
| $V_F = -F$           | $u_D = -\delta = -b^3F/EJ$                     | $EJ_{FC} = EJ$ |
| $W_I = -W = -Fb$     | $EJ_{AB} = EJ$                                 | $EJ_{CG} = EJ$ |
| $W_G = -W = -Fb$     | $EJ_{BC} = EJ$                                 | $EJ_{GH} = EJ$ |
| $q_{GH} = -q = -F/b$ | $EJ_{CD} = EJ$                                 | $EJ_{HI} = EJ$ |
| $q_{EF} = -q = -F/b$ | $EJ_{DE} = EJ$                                 | $EJ_{IE} = EJ$ |

Reazioni iperstatiche in soluzione:  $X=H_A$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi x,y.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 Spostamento orizzontale assoluto u imposto al nodo D.  
 © Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

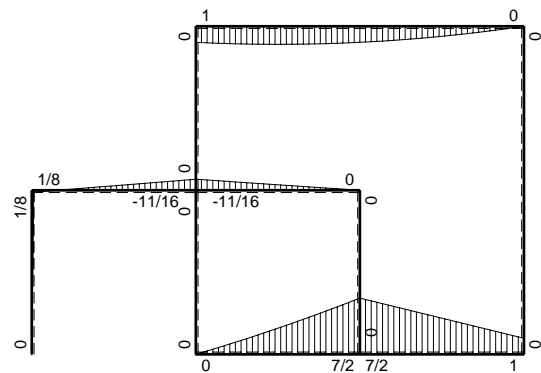




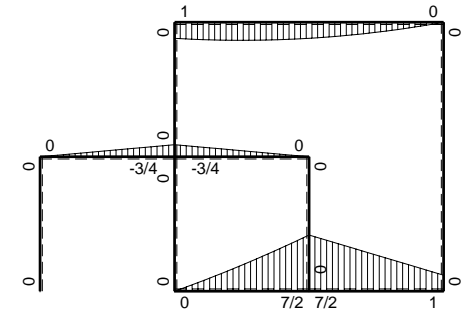
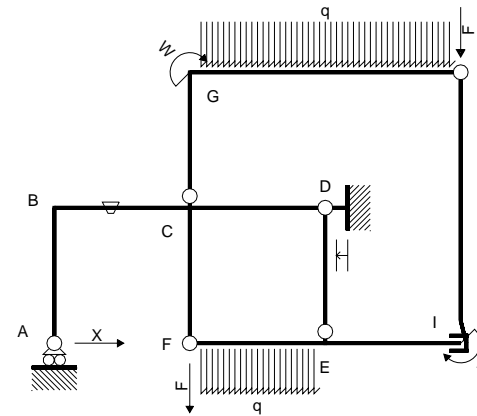
← (+) → F



↑ (+) ↓ F

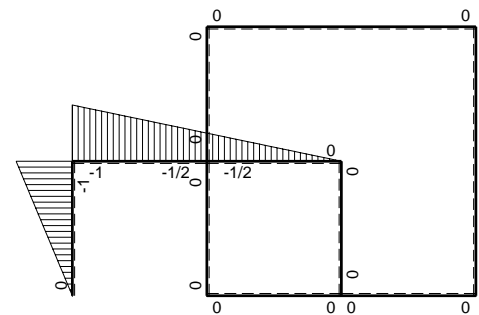


⊕ (+) ⊖ F<sub>b</sub>



Schema di calcolo iperstatico

⊕ (+) ⊖ M<sub>0</sub> flessione da carichi assegnati



⊕ (+) ⊖ M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=H_A$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$           | $M_x M_x$             | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |           |
|-------|-----------------------------|---------------------|----------|--------------------------|------------------------|-----------------------|-----------------------------|------------------------|-----------|
| AB b  | -x                          | 0                   | 0        | 0                        | 0                      | $x^2$                 | 0+0                         | $1/3Xb^3/EJ$           |           |
| BA b  | b-x                         | 0                   | 0        | 0                        | 0                      | $b^2-2bx+x^2$         |                             |                        |           |
| BC b  | -b+1/2x                     | -3/4Fx              | -Fb/EJ   | $3/4Fbx-3/8Fx^2$         | $Fb^2/EJ-1/2Fxb/EJ$    | $b^2-bx+1/4x^2$       | $(1/4+3/4)Fb^3/EJ$          | $7/12Xb^3/EJ$          |           |
| CB b  | $1/2b+1/2x$                 | $3/4Fb-3/4Fx$       | Fb/EJ    | $3/8Fb^2-3/8Fx^2$        | $1/2Fb^2/EJ+1/2Fxb/EJ$ | $1/4b^2+1/2bx+1/4x^2$ |                             |                        |           |
| CD b  | $-1/2b+1/2x$                | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb^2-3/4Fbx+3/8Fx^2$ | 0                      | $1/4b^2-1/2bx+1/4x^2$ | $(1/8+0)Fb^3/EJ$            | $1/12Xb^3/EJ$          |           |
| DC b  | $1/2x$                      | $3/4Fx$             | 0        | $3/8Fx^2$                | 0                      | $1/4x^2$              |                             |                        |           |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$  | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$    | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| IE b  | 0                           | $Fb+5/2Fx$          | 0        | 0                        | 0                      | 0                     | 0+0                         | 0                      |           |
| EI b  | 0                           | $-7/2Fb+5/2Fx$      | 0        | 0                        | 0                      | 0                     |                             |                        |           |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                        |                       |                             | $-Fb^3/EJ$             |           |
|       | totali                      |                     |          |                          |                        |                       |                             | $1/8Fb^3/EJ$           | $Xb^3/EJ$ |
|       | iperstatica $X=H_A$         |                     |          |                          |                        |                       |                             | $-1/8F$                |           |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) b^2 \frac{1}{EJ} dx = \left[ x - x^2/b + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) b^2 \frac{1}{EJ} = \frac{1}{3} b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{7}{12} b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) b^2 \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b b^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) b^2 \frac{1}{EJ} = \frac{1}{12} b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{3}{4} \frac{x}{b} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( 1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{3}{8} \frac{x^2}{b} - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( b - \frac{1}{4} b \right) \theta = Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{3}{8} x - \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ} + \left[ -\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{3}{8} b - \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} + \left( -\frac{1}{2} b - \frac{1}{4} b \right) \theta = Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b \left( \frac{3}{8} - \frac{3}{4} \frac{x}{b} + \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{3}{8} x - \frac{3}{8} \frac{x^2}{b} + \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{3}{8} b - \frac{3}{8} b + \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{3}{8} \frac{x^2}{b^2} \right) Fb^2 \frac{1}{EJ} dx = \left[ \frac{1}{8} \frac{x^3}{b^2} \right]_0^b Fb^2 \frac{1}{EJ}$$

$$= \left( \frac{1}{8} b \right) Fb^2 \frac{1}{EJ} = \frac{1}{8} Fb^3/EJ$$