

$V_I = -F$	$EJ_{AB} = EJ$	$EJ_{GH} = EJ$
$W_F = -W = -Fb$	$EJ_{BC} = EJ$	$EJ_{HI} = EJ$
$p_{ED} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{IF} = EJ$
$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{ED} = EJ$	$EJ_{HB} = EJ$
$v_A = -\delta = -b^3F/EJ$	$EJ_{EF} = EJ$	
$k_{CD} = 4EJ/b$	$EJ_{FG} = EJ$	

ANALISI STRUTTURE IPERSTATICHE
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Riportare sul fronte:

- 1) Declassamento Scelto
- 2) Reazioni calcolate
- 3) Diagrammi finali delle azioni interne

Sul retro:

- 4) Analisi cinematica
- 5) Diagrammi delle strutture M_0 e M^*
- 6) Equazione del PLV
- 7) Reazione vincolare calcolata

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

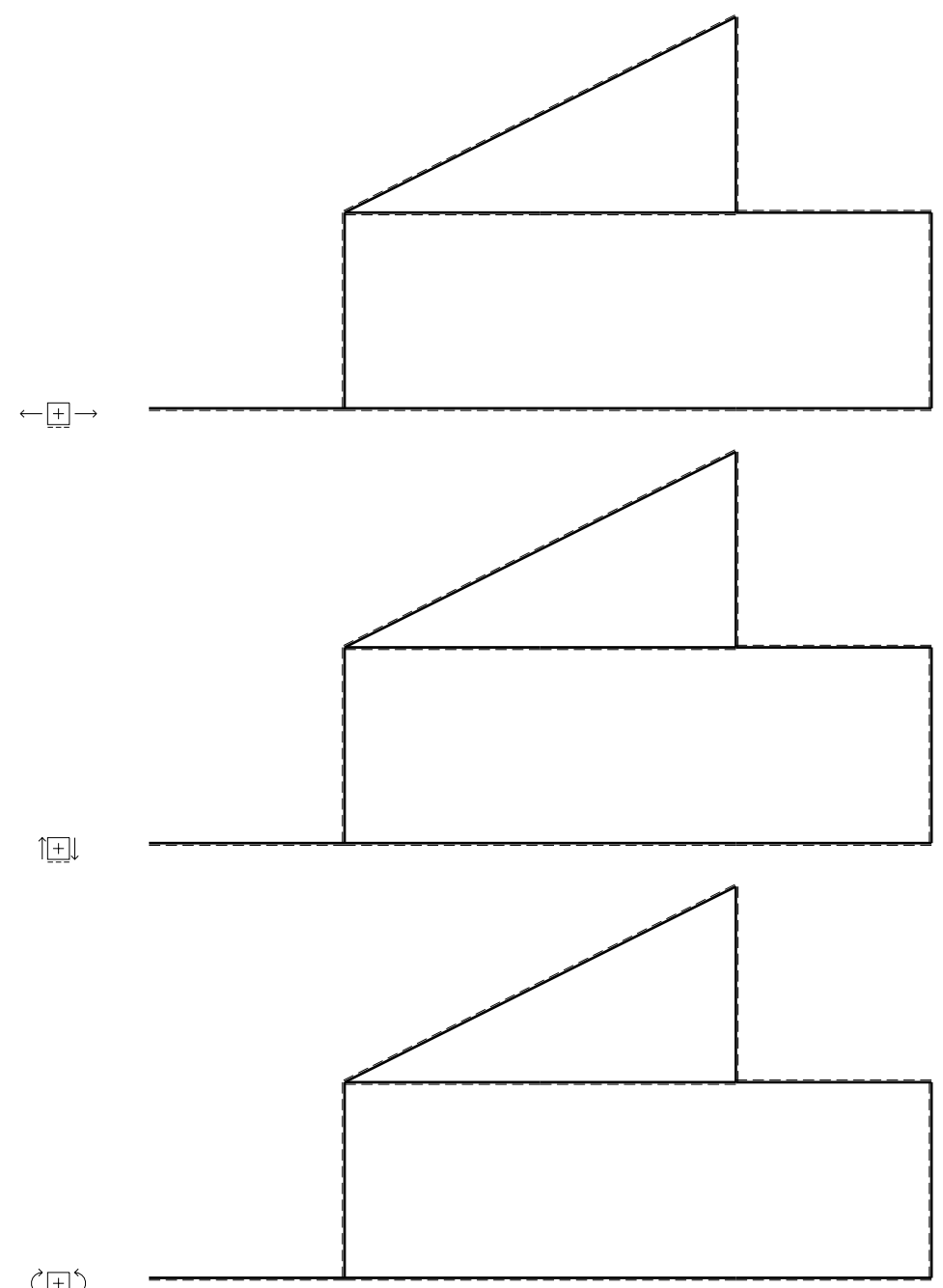
Carichi di aste curve misurati in proiezione sugli assi x, y .

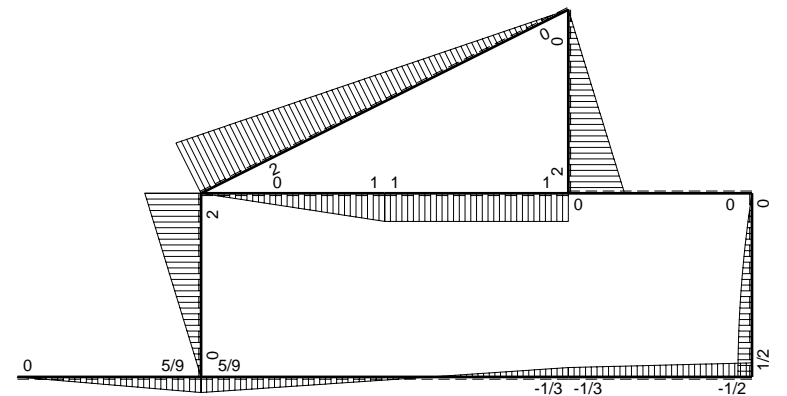
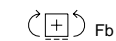
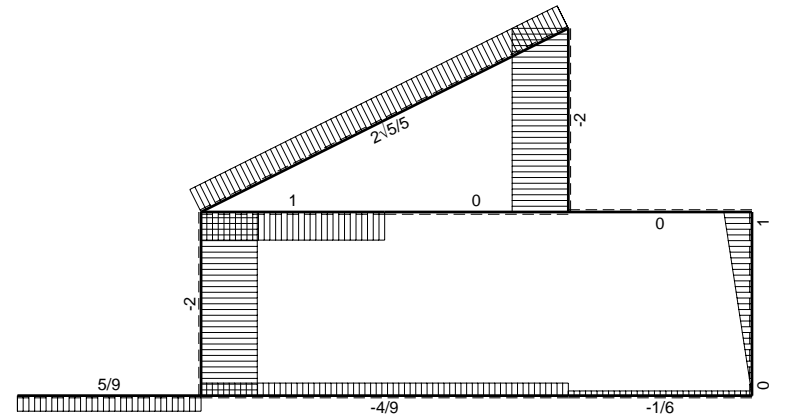
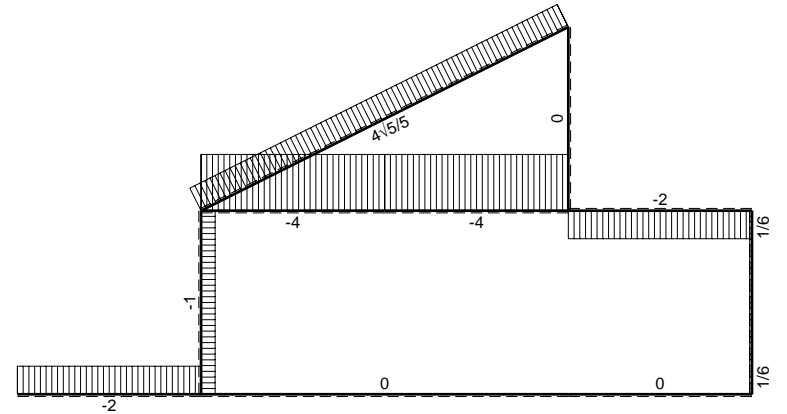
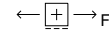
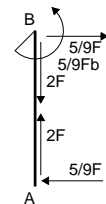
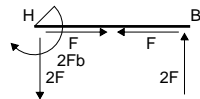
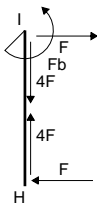
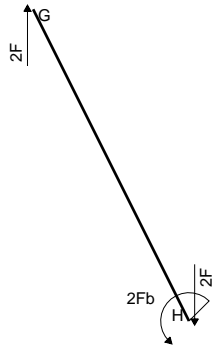
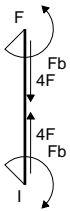
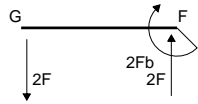
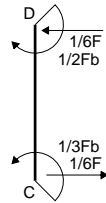
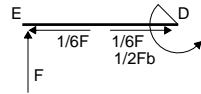
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y .

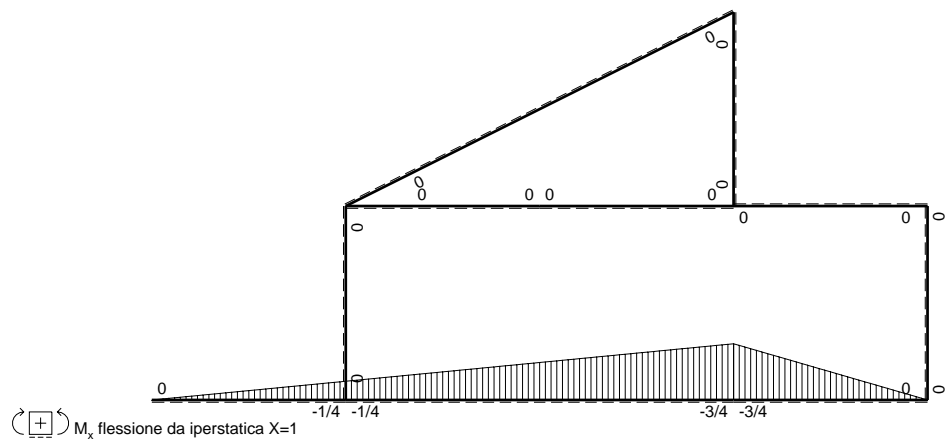
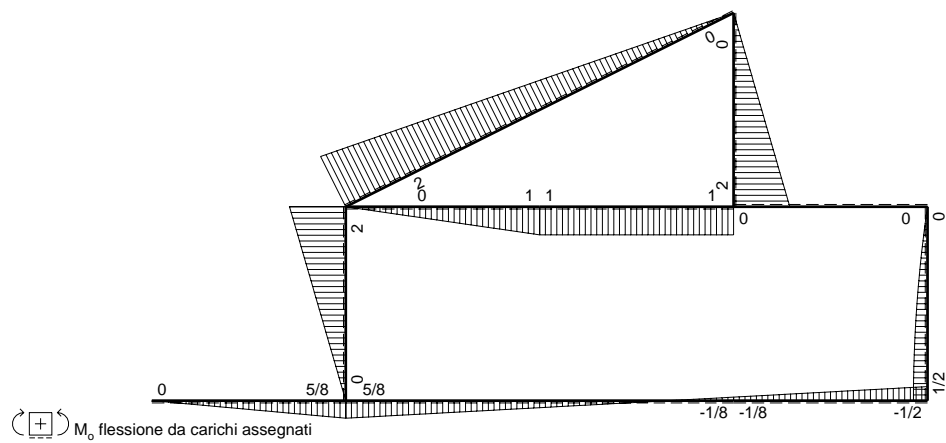
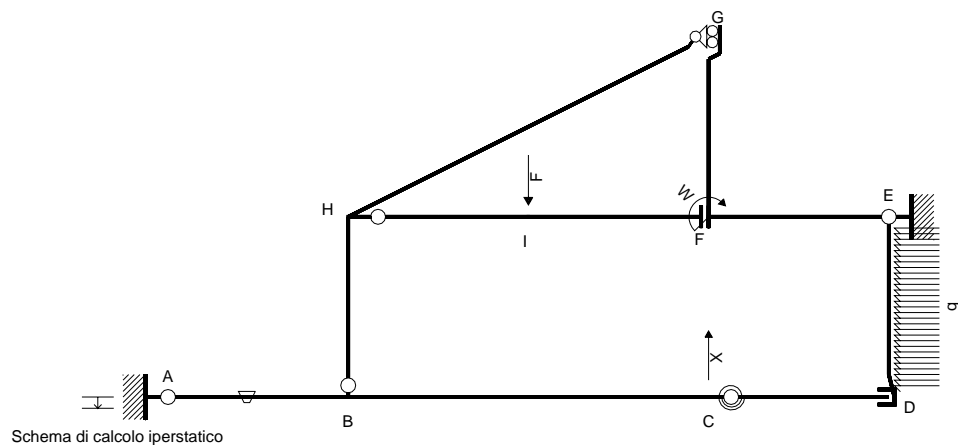
Curvatura θ asta AB positiva se convessa a destra con inizio A .

Spostamento verticale assoluto v imposto al nodo A .

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Quadro contributi PLV per iperstatica $X=V_C$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	$-1/4x$	$5/8Fx$	$-Fb/EJ$	$-5/32Fx^2$	$1/4Fxb/EJ$	$1/16x^2$	$(-5/96+1/8)Fb^3/EJ$	$1/48Xb^3/EJ$	
BA b	$1/4b-1/4x$	$-5/8Fb+5/8Fx$	Fb/EJ	$-5/32Fb^2+5/16Fbx-5/32Fx^2$	$1/4Fb^2/EJ-1/4Fxb/EJ$	$1/16b^2-1/8bx+1/16x^2$			
BC 2b	$-1/4b-1/4x$	$5/8Fb-3/8Fx$	0	$-5/32Fb^2-1/16Fbx+3/32Fx^2$	0	$1/16b^2+1/8bx+1/16x^2$	$(-3/16+0)Fb^3/EJ$	$13/24Xb^3/EJ$	
CB 2b	$3/4b-1/4x$	$1/8Fb-3/8Fx$	0	$3/32Fb^2-5/16Fbx+3/32Fx^2$	0	$9/16b^2-3/8bx+1/16x^2$			
CD b	$-3/4b+3/4x$	$-1/8Fb-3/8Fx$	0	$3/32Fb^2+3/16Fbx-9/32Fx^2$	0	$9/16b^2-9/8bx+9/16x^2$	$(3/32+0)Fb^3/EJ$	$3/16Xb^3/EJ$	
DC b	$3/4x$	$1/2Fb-3/8Fx$	0	$3/8Fbx-9/32Fx^2$	0	$9/16x^2$			
ED b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0	
DE b	0	$-1/2Fb+1/2qx^2$	0	0	0	0			
EF b	0	0	0	0	0	0	0+0	0	
FE b	0	0	0	0	0	0			
FG b	0	$2Fb-2Fx$	0	0	0	0	0+0	0	
GF b	0	$-2Fx$	0	0	0	0			
GH $\sqrt{5}b$	0	$2\sqrt{5}5Fx$	0	0	0	0	0	0	
HI b	0	Fx	0	0	0	0	0+0	0	
IH b	0	$-Fb+Fx$	0	0	0	0			
IF b	0	Fb	0	0	0	0	0+0	0	
FI b	0	$-Fb$	0	0	0	0			
HB b	0	$2Fb-2Fx$	0	0	0	0	0+0	0	
BH b	0	$-2Fx$	0	0	0	0			
CD	molla asta $-W_{1CD}(W_{oCD}+XW_{1CD})/k_{CD}$							$3/128Fb^3/EJ$	$9/64Xb^3/EJ$
A	cedimento nodo $-V_{1A}u_A$							$-1/4Fb^3/EJ$	
	totali							$-95/384Fb^3/EJ$	$57/64Xb^3/EJ$
	iperstatica $X=V_C$							$5/18F$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1/16 x^2/b^2) b^2 1/EJ dx = [1/48 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/48 b) b^2 1/EJ = 1/48 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1/16 - 1/8 x/b + 1/16 x^2/b^2) b^2 1/EJ dx = [1/16 x - 1/16 x^2/b + 1/48 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/16 b - 1/16 b + 1/48 b) b^2 1/EJ = 1/48 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^{2b} (1/16 + 1/8 x/b + 1/16 x^2/b^2) b^2 1/EJ dx = [1/16 x + 1/16 x^2/b + 1/48 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (1/8 b + 1/4 b + 1/6 b) b^2 1/EJ = 13/24 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^{2b} (9/16 - 3/8 x/b + 1/16 x^2/b^2) b^2 1/EJ dx = [9/16 x - 3/16 x^2/b + 1/48 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (9/8 b - 3/4 b + 1/6 b) b^2 1/EJ = 13/24 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (9/16 - 9/8 x/b + 9/16 x^2/b^2) b^2 1/EJ dx + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ$$

$$= [9/16 x - 9/16 x^2/b + 3/16 x^3/b^2]_0^b b^2 1/EJ + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ$$

$$= (9/16 b - 9/16 b + 3/16 b) b^2 1/EJ + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ = 21/64 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (9/16 x^2/b^2) b^2 1/EJ dx + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ = [3/16 x^3/b^2]_0^b b^2 1/EJ + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ$$

$$= (3/16 b) b^2 1/EJ + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ = 21/64 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-5/32 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/4 x/b) \theta dx = [-5/96 x^3/b^2]_0^b Fb^2 1/EJ + [1/8 x^2/b]_0^b \theta$$

$$= (-5/96 b) Fb^2 1/EJ + (1/8 b) \theta = 7/96 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-5/32 + 5/16 x/b - 5/32 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/4 + 1/4 x/b) \theta dx$$

$$= [-5/32 x + 5/32 x^2/b - 5/96 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x + 1/8 x^2/b]_0^b \theta$$

$$= (-5/32 b + 5/32 b - 5/96 b) Fb^2 1/EJ + (-1/4 b + 1/8 b) \theta = 7/96 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^{2b} (-5/32 - 1/16 x/b + 3/32 x^2/b^2) Fb^2 1/EJ dx = [-5/32 x - 1/32 x^2/b + 1/32 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (-5/16 b - 1/8 b + 1/4 b) Fb^2 1/EJ = -3/16 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^{2b} (3/32 - 5/16 x/b + 3/32 x^2/b^2) Fb^2 1/EJ dx = [3/32 x - 5/32 x^2/b + 1/32 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (3/16 b - 5/8 b + 1/4 b) Fb^2 1/EJ = -3/16 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/32 + 3/16 x/b - 9/32 x^2/b^2) Fb^2 1/EJ dx + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ$$

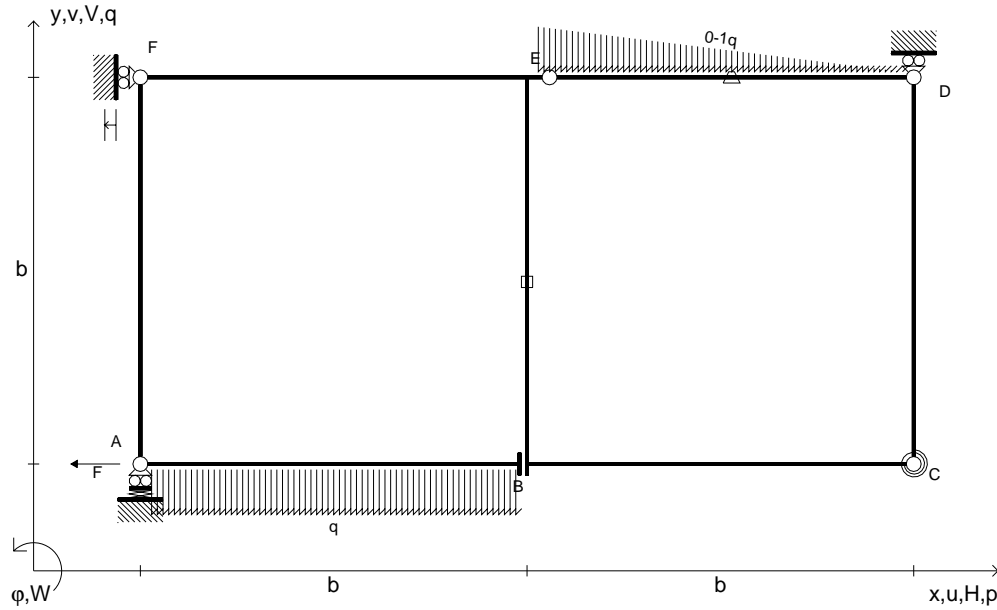
$$= [3/32 x + 3/32 x^2/b - 3/32 x^3/b^2]_0^b Fb^2 1/EJ + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ$$

$$= (3/32 b + 3/32 b - 3/32 b) Fb^2 1/EJ + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ = 15/128 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x/b - 9/32 x^2/b^2) Fb^2 1/EJ dx + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ$$

$$= [3/16 x^2/b - 3/32 x^3/b^2]_0^b Fb^2 1/EJ + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ$$

$$= (3/16 b - 3/32 b) Fb^2 1/EJ + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ = 15/128 Fb^3/EJ$$



$H_A = -F$	$u_F = -\delta = -b^3 F/EJ$	$EJ_{CD} = EJ$
$q_{ED} = -q = -F/b$	$k_{CD} = 4EJ/b$	$EJ_{DE} = EJ$
$q_{AB} = -q = -F/b$	$k_A = 4EJ/b^3$	$EJ_{EB} = EJ$
$\varepsilon_{EB} = -\alpha T = -b^2 F/EJ$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$
$\theta_{DE} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{BC} = EJ$	$EJ_{FA} = EJ$

ANALISI STRUTTURE IPERSTATICHE
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Riportare sul fronte:

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Sul retro:

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- 6) Equazione del PLV
- 7) Reazione vincolare calcolata

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

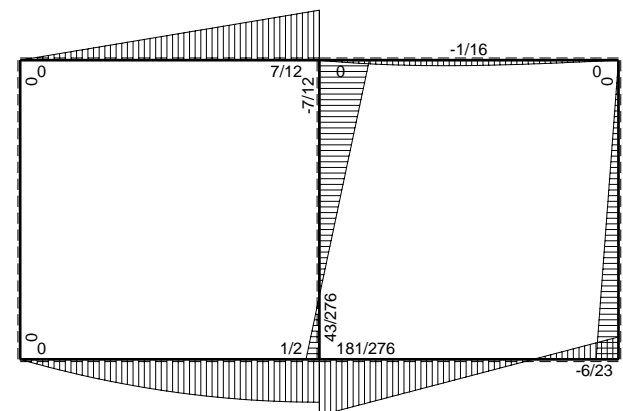
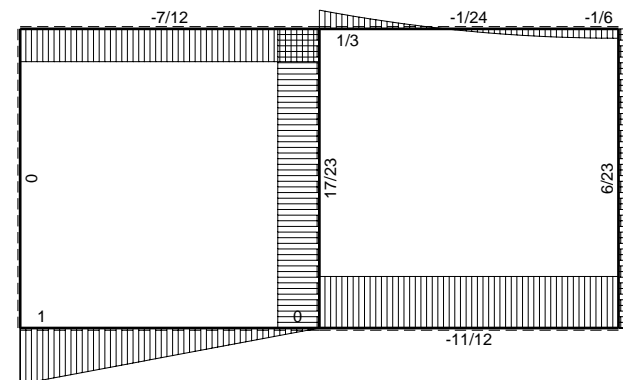
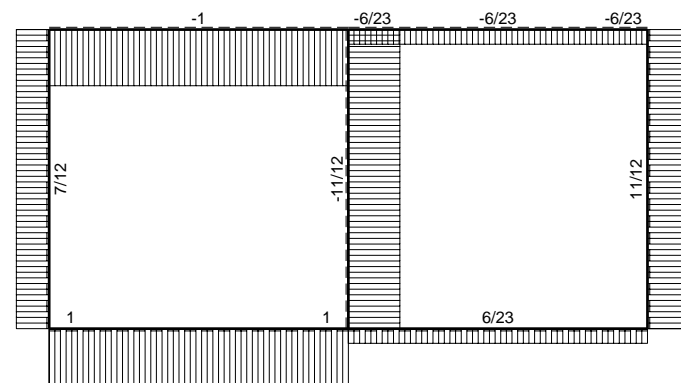
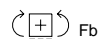
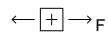
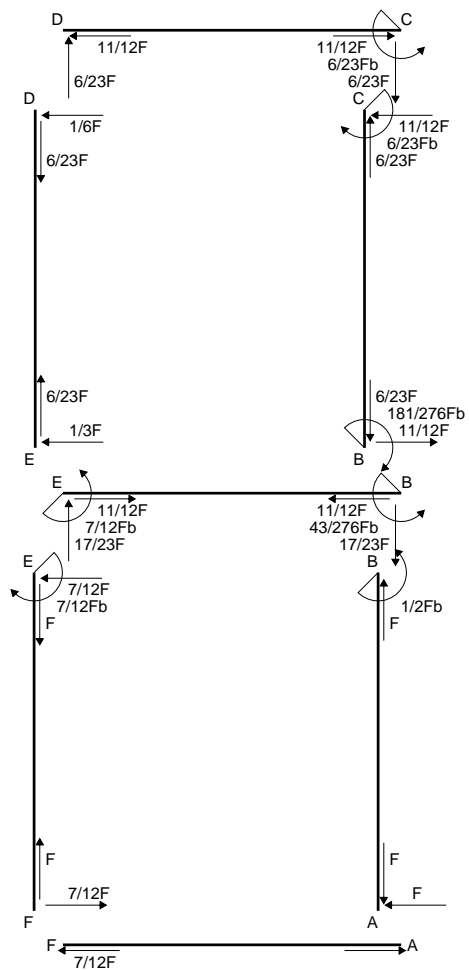
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

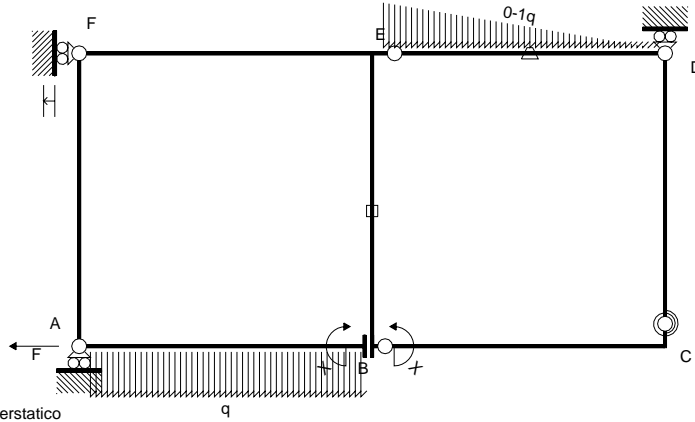
Elongazione termica specifica ε assegnata su asta EB.

Curvatura θ asta DE positiva se convessa a destra con inizio D.

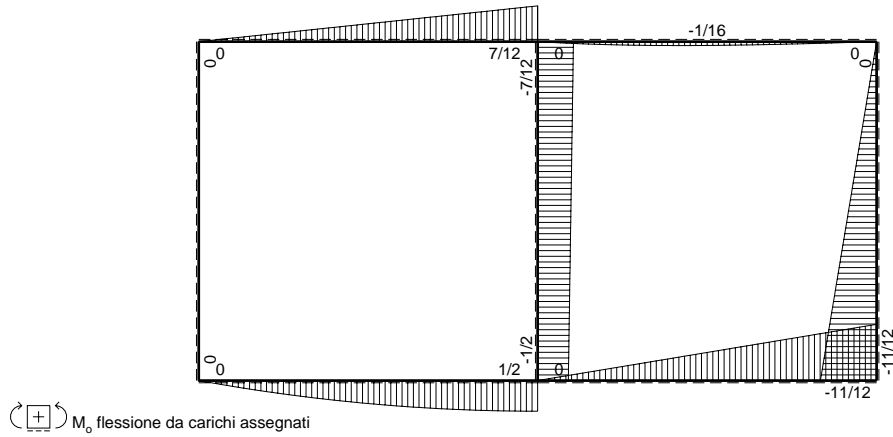
Spostamento orizzontale assoluto u imposto al nodo F.

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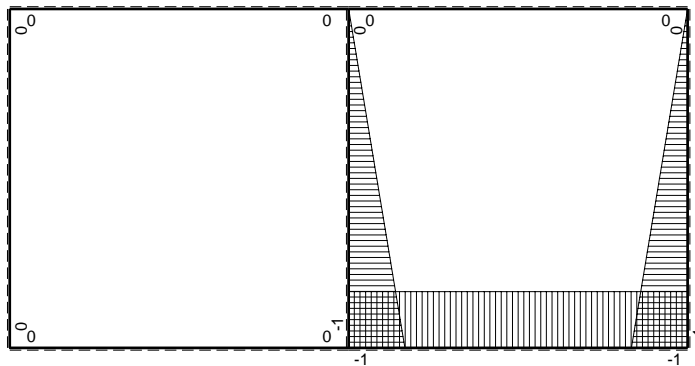




Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{BC}

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x (M_0/EJ+\theta) dx$	$\int X M_x M_x / EJ dx$
AB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fb+1/2qx^2$	0	0	0	0	(11/24+0)Fb ² /EJ	Xb/EJ
BC b	-1	$-11/12Fx$	0	$11/12Fx$	0	1	(11/24+0)Fb ² /EJ	Xb/EJ
CB b	1	$11/12Fb-11/12Fx$	0	$11/12Fb-11/12Fx$	0	1	(11/24+0)Fb ² /EJ	Xb/EJ
CD b	$-1+x/b$	$-11/12Fb+11/12Fx$	0	$11/12Fb-11/12Fx+11/12Fx^2/b$	0	$1-2x/b+x^2/b^2$	(11/36+0)Fb ² /EJ	1/3Xb/EJ
DC b	x/b	$11/12Fx$	0	$11/12Fx^2/b$	0	x^2/b^2	(11/36+0)Fb ² /EJ	1/3Xb/EJ
DE b	0	$-1/6Fx+1/6qx^3/b$	-Fb/EJ	0	0	0	0+0	0
ED b	0	$1/3Fx-1/2qx^2+1/6qx^3/b$	Fb/EJ	0	0	0	0+0	0
EB b	$-x/b$	$-7/12Fb+1/12Fx$	0	$7/12Fx-1/12Fx^2/b$	0	x^2/b^2	(19/72+0)Fb ² /EJ	1/3Xb/EJ
BE b	$1-x/b$	$1/2Fb+1/12Fx$	0	$1/2Fb-5/12Fx-1/12Fx^2/b$	0	$1-2x/b+x^2/b^2$	(19/72+0)Fb ² /EJ	1/3Xb/EJ
EF b	0	$7/12Fb-7/12Fx$	0	0	0	0	0+0	0
FE b	0	$-7/12Fx$	0	0	0	0	0+0	0
FA b	0	0	0	0	0	0	0+0	0
AF b	0	0	0	0	0	0	0+0	0
CD	molla asta $-W_{1CD}(W_{6CD}+XW_{1CD})/k_{CD}$						11/48Fb ² /EJ	1/4Xb/EJ
	totali						181/144Fb ² /EJ	23/12Xb/EJ
	iperstatica X=W _{BC}						-181/276Fb	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (1) \cdot 1/EJ \, dx = [x]_0^b \cdot 1/EJ$$

$$= (b) \cdot 1/EJ = b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1) \cdot 1/EJ \, dx = [x]_0^b \cdot 1/EJ$$

$$= (b) \cdot 1/EJ = b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \cdot 1/EJ \, dx + 1 \cdot 1 \cdot 1/4 \cdot b/EJ = [x - x^2/b + 1/3 x^3/b^2]_0^b \cdot 1/EJ + 1 \cdot 1 \cdot 1/4 \cdot b/EJ$$

$$= (b - b + 1/3 b) \cdot 1/EJ + 1 \cdot 1 \cdot 1/4 \cdot b/EJ = 7/12 \cdot b/EJ$$

$$L_{DC}^{xx} = \int_0^b (x^2/b^2) \cdot 1/EJ \, dx + 1 \cdot 1 \cdot 1/4 \cdot b/EJ = [1/3 x^3/b^2]_0^b \cdot 1/EJ + 1 \cdot 1 \cdot 1/4 \cdot b/EJ$$

$$= (1/3 b) \cdot 1/EJ + 1 \cdot 1 \cdot 1/4 \cdot b/EJ = 7/12 \cdot b/EJ$$

$$L_{EB}^{xx} = \int_0^b (x^2/b^2) \cdot 1/EJ \, dx = [1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (1/3 b) \cdot 1/EJ = 1/3 \cdot b/EJ$$

$$L_{BE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \cdot 1/EJ \, dx = [x - x^2/b + 1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (b - b + 1/3 b) \cdot 1/EJ = 1/3 \cdot b/EJ$$

$$L_{BC}^{xo} = \int_0^b (11/12 x/b) \cdot Fb \cdot 1/EJ \, dx = [11/24 x^2/b]_0^b \cdot Fb \cdot 1/EJ$$

$$= (11/24 b) \cdot Fb \cdot 1/EJ = 11/24 \cdot Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (11/12 - 11/12 x/b) \cdot Fb \cdot 1/EJ \, dx = [11/12 x - 11/24 x^2/b]_0^b \cdot Fb \cdot 1/EJ$$

$$= (11/12 b - 11/24 b) \cdot Fb \cdot 1/EJ = 11/24 \cdot Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (11/12 - 11/6 x/b + 11/12 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx + 1 \cdot 11/12 \cdot 1/4 \cdot Fb^2/EJ$$

$$= [11/12 x - 11/12 x^2/b + 11/36 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ + 1 \cdot 11/12 \cdot 1/4 \cdot Fb^2/EJ$$

$$= (11/12 b - 11/12 b + 11/36 b) \cdot Fb \cdot 1/EJ + 1 \cdot 11/12 \cdot 1/4 \cdot Fb^2/EJ = 77/144 \cdot Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (11/12 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx + 1 \cdot 11/12 \cdot 1/4 \cdot Fb^2/EJ$$

$$= [11/36 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ + 1 \cdot 11/12 \cdot 1/4 \cdot Fb^2/EJ$$

$$= (11/36 b) \cdot Fb \cdot 1/EJ + 1 \cdot 11/12 \cdot 1/4 \cdot Fb^2/EJ = 77/144 \cdot Fb^2/EJ$$

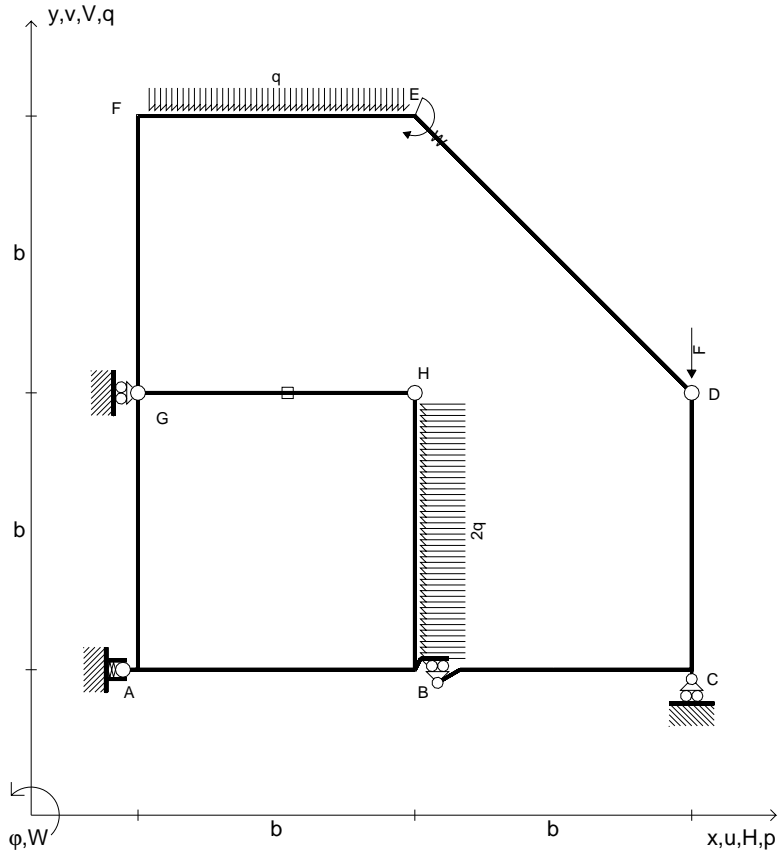
$$L_{EB}^{xo} = \int_0^b (7/12 x/b - 1/12 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [7/24 x^2/b - 1/36 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (7/24 b - 1/36 b) \cdot Fb \cdot 1/EJ = 19/72 \cdot Fb^2/EJ$$

$$L_{BE}^{xo} = \int_0^b (1/2 - 5/12 x/b - 1/12 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [1/2 x - 5/24 x^2/b - 1/36 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (1/2 b - 5/24 b - 1/36 b) \cdot Fb \cdot 1/EJ = 19/72 \cdot Fb^2/EJ$$

- $V_D = -F$
- $W_E = -W = -Fb$
- $q_{EF} = -q = -F/b$
- $p_{HB} = -2q = -2F/b$
- $\varepsilon_{GH} = -\alpha T = -b^2 F/EJ$
- $k_A = 4EJ/b^3$
- $EJ_{AB} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{CD} = EJ$
- $EJ_{DE} = EJ$
- $EJ_{EF} = EJ$
- $EJ_{FG} = EJ$
- $EJ_{GA} = EJ$
- $EJ_{GH} = EJ$
- $EJ_{HB} = EJ$



ANALISI STRUTTURE IPERSTATICHE
PRINCIPIO DEI LAVORI VIRTUALI

Riportare sul fronte:

- 1) Declassamento Scelto
- 2) Reazioni calcolate
- 3) Diagrammi finali delle azioni interne

Sul retro:

- 4) Analisi cinematica
- 5) Diagrammi delle strutture M_0 e M^*
- 6) Equazione del PLV
- 7) Reazione vincolare calcolata

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

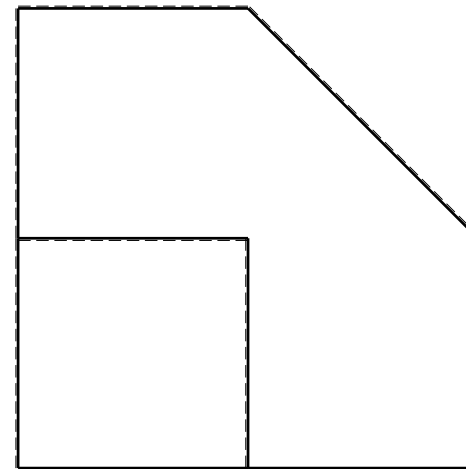
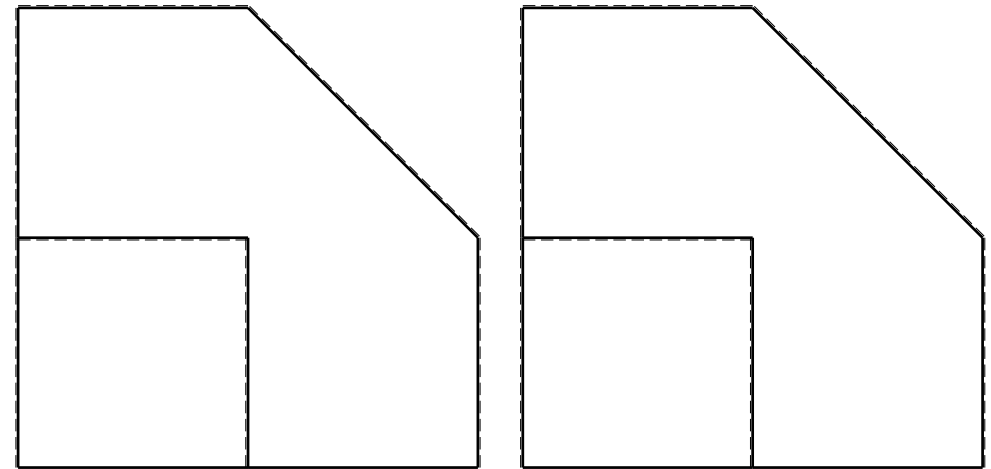
Tracciare i diagrammi quotati delle azioni interne nelle aste.

$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Elongazione termica specifica ε assegnata su asta GH.

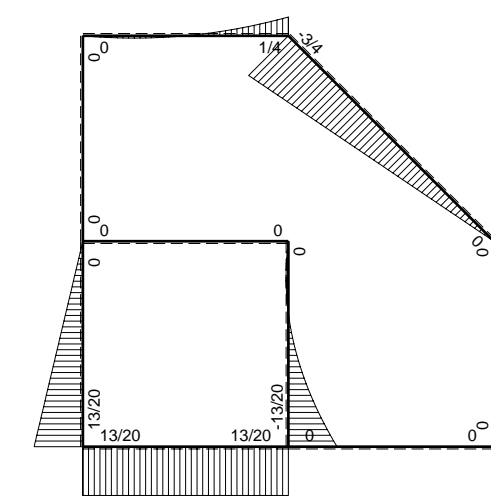
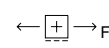
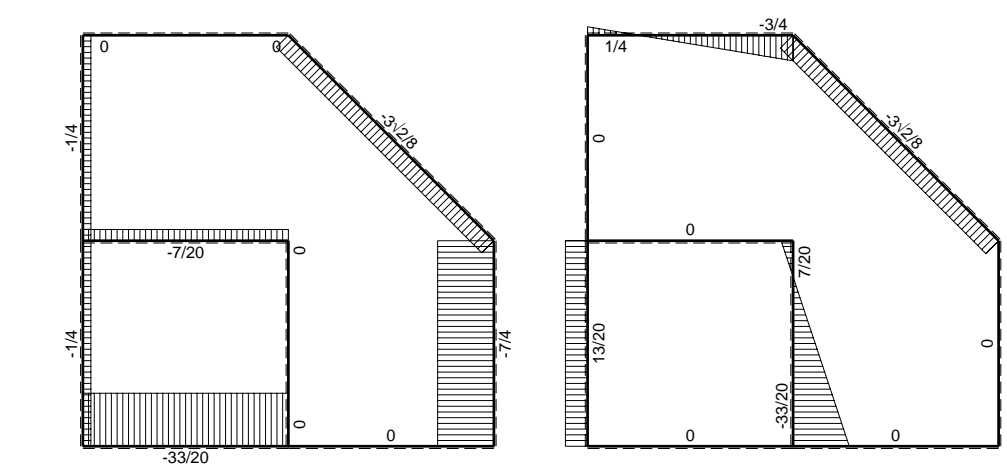
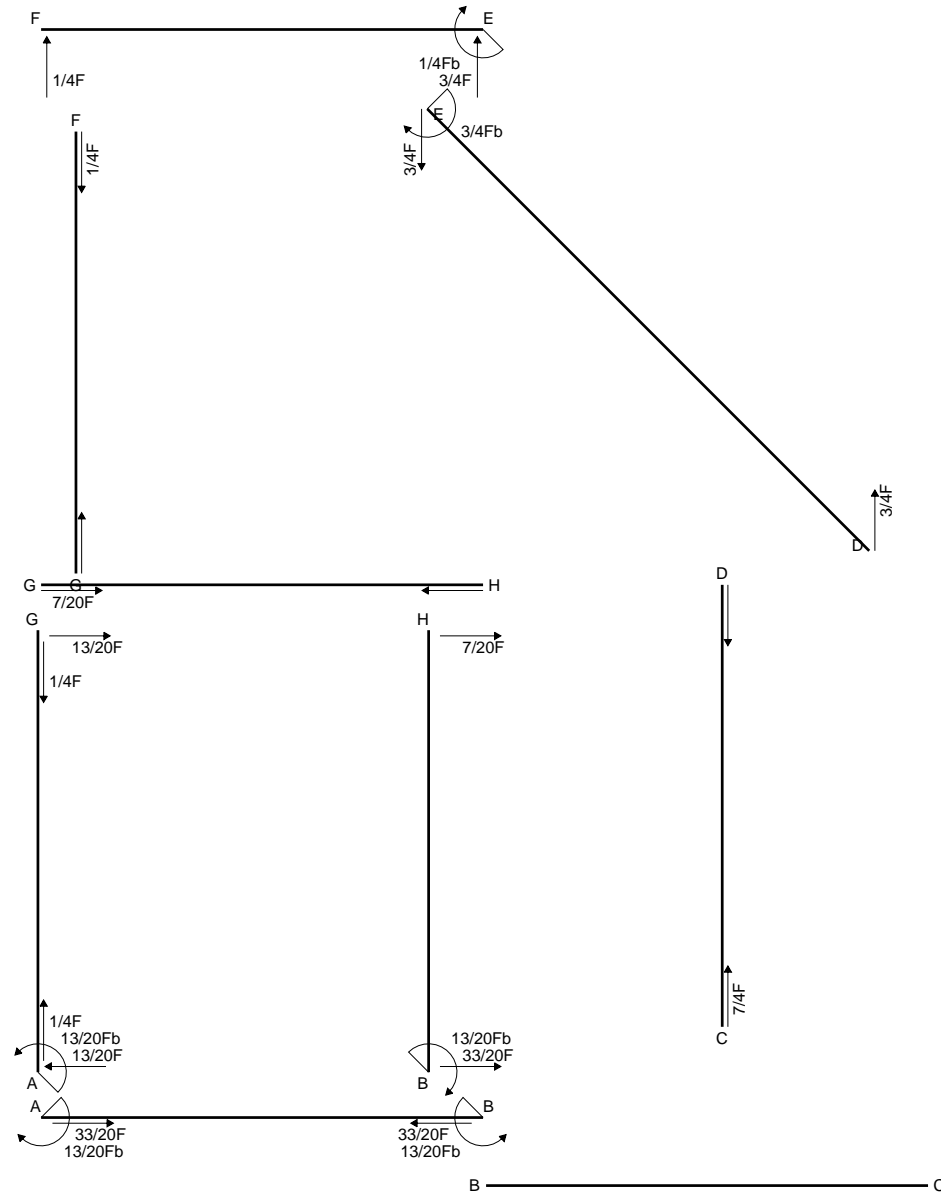
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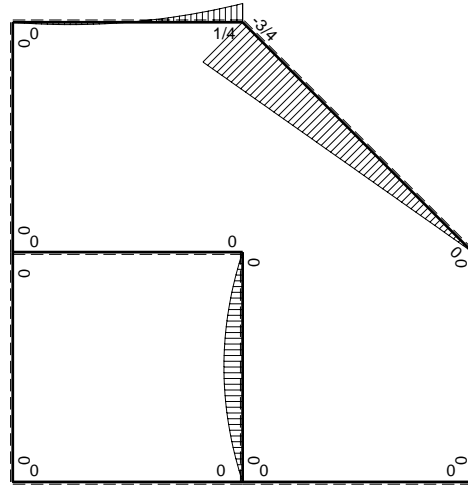
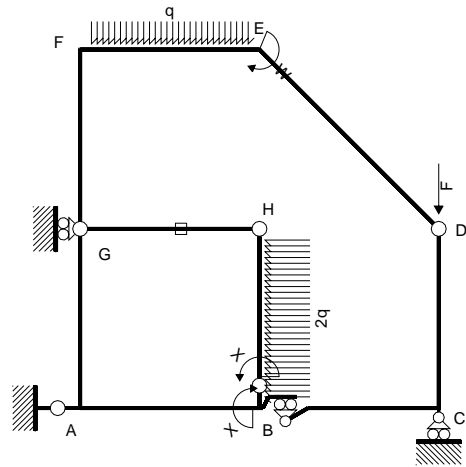
19.11.24



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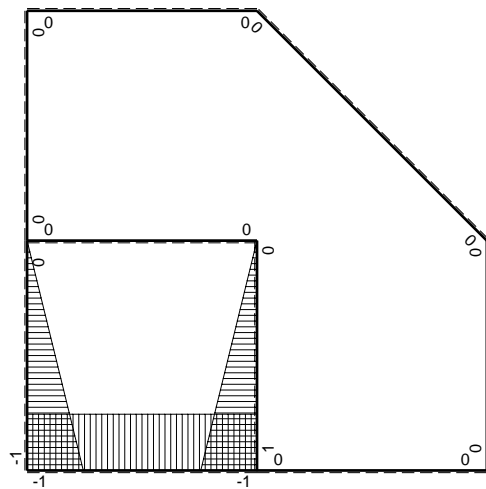
19.11.24





M_o flessione da carichi assegnati

Schema di calcolo iperstatico



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{BH}

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	-1	0	0	1	0	Xb/EJ
BA b	1	0	0	1	0	
BC b	0	0	0	0	0	0
CB b	0	0	0	0	0	0
CD b	0	0	0	0	0	0
DC b	0	0	0	0	0	0
DE $\sqrt{2}b$	0	$-3\sqrt{2}/8Fx$	0	0	0	0
EF b	0	$1/4Fb - 3/4Fx + 1/2qx^2$	0	0	0	0
FE b	0	$1/4Fx - 1/2qx^2$	0	0	0	0
FG b	0	0	0	0	0	0
GF b	0	0	0	0	0	0
GA b	$-x/b$	0	0	x^2/b^2	0	$1/3Xb/EJ$
AG b	$1-x/b$	0	0	$1-2x/b+x^2/b^2$	0	
GH b	0	0	0	0	0	0
HG b	0	0	0	0	0	0
HB b	x/b	$Fx - qx^2$	$Fx^2/b - qx^3/b$	x^2/b^2	$1/12Fb^2/EJ$	$1/3Xb/EJ$
BH b	$-1+x/b$	$-Fx + qx^2$	$Fx - 2Fx^2/b + qx^3/b$	$1-2x/b+x^2/b^2$		
GH	elongazione asta $N_{1GH} \epsilon_{GH} L_{GH}$				Fb^2/EJ	
totali					$13/12Fb^2/EJ$	$5/3Xb/EJ$
iperstatica X=W _{BH}					$-13/20Fb$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GA}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AG}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HB}^{xo} = \int_0^b (x^2/b^2 - x^3/b^3) Fb 1/EJ dx = [1/3 x^3/b^2 - 1/4 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/3 b - 1/4 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{BH}^{xo} = \int_0^b (x/b - 2x^2/b^2 + x^3/b^3) Fb 1/EJ dx = [1/2 x^2/b - 2/3 x^3/b^2 + 1/4 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/2 b - 2/3 b + 1/4 b) Fb 1/EJ = 1/12 Fb^2/EJ$$